Magneto-electronic transport theory in ferromagnets above the Curie temperature and in semiconductors

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Quantitative differences of Lagrange multipliers between standard Fermi-Dirac statistics (FDS) and Ionization energy (E$_i$) based FDS (iFDS) are analyzed in detail to obtain reasonably accurate interpretations without violating the standard FDS. The resistivity and Hall-resistance models in 1D, 2D and 3D are also derived to illustrate the transport phenomena in semiconducting materials. It is shown via calculation that the charge carriers in these materials seem to be strongly correlated in term of electron-ion attraction or simply, fermions in those materials are somewhat gapped due to Coulomb attraction. This Coulomb attraction naturally captures the polaronic effect in manganites. E$_i$ is found to be the only essential parameter that predicts $\rho(T, doping, pressure, magnetic field)$ quite accurately. However, this model as will be pointed out, is not suitable for metals with free-electrons and strong electron-phonon scattering. It is to be noted that iFDS and $\rho(T, doping, pressure, magnetic field)$ models are only valid in the paramagnetic region of ferromagnetic manganites and other doped-semiconductors. Recent X-ray photoemission spectroscopy (XPS) studies have indicated that there exists a critical crossover from Mn$^{3+}$ to Mn$^{2+}$ depending upon Mn’s concentrations in (Ga$_{1-x}$Mn$_x$)As diluted magnetic semiconductors (DMS). Such phenomenon occurring at certain critical concentration directly point towards the applicability of E$_i$ based Fermi-liquid theory (iFLT). As such, iFDS is also discussed with respect to DMS above the Curie temperature.

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1. INTRODUCTION

Doped-compounds including oxides and its electrical and magnetic measurements have contributed enormously to the understanding of electrical properties of ferromagnets at $T > T_C$ (paramagnetic ↔ ferromagnetic transition $T$) and semiconductors. The complete mechanism above $T_C$ for ferromagnets is somewhat vague since the variation of $\rho(T, doping, pressure, magnetic field (H))$ in term of hopping activation energy, $E_p$ is still unclear. I.e., the variation of $E_p$ with doping is not explicitly predictable. Hence, it is essential to study and understand the variation of $\rho(T, doping, pressure, H)$ in order to enhance the predictability of electrical properties that may accelerate the possible applications of these materials. Two-dimensional (2D) $E_i$ based Fermi liquid model was originally used to describe c-axis and ab-planes conduction of overdoped cuprate superconductors [1, 2, 3, 4]. Subsequently, it was further developed to capture both $T_{crossover}$ (c-axis pseudogap) and $T^*$ (spin gap characteristic $T$) in ab-planes peculiar conduction involving spinons and holons, which is known as the hybrid model [2, 5]. Take note that there are three typographical errors in Ref. [2], two of them can be found in Eq. (9) of Ref. [6] where $(m^*_e m^*_h)^{-3/4}$ and $(2\pi \hbar^2 / k_B)^{-3/2}$ should be replaced with $(m^*_e m^*_h)^{-1/4}$ and $(2\pi \hbar^2 / k_B)^{3/2}$ respectively. Therefore, the fitting parameter, $A_2$ is actually equal to $(A_2/2 e^2)^{(m^*_e m^*_h)^{-1/4}/(2\pi \hbar^2 / k_B)^{3/2}}$. One can safely claim that manganites are one of the well studied compound after Cuprates due to its vast applications with hardly any temperature constraints. Literally, La-Nd based manganites have been investigated in almost all aspects of experimental techniques, yet there are still several theoretical rendezvous on magneto-electronic properties both below and above $T_C$. A wide variety of these properties based on doping and Mn’s valence state in manganites were reported to understand the transport mechanism(s) [22, 23, 24, 25, 26, 27, 28, 29, 30] are also regarded as equally important to determine the electrical properties of manganites. Furthermore, metallic conduction below $T_C$ has been studied using double exchange mechanism (DEM) between s and d orbitals [31] and the displacement of hysteresis loop in field-cooled sample with an additional scenario of non-linear spin and charge fluctuations due to magnon [32]. Explanations in term of hopping electrons and DEM [33], and the influence of microstructural transition arises from ionic radius or valence state of Nd in Nd$_2$Sm$_{1-x}$Ca$_x$MnO$_3$ [34] were also reported extensively.

Presently, the discovery of ferromagnetism (FM) in diluted magnetic-semiconductors (DMS) such as Mn doped...
GaAs is expected to have an enormous impact on scientific and technological developments since both electronic and magnetic properties are realized simultaneously. Two primary advantages of DMS over manganites are the ease of controlling electrons’ \( n \) and holes’ \( p \) concentrations and electromigration hardly plays any significant role due to zilch oxygen content. Electromigration in oxides is a process of migration and diffusion of mainly oxygen atoms (weakly bound) in the vicinity of high temperature, \( T \) (random direction), electric, \( E \) and magnetic fields, \( H \) (directional) as well as in high vacuum. The attention for the discovery of DMS by Munekata et al. \[32\] and later by Ohno et al. \[34, 37\] in the early 90s was actually due to the possibility of having a brand new type of magneto-electronic materials other than manganites both in term of experimental facts and theories though it does not mimic the excitement for the discovery of cuprate superconductors in the late 80s. Nevertheless, DMS is indeed unique in a sense that it exposes the potential applications in a wide variety of electronic industries namely, high-density non-volatile magnetic memory integrated with semiconductor integrated-circuits, magnetic sensors, optical isolators integrated with semiconductor lasers for optical communication systems \[38\] and nanotechnology \[39, 40\]. The intensive progress of DMS’s sample growth is apparently fuelled by the advancement of Molecular-Beam-Epitaxy’s (MBE) technique in the 90s \[41, 42, 43, 44\]. Basically, FM arising from the said doped material was argued against the well known Zener DEM \[43, 44\] since DMS requires long-range spin interaction. This long-range spin interaction is necessary to produce FM due to very low concentration of magnetic ion (Mn) in DMS and consequently it is named as such. The highest Curie temperature \( (T_C) \) has been reported to be 110 K for MBE grown, Ga\(_{0.94}\)Mn\(_{0.053}\)As compound \[37\]. Recently however, Teraguchi et al. \[47\] and Saito et al. \[48\] have highlighted that Ga\(_{0.94}\)Ga\(_{0.06}\)N and Zn\(_{0.8}\)Cr\(_{0.2}\)Te have \( T_C \)s of about 300 K and 400 K respectively. As a matter of fact, it is well known that only oxides namely, Pr,La,Nd,Ca,Sr-Mn,Fe-O compounds provide the established phenomenon called FM with \( T_C \) ranging from above and far below room temperature \( (T_r) \). In fact, the discovery of FM in manganites were first reported by Jonker and van Santen \[49, 50\] in early 50s and antiferromagnetism (AFM) in Cd\(_2\)Mn\(_{1-x}\)O\(_3\) by Dey \[51\]. This particular oxide is also captured in detail by Zener DEM \[43, 44\] with slight improvements in term of polaronic effect \[52, 53\] due to Jahn-Teller splitting of Mn\(^{3+}\) ions. However, FM with a \( T_C \) up to 110 K that was observed in Ga-Mn-As magnetic semiconductors \[37\] have invoked theoretical and experimental endeavors due to long-range spin interaction.

Several competing models have attempted to describe FM in Mn doped GaAs that includes Ruderman-Kittel-Kasuya-Yosida (RKKY) model, DEM and antiferromagnetic (AFM) superexchange mechanism as well as the Mn-holes (Mn-\( h \)) complex. Actually the RKKY approach in the presence of the Friedel oscillations can be approximated to DEM for DMS \[54\]. The itinerant character of magnetic electrons and quantum oscillations (Friedel) of the electron’s spin polarization around the localized spins were established for the theory of magnetic metals. Hence, the resulting competition between FM and AFM interactions give rise to spin-glass freezing \[54, 55\]. In DMS however, the mean distance between the carriers are larger than the distance between spins. As a consequence, the exchange interaction mediated by the carriers in DMS favors FM rather than spin-glass freezing, which is for magnetic metals \[54, 55\]. Originally, Van Esch et al. \[56, 57\] have given an interpretation for FM in DMS in term of interaction between Mn ions and \( h \) surrounding it in which, the Mn ions are neutral Mn\(^{3+}\) acceptors associated with 3\(d^5 + h \) configuration. In addition, this neutral Mn\(^{3+}\) acceptors that contribute to magnetic properties could be compensated by As, where for a higher concentrations of Mn, instead of replacing Ga it will form a six-fold coordinated centers with As (Mn\(^{6+}\)) \[54, 55\]. These centers will eventually reduce the magnitude of FM in DMS due to the loss of spin-spin interaction between Mn\((3d^5)\) and \( h \). Interestingly, Dietl et al. \[54, 55\] have successfully incorporated both DEM and AFM superexchange mechanism into the Mn-\( h \) complex in order to capture the overall mechanism involved in III-V DMS. Such incorporations are important since DEM seems to be the only mechanism that predicts \( T_C \) accurately after taking band structure effect into account \[54, 55\]. DMS was assumed to be charge-transfer insulators by Dietl et al. because \( h \) from Mn\(^{3+}\) does not reside on \( d \) orbitals but occupies an effective mass Bohr orbit with large intrasite correlation energy. Moreover, Mn ions also function as a source of localized spins as well as acceptors \[54\]. Dietl et al. \[54, 55\] have further suggested that FM between localized spins \((S = 5/2)\) of Mn is mediated by delocalized or weakly localized holes surrounding it and the resulting spin-spin interaction are indeed long range as it should be. On the other hand, the short-range AFM superexchange (if any) arises from the spin polarization of occupied electron bands (valence band). As such, AFM superexchange is mediated by short-ranged localized spins unlike FM where it requires at least weakly localized spins for long range interaction. The readers are referred to Dietl et al. \[54\] for a thorough review on DMS.

Apart from that, Okabayashi et al. \[58\] have pointed out convincingly that there exists an indeterminacy on pin pointing whether Mn\(^{2+}\) (negative) or Mn\(^{3+}\) (neutral) ion that dominates Ga\(_{0.93}\)Mn\(_{0.07}\)As DMS. Both ions (Mn\(^{2+}\) and Mn\(^{3+}\)) were found to be favorable towards FM. In order to explain this scenario, Ando et al. \[59\] have proposed that Mn\(^{3+}\) ions populate at very low Mn concentrations due to low screening effect or large long-
range Coulomb interaction. I.e. long-range Coulomb interaction will be screened-out for higher Mn concentrations. This interesting effect of transition between Mn$^{2+}$ (negative) ↔ Mn$^{3+}$ (neutral) with Mn doping can also be analyzed in accordance with iFLT. The mechanism that governs DMS above $T_C$ and the predictability of its $\rho(T,\text{doping})$ and $R_H(T,\text{doping})$ as well as the changes in $T_C$ with $H$ are also evaluated. Actually, the transition of Mn$^{2+}$ (negative) ↔ Mn$^{3+}$ (neutral) can be described by simply calculating $E_I$ for each doping from its respective $\rho(T,\text{doping})$.

In this paper, iFDS is re-derived to extract the Lagrange multipliers so as to trigger sufficient interest for applications in other compounds such as ferromagnetic manganites and DMS above $T_C$. $\rho(T)$ curves are simulated at various doping or gap ($E_I$), pressure and $T$ to further enhance its applicability. Basically, we will stress the polaronic effect in semiconducting manganites and DMS in order to obtain accurate predictions of the charge carriers transport mechanisms above $T_C$ where the changes in $E_I$ is well accounted for with doping. In addition, it is also shown with detailed derivation of somewhat different Lagrange multipliers that separately influence the standard FDS and iFDS. In addition, the probability functions of electrons and holes, charge carriers' concentrations, $\rho(T,\text{doping})$ and $R_H(T,\text{doping})$ models in 1D, 2D and 3D are derived as well via the standard quantum statistical method. Interpretations of electrical properties based on these models for manganites and DMS are highlighted in detail. Quantum statistical method is used instead of other approaches because the former method will lead to the final results to understand $\rho(T, \text{doping}, \text{pressure}, H)$ without unnecessary approximations.

2. THEORETICAL DETAILS

2.1. Fermi-Dirac distribution function based on ionization energy

Both FDS and iFDS are for the half-integral spin particles such as electrons and holes. Its total wave function, $\Psi$ has to be antisymmetric in order to satisfy quantum-mechanical symmetry requirement. Under such condition, interchange of any 2 particles ($A$ and $B$) of different states, $\psi_i$ and $\psi_j$ ($j \neq i$) will result in change of sign, hence the wave function for Fermions is in the form of

$$\Psi_{i,j}(C_A,C_B) = \psi_i(C_A)\psi_j(C_B) - \psi_i(C_B)\psi_j(C_A), \quad (1)$$

The negative sign in Eq. (1) that fulfills antisymmetric requirement is actually due to one of the eigenvalue of exchange operator $\mathbf{P}$, $\mathbf{P} = -1$. The other eigenvalue, $\mathbf{P} = +1$ is for Bosons. $C_A$ and $C_B$ denote all the necessary cartesian coordinates of the particles $A$ and $B$ respectively. Equation (1) is nothing but Pauli’s exclusion principle. The one-particle energies $E_1, E_2, E_3, ..., E_m$ for the corresponding one-particle quantum states $q_1, q_2, q_3, ..., q_m$ can be rewritten as $(E_{\text{initial state}} \pm E_I)_1$, $(E_{\text{initial state}} \pm E_I)_2$, $(E_{\text{initial state}} \pm E_I)_3$, ..., $(E_{\text{initial state}} \pm E_I)_m$. Note here that $E_{\text{initial state}} + E_I = E_{\text{electrons}}$ and $E_{\text{initial state}} - E_I = E_{\text{holes}}$. Subsequently, the latter ($E_{\text{initial state}} \pm E_I$), version where $i = 1, 2, 3, ..., m$ with $E_I$ as an additional inclusion will be used to derive iFDS and its Lagrange multipliers. This $\pm E_I$ is inserted carefully to justify that an electron to occupy a higher state $N$ from initial state $M$ is more probable than from initial state $L$ if condition $E_I(M) < E_I(L)$ at certain $T$ is satisfied. As for a hole to occupy a lower state $M$ from initial state $N$ is more probable than to occupy state $L$ if the same condition above is satisfied. $E_{\text{initial state}}$ is the energy of a particle in a given system at a certain initial state and ranges from $+\infty$ to 0 for electrons and 0 to $-\infty$ for holes. In contrast, standard FDS only requires $E_i$ ($i = 1, 2, 3, ..., m$) as the energy of a particle at a certain state. Denoting $n$ as the total number of particles with $n_1$ particles with energy $(E_{\text{initial state}} \pm E_I)_1$, $n_2$ particles with energy $(E_{\text{initial state}} \pm E_I)_2$ and so on implies that $n = n_1 + n_2 + n_3 + ... + n_m$. As a consequence, the number of ways for $q_1$ quantum states to be arranged among $n_1$ particles is given as

$$P(n_1, q_1) = \frac{q_1!}{n_1!(q_1 - n_1)!}. \quad (2)$$

Now it is easy to enumerate the total number of ways for $q$ quantum states ($q = q_1 + q_2 + q_3 + ... + q_m$) to be arranged among $n$ particles, which is

$$P(n, q) = \prod_{i=1}^{\infty} \frac{q_i!}{n_i!(q_i - n_i)!}. \quad (3)$$

The most probable configuration at certain $T$ can be obtained by maximising $P(n, q)$ subject to the restrictive conditions

$$\sum_{i=1}^{\infty} n_i = n, \quad (4)$$

$$\sum_{i=1}^{\infty} dn_i = 0.$$

The method of Lagrange multipliers $\mathbf{B}$ can be employed to maximise Eq. (3). Hence, a new function,
\[ F(x_1, x_2, x_3, \ldots, \mu, \lambda, \ldots) = f + \mu f_1 + \lambda f_2 + \ldots \] is introduced and all its derivatives are set to zero

\[ \frac{\partial F}{\partial x_n} = 0; \quad \frac{\partial F}{\partial \mu} = 0; \quad \frac{\partial F}{\partial \lambda} = 0. \tag{6} \]

As such, one can let the new function in the form of

\[ F = \ln P + \mu \sum_i n_i + \lambda \sum_i (E_{\text{initial state}} \pm E_I) n_i. \tag{7} \]

After applying Stirling’s approximation, \( \partial F/\partial n_i \) can be written as

\[ \frac{\partial F}{\partial n_i} = \ln(q_i - n_i) - \ln n_i + \mu + \lambda(E_{\text{initial state}} \pm E_I) n_i = 0. \tag{8} \]

Thus, the Fermi-Dirac statistics based on ionization energy is simply given by

\[ n_i = \frac{1}{q_i - \exp[\mu + \lambda(E_{\text{initial state}} \pm E_I) i]} + 1. \tag{9} \]

The importance of \( E_I \)’s inclusion is that it can be interpreted as a charge gap that will be described later and also, particularly the \( E_I \) can be used to estimate the resistivity transition upon substitution of different valence state ions.

### 2.2. Lagrange multipliers

By utilizing Eq. (9) and taking \( \exp[\mu + \lambda(E \pm E_I)] \gg 1 \), one can arrive at the probability function for electrons in an explicit form as \[ 2 \]

\[ f_e(k) = \exp \left[ -\mu - \lambda \left( \frac{\hbar^2 k^2}{2m} + E_I \right) \right]. \tag{10} \]

Similarly, the probability function for \( h \) is given by

\[ f_h(k) = \exp \left[ \mu + \lambda \left( \frac{\hbar^2 k^2}{2m} - E_I \right) \right]. \tag{11} \]

The parameters \( \mu \) and \( \lambda \) are the Lagrange multipliers. \( \hbar = h/2\pi \), \( h = \) Planck constant and \( m \) is the charge carriers’ mass. Note that \( E \) has been substituted with \( \hbar^2 k^2/2m \). In the standard FDS, Eqs. (10) and (11) are simply given by, \( f_e(k) = \exp[-\mu - \lambda(\hbar^2 k^2/2m)] \) and \( f_h(k) = \exp[\mu + \lambda(\hbar^2 k^2/2m)] \). Equation (11) can be rewritten by employing the 3D density of states’ (DOS) derivative, \( dn = V k^2 \, dk / 2\pi^2 \), that eventually gives \[ 60 \]

\[ n = \frac{V}{2\pi^2} e^{-\mu} \int_0^\infty \frac{1}{k^2} \exp \left[ -\lambda \frac{\hbar^2 k^2}{2m} \right] \, dk, \tag{12} \]

\[ p = \frac{V}{2\pi^2} e^{-\mu} \int_{-\infty}^0 \frac{1}{k^2} \exp \left[ -\lambda \frac{\hbar^2 k^2}{2m} \right] \, dk. \tag{13} \]

\( n \) is the concentration of electrons whereas \( p \) represents holes’ concentration. \( V \) denotes volume in \( k \)-space. The respective solutions of Eqs. (12) and (13) are given below

\[ \mu_e = -\ln \left[ \frac{n}{V} \left( \frac{2\pi \hbar^2}{m} \right)^{3/2} \right], \tag{14} \]

\[ \mu_h = \ln \left[ \frac{p}{V} \left( \frac{2\pi \hbar^2}{m} \right)^{3/2} \right]. \tag{15} \]

The subscripts \( e \) and \( h \) represent electrons and holes respectively. Separately, Eq. (9) after neglecting \( \pm E_I \) (for FDS) can be written as

\[ E = \frac{V \hbar^2}{4m \pi^2} e^{-\mu} \int_0^\infty \frac{1}{k^4} \exp \left[ -\lambda \frac{\hbar^2 k^2}{2m} \right] \, dk \]

\[ = \frac{3V}{2\lambda} e^{-\mu} \left[ \frac{m}{(2\pi \hbar^2)^{3/2}} \right]. \tag{16} \]

Quantitative comparison \[ 60 \] between Eq. (16) and with the energy of a 3D ideal gas, \( E = 3nk_B T/2 \), after substituting Eq. (16) into Eq. (10) will enable one to conclude \( \lambda_{FDS} = 1/k_B T \). \( k_B \) is the Boltzmann constant. Applying the identical procedure to iFDS, i.e. by employing Eqs. (10) and (11), then Eqs. (12) and (13) are respectively rewritten as

\[ n = \frac{V}{2\pi^2} e^{-\mu - \lambda E_I} \int_0^\infty \frac{1}{k^2} \exp \left[ -\lambda \frac{\hbar^2 k^2}{2m} \right] \, dk, \tag{17} \]

\[ p = \frac{V}{2\pi^2} e^{-\mu - \lambda E_I} \int_{-\infty}^0 \frac{1}{k^2} \exp \left[ \lambda \frac{\hbar^2 k^2}{2m} \right] \, dk. \tag{18} \]

The respective solutions of Eqs. (17) and (18) are

\[ \mu + \lambda E_I = -\ln \left[ \frac{n}{V} \left( \frac{2\pi \hbar^2}{m} \right)^{3/2} \right], \tag{19} \]
Furthermore, using Eq. (5), one can rewrite Eq. (16) as

\[
\mu - \lambda E_I = \ln \left( \frac{p}{V} \left( \frac{2\pi \hbar^2}{m} \right)^{3/2} \right). \tag{20}
\]

Note that Eqs. (19) and (20) simply imply that \( \mu_e + \lambda E_I = \mu_h - \lambda E_I \). Furthermore, using Eq. (5), one can rewrite Eq. (16) as

\[
E = \frac{V \hbar^2}{4m\pi^2} e^{-\mu - \lambda E_I} \int_0^\infty k^4 \exp\left(-\frac{\hbar^2 k^2}{2m}\right) dk \tag{21}
\]

\[
= \frac{3V}{2\lambda} e^{-\mu - \lambda E_I} \left( \frac{m}{2\pi \hbar^2} \right)^{3/2}.
\]

Again, quantitative comparison between Eq. (21) and with the energy of a 3D ideal gas, \( E = 3nk_B T/2 \), after substituting Eq. (19) into Eq. (21) will enable one to determine \( \lambda \). It is found that \( \lambda \) remains the same as \( 1/k_BT \). I.e., \( \lambda_{FDS} = \lambda_{iFDS} \) as required by the standard FDS. Hence, the relationship between FDS and iFDS in term of Lagrange multipliers has been derived and shown clearly.

### 2.3. Resistivity models

Denoting \( \mu = -E_F \) (Fermi level), \( \lambda = 1/k_BT \), \( \hbar^2 k^2/2m = E \) and substituting these into Eqs. (19) and (20) will lead one to write

\[
f_e(E) = \exp\left[ \frac{E_F - E_I - E}{k_BT} \right], \tag{22}
\]

\[
f_h(E) = \exp\left[ \frac{E - E_I - E_F}{k_BT} \right]. \tag{23}
\]

At this point, one might again wonder the reason for \( E_I \)'s inclusion. The unique reason is that it directly determines the kinetic energies of electrons which carry the identity of its origin atom. Detailed experimental implications are given in the discussion. These iFDS probability functions for electrons and holes are unique in a sense that it allow the prediction of charge carriers' concentrations at various \( T \) and doping. It is worth noting that, \( -E_I \) in Eq. (23) for \( h \) follows naturally from the Dirac's theory of antiparticle interpretations \[61\]. Besides, the charge carriers are not entirely free since there exist a gap-like parameter that can be related to electrons-ion or Coulomb attraction. In fact, application of Eqs. (22) and (23) in semiconducting manganites will be explained in section III.B. The general equations to compute charge carriers' concentrations are stated below.

\[
n = \int_0^\infty f_e(E) N_e(E) dE, \tag{24}
\]

\[
p = \int_{-\infty}^0 f_h(E) N_h(E) dE. \tag{25}
\]

Existence of \( E_g \), which is the energy gap due to energy band splitting or lattice based gap is not inserted explicitly thus it is (if any) can be coupled with \( E_I \), which is tied to ions via Coulomb attraction. Having said that, now it is possible to obtain the geometric-mean concentrations of electrons and holes for 1D, 2D and 3D respectively in the forms of (assuming \( n \approx p \))

\[
\sqrt{n \rho}(1D) = \left( \frac{m^* m_h}{\hbar^2} \right)^{1/4} \left( \frac{k_B T}{2\pi} \right)^{1/2} \exp\left[ \frac{-E_I}{k_BT} \right], \tag{26}
\]

\[
\sqrt{n \rho}(2D) = \frac{k_B T}{\pi \hbar^2} \left( m^* m_h \right)^{1/2} \exp\left[ \frac{-E_I}{k_BT} \right], \tag{27}
\]

\[
\sqrt{n \rho}(3D) = \frac{k_B T}{2\pi \hbar^2} \left( m^* m_h \right)^{3/4} \exp\left[ \frac{-E_I}{k_BT} \right]. \tag{28}
\]

In which

\[
n(3D) = 2 \left[ \frac{k_B T}{2\pi \hbar^2} \right]^{3/2} \left( m_e^* \right)^{3/2} \exp\left[ \frac{E_F - E_I}{k_BT} \right], \tag{29}
\]

\[
p(3D) = 2 \left[ \frac{k_B T}{2\pi \hbar^2} \right]^{3/2} \left( m_h^* \right)^{3/2} \exp\left[ \frac{-E_F - E_I}{k_BT} \right]. \tag{30}
\]

It is apparent that \( n(1D,2D,3D) \propto \exp[\E_F - E_I)/k_BT] \) and \( p(1D,2D,3D) \propto \exp[\E_F - E_I)/k_BT] \) while the rest are just typical physical constants except \( T \). As such, the constant, \( E_F \) in the exponential term will appear where appropriate if one assumes \( n \gg p \) or \( p \gg n \) instead of \( \sqrt{n \rho} \). The DOS, \( N(E,1D) = (E^{-1/2}\sqrt{m^*/2})/\pi\hbar , N(E,2D) = m^*/\pi\hbar^2 \) and \( N(E,3D) \)
\[ \rho(1D) = \frac{A_1 \hbar (m^*_e m^*_h)^{1/4}}{e^2} \left( \frac{2 \pi}{k_B} \right)^{1/2} \times T^{3/2} \exp \left[ \frac{E_I}{k_B T} \right], \]  
(31)

\[ \rho(2D) = \frac{A_2 \pi \hbar^2}{e^2 k_B T} \exp \left[ \frac{E_I}{k_B T} \right], \]  
(32)

\[ \rho(3D) = \frac{A_3}{2e^2} \left( \frac{2 \pi \hbar^2}{k_B} \right)^{3/2} (m^*_e m^*_h)^{-1/4} \times T^{1/2} \exp \left[ \frac{E_I}{k_B T} \right]. \]  
(33)

In which

\[ \rho_c(3D) = \frac{A_3}{2e^2} \left( \frac{2 \pi \hbar^2}{k_B} \right)^{3/2} (m^*_e)^{-1/2} \times T^{1/2} \exp \left[ \frac{E_I - E_F}{k_B T} \right]. \]  
(34)

\[ \rho_h(3D) = \frac{A_3}{2e^2} \left( \frac{2 \pi \hbar^2}{k_B} \right)^{3/2} (m^*_h)^{-1/2} \times T^{1/2} \exp \left[ \frac{E_I + E_F}{k_B T} \right]. \]  
(35)

2.4. Hall resistance

The equations of motion for charge carriers under the influence of static \( \mathbf{H} \) and electric field \( \mathbf{E} \) can be written in an identical fashion as given in Ref. \( \text{[62]} \), which are given by

\[ m \left[ \frac{d}{dt} + \frac{1}{\tau_H} \right] v_y = -eE_y - eH_x v_z, \]  
(36)

\[ m \left[ \frac{d}{dt} + \frac{1}{\tau_H} \right] v_z = -eE_z + eH_x v_y. \]  
(37)

The subscripts \( x, y, \) and \( z \) represent the axes in \( x, y, \) and \( z \) directions while the scattering rate, \( 1/\tau_H = A_{D=2,3}^H T^2 \) in which \( A_{D=2,3}^H \) may not be necessarily equals to \( A_{D=2,3} \), though both \( A_{D=2,3}^H \) and \( A_{D=2,3} \) are independent of \( T \). \( A_{H} \) and \( \tau_H \) denote the \( T \)-independent scattering rate constant and scattering rate respectively with applied \( \mathbf{H} \).

In a steady state of a static \( \mathbf{H} \) and \( \mathbf{E} \), \( dv_z/dt = dv_y/dt = 0 \) and \( v_z = 0 \) hence, \( \mathbf{E}_z \) can be obtained from

\[ \mathbf{E}_z = -\frac{eH_x E_y}{m} \tau_H. \]  
(38)

In addition, it is further assumed that \( \rho_x(T) = \rho_y(T) = \rho_z(T) = \rho(T) \). \( R_{H}^{(2)} \) is defined as \( \mathbf{E}_z/j_y \mathbf{H}_x \), \( j_y = \mathbf{E}_y/\rho(T) \) and \( \tan \theta_H^{(z)} = \mathbf{E}_z/\mathbf{E}_y \). Parallel to this,

\[ R_{H}^{(z)} = \frac{\tan \theta_H^{(z)} \rho(T)}{H_x}. \]  
(39)

\( j_y \) is the current due to charge carriers’ motion along \( y \)-axis and \( \theta_H^{(z)} \) is the Hall angle. Furthermore, \( \tan \theta_H^{(z)} \) can be rewritten as \(-e\mathbf{H}_x \tau_H/m \). Therefore, it is easy to surmise that \( \cot \theta_H^{(z)} \propto T^2 \). After employing Eq. (32) with \( E_F \) (\( n \gg p \)) and Eq. (34), then one can respectively arrive at

\[ R_{H}^{(2D)} = -\frac{A_2 \pi \hbar^2}{A_2^H \hbar e m_e k_B} T^{-1} \exp \left[ \frac{E_I - E_F}{k_B T} \right], \]  
(40)

\[ R_{H}^{(3D)} = -\frac{A_3}{2A_3^H \hbar^2} \left( m_e^{*} \right)^{3/2} \frac{k_B}{T^3} \exp \left[ \frac{E_I - E_F}{k_B T} \right]. \]  
(41)

The negative charges in Eqs. (38), (40) and (41) are due to the assumption that the charge carriers are electrons. Note that \( R_{H}^{(T,1D)} \) for any given samples that exhibit purely 1D conduction is obviously irrelevant or simply, could not be derived with above procedures, since Hall effect requires at least 2D conduction.

2.5. Special cases

There are nowhere in the above derivations that take into account any free electrons and \( T \)-dependence of electron-phonon scattering. Hence, the models derived thus far are obviously not suitable for such applications except for semiconducting free electrons above conduction band. In this case, Eqs. (21) and (25) should be integrated from \( E_g \rightarrow \infty \) and \( 0 \rightarrow -\infty \) respectively after replacing \( E_I = 0 \) in Eqs. (22) and (28). As for metals
with free electrons and strong phonon contributions, it is advisable to switch to the well known Bloch-Grüneisen formula given by \[63\],

\[
\rho(T, 3D) = \rho_0 + \lambda_{tr} \frac{128\pi n^* (k_B T)^5}{n e^2 (k_B \Theta_D)^4} \Theta_D^{2/5}T \int_0^T x^7 \sinh^{-2} x \, dx.
\]

\[ \lambda_{tr} = \text{electron-phonon coupling constant, } \rho_0 = \rho(T = 0), \quad m^* = \text{average effective mass of the occupied carrier states, } \Theta_D = \text{Debye temperature, } n = \text{free electrons concentrations.} \]

As a matter of fact, one should not be encouraged to substitute any of the Eqs. \[20\]–\[29\] for \( n \) into Eq. \[12\] just to capture the electron-phonon scattering because the scattering of free electrons considered in Eq. \[12\] may not be compatible with gapped-electrons’ scattering of iFDS, unless proven otherwise.

### 2.6. The rationale behind iFDS

Recall that \( E_I \) is the gap from electron-ion attraction along the easiest path while \( E_g \) is the averaged gap from the energy band splitting. \( E_g \) from the theory of electronic band structure (TEBS) does not give the correct prediction on how certain ionic substitutions affect the transport properties of strongly correlated systems. As I have pointed out in the last paragraph of the introduction, iFDS follows a brand new approach instead of the common TEBS, which is not suitable and/or reliable for the strongly correlated systems namely, superconductors, ferromagnets and ferroelectrics. In fact, application of iFDS in the normal state of high-\( T_c \) superconductors have been carried out while its consequences in ferro-electrics has been reported as well \[64\]. Usually, one needs to incorporate TEBS into FDS to take the lattice properties into account to expose the transport mechanism and one can also include the defects so as to arrive at a comprehensive theory. However, the magnitude of lattice effects and anisotropies calculated from TEBS is obviously not suitable for the transport theory because the charge carriers move along the easiest path complying the principle of least action. This is why TEBS-FDS is not reliable when it comes to the transport properties of strongly correlated systems. It is also important to realize that for simple systems, the lattice properties are isotropic therefore the magnitude of say, \( E_g \) calculated from TEBS is comparable with \( E_g \) from the easiest path, thus simple systems are very well described by TEBS-FDS. Unlike TEBS-FDS, iFDS takes care of the crystalline lattice via \( E_I \) in accordance with the easiest path (transport measurements) since any changes to lattice’s crystallinity due to substitution will be mirrored through the easiest path’s \( E_I \) and subsequently shall be picked up by the electrons, influencing its transport mechanism. Whereas, the TEBS-FDS theory assumes that all the lattice effects including \( E_g \) and its magnitudes are isotropic, which will be imposed on the electrons unnecessarily. In other words, it is logically desirable to let the electrons reveal the obstacles via iFDS rather than forcing \( E_g \) onto the electrons in order to calculate the electronic transport properties.

I would like to stress here that if one substitutes \( E_I \) with \( E_g \) in Eq. \[63\] to comply with the standard FDS, then one has to feed the value of \( E_g \) from elsewhere (such as TEBS) in order to predict the transport properties and apparently, TEBS is not suitable for the reasons described above. Hence, the Eqs. \[20\]–\[33\] and Eqs. \[40\]–\[41\] can be directly employed to analyze the doping effect on the transport properties of ferromagnets above \( T_C \), be it diluted (DMS) or concentrated (manganites). Take note that the proposed theory based on iFDS is in its infancy, thus the correct method to insert defects via \( E_I \) will be developed later on. Consequently, the application of iFDS is to be confined to pure crystalline and polycrystalline samples. The above-stated arguments do not invalidate the importance of TEBS since the lattice properties and its consequences on electronic structure are indeed invaluable to derive lattice based Hamiltonian.

### 3. DISCUSSION

#### 3.1. Simulated curves

Figures 1a, 1b and 1c illustrate the variation of \( \rho(T) \) from Eqs. \[21\]–\[29\] respectively with conduction dimensionality and doping parameter \( (E_I) \). The proportionality of Eqs. \[21\]–\[29\] with \( E_I \) is similar to Eqs. \[20\] and \[33\], thus it does not matter whether one chooses \( \sqrt{n}, n \) or \( \rho \) in the subsequent analysis as well as for manganites. One can also identify the \( \rho(T) \) transition from metallic \( \rightarrow \) semiconducting conduction with increasing \( E_I \). It is also worth noting that \( \rho(T,1D), \rho(T,2D) \) and \( \rho(T,3D) \) are \( \propto T^{3/2}, T \) and \( \sqrt{T} \) respectively if and only if \( E_I \ll T \). Another point worth to extract from these curves are the relationship between \( T_{\text{crossover}} \) and \( E_I \), where \( T_{\text{crossover}} < E_I \) for 1D, \( T_{\text{crossover}} = E_I \) for 2D and \( T_{\text{crossover}} > E_I \) for 3D. Apparently, these relations are again due to the proportionality of \( \rho(T,1D) \propto T^{3/2}, \rho(T,2D) \propto T \) and \( \rho(T,3D) \propto \sqrt{T} \). Figures 2a and 2b plot the simulated \( R_H(T) \) curves in 2D and 3D as well as at different \( E_I \) (0 K, 150 K, 310 K) that follow from Eqs. \[40\] and \[41\]. There are no significant differences of \( R_H \) between 2D and 3D since \( R_H(2D) \) and \( R_H(3D) \) are \( \propto T^{-1} \) and \( T^{-3/2} \) respectively. Besides, the \( T \) from \( \exp [E_I/T] \) also inversely proportional to both \( R_H(2D) \) and \( R_H(3D) \). These scenarios will always lead \( R_H \) to increase with lowering \( T \) without any observable \( T_{\text{crossover}} \).
regardless of $E_i$’s magnitude, unlike $\rho(T)$. It is convenient to directly quantify $\rho(T)$ variation with doping by relating $E_I$ as a doping parameter, as will be discussed in the following paragraph for manganites at $T > T_C$.

3.2. Manganites

As stated previously, Jonker and van Santen \[44, 50\] suggested that FM is due to indirect coupling of $d$-shells via conducting electrons. Subsequently, Zener \[15, 16\], Anderson and Hasegawa \[65\] have provided sufficient theoretical backgrounds on DEM. However, this paper will not discuss the property of DEM below $T_C$, instead the electrical properties above $T_C$ (paramagnetic phase) will be addressed in detail in which, DEM is employed at $T < T_C$ (ferromagnetic phase). It is interesting to observe reduced $\rho(T)$ and increased Curie temperature ($T_C$) at higher $H$ for La$_{1-x}$Ca$_x$Sr$_2$MnO$_3$ compounds \[60, 67, 68\]. The results that larger $H$ giving rise to conductivity at $T > T_C$ point towards the enhancement of conductivity from DEM where the exponential increase of $\rho(T)$ is suppressed with larger $H$. Additionally, small polarons have also been attributed \[62\] to play a significant role on $\rho(T)$ at $T > T_C$. As a matter of fact, this polaronic effect is naturally captured by Eqs. \[43, 44\] in which the gap-parameter, $E_I$ that represents electron-ion attraction is also a measure that the combination of electrons and its strain field due to neighboring ions, which is nothing but polarons. The absolute value of $E_I$ can be obtained from \[62\]

$$E_I = \frac{\epsilon^2}{8\pi\epsilon_0 r_B^2}. \quad (43)$$

$\epsilon$ and $\epsilon_0$ are the dielectric constant and permittivity of free space respectively, $r_B$ is the Bohr radius. Furthermore, the decrement of $E_I$ with $H$ indicates that $r_B$ increases with $H$. Identical relationship was also given between polaronic radius, $r_p$, and $E_p$ by Banerjee et al. \[71\]. Actually, Millis et al. \[53\] have somewhat proved the inadequacy of DEM alone to describe $\rho(T, doping,H)$ and reinforced the need to include small polarons as a consequence of Jahn-Teller (JT) splitting of Mn$^{3+}$ ions. This statement was further justified by experimental work of Banerjee et al. \[70, 71, 72\] in which, they have established the existence of small polarons at $T > T_C$ in La$_{0.5}$Pb$_{0.5}$Mn$_{1-x}$Cr$_x$O$_3$ for $x = 0 \rightarrow 0.45$ using thermoelectric power and positron annihilation lifetime measurements. Banerjee et al. suggested that the substitution of Cr$^{3+}$ into Mn sites localizes $\epsilon^1_g$ electrons that gives rise to $\rho(T)$ \[70, 71, 72\]. However, the increment of activation energy, $E_p$ with $x$ is still unclear in term of $\epsilon^1_g$ electrons' localization. On the other hand, $\rho(T)$ (Eqs. \[45, 46\]) based on iFDS could explain the increment of $E_I$ with $x$ as well as the structural changes accompanied by this doping, which is due to the fact that valence state of Cr and Mn may change with doping that can be calculated with Eq. \[44\] as will be shown later.

In addition, Moskvin \[74, 75\] reforces the importance of considering different charge distribution in MnO$_6$, Mn and Oxygen instead of just considering the DEM. Parallel to this, doping-friendly resistivity models derived from iFDS will indeed lead to identical conclusions with Billinge et al. and Moskvin since iFDS naturally consider polarons and the substitution of Ca$^{2+}$ or Cr$^{2+}$ into La$_{1-x}$Ca$_x$Mn$_{1-y}$Cr$_y$O$_3$ system does indeed modify the overall charge distribution in accordance with the valence state of Ca, La, Mn and Cr. Add to that, Louca and Egami \[70\] invoked the JT distortion to describe the effect of lattice on $T_C$. They have utilized the results of pulsed neutron-diffraction experiments to conclude that the variation in Mn-O bond length with Sr substitution in La$_{1-x}$Sr$_x$MnO$_3$ compound can be related to JT. Again, note here that the change of Mn-O length with Sr substitution implies the valence state of Mn varies with doping. In an identical compound of La$_{1-x}$Ca$_x$MnO$_3$, substitution of Ca into La, will have to satisfy the inequality of average $E_I$ between Ca$^{2+}$ ($867$ kJmol$^{-1}$) and La$^{3+}$ ($1152$ kJmol$^{-1}$) i.e., $E_I$(La$^{3+}$) > $E_I$(Ca$^{2+}$). Here, one can easily fix the valence state of Ca$^{2+}$ and La$^{3+}$ as noted. As a consequence, this will ease the prediction of $\rho(T)$ with doping. If one of the ions is multivalence, then the linear algebraic equation as given below must be used to predict the valence state of the multivalence ion from $\rho(T)$ curves \[2, 3\].

$$\delta \frac{z+j}{j} \sum_{i=z+1}^{z+j} E_I i + \frac{1}{z} \sum_{i=1}^{z} E_I i = \frac{1}{q} \sum_{i=1}^{q} E_I i. \quad (44)$$

The first term, $\delta \frac{z+j}{j} \sum_{i=z+1}^{z+j} E_I i$ above has $i = z + 1, z + 2,..., z + j$ and $j = 1, 2, 3,...$. It is solely due to multivalence ion for example, assume Mn$^{3+}$ is substituted with Nd$^{3+}$ (La$_{0.7}$Ca$_{0.3}$Mn$_{1-2}$Nd$_x$O$_3$) hence from Eq. \[44\], the first term is due to Mn$^{4+}$ ion’s contribution or caused by reaction of the form Mn$^{3+}$ – electron $\rightarrow$ Mn$^{4+}$ (4940 kJmol$^{-1}$), hence $j$ is equals to 1 in this case and $\delta$ represents the additional contribution from Mn$^{4+}$. The second ($i = 1, 2, 3,..., z$ and last ($i = 1, 2, 3,..., q$) terms respectively are due to reaction of the form Mn $–$ 3(electrons) $\rightarrow$ Mn$^{3+}$ and Nd $–$ 3(electrons) $\rightarrow$ Nd$^{3+}$. Recall that $q = z = 3+$ and $\delta = 1, 2, 3,...$ represent the first, second, third, ... ionization energies while $j = 1, 2, 3,...$ represent the fourth, fifth, sixth, ... ionization energies. Therefore, $z + \delta$ gives the minimum valence number for Mn which can be calculated from Eq. \[44\]. Now, it is possible to explain the doping effect in La$_{0.5}$Pb$_{0.5}$Mn$_{1-x}$Cr$_x$O$_3$ system \[70, 72\]. The inequalities of $E_I$s are given as Mn$^{3+}$ ($1825$ kJmol$^{-1}$) > Cr$^{3+}$ ($1743$ kJmol$^{-1}$) and Mn$^{4+}$ ($2604$ kJmol$^{-1}$) > Cr$^{4+}$ ($2493$ kJmol$^{-1}$). These relations strongly indicate
that $\rho(T)$ should decrease with Cr$^{3+}$ content contradicting with experimental data from Refs. 73, 71. The only way to handle this situation is to use Eq. 44, so as to calculate the minimum valence state of Cr$^{3+}$, which is 3.033+. Actually, the valence state of Cr that substitutes Mn$^{4+}$ is Cr$^{2.933+}$ and of course, the valence state of Mn is fixed to be 3+. There is no need to vary it because $\rho(T)$ was found to increase with Cr content 71, 73. Actually, the existence of Mn$^{4+}$ is unlikely and negligible, while Mn$^{3+}$ (if any) will further increase the value of $\delta$ in Cr$^{3+}$, in which both scenarios comply with iFDS via Eq. 44 and the resistivity measurements 70, 71.

The effect of hydrostatic (external) pressure ($P$) and chemical doping (internal $P$) on metal-insulator transition of $\text{Pr}_x\text{La}_{1-x}\text{MnO}_3$, $\text{Pr}_x\text{La}_{1-x}\text{Sr}_{0.99}\text{MnO}_3$, $\text{Pr}_{0.86}\text{La}_{0.12}\text{Ca}_{0.3}\text{MnO}_3$ and $\text{Pr}_{0.54}\text{La}_{0.16}\text{Ca}_{0.3}\text{MnO}_3$ systems were reported by Medvedeva et al. 73. It is found that $\rho(T)$ and $T_C$ are observed to be decreased and increased respectively with increasing pressure ranging from 0 → 15 kbar. As anticipated, $\rho(T)$ above and below $T_C$ have been decreased significantly with $P$ i.e. $P$ affects both Mn-O-Mn bond angle and length. Hence, it is apparent that doping and $P$ give rise to the variation in the valence state of Pr and Mn in order to achieve a certain crystal structure and simultaneously increase the number of charge carriers. As such, changes in $\rho(T)$ above $T_C$ can be very well accounted for with Eqs. (33) → (36), (38) → (39) where $P$ reduces $E_I$ (increases $r_B$) of certain ions in a similar fashion to doping (internal $P$). However, this paper as stated earlier does not attempt to describe the correct mechanism(s) involved below $T_C$ and its variations with $P$ and doping. In short, at $T < T_C$, the interactions among DEM with JT or polaron or all may play a significant role as suggested by Billinge et al. 83 and Medvedeva et al. 73.

### 3.3. Diluted magnetic semiconductors

It is quite straightforward to extend iFLT described above to DMS. Firstly, Oiwa et al. 78, Matsukura et al. 72, Dietl et al. 80 and Iye et al. 81 have done detailed investigations on Ga-As-Mn based DMS. The variation of electrical resistivities with temperature and doping for Ga$_{1-x}$Mn$_x$As in Refs. 78, 72, 81 (sample A1) differ slightly from Ref. 81 (sample B2). It has been reported that the maximum $T_C$ is ~70 K for Ga$_{0.957}$Mn$_{0.043}$As 81 whereas Ga$_{0.947}$Mn$_{0.053}$As had a $T_C$ of ~100 K. Apart from $T_C$8, there is a switch-over in the variation of $\rho(T, x)$ where $\rho(T, x)$ decreases with $x$ initially before switching to increasing $\rho(T, x)$ with $x$. $\rho(T, x)$’s variation for sample A1 is $\rho(T, 0.015) > \rho(T, 0.022) > \rho(T, 0.071) > \rho(T, 0.053) > \rho(T, 0.043) > \rho(T, 0.035)$ while for sample B2, $\rho(T, 0.015) > \rho(T, 0.022) > \rho(T, 0.071) > \rho(T, 0.035) > \rho(T, 0.043) > \rho(T, 0.053)$. The initial reduction of $\rho(T)$ with $x$ is due to $E_I$ (Mn$^{4+}$ = 1825 kJmol$^{-1}$) < $E_I$ (Ga$^{3+}$ = 1840 kJmol$^{-1}$), i.e. it supports that Mn$^{3+}$ substitutes at Ga$^{3+}$ sites. However, $\rho(T)$’s switch-over (from decreasing to increasing with $x$) at critical concentrations, $x_{c1} = 0.035$ and $x_{c2} = 0.053$ for samples A1 and B2 respectively can only be explained if one employs the mechanism proposed by Van Esch et al. 56, 57 and Ando et al. 59 in which, Mn$^{6+}$ formations is substantial above $x_c$ in such a way that Mn$^{3+}$ ions do not substitute Ga$^{3+}$ ions. Therefore, $\rho(T)$ will be influenced by Mn$^{6+}$ clusters, defects and Ga-Mn-As phase simultaneously significantly above $x_c$.

This is indeed in fact in accordance with iFDS based resistivity models since if one assumes Mn$^{2+}$ substitutes Ga$^{3+}$, then $\rho(T)$ should further decrease with $x$ without any switch-overs. Hence, the formation of Mn$^{6+}$ with $x$ is inevitable in III-V Ga$_{1-x}$Mn$_x$As DMS. Again, $T_C$ of DMS is also directly proportional to $H$, identical to manganites. All the values of $E_I$ discussed above (including in section IIIB) were averaged in accordance with $E_I[X^{2+}] = \sum_{i=1}^{70} E_I^i$ and should not be taken literally since those $E_I$s are not absolute values. The absolute values need to be obtained from the $r_B$ dependent $E_I$ equation stated earlier 82, 83. Prior to averaging, the 1$^{st}$, 2$^{nd}$, 3$^{rd}$ and 4$^{th}$ ionization energies for all the elements mentioned above were taken from Ref. 82 and the predictions stated above are only valid for reasonably pure materials without any significant impurity phases as well as with minimal grain boundary effects.

### 4. CONCLUSIONS

In conclusion, the ionization energy based Fermi-Dirac statistics is useful to estimate the transitional progress of $\rho(T, \text{doping}, \text{pressure}, \text{H})$ from metallic → semiconducting or vice versa in both manganites and diluted magnetic semiconductors. This is made possible by an additional unique constraint, which is nothing but the ionization energy that captures the electron’s kinetic energy and maps it to its origin atom. The relation of Lagrange multipliers ($\lambda$ and $\mu$) between FDS and iFDS have been derived explicitly solely to flush-out any misinterpretations that will lead to further complications in describing experimental data on semiconductor ferromagnets and DMS at $T > T_C$. The presented iFDS model however, does not admit completely free-electrons and strong electron-phonon scattering. Importantly, $E_I$ captures the polaronic effect quite naturally to explain the paramagnetic phase electrical properties of the said materials accurately.

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FIG. 1: Simulated $\rho(T, E_I)$ curves from Eqs. $31, 32$ and $33$ are depicted in a), b) and c) respectively. These curves capture 1D, 2D and 3D conduction respectively at various $E_I$ between 0 K and 310 K. The intrinsic $T$-dependence among 1D, 2D and 3D conduction are $T^{1/2}$, $T$ and $\sqrt{T}$ respectively for $T > T_{\text{crossover}}$ whereas all $\rho(T, E_I)$ behave identically as $\rho \propto \exp(1/T)$ at $T < T_{\text{crossover}}$ since $T_{\text{crossover}}$ is less than the numerator in the exponential terms. The latter proportionalties can be verified from Eqs. $31$, $32$ and $33$. 

FIG. 2: Simulated 2D and 3D curves of Hall resistances $(R_H(T, E_I))$ from Eqs. $10$ and $11$ are given in a) and b) respectively. All curves for both dimensionalities (2D and 3D) are also plotted at $E_I = 0$ K, $E_I = 150$ K and $E_I = 310$ K. Note that $R_H(T, E_I)$ curves are inversely proportional to $T$ regardless of the magnitude of $T_{\text{crossover}}$ that can be easily verified from Eqs. $10$ and $11$. Therefore, one could not excerpt sufficient quantitative differences between 2D and 3D $R_H(T, E_I)$ as opposed to $\rho(T, E_I)$.
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The diagrams illustrate the behavior of a parameter with temperature for different initial conditions (Ei).

- **Diagram a** shows data for 310 K (1D), 150 K (1D), and 0 K (1D).
- **Diagram b** displays data for 310 K (2D) and 150 K (2D).
- **Diagram c** includes data for 310 K (3D) and 150 K (3D).

Each diagram plots temperature (T) on the x-axis and density (ρ) on the y-axis, with markers indicating different initial conditions.
\[ E_i = 310 \text{ K} \ (2\text{D}) \]

\[ E_i = 150 \text{ K} \ (2\text{D}) \]

\[ E_i = 0 \text{ K} \ (2\text{D}) \]