πη pair hard electroproduction and exotic hybrid mesons

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We show that hard electroproduction is a promising way to study exotic hybrid mesons, in particular through the hybrid decay channel $H \rightarrow \pi \eta$. We discuss the πη generalized distribution amplitude, calculate the production amplitude and propose a forward-backward asymmetry as a signal for the hybrid meson production.

1. Introduction

Present candidates for exotic hybrid mesons with $J^{PC} = 1^{-+}$ include $\pi_1(1400)$ which is mostly seen through its $\pi \eta$ decay and $\pi_1(1600)$ which is seen through its $\pi \eta'$ and $\pi \rho$ decays \cite{1}. The first experimental investigation of the hybrid with $J^{PC} = 1^{-+}$ as the resonance in $\pi^- \eta$ mode was implemented by the Brookhaven collaboration E852 \cite{2}. Theoretically these states are objects of intense studies \cite{3}, mostly through lattice simulations \cite{4}. We recently studied exotic hybrid meson electroproduction \cite{5} and showed that its cross section is sizable in the kinematics of JLab or HERMES experiments. We emphasize here that an angular asymmetry in the reaction

$$e(k_1) + N(p_1) \rightarrow e(k_2) + \pi^0(p_{\pi}) + \eta(p_{\eta}) + N(p_2)$$

will sign unambiguously the existence of the hybrid meson.

Exotic hybrid mesons are expected to be quite copiously electroproduced since the normalization of its distribution amplitude has been shown to be quite similar to the one for the $\rho$-meson. If an experiment is equipped with a recoil detector, the hybrid production events may be identified through a missing mass reconstruction, and all the decay channels may then be analyzed. If not, one will have to base an identification process through the possible decay products of the hybrid meson $H$. Since the hybrid candidate known as the $\pi_1(1400)$ has a dominant $\pi \eta$ decay mode, we proceed to the description of the electroproduction process \cite{6} or $\gamma^*(q) + N(p_1) \rightarrow \pi^0(p_{\pi}) + \eta(p_{\eta}) + N(p_2)$.

To perform a leading order computation of such process we need to introduce the concept of generalized distribution amplitude \cite{7}. Note that a very similar analysis may be carried for the $\pi \eta'$ decay mode of the candidate $\pi_1(1600)$. 


2. $\pi\eta$ generalized distribution amplitude

Let us briefly introduce and discuss the generalized distribution amplitude related to the $\pi\eta$–to–vacuum matrix element. On the basis of Lorentz invariance, the $\pi^0\eta$ GDA may be defined as:

$$\langle \pi^0(p_\pi)\eta(p_\eta) | \bar{\psi}_3 \left( \frac{-\tau}{2} \right) \gamma^\mu \left[ \frac{-\tau}{2}, \frac{\tau}{2} \right] \gamma^3 f_3 \psi_1 \left( \frac{\tau}{2} \right) | 0 \rangle = p_\pi^\mu \int_0^1 dy e^{i(y-p_\pi\eta-\frac{z^2}{2})} \Phi^{(\pi\eta)}(y, \zeta, m_{\pi\eta}^2),$$

where $m_{\pi\eta}^2$ denotes the modulus of three-dimension momentum of $\pi$ and $\eta$ mesons in the center–of–mass system.

The relation between $\zeta$ and the angle $\theta_{cm}$ are $2\zeta - 1 = \beta \cos \theta_{cm}$, $\beta = 2|\mathbf{p}|/m_{\pi\eta}$, where $|\mathbf{p}|$ denote the modulus of three-dimension momentum of $\pi$ and $\eta$ mesons in the center–of–mass system.

In the reaction under study, the $\pi\eta$ state may have total momentum, parity and charge–conjugation in the following sequence $J^{PC} = 0^{++}, 1^{-+}, 2^{++}, ...$, that corresponds to the following values of the $\pi\eta$ orbital angular momentum $L$: $L = 0, 1, 2, ...$ respectively. We can see that a resonance with a $\pi\eta$ decay mode for odd orbital angular momentum $L$ should be considered as an exotic meson.

The mass region around 1400 MeV is dominated by the strong $a_2(1329)\ (2^{++})$ resonance [7]. It is therefore natural to look for the interference of the amplitudes of hybrid and $a_2$ production, which is linear, rather than quadratic in the hybrid electroproduction amplitude. Such interference arises from the usual representation of the $\pi\eta$ generalized distribution amplitude in the form suggested by its asymptotic expression:

$$\Phi^{(\pi\eta)}(y, \zeta, m_{\pi\eta}^2) = 10y(1-y)C_1^{(3/2)}(2y - 1) \sum_{l=0}^2 B_{1l}(m_{\pi\eta}^2) P_l(\cos \theta).$$

Keeping only $L = 1$ and $L = 2$ terms, and using the description of tensor meson distribution amplitudes suggested by Ref [8], we model the $\pi\eta$ distribution amplitude in the following form:

$$\Phi^{(\pi\eta)}(y, \zeta, m_{\pi\eta}^2) = 30y(1-y)(2y - 1) \left[ B_{11}(m_{\pi\eta}^2) P_1(\cos \theta) + B_{12}(m_{\pi\eta}^2) P_2(\cos \theta) \right],$$

with the coefficient functions $B_{11}(m_{\pi\eta}^2)$ and $B_{12}(m_{\pi\eta}^2)$ related to corresponding Breit–Wigner amplitudes when $m_{\pi\eta}^2$ is in the vicinity of $M_{a_2}^2$, $M_H^2$. We have (see the technical details in Ref [5]):

$$B_{11}(m_{\pi\eta}^2) = \frac{5}{3} \frac{g_{H\pi\eta} f_H M_H \beta}{M_H^2 - m_{\pi\eta}^2 - i\Gamma_H M_H}, \quad B_{12}(m_{\pi\eta}^2) = \frac{10}{9} \frac{i g_{a_2\pi\eta} f_{a_2} M_{a_2}^2 \beta^2}{M_{a_2}^2 - m_{\pi\eta}^2 - i\Gamma_{a_2} M_{a_2}},$$

where $f_H$, $f_{a_2}$, $g_{H\pi\eta}$ and $g_{a_2\pi\eta}$ are the corresponding coupling constants.
3. Differential cross section for $\pi\eta$ electroproduction

The amplitude of reaction (2):

$$T^{\pi\eta} = \bar{u}(k_2, s_2)\gamma_\mu e_L u(k_1, s_1) \frac{1}{q^2} A^{\pi\eta}_{(q)}, \quad |T^{\pi\eta}|^2 = \frac{4e^2(1-y_l)}{Q^2 y_t^2} |A^{\pi\eta}_{(q)}|^2,$$

where

$$A^{\pi\eta}_{(q)} = \frac{e\pi\alpha_s C_F}{N_c Q} \left[ e_u H_{uu} - e_d H_{dd} \right] \left[ B_{11}(m_{\pi\eta}^2) P_1(\cos \theta_{cm}) + B_{12}(m_{\pi\eta}^2) P_2(\cos \theta_{cm}) \right].$$

Finally, the differential cross section of process (2) takes the form

$$\frac{d\sigma^{\pi\eta}}{dQ^2 dy_t dt dm_{\pi\eta} d(\cos \theta_{cm})} = \frac{1}{4(4\pi)^5 y_t^2} \frac{m_{\pi\eta}\beta}{y_t \lambda^2(\hat{s}, -Q^2, m_N^2)} |T^{\pi\eta}|^2.$$

4. Angular asymmetry

Asymmetries are often a good way to get a measurable signal for a small amplitude, by taking profit of its interference with a larger one. In our case, since the hybrid production amplitude may be rather small with respect to a continuous background, we propose to use the supposedly large amplitude for $a_2$ electroproduction as a magnifying lens to unravel the presence of the exotic hybrid meson. Since these two amplitudes describe different orbital angular momentum of the $\pi$ and $\eta$ mesons, the asymmetry which is sensitive to their interference is an angular asymmetry defined by

$$A(Q^2, y_t, \hat{t}, m_{\pi\eta}) = \frac{\int \cos \theta_{cm} d\sigma^{\pi\eta}(Q^2, y_t, \hat{t}, m_{\pi\eta}, \cos \theta_{cm})}{\int d\sigma^{\pi\eta}(Q^2, y_t, \hat{t}, m_{\pi\eta}, \cos \theta_{cm})}$$

as a weighted integral over polar angle $\theta_{cm}$ of the relative momentum of $\pi$ and $\eta$ mesons.
Note that this angular asymmetry is completely similar to the charge asymmetry which was studied in $\pi^+\pi^-$ electroproduction at HERMES [9] and discussed in Ref [10].

Due to the fact that the $\cos \theta_{cm}$-independent factors in both the numerator and denominator of (10) are completely factorized and, on the other hand, these factors are the same, we are able to rewrite the asymmetry (10) as

$$A(m_{\pi\eta}) = \frac{\int d(\cos \theta_{cm}) \cos \theta_{cm} \left| B_{11}(m_{\pi\eta}^2)P_1(\cos \theta_{cm}) + B_{12}(m_{\pi\eta}^2)P_2(\cos \theta_{cm}) \right|^2}{\int d(\cos \theta_{cm}) \left| B_{11}(m_{\pi\eta}^2)P_1(\cos \theta_{cm}) + B_{12}(m_{\pi\eta}^2)P_2(\cos \theta_{cm}) \right|^2}. \quad (11)$$

Our estimation of the asymmetry (11) is shown on Fig 1. Since the numerator of (11), i.e. the real part of the product of $B_{11}(m_{\pi\eta}^2)$ and $B_{12}^*(m_{\pi\eta}^2)$, is proportional to the cosine of the phase difference $\Delta \delta_{1,2} = \delta_{l=1} - \delta_{l=2}$, the zeroth value of (11) takes place at $\Delta \delta_{1,2} = \pi/2$. This is achieved for $m_{\pi\eta} \approx 1.3$ GeV. Besides, one can see from Fig. 1 that the first positive extremum is located at $m_{\pi\eta}$ around the mass of $a_2$ meson while the second negative extremum corresponds to the hybrid meson mass.

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