A Schematic Model of Leptons and Quarks
Based on Majorana Partners

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The possible extension of generation structure is investigated on the basis of a composite model, in which leptons and quarks are composed of rishons $T$ with charge $1/3$ and purely neutral $V$. In this sub-system, we regard $V$ as the origin of Majorana particles. The seesaw mechanism implies the possible existence of heavy $V$, denoted as $V_B$, in addition to ordinary $V$, denoted as $V_s$. If we regard $V_B$ as a constitute block of leptons and quarks, a new generation scheme arises. This new scheme satisfies the anomaly-free condition.

1. Introduction

With the progress made in neutrino physics, the existence of Majorana particles now seems almost certain. It is well known that the extremely small mass of neutrinos is nicely explained by the seesaw mechanism, which is possible only for Majorana particles. Are there any Majorana particles in addition to neutrinos? It should be noted that in a certain kind of composite model leptons and quarks contain purely electrically neutral constituents. In such a case, the additional particles appear in a generation structure, provided that their neutral components can be regarded as Majorana particles. This paper is concerning to this problem. We first give a short review of the rishon model, which contains a purely electrically neutral constituent called a “V rishon”. We regard $V$ as a Majorana particle, and the seesaw mechanism concerning the $V$ rishon is formulated. Then, a possible new structure of the generation is proposed, in which new heavy quarks and neutrinos appear.

In this paper, we suppose the rishon is a purely mathematical entity or an entity whose dynamics are not yet known. In order to approach to its dynamics, it is assumed that some frameworks of conventional field theory are partially applicable. It is also assumed that the gauge structure of the Standard Model appears as a result of the dynamics of a sub-level at the level of leptons and quarks, where GUTs and/or SUSY-like structure is partially realized.

2. Model of leptons and quarks with neutral constituents

It is known that only purely electrically neutral particles can be regarded as Majorana particles. Though the total charge of neutron is zero, it cannot be regarded as a Majorana particle, if ignoring its composite nature, because it has an intrinsic magnetic moment. Thus, among observed leptons and quarks, only neutrinos can be Majorana particles. An additional possible Majorana particle appears in a certain kind of composite model of leptons and quarks. This is the $V$ rishon in the composite model of leptons and quarks proposed by Harari and Shupe. The rishon model is based on the elementarity of the electric charge, which seems to be
absolutely conserved. It should be noted that almost all quantum numbers concerning to conventional conservation laws are not conserved in GUTs except the electric charge. Taking this point into account, the rishon model appears to be worthy of consideration. Furthermore, the three generation structure in the "cold-sector" is naturally explained by this model.\(^3\) In the rishon model, all leptons and quarks are three body systems of rishons T, with charge 1/3, and V, with charge 0, and color degree of freedom of u and d quarks is realized by their configuration.\(^2\),\(^4\) The leptons and quarks in this model are given as

\[
u_e = VV V, \quad e^- = TTV.
\]  

(1)

It should be noted that the V rishon is purely electrically neutral. Therefore it is possible to introduce the Majorana interaction on V.

3. The seesaw mechanism in the V rishon as a Majorana particle

For the V rishon, let us consider the Dirac-Majorana (D-M) mass term\(^5\)–\(^7\) in the simplest case of one generation. We have\(^*\)

\[
\mathcal{L}^{D-M} = -\frac{1}{2} \alpha \beta \gamma \delta \left( V_L \right)^c V_L - m_D \bar{V}_R V_L - \frac{1}{2} m_R \bar{V}_R (V_R)^c \quad + \quad \text{h.c.}
\]

\[
= -\frac{1}{2} \left( (V_L)^c / V_R \right) M \left( \begin{array}{cc} V_L^c & V_R^c \end{array} \right) + \quad \text{h.c.}
\]  

(2)

Here

\[
M = \left( \begin{array}{cc} m_L & m_D \\
& m_R \end{array} \right),
\]  

(3)

where \(m_L, m_D\) and \(m_R\) are parameters. For a symmetrical matrix \(M\), we have

\[
M = U m U^\dagger,
\]  

(4)

where \(U^\dagger U = 1\) and \(m_{jk} = m_j \delta_{jk}\). From Eqs. (2) and (4), we have

\[
\mathcal{L}^{D-M} = -\frac{1}{2} \sum_{\alpha=1}^2 m_\alpha \bar{\chi}_\alpha \chi_\alpha,
\]  

(5)

where

\[
V_L = \cos \theta \chi_1 + \sin \theta \chi_2,
\]  

\[
(V_R)^c = -\sin \theta \chi_1 + \cos \theta \chi_2.
\]  

(6)

Here \(\chi_1\) and \(\chi_2\) are fields of Majorana \(V\) rishon with masses \(m_s\) (a "small" mass), \(m_B\) (a "Big" mass), respectively. Assuming now that

\[
m_L = 0, \quad m_D \simeq m_F, \quad m_R \gg m_F,
\]  

(7)

\(*\) In a classification scheme based on the hyper-color and color group \(SU_3(H) \times SU_3(C)\), the singlet property of the Majorana mass term may be destroyed if the rishon belongs to the fundamental representation. The 8 representation seems to avoid this difficulty. The details of the resolution of this problem will be discussed elsewhere.
where $m_F$ is a typical mass of the rishon, particles with definite masses are distinguished as a very light Majorana $V$ rishon with mass $m_s \ll m_F$ and a very heavy Majorana particle with mass $m_B \simeq m_R$. The current $V$ rishon field, $V_L$, nearly coincides with $\chi_{1L}$, and $\chi_2 \simeq V_R + (V_R)^c$, because $\theta$ is extremely small. It should be noted that $V_L$ and $(V_R)^c$ represent the state corresponding to that concerning the possible weak interaction, while $\chi_1$ and $\chi_2$ are the mass eigenstates. Hereafter, we use the notation

$$V_s \equiv \chi_1, \quad \text{(mass } m_s) \quad V_B \equiv \chi_2, \quad \text{(mass } m_B)$$  \hspace{1cm} (8)

In our scheme, the $V$ rishon $V_s$ is a Majorana particle with masses much smaller than those of the other fermions. The predictions for the $V$ rishon masses depend on the value of the $m_R$ mass.

4. New structure of the generation based on the Majorana partners

The existence of $V_B$ in Eq. (8) implies the new generation structure with additional particles shown in Table I, where $TTV_s$, $\bar{T}V_sV_s$, $V_sV_sV_s$ and $\bar{T}\bar{T}\bar{T}$ represent the observed quarks and leptons, $u$, $d$, $\nu$ and $e^-$, respectively. In addition, new heavy quarks $TTV_B$, $\bar{T}V_sV_B$ and $\bar{T}V_BV_B$ and heavy neutrinos $V_sV_sV_B$, $V_sV_BV_B$ and $V_BV_BV_B$ appear. They are characterized by their configurations containing the heavy $V$ rishon $\bar{V}_B$. The row corresponding to $\nu$ lists the mass eigenstates of neutrinos belonging to the first generation, and $\nu_e$ the state concerning the weak interaction, which is realized as a superposition of mass eigenstates.

It should be emphasized that the new scheme satisfies the anomaly-free condition at the level of leptons and quarks,\footnote{In this paper, we have assumed that the gauge structure appears at the level of leptons and quarks, as a result of the dynamics of the sub-system, and the anomaly-free condition is formally realized at this level. In this sense, the meaning of this condition in the sub-system seems to be rather ambiguous.} that is, the condition

$$\sum Q = 0$$  \hspace{1cm} (9)

is satisfied.

It should be noted that in the seesaw mechanism, if the mass of $V_B$ is of the order of the Planck mass, the generation structure will not be affected in practice. The most interesting case is in which the mass of $V_B$ is not extremely large, for example, on the order of possible super-partners. In such a case, heavy quarks and neutrinos appear at the level of leptons and quarks. In the special case in which the

| flavor | standard | $B$ | $BB$ | $BBB$ |
|--------|----------|-----|------|-------|
| $u$    | $TTV_s$  | $TTV_B$ |       |       |
| $d$    | $\bar{T}V_sV_s$ | $\bar{T}V_sV_B$ | $\bar{T}V_BV_B$ |       |
| $\nu$  | $V_sV_sV_s$ | $V_sV_sV_B$ | $V_sV_BV_B$ | $V_BV_BV_B$ |
| $e^-$  | $\bar{T}\bar{T}\bar{T}$ | | | |

Table I. Configurations of leptons and quarks constituting a generation, where the row corresponding to $\nu$ lists its mass eigenstates belonging to the first generation.
two Majorana particles have exactly the same mass, $m_{V_s} = m_{V_B}$, they behave as if a single Dirac particle did, which is often called a “pseudo-Dirac particle”.

5. Possible creation and decay modes of new particles

Our model of leptons and quarks is an extension of the Standard Model that is based on the possible Majorana property of the constituents of leptons and quarks. As the fundamental dynamics of the sub-system are not yet known, we have assumed, that the gauge structure of the Standard Model appears as a result the dynamics of the sub-system, at the level of leptons and quarks, and assumed that the quantum field theory is partially applicable in that sub-system. Within this picture, the color structure of the Majorana partners $u_{B_1}$, $d_{B_1}$ and $d_{B_2}$ is the same as that of ordinary quarks. Thus, hadrons containing these particles will be created by pair creation such as

\[ P + P \rightarrow \text{hadron}(u_{B_1} \cdots) + \text{hadron}(\bar{u}_{B_1} \cdots) \cdots, \tag{10} \]

where $P$ and $\text{hadron}(u_{B_1} \cdots)$ represent the proton and hadron containing the $u_{B_1}$ quark.

The decay mode of these Majorana partners depends strongly on the possible dynamics of the sub-system. If we naively assume that the electroweak doublet structure is realized in the sub-system ($T \bar{V}$), sub-gauge bosons appear, and our Majorana partners are “sterile” in the sense that they do not take part in the standard weak interaction. Thus, provided that this characteristic property remains unchanged in our system of leptons and quarks, the decay modes are restricted to pair annihilation and other interactions, such as rare processes, except to the standard weak interaction. Therefore, it is expected that our Majorana partners have a considerably long lifetime.

Further, it should be noted that our model explains the proton-decay problem without difficulty. A typical decay mode is known as

\[ P \rightarrow e^+ + \pi_0, \tag{11} \]

which is caused due to the $X$ gauge boson in GUTs. In our model, this process is realized through the same process as in GUTs,

\[ u + u \rightarrow X \rightarrow e^+ + \bar{d}, \tag{12} \]

where we have regarded the 6 rishon system as $X$. It should be noted that the gauge bosons are assumed to be 6-body systems of rishons in the first work on the rishon model. The suppression of this process will be reduced to that of the rearrangement of rishons or, equivalently, the largeness of the $X$ boson mass.

6. Why are light quarks so light?

It is well known that the light $u$ and $d$ quarks have very small masses compared with the typical electroweak mass scale. They are estimated to be on the order of several MeV. Through what mechanism is the smallness of these quark masses realized?

It should be noted that our scheme is consistent with the smallness of the $u$ and $d$ quark masses, though it is not sufficient to explain smallness, because we do
not yet know the dynamics of these composite systems. Furthermore, the seesaw mechanism in our scheme cannot explain the smallness of electron mass, in which no neutral constituent is contained. It should be remembered that the electron has the special configuration $e^- = \bar{T} \bar{T} \bar{T} \bar{T}$ in the rishon model. In our opinion, a certain kind of statistics of order 3 may be concerned with the realization of the small mass, together with the appearance of yet unknown dynamics. Investigation of these points is a future problem.

7. Discussion

In this paper, we have proposed a possible new scheme describing the generation realized as a result of the Majorana property of the $V$ rishon. It should be emphasized that our approach allows for the extension of the knowledge for existence-form of matter found in Majorana neutrinos$^{1,11}$ to other fundamental particles as a universal property. Properties of the fundamental leptons and quarks belonging to the first generation are listed in Table II.

It is noteworthy that the new heavy "Majorana partners" $u_{B1}$, $d_{B1}$, $d_{B2}$, $\nu_{B1}$, $\nu_{B2}$ and $\nu_{B3}$ appear in our model. Their possible existence should be made clear in the near future. Our work is based on the rishon model, which is still at a hypothetical level. It should be emphasized, however, that this model is based on the simple concept of the elementarity of electric charge, and in this sense it is natural. In fact, five types of electric charge appear in GUTs,$^8$ where the 4/3 charge is carried by $X$ gauge boson, which is considered to be the particle responsible for proton decay. Then, as far as the charge units are concerned, it seems that GUTs are too complicated to be the final form of the theory of elementary particles.

It should be noted that the standard formulation is not yet known in the rishon model, and, as mentioned above, it is probable that the rishon is beyond ordinary quantum field theory.$^{12}$ However, it seems to be meaningful to treat it in the framework of conventional field theory. In this paper, we have assumed that the gauge structure of the Standard Model appears as a result of behavior of the level of leptons and quarks. It seems that there is another possible approach, in which, the pre-electroweak structure appears in the rishon level, and a certain kind of confinement of pre-gauge bosons realizes the well-known electroweak structure at the level of leptons and quarks. In this case, the breakdown of symmetry that causes the Majorana mass terms will occur successively after the breakdown of the electroweak structure of the sub-system. It should be emphasized that the seesaw mechanism itself does not cause a small mass, but, instead, it is just a result of breakdown of symmetry. In order to understand the extremely small mass of neutrinos, it may be necessary to assume further breakdown of symmetry at the level of leptons and

| $Q$ | flavor | $B1$ | $B2$ | $B3$ |
|-----|--------|------|------|------|
| 2/3 | $u$    | $u_{B1}$ |      |      |
| −1/3 | $d$ | $d_{B1}$ | $d_{B2}$ |      |
| 0   | $\nu$ | $\nu_{B1}$ | $\nu_{B2}$ | $\nu_{B3}$ |
| −1  | $e^-$ |      |      |      |

Table II. Leptons and quarks belonging to the first generation.
quarks that is responsible for the seesaw mechanism at this level.\textsuperscript{14)}

Finally, is there really a sub-level below the level of leptons and quarks? In our opinion, such a sub-level surely exists, and disclosing it will make it possible to predict theoretically quantities such as the Higgs coupling constants, the magnitudes of symmetry breaking $\langle \phi \rangle$, the mass spectrum and mixing parameters of all particles, etc. We restricted ourselves in this paper to the proposal of a possible new scheme for a generation with "Majorana partners". The possible dynamical properties of rishon systems will be discussed elsewhere. We conclude this paper by emphasizing that the possible existence of Majorana partners is probable, together with that of the super-partners in SUSY.

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