Z_N Orientifolds with Flux

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Abstract

We compute the flux induced tadpole and superpotential in various type IIB Z_N compact orientifolds in order to study moduli stabilization. We find supersymmetric vacua with $g_s < 1$ and describe brane configurations with cancelled tadpoles. In some cases moduli are only partially fixed unless anti D3-branes are included.

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1 Preamble

A most attractive feature of compactifications with fluxes is the existence of a moduli dependent potential with minima at finite values. Flux induced moduli stabilization in type IIB orientifolds on $T^6$, $K3 \times T^2$, $T^6/Z_2 \times Z_2$, and Calabi-Yau threefolds, has been largely discussed in recent times [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. In this note we consider type IIB $\mathcal{N}=1$ orientifolds with internal space $T^6/Z_N$, studied only briefly up to now [6], which in fact provide a simpler setup in which moduli stabilization can be described in detail. Moreover, full models with tadpole cancellation can be constructed. We will focus on the behavior of complex structure moduli and the dilaton-axion $\tau = C_0 + i e^{-\phi}$. We have no new insights about fixing Kähler moduli so they will be dropped from the analysis.

In Type IIB compactifications it is natural to turn on fluxes of the NS-NS and R-R 3-forms. These fluxes must satisfy the quantization conditions

$$\frac{1}{(2\pi)^2\alpha'} \int_{\gamma} F_3 \in \mathbb{Z} \quad ; \quad \frac{1}{(2\pi)^2\alpha'} \int_{\gamma} H_3 \in \mathbb{Z} ,$$

for any 3-cycle $\gamma$ in the internal space $\mathcal{M}$. It is useful to introduce the combination $G_3 = F_3 - \tau H_3$. The equations of motion require that $G_3$ be imaginary self-dual (ISD), i.e. $^*G_3 = iG_3$ [14].

The fluxes induce a tadpole for the R-R 4-form with coefficient

$$N_{flux} = \frac{1}{(2\pi)^4\alpha'^2} \int_{\mathcal{M}} H_3 \wedge F_3 .$$

In $Z_N$ toroidal orientifolds with orientifold action including a reflection $I_6$ of all six internal coordinates there are 64 O3-planes, and typically D3-branes, that contribute to the tadpole. We consider only O3-planes with neither NS-NS nor R-R backgrounds, thus with charge $-1/2$ (in units such that a D3-brane has charge 1). This requires that in the flux quantization [11] all integers be even [2, 3, 6]. The $C_4$ tadpole cancellation condition is then

$$N_{D_3} + \tilde{N}_{flux} = 32 ,$$

where $N_{D_3}$ is the net number of D3-branes. Here $\tilde{N}_{flux}$ is computed on the torus, this is appropriate because the number of dynamical 3-branes is not $32/2N$. In general there
are twisted tadpoles whose cancellation requires a non-zero number of D3-branes pinned down at the origin, i.e. the point fixed by the full orientifold group. For \( N \) even there are further O7-planes and D7-branes and the \( C_8 \) tadpole must be cancelled.

The fluxes also generate a superpotential \[ W = \int_{\mathcal{M}} G_3 \wedge \Omega , \] where \( \Omega \) is the holomorphic \((3,0)\) form. The superpotential depends on the dilaton and the complex structure moduli denoted \( U_\alpha \). The ISD condition on \( G_3 \) is equivalent to demanding \[ D_\tau W \equiv \partial_\tau W + (\partial_\tau K) W = 0 ; \quad D_\alpha W \equiv \partial_\alpha W + (\partial_\alpha K) W = 0 , \] where

\[
K = - \log[-i(\tau - \bar{\tau})] - \log \left( -i \int_{\mathcal{M}} \Omega \wedge \bar{\Omega} \right)
\]

is the tree-level Kähler potential for \( \tau \) and \( U_\alpha \). We will not include Kähler moduli in the analysis. However, we implicitly assume that their Kähler potential is of no-scale form so that \( (5) \) corresponds to the minimum of the supergravity potential. Generically \( W \) does not vanish at the minimum, thus supersymmetry is broken by the F-term of the Kähler moduli. This soft supersymmetry breaking is measured by \( \exp \mathcal{G} \), where \( \mathcal{G} = K + \log |W|^2 \).

As usual we define complex coordinates \( z^a = x^a + ix^{a+3}, a = 1, 2, 3 \), on which the \( \mathbb{Z}_N \) action is \( \theta : z^a \to e^{2\pi i v_a} z^a \). To preserve supersymmetry we specifically choose \( \sum_a v_a = 0 \). The torus lattice is denoted \( \Lambda \), and its basis \( e_i, i = 1, \ldots, 6 \). Since \( \Lambda \) must be \( \mathbb{Z}_N \) symmetric, its deformation parameters are restricted, being actually in one-to-one correspondence with the untwisted Kähler and complex structure moduli \[ [16] [17] \]. In fact, in these orbifolds there is at most one untwisted \( U \) modulus, allowed when \( N \) is even and, say, \( v_3 = -1/2 \).

We use conventions in which

\[
\Omega = \lambda \, dz^1 \wedge dz^2 \wedge dz^3 .
\]

Since the \( x^i \) have flat metric \( \delta_{ij} \), \( \Omega \) is imaginary anti-self-dual, i.e. \( \ast \Omega = -i\Omega \). The normalization factor \( \lambda \) generically has units of \((\text{length})^{-3}\) and will be chosen so that \( K \) and \( W \) do not depend on the Kähler moduli. This is just a matter of convenience. The
physically relevant quantity $G$ is independent of $\lambda$, and the Kähler moduli. When $N$ is even, and $v_3 = -1/2$, there is also an untwisted invariant (2,1) form, namely

$$\xi = \lambda dz^1 \wedge dz^2 \wedge d\bar{z}^3.$$  \hspace{1cm} (8)

Notice that $\xi$ is ISD.

In general, $F_3$ and $H_3$ are linear combinations of all $b_3 = 2 + 2h_{12}$ 3-forms but in this note we only turn on fluxes along untwisted 3-forms such as $\Omega$, $\xi$, and their complex conjugates. Such fluxes are also primitive. The Kähler form, $J = \sum_a J_adz^a \wedge d\bar{z}^a$, satisfies $J \wedge \Omega = 0$ and $J \wedge \xi = 0$.

2 Examples

In this section we determine the flux induced quantities in various $T^6/Z_N$ orientifolds. We will discuss to what extent the fluxes fix the values of the moduli and comment on the effect in the open string spectrum.

We restrict to $Z_N$ orbifolds in which there is no contribution to $h_{21}$ from twisted sectors. In this situation, the symplectic basis of 3-cycles does not include fractional ones that might require flux integers to be multiple of some minimal quantum \[5\]. We only need demand even integers in order to avoid exotic O3-planes as mentioned before.

2.1 $T^6/Z_3$

The torus lattice has nine free parameters, all corresponding to untwisted Kähler moduli. To simplify we choose $\Lambda$ to be the product of three $SU(3)$ root lattices (up to size) orthogonal to each other. This means $e_i \cdot e_{i+1} = -R_i^2/2$, $i = 1, 3, 5$, and all other mixed $e_i \cdot e_j = 0$. A suitable normalization for $\Omega$ is $\lambda^{-1} = R_1R_3R_5$. Since $h_{21} = 0$, there are only two homology 3-cycles and two harmonic 3-forms, namely $\Omega$ and $\overline{\Omega}$. Hence we can write

$$F_3 = a_F\overline{\Omega} + \text{c.c.} \quad ; \quad H_3 = a_H\overline{\Omega} + \text{c.c.}.$$  \hspace{1cm} (9)

To determine the coefficients we must impose the quantization conditions [1] and to this end we need a basis of 3-cycles.
As in [18] we define the toroidal 3-cycles

$$\pi_{ijk} = e_i \otimes e_j \otimes e_k$$,

where the $e_i$ are the 1-cycles corresponding to the lattice basis. Invariant combinations are systematically found by taking $\mathbb{Z}_N$ orbits of the $\pi_{ijk}$ [18]. Given the $\mathbb{Z}_3$ action, $\theta e_i = e_{i+1}$, $\theta e_{i+1} = -e_i - e_{i+1}$, $i = 1, 3, 5$, one readily checks that there are just two invariant independent 3-cycles. A convenient basis is provided by the orbits of $\pi_{135}$ and $\pi_{136}$.

Explicitly,

$$\gamma_1 = -\pi_{136} - \pi_{145} - \pi_{146} - \pi_{235} - \pi_{236} - \pi_{245}$$

$$\gamma_2 = \pi_{136} + \pi_{145} + \pi_{235} + \pi_{135} - \pi_{246}$$.

Besides, $\gamma_1 \cap \gamma_2 = 1$ so that the basis is symplectic. The periods of $\Omega$, i.e. the integrals $Y_A = \int_{\gamma_A} \Omega$, are easily calculated to be $Y_1 = 1$ and $Y_2 = e^{2\pi i/3}$. Using (11) we then obtain

$$a_H = \frac{1}{2\sqrt{3}} [\sqrt{3} \ell_1 + i(2\ell_2 + \ell_1)] \quad ; \quad \ell_1, \ell_2 \in \mathbb{Z}$$.

For $a_F$, just replace $\ell$ by $k$. To avoid cluttering we omit factors of $(2\pi)^2 \alpha'$ that can be reinserted in the end. An equivalent way to proceed is to expand the fluxes in the dual cohomology basis $\Sigma_A$ such that $\int_{\Sigma_B} \Sigma_A = \delta^B_A$. Then, $F_3 = k^A \Sigma_A$ and $H_3 = \ell^A \Sigma_A$. From (12) one can easily deduce the $\Sigma_A$ in terms of $\Omega$ and $\overline{\Omega}$ and compute the intersection $\eta_{12} = -1$, where $\eta_{AB} = \int_M \Sigma_A \wedge \Sigma_B$.

For the flux induced tadpole we find

$$\tilde{N}_{\text{flux}} = 3(k_1\ell_2 - k_2\ell_1)$$.

Clearly, $\tilde{N}_{\text{flux}} = 3\ell^A \eta_{AB} k^B$. The factor of 3 takes into account that $\tilde{N}_{\text{flux}}$ is computed on the torus. The superpotential is $W = i\sqrt{3}(a_H \tau - a_F)$. Imposing the ISD condition as $D_7 W = 0$, or as cancellation of the coefficient of $\Omega$ in $G_3$, gives

$$\tau = \frac{2k_2 + k_1 + i\sqrt{3}k_1}{2\ell_2 + \ell_1 + i\sqrt{3}\ell_1}$$.

Notice that requiring $\text{Im}\tau > 0$ implies $\tilde{N}_{\text{flux}} > 0$. Recall that the flux integers are even, thus $\tilde{N}_{\text{flux}}$ is quantized in multiples of 12.
Another constraint on the fluxes is the validity of the perturbative expansion. Using (13) and (14) we can write the string coupling \( g_s = e^\phi \) as

\[
\frac{2\sqrt{3}(\ell_2^2 + \ell_1^2 + \ell_2 \ell_1)}{\tilde{N}_{flux}}.
\]

This immediately shows that there are no solutions with \( \tilde{N}_{flux} = 12 \) and \( g_s < 1 \). Furthermore, there is basically just one solution with \( \tilde{N}_{flux} = 24 \) and \( g_s < 1 \). For instance, using a vector notation, the fluxes \( \ell = (0, 2), k = (4, -2) \), fix the dilaton at \( \tau_0 = i\sqrt{3} \), and at the minimum \( |W_0| = 4\sqrt{3} \). All other choices are the same up to \( SL(2, \mathbb{Z}) \) transformations. With larger \( \tilde{N}_{flux} \) the number of solutions with \( g_s < 1 \) obviously increases. For instance, for \( \tilde{N}_{flux} = 48 \) there are vacua with \( g_s = \sqrt{3}/6 \) and \( g_s = \sqrt{3}/2 \). However, according to (3), anti D3-branes must be added to absorb the extra charge, thus breaking supersymmetry in an explicit way.

In the \( \mathbb{Z}_3 \) orientifold with \( \Omega' = \Omega(-1)^{F_L}I_6 \), twisted tadpole cancellation requires that at least 8 D3-branes be placed at the origin \( O \). This can be seen directly from the condition \( \text{Tr} \gamma_{\theta,3,O} = -4 \), where \( \gamma_{\theta,3,O} \) is the embedding of the orbifold action on the Chan-Paton labels. In turn this condition can be obtained applying T-duality to the regular \( \Omega \) orientifold with D9-branes in which \( \text{Tr} \gamma_{\theta,9} = -4 \) [19, 20]. The smallest such \( \gamma_\theta \), with \( \gamma_3^3 = 1 \), is of dimension 8.

An alternative view is the following. In the regular \( \Omega \) orientifold the D9 gauge group is \( SO(8) \times U(12) \), which can be broken by Wilson lines [20]. In particular, there is a family of discrete Wilson lines that gives \( SO(8-2n) \times U(12-2n) \times U(n)^3, n = 0, \ldots, 4 \). In the T-dual picture the first two factors arise from \( (32-6n) \) D3-branes staying at the origin while the last factor is due to \( 3n \) branes located at some other fixed point \( P \) of \( \theta \), for which \( \text{Tr} \gamma_{\theta,3,P} = 0 \) (the remaining \( 3n \) branes are at the image of \( P \) under \( I_6 \)). Since all branes are placed at \( \mathbb{Z}_3 \) fixed points the rank of the group is not reduced. Observe that we can remove at most 24 D3-branes from \( O \). These could be arranged as just described or they could be sent fully to the bulk as 4 dynamical 3-branes that could be traded by flux.

The upshot is that with \( \tilde{N}_{flux} = 24 \) all tadpoles can be cancelled by placing 8 D3-branes at the origin. They have gauge group \( U(4) \) and three chiral multiplets in the 6. This chiral content is clearly anomaly free.
With lower $\tilde{N}_{flux}$ more interesting brane configurations can be designed. A 3-family $SU(5)$ model with $\tilde{N}_{flux} = 12$ and 20 D3-branes is described in [6].

2.2 $T^6 / \mathbb{Z}_7$

The torus lattice has three Kähler moduli encoded in the $R_i$ entering in the lattice vectors [10]. As $\Omega$ normalization we take $\lambda = R_1 R_3 R_5$. In this case $h_2(\mathbb{C}P^1) = 0$ so the fluxes are of the form (9). Using $\theta e_i = e_{i+1}$, $i = 1, \ldots, 5$, $\theta e_6 = -e_1 - e_2 - e_3 - e_4 - e_5 - e_6$, shows that there are just two invariant independent 3-cycles. As basis we take $\gamma_1, \gamma_2$ equal to the orbits of $\pi_{123}$ and $\pi_{124}$ respectively. These fulfill $\gamma_1 \cap \gamma_2 = 1$. The periods of $\Omega$ are $Y_1 = -i\sqrt{7}$ and $Y_2 = (7 + i\sqrt{7})/2$. It then follows

$$a_H = \frac{1}{14}(2\ell_2 + \ell_1 - i\sqrt{7}\ell_1),$$
$$\tilde{N}_{flux} = 7(k_1\ell_2 - k_2\ell_1).$$ (16)

The superpotential is $W = i7\sqrt{7}(a_H \tau - a_F)$. The ISD condition leads to

$$\tau = \frac{2k_2 + k_1 + i\sqrt{7}k_1}{2\ell_2 + \ell_1 + i\sqrt{7}\ell_1}.$$ (17)

As before, $\text{Im} \tau > 0$ implies $\tilde{N}_{flux} > 0$. Without anti D3-branes the maximum $\tilde{N}_{flux}$ is 28. There is just one vacuum with $g_s < 1$, achieved choosing, say $\ell = (0, 2)$, $k = (2, 0)$. Then, $\tau_0 = (1 + i\sqrt{7})/2$, $|W_0| = 14$.

According to [10], to cancel the untwisted tadpole, 4 D3-branes are needed. Twisted tadpole cancellation requiring $\text{Tr} \gamma_{\theta,3,O} = 4$ is then satisfied placing the branes at the origin. They give rise to group $SO(4)$ without matter. In the regular orientifold, the D9 gauge group $SO(8) \times U(4)^3$ can be broken by a discrete Wilson line to $SO(4) \times U(2)^7$. In the T-dual picture the $SO(4)$ comes from the 4 branes at the origin and $U(2)^7$ from 14 branes located at another $\theta$ fixed point $P$, for which $\text{Tr} \gamma_{\theta,3,P} = 0$ (the remaining 14 branes being at the orientifold image). The 28 D3-branes liberated from the origin can move completely to the bulk and turn into flux.

The $SO(4)$ group of the D3-branes at the origin is pure $N=1$ super Yang-Mills. This raises the possibility of adding a non-perturbative superpotential $W_{np} \sim \exp(8\pi^2 i\tau/C(G))$ generated by gaugino condensation. However, for $SO(4)$ with Casimir $C(G) = 2$, the flux
superpotential linear in \(\tau\) dominates and the solution of \(D_{\tau}(W + W_{np}) = 0\) stays at \(\tau_0\). For gaugino condensation to produce a bigger effect one needs groups with a larger Casimir.

### 2.3 \(T^6/Z'_6\)

The \(Z'_6\) action has \(v_1 = \frac{1}{6}, v_2 = \frac{1}{3}, v_3 = -\frac{1}{2}\). It can be realized in four different lattices \[17\]. Here we work with \(\Lambda\) of \(SU(6) \times SU(2)\). All possible lattices allow five real deformation parameters, two of them corresponding to one untwisted complex structure modulus. The number of twisted moduli depends however on the lattice. In our case there are no twisted 3-forms, i.e. \(h_{12} = 1\) \[17\].

The lattice vectors satisfy 
\[
\theta e_i = e_{i+1}, \quad i = 1, \ldots, 4, \quad \theta e_5 = -e_1 - e_2 - e_3 - e_4 - e_5 .
\]

Following \[16\] we write them as
\[
e_1 = (R_1, R_3, R_5, 0, 0, 0) ; \quad e_4 = (-R_1, R_3, -R_5, 0, 0, 0) \\
e_2 = \frac{1}{2}(R_1, -R_3, -2R_5, \sqrt{3}R_1, \sqrt{3}R_3, 0) ; \quad e_5 = \frac{1}{2}(-R_1, -R_3, 2R_5, -\sqrt{3}R_1, \sqrt{3}R_3, 0) \\
e_3 = \frac{1}{2}(-R_1, -R_3, 2R_5, \sqrt{3}R_1, -\sqrt{3}R_3, 0) ; \quad e_6 = R_6(0, 0, \cos \beta, 0, 0, \sin \beta)
\]

The untwisted complex modulus is simply
\[
U = \frac{R_6}{\sqrt{3}R_5}e^{i\beta} . \tag{18}
\]

Again, \(\lambda^{-1} = R_1 R_3 R_5\).

The most general 3-form background is of type
\[
F_3 = a_F \Omega + b_F \xi + c.c. ; \quad H_3 = a_H \Omega + b_H \xi + c.c. . \tag{19}
\]

It is easy to show that there are four toroidal invariant 3-cycles. As basis we take \(\gamma_A, A = 1, \ldots, 4\), to be the orbits of \(\pi_{123}, \pi_{124}, \pi_{126}\) and \(\pi_{136}\) respectively. The intersection matrix is unimodular. The periods of \(\Omega\) are \(Y_1 = -i\sqrt{3}\), \(Y_2 = 3\), \(Y_3 = -\sqrt{3}U\) and \(Y_4 = -3iU\). To obtain the periods of \(\xi\) just replace \(U\) by \(U\). Imposing flux quantization leads to
\[
a_H = \frac{1}{12\text{Im}U} \left[ \ell_4 - i\sqrt{3}\ell_3 - U(\sqrt{3}\ell_1 + i\ell_2) \right] \\
b_H = \frac{1}{12\text{Im}U} \left[ -\ell_4 - i\sqrt{3}\ell_3 + U(\sqrt{3}\ell_1 - i\ell_2) \right] \tag{20}
\]

The superpotential is \(W = 12i\sqrt{3}\text{Im}U(a_H \tau - a_F)\). The tadpole coefficient is
\[
\tilde{N}_{\text{flux}} = 6(k_1\ell_4 - k_2\ell_3 + k_3\ell_2 - k_4\ell_1) . \tag{21}
\]
Indeed, the dual 3-forms have intersections $\eta_{14} = -1$, $\eta_{23} = 1$ and $\eta_{AB} = 0$ otherwise. Note that $\tilde{N}_{\text{flux}}$ is quantized in multiples of 24.

The ISD condition reduces to

$$\sqrt{3}k_3 + k_2 U - \sqrt{3}l_3 \tau - \ell_2 \tau U = 0$$

$$k_4 - \sqrt{3}k_1 U - \ell_4 \tau + \sqrt{3}l_1 \tau U = 0$$

We assume that $\text{Im}\tau \neq 0$ and $\text{Im}U \neq 0$, i.e. we exclude infinite string coupling and degenerate torus. Then, equations (22) can generically be solved to obtain the ratio $\text{Im}\tau/\text{Im}U$, as well as the values of $\text{Re}\tau$ and $\text{Re}U$. There is a remaining equation quadratic in $\text{Im}U$ that, depending on the fluxes, might not have real solutions. In fact, for $\tilde{N}_{\text{flux}} = 24$ we find no solutions with $\tau$ and $U$ completely fixed. The best alternative is to obtain a relation between them. Clearly, this can be easily achieved with fluxes such that one of the equations in (22) is trivial. For example, with $k = (2, 0, 0, 0), \ell = (0, 0, 0, 2), \tau = -\sqrt{3}U$. Or with $k = (0, 0, 2, 0), \ell = (0, 2, 0, 0), \tau U = \sqrt{3}$.

With $\tilde{N}_{\text{flux}} = 24$, eq. (3) is satisfied by taking $N_{D_3} = 8$. In fact, one needs 8 D3-branes at the origin to cancel twisted tadpoles. This cancellation condition, as well as the open string spectrum, can be deduced adapting the results in the $\Omega$ orientifold with D9 and D5-branes [20]. The D3-branes at the origin have gauge group $U(4)$ with two hypermultiplets in the $6$. There are 32 D7-branes that can be located at the origin thus giving group $U(4) \times U(4) \times U(8)$ with charged hypermultiplets as in the 55 sector [20]. The 37 matter consists of one hypermultiplet in $(4, 4, 1, 1) + (4, 1, 4, 1)$. The full matter content is anomaly free.

With larger $\tilde{N}_{\text{flux}}$ one does find vacua with $\tau$ and $U$ fixed. For example, for $\tilde{N}_{\text{flux}} = 48$ there is the family $k = (2, 0, 2, 2n), \ell = (0, 2, 0, 2), |n| \leq 3, \text{with } \tau = (n + i\sqrt{12 - n^2})/2, U = \tau/\sqrt{3}$. Notice that $g_s < 1$ for $|n| \leq 2$. For $|n| = 3, g_s > 1$, and $\tau$ is fixed under S-duality. For $|n| \geq 4$ these fluxes could only give a solution with $\text{Im}\tau = \text{Im}U = 0$.

3 Final Comments

The purpose of this note was to work out further examples of $D=4$, $\mathcal{N}=1$, toroidal orientifolds with fluxes. As we have seen, a rather explicit analysis of moduli stabilization
can be carried out. Moreover, D3/D7-brane configurations with cancelled tadpoles, and the corresponding chiral spectrum, can be found. These orientifolds can serve as starting points to build semi-realistic models and study other flux effects such as soft supersymmetry breaking \[21, 22, 7, 23, 24, 25\].

In toroidal orientifolds the fluxes are limited to small values unless anti D3-branes are included to absorb the extra charge. These anti-branes break supersymmetry explicitly, in particular the spectrum is no longer supersymmetric \[26\]. On the other hand, D3-branes could have useful applications as in the proposal to produce de Sitter vacua \[27\]. Note however that in our case the number of D3-branes needed to increase the fluxes appreciably is large, so they are not a small perturbation above a supersymmetric vacuum. Moreover, there are flux-brane annihilation processes that could not be sufficiently suppressed \[28\].

Another possibility to increase the allowed values of \(N_{\text{flux}}\) would be to add magnetized D9-branes that can balance the excess charge without breaking supersymmetry as observed recently \[29\]. Indeed, magnetized D9-branes have been used to construct semi-realistic models in the \(T^6/\mathbb{Z}_2 \times \mathbb{Z}_2\) orientifold with fluxes \[3, 15, 7, 29, 30, 25\].

The fluxed orientifolds in this note furnish neat examples in which a counting of vacua could be performed thoroughly. We have concentrated on small fluxes and \(g_s < 1\), and found that in this situation there are only very few vacua. In the \(\mathbb{Z}_3\) and \(\mathbb{Z}_7\) there are no complex structure moduli and the values of \(\tau\) are indeed determined as in the toy model of a rigid Calabi-Yau discussed in \[8, 10\]. The \(\mathbb{Z}_6'\) is the analog of a Calabi-Yau orientifold with one modulus. For large fluxes the number of solutions of \((22)\) is expected to agree with general predictions \[8\]. In the regime of small fluxes one can just analyze \((22)\) directly.

Finally, for the \(\mathbb{Z}_3\) and \(\mathbb{Z}_7\) type IIB orientifolds a heterotic dual without fluxes is known \[19, 31\]. It would be interesting to track down the effect of fluxes in the heterotic side.

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