A Novel Hybrid Model for Predicting Traffic Flow via Improved Ensemble Learning Combined with Deep Belief Networks

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The intelligent transportation system (ITS) plays an irreplaceable role in alleviating urban traffic congestion and realizing sustainable urban development. Accurate and efficient short-term traffic state forecasting is a significant issue in ITS. This study proposes a novel hybrid model (ELM-IBF) to predict the traffic state on urban expressways by taking advantage of both deep learning models and ensemble learning framework. First, a developed bagging framework is introduced to combine several deep belief networks (DBNs) that are utilized to capture the complicated temporal characteristic of traffic flow. Then, a novel combination method named improved Bayesian fusion (IBF) is proposed to replace the averaging method in the bagging framework since it can better fuse the prediction results of the component DBNs by assigning the reasonable weights to DBNs at each prediction time interval. Finally, the proposed hybrid model is validated with ground-truth traffic flow data captured by the remote traffic microwave sensors installed on the multiple road sections of 2nd Ring Road in Beijing. The experimental results illustrate that the ELM-IBF method can effectively capture sharp fluctuations in the traffic flow. Compared with several benchmark models (e.g., artificial neural network, long short-term memory neural network, and DBN), the ELM-IBF model reveals better performance in forecasting single-step-ahead traffic volume and speed. Additionally, it is proved that the ELM-IBF model is capable of providing stable and high-quality results in multistep-ahead traffic flow prediction.

1. Introduction

With the rapid growth of vehicle ownership, the conflict between traffic demand and traffic supply becomes increasingly acute, and it causes more frequent traffic congestion. Although the construction and improvement of the expressway network alleviate the rapidly increasing traffic demand to some extent, the urban traffic problems cannot be solved only by building or expanding the expressway network because of the restrictions on urban land use and environmental factors [1]. Therefore, to alleviate traffic congestion on the urban expressways with limited space, ITS has gradually become an effective way to manage the traffic flow on expressways and to improve the mobility and safety of expressways.

Short-term traffic flow prediction technology is an essential issue in ITS. It describes the process of estimating the anticipated traffic conditions in the short-term given historical and current traffic information captured by several traffic detectors [2–4]. With the help of traffic information released by the advanced traffic information service system, travelers can understand road conditions and choose time-saving travel paths [5]. At present, the common detectors employed in the ITS include loop detectors, global positioning systems, and remote traffic microwave sensors (RTMS). As one of the most popular nonintrusive traffic detectors, RTMSs transmit microwave beams to both moving and stationary objects and receive the reflected signals as the background signals [6]. When a car enters the detection zone, the reflected signal will be strengthened to
exceed the background signal threshold, and the car will be detected. Meanwhile, RTMSs are installed on the side of the road without causing temporary lane closure or traffic flow interruption during the process of installation [7]. Hence, more traffic management departments regard this kind of non-invasive sensor as an automatic transportation data acquisition method. Due to the high measurement accuracy of RTMSs, which can be up to 95% [8], the continuous travel speed and volume captured by RTMS are used as the dataset in this study.

Because of the complexity and dynamic state of traffic flow, massive approaches including deep learning technologies [9–12] have been proposed to address traffic state estimation issues. However, high-efficiency and robust prediction of traffic conditions are difficult because of the following challenges and problems: (1) though traffic conditions on urban expressways exhibit recurrent patterns over time, recurrent traffic conditions are affected by planned incidents such as road construction and sports events, and unplanned incidents and accidents, resulting in a deviation from the recurrent patterns; (2) deep learning-based models such as convolutional neural network and long short-term memory (LSTM) neural network need big datasets to train thousands of parameters and learn the spatiotemporal characteristics of the traffic flow. It may cause huge time consumption and fail to meet the real-time requirements of forecasting traffic conditions; (3) most of the existing methods focus on predicting the traffic flow in the next time interval. In fact, accurate and stable methods are needed to deal with the task of multistep-ahead traffic flow forecasting [13].

By taking advantage of both deep learning models and ensemble learning framework, this paper builds a novel hybrid model (ELM-IBF) to forecast traffic state. It has been proven that deep belief networks (DBN) are valid in the prediction task [14]. Hence, we utilized DBNs as the subpredictors in this study. Then, a new bagging framework is introduced to decrease the size of the training data and reduce the training time of the DBNs. Finally, we developed the bagging framework by replacing the average combination methods with an improved Bayesian fusion method. Based on the aforementioned discussion, the improvements and contributions of this paper are summarized in the following four aspects:

1. A novel hybrid model (ELM-IBF), which combines the ensemble learning theory and deep belief network, is proposed to make short-term traffic state prediction on urban expressways.
2. Deep belief networks are utilized as the subpredictors in the Bagging algorithm to learn the complicated temporal characteristic of traffic flow.
3. A combination method, named improved Bayesian fusion, is proposed to replace the averaging method in the Bagging framework. It can better fuse the prediction results of the component DBNs by assigning the reasonable weights to component DBNs at each prediction time intervals.

The remainder of this paper is organized as follows: a general overview of existing literature on traffic forecasting is provided in Section 2. The theory of DBN, the improved Bayesian fusion (IBF) methods, and the proposed ELM-IBF model are introduced in Section 3. The experimental dataset, the evaluation index, and the experimental environment are given in Section 4. The discussions of the prediction performance of different methods and relevant analyses are carried out in Section 5. Finally, we present the conclusions and future research efforts in Section 6.

2. Literature Review

Due to the significance and prospective applications of traffic flow prediction, considerable research efforts have been made to enrich traffic prediction approaches. In general, the methodology falls into three major categories: statistical methods, nonparametric methods, and hybrid methods.

2.1. Statistical Methods. The statistical methods mainly include time-series models [15], Kalman filtering methods [16], and Auto-Regressive Integrated Moving Average (ARIMA) methods [17]. Among the various statistical methods, ARIMA [17] and its variants [18, 19] have been proved to be capable of getting promising forecasting results. Many researchers applied the data analysis techniques developed by Box and Jenkins [20] to predict freeway short-term traffic flow, and it is found that the ARIMA model can represent freeway time-series data in a highly accurate manner. It is also shown that the ARIMA model was adequate in reproducing time series of urban arterials [18]. Compared with typical ARIMA forecasting methods, Seasonal ARIMA (SARIMA) enables the extraction of the seasonal variations and reveals implicit periodical characteristics in the time-series data. Hence, SARIMA-based methods [21] have obtained better predictive performances than typical ARIMA methods.

2.2. Nonparametric Methods. Though statistical methods offer explicit formulas to give valuable interpretations of traffic characteristics, they are inferior at predicting the traffic flow with irregular fluctuations considering their definite model structure [22]. During recent years, massive nonparametric methods for traffic flow prediction have emerged such as support vector regression [23, 24], k-nearest neighbor [25], random forest [26], and XGBoost [27]. Different from statistical methods, nonparametric methods have relaxed assumptions for inputs, and they are more capable of processing outliers and noisy data [28]. Artificial neural networks (ANN), which can capture the nonlinear correlations and mine the complex patterns of the measured historical data, are the most popular nonparametric methods. Since Hua and Faghri introduced ANN models to vehicle travel time estimation [29], many other advanced neural network models were shifted to the traffic forecast domain successively, such as feed-forward neural network [30], fuzzy neural network [31], echo state neural network [32], radial basis function neural network (RBFNN) [33], and recurrent neural network (RNN) [34].
component models can be trained or calibrated parallelly, which saves time and is more efficient than some complicated single models.

Therefore, inspired by the powerful data mining capability of deep learning methods and the high robustness of ensemble learning technology, we propose a novel hybrid method (ELM-IBF) for traffic flow prediction through integrating the deep belief networks and an improved ensemble learning framework. Experimental results confirm that the ELM-IBF model enables us to make accurate and stable multistep-ahead traffic flow forecasting.

3. Methodology

In this section, we first introduce the DBNs, which are component predictors of the ELM-IBF model. Then, a description of an IBF method is given. Finally, we introduce the ELM-IBF model based on the bagging framework developed by DBNs and IBF.

3.1. Deep Belief Network (DBN). DBN is a deep artificial neural network with many hidden layers as shown in Figure 1(a). Each hidden layer has a large number of hidden units. The classical DBN is equivalent to the superposition of several Restricted Boltzmann Machines (RBMs) and an output layer. DBN usually utilizes a fast and greedy unsupervised learning algorithm to train RBMs. After the RBM layers training is completed, a supervised fine-tuning method is adopted to adjust the network by the training data [54]. Figure 1(b) illustrates the structure of an RBM, where \( v \) represents a visible layer, and \( h \) represents a hidden layer. In the stacked structure of the DBN shown in Figure 1(c), the hidden layer of one RBM is regarded as the visible layer of the next RBM. An energy function is defined by an RBM according to its parameter set \( \theta = (w, b, a) \) as follows:

\[
E(v, h|\theta) = - \sum_i b_i v_i - \sum_j a_j h_j - \sum_{ij} w_{ij} v_i h_j,
\]

where \( v_i \) and \( h_j \) are the \( i \)th visible layer unit and \( j \)th hidden layer unit, respectively; \( w_{ij} \) is the weight between \( v_i \) and \( h_j \); \( b_i \) and \( a_j \) are the biases of the layers.

The joint probability distribution of visible layer \( v \) and hidden layer \( h \) can be calculated as follows:

\[
p(v, h|\theta) = \frac{\exp(-E(v, h|\theta))}{\sum_{v,h} \exp(-E(v, h|\theta))}.
\]

As shown in Figure 1(a), there are no interconnections between the neurons in the visible layer and the hidden layer. As the binary units are used, \( v_i \) and \( h_j \) belong to \((0, 1)\). The activation probabilities of the neurons in the hidden layer and the neurons in the visible layer are given as follows:

\[
p(h_j = 1|v, \theta) = \sigma(b_j + \sum_i w_{ij}v_i),
\]

\[
p(v_i = 1|h, \theta) = \sigma(a_i + \sum_j w_{ij}h_j),
\]
where $\sigma$ is a sigmoid activation function.

For the whole hidden layer and visible layer, they can be
given by

$$p(h|v) = \prod_j (h_j|v),$$

$$p(v|h) = \prod_i (v_i|h).$$

The marginal probability distribution of input vector $v$
over the hidden units is obtained as

$$p(v|\theta) = \frac{\sum_h \exp(-E(v,h|\theta))}{\sum_v \exp(-E(v,h|\theta))}.$$

Therefore, the objective function can be given as

$$L(\theta, v) = \sum_{v \in D} \log p(v, \theta),$$

where $D$ represents the training dataset.

To obtain the optimal $\theta$ for a single data vector $v$, the
gradient of log-likelihood estimation can be calculated based
on the following equations:

$$\frac{\partial \log p(v|\theta)}{\partial w_{ij}} = E_p(\langle h_j | v_i \rangle) - E_p(\langle h_j | v \rangle),$$

$$\frac{\partial \log p(v|\theta)}{\partial a_j} = E_p(\langle h_j | v \rangle) - E_p(\langle h_j \rangle),$$

$$\frac{\partial \log p(v|\theta)}{\partial b_i} = E_p(\langle v_i | \rangle) - E_p(\langle v_i \rangle),$$

where $E_p(\langle . \rangle)$ and $E_\theta(\langle . \rangle)$ are the expectations of the probability over $v$ under the empirical distribution $P$ and model’s
distribution $\hat{P}$.

The contrastive divergence (CD) learning method [54–56] can be utilized by reconstruction to minimize the
difference of two Kullback-Leibler divergences (KL) to
gain $E_\theta(\langle . \rangle)$. CD learning is proved to be efficiently practical and also can reduce computational cost compared with the
typical Gibbs sampling method. At the beginning of the CD
algorithm, the visible layer is initialized with training data; then, the hidden layer is calculated with the conditional
distribution. Afterward, the visible layer is calculated with conditional distribution according to the hidden layer. The result is a reconstruction of the input. This algorithm only
needs to iterate for $k$ times to obtain the estimation of the
model, and $k$ usually takes the value of 1. The pseudocode of
training an RBM is presented in Algorithm 1.

After obtaining a trained RBM, a regression layer is
introduced to form the DBN model, which enables it to
predict traffic flow data. The whole structure of the DBN
model is shown in Figure 1(c). Afterward, the complete
training process of DBN can be divided into two phases
including a pretraining phase and a fine-tuning phase.

During the pretraining phase, a greedy layer-by-layer is
used to gain the weights. First, training data is employed to
fully train the first RBM as the code in Algorithm 1. Second,
the output layer of the first RBM is taken as the input of the
second RBM. Then, the above steps can be repeated several
times.

After the pretraining phase, the backpropagation (BP)
algorithm, a supervised learning method, is adopted to
adjust the parameters to finish the fine-tuning phase.
Bayesian fusion is a local extreme point because the unlabeled data has provided space. Empirically speaking, starting from these positions, of each layer will be in a better position in the parameter framework of ensemble learning. The linear fusion method with dynamic weights based on their historical one. (U_he principle of Bayesian fusion is to assign dynamic prediction results of several subpredictors to form a better combination of the output of all the subpredictors, which is formulated as

\[
\tilde{y}_{t+1} = \sum_{m=1}^{N} P_t^m y_{t+1}^m,
\]

where \( \tilde{y}_{t+1} \) is the predicted value of the BF at time interval \( t + 1; P_t^m \) is the weight of the subpredictor \( m \) at the time interval \( t + 1; y_{t+1}^m \) is the predicted value of the subpredictor \( m \) at the time interval \( t + 1 \).

However, the BF still has a drawback in the linear combination equation, because the sum of the weights of all subpredictors is equal to one. Hence, if the prediction results of the subpredictors are all larger or less than the actual value, the fusion result will still be larger or less than the actual value, and the prediction error of the BF could be even larger than those of some of the subpredictors. To deal with the problem above, this study replaces equation (13) with a nonlinear equation illustrated as follows:

\[
\tilde{y}_{t+1} = \tau_t \sum_{m=1}^{N} P_t^m y_{t+1}^m,
\]

where \( \tau_t \) is an error compensation factor, which can be calculated in the following equation:

\[
\tau_t = \begin{cases} 
1 + \frac{\sum_{m=1}^{N} \sum_{i=t-1}^{t} e_i^m}{\sum_{m=1}^{N} \sum_{i=t-1}^{t} y_i^m}, & \text{if } \sum_{i=t-1}^{t} e_i^m \geq 0, \forall m \in [1, N], \\
1, & \text{otherwise,}
\end{cases}
\]

\[
1 + \frac{\sum_{m=1}^{N} \sum_{i=t-1}^{t} e_i^m}{\sum_{m=1}^{N} \sum_{i=t-1}^{t} y_i^m}, \sum_{i=t-1}^{t} e_i^m \leq 0, \forall m \in [1, N],
\]

(15)
where $e_m^i$ is the prediction error of the subpredictor $m$ at the time interval $i$ and $e_m^m = y_i - y_m^m$.

From equation (15), it is illustrated that $\tau$, related to the length of the fusion step $\lambda$, the prediction error of the subpredictors $e_m^i$, and the prediction value of the subpredictors $y_m^m$. Based on Wang's study [46], an improved Bayesian fusion (IBF) can be proposed by combining equations (12), (14) and (15).

3.3. Proposed Hybrid Method (ELM-IBF). Ensemble learning, which is also referred to as the multiclassifier system, completes the learning task by building and combining multiple learning devices [58]. Common ensemble learning strategies include AdaBoost [59], bagging [60], and stacking.

The Bagging method is a representative parallel ensemble learning algorithm that mainly uses the Bootstrap sampling approach to create subdatasets. During the training process, a Bagging method takes a certain number of samples from the original samples, and repeated sampling is allowed during training process. In this case, some samples in the original sample may be selected as training set samples many times, while other samples may not appear in the process of Bootstrap sampling. Through this process, the diversity of individual learners can be increased, and the overall generalization ability of the entire model can be improved.

For a given training dataset $D = \{(x^{(i)}, y^{(i)}) : i = 1, 2, \ldots, l\}$, the number of iterations is $T_s = 1, 2, \ldots, T_s$ and the specific process of a general Bagging method for regression problem is shown as follows:

Step 1: Use Bootstrap to sample subdatasets $D(t_s)$ with the size of $k$ from the dataset $D$ for $T_s$ times, $T_s$ training subdatasets with the size of $k$ can be obtained.

Step 2: Each training set is learned with a subpredictor, and the corresponding subpredictor $h_t$ can be obtained.

Step 3: Use each trained subpredictor $h_t$, $t_s = 1, 2, \ldots, T_s$ to make a prediction, and use the averaging method to calculate the final output.

In this paper, an improved hybrid model (ELM-IBF) based on an improved ensemble learning method and DBNs is established to make short-term traffic flow predictions for the expressways. Figure 2 illustrates the structure of the ELM-IBF model. After applying the bagging method as its basic framework, the proposed method introduces DBNs as the subpredictors of the bagging framework, which are trained based on the different subdatasets and used to forecast multistep-ahead traffic flow, respectively. Then, the IBF method, which can dynamically adjust the weights of the subpredictors and adopt the error compensation factor changing the sum of the weights at each prediction time interval, is employed as the combination strategy of the bagging framework. Finally, the prediction results of the ELM-IBF model can be calculated by equation (14).

Algorithm 2 reveals a pseudocode of implementing an ELM-IBF model. As indicated in the tables, the main parameters in the ELM-IBF model are the number of the DBNs $T_s$ and the lengths of the fusion steps $\lambda$, which are determined before the fusion. At each time step, $T_s$ DBNs are assigned appropriate weights automatically according to their performance according to the previous $\lambda$ time steps. The hyperparameters in the DBNs can be set according to previous studies [13].

The input and output of the ELM-IBF model can be written as follows:

$$x_{in} = [x_{t-t_0+1}, x_{t-t_2+2}, \ldots, x_t]^T,
(16)$$

$$x_{out} = [\bar{x}_{t+1}, \bar{x}_{t+2}, \ldots, \bar{x}_{t+\lambda}]^T,
(17)$$

where $t_0$ is the look-back time step of the input vectors; $t_s$ is the look-ahead prediction time step; $x_t$ is the measured traffic flow data (volume or speed) at time interval $t$; $\bar{x}_{t+\lambda}$ is the predicted traffic flow data (volume or speed) at time interval $t+1$.

4. Experiment

4.1. Data Description. To demonstrate the performance of the proposed model, we conduct numerical experiments based on the real-world traffic flow data containing volume and speed data. As revealed in Figure 3, the ground-truth data for measuring varied model performance were captured by multiple remote traffic microwave sensors (RTMS) located at six road sections on the 2nd ring roads of Beijing. These six road sections include the East of Jishuitan Bridge (P1), the North of Fuxingmen Bridge (P2), the West of You’ammen Bridge (P3), the West of Zuo’ammen Bridge (P4), the South of Dongbianmen Bridge (P5), and the North of Chaoyangmen Bridge (P6). The length of the sampling time is two weeks (from January 6, 2014 to January 19, 2014) with a 2 min sampling time interval (720 time intervals in a day). Thus, the total number of records captured by each detector is 10080 with less than 5% missing and error rate. In addition, the whole dataset is divided into two parts including the first nine days for training (from Jan 6, 2014, to Jan 14, 2014) and the next five days for testing (from Jan 15, 2014, to Jan. 19, 2014). Besides, the raw traffic flow data may contain little noises or abnormal data, and the threshold value method was employed to remove the outliers. To ensure a more reliable result, missing and erroneous records were properly remedied by using the temporally adjacent records. For the purpose of examining the performance of the models, especially the stability and robustness with the limited dataset, volume and speed data are both treated as the prediction target during the experiments.

The experimental platform of our research is a Lenovo computer with Intel(R) Core(TM) i7-8700 CPU@3.20 GHz and 8 GB memory. Python 3.6 with TensorFlow 1.0, SciKit-learn, and Keras 0.9 is exploited to implement the relevant models.
Phase I: Training the DBNs

**Input**: Training dataset $D$;
Sampling size $k$;
Number of DBN predictor $T_s$

**Output**: DBNs with the trained parameters

1. for $i = 1$ to $T_s$ do
2. Sample the subdataset $D(i)$ with size $k$ from the dataset $D$ by Bootstrap
3. Train the DBN $h_i$ with subdataset $D(i)$ by Algorithm 1 and BP Algorithm
4. end

Phase II: Forecasting the traffic flow with trained DBNs

**Input**: Testing dataset $D'$;
Length of the testing dataset $D'$: $l$;
Trained DBNs;
Length of the fusion step: $\lambda$

**Output**: Predicted values of the ELM-IBF method

5. for $i = 1$ to $T_s$ do
6. Predict the traffic flow data with testing dataset $D'$ by using DBN $h_i$
7. end
8. for $i$ from 1 to $l$ do
9. Calculate the weights of DBNs $h_j$, $j = 1, 2, \ldots, T_s$ at each prediction time interval by using equation (12)
10. end
11. Calculate the predicted values with equations (14) and (15) in IBF

Algorithm 2: Implementing an ELM-IBF model.
4.2. Baseline Methods. To examine the practicability and effectiveness of the proposed method, ARIMA, radial basis function neural network (RBFNN), BPNN, LSTM, and DBN are chosen as the single benchmark models. Also, the ensemble learning methods ELM-AM and ELM-BF are employed as the hybrid benchmarks.

For the ARIMA, the Akaike information criterion (AIC) is considered to determine the best order \((p, d, q)\) of the model, and the parameters \(p, d, q\) are chosen as 2, 1, and 1, respectively. For the RBFNN and BPNN, they both have an input layer with 20 neurons, a hidden layer with 50 neurons, and an output layer with one neuron. The sigmoid function and Gaussian radial basis function are employed as the activation function in the BPNN and the RBFNN, respectively. Meanwhile, the LSTM, which has the superior capability for time-series prediction with long temporal dependency, is composed of one input layer, one LSTM layer with memory blocks, and one output layer. The number of the hidden units in the LSTM layer is 200, and the largest time lag is 20. The relevant parameters of the benchmark DBN model are shown in Table 1, and the parameters of the DBNs in the ELM-IBF method are set the same as those of the DBN-based benchmark models. It is necessary to notice that the ELM-AM and the ELM-BF share the same DBN structures with the ELM-IBF model. To be specific, the ELM-AM and ELM-BF use the average method combination strategy and the Bayesian fusion strategy, respectively.

Note that the look-back time step \(t_b\) of the NN-based models (RBFNN, BPNN, LSTM, DBN, ELM-AM, ELM-BF, and ELM-IBF) is set as 20, and it means that the traffic condition during the previous 40 min will be considered in the input of the NN-based models. In the proposed method, the sampling size \(k\) is set as the \(2/3\) size of the entire training dataset. The number of DBN predictors \(T_i\) is set as 3, and the fusion step \(\lambda\) in IBF is set as 1. The prediction target of this study is the traffic volume and speed in the testing dataset.

### Table 1: Relevant parameters of the DBNs.

| Description                              | Value |
|------------------------------------------|-------|
| Number of nodes in the input layer       | 20    |
| Number of RBM layers                     | 2     |
| Number of nodes in the first hidden layer| 400   |
| Number of nodes in the second hidden layer| 400   |
| Iteration number of each RBM             | 200   |
| Activation function (RBM)                | Sigm  |
| Learning rate (RBM)                      | 1     |
| Batch size (RBM)                         | 100   |
| Number of epochs (RBM)                   | 200   |
| Batch size (NN)                          | 100   |
| Number of epochs (NN)                    | 500   |
| Activation function (NN)                 | Sigm  |
| Learning rate (NN)                       | 1     |
| Input Zero Masked Fraction (RBM)         | 0.5   |

4.3. Evaluation Indicators. To evaluate the performance of the proposed model and benchmark models, mean absolute error (MAE), root mean square error (RMSE), mean absolute percentage error (MAPE), and Theil inequality coefficient (TIC) are selected as the evaluation index.

\[
\text{MAE} = \frac{1}{s} \sum_{i=1}^{s} |\hat{y}_i - y_i|, \tag{18}
\]

\[
\text{RMSE} = \sqrt{\frac{1}{s} \sum_{i=1}^{s} (\hat{y}_i - y_i)^2}, \tag{19}
\]
\[
\text{MAPE} = \frac{1}{s} \sum_{i=1}^{s} \left| \frac{\hat{y}_i - y_i}{y_i} \right| \times 100, \tag{20}
\]
\[
\text{TIC} = \frac{\sqrt{(1/s) \sum_{i=1}^{s} (\hat{y}_i - y_i)^2}}{\sqrt{(1/s) \sum_{i=1}^{s} (\hat{y}_i)^2 + (1/s) \sum_{i=1}^{s} (y_i)^2}}, \tag{21}
\]
where \(\hat{y}_i\) is the predicted value, and \(y_i\) is the observed value. \(s\) is the length of the prediction time interval.

Note that MAE, RMSE, and MAPE are utilized to represent the accuracy of the prediction models. TIC reflects the fitting degree between the predicted value and the observed value. The numerical distribution of TIC is between 0 and 1. The smaller the TIC is, the higher the fitting degree is.

5. Results and Discussion

5.1. Overall Prediction Performance of Different Methods.

With the look-ahead prediction time step \(t_o\) of the ELM-IBF model set as 1, Tables 2 and 3 compare the overall prediction performance of the ELM-IBF model and its benchmarks in terms of traffic volume and traffic speed prediction by taking the prediction results of the separated six scenarios as a whole.

Generally, the ELM-IBF model demonstrates better predictive performance compared with the single models. In terms of speed prediction illustrated in Table 2, the ELM-IBF model outperforms the DBN, which is the second-best single model with improvements of 28.98%, 30.66%, and 28.66% on MAE, MAPE, and RMSE, respectively. In terms of the volume prediction revealed in Table 3, the MAE and MAPE of the ELM-IBF model are 33.69% and 36.17% less than those of the DBN. Among the hybrid models, the performance advantages of the ELM-IBF model are more obvious under the condition that the ELM-AM, the ELM-BF, and the ELM-IBF share the same component predictors. Meanwhile, the robustness of the ELM-IBF model can be guaranteed since its ensemble learning structure can keep the model robust, and its improved Bayesian fusion mechanism can benefit the accuracy.

It can be observed that the prediction performance of the ELM-AM and the ELM-BF is slightly worse than that of their component model DBN. The possible reasons can be summarized as follows: (1) the training data of DBN in the ensemble learning frameworks are not large enough, which leads to insufficient training. (2) The fusion algorithms, such as AM and BF, are not efficient enough to assign larger weights to the DBN predictors, which illustrate better performance.

Figure 4 gives the overall prediction errors produced by these different methods in several road sections. As indicated in Figure 4, the proposed ELM-IBF method reveals better predictive performance than other models in terms of the maximum, minimum, and median of errors. Besides, it can be found that the ELM-IBF model has a smaller distance between Q1 and Q3, and the error distribution of the ELM-IBF model is more concentrated than that of other models.

Figure 5 illustrates the prediction results of the ELM-IBF model on a weekday (2014.1.16) and a weekend (2014.1.18) on the road section P1. In Figure 5, the ELM-IBF model is capable of capturing the tendency and volatility of the traffic flow for the entire day on the weekdays and the weekends. Even at the morning peak (8:00–11:00) and the evening peak (16:00–20:00) of the weekday when the traffic speeds fluctuate sharply, the proposed method is still able to fit the fierce fluctuation of traffic speeds and make a precise single-step-ahead prediction.

Figures 6 and 7 display the correlation between actual values and predicted values from six models including one statistical method (ARIMA), two traditional nonparametric methods (BPNN and RBFNN), two deep learning methods (DBN and LSTM), and the proposed ELM-IBF model in five days. \(r\) represents the Pearson correlation coefficient to evaluate the relevance between observed and predicted values. The two figures indicate the prediction results in the task of speed prediction and volume prediction, respectively. From the observation of two figures, we can gain several conclusions as follows: the ELM-IBF model produces better prediction results with \(r\) compared with other models. In addition, the deep learning models outperform the statistical model and traditional nonparametric methods, because DBN and LSTM have complex structures and strong learning abilities. Furthermore, the ELM-IBF model works best among these models, because it combines the advantages of deep learning methods and ensemble learning theory.

5.2. Performance of the ELM-IBF Model for Multistep-Ahead Prediction.

Table 4 lists the prediction performance of the ELM-IBF model for multistep-ahead prediction tasks. It can be observed that the MAEs, MAPES, RMSES, and TICs of the proposed method do not rise obviously as the look-ahead
Figure 4: Comparison of the prediction errors for each method at the road section P1. (a) Speed prediction (MAE). (b) Speed prediction (MAPE). (c) Volume prediction (MAE). (d) Volume prediction (MAPE).

Figure 5: Continued.
time step increases. To be specific, the MAEs, MAPEs, and RMSEs fluctuate slightly with the look-ahead time step $t_a$ increasing from 1 to 10, which corresponds to the prediction horizons ranging from 2 min to 20 min. It is interesting to find that the ELM-IBF model becomes more accurate when it is utilized to forecast traffic volume with a longer look-ahead time step. Meanwhile, the TICs rise slowly with $t_a$ becoming larger, and it means that the fitting degree of the ELM-IBF model drops a little, although its prediction accuracy is relatively stable.

Overall, the proposed method exhibits the advantages of making multistep-ahead traffic flow prediction. In the case of both traffic speed and volume prediction, the accuracy and stability of the results predicted by the ELM-IBF model with 10 look-ahead time steps are even higher than those predicted by its benchmark models (e.g., ARIMA, BPNN,

Figure 5: Prediction results of the ELM-IBF model at the road section P1. (a) Speed prediction on a weekday (2014.1.16). (b) Volume prediction on a weekday (2014.1.16). (c) Speed prediction on a weekend (2014.1.18). (d) Volume prediction on a weekend (2014.1.18).

Figure 6: The correlation distribution of predicted results for the traffic speed. (a) ARIMA; (b) RBFNN; (c) BPNN; (d) DBN; (e) LSTM; (f) ELM-IBF.
RBFNN, LSTM, DBN, ELM-AM, and ELM-BF) with only one look-ahead time step shown in Tables 2 and 3.

5.3. Sensitivity Analysis. In this section, we conduct the sensitivity analysis and parameter tuning on the ELM-IBF model, where two kinds of critical parameters are investigated, including the length of the fusion step and the number of subpredictors.

The length of the fusion step $\lambda$ in the IBF is an important parameter that affects the prediction performance of the proposed method. Figure 8 indicates the relationship between the prediction performance of the ELM-IBF model and the length of the fusion step under the condition that the look-ahead prediction time step $t_a$ is chosen to be 1, and the number of DBN predictors is set as 3. Figure 8 reveals that the prediction accuracy and fitting degree decline gradually with the length of the fusion step $\lambda$ increasing from 1 to 10. This may be caused by the phenomenon that the error compensation factor $\tau_t$ in equation (15) becomes closer to 1 when prediction errors during more previous time intervals are taken into the fusion process with the fusion step becoming larger. Therefore, the recommended length of the fusion step is $\lambda = 1$, which can ensure the accuracy and robustness of the ELM-IBF method without costing too much fusion time.

Figure 9 indicates the relationship between the prediction performance of the model and the number of DBN predictors. Tables 4 presents the prediction results of the ELM-IBF model with longer prediction horizon.

![Figure 7: Correlation distribution of predicted results for the traffic volume. (a) ARIMA; (b) RBFNN; (c) BPNN; (d) DBN; (e) LSTM; (f) ELM-IBF.](image)

| Look-ahead time step | Speed prediction | Volume prediction |
|----------------------|------------------|-------------------|
|                      | MAE | MAPE (%) | RMSE | TIC | MAE | MAPE (%) | RMSE | TIC |
| 1                    | 3.21| 6.13      | 4.63  | 0.0354 | 6.91 | 7.27      | 9.21  | 0.0359 |
| 2                    | 3.28| 6.89      | 5.43  | 0.0424 | 6.95 | 7.47      | 9.30  | 0.0364 |
| 3                    | 3.45| 7.81      | 6.53  | 0.0515 | 7.00 | 7.58      | 9.50  | 0.0371 |
| 4                    | 3.84| 9.29      | 7.82  | 0.0616 | 7.03 | 7.66      | 9.63  | 0.0378 |
| 5                    | 3.89| 9.49      | 7.86  | 0.0621 | 7.13 | 7.75      | 9.70  | 0.0384 |
| 6                    | 4.08| 10.06     | 8.31  | 0.0653 | 7.22 | 7.93      | 9.88  | 0.0388 |
| 7                    | 4.24| 10.71     | 8.68  | 0.0682 | 7.36 | 8.22      | 10.18 | 0.0399 |
| 8                    | 4.37| 11.28     | 9.06  | 0.0712 | 7.43 | 8.32      | 10.28 | 0.0402 |
| 9                    | 4.49| 11.58     | 9.23  | 0.0724 | 7.77 | 8.79      | 10.85 | 0.0423 |
| 10                   | 4.54| 11.66     | 9.29  | 0.0729 | 7.97 | 9.02      | 10.90 | 0.0455 |
Figure 8: The influence of the length of the fusion steps on the ELM-IBF model. (a) MAE; (b) MAPE; (c) RMSE; (d) TIC.

Figure 9: Continued.
subpredictors when making a single-step-ahead prediction with the fusion time step set as 1. The number of DBN subpredictors is tuned from 1 to 10 with a step of 1. As illustrated in Figure 9, the MAEs, RMSEs, and MAPEs decrease more than 30% with the number of DBN subpredictors increasing from 1 to 5 in the task of forecasting both traffic speed and volume prediction, implying that the appropriate increase of the subpredictors can significantly improve the accuracy and fitting degree of the ensemble model. In addition, when the number of DBN subpredictors is larger than 5, the performance of the ELM-IBF model remains stable, and it demonstrates that the excessive increase of the number of subpredictors has little effect on the prediction performance probably due to the similarity of the homogeneous models. Therefore, the suitable number of the DBNs is around 3−5 considering the prediction accuracy and the time consumption of training the deep learning models.

Note that though the ELM-IBF model exhibits its capability of predicting traffic flow parameters accurately and stably, the shortcoming of the ELM-IBF may lie in its efficiency since the ensemble learning framework needs to combine several DBNs, and the aggregate time of training several DBNs is much larger than one DBN model. However, with the rapid development of data processing, data storage, and parallel computing technology, the training time consumption of the deep learning model may be shortened dramatically, and the sensitivity analysis of the above two critical parameters is also beneficial to reduce the time consumption and keep the accuracy of the ELM-IBF model.

6. Conclusions

Short-term traffic flow prediction is a significant problem in ITS. This paper establishes a novel hybrid model (ELM-IBF) for short-term traffic flow forecasting based on the combination of ensemble learning theory and deep belief network. At first, the Bagging algorithm is employed to divide the training dataset into several subdatasets and determine the entire structure of the ELM-IBF model. Then, the deep belief networks are introduced as the subpredictors trained by the divided datasets. Afterward, an improved Bayesian fusion approach is proposed to integrate the prediction results of DBNs more efficiently. Finally, the measured traffic flow data collected on the six road sections of expressways in Beijing are utilized to examine the accuracy and robustness of the proposed method.

The main conclusions of this study can be summarized as follows: (1) the overall prediction results demonstrate that the ELM-IBF model outperforms the single model-based benchmarks (e.g., ARIMA, BPNN, RBFNN, LSTM, and DBN) in terms of accuracy and fitting degree when making single-step-ahead traffic volume and speed prediction. (2) Compared with other ensemble learning methods (e.g., ELM-AM and ELM-BF) with the same subpredictors, the ELM-IBF has lower MAE, MAPE, RMSE, and TIC. It proves that the IBF combination methods can improve the performance of the bagging framework significantly and work better than AM and BF methods. (3) The ELM-IBF model illustrates a stable and accurate prediction performance in the task of forecasting multistep-ahead traffic flow. (4) Sensibility analysis confirms that the length of the fusion step and the number of the subpredictors both affect the predictive performance of the ELM-IBF model, and the recommended values of the two aforementioned parameters are 1 and 5, respectively.

In the future, we will concentrate on utilizing multi-source traffic flow data to enrich the input variables of the proposed method to further improve model performance. Larger amounts of data including more abnormal traffic conditions are necessary to be used for training and examining the proposed models. Furthermore, this paper only applies homogeneous component predictors in the ELM-IBF model, while heterogeneous component predictors could be added and investigated to enhance the generalization ability of the ensemble learning model.

Data Availability

The data used to support the findings of this study were provided by the Beijing Municipal Commission of Transport and were not made public.
Conflicts of Interest
The authors declare that there are no conflicts of interest regarding the publication of this paper.

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