Orthogonal Polynomials with Recursion Coefficients of Generalized Bounded Variation

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Abstract. We consider probability measures on the real line or unit circle with Jacobi or Verblunsky coefficients satisfying an $\ell^p$ condition and a generalized bounded variation condition. This latter condition requires that a sequence can be expressed as a sum of sequences $\beta^{(l)}$, each of which has rotated bounded variation, i.e.,

$$\sum_{n=0}^{\infty} |e^{i\phi_l} \beta^{(l)}_{n+1} - \beta^{(l)}_n| < \infty$$

for some $\phi_l$ (note that for $\phi_l = 0$ this becomes the usual bounded variation). For the real line, we impose this condition on sequences $\{a_n - 1\}$ and $\{b_n\}$, where $b_n$ are the diagonal and $a_n$ the off-diagonal Jacobi coefficients, and for the unit circle, we impose it on Verblunsky coefficients.

For the real line, our results state that in the Lebesgue decomposition $d\mu = f dm + d\mu_s$ of such measures, supp$(d\mu_s) \cap (-2, 2)$ is contained in a finite set $S$ (thus, there is no singular continuous part), and $f$ is continuous and non-vanishing on $(-2, 2) \setminus S$. By a theorem of Levin–Lubinsky, this also implies uniform clock behavior of zeros on closed intervals in $(-2, 2) \setminus S$. The results for the unit circle are analogous, with $(-2, 2)$ replaced by the unit circle.