Acoustic waves in polariton wires

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Abstract. The interaction of exciton-polariton condensates with acoustic phonons in a polariton wire is analyzed. For the case of a coherently pumped condensate we demonstrate that above a certain threshold the parametric instability in the system develops and leads to the generation of an acoustic wave and polariton harmonics with a lower frequency.

1. Introduction

In this work, we present the investigation of the interaction between a condensate of exciton-polaritons confined within a 1D microcavity and a coherent phonon field of the same dimensionality. From an experimental point of view, this can be achieved by a ring-shaped polariton microcavity \cite{1} on a substrate that doesn’t allow the generated acoustic waves to leak from the polariton wire to the substrate. The ring should have a large enough radius to make the discreteness of the spectrum in the ring negligible. We employ the model of the interaction between exciton-polaritons and phonons developed before\cite{2, 3}, and consider the case of the coherent pump of a polariton mode. We show that after once a certain threshold of the pump is passed, the parametric instability occurs and additional polariton harmonics of a lower frequency together with a coherent acoustic wave are generated.

2. Analytical model

The coupled equations describing the interaction between excitons or exciton-polaritons and phonons in 1D is given by \cite{2}:

\[
\begin{align*}
\mathcal{L}\Psi &= \left[ -\frac{\hbar^2 \partial_x^2}{2m} + \alpha|\Psi|^2 + g\Phi - i\hbar \gamma \right] \Psi + P(x, t), \\
\partial_t^2 \Phi &= \frac{1}{\rho} \partial_x \left[ Y\Phi + g|\Psi|^2 \right] - \Gamma \partial_t \Phi,
\end{align*}
\]

where $\Psi$ and $\Phi$ are the polariton and phonon scalar fields correspondingly, $m$ is the polariton effective mass, $\alpha$ and $g$ are the polariton-polariton and polariton-phonon interaction constants, $\gamma$ and $\Gamma$ are the polariton and phonon decay rates, $Y$ and $\rho$ are the crystal Young’s modulus and density, which are related to the speed of sound $c = \sqrt{Y/\rho}$. The sketch of the system if plotted in Fig. 1.
We rewrite the Eqs. (1,2) in dimensionless form by redefining $t \rightarrow t/t_0$, $x \rightarrow x/x_0$, $\gamma \rightarrow \gamma t_0$, $\Gamma \rightarrow \Gamma t_0$, $\Psi \rightarrow \Psi \sqrt{x_0}/\hbar$, $\alpha \rightarrow \alpha/\hbar c$, $g \rightarrow g/\sqrt{\hbar \rho c^3}$, $P \rightarrow -it_0 \sqrt{x_0} P/\hbar$, where $t_0 = \hbar/(mc^2)$, $x_0 = \hbar/(mc)$:

$$\partial_t \Psi = \left[ i \frac{\partial^2}{\partial x^2} - ig \Phi - i\alpha |\Psi|^2 - \gamma \right] \Psi + P,$$

$$\partial^2 \Phi = \partial^2 \frac{|\Psi|^2}{\hbar} - \Gamma \Phi,$$  

Performing the transformation $\Psi \rightarrow \Psi \exp(i \kappa x - i \delta t)$ we arrive at

$$\partial_t \Psi = \left[ i \frac{\partial^2}{\partial x^2} - \kappa \partial_x + i \mu - i \alpha |\Psi|^2 - ig \Phi - \gamma \right] \Psi + P,$$

where $\mu = \delta - \kappa^2/2$ is the detuning of the pumping frequency from the dispersion of free polaritons.

We seek for the solution in the form of a perturbed spatially uniform wavefunction $\Psi = \psi_0 + \psi$ with $\psi_0 = \sqrt{N_0}$ chosen real without loss of generality. The phonon field $\Phi = \phi$ produced by the perturbation is of the same order of smallness

$$\partial_t \psi = \left[ i \frac{\partial^2}{\partial x^2} - \kappa \partial_x + i \mu - i \alpha |\Psi|^2 - iG \phi \right] \psi - iA \psi^* + iG \phi,$$

$$\partial^2 \phi = \partial^2 \left[ \phi + G (\psi + \psi^*) \right] - \Gamma \partial_t \phi,$$

where $G = g \psi_0$ and $A = \alpha N_0$. Eqs. (6)-(7) can be expressed in a matrix form

$$\partial_t U = \hat{B} U,$$

where $U = (\text{Re}\{\psi\}, \text{Im}\{\psi\}, \phi, \partial_t \phi)^T$ and the linear matrix differential operator is given by

$$\hat{B} = \begin{pmatrix}
-\gamma - \kappa \partial_x & -\mu - \frac{1}{2} \partial^2_x - A & 0 & 0 \\
\mu + \frac{1}{2} \partial^2_x + 3A & -\gamma - \kappa \partial_x & -G & 0 \\
0 & 0 & 0 & 1 \\
2G \partial^2_x & 0 & \partial^2_x & -\Gamma
\end{pmatrix}.$$  

Having done that, we can find the eigenmodes in the form of plane waves $U \propto \exp(ikx - i \omega t)$ and the corresponding eigenvalues. After some algebraic manipulations we obtain the equation on the dispersion relation $\omega(k)$ for the excitations existing in the system, that can be cast to the convenient form

$$(\omega - \kappa k + Q(k) + i\gamma) (\omega - \kappa k - Q(k) + i\gamma) \times$$

$$\left( \omega + \sqrt{k^2 - \Gamma^2/4 + i\Gamma} \right) \left( \omega - \sqrt{k^2 - \Gamma^2/4 + i\Gamma} \right) =$$

$$G^2 k^2 \left[ k^2 - 2(\mu + A) \right],$$

where $Q(k) = g \psi_0$.
where
\[ Q(k) = \sqrt{(\mu + 3A - \frac{k^2}{2})(\mu + A - \frac{k^2}{2})}. \]  

(11)

Solving this equation numerically we obtained the dispersion curves of the system, plotted in Fig. 2.

\[ \Phi = \frac{-gk_r^2A_1A_r^* \exp(-ik_r x + i\omega_r t)}{k_r^2 - \omega_r^2 + i\Gamma \omega_r} + c.c. \]  

(13)

If the resonant condition \( \omega_r = k_r \) is satisfied, the equation above reduces to

\[ \Phi = igk_r A_1 A_r^* \exp(-ik_r x + i\omega_r t) + c.c. \]  

(14)

Substituting the expression (14) into the first line of Eq. (1) and discarding the terms that aren’t proportional to \( \exp(i k_r x - i \omega_r t) \) we obtain

\[ \gamma A_2 = \frac{g^2 k_r}{\Gamma} |A_1|^2 A_2, \]  

(15)
from where we can get the expression for the population of the directly excited mode \(N_1 = |A_1|^2\) above the instability threshold \(A_2 \neq 0\)

\[
N_1 = \frac{\gamma \Gamma}{g^2 k_r}, \tag{16}
\]

from where we clearly see that in order to make possible an experimental observation of the phenomenon, one should make the losses as small as possible.

3. Conclusion
The mutual dynamics of exciton-polaritons and acoustic waves is studied in the present work. It is demonstrated that there may be a resonant interaction between exciton-polaritons and acoustic waves. The resonant condition was derived analytically and it is shown that polaritons can excite coherent acoustic waves and new polaritons having lower frequencies. This is reminiscent of the parametric processes used in optical parametric oscillators. The states appear in the system are of hybrid nature with the polaritonic and acoustic components.

The theory is developed for the case of coherently driven systems taking into account finite losses in both subsystems (polaritonic and acoustic). The spectra of linear excitations are analyzed and it is demonstrated that the pump changes the positions of the resonances. The formation of the stationary states was analyzed within a framework of perturbation theory.

The discussed effects are not only of fundamental interest but also can have practical importance. For instance, the phenomena described in the work can find application for acousto-polariton lasers.

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