The Reactive Power Compensation Planning for N-1 Line Outage Based on Line Fault Betweenness

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Abstract. An inappropriate compensation capacity can cause oscillation or even collapse of the grid. This paper focuses on the RPP problem considering single line outage in the grid. Firstly, the line fault betweenness (LFB) is proposed to identify the pre-outage and post-outage impact on vulnerable lines. Then, considering the cost and power quality of the grid, a multi-objective optimization is adopted to analyze the result of compensation planning and solved by the multi-objective genetic algorithm and F1-scores. Finally, the IEEE-39 test system is employed to illustrate the important lines with LFB ranking and the decision of reactive power compensation planning. The compensation result reveals the improvement on power factor of generators and load voltage offset reduction.

1. Introduction
In the current power system, the reactive power planning (RPP) is critical for the grid operation. In existing researches, it has been found that the reactive power compensation can reduce the loss in the power transmission, improve the power factor of generators and suppress the harmonic increasing [1]. In general, a rational compensation planning improves the grid operating efficiency and quality.

The objectives of the RPP vary in existing studies. The majority of objectives are to minimize the cost of reactive power supplies and the loss in the grid [2]. Venkatesh took the cost and real power loss as objective functions and solved the optimization problem by simplifying the AC model into a DC model [3].

The vulnerable line detection is another challenging work in grid systems. A cascaded outage of several power lines could be caused by one line overload or open circuit. Ding assessed the vulnerable line of small-world power grid by weighted topological model [4]. Furthermore, some researchers used electricity betweenness and electrical centrality to find out the system vulnerability [5].

In this paper, the line fault betweenness (LFB) is proposed and its calculation will be formulated in section 2. Multi-objective optimization is adopted to decide the RPP, and F index is used to measure the effect of the proposed compensation strategy in section 3. Finally, the IEEE-39 test system is employed to illustrate the important lines with LFB ranking and the decision of reactive power compensation planning.
2. Line Fault Betweenness

The topological edge betweenness [6] and electricity betweenness [7] are normally used to analysis the impact of edges on a complex network. To deal with active power and reactive power transfer in line-outage grid, line fault betweenness (LFB) will be proposed to evaluate the sensitivity of line $ij$ to line $\alpha\beta$ in an steady-state AC model. When line $\alpha\beta$ is reconnected to the grid, line closure distribution factors (LODC) denotes the percent of the post-closure flow showing up on line $ij$ after the closure of line $\alpha\beta$, it defined as:

$$LCDF_{ij,\alpha\beta} = \frac{\Delta P_{ij,\alpha\beta}}{P_{\alpha\beta}}$$  \hspace{1cm} (1)

$P_{\alpha\beta}$ denotes the power of pre-closure flow on line $\alpha\beta$. As for reactive power, learning from concept of LODF in DC model, the reactive influence on line $ij$ when line $\alpha\beta$ outage can be expressed as :

$$LODFR_{ij,\alpha\beta} = \frac{\Delta Q_{ij,\alpha\beta}}{Q_{\alpha\beta}}$$  \hspace{1cm} (2)

and the line closure distribution factors with reactive power is defined as:

$$LCDFR_{ij,\alpha\beta} = \frac{\Delta Q_{ij,\alpha\beta}}{Q_{\alpha\beta}}$$  \hspace{1cm} (3)

with $Q_{\alpha\beta}$ denotes the power of pre-closure flow on line $\alpha\beta$. The LFB can be as:

$$LFB_{\alpha\beta} = \sum_{ij \in E, ij \neq \alpha\beta} \frac{P_{\alpha\beta} \cdot LOCDF_{ij,\alpha\beta}}{|Z_{i\beta}|} + \frac{Q_{\alpha\beta} \cdot LOCDFR_{ij,\alpha\beta}}{|Z_{i\beta}|}$$  \hspace{1cm} (4)

where the $LOCDF$ and $LOCDFR$ is :

$$LOCDF_{ij,\alpha\beta} = LODF_{\alpha\beta,ij} + LCDF_{\alpha\beta,ij}$$

$$LOCDFR_{ij,\alpha\beta} = LODFQ_{\alpha\beta,ij} + LCDFQ_{\alpha\beta,ij}$$  \hspace{1cm} (5)

The $|Z_{i\beta}|$ is the transimpedance of node $i$ to node $\beta$, its represent the shortest electricity path from node $i$ to node $\beta$.

3. LFB-based Reactive Power Compensation Planning

3.1. Assumption

Two common assumptions of RPP and one assumption of reactive power compensation are formulated:

- Generator resources can adjust its active power and reactive power to meet the grid requirements. Its voltage offset is 0.
- Loads requirements do no changes in the simulation.

3.2. Voltage offset and power factor

It needs some efficient indexes to evaluate the level of grid. The first objective function is to evaluate the voltage offset on load nodes and power factor of generator nodes. The voltage offset refers to the difference between the actual voltage of node and the reference voltage. The power factor is the ratio of active power and apparent power. It revels the proportion of active power in generator output and generator is more efficient with greater power factor.
The voltage offset is defined as:

\[
V_{\alpha\beta}^{\text{off}} = \frac{1}{n_L} \sum_l (V_l - V_{\text{imin}})(V_l - V_{\text{imax}}) - (V_{\text{imax}} - 1)(V_{\text{imin}} - 1) \quad l \in L
\]  

(6)

where \(n\) is the node \(n\), \(n_L\) is the set of loads and \(V_l\) is the voltage magnitude (p.u.). The \(V_{\text{imax}}\) and \(V_{\text{imin}}\) is the upper limit and lower limit of voltage offset in the simulation. The \(V_{\alpha\beta}^{\text{off}}\) denotes the voltage offset index with line outage on \(\alpha\beta\), and \(V_{\alpha\beta}^{\text{off}}\) is combined with average voltage offset and max voltage offset.

The power factor in the objective function can be calculated as follow:

\[
P_f^{\alpha\beta} = \frac{1}{n_G} \sum_n \left(1 - \frac{P_n}{S_n}\right) \quad n \in G
\]

(7)

where \(n_G\) is the set of generators and \(P_n\) is the active power of node and \(S_n\) is the apparent power. The \(P_f^{\alpha\beta}\) denotes the power factor index.

The first function can be express as:

\[
F_1 = \sum_{\alpha\beta \in E} LFB_{\alpha\beta} \cdot (V_{\alpha\beta}^{\text{off}} + P_f^{\alpha\beta})(8)
\]

where \(F_1\) denotes the grid steady level when the single line outage occurs on \(\alpha\beta\), and the \(E\) is the set of lines in grid. As for the electricity betweenness function, it defined as:

\[
F_{Be,1} = \sum_{\alpha\beta \in E} EB_{\alpha\beta} \cdot (V_{\alpha\beta}^{\text{off}} + P_f^{\alpha\beta})
\]

(9)

where the \(EB_{\alpha\beta}\) is the weight calculated by the electricity betweenness.

3.3. Cost of loss and compensation power

The second objective function is to minimizes the cost of loss and compensation power [8]. It is defined as:

\[
F_2 = \sum_{\alpha\beta \in E} a_2(P_{\text{loss}} + P_{\text{cp}})^2 + a_1(P_{\text{loss}} + P_{\text{cp}}) + a_0
\]

(10)

where the \(F_2\) denotes the total price of extra electricity. When the single line outage occurs on \(\alpha\beta\), \(P_{\text{loss}}\) is the loss of grid and \(P_{\text{cp}}\) is the compensation capacity of loads. The \(a_2, a_1, a_0\) are the polynomial coefficient of electricity price formula.

3.4. Multi-objective function

In the optimization, the numbers of decision variable is \(n\), which denotes the compensation capacity on loads. It also includes a set of constraints.

The multi-objective optimization function is defined as:

\[
\text{optimize} \quad F = (1 + \beta^2) \cdot \frac{F_1 \cdot F_2}{F_1 + F_2}
\]

s.t.

\[
P_{\text{bus}}(\theta, V_l) + P_d - C_g P_g = 0 \quad P_{\text{gmin}} \leq P_g \leq P_{\text{gmax}}
\]

\[
Q_{\text{bus}}(\theta, V_l) + Q_d - C_g Q_g = 0 \quad Q_{\text{gmin}} \leq Q_g \leq Q_{\text{gmax}}
\]

\[
V_{\text{imin}} \leq V_l \leq V_{\text{imax}}
\]

\[-P_{\text{mcp}} \leq x_i \leq P_{\text{mcp}} \quad i \in 1,2...L\]

(11)
Table 1: The simulation parameter.

| Parameter | Value |
|-----------|-------|
| $a_0$     | 0     |
| $a_1$     | 20    |
| $a_2$     | 0.01  |
| $V_{\text{min}}$ | 0.94 |
| $V_{\text{max}}$ | 1.06 |
| $P_{\text{mcp}}$ | 40 / 80 |
| $\beta$  | 1     |

Table 2: The Comparison of betweenness.

| lines(ID) | LFB   | LFB rank | electricity betweenness rank | topology betweenness rank |
|-----------|-------|----------|------------------------------|---------------------------|
| 5-8(11)   | 5.4129| 1        | 11                          | 24                        |
| 10-13(19) | 4.8524| 2        | 22                          | 25                        |
| 16-21(28) | 4.1407| 3        | 39                          | 15                        |
| 10-32(20) | 3.6960| 4        | 3                           | 36                        |
| 21-22(35) | 3.4424| 5        | 16                          | 23                        |
| 22-35(37) | 3.0163| 6        | 4                           | 33                        |
| 25-37(41) | 2.5277| 8        | 7                           | 31                        |
| 19-33(33) | 2.5088| 9        | 5                           | 35                        |

Where the F is the harmonic mean of two indexes $F_1$ and $F_2$ used F1 scores method and the $\beta$ is the weight [9]. And the constraints considered in the simulation includes the power-flow equation and compensation capacity limit. The power-flow equation here takes care of active power balance, reactive power balance, generation maximum output limit and voltage offset [10].

4. Simulation Result and Analysis

The IEEE-39 system model power system is used in the simulation which is proposed in 1979 by T. Athay [11]. During the simulation, some parameter values shown in Table 1 are following ($P_{\text{gmin}}, P_{\text{gmax}}, Q_{\text{gmin}}, Q_{\text{gmax}}$ and $C_g$ are given by model).

![Normalized electricity betweenness](image1)

![Normalized line fault betweenness](image2)

Figure 1: Normalized electricity betweenness.  
Figure 2: Normalized line fault betweenness.

The Table 2 compares the three kinds of betweenness. As Figure 1 and Figure 2 shown, after the normalization, the sum of betweenness equals the number of lines and the LFB and electricity betweenness are quite similar but some differences.
Figure 3: Comparison with three kinds of compensations.

Table 3: The decision of compensation strategy.

| Compensation Strategy | F1-scores          | Compensation Planning          |
|-----------------------|--------------------|--------------------------------|
| electricity betweenness | 0.44406           | 17.58, 13.02, 6.87, 7.15, 19.23, 14.20, 3.29, 9.20, 12.70, 6.89, 15.68, 6.17, 3.34, 8.42, 11.14, 9.93, 16.34, -1.26, 1.52 |
| LFB-based with 40 Mvar limit | 0.41699           | 29.15, 29.29, 2.47, 9.48, 19.52, 14.86, 11.27, 20.88, 7.30, 6.88, 6.54, 5.55, 13.97, 24.10, 17.05, 9.21, 8.70, 20.55, 12.00, 5.63, 11.77, 7.51, 8.13, 15.28, 13.63, 8.80, 13.44, 5.59, 4.64 |
| LFB-based with 80 Mvar limit | 0.40104           | 8.96, 41.02, 26.02, 18.10, 6.74, 5.93, 8.52, 13.29, 4.85, 18.04, 13.77, 16.35, 27.33, 12.23, 18.29, 31.50, 6.26, 23.27, 2.16, 24.52, 2.18, 4.07, 26.56, 12.01, 12.61, -5.28, 11.62, 10.88, 9.33 |

Figure 3 shows the comparison of three kinds of compensations with different betweenness and different limits of compensation. In this paper, we adopt the F1 scores to measure the multi-objective and the Pareto front. From the Table 3, the decision of compensation plan is made by F1-scores. The capacity of compensation plan also indicates that, the reactive power is transferred from generations to loads and some load require the larger reactive compensation capacity. From the Figure 4 and Figure 5, thanks to the reduction of reactive power in generations, the voltage offset and power factors of three kinds of compensation strategy have been significant improved. It can be observed that the power quality improvement is the LFB strategy with 80 Mvar limit.

5. Conclusion
Considering the line outage occurs in grid, firstly the LFB is defined and formulated section 2 to deal with the active power transfer and reactive power transfer. Then two functions are proposed to measure the level of steady of grid and cost of loss and compensation. Merging the two function by multi-objective to optimize the compensation strategy and tracking the Pareto front using multi-objective GA and F1-scores to make decisions. The result of different capacity limitation demonstrates its influence to the planning decision. It is noteworthy that, we can find out that some loads do need large compensation capacity to promote the system security for the outage.
Figure 4: Voltage offset of loads under three kinds of strategy.

Figure 5: Power factor of generations under three kinds of strategy.

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