Abstract

The production of quarks and antiquarks during a sudden restoration of chiral symmetry, as it might occur in very energetic heavy ion collisions, is considered. If gluons are already present they can assist additively to the overall production: real gluons are partially decaying during such a chiral transition. The total number of produced quarks is calculated and found to be quite sizeable. It is speculated that such a phenomenon could give rise to a significant contribution to the overall quark pair creation in the preequilibrium stage of the heavy ion collision.

25.75.+r, 11.10.Qr, 12.38.Bx, 24.85.+p
One primary goal in the upcoming relativistic heavy ion collision experiments at the Brookhaven Relativistic Heavy Ion Collider (RHIC) and the CERN Large Hadron Collider (LHC) is the temporary formation and potential observation of a new form of matter, the quark gluon plasma, a deconfined phase of QCD [1]. It is expected that this phase is also accompanied by a restoration of chiral symmetry [2]. At high enough temperatures the massive and confined quasiparticles, the constituent quarks, become bare, undressed quarks with current quark masses being much lighter than the constituent quark masses. Such a physical picture is realized in model descriptions of low-energy, effective QCD lagrangians like the Nambu and Jona-Lasinio (NJL) model [3,4]: At large enough coupling chiral symmetry of the vacuum is spontaneously broken and exhibits a nonvanishing scalar quark condensate $\langle \bar{q}q \rangle \simeq -(250 \text{ MeV})^3$. A similar behaviour is experienced in lattice QCD calculations [5]. The scalar condensate attributes a scalar selfinteraction to the fermion propagator resulting in the presence of constituent quarks at low energies, in line with experience from hadronic spectroscopy. For higher temperatures, however, the condensate becomes weaker and vanishes at sufficiently large temperatures (on the order of $T \sim 200 \text{ MeV}$) [6,3], which is also found in recent QCD calculations [7].

Relativistic heavy ion collisions offer the only possible way to gain more insight into the phase structure of nuclear matter at such high temperatures (and densities). However, from the onset the reaction during any such a collision proceeds far off equilibrium, so that a thermal quark gluon plasma may be reached only after some finite time. According to the results of the parton cascade model [8] the quarks will reach, if at all, a chemical saturation only eventually in a central relativistic heavy ion collision at collider energies whereas the gluons saturate on a significantly smaller time scale (in about half a fm/$c$) [9,10]. This ‘hot glue’ scenario was proposed by Shuryak [9]. The reason for this particular behaviour lies in the fact that the quarks are only produced in second order by gluon annihilation and the rate for this process is small compared to the corresponding one for gluon production. The decay of a single gluon into a quark and antiquark is, of course, kinematically not allowed, as long as all partons are treated as effective on-shell particles. In this work we want to propose that
the occurrence of a chiral phase transition can have some profound influence on the quark pair production, if the restoration will happen on a very short timescale. The idea is that if indeed a deconfined state (equilibrated or not) will be formed in the reaction of a heavy ion collision, the ‘undressing’ of the quarks should already have occurred. Being probably still far from equilibrium this chiral restoration may thus happen rather spontaneously. We assume that the scalar quark condensate $\sigma \sim \langle \bar{q}q \rangle$ would melt on a timescale smaller than any scale set by the dynamics. The decay of the vacuum induces a spontaneous creation of quark and antiquark pairs. During the chiral restoration transition the quarks are no longer on-shell particles, their spectral function contains a wide off-shell spectrum, thus permitting also the decay of a single gluon into a quark-antiquark pair. The decay of both the vacuum and a substantial fraction of energetic gluons would accelerate the chemical saturation of the quark phase space.

To describe the effect on quark pair production we construct a simple model for fermionic matter. Denoting the time of chiral restoration at $t = 0$, the light quark masses then are functions of time

$$m(t) = \begin{cases} m_c \sim 350 \text{ MeV} & \text{for } t \leq 0 \\ m_b \sim 10 \text{ MeV} & \text{for } t > 0 \end{cases}$$

(1)

We assume that no further residual interactions act among the quarks. While ansatz (1) may appear quite artificial, it captures the essential spirit of what one would naively call ‘a transition to the phase with restored chiral symmetry’. The constituent vacuum becomes unstable so that during the restoration of the masses occupied states of the negative Dirac continuum are partially transferred into the positive continuum. The amplitude for this is obtained in a straightforward way within the In/Out formalism. A particle with momentum $\mathbf{p}$ sitting in the negative Dirac sea is described by the incoming (constituent) wavefunction

$$\psi_{\downarrow}^{(In)} \equiv \varphi_{p\downarrow}^{(+)} = \varphi_{pl\downarrow}(x, t)$$

(the arrow ‘$\downarrow$’ denotes a Dirac-state in the negative continuum, while ‘$\uparrow$’ denotes one in the positive continuum). After the change of the vacuum structure this wavefunction goes over into

$$\psi_{\uparrow}^{(In)} \rightarrow \varphi_{p\uparrow}^{(+)} = \alpha_p \varphi_{p\downarrow}^{(b)} - \beta_p \varphi_{p\uparrow}^{(b)}$$

where states with the label (b) denote the outgoing (bare) wavefunctions. The number of produced quarks is obtained by
The projection squared on the appropriate outgoing states defining the asymptotic particles and takes here the simple form

\[ N_p^{(Out)} = \langle 0 \text{In} | \hat{b}^{(Out)}_p \hat{b}^{(Out)}_p | 0 \text{In} \rangle \\
= \beta^*_p \beta_p \langle 0 \text{In} | \hat{d}^{(In)}_p \hat{d}^{(In)}_p | 0 \text{In} \rangle = | \beta_p |^2 \\
= \frac{1}{2} \left( 1 - \frac{m_b m_c + p^2}{E_p^{(b)} E_p^{(c)}} \right). \tag{2} \]

The number of produced quarks is depicted in fig. 1. It is found that the spectrum resembles the distribution of a saturated hot quark-antiquark plasma with a temperature between 200 and 300 MeV, although the distribution does not drop off exponentially, but as a power law for large momenta, \( N_p \approx (m_c - m_b)^2 / (4p^2) \). The total number of produced quarks is hence linearly divergent and should be regulated by a typical momentum cutoff \( \Lambda_C \) as in the NJL model which corresponds to a separation between the low and high momentum degrees of freedom. For \( \Lambda_C \approx 1 \text{ GeV} \) the total produced quark density is \( n_q \sim 1.5 - 2 \text{ fm}^{-3} \) which again is similar to the quark densities in a plasma for temperatures between 200 and 300 MeV. In addition we have also shown in fig. 1 the situation for a bare quark mass of 150 MeV \( (m_c = 550 \text{ MeV}) \), a typical value for the strange quark. Although the production is suppressed for small momenta, the total integrated quark density is only minorly affected \( (n_s \sim 0.65 - 0.8 \text{ fm}^{-3}) \).

The mechanism for this multiparticle production can be understood as follows: In the current quark picture, what happens is that the ‘hadronic’ vacuum state \( | 0 \text{In} \rangle \) corresponds to a complex coherent superposition of current quark states which rapidly go out of phase. The time \( \Delta t \) needed for the vacuum to restore chiral symmetry is expected to be proportional to the inverse of the typical nonpertubative QCD energy scale or the chiral symmetry breaking scale \( \Lambda_{QCD}^{-1} \sim \Lambda_C^{-1} \sim 0.2 \text{ fm/c} \). A similar estimate is based on the kinetic equilibration time of the gluons at collider energies predicted to be of the order \( 0.3 \text{ fm/c} \). A small but finite duration \( \Delta t \) suppresses the pair production at sufficiently high momenta, because the wavefunction of the quark resolves the smoothness of the transition. To quantify this we substitute for the time dependent mass \( m(x) \).
\[ m^*(t) = \frac{m_e + m_b}{2} - \frac{m_e - m_b}{2} \operatorname{sgn}(t) \left( 1 - e^{-2|t|/\tau} \right) \]  

(3)

and propose a nontrivial ansatz for the wavefunction

\[ \psi_p(x, t) = \frac{1}{\sqrt{V}} \begin{pmatrix} e^{i\alpha(t)} \cos \varphi(t) U \\ e^{-i\alpha(t)} \sin \varphi(t) \frac{\hat{x} \cdot \vec{p}}{p} U \end{pmatrix} e^{-\bar{\hbar} \left( \varepsilon(t) - \vec{p} \cdot \vec{x} \right)}, \]  

(4)

where \( \alpha, \varphi \) and \( \varepsilon \) are real functions in time. Inserting the ansatz into the time dependent Dirac equation yields the following coupled set of differential equations

\[ \begin{align*}
\dot{\varepsilon} &= \frac{1}{2} p \cos(2\alpha) \frac{1}{\sin \varphi \cos \varphi} \\
\bar{\hbar} \dot{\alpha} &= -m^*(t) + p \cos(2\alpha) \left( \frac{1}{2 \sin \varphi \cos \varphi} - \frac{\sin \varphi}{\cos \varphi} \right) \\
\bar{\hbar} \dot{\varphi} &= p \sin(2\alpha).
\end{align*} \]  

(5)

With the appropriate initial condition for the state in the negative continuum these equations can be integrated numerically and then projected on the outgoing particle state. In fig. 2 the spectrum of particles produced in a ‘smooth’ transition are depicted for various choices of \( \tau \). The total duration of the transition is approximately \( \Delta t \approx 1.5 \tau \). As expected, the particle number at larger momenta depends sensitively on the choice of \( \tau \). However, for \( \tau < 0.2 \text{ fm/c} \) (and thus \( \Delta t < 0.3 \text{ fm/c} \)) only the high momentum yield is affected. We conclude that if for dynamical reasons, as estimated above, the transition from the broken to the unbroken chiral phase occurs rapidly enough, the change of the underlying vacuum structure is accompanied by a substantial nonperturbative production of quarks and antiquarks.

The additional possibility of gluon decay suggests an interesting field theoretical problem and can be phrased as follows: What is the number of quarks (and antiquarks) produced in a first order decay of already *present* gluons propagating in an external time dependent scalar background field? We assume now that gluons interact perturbatively with the quarks, although they are in part responsible for the dynamic quark mass. The formulation we have carried out by using real-time Green functions \([12,13]\) is appropriate for non-equilibrium studies. The number of produced quarks is contained in the ‘\(<\)’-component of the complete
one-particle Green function and is given by the projection on the outgoing state in the distant future

\[ N_p^{(Out)} = \langle \Omega^{(in)} | \hat{U}(-\infty, \infty) \hat{\mathcal{J}}_p^{(Out)} \hat{U}(\infty, -\infty) | \Omega^{(in)} \rangle \]

\[ = \lim_{t_1 = t_2 \to \infty} \int d^3x_1 d^3x_2 \left( (-i) \varphi_{p\uparrow}^{(b)}(1) G^<_{p}(1, 2) (\gamma_0 \varphi_{p\uparrow}^{(b)}(2)) \right). \quad (6) \]

For our purpose we have to specify the fermionic initial conditions. For this the zeroth order Green function \( G_0 \) is determined by the full set of wavefunctions defined in respect to the incoming vacuum state and evolving nonperturbatively in time according to the time dependent scalar background Hamiltonian \( \mathcal{H} \), i.e.

\[ G_0^>(1, 2) = +i \sum_{p_1} \varphi_{p\downarrow}^{(+)}(1) \bar{\varphi}_{p\downarrow}^{(+)}(2) \]

\[ G_0^<(1, 2) = -i \sum_{p_1} \varphi_{p\uparrow}^{(+)}(1) \bar{\varphi}_{p\uparrow}^{(+)}(2). \quad (7) \]

A perturbative expansion is diagrammatically straightforward. The first order decay is contained in the lowest order self-energy insertion \( \Sigma \) for the quarks, where the average distribution \( \langle n_k \rangle \) of the real gluons enters explicitly. The number of produced particles can now be calculated \[16\]. By using some general properties of the self energy and the Green functions, the change in the quark occupation number induced by the self-energy insertion reads

\[ \Delta N_p^{(Out)} = 2 \cdot \lim_{t_1 = t_2 \to \infty} \int d^3x_1 d^3x_2 \int d^4x_3 d^4x_4 \mathbb{R} \left\{ (-i) \varphi_{p\uparrow}^{(b)}(1) G_{0}^{ret}(1, 3) \theta(t_3 - t_4) \right\} \]

\[ \left[ \Sigma^>(3, 4) G_{0}^<(4, 2) - \Sigma^<(3, 4) G_{0}^>(4, 2) \right] \left( \gamma_0 \varphi_{p\uparrow}^{(b)}(2) \right) \). \quad (8) \]

The expression (8) for the amount of produced (or scattered) particles resembles the typical form used in kinetic transport theories \[12,13\]; in particular the two terms in the inner bracket suggest a ‘Gain’ and ‘Loss’ contribution. Two reasons, however, make further analysis cumbersome. The first, purely technical one lies in the fact that the Green functions depend on both time arguments separately due to the time dependent external field. The second one stems from the occurrence of a number of infrared singularities. Besides the pair production the above expression also incorporates the absorption or emission of gluons.
for the quarks produced in lowest order in the vacuum decay (2). A separation of the pair production from these processes is not possible because of quantum mechanical interference. In the end one has to show that these singularities cancel each other, as they indeed do [14]. Finally, (8) can be split into three contributions [14],

$$\Delta N_p^{\text{(Out)}} = \Delta N_p^{\text{Gain}'} + \Delta N_p^{\text{Loss}'} + \Delta N_p^{\text{Masshift}'}$$, 

(9)

where the ‘Gain’ and the ‘Loss’ term both are not simply the squared of some particular amplitude. However, as a numerical investigation shows, the first is mostly positive while the second is negative. The interpretation stems from the fact that for example the ‘Gain’ part contains quarks scattered into the momentum state $p$ either by emission or absorption of a gluon as well as the decay of a gluon into a quark and antiquark. The exotic process of the simultaneous creation of a quark-antiquark pair and a gluon also contributes. The ‘Loss’ term contains the opposite processes. The cancellation of the infrared singularities among both terms is easily understood within this interpretation. The designation of a ‘masshift’ part for the last contribution is based on the known fact that in an equilibrated plasma the quarks become dressed and dynamically massive by the interaction with the thermal gluons [15]. Such a masshift should enter as a correction to the zeroth order vacuum decay (2) [14].

An ultraviolet divergence arising from interactions with the perturbative gluonic vacuum is interpreted as mass renormalization. The vacuum terms can explicitly be separated and should be regarded as the first order vacuum correction to the spontaneous decay. However, we are mainly interested in the overall effect due to the presence of real gluons. In the following numerical study these vacuum contributions are therefore discarded.

The distribution of gluons, at the time where the restoration of chiral symmetry takes place, is probably far from local thermal or kinetic equilibrium. In particular high energetic gluons parallel to the beam axis may be present. However, for our present purpose it is convenient and sufficient to parametrize the incoming gluons by a thermal distribution with a large temperature (in the range of 700 to 1000 MeV) which includes such high momenta. The results for a chosen temperature of $T_g = 700$ MeV are shown in fig. 3. The QCD
coupling constant $\alpha_s = g^2/(4\pi)$ is taken to be 0.3. For low momenta the quark distribution is large and positive and exceeds unity by three orders of magnitude. It then quickly turns and remains negative up to momenta of 400 MeV. At larger momenta the distribution resembles a not fully saturated thermal quark distribution of nearly the same temperature as the gluons. The distribution, however, drops faster at still higher momenta which are not shown here. The low momentum behaviour is obviously unphysical and shows that perturbation theory is not applicable here. If we integrate the calculated distribution over momentum space, it turns out that the behaviour at low momentum is irrelevant. The major amount of particles is created at larger momenta. An integration up to still only moderate momenta $p \simeq 1200$ MeV explicitly demonstrates that about $12 \times 0.36 = 4.3$ quarks per fm$^3$ are produced by the initiated decay of gluons (integrating up to momenta $\simeq 2500$ MeV the number changes to $12 \times 0.84 = 10.1$ quarks per fm$^3$)! Further numerical investigation shows that the importance of this effect increases with a higher temperature employed for the gluons. It turns out that the nearly collinear gluons with high momenta are responsible for the decay [14]. This suggests that such a phenomenon should be of particular importance in the early stages of the gluon evolution when the chiral symmetry restoration takes place. The effect, however, is small if we take a bare mass $m_b$ of 150 MeV. The distribution is shifted towards higher momenta. Integrating up to momenta $\simeq 2500$ MeV the number of produced quarks is evaluated as 0.34 fm$^{-3}$. Consequently, compared to the light quarks, the number of strange quarks produced by gluon decay would be strongly suppressed.

To summarize, the occurrence of a rapid chiral restoration of the vacuum can lead to some particular new and interesting phenomena in very energetic relativistic heavy ion collisions. In this letter we investigated the pair production of quarks within a simple model illustrating such a dynamical transition. The presence of real gluons assist the overall production: During the change of the vacuum structure the direct decay of a gluon into a quark and antiquark pair is kinematically allowed. According to our finding indeed a significant amount of quarks are produced during this process. Such a phenomenon could give rise to a nonperturbative dynamical creation of quarks and antiquarks (as well as
entropy) in the very early stages of the heavy ion reaction. It is tempting to speculate that the total number of produced quarks during such a transition and during the further evolution suffices so that not only the gluonic degrees of freedom are saturated, but also the fermionic. The ‘hot glue’ scenario could turn out to be a hot ‘quark gluon plasma’ after all.

ACKNOWLEDGMENTS

The author wants to thank the Alexander von Humboldt Stiftung for its support with a Feodor Lynen scholarship. This work was also supported in part by the U.S. Department of Energy (grant DE-FG05-90ER40592). The author is thankful for the various and stimulating discussions with Prof. B. Müller. Discussions with A. Schäfer and S. Mrowczynski are appreciated.
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FIGURES

FIG. 1. The spontaneous production of quarks is shown for $m_c = 350$ MeV, $m_b \simeq 0$ MeV (a) and for $m_c = 550$ MeV, $m_b = 150$ MeV (b). To guide the eye a thermal distribution of massless quarks is also drawn (for a temperature of $T = 200$ MeV (c) and $T = 300$ MeV (d)).

FIG. 2. The production of quarks is shown for a ‘smooth’ transition in time of the vacuum (compare text). The parameters given in the figure are to be read from the left to the right. $m_c = 350$ MeV and $m_b$ is taken to be nearly zero.

FIG. 3. The first order corrections on the particle distribution due to the presence of real gluons are depicted. The gluons are taken to be distributed with a temperature of 700 MeV. (a) corresponds to the parameter set $m_c = 350$ MeV, $m_b = 10$ MeV, (b) to the same as in fig. 1. For a comparison a thermal distribution of massless quarks with a temperature of 700 MeV is also drawn (c).
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