QCD sum rules for $\Sigma$ hyperons in nuclear matter

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Abstract

Within finite-density QCD sum-rule approach we investigate the self-energies of $\Sigma$ hyperons propagating in nuclear matter from a correlator of $\Sigma$ interpolating fields evaluated in the nuclear matter ground state. We find that the Lorentz vector self-energy of the $\Sigma$ is similar to the nucleon vector self-energy. The magnitude of Lorentz scalar self-energy of the $\Sigma$ is also close to the corresponding value for nucleon; however, this prediction is sensitive to the strangeness content of the nucleon and to the assumed density dependence of certain four-quark condensate. The scalar and vector self-energies tend to cancel, but not completely. The implications for the couplings of $\Sigma$ to the scalar and vector mesons in nuclear matter and for the $\Sigma$ spin-orbit force in a finite nucleus are discussed.

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I. INTRODUCTION

Large and canceling isoscalar Lorentz scalar and vector self-energies for propagating nucleons in nuclear matter (unlike the conventional nonrelativistic picture) are essential for the successes of relativistic nuclear phenomenology [1–4]. It has been shown recently, within a finite-density QCD sum-rule approach, that this physics might be motivated from quantum chromodynamics (QCD) [5–9]. (Other applications of sum rule methods to finite density problems are discussed in Refs. [10–15].) The predictions of QCD sum rules for the nucleon self-energies are found to be consistent with those obtained from relativistic phenomenological models (e.g., the relativistic optical potentials of Dirac phenomenology [1,2] or Brueckner calculations [3,4]). However, it was found in Refs. [7,9] that the sum-rule predictions for the scalar self-energy are sensitive to assumptions made about the density dependence of certain four-quark condensates. Clearly, definite conclusions are not yet justified and further tests of the approach are needed. To this end, one is naturally led to consider the hyperons in nuclear matter within the same framework.

By studying the hyperons, one can use both experimental data on hypernuclei and relativistic phenomenology to confront with the QCD sum-rule predictions. Various investigators [16–28] have applied the relativistic phenomenology to hypernuclear physics. In these relativistic models, the hyperons are coupled to the same scalar and vector fields as the nucleon, but with different coupling strengths. (These couplings are also of great relevance to other branch of physics [29].) However, these strengths are not well-established. QCD sum-rule predictions for the scalar and vector self-energies may provide valuable insight into these couplings in the nuclear medium.

In Ref. [30], the self-energies of a Λ hyperon propagating in nuclear matter have been studied using the finite-density QCD sum-rule methods. The sum-rule calculations indicate that the self-energies of the Λ are only about 1/3 of the corresponding nucleon self-energies, suggesting a significant deviation from SU(3). In this paper, we further extend the finite-
density QCD sum-rule methods to explore the self-energies of Σ hyperons in an infinite nuclear matter.

One of the compelling successes of relativistic models in describing nucleon-nucleus interactions is the naturally large spin-orbit force for nucleons in a finite nucleus. This force depends on the derivatives of the scalar and vector optical potentials, which add constructively. An analogous prediction for the Λ hyperon would seem to be problematic, if one adopts the *naive* SU(3) prediction that each coupling for the Λ should be \( \frac{2}{3} \) the coupling for the nucleon \([31,32,20–25]\). [That is, if one assumes that the scalar (σ) and vector (ω) mesons couple exclusively to the up and down quarks and not to the strange quark.] In the Λ-nucleus system, recent experiments indicate that the spin-orbit force is small, and even consistent with zero \([33,34]\). This has raised questions about the validity of relativistic nuclear phenomenology for the hyperons.

In Ref. \([16]\), a weak Λ spin-orbit force was achieved by taking the potentials (i.e., the couplings) for the Λ to be a factor of three smaller than for the nucleon. More recently it has been suggested \([17,21,23]\) that larger values of the scalar and vector coupling strengths, consistent with the naive SU(3) prediction, can be used if a new tensor coupling between hyperons and the vector meson (ω) is introduced. In Refs. \([20,23,24]\), it was argued that a quark-model picture implies that the tensor couplings of the hyperons (Λ,Σ,Ξ) to the vector meson differ in their magnitudes and signs and hence their contribution to the spin-orbit force is different. For Λ, this picture leads to a tensor coupling with strength equal in magnitude to the corresponding vector coupling, but with the opposite sign. The net result, in combination with the scalar contribution, is a small spin-orbit force for the Λ. However, the experimental information available is not sufficient to resolve the effects of the tensor couplings. The sum-rule calculations in Ref. \([30]\) suggest that the coupling of the Λ to mesons are only about \( \frac{1}{3} \) of the corresponding nucleon couplings, implying a significant deviation from the naive SU(3) prediction and a weak Λ spin-orbit force; the tensor coupling, however, does not appear in the sum-rule calculations in a uniform nuclear matter.
At the moment, experimental evidence of Σ hypernuclei is insufficient. However, a number of authors \cite{23-26} have extended the relativistic phenomenology to Σ hypernuclei and presented theoretical predictions. In Refs. \cite{23,25}, the naive SU(3) prediction for the coupling strengths of Σ to the scalar and vector mesons were adopted (the vector coupling for Σ is $\frac{2}{3}$ the coupling for nucleon and the scalar coupling for Σ is slightly smaller than $\frac{2}{3}$ the coupling for nucleon); in addition the tensor coupling between the Σ and the vector meson was included. In the quark-model picture of Refs. \cite{23,32,20,24}, the tensor coupling of Σ to ω has the same sign as the corresponding vector coupling, in contrast to the Λ case. With the quark model values for the tensor coupling, the spin-orbit force for the Σ was found to be comparable with the nucleon spin-orbit force. In Ref. \cite{29} the tensor coupling was omitted and universal couplings were assumed for all hyperons; the ratio of Σ to Λ spin-orbit force is about 0.9. The sum-rule approach may offer independent information on the scalar and vector couplings for Σ and new insight into deviations from SU(3) and into the spin-orbit force, although the prediction of a tensor coupling is not tested in the calculations described here.

The finite-density QCD sum-rule approach focuses on a correlation function of interpolating fields, made up of quark fields, which carry the quantum numbers of the hadron of interest. The correlation function is evaluated in the ground state of nuclear matter instead of the QCD vacuum (as in the usual sum rules). For spin-$\frac{1}{2}$ baryons, this function can, in general, be decomposed into three invariant functions of two kinematic invariants. The appearance of an additional invariant function compared to the vacuum case is due to an additional four-vector at finite density, the four-velocity of the nuclear medium. In the rest frame of the medium, the analytic properties of these invariant functions can be studied through a Lehmann representation in energy.

The quasibaryon excitations (i.e., the quasiparticle excitations with baryon quantum numbers) are characterized by the discontinuities of the invariant functions across the real axis, which we use to identify the on-shell self-energies. By introducing a simple phenomeno-
logical ansatz for these spectral densities, we obtain a representation of the correlation function. On the other hand, the correlation function can be evaluated at large space-like momenta using an operator product expansion (OPE). By equating these two different representations using appropriately weighted integrals, one obtains QCD sum rules, which relate the baryon self-energies in the nuclear medium to QCD Lagrangian parameters and finite-density condensates [7].

We find that the finite-density QCD sum rules predict strong Lorentz scalar and vector self-energies for the $\Sigma$ hyperon. In particular, the $\Sigma$ vector self-energy is very similar to the sum-rule prediction for the nucleon vector self-energy. In terms of an effective theory of baryons and mesons, this implies larger couplings than would be indicated by SU(3) symmetry and the quark model. The predictions for the vector self-energy are essentially determined by the nucleon density and the fact $\langle s\bar{s}\rangle_{\rho_N} = 0$, and the predicted ratio of the vector self-energy to the zero-density $\Sigma$ mass is found to be largely insensitive to the details of the calculation. In contrast, the predictions for the scalar self-energy are sensitive to the strangeness content of the nucleon and to the assumed density dependence of four-quark condensate $\langle q\bar{q} \rangle^2_{\rho_N}$. If we assume that the nucleon has a significant strangeness content and the four-quark condensate $\langle q\bar{q} \rangle^2_{\rho_N}$ has weak density dependence, then the magnitude of the $\Sigma$ scalar self-energy is found to be close to the corresponding value for nucleon and tends to cancel the vector self-energy, which is compatible with relativistic phenomenology. On the other hand, if the strangeness content of the nucleon is small or $\langle q\bar{q} \rangle^2_{\rho_N}$ depends significantly on the nucleon density, the magnitude of $\Sigma$ scalar self-energy is very small and the net $\Sigma$ self-energy is sizable and repulsive.

This paper is organized as follows. The sum rules for $\Sigma$ hyperons in nuclear matter are established in Sec. II. The results are presented in Sec. III and discussions are given in Sec. IV. Section V is a summary.
II. FINITE-DENSITY SUM RULES FOR \( \Sigma \)

In this section, we derive the finite-density sum rules for \( \Sigma \). We use the methods developed in Refs. [7,8], and refer the reader to these papers for further details. We work to leading order in perturbation theory. (The leading-logarithmic perturbative corrections are included through anomalous dimension factors.) In the OPE for \( \Sigma \) correlator, we consider all condensates up to dimension four, and the terms up to first order in the strange quark mass \( m_s \). Contributions proportional to the up and down current quark mass are neglected since they give numerically small contributions. In addition, we include the contributions from the dimension-six four-quark condensates (in the first order of \( m_s \), we only consider the terms which do not vanish in the zero-density limit). All other dimension six and higher-dimensional condensates are neglected since we expect their contributions to be small in the region of optimal Borel mass.

QCD sum rules for \( \Sigma \) at finite density study the correlator defined by

\[
\Pi_\Sigma(q) \equiv i \int d^4x \, e^{iq \cdot x} \langle \Psi_0 | T[\eta_\Sigma(x)\eta_\Sigma(0)] | \Psi_0 \rangle ,
\]

where \( \eta_\Sigma(x) \) is a colorless interpolating field, constructed from quark fields, which carries the quantum numbers of \( \Sigma \). The ground state of nuclear matter \( | \Psi_0 \rangle \) is characterized by the nucleon density \( \rho_N \) in the rest frame and the nuclear matter four-velocity \( u^\mu \); it is assumed to be invariant under parity and time reversal. Here we consider the baryon interpolating fields that contain no derivatives and couple to spin-\( \frac{1}{2} \) states only. There are two linearly independent fields with these features. For the nucleon, Ioffe [35,36] has argued that the optimal choice is \( \eta_N(x) = \epsilon_{abc} [u_a^T(x)C\gamma_\mu u_b(x)] \gamma_5\gamma^\mu d_c(x) \) (for the proton). The interpolating field for \( \Sigma^+ \) can be directly obtained by an SU(3)-transformation of \( \eta_N(x) \) [37]:

\[
\eta_\Sigma(x) = \epsilon_{abc} [u_a^T(x)C\gamma_\mu u_b(x)] \gamma_5\gamma^\mu s_c(x) ,
\]

where \( T \) denotes a transpose in Dirac space, \( C \) is the charge conjugation matrix, \( a, b, \) and \( c \) are color indices, and \( u(x) \) and \( s(x) \) are the up and strange quark fields, respectively.
The analogous interpolating field for $\Sigma^-$ follows by changing the up quark field into down quark field. In the calculations to follow, we assume the isospin symmetry and use $q$ to denote either up or down quark field. The interpolating field Eq. (2.2) will be used in our calculations.

Lorentz covariance, parity and time reversal then imply that the correlator can only have three distinct structures [5,7,8]:

$$\Pi_\Sigma(q) \equiv \frac{\Pi_s(q^2, q \cdot u) + \Pi_q(q^2, q \cdot u)}{q} + \frac{\Pi_u(q^2, q \cdot u)}{u}.$$  \hspace{1cm} (2.3)

[A potential invariant function multiplying $(q_\mu u_\nu - q_\nu u_\mu)\sigma^{\mu\nu}$ vanishes due to time-reversal invariance and a potential invariant function multiplying $\epsilon^{\lambda\rho\mu\nu}(q_\lambda u_\rho - q_\rho u_\lambda)\sigma^{\mu\nu}$ vanishes due to parity conservation]. The three invariant functions, $\Pi_s, \Pi_q$ and $\Pi_u$, are functions of the two Lorentz scalars $q^2$ and $qu$. In the zero-density limit, $\Pi_u \to 0$, and $\Pi_s$ and $\Pi_q$ become functions of $q^2$ only. For convenience, we will work in the rest frame of nuclear matter hereafter, where $u^\mu = (1, 0)$ and $q \cdot u \to q_0$; we also take $\Pi_i(q^2, q \cdot u) \to \Pi_i(q_0, |q|)$ ($i = \{s, q, u\}$). To obtain QCD sum rules, we need to construct a phenomenological representation for $\Pi_\Sigma(q)$ and to evaluate $\Pi_\Sigma(q)$ using OPE techniques.

In the rest frame of nuclear matter, the analytic properties of $\Pi_\Sigma(q)$ can be made manifest by a Lehmann representation [38], which leads to a dispersion relation in $q_0$ of the form [7]

$$\Pi_i(q_0, |q|) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega \frac{\Delta \Pi_i(\omega, |q|)}{\omega - q_0} + \text{polynomial} \hspace{1cm} (2.4)$$

for each invariant function $\Pi_i, \ i = \{s, q, u\}$. The polynomial stands for contributions from the contour at large $|q_0|$, which will be eliminated by a subsequent Borel transform (see below). The discontinuity $\Delta \Pi_i$ (which is the spectral density up to a constant) defined by

$$\Delta \Pi_i(\omega, |q|) \equiv \lim_{\epsilon \to 0^+} [\Pi_i(\omega + i\epsilon, |q|) - \Pi_i(\omega - i\epsilon, |q|)], \hspace{1cm} (2.5)$$

contains the spectral information on the quasiparticle, quasihole, and higher energy states.

In QCD sum-rule applications, one parametrizes the spectral density with a small number of spectral parameters characterizing the resonances in the channel of interest (e.g., poles,
residues, etc.). In vacuum, the spectral weights for baryon and antibaryon are related by charge conjugation symmetry and one usually parametrizes the spectral density as a single sharp pole representing the lowest resonance plus a smooth continuum representing higher mass states. At finite density, the ground state is no longer invariant under ordinary charge conjugation. Thus, the spectral densities for baryon and antibaryon are not simply related.

The width of $\Sigma$ in free space is small and can be ignored on hadronic scales. At finite density, the width of the $\Sigma$ will be broadened due to strong conversions. Here we assume that the broadened width is relatively small on the hadronic scale and that a quasiparticle description of the $\Sigma$ is reasonable. In the context of relativistic phenomenology, the $\Sigma$ is assumed to couple to the same scalar and vector fields as the nucleons in the nuclear matter, and are treated as quasiparticles with real Lorentz scalar and vector self-energies. We follow Ref. [7] and assume a pole approximation for the quasibaryon and take the spectral ansatz to be (higher-energy states are included in a continuum contribution, as discussed below)

$$\Delta \Pi_s(\omega, |q|) = -2\pi i \frac{M_{\Sigma}^* \lambda_{\Sigma}^2}{2E_q^*} \left[ \delta(\omega - E_q) - \delta(\omega - E_{q'}) \right],$$

(2.6)

$$\Delta \Pi_q(\omega, |q|) = -2\pi i \frac{\lambda_{\Sigma}^2}{2E_q^*} \left[ \delta(\omega - E_q) - \delta(\omega - E_{q'}) \right],$$

(2.7)

$$\Delta \Pi_u(\omega, |q|) = +2\pi i \frac{\Sigma_v \lambda_{\Sigma}^2}{2E_q^*} \left[ \delta(\omega - E_q) - \delta(\omega - E_{q'}) \right],$$

(2.8)

where $\lambda_{\Sigma}^2$ is an overall residue. Here we have defined $M_{\Sigma}^* \equiv M_\Sigma + \Sigma_s$, $E_q^* \equiv \sqrt{M_{\Sigma}^2 + q^2}$, $E_q \equiv \Sigma_v + \sqrt{M_{\Sigma}^2 + q^2}$, and $E_{q'} \equiv \Sigma_v - \sqrt{M_{\Sigma}^2 + q^2}$, where $M_\Sigma$ is the mass of $\Sigma$ and $\Sigma_s$ and $\Sigma_v$ are identified as the scalar and vector self-energies of a $\Sigma$ hyperon in nuclear matter. The positive- and negative-energy poles are at $E_q$ and $E_{q'}$, respectively. Since we want to focus on the positive-energy $\Sigma$ pole, we approximate the spectral functions for the positive-energy $\Sigma$ by a quasiparticle pole while suppressing contributions from the region of the negative-energy excitations. This is achieved by manipulating the parts of the correlator that are even and odd in $q_0$ (see below).

We now turn to the QCD expansion of $\Pi_\Sigma(q)$, which is obtained by applying the oper-
ator product expansion to the time-ordered operator product in Eq. (2.1). In the present formalism, the correlator is studied in the limit that \( q_0 \) becomes large and imaginary while \(|q|\) remains fixed (in the nuclear matter rest frame). This limit takes \( q^2 \rightarrow -\infty \) with \(|q^2/q \cdot u| \rightarrow \infty \), which satisfies the conditions discussed in Ref. [39] for a short distance expansion. At finite density, the OPE for the invariant functions of spin-\( \frac{1}{2} \) baryon correlator takes the general form [7,8]

\[
\Pi_i(q^2, q \cdot u) = \sum_n C^{i}_{n}(q^2, q \cdot u) \langle \hat{O}_n \rangle_{\rho N},
\]

(2.9)

where \( \langle \hat{O}_n \rangle_{\rho N} \equiv \langle \Psi_0 | \hat{O}_n | \Psi_0 \rangle \). The \( C^{i}_{n}(q^2, q \cdot u) \) \((i = \{s, q, u\})\) are the Wilson coefficients, which depend on QCD Lagrangian parameters such as the quark masses and the strong coupling constant. The most important feature is that the local composite operators are defined such that all density dependence of the correlator resides in the condensates; the Wilson coefficients are then independent of density [8]. The operators \( \hat{O}_n \) are ordered by dimension (measured as a power of mass) and the \( C^{i}_{n}(q^2, q \cdot u) \) for higher-dimensional operators fall off by corresponding powers of \( Q^2 \equiv -q^2 \). Therefore, for sufficiently large \( Q^2 \), the operators of lowest dimension dominate, and the OPE can be truncated after a small number of lower-dimensional operators.

The Wilson coefficients only depend on \( q^\mu \), and the ground-state expectation values of the operators are proportional to tensors constructed from the nuclear matter four-velocity \( u^\mu \), the metric \( g^{\mu\nu} \), and the antisymmetric tensor \( \epsilon^{\kappa\lambda\mu\nu} \). In Eq. (2.9) we incorporate the contraction of \( q^\mu \) (from the OPE) and \( u^\mu \) (from the ground-state expectation values of the operators) into the definition of the Wilson coefficients \( C^{i}_{n}(q^2, q \cdot u) \). Thus the dependence on \( q \cdot u \) is solely in the form of polynomial factors. We have also suppressed the dependence on the normalization point \( \mu \).

The Wilson coefficients can be calculated using the fixed-point gauge [40,41] and standard background-field techniques [42–46,37]. To obtain the Wilson coefficients one can apply Wick’s theorem to the coordinate space time-ordered product in Eq. (2.1), retaining only
those contributions in which the quark fields are fully contracted, and using the quark propagators in the presence of the nonperturbative nuclear medium, given in Refs. [8,30], for the contractions.

Although the four-quark condensates have dimension six, their contribution to the baryon correlator are particularly important since the corresponding Wilson coefficients do not carry the large numerical suppression factors typically associated with loops [47]. By using the quark propagators given in Refs. [8,30] in the calculations, one includes the contributions from the four-quark condensates automatically with the condensates in their in-medium factorized form [7,8]. (See Sec. III for discussion of the factorization approximation).

For convenience we split the invariant function $\Pi_i(q_0, |q|)$ into two pieces that are even and odd in $q_0$:

$$\Pi_i(q_0, |q|) = \Pi^E_i(q_0, |q|) + q_0 \Pi^O_i(q_0, |q|) ,$$

for $i = \{ s, q, u \}$. The results of our calculations are

$$\Pi^E_s = - \frac{m_s}{32\pi^4} (q^2)^2 \ln(-q^2) + \frac{1}{4\pi^2} q^2 \ln(-q^2) \langle \bar{s}s \rangle_{\rho N} - \frac{4m_s q_0^2}{3\pi^2} q^2 \langle q^\dagger iD_0 q \rangle_{\rho N}$$

$$- \frac{4m_s}{3q^2} \langle \bar{q} q \rangle_{\rho N} ,$$

$$\Pi^O_s = \frac{m_s}{2\pi^2} \ln(-q^2) \left( \langle q^\dagger q \rangle_{\rho N} - \langle s^\dagger s \rangle_{\rho N} \right) - \frac{4}{3q^2} \langle \bar{s}s \rangle_{\rho N} \langle q^\dagger q \rangle_{\rho N} ,$$

$$\Pi^E_q = - \frac{1}{64\pi^2} (q^2)^2 \ln(-q^2) - \frac{1}{32\pi^2} \ln(-q^2) \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\rho N}$$

$$- \frac{1}{144\pi^2} \left( \ln(-q^2) - 4 \frac{q_0^2}{q^2} \right) \left\langle \frac{\alpha_s}{\pi} \left[ (u \cdot G)^2 + (u \cdot \tilde{G})^2 \right] \right\rangle_{\rho N}$$

$$- \frac{m_s}{18\pi^2} \left[ 5 \ln(-q^2) - 2 \frac{q_0^2}{q^2} \right] \langle \bar{s}s \rangle_{\rho N} + \frac{4}{9\pi^2} \left( \ln(-q^2) - \frac{q_0^2}{q^2} \right) \langle q^\dagger iD_0 q \rangle_{\rho N}$$

$$+ \frac{1}{9\pi^2} \left( \ln(-q^2) - 4 \frac{q_0^2}{q^2} \right) \langle s^\dagger D_0 s \rangle_{\rho N} - \frac{2}{3q^2} \langle \bar{q} q \rangle_{\rho N}^2$$

$$- \frac{4}{3q^2} \langle q^\dagger q \rangle_{\rho N} \langle s^\dagger s \rangle_{\rho N} ,$$

$$\Pi^O_q = \frac{1}{6\pi^2} \ln(-q^2) \left( \langle q^\dagger q \rangle_{\rho N} + \langle s^\dagger s \rangle_{\rho N} \right) ,$$

$$\Pi^E_u = \frac{1}{12\pi^2} q^2 \ln(-q^2) \left( 7 \langle q^\dagger q \rangle_{\rho N} + \langle s^\dagger s \rangle_{\rho N} \right) .$$
\[ \Pi_u^0 = \frac{1}{9\pi^2} \ln(-q^2) \left( m_u \langle \bar{s} s \rangle_{\rho N} - 16 \langle q \hat{t} D_0 q \rangle_{\rho N} - 4 \langle s \hat{t} D_0 s \rangle_{\rho N} \right) \\
+ \frac{1}{36\pi^2} \ln(-q^2) \left\langle \frac{\alpha_s}{\pi} \left[ (u \cdot G)^2 + (u \cdot \tilde{G})^2 \right] \right\rangle_{\rho N} \\
- \frac{4}{3q^2} \left( \langle q \hat{t} q \rangle_{\rho N} + \langle q \hat{t} q \rangle_{\rho N} \langle s \hat{t} s \rangle_{\rho N} \right), \tag{2.16} \]

where we have introduced the notation \( G^2 \equiv G_{\mu\nu}^\lambda G^{\lambda\mu\nu} \) and \( \tilde{G}^{\lambda\kappa\lambda} \equiv \frac{1}{2} \epsilon^{\kappa\lambda\mu\nu} G_{\mu\nu}^\lambda \), and have omitted the polynomials in \( q^2 \) and \( q_0^2 \), which vanish under the Borel transform. Here we take \( \langle \bar{q} \hat{O} q \rangle_{\rho N} \equiv (\langle \bar{u} \hat{O} u \rangle_{\rho N} + \langle \bar{d} \hat{O} d \rangle_{\rho N})/2 \), with \( \hat{O} \) a combination of Dirac matrices, gluon field tensors, and covariant derivatives. In Eqs. (2.11–2.16), we have neglected the terms proportional to \( \langle q \hat{t} D_0 q \rangle_{\rho N} \) since \( \langle \bar{q} \hat{t} D_0 q \rangle_{\rho N} = \frac{7}{3} (m_u + m_d) \rho N \simeq 0 \) (see Ref. [8]).

In the present calculations, we have neglected all the terms proportional to the dimension-five quark and quark-gluon condensates. There is no difficulty in calculating the Wilson coefficients for these condensates. However, we have no reliable approach to determine these condensates. With the estimated values of the quark and quark-gluon condensates (containing only light quarks) given in Ref. [8], the dimension-five quark and quark-gluon condensates make numerically small contributions to the nucleon sum rules [9]. We expect that the contributions from the dimension-five quark and quark-gluon condensates are also small in the \( \Sigma \) sum rules, so we neglect these terms. Contributions from three-gluon condensates, which are dimension six, are also expected to be numerically small compared to those of four-quark condensates since the four-quark condensates enter at tree level whereas the three-gluon condensates involve loops [8].

The QCD sum rules follow by equating the spectral representation of the correlator to the corresponding OPE representation. To improve the overlap of the two descriptions for QCD sum rules for hadrons in vacuum, one typically applies a Borel transform to both sides of the sum rules. For practical purposes in the present approach, the Borel transform can be applied using the operator \( B \) defined by
\[
\mathcal{B}[f(Q^2, |q|)] \equiv \lim_{Q^2,n \to \infty} \frac{(Q^2)^n + 1}{n!} \left(-\frac{\partial}{\partial Q^2}\right)^n f(Q^2, |q|) \equiv \hat{f}(M^2, |q|),
\]
\[
Q^2 \equiv -q_0^2, \quad M^2 \equiv \frac{Q^2}{n}.
\]

(2.17)

The result \(\hat{f}(M^2, |q|)\) depends on the Borel mass \(M\).

The finite-density QCD sum rules are then given by [7,9]
\[
\mathcal{B}[\Pi_i^E(q_0^2, |q|) - E_q \Pi_i^O(q_0^2, |q|)]_{\text{QCD}} = \mathcal{B}[\Pi_i^E(q_0^2, |q|) - E_q \Pi_i^O(q_0^2, |q|)]_{\text{phen}},
\]

(2.18)

for \(i = \{s, q, u\}\), where the left-hand side is obtained from the OPE and right-hand side from the phenomenological dispersion relations. To see how this manipulation suppresses contributions from the region of negative-energy excitations, we rewrite Eq. (2.4) as
\[
\Pi_i(q_0, |q|) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega \frac{\Delta \Pi_i(\omega, |q|)}{\omega^2 - q_0^2} + \frac{q_0}{2\pi i} \int_{-\infty}^{\infty} d\omega \frac{\Delta \Pi_i(\omega, |q|)}{\omega^2 - q_0^2},
\]

(2.19)

where we have omitted the polynomial term. We then obtain
\[
\mathcal{B}[\Pi_i^E(q_0^2, |q|) - E_q \Pi_i^O(q_0^2, |q|)]_{\text{phen}} = \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega (\omega - E_q) \Delta \Pi_i(\omega, |q|) e^{-\omega^2/M^2}.
\]

(2.20)

The negative-energy pole contribution is now suppressed by the factor \((\omega - E_q)\), which equals zero at \(\omega = E_q\). For Borel \(M\) near the \(\Sigma\) energy, contributions of higher-energy states to the integral are exponentially suppressed.

Perturbative corrections \(\sim \alpha_s^n\) can be taken into account in the leading logarithmic approximation through anomalous-dimension factor, \(L \equiv \frac{\ln M/\Lambda_{\text{QCD}}}{\ln \mu/\Lambda_{\text{QCD}}}[35]\), where \(\mu\) is the normalization point of the operator product expansion and \(\Lambda_{\text{QCD}}\) is the QCD scale parameter. In our numerical calculations, we take \(\mu = 0.5\text{ GeV}\) and \(\Lambda_{\text{QCD}} = 0.1\text{ GeV}[48]\).

Finally, the contributions from higher-energy states are roughly approximated using the leading terms in the OPE, starting at an effective threshold; these contributions can be included by modifying the terms with positive powers of \(M^2\) on the OPE side of each sum-rule equation as follows [33,7,9]:
\[ M^2 \rightarrow M^2 E_0 \equiv M^2 \left( 1 - e^{-s_0^*/M^2} \right), \quad (2.21) \]

\[ M^4 \rightarrow M^2 E_1 \equiv M^4 \left[ 1 - e^{-s_0^*/M^2} \left( \frac{s_0^*}{M^2} + 1 \right) \right], \quad (2.22) \]

\[ M^6 \rightarrow M^6 E_2 \equiv M^6 \left[ 1 - e^{-s_0^*/M^2} \left( \frac{s_0^*}{2M^4} + \frac{s_0^*}{M^2} + 1 \right) \right], \quad (2.23) \]

where we define the continuum threshold \( s_0^* = \omega_0^2 - q^2 \), with \( \omega_0 \) the energy at the continuum threshold. In principle, the effective thresholds are different for positive and negative energies and for the different sum rules. The former differences are critical in some sum rule formulations [50], but are not numerically important in the present formulation. Furthermore, the thresholds are relatively poorly determined by the sum rules and effects due to different thresholds in different sum rules may be absorbed by slight changes in the other parameters. In the present paper, we use a universal effective threshold for simplicity.

With the spectral ansatz Eqs. (2.6)–(2.8) and the OPE results of Eqs. (2.11)–(2.16), we obtain the finite-density sum rules for the \( \Sigma \):

\[ \lambda^2 \Sigma^* \Sigma e^{-(E_3^2-q^2)/M^2} = \frac{m_s}{16\pi^4} M^6 E_2 L^{-8/9} - \frac{M^4}{4\pi^2} E_1 \langle \bar{s}s \rangle_{\rho N} \]

\[ + \frac{m_s}{2\pi^2} \bar{E} q M^2 E_0 \left( \langle q^\dagger q \rangle_{\rho N} - \langle s^\dagger s \rangle_{\rho N} \right) L^{-8/9} \]

\[ + \frac{4m_s}{3\pi^2} q^2 \langle \tilde{q}^\dagger D_0 q \rangle_{\rho N} L^{-8/9} + \frac{4m_s}{3\pi^2} \langle \bar{q} q \rangle_{\rho N}^2 \]

\[ - \frac{4}{3} \bar{E} q \langle \bar{s}s \rangle_{\rho N} \langle q^\dagger q \rangle_{\rho N}, \quad (2.24) \]

\[ \lambda^2 \Sigma^* e^{-(E_3^2-q^2)/M^2} = \frac{M^6}{32\pi^2} E_2 L^{-4/9} + \frac{M^2}{32\pi^2} \left( \frac{\alpha_s}{\pi} G^2 \right)_{\rho N} E_0 L^{-4/9} \]

\[ + \frac{M^2}{144\pi^2} \left( E_0 - \frac{q^2}{M^2} \right) \left( \frac{\alpha_s}{\pi} \left[ (u \cdot G)^2 + (u \cdot \tilde{G})^2 \right] \right)_{\rho N} L^{-4/9} \]

\[ + \frac{m_s}{18\pi^2} M^2 \left( 5E_0 - 2 \frac{q^2}{M^2} \right) \langle \bar{s}s \rangle_{\rho N} L^{-4/9} \]

\[ - \frac{4M^2}{9\pi^2} \left( E_0 - \frac{q^2}{M^2} \right) \langle \bar{q}^\dagger D_0 q \rangle_{\rho N} L^{-4/9} \]

\[ - \frac{M^2}{9\pi^2} \left( E_0 - \frac{q^2}{M^2} \right) \langle s^\dagger D_0 s \rangle_{\rho N} L^{-4/9} \]

\[ + \frac{E}{6\pi^2} M^2 E_0 \left( \langle q^\dagger q \rangle_{\rho N} + \langle s^\dagger s \rangle_{\rho N} \right) L^{-4/9} \]

\[ + \frac{2}{3} \langle \bar{q} q \rangle_{\rho N}^2 L^{4/9} + \frac{4}{3} \langle q^\dagger q \rangle_{\rho N} \langle s^\dagger s \rangle_{\rho N} L^{-4/9}, \quad (2.25) \]
Here $E_0$, $E_1$ and $E_2$ have been defined in Eqs. (2.21)–(2.23). We have ignored the anomalous dimensions of dimension four operators, either because the operators are renormalization-group invariant, because the anomalous dimension is small, because the corresponding condensates give small contributions, or because the accuracy to which the nucleon matrix elements of the operators are known is such that anomalous-dimension corrections represent an unwarrantable refinement (see Sec. III). We will adopt the values of the corresponding condensates at the scale of 1 GeV in our numerical calculations. The four-quark operators are in general not renormalization-covariant, so they mix with one another under the renormalization group \[47\]. In vacuum, the anomalous dimension effects do not violate the factorization assumption to within 10\% \[47\], and thus one usually assumes that the anomalous dimension of a four-quark operator is equal to the sum of the anomalous dimensions of the factorized operators \[35,51\]. In the present paper, we follow this assumption.

III. RESULTS

A. In-medium condensates

To obtain the predictions for the $\Sigma$ self-energies from the finite-density sum rules derived in the previous section, we need to know the in-medium condensates appearing in the sum rules. To first order in the nucleon density, one can write

$$\langle \hat{O} \rangle_{\rho_N} = \langle \hat{O} \rangle_{\text{vac}} + \langle \hat{O} \rangle_{N\rho_N} + \cdots,$$

\[3.1\]
where \( \cdots \) denotes correction terms that are of higher order in \( \rho_N \), and \( \langle \hat{O} \rangle_N \) is the spin averaged nucleon matrix element. (Note that this is not a Taylor series in \( \rho_N \).) For a general operator \( \hat{O} \) there is not a systematic way to study contributions to \( \langle \hat{O} \rangle_{\rho N} \) that are of higher order in \( \rho_N \). Model-dependent estimates in Ref. [6] suggest that the linear approximation to \( \langle \hat{q}q \rangle_{\rho N} \) should be good (higher-order corrections \( \sim 20\% \) of the linear term) up to nuclear matter saturation density. Here we assume the first-order approximation of all condensates to be reasonable for calculating scalar and vector self-energies up to nuclear matter saturation density. Justifying the limits of this type of density expansion is an important topic for further study.

The simplest in-medium condensates are \( \langle \hat{q}^\dagger q \rangle_{\rho N} \) and \( \langle \hat{s}^\dagger s \rangle_{\rho N} \). Since the baryon current is conserved, \( \langle \hat{q}^\dagger q \rangle_{\rho N} \) is proportional to the (net) nucleon and strangeness densities: \( \langle \hat{q}^\dagger q \rangle_{\rho N} = \frac{3}{2} \rho_N \), and \( \langle \hat{s}^\dagger s \rangle_{\rho N} = 0 \). These are exact results. The other dimension three and four quark and gluon condensates have been studied in Refs. [6,10,8,30]. The results are:

\[
\langle \overline{q}q \rangle_{\rho N} = \langle \overline{q}q \rangle_{\text{vac}} + \frac{\sigma_N}{2m_q} \rho_N , \tag{3.2}
\]

\[
\langle \overline{s}s \rangle_{\rho N} = \langle \overline{s}s \rangle_{\text{vac}} + \langle \overline{s}s \rangle_{\text{N}} \rho_N = \langle \overline{s}s \rangle_{\text{vac}} + y \frac{\sigma_N}{2m_q} \rho_N , \tag{3.3}
\]

\[
\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\rho N} = \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\text{vac}} - (650 \text{ MeV}) \rho_N , \tag{3.4}
\]

\[
\left\langle \frac{\alpha_s}{\pi} \left[ (u' \cdot G)^2 + (u' \cdot \tilde{G})^2 \right] \right\rangle_{\rho N} = -(100 \text{ MeV}) \rho_N , \tag{3.5}
\]

\[
\langle \hat{q}^\dagger D_0 q \rangle_{\rho N} = (180 \text{ MeV}) \rho_N , \tag{3.6}
\]

\[
\langle \hat{s}^\dagger D_0 s \rangle_{\rho N} = \frac{m_s}{4} \langle \overline{s}s \rangle_{\rho N} + (18 \text{ MeV}) \rho_N . \tag{3.7}
\]

where \( m_q \equiv \frac{1}{2}(m_u + m_d) \) is the average of the up and down current quark masses, \( \sigma_N \) is the nucleon \( \sigma \) term, and \( y \equiv \langle \overline{s}s \rangle_{\text{N}} / \langle \overline{q}q \rangle_{\text{N}} \) is a real parameter, measuring the strangeness content of the nucleon.

Four-quark condensates are numerically important in both the vacuum and the finite-density baryon sum rules because they contribute in tree diagrams and do not carry the numerical suppression factors typically associated with loops. In the sum rules derived in
Sec. II, we included the contributions from the four-quark condensates in their in-medium factorized forms; however, the factorization approximation may not be justified in nuclear matter \[7–9,30\]. In the nucleon sum rules, the “scalar-scalar” four-quark condensate \( \langle \bar{q}q \rangle_{\rho N}^2 \) gives important contributions; the sum-rule predictions are sensitive to the value of this condensate \[7,9\]. This four-quark condensate also appears in the \( \Sigma \) sum rules. In its factorized form, this scalar-scalar four-quark condensate has a strong density dependence; one might suspect that this strong density dependence is an artifact of the factorization approximation. Thus we follow Ref. \[9\] and parametrize the scalar-scalar four-quark condensate so that it interpolates between its factorized form in free space and its factorized form in nuclear matter:

\[
\langle \bar{q}q \rangle_{\rho N}^2 \longrightarrow \langle \tilde{\bar{q}}\tilde{q} \rangle_{\rho N}^2 = (1 - f)\langle \bar{q}q \rangle_{\rho N}^2 + f\langle \bar{q}q \rangle_{\rho N}^2, \tag{3.8}
\]

where \( f \) is a real parameter. The density dependence of the scalar-scalar four-quark condensate is thus parametrized by \( f \), and the density dependence of \( \langle \bar{q}q \rangle_{\rho N} \) [see Eqs. (3.2)]. The factorized four-quark condensate \( \langle \bar{q}q \rangle_{\rho N}^2 \) appearing in Eqs. (2.24–2.26) will be replaced by \( \langle \tilde{\bar{q}}\tilde{q} \rangle_{\rho N}^2 \) in the calculations to follow. Studies of nucleon self-energies with QCD sum rules yield results in strong contradiction to experiment unless \( 0 \leq f \leq 0.5 \) \[7,9\]; however, there is not an independent determination of the density dependence. Here we will consider only this range of \( f \). The other four-quark condensates have much smaller numerical contributions, so we just use the in-medium factorized form for simplicity.

The values of \( \sigma_N \) and \( y \) remain controversial \[52–54\]. In this paper, we take \( \sigma_N = 45 \text{ MeV} \), which is the value obtained in a recent analysis \[54\], and consider values of \( y \) in the range of \( 0 - 0.6 \), which covers the values discussed in \[52–54\]. For the condensates in vacuum, we use \( \langle \bar{q}q \rangle_{\text{vac}} \approx -(245 \text{ MeV})^3 \) \( (m_q \approx 5.5 \text{ MeV}) \) \[4,9\], and take \( \langle \bar{s}s \rangle_{\text{vac}} = 0.8\langle \bar{q}q \rangle_{\text{vac}} \) \[51,57,55\] and \( \langle (\alpha_s/\pi)G^2 \rangle_{\text{vac}} = (330 \text{ MeV})^4 \) \[17,14\]. We consider the strange quark mass \( m_s \) in the range of \( 100 - 200 \text{ MeV} \).
B. Sum-rule analysis

In principle, the predictions based on the sum rules should be independent of the auxiliary parameter $M^2$. In practice, however, we have to truncate the OPE and use a simple phenomenological ansatz for the spectral density; thus one expects the two descriptions to overlap only in some limited range of $M^2$ (at best). As a result, one expects to see a “plateau” in the predicted quantities as functions of $M^2$. The studies of the vacuum sum rules for the octet baryons and the nucleon sum rules at finite-density show that the sum rules truncated at dimension-six condensates do not provide a particularly convincing plateau \[35,55,7\]. Nevertheless, we will assume that the sum rule actually has a region of overlap, although imperfect. We follow Refs. \[7,9,30\] and rely on the cancellation of systematic discrepancies by normalizing all finite-density self-energies to the zero-density prediction for the mass. One hopes that this might compensate for general limitations of the sum rules. All the finite-density results presented are obtained at nuclear matter saturation density, which is taken to be $\rho_N = (110 \text{ MeV})^3$.

To analyze the sum rules and extract the self-energies, we sample the sum rules in the fiducial region of $M^2$, where the contributions from the highest-order condensates included in the sum rule are small and the continuum contribution is controllable. Here we choose $1.0 \leq M^2 \leq 1.6 \text{ GeV}^2$ as the optimization region [the ratios of the self-energies to the $\Sigma$ mass are insensitive to the choice of the upper bound of the optimal region (see Fig. 3)]. The study of the $\Sigma$ sum rules in vacuum suggests that the sum rules are valid in this region \[51,55\]. To quantify the fit of the left- and right-hand sides, we use the logarithmic measure \[51,55,7,30\]

$$
\delta(M^2) = \ln \frac{\text{maximum}\{\lambda_{\Sigma}^2 e^{-(E_q^2 - q^2)/M^2}, \Pi_q'/M_\Sigma, \Pi_u'/\Sigma_v\}}{\text{minimum}\{\lambda_{\Sigma}^2 e^{-(E_q^2 - q^2)/M^2}, \Pi_s'/M_\Sigma, \Pi_q'/\Sigma_v\}},
$$

(3.9)

1Including direct-instanton effects in nucleon sum rules in vacuum leads to a more convincing plateau \[56,57\].
which is averaged over 150 points evenly spaced within the fiducial region of $M^2$. Here $\Pi'_s$, $\Pi'_q$, and $\Pi'_u$ denote the right-hand sides of Eqs. (2.24)–(2.26), respectively. The predictions for $M^*_\Sigma$, $\Sigma_v$, $\omega^2_0$, and $\lambda^*_{\Sigma}$ are obtained by minimizing the averaged measure $\delta$. To get a prediction for the $\Sigma$ mass in vacuum, we apply the same procedure to the sum rules evaluated in the zero-density limit.

In Fig. [1], we displayed the optimized results for the ratios $M^*_\Sigma/M_\Sigma$ and $\Sigma_v/M_\Sigma$ as functions of $y$ for $m_s = 150$ MeV, $|q| = 270$ MeV, and three different values of $f$. (The momentum dependence of the self-energies for momenta below the Fermi surface is very weak, see Fig. [4]). It can be seen that $\Sigma_v/M_\Sigma$ is insensitive to both $y$ and $f$. However, the $M^*_\Sigma/M_\Sigma$ varies rapidly with $y$ and $f$. Therefore, the sum rule prediction for the scalar self-energy is strongly dependent on the strangeness content of the nucleon and on the density dependence of the four-quark condensate. For $f = 0$ and the values of $y$ in the range $0.4 \leq y \leq 0.6$, the predictions are:

\[
M^*_\Sigma/M_\Sigma \simeq 0.78–0.85, \quad (3.10)
\]
\[
\Sigma_v/M_\Sigma \simeq 0.18–0.19. \quad (3.11)
\]

On the other hand, for $f = 0$ and small values of $y$ ($0 \leq y \leq 0.2$), we find $\Sigma_v/M_\Sigma \sim 0.18$ and $M^*_\Sigma/M_\Sigma \simeq 0.92–0.98$. As $f$ increases, $M^*_\Sigma/M_\Sigma$ increases, which implies an even smaller magnitude of the scalar self-energy. The predictions for the ratios $\lambda^*_{\Sigma}/\lambda^2_{\Sigma}$ and $s_0^*/s_0$ also depend on $y$ and $f$. For $f = 0$ and large values of $y$ ($0.4 \leq y \leq 0.6$), both the continuum threshold and the residue $\lambda^*_{\Sigma}$ are close to their corresponding vacuum values. For $f = 0$ and small values of $y$ ($0 \leq y \leq 0.2$), the continuum threshold increases by about 20% relative to the vacuum value and the residue $\lambda^*_{\Sigma}$ increases by about 50% relative to its vacuum value. As $f$ increases, both the continuum threshold and the residue increase.

One can see, from the sum rules Eqs. (2.24)–(2.26), that the ratios $\Pi'_s/\Pi'_q$ and $\Pi'_u/\Pi'_q$ give $M^*_\Sigma$ and $\Sigma_v$ as functions of Borel $M^2$, and $\Pi'_s/\Pi'_q$ in the zero-density limit yields $M^*_\Sigma$ as a function of $M^2$. In Fig. [2] the ratios $M^*_\Sigma/M_\Sigma$ and $\Sigma_v/M_\Sigma$ are plotted as functions of $M^2$. 

18
for $y = 0.5$ and various values of $f$, with $E_q$, $\overline{E}_q$ and the continuum threshold fixed at their optimized values. The curves for the ratios $M_{\Sigma}^*/M_{\Sigma}$ and $\Sigma_v/M_{\Sigma}$ are flat, and thus imply a weak dependence of the predicted ratios on $M^2$ (though the individual sum-rule predictions before taking ratios are not flat).

We plot $\lambda_{\Sigma}^2 e^{-(E_\Sigma^2-\bar{q}^2)/M^2}$, $\Pi'_q(M^2)/M_{\Sigma}$, $\Pi'_q(M^2)$, and $\Pi'_q(M^2)/\Sigma_v$ as functions of $M^2$ for $y = 0.5$ and $f = 0$ in Fig. 3(a), with the predicted values for $M_{\Sigma}^*$, $\Sigma_v$, $\omega_0^2$, and $\lambda_{\Sigma}^2$. If the sum rules work well, one should expect to see that the four curves coincide with each other. It is seen that their $M^2$ dependence in the Borel region of interest turns out to be equal up to 15%. The overlap of the corresponding vacuum sum rules (i.e., the zero-density limit) is illustrated in Fig. 3(b). We observe that the quality of the overlap for the finite-density sum rules is similar to that of the corresponding sum rules in vacuum.

The sensitivity of our predictions to $m_s$ is illustrated in Fig. 4, where $y$ and $|q|$ are fixed at 0.5 and 270 MeV, respectively. The predictions for $M_{\Sigma}^*/M_{\Sigma}$ and $\Sigma_v/M_{\Sigma}$ are largely insensitive to changes in $m_s$, with a variation of $\Sigma_v/M_{\Sigma}$ less than 10% in the range of $m_s = 0.1 - 0.2$ GeV. Fig. 4 shows the three-momentum $|q|$ dependence of the predicted ratios. The results are only weakly dependent on three momentum in the range of $|q| = 0 - 500$ MeV. Finally, the dependence of the results on the choice of the upper bound of the Borel window is shown in Fig. 6. The two dashed curves are obtained using a fixed Borel window at $1.0$ GeV$^2 \leq M^2 \leq 1.6$ GeV$^2$. The two solid curves are the results obtained by taking an upper bound of the Borel window such that the continuum contributions to the phenomenological sides do not exceed 50% of the total phenomenological contributions to the sum rules (i.e., the sum of the quasiparticle pole and the continuum contributions) while fixing the lower bound at 1.0 GeV$^2$. We have used the same procedure in extracting the $\Sigma$ mass in vacuum. We see that the two set of curves are almost indistinguishable, implying that changing the upper limit of the optimum Borel region does not affect the sum-rule predictions for the two ratios.
IV. DISCUSSION

We note that the sum-rule predictions for the scalar self-energy are quite sensitive to the strangeness content of the nucleon and to the undetermined density dependence of certain four-quark condensate. Therefore, we cannot draw definite conclusions about the \( \Sigma \) scalar self-energy at this point. Nevertheless, we emphasize that the sum-rule prediction for the normalized vector self-energy, \( \Sigma_v/M_\Sigma \), is apparently insensitive to the details of calculations. For typical values of the relevant condensates and other input parameters, \( \Sigma_v/M_\Sigma \sim 0.18–0.21 \). The finite-density nucleon sum rules predict \( \Sigma_v/M_N \sim 0.25–0.30 \) \cite{7,9}. Thus, we find \( (\Sigma_v)_\Sigma/(\Sigma_v)_N \sim 0.8–1.1 \).

This result, if interpreted in terms of a relativistic hadronic model, would imply that the coupling of the \( \Sigma \) to the Lorentz vector field is very similar to the corresponding nucleon coupling in the same ratio. This compares to the naive SU(3) prediction of \( \frac{2}{3} \), which is obtained by assuming that the mesons couple directly to constituent quarks \cite{31,21–22}, and thus suggests a significant deviation from SU(3) symmetry in nuclear matter. This deviation can be attributed to two sources. First, the nuclear matter ground state is not SU(3) symmetric due to the absence of net strangeness. This leads to further deviation of various condensates from SU(3) relative to the deviation in vacuum. The second source originates from the baryon interpolating fields. In the present work, we used the interpolating field defined in Eq. (2.2), corresponding to an axial vector diquark, composed of two up quarks, coupled to a strange quark. However, these quarks contribute to the sum rules differently. This can be seen from the leading-order terms in the sum rules. In Eq. (2.26) there is an extra factor of seven multiplying \( \langle q^\dagger q \rangle_{\rho N} \) relative to the term proportional to \( \langle s^\dagger s \rangle_{\rho N} \). Since the strange quark does not couple to the nuclear vector current (i.e., \( \langle s^\dagger s \rangle_{\rho N} = 0 \)), the leading-order term of Eq. (2.26), which sets the scale for the \( \Sigma \) vector self-energy, is very close to the corresponding leading term for the nucleon.

The dependence of the \( \Sigma \) scalar self-energy on \( y \) comes mainly from the leading-order
term (proportional to $\langle \pi \rangle_{\rho_N}$) of Eq. (2.24) and the parametrization Eq. (3.3). If we assume that the nucleon has a large strangeness content (i.e., $y \sim 0.4 - 0.6$) and the four-quark condensate $\langle \bar{q}q \rangle^2_{\rho_N}$ depends only weakly on the nucleon density (i.e., if $f \sim 0$), we find $M_{\Sigma}^*/M_{\Sigma} \sim 0.77 - 0.84$, which implies $\Sigma_s/M_{\Sigma} \sim -(0.16 - 0.23)$. With the nucleon sum-rule prediction $M_{N}^*/M_{N} \sim 0.65 - 0.70$ [7,9], we obtain $(\Sigma_s)_{\Sigma}/(\Sigma_s)_{N} \sim 0.6 - 1.0$. In a hadronic model, this implies again a coupling of the $\Sigma$ to the Lorentz scalar field close to that for the nucleon. In this case, there is a significant degree of cancellation between the scalar and vector self-energies, which is compatible with that implemented in the relativistic phenomenological models.

In contrast, if the strangeness content of the nucleon is small (i.e., $y \leq 0.2$) or if $\langle \bar{q}q \rangle^2_{\rho_N}$ has a significant dependence on the nucleon density, the predicted ratio $M_{\Sigma}^*/M_{\Sigma}$ is close to unity, implying that the scalar self-energy is very small. The predicted vector self-energy, on the other hand, is still essentially the same as the nucleon vector self-energy. Thus, in this case the sum rules predict incomplete cancellation, and hence a sizable repulsive net self-energy for the $\Sigma$. This result contradicts with that of the relativistic models.

We now turn to the $\Sigma$ spin-orbit force in a finite nucleus. In the present approach, all sum-rule predictions are obtained for uniform infinite nuclear matter; thus one cannot obtain a direct prediction for the $\Sigma$ spin-orbit force. However, we can still get some partial information on the $\Sigma$ spin-orbit force by adopting an approach of Dirac phenomenology [3,22]. In this approach, the coordinate-space potentials entering the Dirac equation for a baryon scattering from a finite nucleus are assumed to follow a Fermi distribution with two overall potential depths (scalar and vector), which are independent of nuclei, and which can be associated with the self-energies in infinite nuclear matter. The spin-orbit force follows by recasting the Dirac equation in Schrödinger form. The resulting spin-orbit potential is proportional to the sum of the magnitudes of the potential depths (that is, the scalar and vector self-energies) multiplied by the derivative of the assumed Fermi distribution [3].

The sum-rule predictions for the $\Sigma$ scalar and vector self-energies in the present paper
imply that the $\Sigma$ spin-orbit force in a nucleus is somewhat weaker than (but comparable with) that felt by a nucleon, but much stronger than that felt by a $\Lambda$. This is consistent with that obtained in Refs. [23–25]. However, we emphasize that in Refs. [23–25] the scalar and vector couplings, consistent with SU(3), have been adopted and it is the extra tensor coupling of $\Sigma$ to the vector meson that enhances the spin-orbit force. Our sum-rule calculations, on the other hand, suggest that it is the strong scalar and vector couplings, deviated from the naive SU(3) prediction, that lead to large spin-orbit force. We also note that our sum-rule predictions in this paper and those in Ref. [30] for $\Lambda$ do not agree with the universal coupling assumption (i.e., all hyperons couple to the scalar and vector fields with the same strength) suggested in Ref. [26].

V. SUMMARY

In this paper, we have applied finite-density QCD sum-rule methods to investigate the self-energies of a $\Sigma$ hyperon in nuclear matter. The approach focuses on a correlator of $\Sigma$ interpolating fields, evaluated in the nuclear matter ground state. A QCD expansion of the correlator is obtained by applying the operator product expansion. We retained the contributions from all condensates up through dimension four and to first-order in the strange quark mass $m_s$, and we also included the contributions from the four-quark condensates. The expansion requires new condensates not present at zero density, as well as information on the density dependence of condensates. In the rest frame of the nuclear matter, a Lehmann representation for the correlator, with fixed three-momentum, leads to a dispersion relation in the energy variable. A simple quasiparticle pole ansatz, with real Lorentz scalar and vector self-energies, is assumed for the spectral functions associated with the three invariant functions comprising the $\Sigma$ correlator. Contributions from higher-energy states are roughly approximated by a perturbative evaluation of the correlator, starting at an effective threshold.
The sum-rule analysis indicates that the $\Sigma$ vector self-energy is similar to the corresponding nucleon self-energy. If we interpret this result in terms of a relativistic hadronic model, it implies that the vector coupling for the $\Sigma$ is similar to the corresponding nucleon coupling to the vector meson. The vector self-energy is largely insensitive to the details of calculations, and is essentially determined by the nuclear matter density and the fact that $\langle s\dagger s \rangle_{\rho_N} = 0$.

The sum-rule predictions for the $\Sigma$ vector self-energy, along with that for the $\Lambda$ in Ref. [30], indicate a significant deviation from SU(3) in nuclear matter. This deviation is mainly due to the violation of SU(3) in the baryon interpolating fields and due to the violation of SU(3) in the nuclear matter ground state. At this stage, it is unclear whether the violation of SU(3) in the baryon interpolating fields is connected to the violation of SU(3) in the baryon wave functions. Further study of the dependence of the deviation on the choice of the interpolating field will be important in understanding the deviation from SU(3).

The scalar self-energy is found to be quite sensitive to the strangeness content of the nucleon and the assumed density dependence of the four-quark condensate $\langle \bar{q}q \rangle^2_{\rho_N}$. We parametrized the density dependence of this four-quark condensate in terms of its factorized form in free space and its factorized form in nuclear matter. If the nucleon has a significant strangeness content and if the four-quark condensate $\langle \bar{q}q \rangle^2_{\rho_N}$ has a weak density dependence, the sum rules predict strong scalar and vector self-energies. This is qualitatively compatible with relativistic models. On the other hand, if the strangeness content of the nucleon is small or $\langle \bar{q}q \rangle^2_{\rho_N}$ depends significantly on the nucleon density, the sum rules predict a very small scalar self-energy and a strong vector self-energy, which differs from the relativistic models. Clearly, further study of the strangeness content of the nucleon and the four-quark condensates in the nuclear medium is very important, along with analyses of the higher-order density dependence of other condensates and the contributions from the condensates with higher dimension.
The sum-rule predictions for the $\Sigma$ scalar and vector self-energies seem to imply a somewhat weaker spin-orbit force for $\Sigma$ in a nucleus than that for a nucleon, but much stronger than that for a $\Lambda$. This is compatible with those obtained from relativistic phenomenological models with an extra tensor coupling between the hyperons and the vector meson. This possible tensor contribution, however, cannot be observed in uniform nuclear matter.

It is a straightforward exercise to study the self-energies of $\Xi$ within the same framework. However, in the $\Xi$ sum rules, the leading order term of $\Pi_u$ is small relative to the higher order terms. Thus, it might be difficult to extract useful information since one does not have much control over the values of the higher order terms.

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FIGURES

FIG. 1. Optimized sum-rule predictions for $M^*_\Sigma/M_\Sigma$ and $\Sigma_v/M_\Sigma$ as functions of $y$. The three curves correspond to $f = 0$ (solid), $f = 0.25$ (dashed), and $f = 0.5$ (dotted). The other input parameters are described in the text.

FIG. 2. Ratios $M^*_\Sigma/M_\Sigma$ and $\Sigma_v/M_\Sigma$ as functions of Borel $M^2$ for $y = 0.5$, with optimized predictions for $E_q$, $\overline{E}_q$, and the continuum thresholds. The three curves correspond to $f = 0$ (solid), $f = 0.25$ (long-dashed), and $f = 0.5$ (dotted). The other input parameters are the same as in Fig. 1.

FIG. 3. (a) The left- and right-hand sides of the sum rules as functions of Borel $M^2$ for $y = 0.5$ and $f = 0$, with the optimized values for $M^*_\Sigma$, $\Sigma_v$, $s^*_0$, and $\lambda^2_{\Sigma}$. The other parameters are the same as in Fig. 1. The four curves correspond to $\Pi'_s/M^*_\Sigma$ (solid), $\Pi'_q$ (dashed), $\Pi'_u/\Sigma_v$ (dot-dashed), and $\lambda^2_{\Sigma}e^{-(E^2-q^2)/M^2}$ (dotted). (b) The left- and right-hand sides of the corresponding vacuum sum rules. The three curves correspond to $\Pi'_s/M_\Sigma$ (solid), $\Pi'_q$ (dashed), and $\lambda^2_{\Sigma}e^{-M^2_\Sigma/M^2}$ (dot-dashed) at the zero-density limit, with the optimized values for $M_\Sigma$, $s_0$, and $\lambda^2_{\Sigma}$.

FIG. 4. Optimized predictions for $M^*_\Sigma/M_\Sigma$ and $\Sigma_v/M_\Sigma$ as functions of $m_s$ for $y = 0.5$. The three curves correspond to $f = 0$ (solid), $f = 0.25$ (dashed), and $f = 0.5$ (dotted). The other input parameters are the same as in Fig. 1.

FIG. 5. Three-momentum dependence of the predicted $M^*_\Sigma/M_\Sigma$ and $\Sigma_v/M_\Sigma$ for $y = 0.5$. The three curves correspond to $f = 0$ (solid), $f = 0.25$ (dashed), and $f = 0.5$ (dotted). The other input parameters are the same as in Fig. 1.

FIG. 6. Optimized sum-rule predictions for $M^*_\Sigma/M_\Sigma$ and $\Sigma_v/M_\Sigma$ as functions of $f$, with $y = 0.5$. The other input parameters are the same as in Fig. 1. The solid curves correspond to the results obtained by requiring the continuum contributions to be less than 50% in the fiducial Borel region and the dashed curves correspond to the results obtained using a fixed Borel window at $1.0\,\text{GeV}^2 \leq M^2 \leq 1.6\,\text{GeV}^2$.  

29
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