Constructing the supersymmetric Standard Model from intersecting D6-branes on the $\mathbb{Z}_6'$ orientifold

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Abstract

Intersecting stacks of supersymmetric fractional branes on the $\mathbb{Z}_6'$ orientifold may be used to construct the supersymmetric Standard Model. If $a, b$ are the stacks that generate the $SU(3)_{\text{colour}}$ and $SU(2)_L$ gauge particles, then, in order to obtain just the chiral spectrum of the (supersymmetric) Standard Model (with non-zero Yukawa couplings to the Higgs multiplets), it is necessary that the number of intersections $a \cap b$ of the stacks $a$ and $b$, and the number of intersections $a \cap b'$ of $a$ with the orientifold image $b'$ of $b$ satisfy $(a \cap b, a \cap b') = (2, 1)$ or $(1, 2)$. It is also necessary that there is no matter in symmetric representations of the gauge group, and not too much matter in antisymmetric representations, on either stack. Fractional branes having all of these properties may be constructed on the $\mathbb{Z}_6'$ orientifold. We provide a number of new examples having these properties, some of which may be extended to give the Standard Model spectrum. Specifically, we construct four-stack models with two further stacks, each with just a single brane, which have the matter spectrum of the supersymmetric Standard Model, including a single pair of Higgs doublets, plus three right-chiral neutrino singlets. Ramond-Ramond tadpole cancellation is achieved by the introduction of background $H_3$ flux, the 3-form field strength associated with the Kalb-Ramond 2-form field $B_2$. There remains a single unwanted gauged $U(1)_{B-L}$. 
1 Introduction

An attractive, bottom-up approach to constructing the Standard Model is to use intersecting D6-branes \[1\]. In these models one starts with two stacks, \(a\) and \(b\) with \(N_a = 3\) and \(N_b = 2\), of D6-branes wrapping the three large spatial dimensions plus 3-cycles of the six-dimensional internal space (typically a torus \(T^6\) or a Calabi-Yau 3-fold) on which the theory is compactified. These generate the gauge group \(U(3) \times U(2) \supset SU(3)_c \times SU(2)_L\), and the non-abelian component of the standard model gauge group is immediately assured. Further, (four-dimensional) fermions in bifundamental representations \((N_a, N_b) = (3, 2)\) of the gauge group can arise at the multiple intersections of the two stacks. These are precisely the representations needed for the quark doublets \(Q_L\) of the Standard Model, and indeed an attractive model having just the spectrum of the Standard Model has been constructed \([2]\). The D6-branes wrap 3-cycles of an orientifold \(T^6/\Omega\), where \(\Omega\) is the world-sheet parity operator. The advantage and, indeed, the necessity of using an orientifold stems from the fact that for every stack \(a, b, \ldots\) there is an orientifold image \(a', b', \ldots\). At intersections of \(a\) and \(b\) there are chiral fermions in the \((3, 2)\) representation of \(U(3) \times U(2)\), where the 3 has charge \(Q_a = +1\) with respect to the \(U(1)_a\) in \(U(3) = SU(3)_{\text{colour}} \times U(1)_a\), and the 2 has charge \(Q_b = -1\) with respect to the \(U(1)_b\) in \(U(2) = SU(2)_L \times U(1)_b\). However, at intersections of \(a\) and \(b'\) there are chiral fermions in the \((3, 2)\) representation, where the 2 has \(U(1)_b\) charge \(Q_b = +1\). In the model of \([2]\), the number of intersections \(a \cap b\) of the stack \(a\) with \(b\) is 2, and the number of intersections \(a \cap b'\) of the stack \(a\) with \(b'\) is 1. Thus, as required for the Standard Model, there are 3 quark doublets. These have net \(U(1)_a\) charge \(Q_a = 6\), and net \(U(1)_b\) charge \(Q_b = -3\). Tadpole cancellation requires that overall both charges, sum to zero, so further fermions are essential, and indeed required by the Standard Model. 6 quark-singlet states \(v^a_L\) and \(d^a_L\) belonging to the \((1, 3)\) representation of \(U(1) \times U(3)\), having a total of \(Q_a = -6\) are sufficient to ensure overall cancellation of \(Q_a\), and these arise from the intersections of \(a\) with other stacks \(c, d, \ldots\) having just a single D6-brane. Similarly, 3 lepton doublets \(L\), belonging to the \((2, 1)\) representation of \(U(2) \times U(1)\), having a total \(U(1)_b\) charge of \(Q_b = 3\), are sufficient to ensure overall cancellation of \(Q_b\), and these arise from the intersections of \(b\) with other stacks having just a single D6-brane. In contrast, had we not used an orientifold, the requirement of 3 quark doublets would necessitate having the number of intersections \(a \cap b = 3\). This makes no difference to the charge \(Q_a = 6\) carried by the quark doublets, but instead the \(U(1)_b\) charge carried by the quark doublets is \(Q_b = -9\), which cannot be cancelled by just 3 lepton doublets \(L\). Consequently, additional vector-like fermions are unavoidable unless the orientifold projection is available. This is why the orientifold is essential if we are to get just the matter content of the Standard Model or of the MSSM.

Actually, an orientifold can allow essentially the standard-model spectrum without vector-like matter even when \(a \cap b = 3\) and \(a \cap b' = 0\) \[3\]. This is because in orientifold models it is also possible to get chiral matter in the symmetric and/or antisymmetric representation of the relevant gauge group from open strings stretched between a stack and its orientifold image. Both representations have charge \(Q = 2\) with respect to the relevant \(U(1)\). The antisymmetric (singlet) representation of \(U(2)\) can describe a neutrino singlet state \(v^c_L\), and 3 copies contribute \(Q_b = 6\) units of \(U(1)_b\) charge. If there are also 3 lepton doublets \(L\) belonging to the bifundamental representation \((2, 1)\) representation of \(U(2) \times U(1)\), each contributing \(Q_b = 1\) as above, then the total contribution is \(Q_b = 9\) which can be cancelled by 3 quark doublets \(Q_L\) in the \((3, 2)\) representation of \(U(3) \times U(2)\). Thus, orientifold models can allow the standard-model spectrum plus 3 neutrino singlet states even when \((a \cap b, a \cap b') = (3, 0)\).

Non-supersymmetric intersecting-brane models lead to flavour-changing neutral-current (FCNC) processes that can only be suppressed to levels consistent with the current bounds by making the string scale rather high, of order \(10^3\) TeV, which in turn leads to fine-tuning problems \[4\]. Further, in non-supersymmetric theories, such as these, the cancellation of Ramond-Ramond (RR) tadpoles does not ensure Neveu Schwarz-Neveu Schwarz (NSNS) tadpole cancellation. NSNS tadpoles are simply the first derivative of the scalar potential with respect to the scalar fields, specifically the complex structure and Kähler moduli and the dilaton. A non-vanishing derivative of the scalar potential signifies that such scalar fields are not even solutions of the equations of motion. Thus a particular consequence of the non-cancellation is that the complex structure moduli are unstable \[5\]. It is well known that the point group of an orbifold fixes the complex structure moduli, so that one way to stabilise these moduli is for
the D-branes to wrap an orbifold $T^6/P$ rather than a torus $T^6$. The FCNC problem can be solved and the complex structure moduli stabilised when the theory is supersymmetric. First, a supersymmetric theory is not obliged to have the low string scale that led to problematic FCNCs induced by string instantons. Second, in a supersymmetric theory, RR tadpole cancellation ensures cancellation of the NSNS tadpoles \[6\,7\]. An orientifold is then constructed by quotienting the orbifold with the world-sheet parity operator $\Omega$. (An orientifold, rather than an orbifold, is required because orientifold 6-planes are needed to allow cancellation of the RR charge of the D-branes without using anti-D-branes which would themselves break supersymmetry.)

In this paper we shall be concerned with the orientifold having point group $P = \mathbb{Z}_6'$. We showed in a previous paper \[8\] that this does have (fractional) supersymmetric D6-branes $a$ and $b$ with intersection numbers $(a \cap b, a \cap b') = (1, 2)$ or $(2, 1)$, which might be used to construct the supersymmetric Standard Model having just the requisite standard-model matter content, and in \[9\] we presented an example of just such an extension. The 6-torus factorises into three 2-torii as $T^6 = T^2_1 \times T^2_2 \times T^2_3$ with $T^2_k$ ($k = 1, 2, 3$) parametrised by the complex coordinate $z_k$. The generator $\theta$ of the point group $P = \mathbb{Z}_6'$ acts on the three complex coordinates $z_k$ as

$$\theta z_k = e^{2\pi i v_k} z_k$$

(1)

where

$$v_1, v_2, v_3 = \frac{1}{6}(1, 2, -3)$$

(2)

This action must be an automorphism of the lattice, and we take $T^2_1$ and $T^2_2$ to be $SU(3)$ root lattices. Thus the complex structure moduli $U_{1,2}$ for $T^2_{1,2}$ are fixed to the values $U_1 = U_2 = e^{i\pi/3}$. However, since $\theta$ acts on $z_3$ as a reflection, the lattice for $T^2_3$, and hence its complex structure $U_3$, is arbitrary. The embedding $\mathcal{R}$ of the world-sheet parity operator $\Omega$ acts on all $z_k$ as complex conjugation

$$\mathcal{R} z_k = \overline{z}_k \quad (k = 1, 2, 3)$$

(3)

This too must be an automorphism of the lattice, and this requires the lattice for each torus $T^2_k$ to be in one of two orientations, $A$ or $B$, relative to the Re $z_k$-axis. It also fixes the real part of the complex structure for $T^2_3$, Re $U_3 = 0$ for $A$ and Re $U_3 = \frac{1}{2}$ for $B$; the imaginary part remains arbitrary. We noted in \[8\] that different orientations of the lattices can give rise to different physics. The realisation of the Standard Model presented in the erratum to \[9\] utilised the AAA configuration. In this paper, we shall present a systematic study of the possibility of constructing just the spectrum of the Standard Model on all orientations of the lattices. However, since starting this work, it has been shown \[10\] that there are no three-generation standard models on this lattice that satisfy the tadpole cancellation conditions.

The fractional branes $\kappa$ with which we are concerned have the general form

$$\kappa = \frac{1}{2} \left( \Pi_{\kappa}^{\text{bulk}} + \Pi_{\kappa}^{\text{ex}} \right)$$

(4)

where

$$\Pi_{\kappa}^{\text{bulk}} = \sum_{p=1,3,4,6} \alpha_p^\kappa \rho_p$$

(5)

is an (untwisted) invariant 3-cycle, and

$$\Pi_{\kappa}^{\text{ex}} = \sum_{j=1,4,5,6} (\alpha_j^\kappa \epsilon_j + \bar{\alpha}_j^\kappa \bar{\epsilon}_j)$$

(6)

is an exceptional 3-cycle associated with the $\theta^3$-twisted sector. It consists of a collapsed 2-cycle at a $\theta^3$ fixed point in $T^2_1 \times T^2_2$ times a 1-cycle in the ($\theta^3$-invariant plane) $T^2_3$. The four basis invariant 3-cycles $\rho_p$, ($p = 1, 3, 4, 6$) and the 8 basis exceptional cycles $\epsilon_j$ and $\bar{\epsilon}_j$, ($j = 1, 4, 5, 6$) are defined in reference \[8\]. Their non-zero intersection numbers are

$$\rho_1 \cap \rho_4 = 4, \quad \rho_1 \cap \rho_6 = -2$$

(7)

$$\epsilon_1 \cap \epsilon_2 = 2, \quad \epsilon_1 \cap \epsilon_4 = -1$$

(8)
The “bulk coefficients” \( A_\kappa^\kappa \) are given by

\[
\begin{align*}
A_1^\kappa &= (m_1^\kappa n_2^\kappa + n_1^\kappa m_2^\kappa + m_1^\kappa n_2^\kappa)n_3^\kappa, \\
A_3^\kappa &= (m_1^\kappa m_2^\kappa + n_1^\kappa m_2^\kappa + m_1^\kappa n_2^\kappa)n_3^\kappa, \\
A_4^\kappa &= (m_1^\kappa n_2^\kappa + n_1^\kappa m_2^\kappa + m_1^\kappa n_2^\kappa)m_3^\kappa, \\
A_6^\kappa &= (m_1^\kappa m_2^\kappa + n_1^\kappa m_2^\kappa + m_1^\kappa n_2^\kappa)m_3^\kappa.
\end{align*}
\]

where \( (n_k^\kappa, m_k^\kappa) \) are the (coprime) wrapping numbers for the basis 1-cycles \( (\pi_{2k-1}, \pi_{2k}) \) of the torus \( T_k^2 \) \( (k = 1, 2, 3) \). The corresponding formulae for the exceptional part are also given in [8].

In the first instance we need two stacks \( a \) and \( b \) of such fractional branes, with \( N_a = 3 \) and \( N_b = 2 \), satisfying

\[
(a \cap b, a \cap b') = (2, 1) \quad \text{or} \quad (1, 2)
\]

\( A \) priori the weak hypercharge \( Y \) is a general linear combination

\[
Y = \sum_\kappa y_\kappa Q_\kappa
\]

of the \( U(1) \) charges \( Q_\kappa \) associated with the stack \( \kappa \). We require that both the \( (3, 2) \) and the \( (3, 2) \) representations that occur respectively at the intersections of \( a \) with \( b \) and with \( b' \) have the correct weak hypercharge \( Y = 1/6 \) of the quark doublets \( Q_L \). It follows that

\[
y_a = \frac{1}{6}, \quad y_b = 0
\]

We also require that both stacks are supersymmetric, which is ensured by two linear conditions \( X^{a,b} > 0 \) and \( Y^{a,b} = 0 \) on the bulk coefficients \( A_\kappa^{a,b} \) for each stack. The precise form of \( X^\kappa \) and \( Y^\kappa \) depends on the lattice used and is given for all eight possibilities in Table 8 of reference [8]. In all cases, both \( X^\kappa \) and \( Y^\kappa \) depend upon \( \text{Im} \ U_3 \), so that the requirement of supersymmetry on these two stacks, as well as the others that we must add, fixes \( \text{Im} \ U_3 \). Supersymmetry also requires that the exceptional part \( \Pi^\text{ex}_\kappa \) of the stack \( \kappa \) is associated with fixed points in \( T_2^2 \) and \( T_3^2 \) that are traversed by the bulk 3-cycle \( \Pi^\text{bulk}_\kappa \). As detailed in [8], the effect of this is that, up to Wilson lines, \( \Pi^\text{ex}_\kappa \) is entirely determined by the wrapping numbers \( (n_2^\kappa, m_2^\kappa) \) of \( \Pi^\text{bulk}_\kappa \) in \( T_2^2 \).

In general, besides the gauge supermultiplets that live on each stack \( \kappa \), there is also chiral matter in the symmetric \( S_\kappa \) and, if \( N_\kappa > 1 \), antisymmetric \( A_\kappa \) representations of the gauge group \( SU(N_\kappa) \). For the \( a \) stack we have that \( S_a = 6 \in SU(3)_\text{colour} \), and for the \( b \) stack \( S_b = 3 \in SU(2)_L \). Both representations are unobserved. Thus we further require that they do not occur. Orientifolding induces topological defects, O6-planes, which are sources of RR charge. The numbers \#(\( S_\kappa \)) of symmetric representations and \#(\( A_\kappa \)) of antisymmetric representations are given by

\[
\begin{align*}
\#(S_\kappa) &= \frac{1}{2}(\kappa \cap \kappa' - \kappa \cap \Pi_\text{O6}), \\
\#(A_\kappa) &= \frac{1}{2}(\kappa \cap \kappa' + \kappa \cap \Pi_\text{O6}),
\end{align*}
\]

where \( \Pi_\text{O6} \) is the homology class of the O6-planes. (The required homology classes for all eight lattices are listed in Table 5 of [8].) Consequently, the absence of symmetric representations on \( a \) and \( b \) requires that

\[
\begin{align*}
a \cap a' &= a \cap \Pi_\text{O6}, \\
b \cap b' &= b \cap \Pi_\text{O6}
\end{align*}
\]

For the \( a \) stack the antisymmetric representation is \( A_a = 3 \in SU(3)_\text{colour} \) with \( Q_a = 2 \) and hence \( Y = 1/6 \).
with $Q_b = 2$ and hence $Y = 0$, so any such states will be neutrino singlets $\nu_L^c$. Clearly, if we are to obtain just the standard-model spectrum, we must not have more than 3 copies of either representation. Hence we must also demand that

\[ 0 \leq \#(A_a) = a \cap a' \leq 3 \quad (22) \]

\[ |\#(A_b)| = |b \cap b'| \leq 3 \quad (23) \]

As shown in Table 10 of [8], for the lattices in which $T_3^2$ is of B-type the constraints (20) and (21) restrict the wrapping numbers $(n_{k}^{a,b}, m_{k}^{a,b}) \mod 2$ for $a$ and $b$ of the basis 1-cycles $(\pi_{2k-1}, \pi_{2k})$ on $T_3^2$ for $a$ and $b$ to be in one of two classes, whereas for lattices in which $T_3^2$ is of A-type they must be in one of three classes. This makes the search for solutions satisfying (14) much easier in the former case than than in the latter. It was for this reason that only the former case was considered in [8]. In the next section we will present solutions satisfying all of the constraints in the cases that $T_3^2$ is of A-type.

As noted earlier, in order to obtain all of the standard-model spectrum, it is necessary to add further stacks $c, d, \ldots$ all consisting of a single D6-brane $N_{c,d,\ldots} = 1$, so that the gauge group acquires no further non-abelian components. The identification of these additional stacks is the main task of this paper. Unlike the (non-abelian) stacks $a$ and $b$, there is no requirement that the symmetric representations $S_{c,d,\ldots}$ on these $U(1)$ stacks are absent. (There is no antisymmetric representation of $U(1)$.) Such representations are singlets with respect to both of the non-abelian components $SU(3)_{colour}$ and $SU(2)_{L}$ of the standard-model gauge group and might therefore describe lepton $\ell_L^c$ or neutrino $\nu_L^c$ singlet states.

## 2 Quark doublets when $T_3^2$ is of A-type

The objective is to find the (coprime) wrapping numbers $(n_{k}^{a,b}, m_{k}^{a,b})$ for the two supersymmetric stacks $a$ and $b$ of fractional D6-branes that satisfy (14), (20), (21), (22) and (23). The intersection numbers are given by

\[ a \cap b = \frac{1}{4} f_{AB} + \frac{1}{4} (i_{1}^{a}, i_{2}^{a})(j_{1}^{a}, j_{2}^{a}) \cap (i_{1}^{b}, i_{2}^{b})(j_{1}^{b}, j_{2}^{b}) \quad (24) \]

\[ a \cap b' = \frac{1}{4} f_{AB'} + \frac{1}{4} (i_{1}^{a}, i_{2}^{a})(j_{1}^{a}, j_{2}^{a}) \cap (i_{1}^{b}, i_{2}^{b})(j_{1}^{b}, j_{2}^{b})' \quad (25) \]

where we are using the notation for the exceptional parts used previously

\[ \Pi_{a(i_{1}^{a}, i_{2}^{a})(j_{1}^{a}, j_{2}^{a})}(n_{k}^{a}, m_{k}^{a}) \rightarrow (i_{1}^{a}, i_{2}^{a})(j_{1}^{a}, j_{2}^{a}) \quad (26) \]

The contributions from the bulk parts are

\[ f_{AB} = \Pi_{a}^{bulk} \cap \Pi_{b}^{bulk} \quad (27) \]

\[ = 4(A_{4}^{b}A_{4}^{b} - A_{4}^{a}A_{4}^{a}) - 2(A_{4}^{a}A_{4}^{b} - A_{4}^{b}A_{4}^{a}) - 2(A_{3}^{a}A_{3}^{b} - A_{3}^{b}A_{3}^{a}) + 4(A_{3}^{a}A_{3}^{b} - A_{3}^{b}A_{3}^{a}) \quad (28) \]

\[ f_{AB'} = \Pi_{a}^{bulk} \cap \Pi_{b}^{bulk'} \quad (29) \]

and the function $-f_{AB'}$ is given in Table 11 of [8] for the various lattices. (The sign change from the Table is a consequence of the overall sign change for intersections of the bulk 3-cycles, as explained in the Erratum.)

As in [8], by acting with the generator $\theta$ of the point group $Z_{0}'$ on the wrapping numbers $(n_{1}^{a,b}, m_{1}^{a,b})$ on $T_3^2$, we may take $(n_{1}^{a}, m_{1}^{a}) = (n_{1}^{b}, m_{1}^{b}) \mod 2$, and likewise for $b$. Since there are three possibilities for $(n_{1}^{a}, m_{1}^{a}) \mod 2$, namely $(1, 0)$, $(0, 1)$, or $(1, 1) \mod 2$ when $(n_{1}^{a}, m_{1}^{a})$ are coprime, there are nine distinct pairs for $(n_{1}^{a}, m_{1}^{a})(n_{1}^{b}, m_{1}^{b}) \mod 2$, three with $(n_{1}^{a}, m_{1}^{a}) = (n_{1}^{b}, m_{1}^{b}) \mod 2$, and six with $(n_{1}^{a}, m_{1}^{a}) \neq (n_{1}^{b}, m_{1}^{b}) \mod 2$. When $T_3^2$ is of B-type, we showed that we need only consider the cases in which $(n_{1}^{a}, m_{1}^{a}) = (n_{1}^{b}, m_{1}^{b}) = (1, 0)$, or $(1, 1) \mod 2$, and the calculation of the contribution $(i_{1}^{a}, i_{2}^{a})(j_{1}^{a}, j_{2}^{a}) \cap (i_{1}^{b}, i_{2}^{b})(j_{1}^{b}, j_{2}^{b})$ of the exceptional branes to $a \cap b$ for these cases is presented in $\S$6 of [8]: the calculation of the corresponding contributions to $a \cap b'$ for the four lattices in which
$T^2_3$ is of B-type is given in the appendices of that paper\(^2\). To deal with the cases in which $T^2_3$ is of A-type, we therefore need only present the contributions from the exceptional branes to $a \cap b$ when $(n^a_1, m^b_1) = (n^a_3, m^b_3) = (0, 1)$ mod 2 and/or $(n^a_1, m^b_1) = (n^a_3, m^b_3) = (0, 1)$ mod 2; the contributions from the exceptional branes to $a \cap b'$ for the four lattices in which $T^2_3$ is of A-type are given in the appendices.

### 2.1 $(n^{a,b}_1, m^{a,b}_1) = (n^{a,b}_3, m^{a,b}_3) = (0, 1)$ mod 2

In this case $(i^a_1, j^a_2) = (46)$ and $(j^a_1, j^a_2) = (15)$ or $(46)$.

\[
\begin{align*}
(46)(15) \cap (46)(15) &= (46)(46) \cap (46)(46) = \\frac{(-1)^{\tau^a + \tau^b + 12}}{1 + (-1)^{\tau^a + \tau^b}} \\
&= \frac{(m^a_2 n^b_2 - n^a_2 m^b_2) [1 + (-1)^{\tau^a + \tau^b}]}{(m^a_2 n^b_2 - n^a_2 m^b_2) [1 + (-1)^{\tau^a + \tau^b}] + (n^a_2 n^b_2 + m^a_2 m^b_2 + n^a_2 m^b_2)} \quad (30)
\end{align*}
\]

### 2.2 $(n^a_1, m^a_1) = (n^a_3, m^a_3) = (1, 1)$ mod 2, $(n^b_1, m^b_1) = (n^b_3, m^b_3) = (0, 1)$ mod 2

In this case $(i^a_1, j^a_2) = (46)$, $(j^a_1, j^a_2) = (15)$ or $(46)$, and $(i^b_1, j^b_2) = (45)$, $(j^b_1, j^b_2) = (16)$ or $(45)$.

\[
\begin{align*}
(45)(16) \cap (46)(15) &= (-1)^{\tau^a + \tau^b} (45)(16) \cap (46)(46) = \\
&= (45)(45) \cap (46)(46) = (-1)^{\tau^a + \tau^b} (45)(45) \cap (46)(15) = \\
&= (-1)^{\tau^a + \tau^b + 12} \left(\frac{(m^a_2 n^b_2 - n^a_2 m^b_2) [1 + (-1)^{\tau^a + \tau^b}]}{(m^a_2 n^b_2 - n^a_2 m^b_2) [1 + (-1)^{\tau^a + \tau^b}] + (n^a_2 n^b_2 + m^a_2 m^b_2 + n^a_2 m^b_2)} + ((-1)^{\tau^a + \tau^b} (n^a_2 n^b_2 + m^a_2 m^b_2 + n^a_2 m^b_2)) \right) \quad (32)
\end{align*}
\]

Interchanging the labels $a \leftrightarrow b$ in this calculation immediately gives the results for the case when $(n^a_1, m^a_1) = (n^a_3, m^a_3) = (0, 1)$ mod 2, $(n^b_1, m^b_1) = (n^b_3, m^b_3) = (1, 1)$ mod 2.

### 2.3 $(n^a_1, m^a_1) = (n^a_3, m^a_3) = (0, 1)$ mod 2, $(n^b_1, m^b_1) = (n^b_3, m^b_3) = (1, 0)$ mod 2

In this case $(i^a_1, j^a_2) = (46)$, $(j^a_1, j^a_2) = (15)$ or $(46)$, and $(i^b_1, j^b_2) = (45)$, $(j^b_1, j^b_2) = (16)$ or $(45)$.

\[
\begin{align*}
(46)(15) \cap (56)(14) &= (-1)^{\tau^a} (46)(15) \cap (56)(56) = \\
&= (-1)^{\tau^a + \tau^b} (46)(46) \cap (56)(56) = (-1)^{\tau^a} (46)(46) \cap (56)(14) = \\
&= (-1)^{\tau^a + \tau^b + 12} \left(\frac{(n^a_2 n^b_2 + m^a_2 m^b_2 + n^a_2 m^b_2) \quad (n^a_2 n^b_2 + m^a_2 m^b_2 + n^a_2 m^b_2)}{[(-1)^{\tau^a + \tau^b + 1}] (n^a_2 n^b_2 + m^a_2 m^b_2 + n^a_2 m^b_2) + (n^a_2 n^b_2 + m^a_2 m^b_2 + n^a_2 m^b_2)} \right) \quad (33)
\end{align*}
\]

As above, interchanging the labels $a \leftrightarrow b$ in this calculation immediately gives the results for the case when $(n^a_1, m^a_1) = (n^a_3, m^a_3) = (1, 0)$ mod 2, $(n^b_1, m^b_1) = (n^b_3, m^b_3) = (0, 1)$ mod 2.

### 3 Computation when $T^2_3$ is of A-type

Using the calculations presented in the previous section and the appendices, we seek wrapping numbers $(n^{a,b}_k, m^{a,b}_k)$ ($k = 1, 2, 3$) for two stacks $a$ and $b$ of fractional branes that yield the required intersection numbers $(a \cap b, a \cap b') = (1, 2)$ or $(2, 1)$, that have no symmetric matter on $a$ or $b$, that satisfy the supersymmetry constraints $Y^a, Y^b = 0$ and $X^a, X^b > 0$, and that do not have more than three copies of matter in the antisymmetric representation on $a$ or $b$.

\(^2\) Again, as explained in the Erratum, there is an overall sign change for all calculations of $(i^a_1, j^a_2)(i^b_1, j^b_2) \cap (i^a_1, j^a_2)(j^b_1, j^b_2)$ presented in the Appendices.
We found solutions with the required properties for four values of $U_3$.

From Table 5 of [8] the $B$-type, it appears that solutions only arise when

\begin{align}
X^a &\equiv 2A_1^a - A_3^a - A_6^a \sqrt{3} \Im U_3 > 0 \\
Y^a &\equiv \sqrt{3}A_3^a + (2A_4^a - A_6^a)\Im U_3 = 0
\end{align}

We found solutions with the required properties for four values of

\[
\Im U_3 = -\frac{1}{\sqrt{3}}
\]

These are displayed in Tables 1, 2, 3, and 4 respectively.

On this lattice and on the others in which $T_3^2$ is of $A$-type, and indeed on the lattices in which $T_3^2$ is of $B$-type, it appears that solutions only arise when $(n_1^a, m_1^a) = (n_3^a, m_3^a) \mod 2 \neq (n_1^b, m_1^b) = (n_3^b, m_3^b) \mod 2$.

### 3.1 AAA lattice

On the AAA lattice the supersymmetry constraints for a general stack $\kappa$ are

\[
\begin{align}
X^\kappa &\equiv 2A_1^\kappa - A_3^\kappa - A_6^\kappa \sqrt{3} \Im U_3 > 0 \\
Y^\kappa &\equiv \sqrt{3}A_3^\kappa + (2A_4^\kappa - A_6^\kappa)\Im U_3 = 0
\end{align}
\]

We found solutions with the required properties for four values of

\[
\Im U_3 = -\frac{1}{\sqrt{3}}
\]

These are displayed in Tables 1, 2, 3, and 4 respectively.

### 3.1.1 Solutions with $\Im U_3 = -1/\sqrt{3}$

The first solution in Table 1 has $SU(3)$ colour stack $a$, with bulk part given by

\[
\Pi_a^{\text{bulk}} = -\rho_2 + 3\rho_6 \\
\Pi_a^{\text{bulk}'} = \rho_2 + \rho_3 + 3\rho_4 + 3\rho_6
\]

From Table 5 of [8] the O6-plane is

\[
\Pi_{O6} = \rho_1 + \rho_4 + 2\rho_6
\]
on the AAA lattice. Hence,

\[ \Pi_b^{\text{bulk}} \cap \Pi_{O6} = 0 \]  
\[ \Pi_b^{\text{bulk}} \cap \Pi_b^{\text{bulk'}} = -12 \]  

Since \((n_a^2, m_b^2) = (1, 0)\), the exceptional part of \(a\) is

\[ \Pi_a^{\text{ex}} = (45)(16)(n_a^2, m_b^2) \]
\[ = (-1)^n_a \left( \left[ -(1)^\tau_a \right] [\epsilon_1 + (1 - (1)^\tau_a \epsilon_6] + [1 - (1)^\tau_a \epsilon_1 + (1)^\tau_a \epsilon_6] \right) \]  

In both cases

\[ \Pi_a^{\text{ex}} \cap \Pi_a^{\text{ex'}} = 4[1 - 2(-1)^\tau_a] \]  

and the absence of symmetric representations on \(a\) is guaranteed provided that

\[ \tau_a^1 = 1 \mod 2 \]  

Hence,

\[ \Pi_a^{\text{ex}} = (-1)^n_a \left( [\epsilon_1 + (1 - (1)^\tau_a \epsilon_6] + 2[\epsilon_1 + (1)^\tau_a \epsilon_6] \right) \]  

The \(SU(2)_L\) stack \(b\) has

\[ \Pi_b^{\text{bulk}} = \rho_1 = \Pi_b^{\text{bulk'}} \]  

Hence

\[ \Pi_b^{\text{bulk}} \cap \Pi_{O6} = 0 \]  
\[ \Pi_b^{\text{bulk}} \cap \Pi_b^{\text{bulk'}} = 0 \]  

Since \((n_b^2, m_b^2) = (1, 0)\), the exceptional part is given by

\[ \Pi_b^{\text{ex}} = (56)(14)(n_b^2, m_b^2) = (-1)^n_b \left( [-(1)^\tau_b] [\epsilon_1 + (1 - (1)^\tau_b \epsilon_4] - [\epsilon_1 + (1)^\tau_b \epsilon_4] \right) \]  

The orientifold image is given by

\[ \Pi_b^{\text{ex'}} = (-1)^n_b + \Pi_b^{\text{ex}} \]  

Hence,

\[ \Pi_b^{\text{ex}} \cap \Pi_b^{\text{ex'}} = 0 \]  

and the absence of symmetric representations on \(b\) is guaranteed independently of the choice of \(\tau_b^1\).

The contributions to \(a \cap b\) and \(a \cap b'\) from the bulk parts are

\[ (\Pi_a^{\text{bulk}} \cap \Pi_b^{\text{bulk}}, \Pi_a^{\text{bulk}} \cap \Pi_b^{\text{bulk'}}) = (6, 6) \]  

so that the required intersection numbers \((a \cap b, a \cap b') = (1, 2)\) or \((2, 1)\) are achieved when

\[ (\Pi_a^{\text{ex}} \cap \Pi_b^{\text{ex}}, \Pi_a^{\text{ex}} \cap \Pi_b^{\text{ex'}}) = \pm(2, -2) \]  

From (45) and (52) with (47) we find that

| \((n_a^2, m_b^2; n_b^2, m_b^2)\) | \((A_1^b, A_3^b, A_1^a, A_3^a)\) | \#\((A_a)\) | \((n_a^2, m_b^2; n_b^2, m_b^2)\) | \((A_1^b, A_3^b, A_1^a, A_3^a)\) | \#\((A_a)\) |
|---|---|---|---|---|---|
| \((1, -2; 1, -1; -1, -2)\) | \((2, 1, 4, 2)\) | -3 | \((-2, 1, 1, -1, 0, 1)\) | \((0, 0, 1, 2)\) | 0 |
| \((1, 0; 1, 1; 1, 2)\) | \((2, 1, 4, 2)\) | -3 | \((-2, 1, 1, -1, 0, 1)\) | \((0, 0, 1, 2)\) | 0 |

**Table 4:** Solutions on the AAA lattice with \(\text{Im } U_3 = -1/2\sqrt{3}\).
Thus \((56)\) requires that
\[
\tau^b_1 = 0
\]  
Thus in this solution the \(SU(2)_L\) stack \(b\) has
\[
\Pi_b^{\text{ex}} = -\Pi_b^{\text{ex}'}
\]
\[
= (-1)^{\tau^b_0 + 1} [\bar{\epsilon}_1 + (\tau^b_2 \bar{\epsilon}_4)]
\]  

The second and third solutions have the same \(SU(2)_L\) stack \(b\) as in the first solution, but different \(SU(3)_{\text{colour}}\) stacks \(a\). For the second solution, proceeding similarly, we find
\[
\Pi_a^{\text{ex}} = (-1)^{\tau^a_0} \left( [\epsilon_1 + (\tau^a_2 \epsilon_6)] + [\bar{\epsilon}_1 + (\tau^a_2 \bar{\epsilon}_6)] \right)
\]  
and for the third
\[
\Pi_a^{\text{ex}} = (-1)^{\tau^a_0 + 1} \left( [\epsilon_1 + (\tau^a_2 \epsilon_6)] + [\bar{\epsilon}_1 + (\tau^a_2 \bar{\epsilon}_6)] \right)
\]

The three solutions displayed in the lower half of Table 1 have \(SU(3)_{\text{colour}}\) stacks \(a\) that (up to a phase) are the orientifold duals of the solutions in the upper half of the Table. We get
\[
\Pi_a^{\text{ex}} = (-1)^{\tau^a_0} \left( [\epsilon_1 + (\tau^a_2 \epsilon_6)] - [\bar{\epsilon}_1 + (\tau^a_2 \bar{\epsilon}_6)] \right)
\]
\[
= (-1)^{\tau^a_0} \left( [\epsilon_1 + (\tau^a_2 \epsilon_6)] + 2[\bar{\epsilon}_1 + (\tau^a_2 \bar{\epsilon}_6)] \right)
\]  
\[
= (-1)^{\tau^a_0 + 1} [\epsilon_1 + (\tau^a_2 \epsilon_6)]
\]
respectively. They have the same \(SU(2)_L\) stack \(b\) with
\[
\Pi_b^{\text{ex}} = -\Pi_b^{\text{ex}'}
\]
\[
= (-1)^{\tau^b_0} [\bar{\epsilon}_1 + (\tau^b_2 \bar{\epsilon}_5)]
\]  

3.1.2 Solution with \(\text{Im } U_3 = -\sqrt{3}\)

The absence of symmetric representations on the \(SU(3)_{\text{colour}}\) stack \(a\) for the solution given in Table 2 requires that
\[
\tau^a_1 = 0 \mod 2
\]  
Then
\[
\Pi_a^{\text{ex}} = (45)(16) (n^a_2, n^a_2) = (-1)^{\tau^a_0} \left( [\epsilon_1 + (\tau^a_2 \epsilon_6)] + [\bar{\epsilon}_1 + (\tau^a_2 \bar{\epsilon}_6)] \right)
\]  

The \(SU(2)_L\) stack \(b\) is identical to that given in (67) for the three solutions in the bottom half of Table 1.

3.1.3 Solution with \(\text{Im } U_3 = -2/\sqrt{3}\)

The absence of symmetric representations on the \(SU(3)_{\text{colour}}\) stack \(a\) for the solution given in Table 3 requires that
\[
\tau^a_1 = 1 \mod 2
\]  
Then
\[
\Pi_a^{\text{ex}} = (-1)^{\tau^a_0} \left( [\epsilon_1 + (\tau^a_2 \epsilon_5)] + 2[\bar{\epsilon}_1 + (\tau^a_2 \bar{\epsilon}_5)] \right)
\]  

The \(SU(2)_L\) stack \(b\) is identical to that given in (60) for the three solutions in the top half of Table 1.
Table 5: Solution on the BAA lattice with $\text{Im } U_3 = -1/\sqrt{3}$.

| $(n_1^a, m_1^a, n_2^a, m_2^a; n_3^a, m_3^a)$ | $(A_1^a, A_2^a, A_3^a, A_4^a)$ | #(A<sub>a</sub>) | $(n_1^b, m_1^b, n_2^b, m_2^b; n_3^b, m_3^b)$ | $(A_1^b, A_2^b, A_3^b, A_4^b)$ | #(A<sub>b</sub>) |
|-----------------------------------------|---------------------------------|-----------------|-----------------------------------------|---------------------------------|-----------------|
| $(1, -1; 0, -1; 0)$                    | $(1, 0, -1, 0)$                 | 2               | $(1, -1; 0, -1; 0)$                    | $(1, -2; 1, 1, 0)$              | 0               |

Table 6: Solutions on the BAA lattice with $\text{Im } U_3 = -\sqrt{3}$.

| $(n_1^a, m_1^a, n_2^a, m_2^a; n_3^a, m_3^a)$ | $(A_1^a, A_2^a, A_3^a, A_4^a)$ | #(A<sub>a</sub>) | $(n_1^b, m_1^b, n_2^b, m_2^b; n_3^b, m_3^b)$ | $(A_1^b, A_2^b, A_3^b, A_4^b)$ | #(A<sub>b</sub>) |
|-----------------------------------------|---------------------------------|-----------------|-----------------------------------------|---------------------------------|-----------------|
| $(1, -1; -2, 1, 1)$                    | $(1, 2, 1, 2)$                 | 0               | $(1, -2; 0, 1, 0)$                    | $(0, 1, 0, 1)$                 | 0               |
| $(1, -1; 1, 1; 1)$                     | $(3, 1, 1)$                    | 0               | $(1, 2; 1, 1, 0)$                     | $(2, 1, 0, 0)$                 | 0               |
| $(1, -1; 0, 1; 3)$                     | $(3, 0, -1)$                   | 0               | $(1, -2; 1, 1, 0)$                    | $(2, 1, 0, 0)$                 | 0               |

3.1.4 Solution with $\text{Im } U_3 = -1/2\sqrt{3}$

The absence of symmetric representations on the $SU(3)_{\text{colour}}$ stack $a$ for the first solution given in Table 4 requires that

$$\tau_1^a = 1 \mod 2$$  \hspace{1cm} (72)

Then

$$\Pi_a^{\text{ex}} = (-1)^{\bar{\tau}_a^b + 1} ( [\epsilon_1 + (-1)^{\bar{\tau}_a^b} \epsilon_4] + 2[\tilde{\epsilon}_1 + (-1)^{\bar{\tau}_a^b} \tilde{\epsilon}_4])$$  \hspace{1cm} (73)

The $SU(2)_L$ stack $b$ is identical to that given in (67) for the three solutions in the bottom half of Table 1.

The second solution in Table 4 differs from the first only in the wrapping numbers $(n_3^b, m_3^b)$ of the $SU(3)_{\text{colour}}$ stack $a$. The absence of symmetric representations on $a$ then requires that $\tau_1^a = 0 \mod 2$ for this solution, but then $\Pi_a^{\text{ex}}$ is identical to that given in (73). Thus this solution is identical to the first.

3.2 BAA lattice

On the BAA lattice the supersymmetry constraints for a general stack $\kappa$ are

$$X^\kappa \equiv \sqrt{3} A_1^\kappa + (A_1^a - 2 A_1^b) \text{ Im } U_3 > 0$$  \hspace{1cm} (74)

$$Y^\alpha \equiv 2 A_3^\alpha - A_1^\alpha + \frac{A_3^a}{\sqrt{3}} \text{ Im } U_3 = 0$$  \hspace{1cm} (75)

We again found solutions with the required properties for four values of

$$\text{Im } U_3 = \frac{-1}{\sqrt{3}}$$  \hspace{1cm} (76)

$$= -\sqrt{3}$$  \hspace{1cm} (77)

$$= -2\sqrt{3}$$  \hspace{1cm} (78)

$$= \frac{\sqrt{3}}{2}$$  \hspace{1cm} (79)

These are displayed in Tables 7, 6, 7 and 8 respectively.

| $(n_1^a, m_1^a, n_2^a, m_2^a; n_3^a, m_3^a)$ | $(A_1^a, A_2^a, A_3^a, A_4^a)$ | #(A<sub>a</sub>) | $(n_1^b, m_1^b, n_2^b, m_2^b; n_3^b, m_3^b)$ | $(A_1^b, A_2^b, A_3^b, A_4^b)$ | #(A<sub>b</sub>) |
|-----------------------------------------|---------------------------------|-----------------|-----------------------------------------|---------------------------------|-----------------|
| $(0, 1; -1, 2; -2, 1)$                   | $(2, -2, -1, 1)$               | -3              | $(1, -2; 1, 1, 0)$                     | $(2, 1, 0, 0)$                 | 0               |

Table 7: Solution on the BAA lattice with $\text{Im } U_3 = -2\sqrt{3}$. 
Hence,

\[ \text{with (84) we find that} \]

Since intersection numbers

Thus (56) requires that (58) is satisfied, so that

The orientifold image is given by

and the absence of symmetric representations on \( a \) is guaranteed provided that

Hence,

The \( SU(2)_L \) stack \( b \) has

Hence

Since \((n^b_2, m^b_2) = (-1, 1)\), the exceptional part is given by

The orientifold image is given by

Hence,

and the absence of symmetric representations on \( b \) is guaranteed independently of the choice of \( \tau^b_1 \).

The contributions to \( a \cap b \) and \( a \cap b' \) from the bulk parts again satisfy (55) so that the required intersection numbers \((a \cap b, a \cap b') = (1, 2) \) or \((2, 1)\) are achieved when (56) is satisfied. From (85) and (89) with (84) we find that

Thus (56) requires that (58) is satisfied, so that

Table 8: Solution on the \textbf{BAA} lattice with \( \text{Im } U_3 = -\sqrt{3}/2 \).

| \( n^a_1, m^a_1, n^a_2, m^a_2; n^a_3, m^a_3 \) | \( A^a_1, A^a_3, A^a_4, A^a_6 \) | \#(A\text{a}) | \( n^b_1, m^b_1, n^b_2, m^b_2; n^b_3, m^b_3 \) | \( A^b_1, A^b_3, A^b_4, A^b_6 \) | \#(A\text{b}) |
|-----------------|---------------------|--------|------------------|---------------------|--------|
| \( (1, 0; 0, 1; 3, 2) \) | \( (3, 3, 2, 2) \) | -3 | \( (0, 1; 0, 1; 0) \) | \( (0, 0, 1) \) | 0 |

| 3.2.1 Solution with \( \text{Im } U_3 = -1/\sqrt{3} \) |  |  |  |  |  |  |
### 3.2.2 Solutions with $\Im U_3 = -\sqrt{3}$

The absence of symmetric representations on the $SU(3)_{\text{colour}}$ stack $a$ of the first solution in Table 6 requires that $\tau_1^a = 0 \mod 2$ so that

$$\Pi^a_{\text{ex}} = (-1)^{\tau_0^b} \left( 2[\epsilon_1 + (-1)^{\tau_2^b} \epsilon_6] + [\tilde{\epsilon}_1 + (-1)^{\tau_2^b} \tilde{\epsilon}_6] \right)$$

(94)

The required intersection numbers fix $\tau_0^b = 0 \mod 2$, and then

$$\Pi^b_{\text{ex}} = -\Pi^{b\prime}_{\text{ex}}$$

(95)

$$\Pi^b_{\text{ex}} = (-1)^{\tau_0^b} [\epsilon_1 + (-1)^{\tau_2^b} \epsilon_5]$$

(96)

Similarly, for the second solution $\tau_1^b = 1 \mod 2$ and we find

$$\Pi^b_{\text{ex}} = (-1)^{\tau_0^b} \left( [\epsilon_1 + (-1)^{\tau_2^b} \epsilon_6] - [\tilde{\epsilon}_1 + (-1)^{\tau_2^b} \tilde{\epsilon}_6] \right)$$

(97)

which is just (minus) the orientifold dual of (94). The $SU(2)_L$ stack $b$ is the same as for the first solution and is given in (96).

The third and fourth solutions, displayed in the bottom half of Table 6 have $SU(3)_{\text{colour}}$ stacks $a$ that are the orientifold duals of the two solutions in the upper half of the table. Thus the exceptional parts are given by (97) and (94) respectively. They have the same $SU(2)_L$ stack $b$, and the required intersection numbers occur when $\tau_1^b = 0 \mod 2$. Thus $\Pi^b_{\text{ex}}$ is the same as that found in 3.2.1 and given in (93).

### 3.2.3 Solution with $\Im U_3 = -2\sqrt{3}$

The absence of symmetric representations on the $SU(3)_{\text{colour}}$ stack $a$ of the solution in Table 7 requires that $\tau_1^a = 0 \mod 2$ so that

$$\Pi^a_{\text{ex}} = (-1)^{\tau_0^b} \left( [\epsilon_1 + (-1)^{\tau_2^b} \epsilon_5] - [\tilde{\epsilon}_1 + (-1)^{\tau_2^b} \tilde{\epsilon}_5] \right)$$

(98)

Again, the $SU(2)$ stack $b$ is the same as that found in 3.2.1 and given in (93).

### 3.2.4 Solution with $\Im U_3 = -\sqrt{3}/2$

The absence of symmetric representations on the $SU(3)_{\text{colour}}$ stack $a$ of the solution in Table 8 requires that $\tau_1^a = 1 \mod 2$ so that

$$\Pi^a_{\text{ex}} = (-1)^{\tau_0^b+1} \left( [\epsilon_1 + (-1)^{\tau_2^b} \epsilon_4] - [\tilde{\epsilon}_1 + (-1)^{\tau_2^b} \tilde{\epsilon}_4] \right)$$

(99)

The $SU(2)$ stack $b$ is the same as that found in for the first two solutions in 3.2.2 and given in (96).

### 3.3 ABA lattice

On the BAA lattice the supersymmetry constraints for a general stack $\kappa$ are the same as for the BAA lattice given in (74) and (75). We found one solution having the required properties with

$$\Im U_3 = -\frac{1}{2\sqrt{3}}$$

(100)

It is displayed in Table 9. From Table 5 of [8] the O6-plane is

| $(n_1^a, m_1^a; n_2^a; m_2^a; n_3^a, m_3^a)$ | $(A_1^a, A_3^a; A_4^a; A_5^a)$ | $\#(\mathcal{A}_a)$ | $(n_1^b, m_1^b; n_2^b; m_2^b; n_3^b, m_3^b)$ | $(A_1^b, A_3^b; A_4^b; A_5^b)$ | $\#(\mathcal{A}_b)$ |
| --- | --- | --- | --- | --- | --- |
| $(1, 0; 1; 0; 1; 2)$ | $(1, 0, 2, 0)$ | -3 | $(-2, 1; -2; 1; 0, 1)$ | $(0, 0, 0, -3)$ | 0 |

Table 9: Solution on the ABA lattice.
Table 10: Solution on the BBA lattice.

\[ \Pi_{O6} = 2 \rho_1 + \rho_3 - 3 \rho_6 \]  

(101)

on the ABA lattice. Hence, for the solution displayed in Table 9

\[ \Pi_{O6}^{\text{bulk}} \cap \Pi_{O6} = -6 \]  

(102)

\[ \Pi_{O6}^{\text{bulk}} \cap \Pi_{O6}' = -8 \]  

(103)

\[ \Pi_{O6}^{\text{ex}} \cap \Pi_{O6}' = 4 [1 - 2(-1)^{\tau_a}] \]  

(104)

and the absence of symmetric representations on \( a \) is guaranteed provided that

\[ \tau_a = 0 \text{ mod } 2 \]  

(105)

Hence,

\[ \Pi_{O6}^{\text{ex}} = (-1)^{\tau_a} [\tilde{\epsilon}_1 + (-1)^{\tau_a} \tilde{\epsilon}_4] \]  

(106)

As before, the absence of symmetric representations on the \( SU(2)_L \) stack \( b \) is guaranteed independently of the choice of \( \tau_b \). However, the required intersection numbers arise only if \( \tau_b = 0 \text{ mod } 2 \), and then

\[ \Pi_{O6}^{\text{ex}} = -\Pi_{O6}' \]

\[ = (-1)^{\tau_a} \left( [\tilde{\epsilon}_1 + (-1)^{\tau_a} \tilde{\epsilon}_4] + 2[\tilde{\epsilon}_1 + (-1)^{\tau_a} \tilde{\epsilon}_4] \right) \]  

(107)

### 3.4 BBA lattice

On the BBA lattice the supersymmetry constraints for a general stack \( \kappa \) are

\[ X^\kappa \equiv A^\kappa_1 + A^\kappa_3 + (A^\kappa_4 - A^\kappa_6) \sqrt{3} \text{ Im } U_3 > 0 \]  

(108)

\[ Y^\kappa \equiv \sqrt{3}(A^\kappa_3 - A^\kappa_1) + (A^\kappa_4 - A^\kappa_6) \sqrt{3} \text{ Im } U_3 = 0 \]  

(109)

We again found one solution with the required properties with

\[ \text{Im } U_3 = 2\sqrt{3} \]  

(110)

\[ \text{Im } U_3 = 2\sqrt{3} \]  

(111)

This is displayed in Table 10. From Table 5 of [8] the O6-plane is

\[ \Pi_{O6} = 3 \rho_1 + 3 \rho_3 + \rho_4 - \rho_6 \]  

(112)

on the BBA lattice. Hence, for the solution displayed in Table 10

\[ \Pi_{O6}^{\text{bulk}} \cap \Pi_{O6} = -6 \]  

(113)

\[ \Pi_{O6}^{\text{bulk}} \cap \Pi_{O6}' = -8 \]  

(114)

\[ \Pi_{O6}^{\text{ex}} \cap \Pi_{O6}' = 4 [1 - 2(-1)^{\tau_a}] \]  

(115)

and the absence of symmetric representations on \( a \) is guaranteed provided that

\[ \tau_a = 0 \text{ mod } 2 \]  

(116)
Hence,

\[ \Pi^\text{ex}_a = (-1)^{\tau^b_0 + 1} [\epsilon_1 + (-1)^{\tau^b_2} \epsilon_5] \]  

The absence of symmetric representations on the \( SU(2)_L \) stack \( b \) is again guaranteed independently of the choice of \( \tau^b_i \), and the required intersection numbers arise only if \( \tau^b_1 = 0 \mod 2 \). Then

\[ \Pi^\text{ex}_b = -\Pi^\text{ex'}_b = (-1)^{\tau^b_0 + 1} \left( [\epsilon_1 + (-1)^{\tau^b_2} \epsilon_4] - [\epsilon_1 + (-1)^{\tau^b_2} \tilde{\epsilon}_4] \right) \]  

4 No-go results when \( T^2_3 \) is of B-type

We must now see whether it is possible to find further stacks \( \kappa = c, d, \ldots \) of fractional branes with \( N_\kappa = 1 \) so that the quark- and lepton-singlet content, as well as the lepton- and Higgs-doublet content of the standard model arises at intersections of \( a \) and \( b \) with these new stacks, and/or at intersections of the new stacks with each other, and/or, for the singlet-matter, as symmetric representations on the some or all of the stacks.

At the \( a \cap \kappa \) intersections of the \( SU(3)_\text{colour} \) stack \( a \) with a \( U(1) \) stack \( \kappa \) there is chiral matter in the \( (3, 1) \) representation of \( SU(3)_\text{colour} \times SU(2)_L \) which must correspond to quark-singlet matter if we are to get just the standard-model spectrum. It follows from (15) and (16) that the weak hypercharge of such matter is \( Y = \frac{1}{3} - y_\kappa \). Thus if \( y_\kappa = \frac{1}{2} \), the colour-triplet matter will be \( d \)-quark singlets, while if \( y_\kappa = -\frac{1}{2} \), it will be \( u \)-quark singlets; no other values of \( y_\kappa \) are permitted if we insist on the standard-model spectrum. Likewise, at the \( a \cap \kappa' \) intersections of \( a \) with the orientifold image \( \kappa' \) of \( \kappa \), there will be colour-triplet \( u \)-quark singlet matter if \( y_\kappa = \frac{1}{2} \), and \( d \)-quark singlet matter if \( y_\kappa = -\frac{1}{2} \). For our purposes, we require that

\[ -3 \leq a \cap \kappa, a \cap \kappa' \leq 0 \]  

corresponding to not more than three \( d^c_L \) or \( u^c_L \) states.

Similarly, using (17), at the intersections of the \( SU(2)_L \) stack \( b \) with \( \kappa \) and \( \kappa' \) there is chiral matter in the \( (1, 2) \) representation of \( SU(3)_\text{colour} \times SU(2)_L \) with \( Y = -y_\kappa \) and \( Y = y_\kappa \) respectively. If \( y_\kappa = \frac{1}{2} \), the former corresponds to lepton \( L \) or Higgs \( H \) doublets and the latter to \( H \) doublets, and vice versa if \( y_\kappa = -\frac{1}{2} \). For the standard-model spectrum we require that there are four doublets with \( Y = -\frac{1}{2} \) and one with \( Y = \frac{1}{2} \), and this requires that for at least one stack \( \kappa \)

\[ b \cap \kappa - b \cap \kappa' = 1 \mod 2 \]  

In many cases it turns out that this is a very restrictive constraint.

In all of the solutions presented in the last section (for lattices in which \( T^2_3 \) is of A-type) the \( SU(2)_L \) stack \( b \) has the property that

\[ \Pi^\text{bulk}_b = \Pi^\text{bulk'}_b, \quad \Pi^\text{ex}_b = -\Pi^\text{ex'}_b \]  

and the same is true for some of the solutions presented in [8] for lattices in which \( T^2_3 \) is of B-type. For the solutions of which this is true it follows that, for any stack \( \kappa \), and in particular any of the \( U(1) \) stacks \( \kappa = c, d, \ldots \)

\[ b \cap \kappa - b \cap \kappa' = \frac{1}{2} \Pi^\text{bulk}_b \cap \Pi^\text{bulk}_\kappa \]  

\[ b \cap \kappa + b \cap \kappa' = \frac{1}{2} \Pi^\text{ex}_b \cap \Pi^\text{ex}_\kappa \]
4.1 BBB and ABB lattices

To see how restrictive (120) is, we consider the solutions presented in §7.4 of [8] for the BBB lattice. All three solutions have the same $SU(3)_{\text{colour}}$ stack $a$ (denoted by $b$ in [8]):

\[
\Pi_{\text{bulk}}^{b} = \rho_{1} - \rho_{4} \quad (125)
\]

\[
\Pi_{\text{ex}}^{b} = (-1)^{\tau_{b}^{0}} \left[ \bar{\epsilon}_{1} + (-1)^{\tau_{b}^{2}} \bar{\epsilon}_{6} \right] \quad (126)
\]

or

\[
\Pi_{\text{ex}}^{b} = (-1)^{\tau_{b}^{0}} \left[ \bar{\epsilon}_{4} + (-1)^{\tau_{b}^{2}} \bar{\epsilon}_{5} \right] \quad (127)
\]

In the first solution the $SU(2)_{L}$ stack $b$ has

\[
\Pi_{\text{bulk}}^{b} = \rho_{1} - \rho_{3} - 2\rho_{4} + 2\rho_{6} = \Pi_{\text{bulk}}^{b'} \quad (128)
\]

\[
\Pi_{\text{ex}}^{b} = -\Pi_{\text{ex}}^{b'} \quad (129)
\]

or

\[
\Pi_{\text{ex}}^{b} = (-1)^{\tau_{b}^{0}} \left( [\epsilon_{1} + (-1)^{\tau_{b}^{2}} \epsilon_{4}] - [\bar{\epsilon}_{1} + (-1)^{\tau_{b}^{2}} \bar{\epsilon}_{4}] \right) \quad (130)
\]

or

\[
\Pi_{\text{ex}}^{b} = (-1)^{\tau_{b}^{0}} ([\epsilon_{5} + \epsilon_{6}] - [\bar{\epsilon}_{5} + \bar{\epsilon}_{6}]) \quad (131)
\]

The supersymmetry constraint on this lattice

\[
Y^{b} \equiv \sqrt{3}(A_{3}^{b} - A_{1}^{b} + \frac{1}{2}A_{6}^{b} - \frac{1}{2}A_{4}^{b}) + (A_{4}^{b} + A_{6}^{b})\text{Im} U_{3} = 0 \quad (132)
\]

then requires that

\[
\text{Im} U_{3} = -\frac{\sqrt{3}}{2} \quad (133)
\]

so that all stacks $\kappa$ are required to satisfy

\[
X^{\kappa} \equiv A_{1}^{\kappa} + A_{3}^{\kappa} - A_{4}^{\kappa} + 2A_{6}^{\kappa} > 0 \quad (134)
\]

\[
\frac{1}{\sqrt{3}} Y^{\kappa} \equiv A_{3}^{\kappa} - A_{1}^{\kappa} - A_{4}^{\kappa} = 0 \quad (135)
\]

It is easy to see that this requires that the bulk wrapping numbers $A_{\kappa}^{b}$ satisfy

\[
(A_{1}^{\kappa}, A_{3}^{\kappa}, A_{4}^{\kappa}, A_{6}^{\kappa}) = (1, 1, 0, 0) \mod 2 \quad (136)
\]

or

\[
(1, 0, 1, 0) \mod 2 \quad (137)
\]

or

\[
(0, 0, 0, 1) \mod 2 \quad (138)
\]

and hence that the wrapping numbers $(n_{1}^{\kappa}, m_{1}^{\kappa})$ on the torus $T_{k}^{2}$ satisfy

\[
(n_{1}^{\kappa}, n_{2}^{\kappa}, n_{3}^{\kappa}, n_{4}^{\kappa}, m_{1}^{\kappa}) = (1, 0; 0, 1; 1, 0) \mod 2 \quad (139)
\]

or

\[
(1, 1; 0, 1; 1, 1) \mod 2 \quad (140)
\]

or

\[
(0, 0; 0, 0, 1) \mod 2 \quad (141)
\]

respectively, when we choose the representative 3-cycle in which $(n_{1}^{\kappa}, m_{1}^{\kappa}) = (n_{5}^{\kappa}, m_{5}^{\kappa}) \mod 2$. In fact, the last two cases are interchanged under the action of $R$, so that we need only consider the first two possibilities. We denote by $c$ stacks with wrapping numbers satisfying (136), (139), and by $d$ stacks with wrapping numbers satisfying (137), (140). It follows from (123) and (128) that for this solution

\[
b \cap \kappa - b \cap \kappa' = 3(2A_{1}^{\kappa} - 2A_{3}^{\kappa} + A_{4}^{\kappa} - A_{6}^{\kappa}) \quad (142)
\]

\[
= -3(A_{4}^{\kappa} + A_{6}^{\kappa}) \quad (143)
\]

using (135), which only satisfies (120) if $\kappa$ is of type $d$. Further, the only solutions that do not entail unwanted vector-like doublets have

\[
A_{4}^{d} + A_{6}^{d} = \epsilon \quad (144)
\]
where $\epsilon = \pm 1$. It is easy to see that the only solution consistent with supersymmetry (134), (135) and the requirement that

$$A^c_6 A^c_6 = A^c_3 A^c_4$$

(145)

is when $\epsilon = -1$ and

$$(A^d_1, A^d_3, A^d_4, A^d_6) = (1, 0, -1, 0)$$

(146)

Then the wrapping numbers are given by

$$(n^d_1, m^d_1; n^d_2, m^d_2; n^d_3, m^d_3) = (\eta \chi, -\eta \chi; 0, \chi; \eta, -\eta)$$

(147)

where $\eta, \chi = \pm 1$. Using the results presented in [8], the general form for the exceptional part of a $d$-type stack is given by

$$\Pi^\text{ex}_d = ( -1)^{\tau_0^d} \left[ m^d_2 - ( -1)^{\tau^d_1} (n^d_2 + m^d_2) \right] [\epsilon_1 + ( -1)^{\tau^d_2} \epsilon_6] + [n^d_2 + m^d_2 - ( -1)^{\tau^d_1} n^d_2] [\bar{\epsilon}_1 + ( -1)^{\tau^d_2} \bar{\epsilon}_6]$$

(148)

or

$$\Pi^\text{ex}_d = ( -1)^{\tau_0^d} \left[ m^d_2 - ( -1)^{\tau^d_1} (n^d_2 + m^d_2) \right] [\epsilon_4 + ( -1)^{\tau^d_2} \epsilon_5] + [n^d_2 + m^d_2 - ( -1)^{\tau^d_1} n^d_2] [\bar{\epsilon}_4 + ( -1)^{\tau^d_2} \bar{\epsilon}_5]$$

(149)

Then, using (147), it follows from (124) and (130) or (131) that

$$b \cap d + b \cap d' = \pm 1 \text{ or } \pm 3$$

(150)

and hence that

$$(b \cap d, b \cap d') = (1, -2), \ (2, -1), \ (3, 0) \text{ or } (0, -3)$$

(151)

In all four cases such a stack will give three $L$ or $H$ doublets with $Y = -1/2$ provided that we choose $y_d = -1/2$. However, there remains to be found a pair of doublets with $Y = 1/2, -1/2$ arising at intersections of the stack $b$ with a different $U(1)$ stack $\kappa$ satisfying

$$(b \cap \kappa, b \cap \kappa') = \pm (1, 1)$$

(152)

(There is no possibility of utilising two further stacks, since it follows from (143) that the only solutions satisfying (120) necessarily have $b \cap \kappa - b \cap \kappa' = 0 \mod 3$.) It follows from (143) that $\kappa$ must be of type $c$. Now, the general form for the exceptional part of a $c$-type stack is given by

$$\Pi^\text{ex}_c = ( -1)^{\tau_0^c} \left[ - (n^c_2 + m^c_2) + ( -1)^{\tau^c_1} n^c_2 \right] [\epsilon_1 + ( -1)^{\tau^c_2} \epsilon_4] - [n^c_2 + ( -1)^{\tau^c_1} m^c_2] [\bar{\epsilon}_1 + ( -1)^{\tau^c_2} \bar{\epsilon}_4]$$

(153)

or

$$\Pi^\text{ex}_c = ( -1)^{\tau_0^c} \left[ - (n^c_2 + m^c_2) + ( -1)^{\tau^c_1} n^c_2 \right] [\epsilon_5 + ( -1)^{\tau^c_2} \epsilon_6] - [n^c_2 + ( -1)^{\tau^c_1} m^c_2] [\bar{\epsilon}_5 + ( -1)^{\tau^c_2} \bar{\epsilon}_6]$$

(154)

Using (124) and (130) or (131) this gives

$$b \cap c + b \cap c' = ( -1)^{\tau_0^c + \tau_0^{c'}} [1 + ( -1)^{\tau^c_2 + \tau^{c'}_2}] \left( m^c_2 - ( -1)^{\tau^c_1} (n^c_2 + m^c_2) \right)$$

(155)

$$= 0 \mod 4$$

(156)

since $(n^c_2, m^c_2) = (0, 1) \mod 2$. It follows that we cannot satisfy (152) using a type $c$ stack, and certainly not using $d$-type. We conclude that this solution cannot produce just the standard-model spectrum. A similar argument shows that the other solutions on the BBB lattice also cannot yield the required doublet spectrum. In fact, the same conclusion, reached by a similar argument, also holds for the three solutions found on the ABB lattice.
4.2 AAB and BAB lattices

For the other two lattices in which $T_2^3$ is of B-type the situation is different. On each of these lattices there is one solution with the property (121) and (122), and an argument similar to that given above shows that they too cannot yield the required doublet spectrum. For the other solution on each of these two lattices it is the $SU(3)$ colour stack $a$ that has the property

$$\Pi_a^{\text{bulk}} = \Pi_a^{\text{bulk}'}$$

(157)

$$\Pi_a^{\text{ex}} = -\Pi_a^{\text{ex}'}$$

(158)

so that

$$a \cap \kappa - a \cap \kappa' = \frac{1}{2} \Pi_a^{\text{bulk}} \cap \Pi_\kappa^{\text{bulk}}$$

(159)

$$a \cap \kappa + a \cap \kappa' = \frac{1}{2} \Pi_a^{\text{ex}} \cap \Pi_\kappa^{\text{ex}}$$

(160)

This is the case for the first solution on the AAB lattice, given in §7.1 of [8], in which

$$\Pi_a^{\text{bulk}} = \rho_1 = \Pi_a^{\text{bulk}'}$$

(161)

$$\Pi_a^{\text{ex}} = -\Pi_a^{\text{ex}'}$$

(162)

$$= (-1)^{\tau_0^a+1} (2[e_5 - e_6] + [\bar{e}_5 - \bar{e}_6])$$

(163)

and

$$\Pi_b^{\text{bulk}} = \rho_1 + \rho_3 - \rho_4 - \rho_6$$

(164)

$$\Pi_b^{\text{ex}} = (-1)^{\tau_0^b+1} \left( [\epsilon_1 + (-1)^{\tau_2^b} \epsilon_6] - [\bar{\epsilon}_1 + (-1)^{\tau_2^b} \bar{\epsilon}_6] \right)$$

or $$= (-1)^{\tau_0^b} \left( [\epsilon_4 + (-1)^{\tau_2^b} \epsilon_5] - [\bar{\epsilon}_4 + (-1)^{\tau_2^b} \bar{\epsilon}_5] \right)$$

(165)

(166)

The supersymmetry constraint on this lattice

$$Y^b \equiv \sqrt{3} (A_b^3 + \frac{1}{2} A_b^6) + (2A_4^b - A_6^b) \text{Im} U_3 = 0$$

(167)

then requires that

$$\text{Im} U_3 = \frac{\sqrt{3}}{2}$$

(168)

so that all stacks $\kappa$ are required to satisfy

$$X^\kappa \equiv 2A_4^\kappa - A_3^\kappa + A_4^\kappa - 2A_6^\kappa > 0$$

(169)

$$\frac{1}{\sqrt{3}} Y^\kappa \equiv A_3^\kappa + A_4^\kappa = 0$$

(170)

This requires that the bulk wrapping numbers $A_p^\kappa$ satisfy

$$(A_1^\kappa, A_3^\kappa, A_4^\kappa, A_6^\kappa) = (1, 0, 0, 0) \text{ mod } 2$$

(171)

or $$= (1, 1, 1, 1) \text{ mod } 2$$

(172)

or $$= (0, 0, 0, 1) \text{ mod } 2$$

(173)

and hence that the wrapping numbers $(n^\kappa_1, m^\kappa_1; n^\kappa_2, m^\kappa_2; n^\kappa_3, m^\kappa_3)$ on the torus $T_k^2$ satisfy

$$(n^\kappa_1, m^\kappa_1; n^\kappa_2, m^\kappa_2; n^\kappa_3, m^\kappa_3) = (1, 0; 1, 0; 1, 0) \text{ mod } 2$$

(174)

or $$= (1, 1; 1, 1; 1, 1) \text{ mod } 2$$

(175)

or $$= (0, 1; 0, 1; 0, 1) \text{ mod } 2$$

(176)

respectively, when we choose the representative 3-cycle in which $(n^\kappa_1, m^\kappa_1) = (n^\kappa_2, m^\kappa_2) \text{ mod } 2$. The last
possibilities. We denote by \( c \) stacks with wrapping numbers satisfying (171), (174), and by \( d \) stacks with wrapping numbers satisfying (172), (175). As noted earlier, the intersections of the \( SU(3) \) colour stack \( a \) with a \( U(1) \) stack \( \kappa \) produce quark-singlet matter. It is obvious from (161) and (163), using (22), that

\[
a \cap a' = 0 = \#(A_a)
\]

(177)

Since there is no antisymmetric matter on \( a \), all of the quark-singlet matter must arise at intersections of \( a \) with such \( U(1) \) stacks. Using (161), (159), (163) and (160) we find that

\[
a \cap c - a \cap c' = 2A^c_4 - A^c_6 = 0 \mod 4
\]

(178)

The last equality follows because \( A^c_1 A^c_6 = A^c_3 A^c_4 = 0 \mod 4 \) for a \( c \)-type stack. The general form for the exceptional part of a \( c \)-type stack is given by (153) or (154). Then, using (160) and (163) or (163), it follows that

\[
a \cap c + a \cap c' = (-1)^{\tau^C_0 + \tau^C_6} [1 + (-1)^{\tau^C_2 + \tau^C_7}] [m^c_2 - n^c_2 - (-1)^{\tau^C_1} (n^c_2 + 2m^c_2)]
\]

(179)

\[
= 0 \mod 4
\]

(180)

Thus, the only solutions satisfying (119) are

\[
(a \cap c, a \cap c') = (0,0) \text{ and } (-2,-2)
\]

(181)

Similarly, we find that

\[
a \cap d - a \cap d' = 2A^d_4 - A^d_6 = 1 \mod 2
\]

(182)

\[
a \cap d + a \cap d' = (-1)^{\tau^d_0 + \tau^d_6} [2n^d_2 + m^d_2 - (-1)^{\tau^d_1} (n^d_2 - m^d_2)] = 1 \mod 2
\]

(183)

which, in principle, allows

\[
(a \cap d, a \cap d') = (-1,0), (1,2), (3,2), \text{ and } (3,0)
\]

(184)

where the underlining signifies that either ordering is allowed. Clearly we cannot obtain the required quark-singlet content without using at least two \( d \)-type stacks. The first three of the above four possibilities require that

\[
2A^d_4 - A^d_6 = \epsilon
\]

(185)

where \( \epsilon = \pm 1 \). The only solution consistent with supersymmetry (169), (170) and the requirement that

\[
A^d_1 A^d_6 = A^d_3 A^d_4
\]

(186)

is when \( \epsilon = -1 \) and

\[
(A^d_1, A^d_3, A^d_4, A^d_6) = (1,1,-1,-1)
\]

(187)

But then

\[
(a \cap d, a \cap d') \neq (-1,0), (2,1), \text{ or } (3,2)
\]

(188)

A similar argument for the fourth possibility in (184) shows that

\[
(a \cap d, a \cap d') \neq (-3,0)
\]

(189)

In all cases, therefore, \( a \cap d > a \cap d' \), and we are unable to achieve equal numbers of \( u^c_L \) and \( d^c_L \) quark singlet states using only \( d \)-type stacks. We noted above that only \( c \)-type stacks with \( a \cap c = a \cap c' = 0 \mod 2 \) are allowed, so it follows that we cannot obtain just the standard-model quark-singlet spectrum from this model. A similar argument applies to the solution on the BAB lattice which also has the property (157) and (158). We conclude that for one reason or the other none of the solutions presented in [8], in which \( T^3_2 \) is of B-type, can yield just the standard model spectrum.
5 Solutions for AAA lattice

Fortunately, some, but not all, of the solutions on lattices in which $T^2$ is of $A$-type can be extended to give the standard-model spectrum.

5.1 Solutions with $\text{Im } U_3 = -1/\sqrt{3}$

Consider first the solutions on the AAA lattice with $\text{Im } U_3 = -1/\sqrt{3}$ presented in Table I. On this lattice the supersymmetry constraint (35) requires that the bulk wrapping numbers for all stacks $\kappa$ satisfy

$$ (A_1^\kappa, A_3^\kappa, A_4^\kappa, A_6^\kappa) = (1, 0, 0, 0) \pmod{2} $$

or

$$ (1, \theta, 1, \theta) \pmod{2} $$

or

$$ (0, 0, 1, 0) \pmod{2} $$

where $\theta = 0, 1$. This restricts the allowed wrapping numbers $(n^\kappa, m^\kappa)$ to the cases

$$ (n^1_1, m^1_1; n^1_2, m^1_2; n^1_3, m^1_3) = (1, 0; 1, 0; 1, 0) \pmod{2} $$

or

$$ (1, 1; \theta, 1; 1, 1) \pmod{2} $$

or

$$ (0, 1; 1, 1; 0, 1) \pmod{2} $$

respectively, in the “gauge” in which $(n^1_1, m^1_1) = (n^1_3, m^1_3) \pmod{2}$. As before, there is also a fourth class which may be obtained by the action of $R$ on (191) and (194), but we need only consider one of them. We denote by $c$ stacks with wrapping numbers satisfying (190), (193), by $d_1$ stacks with wrapping numbers satisfying (191), (194), and by $e$ stacks with wrapping numbers satisfying (192), (195). The three solutions in the lower half of the Table all have the same $SU(2)_L$ stack $b$ with

$$ \Pi_{b}^{\text{bulk}} = \rho_4 + 2\rho_6 = \Pi_{b}^{\text{bulk}'} $$

and $\Pi_b^{\text{ex}}$ given in (66) and (67). It follows from (123) that for this solution

$$ b \cap \kappa - b \cap \kappa' = -3A_3^\kappa $$

$$ = A_6^\kappa - 2A_4^\kappa $$

the last line following from the supersymmetry constraint (35). This only satisfies (120) if $\kappa$ is of type $d_1$. Further, the only possibility that does not entail unwanted vector-like doublets is when

$$ A_3^{d_1} = \epsilon $$

where $\epsilon = \pm 1$. The only solutions satisfying the supersymmetry constraints (34), (35) and the consistency condition, namely

$$ A_1^{d_1} A_6^{d_1} = A_3^{d_1} A_4^{d_1} $$

are

$$ (A_1^d, A_3^d, A_4^d, A_6^d) = (1, 1, 3, 3) $$

or

$$ = (1, -1, -1, 1) $$

The former requires that

$$ (n_1^d, m_1^d) = \eta \chi (-1, 1), (n_2^d, m_2^d) = \chi (1, -1), (n_3^d, m_3^d) = \eta (1, 3) $$

Now, from (60), we have that

$$ b \cap \kappa + b \cap \kappa' = \frac{1}{2} \Pi_b^{\text{ex}} \cap \Pi_{\kappa}^{\text{ex}} $$

Then (203) gives
Hence, we must take

\[ y_d = -\frac{1}{2} \]  

(206)

so that there are three lepton \( L \) or Higgs \( \bar{H} \) doublets with weak hypercharge \( Y = -\frac{1}{2} \). The latter possibility (202) requires that

\[
\begin{align*}
(n_1^d, m_1^d) &= \eta \chi (1, -1), \quad (n_2^d, m_2^d) = \chi (1, 1), \quad (n_3^d, m_3^d) = \eta (1, -1) \\
\text{or} \quad (n_1^d, m_1^d) &= \eta \chi (1, 1), \quad (n_2^d, m_2^d) = \chi (1, -1), \quad (n_3^d, m_3^d) = \eta (1, -1)
\end{align*}
\]  

(207) \hspace{1cm} (208)

These give

\[
(b \cap d, b \cap d') = (0, -3) \text{ or } (3, 0)
\]  

(209) \hspace{1cm} (210)

respectively, and in this case we must take

\[ y_d = \frac{1}{2} \]  

(211)

Both possibilities require that there is precisely one further \( U(1) \) stack \( \kappa \) whose intersections with \( b \) give the remaining lepton/Higgs doublets

\[
(b \cap \kappa, b \cap \kappa') = \pm (1, 1)
\]  

(212)

Then, from (197), (198) and the consistency condition, we infer that \( \kappa \) must be of type \( e \) or of type \( c \) with

\[
(A_e^1, A_e^3, A_e^4, A_e^6) = (2j + 1)(0, 0, 1, 2)
\]  

(213) \hspace{1cm} \text{or} \hspace{1cm} (2j + 1)(1, 0, 0, 0)

(214)

with the integer \( j \geq 0 \) to ensure positivity of \( X_{e,c} \). However,

\[
\Pi_e^{\text{ex}} \cap \Pi_e^{\text{ex}} = (-1)^{j_0 + j_1 + 1}[1 + (-1)^{j_0 + j_1}][m_e^2 + (-1)^{j_1}n_e^2]
\]

(215)

\[ = 0 \text{ mod } 8 \]  

(216)

since an \( e \) stack has \( (n_e^2, m_e^2) = (1, 1) \) mod 2. Hence

\[
\frac{1}{2} \Pi_e^{\text{ex}} \cap \Pi_e^{\text{ex}} \equiv (b \cap e + b \cap e') \equiv 0 \text{ mod } 4
\]

(217) \hspace{1cm} (218)

It follows that we can never satisfy (212) with a \( e \) type stack.

For a \( c \)-type stack, it follows from (214) that

\[
(n_3^c, m_3^c) = \eta (1, 0)
\]  

(219)

where \( \eta = \pm 1 \), and that

\[
\left( \begin{array}{c}
\eta^c \\
\eta^c
\end{array} \right) = \frac{\eta (2j + 1)}{n^c_2 + m^c_2 n^c_2 + m^c_2} \left( \begin{array}{c}
\eta^c + m^c_2 \\
-m^c_2
\end{array} \right)
\]  

(220)

We also require that \( b \cap c + b \cap c' = \pm 2 \). Hence

\[
-(n_2^c + m_2^c) + (-1)^{j_1}n_2^c = 2\phi
\]  

(221)

where \( \phi = \pm 1 \). We define

\[
p_c \equiv n_2^c - (-1)^{j_1} \phi \equiv 0 \text{ mod } 2
\]  

(222)

Then for every pair of (coprime) wrapping numbers of the form
we are guaranteed to generate the required bulk wrapping numbers \(214\) and intersection numbers provided that we choose \(j\) such that

\[
\left( \begin{array}{c}
\frac{n^c_i}{m^c_i}
\end{array} \right) = \frac{\eta(2j + 1)}{[2 - (1)^{\tau^c_i}p_c^2 + [2 + (1)^{\tau^c_i}]} \left( \begin{array}{c}
\frac{(-1)^{\tau^c_i}p_c - \phi}{p_c[1 - (1)^{\tau^c_i}] + [1 + (1)^{\tau^c_i}]\phi}
\end{array} \right)
\]  

(224)

are also coprime integers. For example, if \(p_c = 0\), then

\[
\left( \begin{array}{c}
\frac{n^c_i}{m^c_i}
\end{array} \right) = \frac{\eta(2j + 1)}{2 + (1)^{\tau^c_i}} \left( \begin{array}{c}
\frac{(-1)^{\tau^c_i} + 1}{\phi}
\end{array} \right)
\]

(225)

Thus, we must choose \(j = \frac{1}{2}[1 + (1)^{\tau^c_i}]\). In the cases that \(\tau^c_i = 0 \mod 2\)

\[
\left( \begin{array}{c}
\frac{n^c_i}{m^c_i}
\end{array} \right) = \eta(2j + 1) \left( \begin{array}{c}
\frac{(-1)^{\tau^c_i}}{2\phi}
\end{array} \right)
\]

(226)

and we must choose \(j = 1 + \frac{1}{2}p_c^2\). We conclude that to get the required standard-model doublet spectrum we must have both a \(d\)-type stack, and a \(c\)-type stack.

Since there are no antisymmetric representations \(A_a = 3\) on the \(SU(3)\) colour stack \(a\), all of the quark singlets must arise at intersections of \(a\) with the \(U(1)\) stacks \(d\) and \(c\), unless we introduce further stacks that have no intersections with the \(SU(2)_L\) stack \(b\). Consider the first of the three solutions in the bottom half of Table 1. For the first possibility for \(d\), given in (201), we have

\[
\Pi_{a}^{\text{bulk}} \cap \Pi_{d}^{\text{bulk}} = 0
\]

(227)

\[
\Pi_{a}^{\text{bulk}} \cap \Pi_{d}^{\text{bulk}'} = 12
\]

(228)

and

\[
\Pi_{a}^{\text{ex}} \cap \Pi_{d}^{\text{ex}} = (-1)^{\tau^d_0 + \tau^d_2}2\chi[1 + (1)^{\tau^d_0 + \tau^d_2}][1 + (1)^{\tau^d_0}]
\]

(229)

\[
\Pi_{a}^{\text{ex}} \cap \Pi_{d}^{\text{ex}'} = (-1)^{\tau^d_0 + \tau^d_2}2\chi[1 + (1)^{\tau^d_0 + \tau^d_2}][2 - (1)^{\tau^d_0}]
\]

(230)

Hence if

\[
\tau^d_0 + \tau^d_2 = 1 \mod 2
\]

(231)

then

\[
(a \cap d, a \cap d') = (0, 3)
\]

(232)

and, using (206), we get three \(d^c_L\) quark-singlet states from intersections of \(a\) with \(d'\). To avoid vectorlike quark-single matter, therefore, we must take

\[
\tau^d_0 + \tau^d_2 = 0 \mod 2
\]

(233)

Then if

\[
\tau^d_1 = 0 \mod 2
\]

(234)

then

\[
(a \cap d, a \cap d') = (-2, 2) \text{ or } (2, 4)
\]

(235)

both of which violate the inequalities (119) and are therefore unacceptable. Alternatively, if

\[
\tau^d_1 = 1 \mod 2
\]

(236)

then

\[
(a \cap d, a \cap d') = (0, 6) \text{ or } (0, 0)
\]

(237)

Only the latter does not violate (119), and this occurs when

\[
\tau^d_0 + \tau^d_2
\]
in which case
$$\Pi_{ex}^a = \Pi_{ex}^d$$
(239)

with the latter given in (63). Then \(d = a\), which is unacceptable.

In the second possibility for \(d\), given in (202),

$$\Pi_a^{\text{bulk}} \cap \Pi_d^{\text{bulk}} = 0$$
(240)

$$\Pi_a^{\text{bulk}} \cap \Pi_d^{\text{bulk}}' = -12$$
(241)

Using (207) we get

$$\Pi_a^{\text{ex}} \cap \Pi_d^{\text{ex}} = \frac{1}{\Delta}(-1)^{\frac{\tau^a_0 + \tau^d_0}{2}}[1 + (-1)^{\frac{\tau^a_2 + \tau^d_2}{2}}3(-1)^{\tau^d_2} - 1]$$
(242)

$$\Pi_a^{\text{ex}} \cap \Pi_d^{\text{ex}}' = \frac{1}{\Delta}(-1)^{\frac{\tau^a_0 + \tau^d_0}{2}}[1 + (-1)^{\frac{\tau^a_2 + \tau^d_2}{2}}3(-1)^{\tau^d_2}]$$
(243)

whereas using (208) we get (229) and (230) again. Hence if

$$\tau^a_2 + \tau^d_2 = 1 \mod 2$$
(244)

then

$$(a \cap d, a \cap d') = (0, -3)$$
(245)

and, using (211), we get \(3u_L\) quark-singlet states from intersections of \(a\) with \(d'\). Alternatively, if

$$\tau^a_2 + \tau^d_2 = 0 \mod 2$$
(246)

the only solution that does not entail vector-like quark-singlet matter is

$$(a \cap d, a \cap d') = (0, 0)$$
(247)

which occurs when \(\tau^d_1 = 0 \mod 2\) in the case of (207) and when \(\tau^d_1 = 1 \mod 2\) for (208). In both cases

$$(-1)^{\tau^a_0 + \tau^d_0}[1 + (-1)^{\tau^a_2 + \tau^d_2}] = +1$$
(248)

and

$$\Pi_{ex}^d = -\Pi_{ex}^a$$
(249)

Similarly, with the fourth stack of type \(c\), as given in (214) and (223), then

$$a \cap c = -\frac{3}{2}(2j + 1) + (-1)^{\frac{\tau^c_0 + \tau^d_0}{2}}[p_c((-1)^{\tau^c_1} - 2) + 3\phi]$$
(250)

$$a \cap c' = -\frac{3}{2}(2j + 1) + (-1)^{\frac{\tau^c_0 + \tau^d_0}{2}}[p_c(2 - (-1)^{\tau^c_1}) + 3\phi]$$
(251)

Hence,

$$a \cap c - a \cap c' = p_c((-1)^{\frac{\tau^c_0 + \tau^d_0}{2}}[1 + (-1)^{\tau^c_1} - 2]$$
(252)

$$= 0 \mod 2$$
(253)

If we use the possibility (245), it is therefore impossible to obtain the required \(3d_L\) quark singlets that are needed just from the intersections of the \(SU(3)_c\) stack of type \(c\) with \(d\). To avoid vectorlike matter we must take \(p_c = 0\) or else \(|p_c| = 2\) but then only with \(\tau^c_1 = 0 \mod 2\). For the former,

$$2j + 1 = 2 + (-1)^{\tau^c_1}$$
(254)

so that

$$a \cap c = a \cap c' = \frac{3}{2}[-2 + (-1)^{\tau^c_1} + (-1)^{\tau^c_0 + \tau^d_0}\phi]$$
(255)
For either choice of $y_c = \pm \frac{1}{2}$, we get all six quark-singlet states $3u^c_L + 3d^c_L$, when

$$(-1)^{\tau_1^c} + (-1)^{\rho_0^c + \tau_0^c} \phi = 0$$

(257)

so that

$$\Pi_{c}^{ex} = (-1)^{\tau_0^c} \phi \left( 2[\epsilon_1 + (-1)^{\tau_2^c} \epsilon_4] + [\bar{\epsilon}_1 + (-1)^{\tau_2^c} \bar{\epsilon}_4] \right)$$

(258)

$$\Pi_{c}^{ex'} =$$

(259)

The only other possibility is that $\tau_1^c = 1 \mod 2$ and $|p_c| = 2$. Then $2j + 1 = 7$, and

$$a \cap c, a \cap c' \geq 8$$

(260)

which violate (119) and so are unacceptable.

Our conclusion is that to get the required quark-singlet spectrum, we must ensure that none of them arise at intersections with the $d$-type stack, and that they all arise from intersections with $c$ and $c'$. This requires that $d$ is given by (202), (249) and (63) with (246) and (248). It then follows that

$$\#(S_d) = \frac{1}{2} (d \cap d' - d \cap \Pi_{O6}) = 0$$

(261)

and there are no lepton singlets on $d$. Similarly, the $c$ stack is given by

$$2c = 2c'$$

(262)

$$= (2 + (-1)^{\tau_1^c}) \rho_1 + (-1)^{\tau_0^c} \phi \left( 2[\epsilon_1 + (-1)^{\tau_2^c} \epsilon_4] + [\bar{\epsilon}_1 + (-1)^{\tau_2^c} \bar{\epsilon}_4] \right)$$

(263)

with the constraint (257). It follows that,

$$\#(S_c) = 0$$

(264)

and there are no lepton singlets on $c$ either. Finally, we find

$$d \cap c = d \cap c' = \frac{3}{2} \left[2 + (-1)^{\tau_1^c} + (-1)^{\rho_0^c + \tau_0^c} \chi \phi \right]$$

(265)

$$= 3$$

(266)

using (248) and (257). It follows, using (211), that at these intersections we get the three standard-model charged lepton singlets $3\ell^c_L$ plus three neutral lepton singlets $3\nu_c^c$ for either choice of $y_c$. A similar analysis for the other solutions displayed in Table 1 yields the same conclusions, and the same physics. The first solution in the Table was the one used to illustrate our conclusions in the Erratum to [9].

### 5.2 Solution with $\text{Im} U_3 = -\sqrt{3}$

Consider next the solution presented in Table 2. The supersymmetry constraint with this value of $\text{Im} U_3$ on this lattice, together with the consistency condition $A_0^d A_0^d = A_0^d A_0^d$, allows the same three classes of branes as were found for the lattice with $\text{Im} U_3 = \frac{1}{\sqrt{3}}$, namely those characterised by equations (190), (191), ..., (195).

As noted in §3.1.2, the $SU(2)_L$ stack $b$ is identical to that given in (67) for the three solutions in the bottom half of Table 1. However, in this case, the supersymmetry constraints allow solutions of (120) only when $\kappa$ is $d$-type with

$$(A_1^d, A_3^d, A_4^d, A_6^d) = (1, 1, 1, 1)$$

(267)

or

$$= (0, -1, 0, 1)$$

(268)

These are just orientifold duals of each other, so that we need only consider the first possibility. It requires that

$$(n_1^d, m_1^d) = \eta \chi(-1, 1), \ (n_2^d, m_2^d) = \chi(1, -1), \ (n_3^d, m_3^d) = \eta(1, 1)$$

(269)
which gives the same intersection numbers as in \((205)\), and we must take \(y_d\) as given in \((206)\). Again, we need at least one further \(U(1)\) stack \(\kappa\) satisfying \((212)\). The solution is identical to that given in \((214)\) and \((224)\).

Next we determine the quark-singlet states that arise on the \(SU(3)_{\text{colour}}\) stack \(a\), and at its intersections with \(d\) and \(c\). First, since \(#(S_a) = 0\), it follows that

\[
#(A_a) = a \cap \Pi_{O6} = 2
\]  

(270)

Thus there are 2\(d^c_L\) quark-singlet states on \(a\), and we require one further \(d^c_L\) and 3\(a^c_L\) from the intersections of \(a\) with the \(U(1)\) stacks \(d\) and \(c\). \(d\) is specified in \((267)\) with \((269)\), then

\[
(a \cap d, a \cap d') = (0, 1) + (\ldots) \chi (1 + \ldots) \left((-1)^{\tau^d_1} - 1, -(-1)^{\tau^d_1}\right)
\]  

(271)

With \(y_d\) given by \((206)\), the states at \(a \cap d = 3_{Y=\frac{4}{3}}\), while those at \(a \cap d' = 3_{Y=-\frac{4}{3}}\). Thus, to avoid vector-like quark-singlet matter, we require that \(a \cap d, a \cap d' \leq 0\). The only acceptable solution is therefore when \(a \cap d = 0 = a \cap d'\). Thus, the only possibility is to get all of the required quark-singlets from intersections with \(c\). With \(c\) given by \((214)\) and \((223)\), we find

\[
(a \cap c, a \cap c') = -\frac{1}{2} (2j + 1, 2j + 1) + \frac{1}{2} (\ldots) (2 - (\ldots)) - \phi, -\phi
\]  

(272)

Hence

\[
a \cap c - a \cap c' = -\frac{1}{2} (\ldots) (2 - (\ldots))
\]  

(273)

and to avoid vector-like matter

\[
p_c = 0 \text{ or } |p_c| = 2
\]  

(274)

the latter only being possible when \(\tau^d_1 = 0 \mod 2\). However, \(p_c = 0\) gives \(a \cap c = a \cap c'\) which cannot yield all of the missing quark singlets. The alternative requires that \(2j + 1 = p^2_c + 3 = 7\) so that

\[
a \cap c + a \cap c' = -7 + (\ldots) \phi \geq -6
\]  

(275)

Thus, in this case we cannot obtain the required quark-singlet spectrum.

5.3 Solutions with \(\text{Im } U_3 = -2/\sqrt{3}\) and \(\text{Im } U_3 = -1/2\sqrt{3}\)

All three of the solutions given in Tables 3 and 4 violate the inequality \((22)\). They have \(#(A_a) = a \cap \Pi_{O6} = -3\), corresponding to 3\(d^c_L\) quark singlets on the \(SU(3)_{\text{colour}}\) stack \(a\). Therefore, these models cannot yield just the standard-model quark-singlet spectrum. The same objection applies to the solutions given in Table 9 for the ABA lattice and Table 10 for the BBA lattice.

6 Solutions for BAA lattice

The treatment of the solutions found on the BAA lattice proceeds very similarly to that in the previous section for the solutions on the AAA lattice. Both of the solutions given in Table 7 for the case \(\text{Im } U_3 = -2/\sqrt{3}\) and Table 8 for the case \(\text{Im } U_3 = -\sqrt{3}/2\) violate \((22)\). They also have \(#(A_a) = a \cap \Pi_{O6} = -3\), and so cannot yield just the standard-model quark-singlet spectrum.

6.1 Solution with \(\text{Im } U_3 = -1/\sqrt{3}\)

This is given in Table 5 and its treatment is very similar to that given in \((5.2)\). In this case supersymmetry constrains the bulk wrapping numbers to satisfy

\[
(A^u_1, A^u_3, A^u_4, A^u_6) = (0, 1, 0, 0) \mod 2
\]  

(276)

or

\[
(\theta, 1, \theta, 1) \mod 2
\]  

(277)
where \( \theta = 0, 1 \). This restricts the allowed wrapping numbers \((n^\kappa_k, m^\kappa_k)\) to the cases

\[
(n^\kappa_1, m^\kappa_1; n^\kappa_2, m^\kappa_2; n^\kappa_3, m^\kappa_3) = (1, 0; 1, 1; 1, 0) \mod 2 \tag{279}
\]

or

\[
(1, 1; 1, \theta; 1, 1) \mod 2 \tag{280}
\]

or

\[
(0, 1; 0, 1; 0, 1) \mod 2 \tag{281}
\]

We denote by \( c \) stacks with wrapping numbers satisfying \((276), (279)\), by \( d_\theta \) stacks with wrapping numbers satisfying \((277), (280)\), and by \( e \) stacks with wrapping numbers satisfying \((278), (281)\). With the \( SU(2)_L \) stack \( b \) given by \((86)\), to satisfy \((120)\) requires the existence of a \( d_1 \)-type stack. The unique solution that avoids vector-like doublets again has the form given in \((267)\) and \((269)\) which gives the intersection numbers provided that we choose \( p \) and \((282)\) that

\[
(A^\kappa_1, A^\kappa_3, A^\kappa_4, A^\kappa_6) = (2j + 1)(0, 0, 0, 1) \tag{282}
\]

with the integer \( j \geq 0 \). This requires that

\[
(n^\kappa_3, m^\kappa_3) = \eta(0, 1) \tag{283}
\]

where \( \eta = \pm 1 \). Then for every pair of (co prime) wrapping numbers of the form

\[
(n^\kappa_2, m^\kappa_2) = \left( [-1]^{\tau^\kappa_1} - 1|p_e + [-1]^{\tau^\kappa_1} + 1|\phi, p_e - (-1)^{\tau^\kappa_1} \phi \right) \tag{284}
\]

with \( p_e = 0 \mod 2 \), we are guaranteed to generate the required bulk wrapping numbers \((282)\) and intersection numbers provided that we choose \( j \) such that

\[
\begin{pmatrix}
  n^\kappa_1 \\
  m^\kappa_1
\end{pmatrix}
= \frac{\eta(2j + 1)}{[2 - (-1)^{\tau^\kappa_1}] p_e^2 + [2 + (-1)^{\tau^\kappa_1}]} \begin{pmatrix}
  1 - (-1)^{\tau^\kappa_1} p_e - [1 + (-1)^{\tau^\kappa_1}] \phi \\
  p_e(-1)^{\tau^\kappa_1} + \phi
\end{pmatrix} \tag{285}
\]

are also coprime integers.

Since there are \( \text{two} \) antisymmetric representations \( A_n = \bar{3}_Y \) on the \( SU(3) \) colour stack \( a \), and these correspond to \( 2d^c_L \) quark singlets, all of the remaining \( d^c_L + 3w^c_L \) quark singlets must arise at intersections of \( a \) with the \( U(1) \) stacks \( d \) and \( e \). As in \((5.2)\), the only way to avoid vector-like quark singlets at the intersections of \( a \) with \( d \) and \( d' \) is to ensure that \( a \cap d = 0 = a \cap d' \). It follows from \((282)\) and \((284)\) that

\[
\begin{align*}
a \cap e - a \cap e' &= p_e(-1)^{\tau^\kappa_0 + \tau^\kappa_6}[2 - (-1)^{\tau^\kappa_1}] \\
&\text{and to get the missing quark singlets we therefore require that } |p_e| = 2 \text{ and } \tau^\kappa_1 = 0 \mod 2 \text{. However, in that case, } j = 3 \text{ and }
\end{align*}
\]

\[
a \cap e + a \cap e' = -7 + (-1)^{\tau^\kappa_0 + \tau^\kappa_6} \phi \tag{287}
\]

where \( \phi = \pm 1 \). Hence,

\[
|a \cap e + a \cap e'| \geq 6 \tag{288}
\]

so that we cannot get just the standard-model quark-singlet spectrum.

### 6.2 Solutions with \( \text{Im } U_3 = -\sqrt{3} \)

Supersymmetry on this lattice again allows the same three classes as we found in \((6.1)\). The form of the \( SU(2)_L \) stack \( b \) for the first two solutions given in Table \((6)\) and \((96)\) again requires the existence of a \( d \)-type stack with

\[
(A^d_1, A^d_3, A^d_4, A^d_6) = (1, -1, -1, 1) \tag{289}
\]

or

\[
(3, 3, 1, 1) \tag{290}
\]
Proceeding as before these give

\[
(b \cap d, b \cap d') = (3, 0), (0, -3), (2, -1) \text{ or } (1, -2) \quad (291)
\]

or

\[
\text{or } = (-2, 1) \text{ or } (-1, 2) \quad (292)
\]

respectively. Hence, we must take

\[
y_d = \frac{1}{2} \quad (293)
\]

or

\[
y_d = -\frac{1}{2} \quad (294)
\]

so that there are three (lepton) doublets \( L \) having weak hypercharge \( Y = -\frac{1}{2} \). On this occasion the fourth stack must be of type \( c \) with

\[
(A_1^c, A_3^c, A_4^c, A_6^c) = (2j + 1)(2, 1, 0, 0) \quad (295)
\]

Then

\[
(n_3^c, m_3^c) = \eta(1, 0) \quad (296)
\]

where \( \eta = \pm 1 \), and for every pair of (coprime) wrapping numbers of the form

\[
(n_3^c, m_3^c) = (\phi - (-1)^{\tau_1} p_c, p_c + (-1)^{\tau_1} \phi) \quad (297)
\]

with \( p_c = 0 \text{ mod } 2 \), we are guaranteed to generate the required bulk wrapping numbers (295) and intersection numbers provided that we choose \( j \) such that

\[
\left\{ \begin{array}{l}
\eta(2j + 1) \\
\phi\phi
\end{array} \right\} = \frac{\phi}{2} \frac{(-1)^{\tau_1} p_c + [1 + 2(-1)^{\tau_1}] \phi}{2 + (-1)^{\tau_1} (\phi - (1 + (-1)^{\tau_1}) - p_c [1 + (-1)^{\tau_1}])} \quad (298)
\]

are also coprime integers.

Consider the \( SU(3)_\text{colour} \) stack \( a \) for the first solution presented in Table 6. Since there are no antisymmetric representations on \( a \), all of the \( 3a_L^c + 3a_L^c \) quark singlets must arise at intersections of \( a \) with the \( U(1) \) stacks \( d \) and \( c \). If we use the bulk part of \( d \), given in (289), then the only acceptable result is \( a \cap d = 0 = a \cap d' \). If instead we use (290), then there is the additional possibility that \( (a \cap d, a \cap d') = (-3, 0) \), which corresponds to \( 3a_L^c \) quark singlets at the intersections of \( a \) with \( d \). However, it follows from (297) that \( a \cap c - a \cap c' = 0 \text{ mod } 2 \), which means that we cannot get \( 3d_L^c \) quark-singlet states from the intersections of \( a \) with \( c \) and \( c' \). Thus, as in (5.1) we require that \( a \cap d = 0 = a \cap d' \), and we must get all of the quark-singlet states from the intersections with the fourth stack. The former requires that

\[
(-1)^{\tau_6 + \tau_6^d} \chi = \pm 1 \quad (299)
\]

where \( n_3^d = \chi = \pm 1 \), the upper sign in (299) corresponds to (289), and the lower to (290). The latter requires that

\[
p_c = 0 \quad (300)
\]

and hence that

\[
j = 0 \quad (301)
\]

Then

\[
a \cap c = a \cap c' = -\frac{3}{2}[1 + (+1)^{\tau_6 + \tau_6^d} \phi] \quad (302)
\]

where \( \phi = \pm 1 \), so that \( (a \cap c, a \cap c') = (-3, -3) \) provided that

\[
(-1)^{\tau_6 + \tau_6^d} \phi = 1 \quad (303)
\]

As before, no lepton-singlet states arise as symmetric representations \( S_d \) or \( S_c \) on the \( U(1) \) stacks \( d \) and \( c \), and we find

\[
d \cap c = d \cap c' = \pm \frac{3}{2}[1 \mp (-1)^{\tau_6 + \tau_6^d} \chi \phi] \quad (304)
\]
again with the upper sign corresponding to (289) and (293) and the lower to (290) and (294); the last line follows using (299) and (303). Either way we again get three charged-lepton singlets \(3\ell_L^c\) plus three neutral-lepton singlets \(3\nu_L^c\) for either choice of \(y_a = \pm \frac{1}{2}\).

The treatment of the second solution in Table 6 is almost the same. The two solutions have the same \(SU(2)_L\) stack \(b\), so that to get the correct \(3L + H + H\) doublet content, we need \(U(1)\) stacks \(d\) and \(e\) of the same general form as just found. The two solutions differ only in the form of the \(SU(3)_{\text{colour}}\) stack \(a\). However, it turns out that

\[
\left(\Pi_a^{\text{bulk}} \cap \Pi_d^{\text{bulk}}, \Pi_a^{\text{bulk}} \cap \Pi_d^{\text{bulk}'}\right) = \left(\tilde{\Pi}_a^{\text{bulk}} \cap \tilde{\Pi}_d^{\text{bulk}'}, \tilde{\Pi}_a^{\text{bulk}} \cap \tilde{\Pi}_d^{\text{bulk}}\right)
\]

(306)

where \(\tilde{\Pi}_a^{\text{bulk} (\text{ex})}\) is the bulk (exceptional) part of \(a\) in the first solution. Further, as noted after (97),

\[
\Pi_a^{\text{ex}} = -\mathcal{R}\tilde{\Pi}_a^{\text{ex}}
\]

(307)

It follows that

\[
\left(\Pi_a^{\text{ex}} \cap \Pi_d^{\text{ex}}, \Pi_a^{\text{ex}} \cap \Pi_d^{\text{ex}'}\right) = \left(\tilde{\Pi}_a^{\text{ex}} \cap \tilde{\Pi}_d^{\text{ex}'}, \tilde{\Pi}_a^{\text{ex}} \cap \tilde{\Pi}_d^{\text{ex}}\right)
\]

(308)

and hence that

\[
\langle a \cap d, a \cap d' \rangle = \langle \tilde{a} \cap d', \tilde{a} \cap d \rangle
\]

(309)

where \(\tilde{a}\) denotes the full fractional \(SU(3)\) stack in the first solution. Since the only acceptable solution was \(\langle \tilde{a} \cap d', \tilde{a} \cap d \rangle = (0, 0)\), we conclude that the same is the case for this solution and that \(d\) has exactly the same form as for the first solution. Likewise,

\[
\Pi_a^{\text{bulk} \cap \Pi_c^{\text{bulk}}} = \Pi_a^{\text{bulk} \cap \Pi_c^{\text{bulk}'}} = (2j + 1)(-6)
\]

(310)

exactly as in the first solution. Thus the argument given above for \(d\) follows again for \(c\), and \(c\) too has exactly the same form as before. Consequently the physics of the two solutions is identical.

A similar relationship exists between the two solutions in the bottom half of Table 6. Both have the same \(SU(2)_L\) stack \(b\) as the solution in §6.1. To get the correct \(3L + H + H\) doublet content, we therefore need \(U(1)\) stacks \(d\) and \(e\) of the same general form as found there. It turns out that the requirements of supersymmetry and the absence of vector-like doublets forces \(d\) to have the same form (289) or (290) as found for the first and second solutions, and the same is true of the exceptional part. Likewise, the form of \(e\) is as we found in §6.1 and specified in (282) and (284). The only acceptable solutions again require that all quark-singlet states arise at intersections of the \(SU(3)_{\text{colour}}\) stack \(a\) with \(e\) and \(e'\), which in turn requires that \(p_e = 0 = j\). Then there are no symmetric representations on \(d\) or \(e\) and once again we get \(3\ell_L^c + 3\nu_L^c\) from the intersections of the \(U(1)\) stacks. These two solutions therefore constitute a different realisation of the same physics as in the two models in the upper half of Table 6.

### 7 Tadpole cancellation

The cancellation of RR tadpoles, and hence of the NSNS tadpoles, requires that the overall homology class of the D6-branes and the O6-planes vanishes [11][12][13]:

\[
\sum_{\kappa} N_{\kappa}(\kappa + \kappa') - 4\Pi_{O6} = 0
\]

(311)

where the sum is over all D6-brane stacks \(\kappa\). Both bulk and exceptional parts are required to cancel separately. Since \(\Pi_{O6}\) has no exceptional part, the contributions from the exceptional parts \(\Pi_{\kappa}^{\text{ex}}\) of the various stacks \(\kappa\) must cancel among themselves. In our case, \(\kappa\) ranges over four stacks: the \(SU(3)_{\text{colour}}\) stack \(a\), the \(SU(2)_L\) stack \(b\), and the two \(U(1)\) stacks that are always needed to ensure the correct, supersymmetric standard-model lepton/Higgs doublet content.

We only need consider the models in which \(T_3^2\) is of A type, since only (some of) these have been extended to give the standard-model spectrum. As previously noted, in all such models the \(SU(2)_L\) stack \(b\) has the property (122), and it follows that there is no contribution from this stack to the exceptional part.
other (three) stacks cancel. In the case of the standard-model solution derived in [5.1] the $SU(3)_{\text{colour}}$
stack $a$ is of type $d$, as defined in (191) and (194), and therefore uses the fixed points (16) or (45) in $T^2_3$; then $\Pi^c_{\alpha}$ involves $\epsilon_{1,6}$ and $\bar{\epsilon}_{1,6}$ or $\epsilon_{4,5}$ and $\bar{\epsilon}_{4,5}$, as found in [48]. The $U(1)$ stack denoted $d$ is also of $d$-type (hence the nomenclature), so that $\Pi^d_{\alpha}$ involves the same exceptional cycles. However, the fourth stack $c$ is of $c$-type, defined in (190) and (193). It therefore uses the fixed points (14) or (56) in $T^2_3$, and $\Pi^c_{\alpha}$ involves $\epsilon_{1,4}$ and $\bar{\epsilon}_{1,4}$ or $\epsilon_{5,6}$ and $\bar{\epsilon}_{5,6}$. Clearly, there is no possibility that the contributions to (311) from the exceptional parts of $a$, $d$, and $c$ cancel. In fact, for models in which $T^2_3$ is of $A$ type, such a cancellation requires either that all three stacks are of the same general type $c$, $d$ or $e$, or that they are all of different types. In all of our standard-model solutions $T^2_3$ is of $A$ type, and none of them has the property necessary for cancellation.

In principle there are two ways to remedy the situation. One possibility is to add further branes designed to remove the discrepancy. However, these will certainly generate additional gauge symmetry, and probably extra matter. It might be possible to arrange that the unwanted extra matter and gauge interactions are hidden from the observable sector containing the standard model that we have previously obtained. However, this approach is contrary to our objective which was to obtain just the standard-model spectrum, and we have not explored it further. The alternative is to introduce background fluxes [14, 15, 16, 17, 18, 19]. These do not generate unwanted gauge groups or matter, and, in any case, may be needed to stabilise moduli and to break supersymmetry. The 7-form gauge field $C_7$ to which the D6-branes are electrically coupled also couples to certain background field strengths. The generalised tadpole cancellation condition, in the presence of background fields, may be derived [19] from the contribution of the RR fields to the (massive) type IIA supergravity action, supplemented by the coupling of the D6-branes to $C_7$:

$$S_{\text{IIA}} = -\frac{1}{2\kappa^2} \int \frac{1}{2} F_2 \wedge F_2 + ... + \mu_6 \sum_{\kappa} N_\kappa \int_{\mathcal{M}_4 \times \kappa} C_7$$

(312)

where

$$2\kappa^2 = (2\pi)^7 \alpha'^4$$

(313)

$$\mu_6 = (2\pi)^{-6} \alpha'^{-7/2}$$

(314)

with $2\pi\alpha'$ the inverse string tension. The sum over $\kappa$ is here understood to include the orientifold image $\kappa'$ of $\kappa$, as well as the O6-plane coupled with charge $-4\mu_6$.

$$F_2 = dC_1 + mB_2 + \bar{F}_2$$

(315)

is the field strength of the 1-form gauge field $C_1$ dual to $C_7$

$$dC_7 = F_8 := ^*dC_1$$

(316)

$B_2$ is the Kalb-Ramond 2-form with field strength

$$H_3 = dB_2 + \bar{H}_3$$

(317)

and $\bar{H}_3$ and $\bar{F}_2$ are respectively the possibly non-zero NSNS and RR background field strengths with which we are concerned. Then

$$^*F_2 = dC_7 + m^*B_2 + ^*\bar{F}_2$$

(318)

and

$$F_2 \wedge F_2 = F_2 \wedge (dC_7 + ...) = d(F_2 \wedge C_7) + C_7 \wedge (mH_3 - m\bar{H}_3 + d\bar{F}_2) + ...$$

(319)

The D6-brane contribution to (312) may be rewritten in terms of the Poincaré dual 3-form $\hat{\delta}(\kappa)$ of $\kappa$

$$\int_{\mathcal{M}_4 \times \kappa} C_7 = \int_{\mathcal{M}_4 \times \kappa} C_7 \wedge \hat{\delta}(\kappa)$$

(320)
so that requiring the absence of tadpole terms in a general background field strength gives

$$\frac{1}{4\kappa_{10}^2} (m\bar{H}_3 - d\bar{F}_2) + \mu_6 \sum_{\kappa} N_\kappa \delta(\kappa) = 0$$

(321)

or in terms of the usual homology classes

$$\frac{1}{4\kappa_{10}^2} \Pi_{mH_3 - dF_2} + \mu_6 \left( \sum_{\kappa} N_\kappa (\kappa + \kappa') - 4\Pi_{O6} \right) = 0$$

(322)

where $\Pi_{mH_3 - dF_2}$ is the 3-cycle of which $m\bar{H}_3 - d\bar{F}_2$ is the Poincaré dual, and we have now displayed explicitly the contributions from $\kappa'$ and the O6-plane. In the absence of the background fields $H_3$ and $dF_2$, we retrieve the usual tadpole cancellation constraint (311), which is not satisfied by any of our models. In principle, then, background fields are necessary for a consistent theory. However, the fluxes of these fields through closed cycles are quantised. In general the flux of the background field strength $\bar{F}_{p+2}$ associated with a D$p$-brane satisfies

$$\mu_p \int_{\Sigma_{p+2}} \bar{F}_{p+2} = 2\pi m_p$$

(323)

where

$$\mu_p = (2\pi)^{-p} \alpha'^{(p+1)/2}$$

(324)

is the (electric) charge of the D$p$-brane, $\Sigma_{p+2}$ is a closed $(p+2)$-cycle, and $m_p \in \mathbb{Z}$. The mass $m$ is also a flux, quantised as $F_0 = m = (4\pi^2 \alpha')^{-1/2} n_0$ with $n_0 \in \mathbb{Z}$, and the flux of $H_3$ is quantised like $\bar{F}_3$. The question then is whether we can solve the Diophantine equations that follow from the generalised tadpole cancellation condition (322).

As noted previously, tadpole cancellation must be satisfied separately by the bulk and exceptional brane contributions to (322). This requires that $\Pi_{mH_3 - dF_2}$ is $\mathcal{R}$-invariant. Thus we need the Poincaré duals of the $\mathcal{R}$-invariant combinations of the untwisted bulk 3-cycles $\rho_p$, $p = 1, 3, 4, 6$, and of the $\theta^3$-twisted sector cycles $\bar{e}_j$, $\bar{e}_j$, $j = 1, 4, 5, 6$.

### 7.1 Untwisted sector

The Poincaré dual $\eta$ of a closed 3-cycle $\rho$ in a 6-dimensional manifold $\mathcal{M}$ is a 3-form satisfying

$$\int_\rho i^* \omega_3 = \int_{\mathcal{M}} \omega_3 \wedge \eta$$

(325)

for an arbitrary closed 3-form $\omega_3$. $i^*$ is the pullback mapping the 3-form $\omega_3$ on $\mathcal{M}$ to a 3-form on $\rho$. We may choose

$$\sigma_0 \equiv dz_1 \wedge dz_2 \wedge dz_3$$

(326)

$$\sigma_1 \equiv dz_1 \wedge d\bar{z}_2 \wedge dz_3$$

(327)

$$\sigma_2 \equiv d\bar{z}_1 \wedge d\bar{z}_2 \wedge dz_3$$

(328)

$$\sigma_3 \equiv dz_1 \wedge dz_2 \wedge dz_3$$

(329)

as the 4 elements of the basis of $H^3(T^6/\mathbb{Z}_6')$, the space of invariant untwisted 3-forms. This space is dual to the space $H_3(T^6/\mathbb{Z}_6')$ of (untwisted) 3-cycles. Then the Poincaré duals $\eta_p$, $p = 1, 3, 4, 6$ of the 3-cycles $\rho_p$ are

$$\eta_1 = \frac{3}{4i \text{Vol}(\mathcal{M})} (e_1 \bar{e}_3 \bar{e}_5 \sigma_0 - \bar{e}_1 \bar{e}_3 e_5 \sigma_1 + e_1 e_3 \bar{e}_5 \sigma_2 - e_1 e_3 e_5 \sigma_3)$$

(330)

$$\eta_3 = \frac{3}{4i \text{Vol}(\mathcal{M})} (-\alpha' e_1 \bar{e}_3 e_5 \sigma_0 + \alpha' e_1 \bar{e}_3 e_5 \sigma_1 + \alpha' e_1 e_3 \bar{e}_5 \sigma_2 - \alpha' e_1 e_3 e_5 \sigma_3)$$

(331)

$$\eta_4 = \frac{3}{4i \text{Vol}(\mathcal{M})} (e_1 \bar{e}_3 \bar{e}_5 \tau_3 \sigma_0 - \bar{e}_1 \bar{e}_3 e_5 \tau_3 \sigma_1 + e_1 e_3 \bar{e}_5 \tau_3 \sigma_2 - e_1 e_3 e_5 \tau_3 \sigma_3)$$

(332)

$$\eta_6 = \frac{3}{4i \text{Vol}(\mathcal{M})} (e_1 \bar{e}_3 \bar{e}_5 \sigma_0 - \bar{e}_1 \bar{e}_3 e_5 \sigma_1 + e_1 e_3 \bar{e}_5 \sigma_2 - e_1 e_3 e_5 \sigma_3)$$

(333)
where $e_{2k-1}$ and $e_{2k}$ are complex numbers defining the basis 1-cycles of $T^2_k$, and
\[
\text{Vol}(\mathcal{M}) = \frac{3}{4} |e_1|^2 |e_3|^2 |e_5|^2 \text{Im} \tau_3
\] (334)
since $T^2_1$ and $T^2_2$ are $SU(3)$ lattices; $\tau_3 \equiv e_6/e_5$ is the complex structure of $T^2_3$.

On the AAA lattice,
\[
e_1 = \bar{e}_1 \equiv R_1, \quad e_3 = \bar{e}_3 \equiv R_3, \quad e_5 = \bar{e}_5 \equiv R_5, \quad \text{Re} \tau_3 = 0
\] (335)
and the only model that gives the standard-model spectrum on the AAA lattice has
\[
\text{Im} \tau_3 = -\frac{1}{\sqrt{3}}
\] (336)

Then the Poincaré duals of the $R$-invariant 3-cycles $\rho_1$ and $\rho_4 + 2\rho_6$ are
\[
\eta_1 = \frac{i \sqrt{3}}{R_1 R_3 R_5} (\sigma_0 - \sigma_1 + \sigma_2 - \sigma_3) \quad (337)
\]
\[
\eta_4 + 2\eta_6 = \frac{i \sqrt{3}}{R_1 R_3 R_5} (\sigma_0 + \sigma_1 - \sigma_2 - \sigma_3) \quad (338)
\]
both of which are, like $\bar{H}_3$, odd under the action of $R$. Using the flux quantisation conditions we infer that
\[
\bar{H}_3 = -\frac{\pi^2 \alpha'}{9} [n_3 (\eta_4 + 2\eta_6) + 3n_6 \eta_1]
\] (339)
where $n_{3,6}$ are the integers associated with the flux of $\bar{H}_3$ through the 3-cycles $\rho_{3,6}$.

Similarly, for the solutions on the BAA lattice, for which
\[
\text{Im} \tau_3 = -\sqrt{3}
\] (340)
we find that the 3-forms dual to the $R$-invariant 3-cycles $\rho_6$ and $\rho_3 + 2\rho_1$ are
\[
\eta_6 = \frac{i}{R_1 R_3 R_5} (\sigma_0 + \sigma_1 - \sigma_2 - \sigma_3) \quad (341)
\]
\[
\eta_3 + 2\eta_1 = \frac{i}{R_1 R_3 R_5} (\sigma_0 - \sigma_1 + \sigma_2 - \sigma_3) \quad (342)
\]
Again, both are odd under the action of $R$, like $\bar{H}_3$. In this case flux quantisation gives
\[
\bar{H}_3 = \frac{\pi^2 \alpha'}{9} [3n_1 \eta_6 + n_4 (2\eta_1 + \eta_3)]
\] (343)
with $n_{1,4} \in \mathbb{Z}$ the integers associated with the flux of $\bar{H}_3$ through $\rho_{1,4}$.

### 7.2 $\theta^3$-twisted sector
At each of the 16 fixed points $(k, \ell)$ with $k, \ell = 1, 4, 5, 6$ of $\mathbb{Z}_2$ in $T^2_1 \times T^2_3$ there is a collapsed 2-cycle that we have denoted by $f_{k,\ell}$. To compute the Poincaré duals of these we need to blow up the metric at each fixed point using the Eguchi-Hanson $EH_2$ metric [21]
\[
ds^2 = g_{ij} dz^i d\bar{z}^j
\] (344)
where $i, j = 1, 3$
\[
g_{ij} = A(u) \delta_{ij} + B(u) (z_i - Z_i)(\bar{z}_j - \bar{Z}_j)
\] (345)
with
\[
A(u) = 1, \quad B(u) = u^2 e^{3u^2/2} - \frac{3}{8}
\]
and the fixed point \((k, \ell)\) is at \((z^1, z^3) = (Z^1, Z^3) \in T_1^2 \times T_3^2\). The functions \(A\) and \(B\) are given by

\[
A(u) \equiv u^{-1}(\lambda^4 + u^2)^{1/2} \quad B(u) = A'(u) \tag{347}
\]

For \(u \gg \lambda^2\), \(A(u) \sim 1\) and the metric is flat. However, for \(u \ll \lambda^2\),

\[
g_{ij} \simeq \frac{\lambda^2}{u} \left( \delta_{ij} - \frac{(z_i - Z_i)(\bar{z}_j - \bar{Z}_j)}{u} \right) \tag{349}
\]

and we see that \(ds^2\) is invariant under \((z^i - Z^i) \to \rho e^{i\chi}(z^i - Z^i)\). Thus \(ds^2\) depends only on two parameters, rather than four. We can make this explicit by changing variables:

\[
z^1 - Z^1 := \sqrt{u} \cos \left( \frac{\theta}{2} \right) e^{i(x+\phi)/2} \tag{350}
\]

\[
z^3 - Z^3 := \sqrt{u} \sin \left( \frac{\theta}{2} \right) e^{i(x-\phi)/2} \tag{351}
\]

Then

\[
ds^2 = \frac{\lambda^2}{4} (d\theta^2 + \sin^2 \theta d\phi^2) \tag{352}
\]

which is the metric on \(S^2\) localised at \((k, \ell)\). We may therefore characterise the collapsed 2-cycle

\[
f_{k,\ell} = \{(z^1, z^3) \mid z^1 = Z^1 + \sqrt{u} \cos \left( \frac{\theta}{2} \right) e^{i(x+\phi)/2}, z^3 = Z^3 + \sqrt{u} \sin \left( \frac{\theta}{2} \right) e^{i(x-\phi)/2}, 0 < \theta < \pi/2, 0 < \phi < 2\pi \} \tag{353}
\]

Note, we only need \(0 < \theta < \pi/2\) for a closed 2-cycle because of the \(\mathbb{Z}_2\) symmetry.

There is also an harmonic \((1, 1)\)-form localised at each fixed point \((k, \ell)\):

\[
e_{k,\ell} = \omega_{i,j} dz^i \wedge d\bar{z}^j \tag{354}
\]

where

\[
\omega_{i,j} = \alpha(u)\delta_{i,j} + \beta(u)(z_i - Z_i)(\bar{z}_j - \bar{Z}_j) \tag{355}
\]

with

\[
\alpha(u) \equiv u^{-1}(\lambda^4 + u^2)^{-1/2} \lambda^4 \quad \beta(u) = \alpha'(u) \tag{356}
\]

Note that \(\alpha = O(u^{-2})\) and \(u\beta = O(u^{-2})\) as \(u \to \infty\), so the \((1, 1)\)-form is localised at the fixed point, the origin \(u = 0\). For \(u \ll \lambda^2\),

\[
\omega_{i,j} = \frac{\lambda^2}{u} \left( \delta_{i,j} - \frac{(z_i - Z_i)(\bar{z}_j - \bar{Z}_j)}{u} \right) \tag{358}
\]

Thus on \(f_{k,\ell}\) defined \((353)\) we get

\[
e_{k,\ell} = \frac{\lambda^2}{2} \sin \theta \ d\theta \wedge d\phi \tag{359}
\]

and hence

\[
\int_{f_{k,\ell}} e_{m,n} = \pi i \lambda^2 \delta_{k,m} \delta_{\ell,n} \tag{360}
\]

Similarly, the localisation of the \((1, 1)\)-forms gives

\[
\int_{T^4} e_{k,\ell} \wedge e_{m,n} = \lambda^4 \pi^2 \delta_{k,m} \delta_{\ell,n} \tag{361}
\]
The action of $\theta$ on the collapsed 2-cycles $f_{i,j}$ is
\[ f_{1,j} \rightarrow f_{1,j} \]
\[ f_{4,j} \rightarrow f_{5,j} \rightarrow f_{6,j} \rightarrow f_{4,j} \]
for each $j = 1, 4, 5, 6$. It follows that
\[ \epsilon_j \equiv (f_{6,j} - f_{4,j}) \otimes \pi_3 + (f_{4,j} - f_{5,j}) \otimes \pi_4 \]
\[ \bar{\epsilon}_j \equiv (f_{4,j} - f_{5,j}) \otimes \pi_3 + (f_{5,j} - f_{6,j}) \otimes \pi_4 \]
are invariant (exceptional) 3-cycles. In contrast, the localised harmonic $(1,1)$-forms $\epsilon_{i,j}$ transform as
\[ \epsilon_{1,j} \rightarrow \epsilon_{1,j} \]
\[ \epsilon_{4,j} \rightarrow \epsilon_{6,j} \rightarrow \epsilon_{5,j} \rightarrow \epsilon_{4,j} \]
and
\[ \omega_j \equiv [\alpha(e_{4,j} - e_{5,j}) + (e_{5,j} - e_{6,j})]dz_2 \]
\[ \bar{\omega}_j \equiv [(e_{4,j} - e_{5,j}) + \alpha(e_{5,j} - e_{6,j})]d\bar{z}_2 \]
may be taken respectively as the basis invariant twisted $(2,1)$- and $(1,2)$-forms in this sector. It follows that the Poincaré duals of $\epsilon_j$ and $\bar{\epsilon}_j$ are respectively
\[ \chi_j = \frac{1}{2\pi \lambda' \alpha \text{Vol}(T^2_\mathbb{R})}(-\alpha \bar{e}_3 \omega_j + e_3 \bar{\omega}_j) \]
\[ \bar{\chi}_j = \frac{1}{2\pi \lambda' \alpha \text{Vol}(T^2_\mathbb{R})}(e_3 \omega_j - \alpha e_3 \bar{\omega}_j) \]
where
\[ \text{Vol}(T^2_\mathbb{R}) = \frac{\sqrt{3}}{2} |e_3|^2 \]
On the AAA lattice, the combinations $2\epsilon_j + \bar{\epsilon}_j$ of exceptional 3-cycles are $\mathcal{R}$-invariant and their Poincaré duals $2\chi_j + \bar{\chi}_j$ are odd, like $\tilde{H}_3$. The flux quantisation conditions then give
\[ \tilde{H}_3 = -\frac{4\pi^2 \alpha'}{3} \sum_j \bar{n}_j (2\chi_j + \bar{\chi}_j) \]
with $\bar{n}_j \in \mathbb{Z}$ the integer associated with the flux through $\epsilon_j$. On the BAA lattice the corresponding result is
\[ \tilde{H}_3 = \frac{4\pi^2 \alpha'}{3} \sum_j \bar{n}_j (\chi_j + 2\bar{\chi}_j) \]
where $\bar{n}_j \in \mathbb{Z}$ is the integer associated with the flux through $\bar{\epsilon}_j$.

We may now use the foregoing results to rewrite the generalised tadpole cancellation condition (322). For the AAA and BAA lattices respectively, we get
\[ -\frac{n_0}{36} [n_3(\rho_1 + 2\rho_6) + 3n_6 \rho_1] + \frac{n_0}{3} \sum_j \bar{n}_j (2\epsilon_j + \bar{\epsilon}_j) + \sum_{\kappa} N_{\kappa}(\kappa + \kappa') - 4\pi O_6 = 0 \]
(375)
\[ -\frac{n_0}{36} [n_4(\rho_3 + 2\rho_1) + 3n_1 \rho_3] - \frac{n_0}{3} \sum_j \bar{n}_j (\epsilon_j + 2\bar{\epsilon}_j) + \sum_{\kappa} N_{\kappa}(\kappa + \kappa') - 4\pi O_6 = 0 \]
(376)
For the solution on the AAA lattice given in (35.1) we find
\[ \sum_{\kappa} N_{\kappa}(\kappa + \kappa') - 4\pi O_6 = [1 + (-1)^{\tilde{e}^2}]\rho_1 + 3(\rho_4 + 2\rho_6) + (-1)^{\tilde{\rho}^2} (2[\epsilon_1 + (-1)^{\tilde{\epsilon}^2}]\rho_6 + [\tilde{\epsilon}_1 + (-1)^{\tilde{\rho}^2}]\rho_6) \]
Cancellation of the exceptional parts requires that $|n_0 \hat{n}_4| = 3$. Thus $|n_0| = 1$ or 3. In the first case, if $n_0 = 1$, then

$$
\hat{n}_1 = -3[(-1)^{\hat{e}_0} + (-1)^{\hat{e}_6}] \\
\hat{n}_4 = -3(-1)^{\hat{e}_6 + \hat{e}_2} \\
\hat{n}_6 = -3(-1)^{\hat{e}_0 + \hat{e}_2} \\
n_3 = 108 \\
n_6 = 12[1 + (-1)^{\hat{e}_1}]
$$

whereas, in the second case, if $n_0 = 3$, then

$$
\hat{n}_1 = [(1)^{\hat{e}_0} + (1)^{\hat{e}_6}] \\
\hat{n}_4 = (1)^{\hat{e}_6 + \hat{e}_2} \\
\hat{n}_6 = (1)^{\hat{e}_0 + \hat{e}_2} \\
n_3 = 36 \\
n_6 = 4[1 + (-1)^{\hat{e}_1}]
$$

Similarly, for the solution on the BAA lattice given in §6.2 we find

$$
\sum_{\kappa} N_{\kappa} (\kappa + \kappa') - 4\pi O_6 = 3\rho_6 + (-1)^{\hat{e}_0} \left( [\hat{e}_1 + (-1)^{\hat{e}_2} \hat{e}_6] + 2[\hat{e}_1 + (-1)^{\hat{e}_2} \hat{e}_6] \right) \\
+ (-1)^{\hat{e}_6} \left( [\hat{e}_1 + (-1)^{\hat{e}_2} \hat{e}_4] + 2[\hat{e}_1 + (-1)^{\hat{e}_2} \hat{e}_4] \right)
$$

Thus to achieve the required cancellation of the exceptional parts, we must again take $|n_0| = 1$ or 3, and the solutions are, if $n_0 = 1$ then

$$
\hat{n}_1 = 3[(-1)^{\hat{e}_0} + (-1)^{\hat{e}_6}] \\
\hat{n}_4 = 3(-1)^{\hat{e}_6 + \hat{e}_2} \\
\hat{n}_6 = 3(-1)^{\hat{e}_0 + \hat{e}_2} \\
n_1 = 36 \\
n_4 = 0
$$

whereas if $n_0 = 3$, then

$$
\hat{n}_1 = (-1)^{\hat{e}_0} + (-1)^{\hat{e}_6} \\
\hat{n}_4 = (-1)^{\hat{e}_6 + \hat{e}_2} \\
\hat{n}_6 = (-1)^{\hat{e}_0 + \hat{e}_2} \\
n_1 = -12 \\
n_4 = 0
$$

Thus, in both cases we can choose the background NSNS 3-form fieldstrength $\bar{H}_3$ so as to satisfy the tadpole cancellation conditions. There remains the possibilities that this could also be done using the background $d\bar{F}_2$ that derives from “metric fluxes”, or by a combination of both. We have not explored this further.

## 8 Non-anomalous $U(1)$ groups

Tadpole cancellation generally ensures that any anomalous $U(1)$ gauge symmetries are removed; the associated gauge boson acquires a string-scale mass via the generalised Green-Schwarz mechanism and the $U(1)$ survives only as a global symmetry of the theory. In any case, there remains the possibility that non-anomalous $U(1)$ may survive at low energy, in which case we require it to
the case for the $U(1)_Y$ associated with the weak hypercharge $Y$. The $U(1)$ gauge boson associated with a general linear combination of the $U(1)$ charges $Q_\kappa$ 

$$X = \sum_{\kappa} x_\kappa Q_\kappa$$  \hspace{1cm} (399)

whether anomalous or non-anomalous, does not acquire a mass via the Green-Schwarz mechanism provided that [22, 23, 24]

$$\sum_{\kappa} x_\kappa N_\kappa (\kappa - \kappa') = 0$$  \hspace{1cm} (400)

Consider again the model derived in [5,1] deriving from the fourth entry in Table [1] Using (63) we find that

$$2(a - a') = \rho_1 + 2\rho_3 + 3\rho_4 - 3(-1)^{\tau_0} \tilde{\epsilon}_1 + (-1)^{\tau_2} \tilde{\epsilon}_6$$  \hspace{1cm} (401)

Thus $a \neq a'$, which shows that the gauge boson of $U(1)_a$, associated with $Q_a$, does not remain massless, and $U(1)_a$ survives only as a global symmetry. Since none of the quark-singlet states arise as antisymmetric representations on the stack $a$, baryon number $B = \frac{1}{3} Q_a$. It follows that the global $U(1)_a$ symmetry is just baryon-number conservation. Similarly, from (66) and (67) we find

$$2(b - b') = (-1)^{\tau_0} \tilde{\epsilon}_1 + (-1)^{\tau_2} \tilde{\epsilon}_5$$  \hspace{1cm} (402)

so that $b - b' \neq 0$, the gauge boson of $U(1)_b$ acquires a string-scale mass, and $U(1)_b$ also survives only as a global symmetry. From (202) and (249) we find

$$2(d - d') = -\rho_1 - 2\rho_3 + 3\rho_4 + 3(-1)^{\tau_0} \tilde{\epsilon}_1 + (-1)^{\tau_2} \tilde{\epsilon}_6$$  \hspace{1cm} (403)

Finally, from (214) and (259) we find

$$c - c' = 0$$  \hspace{1cm} (404)

For this solution, using (16), (17) and (211)

$$Y = \frac{1}{6} Q_a + \frac{1}{2} Q_d + \frac{1}{2} Q_c$$  \hspace{1cm} (405)

It follows from these that $U(1)_Y$ does remain massless, as required. However, since $c = c'$, so too does $U(1)_c$. Thus, we have an unwanted $U(1)$ factor in the surviving gauge group, besides the required $SU(3)_{\text{colour}} \times SU(2)_L \times U(1)_Y$ of the standard model. It is easy to verify that $B - L$, where $B$ is baryon number and $L$ is lepton number, is given by

$$B - L = \frac{1}{3} Q_a + Q_d$$  \hspace{1cm} (406)

This gives the correct values for the quark and lepton states, and also ensures that the doublets that arise at the intersections of $b$ with $c$ and $c'$ have $B - L = 0$. Thus these states are the (so far unobserved) Higgs doublets, and the unwanted $U(1)_c$ is just a linear combination of the massless $U(1)_Y$ with $U(1)_{B-L}$. The same defect is present in the other standard-model solutions, on both the AAA and BAA lattices.

9 Conclusions

The $Z'_6$ orientifold is so far the only known compactification of Type IIA string theory that can accommodate intersecting supersymmetric stacks $a$ and $b$ (with $N_a = 3$ and $N_b = 2$) of (fractional) D6-branes satisfying [14], having no matter in symmetric representations, and not too much in antisymmetric representations, on either stack. Stacks having these properties are a useful starting point if we are eventually to obtain just the spectrum of the supersymmetric Standard Model, although in principle $(a \cap b, a \cap b') = (0, 3) \text{ or } (3, 0)$ are also allowed. In a previous publication [8] we presented a number of examples possessing the former properties in cases in which $T'_{3}$ is of B-type, and in this paper we...
extended to give just the (supersymmetric) standard-model spectrum by the addition of extra $U(1)$ stacks $c, d, \ldots$ with $N_{c,d,\ldots} = 1$. In all of the former cases, as detailed in \cite{4}, the answer is negative. The same is immediately true for the solutions found on the ABA and BBA lattices, since they have $d_7^i$ quark-singlet states arising as antisymmetric matter on the $SU(3)_{\text{colour}}$ stack $a$. However, we have found models that give the standard-model spectrum, always accompanied by three neutrino-singlet states $3\nu^c_f$, on the AAA and BAA lattices. In all cases we require two $U(1)$ stacks to get the correct lepton/Higgs doublet content. Baryon number conservation survives as a global symmetry in all of our solutions. Also in all cases, though, besides the standard-model $SU(3)_{\text{colour}} \times SU(2)_L \times U(1)_Y$ gauge group, there is unavoidably an additional (non-anomalous) $U(1)$ factor surviving as a local, rather than a global, symmetry. This is effectively $U(1)_{B-L}$, the $U(1)$ associated with baryon number $B$ minus lepton number $L$. Assuming that it can be broken, such a $U(1)$ is in principle useful \cite{25,26,27,28,29,30} in linking the neutrino masses and non-baryonic dark matter.

In the first instance, the solutions obtained when $T_2^2$ is of A-type do not satisfy the tadpole-cancellation conditions \cite{311}, so that they are not consistent configurations of D6-branes. However, the 7-form gauge potential $C_7$ associated with D6-branes also couples to the background NSNS 3-form field strength $H_3$, and this leads to modification of the tadpole cancellation conditions in the presence of such flux. We have shown that it is possible to choose the background so that the modified tadpole cancellation conditions are satisfied. Nevertheless, there remain unstabilised Kähler and dilaton moduli. It is known that in principle these may be stabilised using RR, NSNS and metric fluxes \cite{14,15,16,17,18}. Models similar to the ones we have been discussing can be uplifted into ones with stabilised Kähler moduli using a "rigid corset" \cite{19,31}, which can be added to any RR tadpole-free assembly of D6-branes in order to stabilise all moduli. Fluxes may also be necessary to break supersymmetry. So far, we have only explored models in which both $T_2^2$ and $T_2^3$ are $SU(3)$ root lattices. Either or both could be $G_2$ root lattices, and the results presented here illustrate amply how different lattices give different physics. We shall explore all of these possibilities in future work.

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A Calculations of $(i^a_1, i^b_2)(j^a_1, j^b_2) \cap (i^b_1, i^a_2)(j^b_1, j^a_2)'$ on the AAA lattice

A.1 $(n^a_{1,2}, m^a_{1,2}) = (n^a_{3,4}, m^a_{3,4}) = (1, 0) \mod 2$

$(56)(14) \cap (56)(14)' = (56)(56) \cap (56)(56)' =
= (-1)^{\tau^a_0 + \tau^b_0}2[1 + (-1)^{\tau^2_0 + \tau^2_2}] \left[ (n^a_2 n^b_2 - m^a_2 m^b_2) - (-1)^{\tau^1_0 + \tau^1_2} (n^a_2 n^b_2 + n^a_2 m^b_2 + m^a_2 n^b_2) + \right] + \left[ (-1)^{\tau^1_0} + (-1)^{\tau^1_2} \right] (m^a_2 m^b_2 + n^a_2 m^b_2 + m^a_2 n^b_2)
\tag{407}

(56)(14) \cap (56)(56)' = (56)(56) \cap (56)(14)' = 0
\tag{408}

A.2 $(n^a_{1,2}, m^a_{1,2}) = (n^a_{3,4}, m^a_{3,4}) = (1, 1) \mod 2$

$(45)(16) \cap (45)(16)' = (45)(45) \cap (45)(45)' =
= (-1)^{\tau^a_0 + \tau^b_0}2[1 + (-1)^{\tau^2_0 + \tau^2_2}] \left[ m^a_2 m^b_2 + n^a_2 m^b_2 + m^a_2 n^b_2 + \right] + \left[ (-1)^{\tau^2_0 + 1} + (-1)^{\tau^2_0 + 1} \right] (n^a_2 n^b_2 + n^a_2 m^b_2 + m^a_2 n^b_2) + (-1)^{\tau^1_0 + \tau^1_2} (n^a_2 n^b_2 - m^a_2 m^b_2)
\tag{409}

(45)(16) \cap (45)(45)' = 0 = (45)(45) \cap (45)(16)'
\tag{410}
A.3 \((n_1^{a,b}, m_1^{a,b}) = (n_3^{a,b}, m_3^{a,b}) = (0, 1) \text{ mod } 2\)
\begin{align*}
(46)(15) & \cap (46)(15)' = (46)(46) \cap (46)(46)' = \\
& = (-1)^{r_6^{a}+r_5^{a}}2[1 + (-1)^{r_5^{a}+r_4^{a}}] \left[ m_2^{a} m_2^{b} + n_2^{a} m_2^{b} + m_2^{a} n_2^{b} - \\
& - (-1)^{r_4^{a}+r_3^{a}}(n_2^{a} n_2^{b} + n_2^{b} m_2^{b} + m_2^{b} n_2^{b}) + [( - 1)^{r_3^{a} + (-1)^{r_2^{a}}}](n_2^{a} n_2^{b} - m_2^{b} m_2^{b}) \right] \quad (411)
\end{align*}

A.4 \((n_1^{a}, m_1^{a}) = (n_3^{a}, m_3^{a}) = (1, 0) \text{ mod } 2, \ (n_1^{b}, m_1^{b}) = (n_3^{b}, m_3^{b}) = (1, 1) \text{ mod } 2\)
\begin{align*}
(56)(14) & \cap (45)(16)' = (-1)^{r_6^{a}+r_5^{a}}(56)(14) \cap (45)(45)' = \\
& = (-1)^{r_6^{a}+r_5^{a}}(56)(56) \cap (45)(45)' = (-1)^{r_5^{a}+r_4^{a}}(56)(56) \cap (45)(16)' = \\
& = (-1)^{r_5^{a}+r_4^{a}}2 \left[ -(n_2^{a} n_2^{b} + n_2^{b} m_2^{b} + m_2^{b} n_2^{b}) + ( - 1)^{r_4^{a}+r_3^{a}}(m_2^{a} m_2^{b} + n_2^{b} m_2^{b} + m_2^{b} n_2^{b}) + \\
& + [(-1)^{r_3^{a} + (-1)^{r_2^{a}}}](n_2^{a} n_2^{b} - m_2^{b} m_2^{b}) \right] \quad (413)
\end{align*}

A.5 \((n_1^{a}, m_1^{a}) = (n_3^{a}, m_3^{a}) = (1, 1) \text{ mod } 2, \ (n_1^{b}, m_1^{b}) = (n_3^{b}, m_3^{b}) = (0, 1) \text{ mod } 2\)

In the 2 cases in which \((n_2^{a}, m_2^{a}) = (n_2^{b}, m_2^{b}) \text{ mod } 2\) we find that \(f_{AB'} = 2 \text{ mod } 4\). In the other 2 cases it is 0 \text{ mod } 4.
\begin{align*}
(45)(16) & \cap (46)(15)' = (-1)^{r_6^{a}+r_5^{a}}(45)(16) \cap (46)(46)' = \\
& = (45)(45) \cap (46)(46)' = (-1)^{r_5^{a}+r_4^{a}}(45)(45) \cap (46)(15)' = \\
& = (-1)^{r_5^{a}+r_4^{a}}2 \left[ (m_2^{a} m_2^{b} + n_2^{a} m_2^{b} + m_2^{a} n_2^{b})[1 + (-1)^{r_4^{a}+r_3^{a}}] + \\
& - ( - 1)^{r_4^{a}+r_3^{a}}(n_2^{a} n_2^{b} + n_2^{b} m_2^{b} + m_2^{b} n_2^{b}) + ( - 1)^{r_3^{a}+r_2^{a}}(m_2^{a} m_2^{b} - n_2^{b} m_2^{b}) \right] \quad (414)
\end{align*}

A.6 \((n_1^{a}, m_1^{a}) = (n_3^{a}, m_3^{a}) = (0, 1) \text{ mod } 2, \ (n_1^{b}, m_1^{b}) = (n_3^{b}, m_3^{b}) = (1, 0) \text{ mod } 2\)

In this case \(f_{AB'} = 0 \text{ mod } 4\) in the case that \((n_2^{a}, m_2^{a}) = (1, 1)(0, 1) \text{ mod } 2,\) and 2 \text{ mod } 4 in the 3 other cases.
\begin{align*}
(46)(15) & \cap (56)(14)' = (-1)^{r_6^{a}+r_5^{a}}(46)(15) \cap (56)(56)' = \\
& = (-1)^{r_6^{a}+r_5^{a}}(46)(46) \cap (56)(56)' = (-1)^{r_5^{a}+r_4^{a}}(46)(46) \cap (56)(14)' = \\
& = (-1)^{r_5^{a}+r_4^{a}}2 \left[ -(n_2^{a} n_2^{b} + n_2^{b} m_2^{b} + m_2^{b} n_2^{b})[1 + (-1)^{r_4^{a}+r_3^{a}}] + \\
& + ( - 1)^{r_4^{a}+r_3^{a}}(n_2^{a} n_2^{b} + n_2^{b} m_2^{b} + m_2^{b} n_2^{b}) + ( - 1)^{r_3^{a}+r_2^{a}}(m_2^{a} m_2^{b} - n_2^{b} m_2^{b}) \right] \quad (415)
\end{align*}

B \textbf{ Calculations of } \((i_1^{a}, j_1^{a})(j_1^{b}, j_2^{b}) \cap (i_1^{b}, i_2^{a})(j_1^{b}, j_2^{b})' \text{ on the } \text{ABA lattice}\)

On the ABA lattice the action of \(\mathcal{R}\) on the bulk 3-cycles \(p_p \ (p = 1, 3, 4, 6)\) is the same as on the BAA lattice. Consequently, the function \(f_{AB'} = \Pi_{bulk}^{u} \cap \Pi_{bulk}'^{u}\) that determines the bulk contribution to \(a \cap b\) is the same on the two lattices. In contrast, the action of \(\mathcal{R}\) on the exceptional 3-cycles \(e_j, e_j' \ (j = 1, 4, 5, 6)\) differs by an overall sign on the two lattices. The relative sign of the bulk and exceptional contributions to \(a \cap b\) and \(a \cap b'\) is controlled by the overall phase \((-1)^{r_5^{a}+r_6^{a}}\). Thus the calculations of these quantities on the ABA lattice may be obtained from those on the BAA lattice by the replacement \(r_6^{a} \rightarrow r_6^{b} + 1\), but only in the expressions for \(a \cap b'\). The results of calculations for the ABA lattice may therefore be obtained immediately from those presented below. This does not mean that we may obtain solutions for the fractional branes \(a\) and \(b\) having the required properties on the ABA lattice trivially from solutions on the BAA lattice. The orientifold planes and the total homology class \(\Pi_{O6}\) are different in the two cases,
\(B.1\) \((n_{1b}^a, m_{1a}^b) = (n_{3b}^a, m_{3a}^b) = (1, 0) \mod 2\)

\[
(56)(14) \cap (56)(14)' = (56)(56) \cap (56)(56)' = \\
= (-1)^{\tau_2^a + \tau_0^b + 12}[1 + (-1)^{\tau_2^a + \tau_2^b}]
[m_{2b}^0 b + n_{2b}^0 b + m_{2b}^0 n_{2b} + (-1)^{\tau_2^a + \tau_2^b}(m_{2b}^0 m_{2b} + n_{2b}^0 m_{2b} + m_{2b}^0 n_{2b}) + \\
+ [(-1)^{\tau_1^a} + (-1)^{\tau_1^b}](m_{2b}^0 m_{2b} - n_{2b}^0 n_{2b})]
\]

\(B.2\) \((n_{1b}^a, m_{1a}^b) = (n_{3b}^a, m_{3a}^b) = (1, 1) \mod 2\)

\[
(45)(16) \cap (45)(16)' = (45)(45) \cap (45)(45)' = \\
= (-1)^{\tau_2^a + \tau_0^b + 12}[1 + (-1)^{\tau_2^a + \tau_2^b}]
[m_{2b}^0 b - n_{2b}^0 b + (-1)^{\tau_2^a + \tau_2^b}(m_{2b}^0 m_{2b} + n_{2b}^0 m_{2b} + m_{2b}^0 n_{2b}) + \\
+ [(-1)^{\tau_1^a + 1} + (-1)^{\tau_1^b + 1}](m_{2b}^0 b + n_{2b}^0 b + m_{2b}^0 b) + ]
\]

\(B.3\) \((n_{1b}^a, m_{1a}^b) = (n_{3b}^a, m_{3a}^b) = (0, 1) \mod 2\)

\[
(46)(15) \cap (46)(15)' = (46)(46) \cap (46)(46)' = \\
= (-1)^{\tau_2^a + \tau_0^b + 12}[1 + (-1)^{\tau_2^a + \tau_2^b}]
[m_{2b}^0 b - n_{2b}^0 b + (-1)^{\tau_2^a + \tau_2^b}(m_{2b}^0 m_{2b} + n_{2b}^0 m_{2b} + m_{2b}^0 n_{2b}) + \\
+ [(-1)^{\tau_1^a} + (-1)^{\tau_1^b}](m_{2b}^0 b + n_{2b}^0 b + m_{2b}^0 b) + ]
\]

\(B.4\) \((n_{1b}^a, m_{1a}^b) = (n_{3b}^a, m_{3a}^b) = (1, 0) \mod 2, (n_{1b}^b, m_{1b}) = (n_{3b}^b, m_{3b}) = (1, 1) \mod 2\)

\[
(56)(14) \cap (45)(16)' = (-1)^{\tau_2^a + \tau_0^b + 12}(56)(14) \cap (45)(45)' = \\
= (-1)^{\tau_2^a + \tau_0^b + 12}[(m_{2b}^0 m_{2b} + n_{2b}^0 m_{2b} + m_{2b}^0 n_{2b})[1 + (-1)^{\tau_2^a + \tau_2^b}]
+ \\
- (-1)^{\tau_1^a}(m_{2b}^0 m_{2b} + n_{2b}^0 m_{2b} + m_{2b}^0 m_{2b}) + (-1)^{\tau_1^b}(n_{2b}^0 m_{2b} + n_{2b}^0 m_{2b} + m_{2b}^0 m_{2b})]
\]

\(B.5\) \((n_{1b}^a, m_{1a}^b) = (n_{3b}^a, m_{3a}^b) = (1, 1) \mod 2, (n_{1b}^b, m_{1b}) = (n_{3b}^b, m_{3b}) = (0, 1) \mod 2\)

\[
(46)(16) \cap (46)(16)' = (-1)^{\tau_2^a + \tau_2^b}(46)(16) \cap (46)(46)' = \\
= (45)(45) \cap (46)(46)' = (-1)^{\tau_2^a + \tau_2^b}(45)(45) \cap (46)(15)' = \\
= (-1)^{\tau_2^a + \tau_2^b + 12}[(m_{2b}^0 m_{2b} + n_{2b}^0 m_{2b} + m_{2b}^0 n_{2b})[1 + (-1)^{\tau_2^a + \tau_2^b}]
+ \\
- (-1)^{\tau_1^a}(m_{2b}^0 m_{2b} + n_{2b}^0 m_{2b} + m_{2b}^0 m_{2b}) + (-1)^{\tau_1^b}(n_{2b}^0 m_{2b} + n_{2b}^0 m_{2b} + m_{2b}^0 m_{2b})]
\]

\(B.6\) \((n_{1b}^a, m_{1a}^b) = (n_{3b}^a, m_{3a}^b) = (0, 1) \mod 2, (n_{1b}^b, m_{1b}) = (n_{3b}^b, m_{3b}) = (1, 0) \mod 2\)

\[
(46)(15) \cap (56)(14)' = (-1)^{\tau_2^a + \tau_2^b}(46)(15) \cap (56)(56)' = \\
= (-1)^{\tau_2^a + \tau_2^b}(46)(46) \cap (56)(56)' = (-1)^{\tau_2^a + \tau_2^b}(46)(46) \cap (56)(14)' = \\
= (-1)^{\tau_2^a + \tau_2^b + 12}[(m_{2b}^0 m_{2b} + n_{2b}^0 m_{2b} + m_{2b}^0 n_{2b})[1 + (-1)^{\tau_2^a + \tau_2^b}]
+ \\
- (-1)^{\tau_1^a}(m_{2b}^0 m_{2b} + n_{2b}^0 m_{2b} + m_{2b}^0 m_{2b}) + (-1)^{\tau_1^b}(n_{2b}^0 m_{2b} + n_{2b}^0 m_{2b} + m_{2b}^0 m_{2b})]
\]
C Calculations of \((i_1^a, i_2^a) (j_1^b, j_2^b) \cap (i_1^b, i_2^b) (j_1^b, j_2^b)'\) on the BBA lattice

C.1 \((n_{1a}^{ab}, m_{1a}^{ab}) = (n_{3a}^{ab}, m_{3a}^{ab}) = (1, 0) \mod 2\)

\[(56)(14) \cap (56)(14)' = (56)(56) \cap (56)(56)' =
\]
\[= (-1)^{\tau_1^a + \tau_2^b} 2[1 + (-1)^{\tau_1^b + \tau_2^b}] \left[ -m_2^a m_2^b + n_2^a m_2^b + m_2^a n_2^b \right] + \]
\[+ [(-1)^{\tau_1^a} + (-1)^{\tau_1^b}] (n_2^a n_2^b + n_2^a m_2^b + m_2^a n_2^b) \quad (425)\]

\[(56)(14) \cap (56)(56)' = (56)(56) \cap (56)(14)' = 0 \quad (426)\]

C.2 \((n_{1a}^{ab}, m_{1a}^{ab}) = (n_{3a}^{ab}, m_{3a}^{ab}) = (1, 1) \mod 2\)

\[(45)(16) \cap (45)(16)' = (45)(45) \cap (45)(45)' =
\]
\[= (-1)^{\tau_1^a + \tau_2^b} 2[1 + (-1)^{\tau_1^b + \tau_2^b}] \left[ n_2^a n_2^b + m_2^a n_2^b - \right]
\[+ (-1)^{\tau_1^a + \tau_1^b} (m_2^a m_2^b + n_2^a m_2^b + m_2^a n_2^b) + [(-1)^{\tau_1^a} + (-1)^{\tau_1^b}] (m_2^a m_2^b - n_2^a n_2^b) \quad (427)\]

\[(45)(16) \cap (45)(45)' = 0 = (45)(45) \cap (45)(16)' \quad (428)\]

C.3 \((n_{1a}^{ab}, m_{1a}^{ab}) = (n_{3a}^{ab}, m_{3a}^{ab}) = (0, 1) \mod 2\)

\[(46)(15) \cap (46)(15)' = (46)(46) \cap (46)(46)' =
\]
\[= (-1)^{\tau_1^a + \tau_2^b} 2[1 + (-1)^{\tau_1^b + \tau_2^b}] \left[ n_2^a n_2^b + m_2^a n_2^b + m_2^a n_2^b \right] +
\[+ (n_2^a m_2^b - n_2^a n_2^b) - [(-1)^{\tau_1^a} + (-1)^{\tau_1^b}] (m_2^a m_2^b + n_2^a m_2^b + m_2^a n_2^b) \quad (429)\]

\[(46)(15) \cap (46)(46)' = 0 = (46)(46) \cap (46)(15)' \quad (430)\]

C.4 \((n_{1a}^a, m_{1a}^a) = (n_{3a}^a, m_{3a}^a) = (1, 0) \mod 2, \ (n_{1b}^b, m_{1b}^b) = (n_{3b}^b, m_{3b}^b) = (1, 1) \mod 2\)

\[(56)(14) \cap (45)(16)' = (-1)^{\tau_1^a} (56)(14) \cap (45)(45)' =
\]
\[= (-1)^{\tau_1^b} (56)(56) \cap (45)(45)' = (-1)^{\tau_1^a + \tau_2^b} (56)(56) \cap (45)(16)' =
\]
\[= (-1)^{\tau_1^a + \tau_2^b} 2 \left[ m_2^a m_2^b - n_2^a n_2^b + (-1)^{\tau_1^a + \tau_1^b} (n_2^a m_2^b + m_2^a n_2^b) \right] +
\[+ [(-1)^{\tau_1^a} + (-1)^{\tau_1^a + \tau_1^b}] (m_2^a m_2^b + n_2^a m_2^b + m_2^a n_2^b) \quad (431)\]

C.5 \((n_{1a}^a, m_{1a}^a) = (n_{3a}^a, m_{3a}^a) = (1, 1) \mod 2, \ (n_{1b}^b, m_{1b}^b) = (n_{3b}^b, m_{3b}^b) = (0, 1) \mod 2\)

\[(45)(16) \cap (46)(15)' = (-1)^{\tau_1^a + \tau_2^b} (45)(16) \cap (46)(46)' =
\]
\[= (45)(45) \cap (46)(46)' = (-1)^{\tau_1^a + \tau_1^b} (45)(45) \cap (46)(15)' =
\]
\[= (-1)^{\tau_1^a + \tau_2^b} 2 \left[ (n_2^a m_2^b + n_2^a m_2^b)[1 + (-1)^{\tau_1^a + \tau_1^b}] +
\[+ (-1)^{\tau_1^a} (m_2^a m_2^b - n_2^a n_2^b) - (m_2^a m_2^b + n_2^a m_2^b + m_2^a n_2^b) \right] \quad (432)\]

C.6 \((n_{1a}^a, m_{1a}^a) = (n_{3a}^a, m_{3a}^a) = (0, 1) \mod 2, \ (n_{1b}^b, m_{1b}^b) = (n_{3b}^b, m_{3b}^b) = (1, 0) \mod 2\)

\[(46)(15) \cap (56)(14)' = (-1)^{\tau_1^a} (46)(15) \cap (56)(56)' =
\]
\[= (-1)^{\tau_1^a + \tau_2^b} (46)(46) \cap (56)(56)' = (-1)^{\tau_1^a} (46)(46) \cap (56)(14)' =
\]
\[= (-1)^{\tau_1^a + \tau_2^b} 2 \left[ (m_2^a m_2^b - n_2^a m_2^b)[1 + (-1)^{\tau_1^a + \tau_1^b}] +
\[+ (-1)^{\tau_1^a} (m_2^a m_2^b + n_2^a m_2^b + m_2^a n_2^b) \right] \quad (433)\]
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