A model for Dirac neutrino mass matrix with only four parameters

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Abstract

We study mu-tau symmetry in the context of Dirac neutrinos. In particular, we assume the hermiticity of the Dirac neutrino mass matrix and demand the invariance of the Lagrangian under interchange of $\nu_{\mu R} \leftrightarrow -\nu_{\tau R}$ followed by a CP transformation of the leptonic sector. This symmetry gives rise to a particular structure of the Dirac Neutrino mass matrix with only four real parameters. We show that for inverted neutrino mass hierarchy, this four parameter Dirac Neutrino mass matrix can explain the observed values for the two squared-mass differences and three mixing angles with particular predictions for the absolute values of the neutrino masses ($m_1 = 4.81 \times 10^{-2}$, $m_2 = 4.89 \times 10^{-2}$ and $m_3 = -1.198 \times 10^{-3}$ eV) and CP violating phase $\delta = \pi/2$.

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The mixing between different neutrino flavors was first hinted by the deficit of solar neutrino flux as measured in Earth. The solar neutrino deficit can be explained if we assume non-zero neutrino masses, mixings and hence, oscillation between different neutrino flavors. During last two decades different experiments on atmospheric ($\nu_\mu$ and $\bar{\nu}_\mu$) neutrinos (Super-K [1], K2K [2], MINOS [3]), solar ($\nu_e$) neutrinos (SNO [4], Super-K [5], KamLAND [6]) as well as reactor/accelerator ($\bar{\nu}_e/\nu_\mu$) neutrinos (Daya Bay [7], RENO [8], Double Chooz [9], T2K [10], NOvA [11]) provided us convincing evidences for non-zero neutrino masses and mixings. All currently available data on the oscillations can be described assuming 3-flavor ($\nu_e$, $\nu_\mu$ and $\nu_\tau$) neutrino mixing in vacuum. In the basis where the weak interaction is flavor diagonal and universal, the mass eigenstates ($\nu_1$, $\nu_2$ and $\nu_3$) are related to the weak (flavors) eigenstates ($\nu_e$, $\nu_\mu$ and $\nu_\tau$) as follows,

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} = U 
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix},
$$

(1)

where, $U$ is the $3 \times 3$ neutrino mixing matrix [12, 13]. For Dirac neutrinos the mixing matrix $U$ can be parametrized by 3 angles ($\theta_{12}$, $\theta_{13}$ and $\theta_{23}$) and one CP violating phase ($\delta$):

$$
U = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix},
$$

(2)

where, $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$. A global analysis of neutrino oscillations data from different experiments give the best fit values for the three mixing angles and two squared-mass differences [14], $\Delta m_{ij}^2 = m_i^2 - m_j^2$. However there are several important parameters yet to be measured. These include the value of the CP phase(s) which will determine the magnitude of CP violation in the leptonic sector and the sign of $\Delta m_{32}^2$ which will determine whether the neutrino mass hierarchy is normal or inverted. Moreover, we also don’t know yet if the neutrinos are Majorana or Dirac particles.

The best fit values of the three mixing angles and two squared-mass differences along with their $3\sigma$ allowed range are presented in Table 1. The experimental data in Table 1 shows two important properties: (i) There is a $O(10^3)$ hierarchy in the squared-mass differences and (ii) The atmospheric and solar mixing angles ($\theta_{12}$ and $\theta_{23}$) are large whereas the reactor mixing angle ($\theta_{13}$) is very small. It is well known that the presence of tiny quantities or hierarchies indicates towards a protection symmetry in underlying scenario [15]. Example of one such well studied symmetry, in the context of neutrino physics, is the invariance of flavor neutrino mass matrix under interchange of $\nu_\mu$ and $\nu_\tau$ [16, 17, 18, 19, 20, 21]. It is easy to see from Eq. 2 that the exact $\mu-\tau$ symmetry of the neutrino mixing matrix demands $s_{23}^2 = 0.5$ and $s_{13} = 0$. Table 1 shows that $s_{23}^2 = 0.5$ is still within the $3\sigma$ of the central value however, $s_{13} = 0$ is already ruled out with more than $5\sigma$ C.L. Moreover, the charged leptons and left handed neutrinos are in the $SU(2)_L$ doublets and thus, the $\mu-\tau$ symmetry respected by the neutrinos should be respected by the charged leptons. However, the charged leptons clearly violate these symmetries at the Lagrangian level. Therefore, one can only impose $\mu-\tau$ symmetry as a symmetry of neutrino mass matrix not as a symmetry of the Lagrangian. This fact apparently disfavors the requirement of the $\mu-\tau$ symmetry.

In this work, we have enlarged the SM field content by introducing three right handed $SU(2)_L$ singlet neutrino fields ($\nu_{eR}$, $\nu_{\mu R}$ and $\nu_{\tau R}$). We have also considered Yukawa terms.
| Parameter | best-fit ($\pm \sigma$) | 3$\sigma$ |
|-----------|------------------------|----------|
| $\Delta m^2_{21}[10^{-5} eV^2]$ | $7.53^{+0.26}_{-0.22}$ | 6.99 - 8.18 |
| $\Delta m^2[10^{-3} eV^2]$ | $2.43^{+0.09}_{-0.10}$ (2.42$^{+0.01}_{-0.11}$) | 2.19(2.17) - 2.62(2.61) |
| $\sin^2 \theta_{12}$ | $0.307^{+0.016}_{-0.016}$ | 0.259 - 0.359 |
| $\sin^2 \theta_{23}$ | $0.386^{+0.021}_{-0.022}$ (0.392$^{+0.009}_{-0.010}$) | 0.331(0.335) - 0.637(0.663) |
| $\sin^2 \theta_{13}$ | $0.0241 \pm 0.0025 (0.0244^{+0.0023}_{-0.0025})$ | 0.0169(0.0171) - 0.0313(0.0315) |

Table 1: The best-fit values and 3$\sigma$ allowed ranges of the 3-neutrino oscillation parameters. The values (values in brackets) correspond to normal neutrino mass hierarchy (NH) i.e., $m_1 < m_2 < m_3$ (inverted neutrino mass hierarchy (IH) i.e., $m_3 < m_1 < m_2$). The definition of $\Delta m^2$ used is $\Delta m^2 = m_3^2 - (m_2^2 + m_1^2)/2$. Thus $\Delta m^2 = \Delta m^2_{31} - m_{21}^2/2$ if $m_1 < m_2 < m_3$ and $\Delta m^2 = \Delta m^2_{32} + m_{21}^2/2$ for $m_3 < m_1 < m_2$.

for the neutrinos in order to give them Dirac masses. In this frame work, we can demand a invariance of flavor neutrino mass terms under the interchange of the right handed muon neutrino ($\nu_{\mu R}$) and tau neutrino ($\nu_{\tau R}$). The RH charged leptons and neutrinos are singlet under $SU(2)_L$ and thus they do not form a multiplet. Therefore, we can invoke any symmetry in the RH neutrino sector without imposing that symmetry in the charged lepton sector. If any symmetry exists in the Dirac neutrino mass matrix under interchange of $\nu_{\mu R}$-$\nu_{\tau R}$ then this will be symmetry of the whole Lagrangian. We have constructed the different Dirac neutrino mass matrices assuming different kinds of symmetries in the $\nu_{\mu R}$ and $\nu_{\tau R}$ sector and tried to fit the experimentally observed quantities. Finally, we end up with a four parameter Dirac neutrino mass matrix which is based on the assumption of the Hermiticity of the Dirac neutrino mass matrix and a particular symmetry between $\nu_{\mu R}$ and $\nu_{\tau R}$. We have also shown that assuming IH in the neutrino sector, this four parameter neutrino mass matrix is consistent with the observed values of the three mixing angles and two squared-mass differences listed in Table.

The most general Dirac neutrino mass matrix contain 9 complex parameters and can be written as:

$$M_{\nu} = \begin{pmatrix} m_{e_L e_R} & m_{e_{L\mu R}} & m_{e_{L\tau R}} \\ m_{\mu_{L e R}} & m_{\mu_{L\mu R}} & m_{\mu_{L\tau R}} \\ m_{\tau_{L e R}} & m_{\tau_{L\mu R}} & m_{\tau_{L\tau R}} \end{pmatrix}.$$  \hspace{1cm} (3)

On this 18 parameter Dirac neutrino mass matrix, we have imposed the following conditions:

- We have assumed the hermiticity of the neutrino mass matrix. As result of this assump-

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4If the neutrinos get mass via the Yukawa couplings with the SH Higgs then the order of the neutrino Yukawa coupling should be about $10^{-12}$. However, there are interesting studies in the literature which assume a discrete $Z_2$ symmetry and a second Higgs doublet with vacuum expectation value in the eV to keV range, in order to generate sub eV scale Dirac type neutrino masses with a Yukawa coupling of the order of charged lepton Yukawa coupling.

5It is important to note that the assumption of Hermiticity is somewhat ad hoc i.e., Hermiticity of neutrino mass matrix is not an outcome of symmetry argument. However, we have shown in the following that with this assumption, the existing neutrino data can completely determine the mass matrix for the Dirac neutrinos with particular predictions for the neutrino masses and the CP violating phase which can be tested at the ongoing and future neutrino experiments. Therefore, in our analysis, the assumption of hermiticity of neutrino mass matrix is a purely phenomenological assumption. However, in the future, there might be some compelling theoretical framework which requires the hermiticity of neutrino mass matrix.
tion, the diagonal elements Eq. 3 become real and off-diagonal elements become complex conjugate of each other: \( m_{\mu L \mu R} = m_{e L \mu R}^* \), \( m_{\tau L \mu R} = m_{e L \tau R}^* \) and \( m_{\mu L \tau R} = m_{\mu L \tau R}^* \). Therefore, after demanding the hermiticity, we have a 9 parameter neutrino mass matrix.

- We have demanded the invariance under interchange of \( \nu_{\mu R} \leftrightarrow -\nu_{\tau R} \) followed by a complex conjugation of the couplings. Complex conjugation of the couplings is equivalent to making a CP transformation. In the rest of this article, the symmetry under interchange of \( \nu_{\mu R} \leftrightarrow -\nu_{\tau R} \) followed by a CP transformation is denoted as \( \nu_{\mu R} - \nu_{\tau R} \) reflection symmetry. As a result of the \( \nu_{\mu R} - \nu_{\tau R} \) reflection symmetry, we can derive three more constraints on the elements of the hermitian Dirac neutrino mass matrix:
  \[
  m_{e L \tau R} = -m_{e L \mu R}^*, \\
  m_{\mu L \tau R} = -m_{\mu L \mu R}^*, \\
  m_{\tau L \tau R} = -m_{\tau L \mu R}^*.
  \]
  These constraints reduces 5 more parameters and the 4 parameter hermitian Dirac neutrino mass matrix which respects \( \nu_{\mu R} - \nu_{\tau R} \) reflection symmetry can be written as:

\[
M_\nu = \begin{pmatrix}
a & b & -b^* \\
b^* & c & -c \\
-b & -c & c
\end{pmatrix},
\]

(4)

where, \( a \) and \( c \) are real and \( b \) is complex parameter.

The hermitian neutrino mass matrix is given in the flavor basis by

\[
M_\nu = U_\nu M^\text{diag}_\nu U^\dagger_\nu,
\]

(5)

where, \( M^\text{diag}_\nu \) is the diagonal neutrino mass matrix in the mass basis. Two squared-mass differences of the neutrinos are known from the experiments. Therefore, \( M^\text{diag}_\nu \) can be constructed with only one mass as unknown. For IH, the diagonal neutrino mass matrix is given by,

\[
M^\text{diag}_\nu = \begin{pmatrix}
\sqrt{m_3^2 + 0.002315} & 0 & 0 \\
0 & \sqrt{m_3^2 + 0.00239} & 0 \\
0 & 0 & m_3
\end{pmatrix},
\]

(6)

where, \( m_3 \) is the unknown mass and we have used the central values of the squared-mass differences listed in Table 1 for IH. In the mixing matrix \( U \), there are three angles and one phase. The mixing angles are already measured (see Table 1 for their central values and 3\( \sigma \) range) with good precision. In our analysis, we have considered the IH central values for the \( s_{12}^2 \) and \( s_{13}^2 \). However, we have considered \( s_{23}^2 = 0.5 \) which is not the central value but well within 3\( \sigma \) of the central value.

If we assume one particular neutrino mass hierarchy, there are still two quantities unknown in for the Dirac neutrinos namely, the mass \( m_3 \) in the diagonal mass matrix and the CP violating phase (\( \delta \)) in the mixing matrix. In our analysis, we have scanned unknown parameters (\( m_3 \) and \( \delta \)) over a range of values and try to fit the 5 experimental results (three mixing angles and two squared-mass differences) in the framework of a 4 parameter neutrino mass matrix (which results as a consequence of the assumption of hermiticity and \( \nu_{\mu R} - \nu_{\tau R} \) reflection symmetry) in Eq. 4. Our results are summarized in the following:

- In Fig. 1, we have presented \( m_{\mu L \mu R} \) and real part of \(-m_{\mu L \tau R}^\prime\) elements of the Dirac neutrino mass matrix in Eq. 3 as a function of \( m_3 \). The other free parameter \( \delta \) was randomly varied between 0 and \( \pi \). Fig. 1 shows that two curves interests each other at \( m_3 = -1.198 \times 10^{-3} \text{eV} \).
Figure 1: The elements $m_{\mu L\mu R}$ and real part of $-m_{\mu L\tau R}$ of the Dirac neutrino mass matrix in Eq. 3 as a function of $m_3$. The other free parameter $\delta$ was randomly varied between 0 and $\pi$. We have used IH central values for the $\Delta m^2_{21}$, $\Delta m^2$, $s^2_{12}$ and $s^2_{13}$ from Table 1 and for $s^2_{23}$, we choose $s^2_{23} = 0.5$.

- In Fig. 2, we have presented real and imaginary parts of the elements $m_{e L\mu R}$ and $m_{e L\tau R}$ (left panel) and diagonal elements $m_{\mu L\mu R}$ and $m_{\tau L\tau R}$ (right panel) of the Dirac neutrino mass matrix in Eq. 3 as a function of $\delta$ for $m_3 = -1.198 \times 10^{-3}$ eV. Fig. 2 shows that the desired structure (see Eq. 4) of the Dirac neutrino mass matrix is hermitian and reflection symmetric, is obtained for $\delta = \pi/2$ and $m_3 = -1.198 \times 10^{-3}$ eV. The numerical form of the mass matrix in the faveour basis for $\delta = \pi/2$ and $m_3 = -1.198 \times 10^{-3}$ eV is given by,

$$
\begin{pmatrix}
4.72 \times 10^{-2} & 2.49 \times 10^{-4} - 5.37 \times 10^{-3} i & -2.49 \times 10^{-4} - 5.37 \times 10^{-3} i \\
2.49 \times 10^{-4} + 5.37 \times 10^{-3} i & 2.43 \times 10^{-2} & -2.43 \times 10^{-2} \\
-2.49 \times 10^{-4} + 5.37 \times 10^{-3} i & -2.43 \times 10^{-2} & 2.43 \times 10^{-2}
\end{pmatrix},
$$

(7)

To summarize, we have considered Dirac neutrino mass matrix and investigated the possible symmetries in the $\mu-\tau$ sector. In order to ensure that the imposed condition is a symmetry of the Lagrangian (not only the symmetry of the neutrino mass matrix in the favlor basis), we have restricted the requirements only to the singlet right-handed muon and tau neutrinos. Assuming the hermiticity of the neutrino mass matrix and demanding the $\nu_{\mu R}-\nu_{\tau R}$ reflection symmetry of the Lagrangian, we have obtained a particular structure of the Dirac neutrino mass matrix with only 4 parameters. This 4 parameter Dirac neutrino mass matrix can explain all five (two squared-mass differences and three mixing angles) experimental results in the neutrino sector with particular predictions for the absolute values of the neutrino masses.
Figure 2: Left panel: The real and imaginary part of the elements $m_{eL_R}$ and $m_{eL_R}$ of the Dirac neutrino mass matrix in Eq. 3 as a function of $\delta$ (in radian) for $m_3 = -1.198 \times 10^{-3}$. Right panel: The diagonal elements $m_{\mu L R}$ and $m_{\tau L R}$ of the Dirac neutrino mass matrix in Eq. 3 as a function of $\delta$ for $m_3 = -1.198 \times 10^{-3}$. We have used IH central values for the $\Delta m^2_{21}, \Delta m^2, s^2_{12}$ and $s^2_{13}$ from Table 1 and for $s^2_{23}$, we choose $s^2_{23} = 0.5$.

(m_1 = 4.81 \times 10^{-2}, m_2 = 4.89 \times 10^{-2} \text{ and } m_3 = -1.198 \times 10^{-3} \text{ eV}) \text{ and CP violating phase } \delta = \pi/2. \text{ These predictions can be tested in future neutrino experiments.}

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