Impedance Control Method for Space Manipulator System In On-orbit Self-assembly Task

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Abstract. To complete the large-load on-orbit self-assembly mission successfully, an impedance control method for the space manipulator system is researched in this paper. Considering the uncertainty of dynamic parameters and assembly environment, the dynamic model of space manipulator system is established. Furthermore, the system dynamics is analyzed for the closed chain structure consisting of the manipulator. In order to realize a stable on-orbit assembly under the disturbance caused by the collision, the compliance control strategy of space manipulator and the stability control strategy of spacecraft base is researched. Finally, taking the typical 7-DOF space manipulator as an example, a simulation is carried out, which shows that the proposed impedance control algorithm is effective.

1. Introduction

On-orbit self-assembly with a heavy payload is an extremely important task in constructing the space station. In the self-assembly task, due to the large arm span, control error, mechanical error and other factors, the end precision of the space manipulator is poor, which may lead to the failure of docking. If the position control only, the errors may produces great force and the structure of the manipulator may be damaged. In view of the above situation, the application of force control in the assistant docking task of space manipulator can solve the problem of excessive force caused by errors. Zhang W studied the compliance control method of manipulator based on force feedback [1]. Pathak P.M studied the application of impedance control method in space manipulator [2]. Wang M studied the impedance control method for grasping non-cooperative target [3]. Huang P.F. studied the adaptive control method of grasping target [4]. Uyama N studied the impedance control method of space manipulator grasping non-cooperative targets in free-floating state [5]. However, these studies are based on open-loop chain structure. In self-assembly task, the assembly-force acts at the end and base of the manipulator simultaneously. If the coupling moment of the manipulator on the base is not considered, the control accuracy of the manipulator will be affected. Therefore, this paper analyses the dynamic characteristics of the closed chain system, then designs the control algorithm to realize the impedance control of the self-assembly task.
2. Dynamic Model of Space Manipulator System In On-orbit Self-assembly Task

The general kinematics equation of the free-floating space manipulator on the base can be expressed as follows [5].

\[
\dot{x}_b = J_b \dot{x}_b + J_m \dot{\theta}
\]  
(1)

In the expression above, \( \dot{x}_b = \begin{bmatrix} v_b \\ \omega_b \end{bmatrix} \) and \( \dot{x}_e = \begin{bmatrix} v_e \\ \omega_e \end{bmatrix} \) are the linear and angular velocities of the end and the base of the manipulator respectively. \( \dot{\theta} = [\dot{\theta}_1, \dot{\theta}_2, \cdots, \dot{\theta}_n]^T \) is the angular velocity of the joint of the space manipulator. \( J_b \) and \( J_m \) are Jacobian matrices describing the mapping relations between \( \dot{x}_e \) and \( \dot{x}_b \), \( \dot{\theta} \) and \( \dot{x}_b \) respectively.

Based on the kinematics analysis of space manipulator, the forward dynamic model of the system is established by Lagrange equation. The total kinetic energy of the system is:

\[
T = \frac{1}{2} \sum_{k=0}^{n} \left( (\omega_k)^T \mathbf{I}_k \omega_k + m_k (v_k)^T v_k \right) = \frac{1}{2} \left[ v_0, \omega_0, \theta \right]^T \mathbf{H} \begin{bmatrix} v_0 \\ \omega_0 \\ \theta \end{bmatrix}
\]  
(2)

For a general space manipulator system, the system is in a free floating state in microgravity environment and is not affected by any other external forces. Without considering the flexibility of the arm and the joint, the system's potential energy is 0. Based on Lagrange equation, the forward dynamic equation of the space manipulator can be obtained as follows.

\[
\begin{bmatrix} \mathbf{H}_b & \mathbf{H}_{bm} \\ \mathbf{H}_{bm}^T & \mathbf{H}_m \end{bmatrix} \begin{bmatrix} \dot{x}_b \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} \mathbf{c}_b \\ \mathbf{c}_m \end{bmatrix} = \begin{bmatrix} \mathbf{c}_b \\ \mathbf{c}_m \end{bmatrix} = \begin{bmatrix} \tau_m \end{bmatrix}
\]  
(3)

Among them, 

\[
\mathbf{H}_b = \begin{bmatrix} ME_3 \\ Mr_0 \sum_{k=0}^{n} (J_{ok}^T)^T \end{bmatrix}, \quad \mathbf{H}_m = \begin{bmatrix} \sum_{k=1}^{n} (m_J J_{vk}) \\ \sum_{k=1}^{n} (I_J J_{ok} + m_k (J_{vk})^T J_{ok}) \end{bmatrix}
\]

\( \mathbf{H}_{bm} \) are expressed as the inertia matrix of the base, the coupling inertia matrix of the base and the manipulator, and the inertia matrix of the manipulator respectively, \( \dot{x}_b = [v_b, \omega_b]^T \) is the acceleration term of the base, \( \mathbf{c}_b, \mathbf{c}_m \) are the non-linear terms of velocity dependence of base and manipulator respectively, \( \tau_m \) is torque of the joints of the manipulator.

In the process of self-assembly, the docking rod on the target cabin will generally collide with the acceptance cone on the base, and then slide to the docking locking device, as shown in Fig. 1. \( F_T \) is the collision force. The vector between the collision point and the origin of the base coordinate system is expressed in the inertial coordinate system as \( r \). The vector between the collision point and the origin of the end-coordinate system is expressed in the inertial coordinate system as \( r_{eh} \).
The force and moment on the end of the manipulator and base are as follows.

\[
\mathbf{F}_e = \begin{bmatrix} F_{ex} \\ \tau_{ex} \end{bmatrix}, \quad \mathbf{F}_b = \begin{bmatrix} F_{be} \\ \tau_{be} \end{bmatrix}
\]

\[
\mathbf{F}_r = \begin{bmatrix} F_r \\ r \times F_r \end{bmatrix}
\]

(4)

In the process of self-assembly, the torque generated by the movement of the large-load cabin on the manipulator is as follows:

\[
\tau_{ml} = J^T_f H_L \ddot{q}_L
\]

(5)

Where, \( H_L \) is the inertia matrix of the cabin to be assembled, and \( \ddot{q}_L \) is the generalized acceleration of the cabin to be assembled.

Substituting Eq.5 into Eq.3, we can get the dynamic equation of the cabin assembly process as follows:

\[
\begin{bmatrix} H_b & \mathbf{H}_{bm} \\ \mathbf{H}_{bm}^T & \mathbf{H}_m \end{bmatrix} \begin{bmatrix} \ddot{x}_b \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} \mathbf{C}_b \\ \mathbf{C}_m \end{bmatrix} = \begin{bmatrix} \mathbf{F}_b \\ \mathbf{r} + \mathbf{r}_{ml} \end{bmatrix} + J^T_b \begin{bmatrix} \mathbf{F}_r \\ \mathbf{r} \times \mathbf{F}_r \end{bmatrix}
\]

(6)

Unfolding Eq.7 and eliminating \( \ddot{x}_b \), the following results can be obtained as follow.

\[
\begin{bmatrix} \mathbf{H}_{bm}^T \mathbf{H}_m - \mathbf{H}_{bm}^{-1} \mathbf{H}_{bm} \end{bmatrix} \dot{\theta} + \mathbf{H}_{bm}^{-1} \mathbf{c}_m - \mathbf{H}_{bm}^{-1} \mathbf{c}_p =
\]

\[
\mathbf{H}_{bm}^{-1} \mathbf{r}_m + \tau_{ml} + \begin{bmatrix} \mathbf{H}_{bm}^T J^T_m - \mathbf{H}_{bm}^{-1} J^T_b \end{bmatrix} \mathbf{F}_e - \mathbf{H}_{bm}^{-1} \mathbf{F}_r
\]

(7)

By simplifying the above formula in a unified form, we can obtain the dynamic equation of the joint space of the self-assembled space manipulator as follows.

\[
\mathbf{\bar{H}} \ddot{\theta} + \mathbf{\bar{c}} = \mathbf{\tau}_m + \mathbf{\tau}_{ml} + J^T_f \mathbf{F}_e + J^T_b \mathbf{F}_b
\]

(8)

Where, \( \mathbf{\bar{H}} = \left( \mathbf{H}_m - \mathbf{H}_{bm}^T \mathbf{H}_b^{-1} \mathbf{H}_{bm} \right) \) is the inertia matrix of the space manipulator relative to the joint, \( \mathbf{\bar{c}} = \left( \mathbf{c}_m - \mathbf{H}_{bm}^T \mathbf{H}_b^{-1} \mathbf{c}_b \right) \) is a joint velocity dependence, \( J_f = \left( J_m - J_b \mathbf{H}_b^{-1} \mathbf{H}_{bm} \right) \) is Jacobian
matrix of free-floating space manipulator, also known as generalized Jacobian matrix, \( J_{bm} = -H_b^{-1}H_{bn} \), \( J_{bm} \) is called the Jacobian matrix of the base-manipulator.

3. Impedance Control
The essence of controller design in impedance control is to maintain the balance of force and position. The higher the order of the impedance function in impedance control, the more accurate the impedance model. Generally, the second-order linear impedance function can be expressed as follow.

\[
M(\ddot{X} - \ddot{X}_r) + B(\dot{X} - \dot{X}_r) + K(X - X_r) = F_e
\]  

(9)

In which, \( M, B \) and \( K \) are the inertia matrix, damping matrix and stiffness matrix of the target impedance, and the acceleration is generally obtained indirectly by the measured force. \( X \) and \( X_r \) are the actual and reference trajectories of the end of the manipulator respectively. Impedance function can control external forces by controlling acceleration, velocity and position deviation.

According to Eq.8, in order to simplify the calculation, ignoring the frictional force and the additional moment caused by non-centric collision, let \( F_b = -F_e \), we can obtain the dynamic equation of the manipulator in joint space under assembly condition as follow.

\[
\ddot{H}\theta + \ddot{c} = \tau + \tau_{ext}
\]  

(10)

In which, \( \tau = \tau_m + \tau_{ml} \) is the control moment of the space manipulator; \( \tau_{ext} = J_f^TF_e - J_{bm}^TF_e \) is the joint moment caused by the contact collision during the self-assembly process.

By multiplying both sides of Eq.10 and combining with Eq.1, the dynamic equation of the space manipulator operating space can be obtained as follow.

\[
\ddot{H}\dot{c} + \dot{c} = F_m + F_e - TF_e
\]  

(11)

Where, \( \ddot{H} = (J_f^T)^{-1}\ddot{H}J_f^{-1} = (J_f\ddot{H}J_f^T)^{-1} \), \( \dot{c} = (J_f^T)^{-1}\dot{c} - \ddot{H}J_f\dot{\theta} \), \( F_m = (J_f^T)^{-1}\tau_m \), \( T = (J_f^T)^{-1}J_{bm}^T \).

Different from the general state of space manipulator model, the base is subjected to contact force during the self-assembly process, which will bring disturbance to the manipulator system, which can not be ignored. \( T \) is defined as the coefficient matrix of the base force on the manipulator.

Combined Eq.12 and Eq.9, the operating-space impedance control model can be set as follow.

\[
F_m = \dot{H}V + \dot{c} + (E - T)F_e
\]  

(12)

In which, \( V = \dot{x}_r + M_i^{-1}(B_i(\dot{x}_r - \dot{x}_e) + K_i(x_r - x_e) + F_e) \), \( M_i, B_i, K_i \) are all diagonal positive definite symmetric matrices, representing the expected inertia, expected damping and expected stiffness in the operating space. \( x_r \) is the reference position of the operating environment, and \( \dot{x}_e \) is the actual position of the end of the manipulator.

According to the impedance model, it can be concluded that:

\[
M_i\ddot{c} + B_i\dot{c} + K_i\dot{c} = -F_e = F_e^T
\]  

(13)
In which, \( e = (x_r - x_e), \dot{e} = (\dot{x}_r - \dot{x}_e), \ddot{e} = (\ddot{x}_r - \ddot{x}_e) \). \( F'_e \) is the force acting on the environment of the manipulator, which is opposite to \( F_e \).

\[
F'_e
\]

Fig. 2 Block diagram of impedance control of self-assembly task

The control structure of the space manipulator using the controller model is shown in Fig. 2. When it is necessary to track the expected collision force, Eq. 13 should be in the following form:

\[
M_i \ddot{e} + B_i \dot{e} + K_i e = F'_e - F_d \tag{14}
\]

From Eq. 14, the impedance control law of the self-assembled space manipulator in the operating space can be obtained as follow.

\[
\tau_m = J^T(\ddot{x_r} + M_i^{-1}(B_i (\dot{x}_r - \dot{x}_e) + K_i (x_r - x_e) + (F_d - F'_e)) + \ddot{e} + (E - T) F'_e) \tag{15}
\]

4. Simulation and verification

In this paper, the 7-DOF space manipulator is taken as the research object. In order to facilitate the follow-up analysis, the initial reference configuration of the space manipulator is given in Fig. 3. The joint angles mentioned in this paper are all taken as reference zeros. In order to be consistent with the coordinate system in the space operator algebraic method, the joint numbers from the base to the end are joint 7 to joint 1 in turn. In addition, the D-H coordinate system of the manipulator is given in Table. 1. The corresponding dimensions in the table are as follows:

\[
\begin{align*}
    d_7 &= 1.2m, \quad d_6 = 0.53m, \quad d_5 = 0.53m, \quad a_5 = 5.8m, \quad d_4 = 0.52m, \quad a_4 = 5.8m, \\
    d_3 &= 0.53m, \quad d_2 = 0.53m, \quad d_1 = 1.2m
\end{align*}
\]
Fig. 3 The D-H coordinate system of 7-DOF manipulator

Σ_b (Installation coordinate system of manipulator) in the base coordinate system, the position and attitude coordinates are \([-0.2m, 0m, 2m, 0^\circ, 0^\circ]\). The D-H parameters of the manipulator are shown in Table 1, and the mass characteristic parameters of the spacecraft base and the manipulator are shown in Table 2.

Table 1. D-H parameters of the manipulator

| Link i | \(\theta_i / (\circ)\) | \(d_i / (m)\) | \(a_i / (m)\) | \(\alpha_{i-1} / (\circ)\) |
|--------|------------------|-------------|-------------|--------------------|
| 7      | \(\theta_7(0)\)  | 0           | 0           | 90                 |
| 6      | \(\theta_6(90)\) | 0           | 0           | -90                |
| 5      | \(\theta_5(0)\)  | 0           | 0           | 0                  |
| 4      | \(\theta_4(0)\)  | \(d_4 + d_5 + d_6\) | 0         | 0                  |
| 3      | \(\theta_3(0)\)  | 0           | 0           | 90                 |
| 2      | \(\theta_2(-90)\)| 0           | 0           | -90                |
| 1      | \(\theta_1(0)\)  | 0           | 0           | 0                  |

Table 2. Mass parameters of 7-DOF space manipulator

| Parameters | Base | Link 7 | Link 6 | Link 5 | Link 4 | Link 3 | Link 2 | Link 1 |
|------------|------|--------|--------|--------|--------|--------|--------|--------|
| Mass(kg)   | 20e+04 | 30     | 30     | 70     | 75     | 30     | 30     | 40     |
| \(m_i\)    |       | 0      | 0.265  | 2.9    | 2.7    | 0      | 0      | 0      |
|            |       | 0      | 0.265  | 0      | 0      | 0      | 0      | 0      |
|            |       | 0      | 0      | 0      | 0.5    | 0.265  | 0.265  | 0.6    |
| Moment of Inertia \(/(kg.m^2)\) | \(I_{xx}\) | 7.5e+05 | 0     | 0.57   | 1.32   | 1.91   | 0.98   | 0.98   | 5.18   |
|            | \(I_{yy}\) | 7.5e+05 | 0.57   | 0.98   | 197.2  | 243.4  | 0.98   | 0.98   | 5.18   |
|            | \(I_{zz}\) | 7.5e+05 | 0.98   | 0.98   | 197.2  | 22.9   | 0.57   | 0.57   | 0.75   |
|            | \(I_{xy}\) | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
|            | \(I_{xz}\) | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
|            | \(I_{yz}\) | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
In the initial state, set the initial configuration to \([-54.05, -182.01, 172.19, -133.40, 141.20, 178.14, -54.05]\) rad. The initial position \(\mathbf{x}_{\text{ini}} = [1, 4, 1] \text{m}\), the end position \(\mathbf{x}_{\text{end}} = [1, 4.25, 1] \text{m}\), expected operating force \(\mathbf{F}_{\text{d}} = [0, 80, 0] N\). The mass of the target capsule is 3e+04 kg, the Inertia tensor is \(I_{xx} = I_{zz} = 8.5 \times 10^5 \text{kg} \cdot \text{m}^2\), \(I_{yy} = 1.65 \times 10^5 \text{kg} \cdot \text{m}^2\). The position of the center of mass in the end-coordinate system is \([0, 1.725, 0] \text{m}\). When a collision happens, adjust the control parameters \(M_i = 30 E\), \(B_i = 50 E\), \(K_i = 0\). The collision force in the assembly process is calculated by Adams simulation and imported into Matlab. The manipulator impedance control effect can be obtained as shown in Fig. 4.

The Fig. 4 shows the position tracking effect achieved by the control strategy. The first 10s are the track tracking control results in free space. When assembly contact occurs, the contact is detected and switched to the contact force control mode. The force tracking control results are shown in figure 13. It only takes about 1.38s to control the right-hand force tracking error within 6%. After that, the manipulator always keeps contact tracking and gradually reduces the force tracking error until it reaches zero.

![Impedance control effect diagram](image)

**Fig. 4** Impedance control effect diagram

### 5. Conclusion

In this paper, the dynamic model of the self-assembled closed chain system is obtained based on the dynamics model of the space manipulator, considering the simultaneous stress of the base and the end. An impedance control method is designed for the dynamics model of the closed chain system to ensure the successful completion of the task. The simulation experiment was designed, and Adams-Matlab simulation verified that the control method in this paper can effectively reduce the contact force on the premise of ensuring a successful docking.

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