Estimating short-time period to break different types of chaotic modulation based secure communications

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Abstract

In recent years, chaotic attractors have been extensively used in the design of secure communication systems. One of the preferred ways of transmitting the information signal is binary chaotic modulation, in which a binary message modulates a parameter of the chaotic generator. This paper presents a method of attack based on estimating the short-time period of the ciphertext generated from the modulated chaotic attractor. By calculating and then filtering the short-time period of the transmitted signal it is possible to obtain the binary information signal with great accuracy without any knowledge of the parameters of the underlying chaotic system. This method is successfully applied to various secure communication systems proposed in the literature based on different chaotic attractors.

Key words: Chaotic cryptosystems, Chaotic attractors, Cryptanalysis, Short-time period, Lorenz, Chua, Rössler

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1 Introduction

During the last decade, there have been many proposals to apply non-linear dynamical systems to cryptography and secure communications under the assumption that chaotic orbits resemble random-number generators and might

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There exist two main approaches to chaotic ciphers design: analog and digital. The first one is based on the concept of chaotic synchronization, first shown by Pecora and Carroll [4]. In these systems, the information can be transmitted by the chaotic signal in a number of ways, including, but not limited to, chaotic masking [5,6,7,8,9], in which the analog message signal $i(t)$ is added to the output of the chaotic generator $x(t)$ in the transmitter; chaotic switching or chaos shift keying (CSK) [10,11], in which a binary message signal is used to choose between two statistically similar chaotic attractors; chaotic modulation (CM) [12,13,14,15,16,17,18], in which a message, most frequently in binary form, modulates a parameter of the chaotic generator; or inverse system approach (ISA), in which the receiver system runs in an exactly inverse way of the transmitter system to exactly recover the message [19,20]. Regardless of the method used to transmit the message signal, the receiver has to synchronize with the transmitter’s chaotic generator to regenerate the chaotic signal $x(t)$ and thus recover the message $i(t)$.

Chaotic modulation has been repeatedly used as an information concealing method throughout the years until very recently [12,13,15,16,17,18]. In the literature, when a binary digital signal is to be encrypted by this means, one of the state variables of the modulated chaotic system is commonly taken as the transmitted encrypted message. The key of the cryptosystem is composed by the unknown internal parameters of the chaotic system. Thus, the only information available to the attacker is the instantaneous value of the transmitted state variable. In [21] it is pointed out that most widely-used chaotic attractors in secure communication systems exhibit an inherent frequency-dependent on the system parameters. As a result, it is reasonable to assume that the period of chaotic signals generated using different sets of parameters must be different. This letter shows that this assumption proves to be true, even for small variations of the parameters, and for different types of synchronization and parameter modulation. A method based on short-time period estimation is described to detect these slight variations in period to be able to discern between different attractors and thus between different values of the binary information signal. The method works for different chaotic attractors, different synchronization, and different modulation techniques.

The rest of this letter is organized as follows. In Sec. 2, the method used to compute the short-time period is explained. In Sec. 3, some examples of how the method works are given. The examples use different modulation techniques to encrypt the message signal. In Sec. 4, our method is compared against other cryptanalytic methods frequently found in the literature. Finally, Sec. 5 concludes the letter.
2 Measuring the short-time period

In this section the method followed to calculate the short-time period of a chaotic scalar signal is explained. It is assumed the use of 3-D chaotic attractors, given as an autonomous continuous dynamical system $\dot{x} = f(x)$. Two trajectories $x(t)$ and $x'(t)$ are said to completely synchronize if:

$$\lim_{t \to \infty} |x(t) - x'(t)| = 0.$$  \hfill (1)

For robust synchronization to be maintained, it is required that all conditional Lyapunov exponents (CLE) of the response subsystem are negative [4].

As is well known, chaotic signals present some properties as sensitive dependence on parameters and initial conditions, ergodicity, mixing, and dense periodic points. These properties make them similar to pseudorandom noise. As a result, this apparent randomness has motivated their use in secure communication applications. The most widely-used chaotic signal generators in this context are based on the double-scroll Lorenz and Chua attractors, and on the single-scroll Rössler attractor. As studied in [21], these chaotic attractors exhibit an inherent frequency uniquely determined by their system parameters. When present, this frequency can be measured over long-time periods. However, in this work we are interested in knowing the fast fluctuations in the frequency in short term in an effort to estimate the attractor’s instantaneous frequency. We try to measure the short-time period as a function of time to unmask the binary modulating signal. If the signal is periodic or nearly-periodic, calculating the short-time zero-crossing rate (STZCR) or a short Discrete Fourier Transform (DFT) would be enough, but chaotic signals are essentially aperiodic. Nevertheless, along the trajectory followed by an initial point in these attractors there are regions where the movement is very close to periodic, thus allowing for a very accurate estimation of the short-time period. The peculiarities of different chaotic signals require the customization of the method for the three different types of attractors under consideration. Once a sufficiently stable region is found as described in the next section, then it is possible to compute the short-time period following the procedure described below.

In parameter modulation based chaotic secure communication systems, one parameter of the attractor is changed according to the binary value of the information signal $i(t)$ regardless of the synchronization method. Usually one state variable of the attractor, $x_i(t)$, is used to convey the concealed information signal. This state variable is the only information available to the attacker. Let us note that the orbit followed by an initial condition is generally non-uniform. However, if a 2-D projection of the chaotic attractor is used, it will be observed that there is always a region in which the average
rotation angular speed is almost constant, i.e., the elapsed time for each visit to this region is almost constant. The part of the signal corresponding to this nearly-periodic region is used to make the measure as accurate as possible. Thus, instead of measuring the whole period, which may lead to inaccurate results, only a fraction of the period is measured, corresponding to the elapsed time within the nearly-periodic region in each rotation. Let us note that this method tries to spot variations in the short-time period and is not concerned with measuring its exact value. Next, a new time signal \( p(t) \) is created, by assigning this measured value to \( p(t) \) for the duration of the whole rotation period of \( x_i(t) \). Once \( p(t) \) has been created, its DC component is removed by subtracting its mean value. The new signal is \( p^*(t) \). Last, an appropriate moving averaging filter with a Hanning window to smooth up the result is used on \( p^*(t) \). As will be seen, the resulting filtered signal, \( f p^*(t) \), suffices to detect the plaintext, although a Schmitt-Trigger with adequate switch-on and switch-off levels might be used to obtain the final recovered signal, \( i^*(t) \).

In the next section, several examples are given where the above process is further explained and successfully applied to the cryptanalysis of different types of chaotic modulation based secure communication systems.

3 Examples

In this section the performance of the short-time period estimation method is analyzed when applied to different secure communication systems proposed in past and recent literature, including the classical parameter modulation method, a phase synchronization method, and an adaptive observer-based chaos synchronization method. We believe such a cryptanalysis method can be further generalized to break other secure communication systems.

3.1 Classical parameter modulation method

The first implementation of parameter modulation [12] uses the well-known double-scroll Lorenz attractor [22] as the chaotic signal generator. The transmitter end is represented by:

\[
\begin{align*}
\dot{x}_1 &= \sigma(x_2 - x_1), \\
\dot{x}_2 &= r x_1 - x_2 - x_1 x_3, \\
\dot{x}_3 &= x_1 x_2 - b(i(t)) x_3,
\end{align*}
\]

(2)

where \( \sigma, r, \) and \( b \) are the internal system parameters. Furthermore, \( i(t) \) is a binary information signal controlling the parameter \( b \) to be one of two different
values $b_0$ and $b_1$. At the receiver end, an identical system is used by tuning the parameter $b$ to be $b_0$ (or $b_1$). When the receiver subsystem synchronizes with the transmitted signal in the sense described by Eq. (1), it is known that $b_0$ (or $b_1$) was used at the transmitter end; when it does not synchronize, the other value is assumed. In such a way, the binary message $i(t)$ is decrypted from $b = b_0$ or $b_1$ at each given time $t$.

When the Lorenz attractor is used, usually either $x_1(t)$ or $x_2(t)$ is transmitted as ciphertext. In [12], $x_1(t)$ is used (see Fig. 2.b), $\sigma = 16.0$, $r = 45.6$, and the modulated parameter is $b$, taking values $b = 4$ or $4.4$ for the binary signal equal to $i(t) = 0$ or $i(t) = 1$, respectively. The message signal $i(t)$ is plotted in Fig. 2.a.

The Lorenz chaotic signal must be first correctly conditioned before computing its short-time period. As can be observed in Fig. 1.a, the orbit followed by an arbitrary initial point spirals around the two scrolls, jumping from one scroll to the other in a chaotic manner. The work with this attractor is simplified if its absolute value is taken: $y_i(t) = |x_i(t)|$, $i = \{1, 2\}$. This operation folds the attractor back on itself due to its symmetry with respect to $x_1 = 0$ and $x_2 = 0$, in such a way that the trajectories spiral around one merged scroll in the same rotation direction, as observed in Fig. 1.b. As discussed in the previous section, to obtain the maximum accuracy in the estimations, the rotation duration is measured on the region where the average rotation angular speed is almost constant. This region, plotted in Fig. 1.b, can be easily computed as the region to the right of the middle value of the maximum value of $y_i(t)$.

In the example, the period is computed as the elapsed time during which $\max(y_1)/2 < y_1(t) < \max(y_1)$ holds.

The rest of the process of measuring the short-time period value is quite similar to the one outlined in the previous section, but proceeding with $y_i(t)$ instead. Following this process, plotted in Figs. 2.c-f, the original message signal is recovered with great accuracy. A Schmitt-Trigger with switch-on level of 0 and switch-off level of $-20$ was used. The recovered signal $i^*(t)$ is slightly delayed with respect to the original $i(t)$ due to the delay introduced by the filter and can be easily removed if desired.

It must be added that this method of parameter modulation has been known to be insecure many years before [23,24,25,26].

### 3.2 Phase synchronization method

Most secure chaotic communication systems are based on complete synchronization in the sense of Eq. (1), whereas new cryptosystems have been proposed based on phase synchronization [18]. This scheme hides binary messages
in the instantaneous phase of the drive subsystem used as the transmitting signal to drive the response subsystem. At the receiver, the phase difference is detected and its strong fluctuation above or below zero allows the plaintext recovering at certain coupling strength. The secure communication process is illustrated in [18] by means of an example based on coupled Rössler chaotic oscillators. In the example, the drive subsystem is formed by two weakly-coupled oscillators. The plaintext is used to modulate the same parameter in both oscillators 1 and 2. The equations of the drive subsystem are:

\[
\begin{align*}
\dot{x}_{1,2} &= -(\omega + \Delta \omega)y_{1,2} - z_{1,2} + \varepsilon(x_{2,1} - x_{1,2}), \\
\dot{y}_{1,2} &= (\omega + \Delta \omega)x_{1,2} + \alpha y_{1,2}, \\
\dot{z}_{1,2} &= 0.2 + z_{1,2}(x_{1,2} - 10).
\end{align*}
\]

The response subsystem is governed by:

\[
\begin{align*}
\dot{x}_3 &= -\omega' y_3 - z_3 + \eta((x_3^2 + y_3^2)^{1/2}) \cos \phi_m - x_3, \\
\dot{y}_3 &= \omega' x_3 + \alpha' y_3, \\
\dot{z}_3 &= 0.2 + z_3(x_3 - 10).
\end{align*}
\]

In the example, the parameter values are: \(\omega = \omega' = 1\), \(\varepsilon = 5 \times 10^{-3}\), \(\eta = 5.3\), and \(\alpha = \alpha' = 0.15\). The parameter \(\omega\) corresponds to the natural frequency of the Rössler oscillator drive subsystems 1 and 2. The parameter \(\omega'\) corresponds to the natural frequency of the Rössler oscillator driven subsystem 3, \(\varepsilon\) corresponds to the weak coupling factor between the oscillators 1 and 2, and \(\eta\) corresponds to the strong coupling factor between the 2 driven oscillators and the response oscillator 3. The parameter mismatch \(\Delta \omega\) is modulated by the plaintext, being \(\Delta \omega = 0.01\) if the bit to be transmitted is “1” and \(\Delta \omega = -0.01\) if the bit to be transmitted is “0”.

The ciphertext consists of the phase of the mean field of the drive oscillators:

\[
\phi_m = \arctan \frac{x_1 + x_2}{y_1 + y_2},
\]

where \(\arctan\) is the arctangent function of the argument, from \(-\pi\) to \(\pi\).

The signal available to the attacker is \(\phi_m(t)\), the instantaneous phase. In the following, without loss of generality, only \(\phi_1(t) = \arctan(x_1/y_1)\) is considered to qualitatively illustrate the behavior of the Rössler attractor. In Fig. 3.a it is observed that for \(x_1(t) < 0\), the rotation angular speed is approximately constant, i.e., the phase increases almost linearly. However, depending on the system parameters chosen, the phase can change abruptly in the first quadrant when \(0 < \phi_1(t) < \pi/2\). Thus, this is the part of the signal to be avoided to compute the short-time period. Although this cryptosystem was already
broken by an economic brute-force attack in [27], Fig. 4 shows the results obtained after applying a more elegant and straightforward avenue of attack using the cryptanalysis described in this letter. In the example analyzed, the signal conditioning is limited to considering \( \pi/2 < |\phi_m(t)| < \pi \) to compute \( p(t) \). In this case, it is not necessary to filter \( p^*(t) \) because each bit of the plain-signal corresponds exactly to one short-time period of \( p^*(t) \). Thus, by simply rescaling \( p^*(t) \) a perfect estimation of \( i(t) \) is obtained. Again, the time delay can be removed if desired.

3.3 Adaptive observer-based chaos synchronization

In [17], the author proposes a symmetric secure communication system based on parameter modulation of a chaotic oscillator acting as a transmitter. The receiver is a chaotic system synchronized by means of an adaptive observer. Two sample implementations are given: one with the Lorenz attractor and another with Chua attractor. In this letter the latter will be broken, to illustrate how our method works with a different double-scroll attractor. It works equally well for Lorenz, though.

Chua’s circuit dynamics can be described by the following equations:

\[
\begin{align*}
\dot{x}_1 &= \alpha(-x_1 + x_2) - f_1(x_1), \\
\dot{x}_2 &= x_1 - x_2 + x_3, \\
\dot{x}_3 &= -\beta x_2.
\end{align*}
\] (6)

where \( f_1(x) = bx + 0.5(a - b)(|x + 1| - |x - 1|) \). In the example the system is implemented with the following parameter values, \((\alpha, \beta, a, b) = (10, 18, -4/3, -3/4)\). The signal used for synchronization of the receiver is \( x_1 \). The encryption process is defined by modulating the parameter \( \beta \) with the binary encoded plaintext, so that it is \( \beta + 1.25 \) if the plaintext bit is "1" and \( \beta - 1.25 \) if the plaintext bit is "0". The duration of the plaintext bits must be much larger than the convergence time of the adaptation law. The uncertain system can be rewritten in a compact form as:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} = \begin{bmatrix}
-10 & 10 & 0 \\
1 & -1 & 1 \\
0 & \beta & 0
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} + \begin{bmatrix}
f_1(x_1) \\
0 \\
0
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} x_2 \theta,
\] (7)

\[y = C \cdot x = x_3,\] (8)

\[C = [0 \ 0 \ 1],\] (9)
\( \theta = \Delta \beta = \pm 1.25. \quad (10) \)

The transmitted ciphertext is the signal \( x_3(t) \).

Again, we are dealing with a double-scroll attractor. In contrast to the Lorenz attractor, the trajectory followed by an arbitrary initial point in the Chua attractor is uniform enough to allow the direct estimation of the (whole) short-time period from the transmitted signal \( x_3(t) \).

After applying this method to \( x_3(t) \), as shown in Figs. 5.c-f, the original message signal is recovered with great accuracy. A Schmitt-Trigger with switch-on and switch-off levels of 10 and 0 respectively was used. The recovered signal \( i^*(t) \) is slightly delayed with respect to the original \( i(t) \) due to the delay introduced by the filter and can be easily removed if desired.

4 Comparison with other attack methods

Throughout the years, different methods have been proposed to attack chaos-based secure communication systems. In this section, the performance of the short-time period estimation method is compared against the most relevant.

The return-map method was initially devised by [28] and further developed by [24]. Given one of the variables in the chaotic system, one or more proper return maps can be constructed allowing for a partial reconstruction of the dynamics. By analyzing the evolution of the signal on the attracting sets of those maps, the message can be extracted under certain conditions. These attacks can be performed without the knowledge of the precise structure of the chaotic system in use. This method not only decrypts ciphertexts encrypted using chaotic modulation, but also using chaotic masking. However, it does not work for phase synchronization cryptosystems. There are some improved cryptosystems [29] which avoid the return map attack by modulating the transmitted signal with an appropriately chosen scalar signal. Our method was checked against this improved method. The results show that it is still able to directly recover the correct signal, and also can be used to identify and remove the modulating signal, thus rendering the return-map attack again possible.

In [23], the short-time zero-crossing rate (STZCR) of the differential of the transmitted signal is used to recover the information digital signal. This method presents the limitation of only working on single-scroll Chua’s circuits proposed in [10], while this letter generalizes this method so that can be used on different chaotic attractors, including the three most frequently-used ones,
i.e., (double-scroll) Lorenz, (single-scroll) Rössler and (double-scroll) Chua attractors. In this letter different conditioning methods are discussed to show the great flexibility of our method. In doing so, we have partially revealed the theory hiding behind the nearly-stable short-time period of many 3-D chaotic attractors.

When two different attractors (or for the same event, the same attractor with two different parameter sets) are switched to encode a binary message, a spectrogram might reveal the evolution of the energy distribution in spectral-time space from the transmitted signal. If the two chaotic attractors have some detectable difference in their spectrums, then the spectrogram can be used to detect this difference and thus unmask the scrambled binary information [25]. This method can be used in chaotic masking too, but does not work for phase synchronization.

In the same way that changing the parameter in an attractor affects its frequency, it is reasonable to assume that also small changes in its amplitude will take place when shifting from one set of parameters to the other. This approach was used in [30], squaring the ciphertext signal and low-pass filtering it, so that the enveloping waveform, i.e., the binary modulating signal, was finally extracted. This method performs well when the difference in amplitude of the two bits in the modulating square waveform is big enough to be observed after the filtering. Obviously, it does not work for phase synchronization where the amplitude does not change.

The generalized synchronization attack, first introduced by [26], assumes that the attacker knows the type of attractor used for the transmission and reception, but ignores the precise value of the parameters, which usually are considered to be the secret key of the cryptosystem. Using the concept of generalized synchronization (GS) defined in [31], the attacker’s receiver uses a set of parameters which is completely different to the secret key and thus will never achieve synchronization. Nevertheless, by measuring the synchronization error over time, it is possible to detect the switching between the two attractors in the transmitter as a variation in the square error. This one is a very powerful technique when complete synchronization is used. It doesn’t work for some other types of synchronization though.

5 Conclusion

A new cryptanalytic method to break parameter modulation based chaotic secure communication systems is presented. The method computes the short-time period of the ciphertext signal to detect slight variations in its frequency. For the method to work in a wide variety of modulation techniques and for
different chaotic attractors, first the transmitted signal must be conditioned according to the structure of the underlying chaotic attractor used for the modulation. The letter describes the different conditioning processes required for different attractors and explains how to calculate the short-time period variation as a function of time of the conditioned signal. The signal processing required to eventually recover the original plaintext is explained. Finally, this method is compared to some other cryptanalytic techniques used in literature. It is shown that it is the first method apart from brute force which recovers the signal when phase synchronization is used. Some important facts about the nearly-stable short-time period of many 3-D chaotic attractors are also revealed by this work.

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Figures and captions

Fig. 1. The Lorenz attractor: a) $x_1 - x_3$ projection; b) $|x_1| - x_3$ projection.
Fig. 2. Breaking classical parameter modulation using Lorenz attractor: a) original binary information signal, $i(t)$; b) the transmitted state variable signal or ciphertext, $x_1(t)$; c) the short-time period signal, $p(t)$; d) the positive value after removing DC component, $p^*(t)$; e) the low-pass filtered signal, $fp^*(t)$, revealing the modulation signal; f) recovered message signal, $i^*(t)$, after adequate detection.
Fig. 3. The well-known Rössler attractor: a) $x_1 - y_1$ projection; b) $x_1 - z_1$ projection.
Fig. 4. Breaking phase synchronization using Rössler attractor: a) original binary information signal, $i(t)$; b) the transmitted phase signal or ciphertext, $\phi_m(t)$; c) the short-time period signal, $p(t)$; d) the positive value after removing DC component, $p^*(t)$; e) recovered message signal, $i^*(t)$, after adequate detection.
Fig. 5. Breaking adaptive observer-based chaos synchronization using Chua attractor: a) original binary information signal, $i(t)$; b) the transmitted state variable signal or ciphertext, $x_3(t)$; c) the short-time period signal, $p(t)$; d) the clipped signal, $p^*(t)$, after removing singular peaks and DC component; e) the low-pass filtered signal, $fp^*(t)$, revealing the modulation signal; f) recovered message signal, $i^*(t)$, after adequate detection.