Fractional Quantum Hall Effect Measurements at Zero $g$-Factor

D.R. Leadley, R.J. Nicholas, D.K. Maude, A.N. Utjuzh, J.C. Portal, J.J. Harris and C.T. Foxon

1 Department of Physics, University of Warwick, Coventry, CV4 7AL, UK
2 Department of Physics, Clarendon Laboratory, Parks Road, Oxford, OX1 3PU, UK
3 Grenoble High Magnetic Field Laboratory, MPI-CNRS, 25 Avenue des Martyrs BP 166, F-38042 Grenoble Cedex 9, France
4 Russian Academy of Sciences, Institute of High Pressure Physics, 142092 Troitsk, Moscow Region, Russia
5 Department of Electronic and Electrical Engineering, University College, London, WC1E 7JE, UK
6 Department of Physics, Nottingham University, University Park, Nottingham, NG7 2RD, UK

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Abstract

Fractional quantum Hall effect energy gaps have been measured in GaAs/Ga$_{0.7}$Al$_{0.3}$As heterojunctions as a function of Zeeman energy, which is varied by applying hydrostatic pressure up to 20 kbar. The gap at $\nu = 1/3$ decreases with pressure until the $g$-factor changes sign when it again increases. The behavior is similar to that seen at $\nu = 1$ and shows that excitations from the 1/3 ground state can be spin-like in character. At small Zeeman energy, the excitation appears to consist of 3 spins and may be interpreted as a small composite skyrmion.

73.40.Hm, 73.20.Dx, 72.20.Jv
The two dimensional electron gas in a high magnetic field is an excellent test bed for studying electron-electron interactions. In recent years our understanding has been greatly simplified by the composite Fermion (CF) model, which maps the fractional quantum Hall effect (FQHE) of electrons onto an integer quantum Hall effect (IQHE) of CFs \([1–3]\). Thus the physics of the state at filling factor \(\nu = 1/3\), where there is one completely occupied CF Landau level (LL), is explained by analogy with \(\nu = 1\). The other principal FQHE states at \(\nu = p/(2p + 1)\) can similarly be explained by the integer states at \(\nu = p\). While the ground states are quite well understood the same is not true of the excited states which are responsible for conduction when the Fermi energy lies in a mobility gap.

The state at \(\nu = 1\) is an itinerant ferromagnet with a spontaneous magnetization, and consequently the activation energy gap deduced from transport measurements was found to be much larger than the single particle Zeeman energy (ZE). Recently it has been shown optically \([4]\) and electrically \([5–7]\) that the excitations at this point are probably spin texture excitations which in the limit of vanishing ZE are skyrmions \([8,9]\). In this paper we examine the CF analogue of this state at \(\nu = 1/3\) as the ZE vanishes. Our measurements suggest that in this limit a new skyrmionic CF excitation occurs.

Although the initial CF model ignored spin it is very important when the Landé g-factor is small. In GaAs the electronic ZE \(g\mu_B B\) has similar magnitude to the gaps between CF LLs, which arise from electron-electron correlations and scale like \(E_c = e^2/4\pi\epsilon l_B\) (\(l_B = \sqrt{\hbar/eB}\) is the magnetic length). CF LLs originate from \(\nu = 1/2\) where there is an offset of \(g\mu_B B_{1/2}\) between fans of each spin, which provides an essential difference from the IQHE. This gives the possibility of level crossing as the ZE and magnetic field are varied and leads to the observed disappearance and re-emergence of fractions \([10–12]\). Although the ground state at \(\nu = 1/3\) will be fully spin polarized, the states at \(\nu = 2/3\) or 2/5 may be either fully polarized or unpolarized depending on the relative sizes of the ZE and the CF LL gaps. Similarly the excitations may involve either spin flips or inter CF LL transitions. At \(\nu = 1/3\) and small ZE, i.e. very low magnetic fields or small g-factor, we expect a spin flip transition to the lowest CF LL state with the opposite spin. The interesting question which we address is whether this is a single spin flip of one CF or a collective phenomenon i.e. a skyrmionic excitation of the CFs which we will refer to as a composite skyrmion.

We have performed experiments where \(g\) can be tuned through zero thus favoring skyrmion formation. The tuning is achieved with hydrostatic pressure of up to 22 kbar \([13]\). In GaAs \(g = -0.44\), as a result of subtracting band structure effects from the free electron value of 2. At higher pressure the band structure contribution reduces, and so does the magnitude of \(g\) which passes through zero at \(\sim 18\) kbar. Previously we used this method to investigate the changing energy gaps of the mixed spin states around \(\nu = 3/2\) \([10]\). Here we demonstrate that the gap at \(\nu = 1/3\) is indeed a spin gap with excitations consistent with flipping \(\sim 3\) spins at small ZE. This shows that composite skyrmions can be formed at \(\nu = 1/3\) when the electron g-factor is sufficiently small. By contrast the gap at \(\nu = 2/5\) is consistent with a single particle excitation.

The samples studied were high quality GaAs/ \(\text{Ga}_0.7\text{Al}_{0.3}\text{As}\) heterojunctions grown by Molecular Beam Epitaxy at Philips Research Laboratories, Redhill. Samples G586, G627 and G902 have undoped spacer layers of 40, 40 and 20 nm. At ambient pressure and 4 K their respective electron densities after photoexcitation are 3.3, 3.5 and \(5.7 \times 10^{15}\) m\(^{-2}\) with corresponding mobilities of 300, 370 and 200 m\(^2\)/Vs. Data from similar samples measured
without applied pressure is included from Ref. [14]. The samples were mounted inside a non-magnetic beryllium copper clamp cell [15] and the pressure was measured from the resistance change of manganin wire. The absolute values quoted at low temperature are accurate to ±1 kbar, but between data points the variation is less than ±0.2 kbar. The pressure cell was attached to a top loading dilution refrigerator probe allowing temperatures as low as 30 mK to be obtained and measured with a ruthenium oxide resistor attached outside the pressure cell, which followed the sample temperature with a negligible time lag.

Increasing the pressure causes the GaAlAs conduction band to move relative to the GaAs conduction band in the well reducing the number of electrons. Above ∼ 13 kbar no electrons were present in the dark at low temperature, but a certain number could be recovered after illumination from a red LED. The illumination time required to obtain a constant number of electrons roughly doubled for every 2 kbar increase in pressure, reaching several hours at 20 kbar. The highest pressure studied was 22 kbar, but no conductivity could be measured despite prolonged illumination. The sample required several hours for the density to stabilize before quantitative measurements could be made during which it varied by less than 1% over the full temperature range. The data from G586 was recorded with a density of 0.44 ± 0.06 × 10^{15} m^{-2} above 13 kbar and slightly higher at lower pressures. This puts ν = 1/3 at 5.4 T. For G627 and G902 the data was recorded in the range 0.77–1.23 × 10^{15} m^{-2} i.e. ν = 1/3 at 9–15T.

The magnetoresistance ρ_{xx} of sample G586 at 40 mK is shown for pressures between 10 and 20 kbar in Fig. 1, plotted against 1/ν to remove the remaining small density variation. The feature at ν = 1/3 weakens as the pressure is increased, it completely disappears at 18.7 kbar and is recovered at the highest pressure. Meanwhile, the feature at ν = 2/3 remains approximately constant, which is an important indication that pressure does not denigrate the sample quality and destroy the FQHE.

Figure 2 shows the temperature and pressure variation of the 1/3 minimum, defined as (ρ_{xx}(∞) − ρ_{xx}(T))/ρ_{xx}(∞), where ρ_{xx}(∞) is the resistivity at the same field taken from a high temperature trace where there is no longer a minimum. From the figure it is clear that at higher pressures progressively lower temperatures are required to see a 1/3 minimum. Thus, however the data is analyzed, the energy gap E_g will decrease strongly with pressure. We have extracted values of E_g by fitting the temperature dependence to the Lifshitz-Kosevich (LK) formula, from which Δρ_{xx} ∝ X/\sinh X where X = 2π^2kT/E_g. This procedure, described in more detail in Ref. [14], has the advantages over finding activation energies from an Arrhenius plot that firstly it measures the gap between LL centers not the mobility gap, and so is less sensitive to changes in disorder, and secondly an accurate zero of resistance is not required, which avoids any problems of parallel conduction and means especially low temperatures are not required. Values of E_g at ν = 1/3, 2/3, and 2/5 are shown in Fig. 3 in units of E_c for G586. In these units a gap of 0.01 is equivalent to 1.17 K for ν = 1/3 at 5.4 T and 0.83 K for ν = 2/3 at 2.7 T. This scaled data shows the same trends as the raw data, but the scatter due to the small density variation between different pressures is removed. Scaling the data also makes comparison with theory easier.

The trends observed in the raw data can now be quantified and it is seen that 1/3 decreases and 2/5 increases with pressure over this range. Experimentally the feature at 1/3 vanishes between 17 and 19 kbar, which is exactly the pressure region where g is predicted to pass through zero. By vanishing we mean that 1/3 is weaker than 2/5 and a separate
minimum cannot be observed, although the 2/5 minimum has a pronounced tail on the high field side from the residual 1/3 feature. Thus an upper limit can be set on the 1/3 gap, although we can not tell if it has completely collapsed. Even though a 1/3 feature could be seen at 20 kbar in the lowest temperature data, it was not possible to obtain an accurate value for the energy gap as the minimum could not be followed to higher temperatures. Taking the temperature dependence of $\rho_{xx}$ at the field where 1/3 occurs at base temperature, an energy gap of 0.017$E_c$ results which is consistent with the fraction being established again once $g$ has changed sign.

As $g$ varies with both pressure and density, the ZE must also be scaled to compare data from different samples. Figure 4 shows the energy gaps at 1/3 and 2/5 for all the samples studied as a function of ZE. Both axes are scaled by $E_c$ so the x-axis is $\eta = g\mu_B B / E_c$, the ratio which determines the skyrmion size and energy [10].

Considering 1/3 first, the data falls into two distinct groups. For $\eta > 0.01$, mostly from data taken at ambient pressure, the gap only scales with the Coulomb energy. This behavior is very similar to that observed at $\nu = 1$ [4] and shows the FQHE state at $\nu = 1/3$ has a Coulomb gap, which may correspond to either the spin-wave or more probably the CF gap. For $\eta < 0.01$, using data taken above 9 kbar, there is a spin gap proportional to ZE. The line on Fig. 4(a), with a gradient of 3, fits the data very well at small $\eta$. A slope of unity cannot account for data at small ZE. The slope of 3 corresponds to an energy gap of $3g\mu_B B$ which indicates an excitation involving the reversal of three spins.

This excitation could be a small composite skyrmion, as predicted by theory. In a rough estimate Sondhi et al. suggested that a skyrmion formed at $\nu = 1/3$ occurring at 1 T should contain ‘a couple of reversed spins’ [4]. They also estimate the skyrmion–antiskyrmion pair gap as 0.024$E_c$ at $g = 0$. The minimum gap we obtain is 0.01$E_c$, which compares well when account is taken of the typical 50% reduction in Coulomb energies found in calculations where finite thickness is included [17]. In a more detailed calculation the energy to creating an antiskyrmion at $\nu = 1/3$, i.e. the energy to remove one spin at fixed magnetic field, was found to be $E_{1/3}/E_c = 0.069 + 0.024 \exp(-0.38R^{0.72}) + \eta R$ [18]. The number of reversed spins $R$ in the composite skyrmion can be found by minimising this expression and we see that $R = 1$ for $\eta > 0.004$; $R = 3$ at $\eta = 0.002$ and $R = 6$ at $\eta = 0.001$. These numbers cannot be directly compared with our experiments at fixed particle number where the excitation is a skyrmion–antiskyrmion pair because they do not include creation of the quasi-particle skyrmion or finite thickness effects. Nonetheless, they allow us to estimate relevant energy and size scales. It is clear that composite skyrmions will always be small for experimentally accessible parameters and that a size of 3 spins provides good agreement between the experiment and theory in the region of $\eta = 0.002$. The experiment suggests however that the minimum gap for skyrmionic CF excitations is much less than half of the gap at large ZE. This is substantially different from the prediction of exactly a 50% reduction in the gap at $\nu = 1$ made in Ref. [11] for infinite sized skyrmions.

Turning now to the full set of scaled data obtained at $\nu = 2/5$ (Fig. 4(b)), there are two distinct regions that cross over at $\eta \approx -0.006$. For $\eta < -0.006$ the gap decreases as the size of the spin splitting decreases and for $\eta > -0.006$ the gap increases again. This suggests a level crossing and finds a straightforward explanation in the CF picture. The $\nu = 2/5$ FQHE gap occurs when two CF LLs are full. When the ZE is small these will be the lowest LLs of the two opposite spin ladders, thus the excitation at $\nu = 2/5$ is a spin
flip from an unpolarized ground state. As the ZE increases the spin reversed ladder moves up relative to the other spin and the 2/5 gap decreases. When the second and third levels cross over there is a transition to a fully polarized ferromagnetic ground state and the gap might be expected to vanish. Further increase of ZE opens the $\nu = 2/5$ gap again, until the spin flip is no longer the lowest excitation and the gap saturates at $\hbar \omega_c^*$. The slope of unity observed on Fig. 4(b) shows that this description in terms of single particle CF energy levels is valid except in the immediate neighborhood of $\eta = -0.006$, where a finite gap remains. For $kl_B = 0$ excitations the cross over would be expected when $g\mu_B B = \hbar \omega_c^*$, which is clearly not the case since, from the gap saturation value and our previous work [14], $\hbar \omega_c^*$ is $\sim 0.03 E_c$ at $\nu = 2/5$. However, the large $kl_B$ excitations, that transport experiments measure, will again be spin waves with a much greater energy than $g\mu_B B$, causing the cross over to occur at a smaller value of $\eta$. Thus there will be an anti-crossing of the levels and a finite gap between the ground and excited states of the ferromagnet formed at $\nu = 2/5$. The formation and excitations out of this state would be very interesting to study theoretically.

While the gap at $\nu = 2/3$ appears to be approximately constant over the range of pressure in Fig. 3, the field at 2/3 is only half that at 1/3 which makes the range of ZE insufficient to draw definitive conclusions. When the scaled data at large ZE from other samples with $\eta < -0.01$ is included, the gap decreases by $g\mu_B B$ in a manner similar to 2/5. In the CF model 2/3 and 2/5 are expected to behave in a very similar way as they both have the same CF LLs structure. We do not see an obvious minimum in the region $-0.01 < \eta < 0$, although the scatter is larger than for 2/5. While we can not see the levels cross over for 2/3 this has previously been observed when the Zeeman energy is increased by tilting the magnetic field, but only for the lowest density samples [19]. Interestingly the tilted field measurements did not see the cross over for 2/5, so it can be seen that a combination of experimental techniques is required for the complete study of the FQHE.

In summary we have measured the FQHE gaps at $\nu = 2/3$, 2/5 and 1/3 under conditions where the Zeeman energy can be tuned through zero. For the ferromagnetic state at $\nu = 1/3$ the energy gap decreased dramatically as the ZE was reduced to zero and recovered again once the sign of the g-factor changed. At small ZE, the excitation appears to consist of 3 reversed spins which we interpret as a small composite skyrmion. The behavior is very similar to that seen for the most easily accessible quantum Hall ferromagnet state at $\nu = 1$, and is in general agreement with theoretical predictions. These experiments lend support to the existence of skyrmionic composite Fermions excitations within the two-dimensional electron gas.

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FIGURES

FIG. 1. Magnetoresistance recorded at 40 mK for sample G586 at pressures between 10 and 20 kbar. Note how the feature at $\nu = 1/3$ becomes weaker relative to $2/5$ with increasing pressure, but is recovered at the highest pressure.

FIG. 2. Temperature dependence of the $\nu = 1/3$ minimum, $\Delta \rho_{xx}/\rho_{xx}$ for sample G586. The energy gap is extracted from fits to the LK formula shown by dashed lines.

FIG. 3. Energy gaps measured for the strongest fractions in sample G586, showing how the gap at $\nu = 1/3$ decreases, $2/5$ increases and $2/3$ remains approximately constant as the applied hydrostatic pressure is increased. Above 19 kbar the gap at $1/3$ increases again but reliable quantitative values cannot be obtained.

FIG. 4. (a) Energy gap at $\nu = 1/3$ for all the samples studied as a function of the Zeeman energy, (both in units of $E_c$). The line shows the energy required to flip 3 spins. (b) The energy gap at $\nu = 2/5$. The slope of the lines now corresponds to a single spin flip.
Figure 1
Figure 2
Figure 3

[Graph showing energy gap $E_g/E_c$ versus pressure (kbar) for different fractions: 1/3 (black squares), 2/5 (red circles), and 2/3 (green triangles).]

Figure 3
Figure 4

(a) \( \nu = 1/3 \)
(b) \( \nu = 2/5 \)

\[ 3 g \mu_B B \]

\[ g \mu_B B \]

\[ -g \mu_B B \]

\( \eta = \text{Zeeman energy} / E_c \)

Ref. 14