Determining the CP of a Higgs Particle at a Future Linear Collider

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In a more general electroweak theory, there could be Higgs particles that are odd under CP. Correlations among momenta of the initial electron and final-state fermions are in the Bjorken process sensitive to the CP parity. Monte Carlo data on the expected efficiency demonstrate that it should be possible to verify the scalar character of an intermediate-mass Standard Model Higgs boson after three years of data taking at a future linear collider. This is most likely not possible at LEP2. Signals of possible presence of CP violation in the Higgs sector are briefly discussed.

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1 Introduction

One of the main purposes of accelerators being planned and built today, is to elucidate the mechanism of mass generation. In the Standard Model mass is generated via an $SU(2)$ Higgs field doublet, associated with the existence of a Higgs particle, whereas in more general models there are typically several such Higgs fields, and also more physical particles.

When some candidate for the Higgs particle is discovered it becomes imperative to establish its properties, other than the mass. While the Standard Model Higgs boson is even under $CP$, extended models may include pseudoscalar Higgs bosons. An example of such a theory is the minimal supersymmetric model ($MSSM$) [1], where there is a neutral $CP$-odd Higgs boson, often denoted $A^0$ and sometimes referred to as a pseudoscalar.

In the context of Higgs production via the Bjorken mechanism [2], we have recently [3] investigated how angular distributions may serve to disentangle a scalar Higgs candidate from a pseudoscalar one. In trying to probe the uniqueness of the scalar character of the Higgs boson as provided by the Standard Model, we have to confront its predictions with those provided by possible extensions of the Standard Model. There is also the possibility that $CP$ violation may be present in the Higgs sector, as first pointed out by Weinberg [4]. We briefly discuss some possible signals of such effects.

Below we postulate an effective Lagrangian which contains $CP$ violation in the Higgs sector. In cases considered in the literature, $CP$ violation usually appears as a one-loop effect. This is due to the fact that the $CP$-odd coupling introduced below is a higher-dimensional operator and in renormalizable models these are induced only at loop level. Consequently we expect the effects to be small and the confirmation of presence of $CP$ violation to be very difficult. Although there may be several sources of $CP$ violation, including the CKM matrix [5], we will here consider a simple model where the $CP$ violation is restricted to the Higgs sector and in particular to the coupling between some Higgs boson and the vector bosons. Specifically, by assuming that the coupling between the Higgs boson $H$ and the $Z$ has both scalar and pseudoscalar components, the most general coupling for the $HZZ$-vertex relevant for the Bjorken process may be written as [6, 7]

\[ i \frac{2^{5/4}}{\sqrt{G_F}} \left[ m_Z^2 g^{\mu\nu} + \xi \left( k_1^2, k_2^2 \right) \left( k_1 \cdot k_2 g^{\mu\nu} - k_1^\mu k_2^\nu \right) + \eta \left( k_1^2, k_2^2 \right) \epsilon^{\mu\nu\rho\sigma} k_1^\rho k_2^\sigma \right], \]

with $k_j$ the vector boson momentum, $j = 1, 2$. The first term is the familiar $CP$-even $Z^\mu Z^\nu H$ tree-level Standard Model coupling. The second term stems from the dimension-5 $CP$-even operator $Z^{\mu\nu} Z_{\mu\nu} H$ with $Z_{\mu\nu} = \partial_{\mu} Z_{\nu} - \partial_{\nu} Z_{\mu}$. The last term is $CP$ odd and originates from the dimension-5 operator $\epsilon^{\mu\nu\rho\sigma} Z_{\mu\nu} Z_{\rho\sigma} H$. Simultaneous presence of $CP$-even and $CP$-odd terms leads to $CP$ violation, whereas presence of only the last term describes a pseudoscalar coupling to the vector bosons. The higher-dimensional operators could be radiatively induced and are likely to be small.

Related studies on how to discriminate $CP$ eigenstates have been reported by [6–14].
We demonstrate that a cut on the angle between the outgoing Z and the incoming electron alters the azimuthal angular distributions significantly [3]. Our study also addresses the problem of separating the signal from the background. Furthermore, we include Monte Carlo data, exploiting the background study by Barger et. al. [10], and demonstrate that it should be possible to verify the scalar character of the Standard Model Higgs after three years of running at a future linear collider. We also study the energy spectrum of one of the final-state fermions in the Bjorken process, as recently suggested [12] in connection with Higgs decay via vector bosons to four fermions. We compare the relative usefulness of the angular and energy distributions.

2 Distinguishing CP eigenstates

We compare the production of a Standard-Model Higgs (h = H) with the production of a ‘pseudoscalar’ Higgs particle (h = A) via the Bjorken mechanism,

\[ e^- (p_1) e^+ (p_2) \rightarrow Z (Q) h (q_3) \rightarrow f (q_1) \bar{f} (q_2) h (q_3). \] (2)

The couplings of H and A to the vector bosons are given by retaining only the first and last term in (1), respectively.

Let the momenta of the two final-state fermions and the initial electron (in the overall c.m. frame) define two planes, and denote by \( \phi \) the angle between those two planes; i.e.

\[ \cos \phi = \frac{(p_1 \times Q) \cdot (q_1 \times q_2)}{|p_1 \times Q||q_1 \times q_2|}. \] (3)

We shall discuss the angular distribution of the cross section \( \sigma \),

\[ \frac{1}{\sigma} \frac{d\sigma}{d\phi} \] (4)

both in the case of CP-even and CP-odd Higgs bosons.

The fermion-vector couplings are given by \( g_V \) and \( g_A \). As a parameterization of these, we define the angle \( \chi \) by \( g_V \equiv g \cos \chi \) and \( g_A \equiv g \sin \chi \). In the present work, the only reference to this angle is through \( \sin 2\chi \) (see table 1 of ref. [8]). The distributions of eq. (4) take the form

\[ \frac{2\pi}{\sigma_H} \frac{d\sigma_H}{d\phi} = 1 + \alpha(s, m) \cos \phi + \beta(s, m) \cos 2\phi, \] (5)

\[ \frac{2\pi}{\sigma_A} \frac{d\sigma_A}{d\phi} = 1 - \frac{1}{4} \cos 2\phi. \] (6)

The coefficients \( \alpha(s, m) \) and \( \beta(s, m) \) given in [8] will at very high energies vanish as \( s^{-1/2} \) and \( s^{-1} \), respectively. Therefore, the Standard-Model distribution (5) will asymptotically
at high energies become flat, whereas the $CP$-odd distribution in eq. (3) is independent of energy and Higgs mass. A representative set of angular distributions is given in fig. 2 for the case $e^+e^- \rightarrow \mu^+\mu^-h$ for both LEP2 and higher energies, and for different Higgs masses. There is seen to be a clear difference between the $CP$-even and the $CP$-odd cases.

![Figure 1](image.png)

**Figure 1.** Angular distributions of the planes defined by incoming $e^-$ and final-state fermions for a $CP$-even Higgs particle (solid) compared with the corresponding distribution for a $CP$-odd one (dashed). Different energies and masses are considered in the $CP$-even case. We assume $\sqrt{s} = 200$ and 500 GeV at LEP2 and NLC, respectively.

Experimentally, however, one faces the challenge of contrasting two angular distributions with a restricted number of events and allowing also for background. We shall here focus on the intermediate Higgs mass range; more specifically, we consider $m \lesssim 140$ GeV where the Higgs decays dominantly to $b\bar{b}$. The main background will then stem from $e^+e^- \rightarrow ZZ$ and also $e^+e^- \rightarrow Z\gamma, \gamma\gamma$. The cleanest channel for isolating the Higgs signal from the background is provided by the $\mu^+\mu^-$ and $e^+e^-$ decay modes of the $Z$ boson.

Let us next limit consideration to the energy range $\sqrt{s} = 300−500$ GeV, as appropriate for a linear collider [15], henceforth denoted NLC. We impose reasonable cuts and constraints as described in [10]; e.g. $|m_{\mu^+\mu^-} - m_Z| \leq 6$ GeV and $|\cos\theta_Z| \leq 0.6$. The signal for
\[ e^+e^- \rightarrow ZH \rightarrow \mu^+\mu^-b\bar{b} \] will then be larger than the background \( e^+e^- \rightarrow ZZ \rightarrow \mu^+\mu^-b\bar{b} \) by an order of magnitude. In the following we shall thus neglect the background in the discussion of (3) versus (4). With \( \sigma(e^+e^- \rightarrow ZH) \sim 200 \text{ fb} \) and an integrated luminosity of 20 fb\(^{-1}\) per year \([10]\), about 4000 Higgs particles will be produced per year, in this intermediate mass range. However, following \([10]\) we have only \( \sim 30 \) signal events \( e^+e^- \rightarrow ZH \rightarrow \mu^+\mu^-b\bar{b} \) left per year for e.g. a NLC operating at \( \sqrt{s} = 300 \text{ GeV} \) and a Higgs particle of mass \( m = 125 \text{ GeV} \). In the case \( e^+e^- \rightarrow ZH \rightarrow e^+e^-b\bar{b} \) we also have a t-channel background contribution from the \( ZZ \) fusion process \( e^+e^- \rightarrow e^+e^- (ZZ) \rightarrow e^+e^- H \). This contribution may be neglected at LEP energies, but it is comparable to the s-channel contribution at higher energies. However, this contribution can be suppressed by imposing a cut on the invariant mass of the final-state electrons, e.g. \( |m_{e^+e^-} - m_Z| \leq 6 \text{ GeV} \). Hence, we can effectively treat the electrons on the same footing as the muons, thereby obtaining a doubling of the event rate.

Imposing the cut \( |\cos \theta_Z| \leq b \), the predictions for the azimuthal correlations of eqs. (3)–(6) get modified. For the \( CP \)-even case we find

\[
\alpha^b(s, m) = \sin 2\chi \sin 2\chi_1 \left( \frac{3\pi}{4} \right)^2 \frac{\sqrt{s} m_Z (s + m^2_{Z} - m^2)}{\xi(b) \lambda(s, m^2_{Z}, m^2) + 12 s m^2_{Z}} \zeta(b),
\]

\[
\beta^b(s, m) = \frac{2 \xi(b) s m^2_{Z}}{\xi(b) \lambda(s, m^2_{Z}, m^2) + 12 s m^2_{Z}},
\]

where \( \lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz) \) is the Källen function and

\[
\xi(b) = \frac{1}{2} \left( 3 - b^2 \right), \quad \xi(1) = 1,
\]

\[
\zeta(b) = \frac{2}{\pi} \left( \frac{\pi - \arccos b}{b} + \sqrt{1 - b^2} \right), \quad \zeta(1) = 1,
\]

whereas for the \( CP \)-odd case

\[
-\frac{1}{4} \rightarrow -\frac{\xi(b)}{3 + b^2}.
\]

In order to demonstrate the potential of the NLC for determining the \( CP \) of the Higgs particle, we show in fig. 3 the result of a Monte Carlo simulation. For this purpose we have used PYTHIA \([16]\), suitably modified to allow for the \( CP \)-odd case. The statistics correspond to 3 years of running\([6]\) using both the \( \mu^+\mu^- \) and \( e^+e^- \) decay modes of the \( Z \) boson. This yields about 200 events in these channels. Although the cut \( b = 0.6 \) makes \( \alpha \) increase as shown in \([4]\), the \( \cos \phi \) term is still too small to show up in the Monte Carlo simulation. For \( \sqrt{s} = 300 \text{ GeV} \) and \( m_H = 125 \text{ GeV} \), the ‘bare’ prediction \([3]\) for \( \beta \) is 0.12. The cut \( b = 0.6 \) increases it slightly to 0.14. Similarly, the ‘\(-1/4\)’ of \([3]\) changes

\[\footnote{The event rate is based on the Standard Model, and could be different for a non-standard Higgs sector.}\]
Figure 2. Monte Carlo data displaying the angular distribution of events $e^+e^- \to ZH \to l^+l^- b\bar{b}$, $l = \mu, e$ for a Standard-Model Higgs versus a CP-odd one. We have taken $\sqrt{s} = 300$ GeV, $m = 125$ GeV, and an angular cut $|\cos \theta| \leq b = 0.6$.

significantly to $-0.39$. Consequently, the cut makes it easier to discriminate between the CP-even distribution and the CP-odd one. From fig. 3 we see that the individual angular Monte Carlo distributions are consistent with the predictions, showing that a three-year data sample is large enough to reproduce the azimuthal distributions. In the Standard-Model case the fit gives $0.92 \pm 0.07$ and $0.2 \pm 0.1$ for the predictions 1.00 and 0.14, respectively, with $\chi^2 = 1.0$. In the CP-odd case the fit gives $0.94 \pm 0.07$ and $-0.4 \pm 0.1$ for the predictions 1.00 and $-0.39$, respectively, with $\chi^2 = 0.7$. More importantly, since the $\cos 2\phi$ terms are more than 4 standard deviations away, a data sample of this size is sufficient to verify the scalar nature of the Standard-Model Higgs. Using likelihood ratios, as described in [17], for choosing between the two hypotheses of CP even and CP odd, we find that less than 3 years of running suffices using similar criteria.

An alternative test has recently been suggested by Arens et. al. [12] in the context of Higgs decaying via vector bosons to four fermions, where one studies the energy spectrum of one of the final-state fermions. Applying this idea to the Bjorken process one would study the energy distribution of an outgoing fermion, e.g. $\mu^-$ or $e^-$. Introducing the
scaled lepton energy, \( x = 4E_l^-/\sqrt{s} \), \( l = \mu, e \), we shall consider the energy distribution of the cross section with respect to this final-fermion energy,

\[
\frac{1}{\sigma} \frac{d\sigma}{dx}
\]

both in the case of \( CP \)-even and \( CP \)-odd Higgs bosons. We are using the narrow-width approximation and the range of \( x \) is given by \( x_- \leq x \leq x_+ \), with \( sx_\pm = s + m_Z^2 - m^2 \pm \sqrt{\lambda} \). Here the distributions are given as second-degree polynomials in \( x \), and, as shown in [4], the coefficients have a non-trivial dependence on the c.m. energy and the Higgs mass, also for the \( CP \)-odd case. A representative set of energy distributions is given in fig. 3 for the case \( e^+e^- \rightarrow \mu^+\mu^- h \) for both LEP2 and NLC energies. There is a clear difference between the \( CP \)-even and the \( CP \)-odd cases.

![Figure 3. Characteristic distributions for the scaled energy of the \( l^- \), \( l = \mu, e \) in the Bjorken process \( e^+e^- \rightarrow l^+l^- h \). Different energies and masses are considered.](image)

In fig. 3 we show the result of a Monte-Carlo simulation for the energy distribution eq. (14) analogous to the one in fig. 3. Again, a cut on the polar angle is imposed. As in the case of angular distributions, the cut makes it easier to discriminate between the \( CP \)-even distribution and the \( CP \)-odd one. Also in this case the data sample reproduces the predicted energy distributions. An analysis of the likelihood ratios demonstrates that less than 3 years of running is sufficient if we require the correct answer with a discrimination
by four standard deviations, but more events seem to be required than in the case of angular distributions.

3 CP violation

As previously mentioned, if we allow for both the Standard-Model and the CP-odd term in the Higgs-vector coupling (2), then there will be CP violation. This situation will be discussed here. It is similar to the case of Higgs decay discussed elsewhere [18]. We discard the higher-dimensional CP-even term since it is likely to be small.

In terms of the invariant mass $s_1$ of the fermion pair, and neglecting terms of $\mathcal{O}((\text{Im } \eta)^2)$, the distribution (4) can be written compactly as

$$
\frac{d^2 \sigma}{d\phi \, ds_1} = \frac{N_1}{144 \sqrt{2} (4\pi)^4} \frac{G_F}{s^2} \sqrt{\lambda(s, s_1, m^2)} D(s, s_1)
$$

$$
\times \left[ \lambda(s, s_1, m^2) + 4ss_1 (1 + 2\rho^2) + 2ss_1 \rho^2 \cos 2(\phi + \delta) \right]
$$

Figure 4. Monte Carlo data displaying the lepton energy distribution for events $e^+e^- \to ZH \to l^+l^-b\bar{b}$, $l = \mu, e$ for a Standard-Model Higgs versus a CP-odd one. We have taken $\sqrt{s} = 300$ GeV and $m = 125$ GeV.
\begin{align}
\sin 2\chi \sin 2\chi_1 \left( \frac{3\pi}{4} \right)^2 \sqrt{ss_1} (s + s_1 - m^2) \rho \cos(\phi + \delta) \right],
\end{align}

with a modulation function
\begin{align}
\rho = \sqrt{1 + (\text{Re} \eta)^2 \lambda(s, s_1, m^2)/(4m_Z^4)},
\end{align}

and an angle
\begin{align}
\delta = \arctan \frac{\text{Re} \eta(s, s_1) \sqrt{\lambda(s, s_1, m^2)}}{2m_Z^2}, \quad -\pi/2 < \delta < \pi/2,
\end{align}

describing the relative shift in the angular distribution of the two planes, due to CP violation. This rotation vanishes at the threshold for producing a real vector boson (where $\lambda = 0$) and, even for a fixed value of Re $\eta$, grows with energy (because of the $\sqrt{\lambda}$-factor).

This relation (13) can be inverted to give for the CP-odd term in the coupling:
\begin{equation}
\text{Re} \eta = \frac{2m_Z^2}{\sqrt{\lambda(s, s_1, m^2)}} \tan \delta.
\end{equation}

This result (14) is completely analogous to the one encountered for the decay of Higgs particles, eq. (12) of [18], if we interchange $\phi$ and $\pi - \phi$.

Above threshold for producing a real vector meson accompanying the Higgs particle, we may integrate over $s_1$ in the narrow-width approximation. Imposing the cut $|\cos \theta_Z| \leq b$, the distribution of eq. (4) takes the compact form
\begin{equation}
\frac{2\pi}{\sigma^b} \frac{d\sigma^b}{d\phi} = 1 + \alpha^b(s, m) \rho \cos(\phi + \delta) + \beta^b(s, m) \rho^2 \cos 2(\phi + \delta).
\end{equation}

with $\rho$ and $\delta$ given by eqs. (12) and (13), substituting $s_1 = m_Z^2$. The details of implementing a cut in polar angle are given in [3]. Any CP violation would thus show up as a “tilt” in the azimuthal distribution, by the amount $\delta$.

A representative set of angular distributions is given in fig. 3 for a broad range of Re $\eta$ values. We have considered a Higgs boson of $m = 200$ GeV accompanied by a $\mu^+\mu^-$-pair in the final state, produced at $\sqrt{s} = 500$ GeV. We observe that for Re $\eta \lesssim 0.1$ and Re $\eta \gtrsim 5$, the deviations from the CP-even and CP-odd distributions, respectively, are small. Experimentally it will be very difficult to disentangle two distributions which differ by such a small phase shift. Thus, observation of a small amount of CP violation would require a very large amount of data. This should be compared with the situation in fig. 2 and fig. 3.

We note that the special cases $\eta = 0$ and $|\eta| \gg 1$ correspond to the CP even and CP odd eigenstates, respectively. Hence, the distribution (13) should be interpreted as being intermediate between those for the two eigenstates.
Figure 5. for different amounts of CP violation, including the CP-even (\( \eta = 0 \)) and CP-odd (\(|\eta| \gg 1\)) eigenstates. We have used \( \text{Re} \ \eta = 0, 0.5, 5 \) for \( \sqrt{s} = 500 \) GeV and \( m = 200 \) GeV.

4 Summary

We have addressed the problem of estimating the amount of data needed in order to distinguish a scalar Higgs from a pseudoscalar one at a future linear collider. This is most likely not possible at LEP2 due to much smaller event rates and background which is not easily suppressed. However, we have demonstrated that one will be able to establish the scalar nature of the Higgs boson at the Next Linear Collider from an analysis of angular or energy correlations. This particular study has been carried out for the case \( \sqrt{s} = 300 \) GeV, \( m = 125 \) GeV. Similar results are expected in other cases as long as the background is small. In cases where the background can not be significantly suppressed a more dedicated study would be required.

In order to establish or rule out specific models, one will also need to compare different branching ratios, in particular to fermionic final states. The methods proposed above instead deal with quite general properties of the models.

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