Partial Spontaneous Breaking of Global Supersymmetry

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We review in detail the recently discovered phenomenon of partial spontaneous breaking of supersymmetry in the case of a $N = 2$ pure gauge $U(1)$ theory, and recall how the standard lore no-go theorem is evaded. We discuss the extension of this mechanism to theories with charged matter, and surprisingly find that the gauging forbids the existence of a magnetic Fayet–Iliopoulos term.

1. Introduction

During the last two years, vast progress has been made in our understanding of the non-perturbative dynamics of supersymmetric quantum field theory and string theory. $N = 2$ supersymmetric theories present a manifold of vacua (so called moduli space) corresponding to the arbitrary values of scalar fields in the flat directions of the potential; while the physics is determined at a given point on that manifold, it admits different (dual) descriptions in each overlapping chart, and the consistency of the chart changing operations, together with holomorphy properties often allows for the exact determination of the physics,

\[ H = \frac{1}{2} \left| \langle Q_A | 0 \rangle \right|^2 + \frac{1}{2} |Q_A |^2 . \]

(2)

Now, if it exists $A_0$ such that the vacuum is not annihilated by the corresponding supersymmetry generator, then the r.h.s. of (2) is strictly positive and so is the Hamiltonian $H$. Then, for any other supersymmetry generator $Q^A$, the r.h.s. of (2) is also strictly positive and $Q^A$ is spontaneously broken to $N = 1$ or $N = 0$, in principle still allowing control of the dynamics of the latter. Even after partial spontaneous breaking of local extended supersymmetry had been demonstrated, it was so far unanimously believed not to occur in globally supersymmetric, based on common sense no-go theorems (recalled in section 2). Common sense nevertheless had to retract when an explicit example of such a phenomenon was found, namely in the simplest case of a globally $N = 2$ SUSY $U(1)$ pure gauge theory. This model is reminiscent of the celebrated one considered by Seiberg and Witten as an effective theory of a pure $N = 2$ $SU(2)$ super-Yang-Mills multiplet Higgsed to $U(1)$, but it uses extra deformations compatible with $N = 2$ SUSY, but not dynamically generated in the case of Seiberg–Witten. These Fayet–Iliopoulos terms, introduced in section 3, lift the flat directions and leave a $N = 1$ supersymmetric vacuum, as reviewed in section 4. It is then legitimate to ask how general is this phenomenon. Another surprise comes in section 5, where we try to extend it to the case of a $U(1)$ theory with charged matter: there appears to be no such deformation compatible with $N = 2$ SUSY anymore.

2. Partial Breaking no-go Theorem

The following arguments are general but we use here four dimensional notations. $N$-extended supersymmetry reads, for all $A$ and $B \in \{1, ..., N\}$

\[ \{Q_A, \tilde{Q}_B \} = H \delta^{AB} \delta_{\dot{a}\dot{a}}, \]

(1)

and therefore, for any fixed $A \in \{1, ..., N\}$,

\[ 2H = \| Q_A | 0 \rangle \|^2 + \| Q_A^A | 0 \rangle \|^2 . \]

(2)

Now, if it exists $A_0$ such that the vacuum is not annihilated by the corresponding supersymmetry generator, then the r.h.s. of (2) is strictly positive and so is the Hamiltonian $H$. Then, for any other supersymmetry generator $Q^A$, the r.h.s. of (2) is also strictly positive and $Q^A$ is spontaneously broken to $N = 1$ or $N = 0$, in principle still allowing control of the dynamics of the latter. Even after partial spontaneous breaking of local extended supersymmetry had been demonstrated, it was so far unanimously believed not to occur in globally supersymmetric, based on common sense no-go theorems (recalled in section 2). Common sense nevertheless had to retract when an explicit example of such a phenomenon was found, namely in the simplest case of a globally $N = 2$ SUSY $U(1)$ pure gauge theory. This model is reminiscent of the celebrated one considered by Seiberg and Witten as an effective theory of a pure $N = 2$ $SU(2)$ super-Yang-Mills multiplet Higgsed to $U(1)$, but it uses extra deformations compatible with $N = 2$ SUSY, but not dynamically generated in the case of Seiberg–Witten. These Fayet–Iliopoulos terms, introduced in section 3, lift the flat directions and leave a $N = 1$ supersymmetric vacuum, as reviewed in section 4. It is then legitimate to ask how general is this phenomenon. Another surprise comes in section 5, where we try to extend it to the case of a $U(1)$ theory with charged matter: there appears to be no such deformation compatible with $N = 2$ SUSY anymore.
broken as well. So, the no-go theorem states that in a N-extended global supersymmetric theory, either all or no supersymmetry is spontaneously broken.

This however assumes that the Hilbert space has a definite positive metric, which fails in the case of local supersymmetry, and also that the SUSY charges $Q^A$ can be defined at all. It is a weaker assumption that a SUSY current algebra exists. As was shown in [3], the familiar form of the current algebra can be extended with a central extension, and thus the Jacobi identity is still verified. For the same reason, the presence of a SUSY representation in terms of fields.

Also, the variation of the superfield $\tilde{Y}$ gives an infinite contribution on the other side. Therefore, whenever $C^{AB} \neq 0$, one cannot write eq. (3) and the no-go theorem breaks down.

3. N=2, U(1) gauge theory in D=4

Here we describe the most general N=2 SUSY action for a $U(1)$ vector multiplet, using first the formalism of $N=2$ superspace. We start by considering a $N=2$ chiral superfield $A$ \[ \mathbb{R} \]. This multiplet is a reducible representation of $N=2$, and one can reduce it to obtain a vector multiplet by imposing the constraint:

$$ (\epsilon_{ij} D^i \sigma_{\mu\nu} D^j)^2 A = -96 \Box A, $$

where the $D^i$ are the $N=2$ supercovariant derivatives. The component fields are then a complex scalar $a$, a $SU(2)_R$ doublet of chiral fermions $\lambda^a$, a triplet of auxiliary fields which verifies $\Box \tilde{Y} = \Box \tilde{Y}^*$, and a self-dual field strength now verifying the Bianchi identity, so that it can be traded for a gauge boson $A_\mu$.

Now, one may consider a general prepotential $F$ which is a holomorphic function, and write a $N=2$ Lagrangian

$$ \mathcal{L} = -\frac{\text{Im}}{8\pi} \int d^2 \theta_1 d^2 \theta_2 F(A) - i \tilde{e} \cdot \tilde{Y} $$

where $\tilde{e} \cdot$ is a real constant vector of Fayet-Iliopoulos terms \[ \mathbb{R} \], $\tilde{Y}$ being real. Instead, for reasons which will become clear in a moment, we prefer to start with a non-reduced $N=2$ chiral superfield $A$ and the following Lagrangian:

$$ \mathcal{L}_0 = -\frac{\text{Im}}{8\pi} \int d^2 \theta_1 d^2 \theta_2 [F(A) - A_D A] $$

where $A_D$ is a reduced superfield, i.e. a vector multiplet. The elimination of this superfield by use of the equations of motion of its component fields imposes all the reduction relations on $A$ which then becomes a vector multiplet as well. Also, the variation of the superfield $A$ gives

$$ [F'(A) - A_D] \delta A = 0, $$

which means that $A_D$ is the magnetic dual of the original electric vector multiplet $A$ [4].

Now, the supersymmetric variation of the auxiliary fields $\tilde{Y}_D$ of $A_D$ are total derivatives. In addition, the variations of $\tilde{Y}$ become total derivatives when the reduction conditions are imposed, which is the case when $A_D$ is eliminated.

Therefore, we can consider the previous Lagrangian to which we add linear terms in $\tilde{Y}$ and $\tilde{Y}_D$:

$$ \mathcal{L}_{\text{tot}} = \mathcal{L}_0 + \mathcal{L}_{\mathcal{F}.I.}, \quad \text{where} \quad \mathcal{L}_{\mathcal{F}.I.} = \frac{\text{Im}}{8\pi} \left[ i(\tilde{e} + i \tilde{p}) \cdot \tilde{Y} + i \tilde{m} \cdot \tilde{Y}_D \right] $$

and $\tilde{e}$, $\tilde{p}$, $\tilde{m}$ are real constants. Rewriting $\mathcal{L}_{\text{tot}}$ in component fields and eliminating the dual ones, one can easily see that

$$ \mathcal{L}_{\text{tot}} = -\frac{\text{Im}}{8\pi} \int d^2 \theta_1 d^2 \theta_2 F(A_\tilde{m}) - i \tilde{e} \cdot \tilde{Y}_\tilde{m} $$

$$ - \frac{\tilde{p} \cdot \tilde{m}}{8\pi}, $$

where $A_\tilde{m}$ is the original field $A$ where the reducing condition is imposed and in addition $\tilde{Y}$ is replaced by $\tilde{Y} + i \tilde{m}$. The point is that the addition of such imaginary constants to the real auxiliary fields $\tilde{Y}$ is still compatible with the reduction condition $\Box \tilde{Y} = \Box \tilde{Y}^*$. Since the addition of a real constant to $\tilde{Y}$ is the result of the
well known Fayet-Iliopoulos terms $\vec{e}$ for the electric $U(1)$ vector multiplet $A$, $\vec{m}$ well deserves the name of magnetic Fayet-Iliopoulos term. Notice that the parameters $\vec{p}$ only appear as a constant term in eq. (8). As a result, we can set it to zero without loss of generality.

In order to make contact with the more familiar $N = 1$ superspace formalism, we have to choose a particular direction in the $SU(2)_R$ automorphism group of $N = 2$. A suitable choice puts $\vec{e}$ and $\vec{m}$ in the form

$$\vec{e} = (0, e, \xi), \quad \vec{m} = (0, m, 0). \quad (10)$$

Then, we can rewrite

$$\mathcal{L}_{tot} = \frac{\text{Im}}{4\pi} [\mathcal{L}_0^1 + \mathcal{L}_{F.I.}], \quad \text{where}$$

$$\mathcal{L}_0^1 = \int d^2\theta d^2\bar{\theta} F' (A) \bar{A} + \int d^2\theta \frac{1}{2} \tau (A) W^\alpha W_\alpha,$$

$$\mathcal{L}_{F.I.} = i \int d^2\theta \left [ e A + m F' (A) \right ] - i\sqrt{2} \int d^2\theta d^2\bar{\theta} \bar{\epsilon} V$$

and $V$ is a $N = 1$ vector superfield whereas $A$ is a $N = 1$ chiral superfield those degrees of freedom are $A_\mu, \lambda_1 \equiv \lambda$ and $a, \lambda_2 \equiv \chi$, respectively. Here, we use the notations and conventions of [8] and [3] where $\tau = F''$. In that case, the fermionic mass matrix and scalar potential take the form :

$$\mathcal{M} = \frac{\tau'}{\sqrt{2} \pi \tau_2} \left [ \begin{array}{cc} -\chi & -\xi \\ -i (e + m \tau) & \xi \end{array} \right ] \left [ \begin{array}{c} \lambda \\ \chi \end{array} \right ]$$

$$\mathcal{V} = \frac{|e + m \tau|^2 + \xi^2}{16 \pi \tau_2}$$

where $\tau = \tau_1 + i \tau_2$ is now a function of $a$, and we have $\langle \tau \rangle = \frac{\theta}{\pi} + i \frac{4\tau}{\pi}$ in terms of the $\theta$-angle and coupling constant $g$.

4. Partial Breaking of Supersymmetry

We now turn to the minimization of $\mathcal{V}$. It has a stationary point at $\langle \tau'(a) \rangle = 0$, but that corresponds to a saddle point and therefore does not give rise to a vacuum. The central charge matrix of eq. (9) was computed in [8]:

$$C^{AB} = 4 \vec{\sigma}^{AB} \cdot (\vec{e} \times \vec{m}) = -4 \xi m \left [ \begin{array}{c} 0 \\ 1 \\ 0 \end{array} \right ] \quad (12)$$

We distinguish between the following cases:

- $\xi m \neq 0$ : the supersymmetric current algebra is modified and therefore a partial breaking can occur in this case. Minimizing $\mathcal{V}$ with respect to $\langle \tau_1 \rangle$ and $\langle \tau_2 \rangle$ gives a vacuum at

$$\langle \tau_1 \rangle = - \frac{e}{m}, \quad \langle \tau_2 \rangle = \left | \frac{\xi}{m} \right | \quad (13)$$

from which a vacuum expectation $\langle a \rangle$ can be found. At this point of the moduli space, one finds the following spectrum

$$\left ( a, \frac{\lambda - s \chi}{\sqrt{2}} \right ) \quad \text{of mass } \frac{sm^2}{2\xi} \langle \tau \rangle,$$

$$\left ( A_\mu, \frac{\lambda + s \chi}{\sqrt{2}} \right ) \quad \text{massless, with } s = \text{sign}(\xi m). \quad (15)$$

which suggests that we end up with an $N = 1$ vacuum. This can be checked on the SUSY transformation of the Goldstino ($\lambda + s \chi)/\sqrt{2}$:

$$\delta \left ( \frac{\lambda + s \chi}{\sqrt{2}} \right ) = im (se^1 - e^2) \quad \delta \left ( \frac{\lambda - s \chi}{\sqrt{2}} \right ) = 0,$$

where $e^{1,2}$ are the two infinitesimal supersymmetry parameters. Therefore, partial $N = 2 \to N = 1$ supersymmetry breaking does occur and we get here a massless $U(1)$ vector multiplet plus a massive chiral multiplet. Notice that, at any other point of the moduli space, the two supersymmetries are non-linearly realized and that only at the minimum of $\mathcal{V}$, we get a combination $sQ^1 - Q^2$ of the initial generators which is restored. Also, $\mathcal{V} = |\xi m|/8\pi > 0$ at the minimum but one could have chosen the parameters $\vec{p}$ in order to shift the scalar potential to a zero vacuum energy (see eq. (8)).

- $m \neq 0, \xi = 0$ : in this case, we deal with the usual supersymmetric algebra. The previous minimum is now at

$$\langle \tau_1 \rangle = - \frac{e}{m}, \quad \langle \tau_2 \rangle = 0 \quad (16)$$

Since $4\pi \mathcal{L}_{tot} = - \langle \tau_2 \rangle \partial^\mu a \partial_\mu a + \cdots$, we are sent to a point $\langle a \rangle$ where the Kähler metric $K_{a\bar{a}} \equiv \langle \tau_2 \rangle$ is singular. Several interpretations are then possible, depending on the underlying theory.

Firstly, if the theory is not asymptotically free, $g(\langle a \rangle) \to \infty$ and $\langle \tau_2 \rangle \to 0$ when $\langle a \rangle \to \infty$. In
that case, the theory has a run away behavior. However, such an eventuality should not occur in a consistent underlying theory.

Secondly, the singularity could be due to the appearance of new massless states at $\langle \tilde{a} \rangle$. For a non-Abelian underlying theory, these states cannot be vector multiplets because an enhanced gauge symmetry would be incompatible with the presence of Fayet–Iliopoulos terms. However, in that case, dyonic states can be present in the theory and therefore we suppose that one of them becomes massless at this point. Let $(m_0, e_0)$ denote its magnetic and electric charges. In order to include it explicitly in our effective theory, we have to perform an electric–magnetic duality transformation which replaces the original photon $A_\mu$ by a “dyonic” one which couples locally to the dyon. Let us define

$$[\tilde{F}^a(\tilde{a})] := \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} [F^a(a)],$$

(17)

where the matrix is of determinant one and chosen so that the BPS mass formula for the dyon takes the form [1]

$$m = \sqrt{2} \left( m_0 \delta - e_0 \gamma = 0, \tilde{q} \right) \left[ \tilde{F}^a(\tilde{a}) \right],$$

(18)

where $\tilde{q} = -m_0 \delta + e_0 \alpha$. Now, since

$$\tilde{\tau} = \frac{\alpha \tau + \beta}{\gamma \tau + \delta} \sim -i \frac{1}{\alpha} \ln \tilde{a} \to \infty$$

(19)

as $\tilde{a} \to 0$ i.e. $\tau \to -\frac{m_0}{e_0}$, from the vanishing of the denominator of $\tilde{\tau}$, one deduces that $(m, e)$ is orthogonal to $(\delta, -\gamma)$. Moreover, from eq. (18), we have $(\delta, -\gamma)$ orthogonal to $(m_0, e_0)$ and therefore it exists a constant $c$ such that

$$(m, e) = c(m_0, e_0).$$

(20)

The charges of the dyon are now $(0, \tilde{q})$ with respect to $\tilde{A}_\mu$. Performing the symplectic transformation on the Lagrangian $\mathcal{L}_{tot}$ in eq. (11) and using eq. (21), one finds

$$S_{tot} = \frac{\text{Im}}{4\pi} \int d^4x \left[ \tilde{L}_0^1 + \tilde{L}_{F.I.} \right],$$

(21)

$$\tilde{L}_0^1 = \int d^2q d^2\bar{q} \tilde{F}^a(\tilde{\tau}) \tilde{A} + \int d^2\theta d^2\bar{\theta} \tilde{\sigma} \tilde{\omega},$$

(22)

to which one must add the coupling of $\tilde{A}_\mu$ to the dyon $(\tilde{\Phi}_+, \tilde{\Phi}_-)$ where $\tilde{\Phi}_\pm$ are two $N = 1$ chiral superfields. For simplicity, we choose the hyperkähler manifold of the hypermultiplet to be flat:

$$S_{dyon} = \frac{1}{4\pi} \int d^4x d^2\theta d^2\bar{\theta} \sum_{\sigma = \pm} \tilde{\Phi}_\sigma e^{\sigma \tilde{q} \tilde{\bar{q}}} \tilde{\Phi}_\sigma$$

$$+ \frac{\text{Im}}{4\pi} \left[ \sqrt{2} \tilde{q} \int d^4x d^2\theta A \tilde{\Phi}_+ \tilde{\Phi}_- \right].$$

(22)

Minimizing the new scalar potential, one finds that at the singular vacuum $\langle \tilde{a} \rangle = 0$, the $\tilde{\Phi}_\pm$ scalar VEV’s $\langle \tilde{\phi}_\pm \rangle$ are such that

$$|\langle \tilde{\phi}_+ \rangle| = |\langle \tilde{\phi}_- \rangle|, \quad \langle \tilde{\phi}_+ \rangle/\langle \tilde{\phi}_- \rangle = \frac{i e}{\sqrt{2}}.$$  

(23)

We have a condensation of dyons which breaks the $U(1)$ gauge symmetry in a zero energy vacuum. Since we deal with the usual supersymmetry algebra, this is enough to show that $N = 2$ is unbroken. This phenomenon is similar to the one found in [1] where such a condensation happened in the case of an explicit breaking of $N = 2$ to $N = 1$. The spontaneous mechanism we have here is also similar to the effect induced by a generic superpotential near the conifold singularity of type II superstrings compactified on a Calabi-Yau threefold [10]. In that case, the massless hypermultiplets are black holes which condense at the conifold points, which leads also to new $N = 2$ vacua. One way to generate such a superpotential is to give a VEV to the 10-form $F.I.$, which should correspond in 4 dimensions to a magnetic Fayet-Iliopoulos term.

- $m = 0, \xi \neq 0$: this case is similar to the previous one except that the metric is now $\langle \tau_2 \rangle = +\infty$ at the minimum. A similar analysis shows that this can happen either when the theory is asymptotically free, which gives rise to a run away behavior, or if electric hypermultiplets become massless at this point.

- $m = \xi = 0, e \neq 0$: in that case, $\mathcal{V} = e^2/16\pi \tau_2 \to 0$ when $\tau_2 \to +\infty$ and the conclusion of the previous case are still valid here.

- $m = \xi = e = 0$: $N = 2$ and the $U(1)$ gauge symmetry are not broken.
5. Gauge theory with matter

Having discussed partial spontaneous SUSY breaking in a pure gauge theory, we shall now consider extending it to a theory with charged matter, the simplest case being one hypermultiplet coupled to a $U(1)$ vector multiplet. Surprisingly, we shall find that the theory does not admit any magnetic Fayet–Iliopoulos term anymore (while it still keeps the electric ones). There are already a few hints that could prevent us from boldly extrapolating from the pure gauge theory. First, Fayet–Iliopoulos terms are intimately related with expectation values of auxiliary fields, whereas the hypermultiplet does not admit a (finite) off-shell formulation. Second, existence of charged matter breaks electric–magnetic duality, so we cannot generate a magnetic Fayet–Iliopoulos term from an electric one by a symplectic transformation. Of course, we will not be able to rigorously prove the inexistence of such a deformation, but we shall at least show why a straightforward generalization fails.

Let us therefore consider the simplest case of hypermultiplet $\Phi_{\pm} \equiv (\phi_{\pm}, \psi_{\pm})$, described by a flat hyperkähler manifold $\mathbb{R}^4$, and couple it with charge $q$ to a vector multiplet $a, A_\mu, \lambda, \chi$ with arbitrary prepotential and Fayet–Iliopoulos terms. The Lagrangian takes the form:

$$L_{v+h} = L_{tot} + L_{hyp} + \frac{1}{2\pi} \int d^2 \theta iM \Phi^\dagger \Phi$$

where $M$ is a complex mass term, $L_{tot}$ is defined in eq. (11), and $L_{hyp}$ is formally given by the Lagrangian of the action in eq. (22) with the tildes removed.

By construction, we obtain a $N = 1$, presumably $N = 2$, supersymmetric Lagrangian, from which we can work out the scalar potential and the fermion mass matrix by eliminating the auxiliary fields. Aiming at proving its $N = 2$ invariance, we display the result in a $SU(2)_R$ invariant way, where $SU(2)_R$ is the automorphism group of $N = 2, D = 4$ supersymmetry algebra:

$$V = V_C + V_{NC}, \text{ with }$$

$$V_C = \frac{1}{4\pi} \left| \frac{iq}{\sqrt{2}} a - M \right|^2 (|\phi_+|^2 + |\phi_-|^2)$$

$$V_{NC} = -\frac{mq\sqrt{2}}{8\pi} \text{ Re}(\phi_+ \phi_-),$$

$$\mathcal{M} = -\frac{iq}{4\sqrt{2\pi}} [-\chi \lambda] \left[ \begin{array}{ll} \phi_+ & -\phi_- \end{array} \right] \left[ \begin{array}{l} 1 \\ -1 \end{array} \right] \left[ \begin{array}{l} \psi_+ \\ \psi_- \end{array} \right]$$

$$+ \frac{1}{8\pi} \left( \frac{iq}{\sqrt{2}} a - M \right) \left[ \begin{array}{ll} -\psi_+ & \psi_- \end{array} \right] \left[ \begin{array}{l} 1 \\ -1 \end{array} \right] \left[ \begin{array}{l} \psi_+ \\ \psi_- \end{array} \right]$$

$$+ \frac{\tau'}{32\pi^2} [-\chi \lambda] \left( \frac{q\bar{D}}{\sqrt{2}} - \bar{e} - \bar{m}\tau \right) \cdot \bar{\sigma} \left[ \begin{array}{l} \lambda \\ \chi \end{array} \right],$$

where $SU(2)_R$ acts on the left of the hypermultiplet vierbein $U = \left[ \begin{array}{ll} \phi_+ & -\phi_- \\ \phi_- & \phi_+ \end{array} \right]$ and the gauginos $[\lambda \chi]$, whereas the gauge group $U(1)_R$, embedded in the direction $\sigma_3$ of the sympletic group $Sp(1) = SU(2)$, acts on the right of the vierbein and on the left of the hyperino doublet $[\psi_+ \psi_-]$. $

\tilde{D} := \frac{i}{16\pi^2} \text{tr} U \sigma_3 U^\dagger \bar{\sigma}$ is the triholomorphic moment map for the hypermultiplets (see [12] for a pedagogical review of hypermultiplet geometry).

In fact, it is not quite invariant under $SU(2)_R$, because of the second term $V_{NC}$ in the scalar potential, proportional to $D_1m_2$: this sheds some doubt on the $N = 2$ SUSY of the model. On the other hand, it may be shown that the covariant piece $V_C$ is precisely obtained when one covariantizes the general potential of a rigid $N = 2$ theory [13] with respect to the symplectic duality group, i.e. by adjoining to the triholomorphic moment map a dual 'magnetic' moment map to make a true symplectic section.

In order to check $N = 2$ SUSY, we have to work out the SUSY variations of the fields:

$$\delta a = -\sqrt{2} [-\chi \lambda] \left[ \begin{array}{l} \epsilon^1 \\ \epsilon^2 \end{array} \right]$$

$$\delta \left[ \begin{array}{l} \lambda \\ \chi \end{array} \right] = i\sqrt{2} \partial_\mu a \sigma^{\mu} \left[ \begin{array}{l} \epsilon^2 \\ \epsilon^1 \end{array} \right] + F_{\mu\nu} \sigma^{\mu\nu} \left[ \begin{array}{l} \epsilon^1 \\ \epsilon^2 \end{array} \right]$$

$$- \frac{i}{\sqrt{2\pi^2}} \left( \frac{q\bar{D}}{\sqrt{2}} - \bar{e} - \bar{m}\tau \right) \cdot \bar{\sigma} \left[ \begin{array}{l} \epsilon^1 \\ \epsilon^2 \end{array} \right]$$

$$+ \frac{2\sqrt{2}}{4\pi} \left( \bar{\epsilon}^1 \lambda \chi + \bar{\epsilon}^2 \lambda \chi \right) \left[ \begin{array}{l} \epsilon^1 \\ \epsilon^2 \end{array} \right]$$
\[ \delta \left[ \frac{\phi^+}{\phi^-} \right] = -\sqrt{2} \left[ \epsilon^1 \right] \left[ -\psi^+ \right] \psi^- \]

\[ + \sqrt{2} \left[ -\epsilon^2 \right] \left[ \bar{\psi}^- \right] \bar{\psi}^+ \]

\[ \delta \left[ -\psi^+ \right] \psi^- = -i \sqrt{2} \sigma^\mu \left[ \epsilon^1 \right] \nabla^\mu \left[ \frac{\phi^+}{\phi^-} \right] \]

\[ + \sqrt{2} \left( \frac{iq}{\sqrt{2}} \bar{\sigma} + \bar{M} \right) \left[ -\epsilon^2 \right] \left[ \frac{\phi^+}{\phi^-} \right] \left[ \frac{1}{1} \right]. \]

These variations are imposed to us by the requirement that they should reduce to the usual variations when restricting to \( N = 1 \) SUSY \((\epsilon^1 = 0)\), to the \( N = 2 \) variations when one forgets the hypermultiplets, and by \( SU(2)_R \) covariance. Alternatively, they can be obtained by symplectic covariantization of the general SUSY variations of [13].

Provisionally forgetting the non-covariant term, we can now evaluate up to three fermion terms the holomorphic \((\epsilon^1 = \epsilon^2 = 0)\) SUSY variation of the Lagrangian:

\[ \delta (M + V_C) = \]

\[ \frac{iq}{2} \left[ -\psi^+ \right] \psi^- \left[ 1 \right] -1 \left[ \frac{\phi^+}{\phi^-} \right] \left[ \frac{\phi^-}{\phi^+} \right] \bar{m} \sigma^\mu \left[ \epsilon^1 \right] \]

\[ = -\delta V_{IM} \text{, with} \]

\[ V_{IM} = -\frac{i q}{8 \pi \sqrt{2}} \bar{\sigma} \cdot \bar{m} = -\frac{\sqrt{2}}{8 \pi} \frac{m q}{2} \text{ Im}(\phi^+ \phi^-). \]

So we see that in order to compensate this variation we would have to add to the covariant Lagrangian \( V_C \) a piece \( V_{IM} \), covariant but purely imaginary. Compensating the antiholomorphic variation instead would require adding the complex conjugate piece \(-V_{IM}\), so there is really no way to implement \( N = 2 \) SUSY! Taking into account \( V_{NC} \), \( N = 1 \) supersymmetry is explicitly checked, since \( V_{IM} \) and \( V_{NC} \) have the same holomorphic part, while the \( N = 1 \) SUSY transformation \( \epsilon^2 = 0 \) involves only the holomorphic derivatives of the potential. Lastly, if one forgets both the non-covariant and imaginary pieces, we find that the naive covariantized version of the general N=2 Lagrangian is not even supersymmetric!

In case the arguments presented above should not be compelling enough, it is still possible to evaluate the spectrum in vacua of the theory with hypermultiplets and determine from the residual SUSY and the number of Goldstinos the original SUSY, and the result confirms that the Lagrangian obtained by minimally coupling the hypers to the pure Fayet-Iliopoulos deformed gauge theory does have N=1 SUSY only.

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