Axially asymmetric fermion scattering off electroweak phase transition bubble walls with hypermagnetic fields

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We show that in the presence of large scale primordial hypermagnetic fields, it is possible to generate an axial asymmetry for a first order electroweak phase transition. This happens during the reflection and transmission of fermions off the true vacuum bubbles, due to the chiral nature of the fermion coupling with the background field in the symmetric phase. We derive and solve the Dirac equation for such fermions and compute the reflection and transmission coefficients for the case when these fermions move from the symmetric to the symmetry broken phase. We also comment on the possible implications of such axial charge segregation processes for baryon number generation.

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I. INTRODUCTION

One of the most challenging problems for particle physics as applied to cosmology is the explanation of the observed excess of baryons over antibaryons in the universe. For this purpose, a theory has to meet the three well-known Sakharov conditions [1], namely: (1) Existence of interactions that violate baryon number; (2) CP violation and (3) departure from thermal equilibrium. The above conditions are met in the standard model (SM) provided the electroweak phase transition (EWPT) is of first order. This has raised the interesting possibility that the cosmological phase transition that gave rise to the mass of particles, which took place at temperatures of order 100 GeV, could also explain the generation of baryon number. Consequently, a great deal of effort has been devoted to explore this possibility [2].

Nowadays, the consensus is that the minimal SM, as such, cannot explain the observed baryon number. The reason is that the EWPT turns out to be only too weakly first order which in turn implies that any baryon asymmetry generated at the phase transition was erased by the same mechanism that produced it, i.e., sphaleron induced processes [3]. Moreover, the amount of CP violation coming from the CKM matrix alone cannot account by itself for the observed asymmetry, given that its effect shows up in the coupling of the Higgs with fermions at a high perturbative order [4], producing a baryon to entropy ratio at least ten orders of magnitude smaller than the observed one.

Nevertheless, it has been recently pointed out that, provided a source of enough CP violation exists, the above scenario could significantly change in the presence of large-scale primordial magnetic fields [5–8] (see however Ref. [9]), which can be responsible for a stronger first-order EWPT. This situation is analogous to the case of a type I superconductor in which the presence of an external magnetic field modifies the order of the phase transition due to the Meissner effect. Though the nature of these fields is a subject of current research, their existence prior to the EWPT epoch cannot certainly be ruled out [8].

Magnetic fields have been observed in many astrophysical objects. Estimation of their strengths require independent knowledge of the local electron density and the spatial structure of the field. Both quantities are reasonably well known for our galaxy, where the average field strength has been measured to be between 3–4 $\mu$G; moreover, various spiral galaxies in our neighborhood present similar magnetic field strengths [10]. At larger scales, only model dependent upper limits can be established and these are also in the few $\mu$G range. Magnetic fields at the $\mu$G level have been observed as well in high-redshift objects. In the intergalactic medium, adopting some reasonable values for the magnetic coherence length, the upper bound of $10^{-6}$G has been estimated [11]. The origin of these fields is nowadays unknown but it is widely believed that, in order to produce them, two ingredients are needed: a mechanism for creating the seed fields and a process for amplifying both their amplitude and their coherence scale.

Generation of the seed field (magnetogenesis) may be either primordial or be produced during the process of structure formation. In the early universe, which is the case of interest here, there are a number of proposed mechanisms that could possibly generate large-scale primordial fields. Among the best suited are first order phase transitions [12,13], which provide favorable conditions such as charge separation, turbulence and departure from equilibrium. In particular, bubble wall collisions produce phase gradients of a complex order parameter that act as a source for gauge fields [13].
ested in larger coherence scales, a plausible scenario is inflation, where super-horizon scale fields are generated through the amplification of quantum fluctuations of the gauge fields. This process needs however a mechanism for breaking conformal invariance of the electromagnetic field [14].

The most promising way to distinguish between primordial and protogalactic fields is through the search of their imprint on the cosmic microwave background radiation (CMBR). Temperature anisotropies from COBE results place an upper bound \( B_0 \sim 10^{-9} \) G for homogeneous fields (\( B_0 \) refers to the intensity that the field would have today under the assumption of adiabatic decay due to the Hubble expansion) [13]. In the case of inhomogeneous fields their effect must be searched for in the Doppler peaks [10] and in the polarization of the CMBR [13]. The future CMBR satellite missions MAP and PLANCK may reach the required sensitivity for the detection of these last signals.

Independently of their origin, primordial fields could have had some influence on physical processes which occurred in the early universe, like big-bang nucleosynthesis and electroweak baryogenesis.

Recall that for temperatures above the EWPT, the SU(2)×U(1)\(_Y\) symmetry is restored and the propagating, non-screened vector modes that represent a magnetic field correspond to the U(1)\(_Y\) group instead of to the U(1)\(_{em}\) group, and are therefore properly called hypermagnetic fields.

In this paper we use a simple model to show that the presence of such fields also provides a mechanism, working in the same manner as the existence of additional CP violation within the SM, to produce an axial charge segregation during the EWPT. This happens in the scattering of fermions off the true vacuum bubbles nucleated during the phase transition and is a consequence of the chiral nature of the fermion coupling to hypermagnetic fields in the symmetric phase.

The outline of this work is as follows: In Sect. II, we write the Dirac equation for the left and right-handed chirality modes propagating in a background hypermagnetic field during the EWPT. In Sect. III, we find the solution and discuss its properties. In Sect. IV, we use this solution to compute reflection and transmission probabilities. We show that these probabilities differ for the two distinct chirality modes. Finally in Sect. V, we conclude by looking out at the possible implications of such axially asymmetric fermion reflection and transmission.

II. DIRAC EQUATION FOR FERMIONS MOVING IN A BACKGROUND HYPERMAGNETIC FIELD

In a first order phase transition, the conversion from one phase to another happens through nucleation. The region separating both phases is called the wall. During the EWPT, the properties of the wall depend on the effective, finite temperature Higgs potential. Under the assumption that the wall is thin and that the phase transition happens when the energy densities of both phases are degenerate, it is possible to find a one-dimensional analytical solution for the Higgs field \( \phi \) called the kink. This is given by

\[
\phi(z) \sim 1 + \tanh(z/\lambda),
\]

where \( z \) is the coordinate along the direction of the phase change and \( \lambda \) is the width of the wall. When scattering is not affected by diffusion, the problem of fermion reflection and transmission through the wall can be casted in terms of solving the Dirac equation with a position dependent fermion mass, proportional to the Higgs field [18]. Let us further simplify the problem by considering the limit when the width of the wall approaches zero. In this case, the kink solution becomes a step function, \( \Theta(z) \), and consequently, the expression for the particle’s mass becomes

\[
m(z) = m_0 \Theta(z).
\]

In terms of Eq. (3), we can see that \( z \leq 0 \) represents the region outside the bubble, that is the region in the symmetric phase where particles are massless. Conversely, for \( z \geq 0 \), the system is inside the bubble, that is in the broken phase and particles have acquired a finite mass \( m_0 \).

In the presence of an external magnetic field, we need to consider that fermion modes couple differently to the field in the broken and the symmetric phases. We start the analysis looking at the unbroken phase.

For \( z \leq 0 \), the coupling is chiral. Let

\[
\Psi_R = \frac{1}{2} (1 + \gamma_5) \Psi
\]

\[
\Psi_L = \frac{1}{2} (1 - \gamma_5) \Psi
\]

represent, as usual, the right and left-handed chirality modes for the spinor \( \Psi \), respectively. Then, the equations of motion for these modes, as derived from the electroweak interaction Lagrangian, are

\[
(i\partial - \frac{y_L g' A}{2}) \Psi_L - m(z) \Psi_R = 0
\]

\[
(i\partial - \frac{y_R g' A}{2}) \Psi_R - m(z) \Psi_L = 0,
\]

where \( y_{R,L} \) are the right and left-handed hypercharges corresponding to the given fermion, respectively, \( g' \) the \( U(1)_Y \) coupling constant and we take \( A^\mu = (0, A) \) representing a, not as yet specified, four-vector potential having non-zero components only for its spatial part, in the rest frame of the wall.

The set of Eqs. (4) can be written as a single equation for the spinor \( \Psi = \Psi_R + \Psi_L \) by adding up the former equations.
\[ \{ i \partial - \mathcal{A} \left[ \frac{y_R}{4} g' (1 + \gamma_5) + \frac{y_L}{4} g' (1 - \gamma_5) \right] - m(z) \} \Psi = 0. \] (5)

Hereafter, we explicitly work in the chiral representation of the gamma matrices where

\[ \gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}, \quad \gamma = \begin{pmatrix} 0 & \sigma \\ -\sigma & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}. \] (6)

Within this representation, we can write Eq. (5) as

\[ \left\{ i \partial - \mathcal{A}_\mu \gamma^\mu - m(z) \right\} \Psi = 0, \] (7)

where we have introduced the matrix

\[ \mathcal{G} = \begin{pmatrix} \frac{y_R}{2} g' I & 0 \\ 0 & \frac{y_L}{2} g' I \end{pmatrix}. \] (8)

We now look at the corresponding equation in the broken-symmetry phase. For \( z \geq 0 \) the coupling of the fermion with the external field is through the electric charge \( e \) and thus, the equation of motion is simply the Dirac equation describing an electrically charged fermion in a background magnetic field, namely,

\[ \left\{ i \partial - eA_\mu \gamma^\mu - m(z) \right\} \Psi = 0. \] (9)

In the following section, we explicitly construct the solutions to Eqs. (7) and (9) with a constant magnetic field, requiring that these match at the interface \( z = 0 \).

### III. SOLVING THE DIRAC EQUATION

Let us first find the solution to Eq. (9), namely, for fermions moving in the symmetric phase, \( z \leq 0 \). For this purpose, we look for a solution of the form

\[ \Psi = \left\{ i \partial - A_\mu \gamma^\mu \mathcal{G} + m(z) \right\} \Phi. \] (10)

Inserting this expression into Eq. (9), we obtain

\[ \left\{ - \partial^2 - i \mathcal{G} \partial \mu A_\mu - \frac{1}{2} \sigma^{\mu\nu} \mathcal{G} F_{\nu\mu} - 2i \mathcal{G} A_\mu \gamma^\mu + \mathcal{G}^2 A_\mu A^\mu + i \gamma^\mu \partial_\mu m(z) \right\} \Phi = 0, \] (11)

where, as usual,

\[ \sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu] \]

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \] (12)

For definiteness, let us consider a constant magnetic field \( \mathbf{B} = B \hat{z} \) pointing along the \( \hat{z} \) direction. In this case, the vector potential \( \mathbf{A} \) can only have components perpendicular to \( \hat{z} \) and the solution to Eq. (11) factorizes as [13]

\[ \Phi(t, \mathbf{x}) = \zeta(x, y) \Phi(t, z). \] (13)

We concentrate on the solution describing the motion of fermions perpendicular to the wall, i.e., along the \( \hat{z} \) axis and, furthermore, look for stationary states, namely

\[ \Phi(t, z) = e^{-iEt} \Phi(z). \] (14)

Therefore, working in the Lorentz gauge, \( \partial^\mu A_\mu = 0 \), Eq. (11) becomes

\[ \left\{ \frac{d^2}{dz^2} + i\gamma^5 \frac{dm(z)}{dz} + E^2 + iB \gamma^1 \gamma^5 \right\} \Phi(z) = 0. \] (15)

Notice that Eqs. (11) and (13) have the appropriate limit when \( y_R = y_L = e \), corresponding to the description of fermions coupled with their electric charge to a background magnetic field [19].

We now expand \( \Phi(z) \) in terms of the eigen-spinors \( u^s_\pm \) (\( s = 1, 2 \)) of \( \gamma^5 \),

\[ u^1_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad u^2_+ = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \] (16)

These spinors have the properties

\[ \gamma^5 u^1_+ = \pm i u^1_+, \quad \gamma^5 u^2_+ = \pm i u^2_+. \] (17)

Writing

\[ \Phi(z) = \phi^+_1(z) u^1_+ + \phi^+_2(z) u^2_+ + \phi^-_1(z) u^1_- + \phi^-_2(z) u^2_- \] (18)

and inserting this expression into Eq. (13), we obtain

\[ \left[ \frac{d^2}{dz^2} + E^2 + g \frac{(y_L + y_R)}{4} B \right] \phi^+_1(z) + g \frac{(y_L - y_R)}{4} B \phi^+_2(z) = \right. \]

\[ m_0 \delta(z) \phi^+_1(z) \]

\[ \left[ \frac{d^2}{dz^2} + E^2 + g \frac{(y_L + y_R)}{4} B \right] \phi^-_1(z) + g \frac{(y_L - y_R)}{4} B \phi^-_2(z) = \right. \]

\[ -m_0 \delta(z) \phi^-_1(z) \] (19)

and

\[ \left[ \frac{d^2}{dz^2} + E^2 - g \frac{(y_L + y_R)}{4} B \right] \phi^+_2(z) - \]
where we use the notation
\[
\alpha_{1}^{R,L} = \sqrt{E^2 + \frac{y_{R,L}g'}{2}B}.
\]
(23)

It is a straightforward exercise to verify that the functions \(\phi_{\pm}^{1(a,b)}(z)\) given by Eqs. (21) and (22) indeed satisfy the system of Eqs. (19).

The corresponding fermion wave functions are given in terms of Eq. (3). Taking \(E > 0\) and in the approximation where we look only at the part of the wave function that describes motion perpendicular to the wall, we obtain, for solutions type (a)
\[
\Psi_{\text{inc}}^{(a)}(z) = -i(\alpha_{1}^{L} - E)(u_{+}^{1} - u_{-}^{1})e^{i\alpha_{1}^{L}z} - i(\alpha_{2}^{L} + E)(u_{+}^{2} - u_{-}^{2})e^{i\alpha_{2}^{L}z}.
\]
(20)

\[
\Psi_{\text{ref}}^{(a)}(z) = \frac{-im_{0}}{4\alpha_{1}^{L}\alpha_{1}^{R} + m_{0}^{2}}\left\{ m_{0}(\alpha_{1}^{L} + E)(u_{+}^{1} - u_{-}^{1})e^{-i\alpha_{1}^{L}z} + 2i\alpha_{1}^{L}(\alpha_{1}^{R} - E)(u_{+}^{1} + u_{-}^{1})e^{-i\alpha_{1}^{R}z} \right\} - \frac{-im_{0}}{4\alpha_{2}^{L}\alpha_{2}^{R} + m_{0}^{2}}\left\{ m_{0}(\alpha_{2}^{L} - E)(u_{+}^{2} - u_{-}^{2})e^{-i\alpha_{2}^{L}z} + 2i\alpha_{2}^{L}(\alpha_{2}^{R} + E)(u_{+}^{2} + u_{-}^{2})e^{-i\alpha_{2}^{R}z} \right\},
\]
whereas for solutions type (b)
\[
\Psi_{\text{inc}}^{(b)}(z) = -i(\alpha_{1}^{R} + E)(u_{+}^{1} + u_{-}^{1})e^{i\alpha_{1}^{R}z} - i(\alpha_{2}^{R} - E)(u_{+}^{2} + u_{-}^{2})e^{i\alpha_{2}^{R}z}.
\]
(21)

\[
\Psi_{\text{ref}}^{(b)}(z) = \frac{-im_{0}}{4\alpha_{1}^{L}\alpha_{1}^{R} + m_{0}^{2}}\left\{ m_{0}(\alpha_{1}^{R} - E)(u_{+}^{1} + u_{-}^{1})e^{-i\alpha_{1}^{L}z} + 2i\alpha_{1}^{R}(\alpha_{1}^{L} + E)(u_{+}^{1} - u_{-}^{1})e^{-i\alpha_{1}^{R}z} \right\} - \frac{-im_{0}}{4\alpha_{2}^{L}\alpha_{2}^{R} + m_{0}^{2}}\left\{ m_{0}(\alpha_{2}^{R} + E)(u_{+}^{2} + u_{-}^{2})e^{-i\alpha_{2}^{L}z} + 2i\alpha_{2}^{R}(\alpha_{2}^{L} - E)(u_{+}^{2} - u_{-}^{2})e^{-i\alpha_{2}^{R}z} \right\},
\]
where, in analogy with Eq. (23), we define
\[
\alpha_{2}^{R,L} = \sqrt{E^2 - \frac{y_{R,L}g'}{2}B}.
\]
(22)

We now turn to finding the solution to Eq. (3), namely, for fermions moving in the broken phase, \(z \geq 0\). This time, we look for a solution of the form
\[
\Psi = \left\{ i\partial_{z} - eA_{\mu}\gamma_{\mu} + m(z) \right\}\Phi.
\]
(27)

By a procedure similar to that leading to Eqs. (19) and (20), the corresponding equations for the functions \(\phi_{\pm}^{1,2}(z)\) in this region become
\[
\left[ \frac{d^2}{dz^2} + E^2 - m_0^2 + eB \right] \phi_\pm(z) = \pm m_0 \delta(z) \phi_\pm(z)
\]
(24)

\[
\left[ \frac{d^2}{dz^2} + E^2 - m_0^2 - eB \right] \phi_\mp(z) = \pm m_0 \delta(z) \phi_\pm(z).
\]
(25)

As expected, when the coupling of the fermion with the external magnetic field is through its electric charge, the equations describing the behavior of the functions \(\phi_{\pm}^{1,2}(z)\) decouple. For our purposes, we look for the scattering states appropriate for the description of transmitted waves. These are
\[ \phi_{\pm}^{1,2}(z) = e^{i\alpha_1 z} \pm \frac{im_0}{2\alpha_{1,2} \pm im_0} e^{i\alpha_{1,2} |z|}, \]  

(29)

where we use the notation

\[ \alpha_1 = \sqrt{E^2 - m_0^2 + eB} \]
\[ \alpha_2 = \sqrt{E^2 - m_0^2 - eB}. \]  

(30)

It is also a straightforward exercise to verify that Eq. (24) indeed satisfies the set of Eqs. (28). The fermion wave function is obtained from Eq. (27). Also, for \( E > 0 \) and in the approximation where we look only at the part describing the motion of particles along \( \hat{z} \) and furthermore, imposing continuity of the fermion wave function at \( z = 0 \), we obtain for solutions type (a)

\[ \Psi_{\text{tra}}^{(a)}(z) = \frac{2\alpha_1^L}{4\alpha_1^L \alpha_1^R + m_0^2} \left\{ m_0(\alpha_1^R - E)(u^1_+ + u^1_-) - i \left[ 2\alpha_1^L(\alpha_1^L - E) + m_0^2 \right](u^1_+ - u^1_-) \right\} e^{i\alpha_1 z} \]
\[ + \frac{2\alpha_1^L}{4\alpha_2^L \alpha_2^R + m_0^2} \left\{ m_0(\alpha_2^R + E)(u^2_+ + u^2_-) - i \left[ 2\alpha_2^L(\alpha_2^L + E) + m_0^2 \right](u^2_+ - u^2_-) \right\} e^{i\alpha_2 z}, \]  

(31)

and for solutions type (b)

\[ \Psi_{\text{tra}}^{(b)}(z) = \frac{2\alpha_1^R}{4\alpha_1^R \alpha_1^L + m_0^2} \left\{ m_0(\alpha_1^L + E)(u^1_+ + u^1_-) - i \left[ 2\alpha_1^R(\alpha_1^R + E) + m_0^2 \right](u^1_+ - u^1_-) \right\} e^{i\alpha_1 z} \]
\[ + \frac{2\alpha_2^R}{4\alpha_2^R \alpha_2^L + m_0^2} \left\{ m_0(\alpha_2^L - E)(u^2_+ + u^2_-) - i \left[ 2\alpha_2^L(\alpha_2^L - E) + m_0^2 \right](u^2_+ - u^2_-) \right\} e^{i\alpha_2 z}. \]  

(32)

Recall that in the absence of the hypermagnetic field, the eigenvalues of the chirality and the helicity operators, \( (\chi \text{ and } h, \text{ respectively}) \) are the same. The presence of the external field lifts such degeneracy and the eigenstates of chirality no longer have a definite helicity. Nevertheless, it is easy to check that for field strengths \( eB \) smaller than \( m_0^2 \), the component with \( h \) that would correspond to a given \( \chi \) in the absence of the external field, dominates over the rest of the components. For \( E > 0 \), this means that, to a good approximation, left (right)-handed particles are transmitted as such (both in chirality and helicity) but become right (left)-handed (both in chirality and helicity) upon reflection. In these cases and to a good approximation, the quantum number conserved during scattering off the wall is the ratio \( \chi / h = 1 \). It can also be shown [24] that to a good approximation, for \( E < 0 \), the corresponding conserved quantum number is \( \chi / h = -1 \).

IV. REFLECTION AND TRANSMISSION PROBABILITIES

The fact that the amplitudes in Eqs. (25) and (32) are not the same as those in Eqs. (24) and (31), means that an axial asymmetry is built during the scattering of fermions off the wall. To quantify the asymmetry, we need to compute the corresponding reflection and transmission coefficients. These are built from the reflected, transmitted and incident currents of each type. Recall that for a given spinor wave function \( \Psi \), the current normal to the wall is given by

\[ J = \Psi_{\text{inc}}^{1,0,3} \]  

(33)

As can be seen from Eqs. (25), (32) and (24), (31), an incident wave with a given chirality (left-handed for waves type (a), right-handed for waves type (b), contains, upon reflection and transmission, both kinds of chirality modes. For waves type (a), the corresponding currents are

\[ J_{\text{inc}}^{(a)} = 4 \left\{ (\alpha_2^L + E)^2 - (\alpha_1^L - E)^2 \right\} \]  

(34)

\[ J_{\text{ref}}^{(a)} = J_{\text{tra}}^{(a)R} + J_{\text{tra}}^{(a)L} \]

\[ J_{\text{ref}}^{(a)L} = -4m_0^2 \left\{ \frac{m_0}{4\alpha_1^L \alpha_1^R + m_0^2} (\alpha_1^L + E)^2 - \frac{2\alpha_1^L}{4\alpha_1^L \alpha_1^R + m_0^2} (\alpha_1^R - E)^2 \right\} \]

\[ J_{\text{tra}}^{(a)L} = 16 \left\{ \frac{m_0^2}{4\alpha_1^L \alpha_1^R + m_0^2} (\alpha_1^L - E)^2 + \frac{2\alpha_1^L}{4\alpha_1^L \alpha_1^R + m_0^2} (\alpha_1^L + E)^2 \right\}. \]  

(34)

whereas for waves of type (b), the corresponding currents are
where

\[
J^{(b)R}_{\text{ref}} = -4m_0^2 \left\{ \left( \frac{m_0}{4\alpha_1^2\alpha_2^2 + m_0^2} \right)^2 (\alpha_2^R + E)^2 - \left( \frac{2\alpha_1^R}{4\alpha_1^2\alpha_2^2 + m_0^2} \right)^2 (\alpha_2^L - E)^2 \right\}
\]

\[
J^{(b)R}_{\text{tra}} = -4m_0^2 \left\{ \left( \frac{2\alpha_1^R}{4\alpha_1^2\alpha_2^2 + m_0^2} \right)^2 (\alpha_2^L + E)^2 - \left( \frac{2\alpha_2^R}{4\alpha_1^2\alpha_2^2 + m_0^2} \right)^2 (\alpha_2^L - E)^2 \right\}
\]

and

\[
J^{(b)R}_{\text{inc}} = \frac{4}{E} \left\{ (\alpha_1^R + E)^2 - (\alpha_2^L - E)^2 \right\}
\]

Therefore, the probabilities for finding a left or a right-handed particle in the symmetric phase after reflection, \( PR_L, PR_R \) are given, respectively by

\[
PR_L = R_{L\rightarrow L} + R_{R\rightarrow L}
\]
\[
PR_R = R_{L\rightarrow R} + R_{R\rightarrow R},
\]

whereas the probabilities for finding a left or a right-handed particle in the symmetry broken phase after transmission, \( PT_L, PT_R \) are given, respectively by

\[
PT_L = T_{L\rightarrow L} + T_{R\rightarrow L}
\]
\[
PT_R = T_{L\rightarrow R} + T_{R\rightarrow R}.
\]

Equations (36) represent the probabilities that a left-handed incident particle bounces off the wall as a left or a right-handed particle or is transmitted through the wall as a left or a right-handed particle, respectively. The corresponding probabilities for the axially conjugate processes are

\[
R_{L\rightarrow L} = -J^{(a)L}_{\text{ref}} / J^{(a)\text{inc}}
\]
\[
R_{L\rightarrow R} = -J^{(a)R}_{\text{ref}} / J^{(a)\text{inc}}
\]
\[
T_{L\rightarrow L} = J^{(a)L}_{\text{tra}} / J^{(a)\text{inc}}
\]
\[
T_{L\rightarrow R} = J^{(a)R}_{\text{tra}} / J^{(a)\text{inc}}.
\]

The reflection and transmission coefficients are given as the ratios of the reflected and transmitted currents, to the incident one, respectively, projected along a unit vector normal to the wall,

\[
J^{(b)R}_{\text{ref}} = -4m_0^2 \left\{ \left( \frac{m_0}{4\alpha_1^2\alpha_2^2 + m_0^2} \right)^2 (\alpha_2^R + E)^2 - \left( \frac{2\alpha_1^R}{4\alpha_1^2\alpha_2^2 + m_0^2} \right)^2 (\alpha_2^L - E)^2 \right\}
\]

\[
J^{(b)R}_{\text{tra}} = -4m_0^2 \left\{ \left( \frac{2\alpha_1^R}{4\alpha_1^2\alpha_2^2 + m_0^2} \right)^2 (\alpha_2^L + E)^2 - \left( \frac{2\alpha_2^R}{4\alpha_1^2\alpha_2^2 + m_0^2} \right)^2 (\alpha_2^L - E)^2 \right\}
\]

Equations (36) represent the probabilities that a left-handed incident particle bounces off the wall as a left or a right-handed particle or is transmitted through the wall as a left or a right-handed particle, respectively. The corresponding probabilities for the axially conjugate processes are

\[
R_{L\rightarrow L} = -J^{(a)L}_{\text{ref}} / J^{(a)\text{inc}}
\]
\[
R_{L\rightarrow R} = -J^{(a)R}_{\text{ref}} / J^{(a)\text{inc}}
\]
\[
T_{L\rightarrow L} = J^{(a)L}_{\text{tra}} / J^{(a)\text{inc}}
\]
\[
T_{L\rightarrow R} = J^{(a)R}_{\text{tra}} / J^{(a)\text{inc}}.
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\[
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\]
\[
R_{L\rightarrow R} = -J^{(a)R}_{\text{ref}} / J^{(a)\text{inc}}
\]
\[
T_{L\rightarrow L} = J^{(a)L}_{\text{tra}} / J^{(a)\text{inc}}
\]
\[
T_{L\rightarrow R} = J^{(a)R}_{\text{tra}} / J^{(a)\text{inc}}.
\]
parameters $\alpha_{1,2}$ in Eqs. (30) are real, which in turn implies that $E \geq \sqrt{m_0^2 + c B}$. It can be numerically checked that $P R_L + P T_L = P R_R + P T_R = 1$ to within a maximum deviation of one part in one thousand. The fact that these probabilities add up to one is equivalent to current conservation

$$J^{(i)}_{\text{tra}} - J^{(i)}_{\text{ref}} = J^{(i)}_{\text{inc}},$$  \hfill (40)

($i = a, b$), as a consequence of the equality of the currents

$$J^{(a)R}_{\text{tra}} = J^{(a)R}_{\text{ref}},$$

$$J^{(b)L}_{\text{tra}} = J^{(b)L}_{\text{ref}},$$  \hfill (41)

as can be checked from the sets of Eqs. (34) and (35).

V. CONCLUSIONS AND OUTLOOK

In this paper we have derived and solved the Dirac equation for fermions scattering off a first order EWPT bubble wall in the presence of a magnetic field directed along the fermion direction of motion. In the symmetric phase, the fermions couple chirally to the magnetic field, which receives the name of hypermagnetic, given that it belongs to the $U(1)_Y$ group. We have shown that the chiral nature of this coupling implies that it is possible to build an axial asymmetry during the scattering of fermions off the wall. We have computed reflection and transmission coefficients showing explicitly that they differ for left and right-handed incident particles from the symmetric phase.

Recall that under the very general assumptions of CPT invariance, together with conservation of unitarity, which are satisfied in the present analysis, the total axial asymmetry (which includes contributions both from particles and antiparticles) is quantified in terms of the particle (axial) asymmetry. Let $\rho_i$ represent the number density for species $i$. The net densities in left-handed and right-handed axial charges are obtained by taking the differences $\rho_L - \rho_L$ and $\rho_R - \rho_R$, respectively. It is straightforward to show that CPT invariance and unitarity imply that the above net densities are given by

$$\rho_L - \rho_L = (f^s - f^b)(P R_L - P R_R),$$

$$\rho_R - \rho_R = (f^s - f^b)(P R_R - P L_L),$$  \hfill (42)

where $f^s$ and $f^b$ are the statistical distributions for particles or antiparticles (since the chemical potentials are assumed to be zero or small compared to the temperature, these distributions are the same for particles or antiparticles) in the symmetric and the symmetry-broken phases, respectively. From Eq. (42), the asymmetry in the axial charge density is finally given by

$$(\rho_L - \rho_L) - (\rho_R - \rho_R) = 2(f^s - f^b)(P R_L - P R_R).$$  \hfill (43)

This asymmetry in the axial charge, built on either side of the wall, is dissociated from non-conserving baryon number processes and can subsequently be converted to baryon number in the unbroken phase where sphaleron induced transitions are taking place with a large rate. This mechanism receives the name of non-local baryogenesis [21] and, in the absence of the external field, it can only be realized in extensions of the SM where a source of CP violation is introduced ad hoc into a complex, space-dependent phase of the Higgs field during the development of the EWPT [24].

![FIG. 2. Reflection and transmission probabilities as a function of the particle’s energy $E$. Figure 2a (upper panel) shows the probabilities $P R_L$ and $P T_L$. Figure 2b (lower panel) shows the probabilities $P R_R$ and $P T_R$. In both cases, the strength of the magnetic field is taken with $b = 1$ and $T = 100$ GeV. Also $m_0 = 175$ GeV, $y_R = 4/3$, $y_L = 1/3$, as corresponds to a top quark.](image)
of the SM, the mechanism advocated in this work has to be considered as acting in the same manner as a source of CP violation that can have important consequences for the generation of a baryon number. These matters will be the subject of an upcoming work [20].

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