Towards room-temperature superconductivity

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Abstract – By taking advantage of a stability criterion established recently, the critical temperature $T_c$ is reckoned with help of the microscopic parameters, characterising the normal and superconducting electrons, namely the independent-electron band structure and a repulsive two-electron force. The emphasis is laid on the sharp $T_c$ dependence upon electron concentration and inter-electron coupling, which might offer a practical route toward higher $T_c$ values and help to understand why high-$T_c$ compounds exhibit such remarkable properties.

Introduction. – The BCS theory [1], despite its impressive success, does not enable one to predict [2] superconductivity occurring in any metallic compound. As shown elsewhere [3], such a drawback ensues from an attractive interaction, assumed to couple electrons together, which is not only at loggerheads with the sign of the Coulomb repulsion, but in addition leads to inconsistencies to be recalled below. Therefore this work is intended at investigating the $T_c$ dependence upon the parameters, characterising the motion of electrons correlated together through a repulsive force, in order to devise, within the framework of a two-fluid picture developed elsewhere [3–6], a single model, valid for all superconductors, including low and high $T_c$ materials, as well. This is in marked contrast with the mainstream approach, tending to seek a particular model, dedicated to each superconducting compound [7–9].

In order to reach the hereabove mentioned goal, a historical survey, aimed at identifying the cause of the BCS failure and its far-reaching consequences, is needed. Moreover, it will enable the reader to understand why our previous work [3–6] is not only unrelated to the mainstream view, but also runs afool at it. As a matter of fact, the original sin was to purport with no scientific proof whatsoever that the many bound electron state, coming out of the BCS variational calculation [1], conveyed the signature of superconductivity, namely it did sustain persistent currents. Actually, the only well-established property of the BCS state [1] is that its energy is lower than that of the Fermi gas [10] of same electron concentration at $T = 0$K, which is of course irrelevant to the issue of persistent currents. Accordingly, since the BCS conclusion requires by all means to assume an attractive two-electron coupling, all authors [7–9], in the wake of BCS, lacking anyhow a reliable criterion of superconductivity, took for granted that any contrived two-electron interaction would ensure superconductivity, provided it is attractive. Illustrative examples of this surmise can be found, in particular, in several extensive studies [11–17] dealing with the electron-phonon coupling that has been believed to mediate the attractive force between electrons since Froehlich’s work [18]. Unfortunately, such wishful thinking proved a fatal mistake, whenever it was shown [3] that superconductivity cannot arise, if the electrons are coupled together through an attractive force. As an appalling consequence, the whole research work in superconductivity decayed eventually into a muddled, religious war, vividly documented by Zaanen [8] and Lederer [9].

This barren bickering might have lasted till doomsday, were it not for the recent discovery [4] of a long-awaited criterion of superconductivity, saying that persistent currents are observed, not because the resistivity vanishes (it has been shown [19–22] actually to be finite, albeit much smaller than in the corresponding normal metallic phase), but rather because their being destroyed by the Joule effect would violate the second law of thermody-
The binding energy of the superconducting state $E_B(T < T_c)$ has been worked out [3, 24] as

$$E_B(T) = \int_T^{T_c} (C_s(u) - C_n(u)) \, du,$$

with $C_s(T), C_n(T)$ being the electronic specific heat of a superconductor, flown through by a vanishing current [3] and that of a degenerate Fermi gas [10]. A stable phase ($\Rightarrow E_B > 0$) requires $C_s(T_c) > C_n(T_c)$, which can be secured only by fulfilling the following conditions [6]

$$\frac{\partial E_F}{\partial c_n}(T_c, c_0) = - \frac{\partial \mu}{\partial c_n}(0), \quad \rho'(E_F(T_c, c_0)) > 0 \quad (3)$$

with $\rho(\epsilon), \epsilon$ being the one-electron density of states and energy [10], respectively, and $\rho' = \frac{d \rho}{d \epsilon}$.

2-Microscopic parameters. – Since the remaining analysis relies heavily on Eqs. (2,3), explicit expressions are needed for $E_F(T_c, c_0), \frac{\partial E_F}{\partial c_n}(T_c, c_0), \varepsilon_B, \frac{\partial \mu}{\partial c_n}(0)$. Because the independent electrons make up a degenerate Fermi gas ($\Rightarrow T << E_F/k_B$ with $k_B$ being Boltzmann’s constant), applying the Sommerfeld expansion [10] up to $T^2$ yields

$$E_F(T,c_0) = E_F(0,0) - \frac{\rho'(0)(\varepsilon_{ka}T_c)^2}{\rho} - \frac{\partial \mu}{\partial c_n}(0)(\rho + \rho''(\varepsilon_{ka}T_c)^2)^{-1} \quad (4)$$

with $\rho = \rho(E_F(0,0)), \rho' = \frac{d \rho}{d \epsilon}(E_F(0,0)), \rho'' = \frac{d^2 \rho}{d \epsilon^2}(E_F(0,0))$. As for $\varepsilon_B, \frac{\partial \mu}{\partial c_n}(0)$, a truncated Hubbard Hamiltonian $H_K$, introduced previously [25–27], will be used. The main features of the calculation [3] are summarised below for self-containment.

The independent electron motion is described by the Hamiltonian $H_d$

$$H_d = \sum_{k,\sigma} \epsilon(k)c^+_k \epsilon \sigma c_k \epsilon \sigma .$$

$\epsilon(k)$, $k$ are the one-electron energy ($\epsilon(k) = \epsilon(-k)$) and a vector of the Brillouin zone, respectively, $\sigma = \pm$ is the electron spin and the sum over $k$ is to be carried out over the whole Brillouin zone. Then $c^+_k \epsilon \sigma, c_k \epsilon \sigma$ are creation and annihilation operators on the Bloch state $|k, \sigma \rangle$,

$$|k, \sigma \rangle = c^+_k \epsilon \sigma |0 \rangle, \quad |0 \rangle = c_k \epsilon \sigma |k, \sigma \rangle$$

with $|0 \rangle$ being the no electron state. The Hamiltonian $H_K$ reads then

$$H_K = H_d + \frac{U}{N} \sum_{k,k',\sigma} c^+_k \epsilon \sigma c_{-k} \epsilon \sigma c_{-k'} \epsilon \sigma c_{k'} + \quad (5)$$

with $N >> 1, U > 0$ being the number of atomic sites, making up the three-dimensional crystal, and the Hubbard constant, respectively. Note that the Hamiltonian used by Cooper [28] is identical to $H_{K=0}$, but with $U < 0$. 

The conduction electrons are taken to comprise [3, 4, 6] bound and independent electrons, in respective temperature dependent concentration $c_s(T), c_n(T)$, such that

$$c_0 = c_s(T) + c_n(T) \quad (1)$$

with $c_0$ being the total concentration of conduction electrons. They are organized, respectively, as a many bound electron [3] (MBE) state, characterised by its chemical potential $\mu(c_s)$, and a Fermi gas [10] of Fermi energy $E_F(T, c_0)$. The Helmholtz free energy of independent electrons per unit volume $F_n$ and $E_F$ on the one hand, and the eigenenergy per unit volume $E_s(c_s)$ of bound electrons and $\mu$ on the other hand, are related [10, 23], respectively, by $E_F = \frac{\partial E}{\partial c_n}$ and $\mu = \frac{\partial E}{\partial c_n}$. Then a stable equilibrium is conditioned [4] by Gibbs and Duhem’s law

$$E_F(T, c_n(T)) = \mu(c_s(T)) \quad (1),$$

which expresses [23] that the total free energy $F_n + E_s$ is minimum at $T$ kept fixed, provided $\frac{\partial E}{\partial c_n} + \frac{\partial \mu}{\partial c_n} > 0$. Note-worthy is that $\frac{\partial \mu}{\partial c_n} < 0$ has been shown to be a prerequisite for persistent currents [4], thermal equilibrium [3], the Josephson effect [5] and a stable [6] superconducting phase. Likewise, Eq.(1) reads [3, 4, 6] for $T = T_c$

$$E_F(T_c, c_0) = \mu(c_s(0)) = \varepsilon_B/2 \quad (2)$$

with $\varepsilon_B$ being the energy of a bound electron pair [3]. Note that Eqs.(1,2) are consistent with the superconducting transition being of second order [23], whereas it has been shown [3] to be of first order at $T < T_c$ ($\Rightarrow E_F(T, c_0 - c_c) > \mu(c_s)$), if the sample is flown through by a finite current ($\Rightarrow c_s < c_s(T)$).
\[ H_K \text{ sustains [3] a single bound pair eigenstate, the energy } \varepsilon_B(K) \text{ of which is obtained by solving} \]
\[
\frac{1}{U} = \frac{1}{N} \sum_k \varepsilon_B(K) - \varepsilon(K, k) = \int_{-t_K}^{t_K} \rho_K(\varepsilon) \varepsilon(B(K) - \varepsilon) \, d\varepsilon. \tag{5}
\]

\( \pm t_K \) are the upper and lower bounds of the two-electron band, i.e., the maximum and minimum of the two-electron energy \( \varepsilon(K, k) = \varepsilon(k) + \epsilon(K-k) \) over \( k \) with \( K \) kept fixed, whereas \( \rho_K(\varepsilon) \) is the corresponding two-electron density of states, taken equal to
\[
\rho_K(\varepsilon) = \frac{2}{\pi t_K} \sqrt{1 - \left( \frac{\varepsilon}{t_K} \right)^2}, \quad -t_K \leq \varepsilon \leq t_K.
\]

The dispersion curves \( \varepsilon_B(K) \) are plotted in Fig.1. Though Eq.(5) is identical to the equation yielding the Cooper pair energy [28], their respective properties are quite different:

- the data in Fig.1 have been calculated with \( U > 0 \), rather than \( U < 0 \) favoured by BCS [1], because, due to the inequality [3] \( U \frac{\partial \mu}{\partial N_c} < 0 \), choosing \( U < 0 \) entails \( \frac{\partial \mu}{\partial N_c} > 0 \), which has been shown not to be consistent with persistent currents [4], thermal equilibrium [3], the Josephson effect [5] and occurrence [6] of superconductivity. As a further consequence of \( U > 0 \), \( \varepsilon_B(K) \) shows up in the upper gap of the two-electron band structure (\( \rightarrow \varepsilon_B(K) > t_K \)) rather than in the lower gap (\( \rightarrow \varepsilon_B(K) = 0 < -t_K \)) in case of the Cooper pair [28]. Nevertheless the bound pair is thermodynamically stable, because every one-electron state of energy \( \varepsilon(k) \leq E_F(T_c, c_0) \) is occupied for a degenerate Fermi gas [10], so that, due to Pauli’s principle, a bound electron pair of energy \( \varepsilon_B(K) = 2E_F(T_c, c_0) \), according to Eq.(2), cannot decay into two one-electron states \( \varepsilon(k) \leq E_F, \varepsilon(K-k) \leq E_F \) such that
\[
\varepsilon(k) + \epsilon(K-k) \leq \varepsilon_B(K).
\]

- a remarkable feature in Fig.1 is that \( \varepsilon_B(K) \rightarrow t_K \) for \( U \rightarrow t_K/2 \), so that there is no bound pair for \( U < t_K/2 \) (accordingly, the dashed curve is no longer defined in Fig.1 for \( \frac{E_F}{k_B} < 0.13 \), in marked contrast with the opposite conclusion drawn by Cooper [28], that there is a Cooper pair, even for \( U > 0 \). This discrepancy results from the three-dimensional Van Hove singularities, showing up at both two-electron band edges \( \rho_K(\varepsilon) \rightarrow \pm t_K \) \( \propto \sqrt{t_K - |\varepsilon|} \), unlike the two-electron density of states, used by Cooper [28] which is constant and thence displays no such singularity. Likewise the width of Cooper’s two-electron band is equal to a Debye phonon energy \( 2t_K = \omega_D \approx 30 \text{meV} < E_F \approx 3eV \). Hence the resulting small concentration of superconducting electrons, \( c_0(T=0) \approx \frac{\omega_D}{E_F} \approx .01 \), entails that London’s length should be at least 10 times larger than observed values [19–22];

- at last Cooper’s assumption \( U < 0 \) implies \( \varepsilon_B/2 < E_F(T_c) \), which is typical of a first order transition but runs afoul at all measurements, proving conversely the superconducting transition to be of second order (\( \varepsilon_B/2 = E_F(T_c) \) in accordance with Eq.(2)).

The bound pair of energy \( \varepsilon_B(K) \) turns [3], at finite concentration \( c_s \), into a MBE state, characterised by \( \mu(c_s) \). Its properties have been calculated thanks to a variational procedure, displaying several merits with respect to that used by BCS [1]:

- it shows that \( \mu(0) = \varepsilon_B/2 \);

- the energy of the MBE state has been shown to be exact for \( |U| \rightarrow \infty \);

- an analytical expression has been worked out for \( \frac{\partial \mu}{\partial c_s}(K, c_s = 0) \) as:
\[
\frac{\partial \mu}{\partial c_s}(K, c_s = 0) = \frac{1}{2} \int_{-t_K}^{t_K} \frac{\rho^c(\varepsilon)}{(\varepsilon_B(\varepsilon) - \varepsilon_S(\varepsilon))^2} \, d\varepsilon. \tag{6}
\]

Note that \( \varepsilon_B(K) \rightarrow t_K \Rightarrow \frac{\partial \mu}{\partial c_s}(K, c_s = 0) \rightarrow \infty \).

At last, focussing on \( H_K \) instead of the Hubbard Hamiltonian \( H \), as done above, deserves a special comment. \( H \) can be written as
\[
H = H_d + \sum_K (H_K - H_d).
\]

The bound electron pair of eigenenergy \( \varepsilon_B(K) \) is indeed an eigenstate of both \( H_K \) and \( H \), whereas the BCS scheme [1], introduced to approximate the many bound electron eigenstate \( \psi_K \) of \( H_K \), cannot be applied to \( H \). However \( H_K \) is not a realistic Hamiltonian, because it involves four-electron forces, whenever it is Fourier transformed back into real space. Thus \( \psi_K \) is seen to be of little significance, unless it happens to be an eigenstate of \( H \) too. Such a claim, which has been demonstrated [25] but within a Hilbert space, containing the usual [10] one \( S \), as a subspace, will be proved in \( S \) too, in a forthcoming publication.

3-T \text{c} versus electron concentration. - The \( T_c \) dependence on \( c_0 \) will be discussed by assigning to the one-electron density of states the expression valid for free electrons [10]
\[
\rho(\varepsilon) = \eta \sqrt{\varepsilon - \varepsilon_b} \Rightarrow c_0 = \frac{2}{3} \eta(E_F(0, c_0) - \varepsilon_b)^{3/2}, \tag{7}
\]

with \( \eta = \frac{\sqrt{m}}{\pi^{3/2}} m^{1/3} \), whereas \( \varepsilon_b, m, V = 17 \text{A}^3 \) stand for the bottom of the conduction band, electron mass and volume of the unit-cell, respectively. With help of Eq.(4), Eqs.(2,3) can be recast into a system of two equations
\[
E_F(0, c_0) = \frac{\rho'}{\rho} (\varepsilon_k T_c)^2 \rho + \rho^c (\varepsilon_k T_c)^2 \frac{\partial c}{\partial c} (K, c_s = 0) (t_K - \varepsilon_S(\varepsilon))^2. \tag{8}
\]
To that end, starting values are assigned to $U$, $t_K$, which gives access to $\varepsilon_B(K)$, $\frac{\partial\varepsilon_B}{\partial c}(K,c_0 = 0)$ and thence to $E_F(0,c_0), \epsilon_B$ and finally to $c_0$, owing to Eqs.(2,3,7). Those values of $c_0,t_K$ are then fed into Eqs.(8) to launch a Newton procedure, yielding the solutions $c_0(T_c),t_K(T_c)$. The results are presented in table 1. Since we intend to apply this analysis to high-$T_c$ compounds [7], we have focused upon low concentrations $c_0 < 0.2$, which entails, in view of Eqs.(4,7), that $\frac{\partial\varepsilon_B}{\partial c}$ takes a high value. This requires in turn $\varepsilon_B(K) \rightarrow t_K$ (see Eq.(6)) and hence [3] $U \rightarrow \frac{t_K}{2}$, in agreement with $\frac{\partial\varepsilon_B}{\partial c} \approx 2$ in table 1.

A remarkable property of the data in table 1 is that $c_0,t_K$ are barely sensitive to large variations of $T_c$, i.e., $|\delta c_0| < 10^{-3},|\delta t_K| < 10^{-5}$ for $T_c \approx 400K$. This can be understood as follows : taking advantage of Eqs.(2,4,7) results into

$$\frac{2E_F(0,c_0)}{\varepsilon_B(K)} - 1 = \frac{\pi^2}{12} \left( \frac{k_BT_c}{\Delta(T_c)} \right)^2,$$

which, due to $\frac{\partial\varepsilon_B}{\partial c} \approx 0$, $\Delta(T_c) \approx 1eV$, $T_c = 400K$, yields indeed $\delta c_0 = c_0(400K) - c_0(1K) \approx 10^{-3}$, in agreement with the data in table 1. Such a result is significant in two respects, regarding high-$T_c$ compounds, for which $c_0$ can be varied over a wide range :

- because of $\frac{\partial\varepsilon_B}{\partial c} \approx 0$, the one-electron band structure can be regarded safely as $c_0$ independent, which enhances the usefulness of the above analysis;

- the large doping rate up to $\approx 0.2$ is likely to give rise to local fluctuations of $c_0$. Hence, in view of the utmost sensitivity of $T_c$ with respect to $c_0$, this will result into a heterogeneous sample, consisting in domains, displaying $T_c$ varying from 0 up to a few hundreds of K. Thus the observed $T_c$ turns out to be the upper bound of a broad distribution of $T_c$ values, associated with superconducting regions, the set of which makes up a percolation path throughout the sample. However, if the daunting challenge of making samples, wherein local $c_0$ fluctuations would be kept well below $10^{-4}$, could be overcome, this might prove the way to superconductivity at room temperature.

**4-$T_c$ versus inter-electron coupling.** — The $T_c$ dependence upon $U$ will be analysed with the one-electron density of states

$$\rho(\epsilon) = \frac{4}{\pi t^2} \left( 1 - \left( \frac{\epsilon - \epsilon_B}{\epsilon_B} \right)^2 \right),$$

where $2t$ stands for the one-electron bandwidth ($\approx 0 \leq \epsilon \leq 2t$). Our purpose is to determine the unknowns $t_K(E_F,T_c), U(E_F,T_c)$ with

$$E_F = E_F(T = 0,c_0) , \quad c_0 = \int_0^{E_F} \rho(\epsilon)d\epsilon.$$
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Fig. 2: Plots of \(E_F(T_c, c_0), t_K(T_c, c_0), U(T_c, c_0)\) calculated for \(T_c = 1\,K\) and \(t = 3\,eV\); the unit for \(c_0\) is the number of conduction electrons per atomic site; \(\delta f\) with \(f = E_F, t_K, U\) is defined as \(\delta f = 1 - \frac{\rho(E_F, c_0)}{\rho_0(E_F, c_0)}\); the scale is linear for \(E_F, t_K, \frac{U}{\rho}\) on the left hand side, but logarithmic for dimensionless \(\delta E_F, \delta t_K, \delta U\) on the right hand side.

But the most significant feature is that \(\delta U\) is almost insensitive to large \(T_c\) variation, except for \(E_F \to 0\), i.e. for \(E_F\) close to the Van Hove singularity, located at the bottom of the one-electron band, which has two consequences:

- \(c_0\) cannot be varied in most superconducting materials, apart from high-\(T_c\) compounds, so that \(U\) is unlikely to be equal to \(U(c_0)\), indicated in Fig.2. Conversely, since high-\(T_c\) compounds allow for wide \(c_0\) variation, \(c_0\) can be tuned so that \(U = U(c_0)\);

- the only possibility for a non high-\(T_c\) material to turn superconducting is then offered at the bottom of the band, because \(\delta U\) becomes large due to \(\frac{\delta U}{U}(E_F \to 0) \propto \frac{1}{E_F}\) in Eq.(4). Such a conclusion, that superconductivity was likely to occur in the vicinity of a Van Hove singularity in low-\(T_c\) materials, had already been drawn independently, based on magnetostricton data [3].

It will be shown now that the one and two-electron densities of states \(\rho(\epsilon), \rho_K(\epsilon)\) cannot stem from the same one-electron band. The proof is by contradiction. As a matter of fact the one-electron density of states should read in that case

\[
\rho(\epsilon) = \frac{4}{\pi t^2} \sqrt{1 - \left(\frac{\epsilon}{t}\right)^2}.
\]

Hence \(U > 0\) entails, in view of Fig.1 and Eq.(2), that there is \(\frac{\delta t}{t} = E_F > 0\), which implies \(\rho'(E_F) < 0\) in contradiction with Eq.(3). Accordingly, since the two different one-electron bands, giving rise to \(\rho(\epsilon), \rho_K(\epsilon)\), respectively, display a sizeable overlap, they should in addition belong to different symmetry classes of the crystal point group, so that superconductivity cannot arise if there are only s-like electrons at \(E_F\) (no alcali or noble metal is indeed superconducting) or if the point group reduces to identity. At last superconductivity is inferred not to occur in case of an almost full conduction band, because it is tantamount to \(E_F\) being located near the upper band-edge and hence implies \(\rho'(E_F(T_c, c_0)) < 0\), in contradiction with Eq.(3). Noteworthy is that all of those hereabove conclusions had already been drawn empirically [2].

Conclusion. – This work, combined with our previous publications [3–6, 19–22], is aimed at presenting the reader with a comprehensive, albeit elementary solution, based mainly on thermodynamics, of the long-standing riddle of superconductivity. The critical temperature \(T_c\) has been calculated for conduction electrons, coupled via a repulsive force, within a model based on two conditions, expressed in Eqs.(2,3) and characterising a second order transition happening at \(T_c\). It should be noted that Eq.(3), unlike Eq.(2), is difficult to fulfil. Likewise, the unfortunate consequences of an attractive inter-electron force, assumed by BCS [1], have been analysed.

Superconductivity occurring in conventional materials has been shown to require \(E_F(T_c)\) being located near a Van Hove singularity of the one-electron band structure, whereas a practical route towards still higher \(T_c\) values has been delineated in high-\(T_c\) compounds, provided the local electron concentration can be controlled accurately. At last, the thermodynamical criterias in Eqs.(2,3) unveil the close interplay between independent and bound electrons in giving rise to superconductivity.

Since magnetic (ferromagnetic, antiferromagnetic, ferrimagnetic, static spin-wave...) transitions are also of second order and involve only conduction electrons [29], it might be tempting to apply the thermodinamical conditions in Eqs.(2,3) to the study of this case. However this cannot be done, because the electrons, responsible for the magnetic long-range order, are localised or Wannier-like [10], unlike the normal and bound electrons which are itinerant and Bloch-like [10]. Hence tackling the magnetic issue will require a dedicated approach.

REFERENCES

[1] SCHRIEFFER J.R., Theory of Superconductivity (Addison-Wesley) 1993
[2] MATTHIAS B. T., GEBALLE T. H. and COMPTON V. B., Rev.Mod.Phys., 35 (1963) 1
[3] SZEFTEL J., SANDEAU N. and ABOU GHANTOUS M., J.Supercond.Nov.Magn., 33 (2020) 1307
[4] SZEFTEL J., SANDEAU N. and ABOU GHANTOUS M., Eur.Phys.J.B, 92 (2019) 67
[5] SZEFTEL J., SANDEAU N. and ABOU GHANTOUS M., arXiv : 2003.07196
[6] Szeftel J., Sandeau N., Abou Ghantous M. and El-Saba M., J. Supercond. Nov. Magn., 34 (2021) 37
[7] Armitage N.P., Fournier P. and Greene R.L., Rev. Mod. Phys., 82 (2010) 2421
[8] Zaanen J., 100 years of superconductivity, edited by Rochalla H. and Kes P.H. (Chapman and Hall) 2011 (also available in arXiv: 1012.5461)
[9] Lederer P., arXiv: 1510.08808
[10] Ashcroft N.W. and Mermin N. D., Solid State Physics (Saunders College) 1976
[11] Carson J.P., Rev. Mod. Phys., 62 (1990) 1027
[12] Pierronero L., Straessler S. and Grimaldi C., Phys. Rev. B, 52 (1995) 10516
[13] Grimaldi C., Pierronero L. and Straessler S., Phys. Rev. B, 52 (1995) 10530
[14] Ruiz H.S., Giraldo J.J. and Baquero R., J. Supercond. Nov. Magn., 21 (2008) 21
[15] Szczesniak D. and Szczesniak R., Phys. Rev. B, 99 (2019) 224512
[16] Schrodi F., Oppeneer P.M. and Aperis A., Phys. Rev. B, 102 (2020) 024503
[17] Sadovskii M.V., J. Supercond. Nov. Magn., 33 (2020) 19
[18] Froehlich H., Phys. Rev., 79 (1950) 845
[19] Szeftel J., Sandeau N. and Khater A., Phys. Lett. A, 381 (2017) 1525
[20] Szeftel J., Sandeau N. and Khater A., Prog. In. Electr. Res. M., 69 (2018) 69
[21] Szeftel J., Abou Ghantous M. and Sandeau N., Prog. In. Electr. Res. L., 81 (2019) 1
[22] Szeftel J., Sandeau N., Abou Ghantous M. and Khater A., EuroPhys. Lett., 131 (2020) 17003
[23] Landau L.D. and Lifshitz E.M., Statistical Physics (Pergamon Press) 1959
[24] Gorter C.J. and Casimir H., Physica, 1 (1934) 306
[25] Szeftel J. and Khater A., Phys. Rev. B, 54 (1996) 13581
[26] Szeftel J., Electron Correlations and Material Properties 2, edited by Gonis A. and Kioussis N. and Ciftan M. (Kluwer Academic, New York) 2003
[27] Szeftel J. and Caffarel M., J. Phys. A, 37 (2004) 623
[28] Cooper L.N., Phys. Rev., 104 (1956) 1189
[29] Lévy L.P., Magnetism and Superconductivity (Springer) 2000