INTEGRATED NAVIGATION SYSTEM OF SINGLE-AXLE WHEELED ROBOT WITH TWO DEGREES OF FREEDOM ROTATING PLATFORM

Aleshin B.S., Kuris E.D., Lelkov K.S., Miheev V.V., Chernomorsky A.I.
Department of Automated Complexes of Orientation and Navigation Systems
Moscow Aviation Institute (National Research University)
125993, 4 Volokolamskoe Highway, Moscow, Russian Federation

ekurs@mail.ru

Abstract. A complex navigation system for a uniaxial wheeled robot with a two degrees of freedom rotating platform, which is a carrier of environmental monitoring equipment, is proposed. The navigation system includes an odometric subsystem, a block of inertial sensors, laser altimeters, and a receiver of the global navigation satellite system. Integration is carried out based on the use of the Kalman filter, taking into account the possibility of deformation of the wheels and their slippage when the robot moves relative to the non-horizontal underlying surface. The results of semi-natural modeling confirmed the effectiveness of the decisions made.

1. Introduction

One of the promising carriers of environmental monitoring equipment are ground uniaxial wheeled robots with a two degrees of freedom rotating platform (UWRRP), on which this equipment is installed [1-5]. Such a robot is actually a ground wheeled gyroscopic stabilizer and provides the ability to transport equipment along a given route, and control of the angular orientation of this equipment relative to the plane of the horizon and in azimuth. The purpose of this work is to present the results of the development of an integrated navigation system (INS) of the UWRRP, taking into account the possibility of deformation of the wheels and their slippage when the robot moves along a non-horizontal underlying surface.

2. Physical model of the UWRRP with INS

The UWRRP scheme is shown in Fig. 1.
Figure 1 – UWRRP scheme.

UWRRP includes a platform, hinged in the frame, on which two wheels with electric motors are installed. Stabilization and control of the frame around the axle of the wheels $y_p$ is carried out with the help of the power stabilizing flywheel 1 during its rotations $\psi_1$ [6-9]; it can simultaneously move along the $x_p$ frame axis to create additional stabilizing moment. The UWRRP platform is stabilized and controlled around the $x_n$ axis using the flywheel 2 during its rotations $\psi_2$; it can also move simultaneously along the $y_n$ axis of the platform. A flywheel 3 is located on the platform, which creates additional moments around the $x_n$, $y_n$ axes of the platform, which compensate for the moments of inertia forces applied at the center of mass of the platform $O_0$ and generated by linear accelerations of the UWRRP [10]. Encoders are installed on the wheel axles; there are three laser altimeters in the corners of the platform that measure the distance to the underlying surface. A number of units are installed on the platform, including a unit of inertial meters and a global navigation satellite system (GNSS) receiver.

In fig. 1 the following coordinate systems (CS) are also indicated: OXYZ - starting CS (OX, OY lie in the horizontal plane); $O_1x_1y_1z_1$ - accompanying CS (its axes are parallel to the axes OXYZ, $O_1$ is the center of the wheelset axle); $O_2x_2y_2z_2$ - trajectory horizontal CS, rotated relative to $O_1x_1y_1z_1$ by $\theta_1$ - heading angle in the horizontal plane ($V_e$ - horizontal component of velocity directed along the $O_1x_1$ axis); $O_3x_3y_3z_3$ - speed CS (axes $O_3x_3$, $O_3y_3$, are located in the plane of the underlying surface, $V_n$ is the speed of movement of the UWRRP, directed along the axis $O_3x_3$); $O_4x_4y_4z_4$ - frame CS; $O_5x_5y_5z_5$ - platform CS. In fig. 1 also indicated: $\alpha_r, \beta_r$ - angles of deviation of the platform from the horizon plane, respectively, around the $y_1$ and $x_r$ axes; $\alpha, \beta$ are the angles of deviation of the platform from the underlying surface, respectively, around the $y_p$ and $x_p$ axes (at $\alpha_r = \beta_r = 0$, the angles $\alpha$ and $\beta$ are the longitudinal and transverse slopes of the underlying surface relative to the horizon plane, respectively); $\gamma_1, \gamma_2$ - angular speeds of rotation of the first and second wheels; $\dot{\theta}_r$ is the angular velocity of the directional turn of the UWRRP in the plane of the underlying surface. The parameters
are also presented here: \( r \) - wheel radius; \( 2b \) - track of a wheelset; \( l_s \) - offset of the GNSS antenna relative to the center \( O_s \).

The fig. 2 shows the structure of the developed INS for UWRRP.

The task of the INS is to determine the coordinates \((x_n, y_n, z_n)\) of the location of the UWRRP in the starting CS \(OXYZ\) in open spaces, in particular in the area of the aerodrome infrastructure. In addition, the INS determines the velocities \(V_r, \dot{\theta}_r\), the heading angle \(\theta_r\) and the angles \(\alpha, \beta\), which are necessary to control the trajectory motion and angular orientation of the UWRRP platform. The main elements of the INS are: odometric subsystem with encoders; GNSS receiver; laser altimeters; a block of inertial meters measuring, in particular, the angles \(\theta\), as well as the angular velocity sensor (AVS) parameter \(\dot{\theta}_{AVS}\) - the speed of the UWRRP in the horizontal plane. In accordance with the structure in Fig. 2, the following information is sent to the calculator of the movement parameters of the UWRRP in the starting CS: about the speeds \(V_d, \dot{\theta}_d\) (the index “d” denotes the measurements of the odometric system); about angles \(\alpha, \beta\) - slopes of the underlying surface; about \(\theta_{AVS}\); \(\delta \omega_{\theta}\) is the estimate of the error in determining the zero shift of the AVS, obtained in the Kalman filter. Note that the slopes \(\alpha, \beta\) are determined based on the following relations:

\[
\alpha = \arctg \frac{l_{n2} - l_{n1}}{l_n}; \quad \beta = \arctg \frac{l_{n3} - l_{n1}}{l_n}, \tag{1}
\]

where \(l_n\) – distance between 1 and 2, 1 and 3 altimeters; \(l_{n1}, l_{n2}, l_{n3}\) – distances to the surface, measured respectively by the first, second and third altimeters.
In the calculator of motion parameters, the parameters \( x_m, y_m, z_m, \theta_t, V_d, \dot{\theta}_d \), entering the Kalman filter, are determined (the index "г" means belonging to the horizontal plane). The calculator also determines the difference in the signal about the averaged value of the velocity \( \hat{\theta}_{\text{гвс}} \), taking into account the zero shift of the gyroscope \( \omega_z \) and the signal from the odometric subsystem. The Kalman filter additionally receives information from the GNSS about the current location of the receiver of signals \( x_p, y_p, z_p \) and its velocities \( \dot{x}_p, \dot{y}_p, \dot{z}_p \) in the global coordinate system. The error estimates obtained in the filter \( \delta x_m, \delta y_m, \delta z_m, \delta \theta_t, \) as well as the error estimates of the wheel circle lengths \( \delta l_1, \delta l_2 \) are fed further to the calculator of the adjusted navigation parameters. These parameters are used in the trajectory motion and angular orientation control system of the UWRRP platform. At the output of the system - the control torques \( M_{\text{гв1}}, M_{\text{гв2}} \), developed by the wheel motors.

3. UWRRP INS algorithms

Let us consider the algorithm of the INS UWRRP with a horizontal platform, which moves in three-dimensional space along a non-horizontal uneven surface. Let us assume that the irregularities are of a spatial long-period character, and at each moment of time, the movement of the UWRRP can be interpreted as movement along an inclined plane with relatively small angles of inclination \( \varphi \) and \( \kappa \).

We will assume that when the UWRRP moves along the trajectory, slippage of its wheels relative to the surface at the points of their contact with it is possible. In this case, the slippage factor will be taken into account in the algorithm of the odometric subsystem by adequately mapping the slippage phenomenon into the component of the error estimates for the wheel circumference. The components of these estimates are also the variations in lengths generated by wheel deformations during the movement of the UWRRP and errors in the preliminary measurement of these lengths.

In accordance with the structure in Fig. 2, the integration will be carried out based on the use of the Kalman filter. Let us first consider the relations that determine the measurements of the odometric subsystem and the block of inertial meters. Based on the measurements of \( \dot{y}_1, \dot{y}_2 \) encoders of the odometric subsystem, the linear \( V_d \) and angular \( \dot{\theta}_d \) velocities of the OCRDP in the inclined plane are calculated:

\[
V_d = \frac{\dot{y}_1 l_1 + \dot{y}_2 l_2}{4\pi} \quad \dot{\theta}_d = \frac{1}{4\pi b} (\dot{y}_2 l_2 - \dot{y}_1 l_1),
\]

where \( l_1 = 2\pi r_1, l_2 = 2\pi r_2 \) are the circumferences of the first and second wheels, respectively.

The expressions for the errors of \( \delta V_d, \delta \dot{\theta}_d \) in the determination of the components \( V_d, \dot{\theta}_d \) are found by varying the relations (2):

\[
\delta V_d = \frac{\dot{y}_1 \delta l_1 + \dot{y}_2 \delta l_2}{4\pi} \quad \delta \dot{\theta}_d = \frac{1}{4\pi b} (\dot{y}_2 \delta l_2 - \dot{y}_1 \delta l_1),
\]

where \( \delta l_1, \delta l_2 \) – errors in determining the lengths of the circumferences of the wheels.

When the platform is horizontal, we will form the expressions for \( V_d \) and \( \dot{\theta}_d \) based on information from laser altimeters, taking into account the longitudinal \( \alpha \) and transverse \( \beta \) slopes of the underlying surface:

\[
V_{rd} = V_d \cos \alpha ; \dot{\theta}_{rd} = \dot{\theta}_d \cos \alpha \cos \beta.
\]

Approximately the zero shift of the heading gyroscope \( \omega_z \), assuming that it constitutes the main component of the difference \( \hat{\theta}_{\text{гвс}} - \theta_{\text{гвд}} \), can be determined as follows:

\[
\omega_z = \hat{\theta}_{\text{гвс}} - \theta_{\text{гвд}},
\]

Then the ratio for the increment of the heading angle \( \Delta \theta_t \), during the time \( \Delta t \), taking into account the compensation of the zero shift of the gyroscope \( \omega_z \), can be written as follows:

\[
\Delta \theta_t = (\hat{\theta}_{\text{гвс}} - \omega_z) \Delta t,
\]

where \( \Delta t \) – the time interval between two successive measurements of the gyroscope; \( \hat{\theta}_{\text{гвс}} \) – is the average value of the angular velocity \( \dot{\theta}_{\text{гвс}} \) for the time \( \Delta t \).

The error in determining the zero shift \( \delta \omega_z \) based on (5) and (6) is as follows:

\[
\delta \omega_z = -\delta \dot{\theta}_d \cos \alpha \cos \beta = -\frac{1}{4\pi b} (\dot{y}_2 \delta l_2 - \dot{y}_1 \delta l_1) \cos \alpha \cos \beta.
\]
The error in determining the course angle of the UWRRP is presented in the form:

$$\delta \theta_r = (\delta n_{r_{up}} + \delta \omega_z) \Delta t,$$

where $\delta n_{r_{up}}$ – noise component of AVS measurements.

Expressions for the lengths of the circles of the first and second wheels, taking into account the errors $\delta l_1, \delta l_2$, are as follows:

$$l_1 = l_{10} - \delta l_1; \quad l_2 = l_{20} - \delta l_2,$$

where $l_{10}, l_{20}$ – pre-measured wheel circumferences.

The current coordinates of the UWRRP location $x_n, y_n, z_n$ are calculated by integrating the linear velocity $V_{rd}$ (4), taking into account the current orientation $\theta_r$, determined based on (6) and relation (1) for the angle of inclination of the underlying surface $\alpha$. For the $m$-th measurement of the AVS and the odometric subsystem, we have:

$$\begin{align*}
(x_n)_m &= (x_n)_{(m-1)_{K}} + (V_{rd} \cos \theta_r)_{m} \Delta t; \\
(y_n)_m &= (y_n)_{(m-1)_{K}} + (V_{rd} \sin \theta_r)_{m} \Delta t; \\
(z_n)_m &= (z_n)_{(m-1)_{K}} + (V_d \sin \alpha)_{m} \Delta t,
\end{align*}$$

where $(x_n)_{(m-1)_{K}}, (y_n)_{(m-1)_{K}}, (z_n)_{(m-1)_{K}}$ – the values of the coordinates of the UWRRP at the $m-1$ step taking into account their correction at the previous step.

The errors in determining the coordinates $\delta x_n, \delta y_n, \delta z_n$, obtained on the basis of variation (10), have the form:

$$\begin{align*}
\delta x_n &= (\delta V_{rd} \cos \theta_r - V_{rd} \delta \theta_r \sin \theta_r) \Delta t; \\
\delta y_n &= (\delta V_{rd} \sin \theta_r + V_{rd} \delta \theta_r \cos \theta_r) \Delta t; \\
\delta z_n &= (\delta V_d \sin \alpha) \Delta t.
\end{align*}$$

The expressions for the corrected values of the coordinates of the UWRRP at the $m$-th step, taking into account the errors $\delta \omega_x$ (7) and $\delta \theta_r$ (8) have the form:

$$(x_n)_{mk} = (x_n)_m - \delta x_n; \quad (y_n)_{mk} = (y_n)_m - \delta y_n; \quad (z_n)_{mk} = (z_n)_m - \delta z_n.$$  

Expressions for the corrected values of the gyroscope zero shift and the heading angle of the UWRRP taking into account the errors (11) have the form:

$$(\omega_x)_{mk} = (\omega_x)_m - \delta \omega_x; \quad (\theta_r)_{mk} = (\theta_r)_m - \delta \theta_r.$$  

GNSS provides obtaining the $m$-th measurement step of the current coordinates $x_s, y_s, z_s$ and the velocities $\dot{x}_s, \dot{y}_s, \dot{z}_s$ of the GNSS receiver installed on the UWRRP (Fig. 1):

$$\begin{align*}
x_s &= (x_n)_{mk} + l_{sx} \cos \theta_r - l_{sy} \sin \theta_r; \\
y_s &= (y_n)_{mk} + l_{sx} \cos \theta_r + l_{sy} \sin \theta_r; \\
z_s &= (z_n)_{mk} + l_{sz},
\end{align*}$$

$$\begin{align*}
\dot{x}_s &= (V_{rd} - \dot{\theta}_r l_{sx}) \cos \theta_r - l_{sy} \sin \theta_r; \\
\dot{y}_s &= (V_{rd} + \dot{\theta}_r l_{sx}) \sin \theta_r + l_{sy} \cos \theta_r; \quad \dot{z}_s = V_d \sin \alpha,
\end{align*}$$

where $l_{sx}, l_{sy}, l_{sz}$ – displacement of the GNSS antenna relative to the center $O_r$ along the axes of the trajectory CS $O_r, x_r, y_r, z_r$.

Then the variations in the differences between the measurements of the GNSS and the odometric subsystem take the form:

$$\begin{align*}
\delta(x_s - (x_n)_{mk}) &= -\theta(\delta l_{sx} \sin \theta_r - \delta l_{sy} \cos \theta_r); \\
\delta(y_s - (y_n)_{mk}) &= \theta(\delta l_{sx} \cos \theta_r + \delta l_{sy} \sin \theta_r); \\
\delta(z_s - (z_n)_{mk}) &= 0.
\end{align*}$$

$$\begin{align*}
\delta(\dot{x}_s - \dot{x}_n) &= \delta V_{rd} \cos \theta_r - \delta \theta_r V_{rd} \sin \theta_r - \theta(\delta l_{sx} \sin \theta_r + \delta l_{sy} \cos \theta_r); \\
\delta(\dot{y}_s - \dot{y}_n) &= \delta V_{rd} \sin \theta_r + \delta \theta_r V_{rd} \cos \theta_r + \theta(\delta l_{sx} \cos \theta_r - \delta l_{sy} \sin \theta_r); \\
\delta(\dot{z}_s - \dot{z}_n) &= \delta V_d \sin \alpha,
\end{align*}$$

where $\dot{\theta}_r$ – angular velocity error $\dot{\theta}_r = \delta n_{r_{up}} + \delta \omega_z$. 

Relations (2), (4), (6), (11) are the basis for constructing equations of the dynamics of the system in the Kalman filter, and relations (13), (15), (16) for constructing measurement equations in it. Let us use the general form of the Kalman filter equation [11]:

\[
\dot{X} = F \times X + Q;
\]

\[
Z = H \times X + v,
\]  \hspace{1cm} (17)

where \(X\) is the system state vector; \(F\) - matrix of system dynamics; \(Q, v\) - vectors of white noise of the system and measurements with zero mean values; \(Z\) - measurement vector; \(H\) - measurement matrix.

With regard to the construction of a filter for determining the estimates of the parameter errors used in the algorithm of the UWRPP navigation system, the state vector looks as follows:

\[
X = [\delta x_n, \delta y_n, \delta z_n, \delta \theta_f, \delta l_1, \delta l_2, \delta \omega_z]^T.
\]  \hspace{1cm} (18)

The system dynamics matrix \(F\) can be represented as follows:

\[
F = \begin{bmatrix}
0 & 0 & 0 & -V_\text{rd} \sin \theta & \frac{\hat{y}_1 \cos \theta \cos \alpha}{4\pi} & \frac{\hat{y}_2 \cos \theta \cos \alpha}{4\pi} & 0 \\
0 & 0 & 0 & V_\text{rd} \cos \theta & \frac{\hat{y}_1 \sin \theta \cos \alpha}{4\pi} & \frac{\hat{y}_2 \sin \theta \cos \alpha}{4\pi} & 0 \\
0 & 0 & 0 & 0 & \frac{\hat{y}_1 \sin \alpha}{4\pi} & \frac{\hat{y}_2 \sin \alpha}{4\pi} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{dt} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{dt} & 0 \\
0 & 0 & 0 & 0 & \frac{\hat{y}_1}{4\pi n b d t} & -\frac{\hat{y}_2}{4\pi n b d t} & 0
\end{bmatrix}.
\]  \hspace{1cm} (19)

The measurement vector has the form:

\[
Z = \begin{bmatrix}
\delta(x_s - x_n) \\
\delta(y_s - y_n) \\
\delta(z_s - z_n) \\
\delta(\hat{x}_n - \hat{x}_n) \\
\delta(\hat{y}_n - \hat{y}_n) \\
\delta(\hat{z}_n - \hat{z}_n) \\
(\hat{\theta}_f - \omega_z) - \theta_\text{rd}
\end{bmatrix}.
\]  \hspace{1cm} (20)

The measurement matrix \(H\) is as follows:

\[
H = \begin{bmatrix}
1 & 0 & 0 & -(l_{sx} \sin \theta_t + l_{sy} \cos \theta_t) & 0 & 0 & 0 \\
0 & 1 & 0 & l_{sx} \cos \theta_t - l_{sy} \sin \theta_t & 0 & 0 & 0 \\
0 & 0 & 1 & \frac{\hat{y}_1 \cos \theta \cos \alpha}{4\pi} & \frac{\hat{y}_2 \cos \theta \cos \alpha}{4\pi} & l_{sx} \cos \theta_t - l_{sy} \sin \theta_t, \\
0 & 0 & 0 & -(V_\text{rd} + \hat{\theta}_t l_{sx}) \sin \theta_t - \hat{\theta}_t l_{sy} \cos \theta_t & \frac{\hat{y}_1 \sin \theta \cos \alpha}{4\pi} & \frac{\hat{y}_2 \sin \theta \cos \alpha}{4\pi} & \frac{\hat{y}_1 \sin \alpha}{4\pi} & \frac{\hat{y}_2 \sin \alpha}{4\pi} & 0 \\
0 & 0 & 0 & V_\text{rd} + \hat{\theta}_t l_{sx} \cos \theta_t - \hat{\theta}_t l_{sy} \sin \theta_t & \frac{\hat{y}_1 \sin \theta \cos \alpha}{4\pi} & \frac{\hat{y}_2 \sin \theta \cos \alpha}{4\pi} & \frac{\hat{y}_1 \sin \alpha}{4\pi} & \frac{\hat{y}_2 \sin \alpha}{4\pi} & 0 \\
0 & 0 & 0 & 0 & \frac{\hat{y}_1 \sin \alpha}{4\pi} & \frac{\hat{y}_2 \sin \alpha}{4\pi} & 0 & 0 & 1
\end{bmatrix}.
\]  \hspace{1cm} (21)

The optimal estimates of the components of the state vector \(\bar{\delta}x_{\mu}, \bar{\delta}y_{\mu}, \bar{\delta}z_{\mu}\) obtained in the Kalman filter based on (12) are used to correct the coordinates of the location of the center of the axle of the wheelset of the UWRPP. The estimate of the component \(\bar{\delta}\theta_f\) according to (13), is used to correct the current heading orientation of the UWRPP, the estimate of the component \(\bar{\delta}\omega_z\) according to (6), (13) is used to form the measurement vector (20), the estimate of the components \(\bar{\delta}l_1, \bar{\delta}l_2\) according to (9) - for the correction of the lengths of the circles of the wheels.

4. UWRPP INS simulation
The task of the semi-natural modeling was to test the performance of the hardware solutions of the developed INS and to obtain numerical estimates of the accuracy of the algorithm of its operation when the UWRRP moves along a given trajectory on a non-horizontal surface. A typical trajectory of movement, set on the ground, is indicated in Fig. 3 with a dotted line in the starting CS OXYZ. A model of the developed navigation system was installed on the UWRRP and in the process of the UWRRP passage along a given trajectory, this system determined the navigation parameters of the robot.

The simulation time was 490 seconds. Preliminarily measured circumferences of the first and second wheels - 0.628 m. The passport estimate of the magnitude of the zero signal drift of the micromechanical AVS was \( \omega_x = 1.5 \, \text{deg/h} \), the standard deviation of the error in measuring the angular velocity by the gyroscope was \( 6 \cdot 10^{-3} \, \text{rad/s} \), the frequency AVS measurements - 200 Hz. The magnitude of the systematic error in GNSS measurements in the vertical channel is 0.3 m, the standard deviation of the errors in the measurements of the GNSS of the horizontal coordinates is 1 m, the standard deviation of the errors in the measurements of the GNSS velocities is 0.2 m/s. The frequency of measurements from GNSS and laser altimeters was 1 Hz. The standard deviation of measurements of the angular velocities of rotation of the wheels by the odometric subsystem is \( 10^{-2} \, \text{rad/s} \).

In Figure 3, solid lines indicate the trajectories constructed from the results of measurements of the navigation system, obtained in the process of three passes of the UWRRP along a given trajectory.

The analysis shows that the errors in measuring coordinates in the plane of the horizon do not exceed 0.4 m, while the errors in measurements of only GNSS reach 1.5 m. The error in measuring the height increases with time, approaching the mark corresponding to the value of the systematic error of measurements in the high-altitude channel GNSS - 0.3 m. The maximum changes in the lengths of the wheels' circumferences, in which the slipping factor is manifested, as well as the deformation factor of these wheels, amounted to 0.016 m.

5. Conclusion
The structure and algorithm of an integrated navigation system for a uniaxial wheeled robot with a two-stage platform, which is a carrier of equipment designed to solve various problems of environmental monitoring, are proposed. The navigation system includes an odometric subsystem, a global navigation satellite system receiver, a block of inertial meters and laser altimeters. The integration of this equipment based on the optimal Kalman filtration provided an effective solution to the problem of determining the navigation parameters of the robot in the conditions of its movement on a non-horizontal underlying surface in the presence of wheel slip relative to it. The results of semi-natural modeling confirmed the effectiveness of the adopted technical solutions.
References

[1] Aleshin B.S., Chernomorskiy A.I., Feshchenko S.V. and others. Orientation, navigation and stabilization of single-axle wheel modules / Ed. B. S. Aleshina, A. I. Chernomorskiy M.: Publishing house MAI, 2012.271 p.

[2] Maksimov V.N., Chernomorsky A.I. Integrated navigation and local mapping system for a uniaxial wheel module // Gyroscopy and navigation. 2016.T.92. No. 1. P. 116-132.

[3] Maksimov V.N., Chernomorskiy A.I. Control system for a nonholonomic uniaxial wheel module for monitoring the geometric parameters of airfield pavements // Izv. RAS. TiSU. 2015. No. 3. S. 156-167.

[4] Aleshin B.S., Kuris E.D., Lel’kov K.S., Maksimov V.N., Chernomorsky A.I. Control of the angular orientation of the platform of a uniaxial wheel module when it moves along the underlying surface without slipping // Izv. RAS. TiSU. # 1. 2017.S. 125-135

[5] Aleshin BS, Maksimov VN, Mikheev VV, Chernomorskii AI Stabilization in the plane of the horizon of a two-degree platform of a uniaxial wheeled module moving along a given trajectory on the underlying surface. Izv. RAS. TiSU. 2017. No. 3. S. 135-147.

[6] Belotelov VN, Martynenko Yu. G. Control of the spatial motion of an inverted pendulum mounted on a wheel pair // Izv. RAS. MTT. 2006. No. 6. P. 10–28.

[7] Beznos A.V., Grishin A.A., Lensky A.V., Okhotsimsky D.Ye., Formalsky A.M. A pendulum controlled by a flywheel. // DAN. No. 6. 2003. T. 392. S. 743-749.

[8] Beznos A.V., Grishin A.A., Lensky A.V., Okhotsimsky D.E., Formalsky A.M. Flywheel control of a pendulum with a fixed suspension point // Izv. RAS. TiSU. No. 4. 2004.S. 27-38.

[9] Formalsky A.M. Motion control of unstable objects // M. Fizmatlit, 2012. 232p.

[10] Ishlinsky A.Yu. Complete compensation of external disturbances caused by maneuvering in gyroscopic systems // Coll. tr. Academy of Sciences of the Ukrainian SSR “Theory of invariance and its application in automatic devices.” M., 1959.S. 81-92.

[11] A.V.Balakrishnan Kalman Filtering Theory, Optimization Inc., Publication Division, new York, 1984