The transverse structure of the pion in momentum space inspired by the AdS/QCD correspondence

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We study the internal structure of the pion using a model inspired by the AdS/QCD correspondence. The holographic approach provides the light-front wave function (LFWF) for the leading Fock state component of the pion. We adopt two different forms for the LFWF derived from the AdS/QCD soft-wall model, with free parameters fitted to the available experimental information on the pion electromagnetic form factor and the leading-twist parton distribution function. The intrinsic scale of the model is taken as an additional fit parameter. Within this framework, we provide predictions for the unpolarized transverse momentum dependent parton distribution (TMD), and discuss its property both at the scale of the model and after TMD evolution to higher scales that are relevant for upcoming experimental measurements.

I. INTRODUCTION

Light-front holographic Quantum ChromoDynamics (QCD) [1, 2] is based on the connection between strongly-coupled QCD in standard Minkowski space-time and a weakly interacting theory of gravity in higher dimensional Anti-de Sitter (AdS) space-time. This connection is usually referred to as AdS/QCD correspondence and is inspired by the analogous AdS/CFT correspondence [3], where CFT stands for Conformal Field Theory. In the so-called “soft-wall” version of AdS/QCD correspondence [4], conformal invariance is broken by introducing a harmonic confining potential (whose strength is determined by a mass parameter κ), corresponding to an infrared distortion of the AdS space.

Light-front holographic QCD methods (see [5] and references therein for a complete review on the topic) have been employed in a number of recent works to obtain new insights into the structure of hadrons [1, 2, 4–7]. One of the remarkable achievements of light-front holographic QCD has been to provide expressions for the light-front wave function (LFWF) of the valence Fock-state component of mesons. This makes it possible to obtain direct information about many hadronic observables, which can be expressed in terms of overlaps of LFWFs.

The expressions of the LFWFs coming from the soft-wall model of the AdS/QCD correspondence were originally derived in two different matching procedures [8, 9]. These two forms for the LFWF have been used as starting point to calculate collinear and transverse-momentum dependent parton distributions (PDFs and TMDs, respectively), generalized parton distributions (GPDs) and other parton densities both for mesons and nucleons (see for instance [10–23]).

The structure of the pion has attracted interest since the pion was predicted and detected experimentally. The most intriguing aspect is the dual nature of the pion [24]. It can be seen as the simplest realization of a QCD bound state of quark and anti-quark as well as the Nambu-Goldstone boson of the dynamically broken Chiral Symmetry in QCD. These complementary pictures emerge when we study different properties of the pion’s interior, such as elastic and transition electromagnetic form factors (see e.g. [25–29]), distribution amplitude (see e.g. [30, 31]), PDFs (see e.g. [32–34]), GPDs (see e.g. [35–41]), TMDs (see e.g. [42–44]), and Fragmentation Functions (see, e.g., [45–48]). The comparison with experiment is crucial to draw definitive conclusions, and the experiments planned at JLab 12 [49], and the new mesonic Drell-Yan measurements at modern facilities [50, 51] can provide valuable information.

In this work, we use the LFWFs from the AdS/QCD correspondence to study the 3D internal structure and dynamics of the pion in momentum space. At leading twist, the pion transverse momentum dependent quark-quark correlator consists of two functions, the unpolarized TMD function \( f_1(x, k^2_T) \) and the Boer-Mulders TMD function \( h_T^e(x, k^2_T) \). We restrict ourselves to discuss the unpolarized TMD, since the Boer-Mulders function would require to

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construct a spin-dependent LFWF, which is not naturally present in the original AdS/QCD approach (see for example the phenomenological pion LFWF of \([\text{[12]}\) and \([\text{[14]}\)).

A crucial ingredient of the calculation is to identify the energy scale where the model is valid. Light-front holographic QCD describes the nonperturbative regime of QCD and therefore is expected to be valid at low energies, approximately of the order of hadron masses. As soon as the energy increases, a gradual transition to the regime of perturbative QCD takes place (see \([\text{[1]}\) \([\text{[52]}\) \([\text{[55]}\) for more details on the transitions from one description to the other). In our work, we assume that the transition between the nonperturbative regime (where the model is applicable) and the perturbative regime (where pQCD is applicable) occurs at a precise scale, which we define as the model scale. We fix this scale by fitting the pion PDF to available phenomenological parametrizations, after applying DGLAP evolution equations \([\text{[56]}\) \([\text{[57]}\). Once the initial scale of the model is fixed, we also discuss the application of pQCD-based TMD evolution equations \([\text{[58]}\) to the unpolarized pion TMD.

The outline of this work is as follows: in Section \([\text{II]}\) we introduce the explicit expressions for two forms of the LFWFs in the AdS/QCD soft-wall model \([\text{[8]}\) \([\text{[9]}\). After introducing the quark mass in a Lorentz invariant way and deriving analytical expressions for the relevant hadronic matrix elements, we fix the free parameters of the LFWFs and the scale of the model by fitting simultaneously the data of the form factor \([\text{[59–62]}\) and phenomenological parametrizations of the pion PDF \([\text{[63]}\). With these sets of parameters, we provide predictions for the unpolarized TMD at the model scale in Section \([\text{III]}\). In Section \([\text{III A]}\) we discuss TMD evolution \([\text{[58]}\). We estimate the effects of the evolution for this scale by fitting the pion PDF to available phenomenological parametrizations, after applying DGLAP evolution equations \([\text{[56, 57]}\). Once the initial scale of the model is fixed, we also discuss the application of pQCD-based TMD evolution equations \([\text{[58]}\) to the unpolarized pion TMD.

II. THE PION LFWF

Thanks to the AdS/QCD correspondence it is possible to relate the gravitational theory defined in the five-dimensional AdS space to the Hamiltonian formulation of QCD on the light-front. This allows one to obtain a suitable first approximation of the valence wave function for mesons. In particular, the direct comparison of the expression for the form factors derived in both formalism offers the possibility of identifying the spinless string modes in five-dimensional AdS space with the meson LFWFs.

In Ref. \([\text{[8]}\), inspired by \([\text{[64, 65]}\), the correspondence is performed by using the expression for the transition matrix element of the free electromagnetic current propagating in the AdS space, evaluated between five-dimensional AdS space with the meson LFWFs.

Using the LFWF overlap representation of the PDF and form factor, we obtain

\[
f_1^V (x; Q_0) = \int_{-\infty}^{+\infty} \frac{d^2 k_\perp}{16\pi^3} |\psi_{q_\perp/\pi}^V (x, k_\perp)|^2 = A^2 e^{\left(\frac{-m^2}{x^2} - \frac{-m^2}{x^2(1-x)}\right)},
\]  

(5)
| LFWF         | \( m \) (GeV) | \( \kappa \) (GeV) | \( Q_0 \) (GeV) | \( \chi^2_{\text{FF}} \) | \( \chi^2_{\text{PDF}} \) | \( \chi^2_{\text{d.o.f.}} \) |
|--------------|----------------|-------------------|-----------------|----------------|----------------|----------------|
| \( \psi^V_{q\pi/\pi} \) | 0.005 (fixed) | 0.397 ± 0.003 | 0.500 ± 0.003 | 228.7 | 139.6 | 3.15 |
|              | 0.200 (fixed) | 0.351 ± 0.003 | 0.491 ± 0.003 | 1064.8 | 311.6 | 11.76 |
|              | 0.0500 ± 0.0004 | 0.371 ± 0.002 | 0.498 ± 0.002 | 213.0 | 47.5 | 2.25 |
| \( \psi^E_{q\pi/\pi} \) | 0.005 (fixed) | 0.261 ± 0.002 | 0.498 ± 0.003 | 469.6 | 139.4 | 5.44 |
|              | 0.200 (fixed) | 0.322 ± 0.002 | 0.630 ± 0.008 | 1349.1 | 167.8 | 12.96 |
|              | 0. (fixed) | 0.262 ± 0.002 | 0.498 ± 0.003 | 487.4 | 142.4 | 5.38 |

TABLE I: Results from the fit of the pure- and effective-valence LFWFs in different quark mass scenarios. The values obtained for the \( \chi^2 \) are displayed separately for the form factor (FF) and PDF case (fifth and sixth column respectively). In the last column \( \chi^2_{\text{d.o.f.}} \) indicates the sum of the FF and PDF total values divided by the total degrees of freedom (total number of points \( N \) minus the number of free parameters \( N_{\text{par}} \)).

\[
F^V_{\pi}(Q^2) = \int_{-\infty}^{+\infty} \frac{d^2k}{16\pi^3} \psi^V_{q\pi/\pi}(x, k_\perp + (1-x)q_\perp) \psi^V_{q\pi/\pi}(x, k_\perp) = \int_0^1 dx A^2 e^{-\frac{m^2}{x^2} - \frac{m^2}{(1-x)^2} + \frac{Q^2(1-x)}{4\kappa x}}. \quad (6)
\]

where \( |q_\perp|^2 = Q^2 \). The condition [4] implies that \( \int_0^1 dx f^V_{\pi}(x; Q_0) = F^V_{\pi}(Q^2 = 0) = 1 \). Throughout this work \( f^V_{\pi}(x) = f^V_{\pi}(x) \) is always consistently understood and we discuss results for the \( \pi^\pm \) hadron, as the distributions for the \( \pi^0 \) and \( \pi^- \) can be related by isospin and charge conjugation symmetry.

An alternative expression for the LFWF has been derived in [9], considering the mapping of the matrix element of a confined electromagnetic current propagating in a warped AdS space to the LFWF overlap representation of the pion form factor. In this case, one obtains a LFWF which incorporates the effects due to non-valence higher-Fock states generated by the "dressed" confined current, and therefore represents an "effective" two-parton state of the pion. It reads

\[
\psi^E_{q\pi/\pi}(x, k_\perp) \sim \sqrt{\frac{\log(\frac{1}{x})}{\kappa(1-x)}} e^{-\frac{\log(1/x)k_\perp^2}{2x^2}}, \quad (7)
\]

where the superscript \( E \) indicates that we are considering an "effective-valence" component of the LFWF. At variance with the pure-valence LFWF, the effective-valence LFWF is not symmetric in the longitudinal variables \( x \) and \( 1-x \) of the active and spectator quark, respectively. Introducing the quark mass dependence as outlined above, the effective-valence LFWF becomes

\[
\psi^E_{q\pi/\pi}(x, k_\perp) = 4\pi A \sqrt{\frac{\log(\frac{1}{x})}{\kappa(1-x)}} e^{-\frac{\log(1/x)k_\perp^2 + m^2}{2x^2}} \cdot \quad (8)
\]

where the parameter \( A \) is once more fixed by demanding the validity of [4]. The corresponding expressions for the PDF and the form factor are given by

\[
f^E_{1}(x; Q_0) = A^2 e^{-\frac{\log(1/x)k_\perp^2}{2x^2}}; \quad F^E_{\pi}(Q^2) = \int_0^1 dx A^2 e^{-\frac{\log(1/x)k_\perp^2 + 4m^2}{2x^2} + \frac{Q^2}{4\kappa x}}. \quad (9)
\]

We fix the parameters of the LWFs [3] and [9] by fitting the available experimental data for the pion electromagnetic form factor \[59\ 62\] and the parametrization of the pion PDF in [63]. For the fit of the PDF, we apply the DGLAP evolution equations at next-to-leading-order (NLO) to evolve the PDF from the (low) scale of the model \( Q_0 \) to the scale \( Q = 5 \text{ GeV} \) of the parametrization, using the HOPPET code \[67\]. We leave in the initial scale \( Q_0 \) as an additional free parameter to be fitted with the data. Starting from the functional form of the parametrization [63], we select 61 equally-spaced points from \( x = 0.2 \) to \( x = 0.8 \) and for each of them we construct error bars by propagation of the errors on the individual parameters. Summing the PDF points and the 58 form factor points, we perform the fit using in total 119 points. In the case of the pure-valence LFWF we consider two different fitting strategies: either we fix the quark mass to a constant value ("current quark" mass \( m = 0.005 \text{ GeV} \) and "constituent quark" mass \( m = 0.2 \text{ GeV} \)) or, alternatively, we let the quark mass entering as an additional fit parameter. For the effective-valence LFWF, we fix the quark mass to the same values as before, but we include also the limit of massless quarks (leaving the quark
mass as a free parameter in this case leads anyway to a vanishing mass). The results of the fit are summarized in Tab. III In the following we discuss the results for two sets of parameters in Tab. III corresponding with the lowest value of the total \( \chi^2_{\text{d.o.f.}} \) for non-vanishing quark mass.

In Fig. 1 we show the results for the form factor of the pure-valence (solid curve) and effective-valence (dashed curve) LFWF. The corresponding results for the PDF are shown in Fig. 2(a) and 2(b), respectively. The solid curves show the results at the hadronic scale, and the dashed curves are obtained after NLO evolution to \( Q = 5 \text{ GeV} \). The shaded band corresponds to the results from the parametrization at \( Q = 5 \text{ GeV} \) of Ref. [63].

The results from the pure-valence LFWF are in good agreement with the available experimental and phenomenological information, while a worst comparison, especially for the form factor, is obtained in the case of the effective-valence LFWF.

The mass parameter \( \kappa \) plays a very important role, as it is originally the only free parameter of the theory and it is related to the strength of the confining harmonic potential in the soft-wall model [2, 68]. The value \( \kappa \approx 0.37 \text{ GeV} \) obtained in the pure-valence LFWF case is similar to what was obtained in Ref. [8], whereas in the study of the effective-valence LFWF we obtain smaller values, \( \kappa \approx 0.26 \text{ GeV} \), compared to previous analyses [12, 16]. Moreover, we point out that a larger value, namely \( \kappa = 0.54 \text{ GeV} \), is needed in order to describe the hadronic mass spectra and the Regge trajectories [69–71] and this value has been quite extensively used (see [5, 14] for a more complete
overview). Recent works \cite{72, 73} quote a value of \( \kappa = 0.523 \) GeV to reproduce Regge slopes for mesons and baryons and to realize the transition from the non-perturbative (described by light-front holography) and the perturbative regimes, which occurs at an energy scale of about 1 GeV.

Our result for the initial scale is \( Q_0 \sim 0.5 \) GeV in the pure-valence case and is consistent with the values obtained in different phenomenological quark models \cite{11, 32}, where the scale is fixed by requiring that the model results for the momentum carried by the valence quarks match the experimental value, after DGLAP evolution. We also notice that the fit of the quark mass provides a value that is quite close to the average effective light-quark mass obtained in LF holographic QCD from the meson spectrum \cite{5}. In the case of the effective-valence LFWF, we expect that the inclusion of the effects of higher-order Fock state components should correspond to a higher hadronic scale. This is the case when comparing the results between the effective-valence and pure-valence LFWF with \( m = 200 \) MeV and similar values of \( \kappa \). However, for the other quark-mass scenarios we find similar values of \( Q_0 \) in the two models, which are compensated by much lower values for the parameter \( \kappa \) in the case of the effective-valence LFWF. Both the values of \( \kappa \) and the initial scale \( Q_0 \) differ with respect to \cite{72, 73}.

III. TMD ANALYSIS

The unpolarized TMD \( f_1(x, k_T^2) \) can be obtained from the following LFWF overlap \cite{42}

\[
f_1(x, k_T^2; Q_0) = \frac{1}{16\pi^3} |\psi_{q\pi}(x, k_\perp)|^2,
\]

which reduces to the PDF in Eq. (5) after integration over \( k_\perp \). Using the expressions in Eqs. (3) and (8), one finds that the TMD in both models is a Gaussian distribution in \( k_\perp \), with an \( x \)-dependent mean square transverse momenta, i.e.

\[
f_1^V(x, k_T^2; Q_0) = \frac{A^2}{\pi \kappa^2 x (1-x)} e^{-\frac{k_T^2 + m^2}{2 \pi x^2 (1-x)}}, \quad \langle k_T^2(x) \rangle^V = \kappa^2 x (1-x),
\]

\[
f_1^E(x, k_T^2; Q_0) = \frac{A^2 \log(x)}{\pi \kappa^2 (1-x)^2} e^{-\log(x) \frac{k_T^2 + m^2}{2 \pi x^2 (1-x)^2}}, \quad \langle k_T^2(x) \rangle^E = \frac{\kappa^2 (1-x)^2}{\log(1/x)},
\]

where \( k_\perp = |k_\perp| \). In Fig. 3 we show the results for the TMD in the two models, as function of \( x \) and \( k_T^2 \). As in the case of the PDF, the pure-valence model is symmetric under the exchange of \( x \to 1-x \), while this symmetry is lost when including effects beyond the valence sector in the effective-valence LFWF. The fall-off in \( k_T^2 \) is Gaussian in both models.

The width of the distribution \( \langle k_T^2(x) \rangle \) is shown as function of \( x \) in Fig. 3. It is slightly larger in the pure-valence model, with a maximum at \( x = 0.5 \) and the characteristic symmetric behaviour around the maximum. Integrating over \( x \), one obtains \( \langle k_T^2 \rangle^V = 0.023 \) GeV\(^2\). In the case of the effective-valence LFWF the maximum is shifted at lower values of \( x \), i.e. \( x = 0.28 \), and the result after \( x \)-integration is \( \langle k_T^2 \rangle^E = 0.020 \) GeV\(^2\).

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig3.png}
\caption{Results for the quark TMD of the pion as function of \( x \) and \( k_T^2 \) from the pure-valence LFWF (left) and the effective-valence LFWF (right) with the two sets of parameters in Tab. II corresponding with the lowest values of \( \chi^2_{d.o.f.} \) for non-vanishing quark mass.}
\end{figure}
A. TMD evolution

As explained before, the AdS/QCD LFWF and the resulting TMDs are obtained at a scale of about 0.5 GeV. In order to be able to compare with data or extractions, TMDs need to be evolved according to TMD evolution equations (see, e.g., Ref. [74]). These equations describe the broadening of the initial TMD due to gluon radiation.

Even though TMD evolution equations are based on perturbative QCD calculations, their implementation requires the introduction of some prescriptions to avoid extending the calculations outside their region of validity. In general, such prescriptions have the effect of inhibiting perturbative gluon radiation at low transverse momentum and at low $Q$, but must be complemented with an additional component of gluon radiation, usually referred to as nonperturbative component of TMD evolution [75]. This component cannot be predicted by perturbative QCD, but has to be extracted from experimental measurements, taking advantage of the fact that it is highly universal (i.e., it is independent of the quark’s flavor and spin, the parent hadron, the type of process, and whether one considers TMD distribution and fragmentation functions). It may be possible to use AdS/QCD correspondence also to compute the nonperturbative components of TMD evolution, but we leave this issue to future studies (for a recent example of a computation of the behavior of the nonperturbative component of TMD evolution see Ref. [76]).

Several prescriptions have been proposed in the literature (see, e.g., Refs. [75, 77–80]). In principle, if complemented with the appropriate nonperturbative components, they should lead to compatible results for the evolved TMDs. However, there is still considerable uncertainty on the nonperturbative components and systematic studies of these uncertainty are still lacking. We therefore choose a specific implementation of TMD evolution equations, which has been successfully applied to the description of data in the range $1.2 \text{ GeV} \lesssim Q \lesssim 80 \text{ GeV}$. Details of this implementation are discussed in Ref. [81]. We summarize here the most important points.

TMD evolution is implemented in the space Fourier-conjugate to $k^2$. Therefore, we first define the Fourier-transformed TMDs

$$\tilde{f}_1(x, b^2; \mu) = \int_0^\infty dk k J_0(b k) f_1(x, k^2; \mu)$$

and we use the following form for the evolved TMDs in $b$ space (see Refs. [55, 56])

$$\tilde{f}_1^a(x, b^2; \mu) = \sum_{i=q, \bar{q}, g} (\hat{C}_{ai} \otimes f_1^i)(x; \mu) e^{S(b; \mu, a)} e^{gK(b)} \ln \frac{\mu}{Q_0} \tilde{f}_1^a(x, b^2; Q_0),$$

where the label $a$ indicates the parton type. We consider the above equation at Next-to-Leading Logarithmic (NLL) approximation and at leading order in $\alpha_S$. In this case, the convolution at the beginning of the evolved formula reduces simply to

$$\sum_{i=q, \bar{q}, g} (C_{ai} \otimes f_1^i)(x; \mu_0^2) \approx f_1^a(x; \mu_0^2),$$
and the expression for the Sudakov form factor \( \tilde{S}(b_\perp; \mu_b, \mu) \) can be found, e.g., in Ref. [83-84]. We further use
\[
\mu_b = \frac{2 e^{-\gamma_E}}{b_\perp}, \quad \quad g_K = -g_2 b_\perp^2/2, \quad \quad Q_0 = 0.5 \text{ GeV}.
\]
We introduced the following variable
\[
b_\perp \equiv b_{\text{max}} \left( \frac{1 - e^{-b_\perp^2/b_{\text{max}}^2}}{1 - e^{-b_\perp^2/b_{\text{min}}^2}} \right)^{1/4},
\]
with
\[
b_{\text{max}} = 2 e^{-\gamma_E}/Q_0 = 2.246 \text{ GeV}^{-1}, \quad \quad b_{\text{min}} = 2 e^{-\gamma_E}/Q.
\]
The above choice guarantee that at the initial scale \( Q = Q_0 \) any effect of TMD evolution is absent. The model results are thus preserved and in particular the relation between TMD and collinear PDF is maintained.

The value of the \( g_2 \) parameter should be extracted from experimental data, keeping all other choices fixed. In a recent analysis, the parameter was found to be \( 0.13 \pm 0.01 \text{ GeV}^2 \) in combination with a \( b_{\text{max}} \) that was half of the value we assume here. Since \( b_{\text{max}} \) and \( g_2 \) are in general anticorrelated, we choose for the present analysis the following three values
\[
g_2 = 0.09, \quad 0.11, \quad 0.13 \text{ GeV}^2.
\]

Figures [5(a) and [6(a)] show the effect of TMD evolution when going from the model scale to 1 GeV (at an illustrative value of \( x = 0.5 \)). The value of \( k_\perp \) corresponding to the position of the peak of the distributions \( k_\perp f_1(x, k_\perp^2) \) can be used as a measure of the width of the TMDs. The peak moves from about 0.1 to 0.3 GeV, showing that there is a broadening of the width of the distributions. Even if this not evident from the plot, the distributions are no longer Gaussian.

Figures [5(b) and [6(b)] show the position of the peak for \( x \) between 0.1 and 0.8 and for three values of \( Q \). At the scale of the model, this is an analytic function which reads:
\[
k_{\perp,\text{MAX}}(x) = \sqrt{\frac{\langle k_\perp^2(x) \rangle}{2}}.
\]
After evolution to 1 GeV, as already observed, the width of the TMD increases to about 0.3 GeV in both versions of the model. The \( x \) dependence of the TMD width is rather flat. The symmetry about \( x = 0.5 \) of the pure-valence model is lost. The two models become quite similar to each other: the position of the peak is the same within a 5% error. At 5 GeV, the width of the TMD increases to about 0.7 GeV at \( x = 0.5 \) and increases at low \( x \), and is again very similar in the two versions of the model.

In summary, TMD evolution from the model scale (0.5 GeV) to a typical experimental scale of 5 GeV increases the width of the TMD of almost one order of magnitude and leads to an \( x \) dependence of the width that is different from the original one, with no strong difference between the two versions of the model.

### IV. CONCLUSIONS

We have performed a study of the pion using light-front holographic QCD, which allows us to construct pion LFWFs. We took into consideration two different versions of the pion LFWFs: pure-valence and effective-valence. For each version, the model contains three free parameters: the mass parameter \( \kappa \) (expressing the strength of the confining harmonic potential that breaks conformal invariance), the quark mass, and the scale of the model. We fix the parameters by comparison to experimental information on pion form factors and PDFs.

We obtain a value of \( \kappa \) in agreement with previous estimate [8], for the pure-valence version of the model. For the effective-valence version, we obtain a smaller value [12, 13]. The best agreement with data in the case of massive quarks is obtained for a quark mass \( m = 0.05 \text{ GeV} \) for the pure-valence version, and \( m = 0.005 \text{ GeV} \) for the effective-valence version. In order to achieve a fair agreement with the pion PDF at 5 GeV, the model scale has to be set to about 0.5 GeV. This turns out to be true both for the pure-valence and the effective-valence LFWF.

The sets of parameters obtained have then been used to study the unpolarized TMD of the pion. At the model scale, the resulting TMD has a Gaussian shape with a width (defined as the position of the peak of the distributions \( k_\perp f_1(x, k_\perp^2) \)) of about 0.1 GeV at \( x = 0.5 \). The \( x \) dependence of this width is different in the two versions of the model: in the pure-valence model the TMD attains its maximal width at \( x = 0.5 \); in the effective model, this happens at \( x = 0.28 \). After the TMD is evolved to a typical experimental scale of about 5 GeV, its width increases by almost one order of magnitude. The \( x \) dependence is different from the one at the model scale: the width grows monotonically as \( x \) decreases, and the differences between the two versions of the model fade away.
FIG. 5: Left panel: results for the quark TMD of the pion, multiplied by $k_\perp$, from the pure-valence LFWF for the $m = 50$ MeV scenario, as function of $k_\perp$ and at fixed $x = 0.5$. The solid curve shows the result at the scale of the model, $Q_0 = 0.5$ GeV, corresponding with the initial scale for the TMD evolution. The shaded band gives the spread of the results after evolution of the TMD to 1 GeV with three different values of $g_2$: 0.09 GeV$^2$ (dashed curve), 0.11 GeV$^2$ (dotted curve) and 0.13 GeV$^2$ (dashed-dotted curve). Right panel: results for $k_{\perp,\text{MAX}}$ as function of $x$, at the scale of the model (solid curve) and after TMD evolution to $Q = 1$ GeV (lower band) and $Q = 5$ GeV (upper band) with three different values of $g_2$: 0.09 GeV$^2$ (dashed curve), 0.11 GeV$^2$ (dotted curve) and 0.13 GeV$^2$ (dashed-dotted curve).

FIG. 6: Left panel: results for the quark TMD of the pion, multiplied by $k_\perp$, from the effective-valence LFWF for the $m = 50$ MeV scenario as function of $k_\perp$ and at fixed $x = 0.5$. The solid curve shows the result at the scale of the model, $Q_0 = 0.5$ GeV, corresponding with the initial scale for the TMD evolution. The shaded band gives the spread of the results after evolution of the TMD to 1 GeV with three different values of $g_2$: 0.09 GeV$^2$ (dashed curve), 0.11 GeV$^2$ (dotted curve) and 0.13 GeV$^2$ (dashed-dotted curve). Right panel: results for $k_{\perp,\text{MAX}}$ as function of $x$, at the scale of the model (solid curve) and after TMD evolution to $Q = 1$ GeV (lower band) and $Q = 5$ GeV (upper band) with three different values of $g_2$: 0.09 GeV$^2$ (dashed curve), 0.11 GeV$^2$ (dotted curve) and 0.13 GeV$^2$ (dashed-dotted curve).

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[1] S. J. Brodsky and G. F. de Teramond, Phys. Rev. Lett. 96 (2006) 201601.
[2] G. F. de Teramond and S. J. Brodsky, Phys. Rev. Lett. 102 (2009) 081601.
[3] J. M. Maldacena, Int. J. Theor. Phys. 38 (1999) 1113 [Adv. Theor. Math. Phys. 2 (1998) 231].
[4] A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D 74 (2006) 015005.
[5] S. J. Brodsky, G. F. de Teramond, H. G. Dosch and J. Erlich, Phys. Rept. 584 (2015) 1.
[6] J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. 95 (2005) 261602.
[7] L. Da Rold and A. Pomarol, Nucl. Phys. B 721 (2005) 79.
[8] S. J. Brodsky and G. F. de Teramond, Phys. Rev. D 77 (2008) 056007.
[9] S. J. Brodsky, F. G. Cao and G. F. de Teramond, Phys. Rev. D 84 (2011) 075012.
[10] J. R. Forshaw and R. Sandapen, Phys. Rev. Lett. 109 (2012) 081601.
[11] A. Vega, I. Schmidt, T. Branz, T. Gutsche and V. E. Lyubovitskij, Phys. Rev. D 80 (2009) 055014.
[12] T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, J. Phys. G 42 (2015) no.9, 095005.
[13] R. Swarnkar and D. Chakrabarti, Phys. Rev. D 93 (2016) no.7, 074023.
[14] M. Ahmady, F. Chihi, and T. Sandapen, [arXiv:1609.07024 [hep-ph]].
[15] D. Chakrabarti and C. Mondal, Phys. Rev. D 89 (2014) no.5, 054033 Erratum: [Phys. Rev. D 92 (2015) no.1, 019902].
[16] C. Mondal and D. Chakrabarti, Eur. Phys. J. C 75 (2015) no.6, 261.
[17] D. Chakrabarti and C. Mondal, Phys. Rev. D 92 (2015) no.7, 074012.
[18] T. Liu and B. Q. Ma, Phys. Rev. D 92 (2015) no.9, 096003.
[19] M. Aghasyan, H. Avakian, E. De Sanctis, L. Gamberg, M. Mirazita, B. Musch, A. Prokudin and P. Rossi, JHEP 1503 (2015) 039.
[20] T. Maji, C. Mondal, D. Chakrabarti and O. V. Teryaev, JHEP 1601 (2016) 165.
[21] T. Maji and D. Chakrabarti, Phys. Rev. D 94 (2016) no.9, 094020.
[22] D. Chakrabarti, T. Maji, C. Mondal and A. Mukherjee, Eur. Phys. J. C 76 (2016) no.7, 409.
[23] T. Horn and C. D. Roberts, J. Phys. G 43 (2016) no.7, 073001.
[24] J. P. B. C. de Melo, T. Frederico, E. Pace, and G. Salme, Phys. Rev. D 83 (2011) 074013.
[25] J. P. B. C. de Melo, T. Frederico, E. Pace and G. Salme, Phys. Lett. B 751 (2016) 74.
[26] A. Mukherjee, I. V. Musatov, H. C. Pauli and A. V. Radyushkin, Phys. Rev. D 67 (2003) 073014.
[27] E. Ruiz Arriola and W. Broniowski, Phys. Rev. D 81 (2010) 094021.
[28] A. E. Dorokhov and E. A. Kuraev, Phys. Rev. D 88 (2013) no.1, 014038.
[29] A. V. Radyushkin, Phys. Rev. D 80 (2009) 094009.
[30] D. G. Dumm, S. Noguera, N. N. Scoccola and S. Scopetta, Phys. Rev. D 89 (2014) no.5, 054031.
[31] N. Chouika, C. Mezrag, H. Moutarde and J. Rodriguez-Quintero, [arXiv:1612.01176 [hep-ph]].
[32] L. Chang, C. Mezrag, H. Moutarde, C. D. Roberts, J. Rodriguez-Quintero and P. C. Tandy, Phys. Lett. B 737 (2014) 23.
[33] C. Chen, L. Chang, C. D. Roberts, S. Wan and H. S. Zong, Phys. Rev. D 93 (2016) no.7, 074021.
[34] A. Mukherjee, I. V. Musatov, H. C. Pauli and A. V. Radyushkin, Phys. Rev. D 77 (2008) 073001.
[35] C. Ronchi, G. A. Miller, Phys. Rev. D 87 (2008) 073040.
[36] C. R. Ji, Y. Mishchenko and A. Radyushkin, Phys. Rev. D 73 (2006) 114013.
[37] T. Frederico, E. Pace, B. Pasquini and G. Salme, Phys. Rev. D 80 (2009) 054021.
[38] A. E. Dorokhov, W. Broniowski and E. Ruiz Arriola, Phys. Rev. D 84 (2011) 074015.
[39] C. Mezrag, L. Chang, H. Moutarde, C. D. Roberts, J. Rodriguez-Quintero, F. Sabati and S. M. Schmidt, Phys. Lett. B 741 (2015) 190.
[40] C. Fanelli, E. Pace, G. Romanelli, G. Salme and M. Salmistraro, Eur. Phys. J. C 76 (2016) no.5, 253.
[41] B. Pasquini and P. Schweitzer, Phys. Rev. D 90 (2014) no.1, 014050.
[42] C. Lorcée, B. Pasquini and P. Schweitzer, Eur. Phys. J. C 76 (2016) no.7, 415.
[43] S. Noguera and S. Scopetta, JHEP 1511 (2015) 102.
[44] A. Bacchetta, R. Kundu, A. Metz and P. J. Mulders, Phys. Rev. D 65 (2002) 094021.
[45] A. Bacchetta, L. P. Gamberg, G. R. Goldstein and A. Mukherjee, Phys. Lett. B 659 (2008) 234.
[46] H. H. Matevosyan, W. Bentz, I. C. Cloet and A. W. Thomas, Phys. Rev. D 85 (2012) 014021.
[47] S. i. Nam and C. W. Kao, Phys. Rev. D 85 (2012) 034023.
[48] J. Dudek et al., Eur. Phys. J. A 48 (2012) 187.
[49] R. J. Holt and P. E. Reimer, AIP Conf. Proc. 588 (2001) 234.
[50] F. Gautheron et al. [COMPASS Collaboration], SPSC-P-340, CERN-SPSC-2010-014.
[51] A. Deur, V. Burkert, J. P. Chen and W. Korsch, Phys. Lett. B 650 (2007) 244.
[52] S. J. Brodsky, G. F. de Teramond and A. Deur, Phys. Rev. D 81 (2010) 096010.
[53] A. Deur, S. J. Brodsky and G. F. de Teramond, Phys. Lett. B 750 (2015) 528.
[54] A. Deur, S. J. Brodsky and G. F. de Teramond, Prog. Part. Nucl. Phys. 90 (2016) 1.
[56] G. Altarelli and G. Parisi, Nucl. Phys. B 126 (1977) 298.
[57] Y. L. Dokshitzer, Sov. Phys. JETP 46 (1977) 641 [Zh. Eksp. Teor. Fiz. 73 (1977) 1216].
[58] J. Collins, (Cambridge monographs on particle physics, nuclear physics and cosmology. 32)
[59] S. R. Amendolia et al. [NA7 Collaboration], Nucl. Phys. B 277 (1986) 168.
[60] P. Brauel et al., Z. Phys. C 3 (1979) 101.
[61] J. Volmer et al. [Jefferson Lab F(pi) Collaboration], Phys. Rev. Lett. 86 (2001) 1713.
[62] C. J. Bebek et al., Phys. Rev. D 17 (1978) 1693.
[63] K. Wijesooriya, P. E. Reimer and R. J. Holt, Phys. Rev. C 72 (2005) 065203.
[64] J. Polchinski and M. J. Strassler, Phys. Rev. Lett. 88 (2002) 031601.
[65] J. Polchinski and M. J. Strassler, JHEP 0305 (2003) 012.
[66] S. J. Brodsky and G. F. de Teramond, Subnucl. Ser. 45 (2009) 139.
[67] G. P. Salam and J. Rojo, Comput. Phys. Commun. 180 (2009) 120.
[68] A. P. Trawiński, S. D. Glazek, S. J. Brodsky, G. F. de Téramond and H. G. Dosch, Phys. Rev. D 90 (2014) no.7, 074017.
[69] P. Colangelo, F. De Fazio, F. Giannuzzi, F. Jugeau and S. Nicotri, Phys. Rev. D 78 (2008) 055009.
[70] H. Forkel, M. Beyer and T. Frederico, JHEP 0707 (2007) 077.
[71] T. Branz, T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D 82 (2010) 074022.
[72] S. J. Brodsky, G. F. de Tramond, H. G. Dosch and C. Lorcè, Int. J. Mod. Phys. A 31 (2016) no.19, 1630029.
[73] A. Deur, S. J. Brodsky and G. F. de Teramond, arXiv:1608.04933 [hep-ph].
[74] T. C. Rogers, Eur. Phys. J. A 52 (2016) no.6, 153.
[75] J. Collins and T. Rogers, Phys. Rev. D 91 (2015) no.7, 074020.
[76] I. Scimemi and A. Vladimirov, arXiv:1609.06047 [hep-ph].
[77] J. C. Collins, D. E. Soper and G. F. Sterman, Nucl. Phys. B 250, 199 (1985).
[78] E. Laenen, G. F. Sterman and W. Vogelsang, Phys. Rev. Lett. 84, 4296 (2000).
[79] U. D’Alesio, M. G. Echevarria, S. Melis and I. Scimemi, JHEP 1411 (2014) 098.
[80] A. Bacchetta, M. G. Echevarria, P. J. G. Mulders, M. Radici and A. Signori, JHEP 1511 (2015) 076.
[81] A. Bacchetta, F. Delcarro, C. Pisano, M. Radici and A. Signori, in preparation.
[82] S. M. Aybat and T. C. Rogers, Phys. Rev. D 83 (2011) 114042.
[83] S. Frixione, P. Nason and G. Ridolfi, Nucl. Phys. B 542 (1999) 311.
[84] M. G. Echevarria, A. Idilbi, A. Schäfer and I. Scimemi, Eur. Phys. J. C 73 (2013) no.12, 2636.