Influence of Heat Transfer on Magnetohydrodynamics Oscillatory Flow for Bingham Fluid with Variable Viscosity Through a Porous channel

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Abstract
This paper aims to study a mathematical model explaining the impact of heat transfer on magnetohydrodynamics oscillatory flow for Bingham fluid with variable viscosity through a porous channel. The governing equations are solved numerically by developing a suitable finite difference scheme. As an application of the theory in the field of magnetohydrodynamics, we used the perturbation method to obtain a clear formula for fluid motion. The results obtained are illustrated by graphs after we reached the momentum equation solution, and used the MATHEMATICA program for plotting the velocity and temperature of the fluid for two types of flow (Poiseuille and Couette).

Keywords: Bingham fluid, MHD, oscillatory flow, variable viscosity porous channel.

1. Introduction

The flow of an electrically conductive fluid has various uses in engineering applications like (MHD) generators, plasma studies, nuclear reactors, geothermal energy extraction, and boundary control in aerodynamics [1]. One of the most important properties of fluids is viscosity. The petroleum products viscosity is a significant physical feature of numerous derivatives, specifically oils and lubricants. Moreover, crude oil viscosity and its derivatives are technically vital and have a prominent part in the calculation of flow. Different papers in the scientific literature have covered the movement of fluids in the duct. For instance, [2] examined the influence of heat transfer on the flow oscillation of the hydrodynamics magnetizing Eyring-Powell fluid via a permeable medium. In [3] studied the effect of transferring heat on (MHD) for oscillatory flow of Williamson fluid with different viscosity models for two types of geometries "Poiseuille flow and Couette flow" via a porous medium passage. By [4] an investigation of peristaltic flow of Bingham plastic fluid in an inclined tapered asymmetric channel with variable viscosity. The study of transferring heat effect on the oscillatory flow of Jeffrey fluid with a different viscosity model via a porous medium by Khafajy [5]. Adnan and Abdulhadi [6] analyzed the influence of an inclined magnetic field on peristaltic flow of Bingham plastic fluid in an inclined symmetric channel with slip situation. In the same year Adnan and Abdulhadi [7] investigated the peristaltic flow of Bingham plastic fluid in a curved channel. Ara et al. [8] explored the Jeffery-Hamel flow of Bingham plastic fluid in converging channel in the presence of external magnetic field. However, Salih [9] illustrated the impact of different temperature and concentration on peristaltic transport (MHD) of a fluid with different viscosity Jeffrey via the porous passage. The effect of variable temperature and focus on peristaltic transport (MHD) of a fluid with variable viscosity Jeffrey through the porous channel. It is naturally existed in human living body such as urine movement from kidney to bladder, food swallowing process and blood flow in the small vessels [10-12].

The study considers a mathematical model for the impact of transferring heat on oscillatory flow (MHD) for Bingham fluid with different viscosity via a porous passage. The series of perturbation technique are used in this study to explain the problem for the two types of flow "Poiseuille flow and Couette flow". The results of the physical parameter problem are considered through using graphs.
2. Mathematical Formulation

Considered the unsteady flow of a Bingham fluid in a porous channel of width \( h \) under the impacts of an electrically applied magnetic field as represented in (Fig. one). Assumed that fluids have produced very slight electromagnetic force and the connection of electricity is poor. It is though that cartesian coordinate system like, \((u(y, \tau), 0, 0)\) is a speed vector where \( u \) is the first constituent of the speed field and the perpendicular to the x-axis is \( y \).

**Fig 1** Channel format: (i) Poiseuille flow and (ii) Couette flow.

The essential equations governing the problem are provided as:

The continuousness equation is given by:

\[
\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0
\]

(1)

The momentum equations are:

- In the \( x \)-direction:
  \[
  \rho \left( \frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \nu \frac{\partial \bar{u}}{\partial y} \right) = -\frac{\partial p}{\partial x} + \frac{\partial \bar{S}_{xx}}{\partial x} + \frac{\partial \bar{S}_{xy}}{\partial y} - \sigma B_0^2 \bar{u} - \frac{\mu(y)}{k} \bar{u}
  \]
  \(\text{(2)}\)

- In the \( y \)-direction:
  \[
  \rho \left( \frac{\partial \bar{v}}{\partial t} + \bar{u} \frac{\partial \bar{v}}{\partial x} + \nu \frac{\partial \bar{v}}{\partial y} \right) = -\frac{\partial p}{\partial y} + \frac{\partial \bar{S}_{xy}}{\partial x} + \frac{\partial \bar{S}_{yy}}{\partial y} - \frac{\mu(y)}{k} \bar{v}
  \]
  \(\text{(3)}\)

The temperature equation is given by:

\[
c_T \rho \left( \frac{\partial T}{\partial t} + \bar{u} \frac{\partial T}{\partial x} + \bar{v} \frac{\partial T}{\partial y} \right) = K_T \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \bar{S} \cdot (\text{grad} \ \bar{V}) - \frac{\partial q}{\partial y}
\]

(4)

where \( T(y, \tau) \) is a fluid temperature, \( B_0 \) is a strength of magnetic field, \( \mu_c \) the magnetic permeability, \( \rho \) is a fluid density, \( \mu_p \) the magnetic permeability, \( \sigma \) is a conductivity of the fluid, \( k \) is a permeability, \( c_T \) is a specific heat at constant pressure, \( K_T \) is a thermal conductivity and \( q \) is a radioactive heat flux.

The temperatures at the walls of the channel as the following:

\[
T = T_0 \text{ at } \bar{y} = 0, \quad \text{and} \quad T = T_1 \text{ at } \bar{y} = h
\]

(5)

The radioactive heat flux [6] is got through:

\[
\frac{\partial q}{\partial y} = 4\eta^2(T_0 - T)
\]

(6)

here \( \eta \) denotes the absorption of radiation.

The basic equation for the Bingham fluid is as the folloowing:

\[
\bar{S} = \begin{cases} 
\mu(y) \bar{X} + \frac{\nu}{\bar{y}} \bar{X} & \text{for } \tau \geq \tau_0 \\
0 & \text{for } \tau < \tau_0 
\end{cases}
\]

(7)

Where \( \mu_y \) is a fluid viscosity dependent on the traveled distance, \( \tau_0 \) is the yield stress, and \( \bar{y} \) is defined as:

\[
\bar{y} = \frac{1}{2} \Sigma_i \Sigma_j \bar{y}_{ij} \bar{y}_{ji} = \frac{1}{2} \bar{\Pi}
\]
here \( \Pi \) is the second invariant strain tensor.

The Rivlin-Ericksen tensors are given as \( \vec{\mathbf{X}} = \nabla \vec{V} + (\nabla \vec{V})^T \), where \( \nabla \vec{V} \) is the velocity field in the cartesian coordinates system \((x, y, z)\) and \((\nabla \vec{V})^T\) is the transpose of the velocity field.

The stress component are given by:

\[
\vec{S}_{xx} = \vec{S}_{yy} = 0, \vec{S}_{xy} = \vec{S}_{yx} = \mu(\vec{y})\vec{u}_y + \tau_0
\]

and \( \vec{S}_{(grad \vec{V})} = \vec{S}_{xy}\vec{u}_y \) (8)

So that the heat equation become

\[
c_T \rho \frac{\partial T}{\partial t} = k_T \frac{\partial^2 T}{\partial y^2} + \vec{S}_{xy}\vec{u}_y - 4\eta^2(T_0 - T)
\]

3. Method of Solution

The governing equations of the motion, we may introduce the non-dimensional conditions are as follows:

\[
x = \frac{x}{h}, y = \frac{y}{h}, u = \frac{u}{h}, t = \frac{t}{\frac{h^2}{v}}, \rho = \frac{\rho h}{\mu_c}, M^2 = \frac{\sigma B^2 h^2}{\mu_c}, D_a = \frac{k}{h}, \gamma = \frac{h}{u}
\]

\[
\Delta T = T_1 - T_0, \theta = \frac{T_1 + T_0}{2}, Bn = \frac{h^2}{\mu_c}, \mu(\gamma) = \frac{\rho h^2 c_p}{K_T}, Ec \approx \frac{\mu}{c_p}, Pr = \frac{\mu c_p}{K_T}, N^2 = \frac{4\eta^2 h^2}{K_T}
\]

where \((U)\) is the mean speed flow, \((D_a)\) is Darcy number, \((Re)\) is Reynolds number, \((M)\) is magnetic parameter, \((Pe)\) is the Peclet number and \((N)\) is the radiation parameter, \((Bn)\) Bingham number, \((Br)\) Brinkman number, \((Pr)\) Prandtl number, \((Ec)\) Eckert number.

Substituting equations (11) into equations (1)-(5), (8), and (10), we will get the non-dimensional equations as follows:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\frac{\partial \theta}{\partial x} = -Re \frac{\partial u}{\partial \theta} + \frac{\partial}{\partial \theta} \left( \mu(\theta) \frac{\partial u}{\partial \theta} + Bn \right) - \left( M^2 + \frac{\mu(\theta)}{D_a} \right) u
\]

\[
Pe \frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} + N^2 \theta + Br \left( \mu(\theta) \frac{\partial u}{\partial \theta} + Bn \right) \frac{\partial u}{\partial \theta}
\]

and \( S_{xy} = \mu(\gamma) \frac{\partial u}{\partial \gamma} + Bn \)

4. Reynold’s Model of Viscosity

The reynold’s model and variant of viscosity with dependent on the traveled distance are identified as:

\[
\mu(\gamma) = e^{-\alpha \gamma}
\]

Through employment the series of maclaurin, we will get:

\[
\mu(\gamma) = 1 - \alpha \gamma, \quad \alpha << 1
\]

In such situation, the viscosity is constant at \( \alpha = 0 \), by substituting equation (16) by equation (13), we get:

\[
\frac{\partial \rho}{\partial x} = -Re \frac{\partial u}{\partial \theta} + \left( 1 - \alpha \gamma \right) \frac{\partial^2 u}{\partial \gamma^2} - \alpha \frac{\partial u}{\partial \gamma} - \left( M^2 + \frac{1-\alpha \gamma}{D_a} \right) u
\]
5. Solution of the Problem

This section contains a solution of the heat equation and motion equation.

5.1 Solution of motion equation

5.1.1 Poiseuille flow

We will find a solution to the equation of motion (13) for two kinds of flows "Poiseuille and Couette". First let

\[ u(y, t) = u_0(y)e^{i\omega t}, \quad -\frac{dp}{dx} = \lambda e^{i\omega t} \]  

(18)

where \( \omega \) is a frequency of the oscillation and \( \lambda \) is a real constant.

Substituting equations (18) into equation (13), we have

\[ \text{Re} \, i\omega \, u_0(y) = \lambda + \left( 1 - \alpha y \right) \frac{\partial^2 u_0(y)}{\partial y^2} - \alpha \frac{\partial}{\partial y} u_0(y) - \left( M^2 + \frac{(1 - \alpha y)}{\Delta a} \right) u_0(y) \]  

(19)

Equation (19) is a non-linear differential equation and it is hard to deduce a precise answer, so perturbation technique will be used to discover an explanation to the problem in below:

\[ u_0(y) = u_{00}(y) + \alpha u_{01}(y) \]  

(20)

Now, by substituting equation (20) in equation (19), we obtain:

\[ \frac{\partial^2 u_{00}}{\partial y^2} + \alpha \frac{\partial^2 u_{01}}{\partial y^2} - \alpha y \frac{\partial^2 u_{00}}{\partial y^2} - \alpha \frac{\partial}{\partial y} u_{00} - \alpha \frac{\partial}{\partial y} u_{01} - \text{Re} \, i\omega u_{00} - M^2 u_{00} - \frac{1}{\Delta a} u_{00} + \]  

\[ \frac{\alpha y}{\Delta a} u_{01} - \text{Re} \, i\omega \alpha u_{01} - M^2 \alpha u_{01} - \frac{\alpha}{\Delta a} u_{01} + \frac{\partial^2 u_{01}}{\partial y^2} = -\lambda \]  

(21)

By equating the like powers of \( \alpha \), we obtain the following results presented in the forthcoming subsections:

1. Zeros-order system(\( \alpha^0 \))

\[ \frac{\partial^2 u_{00}}{\partial y^2} - \left( \text{Re} \, i\omega + M^2 + \frac{1}{\Delta a} \right) u_{00} = -\lambda \]  

(22)

The related boundary situations are: \( u_{00}(0) = u_{00}(1) = 0 \)

2. First - order system(\( \alpha^1 \))

\[ \frac{\partial u_{01}}{\partial y^2} - \left( \text{Re} \, i\omega + M^2 + \frac{1}{\Delta a} \right) u_{01} = \frac{\partial^2 u_{00}}{\partial y^2} + \frac{\partial}{\partial y} u_{00} - y \frac{\partial}{\partial y} u_{00} \]  

(23)

The associated boundary conditions are: \( u_{01}(0) = u_{01}(1) = 0 \).

From the solution of equations (22) and (23), and the resulting substitution in equation (18) after substitution in equation (20), we obtain the solution of the equation of motion in the case of Poiseuille flow

\[ u(y, t) = \left( \frac{\lambda}{A} + \left( -\frac{\lambda}{A(1 + e^{\xi y})} \right) e^{\sqrt{\xi} y} + \left( -\frac{\lambda e^{\sqrt{\xi} y}}{A(1 + e^{\sqrt{\xi} y})} \right) e^{-\sqrt{\xi} y} + \alpha \left( -\frac{K}{A} + e^{\sqrt{\xi} y} \left( \frac{K}{A(1 + e^{\sqrt{\xi} y})} \right) + e^{-\sqrt{\xi} y} \left( \frac{e^{\sqrt{\xi} y}}{A(1 + e^{\sqrt{\xi} y})} \right) \right) \right) e^{i\omega t} \]  

(24)
5.1.2 Couette flow

Assume here that the flow channel is two parallel plates, the below plate is at rest and the higher plate is moving with the speed $U_0$. We obtain the solution of the equation of motion in the case of Couette flow

$$u(y, t) = \left( \frac{\lambda}{\alpha} + e^{\sqrt{\alpha}y} \left( \frac{e^{\sqrt{\alpha}y} - e^{-\sqrt{\alpha}y}}{\alpha} \right) + e^{-\sqrt{\alpha}y} \left( - \frac{e^{\sqrt{\alpha}y}}{\alpha} \right) + \alpha \right) - \left( \frac{\lambda}{\alpha} + e^{\sqrt{\alpha}y} \left( 1 + e^{2\sqrt{\alpha}y} \right) \right)$$

(25)

where

$$K = \sqrt{\frac{\lambda}{\alpha}} - \sqrt{\frac{\lambda}{\alpha}} + \sqrt{\frac{\lambda}{\alpha}} - \sqrt{\frac{\lambda}{\alpha}} + \sqrt{\frac{\lambda}{\alpha}} - \sqrt{\frac{\lambda}{\alpha}}$$

and $A = \left( M^2 + \text{Re} \omega + \frac{1}{\text{Da}} \right)$ in two cases "Poiseuille flow and Couette flow".

5.2 Solution of the temperature equation

We will find a solution to the equation of temperature (14) of two kinds of flows "Poiseuille and Couette". The non-dimensional boundary conditions for heat equation (14) are $\theta(0) = 0, \theta(1) = 1$. To solve the heat equation (14), let

$$\theta(y, t) = \theta_0(y) e^{i\omega t}$$

(26)

where $\omega$ is a frequency of the oscillation. Substituting equation (26) into equation (14), we have

$$Pec \frac{\partial}{\partial t} \left( \theta_0(y) e^{i\omega t} \right) = \frac{\partial^2}{\partial y^2} \left( \theta_0(y) e^{i\omega t} \right) + N^2 \left( \theta_0(y) e^{i\omega t} \right) + \mu(y) Br \left( \frac{\partial u}{\partial y} \right)^2 + Br Bn \left( \frac{\partial u}{\partial y} \right)$$

$$\Rightarrow$$

$$Pec \frac{\partial}{\partial t} \left( \theta_0(y) e^{i\omega t} \right) = \frac{\partial^2}{\partial y^2} \left( \theta_0(y) e^{i\omega t} \right) + N^2 \left( \theta_0(y) e^{i\omega t} \right) + e^{-\alpha y} Br \left( \frac{\partial u}{\partial y} \right)^2 + Br Bn \left( \frac{\partial u}{\partial y} \right)$$

implies that

$$\frac{\partial^2 \theta_0(y)}{\partial y^2} + \left( N^2 - i\omega Pe \right) \theta_0(y) = -e^{-i\omega t} \left( e^{-\alpha y} Br \left( \frac{\partial u}{\partial y} \right)^2 + Br Bn \left( \frac{\partial u}{\partial y} \right) \right)$$

(27)

The solution of equation (27) with boundary conditions $\theta_0(0) = 0, \theta_0(1) = 1$, is divided into two parts according to the type of flow "Poiseuille flow or Couette flow" and thus we will have;

The temperature of Bingham fluid for Poiseuille flow is;

$$\theta_0 = e^{i\omega t} \left( \frac{G}{F} + \left( -\frac{G}{F} \right) \cos(\sqrt{F}y) + \left( -\frac{G \cot(\sqrt{F}) + G \csc(\sqrt{F}) - F \csc(\sqrt{F})}{F} \right) \sin(\sqrt{F}y) \right)$$

(28)

The temperature of Bingham fluid for Couette flow is;

$$\theta_0 = e^{i\omega t} \left( \frac{G}{F} + \left( -\frac{G}{F} \right) \cos(\sqrt{F}y) + \left( -\frac{G \cot(\sqrt{F}) + G \csc(\sqrt{F}) - F \csc(\sqrt{F})}{F} \right) \sin(\sqrt{F}y) \right)$$

(29)
\[ G = -e^{-i\omega t} \left( e^{-ay}Br \right) \left( e^{i\omega t} \left( \frac{e^{\lambda x} - \lambda x}{(1+e^{\lambda x})^2} \right) + BrN \right) \] where F = N^2 - i\omega Pe in two cases "Poiseuille flow and Couette flow".

6. Numerical Results and Discussion

This section considered the effect of transferring heat on the magnetohydrodynamics oscillatory flow of Bingham fluid with various viscosity via a porous medium for Poiseuille flow and Couette flow. Use the MATHEMATICA-12 program, to discussed graphically all solutions obtained (velocity, shear stress and temperature) of the fluid under variations of different parameters relevant, important outcomes are given in figs. (2-27), through area 0 ≤ y ≤ 1 which is the width of the flow channel.

Based on equations (24) and (25), the velocity profile appears in figures (2-8), where each figure is divided into two figures, (a) representing Poiseuille flow, and (b) representing Couette flow. Fig 2 speed profile that introduces u decreases and t decreases. Figure 3 shows the effect on velocity profiles against y. It is found by growing the function of velocity profiles u (decrease). Figure 4 denotes that u increases with maximizing Da. Figure 5 shows that u increases with increasing Re. Figure 6 shows that u decreases with increasing M. Figure 7 denotes the impact of () on speed profiles versus y. It is deduced that by maximizing (o) the velocity profile minimizes. Fig 8 show that with the maximizing of α the velocity increases when 0.45 < y ≤ 1, and minimizes when 0 ≤ y < 0.45 in Poiseuille flow, while in Couette flow the velocity decreases with the increasing α. We noticed from drawings 2-7 that the fluid flow velocity in the case of Couette flow) is more than that of (Poiseuille flow) due to the movement of the upper wall of the channel at a constant velocity (U0 = 0.3), which is why the highest value of the fluid velocity in Poiseuille flow is in the middle of the flow channel, while the highest value of the speed in Couette flow is at the upper wall.

Based on equation (15), the shear stress appears in figures (9-16), where each figure is divided into two figures, (a) representing shear stress when the flow is Poiseuille, and (b) representing shear stress when the flow is Couette. Fig 9 show us that with the increasing of t the shear stress increases when 0.45 < y ≤ 1, and decreases when 0 ≤ y < 0.45 in Poiseuille flow, while in Couette flow the shear stress profile decreases with the increasing t. Figs 10 and 11 show us that with the increasing of λ and Da the shear stress decreases at upper wall channel and increases at lower wall channel in two flows, respectively. Figs 12 and 13 illustrations the influence of the parameters Re and M on the shear stress, here the effect of Re and M is opposite to the effect of the parameters λ and Da on shear stress. Fig 14 show that with the increasing of ω the shear stress increases when 0.45 < y ≤ 1, and decreases when 0 ≤ y < 0.45 in Poiseuille flow, while in Couette flow the shear stress decreases with the increasing ω. Fig 15 shows the shear stress profile rising up by the increasing Bn. Fig 16 show us that with the increasing of α the shear stress decreases when 0.45 < y ≤ 1, and increases when 0 ≤ y < 0.45 in Poiseuille flow, while in Couette flow the shear stress profile increasing with the increasing α.

Based on equations (28) and (29), the temperature profile appears in figures (17-27), where each figure is divided into two figures, (a) representing Poiseuille flow, and (b) representing Couette flow. Fig 17 denotes for us that with the rising of M the temperature escalates when 0.45 < y ≤ 1, and minimizes when 0 ≤ y < 0.45 in Poiseuille flow, while in Couette flow the temperature increasing when 0.65 < y ≤ 1, and decreases when 0 ≤ y < 0.65. Fig 18 shows the temperature profile decreases with the increasing ω. Figs 19 and 20 refer to that with the rising of Bn and Br the temperature minimizes when 0.45 < y ≤ 1, and increases when 0 ≤ y < 0.45 in Poiseuille flow, while in Couette flow the temperature profile increasing with the increasing Bn and Br. Fig 21 denotes the temperature profile
lessens with the maximaization of Pe. Fig 22 shows the temperature profile increasing with the increasing N. Fig 23 displays that with the rising of Da the temperature reduces when $0.45 < y \leq 1$, and increasing when $0 \leq y < 0.45$. Fig 24 shows the temperature profile increasing with the increasing Re in Poiseuille flow, while in Couette flow the temperature decreases when $0.45 < y \leq 1$, and increasing when $0 \leq y < 0.45$. Fig 25 denotes that with the rising of $\lambda$ the temperature reduces when $0.45 < y \leq 1$, and increasing when $0 \leq y < 0.4$. Fig 26 denotes the temperature profile reduces with the $t$ increase. Fig 27 displays that the rising of $\alpha$ the temperature reduces when $0.45 < y \leq 1$, and increases when $0 \leq y < 0.45$ in Poiseuille flow, while in Couette flow the temperature profile increasing with the increasing $\alpha$.

**Fig 2** Velocity profile with various values $t$ for (a) Poiseuille flow and (b) Couette flow, with $Da = 1, M = 1, \omega = 1, \lambda = 1, \alpha = 0.05, Re = 1, U_0 = 0.3$.

**Fig 3** Velocity profile with various values $\lambda$ for (a) Poiseuille flow and (b) Couette flow, with $Da = 1, M = 1, \omega = 1, \alpha = 0.05, Re = 1, U_0 = 0.3, t = 0.5$.

**Fig 4** Velocity profile with different values $Da$ for (a) Poiseuille flow and (b) Couette flow, with $M = 1, \omega = 1, \lambda = 1, \alpha = 0.05, Re = 1, U_0 = 0.3, t = 0.5$. 

Fig 5 Velocity profile with different values Re for (a) Poiseuille flow and (b) Couette flow, with $Da = 1, M = 1, \omega = 1, \lambda = 1, \alpha = 0.05, U_0 = 0.3, t = 0.5$.

Fig 6 Velocity profile with different values M for (a) Poiseuille flow and (b) Couette flow, with $Da = 1, \omega = 1, \lambda = 1, \alpha = 0.05, Re = 1, U_0 = 0.3, t = 0.5$.

Fig 7 Velocity profile with different values $\omega$ for (a) Poiseuille flow and (b) Couette flow, with $Da = 1, M = 1, \lambda = 1, \alpha = 0.05, Re = 1, U_0 = 0.3, t = 0.5$. 
Fig 8 Velocity profile with different values $\alpha$ for (a) Poiseuille flow and (b) Couette flow, with $Da = 1, M = 1, \lambda = 1, Re = 1, U_0 = 0.3, t = 0.5$.

Fig 9 Shear stress profile with different values $t$ for (a) Poiseuille flow and (b) Couette flow, with $Da = 1, M = 1, \omega = 1, \alpha = 0.05, \lambda = 1, Bn = 1, Re = 1, U_0 = 0.3$.

Fig 10 Shear stress profile with different values $\lambda$ for (a) Poiseuille flow and (b) Couette flow, with $Da = 1, M = 1, \omega = 1, \alpha = 0.05, Bn = 1, Re = 1, U_0 = 0.3, t = 0.5$. 
**Fig 11** Shear stress profile with different values Da for (a) Poiseuille flow and (b) Couette flow, with $M = 1, \omega = 1, Bn = 1, \alpha = 0.05, \lambda = 1, Re = 1, U_0 = 0.3, t = 0.5$.

**Fig 12** Shear stress profile with different values Re for (a) Poiseuille flow and (b) Couette flow, with $Da = 1, M = 1, \omega = 1, \alpha = 0.05, Bn = 1, \lambda = 1, U_0 = 0.3, t = 0.5$.

**Fig 13** Shear stress profile with different values M for (a) Poiseuille flow and (b) Couette flow, with $Da = 1, \omega = 1, Bn = 1, \lambda = 1, \alpha = 0.05, Re = 1, U_0 = 0.3, t = 0.5$. 
Fig 14 Shear stress profile with different values $\omega$ for (a) Poiseuille flow and (b) Couette flow, with $Da = 1, M = 1, Bn = 1, \alpha = 0.05, \lambda = 1, Re = 1, U_0 = 0.3, t = 0.5$.

Fig 15 Shear stress profile with different values $Bn$ for (a) Poiseuille flow and (b) Couette flow, with $Da = 1, M = 1, \omega = 1, \alpha = 0.05, \lambda = 1, Re = 1, U_0 = 0.3, t = 0.5$.

Fig 16 Shear stress profile with different values $\alpha$ for (a) Poiseuille flow and (b) Couette flow, with $Da = 1, M = 1, \omega = 1, \lambda = 1, Re = 1, U_0 = 0.3, t = 0.5$. 
**Fig 17** Temperature profile with different values $M$ for (a) Poiseuille flow and (b) Couette flow, with $Bn = 1, Pe = 2, Br = 2, \alpha = 0.05, \omega = 1, N = 1.25, M = 1, t = 0.5$.

**Fig 18** Temperature profile with different values $\omega$ for (a) Poiseuille flow and (b) Couette flow, with $Bn = 1, Pe = 2, Br = 2, \alpha = 0.05, N = 1.25, M = 1, t = 0.5$.

**Fig 19** Temperature profile with different values $Bn$ for (a) Poiseuille flow and (b) Couette flow, with $Pe = 2, Br = 2, \omega = 1, \alpha = 0.05, N = 1.25, M = 1, t = 0.5$. 
Fig 20 Temperature profile with different values $Br$ for (a) Poiseuille flow and (b) Couette flow, with $Bn = 1, Pe = 2, \omega = 1, \alpha = 0.05, N = 1.25, M = 1, t = 0.5$.

Fig 21 Temperature profile with different values $Pe$ for (a) Poiseuille flow and (b) Couette flow, with $Bn = 1, Br = 2, \omega = 1, \alpha = 0.05, N = 1.25, M = 1, t = 0.5$.

Fig 22 Temperature profile with different values $N$ for (a) Poiseuille flow and (b) Couette flow, with $Bn = 1, Pe = 2, Br = 2, \alpha = 0.05, \omega = 1, M = 1, t = 0.5$. 
Fig 23 Temperature profile with different values Da for (a) Poiseuille flow and (b) Couette flow, with Bn = 1, Pe = 2, Br = 2, α = 0.05, ω = 1, M = 1, t = 0.5.

Fig 24 Temperature profile with different values Re for (a) Poiseuille flow and (b) Couette flow, with Bn = 1, Pe = 2, Br = 2, α = 0.05, ω = 1, M = 1, t = 0.5.

Fig 25 Temperature profile with different values λ for (a) Poiseuille flow and (b) Couette flow, with Bn = 1, Pe = 2, Br = 2, α = 0.05, ω = 1, M = 1, t = 0.5.
Fig 26 Temperature profile with different values $t$ for (a) Poiseuille flow and (b) Couette flow, with $Bn = 1, Pe = 2, \alpha = 0.05, Br = 2, \omega = 1, M = 1$.

Fig 27 Temperature profile with different values $\alpha$ for (a) Poiseuille flow and (b) Couette flow, with $Bn = 1, Pe = 2, Br = 2, \omega = 1, M = 1$.

Conclusion

This study, explaining the impacts of mass transfer on MHD oscillatory flow for Bingham fluid through a porous channel. We examined the influences of certain constraints that are active on fluid velocity through scrutinizing the graphs got when we concluded the momentum equation solution, and used the MATHEMATICA program to plot the velocity and temperature of the fluid for two types of flow (Poiseuille and Couette).

- Various values are employed to reach to the outcomes of pertinent parameters, specifically Darcy number ($Da$), Reynolds number ($Re$), Peclet number ($Pe$), magnetic parameter ($M$), Bingham number ($Bn$), Brinkman number ($Br$), radiation parameter ($N$).
- We show that the velocity profiles were increased by the increasing $Da, \lambda$ and $Re$ for the Poiseuille and Couette flow.
- The velocity profiles were decreases by the increasing $t, M$ and $\omega$ for the Poiseuille and Couette flow.
- We explain that the temperature profiles were decreases by the increasing $t, Pe$ and $\omega$ for the Poiseuille and Couette flow while the temperature profile was increased by the increasing $N$.
- The temperature profiles were increased by the increasing $\lambda, Re, Da, Bn, Br$ when $0 \leq y < 0.45$, while in Couette flow the temperature profile increasing with the increasing $Bn$ and $Br$. 

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