A hybrid reliability algorithm using PSO-optimized Kriging model and adaptive importance sampling

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Abstract. This paper aims to reduce the computational cost of reliability analysis. A new hybrid algorithm is proposed based on PSO-optimized Kriging model and adaptive importance sampling method. Firstly, the particle swarm optimization algorithm (PSO) is used to optimize the parameters of Kriging model. A typical function is fitted to validate improvement by comparing results of PSO-optimized Kriging model with those of the original Kriging model. Secondly, a hybrid algorithm for reliability analysis combined optimized Kriging model and adaptive importance sampling is proposed. Two cases from literatures are given to validate the efficiency and correctness. The proposed method is proved to be more efficient due to its application of small number of sample points according to comparison results.

1. Introduction

In recent years, the first-order reliability method (FORM) or the second-order reliability method (SORM) [5-6] is often applied to calculating the reliability, but the accuracy of the estimated reliability is very low because FORM and SORM are based on Taylor expansion, which ignores higher order terms. It is well known that the Monte Carlo simulation (MCS) is a widely accepted and robust method, but it is impossible to calculate the reliability with a great many samples in a short time. Especially, in case that the performance function is a complicated and time-consuming process.

To reduce computations of the expensive performance function, the Kriging surrogate model been used to deal with this issue for many years. Romero [4], the first researcher applied Kriging on structural reliability problems, compared Kriging with polynomial regression and finite-element interpolation by the Latin hypercube sampling method. Kaymaz [5] calculate structural reliability based on Bucher’s design and central composite design, and compared Kriging model method with classic response surface method. Choi [6] proposed an Bayesian reliability algorithm based on the Kriging dimension reduction method. Echard [7] proposed an active learning function for updating the design of experiment (DoE) base on Kriging, and compared it with other metamodells [8]. Dubourg [9] developed a hybrid algorithm based on Kriging model to solve reliability-based design optimization (RBDO). Zhang [10] used Kriging model for geotechnical reliability ansys. Balesdent [11] developed an adaptive importance sampling method based on Kriging when event probability estimation is rare. Tong [12] proposed a hybrid adaptive method by combining subset simulation and Kriging. In recent years, most studies concentrate more on how to apply Kriging model developed by Lophaven [13] to the structural reliability. However, study finds the solution of Kriging model developed by Lophaven is greatly affected by the given initial value $\theta$, and the pattern search method by Lophaven may lead...
to trapping in local minimum if the initial \( \theta \) is selected inappropriately, which can be seen in Section 2.2.2.

In this paper, to reduce the number of calls to the expensive performance function, a hybrid algorithm for the calculation of reliability is presented. Firstly, particle swarm optimization algorithm (PSO) is used to globally search the coefficients of Kriging model, improving the precision of Kriging prediction. Secondly, a hybrid reliability algorithm combined PSO-optimized Kriging model and adaptive importance sampling is proposed, more details on this algorithm are described in Section 3.3. Two cases are given to validate its superiority. In the first one, the proposed method is proved to be more superior than RSM [14] and ABC-optimized Kriging method by comparing the results they calculate a two-dimension performance function, while in another, it’s more efficient than other surrogate models by comparison of different results[7]. More details can be seen in Section 4.2. Section 5 is the conclusion.

2. Establishment of PSO-optimized Kriging Model

2.1. Basic Kriging model

Following [15], system response \( y(x) \) can be expressed as

\[
y(x) = \sum_{i=1}^{p} \beta_{i} f_{i}(x) + z(x) = f(x)^{T} \beta + z(x)
\]

in which \( \beta = [\beta_{1}, \beta_{2}, \ldots, \beta_{p}]^{T} \) is the vector of regression parameters; \( f(x) = [f_{i}(x), f_{i}(x), \ldots, f_{p}(x)]^{T} \) is the vector of basic functions; \( z(x) \) is the random process.

Gaussian process is selected in Kriging model, and the predictor value \( \hat{G}(x) \) and variance \( \sigma_{e}^{2}(x) \) can be written as

\[
\hat{G}(x) = f(x)^{T} \beta' + r(x)^{T} R^{-1} (Y - F \beta')
\]

\[
\sigma_{e}^{2}(x) = \sigma_{e}^{2} (1 + u(x)^{T} (F^{T} R^{-1} F)^{-1} u(x) - r(x)^{T} R^{-1} r(x))
\]

In which \( u(x) = F^{T} R^{-1} r - f \), \( \sigma_{e} = \frac{1}{m} (Y - F \beta')^{T} (Y - F \beta') \).

The optimal coefficients \( \theta' \) corresponding to maximum likelihood estimation should solve

\[
\min_{\theta} \left\{ \psi(\theta) \equiv |R|^{-1} \sigma^{2} \right\}
\]

in which \( R = (R_{ij})_{nm} \) is the correlation matrix of the stochastic process, and \( |R| \) is the determinant of \( R \), where \( R_{ij} = R(\theta, s_{i}, s_{j}) \), \( i, j = 1, \ldots, m \). When \( \theta' \) is determined, Kriging prediction model is built.

2.2. PSO-optimized Kriging Model

Some text.

2.2.1. PSO Algorithm. Particle Swarm Optimization (PSO) is an optimization algorithm based on evolutionary theory [16-17]. The basic PSO algorithm can be express as

\[
\begin{align*}
v_{i}^{k+1} &= \omega v_{i}^{k} + c_{1} r_{1}(p_{i}^{k} - \theta_{i}^{k}) + c_{2} r_{2}(g_{i}^{k} - \theta_{i}^{k}) \\
\theta_{i}^{k+1} &= \theta_{i}^{k} + v_{i}^{k+1}, (j = 1,2,\ldots,n; i = 1,2,\ldots,N)
\end{align*}
\]

Where \( i \) is particle index; \( j \) is jth component of dimension; \( k \) is time index; \( \nu \) is velocity of particle; \( \theta \) is the position of particle; \( p \) is best position of particle; \( g \) is best position of swarm; \( r_{1}, r_{2} \) are random numbers on the interval \([0,1]\); \( \omega \) is inertia function; \( c_{1}, c_{2} \) are acceleration constants.
2.2.2. PSO-optimized Kriging Model. However, Literature [4] showed that the accuracy of Kriging model approach is not higher than that of RSM if $\theta$ is inappropriate. In DACE [13], $\theta^*$ is found by the pattern search method, but the result is greatly affected by the given initial value $\theta$ because pattern search method has to start with the initial $\theta$, which can be seen in Section 2.2.2.2. Thus, this method may lead to trapping in local minimum.

In this paper, PSO [17], an efficient global optimization algorithm is used to search $\theta^*$ in this study. If the optimal parameters $\theta^*$ of correlation function are obtained, the optimized Kriging model will be established by using MATLAB software.

(1) The procedures of PSO-optimized Kriging

The procedures of PSO-optimized Kriging model is as follows:

Step 1 The parameters of PSO algorithm are initialized, and these parameters consist of $c_1$, $c_2$, $r_1$, $r_2$, $\omega$ which is mentioned in Section 2.2.1. Reference [18] shows $c_1$ and $c_2$ are usually set to equal to 2. $\omega$ is taken as a decreasing linear function in $k$ from 0.9 to 0.4. The bound of position is set to $[0, 100]$, the bound of velocity is set to $[0, 20]$, and number of particles in the swarm $N$ is set to equal 24.

Step 2 Initial positions $\theta_0^i (i=1,2,\ldots,n)$ and velocities $v_0^i (i=1,2,\ldots,n)$ of particles are generated by Latin hypercube sampling method over $[0, 100]$ and $[-4, 4]$, respectively.

Step 3 $\psi(\theta)$ is selected as the objective function according to Eq. (7), and the objective function value of each particle (also called fitness value) is calculated. If the fitness value of current particle is better than its local best value (it is named $\text{pbest}_i$), the position of $i$th particle (it is named $p_i = [\theta_1, \theta_2, \ldots, \theta_n]^T, i = 1,2,\ldots,N$) is updated and the $i$th local best value is updated as well. If any of the fitness values of all particles is better than global best value (it is named $\text{gbest}$), the global best position (it is named $p = [\theta_1, \theta_2, \ldots, \theta_n]^T$) is updated.

Step 4 If the number of iterations (it is named $N_{\text{iter}}$) reach the maximum number (it is set to equal 2000 in this paper) or the relative error between current global best value and last global best value is less than minimum error (it is named $\epsilon$ and set to equal $10^{-10}$ in this paper), namely $N_{\text{iter}} < 2000$ or $\frac{\text{gbest}^k - \text{gbest}^{k-1}}{\text{gbest}^{k-1}} < 10^{-10}$, then iteration is stopped, therefore, the optimal position is $\theta^* = p$ and the optimal value is $g$. Otherwise, the velocity and position of every particle is updated according to Eq. (8), go to Step 3.

Step 5 Substituting the optimal $\theta^*$ into Kriging model, and then establishment of the initial PSO-optimized Kriging model is completed.

(2) Validation

To further demonstrate the fitting efficiency of the PSO-optimized Kriging model, the curve $y = \sin(x)$ is fitted by using the optimized and original Kriging model, where the sample points are $\left\{ \frac{i\pi}{10}, \sin\left(\frac{i\pi}{10}\right) \right\}, i = 3,5,7,9,11$ . The parameters $\theta$ and $\psi(\theta)$ reach the steady state after 60 iterations can be seen from Fig.1, the fitted results of optimized and original Kriging model are shown in Fig.2. From Fig.2, it’s obvious that PSO-Kriging model can simulate the curve $y = \sin(x)$ more accurately than original Kriging model because PSO-optimized Kriging enables optimized model to comprehensively possess the exact interpolation property of Kriging model and the global optimization ability of PSO.
3. Proposed method: hybrid reliability algorithm

3.1. Adaptive Importance Sampling (AIS)

Important Sampling (IS) simulates a population centered on the most probable failure point (also called design point, and its calculation can be seen in Section 3.2) rather than the origin. The failure event probability thus enlarges and it accelerates convergence speed of failure probability. Therefore, the number of sample points remains small. According to literature [19]. The failure probability can be written as

\[ P_f = \int_{-\infty}^{\infty} \frac{I[g(x)] f_x(x)}{h_x(x)} d(x) \]  \hspace{1cm} (6)

Where \( h_x(x) \) is importance sampling density function in which samples concentrate in the regions of most probable failure; \( f_x(x) \) is the real probability density function. The failure probability can be estimated by

\[ P_f \approx \hat{P}_f = \frac{1}{N_{IS}} \sum_{i=1}^{N_{IS}} I[g(x_i)] f_x(x_i)\frac{h_x(x_i)}{h_x(x_i)} = \frac{1}{N_{IS}} \sum_{i=1}^{N_{IS}} I[g(x_i)] f_x(x_i)\frac{h_x(x_i)}{h_x(x_i)} \]  \hspace{1cm} (7)

The coefficient of variation is calculated by

\[ \text{Var}[\hat{P}_f] = \frac{1}{N_{IS}} \left[ \frac{1}{N_{IS}} \sum_{i=1}^{N_{IS}} I[g(x_i)] \left( \frac{f_x(x_i)}{h_x(x_i)} \right)^2 \right] - \hat{P}_f^2 \]  \hspace{1cm} (8)

The coefficient of variation is expressed as

\[ \delta_{\hat{P}_f} = \sqrt{\text{Var}[\hat{P}_f]} \]  \hspace{1cm} (9)

However, the most probable failure point must be calculated in advance. The closer the calculated design point is to actual one, the better sampling efficiency will be obtained. To overcome this drawback that IS depends on the design point, the adaptive importance sampling method is good choice. The general steps of adaptive importance sampling (AIS) can be expressed as following [1,2]:

Firstly, a number of sample points centered on an initial point should be generated through IS, and then the next sampling centers on the design point which is screened from the former one. If there are \( N_{1,\text{fail}} \) sample points in the failure domain, the central point moves to the one which is screened from \( \max(f_x(x_i))(i=0,1,\cdots,N_{1,\text{fail}}) \). Otherwise, if there are no any sample points in the failure domain, and then the central point moves to the one which is screened from \( \min(g(x_i))(i=0,1,\cdots,N_{1}) \). Through several iterations with this manner, the central point can gradually close to the real design point.
According to the principle of adaptive importance sampling, it is known that these this sample point is closer to the design point than the others.

Inspired by the above selection rule of the central point in adaptive importance sampling, in this article, these sample points which are screened from a number of population through IS and they are applied to fit performance function by the usage of PSO-optimized Kriging model, the efficiency of this sampling method is validate in Section 4. Details about the sampling process in the hybrid algorithm can be seen in Section 3.1.

3.2. Calculation of the reliability index

According to the study presented by Hasofer and Lind [20], the reliability index can be computed as the following constrained optimization problem.

\[
\begin{align*}
\min \beta &= \sqrt{\frac{(x_1 - \mu_1)^2}{\sigma_1} + \frac{(x_2 - \mu_2)^2}{\sigma_2} + \ldots + \frac{(x_n - \mu_n)^2}{\sigma_n}} \\
g(x) &= g(x_1, x_2, \ldots, x_n) = 0
\end{align*}
\] (10)

To solve Eq.(10), a Lagrange multipliers iterative method is presented in this paper. The method is described in detail as follows. To calculate a minimum value \( L(x) = \beta + \lambda g(x) \), the following equation is obtained

\[
\begin{align*}
\frac{\partial L}{\partial x_i} &= \beta \frac{\partial g}{\partial x_i} + \lambda \\
\frac{\partial L}{\partial \lambda} &= g(x) = 0
\end{align*}
\] (11)

If

\[
G = \left( \frac{\partial g}{\partial x_1}, \ldots, \frac{\partial g}{\partial x_n} \right)
\] (12)

Eq. (11) can also be expressed as

\[
\frac{x}{\beta} + \lambda G^T = 0 \\
\Rightarrow x^T = -\lambda \beta G^T
\] (13)

Substituting Eq.(13) into Eq.(11), the reliability index \( \beta \) can be written as

\[
\beta = \sqrt{x^T x} = \sqrt{(\lambda \beta G)(\lambda \beta G)^T} = \lambda \beta \sqrt{G G^T}
\]

\[
\Rightarrow \lambda = \frac{1}{\sqrt{G G^T}}
\] (14)

Substituting Eq.(14) into Eq.(13), the reliability index \( x^T \) can be written as

\[
x^T = -\frac{\beta G}{\sqrt{G G^T}}
\] (15)

Substituting Eq.(15) into Eq.(10), the reliability index \( x^T \) can be written as

\[
g(x) = g^*(\beta) = 0
\] (16)

\( \beta = \beta_{\min}^* \) can be worked out by solving Eq.(16) with numerical methods, where \( \beta_{\min}^* \) is the minimum positive solution of \( \beta \).

Therefore, the iterative formula of Lagrange multiplier method is composed of Eq.(12), Eq.(15) and Eq.(16). After repeated iterations, the reliability index \( \beta \) and design point \( x \) in convergence state are obtained.

3.3. Procedures of the hybrid reliability algorithm
Fig. 3 shows calculation procedure of the hybrid reliability algorithm, which are illustrated below:

Step 1 The initial sample points \( S_0 \) is generated through design of numerical experiments (DOE), and the performance response \( Y_0 \) is calculated through the true performance function (If the performance function is implicit, numerical calculation such as FEM must be adopted). The initial DOE is preferred to be defined small and to add the best point to DOE step by step, but the number of sample points required \( \frac{(n+1)(n+2)}{2} \) \( (n \) is the number of random variables) at least.

Step 2 An PSO-optimized Kriging prediction model is established on the base of \( S_0 \) and \( Y_0 \), more details can be seen in Section 2.2.

Step 3 According to the PSO-optimized Kriging model, the initial design point \( x_0^* \) and \( \beta_0 \) are calculated by using Lagrange iteration method which is described in Section 3.2.

Step 4 A number of sample points centered on \( x_0^* \) are generated through IS, which are called \( S' \), and the number of \( S' \) is \( N \). At this step, not all of \( S' \) are applied to fit the performance function and only one of them is used to add to DOE according to the selection rule of the central point in AIS (this selection rule can be described in Section 3.1). This sample point is named \( x' \), update of the previous DOE with \( x' \), \( S_{i+1}=[S_i;x'] \). The response \( y \) corresponding to \( x' \) is calculated by using through true performance function, and \( Y_{i+1}=[Y_i;y] \).

Step 5 PSO-optimized Kriging prediction model is established on the base of \( S_{i+1} \) and \( Y_{i+1} \), and the design point \( x_{i+1}^* \) and \( \beta_{i+1} \) are computed by Lagrange iteration method.

Step 6 If \( \left| \beta_{i+1} - \beta_i \right| / \beta_i \leq \varepsilon \), iteration stopped, and the failure probability and the coefficient of variation \( \delta_f \) are computed with Eq. (7) and Eq. (9) based on the PSO-Kriging model, otherwise, \( i=i+1 \) go back to Step 3.

4. Validation of hybrid reliability algorithm

4.1. Case 1: A 2D performance function

The first case is a 2D performance function from literature \([21]\) to examine the efficiency of the hybrid reliability algorithm, and the performance function can be expressed as

\[
g(x_1, x_2) = \exp(0.2x_1 + 6.2) - \exp(4.7x_1 + 5.0)
\]  

(17)

Where the random variables \( x_1 \) and \( x_2 \) are standard normal distribution and independent, namely \( x_1, x_2 \sim N(0,1) \). Fig.3 shows that the performance function is a straight line when \( g(x) = 0 \).
Results of the proposed method, ABC-Kriging and RSM are listed in Table 1. By comparison, it shows that the result of the proposed method is more similar to the one calculated by MCS ($N=10^7$) than ABC-optimized Kriging [21] and RSM [14]. What’s more, the proposed method required fewer number of sampling points than other reliability method. Note that $P_f$, the failure probability; Var[$\hat{P}_f$], the coefficient of variation for $P_f$; $\varepsilon$, the percentage error in comparison with the failure probability of the MCS.

Table 1 Results of different methods

| Method                          | Total number of sampling points | $P_f$ (Var[$\hat{P}_f$]) | $\varepsilon$ /% |
|--------------------------------|--------------------------------|---------------------------|------------------|
| MCS                            | $10^7$                         | 9.3961 $\times 10^{-3}$ (0.325%) | -                |
| Proposed method                | 14(6+8)                        | 9.4033 $\times 10^{-3}$     | 7.6628 $\times 10^{-2}$ |
| ABC-optimized Kriging          | 49                             | 9.4044 $\times 10^{-3}$     | 8.8335 $\times 10^{-2}$ |
| RSM                            | 100                            | 3.6893 $\times 10^{-3}$     | 60736 $\times 10^{-2}$ |

4.2. Case 2: A performance function with a moderate number of random variables

The second example is a nonlinear performance function with a moderate number of random variables. It is used to examine the efficiency of the proposed method. The performance function is given as

$$g(C_1, C_2, M, R, T, F) = 3R - \frac{2F}{M} \sin \left( \frac{\omega_0 T}{2} \right)$$

(18)

where $\omega_0 = \sqrt{\frac{C_1 + C_2}{M}}$. The mean and standard deviation of all random variables are listed in Table 2.

The proposed method is compared with the crude MCS and the surrogate models [7]. It is seen from Table 3 that the proposed method requires less number of samples, and the number of samples is 50 (29+21). Furthermore, the reliability obtained by the proposed method is more similar to the one estimated by MCS ($N=10^7$) than that of other surrogate models.

Table 2 Mean and standard deviation of six variables

| Variable | Distribution type | Mean | Standard deviation |
|----------|-------------------|------|--------------------|
| $C_1$    | Gauss             | 1    | 0.1                |
| $C_2$    | Gauss             | 0.1  | 0.01               |
| $M$      | Gauss             | 1    | 0.05               |
| $R$      | Gauss             | 0.5  | 0.05               |
| $T$      | Gauss             | 1    | 0.2                |
| $F$      | Gauss             | 1    | 0.2                |

Table 3 Results of different methods

| Method                          | Total number of sampling points | $P_f$ (Var[$\hat{P}_f$]) | $\varepsilon$ /% |
|--------------------------------|--------------------------------|---------------------------|------------------|
| MCS                            | $10^7$                         | 2.858 $\times 10^{-3}$ (0.058%) | -                |
| Proposed method                | 29+21                          | 2.856 $\times 10^{-2}$     | 0.066            |
| AK-MCS                         | 58                             | 2.834 $\times 10^{-2}$     | 1.069            |
| Directional Sampling(DS)       | 1281                           | 3.5 $\times 10^{-2}$       | 22.63            |
| DS + Response Surface          | 62                             | 3.4 $\times 10^{-2}$       | 19.13            |
| DS + Spline                    | 76                             | 3.4 $\times 10^{-2}$       | 19.13            |
| DS + Neural Network            | 86                             | 2.8 $\times 10^{-2}$       | 1.892            |
| Importance sampling (IS)       | 6144                           | 2.7 $\times 10^{-2}$       | 5.396            |
| IS + Response Surface          | 109                            | 2.5 $\times 10^{-2}$       | 1.240            |
| IS + Spline                    | 67                             | 2.7 $\times 10^{-2}$       | 5.396            |
| IS + Neural Network            | 68                             | 3.1 $\times 10^{-2}$       | 8.619            |
5. Conclusion
A new hybrid algorithm for reducing the computational cost of reliability analysis is proposed. The proposed approach combines two advantages, namely PSO-optimized Kriging and AIS. The PSO-optimized Kriging model is updated by adding a new sample point to initial DoE step by step, in which selection of the sample point is likely to that of the design point in adaptive importance sampling. Two cases are given to validate its superiority. In the first one, the proposed method is proved to be more superior than RSM and ABC-optimized Kriging by comparing the results they calculate a 2D nonlinear performance function, while in the other, it’s more efficient than other surrogate models by comparison of different results.

The proposed method is independent of the real performance function and treats it as a black box. Therefore, the proposed method can be applied to any other engineering reliability problem.

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