Consistent nonlinear KK reduction of 11d supergravity on $AdS_7 \times S_4$ and self-duality in odd dimensions

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Abstract

We show that there exists a consistent truncation of 11 dimensional supergravity to the 'massless' fields of maximal (N=4) 7 dimensional gauged supergravity. We find the complete expressions for the nonlinear embedding of the 7 dimensional fields into the 11 dimensional fields, and check them by reproducing the d=7 susy transformation laws from the d=11 laws in various sectors. In particular we determine explicitly the matrix U which connects the Killing spinors to the gravitinos in the KK ansatz, and the dependence of the 4-index field strength on the scalars. This is the first time a complete nonlinear KK reduction of the original d=11 supergravity on a nontrivial compact space has been explicitly given. We need a first order formulation for the 3 index tensor field $A_{\Lambda\Pi\Sigma}$ in d=11 to reproduce the 7 dimensional result. The concept of 'self-duality in odd dimensions' is thus shown to originate from first order formalism in higher dimensions. For the AdS-CFT correspondence, our results imply that one can use 7d gauged supergravity (without further massive modes) to compute certain correlators in the d=6 (0,2) CFT at leading order in N. This eliminates an ambiguity in the formulation of the correspondence.

The question whether in general a consistent Kaluza-Klein (KK) truncation exists at the nonlinear level is an old problem. For tori, the consistency is easy to prove, but for more complicated compact spaces little is known. In supergravity (sugra), the truncation of d=11 sugra on $AdS_4 \times S_7$ to maximal $d = 4$ gauged sugra was intensively studied 15 years ago \cite{1}, culminating in a series of papers by de Wit and Nicolai \cite{2}. The interest in those days was to find realistic 4 dimensional models from spontaneous compactification of maximal 11 dimensional sugra. Recent developments in the AdS-CFT correspondence \cite{4,5,6,7,8} have renewed interest in AdS compactifications

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to 5 and 7 dimensions. A crucial question is whether the AdS$_5 \times S_5$ and AdS$_7 \times S_4$ compactifications allow consistent truncations to the massless sector, because if there does not exist a consistent truncation, massive fields have to be considered for computations of correlators in the corresponding CFT [3].

De Wit and Nicolai first studied the KK reduction of the original formulation of d=11 sugra [2], but then they found it advantageous to construct a different formulation of d=11 sugra with a local SU(8) invariance [3]. Because the local SU(8) invariance rotates Bianchi identities into field equations, the action for the SU(8) sugra did not follow directly from the original d=11 action [3]). For this SU(8) formulation the complete nonlinear KK reduction on AdS$_7 \times S_4$ was given: they proposed nonlinear ansätze and checked the consistency of the KK truncation on AdS$_4 \times S_7$ as far as the bosonic and fermionic transformation rules are concerned. It may be that the SU(8) theory will turn out to be important for future research in string theory, but we prefer to work with the original formulation. The connection was formulated in terms of a matrix U for which they derived an equation, but they only could solve this equation in certain sectors. (ref. [2]a, eq. (3.12) and ref. [2]b, eq. (2.14)).

In this article we analyze the KK reduction of d=11 sugra on AdS$_7 \times S_4$. This will allow us to go further than the work on AdS$_4 \times S_7$. As we already mentioned, in ref. [2] d=11 sugra was first reformulated in a form with a local SU(8) invariance [3]. Because the local SU(8) invariance, hence we will directly work with the d=11 sugra as it is usually formulated.

One of the reasons to study the AdS$_7 \times S_4$ case is that we would like to understand the origin of the mysterious 'self-duality in odd dimensions' [10] which appears in various supergravities in odd dimensions. To obtain the action for selfdual tensors one begins with the antisymmetric tensor from d=11 whose action is quadratic in derivatives, and introduces the square of an extra auxiliary antisymmetric tensor field in the lower dimension by hand, rotates both tensors, factorizes the second order field equations into two field equations linear in derivatives, and drops one of the factors. The end product is an action of the form $eFA + A^2$ [10] which is dual to Chern-Simons theory for the abelian case [11] but not for the nonabelian case [12]. In sugra the nonabelian version appears.
Maximal (N=4) sugra in 7 dimensions has a 3 index tensor $S_{\alpha\beta\gamma} (A=1,5)$ with a self-dual action. Because in the linearized KK reduction one needs to introduce an auxiliary field $B_{\alpha\beta\gamma} \sim * B_{\alpha\beta\gamma}$ in d=7 to construct this action \cite{13}, we will start from a first order formulation for $A_{\text{AIHS}}$ in d=11, and deduce self-duality. The 11 dimensional Lagrangian we start from and its supersymmetry transformations rules are given in the Table I, where $\Lambda, \Pi, ... = 0, 10$ are curved vector indices and M,N,...=0,10 are flat vector indices. The action is invariant under the susy laws for any value of the real constants $a$ and $b$. We will fix them later by the requirement of consistent truncation.

The field $F_{\text{AIHS}}$ is an independent field. $F_{\text{AIHS}}$ denotes the usual curl $F_{\text{AIHS}} = 24\partial_{[A} A_{\text{AIHS}]}$ plus the $\Psi\Gamma\Psi$-terms which make it supercovariant. \cite{14} Similarly $\Omega_{\text{MNP}}$ is the usual supercovariant spin connection. \cite{15} Finally, $R_A(\Psi) = \frac{1}{4} \tilde{F}_a$. It is convenient to redefine $F_{\text{AIHS}}$ by introducing an auxiliary tensor density $B_{MNPQ}$:

$$F_{\text{AIHS}} = \partial_A A_{\text{AIHS}} + 23 \text{ terms} + B_{MNPQ} E^{-1/2} E^M \ldots E^Q$$  \hspace{1cm} (1)

The action in table I contains a new first-order formulation as far as the 3-index tensor field $A_{\text{AIHS}}$ is concerned. There exists also a first order formulation of d=11 sugra with an independent spin connection $\Omega$ \cite{15}. Initially we tried to deduce the d=7 selfduality from this field $\Omega$, but this did not work \cite{15}. Instead we will work with a second order formalism for the spin connection.

\begin{equation}
\delta e^1 = -\frac{1}{2} \epsilon \tau^a \psi_a \hspace{3cm} (I.1)
\end{equation}

\begin{equation}
\delta e^i = \frac{1}{2} \delta^{ij} \psi^i \hspace{3cm} (I.2)
\end{equation}

\begin{equation}
\Pi^A \Pi^B \delta B^{AB} = \frac{1}{4} \epsilon \tau^i \psi^i + \frac{1}{8} \epsilon \tau \gamma^i \psi^i \lambda^i \hspace{3cm} (I.3)
\end{equation}

\begin{equation}
\delta S_{\alpha\beta\gamma, A} = -\frac{\sqrt{3} m}{8} \Pi^A \left(2 \epsilon \tau_{ij} k^i \psi^i + \epsilon \tau_{[\alpha} \gamma^i k^i \lambda^i \right) \Pi^B j^i \Pi^C k^j F_{\beta\gamma} \hspace{3cm} (I.4)
\end{equation}

\begin{equation}
\Pi^{-1, A} \Pi^{-1, B} = \frac{1}{2} \left( \epsilon \tau \gamma^i \lambda^i + \epsilon \gamma^i \lambda^i \right) \hspace{3cm} (I.5)
\end{equation}

\begin{equation}
\delta \psi^i = \epsilon \tau_{\alpha} \epsilon_{\alpha} - \frac{1}{12} m \Gamma_{\alpha} \epsilon_{\alpha} - \frac{1}{12} (\tau_{\alpha} \beta) - 8 \delta^{ij} \tau_{\alpha} \beta \gamma^i \gamma^j \epsilon_{\alpha} \Pi^A \Pi^B \Pi^C F_{\beta\gamma} \hspace{3cm} (I.6)
\end{equation}

\begin{equation}
\delta \psi^i = \epsilon \tau_{\alpha} \epsilon_{\alpha} - \frac{2}{20} m \Gamma_{\alpha} \epsilon_{\alpha} - \frac{2}{20} (\tau_{\alpha} \beta - 8 \delta^{ij} \tau_{\alpha} \beta \gamma^i \gamma^j) \epsilon_{\alpha} \Pi^A \Pi^B \Pi^C F_{\beta\gamma} \hspace{3cm} (I.7)
\end{equation}

\begin{table}[h]
\centering
\caption{Table II}
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\begin{table}[h]
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\caption{Table II}
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\footnotetext{By replacing the term $FF$ in (I.1) by $F_F$, the terms $(\Psi\Gamma\Psi)(F + \tilde{F})$ get absorbed. Then the $F$ field equation reads $F = \tilde{F}$ and becomes supercovariant. We have not been able to absorb the remaining four-fermi terms by using our new first order formulation.}
We expand d=11 sugra around the $AdS_7 \times S_4$ background given by $F_{\mu\nu\rho\sigma} = 3/\sqrt{2} m e_{\mu\nu\rho\sigma}$ $det(e^m_\mu) = \eta^{m_1 m_2 m_3}$, where $m^{-1}$ is the radius of $S_4$ and $\hat{e}^m_\mu(x)$ is the background vielbein ($\mu, \nu, ... = 1,4$ are curved indices and $m, n, ...$ are flat indices).

Maximal gauged sugra in $d=7$ has the action and susy laws of Table II. Here $\alpha, \beta, ... = 0, 6$ are curved vector indices and $a, b, ... = 0, 6$ are flat vector indices. The Dirac matrices in $d=7$ and $d=4$ are denoted by $\tau$ and $\gamma^m$, respectively. The model has a local $SO(5)_g$ gauge group for which $A, B, ... = 1, 5$ are vector indices, while $I, J, ... = 1, 4$ are spinor indices. The scalars $\Pi^A = \frac{1}{(d-4)!}$ parametrize the coset $SL(5, \mathbb{R})/SO(5)_c$, but in the gauged model the rigid $SL(5, \mathbb{R})$ symmetry of the action is lost and replaced by the $SO(5)_c$ gauge invariance. The indices $i, j, ... = 1, 5$ are $SO(5)_c$ vector indices and $I', J', ... = 1, 4$ are spinor indices. The model has the following fields: the vielbein $e^a_\alpha$, the 4 gravitinos $\psi^I_\alpha$, the $SO(5)_g$ vector $B^{AB}_\alpha = -B^{BA}_\alpha$, the scalars $\Pi^A$, the antisymmetric tensor $S_{\alpha\beta\gamma}^A$ and the spin $1/2$ fields $\lambda'_\alpha$ (vector-spinors under $SO(5)_c$). They have the correct mass-terms which ensure 'masslessness' in $d=7$ AdS space \cite{16}. In (II.1) $T_{ij} = \Pi^{-1} i \Pi^{-1} \delta^{AB}, \Omega_4[B]$ and $\Omega_3[B]$ are the Chern-Simons forms for $B^{AB}_\alpha$ (normalized to $d\Omega_3[B] = (Tr F^2)^2$ and $d\Omega_5[B] = (Tr F^4))$. The tensor $P_{\alpha ij}$ and the connection $Q_{\alpha ij}$ (appearing in the covariant derivatives $\nabla_{\alpha}$) are the symmetric and antisymmetric parts of $(\Pi^{-1})^A_{ij} (\delta_A^B \partial_\alpha + g B_{AB} \Pi_{Bk} \delta_{kj})$, respectively. Here $D_{\alpha}$ has both a $Q_{\alpha ij}$ and a $P_{\alpha ij}$ piece.

We begin the KK reduction with the usual ansatz for the 11d vielbein:

$$E^A_M = \begin{pmatrix} e^a_\alpha(y) \Delta^{-1/5}(y, x) & B^m_\alpha(y, x) E^m_\mu(y, x) \\ 0 & E^m_\mu(y, x) \end{pmatrix}$$

$$E^M_A = \begin{pmatrix} e^a_\alpha \Delta^{-1/5} & -B^m_\alpha(y, x) e^a_\alpha \Delta^{-1/5} \\ 0 & E^m_\mu E^\mu_n \delta_m^n ; e^a_\alpha e^b_\alpha = \delta^b_a \end{pmatrix}$$

where $B^\mu_\alpha(y, x) = -2 B^{AB}_\alpha V^{AB}(x)$ with $V^{AB}$ Killing vectors on $S_4$. The rescaling by $\Delta^{-1/5}$ where $\Delta = det E^m_\mu(y, x) / det \hat{e}^m_\mu(x)$ brings the $d=7$ Einstein action in canonical form.

We also redefine the $d=11$ gravitino field $\Psi_A$ in terms of a field $\psi_\alpha(y, x)$ and a field $\psi_m(y, x)$ which lead to the canonical gravitino and Dirac Lagrangians in $d=7$.

$$E^A_m \Psi_A = \Delta^{1/10} \gamma_5^{-1/2} e^a_\alpha \psi_\alpha - \frac{1}{5} \tau_a \gamma_5 \gamma^m E^m \Lambda \Psi_A ; \ E^m \Lambda \Psi_A = \Delta^{1/10} \gamma_5^{1/2} \psi_m$$

We formulate the KK reduction of the fermions in terms of $\psi_\alpha$ and $\psi_m$.

$$\psi_\alpha(y, x) = \Delta^{-1/10}(y, x) \gamma_5^{-1/2} e^a_\alpha(y)(\Psi_\alpha(y, x) + \frac{1}{5} \tau_a \gamma_5 \gamma^m \Psi_m(y, x))$$

$$\psi_m(y, x) = \Delta^{-1/10}(y, x) \gamma_5^{1/2} \psi_m(y, x)$$

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3One has $V^{AB}_\mu = Y^{[A} \partial_\mu Y^{B]}$ with strength unity, where $Y^A = \frac{1}{4} (\gamma^A)_{IJ} \eta^I \gamma_5 \eta^J$ is real and satisfies $\sum_A (Y^A)^2 = 1$. We use ‘modified’ Majorana spinors, $\tilde{\eta}^I = \eta^I C_4^{-c} = (\eta^I)^c \gamma_5 \tilde{\Omega}^I K$ with $(C_4^{-c})_c$ numerically equal to $(\tilde{\Omega})_c$ and $\tilde{\Omega}^I$. The matrices $\gamma^A$ and $\gamma^I$ are in both cases given by $\{i \gamma^a \gamma^5, \gamma^5\}$. Furthermore, $\tilde{\epsilon} = \epsilon^T C^{(11)} = \epsilon^I \Gamma^0$ with $C^{(11)} = C^{(7)} \otimes C^{(4)}$, so that $\tilde{\epsilon}(y, x) = \tilde{e}^I(y) U^{IJ} \tilde{\eta}^I(y) \Delta^{-1/10} \sqrt{\gamma_5}$.}
\[
\begin{align*}
\epsilon(y, x) &= \Delta^{1/10}(y, x) \gamma_5^{-1/2} \epsilon(y, x) \text{ with } \sqrt{\gamma_5} = \frac{1}{2}(1 - i)(1 + i \gamma_5) \\
\epsilon(y, x) &= \epsilon_I(y) U^{I'}_I(y, x) \eta^I(x) 
\end{align*}
\]

where \( \eta^I \) are Killing spinors on \( S_4 \) (\( \tilde{D}_6 \eta^I = \frac{i m}{2} \gamma_\mu \eta^I \)). We normalize them to \( \tilde{\eta}^I \eta^I = \tilde{\Omega}^{IJ} \). The expansion into spherical harmonics is the same as in [13] except that we added a matrix \( U^{I'}_I(y, x) \) which interpolates between \( SO(5)_g \) and \( SO(5)_c \). In ref [2], a SU(8) matrix \( U \) was found to be needed to obtain consistency of the KK reduction in certain sectors, but then a reformulation of the theory with full local SU(8) invariance was constructed [3]. We introduce the matrix \( U \) as in [2], but we shall not go to a different formulation of d=11 sugra.

For consistency of our results for the transformation rules the matrix \( U \) needs to satisfy
\[
U^{I'}_I \tilde{\Omega}^{IJJ'} \rightarrow \tilde{\Omega}^{IJJ'} = - (U^{-1})^{I'}_I \tag{8}
\]

For example (II.2) follows from (I.2) only if (8) holds. Here \( \Omega \) and \( \tilde{\Omega} \) are the Usp(4) invariant tensors used to lower and raise the spinor indices, satisfying \( \tilde{\Omega}^{IJJ} \Omega_{KJ} = \delta_k^I \) and \( \tilde{\Omega}^{IJJ} \Omega_{KJ'} = \delta_{K'}^I \). Since \( \Omega \) is the charge conjugation matrix, this restricts \( U \) to be an \( SO(5) \) matrix in the spinor representation.

The ansatz for the expansion of \( E^\mu_m \) into spherical harmonics is found from the result in (II.3) that \( \Pi^A \Pi^B \delta_{AB} \psi_\alpha = \frac{1}{5} \bar{\psi} \gamma^\alpha \psi_\alpha + \frac{1}{5} \bar{\psi} \tau_\alpha \gamma^i \gamma^j \lambda_k \). The first term in (II.3) gives the following result:
\[
\begin{align*}
i E^\mu_m (UV^m U^T)_{I'J'} &= - \Delta^{1/5}(\Pi^{-1})^A_i (\Pi^{-1})^B_j B V^\mu_{AB}(\gamma_{ij})_{I'J'} \\
E^\mu_m &= i \frac{1}{4} \Delta^{1/5}(\Pi^{-1})^A_i (\Pi^{-1})^B_j B V^\mu_{AB} Tr(\gamma_{ij} UV^m U^T \Omega) 
\end{align*}
\]

By substituting \( E^\mu_m \) back into (3), we get a consistency condition on the matrix \( U \),
\[
\frac{1}{4}(\Pi^{-1})^A_i (\Pi^{-1})^B_j B V^\mu_{AB} Tr(\gamma_{ij} UV^m U^T \Omega)(UV^m U^T)_{I'J'} = (\Pi^{-1})^A_i (\Pi^{-1})^B_j B V^\mu_{AB}(\gamma_{ij})_{I'J'} 
\]

where the Killing vector \( V^m IJ \) is given by \( V^m IJ = i V^m_{AB}(\gamma^{AB})_{IJ} = - i V^m_{AB}(\gamma^{AB} \tilde{\Omega})_{IJ} \). We note that \( U=1 \) is not consistent, therefore we indeed need the matrix \( U \). Using this ansatz, the second term in (II.3) also matches the corresponding term on the left hand side, provided one identifies \( \lambda^i_I \) with \( 3i(\gamma^k)_{IJ} \lambda^i_{I'J'} \).

Then, by calculating \( \Delta \), we get
\[
\Delta^{-6/5} = (\Pi^{-1})^A_i (\Pi^{-1})^B_j B \delta_{ij} Y_A Y_B \equiv T^{AB} Y_A Y_B 
\]

where \( Y_A = \frac{1}{3}(\gamma_A)_IJ \tilde{\eta}^I \gamma_5 \tilde{\eta}^J \) is the basic scalar spherical harmonic on \( S_4 \). We can then extract \( \delta \Pi^A_i \) from \( \delta(\Delta^{-6/5}) \) and comparison with (II.5) gives another condition on \( U \):
\[
Y_A(U \gamma^\alpha \tilde{\Omega}(U U^T)_{I'J'}) = \Delta^{3/5}(\Pi^{-1})^A_i (\gamma^i)_{I'J'} Y_A 
\]

Equations (8), (11) and (13) are all we need to know about \( U \) to prove all results on consistency.
At this point we have come in 7 dimensions as far as others in 4 dimensions. However, we have been able to find the solution for $U$. First of all, we have been able to show that (11) follows from (13), so that (13) is the crucial equation. The covariant solution of (13) built out of $Y_i$ and $v_i$ is unique and reads

$$U = -\sqrt{\frac{1 + v_i Y_i}{2}} + \frac{Y_A v_i \delta Bi_{\gamma AB}}{\sqrt{2(1 + v_i Y_i)}}$$

$$v_i = (\Pi^{-1})_i^A Y_A \Delta^{3/5}$$  \hspace{1cm} (14)$$

It was determined by moving one of the $U$ matrices in (13) to the right-hand side yielding $U Y_i = U$ and expanding the $4 \times 4$ matrix $U$ on the basis $1, \gamma_A, Y_A$. Covariance restricts $U$ to $f_1 + f_2 Y_i + f_3 Y_A \gamma B_{\gamma AB}$, where $f_j$ depends only on $Y \cdot v$. Requiring (8) and (13) leads to (14).

Next we turn to the ansatz for $F_{\Lambda \Pi \Sigma \Omega}$. At the linearized level it contains the fluctuations in $g_{\mu \alpha}$ and $g_{\mu \nu}$, and fluctuations in $A_{\alpha \beta \gamma}$. At the nonlinear level, an ansatz for $F_{\Lambda \Pi \Sigma \Omega}$ containing only the fluctuations in $g_{\mu \alpha}$ and yielding the correct Chern-Simons actions in $d=7$ was given in [18, 19]. We now present the complete expression for $F_{\Lambda \Pi \Sigma \Omega}$; it contains the results of [13] and [18, 19] (at the limit when the scalar fields are set to zero) and it satisfies the Bianchi identities.

$$\sqrt{\frac{2}{3m}} F_{\mu \nu \rho \sigma} = \epsilon_{\mu \nu \rho \sigma} \sqrt{\det g} \left[ 1 + \frac{1}{3} \left( \frac{T}{Y_A Y_B T_{AB}} - 5 \right) \right]$$

$$\frac{\sqrt{2}}{3m} F_{\mu \nu \alpha \beta} = \partial_{\mu} \left( \epsilon_{\alpha \beta \gamma \delta} B_{[\alpha \beta Y F B_{\gamma}] C D} \partial_{C D} T_{E F Y G} \right)$$

$$\frac{\sqrt{2}}{3m} F_{\mu \alpha \beta \gamma} = \partial_{\mu} A_{\alpha \beta \gamma}$$

$$\sqrt{2} \frac{2}{3m} F_{\alpha \beta \gamma \delta} = 4 \partial_{(\alpha} A_{\beta \gamma \delta)}$$
Here $T = T_{AB} \delta^{AB}$ and $Y \cdot T \cdot Y \equiv Y_A T^{AB} Y_B, Y \cdot T^2 \cdot Y \equiv Y_A (T^2)^{AB} Y_B$.

This ansatz was obtained by requiring consistency of the susy laws, namely that the the 11-dimensional susy variation law $\delta(d=11) F_{AB} = d\delta(d=11) A_{AB}$ can be written as a total 7-dimensional susy variation $\delta(d=7)$ fields. Our present ansatz reproduces the linearized limit of ref. [13], and it coincides with the geometrical proposal by [18, 19] when we let $T^{AB} = \delta^{AB}$. The $T$-dependent terms in (15) and (16) which are $B$-independent separately satisfy the Bianchi identity, even though they are not an exact form. The terms with $A_{\alpha\beta\gamma}$ as well as the $B$ dependent terms are exact and thus they trivially satisfy the Bianchi identities. The Chern-Simons terms in $d=7$ are not affected by the partial dressing with scalar fields of some spherical harmonics of the ansatz proposed by Freed et al. [14]. The precise expression of the 4-form added in the $F_{\mu\nu\rho\sigma}$ sector is highly constrained. It must reproduce the linearized term in [13], and it must yield the correct scalar potential in $d=7$ after integrating over the compact space. In order to perform this integral to which both the Einstein action and the kinetic action of the 3-index photon contribute, we start with the metric in the internal space and its inverse:

$$g_{\mu\nu} = \Delta^{4/5} C_{\mu} A C_{\nu} B T^{-1}; \quad g^{\mu\nu} = \Delta^{2/5} \left( C_{\mu} A B T^{AB} Y_{C} Y_{D} T^{CD} - C_{\mu} A Y_{B} T^{AB} C_{\nu} Y_{D} T^{CD} \right)$$

(20)

where $C_{\mu} = \partial_{\mu} Y^{A}$ is a conformal Killing vector. We can thus interpret the deformations of the background metric as describing an ellipsoid with the conformal factor $\Delta^{4/5}$, whose axes at a specific point $y$ in the $d=7$ space time are determined by the eigenvalues of $T^{-1}_{AB}$. When setting the gauge fields to zero and disregarding the terms with $d=7$ space time derivatives, the integral over the compact space of the Einstein action is already of the desired form, namely a linear combination of $T^2$ and $Tr(T^2)$.

On the other hand, the integrated kinetic action of the 3 index photon has the form

$$\int d^4x \sqrt{\det g}(x)(Y_E Y_F T^{EF})^2(1+S)^2, \quad \text{where} \quad (3/\sqrt{2}) \sqrt{\det g}(x)\epsilon_{\mu\nu\rho\sigma}S$$

is the extra term we need to add in $F_{\mu\nu\rho\sigma}$ besides its background value. This function $S$ should be of degree zero in $T$ and vanishes in the background. In order that the $d=7$ scalar potential be of the form $(TrT)^2 - 2TrT^2$, we can only admit terms in $S$ of the form $\alpha[TrT/(Y \cdot T \cdot Y) - 5] + \beta[Y \cdot T^2 \cdot Y/(Y \cdot T \cdot Y)^2 - 1]$. Requiring agreement with the linearized ansatz yields $\alpha = 1/3$, while $\beta$ satisfies the quadratic equation $\beta(\beta + 2/3) = 0$ in order to reproduce the $d=7$ scalar potential. The solution $\beta = 0$ does not produce the correct gravitino law, hence consistency requires $\beta = -2/3$.

The ansatz for the independent fluctuations $A_{\alpha\beta\gamma}$ and the auxiliary field $E^{-1/2} B_{\alpha\beta\gamma\delta}$ is found by matching the last term in $\delta\psi_\alpha$ in (II.6) (the term with $S_{\alpha\beta\gamma,A}$):

$$\frac{i\sqrt{3}}{2} A_{\alpha\beta\gamma} = S_{\alpha\beta\gamma,A} Y^{A}$$

(21)

$$\frac{i\sqrt{3}}{2} B_{\alpha\beta\gamma\delta} = \left[ \frac{24k}{5} \nabla_{\alpha} S_{\beta\gamma\delta,A} - \frac{k}{5} \delta_{AC} \Pi^{-1} i_{C} \Pi^{-1} j_{B} \beta \delta_{ij} \epsilon_{\alpha\beta\gamma\delta} \epsilon_{\kappa\lambda} S_{\epsilon\kappa, B} Y^{A} \right]$$

(22)
where the first terms in $B_{\alpha\beta\gamma\delta}$ cancel possible $\partial_\alpha S_{\beta\gamma\delta,A}$ and $B^A_{\alpha} S_{\beta\gamma\delta,B}$ terms in $\delta \psi$.

We also find that $ka = -\frac{5\sqrt{2}}{9}$ and $kb = -\frac{5\sqrt{2}}{72}$, fixing the free constants $a$ and $b$.

However, since $B_{\alpha\beta\gamma\delta} = 0$ has to be an equation of motion we should add to (22) fermion bilinear terms and an FF term to complete the $S_{\alpha\beta\gamma,A}$ equation of motion:

$$\frac{i\sqrt{3}}{2} B_{\alpha\beta\gamma\delta} \frac{1}{\sqrt{E}} = -\frac{k}{5} \epsilon_{\alpha\beta\gamma\delta\eta\kappa} \frac{\delta L^{(7d)}}{\delta S^{\kappa\eta,A}} Y^A$$

(23)

At this moment all the ansätze are fixed, and we can verify the remaining terms in the 7d susy transformation rules (II.4,II.6,II.7). This provides a number of independent nontrivial checks on all our ansätze. The calculations involved in these checks will be published elsewhere [4]. They involve heavy use of the formalism of spherical harmonics [20].

Finally, let us comment on applications to the AdS-CFT correspondence. The fact that there exists a consistent truncation means that we can use the 7 dimensional gauged sugra action for calculations of correlators of the operators in the 6d (0,2) CFT which correspond to the gauged supergravity fields, at leading order in $N$. Indeed, consistency of the truncation means that there are no linear couplings of ‘massive’ fields to the gauged sugra, and so in the tree diagrams of gauged sugra the massive fields will not appear. In [21], a computation of correlators of chiral primary operators in the CFT was performed, following the work for the $AdS_5 \times S_5$ case in [22] (for other calculations of 3- and 4-point functions see [5, 6]). To find the correct CFT behaviour, a nonlinear redefinition of the scalar fields was also needed, which did result in a consistent truncation of the scalar modes to the massless ones. The nonlinear redefinition in $d = 7$ is equivalent to our nonlinear embedding in $d = 11$ for the massless modes, but note that the results of [21] are only up to quadratic order (and only for the scalars) whereas we do find a fully consistent truncation to all massless modes. With our results one can extend the calculations of CFT correlators to the other massless (sugra) bosonic sectors and to the fermionic sector.

We expect that we can also find a consistent truncation in the $AdS_5 \times S_5$ case, in which the same comments apply to the correspondence between $AdS_5 \times S_5$ and $N=4$ d=4 SYM. (Again, a consistent truncation of the scalar modes to the massless ones was implicitly obtained in [22], by imposing the correct CFT behaviour). Perhaps our methods can also be used to complete the explicit expression for the truncation on $AdS_4 \times S_7$.

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