DEVELOPING MATHEMATICAL MODEL OF CROWD BEHAVIOR IN EXTREME SITUATIONS

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Abstract

The article considers the possibility to simulate, using a differential equation, the behavior of the crowd in extreme situations. The author demonstrates the very possibility to develop a mathematical model that describes the changes in the main parameters of the crowd at each time moment. This study is conducted to predict the behavior of the crowd in a particular room, for a more efficient location of escape routes there. The simulation results show that the force of internal friction of the crowd decreases as the speed of the crowd moves from the center to the outskirts. That is, the probability that a person suffers from excessive crowd pressure is higher in the center than on the periphery. This study will be useful in such areas of human activities as building design, engineering, etc. The data obtained by the calculations can be used to arrange emergency exits in buildings to avoid human casualties in case of an emergency.

Keywords: Mathematical modeling, differential equation, crowd linear density, speed, discharge capacity.

I. Introduction

Usually, the behavior of the crowd in case of emergencies - fires, floods, terrorist acts, etc.- causes mass casualties. It is possible to reduce the number of victims by properly designing escape routes in buildings, room layouts, etc. For building design, it is necessary to know the behavior of the crowd in advance in a given room. This can be achieved by simulating the behavior of the crowd.

In the past years, many crowds in connection with incidents occur at places of mass gatherings such as religious places, sport venues, political and social events, and railway stations due to poor crowd management or poor system design. The uncertainties of large-scale evacuation under emergencies have not yet been fully elucidated [XVIII, XX].
Since the ancient times, scientists have believed that the whole universe is governed by mathematical laws. Nowadays, more detailed and in-depth studies have helped scientists prove the validity of this idea [VII, X, XIII, XVII].

People are a part of the universe, as well as all the matter both studied and not. Accordingly, one can try to predict the behavior of a person by mathematical methods. It is rather difficult to predict the behavior of an individual as it is driven by many factors that are both, obvious and hidden. However, the behavior of a group of people such as the crowd is determined by more or less obvious factors, which can be taken into account to predict their behavior. The desired direction of motion is determined by solving an eikonal equation with a density-dependent running cost. The method which contains numbers represented to formulate the first step to compute them defines the accuracy of the results [XI, XIX].

The crowd behavior can be predicted by various mathematical methods. The most accurate model that will coincide with the real crowd behavior can be made using differential equations [XV]. Ordinary differential equations (ODEs) which contains high-order implicit equations are used to describe many different problems in engineering [VIII].

The differential equation describes the relationship between an unknown function and its derivatives. The simplest differential equation is

$$y'(x) = f(x),$$

(1)

There are computational models applicable to molecular or viscous flows; however, the processing of the transitional flow remains underdeveloped. When a time-dependent partial differential equation (PDE) is discretized in space with a spectral approximation, the outcome is a pair system of ordinary differential equations (ODEs) in time. The solution to any well-posed linear problem in strong or weak form can be found by a certain meshless kernel methods to any prescribed accuracy. There are many methods to solve boundary value problems. One of these methods is multilevel augmentation for solving differential equations [IV, VI, IX, XVI].

II. Methodology

II.i. Basic Parameters of the Crowd

It is necessary to determine the key parameters that characterize the crowd to build a mathematical model. The parameters are [XIV]:

- linear crowd density $\rho$, m/person;
- friction force in the crowd $F$, N.

The linear crowd density shows the change in the crowd of people in a certain area at a certain time point. To calculate the crowd density is a task of great importance. The crowd density at the exit in case of a fire makes it possible to judge whether there is a
jam at a given time or whether people are being evacuated in a normal manner without panic [III, II].

II.i. Linear Crowd Density

An algorithm for calculating the linear crowd density was formulated by N. Jacobs who analyzed various crowds of people using aerial photographs [V]. In the course of his research, he concluded that the highest area density was four square feet per at per person.

As shown by N. Jacobs, the police, who provide such assessments most often, tend to exaggerate the crowd density by two or three times.

N. Jacobs’ proposed formula for determining the crowd size is

\[ N = (a + b) \cdot K, \]  

where \( a \) is the length of the territory where the crowd is located, m;
\( b \) is the width of the territory where the crowd is located, m;
\( K \) is the density coefficient.

\( K = 10 \) is for tight crowds;
\( K = 7 \) is for less tight crowds.

He argues that this formula gives results with an accuracy of 20%, which is an excellent result. Of course, there are a lot of factors influencing the crowd assessment, including the area occupied. The error increases with changes in the composition of the crowd and the nature of its movement. However, the Jacobs formula is optimal for calculating simple cases of crowd behavior.

The linear crowd density \( \rho \) in emergencies is a constantly changing value. It is determined by the expression (2) and depends on the unit of time in the interval equal to \( d\tau \).

Using Jacobs formula (2) we get:

\[ \rho(\tau) = \frac{d\rho}{d\tau} = \frac{d(N / S)}{d\tau}, \]

where \( S \) is area of the territory where the crowd is located, m².

Substituting the data from expression (2) into expression (3), we obtain the differential equation of linear crowd density:

\[ \rho(\tau) = \frac{d\rho}{d\tau} = \frac{d([a + b] \cdot K / S)}{d\tau}, \]
III. Results and Discussion

The force of friction in the crowd $F$ describes a so-called jam. A jam in the crowd occurs at the moment of panic. In case of a jam the chance for a person to get killed is several times more probable. Therefore, it is also necessary to know this value along with the linear crowd density.

When studying the behavior of dense crowds over time, one can find some of its similarity with the behavior of fluid molecules. Therefore, it is advisable to apply the law describing the interaction of fluid molecules with each other to the mathematical model of the crowd. Take, e.g., the Newton friction viscosity law.

According to Newton, the force of friction between molecules of a fluid, which is called the internal friction force, varies in proportion to the velocity difference of the molecules of the fluid. Thus, internal friction force $F$ varies in proportion to fluid velocity $v$ in the direction perpendicular to the course of movement. It also depends on $S$ defined as the area of contact of fluid molecules.

The differential equation of the internal crowd friction force at the time of panic in case of an emergency is derived using Newton’s viscosity law and recorded as

$$F'(y) = \frac{dF}{dy} = \eta \cdot S \cdot \frac{dv}{dy},$$

where $\eta$ is the dynamic viscosity coefficient.

$S$ is the contact area of persons in the crowd, m$^2$;

$\frac{dv}{dy}$ is the speed of the crowd moving in a certain direction, m/s.

The Changes in Linear Crowd Density $\rho$ and Crowd Friction Force $F$ Are Simulated in Mathcad v15 [XII].

The behavior of the crowd is simulated within three minutes, or 180 seconds (\(\tau = 180\) sec.). As a rule, this time is sufficient for a significant change in the properties of the crowd in case of an emergency. The length of the room in which the simulation will take place is $a = 200$ m and its width is $b = 100$ m.

The density coefficient is $K = 10$, since an extreme situation for a dense crowd is simulated.

During the simulation the area, in which the crowd is located, constantly decreases, as people move to the escape route.

Algorithm and input data:

\(n := 180\)
\[ \tau := 1..n \]

\[ a_1 := 200 \quad b_1 := 100 \quad K := 10 \]

\[ a_{\tau+1} := a_{\tau} - 1 \quad b_{\tau+1} := b_{\tau} - 0.5 \]

\[ \rho_{\tau} := \frac{K}{a_{\tau} \cdot b_{\tau}} \]

The step of changes in the room length and width will be altered by 1 m for the length and by 0.5 m for the width.

As a result of the simulation, the values of changes in the width and length of the space occupied by the crowd and the linear crowd density are obtained (table 1).
Table 1: Values of changes in the width and length of the space occupied by the crowd and the linear crowd density

| $\tau$ | $a_\tau$ | $b_\tau$ | $\rho_\tau$ |
|--------|----------|----------|-------------|
| 200    | 100      |          | 0.15        |
| 199    | 99.5     |          | 0.151       |
| 198    | 99       |          | 0.152       |
| 197    | 98.5     |          | 0.152       |
| 196    | 98       |          | 0.153       |
| 195    | 97.5     |          | 0.154       |
| 194    | 97       |          | 0.155       |
| 193    | 96.5     |          | 0.155       |
| 192    | 96       |          | 0.156       |
| 191    | 95.5     |          | 0.157       |
| 190    | 95       |          | 0.158       |
| 189    | 94.5     |          | 0.159       |
| 188    | 94       |          | 0.16        |
| 187    | 93.5     |          | 0.16        |
| 186    | 93       |          | 0.161       |
| 185    | 92.5     |          | 0.162       |
| 184    | 92       |          | 0.163       |
| 183    | 91.5     |          | 0.164       |
| 182    | 91       |          | 0.165       |
| 181    | 90.5     |          | 0.166       |
| 180    | 90       |          | 0.167       |
| 179    | 89.5     |          | 0.168       |
| 178    | 89       |          | 0.169       |
| ...    | ...      |          | ...         |

The relation of linear crowd density $\rho$ to time $\tau$ can be depicted as a plot.
Fig. 1: Relation of linear crowd density $\rho$ to time $\tau$.

The plot shows that the linear crowd density becomes critical at the final period of time from 150 to 200 seconds. Therefore, it is important to prevent panic at the very beginning of an emergency.

Then the dynamics of changes in $F$ is simulated. Dynamic viscosity coefficient $\eta$ is equal to 82 because this value is the one most closely reflecting the behavior of people in a crowd. The room length is $a = 20$ m and the room width is $b = 20$ m. The area of the space, where the crowd is located, is $S = 400$ m$^2$.

The speed of the crowd at the initial moment of time is $v_1 = 0.05$ m/s. As the force of internal friction increases toward the center of the crowd, the speed of the crowd will decrease by a step of 0.0001 m/s.

Algorithm and input data:

$$\eta := 82 \quad n := 20$$

$$y := 1 \ldots n$$

$$a := 20 \quad b := 20 \quad v_y := 0.05$$

$$S := a \cdot b \quad S = 400$$

$$v_{y+1} := v_y - 0.0001$$

$$F_y := \eta \cdot S \cdot v_y$$

The values of internal friction force $F$ found by the simulation are presented in the table below.
Table 2: Values of the internal friction force.

The results of the simulation allow us to draw the following plot of the dependence of $F$ on the speed of the crowd moving away from the center.

![Graph showing the relationship between $F$ and $y$.](image)

**Fig. 2:** Changes in internal friction force $F$ dependent on the speed of the crowd moving away from the center.

The simulation results show that the crowd’s internal friction force decreases as the crowd moves from the center to the outskirts. That is, the probability for a person to suffer from excessive crowd pressure is higher in the center than in the periphery.

**IV. Highlights**

- A mathematical model based on two differential equations was developed to describe crowd behavior;
- The calculation of the crowd parameters is suitable for any conditions by changing the values of differential equations;
V. Conclusion

The differential equations obtained in this study can be used to calculate the behavior of the crowd in emergency situations such as fires, terrorist acts, natural disasters, etc. The following conclusions can be drawn based on the study results:
- it is possible to describe the crowd using a mathematical model based on two differential equations;
- it is possible to calculate the parameters of the crowd, suitable for any conditions by changing the values of differential equations;
- this study may be useful in designing premises intended to host large masses of people, such as clubs, cinemas, shopping centers, educational institutions, stadiums, etc.

The approach developed can also be applied in such areas of human activity as building design, engineering, etc. The data obtained by the calculations can be used to arrange emergency exits in buildings to avoid human casualties in case of an emergency.

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