The question of anomalies in the Chern-Simons theory coupled to matter fields

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Abstract

We study the Chern-Simons theory coupled to matter field by means of an effective Lagrangian obtained from the Batalin-Fradkin-Vilkovisky formalism. We show that there is no rotational anomaly for any proper gauge we choose.

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I. Introduction

There has been much controversy on the rule of gauge fixation in the appearance of fractional spin and statistics in the context of Chern-Simons (CS) gauge theory coupled to charged matter fields [1]. Several authors have claimed that these theories exhibit angular momentum anomaly for some specific gauge [2]. In a recent work, however, Banerjee has shown that this kind of anomaly does not exist [3] if an specific gauge choice is not made. He used the Dirac formalism [4], without fixing the gauge and following an approach where the energy momentum tensor is properly modified by introducing first-class constraints by means of Lagrange multipliers [5]. These are fixed by imposing the closure of the Poincaré algebra under the Dirac bracket spectrum. It might be opportune to emphasize that these terms are introduced by hand and that there are no effective Lagrangian that can both lead to it and still correctly describes the initial theory.

The purpose of the present paper is twofold. First we use Hamiltonian path integral formalism due to Batalin, Fradkin and Vilkovisky (BFV) [6] to obtain an effective Lagrangian without choosing any specific form for the gauge fixing function. After that, with this generic Lagrangian and a careful analysis of the Noether currents and their relation to momenta, we show that the closure of the Poincaré algebra can be achieved without anomalies for any gauge condition. The main point it that when one changes the gauge-fixing choice, momenta also change in order to preserve the symmetry.

Our paper is organized as follows: In Sec. II we make a brief analysis of the canonical procedure in order to obtain the interesting structure of constraints of the theory. In Sec. III we discuss the elimination of the second class constraints in the path integral formulation of the BFV formalism and the obtainment of the effective Lagrangian. Sec. IV contains the discussion of the closure of the Poincaré algebra and Sec. V is devoted to some comments and conclusions. We have also included two appendices in order to better clarify some points of the paper.
II. Brief review of the canonical procedure

Pure CS vectorial bosons coupled to complex scalar fields can be described by the Lagrangian density [7]

\begin{equation}
\mathcal{L} = (D_\mu \phi) \dagger (D^\mu \phi) + \frac{\kappa}{4\pi} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho ,
\end{equation}

where \( D_\mu = \partial_\mu + i A_\mu \) is the covariant derivative and \( \epsilon^{\mu\nu\rho} \) is the completely antisymmetric symbol (with \( \epsilon^{012} = \epsilon_{012} = 1 \)). \( \phi, \phi^\dagger \) and \( A_\mu \) represent respectively complex scalar fields and vectorial gauge bosons. We adopt the Minkowsky metric tensor in \( 2 + 1 \) spacetime dimensions as \( \eta_{\mu\nu} = \text{diag} \,(+1, -1, -1) \).

This theory leads to the following set of (primary and secondary) constraints [4, 5]

\begin{align}
\chi^0 &= \pi^0 , \quad (2.2a) \\
\chi^i &= \pi^i - \frac{\kappa}{4\pi} \epsilon^{ij} A_j , \quad (2.2b) \\
\psi &= i(\pi \phi - \pi^\dagger \phi^\dagger) + \frac{\kappa}{2\pi} \epsilon^{ij} \partial_i A_j , \quad (2.2c)
\end{align}

where

\begin{align}
\pi^\mu &= \frac{\kappa}{4\pi} \epsilon^{0\mu\nu} A_\nu , \quad (2.3a) \\
\pi &= (D_0 \phi) \dagger = \dot{\phi}^\dagger - i A_0 \phi^\dagger , \quad (2.3b) \\
\pi^\dagger &= D_0 \phi = \dot{\phi} + i A_0 \phi \quad (2.3c)
\end{align}

are the canonical momenta conjugate to \( A_\mu, \phi \) and \( \phi^\dagger \), respectively. Let us next write down the total Hamiltonian density

\begin{equation}
\mathcal{H} = \mathcal{H}_c + \lambda_0 \chi^0 + \lambda_i \chi^i + \lambda \psi ,
\end{equation}

where \( \mathcal{H}_c \) is the canonical Hamiltonian density.

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\[ H_c = \pi^0 \dot{A}_0 + \frac{\kappa}{4\pi} \epsilon^{ij} A_j \dot{A}_i + (i\pi^\dagger \dot{\phi}^\dagger - i\pi \dot{\phi} - \frac{\kappa}{2\pi} \epsilon^{ij} \partial_i A_j) \Lambda_0 + \pi^\dagger \pi - (D_i \phi)^\dagger (D^i \phi). \]  

(2.5)

Velocities \( \dot{A}_0 \) and \( \dot{A}_i \) cannot be eliminated because the corresponding momentum expressions are constraints. The elimination of \( \dot{\phi} \) and \( \dot{\phi}^\dagger \) by using the expressions (2.3b) and (2.3c) gives

\[ H_c = \pi^0 \dot{A}_0 + (\pi^i - \frac{\kappa}{4\pi} \epsilon^{ij} A_j) \dot{A}_i + (i\pi^\dagger \dot{\phi}^\dagger - i\pi \dot{\phi} - \frac{\kappa}{2\pi} \epsilon^{ij} \partial_i A_j) \Lambda_0 \]

\[ + \pi^\dagger \pi - (D_i \phi)^\dagger (D^i \phi). \]  

(2.6)

Going back with this result to (2.4) and observing the constraint expressions (2.2) we notice that \( \dot{A}_0, \dot{A}_i \) and \( A_0 \) can be respectively absorbed by the Lagrange multipliers \( \lambda_0, \lambda_i \) and \( \lambda \). After that, the component \( A_0 \) disappears. This means that it is effectively like we do not have this degree of freedom any more. In this way it does not make sense to keep the constraint \( \chi_0 \) in the theory. Let us the disregard it (taking \( \lambda_0 = 0 \)) and consider the total Hamiltonian just as

\[ H = \lambda_i \chi^i + \lambda \psi + \pi^\dagger \pi - (D_i \phi)^\dagger (D^i \phi). \]  

(2.7)

The remaining constraints \( \chi^i \) and \( \psi \) are apparently second-class. However, this is not so because the constraint matrix formed with \( \psi, \psi^i \) is obvious singular. Actually, a proper linear combination of them is first-class:

\[ \chi = \partial_i \pi^i + i(\pi \dot{\phi} - \pi^\dagger \dot{\phi}^\dagger) + \frac{\kappa}{4\pi} \epsilon^{ij} \partial_i A_j. \]  

(2.8)

From now on the set of constraints we shall use is formed by \( \chi \) (first-class) and \( \chi^i \) (second-class).

At this stage, quantization can be carried out along some lines. For instance, gauge freedom associated to first-class constraints can be frozen by choosing proper gauge-fixing
conditions and calculating the Dirac brackets in order to get commutators \[4, 5\]. Another
procedure is to keep the gauge freedom until the final stage of the quantization \[3, 5\]. In
this scenario, the physical space of states have to be extracted by imposing that physical
states are annihilated by first-class constraints, written as operators, while second-class
constraints lead to operational identities. We can also mention the procedure due to
Batalin and Fradkin (BF) \[8\] where second class constraints are transformed into first
class by introducing extra degrees of freedom in the theory \[9\].

**III. Effective Lagrangian from the BFV formalism**

The procedure which seems to be relevant for our purposes is the BFV Hamiltonian
path integral formalism, where the gauge choice can be controlled at any stage due to the
powerful Fradkin-Vilkovisky (FV) theorem \[6\]. It assures that the functional generator
is independent of the gauge-fixing since some conditions are satisfied. Our final goal is
to show that Poincaré invariance can be achieved in any gauge, consistent with the FV
conditions.

The implementation of the BFV formalism requires the existence of just first-class
constraints. In the present case, this can be achieved in two ways. The first one is making
use of the BF formalism and transforming second-class constraints into first-class ones.
Another procedure, that we shall develop here, is to eliminate the second-class constraints
with the use of the Dirac bracket definition and at the same time modifying the path
integral measure by means of the Senjanovic procedure \[10\]. This means that the closure
of the algebra of first-class constraints is achieved by using (partial) Dirac brackets instead
of Poisson ones. Let us then write the constraint matrix element \(C^{ij}\)

\[
C^{ij}(\vec{x}, \vec{y}) = \{\chi^i(\vec{x}, t), \chi^j(\vec{y}, t)\},
\]

\[
= -\frac{\kappa}{2\pi} \epsilon^{ij} \delta^{(2)}(\vec{x} - \vec{y}). \tag{3.1}
\]

Hence,
\[ C_{ij}^{-1}(\vec{x}, \vec{y}) = \frac{2\pi}{\kappa} \epsilon_{ij} \delta^{(2)}(\vec{x} - \vec{y}). \] (3.2)

Next, using the Dirac bracket definition, we calculate the fundamental star brackets.

\begin{align*}
\{A^i (\vec{x}, t), A^j (\vec{y}, t)\}^* &= \frac{2\pi}{\kappa} \epsilon^{ij} \delta^{(2)}(\vec{x} - \vec{y}), \quad (3.3a) \\
\{\pi^i (\vec{x}, t), \pi^j (\vec{y}, t)\}^* &= \frac{\kappa}{8\pi} \epsilon^{ij} \delta^{(2)}(\vec{x} - \vec{y}), \quad (3.3b) \\
\{A^i (\vec{x}, t), \pi^j (\vec{y}, t)\}^* &= \frac{1}{2} \eta^{ij} \delta^{(2)}(\vec{x} - \vec{y}). \quad (3.3c)
\end{align*}

Remaining brackets are the same as Poisson ones.

Since constraints \(\chi^i\) were eliminated, the only remaining constraint is \(\chi\). The algebra satisfied by this constraint is obtained by using the fundamental Dirac brackets above. We easily see that it is a rank zero algebra in a sense that

\[ \{\chi(\vec{x}, t), \chi(\vec{y}, t)\}^* = 0. \] (3.4)

Of course, the expression for the constraint \(\chi\) can now be written by eliminating \(\pi_i\) with the use of \(\chi_i\), i.e.

\[ \chi = i(\pi \phi - \pi^\dagger \phi) + \frac{\kappa}{2\pi} \epsilon^{ij} \partial_i A_j. \] (3.5)

In terms of the BFV treatment, extra fields have to be incorporated into the formalism. These are the canonical momentum \(p\) of the Lagrange multiplier \(\lambda\), a pair of ghosts \((c, \bar{c})\) (related to the first-class constraint \(\chi\)) and their corresponding momenta \((\bar{P}, P)\). The expression of the BRST charge [6] reads

\[ \Lambda = \int d^2 \vec{x} \left( c \chi - i \bar{P} p \right). \] (3.6)
The transformations of fields generated by this charge in terms of the star brackets (3.3) are

\[ \delta A_i(x) = \{ A_i(\vec{x}, t), \epsilon \Lambda \}^* , \]
\[ = -\epsilon \partial_i c(x) , \]
\[ \delta \lambda(x) = -i \epsilon P(x) , \]
\[ \delta p(x) = 0 , \]
\[ \delta \phi(x) = i \epsilon \phi(x) c(x) , \]
\[ \delta \phi^\dagger(x) = -i \epsilon \phi^\dagger(x) c(x) , \]
\[ \delta \pi(x) = -i \epsilon \pi(x) c(x) , \]
\[ \delta \pi^\dagger(x) = i \epsilon \pi^\dagger(x) c(x) , \]
\[ \delta c(x) = 0 , \]
\[ \delta \bar{c}(x) = -i \epsilon p(x) . \quad (3.7) \]

Let us now discuss the elimination of the second-class constraints $\chi^i$ in the generating functional of the BFV formalism. Considering that [6]

\[ Z = N \int [d\mu'] e^{iS'_{\text{eff}}} , \quad (3.8) \]

where the effective action reads

\[ S'_{\text{eff}} = \int d^3 x \left[ \dot{A}^i \pi_i + \dot{\phi} \pi + \phi^\dagger \pi^\dagger + \dot{\lambda} P + \dot{\bar{c}} \bar{P} + \dot{\bar{c}} P + (D_i \phi)^\dagger D^i \phi - \pi^\dagger \pi + \{ \Psi, \Lambda \}^* \right] . \quad (3.9) \]

Here, $\Psi$ is the gauge-fixing function. The relative position of velocities in the expression above is because we are using left derivatives for fermionic fields.
The elimination of the second-class constraints is achieved by means of the Senjanovic procedure. This is embodied with the use of the following measure [10]

\[
[d\mu'] = (\det\{|\chi_i, \chi_j|\})^{1/2} \prod_i \delta[\chi_i][dA_i][d\pi_i][d\phi][d\phi^\dagger][d\pi][d\pi^\dagger] \\
\times [d\lambda][dp][dc][d\bar{c}][dP][d\bar{P}]. \tag{3.10}
\]

The determinant factor in the expression above does not involve any field. It is then possible to absorb it into the normalization factor \( N \). We also use the functional integration over \( \pi_i \) to eliminate the second-class constraint. The result is a simpler expression for the vacuum functional

\[
Z = N \int [d\mu] e^{iS_{\text{eff}}}, \tag{3.11}
\]

where

\[
[d\mu] = [dA_i][d\phi][d\phi^\dagger][d\pi][d\pi^\dagger][d\lambda][dp][dc][d\bar{c}][dP], \tag{3.12}
\]

\[
S_{\text{eff}} = \int d^3x \left[ \frac{\kappa}{4\pi} \epsilon^{ij} \dot{A}_i A_j + \dot{\phi} \pi + \dot{\phi}^\dagger \pi^\dagger + \dot{\lambda} p + \dot{c} \bar{P} + \dot{\bar{c}} P \\
+ (D_i \phi)^\dagger D^i \phi - \pi^\dagger \pi + \{\Psi, \Lambda\}^* \right]. \tag{3.13}
\]

The effective action is BRST invariant independently of the choice of \( \Psi \). This can be easily verified by using the Jacobi identity and the nilpotency of the BRST charge.

In order to see if the procedure we have used above to eliminate the second-class constraints is consistent or not we could choose some particular gauge, develop the integration of the vacuum functional and compare the result with the ones found in literature. We show this in the Appendix A, where some comments are also made in order to become clearer what will be developed in the next sections.
IV. Poincaré invariance and absence of anomalies

As it was seen in the Appendix A, to integrate over the momenta it is necessary to choose a particular gauge-fixing condition. Choosing $\Psi = i \bar{c} \left( \frac{1}{2} \alpha p - \nabla \cdot \vec{A} \right) + \bar{P} \lambda$ and integrate over $\pi$, $\pi^\dagger$, $P$, $\bar{P}$ we obtain the effective Lagrangian used by Shim et al. [2] where it was detected anomaly in the angular momentum algebra. In order to make an analysis in a more general point of view, let us consider here the initial effective Lagrangian (3.13), where no particular gauge-fixing function has yet been chosen. Now, the variables $p$, $\pi$, $\pi^\dagger$, $P$ and $\bar{P}$ do not have to be seen as momenta, but just as variables of an extended configuration space. If $\{\Psi, \Lambda\}^*$ does not contain velocities (this is not the case, for example, of the Fock-Schwinger gauge - see Appendix A), we can identify the usual bracket relations involving these variables in terms of the Dirac brackets. For instance, considering that $\{\Psi, \Lambda\}^*$ does not contain $\dot{\lambda}$ we have $\partial L / \partial \dot{\lambda} = p$ and $\partial L / \partial \dot{p} = 0$. Both are second class constraints. Using the Dirac brackets definition we get $\{\lambda(\vec{x}, t), p(\vec{y}, t)\}^* = \delta^{(2)}(\vec{x} - \vec{y})$. If $\{\Psi, \Lambda\}$ had dependence on $\dot{\lambda}$, the effective momentum conjugate to $\lambda$ would be $\partial L / \partial \dot{\lambda} = p + \partial \{\Psi, \Lambda\}^*/\partial \dot{\lambda}$. This is an important fact as we are going to see soon. The absence of anomaly in the Poincaré algebra, for any $\Psi$, will be related to it. If one changes the gauge, momenta may also change in order to preserve the symmetry.

Let us study this point with details. Denoting all fields that appear in (3.13) generically by $\xi_A$ we have that any on-shell variation of these fields leads to (using left-derivatives)

$$\delta L = \partial_\mu \left( \delta \xi_A \frac{\partial L}{\partial (\partial_\mu \xi_A)} \right). \quad (4.1)$$

We first discuss invariance under spacetime translations. Considering infinitesimal translations given by $\delta x^\mu = x'^\mu - x^\mu = \epsilon^\mu$ and that, for any field, $\xi_A(x') = \xi_A(x)$ we have

$$\delta \xi_A(x) = \xi_A'(x) - \xi_A(x),$$

$$= \xi_A'(x) - \xi_A'(x'),$$

$$= - \epsilon^\mu \partial_\mu \xi_A(x) + O(\epsilon^2). \quad (4.2)$$
Of course we have a similar relation for $\delta \mathcal{L}$. Introducing these results into (4.1) one obtains the usual expression for the energy-momentum tensor

$$ T_{\mu\nu} = \partial_{\nu} \xi_{A} \frac{\partial \mathcal{L}}{\partial (\partial_{\mu} \xi_{A})} - \eta_{\mu\nu} \mathcal{L}, $$

which is divergenceless with respect the index $\mu$ (in general, the energy-momentum tensor is not symmetric). Let us then check if

$$ P_{\nu} = \int d^2 x \ T_{0\nu} $$

is actually the generator of spacetime translations. Considering first $P_{0}$ we have

$$ \delta \xi_{A}(x) = e^{0} \{ P_{0}, \xi_{A}(x) \}^{\ast}, $$

$$ = e^{0} \int d^2 \vec{y} \left( \dot{\xi}_{B}(\vec{y}, t) \{ \frac{\partial \mathcal{L}}{\partial \dot{\xi}_{B}(\vec{y}, t)}, \xi(\vec{x}, t) \}^{\ast} + \{ \dot{\xi}_{B}(\vec{y}, t), \xi(\vec{x}, t) \}^{\ast} \frac{\partial \mathcal{L}}{\partial \dot{\xi}_{B}(\vec{y}, t)} - \{ \mathcal{L}(\vec{y}, t), \xi(\vec{x}, t) \}^{\ast} \right), $$

$$ = - e^{0} \dot{\xi}(x). $$

In the last step above we have used the identity

$$ \{ \mathcal{L}(\vec{y}, t), \xi(\vec{x}, t) \}^{\ast} = \frac{\partial \mathcal{L}}{\partial \dot{\xi}_{B}(\vec{y}, t)} \{ \dot{\xi}_{B}(\vec{y}, t), \xi(\vec{x}, t) \}^{\ast}. $$

For pure space translations we have

$$ \delta \xi_{A}(x) = e^{i} \{ P_{i}, \xi_{A}(x) \}^{\ast}, $$

$$ = e^{i} \int d^2 \vec{y} \left( \partial_{i} \dot{\xi}_{B}(\vec{y}, t) \frac{\partial \mathcal{L}}{\partial \dot{\xi}_{B}(\vec{y}, t)}, \xi_{A}(\vec{x}, t) \right)^{\ast}, $$

$$ = - e^{i} \partial_{i} \xi_{A}(x). $$
Since there are no problems with ordering operators, all the (Dirac) brackets above can be replaced by commutators and the corresponding relations can be seen quantically. There is no anomaly. It might be opportune to mention that there was no anomaly for spacetime translations in the previous mentioned paper [2].

Let us now discuss the invariance under spacetime rotations. This cannot be obtained so directly as the translation case. It is necessary to know the Lorentz transformations of fields which appear in (3.13). Since it is written in a manifestly noncovariant form, we have to figure out what must be the their Lorentz transformations in order to preserve $\mathcal{L}$ as a scalar (see Appendix B). The problem here is that there is a lot of fields that we do not know their Lorentz transformation and it is a very difficult task to figure them out. Fortunately, we do not need to know the transformation of all them, only those with spacetime derivatives (see expression (4.1)). Since $p, \pi, \pi^\dagger, P$ and $\bar{P}$ are auxiliary fields (not dynamical), we just need to know the transformations of $A_i, \lambda, \phi, \phi^\dagger, c$ and $\bar{c}$.

The transformation of scalar fields reads (see Appendix B)

$$
\delta \phi(x) = -\omega^{\mu\nu} x_\nu \partial_\mu \phi(x),
\delta \phi^\dagger(x) = -\omega^{\mu\nu} x_\nu \partial_\mu \phi(x)^\dagger.
$$

(4.8)

Concerning the gauge field we have

$$
\delta A_i(x) = A'_i(x) - A_i(x),
= -\omega^{\mu\nu} x_\nu \partial_\mu A_i(x) + \omega_i^\mu A_\mu(x).
$$

(4.9)

As one observes, the transformation of $A_i$ requires the existence of $A_0$, that was disregarded in the beginning. However, this is not a problem because in the BFV formalism one always make $A_0$ appears again by identifying $\lambda$ as $A_0$, independently of the gauge choice (see Appendix A). So, from now on we replace $\lambda$ by $A_0$. 11
Of course, $D_i \phi$ and $(D_i \phi)\dagger$ have the same transformation as $A_i$. Namely,

$$\delta (D_i \phi(x)) = -\omega^{\mu\nu} x_\nu \partial_\mu (D_i \phi(x)) + \omega_i^\mu D_\mu \phi(x),$$

$$\delta (D_i \phi(x))\dagger = -\omega^{\mu\nu} x_\nu \partial_\mu (D_i \phi(x))\dagger + \omega_i^\mu (D_\mu \phi(x))\dagger. \quad (4.10)$$

Concerning the transformation of $c$ and $\bar{c}$, we report back to the particular cases described in the Appendix A. With those particular gauge choices, we see that ghost behave as scalars. Since this intrinsic property cannot depend on gauge choices we conclude that $c$ and $\bar{c}$ must always transform as scalars (*), so

$$\delta c(x) = -\omega^{\mu\nu} x_\nu \partial_\mu c(x),$$

$$\delta \bar{c}(x) = -\omega^{\mu\nu} x_\nu \partial_\mu \bar{c}(x). \quad (4.11)$$

Considering the Lagrangian (3.13) we obtain that expression (4.1) leads to

$$\delta \mathcal{L} = \partial_\mu \left[ \delta A_\nu \frac{\partial \mathcal{L}}{\partial (\partial_\mu A_\nu)} + \delta \phi \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} + \delta \phi^\dagger \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi^\dagger)} + \delta \bar{c} \frac{\partial \mathcal{L}}{\partial (\partial_\mu \bar{c})} + \delta c \frac{\partial \mathcal{L}}{\partial (\partial_\mu c)} \right]. \quad (4.12)$$

Replacing the transformations (4.8)-(4.10) into the expression above, one obtains the following divergenceless quantity (in the $\mu$ index)

$$\mathcal{M}_{\mu\nu\rho} = \left( x_\lambda \partial_\rho - x_\rho \partial_\lambda \right) \left( A_\nu \frac{\partial \mathcal{L}}{\partial (\partial_\mu A_\nu)} + \phi \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} + \phi^\dagger \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi^\dagger)} + c \frac{\partial \mathcal{L}}{\partial (\partial_\mu c)} + \bar{c} \frac{\partial \mathcal{L}}{\partial (\partial_\mu \bar{c})} \right)$$

$$+ \left( \eta_{\lambda\rho} A_\rho + \eta_{\rho\nu} A_\lambda \right) \frac{\partial \mathcal{L}}{\partial (\partial_\mu A_\nu)} + \left( \eta_{\mu\lambda} x_\rho - \eta_{\mu\rho} x_\lambda \right) \mathcal{L}. \quad (4.13)$$

(*) The same argument could not be applied to the auxiliary fields $p, \pi, \pi^\dagger, P$ and $\bar{P}$. For example, for the first gauge choice seen in the Appendix A, $p$ transforms as a scalar. For the second one, where $\{\Psi, \Lambda\}$ contains $\dot{\lambda}$, $p$ does not transform as scalar anymore.
We are interested in the quantities

\[ M_{\lambda\rho} = \int d^2 x \mathcal{M}_{0\lambda\rho}, \quad (4.14) \]

which have to be the generators of the Lorentz algebra. From (4.13) we get

\[
M_{\lambda\rho} = \int d^2 \vec{x} \left[ \left( x_\lambda \partial_\rho - x_\rho \partial_\lambda \right) \left( A_\nu \frac{\partial \mathcal{L}}{\partial A_\nu} + \phi \frac{\partial \mathcal{L}}{\partial \phi} + \phi^\dagger \frac{\partial \mathcal{L}}{\partial \phi^\dagger} + c \frac{\partial \mathcal{L}}{\partial c} + \bar{c} \frac{\partial \mathcal{L}}{\partial \bar{c}} \right) \right.
\]

\[
+ \left( \eta_{\lambda\nu} A_\rho - \eta_{\rho\nu} A_\lambda \right) \frac{\partial \mathcal{L}}{\partial A_\nu}
\]

\[
+ \left( \eta_{\lambda\rho} x_\nu - \eta_{\nu\rho} x_\lambda \right) \mathcal{L} \right]. \quad (4.15)
\]

The quantities \( \partial \mathcal{L}/\partial \dot{A}_\nu, \partial \mathcal{L}/\partial \dot{\phi} \) etc. are the conjugate momenta related to \( A_\nu, \phi \) etc. They are not necessarily the same as \( \pi^\nu, \pi \) etc. So, the usual Lorentz algebra involving \( M_{\lambda\rho} \) can be verified trivially. Since there are no problem with ordering operators, the same algebra can be directly written in terms of commutators. There is no anomaly for any gauge we choice. This result is in agreement with the one recently found by Banerjee [3].

V. Conclusion

We have studied the Abelian CS theory coupled to matter field. Our starting point was the initial effective Lagrangian of the BFV formalism. The importance of this Lagrangian is because the gauge-fixing function has not yet been chosen and we know that the generating functional is independent of this choice if some conditions are satisfied (Fradkin-Vilkovisky theorem). So, the results we have obtained are independent of any gauge choice we make. We have then show that the closure of the full Poincaré algebra is achieved without any anomaly, in agreement with a recent paper of Banerjee [3] and contrarily to previous publications [2]. To show this we have made a careful analysis of the Noether currents and the momenta associated to the dynamical variables of the theory.
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Appendix A

In order to verify the consistency of what we have done in Sec. III, let us first make an usual choice for the gauge-fixing function \( \Psi \), that is to say \[ \Psi = i\bar{c} \left( \frac{1}{2} \alpha p - \nabla \cdot \vec{A} \right) + \bar{P} \lambda \] (A.1) and develop expression (3.11) by integrating over the momenta. We shall obtain a covariant effective action with the Lorentz-like term \((\partial_{\mu} A^{\mu})^2/2\alpha\). First we notice that the gauge choice given by (A.1) leads to

\[
\{\Psi(x), \Lambda\}^* = -\frac{\alpha}{2} p^2 - i\bar{c} \nabla^2 c + p \nabla \cdot \vec{A} - i\bar{P} \lambda - \lambda \left[ i(\pi\phi - \pi^\dagger \phi^\dagger) + \frac{\kappa}{2\pi} \epsilon^{ij} \partial_i A_j \right].
\] (A.2)

With this result the effective action turns to be

\[
S_{\text{eff}} = \int d^3 x \left\{ \frac{\kappa}{4\pi} \epsilon^{ij} \dot{A}_i A_j + \dot{\phi} \pi + \dot{\phi}^\dagger \pi^\dagger + \dot{\lambda} p + \dot{c} \bar{P} + \dot{c} \mathbb{P} + (D_i \phi)^\dagger D^i \phi \\
- \pi^\dagger \pi - \frac{\alpha}{2} p^2 - i\bar{c} \nabla^2 c + p \nabla \cdot \vec{A} - i\bar{P} \lambda \\
- \lambda \left[ i(\pi\phi - \pi^\dagger \phi^\dagger) + \frac{\kappa}{2\pi} \epsilon^{ij} \partial_i A_j \right] \right\}.
\] (A.3)

Going back to the path integral and integrating over the momenta we obtain

\[
Z = N \int [dA_i][d\phi][d\phi^\dagger][d\lambda][dc][d\bar{c}] \exp \left\{ i \int d^3 x \left[ \frac{\kappa}{4\pi} \epsilon^{ij} \dot{A}_i A_j + (D_i \phi)^\dagger D^i \phi - i\bar{c} \Box c \\
+ (\dot{\phi} + i\lambda)(\dot{\phi}^\dagger - i\lambda) + \frac{1}{2\alpha} (\dot{\lambda} + \nabla \cdot \vec{A})^2 - \frac{\kappa}{2\pi} \lambda \epsilon^{ij} \partial_i A_j \right] \right\}. \] (A.4)
In the BFV formalism the manifest covariance is obtained by always letting $\lambda$ to be the missing temporal component of the gauge field. Doing this we finally get

$$Z = N \int [dA_\mu][d\phi][d\phi^\dagger][dc][d\bar{c}] \exp \left\{ i \int d^3x \left[ \frac{\kappa}{4\pi} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho 
+ (D_\mu \phi)^\dagger D^\mu \phi + i\bar{c} \Box c + \frac{1}{2\alpha} (\partial_\mu A^\mu)^2 \right] \right\}. \quad (A.5)$$

If we had not integrated on the momentum $p$ we would have obtained

$$Z = N \int [dA_\mu][d\phi][d\phi^\dagger][dc][d\bar{c}] \exp \left\{ i \int d^3x \left[ \frac{\kappa}{4\pi} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho 
+ (D_\mu \phi)^\dagger D^\mu \phi + i\bar{c} \Box c - \frac{\alpha^2}{2} p^2 + p \partial_\mu A_\mu \right] \right\}. \quad (A.6)$$

This is precisely the Lagrangian used as the starting point in the paper by H. Shin et al. [2] where it was detected an anomaly in the Poincaré algebra.

Although not common, it could have chosen another gauge-fixing function different from (A.1). With the purpose of illustrating a little more this subject and for future reference, let us consider another gauge-fixing function. We choose the one based on the Fock-Schwinger gauge [12], $x_\mu A^\mu = 0$. To implement this gauge in the BFV formalism we have to choose $\Psi$ as [13]

$$\Psi = i\bar{c} \dot{\lambda} - i\dot{c} \lambda + i\bar{c} \left( \frac{1}{2} \alpha p + \frac{x_\mu A^\mu}{x^2} \right) + \mathcal{P} \lambda \quad (A.7)$$

and consider $\lambda$ such that $\{\lambda, \dot{\lambda}\}^* = 0$. Note here that $\Psi$ has dependence on the velocity $\dot{\lambda}$. Integrating over the momenta and identifying $\lambda$ as $A_0$ we get

$$Z = N \int [dA_\mu][d\phi][d\phi^\dagger][dc][d\bar{c}] \exp \left\{ i \int d^3x \left[ \frac{\kappa}{4\pi} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + (D_\mu \phi)^\dagger D^\mu \phi 
+ i\bar{c} \frac{x_\mu \partial_\mu}{x^2} c + \frac{1}{2\alpha x^4} (x_\mu A^\mu)^2 \right] \right\}. \quad (A.8)$$
Appendix B

In order to see how one can figure out the Lorentz transformation of fields in manifestly noncovariant Lagrangians, let us consider a simple example just involving real scalar fields

\[ L = \pi \dot{\phi} + \frac{1}{2} \partial_i \phi \partial^i \phi - \frac{1}{2} \pi^2. \]  

(B.1)

We already know the Lorentz transformation of \( \phi \). Since it is a scalar, we have

\[ \phi'(x') = \phi(x). \]  

(B.2)

For an infinitesimal Lorentz transformation \( \delta x^\mu = \omega^{\mu\nu} x_\nu \) and using (B.2) we obtain

\[ \delta \phi(x) = \phi'(x) - \phi(x), \]
\[ = \phi'(x) - \phi'(x'), \]
\[ = -\omega^{\mu\nu} x_\nu \partial_\mu \phi(x) + 0(\omega^2). \]  

(B.3)

This is the transformation that characterizes scalar fields. Since \( L \) is also supposed to be a scalar it must transform in the same way. So what we have to do is to figure out a transformation for \( \pi \) in order that this occurs. Considering (B.1) and (B.3) we have

\[ \delta L = \delta \pi \dot{\phi} + \pi \delta \dot{\phi} + \partial_i \phi \partial^i \delta \phi - \pi \delta \pi, \]
\[ = -\omega^{\mu\nu} x_\nu \partial_\mu (\pi \dot{\phi} + \frac{1}{2} \partial_i \phi \partial^i \phi) + (\dot{\phi} - \pi) \delta \pi \]
\[ + \omega^{\mu\nu} x_\nu \partial_\mu \pi \dot{\phi} + \omega^{0i} \partial_i \phi (\dot{\phi} - \pi). \]  

(B.4)

Observing the expression above, one concludes that the transformation for \( \pi \) that renders \( L \) a scalar transformation is

\[ \delta \pi = -\omega^{\mu\nu} x_\nu \partial_\mu \pi - \omega^{0i} \partial_i \phi. \]  

(B.5)

We notice that it is the same transformation as \( \dot{\phi} \) as it must be.
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