Density of Bloch states inside a one dimensional photonic crystal

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Abstract
The density of Bloch modes inside a one dimensional photonic crystal (1D PC) is formulated based on its dispersion relations. This density function has applications in thermal emission inside a 1D PC, as well as controlling the dynamics of active materials embedded in them. After deriving the formulations, a practical 1D PC parameters in the visible range are used to calculate the density of transverse electric and transverse magnetic modes. Compared to the alternative methods such as using Dyadic Greens functions, this method is less complex and is exact. The method applies to any anisotropic medium for which the dispersion equations are available, analytically.

Introduction
The density of electromagnetic modes/states (DOS) is an important quantity in statistical physics. It is often used as the degeneracy function for the energy levels in thermal radiation studies such as Planck’s blackbody radiation [1, 2]. Consider the distribution of photons inside a large cavity in thermal equilibrium with a solid matter at temperature T (a.k.a. cloud of photons). The solid matter provides the mechanism to convert photons energies (i.e. annihilate and create them) according to the temperature. Since photons are bosons, the number of photons at each energy level ($\varepsilon_i = \hbar \omega$) follows Bose–Einstein distribution as [3]

$$n_i = \frac{g_i}{e^{\frac{\varepsilon_i - \beta \alpha}{kT}} - 1}$$  (1)

where $g_i$ and $n_i$ are the degeneracy and photon number of each energy level ($\varepsilon_i$), respectively. Equation (1) is obtained by maximizing the number of micro–states in the system (see [3]). The constants $\alpha$ and $\beta$ are determined by enforcing the conditions $\sum n_i = N$, and $\sum \varepsilon_i n_i = U$ where $N$ and $U$ are the total number and energy of bosons, respectively. The later condition leads to $\beta = \frac{1}{kT}$ where $k_B$ is Boltzmann’s constant, and the former does not apply to photons, as they can be created and annihilated by the solid matter. This leads to setting $\alpha = 0$. The modes degeneracy, $g_i$, is where the DOS is required. If there is no photons reflection at the surface of the thermal radiator (i.e. an ideal black body,) the DOS inside the thermal radiator is essentially the same as its ambient medium. Alternatively, we could explain thermal emission by considering the distribution of emitters inside a matter at temperature $T$, and relate it to the radiational modes. An example of such methods is briefly reviewed in the appendix, for self-consistency.

Besides radiative energy transfer, macroscopic quantities such as specific heat, energy density, and thermal conductivity also depend on DOS. Also, thermal near–field radiative transfer between a small particle and large body requires DOS calculations. As another example, Casimir force between objects requires DOS calculations [4]. For this purpose, usually Green’s functions, relating to LDOS, are used to find the Maxwell stress tensor in vacuum in thermal equilibrium. The DOS is closely related to the LDOS and their applications are strongly overlapped. In face, one way of calculating DOS in periodic media is to use LDOS [5].

Engineering DOS inside photonics crystals allows us to control the dynamics of active materials embedded in them. This has applications in microscopic lasers, resonant antennas, optical fibers with photonic crystal cores, and optical switches which are key elements in integrated photonics [6–9]. Other interesting applications of optical property engineering are in light emitting diodes, solar cells, and laser cooling [10].
The DOS inside a 1D PC consists of two contributions: Bloch modes which can propagate and depart a finite-size 1D PC, and the wave-guided modes which trap photons inside the 1D PC (mostly within higher permittivity layers). There are accurate analytical and numerical methods to calculate local density of states inside a 1D PC based on electromagnetics Dyadic Green’s functions [5, 11]. The LDOS can then be used to calculate the total DOS. However, in some studies such as thermal emission, we are interested in differentiating the density of Bloch states (DOS\textsubscript{Bloch}). This is mainly because the wave-guided modes cannot leave the 1D PC and do not couple to the radiating modes in the environment. The separation of (DOS\textsubscript{Bloch}) from the DOS is not a straightforward calculation if we use Greens functions [5]. On the other hand, since the Bloch modes inside a 1D PC have simple analytical dispersion equations, we may calculate DOS\textsubscript{Bloch} with a simpler method, which is discussed here. The DOS\textsubscript{Bloch} is the parameter which, depending on the geometry, can be used directly or indirectly as the degeneracy function in thermal emission from 1D PCs.

Note that we only discuss simple dielectrics with local permittivity and permeability in this paper. More complex materials such as plasma or metals at low temperature exhibit spatial dispersion in which case the indirect approach is more appropriate to describe the density of electromagnetic modes, as the degeneracy function in thermal emission from 1D PCs.

**DOS\textsubscript{Bloch}(\omega) inside a 1D PC**

We start by formulating the DOS inside a medium with known dispersion relations, and general non-spherical iso-frequency surfaces. The conventional method of calculating the DOS is to consider a large rectangular cavity with size L in all dimensions, filled with the medium, and with periodic boundary conditions on its walls. Eigen solutions of the harmonic electromagnetic wave equation, at frequency \(\omega\), in such geometry are modes in the form

\[
F_{\text{med}} = C_{\text{med}}^{p} e^{-i\omega t} \left( e^{i\frac{2\pi m}{L}x} + e^{i\frac{2\pi l}{L}z} \right) \quad (2)
\]

where \(F_{\text{med}}\) is an electric (magnetic) field component of the mode and \(C_{\text{med}}^{p}\) is its associated coefficient (which has both frequency and geometry dependences.) The superscript \(p\) in \(C_{\text{med}}^{p}\) denotes the mode’s polarization and can take two values (any two orthogonal polarizations is correct, but transverse electric and transverse magnetic polarizations along a cartesian coordinate axis are commonly chosen.) The values in the triplet \((m, n, l)\) can be any positive integer number subject to an equation obtained from the eigen value problem

\[
f \left( \frac{2\pi m}{L}, \frac{2\pi n}{L}, \frac{2\pi l}{L}, \omega \right) = 0, \quad (3)
\]

equation (3) is known as the dispersion equation after defining and replacing the wave-vectors as

\[
(k_{x}, k_{y}, k_{z}) = \left( \frac{2\pi m}{L}, \frac{2\pi n}{L}, \frac{2\pi l}{L} \right). \quad (4)
\]

In case of an isotropic homogenous material, equation (3) becomes \(\left( \frac{2\pi m}{L} \right)^{2} + \left( \frac{2\pi n}{L} \right)^{2} + \left( \frac{2\pi l}{L} \right)^{2} = k_{z}^{2} \varepsilon_{0} \mu_{0}.\) The density of electromagnetic modes, DOS\textsubscript{\(\omega\)}, is defined as the spectral density of the modes per cavity volume. That is, DOS\textsubscript{\(\omega\)} \(d\omega\) is the number of supported modes per volume in the interval \([\omega, \omega + d\omega]\),

\[
\text{DOS}(\omega) \, d\omega = \sum_{m,n,l}^{2} \left\{ (m, n, l) : f \left( \frac{2\pi m}{L}, \frac{2\pi n}{L}, \frac{2\pi l}{L}, \omega < \omega' \leq \omega + d\omega \right) = 0 \right\} \quad (5)
\]

where the factor of 2 accounts for the two possible orthogonal polarizations per mode. We used the notation \(\{x : p(x)\}\) which refers to the set of \(x\) for which \(p(x)\) is true. We may write (5) as

\[
\text{DOS}(\omega) \, d\omega = \sum_{m,n,l}^{2} \left\{ (m, n, l) : f \left( \frac{2\pi m}{L}, \frac{2\pi n}{L}, \frac{2\pi l}{L}, \omega < \omega' \leq \omega + d\omega \right) = 0 \right\} \quad (6)
\]

since \(m, n,\) and \(l\) are all integers, that is \(\Delta m = \Delta n = \Delta l = 1.\) Replacing (4) in (6) and converting the summation to the integration (since L is large) gives

\[
\text{DOS}(\omega) \, d\omega = \frac{2}{(2\pi)^{3}} \iiint_{\Delta \omega} d\nu_{k} \quad (7)
\]

where \(d\nu_{k} = dk_{x}dk_{y}dk_{z}\) is the volume differential in \(k\)-space. The integration in (7) is a volume (three-fold) integration between the two 3D surfaces in \(k\)-space, satisfying dispersion equation (3) at \(\omega\) and \(\omega + d\omega\), respectively. These surfaces are also known as the iso-frequency surfaces. Solution of equation (7) gives the DOS.
Equation (7) holds regardless of the shape of the iso-frequency surfaces, and can be used directly for simple surfaces. For example, since the iso-frequency surfaces of a homogeneous isotropic space are spherical, using \[ k_x^2 + k_y^2 + k_z^2 = k^2 \] in (7) gives

\[
\text{DOS}(\omega) d\omega = \frac{2}{(2\pi)^2} \int_{\Omega_{k_r=1}} \int_{0}^{\pi} k^2 \sin \theta d\varphi dk
\]

which after simple manipulation leads to the known equation \( \text{DOS}(\omega) = \frac{\omega^2(\mu_r \varepsilon_r)^{1/2}}{\pi^{3/2}}. \)

We may prepare (7) further for an arbitrary iso-frequency surface in \( k \)-space. This preparation becomes useful in finding \( \text{DOS}_{\text{Bloch}}(\omega) \) in photonic crystals and other non-trivial and anisotropic materials such as hyperbolic metamaterials where iso-frequency surfaces’ shapes depend on the frequency. For example, figure 1 shows the iso-frequency surfaces of a 1D PC at two nearby frequencies.

Each of the iso-frequency surfaces in (7) (and in figure 1) can be parametrized using two independent parameters \( \theta \) and \( \varphi \) as

\[
S(\omega) \colon (k_x(\theta, \varphi), k_y(\theta, \varphi), k_z(\theta, \varphi))
\]

where \( (k_x, k_y, k_z) \) represents a vector in \( k \)-space. In general, the choice of parameters \( \theta \) and \( \varphi \) is arbitrary and depends on the shape of the iso-frequency surface. However, it is convenient to choose \( \theta \) and \( \varphi \) as spherical coordinate’s polar and azimuthal angles for 1D PCs. The surface differential for such arbitrary surface is

\[
dA = \| t_\theta \times t_\varphi \| \, d\theta d\varphi
\]

where \( t_\theta, t_\varphi \) are tangential vectors to the surface as

\[
t_\theta = \left( \frac{\partial k_x(\theta, \varphi)}{\partial \theta}, \frac{\partial k_y(\theta, \varphi)}{\partial \theta}, \frac{\partial k_z(\theta, \varphi)}{\partial \theta} \right),
\]

\[
t_\varphi = \left( \frac{\partial k_x(\theta, \varphi)}{\partial \varphi}, \frac{\partial k_y(\theta, \varphi)}{\partial \varphi}, \frac{\partial k_z(\theta, \varphi)}{\partial \varphi} \right),
\]

and \( \| \| \) and \( \times \) are the vector \( L^2 \) norm the external vector product, respectively. Since \( t_\theta \times t_\varphi \) is a vector normal to the surface, the volume differential between the two iso-frequency surfaces at \( \omega \) and \( \omega + d\omega \) is

\[
dV = |S(\omega + d\omega) - S(\omega)).(t_\theta \times t_\varphi) d\theta d\varphi|,
\]

which simplifies to

\[
dV = \left| \left( \frac{\partial k_x}{\partial \omega}, \frac{\partial k_y}{\partial \omega}, \frac{\partial k_z}{\partial \omega} \right) \right| (t_\theta \times t_\varphi) d\theta d\varphi d\omega.
\]

Note the \( | \cdot | \) operator is necessary in the volume calculations (since the volume between the two surfaces is independent of their order). Replacing (14) into (7) gives

\[
\text{DOS}(\omega) = \frac{2}{(2\pi)^2} \oint S \left| \left( \frac{\partial k_x}{\partial \omega}, \frac{\partial k_y}{\partial \omega}, \frac{\partial k_z}{\partial \omega} \right) \right| (t_\theta \times t_\varphi) d\theta d\varphi d\omega
\]

which can be used for any iso-frequency surface provided that it can be parametrized. As a simple example, the spherical iso-frequency surfaces of a non-magnetic (\( \mu_r = 1 \)) homogeneous isotropic medium can be parametrized as

\[
\text{Figure 1.} \text{ The iso-frequency surfaces of a 1D PC (TE modes) at two different frequencies normalized to } k_1 = \omega \sqrt{\mu_r \varepsilon_r}.
\]
which easily leads to the expected result \( \text{DOS}(\omega) = \frac{\omega^{3/2}}{\pi \varepsilon_0} \).

In a non-magnetic 1D PC, the dispersion relation is \([13, 14]\)

\[
k_x = \frac{1}{T_1 + T_2} \cos^{-1} \left( \cos(k_{x1} T_1) \cos(k_{x2} T_2) \right)
- 0.5 \left( \frac{P_2}{P_1} + \frac{P_1}{P_2} \right) \sin(k_{x2} d_2) \sin(k_{x1} d_1) \right),
\]

where

\[
k_{x1} = \sqrt{k_i^2 - k_j^2 - k_z^2}; \quad P_i = \begin{cases} \frac{k_{x1}}{\omega \mu_0} & \text{TE} \\ \frac{\omega \varepsilon_0 \varepsilon_i}{k_{x1}} & \text{TM} \end{cases}
\]

and transverse electric (TE) and magnetic (TM) modes are defined with respect to the y-z plane (interface plane).

The iso-frequency surface of the 1D PC can be parametrized as

\[
S(\theta, \phi) : \quad k_x(\omega, \theta), \quad \frac{\omega}{c} \sqrt{\varepsilon_i} \cos \theta \sin \phi, \quad \frac{\omega}{c} \sqrt{\varepsilon_i} \cos \theta \cos \phi
\]

where \((\theta, \phi)\) are defined inside the material with the lower permittivity \(\varepsilon_1\) because the tangential wave-vector, \(\sqrt{k_i^2 + k_j^2}\), inside the PC cannot exceed \(\frac{\omega}{c} \sqrt{\varepsilon_{\text{min}}} \). The tangential vectors to the iso-frequency surface are

\[
t_\theta : \frac{dk_x(\omega, \theta)}{d\theta}, \quad -\frac{\omega}{c} \sqrt{\varepsilon_i} \sin \theta \sin \phi, \quad -\frac{\omega}{c} \sqrt{\varepsilon_i} \sin \theta \cos \phi
\]

\[
t_\phi : \quad 0, \quad \frac{\omega}{c} \sqrt{\varepsilon_i} \cos \theta \cos \phi, \quad -\frac{\omega}{c} \sqrt{\varepsilon_i} \cos \theta \sin \phi.
\]

Using (15), (20), and (21), it is straightforward to show the density of Bloch states for TE and TM modes are

\[
\text{DOS}_{\text{Bloch}}(\omega) = \frac{\omega \varepsilon_i}{4c^2 \pi^2} \int_0^{\pi} \left( \cos \theta \frac{\partial k_x(\omega, \theta)}{\partial \theta} + \omega \sin \theta \frac{\partial k_x(\omega, \theta)}{\partial \omega} \right) \cos \theta \, d\theta,
\]

where \(i = \text{TE or TM}\). Equation (22) can be solved, as is, using commercially available solvers and no further analytical expansion is necessary. However, special care should be given to the calculation of \(k_x\) using (17) as it passes through different Riemann sheets, as will be further discussed later.

As an example, figure 2 shows the density of Bloch states inside a 1D PC with parameters \(\varepsilon_1 = 11.9, \varepsilon_2 = 2.1, T_1 = 300 \text{ nm}, \text{and } T_2 = 700 \text{ nm}\). It also includes DOS of an isotropic homogeneous material with \(\varepsilon_1 = 2.1\). Note that Bloch wave’s tangential wave-vector inside a 1D PC is always limited by the material with the lower permittivity (i.e. \(\varepsilon_1\)). The DOS_{\text{Bloch}} in figure 2 shows some derivative discontinuities which are at frequencies near the band edges of TM or TE modes. Some of these local peaks exceed the DOS of an unbounded \(\varepsilon_1\) region.

Note that the inverse cosine function in (17) is a multi-valued function with branch cuts on positive and negative sides of the real axis \([-\infty, -1] \text{ and } [1, +\infty \text{ segments})\). Every pass of the inverse cosine argument across a branch cut requires considering a different (appropriate) Riemann sheet. The argument of the \(\cos^{-1}\) function in (17) for TE modes of the 1D PC example discussed here is shown in figure 3. The crossing points of
this function through branch cuts are where $\frac{\partial}{\partial \omega}(\cdot) = 0$. This leads to the Riemann sheet assignment shown in figure 5(b) so that

$$\cos^{-1}(x) = \begin{cases} (n - 1)\pi + \text{P.V.} & \text{at Riemann } n, \text{ for odd } n \\ n\pi - \text{P.V.} & \text{at Riemann } n, \text{ for even } n \end{cases}$$

(23)

where P.V. is the principal value of $\cos^{-1}(x)$ as $0 \leq P.V. \leq \pi$. The resulting $k^{TE}_x$ is also shown in figure 3.

The DOS$_{\text{Bloch}}$ in figure 2 is also approximated using the Dyadic Greens function method in [5]. The Greens function inside the 1D PC is used to calculate the local density of states (LDOS) as a function of location and the source’s orientation. The average (both in space and orientation) of LDOS is then used to calculate the total DOS, which includes contributions from Bloch modes and the wave-guiding modes. As discussed in [5], it is possible to approximate DOS$_{\text{Bloch}}$ by adjusting the integration limits in the LDOS calculations. The comparison between the two methods shows reasonable agreement in [5]. The discrepancy in the results is due to the approximations used in the Greens function approach (e.g., the 1D PC is truncated to five layers due to the rapid increase of the computational cost by adding more layers). The method here is exact and is much simpler than...
the Greens function approach. However, the Greens function approach can provide more information (such as LDOS) and applies to lossy materials as well.

Conclusion

The density of Bloch states inside a 1D PC was formulated based on its dispersion relations for both TE and TM modes. The quantities were calculated for a practical 1D PC in the visible range.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

Appendix

The ratio of the radiation to absorption, \( F(\omega, T) \), is considered to be a universal function for all solid matters based on Kirchhoff’s law (controversies regarding Kirchhoff’s law are beyond the purpose of this paper, see [15, 16] for details.) There are several methods to obtain \( F(\omega, T) \), a.k.a. the black body radiation spectrum, inside a black body cavity. They all lead to

\[
F(\omega, T) = \rho(\omega) U(\omega, T)
\]  
(A.1)

where \( F(\omega, T) \) is considered to be a continuous function of \( \omega \) with the unit of joules per frequency per volume \( (\text{J} \cdot \text{s}^{-1} \cdot \text{Hz}^{-1} \cdot \text{m}^{-3}) \), and \( U \) is the average total energy of the oscillators inside the black body (or photons in the cavity). In the state of thermal equilibrium, \( U \) can be obtained either using the equipartition theorem in classical statistical mechanics (\( U = kT \))

\[
U = \frac{\hbar \omega}{\exp\left(\frac{\hbar \omega}{kT}\right) - 1}.
\]  
(A.2)

One of the most intuitive methods to obtain \( \rho(\omega) \) in (A.1) (for isotropic homogeneous materials) is to count the supported electromagnetic modes inside the resonator in the interval \([\omega, \omega + d\omega]\), as we did in the main text. It can also be obtained from the emission by classical resonators into an unbounded isotropic medium (modeling a very large cavity) as follows [17]. Consider a particle with mass \( m \) and charge \( e \) acted upon by an elastic restoring force \(-m\omega_0^2 z\) and an external electric field of \( E_z(t) \). For simplicity, assume the particle only moves in one dimension (\( z \)). Newton’s equation of motion for such particle is

\[
z + \omega_0^2 z = \frac{e}{m} E_z(t) + \frac{e}{m} E_{RR}(t)
\]  
(A.3)

where \( E_{RR}(t) \) is the reaction electric field originated from the moving particle itself, and can be shown to be \( E_{RR}(t) = \frac{e^2}{2m} \dot{z}(t) \). If the particle is inside a large rectangular cavity (with size \( L \)), the energy absorption rate by the particle oscillating with frequency \( \omega \) from a cavity resonant mode at frequency \( \omega \) is

\[
W_A = \frac{e^2}{2m} \frac{\gamma \omega^4 E_z^2(\omega)}{(\omega^2 - \omega_0^2)^2 + \gamma^2 \omega^2}; \quad \gamma = \frac{2e^2}{3mc^2}
\]  
(A.4)

where \( E_z(\omega) \) is the \( z \)-directed electric field associated with the electromagnetic mode, and \( c \) is the speed of \( (z \)-polarized) light in the medium. If the radiation has a continuous broadband spectrum, with the \( (z \)-directed electric field) energy density of \( E_z(\omega, T) d\omega = \frac{E_z^2(\omega)}{8\pi} \) in the interval \([\omega, \omega + d\omega]\), it can be shown that [17]

\[
W_A = \frac{4\pi e^2}{m} \int_{\omega} \frac{\gamma E_z(\omega, T) d\omega}{(1 - \omega_0^2/\omega^2)^2 + \gamma^2 \omega^2}.
\]  
(A.5)

For the frequencies of interest in thermal emissions, \( \gamma \omega < 1 \). If \( F(\omega, T) \) is not sharply peaked, we may simplify (A.5) as

\[
W_A = \frac{4\pi e^2 \gamma F_z(\omega_0, T)}{m} \int_{\omega} \frac{d\omega}{4\omega_0^2 (\omega - \omega_0)^2 + \gamma^2 \omega_0^2} = \frac{4\pi e^2 \gamma F_z(\omega_0, T) \omega_0^2}{m} \frac{2\pi}{4\gamma \omega_0^2}.
\]  
(A.6)
The energy emission rate of the oscillating charge is

\[ W_\text{E} = \frac{2e^2 \omega_0^2}{3mc^3} U \]  

(A.7)

where \( c \) is the speed of light in vacuum, and \( U \) is the average total oscillator energy. In the state of thermal equilibrium between the radiational energy and the matter, (A.7) and (A.6) should be equal, leading to

\[ F(\omega_0, T) = \frac{\omega_0^2}{\pi c^3} U, \]  

(A.8)

where \( F(\omega_0, T) = F_f(\omega_0, T) = F_i(\omega_0, T) = F(\omega_0, T) / 3 \) is used. Similar to the resonator mode counting method, (A.8) gives

\[ \rho(\omega) = \frac{\omega^2}{\pi c^3}. \]  

(A.9)

Also, equations (A.1), (A.2), and (A.9) provide the well-known Plank’s emission spectrum inside a blackbody cavity filled with an isotropic homogeneous material.

This method of obtaining \( \rho(\omega) \) provides insight into how radiators inside the matter (e.g. blackbody) couple to the electromagnetic modes in common situations (leading to Plank’s radiation spectrum). Two critical assumptions which simplified (A.5) to (A.6) are (1) radiation spectrum, \( F_f(\omega, T) \), does not have any sharp peaks, and (2) the resonators’ coupling to the radiation is sharply peaked around their natural frequency, \( \omega_0 \). This assumption also implicitly requires linearity of the material. Only with these assumptions we obtained the same \( \rho(\omega) \) as using the other, more fundamental, methods such as BE distribution of photons in the cavity in thermal equilibrium. It appears that (A.5) is a good starting point to study the thermal emission from non-linear (or any other uncommon) material.

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References

[1] Kirchhoff G 1860 On the relation between the radiating and absorbing powers of different bodies for light and heat The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 20 1–21
[2] Planck M 2010 The Theory of Heat Radiation (1914) (Whitefish, MT: Kessinger Publishing)
[3] Reichl L E 1999 A modern course in statistical physics Am. J. Phys. 67 1285–7
[4] Jaffe R 2005 Casimir effect and the quantum vacuum Phys. Rev. D 72 021301
[5] Forati E 2021 Spontaneous emission rate and the density of states inside a one dimensional photonic crystal IEEE J. on Multiscale and Multiphysics Comput. Techniques early view (https://doi.org/10.1109/JMMCT.2022.3153405)
[6] Painter O, Lee R, Scherer A, Yariv A, J O'Brien, Dapkus P and Kim I 1999 Two-dimensional photonic band-gap defect mode laser Science 284 1819–21
[7] Temelkuran B, Bayindir M, Ozbay E, Biswas R, Sigalas M, Tuttle G and Ho K-M 2000 Photonic crystal-based resonant antenna with a very high directivity J. Appl. Phys. 87 603–5
[8] Knight J C, Broeng J, Birks T A and Russell P S J 1998 Photonic band gap guidance in optical fibers Science 282 1476–8
[9] Busch K and John S 1999 Liquid-crystal photonic-band-gap materials: the tunable electromagnetic vacuum Phys. Rev. Lett. 83 967
[10] Chen Y-C, Ghosh I, Schleife A, Carney P S and Bahl G 2018 Optimization of anisotropic photonic density of states for raman cooling of solids Phys. Rev. A 97 043805
[11] Bruck S 2000 Radiation from a dipole embedded in a dielectric slab IEEE J. Sel. Top. Quantum Electron. 6 899–910
[12] Tai C and Collin R E 2000 Radiation of a hertzian dipole immersed in a dissipative medium IEEE Trans. Antennas Propag. 48 1501–6
[13] Qi L and Liu C 2017 Complex band structures of 1d anisotropic graphene photonic crystal Photonics Research 5 543–51
[14] Joannopoulos J, Meade R and Winn J 1995 Photonic crystals: Molding the Flow of Light 1st edn (Princeton, NJ: Princeton Univ. Press)
[15] Robitaille P-M 2009 Kirchhoff’s law of thermal emission: 150 years Progr. Phys. 4 3–13
[16] Robitaille P M 2003 On the validity of kirchhoffs law of thermal emission IEEE Trans. Plasma Sci. 31 1263–7
[17] Milonni P W 2013 The Quantum Vacuum: An Introduction to Quantum Electrodynamics (New York: Academic)