Neutrino oscillation studies and the neutrino cross section

Paolo Lipari
Dipartimento di Fisica, Università di Roma “la Sapienza”,
and I.N.F.N., Sezione di Roma 1, P. A. Moro 2, I-00185 Roma, Italy

The present uncertainties in the knowledge of the neutrino cross sections for \(E_\nu \sim 1\) GeV, that is in the energy range most important for atmospheric and long baseline accelerator neutrinos, are large. These uncertainties do not play a significant role in the interpretation of existing data, however they could become a limiting factor in future studies that aim at a complete and accurate determination of the neutrino oscillation parameters. New data and theoretical understanding on nuclear effects and on the electromagnetic structure functions at low \(Q^2\) and in the resonance production region are available, and can be valuable in reducing the present systematic uncertainties. The collaboration of physicists working in different subfields will be important to obtain the most from this available information. It is now also possible, with the facilities developed for long baseline beams, to produce high intensity and well controlled \(\nu\)–beams to measure the neutrino interaction properties with much better precision that what was done in the past. Several projects and ideas to fully exploit these possibilities are under active investigation. These topics have been the object of the first \(\nu\)–interaction (NUINT01) workshop.

1. Introduction

Neutrino physics is living in a “golden era”. Experiments with atmospheric and solar neutrinos have given strong evidence for the existence of flavor oscillations, or (less likely) some other form of ‘New Physics’ beyond the Standard Model \([1]\). The study of the \(\nu\) flavor transitions offer the possibility to obtain information about the neutrino masses and mixing, and the knowledge about these quantities hopefully represents a fundamental window on the physics of the unification. Theorists are busy trying to make sense of the unexpected results that have been obtained so far, on the other hand new experimental studies are planned to measure with greater precision the properties of the \(\nu\) flavor transitions. These future studies, are designed to determine if the \(\nu\) flavor evolution is completely described by the 3-flavor mixing model \([1]\). The study of the \(\nu\) flavor transitions offer the possibility to obtain information about the neutrino masses and mixing, and the knowledge about these quantities hopefully represents a fundamental window on the physics of the unification. Theorists are busy trying to make sense of the unexpected results that have been obtained so far, on the other hand new experimental studies are planned to measure with greater precision the properties of the \(\nu\) flavor transitions. These future studies, are designed to determine if the \(\nu\) flavor evolution is completely described by the 3-flavor mixing model, or if a more complex dynamics (transitions to additional sterile states, neutrino decay, flavor changing neutral currents) is necessary, and should measure the entire set of parameters (2 squared mass differences, 3 mixing angles and one CP–violating phase) that determine the \(\nu\) flavor evolution in space–time in the 3–flavor scenario.

The discovery of \(\nu\) oscillations has been performed with natural (solar and atmospheric) neutrino sources, however most of the future studies will be performed with artificial neutrinos. In particular, long baseline (LBL) accelerator neutrino beams will play a fundamental role. The first of these projects, the KEK to Kamioka (K2K) \(\nu\)–beam, with \(\langle E_\nu \rangle \simeq 1.8\) GeV and \(L \simeq 250\) Km, has already collected data, that support the existence of the \(\nu_\mu \to \nu_\tau\) transitions as indicated by the atmospheric neutrino data \([1,2]\). Other projects are already funded and in construction (Fermilab to Minos \([3,4]\), and CERN to Gran Sasso \([5,6]\), both with \(L \simeq 730\) Km), or in the design phase (the JHF project \([7,8]\)). Several ‘next generation’ projects are also in discussion, involving “super-beams”, (very intense \(\nu\)–beams produced with the traditional technique: \(p +\) Target \(\to \pi^\pm, K^\pm, \ldots \to \nu_\mu (7_\mu)\)), or neutrino factories (muon storage rings that produce extraordinarily well controlled neutrino beams from muon decay: \(\mu^+ \to 7_\mu + \nu_\tau + e^+\)).

The \(\nu\) energy (and pathlength) in the LBL projects is determined by their scientific goals and the existing technical and experimental constraints. The range of energy \(E_\nu \sim (0.2–10)\) GeV will have crucial importance. In fact the difference in quantum mechanical phase developed by two mass eigenstates after propagation for a dis-
The flavor composition, and energy spectrum of a neutrino beam in a detector (located at a distance \(L\) from the source) is determined from the observation of neutrino interactions, and clearly uncertainties in the knowledge of the neutrino cross sections are a limiting factors in the sensitivity. There are uncertainties about the absolute value of the cross sections, the ratio \(\sigma_{nc}/\sigma_{cc}\) between the neutral and charged current interaction rates, the average multiplicity in the final state, the energy and angular distribution of final state lepton (and hadronic particles). Depending of the detector type and the scientific question asked, these uncertainties can have a different importance.

The NUINT01 workshop\(^1\) has been dedicated to a discussion of the problem of the neutrino interaction properties, attempting to estimate: (i) the size of the existing uncertainties, (ii) their probable impact on studies of the fundamental \(\nu\) properties, and (iii) possible methods to reduce these uncertainties. A list of relevant questions is:

(A) How well have we measured the \(\nu\) cross sections and interaction properties?

(B) How good is our theoretical understanding (and computational capabilities) of the \(\nu\) cross sections and interaction properties?

(C) How well do we need to know \(\sigma_{\nu}\) to determine with the desired accuracy the \(\nu\) flavor evolution and mass matrix?

\(^1\)The CERN Gran Sasso project is designed to study the appearance of \(\tau\) neutrinos, with the process \(\nu_\mu \rightarrow \nu_\tau \rightarrow \tau^+\). Since the threshold for the \(\tau\) production is \(\sim 3.5\) GeV, the beam is designed to have a larger average \(\nu\) energy.

\(^2\)See the web page neutrino.kek.jp/nuint01/
2. Measurements of $\sigma_\nu$

The neutrino cross section can be obtained from data, from the formula:

$$\sigma_\nu(E_\nu) = \frac{N_{\text{int}}(E_\nu)}{\varepsilon_{\text{det}} \Phi_\nu(E_\nu)}$$  \hspace{1cm} (3)

where $N_{\text{int}}$ is the number of detected interactions, $\Phi_\nu$ is the neutrino flux, and $\varepsilon_{\text{det}}$ is detector efficiency. This measurement has been performed in the energy region of interest ($E_\nu \sim 1$ GeV) by several experiments\textsuperscript{[10]}-\textsuperscript{[16]}. The most accurate one were performed with bubble chambers filled with hydrogen or deuterium. A review of this data is given in \textsuperscript{[10]}. The uncertainty of $\sigma_\nu$ is dominated by systematic errors in the determination of the $\nu$ flux. For example for the data of \textsuperscript{[11]} (the 12–foot hydrogen/deuterium bubble chamber at the Argonne National Laboratory), the authors estimated an overall 15% uncertainty for the neutrino beam that was produced by a 12.4 GeV proton beam interacting on a Beryllium target. The systematic uncertainty was estimated as a convolution of different factors: uncertainties on the fundamental $\pi$ production cross section (5%), uncertainties on pion reabsorption in the target (5%), proton beam intensity (2%), and smaller contributions. Recent critical analysis\textsuperscript{[17]} of the uncertainties in the $p$–nucleus cross sections, related to the calculations of the atmospheric neutrino fluxes have concluded that the uncertainties on the hadronic cross sections are larger (of order 15–20%). To be conservative, it could be safer to consider a systematic larger that what was quoted in the original experiments. The facilities developed for the long–baseline experiments can provide $\nu$ beams significantly more intense that the beams used in the 70’s. In order to use these beams to obtain absolute values for the cross sections it is essential to have good control of the normalization and energy shape of the neutrino beam.

(D) What experimental programs can/should be planned in order to improve our knowledge and understanding of the $\nu$ interaction properties?

The K2K experiment has already collected in the near detector a very important sample of events (see section 4), and a problem of considerable interest is to determine how well the $\nu$ beam is known. The same question clearly applies for the near detector in the Fermilab to MINOS project.

In principle it is possible to determine the quasi–elastic cross section on (quasi)–free nucleons without a precise knowledge of the $\nu$ flux, making use of some theoretical input. If the incident $\nu$ energy is known, from a measurement of the energy and direction of both final state particles in the scattering $\nu_p n \rightarrow \mu^- p$ one can determine both the neutrino energy $E_\nu$ and the $Q^2$ of the reaction. It is therefore possible to determine the shape of the differential cross section $d\sigma/dQ^2(Q^2)$, that to a good approximation can be considered as a function of a single parameter the axial mass $M_A$ (see next section), while the absolute normalization, that is the value of the cross section at $Q^2 \rightarrow 0$ can be inferred from $\beta$ decay measurements. This program has been planned in \textsuperscript{[12]} (data from the 7–foot hydrogen/deuterium bubble chamber at the Brookhaven National Laboratory) with the result $M_A = 1.07 \pm 0.06$ GeV, and in \textsuperscript{[11]} (ANL 12–ft bubble chamber) with the result $M_A = 1.01 \pm 0.09$ GeV. The total cross section (and therefore the event rate) is to a good approximation proportional to $M_A^3$. In \textsuperscript{[14]} the analysis of the event rate results in a lower estimate $M_A = 0.75 \pm 0.12$. The poor agreement between the two methods suggests the existence of systematic effects.

Several other experiments have measured $\nu$ interactions in the energy range of interest using nuclear targets. In the published analysis of these works very often the results are corrected for nuclear effects, and the data are expressed as cross sections on free nucleons. Since the nuclear effects themselves are difficult to compute (as discussed in the following) and the subject of some controversy, some careful considerations are needed to use these results. For practical pur-
poses it clearly would be very valuable to have available also the results for interactions on nuclear targets, before the nuclear corrections.

An example of this problem can be seen analysing the results of the two $\nu$ experiments [13] operating with the $\nu$–beam at the Institute for High Energy Physics (IHEP) at Serpukhov (Russia), that were presented at this workshop. The SKAT bubble chamber [13] has collected data on a set of low multiplicity reactions, namely the final states ($\mu^- p$), ($\mu^- p\pi^+$), ($\mu^- p\pi^0$), ($\mu^- n\pi^+$) with an additional non–observed ‘nuclear spectators’ state. Each one of the considered final states can be obtained as the result of nuclear cascading processes, such as $\pi$ absorption ($\pi + A \to A'$), $\pi$ production ($pp \to p\pi^+$, $pp \to p\pi^0$, ...) and $\pi$ charge exchange (for example $\pi^+ n \leftrightarrow \pi^0 n$). The experimentalists have calculated the probabilities for all these processes with montecarlo methods to estimate the rate for exclusive reactions on free nucleons, however these corrections are probably the largest uncertainty in the problem.

3. Cross section Formulae

Only the interactions of neutrinos with nucleons and nuclei are relevant for oscillation studies, since the ratio $\sigma_{\nu_e}/\sigma_{\nu_p(n)}$ is of order $m_e/m_p$; the cross section can naturally be separated into a charged current (CC) and a neutral current (NC) component: $\sigma_{\nu}\ = \sigma_{\nu_{\text{cc}}} + \sigma_{\nu_{\text{nc}}}$. For low energy neutrinos it is also natural to decompose the cross section according to the multiplicity:

$$\sigma_{\nu_{\text{cc}}} = \sigma_{\nu_{\text{cc}}} (\text{qel}) + \sigma_{\nu_{\text{cc}}} (1\pi) + \sigma(2\pi) + \ldots$$

The correct calculation of the quasi–elastic and one pion components has great importance in the analysis of existing data since most of the data on atmospheric neutrinos has been obtained with water Čerenkov detectors (Super–Kamiokande, and before IMB and Kamiokande) that, because of relatively poor pattern recognition capabilities, have limited most of their analysis to the so called single–ring events, where only one particle is visible. This selects quasi–elastic scattering [4] with in addition events with one (or more) charged pion(s) below the Čerenkov threshold. The bottom panel of fig. 1 shows a plot of $\sigma_{\nu_{\text{cc}}}$ separated according to the pion multiplicity and compared with some existing data. The importance of the low multiplicity channels is clearly displayed

3.1. Quasi–elastic scattering

The quasi–elastic cross section can be written in terms of nucleon form factors. The most general matrix element for a nucleon transition can be written [20] in terms of six form factors:

$$\langle \nu(p) | J^\alpha(0) | n(p) \rangle = \bar{u}_n(p) \gamma^\alpha \nu(p)$$

$$\left[ F_V(q^2)\gamma^\alpha + iF_M(q^2)\sigma^{\alpha\beta} q_\beta + F_A(q^2)\gamma^\alpha \gamma_5 + F_P(q^2) \frac{2M}{m} \gamma^\alpha \gamma_5 + iF_A^{\text{sc}}(q^2)\sigma^{\alpha\beta} q_\beta + F_V^{\text{sc}}(q^2) \frac{q^4}{2M^2} \right] \bar{u}_n(p)$$

The hypothesis of CVC allows to relate the vector form factors $F_V$ and $F_A$ to the electromagnetic form factors of the proton and neutron; the contribution of second class currents (that violate $G$–parity) can be assumed to vanish, and one remains with two unknown functions the axial form

4 Most final state protons are below the Čerenkov threshold.
factor $F_A$ and the pseudoscalar form factor $F_P$. The contribution of the pseudoscalar form factor to the cross section is proportional to $m_\nu^2/M^2$, therefore this term is important only for $\nu$, while the axial form factor $F_A$ remains undetermined. The value of $F_A$ at $Q^2 = 0$ can be related to the axial coupling measured in $\beta$-decay experiments ($F_A(0) = g_A = 1.2670 \pm 0.0035$), but the $Q^2$ dependence of $F_A$ has to be determined experimentally. An often used parametrization is a dipolar form:

$$F_A(Q^2) = g_A \left[ 1 + \frac{Q^2}{M_A^2} \right]^{-2} \quad (4)$$

that depends on the single parameter $M_A$. Therefore in first approximation the problem of the determination of the quasi–elastic cross section can be identified with the measurement for $MA$. A review of the status of our knowledge of the nucleon form factors was given in [35] and [36].

3.2. Inelastic channels

The kinematics of a final state lepton is described by two variables, that correspond to its energy $E_\ell$ and angle with respect to the initial neutrino $\theta_{\nu\ell}$. The kinematical variables can be chosen as relativistic invariants such as the adimensional scaling variables $x$ and $y$; of particular dynamical importance are the transfer momentum in the interaction: quantities:

$$Q^2 = -q^2 = -(p_\nu - p_\ell)^2, \quad (5)$$

and the square of the hadronic mass in the final state:

$$W^2 = p_{\text{had}}^2 = (p_\ell + q)^2. \quad (6)$$

The kinematically allowed range of $Q^2$ is the interval between $Q_{\text{min}}^2 \simeq 0$ and $Q_{\text{max}}^2 \simeq 2m_\nu E_\nu$ that grows linearly with energy. For $Q_{\text{max}}^2 \ll M_W^2$ (that is for $E_\nu \ll 3$ TeV) the effects of the $W$ propagator are small and the cross section $\sigma_\nu$ grows approximately linearly with energy. Similarly the allowed range of the squared hadronic mass $W^2$ is between the values $m_\nu^2$ and $W_{\text{max}}^2 - m_\nu^2 \simeq 2m_\nu E_\nu$.

In general [7, 8], the inclusive differential neutrino cross section can be written in terms of 5 structure functions $F_j(x, Q^2)$, according to the general formula:

$$\frac{d\sigma}{dx dy} = G_1^2 M_N E_\nu \left( \frac{M_N^2}{M_N^2 + Q^2} \right)^2 \times \left\{ \begin{array}{l} y (xy + \frac{m_\nu^2}{2E_\nu M_N}) F_1 \\ (1 - y - \frac{M_N^2 xy}{2E_\nu} - \frac{m_\nu^2}{2E_\nu}) F_2 \\ (xy(1 - \frac{y}{2}) - \frac{m_\nu^2}{4M_N E_\nu}) F_3 \\ \frac{m_\nu^2}{2M_N E_\nu} \left[ (xy + \frac{m_\nu^2}{2M_N E_\nu}) F_4 - F_5 \right] \end{array} \right\},$$

where the two signs for the $F_3$ component cor-

Figure 3. Calculated distributions of the differential cc event rates ($dN_{cc}/dW$) in the far detector as a function of $W$ the hadronic mass in the final state. The top panel is for the K2K experiment, the bottom panel for the Fermilab to Miniboone project. The vertical line at $W = m_N$ represents quasi elastic scattering, the dashed line is calculated according to the DIS formula, the solid line is the contribution of the resonances according to the Rein and Sehgal model [3].
respond to $\nu$ and $\overline{\nu}$. The scaling variables $x$ and $y$ are defined as $y = (q \cdot p_f)/(p_i \cdot p_i)$ and $x = Q^2/(2M_N E_{\nu} y)$. Note that the structure functions $F_2$ and $F_3$ contribute terms proportional to $m_p^2/(2M_N E_{\nu})$ and are therefore of practical interest only in the case of $\nu_{\tau} cc$ interactions (where they constitute a source of uncertainty). It is well known that in the Deep Inelastic Scattering (DIS) limit, that corresponds to $Q^2$ and $W^2$ large, the structure functions can be related to the Parton Distribution Functions that can be well measured in different experiments, and have a calculable $Q^2$ evolution. For example in leading order of an $\alpha_s$ expansion:

$$F_{2,3}^{\nu,cc} = \frac{2x F_1^{\nu,cc}}{F_3^{\nu,cc}} = \frac{2x (d + s + \overline{\tau} + \tau)}{2(d + s - \overline{\tau} - \tau)}$$

(7)

Therefore in the DIS region the neutrino cross section is well understood and under good theoretical control. However, essentially the entire phase space relevant for atmospheric neutrino contained events, is well outside of the DIS region, and this remains true for the K2K, Minos to Fermilab and JHF projects. An illustration of this problem is shown in fig. 2, where we plot the cross section for the scattering $\sigma_{\nu_{\tau}} + n \rightarrow \mu^- + X$ at an energy of 3 GeV. At this energy (that is actually rather high for atmospheric $\nu$ and most LBL experiments), the range of possible $W$ is small and extends in the region where hadronic resonances are present. The naive use of the DIS formula with the PDF’s of Gluck Reya and Vogt [21] results in a smooth curve, that predicts a non vanishing cross section also in the interval $m_p \leq W \leq m_p + m_\tau$. Obviously a realistic calculation must predict a more complex structure, as it is the case for the model of Rein and Sehgal [22] whose results are also shown in the figure. Figure 3 is another illustration of the importance of the resonance region in the K2K and Fermilab to MINOS projects. The delta function for $W = m_p$ has an area of ~42% for K2K and of order 11% for MINOS (LE beam).

A central problem for the calculation of the $\nu$ cross section is the correct description of the low $Q^2$, low $W^2$ kinematical range, where perturbative QCD is not valid. Luckily, in recent times, a large body of highly accurate data on electron–proton scattering in this kinematical region has become available, mostly from the Jefferson laboratory (see [23]–[27]). An example of this data is shown in fig. 4, where is plotted the electromagnetic structure function $F_{2}^{\text{e.m.}}$ that in the DIS limit in leading order can be expressed in terms of PDF’s as:

$$F_{2}^{\text{e.m.}} = \sum_j c_j^2 x [q_j(x) + \overline{q}_j]$$

(8)

One can see that the production of resonances plays a very important role, and that three mass peaks are clearly visible at low $Q^2$. Fig. 5 also illustrates the importance of a proper treatment of the relevant effects in the low $Q^2$, region. The physical insight that can be obtained from the
electron scattering data can be applied also to neutrino interactions, even if significant theoretical uncertainties remain. A concept of great importance in the resonance region is the notion of “parton–hadron” duality. Already in 1971, Bloom and Gilman [27] observed empirically, that the scaling structure function \( F_{em}^{e^\pm}(x) \) that characterizes electromagnetic scattering in the DIS (high \( Q^2 \) region), represents a good average over the resonance bumps of the inclusive data observed at much lower \( Q^2 \). The agreement can be significantly improved rescaling the \( x \) variable for target mass correction and using for example the Nachtmann variable \( \xi = 2x/(1 + \sqrt{1 + 4M^2x^2/Q^2}) \).

3.3. Nuclear effects

Neutrino detectors are usually composed of heavy nuclei (the most important nuclear species are oxygen (SK), argon (Icarus), iron (Minos) and lead (Opera)), and in the energy range we are discussing nuclear effects are important and have to be taken properly into account. Schematically one can separate the nuclear effects into two parts: (i) the \( \nu \) interaction with a bound nucleon, and (ii) the propagation of the final state hadrons in the nuclear matter, even if a clear distinction of these two effects is not rigorously valid.

The simplest and most commonly used approximation to describe the interaction with a bound nucleon is the Fermi gas model. In this model the nucleus is described as an ensemble of nucleons with a momentum distribution \( f_{p,n}(\vec{p}_i) \). The energy that corresponds to the 3–momentum \( \vec{p}_i \) is \( E_i = \sqrt{\vec{p}_i^2 + M^2} - \varepsilon_B \) where the quantity \( \varepsilon_B \) is the binding energy that can be different for protons and neutrons\(^5\). In the simplest version of the model [26], the initial nucleon momentum distribution has a “zero temperature” shape:

\[
 f(\vec{p}_i) = \Theta(p_F - |\vec{p}_i|) \tag{9}
\]

that vanishes when \( |\vec{p}_i| \) is larger than the Fermi momentum \( p_F \). In other models it has a more complex shape that includes higher momentum tails. Quasi–elastic scattering can be simply described as an elementary (2 → 2) process such as \( \nu_\ell + n^* \rightarrow \ell^- + p^* \) where \( n^* \) and \( p^* \) are off–shell initial final states nucleons with 3–momenta \( \vec{p}_i \) and \( \vec{p}_i + \vec{q} \) (\( \vec{q} = p_\nu - p_\ell \) is the transfer 4–momentum) and binding energies \( \varepsilon_i \) and \( \varepsilon_f \). This procedure allows naturally to implement Pauli–blocking, forbidding the scattering into occupied nucleon states. This can be done including a factor \( [1 - f_p(\vec{p}_i + \vec{q})] \) in the calculation of the matrix element for the scattering. Some natural questions, that have been discussed at the workshop are: (i) how good is the Fermi gas model as a description of the nuclear effects; (ii) how large are the expected corrections to the Fermi gas model; (iii) what is the best (or most convenient) theoretical framework to describe nuclear effects beyond the Fermi gas model. It is clear that conceptually the Fermi–gas model is a very simple and non–realistic description of the scattering [24,25].

\(^5\)In some implementation of the model \( \varepsilon_B \) is also a function of \( \vec{p}_i \).
Figure 6. Cross section integrated on the muon emission angle as a function of the muon energy. The full lines have been calculated with the Fermi gas model (FG label) and the dashed lines with the model described in [43] that includes final state interactions (FSI label). (from [43]).

However not entirely clear how large is the error introduced by this simplified treatment. Before, during and after the workshop, it was attempted to produce and compare different “beyond the Fermi–gas model” calculations, (see the critical discussion by R. Seki [42] in these Proceedings). Hopefully a generally accepted model will soon emerge from this work. An example of the results for one of these more sophisticated models (by G. Co and collaborators [43]) is shown in fig. 3. This model predicts a suppression of the cross section by approximately 15% with respect to the Fermi gas model, and a softer spectrum for the final state lepton.

The uncertainties due to nuclear effects become larger when discussing the resonance production processes and the propagation of hadronic particles in the nuclear matter. It is important to notice that because of pion reabsorption processes the calculation of pion-less final states must also include contributions from resonance production.

4. The K2K near detector data

The K2K experiment has collected data with the neutrino beam obtained from a 12 GeV proton beam at the KEK accelerator. The goal of the experiment is to study the existence of neutrino flavor transitions comparing the event rates at a near detector, located approximately 300 m from the pion production target, with event rate at a far detector (Super–Kamiokande) located at a distance of 250 km. The event rate recorded at the far detector has been measured to be smaller that the rate extrapolated from the near detector results, with a suppression that is consistent with the $\nu_\mu \leftrightarrow \nu_\tau$ oscillation interpretation of the atmospheric neutrino data [13].

The neutrino interactions collected at the near detector represent a very large sample with potentially extremely useful information. The discussion of this data has been perhaps the highlight of the workshop.

The near detector for the K2K experiment is really a combination of three detectors. The main flux measurement is based on a one-kiloton water Čerenkov detector. This detector uses the same photo-multipliers as Super–Kamiokande, and has essentially the same design (except for scale) and the analysis for the two detectors can use the same reconstruction algorithms. A fine grained detector sits downstream of the water detector. It is made as a scintillating fiber tracker with water targets enclosed between layers of tracking material. The next component of the near detector is a stack of iron plates interleaved with drift tubes to serve as a muon range detector. Both the fine–grained and muon range detectors have transverse pairs of tracking planes so tracks can be reconstructed in three dimensions.

The data of the three near detectors has been discussed in a set of 4 contributions by Ishida, Walter, Mauger and Mine [45–48]. The event sample collected up to now in the near detectors amount to a very significant statistics: 27,000 events in the water detector, 8,300 in the fine
grained detector and 125,000 in the iron detector. A comparison of the data with a Montecarlo simulation shows in general good agreement, demonstrating that there are no large systematic errors in the simulation chain, however small but significant effects are visible.

The fine grained detector as a whole is excellent for event classification studies since it can distinguish quasi-elastic and inelastic events. This is performed measuring the momenta and directions of the muon and the proton in events of type: \( \nu_\mu + A \rightarrow \mu^- + p(\ldots) \). In case of quasi-elastic scattering on a free nucleon, because of 4-momentum conservation, the information obtained from the measurements is redundant, and the measurements of \( E_\mu, \theta_{\mu\nu} \) and \( E_p \) allow to determine the initial neutrino energy and the final state proton direction. For scattering on a bound nucleus, since the spectator nucleons absorb some 4-momentum, the predicted values absorb some recoil, the central values of narrow distributions. The contribution of the inelastic channels can be separated from the quasi-elastic one because it is much broader as illustrated in fig. [7].

A method to discriminate between the hypothesis of \( \nu_\mu \rightarrow \nu_\tau \) and \( \nu_\mu \rightarrow \nu_{\text{sterile}} \) transitions is the study of the neutral current event rate. This event rate is suppressed in the case of transitions of \( \nu_\mu \) into sterile neutrinos, while it remains constant for standard \( \nu_\mu \rightarrow \nu_\tau \) transitions. The most important channel for the study of the neutral current event rate is the reaction: \( \nu_x + A \rightarrow \nu_x + A + \pi^0 \) that has the lowest multiplicity and a clear experimental signature. The problem is that in order to predict the event rate for this process one needs to know with sufficient precision the absolute value of the cross section, or at least the ratio with respect to a measurable charged current process. The K2K near detector is the ideal instrument to measure this cross section (or a cross section ratio such as \( \sigma_{\text{nc}}(1\pi^0)/\sigma_{\text{qel}} \)). The status of these measurements is described in [17]. Approximately 5000 \( \pi^0 \) were reconstructed in a 50 ton fiducial volume in the water Čerenkov near detector, determining the ratio \( \pi^0/\text{FC}_\mu \) with small statistical error and an overall systematic error of order \( \sim 9\% \). This can be applied to the 414 reconstructed \( \pi^0 \) in the atmospheric neutrino sample (1289 days) of SK to obtain a result that disfavors the oscillation into sterile neutrinos.

It is well known that atmospheric neutrinos were initially considered mostly as an unpleasant background for proton decay studies, because for any detector resolution, there is a finite probability that a \( \nu \) interaction will mimic a proton decay event. It is clearly very important to estimate correctly this probability for the interpretation of the existing data, and more important for the planning of future experiments. This probability is clearly strongly dependent on the \( \nu \) interaction properties. The exposition of the 1 kton (50 tons fiducial mass) water Čerenkov detector to the \( \nu \) fluence obtained from \( 3 \times 10^{19} \) protons is roughly equivalent to a \( \sim 1 \) Mton yr exposition to the atmospheric \( \nu \)-flux and therefore allows to
study experimentally the importance of the atmospheric background for $p$ decay studies (see [48] for more discussion).

5. How well do we need to know the $\nu$ cross sections?

It is necessary to estimate carefully the possible impact of systematic uncertainties in our knowledge of the neutrino cross sections for the interpretation of present and future neutrino data.

The effect of uncertainties on $\sigma_\nu$, for the interpretation of the SK atmospheric neutrino data has been addressed at this workshop in the contribution of K. Kaneyuki [24]. This work analysed the effects of three modifications of the neutrino cross section: (a) a change in $\sigma_{NC}$ of $\pm 30\%$, (b) the setting to zero of the cross section for $\pi^0$ coherent production on a nucleus (the process $\nu + A \rightarrow \nu + A + \pi^0$ [38,59]), and (c) a modification of $\pm 30\%$ for the ratio $\sigma_{NQE}/\sigma_{qel}$. The SK simulation was repeated including these modifications, and the allowed region in the plane ($\Delta m^2, \sin^2 2\theta$) was recalculated following the same standard procedure. The allowed intervals calculated with the modified cross section are remarkably similar to the normal ones. The effect of the $\pm 30\%$ increase of the neutral current cross section corresponds to a small ($\sim \pm 4\%$) increase in the allowed range for $|\Delta m^2|$. This can be understood noting that the neutral current events contribution to the single–ring events is larger for $\mu$–like events, therefore a larger neutral current cross section corresponds to a larger ($\mu/e)_{MC}$, to a smaller double ratio ($\mu/e)_{data}/(\mu/e)_{MC}$ and therefore to a larger suppression of the $\mu$–rate, and correspondingly to a larger $|\Delta m^2|$.

A $\pm 30\%$ modification of the multi–pion cross section correspond to a shift of order $\pm 0.02$ in the minimum allowed value of the mixing parameter $\sin^2 2\theta$. This can be qualitatively understood observing that in the presence of a larger contribution of the non–quasi elastic component, the correlation between the muon and the neutrino becomes weaker. The Up/Down asymmetry observed at the muon level is in general smaller than the true asymmetry at the neutrino level, because of finite angular resolution effects, with the dominant effect being the angle $\theta_{\mu\nu}$ between the observed muon and the parent neutrino that is determined by the interaction cross section. Assuming a weaker correlation, the inferred true asymmetry becomes stronger, and the mixing parameter larger.

The effect of neglecting the coherent $\pi^0$ cross section is found to be negligible. These results are very encouraging, in the sense that they demonstrate that the atmospheric neutrino result is very robust, and that the uncertainties in our knowledge of the neutrino cross section is actually very small. This is after all not so surprising and is a consequence of the fact that essentially all effects in atmospheric neutrinos are visible in ratios such as $e/\mu$ or up/down, and the theoretical uncertainties tend to cancel.

A problem that has attracted a fair amount of attention in the past is the question of the absolute value of the atmospheric neutrino fluxes, and the precise shape of their energy dependence.

Future experiments with accelerator beams aiming at more precise determination of the neutrino oscillation parameters will very likely have more stringent requirements on the control of the neutrino cross section. A quantitative discussion of this question is still not complete. For example, possible effects for the Minos experiment have been discussed by Adam Para [4]. In experiments that measure the shape of the survival probability $P_{\nu_{\mu} \rightarrow \nu_{\mu}}(E_{\nu})$ as a function of the neutrino energy $E_{\nu}$ (for a fixed pathlength $L$) it is essential to have a good reconstruction of the neutrino energy. This also depend to a good understanding of the neutrino cross section, to correct the visible energy for the energy absorbed in the excitation of the target nucleus, and the average composition and multiplicity of the final state.
An extremely good knowledge of the neutrino cross section will be required in case the planned neutrino factories will be constructed and used for the determination of CP violating effects in neutrino flavor transitions. In this case one wants to compare the rates of processes such as: $\nu_e \to \nu_\mu \to \mu^-$ to and $\bar{\nu}_e \to \bar{\nu}_\mu \to \mu^+$ and one will need a control of the ratio $\sigma_\nu/\sigma_{\bar{\nu}}$ to a better precision than the size of the expected asymmetries. An interesting contribution was given by Yoshihia Obayashi [8], for the JHF collaboration. The JHF project plans to use the 50 GeV proton beam of the JAE accelerator to produce an intense neutrino beam directed at the Super–Kamiokande detector (with a 295 km baseline). The first phase is directed at the determination of $\theta_{13}$ studying transitions $\nu_\mu \to \nu_e$, while a second phase (with a more massive 10 Mt Hyper–Kamiokande detector) is aimed at a study of CP violating effects. A CP violation asymmetry can be defined as:

$$A_{\text{CP}} = \frac{P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e)}{P(\nu_\mu \to \nu_e) + P(\bar{\nu}_\mu \to \bar{\nu}_e)}$$

(10)

The oscillation probability can be schematically determined as:

$$P(\nu_\mu \to \nu_e) = \frac{N_e}{\sigma_{\nu_e} \varepsilon_e \Phi_{\text{expected}}}$$

(11)

where $N_e$ is the observed rate of $e$–like events, $\varepsilon_e$ is the detection efficiency and $\Phi_{\text{expected}}$ is the expected $\nu_\mu$ fluence at the detector in the absence of oscillations. The probability $P(\bar{\nu}_\mu \to \bar{\nu}_e)$ can be obtained with a neutrino beam of opposite sign. It is clear that for the determination of the asymmetry parameter only the ratio $\sigma_\nu/\sigma_{\bar{\nu}}$ is relevant. In fact, considering that also the determination of the expected flux depends on knowledge of the cross section, the asymmetry can be rewritten as a function of the quantity $r_\sigma = (\sigma_\nu/\sigma_{\bar{\nu}})/(\sigma_{\bar{\nu}}/\sigma_{\nu})$. A part of the systematic uncertainties cancel in this ratio, however it is not clear if, even considering this cancellation, the present uncertainties are sufficiently small to allow the measurement without a special program of cross section measurements.\footnote{In this qualitative discussion we will neglect matter effects.}

6. Montecarlo Codes

In all present and future neutrino experiments the analysis of the data will require detailed montecarlo calculations, therefore the problem of describing the neutrino interactions can be divided into two parts of similar importance:

- The development of a theoretical framework to describe the interactions.
- The development of montecarlo algorithms to implement this understanding.

Of course, since a calculation of the neutrino cross sections with hadrons is not possible from first principles, the first and most important task is to collect experimentally the necessary information. The development of montecarlo algorithms in a non trivial problem, that has probably been given too little attention until this moment. All the neutrino experiments that have recently collected data have used different montecarlo codes, written by members of the collaboration, and the results of the different codes have not been object of a sufficiently careful and detailed comparison. One of the successes of the NUIN01 workshop, has been in fact the discussion between different groups and the comparison between different codes.

The Super–Kamiokande collaboration has used two different montecarlo codes, the NEUT code, originally developed for Kamiokande, and the NUANCE code (with a progenitor developed for IMB). The Soudan–2 detector has used the code NEUGEN that is now developed to be used by MINOS. The Icarus collaboration has performed simulations using a code developed for (and tuned with the data of) the Nomad detector at CERN, and a second one discussed in [53]. The OPERA detector has performed its simulations with the code JETTA developed for CHORUS, that gives special attention to the $\nu_e$ cross section, an additional MC for OPERA was presented at the workshop [54]. The montecarlo code of Miniboone was discussed in [52]. This is not the place for a critical discussion of the results of this comparison, and only few comments are possible. No major discrepancy between the
different codes has been found, however several non–trivial effects have been found, and in fact even non negligible bugs have already been found and corrected for. Most codes tend to use the same theoretical input namely:

(i) Llewellyn Smith \cite{20} for the expression of the quasi–elastic cross section on free nucleons.
(ii) Smith and Moniz for the implementation of the Fermi gas model for quasi–elastic scattering.
(iii) The resonance cross section of Rein and Sehgal.
(iv) The “standard” DIS formula for high \( W \) and \( Q^2 \), with one of the publically available sets of PDF’s.

Even if the theoretical input is the same, there are non–trivial differences that are present, in particular about the implementation of the Fermi–gas model, the “joining” of the resonance production and DIS scattering regimes, and the hadronization of the final hadronic state. The nuclear rescattering effects are treated (when they are) in significantly different ways, so effects such as pion reabsorption, pion elastic and charge–exchange scattering are very different. The field of neutrino physics would certainly benefit significantly if some “standard” codes implementing correctly algorithms recognized as a possible good approximations for the problem, could become publically available, as it is the case for well established event generators in other sub–fields of particle physics. The systematic uncertainties in the neutrino interaction properties are still sufficiently large that it is probably neither possible nor really desirable to have a unique “universal code”, but in the era of precision studies in neutrino physics, when the requirements on the event generators will be more stringent, the development of more sophisticated and better tested instruments is clearly required.

7. Outlook

The study of atmospheric neutrinos with energies around 1 GeV has yielded remarkable results on the neutrino mass matrix. Uncertainties on the the neutrino–nucleus cross section in this energy range are of order \( \sim 20\% \). These uncertainties very likely have a small impact on the interpretation of the data, however for future studies using accelerator neutrinos in approximately the same range of energy, a more precise knowledge of the cross section would be valuable, and in fact it will be required for studies that attempt to detect and measure \( CP \) violation effects in neutrino oscillations. The size of the uncertainties on the \( \nu \) interaction properties can be very likely reduced incorporating in our description of the neutrino cross sections the understanding of nuclear effects, hadronic resonance production, and the behaviour of the nucleon structure functions at low \( Q^2 \) that has been obtained in more recent studies in electron scattering.

The near detector of the K2K experiment has collected the largest existing sample of \( \nu \) interactions in the \( E_\nu \sim 1 \) GeV region. These data have a great potential to clarify several important questions on the neutrino interaction properties. Hopefully a sufficient precise determination of the normalization and energy spectrum of the \( \nu \)–beam will allow the measurement of absolute values for the cross sections; in any case important information about the ratios of cross sections can be obtained. The measurement of the rate for the neutral process \( \nu + A \rightarrow \nu + A + \pi^0 \) relative (for example) to quasi–elastic scattering, is very important to put limits on oscillations of standard neutrinos into sterile states.

Some natural goals for future studies could be:

- A more quantitative estimate of the importance of uncertainties on \( \sigma_\nu \) as limiting factor for the sensitivity of future experiments on neutrino oscillations.
- A complete analysis of the K2K near detector data to obtain the most of these remarkable results.
- The development of a treatment of nuclear effects that incorporates modern understanding and calculation methods, going beyond the Fermi gas approximation.
- More accurate modeling of pion and nucleon rescattering in nuclear matter.
- An improved treatment of hadronic resonance production, with a smooth joining of
this kinematical region to the deep inelastic region, using the insight allowed by the electron-scattering results.

- Development and testing of the montecarlo algorithms that implement our knowledge of the $\nu$ interaction properties.

- Last but not least: the proposal of new experimental studies aimed at the measurement of the most important/interesting properties of neutrino interactions, using the new intense $\nu$-beams. See for example the contribution of Jorge Morfin in these Proceedings [61].

Neutrinos can be a precious window on the physics of the unification (beyond the standard model), but on the other hand they can also be an extraordinarily valuable probe on the properties of QCD and the structure of the nucleons. The $(V-A)$ structure of the neutrino interactions allows to explore with an axial probe the properties of a bound system of quarks in ways that are very difficult to obtain in $e$-scattering. The calculation of the nucleon structure functions and their evolution in the low $Q^2$ region remains an important challenge for future QCD studies. In summary the measurement of the $\nu$ and $\bar{\nu}$ interaction properties can be seen both as an instrument necessary to determine with high precision the $\nu$ masses and mixing and as a a important scientific goal per se. In the golden era of neutrino physics, the motivations are powerful, the difficulties challenging and the opportunities rich e varied.

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