Quantum Spherical Spin Glass.

Supersymmetry and Annealing

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Abstract

We show that the effective action of the quantum spherical spin glass is invariant under a generalized form of the Becchi-Rouet-Stora-Tyutin (BRST) supersymmetry. The Ward identities associated to this invariance indicate that the spin glass order parameter must vanish, and as a result the annealed average is exact in this model. We present new results for the free energy, entropy and specific heat. Due to quantum effects the entropy remains finite and the specific heat vanishes at zero temperature. Results for the phase diagram coincide with those obtained by different formalisms. At zero temperature we derive the scaling behaviour with frequency of the dynamical susceptibility.

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1. Introduction

In a previous publication\cite{1} the classical spherical spin glass was analyzed by using supersymmetry methods\cite{2,3} and it was pointed out that the model was "anomalous" because the order parameter $q$ that measures the replica’s overlap was vanishing and the configurational average of any number of replicas gave the correct known result\cite{4}, then the annealed approximation with only one replica was also correct. More recent work on the calculation of the complexity, or logarithm of the density of states of a certain energy, of the $p$-spherical spin glass, $p \geq 3$, \cite{5,6} and of the Sherrington-Kirkpatrick model \cite{7} shows that the annealed average gives the correct answer when the action is invariant under the generalized Becchi-Rouet-Stora-Tyutin (BRST) supersymmetry\cite{8}.

In the present paper we first demonstrate how the generalized BRST supersymmetry transformation applies to the functional integral formulation of the quantum spherical spin-glass. We also discuss how this internal invariance leads to Ward identities that result in the vanishing of the replica overlap $q$. This result had been obtained previously for the classical spin-glass\cite{1} and justifies the performance of the annealed configurational average\cite{5,6,7} also in the quantum spherical spin-glass. The model was proposed earlier in ref.\cite{9} and was solved by using the distribution of eigenvalues of a gaussian random matrix. Later, the interest in this model was renewed because it corresponds to the infinite-$M$ limit of the spin-glass of $M$-components quantum rotors and it was studied\cite{10} by using the replica method in a replica symmetric theory. Both theories give the same phase diagram, although thermodynamic quantities as free energy and specific heat are not discussed. On the
other hand, the spin-glass model of quantum rotors is equivalent to the Ising spin-glass model in a transverse field\cite{11}. A formulation for quantum spherical spins with a different temporal dependence was presented for multispin interactions\cite{12}. Critical properties were analyzed in ref.\cite{13} and applications to physical systems were presented in ref.\cite{14, 15}. All these papers coincide in that the quantum spherical model presents a finite entropy and vanishing specific heat at low temperature, while the classical model presents a negative infinite entropy and constant specific heat.

We study the quantum spherical spin-glass in the annealed average by using a functional integral formulation derived by using Feynman’s prescription\cite{16, 17}. In sect.2 we present the general formalism and discuss the supersymmetry transformation that leaves invariant the action together with the corresponding Ward identities\cite{18}, while in sect.3 we discuss the results. We are careful to normalize the free energy so as to recover a finite classical limit when the rotors moment of inertia $I \to \infty$. In this way the phase diagram, that is calculated from the spherical condition, coincide with previous studies\cite{9, 10}. In the classical limit $I \to \infty$ the entropy diverges at $T = 0$ in the typical fashion of the classical spherical model\cite{11, 4}, with the important difference that it remains finite in the quantum regime for finite values of the rotors moment of inertia. The specific heat shows the usual discontinuity at the critical temperature but it vanishes at $T = 0$ in the quantum regime. We derive scaling laws for the dynamical susceptibility, in agreement with previous results\cite{10}. We present in the Appendix some useful relations among Grassmann variables without proof, while we refer the reader to the original
work\cite{2} for rigorous results.

2. Supersymmetry and Annealing

We consider a spin glass of quantum rotors\cite{10} with moment of inertia $I$ in the spherical limit with Hamiltonian

$$\mathcal{H}_{SG} + \mu \sum_i S_i^2 = \frac{1}{2I} \sum_i P_i^2 - \frac{1}{2} \sum_{i,j} J_{ij} S_i S_j + \mu \sum_i S_i^2$$  \hspace{1cm} (1)

where the spin variables at each site are continuous $-\infty < S_i < \infty$ and we introduced the canonical momentum $P_i$ with commutation rules:

$$[S_j, P_k] = i \delta_{j,k}$$  \hspace{1cm} (2)

The sum in eq. (1) runs over sites $i, j = 1..N$. The coupling $J_{ij}$ in eq. (1) is an independent random variable with the gaussian distribution

$$P(J_{ij}) = e^{-\frac{J_{ij}^2 N}{2\pi}} \sqrt{\frac{N}{\pi J^2}}$$  \hspace{1cm} (3)

while the chemical potential $\mu$ is a Lagrange multiplier that insures the mean spherical condition

$$- \frac{\partial \langle \ln \mathcal{Z} \rangle}{\partial (\mu)} = \sum_i \int_0^\beta d\tau \langle S_i^2 \rangle = \beta N$$  \hspace{1cm} (4)

and $\beta = 1/T$ is the inverse temperature. We work in units where the Boltzmann constant $k_B = \hbar = 1$ and $\mathcal{Q}$ is the quantum partition function

$$\mathcal{Z} = Tr e^{-\beta (\mathcal{H}_{SG} + \mu \sum_i S_i^2)}$$  \hspace{1cm} (5)
The partition function at zero field can be expressed as a functional integral \[16, 17\]

\[
Z = \int \prod_i D S_i \exp (-\mathcal{A}_\mathcal{O} - \mathcal{A}_\mathcal{SG})
\]  

(6)

where the non interacting action \(\mathcal{A}_\mathcal{O}\) is given by

\[
\mathcal{A}_\mathcal{O} = \int_0^\beta d\tau \sum_i \left( \frac{I}{2} \left( \frac{\partial S_i}{\partial \tau} \right)^2 + \mu S^2_i(\tau) - H_i(\tau) S_i(\tau) \right)
\]  

(7)

and the interacting part

\[
\mathcal{A}_\mathcal{SG} = \frac{1}{2} \sum_{i,j} J_{ij} \int_0^\beta d\tau S_i(\tau) S_f(\tau)
\]  

(8)

After performing a Fourier transformation in time and taking into account the reality of the fields \(S_i(\tau)\) the generating functional can be written in terms of the real components \(R_i(n)\) of the Fourier transforms \(S_i(\omega_n) = S^*_i(-\omega_n)\), where \(\omega_n = \frac{2\pi n}{\beta}\) are boson Matsubara’s frequencies. As it was shown in \[1\] the correlation functions can be obtained from the generating functional for two replicas

\[
\mathcal{W}(H) = \prod_{n \geq 0} \|\Gamma_{ij}(n)\| \int \prod_{i\alpha n \geq 0} dR_{i\alpha}(n) \exp \left( - \sum_{\alpha n \geq 0} \sum_{ij} \Gamma_{ij}(n) R_{i\alpha}(n) R_{j\alpha}(n) + \sum_{i\alpha} H_{i\alpha}(n) R_{i\alpha}(n) \right)
\]  

(9)

where the index \(\alpha = 1, 2\), the \(H_{i\alpha}(n)\) are auxiliary fields and

\[
\Gamma_{ij}(n) = (I\beta \omega_n^2 + 2\beta \mu) \delta_{ij} - J_{ij}
\]  

(10)
By using the results in the Appendix, the determinant $\|\Gamma_{ij}(n)\|$ may be expressed with the help of auxiliary Grassmann fields $\chi^*, \chi$ and $W$ can be written

$$W(H_\alpha, \gamma) = \prod_{n \geq 0} \int \prod_{i, \gamma \geq 0} d\chi_i^*(n) d\gamma_i(n) \prod_\alpha dR_{i\alpha}(n)$$

\[
\exp - \left[ \sum_{n \geq 0} \left[ \sum_{ij} \Gamma_{ij}(n) (\chi_i^*(n) \chi_j(n) + \chi_j^*(n) \chi_i(n)) \right] + \sum_\alpha R_{i\alpha}(n) R_{j\alpha}(n) \right] + \sum_i \left( \gamma_i^*(n) \chi_i(n) + \chi_i^*(n) \gamma_i(n) \right)
\]

\[
+ \sum_\alpha \left( H_{i\alpha}(n) R_{i\alpha}(n) \right)
\]

(11)

where $\chi_i^*(n), \chi_i(n)$ are complex anticommuting Grassmann variables while the $R_{i,\alpha}(n), \alpha = 1, 2$ are real commuting variable and we introduced two extra Grassmann auxiliary fields $\gamma_i$ and $\gamma_i^*$. As the $\Gamma_{ij}$ are symmetric, when the auxiliary fields are set equal to zero the functional is invariant under the supersymmetry transformation

$$\chi_i^*(n)/\sqrt{2} = \chi_i^*(n)/\sqrt{2} + \epsilon^* \sum_\alpha R_{i,\alpha}(n)/\sqrt{2}$$

$$\chi_i(n)/\sqrt{2} = \chi_i(n)/\sqrt{2} + \epsilon \sum_\alpha R_{i,\alpha}(n)/\sqrt{2}$$

$$R_{i,\alpha}(n)' = R_{i,\alpha}(n) - \epsilon^* \chi_i(n)/2 + \epsilon \chi_i^*(n)/2$$

(12)

where $\epsilon$ is a complex Grassmann variable and we adopt the convention for complex conjugation $[2]$ $\epsilon^{**} = -\epsilon$, $(\epsilon \chi_i)^* = \epsilon^* \chi_i^{**} = -\epsilon^* \chi_i$, then $R_{i,\alpha}(n)'$ is effectively a real variable. The transformation differs infinitesimally from unity and $\epsilon^* \epsilon$ can be neglected, although this concept is somehow meaningless in the case of Grassmann variables. Now it is convenient to switch to the superalgebra notation $[2]$ of the Appendix and to introduce the supervectors
for the auxiliary fields

\[
J_i = \begin{pmatrix}
H_{i1} \\
H_{i2} \\
\gamma_i/\sqrt{2} \\
\gamma_i^*/\sqrt{2}
\end{pmatrix}
\]

\[
SJ_i^\dagger = \begin{pmatrix}
H_{i1} & H_{i2} & \gamma_i^*/\sqrt{2} & -\gamma_i/\sqrt{2}
\end{pmatrix}
\tag{13}
\]

and for the field variables

\[
\varphi_i = \begin{pmatrix}
R_{i1} \\
R_{i2} \\
\chi_i/\sqrt{2} \\
\chi_i^*/\sqrt{2}
\end{pmatrix}
\]

\[
S\varphi_i^\dagger = \begin{pmatrix}
R_{i1} & R_{i2} & \chi_i^*/\sqrt{2} & -\chi_i/\sqrt{2}
\end{pmatrix}
\tag{14}
\]

In this way the functional \( W \) in equation(11) may be written in compact form

\[
W(J) = \prod_{n \geq 0} \int d\chi_i^*(n) d\chi_i(n) \prod_{\alpha} dR_{i\alpha}(n)
\exp \left[ -1/2 \sum_{ij} S\varphi_i^\dagger M_{ij} \varphi_j + \sum_{i} SJ_i^\dagger \varphi_i \right]
\tag{15}
\]

that may be formally integrated to give

\[
W(J) = \exp \left[ 1/2 \sum_{ij} SJ_i^\dagger Q_{ij} J_j \right]
\tag{16}
\]

with the 4x4 supermatrices

\[
Q_{ij} = \begin{pmatrix}
q_{11} & q_{12} & \theta_{13} & \theta_{14} \\
qu_{21} & q_{22} & \theta_{23} & \theta_{24} \\
\theta_{31} & \theta_{32} & v_{33} & 0 \\
\theta_{41} & \theta_{42} & 0 & v_{44}
\end{pmatrix}_{ij}
\tag{17}
\]
By symmetry we should have

\[ q_{11} = q_{22} = < R_{i\alpha} R_{i\alpha} > = v_{33} = v_{44} = < \chi^* \chi > \]
\[ v_{34} = v_{43} = < \chi \chi > = < \chi^* \chi^* > = 0 \]

(18)

To derive Ward identities\[18\] we consider that, as the system is invariant when the fields are subject to the supersymmetry transformation in equation\[12\]

\[ \varphi'_i = A \varphi_i \]

where

\[
A = \begin{pmatrix}
1 & 0 & -\epsilon^*/\sqrt{2} & \epsilon/\sqrt{2} \\
0 & 1 & -\epsilon^*/\sqrt{2} & \epsilon/\sqrt{2} \\
\epsilon/\sqrt{2} & \epsilon/\sqrt{2} & 1 & 0 \\
\epsilon^*/\sqrt{2} & \epsilon^*/\sqrt{2} & 0 & 1
\end{pmatrix} = 1 + D
\]

(19)

then it should be also invariant when we transform the external fields \[18\] in equation\[16\]

\[ J' = AJ \]
\[ SJ'' = SJ' SA^\dagger \]

(20)

Where the super-adjoint \( SA^\dagger \) is defined in the Appendix. This condition leads to

\[ SD^\dagger Q + QD = 0 \]

(21)

from where we deduce

\[ v_{33} = q_{11} + q_{12} \]
\[ v_{44} = q_{22} + q_{21} \]

(22)
and from equation (18) we obtain for the overlap between two replicas

\[ q_{12} = q_{21} = 0 \]  \hspace{1cm} (23)

The spin-glass order parameter \( q = q_{12} \) and it vanishes identically from equation (19), thus making the annealed average exact in the quantum spherical spin-glass as it was discussed in [1] for the classical spherical spin-glass.

3. Results

When performing the annealed average we take the configurational average of \( Z \) in equation (6) over the random variables \( J_{ij} \) and after splitting the quadratic term with a gaussian integration we obtain the result

\[ \langle Z \rangle_{ca} = \int DQ(\tau - \tau') \exp \left[ -N \left( \frac{J^2}{4} \int_0^\beta \int_0^\beta d\tau d\tau' Q(\tau - \tau')^2 - \Lambda \right) \right] \]  \hspace{1cm} (24)

where we assumed time translational invariance and

\[ e^\Lambda = \int DS(\tau) \exp \left[ -A_0 + \frac{J^2}{2} \int_0^\beta \int_0^\beta d\tau d\tau' Q(\tau - \tau')S(\tau)S(\tau') \right] \]  \hspace{1cm} (25)

A steepest descent calculation in eq. (24) gives

\[ Q(\tau - \tau') = \frac{1}{N} \sum_i < S_i(\tau)S_i(\tau') > \]  \hspace{1cm} (26)

The partition function in eq. (24) is solved by a time Fourier transformation with the result

\[ \langle Z \rangle_{ca} = N^N \prod_{n=0}^\infty \int dQ(\omega_n) \exp \left\{ -N \left[ \frac{(\beta J)^2}{2} Q(\omega_n)^2 - \frac{\beta H^*(\omega_n)H(\omega_n)}{\beta I \omega_n^2 + 2\beta \mu - 2(\beta J)^2 R(\omega_n)} + \ln \left( \beta I \omega_n^2 + 2\beta \mu - (\beta J)^2 Q(\omega_n) \right) \right] \right\} \]  \hspace{1cm} (27)
where $\omega_n = \frac{2\pi n}{\beta}$ is the boson Matsubara frequency and $Q(\omega_n)$ is the real part of the Fourier transform of $Q(\tau)$. A steepest descent evaluation of the integral in eq.(27) gives, for $H(\omega_n) = 0$

$$Q(\omega_n) = [\beta I \omega_n^2 + 2\beta \mu - (\beta J)^2 Q(\omega_n)]^{-1}$$  \hspace{1cm} (28)$$

with solution

$$Q(\omega_n) = \frac{\beta I \omega_n^2 + 2\beta \mu - \sqrt{(\beta I \omega_n^2 + 2\beta \mu)^2 - 4(\beta J)^2}}{2(\beta J)^2}$$  \hspace{1cm} (29)$$

while the mean spherical condition in eq.(11) becomes

$$\sum_{-\infty}^{\infty} Q(\omega_n) = 1$$  \hspace{1cm} (30)$$

Introducing eq.(29) into eq.(27) we obtain for the free energy per site at the saddle point

$$\beta F = \sum_{-\infty}^{\infty} \left( \frac{1}{4}(\beta J)^2 Q(\omega_n)^2 - \frac{1}{2} \ln (Q(\omega_n)) \right) - \beta \mu - \ln (N)$$  \hspace{1cm} (31)$$

The normalization constant $N = \frac{1}{\sqrt{IJ}}$ is determined so that we recover a finite limit for the free energy when $I \to \infty$. Following standard procedures\[16\] we convert sums over frequencies into integrals with the result for the mean spherical condition

$$\int_{L_-}^{L_+} dy \sqrt{(L_+^2 - y^2)(y^2 - L_-^2)} \coth\left(\frac{\beta y}{2\sqrt{I}}\right) = 2\pi J^2 \sqrt{I}$$  \hspace{1cm} (32)$$

where

$$L_\pm^2 = 2\mu \pm 2J$$  \hspace{1cm} (33)$$

and we obtain for the free energy

$$\beta F = \frac{1}{\pi J^2} \int_{L_-}^{L_+} dy y \ln \sinh\left(\frac{\beta y}{2\sqrt{I}}\right) \sqrt{4J^2 - (2\mu - y^2)^2} - \beta \mu - \ln N$$  \hspace{1cm} (34)$$
In the classical limit $I \to \infty$ the integrals in eq.(32) and eq.(34) can be performed exactly in terms of hypergeometric functions with the result

$$\frac{2\mu_{class}}{J} = \beta J + \frac{1}{\beta J}$$  \hspace{1cm} (35)$$

$$\beta F_{class} = \frac{1}{2} \ln \left[ \beta \mu + \sqrt{(\beta \mu)^2 - (\beta J)^2} \right] + \frac{1}{4} \left( \frac{\beta \mu - \sqrt{(\beta \mu)^2 - (\beta J)^2}}{\beta \mu + \sqrt{(\beta \mu)^2 - (\beta J)^2}} - \mu \beta - \ln \frac{\sqrt{\beta J}}{2} \right)$$  \hspace{1cm} (36)$$

The expression in eq.(35) coincide with the results for the classical spherical spin glass in ref.(1, 4), but the free energy in eq.(36) differs due to the last logarithmic contribution. This originates in the choice of the normalization constant $\mathcal{N}$. If we had chosen $\mathcal{N}$ following Feynman’s prescription[16, 17] we would have obtained unphysical results in the quantum regime for finite values of $I$, like a negative specific heat at low temperatures [19]. The chemical potential $\mu$ decreases with temperature until it reaches the value $\mu_c = J$[20] at the critical temperature $T_c$, for lower temperatures the spherical condition no longer holds and $\mu$ sticks to this value. We show in fig.1 the solution for $\mu$ as a function of $T$ for different values of $I$. The critical temperature is obtained by setting $\mu = \mu_c = J$ in eq.(35). We show in fig.2 the phase diagram for $T_c$ as a function of $I$. $T_c$ decreases with $1/I$ and vanishes at the critical value $I_c = \frac{16}{9\pi^2} J$. This result coincides with previous references[9, 10]. Close to this value we find $T_c^2 = \frac{J(I-I_c)}{3I_c^2}$. We show in fig.3, fig.4, fig.5 results for the free energy, entropy and specific heat as functions of temperature for different values of $I$. We observe that the entropy remains finite
when $T \to 0$ for any finite value of $I$, while we recover the usual divergence $S \to -\infty$ in the classical spherical spin glass model\cite{1,4} when $I \to \infty$. The specific heat vanishes at $T = 0$ for finite values of $I$, as expected in quantum systems\cite{12,13,14,15}. The dynamic susceptibility is the response to the field $H(\omega_n)$ and we obtain from eq.\eqref{eq:27}

$$\chi(\omega_n) = \frac{\omega_n^2 I + 2\mu - \sqrt{(\omega_n^2 I + 2\mu)^2 - 4J^2}}{2J^2}$$

The static susceptibility

$$\chi(0) = \frac{\mu - \sqrt{\mu^2 - J^2}}{J^2}$$

satisfies Curie law for high temperatures and exhibits the usual cusp at the critical temperature. It is shown in fig.6 for different values of $I$. The dynamic susceptibility at $T = 0$ is obtained by setting $i\omega_n \to \omega + i\delta$ in eq.\eqref{eq:37}. For $\Delta = \mu - J \geq 0$ and $I = I_c$ we obtain that the $\text{Im} \chi(\omega)$ satisfies the scaling relation

$$\text{Im} \chi(\omega) = \text{sgn} \omega |\omega|^{\mu'}(\sqrt{I_c}/J)\Phi(\omega \sqrt{I_c/J^2 \Delta})$$

with the exponent $\mu' = 1$ and the scaling function $\Phi(x) = \sqrt{1 - x^{-2}}$, what agrees with the results of ref.\cite{10}.

4. Conclusions

The quantum spherical spin glass was studied within the annealed average, that is exact due to the internal BRST supersymmetry of the model, as it was discussed in sect.1. Our results for the phase diagram coincide with
those obtained by Ye et al[10] with the replica method and by Shukla et al[9] using the semicircular law. However, a substantial difference exists between our results for the free energy, entropy and specific heat and those presented by Shukla et al[9]. The results obtained with quantum functional integrals methods are very sensitive to the use of the correct normalization and measure [16, 17] and we set the normalization constant in our expression for the free energy such as to recover a finite limit when $I \to \infty$. As a result we obtain a finite entropy at low temperatures and a vanishing specific heat for finite values of the rotors moment of inertia $I$, as it is expected in quantum spherical models[12, 13, 14, 15]. In the limit $I \to \infty$ we recover the known negative, infinite entropy and a constant specific heat at $T = 0$. Also at $T = 0$ we derive the scaling behaviour for the dynamical susceptibility that agrees with Ye et al[10].
5. Appendix

We refer here to references [2][3] to give a concise and brief description of Grassmann variables and superalgebra. We indicate complex Grassmann variables by Greek letters, and these anticommuting variables satisfy

\[
[\chi, \gamma]_+ = 0 \quad \chi^2 = \chi^{\ast 2} = 0 \quad \chi^{\ast \ast} = -\chi \\
(\chi\gamma)^* = \chi^*\gamma^*
\]

\[
(\chi^\ast\chi)^* = \chi^{\ast\ast}\chi^* = -\chi\chi^* = \chi^*\chi
\]

(40)

and the integrals

\[
\int d\chi^\ast\chi^* = \int d\chi\chi = 1 \\
\int d\chi = \int d\chi^* = 0
\]

(41)

what leads to

\[
\int \prod_i d\chi_i^* d\chi_i \exp \sum_{ij} \chi_i^* M_{ij} \chi_j = \|M_{ij}\|
\]

(42)

A four-component supervector \( \underline{\varphi} \) and a 4x4 super matrix \( \underline{M} \) have commuting and Grassmann components

\[
\underline{\varphi} = \begin{pmatrix} R_1 \\ R_2 \\ \chi/\sqrt{2} \\ \chi^*/\sqrt{2} \end{pmatrix}
\]

\[
\underline{M} = \begin{pmatrix} a & \bar{a} \\ \bar{a} & \bar{b} \\ p & \bar{b} \end{pmatrix}
\]

(43)
where $a$ and $b$ are $2 \times 2$ matrices of commuting elements while $\sigma$ and $\rho$ are $2 \times 2$ matrices of Grassmann elements. Taking into account the change of sign involved in complex conjugation from eq(40) the super adjoint must be defined

$$S_{\varphi}^\dagger = \begin{pmatrix} R_1 & R_2 \\ \chi^*/\sqrt{2} & -\chi/\sqrt{2} \end{pmatrix}$$

$$SM^\dagger = \begin{pmatrix} a^\dagger & \rho^\dagger \\ -\sigma^\dagger & b^\dagger \end{pmatrix}$$

(44)
6. Figure Captions

Fig.1 Chemical potential \( \mu \) as a function of temperature for different values of the moment of inertia \( I = 0.19J \) (dash-dot), \( I = J \) (dash), \( I = \infty \) (continuous).

Fig.2 Phase diagram and critical line \( T_c(1/I) \) separating the paramagnetic from the spin glass phase in the \( T vs 1/I \) plane.

Fig.3 Free energy as a function of temperature for different values of the moment of inertia \( I = 0.4J \) (dash), \( I = J \) (dash-dot-dot), \( I = 5J \) (dash-dot), \( I = 10J \) (dash-dot-dot), \( I = \infty \) (continuous).

Fig.4 Entropy as a function of temperature for different values of the moment of inertia \( I = J \) (dash), \( I = 5J \) (dash-dot-dot), \( I = 10J \) (dash-dot-dot), \( I = \infty \) (continuous).

Fig.5 Specific heat as a function of temperature for different values of the moment of inertia \( I = 0.4J \) (dash), \( I = J \) (dash-dot), \( I = 10J \) (dash-dot-dot), \( I = \infty \) (continuous).

Fig.6 Static susceptibility as a function of temperature for \( I = 0.4J \) (dash), \( I = J \) (dash-dot), \( I = \infty \) (continuous).
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