Nucleon axial charge from quenched lattice QCD with domain wall fermions and improved gauge action *

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In our previous DWF calculation with the Wilson gauge action at $\beta = 6.0$ ($a^{-1} \approx 1.9$ GeV) on a $16^3 \times 32 \times 16$ lattice, we found that $g_A$ had a fairly strong dependence on the quark mass. A simple linear extrapolation of $g_A$ to the chiral limit yielded a value that was almost a factor of two smaller than the experimental one. Here we report our recent study of this issue. In particular, we investigate possible errors arising from finite lattice volume, especially in the lighter quark mass region. We employ a RG-improved gauge action (DBW2), which maintains very good chiral behavior even on a coarse lattice ($a^{-1} \approx 1.3$ GeV), in order to perform simulations at large physical volume ($> 2(2\text{fm})^3$). Our preliminary results suggest that the finite volume effect is significant.

The nucleon (iso-vector) axial charge $g_A$ is a particularly interesting quantity. We know precisely the experimental value $g_A = 1.2670(35)$ from neutron beta decay. Deviation of this quantity from unity, in contrast to the vector charge, $g_V = 1$, reflects the fact that the axial current is only partially conserved in the strong interaction while the vector current is exactly conserved. However neither lattice-QCD nor any model calculation have successfully reproduced this value. Thus, the calculation of $g_A$ is an especially relevant test of the chiral properties of DWF in the baryon sector. In addition, calculation of $g_A$ is an important first step in studying polarized nucleon structure functions since $g_A = \Delta u - \Delta d$ where $\langle p, s|\bar{q}_f \gamma_5 \gamma_i q_f|p, s\rangle = 2s_\mu \Delta g_f$ with $s^2 = -1$ and $s \cdot p = 0$.

We follow the standard practice \cite{1} for the calculation of $g_V$ and $g_A$. We define the three-point functions for the relevant components of the local vector current $J_V^i = \bar{q}_f \gamma_i q_f$ and axial current $J_A^i = \bar{q}_f \gamma_5 \gamma_i q_f$ ($i = 1, 2, 3$):

\begin{equation}
G_{\nu}^{u,d}(t, t') = \text{Tr}[P_v \sum_{x, x'} \langle T N(x) J_A^i(x') \bar{N}(0)\rangle] \tag{1}
\end{equation}

where $\Gamma = V$ (vector) or $A$ (axial) with $P_v = (1 + \gamma_4)/2$ and $P_A = P_v \gamma_5 \gamma_i$. We use the nucleon interpolating operator $N = \epsilon_{abc}(u_a^T C \gamma_5 d_b) u_c$. For the axial current, the three-point function is averaged over $i = 1, 2, 3$. The lattice estimates of vector and axial charges can be derived from the ratio between two- and three-point functions

\begin{equation}
g_{\Gamma} = \frac{G_{\nu}^{1}(t, t') - G_{\nu}^{2}(t, t')}{G_{\nu}(t)}, \tag{2}
\end{equation}

where $G_{\nu}(t) = \text{Tr}[P_v \sum_{x} \langle T N(x) \bar{N}(0)\rangle]$. Recall that in general lattice operators $O_{\text{lat}}$ and continuum operator $O_{\text{con}}$ are regularized in different schemes. The operators are related by a renormalization factor $Z_C$: $O_{\text{con}}(\mu) = Z_C(\mu)O_{\text{lat}}(a)$. This implies that the continuum value of vector and axial charges are given by $g_{\Gamma} = Z_C g_{\Gamma}^\text{lattice}$. In the case of conventional Wilson fermions, the renormalization factor $Z_A$ is usually estimated in perturbation theory ($Z_A$ differs from unity because of explicit symmetry breaking). For DWF, the conserved axial current receives no renormalization. This is not true for the lattice local current. An important advantage with DWF, however, is that the lattice renormalizations, $Z_V$ and $Z_A$, of the local currents are the same \cite{2} so that
the ratio \((g_A/g_v)^{\text{lattice}}\) directly yields the continuum value \(g_A\) \(^{[3]}\).

Our first DWF results are analyzed on 200 quenched gauge configurations at \(\beta = 6.0\) on a \(16^3 \times 32 \times 16\) lattice with \(M_5 = 1.8\) \(^{[3]}\). We found \(g_A/g_v\) exhibited a strong dependence on the quark mass \(^{[3]}\). A simple linear extrapolation of \(g_A\) to the chiral limit yielded a value that was a factor of two smaller than the experimental one. This issue requires checking related systematic effects arising from finite lattice volume and quenching (for example quenched chiral logarithms, zero modes, and the absence of the full pion cloud). In this work, we mainly focus on the former. Indeed, the above calculation was employed in a rather small physical volume \(\sim (1.6\text{fm})^3\) in comparison with the proton charge radius \(\sim 0.7\text{fm}\).

To determine \(g_A\) in large physical volume \(> (2\text{fm})^3\), we perform our simulations on a coarser lattice. However, it is difficult to maintain good chiral properties on a coarse lattice at fixed \(L_\perp\) with the Wilson gauge action. Recent studies have shown that the Iwasaki gauge action enables studies of quenched DWF at smaller \(L_\perp\) than the Wilson gauge action \(^{[3]}\). In this work, we employ a similar type of renormalization group improved gauge action, DBW2 (\(c_1 = -1.4069\)) \(^{[3]}\):

\[
S_G \propto c_0 \sum_{\text{plaq}} \text{Tr} P(1 \times 1) + c_1 \sum_{\text{rect}} \text{Tr} P(1 \times 2) \tag{3}
\]

with \(c_0 + 8c_1 = 1\). The chiral symmetry of DWF with DBW2 is significantly improved over the Iwasaki action \(^{[3]}\) and also provides good scaling behavior of the light hadron spectrum \(^{[3]}\).

Numerical simulations are performed at \(\beta = 0.87\) (\(a \approx 0.15\text{ fm}\)) on lattice sizes \(8^3 \times 24 \times 16\) and \(16^3 \times 32 \times 16\) with \(M_5 = 1.8\). Our preliminary results are analyzed on 170 quenched gauge configurations for the smaller lattice and 86 configurations for the larger lattice.

First, we check whether or not \(Z_v = Z_A\) is well satisfied on this coarse lattice. The vector renormalization derived from a plateau of \(1/g_v^{\text{lattice}}\) plotted against the location of current insertions. In Fig. 1 we show the dependence of \(Z_v\) on \(m_f\) (open circle). The slight quadratic dependence appears because of the fact that \(V_{\mu}^{\text{conser}} = Z_v V_{\mu}^{\text{local}} + \mathcal{O}(m_f^2 a^2)\). The quadratic chiral extrapolation gives \(Z_v = 0.784(15)\) at \(m_f = 0\), which agrees well with \(Z_A = 0.7777(4)\) \(^{[3]}\). The axial-vector renormalization can be obtained from a completely different calculation of meson two-point correlation functions based on the relation \(\langle A_{\mu}^{\text{conser}}(t)\overline{q}\gamma_5 q(0)\rangle = Z_A \langle A_{\mu}^{\text{local}}(t)\overline{q}\gamma_5 q(0)\rangle\) \(^{[3]}\).

In Fig. 2 we plot the axial charge, \(g_A^{\text{lattice}}\), against the location of current insertions at \(m_f = 0.08\). Filled and open circle symbols correspond to results from the larger volume and the smaller volume respectively. While plateaus are evident for \(15 \leq t \leq 17\) in each case, \(g_A^{\text{lattice}}\) evaluated from the smaller volume is obviously diminished.

Next, we evaluate the continuum value of \(g_A\) from the charge ratios \((g_A/g_v)^{\text{lattice}}\) averaged in the above mentioned time slice range at each \(m_f\). To compare to our previous DWF results, we plot the continuum value of \(g_A\) versus the square of the \(\pi-\rho\) mass ratio in Fig. 3. Our new results using DBW2 gauge action are represented by filled and open circles. The upper points (●) and the lower points (○) are calculated in the larger spatial vol-
Figure 2. The lattice axial charge, $g_{A}^{\text{lattice}}$, at $m_f = 0.08$. The upper filled and the lower open circles represent results obtained on larger and smaller volumes, respectively. A finite volume effect is evident between the observed plateaus.

Figure 3. $g_A$ versus $(m_\pi/m_\rho)^2$. Preliminary results (• and ◦) from DBW2 gauge action ($\beta = 0.87$) and two different physical volumes show the existence of a significant finite volume effect. Our previous results (×) using the Wilson gauge action ($\beta = 6.0$) and a physical volume which is roughly the same as the smaller DBW2 lattice also appears to be affected by finite volume.

In conclusion, using DWF and the DBW2 gauge action, we have studied the effects of finite physical volume on $g_A$ by employing a coarse lattice ($a \approx 0.15$ fm) and two lattice sizes ($V \sim (2.4\text{fm})^3$ and $(1.2\text{fm})^3$). Relevant three-point functions are well behaved (vector, axial and also tensor). We confirmed that $Z_V = Z_A$ is well satisfied even on this coarse lattice. We determined the continuum value of $g_A$ in a fully non-p perturbative way. Although we need more statistics to make a definite conclusion, our preliminary results suggest that the finite volume effect is significant.

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