Reliability assessment of reinforced concrete columns designed by Egyptian code

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In this research, the reliability level of reinforced concrete short column designed according to Egyptian code was investigated. The ultimate limit-state design method considering material strength-reduction factors and partial factors for load combinations given in the Egyptian code was used to obtain the characteristic values of short column capacity. Component reliability program, COMREL V8.1, was used to obtain the reliability index based on first order reliability method. The probabilistic models of basic variables were based on the European probabilistic model code. The reliability index of short column was computed at different reinforcement ratios and compared to the reliability obtained when using Euro code (1992-2004) and ACI code in design of the same member. The sensitivity of the reliability index for different parameters such as concrete and steel strengths and their coefficients of variation and standard deviation of permanent actions, were investigated. The results showed that the design of short column according to the Egyptian code achieved sufficient level of reliability higher than the other compared international codes. Besides, the reliability index is sensitive to the coefficient of variation of reinforcement steel and permanent action.

Key words: Reliability, ultimate limit state design, probabilistic variables.

INTRODUCTION

In the last few decades, extensive work was executed to develop the probability based structural design. The first edition of International Standard ISO 2394 (2015) was issued in 1986 named “General principles on reliability of structures” and replaced in this year by the fourth edition. The first European code “Basis of structural design EN - 1990” (1990-2002) based on reliability level was issued in 2002. Other countries started to establish their national codes like South Africa (SANS, 10160, 2011) and Brazil; these codes refer to the reliability concepts in the international standard ISO 2394. Egypt is going to establish the first Egyptian code of probability basis of structural design to be a mother code of other codes of construction and design with different materials. For this purpose it becomes important to evaluate the level of reliability obtained when designing, using existing Egyptian codes.

In this research, the reliability level of short column designed by Egyptian codes is evaluated and compared with the reliability obtained when designing by other codes. The different parameters affecting the sensitivity of the reliability index are studied.
COMBINATION OF ACTIONS

Design combination of action

In the following analysis, the combination of two actions is considered: permanent action G and imposed load Q, which proves to be the critical combination as found by Holicky and Retief (2000) and Gulvanessian and Holicky (2005). Assuming linear behavior of structural members, actions G and Q and their characteristic values \( G_k \) and \( Q_k \) denote generally appropriate load effects. The fundamental combination of these loads in permanent and transient design situation introduces the following expressions for the design action \( E_d \) according to the Egyptian, Euro and ACI codes.

\[
E_d^{(ECP)} = 1.4 \, G_k + 1.60 \, Q_k \quad \text{ECP 203} \quad (1a)
\]

\[
E_d^{(EUR)} = 1.35 \, G_k + 1.50 \, Q_k \quad \text{EN 1990} \quad (1b)
\]

\[
E_d^{(ACI)} = 1.2 \, G_k + 1.60 \, Q_k \quad \text{ACI 2011} \quad (1c)
\]

Where: \( E_d^{(ECP)} \), \( E_d^{(EUR)} \) and \( E_d^{(ACI)} \) are column design axial resistance by Egyptian, Euro and ACI codes, respectively.

The load ratio \( r \) is applied to investigate load effects under various intensities of variable and permanent actions. Parameter \( r \) denotes the ratio of variable actions \( Q_k \) to the total load \( G_k+Q_k \). The realistic range of \( r \) from 0.1 to 0.6 is found for structures in practice.

Random variable combination of action

The permanent action \( G \), imposed load \( Q \) and the coefficient of model uncertainty of action \( K_e \) are considered as basic random variables. The combination of variable actions \( E \) can be expressed as follows:

\[
E = K_e \, (G + Q) \quad (2)
\]

Resistance of short column

Design resistance

The design value of the axial resistance of reinforced concrete column designed according to the Egyptian code ECP 2007, Euro and ACI codes can be expressed as shown in the following Equations 3a, b and c, respectively:

\[
R_{d(ECP)} = 0.67 \, A_s \, f_y/k + 0.35 \, b \, t \, f_{ck}/0.8 \quad \text{(ECP 203)} \quad (3a)
\]

\[
R_{d(EUR)} = A_s \, f_y/k + 0.8 \, b \, t \, f_{ck}/\gamma_c \quad \text{(EN 1990)} \quad (3b)
\]

\[
R_{d(ACI)} = 0.65 \left( A_s \, f_y/k + 0.85 \, f_{ck} \, (b \, t - A_s) \right) \quad \text{(ACI 2011)} \quad (3c)
\]

Where: \( R_{d(ECP)} \), \( R_{d(EUR)} \) and \( R_{d(ACI)} \) column design axial resistance by Egyptian, Euro and ACI codes, respectively:

- \( A_s \): Reinforcement area
- \( f_y/k \): Characteristic reinforcement steel yield strength
- \( f_{ck} \): Characteristic concrete strength
- \( t \): Column length
- \( b \): Column width
- \( \gamma_s \): Partial factor for reinforcement steel
- \( \gamma_c \): Partial factor for concrete

The column dimensions \( t, b \) are considered here as a deterministic quantity equal to 0.3 m and the partial material factors for steel and concrete strength \( \gamma_s \) and \( \gamma_c \) are considered here to have the values 1.15 and 1.5, respectively. The reinforcement area \( A_s \) is considered as deterministic parameter. The characteristic values of the column length \( t \) and width \( b \) are taken as equal 0.3 m. It is further assumed that the column is designed on the basis of an ‘economic design’, that is, \( R_d = E_d \).

Random variable resistance

The basic random variables of the column resistance are the concrete strength \( f_c \), reinforcement yield strength \( f_y/k \), the section length \( t \) and width \( b \), and the coefficient of model uncertainty of resistance \( K_R \). The variable resistance \( R \) for the short reinforced concrete column assuming cross-sectional dimensions \( t \times b \) and negligible eccentricity can be expressed as given in Equation 4.

\[
R = K_R \left( A_s \cdot f_y/k + 0.80 \cdot b \cdot t \cdot f_{ck} \right) \quad (4)
\]

Reliability analysis

In the reliability analysis a structure is usually considered to be safe if the resistance \( R \) is greater than the load effect \( E \), both considered as random variables. Thus, when the limit state function (reliability margin) \( g(X) = R - E \) is greater than 0, the structure is safe; \( X \) being the vector of basic variables. In the case of a column the limit state function can be written as:

\[
g(X) = K_R \left( A_s \cdot f_y/k + 0.80 \cdot b \cdot t \cdot f_{ck} \right) - K_e \, (G + Q) \quad (5)
\]

Principles

Referring to Holicky and Retief (2000), Gulvanessian and Holicky (2005) and Holicky (2008), the probability of failure of the structure, \( P_f \), is the basic reliability measure used in this research. It can be expressed on the basis of a limit state performance function \( g(X) \) defined in such a way that a structure is considered to survive if \( g(X) > 0 \) and to fail if \( g(X) \leq 0 \). In a general case, the failure probability \( P_f \) can be determined using the integral.
Table 1. Probabilistic models of basic variables

| Type of variable | Variable                      | Symbol | Distribution | Units | Char. value | Mean value $\mu$ | Standard deviation $\sigma$ |
|------------------|-------------------------------|--------|--------------|-------|-------------|------------------|----------------------------|
| Actions          | Permanent action              | G      | Normal       | MN    | $G_k$       | $G_k$            | 0.1 $G_k$                  |
|                  | Imposed load (50 years)      | Q      | Gumbel       | MN    | $Q_k$       | 0.6 $Q_k$        | 0.21 $Q_k$                |
| Material properties | Concrete strength         | $f_c$  | Log Normal   | MPa   | 20          | 30               | 5                          |
|                  | Reinforcement yield strength | $f_y$  | Log Normal   | MPa   | 500         | 560              | 30                         |
| Geometric data   | Reinforcement area           | $A_s$  | deterministic | m$^2$ |             |                  |                            |
|                  | Column length                | $t$    | Normal       | m     | 0.30        | 0.30             | 0.01                       |
|                  | Column width                 | $b$    | Normal       | m     | 0.30        | 0.30             | 0.01                       |
| Model uncertainties | Load uncertainty          | $K_E$  | Log Normal   | -     | 1.0         | 1.0              | 0.05                       |
|                  | Resistance uncertainty      | $K_R$  | Log Normal   | -     | 1.0         | 1.1              | 0.165                      |

$P_f = \text{Prob}(g(X) \leq 0) = \int \phi_g(X) dX \quad (6)$

$\phi_g(X)$ denotes joint probability density distribution of the vector of basic variables $X$. Assume that both the resistance $R(X)$ and the load effect $E(X)$ represent a single variable $Z$ used to analyze structural performance (as axial force, bending moment or stress that is represented by $R(Z)$ and $E(Z)$). Then the integration indicated in expression (6) may be simplified and the probability $P_f$ can then be expressed as:

$P_f = \text{Prob}(g(Z) \leq 0) = \int \phi_E(Z) \Phi_R(Z) dZ \quad (7)$

Where $\Phi_R(Z)$ denotes the cumulative distribution function of the standardized normal distribution. The reliability index $\beta$ is frequently used, as its numerical values are more convenient to handle than values of failure probability $P_f$. EN 1990 recommends a target value of reliability index $\beta_t = 3.8$ that corresponds to the probability of failure $P_f = 7.24 \times 10^{-5}$ for the Ultimate limit states of buildings designed for a fifty-year period.

$P_f = \Phi (-\beta) \quad (8)$

Where $\Phi$ is the cumulative distribution function of the standardized normal distribution. The reliability index $\beta$ is frequently used, as its numerical values are more convenient to handle than values of failure probability $P_f$. EN 1990 recommends a target value of reliability index $\beta_t = 3.8$ that corresponds to the probability of failure $P_f = 7.24 \times 10^{-5}$ for the Ultimate limit states of buildings designed for a fifty-year period.

Method of analysis

Commerically available software COMREL V8.1 (RCP Munich, 1999) is used to compute the reliability index based on first order reliability method.

ANALYSIS OF RESULTS

Reliability of concrete column designed by Egyptian code

All basic variables are considered as random variables, described by a certain type of probability distribution and parameters. The probabilistic models of basic variables $X$ used in this study are summarized in Table 1. The models of basic variables are chosen, taking into account data provided by the JCSS (2001). Where appropriate, characteristic values are also selected to be in agreement with the Egyptian code.

Probabilistic models for basic variables

The reliability of concrete column designed by the Egyptian code is computed at different reinforcement ratios. The probabilistic distribution of the random variables and the deterministic values given in Table 1 are used. Figure 1 shows the relationship between the reliability index $\beta$ and the load ratio $r$ for different reinforcement ratios. It is concluded that the reliability index exceeds the target reliability index $\beta_t$ ($\beta_t = 3.8$ for design period of 50 years) at different values of load and
reinforcement ratios. The increase of reinforcement ratio from 1 to 2%, till 3% slightly increases the reliability index till the load ratio reaches 2.8 and 2.2, respectively, then it began to decrease as the load ratio increases. For the reinforcement ratio above 3% till 6%, the reliability index increases gradually till the load ratio reaches 2.2 then it began to decrease as the load ratio increase. Besides, the reliability index decreased gradually with the increase of reinforcement ratio above 3% till 6%, assuming the same mode of failure occurred.

Comparison between the Egyptian code and international codes

Based on Table 1, the reliability index is computed for concrete column designed by the Egyptian, Euro and ACI codes at different reinforcement ratios. Figures 2 to 7 show the relationships between the reliability index $\beta$ and the load ratio $r$ for different codes at reinforcement ratios of 1% to 6%, respectively. The Egyptian code shows the highest level of reliability and is close to the ACI code at
Figure 3. Reliability index-load ratio relationships designed by different codes at reinforcement ratio 2%.

Figure 4. Reliability index-load ratio relationships designed by different codes at reinforcement ratio 3%.

Figure 5. Reliability index-load ratio relationships designed by different codes at reinforcement ratio 4%.
high level of load ratios as the reinforcement ratio increases. The reliability index of column designed by Euro code is the lowest level of reliability than other codes and is close to ACI code at low level of load ratios as the reinforcement ratio decreased till 2%. For reinforcement ratio of 1%, Euro code is close to ACI code but the reliability index of column designed by Euro code is higher than ACI code only at low level of load ratios till it reaches 0.3, then it began to decrease gradually.

It is concluded that the reliability index for concrete column designed by the Egyptian, Euro and ACI codes exceeds the target reliability index $\beta_t$ ($\beta_t = 3.8$ for design period of 50 years) at different values of load and reinforcement ratios.

**Sensitivity of the reliability index**

The sensitivity of the reliability index of concrete column designed by Egyptian code for the following parameters is studied:

1. Concrete strength
2. Reinforcement yield strengths
3. Permanent action

**Concrete strength**

**Concrete grade**: The reliability index is computed for
concrete columns with different grades based on Table 1. The reliability is computed at concrete grades of characteristic values 15, 20, 25, 35 and 45 MPa. The mean value of concrete strength is taken as equal to $f_{ck}/(1-2C_v)$. Where $C_v$ varies according to JCSS. Figure 8 shows the relationship between the reliability index and the load ratio at different concrete grades. For concrete grade greater than 25 Mpa the reliability index is slightly increased till the load ratio reaches 0.21 and 0.23, respectively, then it began to decrease. After the load ratio reaches 0.3, the reliability level decreases gradually as load ratio and concrete grad increases for all grades. For grades lower than 25 Mpa, the reliability index is slightly decreased as load ratio and concrete grad increases. It is concluded that the reliability index for concrete column designed by the Egyptian code exceeds the target reliability index $\beta_t$ ($\beta_t =3.8$ for design period of 50 years) at different values of load and concrete grad.

**Coefficient of variation of concrete strength:** The reliability index is computed for concrete columns with the same grade ($f_{ck}=20$MPa) based on Table 1, except that the coefficient of variation of concrete strength is varied. Figure 9 shows the relationship between the reliability index and the load ratio at concrete coefficients of variation with range from 0.9 to 1.5 of the standard value.

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**Figure 8.** Reliability index-load ratio relationships at different concrete grades.

**Figure 9.** Reliability index-load ratio relationships at different concrete coefficients of variation.
0.167, according to Table 1. For concrete coefficients of variation less than the standard value 0.167, the reliability level is higher at low level of load ratios and gradually decrease to be closer to the high level of load ratios, implying that as the coefficients of variation is decreased (good quality control) the reliability level increases. For concrete coefficients of variation greater than 0.167 (low quality control), the reliability level decreases as the load ratios and coefficients of variation increases till the load ratio reaches 0.33. After reaching 0.33, the reliability level increases as the coefficients of variation increases, and generally it exceeds the target reliability index $\beta_t$.

**Mean value of concrete strength:** The reliability index is computed for concrete column with the same grade based on Table 1 except that the mean value of concrete strength is equal to the characteristic value. Significant reduction in the reliability index occurs as shown in Figure 10.

**Reinforcement yield strength**

**Reinforcement grade:** The reliability index is computed for concrete columns based on Table 1 except that the reliability is computed at reinforcement grades of characteristic values 500 and 264 MPa. The coefficient of variation $C_v$ is 0.0536 and 0.0565, respectively and the mean value of steel strength is taken as equal to $f_{ck}(1-2C_v)$. Figure 11 shows the relationship between the
reliability index and the load ratio at different steel grades. It is observed that the reliability level is much close for different steel grades.

**Coefficient of variation of yield steel strength:** The reliability index is computed for concrete columns with the same steel grade ($f_{yK} = 500$ MPa) based on Table 1, except that the coefficient of variation of steel strength is varied. Figure 11 shows the relationship between the reliability index and the load ratio at steel coefficients of variation of 0.0536 and 0.107. The reliability level is increased slightly when the value of $C_v$ is doubled.

**Permanent action**

The reliability index is computed for concrete columns based on Table 1 at different values of standard deviation of permanent action ($\sigma = \mu C_v$). Figure 12 shows the relationship between the reliability index and the load ratio at permanent action standard deviation of 0.1, 0.15, 0.20 and 0.25. At low load ratios, significant reduction occurs in the reliability level as the standard deviation of permanent load increases. As the load ratio increases the reliability index becomes very close for different permanent action standard deviation.

**Conclusions**

The following conclusions can be derived:

1. The reliability index for short columns designed by Egyptian code exceeds the target reliability index $\beta_t$ ($\beta_t = 3.8$ for design period of 50 years) at different values of load and reinforcement ratios.
2. The Egyptian code shows the highest level of reliability when compared with international codes and is so close to the ACI code at high reinforcement and load ratios considering no eccentricity.
3. The reliability index of short columns designed by EURO code is extremely lower than other codes but it exceeds the target reliability index $\beta_t$ ($\beta_t = 3.8$ for design period of 50 years) at different values of load and reinforcement ratios.
4. The reliability index decreases with the increase in reinforcement ratio.
5. Different steel grades do not give any significant difference in reliability level.
6. For concrete grade greater than 25 Mpa, the reliability level is slightly increased at low load ratios then it began to decrease as the load ratio increase. After the load ratio reaches 0.3 the reliability level decreases gradually as load ratio and concrete grade increase. For grades lower than 25 Mpa the reliability index is slightly decreased as load ratio and concrete grade increase. The reliability index for concrete column designed by the Egyptian code exceeds the target reliability index $\beta_t$ ($\beta_t = 3.8$ for design period of 50 years) at different values of load and concrete grades.
7. As the concrete coefficients of variation decreased (good quality control) the reliability level increase.
8. For concrete grade 20 Mpa with coefficients of variation greater than 0.167 (low quality control), the reliability level decrease as the load ratios and coefficients of variation increase till the load ratio reaches 0.33. After reaching 0.33, the reliability level increase as the coefficients of variation increase, and generally exceeds the target
reliability index $\beta_t$

9. The reliability level is increased slightly when the coefficient of variation of steel is doubled.

10. At low load ratios, significant reduction occurs in the reliability level as the standard deviation of permanent load increases. As the load ratio increase the reliability index became very close for different permanent action standard deviation.

11. Significant reduction in reliability index occurs when the mean value of concrete strength in field is close to the characteristic value.

CONFLICT OF INTERESTS

The author has not declared any conflict of interests.

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