Bifurcation Characteristics of Airfoil-NESs coupled System

Wenfan Zhang¹, Genbiao Zhou¹, Yafeng Zhou¹, Lei Wang¹, Jiazhong Zhang²

¹ Xi’an ShaanGu Power CO., LTD, Xi’an, Shaanxi 710611, P.R. China
² School of Energy and Power Engineering, Xi’an Jiaotong University, Xi’an, Shaanxi 710049, P.R. China

E-mail: sheen.z@163.com

Abstract. Aeroelastic instability has become a hot research topic. It has increasingly high-lighted its importance in the aircraft and fluid machinery industry and brings challenges to the safe flight of various aircraft and the safe operation of fluid machinery. Numerical simulations were performed to study the flow-induced oscillation of a two-dimensional airfoil with two non-linear energy sinks (NES). The main system has two degrees of freedom - pitch and heave. These two NESs are considered as subsystems, with the first NES at the leading edge and the second NES at the trailing edge. The bifurcation behaviour and the mechanism of the system leading to chaos of the coupled system under different parameters and initial conditions is studied by nonlinear analysis, and the relationship between large amplitude limit cycle oscillation and the chaotic occurrence point of the main system is revealed, this corresponding relationship has important theoretical significance for mastering and adjusting system parameters to make NES work within its ideal oscillation suppression parameters.

1. Introduction

The development of modern aircraft and fluid machinery have been gradually accelerated, and the aeroelastic problems always accompany it. In the classical theory of aeroelasticity, the aeroelasticity problem is simplified into a set of linear equations which are easy to solve. However, when the incoming velocity increases to high subsonic or transonic speeds, this assumption usually results in inaccurate results. In addition, the phenomena of flow separation and shock oscillation are beyond the scope of linear aeroelastic theory. In fact, due to the structural nonlinearity of the system, aeroelastic system may exhibit various phenomena, such as limit cycle oscillation and chaotic oscillation, etc., and the problems caused by structural nonlinearity have gradually attracted the attention of researchers in relative field.

Fung and Dowell et al. modified the basic linear model, studied several unstable oscillation responses caused by system instability [¹, ²], and found that the amplitude increased exponentially. At the same time, due to the nonlinear characteristics of stiffness or damping in the structure, the exponential growth of this response is limited [³], and ultimately, the system maintains LCOs.

In fact, the LCO problem exists for a long time, such as F-16 and F/A-18 under high subsonic and transonic oscillation problem of LCOs, Robert discusses the specific situation to avoid the LCOs is the wing of the method [⁴], reveal some typical similarity and difference between flutter and LCOs, moreover, the LCOs in flight tests on aircraft external system and the ability of the whole task are emphatically studied. Later and Cattarius used numerical methods to study the interactions between the wing and its...
The research of Nayfeh shows that the small imbalance in rotating machinery may also cause the solution to fall into resonance, and the rotating frequency of rotating parts and the frequency of system vibration fall into resonance capture phenomenon [17]. The resonance behavior in dynamic system will lead to complex dynamic behavior, even chaos [18-20]. Sethna has proved that the harmonic excitation of the system can lead to the coupled response among the vibration modes [21], which is caused by the nonlinearity of the system, also known as the mixed mode response. It is also proved that the internal resonance behavior in these systems can explain the interaction relationship between the vibration and the generating conditions of chaotic response [22]. Some other studies involve the dynamic response of arch structure [23, 24], tension chord [25, 26], and forced response of approximate square plate [27]. In these
studies, the natural frequency of the internal vibration mode and the external driving frequency are fixed parameters. Subsequently, some scholars explore the way of resonance capture in the system by studying the phase space geometry of the system, which represents the internal resonance phenomenon in the slowly changing single degree of freedom system [28-30], and these theories have been applied to a series of studies on the rotational phase locking and reentry rolling resonance in the dual rotating spacecraft [31-36]. The limit cycle oscillation (LCO), which is to be suppressed by the NES, is studied from the viewpoint of the TET [37, 38]. The resonance capture (RC) in the coupled nonlinear system is also discussed by the means of the energy and spectrum analysis.

2. Modelling of the system
The rigid airfoil is used as the main system. As shown in Figure 1, the subsystem is a nonlinear energy sink located near the leading edge and trailing edge of the airfoil respectively.

By using the virtual work principle, the governing equation of the coupled system is as follows,

\[
\begin{align*}
\dot{m}\ddot{h} + S_\alpha \dddot{\alpha} + K_h(h + c_1 h^2) + gSC_{L\alpha}(\alpha + \dot{h}/U) + c_{s1}(\dot{h} - d_2 \dddot{\alpha} - \dot{z}_2) + \\
K_{s1}(h - d_2 \dddot{\alpha} - z_2)^3 + c_{s2}(\dot{h} - d_2 \dddot{\alpha} - \dot{z}_2) + k_{s2}(h - d_2 \dddot{\alpha} - z_2)^3) = 0 \\
I_\alpha \dddot{\alpha} + S_\alpha \dot{h} + K_{\alpha}(\alpha + d_2 \dddot{\alpha})^3 - qeSC_{L\alpha}(\alpha + \dot{h}/U) + d_1c_{s1}(d_1 \dddot{\alpha} + \dot{z}_1 - \dot{h}) + \\
d_1k_{s1}(d_1 \dddot{\alpha} + \dot{z}_1 - \dot{h})^3 + d_2c_{s2}(d_2 \dddot{\alpha} + \dot{z}_2 - \dot{h}) + d_2k_{s2}(d_2 \dddot{\alpha} + \dot{z}_2 - \dot{h})^3 = 0 \\
m_{s1}\dot{z}_1 + c_{s1}(\dot{z}_1 + d_1 \dddot{\alpha} - \dot{h}) + k_{s1}(\dot{z}_1 + d_1 \dddot{\alpha} - \dot{h})^3 = 0 \\
m_{s2}\dot{z}_2 + c_{s2}(\dot{z}_2 + d_2 \dddot{\alpha} - \dot{h}) + k_{s2}(\dot{z}_2 + d_2 \dddot{\alpha} - \dot{h})^3 = 0 \\
\end{align*}
\]

(2.1)

And the non-dimensional form is as follows,

\[
\begin{align*}
y'' + \dot{x}_\alpha \dot{\alpha}'' + \Omega^2 y + \dot{x}_y y'' + \mu C_{L\alpha}(\Theta' + \Theta \alpha) + e_1\dot{\lambda}_1(y' - \delta_1 \dot{\alpha}' - v_1') + \\
C_1(y - \delta_2 \dot{\alpha} - v_1)' + e_2\dot{\lambda}_2(y' - \delta_2 \dot{\alpha}' - v_2') + C_2(y - \delta_2 \dot{\alpha} - v_2)' = 0 \\
r_\alpha \dot{\alpha}'' + x_\alpha y'' + r_\alpha \dot{\alpha}'' + \dot{x}_\alpha y'' - \mu C_{L\alpha}(\dot{y}' + \Theta \alpha) + \delta_1\dot{\lambda}_1(\dot{\delta}_1 \alpha' + v_1' - y') + \\
\delta_1C_1(\dot{\delta}_1 \alpha + v_1 - y)'' + \delta_2\dot{\lambda}_2(\dot{\delta}_2 \alpha + v_2 - y)'' + \delta_2C_2(\dot{\delta}_2 \alpha + v_2 - y)'' = 0 \\
\dot{v}_1y'' + \dot{\lambda}_1(v_1' + \delta_1 \dot{\alpha}' - y') + C_1(v_1 + \delta_1 \alpha - y)'' = 0 \\
\dot{v}_2y'' + \dot{\lambda}_2(v_2' + \delta_2 \dot{\alpha}' - y') + C_2(v_2 + \delta_2 \alpha - y)'' = 0 \\
\end{align*}
\]

(2.2)

3. Bifurcation in coupled systems
3.1. Instability of coupled systems
First, the parameters of the coupled system are taken as \( \Theta = 0.9 \), \( \delta_1 = 0.9 \), \( \delta_2 = -0.9 \), \( \lambda_1 = \lambda_2 = 0.1 \), \( C_1 = C_2 = 10 \), \( \varepsilon_1 = \varepsilon_2 = 0.01 \), the data after transient resonance at the initial stage of the system is captured.

As in Fig. 2. \( \phi \) is used as abscissa, and vertical coordinate is the average value of the maximum amplitude of pitch oscillation. There are bifurcation point at \( \phi \) in the system without NESs. At this point, the system changes from a stable point to a periodic oscillation with a large amplitude. It can be known that the main frequency of the heave oscillation of the system before and after the bifurcation point changes from lower frequency to high frequency, while the main frequency of the pitching oscillation is still low, the resonance mode of the two degrees of freedom of the system changes from 1:1 resonance to 3:1 resonance.

From the perspective of system bifurcation, as the front and rear NES are coupled to the airfoil as a subsystem, the mechanism of oscillation suppression and energy absorption is as follows:

First, the NES always stabilized the oscillation of the main system, and the stability of the system in this stage would not change with the increase of the incoming flow velocity.

Secondly, two types of bifurcation phenomena occurred in the system in stage 2: 1) Hopf bifurcation occurs in the system, and the system exhibits the characteristic of periodic oscillation, the oscillation of the system is always suppressed oscillation; 2) With the increase of the incoming flow velocity, the system experiences Neimark-Sacker bifurcation. The oscillation of the system reflects the quasi-periodic characteristics, and the oscillation of the system is a recurrent series of suppressed burst-out.

Finally, at the end of stage 2, with the third bifurcation of the system, the system returned to the periodic oscillation, and the amplitude of the main system suddenly jumped into the oscillation of the limit cycle of extreme large amplitude, at which point the oscillation suppression capacity of the front and rear NES to the main system failed.

3.2. Bifurcation and chaos of coupled systems
As the incoming velocity continues to increase, the system will exhibit more complex dynamic characteristics. The following two groups of representative parameters are taken to study the nonlinear characteristics of the system.

All parameters of the main system remain unchanged, and the parameters of NES1 and the NES2 are taken as \( \delta_1 = 0.9 \), \( \delta_2 = -0.9 \), \( \lambda_1 = \lambda_2 = 0.1 \), \( C_1 = C_2 = 10 \), \( \varepsilon_1 = \varepsilon_2 = 0.01 \). The bifurcation characteristics of the heave oscillation are shown in Fig 3:
Fig. 3 Bifurcation of heave mode
The horizontal sitting is still non-dimensional incoming flow velocity, and the research scope is raised to $\Theta = 3.0$. The vertical coordinate is the point on the Poincare section of the heave oscillation.
According to the characteristics shown in Fig 3, the sections of $\Theta = 0.9 \sim 2.0$ are chosen to study.

Fig. 4 Trajectory change on the Poincare section
As it shows in Fig 4, first, when the non-dimensional incoming flow velocity is $\Theta = 0.9$, the oscillation of the system is quasi-periodic. As the velocity of the incoming flow increases, the oscillation state of the system suddenly develops into chaos, then again into periodic oscillation, and finally into permanent chaotic oscillation. The velocity at which a chaotic system occurs is between $\Theta = 0.9$ and $\Theta = 1$.

The specific location of chaos occurrence point in system 1 is studied. Take a point whose distance is near an exist solution. The sensitivity of the chaotic system to the initial value and the change of the distance between the final solutions are used to determine the location of the chaotic point.
(a) Trajectories on Poincare section  
(b) Sensitivity to initial value 

Fig. 5 Trajectories on Poincare section in system 1 and sensitivity of system to initial value, $\Theta = 0.91$

(a) Trajectories on Poincare section  
(b) Sensitivity to initial value 

Fig. 6 Trajectories on Poincare section in system 1 and sensitivity of system to initial value, $\Theta = 0.92$

(a) Trajectories on Poincare section  
(b) Sensitivity to initial value 

Fig. 7 Trajectories on Poincare section in system 1 and sensitivity of system to initial value, $\Theta = 0.93$
From Fig. 5 to Fig. 7, it can be seen that the three systems under different inflow velocities are all quasi periodic oscillations, which have obvious quasi periodic behaviour on the Poincare section, and the system is not sensitive to the initial value, and the distance between the two adjacent points of the system as the initial value solution has been kept in a limited range. The other parameters remain
unchanged, and the incoming flow speed continues to increase to 0.94, the oscillation state of the system changes into chaos. As can be seen from Fig. 8, the track of the system on the Poincare section no longer keeps the closed curve. And the system becomes very sensitive to the initial value. When two adjacent points are taken as the initial value, the distance between the two system solutions will increase in order of magnitude in a short time and will increase with the development of time. The system represented in Fig. 9 and Fig. 10 have larger range of chaotic attractors, and the maximum distance between the solutions will be the same as the attractor diameter of the chaotic system. In this parameter system, the point of chaos is $\Theta = 0.94$.

4. Conclusions
The bifurcation characteristics of the system in different parameter range are studied, and it is concluded that the system will experience bifurcation twice before developing into large amplitude LCOs.

First, after the system experiences Hopf bifurcation, the system changes from stable point to periodic oscillation. With the increase of the incoming flow velocity, there occurs Neimark-Sacker bifurcation, the oscillation of the system changes from periodic oscillation to quasi-periodic oscillation. As the velocity of incoming flow continues to increase, the oscillation of the system is transformed into large amplitude LCOs. Finally, the system enters chaos when the incoming flow velocity is much higher.

The delay at the chaotic occurrence point of the system also corresponds to the delay at each bifurcation point of the system, that is, the higher the incoming flow velocity corresponding to the chaotic phenomenon, the larger the oscillation suppression parameter range of the NES for the main system.

References
[1] Fung YC 2008 An introduction to the theory of aeroelasticity[M]. New York: Courier Dover Publications.
[2] Dowell EH, Crawley EF, Curtiss HC, et al. 2005 A modern course in aeroelasticity[M]. Netherlands: Springer.
[3] Nayfeh AH, Mook DT, Holmes P. Nonlinear oscillations[J]. Physics Today, 1980, 15(9): 63-65.
[4] Bunton RW, Denegri CM 2000 Limit cycle oscillation characteristics of fighter aircraft[J]. Journal of Aircraft, 37(5): 916-918.
[5] Cattarius J, Cochair JI, Douglas D, et al. 1999 Numerical wingstore interaction analysis of a parametric F-16 wing[D]. Virginia Tech.
[6] Denegri, Charles M. 2000 Limit cycle oscillation flight test results of a fighter with external stores[J]. Journal of Aircraft, 37(5): 761-769.
[7] Lee BH, Leblanc P 1986 Flutter analysis of a two-dimensional airfoil with cubic non-linear restoring force[J]. National Research Council Canada National Aeronautical Establishment.
[8] Lee BH 1987 Flutter analysis of a two-dimensional airfoil containing structural nonlinearities[R]. Canada: National Aeronautical Establishment, Aeronautical Report LR-618, National Research Council.
[9] Singh SN, Brenner M 2003 Limit cycle oscillation and orbital stability in aeroelastic systems with torsional nonlinearity[J]. Nonlinear Dynamics, 31(4): 435-450.
[10] O'Neil T, Strganac TW 1998 Aeroelastic response of a rigid wing supported by nonlinear springs[J]. Journal of Aircraft, 35(4): 616-622.
[11] Gilliatt HC, Strganac TW 2003 Kurdila AJ. An investigation of internal resonance in aeroelastic systems[J]. Nonlinear Dynamics, 31(1): 1-22.
[12] Lee BH, Liu L, Chung KW 2005 Airfoil motion in subsonic flow with strong cubic nonlinear restoring forces[J]. Journal of Sound and Vibration, 281(3-5): 699-717.
[13] Lind R, Snyder K, Brenner M 2001 Wavelet analysis to characterise non-linearities and predict limit cycles of an aeroelastic system[J]. Mechanical Systems and Signal Processing, 15(2): 337-356.
[14] Strganac TW, Ko J, Thompson DE 2000 Identification and control of limit cycle oscillations in
aeroelastic systems[J]. Journal of Guidance Control and Dynamics, 23(6): 1127-33.

[15] Li D, Guo S, Xiang J, et al. 2010 Control of an aeroelastic system with control surface nonlinearity[C]. 51st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Orlando, Florida: 2951.

[16] Li D, Guo S, Xiang J 2012 Study of the conditions that cause chaotic motion in a two-dimensional airfoil with structural nonlinearities in subsonic flow[J]. Journal of Fluids and Structures, 33: 109-126.

[17] Nayfeh AH, Mook DT 2008 Nonlinear oscillations[M]. New Jersey: John Wiley and Sons.

[18] Bajaj AK, Davies P, Chang SI 2018 On internal resonances in mechanical systems[J]. Nonlinear Dynamics and Stochastic Mechanics, 69-94.

[19] Nayfeh AH, Balachandran B 1989 Modal interactions in dynamical and structural systems[J]. Applied Mechanics Reviews, 42: 175-201.

[20] Sethna PR, Bajaj AK 1978 Bifurcations in dynamical systems with internal resonances[J]. Journal of Applied Mechanics, 45: 895-902.

[21] Sethna PR 1963 Coupled in certain classes of weakly nonlinear vibrating systems[C]. International Symposium on Nonlinear Differential Equations and Nonlinear Mechanics. Academic Press: 58-70.

[22] FengZC, Sethna PR 1990 Global bifurcation and chaos in parametrically forced systems with one-one resonance[J]. Dynamics and Stability of Systems, 5(4): 201-225.

[23] Tien WM, Namachchivaya N, Bajaj AK 1994 Nonlinear dynamics of a shallow arch under periodic excitation, part I: 1:2 internal resonance[J]. International Journal of Nonlinear Mechanics, 29(3): 349-366.

[24] Tien WM, Namachchivaya N, Bajaj AK 1994 Nonlinear dynamics of a shallow arch under periodic excitation, part II: 1:1 internal resonance[J]. International Journal of Nonlinear Mechanics, 29(3): 367-386.

[25] Johnson JM, Bajaj AK 1989 Amplitude modulated and chaotic dynamics in resonant motion of strings[J]. Journal of Sound and Vibration, 128: 87-107.

[26] Reilly O, Holmes PJ 1992 Non-linear non-planar and non-periodic vibrations of a string[J]. Journal of Sound and Vibration, 153: 413-435.

[27] Yang XL, Sethna PR 1992 Nonlinear forced vibrations of a nearly square plate-Antisymmetric case[J]. Journal of Sound and Vibration, 155: 413-441.

[28] Henrard J 1982 Capture into resonance: An extension of the use of adiabatic invariants[J]. Celestial Mechanics, 27: 3-22.

[29] Neishtadt AI 1975 Passage through a separatrix in a resonance problem with a slowly-varying parameter[J]. Journal on Applied Mathematical Mechanics, 39(4): 594-605.

[30] Neishtadt AI 1990 Averaging and passage through resonances[C]. Proceedings of the International Congress of Mathematicians, Kyoto: 1271-1283.

[31] Hall CD, Rand RH 1994 Spinup dynamics of axial dual-spin spacecraft[J]. Journal of Guidance, Control, and Dynamics, 17(1): 30-37.

[32] Kinsey RJ, Mingori DL, Rand RH 1990 Spinup through resonance of rotating unbalanced systems with limited torque[C]. AIAA/AAS Astrodynamics Conference, Portland: 805-813.

[33] Rand RH, Kinsey RJ, Mingori DL 1992 Dynamics of spinup through resonance[J]. International Journal of Non-Linear Mechanics, 27(3): 489-502.

[34] Haberman R 1983 Energy bounds for the slow capture by a center in sustained resonance[J]. SIAM Journal on Applied Mathematics, 43(2): 244-256.

[35] Kath WL 1983 Necessary conditions for sustained reentry roll resonance[J]. SIAM Journal on Applied Mathematics, 43(2): 314-324.

[36] Kevorkian J 1974 On a model for re-entry roll resonance[J]. SIAM Journal on Applied Mathematics, 26: 638-669.

[37] Wenfan Zhang, Yan Liu, Shengli Cao, et al. 2017 Targeted energy transfer between 2-D wing and nonlinear energy sinks and their dynamic behaviors[J]. Nonlinear Dynamics, 90(3): 1841-
1850.

[38] Wenfan Zhang, Jiazhong Zhang, Le Wang, et al. 2018 Study on targeted energy transfer and resonance captures in the 2D-wing and nonlinear energy sinks[J]. Journal of Vibration Testing and System Dynamics, 2(4): 297-306.