Non-Gaussianity from Attractor Curvaton

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Abstract

We propose a curvaton model in which the initial condition of the curvaton oscillation is determined by its attractor behavior during inflation. Assuming a chaotic inflation model, we find that the initial condition determined by the attractor behavior is appropriate to generate a sizable non-Gaussianity contribution to the curvature perturbation, which will be tested in the foreseeable future. Implications on the thermal history of the universe and on particle physics models are also discussed.
1 Introduction

The origin of the large scale structures of the universe and the fluctuation of the cosmic microwave background (CMB) radiation can be successfully explained by the primordial density perturbation generated by cosmic inflation [1][2]. In particular, the simplest scenarios in which the density perturbation is dominated by the fluctuation of a single inflaton field has been very successful so far.

In general, however, any light fields other than the inflaton also fluctuate during inflation and may contribute to the primordial density perturbation. In fact, in the curvaton mechanism [4], the primordial density perturbation is mainly generated not by the inflaton fluctuation but by the fluctuation of a light field, the curvaton. The curvaton model is attractive since the inflaton field does not need to provide the whole density perturbation any more, which relaxes the constraints on the inflation models.

One of the interesting observable feature of the curvaton scenario is that it can lead to large non-Gaussianity in the density perturbation when the curvaton energy density is subdominant at its decay time. This is in contrast to the primordial density perturbation generated by the inflaton fluctuation, where the non-Gaussianity is predicted to be highly suppressed [3]. Therefore, if the large non-Gaussianity suggested by WMAP data [5, 6] is confirmed by the forthcoming Planck experiment [7], the curvaton scenario becomes a more plausible explanation for the density perturbation.

There is, however, a drawback in the curvaton scenario if it generates sizable non-Gaussianity. That is the initial condition problem of the oscillation of the curvaton field. As we will briefly review in the next section, the initial field value of the curvaton oscillation is required to be about a hundred times of the Hubble parameter during inflation to generate sizable non-Gaussianity. This initial field value of the curvaton is, however, difficult to be justified since there seems no special meaning on such an intermediate scale.

\footnote{If the required initial field value were at the origin of the curvaton field or at around the Planck scale, for example, it could be rationalized. That is, the origin of the curvaton can be a symmetry enhancement point, which can be chosen during inflation if the symmetry is also respected by the inflaton dynamics. The field value at the Planck scale can be also chosen if the field value during inflation is determined stochastically [9], since the scalar potential of the curvaton is expected to become very steep for the field value larger than the Planck scale.}
In this paper, we propose a curvaton model in which the initial condition of the curvaton oscillation is set by the so-called attractor behavior (See Ref. [8], for example) during inflation. As we will show, the initial condition fixed by the attractor behavior can be appropriate for the curvaton scenario to account for the non-Gaussianity suggested by the WMAP data. We also discuss how high the decay temperature of the curvaton field can be. The implications on the particle physics models are also discussed briefly.

This paper is organized as follows. In section 2, we briefly review the curvaton scenario. In section 3, we discuss the curvaton model which has the attractor behavior. There, we also discuss some implications of the model on the particle physics models.

### 2 Brief Review of The Curvaton Scenario

Before going to discuss the attractor behavior of the curvaton field, let us briefly review the basics of the curvaton scenario. For simplicity, we assume a simple quadratic potential for the curvaton field, $\sigma$,

$$V(\sigma) = \frac{1}{2}m^2\sigma^2,$$  \hspace{1cm} (1)

where $m$ denotes the mass of the curvaton. In the curvaton scenario, the curvaton mass is assumed to be smaller than the Hubble parameter during inflation. We also assume that the Hubble induced mass is suppressed. With such a flat potential, the curvaton field is over-dumped and its field value is frozen to its initial value, $\sigma_i$, during inflation. It should be noted that the field perturbation around $\sigma_i$, $\sigma = \sigma_i + \delta\sigma$, becomes the origin of the primordial density perturbation of the universe eventually.

After inflation, the curvaton starts to oscillate around its vacuum value ($\sigma = 0$) once the Hubble parameter falls below the mass of the curvaton, i.e. $H \lesssim m_\sigma$. Eventually, the coherent oscillation of the curvaton decays into radiation at $t = t_{\text{dec}}$. Hereafter, we assume that the reheating process after inflation completes before the decay of the coherent oscillation of the curvaton.

In the curvaton scenario, the curvature perturbation $\zeta$ on spatial slices of uniform density is evolving until the decay of the curvaton (see e.g. Ref. [11] for a review). After the decay of the curvaton oscillation, the curvature perturbation stops evolving and becomes
constant at the super-horizon scale, which is given by [10],
\[
\zeta = \frac{\rho_\sigma}{4\rho_r + 3\rho_\sigma} \frac{\delta \rho_\sigma}{\rho_\sigma} \bigg|_{t=t_{\text{dec}}} = \frac{\rho_\sigma}{4\rho_r + 3\rho_\sigma} \frac{\delta \rho_\sigma}{\rho_\sigma} \bigg|_{\text{horizon exit}}
\]
\[
= \frac{r_{\text{dec}}}{4 + 3r_{\text{dec}}} \left( \frac{2\delta \sigma_i}{\sigma_i} + \frac{\delta \sigma_i^2}{\sigma_i^2} \right) \bigg|_{\text{horizon exit}} ,
\]
where \(\rho_r\) denotes the energy density of the radiation, \(\rho_\sigma\) and \(\delta \rho_\sigma\) the energy density of the curvaton and its fluctuation on the spatially flat slice. In the above expression, \(r_{\text{dec}}\) is the ratio of the energy densities at the decay time of the curvaton, \(r_{\text{dec}} = \rho_\sigma(t_{\text{dec}})/\rho_r(t_{\text{dec}})\), and we have used the fact that \(\delta \rho_\sigma/\rho_\sigma\) is constant in time from the horizon exit to the decay time. It should be noted that the inflaton contribution to the curvature perturbation is assumed to be negligible.

By using Eq. (2), we obtain the power spectrum of the curvature perturbation,
\[
P_\zeta(k) = \frac{4r_{\text{dec}}^2}{(4 + 3r_{\text{dec}})^2} \left( \frac{H_k}{2\pi \sigma_i} \right)^2 \simeq \frac{r_{\text{dec}}^2}{16\pi^2} \left( \frac{H_k}{\sigma_i} \right)^2 ,
\]
for \(r_{\text{dec}} \ll 1\). Here, \(H_k\) is the Hubble parameter at the horizon exit of the wave number \(k\) and we have used \(\delta \sigma_k = H_k/2\pi\). The nonlinearity parameter \(f_{\text{NL}}\) is, on the other hand, defined by the bispectrum,
\[
B_\zeta(k_1, k_2, k_3) = \frac{6}{5} f_{\text{NL}}(k_1, k_2, k_3)[P_\zeta(k_1)P_\zeta(k_2) + \text{cyclic permutations}] ,
\]
which can be extracted from Eq. (2) by remembering the relation to the Gaussian perturbation \(\zeta_g\),
\[
\zeta = \zeta_g + \frac{3}{5} f_{\text{NL}} \zeta_g^2 .
\]
By comparing Eqs. (2) and (5), we find that
\[
\zeta_g \simeq \frac{r_{\text{dec}} H_k}{4\pi \sigma_i} , \quad f_{\text{NL}} \simeq \frac{5}{3r_{\text{dec}}} ,
\]
for \(r_{\text{dec}} \ll 1\). The above expression shows that the nonlinearity parameter \(f_{\text{NL}}\) can be sizable if the curvaton energy density at its decay time is subdominant.

Now, let us estimate the ratio of the energy densities, \(r_{\text{dec}}\). Here, we assume that the curvaton starts to oscillate before the reheating. At the beginning of the oscillation [14] set by,
\[
H^2 \simeq \left| \frac{1}{5\sigma} \frac{\partial V}{\partial \sigma} \right| = \frac{1}{5} m_\sigma^2 \equiv H^2_{\text{osc}} ,
\]
the ratio of the energy densities of the curvaton and the inflaton, $\phi$, is given by,

$$\frac{\rho_\sigma}{\rho_\phi} \simeq \frac{V(\sigma_i)}{3M_{\text{pl}}^2 H_{\text{osc}}^2} \simeq \frac{5}{6} \frac{\sigma_i^2}{M_{\text{pl}}^2},$$

which is a constant of time until the inflaton decays and the universe is reheated. Here, $M_{\text{pl}}$ denotes the reduced Planck scale. Once the reheating process completes, the energy density of the inflaton is converted to the radiation energy density which is diluted faster than the curvaton density. As a result, the ratio of the energy densities at the curvaton decay time is given by

$$r_{\text{dec}} \simeq \frac{5}{6} \frac{\sigma_i^2}{M_{\text{pl}}^2} \times \frac{T_R}{T_{\text{dec}}}, \text{ (for } T_R > T_{\text{dec}}),$$

where $T_R$ and $T_{\text{dec}}$ are the reheating temperature and the decay temperature of the curvaton, respectively.

As a result, we find that the initial value of the curvaton oscillation should be much smaller than the Planck scale, i.e. $\sigma_i \ll M_{\text{pl}}$, for a sizable nonlinearity parameter to be generated. More explicitly, we may express the required initial value $\sigma_i$ in terms of the power spectrum and the nonlinearity parameter by using Eqs. (3) and (6),

$$\sigma_i \simeq \frac{5}{12\pi} H_k \frac{P_{\zeta}(k)^{1/2}}{f_{NL}} \left(\frac{40 \times 10^{-5}}{P_{\zeta}(k_*)}\right),$$

where $k_* = 0.002 \text{Mpc}^{-1}$ is the pivot scale. Here, we have again assumed $r_{\text{dec}} \ll 1$. As mentioned in the introduction, this initial value of the curvaton is difficult to be rationalized, since it seems no apparent physical meaning on that scale. As we will discuss in the next section, however, the above initial condition is dynamically realized in the model with the attractor behavior.

Before closing this section, let us comment on the decay temperature, $T_{\text{dec}}$. First, let us remember that the baryon asymmetry of the universe and dark matter are required to be generated after the curvaton decay to avoid unacceptably large iso-curvature perturbations [10]. Thus, if the baryon asymmetry is explained by thermal leptogenesis [15], for example, the decay temperature of the curvaton is required to be high, $T_{\text{dec}} \gtrsim 10^9 \text{GeV}$,
since thermal leptogenesis requires a high temperature environment $O(10^9) \text{GeV}$ \cite{16}. On the other hand, from Eqs. (6), (9) and (10), the decay temperature is required to be

$$T_{\text{dec}} \approx \frac{25}{288\pi^2} \frac{1}{P_{\zeta}(k_{NL})} \left( \frac{H_k}{M_{\text{pl}}} \right)^2 T_R,$$

$$\approx 4 \times 10^9 \text{GeV} \left( \frac{H_k}{5 \times 10^{13} \text{GeV}} \right)^2 \left( \frac{40}{f_{NL}} \right) \left( \frac{4.9 \times 10^{-5}}{P_{\zeta}^{1/2}(k_*)} \right)^2 \left( \frac{T_R}{10^{13} \text{GeV}} \right). \quad (11)$$

Thus, a rather high decay temperature, $T_{\text{dec}} \gtrsim 10^9 \text{GeV}$, for example, can be achieved if the Hubble scale during inflation and the reheating temperature are rather high for $f_{NL} = O(10)$.

Unfortunately, however, these requirements are not easily satisfied in explicit inflation models. To make our discussion concrete, let us consider the chaotic inflation model \cite{17} with a quadratic potential $\frac{1}{2} m^2_{\phi} \phi^2$ as an example. In this model, the Hubble parameter is given by $H_k = \sqrt{2N_k/3m_\phi}$ where $N_k$ is the number of $e$-foldings, and hence, the above $T_{\text{dec}}$ is reduced to

$$T_{\text{dec}} \approx \frac{25}{432\pi^2} \frac{N_k}{P_{\zeta}(k_{NL})} \left( \frac{m_{\phi}}{M_{\text{pl}}} \right)^2 T_R,$$

$$\approx 0.7 \times 10^6 \text{GeV} \left( \frac{m_{\phi}}{10^{12} \text{GeV}} \right)^3 \left( \frac{N_k}{60} \right) \left( \frac{40}{f_{NL}} \right) \left( \frac{4.9 \times 10^{-5}}{P_{\zeta}^{1/2}(k_*)} \right)^2 \left( \frac{T_R}{m_{\phi}} \right). \quad (12)$$

It should be noted that the inflaton mass cannot be much larger than $m_{\phi} = 10^{12} \text{GeV}$, since otherwise the inflaton contribution to the curvature perturbation,

$$P_{\zeta}(k_{NL}) \approx \frac{N_k^2 m_{\phi}^2}{6\pi^2 M_{\text{pl}}^2}, \quad (13)$$

cannot be ignored and the non-Gaussianity is suppressed\footnote{If the inflaton contribution to the curvature perturbation is sizable, baryogenesis before the curvaton decay can be consistent with the current constraint on the isocurvature fluctuation \cite{18}.} Therefore, the decay temperature is expected to be rather low for a sizable nonlinearity parameter, $f_{NL} = O(10)$, unless the reheating temperature after inflation is much higher than the inflaton mass (see Refs. \cite{19} for discussions on the reheating temperature and the inflaton mass).
3 Curvaton Model with Attractor Behaviors

Now, let us discuss the curvaton model in which the curvaton field value after inflation is set by the attractor behavior. To be concrete, we consider the chaotic inflation model with a curvaton field whose scalar potential is given by,

\[ V(\phi, \sigma) = \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{4} \lambda \sigma^4 + \frac{1}{2} m_\sigma^2 \sigma^2. \] (14)

where \( \lambda \) denotes a coupling constant. We assume that the Hubble induced mass is negligible. Notice that the spectral index \( n_s \) of the curvature perturbation is red-tilted \( (n_s \approx 0.98) \) in the chaotic inflation scenario, which is consistent with the CMB observations [9].

3.1 Dynamics of the fields during inflation

When the slow-roll conditions are satisfied, the dynamics of the inflaton field and the curvaton field are described by

\[ 3H \dot{\phi} = -\partial_\phi V, \quad 3H \dot{\sigma} = -\partial_\sigma V. \] (15)

In the followings, we assume that the initial value of the inflaton at the beginning of inflation, \( \phi_0 \), is much larger than the Planck scale since we are considering the chaotic inflation scenario. The initial value of the curvaton, \( \sigma_0 \), is, on the other hand, at around the Planck scale, which can be justified if the curvaton potential steeply increases for \( \sigma \gtrsim M_{pl} \).

In the large curvaton field value region, \( \sigma > \sqrt{2} m_\sigma/\sqrt{\lambda} \equiv \sigma_2 \), the potential of the curvaton field is dominated by the quartic term. In this region, the solutions of the equations of motions of the inflaton and the curvaton (Eq. [15]) are interrelated by,

\[ \frac{1}{m_\phi^2} \log \frac{\phi_0}{\phi} = \frac{1}{2\lambda} \left( \frac{1}{\sigma^2} - \frac{1}{\sigma_0^2} \right), \] (16)

where the subscript “0” denotes the initial values at the beginning of inflation. After enough time, \( \sigma \) becomes far smaller than \( \sigma_0 \), and hence, the curvaton field enters into the attractor solution given by

\[ \sigma_{att}(\phi) = \frac{m_\phi}{\sqrt{2\lambda}} \left( \log \frac{\phi_0}{\phi} \right)^{-1/2}. \] (17)
Figure 1: The attractor behavior of the curvaton field during inflation. The magenta, blue, green and red lines correspond to the initial condition $\sigma_0 = 1, 10^{-1}, 10^{-2}, 10^{-3} \times M_{\text{pl}}$, respectively, while the initial condition of $\phi_0$ is $10^2, 10^3, 10^4 \times M_{\text{pl}}$ from left to right. The figure shows that the curvaton becomes insensitive to its initial condition. In the colored region, slow-roll condition for the curvaton field is violated and the curvaton field promptly rolls down until it gets out of the colored region, as is shown with the dotted line. Afterwards, the curvaton field follows the attractor solutions shown in the figure.

In Fig. 1, we show an attractive behavior of the massless curvaton during inflation for a small initial value $\sigma_0 < M_{\text{pl}}$. The figure shows that the curvaton evolves along the attractor solution and becomes insensitive to its initial value $\sigma_0$.

When the curvaton field becomes smaller than $\sigma < \sigma_2$ during inflation, the evolution of the curvaton deviates from the attractor solution, while it keeps following the attractor behavior by the end of inflation if the curvaton field satisfies $\sigma > \sigma_2$ until then. In both cases, the initial condition of the curvaton oscillation after inflation, $\sigma_i$, is insensitive to its initial condition at the beginning of inflation, $\sigma_0$. As we will show shortly, the dynamically chosen $\sigma_i$ can be appropriate to generate a large $f_{NL}$, and hence, the model with the attractor behavior provides us with a solution to the initial condition problem.

Before proceeding to the details of the curvaton behaviors, we comment on the initial condition of the inflaton field. It should be noted that the initial condition of the inflaton cannot be arbitrarily large since the stochastic behavior is significant for $\dot{\phi}H^{-1} < H/2\pi$. 
For the quadratic potential, this condition corresponds to
\[
\phi > (96\pi^2)^{1/4} M_{pl}^{3/2}/m_\phi^{1/2} \sim 10^4 \times M_{pl} \left( \frac{10^{12} \text{GeV}}{m_\phi} \right)^{1/2}.
\] (18)

If the universe begins when the inflaton escapes from the stochastic region, the initial condition is given by the field value which saturates the inequality of Eq. (18). In the followings, we assume that the initial condition of the inflaton field is given by the saturated value,
\[
\phi_0 \sim 10^4 \times M_{pl} \left( \frac{10^{12} \text{GeV}}{m_\phi} \right)^{1/2},
\] (19)
although the attractive behavior is less sensitive to the initial condition of the inflaton, \(\phi_0\) (see Fig. [1]).

3.2 \(\sigma_2 < \sigma_{\text{att}}(\phi_{\text{end}})\)

When the curvaton mass is small enough, it follows the attractor solution by the end of inflation,
\[
\phi \simeq 2M_{pl} \equiv \phi_{\text{end}}.
\] (20)

Thus, in this case, the initial condition of the curvaton after inflation is given by\(^3\)
\[
\sigma_i \simeq \sigma_{\text{end}} \equiv m_\phi \left( \frac{\log \phi_0}{\phi_{\text{end}}} \right)^{-1/2}.
\] (21)

After the end of inflation, the inflaton field starts oscillation around the minimum of the potential. By remembering that
\[
H^2_{\text{end}} \simeq \frac{1}{3M_{pl}^2} \frac{1}{2} m_\phi^2 \phi_{\text{end}}^2 = \frac{2}{3} m_\phi^2, \quad \frac{1}{5\sigma} \frac{\partial V}{\partial \sigma} \bigg|_{\text{end}} \simeq \frac{\lambda}{5} \sigma_{\text{end}}^2 = \frac{1}{10} m_\phi^2 x_{\text{end}}^{-1} < H^2_{\text{end}},
\] (22)
at the end of inflation, we see that the oscillation condition in Eq. (7) is satisfied just a few Hubble times after the end of inflation. Here, we have defined \(x_{\text{end}} = \log(\phi_0/\phi_{\text{end}})\).
Thus, it is reasonable to assume that the curvaton oscillation starts before the reheating of the universe.

When the curvaton field starts oscillation, the potential of the curvaton is governed by the quartic term. In this period, the energy density of the curvaton oscillation scales as the energy density of the radiation. The power spectrum of the curvature perturbation in such a case is given in Ref. [14] (see also the appendix A),

\[
P_\zeta(k) \simeq \frac{9r_{dec}^2}{(4 + 3r_{dec})^2} \frac{\sigma_{osc}^4}{\sigma_k^6} \left( \frac{H_k}{2\pi} \right)^2.
\]  

Here, \(\sigma_{osc}\) and \(\sigma_k\) are the field values of the curvaton at the beginning of the curvaton oscillation and at the horizon exit of the wave number \(k\) during inflation, respectively. As we have mentioned above, the curvaton starts oscillation just after the end of inflation, and hence, \(\sigma_{osc} \sim \sigma_i\), while \(\sigma_k\) is given by \(\sigma_k = m_\phi/\sqrt{2\lambda \log(\phi_0/\phi_k)^{-1/2}}\) with \(\phi_k\) being the field value of the inflaton at the horizon exit of the wave number \(k\). As a result, the power spectrum is given by

\[
P_\zeta(k) = \frac{3r_{dec}^2}{(4 + 3r_{dec})^2} \frac{\lambda}{\pi^2} N_k x_{end}^2 x_k^3.
\]

\[
\simeq 2 \times 10^{-9} \left( \frac{r_{dec}}{0.26} \right)^2 \left( \frac{\lambda}{10^{-8}} \right) \left( \frac{N_k}{60} \right) \left( \frac{9}{x_{end}} \right)^2 \left( \frac{x_k}{7} \right)^3,
\]

where we have defined \(x_k = \log(\phi_0/\phi_k)\) and assumed that \(r_{dec} \ll 1\). The nonlinearity parameter \(f_{NL}\) is also given by

\[
f_{NL} = \frac{16(1 + r_{dec})}{r_{dec}(4 + 3r_{dec})} + \frac{4 + 3r_{dec}}{r_{dec}} \left( 2 - \frac{\sigma_k^2}{\sigma_{osc}^2} \right),
\]

\[
\approx \frac{4}{r_{dec}} \left( 3 - \frac{x_{end}}{x_k} \right),
\]

for \(r_{dec} \ll 1\) (see also the appendix A).

Finally, let us estimate the energy fraction of the curvaton at the decay time, \(r_{dec}\). The energy densities of the radiation and the curvaton at the curvaton decay time are given by

\[
\rho_r|_{dec} = \rho_\phi|_{end} \times \left( \frac{a_{end}}{a_R} \right)^{3} \left( \frac{a_R}{a_{dec}} \right)^{4},
\]

\[
\rho_\sigma|_{dec} = \rho_\sigma|_{end} \times \left( \frac{a_{osc}}{a_2} \right)^{4} \left( \frac{a_2}{a_{dec}} \right)^{3},
\]

(26)
where $a$'s are the scale factors of the universe. The meanings of the subscripts are understood. Therefore, $r_{\text{dec}}$ can be represented in terms of the scale factors as

$$r_{\text{dec}} = \left( \frac{a_{\text{dec}}}{a_R} \right) \left( \frac{a_{\text{osc}}}{a_2} \right) \left( \frac{a_{\text{osc}}}{a_{\text{end}}} \right)^3 \times r_{\text{end}}, \quad (27)$$

with

$$r_{\text{end}} = \frac{\rho_{\sigma}}{\rho_{\text{c}}}. \quad (28)$$

The first factor is given by $a_{\text{dec}}/a_R = T_R/T_{\text{dec}}$, since the temperature of the universe is inversely proportional to the scale factor after the reheating. Between $t_{\text{osc}}$ and $t_2$, the amplitude of the curvaton field is inversely proportional to the scale factor, and hence, we obtain $a_{\text{osc}}/a_2 = \sigma_2/\sigma_i = 2m_\sigma/m_\phi \times \sqrt{x_{\text{end}}}$. The third factor is, on the other hand, given by

$$\left( \frac{a_{\text{osc}}}{a_{\text{end}}} \right)^3 = \left. \frac{H^2_{\text{end}}}{H^2_{\text{osc}}} = H^2_{\text{end}} \left( \frac{1}{5\sigma} \right) \frac{\partial V}{\partial \sigma} \right|_{\text{end}} \simeq 20/3x_{\text{end}}. \quad (29)$$

The curvaton energy fraction at the end of inflation, $r_{\text{end}}$, is simply given by

$$r_{\text{end}} = \frac{1}{4} \lambda \sigma^4_{\text{end}} = \frac{1}{128} \left( \frac{m_\phi}{M_{\text{pl}}} \right)^2 x_{\text{end}}^{-2}. \quad (30)$$

Putting all of them together, $r_{\text{dec}}$ is given by

$$r_{\text{dec}} \simeq \frac{5}{12} \frac{m_\sigma m_\phi}{M_{\text{pl}}} \frac{T_R}{T_{\text{dec}}} x_{\text{end}}^{-1/2}, \quad (31)$$

$$\simeq 0.26 \times \left( \frac{m_\sigma}{10^{11}\text{GeV}} \right) \left( \frac{m_\phi}{10^{12}\text{GeV}} \right) \left( \frac{T_R}{10^{12}\text{GeV}} \right) \left( \frac{10^6\text{GeV}}{10^{-8}/\lambda} \right) \left( \frac{8}{x_{\text{end}}} \right)^{1/2}. \quad (32)$$

As a result, by substituting the above $r_{\text{dec}}$ into Eq. (24), we obtain,

$$P_\zeta(k) \simeq 2 \times 10^{-9} \left( \frac{m_\sigma}{10^{11}\text{GeV}} \right)^2 \left( \frac{m_\phi}{10^{12}\text{GeV}} \right)^2 \left( \frac{T_R}{10^{12}\text{GeV}} \right)^2 \left( \frac{10^6\text{GeV}}{10^{-8}/\lambda} \right)^2 \times \left( \frac{10^{-8}/\lambda}{x_k} \right)^3 \left( \frac{9}{x_{\text{end}}} \right)^3 \left( \frac{N_k}{60} \right), \quad (32)$$

while the nonlinearity parameter in Eq. (25) is of $O(10)$ for the above parameter region. Therefore, we find that the curvaton model where the initial amplitude of its oscillation is set by the attractor behavior can explain the observed power spectrum while predicting a rather large nonlinearity parameter. We should again emphasize that the above result is insensitive to the initial condition of the curvaton field as long as it follows the attractor behavior during inflation.
3.3 $\sigma_{\text{att}}(\phi_{\text{end}}) < \sigma_2$

For a larger curvaton mass, $m_\sigma > m_\phi/(2\sqrt{r_{\text{end}}})$, the curvaton field reaches $\sigma_2$ during inflation. In this case, the curvaton stops following the attractor behavior during inflation and stays at $\sigma \approx \sigma_2$ until it starts oscillation. When the Hubble parameter decreases below the curvaton mass (see Eq. (7)), the curvaton field starts oscillation as a massive field. The power spectrum for such a curvaton is given in the appendix\[ for n = 3, V = \frac{1}{4} \lambda \sigma^4 + \frac{1}{2} m_\sigma^2 \sigma^2$ and $\sigma_{\text{osc}} = \sigma_2$),

$$P_\zeta(k) \approx \frac{r_{\text{dec}}^2}{(4 + 3r_{\text{dec}})^2} \frac{9\lambda}{2m_\sigma^2} \left( \frac{H_k}{2\pi} \right)^2 \approx \frac{3r_{\text{dec}}^2}{4(4 + 3r_{\text{dec}})^2} \frac{\lambda}{\pi^2} \left( \frac{m_\phi}{m_\sigma} \right)^2 N_k,$$

$$\approx 2.5 \times 10^{-9} \left( \frac{r_{\text{dec}}}{0.1} \right)^2 \left( \frac{\lambda}{10^{-6}} \right) \left( \frac{N_k}{60} \right) \left( \frac{m_\phi}{m_\sigma} \right)^2.$$

The nonlinearity parameter is also given by,

$$f_{\text{NL}} \approx \frac{40(1 + r_{\text{dec}})}{3r_{\text{dec}}(4 + 3r_{\text{dec}})} + \frac{5(4 + 3r_{\text{dec}})}{6r_{\text{dec}}} \frac{86}{225},$$

$$\approx \frac{622}{135 r_{\text{dec}}}.$$

By remembering that the energy densities at the curvaton decay time are given by

$$\rho_\phi|_{\text{dec}} \approx \rho_\phi|_{\text{end}} \left( \frac{a_{\text{end}}}{a_R} \right)^3 \left( \frac{a_R}{a_{\text{dec}}} \right)^4,$$

$$\rho_\sigma|_{\text{dec}} \approx \rho_\sigma|_{\text{end}} \left( \frac{a_2}{a_{\text{dec}}} \right)^3,$$

we find that the energy ratio can be expressed in terms of the scale factor of the universe,

$$r_{\text{dec}} \approx \left( \frac{a_{\text{osc}}}{a_{\text{end}}} \right)^3 \left( \frac{a_{\text{dec}}}{a_R} \right) \times R_{\text{end}}.$$

As a result, by using the ratios of the scale factors obtained in the previous section, we find

$$r_{\text{dec}} \approx \frac{10}{9\lambda} \left( \frac{m_\sigma}{M_{\text{pl}}} \right)^2 \frac{T_R}{T_{\text{dec}}} \approx 0.1 \times \left( \frac{10^{-6}}{\lambda} \right) \left( \frac{m_\sigma}{10^{12}\text{GeV}} \right)^2 \left( \frac{T_R}{10^{12}\text{GeV}} \right) \left( \frac{2 \times 10^6 \text{GeV}}{T_{\text{dec}}} \right).$$

\[4\]For $m_\sigma > m_\phi$, the curvaton field is settle to its minimum at $\sigma = 0$ during inflation, and hence, we assume $m_\sigma < m_\phi$ in our analysis.
Therefore, by putting $r_{\text{dec}}$ into the above power spectrum in Eq. (33), we obtain,

\[
P_\xi(k) \simeq 2 \times 10^{-9} \left( \frac{m_\sigma}{10^{12}\text{GeV}} \right)^2 \left( \frac{m_\phi}{10^{12}\text{GeV}} \right)^2 \left( \frac{T_R}{10^{12}\text{GeV}} \right)^2 \left( \frac{2 \times 10^6\text{GeV}}{T_{\text{dec}}} \right)^2 \times \left( \frac{10^{-6}}{\lambda} \right) \left( \frac{N_k}{60} \right),
\]

(38)

while obtaining a rather large nonlinearity parameter $f_{\text{NL}} = O(10)$ (see Eq. (34)).

### 3.4 Implications on particle physics models

Finally, let us discuss some implications of the model. First, let us comment on the decay temperature of the curvaton field. As we have discussed in section 3.2 based on the simplest curvaton model, the decay temperature cannot be very high if we require a rather large nonlinearity parameter, $f_{\text{NL}} = O(10)$. Here, we show that the similar conclusions are reached in the present model. To see such a constraint explicitly, let us express the decay temperature $T_{\text{dec}}$ in terms of $P_\xi$ and $f_{\text{NL}}$ by using Eq. (34), (37) and (38),

\[
T_{\text{dec}} \simeq \frac{311N_k T_R m_\phi^3}{1296\pi^2 m_\phi M_{\text{pl}}^2 P_\xi(k) f_{\text{NL}}},
\]

\[
\simeq 3 \times 10^6 \text{GeV} \left( \frac{m_\phi}{10^{12}\text{GeV}} \right)^3 \left( \frac{40}{f_{\text{NL}}} \right) \left( \frac{T_R}{m_\phi} \right) \left( \frac{N_{k_*}}{60} \right) \left( \frac{4.9 \times 10^{-5}}{P_{\xi}(k_*)^{1/2}} \right)^2. \tag{39}
\]

This shows that the required decay temperature is again rather suppressed for a sizable nonlinearity parameter, although it is slightly larger than the one given in section 3.2 by about a factor of 4.

Secondly, let us discuss a possible interrelation of the present model to another well-motivated new physics model, the see-saw mechanism [21]. For that purpose, let us remember that the required quartic coupling $\lambda$ can be expressed by (see Eqs. (32) and (38)),

\[
\lambda \simeq \frac{97200\pi^2}{96721N_k} f_{\text{NL}}^2 P_{\xi}(k) \left( \frac{m_\sigma}{m_\phi} \right)^2,
\]

\[
\simeq 6 \times 10^{-7} \left( \frac{f_{\text{NL}}}{40} \right)^2 \left( \frac{m_\sigma}{m_\phi} \right)^2 \left( \frac{60}{N_{k_*}} \right) \left( \frac{P_{\xi}(k_*)^{1/2}}{4.9 \times 10^{-5}} \right). \tag{40}
\]

\footnote{Eqs. (32) and (35) show that the higher decay temperature is allowed for the higher curvaton mass. Since we are interested in the upper bound on the required decay temperature, we concentrate on the region $m_\phi/(2\sqrt{x_{\text{end}}}) < m_\sigma < m_\phi$ in the following arguments.}
If we assume that the mass of the curvaton field is generated by the vacuum expectation value of a field $X$ in a supersymmetric model via a superpotential,

$$W = gX\overset{\rightarrow}{\Sigma} \sigma = \frac{1}{\sqrt{2}} \text{Re}(\Sigma),$$

the mass and the quartic coupling of the curvaton are given by $m_\sigma = g \langle X \rangle$ and $\lambda = g^2$, respectively. Thus, for example, to realize $\lambda \sim 10^{-6}$ and $m_\sigma \sim 10^{12}$ GeV, the vacuum expectation value of $X$ is $\langle X \rangle \sim 10^{15}$ GeV. It is quite suggestive that this value is close to the mass scale of the seesaw mechanism. Actually, it has been discussed that the right-handed scalar neutrinos can play a role of the curvaton field in the supersymmetric seesaw mechanism [22]. Detailed studies of the attractor behavior right-handed scalar neutrinos as well as the compatibility with leptogenesis will be given elsewhere [23].

Thirdly, let us comment on an implication on compensated isocurvature perturbations [24]. If the dark matter abundance is created before the curvaton decays, unacceptably large matter isocurvature perturbations are generated and contradicts with the recent observations of the CMB [6]. This can happen in the models of dark matter such as the gravitino produced by thermal scatterings or its decay products [25], the Q-ball or its decay products [26, 27], the WIMPZILLA [28] and the primordial black hole [29]. However, the constraint can be relaxed if the baryon asymmetry is created as the curvaton decays, since the dark matter and the baryon isocurvature perturbations compensate for each other. Such scenario is possible if the right-handed scalar neutrino is the curvaton. The compensation requires $r_{\text{dec}} = \Omega_b/\Omega_{\text{DM}} \simeq 0.2$ [24], where $\Omega_b$ and $\Omega_{\text{DM}}$ are the energy density fraction of the baryon and the dark matter, respectively, and hence the nonlinearity parameter is predicted in each curvaton models. For a curvaton model with a quadratic potential, from Eq. (6), $f_{\text{NL}} \simeq 8$. In the case of an attractor curvaton with a quartic potential, from Eqs. (25) and (34), $f_{\text{NL}}$ can be as large as 40.

As a final remark, let us comment on the Hubble induced mass. In this paper, we have assumed that the Hubble induced mass is negligible. In supergravity, scalar fields obtain soft masses as large as the Hubble scale if inflation is driven by $F$-terms [30], unless there exists a tuning of few percents, a Heisenberg symmetry [31], a discrete $R$ symmetry [32] or a shift symmetry [33], or scalar fields are pseudo Nambu-Goldstone bosons [34]. It will be interesting to construct curvaton models with an attractor behavior but without
fine-tuning in supergravity.

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A General formulas for the power spectrum and the non-Gaussianity

In this appendix, we summarize the formulas for the curvature perturbation and the non-Gaussianity given in Ref. [14]. The assumptions are

· The energy density of the curvaton oscillation is negligible compared to the total energy density at least until the onset of the curvaton oscillation.

· The curvaton field starts oscillation suddenly at \( t = t_{osc} \) and its energy density scales by \( a^{-n} \) in terms of the scale factor of the universe \( a \).

· At some point before the curvaton decays, the quadratic term dominates and the energy density of the curvaton begins to scale by \( a^{-3} \).

· The curvaton oscillation decays suddenly and the curvature perturbation is fixed at that point.

· The curvature perturbation from the fluctuation of the inflaton field is negligible.

· The energy density of the inflaton field scales by \( a^{-3} \) after the end of inflation until the reheating.

· The reheating completes instantaneously at \( t = t_R \) before the curvaton decays.

With these assumptions, the power spectrum of the curvature perturbation is given by

\[
P_\zeta(k) = \frac{r_{dec}^2}{4 + 3r_{dec}} \left( 1 - X(\sigma_{osc}) \right) \frac{V'(\sigma_{osc})}{V'(\sigma_k)} \left( \frac{3}{n} \frac{V'(\sigma_{osc})}{V(\sigma_{osc})} - 3 \frac{X(\sigma_{osc})}{\sigma_{osc}} \right) \frac{H_k}{2\pi},
\] (42)
where the function $X$ is defined by

$$X(\sigma) = \frac{1}{2(c-3)} \left( \frac{\sigma V''(\sigma)}{V'(\sigma)} - 1 \right),$$

$$c = \begin{cases} 
5 & (t_R < t_{osc}) \\
9/2 & (t_{osc} < t_R).
\end{cases}$$

The nonlinearity parameter is given by

$$f_{NL}(k) = \frac{40(1 + r_{dec})}{3r_{dec}(4 + 3r_{dec})} + \frac{5(4 + 3r_{dec})}{6r_{dec}} \left( \frac{3 V''(\sigma_{osc})}{n V(\sigma_{osc})} - \frac{3X(\sigma_{osc})}{\sigma_{osc}} \right)^{-1} \times
$$

$$\left[ \frac{X'(\sigma_{osc})}{1 - X(\sigma_{osc})} + \left( \frac{3 V'(\sigma_{osc})}{n V(\sigma_{osc})} - \frac{3X(\sigma_{osc})}{\sigma_{osc}} \right)^{-1} \left\{ \frac{3 V''(\sigma_{osc})}{n V(\sigma_{osc})} \\
- \frac{3}{n} \left( \frac{V'(\sigma_{osc})}{V(\sigma_{osc})} \right)^2 - \frac{3X'(\sigma_{osc})}{\sigma_{osc}} + \frac{3X(\sigma_{osc})}{\sigma_{osc}}^2 \right\} \right] \frac{V''(\sigma_{osc})}{V'(\sigma_{osc})} - (1 - X(\sigma_{osc})) \frac{V''(\sigma_k)}{V'(\sigma_{osc})}. \quad (44)$$

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