Mirror symmetry breaking in toy models of developed turbulence

Michal Hnatič
Faculty of Sciences, P.J. Šafárik University, Moyzesova 16, 040 01 Košice, Slovakia
Bogoliubov Laboratory of Theoretical Physics, Institute for Nuclear Research, 141980 Dubna, Russia
Institute of Experimental Physics, SAS, Watsonova 47, 040 01 Košice, Slovakia
E-mail: hnatie@saske.sk

Georgii Kalagov
Department of Theoretical Physics, St. Petersburg University, Ulyanovskaya 1, St. Petersburg, Petrodvorets, 198504 Russia
Faculty of Sciences, P.J. Šafárik University, Moyzesova 16, 040 01 Košice, Slovakia
E-mail: kalagov.g@gmail.com

Tomáš Lučivjanský
Faculty of Sciences, P.J. Šafárik University, Moyzesova 16, 040 01 Košice, Slovakia
E-mail: tomas.lucivjansky@upjs.sk

Peter Zalomp
Bogoliubov Laboratory of Theoretical Physics, Institute for Nuclear Research, 141980 Dubna, Russia
Institute of Physics, The Czech Academy of Sciences, Na Slovance 2, 18221 Prague 8, Czech Republic
E-mail: zalomp@fzu.cz

Abstract. Importance of symmetry considerations in high energy physics and statistical systems in critical region is discussed. As a concrete example, the field theoretic renormalization group technique is applied to a general A model of active vector admixture $b$. Parameter $A$ represents a continuous real parameter that governs the interaction structure of the system. The model encompasses the physically most relevant magnetohydrodynamic scenario ($A = 1$), and the model of linearized Navier-Stokes equations ($A = -1$). The environment is assumed to be in a state of fully developed turbulence, i.e. Reynolds number takes an infinite value. Additionally, spatial parity breaking is incorporated via the continuous parameter $\rho$. Studied system exhibits instability with respect to it, and large-scale generation of non-zero mean value $\langle b \rangle$ is observed. This is then explained in terms of spontaneous symmetry breaking mechanism.
1. Introduction

From a general perspective symmetry can be viewed as a main guiding principle in a formulation of modern physical theories. Typical examples can be found in high energy physics. Lorentz covariance, principles of quantum mechanics and local calibration gauge symmetry restrict possible form of theory to such an extent that every such attempt has to end up with a kind of quantum field theory [1]. Further experimental data might lead to additional restrictions on theory, for instance, discrete symmetry of parity, time reversal and charge conjugation play a fundamental role in a formulation of standard model [2]. The hints whether a given interaction is accompanied by a certain symmetry come from experiment, and not from theoretical reasoning.

Non-trivial property of tractable physical theories is their renormalizability. Initially, quantum field theory was believed to be incomplete, because straightforward perturbation calculations were full of infinities. In case of quantum electrodynamics renormalization transformations were identified and formal procedure of eliminating divergences was proposed [1, 3, 4]. Applications to other models in particle physics known at that time (Yukawa model, scalar $\varphi^4$ model, etc.) were not so successful. Further development came from an experimental observation of asymptotic freedom, and applicability of renormalization ides in critical phenomena. Mainly due to K. Wilson, renormalization was put into a larger context as a certain theoretical framework, in which field theory should always be viewed as an effective model for a given scale at which wished physical process is examined [5, 6]. Successive elimination of degrees of freedom leads to a flow of effective Hamiltonians in a functional space of all permissible interaction terms. As it may happen, some flows end up in fixed points, which are then natural candidates for scaling regimes. It is worthwhile to mention that condition of renormalizability is one of main heuristic clues for a construction of theory, because it significantly restricts occurrence of possible terms in Lagrangian.

Renormalization group (RG) is not only applicable to quantum systems. Many models in statistical physics are amenable by RG technique as well. Famous examples include continuous (second order) phase transitions [5]. Other interesting applications can be found in an area of non-equilibrium physics [7], or in theory of stochastic turbulence [6, 8]. In the latter, RG is employed in order to determine and study inertial interval, in which scaling regimes is observed. An overall aim consists in quantitative predictions about correlation and structure functions of velocity field.

Profound connection between quantum field theory and statistical models is revealed through a language of path integrals [5, 6]. Classical random field is then completely analogous to a fluctuating quantum field. However, one has to keep in mind that there is a crucial difference between RG in statistical physics and high energy physics. In high energy physics interest lies in an analysis of scaling in ultraviolet (UV) regime that corresponds to large momenta. On the contrary, in statistical physics the opposite infrared (IR) regimes of small momenta are of interest.

In what follows we will try to demonstrate power of field-theoretic analysis on a important model of magnetohydrodynamics [9, 10, 11, 12]. Being a crucial in an explanation of many effects - magnetic dynamo, convective processes, galaxy formation, etc. [13, 14, 15, 16, 17], it is clear that mutual interplay between velocity field and magnetic field needs to be properly taken into account. Simplified model for a theoretical description of MHD is so-called Kazantsev-Kraichnan kinematic model [18]. Its basic premise is to assume that a magnetic vector field (later in this article referred just as vector) is passively advected by turbulent velocity field, but back influence on the velocity field from magnetic field is negligible. The main point of criticism on Kazantsev-Kraichnan model is an assumption of the velocity field, which states that it is simply given by a Gaussian random variable. More appropriate approach would consider velocity field to be generated by dynamical means. Here, we assume that velocity is governed by a stochastic Navier-Stokes (NS) equation and taking this as an prototype we try
to illustrate essential ideas for a construction of general field-theoretical models of statistical physics. NS equation is mainly employed in order to construct a microscopic model for fully developed turbulence. A quantitative parameter that serves as a measure of turbulent motion is known as Reynolds number $Re \ [8, 19]$. Essentially, it represents a ratio between inertial and dissipative forces. For high enough values of $Re \gg 1$ inertial interval emerges in which just transfer of kinetic energy from outer $L$ (input) to microscopic $l$ (dissipative) scales takes place. One of the microscopic models used for a description of fully developed turbulence in inertial interval is based on a stochastic version of Navier-Stokes equation [6]. The dynamics of a solenoidal incompressible fluid is governed by the following equation [19]

$$\nabla_t v = \nu_0 \nabla^2 v - \nabla Q + f^v,$$

where $v = v(x)$ is a turbulent velocity field, $x = (t, x)$ with $t$ being time and $x$ the position variable, respectively, $\partial_t \equiv \partial / \partial t$, $\partial_i \equiv \partial / \partial x_i$, $\nabla^2 = \Delta = \partial_i \partial_i$ is the Laplace operator, the operator $\nabla_t$ denotes Lagrangian convective derivative $\nabla_t = \partial_t + (v \cdot \nabla)$, and $\nu_0$ is the bare viscosity coefficient. In this work we use a shorthand notation in which summations over repeated vector indices (Einstein summation convention) are always implied. In what follows we employ RG method, in which it is necessary to distinguish between unrenormalized (bare) and renormalized parameters.

Next we consider a passive solenoidal vector field $b \equiv b(x)$ advected by a helical turbulent environment given by an incompressible velocity field $v \equiv v(x)$. Obviously, fields $v$ and $b$ are solenoidal, i.e. they satisfy the incompressibility condition

$$\nabla \cdot b = \nabla \cdot v = 0. \tag{2}$$

The governing equation for vector field $b$ reads

$$\nabla_t b = \nu_0 u_0 \nabla^2 b + A(b \cdot \nabla)v - \nabla P + f^b, \tag{3}$$

where $u_0$ is the bare reciprocal Prandtl number [8, 20], functions $P \equiv P(x)$ and $Q \equiv Q(x)$ in Eqs. (1) and (3) represent the pressure fields and are not important for the future examination. In fact, due to the incompressibility condition (2) it is possible to express functions $P$ and $Q$ in terms of formal Biot-Savart law [8, 20]. The parameter $A$ is a dimensionless parameter of the theory. There are three special values of $A$ that have been comprehensively studied in the past [18]. Namely, value $A = 1$ describes Kazantzev-Kraichnan kinematic dynamo model, when the pressure term in (3) vanishes. Second, the model of passively advected vector impurity obtained for $A = 0$. Last, linearized Navier-Stokes equation with prescribed statistics of the background field $A = 1$. For technical analysis it is thus convenient to keep $A$ as a free parameter and to deal with all possible models at once. Model (3) is sometimes referred as general $A$ model in the literature [18, 21, 22]. Full model thus correspond to two interconnected stochastic equations (1) and (3).

Coarse-grained formulation of turbulence and MHD problem necessarily involves stochastic random forces $f^v$, $f^b$. It is assumed that random variable $f^b$ obeys a Gaussian distribution law with zero mean and correlator (second moment) is chosen in the form

$$D_{ij}^{b}(x; 0) \equiv \langle f_{ij}^{b}(x)f_{ij}^{b}(0) \rangle = \delta(t)C_{ij}(|x|/L), \tag{4}$$

where $L$ is an integral scale of stirring of $b$, $C_{ij}$ is required to be finite for $L \rightarrow \infty$ and for $|x| \gg L$ should rapidly decrease. Otherwise, $C_{ij}$ is not required to be completely specified [18]. On the other hand, $f^v$ simulates the injection of kinetic energy into the turbulent system from the largest spatial scales and is thus constrained by the requirement of real infrared (IR) energy
pumping as employed via its specific RG suitable power-like form [6]. However, the results obtained here do not depend on the specific form due to the universality of the fully developed turbulence, and we just make use of the favorable form for the RG approach. In accordance with these standard considerations [6], we prescribe the following pair correlation function with Gaussian statistics

\[ D^\omega_{ij}(x; 0) = \langle f^\omega_i(x) f^\omega_j(0) \rangle = \delta(t) \int \frac{d^4k}{(2\pi)^d} D_0 k^{d-2\varepsilon} R_{ij}(k)e^{ik\cdot x}, \]

with \( d = 3 \) denoting the spatial dimension of the system; \( k \) is the wave number with \( k = |k| \); \( D_0 \equiv g_0 v_0^2 > 0 \) is the positive amplitude with \( g_0 \) which plays the role of a coupling constant and is moreover related to the characteristic ultraviolet (UV) momentum scale \( \Lambda \) via \( g_0 \approx \Lambda^{2\varepsilon} \).

The parameter \( \varepsilon \) defines the exact form of energy injection at large scales and acquires value of 2 for physically relevant IR energy injection. In the RG approach to critical behavior, \( \varepsilon \) is formally small throughout the calculations and only in the final step its physical value of 2 is inserted into perturbative expansions [6]. The tensor object \( R_{ij} \) controls the spatial parity violation in the present model. In the case of fully symmetric isotropic incompressible turbulent environments, as discussed here, the projector \( R_{ij}(k) \) becomes a sum of the ordinary transverse projector \( P_{ij}(k) = \delta_{ij} - k_i k_j / k^2 \) and a helical term \( H_{ij}(k) = i \rho \varepsilon_{ij} k_k / k \) with \( \varepsilon_{ij} \) being the Levi-Civita tensor, and the real valued helicity parameter \( \rho \) satisfies \(|\rho| \leq 1\) due to the requirement of positive definiteness of the correlation function (5). Obviously, \( \rho = 0 \) corresponds to the fully symmetric (non-helical) case whereas \( \rho = 1 \) to fully broken spatial parity.

2. Field theoretic model for MHD model

Following De Dominicis-Janssen approach [6, 7], the system of stochastic differential Eqs. (3) and (1) is tantamount to a field theoretic model of the double set of fields \( \Phi = \{ \mathbf{v}, \mathbf{b}, \mathbf{v}', \mathbf{b}' \} \), where primed fields stand for the auxiliary response fields [6, 23]. The field theoretic model is then defined via the De Dominicis-Janssen action functional

\[ S[\Phi] = \frac{1}{2} \int dt_1 \int d^d x_1 \int dt_2 \int d^d x_2 [v'_i(x_1)D^\nu_{ij}(x_1; x_2)v'_j(x_2) + b'_i(x_1)D^b_{ij}(x_1; x_2)b'_j(x_2)] \]

\[ + \int dt \int d^d x \{ v'[-D_t + v_0 \nabla^2] + b'[-D_t + v_0 \nabla^2] + (b' \cdot \nabla) v], \]

with \( D^b_{ij} \) and \( D^\nu_{ij} \) specified in Eqs. (4) and (5), respectively; summations over repeating dummy indices \( i, j \in 1, 2, 3 \) are always implied. Similarly to original fields \( \mathbf{v} \) and \( \mathbf{b} \), the response (primed) fields are transverse, i.e. \( \nabla \cdot \mathbf{v}' = \nabla \cdot \mathbf{b}' = 0 \). In a field-theoretic formulation various stochastic quantities (corresponding to Green functions in quantum field theory) are calculable as path integrals with a given weight functional \( \exp(S) \). Main benefits of such approach are transparency of a perturbation theory in Feynman diagrams and feasibility of the other powerful methods such as renormalization group and operator product expansion [6, 7]. In the frequency-momentum representation, free propagators of the model (6) are readily derived

\[ \langle b'_i b_j \rangle_0 = \langle b_i b'_j \rangle_0^* = \frac{P_{ij}(k)}{\omega + v_0 u_0 k^2}, \quad \langle b_i b_j \rangle_0 = \frac{C_{ij}(k)}{|-i\omega + v_0 u_0 k^2|^2}, \]

\[ \langle v'_i v_j \rangle_0 = \langle v_i v'_j \rangle_0^* = \frac{P_{ij}(k)}{\omega + v_0 k^2}, \quad \langle v_i v_j \rangle_0 = \frac{g_0 v_0^2 k^{4-d-2\varepsilon} R_{ij}(k)}{|-i\omega + v_0 k^2|^2}. \]

The function \( C_{ij}(k) \) is the Fourier transform of \( C_{ij}(r/L) \) introduced in Eq. (4). Further, the theory contains three interaction vertices

(i) \( S_{\nu \nu b} \): \( b'_i (-v_j \partial_j b_i + Ab_j \partial_j v_i) = b'_i v_j V_{ij} b_i, \)

}\]
\[ W_{ijk} = \ldots \ldots \quad V_{ijk} = \quad U_{ijk} = \ldots \ldots \]

**Figure 1.** Graphical representation of all interaction vertices of the model related velocity breaking is utilized by a replacement of vector field functional takes the form

\[ S_{\nu'v'b'} = -v_i'v_j\partial_jv_i = v_i'v_jW_{ijk}v_k/2, \]

\[ S_{\nu'b'b} = v_i'v_j\partial_jb_i = v_i'v_jU_{ijk}v_k/2, \]

In the momentum-frequency representation, they correspond to the following vertex factors

\[ V_{ijl} = i(k_j\delta_{il} - k_i\delta_{jl}), \quad W_{ijl} = i(k_i\delta_{ij} + k_j\delta_{il}), \quad U_{ijl} = i(k_i\delta_{ij} + k_j\delta_{il}). \] (9)

In all three cases, momentum \( k \) is flowing into the vertices via the corresponding prime field, i.e. in \( V_{ijl} \) it is the response field \( b' \) and in \( W_{ijl}, U_{ijl} \) the field \( v' \), respectively. Graphical representation of interaction vertices can be found in Fig. 1.

### 3. Field theoretic formulation of general A model with symmetry breaking

Field theoretic model defined by action functional (6) is inherently unstable since 1-irreducible (1-IR) graphs \( \langle \nu'\nu \rangle_{1\text{-IR}} \) and \( \langle b'b \rangle_{1\text{-IR}} \) are not UV finite. In order to stabilize the advection-diffusion system, following [24] we suggest that the vector field \( b \) does not fluctuate around a zero mean but around a spontaneously generated non-zero mean value \( B = (b) \neq 0 \) with magnitude being dependent on the actual underlying mechanism which depends on the parameter \( A \). The response field \( b' \) is assumed to have zero mean value. On a technical level, spontaneous symmetry breaking is utilized by a replacement of vector field \( b \) by a sum \( B + b \) in field-theoretic action (6). Such substitution leads to a new action functional, whose free part now contains two new terms. The interacting part remains intact. Explicitly, the free part of the symmetry broken action functional takes the form

\[ S[\Phi] = \frac{1}{2} \int dt \int d^4x_1 \int dt' \int d^4x_2 \left[ v_i'(x_1)D_{ij}^\nu(x_1;x_2)v_j'(x_2) + b_i'(x_1)D_{ij}^b(x_1;x_2)b_j'(x_2) \right] \]

\[ + \int dt \int d^4x \left\{ v'[-\nabla_t + v_0\nabla^2]v + b'[-\nabla_t + v_0u_0\nabla^2 + v'(B \cdot \nabla)]b + A'B(\nabla \cdot \nabla)v \right\}. \] (10)

Quadratic part of this action determine propagators of the theory and now it clearly has more complicated structure than (6). We observe that now the symmetry breaking gives rise to new cross propagators \( \langle vv' \rangle, \langle bb' \rangle, \langle bv \rangle, \text{ and } \langle bb \rangle \). Moreover, all propagators are more involved and become explicitly depended on a uniform field \( B \). In momentum-frequency representation they take form

\[ \langle v_i v_j \rangle = \frac{\beta(k)\beta^*(k)}{\xi(k)\xi^*(k)}D^\nu(k)R_{ij}(k), \quad \langle b_i b_j \rangle = \frac{A^2(B \cdot k)^2}{\xi(k)\xi^*(k)}D^b(k)R_{ij}(k) \]
\begin{align}
\langle v_i v_j \rangle_0 &= - \quad \langle b_i v_j \rangle_0 = - \\
\langle v'_i v_j \rangle_0 &= - \quad \langle b'_i v_j \rangle_0 = - \\
\langle b_i b_j \rangle_0 &= - \quad \langle v'_i b_j \rangle_0 = - \\
\langle b'_i b_j \rangle_0 &= - \\
\end{align}

**Figure 2.** Graphical representation of all propagators of the model given by the quadratic part of the action (15).

\begin{align}
\langle v_i v'_j \rangle &= \frac{\beta^*(k)}{\xi'(k)} P_{ij}(k), \\
\langle b_i v'_j \rangle &= iA \frac{(B \cdot k)}{\xi'(k)} P_{ij}(k), \\
\langle b'_i b_j \rangle &= \frac{\beta(k)(B \cdot k)}{\xi(k)\xi'(k)} D_v(k)R_{ij}(k),
\end{align}

where for brevity we have introduced following abbreviations \( \alpha(k) = i\omega + \nu^2 \), \( \beta(k) = i\omega + uwk^2 \), \( \xi(k) = A(B \cdot k)^2 + \alpha(k)\beta(k) \). Graphical representation of all propagators is given in Fig. (2). Determination of all relevant UV divergences is facilitated by the usual analysis of canonical dimensions, and we shall omit the details, which are analogous to the corresponding discussion in Ref. [21]. The present model is logarithmic, i.e. all coupling constants are dimensionless, at \( \varepsilon = 0 \) which in the framework of the minimal subtraction (MS) scheme, as used here, fixes all possible UV divergences to be just the corresponding poles in \( \varepsilon \) [6]. Following the results of Ref. [21], we conclude that all UV divergences can be removed by the counterterms of the form \( v' \nabla^2 v \) or \( b' \nabla^2 b \), which leads to the multiplicative renormalization of \( g_0, u_0 \), and \( \nu_0 \) via

\[ \nu_0 = \nu Z_\nu, \quad g_0 = g \mu^{2\varepsilon} Z_g, \quad u_0 = u Z_u, \]

where \( g, u, \) and \( \nu \) are the renormalized counterparts of the corresponding bare ones. The renormalization mass \( \mu \) is introduced to perform dimensional regularization. The quantities \( Z_i = Z_i(g, u; d, \rho; \varepsilon) \) contain poles in \( \varepsilon \). The renormalized action functional reads then as

\begin{align}
S_R[\Phi] &= \frac{1}{2} \int dt \int d^d x_1 \int dt_2 \int d^d x_2 \left[ v'_i(x_1)D^a_{ij}(x_1; x_2)v'_j(x_2) + b'_i(x_1)D^2_{ij}(x_1; x_2)b'_j(x_2) \right] \\
&\quad + \int dt \int d^d x \left\{ v'[-\nabla v + \nu Z_1 \nabla^2 v + Z_3(b \cdot \nabla)b] + b'[-\nabla b + \nu u Z_2 \nabla^2 b] \right. \\
&\quad \left. \quad + A(b \cdot \nabla)v + Z_3 v' \langle B \cdot \nabla \rangle b + Ab' \langle B \cdot \nabla \rangle v \right\},
\end{align}

where \( Z_1 \) and \( Z_2 \) are renormalization constants defined by the equations

\[ Z_\nu = Z_1, \quad Z_g = Z_1^{-3}, \quad Z_u = Z_2 Z_1^{-1}. \]

Each of the renormalization constants \( Z_1 \) and \( Z_2 \) corresponds to a different class of Feynman diagrams (as discussed below) but they share an analogous structure within the MS scheme: the
\[ \Gamma_1 = \quad \Gamma_2 = \quad \Gamma_3 = \quad \Gamma_4 = \quad \Gamma_5 = \quad \Gamma_6 = \quad \Gamma_7 = \quad \Gamma_8 = \]

**Figure 3.** Graphical representation of all Feynman diagrams for two-point one-irreducible Green functions of the action (15). Graphs \( \Gamma_1, \ldots, \Gamma_4 \) represent perturbation expansion for \( \Gamma_{v'v} \) function, and \( \Gamma_5, \ldots, \Gamma_8 \) for \( \Gamma_{b'b} \) function.

\[
\tilde{\Gamma}_{v'bb} = +2 +2 +2 +2 +1 +2 +1 +2 +1 +1
\]

**Figure 4.** Graphical representation of all one-loop Feynman diagrams forces for one-irreducible Green function \( \Gamma_{v'b'b} \).

\( n \)-th order of perturbation theory corresponds to the \( n \)-th power of \( g \) with the corresponding expansion coefficient containing a pole in \( \varepsilon \) of multiplicity \( n \) and less \([6]\).

Feynman diagrams, which have to be analyzed to one-loop order, are depicted in Fig. (3) and Fig. (4). Numerical factors in Fig. (4) stand for total number of ways how a given Feynman diagram can be constructed.

Omitting unnecessary technical details, the isolation of linear divergences in the helicity parameter \( \rho \) is straightforward to perform. In the case of 1-irreducible graphs \( \langle v'v \rangle \) the tensorial structure ensures that no such divergences exist at all as already noticed, for example, in Ref. [21] for a corresponding passive advection limit of the present model. Calculation of uniform field \( B \) in one-loop approximation finally leads to an expression

\[
|B| = \sqrt{\frac{1}{\pi|A|} \frac{\Gamma(d/2 + 3/2)}{\Gamma(d/2 + 1)} u_u \Lambda}, \quad A \notin \{0, -1\}, \tag{17}
\]

where we have retained dependence on the space dimension \( d \). Substituting value for realistic
space dimension $d = 3$ leads to a following prediction

$$|B| = \frac{8}{3\pi \sqrt{|A|}} u_\nu \Lambda. \quad (18)$$

4. Conclusion
In this work, we have discussed the general A model of active vector-like admixture. Active nature of the admixture is required to consistently renormalize the theory in the presence of helical divergences which are linear in $\rho$. It has turned out that the effect of spontaneous symmetry breaking has to be properly incorporated in a field-theoretic formulation. We have briefly discussed aspects of UV renormalization and showed non-trivial appearance of mean “magnetic” field in propagators. Prediction for mean value has been made.

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