On the scaling of turbulence over an irregular rough surface in a transitionally rough regime

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Received: 10 December 2019; Revised: 29 March 2020; Accepted: 9 April 2020

Abstract
In this paper, the characteristics of a transitionally rough turbulent flow over a real rough surface are examined. In our research, high-resolution large eddy simulations over a scanned marine painted rough surface were carried out, and the friction Reynolds number and roughness Reynolds number (inner-scaled roughness height) were systematically varied. Away from the rough surface, the mean velocity and Reynolds stresses are unaffected by the mean roughness height. Further, the similarity away from the rough surface is clearly confirmed when we use the effective wall-normal distance. The effective wall-normal distance is defined as the wall-normal integral of the plane-porosity (void fraction). Moreover, near the rough surface, the Reynolds stresses asymptotically decay toward the bottom of the rough surface when plotted against the inner-scaled effective distance. The profiles of the mean velocity and Reynolds stresses against the effective distance reveal that the mean velocity profile can be characterized by the roughness Reynolds number (inner-scaled roughness length scale). However, the other parameters should be considered when characterizing the near-wall Reynolds stress behavior. When the budget terms in the plane and Reynolds averaged momentum equations are analyzed, it is found that the drag force term dominates the momentum transfer in the vicinity of the bottom of the rough surface. As the roughness Reynolds number increases, the pressure drag contribution to the skin friction coefficient increases, while the viscous contribution decreases.

Keywords: Rough wall, Turbulence, Large eddy simulation, Lattice Boltzmann method

1. Introduction

Turbulence over a rough surface has received much attention over the past few decades, because wall surfaces encountered in engineering, environmental, and geophysical flows are seldom smooth. In the engineering context, wall roughness is inevitably generated in production processes due to the imperfections in the surface finish. In addition, rough surfaces result from time-related deterioration, including erosion, corrosion, and organic and inorganic fouling processes. It is well known that the presence of wall roughness increases skin friction, leading to a decrease in the performance of engineering systems. Accordingly, estimation of the roughness effects is crucial in engineering design and machine maintenance. The first attempt to study the roughness effects on turbulent flows was made by Nikuradse (1933), who measured the pressure drop in pipes with walls covered by sand grains. Based on the effects of the rough surface on the turbulence, he characterized the flow as hydraulically smooth, transitionally rough, and fully rough. When the roughness height is of the same order or smaller than the viscous length, the wall roughness hardly affects turbulence, and the friction drag is unchanged. This regime is referred to as the hydraulically smooth regime. On the other hand, in the fully rough regime, in which the wall roughness height is large enough to effectively destroy the original viscous sublayer, the skin friction coefficient is expressed as a function of the equivalent sand grain roughness. The transitionally rough regime is located between the hydraulically smooth and fully rough regimes.
An increase in the skin friction coefficient due to wall roughness effects results in a decrease in the inner-scaled streamwise mean velocity, which is known as the roughness function. It is well established that in the fully rough regime, a relationship between the roughness function and the inner-scaled equivalent roughness approaches a universal linear asymptote irrespective of the surface geometry. This motivated us to relate the surface topological parameters to the equivalent roughness (Flack and Schultz 2010). However, even if the equivalent roughness is employed, which induces a complete collapse in the fully rough regime, universal flow behavior in the transitionally rough regime has not been observed (Flack et al., 2012). There is much controversy over the characterization of the flow in the transitionally rough regime (Flack et al., 2012). Nikuradse (1933) reported that for a transitionally rough regime, $5 < k^* < 70$, where $k^*$ is the inner-scaled equivalent roughness. The upper limit was reported to be much smaller by Ligrani and Moffat (1986). Flack (2007) and Ligrani and Moffat (1986) reported a range of $15 < k^* < 50$ for a packed sphere, and Flack (2007) reported that $2.5 < k^* < 25$ for a scratched surface. Flack et al. (2012) attempted to identify a roughness length scale that could reasonably scale the roughness function for regimes ranging from the hydraulically smooth to the transitionally rough. Although they found that using the mean peak-to-valley roughness height to predict the onset of the roughness effects was promising, it did not predict the collapse of the roughness function for several types of rough surfaces.

Many other important experimental studies have been performed. However, a full understanding of the roughness effects in the transitionally rough regime has not been achieved due to experimental difficulties in achieving a high fidelity flow within the roughness sublayer. Hence, to obtain detailed turbulence statistics near the wall-roughness, eddy resolving simulations, such as direct numerical simulation (DNS) or large eddy simulation (LES), that faithfully resolve the details of the surface geometry have been employed (Yuan and Piomelli 2014; Busse et al., 2015; Chan et al., 2015; Thakkar et al., 2017; Busse et al., 2017).

Yuan and Piomelli (2014) performed LES of turbulence over sand grains. Their results showed that in the transitionally rough regime, a large distinct roughness peak caused a steep increase in the roughness function with an incase in the inner-scaled equivalent roughness, whereas over a gently undulation surface, the roughness function displayed a more gradual increase. Similar observations based on DNS of turbulent flow in a roughened pipe were made by Chan et al. (2015), who reported that the behavior of the roughness function in the transitionally rough regime was strongly dependent on the slope of the roughness corrugation. The relationship between the surface topology and flow characteristics in a transitionally rough regime were also studied by Thakkar et al. (2017). They performed DNSs of flows in transitionally rough regimes over 17 industrially-relevant irregular surfaces to identify which surface properties were important for characterizing flows. They successfully characterized the roughness function and peak value of the turbulence energy using topological parameters, namely, the solidity, surface skewness, streamwise correlation length, root-mean-square roughness height, and mean peak-to-valley roughness height. While the studies by Thakkar et al. (2017) were limited to the cases in which the Reynolds number was fixed, Busse et al. (2017) examined the Reynolds number dependence of the near-wall flow properties for regimes ranging from the transitionally to the fully rough. They discussed the Reynolds number dependence of the breakdown of the viscous sublayer due to the wall roughness, a process in which the roughness elements gradually destroyed the viscous sublayer; the breakdown was assumed to be complete once the flow regime transitioned to the fully rough regime. They revealed the flows still retained characteristics of the transitionally rough regime even though the flow was assumed to be in the fully rough regime based on established criteria.

There is a large body of literature on transitionally rough turbulence. However, the Reynolds number dependence of turbulence, particularly within the canopy of a rough surface, is not sufficiently understood. Accordingly, in this study, we examine the Reynolds number dependence on the mean velocity, Reynolds stress, and momentum budgets for flow in the transitionally rough regime. The friction Reynolds number and roughness Reynolds number (inner-scaled roughness height) are systematically varied. The goal of this study is to provide information about the scaling of the turbulence statistics that can be used to support future research on turbulence modeling near a rough surface.

Nomenclature

\[ a : \text{acceleration rate} \]
\[ c : \text{particle velocity: } c = \Delta / \Delta t \]
\[ c_s : \text{speed of sound: } c_s / c = 1 / \sqrt{3} \]
\[ f_d : \text{plane-averaged drag force} \]
\[ f : \text{distribution function} \]
\[ f^* : \text{equilibrium distribution function} \]
\( m \): moment  
\( F \): external force term  
\( \hat{g}_i \): inhomogeneous roughness density term  
\( h_m \): mean height of a rough wall  
\( \hat{h}_{\text{rms}} \): standard deviation of roughness elevation  
\( h_t \): mean peak-to-valley height  
\( \hat{h}_i \): roughness Reynolds number: \( \hat{h}_i u_e / \nu \)  
\( k_{\text{SGS}} \): sub-grid-scale turbulence energy  
\( L_x \): streamwise length of the computational domain  
\( L_y \): wall-normal length of the computational domain  
\( L_z \): spanwise length of the computational domain  
\( M \): transformation matrix  
\( n_i \): unit normal vector of the rough surface  
\( p \): pressure  
\( R_{ij} \): plane-averaged Reynolds stress: \( \varphi \langle u'_i u'_j \rangle_f \)  
\( \text{Re}_f \): friction Reynolds number: \( \text{Re}_f = u_e \delta / \nu \)  
\( S \): x-z plane area  
\( S_f \): x-z plane area occupied by the fluid phase  
\( S_k \): skewness of roughness elevation  
\( \hat{S} \): relaxation matrix  
\( t \): time  
\( T_{ij} \): plane-dispersive covariance: \( \varphi \langle \tilde{u}_i \tilde{u}_j \rangle_f \)  
\( u_i \): velocity  
\( u_e \): friction velocity at the rough wall  
\( w_o \): weight parameter  
\( x \): streamwise coordinate  
\( y \): wall-normal coordinate  
\( y_e \): effective wall-normal distance: \( \int_0^y \varphi \text{d}y \)  
\( z \): spanwise coordinate  
\( \delta \): half channel height  
\( \delta_e \): effective half channel height: \( \delta - h_m \)  
\( \delta_{ij} \): Kronecker delta  
\( \delta t \): time step  
\( \Delta \): grid spacing  
\( \Delta U^* \): roughness function  
\( \nu \): kinematic viscosity  
\( \nu_{\text{SGS}} \): sub-grid-scale eddy viscosity  
\( \xi_\alpha \): discrete velocity  
\( \rho \): fluid density  
\( \tau_w \): wall shear stress at the rough wall  
\( \tau_{we} \): wall shear stress at the rough wall due to the viscous force  
\( \tau_{wp} \): wall shear stress at the rough wall due to the pressure force  
\( \varphi \): plane porosity  
\( \psi \): variable  
\( \hat{\varphi} \): Reynolds averaged value of \( \varphi \)  
\( \varphi' \): temporal fluctuation of \( \varphi : \varphi - \hat{\varphi} \)  
\( \langle \varphi \rangle_f \): intrinsic (fluid phase) plane-averaged value of \( \varphi \)  
\( \langle \varphi \rangle \): superficial plane-averaged value of \( \varphi \)  
\( \langle \cdot \rangle_f \): values normalized by the friction velocity at the rough wall
2. Numerical method

Due to the simplicity of its wall treatment, high spatial locality of its calculations, and high accuracy, advantages conferred by its low numerical dissipation and dispersion, the lattice Boltzmann method (LBM) has been used with considerable success in eddy resolving simulations of turbulent flows in complicated geometries, such as flows in porous media (Chukwudzie and Tyagi 2013; Fattahi et al., 2016), flows over porous walls (Kuwata and Suga 2016; Kuwata and Suga 2017), flows over rough walls (Kuwata and Kawaguchi 2016; Kuwata and Kawaguchi 2019), and flows over an urban canopy (Onodera et al., 2013). There are several types of discrete velocity and collision models for three-dimensional simulations. In this study, we used the D3Q27 multiple-relaxation-time lattice Boltzmann method (MRT-LBM), which was developed by our group and has been extensively validated (Suga et al., 2015). This DNS method has been applied especially for turbulent flows over rough walls and porous walls (Kuwata and Suga 2016; Kuwata and Suga 2017; Kuwata and Kawaguchi 2018).

The time evolution of the particle distribution function $f$ of the MRT-LBM can be written as

$$|f(x + \xi_s \delta t, t + \delta t)) - f(x, t)) = - M^{-1} \hat{S} [ |m(x, t)| - |m^e(x, t)|] + M^{-1} \left( I - \frac{\delta}{\Delta} \right) M |F \delta t,$$

where $f$ is the distribution function, $F$ is the external force function, and $\delta t$ is the time step. It should be noted that for the D3Q27 model, $Q = 27$. The matrix $I$ is the identity matrix. The discrete velocity vector components are as follows:

$$[\xi_0, \xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6, \xi_7, \xi_8, \xi_9, \xi_{10}, \xi_{11}, \xi_{12}, \xi_{13}, \xi_{14}, \xi_{15}, \xi_{16}, \xi_{17}, \xi_{18}, \xi_{19}, \xi_{20}, \xi_{21}, \xi_{22}, \xi_{23}, \xi_{24}, \xi_{25}, \xi_{26}]$$

where $c = \Delta / \delta t$ with $\Delta$ is the lattice spacing. The matrix $M$ is a $Q \times Q$ matrix that linearly transforms the distribution functions to the moments as follows:

$$|m| = M|f|$$

The equilibrium moment $m^e$ can be written as $|m^e| = M|f^e|$ with

$$f^e = w_u \left( \rho + \rho_0 \left[ \frac{\xi_s \cdot u}{c_s^2} + \frac{(\xi_s \cdot u)^2 - c_s^2 |u|^2}{2c_s^4} \right] \right),$$

where $u$ is the fluid velocity and $\rho$ is expressed as the sum of the constant and fluctuation values: $\rho = \rho_0 + \delta \rho$. The normalized sound speed is $c_s / c = 1 / \sqrt{3}$, and $w_u$ is the weighted coefficient. The collision matrix $\hat{S}$ is diagonal:

$$\hat{S} = \text{diag}(0, 0, 0, 0, s_4, s_5, s_5, s_6, s_7, s_7, s_7, s_8, s_8, s_9, s_9, s_{10}, s_{10}, s_{10}, s_{11}, s_{11}, s_{11}, s_{12}, s_{12}, s_{12}, s_{12}, s_{13}, s_{13}, s_{13}, s_{14}, s_{14}, s_{14}, s_{15}, s_{15}, s_{15}, s_{16}, s_{16}, s_{16}, s_{17}, s_{17}, s_{17}, s_{18}, s_{18}, s_{18}, s_{19}, s_{19}, s_{19}, s_{20}, s_{20}, s_{20}, s_{21}, s_{21}, s_{21}, s_{22}, s_{22}, s_{22}, s_{23}, s_{23}, s_{23}, s_{24}, s_{24}, s_{24}, s_{25}, s_{25}, s_{25}, s_{26}, s_{26}).$$

The set of relaxation parameters used in this study is as follows:

$$s_4 = 1.54, s_{10} = 1.5, s_{13} = 1.83, s_{16} = 1.4, s_{17} = 1.61, s_{18} = 1.98, s_{23} = s_{26} = 1.74.$$

For a large eddy simulation, the relaxation parameter components $s_4$ and $s_7$ are related to the effective viscosity:

$$v + v_{kgs} = c_s^2 \left( \frac{1}{s_5} - \frac{1}{2} \right) \delta t = c_s^2 \left( \frac{1}{s_7} - \frac{1}{2} \right) \delta t,$$

where $v$ and $v_{kgs}$ are the kinematic viscosity and sub-grid-scale (SGS) eddy viscosity, respectively. The SGS eddy viscosity is obtained via the wall-adapting local eddy-viscosity (WALE) model by Ducros et al., (1998). The term $F$ is the external force term (Guo et al., 2002):

$$F = w_u \rho_0 \left[ \xi_s \cdot a \left( 1 + \frac{\xi_s \cdot u}{c_s^2} \right) - \frac{a \cdot u}{c_s^2} \right].$$

where $a$ is the acceleration rate. As the turbulence pressure is not considered in the lattice Boltzmann method, the influence of the SGS turbulence energy, $k_{SGS}$, is explicitly introduced in the external force term as:

$$a = \frac{\delta}{\delta x} \left( -\frac{2}{3} k_{SGS} \delta t \right),$$

where $k_{SGS}$ is given by the double filtered velocity, as in (Suga et al., 2015). See Suga et al. (2015) for an example of the application of the lattice Boltzmann LES and the details of the parameters for the D3Q27 MRT-LBM.
3. Flow conditions

Figure 1 shows the computational domain of a rough-walled turbulent channel flow where the streamwise, wall-normal, and spanwise directions are designated by $x$, $y$, and $z$, respectively. The domain size is $6\delta(x) \times 2\delta(y) \times 3\delta(z)$ in the streamwise, wall-normal, and spanwise directions, respectively. It has been confirmed that use of this domain size in the streamwise and spanwise directions is enough to capture the turbulence structures over rough walls (Kuwata and Kawaguchi 2019). The standard LBM employs a regular (equal spacing) grid. Hence, the eddy resolving simulations by the LBM require numerous computational cost to resolve the full spectrum of scales, ranging from the near-wall small-scale eddies to the large-scale structures in the outer layer. To circumvent this computational cost, the imbalance-correction grid refinement method (Kuwata and Suga 2016), which is an improvement over the original grid refinement method (Dupuis and Chopard 2003), is used to satisfy the mass and momentum conservation laws. The grid is refined according to the wall-normal coordinate $y$, as shown in Fig.2. Periodic conditions are applied in the streamwise and spanwise directions, and the flow is driven by a streamwise pressure difference. We use a marine paint surface, as shown in Fig.3, as a real irregular surface. A three-dimensional topographical map of the rough surface is obtained using a 3D scanning system (Keyence VR3000).

The measured vertical resolution is 0.1\(\mu m\), and the data is digitized in increments of 11.7\(\mu m\) in the lateral directions. The sampling area is 12\(mm\) \(\times\) 6\(mm\), corresponding to 1024 \(\times\) 512 point data. To enforce periodicity of the rough surface height in the streamwise and spanwise directions, a gradual transition is imposed by applying a sigmoid function at the edge of the scanned surface. Two rough surfaces with different roughness length scales are used for the simulations, as shown in Fig.4: one is denoted “case H,” where a single scanned surface is used; the other is denoted “case L,” where 4 \(\times\) 4 surfaces reduced by a factor of four are used. Hence, the roughness length scale, normalized by $\delta$, is four times smaller for case L than that for case H, which allows us to extensively vary the roughness Reynolds number. The rough surface characteristics, such as the mean peak-to-valley height $h_r$, mean roughness height $h_m$, root-mean-square roughness $h_{rms}$, and skewness $Sk$, are summarized in Table 1. To compute the mean peak-to-valley height, the single rough surface is divided into 5 \(\times\) 5 samples of equal size. Then, the maximum height minus the minimum height for each sample is averaged over the 25 samples, as in Thakkar et al. (2017). Following Thakkar et al. (2017), in the present study, $h_r$ is used as the characteristic length scale for the rough surface. It is noted that in the definition of $h_m$, the lowest position of the deepest valley is defined as the origin of the wall-normal coordinate $y = 0$. As shown in Table 1, the skewness value is found to be positive $Sk = 0.47$. Thus, the surface under consideration has a peak-dominated nature. The mean roughness height is much smaller than the half-channel height $h_{rms}/\delta < 0.05$, suggesting that the flow blockage due to the wall roughness does not affect turbulence in the outer layer (Jiménez 2004). It should be noted that the parameters under consideration (the mean peak-to-valley height, the mean roughness height, the root-mean-square roughness and the skewness) cannot take the effect of roughness pattern into account, and the roughness pattern (or the other surface characteristics) affects turbulence particularly for the surfaces with regularly distributed roughness (Orlandi and Leonard 2006; Foroogphi et al., 2017). However, the goal of this study is not a derivation of the universal correlation functions that scale profiles of the mean velocity, Reynolds stress, and drag force for various types of surface roughness but simply reveal how are those quantities scaled. The effect of the surface topology on the scaling of the turbulence statistics in the transitionally rough regime will be the focus of our future work. The Reynolds number dependence is also considered. The friction Reynolds number, which is based on the effective half-channel height $\delta_e = \delta - h_m$, is varied ($Re_e = 250, 500$ and 1000). Hence, the roughness Reynolds number $h_r^2$, which is based on the friction velocity and $h_r$, ranges from 6 – 91. Here, the friction velocity at the rough surface is evaluated via the momentum balance:

$$\frac{1}{\rho} \frac{\partial p}{\partial x} (\delta - h_m) = \tau_w,$$

where $\tau_w = \rho u^2$ is the wall shear stress at the rough surface. Each simulation case is named to XY-Z, where X can be “H” or “L”, Y denotes the simulated friction Reynolds number, and Z denotes the roughness Reynolds number. The refined level and grid resolutions for each case are determined such that the grid resolutions near the rough surface are smaller than 2.0 wall units. The grid resolution is comparable to those used in the lattice Boltzmann DNS studies (Kuwata and Suga, 2019). Thus, almost all turbulent eddy motion near the rough surface can be directly resolved, and the SGS turbulence model only works effectively away from the rough surface. Parameters describing the flow, such as the friction Reynolds number $Re_e$ and roughness Reynolds number (inner-scaled mean peak-to-valley height) $h_r^2$ and the details of the simulation set up are included in Table 2. The roughness height normalized by $\delta$ for case L is smaller by a factor of four.
than than for case H, implying that a considerably finer grid is required to resolve the small rough surface undulations. Thus, for case L, the level of refinement is increased to four. In the following discussion, the Reynolds averaged value of a variable $\phi$ is denoted as $\overline{\phi}$, and $\phi'$ denotes the fluctuation value: $\phi' = \phi - \overline{\phi}$. In addition to Reynolds averaging, spatial averaging is applied to obtain the flow variable near the rough surface. The averaged value over the flow variable near the rough surface. The averaged value over the fluid region in the $x - z$ plane is denoted by $\langle \phi \rangle_f$, which is referred to as the intrinsic (fluid phase) averaged value. The variable $\phi$ can be decomposed into a contribution from the intrinsic (fluid phase) averaged value $\langle \phi \rangle_f$ and the deviation from the intrinsic averaged value (dispersion) $\tilde{\phi}$.

4. Results and discussion

4.1. Mean velocity

As mentioned above, one of the most important effects of roughness on a turbulent flow is the downward shift of the inner-scaled streamwise mean velocity profile, which is responsible for an increase in the skin friction coefficient at the rough surface. This subsection first focuses on a modification of the streamwise mean velocity due to the presence of wall roughness. Figure 5 shows the inner-scaled streamwise mean velocity $\langle \overline{u} \rangle^+ \phi$ profiles against the effective wall-normal distance $y^*_e$. For comparison, the DNS data for a smooth wall (Lee and Moser 2015) is included. The effective wall-normal distance $y_e$ proposed by Kuwata and Kawaguchi (2019) is defined as follows:

$$y_e = \int_0^y \varphi dy,$$

(9)

where $\varphi$ stands for the $x - z$ plane-porosity (defined as the ratio of the $x - z$ plane area $S$ and the $x - z$ plane area occupied by the fluid phase $S_f$: $\varphi = S_f/S$). The idea of the effective distance was given by integrating the plane and Reynolds averaged momentum equations, and this enable us to account for a virtual origin for the rough surface (Kuwata and Kawaguchi, 2019). Figure 5 confirms that in all cases except L250-6, the $\langle \overline{u} \rangle^+ \phi$ profiles are shifted downward relative to the flow near a smooth wall, which suggests that the skin friction coefficients are higher near the rough wall. For case L250-6, the mean velocity near the rough wall is slightly lower than it is near the smooth wall result at $y_e^* < 10$, while the profile in the log region appears to be unaffected. The $\langle \overline{u} \rangle^+ \phi$ profiles for cases L1000-24 ($h^*_m = 24$) and H250-24 ($h^*_m = 24$) perfectly collapse near the rough wall region of $y_e^* < 50$, suggesting that the mean velocity profile near the rough wall depends on $h^*_m$. The roughness function $\Delta U^+$ is defined as the decrease in mean velocity relative to that for a smooth wall.

![Fig. 1 Computational geometry of a rough-walled channel flow.](image1)  
![Fig. 2 Grid arrangement near a rough wall.](image2)
at the same friction Reynolds number. The roughness function ranges from \( \Delta U^+ = 0 \) for case L250-6 to \( \Delta U^+ = 6.2 \) for case H1000-91. Thus, the simulated flows considered in this study are in the transitionally rough regime based on the criterion developed by Nikuradse (1993), who reported that when \( \Delta U^+ < 7 \), the flow is in the transitionally rough regime. However, it should be noted that there is still much controversy over the range of \( \Delta U^+ \) that defines the transitionally rough regime (Flack et al., 2012).

The roughness function is plotted against the roughness Reynolds number \( h_0^+ \) in Fig.6 together with the empirical correlation between \( \Delta U^+ \) and \( k^*_r \) for sand grain roughness by Nikuradse (1993). It is evident from Fig.6 that \( \Delta U^+ \) can be expressed as a function of the roughness Reynolds number: \( \Delta U^+ \) increases with \( h_0^+ \), but \( \Delta U^+ \) for cases L1000-24 (\( h_0^+ = 24 \)) and H250-24 (\( h_0^+ = 24 \)) are consistent despite the fact that \( \text{Re}_t \) and \( h_0/\delta \) vary. This observation is consistent with the statement by Nikuradse (1993) that \( \Delta U^+ \) is a function of the inner-scaled equivalent roughness \( k^*_r \), namely, the roughness Reynolds number. Figure 6 also shows that the relationship between \( \Delta U^+ \) and \( h_0^+ \) is similar to that between \( \Delta U^+ \) and \( k^*_r \). This observation suggests that the mean peak-to-valley height may be the relevant length scale for determining the equivalent roughness, which partly supports the findings by Flack et al. (2012) that the mean peak-to-valley-height may be an effective parameter for scaling the roughness function. In addition, the increase in \( \Delta U^+ \) with increasing \( h_0^+ \) is similar to that obtained by Nikuradse (1993). This indicates that the behavior of the flow as it transitions to the fully rough regime near the rough surface considered here is similar to that near a real surface with sand grains.

It has been shown that there is a similarity in turbulence beyond a few roughness heights from a wall, which is well known as the similarity hypothesis of Townsend (1976). Accordingly, to discuss the similarity of the profiles of \((\overline{u})^+\), we examine the profiles of the \((\overline{u})^+\) by defect form with reference to \((\overline{u})_m^+\): \((\overline{u})_m^+ = (\overline{u})^+\). Here, \((\overline{u})_m^+\) stands for the centerline velocity \((\overline{u})^+\) at \( y = \delta \). To explore the approximate scaling for the mean velocity defect, the profiles with the distance from the valley \( y/\delta \) and with the effective distance of \( y_c/\delta_c \) are presented in Fig.7 (a) and (b), respectively, and a close up view of the profiles near the rough surface are respectively shown in Fig.7(c) and (d). It is worthwhile to note that when \( y_c \) over the roughness crest coincides with the distance from the position of the mean roughness height, i.e., \( y_c = y - h_m \), the mean velocity scaled with \( y_c \) (above the roughness crest) is identical to that scaled with \( y - h_m \), as in Busse et al., (2015); Chan et al., (2015); Foroogphi et al., (2017); Kuwata and Kawaguchi (2019). In Fig.7 (a) and (b), one can see that away from the rough surface, the mean velocity profiles are similar, which is confirmed with the scaling of \( y/\delta \) and \( y_c/\delta_c \). However, the close up views in Fig.7 (c) and (d) show that the similarity is much clearer when the mean velocity defect is scaled by \( y_c/\delta_c \), which is consistent with the observations in the DNS studies (Chan et al., 2015; Kuwata and Kawaguchi 2019). It is also clear from Fig.7 (c) that when the profiles are plotted against \( y_c/\delta_c \), the dependence of \( \text{Re}_t \) in the region of \( y_c/\delta_c > 0.02 \) is shown: when the \( \text{Re}_t \) values are the same, the profile in \( y_c/\delta_c > 0.02 \) collapses well, despite the differences in \( h_0^+ \) and \( h_0/\delta \). This means that the mean velocity profile away from the rough wall is unaffected by the roughness height but depends only on the friction Reynolds number, supporting the similarity hypothesis of Townsend (1976).

4.2. Reynolds stress

This subsection focuses on the scaling of the Reynolds normal stress profiles. Figure 8 shows profiles of the streamwise and wall-normal Reynolds stresses normalized by the friction velocity. It is noted that as the contribution of the SGS turbulence kinetic energy relative to the resolved one is less than 0.2%, the SGS components are neglected. It is also noted that there is a discontinuity in the grid resolution at the interfaces between the different grid-level blocks, as shown in Fig.2, and that the SGS component rapidly changes at these interfaces. Thus, a small kink in the Reynolds stress profile inevitably occurs in the interface region (Kuwata and Suga 2016). The kink profile is slightly visible in the high Reynolds number case (case H1000-91), i.e., the wall-normal component \( R_{22}^+ \) shows unphysically kink at \( u'_{\tau}^+ \approx 80 \). In general, however, no perceptible kink is found in the Reynolds stress profiles, because the grid resolution used in the present study is fine enough to resolve almost all the eddy motions. We first focus on the profiles away from the rough
wall \((y^*_{e} > 100)\). Here, the Reynolds stress profiles show a dependence on the friction Reynolds number, just as the mean velocity defect profile did. In the region of \(y^*_{e} < 100\), the maximum peak values of \(R_{11}^{+}\) for the low \(h^*_{e}\) cases (cases L250-6 and L500-12) are almost the same while they attenuate significantly as \(h^*_{e}\) increases from 12 to 47. However, the attenuation with the increase in \(h^*_{e}\) is less prominent at \(h^*_{e} < 100\). These results agree with the results obtained by Flack et al., (2007), who reported that the suppression of \(R_{11}^{+}\) increased with increasing \(h^*_{e}\) for flows in the transitionally rough regime, while the near-wall peak of \(R_{11}^{+}\) was absent in the fully rough regime. Jiménez, J. (2004) attributed this observation to the reduction in the low-speed streaks and quasi-streamwise vortices. It should be stressed here that the maximum peak in the \(R_{11}^{+}\) profile cannot be simply expressed as a function of \(h^*_{e}\). In other words, a clear deviation in the Reynolds stress profiles can be seen when the \(h^*_{e}\) values are the same (cases H250-24 and L1000-24), despite the fact that the mean velocity profiles for cases H250-24 and L1000-24 completely collapse, as shown in Fig.5. Thus, the other flow parameters should be considered when determining the scaling of \(R_{11}^{+}\). In contrast, the peak value of \(R_{22}^{+}\) is found to be friction Reynolds number-dependent, as shown in Fig.8(b). The profiles of \(R_{22}^{+}\) perfectly collapse in the lowest Reynolds number cases (L250-6 and H250-24), showing a clear friction Reynolds number dependence. However, even though the friction Reynolds number is the same, for case H1000-91, \(R_{22}^{+}\) near the rough wall region \((y^*_{e} < 50)\) increases more sharply than it does for case L1000-24, indicating that the roughness height \(h_{e}/\delta\) or the roughness Reynolds number affects the near-wall \(R_{22}^{+}\) profile. Although the results are not shown here, we have confirmed that the damping behaviors of the Reynolds
stresses are inconsistent when scaled by the distance from the valley ($y^+$), whereas the profiles are asymptotically damped when scaled by the effective distance, as shown in Fig.8. Hence, scaling with the effective distance is more appropriate when modeling turbulence near rough walls. An understanding of the limiting behaviors of turbulent velocity fluctuations in the immediate vicinity of a wall is a prerequisite for understanding the wall damping effects and progress in turbulence models. The near-wall limiting behaviors of the Reynolds stress components are shown using a log-log scale in Fig.9. It is well established that the streamwise and wall-normal velocity fluctuations in the immediate vicinity of a smooth wall can be expressed by the Taylor series expansion in terms of the distance from a wall $y$ as follows:

\[ u' = a_0 y + b_0 y^2 + c_0 y^3 + \cdots, \]
\[ v' = b_1 y^2 + c_1 y^3 + \cdots, \]  
(10)

This means that the streamwise and wall-normal Reynolds stresses decay with slopes 2 and 4, respectively, with respect to $y$. Fig.9, which gives information about the rough wall turbulence, shows that the Reynolds stresses linearly decay toward the bottom of the rough surface in the log-log graph, suggesting that the Reynolds stresses can be expanded in terms of the effective distance from the wall $y_e$ as follows:

\[ \langle \overline{u' u'} \rangle = d_{u} y_{e} + e_{u} y_{e}^2 + f_{u} y_{e}^3 + \cdots, \]
\[ \langle \overline{v' v'} \rangle = d_{v} y_{e} + e_{v} y_{e}^2 + f_{v} y_{e}^3 + \cdots, \]  
(11)

Fig.9 confirms that the near-wall limiting behaviors depend on the roughness Reynolds number, because the profiles of $R_{11}$ and $R_{22}$ at the same $h^+$ (cases L1000-24 and H250-24) are similar. The slopes of the $R_{11}$ and $R_{22}$ profiles are different: $R_{11}$ and $R_{22}$ decay with slopes 2 and 4, respectively. However, for the rough wall turbulence, both $R_{11}$ and $R_{22}$ decay with a slope of 2 when $h^+$ is small, while the slope is reduced to 1 as $h^+$ increases. The possible reason why the slopes of $R_{11}$ and $R_{22}$ are nearly the same for the rough wall turbulence is as follows. For the smooth wall turbulence, the wall-normal turbulent velocity fluctuations are damped by the wall more rapidly compared with the wall-parallel components, which can be mathematically described from the constraints by the continuity equation as shown in Eq.(10). When the viscous length is much smaller than the roughness length scale, i.e., the length scale of turbulent vortices is so large that the turbulent vortices do not feel undulation of a rough surface, the wall-normal turbulent velocity fluctuations are particularly reduced by the wall due to the constraint from the continuity equation; thus, the limiting behavior is similar to that in the smooth wall turbulence. In contrast, as the roughness Reynolds number representing a ratio of the roughness length scale to the viscous length increases, the undulation of the rough wall affects the turbulent vortices: as in the wall-normal velocity fluctuations near the smooth wall, the surface of the rough wall facing to the streamwise or spanwise directions also reduces the streamwise and spanwise turbulent velocity fluctuations, respectively. In addition, it should be remarked that since the area occupied by the solid obstacles gradually increases below the roughness crest toward the bottom of the rough wall, the reduction in the velocity fluctuations by the wall is more moderate for the rough wall turbulence. Therefore, the presence of the wall-roughness weakens the rapid reduction in the wall-normal turbulent velocity fluctuations coming from the constraint by the continuity equation. Consequently, the wall damping effect of the turbulent velocity fluctuations is weakened by the wall-roughness, and the limiting behavior of the wall-normal component resembles to that of the wall-parallel components as indicated by Eq.(11).
4.3. Momentum transfer

To understand the Reynolds number dependence of the momentum budgets, the $x-z$ plane and Reynolds (double) averaged momentum equation is analyzed. The double averaging operation can be applied to the momentum equation for incompressible flows, yielding the double averaged momentum equation:

$$\frac{\partial (\bar{u}_i)}{\partial t} + \langle \bar{u}_i \rangle \frac{\partial (\bar{u}_j)}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \frac{1}{\rho} \frac{\partial}{\partial x_j} \left( \nu \frac{\partial \langle \bar{u}_i \rangle}{\partial x_j} \right) - \frac{1}{\rho} \frac{\partial}{\partial x_j} \left( \varphi \frac{\partial \langle \bar{u}_i \rangle}{\partial x_j} \right) - \left( \frac{1}{\rho \ell} \int_L n_k \bar{m}_i \bar{n}_k \, dt - \frac{\nu}{\ell} \int_L n_k \frac{\partial \bar{m}_i}{\partial x_k} \, dt \right) \tag{12}$$

where $L$ represents the obstacle perimeter within the $x-z$ plane, $\ell$ represents the circumference of a solid obstacle, and $n_k$ is its unit normal vector pointing outward from the fluid toward the solid phase. In addition to the plane-averaged Reynolds stress term $R_{ij}$, the plane-dispersive covariance $T_{ij}$, which is related to the mean velocity dispersions, is present. The presence of the inhomogeneous roughness density term $\varphi$ is due to the spatial inhomogeneity of the plane-porosity $\varphi$. The plane-averaged drag force $f_i$ represents the viscous and pressure forces acting on the roughness elements. The term $f_i$ consists of two terms: the first term is the line integral of the pressure dispersion, representing the form drag, while the second term is the dispersive mean velocity gradient, representing the viscous drag. By integrating the streamwise double averaged momentum equation (12) over the wall-normal direction from 0 to $y$, the momentum balance, which is non-dimensionalized by the friction velocity, can be written after some manipulation:

$$1 - \frac{y_*}{\delta_e} = \frac{\partial \langle \bar{u}_i \rangle}{\partial y} - R_{12}^i - T_{12}^i + \int_{y_*}^{y_e} \varphi G_{i}^{y+} \, dy^+ + \int_{y_*}^{y_e} \varphi F_{i}^{y+} \, dy^+, \tag{13}$$

where the terms $G_{i}^{y+}$ and $F_{i}^{y+}$ are the inhomogeneous roughness density contribution and drag force contribution terms, respectively. The detailed derivation of Eq.(13) can be found in Kuwata and Kawaguchi 2016; Kuwata and Kawaguchi 2019. The contributions of the second moment terms ($R_{12}$ and $T_{12}$) are first shown in Fig.10, where the plane-dispersive covariance $-T_{12}$ and Reynolds stress $-R_{12}$ are plotted against $y_*$. Figure 10(a) confirms that $-T_{12}$ is negligible when $h_1^* < 47$, while near the rough surface, the term contributes in cases H500-47 and H1000-91. In Fig.10(b), the Reynolds stress $-R_{12}$ is shown to depend on the friction Reynolds number away from the rough wall ($y_*^e > 50$). However, the behavior of $-R_{12}$ is much more complex when it approaches the bottom of the rough wall ($y_*^e < 50$). A comparison of the cases reveals that when the friction Reynolds number is the same, $-R_{12}$ increases as $h_1^*$ decreases near $y_*^e = 30$ but slightly increases near the bottom $y_*^e = 5$ (as $h_1^*$ decreases). However, this trend is unclear in the low Reynolds number cases (L250-6 and H250-24). In the higher Reynolds number cases (Re$_c$ = 500 and 1000), the decrease in $-R_{12}$ near $y_*^e = 20$ can be attributed to the reduction in the streamwise velocity fluctuation, as shown in Fig.8(a), while the increase near the bottom ($y_*^e = 5$) may be due to the increase in the wall-normal velocity fluctuations (Fig.8(b)). The viscous shear stress, inhomogeneous roughness density contribution term, and drag force contribution term are plotted against $y_*^e$ in Fig.11. In Fig.11 (a), the viscous shear stress (i.e., the inner-scaled mean velocity gradient) perfectly collapses away...
Fig. 10 Profiles of the shear stress terms: (a) dispersive covariance $-T_{12}$ and (b) Reynolds shear stress $-R_{12}$.

Fig. 11 Profiles of the stress terms: (a) viscous term $\frac{\partial u_i^+}{\partial y^+}$, (b) inhomogeneous roughness density contribution term $G_i^+$, and (c) drag force contribution term $F_i^+$.

from the rough wall $y_e^+ > 50$, suggesting that the mean velocity gradient away from the rough wall is unaffected by the roughness height. Fig.11 (a) confirms that as in the mean velocity profile, the viscous shear stress profiles are similar when $h^+_t$ is the same. The maximum peak value of the viscous shear stress is close to unity when $h^+_t$ is at its lowest (case L250-6). However, as $h^+_t$ increases, the peak value is reduced and the maximum peak separates from the bottom. In Fig.11 (b) and (c), it is confirmed that the profiles of $G_i^+$ and $F_i^+$ depend on $h^+_t$, because the profiles for cases L1000-24 and H250-24 (when $h^+_t$ is the same) perfectly agree. The contribution terms of $G_i^+$ and $F_i^+$ increase toward the bottom of the rough wall, whereas the other terms, namely, the Reynolds stress, dispersive covariance, and viscous shear stress, decrease.

Finally, we discuss the Reynolds number dependence of the wall shear stress contribution. Since the values of the budget terms in Eq.(13) at the bottom of the wall correspond to the contributions to the wall shear stress, the viscous contribution $\tau_{uv}/\tau_w$ and pressure contribution $\tau_{wp}/\tau_w$ can be defined as follows:

$$\tau_{uv}/\tau_w = \frac{\partial \bar{u}_i}{\partial y^+} \bigg|_{y_e^+ = -1} + \int_{0}^{\delta^+} \varphi \bar{u}_i^+ dy^+ + \int_{0}^{\delta^+} \varphi \bar{v}_i^+ dy^+,$$

$$\tau_{wp}/\tau_w = \int_{0}^{\delta^+} \varphi \bar{p}_i^+ dy^+.$$

(14)

where $\bar{p}$ and $\bar{v}$ represent the pressure and viscous drag forces, respectively.

The contributions of the pressure and viscous forces to the wall shear stress are plotted against $h^+_t$ in Fig.12. In Fig.12, the viscous force contribution $\tau_{uv}/\tau_w$ dominates the wall shear stress at $h^+_t = 3.0$ (case L250-6). As $h^+_t$ increases, $\tau_{uv}/\tau_w$ linearly decreases with $h^+_t$, while the pressure force contribution $\tau_{wp}/\tau_w$ increases. The pressure force contribution overwhelms the viscous force contribution at $h^+_t = 90$, and the intersection point is thus assumed to be near $h^+_t = 80$. The increase/decrease trend of the viscous and pressure forces is consistent with the results of Busse et al., (2017). When the Reynolds number is largest, namely, when $h^+_t = 90$, $\tau_{wp} > \tau_{uv}$ is confirmed. However, even in this case, the viscous contribution remains significant. This observation confirms that the simulated flow considered here remains in the transitionally rough regime at $h^+_t = 90$, because the pressure drag should dominate the wall shear stress in the fully rough regime.
5. Conclusions

A series of large eddy simulations of turbulent flows over a rough painted surface in the transitionally rough regime is carried out. The friction Reynolds number and roughness Reynolds number are systematically varied to discuss a scaling for the turbulence statistics. It is demonstrated that the effective wall-normal distance, which is defined as the wall-normal integral of the plane-porosity, is preferable for scaling profiles of the turbulence statistics over a rough wall. Away from the rough surface, the wall roughness does not affect the mean velocity and Reynolds stress profiles, substantiating the similarity hypothesis of Townsend (1976). By introducing the effective wall-normal distance, the similarity of the mean velocity profile away from the rough wall is improved. It is found that the mean velocity profile can be characterized by the roughness Reynolds number (inner-scaled roughness length scale). However, other characteristic parameters should be considered when characterizing the near-wall Reynolds stress profiles. In the immediate vicinity of the bottom of the rough wall, the wall-blocking effects due to the wall roughness gradually appear, resulting in a more moderate decrease in the Reynolds stresses. In addition, the limiting behaviors toward the bottom of the rough wall are found to be closely related to the roughness Reynolds number. The Reynolds stress profiles near the rough surface asymptotically decay toward the bottom of the rough surface when scaled by the effective distance. Thus, the effective distance is also a suitable scale factor when modeling turbulence near a rough wall. The budget terms in the plane and Reynolds averaged momentum equations are analyzed, and it is found that the drag force term dominates the momentum transfer in the vicinity of the bottom of the rough surface. The pressure drag contribution to the skin friction coefficient increases, while the viscous contribution decreases as the roughness Reynolds number increases.

Acknowledgement

The authors express their gratitude to their colleagues: PhD. K. Suga, Dr. M. Kaneda and Dr. T.Tsukahara for their support. This study was financially supported by research grants No.17K14591 and 16K14162 of the JSPS of Japan. The numerical calculations were carried out on the TSUBAME3.0 supercomputer in the Tokyo Institute of Technology in research project (ID: hp170032).

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