Parity reversion of $^{12}_Λ$Be

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Abstract

The spectrum of $^{12}_Λ$Be is studied by an extended version of antisymmetrized molecular dynamics for hypernuclei. The result predicts the positive-parity ground state of $^{12}_Λ$Be that is reverted to the normal one by the impurity effect of Λ particle. The reversion of the parity is due to the difference of Λ binding energy in the positive- and negative-parity states that originates in the difference of α clustering and deformation.
Despite of their short lifetime, hypernuclei have been a subject of particular interest in nuclear physics. They provide an almost unique opportunity to investigate underlying baryon-baryon interactions. Especially the knowledge of the interaction between Λ and nucleons is greatly increased in these decades [1–6]. This development strongly promotes the physics of hypernuclear many-body problems. A peculiar interest of hypernuclear many-body problems is the dynamical features Λ hypernuclei manifest by the addition of Λ particle such as the stabilization of the system [7, 8], modifications of sizes [9, 10], deformation and clustering [11–16]. Thus, hypernuclear many-body physics can be regarded as an impurity physics. It offers a means to investigate dynamical responses of nuclei to the addition of hyperons or a means to investigate nuclear structure by using hyperons as probe. As the forthcoming experiments will expand domain of hypernuclear physics to neutron-rich or heavier system, many exotic phenomena due to the impurity effect of Λ will be uncovered.

Single Λ hypernuclei of neutron-rich Be isotopes are one of such systems of interest and to be experimentally accessible. More specifically, $^{12}_Λ$Be is of particular interest, since the core nucleus $^{11}$Be is known to have quite exotic structure. It has two bound states with positive- and negative-parity. The ground state is positive parity and the negative-parity state is located at 320 keV above the ground state [17–19]. Since the order of these two states contradicts to the ordinary nuclear shell ordering and the neutron shell gap of $N = 8$ is collapsed, it is called “parity inversion” and “breakdown of $N = 8$ magic number”. Our question is how the “parity reversion” will be affected and modified in $^{12}_Λ$Be by the impurity effect of Λ. This letter reports that the parity inverted in $^{11}$Be will be reverted in $^{12}_Λ$Be by adding Λ particle. This study is based on the theoretical framework of antisymmetrized molecular dynamics (AMD). AMD has been applied to investigate the exotic phenomena in neutron-rich nuclei and has successfully described them such as the breakdown of $N = 8$ and $N = 20$ magic numbers [20–25]. In this study, we use an extended version of AMD for hypernuclei (HyperAMD) to investigate $^{12}_Λ$Be. HyperAMD has already been applied to $p$-$sd$ shell hypernuclei [15, 16] and the reader is directed to them for its detailed formulation. The Hamiltonian used in this study is given as,

$$H = H_N + H_\Lambda - T_g,$$

$$H_N = T_N + V_{NN} + V_C, \quad H_\Lambda = T_\Lambda + V_{\Lambda N},$$

where $T_N$, $T_\Lambda$ and $T_g$ are the kinetic energies of the nucleons, Λ particle and the center-
of-mass motion. The Gogny D1S [26] is used as an effective nucleon-nucleon interaction \( V_{NN} \), and the Coulomb interaction \( V_C \) is approximated by the sum of seven Gaussians. To see the dependence on \( \Lambda N \) interaction, a couple of \( \Lambda N \) effective interactions \( V_{\Lambda N} \) are examined. We have used \( YN \) G-matrix interactions \( YNG-ND \) and \( YNG-NF \) [2] which are respectively derived from the realistic one-boson-exchange potentials of the Nijmegen model-D and model-F [1]. We also used a modified version of \( YNG-NF \) (Improved-NF) suggested by Hiyama et al. [27]. These \( \Lambda N \) interactions have the dependence on the nucleon Fermi momentum \( k_F \) and the value of \( k_F = 0.973 \) fm\(^{-1} \) is applied, that is common to the Ref. [27].

The variational wave function of HyperAMD is the eigenstate of the parity. The intrinsic wave function \( \Psi_{int} \) is represented by the direct product of the \( \Lambda \) single particle wave function \( \varphi_{\Lambda} \) and the wave function of the \( A \) nucleons \( \Psi_N \) which is a Slater determinant of the nucleon wave packets \( \psi_i \),

\[
\Psi^\pm = P^\pm \Psi_{int}, \quad \Psi_{int} = \Psi_N \otimes \varphi_Y, \quad (3)
\]

\[
\Psi_N = \frac{1}{\sqrt{A!}} \det \{ \psi_i (r_j) \}, \quad (4)
\]

where \( P^\pm \) is the parity projector. The nucleon single particle wave packet is represented by a Gaussian,

\[
\psi_i (r) = \phi_i (r) \chi_i \tau_i, \quad (5)
\]

\[
\phi_i (r) = \prod_{\sigma=x,y,z} \left( \frac{2 \nu_{\sigma}}{\pi} \right)^{1/4} \exp \left\{ - \nu_{\sigma} (r - Z_i \sigma)^2 \right\}, \quad (6)
\]

\[
\chi_i = a_i \chi_{\uparrow} + b_i \chi_{\downarrow}, \quad \tau_i = p \text{ or } n. \quad (7)
\]

The \( \Lambda \) single particle wave function is represented by a sum of Gaussians,

\[
\varphi_{\Lambda} (r) = \sum_{m=1}^{M} c_m \varphi_m (r), \quad \varphi_m (r) = \phi_m (r) \chi_m, \quad (8)
\]

\[
\phi_m (r) = \prod_{\sigma=x,y,z} \left( \frac{2 \nu_{\sigma}}{\pi} \right)^{1/4} \exp \left\{ - \nu_{\sigma} (r - \zeta_m \sigma)^2 \right\}, \quad (9)
\]

\[
\chi_m = \alpha_m \chi_{\uparrow} + \beta_m \chi_{\downarrow}, \quad (10)
\]

where the number of Gaussians \( M \) is chosen large enough to achieve the energy convergence. The centroids of Gaussian wave packets \( Z_i \) and \( \zeta_m \), the width of Gaussian \( \nu_{\sigma} \), the coefficients \( c_m \) and spin directions \( a_i, b_i, \alpha_m, \beta_m \) are the variational parameters. They are so determined
to minimize the total energy under the constraint on the matter quadrupole deformation parameter $\beta$ \cite{20}. The constraint is imposed on the value of $\beta$ from 0 to 1.2 with the interval of 0.025.

After the variation, we project out the eigenstate of the total angular momentum $J$ for each value of $\beta$,

$$\Psi_{MK}^{J\pm}(\beta) = \frac{2J + 1}{8\pi^2} \int d\Omega D_{MK}^{J\pm}(\Omega) R(\Omega) \Psi^\pm(\beta).$$  \hspace{1cm} (11)

The integrals over three Euler angles $\Omega$ are performed numerically. The calculation is completed by the generator coordinate method (GCM) \cite{28}. The wave functions that have different values of $K$ and $\beta$ are superposed,

$$\Psi_n^{J\pm} = \sum_p \sum_{K=-J}^J c_{npK} \Psi_{MK}^{J\pm}(\beta_p).$$ \hspace{1cm} (12)

The coefficients $c_{npK}$ are determined by solving Griffin-Hill-Wheeler equation.

![FIG. 1: (color online) (a) Observed spectra of $^{13}$C and $^{11}$Be. (b) Spectrum of $^{11}$Be calculated by AMD. (c) Spectra of $^{12}_\Lambda$Be calculated by HyperAMD with three different $\Lambda N$ interactions.](image)

Before the discussion on $^{12}_\Lambda$Be, it is helpful to overview the structure of $^{11}$Be. Figure 1(a) shows the observed spectra of $N = 7$ isotones, $^{13}$C and $^{11}$Be. The large shell gap between $p_{1/2}$ and $sd$-shell in $^{13}$C is collapsed in $^{11}$Be. Namely, the ground state is positive parity and the order of $p_{3/2}$ and $sd$-shell looks inverted in $^{11}$Be, that is called “parity inversion” \cite{17, 18}. Using the original parameter set of the Gogny D1S, our calculation successfully reproduces the spin-parity of the ground state with the binding energy of 65.32 MeV and the first excited state $1/2^-_1$ is located at 540 keV, while the observed values are 65.48 MeV...
and 320 keV, respectively \[19\]. For more quantitative discussion of $^{12}\Lambda$Be, we have weakened the spin-orbit interaction of Gogny D1S by 5\% to reproduce the observed $1/2^-$ excitation energy exactly. By this modification, the binding energy of $^{11}$Be is calculated as 64.77 MeV and the resulting spectrum is shown in Fig. 1(b). Here, the excited unbound states are calculated within the bound state approximation.

It is known that the low-lying states of Be isotopes have 2\(\alpha\) cluster core and valence neutrons occupying the molecular-orbits around the core which are so-called \(\pi\) and \(\sigma\) orbits \[22\]. The formation of the 2\(\alpha\) cluster core in each state is confirmed in the proton density shown in Fig. 2. The ground state is a member of the \(K^{\pi} = 1/2^+\) band in which two of three valence neutrons occupy \(\pi\)-orbit and the last valence neutron occupies \(\sigma\)-orbit. In terms of the spherical shell model, a neutron is promoted into \(sd\)-shell across the \(N = 8\) shell gap (breakdown of magic number \(N = 8\)). The first excited state is negative parity and belongs to the \(K^{\pi} = 1/2^-\) band. All valence neutrons occupy \(\pi\)-orbit or \(p\)-shell, that corresponds to the normal shell order. As we can see in Fig. 2 and Table II the ground state has more pronounced 2\(\alpha\) clustering and larger quadrupole deformation \(\beta\) than the first excited state. Here, deformation \(\beta\) of each state is defined as that of the basis wave function \(\Psi_{MK}^{J\pm}(\beta)\) which
FIG. 3: (color online) The $\Lambda$ binding energy of the $0^-_1$ and $0^+_1$ states as function of proton quadrupole deformation parameter $\beta$. Lines in the figure denote the deformation parameter of $0^+_1$ states given in Table. [30–32]. We just remark here that the ground state is more deformed than the first excited state and has a neutron in $sd$-shell.

The spectra of $^{12}_\Lambda$Be obtained with three different $\Lambda N$ interactions are shown in Fig. (c). All states shown in the figure have a $\Lambda$ in $s$-orbit and are classified into three bands. They are generated by the coupling of $K^\pi = 1/2^+$, $1/2^-$ and $3/2^-$ bands of $^{11}$Be with $\Lambda$ in $s$-orbit, and therefore, there are always doublet states of $^{12}_\Lambda$Be for each corresponding state of $^{11}$Be. These bands are denoted as $K^\pi = 1/2^+ \otimes \Lambda_s$, $1/2^- \otimes \Lambda_s$ and $3/2^- \otimes \Lambda_s$.

It is found that all of three $\Lambda N$ interactions predicts the negative-parity ground state and give qualitatively same results. Therefore, we focus on the Improved-NF result for a while. The ground doublet is the $0^-_1$ and $1^-_1$ states with the binding energies of 74.69 and 74.66 MeV, that have the configuration of $^{11}$Be$(1/2^-_1) \otimes \Lambda_s$. The first excited doublet is $0^+_1$ and...
TABLE I: Calculated binding and excitation energies $B$ and $E_x$ [MeV], matter quadrupole deformation $\beta$ and root-mean-square radii $r_{rms}$ [fm] for band-head states of $^{11}$Be and $^{12}_{\Lambda}$Be. Numbers in parenthesis shows the observed data [29]. The $\Lambda$ binding energy $B_\Lambda$ [MeV], the $\Lambda$ kinetic energy $T_\Lambda$ [MeV] and $\Lambda N$ potential energy $V_{AN}$ [MeV] are also shown for $^{12}_{\Lambda}$Be.

| $J^\pi$ | $B$ | $E_x$ | $\beta$ | $r_{rms}$ | $B_\Lambda$ | $T_\Lambda$ | $V_{AN}$ |
|----------|-----|------|--------|--------|----------|-----------|--------|
| $1/2_1^-$ | 64.45 | 0.32 | 0.52 | 2.53 | | | |
| | | (65.16) | (0.32) | | | | |
| $^{11}$Be $1/2^+_1$ | 64.77 | 0 | 0.72 | 2.69 | | | |
| | | (65.48) | (0) | | | | |
| $3/2_1^-$ | 62.72 | 2.05 | 0.90 | 2.98 | | | |
| $0_1^-$ | 74.69 | 0 | 0.47 | 2.51 | 10.24 | 6.71 | -16.93 |
| $^{12}_{\Lambda}$Be $0^+_1$ | 74.44 | 0.25 | 0.70 | 2.67 | 9.67 | 6.68 | -16.42 |
| $1/2^-_3$ | 71.32 | 3.37 | 0.87 | 2.94 | 8.60 | 6.36 | -15.08 |

$1^+_1$ states at 250 and 310 keV with the configuration of $^{11}$Be($1/2^+_1$) $\otimes$ $\Lambda_s$. Thus the ground state parity is reverted in $^{12}_{\Lambda}$Be, as if the addition of $\Lambda$ has restored the $N = 8$ shell gap. This parity reversion of $^{12}_{\Lambda}$Be is due to the difference of $\Lambda$ binding energy $B_\Lambda$ in the ground and first excited doublets. As shown in Table, the $0_1^-$ state has larger $B_\Lambda$ than the $0^+_1$ state by about 500 keV. Here $B_\Lambda$ is defined as the difference of binding energies between $^{12}_{\Lambda}$Be and corresponding $^{11}$Be states,

$$B_\Lambda = B(^{12}_{\Lambda}\text{Be}(J^\pi)) - B(^{11}\text{Be}(J^\pi)).$$

The change of the nuclear part $H_N$ by addition of $\Lambda$ is rather small. Therefore the difference of $B_\Lambda$ overwhelms the energy difference between the $1/2^+_1$ states of $^{11}$Be, and the parity reversion is realized in $^{12}_{\Lambda}$Be. The difference in $B_\Lambda$ mainly comes from the difference in the $\Lambda N$ potential $V_{AN}$ as shown in Table, and it originates in the difference of the quadrupole deformation. Figure 3 shows the $\Lambda$ binding energies in $0^+_1$ states as function of quadrupole deformation, that is defined as the expectation value of $H_\Lambda$ by the angular-momentum projected wave functions for each deformation parameter $\beta$,

$$b^\pm_\Lambda(\beta) = -\langle \Psi^0(\beta) | H_\Lambda | \Psi^{0\pm}(\beta) \rangle.$$

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The $b_\Lambda$ of $0^+_1$ and $0^-_1$ states rapidly decrease as deformation becomes larger and their behavior are quite similar to each other. As deformation becomes larger, the overlap between the wave functions of $\Lambda$ and nucleons decreases to reduce $V_{\Lambda N}$. We can confirm that the 500 keV difference in $B_\Lambda$ originates in the different deformation of $0^+_1$ states from Fig. 3. This reduction of $b_\Lambda$ as function of quadrupole deformation is qualitatively common to other $p$-$sd$-shell nuclei discussed in Ref. [15, 16]. Since the $3/2^-_1$ state is most deformed among the band head states of $^{11}$Be, the $1^-_3$ state ($^{11}$Be($3/2^-_1$) $\otimes \Lambda s$) has smallest $B_\Lambda$. Therefore, its excitation energy is also shifted up compared to $^{11}$Be. The reduction of $B_\Lambda$ is common to other member states of $1/2^- \otimes \Lambda s$ and $3/2^- \otimes \Lambda s$ bands.

By the addition of $\Lambda$ particle, structure of core nucleus $^{11}$Be is slightly modified. Due to the attraction of $\Lambda$ particle sitting at the center of the system, the inter-cluster distance between $2\alpha$ clusters is reduced (Fig. 2), and it leads to the reduction of the deformation and radius (Table. I). Dispite of the neutron-halo structure of $^{11}$Be, this reduction if rather small compared to the observed case of $^7\Lambda Li$ and cannot be clearly seen in the density distribution (Fig. 2). This is due to the presence of valence neutrons occupying molecular-orbits in $^{12}\Lambda Be$. When the distance between $2\alpha$ is reduced, the valence neutrons in $\pi$ or $\sigma$-orbits loose their binding energies. Therefore valence neutrons prevent drastic reduction of $2\alpha$ distance. Note that this explains why the change of the expectation value of nuclear part is not large compared to the difference of $B_\Lambda$. Other interactions also predict the parity reversion of $^{12}\Lambda Be$, and the mechanism is common to the case of Improved-NF. Modification of nuclear structure is not large, but the different deformation between the states leads to the difference in $B_\Lambda$ to revert the parity of $^{12}\Lambda Be$. Therefore, the predicted parity reversion of $^{12}\Lambda Be$ suggests a possibility to probe the different deformation between the ground and first excited states $^{11}$Be by adding an impurity of $\Lambda$ particle.

Finally, we discuss the differences between three $\Lambda N$ interactions. It was pointed out by Hiyama et al. [27] that YNG-ND and NF have too attractive odd-parity interaction and does not reproduce the $B_\Lambda$ of $^9\Lambda Be$. They suggested the Improved-NF introducing a repulsive short-range part in the odd-parity interaction. The difference between these $\Lambda N$ interactions can be found in the spectrum of $^{12}\Lambda Be$. We see that the $K^\pi = 1/2^+ \otimes \Lambda s$ and $3/2^- \otimes \Lambda s$ bands are located at higher excitation energies in YNG-NF and ND than Improved-NF. We remind the reader that the $K^\pi = 1/2^-$ band of $^{11}$Be has 7 nucleons in $p$-shell, while the $K^\pi = 1/2^+$ and $3/2^-$ bands have 6 and 5 nucleons. Therefore, the number of the odd-parity
interactions between Λ and nucleons decreases for the $1/2^+_1 \otimes \Lambda_s$, $1/2^-_1 \otimes \Lambda_s$ and $3/2^-_1 \otimes \Lambda_s$ bands in descending order. Consequently, when the odd-parity interaction becomes more attractive, the energy of the $1/2^-_1 \otimes \Lambda_s$ band is lowered relative to other two bands, or in other words, the $1/2^+_1 \otimes \Lambda_s$ and $3/2^-_1 \otimes \Lambda_s$ bands are pushed up. Thus the excitation energies of the $K^\pi = 1/2^+_1 \otimes \Lambda_s$ and $3/2^-_1 \otimes \Lambda_s$ are sensitive to the odd-parity part of ΛN interaction.

In summary, the low-lying states of $^{12}_\Lambda$Be have been investigated by the HyperAMD. We predict the parity reversion of the $^{12}_\Lambda$Be; the ground state parity inverted in $^{11}$Be is reverted in $^{12}_\Lambda$Be. The parity reversion is caused by the different deformation of the ground and first excited states of $^{11}$Be, that produces the difference in $B_\Lambda$. This parity reversion suggests a possibility to probe the different deformation between the ground and first excited states $^{11}$Be by adding Λ particle as impurity. We also point out that the excitation energies of the $K^\pi = 1/2^+_1 \otimes \Lambda_s$ and $3/2^-_1 \otimes \Lambda_s$ are sensitive to the odd-parity part of the ΛN interaction.

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