Tricritical point in strongly coupled U(1) gauge theory with fermions and scalars

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We investigate the tricritical point in the lattice fermion–gauge–scalar model with U(1) gauge symmetry. In the vicinity of this point, in the phase with the broken chiral symmetry, we observe the scaling behavior of the chiral condensate and of the masses of composite fermion and composite scalar, indicating the existence of an interesting continuum limit of the model at this point.

1. Introduction

The lattice 4D fermion-gauge scalar ($\chi U \phi^4$) model with U(1) gauge symmetry has at quite strong gauge coupling, $\beta \simeq 0.64$, a tricritical point [1,2]. The nature of the continuum limit taken at this point is not known. When this point (point E in fig. 1) is approached from the phase with spontaneously broken global chiral symmetry and a massive fermion ($F$), the mass $am_F$ scales. The fermion $F = \phi^\dagger \chi$ is composed of the fundamental fermion ($\chi$) and scalar ($\phi$) fields and is unconfined. This raises the hope that a nonperturbatively renormalizable theory with dynamical fermion mass generation might be obtained [3].

Motivated by this interesting possibility we have started an investigation of the scaling behavior of various observables in the vicinity of the point E. We find that apart from $am_F$ also the mass $am_S$ of a composite scalar boson $S = \sum_{i=1}^3 \phi^\dagger_{x,t} U(x,t) \phi_{x+\vec{t},t}$ scales in the vicinity of E. This, as well as some other properties [4] make the point distinctly different from the (nonrenormalizable [5]) Nambu–Jona-Lasinio model found in the strong coupling limit of the $\chi U \phi^4$ model [4].

This, as well as the experience that tricritical points give rise to the scaling behavior different from that associated with the adjacent critical points [5], contributes to the hope that the point E might correspond to a renormalizable theory.

As reported in [6], in 2D an analogous $\chi U \phi^2$ model seems to belong to the universality class of the 2D Gross-Neveu model and is thus probably renormalizable.

2. The model

The action is

$$S_{\chi U \phi} = S_\chi + S_U + S_\phi ,$$

where

$$S_\chi = \frac{i}{2} \sum_x \sum_{\mu=1}^4 \eta_{\mu x} \bar{\chi}_x \left[ U_{x,\mu} \chi_{x+\mu} - U^\dagger_{x-\mu,\mu} \chi_{x-\mu} \right] + am_0 \sum_x \bar{\chi}_x \chi_x ,$$

$$S_U = \beta \sum_P \left[ 1 - \text{Re} U_P \right] ,$$

$$S_\phi = -\kappa \sum_x \sum_{\mu=1}^4 \left[ \phi^\dagger_{x,\mu} U_{x,\mu} \phi_{x+\mu} + \text{h.c.} \right] .$$

Here $U_P$ is the plaquette product of link variables $U_{x,\mu}$ and $\eta_{\mu x} = (-1)^{x_1+\cdots+x_\mu-1}$. The gauge field link variables $U_{x,\mu}$ are elements of the compact gauge group U(1). The complex scalar field $\phi$ of charge one satisfies the constraint $|\phi| = 1$. The hopping parameter $\kappa$ vanishes (is infinite) when the squared bare scalar mass is positive (negative) infinite.

The staggered fermion field $\chi$ of charge one leads to the global U(1) chiral symmetry of the
Figure 1. Schematic phase diagram of the $\chi U\phi_4$ model with U(1) gauge symmetry. Three critical lines, NE, EE$_\infty$ and EE$_{-\infty}$ meet at the tricritical point E. The NE line is a part of the boundary of the Nambu phase (shadowed region) at $m_0 = 0$, which is actually a sheet of 1$^\text{st}$ order phase transitions, across which $\langle \nabla \chi \rangle$ changes sign. The lines EE$_\infty$ and EE$_{-\infty}$ form a boundary of the “wings” (dashed sheets), which are sheets of 1$^\text{st}$ order phase transitions, corresponding at large $\beta$ to the Higgs phase transition. The dotted part of the $m_0 = 0$ plane is the region of vanishing fermion mass, $am_F = 0$. The vertical sheets containing the points T and C are 1$^\text{st}$ order transitions separating the confinement and Coulomb phases. Line ET is a line of 1$^\text{st}$ order triple points.

model in the chiral limit, i.e. when the bare fermion mass $m_0$ vanishes.

The model is meant in the chiral limit, $m_0 = 0$. However, numerical simulations require nonvanishing $m_0$ and thus also analytic considerations have to include the bare mass. We therefore show in fig. 1 schematic phase diagram of the model in the $(\beta, \kappa, am_0)$ space. It illustrates that the tricritical point E is indeed a point in which three critical lines meet:

- Line NE where $am_F$ and Goldstone boson mass $am_\pi$ simultaneously smoothly vanish.
- Lines EE$_\infty$ and EE$_{-\infty}$ with vanishing $am_S$.

The Nambu phase below the NE line at $am_0 = 0$ is characterized by $am_F > 0$ and $am_\pi = 0$.

The limit cases are

- $\beta = \infty$: 4D XY model and free fermion field.
- $\beta = 0$: Nambu-Jona-Lasinio model with bare mass.
- $m_0 = \pm \infty$: U(1) Higgs model without fermions.
- $\kappa = 0$: compact QED with charged fermion field.
- $\kappa = \infty$: free fermion field.
3. Indications of scaling behavior

As is apparent from fig. 1, an approach to the tricritical point requires in principle tuning of three parameters. Obviously also the lattice sizes have to be varied. But the tricritical points have frequently rather large domains (angles) of dominance [5]. Therefore it is reasonable to fix $\beta = 0.64$, which is our current best estimate of the position of the point E in the $\beta$ direction, and vary only $am_0$ and $\kappa$. Even so an investigation of the scaling behavior is a tremendous task. We can present only some preliminary results obtained for $am_0 = 0.06, 0.04, 0.02$ and, rather incomplete, also for $am_0 = 0.01$, without a proper analytic analysis and in particular without taking into account finite size effects.

In fig. 2 we show the chiral condensate $\langle \chi^2 \rangle$. An onset of the genuine chiral phase transition is indicated with decreasing $am_0$. The fermion mass $am_F$ (fig. 3) behaves similarly. The transition appears to be smooth, i.e. of 2$^{\text{nd}}$ order, close to $\kappa \approx 0.305$.

In fig. 4 we show the scalar boson mass $am_S$. The position of the minimum for each $am_0$ corresponds roughly to the crosssection of a continuation to smaller $\beta$ of the nearly horizontal “wing”
with the $\beta = 0.64$ plane. As $am_0$ decreases, the minimum seems to approach the $\kappa$ value at which the chiral phase transition at $am_0 = 0$ is expected on the basis of figs. 2 and 3. Thus we are indeed in a close vicinity of the tricritical point.

Fig. 5 contains very preliminary results for several $\chi$ (meson) masses. For comparison we show also data for $2am_F$. The Goldstone boson mass $am_\pi$ is small around the transition as expected. The fact that the $\rho$- and $\sigma$- masses cross the $2am_F$ line in the vicinity of the phase transition indicates that in the Nambu phase these mesons can be interpreted as bound states and that their masses scale similarly to $am_F$ and thus may exist also in the continuum limit. This is a hint that the spectrum of the underlying continuum theory might be quite complex.

Our present results suggest that it is worthwhile to continue the investigation of the tricritical point E. For this purpose the full scaling theory of tricritical points and a finite size scaling theory in their vicinity will have to be applied. Some analytic insight into the properties of the model in the relevant coupling region is highly desirable.

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