Dynamical SUSY Breaking in Heterotic M-Theory

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Abstract

It is shown that four-dimensional $N = 1$ supersymmetric QCD with massive flavors in the fundamental representation of the gauge group can be realized in the hidden sector of $E_8 \times E_8$ heterotic string vacua. The number of flavors can be chosen to lie in the range of validity of the free-magnetic dual, using which one can demonstrate the existence of long-lived metastable non-supersymmetric vacua. This is shown explicitly for the gauge group $Spin(10)$, but the methods are applicable to $Spin(N_c)$, $SU(N_c)$ and $Sp(N_c)$ for a wide range of color index $N_c$. Hidden sectors of this type can potentially be used as a mechanism to break supersymmetry within the context of heterotic M-theory.

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Introduction

It has long been known [1] that a four-dimensional $N = 1$ super Yang-Mills theory with purely massive vector-like matter has a non-vanishing Witten index and that this implies the existence of supersymmetry preserving vacua. For a super Yang-Mills theories to have no supersymmetric vacua at all, it must either be chiral [2, 3] or, if non-chiral, it must have massless matter [4, 5]. Theories satisfying these requirements are complicated and their incorporation into realistic particle physics models has been difficult to achieve.

Recent results in various contexts in string theory [6] have stimulated interest in four-dimensional $N = 1$ theories with both supersymmetric and non-supersymmetric vacua, the supersymmetry breaking states being meta-stable and long-lived. With this in mind, the authors of [7] have re-examined four-dimensional $N = 1$ SQCD theories with purely massive vector-like matter. Using the free-magnetic dual description of the theory in the infrared, they can determine the vacuum structure of the strongly coupled gauge theory. They find that, in addition to the requisite supersymmetric vacua, there exist meta-stable supersymmetry breaking vacua that, under the appropriate circumstances, can be long-lived.

Specifically, they show the following. They first consider $N = 1$ supersymmetric QCD with gauge group $SU(N_c)$ and $N_f$ massive flavors in the fundamental representation. Taking $N_f$ to be in the free-magnetic range, $N_c + 1 \leq N_f < \frac{3}{2}N_c$, they prove that, in addition to the $N_c$ expected supersymmetric vacua, there exist supersymmetry breaking meta-stable vacua. Furthermore, in the limit that

$$|\epsilon| = \sqrt{|\frac{m}{\Lambda}|} \ll 1,$$

(1)

where $m$ is the typical scale of the quark masses and $\Lambda$ is the strong-coupling scale, the meta-stable vacua are long-lived. In particular, as the parameter $|\epsilon| \to 0$, the life-time of these non-supersymmetric states can easily exceed the age of the Universe.

The authors of [7] then generalized their results to gauge groups $Spin(N_c)$ and $Sp(N_c)$. In this paper, we will be particularly interested in the $Spin(N_c)$ case. Consider $N = 1$ supersymmetric QCD with gauge group $Spin(N_c)$ and $N_f$ massive flavors in the fundamental representation. Then, in the free-magnetic range (including two special cases),

$$N_c - 4 \leq N_f < \frac{3}{2}(N_c - 2),$$

(2)

it was shown in [7] that, in addition to the $N_c - 2$ expected supersymmetric vacua, there are supersymmetry breaking meta-stable vacua which, when inequality (1) is satisfied, are long-lived.

These results open the door to a re-examination of dynamical supersymmetry breaking in many contexts. In particular, it is of interest to ask whether four-dimensional
$N = 1$ supersymmetric QCD, with a matter spectrum consisting of massive vector-like fundamental representations only and for which the number of flavors lies in the free-magnetic range, can be found as the low energy theory of a string compactification. Furthermore, can such an effective theory be incorporated into a realistic string theory of particle physics? Type II string theory realizations of dynamical supersymmetry breaking were recently obtained in [8, 9]. In this paper, we will show that the answer to both questions is also affirmative in the context of heterotic M-theory. Specifically, we prove the following. Consider the $E_8 \times E_8$ heterotic superstring, either for weak or strong string coupling, and focus on the hidden sector gauge group $E_8$. We will present an explicit elliptically fibered Calabi-Yau threefold $X$ and a slope-stable holomorphic vector bundle $V$ with structure group $SU(4)$ that lead to an $N = 1$ supersymmetric $Spin(N_c)$, $N_c = 10$ theory of QCD in four-dimensions. The matter spectrum of this theory consists entirely of $N_f$ massive fundamental representations, where $N_f$ can take any integer value. Noting that for $N_c = 10$ the constraint eq. (2) becomes

$$6 \leq N_f < 12,$$

we conclude that this hidden sector is a heterotic string vacuum with eight supersymmetric vacua and meta-stable non-supersymmetric vacua. Furthermore, the typical mass parameter $m$ is a linear function of the vector bundle moduli whose vacuum expectation values can be adjusted to satisfy eq. (1). Hence, the supersymmetry breaking vacua are long-lived and of phenomenological interest. Finally, a hidden sector of this type can, in principal, be embedded in heterotic $M$-theory [10] models of the type discussed in [11]. This could provide the mechanism, or one of several mechanisms, for breaking supersymmetry in realistic heterotic string compactifications. The technical details leading to these results will appear in [12].

To summarize, in this paper we present a hidden sector whose low energy theory is $N = 1$ SQCD with gauge group $Spin(10)$ and show that this admits meta-stable, long-lived non-supersymmetric vacua. This gauge group, and the slope-stable holomorphic vector bundle that leads to it, were chosen for mathematical convenience. It is straightforward to generalize this to vector bundles whose low energy theories are $N = 1$ SQCD with gauge groups $SU(N_c)$, $Spin(N_c)$ and $Sp(N_c)$, for a range of values of $N_c$, and $N_f$ massive vector-like fundamental representations where $N_f$ is in the range specified in [7]. These hidden sectors will also admit meta-stable vacua which are long-lived. Such generalized hidden sectors can be used to break supersymmetry in heterotic $M$-theory particle physics models. These generalized theories will be presented elsewhere.
The Heterotic Vacua

The Calabi-Yau Threefold

We construct our hidden sector by compactifying the $E_8 \times E_8$ heterotic string on a simply-connected Calabi-Yau threefold $X$ which is elliptically fibered over the complex surface $dP_9$. It was shown in [13] that $X$ must factorize into the fibered product $X = B_1 \times_{\beta_1} B_2$, where $B_1$ and $B_2$ are both $dP_9$ surfaces. Either can be thought of as the base space and the projection maps are denoted by $\pi_i : X \to B_i$ with $i = 1, 2$ respectively. Each $dP_9$ surface is itself elliptically fibered over the identified projective sphere $\mathbb{P}^1$, with the projection mappings $\beta_i : B_i \to \mathbb{P}^1$ for $i = 1, 2$. It is straightforward to show that the number of Kähler and complex structure moduli of $X$ is given by $h^{1,1} = 19$ and $h^{1,2} = 19$.

The homology of any $dP_9$ surface is easily computed. In particular, $h_2(dP_9, \mathbb{Z}) = 10$ and

$$H_2(dP_9, \mathbb{Z}) = \text{span}\{l, e_1, e_2, \ldots, e_9\},$$

where $l$ is the hyperplane divisor and each $e_i$ is an exceptional divisor. In this basis, the fiber class of the $dP_9$ elliptic fibration is given by $f = 3l - \sum_{i=1}^{9} e_i$. Furthermore, we arbitrarily choose $e_9$ to be the zero section $\sigma$. Which $dP_9$ surface we are referring to, either $B_1$ or $B_2$, will be clear from context so we will not further label their curves.

The Vector Bundle

In addition to presenting the Calabi-Yau threefold $X$, it is necessary to specify a gauge connection, indexed in a subgroup of $E_8$, which satisfies the Hermitian Yang-Mills equations. It follows from the work of Donaldson [14] and Uhlenbeck/Yau [15] that this is equivalent to specifying a holomorphic vector bundle $V$ on $X$ which is slope-stable. In this paper, we construct this bundle in the following way.

First consider a line bundle

$$L = \mathcal{O}_{B_1}(e_1 - e_9)$$

on the surface $B_1$. Next, we construct a rank 3 vector bundle $W$ on $B_2$ as follows. Let $C_W \in \Gamma \mathcal{O}_{B_2}(l + f)$ be a spectral cover and choose the line bundle $\mathcal{N}_{c_w} = \mathcal{O}_{c_w}$ over $c_W$. Then $W$ is constructed from this data via the Fourier-Mukai transform of $(C_W, \mathcal{N}_{c_w})$ [16],

$$W = FM_{B_2}(\mathcal{O}_{c_w}).$$

Note that $W$ has structure group $U(3)$ with a non-trivial determinant line bundle. Combining eqns. (5) and (6), one can construct a rank 3 vector bundle $V_3$ on $X$ as

$$V_3 = \pi_1^*(L) \otimes \pi_2^*(W).$$

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Now define a line bundle $V_1$ on $X$ to be
\[ V_1 = \pi_1^*(L^{-3}) \otimes \pi_2^*(\text{det}^{-1} W). \]
(8)

Given $V_3$ and $V_1$, we construct the requisite vector bundle $V$ on $X$ as a non-trivial extension
\[ 0 \to V_1 \to V \to V_3 \to 0. \]
(9)

Note that because of our explicit choice of $V_1$, any such extension has structure group $SU(4)$.

The vector bundles $V$ satisfying eq. (9) correspond to directions in the linear space of extensions, denoted $\text{Ext}^1(V_3, V_1)$. In terms of sheaf cohomology, this is
\[ \text{Ext}^1(V_3, V_1) = H^1(X, V_1 \otimes V_3^*). \]
(10)

Our construction will only be useful if this space is non-trivial. $\text{Ext}^1(V_3, V_1)$ can be computed as follows. First, using eqns. (7), (8) and pushing the the cohomology down to the projective plane $\mathbb{P}^1$, one can show that
\[ H^1(X, V_1 \otimes V_3^*) = H^0(\mathbb{P}^1, \text{Hsupp}(L^{-4}) \cap \text{Hsupp}(W^* \otimes \text{det}^{-1} W)), \]
(11)

where $\text{Hsupp}(S)$ stands for the set of points in $\mathbb{P}^1$ supporting the skyscraper sheaf $R^1\pi^*_i S$.

We find that
\[ \text{Hsupp}(L^{-4}) = \{q_1, q_2, q_3, s_1, \ldots, s_{12}\} \]
(12)
and
\[ \text{Hsupp}(W^* \otimes \text{det}^{-1} W) = \{p_1, p_2, r_1, \ldots, r_{19}\}, \]
(13)

where, a priori, the $q_i$, $s_j$, $p_k$ and $r_l$ are independent points. It is clear from eq. (11) that if $\{q_i, s_j\} \cap \{p_k, r_l\} = \emptyset$ then $\text{Ext}^1(V_3, V_1)$ is trivial and there are no extensions $V$. This situation is not of interest so, henceforth, we must always choose $L$ and $W$ so that at least one point in $\{q_i, s_j\}$ is identified with at least one point in $\{p_k, r_l\}$. First assume only two points are identified. For the purposes of this paper will always assume that $s_1 = r_1$. Then, it follows from eq. (11) that

- $s_1 = r_1$: \[ \text{Ext}^1(V_3, V_1) = \mathbb{C}, \]
(14)

where $\mathbb{C}$ is the complex numbers. Next let us assume that, in addition to $s_1 = r_1$, a second and third pair of points are identified, which we choose to be $q_1 = p_1$ and $q_2 = p_2$ respectively. Then

- $s_1 = r_1, q_1 = p_1$: \[ \text{Ext}^1(V_3, V_1) = \mathbb{C}^6 \]
(15)
and
\[ s_1 = r_1, \ q_1 = p_1, \ q_2 = p_2: \quad \text{Ext}^1(V_3, V_1) = \mathbb{C}^{\otimes 3}. \] (16)

Three pairs of points, each pair consisting of one point from the base of \( B_1 \) and one from the base of \( B_2 \), can always be identified without restricting the moduli space. It follows that non-trivial extension spaces of the form eqns. (14), (15) and (16) always exist. Hence, non-trivial holomorphic vector bundles \( V \) with structure group \( SU(4) \) over \( X \) can be constructed. The final step is to demonstrate that these bundles are slope-stable. Although this is not difficult to show in this case, the proof requires a longer discussion than is suitable for this paper. Here we simply state that these holomorphic bundles are indeed slope-stable. The complete proof will be presented in [12].

Given eqns. (7), (8) and (9), it is straightforward to compute the Chern classes of \( V \). Relevant to this paper is the fact that \( c_1(V) = 0 \) and \( c_3(V) = 0 \). Compactifying the hidden sector of the \( E_8 \times E_8 \) heterotic string on the Calabi-Yau threefold \( X \) with structure group \( SU(4) \) vector bundle \( V \) will lead to an \( N = 1 \) supersymmetric \( \text{Spin}(10) \) gauge theory in four-dimensions. It remains to compute the low energy matter spectrum of this theory.

**The Spectrum**

With respect to \( SU(4) \times \text{Spin}(10) \) the \( 248 \) representation of the of the hidden sector \( E_8 \) gauge group decomposes as

\[ 248 = (1, 45) \oplus (15, 1) \oplus (4, 16) \oplus (4, \overline{16}) \oplus (6, 10). \] (17)

The \( (1, 45) \) are the gauginos of \( \text{Spin}(10) \), the \( (15, 1) \) corresponds to the vector bundle moduli and the remaining representations are the matter fields. The number of \( 45 \) representations in the low-energy theory is given by \( h^0(X, \mathcal{O}_X) = 1 \). Let \( R \) be any of the remaining representation of \( \text{Spin}(10) \) in eq. (17) and denote the corresponding \( V \) bundle by \( U_R(V) \). As discussed in [17], the multiplicity of representation \( R \) in the low-energy theory is given by \( n_R = h^1(X, U_R(V)) \). The number of vector bundle moduli is \( n_1 = H^1(X, \text{ad}(V)) \), which can be computed using the methods presented in [18]. Here, it suffices to say that \( n_1 > 0 \).

Next we turn to the \( 16 \) and \( \overline{16} \) representations. Using the fact that \( c_3(V) = 0 \), the Atiyah-Singer index theorem tells us that \( n_{16} = n_{\overline{16}} \). Furthermore, it follows from the fact that \( h^1(X, V_1) = h^1(X, V_2) = 0 \) that

\[ n_{16} = n_{\overline{16}} = 0. \] (18)

Hence, no \( 16 \) and \( \overline{16} \) representations appear in the low-energy theory. Now consider the multiplicity of the \( 10 \) representation. This is given by

\[ n_{10} = h^1(X, \wedge^2 V). \] (19)
Using eqns. (7), (8), (9) and pushing the cohomology down to the base \( \mathbb{P}^1 \), one can show that
\[
h^1(X, \wedge^2 V) = 2h^0(\mathbb{P}^1, \text{Hsupp}(L^2) \cap \text{Hsupp}(\wedge^2 W)),
\]
(20)

where
\[
\text{Hsupp}(L^2) = \{q_1, q_2, q_3\}
\]
(21)

and
\[
\text{Hsupp}(\wedge^2 W)) = \{p'_1, p'_2, p'_3, p'_4, p'_5\}.
\]
(22)
The \( q_i, i = 1, 2, 3 \) appeared in eq. (12) and \( p'_j, j = 1, \ldots, 5 \) are five new independent points in \( \mathbb{P}^1 \). To assure the existence of \( V \), we will always take \( s_1 = r_1 \) and, hence, from eq. (14) \( \text{Ext}^1(V_3, V_1) \) is at least \( \mathbb{C} \). Let us first assume that the points \( p'_j \) are all independent of each other and are not identical with any point \( q_i \). Then it follows from eqns. (19) and (20) that

- \( p'_j \neq p'_j', p'_j \neq q_i \):
  
  \[ n_{10} = 0. \]

(23)

Let us now assume that one point \( p'_j \) is identical with one point \( q_i \). Without loss of generality we can choose \( p'_1 = q_1 \). Then

- \( p'_1 = q_1 \):
  
  \[ n_{10} = 2. \]

(24)

Now take two independent points in \( p'_j \) and identify each with a different point in \( q_i \). We can, without loss of generality, choose \( p'_1 = q_1 \) and \( p'_2 = q_2 \). Then from eqns. (19) and (20) we find

- \( p'_1 = q_1, p'_1 = q_1 \):
  
  \[ n_{10} = 4. \]

(25)

Note that for all these cases \( \text{Ext}^1(V_3, V_1) = \mathbb{C} \).

We have now used all the freedom to align up to three pairs of points on the sphere \( \mathbb{P}^1 \). The simplest way to enlarge the number of 10 representations is as follows. Let us choose our sub-bundles so that the \( p'_j \) are no longer all independent. To begin, let us arrange the bundles so that \( p'_1 = p'_2 \), which can be done. When this happens it turns out that this point must also be identical to \( p_1 \). Now take \( p_1 = q_1 \). It follows from eq. (15) that, in this case, the space of extensions enlarges to \( \text{Ext}^1(V_3, V_1) = \mathbb{C}^{\oplus 2} \). If, in addition, we take \( p'_3 = q_2 \), then it follows from eqns. (19) and (20) that

- \( p'_1 = p'_2 = p_1 = q_1, p'_3 = q_2 \):
  
  \[ n_{10} = 6. \]

(26)

Continuing in this way, let us choose \( p'_1 = p'_2 \) and \( p'_3 = p'_4 \). It turns out that these points must also equal \( p_1 \) and \( p_2 \) respectively. Now take \( p_1 = q_1 \) and \( p_2 = q_2 \). We see from eq. (16) that, in this case, \( \text{Ext}^1(V_3, V_1) = \mathbb{C}^{\oplus 3} \). Furthermore, it follows from eqns. (19) and (20) that
\[ p_1' = p_2' = p_1 = q_1, \quad p_3' = p_4' = p_2 = q_2: \quad n_{10} = 8. \] (27)

This is as far as we can go using the five points \( p_j' \) in eq. (22). However, by generalizing the spectral cover \( C_W \) to \( C_W \in \Gamma\mathcal{O}_{B_2}(l + nf) \) for integer \( n > 1 \), the number of points \( p_j' \) can be increased arbitrarily. In this case, we can easily find vector bundle \( V \) for which \( n_{10} = 10, 12, \ldots \) as well. It is also possible to amend the construction of \( V \) so as to obtain any odd value of \( n_{10} \). This is beyond the scope of this paper and will be discussed in [12].

We conclude that \( V \) can be chosen so that the matter spectrum of the four-dimensional \( N = 1 \) supersymmetric \( \text{Spin}(10) \) theory contains \( 10 \) representations only and that \( n_{10} \) satisfies the requisite condition eq. (3). That is, \( n_{10} \) can take any value in the interval
\[ 6 \leq n_{10} < 12. \] (28)

The \( 10 \) Mass

The above analysis proves that one can construct vector bundles \( V \) whose massless matter spectrum consists of \( n_{10} \) fundamental \( 10 \) representations satisfying eq. (28) on a subvariety of its moduli space. However, it follows from eq. (23) that as one moves off this subvariety to generic points in moduli space these multiplets must all become massive. From the point of view of the four-dimensional theory, this can only occur if quadratic pairs of \( 10 \) representations have non-vanishing couplings to vector bundle moduli. This is indeed the case. In [12] we show that non-vanishing cubic couplings of the form
\[ \lambda \phi \text{Tr} 10 \cdot 10, \] (29)
where \( \phi \) are vector bundle moduli, occur in the superpotential. For non-vanishing vacuum expectation values \( \langle \phi \rangle \), this leads to mass terms for each of the \( n_{10} 10 \) representations where
\[ m = \lambda \langle \phi \rangle. \] (30)

The exact values of the parameters \( \lambda \), as well as the generic size of the moduli expectation values \( \langle \phi \rangle \), are model dependent and beyond the scope of this paper. Be that as it may, it is clear from discussions in the literature that there is no obstruction to choosing these so that the typical mass \( m \) satisfies constraint eq. (1), where \( \Lambda \) is the \( \text{Spin}(10) \) strong-coupling scale. Hence, the low-energy theory constructed here will have non-supersymmetric, meta-stable vacua with a long life-time.

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