Imaging of current density distributions with a Nb weak-link scanning nano-SQUID microscope

Yusuke Shibata¹, Shintaro Nomura¹, Hiromi Kashiwaya², Satoshi Kashiwaya², Ryosuke Ishiguro³,⁴ & Hideaki Takayanagi⁴

Superconducting quantum interference devices (SQUIDs) are accepted as one of the highest magnetic field sensitive probes. There are increasing demands to image local magnetic fields to explore spin properties and current density distributions in a two-dimensional layer of semiconductors or superconductors. Nano-SQUIDs have recently attracting much interest for high spatial resolution measurements in nanometer-scale samples. Whereas weak-link Dayem Josephson junction nano-SQUIDs are suitable to miniaturization, hysteresis in current-voltage (I-V) characteristics that is often observed in Dayem Josephson junction is not desirable for a scanning microscope. Here we report on our development of a weak-link nano-SQUIDs scanning microscope with small hysteresis in I-V curve and on reconstructions of two-dimensional current density vector in two-dimensional electron gas from measured magnetic field.

Superconducting quantum interference devices (SQUIDs) utilize phase difference of two-weakly coupled superconducting electrodes with Josephson junctions or weak links forming a superconducting loop. The supercurrent flowing in a SQUID is a periodical function of the flux penetrating the superconducting loop \( \Phi \) divided by the magnetic flux quantum \( \Phi_0 = h/2e = 2.0679 \times 10^{-15} \) Wb. SQUIDs have been widely used as one of the most sensitive magnetic sensors. Images of magnetic flux are captured by a SQUID probe scanned on the surface of a sample. Since there has been increasing interests in detecting magnetic field of small area in order to characterize nanostructures or nano-devices, nano-meter scale SQUIDs have been actively studied recently. Decreasing the size of the superconducting loop favours sensitivity to local magnetic dipoles due to decrease in the limiting flux spectral density and due to reduced distance between magnetic dipoles and the SQUID loop. The size of the superconducting loop was typically 2–100 \( \mu \)m. Recently, a self-aligned SQUID has been fabricated on a tip of a sharpened quartz tube in the range of 40–300 nm in diameter and has been used for scanning probe magnetometry. This method has the highest spatial resolution to date, but the high flexibility of the design and configuration of SQUIDs makes weak-link nano-SQUIDs fabricated by a focused ion beam (FIB) process attractive for use as local probe of magnetic flux.

Weak-link superconducting junctions are fabricated by direct milling using \( \text{Ga}^+ \) ions without lithography process. Simultaneous observations of milled area enable us to precisely position the SQUID and control the properties of the weak-link superconducting junctions. This is especially advantageous to fabricate a weak-link nano-SQUID on a tip of a scanning probe. Multiple SQUIDs may be fabricated on a scanning probe by FIB process, which may find applications for mappings of magnetic field vector. Weak-link SQUIDs have a number of advantages over tunnel junction based SQUIDs. Since the size of

¹Division of Physics, University of Tsukuba, Tennodai, Tsukuba, 305-8571, Japan. ²National Institute of Advanced Industrial Science and Technology (AIST), Umezono, Tsukuba, 305-8568, Japan. ³Department of Mathematical and Physical Sciences, Faculty of Science, Japan Women’s University, Mejirodai, Bunkyo-ku, Tokyo 112-8681, Japan. ⁴Department of Applied Physics, Faculty of Science, Tokyo University of Science, Nlijuku, Katsushika-ku, Tokyo 125-8585, Japan. Correspondence and requests for materials should be addressed to S.N. (email: nomura.shintaro.ge@u.tsukuba.ac.jp)
weak-link junctions can typically be made smaller than the size of tunnel junction, the capacitance and
the inductance are smaller in weak-link junctions than those in tunnel junction based SQUIDs. This
increases sensitivity to magnetic flux and tolerance for high magnetic field environments. However, the
presence of hysteresis in current-voltage (I-V) characteristics has been an obstruct for a weak-link
nano-SQUID to be used in scanning probe measurements.

In this paper, we describe construction and characterization of our weak-link scanning nano-SQUID
microscope (Fig. 1(a–e)) with small hysteresis in I-V characteristics suitable for imaging. We present
results of imaging of magnetic field created by current in a Hall-bar structure of a GaAs/Al0.3Ga0.7As
modulation-doped single heterojunction to evaluate performance of the weak-link scanning nano-SQUID
microscope, and report on reconstruction of the two-dimensional current density vector by a Fourier
analysis.

Fabrication of weak-link nano-SQUID probes
Nb/Au thin films were deposited on a Si substrate. Nb film with the thickness of 23 nm was deposited
by an RF-sputtering and subsequently Au film with the thickness of 70 nm was deposited by an
electron-beam deposition at the base pressure of $6 \times 10^6$ Pa. The Au thin film was used to protect the Nb
film from Ga⁺ ion beam exposure. The Au thin film was also important to reduce the hysteresis in the
I-V curve of a nano-SQUID by reducing the self-heating effect at the weak-link junctions. Superconducting four-terminal configurations were patterned by a maskless laser lithography using 405 nm light (DL-1000, NanoSystem Solution, Inc). Silicon probes for scanning microscopy with a thickness of 100 μm and the size of 2000 × 600 μm² were defined by a maskless laser lithography. Deep etching of silicon substrate with high aspect ratio was performed by 200 cycles of a pulsed reactive ion etching (RIE) by SF₆ and deposition of a C₇F₆ passivation layer. About a hundred pieces of Si probes were fabricated by the above process. (Fig. 1(c)) The tip of a part of Si probes was mechanically polished (Supplementary Fig. 1, and Supplementary Note 1) to remove Nb/Au film near the edge damaged by the RIE process using Al₂O₃ powder with an average diameter of 100 nm dispersed in 1-octanol. Each Si

Figure 1. Schematics of scanning nano-SQUID system and images of nano-SQUID probes. (a) Schematic
illustration of a scanning nano-SQUID system in a cryogen-free 4He refrigerator. Upper green part indicates
a holder for a scanning SQUID probe attached to a quartz tuning fork. A sample stage is placed on triaxial
piezoelectric inertial stages. The SQUID probe and the piezoelectric inertial stages are settled in a stainless
housing which is connected to the 4 K plate with four springs. (b) Optical image of a SQUID probe attached
to a tuning fork and a sample chip carrier. (c) Optical image of probes fabricated by a laser-lithography and
deep etching of a silicon substrate. After detaching each pieces, a nano-SQUID was fabricated at the tip of
the probe by an FIB. (d) Scanning ion microscope images of SQUID probes without mechanical polishing
and (e) after mechanical polishing of the tip of the probe. SQUID loop and weak link junctions were
fabricated by an FIB milling system.
probe was cut into a piece. A Si probe was mounted on a quartz tuning fork with a resonance frequency of 32,768 Hz. After gluing the Si probe to a quartz tuning fork, the resonance frequency decreases to typically 30.8 KHz with the Q-factor of about 10,000 at 4 K. Finally, the superconducting loop and weak-link junctions were milled by the FIB process with typical beam voltage and current of 31 kV and 5 pA, respectively. The SQUID loop with the size of 1 \( \mu \text{m} \) was located within 4.8 \( \mu \text{m} \) from the tip of the Si probe. The dimensions of the SQUID loop and the weak-link width were 1.0 \( \mu \text{m} \) and 80 nm, respectively. (Fig. 1(d,e))

The geometrical inductance \( \mu = L \cdot C \) where \( C \) is the inner circumference of the SQUID\(^{13}\) was 1.6 pH. The effect of the self-induced field as given by \( \beta = \frac{\Phi}{I L^2 c_0} \) was 0.32.

**Results and Discussion**

**Characteristics of SQUID probes.** Figure 2(a,b) show \( I-V \) characteristics of nano-SQUID probes without and after mechanical polishing, respectively, at temperatures between \( T = 3.6 \) and 6.8 K at zero applied magnetic field. The current was swept in both negative and positive directions, and no hysteresis in the \( I-V \) characteristics was observed. (a) Current-voltage characteristics of a SQUID probe without mechanical polishing at \( T = (i) 3.6, (ii) 3.8, (iii) 4.0, (iv) 4.2, (v) 5.0, (vi) 5.2, (vii) 5.4, (viii) 5.6, (ix) 5.8, (x) 6.0, (xi) 6.2, (xii) 6.4, (xiii) 6.6, (xiv) 6.8 \) K. The current was swept for both negative and positive directions, and no hysteresis in the \( I-V \) characteristics was observed. (b) Current-voltage characteristics of a SQUID probe after mechanical polishing at \( T = (i) 3.6, (ii) 3.8, (iii) 4.0, (iv) 4.2, (v) 5.0, (vi) 5.2, (vii) 5.4, (viii) 5.6, (ix) 5.8, (x) 6.0, (xi) 6.2 \) K. No hysteresis in the \( I-V \) characteristics was observed.

![Temperature dependence of current-voltage characteristics of SQUID probes.](image)

Figure 2. Temperature dependence of current-voltage characteristics of SQUID probes. (a) Current-voltage characteristics of a SQUID probe without mechanical polishing at \( T = (i) 3.6, (ii) 3.8, (iii) 4.0, (iv) 4.2, (v) 5.0, (vi) 5.2, (vii) 5.4, (viii) 5.6, (ix) 5.8, (x) 6.0, (xi) 6.2, (xii) 6.4, (xiii) 6.6, (xiv) 6.8 \) K. The current was swept for both negative and positive directions, and no hysteresis in the \( I-V \) characteristics was observed. (b) Current-voltage characteristics of a SQUID probe after mechanical polishing at \( T = (i) 3.6, (ii) 3.8, (iii) 4.0, (iv) 4.2, (v) 5.0, (vi) 5.2, (vii) 5.4, (viii) 5.6, (ix) 5.8, (x) 6.0, (xi) 6.2 \) K. No hysteresis in the \( I-V \) characteristics was observed.

The critical temperature \( T_c \) of the nano-SQUID without mechanical polishing was 6.0 K, which is reduced from typical \( T_c \) of Nb of 9.2 K due to small thickness of the Nb film and the proximity effect between Nb and Au\(^{10}\). Figure 3(a) shows a typical \( I-V \) characteristics of the nano-SQUID probe without mechanical polishing (Fig. 1(d)) at applied magnetic fields between 0.1 and 0.5 mT at 4.2 K. The critical current \( I_c \) is seen to change with the applied magnetic field. The SQUID loop of the probe after mechanical polishing of the tip of the nano-SQUID probe (Fig. 1(e)) was located very close to the edge of the Si substrate and the probe did not exhibit zero resistance as shown in Fig. 2(b) above 3.6 K because of the damage of Nb film during the fabrication process. Nevertheless, the change of the voltage with the applied magnetic field was observed as shown in Fig. 3(b), indicating that this nano-SQUID probe is useful as a magnetic field sensor. The non hysteretic behaviour of our nano-SQUIDs enables us to measure the voltage of a nano-SQUID at a constant bias current to obtain magnetic flux threading the SQUID loop. This is particularly advantageous to use a nano-SQUID as a magnetic sensor for a scanning microscope. If a nano-SQUID shows a hysteretic \( I-V \) characteristic, \( I_c \) has to be measured by sweeping \( I \) at each point. Typical voltage-magnetic field (\( V_{SQ} - B_{ext} \)) characteristics in Fig. 3(c,d) show oscillations of SQUID voltage \( V_{SQ} \) with \( B_{ext} \) at constant current-bias. The noise of the probe without mechanical polishing was estimated to be 3.1 nT/\( \sqrt{\text{Hz}} \) at 2 kHz. The noise
of the probe after mechanical polishing was degraded to 40 nT/√Hz at 2 kHz while the spatial resolution was improved.

**Mapping of magnetic field distribution around Nb/Au strips.** Mapping of magnetic field distribution around Nb/Au strips was performed to test spatial resolution of our nano-SQUID microscope. The thicknesses of Nb and Au films were 600 and 30 nm, respectively. The width and the spacing of Nb/Au strips were 2 and 5 μm, respectively, as shown in Fig. 4(a). A nano-SQUID probe after mechanical polishing was used at a constant distance between the tip of the SQUID probe and the sample surface of 1.2 μm. The obtained mapping of magnetic field distribution of Nb/Au strips at an applied magnetic field of 1.05 mT is shown in Fig. 4(b). A strip of reduced magnetic field because of the Meissner effect of the Nb strips can be identified at 5 < x < 7 μm. The reduction of magnetic field is more clearly seen in the lineprofiles of magnetic field along y = 10, 15, and 20 μm as shown in Fig. 4(c), indicating that the spatial resolution of our nano-SQUID microscope is better than 2 μm.

**Mappings of magnetic field distribution induced by current in a modulation-doped single heterojunction sample.** Results of mappings of magnetic field created by ac current between the voltage probes 1 and 2 of the Hall-bar structure (Fig. 5(a,b)) at $I_{\text{sample}} = 70$ and 2.8 μA are shown in Fig. 5(c,d), respectively, at $T = 4$ K at the applied magnetic field by a superconducting magnet of 0.0 mT using a probe after mechanical polishing. The measurements were performed in unshielded laboratory environment, and hence the sample and the nano-SQUID probe were subject to the environmental magnetic field. The size of the scanning area was 80 × 80 μm² with a step size of 2 μm for x- and y-directions. The height of the SQUID probe from the sample surface was $z_0 = 1.5$ μm. The voltage of a SQUID at a constant current bias of 12 μA changes due to magnetic field created by the current in the Hall-bar structure. This voltage change was synchronously detected with a lock-in amplifier at a time constant of 1 s by scanning the position of the nano-SQUID probe on the surface of the sample. The bias current of $I_{\text{sample}} = 70$ and 2.8 μA corresponds to current density of 7.0 and 0.28 A/m, respectively. The mapping in Fig. 5(c) (Fig. 5(d)) corresponds to the case of the current density above (below) the condition for the breakdown of the quantum Hall effect. The maximum source-drain voltage ($V_{\text{SD}}$) was
8.0 mV at \( I_{\text{sample}} = 70 \mu\text{A} \). The Fermi energy of the electrons is \( \varepsilon_F = 12 \text{ meV} \) at the electron density of the sample of \( n_s = 3.3 \times 10^{15} \text{ m}^{-2} \), which gives \( eV_{\text{SD}} < \varepsilon_F \). The positive and negative magnetic fields are observed near the edges of the stem of the Hall-bar structure with the width of 10 \( \mu\text{m} \) as shown in Fig. 5(c). The magnetic field distributions are seen to be broadened in the center region where the 10 \( \mu\text{m} \) width stem crosses with the bar with the width of 25 \( \mu\text{m} \). Similar structures are seen in the magnetic field distribution at \( I_{\text{sample}} = 28 \mu\text{A} \) (Fig. 5(d)) although the signal-to-noise ratio was degraded.

**Reconstruction of two-dimensional current density distribution.** Two-dimensional current density distribution \( J(x, y) \) can be reconstructed from the measured magnetic flux distribution \( B(x, y, z) \) based on a Fourier analysis\(^\text{17}\). The two-dimensional Fourier transform of the current density and magnetic field is defined by

\[
j_y(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} j_y(x, y) e^{i(k_x x + k_y y)} dx dy
\]

and

\[
b_x(k_x, k_y, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} B_x(x, y, z) e^{i(k_x x + k_y y)} dx dy,
\]

respectively. By using the two-dimensional Fourier transform of the Green's function

\[
g(k_x, k_y, z) = \frac{\mu_0 d}{2} e^{-k_0 z},
\]

we have

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**Figure 4. Image and mapping of magnetic field distribution of Nb/Au strips.** (a) Scanning electron microscope image of Nb/Au strips with a width of 2 \( \mu\text{m} \) and a spacing of 5 \( \mu\text{m} \) at applied magnetic field of 1.05 mT at 4 K. (b) Mapping of magnetic field distribution of Nb/Au strips. (c) Line profiles of magnetic field distribution along (i) \( y = 10 \), (ii) 15, and (iii) 20 \( \mu\text{m} \).
where \( d \) is the thickness of the current and we defined \( k_0 = \sqrt{k_x^2 + k_y^2} \). Similarly, \( b_z \) is given by

\[
b_z(k_x, k_y, z) = ig (k_x, k_y, z) \left( k_y \frac{j_y}{k_0} (k_x, k_y) - k_x \frac{j_x}{k_0} (k_x, k_y) \right),
\]

(5)

We assume quasi-stationary current density \( \nabla \cdot \mathbf{J}(x, y) = 0 \) in real-space and \(-ik_x j_x(k_x, k_y) - ik_y j_y(k_x, k_y) = 0 \) in \( k \)-space. A nano-SQUID probe facing \( \theta \) to the surface of the sample detects \( B_M(x, y, z) = B_x(x, y, z) \sin \theta + B_y(x, y, z) \cos \theta \). Because a nano-SQUID detects magnetic field averaged over the square SQUID loop, the Fourier transform of measured magnetic field \( b_M(k_x, k_y, z) \) should be divided by \( z_{\text{eff}} \)

\[
h(k_x, k_y) = \frac{\sin(k_x L_{\text{SQ}}/2)}{k_x L_{\text{SQ}}/2} \frac{\sin(k_y L_{\text{SQ}}/2)}{k_y L_{\text{SQ}}/2},
\]

(6)

where \( L_{\text{SQ}} \) is the size of the SQUID loop to take into account the finite size of the SQUID in \( x \)- and \( y \)-directions. In the case of our nano-SQUID, \( h(k_x, k_y) \) does not introduce noticeable difference because \( L_{\text{SQ}} \) is smaller than the step size of the measurement. For the \( z \)-direction, we assume a SQUID probe detects the average magnetic field \( B_{\text{ave}} \) at the effective height \( z_{\text{eff}} \) satisfying

\[
B_{\text{ave}} = \frac{\mu_0 I}{2\pi} \frac{1}{L_{\text{SQ}}} \frac{1}{z_{\text{eff}}} \int_{z_0}^{z_{0}+L_{\text{SQ}}} \sin \theta \, dz = \frac{\mu_0 I}{2\pi} \frac{1}{z_{\text{eff}}},
\]

(7)

For \( z_0 = 1.5 \mu m \), \( L_{\text{SQ}} = 1.0 \mu m \), and \( \theta = 51^\circ \), we obtain \( z_{\text{eff}} = 1.9 \mu m \).

We substitute \( j_x = -\frac{k_y}{k_x} j_y \) into equation (5), and using equation (4), we obtain

\[
b_M(k_x, k_y, z_{\text{eff}}) = g(k_x, k_y, z_{\text{eff}}) (\sin \theta - i \frac{k_0}{k_x} \cos \theta) j_y(k_x, k_y).
\]

(8)

The current density can be readily obtained as

Figure 5. Hall-bar structure and mappings of magnetic field distribution by the current flowing in the Hall-bar sample. (a) Schematic structure of a sample Hall-bar. (b) Optical micrograph of a sample Hall-bar structure. Red square indicates the scanning area of 80 \( \mu m \times 80 \mu m \) for (c). Mappings of magnetic field distribution induced by current in a GaAs/AlGaAs modulation-doped single heterojunction sample at \( T = 4 K \) for current (c) \( I_{\text{sample}} = 70 \) and (d) 2.8 \( \mu A \) at the applied magnetic field by a superconducting magnet of 0.0 mT using a nano-SQUID probe after mechanical polishing of the tip of the probe. The height of the SQUID probe from the sample surface was 1.5 \( \mu m \).
\[ j_y(k_x, k_y) = \frac{2}{g(k_x, k_y, \varepsilon_{\text{eff}})} \sin \theta + i \frac{k_x}{k_z} \cos \theta b_M(k_x, k_y, \varepsilon_{\text{eff}}) \sin^2 \theta + i \frac{k_z}{k_y} \cos^2 \theta \]

in \( k \)-space and

\[ I_y(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} j_y(k_x, k_y) e^{-i(k_x x + k_y y)} dk_x dk_y \]

in real-space. \( I_y(x, y) \) may be similarly obtained by using \( j_x = -\frac{k_y}{k_z} j_y \). For the case of \( k_x = k_y = 0, j_y(0, 0) \) cannot be determined from \( b_M(k_x, k_y, \varepsilon_{\text{eff}}) \). The obtained \( I_x \) and \( I_y \) have ambiguity by uniform current density distribution. In the following analysis, we set this uniform current density as zero.

Figure 6(a–d) show reconstructed current density distributions \( I_x(x, y) \) and \( I_y(x, y) \) from the measured magnetic flux in Fig. 3(b,c) at \( I_{\text{sample}} = 70 \) and 2.8 \( \mu \)A, respectively, for \( \theta = 51^\circ \). Calculations were performed on a mesh of 128 \( \times \) 128 in the \( x \) and \( y \) directions. A Parzen window\(^{17,18}\)

\[ W(k) = \begin{cases} 1 - k/k_{\text{max}}, & k < k_{\text{max}} \\ 0, & k > k_{\text{max}} \end{cases} \]

was used to eliminate high-spatial-frequency components of measured mappings\(^{17}\). We empirically chose \( k_{\text{max}} = 1.5 \times 10^6 \) and 6.9 \( \times \) 10^4 \( m^{-1} \) for \( I_{\text{sample}} = 70 \) and 2.8 \( \mu \)A, respectively, so that high-spatial-frequency noise is effectively reduced with minimum loss of spatial resolution. In the center region, where the 10 \( \mu \)m-stem crosses the bar with the width of 25 \( \mu \)m, the current density \( I_x(x, y) \) is seen to spread to the wider bar. This can be more clearly indicated in \( I_y(x, y) \) in Fig. 6(b) by the positive and negative current density near the corners of the mesa structure of the Hall-bar. At smaller current of \( I_{\text{sample}} = 2.8 \mu \)A, the main features of the current densities can still be resolved as shown in Fig. 6(c,d).

Although the signal-to-noise ratio of the mapping of magnetic field in Fig. 5(d) is heavily degraded as compared to the case of \( I_{\text{sample}} = 70 \mu \)A in Fig. 5(c), the current densities are recovered at the expense of degraded spatial resolution by using \( k_{\text{max}} = 6.8 \times 10^5 \) \( m^{-1} \) used for the Parzen window. The mappings of the magnetic field (Fig. 5(c)) and the reconstructed current density distributions (Fig. 6(a,b)) using the nano-SQUID probe after mechanical polishing show remarkably better spatial resolution as compared to the mappings using the nano-SQUID probe without mechanical polishing (Supplementary Fig. 2). This shows the advantage of the nano-SQUID probe after mechanical polishing over the unpolished nano-SQUID probe although the zero resistance was not observed due to the damage of Nb film in the polished nano-SQUID probe. The current obtained by integrating current densities \( I_y(x, y) \) on the white bars in Fig. 6(a) is 84 and 91 \( \mu \)A for the crosssection A-B and C-D, respectively, in reasonable agreement with \( I_{\text{sample}} = 70 \mu \)A. Similarly, current across the white bars in Fig. 6(c) is 4.5 and 4.6 \( \mu \)A for the crosssection A-B and C-D, respectively, for \( I_{\text{sample}} = 2.8 \mu \)A.

Comparison of the measured current density with results of a numerical calculation.

Two-dimensional quasi-stationary current density is calculated by assuming that the current density instantaneously responds to the electric field at the position of the electron as given by\(^{19}\)

\[ J(x, y) = -\sigma(x, y) \nabla \phi(x, y), \]

which is applicable to a low mobility limiting case. Clearly, this model is not directly applicable to high mobility electron gas, nonetheless this model is useful to assist the understandings of the observed current density mappings by our SQUID microscope. One may refer to semiclassical\(^{20,21}\) or quantum mechanical\(^{22}\) ballistic electron transport theories for more realistic descriptions. The measured current density \( J(x, y) \) may be described by the statistical or quantum mechanical average of the local electric current operator, which is described by the momentum \( \mathbf{p} \) and the position \( \mathbf{x} \) of an electron. Then ballistic electron transport theories take into account the scatterings of the electron from \( \mathbf{p} \) to \( \mathbf{p}' \) by such as the external electric field, the impurities, and the phonons. The description implied by equation (12) ignores these effects, in particular, the inertial motion of the current carrying electrons.

We also assume slow modulation frequency of the current with negligible displacement current \( |\varepsilon \partial \mathbf{E}/\partial t| < < |J| \) and isotropic conductivity \( \sigma(x, y) = \sigma_{xx}(x, y) = \sigma_{yy}(x, y) \). Then we solve numerically

\[ \nabla \cdot (\sigma(x, y) \nabla \phi(x, y)) = 0 \]
by a finite-difference method on a mesh of $400 \times 400$ with $\sigma_0 = 0.05 \, \text{S}$, where $\phi(x, y)$ is the electrostatic potential. Conductivity was assumed to be $\sigma(x, y) = \sigma_0$ inside the Hall bar structure and $\sigma(x, y) = 0$ elsewhere as shown in Fig. 6(e). Dirichlet boundary conditions were applied for $x$-direction, and the voltage was set to be $-0.798$ and $0 \, \text{meV}$ at $x = 0$ and $80 \, \mu\text{m}$, respectively. A periodic boundary condition was applied to the $y$-direction. The electrostatic potential $\phi(x, y)$ was converged to an accuracy of less than $10^{-6} \, \text{V}$.

Figure 6(f,g) show calculated current density. The observed main features in Fig. 6(a,b) are reproduced in Fig. 6(f,g). In particular, the spread of $I_x(x, y)$ to the wider stem of the Hall-bar in Fig. 6(a) is reproduced in Fig. 6(f), and the local maxima and minima in $I_y(x, y)$ near the corners of the mesa structure of the Hall-bar in Fig. 6(b) are reproduced in Fig. 6(g). The distances between the local maxima and minima in $I_y(x, y)$ are $w_{x1} = 16 \, \mu\text{m}$ in Fig. 6(b) and $w_{x2} = 25 \, \mu\text{m}$ in Fig. 6(g), indicating a sizable disagreement with $w_{x1} < w_{x2}$, whereas the difference in $w_{y1}$ in Fig. 6(b) and $w_{y2}$ in Fig. 6(g) is small. This disagreement is not explained by the finite $k_{\text{max}}$ for the truncation of high-frequency components.
in the Fourier analysis. We calculated the magnetic field due to the calculated current density as given by equation (12), and reconstructed the current density distributions by changing \( k_{\text{max}} \) by the method described by equations (1)-(11) and checked that \( w_{x2} \) and \( w_{y2} \) did not depend on \( k_{\text{max}} \). Consequently the disagreement between the observed and the calculated distances between the local maxima and minima in \( J_x (x, y) \) is understood by the finite length that the electrons travel before they change momenta following the direction of the gradient of the electrostatic potential \( \phi (x, y) \). The difference \( w_{x2} - w_{x1} = 9 \mu m \) is reasonably explained by the mean free path of the sample of 8.7 \( \mu m \). Thus the disagreement between \( w_{x1} \) and \( w_{x2} \) is a manifestation of ballistic transport of the electrons. The reconstructed \( J_x (x, y) \) in Fig. 6(a) is nearly symmetric and four main peaks in \( 20 < x < 50 \mu m \) in Fig. 6(b) are nearly antisymmetric with respect to the center of the 25 \( \mu m \)-width bar, because ac bias current was applied to the Hall-bar structure and the magnetic field was detected synchronously using a lock-in amplifier. The effect of the limited spatial resolution due to small \( k_{\text{max}} \) is observed in Fig. 6(a) where the current density is larger at the center of the Hall-bar than the edge unlike the case in Fig. 6(f) where the current density is nearly homogeneous across the stem of the Hall-bar with the width of 10 \( \mu m \). The depletion layer thickness was estimated to be 134 nm \(^{23,24} \) for similar GaAs single heterojunction sample with the electron density of 4.6 \( \times 10^{15} \text{m}^{-2} \). The height of the mesa structure of the sample was about 100 nm. Thus the depletion layer thickness of the two-dimensional electron gas and the widths of the lateral etching are too small to explain the measured current density distribution. Improvements in the signal-to-noise ratio of the magnetic flux measurements are required to increase \( k_{\text{max}} \) to obtain better spatial resolution.

In summary, we have constructed a weak-link scanning nano-SQUID microscope using a SQUID probe with small hysteresis in \( I-V \) curve suitable for a magnetic sensor for scanning measurements. We have measured magnetic field distribution created by the current in the Hall-bar structure. Two-dimensional current density components \( J_x (x, y) \) and \( J_y (x, y) \) were reconstructed from measured \( B \) based on a Fourier analysis. The reconstructed two-dimensional current density reproduced most of the features of current density calculated by solving Laplace equation, however, a significant deviation was found near the corners of the Hall-bar structure and was explained by ballistic electron transport. Our newly developed scanning nano-SQUID microscope may contribute to characterize, for example, chiral or helical superconductors and current density in the quantum anomalous Hall effect or in the quantum spin Hall effect.

Methods

**Low temperature scanning SQUID microscope.** A nano-SQUID probe was accommodated in a rigid stainless housing connected with four springs to the 4 K plate of a cryogen-free pulse tube refrigerator (Optistat PT, Oxford Instruments) to reduce vibrations due to cooling cycle. A sample was mounted on closed loop inertially actuated triaxial stepping piezoelectric-stages with resistive position encoders (ANPx101/RES and ANPz101/RES, Attocube systems). The piezoelectric-stages were operated in the coarse positioning mode with typical minimum step size of 10 nm and in the fine positioning mode. The resistive position encoder. The voltage of a SQUID was amplified by a preamplifier (LI-75A, NF Corporation) at room temperature. The measurements were performed in unshielded laboratory environment. Because the size of the SQUID loop is small, the contribution from the environment magnetic field noise is relatively small. The noise of the SQUID probes is mainly limited by the equivalent input noise of the preamplifier.

**Sample.** The sample GaAs/Al\(_{0.3}\)Ga\(_{0.7}\)As modulation-doped single heterojunction consists of a GaAs/AlAs superlattice buffer layer, 200 nm-thick undoped GaAs layer, 40 nm-thick undoped Al\(_{0.3}\)Ga\(_{0.7}\)As layer, 30 nm-thick Si-doped Al\(_{0.3}\)Ga\(_{0.7}\)As layer, and 10 nm-thick Si-doped GaAs capping layer. The density and the mobility of the two-dimensional electron gas were \( 3.3 \times 10^{15} \text{m}^{-2} \) and 91 m\(^2\)/Vs at 2.8 K, giving the mean free path of electrons of 8.7 \( \mu m \). A Hall-bar structure of the width and the length of 25 and 300 \( \mu m \) was fabricated by photolithography. The distance between the voltage probes was 100 \( \mu m \). Current between ohmic contacts of the Hall-bar structure was modulated at 1873 Hz. The voltage of a SQUID probe at a constant current bias changes due to magnetic field created by the current in the Hall-bar structure. This voltage change was detected synchronously with a lock-in amplifier at a time constant of 1 s by scanning the nano-SQUID probe on the surface of the sample.
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Author Contributions

S.N., S.K., R.I. and H.T. conceived the experiments, Y.S. and S.N. conducted the experiments, Y.S., H.K. and S.K. prepared SQUID probes, R.I. prepared a Au/Nb strip sample, Y.S. and S.N. analyzed the results. All authors reviewed the manuscript.

Additional Information

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