General Analysis of Two Products Inventory Model – With Production by Two Units System and Sales

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Abstract: In this paper two units system engenders two different products for sale. The two products are engendered piecemeal in pairs. The sale time commences when k dyads are engendered or when the two units system fails whichever occurs first. Postulating the engenderment and sale time of products have general distribution and the failure and rehabilitate rates of the two units are constants, the double Laplace Stieltjes transform of the joint distribution function of the time to sale and sale time are obtained. Their prospect of engenderment time E(T), sales time E(R) is derived by varying the number of engenderment k. To illustrate the application of Numerical results are distribution by giving different values for λ₁ λ₂ μ.

Mathematics subject classification: 30C45, 30C80.

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1 Introduction. Several researchers studied single commodity inventory systems of (s,S) type. Arrow, Karlin and Scart [1] first analysed such inventory systems. Daniel and Ramanarayanan [2] discussed (s, S) inventory system with desultory lead times and unit demand models. Murthy and Ramanarayanan [3, 4, 5, 6] have considered several (s,S) inventory systems. RajaRao.B [7] Life expectancy for setting the clock back to zero property Usha., Eswariprem, Ramanarayanan [8,9,10] have engendered some results in stochastic Analysis of Time to Vital Organs Failure of Gestational Diabetic Person. Wu and Liang-Yuh ouyang [11] studied (Q, R, L) inventory model with defective items. In authentic life inventory models variants of pairs of products are engendered for sales by the companies. For example drug manufacturing companies engender antibiotic and vitamin tablets. Oil refining companies engender fuel oils like petrol and engine oil to reduce friction and auto mobile companies engender engine and body of conveyances. Textiles mills slake the authoritative ordinance for shirting cloth and pant cloth, inner garments and outer garments land so on. In this paper we consider an inventory system in which two different products are engendered in pairs by a two-unit system. The sale time commences when k dyads are engendered a two- unit system fails. The Laplace Stieltjes Transform of distribution function of time to sale and sale time is derived. Their prospect times and numerical examples are presented.

2. Model

Assumptions: The company engenders two different products A & B and at a time only one type is engendered. The engenderment time of product A is arbitrary variable with Cdf \( G_P(p) \) and that of product B has cdf \( G_Q(q) \). Products A & B are engendered in successive manner. The engenderment time P+Q of a dyad has cdf \( G(P) \).
The products are engendered by a two unit system which fails when the two units are down and it works when at least one unit is good. Let the probability that either of two units fails during \((t, t+\Delta t)\) given that the two units are operating at time \(t\), be \(\lambda_1 \Delta t + o(\Delta t)\) and the probability that one fine-tuned unit fails during \((t, t+\Delta t)\) given that it is operating at time \(t\), be \(\lambda_2 \Delta t + o(\Delta t)\). The rehabilitation rate of the failed unit is \(\mu\).

Sale time commences when \(k\) numbers of pairs of products are engendered or when the two unit system fails.

The products are sold in pairs and the selling time of a dyad is arbitrary variable with Cdf \(R(q)\) and the selling time of product A is \(R_A(q)\).

**Analysis**

We note that the probability of \(n\) number of pairs produced in \((0, t)\)

\[ = G_n(t) - G_{n+1}(t) \text{ where } G_n(t) \text{ is the Cdf of } \sum_{i=1}^{n} (P_i + Q_i). \]

When the \(n^{th}\) pair is produced at time \(P < t\) during \(t - P\) there are two possibilities. After the \(n^{th}\) pair production,

(i) the production for product A is over but for B is not over or

(ii) the production for product A is also not over.

Their respective probabilities are given below. We note that the probability of \(n\) number of pairs and one production of A is over before \(t\)

\[ = \int_0^t g_n(x) \int_0^{t-x} p_\rho(u) \bar{G}_Q(t-x-u) du dx \]

\[ = P[ (n+1) \text{ number of A products and } n \text{ number of B products are produced during } (0,t)] \text{. Here } \bar{G}(x) = 1 - G(P) \]

Here \(g_n(x)\) is the pdf of \(\sum_{i=1}^{n} (P_i + Q_i)\) are produced in \((0,t)\). Also the probability of \(n\) number of pairs in \((0, t)\) and the production of A is not completed before \(t\) is

\[ = \int_0^t g_n(x) \bar{G}_P(t-x) dx. \]

Since the selling time starts when \(k\) pairs are produced or when the two unit system failed, \(T\) the time to start sales is given by

\[ T = \text{min (time to produce } k \text{ pairs, the time at which the two- unit system fails)} \]

To find the distribution function of the time to fail time to failure of the two units system, we need the functions \(P_{0,0}(u)=P\) (At time \(u\) the two units system is working, The system does not fail during \((0,u)\) / at time 0 the two units of the system are working). \(P_{0,1}(u)=P\) (at time \(u\) one unit is under repair, system does not fail during \((0,u)\) \(\downarrow\) (at time 0 the units are working). \(P_{0,2}(u)\) be the pdf of time to failure of the two units system, \(P_{0,2}(u)=p(\text{the two -unit system fails during } (u,u+du),\text{ it does not fail during } (0,u) \downarrow \text{the two units are working at time } 0)\).



We now calculate the \(P(.)\) functions. \(P_{0,0}(P)\) satisfies the following,

\[ P_{0,0}(P) = e^{-\lambda_1 x} + \int_0^x \lambda_1 e^{-\lambda_1 x} P_{1,0} \text{ (P-u) du} \]

Here \(P_{1,0}(P) = P(\text{At time } P \text{ the two units are working } / \text{at time } 0 \text{ one unit is in failed state}) \)

\[ P_{1,0}(P) = \int_0^x \int_0^\nu \mu e^{-\mu u} e^{-\lambda_2 u} \lambda_1 e^{-\lambda_1 (x-u)} du + \int_0^x \int_0^\nu \mu e^{-\mu u} e^{-\lambda_2 u} \lambda_1 e^{-\lambda_1 (v-u)} du \bar{P}_{1,0} \text{ (P-v) dv}. \]

The first term is the probability that the failed unit is repaired at \(u\) and no unit of the system fails during \((0,P)\) and the second term is the probability that the failed unit is repaired at \(u\) and other unit does not fail during \((0,u)\), a unit fails at \(v>u\) and at \(P\) all the two units are working. Laplace transforms of the above equations give,

\[ P^{*}_{0,0}(s) = (\lambda_2 + \mu + s)/[s^2 + s(\lambda_1 + \lambda_2 + \mu) + \lambda_1 \lambda_2] \]  

(3)
Here* indicates Laplace transform.

Using a similar argument we find

\[ P_{0,1}(x) = \int_0^x \lambda_1 e^{-x \lambda_1} d_u + \int_0^x \int_0^u \lambda_1 e^{-x \lambda_1} \mu e^{-(x-u) \lambda_2} du P_{0,1}(P-v)dv. \]

Taking Laplace Transform \( P^*_{0,1}(s) = \frac{\lambda_1}{s^2 + s(\lambda_1 + \lambda_2 + \mu) + \lambda_1 \lambda_2} \]

The failure density \( p_{0,2}(P) \) satisfies

\[ P_{0,2}(x) = \int_0^x \lambda_1 e^{-x \lambda_1} P_{1,2}(P-u)du \]

where \( P_{1,2}(x) dx = p(\text{the two units system fails during } (x, x+dx) \text{ at time 0 one unit of the system is under repair}) \]

\[ P_{1,2}(x) = \lambda_2 e^{-\lambda_2 x} e^{-\mu x} + \int_0^x e^{-\lambda_2 u} \mu e^{-\mu u} P_{0,2}(P-u)du \]

By Laplace transformation,

\[ P^*_{0,2}(s) = \frac{\lambda_1 \lambda_2}{s^2 + s(\lambda_1 + \lambda_2 + \mu) + \lambda_1 \lambda_2} \]

Equation (3) – (5) can be inverted easily \( \beta \)

\[ P_{0,0}(t) = \frac{1}{2} e^{-\alpha t} \left[ e^{\rho t} + e^{\beta t} \right] + \frac{1}{4\beta} \left( \lambda_2 - \lambda_1 + \mu \right) e^{-\alpha t} \left[ e^{\rho t} - e^{\beta t} \right] \]

\[ P_{0,1}(t) = \frac{\lambda_1}{2\beta} e^{-\alpha t} \left[ e^{\rho t} - e^{\beta t} \right] \]

\[ P_{0,2}(t) = \frac{\lambda_1 \lambda_2}{2\beta} e^{-\alpha t} \left[ e^{\rho t} - e^{\beta t} \right] \]

Here \( a = (\lambda_1 + \lambda_2 + \mu)/2 \)

and \( b = \frac{1}{2} \sqrt{((\lambda_1 - \lambda_2)^2 + (\mu)^2 + 2\mu(\lambda_1 + \lambda_2)} \)

\[ P_{0,0}(t) + P_{0,1}(t) = \text{survival function of two unit system.} \]

\[ = e^{-\alpha \beta t} \left[ \frac{\lambda_1}{2} \frac{1}{4\beta} \frac{\lambda_1}{\rho t} + e^{-\alpha \beta t} \left[ \frac{1}{2} \frac{\lambda_1}{4\beta} \frac{\lambda_1}{4\beta} \right] \right] \]

\[ = \frac{\lambda_1 \lambda_2}{2\beta} e^{-\alpha \beta t} \left[ e^{-(\alpha + \beta) t} \right] + \frac{\lambda_1 \beta}{2b} e^{-\alpha t} \left[ e^{-(\alpha + \beta) t} \right] \]

\[ = \frac{\lambda_1 \lambda_2}{2\beta} e^{-\alpha \beta t} - e^{-(\alpha + \beta) t} \]

The pdf of \( T \) is

\[ f_T(t) = \sum_{k=0}^{\infty} \int_0^t g_i(x) \int_0^x g_p(u) \hat{g}(t - x - u) du dx + \sum_{i=0}^{k-1} \int_0^t g_i(x) \hat{g_i}(t - x - u) du dx \]

\[ = \sum_{k=0}^{\infty} \int_0^t g_i(x) \int_0^x g_p(u) \hat{g}(t - x - u) du dx + \sum_{i=0}^{k-1} \int_0^t g_i(x) \hat{g_i}(t - x - u) du dx \].

The first term of the right side of (6) is the part of the pdf that the time to produce \( k \) number of pairs is \( t \) and the two units system has not failed up to time \( t \). One part of the second term is the part of the pdf that the two unit system fails at time \( t \), the time to produce \( i \) number of A and B pairs is \( P \), the \( (i+1)^{th} \) A product is produced at time \( P + u \) and the production of the \( (i+1)^{th} \) B product is not over during \((0, t-P-u)\) for \( 0 \leq u \leq k - 1 \). The other part of the second term is part of the pdf that the two units system fails at time \( t \), the time to produce \( i \) number of A and B pairs is \( P \) and the \( (i+1)^{th} \) production of product A is not over during \((0, t-P)\), \( 0 \leq u \leq k - 1 \).

This gives the joint pdf of time to start sales \( T \) and total sales time of pairs \( R \) as follows considering the sales time of the pairs.

\[ f_{T,R}(x,y) = g_k(x)(c_1 e^{-(\alpha + \beta) x} + c_2 e^{-(\alpha + \beta) x} r_k(y) + c_3 [e^{-(\alpha + \beta) x} - e^{-(\alpha + \beta) x}] \]
\[
\int_{0}^{x} g_i(u) \int_{0}^{x-u} g_p(v) \overline{g}_Q(x - u - v) dv du \int_{0}^{\gamma} r_i(w)r_A(y - w) dw + \sum_{i=0}^{k-1} \int_{0}^{x} g_i(x) \overline{g}_p(x - u) du r_i(y)
\]  

(7)

The double Laplace transform of the pdf is given by

\[
f^*_T_R(\epsilon, \eta) = \int_{0}^{\infty} \int_{0}^{\infty} e^{-x\epsilon} e^{-y\eta} f_{T,R}(x,y) dx dy
\]

\[
f^*_T_R(\epsilon, \eta) = \int_{0}^{\infty} \int_{0}^{\infty} e^{-x\epsilon} g_k(x)(c_1 e^{-(\alpha-\beta)x} + c_2 e^{-(\alpha-\beta)x} r_k(y)) dx dy
\]

\[
+ \int_{0}^{\infty} \int_{0}^{\infty} e^{-x\epsilon} c_3(e^{-(\alpha-\beta)x} - e^{-(\alpha+\beta)x}) \sum_{i=0}^{k-1} \int_{0}^{\infty} g_i(u) \int_{0}^{x-u} g_p(v) \overline{g}_Q(x - u - v) dv du \int_{0}^{\gamma} r_i(w)r_A(y - w) dw dx dy + \int_{0}^{\infty} \int_{0}^{\infty} e^{-x\epsilon} e^{-y\eta} c_3 e^{-(\alpha-\beta)x} \sum_{i=0}^{k-1} \int_{0}^{\infty} g_i(u) \overline{g}_p(x - u) du r_i(y) dx dy
\]

We get

\[
f^*_T,R(\epsilon, \eta) = c_1 g^k(\epsilon + \alpha - \beta)r^k(\eta) + c_2 g^k(\epsilon + \alpha + \beta)r^k(\eta) + c_3 \sum_{i=0}^{k-1} g^{r^i(\epsilon + \alpha - \beta)g_p((\epsilon + \alpha - \beta)\overline{g}_Q((\epsilon + \alpha - \beta)\epsilon + \alpha - \beta)r_A(\eta)
\]

\[
- c_3 \sum_{i=0}^{k-1} g^{r^i(\epsilon + \alpha + \beta)\overline{g}_Q((\epsilon + \alpha + \beta)\epsilon + \alpha + \beta)r_A(\eta)
\]

\[
f^*_T,R(\epsilon, \eta) = c_1 g^k((\epsilon + \alpha - \beta)r^k(\eta) + c_2 g^k((\epsilon + \alpha + \beta)r^k(\eta) +
\]

\[
\left[ \frac{1 - (g^e(\epsilon + \alpha - \beta))r(\eta)}{1 - g^e(\epsilon + \alpha - \beta)r(\eta)} \right]^k \left[ \frac{g_p(\epsilon + a - b)\overline{g}_Q((\epsilon + a - \beta))r_A(\eta) + \overline{g}_p(\epsilon + a - \beta)}{g^e(\epsilon + a - \beta)r(\eta)} \right] -
\]

\[
\left[ \frac{1 - (g^e(\epsilon + \alpha + \beta))r(\eta)}{1 - g^e(\epsilon + \alpha + \beta)r(\eta)} \right]^k \left[ \frac{g_p(\epsilon + a + \beta)\overline{g}_Q((\epsilon + a + \beta))r_A(\eta) + \overline{g}_p(\epsilon + a + \beta)}{g^e(\epsilon + a + \beta)r(\eta)} \right]
\]

(8)

The Laplace transform of T is
\[ f_{T,R}^*(\varepsilon, 0) = c_1 g^k((\varepsilon + \alpha - \beta)) + c_2 g^k((\varepsilon + \alpha + \beta)) + c_3 \left[ \frac{1-(g^r((\varepsilon+\alpha-\beta)))^k}{1-g^r((\varepsilon+\alpha-\beta))} \right] g_p^r((\varepsilon + \alpha - \beta)) + \tilde{g}_Q^r((\varepsilon + \alpha - \beta)) + \tilde{g}_p^r((\varepsilon + \alpha + \beta)) \]

\[ \frac{\partial}{\partial \varepsilon} f_{T,R}^*(0,0) = -E(T) \quad \text{and we obtain} \]

\[ E(T) = -c_1 g^k(\alpha - \beta) g''(\alpha - \beta) - c_2 g^k(\alpha + \beta) g''(\alpha + \beta) + c_3 \left[ \frac{1-g^r((\alpha+\beta))}{1-g^r((\alpha+\beta))} \right] g_p^r(\alpha - \beta) + \tilde{g}_Q^r(\alpha - \beta) \]

\[ \frac{\partial}{\partial \eta} f_{T,R}^*(0,0) = -E(R) \quad \text{and we obtain} \]

\[ E(R) = c_1 g^k(\alpha - \beta) kE(R_1) + c_2 g^k(\alpha - \beta) kE(R_1) - c_3 kE(R_1) \left[ \frac{1-g^r((\alpha+\beta))}{1-g^r((\alpha+\beta))} \right] g_p^r(\alpha - \beta) + \tilde{g}_Q^r(\alpha - \beta) \]

\[ \text{Section 3: Production Time } E(T), \text{ Sales Time } E(R), \]

On differentiation of equation (6) we get, \( \alpha \) 

\[ \frac{\partial}{\partial \varepsilon} f_{T,R}^*(0,0) = -E(T) \quad \text{and we obtain} \]

\[ E(T) = -c_1 g^k(\alpha - \beta) g''(\alpha - \beta) - c_2 g^k(\alpha + \beta) g''(\alpha + \beta) + c_3 \left[ \frac{1-g^r((\alpha+\beta))}{1-g^r((\alpha+\beta))} \right] g_p^r(\alpha - \beta) + \tilde{g}_Q^r(\alpha - \beta) \]

Similarly we note \( \frac{\partial}{\partial \eta} f_{T,R}^*(0,0) = -E(R) \) and we obtain

\[ E(R) = c_1 g^k(\alpha - \beta) kE(R_1) + c_2 g^k(\alpha - \beta) kE(R_1) - c_3 kE(R_1) \left[ \frac{1-g^r((\alpha+\beta))}{1-g^r((\alpha+\beta))} \right] g_p^r(\alpha - \beta) + \tilde{g}_Q^r(\alpha - \beta) + c_3 E(R_1) \left[ \frac{1-g^r((\alpha+\beta))}{1-g^r((\alpha+\beta))} \right] g_p^r(\alpha - \beta) + \tilde{g}_Q^r(\alpha - \beta) \quad (9) \]

\[ \text{Section: 4: Special case of this Model} \]

We now consider the special case in which P & Y re exponential random variables with parameters \( \alpha \) and \( \beta \) respectively, this gives

\[ E(T) = -c_1 g^k(\alpha - \beta) g''(\alpha - \beta) - c_2 g^k(\alpha + \beta) g''(\alpha + \beta) + c_3 \left[ \frac{1-g^r((\alpha+\beta))}{1-g^r((\alpha+\beta))} \right] g_p^r(\alpha - \beta) + \tilde{g}_Q^r(\alpha - \beta) \]

\[ E(R) = c_1 g^k(\alpha - \beta) kE(R_1) + c_2 g^k(\alpha - \beta) kE(R_1) - c_3 kE(R_1) \left[ \frac{1-g^r((\alpha+\beta))}{1-g^r((\alpha+\beta))} \right] g_p^r(\alpha - \beta) + \tilde{g}_Q^r(\alpha - \beta) + c_3 E(R_1) \left[ \frac{1-g^r((\alpha+\beta))}{1-g^r((\alpha+\beta))} \right] g_p^r(\alpha - \beta) + \tilde{g}_Q^r(\alpha - \beta) \quad (9) \]
\[ g_p^*(\alpha - \beta) = \frac{r}{r + \alpha - \beta} \]
\[ \bar{G}_Q^*(\alpha - \beta) = \frac{1}{s + \alpha - \beta} \]
\[ \bar{G}_P^*(\alpha - \beta) = \frac{1}{r + \alpha - \beta} \]

Using (7), (8) & (9) we find \( E(T) \) & \( E(R) \), when \( P \) & \( Y \) re exponentials with parameters \( \alpha \) & \( \beta \) as follows.

\[
E(T) = k \left( \frac{rs}{(r + \alpha - \beta)(s + \alpha - \beta)} \right)^k \left( \frac{r + s + 2\alpha - 2\beta}{(r + \alpha - \beta)(s + \alpha - \beta)} \right)^{\frac{-\lambda_1}{2\beta}} \left[ 1 - \left( \frac{\alpha + \beta}{(r + \alpha - \beta)(s + \alpha - \beta)} \right)^k \right] \]
\[
+ k \left( \frac{\lambda_1}{2\beta} \right) \left( \frac{r}{(r + \alpha - \beta)(s + \alpha - \beta)} \right)^{\frac{\lambda_1}{2\beta}} \left[ 1 - \left( \frac{\alpha + \beta}{(r + \alpha - \beta)(s + \alpha - \beta)} \right)^k \right]
\]

\[
E(R) = kE(R_1) \left( \frac{rs}{(r + \alpha - \beta)(\beta + \alpha - \beta)} \right)^k \left( \frac{-\lambda_1}{2\beta} \right) \left( \frac{\alpha + \beta}{(r + \alpha - \beta)(s + \alpha - \beta)} \right)^{\frac{\lambda_1}{2\beta}} \left[ 1 - \left( \frac{\alpha + \beta}{(r + \alpha - \beta)(s + \alpha - \beta)} \right)^k \right]
\]

3. **Numerical Example**

To illustrate the applications of the above result we give different values for \( \lambda_1, \lambda_2, \mu, \) and \( k \) and we obtain \( E(T), E(R) \) in the following table. For \( E(R_1) = 10 \) and \( E(R) = 20 \)

| b   | k   | E(T)     | E ( R )    |
|-----|-----|----------|-----------|
| 10  | 8.062 | 1        | 0.451925  | 0.092711  |
| 10  | 8.062 | 2        | 0.699881  | 0.513884  |
| 10  | 8.062 | 3        | 0.838209  | 0.771805  |
| 10  | 8.062 | 4        | 0.914601  | 0.879229  |
| 10  | 8.062 | 5        | 0.956512  | 0.917767  |
| 10  | 8.062 | 6        | 0.979385  | 0.930559  |
Keeping $\alpha=4$, $\beta=6$ and varying $k$ values, Production $E(T)$ increases and sales $E(R)$ increases, in 1 & 2 dimensional graph. This graph insist that while season starts, the sales increases and production also increases.

| $k$ | $\alpha=4$, $E(T)$ | $\alpha=5$, $E(T)$ | $\alpha=6$, $E(T)$ | $\alpha=7$, $E(T)$ | $\alpha=8$, $E(T)$ | $\alpha=9$, $E(T)$ |
|-----|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| 1   | 0.452               | 0.417               | 0.392               | 0.372               | 0.357               | 0.344               |
| 2   | 0.700               | 0.660               | 0.629               | 0.604               | 0.584               | 0.567               |
| 3   | 0.838               | 0.804               | 0.776               | 0.753               | 0.733               | 0.717               |
| 4   | 0.915               | 0.889               | 0.866               | 0.847               | 0.831               | 0.816               |
| 5   | 0.957               | 0.938               | 0.922               | 0.907               | 0.893               | 0.882               |
| 6   | 0.979               | 0.967               | 0.955               | 0.944               | 0.934               | 0.925               |
Varying $\alpha$ and $k$, the 3-d graph of $E(T)$ is obtained above.

\begin{center}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
 & $\alpha=0.4$, $E(R)$ & $\alpha=0.5$, $E(R)$ & $\alpha=0.6$, $E(R)$ & $\alpha=0.7$, $E(R)$ & $\alpha=0.8$, $E(R)$ & $\alpha=0.9$, $E(R)$ \\
\hline
$k$ & & & & & & \\
1 & 0.093 & 0.111 & 0.128 & 0.144 & 0.159 & 0.172 \\
2 & 0.514 & 0.524 & 0.507 & 0.468 & 0.412 & 0.344 \\
3 & 0.772 & 0.848 & 0.881 & 0.872 & 0.828 & 0.752 \\
4 & 0.879 & 1.014 & 1.107 & 1.156 & 1.162 & 1.125 \\
5 & 0.918 & 1.086 & 1.221 & 1.320 & 1.378 & 1.393 \\
6 & 0.931 & 1.115 & 1.274 & 1.406 & 1.504 & 1.564 \\
\hline
\end{tabular}
\end{center}

Varying $\alpha$ and $k$, the 3-d graph of $E(R)$ is obtained above.
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