Relaxation of the cosmological constant at inflation?

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Abstract

We suggest that the cosmological constant has been relaxed to its present, very small value during the inflationary stage of the evolution of the Universe. This requires relatively low scale, very long duration and unconventional source of inflation. We present a concrete mechanism of the cosmological constant relaxation at the inflationary epoch.

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The cosmological constant problem is one of the most challenging problems in fundamental physics (for a review see, e.g., Ref.\(^1\)). It would be natural to estimate, on dimensional grounds, that the vacuum energy density \(\epsilon_{\text{vac}}\) is of the order of \(M_{\text{Pl}}^4\). Supersymmetry may help to reduce this estimate by many orders of magnitude, but even the QCD contribution to \(\epsilon_{\text{vac}}\), which is of the order of \(\Lambda_{\text{QCD}}^4 \sim 10^{-3} \text{ GeV}^4\), is much greater than the observationally allowed value \(\epsilon_{\text{vac}} \lesssim 10^{-47} \text{ GeV}^4\). It is hard to imagine any symmetry that would ensure (almost) zero present value of \(\epsilon_{\text{vac}}\) (what symmetry can possibly take care of the details of the structure of QCD vacuum?), so it is natural to search for dynamical explanations of this huge discrepancy. Among the latter, most appealing would be a mechanism that would lead to the relaxation of the cosmological constant from fairly arbitrary value towards zero in the course of the evolution of the Universe.

To the best of author’s knowledge, no relaxation mechanism close to be successful has been suggested so far; there even has been formulated the corresponding “no-go theorem”\(^1\). Existing attempts (see Refs.\(^2,3\) for recent discussion and Ref.\(^1\) for an account of earlier works) are grossly inconsistent with Newtonian gravity, as they lead to exceedingly large values of the effective Planck mass. Besides the requirement of consistency with Newtonian gravity, there are other constraints that make the problem difficult. Namely, the theory of primordial nucleosynthesis requires that much of the vacuum energy density was already absent at the nucleosynthesis epoch, and also that the effective gravitational constant at that epoch was the same as today to about 10 per cent accuracy. Thus, the relaxation of the cosmological constant should have occurred, at least partially, at some earlier cosmological stage. The theory of structure formation in the Universe, that requires long matter dominated epoch, also points in the same direction.

On the other hand, when trying to invent a relaxation mechanism operative at the radiation dominated era, one faces the problem of what was special about vacuum energy density at that time. At first glance, the difference between the energy-momentum tensors of vacuum and radiation is that \(T^{(\text{vac})\mu}_\mu \neq 0\), so one might wish to consider the relaxation of
$T_{\mu}^{\mu}$. However, the trace of energy-momentum tensor of relativistic matter did not vanish in the early Universe because of interactions between particles, so the relaxation of $T_{\mu}^{\mu}$ to zero at the radiation dominated stage would not mean the relaxation of $\epsilon_{\text{vac}}$ to an acceptable value.

Although these observations do not necessarily rule out other options, they suggest that the relaxation of the energy density of vacuum of conventional fields may have occurred during an inflationary epoch. Such a scenario requires, of course, some non-standard mechanism of inflation, in which inflation is driven not by a scalar field, inflaton, with conventional properties. Once this exotic possibility is accepted, the problem to understand what is special about vacuum energy density disappears: vacuum is the only component of conventional matter that does not get inflated away. Hence, a possible scenario is that during the inflationary stage, the energy density of the vacuum of conventional fields relaxes to (almost) zero, whereas the gravitational “constant” (and, maybe, other coupling “constants”) settles down to its present value; these are frozen at later stages, so the post-inflationary evolution proceeds in the standard way.

This scenario in several respects resembles the pre-Big-Bang scenario of Ref.\textsuperscript{4}. Unlike the latter, however, the relaxation of the cosmological constant needs low scale of inflation, for the following reason. The quantity that one wishes to be relaxed to (almost) zero during the inflationary stage is the energy density of the present day vacuum. Hence, to a very good accuracy the vacuum of conventional matter must be the same at the inflationary stage as it is today. This requires sufficiently low Gibbons–Hawking temperature, $T_{GH} \sim H$, where $H$ is the Hubble parameter at inflation. Almost certainly, $T_{GH}$ must be much smaller than the QCD scale, and presumably it must be well below the electron mass. Taking, as a crude estimate, $T_{GH} < 10^{-4}$ GeV and writing $H \sim M_{\text{infl}}^2/M_{\text{Pl}}$, where $M_{\text{infl}}$ is the energy scale of inflation, one obtains $M_{\text{infl}} < 10^7$ GeV. In fact, a particular mechanism presented below may require, depending on parameters, even lower scale of inflation. In this, and a number of other respects, our scenario is similar to brane-Universe one\textsuperscript{5}; in fact, the brane Universe picture may turn out to be a natural framework beyond our phenomenological approach.
As far as the relaxation itself is concerned, we propose to make use of the observation made in the context of hyperextended inflation\textsuperscript{6} that singular kinetic terms of scalar fields tend to terminate the evolution of these fields. This freezing out may occur at values where the scalar potential has non-vanishing slope; we will see that in a class of models the fields freeze out in such a way that the value of the scalar potential, with the energy density of the vacuum of conventional fields included, is indeed very small.

Any model with the above properties will be clearly rather complicated, and will invoke several fields absent in the Standard Model and many of its extensions. At the very least, such a model provides a counter-example to the no-go theorem of Ref.\textsuperscript{1}; optimistically, it may reflect interesting physics beyond (almost) zero cosmological constant. It is encouraging that the energy scales involved are necessarily much smaller than the Planck scale.

2. As a concrete example, let us consider a model in which the gravitational interactions of conventional matter fields are of scalar-tensor type, $\phi$ being the Brans–Dicke scalar field. We will need another scalar field $\chi$ with the scalar potential $V(\chi)$ and $\phi$-dependent kinetic term, and we also include an inflaton sector. In the Einstein conformal frame, the Lagrangian of this model is

\begin{equation}
L = -\frac{1}{16\pi G_0}R\sqrt{-g} + L_{\text{conv}}(\psi; V(\chi); A^2(\phi)g_{\mu\nu}) + (L_{k,\phi} + L_{k,\chi} + L_{\text{infl}}) \tag{1}
\end{equation}

Here $G_0$ is the present value of the gravitational constant, $\psi$ stands for all conventional matter fields and $A^2(\phi)$ is a conformal factor which is assumed to be positive at all $\phi$. The Brans–Dicke field has canonical kinetic term, $L_{k,\phi} = (1/2)\sqrt{-g}g^{\mu\nu}\partial_{\mu}\phi\partial_{\nu}\phi$, and is defined in such a way that the Einstein gravity is restored at $\phi = 0$. At this value of $\phi$ the conformal factor $A^2(\phi)$ is equal to 1, and we assume that in the vicinity of this point $A^2(\phi)$ has the form

\begin{equation}
A^2 = 1 - \frac{1}{2\mu^2}\phi^2 \tag{2}
\end{equation}

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where $\mu$ is a parameter of dimension of mass. We take $\mu \sim M_{Pl}$, as usual in scalar-tensor theories, and will not need to further fine tune this parameter. These properties of $A^2(\varphi)$, i.e., its positivity at all $\varphi$, the absence of a linear term near $\varphi = 0$ and negative $A''(0)$, will be important in what follows.

The scalar potential of the field $\chi$ enters $L_{\text{conv}}$ in the usual way, but the kinetic term $L_{k,\chi}$ is unconventional,

$$L_{k,\chi} = \frac{1}{2} F(\varphi) \partial_\mu \chi \partial^\mu \chi$$

The relaxation mechanism is based on the assumption that $F(\varphi)$ is singular at $\varphi = 0$,

$$F(\varphi) = \frac{\mu^{2p}}{\varphi^{2p}} \quad \text{at} \quad \varphi \to 0 \quad (3)$$

with some integer exponent $p$ (a numerical coefficient here is a matter of normalization of $\chi$). In what follows we take $p \geq 2$. At the moment it is entirely unclear whether fields with such exotic kinetic terms may have natural particle physics interpretation.

Finally, $L_{\text{infl}}$ describes an inflaton sector. We require that (i) the inflaton sector produces inflation with small enough Hubble parameter $H$, (ii) inflaton energy-momentum tensor (almost) vanishes today, and (iii) $L_{\text{infl}}$ does not contain the fields $\varphi$ and $\chi$.

The properties (ii) and (iii) are rather problematic in the case of the usual, potential-driven inflation. Indeed, the property (ii) implies that the inflaton potential is zero at its minimum; the mechanism of relaxation to be described below works for the energy density of the vacuum of conventional fields only, and does not work for the inflaton sector. So, one has to assume some symmetry ensuring this property. The property (iii) means that the inflaton field is indeed unconventional: the Einstein-frame metrics enters $L_{\text{infl}}$ on its own, and not in the combination $A^2(\varphi)g_{\mu\nu}$. We note in this regard that matter which interacts unconventionally with metrics and Brans–Dicke field has been discussed from a different point of view in Ref.\textsuperscript{7}, and that such matter (bulk fields) appears in effective four-dimensional descriptions of brane world.

The properties (ii) and (iii) seem more natural if inflation is driven by higher-order terms in the gravitational action.\textsuperscript{8} Also, the property (ii) is inherent in models of $k$-inflation.\textsuperscript{9}
3. Let us now consider the behavior of the system at the inflationary epoch. As the matter particles have been inflated away, $L_{\text{conv}}$ effectively reduces to $[-\epsilon_{\text{vac}} - V(\chi)]A^4(\varphi)\sqrt{-g}$ at this epoch, where $\epsilon_{\text{vac}}$ is the energy density of vacuum of conventional fields. As discussed above, this vacuum at the inflationary epoch is the same as today, so our aim is to see whether $V_{\text{eff}}(\chi) = [\epsilon_{\text{vac}} + V(\chi)]$ relaxes to a very small value. We assume that $V_{\text{eff}}(\chi)$ takes both positive and negative values, depending on $\chi$, and consider initial conditions with $V_{\text{eff}}(\chi) > 0$.

For a very wide class of initial data, the field $\varphi$ at the beginning undergoes fast non-linear oscillations, whereas $\chi$ slides along the potential $V_{\text{eff}}$. To see this, let us write the equations for homogeneous scalar fields,

\begin{equation}
\frac{d}{dt} \left( F(\varphi) \frac{d\chi}{dt} \right) + 3HF(\varphi) \frac{d\chi}{dt} = -A^4(\varphi) \frac{\partial V_{\text{eff}}}{\partial \chi} \quad (4)
\end{equation}

\begin{equation}
\frac{d^2 \varphi}{dt^2} + 3H \frac{d\varphi}{dt} = -\frac{\partial A^4}{\partial \varphi} V_{\text{eff}} + \frac{1}{2} \frac{\partial F}{\partial \varphi} \left( \frac{d\chi}{dt} \right)^2 \quad (5)
\end{equation}

During the initial stage, the Hubble damping is negligible, and eq.(4) implies that $\dot{\chi} \sim f_1(t)F^{-1}(\varphi)$ where $f_1$ is a slowly varying function of time. Equation (5) is then an equation for a particle with coordinate $\varphi$ in a potential $[f_2(t)A^4(\varphi) + (f_1(t)/2)F^{-1}(\varphi)]$ where $f_2$ is another slowly varying function. From eqs.(2) and (3) one finds that the latter potential behaves near $\varphi = 0$ as $[-f_2\mu^2\varphi^2 + (f_1^2/2)\varphi^{2p} + \text{const}]$. Under mild assumptions about the behavior of $A^2(\varphi)$ and $F(\varphi)$ at large $\varphi$, this potential increases towards $|\varphi| \to \infty$, so the Brans–Dicke field does not run away to infinity. Depending on parameters and initial data, $\varphi$ indeed oscillates either about zero or about a non-vanishing value.

These oscillations are damped because of the expansion of the Universe, and after several Hubble times the slow roll regime sets in. The field $\chi$ rolls down the potential $V_{\text{eff}}(\chi)$, whereas $\varphi$ moves towards $\varphi = 0$ without oscillations. If $V_{\text{eff}}(\chi)$ is initially large, it dominates at the first stage of inflation. Ultimately $V(\chi)$ becomes relatively small, inflation becomes driven by $L_{\text{infl}}$, and the Hubble parameter $H$ becomes approximately constant and independent of $\varphi$ and $\chi$. Let us consider explicitly the final stages of the evolution of $\varphi$. 
and $\chi$, at which $V_{\text{eff}}(\chi)$ approaches zero (being initially positive), and $\varphi$ is close to zero. In a neighbourhood of the point at which $V_{\text{eff}}(\chi) = 0$, the potential may be approximated by a linear function; by redefining $\chi$ (in a way that depends on the value of $\epsilon_{\text{vac}}$) we set $V_{\text{eff}}(\chi) = r \chi$, where the slope $r = V'$ is positive and has dimension $(\text{mass})^3$. Again, we will not need to fine tune $r$.

In the slow roll approximation, which is very good at the stage we discuss, the field equations at small $\varphi$ and $\chi$ are

$$3HF\dot{\chi} = -r$$

$$3H\dot{\varphi} = \frac{2r}{\mu^2} \chi \varphi - \frac{p \mu^{2p}}{\varphi^{2p+1}} \chi^2$$

(7)

Let us first consider the case $p > 2$. We will see that the relaxation of the vacuum energy density requires fairly small $H$. Under this assumption, the fields for long time follow the power-law attractor solution,

$$\chi = \frac{1}{p-1} \left[ \frac{p(p-1)}{2} \right]^p \frac{r}{(3H)^2 (3H t)^{p-1}}$$

(8)

$$\frac{\varphi^2}{\mu^2} = \frac{p(p-1)}{2} \frac{1}{3H t}$$

(9)

This solution is valid until $\chi$ gets very close to zero. For this solution, the left hand side of eq.(7) is negligible, and the two terms on the right hand side cancel each other.

The regime (8), (9) terminates when the left hand side of eq.(7) becomes comparable to $r \chi \varphi / \mu^2$. This occurs at the time determined by $(3H t)^{p-2} \sim \frac{r^2}{(3H)^4 \mu^2}$. At this time the effective vacuum energy density $V_{\text{eff}}(\chi) = r \chi$ is of order

$$\epsilon_* = \frac{r^2}{(3H)^2 \delta^{p-1}}$$

(10)

where

$$\delta = \left[ \frac{(3H)^4 \mu^2}{r^2} \right]^{\frac{1}{p-2}}$$
is dimensionless and small at small $H$. The evolution at later times is more complicated. The field $\chi$ slightly overshoots the point where $V_{\text{eff}} = 0$, so that the effective vacuum energy density becomes negative. Then the two terms on the right hand side of eq. (7) work in the same direction and push $\varphi$ towards zero. The dynamics thus freezes out. The final value of $V_{\text{eff}}$ and the relevant time scale become clear after rescaling,

$$\chi = \mu \delta^{1/2} \cdot \tilde{\chi}, \quad \varphi = \mu \delta^{1/2} \cdot \tilde{\varphi}, \quad t = (3H)^{-1} \delta^{-1} \cdot \tilde{t}.$$  

Written in terms of variables $\tilde{\chi}$, $\tilde{\varphi}$ and $\tilde{t}$, equations (6) and (7) do not contain any parameters. Hence, the final value of $\tilde{\chi}$ is of order 1, and the residual vacuum energy density is of order $\epsilon_{\text{res}} \sim -\epsilon_*$, where $\epsilon_*$ is given by eq.(10). [The property that $\tilde{\chi}$ is finite at $t \to \infty$ can be seen by omitting the second term on the right hand side of eq.(7), which only diminishes the final value of $|\tilde{\chi}|$; then eqs.(6) and (7) are straightforward to solve explicitly. Needless to say, all above properties are straightforward to check by numerical calculations.]

At $p = 3$ and $p = 4$ the residual vacuum energy density $\epsilon_{\text{res}} \sim -\epsilon_*$ is naturally very small. Indeed, at $p = 4$ one has $\epsilon_* = (\mu^2/r)(3H)^4$ which is of order $H^4$ for $r \sim \mu^3$. With $H \sim M_{\text{infl}}^2/M_{\text{Pl}}$, this is consistent with the observational bound provided that the energy scale of inflation is sufficiently low, $M_{\text{infl}} \lesssim$ (a few) TeV. At $p = 3$ the residual vacuum energy density is suppressed even stronger, $\epsilon_* = (\mu^4/r^2)(3H)^6$, so our relaxation mechanism is consistent with larger scales of inflation.

It is worth noting that the relaxation of the vacuum energy density to its present, very small value occurs only if the inflationary stage lasts very long. The above scaling argument implies that the time scale of the relaxation is of order $H^{-1} \delta^{-1}$, so the duration of inflation should be large enough, $t_{\text{infl}} \gtrsim H^{-1} \delta^{-1}$. On the other hand, the estimate for the residual vacuum energy, eq.(10), can be written also as $|\epsilon_{\text{res}}| \sim H^2 \mu^2 \delta$. Requiring that $|\epsilon_{\text{res}}| \lesssim \rho_{\text{crit}} \sim H_0^2 M_{\text{Pl}}^2$, where $H_0$ is the present value of the Hubble parameter, one finds at $\mu \sim M_{\text{Pl}}$ that $\delta \lesssim H_0^2/H^2$, and

$$t_{\text{infl}} \gtrsim \frac{H}{H_0} t_0$$

where $t_0 \sim H_0^{-1} \sim 10^{10}$ yrs. Thus, the relaxation mechanism works only if inflation lasts
many orders of magnitude longer than the entire post-inflationary evolution of the Universe. This bizarre requirement is of course a reflection of the extraordinarily small residual value of the vacuum energy density.

The very large number of inflationary e-foldings, \( n_e \sim (H_{\text{infl}}) \sim \delta^{-1} \), ensures also that \( \varphi \) gets very close to zero by the end of inflation, so that \( A^2(\varphi) \) does not evolve at later stages (provided that \( \mu \sim M_{\text{Pl}} \)), and interactions of the Brans–Dicke field \( \varphi \) with matter are weak enough to satisfy numerous constraints.

The case \( p = 2 \) is even simpler to treat. At relatively small \( H \) the attractor solution (8), (9) (with multiplicative corrections of order \( [1 + O(H^4\mu^2r^{-2})] \) ) describes the evolution of \( \varphi \) and \( \chi \) all the way to the end of inflation. The residual vacuum energy density is positive in this case, and is determined mostly by the number of inflationary e-foldings, \( \epsilon_{\text{res}} \sim (r/3H)^2n_e^{-1} \). Hence, at \( p = 2 \) our compensation mechanism is not particularly sensitive to the scale of inflation, but needs a very large number of e-foldings. The estimate (11) holds at \( p = 2 \) as well.

4. The mechanism just described is capable to relax the cosmological constant to a very small, but non-zero value. This value may be either negative (\( p > 2 \)) or positive (\( p = 2 \)). We note in passing that positive cosmological constant is obtained also at \( p > 2 \) if inflation terminates when the fields \( \chi \) and \( \varphi \) still evolve in the attractor regime (8), (9). It may seem encouraging, in view of observational data (see Ref.11 and references therein), that non-zero and positive cosmological constant comes out naturally in our scenario; the expectation then is that the cosmological constant is time-independent after inflation until and long after the present epoch (it is straightforward to see that, if parameters are not fine tuned, the evolution of the field \( \chi \) is negligible at post-inflationary stages). The problem, however, is that there does not seem to be any chance to address in this context the issue of cosmic coincidence (why \( \Omega_\Lambda \) is presently of the order of \( \Omega_{\text{Matter}} \), and not much greater or much smaller than \( \Omega_{\text{Matter}} \)?). Hence, the relaxation of the cosmological constant at inflation is a less attractive possibility if the cosmological constant is indeed non-zero today.

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