Abstract

We compute the ratio $\Lambda_L/\Lambda_{\overline{MS}}$, where the scale parameter $\Lambda_L$ is associated with a lattice formulation of QCD. We consider a 3-parameter family of gluon actions, which are most frequently used for $O(a)$ improvement à la Symanzik. The gluon action is put together with standard discretizations for fermions (Wilson/clover, overlap), to provide $\Lambda_L$ for several possible combinations of fermion and gluon actions. We employ the background field technique in order to calculate the 1PI 2-point function of the background field; this leads to the coupling constant renormalization function, $Z_g$, at 1-loop level.

Our results are obtained for an extensive range of values for the Symanzik coefficients.

Keywords: Lattice QCD, Lattice perturbation theory, Lambda parameter, Improved actions.

PACS numbers: 11.15.Ha, 12.38.Gc, 11.10.Gh, 12.38.Bx
I. INTRODUCTION

The Λ parameter of QCD has been a subject of interest for almost three decades, since it is the necessary “yardstick” needed to convert dimensionless quantities coming from numerical simulations into measurable predictions for physical observables.

Ever since improved gluon and fermion actions started being employed more frequently in numerical simulations, a number of calculations of the Λ parameter on the lattice have been carried out, using various techniques and discretization prescriptions. Older results involving Wilson gluons [1], Wilson/clover fermions [2,3], overlap fermions [4] can be found in the literature. Some recent results regarding domain wall fermions can be found in Ref. [5].

A calculation of the Λ parameter which is missing is the one involving the Symanzik improved gluon actions which are widely used in recent simulations. The task of the present work is to fill this gap, while at the same time we confirm some of the existing results mentioned before. The contribution of fermions to the computation at hand is independent of the choice of gluon action; similarly, gluon contributions do not depend on the fermion action. This fact will enable us to combine our results with previous findings regarding Wilson/clover fermions [3] and overlap fermions [4].

The scale parameter, $\Lambda_L$, associated with a lattice formulation of QCD provides a relation between the lattice spacing, $a$, and the bare coupling constant $g_0$. It is a particular solution of the renormalization group equation, taking the form

$$ a\Lambda_L = \exp \left[ - \int_{g_0}^{g_L} \frac{dg}{\beta_L(g)} \right] = \exp \left( - \frac{1}{2b_0 g_0^2} \right) (b_0 g_0^2)^{-b_1/2b_0} \left[ 1 + \mathcal{O}(g_0^2) \right] $$

(1)

where $\beta_L(g_0)$ is the lattice $\beta$-function, and $b_0$, $b_1$ the first two coefficients of its perturbative expansion

$$ b_0 = \frac{1}{(4\pi)^2} \left( \frac{11}{3} N_c - \frac{2}{3} N_f \right) $$

$$ b_1 = \frac{1}{(4\pi)^4} \left[ \frac{34}{3} N_c^2 - N_f \left( \frac{13}{3} N_c - \frac{1}{N_c} \right) \right] $$

(2)

(3)

($N_f$: the number of fermion flavors, $N_c$: the number of colors.)

The Λ parameter is a dimensionful quantity; as such it cannot be directly obtained from the lattice. Instead, the quantity which is calculable is the ratio between $\Lambda_L$ and the scale parameter in some continuum renormalization scheme such as $\overline{\text{MS}}$: $\Lambda_L/\Lambda_{\overline{\text{MS}}}$. To this end, it suffices to compute the coupling constant renormalization function $Z_g$, relating the bare lattice coupling $g_0$ to the $\overline{\text{MS}}$-renormalized coupling $g$.

For the purposes of this calculation, we employ the background field technique [6–8]. This technique lends itself particularly well to evaluating $Z_g$, since it obviates the need to consider any 3-point functions.

The rest of the paper is organized as follows: In section II we present all the necessary background and set up. Our results are shown in section III and, finally, a brief discussion regarding some aspects of our calculation and findings is contained in section IV.
II. FORMULATION OF THE PROBLEM

We use the Symanzik improved gauge field action, involving Wilson loops with 4 and 6 links\(^1\). In standard notation, it reads [9]

\[
S_G = \frac{2}{g^2} \left[ c_0 \sum_{\text{plaq}} \Re \text{Tr} \left( 1 - U_{\text{plaq}} \right) + c_1 \sum_{\text{rect}} \Re \text{Tr} \left( 1 - U_{\text{rect}} \right) \\
+ c_2 \sum_{\text{chair}} \Re \text{Tr} \left( 1 - U_{\text{chair}} \right) + c_3 \sum_{\text{paral}} \Re \text{Tr} \left( 1 - U_{\text{paral}} \right) \right] \tag{4}
\]

The lowest order expansion of this action (together with the gauge fixing term, with gauge parameter \(\xi\), see Eq.(10)), leading to the gluon propagator, is [10]

\[
S_{G}^{(0)} = \frac{1}{2} \int_{-\pi/a}^{\pi/a} \frac{d^4 k}{(2\pi)^4} \sum_{\mu\nu} A^a_{\mu}(k) \left[ G_{\mu\nu}(k) - \frac{\xi}{\xi - 1} \hat{k}_{\mu} \hat{k}_{\nu} \right] A^a_{\nu}(-k) \tag{5}
\]

where:

\[
G_{\mu\nu}(k) = \hat{k}_{\mu} \hat{k}_{\nu} + \sum_\rho \left( \hat{k}_{\rho}^2 \delta_{\mu\nu} - \hat{k}_{\mu} \hat{k}_{\rho} \delta_{\rho\nu} \right) d_{\mu\rho}
\]

and:

\[
d_{\mu\nu} = (1 - \delta_{\mu\nu}) \left[ C_0 - C_1 a^2 \hat{k}^2 - C_2 a^2 (\hat{k}_{\mu}^2 + \hat{k}_{\nu}^2) \right]
\]

\[
\hat{k}_{\mu} = \frac{2}{a} \sin \frac{ak_\mu}{2}, \quad \hat{k}^2 = \sum_\mu \hat{k}_{\mu}^2
\]

The coefficients \(C_\lambda\) are related to the Symanzik coefficients \(c_\lambda\) by

\[
C_0 = c_0 + 8c_1 + 16c_2 + 8c_3, \quad C_1 = c_2 + c_3, \quad C_2 = c_1 - c_2 - c_3 \tag{6}
\]

The Symanzik coefficients must satisfy: \(C_0 = 1\), in order to reach the correct classical continuum limit.

Regarding the fermion part of the action, a variety of discretizations are presently used in Monte Carlo simulations. The contribution of fermions to 1 loop is independent of the regularization chosen for the gluonic part; vice versa, gluon contributions do not depend on the fermion action. Consequently, the results of the present work can be directly combined with those of previous calculations regarding Wilson/clover fermions [3] and overlap fermions [4], yielding the \(\Lambda\) ratio for a variety of possible combinations of fermion and gluon actions.

In the background field method, link variables are decomposed as [6]

\[
U_\mu(x) = V_\mu(x) U_{cp}(x) \tag{7}
\]

in terms of links for a quantum field and a classical background field, respectively

\(^{1\times 1\ plaquettes, \ 1\times 2\ rectangles, \ 1\times 2\ chairs\ (bent\ rectangles), \ and\ parallelograms\ wrapped\ around\ elementary\ cubes.\)
\[ V_\mu(x) = e^{i g Q_\mu(x)}, \quad U_{c\mu}(x) = e^{i a B_\mu(x)} \] (8)

The \( N_c \times N_c \) Hermitian matrices \( Q_\mu \) and \( B_\mu \) can be expressed as
\[ Q_\mu(x) = t^a Q^a_\mu(x), \quad B_\mu(x) = t^a B^a_\mu(x), \quad Tr[t^a t^b] = \frac{1}{2} \delta^{ab} \] (9)

A choice of gauge is required for the perturbative expansion; an appropriate gauge-fixing term is
\[ S_{gf} = \frac{1}{1 - \xi} \sum_{\mu,\nu} \sum_x Tr[D^-_\mu Q_\mu D^-_\nu Q_\nu] \] (10)

This term breaks gauge invariance with respect to \( Q_\mu \), as it should, but succeeds in keeping the path integral as a gauge invariant functional of \( B_\mu \). The definition of the lattice derivative, which is covariant with respect to background gauge transformations, is
\[ D^-_\mu(U_c)Q_\nu(x) = U_{c\mu}^{-1}(x - e_\mu)Q_\nu(x - e_\mu)U_{c\mu}(x - e_\mu) - Q_\nu(x) \] (11)

Since the quantities we will be studying are gauge independent, we chose, for convenience, to work in the Feynman gauge, \( \xi = 0 \). Covariant gauge fixing produces the following action for the ghost field \( \omega \)
\[ S_{gh} = 2 \sum_x \sum_\mu \text{Tr} \left( D^+_\mu \omega(x) \right) \left( D^+_\mu \omega(x) + i g_0 [Q_\mu(x), \omega(x)] + \frac{i}{2} g_0 [Q_\mu(x), D^+_\mu \omega(x)] \right) \]
\[ - \frac{1}{12} g_0^2 \left[ Q_\mu(x), [Q_\mu(x), D^+_\mu \omega(x)] \right] + \cdots, \] (12)

where \( D^+_\mu \omega(x) \equiv U_{c\mu}(x) \omega(x + \hat{\mu})U_{c\mu}^{-1}(x) - \omega(x) \).

Finally the change of integration variables from links to vector fields yields a Jacobian that can be rewritten as the usual measure term \( S_m \) in the action
\[ S_m = \sum_{x,\mu} \left\{ \frac{N g_0^2}{12} \text{Tr} \left( Q_\mu(x)^2 \right) + \cdots \right\} \] (13)

The measure part will not contribute to the present calculation.

In order to compute \( \Lambda_L \) we need to evaluate the renormalization function \( Z_g \) for the coupling constant, up to 1 loop
\[ g_o = Z_g(g_o, a\bar{\mu}) g \] (14)

where \( \bar{\mu} \) is the renormalization scale in the \( \overline{MS} \) scheme. Writing
\[ Z_g(g_o, a\bar{\mu})^2 = 1 + g_o^2 (2 b_0 \ln(a\bar{\mu}) + l_o) + O(g_o^4) \] (15)
one has
\[ l_o = 2 b_0 \ln(\Lambda_L/\Lambda_{\overline{MS}}) \] (16)
To obtain $Z_g$ we only need to calculate the one-particle irreducible (1PI) 2-point function of the background field, $\Gamma^{(2,0,0)}(p, -p)_{\mu\mu}$, on the lattice, to one loop. Color symmetry and lattice rotational invariance allow one to write [11]

$$\sum_\mu \Gamma^{(2,0,0)}(p, -p)_{\mu\mu} = -3\delta^{ab}p^2 [1 - \nu(p)] / g_o^2$$

where $\nu(p)$ is a Lorentz invariant amplitude on the lattice, up to terms which vanish as $a \to 0$; $\nu(p)$ is perturbatively expanded as

$$\nu(p) = \sum_{i=1}^{\infty} g_o^{2i} \nu^{(i)}(p)$$

The background field formalism has the advantage that $Z_g$ is directly related to the background field renormalization function $Z_A$, through: $Z_g(g_o, a\bar{\mu})^2 Z_A(g_o, a\bar{\mu}) = 1$. Consequently, no 3-point functions are needed for the evaluation of $Z_g$. In terms of $\nu(p)$, one can express $Z_g$ as

$$Z_g(g_o, a\bar{\mu})^2 = 1 + g_o^2 \left( \nu^{(1)}(p/\bar{\mu}) - \nu^{(1)}(ap) \right) + O(g_o^4)$$

where

$$\nu^{(1)}(p/\bar{\mu}, \xi) = \frac{N_c}{16\pi^2} \left[ -\frac{11}{3} \ln \frac{p^2}{\bar{\mu}^2} + \frac{205}{36} + \frac{3}{2(1 - \xi)} + \frac{1}{4(1 - \xi)^2} \right] + \frac{N_f}{16\pi^2} \left[ \frac{2}{3} \ln \frac{p^2}{\bar{\mu}^2} - \frac{10}{9} \right]$$

is the analogous 1-loop amplitude in the $\overline{MS}$ scheme.

**III. COMPUTATION AND RESULTS**

The Feynman diagrams shown in Fig. 1 contribute to $\nu^{(1)}(p)$. All algebraic manipulation of these diagrams was performed automatically using our software written in Mathematica. Once we have computed $\nu^{(1)}(p)$, we use Eqs.(15) and (19) in order to obtain $l_o$, which is the sum of a part involving only the gluon and ghost action, and a part involving the fermion action, i.e.,

$$l_o = l_o^g + N_f \cdot l_o^f$$

![Fig. 1. One-loop diagrams contributing to $\Gamma^{(2,0,0)}$. A wavy (solid, dashed) line represents gluons (fermions, ghosts). The letter B stands for the external background field.](image)
The dependence of $l_g$ on the Symanzik coefficients is rather complicated and cannot be given in closed form. However, given that the gluon propagator depends only on the combinations $C_1, C_2$ (c.f. Eqs.(4), (6)) we can reexpress all diagrams in terms of $C_1, C_2$ and one additional parameter, say, $c_2$; in this case the dependence on $c_2$ (at fixed $C_1, C_2$) is polynomial. Thus, the part of $l_o$ involving gluon and ghost fields, $l_g$, can be written as

$$l_g = a_1 \frac{1}{N_c} + a_2 N_c + a_3 \frac{c_2}{N_c} + a_4 c_2 N_c + a_5 c_2^2 N_c$$

(22)

where $a_i$ are numerical constants (dependent on $C_1, C_2$) evaluated via numerical integration over loop momenta. We consider ten sets of different values for the Symanzik coefficients, corresponding to the most commonly used actions, shown in Table I: The plaquette action, the tree-level Symanzik improved action, the Lüscher-Weisz tadpole improved actions (TILW), the Iwasaki action and the DBW2 action (see [12–17]). The quantities $a_i$, for each one of the ten sets of parameters, are presented in Table II. The variable $c_2$ can be freely varied; $c_0, c_1$ and $c_3$ are then adjusted accordingly so as to keep $C_1, C_2$ fixed.

The fermionic part of $l_o$, denoted by $l_f$, was calculated in [4] using Neuberger’s overlap formulation of chiral fermions [18], leading to

$$l_f = -\frac{5}{2}\pi^2 - k_f(\rho)$$

(23)

where $k_f(\rho)$ varies from 0.07 to 0.08 in a typical range of the overlap parameter $\rho$. For an extended list of values of $k_f(\rho)$ see Table I of Ref. [4].

In order to assess quantitatively the effect of the Symanzik improved actions on the $\Lambda$ parameter, one may consider the ratio

$$r_\Lambda \equiv \frac{(\Lambda_L/\Lambda_{\overline{MS}})_{\text{Symanzik}}}{(\Lambda_L/\Lambda_{\overline{MS}})_{\text{Wilson}}} = \exp \left[ \frac{1}{2b_0} \left( l_o^{\text{Symanzik}} - l_o^{\text{Wilson}} \right) \right]$$

(24)

This quantity is independent of the fermion action but still depends on the number of flavors, $N_f$, through $b_0$. For completeness, we report the value of $l_o$ found in the literature for Wilson gluons and Wilson/clover fermions [1,3]

$$l_o = 1/(8N_c) - 0.169955999 N_c + N_f l_{o1}$$

(25)

where:

$$l_{o1} = 0.0066696001(5) - 0.00504671402(1) c_{SW} + 0.02984346720(1) c_{SW}^2$$

(26)

In the above, the Wilson parameter is set to $r = 1$ and the clover parameter, $c_{SW}$, can be chosen arbitrarily; the dependence on $c_{SW}$ is seen to be polynomial.

In Table III we list the values of the ratio $r_\Lambda$ for $N_f = 0$ and $N_f = 2$. We present $r_\Lambda$ for each set of parameters shown in Table I, setting $N_c = 3$ and $c_2 = 0$. We also list the $\Lambda$ ratio, $\Lambda_L/\Lambda_{\overline{MS}}|_{N_f=0}$, in the pure gauge theory, and with two flavors of Wilson fermions, $\Lambda_L/\Lambda_{\overline{MS}}|_{N_f=2}$. We stress that $r_\Lambda(N_f = 2)$ is the same for all types of fermion actions.

In Fig. 2 we present our results for $l_o^g$ as a function of both $C_1$ and $C_2$, for $N_c = 3$ and $c_2 = 0$. The range of values for $C_1$ and $C_2$ was selected so as to encompass all values used in current simulations. We can see that the dependence on $C_1$ is almost linear while dependence
on $C_2$ is more complicated. The crosses correspond to the ten actions shown in Table I. In
Fig. 3 we plot the ratio $r_\Lambda$ defined in Eq.(24) as a function of $C_2$. Once again, we have set $N_c = 3, c_2 = c_3 = 0$ and thus $C_2 = c_1$.

Fig. 2. $l_o$ as a function of the parameters $C_1$ and $C_2$ ($N_c = 3, c_2 = 0$). The crosses
denote the 10 set of parameters, identified by their $C_1, C_2$ values, as shown in Table I.

Fig. 3. $r_\Lambda$ as a function of $C_2$, for $N_f = 0, 2$.
We have set $c_2 = c_3 = 0$ (and therefore $C_2 = c_1$) and $N_c = 3$. 

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For easier comparison we report some existing numbers for the ratio $\Lambda_L/\Lambda_{\overline{MS}}$ using Wilson gluons and Wilson or overlap fermions (see e.g. Refs. [3,4])

| Configuration                      | $\Lambda_L/\Lambda_{\overline{MS}}$   |
|------------------------------------|----------------------------------------|
| Wilson gluons, Wilson fermions     | 0.0243589                              |
| Wilson gluons, overlap fermions ($\rho = 1.0$) | 0.0172702                              |
| Wilson gluons, overlap fermions ($\rho = 1.4$) | 0.0172317                              |

**IV. DISCUSSION**

In the present work we evaluated $\Lambda_L/\Lambda_{\overline{MS}}$ for a 3-parameter family of Symanzik improved gluon actions; to this end, we computed $Z_g$, up to 1 loop, using the background field technique. Only diagrams with two external background fields, corresponding to the 1PI two-point function of the background field, were involved in the calculation, as shown in Fig. 1. Alternatively, one could study $Z_g$ by considering the gluon-gluon-gluon, gluon-ghost-antighost or gluon-fermion-antifermion three-point functions, together with the self-energy diagrams for the gluon, ghost and fermion fields; of course, the computation would be much more cumbersome in this case, due to the complexity of the Symanzik improved actions, resulting in lengthier algebraic expressions. It is this very fact pointing out the advantage of the background field technique.

All calculations have been performed in the Feynman gauge ($\xi = 0$), and the conversion of lengthy integrands ($\sim 100,000$ terms) into an efficient Fortran code was carried out by our “integrator” program, a **metacode** written in Mathematica. The numerical integrals were evaluated for lattices up to $L = 128$; the results were then extrapolated to $L \rightarrow \infty$. Given that only a restricted set of functional forms is sufficient to describe the behavior of the results with $L$, the systematic error resulting from such an extrapolation can be estimated quite accurately.

Special attention was given to the extraction of the dependence on the external momentum $p$. The algebraic expressions coming from the evaluation of Feynman diagrams were split into a logarithmically divergent part, comprised of a limited set of tabulated lattice integrals, and a (much larger) part which is Taylor expandable up to second order in $p$. We have seen explicitly that terms of order $O(p^0)$ cancel upon summation of gluon, ghost and fermion diagrams separately, compatibly with gauge invariance.

Our results are functions of the Symanzik coefficients $C_1$, $C_2$ and $c_2$. At fixed $C_1$ and $C_2$, the dependence on $c_2$ is seen to be a second order polynomial, thus no particular values of $c_2$ have to be chosen a priori; conversely, to investigate the effect of the remaining coefficients, we selected a mesh of $25 \times 27$ values of $C_1, C_2$ for numerical integration. The dependence on $C_1$ turns out to be almost linear, while the $C_2$ dependence is more complicated (see Fig. 2).

Given that the gluon and fermion parts of the action give disjoint contributions to $l_0$, our present result can be directly combined with contributions from a variety of different fermion actions, to yield the complete effect on $\Lambda_L$. The number of colors, $N_c$, and the number of fermion flavors, $N_f$ can be chosen arbitrarily.

Through Eq.(24) one can assess the effect of the Symanzik improved actions on the $\Lambda$ parameter. All of the actions shown in Table. I, with the exception of the DBW2 action, give similar results (of order $10^1$) for $r_\Lambda$. The most drastic effect on the $\Lambda$ parameter originates from the DBW2 improved action, where $r_\Lambda$ is of order $10^3$. 8
### TABLE I

The coefficients $c_0$, $c_1$, $c_3$ ($c_2 = 0$), corresponding to some of the most commonly used actions, along with the respective values for $C_1$, $C_2$.

| Action         | $c_0$ | $c_1$ | $c_3$ | $C_1$   | $C_2$   |
|----------------|-------|-------|-------|---------|---------|
| Set 1: Plaquette | 1.0   | 0.0   | 0.0   | 0.0     | 0.0     |
| Set 2: Symanzik  | 1.6666666 | -0.083333 | 0.0   | 0.0     | -0.083333 |
| Set 3: TILW, $\beta = 8.60$ | 2.3168064 | -0.151791 | -0.0128098 | -0.0128098 | -0.138981 |
| Set 4: TILW, $\beta = 8.45$ | 2.3460240 | -0.154846 | -0.0134070 | -0.0134070 | -0.141439 |
| Set 5: TILW, $\beta = 8.30$ | 2.3869776 | -0.159128 | -0.0142442 | -0.0142442 | -0.144884 |
| Set 6: TILW, $\beta = 8.20$ | 2.4127840 | -0.161827 | -0.0147710 | -0.0147710 | -0.147056 |
| Set 7: TILW, $\beta = 8.10$ | 2.4465400 | -0.165353 | -0.0154645 | -0.0154645 | -0.149889 |
| Set 8: TILW, $\beta = 8.00$ | 2.4891712 | -0.169805 | -0.0163414 | -0.0163414 | -0.153464 |
| Set 9: Iwasaki   | 3.648   | -0.331 | 0.0    | 0.0     | -0.331  |
| Set 10: DBW2     | 12.2688 | -1.4086 | 0.0    | 0.0     | -1.4086 |

### TABLE II

Values of the coefficients $a_1$, $a_2$, $a_3$, $a_4$, $a_5$. Set 1 through Set 10 correspond to $C_1$, $C_2$ shown in Table I.

| Set | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ |
|-----|-------|-------|-------|-------|-------|
| 1   | 0.12499999997(6) | -0.1699559990(1) | 0.43112525414(6) | -0.0958290656(4) | -0.657621162(8) |
| 2   | 0.04217165191(7) | -0.0833756545(3) | 0.31095652446(7) | -0.031584124(1) | -0.402576126(2) |
| 3   | -0.0082581838(1) | -0.0310360175(6) | 0.24637322205(3) | 0.000854777(6)  | -0.283219286(3) |
| 4   | -0.010122622374(4) | -0.0290818603(4) | 0.244114928978(2) | 0.000128847(6)  | -0.279256305(1) |
| 5   | -0.01268965657(2) | -0.0263884374(3) | 0.241020561259(1) | 0.001465153(3)  | -0.2738512399(6) |
| 6   | -0.0142802781(2) | -0.0247177255(2) | 0.23911193181(8)  | 0.0022825409(6) | -0.270531792(3) |
| 7   | -0.01632979450(5) | -0.0225635251(5) | 0.2366619590(1)   | 0.003324978(1)  | -0.2662885752(6) |
| 8   | -0.01886971906(3) | -0.01989088288(9) | 0.23364084600(6)  | 0.004598716(5)  | -0.2610823949(1) |
| 9   | -0.07528696825(4) | 0.0433593330(5)  | 0.17401920011(2)  | 0.021865609(1)  | -0.159911864(1)  |
| 10  | -0.204424737(1)  | 0.19876966(7)    | 0.0610275834(6)   | 0.03791059(2)   | -0.027913991(2)  |
TABLE III. The ratio $r_\Lambda$ defined in Eq. (24) for each set of parameters and for $N_f = 0, 2$, along with the respective values for $\Lambda_L/\Lambda_{MS}|_{N_f=0}$ and $\Lambda_L/\Lambda_{MS}|_{N_f=2}$. We have set $N_c = 3$ and $c_2 = 0$.

| Action       | $r_\Lambda(N_f = 0)$ | $\Lambda_L/\Lambda_{MS}|_{N_f=0}$ | $r_\Lambda(N_f = 2)$ | $\Lambda_L/\Lambda_{MS}|_{N_f=2}$ |
|--------------|----------------------|-----------------------------------|----------------------|-----------------------------------|
| Set 1: Plaquette | 1.00000              | 0.034711                          | 1.00000              | 0.024359                          |
| Set 2: Symanzik   | 5.29210              | 0.18369                           | 6.65946              | 0.16222                           |
| Set 3: TILW, $\beta = 8.60$ | 14.4779              | 0.50254                           | 20.9316              | 0.50987                           |
| Set 4: TILW, $\beta = 8.45$ | 15.0329              | 0.52181                           | 21.8471              | 0.53217                           |
| Set 5: TILW, $\beta = 8.30$ | 15.8330              | 0.54958                           | 23.1751              | 0.56452                           |
| Set 6: TILW, $\beta = 8.20$ | 16.3507              | 0.56755                           | 24.0392              | 0.58557                           |
| Set 7: TILW, $\beta = 8.10$ | 17.0432              | 0.59159                           | 25.2012              | 0.61387                           |
| Set 8: TILW, $\beta = 8.00$ | 17.9435              | 0.62284                           | 26.7214              | 0.65090                           |
| Set 9: Iwasaki   | 61.2064              | 2.1245                            | 107.957              | 2.6297                            |
| Set 10: DBW2     | 1276.44              | 44.306                            | 3423.05              | 83.382                            |
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Acknowledgements: This work is supported in part by the Research Promotion Foundation of Cyprus (Proposal Nr: ENIΣΧ/0506/17).