The Kinematic Study of a Window Cleaning Robot in Residence

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ABSTRACT

The paper introduced a window cleaning robot designed for utilizing in residential environment. Bifurcation theory was used to study the constraint conditions of the length of rods. The kinematic analysis and research were carried out. Redundant DOF manipulators’ motion performance was optimized and inverse kinematics was solved with extended Jacobi method.

KEYWORD

plane three rods; extended Jacobi method; redundant DOF robot

INTRODUCTION

With the development of urbanization and the improvement of resident’s living standard, a large amount of glass was applied to resident's house. When it comes to window cleaning in high-rise residential buildings, the glass inside of the window is cleaned easily but the cleaning of glass outside of window always make residents feel troubled. The most common situation is that residents clean the glass outside of the window with a wooden stick with a cleaning cloth. However, this approach is ineffective and dangerous. In addition, there is a kind of glass cleaning device consisting of two magnets but the situation always happens that magnetic cleaning wood plate falls down due to lack of attraction or too much resistance. Therefore, the development of a kind of automated cleaning robot is particularly important.

THE KINEMATICS AND JACOBIAN MATRIX ANALYSIS FOR THE WINDOW CLEANING ROBOT

There is the picture of window needed to clean as Figure1.
Coordinate system of the robot was established using the D-H method (Tan & Xu (2007)) in Figure 2.

Table 1. D-H parameters of the window cleaning robot.

| Rod i | Joint angle θi | Twist angle αi | Rod length ai | Rod offset di |
|-------|----------------|----------------|---------------|---------------|
| 1     | θ1            | 0              | L1            | 0             |
| 2     | θ2            | 0              | L2            | 0             |
| 3     | θ3            | 0              | L3            | 0             |

 Coordinates transformation as follows:

\[
A_i = \begin{bmatrix} C_i & -\sin \theta_i & 0 & L_i \cos \theta_i \\ \sin \theta_i & C_i & 0 & L_i \sin \theta_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} C_2 & -\sin \theta_2 & 0 & L_2 \cos \theta_2 \\ \sin \theta_2 & C_2 & 0 & L_2 \sin \theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} C_3 & -\sin \theta_3 & 0 & L_3 \cos \theta_3 \\ \sin \theta_3 & C_3 & 0 & L_3 \sin \theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

Where \( C_1 \) represent \( \cos \theta_1 \); \( S_1 \) represent \( \sin \theta_1 \) and so on.

So the terminal manipulator position and orientation matrix was obtained as Equation (1).

\[
T = A_1 A_2 A_3
\]

Where \( C_{12} \) represent \( \cos(\theta_1+\theta_2) \); \( C_{123} \) represent \( \cos(\theta_1+\theta_2+\theta_3) \); \( S_{12} \) represent \( \sin(\theta_1+\theta_2) \); \( S_{123} \) represent \( \sin(\theta_1+\theta_2+\theta_3) \) and so on.

Jacobian matrix was obtained from the approach of the product of vectors (Cai 2000).

\[
J = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ 0 & 0 & 0 \end{bmatrix}
\]

Where

\[
f_{11} = -L_1 \sin \theta_1 - L_2 \sin(\theta_1 + \theta_2) - L_3 \sin(\theta_1 + \theta_2 + \theta_3),
\]

\[
f_{12} = -L_2 \sin(\theta_1 + \theta_2) - L_3 \sin(\theta_1 + \theta_2 + \theta_3),
\]

\[
f_{21} = L_1 \cos \theta_1 + L_2 \cos(\theta_1 + \theta_2) + L_3 \cos(\theta_1 + \theta_2 + \theta_3),
\]

\[
f_{22} = L_2 \cos(\theta_1 + \theta_2) + L_3 \cos(\theta_1 + \theta_2 + \theta_3),
\]

\[
f_{23} = L_3 \cos(\theta_1 + \theta_2 + \theta_3).
\]

### IDENTIFYING THE SIZE OF THREE RODS

For redundant DOF robot, its flexible property is affected by the size of rods and then the motion characteristics of the robot are affected. Bifurcation theory was used to study the nonlinear system characteristics that changing process of solutions’ structure and stability caused by the change of parameters (Lu 1989). In order to identify the length and to avoid the bizarre situation for the robot, this paper analyzed the characteristics of 3 DOF robot parameters with the bifurcation theory.

Jacobian matrix’s null space vector as Equation (3).

\[
V = \begin{bmatrix} L_1 L_2 \sin \theta_1 \\ -L_2 L_3 \sin \theta_1 - L_1 L_3 \sin(\theta_1 + \theta_2) \\ L_1 L_2 \sin \theta_1 + L_2 L_3 \sin(\theta_1 + \theta_2) \end{bmatrix}
\]

If \( v = 0 \), then \( \sin \theta_2 = \sin \theta_3 \). So the following singular points were obtained.

\[
\theta_s = (\theta_1, 0/\pm \pi, 0/\pm \pi)
\]
The first derivative matrix of $v$ follows:

$$H = \begin{pmatrix}
0 & 0 & L_l \cos \theta_l \\
0 & -L_l \cos(\theta_l + \theta) & -L_l \cos(\theta_l - \theta) \\
0 & L_l \sin \theta_l + L_l \cos(\theta_l + \theta) & -L_l \cos(\theta_l + \theta)
\end{pmatrix}$$

The eigenvalues of $H$ as follows: $\lambda_1 = 0$;

$$\lambda_2^2 = \lambda_3^3 = -L_l L_2 L_3 (L_2 \cos \theta_2 \cos \theta_3 + L_3 \cos \theta_2 + L_3 \cos \theta_3)$$

There are many cases of the eigenvalues needed to be discussed as follows:

(1) When $\theta_s = (\theta_1 0 0)$, then $\lambda_{2,3}^2 = -L_1 L_2 L_3 (L_1 + L_2 + L_3)$.

(2) When $\theta_s = (\theta_1 \pm \pi 0)$, then $\lambda_{2,3}^2 = -L_1 L_2 L_3 (L_1 - L_2 - L_3)$.

(3) When $\theta_s = (\theta_1 0 \pm \pi)$, then $\lambda_{2,3}^2 = L_1 L_2 L_3 (L_1 + L_2 - L_3)$.

(4) When $\theta_s = (\theta_1 \pm \pi \pm \pi)$, then $\lambda_{2,3}^2 = L_1 L_2 L_3 (L_1 - L_2 + L_3)$.

For case (1), $\lambda_{2,3}^2 < 0$. It is the topology of robot mutated in the boundary and there are inevitable bizarre points regardless of the length of rods.

For case (2), there are three situations: $\lambda > 0, \lambda < 0$ and $\lambda = 0$. We can make the bifurcation parameter

$$u = 1 - \frac{L_1}{L_1 + L_3}$$

When $\lambda_{2,3}^2 > 0$, then $L_1 < L_2 + L_3$. It’s the equilibrium point of bifurcation of codimension one.

So the singular point is avoidable.

When $\lambda_{2,3}^2 < 0$, then $L_1 > L_2 + L_3$. This is the Hopf bifurcation of codimension one and the singular point is inevitable.

When $\lambda_{2,3}^2 = 0$, it’s higher-order singularity and the singularity point is inevitable.

The other cases can be analyzed in accordance with the case (1).

So, constraint conditions of the length of link as follows:

$$\begin{cases}
L_1 + L_2 > L_3 \\
L_1 + L_3 > L_2 \\
L_2 + L_3 > L_1 \\
L_1 + L_2 + L_3 \geq 1560 \\
L_1, L_2, L_3 > 0
\end{cases}$$

The length of three rods can be obtained as follows:

$$L_1 = 520 \text{mm}, L_2 = 520 \text{mm}, L_3 = 520 \text{mm}$$

In addition, in order to ensure each jointpoint moves within the window frame. The Figure 3. was obtained through simulation of ADAMS. The semicircular curve is the trajectory of the jointpoint 2. The irregular closed curve is the trajectory of the jointpoint 3. The rectangular frame is the trajectory of the robot.

![Figure 3. Simulation trajectory of boundary condition.](image-url)
INVERSE KINEMATICS ANALYSIS

The cleaning robot with redundant DOF has infinite number of inverse kinematics solutions. Working in the plane, the robot’s posture and the position are considered only. Then

\[
\begin{bmatrix}
C_{10} & -S_{10} & 0 & X \\
S_{10} & C_{10} & 0 & Y \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

The Equation (7) was obtained as follows:

\[
\begin{cases}
X = L_1C_1 + L_2C_{12} + L_3C_{123} \\
Y = L_1S_1 + L_2S_{12} + L_3S_{123}
\end{cases}
\]

The flexibility of the cleaning robot proposed by Yoshikawa was regarded as optimization target with extended the Jacobi method (Khatib 1986). The optimization target was defined as follows: 

\[ W = \sqrt{\det(JJ^T)} \]

The value of W is bigger, the flexibility of the robot is better.

Theorem (Baillieu1985): x is the terminal position and orientation of robot and when \( \theta = \theta_0 \), H(\( \theta \)) is the extreme value and x would make f(\( \theta \)) = x. Then

\[ \nabla H^T(\theta_0)N(J(\theta)) = 0. \]

The objective function as follows:

\[ W = \sqrt{\det(JJ^T)} = \sqrt{g_1^2 + g_2^2 + g_3^2} \]

Where

\[ g_1 = L_2L_3 \sin \theta_3 ; \]
\[ g_2 = L_1L_3 \sin(\theta_2 + \theta_3) + L_2L_1 \sin \theta_3 ; \]
\[ g_3 = L_1L_2 \sin \theta_2 + L_2L_3 \sin(\theta_2 + \theta_3) . \]

It is too difficult to process, so we regard H = sin2 \( \theta \) + sin23 as the objective function not \[ W = \sqrt{\det(JJ^T)} \]. The new function defined as

\[ G(\theta) = \nabla H^T N(J) . \]

If x given above is the terminal position and orientation of the robot and H(\( \theta \)) is the extreme value, Then G(\( \theta \)) = 0.

\[ \begin{bmatrix}
f(\theta) \\
G(\theta)
\end{bmatrix} = \begin{bmatrix}
x \\
0
\end{bmatrix} \]

(8)

\[ \frac{\partial}{\partial \theta} f(\theta) \bigg|_{\theta = x} = \begin{bmatrix}
x \\
0
\end{bmatrix} \]

(9)

\[ J = \left[ \begin{array}{c}
\frac{\partial}{\partial \theta} f(\theta) \\
\frac{\partial}{\partial \theta} G(\theta)
\end{array} \right] = \begin{bmatrix}
f_{11} & f_{12} & f_{13} \\
f_{21} & f_{22} & f_{23} \\
0 & G_1 & G_2
\end{bmatrix} \]

(10)

Where \( f_{11}, f_{12}, f_{13}, f_{21}, f_{22}, f_{23} \) was given above;

\[ G_i = L_1L_i(\sin^2 \theta_2 - \cos^2 \theta_2) \sin(\theta_1 + \theta_3) - L_4 \sin \theta_4 \cos \theta_4 \cos(\theta_2 + \theta_3) \]
\[ + L_2L_3 (\sin^2 \theta_2 - \cos^2 \theta_2) \sin \theta_4 + L_1L_2 \cos \theta_2 \sin \theta_4 \cos \theta_4 + L_4L_2 \sin \theta_4 \cos \theta_4 \cos(\theta_2 + \theta_3) ; \]
\[ G_2 = -L_4 L_2 \sin \theta_2 \cos \theta_2 \cos(\theta_2 + \theta_3) - L_5 \sin \theta_2 \cos \theta_3 \]
\[ -L_4 L_2 \sin \theta_2 (\sin^2 \theta_2 - \cos^2 \theta_3) + L_5 \sin \theta_3 \cos(\theta_2 + \theta_3) \]
\[ -L_4 L_2 (\sin^2 \theta_2 - \cos^2 \theta_3) \sin(\theta_2 + \theta_3). \]

Null space vector as follows:
\[
N(J) = \begin{bmatrix}
L_4 L_2 \sin \theta_3 \\
- L_5 \sin \theta_3 - L_5 \sin(\theta_2 + \theta_3) \\
L_5 \sin \theta_3 + L_5 \sin(\theta_2 + \theta_3)
\end{bmatrix}
\]
\[
\theta_2 = \theta_3
\]

Equation (11) can be obtained from \( G(\theta) = \nabla H^T N(J) = 0 \).

The new equations can be obtained as follows:
\[
\begin{bmatrix}
X = L_4 C_1 + L_4 C_{12} + L_5 C_{123} \\
Y = L_5 S_1 + L_5 S_{12} + L_5 S_{123}
\end{bmatrix}
\]
\[
\theta_2 = \theta_3
\]

Through a series of transformations and calculations, the inverse solution of the robot as follows:
\[
\theta_1 = \arctan 2(\pm \sqrt{1 - (\frac{x}{\rho})^2}, \frac{x}{\rho}) - \arctan 2(k_x, k_y)
\]
\[
\theta_2 = \theta_3 = \arctan 2(\pm \sqrt{1 - k_z^2}, k_2)
\]

where
\[
k_1 = L_4 + L_5 \cos \theta + L_5 \cos(2\theta) \]
\[
k_2 = L_5 \sin \theta + L_5 \sin(2\theta)
\]
\[
\rho = \sqrt{k_1^2 + k_2^2}
\]

In order to ensure the correctness of solving, the value of x and y was given as follows: \( x=500, y=600 \).

Then \( \theta_1 \approx -25.26^\circ \) or \( \theta_1 \approx -125.73^\circ \); \( \theta_2 = \theta_3 \approx 75.47^\circ \).

The solutions as above can be obtained from Equation (12) and Equation (13). Through calculation, \( \theta_1 \approx -25.26^\circ \) and \( \theta_2 = \theta_3 \approx 75.47^\circ \) were suitable. Then \( x=500, y=600 \) can be obtained from Equation (12). Ignoring rounding errors, the robot’s inverse solution was correct.

CONCLUSIONS

Some researches were done for a window cleaning robot as follows:

(1) The coordinate system was built using the D-H method for glass cleaning robot to analyze kinematics of the robot.

(2) The constraint conditions of the length of rod were built using the bifurcation theory to identify the length of rods and to avoid the bizarre situations.

(3) Flexibility of robot was optimized using extended Jacobi method and the problem of redundant robot’s inverse solution was solved.

REFERENCES

1. Baillieul, J. 1985. Kinematic Programming Alternatives for Redundant Manipulators. \textit{Proc. of IEEE Int. Conf. on Robotics and Automation}: 722-728.
2. Cai Z.X. 2000. *Robotics*, Beijing: Tinghua University press.
3. Khatib O. 1986. Real-Time Obstacle Avoidance for Manipulators and Mobile Robots, *Int. J. of Robotics Research* 5(1)90-98.
4. Lu Q.S. 1989. *Qualitative Methods of Ordinary Differential Equations and Bifurcation*, Beijing: Bei Hang University press.
5. Tan M. and Xu D. 2007. *Advanced Robot Control*, Beijing: Higher education press.
6. Zhou D.H. 1994. *Redundant DOF Robot Mechanism Study*, Beijing: Bei Hang University press.