Impact of Residual Energy on Solar Wind Turbulent Spectra

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Abstract

It is widely reported that the power spectra of magnetic field and velocity fluctuations in the solar wind have power-law scalings with inertial-range spectral indices of $-5/3$ and $-3/2$, respectively. Studies of solar wind turbulence have repeatedly demonstrated the impact of discontinuities and coherent structures on the measured spectral index. Whether or not such discontinuities are self-generated by the turbulence or simply observations of advected structures from the inner heliosphere has been a matter of considerable debate. This work presents a statistical study of magnetic field and velocity spectral indices over 10 years of solar wind observations; we find that anomalously steep magnetic spectra occur in magnetically dominated intervals with negative residual energy. However, an increase in negative residual energy has no noticeable impact on the spectral index of the velocity fluctuations, suggesting that these intervals with negative residual energy correspond to intermittent magnetic structures. We show statistically that the difference between magnetic and velocity spectral indices is a monotonic function of residual energy, consistent with previous work that suggested that intermittency in fluctuations causes spectral steepening. Additionally, a statistical analysis of cross-helicity demonstrates that when the turbulence is balanced (low cross-helicity), the magnetic and velocity spectral indices are not equal, which suggests that our observations of negative residual energy and intermittent structures are related to nonlinear turbulent interactions rather than the presence of advected pre-existing flux-tube structures.

Key words: plasmas – solar wind – turbulence

1. introduction

Observations of power-law spectral distributions of magnetic and kinetic energy in the solar wind, i.e., $E \propto k^n$, have led to the development of various theories of magnetohydrodynamic (MHD) turbulence. It is widely reported that magnetic energy in the inertial range follows a power-law spectrum with $E_B \propto k^{-5/3}$, while kinetic energy follows a shallower power-law spectrum of $E_v \propto k^{-3/2}$ (Mangeney 2001; Podesta et al. 2007; Salem et al. 2009; Borovsky 2012). These spectral indices respectively support the theories of critically balanced turbulence and subsequent modifications accounting for the alignment between velocity and magnetic fluctuations (Goldreich & Sridhar 1995; Boldyrev 2006). The presence of $E \propto k^{-3/2}$ spectral distributions has been recovered in many subsequent numerical simulations (Perez & Boldyrev 2009; Chandran et al. 2015; Mallet et al. 2017). The existence of a truly scale-invariant, self-similar, MHD inertial range would require identical spectral indices for the turbulent magnetic, velocity, and residual energy spectra; accordingly, the observed differences in inertial-range magnetic and kinetic energy spectra remain a relevant topic in studies of MHD turbulence.

It is known that discontinuities and intermittency in observations of turbulence affect measured spectral indices. Roberts & Goldstein (1987) identified large amplitude coherent and discontinuous structures resulting in steep $k^{-2}$ spectra. Li et al. (2011) showed that excluding intermittent current sheets from Ulysses magnetometer data led to the measurement of a $E_B \propto k^{-3/2}$ scaling, rather than the typically reported $E_B \propto k^{-5/3}$ scaling. Borovsky (2010) reconstructed the spectral distribution of magnetic field discontinuities of Advanced Composition Explorer (ACE) observations using a synthetic time-series, finding a $E_B \propto k^{-5/3}$ scaling. There are two dominant explanations for discontinuities and intermittency in the solar wind. The first suggests that discontinuities arise dynamically from the turbulent evolution of the plasma into current sheets (Chang et al. 2004; Mininni & Pouquet 2009; Salem et al. 2009; Boldyrev et al. 2011; Li et al. 2011; Matthaeus et al. 2015). The second suggests that observations of discontinuities correspond to advected flux-tube structures from the inner heliosphere (Mariani et al. 1973; Tu & Marsch 1993; Bruno et al. 2001, 2007; Borovsky 2008).

It is also known that the solar wind contains statistically more magnetic than kinetic energy (Bruno et al. 1985; Roberts et al. 1987; Bavassano et al. 1998; Salem et al. 2009). Various models of MHD turbulence under a range of physical conditions show the growth of negative residual energy, defined as $E_r = E_v - E_b$ (Müller & Grappin 2005; Perez & Boldyrev 2009; Boldyrev et al. 2011, 2012; Gogoberidze et al. 2012; Grappin et al. 2016). The normalized residual energy,

$$\sigma_r = \frac{\langle v^2 \rangle - \langle b^2 \rangle}{\langle v^2 \rangle + \langle b^2 \rangle} = \frac{2\langle z_+ \cdot z_- \rangle}{\langle z_+^2 \rangle + \langle z_-^2 \rangle},$$

is understood to quantify the relative dominance of magnetic or kinetic energy, or equivalently, the alignment between the Elsässer variables defined as $z_\pm = v \pm b/\sqrt{\rho_0 \mu_0}$, where $v$ and $b$ are the fluctuating velocity and magnetic fields and $\rho_0$ is the mean mass density.

A power-law spectrum for $E_r$ was derived by Grappin et al. (1983), with $E_r \propto k^{-2}$ under the assumption of weak turbulence. Müller & Grappin (2005) have subsequently suggested $E_r \propto k^{-7/3}$ spectra for decaying isotropic turbulence and $E_r \propto k^{-2}$ scaling for forced anisotropic turbulence. Numerical experiments of Grappin et al. (2016) explore the growth of residual energy from an Alfvén dynamo effect,
deriving expressions for the scaling of residual energy with total energy based on linear and nonlinear timescales. Chen et al. (2013) used a statistical study of Wind observations to explore connections between spectral index and residual energy, reporting a mean value of $\alpha_v = -1.91$ and a significant correlation between $\alpha_v$ and $\alpha_b$. In a study demonstrating scale invariance of normalized cross-helicity,

$$\alpha_c = \frac{2\langle \mathbf{b} \cdot \delta \mathbf{v} \rangle}{\langle \mathbf{v}^2 \rangle + \langle \mathbf{b}^2 \rangle} = \frac{\langle z_2^2 \rangle - \langle z_1^2 \rangle}{\langle z_1^2 \rangle + \langle z_2^2 \rangle},$$

Podesta & Borovsky (2010) reported $\alpha_c = -1.75$. Both studies demonstrated correlations between cross-helicity and spectral indices for magnetic fields, velocity, and total energy.

The connection between cross-helicity and residual energy is well-established. Bruno et al. (2007) showed that as a fast solar wind evolves from 0.3 to 1 au, the distribution of Helios measurements moves from a highly cross-helical (imbalanced) state to a state with low cross-helicity (balanced) and high negative residual energy. Wicks et al. (2013a) studied the evolution of cross-helicity and residual energy over injection and inertial scales, arguing that the mean angle between the Elsässer variables is scale-dependent and maximized at the outer scale. Wicks et al. (2013b) showed that observations of turbulence tend to be either strongly cross-helical, or have strong residual energy.

In this study, we use 10 years of Wind observations to examine statistical connections between intermittency, magnetic discontinuities, residual energy, and spectral index. We demonstrate that discontinuous events are associated with magnetically dominated intervals with large negative residual energies. Intermittent discontinuities steepen the magnetic spectral index, but have little effect on the measured velocity spectra. Our observations are consistent with the generation of residual energy and intermittency through turbulence, and suggest a close link between residual energy and intermittency.

2. Data

We use observations from several instruments on the Wind mission dating from 1996 January 1 through 2005 December 31: Magnetic Field Investigation (MFI; Lepping et al. 1995), Solar Wind Experiment (SWE; Ogilvie et al. 1995), and Three Dimensional Plasma experiment (3DP; Lin et al. 1995). Data are separated into non-overlapping 1 hr intervals. Intervals are excluded if any of several conditions are met: Wind’s geocentric distance is less than 35R_E, the average solar wind speed is <250 km s^{-1}, or if more than 5% of observations are missing from any one instrument. Linear interpolation is implemented across small data gaps when <5% of an interval is missing. The resulting data consists of 39,415 intervals of 1 hr.

The 3 s cadence 3DP “on board” proton moment measurements are interpolated to the MFI time-base. Velocity and magnetic field measurements ($\mathbf{v}$ and $\mathbf{B}$) into mean and fluctuation quantities using time-averaged values, denoted as $\langle \ldots \rangle$. For example, the mean magnetic field, $\mathbf{B}_0$, is determined by $\langle \mathbf{B} \rangle = \mathbf{B}_0$ with the fluctuation quantities as $\delta \mathbf{B} = \mathbf{B} - \mathbf{B}_0$. We normalize the magnetic field to Alfvén units using $\delta \mathbf{b} = \delta \mathbf{B} / \sqrt{\mu_0 \rho_0}$, where $\rho_0$ is the mean mass density.

Each interval is characterized by energies associated with the velocity and magnetic field fluctuations $E_v = \frac{1}{2} \langle \delta \mathbf{v}^2 \rangle$ and $E_b = \frac{1}{2} \langle \delta \mathbf{b}^2 \rangle$, normalized cross-helicity,

$$\sigma_c = \frac{2\langle \mathbf{b} \cdot \delta \mathbf{v} \rangle}{\langle \mathbf{v}^2 \rangle + \langle \mathbf{b}^2 \rangle},$$

and normalized residual energy

$$\sigma_r = \frac{E_v - E_b}{E_v + E_b}.$$

A minimum variance analysis (MVA) is performed on $\delta \mathbf{v}$ and $\delta \mathbf{b}$ to decompose each interval into eigenvectors corresponding to directions of minimum, maximum, and intermediate variance (Sonnerup & Cahill 1967). Intervals with maximum magnetic energy largely distributed along a single direction likely contain strong discontinuities or a linear polarization to the fluctuations; i.e., if the maximum eigenvalue of the magnetic fluctuations, $\lambda_{\text{max}}^b$, is approximately equal to $E_b$ (Bruno et al. 2001).

Intermittency in the magnetic field is often associated with current sheets (as found by Matthaeus et al. 2015; Veltri & Mangeney 1999; Mininni & Pouquet 2009; Mallet et al. 2016). Using Ampere’s law,

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J},$$

and invoking the Taylor hypothesis, $\frac{\partial}{\partial t} \sim \mathbf{V} \cdot \nabla$, allows the time derivative of magnetic field observations in the spacecraft frame to be used as a proxy for current (Podesta & Roytershteyn 2017). Because single-spacecraft observations constrain spatial derivatives to the bulk solar wind flow direction, the full curl cannot be computed. To estimate the magnitude of currents we implement the reduced curl:

$$\nabla_s \times \mathbf{B} = -\frac{\partial}{\partial x} B_y\hat{y} + \frac{\partial}{\partial x} B_z\hat{z},$$

where the solar wind flow is along $\hat{x}$. Applying the Taylor hypothesis gives an estimate of the current magnitude,

$$J = \frac{1}{\mu_0 V_{sw}} \sqrt{\left(\frac{\partial B_x}{\partial t}\right)^2 + \left(\frac{\partial B_z}{\partial t}\right)^2}.$$

A reduced estimate for the vorticity magnitude $\omega = \nabla \times \mathbf{v}$ is similarly computed.

Intermittency is frequently quantified using the kurtosis,

$$\kappa_x = \frac{\langle x^4 \rangle}{\langle x^2 \rangle^2}$$

(Frisch & Kolmogorov 1995; Veltri & Mangeney 1999; Bruno et al. 2001, 2003; Mangeney 2001; Salem et al. 2009; Matthaeus et al. 2015). Gaussian distributions have $\kappa = 3$, with $\kappa > 3$ indicating heavy-tailed, non-Gaussian statistics. As a simple statistic to quantify intermittency in the magnetic and velocity fluctuations, we measure the kurtosis of the reduced curl estimations of the current and vorticity, $\kappa_J$ and $\kappa_\omega$, for each interval, subtracting 3 to compare with Gaussian statistics.

3. Spectral Fitting

Trace spectral indices for the magnetic and velocity fluctuations in the inertial turbulent range are estimated by performing a linear least-squares fit of the power spectra to a line in logarithmic space. Power spectra for $b(t)$ and $v(t)$ are estimated with a fast Fourier transform (FFT). The trace power
spectra, $E_b$ and $E_a$ are calculated as the sum power spectra from each direction axis.

To prevent overlapping with injection scales, our fits only consider frequencies above $\sim 0.00277$ Hz (6 minutes). To avoid spectral steepening associated with the dissipative scales at high frequencies, we only consider the subsequent 190 frequency bins (up to 0.0555 Hz, or 18 s). The trace spectra are linearly interpolated to an abscissa of 50 logarithmically spaced frequencies (linearly spaced in the logarithmic domain) between 0.00277 and 0.0555 Hz. The power spectra are estimated using a linear least square fit of the interpolated spectra and frequencies in log–log space, with the slope of the best-fit line giving the spectral index (Chen et al. 2013; Podesta et al. 2016). The high-frequency limit helps to minimize flattening effects due to Gaussian noise in low-amplitude 3DP velocity measurements. Though the range of our spectral fits extends to slightly higher frequencies than what previous authors have used, we find good agreement with their estimates for mean values of $\alpha_v$ and $\alpha_b$ (Podesta & Borovsky 2010; Chen et al. 2013; Wicks et al. 2013a).

The spectral index of the trace residual energy is calculated by fitting $[\overline{E}_i] = [\overline{E}_v - \overline{E}_a]$ with the same interpolation and least-squares fitting scheme. The sum of two power-law spectra with different spectral indices cannot itself be a true power-law; we do not intend to claim that our results show precise power-laws, but we use the fitted scaling exponents to quantify the wavenumber dependence of the relevant quantities.

The uncertainty of our estimated spectral indices is found through propagation of error (Press et al. 1992). The variance associated with a single FFT estimation of spectral density is equal to the power spectral density itself. Typically, variance is reduced through averaging over an ensemble of spectra, or windowing the autocorrelation function of a time-series. Here, we derive the uncertainty in spectral index associated with least-squares fitting of a single FFT estimation of spectral density. For spectral density $S_i$, where index $i$ refers to a given frequency bin, $f_i$, Stoica & Moses (2005) give the variance of the spectral density as

$$\text{Var}[S_i] = \sigma^2_i \approx S_i^2. \quad (9)$$

Propagating the variance $\sigma^2_i$ to the logarithm of the spectral density $\log_{10}(S_i)$ gives

$$\text{Var}[\log_{10}(S_i)] = \sigma^2_L = \left( \frac{1}{\ln 10} \right)^2 \sigma^2_i \approx 0.19. \quad (10)$$

The scaling of $\text{Var}[S_i] = S_i$ leads to constant variance in the estimation of the logarithm of spectral density.

For power-law spectra $S_i = \beta f^{\alpha i}$ minimizing

$$\chi^2 = \sum_{i=0}^{N-1} \left( \frac{y_i - \beta - \alpha x_i}{\sigma_i} \right)^2, \quad (11)$$

where $y_i = \log_{10} S_i$ and $x_i = \log_{10} f_i$, with respect to $\alpha$ and $\beta$, gives the least-squares best fits for the spectral index and scaling amplitude. Following Press et al. (1992), propagation of errors gives the uncertainty in $\alpha$ as

$$\sigma^2_{\alpha i} = \sum_i \left( \frac{\partial \alpha}{\partial y_i} \right)^2 \sigma^2_L = \sigma^2_L \frac{\sum x_i^2}{N \sum x_i^2 - (\sum x_i)^2}. \quad (12)$$

The uncertainty in the estimated spectral index, a function of $\sigma^2_L$ and the uniformly used frequency abscissa, is constant for each interval with $\sigma_{\alpha} = \pm 0.16$.

### 4. Results

Figure 1 shows the probability distributions of $\alpha_b$, $\alpha_v$, and $\alpha_r$, with respective means of $-1.66$, $-1.47$, and $-1.73$. Our fits for the the velocity and magnetic energy spectra agree with the spectral indices given in previous studies (Mangeney 2001; Podesta et al. 2007; Salem et al. 2009; Boldyrev et al. 2011; Borovsky 2012). The mean value of $\alpha_r$ is slightly shallower than that in the observations in Chen et al. (2013) but is
consistent with Podesta & Borovsky (2010). Our observations of \( \alpha_r \) are also shallower than predictions of various models of MHD turbulence (Grappin et al. 1983, 2016; Müller & Grappin 2005; Boldyrev et al. 2012; Gogoberidze et al. 2012); however, these models are conducted using assumptions that are not satisfied by solar wind turbulence, e.g., weak turbulence, isotropy, and quasi-normal closure. Deviation in our measurements of \( \alpha_r \) from Chen et al. (2013) likely occur due to differences in the fitting technique and normalization of the magnetic field. Our work directly fits \( \tilde{E}_r \), as the difference in observed velocity and magnetic spectra, and implements MHD normalization of the magnetic field. Chen et al. (2013) use fitted spectra to calculate \( \alpha_r \) and implement a kinetic normalization of the magnetic field. The right panels of Figure 1 show examples of \( \tilde{E}_b, \tilde{E}_v, \) and \( \tilde{E}_r \) with our fits.

The left panel of Figure 2 shows the joint distribution of \( \alpha_b \) and \( \alpha_v \). The mean value of the velocity spectral index is \( \alpha_v \sim -3/2 \). A tendency for \( \alpha_b < \alpha_v \) is evident in the distribution. The right panel of Figure 2 shows the joint distribution of magnetic and velocity indices colored by the mean value of \( \sigma_r \). The statistical preference for negative residual energy is clearly evident in our observations. The residual energy becomes more negative as the spectral indices \( \alpha_v \) and \( \alpha_b \) diverge, i.e., as the magnetic spectral index steepens. Particularly interesting is the consistent level of residual energy along the line \( \alpha_v = \alpha_b \) throughout the range of observations.

The left panel of Figure 3 shows the joint distribution of \( \sigma_r \) and \( \alpha_b \). The secular trend suggests that the residual energy plays a significant role in setting the spectral index of the magnetic field. Specifically, it is evident that magnetically

**Figure 2.** (Left) Joint distribution of the fitted spectral indices for magnetic field, \( \alpha_b \), and velocity, \( \alpha_v \), fluctuations in the inertial range. The black lines show the mean values of \( \alpha_v \sim -3/2 \) and \( \alpha_b \sim -5/3 \). (Right) The distribution of \( \alpha_v \) and \( \alpha_b \) colored by the mean residual energy in each bin. The black line shows \( \alpha_v = \alpha_b \). Deviations from \( \alpha_v \approx \alpha_b \) lead to an increase in negative residual energy (\( \tilde{E}_b > \tilde{E}_v \)).

**Figure 3.** (Left) Joint distribution of normalized residual energy \( \sigma_r \) and spectral index of the inertial-range magnetic fluctuations \( \alpha_b \). Data are column-normalized to the maximum number of counts in each \( \sigma_r \) bin. (Right) Joint distributions of fitted power-law spectral indices of residual energy spectra \( \alpha_r \) and inertial-range magnetic field fluctuations \( \alpha_b \). The solid black line shows the least square linear fit to the data with correlation 0.77 and slope of 0.56. The dashed line shows \( \alpha_r = \alpha_b \). Contours in either panel show 20, 100, and 200 level counts of the joint distributions.
dominated intervals, with $\sigma_v \approx -1$, exhibit steeper magnetic spectra. The right panel of Figure 3 shows the joint distribution of $\alpha_r$ and $\alpha_v$; these variables are highly correlated, with a Pearson correlation value of 0.78. A linear best fit gives $\alpha_b \propto 0.56 \alpha_v$. These results imply that spectral indices of the magnetic fluctuations and residual energy are largely determined by the average residual energy over each interval.

The left panel of Figure 4 shows the joint distribution of $\sigma_r$ and $\alpha_v$. Unlike the spectral index of the magnetic fluctuations, $\alpha_v$ exhibits little dependence on $\sigma_r$, suggesting that residual energy is mostly determined by magnetic fluctuations. The middle panel of Figure 4 shows the difference between the magnetic and velocity spectral indices as a function of residual energy. As the residual energy increases, i.e., the plasma becomes less magnetically dominated, the spectral index of the magnetic field approaches that of the velocity spectra. Though our fit spectra do not distinguish between true and approximate power-law spectral distributions, the lack of effect of the residual energy on the velocity fluctuations, as well as the fact that $\alpha_b \approx \alpha_v \approx -3/2$ when $\sigma_r \approx 0$, suggest that the $-3/2$ spectral index may be the fundamental universal scaling index related to solar wind inertial-range MHD turbulence.

The right panel of Figure 4 shows the joint distribution of the difference between magnetic and velocity spectral indices, $\alpha_v - \alpha_b$, and cross-helicity, $\sigma_r$, suggesting that high cross-helicity measurements occur only when $\alpha_v = \alpha_b$. These results agree with Podesta & Borovsky (2010), who previously reported that the magnetic field spectrum becomes steeper than the velocity spectrum as cross-helicity decreases. A geometrical consideration of cross-helicity and residual energy gives the constraint of $\sigma_r^2 + \sigma_v^2 < 1$ (Wicks et al. 2013b). Clearly, the decrease of $|\sigma_r|$ with large negative $\sigma_r$ and $|\alpha_b| > |\alpha_v|$ is inevitable. However, there is no such geometric argument that demands balanced turbulence (i.e., $|\sigma_r| < 1$) to coincide with unequal spectral indices such that $\alpha_b \approx \alpha_v$. The observations in Figure 4 (right), in which the joint distribution of $\alpha_v - \alpha_b$ is conditioned on $\sigma_r$, suggests that balanced turbulence (i.e., $|\sigma_r| < 1$) coincides with $|\alpha_b| > |\alpha_v|$. There is no a priori reason that we expect balanced turbulence (i.e., $|\sigma_r| < 1$) to have different spectral indices, $\alpha_b \approx \alpha_v$. This result is consistent with the generation of negative residual energy through turbulence, i.e., that nonlinear interactions between the Elsässer variables lead to the growth of intermittent structures with negative residual energy, since at fixed total energy the nonlinear interaction term $\mathbf{z}_\perp \cdot \nabla \mathbf{z}_\perp$ is stronger when $\sigma_r = 0$.

Using the MVA analysis, the value $\lambda_{\text{max}}$, corresponding to the fraction of energy associated with the maximum variance direction, is calculated for both the magnetic and velocity fluctuations. The top panels of Figure 5 show the joint distributions of $\lambda_{\text{max}}^b$ (left) and $\lambda_{\text{max}}^v$ (right) with the residual energy. There is a strong dependence of $\lambda_{\text{max}}^b$ on the residual energy, which is not observed for $\lambda_{\text{max}}^v$. This suggests that large negative residual energy occurs as the result of discontinuous/coherent structures in the magnetic field. In fact, the most negative values of residual energy seem to demonstrate the smallest values of $\lambda_{\text{max}}^v$, which suggest more isotropic velocity fluctuations; however, this could be due to sampling bias toward very-low-amplitude velocity fluctuations subject to noise.

To further connect the negative residual energy with magnetic intermittency, we examine the kurtosis of the reduced curl estimates for current and vorticity, $\kappa_J$ and $\kappa_v$, as proxies for intermittent features. The bottom panels of Figure 5 show the joint distribution of the residual energy and $\kappa_J$. A decrease in $\kappa_J$ is observed with increasing residual energy, suggesting that the negative residual energy is caused by magnetic discontinuities with associated bursty currents. At low residual energy the velocity fluctuations appear more Gaussian, which may indicate low-amplitude velocity fluctuations that are possibly subject to noise. Regardless, we uniformly observe $\kappa_v < \kappa_J$, suggesting less intermittency in the velocity fluctuations.
5. Discussion

Many authors recover $k^{-3/2}$ spectra in simulations (Maron & Goldreich 2001; Müller & Grappin 2005; Perez & Boldyrev 2009; Mallet et al. 2016). This scaling is in agreement with analytic predictions of strong, three-dimensional, anisotropic turbulence, appropriate for the solar wind (Boldyrev 2006; Chandran et al. 2015; Mallet & Schekochihin 2017). Our observations here suggest that the observed difference between the spectral indices of velocity and magnetic field turbulent fluctuations occurs due to the presence of negative residual energy in the form of intermittent current sheets. When the magnetic and kinetic energies are in equipartition, the spectral slope of the magnetic fluctuations approaches the velocity spectral index. The velocity spectral index is insensitive to the residual energy, with a mean value of $\alpha_v = -3/2$. This picture is consistent with the numerical model of Mininni & Pouquet (2009), which demonstrates the formation of thin current sheets in the magnetic field through decaying turbulence, leading to enhanced intermittency and steepening of the magnetic spectral index of $\alpha_b = -5/3$. In this interpretation, the magnetic fluctuations form thinner structures than the velocity fluctuations, which then dissipate energy more quickly. Additionally, Grappin et al. (2016) recovered $\alpha_v = -3/2$ and $\alpha_b = -5/3$ scalings using an Alfvén dynamo model to generate negative residual energy. We note that these descriptions do not address the collisionless and kinetic nature of dissipation in the solar wind; a full explanation requires a more complex account of the physical mechanisms of dissipation.

Our results are congruent with Li et al. (2011), who interpreted their results as indicative of flux-tube crossings. However, for several reasons, we believe our results support the idea of intermittency through turbulence rather than observations of advected flux tubes. First, we have identified the presence of intermittent events in the magnetic field contributing to negative residual energy; these events have no accompanying signature in the velocity fluctuations. Observed intervals with intermittent signatures present in both velocity and magnetic fluctuations are likely contained along the $\alpha_v = \alpha_b$ line, where steepening may occur in both the magnetic and velocity spectra. If observations of flux-tube crossings are present in the data set, they likely exist in this region. Second, our results agree with Salem et al. (2009), who noted that heavy tails in the distributions of fluctuation amplitudes affect measurements of spectral indices at lower amplitudes in the magnetic field than in the velocity measurements. This again suggests the presence of intermittent magnetic fluctuations with no velocity component.

Quantification of the variance in spectral density estimates demonstrates that our fitted spectral indices are accurate to

![Figure 5](image-url)
10%. The implementation of this variance estimate will help constrain observations made of the inner heliosphere by the FIELDS instrument on the Parker Solar Probe (Bale et al. 2016). Additionally, further studies of Wind observations may benefit from our quantitative analysis of spectral variance, e.g., by determining the nature of compressive fluctuations in the solar wind (Bowen et al. 2018).

The joint distribution of the difference of spectral slopes, \( \alpha_b - \alpha_a \), and the cross-helicity \( \sigma_b \), suggests that intervals of balanced turbulence preferentially occur with \( \alpha_b \approx \alpha_a \). Though unequal spectral slopes, associated with non-equipartitioned \( E_b \) and \( E_p \), geometrically preclude the observation of imbalanced fluctuations with \( |\sigma_b| \sim 1 \), the observations in Figure 4 (right) suggest the stronger statement that balanced turbulence occurs only with unequal spectral indices. We interpret the lack of balanced turbulence with \( \alpha_b = \alpha_a \) as evidence for the generation of residual energy through nonlinear turbulent interactions (Boldyrev et al. 2011). The observation that solar wind turbulence is either highly imbalanced, \( |\sigma_b| = 1 \), or highly anti-aligned, \( \sigma_b = -1 \), has been noted by previous authors (Bruno et al. 2007; Wicks et al. 2013b); however, observations of low cross-helicity, directly corresponding directly to deviations in turbulent spectral indices, suggest that the residual energy is closely connected with nonlinear turbulent interactions.

We have omitted the scaling of total energy \( E_T = E_b + E_p \), as it is degenerate with our results for \( E_p \). When \( \sigma_b \approx -1 \), the magnetic energy dominates the kinetic energy; accordingly, the total energy is approximately equal to the magnetic energy. When there is partition in kinetic and magnetic energy, the scalings of kinetic and magnetic energy spectra are equal (see Figure 4); in which case, the scaling of the total energy is again identical to the magnetic energy. Thus, in both limiting cases, the spectrum of total energy scales with the magnetic energy. This observation implies that the scaling of total energy is related to the residual energy of the solar wind, scales similarly to the energy contained in the magnetic field, and reinforces the argument that residual energy plays an active role in the solar wind turbulence.

In summary, this work quantifies the scaling of magnetic, kinetic, and residual energy throughout the inertial range. We demonstrate that intermittency in the solar wind causes deviations in spectral indices between magnetic and kinetic energy spectra. However, we reiterate that the sum of two power-law spectra with different spectral indices cannot itself be a true power law. Strictly speaking, the observed deviation of \( \alpha_b \neq \alpha_a \) when \( \sigma_b \approx -1 \) indicates different distributions in magnetic and kinetic energies across scales rather than the existence of true power-law spectra with unequal indices. However, we note that the spectral indices of the magnetic and kinetic energies converge to \(-3/2\) at low values of residual energy; this fact, coupled with the weak effect of intermittency on the kinetic energy spectral index, suggests that the \(-3/2\) value may represent the universal turbulent inertial-range scaling index of Alfvénic solar wind for \( \sigma_b \sim 0 \).

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