Constraining the nuclear symmetry energy and properties of the neutron star from GW170817 by Bayesian analysis

Yuxi Li, Houyuan Chen, Dehua Wen, Jing Zhang

1 School of Physics and Optoelectronics, South China University of Technology, Guangzhou 510641, People’s Republic of China
2 School of Physics and Astronomy, Sun Sat-Sen University, Zhuhai 519082, People’s Republic of China

Abstract Based on the distribution of tidal deformabilities and component masses of binary neutron star merger GW170817, the parametric equation of state (EOS) is employed to probe the nuclear symmetry energy and the properties of the neutron star. To obtain a proper distribution of the parameters of the EOS that is consistent with the observation, Bayes’ian analysis is used and the constraints of causality and maximum mass are considered. From this analysis, it is found that the symmetry energy and pressure at twice the saturation density of nuclear matter can be constrained within $E_{\text{sym}}(2\rho_0) = 34.5_{-20.3}^{+20.5}$ MeV and $P(2\rho_0) = 3.81_{-1.32}^{+1.18} \times 10^{34}$ dyn cm$^{-2}$ at 90% credible level, respectively. Moreover, the constraints on the radii and dimensionless tidal deformabilities of canonical neutron stars are also demonstrated through this analysis, and the corresponding constraints are $10.80 \text{ km} \leq R_{1.4} \leq 13.20 \text{ km}$ and $133 \leq \Lambda_{1.4} \leq 686$ at 90% credible level, with the most probable value of $R_{1.4} = 12.60 \text{ km}$ and $\Lambda_{1.4} = 500$, respectively. With respect to the prior, our result (posterior result) prefers a softer EOS, corresponding to a lower expected value of symmetry energy, a smaller stellar radius, and a smaller tidal deformability.

1 Introduction

On August 17, 2017, the Advanced LIGO and Advanced Virgo first observed the merger of two neutron stars GW170817 [1]. Through continual research based on the data of this observation, the understanding of properties of the neutron star (such as the radius, the tidal deformability, etc.) and the state of dense nuclear matter is improved continually [2–7]. Considering the case that the detection of gravitational radiation from the coalescence of a neutron star binary system is occasional [1,8], Bayesian inference becomes a popular method to analyze the observational data. In fact, the Bayesian analysis has frequently been used to investigate the properties and the state of the compact-star matter in recent years [9–22].

Based on Bayesian analysis with equation of states described by chiral effective field theory, Lim et al. constrained the dimensionless tidal deformability of a $1.4 M_\odot$ neutron star in a range of $136 \leq \Lambda_{1.4} \leq 519$ at 95% credible level. Moreover, they found an empirical relation between the tidal deformability of a canonical neutron star and the pressure at twice nuclear saturation density, which provides a useful clue to investigating the state of the dense nuclear matter [11]. By performing a Bayesian analysis with the distance and source location derived by electromagnetic observations of the GW170817 event, De et al. constrained $\tilde{\Lambda}$ ($\tilde{\Lambda}$ is defined by Eq. (3) in Ref. [12]) at 90% credible level as follows: $84 \leq \tilde{\Lambda} \leq 642$ for uniform component mass prior, $94 \leq \tilde{\Lambda} \leq 698$ for the distribution of component mass prior deduced from radio observations of Galactic binary neutron stars and $89 \leq \tilde{\Lambda} \leq 681$ for a component mass prior derived by radio pulsars [12].

As is well known, the density of the neutron star matter covers a large range of magnitudes, from a density far lower than the saturation density at the outer crust to a density close to 10 times the saturation density at the stellar center [23,24]. At present, there is relatively small discrepancy in the EOS at density near or lower than the saturation density [25–27]. But for the matter in the core with supra-saturation density, the EOS is far from certain. In nuclear theory, there are too many EOS predictions based on various nuclear theories by using different interactions, and the predicted EOSs often diverge at the supra-saturation density. In fact, the uncertainty of the symmetry energy at supra-saturation density is the main factor leading to the divergency of the EOS [23]. With the aid of the astronomical observations, people find

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*Corresponding author.*

Dehua Wen (e-mail: wendehua@scut.edu.cn)

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a practical way to narrow the divergency. For example, by using the representative stellar radius data of the canonical neutron star, Xie and Li [28] inferred the high-density nuclear symmetry energy through Bayesian inference by employing an isospin-dependent parametric EOS model for neutron star matter. They obtained constraint on the symmetry energy at twice the saturation density of nuclear matter as $E_{\text{sym}}(2\rho_0) = 39.2^{+12.1}_{-8.2}$ MeV at 68% credible level.

Motivated by the above interesting work, we will investigate the constraints on the nuclear symmetry energy and some of the properties of the canonical neutron star through Bayesian inference based on the distribution of tidal deformabilities and component masses of GW170817 in this work.

The paper is organized as follows. In the next section, the isospin-dependent parametric EOS for dense neutron-rich nucleonic matter and properties of the neutron star are outlined. In Sect. 3, Bayesian inference approach employed in this work is discussed in detail. In Sect. 4, through performing Bayesian analysis by correlating the EOS with the GW170817 data released by LIGO and VIRGO, the posterior distribution of the parameter space of the EOS and the symmetry energy of the superdense nuclear matter are presented. Then, in Sect. 5, we present the constraints on the radii and tidal deformabilities of a canonical neutron star through the corresponding posterior distribution. A brief summary is given at the end.

Unless otherwise stated, we use the gravitational units ($G = c = 1$) in the analytic expressions.

2 Isospin-dependent parametric EOS and neutron star properties

Here we give a brief outline of the isospin-dependent parametric EOS, where the dense nuclear matter is supposed to be composed of neutrons, protons, electrons, and muons at $\beta$-equilibrium and charge neutrality [28,29].

The energy density of dense nuclear matter with isospin asymmetry $\delta = (\rho_n - \rho_p)/\rho$ at density $\rho$ can be expressed as

$$\epsilon(\rho, \delta) = \epsilon(\rho) + E_{\text{sym}}(\rho) \cdot \delta^2 + O(\delta^4), \quad (3)$$

where $E(\rho, \delta) = E_0(\rho) + E_{\text{sym}}(\rho) \cdot \delta^2 + O(\delta^4)$,

$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + K_0 \frac{(\rho - \rho_0)}{3\rho_0}^2 + \frac{J_0}{6} \frac{(\rho - \rho_0)}{3\rho_0}^3 + O(\frac{(\rho - \rho_0)}{3\rho_0}^4), \quad (4)$$

$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + L \frac{\rho - \rho_0}{3\rho_0} + \frac{K_{\text{sym}}}{2} \frac{(\rho - \rho_0)}{3\rho_0}^2 + \frac{J_{\text{sym}}}{6} \frac{(\rho - \rho_0)}{3\rho_0}^3 + O(\frac{(\rho - \rho_0)}{3\rho_0}^4). \quad (5)$$

where $\rho_0$ ($\rho_0 \approx 2.7 \times 10^{17}$ kg/m$^3$) is the nuclear saturation density. According to the research near nuclear saturation density, the most probable values of parameters in Eqs. (4) and (5) are as follows: $K_0 = 240 \pm 20$ MeV, $E_{\text{sym}}(\rho_0) = 31.7 \pm 3.2$ MeV, $L = 58.7 \pm 28.1$ MeV, and $-300 \leq J_0 \leq 400$ MeV, $-400 \leq K_{\text{sym}} \leq 100$ MeV, $-200 \leq J_{\text{sym}} \leq 800$ MeV [31–36]. It is shown that the parameters $K_0$, $E_{\text{sym}}(\rho_0)$, and $L$ have already been constrained in a very narrow range, while $J_0$, $K_{\text{sym}}$, and $J_{\text{sym}}$ have large uncertainties. To simplify the calculation, here we choose the most probable values for $K_0$, $E_{\text{sym}}(\rho_0)$, and $L$ as $K_0 = 240$ MeV, $E_{\text{sym}}(\rho_0) = 31.7$ MeV, and $L = 58.7$ MeV. For more details about this EOS please refer to Ref. [29].

Through varying the parameters $J_0$, $K_{\text{sym}}$, and $J_{\text{sym}}$ within their allowed ranges, we can generate sufficiently large number of EOSs to perform the Bayesian analysis. Compared with the multisegment polytropic EOS, the parametric EOS model leads to a more convenient way to extract the symmetry energy of the asymmetric nuclear matter from the astronomical observations. In this work, the core matter of the neutron star is described by the parametric EOS model, while the inner crust and the outer crust of neutron star are described by the NV EOS model [37] and BPS EOS model [38], respectively. In order to simplify the calculation, we use a fixed crust–core transition density (0.085 fm$^{-3}$) in this work. Our sampling calculation shows that the differences of stellar radii and tidal deformabilities of canonical neutron stars calculated by the fixed crust–core transition density and by the thermodynamic stability condition are no more than 1%. By the way, we choose the resolution for the EOS tables as in Ref. [39].

The structure of the neutron star is governed by the Tolman–Oppenheimer–Volkoff (TOV) equations [40,41],
\[ \frac{dP}{dr} = - \left[ m(r) + 4\pi r^3 P(r) \right] \left[ \epsilon(r) + P(r) \right] \frac{r}{r(2m(r))}, \]  
(6)

\[ \frac{dm}{dr} = 4\pi r^2 \epsilon(r), \]  
(7)

where \( \epsilon(r) \) and \( P(r) \) are the energy density and pressure at radius \( r \), and \( m(r) \) denotes the mass enclosed within radius \( r \). For a given EOS, the TOV equations can be numerically integrated from the origin \( (r = 0) \) to the surface \( (r = R) \), where the pressure vanishes, to obtain the \( M-R \) relation of the neutron star.

The tidal deformability describes how a neutron star deforms under an external gravitational field produced by its companion star. It can be given by \[ \Lambda = \frac{2}{3} k_2 \left( \frac{R}{M} \right)^5, \]  
(8)

where \( k_2 \) denotes the second tidal Love number which has to be solved together with the TOV equations [43].

From terrestrial experiments, there are rudimentary constraints on \( K_{\text{sym}} \), \( J_{\text{sym}} \), and \( J_0 \) [48]. One of our prior assumptions is that the parameters \( K_{\text{sym}} \), \( J_{\text{sym}} \), and \( J_0 \) are independent and uniform in parameter space. In this work, we use Monte Carlo random sampling method to generate two million EOSs. Then we screen the generated EOSs by causality and by supporting the recently observed heaviest stellar mass 2.14 \( M_\odot \) of neutron star J0740+6620 [49]. The remaining EOSs are about 1.6 million.

Calculating the TOV equations by inputting the remaining EOSs, we obtain the prior distribution of the \( M-R \) relation, as shown in Fig. 1. It is worth noting that the prior distribution reflects the general features of the parametric EOS, but not including the impact from GW170817. According to Eq. (5) and Eq. (2), we can obtain the prior probability density function (PDF) of \( E_{\text{sym}}(2\rho_0) \) and \( P(2\rho_0) \), as shown in Figs. 2 and 3, respectively. Within 90% credible level, the prior results of \( E_{\text{sym}}(2\rho_0) \) and \( P(2\rho_0) \) are constrained in a range of \( E_{\text{sym}}(2\rho_0) = 54.5^{+4.0}_{-21.5} \) MeV and \( P(2\rho_0) = 4.45^{+0.97}_{-2.63} \times 10^{34} \) dyn cm\(^{-2} \), respectively.

Normally, we call a neutron star with mass of 1.4 \( M_\odot \) a canonical neutron star as most observed neutron stars have stellar masses near 1.4 \( M_\odot \) [50,51]. There has been massive research on the properties of canonical neutron stars in recent years [52–56], especially after the GW170817 event [2,5,7,11,57–62]. Here we will also focus on the proper-
ties of canonical neutron stars. In Fig. 4, we show the prior distribution of \( R_{1.4} \) (a) and \( \Lambda_{1.4} \) (b), where \( R_{1.4} \) and \( \Lambda_{1.4} \) denote the radius and the dimensionless tidal deformability of a canonical neutron star, respectively. It is shown that in the prior distribution and at 90% credible level, the radius is constrained in a range of 11.30 km \( \leq R_{1.4} \leq 13.30 \) km and the dimensionless tidal deformability is constrained in a range of \( 217 \leq \Lambda_{1.4} \leq 723 \), where the most probable value of \( R_{1.4} \) is 12.9 km and the most probable value of \( \Lambda_{1.4} \) is 620, respectively. The relatively larger values of the expected \( E_{\text{sym}}(2\rho_0) \), radius and the tidal deformability indicate that the prior results, which are consistent with the former studies [55,63–72], prefer a stiffer EOS for a neutron star.

3 Bayesian inference approach

In this work, we use the following Bayesian inference to calculate the posterior probability:

\[
P(\theta \mid D) \approx P(D \mid \theta) P(\theta),
\]

(9)

where \( P(D \mid \theta) \) is the likelihood function of the EOS parameter \( \theta \), linked with the reference event \( D \), and \( P(\theta) \) is the prior probability of \( \theta \).

The inference of GW170817 is given by the probability density \( P(\Lambda) \), where \( \Lambda \) is the combined tidal deformability depending on the component masses \( m_{1,2} \) and the corresponding tidal deformabilities \( \Lambda_{1,2} \) as

\[
\Lambda = \frac{16 (m_1 + 12m_2) m_1^4 \Lambda_1 + (m_2 + 12m_1) m_2^4 \Lambda_2}{(m_1 + m_2)^5}.
\]

(10)

On the observation, the well-constrained parameter in GW170817 is the chirp mass

\[
M_c = \frac{(m_1m_2)^{3/5}}{(m_1 + m_2)^{7/5}}.
\]

(11)

Here we adopt the intermediate value \( M_c = 1.186M_\odot \) of the chirp mass. The mass ratio \( q = m_2/m_1 \) of the two component neutron stars follows a specified distribution density \( f(q) \), which is taken from the slow-spin prior in Ref. [1]. The component masses \( m_1 \) and \( m_2 \) are in a range of 1.36–1.75 \( M_\odot \) and 1.07–1.36 \( M_\odot \), respectively [1], which leads to the ratio \( q \) ranging from 0.7 to 1.0. Fixing the chirp mass, the two component masses can be uniquely expressed by the ratio \( q \) as \( m_1(q) \) and \( m_2(q) \).

For a specific EOS parameter \( \theta \), the tidal deformability is a function of the stellar mass, that is, \( \Lambda(\theta, m) \). Combining the above definitions, we can give the implicit function of the combined tidal deformability as

\[
\tilde{\Lambda} = \Lambda(\theta, q).
\]

(12)

Therefore, the likelihood function for each EOS (specified by \( \theta \)) can be derived by using the inference

\[
P(D \mid \theta) = C_\infty \cdot \int_{0.7}^{1.0} dq \cdot P(\tilde{\Lambda}(\theta, q)) \cdot f(q),
\]

(13)

where \( C_\infty \) is the normalized constant that can be eliminated during the posterior calculation. The posterior probability of each parametric EOS can be calculated by correlating Eq. (13) with Eq. (9).
Fig. 5 The posterior probability distribution in parameter space, where the color from red to blue indicates the probability density from high to low. The white areas are the forbidden parameter space, where the maximum mass $M_{\text{max}} = 2.14 \, M_\odot$ cannot be supported. The black dash lines denote the 90% credible interval. The Bayesian analysis is based on the distribution of stellar masses and tidal deformabilities of the binary neutron star in GW170817.

4 Constraints on the parameter space of EOS and the symmetry energy at twice saturation density through the GW170817

Based on the distribution of stellar masses and tidal deformabilities of the binary neutron star in GW170817 [7], by following the Bayesian method described in Sect. 3 and defining the likelihood function of each set of parameters of EOS, we first calculate the posterior PDFs of the parameter space of parametric EOS model, where the Monte Carlo random sampling method is employed to generate two million EOSs. And then we further calculate the posterior PDFs of symmetry energy, pressure, $M-R$ relation, radius, and tidal deformability. The results are presented in Figs. 5, 6, 7, 8, 9 and 10, respectively.

For convenience in showing the posterior probability of the EOS parameters, we plot the posterior probability of $K_{\text{sym}}$ and $J_{\text{sym}}$ in a two-dimensional diagram by fixing the parameter $J_0$, as shown in Fig. 5, where four values of $J_0$: (a) $-100$ MeV, (b) $100$ MeV, (c) $300$ MeV, and (d) $400$ MeV are adopted. From Fig. 5, we can see that the higher probability density area is located in the left area. Moreover, it is also shown that, for a given $J_0$, lower values of $J_{\text{sym}}$ and $K_{\text{sym}}$ are preferred to support the observed data of GW170817. From the definitions of the parameters $K_{\text{sym}}$, $J_{\text{sym}}$, and $J_0$ in Eqs. (4) and (5) and according to Eq. (3), it is easy to understand that higher values of the set of parameters correspond to a stiffer EOS. Therefore, the posterior probability of the parameters of EOS as shown in Fig. 5 indicates that the observation of GW170817 prefers a relatively softer EOS. In addition, the white areas at the left bottom in Fig. 5 are forbidden areas, where the related EOS cannot support the maximum mass of $M_{\text{max}} = 2.14 \, M_\odot$. The black dash lines denote the boundary of 90% credible interval.
The posterior distribution of $K_{\text{sym}}$, $J_{\text{sym}}$, and $J_0$, where the solid vertical lines represent the 90% credible interval for $K_{\text{sym}}$, $J_{\text{sym}}$, and $J_0$, respectively. The posterior distributions are obtained by considering the GW170817 event.

In addition, according to posterior probability distribution of parameter space, we can get a roughly linear constraint of the parameter space (from the boundary of 90% credible interval). That is, the upper boundary (located in the right area) can be approximately expressed as

$$8.06K_{\text{sym}} + 1.21J_{\text{sym}} + J_0 \leq 676.25 \text{ MeV,} \quad (14)$$

and the lower boundary (located in the left area) can be approximately expressed as

$$1.42K_{\text{sym}} + 1.04J_{\text{sym}} + J_0 \geq -177.00 \text{ MeV,} \quad (15)$$

where $K_{\text{sym}}$, $J_{\text{sym}}$, and $J_0$ are the values of corresponding parameters in units of MeV. The two constraints can reduce the parameter space to about 50% of the original parameter space.

The posterior distributions of $K_{\text{sym}}$, $J_{\text{sym}}$, and $J_0$ are presented in Fig. 6. At 90% credible level, it is shown that the parameters $K_{\text{sym}}$, $J_{\text{sym}}$, and $J_0$ are constrained in $-400.0 \leq K_{\text{sym}} \leq 22.0 \text{ MeV}$, $-30.0 \text{ MeV} \leq J_{\text{sym}} \leq 800.0 \text{ MeV}$, and $-98.0 \text{ MeV} \leq J_0 \leq 400.0 \text{ MeV}$, respectively.

It is well known that the radius of normal neutron star is essentially determined by the pressure around twice the saturation density $(2\rho_0)$ of nuclear matter [53]. Thus the knowledge of the EOS, especially the symmetry energy around twice the saturation density, is very important for understanding the radius of the neutron star. Up to now, a lot of research work has been done to constrain the symmetry energy at twice the saturation density [28, 52, 73–86]. For example, through employing three sets of observational related radii data and three sets of imaginary radii data of canonical neutron star to perform Bayesian analysis, Xie and Li [28] inferred the nuclear symmetry energy $E_{\text{sym}}(\rho)$ by the parametric EOS and constrained the symmetry energy at twice the saturation density as $E_{\text{sym}}(2\rho_0) = 39.2^{+12.1}_{-8.2} \text{ MeV}$ at 68% credible level. Based on the oscillation modes of the canonical neutron stars, Wen et al. [52] predicted that the symmetry energy at twice the saturation density $E_{\text{sym}}(2\rho_0)$ should be in a range of $54.5^{+6.5}_{-6.5} \text{ MeV}$ if the frequency of the $f$-mode takes
Fig. 7 The posterior distribution of $E_{\text{sym}}(2\rho_0)$, where the solid vertical lines represent the 90% credible interval for $E_{\text{sym}}(2\rho_0)$. The posterior distribution is obtained by considering the GW170817 event.

Fig. 8 The posterior distribution of $P(2\rho_0)$, where the solid vertical lines represent the 90% credible interval for $P(2\rho_0)$. The posterior distribution is obtained by considering the GW170817 event.

By employing the posterior distribution of the parameter space and Eq. (5), we obtain the posterior probability density of $E_{\text{sym}}(2\rho_0)$, as shown in Fig. 7. It is shown that the symmetry energy at twice the saturation density of nuclear matter is constrained in a range of $E_{\text{sym}}(2\rho_0) = 34.5^{+20.3}_{-2.3}$ MeV at 90% credible level. Obviously, there is a big difference of the maximum probability point between the prior and posterior distribution. The latter one prefers a relatively lower symmetry energy. Moreover, both of the prior and posterior distribution of $E_{\text{sym}}(2\rho_0)$ are consistent with the conclusions of the above literature. The posterior distribution of $P(2\rho_0)$ is shown in Fig. 8. At 90% credible level, the pressure at twice the saturation density of nuclear matter is constrained in a range of $P(2\rho_0) = 3.81^{+1.18}_{-2.32} \times 10^{34}$ dyn cm$^{-2}$, which corresponds to a softer EOS.

It is worth noting that recently Güven et al. have performed a similar Bayesian inference to investigate the constraint on the parametric EOS [20]. They use two sets of empirical parameters in the parametric EOS model as prior to estimate the posterior, and yet involving three different PDFs of tidal deformability as the inference of GW170817. Both prior sets in their work finally lead to a similar result: that the slope parameter $L$ was extremely overestimated in the previous understanding. According to their investigation, a smaller value of $L$ is needed to be compatible with GW170817.

5 The constraints on the radii and tidal deformabilities through the GW170817

For comparison with the prior distribution of the $M–R$ relation, here we present the corresponding posterior distribution in Fig. 9. We use the Monte Carlo random sampling method...
the radius, it is easy to understand that the two peaks in Fig. 10 are consistent with the two higher probability density areas in Fig. 5.

According to our calculations, at 90% credible level in the posterior distribution, the radius of a canonical neutron star is constrained in a range of $R_{1.4} = 12.6_{-1.8}^{+0.6}$ km and its dimensionless tidal deformability is constrained in a range of $\Lambda_{1.4} = 500_{-367}^{+186}$. Comparing with the prior distribution of radius ($R_{1.4} = 12.9_{-0.4}^{+0.4}$ km) and the tidal deformability ($\Lambda_{1.4} = 620_{-403}^{+103}$), we can find that the most probable values of the radius and the tidal deformability of the posterior distribution are smaller than that of the prior distribution, which means that the posterior distribution prefers a relatively softer EOS.

6 Summary

The detection of the gravitational wave of the binary neutron star merger event GW170817 provides us important information, such as the distribution of the tidal deformabilities and the stellar masses of the binary neutron star, to further investigate the properties and the state of matter of neutron stars. In this work, we investigate the radius and tidal deformability of the canonical neutron star and the symmetry energy of the superdense matter through the Bayesian analysis based on the distribution of component masses and tidal deformabilities of binary neutron star merger GW170817 released by LIGO and VIRGO. To perform the Bayesian analysis, one need to generate a large number of EOSs. Normally, the polytropic EOS model is adopted to generate the EOSs. Here we adopt the isospin-dependent parametric EOS model as this kind of model can provide a more convenient way to extract the symmetry energy of the asymmetric nuclear matter from the astronomical observations. In this work, two million isospin-dependent parametric EOSs are generated by the Monte Carlo random sampling method, and the generated EOSs are further screened by the recently observed heaviest stellar mass $2.14\, M_{\odot}$ of J0740+6620 and the causality to perform the Bayesian analysis. From this analysis, we find that the parameter space of EOS can be reduced to about 50% of the original parameter space at 90% credible level. In the posterior distribution, the symmetry energy and the pressure at twice the saturation density of nuclear matter can be constrained within $E_{\text{sym}}(2\rho_0) = 34.5_{-2.3}^{+20.5}$ MeV, and $P(2\rho_0) = 3.81_{-1.18}^{+1.18} \times 10^{34}$ dyn cm$^{-2}$ at 90% credible level. The radius is constrained in a range of $R_{1.4} = 12.6_{-1.8}^{+0.6}$ km. The dimensionless tidal deformability is constrained in a range of $\Lambda_{1.4} = 500_{-367}^{+186}$ at 90% credible level. Comparing with the prior distribution of $E_{\text{sym}}(2\rho_0)$ ($E_{\text{sym}}(2\rho_0) = 54.5_{-21.5}^{+4.0}$ MeV), $P(2\rho_0)$ ($P(2\rho_0) = 4.45_{-2.63}^{+2.63} \times 10^{34}$ dyn cm$^{-2}$), radii ($R_{1.4} = 12.9_{-0.4}^{+0.4}$ km), and the tidal deformability (by the probability density of posterior parameter space) to generate two million EOSs, and then $M-R$ relation, $R_{1.4}$, and $\Lambda_{1.4}$ are calculated statistically. The distribution of the posterior $M-R$ relation is shown in Fig. 9. Comparing with the prior $M-R$ relation distribution, the posterior $M-R$ relation distribution moves slightly to the left in the $M-R$ graph.

The posterior distributions of the radii $R_{1.4}$ and the dimensionless tidal deformabilities $\Lambda_{1.4}$ of canonical neutron stars are presented in Fig. 10a and b, respectively. Unlike the prior distribution (see Fig. 4), there are two peaks in the posterior distribution of $R_{1.4}$ and $\Lambda_{1.4}$ in Fig. 10. We notice that in Fig. 5, each subgraph has two higher probability density areas of the posterior probability in the EOS parameter space and one of them has a relatively lower probability density. Considering a certain relevance between the tidal deformability and the radius, it is easy to understand that the two peaks in Fig. 10 are consistent with the two higher probability density areas in Fig. 5.
bilities ($\Lambda_{1,4} = 620^{+103}_{-403}$), one can see that the posterior distribution prefers a softer EOS.

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