The SM extensions with additional light scalar singlet, nonrenormalizable Yukawa interactions and $(g - 2)_{\mu}$

S.N.Gninenko$^1$ and N.V. Krasnikov$^{1,2}$

$^1$ INR RAS, 117312 Moscow

$^2$ Joint Institute for Nuclear Research, 141980 Dubna

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Abstract

We consider the SM extension with additional light real singlet scalar, right-handed neutrino and nonrenormalizable Yukawa interaction for the first two generations. We show that the proposed model can explain the observed $(g - 2)_{\mu}$ muon anomaly. Phenomenological consequences as flavour violating decays $\tau \to \mu\mu\mu, \mu\mu e, \mu ee$ are briefly discussed. We also propose the $U_R(1)$ gauge generalization of the SM with complex scalar singlet and nonzero right-handed charges for the first two generations.
1 Introduction

The discovery of the neutrino oscillations \[1, 2\] means that at least two neutrino have nonzero masses. The minimal extension of the SM with nonzero neutrino masses is the $\nu$MSM \[3, 4\]. In this model one adds to the SM three additional massive Majorana(right-handed) fermions $\nu_{Ri}$, $i = 1, 2, 3$. Due to seesaw mechanism \[3, 5\] after the spontaneous $SU_L(2) \otimes U(1)$ electroweak symmetry breaking the neutrinos acquire masses $m_{\nu_i} = \frac{m_D^2}{M_{Ri}}$. Here $m_{Di}$ are the Dirac neutrino masses and $M_{Ri}$ are the masses of the $\nu_{Ri}$ neutrinos. The $\nu$MSM has a candidate - the lightest Majorana neutrino with a mass $M_{\nu_R} \leq O(50)$ KeV - for dark matter. Besides, the model with light Majorana neutrino can solve the problem of the baryon asymmetry in our Universe \[4\].

In this report which is based mainly on Refs.\[6\] we consider the extension of the $\nu$MSM with additional scalar field and nonrenormalizable Yukawa interaction for the first two generations. We show that the SM extension with additional light real singlet field can explain the $(g - 2)$ muon anomaly. Phenomenological consequences of the proposed model as flavour violating decays $\tau \rightarrow \mu \mu \mu, \mu ee$ are briefly discussed. We also propose the $U_R(1)$ gauge generalization of the SM with complex scalar singlet and nonzero righthanded charges for the first two generations.

2 The $\nu$MSM extension with additional real scalar isosinglet and nonrenormalizable Yukawa interaction

In this section we consider the extension of the $\nu$MSM with additional scalar field and nonrenormalizable Yukawa interaction for the first two generations \[6\]. The Lagrangian of the model has the form

$$L_{tot} = L_{SM} + L_{Qd\phi} + L_{Qu\phi} + L_{Le\phi} + L_{\phi} + L_{\nu_R}.$$ (1)
Here

\[ L_{Qd\phi} = -\frac{h_{Qd\phi,i}^k}{M} \bar{Q}_{Li} \bar{H} \phi d_{Rk} + H.c. \]  
(2)

\[ L_{Qu\phi} = -\frac{h_{Qu\phi,i}^k}{M} \bar{Q}_{Li} H \phi u_{Rk} + H.c. \]  
(3)

\[ L_{Le\phi} = -\frac{h_{Le\phi,i}^k}{M} \bar{L}_{Li} \bar{H} \phi e_{Rk} + H.c. \]  
(4)

\[ L_\phi = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{M_\phi^2 \phi^2}{2} - \lambda \phi^4 \]  
(5)

\[ L_{\nu R} = i \bar{\nu}_{Rj} \gamma_{\nu Rj} (\frac{M_{\nu Rj}}{2} \nu_{Rj} \nu_{Rj} + h_{Lij} \bar{L}_i H \nu_{Rj} + H.c.) \]  
(6)

where \( L_{SM} \) is the SM Lagrangian, \( \nu_{Rj} \) are the Majorana neutrinos and \( L_1 = (\nu_{eL}, e_L) \), \( L_2 = (\nu_{\mu L}, \mu_L) \), \( L_3 = (\nu_{\tau L}, \tau_L) \), \( e_{R1} = e_R \), \( e_{R2} = \mu_R \), \( e_{R3} = \tau_R \), \( Q_{L1} = (u_L, d_L) \), \( Q_{L2} = (c_L, s_L) \), \( Q_{L3} = (t_L, b_L) \), \( d_{R1} = d_R \), \( d_{R2} = s_R \), \( d_{R3} = b_R \), \( u_{R1} = u_R \), \( u_{R2} = c_R \), \( u_{R3} = t_R \), \( \bar{H} = (-H^-)^*, (H^0)^* \). The main peculiarity of the model (1-6) is the use of nonrenormalizable Yukawa interactions (2-4). Here we consider the particular case of the general model (1 - 6) with nonzero renormalizable Yukawa interaction only for the third fermion generation. We assume that the masses of the first two light generations arise due to nonrenormalizable interactions (2-4). We impose the discrete symmetry

\[ \phi \rightarrow -\phi \]  
(7)

\[ e_{Rk} \rightarrow -e_{Rk} \quad (k = 1, 2) \]  
(8)

\[ d_{Rk} \rightarrow -d_{Rk} \quad (k = 1, 2) \]  
(9)

The discrete symmetry (7 - 9) restricts the form of nonrenormalizable interactions (2-4), namely

\[ h_{Qd\phi,i3} = h_{Qu\phi,i3} = 0 \]  
(10)

\[ h_{Le\phi,i3} = 0 \]  
(11)

As a consequence of the (7-9) the renormalizable SM Yukawa interaction with the first two quark and lepton generations vanishes and the fermions of the first two generations acquire masses only due to nonrenormalizable interactions (2-4).

\[^1\text{Here } H = (H^0, H^-) \text{ is the SM Higgs doublet and } M \text{ is some high energy scale.}\]

\[^2\text{The nonrenormalizable Yukawa interactions have been considered in Refs.\cite{7}.}\]
We can consider the nonrenormalizable interactions (2 - 4) as some effective interactions arising from renormalizable interactions. For instance, the interaction (2) can be realized in renormalizable extension of the SM with additional scalar field $\phi$ and new massive quark $SU(2)_L$ singlet fields $D_R$, $D_L$ with a mass $M_D$ and $U(1)$ hypercharges $Y_{D_L} = Y_{D_R} = -\frac{1}{3}$. The interaction of new quark fields $D_R$, $D_L$ with ordinary quarks and the neutral scalar field $\phi$ is

$$L_{qD\phi} = -c_i\bar{Q}_L^i\bar{H}D_R^i - k_j\bar{D}_L^j d_{Rj}\phi + H.c.$$ \hspace{1cm} (12)

In the heavy $D$-quark mass limit $M_D \to \infty$ we obtain the effective interaction (2) with

$$\frac{h_{Qd\phi,ij}}{M} = \frac{c_ik_j}{M_D}.$$ \hspace{1cm} (13)

Analogously we can consider nonrenormalizable interaction (4) as an effective interaction which arises in renormalizable extension of the SM with additional scalar field and new massive lepton $SU(2)_L$ singlet fields $E_R$, $E_L$ with $Y_{E_L} = Y_{E_R} = 1$ The interaction of new lepton fields $E_R$, $E_L$ with ordinary quarks and the neutral scalar field $\phi$ is

$$L_{eE\phi} = -d_i\bar{L}_L^i\bar{H}E_R^i - f_j\bar{E}_L^j e_{Rj}\phi + H.c.$$ \hspace{1cm} (14)

In the heavy $E$-lepton mass limit $M_E \to \infty$ we obtain the effective interaction (4) with

$$\frac{h_{LE\phi,ij}}{M} = \frac{d_if_j}{M_E}.$$ \hspace{1cm} (15)

After the spontaneous $SU(2)_L \otimes U(1)$ electroweak symmetry breaking the Yukawa interaction of the scalar field with charged leptons takes the form

$$L_{ll\phi} = -\bar{\ell}_L^{ik}\bar{\ell}_L^k\phi e_{Rk} + H.c.,$$ \hspace{1cm} (16)

where

$$\bar{h}_{Le,ik} = h_{Le\phi,ik}\frac{<H>}{M},$$ \hspace{1cm} (17)

$i = 1, 2, 3$, $k = 1, 2$ and $e_{L1} = e_L$, $e_{L2} = \mu_L$, $e_{L3} = \tau_L$, $<H> = 174 \text{ GeV}$. Nonzero vacuum expectation value for the real field $\phi$ generates nonzero lepton masses for electrons and
muons while the mass of $\tau$-lepton arises due to renormalizable Yukawa coupling. The lepton mass matrix and the Yukawa lepton $\phi' = \phi - <\phi>$ interactions are

$$L_{ll} = -\bar{h}_{Le,ik} <\phi> \bar{e}_{Li} e_{Rk} - m_\tau \bar{e}_{L3} e_{R3} + H.c.,$$  \hspace{1cm} (18)

$$L_{l\phi} = -\bar{h}_{Le,ik} \phi' \bar{e}_{Li} e_{Rk} + H.c.,$$  \hspace{1cm} (19)

where $i = 1, 2, 3$ and $k = 1, 2$. The mass terms (18) and the interaction (19) have different flavour structure that leads to the tree level flavour changing transitions like $\tau \to \mu + \phi$, $\tau \to e + \phi$ and as a consequence to the flavour violating decays like

$$\tau^- \to \mu^- + \phi^+ \to \mu^- \mu^+ \mu^-,$$  \hspace{1cm} (20)

$$\tau^- \to \mu^- + \phi^+ \to \mu^+ e^- e^-,$$  \hspace{1cm} (21)

$$\tau \to e^- + \phi^+ \to e^- \mu^+ \mu^-,$$  \hspace{1cm} (22)

$$\tau \to e^- + \phi^+ \to e^- e^- e^-.$$  \hspace{1cm} (23)

At present state of art we can’t predict the value of flavour violating Yukawa couplings $\bar{h}_{Le31}, \bar{h}_{Le32}$.

The interaction (19) leads, in particular, to the additional one loop contribution to muon magnetic moment due to $\phi'$ scalar exchange, namely [8]

$$\Delta a_\mu = \frac{h_{Le,22}^2 m_\mu^2}{8\pi^2 M_\phi^2} \int_0^1 \frac{x^2(2-x)}{(1-x)(1-\lambda^2 x) + \lambda^2 x},$$  \hspace{1cm} (24)

where $\lambda = \frac{m_\mu}{m_\phi}$. In the limit $M_\phi >> m_\mu$

$$\Delta a_\mu = \frac{1}{4\pi^2} \frac{m_\mu^2}{M_\phi^2} \bar{h}_{Le,22}^2 [ln\left(\frac{M_\phi}{m_\mu}\right) - \frac{7}{12}].$$  \hspace{1cm} (25)

The precise measurement of the anomalous magnetic moment of the positive muon from the Brookhaven AGS experiment [9] gives a result which is $3.6\sigma$ higher than the Standard Model (SM) prediction

$$a_\mu^{exp} - a_\mu^{SM} = (288 \pm 80) \cdot 10^{-11},$$  \hspace{1cm} (26)

where $a_\mu = \frac{g-2}{2}$. Using the formulae (25, 26) we find that for $m_\Phi = (100, 10, 1, 0.5) GeV$ the muon $g - 2$ anomaly can be explained for
\[ h_{Le,22}^2 = (1.6 \pm 0.5) \cdot 10^{-2} \text{ for } m_\phi = 100 \text{ GeV} , \]  
(27)

\[ h_{Le,22}^2 = (2.6 \pm 0.8) \cdot 10^{-4} \text{ for } m_\phi = 10 \text{ GeV} , \]  
(28)

\[ h_{Le,22}^2 = (6.2 \pm 1.9) \cdot 10^{-6} \text{ for } m_\phi = 1 \text{ GeV} . \]  
(29)

\[ h_{Le,22}^2 = (2.6 \pm 0.8) \cdot 10^{-6} \text{ for } m_\phi = 0.5 \text{ GeV} . \]  
(30)

For the opposite limit \( m_\mu \gg m_\phi \)

\[ \Delta a_\mu = \frac{3 h_{Le,22}^2}{16\pi^2} \]  
(31)

and as a consequence of (26, 31) we find that

\[ h_{Le,22}^2 = (1.5 \pm 0.5) \cdot 10^{-7} . \]  
(32)

As in the SM the Yukawa couplings \( h_{Le,ii} \) are proportional to the lepton masses. It means that the interaction of the \( \phi \) scalar with electrons is weaker than the interaction of the \( \phi \) scalar with muons by factor \( m_\mu/m_e \approx 200 \) and the contribution of the \( \phi \) scalar to the electron magnetic moment is suppressed at least by factor \( (m_e/m_\mu)^2 \) in comparison with the muon magnetic moment even for superlight \( m_\phi \ll m_e \) scalar. For instance, for \( m_\phi = 1 \text{ GeV} \) the contribution of the \( \phi \) scalar to the electron magnetic moment is

\[ (\Delta a_e)_\phi = (0.16 \pm 0.05) \cdot 10^{-17} \]  
(33)

that is much smaller the bound from \( a_e \) [10]

\[ \Delta a_e = a_e^{exp} - a_e^{SM} = (-1.06 \pm 0.82) \cdot 10^{-12} \]  
(34)

Due to the suppression factor \( m_e/m_\mu \) for electron Yukawa coupling in comparison with muon Yukawa coupling the search for light \( \phi \) scalar in electron fixed target experiments or \( e^+e^- \) experiments is very problematic but not hopeless\(^3\). The search for very light \( \phi \) scalar in \( \pi \rightarrow (\phi \rightarrow e^+e^-)\gamma \) decay is possible but again we have additional suppression factor \( (m_e/m_\mu)^2 \sim O(10^{-2}) \). Light scalar particle \( \phi \) with a mass \( m_\phi \lesssim 1 \text{ GeV} \) decaying into muon pair can be searched for at CERN SPS secondary muon beam in full analogy with the search for new light vector boson \( Z^\prime \) [11].

\(^3\)Roughly speaking we have to improve the discovery potential by 3-4 orders of magnitude.
3 The $U(1)$ gauge generalization of the model with real scalar field

Here we outline one of possible generalizations of the model (1-6). In the proposed generalization instead of real scalar $\phi$ we use complex scalar $\Phi$ and new abelian gauge group $U_R(1)$ with nonzero charges for right handed fermions of the first and second generations, namely

$$Q_X(u_R) = Q_X(c_R) = Q_X(\nu_{eR}) = Q_X(\nu_{\mu R}) = -Q_X(d_R) = -Q_X(s_R) = -Q_X(e_R) = -Q_X(\mu_R).$$  \hspace{1cm} (35)

The nonrenormalizable Yukawa interactions of the first and second generation fermions in full analogy with the (2-4) interactions take the form

$$L_{Q_d \Phi} = -\frac{h_{Q_d \Phi,i} k}{M} \bar{Q}_{Li} \tilde{H} \Phi d_{Rk} + H.c.,$$ \hspace{1cm} (36)

$$L_{Q_u \Phi} = -\frac{h_{Q_u \Phi,i} k}{M} \bar{Q}_{Li} \tilde{H} \Phi^* u_{Rk} + H.c.,$$ \hspace{1cm} (37)

$$L_{L_e \Phi} = \frac{h_{L_e \Phi,i} k}{M} \bar{L}_{Li} \tilde{H} \Phi e_{Rk} + H.c.,$$ \hspace{1cm} (38)

$$L_{L_{\nu} \Phi} = -\frac{h_{L_{\nu} \Phi,i} k}{M} \bar{L}_{Li} \tilde{H} \Phi^* \nu_{Rk} + H.c.,$$ \hspace{1cm} (39)

Note that proposed model is free from $\gamma_5$-anomalies and we can consider the origin of the $U_R(1)$ gauge group as a result of the gauge symmetry breaking $SU_L(2) \otimes SU_R(2) \otimes U(1) \rightarrow SU_L(2) \otimes U_R(1) \otimes U(1)$. We assume that in the considered model $<\Phi \neq 0$ that leads to nonzero $X$ gauge boson mass and nonzero fermion masses for the first and second generations. In the unitaire gauge $\Phi = \phi + <\Phi>$, where $\phi = \phi^*$ is real scalar field as in previous section plus we have massive vector boson $X$. So in this model after $U_R(1)$ gauge symmetry breaking in addition to the $\nu$MSM spectrum we have both scalar and

\footnote{In Refs. [12] new light vector boson interacting with the $L_\mu - L_\tau$ current has been proposed for $(g-2)_\mu$ anomaly explanation, see also Ref.[13] where the model with new light gauge boson interacting with the SM electromagnetic current has been proposed for the $(g-2)_\mu$ anomaly explanation and Ref.[14] where the interaction of light gauge boson with $(B - L) + xY$ current has been considered.}
vector particles. The one loop contribution to the anomalous muon(electron) magnetic moment due to the $\phi$ and $X$ exchanges is

$$\Delta a_\mu = \Delta a_\mu(\phi) + \Delta a_\mu(X),$$

(40)

where the $\Delta a_\mu(\phi)$ contribution is given by the formulae (24,25,31) and the vector $X$ boson contribution for $(V + A)$ right-handed coupling\(^5\) with fermions is

$$\Delta a_\mu = \frac{g_X^2 m_\mu^2}{8\pi^2 M_{X\mu}^2} \int_0^1 \frac{2x^2(2-x) + 2x(1-x)(x-4) - 4\lambda^2 x^3}{(1-x)(1-x^2) + \lambda^2 x},$$

(41)

$$\Delta a_\mu = -\frac{g_X^2 m_\mu^2}{3\pi^2 M_X^2} \quad (for \ M_X \gg m_\mu).$$

(42)

The $X$ boson contribution (41) to the $(g-2)$ is negative. For instance, for $\alpha_X = g_X^2/4\pi = 10^{-8}$ and $M_X = 500 \text{ MeV}$ the $X$-boson contribution to $\Delta a_\mu$ is $\Delta a_\mu(X) = -1.8 \cdot 10^{-10}$. The positive contribution due to $\phi$ boson exchange is positive and cancels the negative contribution from the $X$ boson exchange. The most perspective experiments for the search for light vector $X$-boson with the electron coupling constant $\alpha_X = O(10^{-8})$ are the electron fixed target experiments or $e^+e^-$ experiments. The $X$ boson can decay into electron-positron or muon-antimuon pairs, also invisible decays of the $X$ boson into light sterile neutrino are possible. The experiment NA64\(^15\) at CERN will be able to search for both invisible and visible $X$ boson decay modes with the $\alpha_X \geq O(10^{-12})$\(^16\).

4 Conclusion

The $\nu$MSM with additional scalar field and nonrenormalizable interaction for the first two generations can explain the observed muon $(g-2)$ anomaly. The model predicts the existence of flavour violating quark and lepton decays like $\tau \to \mu\mu\mu, \mu\mu\epsilon, \mu\epsilon\epsilon$. Besides the $U(1)$ gauge generalization of the model with real isosinglet scalar field is also able to explain muon $(g-2)$ anomaly.

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\(^5\)The interaction Lagrangian of the vector $X$ field with muons is $L_{X\mu} = g_X X^\nu \bar{\mu} \gamma_\nu (1 + \gamma_5) \mu$. 

8
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