Renormalized Wolfram model exhibiting non-relativistic quantum behavior

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Abstract

We show a Wolfram model whose renormalization generates a sequence of approximations of a wave function having the Pauli-x matrix as Hamiltonian.

1 Introduction

According to the principle of computational irreducibility, [Wol02], generically, computations that are not obviously simple cannot be made shorter. This result prevents developing a theory powerful enough to verify any possible prediction in a complex system before its execution. Nevertheless, by coarse-graining the complex system, it may be possible to get a new system that is simple enough to be predictable. For example, Navot Israeli and Nigel Goldenfeld [IG06] were able to make predictions in computationally irreducible cellular automata after applying coarse-graining. This technique, known as renormalization, was widely used by Didier Sornette et al. [SS95, SS96, ALFSS95, SS06, SS99, ZS03, Sor89, GS02] to make predictions in multidisciplinary fields. For an introduction to renormalization, the author recommends Didier Sornette’s book [Sor06].

In the present work, we propose an alternative approach to the quantum mechanics of the Wolfram model based on renormalization. In this approach,
we apply coarse-graining to the structures presented by Stephen Wolfram [Wol20] in his physics project, and we obtain a sequence of approximations of a wave function. In principle, this approach differs from previous attempts of mathematical formalization of quantum mechanics in the Wolfram model [Gor20, GNA21, GNA20]. Furthermore, suppose the mathematical model that we are presenting describes the ultimate reality of the universe. In that case, it will be incompatible with ’t Hooft model of quantum mechanics [tH16], since we are working with templates. In contrast, ’t Hooft considers that the universe evolves from one ontic state to another ontic state. A physical interpretation of the approach to the Wolfram model that we are proposing is likely to be related to the many-worlds school.

2 Definition of the Wolfram model

Fix a positive integer $K$. Consider the alphabet $\Sigma = \{a_0, a_1, a_2, ..., a_K\}$. Define a Wolfram model where the states are words over $\Sigma$ and the non-deterministic evolution rule is the set of all concatenations of the argument and any symbol from $\Sigma$, i.e.,

$$\Omega = \begin{cases} 
  w \mapsto w a_0 \\
  w \mapsto w a_1 \\
  \vdots \\
  w \mapsto w a_K 
\end{cases}$$

where $w$ is an arbitrary word generated by the alphabet $\Sigma$.

For example, for $K = 2$ and initial condition $a_0$, we get the following multiway system (shown until level 3, starting by level 0). We omitted the $a$’s in the label of the vertices and only wrote their subindex to improve visibility.
3 Renormalization

Any state $w = w_1w_2...w_\ell$, where $w_1, w_2, ..., w_\ell \in \Sigma$, of our Wolfram model will be coarse-grained as

$$( -i )^m | m \mod 2 \rangle ,$$

where $m$ is the number of times that the character $K$ appears in the list $w_1, w_2, ..., w_\ell$. Notice that the renormalization of $\Omega$ is the multiset of rules

$$\omega = \begin{cases} 
|0\rangle \mapsto |0\rangle & K \text{ times} \\
|0\rangle \mapsto |0\rangle \\
|1\rangle \mapsto |1\rangle & K \text{ times} \\
|1\rangle \mapsto |1\rangle \\
|0\rangle \mapsto -i |1\rangle \\
|1\rangle \mapsto -i |0\rangle 
\end{cases}$$

The multiway system of the renormalized Wolfram model looks as follows (the initial condition and the value of $K$ are the same as before).

Instead of $|0\rangle$ and $|1\rangle$, we have chosen to write $x$ and $y$ respectively, since the graph is easier to visualize in this way. The initial condition and the value of $K$ are the same as before.

The sum of the elements of the $k$-th level of the multiway system of the renormalized Wolfram model will be called the $k$-th template and will be denoted as $|T_k\rangle$.

4 Schrödinger equation

Let $|\Psi(t)\rangle$ be the solution of the Schrödinger equation (for $\hbar = 1$)

$$i \frac{d}{dt}|\Psi(t)\rangle = H|\Psi(t)\rangle,$$
satisfying the initial condition

\[ |\Psi(0)\rangle = |0\rangle, \]

where the Hamiltonian is the Pauli-x matrix

\[ H = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \]

In this case, it is possible to find an explicit expression for the solution,

\[ |\Psi(t)\rangle = (\cos t |0\rangle - i \sin t |1\rangle). \]

The wave function \( |\Psi(t)\rangle \) is the continuous limit of the sequences of normalized templates,

\[ \lim_{K \to \infty} \left( \frac{1}{K} \right)^t |T_{tK}\rangle = |\Psi(t)\rangle. \]

This result is a trivial consequence of the following formula for the exponential of a matrix \( M \),

\[ \lim_{n \to \infty} \left( 1 + \frac{1}{n} M \right)^n = e^M. \]

**Conclusions**

Despite the widespread belief that quantum systems cannot be simulated by classical systems, we have shown that, after renormalization, the Wolfram model can be used to approximate a solution of the Schrödinger equation, having the Pauli-x matrix as Hamiltonian. This result motivates the study of the renormalization of Wolfram models as a method to describe the (non-relativistic) quantum systems. Whether there are some advantages in using a discrete template generated by a renormalized Wolfram model instead of a continuous wave function is an empirical question that can be answered by measuring quantum systems and comparing the prediction of the renormalized Wolfram model with those of mainstream quantum mechanics.

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