Propagation properties of the accelerating beams generated by discrete Airy-Vortex phase mask

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Abstract. We present a novel type of accelerating beam generated by a discrete Airy-Vortex phase mask based on the digital holographic technology. The study shows that the main lobe and the side lobes of such beam rotate with different angular momentums and the whole beam evolves into two separated Airy-like beams in the far field. The intensity distribution of the main lobe and the side lobes in the near field can be modulated by tuning the topological gradient $\Delta L_1$ and $\Delta L_2$ independently. The propagation path of the main lobe follows a parabolic trajectory. The experimental results are consistent with the numerical simulations.

1. Introduction

In recent years, the study of the accelerating beams has attracted tremendous attentions. The airy beam, a most representative accelerating beam, was first theoretically demonstrated by Berry and Balazs in 1979 and experimentally observed by Christodoulides et al. in 2007 [1, 2, 3]. In 2010, the projectile motion of two-dimensional truncated airy beams in a general ballistic trajectory with controllable range and height was reported by Chen et al [4]. Fan et al studied a pair of symmetrically inverted twin Airy beams in which the slope factor can regulate the spacing between the two Airy beam peaks in 2013 [5]. The accelerating quasi-Airy beam propagates along arbitrarily appointed parabolic trajectories and can be readily manipulated with tunable phase patterns, which was reported in 2016 [6]. Lately, Ren et al constructed and generated symmetric accelerating caustic beams which are a type of bimodal accelerating caustic beam with two quasi-constant intensity peaks, very similar to the combination of two face-to-face Airy-like beams judging by appearance [7]. All the studies made it possible for the acceleration beams to be employed in many applications, such as optical trapping [8], self-bending plasma channels [9], and electron capture and acceleration driven [10]. It is worth mentioning that a compound accelerating beam, Airy-Vortex beam, can be generated by introducing the vortex term into a Airy beam. The propagation of such compound accelerating beam in free space has been studied [11]. The results show that the vortex will affect the intensity distribution and the phase distribution. Recently, Ref. [12] reported a new type of accelerating beam generated by a discrete phase mask, in which the factors of the discrete phase mask can modulate the intensity distribution of the whole beam.
In this paper, a novel method to modulate the different parts of an accelerating beam generated by a discrete Airy-Vortex phase mask (DAVPM) based on the holographic technology is proposed. Furthermore, the propagation properties of the accelerating beam is studied numerically and experimentally.

2. Theory
To obtain the accelerating beam and modulate the main lobe, we propose a discrete Airy-Vortex phase mask (DAVPM),

\[
\varphi(k_r) = \begin{cases} 
\exp(i l_{n_1} \theta), & k_R \frac{n_1-1}{2N} < k_r < k_R \frac{r}{2N} \\
\exp(ik_x^3/8 + ik_y^3/8)\exp(i l_n \theta), & k_R \frac{n-1}{N} < k_r < k_R \frac{r}{N} \\
0, & \text{others,}
\end{cases}
\] (1)

where \(n_1\) is the layer numbers of central circle part of the DAVPM and equals to 1,2, the central part splits into two layers. The \(\exp(i l_{n_1} \theta)\) denotes the discrete phase term of the optical vortice with different topological charges \(l_{n_1}\) at two layers of the central circle part, where \(l_{n_1} = l_1 + \Delta L_1, l_1 = 1, \Delta L_1\) is the azimuthal discrete parameter representing the topological gradient and \(\theta\) is the azimuthal coordinate. The \(\exp(ik_x^3/3 + ik_y^3/3)\) is the cubic phase of the Fourier spectrum of Airy beam expressed in \(k_x\)-\(k_y\) space, \(k_r = \sqrt{k_x^2 + k_y^2}\) and the value of \(k_R\) is set to 12. The \(\exp(i l_n \theta)\) denotes the discrete phase term of the rest part, which has the same meaning with the discrete phase term of the central circle part. Where \(N\) is the radial discrete parameter representing the total layer number, \(l_n = 1 + (n - 1)\Delta L\), the integer \(n\) is the layer number of the rest part and equals to 2 to \(N\), the \(\Delta L\) has the same meaning of \(\Delta L_1\). Consequently, \(\Delta L_1\) can modulate the central circle phase distribution of the DAVPM and \(\Delta L\) can modulate the rest part.

Interfered with a plane wave, such a DAVPM can be used to fabricate a digital hologram [13, 14] with a transmission \(T = \exp\{i\frac{[\varphi(k_r)]\arg[\varphi(k_r)]}{\max[|\varphi(k_r)|]} + iP(k_x + k_y)\}\), where \(P\) is used to separate the diffraction orders and set as 10.

Clearly, the first-order diffraction of the digital hologram, \(U_{F1}\) (in \(k\) space), contains all the information of the DAVPM after a plane wave is launched onto the hologram based on the holographic technology. According to the classical optics, the generation of the accelerating beam can be obtained by an inverse Fourier transform through \(L_\Lambda\) and the propagation of the beam can be described by the Fresnel diffraction. The light field \(u_1(x, y, z)\) in real space follows,

\[
u_1(x, y, z) = \text{FresT}_{f+z}(\text{FresT}_f(U_{F1}) \times F),
\] (2)

where \(x\) and \(y\) are the transverse coordinates, \(z\) is the longitudinal coordinate, \(f\) and \(F = \exp[-ik(x^2+y^2)]\) is focal length and modulated function of the Fourier transform lens respectively, and \(k\) is the wave vector. The operator \(\text{FresT}_z\) denotes the Fresnel diffraction integral from \(u_0(x_0, y_0, 0)\) to \(u(x, y, z)\),

\[
\text{FresT}_z(u_0) = \frac{ik}{2\pi|z|} \int \int u_0 \exp\{-\frac{ik}{2|z|}[(x-x_0)^2+(y-y_0)^2]\}dxdy.
\] (3)

3. Experimental setup
Figure 1 shows the schematic of the experiment setup. In the experiment, a 532 nm plane wave is launched onto the spatial light modulator (SLM) through a beam splitter (BS) after expansion by lenses \(L_1\) and \(L_2\). The hologram which are computer calculated by the DAVPM (see the inset in Fig. 1) is exerted on the SLM. To perform the Fourier transform, the SLM is
put on the front focal plane of the Fourier transform lens $L_3$ with a focal length of $f = 150$ mm. A spatial filter (F) on the back focal plane is used to preserve the first-order diffraction and block the other-orders. The propagation dynamics of the produced beam can be monitored by a charge-coupled device (CCD) camera.

**Figure 1.** The schematic of the experimental setup. L, Fourier transform lens; SLM, spatial light modulator; BS, beam splitter; M, plane mirror; F, filter; CCD, charge-coupled device;

4. **Experimental results and numerical simulations**

4.1. **Modulation to the main lobe**

Figure 2 show the impact of $\Delta L_1$ on the light field. When $\Delta L_1 = 2$, $\Delta L = 5$ and $N = 12$, the simulation (the first row) and experimental results (the second row) of the beam along the longitudinal direction $z$ are given, respectively. At $z_0 = 0$mm, the main lobe of the produced beam is a dipole mode and the side lobes are consisted of four discontinuous arms. In the near field, the dipole mode main lobe rotates slowly just like the rotating phenomena in [15]. During the propagation along the $z$ axis, the vortex in the main lobe vanishes gradually [see (a1)-(a4) and (b1)-(b4) in Fig. 2]. Simultaneously, the four discontinuous side lobes also rotate slowly around the main lobe. Apparently, the rotating angles $\beta_1 \neq \alpha_1$ shows that the main lobe and the side lobes rotate with different angular momentums. In the far field ($z = 81$mm), the produced beam evolves finally into two Airy-like beams [see (a6) and (b6) in Fig. 2]. The transmission trajectory of the produced beam can be described in Fig. 3, where the black solid line denotes the simulation result and the red cross symbol (same as the white cross in second row in Fig. 2) denotes the experimental results. From Fig. 2 and Fig. 3, it is obvious that the main lobe of the beam accelerates during the propagation on a parabolic path. Our simulation results are in good agreement with the experimental results.

The third and bottom rows in Fig. 2 show the simulation results and corresponding experimental results of the evolutionary patterns of the generated beam by the DAVPM with $\Delta L_1 = 3$, $\Delta L = 5$ and $N = 12$ along the longitudinal direction $z$, respectively. By changing $\Delta L_1$ from 2 to 3, the main lobe changes to tripple mode which also rotates slowly during the near field while the side lobes is unchanged [see Fig. 2 (c1)-(c3) and (d1)-(d3)]. Furthermore, the produced beam also evolves into two Airy-like beams after the vortex vanished in the far field ($z = 81$mm). This is almost the same as that in the aforementioned case with $\Delta L_1 = 2$ [see Fig. 2 (c4)-(c6) and (d4)-(d6)].
4.2. Modulations to the side lobe

Figure 4 shows the impact of $\Delta L$ on the side lobes. The parameters $\Delta L$, $N$ and $\Delta L_1$ respectively equal to 2, 12, and 2. At $z = 0\text{mm}$, the opening angle between the adjacent side lobes $\alpha 3$ decrease clearly compared with $\alpha 1$ and $\alpha 2$. In the near field, the rotation angle $\beta 3 \approx \beta 1$, denoting that the rotation angle of the dipole mode main lobe is independent on $\Delta L$ [see Fig. 4 (a1)-(a3) and Fig. 2 (a1)-(a3)]. Furthermore, after the vortex of the main lobe vanished in the far field ($z = 81\text{mm}$),
the produced beam finally evolves into two Airy-like beams, too. Note that the light intensity of the inner Airy-like beam is pretty stronger than that of the outer one due to the impact of the smaller $\Delta L$ [see Fig. 4 (a4)-(a6) and (b4)-(b6)].

**Figure 4.** The evolutionary patterns of the accelerating beam with $\Delta L_1 = 2$, $\Delta L = 2$ and $N = 12$ along the $z$ axis. Top row, simulations. Top right in (a6), corresponding DAVPM. Bottom right in (a6), corresponding three dimensional field distribution at $z_3$. Bottom row, corresponding experiments.

5. Conclusion
In conclusion, we have proposed a novel kind of accelerating beam which can be generated by a tunable DAVPM with holographic technology. In the near field, the main lobe and the side lobes rotate slowly with different angular momentums. Such accelerating beam finally split into two separated Airy-like beams in the far field ($z = 81$mm). The whole propagation of the main lobe follows along a parabolic trajectory. Particularly, the intensity distribution of the main lobe in the near field can be controlled by tuning the factor $\Delta L_1$ of the DAVPM, and the opening angle between side lobes can be modulated by the factor $\Delta L$. Furthermore, after the vortex of the main lobe vanished in the far field ($z = 81$mm), the factor $\Delta L$ mainly modulates the intensity distribution of two airy-like beams. Our experimental observations are in accord with the numerical simulations. The intriguing characteristics of the accelerating beam may open up new avenues for optical field manipulation.

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