Proper and improper density matrices and the consistency of the Deutsch model for Local Closed Timelike Curves

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Abstract

We discuss the concept of proper and improper density matrixes and we argue that this issue is of fundamental importance for the understanding of the quantum-mechanical CTC model proposed by Deutsch. We arrive at the conclusion that under a realistic interpretation, the distinction between proper and improper density operators is not a relativistically covariant notion and this fact leads to the conclusion that the D-CTC model is physically inconsistent.

I. INTRODUCTION

Though quantum mechanics has been indisputably recognized for a long time now as much as a counter-intuitive theory as it is an extraordinarily effective one - it may be said that the issue of its “rigidity” as a physical model has arisen only in more recent times [1]. The “rigidity” of a theory can be thought as a qualitative measure of how much the entire theoretical construction is somehow constrained by its principles in the sense that by tampering with one of them would “bring down” the whole structure, making the theory incoherent or inconsistent.

In 1989, Weinberg [2, 3] initiated an investigation on how far one could modify ordinary quantum mechanical principles by adding some extent of non-linearity to its axioms and surprisingly it turned out to be a very difficult enterprise. Almost immediately it was pointed out that this would lead to superluminal communication or signaling [4, 5]. As is well known, this leads to paradoxes and is considered by most physicists to be non-physical.

In fact, in 1949, Gödel found a solution of general relativistic field equations that exhibits closed time-like curves (CTCs) [6]. Most physicists at the time (including Einstein) found this result quite disturbing, but disregarded it as probably nonphysical, preferring to believe maybe that a future more complete understanding of the physics involved should somehow rule out such kind of
Yet, in 1991, Deutsch presented a computational analysis of the problem – both classical and quantum mechanical [7]. He addressed the inherent free-will kind of problems that stems from grandfather-like paradoxes in a more technical and less anthropomorphic way than usual. He assumed the existence of a region of space-time (CTC) which violates the usual chronological respecting space-time (CR) and considered a computational circuit where one or more bits or qubits may enter the CTC and travel back in time meeting itself at an earlier moment. He also conceived that a CR system may interact with the CTC system by some unspecified unitary interaction. He showed that, in general, classical computational logic is inconsistent with the existence of CTCs in the sense that not all possible input states have a consistent solution – this is the technical way of stating the free-will problem. Yet, he suggested a quantum mechanical model for systems (allowing for mixture states), that exhibits output solutions for every input state, seemingly circumventing the paradoxes. Let \( \hat{\rho}^{(i)} \) be the initial state of the chronology respecting system and \( \hat{\rho}_{CTC} \) the state captured in the CTC loop, then he proposed the following self consistent relation that \( \hat{\rho}_{CTC} \) must obey:

\[
\hat{\rho}_{CTC} = \text{tr}_{CR}[\hat{U}(\hat{\rho}^{(i)} \otimes \hat{\rho}_{CTC})\hat{U}^\dagger] \tag{1}
\]

The physical meaning of the above equation is clear. The CR system (Alice) comes close enough to the CTC system (Bob), interacts with it in a limited region of space-time and becomes part of a larger entangled state \( \hat{U}(\hat{\rho}^{(i)} \otimes \hat{\rho}_{CTC})\hat{U}^\dagger \) under the interaction modeled by the global unitary operator \( \hat{U} \). After a while, the system moves away and Bob’s system is obtained by partial tracing out Alice’s system. The above equation is a way to impose that the output state is the same as the input one, obeying the CTC criterion with no paradoxes as Deutsch showed that there is always at least one solution (there may be more than one.) Note that \( \hat{\rho}_{CTC} \) depends on the initial state \( \hat{\rho}^{(i)} \) and on the unitary operator \( \hat{U} \) (the interaction). Alice’s system will have changed to

\[
\hat{\rho}^{(f)} = \text{tr}_{CTC}[\hat{U}(\hat{\rho}^{(i)} \otimes \hat{\rho}_{CTC})\hat{U}^\dagger] \tag{2}
\]

The two equations above clearly imply a non-linear evolution for Alice’s system because (2) means that \( \hat{\rho}^{(f)} \) depends both on \( \hat{\rho}^{(i)} \) and \( \hat{\rho}_{CTC} \), (and on the interaction) but the latter also depends on \( \hat{\rho}^{(i)} \). This feature is a novel ingredient that goes beyond the linear evolution described by Schrödinger’s equation. Since Deutsch’s proposal, many results appeared in the literature where it
was argued that quantum mechanics together with Deutsch’s model for closed time-like curves (D-CTC) is more powerful than ordinary quantum physics. Claims as cloning of quantum states [8], solving NP-complete problems in polynomial time [9] and distinguishing non-orthogonal states [10] have been reported.

In this paper we discuss the fact that under a certain interpretation of quantum mechanics, the ability to distinguish non-orthogonal states leads directly to inconsistencies in the Deutsch protocol for quantum systems traversing CTCs. We argue that the concept of a density matrix to be proper or improper is a relativistically non-covariant notion and that this result leads us to the above conclusion after we examine in detail a specific instance of a protocol for non-orthogonal state discrimination presented by Brun et al.

The paper is structured as follows: In Section II, we will briefly review the non-local properties of a EPR-like pair of qubits and discuss why by the ordinary interpretation of quantum mechanics, the non-distinguishability of non-orthonormal states protects the theory from superluminal communication despite the non-local information contained in the entangled states. In Section III, we discuss some interpretational issues on the difference between proper and improper density matrices in quantum mechanics that are crucial to our main conclusions. In Section IV, we briefly review the results in [10] that allows the discrimination of non-orthogonal states with the Deutsch model and how this implies that the D-CTC model is not consistent. In Section V, we conclude our work with a discussion about the physical meaning of these results and we also set stage for further work.

II. NON-Discrimination of Non-Orthogonal States In Ordinary QM

The celebrated 1935 EPR paper showed, for the first time, what Einstein coined as “spooky action at distance” of Bell-like states even though it was early recognized that it was impossible to use entangled states for superluminal communication [11]. Suppose two parties, Alice and Bob meet to establish an interaction between their qubits to create a maximal entangled pair of qubits in the anti-correlated form

\[ |\Psi⟩ = \frac{1}{\sqrt{2}} (|z+⟩ \otimes |z−⟩ − |z−⟩ \otimes |z+⟩) \]  

After this, they go apart and each one has access only to his own qubit. Both parts will describe their own system by the density matrix
\[ \hat{\rho}_{\text{Alice}} = \hat{\rho}_{\text{Bob}} = \frac{1}{2} I \] (4)

This means that (3) can be written as

\[ |\Psi\rangle = \frac{1}{\sqrt{2}} (|\hat{n}\rangle \otimes |\hat{n}\rangle - |\hat{n}\rangle \otimes |\hat{n}\rangle) \] (5)

Where the state \(|\hat{n}(\theta, \varphi)\rangle\) defined as

\[ |\hat{n}(\theta, \varphi)\rangle = \cos \left( \frac{\theta}{2} \right) |z+\rangle + e^{i\varphi} \sin \left( \frac{\theta}{2} \right) |z-\rangle \] (6)

is projected to an arbitrary point with \((\theta, \varphi)\) coordinates on the surface of the Bloch sphere. Note that Hilbert space orthonormality of two states means that the vectors are projected to antipodal points on the Bloch sphere. To convey superluminal information, one part should be able to distinguish different directions in the Bloch sphere geometry, which means the ability to discriminate non-orthogonal states.

Thus, Alice may “collapse” the mixed state by measuring her state in any direction of the Bloch Sphere, transforming non-locally Bob’s description to a pure state. But since we may assume that Bob is far away, he has no way to find out about Alice’s measurement without receiving a message from her. Alice and Bob could agree before hand to measure only over two different directions, let us say the \(x\) and \(z\) directions. Alice could hopelessly try to code one classical bit of information in these distinguished directions, by assigning logical values to them, for instance, 0 to the \(z\) direction and 1 to the \(x\) direction. By choosing which direction to measure her qubit, she would collapse the whole system instantaneously (in some inertial reference system).

Suppose she chooses to send the 0 bit and after her \(z\) measurement, she “collapses” her state to \(|z+\rangle\). There is no way that Bob can receive this classical piece of information. If he chooses to measure the \(z\) direction, he will certainly obtain \(|z-\rangle\), but there is no way for him to know if his collapsed pure state was in the \(x\) or \(z\) directions without receiving information from Alice in the usual (subluminal) manner. The intrinsic randomness of quantum mechanics avoids superluminal communication.

By the other way around, the ability of discrimination of non-orthogonal states leads necessarily to signaling as can be easily seen. Most papers on no-signaling and quantum mechanics discuss this issue in terms of the
no-cloning theorem \([12, 14]\) because both these abilities are actually equivalent \([15]\). Curiously, we were incapable of finding in the literature any explicit discussion of the no-signaling property of quantum mechanics in terms of the indistinguishability of non-orthogonal states. This issue seems to have solely been discussed in terms of the no-cloning theorem. Maybe this is because signaling is such an obvious consequence of non-orthogonal state distinguishability that most authors just take it for granted. On possible exception is Gisin’s 1989 paper where he refutes Weinberg’s attempt to construct a non-linear quantum mechanical theory \([4]\). Indeed in his paper, he manages to ultimately distinguish non-orthogonal states, but only after a quite elaborate construction where there must be a third party besides Alice and Bob that must continuously provide a stream of entangled qubits, each one sent to one of the other parts. (Alice and Bob). We show in our above analysis that any “non-unitary machine” capable of discriminating non-orthogonal states immediately implies in superluminality, a result that seems quite obvious, but which seems that has not been explicitly stated anywhere before in the literature.

In fact, going back to the EPR protocol discussed above, if Bob has access to some device capable of discriminating between the four state vectors of their common alphabet, Alice will clearly be able to communicate a classical bit in a superluminal way by choosing in which of the two agreed directions she performs her measurement.

III. SOME INTERPRETATIONAL ISSUES OF QUANTUM MECHANICS

A. The interpretation of the state vector

The picture of entanglement presented in the last Section goes along with the most usual interpretation of quantum physics. This interpretation (let us agree to call it a realistic interpretation) views the entanglement phenomenon as exhibiting genuine non-local properties even if this non-locality does not lead to signaling. In this interpretation, the entangled state \([5]\) can be seen as a non-causal channel connecting two distant observers such that if a pair of measurements are performed by Alice and Bob (one measurement each - at space-like separated events) then it follows that there is no consistent way to assign any causal relation between them.

One may say that in this view, one thinks of the state vector (or its projection onto the space of rays) as a real objective property of the system. These quantum channels are
behind many of the modern applications of quantum information as quantum teleportation and cryptography.

A very different view is taken by some physicists like [16] or [17] for example. This approach to quantum mechanics may be described as *epistemic* in the sense that one assigns to the state vector the subjective property of describing only the *knowledge* of an observer about the system. For those who subscribe to this interpretation, there is no such thing as non-local phenomena in quantum mechanics, because the non-local collapse of the global state vector is a non-physical process. In [17], for instance, the authors conclude that the Deutsch scheme manages only to *conceal* the paradoxes involved in time-travelling instead of resolving them.

Yet, regardless of these different interpretations, for linear quantum mechanics, it is noncontroversial that one arrives at the same physical predictions. One could wonder if these subtle philosophical distinctions are then actually physically relevant. But something quantum mechanics has taught us during the last century is that one should be specially careful before jumping to such a conclusion. After all, the Bohr-Einstein debate was generally considered (for three decades) as being of a philosophical character before Bell introduced in 1964 his inequalities for local hidden variable correlations and showed that the dispute was physically verifiable. One may even make the case that this was the moment when the embryo of modern quantum information science was laid. Our opinion is that indeed for linear quantum mechanics, there may not be any physical differences between these two approaches. But if any non-linearity is introduced as some physicists expect for a full consistent quantum theory of gravity (see [18] and [19] for opposite argumentations on this issue), then the situation will probably be very different as we argue in the next sections.

### B. The interpretation of the density matrix

There are two very different approaches to the density matrix concept. The first may be called the text-book concept of a density matrix and it is a reminiscent of the historical way that statistical concepts were introduced in classical physics. One is given a large ensemble of $N$ identical quantum systems and one supposes that the ensemble can be partitioned into many sub-ensembles labeled by an index $\alpha = 1, 2, ..., m$ where $m$ is the number of different sub-ensembles $\alpha$ characterized by the fact that every quantum system that belongs to it is in a pure state $|\psi_\alpha\rangle$. Now suppose further that an observer may “pick” one
system from the ensemble in a random way. Notice that this randomness is purely “classical” in some sense. The probability that the observer picks up a system from a subensemble $\alpha$ is clearly $P_\alpha = n_\alpha/N$ where $n_\alpha$ is the number of systems in sub-ensemble $\alpha$. Of course it clearly holds that the probability $P_\alpha$ is automatically normalized as it indeed must be. Given an arbitrary observable $\hat{O}$, its expectation value for $|\psi_\alpha\rangle$ is $\langle \psi_\alpha | \hat{O} | \psi_\alpha \rangle$ and if the observer repeats the procedure many times, the average expectation value is clearly

$$\sum_\alpha P_\alpha \langle \psi_\alpha | \hat{O} | \psi_\alpha \rangle = \text{tr} \left( \hat{\rho} \hat{O} \right)$$

where

$$\hat{\rho} = \sum_\alpha P_\alpha |\psi_\alpha\rangle \langle \psi_\alpha|$$

is defined as the density matrix. This definition is easily seen as an epistemic definition where one considers $P_\alpha$ a classical probability distribution that is subjective in the sense that it reveals the “classical ignorance” of the observer about which pure state $|\psi_\alpha\rangle$ the system actually belongs to. This epistemic definition is known also as a proper density matrix [20].

A second conception of density matrix has a purely quantum mechanical origin. Given an entangled state $|\Psi\rangle \in W = W^{(a)} \otimes W^{(b)}$ of two subsystems (Alice and Bob’s subsystems), suppose (after the global entangled state has been created locally) each one has physical access only to its own subsystem. Let $\hat{O}$ be again an arbitrary observable of Alice’s subsystem, then she can define a density matrix $\hat{\rho}_{|\Psi\rangle}$ by the following equation

$$\langle \Psi | \hat{O} \otimes \hat{I} | \Psi \rangle = \text{tr} \left( \hat{\rho}_{|\Psi\rangle} \hat{O} \right) \quad \text{(for all $\hat{O}$)}$$

The above equation defines a mapping from the space of rays (the projective space of the full quantum space $W$) to the space of linear operators in $W^{(a)}$. This (non-linear) mapping is called the partial trace and it can be conducted in the same manner for Bob’s subspace. This definition of density matrix is known as an improper mixture and it is commonly thought as an ontological description of a mixed state in some interpretations and should then be seen as essentially different from the former case.

What is remarkable is that mathematically, both definitions lead to equivalent descriptions. Both (proper and improper mixtures) are hermitian, positive and unit trace operators. Some authors deny the existence of proper mixtures in the sense that all density matrices can be thought as resulting from a partial trace. There is some controversy on this matter. (See [21] and [22] for opposite opinions on this issue.) But consider now the following two experiments:

1. Alice tosses a fair coin in her lab to decide if she produces a pure $|z+\rangle$ or $|z-\rangle$ state and then sends the state to Bob.
2. Alice initially produces an entangled 2-qubit state as in (5) and then measures one of the qubits in the z direction and sends the other qubit to Bob.

It is clear that both preparations must lead to physically indistinguishable states for Bob (at least for usual linear quantum mechanics) represented both by the same proper density matrix given by (4). If one introduces elements of non-linearity as is the case of the D-CTC prescription, things are not as simple as we shall see in the next Section.

IV. NON-ORTHOGONAL STATE DISCRIMINATION PROTOCOL WITH DEUTSCH CTC’S

In [10], Brun et al exhibited a quantum computational protocol with access to a Deutsch CTC that allows discrimination of arbitrary non-orthogonal state vectors.

Let \( C = \{ |\psi_k\rangle \}, \quad (k = 1, \ldots, n) \) be a set of \( n \) non-orthogonal normalized state vectors belonging to a finite \( n \)-dimensional Hilbert space \( W \) of the CR system. Suppose that the input state is given by the following pure density matrix \( \hat{\rho}_{CR} = \hat{\pi}_{\psi_s} = |\psi_s\rangle \langle \psi_s| \) chosen from \( C \). The protocol starts by applying the swap operator on the \( W \otimes W_{CTC} \) system followed by the controlled \( \hat{U} \) operator

\[
\hat{U} = \sum_j \hat{\pi}_j \otimes \hat{O}_j
\]

where \( \{ \hat{\pi}_j \} \) is a family of one-dimensional projection operators over an orthonormal basis \( \{ |u_j\rangle \} \) of \( W \) and \( \{ \hat{O}_j \} \) is a family of unitary operators such that \( \hat{O}_k |\psi_k\rangle = |u_k\rangle \). Brun et al have shown that it is always possible to find a set \( \{ \hat{O}_j \} \) such that there is only one single self-consistent solution given by \( \hat{\rho}_{CTC} = \hat{\pi}_s \). This allows the existence of a bijective (non-linear) map between a set of non-orthogonal states in \( W \) and a set of orthogonal states \( \{ |u_j\rangle \} \) which implies the discrimination of the elements of \( C \).

With this result, one may approach again the EPR problem and conclude that superluminal communication seems to be possible. In fact, suppose Alice and Bob share a single pair of qubits in the Bell state given by (5) where the first qubit stays in Alice’s possession and the second qubit travels with Bob. Suppose they have agreed previously over the code described in Section I. Bob now must join his qubit with another qubit system in a known state \(|z+\rangle\) for instance. In this way, after Alice measures her qubit in one of the two possible directions, Bob’s system will be in a state \(|\xi_j\rangle \otimes |z+\rangle\), with the state \(|\xi_j\rangle\) among the following four possibilities: \(|\xi_0\rangle = |z+\rangle, |\xi_1\rangle = |z-\rangle, |\xi_2\rangle = |x+\rangle\) or \(|\xi_3\rangle = |x-\rangle\). Bob then uses the D-CTC to distinguish these non-orthogonal states. The input state is the pure state \(|\xi_j\rangle \otimes |z+\rangle\) and the circuit will swap the CR and CTC system
followed by the controlled unitary operation
(7) with $|u_0\rangle = |z^+\rangle \otimes |z^+\rangle$, $|u_1\rangle = |z^-\rangle \otimes |z^+\rangle$, $|u_2\rangle = |z^+\rangle \otimes |z^-\rangle$, $|u_3\rangle = |z^-\rangle \otimes |z^-\rangle$
and $\hat{O}_i(|\xi_i\rangle \otimes |z^+\rangle) = |u_i\rangle$. In this way, Bob
manages to know which direction Alice decided to measure and her classical bit is com-
municated with infinite speed.

In a recent paper [23], the authors reach a
different conclusion. They show (as Deutsch
assumes in his seminal paper) that quantum
correlations are lost in the D-CTC circuit by
using a “infinite loop” equivalent circuit for-
mulation where the problem is seen by the
perspective of the CTC system that is “cap-
tured for eternity” in the loop. We subscribe
to this view but the authors then make a
claim that we believe is mistaken: they claim
that this fact implies that superluminal phe-
nomena cannot happen. It seems that they
believe that the “collapse” of the global state
of a entangled state must happen only locally.
But this belief is extraneous to the usual view
of quantum mechanics. And neither have we
found elements in Deutsch’s original paper
that supports such extreme notion. To sub-
scribe to this idea would be the same as in-
troducing new elements in quantum physics
that indicate some kind of physical medium
that would propagate the “wave-function col-
lapse” with finite speed. This is contrary
to everything we know about quantum me-
chanics including the well known results that
exhibit non-locality in the EPR experiments
[24].

Notice that Alice and Bob can arrange
things such that for a certain moment just
before Bob interacts with the D-CTC, Alice
measures the system in the way we described
so that Bob is sure that his state is pure. His
description would be given by a proper den-
sity matrix so that he can treat his state in
the same way that Brun et al and Ralf et
al treat the states from the non-orthogonal
alphabet of states chosen between him and
Alice. At this point - a physicist that be-
lieves in a radically epistemic interpretation
of quantum mechanics might reach a differ-
ent conclusion. He may say that there is no
difference between proper and improper mix-
tures and that they should be treated in the
same way. This is the “linearity trap argu-
ment” put forward in [25] but convincingly
refuted in [26] with a sound argument of veri-
fiability for any non-linear evolution. Indeed,
Ralph et al also subscribe to Cavalcanti and
Menicucci’s opinion but seem not to recog-
nize that the same argument leads to signal-
ing by the argument that we present here.

An important point that must be made is
that of taking seriously the fact that the mea-
urements are space-like separated events.
This means that there is a reference system
for which Alice has not yet “collapsed” the
global state and in this case indeed Bob will
insert his qubit as an improper state (4).

This result implies that the concept of proper and improper mixtures is not a relativistically covariant notion. But one should note that for usual linear quantum mechanics this is irrelevant because both concepts are equivalent. In this case we have a completely different physical picture. We agree that indeed in this scenario, the D-CTC should destroy all quantum correlations and the output would again be the maximum entangled state (4).

How can this be possible? The only conceivable answer to this question is that it cannot be. We summarize our analysis in the following way: In one reference system, Bob’s state is pure and he manages to receive superluminal communication which ultimately leads to those same paradoxes that Deutsch was originally trying to prevent and so it is then inconsistent. In another reference state, his state is an improper mixture and the signaling protocol fails. But this is inconsistent with the first scenario. We conclude then that Deutsch’s CTC model itself is overall inconsistent.

V. CONCLUDING REMARKS

Does this analysis imply necessarily the conclusion that the Deutsch CTC model is irreparably inconsistent after all? There are some fundamental questions about the model that may be addressed. For instance, in Deutsch’s original scheme, the unitary operator to be applied in the interaction with the CTC is supposed to be arbitrary. Is this a reasonable supposition on physical grounds? After all, shouldn’t one expect that under the very likely extreme physical conditions that material particles suffer under closed time-like curves as their world-lines, that the physically possible unitary quantum evolutions should be constrained by the rules of a (still unknown) consistent quantum theory of gravitation?

A full description of the largest class of unitary evolutions that not allow arbitrary non-orthogonal state discrimination might turn out to be rewarding in the sense that this information could be used as a clue for what kind of interaction are or not permitted in such extreme scenarios. Even if such a programme could be carried out successfully, the issue of the relativistically noncovariance of the density matrix concept and the resulting lack of inner consistency that it implies would also have to be tackled – see [27] for a recent discussion on related matters. Maybe there is a manner of restraining the set of unitary operations that could at the same time guarantee an equivalent physical description for all reference frames. Of course, the fact that the many tasks that have
been recently claimed to be possible of implement-

ation with Deutsch CTCs may be seen as a sign that this model is inherently incon-
sistent after all.

It should also be noted that there is an alternative model to Deutsch’s CTC. The so called post-selection CTC (see [28], [29], [30], [31] and [32]) is a recent example of a model that allows non-orthogonal discrimination only for a set of linearly independent states. Notice that this clearly forbids the implementation of our signaling protocol between Alice and Bob because the set \( C \) of alphabet states is linearly dependent. This seems to imply that the P-CTC model may indeed be more appropriate physically than the D-CTC model. It is our intention to publish in the near future a paper specifically on the P-CTC model in quantum mechanics.

That the ability to post-select may present counter-intuitive non-local time properties for quantum mechanics has been noticed before [33], [34]. It would not be surprising if Aharonov’s concepts of modular variables and weak values turn out to play an essential role on this issue [35], [36]. It is our opinion that the importance of research on this topic is that of presenting toy models so that one may probe theoretically a very elusive long-sought consistent quantum theory of gravity. Our work also suggests that the construction of such a future theory may require that some important foundational issues (as the intrinsic difference between proper and improper density matrices) be addressed and adequately resolved first.

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