Parameter Governing of Wave Resonance in Water Chamber and Its Application

Husain F¹, Alie M Z M² and Rahman T³
1,2,3Ocean Engineering Department, Engineering Faculty, Hasanuddin University, Makassar, Indonesia

E-mail: firman.husain@unhas.ac.id

Abstract. It has become known that the oscillating water column (OWC) device is very popular as one of wave energy extraction facilities installed in coastal and ocean structures. However, it has not been clarified sufficiently how to obtain an effective cross section design of the structure until now. This paper describes theoretical procedure to yield effective cross section of water chamber type of sea wall, which is similar to the OWC type structure in relation to wave period or wave length. The water chamber type sea wall has a water chamber partitioned by a curtain wall installed in front of part of the structure. This type of sea wall also can be applied to extract wave power same as of OWC function. When the wave conditions on site are known, the dimensions especially the breadth of water chamber type sea wall can be determined.

1. Introduction
The oscillating water column (OWC) is very old type of wave energy extraction device installed in coastal area around the world. This type is most popular among other types of wave energy converter [1]. However, it has not been clarified sufficiently how to obtain and design an effective cross section of the structure until now. Bouali and Larbi [2] studied the geometry and dimension of an OWC type to extract wave power from the wave, but it also not clear how to design an OWC given the wave condition on site. In this paper a theoretical procedure to yield the effective cross section of the OWC type structure in relation to wave period or wave length is proposed. It may become a significant guidance to design the cross section of OWC type sea wall structure in a practical field application.

In this study, a type of sea wall similar to the OWC structure with a water chamber partitioned by a curtain wall in the front is investigated. It has become known that the water chamber type sea wall like an OWC plays a role of an effective dissipater of reflected waves from vertical walls as well as a wave energy extraction device. The dissipation mechanism of the reflected waves is to generate strong vortex flows around a lower edge of the front curtain wall [3-4]. The strong vortex flows are generated by excitation of resonance of pumping or piston mode wave motions in the water chamber. In order to find out the governing parameter of the piston mode resonance, we have dealt with the wave resonance of piston mode theoretically by a vibration model with one degree of freedom. Resonant wave condition is important in order to extract optimum wave power of OWC type [5]. In the analysis, we have also examined the characteristics of added mass of an oscillating water column in a water chamber. Using various dimensions of the dissipater, the governing parameters of the added mass are examined precisely. By taking account of the resonance condition of the piston mode motion, a new parameter governing wave reflection characteristic is deduced theoretically. It was confirmed that the new parameter is useful to determine an effective cross section of the dissipater, such as a draft of the...
2. Piston mode resonance of wave motion in water chamber

Figure 1(a) shows a model of the sea wall with a water chamber. The sea wall model assumed here is comprised of a curtain wall and a vertical backward wall. A curtain wall is generally placed at the definite location with some distance apart from the backward wall. Here the distance may be chosen in such a way that wave resonances take place in the water chamber. The distance between the curtain wall and vertical wall is the width of the water chamber denoted by $B$. A draft depth of the curtain wall, $d$, is a submergence of the curtain. These two dimensions may be very important to determine the resonance condition in the water chamber in relation to incoming wave conditions, typically wave period $T$ or wave length $L$, as well as the water depth $h$.

Figure 1 (b) shows an equivalence of an oscillation model of water mass in the chamber of the real model as shown in figure 1(a). In the equivalence model, the oscillation mode of the wave motion in the chamber is assumed to be piston mode that is like a heave mode motion of a rigid plate, where the water surface moves up and down simultaneously like a piston motion. This kind of model has been adopted in the previous study of an OWC type of wave power extraction devices [6].

As shown in figure 1 (b), there is an idealistically thin horizontal plate without a mass at the still water surface in the water chamber. However, the plate is idealistically strong enough not to be deformed by wave actions except for heave mode motions. In the previous study, a rectangular body is often used instead of a thin plate. When we consider heave mode motions of the thin plate by wave actions, vertical oscillations of water mass in the chamber can be analysed approximately as a vibration model with a single degree of freedom. Here, we adopted the equivalent model to be able to estimate an added mass due to piston-mode wave motions in the chamber.

In the theoretical analysis, the exact analytical model of the water wave problem around the chamber-type dissipater is also used to confirm the validity of the equivalent model, in which there is no assumption about the wave motion in the chamber and the latter analysis, we applied the damping wave model. In addition, for analysing the heave mode motion of the thin plate in the chamber, we have also adopted the extended damping wave model for floating body dynamics proposed by Nakamura and Ide [7]. By the use of the extended model, we can estimate hydrodynamic properties, such as an added mass coefficient and a wave making resistance coefficient of the thin plate. As a final output of the analysis, we can also obtain the reflection coefficient under heave motions of the thin plate by wave actions.

At the first theoretical examinations of model is calculated. It related to the resonance phenomena in the water chamber, such as a piston mode resonance and closely related also to the appearance of the minimum wave reflection from the model body. Figure 2 shows the calculation result of $Cr$ as a function of $L/d$ for various values of $B$ by the equivalent vibration model. The equivalent vibration model is valid for estimating reflection coefficients of the dissipater except for the case of very long...
water chamber width $B$, that is $B=100$ cm. In case of very long water chamber width, wave motions in the water chamber may be different from the piston mode as assumed in the equivalent vibration model. Therefore, it may be possible to use the equivalent vibration model to estimate the dynamic properties of the thin plate located on the still water in the water chamber, especially for comparatively longer wave conditions as compared to the wavelength. Figure 3 is the calculation results of $Cr$ as a function of $L/B$ for various values of draft $d$ under fixed conditions of $h$ and $B$. From the figure, we can see that the minimum reflection point of $L/B$ increases with the draft $d$. It is clear that the reflected wave for longer wave period conditions is effectively dissipated for the model of deeper draft depth.

Figure 2. Reflection coefficient for various water chamber widths $B$ as a function of $L/d$ (Calculation results by the equivalent vibration model, $h=40$cm, $d=25$cm)

Figure 3. Reflection coefficient for various draft depths of the curtain wall as a function of $L/B$ (Calculation results by the equivalent vibration model)

Figure 4 shows the calculation results of heave mode amplitude $X2$ of a thin plate placed in the water chamber as a function of $L/d$ for various water chamber widths. The equivalent vibration model was employed in the calculations. In the figure, the heave mode amplitude is normalized by incident wave amplitude $H/2$. It can be seen that the maximum heave mode amplitude appears at larger values of $L/d$ with increasing the water chamber width $B$. This tendency is very similar to the wave amplifications in the water chamber.

Figure 4. Heave mode amplitudes of the thin plate in the water chamber for various water chamber width $B$. (Calculation results by the equivalent vibration model, $d=25$cm $h=40$cm)

Figure 5. Added mass coefficients $C_A$ for various $B$. (Calculation results by equivalent vibration model, $h=40$cm, $d=25$cm)

Figure 5 shows the calculation result of added mass coefficients $C_A$ as a function of $L/d$ and figure 6 shows wave making resistance coefficients $C_W$ as a function of $L/d$ for various $B$. The water depth $h$
and the draft depth of curtain wall are kept constant. The definitions of $C_A$ and $C_W$ are specified in the corresponding figures, where $\rho$: fluid density, $L_b$: transverse width of the thin plate, $\omega$: angular frequency of wave, $F_A$: added mass force, $F_W$: wave making resistance force. From these figures, it can be seen that both $C_A$ and $C_W$ increase with $B$ for various $L/d$. Such a tendency may be interesting because these hydrodynamic coefficients are obtained after normalization by the use of $B$. It means that both $C_A$ and $C_W$ may have a dependency on the water chamber width $B$. The fact of this dependency may be a major cause of tendency for the shift of minimum reflection point to longer wave conditions with increasing $B$, as seen in figure 2.

Figure 6. Wave making resistance coefficient $C_W$ for various $B$ as a function of $L/d$. (Calculation results by the equivalent vibration model, $h=40\text{cm}$, $d=25\text{cm}$)

Figure 7. Added mass coefficients $C_A$ for various $d$ as a function of $L/B$. (Calculation results by the equivalent vibration model, $h=40\text{cm}$, $B=25\text{cm}$)

Figure 7 is the calculation result of $C_A$ for various draft depths $d$ as a function of $L/B$. The water depth $h$ and the water chamber width $B$ are kept constant. From this figure, it is seen that the dependency of $C_A$ on the draft depth $d$ is not so strong as compared to that of $B$. We can see the weak dependency of $C_A$ on the draft depth $d$ and the added mass is roughly proportional to the draft depth $d$.

3. Theoretical consideration on the piston mode resonance

Here, we would like to consider the governing parameter that gives the wave period condition for dissipating reflected waves most effectively, typically the minimum reflection points. In the following, we would like to think of the occurrence of piston mode resonance in the water chamber. Especially focusing on characteristics of the added mass due to piston mode motion of the water surface in the water chamber, we would like to reconsider the governing parameter.

As it is well known, the natural period $T_n$ of the vibration model of figure 1(a) is given by Equation (1) if the damping term is ignored

$$T_n = 2\pi \sqrt{M_A / K}$$

$$K = \rho g B L_b$$

(1)

Here, $M_A$ is an added mass and $K$ is a spring constant. The spring constant can be obtained by considering hydrostatic force on the idealized thin plate in the chamber, where $\rho$ is a fluid density, $g$ is a gravitational acceleration, $B$ is an inline length of the water chamber and $L_b$ is a transverse length of the water chamber. The following equation is generated after substitution

$$T_n = 2\pi \sqrt{m_b / \rho g B}$$

(2)
where, \( m_A \) is an added mass of the thin plate per unit length of the body. When we use a wave length \( L_n \) instead of \( T_n \) through the use of dispersion relationship of the linear wave theory, the following equation can be derived,

\[
L_n = \frac{2m_A}{\rho B} \tanh\left(\frac{2\pi h}{L_n}\right)
\]  

(3)

Since it is hard to solve Equation (3) analytically, the second approximation for \( L_n \) is obtained by the use of Newton’ method. It is given by

\[
L_n^{(2)} = \frac{2m_A}{\rho B} \tanh\left(\frac{h\rho B}{m_A}\right)
\]  

(4)

where, \( m_A \) is given by the following equation,

\[
m_A = \rho \alpha B^x d^y
\]  

(5)

where, \( \alpha \) is a constant, and \( x \) and \( y \) must satisfy the relation \( x+y=2 \) from the dimensional restriction. Substituting Equation (5) into Equation (4) gives

\[
L_n^{(2)} = 2\pi \alpha B^{x-1} d^y \tanh\left(\frac{h}{\alpha B^{x-1} d^y}\right)
\]  

(6)

This equation gives the definite wave length corresponding to the piston mode resonance in the water chamber. In order to proceed further, we have to determine the functional form of Equation (6), typically \( x, y \) and \( \alpha \). After several trials, we found out that a new dimensionless added mass \( A_D^* \) defined by the following equation is almost constant for various \( B, d, h \) and \( T \).

\[
A_D^* = \frac{m_A}{\rho \alpha B^{1.4} d^{0.05} (h-d)^{0.05}} \tanh\left(\frac{2\pi h}{2\pi \alpha B^{0.4} d^{0.05} (h-d)^{0.05}}\right)
\]  

(7)

The proof of the finding is typically shown in figures 8 and 9. In those figures, the previous results shown in figures 5 and 7 are replotted by the use of \( A_D^* \). From these figures, it can be reconfirmed that \( A_D^* \) defined by Equation (7) is almost constant for various dimensions of the model body. In the above trials, the value of \( \alpha \) is clarified as about 1.6 from additional examinations and this value is applied. From the above examinations on the added mass, it may be concluded that the added mass under the piston mode motion of the water chamber is proportional to \( B^{1.4} \) and \( d^{0.6} \).

**Figure 8.** Examinations of the functional form of the added mass for various \( B \) as a function of \( L/d \). (\( h=40\text{cm}, d=25\text{cm} \))

**Figure 9.** Examinations of the functional form of the added mass for various \( d \) as a function of \( L/B \). (\( h=40\text{cm}, B=25\text{cm} \))
Considering the fact that $A_D^*$ defined by Equation (7) is almost constant for various dimensions of the model body, Equation (6) may be transformed as

$$\frac{L_n^2}{(B^{0.4}d^{0.65}/(h\cdot d)^{0.05})} = \frac{L_n^2}{l_c} = 2$$

where, $l_c$ is a kind of characteristic length of the water chamber and is defined as the denominator of the above equation. Since Equation (9) shows the resonant condition of the water chamber for the piston mode, the minimum reflection also may appear under the corresponding resonant condition.

**Figure 10.** Examinations of the effectiveness of the new parameter as a governing reflection dissipater ($h=40cm$)

**Figure 11.** Experimental verifications of the effectiveness of the new parameter as a governing reflection dissipater

Figure 10 shows the calculation results of reflection coefficient $Cr$ for various $B$ and $d$ of the single water chamber type sea wall as a function of $L/l_c$, defined by Equation (8). Figure 11 shows the corresponding experimental result of reflection coefficient. From these two figures, it can be seen that the minimum reflection point appears at the definite condition of $L/l_c=10$ as predicted analytically. Also, we can confirm that such a tendency is not dependent on the various dimensions of the cross section of the body. Thus, parameter $L/l_c$ is universal for a single water chamber sea wall. Based on the results shown in figure 10 and 11, it may be very easy to determine each dimension of the cross section of the single water chamber type sea wall. Typically, a water chamber width $B$ can be determined by using Equation (8) or its simplified one, i.e. Equation (9), for the given design conditions, such as wave period $T$, water depth $h$, and a draft depth of the front curtain wall $d$.

$$L_s/l_c = L_s/\sqrt{Bd} = 10$$

4. The water chamber type seawall experiment

Physical experiments have been conducted to validate the use of the newly introduced wave resonance parameter. Figure 1 shows the model of water chamber sea wall without a mound. The dimensions of the water chamber sea wall model is summarised in table 1.

| Table 1. Principal dimensions of the model. |
|---------------------------------------------|
| Model breadth | $(B)$ | 29 cm |
| Wall thickness of model | $(t)$ | 1 cm |
| Draft depth | $(d)$ | 20 cm |
| Gap height of water channel | $(e)$ | 16 cm |
| Mound water depth | $(hm)$ | 37 cm |
| Water depth | $(h)$ | 50 cm |
A water chamber type of sea wall has been examined in flume tank using four wave gauges to measure the wave height. The first gauge was set in front of the wave maker. The second and third are placed in front of the model to measure the incident and reflected waves. The fourth gauge installed inside the chamber to measure wave amplifications in the chamber. In the experiment, the range of wave period was from 0.8 to 3.0 s. For each wave period condition, two different wave height conditions, \( H = 6 \) cm and 12 cm are used. For comparatively short wave period conditions, only the wave condition of \( H = 6 \) cm was adopted because of wave breaking. Depth of water measured from the bottom of the wave flume is 50 cm and from above the mound is 37 cm. The water depth is kept constant through experiments.

Figure 12. Reflection coefficient of the water chamber as a function of \( L/B_s \).

Figure 13. Wave amplifications of the water chamber sea wall as a function of \( L/B_s \).

Figure 12 shows the experimental result of wave reflection \( Cr \) of the water chamber model of sea wall, where \( L \) is wave length corresponding to the water depth at the toe of a sloped mound and \( B \) is breadth of a single water chamber type sea wall. The wave height in the experiment was 6 cm and linear damping coefficient \( f_c = 0.5 \) was adopted after some computational trials.

It can be seen that the model structure plays a role of a low reflective sea wall especially under the condition of \( L/B_s \) about 10, i.e. \( Cr = 0.1 \). However, except for the minimum reflection condition, the reflection coefficient increases sharply, especially for shorter waves and longer waves. It shows that the effective range of wave period of the single water chamber type of dissipater is comparatively narrow and its shortcoming must be improved. The minimum condition as shown is related to the produced wave resonance, where the flow velocity is maximum under the vertical wall. After the flow separation occurs, big vortex is formed and energy is released. The wave amplification which appears in the chamber is related to the minimum wave reflection point, as shown in figure 13.

5. Conclusion

Considering the resonance condition of piston mode motion, a new parameter governing wave reflection characteristic is introduced. It has been shown that the new parameter is useful in determining the effective cross section of the dissipater, i.e. the draft of front curtain wall and width of water chamber for given wave conditions. Based on the experiment, it can be concluded that minimum \( Cr \) occurs when \( L/B_s \approx 10 \), where the minimum wave reflection point cause wave amplification.

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