CHAOTIC INFLATION IN F(R) SUPERGRAVITY

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Abstract

The bosonic $f(R)$ gravity function is derived from a chiral $F(R)$ supergravity model for the first time. We find the existence of the upper limit (or AdS bound) on the scalar curvature, as well as a solution with the vanishing cosmological constant. We compare our simple model of $F(R)$ supergravity to the well known Starobinsky model of chaotic inflation.

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1 Introduction

Revealing the identity of inflaton and unification of cosmological inflation with High-Energy Physics remain the outstanding problems beyond the Standard Model of elementary particles and Einstein gravity. One of the easy ways of realization of an inflationary universe is provided by the popular theories of $f(R)$ gravity, whose Lagrangian is a function $f(R)$ of the scalar curvature $R$ in four space-time dimensions (see eg., ref. [1] for some recent reviews). The use of those theories in inflationary cosmology was pioneered by Starobinsky [2].

Any $f(R)$ theory of gravity is known to be equivalent to a scalar-tensor theory of gravity [3]. In view of that equivalence, a dynamics of the spin-2 part of a metric (i.e. gravity itself) is not modified at all, but there is the extra propagating scalar field given by the conformal mode of the metric. The latter plays the role of inflaton in the inflationary models based on $f(R)$ gravity. Being unrelated to any fundamental theory of gravity, those inflationary models are truly phenomenological and have no connection to High-Energy Physics. Moreover, there is no mechanism inside the $f(R)$ gravity theories, that would protect a particular choice of the function $f(R)$ against quantum corrections that may destabilize inflation or exclude its slow roll.

In our recent papers [4] we constructed the new supergravity theory that can be considered as the $N = 1$ locally supersymmetric extension of the $f(R)$ gravity. 2 Supergravity is well-motivated in High-Energy Physics Theory beyond the Standard Model of elementary particles. Supergravity is also the low-energy effective action of Superstrings. 3 Unlike the $f(R)$ theories of gravity, the $F(R)$ supergravity is highly constrained by local supersymmetry and consistency. Moreover, our superspace construction of $F(R)$ supergravity [4] leads to a chiral action in curved $N = 1$ superspace, which may be naturally stable against quantum corrections that are usually given by full superspace integrals. Our supersymmetric extension of $f(R)$ gravity is non-trivial because the supergravity auxiliary fields do not propagate (this feature is called the auxiliary freedom [7]). However, the superconformal mode of the supergravity supervielbein becomes dynamical in $F(R)$ supergravity. As was proven in ref. [4], an $F(R)$ supergravity is equivalent to the standard $N = 1$ Poincaré supergravity coupled to the dynamical chiral superfield whose Kähler potential and superpotential are dictated by a single holomorphic function. That chiral superfield is precisely the superconformal mode of the supervielbein. It was argued in ref. [4] that the leading field component of the chiral superfield may be identified with the dilaton-axion field in Superstring Theory.

The component structure of $F(R)$ supergravity is very complicated, and some of its general features were outlined in ref. [4]. However, no explicit derivation

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2 Another (unimodular) $F(R)$ supergravity theory was proposed in ref. [5].

3 Some applications of $F(R)$ supergravity to Loop Quantum Gravity were given in ref. [6].
of a bosonic (real) function $f(R)$ out of the supergravity (holomorphic) function $F(R)$ was given. In this Letter we fill out this gap, by giving the first explicit example of such calculation.

In sec. 2 we briefly review our superspace construction of $F(R)$ supergravity along the lines of ref. [4], and formulate the equation for the auxiliary fields. In sec. 3 we propose the simplest non-trivial model of $F(R)$ supergravity and derive its corresponding bosonic function $f(R)$. An application of our supergravity model to chaotic inflation [8] in early universe is discussed in sec. 4. Our conclusion is sec. 5.

2 F(R) supergravity and its auxiliary fields

A concise and manifestly supersymmetric description of supergravity is given by superspace [9]. In this section we limit our presentation to a few basic equations. We use the units $c = \hbar = 1$ and $\kappa = M_{\text{Pl}}^{-1}$ in terms of the (reduced) Planck mass $M_{\text{Pl}}$, with the spacetime signature $(+, -, -, -)$.

The chiral superspace density (in the supersymmetric gauge-fixed form) is

$$E(x, \theta) = e(x) \left[ 1 - 2i\theta \sigma_\alpha \bar{\psi}_a(x) + \theta^2 B(x) \right],$$

(2.1)

where $e = \sqrt{-\det g_{\mu\nu}}$, $g_{\mu\nu}$ is a spacetime metric, $\psi_\alpha = e_\mu^a \psi^\mu_\alpha$ is a chiral gravitino, $B = S - iP$ is the complex scalar auxiliary field. We use the lower case middle greek letters $\mu, \nu, \ldots = 0, 1, 2, 3$ for curved spacetime vector indices, the lower case early latin letters $a, b, \ldots = 0, 1, 2, 3$ for flat (target) space vector indices, and the lower case early greek letters $\alpha, \beta, \ldots = 1, 2$ for chiral spinor indices.

The solution of the superspace Bianchi identities and the constraints defining the N=1 Poincaré-type minimal supergravity results in the three relevant superfields $\mathcal{R}, \mathcal{G}_a$ and $\mathcal{W}_{\alpha\beta\gamma}$ (as the parts of supertorsion), subject to the off-shell relations [9]

$$\mathcal{G}_a = \mathcal{G}_a, \quad \mathcal{W}_{\alpha\beta\gamma} = \mathcal{W}_{(\alpha\beta\gamma)}, \quad \nabla_\alpha \mathcal{R} = \nabla_\alpha \mathcal{W}_{\alpha\beta\gamma} = 0,$$

(2.2)

and

$$\nabla_\alpha^* \mathcal{G}_{\alpha\alpha} = \nabla_\alpha \mathcal{R}, \quad \nabla^\gamma \mathcal{W}_{\alpha\beta\gamma} = \frac{i}{2} \nabla_\alpha^* \mathcal{G}_{\beta\alpha} + \frac{i}{2} \nabla_\beta^* \mathcal{G}_{\alpha\alpha},$$

(2.3)

where $(\nabla_\alpha^*, \nabla_\alpha^*, \nabla_\alpha^*)$ represent the $N = 1$ supercovariant derivatives in curved superspace, and bars denote complex conjugation.

The covariantly chiral complex scalar superfield $\mathcal{R}$ has the scalar curvature $R$ as the coefficient at its $\theta^2$ term, the real vector superfield $\mathcal{G}_{\alpha\alpha}$ has the traceless Ricci tensor, $R_{\mu\nu} + R_{\nu\mu} - \frac{1}{2} g_{\mu\nu} R$, as the coefficient at its $\theta \sigma^\alpha \theta$ term, whereas the covariantly chiral, complex, totally symmetric, fermionic superfield $\mathcal{W}_{\alpha\beta\gamma}$ has the Weyl tensor $W_{\alpha\beta\gamma\delta}$ as the coefficient at its linear $\theta^3$-dependent term.

As regards a large-scale evolution of the FRLW Universe in terms of its scale factor, it is the scalar (super)curvature dependence of the gravitational action
that plays the most relevant role. The chiral $F(R)$ supergravity action, proposed in ref. [4], reads
\[ S_{\text{ch}} = \int d^4x d^2\theta \mathcal{E} F(R) + \text{H.c.} \tag{2.4} \]
in terms of a holomorphic function $F(R)$ of the scalar curvature superfield $\mathcal{R}$. Besides manifest local $N = 1$ supersymmetry, the action (2.4) also possess the auxiliary freedom [7], since the auxiliary field $B$ does not propagate. In addition, the action (2.4) gives rise to the spacetime torsion fueled by the gravitino field.

A bosonic $f(R)$ gravity action is given by
\[ S_f = \int d^4x \sqrt{-g} f(R) \tag{2.5} \]

In order to establish a connection between the master chiral superfield function $F(R)$ in eq. (2.4) and the corresponding bosonic function $f(R)$ in eq. (2.5), we use the chiral density integration formula in superspace ($\psi_\mu = 0$),
\[ \int d^4x d^2\theta \mathcal{E} \mathcal{L} = \int d^4x e \left\{ \mathcal{L}_{\text{last}} + B\mathcal{L}_{\text{first}} \right\} \tag{2.6} \]
where we have introduced the field components of the covariantly chiral superfield Lagrangian $\mathcal{L}(x, \theta)$, $\nabla^a\mathcal{L} = 0$, as follows (the vertical bars denote the leading component of a superfield):
\[ \mathcal{L} = \mathcal{L}_{\text{first}}(x), \quad \nabla^2\mathcal{L} = \mathcal{L}_{\text{last}}(x). \tag{2.7} \]

In particular, we have
\[ R| = \frac{\kappa}{3} \bar{B} = \frac{\kappa}{3} (S + iP), \quad \nabla^2 R| = \frac{1}{3} \left( R - \frac{i}{2} \varepsilon^{abcd} R_{abcd} \right) + \frac{4\kappa^2}{9} \bar{B} B, \tag{2.8} \]
The term $\frac{i}{2} \varepsilon^{abcd} R_{abcd}$ does not vanish in supergravity because of the gravitino-indiced torsion.

Applying the chiral density formula (2.6) to our eq. (2.4) yields the purely bosonic Lagrangian in the form
\[ L_{\text{bos}} = F'(X) \left[ \frac{\kappa}{3} R_* + 4XX \right] + 3XF(X) + \text{H.c.} \tag{2.9} \]
where the primes denote differentiation. We have also introduced the notation
\[ X = \frac{\kappa}{3} B \quad \text{and} \quad R_* = R - \frac{i}{2} \varepsilon^{abcd} R_{abcd}. \tag{2.10} \]

Varying eq. (2.9) with respect to the complex auxiliary fields $X$ and $\bar{X}$ gives rise to the algebraic equations on the auxiliary fields,
\[ 3F + X(4F' + 7F'') + 4XXF''' + \frac{i}{2} F'' R_* = 0 \tag{2.11} \]
and its conjugate
\[ 3\bar{F} + \bar{X}(4\bar{F}' + 7\bar{F}'') + 4\bar{X}\bar{F}''' + \frac{i}{2} \bar{F}'' \bar{R}_* = 0 \tag{2.12} \]
where $F = F(X)$ and $\bar{F} = F(\bar{X})$. The algebraic equations (2.11) and (2.12) cannot be explicitly solved for $X$ in a generic $F(\mathcal{R})$ supergravity.
3 Our model

Let’s consider the simplest non-trivial Ansatz for the $F(R)$ supergravity function as

$$F(R) = -\frac{1}{2} f_1 R + \frac{1}{2} f_2 R^2$$ \hspace{1cm} (3.13)

with some real constants $f_1$ and $f_2$, where the first term is supposed to represent the standard (pure) $N = 1$ Poincaré supergravity and the second term is a ‘quantum correction’. As regards mass dimensions of the various quantities introduced, we have

$$[F] = 3, \quad [f_1] = 2, \quad [R] = 2, \quad [f_2] = 1, \quad [\mathcal{R}] = 1$$ \hspace{1cm} (3.14)

Being interested in the bosonic action that follows from eqs. (2.4) and (3.13), we set gravitino to zero, $\psi_\mu = 0$, which also implies $R_* = R$ and a real $X$. Equation (2.9) is now greatly simplified to

$$L_{bos} = 11 f_2 X^3 - 7 f_1 X^2 + \frac{2}{3} f_2 RX - \frac{1}{3} f_1 R$$ \hspace{1cm} (3.15)

In the limit of $f_2 \to 0$ we thus have $X = 0$, as it should. Hence, we recover the Einstein-Hilbert Lagrangian

$$L_{EH} = -\frac{1}{3} f_1 R = -\frac{1}{2\kappa^2} R = -\frac{M_{Pl}^2}{2} R$$ \hspace{1cm} (3.16)

provided that

$$f_1 = \frac{3}{2} M_{Pl}^2$$ \hspace{1cm} (3.17)

For a later use, we trade the parameter $f_2$ for a mass parameter $m$ as

$$f_2 = \frac{M_{Pl}^2}{m}$$ \hspace{1cm} (3.18)

where $m$ is the new scale introduced in eq. (3.13) (in addition to $M_{Pl}$).

The algebraic field equation (2.11) in our case (3.13) takes the form of a quadratic equation,

$$11X^2 - 7mX + \frac{2}{9} R = 0$$ \hspace{1cm} (3.19)

whose solution is given by

$$X_\pm = \frac{7m}{22} \left[ 1 \pm \sqrt{1 - \frac{8 \cdot 11 R}{3^2 \cdot 7^2 m^2}} \right] = \frac{7m}{22} \left[ 1 \pm \sqrt{1 - \frac{R}{R_{\max}}} \right]$$

$$= \left( \frac{2R_{\max}}{99} \right)^{1/2} \left[ 1 \pm \sqrt{1 - \frac{R}{R_{\max}}} \right]$$ \hspace{1cm} (3.20)
where we have introduced the maximal scalar curvature

\[ R_{\text{max}} = \frac{99}{2} \left[ \frac{7m}{22} \right]^2 \]  

(3.21)

The surprising existence of the built-in maximal scalar curvature is a nice bonus of our construction. It comes for free, and it is very welcome for screening our theory of inflation from the Big Bang singularity of General Relativity, since eq. (3.20) implies \( R \leq R_{\text{max}} \). This striking property is similar to the factor \( \sqrt{1 - v^2/c^2} \) of Special Relativity. Yet another close analogy comes from the Born-Infeld nonlinear extension of Maxwell electrodynamics, whose (dual) Hamiltonian is proportional to \( \frac{1}{3} - \frac{\sqrt{1 - \vec{E}^2/E_{\text{max}}^2 - \vec{H}^2/H_{\text{max}}^2}}{c} + \frac{(\vec{E} \times \vec{H})^2}{E_{\text{max}}^2 H_{\text{max}}^2} \) in terms of the electric and magnetic fields \( \vec{E} \) and \( \vec{H} \), respectively, with their maximal values (see eg., ref. [10]) for details). For instance, in string theory, one has \( E_{\text{max}} = H_{\text{max}} = \frac{(2\pi\alpha')^{-1}}{m} \).

As is clear from eq. (3.20), the upper bound on the scalar curvature exists only for \( R > 0 \) and, in particular, for the AdS-spacetimes (in our notation). Equation (3.20) does not imply an upper limit on \( |R| \) for \( R < 0 \), in particular, for the dS-spacetimes.

Equation (3.19) can be used to reduce the Lagrangian (3.15) to a linear function of \( X \) by double iteration. Then a substitution of the solution (3.20) into the Lagrangian gives us a bosonic \( f(R) \) gravity Lagrangian (2.5) in the form

\[ f_{\pm}(R) = \frac{-5 \cdot 17 M_{\text{Pl}}^2}{2 \cdot 3^2 \cdot 11} R + \frac{2 \cdot 7}{3^2 \cdot 11} M_{\text{Pl}}^2 (R - R_{\text{max}}) \left[ 1 \pm \sqrt{1 - R/R_{\text{max}}} \right] \]  

(3.22)

By construction, in the limit \( m \to +\infty \) (or \( R_{\text{max}} \to +\infty \)) both functions \( f_{\pm} \) reproduce General Relativity. In another limit \( R \to 0 \), we find a cosmological constant,

\[ f_{-}(0) \equiv \Lambda_{-} = 0 \quad , \quad f_{+}(0) \equiv \Lambda_{+} = -\frac{7^3}{2^2 \cdot 11^2} M_{\text{Pl}}^2 m^2 = -\frac{14}{99} M_{\text{Pl}}^2 R_{\text{max}} \]  

(3.23)

To the end of this Letter we would like to concentrate on the first solution with the vanishing cosmological constant, so in what follows we identify \( f_{-}(R) = f(R) \).

4 \quad (R + R^2) supergravity model vs. \((R + R^2)\) model of inflation

Any \( f(R) \) gravity (2.5) is known to be equivalent to the scalar-tensor gravity

\[ S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left\{ -\frac{R}{2\kappa^2} + \frac{1}{2} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi - V(\phi) \right\} \]  

(4.24)
where we have introduced the scalar (inflaton) field \( \phi(x) \) with its scalar potential \( V(\phi) \). The equivalence is established via a Legendre-Weyl transform [3]. In our notation we have \( 4 \)

\[
f(R) = R e^y - Z(e^y), \quad R = Z'(e^y), \quad f'(R) = e^y, \quad y = \sqrt{\frac{2}{3} \frac{\phi}{M_{Pl}}} \quad (4.25)
\]

so that the inflaton scalar potential is given by \( 11 \)

\[
V(y) = -\frac{1}{2} M_{Pl}^2 e^{-2y} Z(e^y) \quad (4.26)
\]

When keeping only the leading correction (beyond Einstein-Hilbert term) in eq. (3.22), we get the low-curvature Lagrangian \(|R/R_{max}| \ll 1 \) in the form

\[
f(R) = -\frac{1}{2} M_{Pl}^2 R + \alpha R^2 \equiv -\frac{1}{2} M_{Pl}^2 (R - R^2/M^2) \quad (4.27)
\]

where

\[
\alpha = \frac{1}{2} \frac{M_{Pl}^2}{M^2} = \frac{2}{3^3 \cdot 7} \frac{M_{Pl}^2}{m^2} \quad (4.28)
\]

It is known as the Starobinsky model of chaotic inflation [2]. The corresponding (inflaton) scalar potential (4.26) is well-defined and is given by \( 11 \)

\[
V(y) = V_0 \left( e^{-y} - 1 \right)^2 \quad (4.29)
\]

where \( V_0 = \frac{1}{8} M_{Pl}^2 M^2 \). The constant term in eq. (4.29) is the vacuum energy that drives inflaton towards the minimum of the scalar potential (so that the inflation has an end). The conditions for a slow-roll (chaotic) inflation in the Starobinsky model were studied a long time ago [2]. In terms of the equivalent scalar-tensor gravity (4.24) with the scalar potential (4.29) we find the standard slow-roll parameters \( 12 \) as follows \( 11 \):

\[
\varepsilon = \frac{1}{2} M_{Pl}^2 \left( \frac{V'}{V} \right)^2 = \frac{4 e^{-2y}}{3 \left( e^{-y} - 1 \right)^2} = \frac{3}{4N_e^2} + \mathcal{O} \left( \frac{\ln^2 N_e}{N_e^3} \right) \quad (4.30)
\]

and

\[
\eta = M_{Pl}^2 \frac{V''}{V} = \frac{4 e^{-y} (2e^{-y} - 1)}{3 \left( e^{-y} - 1 \right)^2} = -\frac{1}{N_e} + \frac{3 \ln N_e}{4N_e^2} + \frac{5}{4N_e^2} + \mathcal{O} \left( \frac{\ln^2 N_e}{N_e^3} \right) \quad (4.31)
\]

where the primes denote the derivatives with respect to the inflaton field \( \phi \), and the e-foldings number \( N_e \) is defined by \( 12 \)

\[
N_e = \int_t^{\phi_{end}} H dt \approx \frac{1}{M_{Pl}^2} \int_{\phi_{end}}^{\phi} \frac{V}{V'} d\phi \approx \frac{3}{4} (e^y - y) - 1.04 \quad (4.32)
\]

\( ^4 \)See ref. [11] for more details. Compared to ref. [11], we changed here our notation \( y \rightarrow -y \).
According to the CMB observations, the primordial spectrum in the power-law approximation takes the form of $k^{n_s-1}$ in terms of the comoving wave number $k$ and the spectral index $n_s$. For instance, the recent WMAP5 data [13] yields

$$n_s = 0.960 \pm 0.013 \quad \text{and} \quad r < 0.22 \quad (4.33)$$

where $r$ is the scalar-to-tensor ratio. On the theoretical side, one has [12]

$$n_s = 1 + 2\eta - 6\varepsilon \quad \text{and} \quad r = 16\varepsilon \quad (4.34)$$

In our case, eqs. (4.30), (4.31) and (4.34) imply [11]

$$n_s = 1 - \frac{2}{N_e} + \frac{3\ln N_e}{2N_e^2} - \frac{2}{N_e^2} + \mathcal{O}\left(\frac{\ln^2 N_e}{N_e^3}\right) \quad (4.35)$$

and

$$r = \frac{12}{N_e^2} + \mathcal{O}\left(\frac{\ln^2 N_e}{N_e^3}\right) \quad (4.36)$$

whose leading terms agree with the earlier estimates [14]. It also agrees with the WMAP5 observations (4.33) provided that $N_e$ lies between 36 and 71, with the average value $\bar{N}_e = 54$.

The amplitude of the initial perturbations, $\Delta^2_R = M_{Pl}^4 V/(24\pi^2\varepsilon)$, is another physical observable, whose experimental value is given by [12]

$$\left(\frac{V}{\varepsilon}\right)^{1/4} = 0.027 M_{Pl} \quad (4.37)$$

Then eq. (4.37) determines the normalization of the $R^2$-term in eq. (2.4), in agreement with earlier calculations (see eg., ref. [15]),

$$\frac{M}{M_{Pl}} = (3.5 \pm 1.2) \cdot 10^{-5} \quad (4.38)$$

where we have used $N_e = \bar{N}_e = 54$. In the case (4.28) we find

$$m = \frac{2}{3\sqrt{21}} M \approx 0.15 M \approx 5 \cdot 10^{-6} M_{Pl} \quad \text{and} \quad R_{\text{max}} \approx 10^{-10} M_{Pl}^2 \quad (4.39)$$

Unfortunately, it also implies that we cannot embed the Starobinsky ($R + R^2$)-type inflationary model into our ($\mathcal{R} + R^2$) supergravity, because the higher-order curvature terms cannot be ignored in eq. (3.22), i.e. the $R^n$-terms with $n \geq 3$ are not small against the $R^2$-terms, and $|R/R_{\text{max}}| \sim \mathcal{O}(1)$ during inflation. For example, in the expansion

$$f_-(R) = -\frac{1}{2} M_{Pl}^2 \left( R - \frac{R^2}{M^2} - \frac{R^3}{7M^4} \right) + \mathcal{O}(R^4) \quad (4.40)$$
the $R^2$-term is already not negligible.

The exact gravitational function $f_-(R)$ in eq. (3.22) also leads to a well-defined (single-valued, non-singular, bounded from below) inflaton scalar potential,

$$V(y) = V_0 (11e^y + 3)(e^{-y} - 1)^2$$

where $V_0 = (3^3/2^6)M_{Pl}^2$. The associated slow-roll inflation parameters are given by

$$\varepsilon(y) = \frac{1}{3} \left[ \frac{e^y (11 + 11e^{-y} + 6e^{-2y})}{(11e^y + 3)(e^{-y} - 1)} \right]^2 \geq \frac{1}{3}$$

and

$$\eta(y) = \frac{2}{3} \left[ \frac{11e^y + 5e^{-y} + 12e^{-2y}}{(11e^y + 3)(e^{-y} - 1)^2} \right] \geq \frac{2}{3}$$

which are not small enough for matching the observational data (WMAP). Unlike the potential (4.29), the potential (4.41) is too steep to support a slow-roll inflation. A possibility of destabilizing the Starobinsky cosmological scenario (based on adding the $R^2$-term to the Einstein-Gilbert term) against the terms of the higher order with respect to the scalar curvature was observed earlier in ref. [16].

5 Conclusion

Our main new result is given by eq. (3.22). The $f(R)$ gravity with that function can be locally $N = 1$ supersymmetrized to the $F(R)$ supergravity described by eq. (3.13). That $F(R)$ supergravity model has the upper bound on the scalar curvature — see eq. (3.21). The possible existence of such bound in supergravity was conjectured in ref. [17].

Unfortunately, the function (3.22) is not suitable for a slow-roll inflation. It is worth noticing that it does not mean the failure of the whole approach ($F(R)$ supergravity), because the function (3.13) was chosen ad hoc, due to its simplicity only. It is conceivable that there exist many other functions $F(R)$ leading to slow-roll inflation in agreement with the observations.

Revealing the (quantum) origin of the higher-order scalar supercurvature terms in the supergravity function $F(R)$ is beyond the scope of this paper. For instance, they may come through the radiative corrections responsible for the anomalies of some classical symmetries (like Kähler symmetry) in the matter-coupled supergravities [18], or they may come from Superstrings [4].

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