Load Sharing Behavior of Double-Pinion Planetary Gear Sets Considering Manufacturing Errors

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Abstract. The unequal load sharing of double-pinion planetary gear sets (PGSs) would take place because of manufacturing errors. A quasi-static model, suitable for any planet train number and support condition, was proposed considering manufacturing errors. The load sharing ratio of double-pinion PGSs with four planet trains was simulated for various planet position errors, whose results show that the radial planet position error has different effects on the load sharing behavior from simple PGSs. The mechanics for this phenomenon was revealed. The effect of the planet train number on the load sharing behavior in the presence of different planet position errors was then simulated by introducing the equivalent error variable. Finally, the effect of gear eccentricity errors on the load sharing behavior were simulated and some important characteristics were revealed such that the planet eccentricity error has worse impacts than position errors, and the effect of the eccentricity error of sun gear could be eliminated by floating the sun gear.

1. Introduction

Planetary gear sets (PGSs) can be classified into simple or compound based on the number of pinions in the planet train [1, 2]. Simple PGSs have only one pinion in the planet train as shown in Figure 1(a), while two or more pinions exist in the planet train for compound PGSs as shown in Figure 1(b). The compound PGSs can be categorized into three distinct structures: meshed-planet, stepped-planet, and multi-stage PGSs [2]. The multi-stage structure is obtained by combining several simple, meshed-planet or stepped-planet PGSs. Generally, two or three pinions in the planet train can be typically found in meshed-planet PGSs [3]. When the carrier is fixed, the ring gear and the sun gear rotates reversely for simple and triple-pinion PGSs, contrary to double-pinion PGSs. Structures by the combination of simple and double-pinion PGSs can be commonly found in multi-speed automotive transmission for the design of compactness [4, 5]. The Double-pinion PGS offers advantages of compactness and high torque-to-weight ratio like the simple PGS. However, these advantages would be weaken when unequal load...
sharing among planets trains takes place due to manufacturing and assembling errors, such as planet position error and gear eccentricity error.

Much experimental [6-8] and theoretical work [9-19] have been conducted on the load sharing behavior of simple PGSs so far. Bodas et al. [9] summarized various manufacturing errors into three distinct groups: (i) time-invariant, assembly-independent errors, (ii) time-invariant, assembly-dependent errors, (iii) and time-varying, assembly-dependent errors. The planet position errors belong to the first group and the gear eccentricity errors are within the third group. They built a two-dimensional finite element model to predict the load sharing behavior of a four planet PGS with a floating sun gear. The simulated results showed that opposite planets carry equal load. An effective planetary error was obtained by combing different types of planet errors. Leque et al. [10] studied the effects of these three error groups by a three-dimensional quasi-static model. Ligata et al. [11] analyzed the load sharing behavior considering different number of planets with position errors. With more planets, the maximum load sharing factor becomes more sensitive to planet position errors. Similar results were also obtained by Seager [12]. Singh [13] concluded that the effect of the tangential planet position error is critical while the radial error could be negligible. Mo et al. [14] analyzed and tested the load sharing behavior of two-stage planetary systems considering various errors. Iglesias et al. [15] investigated the load sharing ratio of a three-planet PGS with planet position and eccentricity errors by a quasi-static model. Gu et al. [16, 17] discussed gyroscopic and centrifugal effects under dynamic conditions considering various planet errors. Kahraman [18] concluded that a floating sun gear could improve its load sharing behavior of four-planet PGSs by a dynamic model. This model was further extended to examine the modulation sidebands by Inalpolat et al. [19].

Previous work indicates that manufacturing errors, especially planet position errors and gear eccentricity errors, have significant impact on the load sharing behavior of simple PGSs. Double-pinion PGSs, in which one pinion is inner planet gear and the other one is outer planet gear, would cause more complex load sharing behavior due to various errors. However, to our author’s knowledge, there is little public information about this.

**Figure 1.** Configuration of (a) a simple planetary gear set, (b) a double-pinion planetary gear set.

This paper aims at investigating the load sharing behavior of double-pinion PGSs considering planet position errors and gear eccentricity errors. A generalized quasi-static model, extended from Gu et al.
[16, 17], is proposed in Sec.2 and Sec.3, in which time-varying mesh stiffness is involved. The model is available for different errors, any number of planet trains and support conditions. A load sharing ratio for double-pinion PGSs is defined in Sec.4. The effects of planet position errors, number of planet trains, and gear eccentricity errors on the load sharing behavior are discussed in Sec.5. Conclusions show the value of guiding engineers to design double-pinion PGSs with optimal load sharing behavior.

2. Rigid-body motions

A classic double-pinion PGS is consisted of a sun gear $(s)$, a ring gear $(r)$, a carrier $(c)$ and $N$ planet trains (PTs) with double pinions as shown in Figure 1(b). The pinion meshing with the sun gear is defined as the inner planet gear (IP), while the pinion meshing with the ring gear is defined as the outer planet gear (OP).

When there are planet position and gear eccentricity errors in double-pinion PGS systems, the rigid motion of each component is possible to suffer a perturbation which could generate an additional infinitesimal rotational angle according to the principle of equilibrium. The gear eccentricity errors on central members (sun gear $s$, ring gear $r$, carrier $c$) are defined in Figure 2, and the perturbation at any given time measured from its errorless configuration can be expressed by screws of infinitesimal displacements [17]:

$$\{D^R_k\} \begin{cases} u^R_k(G_k) = e_k T_k \\ \omega^R_k = \phi_k z \end{cases} \quad k = s, r, c$$

Where $u^R_k$ is the rigid translational displacement vector. $\omega^R_k$ is the angular displacement vector. Superscript $R$ represent the errorless system reference coordinate $O_{0uv}$. $\phi_k$ is the possible additional rigid body angle induced by errors. $O_0$ and $G_k$ donate the theoretical and practical center of solid $k$, respectively. $e_k$ and $T_k$ are the amplitude and the direction of the eccentricity error, respectively. $z$ is outward perpendicular to the $O_{0uv}$ plane.

![Figure 2. Definition of eccentricity errors of the central members.](image)

Planet position and eccentricity errors are defined in Figure 3. Planet position errors can be divided into tangential and radial parts. The radial direction $x_m$ is defined from the practical rotation center of carrier $G_i$ to the theoretical rotation central of the planet $O_m$, while the tangential direction $y_m$ is perpendicular to the radial direction $(m = ij, oj)$, where $ij$ and $oj$ represent inner and outer planet in planet
train \#j, respectively, \(j = 1, 2, ..., N\). Similar to the central components, perturbations of planets caused by errors respecting the carrier can also be described by screws of infinitesimal displacements [16]:

\[
\{D_m^c\} \left\{ \begin{array}{l}
\mathbf{u}_m^c(G_m) = e_{xm}x_m + e_{ym}y_m + e_{zm}T_m \\
\omega_m^c = \phi_m^c z 
\end{array} \right\} \quad m = ij, oj
\]  

(2)

Where superscript \(c\) represents the carrier, \(\phi_m^c\) is possible additional rigid body angle. \(e_{xm}\) and \(e_{ym}\) stand for the radial and tangential position error, respectively. \(e_m\) and \(T_m\) donate the amplitude and the direction of the planet eccentricity error, respectively.

Rigid body motions of inner and outer planets relative to the errorless system reference coordinate can be described as:

\[
\{D_m^R\} = \{D_m^c\} + \{D_m^e\} \quad m = ij, oj
\]  

(3)

In what follows, the sun gear is taken as the driving member and its rotational speed is constant so that the additional angle \(\phi_s = 0\). Similarly, the additional angle for the reaction member will be set to zero.

Figure 3. Definition of position errors and eccentricity errors for inner and outer planets.

Here it is assumed that inner planets are always in contact with the sun gear and outer planets in any planet train, while the ring gear is assured to be in contact with at least one outer planet. \(2N+1\) unknown additional angles exit once the driving and reaction member are determined. The constant contact condition of s-IP and IP-OP provides \(2N\) constraints, and the ring gear in contact with at least one OP guarantees one constraint. Therefore, all additional angles can be calculated. Figure 4 shows the meshing relationship of a double-pinion PGS and base plane of different meshes. The contact conditions for s-IP and IP-OP are:

\[
e_{\text{side}}^{ij} = \left[ \mathbf{u}^{e_s}(M_{sij}) - \mathbf{u}^{e_s}(M_{wij}) \right] \mathbf{n}_{wij} = 0
\]

\[
= \left[ \mathbf{u}^{e_s}(G_s) - \zeta \mathbf{u}^{e_s}(G_s) - \zeta \phi_f z \times G_x G_y \right] \mathbf{n}_{wij} = 0
\]

(4a)
\begin{equation}
\begin{aligned}
e_{M_{sij}} &= \left[ u_y^s(G_{sij}) - u_{aj}^s(M_{sij}) \right] \cdot n_{sij} \\
&= \left[ u_y^s(G_{sij}) + (\varphi_j + \zeta \varphi_c) z \times \bar{G}_y M_{sij} - \right. \\
&\left. \zeta \varphi_c z \times \bar{G}_c G_{sij} - u_y^s(G_{aj}) - (\varphi_{aj} + \zeta \varphi_c) z \times \bar{G}_y M_{sij} \right] \cdot n_{sij} = 0
\end{aligned}
\end{equation}

Where \( e_{M_{sij}}, e_{M_{ioj}} \) are the normal deviation at \( M_{sij}, M_{ioj} \), respectively. \( n_{sij}, n_{ioj} \) are the corresponding unit normal vector. \( \zeta \) equals zero if the carrier is not allowed to rotate, otherwise \( \zeta = 1 \).

For the meshing between the ring gear and the outer planet, the normal deviation \( e_{M_{rroj}} \) for any potential contact point \( M_{rroj} \) on the base plane of r-OP can be expressed as [20]:

\begin{equation}
\begin{aligned}
e_{M_{rroj}} &= \left[ \eta u_y^s(M_{rroj}) - u_y^s(M_{rroj}) \right] \cdot n_{rroj} \\
&= \left[ \eta u_y^s(G_{rroj}) + \eta \varphi_r z \times \bar{G}_c M_{rroj} - \zeta u_y^s(G_{rroj}) - \right. \\
&\left. \zeta \varphi_r z \times \bar{G}_c G_{rroj} - u_y^s(G_{rroj}) - (\varphi_{rroj} + \zeta \varphi_r) z \times \bar{G}_y M_{rroj} \right] \cdot n_{rroj}
\end{aligned}
\end{equation}

Where \( n_{rroj} \) is a unit normal vector at \( M_{rroj} \). \( \eta \) equals zero if the ring gear is not allowed to rotate, otherwise \( \eta = 1 \).

The unknown \( \varphi_j + \zeta \varphi_c \) in Eq.(4a) is determined by geometrical errors (gear eccentricity errors and planet position errors) and additional carrier rotational angle \( \varphi_c \), as well as \( \varphi_{aj} + \zeta \varphi_c \) in Eq.(4b). Therefore, \( e_{M_{rroj}} \) is independent of additional rotational angle of inner or outer planets and can be further written as:

\begin{equation}
e_{M_{rroj}} = E + E' \varphi_r + E'' \varphi_r
\end{equation}

Where \( E_j \) is induced by geometrical errors, \( E' \) and \( E'' \) are determined by the double-pinion PGS’s configuration parameters. More details are listed in Appendix 1.

\( e_{M_{rroj}} \) is necessary equal or greater than zero, otherwise interpenetration between mesh interfaces occurs. The initial separations between different mesh gear pairs are obtained by [16, 17]:

\begin{equation}
\begin{aligned}
\delta e_{sij} &= e_{M_{sij}} - \min_{L=1,2,...,N} (e_{Mod.}) = 0 \\
\delta e_{ioj} &= e_{M_{ioj}} - \min_{L=1,2,...,N} (e_{Mod.}) = 0 \\
\delta e_{rroj} &= e_{M_{rroj}} - \min_{L=1,2,...,N} (e_{Mod.}) = E_j - \min_{L=1,2,...,N} (E_j)
\end{aligned}
\end{equation}

Where \( \delta e \) is initial separation, subscripts \( sij, ioj, roj \) are sun gear-inner planet (s-IP), inner planet - outer planet (IP-OP), and ring gear-outer planet (r-OP) mesh for planet train \#j, respectively.
Figure 4. (a) Parameters for mesh relationship of double-pinion PGS, and geometrical parameters in the base plane of (b) sun gear-inner planet, (c) inner-outer planet, and (d) ring gear-outer planet mesh.

Figure 5 shows initial separations of $r_{oj}$, additional angles of IP, OP and carrier with a fixed ring gear of a double-pinion PGS example with four planet trains in an automatic transmission. The sun gear and ring gear are the input and reaction component, respectively. Its basic parameters are listed in Table 1. Figure 5(a) reveals that at least one outer planet is in contact with ring gear at any time.

Figure 5. Rigid-body characteristics of a double-pinion PGS with errors of $e_s=30\mu m$ on sun gear, $e_{r1}=20\mu m$, $e_{r1}=30\mu m$ on inner planet #1, $e_{r2}=10\mu m$ on outer planet #2, and $e_c=25\mu m$ on carrier.
Table 1. Gear parameters of a double-pinion PGS example.

| Parameters | Carrier | Sun gear | Inner planet | Outer planet | Ring gear |
|------------|---------|----------|--------------|--------------|----------|
| Module (m) | 2×10⁻³  |          |              |              |          |
| Pressure angle (°) | 25 |          |              |              |          |
| Tooth number | 48 | 21 | 22 | 108 |
| Face width (m) | 2×10⁻² |          |              |              |          |
| Bearing stiffness (N/m) | 1×10⁸ | 1×10⁸ | 1×10⁸ | 1×10⁸ |
| Torsional reaction stiffness (N·m/rad) | 0 | 0 | 0 | 1×10¹² |
| Relative position angle Φio (°) | 29.71 |
| Angle on outer planet Λ (°) | 52.68 |

3. Double-pinion PGs quasi-static model

All components are taken as rigid with 3 degrees of freedom including two translational (x, y) and one rotational (θ), shown in Figure 6. The coordinate Ox₁y₁ of central gears rotates about the coordinate O₁uv (Figure 2) with the carrier’s angular velocity, where x₁ is directed toward the theoretical center of IP #1. The coordinates of planets Omₓᵧm are fixed to the carrier with xₘ originating from O₀ and towards Oₘ (m = ij, oj). R is introduced to represent the rotating frame of the quasi-static model shown in Figure 6. In such conditions, elastic displacements of each solid in the vicinity of the system rotating frame R can be described by screws of coordinates [20]:

$$
\{D^R_k\} \{u_k^R (O_k) = x_k x_1 + y_k y_1 \}_{o_k}^R = \theta_k z_k \quad (k = s, r, c) \quad (8a)
$$

$$
\{D^R_m\} \{u_m^R (O_m) = x_m x_m + y_m y_m \}_{o_m}^R = \theta_m z_m \quad (m = ij, oj) \quad (8b)
$$

The deflection of different meshing gear pairs on the base plane can be expressed as:

$$
\delta_{pq} = \sum_{i} \left[ u_i^R (O_i) + \omega_i^R \times \overrightarrow{OM_{pq}} \right] n_{pq} - \delta e_{pq}
\quad = \left( V_{pq} \right)^T X_{pq} - \delta e_{pq} \quad (9)
$$

Where (p, q) can be (s, ij), (ij, oj) or (r, oj). δe_{pq} is the initial separation of p-q mesh. X_{pq} is the degrees of freedom vector and V_{pq} is the structure vector. Superscript T donates transpose of the vector. More details are listed in Appendix 2.
Figure 6. Lumped parameter model of double-pinion PGSs.

Therefore, the time-varying mesh stiffness matrix $K_{pq}(t)$ and forcing term vector $E(t)$ caused by the errors are written as:

$$
K_{pq}(t) = \sum_{j=1}^{N} k_{pq}(t) V_{pq} V_{pq}^T + k_{pq}(t) V_{pq} V_{pq}^T
$$

$$
E(t) = \sum_{j=1}^{N} k_{pq}(t) \delta V_{pq} + k_{pq}(t) \delta V_{pq} V_{pq}^T
$$

Where $k_{pq}(t)$ is the time-varying mesh stiffness calculated with the potential energy method [21]. Considering the contact condition, the following is obtained:

$$
k_{pq}(t) = 0 \quad \text{if} \quad \delta_{pq} \leq 0
$$

It is supposed that all shaft support bearings are isotropic, and its stiffness for each IP in different planet trains is identical as well as for each OP. The stiffness matrix for IP- and OP-carrier connections is written in the form of:

$$
K_{rr} = k_{rr}
$$

$$
\begin{bmatrix}
1 & 0 & -R_{cm} \sin \Phi_m & -\cos \Phi_m & \sin \Phi_m & 0 \\
1 & R_{cm} \cos \Phi_m & -\sin \Phi_m & -\cos \Phi_m & 0 & 0 \\
R_{cm}^2 & 0 & -R_{cm} & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
$$

Where $k_{rr}$ is the bearing stiffness of the planet, $\Phi_m$ is the position angle of the planet, $R_{cm}$ is the center distance between the carrier and the planet.
For central gears, their support stiffness matrices are written as:

\[ K_{kb} = \text{diag}[k_{kx}, k_{ky}, k_{k\theta}] \quad k = s, r, e \]  

(14)

Therefore, the overall support stiffness of the double-pinion PGS can be given by:

\[ K_s = \sum_k K_{kb} + \sum_{j=1}^{N} \left( K_{c_{ij}} + K_{c_{oj}} \right) \]

(15)

Similarly, the overall quasi-static systematic equations can be written as:

\[ \left[ K_{pq}(t) + K_s \right] \mathbf{X} = \mathbf{F} + \mathbf{E}(t) \]  

(16)

Where \( \mathbf{X} \) is the overall displacement vector, \( \mathbf{F} \) is the applied external torque.

**4. Load sharing ratio**

The load sharing ratio of a given planet train \( j \), \( LSR_j \), is defined as:

\[ LSR_j = \frac{Rb \cdot F_{sij}}{T_s} \times 100\% \]  

(17)

Where \( F_{sij} \) is the meshing force of \( s \)-IP \#j, \( T_s \) is the external torque at the sun gear.

The simulation process is presented in Figure 7. The calculated \( LSR_j \) for each planet train will be discussed in the following sections, concerning planet position errors, number of planet trains, and eccentricity errors.
5. Results and discussions
The parameters of the simulated double-pinion PGS in the following are listed in Table 1. All planets are assumed to be equally spaced. The ring gear is stationary, and the power flows from the sun gear to the carrier. The input torque $T_s$ is 400Nm.

5.1. Planet position error
The planet position error, defined as the actual planet shaft position deviation from the theoretical value, takes place due to manufacturing tolerances of the carrier. Here, the tangential and radial position errors are discussed for IPs and OPs. Only the position error in one IP or OP are discussed while other planets are assumed to be located in its theoretical position.

**Case i: tangential position error**
Figure 8 shows the effects of the tangential position error ranging from $-100\mu m$ to $100\mu m$ of the IP #1 and OP #1 on the load sharing behavior of the system under the fixed and floating sun gear conditions.

When the sun gear is fixed, the $\text{LSR}$ of PT1 suffers the most while PT3 the least for the tangential error at IP #1 or OP #1. The $\text{LSR}$ of PT2 and PT4 is identical and almost irrelevant whether the sun gear is fixed or not. When the sun gear is fixed and PT1 carries extra load, other planet trains would carry less load than ideal condition. However, when the sun gear is floating, opposite planet trains carry equal load and PT1 suffers less. Therefore, it is identified that a floating sun gear can improve the load sharing behavior of the system. Similar behavior is revealed in Refs.[9-13, 18]. Moreover, the influence of IP tangential position error on the load sharing behavior is contrary to the OP tangential position error. For instance, positive tangential position error of IP #1 leads PT1 to carry less load than ideal condition, but OP #1 with positive tangential position error makes PT1 carry more load.

![Figure 8](image_url)
*Figure 8. LSR of inner planet #j tangential position error $e_{yi}$ for the four planet train double-pinion PGSs having (a) fixed, (b) floating sun gear and outer planet #j tangential position error $e_{yo}$ for the system having (c) fixed sun gear, (d) floating sun gear.*

**Case ii: radial position error**
A radial position error ranging from $-100\mu m$ to $100\mu m$ on IP #1 and OP #1 is simulated and the results are listed in Figure 9. The maximum LSR variation reaches 5.16% for $e_{yo1}=100\mu m$ and 4.95%
for $e_{x1} = 100 \mu m$ under floating sun gear condition, while it even reaches 11.44% for $e_{x1} = 100 \mu m$ and 10.97% for $e_{x1} = 100 \mu m$ under fixed sun gear condition. Furthermore, contrary to the effects of tangential error, the positive radial error of IP or negative radial error of OP make its planet train carry more load. Although the radial position error has less impact on the LSR than tangential position error, its effects on the LSR still could be omitted, which is different from simple PGSs.

**Figure 9.** LSR of inner planet $#j$ radial position error exi1 for the four planet train double-pinion PGSs having (a) fixed, (b) floating sun gear and outer planet $#j$ radial position error exo1 for the system having (a) fixed, (b) floating sun gear.

**Graphic explanation**

Figure 10 illustrates the tangential and radial error of IP $#j$. The dash line represents the theoretical position and the red line the actual position caused by position errors. Separation distance variation of the meshing surface takes place along LOA because of errors. Positive tangential error increases the separation distance of the s-IP mesh ($\Delta_{sij}^{tan}$) and IP-OP mesh ($\Delta_{ioj}^{tan}$). Regarding to the positive radial error, separation distance increases for the s-IP mesh ($\Delta_{sij}^{rad}$), and decreases for the IP-OP mesh ($\Delta_{ioj}^{rad}$). Therefore, the IP comes closer to its mating surfaces by an amount:

\[
\Delta_{ij}^{tan} = \frac{\Delta_{sij}^{tan} + \Delta_{ioj}^{tan}}{2} = -e_{ij} \cos \varphi_{s} + \varphi_{s} \cos \frac{\varphi_{s} - \varphi_{o}}{2} \tag{18a}
\]

\[
\Delta_{ij}^{rad} = \frac{\Delta_{sij}^{rad} + \Delta_{ioj}^{rad}}{2} = e_{ij} \cos \frac{\varphi_{s} + \varphi_{o}}{2} - \sin \frac{\varphi_{s} - \varphi_{o}}{2} \tag{18b}
\]

Where $\Delta_{sij}^{tan} = -e_{ij} \cos \varphi_{s}, \Delta_{sij}^{rad} = -e_{ij} \cos \varphi_{o}, \Delta_{ioj}^{tan} = -e_{ij} \sin \varphi_{s}, \Delta_{ioj}^{rad} = e_{ij} \sin \varphi_{o}, \varphi_{s} = \alpha_{s}, \varphi_{o} = \Lambda + \Phi_{io} + \alpha_{io}$. 

11
Figure 11 shows the influence of position errors on OP. In general, the OP comes closer to its mating surfaces by an amount:

\[
\Delta_{op}^m = \Delta_{op}^{\text{mav}} + \Delta_{op}^{\text{heav}} \\
= e_{op} \cos \frac{\phi_o + \phi_m}{2} \cos \frac{\phi_o' - \phi_m'}{2}
\]

(19a)

\[
\Delta_{op}^{\text{rad}} = \Delta_{op}^{\text{tav}} + \Delta_{op}^{\text{heav}} \\
= -e_{op} \cos \frac{\phi_o' + \phi_m'}{2} \sin \frac{\phi_o' - \phi_m'}{2}
\]

(19b)

Where \( \Delta_{op}^{\text{mav}} = e_{op} \cos \phi_o^{*}, \Delta_{op}^{\text{heav}} = e_{op} \cos \phi_m^{*}, \Delta_{op}^{\text{tav}} = -e_{op} \sin \phi_o^{*}, \Delta_{op}^{\text{heav}} = e_{op} \sin \phi_m^{*}, \phi_{o} = \alpha_s + \alpha_{io}, \phi_{o}' = \Lambda_t + \phi_{io}. \)

If position errors are equal, that is \( e_{op}^{*} = e_{op}^{*} = e_{op} = e_{op} = e_{ij}, \) the proportion of separation distances would satisfy \( \Delta_{op}^{\text{mav}} : \Delta_{op}^{\text{tav}} : \Delta_{op}^{\text{heav}} : \Delta_{op}^{\text{heav}} = 1 : -0.495 : -0.543 : 0.475 \) for the double-pinion PGS example. When the position error is 100 \( \mu \)m, that is \( e_{op} = e_{op} = e_{op} = e_{op} = 100 \mu \)m, variation of LSR of PT1 equals 10.43\%, -5.16\%, -5.66\% and 4.95\% under the floating sun gear condition, respectively. Therefore, these simple graphics are helpful to understand the effects of different position errors qualitatively. Furthermore, other results can also be obtained:

1) For positive IP tangential position error, \( \Delta_{op}^{\text{mav}} \) is negative and its planet train will carry less load than the other planet trains. And for positive OP tangential position error, \( \Delta_{op}^{\text{mav}} \) is positive and its planet train will carry the most load.

2) The effects of radial and tangential position errors on the LSR are relative with the configuration of the double-pinion PGS.

3) With positive radial position error, \( \Delta_{op}^{\text{rad}} \) is positive and \( \Delta_{op}^{\text{rad}} \) is negative if \( \Phi_{io} > 0 \), while \( \Delta_{op}^{\text{rad}} \) is negative and \( \Delta_{op}^{\text{rad}} \) is positive if \( \Phi_{io} < 0 \).

4) When the center line of IP and OP goes through the center of central members, that is \( \Phi_{io} = 0, \Delta_{op}^{\text{rad}} = \Delta_{op}^{\text{rad}} = 0, \) the effects of radial errors of IP and OP will be very tiny to be neglected.
5.2. Number of planet trains

The analyses presented previously showed the effects of position errors on the four planet train double-pinion PGS. Different position errors have a certain relationship on the LSR of the planet train. As a result, an equivalent position error $E_{eq}$, combining different position errors based on the proportional relationship, is defined as:

$$E_{eq} = e_{ox} - 0.495e_{yo} - 0.543e_{xi} + 0.475e_{yi}$$

(20)

Figure 12 shows the LSR curves of the equivalent position error for a three, four, five and six planet train in double-pinion PGSs with a floating sun gear when $-400 \mu m \leq E_{eq} \leq 600 \mu m$. The basic gear parameters are the above example double-pinion system shown in Table 1. The position error is supposed to be only applied at the first planet train. For the three-planet train system, all planet trains are always equally loaded. Loaded planet trains can be two or four of the four-planet train system. When $-130 \mu m \leq E_{eq} \leq 150 \mu m$, all planet trains are in contact, however, only two planet trains are in contact if there is LSR=50% for each planet train. With the increasing of the number of planet trains, LSR is more sensitive to position errors. For five-planet train system, all planet trains are in contact at $-130 \mu m \leq E_{eq} < 120 \mu m$. However, PT1 would be out of contact at $-400 \mu m < E_{eq} < -130 \mu m$, which makes the other four carry load. PT1, PT3 and PT4 are in contact at $150 \mu m < E_{eq} < 600 \mu m$. Furthermore, PT3 and PT4 always carry equal load, as well as PT2 and PT5. The six-planet train system has five loaded planet trains at $-400 \mu m \leq E_{eq} < -80 \mu m$ while four planet trains are in contact at $120 \mu m \leq E_{eq} < 500 \mu m$. Only opposite planet trains PT1 and PT4 are in contact at $500 \mu m < E_{eq} \leq 600 \mu m$. Due to the axial symmetry, PT2 and PT6 always carry equal load, as well as PT3 and PT5. LSR of PT3 and PT5 are independent of the position errors at $-400 \mu m \leq E_{eq} < 120 \mu m$. However, when PT2 and PT6 are out of contact, LSR of PT3 and PT5 decrease with the increasing of $E_{eq}$.

Under the worst case, the maximum LSR of the four-planet train system is 2 times of the nominal LSR while 2.236 times for the five-planet train system and 3 times for the six-planet train system. Furthermore, the four-planet train and six-planet train system have two loaded planet trains, and the five-planet system has three loaded planet trains. This phenomenon can also be found in simple PGSs [22].

![Figure 12. LSR for different planet train in double-pinion PGSs with position errors.](image-url)
5.3. Eccentricity error

The eccentricity error is the deviation of the actual rotation center from the theoretical. Figure 13 shows the value of $LSR$ of the double-pinion PGS with four planet trains, in which there is an eccentricity error of $100\mu m$ on the sun gear. $LSR$ of any planet train is sinusoidal variation. The maximum $LSR$ is approximately $25\%$ with floating sun gear while $34.32\%$ with fixed sun gear. Therefore, floating sun gear can greatly eliminate the influence of its eccentricity error on the load sharing behavior.

![Figure 13. Variation of LSR of four planet trains with eccentricity error $e_o=100\mu m$ of (a) fixed and (b) floating sun gear.](image_url)

Figure 14 shows the calculated $LSR$ of the double-pinion PGS, in which the eccentricity error $e_o$ is $100\mu m$ and the initial angle $\lambda_o$ is $\pi/2$. Sinusoidal $LSR$, because of the eccentricity error on the planet, could induce modulation sidebands of the system\textsuperscript{19}. Initially, its influence on $LSR$ is equivalent as the tangential error of OP #1 for either fixed or floating sun gear. Positions of OP #1 are accordant. Therefore, the planet eccentricity error can be regarded as a combination of radial and tangential position error which varies with the carrier angular position. It is evident that floating sun gear can improve the load sharing behavior, but could not eliminate the impacts of the eccentricity error on the planet. Similar behavior can also be found in simple PGSs\textsuperscript{10}. In addition to those, planet eccentricity errors have worse effects than position errors. For instance, with the fixed sun gear, the contact would disappear and the maximum $LSR$ is $51.64\%$ when $e_o=100\mu m$ shown in Figure 14(a), while all planet trains are in contact and the maximum $LSR$ is $48.10\%$ at $e_o=100\mu m$ presented in Figure 8(c).

![Figure 14. Variation of LSR of four planet trains with $e_o=100\mu m$, $\lambda_o=\pi/2$ for (a) fixed sun gear, and (b) floating sun gear.](image_url)
6. Conclusion
A quasi-static load sharing model for double-pinion PGSs is presented considering the time-varying mesh stiffness and manufacturing errors. Effects of planet position errors, planet train number, and eccentricity errors on the load sharing behavior of the double-pinion PGSs are simulated and analyzed. Some main conclusions can be summarized as follows:

1) Floating sun gear can improve the load sharing behavior of double-pinion PGSs with four planet trains greatly. The radial position error has less impact on the load sharing behavior than the tangential error, but its effects are associated with the configuration of the system and could not be ignored.

2) Adding planet trains would make the double-pinion PGS system more sensitive to position errors if the sun gear is floating. Three-planet train systems share the load equally even in the presence of position errors. For the worst case, only two opposing planet trains carry the load for four- or six-planet train systems, and three planets for five-planet train systems.

3) The load sharing of double-pinion PGSs varies in sinusoidal shape if there are eccentricity errors. Floating support of the sun gear can eliminate the influence from its eccentricity errors effectively. The planet eccentricity error can be regarded as the combination of radial and tangential position errors, which has worse effect than position errors on the load sharing behavior.

Anyway, the noise and vibration problem is a critical issue for double-pinion PGSs, experimental tests would be conducted in the future in order to understand the mechanics of the dynamic behavior of double-pinion PGSs.

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Appendix 1
Calculation of $E$, $E'$ and $E''$
Based on Eq.(4a,b), one can obtain that:

$$
\varphi_0 + \zeta\varphi_0 = \frac{[u^*_s(G_j) - \zeta u^*_s(G_j) - \zeta \varphi_0 z \times \vec{G}_j - u^*_o(G_j)]n_i}{(z \times G_j, M_{ij})n_{ij}}
$$

$$
= \frac{A}{B} + A'\varphi_0
$$

(21a)

$$
\varphi_{ij} + \zeta \varphi_{ij} = \frac{[u^*_s(G_j) + (\varphi_0 + \zeta \varphi_0) z \times \vec{G}_j + u^*_o(G_j)]n_i}{(z \times \vec{G}_j, M_{ij})n_{ij}}
$$

$$
= \frac{C}{D} + C'\varphi_0
$$

(21b)

Where expression $A, B, A', C, D, C'$ can be calculated as:

$$
A = \cos \beta_h \left\{ \begin{array}{c}
e_s \cos (\gamma_{ij} - \psi_j) - \zeta e_s \cos (\gamma_{ij} - \lambda_s) \\
e_s \cos \gamma_{ij} + e_{ij} \sin \gamma_{ij} + e_s \cos (\gamma_{ij} - \psi_j) \\
\end{array} \right\}
$$

(22a)

$$
B = -e_s R_{ij} \cos \beta_h
$$

(22b)

$$
A' = \frac{\zeta R_{ij} \cos \alpha_s}{R_{ij}}
$$

(22c)
\[ C = \cos \beta_b \left\{ \begin{array}{l} e_{ij} \cos \gamma_{ij} + e_{ij} \sin \gamma_{ij} + e_i \cos (\gamma_{ij} - \psi_{ij}) \\ + A \cos \beta_b \end{array} \right\} \]  

(22d)

\[ D = e_{ij} R_{ij} \cos \beta_b \]  

(22e)

\[ C' = \zeta R_{ij} \sin \gamma_{ij} - e_{ij} R_{ij} \cos \alpha_i - R_{ij} \sin \gamma_{ii} \]  

(22f)

By substituting Eq.(22a-f) into Eq.(5) and Eq.(6), one can derive:

\[ E = \cos \beta_b \left\{ \begin{array}{l} e_{ij} \cos \gamma_{ij} + e_{ij} \sin \gamma_{ij} - e_i \cos (\gamma_{ij} - \lambda_i) \\ + e_i \cos (\gamma_{ij} - \psi_{ij}) - C \cos \beta_b \end{array} \right\} \]  

(23a)

\[ E' = -\zeta \cos \beta_b \left\{ \begin{array}{l} e_{ij} R_{ij} \cos \alpha_i - R_{ij} \sin \gamma_{ii} + \\ - R_{ij} \sin \gamma_{ii} - e_i R_{ij} \cos \alpha_i \end{array} \right\} \]  

(23b)

\[ E^* = e_{ij} \eta R_{ij} \cos \beta_b \]  

(23c)

Where \( \beta_b \) is the base helix angle. \( \gamma_{ij} = \Phi_{ij} - e_i (\alpha_i - \pi/2) \), \( \gamma_{ij} = -e_i (\alpha_i - \pi/2) \), \( \gamma_{ii} = \Phi_{ii} + e_i (\alpha_i - \pi/2) \), \( \gamma_{oi} = \Phi_{oi} - e_i (\alpha_i - \pi/2) \), \( \gamma_{oj} = \Phi_{oj} + e_i (\alpha_i - \pi/2) \), \( \lambda_i \) is the angle of OP relative to the sun gear and IP as shown in Fig.4. \( \alpha_i, \alpha_o, \alpha_r \) are the pressure angle of s-IP, IP-OP and r-OP mesh, respectively. \( \Phi_{ij} \) is the position angle of IP \#j, \( \Phi_{ij} \) is the position angle of OP \#j, \( \Phi_{ii} = \Phi_{ij} - \Phi_{ij} \), \( e_i = 1 \) if the sun gear is counter-clockwise rotation and \( e_i = -1 \) if the sun gear is clockwise rotation. \( \psi_{ij} = \Omega_{ij} t + \lambda_{ij} \) is the rotation angle of component \( g \) (\( g = s, r, ij, of \)) relative to the carrier. \( \Omega_{ij} \) is the angular speed of the component \( g \) relative to the carrier \( e_i \). \( \lambda_{ij} \) is initial angle of gear eccentricity errors as shown in Fig.2 and Fig.3. \( R_{bg} \) is the base radius of the gear. \( R_{cm} \) is the center distance of carrier and planet \( m \) (\( m = ij, of \)).

**Appendix 2**

**Vectors in Eq.(9)**

\[ V_{sij} = \cos \beta_b \left\{ \begin{array}{l} \cos \gamma_{ij} \\ \sin \gamma_{ij} \\ e_i \sin (\gamma_{ij} - \psi_{ij}) + e_i R_{ij} \\ -\cos \gamma_{ij} \\ -\sin \gamma_{ij} \\ -e_i e_{ij} \cos (\alpha_i + \psi_{ij}) + e_i R_{ij} \end{array} \right\} \]  

\[ X_{sij} = \left\{ \begin{array}{l} x_i \\ y_i \\ \theta_i \end{array} \right\} \]  

(24a)
\[ \mathbf{V}_{\text{aqj}} = \cos \beta_v \begin{bmatrix} \cos \gamma_{io} \\ \sin \gamma_{io} \\ e_i \sin(\gamma_{io} - \psi_{ij}) - e_x R_i j \\ -\cos \gamma_{io} \\ -\sin \gamma_{io} \\ -e_i \sin(\gamma_{io} - \psi_{ij}) - e_x R_i j \end{bmatrix} \quad \mathbf{X}_{\text{aqj}} = \begin{bmatrix} x_j \\ y_j \\ \theta_j \end{bmatrix} \] (24b)

\[ \mathbf{V}_{\text{aqj}} = \cos \beta_v \begin{bmatrix} \cos \gamma_{roj} \\ \sin \gamma_{roj} \\ e_r \sin(\gamma_{roj} - \psi_{rj}) + e_x R_{rj} \\ -\cos \gamma_{roj} \\ -\sin \gamma_{roj} \\ -e_r \sin(\gamma_{roj} - \psi_{rj}) - e_x R_{rj} \end{bmatrix} \quad \mathbf{X}_{\text{aqj}} = \begin{bmatrix} x_{ij} \\ y_{ij} \\ \theta_{ij} \end{bmatrix} \] (24c)