Reflective Thinking of Mathematics Prospective Teachers' for Problem Solving

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Abstract. Reflective thinking is a thinking activity marked by the appearance of confusion and efforts to overcome it through planned steps based on knowledge, experience, and problem-solving skills. Reflective thinkers involve specific skills to identify problems, examine, evaluate, formulate problems, and draw conclusions. There are three categories in reflective thinking, including clarification reflective thinking, connective reflective thinking, and productive reflective thinking. The difference lies in the way individuals deal with confusion. This research describes the reflective thinking process of mathematics prospective teachers in solving analytical geometry in term of plane content based on aspects of technique, monitoring, insight, conceptualization. The qualitative research employed prospective mathematics teacher in Java - Indonesia as research subjects. Data collection taken by using test techniques, field notes, interviews, and audio-visual recordings. The results showed that subjects with the category of clarification reflective thinking, connective reflective thinking, and productive reflective thinking did all four aspects even though not all indicators of each aspect. Thus, a training is necessary to develop mathematics prospective teachers' reflective thinking.

1. Introduction

Reflective thinking is a process of individual activity in learning through a process of analysis, evaluation, motivation, and obtaining deep meaning [1]. Reflective thinking occurs when individuals experience doubts in solving problems but there is still enthusiasm to find solutions [2]. Moreover, there are three categories of reflective thinking in solving mathematical problems including categories of clarification, connective, and productive. The difference lies in the way of the problem solver in overcoming confusion. Productive reflective thinkers overcome confusion by using different ways of solving problems. Connective reflective thinkers overcome confusion by connecting all mathematical concepts, principles, and processes related to mathematical problems or solutions. Meanwhile, clarifying reflective thinkers overcome confusion by relating solutions with some concepts. There are four aspects in reflecting thinking, among others techniques, monitoring, insight, and conceptualization [3]. Techniques are aspects where problem solvers think of ways or strategies to understand and solve problems effectively and efficiently. Monitoring is an aspect where the problem solver observes and controls every process or solution to a problem. Insight is defined as an aspect where the problem solver employee knowledge and emotions to solve the problem. Conceptualization arises when the problem solver relates the knowledge and experience for making decisions. The authors conducted preliminary research to explore the indicators of reflective thinking in each aspect. Preliminary research shows that...
there are three indicators in each aspect. Thus, there are twelve indicators of reflective thinking that employed as guidelines in the research. Table 1 depicts the indicator of reflective thinking in four aspects.

| Techniques | Understanding information |
|------------|---------------------------|
|            | Understanding question    |
|            | Selecting effective and efficient strategies in solving problem |
| Monitoring | Monitoring the steps of solution |
|            | Monitoring whether the answers correct or not |
|            | Taking into consideration for making decision |
| Insight    | Being enthusiastic in solving problems |
|            | Being ready to correct the wrong answers |
|            | Finding strategies to avoid difficulties |
| Conceptualization | Understanding the alternative solution |
|            | Relating relevant problems |
|            | Relating relevant concepts |

This research describes the reflective thinking process of mathematics prospective teachers in solving analytical geometry based on aspects of technique, monitoring, insight, conceptualization. Mathematics prospective teachers divided into three reflective thinking categories among others productive, connective, and clarifying. Reflective thinking research is a sub focus of educational research. In 2017-2045 research in Indonesia will focus on theme of education, social humanities, arts, and cultures [4]. Thus, the study of reflective thinking as a sub focus of educational research aims to produce highly qualified and competitive human resources. This indicates that reflective thinking research needs attention from the state as the decision maker and for education practitioners.

2. Methodology
2.1 Research Design
The research is an explorative qualitative research because the data is described according to facts without any manipulation [5]. Data is the reflective thinking process of prospective mathematics teachers based on four aspects in the three reflective thinking categories.

2.2 Participants
Participants are prospective mathematics teachers in Java - Indonesia. They come from Mathematics Education Study Program Universitas Muhammadiyah Surakarta – Indonesia, and Mathematics Education Study Program Universitas PGRI Madiun – Indonesia. Participants are reflective thinkers employed think-aloud method in solving the problems. Non reflective thinkers are not employed. Authors categorized participants into three categories of reflective thinking and not reflective thinkers. The number of productive reflective thinkers are 12 people, the connectives are 19, the clarifying are 23, and not reflective thinkers are 23.

2.3 Instruments
Instruments employed in the study are test, in-depth interview guidelines, and observation sheets. The instruments validated by two experts. They are experts in mathematicians and education. Revision process conducted to improve the quality of instrument. The test consists of two items presented in Figure 1.

![THE ANALYTICAL GEOMETRY PROBLEM](image)

**Information:**
Given \( P(1,2,3) \), \( Q(2,3,1) \), \( R(3,2,1) \), and \( S(1,3,2) \).

**Question:**
1. Investigate, are \( PQRS \) coplanar?
2. If \( PQRS \) are coplanar, determine the plane.

Figure 1. The instrument tests
2.4 Data Collection
Data collected by tests, observation sheets, in-depth interviews, and video recordings methods. Researchers observe the participants solving the test with think aloud techniques. The findings wrote down into observation sheets. At the same time, audio-visual recording employed to record the test process.

2.5 Data Analysis
The stages of data analysis are reduction, presentation, and conclusion. The reduction phase to select important data to achieve the research objectives. Then, the reduced data presented to describe pupils' reflective thinking data based on aspects of techniques, monitoring, insight, and conceptualization. The last one, researchers draw a conclusion of the data to answer the research question.

3. Result
In this section, the authors describe data from one subject in each category of reflective thinking. The data of one subject has represented the entire data in each category. Subject 1 (S-1) is a subject with a productive category, subject 2 (S-2) is a subject with a connective category, and subject 3 (S-3) is a subject with a clarifying category.

3.1 Data S-1
S-1 starts solving question 1 by remembering how to solve it. The subject was silent for a few seconds as experience of confusion until the subject remembered the determinant method. Confusion arises when S-1 determines cofactors and conclusions. On completing question 1, S-1 experienced confusion several times which indicated a graffiti on the answer sheet. Monitoring conducted to ensure the steps and answers written are correct. In solving question 1, S-1 also tries to prevent difficulties. S-1 concluded that the coplanar PQRS point. In solving question 2, S-1 employee the cofactor method to determine the field equation. The cofactor method is considered easier for S-1. Then the efforts employed by the subject in convincing the answer and to overcome doubts is to try to employee another method. This is consistent with the following interview excerpt.

Researcher: When you find doubts about solving a problem, what will you do?
Subject S-1: By trying another solution or method.

Figure 2. The productive reflective thinking structure
To ensure the answer is correct, S-1 employee more than one method. The second method, S-1 supposes that the plane equation is \( Ax + By + Cz + D = 0 \). The \( PQRS \) points are substituted in the equation so that equations (1), (2), (3), and (4) are obtained. The elimination method employed to
determine the values of \( A, B, C, \) and \( D. \) Thus, S-1 got the equation \( x + y + z - 6 = 0. \) The subject also monitors whether the conclusions in the first and second ways are the same. The productive reflective thinking structure of S-1 presented in Figure 2.

3.2 Data S-2

The first step taken by S-2 is to write down information and questions on the answer sheet. S-2 starts solving question 1 by making a plan. This is based on the following think-aloud quote, "first, we determine the determinant of the PQRS point. if the determinant is equal to 0, then PQRS coplanar". This quote also shows that the subject has a strategy for solving questions 1. The subject is silent for a few seconds, this shows the subject is experiencing a confusion. The effort employed by the subject in overcoming confusion is by relating the concepts and information needed. The subject made a mistake in doing algebraic calculations. The process of monitoring steps and answers is employed to ensure that the answers are correct. In solving question 2, the subject writes down the information. Then, the subject takes a few seconds due to confusion. The effort employed to overcome confusion is to recall strategies that can be employed to answer question 2. The elimination method employed to obtain the plane equation. S-2 obtained the plane equation as \( Ax + Ay + Az - 6A = 0. \) Thus, the plane equation is \( x + y + z - 6 = 0. \) The connective reflective thinking structure of S-2 presented in Figure 3.

3.3 Data S-3

S-3 begins solving question 1 by identifying information and questions. S-3 was silent for a moment, this shows the subject experiencing confusion. Efforts to overcome the difficulty is to recall the formula that is needed. A trial and error attempt employed. S-3 says "The first step, compiling a matrix containing PQRS points, in the fourth column is given the number 1 for all rows. Then the cofactors are arranged". The subject undergoes confusion again, when the subject is silent for a few seconds. The subject overcomes his doubts by remembering the step of completion. S-3 decided to monitor the steps until an error was found. Improvement efforts were employed to obtain the right answer. Doubt is indicated by graffiti on the answer sheet. S-3 concluded that PQRS coplanar. In solving question 2, S-3 begins by assuming the plane equation as \( Ax + By + Cz + D = 0. \) The PQRS point substituted in the plane equation so that equations (1), (2), (3) and (4) are obtained. The elimination method employed to obtain the values of \( A, B, C, \) and \( D. \) However, S-3 had experienced confusion that was indicated by a graffiti on the answer sheet. Confusion is resolved by changing the method of settlement to obtain the values \( A = B = C = 1 \) and \( D = -6. \) Thus, the plane equation is \( x + y + z - 6 = 0. \) The reflective thinking structure of S-3 presented in Figure 4.
4. Discussion

In techniques aspect, the prospective mathematics teachers with productive reflective thinking category are able to understand the problem through the process of analyzing the problem, filtering out the necessary information, and determining how to obtain new information. It may be critical thinking stage [6], which is the thinking process that begins by analyzing individual problems, completing missing data, and being able to find the necessary information. Connective reflective thinkers are able to relate the information. They relate concepts from given information to find new information for obtaining a solution. Mathematical connections are the ability of individuals to identify problems through connections between mathematical information [7]. The clarifying reflective thinkers able to understand the problem by identifying information and question clearly and precisely. Understanding the problem stage is a process where individuals are able to identify problems through information [6]. In monitoring aspect, the productive reflective thinkers re-monitor their calculations in correcting errors with their capabilities. It may be called reflectivity level. Reflectivity is the stage at the individual level to solve problems with approaches and investigations based on the knowledge and experiences from various points of view. The connective reflective thinkers overcome doubts by re-checking the steps and results of calculations. They are willing to conduct re-calculation when making mistakes. It may be called rationalization. Rationalization is the relationship between experience and the supportive situation. Meanwhile, the clarifying reflective thinkers experience confusion, and employing re-monitors the steps and answers. It conducted in making sure that the steps and answers are correct.

In insight aspect, all prospective teachers were very enthusiastic in solving problems. They are also very willing to correct the mistake or errors. All participants able to prevent difficulties. The way to prevent difficulties are differences. In conceptualization aspect, the productive reflective thinkers overcome difficulties by employing more than one method in convincing the solutions and the answers. It means they prefer employing a unique or different way to solve the problem [8]. The connective reflective thinkers try to remember the problems that were obtained in the class. They relate concepts that have been obtained with the problems faced. It is relevant to the statement that the involvement of reflective thinking is influenced by obstacles, difficulties, confusion, individual assumptions, willingness to make decisions, actions, findings, and settlement to dispel doubts experienced [9]. Meanwhile, the clarifying reflective thinkers think about a strategy by clarifying information and questions. It may be recall step [10]. Recall stage is the problem-solving process conducted by clarifying based on memory, experience, and observed solution.

Figure 4. The clarifying reflective thinking structure
5. Conclusion
The productive reflective thinkers begin problem solving by experience doubts, difficulties, or obstacles. They overcome the confusion by doing re-monitoring and employing more than one method/solution. They also understand the problem by developing new information through the process of analyzing the problem, filtering out the information, and determining how to obtain new information. The connective reflective thinkers begin problem solving by experience doubts, difficulties or obstacles. They overcome confusion by trying to remember the geometry problems that have been obtained before. They also able to relate some concepts and formulas. In addition, they conduct re-monitoring when experiencing doubts, and very willing to correct wrong answers by employing knowledge and experience. The clarifying reflective thinkers begins problem solving by doubts, difficulties, or obstacles. They overcome confusion by re-monitoring the steps and answers in making sure whether the answers correct or not. They also able to understand the problem by clarifying information and question. In addition, the subject is willing to correct answers.

It is good for mathematics teacher to develop students’ reflective thinking in increasing students’ performances. Because reflective thinking takes an important role in problem solving. The further research can take a focus in reflective thinking level.

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