Cosmological black holes on Taub–NUT space in five-dimensional Einstein–Maxwell theory

Daisuke Ida\textsuperscript{1}, Hideki Ishihara\textsuperscript{2}, Masashi Kimura\textsuperscript{2}, Ken Matsuno\textsuperscript{2}, Yoshiyuki Morisawa\textsuperscript{3} and Shinya Tomizawa\textsuperscript{2}

\textsuperscript{1} Department of Physics, Gakusyuin University, Tokyo 171-8588, Japan
\textsuperscript{2} Department of Mathematics and Physics, Graduate School of Science, Osaka City University, 3-3-138 Sugimoto, Sumiyoshi, Osaka 558-8585, Japan
\textsuperscript{3} Faculty of Liberal Arts and Sciences, Osaka University of Economics and Law, Yao City, Osaka 581-8511, Japan

E-mail: daisuke.ida@gakushuin.ac.jp, ishihara@sci.osaka-cu.ac.jp, miki@sci.osaka-cu.ac.jp, matsuno@sci.osaka-cu.ac.jp, morisawa@keiho-u.ac.jp and tomizawa@sci.osaka-cu.ac.jp

Received 23 February 2007, in final form 13 April 2007
Published 12 June 2007
Online at stacks.iop.org/CQG/24/3141

Abstract

The cosmological black hole solution on the Gibbons–Hawking space has been constructed. We also investigate the properties of this solution in the case of a single-black hole. Unlike the Kastor–Traschen solution, which becomes a static solution in a single-black hole, this solution is not static even in a single-black hole case.

PACS numbers: 04.50.+h, 04.70.Bw

1. Introduction

The Majumdar–Papapetrou solution [1] in Einstein–Maxwell theory describes an arbitrary number of extremely charged black holes in static equilibrium. The corresponding solution in Einstein–Maxwell theory with the positive cosmological term was found by Kastor and Traschen [2]. The Kastor–Traschen solution describes the multi-black hole configuration in the de Sitter background. In particular, it realizes the analytic description of the black hole collisions. This implies that the Kastor–Traschen solution in general describes dynamical spacetimes in the sense that there is no time-like Killing vector field. Nevertheless, it becomes static in the single-black hole case, where it coincides with the extreme Reissner–Nordström–de Sitter space. The higher dimensional generalization of the Kastor–Traschen solution is also known [3].

Recently, the Kaluza–Klein black hole solution with a squashed-event horizon in five-dimensional Einstein–Maxwell theory was found by two of the present authors [4]. Thermodynamical properties of the Kaluza–Klein black holes are studied in [5], and some
generalizations are discussed in [6–8]. The Kaluza–Klein black holes also admit multi-black hole configuration when black holes are extremely charged [9]. Though this Kaluza–Klein multi-black hole spacetime belongs to the five-dimensional Majumdar–Papapetrou class, the four-dimensional base space becomes the Gibbons–Hawking multi-instanton space rather than the usual flat space. The spatial cross section of each black hole can be diffeomorphic to the various lens spaces \( L(n; 1) \) (\( n \): natural number) in addition to \( S^3 \). In addition, a pair of static black holes on the Eguchi–Hanson space can also be considered [10].

The purpose of this paper is to consider the effect of the cosmological constant on the multi-black hole on the Gibbons–Hawking base space in five-dimensional Einstein–Maxwell theory and to investigate the global structure of this cosmological spacetime.

The solution in consideration is reduced to the Kaluza–Klein multi-black hole solution [9] by taking the limit of zero cosmological constant, which is contained within supersymmetric solutions classified by Gauntlett et al [11]. It is found that, unlike the Reissner–Nordström black hole in de Sitter background, the spacetime becomes dynamical even in the single-black hole case.

This paper is organized as follows. In section 2, we give a multi-black hole solution on the Gibbons–Hawking multi-instanton space in the five-dimensional Einstein–Maxwell theory with a positive cosmological constant. In section 3, we investigate the properties of this solution in the case of a single-black hole. First, we show that there is no time-like Killing vector field. Next, we study the global structure of this solution. We give an example of the black hole spacetime.

2. Solutions

At first, we give a cosmological solution on the Gibbons–Hawking multi-instanton space. We consider a five-dimensional Einstein–Maxwell system with a positive cosmological constant, which is described by the action

\[
S = \frac{1}{16\pi G_5} \int \sqrt{-g} \left( R - 4\Lambda - F_{\mu\nu} F^{\mu\nu} \right),
\]

(1)

where \( R \) is the five-dimensional scalar curvature, \( F_{\mu\nu} \) is the five-dimensional Maxwell field strength tensor, \( \Lambda \) is the positive cosmological constant and \( G_5 \) is the five-dimensional Newton constant.

From this action, we write down the Einstein equation

\[
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + 2 g_{\mu\nu} \Lambda = 2 \left( F_{\mu\lambda} F^{\lambda\nu} - \frac{1}{4} g_{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} \right),
\]

(2)

and the Maxwell equation

\[
F^{\mu\nu \gamma} = 0.
\]

(3)

Equations (2) and (3) admit the solution whose metric and gauge potential 1-form are

\[
\text{d}s^2 = -H^{-2} \text{d}t^2 + H e^{\lambda t} \text{d}s_{\text{GH}}^2,
\]

(4)

\[
A = \pm \frac{\sqrt{3}}{2} H^{-1} \text{d}t,
\]

(5)

with

\[
H = 1 + \sum_i \frac{M_i}{e^{\lambda t} |x - x_i|}.
\]

(6)
\[ ds_{GH}^2 = V^{-1}(d\mathbf{x} \cdot d\mathbf{x}) + V(d\xi + \omega)^2, \]  
(7)

\[ V^{-1} = \epsilon + \sum_i \frac{N_i}{|x - x_i|}, \]  
(8)

\[ \omega = \sum_i N_i \left( \frac{z - z_i}{|x - x_i|} (x - x_i) dy - (y - y_i) dx, \right. \]  
(9)

where \( ds_{GH}^2 \) denotes the four-dimensional Euclidean Gibbons–Hawking space \([12]\), \( x_i = (x_i, y_i, z_i) \) denotes the position of the \( i \)th NUT singularity with NUT charge \( N_i \) in the three-dimensional Euclidean space and \( M_i \) is a constant. The constant \( \lambda \) is given by \( \lambda = \pm \sqrt{4\Lambda/3} \). The parameter \( \epsilon \) is either 0 or 1. The base space is the multi-centre Eguchi–Hanson space for \( \epsilon = 0 \), and the multi-Taub–NUT space for \( \epsilon = 1 \). The \( \epsilon = 0 \) solution describes the coalescence of black holes \([13]\). Here, we study the \( \epsilon = 1 \) solution and show that it describes a black hole spacetime. This solution is a generalization of five-dimensional Kaluza–Klein multi-black holes \([9]\), and it is analogous to the Kastor–Traschen solution \([2, 3]\).

3. Properties of the single-black hole

Let us consider the single-black hole case. Then, the metric takes the following simple form:

\[ ds^2 = -\left( 1 + \frac{M}{e^{\lambda t} R} \right)^{-2} dt^2 + \left( 1 + \frac{M}{e^{\lambda t} R} \right) e^{\lambda t} ds_{T-NUT}^2, \]  
(10)

\[ ds_{T-NUT}^2 = \left( 1 + \frac{N}{R} \right) (dR^2 + R^2 d\theta^2 + R^2 \sin^2 \theta d\phi^2) + \left( 1 + \frac{N}{R} \right)^{-1} N^2 (d\psi + \cos \theta d\phi)^2, \]

where \( ds_{T-NUT}^2 \) is the four-dimensional Euclidean self-dual Taub–NUT space. The range of parameters is given by \( 0 \leq \theta \leq \pi, 0 \leq \phi \leq 2\pi \) and \( 0 \leq \psi \leq 4\pi \). The Kastor–Traschen spacetime which possesses the flat base space becomes static in the single-black hole case. We show however that this property does not hold in the present case.

3.1. Nonsotatinarity

The Kastor–Traschen solution in the single-black hole case is the extreme Reissner–Nordström–de Sitter solution, which is static and spherically symmetric \([14, 15]\). In contrast, we show that the single-black hole solution (5) is not stationary, but dynamical. Let us seek for the stationary Killing vector field of the geometry (10). For the present purpose, it is enough to exhaust the Killing vector fields for the Gross–Perry–Sorkin (GPS) monopole solution with a cosmological constant

\[ ds^2 = -dr^2 + e^{2r} ds_{T-NUT}^2, \]  
(11)

which is obtained by taking the limit \( M \to 0 \) of equation (10). This is because the metric (11) describes the far region \( (R \to +\infty) \) from the black hole; if the original black hole has a stationary Killing vector field, then the asymptotic form of the Killing vector would coincide with that of (11).

All Killing vectors of the metric (11) are computed as follows:

\[ \xi_1 = \frac{\partial}{\partial \psi}, \]  
(12)

\[ \xi_2 = \frac{\partial}{\partial \phi}. \]  
(13)
\[\xi_3 = \csc \theta \sin \phi \frac{\partial}{\partial \psi} + \cos \phi \frac{\partial}{\partial \theta} - \cot \theta \sin \phi \frac{\partial}{\partial \phi}, \quad (14)\]

\[\xi_4 = \csc \theta \cos \phi \frac{\partial}{\partial \psi} - \sin \phi \frac{\partial}{\partial \theta} - \cot \theta \cos \phi \frac{\partial}{\partial \phi}. \quad (15)\]

Thus, all the Killing vector fields are everywhere space-like. Therefore, the single-black hole solution (10) is not stationary.

3.2. GPS monopole with a cosmological constant

Here, we focus on the case \(\lambda > 0\), which is called the expanding chart. The contracting chart \((\lambda < 0)\) corresponds to its time reversal \((t \rightarrow -t)\). Let us introduce that the Regge–Wheeler tortoise-type coordinate \(R_*\) and a time coordinate \(T\) are given by

\[R_* = \sqrt{R(R + N) + N \ln \left(\frac{\sqrt{R + N} + \sqrt{R}}{\sqrt{N}}\right)}, \quad T = \frac{2}{\lambda} e^{-\lambda t/2}, \quad (16)\]

where \(R_*\) and \(T\) range \(0 \leq R_* \leq \infty\) and \(0 \leq T \leq \infty\), respectively. Then, the metric (11) takes the following form:

\[ds^2 = \frac{4}{\lambda^2 T^2} \left[ -dT^2 + dR_*^2 + \left(1 + \frac{N}{R}\right)^{-1} R^2 d\Omega_3^2 + \left(1 + \frac{N}{R}\right) N^2 (d\psi + \cos \theta \, d\phi)^2 \right]. \quad (17)\]

Here, we define the double null coordinate \((u, v)\) such that

\[v := T + R_*, \quad u := T - R_*, \quad (18)\]

where \(v\) is the retarded outgoing null coordinate and \(u\) is the retarded ingoing null coordinate.

Then, the metric can be written in the form

\[ds^2 = \frac{1}{\lambda^2 (u + v)^2} \left[ -du \, dv + R(R + N) d\Omega_3^2 + \frac{R N^2}{R + N} (d\psi + \cos \theta \, d\phi)^2 \right]. \quad (19)\]

To make coordinate ranges finite, define

\[v = \tan \tilde{V}, \quad u = \tan \tilde{U}, \quad (20)\]

where \(\tilde{U}\) and \(\tilde{V}\) run the ranges of \(-\pi/2 \leq \tilde{U} \leq \pi/2\) and \(-\pi/2 \leq \tilde{V} \leq \pi/2\), respectively. The metric becomes

\[ds^2 = \frac{1}{\lambda^2 \sin^2(U + V)} \left[ -d\tilde{U} \, d\tilde{V} + R(R + N) \cos^2 \tilde{U} \cos^2 \tilde{V} \, d\Omega_3^2 + \frac{R N^2}{R + N} \cos^2 \tilde{U} \cos^2 \tilde{V} (d\psi + \cos \theta \, d\phi)^2 \right]. \quad (21)\]

The \((R, t)\) chart covers the region \(O = \{(\tilde{U}, \tilde{V})|\tilde{U} \leq \tilde{V}, -\tilde{U} \leq \tilde{V}, \tilde{V} \leq \pi/2\}\). The conformal diagram is drawn in figure 1. On the null surface \(t \rightarrow -\infty, R \rightarrow \infty\), the \(S^1\) fibre seems to shrink to zero; the possibility of extension will be discussed in a future article.

3.3. Single-black hole solution

To study the \(M > 0\) case, introducing the coordinate \(\tau := \lambda^{-1} e^{\lambda t} (\tau > 0)\), we rewrite the metric (10) as

\[ds^2 = -\left(\lambda \tau + \frac{M}{R}\right)^{-2} \, d\tau^2 \]

\[+ \left(\lambda \tau + \frac{M}{R}\right) \left[ (1 + \frac{N}{R}) (dR^2 + R^2 d\Omega_3^2) + \left(1 + \frac{N}{R}\right)^{-1} N^2 (d\psi + \cos \theta \, d\phi)^2 \right]. \quad (22)\]
Cosmological black holes on Taub–NUT space

The boundaries of this spacetime consist of the time-like infinity $\tilde{V} = -\tilde{U}(0 < \tilde{V} < \pi/2)$, $\tilde{U} = \tilde{V}(0 < \tilde{V} < \pi/2)$, and the null surface $\tilde{V} = \pi/2$, $|\tilde{U}| < \pi/2$, which is not conformal infinity and cannot be extended analytically.

This shows that the range of $\tau$ can be extended to $-\infty < \tau < +\infty$. The conformal diagram of this spacetime is drawn in figure 2. The sign choice of $\lambda$ corresponds to time reversal, so that here we set $\lambda > 0$, which is called the expanding chart. The boundaries of this chart consist of (1) $\tau = \infty$ and $0 < R < \infty$, (2) $\tau = \infty$ and $R = 0$, (3) $\tau = -\infty$ and $R = 0$, (4) $R = -M/(\lambda \tau)$, and (5) $\tau = 0$ and $R = \infty$. The space-like hypersurface, $\tau = \infty$ and $0 < R < \infty$, is the time-like infinity. $R = -M/(\lambda \tau)$ is a curvature singularity. The null hypersurfaces, $\tau = \infty$, $R = 0$ and $\tau = -\infty$, $R = 0$, and the null surface $\tau = 0$ and $R = \infty$ are not conformal boundary.
not static but dynamical as mentioned above, and one cannot solve the null geodesic equations easily. Now, we give one of the extensions, which can be regarded as a black hole spacetime, at least, within the restricted ranges of the parameters.

In the single-black hole solution case, the form of the metric resembles that of the five-dimensional Reissner–Nordström–de Sitter in the neighbourhood of \( R = 0 \), as mentioned below. For this reason, we apply the Edinton–Finkelstein-like coordinate to our solution to extend the spacetime across \( R = 0 \). The metric of the five-dimensional Reissner–Nordström–de Sitter solution with \( m = \sqrt{3}|Q|/2 \) can be written in the cosmological coordinate \([3]\) as follows:

\[
\begin{align*}
&d\sigma^2 = -(\alpha + \frac{m}{r^2})^{-2} d\tau^2 + \left(\alpha + \frac{m}{r^2}\right) \left[ dr^2 + \frac{r^2}{4} d\Omega^2_{S^2} + \frac{r^2}{4} (d\psi + \cos \theta \, d\phi)^2 \right].
\end{align*}
\]

(23)

To compare this metric with that of our solution, let us introduce the coordinate \( N^2 \):

\[
N^2 = -\frac{r^2}{4N},
\]

(24)

where it should be noted that the parameter \( N \) does not have a physical meaning. In fact, it can be absorbed in the rescale of \( R \). The metric (23) takes the form

\[
\begin{align*}
&d\sigma^2 = -(\alpha + \frac{M}{R})^{-2} d\tau^2 + \left(\alpha + \frac{M}{R}\right) \left[ \left(\frac{N}{R}\right) (dR^2 + R^2 d\Omega^2_{S^2}) - \left(\frac{N}{R}\right)^{-1} N^2 \left(\frac{1}{2} d\psi + \cos \theta \, d\phi\right)^2 \right].
\end{align*}
\]

(25)

where we introduced \( M := m/(4N) \). We note that compared with equations (22) and (25), the behaviour of our solution in the neighbourhood of \( R = 0 \) is equal to that of the five-dimensional Reissner–Nordström–de Sitter solution (25). In the case of the five-dimensional Reissner–Nordström–de Sitter solution (25), there exits the coordinate across the horizon \( (\tilde{R}, v) \) as follows [3]:

\[
\begin{align*}
&\tilde{R}^2 := 4N\alpha + 4NM, \\
&v := \ln \frac{\alpha}{\lambda} + \int \frac{d\tilde{R}}{W} + \Delta(\tilde{R}),
\end{align*}
\]

(26-27)

where the functions \( \Delta(\tilde{R}) \) and \( W(\tilde{R}) \) are defined by the equations

\[
\begin{align*}
&\frac{d\Delta(\tilde{R})}{d\tilde{R}} = -\frac{\lambda}{2(R^2 - m)} \frac{1}{W(\tilde{R})}, \\
&W(\tilde{R}) := \left(1 - \frac{m}{R^2}\right)^2 - \frac{\lambda^2}{4} \tilde{R}^2 = \left(1 - \frac{m}{R^2} - \frac{\lambda}{2R} \right) \left(1 - \frac{m}{R^2} - \frac{\lambda}{2} \right).
\end{align*}
\]

(28-29)

Then, in the coordinate \( (\tilde{R}, v) \), the metric can be written in the form

\[
\begin{align*}
&d\sigma^2 = -W \, dv^2 + 2 \, dv \, d\tilde{R} + \frac{\tilde{R}^2}{4} \left[ d\Omega^2_{S^2} + (d\psi + \cos \theta \, d\phi)^2 \right].
\end{align*}
\]

(30)

In this coordinate \( (\tilde{R}, v) \), three horizons correspond to the three positive roots of the equation \( W = 0 \), i.e. the cubic equations

\[
\pm \lambda R^3 - 2\tilde{R}^2 - 2m = 0.
\]

(31)
This equation has three positive roots when the inequalities
\[ 0 < m\lambda^2 < \frac{16}{27} \]  
(32)
are satisfied. Here, we denote these three roots as \( \tilde{R}_I < \tilde{R}_H < \tilde{R}_C \), where \( \tilde{R} = \tilde{R}_H \) correspond to the black hole horizon. Since the metric on the black hole horizon \( \tilde{R} = \tilde{R}_H \) becomes

\[ ds^2 = 2dv d\tilde{R} + \frac{\tilde{R}_H^2}{4} \left[ d\Omega_4^2 + (d\psi + \cos \theta \, d\phi)^2 \right], \]  
(33)
whose component takes the finite value there, then the spacetime is extended to the region \( \tilde{R} < \tilde{R}_H \) inside the black hole horizon.

Next, let us focus on the extension of the region covered with the chart \((\tau, R)\) in our solution (22), in particular, the extension across \( R = 0 \) and \( \tau = \infty \). To do so let us introduce the same coordinate as \((\tilde{R}, v)\) given by (26) and (27). Then the metric of our solution (22) takes the form

\[ ds^2 = -W dv^2 + 2dv d\tilde{R} + \frac{\tilde{R}_H^2}{4} \left( f d\Omega_4^2 + \frac{1}{f} (d\psi + \cos \theta \, d\phi)^2 \right) + (f - 1) \frac{\lambda^2 \tilde{R}_H^2}{4} \left( \frac{1 - m/\tilde{R}_H^2}{\lambda/2} \frac{1}{W} d\tilde{R} - dv \right)^2, \]  
(34)
where \( f \) is given by

\[ f := 1 + \frac{R}{N}. \]  
(35)
Here we note that \( f \simeq 1 \) near \( R \simeq 0 \). This coordinate seems to be spanned across the horizon, but actually it is a good coordinate only when \( \frac{8}{27} \leq m\lambda^2 < \frac{16}{27} \) holds. To see this we investigate the behaviour of the metric component near the horizon which is located \( \tilde{R} = \tilde{R}_H \), where \( \tilde{R}_H \) is given by the root of equation (31). Then near the horizon, a function \( W \) behaves as

\[ W \sim (\tilde{R} - \tilde{R}_H), \]  
(36)
and the behaviour of the function \( f - 1 \) is

\[ f - 1 = \frac{1}{4N^2} (R^2 - m) \exp \left[ -\lambda \left( v - \int \frac{d\tilde{R}}{W} + \Delta(\tilde{R}) \right) \right] \sim (\tilde{R} - \tilde{R}_H)^\alpha, \]  
(37)
where the exponent \( \alpha \) is given by

\[ \alpha = \lim_{R \to \tilde{R}_H} \frac{\lambda}{\left[ (1 - m/\tilde{R}_H^2) - \frac{i}{2} \right] (R^2 - m)} (\tilde{R} - \tilde{R}_H). \]  
(38)
If the component \( g_{RR} \sim (\tilde{R} - \tilde{R}_H)^{\alpha - 2} \) does not diverge on the horizon \( \tilde{R} = \tilde{R}_H \), it is also the case for the other components. In fact, the other components behave as \( O((\tilde{R} - \tilde{R}_H)^{\alpha - 1}) \) at most. Hence, it is sufficient to investigate the behaviour of the component \( g_{RR} \). Figure 3 shows how the value of \( \alpha \) depends on the parameter \( \lambda^2 m \). From this graph, we see that \( \alpha \) can take the value of \( \alpha \geq 2 \) for \( \frac{8}{27} \leq \lambda^2 m < \frac{16}{27} \), where the component \( g_{RR} \) becomes finite at \( \tilde{R} = \tilde{R}_H \). In particular, the metric is analytic at \( \tilde{R} = \tilde{R}_H \) if \( \alpha \) is a natural number greater than 1. Figure 4 shows the conformal diagram of the extended spacetime. We see that \( \tilde{R} = \tilde{R}_H (R = 0) \) corresponds to an event horizon, which is the boundary of the causal past of \( I^+ \); therefore our solution describes a black hole spacetime.
3.148 Dida et al.

Figure 3. Shows how $\alpha$ depends on the parameter $\lambda^2 m$ for $0 \leq \lambda^2 m < 16/27$.

Figure 4. Global structure of extended spacetime. The null hypersurface $R = 0, \tau = \infty$ is the event horizon.

4. Summary and discussion

In this paper, we have constructed the black hole solution on the self-dual Gibbons–Hawking space in the five-dimensional Einstein–Maxwell theory with a positive cosmological constant. In particular, we have studied the solution on the self-dual Taub–NUT base space in expanding phase and given an extension of spacetime which describes a black hole spacetime.

We have also investigated the geometrical structure of this spacetime within the certain range of the parameters. In the neighbourhood of the event horizon, the metric of our solution is equal to that of the five-dimensional Reissner–Nordström–de-Sitter solution. Therefore, the spatial topology of a black hole horizon is $S^3$. The asymptotic structure is described by the GPS monopole with a positive cosmological constant. Remarkably, the solution has no time-like Killing vector in even a single-black hole case and a cosmological GPS-monopole
case, unlike the Kastor–Traschen solution, which reduces to the static Reissner–Nordström–de Sitter solution in a single-black hole case.

Though it is expected that the boundaries of the chart \( \tau = -\infty, R = 0 \) and \( \tau = 0, R = \infty \) should be extended, we have had no successful attempts in extending the spacetime across the surfaces. It is an open issue and may be interesting for future work. In this paper, we have focussed on the single-horizon solution. While it is expected that the multi-black hole solution describes coalescing black holes. The global structure on higher-dimensional Kastor–Traschen solutions, which describe coalescing multi-black holes, is discussed in [17]. This case in our solution will be discussed in a future article.

Acknowledgments

We are grateful to K Nakao and H Kodama for useful discussions. This work is supported by the grant-in-aid for Scientific Research nos.14540275 and 13135208.

References

[1] Majumdar S D 1947 Phys. Rev. 72 390
[2] Papapetrou A 1947 Proc. R. Ir. Acad. A 51 191
[3] Kastor D and Traschen J 1993 Phys. Rev. D 47 5370
[4] London L A J 1995 Nucl. Phys. B 434 709
[5] Ishihara H and Matsuno K 2006 Prog. Theor. Phys. 116 417
[6] Wang T 2006 Nucl. Phys. B 756 86
[7] Yazadjiev S S 2006 Phys. Rev. D 74 024022
[8] Brihaye Y and Radu E 2006 Phys. Lett. B 641 212
[9] Ishihara H, Kimura M, Matsuno K and Tomizawa S 2006 Class. Quantum Grav. 23 6919
[10] Ishihara H, Kimura M, Matsuno K and Tomizawa S 2006 Phys. Rev. D 74 047501
[11] Gauntlett J P, Gutowski J B, Hull C M, Pakis S and Reall H S 2003 Class. Quantum Grav. 20 4587
[12] Gibbons G W and Hawking S W 1978 Phys. Lett. B 78 430
[13] Ishihara H, Kimura M and Tomizawa S 2006 Class. Quantum Grav. 23 L89
[14] Brill D, Horowitz G, Kastor D and Traschen J 1994 Phys. Rev. D 49 840
[15] Brill D and Hayward S 1994 Class. Quantum Grav. 11 359
[16] Walker M 1970 J. Math. Phys. 11 2290
[17] Astefanesei D, Mann R and Radu E 2004 J. High Energy Phys. JHEP01(2004)029