Boosting Standard Classification Architectures Through a Ranking Regularizer

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Abstract

We employ triplet loss as a feature embedding regularizer to boost classification performance. Standard architectures, like ResNet and Inception, are extended to support both losses with minimal hyper-parameter tuning. This promotes generality while fine-tuning pretrained networks. Triplet loss is a powerful surrogate for recently proposed embedding regularizers. Yet, it is avoided due to large batch-size requirement and high computational cost. Through our experiments, we re-assess these assumptions.

During inference, our network supports both classification and embedding tasks without any computational overhead. Quantitative evaluation highlights a steady improvement on five fine-grained recognition datasets. Further evaluation on an imbalanced video dataset achieves significant improvement. Triplet loss brings feature embedding capabilities like nearest neighbor to classification models. Code available at http://bit.ly/2LNYEqL

1. Introduction

Standard convolutional architectures \cite{8, 38} learn powerful representation for classification. Pretrained ImageNet \cite{4} weights scale their strength through fine tuning to novel domains and relax the large labeled dataset requirement. Yet, the learned representation through softmax attains limited intra-class compactness and inter-class separation. To advocate for a better embedding quality, we propose a two-head architecture. We leverage triplet loss \cite{32} as a classification regularizer. It promotes a better feature embedding by attracting similar and repelling different classes as shown in Figure 1. This embedding also raises classification model interpretability by enabling nearest neighbor retrieval.

Embedding losses have been successfully applied in conjunction with softmax loss as regularizers. For example, center loss \cite{42} was proposed for better face recognition efficiency. Magnet loss \cite{29} generalizes the unimodality assumption of center loss. A recent triplet-center loss (TCL) \cite{9} uses only a unimodal embedding but introduced a repelling force between class centers, \textit{i.e.}, inter-class margin maximization. All these methods assume a fixed number of class centers (embedding modes) for all classes.

Unlike the aforementioned approaches, the standard triplet loss requires no explicit number of embedding modes. Thus, it avoids computing class centers while promoting intra-class compactness and inter-class margin maximization. Surprisingly, recent papers \cite{42, 9} do not report the softmax+triplet loss quantitative evaluation. Assumptions about large training batch requirement \cite{32} for faster convergence or high batch-processing complexity, to compute pairwise distance matrix, have hindered triplet loss’s adoption. Our experiments reassess these assumptions through multiple triplet loss sampling strategies.

To incorporate embedding losses, previous approaches employ loss-specific architectures. This custom setting is imperfect for the softmax baseline as it omits the pre-trained ImageNet weights. Through our proposed seamless integration into standard CNNs, we push our baselines’ limits. We introduce an embedding head similar to the classification head. Each head applies a single fully connected (FC) layer on the pre-logit convolutional layer features. Figure 2 shows our two head architecture where the pre-logit convolutional features support both softmax and triplet losses for classification and embedding respectively. This integration boosts classification performance while promoting better embedding.

We evaluate our approach on various classification domains. The first is a fine-grained visual recognition (FGVR)
across five datasets. The second domain is an ego-motion action recognition task with high class imbalance. Large improvements (1-4\%) are achieved in both domains. Evaluation on multiple architectures with the same hyperparameters highlights our approach’s generality. The large batch size requirement represents a key challenge for triplet loss adoption; Schroff et al. \[32\] use a batch-size $b = 1800$ and trained on a CPU cluster for 1,000 to 2,000 hours. In our experiments, we show that using a small batch size $b = 32$ still improves performance. A further qualitative evaluation highlights beneficial qualities like nearest neighbor retrieval added to standard classification architectures. In summary, the key contributions of this paper are:

1. A two-head architecture proposal that uses triplet loss as a regularizer to boost standard architectures’ performance through promoting a better feature embedding.
2. A re-evaluation of the large batch size requirement and high computational cost assumptions for triplet loss.
3. Enable better nearest neighbor retrieval on standard classification architectures.

### 2. Related Work

Visual recognition deep networks employ softmax loss as follows

$$L_{\text{soft}} = - \frac{1}{N} \sum_{i=1}^{b} \log \frac{e^{x_{i}^{T}x_{i}}}{\sum_{j=1}^{n} e^{W_{i,j}^{T}x_{i}}}, \quad (1)$$

where $x_{i} \in \mathbb{R}^{d}$ denotes the $i$th deep feature, belonging to the $y_{i}$th class. In standard architectures, $x_{i}$ is the pre-logit layer; the result of flattening the pooled convolutional features as shown in Figure 2. $W_{i,j} \in \mathbb{R}^{d}$ denotes the $j$th column of the weights $W \in \mathbb{R}^{d \times n}$ in the last fully connected layer. $b$ and $n$ are the batch size and the number of class respectively. The softmax loss only cares about separating samples from different class. It disregards properties like intra-class compactness and inter-class margin maximization. Embedding regularization is one way to tackle this limitation. Figure 3 depicts different embedding regularizers; all require an explicit number of embedding modes.

#### 2.1. Center Loss

Wen et al. \[42\] propose center loss to minimize intra-class variations. By maintaining a per class representative feature vector $c_{y_{i}} \in \mathbb{R}^{d}$, the novel loss term in equation 2 is proposed. The class centers are computed by averaging corresponding class features. They are updated after every training mini-batch. To avoid perturbations caused by noisy samples, a hyper-parameter $\alpha$ controls the learning rate of the centers, i.e., moving average.

$$L_{\text{cen}} = \frac{1}{2} \sum_{i=1}^{b} \| x_{i} - c_{y_{i}} \|^{2}. \quad (2)$$

#### 2.2. Magnet Loss

Rippel et al. \[29\] propose a center loss term supporting multi-modal embedding, dubbed magnet loss. It computes $K$ class representatives, i.e., $K$-clusters per class. Each sample is iteratively assigned to one of the $K$ clusters and pushed towards its center. The magnet loss adaptively sculpts the representation space by identifying and enforcing intra-class variation and inter-class similarity. This is formulated as follows

$$L_{M} = \frac{1}{N} \sum_{i=1}^{N} \log \frac{\exp(-\frac{1}{2\sigma^{2}} \| x_{i} - \mu_{k} \|^{2} - \alpha)}{\sum_{i \in C_{k}} \sum_{k=1}^{K} \exp(-\frac{1}{2\sigma^{2}} \| x_{i} - \mu_{k} \|^{2} - \alpha)}, \quad (3)$$

where $N$ and $K$ are the number of samples and clusters per class respectively. $x_{i}^{k} \in \mathbb{R}^{d}$ denotes the $i$th deep feature, belonging to cluster $k$ in the $y_{i}$th class, $\mu_{k}^{c} \in \mathbb{R}^{d}$ is the $k$th cluster center belonging to class $c$. Finally $\alpha^{2} = \frac{1}{\sigma^{2}}$.\[\[\text{end}]}
Figure 3: Visualization of softmax and feature embedding regularizers. Softmax separates samples with neither class compactness nor margin maximization considerations. Center loss promotes unimodal compact class while magnet loss supports multi-modal embedding. Triplet center loss strives for unimodal, margin maximization and class compactness. The computed classes’ centers are depicted using a star symbol.

\[ \frac{1}{N-1} \sum \| x^k_i - \mu^k_c \|^2 \] is the variance of all samples from their respective centers. One criticism of magnet loss is the complexity overhead to maintain multiple clusters per class and their assigned samples. Moreover, the constant number of clusters per-class disputes with imbalanced data distributions.

### 2.3. Triplet Center Loss

While promoting class compactness, the center loss depends on the softmax loss supervision signal to push different classes apart. The learned features optimized with the softmax loss supervision signal are not discriminative enough, i.e., no explicit repelling force pushes different classes apart. Inter-class clusters can overlap due to missing an explicit inter-class repelling incentive. He et al. [9] propose triplet center loss (TCL) to avoid this limitation. By maintaining a per class center \( c_{yi} \in \mathbb{R}^d \) similar to [42], TCL is formulated as follows:

\[ L_{tcl} = \frac{1}{b} \sum_{i=1}^{b} \left[ (D(x_i, c_{yi}) - \min_{j \neq i} D(x_i, c_{yj}) + m) \right]_+, \quad (4) \]

where \( m \) is a separating margin, \([x]_+ = \max(0, x)\) and \( D(\cdot) \) represents the squared Euclidean distance function.

Triplet loss is a well-established surrogate for TCL. It achieves the intra and inter-class embedding objectives without computing class centers. Yet, it is largely avoided for its computational complexity and large training batch requirement assumptions. In the experiment section, we address these concerns and evaluate the utility of triplet loss as a regularizer. Our approach is evaluated on the challenging FGVR task where intra-class overwhelm inter-class variations. Further evaluation on the Honda driving dataset (HDD) demonstrates our approach’s competence on an imbalanced video dataset. Triplet loss regularization not only lead to higher classification accuracy but also enables better feature embedding.

### 3. The Triplet Loss Regularizer

The next subsection introduces triplet loss [32] as a softmax loss regularizer. Then, we explain our standard architectural extension to integrate an embedding loss.

#### 3.1. Triplet Loss

Triplet loss [32] has been successfully applied in face recognition [32, 31] and person re-identification [3, 35, 30]. In both domains, it is used as a feature embedding tool to measure similarity between objects and provide a metric for clustering. In this work, we utilize triplet loss as a classification regularizer. It is more efficient than contrastive loss [7, 20], and less computationally expensive than quadruplet [13, 2] and quintuplet [12] losses. While the pre-logits layer learns better representations for classification using the softmax loss, triplet loss promotes a better feature embedding. Equation 5 shows the triplet loss formulation

\[ L_{tri} = \frac{1}{b} \sum_{i=1}^{b} \left[ (D(a_i, p_i) - D(a_i, n_i) + m) \right]_+, \quad (5) \]

where an anchor image’s embedding \( a \) of a specific class is pushed closer to a positive image’s embedding \( p \) from the same class than it is to a negative image’s embedding \( n \) of a different class. Equation 6 is our loss function with a balancing hyper-parameter \( \lambda \).

\[ L = L_{soft} + \lambda L_{tri}. \quad (6) \]

**Sampling:** Triplet loss performance is dependent on its sampling strategy. We evaluate both the hard [10] and semi-hard [32] sampling strategies. In semi-hard negative sampling, instead of picking the hardest positive-negative samples, all anchor-positive pairs and their corresponding semi-hard negatives are considered. Semi-hard negatives satisfy equation [7]. They are further away from the anchor than the
3.2. Two-Head Architecture

Standard convolutional architectures, with ImageNet [4] weights, are employed in various applications for their powerful representation. We seek to leverage pre-trained standard networks for their advantages in tasks like fine-grained visual recognition [22, 18, 17]. This key integration promotes the generality of our approach and distances our work from [42, 9, 36] which use custom architectures. Through experiments, we demonstrate how triplet loss achieves superior classification efficiency compared to center loss.

Unlike VGG [34], recent architectures [8, 39, 14] employ a convolutional layer before the classification head. To generate logits, the classification head pools the convolutional layer features, flatten them, then utilize a customizable fully connected layer to support various numbers of classes. Similarly, we integrate triplet loss to regularize embedding as shown in Figure 6. Before pooling, we flatten the convolutional layer features then apply another fully connected layer $W_{emb}$ to generate embeddings as illustrated in equation (11).

$$Logits = W_{logits} \ast \text{flatten}(x) \quad (8)$$

$$Embedding = W_{emb} \ast \text{flatten}(h), \quad (9)$$

where $x = \text{pool}(h)$. Orderless pooling, like averaging, disregard spatial information. Thus, a fully connected layer $W_{emb}$ applied on $h$ has a better representation power. The final embedding is normalized to the unit-circle and the square Euclidean distance metric is employed. During inference, the two-head architecture enables both classification and retrieval with negligible overhead.
We evaluate our approach on five FGVR datasets. We evaluate our approach against two baselines:

- The **function** max(softplus function)
- the hard margin

A soft margin between classes is imposed by the hyper-parameter \( \lambda \). We evaluated hard sampling, hard \([32]\) sampling variants are evaluated. By default, our approach is complete. Table 2 presents our fine-tuning quantitative evaluation on the five datasets. Our two-head architecture with hard triplet loss achieves large steady (1-4\%) improvement on ResNet-50. Similar trend appears with Inception-V4 but suffers an interesting fluctuation between hard and semi-hard triplet loss. Section 4.3 reflects on this phenomenon through a quantitative embedding analysis. Vanilla DenseNet-161 achieves comparable state-of-the-art results on all FGVR datasets, yet triplet loss regularizer maintains a steady trend of performance improvement.

Center loss achieves an inferior classification performance especially on the Dogs dataset – a lag \( \approx 4\% \) behind vanilla softmax on Inception-V4 and DenseNet-161. The single mode embedding assumption is valid for face recognition \([42]\) and vehicle re-identification \([23]\) because different images for the same identity belong to a single cluster. However, when working with categories of high intra-class variations, this assumption degenerates the feature embedding instability or collapse.

Our simple but vital integration into standard architectures distorts our approach from similar softmax+clustering formulations. In addition, all recent convolutional architectures share similar ending structure; the last convolutional layer is followed by an average pooling, and then a single fully connected layer. Thus, apart from the studied architectures, our secondary embedding head proposal can be applied to other architectures, e.g., MobileNet \([11]\).

### 4.2. Task Generalization

For further evaluation, we leverage the Honda Research Institute Driving Dataset (HDD) \([28]\) for action recognition. HDD is an ego-motion video dataset for driver behavior understanding and causal reasoning. It contains 10,833 events spanning eleven event classes. Moreover, the HDD event class distribution is long-tailed which poses an im-

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**Table 1**: Statistics of five FGVR datasets and their corresponding train, validation and test splits.

| Dataset     | Num Classes | Avg samples Per Class | Train Size | Val Size | Test Size | Total Size |
|-------------|-------------|-----------------------|------------|----------|-----------|------------|
| StanfordCars| 102         | 10                    | 1020       | 1020     | 6149      | 8189       |
| Aircrafts   | 100         | 100                   | 3334       | 3334     | 3333      | 10000      |
| Flower-102  | 550         | 43.5                  | 23929      | 23929    | 24633     | 48562      |
| StanfordDogs| 196         | 41.55                 | 8144       | 8144     | 8041      | 16185      |
| Aircrafts   | 120         | 100                   | 12000      | N/A      | N/A       | 20580      |

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Table 2: Quantitative evaluation on the five FGVR datasets using ResNet-50, Inception-V4, and DenseNet-161.

Table 3: Action recognition quantitative evaluation on the Honda dataset. \( b \) indicates the batch-size used. Macro average accuracy highlights performance on minority classes.

Table 4: Detailed evaluation on the Honda driving dataset. Our two-head architecture using semi-hard triplet loss achieves better performance on minority classes.

They are converted to grayscale, and then every consecutive pair is subtracted to create a stack of difference \( \in Z^{W \times H \times 5} \) as depicted in Figure 8. Standard architectures are easily adapted to this input representation by treating the SOD input as a five-channel image instead of three. Unlike FGVR input \( \in [0, 255] \), SOD \( \in [-255, 255] \). Thus, a ResNet-50 [8] architecture initialized with random weights is employed. It is trained for 10K iterations with \( \lambda = 1 \) and a polynomial decaying learning rate \( lr = 0.01 \). Batch sizes 33 and 63 are used to compare the vanilla softmax against our approach. To highlight performance on minority classes, both micro and macro average accuracies are reported in Table 3. Macro-average computes the metric for each class independently before taking the average. Micro-average is the traditional mean for all samples. Macro-average treats all classes equally while micro-averaging favors majority classes. Table 4 highlights the efficiency of our approach on minority classes.

4.3. Retrieval Evaluation on FGVR

In the two-head architecture, the secondary embedding head brings values like an enhanced feature embedding, nearest neighbor retrieval and interpretability. Following Song et al. [27], we evaluate the quality of feature embedding using Recall@K metric on the test split. We also leverage the Normalized Mutual Info (NMI) score to evaluate the quality of cluster alignments. NMI = \( \frac{I(C;I)}{\sqrt{H(I)H(C)}} \), where \( \Omega = \{ c_1, ..., c_n \} \) is the ground-truth clustering while \( C = \{ c_1, ..., c_n \} \) is a clustering assignment for the learned
Our approach has two hyper-parameters: $\lambda$ and the embedding dimensionality $d_{emb}$. $\lambda$ is tuned on the Flowers-102 dataset through the validation split. All hyper-parameter tuning experiments are executed for 2000 iterations. Figure 11 highlights $\lambda$ stability within $[0.1, 2]$. A larger $\lambda$ making triplet loss dominant is discouraged. Intuitively, further hyper-parameters tuning can achieve better performance.

### 4.4. Ablation Analysis

#### Hyper-Parameter Stability

Our approach has two hyper-parameters: $\lambda$ and the embedding dimensionality $d_{emb}$. $\lambda$ is tuned on the Flowers-102 dataset through the validation split. All hyper-parameter tuning experiments are executed for 2000 iterations. Figure 11 highlights $\lambda$ stability within $[0.1, 2]$. A larger $\lambda$ making triplet loss dominant is discouraged. Intuitively, further hyper-parameters tuning can achieve better performance.

#### Two-Head Time Complexity

The computational cost of the embedding head is negligible. Both sampling and back-propagation are implemented on GPU. Training time in-
Figure 9: Retrieval qualitative evaluation on three FGVR datasets: Flowers-102, Aircrafts and Cars. Given a query image, the three nearest neighbors are depicted. The three consecutive rows show search results using center loss, semi-hard and hard triplet regularizers. Green and red outlines denote match and mismatch between the query and it’s result respectively.

Figure 10: Qualitative misclassification interpretation. The odd columns show a misclassified test image while the even columns show the nearest neighbor from the training split.

Figure 11: Hyper-parameter \( \lambda \) tuning on the Flowers-102 dataset.

Increases by 1%, 3%, and 2% for semi-hard, hard and center losses on Titan XP GPU, respectively. Figure 12 shows a time complexity analysis in terms of batch processing time (secs). Please note that triplet loss approaches retain from computing classes centers or enforcing a specific number of modes.

4.5. Discussion

Our experiments demonstrate how a two-head architecture with triplet loss outperforms a vanilla single-head softmax network. Triplet loss attains the center loss, triplet center loss and magnet loss objectives without enforcing explicit class representatives. It promotes both intra-class compactness and inter-class margin maximization. Semi-hard triplet loss relaxes the unimodal embedding constraint while maintaining stabler learning curve. Hard triplet loss achieves larger improvement margins but can suffer model collapse. Triplet loss effectively regularizes softmax and promote better feature embedding.

The two-head architecture with triplet loss is the main scope of this paper. Investigating other recent ranking losses, e.g. Margin loss [43], and comparing their benefits to softmax remains an open question.

5. Conclusion

We propose a seamless integration of triplet loss as an embedding regularizer into standard classification architectures. The regularizer competence is illustrated on multiple datasets, architectures and recognition tasks. Triplet loss, without the large batch requirement, boosts standard architectures’ performance. With minimal hyper-parameter tuning and a single fully connected layer on top of pretrained standard architectures, we promote generality to novel domains. Promising results are achieved on an imbalanced dataset. We incur a minimal computational overhead during training, but raise classification model efficiency and interpretability. Our architectural extension enables both retrieval and classification tasks during inference.
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6. Supplementary Material

The next subsections provide more details about our architecture and training procedure’s technicalities. Further quantitative evaluations on fine-grained visual recognition (FGVR) are presented. Finally, we demonstrate the training procedure for the Honda Research Institute Driving Dataset.

6.1. Fine-Grained Visual Recognition

Figure 2 in the main paper presents our two-head architecture. The pre-logit layer $x$ supports the softmax loss. Similarly, triplet loss utilizes $h$, where $x = \text{pool}(h)$. The network outputs, both logits and embedding, are formulated as follows.

\[
\text{logits} = W_{\text{logits}} \ast \text{flat}(x) \quad (10)
\]

\[
\text{embedding} = W_{\text{emb}} \ast \text{flat}(h). \quad (11)
\]

Orderless pooling, like averaging, reduces $h$ dimensionality but loses spatial information. For example, in DenseNet161, $h \in \mathbb{R}^{7 \times 7 \times 2208}$ while $x \in \mathbb{R}^{1 \times 1 \times 2208}$. Thus, $W_{\text{emb}}$ employs $h$, instead of $x$, to improve feature embedding. Figure S1 illustrates how $h$ provides a finer control level while learning $W_{\text{emb}}$.

Figure S2 shows a t-SNE visualization for Flowers-102 embedding using 50 random classes, 20 samples per class. In the main paper, the inferior performance of triplet loss with hard-mining is associated with convergence to bad local minima, i.e., a collapsed model (i.e. $f(x) = 0$) [32]. To examine such assumption, we train a DenseNet for 400K iterations on Stanford Dogs. This large number of iterations increases the chances of a model collapse. Figure S3 presents the performance on the test split after every 50K iterations. Triplet loss with hard-mining is evaluated with both soft and hard margin. Soft margin applies the softplus function $\ln(1 + \exp(x))$ while hard margin uses a fixed margin $m = 0.2$. The triplet loss with hard-mining deteriorates with soft margin when trained for a large number of iterations. Hard-mining with hard margin is more robust. We found similar behavior on other datasets like Stanford Cars and Aircrafts datasets.

Table 5 in the main paper presents a quantitative analysis for the feature embedding learned by the second head in our proposed architecture. Similarly, Table S1 presents feature embedding quantitative analysis using the architecture penultimate layer, i.e., layer $x$ (Figure 2 in the main paper). This layer is present in both our proposed two-head and single-head (vanilla softmax) architecture. Similar to Table 5, the triplet loss embedding is superior to the softmax embedding. Triplet loss with hard-mining achieves the best results on ResNet-50 but degrades on Inception-V4 trained for 80K iterations. Center loss achieves good results with DenseNet161 on NABirds but generally fluctuates and suffers with Inception-V4. Triplet loss with semi-hard margin achieves sub-optimal embedding but maintains the highest stability compared to center and hard-mining approaches.

Figure S4 graphically summarizes Table S1. It provides a comparative embedding evaluation between the single-head softmax versus the two-head with semi-hard triplet loss using recall@1 metric. Triplet loss improvements, over the softmax model, are reported as ($\Delta$). The Flowers-102 dataset has the smallest training split with 1020 images only. With this limited data, the head-two architecture
achieves marginal improvement if any.

Table S2 compares our proposed two-head architecture, using DenseNet161, with state-of-the-art approaches on the five FGVR datasets. Our two-head architecture with the semi-hard triplet loss regularizer achieves competitive results.

|   | NMI | R@1 | R@4 | R@8 | R@16 |
|---|-----|-----|-----|-----|------|
| Cars - ResNet | Vanilla | 0.791 | 77.88 | 91.17 | 94.65 | 98.07 |
|   | CNTR | 0.756 | 79.04 | 91.12 | 94.58 | 98.38 |
|   | SEMI | 0.823 | 81.41 | 92.79 | 95.91 | 97.74 |
|   | HARD | 0.853 | 83.31 | 94.30 | 96.82 | 98.07 |
| Flowers - ResNet | Vanilla | 0.810 | 88.76 | 95.21 | 97.47 | 98.49 |
|   | CNTR | 0.807 | 88.79 | 95.58 | 97.32 | 98.49 |
|   | SEMI | 0.818 | 89.48 | 95.82 | 97.37 | 98.37 |
|   | HARD | 0.742 | 90.78 | 95.56 | 96.93 | 97.98 |
| Dogs - ResNet | Vanilla | 0.526 | 48.74 | 71.90 | 80.92 | 87.81 |
|   | CNTR | 0.562 | 54.18 | 76.39 | 84.50 | 91.10 |
|   | SEMI | 0.794 | 77.26 | 90.65 | 93.10 | 95.78 |
|   | HARD | 0.829 | 84.01 | 91.63 | 94.21 | 95.65 |
| Aircrafts - ResNet | Vanilla | 0.697 | 62.17 | 87.85 | 92.26 | 94.99 |
|   | CNTR | 0.726 | 75.84 | 89.85 | 92.26 | 94.99 |
|   | SEMI | 0.767 | 77.84 | 90.79 | 93.91 | 95.85 |
|   | HARD | 0.839 | 84.01 | 91.63 | 94.21 | 95.65 |
| NABirds - ResNet | Vanilla | 0.697 | 62.17 | 87.85 | 92.26 | 94.99 |
|   | CNTR | 0.726 | 75.84 | 89.85 | 92.26 | 94.99 |
|   | SEMI | 0.767 | 77.84 | 90.79 | 93.91 | 95.85 |
|   | HARD | 0.839 | 84.01 | 91.63 | 94.21 | 95.65 |
| Cars - Inc-V4 | Vanilla | 0.690 | 72.47 | 86.77 | 90.88 | 93.55 |
|   | CNTR | 0.496 | 61.55 | 79.09 | 85.09 | 89.69 |
|   | SEMI | 0.788 | 81.46 | 92.14 | 94.64 | 96.37 |
|   | HARD | 0.566 | 63.70 | 82.04 | 87.54 | 91.42 |
| Flowers - Inc-V4 | Vanilla | 0.707 | 85.62 | 93.74 | 95.95 | 97.56 |
|   | CNTR | 0.707 | 85.62 | 93.74 | 95.95 | 97.56 |
|   | SEMI | 0.801 | 89.58 | 95.23 | 96.91 | 97.84 |
|   | HARD | 0.731 | 92.68 | 96.21 | 97.27 | 98.32 |
| Dogs - Inc-V4 | Vanilla | 0.423 | 24.11 | 62.77 | 74.29 | 81.53 |
|   | CNTR | 0.453 | 27.66 | 69.44 | 79.78 | 86.05 |
|   | SEMI | 0.609 | 55.03 | 76.50 | 84.44 | 90.23 |
|   | HARD | 0.568 | 63.09 | 81.71 | 87.49 | 90.88 |
| Aircrafts - Inc-V4 | Vanilla | 0.460 | 60.79 | 85.18 | 89.23 | 91.97 |
|   | CNTR | 0.546 | 61.60 | 79.75 | 83.33 | 89.53 |
|   | SEMI | 0.751 | 78.13 | 89.20 | 91.78 | 94.47 |
|   | HARD | 0.831 | 86.26 | 91.87 | 93.49 | 94.72 |
| NABirds - Inc-V4 | Vanilla | 0.438 | 24.30 | 40.43 | 49.38 | 58.78 |
|   | CNTR | 0.438 | 24.30 | 40.43 | 49.38 | 58.78 |
|   | SEMI | 0.638 | 52.42 | 72.38 | 79.57 | 85.60 |
|   | HARD | 0.433 | 23.68 | 39.28 | 47.48 | 57.10 |
| Cars - Dense | Vanilla | 0.813 | 85.08 | 94.47 | 96.84 | 98.27 |
|   | CNTR | 0.787 | 87.36 | 93.17 | 94.64 | 95.97 |
|   | SEMI | 0.875 | 88.57 | 96.08 | 97.66 | 98.71 |
|   | HARD | 0.839 | 89.44 | 96.38 | 97.86 | 98.76 |
| Flowers - Dense | Vanilla | 0.838 | 89.28 | 98.25 | 98.94 | 99.58 |
|   | CNTR | 0.812 | 95.87 | 98.16 | 98.75 | 99.22 |
|   | SEMI | 0.864 | 95.40 | 98.39 | 99.09 | 99.46 |
|   | HARD | 0.865 | 95.79 | 98.50 | 99.14 | 99.50 |
| Dogs - Dense | Vanilla | 0.494 | 32.18 | 60.72 | 73.25 | 81.72 |
|   | CNTR | 0.720 | 76.96 | 84.00 | 88.19 | 91.96 |
|   | SEMI | 0.725 | 68.55 | 68.12 | 72.35 | 85.63 |
|   | HARD | 0.756 | 70.63 | 82.00 | 89.25 | 96.22 |
| Aircrafts - Dense | Vanilla | 0.708 | 79.06 | 91.06 | 94.06 | 98.49 |
|   | CNTR | 0.792 | 86.20 | 91.63 | 93.16 | 94.48 |
|   | SEMI | 0.853 | 84.49 | 94.15 | 95.68 | 96.97 |
|   | HARD | 0.896 | 85.51 | 95.70 | 98.83 | 99.49 |
| NABirds - Dense | Vanilla | 0.606 | 52.91 | 73.03 | 80.70 | 86.44 |
|   | CNTR | 0.818 | 75.28 | 86.86 | 90.85 | 93.69 |
|   | SEMI | 0.677 | 61.82 | 80.70 | 87.07 | 91.62 |
|   | HARD | 0.674 | 61.64 | 80.21 | 86.77 | 91.37 |

Figure S4: Comparative embedding evaluation between single-head softmax and two-head with semi-hard triplet loss using the penultimate layer in ResNet-50, Inception-V4 and DenseNet161 respectively. Triplet loss semi-hard improvements over the softmax model are reported as (Δ).

6.2. Autonomous Car Driving

The Honda Research Institute Driving Dataset (HDD) contains 137 sessions S. Each session S represents a navigation task performed by a driver. S is divided into 93, 5, and 36 sessions for training, validation and testing splits respectively. Three sessions are removed for missing annotations. HDD has four annotation layers to study the drivers’ actions: (1) Goal-oriented, (2) stimulus-driven, (3) cause and (4) attention. The Goal-oriented layer, utilized in our experiments, defines drivers’ actions to reach their destinations, e.g., left-turn and intersection passing. Ramanishka et al. [28] provides further details for the other three annotation layers.

Triplet loss mini-batches require both positive and negative samples. The FGVR datasets have uniform class distribution. Thus, training batches’ construction is straightforward by sampling random classes and their corresponding images as outlined in the main paper. On the other hand, HDD suffers class imbalance. A different batch construc-
Table S2: Quantitative evaluation on the five FGVR datasets using DenseNet161. Our two-head architecture with semi-hard triplet loss regularizer compares favorably with state-of-the-art results.

Algorithm 1 outlines our training procedure. First, $N_B$ mini-batches are constructed, each containing $b$ random actions. The batches’ embeddings are computed using $N_B$ feed forward passes. The $2D$ matrix $D_\phi$ stores the pairwise distance between the $N_B \times b$ actions. All positive pairs $(a, p)$ and their corresponding semi-hard negatives $n$ are identified. For a fair comparison with vanilla softmax approach, only $(b/3)$ random triplets $(a, p, n)$ are utilized for back-propagation. This process repeats for $N$ training iterations.

**Algorithm 1** HDD training procedure. In our experiments, $b = \{33, 36\}$ is the mini-batch size, $N_B = 3$ is the number of mini-batches, and $N = 10K$ is number of training iterations.

```
for all iteration $i$ in $N$ do
    $S_\phi = \Phi$
    for all $j$ in $N_B$ do
        Add a random batch, of size $b$, to $S_\phi$
    end for
    Compute action embeddings $E_\phi$ for $S_\phi$
    Compute pairwise distance matrix $D_\phi$ using $E_\phi$
    $T_{tri} = \Phi$
    Construct all positive pairs $(a, p)$
    for all $(a, p)$ in positive pairs do
        Find nearest semi-hard negative $n$ using $D_\phi$
        append $(a, p, n)$ to $T_{tri}$
    end for
    if $\text{len}(T_{tri}) > b/3$ then
        $T_{tri} = \text{shuffle}(T_{tri})[0 : b/3]$
    end if
    // $T_{tri}$ contains $b$ actions
    Feed-forward $T_{tri}$
    compute softmax + triplet losses and back-propagate.
end for
```