Pair production and ionizing radiation from superconductors

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We show that an alternative theory of superconductivity recently proposed (theory of hole superconductivity) leads to the surprising consequence that real electron-positron pair production will occur for superconductors larger than a critical size. High frequency radiation with frequencies up to 0.511 MeV/h is predicted to be emitted from superconductors out of equilibrium. Attention to the possibility of harmful consequences is called for.

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I. INTRODUCTION

The theory of hole superconductivity proposes that charge asymmetry is at the root of the phenomenon of superconductivity\(^{3}\), and that this charge asymmetry manifests itself at a macroscopic level in the fact that superconductors expel negative charge from their interior towards the surface\(^{4}\). No definitive experimental evidence for or against this prediction exists so far, however the otherwise unexplained ‘Tao effect’\(^{3}\) has been argued to constitute strong evidence in its favor\(^{4}\). Because the theory of hole superconductivity is at odds with the generally accepted BCS-London theory of conventional superconductivity\(^{5}\), it is important to find unambiguous experimental evidence for or against it.

At a qualitative level the theory predicts that superconducting bodies look like ‘giant atoms’\(^{6}\), with excess negative charge near the surface and excess positive charge in the interior. The fact that the size of these giant atoms is at the experimentalist’s disposal then would appear to allow for a remarkably simple check of the theory. Namely, it has been predicted that for superheavy atoms or molecules spontaneous electron-positron pair production will occur\(^{7}\), when the binding energy of a K-shell electron becomes equal to twice its rest mass. Hence we are led to the surprising conclusion that pair production should also occur for superconductors of sufficiently large size.

Furthermore, the theory predicts that spin currents exist in the ground state of superconductors\(^{3}\), with some electrons moving at speeds approaching the speed of light in macroscopic samples\(^{4}\). These electrons should give rise to high frequency radiation when the superconducting state is destroyed and the spin current stops. In addition, annihilation of electron-positron pairs under suitable non-equilibrium conditions should give rise to 0.511 MeV γ-ray emission, that should be experimentally detectable.

We call attention to the fact that if the theory is indeed correct, this ionizing radiation could constitute a dangerous health hazard for humans in the vicinity. Thus the determination of whether the theory is correct or not acquires an urgency that goes beyond the academic interest of deciding between competing theoretical viewpoints, since with increasing use of superconductors in society this unexpected effect could have potentially harmful consequences.

II. PAIR PRODUCTION IN SUPERHEAVY ATOMS

Shortly after the introduction of Dirac theory it was pointed out by Saunt\(^{11}\) that in a sufficiently strong electric field electron-positron pair creation from the vacuum should take place. For an atom, an electron in the field of a point nucleus of charge \(Z|e|\) has binding energy

\[
E_b = -13.6eV Z^2
\]

neglecting relativistic effects. It is generally believed that when the binding energy becomes degenerate with the Dirac continuum of negative energy states, i.e. \(E_b = -m_e c^2\), spontaneous pair production will occur\(^9\) \((m_e =\text{electron mass})\). From Eq. (1), the condition for this to occur is \(Z > 274\). In fact, when relativity is taken into account the 1s state for a point nucleus becomes unstable already for \(Z > Z_c = 137\) in Dirac theory. This can also be simply seen from a Bohr atom model\(^{11}\), using the relativistic equations

\[
pv = \frac{Ze^2}{r}
\]

\[
p = \gamma m_e v
\]

\((\gamma = 1/\sqrt{1 - v^2/c^2})\) which together with angular momentum quantization \(pr = \hbar\) lead to the condition

\[
\frac{pc}{\sqrt{p^2c^2 + m_e^2c^4}} = \frac{Ze^2}{\hbar c}
\]

so that for \(Z = Z_c = \hbar c/e^2 = 137\), \(p \to \infty\) and \(r \to 0\). Solution of the Dirac equation for a nucleus of finite size shows that at a critical \(Z_c = 172\) spontaneous autoionization will occur for an empty shell\(^{9}\): an electron-positron pair pops out of the Dirac sea, the electron will occupy
FIG. 1: Schematic picture of charge distribution in a spherical superconductor of radius smaller than the critical radius. Excess negative charge exists within a London penetration depth of the surface, and excess positive charge in the interior.

FIG. 2: Schematic picture of charge distribution in a spherical superconductor of radius larger than the critical radius. An intermediate region between the inner positively charged region and outer negatively charged layer exists, where electron-positron pairs pop out of the Dirac sea.

the lowest orbit and the positron is emitted. Experimentally it has been attempted to reach the supercritical region $Z > Z_c$ by colliding two heavy nuclei, however results to date have not been conclusive.[9]

III. CRITICAL RADIUS OF SUPERCONDUCTORS

In a similar vein we argue that in superconductors spontaneous pair production will become possible for superconductors larger than a critical size, within the theory of hole superconductivity. As discussed in [2], the theory predicts that electrons carrying a total negative charge

$$ q = 4\pi R^2 \lambda_L \rho_- \quad (4a) $$

get expelled from the interior of the superconductor towards the surface, giving rise to a negative charge density $\rho_-$ within a London penetration depth $\lambda_L$ of the surface (for simplicity we assume a sample of spherical shape of radius $R$), as shown schematically in Figure 1. The charge density $\rho_-$ is given by[2]

$$ \rho_- = n_s \left( \frac{10}{3} \frac{e}{m_e c^2} \right)^{1/2} \quad (4b) $$

with $n_s$ the superfluid density, $e$ the negative electron charge, and $\epsilon$ the condensation energy per electron. As the size of the sample becomes larger the amount of expelled charge increases, and we argue that when the potential energy of an electron at the edge of the positive charge distribution becomes larger in magnitude than twice the electron rest energy

$$ \frac{qe}{r} = 2m_e c^2 \quad (5) $$

the condition of dynamic equilibrium for an electron orbiting at radius $r$ in the field of a positive charge ($-q$)

$$ \frac{m_e v^2}{r} = \frac{qe}{r^2} \quad (7) $$

yields that the speed $v$ approaches the speed of light $c$ for $qe/r = m_e c^2$, similar to Eq. (5). Of course the condition Eq. (7) ceases to be valid for relativistic speeds and is replaced by Eq. (2) with $q = Ze$, which yields $v/c = 0.91$ for $q$ given by Eq. (5).

As an example we consider Niobium. It was estimated in Ref. [2] that the maximum electric field near the surface is $E_{max} \sim 0.77 \times 10^8$ V/cm, which already indicates that for samples of size of order $cm$ the electron electrostatic energy becomes of the order of the electron rest mass. The London penetration depth for Nb is $\lambda_L = 400 \mu m$ and the thermodynamic critical field $H_c = 1980$ G, from which we find condensation energy per unit volume $\tilde{\epsilon} = 1.56 \times 10^5$ ergs/cm$^3$, $\epsilon = 5.54 \mu eV$, and from Eq. (6)

$$ R_c = 3.33 \times 10^5 \lambda_L \quad (8) $$

hence $R_c = 1.33 cm$. 

$$ R_c \sim 3.33 \times 10^5 \lambda_L $$

IV. DYNAMICAL EQUILIBRIUM IN APPLIED MAGNETIC FIELD

In the previous section we argued that for sufficiently large superconductors the electric field resulting from negative charge expulsion will lead to pair production. Here we show that this expectation is consistent with the Meissner effect and the requirement of dynamical equilibrium for superfluid electrons.

We assume the validity of London’s equation

\[ \vec{\nabla} \times \vec{J} = -\frac{c}{4\pi\lambda_L^2} \vec{B} \]  

(9)

for the screening charge current \( \vec{J} \) in the presence of an applied magnetic field \( \vec{B} \), with \( \vec{J} \) given by

\[ \vec{J} = \rho \vec{v}_\phi \]  

(10)

Here, \( \rho \) is the superfluid transport charge density and \( \vec{v}_\phi \) its azimuthal velocity induced by the applied magnetic field. In conventional London theory it is assumed that \( \rho = e n_s \), with \( n_s \) the total superfluid charge density, independent of the volume of the sample, however we show here that this is untenable in the present context.

In the presence of the electric field \( \vec{E} \) resulting from charge expulsion (Fig. 1), the expelled electrons near the surface will carry a spin current even in the absence of an applied magnetic field, to satisfy dynamical equilibrium, with

\[ \frac{m_e v_0^2}{r} = |e|E \]  

(11)

for electrons at radius \( r \) (neglecting relativistic corrections). Electrons of opposite spin orbit with opposite velocities of equal magnitude \( v_0 \). When a magnetic field is applied, velocities of spin up and down electrons change according to

\[ \vec{v}_\uparrow = \vec{v}_0 + \vec{v}_\phi \]  

(12a)

\[ \vec{v}_\downarrow = -\vec{v}_0 + \vec{v}_\phi \]  

(12b)

and in particular for equatorial orbits

\[ v_\sigma = \sigma v_0 + v_\phi \]  

(12c)

so that the requirement of dynamical equilibrium for equatorial orbits is

\[ \frac{m_e v_\sigma^2}{r} = |e|E + \frac{|e|c}{c} v_\sigma B \]  

(13)

which implies

\[ v_\phi = -\frac{e}{m_e c} \frac{B r}{2} \]  

(14a)

The azimuthal velocity \( v_\phi \) induced by the magnetic field Eq. (14a) is much larger than what would result if \( \rho = e n_s \) in Eq. (10), namely

\[ v_\phi \sim -\frac{e}{m_e c} B \lambda_L. \]  

(14b)

From Eq. (9), since \( B \) is non-zero only in a region of thickness \( \lambda_L \) near the surface,

\[ J = -\frac{c}{4\pi\lambda_L} B \]  

(15)

and from Eqs. (10), (14a) and (15)

\[ \rho = \frac{m_e c^2}{2\pi\lambda_L e r} \]  

(16)

hence the transport charge density decreases inversely with the radius \( r \). Using for the London penetration depth

\[ \frac{1}{\lambda_L^2} = \frac{4\pi n_s e^2}{m_e c^2} \]  

(17)

Eq. (16) becomes

\[ \rho = 2e n_s \frac{\lambda_L}{r} \]  

(18)

which shows that the transport charge density approaches the conventional value \( en_s \) for small samples, but is much smaller than the conventional value for samples much larger than the London penetration depth.

Now it is reasonable to assume that for macroscopic samples the transport charge density cannot be smaller than the expelled charge density \( \rho_- \). From Eqs. (4a) and (16) we find, setting \( \rho = \rho_- \) and \( r = R \)

\[ \frac{qe}{R} = 2m_e c^2 \]  

(19)

i.e. the same condition as Eq. (5). We conclude that at the critical radius given by Eq. (6) the transport charge becomes equal to the excess expelled negative charge Eq. (4).

V. INTERPRETATION OF THE MEISSNER EFFECT

Here we argue that the conclusion reached in the previous section that the transport charge in macroscopic samples is only the excess charge \( \rho_- \) rather than the full superfluid charge density \( e n_s \) also leads to an understanding of the Meissner effect. Indeed, consider cooling a superconductor in the presence of an applied magnetic field \( \vec{B} \). The charge that is expelled from the interior towards the surface experiences a Lorentz force due to the magnetic field

\[ \frac{d\vec{v}}{dt} = \frac{e}{m_e c} \vec{v} \times \vec{B} \]  

(20)
and the azimuthal velocity builds up as electrons move radially outward due to this force. Using $\vec{v} = d\vec{r}/dt$ we find on integrating Eq. (20)

$$\vec{v}_\phi (t = \infty) = \frac{e}{m_e c} [\vec{r}(t = \infty) - \vec{r}(t = 0)] \times \vec{B}$$

(21)

The charge expulsion process results in a total excess negative charge

$$q = 4\pi R^2 \lambda_L \rho_-$$(22)

residing in the layer of thickness $\lambda_L$ at the surface. This negative charge moved outwards upon cooling from above to below $T_c$, from an initial spherical volume of radius ($R - \lambda_L$) to the spherical shell of inner radius ($R - \lambda_L$) and outer radius $R$, as depicted in Figure 3. As shown in Fig. 3, an electron initially near the center of the sphere (denoted as 1) moves from radius $r(t = 0) \sim 0$ to radius $r(t = \infty) \sim R - \lambda_L$, and in so doing acquires an azimuthal speed

$$v_{1\phi} \sim -\frac{e}{m_e c} B R$$

(23a)

together with Eqs. (16), (27) and (28) and demanding that the resulting magnetic field $B$ be the London field Eq. (24) leads to

$$\delta v_\phi \sim \omega R \sin \theta$$

(30)

VI. INTERPRETATION OF THE LONDON FIELD OF ROTATING SUPERCONDUCTORS

In superconductors rotating with angular velocity $\vec{\omega}$ a uniform magnetic field

$$\vec{B} = -\frac{2m_e c}{e} \vec{\omega}$$

(24)

exists in the interior (London field) [13, 14, 15]. This is conventionally interpreted as arising from a 'lagging' of the superfluid rotation within a London penetration depth of the surface: superfluid electrons rotate slower than the rigid rotation speed $v_\phi = \omega R \sin \theta$ by an amount [14]

$$\delta v_\phi = -\omega R \sin \theta$$

(25)

and the azimuthal current density is of order

$$J_\phi \sim e n_s \delta v_\phi \sim e n_s \omega \lambda_L$$

(26)

The total current $I$ due to the superfluid electrons in a surface shell of thickness $\lambda_L$ is of order

$$I \sim 4\pi R^2 \lambda_L J_\phi$$

(27)

and the magnetic field due to a ring current of radius $R$ is of order

$$B \sim \frac{2I}{cR}$$

(28)

Replacing Eq. (27) in (28) and using Eq. (17) for the London penetration depth, the magnetic field Eq. (24) results.

The problem with this argument is that it does not provide a rationale for why the superfluid electrons near the surface would suddenly 'lag behind' when a rotating normal metal is cooled into the superconducting state. Instead, our alternative description does.

In our scenario the charge that 'lags behind' is the transport charge density Eq. (16), which for macroscopic samples is the same as the excess charge density $\rho_-$ residing within a London penetration depth of the surface.

Using

$$J_\phi = \rho_+ \delta v_\phi$$

(29)

together with Eqs. (16), (27) and (28) and demanding that the resulting magnetic field $B$ be the London field Eq. (24) leads to

$$\delta v_\phi \sim \frac{\omega R}{2} \sin \theta$$

(30)
If the transport charge density was expelled from the interior of the superconductor at an average radius $R_c/2$, it will lag the azimuthal velocity at the surface by precisely the amount $Eq. (30)$. Hence our point of view provides a simple rationale for how the London field develops when a rotating normal metal is cooled into the superconducting state: the electrons near the surface that suddenly ‘lag behind’ are electrons originating deep in the interior of the superconductor where much smaller rigid rotation speeds prevail.

VII. SAMPLES LARGER THAN CRITICAL: THE INTERMEDIATE LAYER

Consider now a sample of radius $R$ that is larger than the critical radius $R_c$ given by $Eq. (6)$. Let $q_1$ be the charge expelled from the inner region, that satisfies

$$\frac{q_1e}{R_c} = 2m_e c^2,$$  \hspace{1cm} (31)

and let $(q_1 + q_2)$ be the total negative charge in the outer layer of thickness $\lambda_L$ that is responsible for the charge transport. According to the condition of dynamical equilibrium derived in the previous section $Eq. (19)$ we have

$$\frac{(q_1 + q_2)e}{R} = 2m_e c^2$$  \hspace{1cm} (32)

hence from Eqs. (31) and (32) we conclude that the electric potential is constant in the intermediate region $R_c < r < R$, consequently that no electric field exists in the intermediate region. Clearly, screening of the electric field has occurred through creation of real electron-positron pairs. The positrons reside at the outer surface of the intermediate layer, and the newly created electrons move out and add to the surface layer negative excess charge $\rho_-$, now satisfying

$$q_1 + q_2 = 4\pi R^2 \lambda_L \rho_-.$$  \hspace{1cm} (33)

Hence the positive charge $(-q_2)$ is due to real positrons created from the Dirac vacuum, and its magnitude is

$$|q_2| = |q_1|(\frac{R}{R_c} - 1)$$  \hspace{1cm} (34)

with $q_1$ given by $Eq. (4)$ with $R = R_c$ as given by $Eq. (6)$.

For the case of $Nb$, $\rho_- = 0.017C/cm^3$, yielding $|q_1| = 1.5 \times 10^{-6}C$. For example, in a sample of radius $R = 2R_c$ = 2.66cm, $Eq. (34)$ predicts that $9.4 \times 10^{12}$ real electron-positron pairs created from the Dirac vacuum exist!

Finally, we note that in the intermediate region there is no net electric field, and that because of the electron-positron pair creation the system cannot be described with a wavefunction with a fixed number of particles, but rather requires a description that allows for superposition of charge-neutral states with different numbers of particles. This is precisely the physical situation described by the conventional BCS wave function.

VIII. PAIR PRODUCTION FROM EXTERNALLY APPLIED ELECTRIC FIELD

We have seen in the previous section that creation of real electron-positron pairs is expected to occur for large superconducting samples when the internal electric field exceeds a critical value, to prevent the internal field from becoming larger. Here we show that application of an external electric field is another mechanism leading to pair production.

Indeed, the relation between charge density $\rho(\vec{r})$ and electrostatic potential $\phi(\vec{r})$ in our theory is

$$\rho(\vec{r}) - \rho_0 = -\frac{1}{4\pi \lambda_L^2} [\phi(\vec{r}) - \phi_0(\vec{r})]$$  \hspace{1cm} (35)

where $\rho_0$ and $\phi_0(\vec{r})$ are the interior positive charge density and the associated electrostatic potential respectively. Under an externally applied field the induced charge density is

$$\rho_{ind}(\vec{r}) = -\frac{1}{4\pi \lambda_L^2} \delta\phi(\vec{r})$$  \hspace{1cm} (36a)

where $\delta\phi(\vec{r})$ is the change in the total electrostatic potential due to the applied electric field. $Eq. (36a)$ shows that externally applied electric fields are screened over a London penetration depth, just as magnetic fields. Using $Eq. (17)$ for $\lambda_L$,

$$\rho_{ind}(\vec{r}) = -\frac{n e}{m_e c^2} \frac{\delta\phi(\vec{r})}{\lambda_T^2}$$  \hspace{1cm} (36b)

In contrast, the induced charge density in a normal metal when an external electric field is applied is

$$\rho_{ind}(\vec{r}) = -\frac{1}{4\pi \lambda_{TF}^2} \delta\phi(\vec{r})$$  \hspace{1cm} (37a)

with the Thomas-Fermi screening length given by (for free electrons)

$$\frac{1}{\lambda_{TF}^2} = \frac{6\pi n e^2}{\epsilon_F}$$  \hspace{1cm} (37b)

with $n$ the electron density and $\epsilon_F$ the Fermi energy, so that $Eq. (37a)$ is

$$\rho_{ind}(\vec{r}) = -\frac{3}{2n} \frac{e\delta\phi(\vec{r})}{\epsilon_F}$$  \hspace{1cm} (37c)

$Eq. (37c)$ for the normal metal shows that the fractional change in the local charge density $(ne)$ induced by the external field is the ratio of the energy gain per electron $e\delta\phi$ to the energy cost in putting an extra electron
at the top of the Fermi distribution, \( \epsilon_F \). Analogously, for
the superfluid Eq. (36b) shows that the fractional change in the local superfluid charge density induced by the ex-
ternal field is the ratio of \( \epsilon_0 \delta \phi \) to the energy cost in creating
charges from the Dirac vacuum, \( n_e e^2 \). Because this
cost is much greater than \( \epsilon_F \), the London length is much
larger than the Thomas Fermi length. We conclude that
for the superfluid, screening of externally applied electric
fields occurs through pair production from the Dirac sea,
because the ‘rigidity’ associated with the coherence of the
superfluid wavefunction over the macroscopic sample
prevents screening through local shifts of the superfluid
density.

Consequently, Eq. (36) directly furnishes the num-
ber of positrons created under application of an external
electric field. The change in potential under an applied
electric field \( E_{app} \) that decays to zero over a distance \( \lambda_L \)
is

\[
\delta \phi \sim E_{app} \lambda_L
\]

(38)
giving rise to an induced charge density

\[
\rho_{ind} \sim -\frac{1}{4\pi \lambda_L} E_{app}
\]

(39)
generated by pair production over a layer of thickness \( \lambda_L \).
The total charge created for a sample of surface area \( A \)
exposed to the electric field is

\[
q_{ind} \sim \rho_{ind} \lambda_L A = \frac{E_{app} A}{4\pi} A
\]

(40)

hence the number of positrons created is

\[
N_p \sim \frac{E_{app} A}{4\pi|e|}
\]

(41)

For example, for \( E_{app} = 1kV/mm \) and \( A = 1cm^2 \), \( N_p \sim 5.5 \times 10^6 \).

IX. OBSERVABLE CONSEQUENCES OF PAIR
PRODUCTION

We have seen in Sect. VI that in a spherical super-
conductor of radius \( R \) larger than the critical radius \( R_c \)
given by Eq. (6), the amount of negative charge in the
surface layer of thickness \( \lambda_L \) exceeds the negative charge
expelled from the interior by an amount

\[
q_2 = q\left(\frac{R}{R_c} - 1\right)
\]

(42)

with \( q \) given by Eq. (4). This excess negative charge
originates in pair creation in the intermediate layer and
reflects the existence of real positrons in the intermediate
layer. Of course the same phenomenon is expected in
large samples of non-spherical shape. Similarly we have
argued in Sect. VII that when an external electric field is
applied on a surface area \( A \), real electron-positron pairs
are created in a surface layer of thickness \( \lambda_L \) to screen the
applied field.

When electrons and positrons collide they annihilate and two \( \gamma \)-rays of energy \( m_e c^2 = 0.511 MeV \) are emit-
ted in opposite directions. Of course in a supercon-
ductor in equilibrium or under stationary conditions no such \( \gamma \)-ray emission is expected. We propose however
that under suitable non-stationary conditions \( 0.511 MeV \)
\( \gamma \)-rays will be emitted from superconductors, and sug-
gest that an experimental effort to detect this effect
should be undertaken. Because it is an unexpected ef-
fect within the conventional theory of superconductivity,
if this phenomenon is detected it will shed important new
light onto the physics of superconductivity.

We suggest the following experimental tests. For
a large superconducting sample at low temperatures, positrons should exist in dynamical equilibrium with elec-
trons in the intermediate layer. Rapid destruction of su-
perconductivity by heating, or application of ultrafast ultraslow magnetic field pulses that drive the mate-
rial normal, should cause electron-positron annihilation
and emission of \( 0.511 MeV \) \( \gamma \)-rays that can be detected with appropriate detectors. Similarly, in the presence of an externally applied electric field positrons will exist
which will annihilate and give rise to \( \gamma \)-ray emission if
the electric field is suddenly switched off. We suggest that
placing a superconducting sample between the plates of
a capacitor and increasing the electric field until break-
down occurs will give rise to \( \gamma \)-ray emission when the
capacitor discharges through the superconductor. Alter-
natively, \( \gamma \)-ray emission should also occur upon applica-
tion of a strong rapidly varying ac electric field.

However, \( \gamma \)-radiation can be hazardous to humans.
Exposure to 5 rads per year is usually regarded as the
limit of safety, and \( 5 \times 10^9 \) photons of energy \( 0.511 MeV \)
per square cm of tissue is 1 rad. For the example dis-
cussed in Sect. VI, a sample of \( Nb \) of radius \( R = 2R_c = 2.66cm \) has \( \sim \times 10^{13} \) electron-positron pairs; if each pair emits 2 photons when the sample goes normal it results in \( 2 \times 10^{13} \) photons and a radiation dose of 0.13 rads
at a person 50 cm away. In cycling the sample from below to above \( T_c \) several times, very quickly the limit of
safety for this individual is met! For larger samples the
danger becomes rapidly larger as seen from Eqs. (4) and
(27). Similarly, application of large electric fields
to superconductors can result in significant numbers of
electron-positron pairs created, as discussed in Sect. VII,
and experiments with large time-varying electric fields
could also result in hazardous amounts of \( \gamma \)-radiation.

X. BREMSSTRAHLUNG

In addition to 0.511 MeV radiation, we expect that
high energy radiation with a broad spectral range will be
emitted from superconductors that are rapidly driven
normal through sudden changes in temperature or ap-
pplied magnetic field.
Indeed, the electrons expelled from the interior that give rise to the outer negative charge density \( \rho_- \), carry a spin current in the superconducting state. At radius \( r \) the kinetic energy of these electrons is, from Eq. (7):

\[
\frac{1}{2} m_e v^2 = \frac{qe}{2r}
\]

(43)

where \( q \) is the net charge inside \( r \). In particular, the fastest speeds occur for electrons at the edge of the positive charge distribution shown in Fig. 1. If the superconductor is suddenly driven normal these electrons carrying the macroscopic spin current will stop and emit bremsstrahlung, of maximum frequency \( \omega_m \) determined by conversion of the entire kinetic energy of the electron into a single photon:

\[
\hbar \omega_m(r) = \frac{qe}{2r}
\]

Hence at the critical radius given by Eq. (31) we obtain from Eq. (44)

\[
\hbar \omega_m(R_c) = 0.511 MeV
\]

(45)

which is the same as the photon energy resulting from electron-positron destruction. For radius \( R \) smaller than the critical radius we have simply

\[
\hbar \omega_m = \frac{R}{R_c} m_e c^2
\]

(46)

Consequently we expect a broad spectrum of high energy radiation, with the upper limit frequency \( \omega_m \) increasing with sample size: samples of radius smaller than \( 2 \times 10^{-4} R_c \) will emit in the UV (\( \hbar \omega_m < 100 eV \)), samples of radius up to \( R = 0.2 R_c \) will also emit X-rays (\( 100 eV < \hbar \omega_m < 100 keV \)) and samples with \( R > 0.2 R_c \) will in addition emit \( \gamma \)-rays (\( \hbar \omega_m > 100 keV \)), up to a maximum frequency 511 keV/\( h \) when the radius reaches the critical radius \( R_c \). The radiation will originate predominantly from the region at distance \( \lambda_L \) from the surface of the sample, where the fastest electrons in the spin current reside. When the system becomes normal, these “undressed” electrons in Cooper pairs will suddenly unbind and experience scattering by the discrete ionic potential, and emit a bremsstrahlung spectrum as given by the Bethe-Heitler formula. The spectral distribution detected will depend both on the bremsstrahlung processes and on the scattering processes that occur in the path of the photon towards the surface. When the sample radius becomes larger than \( R_c \), a peak will grow at the maximum frequency 511 keV/\( h \) reflecting the electron-positron annihilation processes. This is schematically depicted in Fig. 4.

The integrated intensity of the bremsstrahlung radiation should be proportional to the negative charge in the outer layer shown in Fig. 1. Hence we expect the integrated intensity to grow proportionally to \( R^2 \) for samples of radius smaller than \( R_c \), according to Eq. (4). For \( R > R_c \), the negative charge density \( \rho_- \) starts to decrease according to Eq. (16), so the integrated intensity (excluding the peak at 0.511 MeV) should grow proportionally to \( R \). The peak at 0.511 MeV should increase proportionally to \( (R/R_c - 1) \) according to Eq. (34).

We conclude from these considerations that experiments with and practical uses of large superconducting samples, as well as processes involving application of large electric fields to superconductors, are potentially dangerous. The level of ionizing radiation generated in these situations should be ascertained before they can be safely carried out in an environment where humans are in danger of exposure.

XI. CONCLUSIONS

Superconductivity has been traditionally regarded as a low energy phenomenon, because low temperatures are involved and because in the conventional theory phonons, that are low energy excitations in the solid, are thought to play the dominant role. Only recently, evidence from optical experiments in high \( T_c \) superconductors has suggested that higher energy scales, in the mid-infrared and visible range (of order \( eV \)) play a role at least in those materials. We have also recently suggested changes in the plasmon dispersion relation, which for conventional superconductors can be above 10 eV; this prediction has not yet been put to experimental test. Continuing this trend, in this paper we have suggested that energy scales relevant to superconductivity extend even much higher, to the range of millions of \( eV \).

How can \( MeV \) energies possibly be relevant for a phenomenon where the local energies involved are of order \( \mu eV \), i.e. a factor \( 10^{12} \) smaller? Qualitatively, the key lies in the quantum coherence of the superconducting state over macroscopic distances. In a sample of volume 1 cm\(^3\) there are of order \( 10^{23} \) atoms in the bulk, and of order \( 10^{18} \) atoms in the surface layer of thickness \( \lambda_L \). Hence a fraction \( 10^{-7} \) of the surface layer atoms could each display a phenomenon at an energy scale of \( 10^6 eV \) if they
are able to harness an energy of $10^{-6}\text{eV}$ from each of the atoms in the bulk. Therein lies the remarkable nature of the macroscopic quantum coherence that is the hallmark of superconductivity.

The importance of relativity in the theory of hole superconductivity was already foreshadowed early on in the lattice formulation of the theory, which describes pairing and superconductivity as driven by an off-diagonal Coulomb interaction term in the Hamiltonian, $\Delta$. This term gives rise to a reduction of the mass of the carriers when they form a Cooper pair, evidence for which has recently been seen experimentally. In relativity a bound state of two particles necessarily has a smaller mass than the sum of its constituent’s masses due to the energy-mass relation $E = mc^2$.

The work discussed here also sheds new light on the meaning of the BCS wave function. The fact that the BCS wave function describes the superconducting state as a superposition of states with different number of particles has until now been regarded merely as a convenient calculational device, without physical content. Indeed, there is no physical reason in the conventional theory for why a wave function describing pairing of electrons could not be described with a fixed number of pairs. Furthermore there is something profoundly unphysical about the BCS wave function in the conventional context: each Cooper pair carries a mass of $1.022\text{MeV}/c^2$ and a charge of $2e$, so that the BCS wavefunction superposes states with widely different electric charges and energies. Why would the description of a low-energy phenomenon require the mixing of such very different states? Instead, in the present context the superposition of states with different number of electrons and positrons (but the same total electric charge) arises as a consequence of the physics and indicates that a BCS-like wavefunction that superposes different occupation number sectors is in fact required to describe the physical reality.

In physics, the first “hole theory” proposed was that of Dirac, to deal with the negative energy states that he encountered in formulating the relativistic quantum theory of the electron. In this paper we have shown that the theory of hole superconductivity leads unavoidably to the inclusion of Dirac’s holes (positrons) in the description of the superconducting state.

The theory discussed here predicts that ionizing radiation with a continuum of frequencies all the way up to $0.511\text{MeV}/h$ will be emitted from large superconductors in non-equilibrium processes. No other theory of superconductivity predicts this effect, hence detection of such radiation will support the theory of hole superconductivity, or call for alternative explanations.

To conclude we emphasize again that in the process of advancing science, incomplete or erroneous understanding can be dangerous. Before the discovery of capacitors it was thought that the bigger an object the more electricity it could store; then, Müsschenbrock described his experiment with a small Leyden jar with the words “suddenly I received in my right hand a shock of such violence that my whole body was shaken as by a lightning stroke”. William Roentgen died of bone cancer and Marie Curie of leukemia, presumably triggered by exposure to X-rays and radioactivity respectively in the course of performing their experiments. If the widely accepted BCS-London theory is correct for conventional superconductors, and d-wave superconductivity for high $T_c$ cuprates, no danger from harmful ionizing radiation from superconductors should be expected; nevertheless, no matter how small the perceived chance to the contrary, I suggest that it behooves scientists to rule out the scenario proposed in this paper.

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