Dust as a surfactant

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Abstract. We argue that dust immersed in a plasma sheath acts as a surfactant. By considering the momentum balance in a plasma sheath, we evaluate the dependence of the plasma surface pressure on the dust density. It is shown that the dust may reduce the surface pressure, giving rise to a sufficiently strong tangential force. The latter is capable of confining the dust layer inside the sheath in the direction perpendicular to the ion flow.

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1. Introduction

One of the most interesting phenomena in dusty plasma physics discovered for some time past is a void that is a dust-free region in a central part of a discharge chamber. Voids have been observed in both ‘earthly’ [1, 2] and microgravity conditions [3]; the theory of the void formation was
given in [4, 5]. It is a common belief nowadays that voids emerge due to the ionization instability of the homogeneous state of a dusty plasma. It should be stressed that the onset of this instability requires strong influence of the dust component upon the discharge structure; in particular, the net number of dust grains should be sufficiently large. The essential feature of various dust structures and, in particular, voids is their sharp boundaries, which are maintained by an ion wind blowing towards the dust density gradient.

On the other hand, numerous laboratory experiments deal with a relatively small number of dust grains levitating above a horizontal rf-powered or dc-biased electrode. Various aspects of the dust–plasma interaction were recently reviewed in [6]. The levitation is provided by a strong electric field incident to a plasma–electrode sheath that counterbalances the gravity force. Dust suspended in a plasma sheath self-organizes itself in various single- or multi-layered structures. In particular, recent experiments [7] demonstrated that with growing rf power or gas pressure the central void emerges inside a single dust layer. Since the conditions in the bulk of the plasma and the plasma sheath are fundamentally different, the ionization instability responsible for the formation of three-dimensional voids cannot develop in the case of a two-dimensional dust layer with a small number of grains. The qualitative theory given in [7] implies the existence of some attractive interaction between dust grains. However, the nature of the interaction is unclear. Seemingly, none of the processes discussed in the literature e.g. the wake field [8], the cohesive force [9] or the LeSage gravity [10]) can provide mutual attraction of dust grains hovering at the same height over the electrode. It is also doubtful whether the radial ion wind may be strong enough to form a void.

In the present paper, we investigate the influence of charged dust on the plasma surface pressure (that is, the surface tension with the minus sign). In accordance with other areas of physics, we define surface pressure as a tangential force (per unit length) acting between two parts of a plasma–electrode interface, that is, a plasma sheath; a more formal definition will be given below. One of our objectives is to demonstrate the availability of this concept in application to plasma physics.

There are several factors providing strong surface pressure at the plasma–electrode interface. Among them are the anisotropic bulk pressure of the electric field inside a sheath and the isotropic bulk electron pressure. However, here we intentionally use the simplest sheath model based on the Child–Langmuir equation [11, 12], i.e. we discount the influence of electrons. The are several reasons for this oversimplification. First, the Child–Langmuir equation provides a qualitatively reasonable description of a plasma sheath, particularly in a dc discharge, and it is often used for estimations e.g. [13]). Second, with the model described below, we are able to carry out most calculations analytically, which always makes the physics more robust. Finally, the dynamics of dust in a space-charge-limited flow is of interest in view of experiments on hard x-ray emission from a dusty plasma [14].

We consider a thin charged layer oriented perpendicular to a space-charge-limited ion flow. By solving the Poisson equation we evaluate the electric field distribution and, therefore, the electric field pressure. Integrating the field pressure over the sheath width, we get the surface pressure. Our main finding is that the negatively charged dust layer under certain conditions reduces the surface plasma pressure, i.e. dust is a surfactant. As a result, the dust-free area of the plasma sheath tends to squeeze the dust layer. The force acting upon the edge of the layer turns out to be strong enough to confine the layer in the horizontal direction.

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2. The model

The model used here is depicted in figure 1. We consider the sheath–presheath boundary \((z = 0)\) as an ideal emitter of positive ions. The conducting planar electrode is placed at \(z = z_0\); the voltage drop between the plasma and the electrode is \(V > 0\). We assume that the velocity of ions entering the sheath, \(u\), is small compared to the characteristic velocity inside the sheath; this may be achieved, for example, by applying the negative bias to the electrode. The conservation of number of ions

\[
en(z)u(z) = j_0,
\]

where \(n(z)\) is the ion density and \(j_0\) is the ion current density, together with the energy conservation

\[
mu^2(z) + e\varphi(z) = 0
\]

result in the Child–Langmuir equation for the potential, \(\varphi(z)\), inside the sheath:

\[
\frac{d^2\varphi(z)}{dz^2} = -4\pi j_0 \sqrt{-\frac{m}{2e\varphi(z)}}.
\]

In the following, we consider the space-charge-limited flow [13], that is, the ion current density, \(j_0\), is the eigenvalue of the problem and the boundary conditions to (3) are

\[
\varphi(0) = 0, \quad \varphi'(0) = 0, \quad \varphi(z_0) = -V.
\]

Now suppose that there is a thin negatively charged dust layer at \(z = z_1 < z_0\). We do not consider here the origin of its charge. We only note that the vertical force balance, cf, e.g., [15], determines the layer position, \(z_1\). Then, the layer is described by introducing the additional boundary conditions to (3)

\[
\varphi(z_1 + 0) - \varphi(z_1 - 0) = 0, \quad \varphi'(z_1 + 0) - \varphi'(z_1 - 0) = 4\pi q\sigma,
\]

where \(\sigma\) is the surface density of the layer and \(-q\ (q > 0)\) is the charge of a single grain. Implementing equations (3)–(5), we also ignore the absorption of ions by the dust layer.
It is convenient to use normalized quantities. Introducing the dimensionless potential $\psi(\zeta) = -\varphi(z)/V$ and the coordinate $\zeta = z/z_0$, we rewrite (3)–(5) as

$$\frac{d^2 \psi(\zeta)}{d\zeta^2} = \frac{\lambda}{4 \sqrt[3]{\psi(\zeta)}},$$

(6)

$$\psi(0) = 0, \quad \psi'(0) = 0,$$

(7)

$$\psi(\zeta_1 - 0) = \psi(\zeta_1 + 0), \quad \psi'(\zeta_1 + 0) - \psi'(\zeta_1 - 0) = -\eta,$$

(8)

$$\psi(1) = 1,$$

(9)

where the normalized current is

$$\lambda = \frac{16\pi z_0^2 j_0}{V^{3/2} \sqrt{2e}},$$

(10)

and the normalized surface density of the layer is

$$\eta = \frac{4\pi q \sigma z_0}{V}.$$  

(11)

3. Electric field

Our next step is to solve the Child–Langmuir equation (6) supplemented with the boundary conditions (7)–(9). The first integral of (6) is

$$\left( \frac{d\psi(\zeta)}{d\zeta} \right)^2 = \lambda \sqrt[3]{\psi(\zeta)} + C,$$

(12)

where $C$ is the integration constant, which is actually proportional to the $zz$-component of the momentum flux. In the interval $0 < \zeta < \zeta_1$, this integration constant is zero, $C = 0$, due to the first two boundary conditions (7). Assuming that $\psi'(\zeta) > 0$, we easily get

$$\psi(\zeta) = \psi^-(\zeta) = \lambda^{2/3} \left( \frac{3}{4} \zeta \right)^{4/3}, \quad 0 < \zeta < \zeta_1.$$  

(13)

Due to the conditions (8), the integration constant in the gap between the layer and the electrode ($\zeta_1 < \zeta < 1$) is

$$C = -2\eta \lambda^{2/3} (3\zeta_1/4)^{1/3} + \eta^2.$$  

(14)

Then, assuming that the layer density is not too large, $\eta < \psi^-(\zeta_1)$, i.e. the electric field above the layer is positive, $\psi'(\zeta) > 0$, we get

$$\zeta - \zeta_1 = \int_{\psi_1}^{\psi} \frac{dy}{\sqrt{\lambda \sqrt[3]{y} + C}} = \left[ \frac{4}{3\lambda^{2/3} (\lambda \sqrt[3]{y} - 2C) \sqrt{\lambda \sqrt[3]{y} + C}} \right]_{\psi_1}^{\psi},$$

(15)

where $\psi_1 = \psi^-(-\zeta_1)$ is the layer potential. Denote the solution of equation (15) with respect to $\psi$ as $\psi^+(\zeta)$.

Finally, making use of the last boundary condition (9) we evaluate the dependence of the normalized current, $\lambda$, on the normalized layer density, $\eta$.

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Although the solution of (15) results in an enormous expression for $\psi^{(+)}(\zeta)$, it is an easy matter to obtain its expansion in powers of $\eta$. Explicitly, the first two terms of the expansion are

$$\psi^{(+)}(\zeta) = \psi^{(-)}(\zeta) - 3\eta(\zeta \zeta_1)^{1/3} \left( \zeta^{1/3} - \zeta_1^{1/3} \right) + \cdots, \quad \zeta_1 \leq \zeta \leq 1. \quad (16)$$

With this expression in hand, we easily get the limiting current

$$\lambda = \frac{16}{9} + 8\eta \left( \zeta_1^{1/3} - \zeta_1^{2/3} \right) + \cdots. \quad (17)$$

For larger values of $\eta$, the potential, $\psi^{(+)}$, and the current, $\lambda$, are found by numeric solution of (9), (15). An example is shown in figure 2, where both branches of the potential $\psi^{(\pm)}(\zeta)$ are depicted by the solid line and the dashed line represents the extrapolation of the solution (13) to the interval $\zeta_1 < \zeta < 1$. It was found that the dependence of the current, $\lambda$, on $\eta$ is almost linear and it is well approximated by the expansion (17).

Qualitatively, the effect of the charged layer upon the potential profile is very simple: it just reduces the electric field between the layer and the electrode. It should be pointed out that our calculations are valid for sufficiently small values of the surface density, $\eta$. If $\eta > \psi^{(-)}(\zeta_1)$, then the field at $\zeta \to \zeta_1 + 0$ changes the sign and the solution given by (15) fails. We will not investigate this situation here.

### 4. Surface pressure

Generally, the conservation of the linear momentum is expressed as $\nabla \cdot T_{ij} = F_{i}^{\text{ext}}$, where $T_{ij}$ is a stress tensor of a medium and $F_{i}^{\text{ext}}$ is an external force density acting upon a system. Within the model adopted here, a force, $F_{i}^{\text{ext}}$, supports the layer at a fixed position and it is nonzero at $z = z_1$ only. This may be, for example, the Earth gravity force. The net stress tensor is a sum of the ion momentum flux and the Maxwellian stress; its nonzero components are:

$$T_{zz} = mn(z)u(z)^2 - \frac{E(z)^2}{8\pi}, \quad (18)$$

$$T_{xx} = T_{yy} = \frac{E(z)^2}{8\pi}, \quad (19)$$

where $E(z) = -\psi^{\prime}(z)$ is the electric field.

Figure 2. Potential profile; $\eta = 1.2, \zeta_1 = 0.5, \lambda = 3.5545$. 

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The balance of the horizontal components of the linear momentum in the sheath is characterized by the surface pressure defined as the average value of the tangential stress (19):

\[ S = \int_{0}^{z_0} dz \frac{E(z)^2}{8\pi}. \]  

(20)

The physical meaning of this quantity is fairly evident. Let us bisect mentally the sheath along, say, \( x = x_0 \) plane (figure 1). The surface pressure (20) is the force per unit length that repels the right part of the sheath. Notice that surface tension in normal liquids acts in the opposite way: it attracts two parts of a surface minimizing the net surface area.

Making use of the first integral (12), which actually expresses the conservation of the \( z \)-component of the momentum (cf (18)), and integrating by parts we rewrite the surface pressure (20) as

\[ s(\eta) = \int_{0}^{1} d\zeta (\psi'(\zeta))^2 = \frac{4}{5} \lambda^{4/3} \left( \frac{3}{4} \zeta_1 \right)^{5/3} + \int_{\psi_1}^{1} d\psi \sqrt{\lambda \psi} + C, \]  

(21)

where the normalized pressure is \( s = z_0 S/(8\pi V^2) \) and \( C \) is given by (14).

The exact expression for the surface pressure (21) is too bulky to be written down, but its expansion in powers of \( \eta \) is

\[ s(\eta) = \frac{16}{15} + \frac{4}{15} \eta \left( 4\zeta_1^{1/3} - 9\zeta_1^{2/3} + 5\zeta_1^{4/3} \right) + \cdots. \]  

(22)

In evaluating this expansion we also took into account the dependence of the current on \( \eta \) (17).

Of great importance is that the surface pressure (21), (22) may decrease with the increasing surface density, \( \eta \). Figure 3 shows the dependence of the derivative \( s'(\eta) \) at \( \eta = 0 \) on the layer position, \( \zeta_1 \), which is negative if \( \zeta_1 > 0.1445 \). The same behaviour is illustrated also by figure 4, where the numerically evaluated surface pressure (21) is depicted for various values of \( \zeta_1 \).

The dependence of the surface pressure on the density may be called the equation of state of the composite system consisting of the charged layer and the plasma sheath. Usually, the equation of state appears in the context of thermodynamic potentials like the free energy. We cannot use thermodynamic concepts in application to the plasma sheath, which is an open system. However, the pressure is related to the conservation of linear momentum and, in this sense, it is a well-defined entity.

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Here we treat the dust layer as a rigid charged sheet, that is, we ignore its inner structure. Evidently, short-range interactions between dust grains should add a contribution to the equation of state (e.g. [16]). The evaluation of the net equation of state of the dust-laden plasma sheath must rely upon more sophisticated models and will be discussed elsewhere. The above calculations show that with sufficiently low dust density, when the dust is in the gaseous phase and the inter-grain interactions are of minor importance, the surface pressure may become a decreasing function of the layer density. In other words, the compressibility of this composite medium is negative. Such a medium is inevitably unstable; it should shrink to some equilibrium size. In this respect, there is probably no need for an external potential confining the dust layer in the horizontal direction. We wonder if anybody has observed a stable dust layer over a flat electrode without a confining ring.

5. Force

Now, let us consider the semi-infinite charged layer occupying the half-plane, \( z = z_1, \ x < 0 \), as depicted in figure 1. Our purpose here is to figure out the horizontal force exerted upon the edge of the layer. The evaluation of the electric field in such a system requires solution of two-dimensional nonlinear equations. However, to get the \( x \)-component of the force we may circumvent this arduous task by using the solution of the one-dimensional problem. Due to the momentum conservation outside the layer, the \( x \)-component of the force may be evaluated by integrating the stress tensor, \( T_{xi} \) over the arbitrary surface enclosing the edge of the layer. In particular, the integration surface may be composed of the planes \( x = x_0, \ x = x_1, \ z = 0, \ z = z_0 \) (see figure 1). Assuming that there is no tangential electric field, \( E_x, \) at \( z = 0 \) and \( z = z_0, \) the shear stress is also zero, \( T_{xz}(z = z0) = T_{xz}(z = 0) = 0. \) Then, the horizontal force per unit length exerted upon the edge is written as the difference \( f = s_1 - s_0, \) where the values of surface pressure \( s_{0,1} \) are evaluated at points \( x = x_{0,1}, \ x_0 > 0 \) and \( x_1 < 0. \) By choosing the coordinates \( x_{0,1} \) far enough from the edge, so that the electric field is independent of \( x, \) we can use the expressions (21), (22) to evaluate the surface pressure. Therefore, assuming that the layer density, \( \eta, \) is constant for \( x < 0 \) and it is zero for \( x > 0 \) we get

\[
f = s(\eta) - s(0).\tag{23}
\]
As we have already pointed out, the surface pressure, \( s(\eta) \), may be a decreasing function of \( \eta \). In this case, the force (23) is negative, that is, the sheath expels the additional negative charge carried by the layer. The reason for this is fairly general and independent of the details of the sheath model. It is readily seen from figure 2 that the layer reduces the electric field; in its turn, the electric field pressure (19) also drops. This pressure differential results in the force (23). The process resembles squeezing toothpaste out of a tube; the only essential difference is that pressure inside a tube is isotropic.

Of interest is that similar reasoning may be applied to evaluate the force upon a charged semi-infinite layer in the absence of ions, i.e. for a planar capacitor. The force then acts in the opposite direction, \( f > 0 \), as is suggested by physicist’s intuition. Therefore, the force (23) is a result of self-consistent restructuring of the sheath under the influence of the charged layer.

6. Estimations and conclusions

Here we give a simple, model-independent estimation of the magnitude of the force applied to the sheath edge. In the absence of dust, the electric field inside the sheath is of the order of magnitude \( E_0 \sim V/z_0 \), where \( V \) is the voltage drop over the sheath and \( z_0 \) is the sheath width. Then the surface pressure (20) at \( x = x_0 \) (figure 1) is estimated as \( S_0 \sim E_0^2 z_0/(8\pi) \).

The electric field of the dust layer is estimated as \( E_d \sim 4\pi q \sigma \sim 4\pi q/a^2 \), where \( a \) is the average distance between grains. The surface pressure at \( x = x_1 < 0 \) is \( S_1 \sim (E_0 - E_d)^2 z_0/(8\pi) \); with sufficiently small dust density \( (E_d \ll E_0) \), the pressure drop is \( S_1 - S_0 \sim E_0 E_d z_0 \). Finally, the horizontal force acting upon a single grain at the edge of the layer is \( F_h \sim (S_1 - S_0)a \sim q V/a \). Of interest is to compare this force with vertical forces exerted upon a grain. Maximal vertical forces are the \( z \)-component of the electric force and the gravity force; both are nearly equal and of the order of magnitude \( F_z \sim q V/z_0 \). Therefore, the horizontal force provided by the inhomogeneous surface pressure is much larger than vertical forces, \( F_h/F_z \sim z_0/a \gg 1 \).

In a real sheath, there are a number of factors that may influence our reasoning. The electron pressure should reduce the force (23). Variation of the layer density results in variation of the sheath width; redistribution of the potential profile alters the charge of the grains and the levitation height. Evaluation of the surface pressure under these conditions requires more sophisticated sheath models. However, assuming that the surface pressure squeezes the layer, the equilibrium surface density near the edge may be estimated in the following way. Suppose that the sheath rigidity is provided by some short-range interactions between dust grains, e.g. by the screened Coulomb repulsion. Then, the layer pushes away the grain situated on its extreme edge with the force \( F_l \sim q^2/a^2 \exp(-a/\lambda_D) \), where \( \lambda_D \) is the linearized Debye length. In equilibrium, this force is compensated by the surface pressure, \( F_l \approx F_s \), that is, \( q/a \exp(-a/\lambda_D) \sim V \). The grain charge is \( q = d \varphi_0 \), where \( d \) is the grain radius and \( \varphi_0 \) is its potential relative to the bulk of the ambient plasma. If there is no additional bias, then the floating potential, \( \varphi_0 \), and the potential drop over the sheath, \( V \), are of the same order of magnitude, that is, the average equilibrium distance between grains is \( a \leq \lambda_D \). Therefore, the surface pressure may be strong enough to confine a closely packed dust layer. It should be stressed that this estimation gives us the minimum possible interparticle distance, \( a \). The various factors mentioned above may substantially reduce the surface pressure.
As we have already pointed out, the homogeneous dust layer may become unstable. It is hardly worth investigating this instability in the framework of a simple sheath model adopted here. However, if the hole arises in a dust layer due to its negative compressibility or by some other means, then the additional electric field pressure inside the hole is strong enough to prevent it from closing up. Probably, this mechanism is responsible for the formation of the two-dimensional void observed in [7].

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References

[1] Samsonov D and Goree J 1999 Phys. Rev. E 59 1047
[2] Rothermel H et al 2002 Phys. Rev. Lett. 89 175001
[3] Morfill G E et al 1999 Phys. Rev. Lett. 83 1598
[4] Goree J et al 1999 Phys. Rev. E 59 7055
[5] Akdim M R and Goedheer W J 2001 Phys. Rev. E 65 015401(R)
[6] Kersten H et al 2003 Int. J. Mass Spectr. 223–224 313
[7] Dahiya R P et al 2002 Phys. Rev. Lett. 89 125001
[8] Nambu M et al 1995 Phys. Lett. A 203 40
[9] Hamaguchi S and Farouki R T 1994 J. Chem. Phys. 101 9876
[10] Ignatov A M 1996 Plasma Phys. Rep. 22 585
[11] Child C D 1911 Phys. Rev. 32 492
[12] Langmuir I 1913 Phys. Rev. 2 450
[13] Forrester A T 1988 Large Ion Beams: Fundamentals of Generation and Propagation (New York: Wiley)
[14] Kurilenkov Y K et al 2000 Frontiers in Dusty Plasmas (Amsterdam: Elsevier) p 543
[15] Vladimirov S V and Cramer N F 2000 Phys. Rev. E 62 2754
[16] Hebner G A et al 2002 Phys. Rev. E 66 046407