Interval of restitution coefficient for chattering in impact damper

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Abstract
Based on the impact damper, a dynamic model of a non-fixed constrained collision system was established. The coefficient of restitution is used as the main control parameter to analyze the system’s periodic movement and its bifurcation region. The chattering movement characteristics of the system were revealed. The interval of restitution coefficient for the chattering of collision system under various mass ratio and frequency ratio was obtained. The results show that the chattering phenomenon occurs in the collision system when the coefficient of restitution is greater than 0.5; as the mass ratio decreases, the interval of restitution coefficient for chattering continues to expand; as the frequency increases, the interval of restitution coefficient for chattering narrows.

Keywords
Impact damper, restitution coefficient, bifurcation, chattering

Introduction
Impact damper uses the collision between the free mass and the main system during the vibration process to control the main system’s responses.1–4 Usually, the current research findings on impact damper employ symmetry collision twice per period as the typical motion model.5,6 Based on the numerical stimulation, the author discovered that there were observable chattering phenomenon in the motion trajectory under the condition of better damping performance, that is, chattering might probably be a new, effective approach for the impact damper to achieve good damping performance.7 The term chattering refers to a condition when there are multiple or even endless times of collisions between the free mass and the main system in a short period of time. There is no available guidance on chattering studies in impact damper, at present.

In regard to studies on chattering, Budd and Dux8 conducted a systematical research on the chattering and viscosity motions of the single-freedom shock primary system under periodical excitation and deduced the existence of the system’s periodical chattering behavior as well as the relationship between the periodical chattering behavior and the chaotic motion. In Toulemonde and Gontier’s study9 on dynamic behavior of the impact primary system of the harmonious excitation, the researchers identified viscosity motion and thereby conducted a study on the periodical viscosity motion for both single freedom and multiple freedom. Wagg and Bishop10 researched the periodical movement, the chaotic chattering motion, and the viscosity motion in a harmonious-excitation two-freedom impact vibration system. Demeio and Lenci11 studied an approximate calculation method of the chattering duration time in an inverted pendulum collision, and their study revealed that the chattering duration time was primarily associated with the amplitude instead of the excitation frequency or the damping ratio. Using experimental and numerical approach, Alzate and Bernardo12 conducted a research on the chattering behavior of the impact model based on the gear transmission system. By introducing the local discontinuity mapping algorithm, Nordmark and Piironen13 studied the stability and bifurcation of chattering behavior in collision system. Quintana and Ciurana14 summarized the research progress, methodology, and classification of chattering in both industry

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and academic research. Besides, Hős and Champneys\textsuperscript{15} studied the impact model based on the pressure relief valve, discussing the model’s bifurcation, chattering behavior, and the path from bifurcation to chattering. Feng et al.\textsuperscript{16} studied the system’s complete chattering and non-complete chattering phenomena and in the meantime analyzed the chattering bifurcation in the system. Ding et al.\textsuperscript{17} carried out an analysis on the sticking motion in a vibro-impact system with multiple constraints. By introducing a discontinuous pull-back mapping method, Wang et al.\textsuperscript{18} studied the complete chattering of two-degree-of-freedom vibration system and obtained the chattering completion point and the chattering time. Zhu et al.\textsuperscript{19} applied the numerical simulation method to their study on the chattering motion of two-degree-of-freedom with soft impacts.

At present, most of the research on the dynamics of collision vibration system was based on the multi-degree-of-freedom system with fixed constraints, and there was no research on the chattering of the nonlinear system without fixed constraints and the change law with the coefficient of restitution. Since the constraint in impact damper is a free mass, a non-fixed constraint model is needed. Therefore, the study of the chattering in impact damper in this paper differed significantly with the existing ones. In this paper, the dynamic model of the collision vibration system with non-fixed constraint was established. The periodic motion and bifurcation region of the system were analyzed with the coefficient of restitution as the main control parameter, and the chattering characteristics of the system were revealed. The restitution coefficient range of chattering of impact damper under different mass ratio and frequency ratio was obtained.

System dynamic model and differential equation of motion

Figure 1 shows a dynamic model of an impact damper. Combining the same mechanical properties of parts, it uses mechanical symbols: M\textsubscript{1} as the mass of the chamber and external support, M\textsubscript{2} as free mass, K as system stiffness, C as system damper, F as system excitation force, X as the displacement of M\textsubscript{1}, Y as the displacement of M\textsubscript{2}. M\textsubscript{2} is inside M\textsubscript{1}. When F acts on M\textsubscript{1}, M\textsubscript{1} and M\textsubscript{2} will have relative collision, indicating the confirmation of displacement difference between them. As K and C act on M\textsubscript{1} and exert controlling force on the movement of M\textsubscript{1}, energy exchange occurs one another among C/K/M\textsubscript{1}/M\textsubscript{2}. Energy consumption of plastic transformation occurs between M\textsubscript{1} and M\textsubscript{2}, and C also consumes part of energy.

For the convenience of calculation, several basic hypotheses are proposed:

1. Free mass (the impactor) only moves horizontally along the chamber, with friction coefficient f = 0;
2. The friction coefficient between M\textsubscript{1} and the ground f = 0;
3. At this moment, the free mass is regarded as mass point;
4. The collision process is determined by the restitution coefficient, r, and the collision is instantaneous.

When there is no collision, the movement differential equation of vibrator M\textsubscript{1} is

\[ M\ddot{x} + C\dot{x} + Kx = F_0\sin \omega t \] (1)

In this equation, “\ddot{x}” means to find the derivative of time t, when \( F_0 \) is the excitation force amplitude, and \( \omega \) is excitation frequency. The dimensionless disposal of the equation is

\[ \ddot{x} + 2\zeta \dot{x} + x = \sin (\omega t + \phi) \] (2)

The dimensionless number is

\[ F_0\sin(\omega t) \]

Figure 1. Non-fixed constrained collision vibration system.
\[ \mu_m = \frac{M_2}{M_1} \]

\[ t = T \sqrt{\frac{K}{M_1}} \]

\[ x = \frac{KX}{F_0} \]

\[ y = \frac{KY}{F_0} \]

\[ \xi = \frac{C}{2 \sqrt{KM_1}} \]

\[ \delta = \frac{KD}{F_0} \]

\[ \omega = \Omega \sqrt{\frac{M_1}{k}} \]

Before collision, the small ball is regarded in uniform linear motion

\[ y(t) = y_0 + v_0 t \] (3)

In this equation, \( y(t) \) is the ball’s displacement, while \( y_0 \) and \( v_0 \) are the initial displacement and the initial speed (or the speed after collision) of the ball. The dimensionless disposal of equation (3) obtains

\[ \ddot{y} = 0 \] (4)

When \( |y-x| = D/2 \), there is collision between the vibrator and the small ball, and the collision is idealized as a discontinued process. Based on the law of conservation of momentum and the definition of the collision restitution coefficient, \( r \), there is

\[ \dot{x}^- + \mu m \dot{y}^- = \dot{x}^+ + \mu m \dot{y}^+ \] (5)

\[ r = \frac{\dot{y}^+ - \dot{x}^+}{\dot{y}^- - \dot{x}^-} \] (6)

Here, \( \mu_m = \frac{M_2}{M_1} \) is the mass ratio, while \( \dot{x}^- \) and \( \dot{y}^- \) are, respectively, the dimensionless instant speed of the vibrator \( M_1 \) and the small ball \( M_2 \) before the collision. \( \dot{x}^+ \) and \( \dot{y}^+ \) are, respectively, the dimensionless instant speed of the vibrator \( M_1 \) and the small ball \( M_2 \) after the collision.

Based on equations (5) and (6), the analytical expression of the speed after collision can be obtained

\[ \dot{x}^+ = \left( \frac{1 - \mu m r}{1 + \mu m} \right) \dot{x}^- + \mu m \left( \frac{1 + r}{1 + \mu m} \right) \dot{y}^- \] (7)

\[ \dot{y}^+ = \left( \frac{1 + r}{1 + \mu m} \right) \dot{x}^- + \left( \frac{\mu m - r}{1 + \mu m} \right) \dot{y}^- \] (8)

In the impact damper model made from the vibrator and the small ball, the movement equation (2) of the vibrator between two collisions is a 2-order non-homogeneous linear differential equation with constant coefficient. Therefore, the solution of equation (2) consists of the general solution of the corresponding homogeneous equation and the special solution of the non-homogeneous equation, which is

\[ x(t) = x_1(t) + x_2(t) \] (9)

In equation (9), \( x_1(t) \) is the system free vibration with damper. Under small damper circumstance, there is

\[ x_1(t) = e^{-\omega_n t}(c_1 \cos \omega_d t + c_2 \sin \omega_d t) \] (10)

Here, \( \omega_n = \sqrt{\frac{k}{M_1}} \), \( \omega_d \) is the natural frequency of the vibrator with damper. \( c_1 \) and \( c_2 \) are determined by the vibrator’s initial displacement and speed. \( x_1(t) \) is vibration attenuation, seen as transient vibration or transient response. In equation (2.9), \( x_2(t) \) represents the forced vibration of the system with damper, assumed as

\[ x_2(t) = Bs \sin(\omega t - \theta) \] (11)

In this equation, \( B \) is response amplitude; \( \theta \) is phase difference. By bringing equation (11) into (2), there are

\[ B = \frac{F_0}{k} \frac{1}{\sqrt{(1 - \lambda^2)^2 + (2\sigma \lambda)^2}} \] (12)

\[ \theta = \frac{2\sigma \lambda}{1 - \lambda^2} \] (13)
In equations (12) and (13), $\omega$ is the frequency of the excitation force, while $\lambda = \frac{\omega}{\omega_n}$ as the frequency ratio and $\sigma$ as the damper ratio. The total motion form represented by equation (2) is intermittent motion, while equation (9) shows that the movement is made of two parts. They are both intermittent motions of which $x_1(t)$ is the free vibration and $x_2(t)$ indicates the vibration is forced, that is, vibration with excitation. Its angular frequency is the angular frequency of the excitation force. When the excitation frequency approaches close to the system natural frequency, it enters the region around the resonance point. At that moment, there occurs a resonance phenomenon and the amplitude will increase dramatically. Compared to the forced vibration, the free vibration happens upon a longer timeline and declines constantly. The forced vibration constantly remains or increases the amplitude due to the energy input. Thus, the free vibration part is negligible, that is, $x_1(t)$ is ruled out to some extent.

Before the collision, the movement status of the vibrator and the small ball is shown in the equations. When the collision occurs, the movement will change, and thus, it is acceptable to define the time before and after the collision. For example, $t_0 = t_+$. The initial conditions for the displacement and velocity of the built model and the state at the moment after the collision are determined by the restitution coefficient $r$. The duration of the collision is difficult to quantify, but for the study of chattering, as the chattering duration, this process must be quantified. Therefore, in the solution to equation (10), it is assumed that there is a short period of time, $\Delta t$, after which the collision happens again. This approach addresses the collision interval problem. Each collision is in concert with the speed changing condition of the coefficient of restitution. Through these methods, we can calculate the changes of displacement and speed, then obtain the variation circumstance of the collision, and therefore acquire the system collision principles.

**Figure 2.** Bifurcation diagram of $\mu_m = 0.05$.

**Figure 3.** Bifurcation diagram of $\mu_m = 0.001$. 
Analysis on the system bifurcation movement

A collision vibration system is usually a multi-parameter system. When one or several parameters change to a certain critical value, the number and stability of the periodic solution will change, resulting in the so-called bifurcation phenomenon. This phenomenon can only be calculated and observed through computer. By numerous bifurcations, the system may enter a chaos state. As the control parameter during collision, the coefficient of restitution helps simplify many

Figure 4. Bifurcation of $r$ as bifurcation parameter ($\mu_m = 0.05$). (a) Phase plane diagram when $r = 0.2$ (b) Poincare mapping diagram when $r = 0.2$ (c) phase plane diagram when $r = 0.8$ (d) Poincare mapping diagram when $r = 0.8$.

Figure 5. $R$ as the bifurcation parameters diagram ($\mu_m = 0.001$). (a) Phase plane portrait when $r = 0.2$, (b) Poincare mapping diagram when $r = 0.2$, (c) phase plane portrait when $r = 0.8$, and (d) Poincare mapping diagram when $r = 0.8$. 
problems. To determine the collision process by the coefficient of restitution means the changes of speed and energy will generate a jump, which has an enormous influence on both bifurcation and chattering.

The bifurcation and chaos actions in the collision vibration system use $r$ as the bifurcation parameter. The results based on numerical integration are the bifurcation diagram of the system response, Poincare mapping figure, phase plane portrait, and time-domain waveform figure.

Figures 2 and 3 are the global bifurcation diagrams with different mass ratio. It can be seen from the figures that, after $r = 0.5$, the system generally has a large area of bifurcation and the area enlarges constantly.

In order to better observe the bifurcation condition, phase plane portraits and Poincare mapping diagrams are shown in Figures 4 and 5. To select a specific point before and after the bifurcation, when $r = 0.2$ and $r = 0.8$, it can be seen respectively that the system bifurcation has a significant difference before and after $r = 0.5$, while the system is under periodic or quasi periodic movement. The system enters a chaos movement after $r = 0.5$. The chattering phenomenon happens mostly in areas where bifurcation is intensive. Below is a study on the chattering behavior in the bifurcation interval.

**Figure 6.** Different $r$ of time-domain waveform figure ($\mu_m = 0.25$, $\omega = 1$). (a) $r = 0.5$ (b) $r = 0.6$ (c) $r = 0.7$ (d) $r = 0.8$ (e) $r = 0.9$. 
Analysis of the system chattering movement

In the previous section, a thorough analysis of the bifurcation phenomenon of a bilateral dynamic constraint system was conducted, and this section will study the influence of the coefficient of restitution on chattering. The chattering phenomenon is a branch of non-smooth dynamic system study, as well as a common phenomenon of non-smooth system. In order for the chattering in a collision vibration system, the impactor must remain a series of low-speed collision motions. If the speed of this series of collision motions eventually approaches zero and remains relatively still compared to the

![Graphs showing different r values](image)

**Figure 7.** Different r of time-domain waveform figure ($\mu_m = 0.25$, $\omega = 3$). (a) $r = 0.5$ (b) $r = 0.6$ (c) $r = 0.7$ (d) $r = 0.8$ (e) $r = 0.9$. 
constraint, the chattering turns into viscosity. As the system relative collision happens over 5 times (including 5), it is considered as chattering. When there is viscosity after chattering, it is called complete chattering and otherwise is non-complete chattering. To set up the parameter, \(\mu_m = 0.25, 0.1, 0.05, 0.001, \delta = 0.005, dt = 0.001, r \in (0, 1)\), when \(\omega = 1, 3, 4\), a continuous variation is conducted through \(r\), and the time-domain waveform figures of different coefficient of restitutions of the vibration system are below.

Figures 6–8 are the time-domain waveforms with different coefficient of restitution when the mass ratio is 0.25.

![Figures 6–8](image)

**Figure 8.** Different \(r\) of time-domain waveform figure (\(\mu_m = 0.25, \omega = 4\)). (a) \(r = 0.5\) (b) \(r = 0.6\) (c) \(r = 0.7\) (d) \(r = 0.8\) (e) \(r = 0.9\).
Figure 6 is the time-domain waveform with different coefficient of restitution when $\omega = 1$. It can be seen from the figures that Figure 6(a)–(e) are mainly based on 1–2 collisions in a single cycle, so no chattering occurs. After the collision, the system is in a long-term viscous state and then enters the next cycle of movement. As the coefficient of restitution increases, the relative displacement of the initial collision tends to increase.

Figure 6 is the time-domain waveforms with different coefficient of restitution when $\omega = 3$. From Figure 7(a)–(e), the growth of coefficient of restitution leads to the increase of the initial collision displacement and more continuous collisions. In Figure 7(c), when $t = 1.75$, there are 5 times of continuous collisions in a single period, which means there is chattering.

Figure 9. Different $r$ of time-domain waveform figure ($\mu_m = 0.1, \omega = 1$). (a) $r = 0.5$ (b) $r = 0.6$ (c) $r = 0.7$ (d) $r = 0.8$ (e) $r = 0.9$. 
and a short term of viscosity after the chattering, which is complete chattering. In Figure 7(d), when \( t = 0.25, 1.25, \) and \( 2.25, \) there are all over-5-time continuous collisions with viscosity, representing complete chattering. In Figure 7(e), there is chattering phenomenon with viscosity when \( t = 0.25, \) meaning complete chattering. Therefore, the coefficient of restitution range is \( r \in (0.7, 0.9) \), and the coefficient of restitution of complete chattering has a range of \( r \in (0.7, 0.9) \).

Figure 8 is the time-domain waveform with different coefficient of restitution when \( \omega = 4 \). It can be seen from Figures 8(a)–(e) that the increase of the coefficient of restitution leads to the increase of the initial collision displacement and more continuous collisions. Figures 8(a)–(c) primarily consist of twice collisions in a single period. Figures 8(d) and (e) are

![Figure 10](image)

**Figure 10.** Different \( r \) of time-domain waveform figure \( (\mu_m = 0.1, \ \omega = 3) \). (a) \( r = 0.5 \) (b) \( r = 0.6 \) (c) \( r = 0.7 \) (d) \( r = 0.8 \) (e) \( r = 0.9 \).
mostly three times of collision per period. Thus, under such condition, there is no chattering phenomenon in the coefficient of restitution interval.

Figures 9–11 are the time-domain waveforms with different coefficient of restitution when the mass ratio is 0.1. Figure 9 is the time-domain waveforms with different coefficient of restitution when \( \omega = 1 \). In Figures 9(a) and (b), the collision time increases constantly, and the initial collision displacement grows. There are 1–4 times of collisions without chattering. In Figure 9(c), when \( t = 0.2 \) and 1.2, there are more than 5 times of collision with viscosity, showing complete

![Time-domain waveforms](image)

**Figure 11.** Different \( r \) of time-domain waveform figure (\( \mu_m = 0.1, \ \omega = 4 \)). (a) \( r = 0.5 \) (b) \( r = 0.6 \) (c) \( r = 0.7 \) (d) \( r = 0.8 \) (e) \( r = 0.9 \).
chattering. In Figure 9(d), when t = 0.3 and 1.3, there are more than 5 times of collision with viscosity, indicating complete chattering. Thus, the coefficient of restitution range is \( r \in (0.7, 0.9) \), and the coefficient of restitution of complete chattering has a range of \( r \in (0.7, 0.9) \).

Figure 10 is the time-domain waveforms with different coefficient of restitution when \( \omega = 3 \). In Figures 10(a) and (e), the collision time increases constantly, and the initial collision displacement grows. Figures 10(a)–(c) consist of primarily 1–4 times collisions per cycle, and there is no chattering. In Figure 10(d), when t = 0.5, 0.8, 1.1, 1.4, 1.7, 2.1, and 2.4, there are more than 5 times of collision without viscosity, and the next cycle happens directly after collision, demonstrating non-complete chattering. Therefore, the interval of the coefficient of restitution for chattering is \( r \in (0.8, 0.9) \), and the coefficient of restitution for the complete chattering has a range around \( r = 0.8 \).

Figure 11 is the time-domain waveforms with different coefficient of restitution when \( \omega = 4 \). In Figures 11(a)–(e), the largest number of collision is 3 times in a single cycle, and the initial collision displacement increases along with the growing coefficient of restitution. Thus, under such conditions, there is no chattering.

**Figure 12.** Different r of time-domain waveform figure (\( \mu_m = 0.05 \), \( \omega = 1 \)). (a) \( r = 0.5 \) (b) \( r = 0.6 \) (c) \( r = 0.7 \) (d) \( r = 0.8 \) (e) \( r = 0.9 \).
Figures 12–14 are the time-domain waveform figures of different coefficient of restitution when the mass ratio is 0.05.

Figure 12 is the time-domain waveform with different coefficient of restitution when $\omega = 1$. In Figure 12(a), there are more than 5 times of collisions with viscosity when $t = 0.2$ and 1.2, referring to complete chattering. In Figure 12(c), when $t = 0.3$ and 1.3, there are also over 5 times of collisions with viscosity, indicating complete chattering. In Figure 12(d), over 5 times of collisions with viscosity happen when $t = 0.5$ and 1.5, which are also complete chattering. In Figure 12(e), when $t = 1$ and 3, there are also over 5 times of collisions with viscosity, indicating complete chattering. Thus, the range of the coefficient of restitution of chattering is $r \in (0.6, 0.9)$, and the coefficient of restitution of complete chattering has a range $r \in (0.6, 0.9)$.

Figure 13 is the time-domain waveform with different coefficient of restitution when $\omega = 3$. In Figure 13(a), the collisions are mostly 1 to 4 times in a single period, with no indication of chattering. In Figure 13(b), when $t = 0.2, 0.5, 1.2, 1.7,$ and 2.2, there are over 5 times of collisions with viscosity, meaning complete chattering. In Figure 13(c), 5 times of collisions with viscosity happen when $t = 0.2, 0.5, 1.2, 1.7,$ and 2.2, showing complete chattering. In Figure 13(d), as $t = 0.6$,...
1.2, 1.6, and 2.2, there are over 5 times of collisions with viscosity, meaning complete chattering. In Figure 13(e), when $t = 0.5, 1.1, 1.6, \text{ and } 2.2$, there are over 5 times of chattering without viscosity, and the system enters next period after the collision, which means non-complete chattering. Thus, the coefficient of restitution of chattering has a range of $r \epsilon (0.6, 0.9)$, and the range of the coefficient of restitution of complete chattering is $r \epsilon (0.6, 0.8)$.

Figure 14 is the time-domain waveform with different coefficient of restitution when $\omega = 4$. In Figure 14(a), the largest number of collision is 3 times per period, and the initial collision displacement grows as the coefficient of restitution goes up. In Figure 14(b), when $t = 0.2, 0.5, 0.8, 1.3, 1.7, \text{ and } 2.2$, there are over 5 times of collisions with viscosity, indicating

![Figure 14](image)

**Figure 14.** Different $r$ of time-domain waveform figure ($\mu_m = 0.05, \omega = 4$). (a) $r = 0.5$ (b) $r = 0.6$ (c) $r = 0.7$ (d) $r = 0.8$ (e) $r = 0.9$. 
complete chattering. In Figure 14(c), over 5 times of collisions with viscosity happen when $t = 0.5, 0.8, 1.3, 1.7,$ and 2.2, meaning complete chattering. In Figure 14(d), when $t = 0.5, 1.0, 1.3, 1.7,$ and 2.2, there are more than 5 times of collision without viscosity, which means non-complete chattering. In Figure 14(e), 5 times of collisions only occur when $t = 0.5$ and 1.3, showing no chattering. Therefore, the coefficient of restitution of chattering has a range of $r e(0.6, 0.9)$, and the range of the coefficient of restitution of complete chattering is $r e(0.6, 0.7)$.

Figures 15–17 are the time-domain waveforms with different coefficient of restitution when the mass ratio is 0.001. Figure 15 indicates the time-domain waveform figure of different coefficient of restitution when $\omega = 1$. In Figure 15(a), there are 5 times of collisions with viscosity at $t = 1.2$, which means complete chattering. In Figure 15(b), over 5 times of collisions with viscosity take place when $t = 0.3$ and 1.3, signifying complete chattering. In Figure 15(c), when $t = 0.5$ and 1.5, there are over 5 times of collisions with viscosity, meaning complete chattering. In Figure 15(e), more than 5 times of collisions without viscosity occur at $t = 1.5$, representing non-complete chattering. So, the coefficient of restitution range of chattering is $re(0.5, 0.9)$, and the coefficient of restitution of complete chattering has a range of $re(0.5, 0.8)$.

![Figure 15](image-url)
Figure 16 indicates the time-domain waveform figure of different coefficient of restitution when $\omega = 3$. In Figures 16(a)–(e), as the collision time increases, the initial collision displacement continuously increases. In Figure 16(a), when $t = 0.2, 0.5, 1.2, 1.7, \text{and} 2.2$, there are over 5 times of collisions with viscosity, indicating complete chattering. In Figure 16(b), when $t = 0.5, 1.2, 1.7, \text{and} 2.2$, there are more than 5 times of collisions with viscosity, demonstrating complete chattering. In Figure 16(c), as $t = 0.5, 1.2, 1.6, \text{and} 2.2$, more than 5 times of collisions without viscosity take place, showing non-complete chattering. In Figure 16(d), there are 5 times of collisions without viscosity, indicating non-complete chattering. In Figure 16(e), there are also 5 times of collisions without viscosity, representing non-complete chattering. Thus, the range of
the coefficient of restitution of chattering is \( r_\varepsilon(0.5, 0.9) \), and the coefficient of restitution of complete chattering has a range of \( r_\varepsilon(0.5, 0.6) \).

Figure 17 indicates the time-domain waveform figure \( (\mu_m = 0.001, \, \omega = 4) \). (a) \( r = 0.5 \) (b) \( r = 0.6 \) (c) \( r = 0.7 \) (d) \( r = 0.8 \) (e) \( r = 0.9 \).

Figure 17. Different \( r \) of time-domain waveform figure \( (\mu_m = 0.001, \, \omega = 4) \). (a) \( r = 0.5 \) (b) \( r = 0.6 \) (c) \( r = 0.7 \) (d) \( r = 0.8 \) (e) \( r = 0.9 \).
non-complete chattering. In Figure 17(c), more than 5 times of collisions without viscosity take place when $t = 0.5, 0.8, 1.2, 1.6,$ and $2.2$, demonstrating non-complete chattering. In Figure 17(d), 5 times of collisions without viscosity happen at $t = 0.8$, indicating non-complete chattering. In Figure 17(e), there is no indication of over-5-time collision, which means the lack of chattering. Therefore, the range of the coefficient of restitution of chattering is $r = (0.5, 0.8)$, and the coefficient of restitution of complete chattering has a range around $r = 0.5$.

Based on above analysis and the obtained ranges of the coefficient of restitution, the following table is shown. It can be seen from Table 1 that under the conditions of different mass ratio and frequency ratio, the coefficient of restitution for chattering is between 0.5 and 0.9, so large coefficient of restitution is prone to chatter. Since the coefficient of restitution of most impact materials is between 0.5 and 0.9, chattering is a frequent phenomenon in impact dampers. And small mass ratio is prone to chatter.

### Conclusion

In this paper, a global bifurcation map is obtained by continuously changing the parameter value of the restitution coefficient by establishing a mechanical model of a non-fixed constrained collision vibration system, and from this, a phase diagram, a Poincare map, and a time-domain diagram under each restitution coefficient are obtained. The interval of restitution coefficient for the chattering of impact damper under different mass ratio and frequency ratio is obtained. (1) Based on the bifurcation diagrams of the coefficient of restitution, when coefficient of restitution is greater than 0.5, there is obvious bifurcation phenomenon occurring in the system, and the bifurcation area constantly expands. (2) By analyzing the system time-domain waveform figure under different coefficient of restitution, it shows that the coefficient of restitution for chattering is between 0.5 and 0.9, and as the mass ratio decreases, the interval of restitution coefficient for chattering continuously expands; as the frequency ratio increases, the interval of restitution coefficient for chattering narrows. (3) The material with large coefficient of restitution is easy to chatter under the condition of small mass ratio.

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