A quantitative analysis of concepts and semantic structure in written language: Long range correlations in dynamics of texts

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Understanding texts requires memory: the reader has to keep in mind enough words to create meaning. This calls for a relation between the memory of the reader and the structure of the text. To investigate this interaction, we first identify a connectivity matrix defined by co-occurrence of words in the text. A vector space of words characterizing the text is spanned by the principal directions of this matrix. It is useful to think of these weighted combinations of words as representing ‘concepts’. As the reader follows the text, the set of words in her window of attention follows a dynamical motion among these concepts. We observe long range power law correlations in this trajectory. By explicitly constructing surrogate hierarchical texts, we demonstrate that the power law originates from structural organization of texts into subunits such as chapters and paragraphs.

INTRODUCTION

Language is a central link through which we interact with other people. As a channel of communication it is limited by our physical ability to speak only one word at a time. The question arises therefore how the complex products of our brain are transformed into the linear string of words that comprise speech or text. Since our mental processes are far from being one dimensional, the use of memory is essential, as is the existence of some type of correlations in time.

Such questions have a long and intense history. Bolzano already noted on the need for specific organization in scientific texts, while Ingardenvon the book”to understand the process by which a text is understood and assimilated. Modern methods combine the work of linguists with those of computer scientists, physicists, psychologists and researchers from many other fields to cover a wide range of topics: from the phoneme, going on to words and grammar, all the way to global text analysis and the evolution of language.

Recent interest has focused on applying methods of statistical physics to identify possible trends and correlations in text. In, for example, the authors study the distribution of words across different works by the same authors, combining notions of information, entropy and statistics to define a random walk on the text. Long ranged correlations have been found in the writings of Shakespeare and Dickens, and a number of hypotheses as to their origin have been proposed. These include the overall existence of ideas and meaning, or of some semantic hierarchy.

Here we aim at a middle ground both in methods of analysis and in ranges of text, based on geometric intuition developed in. We use a variant of Latent Semantic Analysis to uncover semantic connections in the text. This method leads to the identification of cohesive groups of words, and we find it useful to think of these groups as describing “concepts”. The most important of these groups are singled out, and form an (orthogonal) basis of a vector space defined by the text. We then introduce a dynamics on the space of concepts and find long range power-law correlations in time. We end by showing that the origin of these correlations can be found in the hierarchical structure of the text. In this way, we are able to connect with the classic work of Bolzano and Ingarden on the intelligibility of texts.

THE CONCEPT SPACE

We define on the set \( \{ w \} \) of words in the text an arbitrary order, \( W = \{ w_1, w_2, \ldots \} \), for example using their rank, which is the number of times \( m_i \) that word \( w_i \) appears in the text under consideration. We associate with \( w_1 \) the vector \( (1, 0, 0, \ldots) \), with \( w_2 \) the vector \( (0, 1, 0, \ldots) \), and with \( w_i \) the vector which has a “1” at position \( i \) and all others “0”. These vectors span a vector space in which each axis corresponds to one single word.

The analysis proceeds with the construction of a symmetric connectivity matrix \( M \) based on co-occurrence of words. This matrix has rows and columns indexed by words, and the entry \( M_{ij} \) counts how often word \( w_i \) and \( w_j \) co-occur within a distance \( d \) in a given text. We suggestively term \( d \) as the “window of context”, and typically take it to be of size 100.

The connectivity matrix \( M \) is then normalized to take into account the abundance of words. If the \( m_i \) occurrences of \( w_i \) are randomly distributed and are not closer to each other than \( 2d \) (a reasonable assumption if \( d \ll L \)) then the probability that any of the occurrences \( m_j \) of word \( w_j \) will randomly fall within a distance \( d \) them is given by

\[
R_{ij} = \frac{2dm_i m_j}{L}
\]
| Book | Length | $m_{thr}$ | P | T | $S_{conv}$ | Exponent |
|------|--------|----------|---|---|------------|-----------|
| MT   | 22375  | 4        | 342 | 16.2 | 25 | 0.45 (0.05) |
| HM   | 32564  | 5        | 425 | 15.5 | 30 | 0.92 (0.03) |
| NK   | 62190  | 8        | 750 | 20.4 | 60 | 0.81 (0.03) |
| TS   | 73291  | 8        | 644 | 16.8 | 45 | 0.50 (0.03) |
| DC   | 77728  | 8        | 781 | 19.8 | 75 | 0.43 (0.07) |
| IL   | 152400 | 12       | 774 | 22.0 | 75 | 0.40 (0.05) |
| MD   | 213682 | 14       | 1162| 20.0 | 75 | 0.45 (0.04) |
| QJ   | 402870 | 20       | 1246| 18.7 | 100| 0.36 (0.03) |
| WP   | 529547 | 23       | 1498| 22.9 | 300| 0.43 (0.03) |
| EI   | 30715  | 5        | 448 | 25.3 | 50 | 0.75 (0.10) |
| RP   | 118661 | 11       | 609 | 14.5 | 75 | 0.60 (0.06) |
| KT   | 197802 | 14       | 661 | 25.4 | 50 | 0.30 (0.05) |

**TABLE I:** Table of book parameters and results. $m_{thr}$ is the threshold for the number of occurrences and $P$ is the number of words kept after thresholding. $T$ is the percentage of the words in the book that pass the threshold, $T = \frac{\sum_{i=1}^{P} m_i}{L}$. $S_{conv}$ is the dimension at which a power law is being fit. The exponent of the fit is given in the last column, together with its error in parenthesis.

The idea behind this choice of principal directions is that the most important vectors in this decomposition (those with highest singular value) describe concepts. A connectivity matrix similar to the one we use has been introduced before \[11, 20\], based on adjacency of words rather than our looser requirement that words appear together within a wider window. This resulted in the ability to cluster words according to context and identify ambiguity in words \[12\]. What we derive here may be viewed as a large-scale version of the assignment of meaning by co-occurrence, in comparison to the local result obtained previously \[12\].

We now have a basis of the SVD such that every word can be described as a unique superposition of the basis vectors. Thus,

$$e_i = \sum_{j=1}^{S} a_{ij} v_j ,$$

where $e_i$ is the vector of all zeros except at position $i$ (representing the word $w_i$) while the $v_j$ are the vectors of the SVD.

**TEXTS**

We used twelve books—in their English version—for our analysis. Nine of them are novels: “War and Peace” by Tolstoi (WP), “Don Quixote” by Cervantes (QJ), “The Iliad” by Homer (IL), “Moby Dick” (MD) by Melville, “David Crocket” by Abbott (DC), “The Adventures of Tom Sawyer” by Twain (TS), “Naked Lunch” by Burroughs (NK), “Hamlet” by Shakespeare (HM) and “Metamorphosis” by Kafka (MT). They span a variety of periods, styles and also have very different lengths (see Table I). Besides the nine novels we analyzed the scientific didactic book “Special theory of relativity ” by Einstein (EI), the philosophical treatises “The Critique of Pure Reason” by Kant (KT) and “The Republic” by Plato (RP).

Each of the books is processed by eliminating punctuation and extracting the words. Each word is “stemmed” by querying the on-line database WordNet 2.0, and the leading word for this query is retained without keeping the information on whether it was originally a noun, a verb or an adjective .

All the stop words, i.e., words that carry no significant meaning, are assigned a value of zero. The list of these words consists of determiners, pronouns, and the like. This standard list is supplemented with a list of broadly used words which are abundant independently of the text. In practice we reject those words whose occurrences are above $m_{thr}$ in over 90% of the books.

Books are thus transformed into a list of stemmed words with which the connectivity matrix is defined, and to which the SVD process is applied.

Examples of the concept vectors from the different books are illuminating (see Table I). The first ten words in the principal component with highest singular value in Moby Dick immediately carry us into the frame of the story and introduce


| MD(1) | MD(5) | EI(1) | EI(2) | TS(1) | TS(2) |
|-------|-------|-------|-------|-------|-------|
| whale | bed   | surface | planet | spunk | ticket |
| ahab  | room  | Euclidean | sun    | wart  | bible  |
| starbuck | queequeg | being | ellipse | huck  | verse  |
| sperm | dat   | universe | mercury | bigger | blue  |
| cry    | aye   | rod    | orbital | reckon | yellow |
| aye    | moby  | spherical | orbit  | stump | pupil  |
| sir    | dick  | plane  | star    | bet   | ten    |
| boat   | landlord | geometry | arc | midnight | spunk |
| stubb  | ahab  | continuum | angle | johnny | red    |
| leviathan | whale | sphere | second | em | thousand |

**TABLE II:** Examples of the highest singular components for three books. Given are component one and five of Moby Dick (MD), one and two of Einstein (EI) and of Tom Sawyer (TS). The coefficients of the words in the singular component may be positive or negative and their absolute values range from 0.13 to 0.3.

many of its protagonists. The next three principal components are somewhat similar, with the addition of familiar words such as white, shark, captain, ship. By the fifth largest principal component a change of scene occurs as the story takes a detour indoors, and this is evidenced by the second column of Table II.

Similarly, the first ten words of the principal component with highest singular value of Einstein’s “Special relativity” launch us immediately into the subject matter of special relativity, while its second component brings in the applications to astrophysics. It is perhaps amusing to recall the tales of Tom Sawyer by viewing the principal component with highest singular value. These deal with Tom’s various escapades, for example the bible competition which Tom wins by procuring tickets by various trades and bargains.

We can conclude that the “concepts” we defined by using singular vectors do indeed capture much of the content of the text.

**DYNAMIC ANALYSIS**

Having found a representative basis for each of the texts, our main interest is in the dynamics of reading through the text. What is new here in comparison with earlier statistical analysis [18] or linguistic research [4] is that the basic ingredient is not the byte (as in the statistical studies) nor the word, but rather a contextual collection of words (our concept vector). In this way, our study links the word connectivity matrix to semantic meaning.

Basically, we again slide a “window of attention” of fixed size $A$ along the text and observe how the corresponding vector moves in the vector space spanned by the SVD. If this vector space is irrelevant to the text, then the trajectory defined in this space would probably be completely stochastic, and would perform a random walk. However, if the evolution of the text is reflected in this vector space, then the trajectory should trace out the concepts alluded to earlier in a systematic way, and some evidence of this will be observed.

To keep the algorithm reasonably simple, we divide the length of the text into $L/A$ non-overlapping windows, where a value of $A = 150$ words is a good choice. We can gain some intuition in this vector space by replacing the notion of distance along the text (measured in segments of $A$ words) with the concept of time (measured by the time it takes a hypothetical reader to read $A$ words). We define time as $t = \ell \times \delta t$, with $\ell$ the index of the window and $\delta t$ the average time it takes to read $A$ words. For each window we obtain a vector $V(t)$ which is decomposed as:

$$V(t) = \sum_{j=1}^{S} a_j(t)v_j,$$

with the SVD basis $\{v_j\}$ chosen as before.

The moving vector $V(t) \in \mathbb{R}^S$ is a dynamical system and we proceed to study its autocorrelation function in time $C(\tau) = \langle V(t)V(t + \tau) \rangle_t - \langle V(t) \rangle_t^2$, where $\langle \rangle_t$ is the time average. Fig. shows the correlation function of the concept vector in time for “Tom Sawyer” given in a log-log scale. The different lines correspond to different values of $S$. The function is non-zero over a large range, on the order of more than $10^3$ words. This range is much longer than what we found when measuring correlations among sentences, without using the concept vectors (data not shown).

As the dimension $S$ increases the correlation function in the log-log representation converges to a straight line, indicating a very clear power law behavior. The convergence to a power-law behavior and the dimension necessary to produce it depend on the book.

The range at which the correlation is significant and above the noise depends both on the exponent and on the natural noise in the system. The noise in turn depends both on the
quality of the expansion in terms of the SVD and on the cohesiveness of the text.

The results presented in Table I (see Fig. 2) are given for the lowest value of $S$ at which the convergence to a power-law behavior is clearly discerned.

![Power-law fit](image)

**FIG. 2**: Autocorrelation functions and fits for seven of the books listed. The autocorrelation functions are truncated at the level where the noise sets in.

The long range correlations uncovered in this fashion are in line with previous measurements obtained using the random walk approach of [1, 18, 21]. However, the range over which we find correlations is much larger, and the quality of the power law fit is accordingly significantly better.

### CONTROLS

The methods we have described above require a certain number of parameters, such as the threshold rank value $m_{th}$ of the matrix, or the size of the windows that are being moved along the text. We describe here some tests which were performed to check the robustness of the method when these parameters are changed, summarizing the most relevant findings:

1) The threshold must be chosen carefully. By lowering the threshold $m_{th}$ of accepted words, and therefore increasing the number of accepted words, one observes a systematic decrease of correlations. One can take a matrix of $P_{rows}$ rows and $P_{col} = P_{rows}$ columns. The results are shown in Table III and should be compared to those of Table I.

2) A change in the size of the window of attention (the variable $A$) does not affect the results significantly as long as it is kept above 100 words and not much bigger than the window of context ($d = 200$ words). A lower value of $A$ means a lower number of words per window and a correspondingly higher noise.

3) We checked, to some extent, the language dependence of the method, by comparing “Don Quixote” in Spanish and English. While languages can have quite different local syntactic rules, the long term correlations practically do not depend on the language. This is perhaps related to the importance of nouns in creating the correlation function, and these are translated more or less one for one.

### HIERARCHY AND THE ORIGIN OF SCALING

The existence of power laws is often traced to hierarchical structures [23]. We put forward the hypothesis that in our case these structures are parts of the texts (such as “volumes”, “parts”, “chapters” within parts, “sections” in chapters, “paragraphs” in sections, and so on). This is a hierarchy of $K$ levels, each containing several parts. For example, a book may be in 3 volumes that each have about 10 chapters, each of which is divided in 8 or so sections, etc. For simplicity, we assume that each level contains the same number of parts $H$. Typical values are $K = 4$ and $H ≈ 7$. The important point is that the text has the structure of a tree.

We now show that the power law we found earlier for the text is not changed if words are permuted in the text, provided one respects as much as possible the structure of the book as a whole. As discussed above, if the structure is not kept, the randomized text has no correlations.

We prepare an (initially empty) hierarchical book as a template into which we will insert the words from the original book. The empty book is divided in $H$ roughly equal parts, each subdivided again in $H$ roughly equal parts. This subdivision is repeated $K$ times. We end up with the book divided in $K$ levels and a total of $H^K$ subdivisions at the smallest scale. $K$ and $H$ are chosen so that the lowest level corresponding to “paragraph” will have around 100 words.

We place each word into the hierarchical book individually.

| Book | $m_{row}$ | $m_{col}$ | $P_{rows}$ | $P_{col}$ | $T$   | Exponent |
|------|-----------|-----------|------------|-----------|-------|----------|
| MT   | 4         | 4         | 1709       | 342       | 24.9  | 0.65 (0.05) |
| HM   | 5         | 5         | 3440       | 245       | 29.4  | 1.40 (0.20) |
| NK   | 8         | 8         | 3665       | 750       | 35.8  | 1.00 (0.10) |
| TS   | 8         | 8         | 3748       | 644       | 26.4  | 0.70 (0.10) |
| DC   | 8         | 8         | 3071       | 781       | 30.1  | 0.60 (0.08) |
| IL   | 3         | 3         | 2392       | 774       | 27.8  | 0.48 (0.05) |
| MD   | 4         | 4         | 4007       | 1162      | 29.1  | 1.00 (0.10) |
| QJ   | 5         | 5         | 3574       | 1246      | 24.2  | 0.45 (0.03) |
| WP   | 6         | 6         | 4130       | 1498      | 28.6  | 0.50 (0.05) |
| EI   | 5         | 5         | 1721       | 448       | 32.0  | 1.00 (0.20) |
| RP   | 3         | 3         | 2020       | 609       | 20.5  | 0.70 (0.05) |
| KT   | 4         | 4         | 1549       | 661       | 28.6  | 0.37 (0.03) |
Assume the word $w_i$ appears $m_i$ times in the original text. Fix a parameter $E > 1$ (for concreteness take $E = 5$). We define weights recursively for each subdivision. At the top level, each part has the initial weight $J$. Choose one part randomly and confer on it weight $J \cdot E$. The next level inherits the weights introduced before. Now repeat the choice of a random part from the second level, and multiply its weight by $E$. Depending on which slot has been chosen, at the second level there may be one slot with weight $JE^2$, and the others have weight $JE$ or $J$, or there are weights $JE$ and $J$ only. Going on in this fashion we fill all the levels, reaching finally a range of weights $JE^k$, with $k \in \{0, \ldots, K\}$ at the lowest level. We then choose $J$ so that the sum of weights is one, and distribute the $m_i$ copies of word $w_i$ randomly according the weights in the finest subdivision. This procedure is applied to all words and produces a hierarchical randomized text, that preserves the word distribution and resembles the structural hierarchy of the book.

As seen in Fig. 3, performing this hierarchical randomization process on the book preserves the power law behavior of correlations found in the original text. Since the simple randomization destroys the power law see Fig. 3, we can conclude that the power laws of the original text do indeed originate in their hierarchical structure.

One can improve the fit of the power law by introducing further parameters: For example, skipping randomly some levels in the construction some of the weights $JE^k$ gives (this corresponds to pruning some branches of the tree) or by changing the value of $E$ for each word, specifically, by increasing the value of $E$ for some words with lower rank (this comes from the observation than lower ranks words tend to be more concentrated around certain paragraphs or chapters than higher rank words).

CONCLUSIONS

Many questions remain to be addressed, for example applying the dynamic approach to spoken text, in which repetitions are known to be of importance, and comparing the results to those of written text. It may also be of interest to characterize different types of text or of authors according to the correlation exponent. It remains to be seen whether the hierarchical organization we have identified in texts is related to a hierarchical organization in our thought processes.

Our approach enables the quantification and rigorous examination of concepts that have been introduced long ago and discussed heuristically by the great classics in the field. Bolzano, in his Wissenschaftslehre, written in 1837, studies the theory of scientific writing, and points out in great detail how such writing should proceed. In particular, in Vol III, he points out that, starting from “symbols” (he probably thinks of Mathematics) one works one’s way to a fully structured text, containing paragraphs, sections, chapters, and so on. He clearly instructs the reader of how to maintain the intelligibility of the text. Ingarden, in his “vom Erkennen des literarischen Kunswerks” talks, from his philosopher point of view about the activity of the brain which compresses parts of texts so that they may be more easily recalled. The entities he has in mind are “layers of understanding” (16, p.111: …not every layer of an already read part of a text is kept in the same way in memory, …The reader keeps bigger and smaller text-connections—Satzzusammenhänge—in his living memory …)

Our study allows to measure the degree to which the insights of authors like these can be understood. It adds therefore a new piece to the puzzle of understanding the nature of language.

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