Model for two generations of fermions

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In the model with the spontaneous breaking of chiral gauge symmetry, the vacuum structure for the pair of Higgs fields can provide the introduction of two generations of fermions. The mixing matrix of charged currents is determined.

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I. INTRODUCTION

Two generations of fermions, say, electron and muon, have identical properties with respect to a gauge interaction and differ by the mass values yet. In the Standard Model [1] the masses are caused by the spontaneous breaking of gauge symmetry [2], so that the difference between the masses corresponds to the variation of Yukawa constants for the interaction between the fermions and Higgs fields. The values of those constants and the origin for the replication of generations are the problems, which are studied beyond the Standard Model.

At present, the origin of generations is usually described [3] in the framework of

1. composite models with a discrete symmetry of preons [4], which results in a general form of mass-matrices for the effective fermions coupled to the complex bosonic fields,

2. random lattice dynamics of gauge interactions [5],

3. geometric nature of generations: superstring models [6] in a space of extended dimensions compactified at low energies [7].

Thus, the appearance of fermion replications is caused by the dynamics of some extended set of fundamental fields generally including effective Higgs fields, which have the same number of generations as the fermions.

In this work we describe the model of fermions possessing a chiral gauge interaction and coupled to two Higgs fields. The latter ones have a self-action potential with a particular properties of minimal energy corresponding to the special symmetry of the vacuum. So, there are only two configurations of minimum, which are not related by the gauge transformation and give two values of fermion masses, correspondingly. The full vacuum state including these two configurations is symmetric over the action of discrete cyclic group. There is no physical principle to restrict the theory by the only configuration. Thus, the symmetric vacuum provides the introduction of two generations of fermions interacting with the gauge field.
The model of fermion interaction and the Higgs potential are constructed in Section II. The introduction of generations is described. The model with the charged currents is considered in Section III, where the mixing matrix is determined. The results are summarized in Conclusion.

II. MODEL

Consider the chiral interaction of dirac spinor with the abelian gauge field $A_\mu$ and Higgs field $h$

$$L = \bar{\psi}_R p_\mu \gamma^\mu \psi_R + \bar{\psi}_L (p_\mu - e A_\mu) \gamma^\mu \psi_L + g \bar{\psi}_R \psi_L h + g \bar{\psi}_L \psi_R h^* + L_{GF} + L_0 (h, A),$$  

(1)

where $L_{GF}$ is the Lagrangian of free gauge field (with a possible introduction of term fixing the gauge and the corresponding Lagrangian for the Faddeev–Popov ghosts), $L_0$ is the Lagrangian of Higgs field with a self-action and the gauge interaction, $g$ is the Yukawa constant. At the spontaneous breaking of symmetry, the vacuum expectation of Higgs field is not equal to zero

$$\langle 0 | h | 0 \rangle = \eta e^{i\phi}.$$  

(2)

In the unitary gauge $\phi = 0$, the fermion becomes massive, so $m = g\eta$

$$L_m = g\eta (\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R).$$  

(3)

In the arbitrary gauge, it is convenient to use the method of auxiliary fields, so that, for example,

$$f_L = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \psi_L \\ e^{i\phi} \psi_L \end{array} \right), \quad f_R = \frac{1}{\sqrt{2}} \left( \begin{array}{c} e^{-i\phi} \psi_R \\ \psi_R \end{array} \right),$$  

(4)

and

$$L_m = \tilde{f}_R M_{RL} f_L + \tilde{f}_L M_{LR} f_R,$$  

(5)

where

$$M_{RL} = M_{LR} = g\eta \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right).$$  

(6)

By the procedure, the kinetic term and the gauge interaction of $f$ with the $A$ field are not changed. Thus, one can see in (3), that we have the field with the mass $m = g\eta$, as one must expect from the unitary gauge (3).

It is convenient to introduce the following fields

* The appearance of mass for the gauge field is not an object under consideration here.
\[ \tilde{f}_L = \frac{1}{\sqrt{2}} \left( \psi_L + i\psi_L \right), \quad \tilde{f}_R = \frac{1}{\sqrt{2}} \left( \psi_R - i\psi_R \right), \]  

(7)

so that one has

\[ \tilde{M}_{RL} = \tilde{M}_{LR} = g\eta \begin{pmatrix} \cos\phi & \sin\phi \\ \sin\phi & -\cos\phi \end{pmatrix}, \]  

(8)

with the eigenvalues \(|\lambda| = m = g\eta\).

Now consider the analogous interaction of fermions with two higgses \(h_1\) and \(h_2\). For the latter ones at the spontaneous breaking of gauge symmetry, the vacuum expectation values are equal to

\[ \langle 0| h_1 |0 \rangle = \eta_1 e^{i\alpha}, \quad \langle 0| h_2 |0 \rangle = \eta_2 e^{i\alpha + i\phi}, \]  

(9)

where \(\alpha\) is an arbitrary gauge parameter, \(\phi\) is the difference between the phases of VEV, so that this difference can be observable. Further, the introduction of fields (7) results in the appearance of fermion mass \(m\) depending on \(\phi\)

\[ \frac{1}{g^2}m^2 = (\eta_1 + \eta_2 \cos\phi)^2 + \eta_2^2 \sin^2\phi = (\eta_1 + \eta_2)^2 - 4\eta_1\eta_2 \sin^2\phi. \]  

(10)

The following situation is of a special interest: the phase difference \(\phi\) is constrained due to the form of higgs self-action, so that the vacuum minimum of the potential takes a place at some discrete values of \(\phi\).

In the model under consideration, the higgs potential has the form

\[ V = \frac{\lambda_{11}}{4} (h_1 h_1^*)^2 - \frac{\mu^2}{2} h_1 h_1^* + \frac{\lambda_{22}}{4} (h_2 h_2^*)^2 - \frac{\lambda_{12}}{8} [h_1 h_1^* + h_2 h_2^*]^2, \]  

(11)

where all of \(\lambda_{ij}\) and \(\mu^2\) are greater than zero, so that the constraint of the potential stability \((V > 0 \text{ at infinity})\) gives \(\lambda_{11}\lambda_{22} > \lambda_{12}^2\).

Then the constraints on a stable minimum of static energy,

\[ \frac{\partial V}{\partial |h_1|} = 0, \quad \frac{\partial V}{\partial |h_2|} = 0, \]  

result in the vacuum expectation values equal to

\[ \eta_1^2 = \frac{\mu^2}{\lambda_{11} - \frac{\lambda_{12}^2}{\lambda_{22}} \cos^4\phi}, \]  

(12)

\[ \eta_2^2 = \frac{\lambda_{12}}{\lambda_{22}} \eta_1^2 \cos^2\phi. \]  

(13)

Then the minimum energy is equal to

\[ V_{\text{min}} = -\frac{1}{4} \frac{\mu^4}{\lambda_{11} - \frac{\lambda_{12}^4}{\lambda_{22}} \cos^4\phi}. \]  

(14)
Therefore, in the vacuum state one has \( \cos^2 \phi = 1 \). Thus, in the specified model of higgs self-action the phase difference between the vacuum expectations runs over the discrete values

\[ \phi = \pi n, \quad n \in \mathbb{Z}. \]  

(15)

Then introducing fields (4), one gets

\[ M_{RL} = M_{LR}^\dagger = g \begin{pmatrix} \eta_2 & \eta_1 e^{-2i\phi} \\ \eta_1 & \eta_2 \end{pmatrix}, \]  

(16)

and at fixed \( \phi \) (15) the real matrix \( M_{RL} = M_{LR} \) has the eigenvalues corresponding to the fermion masses

\[ m_{1,2} = g|\eta_1 \pm \eta_2|. \]  

(17)

Further, introducing

\[ U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, \]  

(18)

one gets the diagonal form of the mass-matrix

\[ M_{LR}^U = M_{RL}^U = U \cdot M_{LR} \cdot U^\dagger = g \begin{pmatrix} \eta_2 + \eta_1 & 0 \\ 0 & \eta_2 - \eta_1 \end{pmatrix} \]

with the eigen-vectors \( f_{L,R}^U = U f_{L,R} \).

Let us emphasize that, first, the introduced auxiliary fields have the particular form

\[ f_L^U = \begin{pmatrix} \psi_L^{(1)} \\ 0 \end{pmatrix}, \quad f_R^U = \begin{pmatrix} \psi_R^{(1)} \\ 0 \end{pmatrix}, \]

at \( \phi = \phi^{(1)} = 0 \), so that acting on the minimum-energy configuration \( |0^{(1)}\rangle \) the field \( \psi^{(1)} \) results in the state with the mass \( m^{(1)} = g(\eta_1 + \eta_2) \). Second, one obtains

\[ f_L^U = \begin{pmatrix} 0 \\ -\psi_L^{(2)} \end{pmatrix}, \quad f_R^U = \begin{pmatrix} 0 \\ \psi_R^{(2)} \end{pmatrix}, \]

at \( \phi = \phi^{(2)} = \pi \), so that the \( \psi^{(2)} \) field has the mass \( m^{(2)} = g|\eta_1 - \eta_2| \) over the configuration \( |0^{(2)}\rangle \). Therefore, if one constructs the model vacuum as

\[ |\text{vac}\rangle = |0^{(1)}\rangle \otimes |0^{(2)}\rangle = \begin{pmatrix} |0^{(1)}\rangle \\ |0^{(2)}\rangle \end{pmatrix}, \]  

(19)

which is symmetric over the action by the discrete second-order cyclic group, changing the complex phase differences between the VEV of higgses, then one can use two kinds of auxiliary fields at \( \phi = 0 \) and \( \phi = \pi \), having the mentioned particular form and the common mass-matrix independent of the chosen values of \( \phi \), to introduce the common physical field
with the components corresponding to the kind of generation. So, the extended Lagrangian contains the physical field $f$ over the vacuum $|\text{vac}\rangle$ with two generations.

The structure of the Higgs fields vacuum cannot change the number of fermionic degrees of freedom, so, it cannot be the origin of generations, since it cannot produce these fermion fields. Two generations are introduced by construction. However, the considered model of Higgs vacuum certainly provides the introduction of two generations: the Higgs vacuum is arranged for the introduction of two generations of fermions. So, the number of fermionic degrees of freedom is not changed in an original Lagrangian, since it is introduced in the self-consistent way.

It is correct that in a gauge theory the only possible vacuum configuration can be chosen because the physical meaning of gauge invariance is the equivalence of those configurations: the physics in each configuration is the same, and the only configuration must be chosen, the others are produced by the gauge transformations. So, the physical principle for the restriction by the only configuration is the gauge invariance. However, that is not the case of the paper model under consideration. The constructed configurations of minimal energy are not gauge equivalent, since they differ by the phase difference which is the observable physical quantity. Therefore, the full vacuum of the theory must contain all of configurations because there is no physical principle ("a rule of superchoice") to cancel some configuration and to restrict the state. So, the paper describes the theory, where all of configurations are symmetrically presented in the vacuum.

Thus, the pair of fermion species, possessing the identical properties over the gauge interaction and having the different masses, are introduced due to two configurations of symmetric vacuum for the pair of Higgs fields, so that in this model the effect of two generations of fermions is provided by the structure of vacuum for the Higgs fields.

From the practical point of view, equation (17) shows that the large splitting of fermion masses for two generations ($e, \mu$) can be caused by no difference between the values of Yukawa constants for the interaction between the fermions and higgs. The mass difference can be the result of small splitting between the vacuum expectation values† (at $\eta = \eta_1 > \eta_2 > 0$, $\Delta\eta = \eta_1 - \eta_2 \ll \eta$)

$$\frac{m_1}{m_2} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} \approx \frac{\Delta\eta}{2\eta} \ll 1.$$  \hspace{1cm} (20)

Thus, we have shown that at the spontaneous breaking of symmetry in the interaction of fermion field with the pair of higgses, the structure of higgs vacuum can provide the introduction of two generations of fermions.

† Supposing $m_1/m_2 = m_e/m_\mu$, one finds that $\Delta\eta/\eta \approx 1/103 \sim \alpha_{\text{em}}$, so that, probably, the splitting is of the order of radiative corrections.
III. MIXING OF CHARGED CURRENTS

The introduction of auxiliary fields \((\bar{f})\) with the symmetric mass-matrix at \(\phi = 0, \pi\) contains the uncertainty related with an additional term

\[
\Delta M = \begin{pmatrix} -a & 0 \\ 0 & a \end{pmatrix},
\]

which does not contribute to the determination of the mass values, since it is canceled in the Lagrangian expressed through the initial single-generation fields. This uncertainty corresponds to the rotation of the introduced physical fields in the Lagrangian with two generations. So, considering the general form of extended Lagrangian with different Yukawa constants \(g_1\) and \(g_2\) for the Higgs fields \(h_1\) and \(h_2\), one gets the following expression for the mass-matrix\(^\dagger\)

\[
M = \begin{pmatrix} v_2 - v_1 \sin 2\theta & v_1 \cos 2\theta \\ v_1 \cos 2\theta & v_2 + v_1 \sin 2\theta \end{pmatrix},
\]

(21)

where \(v_i = g_i\eta_i\), so that the eigenvalues of the matrix are the same \(m_{1,2} = |v_1 \pm v_2|\), which are independent of the \(\theta\) value. However, there is a particular state, when \(\theta\) is determined by the physical quantities of model. So, we find\(^\sharp\)

\[
M = \begin{pmatrix} v - a & v \\ v & v + a \end{pmatrix},
\]

(22)

where \(v = v_2\), \(a = v \tan 2\theta\),

\[
\cos 2\theta = v_2/v_1 = \frac{g_2}{g_1}\sqrt{\lambda_{12}/\lambda_{22}}.
\]

Note, that matrix (22) has the symmetric texture:

\[
M(1, 2) = M(2, 1) = \frac{1}{2}[M(1, 1) + M(2, 2)].
\]

The \(\theta\) value is related with the masses of fermions

\[
\tan 2\theta = 2 \frac{\sqrt{m_1m_2}}{m_1 - m_2}.
\]

(23)

The matrix gets the ”see-saw” form in the ”heavy” basis \(^\S\)

\(^\dagger\)The number of relative phases in the set of couplings \(\{g_1, g_2, \ldots, g_n\}\) and fields \(\{h_1, h_2, \ldots, h_n\}\) is equal to \(n_\delta = (n - 1)(n - 2)/2\), so that for \(n = 2\) we have the situation, when \(n_\delta = 0\), and \(\{g_1, g_2\}\) are real and positive.

\(^\sharp\)We consider the case of \(v_2/v_1 \leq 1\). A description of the inverse condition is quite evident after a suitable transformation of \(\psi_R\).
\[ M^U = U \cdot M \cdot U^\dagger = \begin{pmatrix} 2v & a \\ a & 0 \end{pmatrix}, \] (24)

where \( U \) is defined in eq.(18). At \( v_1 = v_2 \) we have \( a = 0 \), and the mass-matrix (22) is a "democratic" one \[8\].

Next, the model matrix (24) takes the diagonal form after the action by the rotation to the angle \( \theta \).

Let us consider now the model with two kinds of the chiral fields \( \psi^{(u)} \) and \( \psi^{(d)} \) possessing different charges, so that the charge current interaction between the latter ones has the form

\[ L_{cc} = e \bar{\psi}_L^{(u)} W^\mu \gamma^\mu \psi_L^{(d)} + \text{h.c.}, \]

where the initial mass-matrices have the form (22) with

\[ \cos 2\theta^{(u)} = \frac{g_2^{(u)}}{g_1^{(u)}} \sqrt{\frac{\lambda_{12}}{\lambda_{22}}}, \quad \cos 2\theta^{(d)} = \frac{g_2^{(d)}}{g_1^{(d)}} \sqrt{\frac{\lambda_{12}}{\lambda_{22}}}. \]

Then the Cabibbo mixing matrix has the form

\[ V = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix}, \]

where \( \theta_c = \theta^{(u)} - \theta^{(d)} \). At \( \frac{m_2^{(u)}}{m_1^{(u)}} \ll \frac{m_2^{(d)}}{m_1^{(d)}} \ll 1 \) one gets the Cabibbo angle

\[ \sin \theta_c \approx \sqrt{\frac{m_2^{(d)}}{m_1^{(d)}}}. \]

Thus, the offered model provides the introduction of two generations due to the structure of vacuum for two Higgs fields. It describes also the mixing matrix of charged currents through the physical quantities, that allows one to relate the mixing angle with the masses of fermions.

**IV. CONCLUSION**

In this paper we have constructed the potential of two Higgs fields, which have the discrete symmetry of the vacuum state. Two observable differences between the phases of the vacuum expectation values exist only, which results in two possible values of mass for the fermion fields coupled to the higgses. These two configurations of symmetric vacuum provide the introduction of two generations of fermions.

In the presence of charged currents the mixing angle can be related with the mass ratios determined by the coupling constants of the model.

Certainly, the construction of a realistic model for three generations of leptons and quarks is of a special interest beyond the scope of this paper. Note, that at \( \eta_1 = \eta_2 \) the mass matrix becomes symmetric under the permutations of its elements. This texture is usually referred to the "democracy" pointing out the equal contribution of different flavors \[8\].
form provides the observed regularity of masses for the lepton and quark generations, as the single generation is heavy only, when two others can be considered to be massless in the leading approximation. An introduction of corrections breaking down the permutation symmetry allows one to get some relations for both the fermion masses and the elements of Cabibbo–Kobayashi–Maskawa matrix.

[1] S.Weinberg, Phys. Rev. Lett. 19, 1264 (1967);
   A.Salam, In Proceedings of 8-th Nobel Symp., Stokholm, 1968, p.367;
   S.L.Glashow, J.Iliopoulos, I.Maiani, Phys. Rev. D2, 1285 (1970).
[2] P.N.Higgs, Phys. Lett. C12, 132 (1964);
   F.Englert, R.Brout, Phys. Rev. Lett. 13, 321 (1964);
   D.S.Guralnik, C.R.Hagen, T.W.Kibble, Phys. Rev. Lett. 13, 385 (1964).
[3] R.D.Peccei, preprint UCLA-97-TEP-31 [hep-ph/9712422] (1997).
[4] H.Harari, N.Seiberg, Phys. Lett. B102, 263 (1981);
   S.Weinberg, Phys. Lett. B102, 401 (1981);
   S.Adler, preprint IASSNS-HEP-96/104 [hep-th/9610190] (1996), preprint IASSNS-HEP-97/125 [hep-ph/9711393] (1997).
[5] C.D.Froggatt, H.B.Nielesen, Nucl. Phys. B147, 727 (1979);
   C.D.Froggatt, H.B.Nielesen, Origin of Symmetries (World Scientific, Singapore, 1991);
   H.B.Nielesen, N.Brene, Nucl. Phys. B236, 167 (1984);
   D.L.Bennet, H.B.Nielesen, I.Picek, Phys. Lett. B208, 275 (1988).
[6] M.Green, J.Schwarz, E.Witten, Superstring Theory, Vols. 1 & 2 (Cambridge University Press, Cambridge, U.K. 1987).
[7] C.Wetterich, Nucl. Phys. B223, 109 (1983);
   E.Witten, in Proceedings of the 1983 Shelter Island Conference II (MIT Press, Cambridge, Mass., 1984);
   A.Candelas, G.Horowitz, A.Strominger, E.Witten, sf Nucl. Phys. B258, 46 (1985);
   B.R.Greene, K.H.Kirlin, P.J.Miron, G.G.Ross, Nucl. Phys. B278, 667 (1986), B278, 606 (1987).
   V.Lucas, S.Raby, Phys. Rev. D55, 6986 (1997).
[8] H.Fritzsch, Phys. Lett. 73B, 317 (1978), Nucl. Phys. B155, 189 (1979);
   H.Harari, H.Hant, J.Weyers, Phys. Lett. B78, 459 (1978);
   H.Fritzsch, J.Plankl, Phys. Lett. B237, 451 (1990);
   P.Kaus, S.Meshkov, Mod. Phys. Lett. A3, 1251 (1988), A4, 603, Phys. Rev. D42, 1863 (1990);
   Y.Koide, Phys. Rev. D28, 252 (1983), D39, 1391 (1989), D46, 2121 (1992);
   Y.Koide, Phys. Rev. D28, 252 (1983), D39, 1391 (1989), D46, 2121 (1992);
   P.S.Gill, M.Gupta, Phys. Rev. D56, 3143 (1997);
   P.M.Fishbane, P.Kaus, Z. Phys. C75, 1 (1997).