Focus on topological physics: from condensed matter to cold atoms and optics

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Abstract
The notions of a topological phase and topological order were first introduced in the studies of integer and fractional quantum Hall effects, and further developed in the study of topological insulators and topological superconductors in the past decade. Topological concepts are now widely used in many branches of physics, not only limited to condensed matter systems but also in ultracold atomic systems, photonic materials and trapped ions. Papers published in this focus issue are direct testaments of that, and readers will gain a global view of how topology impacts different branches of contemporary physics. We hope that these pages will inspire new ideas through communication between different fields.

Introduction
These articles highlight the most recent advances in the study of topological matter, which include (i) the interesting aspects of geometry and energetics in the fractional quantum Hall effect; (ii) interaction effects of topological insulators beyond band theory; and (iii) generalization of static topological states to time-periodic Floquet topological states and their realizations in cold atomic and photonic materials. The articles published here can be roughly divided into following four categories:

1. New developments in the fractional quantum Hall effect.
2. Topological insulators with interactions.
3. Topological physics with cold atoms.
4. Topological phases with photonic structures.

New development in the fractional quantum Hall effect
The first category is the new development in the fractional quantum Hall effect. Topological order gives rise to fascinating universal physics at low energy, including fractional charge and statistics of quasiparticles, and of course, quantization of Hall conductance itself. However, one should be cautious not to be misled into the prejudice that all quantum Hall physics is of topological nature and thus ‘universal’, or only topological and universal aspects of the physics are interesting. In fact, articles in this issue about the fractional quantum Hall effect point to the opposite—there is very interesting physics associated with geometry and energetics, and it is not necessarily universal.

The importance of geometry (in addition to topology) in the fractional quantum Hall (FQH) effect was first emphasized by Haldane [1]. One observation that follows is that contrary to common belief, the Laughlin state has a (hidden) geometrical parameter, which needs to be adjusted according to geometry to minimize its energy. This
family of Laughlin states with uniform geometry was explicitly constructed in [2]. In a paper published here [3], Johri et al explored the response of the Laughlin state to (non-uniform) local geometry, and proposed various ways to probe the geometry of FQH liquids. Their proposals are complementary to those made earlier [4].

Another context in which energetics and non-universal physics becomes important is the edge states of FQH liquids. Two articles in this issue touch upon edge physics. In [5], Zucker and Feldman proposed ways to measure velocities of edge modes. Paradoxically, such quantitative (and thus non-universal) information is actually very important for the proper understanding of experiments designed to probe universal properties like fractional statistics of quasiparticles, as demonstrated in [6]. In [7], Li et al studied how the quasihole tunneling matrix elements between two parallel edges scales with the length of the edge using a cylindrical geometry, and obtained better scaling behavior than an earlier study using a disc geometry [8].

A natural generalization of the FQH liquid is fractional Chern insulator (FCI), which can be viewed as FQH liquid stabilized by a crystalline potential that breaks time-reversal symmetry. This is the subject of [9], in which He et al studied how to map the wave functions between an FCI and a FQH liquid. They tested their mapping numerically using the disc geometry, and obtained excellent results.

A potential application of topological phases in general and quantum Hall systems in particular, is using their non-Abelian anyons for topological quantum computation. In [10], the authors discuss improved decoders for such quantum computers.

### Topological insulators with interactions

The second category is topological insulators with interactions. One of the most significant theoretical challenges in the study of topological insulators is going beyond band theory to study the effect of interactions. More precisely, how do strong interactions between electrons modify the characteristic features of TIs, such as their gapless surface states? Can electronic interactions drive a metallic system into a TI? In this focus issue, You et al [11] showed that certain TIs are not stable against interactions and can be continuously connected to a trivial atomic insulator, by establishing a general connection between TIs of non-interacting fermions and Symmetry Protected Topological states (SPTs) of interacting bosons. Meanwhile, Tsai et al [12] demonstrated that in a two-dimensional semimetal with quadratic band crossing points on the Lieb lattice, interactions between electrons can drive the system into various topological phases. Regarding quantum criticality of interacting TIs, Li et al [13] numerically studied quantum phase transitions of spinless Dirac fermions on a honeycomb lattice using sign-problem-free Monte Carlo simulations. Related to realizations and detections of TIs with cold atoms in optical lattices, Mazza et al [14] designed an experimental scheme to engineer TIs in a ladder system and two protocols to measure their topological properties in the presence of interactions.

Global symmetries, such as time reversal, play a crucial role in the topology of band insulators and superconductors. Recently this has been generalized to (i) interacting topological phases that support fractional statistics and (ii) spatial symmetries such as parity (i.e. inversion), leading to many exciting new results. Two highlights along this line appeared in this focus issue. Tarantino et al [15] established a generic framework to classify and construct two-dimensional symmetry-enriched topological states (SETs), i.e. fractionalized topological phases enriched by symmetries. Meanwhile, Chan et al [16] studied exotic signatures of parity-protected topological phases on a non-orientable manifold: the Klein bottle.

In parallel to TIs, topological superconductors (TSCs), which host Majorana fermions, have attracted lots of attention for their potential application in fault-tolerant topological quantum computation. This focus issue also features theoretical efforts along this line. Lobos et al [17] demonstrated how transport properties of a metal-TSC-metal junction can be used to detect topological phase transitions in the TSC region. Das Sarma et al [18] revealed the dependence of Majorana fermion localization length and the transverse size of a TSC nanowire, providing useful guides to future experiments.

In addition to the above theoretical progress, our focus issue also features experimental efforts towards the application of TIs as excellent thermoelectric materials. Guo et al [19] showed that TI thin films of Bi$_2$Se$_3$ exhibit decreasing thermoelectric power factor with the reduction of film thickness, and provided a theoretical explanation. These results will shed light on future improvement of thermoelectric performance in TI materials.

### Topological physics with cold atoms

The third category is topological physics with cold atoms. Realizing topological matter is one of the most actively pursued research topics in contemporary cold atom physics. This can further drive understanding on interactions and non-equilibrium physics in topological matter, by utilizing the controllability of interactions and the tunability of cold atom systems. As of now, Raman coupling and periodic driving are two major schemes implemented in cold atom experiments.
Raman coupling between laser light and atoms drives tunneling or spin flipping processes with a spatial varying phase, which leads to either synthetic magnetic fields or spin–orbit coupling. The NIST group (led by Spielman) was the first to implement this scheme experimentally with bosons in 2010, and later a collaboration between Shan Xi University group (led by Zhang) and Tsinghua University group (led by Zhai) in China, as well as the MIT group (led by Zwerlein), generalized this scheme to fermions in 2012. The paper by LeBlanc et al [20] reports the latest experimental progress from the NIST group. They studied the mechanism of vortex nucleation in a Bose condensation subjected to a synthetic magnetic field. The paper by Zhihao Xu et al [21] points out the possible strongly correlated phase for interacting bosons with a Rashba spin–orbit coupling. The paper by Xiong-Jun Liu et al [22] reports a proposal of generalizing the Raman coupling scheme to optical lattices to realize chiral topological orders. By viewing the internal spin states as another ‘synthetic dimension’, spin–orbit coupling in the real space can be viewed as a fictitious magnetic field in synthetic dimensions. The paper by Boada et al [23] further pursues this idea to simulate non-trivial topology.

Periodic shaking of optical lattices can be used to realize a Floquet topological state, and the ETH group (led by Esslinger) used this scheme to realize a topological Haldane model in the honeycomb lattice in 2014. Similar techniques were also employed by the Munich group (led by Bloch) to achieve topological behavior. The paper by Furukawa and Ueda [24] studied interacting bosons in the Haldane model, and shows how the band topology affects the Bogoliubov quasiparticles in a superfluid. In addition to cold atomic systems, trapped ions represent another platform to simulate topological matter. The paper by Nath et al [25] reports new developments in that domain.

**Topological phases with photonic materials**

The fourth category is topological phases with photonic materials. The field of photonic topological insulators has become a major research effort, with developments emerging rapidly. In this issue, there are a number of important contributions, including theoretical analyses that apply to both condensed matter and photonic systems. For example, there are new proposed designs to achieve optical topological protection: Ma and Shvets [26] discuss an analogue of the valley hall effect in photonics; Jacobs et al [27] propose using tellegen metamaterial systems to break Lorentz reciprocity to realize Chern insulators; and Dubcek et al [28] present a novel design to realize the Harper–Hofstadter (i.e., integer quantum–Hall-like) Hamiltonian in waveguide arrays. In the realm of Floquet topological systems, Nathan and Rudner [29] show new theoretical insights by looking at the spectra of instantaneous Hamiltonians and relating their properties to the overall topology of the stroboscopic Floquet Hamiltonian; Sommer and Simon [30] cast their experimental effort on resonator-based realizations of fractional quantum Hall systems in terms of Floquet theory; Tauber and Delplace [31] use transfer matrices to gain new analytical insight into the Floquet problem. On the topic of photonic artificial gauge fields, Mukherjee et al [32], demonstrate modulation-assisted tunneling in a photonic lattice structure for gauge field engineering; Lin and Fan [33] propose a novel photonic system where an artificial magnetic field can be used to introduce topological behavior. Their system is not based on coupled resonators as in their previous work but on a network of waveguides; they use temporal coupled mode theory to model the artificial field. These contributions mark progress towards the vision by many in the field, namely that ultimately the physics of the integer quantum Hall effect and topological insulators will find more application in photonic devices.

**Concluding remark**

We conclude this editorial by presenting our personal future perspective on topological physics in these directions.

Quantum Hall effect continues to be a frontier in the search for new topological phases of matter. At this point nature of the $5/2$ state, which is our best candidate for non-Abelian state of matter, remains controversial. Experiments complementary to those involving edge tunneling or interferometry, like thermopower and other bulk thermal probes, may be needed to finally settle this issue. More generally, experiments beyond electronic transport may push the field forward.

In spite of rapid theoretical progress on strongly interacting topological insulators in boson systems, both their material realizations and experimental detections remain major challenges. Meanwhile, we know relatively little about quantum criticality beyond Landau–Ginzberg paradigm in these interacting topological phases. Finally, gapless topological phases of strongly interacting bosons/fermions such as various non-Fermi liquids are much less understood compared to gapped topological phases, and remain a major theoretical challenge.

On cold atom physics side, the major challenge nowadays is the heating problem, which plays a major role in current experimental schemes, and which prevents topological systems with cold atoms entering lower temperature regime with profound many-body effects. Several different ideas are now being implemented to
overcome this problem. Once this issue is solved, it will bring this field into a new avenue where the interplay between interactions and topology can be studied experimentally.

In photonics, there are a number of effects that go beyond condensed matter systems and that are just now starting to be explored in relation to topological states. One example is non-Hermiticity: optical systems (like lasers, for example) have gain and loss. So it is natural to ask: what do topological invariants mean when Hermiticity is lost? Exploring the effects of optical nonlinearity (e.g., the Kerr effect, thermal nonlinearities, spectral hole burning, among others) on photonic topological insulators is a natural next step. This is the photonic equivalent of mean-field bosonic interactions in cold atomic systems. On the technological side, perhaps the most compelling question is: can we use topological edge states to realize optical isolators (i.e., diodes for light) that have a smaller on-chip footprint than conventional isolators? These are just several of the many directions that the field of topological photonics is branching into of late, with new ideas consistently emerging from this young field.

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