Renormalization group improvement of the effective potential in six dimensions

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Using the renormalization group improvement technique, we study the effective potential of a model consisting of \(N\) scalar fields \(\phi^i\) transforming in the fundamental representation of \(O(N)\) group coupled to an additional scalar field \(\sigma\) via cubic interactions, defined in a six-dimensional spacetime. We find that the model presents a metastable vacuum, that can be long-lived, where the particles become massive. The existence of attractive and repulsive interactions plays a crucial role in such phenomena.

I. INTRODUCTION

Toy models have been intensely explored in scientific literature, since they provide good theoretical laboratories to discuss key concepts of quantum field theory. Although we might have an unrealistic theory, it may highlight some interesting features we want to study.

An instance of such toy models is the theory of a scalar field with cubic interaction in six dimensions. The \(\phi^3_6\) model has been used to discuss a wide variety of topics. For example, this model shares with QCD the interesting phenomenon of asymptotic freedom [1], but is considerably simpler than the latter thus providing a useful tool to explore this phenomenon [2]. Unlike QCD, however, this model has an unbounded potential from below and, although we might arrange for a stable local minimum, this stability is lost at a critical temperature [3]. This model was also used to study the behaviour of quantum gravity models with thermal instability [4]. Moreover, some variations of this model are also fruitful in ideas. In [5], for example, the authors quantized and solved the noncommutative \(\phi^3_6\) and were also able to compute the exact renormalization of the wave-function and coupling constant by mapping it to the Kontsevich model.

In more recent years, the interest in a particular model with \(N + 1\) scalar fields in \(d = 6 - \epsilon\)

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coupled via cubic interactions has grown [6–9]. This model is described by the Lagrangian
\[
L = \frac{1}{2}(\partial_\mu \phi^i)^2 + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{g_1}{2}(\sigma \phi^i \phi^i) + \frac{g_2}{6} \sigma^3, \quad (i = 1, 2, \cdots, N),
\]
(1)
and it was argued in [6] that it provides an UV completion to the \(O(N)\) symmetric scalar field
theory with interaction \((\phi^i \phi^i)^2\) in the dimension range \(4 < d < 6\), at least for large \(N\).

As it is well known, spontaneous symmetry breaking is one of such key concepts in particle
physics, with Higgs mechanism playing a fundamental role in the Standard Model. In that case, the
symmetry breaking requires a mass parameter in the Lagrangian but S. Coleman and E. Weinberg
demonstrated in [10] that a spontaneous symmetry breaking may occur due to radiative corrections
when a quadratic mass term is absent from the Lagrangian, as it is the case in conformally-invariant
theories, such as the \(\phi_6^3\) model, where we have a dimensionless coupling constant.

In order to discuss the Coleman-Weinberg (CW) mechanism, the standard procedure is to com-
pute the effective potential, a powerful and convenient tool to explore many aspects of the the
low-energy sector of a quantum field theory. In several situations, the one-loop approximation is
good enough, but of course we want sometimes to improve it, adding higher-order contributions
in the loop expansion. However, since calculations become very complicated already at two-loop,
some techniques were developed to improve the calculation of the effective potential. In particular,
we cite [11], where the effective action for the \(\phi_6^3\) model was explicitly computed observing that
the appearance of an arbitrary mass scale \(\mu^2\) introduced by renormalization imposes some condi-
tions for the quantum corrections to the classical potential. The so-called renormalization group
improvement has been intensely used to go beyond one-loop approximation [12–20]. The general
idea is to use the renormalization group equations (RGE) to sum up sub-series of the effective
potential.

In this work, we compute the improved effective potential and use it to discuss the vacuum
structure of a massless theory of scalars with cubic interaction in six dimensions. Our model
consists of \(N\) scalar fields \(\phi^i\) transforming in the fundamental representation of \(O(N)\) coupled to
an additional scalar field \(\sigma\) via cubic interactions, described by the Lagrangian [1]. This theory has
a potential unbounded from below, but it is nevertheless possible that radiative corrections might
generate a stable false vacuum [21]. Our results indicate that the CW mechanism does indeed
provide a metastable vacuum and a generation of mass.

This work is organized as follows, in section II we compute the effective potential using the
Renormalization Group Equation and explore some of its properties in \(d = 6\) dimensions. In
section III, we draw our conclusions.
II. THE EFFECTIVE POTENTIAL IN $d = 6$ DIMENSIONS

We start by using the RGE to evaluate the effective potential for the model defined by the Lagrangian [1] in $d = 6$ dimensions. The effective potential will be computed to the $\sigma$ field, including quantum fluctuations due to $\phi_i$ and $\sigma$ interactions, but we are assuming that $\langle \phi_i \rangle = 0$ (so the $O(N)$ symmetry of this sector of the theory is kept manifest). That means $\sigma$ is the only degree of freedom in the effective potential.

Following the prescription for the RG improvement technique [11], we start assuming that the effective potential has to satisfy the RGE:

$$\left( \mu \frac{\partial}{\partial \mu} + \beta_{g_1} \frac{\partial}{\partial g_1} + \beta_{g_2} \frac{\partial}{\partial g_2} + \gamma_\sigma \frac{\partial}{\partial \sigma} \right) V_{\text{eff}}(\sigma) = 0, \quad (2)$$

where $\beta_{g_1}$ and $\beta_{g_2}$ are the two $\beta$-functions to this model and $\gamma_\sigma$ is the anomalous dimension for the scalar field $\sigma$.

In order to determine the effective potential, it is useful to write $V_{\text{eff}}$ as

$$V_{\text{eff}} = \frac{1}{6} \sigma^3 S_{\text{eff}}(g_1, g_2, L(\sigma)), \quad (3)$$

where $S_{\text{eff}}(g_1, g_2, L(\sigma))$ is a function of the coupling constants and $L(\sigma) = \ln \frac{\sigma^2}{\mu^2}$.

Now we observe that

$$\mu \frac{\partial V_{\text{eff}}}{\partial \mu} = -2 \frac{\sigma^3}{6} \frac{\partial S_{\text{eff}}}{\partial L} = -2 \frac{\partial V_{\text{eff}}}{\partial L} \quad (4)$$

$$\sigma \frac{\partial V_{\text{eff}}}{\partial \sigma} = \frac{1}{6} \sigma^3 \left( 3 + 2 \frac{\partial}{\partial L} \right) S_{\text{eff}} = \left( 3 + 2 \frac{\partial}{\partial L} \right) V_{\text{eff}}, \quad (5)$$

so we can rewrite (2) in terms of derivatives with respect to $L$ and thus we find the RGE for $S_{\text{eff}}$ to be

$$\left[ 2(-1 + \gamma_\sigma) \frac{\partial}{\partial L} + \beta_{g_1} \frac{\partial}{\partial g_1} + \beta_{g_2} \frac{\partial}{\partial g_2} + 3 \gamma_\sigma \right] S_{\text{eff}} = 0. \quad (6)$$

The one-loop renormalization group functions for the model [1] were computed in [6], namely,

$$\gamma_\sigma = \frac{1}{(4\pi)^3} \frac{Ng_1^2 + g_2^2}{12},$$

$$\beta_{g_1} = \frac{(N - 8)g_1^3 - 12g_2^2g_1 + g_1g_2^2}{12(4\pi)^3},$$

$$\beta_{g_2} = -\frac{4Ng_1^3 + Ng_1^2g_2 - 3g_2^3}{4(4\pi)^3}. \quad (7)$$

In order to solve (6) and thus find the effective potential, we first observe that when $V_{\text{eff}}$ is calculated perturbatively the result can be organized as a power series in $L(\sigma) = \ln \frac{\sigma^2}{\mu^2}$, so we will
assume the following Ansatz

\[ S_{\text{eff}} = A + BL + CL^2 + DL^3 + \ldots \]

where the coefficients are power series of the coupling constants \( g_i \), that is,

\[
A = \sum_{n=1}^{\infty} A_n, \quad \text{with} \quad \begin{cases}
A_1 = a_{11}g_1 + a_{12}g_2 \\
A_2 = a_{21}g_1^2 + a_{22}g_1g_2 + a_{23}g_2^2 \\
A_3 = a_{31}g_1^3 + a_{32}g_1^2g_2 + a_{33}g_1g_2^2 + a_{34}g_2^3 \\
\vdots
\end{cases}
\]

and similarly for the other coefficients, \( B, C, \) etc.

The core idea behind the method is the observation that the coefficients in (8) are not all independent, since changes in \( \mu \) must be compensated for by changes in the other parameters, according to the renormalization group. Let us then first reorganize the perturbative expansion (8) alternatively in the so-called leading-log series expansion. By simple power counting, we assemble the effective potential as follows

\[
V_{\text{eff}} = \frac{\sigma^3}{6} \left( \sum_{n=0}^{\infty} C_n^{LL} g^{2n+1} L^n + \sum_{n=1}^{\infty} C_n^{NLL} g^{2n+3} L^n + \ldots + \delta \right),
\]

where \( C_n^{LL} \) and \( C_n^{NLL} \) are respectively the coefficients to the leading logarithms (LL) and next-to-leading logarithms (NLL) contributions, dots represent higher order contributions and \( \delta \) is the counter-term defined by a renormalization condition. In the above expression, \( g^{2n+1} \) denotes some combination of \( g_1 \) and \( g_2 \) at that order, such that \( g^3 \), for example, includes \( g_1^3, g_1^2g_2, g_1g_2^2 \) and \( g_2^3 \).

To compute the leading-log contributions to the effective potential, we consider only the LL series,

\[
V_{\text{eff}} = \frac{\sigma^3}{6} \left( \sum_{n=0}^{\infty} C_n^{LL} g^{2n+1} L^n + \delta \right).
\]

In order to find the coefficients \( C_n^{LL} \), we plug (8) in (6) and consider each order in the expansion in \( L \) to obtain the set of equations

\[
\begin{align*}
\left( \beta_1 \frac{\partial}{\partial g_1} + \beta_2 \frac{\partial}{\partial g_2} + 3\gamma_\sigma \right) A + 2(-1+\gamma_\sigma) B &= 0, \quad (\text{order } L^0) \\
\left( \beta_1 \frac{\partial}{\partial g_1} + \beta_2 \frac{\partial}{\partial g_2} + 3\gamma_\sigma \right) B + 2(-1+\gamma_\sigma)(2C) &= 0, \quad (\text{order } L^1) \\
\left( \beta_1 \frac{\partial}{\partial g_1} + \beta_2 \frac{\partial}{\partial g_2} + 3\gamma_\sigma \right) C + 2(-1+\gamma_\sigma)(3D) &= 0, \quad (\text{order } L^2) \\
\vdots
\end{align*}
\]
Now, each equation can also be expanded in powers of the coupling constants and thus we find:

\[ 2B_3 = (\beta_1 \frac{\partial}{\partial g_1} + \beta_2 \frac{\partial}{\partial g_2} + 3\gamma_\sigma)A_1 \quad \text{(order } g^3 L^0) \),
\[ 4C_5 = (\beta_1 \frac{\partial}{\partial g_1} + \beta_2 \frac{\partial}{\partial g_2} + 3\gamma_\sigma)B_3 \quad \text{(order } g^5 L^1) \),
\[ 6D_7 = (\beta_1 \frac{\partial}{\partial g_1} + \beta_2 \frac{\partial}{\partial g_2} + 3\gamma_\sigma)C_5 \quad \text{(order } g^7 L^2) \),

(13)

where we have considered that \( \gamma_\sigma \sim g^2, \beta_i \sim g^3, A_n \sim g^n, B_n \sim g^n, \) etc. (cf. Eqs. (7) and (9)).

The above set of equations allows us to identify the following recurrence relation for the LL coefficients

\[ C_{n+1}^{LL} = \left( \beta_1 \frac{\partial}{\partial g_1} + \beta_2 \frac{\partial}{\partial g_2} + 3\gamma_\sigma \right) \frac{C_n^{LL}}{2(n+1)}. \]  

(14)

We are now able to compute the LL effective potential up to any order. In particular, it is important to note that the LL effective potential up to \( g^3 L \) order represents the full one-loop effective potential.

A. The effective potential at one-loop order

We can now use (14) for \( n = 0 \) to compute \( C_1^{LL} \), with \( C_0^{LL} = g_2 \) being an input established from tree-level potential:

\[ C_0^{LL} = g_2, \]
\[ C_1^{LL} = \left( \beta_1 \frac{\partial}{\partial g_1} + \beta_2 \frac{\partial}{\partial g_2} + 3\gamma_\sigma \right) \frac{C_0^{LL}}{2} = -\frac{2g_1^3 N - g_1^2 g_2 N + g_2^3}{256\pi^3}. \]  

(15)

The one-loop effective potential \( V_{eff} \) is then given by

\[ V_{eff} = \frac{\sigma^3}{6} \left[ g_2 + \delta + \frac{1}{2} \left( \frac{g_2 (g_1^2 N + g_2^2)}{256\pi^3} + \frac{-4g_1^3 N + N^2 g_2 N - 3g_3^3}{256\pi^3} \right) \ln \left( \frac{\sigma^2}{\mu^2} \right) \right]. \]  

(16)

In order to fix the counter-term \( \delta \), we use the Coleman-Weinberg renormalization condition,

\[ \frac{d^3 V_{eff}}{d\sigma^3} \bigg|_{|\sigma| = \mu} = g_2, \]  

(17)

where \( \mu > 0 \) is the renormalization scale. Thus we find that the renormalized effective potential is

\[ V_{eff} = \frac{\sigma^3}{6} \left[ g_2 + \frac{11}{768\pi^3} \left( \frac{2g_1^3 N - g_1^2 g_2 N + g_2^3}{256\pi^3} - \frac{2g_1^3 N - g_1^2 g_2 N + g_2^3}{256\pi^3} \right) \ln \left( \frac{\sigma^2}{\mu^2} \right) \right]. \]  

(18)
The classical potential is unbounded from below, but it is possible to have a metastable vacuum due to radiative corrections. Let us assume we have a local minimum and explore this possibility by imposing the renormalization scale to be around the (possible) local minimum of the effective potential. The conditions for its existence are given by

\[
\frac{dV_{\text{eff}}}{d\sigma} \bigg|_{\sigma = \mu} = 0, \quad (19a)
\]
\[
\frac{d^2 V_{\text{eff}}}{d\sigma^2} \bigg|_{\sigma = \mu} = m_\sigma^2 > 0, \quad (19b)
\]

where \( m_\sigma^2 \) is the mass for the \( \sigma \) field (possibly) generated by the radiative corrections.

Equation (19a) imposes that

\[
g_2 = -\frac{3}{256\pi^3} \left( 2g_1^3 N - g_1^2 g_2 N + g_2^3 \right), \quad (20)
\]

and therefore the conditions (19) are perturbatively satisfied for \( \sigma = -\mu \) and \( g_2 \approx -\frac{3g_1^3 N}{128\pi^3} \). Around the metastable vacuum, \( V_{\text{eff}} \) can be written as

\[
V_{\text{eff}} = \frac{g_1^3 N \sigma^3}{2304\pi^3} \left[ 2 - 3 \ln \left( \frac{\sigma^2}{\mu^2} \right) \right], \quad (21)
\]

where the generated masses are given by

\[
m_\sigma^2 = \frac{d^2 V_{\text{eff}}}{d\sigma^2} \bigg|_{\sigma = -\mu} = \frac{g_1^3 N}{128\pi^3} \mu; \quad (22a)
\]
\[
m_\phi^2 = -g_1(\sigma) = g_1 \mu. \quad (22b)
\]

We can see that both masses are positive, assuming \( g_1 > 0 \). The effective potential is plotted for different values of \( N \) in figure 1.

Figure 1: One-loop effective potential for different values of \( N \). As large is \( N \) as deeper is the valley of \( V_{\text{eff}} \).
The vacuum induced by radiative corrections is a local minimum and thus we have a metastable vacuum state and at some time it will decay to the real vacuum. However, the potential is unbounded from below, which means that there is no global minimum and therefore no stable solution to the potential with energy smaller than \(-\frac{g_2^3 N \mu^3}{1152\pi^3}\).

Our results to the effective potential reveal three interesting phenomena. First, the model exhibit a dimensional transmutation, since the potential was initially described by two dimensionless parameters \((g_1\) and \(g_2\)) and now it is described by a dimensionless parameter and a dimensionful one \((g_1\) and \(\mu\), respectively). Second, there is generation of mass to both fields in the \(O(N)\)-symmetric phase. Third, these phenomena are due to the appearance of a metastable vacuum.

The decay rate of the vacuum is in general computed through the Callan-Coleman formalism [22], but this formalism can not be used in theories in which the symmetry breaking is due to radiative corrections, since it assumes a bounce solution to the classical potential. In order to compute such decays in theories in which spontaneous symmetry breaking is induced by radiative corrections, we apply a slightly changed form of the Callan-Coleman formalism developed by E. Weinberg [23].

In the case where there is no bounce solution (such as a potential unbounded from below) and the interactions are attractive, J. A. González et al. [21] showed that there is no vacuum decay and the metastable vacuum is indeed the true vacuum. The authors carried out the analysis considering the Callan-Coleman formalism, but the results should be the same for Weinberg’s formalism.

Physically, the tunnelling between false and true vacuum states occurs because when the system is in the false vacuum, quantum fluctuations creates bubbles of the true vacuum, continually. Now thinking about the tunnelling of the state as a phase transition, the bubble must be large enough to grow, i.e. a bubble with a sufficiently large radius to enclose the true vacuum solution.

However, the negative value of \(g_2\) (assuming \(g_1 > 0\)) plays a central role in this analysis because, as the bubble grows, the repulsive interaction becomes more relevant. To see this, let us study the behaviour of the potential near the metastable vacuum, by Taylor expanding it to obtain,

\[
V_{\text{eff}} = -\frac{g_1^3 N \mu^3}{1152\pi^3} + \frac{g_2^3 \mu N (\sigma + \mu)^2}{256\pi^3} - \frac{g_2^3 N (\sigma + \mu)^3}{256\pi^3} + \mathcal{O} ((\sigma + \mu)^4) .
\] (23)

For small fluctuations around the local minimum of the effective potential \(\sigma = -\mu\), this potential is similar to the discussed in [24], in this case the potential can simulate the dynamics of a long chain. In this way, when the bubble is large enough, the repulsive interaction becomes dominant and we observe the fracture of the chain. As expected, as \(N\) grows, the metastable vacuum becomes more stable, once the \(\phi\) fields interacts via an attractive interaction. This feature can be viewed
graphically, because when \( N \) is larger, the metastable vacuum is deeper, as showed in figure \( \text{[1]} \)

### B. The leading log effective potential

Using the the recurrence relation \( [14] \), we can determine higher order corrections to the LL effective potential. The relevant observables of the theory around the metastable vacuum are sensitive up to \( g^7 L^3 \) order, since the counter-term is determined up to \( g^7 \) order because renormalization condition \( [17] \). Therefore, in order to obtain the radiative generated masses it is enough to get only the first four terms in \( [14] \). Following the prescription described in the previous section, the renormalized LL effective potential up to \( \mathcal{O}(g^7) \) is given by

\[
V_{\text{eff}} = \frac{\sigma^2}{6} \left[ A + B \ln \left( \frac{\sigma^2}{\mu^2} \right) + C \ln \left( \frac{\sigma^2}{\mu^2} \right)^2 + D \ln \left( \frac{\sigma^2}{\mu^2} \right)^3 \right],
\]

where

\[
A = g_2 + \frac{g_1^2 N^3}{9437184 \pi^9} - \frac{7 g_1^4 N^2}{2831552 \pi^9} + \frac{5 g_1^6 N}{7077888 \pi^9} - \frac{5 g_1^6 g_2 N^3}{169869312 \pi^9} - \frac{109 g_1^6 g_2 N^2}{169869312 \pi^9} + \frac{73 g_1^6 g_2 N}{42467328 \pi^9} + \frac{g_1^6 g_2 N^2}{1179648 \pi^9} + \frac{g_1^6 g_2 N^2}{24576 \pi^6} - \frac{g_1^5 N}{12288 \pi^6} + \frac{5 g_1^6 g_2 N^2}{56623104 \pi^9} - \frac{1887368 \pi^9}{g_1^2 g_2 N^2} - \frac{73 g_1^4 g_2 N}{73728 \pi^6} + \frac{11 g_1^3 g_2 N^2}{2831552 \pi^9} + \frac{11 g_1^3 N}{2304 \pi^6} - \frac{49 g_1^3 g_2^5 N}{169869312 \pi^9} + \frac{g_1^2 g_2^3 N}{36864 \pi^6} - \frac{11 g_1^2 N}{4608 \pi^3} + \frac{11 g_2^5}{24576 \pi^6} + \frac{g_2^5}{4608 \pi^3} \]
\]

\[
B = \frac{g_2^2 N (g_2 - 2 g_1) - g_2^3}{1536 \pi^6}
\]
\[
C = -\frac{3 g_1^5 (N - 2) N + g_1^4 g_2 N (N + 7) - 2 g_1^2 g_2^3 N + g_1^5}{1536 \pi^6}
\]
\[
D = -\frac{g_1^7 N^3}{75497472 \pi^9} + \frac{7 g_1^5 N^3}{226492416 \pi^9} + \frac{5 g_1^7 N^2}{56623104 \pi^9} + \frac{5 g_1^6 g_2 N^3}{1358954496 \pi^9} + \frac{109 g_1^6 g_2 N^2}{1358954496 \pi^9} + \frac{73 g_1^6 g_2 N}{339738624 \pi^9} - \frac{g_1^5 N}{452984332 \pi^9} + \frac{g_1^4 g_2 N}{150994944 \pi^9} + \frac{g_1^3 g_2^2 N}{226492416 \pi^9} + \frac{1358954496 \pi^9}{g_1^2 g_2 N^2} - \frac{g_1^5}{75497472 \pi^9} - \frac{9437184 \pi^9}{150994944 \pi^9}
\]

Just as in the one-loop case, the conditions \( [19] \) are perturbatively satisfied for \( \sigma = -\mu \), but the coupling constant \( g_2 \) receives corrections up to \( \mathcal{O}(g_1^7) \) given by

\[
g_2 = \frac{3 g_1^3 N}{128 \pi^3} - \frac{g_1^5 N (17 N - 16)}{32768 \pi^6} - \frac{g_1^5 N (651 N^2 + 464 N + 320)}{75497472 \pi^9}.
\]

Therefore, the LL effective potential is

\[
V_{\text{eff}} = \frac{g_1^3 N \sigma^3}{2304 \pi^7} \left[ 2 + \frac{g_1^4 N (17 N - 16) + 768 \pi^3 g_1^2 N}{32768 \pi^6} - \left( 3 + \frac{3 g_1^2 N (g_1^4 (17 N - 16) + 768 \pi^3)}{65536 \pi^6} \right) \ln \left( \frac{\sigma^2}{\mu^2} \right) - \frac{3 (g_1^4 N (N + 7) + 128 \pi^3 g_1^2 (N - 2))}{32768 \pi^6} \ln \left( \frac{\sigma^2}{\mu^2} \right) - \frac{g_1^4 (7 - 3 N) N (N - 20)}{98304 \pi^6} \ln \left( \frac{\sigma^2}{\mu^2} \right) \right].
\]
The fields acquire mass induced by radiative corrections given by

\[ m^2_\sigma = \left. \frac{d^2 V_{\text{eff}}}{d\sigma^2} \right|_{\sigma = -\mu} = \frac{g_1^2 N \mu}{128 \pi^5} \left[ 1 + \frac{g_1^2 (13N - 8)}{768 \pi^3} + \frac{g_4^4 N (59N + 8)}{196608 \pi^6} \right], \quad (27) \]

where \( m^2_\phi \) is the same as (22b). The LL corrections to \( m^2_\sigma \) become larger as \( N \) grows. For instance, if we have \( g_1 \sim 0.2 \) and \( N \sim 10^3 \), the corrections to the one-loop mass is of order of 2%, and for \( N \sim 10^4 \) the corrections is about 27%. Therefore, the LL corrections becomes very relevant in the large \( N \) limit of the effective potential. In the figure 2 we plot the comparison between one-loop (21) and LL (26) effective potentials for \( N = 10^4 \) and \( g_1 = 0.2 \).

![Figure 2: Comparison between one-loop Eq. (21) and LL Eq. (26) effective potentials for \( N = 10^4 \) and \( g_1 = 0.2 \). Leading Log corrections become relevant for large \( N \).](image)

III. CONCLUSIONS

In this work, we studied the possibility of a spontaneous generation of mass, induced by radiative corrections via Coleman-Weinberg mechanism, in a model consisting of \( N \) scalar fields \( \phi^i \) transforming in the fundamental representation of \( O(N) \) coupled to an additional scalar field \( \sigma \) via cubic interactions, defined in a six dimensional spacetime. We computed the improved effective potential and use it to discuss the vacuum structure of the model. This model has a potential unbounded from below, but it is nevertheless possible that radiative corrections might generate a stable false vacuum, as discussed in [21]. Our results indicate that the Coleman-Weinberg mechanism does indeed provide a metastable vacuum and a generation of mass in the model presented here.

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