Reforms of the University Mathematics Education for Non-mathematical Specialties

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Abstract

This article is a part of the report for the research project “Reform of the Course System and Teaching Content of Higher Mathematics (For Non-Mathematical Specialties)” in 1995, supported by the National Ministry of Education. There are thirteen universities participated in this project. The Report not only reflects results of our participants, but also includes valuable opinions of many colleagues in the mathematical education circles.

In this article, after a brief description on the history and reform situation of the higher mathematics education in China, attention concentrates to three aspects. They are: main problems in this field existing; the functions of mathematics accomplishment for college students; these concern course system, teaching and learning philosophy, such as overemphasized specialty education, overlooking to arouse rational thinking and aesthetic conceptions, etc. The last aspect contains a discussion on several important relationships, such as: knowledge impartment and quality cultivation, inherence and modernization of the mathematical knowledge, teacher’s guidance and students’ initiative, mathematical basic training and mathematical application consciousness and ability cultivation, etc.

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1. Introduction

This is a part of the report for the research project of “Reforms on the Course System and Teaching Content of Higher Mathematics (For Non-Mathematical Specialties)” in 1995, supported by National Ministry of Education. There are thirteen universities participated in this research project. They worked together cooperatively. Besides, many experts and scholars in mathematical and educational circles
had been consulted in many aspects. This essay not only reflects the research results of our participants-universities in more than four years, but also includes valuable opinions of many colleagues in the mathematical education circle.

2. Reform situation of the university mathematics education in China and the main problems existing

For the past half a century, our modern university education, accompanying the progress process of our socialist political and economic development, experiencing the stage of “learning from the Soviet Union in all respects”, many times of “educational reforms” and today’s “reform and opening to the world”, has grown in wave-like development and formed today’s scale. In this process, the teaching reform on the university non-mathematics class specialties higher mathematics has almost not been stopped. It can be said that mathematics teaching in this period, generally speaking, adapted itself to the requirements for training professionals in various fields under the planned economic system and made important contributions for the society. In this respect, several generations of educators of mathematical teaching devoted their wisdom and hardworking.

Historically, the focus of the reform of the university mathematical education in China is always concentrated on the problem of the combination of mathematics with practice. In a long period before the reforms and opening to the world, the basic task of the non-mathematics class specialties mathematical teaching is for specialties, or mathematics courses should serve courses of specialties. As mentioned earlier, in the tendency of the extreme situation of putting too much emphasis on specialties, such a saying is natural; but we went even further. In addition, we were in a situation of long-period of isolation. We didn’t know the situation of mathematics education in other countries, even were not clear on the reform in the Soviet Union. Without comparison, it is difficult to discover the problems. Moreover, different opinions on such a problem as “the relationship between the teaching of fundamental mathematics and that of specialties,” should have been a problem of academic views in the first place and can be discussed and tested; but such opinions were always improperly treated as ideological problems even the political ones. Therefore, it was difficult to express different views. Since the 1980s, our policies of the reform and opening to the world have brought spring for the educational reforms. When we learned of the changes in those decades of years, we realized that we had lagged behind and we should do our best to catch up. At that time, people found that, as a developing country, China was confronted with a series of serious challenges in the socialist market economy. In addition, the rapid appearance of the knowledge economy based on the intelligent resources and innovative competition, we are forced to reconsider problems, such as educational concepts, teaching systems, course systems, teaching content and methods etc., which are formed in the original planned economy.

Viewing from the angle of the university mathematics education, the existing
main problems may be summarized as follows:

First, the “specialty education” is overemphasized and thus a one-sided understanding of the role of the university mathematics education “serving specialties” is formed. Such an understanding, as the guiding idea of education, is embodied in every link of teaching. Due to its long duration, the depth and extension of its effects cannot be underestimated. Even now, among quite a large part of teaching cadres and teachers, such an understanding may still work, because in the past the problem of “what is the role of university mathematical courses in the university education” had not been studied properly.

Secondly, as the cultivation target was training “specialists and engineers who could work directly after their graduation,” the teaching process was speeded up. Moreover, we have the tradition of putting emphasis on the classroom teaching, and the course of mathematics itself has its own specific strict logical system, the instillation-type of teaching methods can be said to be deep-rooted.

Thirdly, as mathematics teachers of non-mathematical class specialties usually undertake heavy teaching tasks, and in the technological and engineering colleges and universities, the mathematical scientific research not being combined with practice directly is viewed as being isolated from practice, the teachers engaged in teaching basic courses of mathematics have not touched scientific research for a long time. Under this situation it is difficult to improve their professional skills and their teaching levels.

These problems, facing the serious challenges of the 21st century, cause the inadaptability of the university mathematics education to the current education development situation to be more serious. The results are mainly reflected in the following respects.

For students, too specific majors and their one-sided understanding of the role of basic courses lead to their narrow scopes of knowledge (especially knowledge on mathematics), narrow field of vision, lack of creation and all this is usually expressed in insufficient “aftereffects”.

For teachers, a single-pattern course content, rigid teaching plans and programs lead to the fact that they could only be responsible for textbooks and exams and it is difficult for them to attend to the cultivation of abilities and qualities of students. The development of teachers’ active roles and their growth are both influenced.

For textbooks, most of the content is rather old and their system is stereotyped. There is no mechanism to encourage teachers to edit new teaching textbooks and materials, which gives people an impression of “one thousand people having one face” and results in difficulties for students to grasp mathematical thinking and method and to learn new knowledge in mathematics.

3. Several aspects that should be grasped in doing a good job of our college mathematics education

In recent years, with the rapid development of our economy, higher education in our country has met with an excellent situation of brisk discussions and great
reforms. This time of educational reform, whether from the emphasis degree of the government, or from the realistic spirit of policies, the strength of the funds input and the depth and breadth of studying problems, they are all unprecedented. Just as the case in previous educational reforms, the reform of mathematics teaching is still among one of the key problems. What is different from the past is that the reform is not restricted to the long confusing problem of the relationship between mathematics and practice, but rather, the mathematics education is first put under the background of social development and is raised at the height of university quality education. The past is reviewed, the present is examined and the future is planned. In particular, the role of mathematics education in higher education has been clarified. There emerge schemes, new textbooks and new courses on mathematics course content, structure and system reforms that have creative intention and face the epoch. The research and practice on reforming teaching methods of mathematics and introducing new means of teaching have attracted the attention of broad masses of mathematics teachers; but the development is not balanced. There is not enough systematic research and comprehensive experiments. Especially, viewing from the great part of the circle, there are no great changes of the traditional mathematics teaching modes and methods. As for the renewal of the teaching content of mathematics, it is like “walking with the unsteady steps.” The main problem still lies in the change of educational ideology and teaching concepts. For the teaching of the basic mathematical courses in non-mathematics class specialties, in our opinion, it is necessary to clarify the following points in understanding.

(1) **It is necessary to have a rather comprehensive knowledge of the role of mathematics education in university**

In the educational mode in the planned economy, training of professionals in universities was of an upside-down type. The national plan decided the arrangement of specialties, and teaching programs were drawn up according to the requirements of specialties. Courses were arranged according to the procedure of specialty courses - fundamental specialty courses - basic courses. It was emphasized that the latter shall serve the former. For the basic education of mathematics, the excessive emphasis of the aspect of “serving specialties”, and the neglect of the inherent unity of mathematics as a rational reasoning system and the specific role of mathematics in the comprehensive quality of students, led to the lack of a comprehensive knowledge of the role of mathematics education in university education. In fact, mathematics is the common foundation for cultivating and training various levels of special professionals. For students of non-mathematics class specialties, the role of university basic mathematics courses lies at least in the following three aspects.

**It is the main course for students to grasp mathematical tools.** Such a role is very important for students of non-mathematics class specialties and is an important content of “specialty quality.” The current problems are as follows: on the one hand, teachers should study how to effectively enable students to grasp and use this tool in the whole course of mathematical teaching and how to lay a foundation in this field in the stage of basic courses; on the other hand, it is necessary to prevent the narrow understanding of “tool”, viewing the basic mathematics course only as the tool for serving certain specialty courses, and even the tool specifically
for dealing with certain examinations.

**It is an important carrier for students to cultivate their rational thinking.** What mathematics studies is the model structures of “numerals” and “forms”, and what it uses is such rational thinking methods as logic, reasoning and deduction. Large amounts of facts show that it is not a useless theory that is “isolated from practice”, but is a thinking creation, which originates from practice and guides practice. The role of such a training of rational thinking could hardly be replaced by other courses. Such a cultivation of rational thinking is of utmost importance for students to improve their quality, to enhance their analytical abilities and to enlighten their creation consciousness. The current problems lie in that we lack the mature teaching experience and excellent textbooks in this respect.

**It is a way for students to receive the nurture of beauty sense.** The mathematical aesthetics is part of people’s quality of the appreciation of beauty. With the development of human civilization and the progress of science, such a fact is being gradually recognized by people. In fact, targets mathematics is striving for — the arrangement of chaos into order, the sublimation of experience to laws and the search for the concise and uniform mathematical expression of motions of substances — are the embodiment of mathematical beauty, and are also man’s pursuit for beauty sense. Such a pursuit acts as a subtle influence for the nurture of man’s mental world and is always a motive force of innovation. At present we can only say that it is necessary to pay attention to the role of mathematics in aesthetic education. And further search and attempts should be made in its embodiment in teaching and textbooks.

The roles of the three aspects are unified, but in concrete requirements, according to the different kinds of colleges, specialties and students, there should be different sides to be emphasized, so that the comprehensive realization of the knowledge, abilities and quality in mathematics teaching can be expected.

**2) Emphasis should be put on the solution of problems of the renewal of college mathematics course system and content**

From the abolition of the imperial civil examination system at the end of Qing Dynasty, the establishment of modern school and up to the liberation of China, the education of science and technology in colleges and universities basically followed modes in Europe and America. For the content of mathematical teaching, before the 1950s, mathematics as a compulsory course in most specialties of science and technology except in few specialties, such as physics, was only limited to simple calculus and differential equations. In the 1950s, the Soviet modes were comprehensively introduced in higher education institutes. A kind of specialty education with the aim of training professional talents according to requirements of the trades was formed. The arrangement of the courses in the specialties was aimed at the requirements of the professional knowledge. Basic courses such as mathematics were required to serve specialty courses. Such a system and guiding ideas have actually continued till now. Though in the 1980s, due to the need to employ computers, linear algebra (taking calculation as the chief content) and numerical methods became compulsory mathematics courses, viewing from the guiding ideas, the main point was still “specialty education” and the changes of teaching content were very
limited. Later, there occurred improper evaluations, in addition to the influences of "examination-oriented education" induced by taking the post-graduate entrance examinations, the tendency of one-sided pursuit of the problem-solving techniques was promoted. As a result, the train of thought about reforms became more vague. However, due to the rapid development of the computer technique in the later half of last century, people had more knowledge on the role of modern mathematics. Especially in the recent 30 years, there appears the so-called “modern mathematics technique” which displays its prowess fully in economy and industries. For example, optimization, engineering control, information processing, fuzzy recognition and image reconstruction, etc. they are produced due to the combination of the principles and methods of modern mathematics with computers. They penetrate and are applied in all sectors and trades, and are combined with relevant techniques to form the so-called high and new techniques in those areas. In current developed countries, mathematics is applied to improve the organizational level of economy, and from drawing up macroscopic strategic planning to the storage, distribution and transportation of products and to the market prediction, analysis of finance and insurance businesses, significant progress has been attained. All the facts mentioned above mean that mathematics has turned from part of the traditional natural sciences and engineering techniques to further penetrating into many areas of the modern society and economy and has gradually become one of their indispensable columns. On the other hand, the development of science and technology brings about a series of problems and it is necessary for people to reexamine the relationship between man and society and between man and Nature in a rational way. All this requires that training on the tool property and rationality should be emphasized in college mathematics education. Therefore, it is an urgent task to adjust the basic course system of college mathematics and appropriately renew teaching content.

(3) Reforming the examination-oriented “instillation-type” teaching

In recent years, various kinds of factors have induced a type of examination-oriented teaching. Marks play almost decisive roles for students in many respects such as evaluation in the prizes, graduate entrance examinations and the assignment for jobs. In some occasions, students’ marks are the main factor for evaluating teachers. Such a situation causes teachers to teach for exams and students study for exams. In order to raise the average marks of a class, teachers spend a lot of time and energy on the teaching of problem patterns. Accordingly, the “instillation-type” of teaching methods are in a dominating position in our college teaching. Characteristics of such methods are as follows: on the one hand, various kinds of problem patterns are explained in every detail in the classroom and efforts are made to enable students to understand them as soon as they listen to the explanations. In such a way, besides the small amount of classroom information quantity, the psychology of dependence of the students will certainly be enhanced and they will have a bad habit of being lazy in thinking, which will seriously hinder the cultivation of the innovation sense and innovation capability. On the other hand, students are busy problem solving after the class and problems are substituting learning. They rarely read books and pay little attention to the mathematical thinking and
mathematical applications, let alone the cultivation of the innovation sense.

(4) Devoting great efforts to the construction of teachers’ teams

This is the crux matter of our reform. In recent years, compared to the past, the situation of teachers’ teams engaged in non-mathematics class specialty mathematics teaching has turned to the better. Quite a few of young teachers who have obtained Doctor’s degrees join the career. In common colleges and universities, proportions of mathematics teachers who have gained Master’s degrees are increasing. This is an encouraging phenomenon, indicating that there will be qualified successors to carry on mathematics teaching, but the authorities of colleges and universities have not timely put such a strategic task at a deserved height. For example, knowing clearly that the experienced teachers with high academic levels should be appointed to teach the basic courses, some colleges and universities still classify the teachers according to the order of the postgraduate teaching, senior students teaching, and the basic courses teaching. Such a guiding direction is very unfavorable for cultivating high-quality talents. In addition, viewing from the young teachers’ teams, there also exists something worrying. First, their ideas are not so stable; secondly, they cannot deal well with the relationship between scientific research and teaching. In the key universities, there usually exists the atmosphere of paying more attention to scientific research than that to teaching. In common colleges and universities, there always exists the problem of only teaching without scientific research. Therefore, creating certain conditions and asking the young teachers to participate in scientific research are important measures to improve the levels of college mathematics teachers. Thirdly, not enough efforts have been made in conducting education on teacher morality and on superior teaching traditions for the young teachers.

4. Several important relationships in the university mathematics education reforms

The college mathematics education reform is a very careful job. The treatment of many problems in this respect will not only rely on principles, but also be flexible. According to the experience and lessons from the past mathematics education reform, I would like to discuss some problems concerning principles in mathematics teaching reform from the correct treatment of the relationships in several aspects as follows:

(1) Embodying quality education, paying attention to well treating the relationships between knowledge impartment and quality cultivation

In recent years, there are a large amount of discussions on “quality” and “quality education”. We assume that quality is a mental and physical attribute expressed when people understand and treat things and events. It is based on the congenital psychological conditions and is gradually formed under the influence of the postnatal environment. Viewing from the angle of education, an individual’s quality in a certain aspect is shown in his power of understanding and potential energy in such an aspect. Quality education is a process during which excellent qualities of people are cast through the smelting of systematic knowledge. Any objective ex-
istence in the world has its attribute characters of “numerals” and “forms”. With
the progress of the science of mathematics, people’s knowledge of “numerals” and
“forms” has enhanced from directly-visual quantity relationships and space forms
to the abstract “mathematical structures” and “space concepts” with deeper conno-
tation and wider extension. Such attributes of “numerals” and “forms” in people’s
understanding things and their comprehension and potential ability in handling the
corresponding relationships are obviously a kind of people’s quality and we call such
a quality mathematical quality. Under the background of the tendency of digital-
ization and informationalization in the knowledge economy society, the importance
of possessing such a quality is very great. In the quality education, viewing from
the main body of education (i.e. the main targets of education and corresponding
teaching behaviors), the soul of the college mathematics education is exactly the
mathematical quality, and is what has been discussed in the previous part — qual-
ity property. Viewing from mathematics itself, especially the modern mathematics,
itself is a rational thinking system extracted abstractly from large amounts
of objective phenomena. Therefore, in its education, besides displaying how it ab-
sorbs nutrition from laws of vivid objective things and provides tools, main focus is
the training of the rational thinking techniques and of the mastering mathematical
tools. That is the content of the education of mathematical quality. Though it
imparts “knowledge” as well, “knowledge” of mathematics acts as a carrier of im-
parting quality (such a role was always neglected by people in the past), besides one
side that it is combined with the material content and thus acts as a tool to serve
other disciplines. The quality education in the mathematical teaching is that the
teachers put the lively and rational ways of thinking via the knowledge carrier to
implement the motivational, psychological and mental guidance for their students.

Mathematics has become a fundamental component of the culture of the con-
temporary social culture via its ability of tools, rational spirit and sense of beauty.
In the society of the 21st century if one has no idea what mathematical technology
means, lacks both the rational thinking and the sense of beauty appreciation; then
his total quality will be affected. His abilities in insight, judgement and originality
would be greatly restricted. Therefore, the accomplishment of mathematical culture
is not a kind of “fashion” for one in the ever-increasingly fierce competition among
talents of the society; it is indeed a kind of actual necessity for one in his work,
study and social communication.

The specific content of the mathematical quality is quite rich, the most out-
standing specific characteristics that are generally acknowledged are summarized as
follows:

- The sharp consciousness of extracting the attributes of “numerals and forms”
of things.
- Thinking mode of using “abstract models and structures” to study things.
- Exploring habits of conducting compact deduction by means of signs and
  logical systems.

Features of mathematical qualities in these aspects are connected with each
other and cannot be separated in the actual educational processes. All college stu-
dents should be cultivated and educated in such aspects. Of course, the education
and cultivation of mathematical qualities are different in emphasis points, depth, and breadth between mathematics major students and non-mathematics class specialties students, and in non-mathematics class specialties, such differences do exist between students of science, engineering, agriculture and medicine and those of humanities majors.

(2) Grasping course systems and content renewal and treating well relationships between the inherence and modernization of the mathematical knowledge

The main body of the textbooks of current basic mathematics courses is mostly mathematics before the 19th century. This is in contrast with other basic textbooks of physics, chemistry and biology that began mostly from the 19th century. Therefore, the problem of the modernization of basic university mathematics textbooks is to be stressed. But we should be cautious in this problem. It is unavoidable to appropriately add the commonly accepted and fundamental content of modern mathematics; but it is necessary to consider another aspect: as we have mentioned above, mathematics is a kind of thinking science and has its own specialties. Its system is constructed by logic and is a structure of series established one layer after another from the bottom to the top. The mathematical knowledge important in the past is the “logical basis” of the current mathematics and throwing away the former will influence the later studies. For example, calculus has a history of over three hundred years, but it still is the foundation stone of modern mathematics and can’t be thrown away casually. Such a situation differs from other sciences. For other sciences, if the theories before the 19th century are out-of-date, they can be cancelled, though proceeding and subsequent knowledge has inherent property. The preceding knowledge is not necessarily the direct logical foundation of the subsequent knowledge, so the abandonment of the preceding knowledge will not influence the learning of the subsequent knowledge. In addition, there is still an important reason. Such a part of mathematics content, for example, calculus, has still a rather wide application today. Therefore the idea of “modernization” of the content of college mathematics textbooks is somewhat different from the modernization of other disciplines and thus how much modern content has been listed in the textbook cannot be simply taken as the standard of measurement. We assume that the content modernization of the college mathematics textbooks can mainly include the following aspects. First, the classical mathematics content should be governed by viewpoints and languages of modern mathematics as far as possible; at the same time “roots” in the classical mathematics for some modern mathematics should be appropriately introduced. Secondly, significant results of modern mathematics that have formed the basic part of relevant disciplines should be put into textbooks as far as possible and a popular introduction should be made. Lastly, it is necessary for students to possess essential basis of modern mathematics necessary for students to further study relevant specialties by themselves.

What is similarly important is to cancel some contents such as reasoning loaded down with trivial details and those calculations that can be done by calculators, and concepts and methods that are relatively old and thus have no development prospect in modern science. In summary, facing an accumulation of nearly two
centuries, how to deal with the “metabolism” of the basic content of mathematics is a quite difficult problem. Depending only on “outside extension type” to increase academic hours will not work and efforts should be made on reforms on the structure and content of the course.

(3) Paying attention to the reform of teaching methods and handling well relationships between teachers’ guidance and students’ initiative

Once the teaching system and content of the course of college mathematics have been determined, teaching methods become the key problem of teaching quality. Obviously, the teaching process of basic mathematics is absolutely not man’s repetition of cognition processes for numerals and forms laws, but basic laws of cognition processes should be observed. In addition, the teaching process cannot be the simulation of research process of mathematical problems, but should embody research methods and thinking modes specific for mathematics. The teaching of mathematics includes two sides of teaching and learning and is a comprehensive process in which many teaching activities guided by teachers lead to the initiative of students to study mathematics. The so-called initiative has at least the following meanings. First, students have the interest and dynamic force to learn mathematics. Secondly, students can learn by themselves under the proper guidance of teachers. Thirdly, students can work at and put forward problems independently during the study process. Fourthly, students can use the knowledge their teachers impart and related knowledge gained through self-study after processing and digestion to construct a knowledge system that, even if it is simple and not perfect, actually belongs to themselves.

(4) Emphasizing the practical links of mathematics and paying attention to handling well relationships between mathematical basic training and mathematical application consciousness and ability cultivation

The widespread use of computers and the emergence of a series of powerful mathematical software systems have resulted in profound changes in the roles of mathematics and in mathematical teaching. They make it possible to collect and process large amounts of data, also make mathematical model a means of experiment, and thus greatly promote applications of mathematics in all fields. The combination of mathematical thinking with computers has become an important mode of modern mathematical teaching. For example, since the establishment of the course of “mathematical models” in colleges in the 1980s, there have been hundreds of colleges and universities where this course has been set up, and the course is a favorite of students. Especially the annual national contest of college mathematics model-building attracts students of various specialties and promotes reforms in mathematics teaching. At present, the course “mathematical experiments” aiming at strengthening practice links of mathematics is being tested in several colleges and universities and the preliminary results are encouraging.