Is the exotic $X(5568)$ a bound state?

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Abstract. Stimulated by the recent observation of the exotic $X(5568)$ state by D0 Collaboration, we study the four-quark system $usb\bar{d}$ with quantum numbers $J^P = 0^+$ in the framework of chiral quark model. Two structures, diquark-antidiquark and meson-meson, with all possible color configurations are investigated by using Gaussian expansion method. The results show that energies of the tetraquark states with diquark-antidiquark structure are too high to the candidate of $X(5568)$, and no molecular structure can be formed in our calculations. The calculation is also extended to the four-quark system $us\bar{c}\bar{d}$ and the same results as that of $usb\bar{d}$ are obtained.

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1 Introduction

Since the charmonium-like resonance $X(3872)$ is observed by Bell collaboration [1] in 2003, a lot of experiments spring up to study the exotic states-XYZ particles from Belle, BaBar, BESIII, LHCb, CDF, D0 and other collaborations. And people believe that the traditional convention, the meson is made up of quark and antiquark as well as baryon is made up of three quarks, is broken. The exotic states were observed in $B$ meson decays, in $e^+e^-$ and $pp$ annihilations. In study of $B$ decays, the phenomenon of $CP$ violation has been studied by experimental collaborations. Many predictions of Standard Model are confirmed and some hints beyond Standard Model are exposed.

Very recently, the D0 Collaboration observed a narrow structure, named $X(5568)$, in the $B_0^0\pi^\pm$ invariant mass spectrum with 5.1$\sigma$ significance [2]. The mass and width measured is $M = 5567.8 \pm 2.9_{-1.9}^{+0.9}$ MeV and $\Gamma = 21.9 \pm 6.4_{-2.5}^{+5.0}$ MeV, respectively. Its decay mode $B_0^0\pi^\pm$ indicates that $X(5568)$ is consist of four different flavors: $u, d, s, b$. $X(5568)$ must be a $suds$, or $sdb\bar{u}$ tetraquark state. The D0 Collaboration suggests that the quantum numbers of $X(5568)$ may be $J^P = 0^+$ because $B_0^0\pi^\pm$ is produced in S-wave. However, the preliminary results of the experimental search of the state by LHCb collaboration is negative [3].

The discovery of the exotic state $X(5568)$ stimulated the theoretical interest. Many theoretical work has been done, such as approaches based on QCD sum rules [4, 5, 6, 7], quark models [10, 11, 12], etc. Agaev et al. studied the state $X(5568)$ within the two-point sum rule method using the diquark-antidiquark interpolating current [4, 5], and meson molecule structure [13], their results preferred diquark-antidiquark picture rather than molecule and a nice agreement with experimental data is obtained. QCD sum rule method was also employed by other groups to investigate the state $X(5568)$ as the diquark-antidiquark type scalar and axial-vector tetraquark states [5, 6, 7, 8, 9, 10, 11]. In Ref. [10], a tetraquark interpretation of the $X(5568)$ was proposed based on the diquark-antidiquark scheme, the identification is possible when the systematic errors of the model is taken into account. This result is supported by simple quark model estimations [12, 13]. The hadronic molecule scenarios of the $X(5568)$ is also possible according to the calculation of Ref. [11]. However, there are several theoretical calculations with negative results. Burns and Swanson examined the various interpretations of the state $X(5568)$ and concluded that the threshold, cusp, molecular and tetraquark models are all unfavored [14]. F. K. Guo et al. provided additional arguments using general properties of QCD and obtained the same conclusion [15]. Although the state $X(5568)$ can be reproduced in the coupled channel analysis in Ref. [15], the momentum cutoff used is much larger than the normal one.

Considering the quantum numbers $J^P = 0^+$ of the state $X(5568)$, the spin and orbit angular momentum can be both taken as zero. For meson molecule structure, the possible channels are $B^0_s\pi, B^0_s\rho, B^+K^0$ and $B^{*+}K^{*0}$. For diquark-antidiquark structure, the only possible state is $sud\bar{b}$ for $X(5568)^+$ or $sdb\bar{u}$ for $X(5568)^-$. In the present work, we compute all these states including molecule and diquark-antidiquark structures using chiral quark model under the flavor $SU(3)$ and $SU(4)$ symmetry, respectively. Besides, we extend our investigation to the new family of the four flavor exotic states $X_c$ with $u, d, s, c$ by replacing the $b$ quark with $c$ quark. We hope that we can find some

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2 GEM and Chiral Quark Model

In the chiral quark model, the mass of the tetraquark state is obtained by solving the Schrödinger equation

$$H \Psi_{M_1M_2}^{IJ} = E^{IJ} \Psi_{M_1M_2}^{IJ}$$

where $\Psi_{M_1M_2}^{IJ}$ is the wave function of the tetraquark state, which can be constructed as follows. First, we write down the wave functions of two clusters (Taking meson-meson configuration as an example),

$$\Psi_{M_1M_2}^{IJ}(12) = [\psi_1(r_{12}) \chi_1(12)] \frac{1}{\sqrt{2}} \phi^{c_i(12)} \phi^{c_j(12)},$$

$$\Psi_{M_1M_2}^{IJ}(34) = [\psi_2(r_{34}) \chi_2(34)] \frac{1}{\sqrt{2}} \phi^{c_1(34)} \phi^{c_2(34)},$$

where $\chi$, $\phi$, $\psi$ are spin, color and flavor wavefunctions of the quark-antiquark cluster (the quarks are numbered as 1, 2 and antiquarks 3, 4, see Fig.1). [ ] denotes the angular momentum coupling. Then the total wave function of tetraquark state is obtained,

$$\Psi_{M_1M_2}^{IJ} = A \left[ [\psi_1(r_{12}) \chi_1(12)] \frac{1}{\sqrt{2}} \phi^{c_i(12)} \phi^{c_j(12)} \psi_L(r_{1234}) \right]_{M_1M_2}^{IJ},$$

where $\psi_L(r_{1234})$ is the relative wave function between two clusters with the relative orbit angular momentum $L_r$. $A$ is the antisymmetrization operator. If all quarks (antiquarks) are taken as identical particles, we have

$$A = \frac{1}{2} (1 - P_{13} - P_{24} + P_{13}P_{24}).$$

In GEM, the orbital wave function is written as the product of radial one and spherical harmonics, and the radial part of the wavefunction is expanded by gaussians,

$$\psi_{n\ell m}(r) = \sum_{n=1}^{n_{max}} c_n \psi_{n\ell m}^{G}(r),$$

$$\psi_{n\ell m}^{G}(r) = N_{n\ell m} r^\ell e^{-\nu_n r^2} Y_{\ell m}(r).$$

Gaussian size parameters are taken as the following geometric progression numbers

$$\nu_n = \frac{1}{r_n}, \quad r_n = r_1 a^{n-1}, \quad a = \left( \frac{r_{max}}{r_1} \right).$$

Noting that the gaussians are not orthogonal, the Rayleigh-Ritz variational principle for solving the Schrödinger equation leads to a generalized eigenvalue problem

$$\sum_{n' \ell' \alpha'} (H_{n\ell\alpha}^{IJ} - E^{IJ} N_{n'\ell'\alpha'}^{IJ}) C_{n'\ell'\alpha'}^{IJ} = 0,$$

$$H_{n\ell\alpha}^{IJ} = \langle \phi^{IJ} \mid H \mid \phi^{IJ} \rangle, \quad N_{n'\ell'\alpha'}^{IJ} = \langle \phi^{IJ} \mid \phi^{IJ} \rangle.$$

where $\alpha$ denotes channels.

The Hamiltonian of the chiral quark model includes three parts, the rest masses of quarks, the kinetic energy and the potential energy. The potential energy is composed of color confinement, one-gluon-exchange and one Goldstone boson exchange. The detailed form for tetraquark states is shown below [20]

$$H = \sum_{i=1}^{4} m_i + \frac{p_i^2}{2\mu_{12}} + \frac{p_i^2}{2\mu_{34}} + \frac{p_i^2}{2\mu_{1234}}$$

$$+ \sum_{i,j=1}^{4} \left( V_{ij}^{C} + V_{ij}^{G} + \sum_{\chi=\pi,K} V_{ij}^{\chi} + V_{ij}^{\sigma} \right),$$

where the $V_{ij}^{C}, V_{ij}^{G}, V_{ij}^{\chi}, V_{ij}^{\sigma}$ are the color confinement, one-gluon-exchange, one Goldstone boson exchange and other terms, respectively.

$$V_{ij}^{C} = \frac{4}{a^2} \chi_i \cdot \chi_j \left[ \frac{1}{r_{ij}} - \frac{2}{3} \sum_{m} \frac{1}{m} \right] \delta(r_{ij}),$$

$$V_{ij}^{G} = \left\{ \begin{array}{ll}
\frac{2}{a^2} m_i m_j A_{i}^2 & \text{for } i \neq j, \\
0 & \text{for } i = j.
\end{array} \right.$$
where \( m_i \) is the mass of quarks and antiquarks, and \( \mu_{ij} \) is their reduced mass, \( r_0(\mu_{ij}) = r_0/\mu_{ij} \). \( \sigma \) are the SU(2) Pauli matrices, \( \Lambda \), \( X^i \) are SU(3) flavor, color Gell-Mann matrices, \( g_{cb}^2/4\pi \) is the chiral coupling constant, determined from \( \pi \)-nucleon coupling constant. \( \alpha_s \) is the effective scale-dependent running quark-gluon coupling constant \( \alpha_s(\mu) = \frac{\alpha_0}{\ln [(\mu^2 + \mu_0^2)/\Lambda_0^2]} \) (10).

All model parameters are determined by fitting the meson spectrum and shown in Table 1. The calculated masses of the mesons involved in the present work are shown in Table 2.

### Table 1. Quark Model Parameters.

| Quark masses | \( m_u = m_d \) (MeV) | 313 |
|--------------|----------------------|-----|
| \( m_s \)    | 536                  |
| \( m_c \)    | 1728                 |
| \( m_b \)    | 5112                 |

| Goldstone bosons | \( m_s(f^{-1}) \) | 0.70 |
|------------------|------------------|-----|
| \( m_s(f^{-1}) \) | 3.42             |
| \( m_s(f^{-1}) \) | 2.77             |
| \( m_s(f^{-1}) \) | 2.51             |
| \( \Lambda_r = \Lambda_s(f^{-1}) \) | 4.2 |
| \( \Lambda_q = \Lambda_s(f^{-1}) \) | 5.2 |
| \( g_{cb}^2/4\pi \) | 0.54 |
| \( \theta(\pi) \) | -15               |
| Confinement      | \( \alpha_c \) (MeV) | 101 |
|                  | \( \Delta(m) \) (MeV) | -78.3 |
| OGE              | \( \alpha_0 \) | 3.67 |
|                  | \( \Lambda_0(f^{-1}) \) | 0.033 |
|                  | \( \mu_0 \) (MeV) | 36.976 |
|                  | \( r_0 \) (MeV) | 28.17 |

### Table 2. Meson Spectrum (unit: MeV).

| Meson   | Energy | Experimental value |
|---------|--------|--------------------|
| \( B_s^- \) | 5368 | 5366 |
| \( \pi^- \) | 139 | 139 |
| \( B_s^+ \) | 5410 | 5415 |
| \( \rho \) | 772 | 770 |
| \( B_s^+ \) | 5281 | 5279 |
| \( K^- \) | 497 | 497 |
| \( K_{s}^+ \) | 5320 | 5325 |
| \( K_{s}^- \) | 914 | 892 |
| \( D_s^0 \) | 1953 | 1968 |
| \( \bar{D} \) | 1862 | 1864 |

### 3 Numerical Results

In the present calculation, two structures of four-quark states, diquark-antidiquark and meson-meson, are investigated. And in each structure, all possible states are considered. For diquark-antidiquark structure, two color configurations, color antitriplet-triplet \( (3 \times 3) \) and sextet-antisextet \( (6 \times 6) \) are taken into account. For meson-meson structure, two color configurations, color singlet-singlet \( (1 \times 1) \) and octet-octet \( (8 \times 8) \) are employed.

The calculation with the ordinary flavor symmetry, \( SU(3) \) is first performed, i.e., no Goldstone boson exchanges between \( u, d, s \) and \( b \) quark. In this case, the antisymmetrization operator used is

\[
\mathcal{A} = \sqrt{\frac{1}{2} (1 - P_{13})}
\] (11)

The results in this case are listed in Table 3.

### Table 3. The energies of tetraquark system \( su\bar{d}\bar{b} \) with flavor \( SU(3) \) symmetry. \( E_{th}^{exp} \) is the theoretical threshold value and \( E_{th}^{exp} \) represents the experimental threshold value. (unit: MeV)

| \( qq - \bar{q} \bar{q} \) | \( E_{th}^{exp} \) | \( E_{th}^{exp} \) | \( E_{th}^{exp} \) |
|--------------------------|------------------|------------------|------------------|
| \( 3 \times 3 \)         | 5509.5           | 5509.5           | 5507             |
| \( 3 \times 6 \)         | 5600.5           | 5600.5           | 5597             |
| \( 6 \times 3 \)         | 6185.5           | 6185.5           | 6182             |
| \( 6 \times 6 \)         | 6324.3           | 6324.3           | 6317             |
| \( 1 \times 1 \)         | 5776.8           | 5776.8           | 5774             |
| \( 1 \times 8 \)         | 6343.0           | 6343.0           | 6343.0           |
| \( 8 \times 1 \)         | 6376.9           | 6376.9           | 6376.9           |
| \( 8 \times 8 \)         | 6774            | 6774            | 6774             |

From the Table 3, we can see that the two configurations of diquark-antidiquark structure, \( 3 \times 3 \) and \( 6 \times 6 \), have similar energies, and the coupling between the two configuration is rather strong. Nevertheless, the energy for diquark-antidiquark structure is too large to be a natural candidate of the state X(5568) in our calculation, although it could be a resonance because of its color structure. With regard to meson-meson structure, the calculated energies approach to the theoretical thresholds in all case, so no molecular structure formed in our model calculation. In our calculations, the color singlet-singlet configurations always have the lower energies than that of color octet-octet ones. The coupling between the two configurations is very small. The reason for small coupling can be understood as follows. The effect of K-meson exchange is too weak to push the energy of color singlet-singlet below the threshold, so the two colorless clusters tend to stay apart. While two colorful clusters prefer stay close, the overlap between two configurations is small, so the coupling from the exchange term of K-meson is small.

In the study of \( N^* \) with hidden charm, the flavor \( SU(4) \) symmetry plays an important role. To see the effect of flavor \( SU(4) \) symmetry, we extend our calculation from flavor \( SU(3) \) symmetry to \( SU(4) \). In this case, the Goldstone boson exchanges including \( \pi, K, \eta, B, B_s, \eta_q \), totally fifteen pseudo-scalar mesons. For scalar mesons, we use effective \( \sigma \)-meson exchange instead of sixteen scalar
Table 4. The energies of tetraquark system $sudb$ with flavor $SU(4)$ symmetry. $E_{th}^{(k)}$ is the theoretical threshold value and $E_{exp}^{(k)}$ represents the experimental threshold value. (unit: MeV)

| $qq - ar{q}ar{q}$ | $E_{th}^{(3)}$ | $E_{th}^{(6)}$ | $E_{th}^{(cc)}$ | $E_{th}^{(b)}$ | $E_{exp}^{(b)}$ |
|----------------------|----------------|----------------|----------------|----------------|----------------|
| $sudb$               | 6397.6         | 6466.4         | 6351.0         |                |                |
| $qar{q} - qar{q}$| $B_{s}^{+}$    | 5522.0         | 5631.1         | 5522.0         | 5518           |
|                      |                 |                |                |                | 5505           |
|                      | $B_{c}^{+}$    | 6282.7         | 6324.3         | 6182.5         | 6177           |
|                      |                 |                |                |                | 6185           |
|                      | $B_{s}^{+} - B_{c}^{+}$ | 5522.0         | 6036.1         | 5521.0         | 5518           |
|                      |                 |                |                |                | 5505           |
|                      | $B^{+} K^{0}$  | 5717.6         | 6440.1         | 5717.6         | 5715           |
|                      |                 |                |                |                | 5776           |
|                      | $B^{+} K^{0} - K^{+} K^{0}$ | 6204.6         | 6277.2         | 6204.5         | 6202           |
|                      |                 |                |                |                | 6217           |

Table 5. The energies of tetraquark system $sucd$ with flavor $SU(3)$ symmetry. $E_{th}^{(k)}$ is the theoretical threshold value and $E_{exp}^{(b)}$ represents the experimental threshold value. (unit: MeV)

| $qq - ar{q}ar{q}$ | $E_{th}^{(3)}$ | $E_{th}^{(6)}$ | $E_{th}^{(cc)}$ | $E_{th}^{(b)}$ | $E_{exp}^{(b)}$ |
|----------------------|----------------|----------------|----------------|----------------|----------------|
| $sucd$               | 3059.0         | 3073.9         | 2983           |                |                |
| $SU(3)$              |                |                |                |                |                |
| $qar{q} - qar{q}$| $B_{c}^{+}$    | 2095.1         | 3080.6         | 2095.1         | 2092           |
|                      |                 |                |                |                | 2107           |
|                      | $K^{0} D^{0}$  | 2358.7         | 3133.8         | 2358.7         | 2355           |
|                      |                 |                |                |                | 2361           |

are investigated. We find that the masses of usbd with diquark-antidiquark structure are too high to be candidate of the state $X(5568)$ and no molecular structure can be formed. The calculation is extended to $uscd$ system, the same conclusion is obtained.

Because of the quark contents of the system, the pseudo-scalar mesons are involved. The extraordinary small masses of these Goldstone bosons disfavors the existence of the exotic. Our results agree with the analysis of Burns and Swanson. The recent preliminary results of LHCb collaboration do not confirm the state $X(5568)$, So more experimental and theoretical work are needed to clarify the situation.

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References

1. S.-K. Choi et al. [Belle Collaboration], Phys. Rev. Lett.91, 262001 (2003).
2. V. M. Abazov et al. [D0 Collaboration], arXiv:1602.07588 [hep-ex].
3. The LHCb Collaboration [LHCb Collaboration], LHCbCONF-2016-004, CERN-LHCb-CONF-2016-004.
4. S. S. Agaev, K. Azizi and H. Sundu, arXiv:1603.02498 [hep-ph].
5. Z. G. Wang, arXiv:1602.08711 [hep-ph], arXiv:1603.02498 [hep-ph].
6. C. M. Zanetti, M. Nielsen and K. P. Khemchandani, arXiv:1602.09041 [hep-ph].
7. W. Chen, H. X. Chen, X. Liu, T. G. Steele and S. L. Zhu, arXiv:1602.08916 [hep-ph].
8. J. M. Dias, K. P. Khemchandani, A. M. Torres, et al., arXiv:1603.02749 [hep-ph].
9. W. Wang and R. L. Zhu, arXiv:1602.08806 [hep-ph].
10. C. J. Xiao and D. Y. Chen, arXiv:1603.002282 [hep-ph].
11. Y. R. Liu, X. Liu and S. L. Zhu, arXiv:1603.01131 [hep-ph].
12. Fl. Stancu, arXiv:1603.03322 [hep-ph].

4 Summary

In this paper we have studied the new exotic resonance state $X(5568)$ with the quantum numbers $J^{P} = 0^{+}$, which was observed recently by D0 Collaboration utilizing the collected data of $par{p}$ collision. The chiral quark model, which describes the meson spectrum well, is employed to do the calculation. Two structures: diquark-antidiquark and meson-meson, with flavor symmetries, $SU(3)$ and $SU(4)$, are investigated.

The masses of $usbd$ are investigated. We find that the masses of $usbd$ with diquark-antidiquark structure are too high to be candidate of the state $X(5568)$ and no molecular structure can be formed. The calculation is extended to $uscd$ system, the same conclusion is obtained.

Because of the quark contents of the system, the pseudo-scalar mesons are involved. The extraordinary small masses of these Goldstone bosons disfavors the existence of the exotic. Our results agree with the analysis of Burns and Swanson. The recent preliminary results of LHCb collaboration do not confirm the state $X(5568)$, So more experimental and theoretical work are needed to clarify the situation.

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14. X. H. Liu and G. Li, [arXiv:1603.00708 [hep-ph]].
15. S. S. Agaev, K. Azizi and H. Sundu, [arXiv:1603.00290 [hep-ph]].
16. S. S. Agaev, K. Azizi and H. Sundu, [arXiv:1603.02708 [hep-ph]].
17. T. J. Burns, E. S. Swanson, [arXiv:1603.04366 [hep-ph]].
18. F. K. Guo, U. G. Meissner and B. S. Zou, [arXiv:1603.06316 [hep-ph]].
19. M. Albaladejo, J. Nieves, E. Oset, Z. F. Sun and X. Liu, [arXiv:1603.09230 [hep-ph]].
20. A. Valcarce, H. Garcilazo, F. Fernandez, and P. Gonzalez, Rep. Prog. Phys. 68, 965 (2005) references therein.
21. J. J. Wu, R. Molina, E. Oset and B. S. Zou, Phys. Rev. Lett. 105, 232001 (2010); Phys. Rev. C 84, 015202 (2011).
22. H. Garcilazo, T. Fernández-Caramés, and A. Valcarce, Phys. Rev. C. 75, 034002 (2007).