Optimal Inverter-Based Resources Placement in Low-Inertia Power Systems

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Abstract—The placement of fast-acting Inverter-Based Resources (IBRs) in the grid can substantially impact frequency response, by rapidly injecting or removing power from the grid. In this work, we present an optimal placement algorithm that maximizes the benefits of utilizing IBRs in providing frequency response services. That is, we minimize the overall system frequency deviation while using a minimal amount of electric power injection from the IBRs. The proposed algorithm uses the resistance distance to place the IBRs at nodes that are central to the rest of the nodes in the network thus minimizing the distance of power flow. The proposed greedy algorithm achieves a near optimal performance and relies on the supermodularity of the resistance distances. We validate the performance of the placement algorithm on three IEEE test systems of varying sizes, by comparing its performance to an exhaustive search algorithm. We further evaluate the performance of the placed IBRs on an IEEE test system, to determine their impact on frequency (transient) stability.

Index Terms—Frequency Stability, Inverter-Based Resources, Low-Inertia, Optimal Placement, Power System Dynamics.

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I. INTRODUCTION

The large-scale integration of Inverter-Based Resources (IBRs) [1], [2] will lead to a corresponding decrease in the number of connected synchronous generators, resulting in a decline in the available rotational inertia system-wide. This can potentially result in more pronounced frequency deviations from the nominal when there are disturbances and faults in the grid. As a response, system regulators have started mandating that IBRs participate in providing essential grid services such as frequency response [3], [4]. Due to the fast actuation capability of the IBRs, they can participate in frequency regulation services by rapidly injecting or removing active power to or from the grid [5], [6].

An example that demonstrates the potentials of IBRs in grid services is the South Australia’s Hornsdale Power Reserve, which utilizes Tesla batteries in providing frequency response services with an estimated cost savings of $116 million in 2019 [7], [8]. The benefits accrued can be further maximized by strategically locating these IBRs in the network. Some IBRs, such as solar and wind, are more flexible and can either be paired with a solar or wind resource, or can be located strategically to maximum their benefit to the entire grid. Optimally placing IBRs in the system can result in a more cost effective and efficient participation in frequency response services by reducing frequency deviations using the minimal overall active power injection from the IBRs into the grid.

Varying techniques have been proposed to solve this problem. These techniques typically rely on treating the frequency dynamics of the IBRs similarly as synchronous machines, in a configuration known as virtual synchronous machines (VSMs) [9], [10]. Under this configuration, the optimal IBR placement will be located where the virtual inertia and damping gains of the virtual synchronous machines are maximized [11]. The drawback of this approach is that the fast acting capabilities of the IBR are limited to only provided virtual inertia, ignoring that they can provide more adaptive controls [6]. In addition, the network is often reduced to nodes containing either machines or IBRs, using the Kron reduction method. This results in only a subset of the nodes being considered for IBR placement, which can be suboptimal.

In this paper, we present an optimal IBR placement algorithm that minimizes the overall system frequency deviation while utilizing a minimal amount of power injection from the IBRs, in the event of a disturbance to the network. This algorithm functions by selecting the most central nodes, that is, the nodes that minimize the distance between themselves and the remaining nodes in the network, thereby minimizing the distance of power flow and losses. Under this algorithm, all the nodes in the network can be considered as potential IBR placement points instead of only the nodes with generators. To find the optimal nodes quickly, especially for larger networks, we show that a cost function based on the resistance distance matrix for a power systems network is supermodular. We can therefore adopt a greedy algorithm to determine the optimal nodes efficiently [12]. We also provide a comparison between the exhaustive search algorithm and our placement algorithm for three different sized power systems. The results show that our algorithm returns similar results to using exhaustive searches, but is orders of magnitude more efficient.

To validate the optimality of the IBR placement algorithm on system frequency dynamics, we show that the placed IBRs,
which are configured in a grid-forming mode, effectively reduce frequency deviations after a fault in the system. The exact frequency control algorithm used for the IBRs is a MPC-based inverter power control algorithm that determines the active power set-point of an IBR [6].

The remainder of this paper is organized as follows: Section II defines the models used in this paper. Section III presents the optimal placement algorithm and Section IV shows the results of the placement algorithm tested on the IEEE New England 10 machines 39 bus system and the IEEE 69 machines 300 bus system. The frequency response performance is validated on the 39 bus system.

II. SYSTEM STRUCTURE AND DYNAMICS

We denote the real line by \( \mathbb{R} \) and the cardinality of a finite set \( S \) by \( |S| \). Matrices and vectors are denoted by bold-faced uppercase and lowercase variables, respectively. The \( n \times n \) identity matrix is represented as \( I_n \), the \( n \times n \) zero matrix as \( 0_n \), and the \( i^{th} \) standard basis vector as \( e_i \).

A. Power Flow

For IBR placement, we linearize the AC power flow equations to obtain the standard DC power flow model as described in [6]. During simulations, we will use the full AC power flow. The DC power flow, in vector form, is:

\[
p^i = B\theta^i,
\]

where \( p \) is a vector of the real power at each bus, \( B \) is matrix of admittances between buses, and \( \theta \) is the voltage angles referenced to slack.

B. Resistance Distance and Centrality

The topology of the electric power grid interconnection can be represented as a weighted undirected graph \( G = (\mathcal{V}, \mathcal{E}, \mathcal{W}) \). Under this graph representation, \( \mathcal{V} \), which is the set of nodes represents the electric buses in the grid; \( \mathcal{E} \), which is the set of edges represents the transmission lines which are bidirectional (hence undirected); and \( \mathcal{W} \), which is the set of weights assigned to each edge of the graph, represents the admittance of the transmission lines. Let \( n = |\mathcal{V}| \) represents the number of nodes in \( G \). The Laplacian matrix of \( G \) is a \( n \times n \) real symmetric matrix defined as \( L = D - A \) where \( D \) is the node degree diagonal matrix and \( A \) is the weighted adjacency matrix [13].

**Definition 1.** For the graph \( G \), the resistance distance [14] between two nodes \( i, j \in \mathcal{V} \) is given as:

\[
R(i,j) = (e_i - e_j)^T L^T_i (e_i - e_j) = L_{ii}^T - 2L_{ij}^T + L_{jj}^T,
\]

where \( L \) is the Moore-Penrose pseudo-inverse of \( L \) and is also a \( n \times n \) real symmetric matrix [15]. The pseudo-inverse of \( L \) is used because \( L \) is singular and therefore has no inverse.

**Definition 2.** The resistance distance of a node \( i \in \mathcal{V} \) is the sum of the resistance distance between the node \( i \) and all other nodes in \( \mathcal{V} \), and can be expressed as [16]:

\[
R(i) = \sum_{j \in \mathcal{V}} R(i,j) = nL_{ii}^T + \text{Tr}(L_i^T)
\]

The resistance distance is the distance function on \( G \) [17] and is a measure of the centrality/closeness of the node to other nodes in the network. In the case of an electric power system, a node with a lower resistance distance implies an easier flow of power/current from that node to other nodes.

C. Supermodularity

Supermodularity of a function on a discrete set is analogous to the notion of concavity for functions over continuous sets. It characterizes the idea of diminishing returns, where adding an element to a smaller set gives a larger change in the function than adding the same element to a larger set. A formal definition of this term is given below:

**Definition 3** (Supermodularity). Let \( \mathcal{V} \) be a finite set and \( f : 2^\mathcal{V} \to \mathbb{R} \) be a set function on \( \mathcal{V} \). The function \( f \) is supermodular if \( f(S \cap T) + f(S \cup T) \geq f(S) + f(T) \) or \( f(S) - f(\{u\} \cup S) \geq f(T) - f(\{u\} \cup T) \), for any subsets \( S \subseteq \mathcal{T} \subseteq \mathcal{V} \), and any element \( u \in \mathcal{V} \setminus T \).

If a function over a set is supermodular, then broadly speaking, a greedy algorithm can minimize this function over the set to within a bounded gap of the optimal solution [12], [18]. And if the function \( f \) is supermodular, then \(-f\) is submodular.

In addition to supermodularity, we will use the notion of a non-increasing function over a finite set, defined as:

**Definition 4** (Monotonicity). The set function \( f : 2^\mathcal{V} \to \mathbb{R} \) is monotonically non-increasing if \( f(\mathcal{S} \cup \mathcal{T}) \leq f(\mathcal{S}) \), for all \( \mathcal{S} \subseteq \mathcal{T} \).

The concept of set ordering between sets can be used to show the relational structure between sets and can be defined as:

**Definition 5** (Ordered Set). A set \( \mathcal{S} \) is linearly ordered if the relation \( \leq \) on \( \mathcal{S} \) for all \( s, t, u \in \mathcal{S} \) satisfies the properties of reflexivity \( s \leq s \), anti-symmetry (if \( s \leq t \) and \( t \leq s \), then \( s = t \)), transitivity (if \( s \leq t \) and \( t \leq u \), then \( s \leq u \)), and Trichotomy law (either \( s \leq t \) or \( t \leq s \)).

III. OPTIMAL PLACEMENT ALGORITHM

A. Problem Formulation

The objective of this work is to determine the best location in the grid to place a specified number of IBRs, such that when there is a disturbance to the grid, these IBRs can participate in efficiently stabilizing the grid frequency. To achieve this, we will capitalize on the relationship between frequency and electric power flow in the grid.

From the swing equation that governs frequency dynamics of the grid, it can be observed that the rotor speed deviation and as such the frequency deviation is proportional to the power imbalance. This implies that frequency deviations and rate of change of frequency (ROCOF) can be curtailed by minimizing the power imbalance. One of the advantages of IBRs is their fast actuation capabilities which can enable them participate in frequency control by rapidly injecting or absorbing active power in the grid. Since the disturbance to the grid can occur at any location, typically unknown beforehand, the impact of the
IBRs on frequency control can be maximized by strategically locating them at the “central” nodes in the system.

Equation (1) shows that the power flow in an electric grid is dependent on the topology of the grid, that is, dependent on the susceptance matrix $B$. This matrix is a graph Laplacian and we can define the “resistance” between two buses through (2). Of course, since we work with the DC power flow, there are no losses and these are the not the actual resistance on a line. Rather, they serve as a distance measure as defined in Section II-B. For consistent terminology, we still refer to the quantities computed from $B$ using (2) and (3) as resistances.

Let the set of nodes containing IBRs be denoted as $S$ and the remaining nodes in the network as $V := S \cup T$. The placement problem can be stated as selecting $k$ number of nodes to place the IBRs that minimizes the resistance distance $R(I)$ between nodes with IBRs and nodes without IBRs. Mathematically, this can be written as:

$$\min_{I \subseteq V} R(I) = \sum_{j \notin I} \min_{i \in I} R(i,j)$$

subject to $|I| = k$

For each node $i \in I$ and $j \notin I$, $R(i,j)$ is the resistance distance between node $i$ and $j$ as defined in (2).

B. Supermodularity of Distance Function

To efficiently solve the optimization problem in (4), we use the following properties of the resistance distance function $R$.

**Theorem 1.** The resistance distance function $R(S)$ is a monotonically non-increasing function of the set of vertices $S$. That is, $R(T) \leq R(S)$ for any three sets $S \subseteq T \subseteq V$.

**Proof.** Let $S$ and $T$ be linearly ordered sets and $Q := T \setminus S$ be defined as a linearly ordered set such that $Q = T \setminus S$. From the definition of resistance distance, we have that:

$$R(S \cup Q) = R(S \cup \{q_1\}) = \min_{j \in J} \{ R(S,j), R(q_1,j) \} \leq \sum_{j \in J} R(S,j) = R(S)$$

Adding the elements of set $Q$ sequentially to set $S$ up to the set $T$ to obtain:

$$R(T) = \sum_{j \notin J} \min_{j \in J} R(S,j), R(q_1,j), \ldots, R(q_m,j) \leq \sum_{j \in J} R(S,j) = R(S)$$

Intuitively, this means that as the number of selected nodes increases, the resistance distance between those nodes and the remaining nodes in the network decreases.

**Theorem 2.** If the resistance distance function $R(S)$ is a monotonically non-increasing function of the set of vertices $S$, then $R(S)$ is supermodular, thus

$$R(S \cap T) + R(S \cup T) \geq R(S) + R(T)$$

for any subsets $S \subseteq T \subseteq V$.

**Proof.** Let $S$ and $T$ be linearly ordered sets and $Q := T \setminus S$ be defined as a linearly ordered set such that $Q = T \setminus S$. We have that:

$$R(S \cap T) + R(S \cup T) = R(S \cap T) + R(T) = R(Q) + R(T) \geq R(S) + R(T)$$

The inequality on the last line follows from Theorem 1 since $S$ is a linearly ordered set such that $R(Q) \geq R(S)$.

**Remark.** While results similar to those in Theorems 1 and 2 have appeared in the literature, for example, see [19], [20], the proof presented here are considerably shorter and well-suited for our purpose.

C. Greedy Algorithm

With $R(I)$ proven to be a supermodular set function, equation (4) is the minimization of a supermodular function with cardinality constraint. This problem is NP-hard in general [21]. However, efficient approximate algorithms have been established. For example, the simple greedy algorithm proposed in [12], [21] will find solutions that are within $37\%$ of the optimal solution. Using this algorithm, let the optimal set of IBR placements be $I^*$ and the number of IBRs to be placed be $k$. The optimal node $i^*$ at each iteration is the node which when added to $I^*$, that is $R(I^* \cup \{i^*\}, j)$ minimizes the resistance distance between $I^* \cup \{i^*\}$ and $j \in V \setminus I$. The pseudocode of this algorithm is shown in Algorithm 1.

### Algorithm 1: IBR Placement Algorithm

**Input:** Graph $G = (V, E, W)$

No. of IBRs $k$

**Output:** Optimal set of IBR locations $I^*$

**Initialization:** $I^* \leftarrow \emptyset$

1. Compute $R(i, j)$ using (2)

2. while $|I^*| < k$

3. $i \leftarrow \text{argmin}_{j \in V \setminus I^*} R(I^* \cup i, j)$

4. $I^* \leftarrow I^* \cup \{i\}$

5. **return** $I^*$

IV. CASE STUDIES

A. Optimal Placement

The efficacy of the placement algorithm in section III is validated on the IEEE New England 10 machine 39 bus (IEEE39) and the IEEE 69 machine 300 Bus System (IEEE300) [22], [23]. For each test system, the specified number of IBRs to be placed ranges from one IBR to four IBRs. The goal of the algorithm is to determine the optimal nodes at which to place the specified amount of IBRs, based on its resistance distance to other nodes in the network.

The performance of this placement algorithm is compared to the exhaustive search algorithm (ESA), which tries out every combination of nodes in the network to determine the optimal
node placement. The performance metrics will be based on the accuracy in determining the optimal nodes and computation speed. The simulations were carried out using Macbook Pro 2.7 GHz Dual-Core Intel Core i5 and MATLAB version 2019b.

Table I shows the placement results of 1 to 4 IBRs in the IEEE39 system. The placement algorithm has a 100% accuracy in determining the optimal nodes and accomplishes this at a fraction of the time compared to an exhaustive search. The placement algorithm also scales well to larger systems with a higher amount of nodes. Tables II shows the placement results of 1 to 4 IBRs in the IEEE300 system. The greedy algorithm again finds the optimal solutions with much less time than an exhaustive search.

### B. Frequency Response

The eventual goal of optimally placing the IBRs is to enable them effectively participate in providing frequency response services in the system. We test the frequency response performance by placing 2 grid-forming IBRs, equipped with the MPC-based inverter power controller discussed in [6], in a modified IEEE39 system.

The results of optimally placing 2 IBRs in the IEEE39 system are detailed in table I where the optimal nodes are nodes 6 and 16. The IBRs located at buses 32 and 34 are then relocated to buses 6 and 16. After this placement, the rest of the system can then be reduced to an equivalent network using Kron reduction [24]. It should be noted that the generators in the network are equipped with droop and automatic governor control to enable them effectively participate in providing frequency regulation services.

The total simulation duration is for 30 s and a disturbance in the form of a partial generating capacity loss (60% loss of capacity) is applied to all the generator in the network one at a time, from 0.5 s to 5 s, to initiate an event that can lead to a marked frequency decline. The IBRs will be configured in a grid-forming mode and controlled using the algorithm in [6] to minimize frequency deviation and ROCOF.

The performance metrics that will be used is the time-step scaled $L_1$ norm and is defined for a vector $x$ as $\|x\|_{1,h} = h \cdot \sum_{t=1}^{T} |x^t - x^0|$, where $x^t$ is its value at time $t$, $x^0$ is the nominal value at $t = 0$ and $h$ is the simulation time step. This performance metrics is evaluated on the generator frequency $\|\mathbf{f}\|_{1,h}$, generator power $\|P_{gen}\|_{1,h}$ and IBR power $\|P_{ibr}\|_{1,h}$. The overall performance metrics is the total over all the individual metrics for each disturbed generator. The total overall performance is used because for each individual case, the performance might be better due to proximity to the disturbed generator but worse when far away. For best performance, we expect the optimal placement to have the smallest overall performance metrics value, since a minimal IBR power deviation implies that the IBRs utilize a minimum amount of power to restore the frequency to nominal.

The frequency response performance of the optimal node placement is compared to the next optimal node placement as determined by the placement algorithm, in this case, nodes 5 and 16, and a random node selection placement, chosen as node 3 and 23. Table III shows the performance in frequency response (in select number of buses) of the optimal, next optimal and random bus placement of the IBRs. The overall $\|\mathbf{f}\|_{1,h}$ is 14.83 Hz, 14.89 Hz, and 15.44 Hz, for the optimal, next optimal and random nodes respectively. It can be observed that placing the IBRs at the optimal node as determined by the placement algorithm results in the least overall $\|\mathbf{f}\|_{1,h}$ while randomly selecting a node for placement results in the worst overall $\|\mathbf{f}\|_{1,h}$. The same observations can be made for the total power deviations (both generators and IBRs). Therefore placing IBRs optimally can result in a lot of savings and better performance in providing frequency response services.

### V. Conclusion

In this paper, we proposed an optimal IBR placement algorithm to place IBRs in an electric grid to enable the IBRs participate effectively in frequency regulation services. The algorithm selects as placement nodes, the nodes most central to other nodes in the system using the resistance distance.
function. The proposed algorithm relies on the supermodularity of the resistance distance function to enable computation within a limited time by using a greedy algorithm. We show via simulation on varying system sizes, the efficacy and time saving benefits of our algorithm. We also show via simulation on a test system, the frequency response benefits of placing the grid-forming IBRs, using the proposed algorithm compared to arbitrarily placing the IBRs in the network.

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