Remarks on the Quark-diagram Description of Two-body Nonleptonic $B$-meson Decays

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Abstract

To lowest-order weak interactions in the standard model, we point out that a complete graph description of two-body mesonic $B$ (or $D$) decays needs ten topologically different quark diagrams. The two-body baryonic $B$ decays can be illustrated in terms of five diagrams. We remark a variety of features of the graph language. Some pure channels of mesonic $B$ decays, which can be used to test the quark-diagram scheme, are discussed in some detail.

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The nonleptonic weak decays of $B$ mesons appear to be a valuable window for determining the quark mixing parameters, probing the origin of $CP$ violation, and investigating the non-perturbative confinement forces. To date, some experimental data on many specific channels of this nature have been accumulated [1]. In the framework of the standard model, the dynamics of exclusive nonleptonic decays is not yet well known. Hence one has to rely, in most cases, on approximate methods or models for quantitative studies. In the literature, the quark-diagram language has been extensively applied to the phenomenology of weak $B$ transitions [2,3]. It proves a simple and intuitive approach, which enables one to gain some salient features of the decays under discussion before carrying out realistic calculations.

This short note is to give some new and non-trivial remarks on applications of the quark-diagram method to nonleptonic weak decays of $B$ mesons. Improving the six-graph scheme presented by Chau [3], we show that a complete description of two-body mesonic $B$ (or $D$) decays needs ten topologically different quark diagrams. The two-body baryonic $B$ decays can be described in terms of five quark diagrams. We point out a variety of features of the graph language. Some pure channels of mesonic $B$ decays, which can be used to test the quark-diagram scheme, are discussed in some detail. We emphasize that a comparison between theoretical predictions and experimental data on the single-graph induced $B$ decays could serve as a useful probe of the dynamics of nonleptonic weak transitions.

To lowest-order weak interactions in the standard model, the nonleptonic weak decays are governed by a single $W$-exchange current with flavour changes. It is known that the inclusive nonleptonic decays of $B$ (or $D$) mesons can be illustrated by six topologically distinct quark diagrams [3]. For the exclusive case, the quarks and antiquarks occurring through $\Delta B = \pm 1$ (or $\Delta C = \pm 1$) transitions are combined by final-state strong interactions into specific hadron (meson or baryon) states. To describe these hadrons in a pictorial approach, the quark-antiquark pairs need to be created from the vacuum for some inclusive graphs. This ensures that the final-state valence quarks can assemble and hadronize themselves in all possible ways to form a two-body or multi-body product [2,3]. In this picture, one has assumed that all strong-interaction effects (in the form of all possible gluon lines) are included in the quark diagrams.

Based on the above arguments, we present in Fig. 1 a set of ten topologically different quark diagrams for a complete description of two-body mesonic $B$ (or $D$) decays. The charged
mesons $B^\pm_u, B^\pm_c, D^\pm$, and $D^\pm_s$ can decay via the graphs 1 ($1'$), 3 ($3'$), and 4 ($4'$); while the neutral mesons $B^0_d, B^0_s$, and $D^0_s$ can decay through the graphs 1 ($1'$), 2 ($2'$), 4 ($4'$), and 5 ($5'$). In a similar way one can present a graph scheme for describing the two-body baryonic decays of $B$ mesons. There are five quark diagrams for a $B$ meson decaying to two light baryons, as illustrated in Fig. 2. The mesons $B^\pm_u$ and $B^\pm_c$ can decay through the graphs (a), (c), and (d); while $(−)B^0_d$ and $(−)B^0_s$ can decay via the graphs (a), (b), (d), and (e). Some useful remarks on the above graph scheme are in order.

(i) The diagrams $n$ and $n'$ ($n = 1, 2, 3, 4,$ or $5$) are different from each other in the final-state hadronization of valence quarks. One of them is colour-suppressed. It should be noted that the graphs $2' − 4'$ apply only to the decays where one final-state meson is a flavour singlet, and that $5'$ applies only to the processes where both final-state mesons are flavour singlets.

(ii) Improving the six-graph scheme given previously by Chau [3], here we have taken four additional diagrams ($2', 3', 4'$, and $5'$) into account. Our present scheme can give a complete graph description of two-body mesonic $B$ (or $D$) decays. In fact, there are not enough dynamic or empiric grounds to justify that contributions from the graphs $2' − 4'$ should be negligibly smaller than those from the corresponding graphs $2 − 4$.

(iii) The diagrams $4$ ($4'$) and $5$ ($5'$) are the loop-induced penguin transitions. The quark-antiquark pairs in these graphs can be produced via both strong (hard and soft gluons) and electroweak ($\gamma, Z^0, \text{and } H^0$) interactions. The latter is in general sensitive to the flavour of quarks to which the $\gamma, Z^0, \text{or } H^0$ couples [4].

(iv) In many practical calculations, the $W$-exchange ($2$ and $2'$) and annihilation ($3, 3'$ and $5, 5'$) diagrams are argued to be helicity unfavoured or formfactor suppressed. However, the helicity suppression is possible to be lifted when soft gluon effects are taken into account [5]. On the other hand, the existing evaluation of annihilation formfactors for exclusive $B$ and $D$ decays has many uncertainties. It is quite possible that significant final-state rescattering effects boost the $W$-exchange or annihilation channels of $D$ and $B$ decays [6].

(v) It should be noted that $B^\pm_c$ mesons can decay through either the $c$-quark spectator graph or the $b$-quark one. The latter case is illustrated in Fig. 3, where the light $B$ mesons ($B^\pm_u, B^0_d$, and $B^0_s$) are produced via the semileptonic or nonleptonic $B^\pm_c$ transitions.

(vi) All five graphs in Fig. 2 are colour-suppressed. For a colour-singlet baryon in the final
state, the colour states of its three valence quarks should be different from one another. This
leads to a colour-suppression factor $2/9$, included in every quark-diagram amplitude [7].

The direct way for testing the above graph scheme is to measure the pure decay modes
induced by a single quark diagram, in which the uncertainty from graph mixing is avoidable or
relatively small. In Table 1, we list some pure channels of two-body mesonic $B$ decays. Clearly
the quark diagrams 1–3 and $1' - 3'$ can be across-checked by detecting the respective pure
decay modes of $B_u, B_d, B_s$, and $B_c$ mesons. Note that there is no pure channel occurring only
through the graph 4 ($4'$) or 5 ($5'$). Measuring $\bar{B}_d^0 \rightarrow K^0\phi$ or $B_s^0 \rightarrow K^0\phi$ may test the graphs 4
and 4' as a whole. Similarly one can look for the pure two-body baryonic decays of $B$ mesons.
Unfortunately, it is still a long run to study baryonic $B$ decays in a quantitative and systematic
way, because the relevant data have not been accumulated.

Phenomenologically it is worth while to pay more attention to the pure decay modes listed
in Table 1. Those channels occurring through the spectator graphs 1 and $1'$ can be calculated
by using the tree-level effective Hamiltonian [6,8] and the factorization approximation [9]. For
illustration, we estimate their branching ratios in the context of the Bauer-Stech-Wirbel (BSW)
model [6]. Our numerical results, accompanied by the current experimental limits to these pro-
cesses, are listed in Table 2. One can observe that the pure $B_u^-$ and $\bar{B}_d^0$ decays via the graph 1
are measurable in the near future. The decay modes $\bar{B}_s^0 \rightarrow D_s^{(*)+} + (\pi^-, \rho^-, a_1^-)$ are also promis-
ing for experimental observation. In contrast, the transitions via the graph $1'$ have relatively
smaller branching ratios. It should be emphasized that the pure channels $\bar{B}_d^0 \rightarrow D^{(*)0}K^{(*)0}$ and
their $CP$-conjugate processes are of large interest for studying $CP$ violation and testing the
Cabibbo-Kobayashi-Maskawa (CKM) unitarity triangle [11,12].

It is difficult to estimate branching ratios of the decay modes via the graphs 2 ($2'$) and 3
($3'$), since the relevant annihilation formfactors are not known. However, theoretical difficulties
cannot obstruct experimental attempts to detect the pure channels occurring only through the
$W$-exchange or annihilation diagrams. For instance, the existing data give $\text{Br}(B_u^- \rightarrow D_s^- K^0) < 1.1 \times 10^{-3}$ and $\text{Br}(B_d^0 \rightarrow D_s^{+}K^-) < 2.4 \times 10^{-4}$ [1]. In comparison with the tree-level transition,
those processes via the diagrams 4 ($4'$) and 5 ($5'$) are more difficult to control in phenomenology.
Using the penguin effective Hamiltonian [13] and the factorization approximation, one can
roughly estimate the branching ratios of $\bar{B}_d^0 \rightarrow \bar{K}^0\phi$ etc. For example, it is expected that
$\text{Br}(\bar{B}_d^0 \rightarrow \bar{K}^0\phi) \sim 10^{-5}$ [14]. Certainly such quantitative calculations involve large uncertainties
Note that the pure penguin-induced decay modes $\bar{B}^0 \rightarrow \bar{K}^0\phi$ and $\bar{B}_s^0 \rightarrow K^0\phi$ are good candidates for probing direct $CP$ violation in the neutral $B$-meson system. They are measurable in the forthcoming $B$ factories. Finally, let us remark the importance to study baryonic $B$ decays. We believe that a comparison between the theoretical predictions and experimental measurements of two-body baryonic $B$ decays, once it is possible, should provide valuable information about the creation processes of quark-antiquark pairs inside hadrons.

To lowest-order weak interactions in the standard model, we have presented a complete quark-diagram scheme for the two-body mesonic or baryonic decays of $B$ mesons. Some new remarks are given on applications of this graph language to specific processes. We discuss some pure channels of mesonic $B$ decays in detail, which can be used to test the graph scheme given here. In view of the fact that two-body nonleptonic $B$ decays are supplying valuable opportunities for the study of flavour mixing and $CP$ violation, much more attention is worth paying to them in both theory and experiments.

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| Quark graph(s) | Pure decay modes |
|----------------|------------------|
| $B_u^- \to f$ | $\bar{B}_d^0 \to f$ | $\bar{B}_s^0 \to f$ | $B_c^- \to f$ |
| 1 | $D_s^- \pi^0$ | $D^+ K^-$ | $D^- K^+$ |
| | $D^- K^+$ | $D_s^- \pi^+$ | $D_s^+ \pi^-$ |
| 1' | $\bar{D}^0 K^0$ | $D^0 K^0$ | $D_s^- \pi^0$ |
| | $\bar{D}^0 K^0$ | $\bar{D}^0 K^0$ |
| 2 | $D_s^+ K^-$ | $D^- \pi^+$ |
| | $D_s^- K^+$ | $D^+ \pi^-$ |
| | $D^0 \pi^0$ | $D^0 \pi^0$ |
| 2' | $D^0 \phi$ | $J/\psi \pi^0$ |
| | $\bar{D}^0 \phi$ |
| 3 | $D^- K^0$ | $K^- K^0$ |
| | $D^- K^0$ | $K^- \pi^0$ |
| | $D_s^- K^0$ | $K^- \pi^0$ |
| | $\bar{K}^0 \pi^-$ | $\pi^- \pi^0$ |
| 3' | $D^- \phi$ | $\pi^- \phi$ |
| 4-4' | $\bar{K}^0 \phi$ | $K^0 \phi$ |
| 5-5' | $\phi \phi$ |

Table 1: Some two-body mesonic $B$ decays induced by a single quark diagram or a couple ones. Here the pseudoscalar mesons $\pi, K$, and $D$ can be replaced by their corresponding vector (or axial-vector) counterparts $\rho (a_1), K^*,$ and $D^*$. 
Table 2: Predicted branching ratios for some two-body mesonic $B$ decays induced by a single spectator ($1$ or $1'$) quark diagram in the context of the BSW model. We have used $|V_{cb}| \approx 0.04$ and $|V_{ub}| \approx 0.0035$. The values of meson masses are quoted from Ref. [1], and the values of decay constants and formfactors are quoted from Refs. [1,6,10].

| Pure channel | CKM factor | Branching ratio (BSW model [6]) | Branching ratio (Experiments [1]) |
|--------------|------------|---------------------------------|----------------------------------|
| $B^- \rightarrow D^- \pi^0$ | $V_{ub}V_{cs}^*$ | $2.7 \times 10^{-5}$ | $< 2.1 \times 10^{-4}$ |
| $B^- \rightarrow D^- \rho^0$ | $V_{ub}V_{cs}^*$ | $1.3 \times 10^{-5}$ | $< 4 \times 10^{-4}$ |
| $B^- \rightarrow D^- a_0^0$ | $V_{ub}V_{cs}^*$ | $1.1 \times 10^{-5}$ | $< 2.3 \times 10^{-3}$ |
| $B^- \rightarrow D^- \omega$ | $V_{ub}V_{cs}^*$ | $1.3 \times 10^{-5}$ | $< 5 \times 10^{-4}$ |
| $B^- \rightarrow D^+ \pi^0$ | $V_{ub}V_{cs}^*$ | $1.9 \times 10^{-5}$ | $< 3.4 \times 10^{-4}$ |
| $B^- \rightarrow D^+ \rho^0$ | $V_{ub}V_{cs}^*$ | $3.6 \times 10^{-5}$ | $< 5 \times 10^{-4}$ |
| $B^- \rightarrow D^+ a_0^0$ | $V_{ub}V_{cs}^*$ | $2.6 \times 10^{-5}$ | $< 1.7 \times 10^{-3}$ |
| $B^- \rightarrow D^+ \omega$ | $V_{ub}V_{cs}^*$ | $3.5 \times 10^{-5}$ | $< 7 \times 10^{-4}$ |
| $B^0 \rightarrow D^+ K^-$ | $V_{cb}V_{us}^*$ | $2.5 \times 10^{-4}$ | — |
| $B^0 \rightarrow D^+ K^-$ | $V_{cb}V_{us}^*$ | $4.1 \times 10^{-4}$ | — |
| $B^0 \rightarrow D^+ K^-$ | $V_{cb}V_{us}^*$ | $1.9 \times 10^{-4}$ | — |
| $B^0 \rightarrow D^+ K^-$ | $V_{cb}V_{us}^*$ | $3.9 \times 10^{-4}$ | — |
| $B^0 \rightarrow D^+_s \pi^+$ | $V_{ub}V_{cs}^*$ | $5.6 \times 10^{-5}$ | $< 2.9 \times 10^{-4}$ |
| $B^0 \rightarrow D^+_s \rho^+$ | $V_{ub}V_{cs}^*$ | $2.7 \times 10^{-5}$ | $< 7 \times 10^{-4}$ |
| $B^0 \rightarrow D^+_s a_0^+$ | $V_{ub}V_{cs}^*$ | $2.1 \times 10^{-5}$ | $< 2.7 \times 10^{-3}$ |
| $B^0 \rightarrow D^+_s \pi^+$ | $V_{ub}V_{cs}^*$ | $7.2 \times 10^{-5}$ | $< 5 \times 10^{-4}$ |
| $B^0 \rightarrow D^+_s \rho^+$ | $V_{ub}V_{cs}^*$ | $1.4 \times 10^{-4}$ | $< 2.2 \times 10^{-4}$ |
| $B^0 \rightarrow D^+_s a_0^+$ | $V_{ub}V_{cs}^*$ | $1.0 \times 10^{-3}$ | $< 2.2 \times 10^{-3}$ |
| $B^0 \rightarrow D^- K^+$ | $V_{ub}V_{cs}^*$ | $1.6 \times 10^{-6}$ | — |
| $B^0 \rightarrow D^- K^+$ | $V_{ub}V_{cs}^*$ | $8.3 \times 10^{-7}$ | — |
| $B^0 \rightarrow D^- K^+$ | $V_{ub}V_{cs}^*$ | $1.5 \times 10^{-6}$ | — |
| $B^0 \rightarrow D^- K^+$ | $V_{ub}V_{cs}^*$ | $1.9 \times 10^{-6}$ | — |
| $B^0 \rightarrow D^+ K^+$ | $V_{ub}V_{cs}^*$ | $4.0 \times 10^{-3}$ | — |
| $B^0 \rightarrow D^+ K^+$ | $V_{ub}V_{cs}^*$ | $1.0 \times 10^{-2}$ | — |
| $B^0 \rightarrow D^+ K^+$ | $V_{ub}V_{cs}^*$ | $8.7 \times 10^{-3}$ | — |
| $B^0 \rightarrow D^+ K^+$ | $V_{ub}V_{cs}^*$ | $3.0 \times 10^{-3}$ | — |
| $B^0 \rightarrow D^+ K^+$ | $V_{ub}V_{cs}^*$ | $9.0 \times 10^{-3}$ | — |
| $B^0 \rightarrow D^+ K^+$ | $V_{ub}V_{cs}^*$ | $9.6 \times 10^{-3}$ | — |
| $B^0 \rightarrow D^0 K^0$ | $V_{ub}V_{cs}^*$ | $2.1 \times 10^{-6}$ | — |
| $B^0 \rightarrow D^0 K^0$ | $V_{ub}V_{cs}^*$ | $1.0 \times 10^{-6}$ | — |
| $B^0 \rightarrow D^0 K^0$ | $V_{ub}V_{cs}^*$ | $1.5 \times 10^{-6}$ | — |
| $B^0 \rightarrow D^0 K^0$ | $V_{ub}V_{cs}^*$ | $2.4 \times 10^{-6}$ | — |
| $B^0 \rightarrow D^0 K^0$ | $V_{ub}V_{cs}^*$ | $1.3 \times 10^{-5}$ | — |
| $B^0 \rightarrow D^0 K^0$ | $V_{ub}V_{cs}^*$ | $6.5 \times 10^{-6}$ | — |
| $B^0 \rightarrow D^0 K^0$ | $V_{ub}V_{cs}^*$ | $9.3 \times 10^{-6}$ | — |
| $B^0 \rightarrow D^0 K^0$ | $V_{ub}V_{cs}^*$ | $1.5 \times 10^{-5}$ | — |
| $B^0 \rightarrow D^0 K^0$ | $V_{ub}V_{cs}^*$ | $3.2 \times 10^{-4}$ | — |
| $B^0 \rightarrow D^0 K^0$ | $V_{ub}V_{cs}^*$ | $1.6 \times 10^{-4}$ | — |
| $B^0 \rightarrow D^0 K^0$ | $V_{ub}V_{cs}^*$ | $2.3 \times 10^{-4}$ | — |
| $B^0 \rightarrow D^0 K^0$ | $V_{ub}V_{cs}^*$ | $3.8 \times 10^{-4}$ | — |
| $B^0 \rightarrow D^0 K^0$ | $V_{ub}V_{cs}^*$ | $1.2 \times 10^{-7}$ | — |
| $B^0 \rightarrow D^0 K^0$ | $V_{ub}V_{cs}^*$ | $5.9 \times 10^{-8}$ | — |
| $B^0 \rightarrow D^0 K^0$ | $V_{ub}V_{cs}^*$ | $8.5 \times 10^{-8}$ | — |
| $B^0 \rightarrow D^0 K^0$ | $V_{ub}V_{cs}^*$ | $1.4 \times 10^{-7}$ | — |
Figure 1: Quark diagrams for two-body mesonic $B$ (or $D$) decays.
Figure 2: Quark diagrams for two-body baryonic $B$ decays.

Figure 3: Examples for $B_c^+$ decays to $B_u^+$, $B_d^0$, and $B_s^0$ mesons.