Thermo-convective Arrhenius reactive fluid flow between two parallel plates

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Abstract
A nonlinear thermo-convective Arrhenius Casson fluid flow between two vertical parallel plates is modeled under the actions of exothermic chemical reaction, linear and nonlinear volumetric thermal and concentration expansions. The non dimensional coupled system of ordinary differential equations are solved numerically by using Parametric Continuation Method (PCM). The numerical results of (PCM) method are graphically justified with the solutions of bvp4c package, which presented an excellent agreement with each other. For further validation of (PCM) method, the obtained results are also compared with previously published work and found an accuracy upto two decimal places. It has been found that Frank-Kameneski's parameter, depends on Arrhenius kinetics, has the tendency to enhanced the exothermic chemical reaction, which give rise in internal heat generation and heat transfer rate. The findings of the present study may be used for making energy source materials and in biological systems. Furthermore, the increasing values of $Kr$ are responsible to drop the exothermic reaction level within the flow system, due to which the mass transfer is reduced and make a reduction in concentration profile.

Keywords
Arrhenius kinetics, exothermic chemical reaction, reactant consumption rate, Frank-Kameneski's reaction rate parameter, activation energy parameter

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Introduction
In the modern world and specially in the field of research chemical reaction performs an excellent role. As we know that there are two types of chemical reaction exothermic and endothermic chemical reaction. In exothermic chemical reaction heat and light which is the form energy is released to the environment, while in endothermic chemical reaction energy is observed from it is environment. In chemical reaction, the reaction rate mostly depend on the concentration of the fluids. These reactions have numerous application in industries, Animasaun.1 Order of reaction is very important terminology in the field engineering and industries, in which the reaction rate varies directly to the chemical reactant concentration rate of order one. The reactions in which the reaction rate is proportional to the nth power of the reactant concentration is called nth order chemical reaction. As we discussed exothermic and endothermic chemical reaction, common example of exothermic reaction is flame, which gives us heat as well as light. In this phenomenon dissipated energy is

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released to the surrounding, in which exothermic reaction enthalpy is always negative. Self ignition process is remarkable example of exothermic chemical reaction. Numerous papers have been published, concerning exothermic chemical reaction on a combustible material in the literature.2–4

In recent time researcher are trying to enhanced the thermal conductivity of the fluids, which is possible only to add some special types of nanoparticles to the fluids. In this regard Choi and Eastman5 was first to performed experiment and add some nanoscale particle in desired fluid, he found that these nanoscale particles produced increment in thermal behavior of the base fluids. Viscoelastic fluids behaves as elastic solids for a specific threshold magnitude, after removing of this stress it becomes liquids. It has so many applications in the engineering operations systems, foods preparing and preservation, drilling operations, and so forth. Hayat et al.6 elucidated heat transfer analysis of Viscoelastic Casson fluids with supplying convective heating at the given surface. Likewise heat and mass transfer have numerous application in the field of engineering, heat and mass transfer phenomena is very essential for various thermal engineering features. Concentration gradient between fluid and its surrounding produce natural convective flow. In pours medium heat and radiation effect on the Magneto hydrodynamics (MHD) was first presented by Mohiddin et al.7 Shehzad et al.8 explain measurement of heat and mass exchange, which is magnetically stream flow of Casson liquid. Mustafa and Khan9 probed magnetized Casson liquid along with suspension of nanoparticles on the nonlinear stretchable surfaces.

The study of combined effects of the magnetic field behavior on natural convection with a square cavity, and the presence of nanoparticles have adverse effects on the heat transfer at high volume fraction of the nanoparticles was studied by Mahmoudi and Abu-Nada.10 Adeosun et al.11 studied about MHD of a non-linear convective flow of Arrhenius exothermic reactive fluids, with considering the effect of special parameters, within two vertical long plates. The purpose of this task was to investigate the influence of nonlinear convective parameter on heat and mass transfer with porous materials.

According to the boundary layer theory, non-Newtonian flow on heated surface which is exponentially stretching was examined by Hafeez and Chu.12 Mustafa and Khan13 have explained the magnetic and heat effects on MHD casson nano fluid, over nonlinear temperature across the stretching sheet. Bilal et al.14 have explained thermal radiation upon the laminar free convection boundary layer of a vertical plates, dealing with nonabsorbent gas for constant heat flux condition. They have studied about time varying hydro magnetic radiating fluid passed in infinite oscillating vertical plates, in which deeply explain some parameters such as Soret and Dufour. They have examined very big change in their heat conduction power after adding very small amount of nanoparticles as compared to common fluids.14–22

MHD nanoparticles are playing significant role in numerous industrial processes. Such fluids have broad range applications in optical industry as gratings, fiber filters, modulator, and polymer processes. MHD nanoparticles have great applications in the loud speaker/vehicles cooling, magnetic cell, generating hybrid fuels and electricity plants, etc.23–38

The motion of temperature dependent plastic dynamic viscosity and thermal conductivity of consist-ent incompressible laminar free convective magnetohydrodynamic (MHD) Casson fluid go with the flow over an exponentially stretching floor with suction and exponentially decaying internal warmth generation. It’s far assumed that the natural convection is pushed by way of buoyancy and space dependent heat era. The viscosity and thermal conductivity of Casson fluid is assumed to differ as a linear characteristic of temperature. by way of using appropriate transformation, the governing partial differential equations similar to the momentum and strength equations are transformed into non-linear coupled regular differential equations and solved by means of the Homotopy analysis approach. A new kind of averaged residual mistakes is followed and used to locate the premier convergence manage parameter.29–34

In the modern world most of the researchers use PMC strategy, that is why Grigolyuk and Shalashilin35 introduced and apply to many problems, which is considered difficult. PCM strategy is exceptionally delicate on to settle on a decision a proper continuation parameter. Vorovich and Zipalova36 and Riks37 presented a report to pick an proper heading for continuation parameter to get appropriate conditioning of the solution for linearized system of \( N \times N \) conditions. Varies application of nonlinear mechanics is carried in the geometrical elucidation.38,39 Watson and Holzer40 bring together PCM on Navier-Stokes equations. Most of the engineering problems contain nonlinear ordinary or partial differential equations. That is why, the analysts apply different technique to solve the given problems.

Some recent work on thermo-convective and magnetic have been included in the present article, in which Chilton-Colburn study about thermo convective fluid property in mini channel.41 Variables temperature and mass diffusion with arbitrary velocity for viscoelastic fluid flow has been recently studied by Saeed et al.42 Ionic liquid across micro channels of the peristaltic principle of electro osmotic flow (EOF) has been studied by Humaira and Yasmin.43
For further validation of (PCM) method, the obtained results are also compared with previous published work and found an accuracy up to two decimal places. It has been found that Frank-Kamenekskii’s parameter, depends on Arrhenius kinetics, has the tendency to enhanced the exothermic chemical reaction, which give rise in internal heat generation and heat transfer rate. The findings of the present study may be used for making energy source materials and in biological systems.

The present problem is concerned with the modeling and numerical solutions of Arrhenius reactive casson fluid flow between to verticals stationary plates, under the effects of linear and non linear volumetric thermo-convective along with magnetic effect. The concentration equation is coupled with temperature by introducing Arrhenius reactive term. The findings of the present study may be used for making energy source materials and in biological systems. According to our knowledge no literature is available for such types of materials and in biological systems.

Therefore, the aim of the present article is to investigate and model the behavior of fully developed, incompressible, electrically conducting, steady, linear and nonlinear thermo-convective, exothermic Arrhenius Casson fluid flow between two vertically porous parallel plates. The modeled governing equations in their PDEs form are dimensionalized by introducing some suitable dimensionless quantities, due to which the system of PDEs transformed in a system of nonlinear ODEs. The dimensionless system of nonlinear ODEs are then solved numerically by using Parametric Continuation Method (PCM) and bvp4c package.

**Mathematical formulation of the problem**

A fully developed in-compressible, electrically conducting steady, non linear Arrhenius convective, fluid flow is considered between two parallel vertically long plates, separated by distance of $2h$ from each other. The space between two homogeneous plates is filled with non-Newtonian Casson fluid driven by a constant pressure gradient $\frac{dp}{dy}$ against the gravitational force as shown in Figure 1. The Cartesian co-ordinates system $(x, y)$ is taken at the center of the plates, in which $y$-axis along the flow direction and $x$-axis is perpendicular to the flow. Both plates are kept constant temperature and concentration $T_0$ and $C_0$ respectively. A constant field of strength $B_0$ is applied is perpendicular to $y$-axis. A uni-direction and two dimension velocity field $U(0, v(x, y))$ is considered for the problem.

For rheological equation of in compressible casson fluid as follow:

$$
\frac{\tau_x}{\varepsilon_x} = (\mu_b + \frac{p}{\sqrt{2\pi}}) \quad \text{when} \quad \pi > \pi_c,
$$

$$
\frac{\tau_y}{\varepsilon_y} = (\mu_b + \frac{p}{\sqrt{2\pi}}) \quad \text{when} \quad \pi < \pi_c,
$$

where, $p_c = \mu_b \sqrt{2\pi}/\beta$. By incorporating this value of $p_c$, in equation (1), the kinematic viscosity of Casson fluid becomes as follow:

$$
\nu = \left(\frac{\mu_b}{\rho}\right) \left(1 + \frac{1}{\beta}\right)
$$

where $\mu_b$ is known as plastic dynamic viscosity of the non-Newtonian fluid, $\pi = \varepsilon_x \varepsilon_y$ is the product of the component deformation rate with itself, where $\pi_c$ is critical value base on non Newtonian fluid.

The Basic equation governing the fluid flow are given as:

**Continuity Equation:**

$$
\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0
$$

**Momentum Equation:**

$$
\left(1 + \frac{1}{\beta}\right) \mu \frac{\partial^2 U}{\partial x^2} + \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 V}{\partial y^2} - \rho g \beta_0 (T - T_0) - \rho g \beta C_0 (C - C_0)^2 + \alpha \beta_0^2 V
$$

**Temperature Equation:**

$$
\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \frac{1}{\rho C_v} \left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right)
$$
\[
\frac{d^2T}{dx^2} = -\frac{\partial(T^2-C_w)}{\partial k_w} \exp\left(\frac{-F}{RT}\right) \left(\frac{\delta}{\mu T^2}\right)^n \\
- \frac{\delta}{\mu T^2} \left(1 + \frac{1}{\beta}\right) \frac{d^2T}{dx^2}^2 \\
- \frac{\delta}{\mu T^2} \left(\frac{1}{\mu} + \frac{1}{\beta}\right) \left(\frac{d^2T}{dx^2}\right)^2 \\
\left(\frac{1}{\mu} + \frac{1}{\beta}\right) \left(\frac{d^2T}{dx^2}\right)^2
\]
\]

(5)

Concentration Equation:
\[
\frac{d^2C}{dx^2} = \frac{A}{D} \left(\frac{C - C_0}{C_w}\right) \exp\left(-\frac{E}{RT}\right) - \frac{D}{T_0} \left(\frac{d^2T}{dx^2}\right)
\]
\[
(6)
\]

The appropriate boundary conditions for the flow induced by constant pressure \( p \) with vertical long plates are given by \(^1\)
\[
T = T_w, \quad \bar{v} = 0, \quad \bar{C} = C_w, \quad \text{at} \quad \bar{x} = -h \\
\bar{C} = T_w, \quad \bar{v} = 0, \quad \bar{C} = C_w, \quad \text{at} \quad \bar{x} = +h
\]
\[
(7)
\]

In the above equation we have \( P \)-modified pressure, \( K \)-permeability of the porous medium, \( \mu \)-fluid dynamics viscosity \( \nu \) is axial velocity \( \bar{T} \) absolute temperature, \( C \) is concentration of the reactant, \( T_0 \)-initials fluid temperature, \( T_w \)-fluids temperature at the wall, \( \beta \)-linear volumetric coefficient in thermal expansion, \( \beta_{nl} \)-nonlinear volumetric coefficient in thermal expansion, \( \beta \)-linear volumetric coefficient of concentration expansion, \( \beta_{nl} \)-nonlinear volumetric coefficient of concentration expansion, \( A \)-rate constant, \( E \)-activation energy, \( R \)-universal gas constant, \( Q \)-heat of reaction, \( D \)-reactant diffusivity, \( D \)-thermotropophoretic diffusion coefficient, \( \sigma \)-electrical conductivity, \( \phi \)-magnetic field strength. The reaction kinetics type is expressed in terms of the symbol \( m \), where \( m \) is numerical exponent value such that \( m(0.2, 0.5) \), where \( m = -2 \) represent the Sensitized (laser induced) Kinetics, \( m = 0 \) the Arrhenius kinetics, and \( m = 0.5 \) the Bi molecular kinetics.

The following dimensionless variables are introducing in the equations (3–7):
\[
\begin{align*}
\theta &= \frac{T(T_w-T_0)}{RT_0}, \quad \theta_s = \frac{T(T_w-T_0)}{RT_0}, \quad x = \frac{\bar{T}}{h}, \\
\phi &= \frac{\bar{C} - C_w}{C_w}, \quad \nu = \frac{\bar{v}}{\bar{C}}, \quad M = -\frac{\mu^2 \phi}{\mu^2 \phi} \\
\epsilon &= \frac{R_T}{\bar{E}}, \quad \sigma = \rho gh^2 \beta \frac{C_w}{\mu VME}, \\
G_r &= \frac{\rho g h^2 \beta R_T^2}{\mu VME}, \\
\sigma_1 &= \frac{\beta_{nl} R_T^2}{\mu VME}, \quad \sigma_2 = \frac{\beta_{nl} (C_w - C_0)}{\beta_{nl} C_w} \\
S_0 &= \frac{\beta_{nl} R_T^2}{\mu VME}, \\
\lambda &= \frac{\beta_{nl} R_T^2}{\mu VME}, \\
K_r &= \frac{\beta_{nl} R_T^2}{\mu VME}, \\
\delta &= \frac{\beta_{nl} R_T^2}{\mu VME}, \quad \sigma_1 = \frac{\beta_{nl} (C_w - C_0)}{\beta_{nl} C_w} \\
B &= \frac{1}{D_0}, \quad D_a = \frac{D_0}{D_0}, \quad H_a = \frac{\sigma_1 \beta_{nl}^2}{\mu}
\end{align*}
\]
\[
(8)
\]

Now dimensionless boundary condition of the problem
\[
\begin{align*}
\nu &= 0, \quad \theta = \theta_s, \quad \phi = 1, \quad \text{at} \quad x = -1 \\
\nu &= 0, \quad \theta = \theta_s, \quad \phi = 1, \quad \text{at} \quad x = 1
\end{align*}
\]
\[
(12)
\]

where \( G_r, G_c, \sigma_1, \) and \( \sigma_2 \) have thermal buoyancy parameter, concentration buoyancy parameter, nonlinear thermal convection parameter, and nonlinear concentration convection parameter respectively. Other important parameter \( B, \lambda, \epsilon, \delta, K_r, \theta_s, \) and \( H_a \) are porous medium permeability parameter, Frank-Kamenetskii parameter, activation energy parameter, viscous heating parameter, Concentration consumption rate parameter, wall temperature, Soret number, and Hartmann number respectively.

**Parametric continuation method for solution**

The necessary technique of parametric continuation method(PCM), apply to the systems of ODE’s (9–11), along with the boundary conditions (12) are presented with the following steps \(^45-48\).

**Step 1: Obtaining first order ODE after converting the system of BVP**

We introduce special functions for this purpose:
\[
\begin{align*}
F_1(x) &= \nu(x), \quad F_2(x) = \frac{d\nu}{dx}, \quad F_3(x) = \theta(x), \\
F_4(x) = \frac{d\theta}{dx}, \quad F_5(x) = \phi(x), \quad F_6(x) = \frac{d\phi}{dx}
\end{align*}
\]
\[
(13)
\]

By using transformation (13) into the BVP (9–11) and (12) which take the form as:
\[
\begin{align*}
\frac{dF_2}{dx} &= \left(1 + \frac{1}{\beta}\right)^{-1}\left(-G_r F_3 + \sigma_1 F_3^2\right) \\
&- G_c (F_5 + \sigma_2 F_5^2) + B^2 F_1 + H_a F_1^2 - 1, \\
F_2'(x) &= -\lambda \left[F_2 \exp\left(\frac{F_1}{1 + \epsilon F_1}\right) + \sigma\left(1 + \frac{1}{\beta} F_2 + \beta^2 F_1 + H_a F_1^2\right)\right] \\
&- G_c (F_5 + \sigma_2 F_5^2) + B^2 F_1 + H_a F_1^2 - 1, \\
F_3'(x) &= 1 + \frac{1}{\beta} F_2 + \beta^2 F_1 + H_a F_1^2
\end{align*}
\]
\[
(14) \quad (15) \quad (16)
\]
with the corresponding boundary conditions
\[
\begin{align*}
F_1(x) = 0, &\quad F_3(x) = \theta_0, &\quad F_2(x) = 1, &\quad \text{at } x = -1, \\
F_1(x) = 0, &\quad F_3(x) = \theta_0, &\quad F_2(x) = 1, &\quad \text{at } x = +1.
\end{align*}
\]
(17)

**Step 2: Introducing the important parameter p**

For obtaining a system of ODE in a p-parametric family, introducing the continuation parameter p in the system (14–17) carefully as follow:

\[
\begin{align*}
\frac{\partial F_1}{\partial x} &= (1 + \frac{1}{\beta})^{-1}(- G_1(\frac{\partial F_1}{\partial x} + 2\sigma_1 F_3 \frac{\partial F_3}{\partial x} \\
&- G_2(\frac{\partial F_3}{\partial x} - 2\sigma_2 F_5 \frac{\partial F_5}{\partial x}) + 2\beta_1^2 \frac{\partial F_5}{\partial x} - 1) + (F_2 - (F_2 - 1))p \\
\frac{\partial F_3}{\partial x} &= -\lambda_0(\frac{F_3}{1 + eF_3}) + \exp(\frac{F_3}{1 + eF_3}) \frac{\partial F_3}{\partial x} + 2\beta_1^2 \frac{\partial F_1}{\partial x} + (F_4 - (F_4 - 1))p \\
\frac{\partial F_5}{\partial x} &= K_0(1 + eF_5)^{\alpha} \exp(\frac{F_5}{1 + eF_5})(1 + eF_5) \frac{\partial F_5}{\partial x}
\end{align*}
\]

(18)

**Step 3: Differentiating by parameter “p”**

Differentiate by parameter p, equations (18)–(20) take the following form with the parameter p

\[
P' = AP + R,
\]

(21)

Here, R is the remainder, A is the coefficient matrix, and

\[
P = \frac{\partial F_i}{\partial \tau},
\]

(22)

Here \(i = 1, 2, \ldots, 6\).

**Step 4: Applying specify Cauchy problem and superposition principle for each component**

\[
P = aS + T,
\]

(23)

where \(a\)-unknown blend coefficient and \(S, T\)-unknown vector functions.

Solving Cauchy problems for each components.

\[
S' = AS,
\]

(24)

\[
T' = AT + R,
\]

(25)

putting the approximate solution equation (23) into the original equation (21), we obtain

\[
(aS + T)' = A(aS + T) + R,
\]

(26)

**Step 5: Solving the Cauchy problems**

For this task we used Numerical implicit scheme as follows.

From equations (24) and (25)

\[
\frac{S' + 1 - U_i}{\Delta \eta} = AS' + 1, \quad \text{or}
\]

(27)

\[
\frac{(I - \Delta \eta A)S' + 1 = S'}, \quad \text{or}
\]

(28)

\[
\frac{T' + 1 - T'}{\Delta \eta} = AT' + 1 + R, \quad \text{or}
\]

(29)

\[
(I - \Delta \eta A)^T + 1 = T' + \Delta \eta R,
\]

(30)

from where obtaining the iterative form of the solution as:

\[
S' + 1 = (I - \Delta \eta A)^{-1} S',
\]

(31)

\[
T' + 1 = (I - \Delta \eta A)^{-1}(T' + \Delta \eta R).
\]

(32)

**Results and discussion**

The Arrhenius reactive fluid flow between two vertical plates is modeled in the form of nonlinear equations (9)–(11), along with boundary conditions equation (12). The velocity, temperature and concentration profiles, are obtained by using two different numerical schemes PCM and \(bvp4c\), and are displayed through Figures 2 to 11. A comparative analysis has also been presented through figures between PCM and \(bvp4c\) and for the validity of present numerical scheme, the obtained results are compared with previously published work in Table 1. The physical important quantities such as mass transfer, heat transfer rate, and wall shear stress, are calculated against different parameters and depicted through graphs.

Figure 2(a) to (d) depicts the variation of velocity profile for different physical parameters. Due to the dual nature of Casson fluid parameter \(\beta\), it plays an important role in fluid dynamics. For larger values of \(\beta \rightarrow \infty\), it behaves like Newtonian fluid, while for \(\beta = 2\) the fluid becomes as non-Newtonian. Figure 2(a) shows the variation in velocity component due to the increasing values of \(\beta\). The Newtonian characteristics can be observed for the increasing value of \(\beta\), and as a result yield stress drops gradually which enhance the fluid velocity. Figure 2(b) illustrate the impact of thermal buoyancy parameter \(Gr\) on velocity
component of the fluid. Thermal buoyancy is the heat source term in velocity component of the fluid, which is responsible for the thinning effect in the fluid viscosity and as a result of the enhancement in velocity profile. The nonlinear thermal convection parameter $s_1$ measures the non-linearity in density-temperature relation. Increasing the value of $s_1$ means to increase the non-linearity in terms of thermal convection which accelerate the fluid velocity Figure 2(c). A likewise behavior of the velocity component can be observed for nonlinear concentration parameter $s_2$ in Figure 2(d), which give rise in velocity component

\[ \frac{\partial v}{\partial x} = \frac{\partial v}{\partial x} \]

\[ \frac{\partial u}{\partial x} = \frac{\partial f}{\partial x} \]

\[ \frac{\partial v}{\partial x} = \frac{\partial v}{\partial x} \]

\[ \frac{\partial u}{\partial x} = \frac{\partial f}{\partial x} \]

**Table 1.** Comparison of the present result with previous published work, when $Gr = Gc = \sigma_1 = \sigma_2 = B = 1$, $\epsilon = \delta = Kr = f_w = \lambda = 0.1, Ha = 0.5$, and for present work $\beta = So = 0$.

| Adeosun et al.\(^\text{11}\) | Present work |
|------------------------|--------------|
| $\frac{\partial v}{\partial x} = -0.137041$ | $\frac{\partial v}{\partial x} = 0.011381$ |
| $\frac{\partial u}{\partial x} = 0.278365$ | $\frac{\partial u}{\partial x} = 0.01487$ |
| $\frac{\partial f}{\partial x} = 0.274001$ | $\frac{\partial f}{\partial x} = 0.01488$ |
| $\frac{\partial v}{\partial x} = 0.274003$ | $\frac{\partial v}{\partial x} = -2.282311$ |
| $\frac{\partial u}{\partial x} = 0.274004$ | $\frac{\partial u}{\partial x} = -2.293811$ |

**Figure 2.** Velocity profile $v(x)$ for fixed values of physical parameters $\lambda = 0.1, Kr = 1.0, Gr = 0.4, Gc = 0.05, \sigma_1 = 2.1, \sigma_2 = 18.0, \delta = 6.4, B = 0.3, Ha = 0.7, \epsilon = 0.01, So = 7.0, f_w = 1.0$, and $\beta = 2.5$. 
due to non-linearity effect in density-concentration relation.

The predictive behavior of the velocity component $v(x)$, against physical parameters $Gc$, $B$, $Ha$, and $e$ has been examined in Figure 3(a) to (d). Figure 3(a) elucidate the variations in velocity component against different values of concentration buoyancy parameter $Gc$. The mass buoyancy is the ratio of concentration to inertial forces, which dominates the inertial forces by the increasing values of $Gc$. Both thermal-convective buoyancy forces are responsible for downward motion of the fluid due to gravitational acceleration, but this downward velocity is intercepted by the porous nature of the medium, which takes away the fluid particle in $x$-direction. Figure 3(b) shows the impact of porosity parameter $B$ on the velocity component of the fluid. An increase in porosity parameter means to decrease porosity permeability, which brings about a reduction in flow resistance and that is the reason that the velocity component is increased. As a perpendicular constant magnetic field is applied to the system of vertical plates, which creates magnetic field lines in flow direction, according to right hand rule. These magnetic field lines produce Lorentz force, which has greater tendency to enhance the fluid velocity within the system. This prediction of velocity enhancement, due to Hartmann number $Ha$, can be seen in Figure 3(c). Figure 3(d) shows the effect of activation energy parameter $e$ on velocity profile. The increasing value of $e$ brings about a reduction in velocity component, it may be due to the fact that the function $\exp(\frac{\theta}{1+\theta})$ value is decreases.

Frank-Kamenetskii’s reaction rate parameter $\lambda$, depends on Arrhenius kinetics, has the tendency to exaggerate the exothermic chemical reaction, which give rise in internal heat generation. This effect of reaction rate parameter $\lambda$ can be seen in Figure 4(a).
increasing values of \( \lambda \) enhances the ability of internal heat generation due to which the temperature profile is increasing. The activation energy parameter \( e = \frac{RT_0}{E} \) has a direct relation with temperature and it has also the ability to simulate the internal energy of fluid molecules, due to the exothermic reaction, which give rise the internal heat of the fluid, and it can be reason out for the enhancement of temperature profile, Figure 4(b). Figure 4(c) is concerned with the plot of temperature against the increasing values of viscous heating parameter. This can be reason out that an additional heat generation can be detected by the frictional interaction among the fluid particles and with the channel walls, which increases the internal temperature of the fluid. In the present fluid flow modeling the temperature equation (3) has been coupled with velocity equation (2) by adding the viscous term to the temperature equation. Due to this coupling of temperature and velocity, one can observed that the Casson parameter has an inverse relation with fluid’s viscosity within temperature equation. Its means that whenever the value of Casson parameter \( \beta \) is increases the dynamic viscosity will be decrease and as a result the temperature profile will be increased, and these predictions can be easily observed in Figure 4(d).

The impact of porosity parameter, \( B \), Hartmann number, \( Ha \), concentration buoyancy parameter, \( Gc \), and thermal buoyancy parameter, \( Gr \), are sketched, respectively, in Figure 5(a) to (d). Figure 5(a) shows that the temperature profile is increasing with the increasing value of porosity parameter, \( B \). By the increasing value of porosity parameter, the exothermic reaction is created, and as a result the internal heat is generated, which becomes a source of the enhancement of temperature profile. The impact of Hartmann Number, \( Ha \), can be seen in Figure 5(b). This figure

\[ \text{Figure 4. Temperature profile } \theta(x) \text{ for fixed values of physical parameters } \lambda = 0.1, \; Kr = 1.0, \; Gr = 0.4, \; Gc = 0.05, \; \sigma_1 = 2.1, \; \sigma_2 = 18.0, \; \delta = 6.4, \; B = 0.3, \; Ha = 0.7, \; e = 0.01, \; So = 7.0, \; f_w = 1.0, \; \text{and } \beta = 2.5. \]
shows that Hartmann number has an enhancement effect on temperature profile. By applying an external magnetic field on the fluid flow, a drag like force called Lorentz force is generated, that has a tendency to prevents its generation. Higher value of Hartmann number is a source of Lorentz force and that is the reason that higher temperature gradient can be observed at the middle of the domain, due to which the isothermal lines moves upward and indicates higher heat transfer rate in that region. Figures 5(c) and (d) revealed the impact of concentration and thermal buoyancy parameters, respectively, on temperature profile. Both concentration and thermal buoyancy parameters have an inverse relation with viscous dissipation, this means that with the increasing values of both these parameters the thinning property of the fluid is encouraged which leads to the enhancement of temperature profile. This physical argument can be accomplished by Figure 5(c) and (d).

Figure 6(a) to (d) demonstrate the physical behavior of concentration profile under the effects of different parameters. The reactant consumption rate parameter, $Kr$, has an inverse relation with both heat reaction and reactant diffusivity. The increasing values of $Kr$ are responsible to drop the exothermic reaction level within the flow system, due to which the mass transfer is reduced and make a reduction in concentration profile Figure 6(a). Figure 6(b) illustrate the impact of Frank-Kamenetskii reaction rate parameter, $l$, on concentration profile. An increasing values of $l$ cause a reduction in reactive concentration profile. As described earlier, Frank-Kamenetskii’s parameter, depends on Arrhenius kinetics, has an ability to enhance the exothermic chemical within the fluid flow system, which exaggerate the internal heat generation. More fluid’s concentration is consumed for the enhancement of exothermic chemical reaction and it can be reasoned out for the decline of
A similar result has been detected in Figure 6(c), for the increasing values of activation energy parameter, $e$, on concentration profile. The activation energy parameter enhances the internal energy of the fluid particles, and as a result more heat transfer occurs, which may use more fluid’s concentration and eventually a reduction in concentration profile can be observed. The effect of Soret number $So$ is shown in Figure 6(d), which implies a decline in concentration profile with the increasing values of Soret number. This is due to the fact that Soret number has an inverse relation with concentration difference. Hence, the greater values of Soret number leads to lower mass transfer.

Figure 7(a) to (d) also gives a geometrical sketch to concentration profile against physical important parameters. Figure 7(a) and (b) depict the effects of both concentration and thermal buoyancy parameters, $Gc$ and $Gr$, respectively, on concentration profile. Both these parameters are capable to increase the exothermic chemical reaction within the fluid flow that dissipate more concentration and put down the concentration profile. The effects of both nonlinear thermal and concentration convection, $\sigma_1$ and $\sigma_2$, respectively, are shown in Figure 7(c) and (d). Both these nonlinear convection are involved in velocity equation, which is a reason for the enhancement of velocity profile, but it may required more reactant concentration and it can be elucidated for the decline of concentration profile.

The physical interest quantities, such as wall shear stress, heat and mass transfer rates, plays an important role in the field engineering and industry. Figure 8(a) to (d) shows the effects of wall shear stress against various physical parameters. Wall shear stress shows the uniform distribution of fluid flux throughout the surface of the plates. This uniform flux is produced by the electromagnetic induction which carries a uniform time.
varying current. The Casson parameter $b$ behaves like a non-Newtonian liquid, which create a resistance to fluid flux distribution, due to fluid’s viscosity. It can be reason out for the decrease of wall shear stress for the increasing values of $b$ versus concentration buoyancy $Gc$, see Figure 8(a). Figure 8(b) also shows the impact of nonlinear thermal convection $s_1$ versus thermal buoyancy $Gr$ on wall shear stress. When the values of $s_1$ is increases against $Gr$, wall shear stress is decreases, perhaps this is due to the nonlinear nature of $s_1$. The variation of wall shear stress versus Hartmann and nonlinear concentration parameter, for various values of $B$ and $Gr$, are respectively plotted in Figure 8(c) and (d). It is clear that, when the values of porosity parameter $B$ are increased, it create an interception of the fluid velocity, which reduce the wall shear stress Figure 8(c), while the increasing values of thermal buoyancy parameter also decrease the wall shear stress.

From Figure 9(a) and (b) it can be observed that the heat transfer rate is reduce by the increasing values of thermal activation parameter $e$ versus $l$, whilst the heat transfer rate is enhanced by the increasing values of porosity parameter $B$ against viscous heat parameter $d$. The porosity parameter $B$ creates an opposing velocity against thermal buoyancy and hence a reduction in thermal activation energy is detected, which cause a decrease in heat transfer rate for $e$ and an increase for $B$.

Figure 10(a) and (b) shows a decreasing profile of the mass transfer rate for both thermal activation energy parameter $e$ and Soret number $So$, against concentration consumption rate $Kr$ and reaction rate parameter $l$, respectively. The internal energy of fluid particles is enhanced by activation parameter $e$, due to which the heat transfer rate is increased and as a result consume more concentration, for the sake of this
Figure 8. Variation of wall shear stress against different parameters.

Figure 9. Variation of Nusselt number against different physical parameters.
reason the mass transfer rate is reduced. Soret number $So$ has an inverse relation with concentration difference, which cause a decline in mass transfer rate with the increasing values of Soret number.

For further validation of (PCM) method, the obtained results are also compared with previously published work and found an accuracy up to two decimal places. It has been found that Frank-Kameneskii’s parameter, depends on Arrhenius kinetics, has the tendency to enhance the exothermic chemical reaction, which give rise in internal heat generation and heat transfer rate. In order to check the accuracy and reliability of the present results, the modeled equations (7)–(9) along with boundary conditions (10), are solved by two different numerical schemes, PCM and $bvp4c$ package. Figure 11(a) to (c) gives a comparative sketches of velocity, temperature and concentration profiles. Both the numerical results are very similar to each other, but PCM gives a very accurate and more rapid converges results in the sense of less time consumption than $bvp4c$. The validity and accuracy of the present numerical schemes are also reported in Table 1. The present numerical results are also compared with previously published work, and a very excellent agreement is exhibited in Table 1.

**Conclusion**

In the present article, a non-Newtonian Casson, linear and nonlinear thermo-convective Arrhenius reactive fluid flow is modeled between two vertical porous plates under the effects of exothermic chemical reaction and perpendicular applied magnetic field. The dimensionless governing equation of the flow are solved by two different numerical schemes Parametric Continuation Method (PCM) and $bvp4c$ package. The following results has been drawn on behalf of these simulation.

- Frank-Kameneskii’s reaction rate parameter $\lambda$ has the tendency to elevate the exothermic chemical reaction, which give rise in internal heat generation.
- The activation energy parameter $\epsilon = \frac{RT_0}{E}$ is significantly boosts the energy profile.
- The increasing values of $Kr$ are responsible to drop the exothermic reaction level within the flow system, due to which the mass transfer is reduced and make a reduction in concentration profile.
- The porosity parameter $B$ creates an opposing velocity against thermal buoyancy and hence a reduction in thermal activation energy, which cause a decrease in heat transfer rate for $\epsilon$ and an increase for $B$.
- Soret number $So$ has an inverse relation with concentration difference, which cause a decline in mass transfer rate.
- The present research can be extended to sensitized and Bimolecular kinetics by taking the value of $m = -2$ and $m = 0.5$, respectively.
- The proposed model may be extended by taking different types of fluid over numerous type of

![Figure 10. Variation of Sherwood number against different physical parameters.](image)
geometries, or by modifying the proposed idea to nanofluid and hybrid nanofluid. Furthermore, the present model may more generalized by reforming it to fractional derivative.

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Appendix

Notation

| Symbol | Description |
|--------|-------------|
| \( \eta \) | Similarity variable |
| \( \nu \) | Kinematic coefficient of viscosity (m\(^2\)s\(^{-1}\)) |
| \( \alpha \) | Thermal diffusivity |
| \( \rho \) | Density (kg/m\(^3\)) |
| \( \beta \) | Casson fluid Parameter |
| \( \sigma \) | Electrical conductivity (S/m) |
| \( \lambda \) | Heat source or sink parameter |
| \( \theta \) | Dimensionless temperature (K) |
| \( \tau \) | Heat capacity ratio (JK\(^{-1}\)) |
| \( \infty \) | Conditions far away from the surface |
| \( Gr \) | Thermal buoyancy parameter |
| \( Gc \) | Concentration buoyancy parameter |
| \( B_0 \) | Magnetic induction (tesla(B)) |
| \( C \) | Concentration |
| \( C_a \) | Ambient concentration |
| \( C_f \) | Skin-friction coefficient |
| \( c_p \) | Specific heat capacity (JKgK\(^{-1}\)) |
| \( D_B \) | coefficient of Brownian diffusion |
| \( D_T \) | coefficient of Thermophoretic diffusion |
| \( f \) | Dimensionless stream function |
| \( K \) | Porous media parameter |
| \( k \) | Permeability of the porous medium |
| \( M \) | Magnetic field parameter (tesla(T)) |
| \( N_b \) | Parameter of Brownian motion |
| \( N_T \) | Parameter of Thermophoresis |
| \( Nu_x \) | Nusselt number |
| \( Pr \) | Prandtl number |
| \( Q_0 \) | Absorption coefficient (J.kg\(^{-1}\)) |
| \( R \) | Chemical reaction parameter |
| \( R_0 \) | Chemical reaction coefficient |
| \( \phi \) | Dimensionless concentration (mol/m\(^3\)) |
| \( T_w \) | Reference temperature |
| \( So \) | Soret number |