A model of large quintessence isocurvature fluctuations and low CMB quadrupole

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Abstract. Based on quintessence models with anticorrelation between the quintessence fluctuations and adiabatic perturbations in the other matter components, we show that if the quintessence potential is sufficiently steep in the initial era, i.e., at the beginning of radiation dominated epoch, the quintessence fluctuations can be large enough to sufficiently suppress the CMB power spectrum at low multipoles without excess gravitational waves. The leaping kinetic term quintessence and the crossover quintessence models, can realize such steep potential at the initial stage, and therefore can give rise to a low CMB quadrupole as required by observations.

Keywords: Dark Energy, CMBR, Cosmological perturbation theory.
1. Introduction

The power spectrum of CMB temperature fluctuations, measured by COBE [1] and WMAP [2], has a smaller amplitude than expected theoretically on large angular scales. A small amplitude of the CMB fluctuations on large scales may be a consequence of the data analyses and limits of observation [3, 4, 5]. However, this may also be due to new physics for suppressing the CMB power spectrum at low multipoles. Many models of suppressing the CMB power spectrum at low multipoles have been proposed [6, 7, 8, 9, 10]. One possible model is based on the modification of the integrated Sachs-Wolfe (ISW) effect by dark energy perturbations [11, 12]. Dark energy perturbations can suppress the CMB power spectrum at low multipoles if their amplitude is large enough to modify the evolution of metric perturbations during dark energy domination. Moreover, they must have anticorrelation with the curvature perturbations, otherwise they enhance the low multipoles of the CMB spectrum [13].

In this work, we concentrate on the scalar field model of dark energy, called quintessence [14, 15, 16]. We show that if the potential of the quintessence field is steep enough at the initial stage, i.e., at the beginning of radiation dominated era, the amplitude of the quintessence fluctuations can be large enough to enhance the ISW contribution. As a result, the amplitude of the CMB power spectrum at low multipoles can be suppressed. We also show that the leaping kinetic term quintessence and crossover quintessence models can realize such steep potential at initial stage.

2. The enhancement of ISW contribution by the quintessence fluctuations

Since the quintessence fluctuations are damped inside the horizon, we concentrate on the large scales fluctuations. On large scales, the transfer functions of the CMB temperature fluctuations can be written in conformal Newtonian gauge as [12, 17]

\[ T_\ell(k, \eta_0) = \frac{\sqrt{2\ell(\ell+1)}}{\zeta_i} \left[ (\Theta_s + \Phi_s) j_\ell(kD_s) + \int_{\eta_s}^{\eta_0} d\eta (\Psi' - \Psi') j_\ell(kD) \right] , \]

\[ \approx - \frac{\sqrt{2\ell(\ell+1)}}{\zeta_i} \left[ \frac{1}{3} \Psi_s j_\ell(kD_s) + 2 \int_{\eta_s}^{\eta_0} d\eta \Psi' j_\ell(kD) \right] , \]  

(1)

where a prime denotes a derivative with respect to the conformal time \( \eta \), \( \Theta \) is the temperature monopole, \( \Phi \) is the gravitational potential, \( \Psi \) is the curvature fluctuation, \( j_\ell \) is the spherical Bessel function, \( \zeta_i \) is the initial value of the curvature perturbation on comoving hypersurfaces, \( D = \eta_0 - \eta \) and the subscripts \( * \) and 0 denote evaluation at recombination and present respectively. The angular power spectrum of the CMB can be expressed in terms of the transfer function as

\[ \frac{\ell(\ell+1)C_\ell}{2\pi} = \int \frac{dk}{k} T^2_\ell(k, \eta_0) P_{\zeta_i}(k), \]  

(2)

here \( P_{\zeta_i}(k) \) is the power spectrum of \( \zeta_i \), defined as

\[ \langle \zeta_i(k) \zeta_i(k') \rangle = (2\pi)^3 \delta(k - k') \frac{2\pi^2}{k^3} P_{\zeta_i}(k). \]  

(3)
The first term on the right hand side of eq. (1) corresponds to the gravitational redshift effects due to the photon’s climb out of the potential well at last scattering. This is the ordinary Sachs Wolfe (SW) effect. The second term describes the fluctuations induced by the passage of CMB photons through the time evolving gravitational potential. This is the Integrated Sachs Wolfe (ISW) effect. To obtain the second line, we assume that the density fluctuations are adiabatic before quintessence domination, and that $\Phi = -\Psi$, i.e. anisotropic stress is negligible.

It follows from eq. (1) that the ISW contribution can cancel the ordinary Sachs Wolfe contribution if after last scattering $\Psi'$ and $\Psi$ have opposite signs, i.e. the amplitude of $\Psi$, $|\Psi|$, decreases with time. Because of the expansion of the universe, the amplitude of $\Psi$ decreases with time, leading to a partial cancellation of the ISW effects and the ordinary Sachs Wolfe effect. The contributions from the density perturbations can change the decay rate of $|\Psi|$, i.e. the ISW contribution. For quintessence, the quintessence fluctuations can increase the decay rate of $|\Psi|$, leading to a small amplitude of $\Theta_\ell$ and also $c_\ell$ at low multipoles. The quintessence fluctuations can influence the evolution of $\Psi$ when quintessence contributes a significant fraction of the total energy density. We assume that about the end of matter domination radiation can be neglected, so that the perturbed Einstein equation can be written as

$$\Psi' + \mathcal{H} \Psi + \frac{k^2}{3 \mathcal{H}} \Psi = \frac{1}{2} \mathcal{H} (\Omega_m \delta_m + \Omega_Q \delta_Q),$$

(4)

where the subscripts $m$ and $Q$ denote matter and quintessence, $\Omega$ is the density parameter, $\delta$ is the density contrast, $\mathcal{H} = a'/a$, and $a$ is the scale factor. The above equation shows that the quintessence fluctuations can affect the evolution of $\Psi$ if $|\delta_Q|$ is large enough. The decay rate of $|\Psi|$ increases when $\delta_Q$ and $\Psi$ have opposite signs, and decreases when $\delta_Q$ and $\Psi$ have the same signs, Hence, the quintessence fluctuations can lead to the suppression of the CMB power spectrum at low multipoles if $|\delta_Q|$ is large enough and $\delta_Q$ and $\Psi$ have opposite signs, i.e. they are anticorrelated.

3. A model of large quintessence fluctuations

If the quintessence field $Q$ is light during inflation, it is nearly frozen and acquires quantum fluctuations [19]

$$\delta Q(k)_{\text{inf}} = \frac{H_e}{\sqrt{2k^3}},$$

(5)

where $H_e$ is the Hubble parameter evaluated at the time of horizon exit, the quintessence field fluctuations $\delta Q$ is in the conformal Newtonian gauge and $k$ denotes the wavenumber of the perturbation mode. When the quintessence field is nearly frozen, its density contrast $\delta_Q$ in conformal Newtonian gauge is given by

$$\delta_Q = \frac{\delta \rho_Q}{\rho_Q} \simeq \frac{1}{V(Q)} \frac{dV(Q)}{dQ} \delta Q,$$

(6)

where $\rho_Q$ is the energy density of quintessence and $V(Q)$ is the potential of the quintessence field. As long as the quintessence field is nearly frozen, its fluctuation
\( \delta Q \) is approximately constant. Thus if the quintessence field is nearly frozen until the beginning of the radiation dominated epoch, the initial value of \( \delta Q \) in the radiation dominated era is given by

\[
\delta Q_i \approx \frac{1}{V} \frac{dV}{dQ} \delta Q_{\text{inf}}.
\]  

(7)

The evolution of \( \delta Q \) during the radiation and matter domination depends on the evolution of the quintessence field. Here, we consider the evolution of \( \delta Q \) only on superhorizon scales because \( |\delta Q| \) rapidly decreases with time on subhorizon scales. The evolution of the quintessence field is characterized by the equation of state parameter \( w_Q \) which is the ratio of the pressure to the energy density. The quintessence fluctuations can be classified into adiabatic and isocurvature modes, according to the initial conditions. The adiabatic mode is the mode for which the entropy perturbations vanish, while the isocurvature modes correspond to the modes for which the curvature perturbations vanish or approximately vanish \[20\]. On large scales, the adiabatic mode is constant \[21, 22\] if \( w_Q \) is nearly constant. Nevertheless, its amplitude is limited by the adiabatic conditions, so that \( |\delta Q| \) cannot be adjusted to suppress the CMB power spectrum at low multipoles. For the isocurvature modes, their evolution depend on the value of \( w_Q \). These perturbations modes decrease rapidly on large scales when \( |w_Q| \) is approximately constant and smaller than 1. In the context of tracking quintessence, this corresponds to the case when the quintessence field is in the tracking regime \[23, 24\].

To avoid the coincident problem, the tracking quintessence is in the tracking regime at present. The expansion of the universe is accelerating today because \( w_Q \) in the tracking regime is smaller than \(-1/3\). Before the quintessence field starts the tracking regime, it is nearly frozen because \( w_Q \) is close to \(-1\). The transition from the potential regime to tracking regime occurs between the radiation dominated and quintessence dominated epochs. The redshift at which the transition takes place depends on the initial conditions of the quintessence field. Since \( w_Q \approx -1 \) when the potential energy of quintessence dominates the kinetic energy, the regime in which the quintessence field is frozen is called the potential regime. During this regime, the isocurvature modes are approximately constant \[21, 12\]. In general, the quintessence field can be frozen if the Hubble drag term \( 3H \dot{Q} \) in its evolution equation dominates the potential term \( dV/dQ \). Here, a dot denotes the time derivative and \( H \) is the Hubble parameter. In the case of tracking quintessence, there may exist a short period of kinetic regime between inflation and potential regime. In this regime, the kinetic energy of the quintessence field dominates the potential energy. The existence of this regime depends on the choice of the initial conditions for the quintessence field. The transition from the kinetic regime to potential regime occurs deep in the radiation dominated era. The isocurvature modes increase during this regime. Since the kinetic regime is short and \( |\delta Q| \) decreases during the transition from the kinetic regime to the potential regime \[21, 24\], the total growth of \( |\delta Q| \) is small.

We are interested in the suppression of CMB spectrum at low multipoles via the quintessence fluctuations, which can occur when \( |\delta Q| \) is large enough during quintessence
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domination. From the above summary, we see that the magnitude of $\delta Q$ can be large
during quintessence domination if its initial value is large and the quintessence field
is nearly frozen initially and remains frozen until quintessence domination. If the
quintessence field is nearly frozen until quintessence domination, the amplitude of $\delta Q$
during quintessence domination will be approximately equal to its initial value, given in
eq. (7). Thus, the amplitude of $\delta Q$ during quintessence domination can be large if the
amplitude of $\delta Q_{inf}$ or the initial value of $\frac{1}{V} \frac{dV}{dQ}$ is large. A large initial value of $\frac{1}{V} \frac{dV}{dQ}$
implies a large $\frac{1}{V} \frac{dV}{dQ}$ near the present epoch because $\frac{1}{V} \frac{dV}{dQ}$ is approximately constant
as long as the quintessence field is nearly frozen. The amplitude of $\delta Q_{inf}$ usually depends
on $H_e$, while the initial value of $\frac{1}{V} \frac{dV}{dQ}$ depends on the quintessence model. For simple
quintessence models, whose potential slope does not change much during quintessence
evolution, the ratio $\frac{1}{V} \frac{dV}{dQ}$ cannot be large initially because the quintessence potential
must be flat enough at present to drive an accelerated expansion of the universe today.
Hence, the CMB spectrum at low multipoles can be suppressed sufficiently if $\delta Q_{inf}$ is
large enough. In the simplest case, this requires $H_e$ to be larger than the observational
bound [12], i.e. there are excess gravitational waves. This excess gravitational waves
problem can be solved if the field fluctuation $\delta Q$ is amplified between the inflationary and
present epoch. This amplification can occur if the kinetic coefficient of the quintessence
field varies in time [25].

Alternatively, the problem of excess gravitational waves can be avoided if the
quintessence potential is steep enough initially. This is because the steep potential
can lead to a large $|\delta Q_i|$ although the amplitude of $\delta Q_{inf}$ is small. However, the
quintessence potential must be flat enough at present to drive an accelerated expansion
of the universe. We now consider whether the quintessence potential can be sufficiently
steep initially and flat enough to drive an accelerated expansion of the universe today.
Let us consider the simple exponential quintessence model, whose potential can be
written as $V(Q) = \bar{m}_p^4 \exp(-\lambda Q/\bar{m}_p)$, where $\bar{m}_p = (8\pi G)^{-1/2}$ is the reduced Planck
mass. Thus, $\frac{1}{V} \frac{dV}{dQ} = -\frac{\lambda}{\bar{m}_p}$, and therefore the magnitude of $\delta Q_i$ can be large if $\lambda$ is
large. In the early epoch, the quintessence field can be frozen although its potential is
steep, because quintessence is not a dominant component. Using the evolution equation
for the quintessence field, one can show that [23]

$$\left| \frac{1}{V} \frac{dV}{dQ} \right| = \frac{\lambda}{\bar{m}_p} = \frac{\sqrt{3}}{\bar{m}_p \sqrt{\Omega_Q} \sqrt{1 + w_Q}} \frac{1 + \frac{\ln x}{6 \ln a}}{1 + \frac{1}{6 \ln a}},$$

(8)

where $x = (1 + w_Q)/(1 - w_Q)$. The above equation shows that, although $\lambda$ is large, the
quintessence field can be frozen, i.e., $w_Q \approx -1$, if $\Omega_Q$ is sufficiently small. In order to
drive an accelerated expansion of the universe, the quintessence field has to be slowly
rolling at the present epoch. When the quintessence field is slowly rolling, its evolution
equation yields
\[ \frac{1}{3H^2} \frac{d^2V}{dQ^2} \simeq -\frac{\dot{H}}{H^2}. \]  
This is a slow-roll condition for the quintessence field. During the present epoch, quintessence is the dominant component, so that \( \dot{H} = -\dot{Q}^2/(2\bar{m}_p^2) \). Since \( V > \dot{Q}^2/2 \) and \( H^2 \simeq V/(3\bar{m}_p^2) \) when the quintessence field is slowly rolling, the slow-roll condition requires \( \lambda^2 < 1 \), i.e. the quintessence potential should not be too steep. Unfortunately, the amplitude of CMB quadrupole will be in agreement with the observation if \( \lambda \gg 1 \). However, if \( \lambda \) is a time-dependent parameter, the quintessence potential might be able to be sufficiently steep initially and flat enough at present. An exponential quintessence model with time dependent \( \lambda \) is conveniently described by a leaping kinetic term model \[26\]. The Lagrangian of this quintessence is
\[ \mathcal{L}(\chi) = \frac{1}{2} (\partial\chi)^2 \kappa^2(\chi) - \bar{m}_p^4 \exp[-\chi/\bar{m}_p]. \]  
Using the field variable \( Q = K(\chi) \), where \( \kappa(\chi) = \partial Q/\partial \chi \), the above Lagrangian becomes
\[ \mathcal{L}(Q) = \frac{1}{2} (\partial Q)^2 - \bar{m}_p^4 \exp[-K^{-1}(Q)/\bar{m}_p]. \]  
It is convenient to use the field \( Q \) to study the evolution of quintessence because its kinetic term has a canonical form. The evolution of this field depends on the form of \( \kappa(\chi) \). We use
\[ \kappa(\chi) = \kappa_{\text{min}} + \tanh \left( \frac{\beta}{\bar{m}_p} [\chi - \chi_1] \right) + 1, \]  
where \( \kappa_{\text{min}}, \chi_1 \) and \( \beta \) are constant. In the early epoch, \( \chi \ll \chi_1 \) so \( \kappa \simeq \kappa_{\text{min}} \). Therefore, we have
\[ \frac{1}{V} \frac{dV}{dQ} \simeq -\frac{\bar{m}_p \kappa_{\text{min}}}{1}. \]  
In our consideration, the field \( \chi \) and \( Q \) are nearly frozen during the initial stage. As the universe evolves, the Hubble drag term decreases due to decreasing Hubble parameter, while the potential term is approximately constant because \( Q \) and \( \chi \) are nearly frozen. When the Hubble drag term is small enough, the fields \( Q \) and \( \chi \) are able to roll down their potentials. Consequently, the field \( \chi \) increases and becomes larger than \( \chi_1 \) during matter domination. As a result, the kinetic coefficient \( \kappa \) increases and therefore \( w_Q \) decreases towards \( -1 \) again. The evolution of this quintessence field has a tracking behavior. The present value of \( w_Q \) depends on \( \beta \), while the present value of \( \Omega_Q \) depends on \( \chi_1 \) if \( \beta \) is fixed. The evolution of \( w_Q \) is shown in figure 1.

It is easy to see that \( |\delta_Q| \) is large when \( \kappa_{\text{min}} \) is small. At first sight, this looks similar to the existing literature because the magnitude of the quintessence fluctuations is enhanced by a kinetic coefficient \[25\]. Nevertheless, the physical motivation is different. In the literature, the quintessence field fluctuation is amplified between the inflationary and present epoch by a varying kinetic coefficient. In our analysis, the exponential quintessence can give rise to a large \( |\delta_Q| \) if \( \lambda \) is large. The varying kinetic coefficient is used to push \( w_Q \) towards \( -1 \) at the present epoch without amplifying the fluctuations in the quintessence field. The high energy physics model of this type of quintessence, called
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Figure 1. Evolution of $w_Q$. The solid line and long dashed line show the tracking solutions, where $\beta = 1$ for the solid line and $\beta = 500$ for the long dashed line. The dashed line corresponds to the case when the quintessence field starts in the potential regime and $\beta = 500$. In this plot, we set $\kappa_{\text{min}} = 8.7 \times 10^{-4}$ and $\Omega_Q = 0.7$ at the present epoch.

crossover quintessence, has been proposed by Wetterich [27]. After suitable redefinition of dynamical variables, the action of crossover quintessence takes the form as shown in eq. (10). The evolution of crossover quintessence is not exactly the same as the evolution of leaping kinetic term quintessence because their kinetic coefficients have different forms. However, both quintessence models evolve in the same manner, i.e., they evolve in the early epoch as a simple exponential quintessence model and their equation of state parameter is pushed towards $-1$ at present by rapid increase of the kinetic coefficient. Since the form of the kinetic coefficient of crossover quintessence is more complicated than that of the leaping kinetic term quintessence, we use a leaping kinetic term quintessence model in our consideration.

4. The suppression of the CMB power spectrum

We first consider the initial conditions for the quintessence field and the quintessence fluctuations. Since we suppose that the quintessence field $Q$ is frozen initially, it will be frozen until quintessence domination if its initial value $Q_i$ is larger than the tracking solution value [23]. In our consideration, the value of $Q_i$ is chosen such that $w_Q$ evolves as shown in figure 2. It can be seen in this figure that the redshift $z_c$ at which the quintessence field leaves the potential regime decreases if $Q_i$ increases. The evolution of $\Omega_Q$ is approximately the same for all chosen $Q_i$ so we do not plot it.

To compute the initial conditions for the quintessence fluctuations, we suppose that the isocurvature fluctuations in quintessence have (anti-) correlation with adiabatic fluctuations in the other matter, so that the initial conditions for quintessence fluctuations can be written as [12] $\delta_{Q_{\text{int}}} = \delta_{Q_{\text{ad}}} + \delta_{Q_{\text{iso}}}$ and $u_{Q_{\text{int}}} = u_{Q_{\text{ad}}} + u_{Q_{\text{iso}}}$. Here,
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Figure 2. Evolution of \( w_Q \). The long dashed line represents the case where the quintessence fluctuations have no effect on the evolution of \( \Psi \), the dashed line represents the case where the quintessence fluctuations lead to a large ISW contribution and the solid line represents the case where the CMB power spectrum at low multipoles is sufficiently suppressed by the quintessence fluctuations. For these line styles, we set \( \kappa_{\text{min}} = 8.7 \times 10^{-4} \) and \( \beta = 500 \). The value of \( Q_i \) for the long dashed lines is smallest, while the one for the dashed lines is largest. These representations of line styles will be used in all subsequent figures.

The subscripts int, ad and iso denotes the mixed initial conditions, adiabatic initial conditions and isocurvature initial conditions respectively. The density contrast \( \delta_Q \) and momentum density \( u_Q \) in these expressions are in conformal Newtonian gauge. When the quintessence field is nearly frozen, the density contrast \( \delta_Q \) is roughly equivalent to the gauge invariant density contrast \( \Delta_Q \) because \( \Delta_Q = \delta_Q + 3(1 + w_Q)\Psi \). Moreover, the momentum density \( u_Q \) and the gauge invariant momentum density \( U_Q \) are the same. Thus, we can use the gauge invariant formulas in [18] to compute the adiabatic initial conditions for quintessence and the other species, e.g., photon, baryon, neutrino and CDM. These initial conditions are written in terms of \( \zeta_i \), where \( \zeta_i = 1 \). For isocurvature modes, the value of \( \Delta_{Q_{\text{iso}}} \) is computed using eq. (7). The field fluctuation \( \delta_Q_{\text{inf}} \) is written in terms of curvature perturbation as [19] \( \Delta_{Q_{\text{iso}}} = U_{Q_{\text{iso}}} \left( s - 3w_Q + 3c_{aQ}^2 - 1 \right) \), where \( s = \frac{1}{2} \left[ 6w_Q - 3c_{aQ}^2 - 2 \pm \sqrt{9c_{aQ}^4 + 12c_{aQ}^2 - 20} \right] \) and \( c_{aQ}^2 \) is the adiabatic sound speed of quintessence. In our case, the quintessence field is frozen initially so \( c_{aQ}^2 = -7/3 \) [24] and therefore \( U_{Q_{\text{iso}}} = -\Delta_{Q_{\text{iso}}}/5 \).

We use CMBEASY [29] to compute the evolutions of \( \Delta_Q \) and \( \Psi \) and plot them.
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Figure 3. Evolution of $\Delta Q$ (thin lines) and $\Psi$ (thick lines) for the models whose $w_Q$ is plotted in figure 2. The left panel shows a mode whose wavelength is larger than the horizon today ($k = 2 \times 10^{-4}$ Mpc$^{-1}$), while the right panel shows a mode which enters the horizon about the end of matter domination ($k = 1 \times 10^{-3}$ Mpc$^{-1}$).

In figure 3, from this figure, we see that the magnitude of $\Delta Q$ decreases when $w_Q$ increases from $-1$. For a given redshift, the decay rate of $\Psi$ increases with increasing $|\Delta Q|$. Thus, the ISW contribution increases with decreasing $z_c$ (or equivalently with increasing $Q_i$). This is because $|\Delta Q|$ will start to decrease at lower redshift if $z_c$ decreases. On small scales, the quintessence fluctuations decrease after horizon crossing. Thus, the quintessence fluctuations lead to less modifications of $\Psi'$ compared with the case of large scales. Because of the oscillation of the quintessence fluctuations, $\Psi'$ oscillates after horizon crossing. Therefore, on small scales, the amount of the ISW contribution does not increase with decreasing $z_c$.

The CMB power spectrum is computed using CMBEASY and shown in figure 4. In this figure, we set $\Omega_b h^2 = 0.0223$, $\Omega_m h^2 = 0.127$, $\Omega_Q = 0.73$, $h = 0.73$, $\tau = 0.09$, $n_s = 0.95$ and $\beta = 500$.

From this figure, we see that the amplitude of the CMB power spectrum at low multipoles decreases with decreasing $z_c$ for a suitable range of $z_c$. This is because the ISW contribution increases when $z_c$ decreases. The value of $z_c$ must be smaller than the redshift of matter-radiation equality, otherwise the amount of the quintessence fluctuation is not large enough to modify the ISW contribution. Nevertheless, if $z_c$ is too small, the CMB quadrupole increases but a few higher multipoles decrease when $z_c$ decreases.

Since the ISW contribution depends on the amount of the quintessence fluctuations at low redshift, the amplitude of the CMB power spectrum at low multipoles also depends on $\kappa_{\text{min}}$ and $\beta$. For appropriate values of $z_c$ and $\beta$, the amplitude of the CMB spectrum decreases with decreasing $\kappa_{\text{min}}$ due to the increasing $|\Delta Q_i|$. The different values of $\kappa_{\text{min}}$ can lead to the same amplitude of the CMB power spectrum if the value
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Figure 4. The CMB power spectrum for different choices of $Q_i$. The data points from WMAP 3-year results are denoted by the solid triangles. The error bars show the 68% confidence level.

of $z_c$ for the case of small $\kappa_{\text{min}}$ is larger than the one for the case of large $\kappa_{\text{min}}$. However, the value of $\kappa_{\text{min}}$ must not be too large or too small compared with the value which is chosen here. If $\kappa_{\text{min}}$ is too large, the quintessence fluctuation will have no effect on the CMB power spectrum because $|\Delta Q_i|$ is too small. If $\kappa_{\text{min}}$ is too small, the CMB power spectrum at low multipoles will be enhanced by the quintessence fluctuations due to the large ISW contribution. Furthermore, for appropriate values of $z_c$, $\kappa_{\text{min}}$, and a suitable range of $\beta$, the value of $w_Q$ in the tracking regime gets closer to $-1$ as $\beta$ increases. Since the amplitude of $\Delta Q$ in the tracking regime decreases slower when $w_Q$ is closer to $-1$, the amplitude of $\Delta Q$ at particular redshift in this regime increases with increasing $\beta$. Hence, the amplitude of CMB power spectrum at low multipoles decreases with $\beta$.

The enhancement of the CMB quadrupole when $z_c$ is too small can be understood by considering the CMB transfer function. We plot the CMB transfer functions in figure 5. This figure shows that the ISW contribution for $\ell = 2$ is much enhanced around its peak due to the quintessence fluctuations. If the amount of ISW contribution is too large, it leads to an enhancement of the CMB quadrupole. For $\ell = 4$, quintessence fluctuations lead to less ISW contribution because the ISW contribution peaks at a scale which is smaller than the horizon size today and the quintessence fluctuations decrease inside the horizon. Therefore, when $z_c$ is too small, the CMB quadrupole increases with decreasing $z_c$ although the CMB spectrum at $\ell = 4$ still decreases with decreasing $z_c$. 

$$C_\ell = \frac{(\ell + 1)C_\ell}{\pi (\mu K)^2}$$
5. Conclusions

The quintessence fluctuation can lead to the suppression of the CMB power spectrum at low multipoles if its amplitude is large enough during quintessence domination and it has an anticorrelation with the adiabatic fluctuations in the other matter. Assuming that the quintessence fluctuation is generated during inflation, its amplitude can be large during quintessence domination if the amplitude of its initial value $\Delta Q_i \simeq \frac{1}{V \frac{dV}{dQ}} \delta Q_{\text{inf}}$ is large and the quintessence field is nearly frozen until quintessence domination. It has been shown that the quintessence field may leave the potential regime at low redshift because its coarse-grained part is driven towards a large value by its quantum fluctuations \cite{30, 31}. Thus, if the initial amplitude of the quintessence fluctuation is large, the amplitude of quintessence fluctuations can be large enough to enhance the ISW contribution, and therefore suppress the CMB power spectrum at low multipoles. For simple quintessence models, $|\Delta Q_i|$ can be large if $\delta Q_{\text{inf}}$ is large, leading to excess gravitational waves. In this work, we avoid the excess gravitational waves by keeping $\delta Q_{\text{inf}}$ below observational bound. The large $|\Delta Q_i|$ can be obtained by enhancing the initial value of $\left| \frac{1}{V \frac{dV}{dQ}} \right|$. However, To drive an accelerated expansion of the Universe, the quintessence potential must be flat today. Hence, the quintessence field usually has small $\left| \frac{1}{V \frac{dV}{dQ}} \right|$ in the early epoch. We found that the potential of leaping kinetic term quintessence can be sufficiently steep initially and be nearly flat today. Thus, this quintessence model can produce large fluctuations to explain the observed low CMB quadrupole if the value of
κ_{min} is chosen appropriately. This quintessence model has a tracking behaviour but the amplitude of the CMB power spectrum at low multipoles depends on z_{c}, or equivalently the initial value of the quintessence field.

Finally, we note that one of the basic assumption in this work is that the quintessence fluctuations and the adiabatic perturbations in the other matter must have an anticorrelation. The general idea for generating the correlated fluctuations has been considered in [32, 33]. A specific model for generating correlated fluctuations in particular quintessence model is proposed in [11]. Here, we just suppose that the quintessence fluctuations and the adiabatic fluctuations in the other matter can have an anticorrelation. We do not propose a specific model for generating the anticorrelated fluctuations in the considered quintessence model.

Acknowledgments

This work is a part of my Ph.D. thesis at University of Heidelberg. I would like to thank C. Wetterich who is my thesis advisor. I would also like to thank M. Doran and A. Ungkitchanukit for comments on the manuscript. I am also grateful for financial support from DAAD (German Academic Exchange Service) during my Ph.D. study.

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