\[ B_{d,s}^0 - \bar{B}_{d,s}^0 \] mixing and Lepton Flavour Violation in SUSY GUTs: impact of the first measurements of \( \phi_s \)

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In this work we re-examine the correlation between \( B_{d,s}^0 - \bar{B}_{d,s}^0 \) mixing and Lepton Flavour Violation in the light of recent experimental measurements in the \( B_s \) system. We perform a generic SUSY analysis of the allowed down squark mass insertion parameter space. In the SUSY GUT scenario this parameter space is then used to make predictions for LFV branching ratios. We find that the recent measurement for the CP phase \( \phi_s \) excludes the lowest rates for \( \tau \to \mu \gamma \) and provides a lower bound of \( \sim 3 \times 10^{-9} \) for \( \tan \beta = 10 \). Future experimental improvements in the bound on \( \tau \to \mu \gamma \) and the measurement of \( \phi_s \) will constitute a strong test of the SUSY GUT scenario.

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I. INTRODUCTION

In recent years there have been great efforts made in the search for New Physics (NP) in the B system which has now been seen as an extremely promising place to find NP effects as deviations of flavour-changing neutral-currents (FCNC) from their Standard Model(SM) expectations. The first measurements of \( B_s - \bar{B}_s \) mixing have been observed at DØ [1] and CDF [2] experiments reporting a mass difference,

\[ 17 \text{ ps}^{-1} < \Delta M_s < 21 \text{ ps}^{-1} \quad (90\% \text{ C.L. DØ}) \quad (1) \]

\[ \Delta M_s = 17.77 \pm 0.10 \pm 0.07 \text{ ps}^{-1} \quad (\text{CDF}) \quad (2) \]

The constraints from eq. (2) strongly restrict the allowed parameter space for the NP contribution to \( B_s - \bar{B}_s \) mixing \[ 3, 9, 11, 12, 13, 14, 15 \].

The first measurement of the CP phase associated with \( B_s - \bar{B}_s \) mixing, \( \phi_s \), was reported by the DØ collaboration [4]. More recently, flavour-tagged measurements have now been made by both the CDF collaboration [4] and the DØ collaboration [5]. Combining these latest results the UTfit group [6] found that the new physics phase deviates from zero by about 3\( \sigma \),

\[ \phi_s = (\phi_s^{NP} - 2\beta_s) \quad (3) \]

with \( \beta_s = 0.018 \pm 0.001 \) and,

\[ \phi_s^{NP} = [-60.9, -18.58]^{90}_\circ \cup [-156.90, -106.40]^{90}_\circ \quad (4) \]

at 95% C.L. There still remains a two fold ambiguity in the measurement due to the symmetry in \( \phi_s \to (\pi - \phi_s) \).

In the \( B_d \) system the measured mass difference given by the “Heavy Flavour Averaging Group” (HFAG) [17] is,

\[ \Delta M_d = 0.507 \pm 0.004 \text{ ps}^{-1} \quad (5) \]

and the phase, \( \phi_d \), has been measured as,

\[ \phi_d = 0.757 \pm 0.044 \quad (6) \]

A very appealing model of NP is the supersymmetric grand unified theory (SUSY GUT). In such models there is a deep underlying connection between quarks and leptons above the GUT scale. For example in SU(5) the singlet down quarks are related to the lepton doublet as they live in the same \( 5 \) representation. This deep relation between down quarks and leptons is broken at the GUT scale but its presence may still be felt all the way down at the Electroweak scale. As a result of this relationship the right-handed down squark and left-handed sleptons mixings are related to each other. In such SUSY GUTs there is hence a deep connection between LFV rates such as \( BR(\tau \to \mu \gamma) \) and FCNCs such as \( B_s - \bar{B}_s \) mixings and its associated CP phase [18, 19, 20, 21].

In this work we re-examine the correlation between FCNCs and LFV rates in SUSY GUTs. Specifically we are interested in the connection between the allowed NP parameter space of \( B_s - \bar{B}_s \) mixing and the rate of \( \tau \to \mu \gamma \). To this end we first perform a generic SUSY analysis, of the allowed parameter space of down squark mass insertions \( g_R^{\Delta R} \). Our analysis in generic in that no particular SUSY-breaking mechanism shall be assumed. Having determined the allowed parameter space we then exploit the GUT relationship of squark and slepton mass insertions to make realistic predictions for the rates of \( \tau \to \mu \gamma, \tau \to e \gamma \) and their ratio. Recently a similar study was undertaken [19] on the correlation between LFV and \( B_q \) mixing. We believe that the NP contribution to \( B_d \) mixing was under-estimated in that work leading to over-estimates for the branching ratio of \( \tau \to e \gamma \). In this work we use an analytic form for the gluino contribution to \( B_d \) mixing derived in appendix A in the mass insertion approximation. This allows us to correctly compute the NP contribution to both \( B_d \) and \( B_s \) mixing. Furthermore the recent measurement of the CP phase \( \phi_s \) has prompted a re-examination of this correlation. Crucially we find that these recent measurements severely restrict the allowed parameter space and provide a lower bound
for the rate of $\tau \rightarrow \mu \gamma$. Finally, we explore an example
of an SO(10) inspired model and discuss the tension between
the constraints of LFV and FCNCs.

II. CONSTRAINTS ON THE SUSY CONTRIBUTION TO $B_q$ MIXING

The $\Delta B = 2$ transition between $B_q$ and $\bar{B}_q$ mesons is
defined as,

$$\langle B_q | \hat{H}_{eff} | \bar{B}_q \rangle = 2 M_{B_q} M_{12}^q \ (7)$$

where $M_{B_q}$ is the mass of the $B_q$ meson. We can then
define the $B_q$ mass eigenstate difference as,

$$\Delta M_q \equiv M_q^r - M_q^l = 2 |M_{12}^q| \ (8)$$

and its associated CP phase,

$$\phi_q = arg(M_{12}^q) \ (9)$$

In the Standard Model $M_{12}^{q,SM}$ is given by,

$$M_{12}^{q,SM} = \frac{G_F^2 M_W^2}{12\pi^2} M_{B_q} \hat{\eta}_B \ f_{B_q} \ B_{(V_{tq}V_{tb})^2 S_0(x_t)} \ (10)$$

where $G_F$ is Fermi’s constant, $M_W$ the mass of the
$W$ boson, $\hat{\eta}_B$ is a short-distance QCD correction identical for
both the $B_q$ and $B_d$ systems. The bag parameter $\hat{B}_B$, and decay constant $f_{B_q}$ are non-perturbative quantities and contain the majority of
the theoretical uncertainty. $V_{tq}$ and $V_{tb}$ are elements of the
Cabibbo-Kobayashi-Maskawa (CKM) matrix, and $S_0(x_t) \equiv m_t^2/M_{W}^2 = 2.32 \pm 0.04$.

The NP contribution to $B_q$ mixing may be parameterized in a model independent way as,

$$\Delta M_q = \Delta M_q^{SM} |1 + R_q| \ (11)$$

$$\phi_q = \phi_q^{SM} + \phi_q^{NP} = \phi_q^{SM} + arg(1 + r_q e^{i \sigma_q}) \ (12)$$

where $R_q \equiv r_q e^{i \sigma_q}$ denotes the relative size of the NP contribution. The dominant SUSY contribution to $B_q$ mixing comes from the gluino contribution which may be written as,

$$R_q^2 = a_1^L(m_{\tilde{g}}, x) \left[ (\delta_{q3})_{RR}^d \right] + a_2^L(m_{\tilde{g}}, x) \left[ (\delta_{q3})_{LL}^d \right] + \cdots \ (13)$$

Here we have ignored terms proportional to $\delta_{q3}^{LR,RL}$ mass insertions as they are expected to be heavily suppressed due to constraints from $b \rightarrow s \gamma$ [13]. In appendix A we give details of the functions $a_1^L$ and $a_2^L$.

We can now constrain both the magnitude and phase of the NP contribution, $r_q$ and $\sigma_q$, through the comparison of the experimental measurements with SM expectations. From the definition of eq. (11) we have the constraint,

$$\rho_q \equiv \frac{\Delta M_q}{\Delta M_q^{SM}} = \sqrt{1 + 2 r_q \cos \sigma_q + r_q^2} \ (14)$$

for $r_q$ and $\sigma_q$. The values for $r_q$ given by the UTfit analysis [6,7] at the 95% C.L. are,

$$\rho_d = [0.53, 2.05] \ (15)$$

$$\rho_s = [0.62, 1.93] \ (16)$$

From eq. (12) the phase associated with NP can also be written in terms of $r_q$ and $\sigma_q$,

$$\sin \phi_q^{NP} = \frac{r_q \sin \sigma_q}{\sqrt{1 + 2 r_q \cos \sigma_q + r_q^2}} \ (17)$$

$$\cos \phi_q^{NP} = \frac{1 + r_q \cos \sigma_q}{\sqrt{1 + 2 r_q \cos \sigma_q + r_q^2}} \ (17)$$

Here [6,7] gives the 95% C.L. constraints,

$$\phi_d^{NP} = [-16.6, 3.2]^\circ \ (18)$$

$$\phi_s^{NP} = [-60.9, -18.58]^\circ \cup [-156.90, -106.40]^\circ \ (19)$$

In order to consistently apply the above constraints all input parameters are chosen to match those used in the analysis of the UTfit group [6,7] with the non-perturbative parameters,

$$f_{B_q} \tilde{B}_{B_q}^4 = 262 \pm 35 \text{ MeV} \ (20)$$

$$\xi = 1.23 \pm 0.06 \text{ MeV} \ (21)$$

$$f_B = 230 \pm 30 \text{ MeV} \ (22)$$

$$f_{B_s} = 189 \pm 27 \text{ MeV} \ (23)$$

Now we have a clear picture of the constraints to be imposed on the $r_q - \sigma_q$ plane as listed in eq. (14-19). In the following sections these constraints shall be used in a generic SUSY analysis of the mass insertion parameter space, followed by a study of the correlation of $B_q$ mixing and LFV rates in the minimal SUSY GUT.

III. CORRELATION BETWEEN $B_q$ MIXING AND LFV RATES IN SUSY GUTS

In many ways the prototypical GUT is SU(5) as it is the
minimal group which can unify the gauge group of
the Standard Model. In the SU(5) GUT the quarks and
leptons are placed into $10 = (Q, u^c, e^c)$ and $\tilde{5} = (L, d^c)$
representations. Due to the symmetry of this simple
SUSY GUT there exists relations amongst the slepton and
squark soft SUSY breaking masses,

$$m_{10}^2 = m_{Q}^2 = m_{U}^2 = m_{E}^2, \quad m_{5}^2 = m_{L}^2 = m_{D}^2 \ (24)$$

These relations hold for scales at and above the GUT scale. Interestingly this implies that left-handed slepton mixing and right-handed down squark mixing are related.

This relation can still be felt very strongly at the Electro-
weak scale in the correlation of LFV rates and FCNCs.

Further renormalization group(RG) evolution down to
the Electroweak scale has very little effect on the size
of these off-diagonal elements [18]. So we may assume that these GUT scale values are approximately equal to their values at the electroweak scale. Hence it is a fair approximation to assume that \((m_D^2)_{ij} \approx (m_L^2)_{ij}\) at the electroweak scale. Then the \((\delta_{ij}^d)_{RR} \equiv (m_D^2)_{ij}/m_q^2\) contributions to FCNC and \((\delta_{ij}^s)_{LL} \equiv (m_L^2)_{ij}/m_q^2\) contributions to LFV are clearly correlated. We may explicitly write the rate of \(l_i \rightarrow l_j \gamma\) in the very suggestive form [19],

\[
BR(l_i \rightarrow l_j \gamma) \approx \frac{\alpha^3}{G_F^2 M_S^8} \left| (\delta_{ij}^d)_{RR} \right|^2 \tan^2 \beta \tag{25}
\]

where \(m_q\) is the average squark mass and \(M_S\) is the typical SUSY scale.

We also need to consider the RGE effects of the CKM mixings in the left-handed down squark matrices. These effects can be approximated as,

\[
(\delta_{ij}^s)_{LL} \approx \frac{1}{8 \pi^2} V_{ti}^* V_{tj} \left( \frac{3m_0^2 + A_0^2}{m_q^2} \right) \ln \frac{M^*}{M_W} \tag{26}
\]

Here \(m_0\) is the universal scalar mass, \(A_0\) the universal A-term and \(M^*\) is the scale at which the flavour blind soft SUSY breaking is communicated. These effects may be quite important in the \((\delta_{LL}^d, \delta_{RR}^d)\) contribution to eq. (13) due to the typically large value of \(a_4 \approx -100 a_1\). We shall take a minimal approach and assume \(M^*\) to be the GUT scale \(\sim 2 \times 10^{14}\) GeV, in which case the mass insertions are of the order, \((\delta_{23}^d)_{LL} \sim \lambda^2\) and \((\delta_{13}^s)_{LL} \sim \lambda^3\). Larger values of \(M^*\) will lead to an even more restricted parameter space for \((\delta_{13,23}^d)_{RR}\).

If we assume that the second term of eq. (13) dominates then we may derive the relation,

\[
\left| \frac{R_e}{R_d} \right|^2 \approx \frac{|a_4^1/|a_4^d|^2| V_{td}^2/V_{ts}^2 \approx \lambda^2} \left( \frac{\delta_{23}^d)_{LL}}{\delta_{13}^s)_{LL}} \right|^2 \frac{\left( \delta_{23}^d)_{RR}}{\left( \delta_{13}^s)_{RR}} \right|^2 \frac{BR(\tau \rightarrow \mu \gamma)}{BR(\tau \rightarrow e \gamma)} \tag{27}
\]

Here we have made use of the ratio, \(|a_4^1/|a_4^d|^2| \approx V_{td}^2/V_{ts}^2 \approx \lambda^2\), as derived in appendix A. This relation differs from that derived in [19] where it was assumed that, \(a_4^1/a_4^d = 1\).

In appendix A we show the full form of the functions \(a_1\) and \(a_4\) where it is shown that the correct ratio is \(|a_4^1/|a_4^d|^2| \approx V_{td}^2/V_{ts}^2 \approx \lambda^2\). As a result the allowed parameter space for the mass insertion \((\delta_{13}^s)_{RR}\) is suppressed relative to \((\delta_{23}^d)_{RR}\) by a factor \(\lambda^2\) as shown above.

**IV. NUMERICAL RESULTS AND DISCUSSION**

In this section we shall show the results of numerical calculations of our generic SUSY analysis of the allowed parameter space for the mass insertions \((\delta_{23,23}^d)_{RR}\). We then make use of this allowed parameter space to study the correlation of FCNCs and LFV rates.

For the numerical analysis we fixed the values of \(\tan \beta = 10, M_S = m_0 = M_{1/2} = m_q = (300, 500)\) GeV, \(A_0 = 0\) and the ratio \(x \equiv m_2^q/m_1^q = 1\). Taking these values we scan over \((\delta_{23,23}^d)_{RR}\) and require fits to the values of \(\rho_q\) and \(\phi_q^{NP}\) as described in the previous section.

![FIG. 1: Allowed parameter space for the mass insertion \((\delta_{23}^d)_{RR}\) as dictated by the constraints imposed from \(\Delta M_d\) and \(\phi_q\). The allowed region parameter space form loops in the \(r_q - \sigma_q\) plane. The constraints from the CP phases represent solid black lines as shown in fig. 4.](image)
slice shown by the green points in fig. 3. On the other hand, the present measurement of φd has a two fold ambiguity shown by the two green wedges in fig. 4.

Next we would like to discuss the implications of this allowed parameter space on predictions for the LFV rates of τ −→ μγ and τ −→ eγ. We have shown that in a SUSY GUT the mass insertions associated with the off-diagonal down squark mass matrix elements and those of the sleptons are clearly related. This relation leads to a strong correlation between the Bq mixing and LFV rates. Here we would like to expose this correlation in making realistic predictions for LFV rates and to extract information about the NP CP phase.

Let us first consider the 23 sector. Fig. 6 shows the predicted rate for τ −→ μγ plotted against the NP phase φs. The possibility of φs = 0 is excluded at the 3σ level [6], which also excludes the lowest rates for τ −→ μγ at the centre of the plot where φsNP → 0. The general feature of the plot is that larger LFV rates are predicted for larger NP phases. Without the input of φs this plot is symmetric in φsNP → −φsNP and extends for all φsNP ∈ (−π, π). The inclusion of the φs constraint drastically reduces the allowed parameter space and the resulting prediction space, as shown in fig. 5. These predictions are independent of the choice of Ms = m3 due to the 1/M2 dependence of both BR(li −→ ljγ) and the functions a1,4 shown in appendix A. There are two allowed regions, one with large NP phase and one with smaller NP phase. From fig. 5 it is clear that the upper bound on τ −→ μγ disfavours the large phase solution. For the second region, there is a lower bound on the BR(τ −→ μγ) > 3 × 10−9. This second allowed region is just below the present experimental bounds for the branching ratio, BR(τ −→ μγ) > 6.8 × 10−8 BaBar [22, 23] and BR(τ −→ μγ) < 4.5 × 10−8 Belle [24]. From these plots it is clear that improvements of the bound on τ −→ μγ in conjunction with an improved measurement of φs will provide a much stricter test of the SUSY GUT scenario.

In the 13 sector, we can see from fig. 2 that the allowed parameter space for the mass insertion (δ13)RR is far more restricted. This is due to the smaller mass difference in the Bq system, as a result the functions a1 and a4 are enhanced by the ratio, ∆Md/∆Ms ≈ V23/V132 ∼ λ2, see appendix A. This restriction leads to a much more suppressed prediction for the rates of τ −→ eγ. These rates are plotted in fig. 6 against the allowed values of the NP phase φd. The predicted values are in the region of 10−14 − 10−9. The present experimental bounds for the branching ratio are BR(τ −→ eγ) < 1.1 × 10−7 BaBar [22, 23] and BR(τ −→ eγ) < 1.2 × 10−7 Belle [24]. Looking at fig. 6 we can see that the allowed region of the (δ13)RR parameter space predicts rates well below the present experimental bounds.

FIG. 2: Allowed parameter space for the mass insertion (δ13)RR, with m3 = 300(smaller) and 500(larger) GeV respectively. Red/Blue points are constrained by the measurement of ∆Md while green/pink points have the extra constraint from the measurement of φd both at the 95% C.L..

FIG. 3: Allowed region of ra − σa parameter space for the NP contribution to Bd mixing. Red points agree with the ∆Md constraint while green points also agree with that of φd both at the 95% C.L..

FIG. 4: The allowed region of the ra − σa parameter space for the NP contribution to Bd mixing. Points in Red agree with the constraint from ∆Md, while green points also agree with the φd measurement both at the 95% C.L..
bounds. Therefore the present bounds have no impact on the allowed parameter space.

\[ \phi \rightarrow \text{solid black horizontal line.} \]

FIG. 5: Predictions for BR(\(\tau \rightarrow \mu \gamma\)) from constraints on \(B_s\) mixing. The red points conform to the \(\Delta M_s\) constraint while green points show the additional constraint from \(\phi_s\) both at 95\% C.L. The present experimental bound is shown by the solid black horizontal line.

The ratio of the branching fraction of \(\tau \rightarrow \mu \gamma\) and \(\tau \rightarrow e \gamma\) is less dependent on the SUSY parameter space and so it is interesting to consider the allowed size of this ratio. Using the parameter space of \((\delta_{13}^d)_{RR}\) and \((\delta_{23}^d)_{RR}\) allowed by the constraints of \(\rho_{d,s}\) and \(\phi_{d,s}^{NP}\) we can make predictions for the ratio BR(\(\tau \rightarrow \mu \gamma\))/BR(\(\tau \rightarrow e \gamma\)). Fig. 7 shows the resulting plot of BR(\(\tau \rightarrow \mu \gamma\))/BR(\(\tau \rightarrow e \gamma\)) plotted against the prediction for the NP CP phase \(\phi_{s}^{NP}\). We can see that the numerical results show this ratio ranging from 0.01 up to tens of thousands. Applying the constraints from the CP phases \(\phi_{d,s}\) implies that this ratio must lie in the region \(\gtrsim 10\). This large ratio is again due to the suppression of the \((\delta_{13}^d)_{RR}\) allowed parameter space due to the smallness of the mass difference in the \(B_d\) system as opposed to the larger mass difference in the \(B_s\) system. Here our results disagree with those presented in [19] due to their erroneous assumption of the ratio \(a_1^s/a_1^d = 1\).

Let us now look at a specific example of a simple SUSY GUT. The example we pick is an SO(10) inspired model as introduced in [20] [24]. In this model there are two Higgs representations one for the up/neutrino and down/charged lepton Yukawa couplings separately. The neutrino Yukawa coupling is then related to the up quark Yukawa, where two limiting cases are possible: (i) \(Y_c\) has CKM like mixing, (ii) \(Y_c\) has MNS like mixing. It is assumed that SO(10) breaks to SU(5) at the scale \(M_{10}\) and further breaks to the SM gauge group at the scale \(M_{\text{GUT}}\). Assuming that the flavour blind SUSY breaking occurs at the scale \(M_{s} = M_{10} = 10^{17}\) GeV, then the mass insertions can be written as,

\[
(\delta_{ij}^d)_{RR} \simeq -\frac{3}{8\pi^2} v_{ij}^2 \left| V_{ij} \right| \ln \frac{M_{10}}{M_{\text{GUT}}} \tag{28}
\]

\[
(\delta_{ij}^d)_{LL} \simeq -\frac{3}{8\pi^2} v_{ij}^2 \left| V_{ij} \right| \ln \frac{M_{10}}{M_{R}} \tag{29}
\]

where the matrix \(V\) is \(V_{\text{CKM}}\) and \(U_{\text{MNS}}\) for case (i) and (ii) respectively. The scale \(M_{R} \sim 10^{15}\) GeV is the scale of the right-handed neutrinos. Case (i) produces relatively small LFV rates with \((\delta_{23}^d)_{LL} \sim 0.008\) and \((\delta_{23}^d)_{RR} \sim 0.003\). The present bound on the rate of \(\tau \rightarrow \mu \gamma\) leads to the weak bound \(m_{0} \gtrsim 200\) GeV. Alternatively the contribution to \(B_s\) mixing is also very small, for \(m_{0} \gtrsim 200\) GeV we have too small a value \(|R_s| \lesssim 0.2\). Case (ii) produces large LFV rates with \((\delta_{23}^d)_{LL} \sim 0.25\) and \((\delta_{23}^d)_{RR} \sim 0.08\). Here the present bound on the rate of \(\tau \rightarrow \mu \gamma\) leads to the strong bound \(m_{0} \gtrsim 1000\) GeV. Here again the contribution to \(B_s\) mixing is very small, for \(m_{0} \gtrsim 1000\) GeV we once again have too small a value \(|R_s| \lesssim 0.2\).

\[ \phi_{s}^{NP} \]

FIG. 6: Predictions for BR(\(\tau \rightarrow e \gamma\)) from constraints on \(B_d\) mixing.

FIG. 7: BR(\(\tau \rightarrow \mu \gamma\))/BR(\(\tau \rightarrow e \gamma\)) from constraints on \(B_s\) mixing. Red points are from the \(\Delta M_{d,s}\) constraint only, green points are for both \(\Delta M_{d,s}\) and \(\phi_{d,s}\) constraints.
V. SUMMARY

In this work we have re-examined the constraints imposed on the parameter space of NP contributions to $B_s$ and $B_d$ mixing in the light of recent measurements of $\Delta M_s$ and its associated CP phase $\phi_s$. These constraints were then imposed to make a generic SUSY analysis of the allowed parameter space of the mass insertions $(\delta_{23}^d)_{RR}$ and $(\delta_{23}^q)_{RR}$.

In SUSY GUTs there is a deep underlying connection between (s)quarks and (s)leptons. In such a theory we should therefore expect that the flavour mixings observed in the quark and lepton sectors are correlated. Assuming that such a SUSY GUT exists, we have re-examined the correlation between LFVs and FCNCs.

From our numerical analysis we have found that the allowed parameter space for the mass insertion $(\delta_{23}^d)_{RR}$ is particularly restricted by the present measurements of $\Delta M_d$ and $\phi_d$. This is due to the small mass difference in the $B_d$ system compared to the $B_s$ system. As a result, the predicted branching ratio for $\tau \to e\gamma$ is particularly suppressed. On the other hand, the present experimental determination of $\Delta M_s$ and particularly $\phi_s$ are not yet so restrictive on the allowed values of $(\delta_{23}^q)_{RR}$. The larger mass difference in the $B_s$ system also means that the predicted values of the branching ratio of $\tau \to \mu\gamma$ are much larger and close to the present bounds from the B factories. The main conclusion of this work is that the recent measurement of $\phi_s$ substantially restricts the allowed parameter space and excludes the region where $\mu\gamma$ is to be found, leading to a lower bound of $\sim 3 \times 10^{-9}$ for tan$\beta = 10$. Future experimental improvement on the bound on the decay rate $\tau \to \mu\gamma$ and the measurement of $\phi_s$ will lead to a strong test of the SUSY-GUT scenario.

We have also considered a specific example of a SUSY SO(10) GUT and found there to be significant tension between the constraints from FCNCs and LFV decay rates.

APPENDIX A: $B^0_{s,d} - \bar{B}^0_{s,d}$ MIXING IN SUSY MODELS

To examine the contributions from new physics we may write the $\Delta B = 2$ effective Hamiltonian in the following form,

$$H_{e_{eff}}^{\Delta B=2} = \sum_{i=1}^{5} C_i(\mu) Q_i(\mu) + h.c.$$ (A1)

The operators are defined as,

$$Q_1 = \bar{q}_L^i \gamma_\mu q_R^i \bar{q}_R^j \gamma_\nu b_L^j$$ (A2)

$$Q_2 = \bar{q}_L^i \gamma_\mu q_R^i \bar{q}_R^j \gamma_\nu b_L^j$$ (A3)

$$Q_3 = \bar{q}_R^i \gamma_\mu q_L^i \bar{q}_R^j \gamma_\nu b_L^j$$ (A4)

$$Q_4 = \bar{q}_L^i \gamma_\mu q_R^i \bar{q}_L^j \gamma_\nu b_L^j$$ (A5)

$$Q_5 = \bar{q}_R^i \gamma_\mu q_L^i \bar{q}_L^j \gamma_\nu b_L^j$$ (A6)

We shall also need to define the hadronic matrix elements of the above operators such that,

$$\langle B_q|Q_i|\bar{B}_q\rangle = -\frac{1}{3} M_{B_q} f_{B_q}^2 B_1(\mu)$$ (A7)

$$\langle B_q|Q_2|\bar{B}_q\rangle = \frac{5}{24} R_{B_q} M_{B_q} f_{B_q}^2 B_2(\mu)$$ (A8)

$$\langle B_q|Q_3|\bar{B}_q\rangle = -\frac{1}{24} R_{B_q} M_{B_q} f_{B_q}^2 B_3(\mu)$$ (A9)

$$\langle B_q|Q_4|\bar{B}_q\rangle = -\frac{1}{4} R_{B_q} M_{B_q} f_{B_q}^2 B_4(\mu)$$ (A10)

$$\langle B_q|Q_5|\bar{B}_q\rangle = -\frac{1}{12} R_{B_q} M_{B_q} f_{B_q}^2 B_5(\mu)$$ (A11)

where we have defined $R_{B_q} = \left(\frac{M_{B_q}}{m_{(\mu)} + m_{(\mu)}}\right)^2$. The values of the bag parameters have been calculated on the lattice and can be found in [25].

The dominant SUSY contribution is from the gluino, which gives the following Wilson coefficients [27, 28, 29, 30, 31, 32].

$$C_1(\mu_S) = -\frac{a_2^2(\mu_S)}{216 m_{\tilde{q}}^2} \times \left[ 24 x f_6(x) + 66 f_6(x) \right] (\delta_{33}^d)^2_{LL}$$ (A12)

$$C_4(\mu_S) = -\frac{a_2^2(\mu_S)}{216 m_{\tilde{q}}^2} \times \left[ 504 x f_6(x) - 72 f_6(x) \right] (\delta_{33}^d)^2_{LL} (\delta_{33}^d)^2_{RR}$$ (A13)

$$C_5(\mu_S) = -\frac{a_2^2(\mu_S)}{216 m_{\tilde{q}}^2} \times \left[ 24 x f_6(x) + 120 f_6(x) \right] (\delta_{33}^d)^2_{LL} (\delta_{33}^d)^2_{RR}$$ (A14)

Here only the LL and RR terms have been kept as the contributions from RL and LR are suppressed. The above Wilson coefficients are evaluated at the scale $\mu_S = (\mu_g + \mu_q)/2 = (\sqrt{x} + 1)\mu_q/2$.

These Wilson coefficients need to be renormalization group (RG) evolved to the scale of the bottom quark. Using the magic numbers from [32], we have,

$$C_\alpha(\mu_b) = \sum_i \sum_\beta \left( b_i^{(\alpha,\beta)} + \eta_i^{(\alpha,\beta)} \right) \eta^\alpha \eta^\beta C_\beta(\mu_S)$$ (A15)

with $\eta = \alpha_s(\mu_S)/\alpha_s(\mu_b)$. Let us now write the $B_s - \bar{B}_s$ mixing parameter

$$\Delta M_q = 2|M_{q_{12}}^q| = 2|M_{q_{12}}^{SM} (1 + R_q)|$$

$$= |\Delta M_q^{SM}| |1 + R_q^g + \ldots|$$ (A16)

where $R_q \equiv r_q e^{i\phi_q} = M_{q_{12}}^{NP} / M_{q_{12}}^{SM}$ parameterizes the contribution from new physics. We consider the dominant effect to come from the gluino $R_q^g = M_{q_{12}}^{NP} / M_{q_{12}}^{SM}$. From the above effective Hamiltonian, Wilson coefficients, operator matrix elements and RG evolution we
can write,
\[ R_q^2 = a_i^2(m_{\tilde{g}}, x) \left[ (\delta_{3}^{d})_{RR} + (\delta_{3}^{d})_{LL} \right] + a_4^2(m_{\tilde{g}}, x) (\delta_{3}^{d})_{LL}(\delta_{3}^{d})_{RR} + \ldots \]  
(A17)

have we have only kept the LL, RR terms as they will dominate. The complete form of the functions \( a_1(m_{\tilde{g}}, x) \) and \( a_4(m_{\tilde{g}}, x) \) are as follows,

\[ M_{12}^{SM} a_1^2(m_{\tilde{g}}, x) = \frac{-a_2^2(M_S)}{216 m_\mu^2} \times \frac{1}{3} m_{B_s} f_{B_r}^2 B_1(\mu) \sum_i (b_i^{(4,4)} + \eta c_i^{(4,4)}) \eta_a^i \frac{1}{4} B_4(\mu) \]

\[ + A \sum_i (b_i^{(4,5)} + \eta c_i^{(4,5)}) \eta_a^i \frac{1}{4} B_4(\mu) \]

\[ + B \sum_i (b_i^{(5,4)} + \eta c_i^{(5,4)}) \eta_a^i \frac{1}{12} B_5(\mu) \]

\[ + A \sum_i (b_i^{(5,5)} + \eta c_i^{(5,5)}) \eta_a^i \frac{1}{12} B_5(\mu) \]

(A18)

Where we have defined, \( A = 24 x f_6(x) + 66 \tilde{f}_6(x) \) and \( B = 504 f_6(x) - 72 \tilde{f}_6(x) \) where,

\[ f_6(x) = \frac{6(1 + 3x) \ln x - 9x - 9x + 17}{6(x - 1)^5} \]

\[ \tilde{f}_6(x) = \frac{6x(1 + x) \ln x - 9x - 9x + 9x + 1}{3(x - 1)^5} \]

(A20)

(A21)

and \( x = m_{\tilde{g}}^2/m_{\mu}^2 \).

The dominant contributions to the above functions come from \( b_i^{(1,1)} = 0.865, b_i^{(4,4)} = 2.87, b_i^{(5,4)} = 0.961, b_i^{(5,5)} = 0.863 \), with \( a_i = (0.286, -0.692, 0.787, -1.143, 0.143) \).

So we can write the approximate functions as,

\[ M_{12}^{SM} a_1^2(m_{\tilde{g}}, x) \approx \frac{-a_2^2(M_S)}{216 m_\mu^2} \frac{1}{3} m_{B_s} f_{B_r}^2 B_1(\mu) \times A 0.865 \eta^{0.286} \]

\[ M_{12}^{SM} a_4^2(m_{\tilde{g}}, x) \approx \frac{-a_2^2(M_S)}{216 m_\mu^2} R_{B_s} m_{B_s} f_{B_s}^2 \times \left[ B 2.87 \eta^{-1.143} + A 0.961 \eta^{-1.143} \right] \frac{1}{4} B_4(\mu) \]

\[ + B 0.09 \eta^{-1.143} + A 0.863 \eta^{0.143} \]  

(A22)

(A23)

We can see that the values of the functions \( a_1, a_4 \) are not the same for the \( B_s \) and \( B_d \) systems. From eq. \( A18-A19 \) we can approximate the ratio of these functions as,

\[ \frac{|a_1|}{|a_4|} \approx \frac{|a_1^a|}{|a_4^a|} \sim \frac{\Delta M_{d}^{SM}}{\Delta M_{s}^{SM}} \sim \frac{V_s^2}{V_d^2} \sim \lambda^2 \]

(A24)

so that the value of the functions \( a_1 \) and \( a_4 \) are enhanced in the \( B_d \) relative to those derived in the \( B_s \) system. This enhancement will lead to a more constrained parameter space of the \( 13 \) mass insertion relative to the \( 23 \) sector. It is also important to notice that \( a_1 \) and \( a_4 \) are complex parameters. This is particularly important in the case of the \( B_d \) system where the phase of \( M_{12}^{SM}, \phi_d^{SM} = 2\beta \), is large.

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