Computational Complexity of Traffic Hijacking under BGP and S-BGP

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Abstract. Harmful Internet hijacking incidents put in evidence how fragile the Border Gateway Protocol (BGP) is, which is used to exchange routing information between Autonomous Systems (ASes). As proved by recent research contributions, even S-BGP, the secure variant of BGP that is being deployed, is not fully able to blunt traffic attraction attacks. Given a traffic flow between two ASes, we study how difficult it is for a malicious AS to devise a strategy for hijacking or intercepting that flow. We show that this problem marks a sharp difference between BGP and S-BGP. Namely, while it is solvable, under reasonable assumptions, in polynomial time for the type of attacks that are usually performed in BGP, it is NP-hard for S-BGP. Our study has several by-products. E.g., we solve a problem left open in the literature, stating when performing a hijacking in S-BGP is equivalent to performing an interception.

1 Introduction and Overview

On 24th Feb. 2008, Pakistan Telecom started an unauthorized announcement of prefix 208.65.153.0/24 [14]. This announcement was propagated to the rest of the Internet, which resulted in the hijacking of YouTube traffic on a global scale. Incidents like this put in evidence how fragile is the Border Gateway Protocol (BGP) [11], which is used to exchange routing information between Internet Service Providers (ISPs). Indeed, performing a hijacking attack is a relatively simple task. It suffices to issue a BGP announcement of a victim prefix from a border router of a malicious (or unaware) Autonomous System (AS). Part of the traffic addressed to the prefix will be routed towards the malicious AS rather than to the intended destination. A mischievous variation of the hijacking is the interception when, after passing through the malicious AS, the traffic is forwarded to the correct destination. This allows the rogue AS to eavesdrop or even modify the transit packets.

In order to cope with this security vulnerability, a variant of BGP, called S-BGP [9], has been proposed, that requires a PKI infrastructure both to validate

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the correctness of the AS that originates a prefix and to allow an AS to sign its announcements to other ASes. In this setting an AS cannot forge announcements that do not derive from announcements received from its neighbors. However, [4] contains surprising results: (i) simple hijacking strategies are tremendously effective and (ii) finding a strategy that maximizes the amount of traffic that is hijacked is NP-hard both for BGP and for S-BGP.

In this paper we tackle the hijacking and interception problems from a new perspective. Namely, given a traffic flow between two ASes, how difficult is it for a malicious AS to devise a strategy for hijacking or intercepting at least that specific flow? We show that this problem marks a sharp difference between BGP and S-BGP. Namely, while it is polynomial time solvable, under reasonable assumptions, for typical BGP attacks, it is NP-hard for S-BGP. This gives new complexity related evidence of the effectiveness of the adoption of S-BGP. Also, we solve an open problem [4], showing when every hijack in S-BGP results in an interception. Tab. 1 summarizes our results. Rows correspond to different settings for a malicious AS $m$. The origin-spoofing setting (Sect. 2) corresponds to a scenario where $m$ issues BGP announcements pretending to be the owner of a prefix. Its degree of freedom is to choose a subset of its neighbors for such a bogus announcement. This is the most common type of hijacking attack to BGP [1]. In S-BGP (Sect. 3) $m$ must enforce the constraints imposed by S-BGP, which does not allow to pretend to be the owner of a prefix that is assigned to another AS. Columns of Tab. 1 correspond to different assumptions about the Internet. In the first column we assume that the longest valley-free path (i.e. a path enforcing certain customer-provider constraints) in the Internet can be of arbitrary length. This column has a theoretical interest since the length of the longest path (and hence valley-free path) observed in the Internet remained constant even though the Internet has been growing in terms of active AS numbers during the last 15 years [8]. Moreover, in today’s Internet about 95% of the ASes is reached in 3 AS hops [8]. Hence, the second column corresponds to a quite realistic Internet, where the AS-path length is bounded by a constant. In the third column we assume that the number of neighbors of $m$ is bounded by a constant. This is typical in the periphery of the Internet. A “P” means that a Polynomial-time algorithm exists. Since moving from left to right the setting is more constrained, we prove only the rightmost NP-hardness results, since they imply the NP-hardness results to their left. Analogously, we prove only the leftmost “P” results.

|                  | AS-paths of any length | Bounded AS-path length | Bounded AS-path length and AS degree |
|------------------|------------------------|------------------------|--------------------------------------|
| Origin-spoofing  | NP-hard (Thm. 1)       | P (Thm. 2)             | P                                    |
| S-BGP            | NP-hard                | NP-hard (Thm. 3)       | P (Thm. 4)                           |

Table 1: Complexity of finding a hijack strategy in different settings.
1.1 A Model for BGP Routing

As in previous work on interdomain hijacking [4], we model the Internet as a graph \( G = (V, E) \). A vertex in \( V \) is an Autonomous System (AS). Edges in \( E \) are peerings (i.e., connections) between ASes. A vertex owns one or more prefixes, i.e., sets of contiguous IP numbers. The routes used to reach prefixes are spread and selected via BGP. Since each prefix is handled independently by BGP, we focus on a single prefix \( \pi \), owned by a destination vertex \( d \).

BGP allows each AS to autonomously specify which paths are forbidden (import policy), how to choose the best path among those available to reach a destination (selection policy), and a subset of neighbors to whom the best path should be announced (export policy). BGP works as follows. Vertex \( d \) initializes the routing process by sending announcements to (a subset of) its neighbors. Such announcements contain \( \pi \) and the path of \( G \) that should be traversed by the traffic to reach \( d \). In the announcements sent from \( d \) such a path contains just \( d \). We say that a path \( P = (v_n \ldots v_0) \) is available at vertex \( v \) if \( v_0 \) announces \( P \) to \( v \). Each vertex checks among its available paths that are not filtered by the import policy, which is the best one according to its selection policy, and then it announces that path to a set of its neighbors in accordance with the export policy. Further, BGP has a loop detection mechanism, i.e., each vertex \( v \) ignores a route if \( v \) is already contained in the route itself.

Policies are typically specified according to two types of relationships [7]. In a customer-provider relationship, an AS that wants to access the Internet pays an AS which sells this service. In a peer-peer relationship two ASes exchange traffic without any money transfer between them. Such commercial relationships between ASes are represented by orienting a subset of the edges of \( E \). Namely, edge \( (u, v) \in E \) is directed from \( u \) to \( v \) if \( u \) is a customer of \( v \), while it is undirected if \( u \) and \( v \) are peers. A path is valley-free if provider-customer and peer-peer edges are only followed by provider-customer edges.

The Gao-Rexford [3] Export-all (GR-EA) conditions are commonly assumed to hold in this setting [4].

- **GR1**: \( G \) has no directed cycles that would correspond to unclear customer-provider roles.
- **GR2**: Each vertex \( v \in V \) sends an announcement containing a path \( P \) to a neighbor \( n \) only if path \( (n, v) \) is valley-free. Otherwise, some AS would provide transit to either its peers or its providers without revenues.
- **GR3**: A vertex prefers paths through customers over those provided by peers and paths through peers over those provided by providers.

**Shortest Paths**: Among paths received from neighbors of the same class (customers, peers, and provider), a vertex chooses the shortest ones.

**Tie Break**: If there are multiple such paths, a vertex chooses according to some tie break rule. As in [4], we assume that the one whose next hop has lowest AS number is chosen. Also, as in [2], to tie break equal class and equal length simple paths \( P_1^u = (u, v)P_1^v \) and \( P_2^u = (u, v)P_2^v \) at the same vertex \( u \) from the same neighbor \( v \), if \( v \) prefers \( P_1^v \) over \( P_2^v \), then \( u \) prefers \( P_1^u \) over \( P_2^u \). This choice is called policy consistent in [2] and it can be proven that it has the nice property of making the entire set of policies considered in this paper policy consistent.

**NE policy**: a vertex always exports a path except when GR2 forbids it to do so.
Since we assume that the GR-EA conditions are satisfied, then a (partially directed) graph is sufficient to fully specify the policies of the ASes. Hence, in the following a BGP instance is just a graph.

1.2 Understanding Hacking Strategies

We consider the following problem. A BGP instance with three specific vertices, $d$, $s$, and $m$ are given, where such vertices are: the AS originating a prefix $\pi$, a source of traffic for $\pi$, and an attacker, respectively. All vertices, but $m$, behave correctly, i.e., according to the BGP protocol and GR-EA conditions. Vertex $m$ is interested in two types of attacks: hijacking and interception. In the hijacking attack $m$’s goal is to attract to itself at least the traffic from $s$ to $d$. In the interception attack $m$’s goal is to be traversed by at least the traffic from $s$ to $d$.

In Fig. 1 (2, 6) is peer-to-peer and the other edges are customer-provider. Prefix $\pi$ is owned and announced by $d$. According to BGP, the traffic from $s$ to $d$ follows ($s \ 6 \ 2 \ 1 \ d$). In fact, 2 selects (1 $d$). Vertex 6 receives a unique announcement from $d$ (it cannot receive an announcement with (5 4 3 $m \ 2 \ 1 \ d)$ since it is not valley-free). By cheating, (Example 1) $m$ can deviate the traffic from $s$ to $d$ attracting traffic from $s$. In fact, if $m$ pretends to be the owner of $\pi$ and announces it to 2, then 2 prefers, for shortest-path, (2 $m$) over (2 1 $d$). Hence, the traffic from $s$ to $d$ is received by $m$ following ($s \ 6 \ 2 \ m$). A hijack!

Observe that $m$ could be smarter (Example 2). Violating GR2, it can announce (2 1 $d$) to 3. Since each of 3, 4 and 5 prefers paths announced by customers (GR3), the propagation of this path is guaranteed. Therefore, 6 has two available paths, namely, (2 1 $d$) and (5 4 3 $m \ 2 \ 1 \ d$). The second one is preferred because 5 is a customer of 6, while 2 is a peer of 6. Hence, the traffic from $s$ to $d$ is received by $m$ following path ($s \ 6 \ 5 \ 4 \ 3 \ m$). Since after passing through $m$ the traffic reaches $d$ following (m 2 1 $d$) this is an interception.

Fig. 2 allows to show a negative example (Example 3). According to BGP, the traffic from $s$ to $d$ follows ($s \ 4 \ d$). In fact, $s$ receives only paths (4 $d$) and (1 2 3 $d$), both from a provider, and prefers the shortest one. Suppose that $m$ wants to hijack and starts just announcing $\pi$ to 6. Since all the neighbors of $s$ are providers, $s$ prefers, for shortest path, (4 $d$) over (5 6 $m$) (over (1 2 3 $d$) over (4 9 8 7 $m$)) and the hijack fails. But $m$ can use another strategy. Since (s 5 6 $m$) is shorter than (s 1 2 3 $d$), $m$ can attract traffic if (4 $d$) is “disrupted” and becomes not available at $s$. This happens if 4 selects, instead of (d), a path received from its peer neighbor 9 ($m$ may announce that it is the originator of $\pi$ also to 7). However, observe that if 4 selects path (4 9 8 7 $m$) then 5 selects path (5 9 8 7 $m$) since it is received from a peer and stops the propagation of (s 5 6 $m$). Hence, $s$ still selects path (s 1 2 3 $d$) and the hijack fails.

In order to cope with the lack of any security mechanism in BGP, several variations of the protocol have been proposed by the Internet community. One of the most famous, S-BGP, uses both origin authentication and cryptographically-signed announcements in order to guarantee that an AS announces a path only if it has received this path in the past.
The attacker $m$ has more or less constrained cheating capabilities. 1. With the origin-spoofing cheating capabilities $m$ can do the typical BGP announcement manipulation. I.e., $m$ can pretend to be the origin of prefix $\pi$ owned by $d$, announcing this to a subset of its neighbors. 2. With the S-BGP cheating capabilities $m$ must comply with the S-BGP constraints. I.e.: (a) $m$ cannot pretend to be the origin of prefix $\pi$; and (b) $m$ can announce a path $(m \ u) P$ only if $u$ announced $P$ to $m$ in the past. However, $m$ can still announce paths that are not the best to reach $d$ and can decide to announce different paths to different neighbors. In Example 2, $m$ has S-BGP cheating capabilities.

In this paper we study the computational complexity of the hijack and of the interception problems. The hijack problem is formally defined as follows. **Instance:** A BGP instance $G$, a source vertex $s$, a destination vertex $d$, a manipulator vertex $m$, and a cheating capability for $m$. **Question:** Does there exist a set of announcements that $m$ can simultaneously send to its neighbors, according to its cheating capability, that produces a stable state for $G$ where the traffic from $s$ to $d$ goes to $m$? The interception problem is defined in the same way but changing “the traffic from $s$ to $d$ goes to $m$” to “the traffic from $s$ to $d$ passes through $m$ before reaching $d$”.

### 1.3 Notation and Definitions

We introduce some technical notation in order to prove our lemmas and theorems. A ranking function determines the level of preference of paths available at vertex $v$. If $P_1, P_2$ are available at $v$ and $P_1$ is preferred over $P_2$ we write $P_1 \prec P_2$. 
The concatenation of two nonempty paths \( P = (v_k v_{k-1} \ldots v_i), k \geq i, \) and \( Q = (v_i v_{i-1} \ldots v_0), i \geq 0, \) denoted as \( PQ, \) is the path \( (v_k v_{k-1} \ldots v_{i+1} v_i v_{i-1} \ldots v_0). \) Also, let \( P \) be a valley-free path from vertex \( v. \) We say that \( P \) is of class 3, 2, or 1 if its first edge connects \( v \) with a customer, a peer, or a provider of \( v, \) respectively. We also define a function \( f^c \) for each vertex \( v, \) that maps each path from \( v \) to the integer of its class. Given two paths \( P \) and \( P' \) available at \( v \) if \( f^c(P) > f^c(P') \) we say that the class of \( P \) is better than the class of \( P'. \) In stable routing state \( S, \) a path \( P = (v_1 \ldots v_n) \) is disrupted at vertex \( v_i \) by a path \( P' \) if there exists a vertex \( v_i \) of \( P \) such that \( v_i \) selects path \( P'. \) Also, if \( P' \) is preferred over \( (v_1 \ldots v_n) \) because of the GR3 condition, we say that path \( P \) is disrupted by a path of a better class. Otherwise, if \( P' \) is preferred over \( (v_1 \ldots v_n) \) because of the shortest-paths criterion, we say that path \( P \) is disrupted by a path of the same class.

1.4 Routing Stability under Manipulator Attacks

BGP policies can be so complex that there exist configurations that do not allow to reach any stable routing state (see, e.g., \([6]\)). A routing state is stable if there exists a time \( t \) such that after \( t \) no AS changes its selected path. If the GR-EA conditions are satisfied \([3]\), then a BGP network always converges to a stable state. However, there is a subtle issue to consider in attacks. As we have seen in the examples, \( m \) can deliberately ignore the GR-EA conditions. Anyway, the following lemma makes it possible, in our setting, to study the Hijack and the interception problem ignoring stability related issues. First, we introduce some notation.

**Lemma 1.** Let \( G \) be a BGP instance and suppose that at a certain time a manipulator \( m \) starts announcing steadily any set of arbitrary paths to its neighbors. Routing in \( G \) converges to a stable state.

**Proof.** Suppose, for a contradiction, that, after \( m \) starts its announcements, routing in \( G \) is not stable. Let \( u_0, \ldots, u_n \) be a circular sequence of vertices such that:

1. each \( u_i \) does not steadily announce a path; (2) the most preferred path \( P^\mu_i = R_i Q_i = (r^\mu_i 1 \ldots r^\mu_n) Q_i, \) at \( u_i \) that is available infinitely many times is such that each vertex in \( Q_i \) but \( r^\mu_i \) steadily announces a path; and (3) \( r^\mu_i = u_{i+1}, \) where \( i \) has to be interpreted modulo \( n. \) Such a circular sequence is called dispute-wheel and it has been proved in \([5]\) that if a system (with no manipulators) is not stable then it contains a dispute-wheel. We prove that the presence of a dispute-wheel in a GR-EA instance leads to a contradiction. Hence, a GR-EA instance always converges to a stable state. Observe that \( |R_i| \geq 2, \) otherwise \( u_i \) would be stable. Since \( m \) and \( d \) steadily announce some paths and because \( G \) is finite, such a sequence exists. Observe that for each \( i, \) we have that \( Q_{i-1} > u_i P^\mu_i. \)

Suppose that for each \( u_i \) we have that \( P^\mu_i \) is preferred over \( Q_{i-1} \) either by shortest path or by tie-break, i.e., \( f^u_i(Q_{i-1}) = f^u_i(P^\mu_i) \) and \( |Q_{i-1}| \geq |P^\mu_i|. \) Inequality \( |R_i| \geq 2 \) implies \( |P^\mu_i| > |Q_i|. \) Hence, we have \( |Q_{i-1}| > |P^\mu_i| > |Q_i| \geq |P^{\mu_{i+1}}|. \) Following the cycle of inequalities we have a contradiction as we obtain \( |Q_i| > |Q_i|. \)
Conversely, let $u_m$ be a vertex that prefers $P_{u_m}$ over $Q_{m-1}$ for better class, that is $f_{u_m}(P_{u_m}) > f_{u_m}(Q_{m-1})$. For each $i = 0, \ldots, m - 1, m + 1, \ldots, n$ we have $f_{u_i}(P_{u_i}) \geq f_{u_i}(Q_{i-1}) \geq f_{u_{i-1}}(P_{u_{i-1}})$ because of the GR2 and GR3 conditions. Following the cycle of inequalities we have a contradiction as we obtain $f_{u_m}(P_{u_m}) > f_{u_m}(P_{u_m})$. 

The existence of a stable state (pure Nash equilibrium) in a game where one player can deviate from a standard behavior has been proved, in a different setting in [24]. Such a result and Lemma 1 are somehow complementary since the export policies they consider are more general than Export-All, while the convergence to the stable state is not guaranteed (even if such a stable state is always reachable from any initial state).

2 Checking if an Origin-Spoofing BGP Attack Exists

In this section, we show that, in general, it is hard to find an attack strategy if a manipulator has an origin-spoofing cheating capability (Theorem 1), while the problem turns to be easier in a realistic setting (Theorem 2).

A hijacking can be obviously found in exponential time by a simple brute force approach which simulates every possible attack strategy and verifies its effectiveness. The following result in the case the Internet graph has no bound constraints may be somehow expected.

**Theorem 1.** If the manipulator has origin-spoofing cheating capabilities, then problem Hijack is NP-hard.

**Proof.** We prove that Hijack is NP-hard by a reduction from the 3-SAT problem. Let $F$ be a logical formula in conjunctive normal form with variables $X_1 \ldots X_n$ and clauses $C_1 \ldots C_m$ where each clause $C_i$ contains three literals. We construct a GR-EA compliant BGP instance $G$ as follows.

Graph $G$ consists of 4 structures: the Intermediate structure, the Short structure, the Long structure, and the Disruptive structure. See Fig. 3.

The Intermediate structure is the only portion of $G$ containing valley-free paths joining $s$ and $m$ that are shorter than the one contained in the Long structure. It is composed by edge $(m, q_1)$ and two directed paths from $s$ to $q_1$ of length $2n$: the first path is composed by edges $(s, l_n)$, $(l_n, q_n)$, $(q_n, l_{n-1})$, $(l_{n-1}, q_{n-1})$, $(q_{n-1}, t_{n-2})$, $\ldots$, $(l_2, q_2)$, $(q_2, l_1)$, and $(l_1, q_1)$ while the second path is composed by edges $(s, l_n)$, $(l_n, q_n)$, $(q_n, l_{n-1})$, $(l_{n-1}, q_{n-1})$, $(q_{n-1}, t_{n-2})$, $\ldots$, $(l_2, q_2)$, $(q_2, l_1)$, and $(l_1, q_1)$. Obviously, these two paths can be used to construct an exponential number of other paths. We say that a path traverses the Intermediate structure if it passes through vertices $s$ and $q_1$.

The Short structure consists of $h$ paths joining $s$ and $d$. Each path has length 4 and has edges $(s, c_{i,1})$, $(c_{i,1}, c_{i,2})$, $(c_{i,2}, c_{i,3})$, and $(c_{i,3}, d)$ ($1 \leq i \leq h$). The Long structure is a directed path of length $2n + 3$ with edges $(s, w_1)$, $(w_1, w_2)$, $\ldots$, $(w_{2n+1}, w_{2n+2})$, and $(w_{2n+2}, d)$. The Disruptive structure is composed by 2$n$ paths plus 3$h$ edges. The 2$n$ paths are defined as follows. For $1 \leq i \leq n$ we define
two paths. The first path contains a directed subpath of length $2n + 2$ from $m$ to $x_i$ (dotted lines in Fig. 3), plus the undirected edge $(x_i, t_i)$. The second path contains a directed subpath of length $2n + 2$ from $m$ to $\bar{x}_i$ (dotted lines in Fig. 3) plus the undirected edge $(\bar{x}_i, \bar{t}_i)$. The $3h$ edges are added to $G$ as follows. For each clause $C_i$ and each literal $L_{i,j}$ of $C_i$, which is associated to a variable $x_k$, if $L_{i,j}$ is positive, then we add $(x_k, c_{i,j})$, otherwise we add $(\bar{x}_k, c_{i,j})$. We say that a path traverses the DISRUPTIVE structure if it traverses it from $m$ to $s$.

Vertices $s$, $d$, and $m$ have source, destination, and manipulator roles, respectively.

Intuitively, the proof works as follows. The paths that allow traffic to go from $s$ to $m$ are only those passing through the DISRUPTIVE structure and the INTERMEDIATE structure. Also, the paths through the INTERMEDIATE structure are shorter than the one through the LONG structure, which is shorter than those through the DISRUPTIVE structure.

If $m$ does not behave maliciously, $s$ receives only paths that traverse the SHORT structure and the LONG structure. In this case $s$ selects one of the paths in the SHORT structure according to its tie break policy.

Observe that if $m$ wants to attract traffic from $s$, then: (i) a path from $m$ traversing entirely the INTERMEDIATE structure has to reach $s$ and (ii) all paths contained in the SHORT structure have to be disrupted by a path announced by $m$.

Observe that only valley-free paths contained in the INTERMEDIATE structure, which have length at least $2n + 2$, can be used to attract traffic from $s$. If (i) does not hold, then $s$ selects the path contained in the LONG structure or a path contained in the SHORT structure. If (ii) does not hold, then $s$ selects a path contained in the SHORT structure.

Our construction is such that the 3-sat formula is satisfiable iff $m$ can attract the traffic from $s$ to $d$. To understand the interplay between our construction and the 3-sat problem, consider (see Fig. 3) the behavior of $m$ with respect to neighbors $x_2$ and $\bar{x}_2$. If $m$ wants to disrupt path $(s, c_{1,1}, c_{1,2}, c_{1,3}, d)$ (which corresponds to making clause $C_1$ true) it might announce the prefix to $x_2$. This would have the effect of disrupting $(s, c_{1,1}, c_{1,2}, c_{1,3}, d)$ by better class. Observe that at the same time this would disrupt all the paths through $t_2$. If $m$ is able to disrupt all the paths in the SHORT structure, then $s$ has to select a path in the INTERMEDIATE structure. However, $m$ has to be careful for two reasons. First, $m$ has to announce the prefix to $q_1$ (otherwise no path can traverse the INTERMEDIATE structure). Second, $m$ cannot announce the prefix both to $x_2$ and to $\bar{x}_2$ (variable $X_2$ cannot be true and false at the same time). In this case, all the paths through $t_2$ and $\bar{t}_2$ are disrupted. Also, consider that the paths that reach $s$ through $t_2$ and $x_2$ ($t_2$ and $\bar{t}_2$) and that remain available are longer than the one in the LONG structure.

Now we show that if $F$ is satisfiable, then $m$ can attract traffic from $s$. Let $M$ be a truth assignment to variables $X_1, \ldots, X_n$ satisfying formula $F$. Let $m$ announce to its neighbors paths as follows: if $X_i$ ($i = 1, \ldots, n$) is true then $m$
announces the prefix to $x_i$ and does not announce anything to $\bar{x}_i$; otherwise $m$ does the opposite. Also, the prefix in announced to $q_1$ in all cases.

We have that: 1. all paths (one for each clause) in the Short structure are disrupted by better class from the paths in the Disruptive structure; 2. one path belonging to the Intermediate structure is available at $s$; 3. the path in the Long structure, available at $s$, is longer than the path in the Intermediate structure. Hence, $m$ can attract traffic from $s$.

Now we prove that if manipulator $m$ can attract traffic from $s$, then $F$ is satisfiable.

We already know from the above discussion that $m$ can attract traffic from $s$ only using paths that traverse the Intermediate structure entirely. We also know that these paths are longer than paths contained in the Short structure and therefore, every path contained in the Short structure has to be disrupted. We have that paths contained in the Short structure can be disrupted only by using paths contained in the Disruptive structure. Let $V^*$ be the set of neighbors of $m$ different from $q_1$ that receive an announcement of the prefix from $m$. Observe that $s$, to attract traffic from $m$, has to announce the prefix to $q_1$. From the above discussion we have that for $i = 1, \ldots, n$ it is not possible both for $x_i$ and for $\bar{x}_i$ to receive the announcement. Also, since all paths in the Short structure have been disrupted, for $j = 1, \ldots, h$ at least one of the $c_{j,k}$ ($k = 1, 2, 3$) receives an announcement of the prefix from $m$. Hence, we define an assignment $M$, which satisfies formula $F$, as follows: for each $i = 1, \ldots, n$, if $x_i \in V^*$, then $M(X_i) = \top$, otherwise $M(X_i) = \bot$.

Surprisingly, in a more realistic scenario, where the length of valley-free paths is bounded by a constant $k$, we have that in the origin-spoofing setting an attack strategy can be found in polynomial time ($n^{O(k)}$, where $n$ is the number of vertices of $G$). Let $N$ be the set of neighbors of $m$. Indeed, the difficulty of the Hijack problem in the origin-spoofing setting depends on the fact that $m$ has to decide to which of the vertices in $N$ it announces the attacked prefix $\pi$, which leads to an exponential number of possibilities. However, when the longest valley-free path in the graph is bounded by a constant $k$, it is possible to design a polynomial-time algorithm based on the following intuition, that will be formalized below. Suppose $m$ is announcing $\pi$ to a subset $A \subseteq N$ of its neighbors and path $p = (z \ldots n \ m)$ is available at an arbitrary vertex $z$ of the graph. Let $n_1,n_2$ be two vertices of $N \setminus A$. If $p$ is disrupted (is not disrupted) by better class both when $\pi$ is announced either to $n_1$ or to $n_2$, then $p$ is disrupted (is not disrupted) by better class when $\pi$ is announced to both $n_1$ and $n_2$. This implies that once $m$ has a candidate path $p^*$ for attracting traffic from $s$, it can check independently to which of its neighbors it can announce $\pi$ without disrupting $p^*$ by better class, which guarantees that a path from $m$ to $z$ longer than $p$ cannot be selected at $z$.

In order to prove Theorem we introduce the following lemmata that relate attacks to the structure of the Internet.
Lemma 2. Consider a valley-free path $p = (v_n \ldots v_1)$ and consider an attack of $m$ such that $v_1$ announces a path $p_{v_1}$ to $v_2$ to reach prefix $\pi$ and $p$ is possibly disrupted only by same class. Vertex $v_n$ selects a path $p_n \leq^v \lambda p_{v_1}$.

Proof. We prove inductively that each vertex $v_i$ in $p$ selects a path $p_i \leq^v \lambda (v_i \ldots v_1)$ such that $|p_i| \leq |(v_i \ldots v_1)|$. In the base case ($n = 1$), the statement holds since $p_1 \leq^v \lambda p_1$ and $|p_1| \leq |p_1|$. In the inductive step ($n > 1$), by induction hypothesis and NE policy, vertex $v_i$ receives a path $p_{i-1}$ from vertex $v_{i-1}$ such that $p_{i-1} \leq^v_{i-1} (v_{i-1} \ldots v_1)$ and $|p_{i-1}| \leq |(v_{i-1} \ldots v_1)|$. Two cases are possible: $p_{i-1}$ contains $v_i$, or not. In the second case, $v_i$ selects a path $p_i \leq^v (v_i v_{i-1})p_{i-1}$ and since path $(v_i \ldots v_1)$ is disrupted only by same class, we have also $|p_i| \leq |((v_i v_{i-1})p_{i-1})| \leq |(v_i \ldots v_1)|$. In the first case, let $p'$ be the subpath of $p_{i-1}$ from $v_i$. Observe that since $(v_i v_{i-1})p_{i-1}$ is a valley-free path and vertex $v_i$ is repeated in that path, we have that $f^v(p') > f^v((v_i v_{i-1})p_{i-1}) = f^v(v_i \ldots v_1)$, which is not possible since $(v_i \ldots v_1)$ cannot be disrupted by better class.

Lemma 3. Consider a successful attack for $m$ and let $p_{sm}$ be the path selected at $s$. Let $p_{sd}$ be a valley-free path from $s$ to $d$ such that it does not traverse $m$ and such that $p_{sd} \leq^v \lambda p_{sm}$. Path $p_{sd}$ is disrupted by a path of better class.

Proof. Suppose by contradiction that there exists a valley-free path $p_{sd}$ from $s$ to $d$ such that $p_{sd} \leq^v \lambda p_{sm}$ and $p_{sd}$ is not disrupted by a path of better class. If $p_{sd}$ is not disrupted, then it is available at vertex $s$. It implies that $s$ selects $p_{sd}$ as its best path, which leads to a contradiction. Otherwise, suppose $p_{sd}$ is disrupted only by same class. By Lemma 2 we have a contradiction since $s$ selects a path $p \leq^v (p_{sd}) \leq^v (p_{sm})$ different from $p_{sm}$.

Lemma 4. Let $p = (v_n \ldots v_1)$ be a valley-free path. Consider an attack where $v_1$ announces a path $p_{v_1}$ to $v_2$. Vertex $v_n$ selects a path of class at least $f^v(p)$.

Proof. We prove that each vertex $v_i$ in $p$ selects a path $p_i$ such that $f^v(p_i) \geq f^v(v_i \ldots v_1)$. In the base case ($n = 1$), the statement holds since $f^v(p_1) \geq f^v(v_1 \ldots v_1)$. In the inductive step ($n > 1$), by induction hypothesis and NE policy, vertex $v_i$ receives a path $p_{i-1}$ from vertex $v_{i-1}$ such that $f^v_{i-1}(p_{i-1}) \geq f^v_{i-1}(v_{i-1} \ldots v_1)$. Two cases are possible: $p_{i-1}$ contains $v_i$ or not. In the second case, $v_i$ selects a path $p_i \leq^v (v_i v_{i-1})p_{i-1}$ which implies that $f^v(p_i) \geq f^v(v_i \ldots v_1)$. In the first case, let $p'$ be the subpath of $p_{i-1}$ from $v_i$. Observe that since $(v_i v_{i-1})p_{i-1}$ is a valley-free path and vertex $v_i$ is repeated in that path, we have that, $f^v(p') > f^v((v_i v_{i-1})p_{i-1}) = f^v(v_i \ldots v_1)$, and the statement holds also in this case.

Theorem 2. If the manipulator has origin-spoofing cheating capabilities and the length of the longest valley-free path is bounded by a constant, then problem HIJACK is in $P$.

Proof. We tackle the problem with Alg. 1. First, observe that line 9 tests if a certain set of announcements causes a successful attack and, in that case, it
Algorithm 1 Algorithm for the HIJACK problem where $m$ has origin-spoofing capabilities and the longest valley-free path in the graph is bounded.

1: **Input**: instance of HIJACK problem, $m$ has origin-spoofing cheating capabilities;
2: **Output**: an attack pattern if the attack exists, fail otherwise;
3: let $P_{sm}$ be the set of all valley-free paths from $s$ to $m$;
4: **for all** $p_{sm}$ in $P_{sm}$ **do**
5: \hspace{1em} let $w$ be the vertex of $p_{sm}$ adjacent to $m$; let $A$ be a set of vertices and initialize $A$ to $\{w\}$; let $N$ be the set of the neighbors of $m$;
6: \hspace{1em} **for all** $n$ in $N \setminus \{w\}$ **do**
7: \hspace{2em} if there is no path $p$ through $(m, n)$ to a vertex $x$ of $p_{sm}$ such that $f^x(p) > f^x(p_{sm})$, where $p_{sm}$ is the subpath of $p_{sm}$ from $x$ to $m$ **then**
8: \hspace{2em} insert $n$ into $A$
9: \hspace{1em} **if** the attack succeeds when $m$ announces $\pi$ only to the vertices in $A$ **then**
10: \hspace{1em} **return** $A$
11: **return** fail

returns the corresponding set of neighbors to whom $m$ announces prefix $\pi$. Hence, if Alg. 1 returns without failure it is trivial to see that it found a successful attack. Suppose now that there exists a successful attack $a^*$ from $m$ that is not found by Alg. 1. Let $p_{sm}^*$ be the path selected by $s$ in attack $a^*$. Let $A^*$ be the set of neighbors of $m$ that receives prefix $\pi$ from $m$ in the successful attack.

Consider the iteration of the Alg. 1 where path $p_{sm}^*$ is analyzed in the outer loop. At the end of the iteration Alg. 1 constructs a set $A$ of neighbors of $m$. Let $a$ be an attack from $m$ where $m$ announces $\pi$ only to the vertices in $A$.

First, we prove that $A^* \subseteq A$. Suppose by contradiction that there exists a vertex $n \in A^*$ that is not contained in $A$. It implies that there exists a valley-free path $p$ through $(m, n)$ to a vertex $x$ of $p_{sm}^*$ such that $f^x(p) > f^x(p_{sm})$, where $p_{sm}$ is the subpath of $p_{sm}^*$ from $x$ to $m$. Since $m$ announces $\pi$ to $n$, by Lemma 4 we have that $x$ selects a path $p'$ of class at least $f^x(p)$, that is a contradiction since $p_{sm}^*$ would be disrupted by better class. Hence, $A^* \subseteq A$.

Now, we prove that attack $a$ is a successful attack for $m$. Consider a valley-free path $p_{sd}$ from $s$ to $d$ that does not traverse $m$ and is preferred over $p_{sm}$. By Lemma 3 it is disrupted by better class in attack $a^*$. By Lemma 4 since $A^* \subseteq A$, we have that also in $a$ path $p_{sd}$ is disrupted by better class. Let $x$ be the vertex adjacent to $s$ in $p_{sd}$. Observe that, vertex $s$ cannot have an available path $(s x)p$ to $d$ such that $(s x)p <_A p_{sm}^*$, because $(s x)p$ must be disrupted by better class.

Moreover, consider path $p_{sm}$. Since in $a^*$ path $p_{sm}^*$ is not disrupted by better class by a path to $d$, by Lemma 4 there does not exist a path $p_{sd}$ from a vertex $x$ of $p_{sm}^*$ to $d$ of class higher than $p_{sm}$, where $p_{sm}$ is the subpath of $p_{sm}^*$ from $x$ to $m$. Hence, path $p_{sm}^*$ cannot be disrupted by better class by a path to $d$.

Also, observe that for each $n \in A$ there is no path $p$ through $(m, n)$ to a vertex $x$ of $p_{sm}$ such that $f^x(p) > f^x(p_{sm})$, where $p_{sm}$ is the subpath of $p_{sm}$ from $x$ to $m$. Hence, $p_{sm}^*$ can be disrupted only by same class. By Lemma 2 we have that $s$ selects a path $p$ such that $p <_A p_{sm}^*$. Since path $p$ cannot be a path to $d$, attack $a$ is successful. This is a contradiction since we assumed that Alg. 1 failed.
Finally, since the length of the valley-free paths is bounded, the iterations of the algorithm where paths in $P_{sm}$ are considered require a number of steps that is polynomial in the number of vertices of the graph. Also, the disruption checks can be performed in polynomial time by using the algorithm in [12].

\[3\] S-BGP Gives Hackers Hard Times

We open this section by strengthening the role of S-BGP as a security protocol. Indeed, S-BGP adds more complexity to the problem of finding an attack strategy (Theorem 3). After that we also provide an answer to a conjecture posed in [4] about hijacking and interception attacks in S-BGP when a single path is announced by the manipulator. In this case, we prove that every successful hijacking attack is also an interception attack (Theorem 5).

**Theorem 3.** If the manipulator has S-BGP cheating capabilities and the length of the longest valley-free path is bounded by a constant, then problem hijack is NP-hard.

**Proof.** We reduce from a version of 3-SAT where each variable appears at most three times and each positive literal at most once [10]. Let $F$ be a logical formula in conjunctive normal form with variables $X_1, \ldots, X_n$ and clauses $C_1, \ldots, C_h$. We build a BGP instance $G$ (see Fig. 1) consisting of 4 structures: Intermediate, Short, Long, and Disruptive. The Long structure is a directed path of length 6 with edges $(s, w_1), (w_1, w_2), \ldots, (w_4, w_5)$, and $(w_5, d)$. The Intermediate structure consists of a valley-free path joining $m$ and $s$. It has length 4 and is composed by a directed path $(s, j_3, j_2, j_1)$, and $(m, j_1)$. The Short structure has $h$ directed paths from $s$ to $d$. Each path has length at most 4 and has edges $(s, c_{i,1}), (c_{i,1}, c_{i,2}), \ldots, (c_{i,v(C)}, d)$ (1 \(\leq i \leq h\), where \(v(C)\) is the size of $C_i$. The Disruptive structure contains, for each variable $X_i$ vertices, $r_i$, $t_i$, $x_i$, $p_i$, and $p_i'$. Vertices, $r_i$, $t_i$, and $x_i$, are reached via long directed paths from $m$ and are connected by $(t_i, p_i), (x_i, p_i), (x_i, p_i'), (r_i, j_3), (p_i, j_3)$, and $(p_i, d)$. Finally, suppose $X_i$ occurs in clause $C_j$ with a literal in position $l$. If the literal is negative the undirected edge $(p_i, c_{j,l})$ is added, otherwise, edges $(p_i, c_{j,l}), (r_i, c_{j,l}), (c_{j,l}, j_3)$, and undirected edge $(p_i', c_{j,l})$ are added. An edge connects $m$ to $d$. Vertices $s, d$, and $m$ have source, destination, and manipulator roles, respectively.

Intuitively, the proof works as follows. The paths that allow traffic to go from $s$ to $m$ are only those passing through the Disruptive structure and the one in the Intermediate structure. Also, the path through the Intermediate structure is shorter than the one through the Long structure, which is shorter than those through the Disruptive structure. If $m$ does not behave maliciously, $s$ receives only paths traversing the Short structure and the Long structure. In this case $s$ selects one of the paths in the Short structure according to its tie break policy. If $m$ wants to attract traffic from $s$, then: (i) path $(j_3, j_2, j_1, m, d)$ must be available at $s$ and (ii) all paths contained in the Short structure must be disrupted by a path announced by $m$. If (i) does not hold, then $s$ selects the
path contained in either the LONG structure or the SHORT structure. If (ii) does not hold, then $s$ selects a path contained in the SHORT structure.

Our construction is such that the 3-sat formula is true iff $m$ can attract the traffic from $s$ to $d$. To understand the relationship with the 3-sat problem, consider the behavior of $m$ with respect to variable $X_1$ (see Fig. 4) that appears with a positive literal in the first position of clause $C_1$, a negative literal in the first position of $C_2$, and a negative literal in the second position of $C_h$.

First, we explore the possible actions that $m$ can perform in order to disrupt paths in the SHORT structure. Since $m$ has S-BGP cheating capabilities, $m$ is constrained to propagate only the announcements it receives. If $m$ does not behave maliciously, $m$ receives path $(d)$ from $d$ and paths $P_{r_1}$, $P_{t_1}$, and $P_{x_1}$ from $r_1$, $t_1$, and $x_1$, respectively. These paths have the following properties: $P_{r_1}$ contains vertex $c_{1,1}$ that is contained in the path of the SHORT structure that corresponds to clause $C_1$; paths $P_{t_1}$ and $P_{x_1}$ both contain vertex $p_1$ and do not contain vertex $c_{1,1}$ since $p_1$ prefers $(p_1, d)$ over $(p_1, c_{1,1}, c_{1,2}, c_{1,3}, d)$.

Now, we analyze what actions are not useful for $m$ to perform an attack. If $m$ issues any announcement towards $t_1$ or $r_1$, the path traversing the INTERMEDIATE structure is disrupted by better class. Also, if $m$ sends a path $P_{r_1}, P_{t_1},$ or $P_{x_1}$ towards $r_j$, $t_j$, or $x_j$, with $j = 2, \ldots, n$, the path traversing the INTERMEDIATE structure is disrupted by better class. Also, if $m$ sends $(m, d)$ to $x_1$, then the path traversing the INTERMEDIATE structure is disrupted from $c_{1,1}$ by better class. If $m$ sends $P_{x_1}$ to $x_1$, then it is discarded by $x_1$ because of loop detection. In each of these cases $m$ cannot disrupt any path traversing the SHORT structure without disrupting the path traversing the INTERMEDIATE structure. Hence, $m$ can disrupt path in the SHORT structure without disrupting the path traversing the INTERMEDIATE structure only announcing $P_{r_1}$ and $P_{t_1}$ from $m$ towards $x_1$.

If path $P_{t_1}$ is announced to $x_1$, then $p_1$ discards that announcement because of loop detection and path $(s, c_{1,1}, c_{1,2}, c_{1,3}, d)$ is disrupted from $p_1$ by better class. Also, the path through the INTERMEDIATE structure remains available because the announcement through $p_1$ cannot reach $j_3$ from $c_{1,1}$, otherwise valley-freeness would be violated. Hence, announcing path $P_{t_1}$, corresponds to assigning true value to variable $X_1$, since the only path in the SHORT structure that is disrupted is the one that corresponds to the clause that contains the positive literal of $X_1$.

If path $P_{r_1}$ is announced to $x_1$, then $c_{1,1}$ discards that announcement because of loop detection and both paths $(s, c_{2,1}, c_{2,2}, c_{2,3}, d)$ and $(s, c_{h,1}, c_{h,2}, c_{h,3}, d)$ are disrupted by better class from $p_1$. Also, the path through the INTERMEDIATE structure remains available because the announcement through $p_1$ cannot reach $j_3$ from $c_{2,1}$ or $c_{h,2}$, otherwise valley-freeness would be violated. Hence, announcing path $P_{r_1}$, corresponds to assigning false value to variable $X_1$, since the only paths in the SHORT structure that are disrupted are the ones that correspond to the clauses that contain a negative literal of $X_1$.

Hence, announcing path $P_{t_1}$ ($P_{r_1}$) from $m$ to $x_1$ corresponds to assigning the true (false) value to variable $X_1$. As a consequence, $m$ can disrupt every path in the SHORT structure without disrupting the path in the INTERMEDIATE structure iff formula $F$ is satisfiable. □
Theorem 4. If the manipulator has S-BGP cheating capabilities and its degree is bounded by a constant, then problem HIJACK is in P.

Proof. Observe that if the manipulator \( m \) has S-BGP cheating capabilities, the degree of the manipulator’s vertex is bounded by a constant \( k \), then problem HIJACK is in P. In fact, since \( m \) has at most \( k \) available paths plus the empty path, a brute force approach approach needs to explore \((k + 1)^k\) number of possible cases.

To study the relationship between hijacking and interception we introduce the following technical lemma.

Lemma 5. Let \( G \) be a GR-EA compliant BGP instance, let \( m \) be a vertex with S-BGP cheating capabilities, and let \( d \neq m \) be any vertex of \( G \). All vertices that admit a class \( c \) valley-free path to \( d \) not containing \( m \) have an available path of class \( c \) or better to \( d \), irrespective of the paths propagated by \( m \) to its neighbors.

Proof. Let \( p = (v_n \ldots v_1) \) be a valley-free path to \( d \) not containing \( m \). We prove by induction on vertices \( v_1, \ldots, v_n \) that each vertex \( v_i \) has an available path of class \( f^{v_i}(v_i \ldots v_1) \) or better. In the base case \( i = 2 \), \( v_2 \) is directly connected to \( d \) and the statements trivially holds. Suppose that vertex \( v_i \), with \( i > 2 \), has an available path of class \( f^{v_i}(v_i \ldots v_1) \). Hence, \( v_i \) selects a path \( p^* \) such that \( f^{v_i}(p^*) \geq f^{v_i}(v_i \ldots v_1) \). Also, since \((v_{i+1} v_i \ldots v_1)\) is valley-free even \((v_{i+1} v_i)\) is valley-free. Then, \( v_i \) announces (because of the NE policy) its best path \( p^* \) to \( v_{i+1} \). There are two possible cases: either \( p^* \) does not contain \( v_{i+1} \) or not. In the first case, path \((v_{i+1} v_i)p^* \) is available at \( v_{i+1} \) and the statement holds. In the second case, consider the subpath \( p^*_{v_{i+1}} \) of \( p^* \) from \( v_{i+1} \) to \( d \). The statement easily follows because \( f^{v_{i+1}}(p^*_{v_{i+1}}) \geq f^{v_{i+1}}((v_{i+1} v_i)p^*) \).

Theorem 5. Let \( m \) be a manipulator with S-BGP cheating capabilities. If \( m \) announces the same path to any arbitrary set of its neighbors, then every successful hijacking attack is also a successful interception attack. If \( m \) announces different paths to different vertices, then the hijacking may not be an interception.

Proof. We prove the following more technical statement that implies the first part of the theorem. Let \( G \) be a BGP instance, let \( m \) be a vertex with S-BGP cheating capabilities. Let \( p \) be a path available at \( m \) in the stable state \( S \) reached when \( m \) behaves correctly. Suppose that \( m \) starts announcing \( p \) to any subset of its neighbors. Let \( S' \) be the corresponding routing state. Path \( p \) remains available at vertex \( m \) in \( S' \). The truth of the statement implies that \( m \) can forward the traffic to \( d \) by exploiting \( p \).

Suppose for a contradiction that path \( p \) is disrupted in \( S' \) when \( m \) propagates it to a subset of its neighbors. Let \( x \) be the first vertex of \( p \) that prefers a different path \( p_x \) (\( p \) is disrupted by \( p_x \)) in \( S' \) and let \( p' \) be the subpath of \( p \) from vertex \( d \) to \( x \) (see Fig. 5). Observe that \( p \) is not a subpath of \( p_x \) as \( x \) cannot select a path that passes through itself. Since \( p_x \) is not available at \( x \) in \( S \), let \( y \) be the vertex in \( p_x \) closest to \( d \) that selects a path \( p_y \) that is preferred over \( p' \) in \( S \), where \( p' \) is the subpath of \( p_x \) from \( y \) to \( d \).
We have two cases: either \(f^x(p_x) > f^x(p')\) or \(f^x(p_x) = f^x(p')\) (i.e., \(p_x\) is preferred to \(p'\) by better or by same class).

Suppose that \(f^x(p_x) > f^x(p')\). By Lemma 5, since there exists a valley-free path \(p_x\) from \(x\) to \(d\) that does not traverse \(m\), then \(x\) has an available path of class at least \(f^x(p_x)\). Hence, \(x\) cannot select path \(p'\) in \(S\), a contradiction.

Suppose that \(f^x(p_x) = f^x(p')\). Two cases are possible: either \(p_y\) contains \(x\) or not. In the first case either \(f^y(p_y) > f^y(p'_y)\) or \(f^y(p_y) = f^y(p'_y)\). If \(f^y(p_y) > f^y(p'_y)\), then we have that \(f^y(p_y) \leq f^y(p') = f^y(p_x) \leq f^y(p'_y)\), a contradiction.

If \(f^y(p_y) = f^y(p'_y)\), we have that \(|p'_y| < |p_x| \leq |p'| < |p_y|\). A contradiction since a longer path is preferred.

The second case \((f^x(p_x) = f^x(p')\) and \(p_y\) does not contain \(x\)) is more complex.

We have that \(|p'| \geq |p_x|\). Also, by Lemma 5, since \(p_y\) and \(p'_y\) do not pass through \(m\), then \(y\) has an available path of class at least \(\max\{f^y(p_y), f^y(p'_y)\}\). As \(y\) alternatively chooses \(p_y\) and \(p'_y\) we have that \(f^y(p_y) = f^y(p'_y)\), which implies that \(|p'_y| \geq |p_y|\). Denote by \(p_x\) the subpath \((v_m \ldots v_0)\) of \(p_x\), where \(v_0 = y\) and \(v_m = x\). Consider routing in state \(S\). Two cases are possible: either \(p_x p_y\) is available at \(x\) or not. In the first case, since \(|p'| \geq |p_x| = |p_x p'_y| \geq |p_x p_y|\), we have a contradiction because \(p'\) would not be selected in \(S\). In the second case, we will prove that for each vertex \(v_h \neq x\) in \(p_x\) we have that \(|p_h| \leq |(v_h \ldots v_0)p_y|\), where \(p_h\) is the path selected by \(v_h\) in \(S\). This implies that \(|(v_m v_{m-1})p_{m-1}| \leq |p_x p_y| \leq |p_x| \leq |p'|\) and this leads to a contradiction. In fact, if \(|(v_m v_{m-1})p_{m-1}| < |p'|\), then we have a contradiction because \(p'\) would not be selected in \(S\). Otherwise, if \(|(v_m v_{m-1})p_{m-1}| = |p'|\), we have that \(|p_x| = |p'|\). Then, \(x\) prefers \(p_x\) over \(p'\) because of tie break. We have a contradiction since also \((v_m v_{m-1})p_{m-1}\) is preferred over \(p'\) because of tie break in \(S\).

Finally, we prove that for each vertex \(v_h \neq x\) in \(p_x\) we have that \(|p_h| \leq |(v_h \ldots v_0)p_y|\). This trivially holds for \(v_0 = y\). We prove that if it holds for \(v_i\),
then it also holds for \( v_{i+1} \). If \( v_{i+1} \) selects \((v_{i+1} \ v_i)p_i\), then the property holds. Otherwise, \((v_{i+1} \ v_i)p_i\) is disrupted either by better class or by same class by a path \( p_{i+1} \). In the first case, we have that either \( p_{i+1} \) traverses \( m \) or not. Suppose \( p_{i+1} \) traverses \( m \) and let \( q' \) be the neighbor of \( v_{i+1} \) on \( p_{i+1} \). Since \( p_{i+1} \) disrupts \((v_{i+1} \ v_i)p_i\) by better class, then \( p_{i+1} \) is composed by a directed path from \( d \) to \( q' \) and an edge \(((q', \ v_{i+1})\) that can be either an oriented edge from \( q' \) to \( v_{i+1} \) or an unoriented edge. Let \( n \) be the neighbor of \( m \) on \( p \) and \( n' \) be the neighbor of \( n \) on \( p \) different from \( m \). Consider the relationship between \( n \) and \( n' \). Suppose \( n \) is a customer or a peer of \( n' \). If \( m \) is a provider or a peer of \( n \), then \( p \) is not valley-free and \( p \) cannot be available at \( m \) in \( S \), which leads to a contradiction. Otherwise, if \( m \) is a customer of \( n \), then \( n \) would have preferred the best path from its customer \( m \) rather than the path learnt from its provider \( n' \). It implies that \( p \) would not be available at \( m \) in \( S \), that is a contradiction. Hence, \( n \) is a provider of \( n' \) and the subpath of \( p \) from \( d \) to \( n \) is a directed path. Since \( f^x(p_x) = f^x(p') \), we have that also \( p_x \) is a directed path from \( d \) to \( x \). Therefore, \( v_{i+1} \) is a provider of \( v_i \) and so \((v_{i+1} \ v_i)p_i\) would not be disrupted by class in \( S \), which is a contradiction. Hence, \( p_{i+1} \) does not traverse \( m \). By Lemma 5 a path of a class better than \((v_{i+1} \ldots \ v_0)p'_x \) is available at \( v_{i+1} \) and so \( v_{i+1} \) cannot select \((v_{i+1} \ldots \ v_0)p'_x \) in \( S' \), a contradiction. In the second case \( ((v_{i+1} \ v_i)p_i \) is disrupted by same class by a path \( p_{i+1} \) we have that \(|p_{i+1}| \leq |(v_{i+1} \ v_i)p_i| \leq |(v_{i+1} \ldots \ v_0)p_y| \). The second inequality comes from the induction hypothesis.

This concludes the first part of the proof. For proving the second part we show an example where \( m \) announces different paths to different neighbors and the resulting hijacking is not an interception. Consider the BGP instance in Fig. 6. In order to hijack traffic from \( s \), vertices 1 and 4 must be hijacked. Hence, \( m \) must announce \((m \ 3 \ 4 \ d) \) to 2 and \((m \ 2 \ 1 \ d) \) to 3. However, since \((3 \ 4 \ d) \) and \((2 \ 1 \ d) \) are no longer available at \( m \) the interception fails.

\[ \square \]

## 4 Conclusions and Open Problems

Given a communication flow between two ASes we studied how difficult it is for a malicious AS \( m \) to devise a strategy for hijacking or intercepting that flow. This problem marks a sharp difference between BGP and S-BGP. Namely, while in a realistic scenario the problem is computationally tractable for typical BGP attacks it is NP-hard for S-BGP. This gives new evidence of the effectiveness of the adoption of S-BGP. It is easy to see that all the NP-hardness results that we obtained for the hijacking problem easily extend to the interception problem. Further, we solved a problem left open in [4], showing when performing a hijacking in S-BGP is equivalent to performing an interception.

Several problems remain open: 1. We focused on a unique \( m \). How difficult is it to find a strategy involving several malicious ASes [4]? 2. In [13] it has been proposed to disregard the AS-paths length in the BGP decision process. How difficult is it to find an attack strategy in this different model?
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