Measurement of angle $\beta$ with time-dependent CP asymmetry in $B^0 \to K^+K^-K^0$ decays

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We present recent results on CP-violation, and the determination of CKM angle $\beta$, with the decay $B^0 \to K^+K^-K^0$, with BABAR and Belle detectors.

I. INTRODUCTION

In the Standard Model (SM) of particle physics, the phase of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix $[1, 2]$ is the only source of CP violation in the quark sector. Due to the interference between mixing and decay, this phase can be observed in measurements of time-dependent CP asymmetries of $B^0$ mesons. In the SM, CP asymmetries in $\to s\bar{s}s$ decays, such as $B^0 \to K^+K^-K^0$, are expected to be nearly equal to those observed in tree-dominated $\bar{\tau}s$ decays $[3]$. However, because in the SM the former are dominated by loop amplitudes, new particles in those loops potentially introduce new physics at the same order as the SM process. Within the SM, deviations from the expected CP asymmetries in $B^0 \to K^+K^-K^0$ decays depend on the Dalitz plot position, but are expected to be small and positive $[4]$. In particular, for the decay $B^0 \to \phi K^0$ they are expected to be less than 4%. BABAR extracts the time-dependent CP-violation parameters by taking into account different amplitudes and phases across the $B^0$ and $\bar{B}$ Dalitz plots, while Belle measures it separately for $B^0 \to \phi K^0$ and the rest of $K^+K^-K^0$ events, neglecting interference between intermediate states.

The analyses presented here are based on 347 (535) million $B\bar{B}$ pairs collected with the BABAR (Belle) detector at the SLAC PEP-II (KEKB) $e^+e^-$ asymmetric-energy collider. Data are collected on the $\Upsilon(4S)$ resonance, while a fraction of about 10% is collected at approximately 40 MeV below the $\Upsilon(4S)$ resonance, and it is used to study the background arising from $e^+e^- \to \phi(q = u, d, s, c)$ continuum events. The BABAR and Belle detectors are described in detail elsewhere $[5]$.

II. EVENT RECONSTRUCTION

Events are fully reconstructed combining tracks and neutral clusters in the detector to form $B^0 \to K^+K^-K^0$, with a $K^0$ reconstructed as $K^0_s \to \pi^+\pi^- (B^0 \to K^+K^-) (\text{BABAR and Belle})$ and $K^0_s \to \pi^0\pi^0 (B^0_{(00)})$, or $K^0_s \to \pi^0\pi^0 (B^0_{(465)})$ (BABAR only). In order to select $B$ candidates we use a set of two kinematic variables: the beam-energy-substituted mass $m_{ES} = \sqrt{(s/2 + p_B^2)/(E_1^2 + p_B^2)} (M_{ES}$ for Belle), and the energy difference $\Delta E = E_B^* - \sqrt{s}/2$. Here, $(E_1, p_1)$ is the four-vector of the initial $e^+e^-$ system, $\sqrt{s}$ is the center-of-mass energy, $p_B$ is the reconstructed momentum of the $B^0$ candidate, and $E_B^*$ is its energy calculated in the $e^+e^-$ rest frame. For signal decays, the $m_{ES}$ distribution peaks near the $B^0$ mass with a resolution of about 2.5 MeV/$c^2$, and the $\Delta E$ distribution peaks near zero with a resolution of 10 ~ 50 MeV, depending on the final state. For decays with a $K^0_s$, $K^0_s$ momentum is not measured, but evaluated by constraining the $B^0$ mass to the nominal value $[3]$. In this case only $\Delta E$ is used, and it has a resolution of about 3 MeV. The main background comes from random combinations of particles produced in continuum $e^+e^- \to q\bar{q}$. In the CM frame, these events have a jet-like structure, while $B$ decays have a nearly isotropic topology. We parameterize this difference using several variables, providing additional discrimination between signal and background. Another source of background comes from decays of $B$ mesons which mimic the signal. This background is typically more difficult to suppress. The contribution of these decays is estimated from Monte Carlo simulations.

For each fully reconstructed $B^0$ meson ($B_{CP}$), we use the remaining particles in the event to reconstruct the decay vertex of the other $B$ meson ($B_{tag}$) and identify its flavor $q_{tag}$. A multivariate tagging algorithm determines the flavor of the $B_{tag}$ meson in BABAR data and classifies it in one of seven mutually exclusive tagging categories depending on the presence of prompt leptons, one or more charged kaons and pions $[7]$. The performances of this algorithm are measured with a data sample of fully reconstructed $B^0$ decays into flavor eigenstates ($B_{nav}$): $B^0 \to D^{(*)-}\pi^+/\rho^+/(\phi^+)$. The effective tagging efficiency is $Q \equiv \sum_e e(1-2w_e)^2 = 0.304\pm0.003$ for BABAR (similar for Belle), where $e(\bar{w})$ is the efficiency (mistag probability) for events tagged in category $c$. For Belle, the tagging algorithm returns $q_{tag}$ and the tag quality $r$, which varies from $r = 0$ for no flavor discrimination to $r = 1$ for unambiguous flavor assignment. Events with $r \leq 0.1$ are discarded for the CP-asymmetry measurement, and the others are sorted into six intervals. The difference $\Delta t$ is $t_{CP} - t_{tag}$ of the proper decay times of the $B_{CP}$ and $B_{tag}$ mesons is calculated from the measured distance between the reconstructed decay vertices and the boost ($\beta\gamma = 0.56(0.465)$ for BABAR (Belle)) of the $\Upsilon(4S)$. The error on $\Delta t$, $\sigma_{\Delta t}$, is also estimated for each event. Events are accepted if the calculated $\Delta t$ uncertainty is less than 2.5 ps and $|\Delta t| < 20$ ps. The fraction of events which satisfy these requirements is 95%.

The BABAR analysis strategy is to perform a maximum likelihood fit to the selected $K^+K^-K^0$ events with a likelihood function $L$, which uses as probability den-
sity function (PDF) for each event, \( \mathcal{L} \equiv \mathcal{P}(m_{ES}, \Delta E) \cdot \mathcal{P}_{\text{Low}} \cdot \mathcal{P}_{\text{DP}}(m_{K^+K^-}, \cos \theta_H, \Delta t, n_{sag}, \sigma_{\Delta t}) \) where \( n_i \) is the yield for each category \((i = \text{signal}, \text{continuum background}, \text{and } B\overline{B} \text{ backgrounds})\), and \( \mathcal{R} \) is a \( \Delta t \) resolution function with parameters determined in the \( B_{\text{low}} \) data sample. \( \mathcal{P}_{\text{Low}} \) is a supplementary PDF used only in the fits to the region with \( m_{K^+K^-} < 1.1 \text{ GeV}/c^2 \) discussed below. It depends on the event shape variables and, for \( B^0_{(\ell)} \) only, the missing momentum of the event. This PDF accounts for the fact that signal decays have a missing momentum consistent with the reconstructed \( K^0_L \) direction, while for background events it is more isotropically distributed. We characterize events on the Dalitz plot in terms of the invariant \( K^+K^- \) mass, \( m_{K^+K^-} \), and the cosine of the helicity angle between the \( K^+ \) and the \( K^0 \) in the CM frame of the \( K^+K^- \) system, \( \cos \theta_H \).

In the Belle approach the likelihood for the event selection is the same, without \( \mathcal{P}_{\text{DP}} \). A loose requirement on the likelihood ratio \( R_{\text{sig}} \equiv \mathcal{L}_{\text{sig}}/(\mathcal{L}_{\text{sig}} + \mathcal{L}_{\text{bkg}}) \) is applied, and a maximum-likelihood fit to observed \( \Delta t \) distribution is performed to the selected events.

III. ANALYSIS OF DALITZ PLOT

Accounting for the experimental efficiency \( \varepsilon \), the Dalitz plot PDF for signal events is

\[
P_{\text{DP}}(\Delta t, \cos \theta_H) \cdot |J(m_{K^+K^-})|,
\]

where the final term is the Jacobian of the transformation for our choice of Dalitz plot coordinates.

The time- and flavor-dependent decay rate over the Dalitz plot is

\[
d\Gamma \propto \frac{e^{-|\Delta t|/\tau}}{4\tau} \times \left[ |A|^2 + |\bar{A}|^2 \right. \\
\left. + \eta_{CP} \, q_{\text{ag}} \, 2Im (A^*A^0 e^{-2i\beta}) \sin \Delta m_{\phi} \Delta t \right. \\
\left. - q \left(|A|^2 - |\bar{A}|^2\right) \cos \Delta m_{\phi} \Delta t \right],
\]

where \( \eta_{CP} = 1 \) (\(-1\)) for \( B^0_{(\ldots)} \), \( B^0_{(\ldots)} \), \( B^0 \), \( \overline{B}^0 \), \( \tau \). and \( \Delta m_{\phi} \) are the lifetime and mixing frequency of the \( B^0 \) meson, respectively. The CKM angle \( \beta \) enters through \( B^0 \), \( \overline{B}^0 \) mixing. Actual world average is \( \beta = 0.38 \). We define the amplitude \( A(\bar{A}) \) for \( B^0 \), \( \overline{B}^0 \) decay as a sum of isobar amplitudes,

\[
A(\bar{A}) = \sum_r c_r (1 \pm b_r) e^{i(\phi_r + \delta_r)} \cdot f(\bar{f}),
\]

where the parameters \( c_r \) and \( \phi_r \) are the magnitude and phase of the amplitude of component \( r \), and \( b_r \) allow for different isobar coefficients for \( B^0 \), \( \overline{B}^0 \) decays through the asymmetry parameters \( b_r \) and \( \delta_r \). Our model includes the vector meson \( \phi(1020) \). We include also decays into intermediate scalar mesons: \( f_0(980) \), \( X_0(1550) \), and \( \chi_{c0} \). The angular distribution is constant for scalar decays, whereas for vector decays is \( Z_v = -4\bar{q} \cdot \bar{p} \), where \( \bar{q} \) is the momentum of the resonant daughter, and \( \bar{p} \) is the momentum of the third particle in the resonance frame. We describe the line-shape for the \( \phi(1020), \chi(1550) \), and \( \chi_{c0} \) using the relativistic Breit-Wigner function. For the \( \phi(1020) \) and \( \chi_{c0} \) parameters, we use average measurements. For the \( X(1550) \) resonance, we use parameters from our analysis of the \( B^+ \to K^+K^-\overline{K}^0 \) decay. The \( f_0(980) \) resonance is described by the coupled-channel (Flatté) function, with the coupling strengths for the \( KK \) and \( \pi\pi \) channels taken as \( g_{\pi} = 0.165 \pm 0.018 \text{ GeV}/c^2 \), \( g_{\pi} = 4.21 \pm 0.33 \), and the resonance pole mass \( m_{\pi} = 0.965 \pm 0.010 \text{ GeV}/c^2 \). In addition to resonant decays, we include three non-resonant amplitudes. The existing theoretical models, which consider contributions from contact term or higher-resonance tails do not reproduce well the features observed in data. Therefore we adopt a phenomenological parameterization, and describe the non-resonant terms as an exponential decay:

\[
A_{NR} = \sum_{i \neq j} g_{ij} e^{i\phi_{ij}} e^{-\alpha m_{ij}^2} \cdot (1 + b_{NR}) \cdot e^{i(\beta + \delta_{NR})}
\]

and similarly for \( \bar{A}_{NR} \).

We compute the CP-asymmetry parameters for component \( r \) from the asymmetries in amplitudes \( (b_r) \) and phases \( (\delta_r) \) given in Eq. 3. The rate asymmetry is

\[
A_{CP_r} = \left| A_r \right|^2 - \left| \bar{A}_r \right|^2 = \frac{-2b_r}{1 + b_r^2},
\]

and \( \beta_{eff} = \beta + \delta_r \) is the phase asymmetry.

The fraction for resonance \( r \) is computed

\[
\mathcal{F}_r = \frac{\int d\cos \theta_H \, dm_{K^+K^-} \cdot |J| \cdot (\left| A_r \right|^2 + \left| \bar{A}_r \right|^2)}{\int d\cos \theta_H \, dm_{K^+K^-} \cdot |J| \cdot (\left| A_r \right|^2 + \left| \bar{A}_r \right|^2)}.
\]

The sum of the fractions can be larger than one due to negative interference in the scalar sector.

The fit to \( B_{\text{Babar}} \) data returns \( 879 \pm 36 \text{ GeV}^2 \), \( 138 \pm 17 \text{ GeV}, \) \( 499 \pm 52 \text{ GeV}^2 \), \( 52 \text{ GeV}^2, \) \( 2 \) signal candidates. The isobar amplitudes, phases and fractions are listed in Table.

Signal weighted distribution for the Dalitz plot projections in the entire phase space and in a reduced region \( m_{K^+K^-} < 1.1 \text{ GeV}/c^2 \), where we extract separate CP asymmetry parameters, are shown in Fig.

The fit to \( B_{\text{Belle}} \) data returns \( 840 \pm 34 \text{ signal } B_{(\ldots)} \) candidates, after vetoing the \( \phi \) with the requirement \( m_{K^+K^-} < m_{\phi} > 15 \text{ MeV}/c^2 \).

As a cross-check of the Dalitz model extracted by the fit, we compute angular moments and extract strengths of the partial waves in \( m_{K^+K^-} \) bins using the \( B_{(\ldots)} \) sample. In this approach we only assume that the two lowest partial waves are present. We validated this assumption determining that the higher angular moments \( (\langle P_{3-5}\rangle) \) are consistent with zero. In our model, the \( P \)-wave contribution comes from \( \phi(1020)K^0 \) decays and from non-resonant events with \( K^+K^- \) and \( K^-K^0 \) mass dependence. We find that the total fraction of \( P \)-wave in the entire Dalitz plot is \( f_P = 0.29 \pm 0.03(\text{stat}) \).
TABLE I: Isobar amplitudes, phases, and fractions from the fit to BABAR data. The fraction for non-resonant amplitude is given for the combination of the three contributions.

| Decay     | Amplitude $c_r$ | Phase $\phi_r$ | Fraction $F_r$ (%) |
|-----------|-----------------|----------------|-------------------|
| $\phi(1020)K^0$ | 0.0098 ± 0.016 | $-0.11 \pm 0.31$ | 12.9 ± 1.3 |
| $f_0(980)K^0$  | 0.528 ± 0.063  | $-0.33 \pm 0.26$ | 22.3 ± 8.9 |
| $X_0(1550)K^0$ | 0.130 ± 0.025  | $-0.54 \pm 0.24$ | 4.1 ± 1.8 |
| NR ($K^+K^-$) | 1 (fixed)       | 0 (fixed)       |                   |
| ($K^+K^0$)    | 0.38 ± 0.11    | 2.01 ± 0.28    | 91 ± 19    |
| ($K^-K^0$)    | 0.36 ± 0.16    | $-1.19 \pm 0.37$ |                   |

$\chi^2/K^0$ | 0.0343 ± 0.0067 | 1.29 ± 0.41 | 2.84 ± 0.77 |
| $D^+K^-$    | 1.18 ± 0.24    | -            | 3.18 ± 0.89 |
| $D^+K^-$    | 0.85 ± 0.20    | -            | 1.72 ± 0.65 |

FIG. 1: Distributions of the Dalitz plot variables (left) $m_{K+K^-}$ and (right) $\cos \theta_B$ for signal events (points) compared with the fit PDF for $B^0(\pm\pm)$ candidates. Top distributions are for the entire phase space, bottoms for a reduced region $m_{K+K^-} < 1.1$ GeV/c^2.

IV. CP ASYMMETRY

In the Belle “quasi-two-body” approach, the time-dependent decay rate in Eq. [3] is simplified because the interference effects are neglected:

$$f_\pm(\Delta t) = \frac{e^{-|\Delta t|/\tau}}{4\tau} \times [1 \pm S \sin(\Delta m_\ell \Delta t) + C \cos(\Delta m_\ell \Delta t)] \ ,$$

(7)

The parameters $C$ and $S$ describe the amount of CP violation in decay and in the interference between decay with and without mixing, respectively. The SM expectations for $B^0 \to K^+K^-$ are $S = -(2 f_+ - 1) \sin 2\beta$, where $f_+$ is the CP-even fraction. Using isospin relations, $f_+$ has been measured on 357 fb$^{-1}$ data sample, and gives $f_+ = 0.93 \pm 0.09$(stat) ± 0.05(syst) [13]. This result is confirmed by partial wave analysis performed on BABAR data [14].

In the BABAR analysis, the time-dependent fit to the Dalitz plot allows to extract $\beta_{\text{eff}}$ removing the trigonometrical ambiguity $\beta_{\text{eff}} \to \pi/2 - \beta_{\text{eff}}$. In this analysis

the reflection is suppressed from the interference between CP-even and CP-odd decays that give rise to a $\cos(2\beta_{\text{eff}})$ term in Eq. [3], in addition to the $\sin(2\beta_{\text{eff}})$ terms that come from the interference decays with and without mixing. In this case we measure an average $\beta$ and $A_{\text{CP}}$ for the full $K^+K^-K^0$ phase space.

We measure also CP-asymmetry parameters in the region $m_{K+K^-} < 1.1$ GeV/c^2, where the $\phi$ model dependence from the rest of the Dalitz plot is highly reduced. In this region, we fit the CP time-dependent asymmetries for the $\phi(1020)$ and $f_0(980)$ components, while we fix the ones for the low-$K^+K^-$ tail of the non-resonant decays to the SM expectation. The Dalitz plot model is fixed to the one measured in the full phase space and reported in Table II with the exception of the $\phi(1020)$ isobar coefficients, which are fitted simultaneously with CP-asymmetry parameters. In this reduced region of the phase space we find 252 ± 19, 35 ± 9, 195 ± 33 signal events for $B^0(\pm\pm)$, $B^0(\pm\pm)$, and $B^0(\pm\pm)$, respectively. The results on the CP asymmetry are shown in Table II. The sources of systematic uncertainties are briefly described below.

The significance of the nominal result for $\beta_{\text{eff}}$ in the entire Dalitz plot, compared to the trigonometrical reflection is of 4.6$\sigma$. The significance of the CP-violation is 4.5$\sigma$. In Fig. 2 the distributions of $\Delta t$ for $B^0$-tagged and $\bar{B}^0$-tagged events, and the asymmetry $A(\Delta t) = (N_{B^0} - N_{\bar{B}^0})/(N_{B^0} + N_{\bar{B}^0})$, for background subtracted BABAR data are shown. In Fig. 3 the asymmetries for good-tagged ($r > 0.5$) $K^+K^-K^0$ events in Belle data are shown.

Systematic effects are associated to parameterization of the signal PDFs, possible fit bias, fixed $\Delta t$ resolution parameters, $B^0$ lifetime, $B^0$-$\bar{B}^0$ mixing and flavor tagging parameters. Smaller errors due to beam-spot position uncertainty, detector alignment, and the boost correction are based on studies done in charmion decays. In BABAR analysis, an uncertainty is assigned to the resonant and non-resonant line-shapes. We try several alternative non-resonant models which omit some

TABLE II: Time-dependent CP-asymmetries $A_{\text{CP}}$ and $\beta_{\text{eff}}$ for $B^0 \to K^+K^-K^0$ in $m_{K^+K^-} < 1.1$ GeV/c^2 and in the whole phase space (BABAR), and $C$ and $S$ for $B^0 \to K^+K^-K^0$ in $m_{K^+K^-} > 1.034$ GeV/c^2 (Belle). The first error is statistic, the second systematic. The third error in $S$ is systematic effect due to the $C$P-content uncertainty.

| Decay     | CP asymmetry |
|-----------|--------------|
| $A_{\text{CP}}(\phi K^0)$ | $-0.18 \pm 0.20 \pm 0.10$ |
| $\beta_{\text{eff}}(\phi K^0)$ | $0.06 \pm 0.16 \pm 0.05$ |
| $A_{\text{CP}}(f_0 K^0)$ | $0.45 \pm 0.28 \pm 0.10$ |
| $\beta_{\text{eff}}(f_0 K^0)$ | $0.18 \pm 0.19 \pm 0.04$ |
| $A_{\text{CP}}(K^+K^-K^0)$ | $-0.034 \pm 0.079 \pm 0.025$ |
| $\beta_{\text{eff}}(K^+K^-K^0)$ | $0.361 \pm 0.079 \pm 0.037$ |
| $C(K^+K^-K^0)$ | $0.09 \pm 0.10 \pm 0.05$ |
| $\sin(2\beta_{\text{eff}})(K^+K^-K^0)$ | $0.68 \pm 0.15 \pm 0.09^{+0.21}_{-0.13}$ |
of the dependencies on $K^+K^0$ and $K^-K^0$ masses (see Eq. [4]). We also study the effect of the uncertainty of the shape parameter $\alpha$ on the $CP$ parameters. The non-resonant events contribute to the background under the $\phi$ but their shape is determined from the high-mass region. We therefore omit the non-resonant terms and take the difference from the reference fit as a systematic error for $\phi K^0$ and $f_0(980)K^0$ $CP$-asymmetries. In Belle measurement, by far the largest systematic uncertainty on $CP$ parameters is associated to the knowledge of the $CP$-even fraction $f_+$.

V. CONCLUSIONS

We have measured the time-dependent $CP$-asymmetries in $B^0 \to K^+K^-K^0$ decays, with a simultaneous analysis of the Dalitz plot distribution of the intermediate states ($BABAR$) or with a “quasi-two-body” approach (Belle). The measured value of $CP$-asymmetries in the entire Dalitz plot is $\beta_{eff} = 0.361 \pm 0.079 \pm 0.037$, which is consistent with the SM expectations ($\beta \sim 0.38$). The trigonometrical ambiguity in $\beta_{eff}$ is removed at 4.6a. This is the first such measurement in penguin modes. Additionally, we extracted the $CP$-asymmetry parameters for $B^0 \to \phi K^0$ and $B^0 \to f_0(980)K^0$, to be $\beta_{eff} = 0.06 \pm 0.16 \pm 0.05$ and $\beta_{eff} = 0.18 \pm 0.19 \pm 0.04$, respectively. Therefore we do not observe significant deviations from the SM predictions.

In Belle measurement, the measured $\sin(2\beta_{eff}) = 0.68 \pm 0.15 \pm 0.03^{+0.21}_{-0.13}$ for $B^0 \to K^+K^-K^0_s$ decays with exclusion of $\phi K^0_s$ events is also consistent with SM predictions.

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