Reply to Comment on ‘Do electromagnetic waves always propagate along null geodesics?’

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Abstract
A reply to the previous article commenting on non-geodesical propagation of electromagnetic fields on gravitational backgrounds and the eikonal limit are presented.

Keywords: couplings, electromagnetic wave, propagation, consistency, geometrical, eikonal limit

Geodesic motion is one of the cornerstones of the construction of general relativity, tightly related to the equivalence principle and general covariance. While those fundamental principles may be safely applied to point-like objects (particles in mechanics or rays in optics) it is not clear that they apply to extended objects such as particles with structure (spinning or otherwise) and/or fields. Numerous theoretical and experimental results obtained by multiple authors, clearly show that (both classical and quantum) dynamics described by fields such as optics and quantum mechanics, exhibit unexpected and surprising non-geodesic propagation (see, for instance, references [1–5] among others).

Two of our articles [6, 7] are critically reviewed in [8] (preceding article). While most of the results found in [8] agree with ours, there is one of them, the problem of the eikonal limit
of light waves propagating on a cosmological Gödel background, in which the results seem to be at odds. In the present reply article we would like to address this point. We focus on the problem of the appropriate definition of the eikonal limit for light waves propagating on a given gravitational background.

The eikonal limit of the dispersion relation for light propagating in a Gödel spacetime is discussed in reference [8]. In it, it is argued that in reference [7] we state the inconsistency of light propagation on this background in the high-frequency limit, when $\omega \to \infty$. However, this is not so, and the claim made in reference [8] results in a misinterpretation about the meaning of the high-frequency limit. It is important to remark that the preceding article has given us the chance to clarify this point and we thank the authors for their very careful scrutiny of our previous work.

We use the nomenclature of optics and plasma physics where it is a standard practice to refer to the high-frequency limit as the case in which the phase variables of the wave, such as the frequency or the wavevector, are larger than other relevant physical variables involved in the process, such as the amplitude variations of the wave or the frequency response of the medium. In the case of light propagating on a Gödel spacetime, studied in reference [7], the dispersion relation for light is given by

$$K_\mu K^\mu = g^{00} \omega^2 + K_1^2 = -\frac{\Omega^2}{2} - \frac{K_0^2}{2K_x} + \frac{3K_x^2}{4K_1^2},$$

where $K_\mu$ is the covariant four-wavevector of light, $K_0 = \omega$ is its frequency, $K_x$ is the spatial wavevector along $x$ (the propagation direction), $\Omega$ is the constant related to the angular velocity of the rotating Gödel Universe, and the symbol $'$ denotes the spatial derivative along the propagation direction $x$. Note that this dispersion relation is written without the $g^{00}$ factor in reference [8].

The null geodesic motion of light, $K_\mu K^\mu = 0$ [9], is achieved in the high-frequency limit, i.e., when

$$\omega \text{ or } K_x \gg \Omega \text{ or } \frac{1}{L},$$

where $L = K_x/K'_x$ are the scale-length of variations of the wavevector. In reference [7], it is proved that the amplitude of a light wave in this spacetime is proportional to $(-gK_1^2)^{-1/4}$. Therefore, the scale-length of variation of the amplitude is anew proportional to $1/\Omega$ and $L$. In this way, the high-frequency limit is achieved when the wavelength of light is much smaller to any variation of the amplitude or the scale $1/\Omega$ associated to Gödel spacetime. This is explicitly said after equation (18) in reference [7]. In this sense, the correct null geodesic limit is obtained when $\Omega/\omega \ll 1$, as it is stated in reference [8]. Therefore, the expression $\omega \to \infty$ was used to indicate that frequency is the larger relevant physical quantity in the propagation problem.

In our work [7] we do not claim that the high-frequency limit is inconsistent. The title of one of our papers clearly implies that electromagnetic waves do not always propagate along null geodesics [7]. We acknowledge that our choice of words was inappropriate.

On the contrary, in reference [6] we stress that in the geometrical optics approximation (the eikonal or high-frequency limit) the null geodesic behavior of light is always possible on any curved spacetime. In both of our articles, we simply wanted to state that non-null geodesic propagation of light is just one of the possible solutions for light in some curved spacetimes,
which occurs due to the coupling of the extended properties of light waves with the curvature of spacetime.

Finally, we would like to emphasize that we agree with the three criteria cited in the preceding article showing that light may propagate along null geodesics, on any spacetime, in the eikonal limit. Our main point is that, besides null geodesic propagation of electromagnetic waves, there are other electromagnetic waves solutions which behave differently, as it can be easily understood in the case of birefringence [6], where at least one of the waves do not present null geodesic behavior.

Thus, the results of reference [8] confirm the possibility of non-null geodesic propagation of light discussed in references [6, 7].

It is important to state that our results concerning non-geodesic behavior of electromagnetic waves propagation on gravitational backgrounds, hold when the propagation is described beyond the geometric optics/eikonal/high frequency limit, where electromagnetic waves do not behave as it is usually expected, according to conventional wisdom.

Data availability statement

No new data were created or analysed in this study.

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