A carrier frequency estimation algorithm for Multi-h CPM signals

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Abstract. Considering the problem that Multi-h CPM signal transmission performance is excellent but carrier frequency synchronization is complex, the article proposed a multi-h CPM non-data assisted carrier frequency estimation algorithm based on maximum likelihood. Firstly, the importance of carrier frequency synchronization is explained by simulating the influence of carrier frequency offset on transmission error performance. Then, the carrier phase deviation, frequency deviation, propagation delay, modulation index error and symbol error joint likelihood function in signal transmission are given. The likelihood function is derived and the maximum likelihood carrier frequency estimation is derived in detail. Then the flow chart of the carrier frequency synchronization algorithm based on maximum likelihood is designed. Finally, the ARTM Tier2 signal is simulated by Matlab simulation. The results show that the proposed algorithm is suitable for Multi-h CPM signals and has good carrier frequency estimation performance, and is insensitive to symbol timing deviation and modulation index synchronization error.

1. Introduction
Continuous Phase Modulation (CPM) is a general term for a series of phase-continuous and strictly constant modulation schemes. It is widely used in the field of communication due to its high frequency band utilization and small influence on channel nonlinearity\(^1\)[2]. Unlike the traditional Single-h CPM, the Multi-h CPM has \(N_h\) \((N_h \geq 2)\) modulation index that varies cyclically with time and remains constant for each symbol interval. The cyclic effect of the multi-modulation index greatly increases the difficulty of multi-h CPM signal carrier frequency estimation while making the Multi-h CPM have better transmission performance \(^3\).

At present, there are few algorithms for multi-h CPM signal carrier frequency offset estimation, and mainly focus on data assisted methods. That is to say, by inserting a preamble symbol as a pilot signal at a determined position, to assist in generating a coherent carrier frequency. This type of method is widely used in satellite communications\(^4\). Although the auxiliary pilot structure can better perform carrier synchronization of Multi-h CPM, the introduction of preamble symbols occupies additional signal bandwidth and affects bandwidth efficiency. In addition, the FFT frequency discrimination method is a common carrier frequency estimation method for CPM. This method can also be applied to Multi-h CPM signals. However, this algorithm has limited accuracy in suppressing the carrier signal frequency \(^5\). Andrea proposed a non-data-assisted carrier frequency estimation algorithm with good synchronization effect, but the method is not suitable for Multi-h CPM signal\(^6\). The paper proposes a carrier frequency estimation algorithm for Multi-h CPM signals.
2. Multi-\( h \) CPM signal model

The complex baseband expression for the Multi-\( h \) CPM signal is:

\[
s(t, \alpha) = \sqrt{\frac{2E_s}{T}} \exp\{j\phi(t, \alpha)\}
\]

(1)

Where, \( E_s \) is the symbol energy, \( T \) is the symbol interval, and \( \phi(t, \alpha) \) is the phase function of the bearer information, and its expression is:

\[
\phi(t, \alpha) = 2\pi \sum_{i=-\infty}^{\infty} h_i \alpha_i q(t - iT)
\]

(2)

Where, \( \alpha = (\alpha_0, \alpha_1, \ldots, \alpha_N) \) is the sequence of M-ary information symbols at the transmitting end, \( h \) is the modulation index, \( N \) modulation indices \( \{h_1, h_2, \ldots, h_N\} \) are cyclically changed symbol by symbol; \( q(t) \) is the phase pulse function, the derivative of which is the frequency pulse function \( g(t) \), and \( g(t) \) is smooth in \( 0 \leq t \leq LT \) pulse shape, parameter \( L \) is the associated length.

3. Influence of carrier frequency offset on bit error rate performance

In a coherent demodulation system, the carrier synchronization module is an indispensable structure for coherent demodulation. The role of the carrier synchronization module is to make the local oscillation and the received signal in phase with each other in order to facilitate the accurate detection of the back-end sequence. Therefore, the quality of the carrier synchronization module directly has a great impact on the demodulation results. Carrier synchronization is divided into carrier phase synchronization and carrier frequency synchronization. This section mainly simulates the ARTM Tier 2 signal, mainly analyzes the influence of carrier frequency offset on the error performance of the demodulation system.

The simulation assumes that other demodulation parameters have been ideally synchronized, and the emulation channel is AWGN. Figure 1 shows the bit error rate performance curves for different normalized carrier frequency offsets. It can be seen from the figure that when the carrier frequency offset is \( fT = 10^{-4} \), the error performance has a loss of 2.8 dB compared with the ideal synchronization. As the carrier frequency offset further increases, the loss of error performance is more serious. When the carrier frequency offset is \( fT = 10^{-2} \), the receiver has completely lost its effect. It can be seen that the carrier frequency offset has a great influence on the demodulation error performance. It is necessary to eliminate the influence of the carrier frequency offset on the receiver performance through synchronization processing.

![Figure 1 Influence of carrier frequency offset on error performance of ARTM tier2 system](image)

4. Carrier frequency estimation algorithm

In the actual signal transmission process, the signal from the transmitting end will be affected by the complex transmission environment, causing delay, frequency offset, phase deviation and other
deviations. Therefore, it is necessary to re-establish the signal model of the Multi-h CPM received signal to accurately estimate the unknown parameters. Taking the AWGN channel as an example, we can express the signal model of the received Multi-h CPM signal after down-conversion processing as:

\[
r(t) = \sqrt{\frac{E}{T}} \exp \left\{ i \theta + 2\pi ft + 2\pi \sum_{j} \alpha_{j} h_{j} q(t - iT - \tau - \xi T) \right\} + n(t)\]

(3)

Where, \( \alpha \) is a sequence of symbols, \( \theta \) is the carrier phase deviation, and \( \theta = \theta_{0} + 2\pi f \tau \). \( \theta_{0} \) is the carrier initial phase, \( \tau \) is the timing error caused by the propagation delay, \( f \) is the carrier frequency offset of the received signal, \( \xi \) is the modulation index error of the received signal, and \( n(t) \) is the additive white Gaussian noise. It is worth mentioning that the modulation index error \( \xi \) is a specific error parameter to Multi-h CPM signal transmission, which is the reason that the start and stop timings of the multiple modulation exponential cycles of the received signal deviate from the transmitted signal. Therefore, the range of the modulation index synchronization error is related to the number of modulation indices \( N_{h} \). That is \( 0 \leq \xi < N_{h} \). It can be seen from equation (3) that the quality of the received signal synchronization system is mainly related to the accurate estimation of the five unknown parameters of carrier phase deviation \( \theta \), carrier frequency deviation \( f \), propagation delay \( \tau \), modulation index deviation \( \xi \) and decision symbol \( \alpha \).

According to the maximum likelihood principle, it is assumed that the five parameters \( \theta, f, \tau, \xi \) and \( \alpha \) in equation (3) are unknown and independent of each other, then in the \( 0 \leq t \leq L_{t} T \) time interval (\( T \) is the unit-matching interval, \( L_{t} \) is the observation symbol interval), regarding these five parameters for the joint likelihood function of the estimated parameters can be expressed as:

\[
\Lambda(r | \hat{\alpha}, \hat{\theta}, \hat{\tau}, \hat{\xi}, \hat{f}) = \exp \left\{ \frac{1}{N_{0}} \sqrt{\frac{E}{T}} \text{Re} \left[ e^{i(\hat{\theta} - 2\hat{\pi} f \hat{\tau})} \int_{0}^{L_{t} T} r(t) e^{-j2\pi \sum_{j} \alpha_{j} h_{j} q(t - iT - \xi T)} dt \right] \right\}
\]

(4)

In order to obtain the carrier frequency offset estimation value \( \hat{f} \) by the maximum likelihood method, it is necessary to obtain the edge likelihood function \( \Lambda(r | \hat{f}) \) about \( \hat{f} \) by calculating the expectation of the four estimated parameters of \( \hat{\theta}, \hat{\alpha}, \hat{\tau} \) and \( \hat{\xi} \) respectively according to the corresponding probability distribution; Then, the partial derivative of the carrier frequency offset estimation \( \hat{f} \) is obtained for \( \Lambda(r | \hat{f}) \), so that the derivative function value is zero, that is, the maximum likelihood frequency offset estimation value. Next, the detailed derivation process is introduced.

1) Firstly, we should obtain the expectation of carrier phase estimation \( \hat{\theta} \). In order to facilitate the calculation, the item containing \( \hat{\theta} \) is separated, so

\[
X = \int_{0}^{L_{t} T} r(t) e^{-j2\hat{\pi} f \hat{\tau}} e^{-j2\pi \sum_{j} \hat{\alpha}_{j} h_{j} q(t - iT - \xi T)} dt
\]

(5)

The complex variable \( X \) can be simply expressed as:

\[
X = |X| e^{j\phi_{X}}
\]

(6)

Where, \( |X| \) and \( \phi_{X} \) are the magnitude and phase angle of \( X \). Substituting equation (5) into (4) can rewrite the joint likelihood distribution to:

\[
\Lambda(r | \hat{\alpha}, \hat{\theta}, \hat{\tau}, \hat{\xi}, \hat{f}) = \exp \left\{ \frac{1}{N_{0}} \sqrt{\frac{E}{T}} |X| \cos(\phi_{X} - \hat{\theta}) \right\}
\]

(7)

Carrier phase estimation \( \hat{\theta} \) can be considered to be uniformly distributed in the interval \([0, 2\pi]\). With respect to equation (7), the expectation for \( \hat{\theta} \) is available:
\[ \Lambda(r \mid \tilde{a}, \tilde{\Theta}, \tilde{\tau}, \tilde{\xi}, \tilde{f}) = I_0 \left( \frac{1}{N_0} \sqrt{\frac{E_s}{T}} |X| \right) \]  

(8)

Where, \( I_0(x) = \frac{1}{2\pi} \int_0^\infty e^{ix\cos\theta} d\theta \), represents the zero-order modified first-class Bessel function. Taylor expansion of the Bessel function can be approximated as:

\[ I_0(x) \approx 1 + \frac{1}{4} x^2 \]  

(9)

Then equation (8) is equivalent to:

\[ \Lambda(r \mid \tilde{a}, \tilde{\Theta}, \tilde{\tau}, \tilde{\xi}, \tilde{f}) \approx |X|^2 \]  

(10)

Where \(|X|^2\) is represented by a discrete value as:

\[ |X|^2 = \sum_{k_j=0}^{N_{\Delta x}-1} \sum_{k_j=0}^{N_{\Delta x}-1} (k_i) (k_j) e^{-j2\pi(k_i-k_j)T^*} R_x \]  

(11)

Where \(*\) means conjugate, and the expression of \( R_x \) is:

\[ R_x = e^{[i\omega k T_\tau + i\omega T_\tau - \tilde{\xi} T_\tau + i\omega T_\tau \tilde{\xi}]} \]  

(12)

(2) Next, we should obtain the expectation of Symbol estimation \( \tilde{a} \). It can be seen that the \( R_x \) term contains the remaining three unknown parameters \( \tilde{a}, \tilde{\tau} \) and \( \tilde{\xi} \). For ease of presentation, we can first calculate the expectation of \( R_x \) versus \( \tilde{\alpha} \).

\[ E_{\hat{\alpha}}(R_x) = \prod_{i=0}^{+\infty} \frac{1}{M} \sum_{k_j=0}^{N_{\Delta x}-1} \frac{\sin \left( 2\pi h \sum_{i=0}^{N_{\Delta x}-1} (k_i) \right)}{\sin \left( 2\pi h \sum_{i=0}^{N_{\Delta x}-1} (k_i) \right)} \]  

(13)

Where, \( q(\Delta t, t) \) is related to the phase pulse function \( q(t) \) is \( q(\Delta t, t) = q(t) - q(t - \Delta t) \).

(3) Then, we should obtain the expectation of Symbol error estimation \( \tilde{\xi} \). Hypothesis \( \tilde{\tau} \) assumes a uniform distribution within interval \([-0.5T, 0.5T]\), then the mathematical expectation of equation (13) versus \( \tilde{\tau} \) can be expressed as:

\[ E_{\hat{\tau}}(E_{\hat{\alpha}}(R_x)) = \int_{-0.5T}^{+0.5T} \prod_{i=0}^{+\infty} \frac{1}{M} \sum_{k_j=0}^{N_{\Delta x}-1} \frac{\sin \left( 2\pi h \sum_{i=0}^{N_{\Delta x}-1} (k_i) \right)}{\sin \left( 2\pi h \sum_{i=0}^{N_{\Delta x}-1} (k_i) \right)} dt \]  

(14)

At this time, equation (14) is only related to the modulation index estimate \( \tilde{\xi} \), which is recorded \( H_{\tilde{\xi}} [(k_i - k_j)T_j] \). For ease of representation, we order

\[ F(\Delta t, t) = \prod_{i=0}^{+\infty} \frac{1}{M} \sum_{k_j=0}^{N_{\Delta x}-1} \frac{\sin \left( 2\pi h \sum_{i=0}^{N_{\Delta x}-1} (k_i) \right)}{\sin \left( 2\pi h \sum_{i=0}^{N_{\Delta x}-1} (k_i) \right)} \]  

(15)

Equation (14) can be expressed as:

\[ H_{\tilde{\xi}} [(k_i - k_j)T_j] = \frac{1}{T} \int_{-0.5T}^{0.5T} F(\Delta t, t)dt \]  

(16)

Substituting equation (15) and equation (16) into equation (10), the expression of the edge joint likelihood function of Multi-h CPM with respect to carrier frequency offset estimation \( \tilde{f} \) and modulation index estimation \( \tilde{\xi} \) is:

\[ \Lambda(r \mid \tilde{\xi}, \tilde{f}) \approx \sum_{k_i=0}^{N_{\Delta x}} \sum_{k_j=0}^{N_{\Delta x}} (k_i) (k_j) e^{-j2\pi(k_i-k_j)T^*} H_{\tilde{\xi}} [(k_i - k_j)T_j] \]  

(17)
Then, we should obtain the expectation of modulation index estimation $\bar{\xi}$. The modulation index of the Multi-h CPM signal is cyclically changed, so the modulation index error $\xi$ is a discrete equivalence distribution. Obtain the mathematical expectation of equation (17) to $\bar{\xi}$. Then the expression of the likelihood function of the Multi-h CPM signal with respect to the carrier frequency of $f_{st}$ estimate $f$ is:

$$
\Lambda(r | \bar{f}) = \sum_{k_0}^{M_0-1} \sum_{k_j=0}^{N_{T}-1} r(k_0) r^* (k_j) e^{-i 2\pi k_0 k_j T} H \left[ (k_2 - k_1) T \right]
$$

(18)

Where,

$$
H \left[ (k_2 - k_1) T \right] = \frac{1}{N_h} \sum_{\xi=0}^{N_h-1} H_\xi \left[ (k_2 - k_1) T \right]
$$

(19)

Finally, we should obtain the Partial derivative of Carrier frequency offset estimation,

$$
\frac{\partial \Lambda(r | \bar{f})}{\partial f} = -2\pi T \text{Im} \left\{ \sum_{k_0=0}^{M_0-1} \sum_{k_j=0}^{N_{T}-1} y(k_0) y^*(k_j) h(k_2 - k_1) \right\}
$$

(20)

$$
y(k) = r(k) e^{-i 2\pi f T}
$$

(21)

$$
h(k) = k H(k)
$$

(22)

It can be seen that there is only one term in equation (20) related to the Multi-h CPM signal. As we know, $h(k)$ is a non-causal filter. Therefore, we need to shift $h(k)$ to the right by ND sampling points ($D=4$ at this time), making it a physically achievable causal filter. Then equation (20) can be rewritten as:

$$
\frac{\partial \Lambda(r | \bar{f})}{\partial f} = -2\pi T \sum_{k=ND}^{(i\epsilon+D)N-1} \text{Im} \left\{ y \left[ (k - ND) \right] w^* (k) \right\}
$$

(23)

Where,

$$
w(k) = y(k) \otimes h(k-ND)
$$

(24)

And $\otimes$ indicates the real part. If $\frac{\partial \Lambda(r | \bar{f})}{\partial f} = 0$, the obtained $\bar{f}$ is the maximum likelihood frequency offset estimate. Therefore, equation (23) can be used as a carrier frequency offset error extractor (FED), i.e.

$$
e_f(i) = \sum_{k=ND}^{(i\epsilon+D)N-1} \text{Im} \left\{ y \left[ (k - ND) \right] w^* (k) \right\}
$$

(25)

Where, $i = \text{floor}(k/N)$, Carrier frequency offset error signal $e_f(i)$ is updated continuously with time. In order to track the carrier frequency offset, it can be iteratively estimated by using a first-order phase-locked loop, and its iterative update equation can be expressed as:

$$
\hat{f}(i+1) = \hat{f}(i) + \gamma_f e_f(i)
$$

(26)

Where, $\gamma_f = 4B_T k_d$ represents the iteration step factor, $B_T$ is the normalized equivalent loop noise bandwidth, $k_d$ is the slope of the frequency S curve, indicating the frequency gain.
Figure 2 shows a flow chart of a carrier frequency synchronization algorithm based on maximum likelihood. It can be seen that the discretized signal \( r(k) \) obtained by the down-converted processing of the received signal is processed by two branches respectively. One of the branches is processed by a filter with an impulse response of \( h(k-ND) \), and is conjugated to obtain \( w^*(k) \); The other branch obtains the signal \( y(k-ND) \) by delaying the ND sampling period. The outputs \( w^*(k) \) \( y(k-ND) \) of the two branches are sent to the discriminator to extract the carrier frequency offset error \( f_{ei} \). After the carrier frequency offset error value \( e_{f_{ei}}(i) \) is suppressed by the loop filter, the carrier frequency offset estimation value \( \tilde{f}(i) \) is obtained through iteration, and the carrier frequency offset estimation \( \tilde{f}(i) \) controls the voltage controlled oscillator VCO to generate the carrier instantaneous phase offset \( e^{-j\omega(k)} \), and the discretized received signal \( r(k) \) multiply, and finally achieve carrier frequency synchronization.

5. Simulation analysis and verification

There is always a certain deviation between the parameter estimation value and the real data, and this deviation cannot be completely eliminated. Therefore, it is usually used to evaluate the quality of the unbiased estimation method using the Cramer Barley Circle (CRLB). However, for the derivation of the synchronization parameter estimation of Multi-h CPM signal, it is difficult to obtain CRLB. Therefore, it is usually more convenient to modify the Cramer Mercury (MCRB) as the most evaluation standard. It can be expressed as:

\[
T^2 \times MCRB(f) = \frac{3}{2\pi^2 L_0} \times \frac{1}{E_i/N_0}
\]

(27)

In order to verify the performance of the maximum likelihood carrier frequency offset estimation, we simulate the ARTM Tier 2 signal by Matlab software.

Figure 3 shows the normalized variance performance curve. It can be seen from Figure 5 that the estimated variance based on the maximum likelihood carrier frequency offset estimation method becomes smaller as the symbol signal to noise ratio increases. However, as the symbol-to-noise ratio increases, the estimation performance of the proposed method will be flattened by the variance error. This phenomenon is mainly due to the characteristics of the Multi-h CPM signal and the maximum likelihood synchronization algorithm, which is equivalent to self-generated crosstalk between codes. In the middle-high signal-to-noise ratio, the external noise interference is small, and its estimation performance is mainly affected by the signal and the inter-code crosstalk of the algorithm itself, so that the estimated variance curve is basically stable at the magnitude \(10^{-3}\). It can be seen that the carrier frequency offset estimation algorithm based on maximum likelihood has better estimation effect and can basically meet the actual carrier frequency offset estimation requirements.
In actual signal reception, carrier frequency synchronization is only a part of the receiver synchronization module, and other synchronization modules may have an impact on the carrier frequency offset estimation performance. In order to study the influence of symbol timing deviation and modulation index deviation on the maximum likelihood carrier frequency offset estimation, Fig. 4 shows the normalized frequency error variance curve of the ARTM Tier2 signal with different symbol timing deviation and modulation index deviation. It can be seen from Fig. 6 that when the symbol timing deviation is $\tau = 0.5T$ and the modulation index deviation is $\xi = 1$, the performance of the carrier frequency offset estimation does not deteriorate. Therefore, it can be explained that the maximum likelihood-based carrier frequency offset estimation method is not sensitive to the influence of timing deviation and modulation index deviation. In practical applications, we can make the maximum likelihood carrier frequency offset synchronization module before the symbol timing synchronization algorithm, thereby avoiding the carrier frequency offset in the received data from having a large impact on the symbol timing synchronization module.

6. Conclusion
Considering the difficulty of multi-h CPM signal carrier frequency synchronization, based on the single-modulation exponential CPM carrier frequency synchronization, the likelihood function of the carrier frequency estimation of Multi-h CPM signal is derived in detail, and the maximum likelihood estimation of carrier frequency is obtained. The carrier frequency synchronization implementation structure is designed. The simulation results show that the proposed method has good synchronization performance and is insensitive to symbol timing deviation and modulation index synchronization deviation, which can basically meet the actual carrier frequency offset estimation requirements.

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