A Scheffe’s Predictive Model for Modulus of Elasticity of Sawdust Ash - Sand Concrete

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ABSTRACT
The industrial waste, Saw Dust Ash (SDA) has been explored by several concrete related researches to achieve environmental and economic sustainability. In this study, 5% of sand was replaced with SDA to produce concrete with five different mix ratios. Scheffe’s simplex theory was used for five mix ratios in a [5,2] experimental design which resulted in additional ten mix ratios. For purposes of verification and testing, additional fifteen mix ratios were generated from the initial fifteen. Concrete cubes of 150mmX150mmX150mm were formed using the thirty concrete mix ratios generated, and cured in water for 28days. The compressive strengths of cubes from each mix ratio were determined. The static moduli of elasticity were also determined with a mathematical relationship. The results of the first fifteen static moduli of elasticity values were used for the calibration of the model constant coefficients, while those from the second fifteen were used for the model verification and testing using Scheffe’s simplex lattice design. A mathematical regression model was formulated from the results, with which the static moduli of elasticity were predicted. The model was then subjected to a two-tailed t-test with 5% significance, which confirms the model adequate and fit with an R2 of 0.8536. The study also revealed that SDA can be used to replace 5% of sand and promote environmental sustainability without significantly decreasing the static modulus of elasticity.

Keywords – Saw Dust Ash, Scheffe’s Simplex Lattice, Sustainability, Modulus of Elasticity of Concrete

I. INTRODUCTION
The modulus of Elasticity of an engineering material is the stress per unit strain of that material. Hardened concrete materials, as indicated by[1] undergo non-linear and non-elastic deformation which is a permanent deformation that occurs after removal of loads. A static Modulus of Elasticity is usually referred to when a laboratory experiment is carried out to determine the Modulus of Elasticity. However, from the chart in Figure 1, the initial tangent modulus of elasticity, according to[1] is approximately referred to as the dynamic modulus of elasticity.

The relationship between the static Modulus of Elasticity, $E_c$ and the dynamic Modulus of Elasticity, $E_d$ is:

$$E_c = 1.25E_d - 19 \quad \quad (1)$$

$$E_c = 1.25E_d - (2.75 \times 10^6) \quad \quad (2)$$

The above equations only apply when less than or equal to 500kg of cement per cubic metres is used. When otherwise, the following applies:

$$E_c = 1.04E_d - 4.1 \quad \quad (3)$$

$$E_c = 1.04E_d - (0.59 \times 10^6) \quad \quad (4)$$

Equations (1) and (3) are in SI units (GPa) while Equations (2) and (4) are in Imperial units (psi). The static modulus of Elasticity $E_c$, according to[1] when relating it with the compressive strength can be determined by the equations:

$$E_c = 9.1f_c^{0.33} \quad \quad (5)$$

$$E_c = 0.255f_c^{0.33} \times 10^6 \quad \quad (6)$$

Equation (5) is in SI units (GPa) while equation (6) is in Imperial units (psi) and the density of concrete is 2320kg/m³. When the density of concrete is between 1400kg/m³ and 2320kg/m³, equations (7) and (8) are used in SI units and Imperial units respectively.

$$E_c = 1.7\rho_f^{0.33} \quad \quad (7)$$

$$E_c = 12.24\rho_f^{0.33} \times 10^6 \quad \quad (8)$$

where, $\rho_f$ is the density of concrete (kg/m³) and $f_c$ is the compressive strength (N/mm² or MPa).

However,[2] stated that the relationships between modulus of elasticity and compressive strength...
are equations (9) and (10) for SI (GPa) and Imperial (psi) units respectively.
\[ E_s = 4.731 \sqrt{f_c} \] (9)
\[ E_s = 57000 \sqrt{f_c} \] (10)
These equations are slightly different from those given by [1].

A study carried out by [3] revealed that 10% replacement of Portland cement with Hydrated Lime (HL) and Saw Dust Ash (SDA) gave acceptable compressive strength and modulus of elasticity. Another study on cement mortar [4] affirmed that addition of 0.5% SDA to the mixture increased the dynamic modulus of elasticity by 2-10%, while a 3% addition caused about 23% decrease in the dynamic modulus of elasticity. With reference to equations (1) to (4), these changes will also affect the dynamic modulus of elasticity.

**A. Saw Dust Ash in Concrete**

Saw Dust Ash is the pulverised form of saw dust produced as waste from saw mills. It has been used in concrete construction for over 30 years [5]. In addition, cement production has been a major source of environmental degradation, as about 400kg of CO₂ are emitted from every 600kg of cement produced [6]. Investigations [7] showed that SDA has a specific gravity of 2.5, water absorption of 0.56%, fineness modulus of 1.78, and bulk dry density of 13000kg/m³, while sand has specific gravity of 2.65, water absorption of 0.45%, fineness modulus of 2.21, and bulk dry density of 1512 kg/m³. A 10% replacement of sand with SDA, modified the properties to 2.67, 0.5%, 2.2, and 1436kg/m³ for specific gravity, water absorption, fineness modulus, and bulk dry density respectively. This is a strong indication that sand and SDA mixture did not significantly change the physical properties of sand, making the mixture adequate for a fine aggregate.

The chemical compositions of SDA [7] by mass are: 65.3% SiO₂, 4% Al₂O₃, 2.23% Fe₂O₃, 9.6% CaO, 5.8% MgO, 0.01% MnO, 0.07% Na₂O, 0.11% K₂O, 0.43% P₂O₅, and 0.45% SO₂. The summation of SiO₂, Al₂O₃, and Fe₂O₃ gives 71.53%. A similar investigation by [8] found 67.95% SiO₂, 4.29% Al₂O₃, 2.15% Fe₂O₃, 9.47% CaO, 5.84% MgO, 0.01% MnO, 0.06% Na₂O, 0.11% K₂O, and 0.56% SO₂. The summation of SiO₂, Al₂O₃, and Fe₂O₃ gave 74.39. These, in accordance with [9] indicate that SDA is a good pozzolanic material. The chemical compositions of SDA as found by [7], [8] all indicate that SDA has a high percentage of SiO₂ and small percentages of Al₂O₃ and Fe₂O₃, which are similar to those of sand with high percentage of about 95% SiO₂. Hence SDA can be used with sand as fine aggregate.

**B. Scheffe’s Simplex Theory**

Several authors [10], [11], [20]–[23], [12]–[19] have carried out concrete mixture researches with development of mathematical models, most of which were based on Scheffe’s Simplex theory. Scheffe’s model is based on the simplex lattice and simplex theory or approach [24]. The simplex approach considers a number of components, q, and a degree of polynomial, m. The sum of all the \( i \)th components is not greater than 1. Hence,
\[ \sum_{i=1}^{q} x_i = 1 \] (11)
\[ x_1 + x_2 + \ldots + x_q = 1 \] (12)
with \( 0 \leq x \leq 1 \), the factor space becomes \( S_{q-1} \).

According to [24] the \( \{ q,m \} \) simplex lattice design is a symmetrical arrangement of points within the experimental region in a suitable polynomial equation representing the response surface in the simplex region.

The number of points \( C^q_m \) (m+1) equally spaced values of \( x_i = 0, 1/m, 2/m, \ldots, m/m \) for a 3-component mixture with degree of polynomial 2, the corresponding number of points will be \( C^q_m \) which gives 6 (eq. 13 or eq. 14 below) with number of spaced values, 2+1 = 3, that is \( x_i = 0, 1/2, 1 \) and 1 design points of (1,0,0), (0,1,0), (0,0,1), (1/2,1/2,0), (1/2,0,1/2), (1/2,0,0,1/2), (0,1/2,1/2,0), (0,1/2,0,1/2), (0,0,1/2,1/2), (0,1/2,0,1/2), (0,1/2,0,1/2). Similarly, for a \( \{ 5,2 \} \) simplex, there will be 15 points with \( x_i = 0, 1/2, 1 \) as spaced values. The 15 design points are (1,0,0,0,0), (0,1,0,0,0), (0,0,1,0,0), (0,0,0,1,0), (1/2,1/2,0,0,0), (1/2,0,1/2,0,0), (1/2,0,0,1/2,0), (1/2,0,0,0,1/2), (0,1/2,1/2,0,0), (0,1/2,0,1/2,0), (0,0,1/2,1/2,0), (0,1/2,0,0,1/2).

\[ N = C^q_m \] (13)
\[ N = \frac{(q+n-1)!}{(q-1)!(n-1)!} \] (14)

For a polynomial of degree \( m \) with \( q \) component variables where eq. (12) holds, the general form is:
\[ Y = b_0 + \sum b_i x_i + \sum b_{ij} x_i x_j + \sum b_{ijk} x_i x_j x_k + \ldots + \sum b_{i_1 i_2 \ldots i_n} x_{i_1} x_{i_2} x_{i_3} \ldots x_{i_n} \] (15)
Where \( 1 \leq i \leq q, 1 \leq j \leq q, 1 \leq i \leq j \leq k \leq q \), and \( b_0 \) is the constant coefficient.

\( x \) is the pseudo component for constituents \( i, j, k \). When \( \{ q,m \} = \{ 5,2 \} \), eq. (15) becomes:
\[ Y = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 + b_5 x_5 + b_{12} x_1 x_2 + b_{13} x_1 x_3 + b_{14} x_1 x_4 + b_{15} x_1 x_5 + b_{23} x_2 x_3 + b_{24} x_2 x_4 + b_{25} x_2 x_5 + b_{34} x_3 x_4 + b_{35} x_3 x_5 + b_{45} x_4 x_5 + b_{11} x_1^2 + b_{22} x_2^2 + b_{33} x_3^2 + b_{44} x_4^2 + b_{55} x_5^2 \] (16)
and eq. (12) becomes
\[ x_1 + x_2 + x_3 + x_4 + x_5 = 1 \] (17)
Multiplying eq. (17) by \( b_0 \) gives
\[ b_0 x_1 + b_0 x_2 + b_0 x_3 + b_0 x_4 + b_0 x_5 = b_0 \] (18)
Multiplying eq. (17) successively by \( x_1, x_2, x_3, x_4, \) and \( x_5 \) and making \( x_1, x_2, x_3, x_4, \) and \( x_5 \) the subjects of the respective formulas:
\[ x_1^2 = x_1 - x_1 x_2 - x_1 x_3 - x_1 x_4 - x_1 x_5 \] (19)
\[ x_2^2 = x_2 - x_2 x_3 - x_2 x_4 - x_2 x_5 \] (19)
\[ x_3^2 = x_3 - x_3 x_4 - x_3 x_5 \] (19)
\[ x_4^2 = x_4 - x_4 x_5 \] (19)
\[ x_5^2 = x_5 \] (19)
Substituting eq. (18) and eq. (19) into eq. (16) we have:

\[
Y = b_0 x_1 + b_0 x_2 + b_0 x_3 + b_0 x_4 + b_0 x_5 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 + b_5 x_5 + b_{12} x_1 x_2 + b_{13} x_1 x_3 + b_{14} x_1 x_4 + b_{15} x_1 x_5 + b_{23} x_2 x_3 + b_{24} x_2 x_4 + b_{25} x_2 x_5 + b_{34} x_3 x_4 + b_{35} x_3 x_5 + b_{45} x_4 x_5 + b_{14} x_4 x_5 + b_{15} x_4 x_5 + b_{23} x_2 x_5 + b_{24} x_2 x_5 + b_{25} x_2 x_5 + b_{34} x_3 x_5 + b_{35} x_3 x_5 + b_{45} x_4 x_5 + b_{55} x_5 x_5
\]

This can be rewritten as:

\[
Y = \sum_{i=1}^{5} \beta_i x_i + \sum_{1 \leq i < j \leq 5} \beta_{ij} x_i x_j
\]

Where the response, \(Y\) is a dependent variable (Water Absorption of concrete). Eq. (22) is the general equation for a \([5,2]\) polynomial, and it has 15 terms, which conforms to Scheftee’s theory in eq. (13).

Let \(Y_i\) denote response to pure components, and \(Y_{ij}\) denote response to mixture components in \(i\) and \(j\). If \(x_i = 1\) and \(x_j = 0\), since \(j \neq i\), then

\[
Y_i = \beta_i
\]

Which means

\[
\sum_{i=1}^{5} \beta_i x_i = \sum_{i=1}^{5} Y_i x_i
\]

Hence, from eq. (24)

\[
Y_i = \beta_i
\]

\[
Y_j = \beta_j
\]

\[
Y_{ij} = \beta_{ij}
\]

According to [24],

\[
\beta_{ij} = 4 Y_{ij} - 2 \beta_i - 2 \beta_j
\]

Substituting eq. (24)

\[
\beta_{ij} = 4 Y_{ij} - 2 \beta_i - 2 \beta_j
\]

II. MATERIALS AND METHODS

The materials used for the production of the concrete for the study were water, cement, sand, SDA, and granite. These are the five components in the concrete mix, with SDA used to partially replace 5% of the fine aggregate (sand).

The first five concrete mix ratios derived from different mix design methods [17], [18] are given as:

- BRE 12 = [0.54 11.9475 0.1025 2.95];
- BRE 22 = [0.58 2.1185 0.1115 3.21];
- USBR 22 = [0.58 12.2515 0.1185 3.29];
- BIS 12 = [0.43 11.2065 0.0635 2.88];
- ACI 12 = [0.55 11.8335 0.0965 3.09]

These can be put in matrix form as follows:

\[
Y = \begin{bmatrix}
0.54 & 0.58 & 0.58 & 0.43 & 0.55 \\
1 & 1 & 1 & 1 & 1 \\
1.9475 & 2.1185 & 2.2515 & 1.2065 & 1.8335 \\
0.1025 & 0.1115 & 0.1185 & 0.0635 & 0.0965 \\
2.95 & 3.21 & 3.29 & 2.88 & 3.09
\end{bmatrix}
\]

Their corresponding pseudo components are given as:

\[
S = \begin{bmatrix}
0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]

with centre points

\[
X = \begin{bmatrix}
X_{12} & X_{13} & X_{14} & X_{15} & X_{23} & X_{24} & X_{25} & X_{34} & X_{35} & X_{45}
\end{bmatrix}
\]

According to [24],

\[
S_j = X S_j
\]

Substituting,

\[
S_{12} = 0.5 0.5 0.5 0.5 0.5
\]

This process is repeated for \(S_{13}, S_{14}, S_{15}, S_{23}, S_{24}, S_{25}, S_{34}, S_{35},\) and \(S_{45}\). Similarly, this process is repeated for an additional 15 (control) points that will be used for the verification of the formulated model. The regular
pentagons for the actual components with their corresponding pseudo components are given in Figures 2 and 3 respectively. Tables 1 and 2 mix ratio data were generated for the main and verification purposes respectively from [25].

![Figure 2: Simplex Plot for Actual Components](image1)

![Figure 3: Simplex Plot for Pseudo Components](image2)

**Table 1: Model Mix Ratios**

| Sample Points | w-c ratio | Actual Components | Response Y exp | Pseudo Components |
|---------------|-----------|-------------------|----------------|-------------------|
|               | w-c ratio | Cement | Sand | SDA | Granite | X1 | X2 | X3 | X4 | X5 |
| BRE12         | 0.54      | 1      | 1.9475 | 0.1025 | 2.95 | Y1 | 0  | 0  | 0  | 0  |
| BRE22         | 0.58      | 1      | 2.1185 | 0.1115 | 3.21 | Y2 | 0  | 1  | 0  | 0  |
| USBR22        | 0.58      | 1      | 2.2515 | 0.1185 | 3.29 | Y3 | 0  | 0  | 1  | 0  |
| BIS12         | 0.43      | 1      | 1.2065 | 0.0635 | 2.88 | Y4 | 0  | 0  | 0  | 1  |
| ACH12         | 0.55      | 1      | 1.8335 | 0.0965 | 3.09 | Y5 | 0  | 0  | 0  | 0  |
| N1            | 0.56      | 1      | 2.033  | 0.107  | 3.08 | Y6 | 0.5| 0.5| 0  | 0  |
| N2            | 0.56      | 1      | 2.0995 | 0.1105 | 3.12 | Y7 | 0.5| 0  | 0  | 0  |
| N3            | 0.485     | 1      | 1.577  | 0.083  | 2.915| Y8 | 0.5| 0  | 0  | 0  |
| N4            | 0.545     | 1      | 1.8905 | 0.0995 | 3.02 | Y9 | 0.5| 0  | 0  | 0  |
| N5            | 0.58      | 1      | 2.185  | 0.115  | 3.25 | Y10| 0  | 0  | 0  | 0  |
| N6            | 0.505     | 1      | 1.6625 | 0.0875 | 3.045| Y11| 0  | 0  | 0  | 0  |
| N7            | 0.565     | 1      | 1.976  | 0.104  | 3.15 | Y12| 0  | 0  | 0  | 0  |
| N8            | 0.505     | 1      | 1.729  | 0.091  | 3.085| Y13| 0  | 0  | 0  | 0  |
| N9            | 0.565     | 1      | 2.0425 | 0.1075 | 3.19 | Y14| 0  | 0  | 0  | 0  |
| N10           | 0.49      | 1      | 1.52   | 0.08   | 2.985| Y15| 0  | 0  | 0  | 0  |

**Table 2: Control Points**

| Sample Points | w-c ratio | Actual Components | Response Y exp | Pseudo Components |
|---------------|-----------|-------------------|----------------|-------------------|
|               | w-c ratio | Cement | Sand | SDA | Granite | X1 | X2 | X3 | X4 | X5 |
| C1            | 0.558     | 1      | 2.0463 | 0.1077 | 3.114| YC1| 0  | 0  | 0  | 0  |
| C2            | 0.52      | 1      | 1.7537 | 0.0923 | 3.078| YC2| 0  | 0.6| 0  | 0  |
| C3            | 0.548     | 1      | 2.0083 | 0.1057 | 3.018| YC3| 0  | 0  | 0  | 0  |
| C4            | 0.49      | 1      | 1.5713 | 0.0827 | 3.012| YC4| 0  | 0  | 0  | 0  |
| C5            | 0.544     | 1      | 1.9019 | 0.1001 | 3.006| YC5| 0  | 0  | 0  | 0  |
| C6            | 0.55      | 1      | 2.0425 | 0.1075 | 3.208| YC6| 0  | 0  | 0.8| 0  |
| C7            | 0.55      | 1      | 1.9589 | 0.1031 | 3.03 | YC7| 0  | 0  | 0  | 0  |
| C8            | 0.514     | 1      | 1.6967 | 0.0893 | 3.054| YC8| 0  | 0  | 0  | 0  |
| C9            | 0.548     | 1      | 1.8563 | 0.0977 | 3.062| YC9| 0  | 0  | 0  | 0  |
| C10           | 0.46      | 1      | 1.4155 | 0.0745 | 2.962| YC10| 0  | 0  | 0  | 0  |
| C11           | 0.566     | 1      | 2.1071 | 0.1109 | 3.182| YC11| 0  | 0  | 0.6| 0  |
A. Static Modulus of Elasticity of Concrete

Two replicate concrete cubes were made for each of the thirty mix ratios in 150mmX150mmX150mm moulds and allowed to harden. The concrete cubes were removed from the moulds after 24 hours and cured in water for 28 days after which the compressive strengths were determined with results from [18]. The static modulus of Elasticity $E_c$, was determined by equation (33) for each mix ratio as follows:

$$E_c = 9.1f_c^{0.33} \times 10^6 \text{ (KN/mm}^2)$$  \hspace{1cm} (33)

where $f_c$ is the compressive strength (N/mm$^2$).

The average was taken and recorded.

Sieve analysis was carried out on the fine aggregate mixed with 5% SDA as a preliminary investigation. The particle size distribution of the 5% replacement of sand with SDA is shown in Figure 4, and the fineness modulus calculated below. Fineness modulus,

$$FM = \frac{0.73 + 4.24 + 14.08 + 43.61 + 80.48 + 97.88}{100} = 2.41$$

This value indicates that the material is a fine aggregate as it ranges between 2.3 and 3.1 [2].

### III. RESULTS AND DISCUSSIONS

![Particle Size Distribution for Fine Aggregate with 5% SDA replacement](image)

**Figure 4:** Particle Size Distribution for Fine Aggregate with 5% SDA replacement

The results of the 28 days Static Modulus of Elasticity are shown in Table 3 below.

| Sample Points | Sample Compress. Strength (N/mm$^2$) | Modulus of Elasticity (KN/mm$^2$) |
|---------------|-------------------------------------|----------------------------------|
|               | A                                   | B                                 | A       | B       | Average  |
| C12           | 28.444                              | 28.444                            | 27.470  | 27.470  | 27.470   |
| C13           | 23.111                              | 24.444                            | 25.651  | 26.130  | 25.890   |
| C14           | 26.191                              | 26.313                            | 26.732  | 26.773  | 26.753   |
| C15           | 36.222                              | 35.778                            | 29.751  | 29.630  | 29.691   |
| ACI12         | 31.564                              | 31.956                            | 28.430  | 28.546  | 28.488   |
| N1            | 29.333                              | 28.489                            | 27.750  | 27.484  | 27.617   |
| N2            | 21.956                              | 22.489                            | 25.220  | 25.421  | 25.321   |

| Sample Points | Actual Components | Pseudo Components |
|---------------|-------------------|-------------------|
|               | w-c ratio Cement | Sand | SDA | Granite | w-c ratio Cement | Sand | SDA | Granite |
|               | $S_1$ $S_2$ $S_3$ $S_4$ $S_5$ | $X_1$ $X_2$ $X_3$ $X_4$ $X_5$ |
| C12           | 0.544 1.9323 0.1017 3.152 | YC$_{12}$ 0.2 0.4 0.2 0.2 |
| C13           | 0.58 2.1451 0.1129 3.226 | YC$_{13}$ 0.8 0.2 0.2 0 |
| C14           | 0.532 1.7651 0.0929 3.072 | YC$_{14}$ 0.2 0.2 0.2 0.6 |
| C15           | 0.536 1.8715 0.0985 3.084 | YC$_{15}$ 0.2 0.2 0.2 0.2 |
A. Model Formulation

The coefficients of polynomial from Table 3, equation (26), and equation (28) are:
\[
\begin{align*}
\beta_1 &= 27.47, \quad \beta_2 = 25.89, \quad \beta_3 = 26.753, \quad \beta_4 = 29.691, \quad \beta_5 = 28.488, \\
\beta_{12} &= 4Y_{12} - 2Y_1 - 2Y_2, \\
\beta_{11} &= 4 \times 27.617 - 2 \times 27.47 - 2 \times 25.89 = 3.748 \\
\text{Similarly,} \quad \beta_{12} &= -7.162, \quad \beta_{22} = -2.384, \quad \beta_{23} = 4.726, \quad \beta_{24} = -0.298, \quad \beta_{25} = -0.912, \quad \beta_{34} = -0.632, \quad \beta_{35} = -6.958, \quad \beta_{15} = -6.934.
\end{align*}
\]

Substituting the above coefficients into equation (22) gives

\[
Y = 27.47x_1 + 25.89x_2 + 26.753x_3 + 29.691x_4 + 28.488x_5 + 3.748x_1x_2 - 7.162x_1x_3 - 7.054x_1x_4 - 2.384x_1x_5 + 4.726x_2x_3 - 0.298x_2x_4 - 0.912x_2x_5 - 6.958x_3x_5 + 6.934x_3x_5
\]  \hspace{1cm} (34)

Equation (34) is the mathematical model to predict the 28 days static modulus of elasticity of concrete using SDA to replace 5% of fine aggregate. Table 4 shows the predictions, while Figure 5 shows the comparison between the predicted and experimented values of 28days static modulus of elasticity using the control (verification) data.

### Table 4: Experimental and Predicted values of 28days Static Modulus of Elasticity of Concrete

| Sample Points | Response Y | W-c ratio | Pseudo Components | Static Mod. of Elasticity $Y_{exp}$ (KN/mm²) | Static Mod. of Elasticity $Y_{pred}$ (KN/mm²) |
|---------------|------------|-----------|-------------------|--------------------------------------------|--------------------------------------------|
| BRE12         | Y1         | X1 1      | Cement X2 0       | 0 X3 0                                    | 27.470                                     |
| BRE22         | Y2         | X2 0      | 1 Sand X3 0       | 0 X4 0                                    | 25.890                                     |
| USBR22        | Y3         | X3 0      | 0 Granite X4 0    | 1 X5 0                                    | 26.753                                     |
| BIS12         | Y4         | X4 0      | 0 Granite X5 0    | 1 0                                       | 29.691                                     |
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**Figure 5:** Comparison between Experimental and Predicted 28 days Static Modulus of Elasticity

**B. Test of Adequacy of the Model**

A two-tailed student t-test was carried out at 95% confidence level, which implies $100 - 95 = 5\%$ significance. Since it is a two-tailed, significance $= 5/2 = 2.5\%$

Hence significance level $= 100 - 2.5 = 97.5\%$
Let D be the difference between the experimental and predicted responses.

The mean of the difference,
\[ D_a = \frac{1}{n} \sum_{i=1}^{n} D_i \]  
(35)

The variance of the difference,
\[ S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (D_i - D_a)^2 \]  
(36)

\[ t_{\text{calculated}} = \frac{D_a \sqrt{n}}{S} \]  
(37)

Where n = number of observations with degree of freedom n – 1. Table 5 shows the details of the t-test results.

### Table 5: Student t-test for 28 days Static Modulus of Elasticity of Concrete

| Sample | Static Moduli of Elasticity (kN/m²) | t-test |  |
|--------|-------------------------------------|--------|---|
|        | Y_{\text{experimental}} | Y_{\text{predicted}} | D = Y_{\text{exp}} - Y_{\text{pred}} | D_a - D | (D - D_a)^2 |
| C1     | 25.915                           | 25.494  | 0.421                           | -0.323  | 0.104      |
| C2     | 27.569                           | 27.339  | 0.230                           | -0.132  | 0.017      |
| C3     | 26.216                           | 26.181  | 0.035                           | 0.063   | 0.004      |
| C4     | 28.433                           | 28.099  | 0.334                           | -0.236  | 0.056      |
| C5     | 27.534                           | 27.305  | 0.229                           | -0.131  | 0.017      |
| C6     | 27.213                           | 27.239  | -0.026                          | 0.124   | 0.015      |
| C7     | 27.434                           | 27.485  | -0.051                          | 0.149   | 0.022      |
| C8     | 27.888                           | 27.255  | 0.633                           | -0.535  | 0.286      |
| C9     | 27.283                           | 27.903  | -0.620                          | 0.718   | 0.516      |
| C10    | 28.500                           | 29.002  | -0.502                          | 0.600   | 0.360      |
| C11    | 25.626                           | 25.454  | 0.172                           | -0.074  | 0.005      |
| C12    | 27.313                           | 26.960  | 0.353                           | -0.255  | 0.065      |
| C13    | 26.569                           | 26.819  | -0.250                          | 0.348   | 0.121      |
| C14    | 27.750                           | 27.256  | 0.494                           | -0.396  | 0.156      |
| C15    | 26.728                           | 26.704  | 0.024                           | 0.074   | 0.006      |
| **TOTAL** | 1.476                           | 1.752   | 0.098                           |        |           |

From the t-table, with \( v = 15 - 1 = 14 \), and \( \beta \) = significance level, \( t_{0.975,14} = 2.145 \).

Since \( t_{\text{calculated}} \leq t_{0.975,14} \), and lies between -2.145 and 2.145, there is no significant difference between the experimental and predicted responses, \( H_0 \) is accepted, and \( H_a \) is rejected. The model is confirmed to be adequate.

The \( R^2 \) value of 0.8536 in Figure 6 indicates that the experimental results are highly correlated to the predicted results. This is also an indication that the model is fit and adequate.

![Figure 6: Scatterplot of Predicted vs. Experimental Water Absorption](image)
IV. CONCLUSIONS

Partial replacement of fine aggregate (sand) with 5% SDA was carried out to produce concrete in which cement, granite, and water were the other ingredients. Five different concrete mix ratios were used for the batching. The static moduli of elasticity determined from the 28days compressive strengths were between 25.321KN/mm² and 29.691KN/mm². A multiple regression model was generated from the resulting 28days static moduli of elasticity values determined from the compressive strength experiments, using Scheffe’s simplex theory for a {5,2} simplex lattice. A two-tailed student t-test was carried out at 5% significance level, which confirmed the model adequate with an R² of 0.8536. The results from the study show that Scheffe’s simplex approach is very effective and has a high predictive accuracy for modulus of elasticity of concrete having about 5% sand replaced with SDA. The results also confirmed that SDA is a suitable material to replace up to 5% of sand in the production of concrete in a bid to promote environmental sustainability. However, further research is recommended with different percentages of SDA-sand replacement.

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