Radiative Majorana Neutrino Masses

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We present new radiative mechanisms for generating Majorana neutrino masses, within an extension of the standard model that successfully generates radiative charged lepton masses, order by order, from heavy sequential leptons. Only the new sequential neutral lepton has a right-handed partner, and its Majorana mass provides the seed for Majorana neutrino mass generation. Saturating the cosmological bound of 50 eV with $m_{\nu_\tau}$, we find that $m_{\nu_\mu}$ and $m_{\nu_e}$ could be at most $10^{-2}$, and $10^{-3}$ eV, respectively. The electron neutrino mass may vanish in the limit of degenerate charged Higgs bosons. Unfortunately, $\nu_e - \nu_\tau$ mixing is also radiatively induced, and is too small for sake of solving the solar neutrino problem via the Mikheyev–Smirnov–Wolfenstein effect.
There is no fundamental principle requiring the neutrinos to be massless. However, if neutrinos are not exactly massless, their near masslessness is a real mystery. The so-called seesaw mechanism \[1\] was invented, usually in the context of grand unified theories (GUT), to give a natural framework for almost massless neutrinos. It also provides a natural framework for explaining the solar neutrino puzzle via the Mikheyev-Smirnov-Wolfenstein effect \[2\]. Recent neutrino counting experiments \[3\] show that there exists three and only three species of light neutrinos. On the other hand, direct search for new sequential leptons at $Z^0$ resonance have been conducted and yields the limits \[3\]

$$m_E, m_N \gtrsim M_Z/2,$$

where we denote new charge and neutral leptons as $E$ and $N$, respectively, and sequential means that left-handed leptons come in weak doublets. Although there is as yet no evidence for more sequential leptons, it has been pointed out \[4\] that if they are found with weak scale masses, the traditional seesaw mechanism and $SO(10)$ based GUT theories could be in jeopardy, and one may also face a serious challenge with the solar neutrino problem. It is therefore of interest to investigate whether very small neutrino masses can still be naturally generated under such circumstances. In this regard, Babu and Ma \[5\] have built a radiative seesaw model in which the three known neutrinos acquire radiative Majorana masses through the exchange of two W bosons, and could provide a natural explanation of the solar neutrino problem. Recently, we have successfully constructed a $Z_8$ model \[6\] where the charged leptons acquire mass radiatively order by order, with all Yukawa couplings of order unity. The three known neutrinos have no right-handed counterparts. They are held strictly massless if one requires the Majorana mass $m_R$ for the single right-handed neutral lepton $N_R$ to vanish. In this letter, we remove this \textit{ad hoc} condition and investigate the question of light neutrino Majorana mass.

Let us briefly review the model. With minimal “3+1” generations \[4\], there is only one right-handed neutral lepton $N_R$. Consider a discrete $Z_8$ symmetry ($\omega^8 = 1$). We assign both $\bar{\ell}_i L = (\bar{\nu}_i L, \bar{e}_i L)$ and $e_{i R}$ to transform as $\omega^3, \omega^2, \omega^1, \omega^4$ for $i = 1 - 4$, respectively, while $N_R$
transforms as $\omega^4$. The scalar sector consists of three doublets, $\Phi_0$, $\Phi_3$ and $\Phi_5$, transforming as 1, $\omega^3$ and $\omega^5$, respectively. Thus, except for $E \simeq e_4$ and $N$, only nearest-neighbor Yukawa couplings are allowed,

$$-\mathcal{L}_Y = f_{44} \bar{\ell}_4 L e_4 R \Phi_0 + \tilde{f}_{44} \bar{\ell}_4 L N_R \tilde{\Phi}_0$$
$$+ f_{43} \bar{\ell}_4 L e_3 R \Phi_3 + f_{34} \bar{\ell}_3 L e_4 R \Phi_3 + \tilde{f}_{34} \bar{\ell}_3 L N_R \tilde{\Phi}_5$$
$$+ f_{32} \bar{\ell}_3 L e_2 R \Phi_5 + f_{23} \bar{\ell}_2 L e_3 R \Phi_5$$
$$+ f_{21} \bar{\ell}_2 L e_1 R \Phi_3 + f_{12} \bar{\ell}_1 L e_2 R \Phi_3 + H.c.$$ (2)

where $\tilde{\Phi} \equiv i \sigma_2 \Phi^* = (\phi^0, -\phi^-)$ as usual. We assume $CP$ invariance for sake of simplicity.

If only $\langle \phi_0^0 \rangle = v/\sqrt{2}$, $E$ and $N$ become massive and are naturally at $v$ scale if $f_{44}$, $\tilde{f}_{44} \sim 1$.

The lower generation leptons remain massless at this stage, protected by the $Z_8$ symmetry. To allow for radiative mass generation, the $Z_8$ symmetry is softly broken down to $Z_2$ in the Higgs potential by $\Phi_3$-$\Phi_5$ mixing. Explicitly,

$$V = \sum_i \mu_i^2 \Phi_i^\dagger \Phi_i + \sum_{i,j} \lambda_{ij}(\Phi_i^\dagger \Phi_i)(\Phi_j^\dagger \Phi_j) + \sum_{i \neq j} \eta_{ij}(\Phi_i^\dagger \Phi_j)(\Phi_j^\dagger \Phi_i)$$
$$+ [\tilde{\mu}^2 \Phi_3^\dagger \Phi_5 + \zeta(\Phi_0^\dagger \Phi_3)(\Phi_0^\dagger \Phi_5) + H.c.]$$ (3)

where $\lambda_{ij}$ and $\eta_{ij}$ are symmetric. Note that the $\zeta$ term is $Z_8$ invariant, while the gauge invariant “mass” $\tilde{\mu}^2$ transforms as $\omega^2$. Since only $\mu_0^2 < 0$, while $\mu_3^2$ and $\mu_5^2 > 0$, $\phi_0^0 \rightarrow (v + H_0 + i \chi_h)/\sqrt{2}$, and $\phi_i^0 \rightarrow (h_i + i \chi_i)/\sqrt{2}$ for $i = 3, 5$. The quadratic part of $V$ is

$$V^{(2)} = \lambda_{00} v^2 H_0^2 + \sum_{i \neq 0} \left( \frac{1}{2}(\mu_i^2 + \lambda_{0i} v^2 + \eta_{0i} v^2)(h_i^2 + \chi_i^2) + (\mu_i^2 + \lambda_{0i} v^2)|\phi_i^+|^2 \right)$$
$$+ \tilde{\mu}^2(h_3 h_5 + \chi_3 \chi_5 + \phi_3^+ \phi_3^+ + \phi_5^+ \phi_5^+) + \frac{1}{2} \zeta v^2(h_3 h_5 - \chi_3 \chi_5).$$ (4)

The standard Higgs boson $H_0$ couples only diagonally to heavy particles. The nonstandard scalars ($\phi_3^\pm, \phi_5^\pm$), ($h_3, h_5$) and ($\chi_3, \chi_5$) mix via $\tilde{\mu}^2$ and $\zeta$ terms. Rotating by $\theta_+ \equiv \theta_H$ and $\theta_A$, we obtain the mass basis ($H_1^+, H_2^+$), ($H_1, H_2$) and ($A_1, A_2$), respectively. It is clear that $\sin \theta_+ \rightarrow 0$, ($\theta_A, m_{A_1}, m_{A_2}) \rightarrow (-\theta_H, m_{H_1}, m_{H_2})$ as $\tilde{\mu}^2 \rightarrow 0$, while in the limit $\zeta \rightarrow 0$, one has ($\theta_A, m_{A_1}, m_{A_2}) \rightarrow (+\theta_H, m_{H_1}, m_{H_2})$. These two limits restore the two extra $U(1)$ symmetries of the doublets $\Phi_3$ and $\Phi_5$. 

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Since $N_R$ has $Z_8$ charge assignment of $\omega^4$, a Majorana mass term $m_R$ is in fact permitted. We have previously set this term to zero [6] for sake of simplicity. This was in part also for the reason of “naturalness”, since cosmological considerations [7] suggest that $m_R/v \ll 1$, in strong contrast to other dimensionless parameters of the model. If we do permit nonzero $m_R$, however, we see that it breaks the chiral symmetry of the first three generation of massless neutrinos. It provides the seed for generating tiny Majorana neutrino masses through mixing and nondegeneracy of the four real scalar fields, as well as the two charged scalars.

Radiative mass generation for charged leptons has already been discussed in ref. [6]. Here, we concentrate on neutrino mass generation. The tau neutrino $\nu_\tau$ acquires Majorana mass via the one-loop diagram shown in Fig. 1, which is very similar to the charged lepton mass generation mechanism. We find

$$
(m_\nu)_{33} = \left(\frac{f_{34}^2}{32 \pi^2}\right) \left[\sin^2 \theta_H \ G(m_{H_1}/m_R) + \cos^2 \theta_H \ G(m_{H_2}/m_R) \right. \\
- \left. \sin^2 \theta_A \ G(m_{A_1}/m_R) - \cos^2 \theta_A \ G(m_{A_2}/m_R)\right] m_R,
$$

where $G(x) = (x^2 \ln x^2)/(x^2 - 1)$, while $(m_\nu)_{34} = (m_\nu)_{43} = 0$. Note that the loop-induced Majorana mass is proportional to $m_R$.

Light neutrinos are subject to the cosmological upper bound [7] of 50 eV on their masses, unless they have fast decay channels. In absence of the latter, we saturate this bound with $m_{\nu_\tau}$, assuming that the quantity in brackets is of order one, and that all Yukawa couplings are of order unity in this model [6]. We find that the tree level Majorana mass $m_R$ should be less than a few hundred keV. This may appear unnatural at first sight. We will return for discussions later.

The muon neutrino $\nu_\mu$ acquires mass via the exchange of two $\phi^\pm_5$ bosons (in gauge basis) with overlapping loops, as shown in Fig. 2. Because the three left-handed neutrinos do not have right-handed partners, one cannot have analogous two-loop nested diagrams as for charged lepton mass generation [6]. Fig. 2 is generated as follows. From nearest neighbor Yukawa couplings, which follow from the $Z_8$ charge assignments, the left-handed $\nu_\mu$ first couples to a right-handed $\tau$ via the $f_{23}$ term. An $m_\tau$ insertion is necessary to flip the
chirality of $\tau$, before another charged $\phi_5$ scalar emission flips $\tau_L$ into $N_R$ via the $\tilde{f}_{34}$ term. After $m_R$ insertion, the sequence is repeated in reverse order. In all, two $m_\tau$ insertions are necessary, but ultimately $m_R$ provides the seed. It is also clear that only the $\phi_5^{\pm}$ boson enters the diagram, hence the process does not depend on scalar mixing in any crucial way. The contribution from Fig. 2 is

$$\langle m_\nu \rangle_{22} = -(f_{23}\tilde{f}_{34})^2 m_R m_\tau^2 \int \frac{d^4 p}{(2\pi)^4}\frac{1}{p^2 - m_\tau^2} \left[ \frac{s_+^2}{p^2 - m_{H_1^+}^2} + \frac{c_+^2}{p^2 - m_{H_2^+}^2} \right] \times$$

$$\int \frac{d^4 q}{(2\pi)^4}\frac{1}{q^2 - m_\tau^2} \left[ \frac{s_+^2}{(p + q)^2} + \frac{c_+^2}{q^2 - m_{H_1^+}^2} \right].$$

(6)

In the above $s_+^2$ and $c_+^2$ are short-hand for $\sin^2 \theta_+$ and $\cos^2 \theta_+$, respectively, and we have also ignored $m_R$ in the denominator since $m_R \ll m_\tau$. This approximation simplifies the calculation. The presence of two tau mass insertions makes the diagram finite. Note that the two-loop Dirac mass term $(m_\nu')_{24}\bar{\nu}_L N_R$ can be rotated away and does not contribute to $m_\nu'$ directly, but modifies the Dirac mass for $N$ slightly. Since each power of $m_\tau$ is at one-loop order [6], Fig. 2 is effectively at four-loop order. In contrast, the contribution from the exchange of two W bosons [5] is effectively at six-loop order within the model, since mixing in leptonic charged current is also loop induced [4]. Hence, numerically it can be neglected in our model.

Eq. (6) can be reduced to

$$\langle m_\nu \rangle_{22} = -\left(\frac{f_{23}\tilde{f}_{34}}{16\pi^2}\right)^2 m_R m_\tau^2 \sum_{i,j=H_1^+,H_2^+} \frac{g_{ij}^{(\mu)}}{(m_i^2 - m_{H_1^+}^2)(m_j^2 - m_{H_2^+}^2)} \times$$

$$\int_0^1 dx \left\{ m_j^2 \left[ -Li_2(-u)\right]_{u=(a_{ij} - x)/x} - m_j^2 \left[ -Li_2(-u)\right]_{u=(a_{i\tau} - x)/x} \right\},$$

(7)

where

$$g_{ij}^{(\mu)} = \begin{pmatrix} s_+^4 & s_+^2 c_+^2 \\ s_+^2 c_+^2 & c_+^4 \end{pmatrix},$$

$$a_{xy} = m_x^2/m_y^2, \quad (x, y = i, j, \tau).$$

(8)

Here, $Li_2(u)$ is the dilogarithm (polylogarithmic function of order 2). The integral in eq. (7) is evaluated numerically. Note that the Majorana mass $(m_\nu')_{24}$ is of the same loop order as
Nevertheless, it is still reasonable to use \((m_\nu)_{22}\) to represent \(m_\nu\), since these higher order masses are much less than the tree level mass \(m_R\). To estimate the range of \(m_\nu\), we will take the maximal value \(m_R = 0.5\) MeV (the electron mass!) in eq. (7). As mentioned in ref. [8], the nonstandard Higgs bosons cannot be too far above the electroweak scale while all Yukawa couplings should be of order unity. We therefore take \(f_{23} = \tilde{f}_{34} = \sqrt{2}\) and fix \(\sin \theta_+ = 0.2\). We plot in Fig. 3 \(m_\nu\) versus \(m_{H_2^+}\)∈\((125, 4000)\) GeV, for \(m_{H_1^+} = 125, 250, 500, 1000\) GeV. We see that \(m_\nu\) can reach up to the order of \(10^{-2}\) eV. It is likely to be within the range of \(10^{-3} - 10^{-2}\) eV for \(m_{H_2^+}\) below 2 TeV. In general, it is insensitive to \(m_{H_1^+}\) if \(\sin \theta_+\) is not too large, since Fig. 2 is due to \(\phi_5^+\) alone in gauge basis.

The electron neutrino \(\nu_e\) acquires mass via the exchange and mixing of \(\phi_3^-\) and \(\phi_5^-\), as shown in Fig. 4. The left-handed electron neutrino flips to a right-handed muon by emitting a \(\phi_3^-\) boson via the \(f_{12}\) coupling. The right-handed muon then emits a \(\phi_5^-\) boson and changes into left-handed tau neutrino. Since \(\nu_\tau\) has acquired Majorana mass via Fig. 1, \(m_\nu\) now effectively functions as seed. The sequence is then repeated in reverse order. It is clear that \(\phi_3^+ - \phi_5^+\) mixing is needed. Since \(m_\nu\) is effectively at one-loop order, this is effectively a three-loop diagram. The contribution from Fig. 4 is

\[ (m_\nu)_{11} = -(f_{12} f_{32})^2 s^2_+ c^2_+ m_\nu \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{1}{p^2 - m_{H_1^+}^2 + m_{H_2^+}^2} \times \]

\[ \int \frac{d^4 q}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - m_{H_1^+}^2 + m_{H_2^+}^2} \times \]

\[ \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 - m_{H_1^+}^2 + m_{H_2^+}^2} \times \]

where we have ignored \(m_\nu\) in the denominator since \(m_\nu \ll m_{H_1^+}, m_{H_2^+}\). After some calculation the above equation becomes

\[ (m_\nu)_{11} = \left( \frac{f_{12} f_{32}}{16\pi^2} \right)^2 s^2_+ c^2_+ m_\nu \sum_{i,j=H_1^+, H_2^+} \frac{g_{ij}^{(e)} m_j^2}{m_i^2} \times \]

\[ \int_0^1 x dx [-Li_2(-u)]_{u=\{a_{ij} - x\}/x}, \]

where

\[ g_{ij}^{(e)} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}. \]
The diagram is logarithmically divergent, but the GIM-like mechanism of $\phi_3^\pm - \phi_5^\pm$ mixing renders the contribution finite. To evaluate the plausible range for $m_{\nu_e}$, we take $f_{12} = f_{32} = \sqrt{2}$, $m_{\nu_e} \sim 50$ eV, and plot, in Fig. 5, $m_{\nu_e}$ versus $m_{H_2}^+$ with $m_{H_1}^+ = 125, 250, 500, 1000$ GeV, fixing $\sin \theta_+ = 0.2$. We see that $m_{\nu_e}$ can reach up to order $10^{-3}$ eV if $H_1^+$ and $H_2^+$ are far from being degenerate. However, as expected, $m_{\nu_e}$ vanishes for $m_{H_1^+} = m_{H_2^+}$ and we see that this GIM-like cancellation effect is rather strong, and $m_{\nu_e}$ could be far below $10^{-3}$ eV.

Some discussion is in order. It may be asked why the effectively four-loop $\nu_\mu$ mass generation diagram is seemingly larger than the effectively three-loop $\nu_e$ mass generation diagram. The reason is because Fig. 2 depends on $\phi_5^\pm$ alone, and is not sensitive to charged scalar mixing. Indeed, we have taken the mixing angle $\sin \theta_+$ as “small” and the result depends largely on $m_{H_2^+}$, the physical charged scalar that is mostly the $\phi_5^+$ in gauge basis. In contrast, the mechanism in Fig. 4 for $\nu_e$ mass generation depends crucially on $\phi_3^\pm - \phi_5^\pm$ mixing. It provides a GIM-like mechanism that not only leads to divergence cancellation, it further suppresses the three loop contribution in an interesting parameter domain. From the perspective of the model, $H_1^+$ and $H_2^+$ mass should not be too widely apart [6], hence for the plausible range of $m_{H_{1,2}^+} \in (0.5, 2)$ TeV, $m_{\nu_e}$ is rather suppressed. But outside of this domain, if $H_1^+ - H_2^+$ splitting is rather sizeable, one in general finds Fig. 4 dominating over Fig. 2, as expected.

Second, although all light neutrino Majorana masses vanish with $m_R$, note that $\nu_\mu$ mass is directly related to $m_R$ and suffers from no cancellation mechanism. In contrast, $m_{\nu_\tau}$ and $m_{\nu_e}$ have separate cancellation mechanisms in the form of scalar mixing, and in fact $m_{\nu_e}$ depends on $m_R$ through an $\nu_\tau$ mass insertion. The reason behind the difference between $m_{\nu_\mu}$ vs. $m_{\nu_e}$, $m_{\nu_\tau}$ is the remnant $Z_2$ symmetry [8] after soft-breaking of $Z_8$: $\nu_\mu$ and $N_L$ is even, while $\nu_e$ and $\nu_\tau$ are odd, as are the nonstandard scalar bosons. Thus, $\nu_e$ and $\nu_\tau$ belong to the same class, while $\nu_\mu$ is more closely related to $N_L$. This dichotomy of leptons is very useful in suppressing dangerous transitions like $\mu \to e\gamma$. Third, it is of interest to look into questions related to neutrino oscillations. Inspection of Fig. 4 suggests that one has a $\nu_e - \nu_\tau$ Majorana mixing term also of three-loop order. The only difference is that for one of
the scalar loops, one has $\phi_5^\pm$ throughout, hence one is less sensitive to the GIM like $\phi_3^+ - \phi_5^+$ mixing effect, although it is still needed for sake of divergence cancellation. Note however that this off-diagonal term is still at three-loop order and proportional to the one-loop $m_{\nu_\tau}$ mass. Hence, we estimate the $\nu_e - \nu_\tau$ mixing angle to be of order $10^{-5} - 10^{-4}$, which is almost independs of $m_{\nu_\tau}$ value, but depends on the fact that there are two extra loops compared to Fig. 1. The mass eigenvalue is barely changed from that of eq. (10). The mass difference between $\nu_e$ and $\nu_\tau$ is basically just $m_{\nu_\tau}$. We have used it to saturate the cosmological bound, but it can be lowered if one is willing to entertain a lower $m_R$ value. However, since the tiny mixing angle is more or less independent of $m_{\nu_e}$, we find that our model cannot provide a basis for a MSW-type solution to the solar neutrino problem. Finally, we have to face the issue of naturalness of the $m_R$ scale. One of the most interesting aspects of our radiative model for charged lepton mass generation is that all dimensional parameters are of order weak scale, while all dimensionless parameters are of order one. To allow for Majorana neutrino mass generation, however, we need $m_R$ to be less than one MeV, which seems to run against the spirit of the model. We note, however, that $m_R$ in the model is a very different parameter from all the others. It is not related in anyway to the Higgs bosons introduced. It is basically a free parameter in the same way that $m_e$ is a parameter for QED. It is clear that setting it to zero one restores a larger chiral symmetry. We therefore would like to advocate a lenient attitude towards the naturalness question for $m_R$.

In summary, if new sequential leptons are found at the weak scale, the traditional seesaw mechanism for neutrino mass generation, as well as $SO(10)$ based GUT theories, may be in jeopardy. One may therefore face a serious challenge with the solar neutrino problem. We provide a new radiative mechanism for generating tiny Majorana neutrino masses. Because of remnant $Z_2$ symmetry, $\nu_\tau$ and $\nu_e$ receive Majorana mass in a different way than $\nu_\mu$, although in each case the tree level Majorana mass for heavy neutral lepton, $m_R$, provides the seed. If one saturates the cosmological bound with $m_{\nu_\tau}$, then $m_{\nu_\mu}$ and $m_{\nu_e}$ are typically of order $10^{-2}$ and $10^{-3}$ eV, respectively, although $m_{\nu_e}$ can be considerably smaller. Unfortunately, $\nu_e - \nu_\tau$ mixing is loop induced and is too small to sustain an MSW solution to the solar
neutrino problem.

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FIGURES

FIG. 1. Radiative mechanism for $m_{\nu_e}$.

FIG. 2. Radiative mechanisms for $m_{\nu_\mu}$.

FIG. 3. $m_{\nu_\mu}$ vs. $m_{H_2}^+$ for (top to bottom) $m_{H_1}^+ = 125, 250, 500, 1000$ GeV at fixed $\sin \theta_+ = 0.2$.

FIG. 4. Radiative mechanisms for $m_{\nu_e}$.

FIG. 5. $m_{\nu_e}$ vs. $m_{H_2}^+$ for (top to bottom) $m_{H_1}^+ = 125, 250, 500, 1000$ GeV at fixed $\sin \theta_+ = 0.2$. 