Three-Dimensional $\mathcal{N} \geq 5$ Superconformal Chern-Simons Gauge Theories And Their Relations

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Abstract

We propose three-dimensional $\mathcal{N} = 6$ superconformal $U(N) \times U(M)$ and $SU(N) \times SU(N)$ Chern-Simons gauge theories with two pairs of bifundamental chiral superfields in the $(N,M)$ and $(\bar{N},\bar{M})$ representations and in the $(N,N)$ and $(\bar{N},\bar{N})$ representations, respectively. We also propose the superconformal $U(1) \times U(1)$ gauge theories that have $n$ pairs of bifundamental chiral superfields with $U(1) \times U(1)$ charges $(\pm 1, \mp 1)$ or $(\pm 1, \pm 1)$. Although these $U(1) \times U(1)$ gauge theories have global symmetry $SU(2n)$, the R-symmetry is $SO(6)$ for $n = 2$, and might be $SO(2n)$ or $SO(2n+1)$ for $3 \leq n \leq 8$. In addition, we show that from either the generalized ABJM theories, or our $U(N) \times U(M)$ theories, or the $\mathcal{N} = 5$ superconformal $O(N) \times USp(2M)$ gauge theories, we can derive all the $\mathcal{N} \geq 5$ superconformal Chern-Simons gauge theories except the $\mathcal{N} = 5$ superconformal $G_2 \times USp(2)$ gauge theory and our $U(1) \times U(1)$ gauge theories with $n \neq 2$ and 4. Furthermore, we derive the three-dimensional $\mathcal{N} = 8$ superconformal $U(1) \times U(1)$ gauge theory from the BLG theory, and study the corresponding moduli space. With the novel Higgs mechanism in the unitary gauge, we suggest that it describes a D2-brane and a decoupled D2-brane.

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1. INTRODUCTION

A Lagrangian description for the worldvolume of multiple M2-branes is very important to understand the M-theory. Especially, the superconformal field theory on the worldvolume of multiple M2-branes is dual to the M-theory on $AdS_4 \times S^7$ in the AdS/CFT correspondence. The superconformal Chern-Simons gauge theories without Yang-Mills kinetic terms have been studied for this purpose [1], but, they did not have enough supersymmetry. Along this approach, Barger and Lambert [2, 3], as well as Gustavsson [4] (BLG) have successfully constructed three-dimensional $\mathcal{N} = 8$ superconformal Chern-Simons gauge theory with manifest $SO(8)$ R-symmetry based on three algebra. And then there is intensive research on three-dimensional superconformal gauge theories and their relations to the low energy theory on M2-branes [5, 6, 7, 8, 9, 10, 11, 12]. Although the BLG theory is expected to describe any number of M2-branes, its gauge group can only be $SO(4)$ for the positive definite metric [9, 10, 11]. At the Chern-Simons level one, the BLG $SO(4)$ gauge theory describes two M2-branes on a $R^8/Z_2$ orbifold [7, 8].

To generalize the BLG theory so that it can describe an arbitrary number of M2-branes, Aharony, Bergman, Jafferis and Maldacena (ABJM) have constructed three-dimensional $\mathcal{N} = 6$ superconformal Chern-Simons gauge theories with groups $U(N) \times U(N)$ and $SU(N) \times SU(N)$ [13]. (For Chern-Simons gauge theories with $\mathcal{N} = 3$ and 4 supersymmetries, see Refs. [14, 15].) Using brane constructions they argued that the $U(N) \times U(N)$ theory at Chern-Simons level $k$ describes the low-energy limit of $N$ M2-branes on a $C^4/Z_k$ orbifold. In particular, for $k = 1$ and 2, ABJM conjectured that their theory describes the $N$ M2-branes respectively in the flat space and on a $R^8/Z_2$ orbifold, and then might have $\mathcal{N} = 8$ supersymmetry. Especially, the $SU(2) \times SU(2)$ theory has enhanced $SO(8)$ R-symmetry and is the same as the BLG theory [13]. And it was proposed that the $U(2) \times U(2)$ theory might also have the enhanced $\mathcal{N} = 8$ supersymmetry at Chern-Simons level one and two [16]. Moreover, the ABJM theories can be easily generalized to have the $U(N) \times U(M)$ gauge symmetry [13, 17]. Thus, in this paper, we will define the ABJM theories as the generalized ABJM theories that can have gauge groups $U(N) \times U(M)$ or $SU(N) \times SU(N)$. In addition, the superconformal Chern-Simons gauge theories with $\mathcal{N} = 5$ and 6 supersymmetries have been classified recently [17, 18, 19, 20, 21]: the gauge groups for $\mathcal{N} = 6$ supersymmetry are $SU(N) \times SU(N)$, $U(N) \times U(M)$, and $U(1) \times USp(2N)$; and the gauge groups for $\mathcal{N} = 5$ supersymmetry are $O(N) \times USp(2M)$ and $G_2 \times USp(2)$. We would like to emphasize that for all the $\mathcal{N} = 6$ superconformal $U(N) \times U(M)$ and $SU(N) \times SU(N)$ Chern-Simons gauge theories that have been constructed so far [13, 17, 19], there are two pairs of bifundamental chiral superfields in the $\mathcal{(N, \overline{M})}$ and $\mathcal{(N, M)}$ representations, and in the $\mathcal{(N, \overline{N})}$ and $\mathcal{(\overline{N}, N)}$ representations, respectively.
As we know, the gauge symmetry for $N$ stacks of D-branes in Type II theories is $U(N)$. With orientifold actions, we can obtain the $SO(M)$ or $USp(2N)$ gauge symmetry for the D-branes on the top of orientifold planes via orientifold projections. Moreover, from the $SO(M)$ or $USp(2N)$ gauge symmetry for the D-branes on the top of orientifold planes, we can obtain back the $U(N)$ gauge symmetry by putting the same number of D-branes on all the orientifold images (for an example, see Ref. [22]). Therefore, it seems to us that the generic three-dimensional $\mathcal{N} \geq 5$ superconformal Chern-Simons gauge theories may be derived from the $\mathcal{N} = 6$ superconformal $U(N) \times U(M)$ gauge theories and the $\mathcal{N} = 5$ superconformal $O(N) \times USp(2M)$ gauge theories.

In this paper, we first propose the three-dimensional $\mathcal{N} = 6$ superconformal $U(N) \times U(M)$ gauge theories with two pairs of bifundamental chiral superfields in the $(N, M)$ and $(\bar{N}, \bar{M})$ representations, and the $\mathcal{N} = 6$ superconformal $SU(N) \times SU(N)$ gauge theories with two pairs of bifundamental chiral superfields in the $(N, N)$ and $(\bar{N}, \bar{N})$ representations. For our $SU(N) \times SU(N)$ theory, we obtain the BLG theory when $N = 2$. Moreover, we propose the superconformal $U(1) \times U(1)$ gauge theories that have $n$ pairs of bifundamental chiral superfields with the $U(1) \times U(1)$ charges $ (+1, -1)$ and $(-1, +1)$, or the $U(1) \times U(1)$ charges $(+1, +1)$ and $(-1, -1)$. These theories have global symmetry $SU(2n)$, and it seems to us that the R-symmetry is $SO(6)$ for $n = 2$, and might be $SO(2n)$ or $SO(2n+1)$ for $3 \leq n \leq 8$.

With the similar mechanism for Wilson line gauge symmetry breaking, we show that in the ABJM $U(N) \times U(M)$ theories and our $U(N) \times U(M)$ theories, the $\mathcal{N} = 6$ superconformal $U(N') \times U(M')$ Chern-Simons gauge theories can be obtained from the $\mathcal{N} = 6$ superconformal $U(N) \times U(N)$ Chern-Simons gauge theories, and the $\mathcal{N} = 6$ superconformal $U(N') \times U(N')$ Chern-Simons gauge theories can be obtained from the $\mathcal{N} = 6$ superconformal $U(N) \times U(M)$ Chern-Simons gauge theories, where $N' \leq N$, $M' \leq N$, and $N' \leq M$. In addition, we prove that the $\mathcal{N} = 5$ superconformal $O(N) \times USp(2M)$ Chern-Simons gauge theories can be derived from the ABJM $U(N) \times U(2M)$ theories and our $U(N) \times U(2M)$ theories. Also, both the ABJM $U(N) \times U(M)$ theories and our $U(N) \times U(M)$ theories can be derived from the $\mathcal{N} = 5$ superconformal $O(2N) \times USp(2M)$ Chern-Simons gauge theories. Moreover, we point out that the $\mathcal{N} = 5$ superconformal $O(2) \times USp(2N)$ gauge theories have enhanced $SO(6)$ R-symmetry, and become the $\mathcal{N} = 6$ superconformal $U(1) \times USp(2N)$ gauge theories. And it seems to us that the $\mathcal{N} = 5$ superconformal $G_2 \times USp(2)$ Chern-Simons gauge theory might be obtained from the $\mathcal{N} = 5$ superconformal $O(7) \times USp(2)$ Chern-Simons gauge theory since $G_2$ is a special maximal subgroup of $SO(7)$.

Furthermore, we derive the three-dimensional $\mathcal{N} = 8$ superconformal $U(1) \times U(1)$ gauge theory from the BLG theory, which can be considered as our above superconformal $U(1) \times U(1)$ gauge theory with four pairs of chiral superfields whose $U(1) \times U(1)$ charges are $(+1, -1)$ and $(-1, +1)$. And the moduli space has been studied in details. With the novel
Higgs mechanism in the unitary gauge, we show explicitly that this theory may describe a D2-brane and a decoupled D2-brane, and we present the concrete physics picture as well. Although the R-symmetry in our theory should be \( SO(8) \), the global symmetry is indeed \( SU(8) \). We emphasize that this superconformal \( U(1) \times U(1) \) gauge theory is different from the ABJM \( U(1) \times U(1) \) theory since we have eight bifundamental chiral superfields.

This paper is organized as follows. In Section II, we propose the three-dimensional \( \mathcal{N} = 6 \) superconformal \( U(N) \times U(M) \) and \( SU(N) \times SU(N) \) gauge theories, and the \( \mathcal{N} \geq 6 \) superconformal \( U(1) \times U(1) \) gauge theories. In Section III, we study the relations among the \( \mathcal{N} \geq 5 \) superconformal Chern-Simons gauge theories. In Section IV, we consider three-dimensional \( \mathcal{N} = 8 \) superconformal \( U(1) \times U(1) \) gauge theory in details. Our discussion and conclusions are given in Section V.

II. NEW THREE-DIMENSIONAL SUPERCONFORMAL CHERN-SIMONS GAUGE THEORIES

In this Section, we will propose the three-dimensional \( \mathcal{N} = 6 \) superconformal \( U(N) \times U(M) \) and \( SU(N) \times SU(N) \) gauge theories, and the \( \mathcal{N} \geq 6 \) superconformal \( U(1) \times U(1) \) gauge theories. The detail study of our theories will be presented elsewhere. Although the Chern-Simons level \( k \) for the \( \mathcal{N} = 6 \) superconformal \( U(N_k) \times U(M_{-k}) \) gauge theories should be equal to or larger than \( |N - M| \) \( [21] \), i.e., \( k \geq |N - M| \), we will neglect this subtlety for simplicity in the following discussions.

A. \( \mathcal{N} = 6 \) Superconformal \( U(N) \times U(M) \) Theories

Following the ABJM construction \([13] \), we consider the \( \mathcal{N} = 3 \) supersymmetric \( U(N) \times U(M) \) theories. Starting from the field content of an \( \mathcal{N} = 4 \) supersymmetric theory, we need to add to the \( U(N) \) and \( U(M) \) vector multiplets the auxiliary chiral multiplets \( \Phi \) and \( \Phi' \) in the \( U(N) \) and \( U(M) \) adjoint representations, respectively. Moreover, we introduce two hypermultiplets whose chiral superfields are in the bifundamental representations \( (\mathbf{N}, \mathbf{M}) \) and \( (\mathbf{N}, \mathbf{M}) \). To be concrete, we denote the bifundamental chiral superfields as \( (Z_i)_{\alpha\bar{\alpha}} \) and \( (W_i)_{\alpha\bar{\alpha}} \) with \( i = 1, 2 \), where \( a \) (\( \bar{a} \)) and \( \alpha \) (\( \bar{\alpha} \)) are (anti-)fundamental indices for \( U(N) \) and \( U(M) \), respectively. We also choose the Chern-Simons levels of the two gauge groups to be equal but opposite in sign. The superpotential in our theories is

\[
W = \frac{k}{8\pi} \left( \text{tr}\Phi^2 - \text{Tr}\Phi^2 \right) + \text{tr} (W_i \Phi Z_i) + \text{Tr} (W_i^T \Phi' Z_i^T),
\]

where \( \text{Tr} \) and \( \text{tr} \) are the traces for \( U(N) \) and \( U(M) \) gauge groups, respectively, and the upper index \( T \) means transpose. Because there are no kinetic terms for the auxiliary fields
Φ and Φ′, we can integrate them out and obtain the superpotential
\[ W = \frac{2\pi k}{\kappa} \left( \text{Tr}(Z_i W_i Z_j W_j) - \text{tr}(Z_i^T W_i^T Z_j^T W_j^T) \right) \]
\[ = \frac{2\pi k}{\kappa} \text{Tr} \left( Z_i W_i Z_j W_j - Z_i W_j Z_j W_i \right). \]  
(2)

So, we can rewrite it as follows
\[ W = \frac{2\pi k}{\kappa} \epsilon^{\rho\sigma} \epsilon^{\dot{\rho}\dot{\sigma}} \text{Tr} \left( Z_{\rho} W_{\rho} Z_{\sigma} W_{\sigma} \right). \]  
(3)

Thus, there are explicit SU(2) × SU(2) symmetry acting respectively on the Z_i and W_i. In addition, we have SU(2)_R symmetry under which the bosonic components z_i and w^*_i of Z_i and W_i transform as a doublet. Because the SU(2)_R symmetry does not commute with the above SU(2) × SU(2) symmetry, we can have the SU(4) global symmetry under which the four bosonic fields (z_1, z_2, w^*_1, w^*_2) transform in the fundamental 4 representation. The supercharges can not be singlets under this SU(4) because SU(2)_R is R-symmetry. Note that the generic three-dimensional superconformal theories have SO(N) R-symmetry with the supercharge in the fundamental representation, we should have at least \( \mathcal{N} = 6 \) superconformal symmetry.

If \( N \) is equal to \( M \), similar to the above discussions, we can construct the three-dimensional \( \mathcal{N} = 8 \) superconformal SU(N) × SU(N) theories as well. In this case, we can have another global U(1)_b baryon number symmetry under which \( Z_i \) and \( W_i \) have charges +1 and −1, respectively. And we would like to emphasize that our SU(2) × SU(2) theory has enhanced SO(8) R-symmetry and then is the same as the BLG theory.

Furthermore, our U(N) × U(M) theories are related to the ABJM U(N) × U(M) theories by the following transformation on the generators \( \hat{T}^a \) of the second gauge group U(M)
\[ \hat{T}^a \rightarrow -\hat{T}^{aT}. \]  
(4)

B. Superconformal U(1) × U(1) Gauge Theories with SU(2n) Global Symmetry

Similar to the above subsection, we consider the \( \mathcal{N} = 3 \) supersymmetric U(1) × U(1) gauge theories and begin from the field content of an \( \mathcal{N} = 4 \) supersymmetric theory. The auxiliary chiral multiplets for the first and second U(1) are Φ and Φ′ respectively. For the matter fields, we consider two scenarios

(I) We introduce \( n \) hypermultiplets whose chiral superfields have U(1) × U(1) charges (+1, −1) and (−1, +1). To be concrete, we denote the bifundamental chiral superfields as \((Z_i)_{a\bar{a}}\) and \((W_i)_{a\bar{a}}\) with \( i = 1, 2, ..., n \), where \( a (\bar{a}) \) and \( \alpha (\bar{\alpha}) \) are the charge +1 (−1) indices for the first and second U(1)s, respectively. We also choose the Chern-Simons levels of the
two gauge groups to be equal but opposite in sign. The superpotential is

\[ W = \frac{k}{8\pi} (\Phi'^2 - \Phi^2) + W_i \Phi Z_i - W_i \Phi' Z_i. \]  

(5)

In general, we only need to require that the Chern-Simons level of \( U(1) \)s be a rational number, i.e., \( k = p/q \) where \( p \) and \( q \) are relatively coprime.

Integrating out \( \Phi \) and \( \Phi' \), we obtain

\[ W = \frac{2\pi}{k} (Z_i W_i Z_j W_j - Z_i W_i Z_j W_j) \equiv 0. \]  

(6)

Interestingly, the superpotential vanishes. Therefore, we have \( SU(n) \times SU(n) \) global symmetry acting on \( Z_i \) and \( W_i \), respectively. Because \( SU(2)_R \) symmetry does not commute with this \( SU(n) \times SU(n) \) symmetry, we should have an \( SU(2n) \) global symmetry under which the bosonic fields \( (z_1, z_2, ..., z_n, w_1^*, w_2^*, ..., w_n^*) \) transform in the \( 2n \) fundamental representation of \( SU(2n) \). Note that the supercharges can not be singlets under this \( SU(4) \) because its subgroup \( SU(2)_R \) is R-symmetry, we conjecture that for \( n = 2 \), the R-symmetry is \( SU(4) \) or \( SO(6) \); for \( n = 3 \), the R-symmetry is \( SO(7) \) (\( SU(6) \supset USp(6) \sim SO(7) \)); and for \( 4 \leq n \leq 8 \), the R-symmetry is \( SO(2n) \) or \( SO(2n + 1) \). In Section IV, we will derive the \( U(1) \times U(1) \) gauge theory with \( n = 4 \) from the BLG theory, and then it has at least \( SO(8) \) R-symmetry.

(II) We can introduce \( n \) hypermultiplets whose chiral superfields have \( U(1) \times U(1) \) charges \((+1,+1)\) and \((-1,-1)\). We denote the bifundamental chiral superfields as \((Z'_i)_{\alpha \alpha}\) and \((W'_i)_{\bar{\alpha} \bar{\alpha}}\). And the Chern-Simons levels of the two gauge groups are chosen to be equal but opposite in sign. The superpotential is

\[ W = \frac{k}{8\pi} (\Phi'^2 - \Phi^2) + W'_i \Phi Z'_i + W'_i \Phi' Z'_i. \]  

(7)

Integrating out \( \Phi \) and \( \Phi' \), we obtain

\[ W = \frac{2\pi}{k} (Z'_i W'_i Z'_j W'_j - Z'_i W'_i Z'_j W'_j) \equiv 0. \]  

(8)

Because all the rest discussions are the same as those in the above scenario, we will not repeat them here.

Furthermore, the \( U(1) \times U(1) \) gauge theories with \( n \) pairs of chiral superfields whose \( U(1) \times U(1) \) charges are \((\pm 1, \mp 1)\) are related to the \( U(1) \times U(1) \) gauge theories with \( n \) pairs of chiral superfields whose \( U(1) \times U(1) \) charges are \((\pm 1, \pm 1)\) via the following transformation on the gauge field \( \hat{A}_\mu \) of the second \( U(1) \)

\[ \hat{A}_\mu \rightarrow -\hat{A}_\mu. \]  

(9)
III. RELATIONS AMONG THE THREE-DIMENSIONAL \( N \geq 5 \) SUPERCONFORMAL GAUGE THEORIES

Let us briefly review the ABJM theories with generic gauge groups \( U(N) \times U(M) \) or \( SU(N) \times SU(N) \) [13]. Following the convention in Ref. [23], we can write the explicit Lagrangian as follows

\[
\mathcal{L} = 2K \epsilon^{\mu\nu\lambda} \text{Tr} \left( A_\mu \partial_\nu A_\lambda + \frac{2i}{3} A_\mu A_\nu A_\lambda - \tilde{A}_\mu \partial_\nu \tilde{A}_\lambda - \frac{2i}{3} \tilde{A}_\mu \tilde{A}_\nu \tilde{A}_\lambda \right) - \text{Tr} \left( (D_\mu Z) D^\mu Z + (D_\mu W) D^\mu W + i \zeta^i \gamma^\mu D_\mu \zeta + i \omega^i \gamma^\mu D_\mu \omega \right) - V_{\text{Bos}} - V_{\text{Ferm}} ,
\]

where

\[
K = \frac{k}{8\pi} ,
\]

\[
Z^1 = X^1 + iX^5 , \quad Z^2 = X^2 + iX^6 , \quad W_1 = X^{3\dagger} - iX^7 \dagger , \quad W_2 = X^{4\dagger} - iX^8 \dagger ,
\]

where \( X^i \) belongs to the bifundamental \((N, \overline{M})\) representation of \( U(N) \times U(M) \), or the bifundamental \((N, N)\) representation of \( SU(N) \times SU(N) \). Also, \((Z^i, \zeta^i)\) and \((W_i, \omega_i)\) form chiral superfields. For simplicity, we will not distinguish the trace between the \( U(N) \) and \( U(M) \). To write the potential \( V_{\text{Bos}} \) and \( V_{\text{Ferm}} \) that are invariant explicitly under \( SU(4) \) R-symmetry, we define

\[
Y^A = \{ Z^A, W^{\dagger A} \} , \quad Y_\dagger^A = \{ Z_\dagger^A, W_A \} ,
\]

\[
\psi_A = \{ \epsilon_{AB} \zeta^B e^{-i\pi/4} , - \epsilon_{AB} \omega^B e^{i\pi/4} \} , \quad \psi_A^{\dagger} = \{ -\epsilon^{AB} \zeta_B^{\dagger} e^{i\pi/4} , \epsilon^{AB} \omega_B e^{-i\pi/4} \} .
\]

And then, we have

\[
V_{\text{Bos}} = -\frac{1}{48K^2} \text{Tr} \left( Y^A Y_\dagger^B Y^C Y_\dagger^C Y_\dagger^A Y^B Y^C + Y_\dagger^A Y^B Y_\dagger^C Y^C Y_\dagger^A Y^B Y^C \right.
\]

\[
+4Y^A Y_\dagger^B Y^C Y_\dagger^B Y^C - 6Y^A Y_\dagger^B Y^B Y^C Y_\dagger^C Y^C ) ) ,
\]

\[
V_{\text{Ferm}} = \frac{i}{4K} \text{Tr} \left( Y_\dagger^A Y^A \psi_A^{\dagger} \psi_B - Y^A Y_\dagger^A \psi_B \psi_B^{\dagger} + 2Y^A Y_\dagger^B \psi_A \psi_B^{\dagger} - 2Y_A Y_B \psi_A \psi_B - \epsilon^{ABCD} Y_\dagger^A \psi_B Y_B^{\dagger} \psi_D + \epsilon_{ABCD} Y^A \psi_B^{\dagger} Y^C \psi_B^{\dagger} \right) .
\]

For convention, we choose

\[
\text{Tr}(T^a T^b) = \frac{1}{2} \delta_{ab} , \quad \{ T^a , T^b \} = if_{abc} T^c ,
\]
where $T^{a,b,c}$ are the generators of the corresponding gauge group.

The Lagrangians for our $\mathcal{N} = 6$ superconformal $U(N) \times U(M)$ and $SU(N) \times SU(N)$ theories are similar to the above ABJM theories except that $X^i$ belongs to the bifundamental $(N,M)$ representation for $U(N) \times U(M)$ or the bifundamental $(N,N)$ representation for $SU(N) \times SU(N)$. Moreover, we obtain the Lagrangians for the $\mathcal{N} = 5$ superconformal $O(N) \times USp(2M)$ gauge theories from the above ABJM theories by changing the gauge groups to $O(N) \times USp(2M)$ and requiring the reality conditions

$$(W^{A\dot{j}})_{a\dot{a}} = \delta_{a\dot{a}} \omega_{\dot{a}\beta} \epsilon^{AB}(Z^{B*})_{\hat{b}\beta},$$

where $\omega_{\dot{a}\beta}$ is the anti-symmetric invariant tensor of $USp(2M)$.

In the ABJM and our theories, the $U(N) \times U(N)$ theories are related to the $SU(N) \times SU(N)$ theories. Roughly speaking, the ABJM $U(N) \times U(N)$ theories are the direct sum of the ABJM $SU(N) \times SU(N)$ theories and our $U(1) \times U(1)$ theories that have $n = 2N^2$ pairs of bifundamental chiral superfields with $U(1) \times U(1)$ charges $(\pm 1, \mp 1)$. And our $U(N) \times U(N)$ theories are the direct sum of our $SU(N) \times SU(N)$ theories and our $U(1) \times U(1)$ theories that have $n = 2N^2$ pairs of bifundamental chiral superfields with $U(1) \times U(1)$ charges $(\pm 1, \pm 1)$. Exactly speaking, these statements are not completely correct since the matter fields are the same matter fields for both $SU(N) \times SU(N)$ and $U(1) \times U(1)$ theories. Thus, in the following discussions, we will concentrate on the $U(N) \times U(M)$ theories proposed by ABJM and us for simplicity.

To study the relations among the $\mathcal{N} \geq 5$ superconformal Chern-Simons gauge theories, similar to the Wilson line gauge symmetry breaking, we introduce a discrete symmetry to the ABJM $U(N) \times U(M)$ theories, our $U(N) \times U(M)$ theories, and the $\mathcal{N} = 5$ superconformal $O(N) \times USp(2M)$ Chern-Simons gauge theories. Requiring that the theories be invariant under the discrete symmetry, we obtain the new three-dimensional superconformal Chern-Simons gauge theories. For simplicity, we shall only consider the $Z_2$ symmetry in this paper.

### A. Relations Among the $\mathcal{N} = 6$ Superconformal $U(N) \times U(M)$ Theories

We consider the ABJM theories and our theories with $U(N) \times U(M)$ group. And we introduce the following $Z_2$ transformations that act on the gauge fields $A_\mu$ and $\hat{A}_\mu$

$$A_\mu \rightarrow \Omega A_\mu \Omega^\dagger, \quad \hat{A}_\mu \rightarrow \hat{\Omega} \hat{A}_\mu \hat{\Omega}^\dagger,$$

where $\Omega^2 = 1$, and $\hat{\Omega}^2 = 1$. For the ABJM theories, the matter fields $(Z^i, \zeta^i)$ and $(W_i, \omega_i)$ transform as

$$(Z^i, \zeta^i) \rightarrow (\Omega Z^i \hat{\Omega}^\dagger, \Omega \zeta^i \hat{\Omega}^\dagger), \quad (W_i, \omega_i) \rightarrow (\hat{\Omega} W_i \Omega^\dagger, \hat{\Omega} \omega_i \Omega^\dagger).$$
And for our $U(N) \times U(M)$ theories, the matter fields transform as

$$(Z^i, \zeta^i) \longrightarrow (\Omega Z^i \hat{\Omega}^T, \Omega \zeta^i \hat{\Omega}^T), \quad (W_i, \omega_i) \longrightarrow (\hat{\Omega}^T W_i \Omega, \hat{\Omega}^T \omega_i \Omega). \quad (21)$$

To derive the new theories, we choose the following representations for $\Omega$ and $\hat{\Omega}$

$$\Omega = \begin{pmatrix} I_{N1 \times N1} & 0 \\ 0 & -I_{N2 \times N2} \end{pmatrix}, \quad \hat{\Omega} = \begin{pmatrix} I_{M1 \times M1} & 0 \\ 0 & -I_{M2 \times M2} \end{pmatrix}, \quad (22)$$

where $I_{n \times n}$ is the $n$ by $n$ identity matrix. In addition, $N1 > 0$, $N2 \geq 0$, $M1 > 0$, $M2 \geq 0$, $N1 + N2 = N$, and $M1 + M2 = M$. Therefore, the three-dimensional $\mathcal{N} = 6$ superconformal Chern-Simons gauge theories with group $U(N) \times U(M)$ are broken down to two decoupled three-dimensional $\mathcal{N} = 6$ superconformal Chern-Simons gauge theories with groups $U(N1) \times U(M1)$ and $U(N2) \times U(M2)$. Also, if $N2 = 0$ (or $M2 = 0$), we will have pure Chern-Simons gauge theories with group $U(M2)$ (or $U(N2)$) which can have any desired amount of supersymmetry [1]. In short, we have showed that in the ABJM theories and our theories, the $\mathcal{N} = 6$ superconformal $U(N') \times U(M')$ Chern-Simons gauge theories can be obtained from $\mathcal{N} = 6$ superconformal $U(N) \times U(M)$ Chern-Simons gauge theories, and the $\mathcal{N} = 6$ superconformal $U(N') \times U(N')$ Chern-Simons gauge theories can be obtained from the $\mathcal{N} = 6$ superconformal $U(N) \times U(M)$ Chern-Simons gauge theories, where $N' \leq N$, $M' \leq N$, and $N' \leq M$.

**B. Relations between the $\mathcal{N} = 6$ Superconformal $U(N) \times U(M)$ Chern-Simons Gauge Theories and the $\mathcal{N} = 5$ Superconformal $O(N) \times USp(2M)$ Chern-Simons Gauge Theories**

First, we would like to derive the $\mathcal{N} = 5$ superconformal $O(N) \times USp(2M)$ Chern-Simons gauge theories from the $\mathcal{N} = 6$ superconformal $U(N) \times U(2M)$ gauge theories proposed by ABJM and us. We introduce the following $Z_2$ transformations that act on the gauge fields $A_\mu$ and $\hat{A}_\mu$

$$A_\mu \longrightarrow -A_\mu^T, \quad \hat{A}_\mu \longrightarrow J \hat{A}_\mu J, \quad (23)$$

where

$$J = \begin{pmatrix} 0 & I_{M \times M} \\ -I_{M \times M} & 0 \end{pmatrix}. \quad (24)$$

For the ABJM theories, the matter fields $Y^A$ transform as

$$(Y^A)_{a\bar{a}} \longrightarrow \tilde{J}^{AB} \delta_{a\bar{b}} (Y^B)_{b\bar{b}} J_{\bar{b}\bar{a}}, \quad (25)$$
and for our theories, the matter fields transform as

\[(Y^A)_{\alpha\bar{a}} \rightarrow \tilde{J}^{AB\delta}_{\alpha\bar{b}}(Y^{B*})_{\bar{b}\delta}J_{\beta\alpha},\]

where \(a\) and \(b\) are \(U(N)\) indices, and \(\alpha\) and \(\beta\) are \(U(2M)\) indices, and

\[
\tilde{J} = \begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0 \\
0 & 1 & 0 & 0 \\
-1 & 0 & 0 & 0
\end{pmatrix}.
\]

Also, the transformations for \(\psi_A\) are similar to those for \(Y^A\).

Similar to the D-branes on the orientifold planes, we break the \(U(N) \times U(2M)\) gauge symmetry down to the \(O(N) \times USp(2M)\) gauge symmetry. In addition, the \(SU(4)\) R-symmetry is broken down to the \(USp(4)\) due to \(\tilde{J}\) projections. And we only have a single bifundamental hypermultiplet, or equivalently a pair of bifundamental chiral superfields. Therefore, requiring that the new theory be invariant under the transformations in Eqs. (23) and (25) or (26), we obtain the three-dimensional \(\mathcal{N} = 5\) superconformal \(O(N) \times USp(2M)\) Chern-Simons gauge theories since \(USp(4) \simeq SO(5)\).

Note that \(SO(2) \simeq U(1)\), the \(\mathcal{N} = 5\) superconformal symmetry in the \(O(2) \times USp(2M)\) Chern-Simons gauge theories is enhanced to the \(\mathcal{N} = 6\) superconformal symmetry. The point is that under the gauge symmetry \(U(1) \times USp(2M)\), we will have two bifundamental hypermultiplets, or equivalently two pairs of bifundamental chiral superfields. And then, the R-symmetry is indeed \(SU(4)\). Thus, we can derive the \(\mathcal{N} = 6\) superconformal \(U(1) \times USp(2M)\) Chern-Simons gauge theories from the ABJM theories and our theories with gauge group \(U(2) \times U(2M)\). Although \(SO(3) \simeq SU(2)\) and \(SO(6) \simeq SU(4)\), we can show that there is no enhanced \(SO(6)\) superconformal symmetry in the \(\mathcal{N} = 5\) superconformal \(O(3) \times USp(2M)\) and \(O(6) \times USp(2M)\) gauge theories.

For \(\mathcal{N} = 5\) superconformal \(G_2 \times USp(2)\) Chern-Simons gauge theory, it seems to us that we might not have the multiple M2-branes’ interpretation. However, we indeed might obtain such theory from the \(\mathcal{N} = 5\) superconformal \(O(N) \times USp(2M)\) Chern-Simons gauge theories. Note that \(G_2\) is a special maximal subgroup of \(SO(7)\), it seems to us that the \(\mathcal{N} = 5\) superconformal \(G_2 \times USp(2)\) Chern-Simons gauge theory might be derived from the \(\mathcal{N} = 5\) superconformal \(O(7) \times USp(2)\) Chern-Simons gauge theory.

In short, from the ABJM \(U(N) \times U(M)\) theories and our \(U(N) \times U(M)\) theories, we can derive the \(\mathcal{N} = 5\) superconformal \(O(N) \times USp(2M)\) Chern-Simons gauge theories and the \(\mathcal{N} = 6\) superconformal \(U(1) \times USp(2N)\) Chern-Simons gauge theories, and might derive the \(\mathcal{N} = 5\) superconformal \(G_2 \times USp(2)\) Chern-Simons gauge theory.
Next, we will derive the ABJM $U(N) \times U(M)$ theories and our $U(N) \times U(M)$ theories from the $\mathcal{N} = 5$ superconformal $O(2N) \times USp(2M)$ Chern-Simons gauge theories. We denote the gauge fields of $O(2N)$ and $USp(2M)$ as $A_\mu$ and $\tilde{A}_\mu$, and the bifundamental hypermultiplet in terms of two bifundamental chiral superfields $\Sigma_{a\alpha}$ and $\tilde{\Sigma}_{\alpha a}$ where $a$ and $\alpha$ are indices for $O(2N)$ and $USp(2M)$, respectively. Following the Ref. [24], we choose the following generators for $U(N)$

$$(T_{ab})_{ij} = (e_{a,b})_{ij} \equiv \delta_{ai}\delta_{bj},$$

where $a$ and $b$ are from 1 to $N$. In addition, we choose the generators for $SO(2N)$ as follows

$$T^1_{ab} \equiv e_{a,b} - e_{a+N,b+N},$$

$$T^2_{ab} \equiv e_{a,b+N} - e_{b,a+N}, \quad \text{where } a < b,$$

$$T^3_{ab} \equiv e_{a+N,b} - e_{b+N,a}, \quad \text{where } a < b,$$

where $a$ and $b$ are from 1 to $N$ with possible extra conditions on $a$ given above. And we choose the following generators for $USp(2M)$

$$T^1_{ab} \equiv e_{a,b} - e_{a+M,b+M},$$

$$T^2_{ab} \equiv e_{a,b+M} + e_{b,a+M},$$

$$T^3_{ab} \equiv e_{a+M,b} + e_{b+M,a},$$

where $a$ and $b$ are from 1 to $M$.

The fundamental representation of $SO(2N)$ can be decomposed as the representations of $SU(N) \times U(1)$ as follows

$$2N \rightarrow (N, +1) \oplus (\overline{N}, -1),$$

where we have normalized the $U(1)$ properly. Also, the fundamental representation of $USp(2M)$ can be decomposed as the representations of $SU(M) \times U(1)$ as follows

$$2M \rightarrow (M, +1) \oplus (\overline{M}, -1).$$

Similar to the above discussions, we introduce the following $Z_2$ transformations that act on the $O(2N)$ and $USp(2M)$ gauge fields $A_\mu$ and $\tilde{A}_\mu$, and the matter fields $\Sigma$ and $\tilde{\Sigma}$

$$A_\mu \rightarrow \Omega A_\mu \Omega, \quad \tilde{A}_\mu \rightarrow \tilde{\Omega} \tilde{A}_\mu \tilde{\Omega},$$

$$\Sigma \rightarrow \Omega \Sigma \tilde{\Omega}, \quad \tilde{\Sigma} \rightarrow \tilde{\Omega} \tilde{\Sigma} \Omega.$$
To derive the ABJM theories with $U(N) \times U(M)$ gauge group, we choose
\[ \Omega = \begin{pmatrix} I_{N \times N} & 0 \\ 0 & -I_{N \times N} \end{pmatrix}, \quad \hat{\Omega} = \begin{pmatrix} -I_{M \times M} & 0 \\ 0 & I_{M \times M} \end{pmatrix}. \tag{35} \]

So, the $O(2N) \times USp(2M)$ gauge symmetry is broken down to the $U(N) \times U(M)$ symmetry. Also, we obtain a pair of chiral superfields in the $U(N) \times U(M)$ bifundamental representations $(N, \overline{M})$ and $(\overline{N}, M)$ from $\Sigma$, and another pair of chiral superfields in the same representations from $\hat{\Sigma}$. Therefore, we can have the enhanced $SU(4)$ R-symmetry and indeed derive the $\mathcal{N} = 6$ superconformal ABJM theories with gauge group $U(N) \times U(M)$.

Furthermore, to derive our theories with $U(N) \times U(M)$ gauge group, we choose
\[ \Omega = \begin{pmatrix} I_{N \times N} & 0 \\ 0 & -I_{N \times N} \end{pmatrix}, \quad \hat{\Omega} = \begin{pmatrix} I_{M \times M} & 0 \\ 0 & -I_{M \times M} \end{pmatrix}. \tag{36} \]

Thus, the $O(2N) \times USp(2M)$ gauge symmetry is broken down to the $U(N) \times U(M)$ symmetry. Moreover, we obtain a pair of chiral superfields in the $U(N) \times U(M)$ bifundamental representations $(N, M)$ and $(\overline{N}, \overline{M})$ from $\Sigma$, and another pair of chiral superfields in the same representations from $\hat{\Sigma}$. Therefore, we can also have the enhanced $SU(4)$ R-symmetry and derive our $\mathcal{N} = 6$ superconformal theories with gauge group $U(N) \times U(M)$.

### IV. THREE-DIMENSIONAL $\mathcal{N} = 8$ SUPERCONFORMAL $U(1) \times U(1)$ THEORY FROM THE BLG THEORY

We shall briefly review the BLG theory in the product gauge group formulation by van Raamsdonk [6]. In this formulation, the BLG theory is rewritten as a superconformal Chern-Simons theory with the $SU(2) \times SU(2)$ gauge group and bifundamental matters, which has a manifest global $SO(8)$ R-symmetry. The Lagrangian is given by [6]

\[ \mathcal{L} = \text{Tr} \left( \frac{1}{2f} \epsilon^{\mu\nu\lambda}(A_\mu \partial_\nu A_\lambda + \frac{2i}{3} A_\mu A_\nu A_\lambda) - \frac{1}{2f} \epsilon^{\mu\nu\lambda}(\hat{A}_\mu \partial_\nu \hat{A}_\lambda + \frac{2i}{3} \hat{A}_\mu \hat{A}_\nu \hat{A}_\lambda) \right. \\
\left. \quad - (D^\mu X^I)^\dagger D_\mu X^I + i \bar{\Psi} \Gamma^\mu D_\mu \Psi - \frac{8f^2}{3} X^{I[I}X^{J]K}X^{I]}X^{J]X^{I]}X^{I]} \\
\left. \quad - \frac{2if}{3} \bar{\Psi} \Gamma_{IJ}(X^I X^J \Psi + X^J \Psi^I X^I + \Psi X^I X^I) \right) , \tag{37} \]

where the fermions $\Psi$ are represented by 32-component Majorana spinors of $SO(1,10)$ subject to a chirality condition on the world volume which leaves 16 real degrees of freedom. And the covariant derivative is

\[ D_\mu X = \partial_\mu X + i A_\mu X - iX \hat{A}_\mu . \tag{38} \]
The Chern-Simons level \( k \) is related to \( f \) as follows
\[
f = \frac{2\pi}{k}.
\] (39)

The bifundamental scalars \( X' \) are related to the original BLG variables \( x_a^I \) with \( SO(4) \) index \( a \) through
\[
X' = \frac{1}{2} (x_4^I \mathbb{I}_{2\times2} + i x_i^I \sigma^i),
\] (40)
where \( \sigma^i \) are the Pauli matrices. And there is a similar expression for spinor \( \Psi \). Also, the scalars need to satisfy the reality condition
\[
X^I_{\alpha\beta} = \epsilon_{ab} \epsilon_{\alpha\beta} (X^I)^{ab}.
\] (41)

To be concrete, we define \( X' \) as following
\[
X' \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} z' & w' \\ -\bar{w}' & \bar{z}' \end{pmatrix},
\] (42)
where
\[
z' = \frac{1}{\sqrt{2}} (x_4^I + ix_3^I), \quad w' = \frac{1}{\sqrt{2}} (x_2^I + ix_1^I).
\] (43)

Moreover, the \( SU(2) \times SU(2) \) gauge transformations are
\[
A_\mu \rightarrow UA_\mu U^\dagger - iU \partial_\mu U^\dagger, \quad \hat{A}_\mu \rightarrow \hat{U}\hat{A}_\mu \hat{U}^\dagger - i\hat{U} \partial_\mu \hat{U}^\dagger, \quad X' \rightarrow UX' \hat{U}^\dagger.
\] (44)

Similar to the last Section, we consider the \( Z_2 \) discrete symmetry. Under \( Z_2 \) symmetry, the gauge fields and matter fields transform as follows
\[
A_\mu \rightarrow \Omega A_\mu \Omega^\dagger, \quad \hat{A}_\mu \rightarrow \hat{\Omega}\hat{A}_\mu \hat{\Omega}^\dagger, \quad X' \rightarrow \Omega X' \hat{\Omega}^\dagger.
\] (45)

The transformations for \( \Psi \) are similar to these for \( X' \). And there are three kinds of independent and non-trivial representations for \( \Omega \) and \( \hat{\Omega} \)
\[
\Omega = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \hat{\Omega} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},
\] (46)
\[
\Omega = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \hat{\Omega} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},
\] (47)
\[
\Omega = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \hat{\Omega} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\] (48)
For the first case with $\Omega$ and $\hat{\Omega}$ in Eq. (46), the gauge group $SU(2) \times SU(2)$ is broken down to $SU(2) \times U(1)$. In addition, the components $w^I$ and $\bar{w}^I$ of $X^I$ are projected out. So the reality condition in Eq. (41) cannot be satisfied. Note that only the components $z^I$ and $\bar{w}^I$ of $X^I$ can not form the complete chiral superfields under the remaining gauge symmetry $SU(2) \times U(1)$. In order to have the complete complex representation under $SU(2) \times U(1)$, we redefine

$$Y^i = \left( \begin{array}{c} z^i + iz^{i+4} \\ -\bar{w}^i + i\bar{w}^{i+4} \end{array} \right),$$

(49)

where $i = 1, 2, 3, 4$. The discussions for the spinor are similar. Thus, we have four chiral superfields in the bifundamental representation of $SU(2) \times U(1)$. And then, we obtain the three-dimensional $\mathcal{N} = 6$ superconformal $SU(2) \times U(1)$ theory, i.e., the ABJM theory and our theory with group $SU(2) \times U(1)$ since these two theories are identical in this case. The discussions for the second case with $\Omega$ and $\hat{\Omega}$ in Eq. (47) are the same as the first case since they are related by the parity symmetry.

The most interesting case is the third one with $\Omega$ and $\hat{\Omega}$ in Eq. (48). We break the $SU(2) \times SU(2)$ gauge symmetry down to its Cartan subgroup $U(1) \times U(1)$, and we project out the components $w^I$ and $\bar{w}^I$. Thus, we have eight complex scalar fields $z^I$ with charge $(+1, -1)$ under gauge group $U(1) \times U(1)$. The discussions for the spinor $\Psi$ are similar, and we get a complex Dirac fermion $\psi$ with charge $(+1, -1)$ under gauge group $U(1) \times U(1)$. Similar to Ref. [23], we can rewrite $(z^i, \psi)$ as 8 chiral superfields, which will not be discussed here. Because we do not break the supersymmetry, we obtain the three-dimensional $\mathcal{N} = 8$ superconformal $U(1) \times U(1)$ gauge theory. And the Lagrangian is

$$\mathcal{L} = \frac{1}{4f} \epsilon^{\mu\nu\lambda} \left(A_\mu^3 \partial_\nu A_\lambda^3 - \hat{A}_\mu^3 \partial_\nu \hat{A}_\lambda^3 \right) - (D^\mu z^I)^\dagger D_\mu z^I + i\psi^\dagger \Gamma^\mu D_\mu \psi ,$$

(50)

where the covariant derivative is

$$D_\mu z^I = \partial_\mu z^I + i\frac{1}{2}(A_\mu^3 - \hat{A}_\mu^3)z^I .$$

(51)

We define

$$a_\mu \equiv \frac{1}{2} \left( A_\mu^3 + \hat{A}_\mu^3 \right) , \quad \hat{a}_\mu \equiv \frac{1}{2} \left( A_\mu^3 - \hat{A}_\mu^3 \right) .$$

(52)

And then we rewrite the above Lagrangian in Eq. (50) as follows

$$\mathcal{L} = \frac{1}{2f} \epsilon^{\mu\nu\lambda} \hat{a}_\mu f_{\nu\lambda} - (D^\mu z^I)^\dagger D_\mu z^I + i\psi^\dagger \Gamma^\mu D_\mu \psi ,$$

(53)

where

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu , \quad D_\mu z^I = \partial_\mu z^I + i\hat{a}_\mu z^I .$$

(54)
So, only the $\hat{a}_\mu$ gauge field couples to the matter fields. In particular, because the gauge symmetry in our theory is $U(1) \times U(1)$, the level $k$ can be a rational number in general, i.e., $k = p/q$ where $p$ and $q$ are relatively prime. We would like to point out that this $U(1) \times U(1)$ gauge theory derived from the BLG theory is the same as our $U(1) \times U(1)$ theory with four pairs of chiral superfields whose $U(1) \times U(1)$ charges are $(\pm 1, \mp 1)$.

A. Moduli Space

We will study the moduli space of our $U(1) \times U(1)$ theory, and focus on the gauge fields and scalar fields. The discussions are similar to those in Refs. [7, 8]. The gauge transformations for $A_\mu^3$ and $\hat{A}_\mu^3$ are

$$A_\mu^3 \longrightarrow A_\mu^3 - \partial_\mu \theta \ , \quad \hat{A}_\mu^3 \longrightarrow \hat{A}_\mu^3 - \partial_\mu \hat{\theta} \ ,$$

so we obtain the gauge transformations for $a_\mu$, $\hat{a}_\mu$, and $z^I$

$$z^I \longrightarrow e^{i(\theta - \hat{\theta})/2} z^I \ , \quad a_\mu \longrightarrow a_\mu - \frac{1}{2}(\partial_\mu \theta + \partial_\mu \hat{\theta}) \ , \quad \hat{a}_\mu \longrightarrow \hat{a}_\mu - \frac{1}{2}(\partial_\mu \theta - \partial_\mu \hat{\theta}) \ .$$

We define

$$\alpha \equiv \frac{1}{2}(\theta + \hat{\theta}) \ , \quad \hat{\alpha} \equiv \frac{1}{2}(\theta - \hat{\theta}) \ ,$$

Then we obtain the gauge transformations for $a_\mu$, $\hat{a}_\mu$, and $z^I$

$$z^I \longrightarrow e^{i\hat{\alpha}} z^I \ , \quad a_\mu \longrightarrow a_\mu - \partial_\mu \alpha \ , \quad \hat{a}_\mu \longrightarrow \hat{a}_\mu - \partial_\mu \hat{\alpha} \ .$$

We emphasize that both $\alpha$ and $\hat{\alpha}$ have period $2\pi$, which is consistent with the $2\pi$ period for $z^I$.

Moreover, we introduce a Lagrange multiplier term

$$\mathcal{L}_\sigma = \frac{1}{4\pi} \sigma \epsilon^{\mu\nu\lambda} \partial_\mu f_{\nu\lambda} \ ,$$

where $\sigma$ is a new scalar field and has period $2\pi$. And then we can treat $f_{\mu\nu}$ as an independent variable.

Therefore, the relevant Lagrangian for the gauge and scalar fields is

$$\mathcal{L} = -|\partial_\mu z^I + i\hat{a}_\mu z^I|^2 + \frac{k}{4\pi} \epsilon^{\mu\nu\lambda} \partial_\mu \hat{a}_{\nu\lambda} + \frac{1}{4\pi} \sigma \epsilon^{\mu\nu\lambda} \partial_\mu f_{\nu\lambda} \ .$$

From the equation of motion for $f_{\mu\nu}$, we have

$$\hat{a}_\mu = \frac{1}{k} \partial_\mu \sigma \ .$$
Using this, we obtain the reduced action

$$\mathcal{L} = -|\partial_\mu z^I + \frac{i}{k}(\partial_\mu \sigma)z^I|^2.$$  \hfill (62)

From the Eqs. (58) and (61), we obtain the new gauge transformations for $z^I$ and $\sigma$

$$z^I \rightarrow e^{i\hat{\alpha}}z^I, \quad \sigma \rightarrow \sigma - k\hat{\alpha}.$$ \hfill (63)

Fixing the gauge $\sigma = 0$, we still have the residual gauge transformation

$$\hat{\alpha}(x) = \frac{2n\pi}{k}.$$ \hfill (64)

Thus, the moduli space is characterized by a set of eight complex numbers $z^I$, and the residual symmetry transformations are

$$z^I \rightarrow e^{i2n\pi/k}z^I,$$ \hfill (65)

and

$$z^I \rightarrow \bar{z}^I.$$ \hfill (66)

Therefore, we conclude that for $k = 1$, the moduli space is

$$(\mathbb{R}^8 \times \mathbb{R}^8)/Z_2,$$ \hfill (67)

for $k = 2$, the moduli space is

$$(\mathbb{R}^8 \times \mathbb{R}^8)/(Z_2 \times Z_2),$$ \hfill (68)

and for $k > 2$, the moduli space is

$$(\mathbb{R}^8 \times \mathbb{R}^8)/\text{Dih}_k,$$ \hfill (69)

where $\text{Dih}_k$ is the dihedral group.

For $k = p/q$ in general, the discussions of moduli space are similar to the above, so we will not present them here.

B. Novel Higgs Mechanism

Similar to the Refs. [5, 12], we can obtain the D-brane action via the novel Higgs mechanism. For simplicity, we focus on the gauge fields here. Because we have the $SU(8)$ global symmetry, we can always make the rotation so that only the scalar field $z^8$ develops a vacuum expectation value (VEV)

$$\langle z^8 \rangle = v.$$ \hfill (70)
Then \( \hat{a}_\mu \) becomes massive, and we obtain the relevant Lagrangian for gauge fields

\[
\mathcal{L} = \frac{k}{4\pi} \epsilon^{\mu\nu\lambda} \hat{a}_\mu f_{\nu\lambda} - v^2 \hat{a}_\mu^2 , \tag{71}
\]

where the second term comes from the kinetic term of \( z^8 \). Integrating out \( \hat{a}_\mu \), we obtain

\[
\mathcal{L} = -\frac{k^2}{32\pi^2 v^2} f^{\mu\nu} f_{\mu\nu} . \tag{72}
\]

Thus, the gauge field \( a_\mu \) becomes a dynamical field, and its gauge coupling is

\[
g = 2\sqrt{2}\pi \frac{v}{k} . \tag{73}
\]

So, for very large \( v \) and \( k \), we can still fix the gauge coupling \( g \) as a constant.

Because we have eight chiral superfields from \( z^I \) and \( \psi \), we should have another decoupled \( U(1) \) gauge field in addition to \( a_\mu \) if we only have \( \mathcal{N} = 8 \) superconformal symmetry. To be concrete, we shall choose the unitary gauge, and define \( z^8 \) as follows

\[
z^8 \equiv \frac{1}{\sqrt{2}} \rho e^{i\phi} . \tag{74}
\]

From the kinetic term of \( z^8 \), we obtain the kinetic term for \( \rho \)

\[
\mathcal{L} = -\frac{1}{2} |\partial_\mu \rho + iB_\mu \rho|^2 , \tag{75}
\]

where

\[
B_\mu = \hat{a}_\mu + \partial_\mu \phi . \tag{76}
\]

And the Chern-Simons term for the gauge fields becomes

\[
\frac{k}{4\pi} \epsilon^{\mu\nu\lambda} \hat{a}_\mu f_{\nu\lambda} = \frac{k}{4\pi} \epsilon^{\mu\nu\lambda} (B_\mu f_{\nu\lambda} - \partial_\mu \phi f_{\nu\lambda}) . \tag{77}
\]

Using the Bianchi identity for \( f_{\nu\lambda} \), the last term in the above Lagrangian vanishes. Giving the following VEV to \( \rho \)

\[
\langle \rho \rangle = \sqrt{2}v , \tag{78}
\]

we obtain the relevant Lagrangian for gauge fields

\[
\mathcal{L} = \frac{k}{4\pi} \epsilon^{\mu\nu\lambda} B_\mu f_{\nu\lambda} - v^2 B_\mu^2 . \tag{79}
\]

Integrating out the gauge field \( B_\mu \), we obtain the same action for \( a_\mu \) given in Eq. (72).

Therefore, the physical picture is the following: the field \( \phi \) is eaten by the gauge field \( B_\mu \), and the gauge field \( a_\mu \) becomes dynamical after \( B_\mu \) is integrated out. Equivalently speaking, the field \( \phi \) is eaten by the gauge field \( a_\mu \). In addition, \( \rho \) is dual to the decoupled \( U(1) \) gauge field, which explains why we have eight chiral superfields from \( z^I \) and \( \psi \). In the unitary gauge, similar discussions can be applied to the novel Higgs mechanism in all the \( \mathcal{N} \geq 5 \) superconformal Chern-Simons gauge theories.
V. DISCUSSION AND CONCLUSIONS

In this paper, we have proposed the three-dimensional $\mathcal{N} = 6$ superconformal $U(N) \times U(M)$ gauge theories with two pairs of bifundamental chiral superfields in the $(N, M)$ and $(\overline{N}, \overline{M})$ representations, and the $\mathcal{N} = 6$ superconformal $SU(N) \times SU(N)$ gauge theories with two pairs of bifundamental chiral superfields in the $(N, N)$ and $(\overline{N}, \overline{N})$ representations. For our $SU(N) \times SU(N)$ theory with $N = 2$, we reproduced the BLG theory. We also proposed the superconformal $U(1) \times U(1)$ gauge theories that have $n$ pairs of bifundamental chiral superfields with the $U(1) \times U(1)$ charges $(\pm 1, \mp 1)$, or the $U(1) \times U(1)$ charges $(\pm 1, \pm 1)$. Although these theories have global symmetry $SU(2n)$, the R-symmetry is $SO(6)$ for $n = 2$ and might be $SO(2n)$ or $SO(2n + 1)$ for $3 \leq n \leq 8$.

In addition, we showed that in the ABJM $U(N) \times U(M)$ theories and our $U(N) \times U(M)$ theories, the $\mathcal{N} = 6$ superconformal $U(N') \times U(N')$ Chern-Simons gauge theories can be obtained from the $\mathcal{N} = 6$ superconformal $U(N) \times U(M)$ Chern-Simons gauge theories, and vice versa. Moreover, we proved that the $\mathcal{N} = 5$ superconformal $O(N) \times USp(2M)$ Chern-Simons gauge theories can be derived from the ABJM $U(N) \times U(2M)$ theories and our $U(N) \times U(2M)$ theories. Also, both the ABJM $U(N) \times U(M)$ theories and our $U(N) \times U(M)$ theories can be derived from the $\mathcal{N} = 5$ superconformal $O(2N) \times USp(2M)$ Chern-Simons gauge theories as well. Moreover, we explained that the $SO(5)$ R-symmetry in the $\mathcal{N} = 5$ superconformal $O(2) \times USp(2N)$ gauge theories is enhanced to $SO(6)$, and then we obtained the $\mathcal{N} = 6$ superconformal $U(1) \times USp(2N)$ gauge theories. Because $G_2$ is a special maximal subgroup of $SO(7)$, it seems to us that the $\mathcal{N} = 5$ superconformal $G_2 \times USp(2)$ Chern-Simons gauge theory might be obtained from the $\mathcal{N} = 5$ superconformal $O(7) \times USp(2)$ Chern-Simons gauge theory.

Furthermore, we derived the three-dimensional $\mathcal{N} = 8$ superconformal $U(1) \times U(1)$ gauge theory from the BLG theory, which can be considered as our superconformal $U(1) \times U(1)$ gauge theory with four pairs of chiral superfields whose $U(1) \times U(1)$ charges are $(\pm 1, \mp 1)$. Also, we have studied the moduli space in details. Considering the novel Higgs mechanism in the unitary gauge, we suggested that this theory may describe a D2-brane and a decoupled D2-brane.
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