Three-parameter bivariate gamma regression model for analyzing under-five mortality rate and maternal mortality rate

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Abstract. Gamma regression is often used to model continuous, right-skewed and strictly positive data. This paper discusses three-parameter gamma regression, namely scale, shape, and location parameter, with two correlated response variables. The purpose of this study is to determine the estimator parameters, perform the testing for the parameters using Maximum Likelihood Ratio Test (MLRT) and its application in the cases of the Under-five Mortality Rate (U5MR) and the Maternal Mortality Rate (MMR) in North Sulawesi, Gorontalo, Central Sulawesi Provinces in 2016. The parameter estimation in this global model obtained through MLE. Nevertheless, the results showed an unsolvable equation in closed-form, then we used numerical optimization. In this study, we were using the Berndt-Hall-Hall-Hausman optimization method. After the estimation results are obtained, the estimation parameters need to be tested with a simultaneous test with the MLRT, while for test partially, we used the Z test. Moreover, the factors that influence U5MR and MMR are the poor population, the obstetric complications handled, the mothers received Fe3 during pregnancy, the teenage pregnancy, the children under-five received vitamin A and the households with clean and healthy life behavior.

1. Introduction

The normal distribution is the continuous probability distribution that is most often used in statistics. Many problems that occur in nature, in the fields of science and engineering can be solved using a normal distribution, but there are some cases that do not provide enough explanation when forced to use a normal distribution such as rainfall or lifetime data, so we need to try other probability distributions such as the gamma distribution. The research of the gamma distribution has been developed, such as estimation parameter in k-variate gamma distribution with a heuristic approach where the form of the multivariate gamma distribution function used is that introduced by [1]. Instead of using the complex structure of the k-variate gamma distribution as the research methodology, researchers proposed methodology use a marginal distribution. They were using simulation studies performed to obtain estimated parameters through the proposed methodology and the other method, Maximum Likelihood, Maximum Product of Spacings Estimation and Least Square Estimation [2].

Gamma distribution is also used in regression analysis, it can be seen from these researchs, modeling gamma regression with two response variables, where researchers used the MLE method to determine the parameter estimators in the Bivariate Gamma Regression model with two-parameter and applied to river water pollution data in Surabaya in 2016 [3]. Moreover, [4] conducted a study of
three-parameter gamma regression, respectively, scale, shape, and location parameter with the trivariate response variable. The result of this study is the parameter estimators using MLE are not closed-form, so numerical optimization such as BFGS quasi-Newton is used.

Maternal Mortality Rate (MMR) and Under-five Mortality Rate (U5MR) are examples of continuous data. MMR and U5MR are issues of concern to almost all the country including in Indonesia because MMR and U5MR are not only able to assess maternal health programs but can assess the health and economic difficulties of the population. Even these issues are included in the Sustainable Development Goals (SDGs) set by the United Nations General Assembly in 2015, also known as Global Goals. The target that related to maternal and children health by 2030 reduces the global Maternal Mortality Rate to be as low as 70 per 100,000 live births and the Under-five Mortality Rate to be less than 25 per 1,000 live births [5].

MMR in the provinces of North Sulawesi, Gorontalo and Central Sulawesi is higher than the target to be achieved by the government, namely for MMR in the three provinces respectively, 129, 302 and 155 per 100,000 KH and for U5MR: 37, 78 and 85 per 1,000 KH, where these figures are higher than the national average and are still far from the SDGs target, so it is necessary to pay attention to the factors that influence the MMR and U5MR in these three provinces so that they can achieve the SDGs target in a way evenly distributed for each province [6, 7, 8].

A study in Indonesia using multilevel logistic regression to analyze the factors that affect maternal mortality rate showed that there are some important contributors to reduce the maternal mortality rate in Indonesia. Such as the number of doctors working at the community health center, the number of doctors in the village and the distance to the nearest hospital [9].

The objective of this study is to obtain a regression model with two mutually correlated response variables that follow a gamma distribution with three parameters. We need to estimate the parameter and to test the estimated parameter. In order to estimate the parameter, we use MLE, when the results showed an unsolvable problem in closed-form, then we have to use iterative procedures. Then we applied to the U5MR and MMR cases in North Sulawesi, Gorontalo, and Central Sulawesi Provinces. To perform tests of hypotheses on parameters that been estimated by ML simultaneously, we are using the MLRT and Z test to test the parameter partially.

2. Material and Methodology
In probability theory, the gamma distribution is a family of continuous distributions. The gamma distribution is a common form of the exponential distribution, which was first introduced in the 18th century by Swiss mathematician Leonard Euler. Gamma distribution has often been used because of the relationship between the gamma distribution and the distribution most commonly used, such as the normal distribution and the exponential distribution. The gamma distribution becomes one of the alternative models that is widely used for data that strictly positive, and right-skewed such as in the case of rainfall, climatology, and socio-economic models.

2.1 Three-Parameter Gamma Distribution
A random variable \( Y \), which follows a gamma distribution with parameters scale, shape and location: \( \alpha, \theta, \gamma \), respectively. The probability density function of \( Y \) is then defined as [1].

\[
f(y) = \begin{cases} 
\frac{(y-\gamma)^{\alpha-1} e^{-(y-\gamma)\theta}}{\theta^\alpha \Gamma(\alpha)} , & \alpha > 0; \theta > 0; y > \gamma \\
0, & \text{otherwise}
\end{cases}
\]

If \( Y_1, Y_2 \sim \text{Gamma} (\alpha_1, \alpha_2, \theta_1, \gamma_1, \gamma_2) \), the pdf of \( Y_1, Y_2 \) is given by:
The test statistic can be written as:

\[ G^2 = -2 \ln L(\hat{\phi}) + 2 \ln L(\hat{\Omega}) \]  

The rejection region is \( G^2 > \chi^2_{\alpha,k} \) with \( k \) is the difference between the number of parameters under the population and number of the parameter under the null hypothesis. Equation (4) shows that we need to find \( L(\hat{\phi}) \) and \( L(\hat{\Omega}) \) in order to define the likelihood ratio, and it will be discussed in section 3. Moreover, for the partial test, we are using the Z test with the following hypothesis.

\[ H_0 : \beta_{i1} = \beta_{i2} = \ldots = \beta_{ik} = 0 \]  
\[ H_1 : \text{at least one } \beta_{ij} \neq 0; i = 1,2; j = 1,2,\ldots,k \]
\[ H_0 : \beta_j = 0 \]
\[ H_1 : \beta_j \neq 0; j = 1, 2, \ldots, k; l = 1, 2. \]

Test statistic:
\[ Z = \frac{\hat{\beta}_j}{\text{se}(\hat{\beta}_j)} \]

with \( \text{se}(\hat{\beta}_j) = \sqrt{\text{var}(\hat{\beta}_j)} \), \( \text{var}(\hat{\beta}_j) \) is the diagonal element that corresponds with \(-H^{-1}(\lambda)\). The rejection region is \( |Z| > Z_{\alpha/2} \).

2.4 Under-Five Mortality

The under-five children mortality rate is the probability per 1,000 live births that in a specific year, a child born dying before reaching the age of five if subjected to current-age specific mortality rates [10]. The under-five mortality rate formula is given by.

\[ \text{U5MR} = \frac{\text{under-five mortality}}{\text{live births}} \times 1,000 \]

2.5 Maternal Mortality

According to [10], maternal mortality is the death of a woman while pregnant or death within 42 days of termination of pregnancy, regardless of the length of the pregnancy or place of delivery, i.e., deaths due to pregnancy or its management, but not due to other causes such as accidents, falls, etc. The definition of maternal death explicitly explains that the scope of maternal death is vast, not only due to deaths during childbirth but also includes deaths during pregnancy and childbirth. The indicator to measure maternal mortality is the Maternal Mortality Rate (MMR), which can be calculated using the following formula.

\[ \text{MMR} = \frac{\text{Maternal mortality}}{\text{live births}} \times 100,000 \]

2.6 The Factors That Influenced U5MR and MMR

A comprehensive and integrated framework in analyzing cultural, social, economic, behavioral, and biological factors that affect infant and maternal mortality was constructed by [11]. From the framework with a modified, we can take several variables that may influence U5MR and MMR, such as the poor population, the obstetric complications handled, and the mothers that received Fe3 during pregnancy. In addition, health care utilization, under-five children received vitamin A, and households having clean, healthy life behavior that influence infant mortality and maternal mortality.

2.7 Methodology

In this study, we use the secondary data obtained from the Health Profile Publications of North Sulawesi, Gorontalo, and Central Sulawesi Provinces in 2016 and the Publication Statistics of the Welfare Provinces of North Sulawesi, Central Sulawesi and Gorontalo (Sulutenggo) in 2016. Response variables used are U5MR \((Y_1)\) and MMR \((Y_2)\) with predictor variables, namely percentage of poor population \((X_1)\), the percentage of obstetric complications handled \((X_2)\), percentage of pregnant women who received Fe3 \((X_3)\), percentage of teenager pregnancy (under 17 years) \((X_4)\), percentage of use of health facilities \((X_5)\), percentage of children Under-five received vitamin A \((X_6)\) and the percentage of households having clean, healthy life behavior \((X_7)\). The observation units were 34 districts/cities consisting of ten districts and four cities in North Sulawesi Province, five districts and one city in Gorontalo Province and fifteen districts, one city in Central Sulawesi Province. Moreover, the procedures of this study can describe as follows.
1. Estimating the parameters for the bivariate gamma regression model using MLE. In the first step, we have to define the likelihood and the In likelihood function by (4). Then Maximizing In likelihood function by computing the first partial derivative and setting the partial derivative equal to zero. When the results showed in not a closed-form, then we have to use numerical optimization (e.g., by BHHH) to obtain the estimation parameter. BHHH’s algorithm begins with setting the initial guess: \( \hat{\lambda}_n = \left[ \hat{\lambda}_{00}, \hat{\gamma}_{00}, \hat{\beta}_{00} \right] \), then computing the gradient vector and the Hessian matrix (approximated as the negative of the sum the outer of the gradient). The stopping criterion is \( \| \hat{\lambda}_{n} - \hat{\lambda}_{n-1} \| < \varepsilon \).

2. Establishing the test statistic for the BGR model using MLRT. We first need to find the maximum of the likelihood function with respect to the parameters when the parameters are in the null parameter space and the population parameter space. Then constructing the test statistic \( G^2 \).

3. Determining the factors that influence the U5MR and MMR with the BGR model, we need to perform descriptive statistics for each variable, testing gamma distribution on the response variables and calculating the correlation value between response variables then performing multicollinearity tests using VIF values. And then analyzing the data using the three-parameter bivariate gamma regression model and determining the deviance value. When we obtain the coefficient of the global model, we need to calculating the Mean Square Error (MSE) of estimation and testing simultaneously and partially. After the model is obtained, the next step is calculating the correct AIC (AICc) in order to perform the goodness of fit a model and then determining the conclusions.

3. Results and Discussion

The results of this research demonstrated in this section that consists of estimation parameters using MLE, parameter testing and BGR model application in U5MR and MMR in North Sulawesi, Gorontalo dan Central Sulawesi in 2016.

3.1 Parameter Estimation and Test Statistic of BGR Model

To estimate the parameters using MLE, we can follow the steps outlined in Section 2.7. First, we need to define the ln-likelihood and can be written as.

\[
L(\theta, \gamma_1, \gamma_2, \beta_1, \beta_2) = \prod_{i=1}^{n} f(y_{1i}, y_{2i}) = \prod_{i=1}^{n} \left( \frac{e^{\gamma_1 y_{1i} - \gamma_2 y_{2i} - \gamma_1 - \gamma_2}}{\theta} \right) \left( \frac{\gamma_1 y_{1i} - \gamma_2 y_{2i} - \gamma_1 - \gamma_2}{\theta} \right) \left( \frac{\gamma_1 y_{1i} - \gamma_2 y_{2i} - \gamma_1 - \gamma_2}{\theta} \right)
\]

\[
\ln L(\theta, \gamma_1, \gamma_2, \beta_1, \beta_2) = \sum_{i=1}^{n} \left( \frac{e^{\gamma_1 y_{1i} - \gamma_2 y_{2i} - \gamma_1 - \gamma_2}}{\theta} \ln(y_{1i} - y_{1i}) \right) + \sum_{i=1}^{n} \left( \frac{e^{\gamma_1 y_{1i} - \gamma_2 y_{2i} - \gamma_1 - \gamma_2}}{\theta} \ln(y_{2i} - y_{1i} - y_{2i}) \right) - \sum_{i=1}^{n} \left( \frac{(y_{1i} - y_{1i}) - \gamma_1}{\theta} \right) - \sum_{i=1}^{n} \left( \frac{(y_{2i} - y_{1i} - y_{2i}) - \gamma_2}{\theta} \right) \ln \theta - \sum_{i=1}^{n} \ln \left( \frac{e^{\gamma_1 y_{1i} - \gamma_1}}{\theta} \right) - \sum_{i=1}^{n} \ln \left( \frac{e^{\gamma_2 y_{2i} - \gamma_2}}{\theta} \right)
\]

(6)
To estimate the parameters of the bivariate gamma regression model, we need to find the derivative of equation (6) with respect to each parameter and setting these to zero. It can be written as follow.

\[
\frac{\partial \ln L(\theta, \gamma_1, \gamma_2, \beta_1, \beta_2)}{\partial \theta} = 0, \quad \frac{\partial \ln L(\theta, \gamma_1, \gamma_2, \beta_1, \beta_2)}{\partial \gamma_1} = 0, \quad \frac{\partial \ln L(\theta, \gamma_1, \gamma_2, \beta_1, \beta_2)}{\partial \gamma_2} = 0, \quad \frac{\partial \ln L(\theta, \gamma_1, \gamma_2, \beta_1, \beta_2)}{\partial \beta_1} = 0, \quad \frac{\partial \ln L(\theta, \gamma_1, \gamma_2, \beta_1, \beta_2)}{\partial \beta_2} = 0
\]

And we obtain the first derivative of ln likelihood function with respect to \( \theta \):

\[
\begin{align*}
\frac{\partial \ln L(\theta, \gamma_1, \gamma_2, \beta_1, \beta_2)}{\partial \theta} &= -\sum_{i=1}^{n} \frac{\ln(y_{2i} - y_{1i} - \gamma_2)(\frac{e^{\beta_1} - e^{\beta_2}}{\theta})}{\theta^2} - \sum_{i=1}^{n} \frac{n(y_{1i} + \gamma_2)}{\theta^2} - \\
&\quad + \sum_{i=1}^{n} \left( \psi \left( \frac{e^{\beta_1} - e^{\beta_2}}{\theta} \right) \right) \left( \frac{e^{\beta_1} - e^{\beta_2}}{\theta} \right) - \sum_{i=1}^{n} \left( \ln(\theta_1 + \ln(\theta_2)) + \frac{\ln(\theta_2)}{\theta^2} \right) \left( \frac{e^{\beta_1} - e^{\beta_2}}{\theta} \right) \left( \frac{e^{\beta_1} - e^{\beta_2}}{\theta} \right) + \\
&\quad + \sum_{i=1}^{n} \left( \psi \left( \frac{e^{\beta_1} - e^{\beta_2}}{\theta} \right) \right) \left( \frac{e^{\beta_1} - e^{\beta_2}}{\theta} \right)
\end{align*}
\]

where, \( \psi(\bullet) = \frac{\partial \ln \Gamma(\bullet)}{\partial (\bullet)} \)

The first partial derivative of ln likelihood function with respect to \( \gamma_1 \) and \( \gamma_2 \):

\[
\begin{align*}
\frac{\partial \ln L(\theta, \gamma_1, \gamma_2, \beta_1, \beta_2)}{\partial \gamma_1} &= \sum_{i=1}^{n} \left( -\frac{\ln(y_{1i} - \gamma_1)}{\theta} - \frac{e^{\beta_1} - e^{\beta_2}}{(y_{1i} - \gamma_1)\theta} \right) + \frac{n}{\theta} \ln \theta + \sum_{i=1}^{n} \frac{\psi}{\theta} \\
&\quad + \sum_{i=1}^{n} \left( \frac{e^{\beta_1} - e^{\beta_2}}{\theta} \right) \left( \frac{e^{\beta_1} - e^{\beta_2}}{\theta} \right) \left( \frac{e^{\beta_1} - e^{\beta_2}}{\theta} \right) \\
\frac{\partial \ln L(\theta, \gamma_1, \gamma_2, \beta_1, \beta_2)}{\partial \gamma_2} &= \sum_{i=1}^{n} \left( -\frac{\ln(y_{2i} - y_{1i} - \gamma_2)}{\theta} - \frac{(e^{\beta_1} - e^{\beta_2}) - (e^{\beta_1} - e^{\beta_2} - \gamma_2)}{(y_{2i} - y_{1i} - \gamma_2)\theta} \right) + \frac{n}{\theta} \ln \theta + \sum_{i=1}^{n} \frac{\psi}{\theta} \\
&\quad + \sum_{i=1}^{n} \left( \frac{e^{\beta_1} - e^{\beta_2}}{\theta} \right) \left( \frac{e^{\beta_1} - e^{\beta_2}}{\theta} \right) \left( \frac{e^{\beta_1} - e^{\beta_2}}{\theta} \right)
\end{align*}
\]

The first partial derivative of the ln likelihood function with respect to \( \beta_1 \) and \( \beta_2 \):

\[
\begin{align*}
\frac{\partial \ln L(\theta, \gamma_1, \gamma_2, \beta_1, \beta_2)}{\partial \beta_1} &= \sum_{i=1}^{n} \frac{x_{i}^T e^{\beta_1} \ln(y_{2i} - y_{1i} - \gamma_2)}{\theta} + \sum_{i=1}^{n} \frac{x_{i}^T e^{\beta_1} \ln(y_{1i} - y_{2i} - \gamma_2)}{\theta} - \\
&\quad + \sum_{i=1}^{n} \left( \frac{\psi}{\theta} \right) \left( \frac{e^{\beta_1} - e^{\beta_2}}{\theta} \right) \left( \frac{e^{\beta_1} - e^{\beta_2}}{\theta} \right) \\
\frac{\partial \ln L(\theta, \gamma_1, \gamma_2, \beta_1, \beta_2)}{\partial \beta_2} &= \sum_{i=1}^{n} \frac{x_{i}^T e^{\beta_2} \ln(y_{2i} - y_{1i} - \gamma_2)}{\theta} - \sum_{i=1}^{n} \frac{x_{i}^T e^{\beta_2} \ln(y_{1i} - y_{2i} - \gamma_2)}{\theta} - \\
&\quad - \sum_{i=1}^{n} \left( \frac{\psi}{\theta} \right) \left( \frac{e^{\beta_1} - e^{\beta_2}}{\theta} \right) \left( \frac{e^{\beta_1} - e^{\beta_2}}{\theta} \right)
\end{align*}
\]
(7), (8), (9), (10), and (11) show that these equations are not closed-form so that numerical analysis is needed to provide a solution. As in the settlement for the previous univariate regression parameter estimation, which uses a numerical approach with BHHH. For testing the parameter estimation with the hypothesis in Section 2.3, the test statistics are obtained for simultaneous testing and the Z test in (5) for partial testing. To obtain the $L(\hat{\omega})$ and $L(\hat{\Omega})$ for the test statistic, we need to find the first partial derivative of ln likelihood under the population and ln likelihood under the null hypothesis. Thus, the test statistic is shown in the equation below.

$$G^2 = -2 \ln \Lambda = -2 \ln (L(\hat{\omega}) - L(\hat{\Omega}))$$

with,

$$\ln L(\hat{\omega}) = \sum_{i=1}^{n} \frac{e^{\hat{\beta}_1} - \hat{\gamma}_1 - \hat{\theta}}{\hat{\theta}} \ln (y_{ii} - \hat{\gamma}_1) + \sum_{i=1}^{n} \frac{e^{\hat{\beta}_0} - e^{\hat{\beta}_1} - \hat{\gamma}_2 - \hat{\theta}}{\hat{\theta}} \ln (y_{ii} - y_{ii} - \hat{\gamma}_2) -$$

$$\sum_{i=1}^{n} \frac{y_{ii} - \hat{\gamma}_1 - \hat{\gamma}_2 - \hat{\theta}}{\hat{\theta}} \ln \theta - \sum_{i=1}^{n} \ln \left( \frac{e^{\hat{\theta} - \hat{\gamma}_1}}{\hat{\theta}} \right) - \sum_{i=1}^{n} \ln \left( \frac{e^{\hat{\theta} - \hat{\gamma}_2}}{\hat{\theta}} \right) - \sum_{i=1}^{n} \ln \left( \frac{e^{\hat{\theta} - \hat{\gamma}_1 - \hat{\gamma}_2}}{\hat{\theta}} \right)$$

$$\ln L(\hat{\Omega}) = \sum_{i=1}^{n} \frac{e^{\hat{x}_i \hat{\beta}_1} - \hat{\gamma}_1 - \hat{\theta}}{\hat{\theta}} \ln (y_{ii} - \hat{\gamma}_1) + \sum_{i=1}^{n} \frac{e^{\hat{x}_i \hat{\beta}_0} - e^{\hat{x}_i \hat{\beta}_1} - \hat{\gamma}_2 - \hat{\theta}}{\hat{\theta}} \ln (y_{ii} - y_{ii} - \hat{\gamma}_2) -$$

$$\sum_{i=1}^{n} \frac{y_{ii} - \hat{\gamma}_1 - \hat{\gamma}_2 - \hat{\theta}}{\hat{\theta}} \ln \hat{\theta} - \sum_{i=1}^{n} \ln \left( \frac{e^{\hat{x}_i \hat{\beta}_1 - \hat{\gamma}_1}}{\hat{\theta}} \right) - \sum_{i=1}^{n} \ln \left( \frac{e^{\hat{x}_i \hat{\beta}_0 - e^{\hat{x}_i \hat{\beta}_1} - \hat{\gamma}_2}}{\hat{\theta}} \right)$$

3.2 Descriptive Statistics of Response and Predictor Variable

Table 1 provides descriptive statistics for the response variables and predictor variables. It clearly shows the average U5MR in the provinces of North Sulawesi, Gorontalo, and Central Sulawesi is 10.463 per 1,000 live births, and the average of MMR is 209.6 per 100,000 live births. Variable $X_7$, that is, percentage of households with clean and healthy life behavior, with the minimum value of 27.12% and the maximum value of 93.90% make the range large with an average of 57.06%. Also, we can see that the mean of the percentage of teenage pregnancy (under 17 years) in North Sulawesi, Gorontalo, and Central Sulawesi is 11.851% with the maximum value is 20.85%.

| Variable | Mean | StDev | Minimum | Median | Maximum |
|----------|------|-------|---------|--------|---------|
| U5MR     | 10.463 | 5.558 | 1.139   | 10.682 | 28.571  |
| MMR      | 209.6 | 125.8 | 66.3    | 182.8  | 655.2   |

| Predictor Variable | Mean | StDev | Minimum | Median | Maximum |
|--------------------|------|-------|---------|--------|---------|
| $X_1$              | 15.95| 12.96 | 5.24    | 14.09  | 78.36   |
| $X_2$              | 76.55| 28.92 | 7.24    | 79.28  | 154.85  |
| $X_3$              | 71.23| 18.45 | 21.78   | 71.57  | 106.83  |
| $X_4$              | 11.851| 4.239| 4.770   | 12.645 | 20.850  |
| $X_5$              | 65.52| 25.62 | 0.00    | 71.93  | 98.15   |
| $X_6$              | 83.90| 21.46 | 28.29   | 86.97  | 130.11  |
| $X_7$              | 57.06| 17.29 | 27.12   | 57.36  | 93.90   |

Predictor variables need to be tested the multicollinearity using VIF statistics. VIF values greater than 10 indicate that there is a correlation on the predictor variable. Based on Table 2, the VIF value
for each variable is smaller than 2. It means that there is no multicollinearity in the predictor variable so that the variables can be included in the BGR model testing.

| Table 2. Multicollinearity Test |
|---------------------------------|
| VIF | X₁ | X₂ | X₃ | X₄ | X₅ | X₆ | X₇ |
|-----|----|----|----|----|----|----|----|
| 1.26| 1.61| 1.38| 1.66| 1.77| 1.29| 1.22|

**Figure 1.** Histogram of of Y₁ and Y₂

The graphs in Figure 1 show that the data are skewed to the right, also known as positively skewed. Besides, we were testing the distribution for each of the response variables and produce a statistical value for U5MR: 0.3374 with P-value 0.8643 and for MMR: 0.2274 with P-value: 0.9919. It is clear that the two variables produce a P-value greater than α = 0.05, it can be concluded that U5MR and MMR follow a gamma distribution. Then we obtained a correlation value between the U5MR and MMR response variables that is equal to 0.4948.

### 3.3 BGR Model for U5MR and MMR

As noted in the previous section, the BGR model produces global parameters, and it means that the estimated parameters obtained to apply for all districts/cities in the provinces of North Sulawesi, Gorontalo, and Central Sulawesi. The following is a model of U5MR and MMR with a value of $G^2 = 11866.87 > \chi^2$ table = 6.571 with a significance level 0.05 for test the estimated parameter simultaneously. The conclusion is to reject $H_0$, and it means that there is at least one predictor variable influence in the model. Then, the partial test is needed to know the variable that significantly influences the model using the Z test.

| Table 3. Estimation Parameter for BGR Model |
|---------------------------------------------|
| Parameter Estimate | Z | P-Value | Parameter Estimate | Z | P-Value |
| $\beta_{10}$ | -3.3187 | 210148.6 | 0.000 | $\beta_{20}$ | -5.7607 | 13.7x10³ | 0.000 |
| $\beta_{11}$ | -0.0053 | -46.1449 | 0.000 | $\beta_{21}$ | -0.0041 | 34.5751 | 0.000 |
| $\beta_{12}$ | 0.0047 | -14.5403 | 0.000 | $\beta_{22}$ | -0.0130 | 16.2359 | 0.000 |
| $\beta_{13}$ | -0.0011 | 3.1434 | 0.002 | $\beta_{23}$ | -0.0060 | 23.1976 | 0.000 |
| $\beta_{14}$ | 0.0719 | -103.4387 | 0.000 | $\beta_{24}$ | -0.0127 | -177.446 | 0.000 |
| $\beta_{15}$ | -0.0010 | 4.4921 | 0.000 | $\beta_{25}$ | 0.0160 | 98.8251 | 0.000 |
| $\beta_{16}$ | 0.0088 | -56.9205 | 0.000 | $\beta_{26}$ | 0.0030 | -5.5056 | 0.000 |
| $\beta_{17}$ | 0.0173 | -131.825 | 0.000 | $\beta_{27}$ | 0.0089 | -47.8628 | 0.000 |

Table 3 shows the estimated results of BGR model parameters with the Z test statistic. The P-value of these estimated parameters showed that the factors that influence the U5MR and MMR are the percentage of poor population ($X₁$), the percentage of obstetric complications handled ($X₃$), the percentage mothers received Fe3 during pregnancy ($X₅$), the percentage of teenage pregnancy (under
17 years) \( (X_4) \), the percentage of health care utilization \( (X_5) \), the percentage of children Under-five received vitamin A \( (X_6) \) and the percentage of households with clean and healthy life behavior \( (X_7) \). In other words, all variables are significant influence U5MR and MMR with a very small p-value of 0.00. Based on table 3, the BGR model can be formed as follows.

\[
\hat{\mu}_1 = \exp(-3.32 - 0.0053x_{11} + 0.0047x_{12} - 0.0011x_{13} + 0.0719x_{14} - 0.0010x_{15} - 0.0088x_{16} - 0.0173x_{17}) \\
\hat{\mu}_2 = \exp(-5.76 - 0.004x_{21} - 0.0130x_{22} - 0.006x_{23} - 0.0127x_{24} + 0.016x_{25} - 0.003x_{26} - 0.0089x_{27})
\]

Based on the model above, it can be interpreted that an increase of 1% the percentage of mothers received Fe3 during pregnancy \( (X_2) \) will decrease the under-five mortality rate of \( \exp(-0.0011) = 0.998 \) times and decrease maternal mortality of \( \exp(-0.006) = 0.994 \) times as long as the other predictor variables are constant. The coefficient percentage of households with clean and healthy life behavior \( (X_7) \) indicates that for a 1% additional in \( X_7 \), we expect to see about an \( \exp(-0.0173) = 0.983 \) times decrease in maternal mortality and decrease the maternal mortality rate of \( \exp(-0.0089) = 0.991 \) times. The MSE obtained for U5MR and MMR were 99.83 and 24678.66. Furthermore, the goodness of the BGR model seen from the AICc value was 1108.88.

4. Conclusion

The three-parameter BGR model proposed to obtain the factors that influencing U5MR and MMR globally. In other words, the coefficient values for every location are the same. Furthermore, the result shows that the percentage of poor population, percentage of obstetric complications handled, percentage of mothers received Fe3 during pregnancy, percentage of teenage pregnancy (under 17 years), percentage of children under-five received vitamin A and the percentage of households with clean and healthy life behavior are significantly influencing U5MR and MMR. Therefore, it is necessary to consider by the provincial government to make these factors as a reference for improving the quality of health of under-five children and mothers in the provinces of North Sulawesi, Gorontalo, and Central Sulawesi. Moreover, the recommendation to further research to modeling the U5MR and MMR with geographically weighted regression, so we can get more information about the factors that affect the U5MR and MMR for every district and city in Sulawesi Utara, Gorontalo and Sulawesi Tengah.

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