Complex index of a system’s quality for a set of observations

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Abstract. This paper discusses the solution to the problem of constructing latent complex indexes of a change in a system’s quality for several observations in the absence of training. The algorithm for constructing complex indexes is implemented with the definition of non-random variables of the principal component characterizing the structure of the system under discussion. The algorithm uses a new approach to choose the principal component number, determine the weights of the considered variables and subsystems, and to determine the information content of the complex index based on the selected signal-to-noise ratio parameter. The algorithm was used to obtain complex indexes of quality of life for Russia’s constituent entities for 2007-2016.

1. Introduction
Comparison of composite indexes and ratings of objects determined by these indexes allows evaluating the degree of achieving the management goal. The scientific community did not come to a consensus, if, in principle, one can characterize a multi-aspect phenomenon with a single scalar. According to the UN data, 290 indexes for ranking or complex assessment of countries were developed to 2011 [1]. A variety of methods used to assess the hidden characteristics of poorly formalizable systems of various kinds emphasize the dissatisfaction with results, and the necessity of further investigations in this field.

Rapid growth of the number of complex indexes is a marked sign of their importance in politics and economics. Every large international organization, such as the Organization of Economic Cooperation and Development (OECD), the European Union, the World Economic Forum, or the International Monetary Fund, develops summary indicators in different fields [1, 2].

The common goal of the majority of these indicators is ranking of objects (countries) and their comparative analysis for a certain aggregate measure [1–5].

The use of a single indicator characterizing poorly formalizable processes of social systems (the quality of life, the demographic situation, etc.) is the only possible solution to this problem. That is why the increase of the quality of composite indicators is relevant from both the theoretical and practical point of view. The discussion of advantages and disadvantages of composite indicators is given in [2, 3].

The Organization of Economic Cooperation and Development (OECD) has been working, making the methods of constructing complex indexes more perfect [2, 4, 6, 7]. In 2008, the OECD in cooperation with the European Commission Joint Research Center prepared the Reference Book [8] which became the result of years of the research in this field [2, 4, 6, 7, 9].

The Reference Book gives the set of technical principles of forming composite indicators. Authors select the linear convolution of indicators for the main method of aggregating data, while the main tool to make composite indicators is a factor analysis.
2. Problem Statement

Let us consider the construction of integral evaluation of a system from $m$ of objects, for which the tables of description of objects for a number of observations are known – matrices with the dimension $m \times n$. $A^t = \{a_{ij}^t\}_{i,j=1}^{n,m}$, $t = 1, \ldots, T$. The element of the matrix $a_{ij}^t$ is the value of $j$ indicator of $i$ object, the vector $a_{i}^t = (a_{i1}^t, \ldots, a_{in}^t)$ is the description of $i$ object in the moment $t$. For each moment $t$, the vector of integral indicators looks like

$$q^t = A^t \cdot w^t,$$

Or, for $i$ object in the moment $t$

$$q^t_i = \sum_{j=1}^{n} w^t_j \cdot a^t_{ij},$$

where $q^t = (q_{1}^t, q_{2}^t, \ldots, q_{m}^t)^T$ is a vector of integral indicators of the moment $t$, $w^t = (w_{1}^t, w_{2}^t, \ldots, w_{m}^t)^T$ is a vector of weights of indicators for the moment $t$, $A^t$ is a matrix of the previously processed data for the moment $t$.

Numeric characteristics of the system have been previously unified – the values of variables were collected on the segment $[0, 1]$. If an initial indicator is related to the analyzed integral quality property by monotonous dependence, then initial variables $x_{ij}^t$ for every observation moment are transformed according to the rule:

$$a^t_{ij} = s_j + (-1)^{s_j} \cdot \frac{x_{ij}^t - m_j}{M_j - m_j},$$

where $s_j = 0$, if an optimal value of $j$ indicator is maximal, and $s_j = 1$, if an optimal value of $j$ indicator is minimal; $m_j$ is the lowest value of $j$ indicator across the sample (a global minimum), $M_j$ is the highest value of $j$ indicator across the sample (a global maximum).

If an initial indicator is related to the analyzed integral quality property by non-monotonous dependence (i.e., within the change range of this indicator there is a value $x_j^{opt}$, at which the highest value can be achieved), then the value of a corresponding unified indicator is calculated by the formula:

$$a^t_{ij} = \left\{ \begin{array}{ll}
1 - \frac{|x_{ij}^t - x_j^{opt}|}{\max ((M_j - x_j^{opt}), (x_j^{opt} - m_j))}, & \\
\end{array} \right.$$
interpretation of weight indicators removes one of main uncertainties, when constructing an integral indicator.

3. Application of principal component analysis to calculate complex index of system quality

One of the simplest methods of analyzing the structure of the system being investigated is Principal Component Analysis (PCA). The space of principal components is optimal to model the inner structure of data. In the work [10] the PCA is applied to analyze a relief structure, in the work [11] – to investigate structural properties of carbon nanotubes, in [12] – to analyze geometric properties of chemical compounds, in the work [14] the PCA is applied to analyze structures of biomolecular objects. Among advantages of the PCA which allow to analyze the data flow, authors [14] note a high effectiveness of this method to solve the task of noise filtration and search for the most characteristic features in the data. Successful application of the PCA to describe structures of different systems allows suggesting that this method will give adequate results for description of social systems as well.

When calculating integral indicators by the principal component method, the projections of objects to a new system of coordinates are being constructed, this system being called principal components. Herewith, the sum of squared distances from the objects to their projections onto the first principal component is minimal. Let us consider an orthogonal matrix \( W \) in a linear combination \( Z^T = A^T \cdot W \) of those vectors of lines of matrix \( A \), where the vectors columns of matrix \( Z \) would have the maximal sum of dispersions

\[
\sum_{j=1}^{n} \sigma^2(z_j) \rightarrow \max, \quad \text{where and} \quad \bar{z} = \frac{1}{m} \sum_{i=1}^{m} z_i.
\]

In [15, 16], it is shown that the columns of the matrix \( W \) are eigenvectors of a covariance matrix of the data \( \Sigma = A^T \cdot A \). Usually the vector of integral indicators is calculated as the projection of vectors-lines of the matrix of unified data \( A \) onto the first principal component: \( q = A \cdot w \), where the eigenvector \( w \) matches the maximal eigenvalue of the covariance matrix [17–20]. If the projection onto the first component could not be considered as a successful evaluation, then the situation can be corrected, if, instead of one component, one selects \( l \) of components so that the relative spread \( \gamma_1 \), per the first \( l \) \((l \leq n)\) of principal components:

\[
\gamma_1 = \frac{\lambda_1 + \lambda_2 + \ldots + \lambda_l}{\lambda_1 + \lambda_2 + \ldots + \lambda_n} \geq \Theta
\]

is not less than a certain value \( \Theta \). Selected components are the columns of the matrix \( W \). Using the PCA, the selected principal components form the matrix \( W \), for each sign the impact effect of the selected components is summed up, thus the weights are defined in (2):

\[
w'_{ij} = \sum_{i=1}^{l} w_{ij}
\]

Using the factor analysis, the selected components are exposed to rotation, then significant and insignificant loadings are defined in them. Usually the loadings more than 0,5 are considered as significant. Insignificant loadings reset to zero. Usually the criterion of a satisfactory solution is the possibility of a clear meaningful interpretation of the resulting factors. The last also form the matrix \( W \), and for every sign the impact effect of the selected factors is summed up by the same way [15–16].

4. Calculation of complex index of system quality for a number of observations

Any measurement, including a statistic one, is related to accuracy of a measuring device, so the measurement result inevitably contains a fatal error. Constructing a complex index of a system using statistic data containing errors may be considered as the task of highlighting a useful signal against a background of noise. To obtain accurate characteristics of an object (including weights of composite indexes) on the base of a single measurement inevitably containing an unknown error does not seem
possible. However, calculation of an unknown characteristic is quite probable from a series of such measurements. In particular, a similar task may be successfully solved by astrophotometry which defines main numeric parameters of astronomic objects from more than one observation (image), using a series of noisy images.

The task of searching for the best way to detect a signal in the presence of interferences is one of main and complex problems in fields of measuring technique, radiolocation, astronomy, optical communication, location, navigation, television automation, and many other fields of science and technology. Ideal receiving of signals in the conditions of noise and interference impact is based on simple and deep ideas stated in the most consistent and clear form by P.M.Woodward [21]. From this idea it follows that the task of an ideal device receiving at its input a signal mixed with noise is a complete destruction of unnecessary information which the mixture contains, as well as saving of useful information about those signal parameters which are interesting to the user of the system. In any communication system the most important parameter characterizing the interference level is the ratio of a signal level to a noise level (S/N). This value most fully describes the quality of a signal playback in television systems, mobile communication systems, astrophotometry. Acceptable quality of a reproduced signal is determined by a threshold value of the ratio signal/noise. The threshold ratio S/N is practically similar for different systems and is about 2.2 in dimensionless units.

The technique of the multi-dimension analysis which works excellently to evaluate technical systems gives a dissatisfactory result, when making composite indexes of social systems for consecutive observations. Particularly, computed composite indexes are extremely unstable. Ratings defined by these indexes have a wide spread. Modification of the principal component analysis considering the presence of errors in the data used will be the way out of this situation. The principal component analysis allows highlighting a structure in the noisy data array, and, particularly, it is successfully used to suppress noise, when detecting images. This allows hoping that the PCA may be successfully applied to make composite indexes of poorly formalizable systems.

Using main ideas underlying astrophotometry, one may consider the making of an integral characteristic of the quality change of a complex system as the solution of a task of extracting a useful signal from a series of observations. The set of observations contains the description of an unknown parameter in a multi-dimension mass of noisy data in the conditions of a priori uncertainty about properties of a useful signal. A signal is extracted from the noisy data on the base of a predetermined threshold signal/noise ratio. Weighting coefficients $w_j$ of a linear convolution (1–2) which characterize the structure of the system under consideration on the observation interval are the determined signal in this case. Weighting factors are determined by the set of initial data – matrices

$$A' = \{a'_{ij}\}_{n,m}^{n,m}.$$  

This task is similar to that of reproducing digital images distorted by Gaussian noise over a series of noisy images.

Data change over time can be caused both by the situation and by random errors. The principal component analysis describes an invariable structure of a system, based on eigenvectors and eigenvalues various for different observations. So, it is undistorted values of eigennumbers (empirical eigennumbers) and of eigenvectors (empirical eigenvectors) determined by the data set that will characterize the structure of the system under consideration and be that signal that needs to be extracted from available realizations from the noisy data.

A mean value is this signal for eigenvalues. Averaging works under the assumption about the random nature of noise. It is averaging of values that is used by astrophotography to reduce noise. The assumption about a general trend of change of input data is illustrated in figure 1, where the eigenvalues of covariance matrices for different observations, sorted in descending order, are presented. The signal (an average value) and the deviation from it (noise) are clearly seen.

In the PCA, eigenvectors can be defined with accuracy to the direction, unlike the eigenvalues that are defined with certainty. An average value of factor loadings of variables depends on the chosen direction and it cannot characterize the signal. It is necessary to detect random and nonrandom
components of eigenvectors. We assume that the nonrandom (significant) contribution of a variable into the structure of principal components is rather the invariance of factor loading when data change than the high value of factor loading after the rotation. The sign of the invariance of a variable is a signal/noise ratio which is calculated by the values of factor loadings of this variable. Amplitude of the signal is an average value of the loading coefficients, amplitude of the noise is a standard deviation of the loading coefficients.

Figure 1. The eigenvalues of the covariance matrix of the variables for different observation times. Right – is the mean of the eigenvalues.

Figure 2 presents the choice of the direction of the set of principal components which allows defining random and significant random and nonrandom components of eigenvectors. If the signal/noise ratio is higher than the threshold value, then such a variable is considered as nonrandom. Otherwise, the variable characterizes the noise component of the signal and do not participate in further consideration. The criterion of choice of the direction of eigenvectors will be maximization of the signal level – the sum of calculated values of the SNR for significant variables. A principal component obtained by this way will be called an empiric principal component (EPC). Significant variables of the EPC, as in factor analysis, will be considered further, and insignificant variables are ignored.

Figure 2. Coordination of eigenvectors directions for the first principal component. Left – the original directions of the eigenvectors, on the right – the choice of the direction of the eigenvectors, which ensures the maximization of the signal level.
Table 1 gives the example of defining the EPC over eight observations. The second, fifth, eighth and ninth variables turned out to be significant (the computed SNR is higher than the threshold value 2.2), and the loading values for these variables will be non-zero in this EPC.

The choice of a required number of principal components is performed, considering the noise level of definite selective principal components. The selected empiric principal components form the matrix \( W \), and for every sign the impact effect of the selected empiric components is summed up, thus defining the sought-for weights in (2).

| Years | Variables | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|-----------|---|---|---|---|---|---|---|---|---|
| 2007  | -0.07     | -0.1| 0.26| 0.38| 0.08| -0.21| 0.67| -0.28| -0.44|
| 2008  | -0.05     | 0.19| 0.04| -0.14| -0.33| -0.18| -0.41| 0.2    | 0.77 |
| 2009  | -0.16     | 0.22| 0.19| -0.06| -0.49| -0.2  | -0.24| 0.04   | 0.74 |
| 2010  | -0.17     | 0.22| 0.24| -0.06| -0.37| -0.25| -0.14| 0.16   | 0.79 |
| 2011  | -0.24     | 0.18| 0.31| 0.07 | -0.23| -0.2  | 0    | 0.21   | 0.82 |
| 2012  | -0.25     | 0.18| 0.28| 0.07 | -0.4  | -0.23 | -0.03| 0.17   | 0.76 |
| 2013  | -0.2      | 0.17| 0.2  | 0.01 | -0.47| -0.17 | -0.03| 0.12   | 0.79 |
| 2014  | -0.3      | 0.14| 0.18| 0    | -0.38| -0.22 | 0.04 | 0.18   | 0.79 |

Mean value, \( m \) = 0.18, Standard deviation, \( s \) = 0.15, Signal/noise Ratio, SNR = 2.1, Sum of significant SNRs = 18.13

| Years | Variables | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|-----------|---|---|---|---|---|---|---|---|---|
| 2007  | 0.07      | 0.1 | -0.26| -0.38| -0.08| 0.21 | -0.67| 0.28  | 0.44 |
| 2008  | -0.05     | 0.19| 0.04| -0.14| -0.33| -0.18| -0.41| 0.2    | 0.77 |
| 2009  | -0.16     | 0.22| 0.19| -0.06| -0.49| -0.2  | -0.24| 0.04   | 0.74 |
| 2010  | -0.17     | 0.22| 0.24| -0.06| -0.37| -0.25| -0.14| 0.16   | 0.79 |
| 2011  | -0.24     | 0.18| 0.31| 0.07 | -0.23| -0.2  | 0    | 0.21   | 0.82 |
| 2012  | -0.25     | 0.18| 0.28| 0.07 | -0.4  | -0.23 | -0.03| 0.17   | 0.76 |
| 2013  | -0.2      | 0.17| 0.2  | 0.01 | -0.47| -0.17 | -0.03| 0.12   | 0.79 |
| 2014  | -0.3      | 0.14| 0.18| 0    | -0.38| -0.22 | 0.04 | 0.18   | 0.79 |

Mean value, \( m \) = 0.16, Standard deviation, \( s \) = 0.18, Signal/noise Ratio, SNR = 3.6, Sum of significant SNRs = 19.7

5. Informational criterion of the PCA for noisy data
In calculation algorithms of composite indexes by means of the PCA [5–8], an informational concept, conventional for the PCA and defined (3), is used. It defines the number of principal components \( l \) used to calculate complex indexes.

However, the dimension of the space of signs in the tasks of calculating composite indexes of the quality of a complex system is not too large. Herewith, there are no difficulties, when defining eigenvalues and vectors. A qualitative description of the structure of a system requires either all principal components or the quite large number of them. It may turn out that the information valuable for a specific task is contained exclusively in the last main components. For example, when making a digital model of a terrain based on digitized images, a required contour is set with the eighth and ninth main
components, and principal components 12 and 13 in the method “Gusenitsa” (“Caterpillar”) indicate availability of periodic editions in the fractional period in the analyzed data [22].

Approaches to the evaluation of the number of principal components relative to the required share of explained dispersion are formally applicable, but they implicitly suppose that there is no division into “signal” and “noise”, and any previously defined accuracy has the meaning. When data are divided into a useful signal and noise, the indicated accuracy loses meaning, and it is required to redefine the notion of informativeness. Similar to the dispersion of information, according to (3), one may define SNR-informativeness for the selected number of empiric principal components $N$:

$$\gamma_{SNR} = \frac{S_{1k} + S_{12} + \ldots + S_{1N}}{S_{21} + S_{22} + \ldots + S_{2N}}$$

where $S_{1k}$ is the sum of values of the SNR of non-random variables of $k$ EPC, $S_{2k}$ is the sum of values of the SNR of all variables of $k$ EPC. This value will be a posteriori evaluation (from above) of the SNR-informativeness. Unlike the dispersion information, the SNR-informativeness cannot reach 100% in accordance with the logic of construction. Complete informativeness of a system can be defined by dispersion and SNR-informativeness:

$$\gamma = \gamma_\sigma \cdot \gamma_{SNR}$$

![Figure 3](image_url)  
**Figure 3.** Different types of content calculated for 85 empirical principal component.  

![Figure 4](image_url)  
**Figure 4.** Determination of the number of empirical principal components for calculating the complex index of system.

The number of selected EPC taking part in calculation of a composite index maximizes the general informativeness of a decision defined both by the traditional total dispersion information content and by the accumulated SNR – informativeness which characterizes the EPC signal level relative to the background level. The informativeness of dispersion increases as the number of used EPCs increases, and the informativeness of the SNR decreases, since minor components are more noisy. The SNR – informativeness of EPCs, defined in table 1, amounts to $15.37 / 19.7 = 0.78$.

Figures 3 and 4 show the definition of the number of empiric principal components to calculate the complex index of a system described by 85 statistic indicators. Maximal informativeness $\gamma = 60.52\%$ is achieved, if, to calculate the complex index of a system, nine EPCs are used. Detailed description of the algorithm is in [23, 24].
6. Calculation of composite indicator of the life quality of Russian population
Using the suggested algorithm, complex indicators of the life quality of subjects of the Russian Federation for 2007-2016 were calculated. For the study the valuables from [25] were used. All values of the variables were taken from the website of Rosstat (Federal State Statistics Service).

The change of the life quality of some subjects of the Russian Federation is shown in Figure 5. It is interesting to trace the reflection of recent political events from the values of the calculated composite index. The events of 2014 mostly impacted on the life quality in coastal regions – Kaliningrad and Murmansk, and financial capitals – Moscow and Saint-Petersburg. In other regions the life quality index is less prone to fluctuations in changing political situation. However, over time, the gap in the life quality with the leaders practically does not narrow.

7. Conclusion
Constructing an integral characteristic of a system can be considered as the task of extracting a useful signal against the noise background. In this case, linear convolution weight coefficients of indicators are the signal. Definable weight coefficients must reflect the structure of the system being evaluated. Successful application of the PCA to describe structures of different systems allows suggesting that this method will also give adequate results to describe social systems. However, the principal component analysis and factor analysis (even with fixed methods for extracting factors and rotation way) define differently the structure of principal components and principal factors for different observations. So, the technique of defining weight coefficients by means of the multi-dimension analysis cannot be applied to compare characteristics of objects in dynamics.

The cause of this may be the presence of inevitable errors of the data used. Even small perturbation of source data can cause a significant change in weighting factors, when using the multi-dimension analysis methods. Modification of the principal component analysis considering the presence of errors in the used data has been suggested as a way out of the situation. The algorithm uses a new approach to selection of the number of principal components, and the definition of weights of subsystems under consideration, as well as that of informativeness of the obtained characteristic, based on the selected parameter of the signal/noise ratio. Integral indicators calculated with the help of the suggested modification of the principal component analysis which considers the presence of errors in the used data show a high reliability and a good resistance to input data change.

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