Artificial satellites orbiting planetary satellites: critical inclination and sun-synchronous orbits

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Abstract. The behavior of critical inclinations and sun-synchronous orbits of artificial satellites orbiting planetary satellites are analyzed considering, simultaneously, the influence of the harmonics $J_2$ and $C_{22}$, both due to the non-uniform mass distribution of the natural satellite. In the present research, the central bodies of interest are the Moon and two of the Galilean moons: Io and Europa.

1. Introduction

Recent space missions show interest in the exploration of planetary satellites of our Solar System. It is interesting to note that the internal mass distribution for some of the lunar satellites differ from the Earth. This fact, in general, implies in a change of the hierarchical order of the harmonic coefficients, as is the case of the Moon (see table 1).

Table 1. Spherical harmonic coefficients.

| Body | $J_2$ ($10^{-6}$) | $C_{22}$ ($10^{-6}$) | $\sigma \equiv J_2/C_{22}$ | $J_3$ ($10^{-6}$) | $J_4$ ($10^{-6}$) | $J_5$ ($10^{-6}$) | $J_6$ ($10^{-6}$) |
|------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| Moon¹ | 203.237 | 22.35700 | 9.090531 | 8.47590 | -9.59193 | 0.715409 | -13.5777 |
| Io² | 1859.5 | 558.8 | 3.327666 | — | — | — | — |
| Europa² | 435.50 | 131.5 | 3.311787 | — | — | — | — |

Due to the characteristics of the gravity field of the Moon, the potential truncated up to the $J_9$ term produces different effects when compared to the frozen orbits which were obtained using the potential truncated up to $J_7$ [3, 4]. Furthermore, as an example of this effect, it can be noted in the time variation of the eccentricity that its amplitude increases as a consequence of considering higher order zonal harmonics as shown in Fig. 1. For a Moon’s orbiter in an altitude of about 110 km, the amplitude of the time variation of the eccentricity decreases when the first order perturbation due to the gravitational attraction of the Earth is taken into account as well.
Critical inclination is a special value of the inclination of the orbital plane that makes the argument of the periapsis stay fixed when some perturbations are considered. This condition can be helpful when applying impulses in orbital maneuvers. Sun-synchronous orbits are those where the artificial satellite has a constant perspective on the Sun and, consequently, the orbital plane makes a constant angle with the radius Sun-satellite. Such orbits are of practical interest because the ascending node will lie at a fixed local time [5].

Since these two kinds of orbits are important and desirable for determined space missions, in the present work, a study on the orbital behavior of lunar artificial satellites at low altitude is made considering the action of perturbative forces as a consequence of the non-uniform distribution of mass of the central body, particularly, the oblateness \( J_2 \) and the equatorial ellipticity \( C_{22} \).

2. The potential considered

According to [6], the disturbing potential thanks to the gravitational attraction of the primary body, taking into account secular terms up to \( J_2 \), long period terms up to \( C_{22} \) and eccentricity up to \( e^2 \), is expressed by

\[
R = \frac{1}{8} \frac{\mu}{a^3} \Delta a_e \left[ 6J_2 \cos(i) e^2 - 3J_2 e^2 - 2J_2 - 18C_{22} \cos(2\Omega) e^2 + 18C_{22} \cos(2\Omega) \cos(i)^2 e^2 - 12C_{22} \cos(2\Omega) \right] + 12C_{22} \cos(2\Omega) \cos(i)^2 + 9J_2 \cos(i)^2 e^2, \tag{1}
\]

where \( \mu \) is the gravitational parameter; \( a, e, i \) and \( \Omega \) are the keplerian orbital elements and \( a_e \) is the equatorial radius of the moon orbited.

3. Critical inclination

Critical inclination \( i \triangleq i_c \) is the value of the inclination that makes the time derivative of the argument of periapsis (\( \omega \)) to be zero. Thus, substituting Eq. (1) in the Lagrange Planetary Equations [7] and solving for \( d\omega/dt = 0 \), we obtain

\[
\cos(i_c)^2 = \frac{J_2 - 6C_{22} \cos(2\Omega)}{5J_2 - 10C_{22} \cos(2\Omega)}. \tag{2}
\]
If $C_{22}$ is neglected, the critical inclination for direct orbits is $i_c = 63.43^\circ$ and for retrograde (i.e., orbits with inclination between 90 and 180 degrees, so the spacecraft moves in the counterclockwise sense) ones, $i_c = 116.56^\circ$. Otherwise, the critical inclination is a function of the ascending node $\Omega$. If the coefficients $J_2$ and $C_{22}$ are close enough to each other the critical inclination does not exist for some values of $\Omega$. This fact is pointed by [8, 9].

For initial conditions of the ascending node in the interval $[0, 2\pi]$, Figs. 2 and 3 shows ranges of values where it is possible to find critical inclinations for spacecrafts orbiting the Moon, Io and Europa, for direct and retrograde orbits, respectively.

![Figure 2. Critical inclinations for direct orbits.](image1)

![Figure 3. Critical inclinations for retrograde orbits.](image2)

Observe that the curve of the critical inclination is the same for orbits around Io and Europa. In fact, for this model, this occurs for all bodies such that the ratio $\sigma = 3.3333\bar{3}$, i.e., bodies in hydrostatic equilibrium (a balance between the gravitational field and the pressure gradient). Therefore, differently of the Moon’s case, the critical inclination does not exist for all values of $\Omega$ of the domain for Io and Europa.

4. Sun-synchronous orbits

For a sun-synchronous orbit to develop, the orbital plane must rotate in inertial space with the angular velocity of the central body in its orbit around the Sun, in other words, the orbital plane must precess at this rate [5]. Thus, it follows that the precession rate is given by

$$\Delta \Omega \triangleq \frac{d\Omega}{dt} = \left[ \frac{T}{y} \right] 360^\circ / \text{lunar day},$$

(3)

where $T$ is the orbital period of the moon around its planet and $y$ is the orbital period of the moon around the Sun. Substituting Eq. (1) in $d\Omega/dt$ from the Lagrange Planetary Equations [5] and solving for $i = i_s$ (sun-synchronous inclination), considering terms up to the order of $e^2$, an expression to compute $i_s$ is found.
\[
\cos(i_s) = \frac{2a^2 \Delta \Omega}{3a^2 \varepsilon \left( 1 + 2\varepsilon^2 \right) \left[ J_2 + 2C_{22} \cos(2\Omega) \right]} 
\] (4)

For particular values of semi-major axis \(a\) and eccentricity \(e\), the combined effect of \(J_2\) and \(C_{22}\) on the sun-synchronous orbits around the Moon, Io and Europa are shown in Figs. 4-9.

![Figure 4](image1.png) ![Figure 5](image2.png)

**Figure 4.** Sun-sync. orbits around the Moon for \(e = 0.01\).

**Figure 5.** Sun-sync. orbits around the Moon for \(e = 0.05\).

![Figure 6](image3.png) ![Figure 7](image4.png)

**Figure 6.** Sun-sync. orbits around Io for \(e = 0.01\).

**Figure 7.** Sun-sync. orbits around Io for \(e = 0.05\).
Figure 8. Sun-sync. orbits around Europa for $e = 0.01$.

Figure 9. Sun-sync. orbits around Europa for $e = 0.05$.

It can be seen for Io and Europa that the sun-synchronous inclination exists for all the domain of the longitude of the ascending node ($\Omega$) at the low altitude orbits analyzed as shown in Figs. 6-9. For the Moon, see Figs. 4 and 5, the behavior of the sun-synchronous inclination changes with the chosen values of the semi-major axis ($a$) and the eccentricity ($e$).

5. Conclusion

The motion of artificial satellites at low orbits are strongly affected by the central body oblateness and equatorial ellipticity. It is well known that these natural effects can be used as an advantage in the moons exploration of our Solar System. In this way, it is important to study and find critical and sun-synchronous inclinations for cases where these effects can be used as an advantage.

In the analysis of the critical inclination, it is verified the existence of direct and retrograde orbits around the Moon, Io and Europa through the Figs. 2 and 3. It is known that the critical inclination expression, see Eq. (2), is a function of the ascending node and it depends on the mass and the constants $J_2$ and $C_{22}$, which dictate the behavior of the curve by the ratio $\sigma$ [8]. The larger the value of $\sigma$, the more well behaved is the curve. Otherwise, the range of ascending nodes values that does not exist critical inclination $i_c$ is larger.

From table 1 it is seen that the ratio of the Moon is $\sigma = 9.090531$ and there are critical inclinations for all values of $\Omega$ in the domain as shown in Figs. 2 and 3. By contrast, the values of $\sigma$ for Io and Europa are smaller, so the curves have discontinuities. When the central body has $\sigma = 3.33333$ or close enough, as is the case for Io and Europa (table 1), the curve of critical inclination is about the same. This fact can be verified in other moons with hydrostatic equilibrium in our Solar System [10].

For the case of the sun-synchronous orbits, the expression for $i_s$, besides depending on $J_2$, $C_{22}$ and $\Omega$, also depends on the semi-major axis $a$ and eccentricity $e$ of the orbit. Therefore, to find convenient sun-synchronous inclinations, beyond analysing the ratio $\sigma$, it is needed to choose proper initial conditions for $a$ and $e$ as well.

For the case of bodies with ratio $\sigma = 3.33333$ or close enough, the sun-synchronous inclinations exist for all the domain of the longitude of the ascending node at the low altitude orbits analyzed in this research, as can be seen in Figs. 6-9. This fact can be verified for other moons in hydrostatic equilibrium [10]. When $\sigma$ is not close to 3.33333, the behavior of the sun-synchronous inclinations change with the values of the semi-major axis and eccentricity. It may not exist for certain values of these parameters. In the Moon’s case this is clearly seen, as shown in Figs. 4 and 5.
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