A number-state filter for pulses of light

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We present a detailed theoretical analysis of a Fock-state filter based on the photon-number dependent group delay in cavity induced transparency proposed in Phys. Rev. Lett. 105, 013601 (2010). We derive a general expression for the propagation velocity of different photon-number components of a light pulse interacting with an optically dense ensemble of three-level atoms coupled to a resonator mode under conditions of cavity induced transparency. These predictions are compared to numerical simulations of the propagation of few photon wave packets, and an estimation for experimental realization is made.

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I. INTRODUCTION

Creation of non classical states of light is one of the central topics in quantum optics. Photon number states, also called Fock states of light are maybe the most prominent representatives of those. They are of particular interest for quantum information processing as they can be used as a carrier of discrete bits of quantum information [1-3]. Despite the fact that by now there are many successful experimental realizations for creation of single-photon states, e.g. in cavity-QED systems [4-10], an ideal single-photon source, a device that efficiently provides propagating single-photons on demand, is still missing. An extremely useful tool would be a filter that allows to extract different photon-number components of a propagating wave packet. Such a system was proposed in [11], where a possibility of spatial separation of different photon-number components of an initially coherent pulse was shown. However, the theoretical analysis performed in [11] was based on nonlinear operator equations. To handle these a couple of simplifying assumptions and approximations on operator level had to be made. These are known to be difficult to justified in general. In the present paper we re-examine this system and provide a rigorous and quantitative analysis of the scheme including an assessment of experimental requirements.

The proposed Fock-state filter is based on a phenomenon called cavity induced transparency (CIT). It occurs in an ensemble of three level atoms with a Λ-type configuration of couplings to two electromagnetic fields and is closely related to the well-known effect of electromagnetically induced transparency (EIT) [12]. The difference between the two systems is the replacement of the coherent control field in EIT by a quantized cavity mode. The coupling of the atomic ensemble to the cavity mode, even if it is in the vacuum state, can lead to transparency for the propagating probe field in otherwise opaque medium. Transparency induced by an empty cavity, called vacuum induced transparency (VIT), was theoretically proposed in [13] and has been demonstrated experimentally in [14]. The interaction of the probe field with the coupled atom-cavity system leads to a temporary transfer of photons from the probe field to the cavity mode. The number of cavity photons is determined by the number of the probe field photons, and therefore is proportional to the probe field intensity. The back-action of the hybrid atom-cavity system onto the probe field, in particular its effect onto the group velocity, depends on the strength of the cavity field, i.e. on the number of cavity photons. As a consequence different photon-number components of the probe field propagate with different velocities causing a photon-number dependent group delay of the probe field. This process is analyzed in detail in this paper.

The paper is organized as follows. In Sect. II we introduce the model, discuss the underlying principle of the system and summarize the expected results based on intuition. Then in Sect. III we derive a general expression for the photon-number dependent group velocity using the concept of the dark states. To confirm these results we have performed numerical wave-function simulations for up to two photons in the initial pulse, which we present in Sec. IV. Sec. V discusses consequences for experimental implementations and Sec. VI gives summary and conclusions.

II. MODEL

We consider a gas consisting of three level atoms in a Λ-type configuration (see Fig. 1b). A cavity mode \( \hat{a}_c \) couples the \( |s\rangle - |e\rangle \) transition of the atoms, the adjacent \( |g\rangle - |e\rangle \) transition is coupled to a propagating quantum field described by the slowly varying operator

\[
\hat{E}(r, t) = \frac{1}{\sqrt{V}} \sum_k \hat{b}_k e^{i(k \cdot r - \omega_k t)} e^{-i(k_p z - \omega_p t)}, \tag{1}
\]

where \( \hat{b}_k \) is annihilation operator of a photon in mode \( k \) with corresponding frequency \( \omega_k \), \( V \) is the quantization volume, and \( \omega_p \) is the carrier frequency of the quantized probe field with corresponding wave number \( k_p = \omega_p / c \).
where we introduced continuous atomic operators
\[ \sigma_{m}(r) = \frac{1}{\Delta N} \sum_{j} |j\rangle\langle m| \]
by averaging over small volume around \( r \) containing \( \Delta N \) particles [14]. \( n(r) \) is the atomic density, and \( \Delta \) and \( \delta \) denote the single photon and two photon detunings respectively. \( g = d_{ge} \sqrt{\frac{2}{\gamma_{\text{cav}}}} \) and
\[ G = d_{se} \sqrt{\frac{e^2}{2\varepsilon_{0}V_{c}}} \]
are the single atom coupling constants for the probe field and cavity field respectively, where \( d_{ge} \) and \( d_{se} \) are dipole matrix elements of the \( |g\rangle - |e\rangle \) and \(|s\rangle - |e\rangle \) transitions, and \( V_{c} \) is the cavity quantization volume.

The first line in (2) describes the free evolution of the atomic system and the propagating probe field and the second line describes the interaction between them. Note that the free Hamiltonian of the cavity mode vanishes in the chosen rotating frame.

\[ \hat{H} = \sum_{k} \hbar(\omega_k - \omega_p)b_{k}^\dagger b_{k} + \hbar \int d^3r \ n(r) \left( \Delta \sigma_{se}(r) + \delta \sigma_{se}(r) \right) - \int d^3r \ n(r) \left( g\sigma_{ge}(r)\hat{\mathcal{E}}(r) + G\sigma_{se}(r)\hat{a}_{c}^\dagger + \text{H.c.} \right), \]

where the sum runs over all interacting atoms \( N_{a} \) and \( G = G_{\text{cav}} \sqrt{N} \) where \( N \) is the number of photons in the cavity. Thus we expect that for strong back-action of the atom-cavity system onto the probe field, which happens in the strong coupling regime, i.e. for a single-atom cooperativity \( C = G^2/\gamma_{\text{cav}} > 1 \), different photon number component will propagate with different group velocities. Here \( \kappa \) and \( \gamma \) are the decay rates of the cavity and the atomic polarization, respectively.

### III. GROUP VELOCITY

To become more acquainted with the system and to introduce the key concept of dark states let us first consider a related toy model, where we consider the probe field as a single mode cavity field \( \hat{\mathcal{E}} = \hat{b}/\sqrt{V} \). For the sake of simplicity we set all detunings to zero. The corresponding Hamiltonian is then given by

\[ \hat{H} = -\hbar \sum_{i=1}^{N_{a}} (g\sigma_{ge}^i \hat{b}_{i}^\dagger + G\sigma_{se}^i \hat{a}_{c}^i + \text{H.c.}) \]

where the sum runs over all interacting atoms \( N_{a} \). It is easy to verify that this Hamiltonian conserves the total number of excitations, i.e. the Hilbert space splits into decoupled manifolds each of which contains all states with fixed excitation number (Fig. 2), and we can treat each manifold separately. By looking on the spectrum of (4) in different manifolds we note that all of these manifolds have in common an existence of an eigenstate with eigenvalue zero, so called dark state. It is convenient to introduce the following notation for the interacting states \( |n_{s},e^{l}s^{m},n_{c}\rangle \), where \( n_{s}\) and \( n_{c}\) denote the number of the probe field photons and cavity photons respectively and \( e^{l}s^{m}\) denotes the atomic state that interacts with the probe and cavity fields and contains \( l \) atoms in state \( |e\rangle \), \( m \) atoms in state \( |s\rangle \) and all the other atoms in the ground state \( |g\rangle \). Using this notation we can express the dark state in the single excitation manifold as

\[ |\psi_{D}^{(1)}\rangle = \frac{G}{\sqrt{G^2 + g^2n}} |1_{s},g,0_{c}\rangle - \frac{g\sqrt{n}}{\sqrt{G^2 + g^2n}} |0_{s},s,1_{c}\rangle. \]
Here \( g\sqrt{n} \) is the collective coupling strength of \( N_a \) atoms in a volume \( V \) with homogeneous density \( n = N_a/V \) to the mode \( \hat{b} \). The dark state for the subspace containing two excitations is given by

\[
|\psi_D^{(2)}\rangle = \frac{1}{\mathcal{N}} (\sqrt{2}G^2 |2_s, g, 0_c\rangle - 2Gg\sqrt{n}|1_s, s, 1_c\rangle + \sqrt{n} |0_s, ss, 2_c\rangle)
\]  

(6)

where \( \mathcal{N} = \sqrt{2G^2 + 4G^2g^2n + g^4n^2} \) is the normalization constant.

Note that in the low excitation limit, i.e. if the number of excitations is much smaller than the number of atoms, atomic excitation can be treated as a bosonic excitation. As a consequence, the coupling of the state \(|0_s, ss, 2_c\rangle\) to the upper state \(|0_s, es, 1_c\rangle\) experiences a two-fold bosonic enhancement leading to a term \(2Gg\sqrt{n}\) instead of \(\sqrt{2}G\). Because of this two-fold enhancement we can not write the double-excitation dark state as a direct product of two single-excitation dark states. This is different from usual EIT, where the quantization of the control field is not considered and dark states can be represented as number states of a polariton operator [15].

The general expression for the dark state in the \( N \) excitations subspace reads

\[
|\psi_D^{(N)}\rangle = \frac{1}{\mathcal{N}} \sum_{M=0}^{N} f^M_s^{N-M} |M_s, s^{N-M}, (N-M)c\rangle,
\]

(7)

where \( \mathcal{N} \) is normalization constant and the coefficients are given by

\[
f^M_s^{N-M} = (-1)^M \frac{N!}{(N-M)!} \frac{1}{\sqrt{M!}} \left( \frac{G}{g\sqrt{n}} \right)^M.
\]

(8)

Taking into account a possible decay from the excited atomic state \(|e\rangle\) we find another feature of the dark states. Due to fact that all of them do not contain contributions from the atoms in the excited state \(|e\rangle\), the dark states are not affected by the decay from this state, which is the origin of their name. As a consequence, the dark states make up stationary states of the system in that case.

Let us proceed and consider the propagation of the probe field. To describe the propagation we have to include many modes with different wave numbers \(k\), i.e. for a probe field propagating in the \(z\)-direction we can write \( \hat{E}(z) = \frac{1}{\sqrt{V}} \sum_k \hat{b}_k e^{ikz} \). Replacing the single mode operator in (4) by this expression leads to a modification of the Hamiltonian according to

\[
H = -\hbar \sum_{i=1}^{N} g\sigma_{ge}^i \frac{1}{\sqrt{V}} \sum_k \hat{b}_k^\dagger e^{-ikz} + G\sigma_{se}^i \hat{a}_c^\dagger + H.c.
\]

(9)

where the additional part corresponds to the energy of the free probe field and gives rise to field propagation. We assume here an infinitely extended medium and ignore boundary effects. In this case the system is translationally invariant and the Hamiltonian (9) does not couple modes with different \(k\)’s.

We start again with a single excitation. Due to translational invariance we can treat all \(k\)-modes independently, i.e. for every mode \(k\) we have three states \( |1_k, g, 0_c\rangle, |0_k, es, 0_c\rangle \) and \( |0_k, sk, 1_c\rangle \) coupled in a \(\Lambda\) configuration (compare Fig. 2), where we modified the previous notation by labeling it with the mode wave number \(k\). Similar to the single mode case we can write for the dark state of mode \(k\)

\[
|\psi_D^{k}\rangle = \frac{G}{\sqrt{G^2 + g^2n}} |1_k, g, 0_c\rangle - \frac{g\sqrt{n}}{\sqrt{G^2 + g^2n}} |0_k, sk, 1_c\rangle.
\]

(10)

States belonging to different \(k\)’s are linearly independent and thus the dark state for the entire single-excitation manifold is given by

\[
|\psi_D\rangle = \sum_k C_k |\psi_D^{k}\rangle
\]

(11)

with some constants \(C_k\) which fulfill the normalization condition \(\sum_k |C_k|^2 = 1\).

To calculate the group velocity we assume that the spectral width of the incoming photon is smaller than the transparency width \(\Delta k_s = \Delta \omega_s < \omega_{tr}\), which is defined by \(\omega_{tr} = \frac{G^2}{\gamma c^2} \) [15]. Here \(OD = L/L_{abs}\) is the optical depth and \(L_{abs} = \gamma c/g^2n\) is the resonant absorption length of the medium. Since within the transparency width each \(k\) excitation is described by the corresponding dark state \(|\psi_D^{k}\rangle\), fulfilling this condition ensures that the entire excitation will propagate as a dark state. In this case we can treat the propagation perturbatively by calculating the first order energy correction resulting from the last term in (9). The group velocity is then given by

\[
\nu_g^{(1)} = \frac{\partial \langle \psi_D | \sum_k \omega_k \hat{b}_k^\dagger \hat{b}_k | \psi_D \rangle}{\partial k} = c \frac{G^2}{g^2n}.
\]

(12)

where we used the fact that for a free field \(\omega_k = ck\).

-FIG. 3. (Color online) Bare levels and coupling diagram in the case of two propagating excitations.

In the next step we consider two excitations. The corresponding coupling scheme is shown in Fig. 3. At
first glance it looks different to the corresponding coupling diagram in the single mode case (see Fig. 2). For example, the state that contains two photonic excitations $|1_k, 1_k', g, 0\rangle$ couples now to two different states which contain an atomic excitation, namely $|1_k, e_k', 0\rangle$ and $|1_k, e_k, 0\rangle$. Also the coupling constants are changed. However, if we combine these two states to a symmetric superposition state $|1, e, 0\rangle_{S}^{kk'} = 1/\sqrt{2}(|1_k, e_k', 0\rangle + |1_k', e_k, 0\rangle)$ and apply this procedure to all other degenerate states, we again end up with a coupling scheme that is identical to that of the single mode case. Most importantly the resulting effective coupling constants between the symmetric states are the same as in the single-mode case. The corresponding dark state is then given by equation (6) and reads

$$|\psi_{D}^{kk'}\rangle = \frac{1}{N} \left( \sqrt{2}G^2 |1_k 1_k', g, 0\rangle + g^2 n |0_s, s_k s_{k'}, 2_c\rangle - 2 G v \sqrt{n} \frac{1}{\sqrt{2}} (|1_k, s_{k'}', 1_c\rangle + |1_k', s_k, 1_c\rangle) \right). \quad (13)$$

A general dark state containing two excitations then reads

$$|\psi_{D}^{(2)}\rangle = \sum_{kk'} C_{kk'} |\psi_{D}^{kk'}\rangle, \quad (14)$$

in complete analogy to equation (11). The group velocity can be determined in a similar manner as in the single-excitation case

$$v_g^{(2)} = \frac{\partial \langle \psi_{D}^{kk'} | \sum_k \omega_k \hat{b}_k \hat{b}_k^\dagger | \psi_{D}^{kk'} \rangle}{\partial K} = c \cdot \frac{2G^4 + 2G^2 g^2 n}{g^4 n^2 + 4G^2 g^2 n + 2G^4}, \quad (15)$$

where the differentiation is now made with respect to the center of mass momentum $K = k + k'$. The generalization to $N$ excitations is straightforward. Start with the $N$ excitation dark state (7), replace the state $|M_s, s^{N-M}, (N-M)_c\rangle$ by the symmetric state of all $\binom{N}{M}$ degenerate states. Calculate the first order energy correction and differentiate this with respect to the center of mass momentum $K = k_1 + k_2 + \cdots + k_N$ to obtain the group velocity. This yields

$$\frac{v_g^{(N)}}{c} = \sum_{M=0}^{N} \frac{M}{N} \left( \frac{f_{M}^{s^{N-M}}}{N} \right)^2. \quad (16)$$

The factor $\frac{M}{N}$ in the nominator results from the symmetrization procedure and can be interpreted as a weighting factor of the corresponding state to the group velocity. This means for example that the component of the state containing $N$ photons contributes fully and the symmetric state with only one photon contributes with relative weight $1/N$ to the propagation velocity.

The dependence of the group velocity on the number of incoming photons according to equation (16) is plotted in Fig. 3. One notices that the group velocity for $N$ photons is always smaller than the group velocity for $N+1$ photons and that in the limit of large $N$ the group velocity approaches the vacuum speed of light, as one would expect.

![FIG. 4. (Color online) The dependence of the group velocity on the number of incoming photons in the case $G=g\sqrt{n}$. The group velocity for some fixed photon number $G$ is always larger than the one for the preceding numbers and approaches the vacuum speed of light in the limit of large photon numbers $N$.](image)

In the limit $G \ll g\sqrt{n}$, which is typically the case in experiments, we can give an analytic approximation for the group velocity

$$\frac{v_g}{c} \approx \frac{G^2}{g n} N, \quad (17)$$

i.e. the group velocity scales linearly with the number of photons. A comparison between this approximation and the full expression (16) is shown in Fig. 5.

![FIG. 5. (Color online) The dependence of the group velocity on the number of incoming photons in the case $G=0.01g\sqrt{n}$ and the corresponding fit of the approximation (17).](image)
IV. NUMERICAL RESULTS

To confirm the results derived in the previous section and to take into account boundary effects associated with finite spatial extend of the medium we numerically simulate the propagation of pulses with up to two photons. We perform the simulations using Hamiltonian (2) by making a wave function ansatz and numerically integrating the corresponding Schrödinger equations for the amplitudes of the different components. The single excitation wave function reads

\[ |\psi(t)\rangle = \int d^3r \frac{f(z,t)}{\sqrt{V}} \hat{E}^\dagger(r) |0\rangle + \int d^3r \frac{e(z,t)}{\sqrt{V}} \sigma_{eg}(r) |0\rangle + \int d^3r \frac{s(z,t)}{\sqrt{V}} \sigma_{eg}(r) a_1^\dagger |0\rangle, \]

where \(|f(z,t)|^2\) corresponds to probability of finding a photon at position \(z\), and the probability of finding an atom at the same position in state |\(e\rangle\) and |\(s\rangle\) is given by |\(e(z,t)|^2\) and |\(s(z,t)|^2\) respectively. Using the commutator relations

\[ [\hat{E}(r), \hat{E}^\dagger(r')] = \delta(r - r'), \]

\[ [\sigma_{ij}(r), \sigma_{kl}(r')] = \frac{1}{n(r)} (\delta_{jk} \sigma_{il}(r) - \delta_{il} \sigma_{kj}(r)) \delta(r - r') \]

we obtain the corresponding equations of motion

\[ \partial_t f(z,t) = -c \partial_z f(z,t) + ig \sqrt{n(r)} e(z,t), \]
\[ \partial_t e(z,t) = -\gamma e(z,t) + ig \sqrt{n(r)} f(z,t) + iG^* s(z,t), \]
\[ \partial_t s(z,t) = iG e(z,t), \]

where we include the decay from the excited state |\(e\rangle\). These equations are equivalent to the propagation equations in the EIT case. From the EIT case we know that if the pulses are long enough, i.e., if they fulfill the adiabaticity condition \(T_p > 1/\omega_r\), we can adiabatically eliminate the \(e(z,t)\) component. This allows us to recast the equations of motion to a single propagation equation for the \(f(z,t)\) component, namely

\[ (\partial_t + v_g \partial_z) f(z,t) = 0 \]

i.e. the single photon pulse travels through the medium without being absorbed with reduced group velocity \(v_g = cG^2/(G^2 + g^2n)\), which coincides with the propagation velocity \(v_g^{(1)}\) derived in the last section for the single excitation. The corresponding time delay after propagation reads then \(\Delta T = L/v_g^{(1)}\), where \(L\) is the medium length. Using the EIT analogy we can also describe the behavior of the pulse on the medium boundary, where the group velocity changes from \(c\) to \(v_g\). Such a change leads to a pulse compression inside the medium by the factor \(v_g/c\).

Let us move on to two-photon pulses. Here the wave

FIG. 6. The time evolution of the two photon component \(ff(z_1, z_2)\) of an initial Gaussian pulse. The parameters are \(G = g\sqrt{n} = 500c/L\). The white lines denote the medium boundaries.
function can be written as
\[
|\psi(t)\rangle = \frac{1}{\sqrt{2}} \int d^3r \int d^3r' \frac{f(z,z',t)}{V} \hat{c}_\uparrow(r)\hat{c}_\downarrow(r') |0\rangle
\]

\[ + \int d^3r \int d^3r' \frac{e(z,z',t)}{V} \sqrt{n(r)}\sigma_g(r)\hat{c}_\uparrow(r') |0\rangle \]

\[ + \frac{1}{\sqrt{2}} \int d^3r \int d^3r' \frac{e_g(z,z',t)}{V} \sqrt{n(r)}\sqrt{n(r')}\sigma_g(r)\sigma_g(r') |0\rangle \]

\[ + \int d^3r \int d^3r' \frac{s(z,z',t)}{V} \sqrt{n(r)}\sqrt{n(r')}\sigma_g(r)\sigma_g(r') a_\sigma a_\sigma† |0\rangle \]

\[ + \frac{1}{2} \int d^3r \int d^3r' \frac{ss(z,z',t)}{V} \sqrt{n(r)}\sqrt{n(r')}\sigma_g(r)\sigma_g(r') a_\sigma a_\sigma† |0\rangle \]

Similar to single excitation the absolute value squared of the coefficients gives the probability of finding the system in the corresponding state. In Fig. 4 we plot the quantity \(|ff(z,z')|^2\), which is proportional to the probability of finding two photons at positions \(z\) and \(z'\). We see that just as in the single-photon case the two-photon pulse is compressed inside the medium. However, in addition we recognize that the shape of the wave function is distorted after propagation through the medium. To understand how this distortion comes about let’s consider some component \(ff(z_1,z_2)\). As already mentioned the absolute value squared gives the probability of finding two photons with mutual distance \(d = |z_1 - z_2|\). Initially both photons are outside of the medium and travel with the speed of light. Then the first photon enters the medium and propagates now with the reduced group velocity \(v_g^{(1)}\) until after the time \(t = d/c\) the second photon enters the medium. Since now there are two photons inside the medium they both propagate with the group velocity \(v_g^{(2)} > v_g^{(1)}\). Due to pulse compression in the medium the distance of the two photons is reduced to \(d' = d v_g^{(1)}/c\). Then after the time \(t' = (L - d')/v_g^{(2)}\), where \(L\) is the length of the cavity, the first photon will leave the medium and the remaining photon will now propagate with the group velocity \(v_g^{(1)}\) until it leaves the medium. Afterwards both photons will again propagate with the speed of light. This shows that the amount of time that both photons propagate with the two-photon group velocity \(v_g^{(2)}\) depends on their mutual distance inside the medium \(d'\). Taking this into account we can explain the shape distortion. The components on the first bisectrix have the smallest possible distance \(d = 0\) and travel at all times with the larger group velocity \(v_g^{(2)}\) and hence are more advanced in comparison to other components with non vanishing mutual distance. The maximal time delay between the single and two photon component is then
\[
\Delta \tau_{12} = L \left( \frac{1}{v_g^{(1)}} - \frac{1}{v_g^{(2)}} \right) \approx \frac{1}{2} \frac{g^2 n}{G^2} = \gamma G^2 \text{OD},
\]

where OD is the optical depth of the medium. The other extreme case is when the mutual distance between two photons inside the medium becomes larger than the medium size \(L\). Obviously these components propagate only with the velocity \(v_g^{(1)}\) and therefore can not be separated from the single photon components. This puts a limitation on the maximal pulse length. On the other hand one can not use arbitrary short pulses, since those would violate the adiabaticity condition and lead to pulse absorption. Rewriting the adiabaticity condition in terms of maximal delay time we can give an upper bound for the ratio of the maximal delay time to the pulse time
\[
\frac{\Delta \tau_{12}}{T_p} < \frac{1}{2} \sqrt{OD}.
\]

In order to be able to effectively separate the single photon component this ratio should be larger than 1. Both conditions can only be satisfied at large optical depths.

In this section we want to investigate the possibility for an experimental realization of our proposal. State of the art cavities can reach single atom coupling strength of about \(G \approx (2\pi)3\text{MHz}\) with cavity decay rates of roughly \(\kappa \approx (2\pi)0.1\text{MHz}\) [16]. Using a Bose-Einstein condensate as our three level medium allows us to obtain the required optical depths. For example using Rb BEC one can reach

V. ESTIMATION FOR EXPERIMENTAL REALIZATION

In this section we want to investigate the possibility for an experimental realization of our proposal. State of the art cavities can reach single atom coupling strength of about \(G \approx (2\pi)3\text{MHz}\) with cavity decay rates of roughly \(\kappa \approx (2\pi)0.1\text{MHz}\) [16]. Using a Bose-Einstein condensate as our three level medium allows us to obtain the required optical depths. For example using Rb BEC one can reach
optical depths of $\OD \approx (10 - 100)$ \cite{17, 18} and a single atom cooperativity of $C \gtrsim 15$ \cite{16, 19}. A weak laser pulse can be used as the propagating probe field, i.e. we can approximate the state of the incoming field as

$$|\alpha\rangle \approx \left(1 - \frac{|\alpha|^2}{2}\right)|0\rangle + \alpha \int dz f(z) \hat{E}^\dagger(z)|0\rangle + \frac{\alpha^2}{2} \int dz_1 \int dz_2 f(z_1) f(z_2) \hat{E}^\dagger(z_1) \hat{E}^\dagger(z_2)|0\rangle,$$

which is a good approximation for a weak coherent pulse. This allows us to utilize the results of our calculations and extract all relevant quantities. In Fig. 7 we plot the intensity $\langle \hat{E}^\dagger(t)\hat{E}(t) \rangle$ of the field after propagation through the medium calculated using experimental realistic numbers from above and setting all detunings to zero. Already in this intensity plot we can recognize the spatial separation of different components. To make this separation more evident we also plot the expectation value of the two photon component $\langle \hat{E}^\dagger(t) \hat{E}^\dagger(t) \hat{E}(t) \hat{E}(t) \rangle$, here we can clearly see that this component is about $\Delta\tau_{12}$ ahead of the single photon component.

Until now, we completely disregarded cavity damping and the excited state decay in our considerations. While we can safely neglect the excited state decay as long as we fulfil the adiabaticity condition, the cavity decay could be a practical limitation, since it will destroy the dark states and induce coupling between different excitation manifolds. Its influence can be neglected if the cavity lifetime is larger than the propagation time of the single photon, since it is the slowest component, i.e.

$$\kappa < \frac{v_g^{(1)}}{L} \approx \frac{C^2}{\gamma \OD}.$$  \hspace{1cm} (29)

This condition can be rewritten in terms of the cavity cooperativity leading to more restrictive condition $C > \OD$ on the cavity than the usual strong coupling condition $C > 1$. This represents a major limitation for the experimental realization.

VI. CONCLUSION

In conclusion, we presented a detailed analysis of our proposal for a number state filter for propagating light pulses based on cavity induced transparency. Assuming adiabaticity and an infinite homogeneous medium we derived a general expression for the dependence of the group velocity on the number of incoming photons. To take into account the effects associated with the finite medium size we performed numerical simulations for few-photon wave packets. Using the results of these simulations we could explain the behavior of the light pulse components with different photon numbers at the medium boundaries and derive a condition for the separation of the single photon component from the rest. Finally we investigated a possibility for an experimental realization of our proposal. We found that for successful implementation we have to modify the usual strong coupling condition in terms of the cavity cooperativity $C > 1$ to the more restrictive condition $C > \OD$, where $OD$ is the optical depth of the medium.

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