Spontaneous $N = 2 \to N = 1$ Supergravity Breaking in Three Dimensions

Olaf Hohm$^a$ and Jan Louis$^{a,b}$

$^a$II. Institut für Theoretische Physik
Universität Hamburg
Luruper Chaussee 149
D-22716 Hamburg, Germany

$^b$Laboratoire de Physique Théorique de l’Ecole Normale Supérieure
24 rue Lhomond,
75231 Paris Cedex, France

email: olaf.hohm@desy.de, jan.louis@desy.de

ABSTRACT

We study models of spontaneous $N = 2 \to N = 1$ supergravity breaking in three space-time dimensions and discuss the topological Higgs- and super-Higgs mechanism which generates the masses for the spin-3/2 gravitino multiplet. The resulting $N = 1$ spectrum and its effective action is analysed.
1 Introduction

Partial supersymmetry breaking is of interest in its own right but also within string theory. The early no-go theorems stated that partial supersymmetry breaking is impossible and either all supercharges are preserved or all supercharges are broken [1]. These theorems were later on modified in global supersymmetry [2, 3], supergravity [4] and in string theory [5, 6, 7, 8, 9]. However, a satisfactory and conceptual understanding of partial supersymmetry breaking is still lacking.

In supergravity the main focus so far was on spontaneous $N = 2 \rightarrow N = 1$ in four space-time dimensions ($D = 4$) [4, 10, 11, 12]. In this paper we concentrate on the somewhat simpler situation of $N = 2 \rightarrow N = 1$ breaking in $D = 3$. These $N = 2, D = 3$ theories are closely related to dimensionally reduced $N = 1$ theories in $D = 4$ as they also feature four supercharges. Partial breaking to $N = 1$ (in $D = 3$) breaks two of the four supercharges leaving an $N = 1$ in $D = 3$ intact. In the context of global supersymmetry this breaking has also been discussed in [13].

The reason to concentrate on $D = 3$ vacua is on the one hand their simplicity. On the other hand they also arise in the context of string theory by compactifying M-theory on Calabi-Yau fourfolds with non-trivial background fluxes [14, 15, 16, 17, 18, 19, 20]. The background fluxes appear in the low energy effective action as gauge or mass parameters and generically turn an ordinary supergravity into a gauged or massive supergravity. The fluxes induce a potential for the scalar fields which only under certain conditions preserves some of the supercharges. A class of models where Peccei-Quinn isometries are gauged and $N = 2 \rightarrow N = 1$ breaking occurs was recently found in ref. [20] and in our analysis we closely follow this work.

The purpose of this paper is to further analyse the mechanism of $N = 2 \rightarrow N = 1$ breaking in $D = 3$. In particular we study the details of the Higgs and super-Higgs effect which gives masses to the gauge bosons and the gravitino. We find that contrary to $D = 4$ a topological Higgs mechanism is at work in $D = 3$ where no Goldstone boson is eaten [21, 22]. The presence of a Chern-Simons term in addition to the standard Yang-Mills terms renders a gauge field massive without adding any physical degree of freedom to the gauge field. Alternatively in a ‘dual’ version of this mechanism the gauge theory has no standard Yang-Mills term but instead a Chern-Simons term and a conventional mass term [23, 24] or in other words a Chern-Simons term and a coupling of the gauge field to a Goldstone boson [25].

More specifically the paper is organized as follows. In section 2 we recall the necessary facts about gauged $N = 2$ supergravity in $D = 3$ following refs. [26, 27, 28] with particular emphasis on gauged Peccci-Quinn isometries [20]. In section 3 we review the class of models considered in [20] where spontaneous $N = 2 \rightarrow N = 1$ supergravity breaking occurs. The super-Higgs mechanism and the reorganization in $N = 1$ supermultiplets are discussed in section 4. Section 5 contains our conclusions and our spinor conventions are given in appendix A. In appendix B we review the construction of the massive supermultiplets for arbitrary $N$ in $D = 3$ which are necessary for the analysis of the possible multiplets resulting from partial supergravity breaking.
2 Gauged $N = 2$ Supergravity in $D = 3$

2.1 Generalities

Let us start by first recalling a few facts about $N = 2$ supergravity in $D = 3$ \cite{26, 27}. This theory is closely related to a dimensionally reduced $N = 1$ supergravity in $D = 4$ and as a consequence similar multiplets and geometrical structures appear.

The gravity multiplet $(g_{\mu\nu}, \psi^{I}_{\mu})$, $\mu, \nu = 0, 1, 2$, contains besides the spacetime metric $g_{\mu\nu}$ two Majorana gravitini $\psi^{I}_{\mu}$. (See appendix A for our spinor conventions and appendix B for a summary of the supermultiplets.) In contrast to higher-dimensional theories these fields carry no propagating degrees of freedom. In order to get dynamically non-trivial theories one can add $d$ $N = 2$ scalar multiplets $(\phi^{i}, \chi^{i}_{1}, \chi^{i}_{2})$, $i = 1, \ldots, d$, consisting of $d$ complex scalar fields $\phi^{i}$ and $2d$ real Majorana fermions $\chi^{i}_{1}, \chi^{i}_{2}$. The bosonic Lagrangian is given by

$$L_{B} = \frac{1}{2} e R - eg_{ij}(\phi, \bar{\phi}) \partial_{\mu}\phi^{i} \partial^{\mu}\bar{\phi}^{j} + eV , \quad (2.1)$$

where $g_{ij}$ is a metric on a Kähler manifold which can be expressed in terms of a Kähler potential as $g_{ij} = \partial_{i}\partial_{j}K$. $V$ is determined in terms of a holomorphic superpotential $W(\phi)$ and its Kähler covariant derivative $D_{j}W = \partial_{j}W + (\partial_{j}K)W$ via

$$V = \frac{g^{2}}{4} e^{K} (g^{ij}D_{i}WD_{j}\bar{W} - 4|W|^{2}) . \quad (2.2)$$

It is possible to couple gauge fields to this theory by gauging isometries of the metric $g_{ij}$. Such isometries are generated by $n$ holomorphic Killing vector fields $X_{A}^{i}$ satisfying

$$\partial_{j}X_{A}^{i} = 0 , \quad \nabla_{i}X_{jA} + \nabla_{j}X_{iA} = 0 , \quad A = 1, \ldots, n . \quad (2.3)$$

These Killing equations determine $X_{A}^{j}$ in terms of Killing prepotentials (or momentum maps) $P_{A}$ via

$$g_{k\bar{j}}X_{A}^{\bar{j}} = -2i\partial_{k}P_{A} . \quad (2.4)$$

Under the isometries the scalar fields transform according to

$$\delta\phi^{i} = \alpha^{A}X_{A}^{i}(\phi) , \quad (2.5)$$

where $\alpha^{A}$ are the (local) gauge parameters for the $n$ linearly independent isometries. Gauge invariance is ensured by introducing $n$ vector fields $A_{\mu A}$ and promoting the ordinary derivative $\partial_{\mu}\phi$ to covariant derivatives

$$D_{\mu}\phi^{i} = \partial_{\mu}\phi^{i} + g\Theta^{AB}A_{\mu A}X_{B}^{i} , \quad (2.6)$$

where $g$ is the gauge coupling and $\Theta^{AB}$ an arbitrary constant symmetric matrix.

Introducing the covariant derivative alone is not sufficient but in addition the supersymmetry transformation laws have to be modified which in turn requires adding

\footnote{Following \cite{27} we have included a factor $g^{2}$ in the potential for later convenience. In a more standard notation it would be reabsorbed in the definition of $W$.}
Yukawa-type couplings for the fermionic fields and a scalar potential. Furthermore, a “kinetic” term for the vector fields has to be supplied.

The usual procedure would be to promote $A_{\mu}A$ to a supermultiplet and add a standard supersymmetric Yang-Mills kinetic term to the action. However, in $D = 3$ there is another peculiar option advisable [29]. It is necessary to add instead a Chern-Simons term for the gauge fields

$$L_{CS} = \frac{1}{4}g\Theta^{AB}\varepsilon^{\mu\nu\rho}A\mu F_{B\nu\rho},$$

which is topological and therefore introduces no propagating gauge degrees of freedom. Or in other words the gauge fields are auxiliary fields which nevertheless ensure the gauge invariance of the theory. Since $A_{\mu}A$ with the interaction (2.7) and (2.6) has no physical degree of freedom it is not necessary to introduce additional fermionic partners in order to balance the boson-fermion degeneracy. Indeed, the consistency with supersymmetry has been established in [27].

The scalar potential $V$ given in (2.2) has to be modified in gauged supergravities according to [20, 27]

$$V = g^{2}(4g^{i\bar{j}}\partial_{i}T\partial_{j}T - 4T^{2} + \frac{1}{4}e^{K}(g^{i\bar{j}}D_{i}WD_{j}\bar{W} - 4|W|^{2})),$$

where $T = P_{A}\Theta^{AB}P_{B}$ ($P_{A}$ is the momentum map defined in (2.4)). $T$ and $W$ are not completely independent but gauge invariance of $W$ imposes the constraint [27]

$$X_{A}^{i}D_{i}W = 2iWP_{A}.$$

To summarize, the bosonic part of the Lagrangian for an $N = 2$ action for gauged supergravity has the generic form

$$e^{-1}L = R - g_{ij}\partial_{i}\phi^{j}\partial_{j}\phi - \frac{1}{4}g\Theta^{AB}\varepsilon_{\mu\nu\rho}A^{\mu}_{A}F^{\nu\rho}_{B} + V.$$

Let us now turn to the fermionic couplings. It is more convenient to treat the 2d Majorana fermions $\chi^{i}$ as real fermions (i.e. not assemble them in a complex notation as we did for the bosons) and label them by the ‘real’ indices $i, j = 1,..., 2d$. In this notation the Kähler metric is denoted by $g^{i\bar{j}}$ which when written in complex coordinates ($\hat{i} = (i, \bar{i})$) only has the non-vanishing components $g_{ij} = g_{ij}$ known from (2.1). In this paper we only need the fermion bilinear terms which read [27]

$$e^{-1}L_{F} = -\frac{i}{2}\varepsilon^{\mu\nu\rho}\bar{\psi}_{\mu}^{I}\nabla_{\nu}\psi_{\rho}^{I} - \frac{1}{2}g_{ij}\bar{\chi}^{i}\gamma^{j}\nabla_{\mu}\chi^{j}$$

$$+ \frac{1}{2}gA_{1}^{I}(\phi)\bar{\psi}_{\mu}^{I}\gamma^{\mu}\psi_{\nu}^{J} + gA_{2}^{I}(\phi)\bar{\psi}_{\mu}^{I}\gamma^{\mu}\chi^{j} + 2igA_{3}^{I}(\phi)\bar{\chi}_{\mu}^{I}\gamma^{\mu}\chi^{j}.$$ 

Furthermore, supersymmetry demands that the tensors $A_{1}$ and $A_{2}$ satisfy a quadratic identity which also determines the potential. It reads

$$A_{1}^{IK}A_{1}^{KJ} - g_{ij}A_{2}^{Ij}A_{2}^{Ij} = -g^{-2}V\delta^{IJ}.$$

In addition, at the stationary points of the potential, i.e. at the points where the first derivatives vanish, the following identity is valid [27]

$$3A_{1}^{IJ}A_{2}^{Ij} + g_{jk}A_{2}^{Ij}A_{3}^{jk} = 0.$$
Explicitly, in our conventions the gravitino mass matrix $A_1$ is given by

$$A_1^{IJ} = \begin{pmatrix} -2T & 0 \\ 0 & -2T \end{pmatrix} + e^{K/2} \begin{pmatrix} -\text{Re}W & \text{Im}W \\ \text{Im}W & \text{Re}W \end{pmatrix},$$

(2.14)

while $A_2$ and $A_3$ read in complex notation

$$A_2^{ij} = -\frac{1}{2} (\partial_i T + e^{K/2} D_i W),$$

$$A_3^{ij} = \frac{1}{2} e^{K/2} D_i D_j W,$$

$$A_3^{i\bar{j}} = -D_i \partial_j T - \frac{1}{2} g_{ij} T + \partial_i \partial_j T.$$ 

(2.15)

### 2.2 Gauged Peccei-Quinn-Isometries

A special class of isometries are translational or Peccei-Quinn (PQ) isometries. They correspond to constant Killing vectors $X_A^i$ and as we see from (2.5) they act as (local) shifts $\phi \to \phi + \alpha$ on a subset of the scalar fields. Such isometries typically appear in the effective low energy supergravities arising from string theory when background fluxes are turned on. For this reason they were studied in ref. [20] and in this section we follow their analysis.

Since the PQ isometries act on a subset of the scalar fields it is convenient to introduce a notation where the scalar fields $\phi^i$ are split into

$$\phi^i = (\phi^a, \phi^A) = (\phi^a, \varphi^A + i \hat{\varphi}^A), \quad a = 1, \ldots, d - n, \quad A = 1, \ldots, n.$$ 

(2.16)

We define the PQ symmetries to act only on the $\hat{\varphi}^A$ but that they leave the $\phi^a$ and $\varphi^A$ invariant. Using (2.5) this corresponds to the Killing vectors

$$X^i_B = (0, i \delta^A_B).$$

(2.17)

Inserted into (2.6) yields

$$\mathcal{D}_\mu \hat{\varphi}^A = \partial_\mu \varphi^A + g \Theta^{AB} A_{\mu B},$$ 

(2.18)

while the covariant derivatives of all other scalars reduce to ordinary partial derivatives.

The PQ-symmetries severely constrain the possible couplings of the charged scalar fields. In particular the potential $V$ (and thus $W$ and $T$) and the Kähler potential $K$ cannot be arbitrary. For simplicity ref. [20] assumed that all these couplings are independent of $\hat{\varphi}^A$. For the holomorphic superpotential this implies by the constraint (2.9) that $W$ is only a function of the $\phi^a$, while the Kähler potential obeys $K = K(\phi^a, \bar{\phi}^a, \phi^A + \bar{\phi}^A)$. This in turn implies

$$g_{\bar{i}j} = \begin{pmatrix} g_{ab} & g_{aB} \\ g_{\bar{a}b} & g_{\bar{a}B} \end{pmatrix} = \begin{pmatrix} g_{ab} & g_{aB} \\ g_{\bar{a}b} & g_{\bar{a}B} \end{pmatrix},$$

(2.19)

This situation certainly has a PQ-symmetry but it is not the most general case.
where \( g_{ab} = \partial_a \partial_b K \), \( g_{aA} = \frac{1}{2} \partial_a \partial_A K \), \( g_{AB} = \frac{1}{4} \partial_A \partial_B K =: \frac{1}{2} G_{AB} \) and \( \partial_A \) denotes a real derivative.

In \( D = 3 \) a massless propagating vector is Hodge-dual to a massless scalar. Under certain condition this duality can be inverted in that a massless scalar can be dualized ‘back’ to a vector. In [20] it was shown that this is possible precisely for the scalars \( \hat{\varphi}^A \) that are charged under the PQ-symmetry and that the duality relation is given by

\[
F_{A\mu\nu} = -\varepsilon_{\mu\nu\rho}(G_{AB} D^\rho \hat{\varphi}^B + 2 \text{Im}[g_{AB} \partial^\rho \hat{\varphi}^A]) + \text{fermions} \,
\]

If one assumes that \( G_{AB} \) is invertible, (2.20) can be used to express \( D^\rho \hat{\varphi}^B \) in terms of \( F_{A\mu\nu} \) such that the entire Lagrangian no longer depends on \( \hat{\varphi}^B \). Instead, the vector fields \( A_{\mu A} \) obtain proper Yang-Mills kinetic terms and become propagating degrees of freedom. In terms of supermultiplets the original scalar multiplet is dualized into a vector multiplet which contains a vector \( A_\mu \), two Majorana gauginos \( \chi^1, \chi^2 \) and a real scalar \( \varphi \). In these field variables the bosonic Lagrangian reads [20]

\[
e^{-1} \mathcal{L} = \frac{1}{2} R - \frac{1}{2} G^{AB} \partial_\mu M_A \partial^\mu M_B + \frac{1}{4} G^{AB} F_{A\mu\nu} F_{B\mu\nu} - G_{\bar{a}b} \partial_\mu \hat{\varphi}^a \partial^\mu \hat{\varphi}^b \\
+ \varepsilon_{\mu\nu\rho} F_{A\mu\nu} [G^{AB} g_{\bar{a}b} \partial^\rho \hat{\varphi}^a] + \frac{1}{4} g \Theta^{AB} \varepsilon_{\mu\nu\rho} A^\mu F_{B\rho} + V, \tag{2.21}
\]

where in addition the \( \varphi^A \) are eliminated in favor of the (real) coordinates \( M_A := \frac{1}{2} \partial_A K \). Furthermore, the Kähler metric is redefined such that [20]

\[
G_{ab} = (G^{ab})^{-1} := g_{ab} - 2 g_{aA} G^{AB} g_{bB} = (g^{ab})^{-1}, \quad G^{ab} = g^{ab}. \tag{2.22}
\]

Finally, using (2.4) and (2.17) the Killing prepotential \( \mathcal{P}_A \) is found to be

\[
\mathcal{P}_A = -\partial_A K = \frac{1}{2} M_A, \tag{2.23}
\]

resulting in

\[
T = \frac{1}{4} M_A \Theta^{AB} M_B. \tag{2.24}
\]

Inserted into (2.24) one arrives at

\[
g^{-2} V = \frac{1}{2} M_A \Theta^{AC} G_{CD} \Theta^{DB} M_B - \frac{1}{4} (M_A \Theta^{AB} M_B)^2 \\
+ \frac{1}{4} e^K G^{\bar{a}b} D_a W D_b \bar{W} - e^K (1 - \frac{1}{2} M_A G^{AB} M_B) |W|^2, \tag{2.25}
\]

where the Kähler-covariant derivative is given by \( D_a W = \partial_a W + (\partial_a K) W \) and \( \partial_a K \) depends in general also on \( M_A \).

### 3 Conditions for Partial \( N = 2 \to N = 1 \) Breaking

Let us now determine the necessary condition such that the \( N = 2 \) theory can show partial supersymmetry breaking to \( N = 1 \) [20]. We start from a generic \( N = 2 \) supergravity spectrum with a gravitational multiplet, \( n \) vector multiplets and \( d - n \) scalar multiplets

\[
[g_{\mu\nu}, \psi^{1}_\mu, \psi^{2}_\mu] \oplus [A_{A\mu}, \chi^{1}_A, \chi^{2}_A; M_A] \oplus [\phi^a, \chi^{1}_a, \chi^{2}_a]. \tag{3.1}
\]

\(^3\)Note that \( K \) is still the Kähler potential for the metric \( g_{ab} \) but not for the redefined \( G_{ab} \).
The condition for unbroken supersymmetry is the vanishing of the fermionic supersymmetry variations in the (Lorentz-invariant) ground state. For the gravitinos this amounts to

$$\langle \delta \psi^I \rangle = \langle \nabla_\mu \epsilon^I + i g A_1^{Ij} \gamma_\mu \epsilon^j \rangle = 0 .$$

(3.2)

In order to solve this Killing spinor equation for an AdS or Minkowski ground state, it is sufficient to make a product ansatz for $\epsilon^I$ in terms of an eigenvector of $A_1$ and a three-dimensional AdS/Minkowski Killing spinor. Using

$$\nabla_\mu \nabla_\nu \epsilon^I = i \frac{2}{R} \epsilon^{ab} \gamma^{ab} \epsilon^I$$

and

$$R_{\mu\nu\rho\sigma} = 4 V_0 (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho})$$

we see, that eq. (3.2) can only be solved if the following integrability relation is satisfied

$$g |\lambda| = -V_0 .$$

(3.3)

Here $\lambda$ is an eigenvalue of $A_1$ and $V_0$ denotes the cosmological constant, i.e. the value of the potential in the ground state. Thus each eigenvector of $A_1$ whose eigenvalue satisfies (3.3) yields a solution of the Killing spinor equation (3.2).

The supersymmetry transformations of the other fermions $\chi^i$ evaluated in the ground state are given by

$$\langle \delta \chi^i \rangle = -2g \langle \bar{g}^A A_2^j \epsilon^I \rangle \epsilon^J .$$

(3.4)

We see from (3.2) and (3.4) that each conserved supersymmetry corresponds to a spinor parameter $\epsilon^I$ which is a common eigenvector of $A_1$ and $A_2$, for $A_1$ with an eigenvalue related to the cosmological constant by (3.3) and for $A_2$ with a zero eigenvalue. Using the identity

$$\nabla_\mu \nabla_\nu \epsilon^I = i \frac{2}{R} \epsilon^{ab} \gamma^{ab} \epsilon^I$$

one infers that both conditions or in other words (3.2) and (3.4) are necessarily satisfied simultaneously. Put differently, each eigenvalue of $A_1$ satisfying (3.3) is automatically an eigenvector of $A_2$ with vanishing eigenvalue. Therefore, one only has to determine the eigenvectors of $A_1$ with eigenvalue (3.3) in order to find the unbroken supercharges. Using (2.14) one obtains

$$\lambda_{\pm} = -2T \pm e^{K/2} |W| = -\frac{1}{2} M_A \Theta^{AB} M_B \pm e^{K/2} |W| ,$$

(3.5)

where the second equation used (2.24). Inserted into (3.3) and using (2.25) one arrives at

$$\pm 2 e^{K/2} M_A \Theta^{AB} M_B |W| = M_A \Theta^{AC} G_{CD} \Theta^{DB} M_B + e^{K} |W|^2 M_A G^{AB} M_B$$

$$+ \frac{1}{2} e^K G^{ab} D_a W D_b W .$$

(3.6)

For the ground state to respect the full $N = 2$ supersymmetry, this relation has to be satisfied for both eigenvalues $\lambda_{\pm}$, i.e. for both signs in (3.6). Since $\Theta^{AB}$ and $G_{AB}$ are non-vanishing and $G_{AB}$ is positive definite each term in (3.6) has to vanish separately. This implies the two solutions

$$\Theta^{AB} M_B = W = D_a W = 0 , \quad \text{or} \quad M_A = D_a W = 0 , \quad W \neq 0 .$$

(3.7)

Both solutions correspond to stationary points of $V$ in that they satisfy $\partial_A V = \partial_a V = 0$ as can be checked from the expression (2.25). The first solution has vanishing cosmological

---

4Strictly speaking, $A_2$ cannot have an eigenvector, because it is a rectangular matrix. However, we just mean by this the statement $A_2^{Ij} \epsilon^I = 0$. 
constant since it satisfies $T = V_0 = 0$. The second solution has $T = 0, V_0 = -g^2 e^K |W|^2$ and hence a negative cosmological constant.

For partial $N = 2 \to N = 1$ breaking the situation is more involved. For this case the condition (3.6) should only be satisfied for one eigenvalue but not both. Thus the right hand side of (3.6) cannot vanish or equivalently the left hand side must be non-vanishing. This implies

$$|W| \neq 0 \quad \text{and} \quad \Theta^{AB} M_B \neq 0 . \quad (3.8)$$

Without additional input (3.6) cannot be further simplified. However, if we impose a vanishing cosmological constant of the $N = 1$ ground state, i.e. $V_0 = 0$, eqs. (2.25) and (3.6) imply

$$\pm 2 e^{K/2} |W| = M_A \Theta^{AB} M_B , \quad (3.9)$$

for precisely one choice of the signs. In order to simplify the analysis we follow [20] and assume $M_A G^{AB} M_B = 2$. In this case the potential (2.25) is manifestly positive-definite

$$g^{-2} V = \frac{1}{2} (M_A \Theta^{AC} - 2T M_A G^{AC}) G_{CD} (\Theta^{DB} M_B - 2T G^{DB} M_B)$$

$$+ \frac{1}{4} e^K G^{ab} D_a W D_b \overline{W} , \quad (3.10)$$

and a sufficient condition for the minimum is given by

$$\Theta^{DB} M_B - 2T G^{DB} M_B = 0 \quad \text{and} \quad D_a W = 0 . \quad (3.11)$$

For the positive-definite potential (3.10) the two conditions (3.11) and (3.9) are necessary and sufficient for a $N = 1$ Minkowski ground state [20].

4 The $N = 1$ Supermultiplets

In this section we study the spectrum of a spontaneously broken $N = 2$ supergravity or in other words the rearrangement of the $N = 2$ multiplets in terms of $N = 1$ multiplets. We only discuss the case of Minkowskian ground states leaving the study of the AdS case to a separate publication.

As we observed in the previous section an $N = 1$ Minkowskian ground state is characterized by one zero and one non-zero eigenvalue of $A_1$. From eq. (2.11) we see that $A_1$ is the mass matrix of the two gravitini and thus one zero and one non-zero eigenvalue corresponds to a massless and a massive gravitino. Which of the eigenvalues given in (3.5) is zero is a matter of convention and without loss of generality we choose to discuss the situation $\lambda_+ \neq 0, \lambda_- = 0$. We will see in the following that the gravitino mass is related to the non-vanishing eigenvalue by $\lambda_+ = 2g^{-1} m_\psi$. Together with (3.9) this implies

$$m_\psi = \frac{1}{2} g \lambda_+ = g e^{K/2} |W| = -\frac{1}{2} g M_A \Theta^{AB} M_B = -2gT , \quad \lambda_- = 0 . \quad (4.1)$$

A massive gravitino requires a super-Higgs effect or in other words the presence of a Goldstone fermion $\eta$ which by an appropriate redefinition of the fields can be eliminated from the action (i.e. ‘eaten’ by the gravitino). In section 4.1 we discuss the details of this super-Higgs effect and in section 4.2 we show how the supersymmetric partner of the gravitino receives its mass by a topological Higgs mechanism.
4.1 The super-Higgs effect in $D = 3$

In analogy with the situation in $D = 4, N = 1$ we define $\eta^I = \langle A^I_2 \rangle \chi^I$, $I = 1, 2$. Using (3.11) one immediately infers the transformation law of $\eta$ (evaluated in the ground state) to be

$$\langle \delta \eta^I \rangle = -2g\langle A^{I}_2 g^{j} A^{K}_{2k} \rangle \epsilon^K.$$  \hspace{1cm} (4.2)

For the conserved supersymmetry $\epsilon^K$ is an eigenvector of $A_2$ with vanishing eigenvalue and hence $\langle \delta \eta^I \rangle = 0$ holds. For the broken supersymmetry we have instead

$$\langle \delta \eta^I \rangle = -2g\langle A^{I}_{2j} g^{j} A^{K}_{2k} \rangle \epsilon^K = -2g\langle A^{I}_{1} A^{I}_{1} \rangle \epsilon^K = -8g^{-1}m_\psi^2 \epsilon^I,$$ \hspace{1cm} (4.3)

where we used (2.12) and (4.1). Eq. (4.3) shows that $\eta^I$ transforms inhomogeneously (by a shift) exactly as required by a Goldstone fermion.

In the ground state the matrix $A_2$ considerably simplifies as can be seen by inserting (3.11) and (2.24) into the definition given in (2.15). Since $T$ is only a function of the $M_A$ we see that in the direction of the chiral multiplets $\langle A_{2n} \rangle = 0$ holds while in the direction of the vector multiplets we have $\langle A_{2A} \rangle \neq 0$. This implies that the Goldstone fermion $\eta^I$ is a linear combination of only the fermions in the vector multiplets but does not contain any fermions in chiral multiplets.

Exactly as in $D = 4, N = 1$ it is possible to redefine the massive gravitino and absorb the Goldstone fermion. Let us first perform an $SO(2)$ transformation on the $\psi_\mu^I$ such that $A^{I}_{1j}$ in (2.11) is diagonalized. Since $A_1$ and $A_2$ have common eigenvectors we can rotate $\eta^I$ accordingly. In this rotated basis $\eta^1$ vanishes from the Lagrangian since $A^{1}_{2i} = 0$ holds as a consequence of the conserved supersymmetry.\(^5\) Thus we arrive at

$$e^{-1}L_Y = m_\psi \bar{\psi}_\mu^2 \gamma^\mu \psi^2 + 2g\bar{\psi}_\mu^2 \gamma^\mu \eta^2 + 2\imath g A_{3ij} \bar{\chi}^i \chi^j.$$ \hspace{1cm} (4.4)

As expected the massless gravitino $\psi_\mu^1$ disappeared from the mass terms and we are left with an off-diagonal fermionic mass matrix. Before diagonalizing the Yukawa coupling let us split off the physical spin-1/2 fermions according to

$$\chi_\perp^i := \chi^i - \frac{g^2}{\sqrt{2}m_\psi} \langle g^{ij} A^{2}_{2j} \rangle \eta^2,$$ \hspace{1cm} (4.5)

such that the fermionic part of the action looks together with the kinetic terms like

$$e^{-1}L_F = -\frac{1}{2} \epsilon^{\mu\nu} \psi_\mu^2 \partial_\nu \psi^2 - \frac{1}{2} \imath A_{3ij} \bar{\chi}_\perp^i \gamma^\nu \partial_\nu \chi_\perp^j - \frac{g^2}{m_\psi^2} \eta^2 \gamma^\mu \partial_\mu \eta^2$$ \hspace{1cm} (4.6)

$$+m_\psi \bar{\psi}_\mu^2 \gamma^\mu \psi^2 + 2g\bar{\psi}_\mu^2 \gamma^\mu \eta^2 + 2\imath g A_{3ij} \bar{\chi}_\perp^i \chi_\perp^j,$$

where we have used (2.12). By an appropriate redefinition of the massive gravitino $\psi_\mu^2$ the Goldstone fermion $\eta^2$ can be removed from the entire action. This redefinition is inspired by the supersymmetry transformations of $\eta$ given in (4.2) with $\epsilon^I = \frac{g}{s m_\psi} \eta^I$, including a

\(^5\)Note that in the case that both supersymmetries are broken also $A^{1}_{2i}$ would be different from zero, corresponding to the second Goldstone fermion.
term proportional to $\partial_\mu \eta$ in order to remove the kinetic term for the Goldstone fermion. It reads (omitting the index 2)

$$\hat{\psi}_\mu = \psi_\mu + \frac{g}{m_\psi} \partial_\mu \eta - \frac{g}{m_\psi} i\gamma_\mu \eta ,$$

(4.7)

resulting in

$$e^{-1} \mathcal{L}_F = - \frac{1}{2} i \varepsilon^{\mu \nu \rho} \hat{\psi}_\mu \partial_\nu \hat{\psi}_\rho - \frac{1}{2} g_{ij} \hat{\psi}^j \gamma^\mu \partial_\mu \hat{\chi}_i + m_\psi \hat{\psi}_\mu \gamma^{\mu \nu} \psi_\nu + i (m_\chi)_{ij} \hat{\psi}^j \hat{\chi}_i ,$$

(4.8)

where

$$(m_\chi)_{ij} = 2 g A_{i2j} - \frac{3 g^2}{m_\psi} A_{2i} A_{2j} .$$

(4.9)

As promised $\eta$ has disappeared from the action and left a zero eigenvalue in the mass matrix $(m_\chi)_{ij}$. This zero eigenvalue is most easily seen by observing that $g^{jk} A_{2k}$ is a null vector of (4.9)

$$m_{ij} g^{jk} A_{2k} = 2 g (A_{3ij} g^{jk} A_{2k} + \frac{3 g}{2 m_\psi} A_{2i} g^{jk} A_{2j} A_{2k})$$

(4.10)

$$= 2 g (g^{jk} A_{3ij} A_{2k} + \frac{6 m_\psi}{g} A_{2i}) = 0 ,$$

where we used (2.14) and (2.12) together with (4.1) and (2.13). The use of (2.13) is justified by the results of [1] in that a $N = 1$ configuration is necessarily a stationary point of the potential. We see that after the redefinition (4.7) the Goldstone fermion disappeared from the action and left a properly normalized massive gravitino with one physical degree of freedom. This propagating degree of freedom can be seen by applying the differential operator $(-i \varepsilon^{\mu \lambda \sigma} \partial_\lambda - 2 m_\psi \gamma^{\mu \nu} \psi_\nu)$ to the equation of motion for the redefined gravitino (we are dropping the ‘hats’ henceforth)

$$i \varepsilon^{\mu \nu \rho} \partial_\nu \psi_\rho - 2 m_\psi \gamma^{\mu \nu} \psi_\nu = 0 .$$

(4.11)

Up to linear terms in the derivatives this results in

$$0 = (\Box + m_\psi^2) \psi_\mu - \partial_\mu (\partial \cdot \psi) + \ldots ,$$

(4.12)

which shows that the gravitino has become a massive propagating field. Altogether it carries now one fermionic degree of freedom. Thus the originally massless and topological gravitino (with no physical degree of freedom) becomes propagating due to the presence of the mass term. The physical degree of freedom of the massive gravitino coincides with the physical degree of freedom of the Goldstone fermion it has eaten. A similar situation occurs in the topological Higgs mechanism for vector fields which we turn to now.

6 Counting the physical degrees of freedom is also consistent with the reduction from $D = 4$. In $D = 4$ a massive gravitino has four degrees of freedom and it splits in the reduction as $\psi_m = (\psi_{10} , \psi_{30}) = (\psi^{1a}_1 + i \psi^{2a}_1 , \psi^{1a}_3 + i \psi^{2a}_3)$, where $m = 0 , \ldots , 3$. From a three-dimensional point of view the real and imaginary parts of $\psi_m$ are real Majorana spinors of the $D = 3$ Lorentz group $SL(2, \mathbb{R})$. Thus we see that each of the $(\psi^{1a}_1 , \psi^{2a}_1 , \psi^{1a}_3 , \psi^{2a}_3)$ carries one degree of freedom.
4.2 The topological Higgs mechanism

The spontaneous supersymmetry breaking we are considering in this paper leaves an $N = 1$ unbroken. Therefore, after the breaking the original $N = 2$ multiplets have to rearrange in $N = 1$ multiplets. This means in particular that the massive gravitino must be the member of a massive $N = 1$ multiplet and in this section we identify the supersymmetric partner of the massive gravitino. In appendix B we review the general structure of the massive $N = 1$ supermultiplets in $D = 3$ and show that the irreducible multiplets contain the spins $(j, j + \frac{1}{2})$. Because a massive gravitino even in $D = 3$ has spin $\frac{3}{2}$ [32], we expect a massive vector to be the supersymmetric partner of the massive gravitino. This is also suggested by the fact that the Goldstone fermion arises solely from the vector multiplets leaving the chiral multiplets untouched. Therefore the supersymmetric partner of the massive gravitino should also come out of the vector multiplets.

However, this proposition is somewhat puzzling at first sight. The reason is that as we just argued the massive gravitino has only one physical degree of freedom which is the same as a massless vector. In the standard Higgs mechanism the vector acquires a mass by ‘eating’ a (spin-0) Goldstone boson which adds one degree of freedom to the vector. A second problem is that in the action (2.21) there is no candidate for a Goldstone boson which has the appropriate couplings. The resolution of this apparent paradox is the possibility of a topological Higgs effect which does exist in $D = 3$ [21, 22]. Let us briefly review the mechanism.

This topological Higgs effect arises when one adds a Chern-Simons term to the standard Yang-Mills term or in other words the gauge invariant action is given by

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \xi \varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda.$$  \hfill (4.13)

From the equations of motion one shows that the theory defined by (4.13) has a massive excitation with mass $\xi$ [21, 22]. The proof is analogous to the argument presented at the end of the previous section for the gravitino. The equation of motion derived from the action (4.13) reads

$$\xi F^\mu - \varepsilon^{\nu\lambda\mu} \partial_\nu F_\lambda = 0,$$  \hfill (4.14)

where $F^\mu \equiv \varepsilon^{\mu\nu\rho} F_{\nu\rho}$. Applying to (4.14) the first-order differential operator $\xi \varepsilon_{\sigma\rho} + \varepsilon_{\sigma\mu} \partial^\rho$ and using the Bianchi identity $\partial_\mu F^\mu = 0$ one derives\footnote{Note that the sign in front of the Chern-Simons term is in view of the massive excitation arbitrary, because it cannot affect the quadratic mass term in (4.15). But, according to appendix B, the spin could be +1 or -1 and which one is realized is determined by this sign [28].}

$$(\square + \xi^2) F^\mu = 0,$$  \hfill (4.15)

which proves the massive excitation. Since no Goldstone boson was eaten, $A_\mu$ still has only one physical degree of freedom and furthermore the gauge invariance is unbroken.

The action (2.21) includes the two terms given in (4.13). Thus it possibly generates at least one massive vector, because the gauge coupling functions $G^{AB}$ get a non-vanishing vev. The only thing left to show is that among the massive vectors there is one which is degenerate with the gravitino and has a mass $m_\psi$. In order to display this vector we need
to canonically normalize the Yang-Mills action in the ground state. This can be achieved by rotating the vector fields according to an $SO(n)$-transformation $A_{\mu A} \rightarrow S_A^B A_{\mu B}$ such that the $G^{AB}$ are diagonalized and normalized to be the identity matrix. In this basis the physical mass matrix for the vector fields is given in view of (4.13) by

$$m^{AB} = -g (S^{-1} \Theta S)^{AB} .$$

Now we need to show that $m^{AB}$ has at least one eigenvalue given by $m_\psi$. It can be most easily seen by multiplying (4.16) by $S_{BC}^{-1} M_C$ and using (3.11). This implies

$$m^{AB} S_{BC}^{-1} M_C = -g (S^{-1} \Theta)^{AC} M_C = -2g T (S^{-1} G)^{AB} S_{BC}^{-1} M_C = -2g T S_{AB}^{-1} M_B .$$

This equation indeed shows that $S_{BC}^{-1} M_C$ is an eigenvector of $m^{AB}$ with eigenvalue $-2g T = m_\psi$ (here we used (4.1)). Thus the mass matrix (4.16) for the vectors has at least one eigenvalue $m_\psi$ as required by the unbroken $N = 1$ supersymmetry.

Before we continue let us note that this discussion can also be carried out in terms of the original Lagrangian (2.10) where no Yang-Mills kinetic term is present but instead a set of charged scalar fields $\hat{\varphi}^A$ with covariant derivatives given in (2.18). In this field basis the Chern-Simons term acts as a kinetic term and the $\hat{\varphi}^A$ are the Goldstone bosons giving a mass for the gauge fields. (Compare with the discussion in ref. [33].) This version of the topological Higgs mechanism has also been discussed in the literature [23, 24] and we will briefly review it.

Starting from the action

$$\mathcal{L} = \frac{1}{2} \varepsilon^{\mu \nu \rho} A_\mu \partial_\nu A_\rho - \frac{1}{2} \xi A_\mu A^\mu$$

with a conventional mass term, also leads to a massive excitation of one propagating degree of freedom [24]. Namely the equation of motion states that the vector field is Hodge-dual to the field strength, $A_\mu = -\frac{1}{2g} \varepsilon_{\mu \nu \rho} F^{\nu \rho}$, which in turn implies the Proca equation for a massive vector

$$\partial^\mu F_{\mu \nu} + \xi^2 A_\nu = 0 ,$$

together with the Lorentz condition $\partial_\mu A^\mu = 0$. Furthermore, (4.13) and (4.18) are equivalent in the sense that both can be derived from a “master Lagrangian” by varying different fields [23]. But in this version it is possible to relate this topological effect in a more standard way to the notion of spontaneous symmetry breaking.

In the original action (2.10), where no scalar fields have been dualized, the relevant part can be written as

$$\mathcal{L} = \frac{1}{2} G(M) D^\mu \hat{\varphi} D_\mu \hat{\varphi} + \frac{1}{2} g \Theta \varepsilon^{\mu \nu \rho} A_\mu \partial_\nu A_\rho$$

$$= \frac{1}{2} g^2 \Theta^2 G(M) A_\mu A^\mu + \frac{1}{2} g \Theta \varepsilon^{\mu \nu \rho} A_\mu \partial_\nu A^\rho ,$$

where $D^\mu \hat{\varphi} = \partial^\mu \hat{\varphi} + g \Theta A_\mu$ and for simplicity we have restricted the discussion to one vector field. In the second equation the scalar field has been absorbed by a gauge transformation into the gauge field, leaving only a $(\text{Stückelberg})$ mass term for $A_\mu$. Due to
we have $-g\Theta(G(M)) = m_\psi$, such that $A_\mu$ gets the gravitino mass. Hence $\hat{\varphi}$ plays
the role of the standard Goldstone boson which provides one physical degree of freedom to a previously
topological gauge field. Note that the potential does not depend on $\hat{\varphi}$, which means that $\hat{\varphi}$ remains massless with respect to an arbitrary groundstate, as required for Goldstone bosons. Also in this field basis one shows in general that the mass
matrix has at least one eigenvalue given by $m_\psi$.

Let us note that this situation is a little bit different from the one considered in [25]. In our case the PQ-isometries form a non-compact gauge group isomorphic to $\mathbb{R}$ acting only on the real scalar fields $\hat{\varphi}$. In contrast one can start like in [25] from a Chern-
Simons $U(1)$-gauge theory coupled to a complex scalar and break the gauge invariance by adding a usual Higgs potential. Gauging the phase factor of the complex scalar away, the action reduces to (4.18), together with the kinetic term for the left-over real Higgs field. Therefore the phase factor plays the role of the Goldstone boson, which is absorbed into
the gauge field by a phase transformation. In this setup one has the standard connection
of the mass of a gauge boson with spontaneous symmetry breaking.

4.3 The $N = 1$ mass spectrum

We started our analysis from a generic $N = 2$ spectrum given in (3.1) containing a
gravitational multiplet, $n$ vector multiplets and $d - n$ scalar multiplets. Let us now show
how after spontaneous $N = 2 \rightarrow N = 1$ breaking this spectrum arranges into $N = 1$
multiplets. In $D = 3$ the possible irreducible massive or massless $N = 1$ multiplets are
a chiral multiplet $(\varphi, \chi)$ containing one real scalar $\varphi$ and one real Majorana fermion $\chi$, a vector multiplet $(A_\mu, \chi)$ containing a vector $A_\mu$ and a Majorana fermion, the massive
gravitino multiplet $(\psi_\mu, A_\mu)$ containing a massive gravitino $\psi_\mu$ and a massive vector $A_\mu$ and the gravitational multiplet $(g_{\mu\nu}, \psi_\mu)$ containing the metric and a massless gravitino. All multiplets have one fermionic and one bosonic degree of freedom except that the
gravitational multiplet contains no degrees of freedom.

From this representation theory and the consideration of the previous section we see
that after partial supersymmetry breaking the original $N = 2$ spectrum assembles into
one gravitational multiplet, one massive gravitino multiplet, $n - 1$ vector multiplets, $n$
chiral multiplets coming out of the $N = 2$ vector multiplets and $2(d - n)$ chiral multiplets
from the sector of complex scalars. More precisely we have

$$\begin{align*}
[g_{\mu\nu}, \psi^1_\mu, \psi^2_\mu] & \oplus n \times [A_\mu, \chi^1, \chi^2, M_A] \oplus (d - n) \times [\varphi, \chi_c] \longrightarrow (4.21) \\
[g_{\mu\nu}, \psi^1_\mu] & \oplus [\psi^2_\mu, A_\mu] \oplus (n - 1) \times [A_\mu, \chi] \oplus n \times [M, \chi] \oplus 2(d - n) \times [\varphi, \chi].
\end{align*}$$

We already argued that the gravitino multiplet is massive and so we are left to de-
termine the masses of the other vector- and chiral multiplets.

For the vectors we already recorded their mass matrix in (4.16) and the number of
massive vector fields will be given by the rank of $\Theta^{AB}$. For the scalar fields coming out
of the vector multiplets one computes the mass matrix as the second derivative of the
scalar potential, $m_A^{AB} = \partial_M G^{AB} M_B$. We focus on the positive-definite potential (3.10)
and first note that in this case the condition $M_A G^{AB} M_B = 2$ implies that $\partial_a \partial_{M_B} K = 0$.
This again shows together with $W = W(\phi^a)$ that the Kähler-covariant derivative $D_a W$
is independent of $M_A$. As a consequence the mixed derivative of \((3.10)\) with respect to $M_A$ and $\phi^a$ vanishes in view of \((3.11)\), i.e. the mass spectrum decouples between the scalars belonging to the chiral and the vector multiplets. We will not reproduce the full expression for $m^{AB}$, but just state that depending on the geometry given by $G_{AB}$ some of the $M_A$ remain massless and some of them become massive with masses which are of the order of $m_\psi$.

For the fermions belonging to the scalar multiplets $[M_A, \chi_A]$ as well as the fermions of the vector multiplets $[A_\mu, \chi]$ the masses can be computed from $A_3$ and $A_2$ using \((4.9)\) upon making the coordinate transformation to $(\phi^a, M_A)$. However, we will not work out the details here.

Let us now turn to the chiral multiplets $[\varphi, \chi]$. As we have seen there is no coupling between the scalars belonging to vector and scalar multiplets, such that the mass matrix of the $\varphi^a$ is determined by the second derivatives

\[
M_{\varphi\varphi}^2 = \left( \frac{\partial^2 V}{\partial \phi^a \partial \phi^b} \right) = \frac{g^2}{4} e^K W \left[ D_a (D_b W) + D_b (D_a W) \right],
\]

\[
M_{\varphi\chi}^2 = \left( \frac{\partial^2 V}{\partial \phi^a \partial \chi^b} \right) = \frac{g^2}{4} e^K \left[ g^{cd} D_a (D_c W) D_b (D_d W) + g_{ab} |W|^2 \right],
\]

where we used \((3.11)\).

As a next step we calculate the corresponding fermion masses. We first note that as for the scalar fields there is no coupling between the fermions belonging to the vector multiplets and the ones belonging to the scalar multiplets. This can be seen by inspecting $A_{3aA}$ in \((2.15)\) which shows that it is proportional to $D_a D_A W \sim M_A D_a W$ and which vanishes in the ground state due to \((3.11)\). Altogether, we see that the rearrangement of the multiplets runs independently between the vector and scalar multiplets, as indicated by \((4.21)\).\(^8\)

The masses of the fermionic partners of the $\phi^a$ are directly given by $A_{3ab}$ since these fermions do not mix with the Goldstone fermion (see \((4.1)\)) and thus there is no coupling with the gravitinos. From \((4.9)\) we learn that the unnormalized mass matrix of these fermions is given by $m_{ab} = 2g \langle A_{3ab} \rangle$. With \((2.15)\) these matrices can be written in a complex notation as

\[
m_{\varphi\varphi} = \frac{g}{2} e^{K/2} D_a (D_b W), \quad m_{\varphi\chi} = -gg_{ab} T = \frac{g}{2} e^{K/2} g_{ab} |W|, \tag{4.23}
\]

where we have used for the second equation \((3.9)\). To compare these mass matrices with the ones for the scalars, we compute their squares $M_{\varphi\varphi}^2 = \delta^{ij} m_{\varphi i} m_{\varphi j}$ and arrive at

\[
M_{\varphi\varphi}^2 = g^{cd} m_{\varphi c} m_{\varphi d} + g^{cd} m_{\chi c} m_{\chi d} = \frac{g^2}{4} e^K W \left[ D_a (D_b W) + D_b (D_a W) \right],
\]

\[
M_{\varphi\chi}^2 = g^{cd} m_{\varphi c} m_{\chi d} + g^{cd} m_{\chi c} m_{\varphi d} = \frac{g^2}{4} e^K \left[ g^{cd} D_a (D_c W) D_b (D_d W) + g_{ab} |W|^2 \right], \tag{4.24}
\]

\(^8\)Note that this is only true for the positive-definite potential \((3.10)\) and the corresponding conditions \((3.11)\). In general the scalar and vector multiplets can couple in a complicated way in which case the analysis would be much more involved.
where in the first equation we have performed an appropriate Kähler transformation to write $\overline{W}$ instead of $|W|$. As demanded by $N = 1$ supersymmetry we find mass degenerate chiral multiplets or in other words agreement between (4.22) and (4.24).

4.4 The $N = 1$ effective action

Well below the scale of partial $N = 2 \to N = 1$ supersymmetry breaking set by $m_\psi$, the dynamics of the light $N = 1$ multiplets can be best described by an effective action $\mathcal{L}_{\text{eff}}^{N=1}$. Since $N = 1$ is unbroken this action should be manifestly $N = 1$ supersymmetric. It is calculated by “integrating out” the massive gravitino multiplet together with all other multiplets with masses of order $m_\psi$. The resulting effective action can be obtained as a power series expansion in $p/m_\psi$ where $p$ is a typical momentum scale satisfying $p \ll m_\psi$. To lowest non-trivial order in $p/m_\psi$ this amounts to setting all massive multiplets equal to zero keeping only the left-over light $N = 1$ multiplets. In effect this truncates the scalar manifold to a subspace of the original Kähler manifold and projects out a set of heavy vector multiplets. Due to the topological nature of the Higgs mechanism in $D = 3$ this is very different from the situation in $D = 4$ where the mixing of the Goldstone bosons results in taking a certain quotient space of the original scalar manifold [11]. Here the scalar manifold is merely truncated to a submanifold and one is left with a standard (ungauged) $N = 1$ supergravity action coupled to light chiral multiplets.

Let us further note that due to this truncation the geometry loses generically some of its structure. According to the fact that the $N = 1$ multiplets contain at most one real scalar, setting the massive multiplets equal to zero in general distinguishes between real and imaginary parts. As a consequence the holomorphicity property of the superpotential as well as the complex structure of the scalar manifold is lost. But, this is in turn consistent with $N = 1$ supersymmetry which only demands the scalar manifold to have a Riemannian structure and the superpotential to be a real function [27].

5 Conclusions

In this paper we analyzed models of partial $N = 2 \to N = 1$ breaking in three-dimensional $N = 2$ supergravity. These models are inspired by string compactifications of M-theory on Calabi-Yau fourfolds [20] and correspond to supergravities where translational (Peccei-Quinn) isometries are gauged. We saw that the massive gravitino multiplet consists of a gravitino and a vector boson degenerate in mass which both carry one degree of freedom. This is possible in $D = 3$ due to the topological Higgs mechanism where the gauge boson mass is generated by a Chern-Simons term without the necessity of any Goldstone boson. Alternatively, in a dual description this topological Higgs mechanism can be described with a standard coupling to a Goldstone boson, which was precisely the scalar field that could be dualized into the vector before a spontaneous breaking. But in this field basis, where all degrees of freedom reside in the scalar sector, the Yang-Mills kinetic term is absent and instead only a topological Chern-Simons terms is present.

We argue now that in $D = 3$ the presented picture is in general valid. Indeed, it was shown in [29] that gauged supergravity with Chern-Simons kinetic term on the one hand and Yang-Mills kinetic term on the other hand (Yang-Mills or Chern-Simons gauging) are
equivalent in the following sense. Using the on-shell duality between scalars and vectors, a Chern-Simons theory with gauge group $G' = G \rtimes T$, where $T$ denotes an abelian group of translations transforming non-trivially under $G$, can be transformed into a Yang-Mills theory with gauge group $G$, i.e. in the Yang-Mills picture the gauge group is broken to a smaller one.\footnote{In the model discussed here we had the situation that $G$ is trivial, i.e. that there are no charged scalars in the Yang-Mills picture.} Therefore they are now two ways of obtaining massive vector fields with respect to a non-trivial groundstate. In the picture of pure Chern-Simons gauging the subgroup $T$ of the gauge symmetry is broken, leading to massive vectors via eating the Goldstone bosons corresponding to the broken symmetry in analogy with (4.18). Otherwise, in the picture of Yang-Mills gauging the gauge group is still the subgroup $G \subset G'$, that means that the gauge symmetry remains unbroken giving rise to topologically massive vectors corresponding to (4.13). In particular no Goldstone bosons get eaten because the corresponding degrees of freedom are still dualized into the gauge fields.\footnote{Compare the situation in the context of Kaluza-Klein supergravity analyzed in [34].}

The class of models analyzed in this paper contained gauged translational isometries and only Chern-Simons kinetic terms for the gauge fields and thus the possibility of a topological Higgs mechanism existed right from the beginning. However, our analysis of the super-Higgs effect suggests that this is of more general validity. Since the massive $N = 1$ gravitino multiplet has one fermionic and one bosonic degree of freedom, an ordinary Higgs mechanism which raises the number of degrees of freedom of a gauge boson from one to two is not suitable.

Following this line of reasoning we would like to clarify the following characteristic property of supersymmetric field theories in 2 + 1 dimensions, in order to consider further constraints on different scenarios of supersymmetry breaking. As it is explained in appendix B, the massive spin in $D = 3$ is roughly the same as helicity in $D = 4$ and the massive supermultiplets in $D = 3$ are formally the same as the massless ones in $D = 4$. In the latter case massless fields of arbitrary spin $s$ carry always two degrees of freedom, corresponding to the two helicity states $\pm s$. Depending on the spin and the amount of supersymmetry one has therefore possibly to double the massless supermultiplets to organize the fields of a supersymmetric theory into (no longer irreducible) representations of the superalgebra $[30]$, e.g. for an $N = 1$ chiral multiplet one has to take $(0, \frac{1}{2}) \oplus (-\frac{1}{2}, 0)$. This is in contrast to the massive case in $D = 3$. Here, e.g., a scalar multiplet consists of a real scalar and a real Majorana fermion, without the need to double any multiplets. For the fermions this reduction of degrees of freedom is not surprising, because a complex Weyl spinor in $D = 4$ splits in a Lorentz-invariant fashion into its real parts (compare sec. 4.1). On the other hand, if one simply takes the dimensional reduction of massive bosonic theories in $D = 4$, the fields in $D = 3$ would also have more than one degrees of freedom and therefore would not be well adapted for supersymmetry. (Compare the standard massive spin 1 action, which leads to two degrees of freedom [24].) Happily, as we have seen, with the topological mass term there exists a kind of “square root” action for a massive vector, which turned out to be necessary for supersymmetry. Furthermore, in $D = 3$ there is also a topologically massive spin-2 extension of gravity [22] and a spin-$\frac{3}{2}$ extension of the Rarita-Schwinger field [32], which together constitute topologically massive supergravity [35]. In addition, this possibility of describing massive fields in an irreducible way, giving them one degree of freedom, has been generalized to fields
of arbitrary high spin \[36\]. One sees that these type of actions are much more natural for supersymmetry in \(D = 3\) and one should expect them to be of general importance. Keeping this in mind, similar partial breakings of supersymmetry or supergravity could be analyzed using the massive supermultiplets constructed in appendix B.

Acknowledgments

This work is supported by DFG – The German Science Foundation, the European RTN Programs HPRN-CT-2000-00148, HPRN-CT-2000-00122, HPRN-CT-2000-00131 and the DAAD – the German Academic Exchange Service.

We have greatly benefited from conversations with H. Jockers and especially H. Samtleben.

J.L. thanks E. Cremmer and the L.P.T.E.N.S. in Paris for hospitality and financial support during the final stages of this work.

Appendix

A Spinor conventions

We use a space-time metric with signature \(\eta_{\mu\nu} = \text{diag}(1, -1, -1)\) and a Clifford algebra representation given by

\[
\gamma^0 = -\sigma_2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \quad \gamma^1 = i\sigma_3 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad \gamma^2 = -i\sigma_1 = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}. \tag{A.1}
\]

Furthermore, we choose the charge-conjugation matrix to be

\[
C = \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tag{A.2}
\]

such that the defining relation

\[
C(\gamma^\mu)^i_C^{-1} = -\gamma^\mu \tag{A.3}
\]

is satisfied and the Majorana spinors are the real Dirac spinors. The Lorentz generators in the spinor representation are given by \(\gamma^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]\) and the group generated by these is simply \(SL(2, \mathbb{R})\), the double covering \(\text{Spin}(1, 2)\) of the Lorentz group.

The important relations of the super algebra are given by

\[
\{Q^I_\alpha, Q^J_\beta\} = 2(\gamma^\mu)_{\alpha\beta} P_\mu \delta^{IJ}, \tag{A.4}
\]

\[
[M^{\mu\nu}, Q^I_\alpha] = (\gamma^{\mu\nu})^\beta_\alpha Q^I_\beta, \tag{A.5}
\]

where the supercharges \(Q^I_\alpha\) are Majorana spinors, \(P_\mu\) denotes the energy-momentum operator and \(M^{\mu\nu}\) the Lorentz generators. Using (A.3), (A.1) can be written as

\[
\{Q^I_\alpha, Q^J_\beta\} = -2(\gamma^\mu C)_{\alpha\beta} P_\mu \delta^{IJ}. \tag{A.6}
\]
B Supermultiplets in $D = 3$

In this appendix we review the massive and massless supermultiplets in $D = 3$ for arbitrary $N$. Let us first mention that it is not straightforward to use the spin as organizing principle in $D = 3$. This is due to the fact that there is at least in the massless case no good concept of spin or helicity [37], which is reflected by the fact that half of the supercharges vanish. (See [26], where the supermultiplets in the massless case were constructed.) For completeness we give the massless multiplets in table 1, where $d_n$ denotes the number of bosonic and fermionic degrees of freedom separately.

| $N$ | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | $n + 8$ |
|-----|----|----|----|----|----|----|----|----|---------|
| $d_n$ | 1  | 2  | 4  | 4  | 8  | 8  | 8  | 8  | 16$d_n$ |

Table 1: Massless Supermultiplets according to [26]

In order to write a standard Lagrangian one also needs supermultiplets with no physical degree of freedoms. In the main text we use for $N = 2$

- **$N = 2$**
  - gravitational multiplet $(g_{\mu\nu}, \psi^I_\mu)$ (0 + 0)
  - vector multiplet $(A_\mu, \lambda^I, M)$ (2 + 2)
  - chiral multiplet $(\phi, \chi^I)$ (2 + 2)

where $I = 1, 2$. $\phi$ is a complex scalar while $M$ is a real scalar. The fermions are all real Majorana.

For $N = 1$ we use

- **$N = 1$**
  - gravitational multiplet $(g_{\mu\nu}, \psi_\mu)$ (0 + 0)
  - gravitino multiplet $(\psi_\mu, A_\mu)$ (1 + 1)
  - vector multiplet $(A_\mu, \lambda)$ (1 + 1)
  - chiral multiplet $(\varphi, \chi)$ (1 + 1)

$\varphi$ is real and all fermions are again real Majorana.

In the massive case the little group is $SO(2)$, the same as in the massless case in $D = 4$. Therefore we can adopt the representation theory and assert that the massive spin in $D = 3$ is the same as helicity in $D = 4$. The only difference is, that one cannot exclude continuous or anyonic spin states by topological considerations. Since half of the supercharges vanish in the massless $D = 4$ representations [30] leaving the same number of supercharges as for massive representations in $D = 3$ we expect that for any $N$ the supermultiplets are the same. This will be shown in the following.

To construct the massive multiplets, it is convenient to introduce the following linear combinations of the supercharges

$$ R^I := \frac{1}{\sqrt{2}} (Q^I_1 - iQ^I_2) , \quad (R^I)^\dagger := \frac{1}{\sqrt{2}} (Q^I_1 + iQ^I_2) . \quad (B.1) $$

These charges have well-defined spin-properties in the sense that they raise and lower the spin by an amount of $\frac{1}{2}$, which can be seen as follows. For the single spin operator $S$ in $D = 3$ one has by use of $S = M^{12}$ and \[A.5\]

$$ [S, R^I] = -\frac{1}{2} R^I , \quad [S, (R^I)^\dagger] = \frac{1}{2} (R^I)^\dagger , \quad (B.2) $$

17
from which we conclude, that for a state $|j\rangle$ with spin $j$, i.e. with $S|j\rangle = j|j\rangle$ one has

$$S(R^I)^\dagger |j\rangle = (j + \frac{1}{2})(R^I)^\dagger |j\rangle, \quad SR^I |j\rangle = (j - \frac{1}{2})R^I |j\rangle.$$  \hfill (B.3)

Therefore the operators $R^I$ and $(R^I)^\dagger$ transform bosons and fermions into each other and anyons into themselves.

In terms of $R^I$ and $(R^I)^\dagger$ the algebra (A.6) can be rewritten as

$$\{R^I, (R^J)^\dagger\} = 2\mathcal{P}_0 \delta^{IJ},$$
$$\{R^I, R^J\} = -2(P_2 - iP_1)\delta^{IJ},$$
$$\{(R^I)^\dagger, (R^J)^\dagger\} = -2(P_2 + iP_1)\delta^{IJ}. \hfill (B.4)$$

In the massive case we can boost into the rest frame $P_\mu = (m, 0, 0)$ such that after the rescaling $a^I := \frac{1}{\sqrt{2m}}R^I$ and $(a^I)^\dagger := \frac{1}{\sqrt{2m}}(R^I)^\dagger$ the algebra (B.4) reduces to the well known algebra of fermionic creation and annihilation operators

$$\{a^I, (a^J)^\dagger\} = \delta^{IJ}, \quad \{a^I, a^J\} = \{(a^I)^\dagger, (a^J)^\dagger\} = 0, \hfill (B.5)$$

where again $(a^I)^\dagger$ raises the spin by $\frac{1}{2}$ and $a^I$ lowers the spin by $\frac{1}{2}$. Now the construction of the supermultiplets is straightforward. We introduce the Clifford vacuum $\Omega$ defined by $a^I \Omega = 0$ for all $I = 1, ..., N$ and construct the representation by application of $(a^I)^\dagger$ in the standard fashion [30].

For $N = 1$ we get only the two linear independent states $\Omega$ and $a^I \Omega$. If $\Omega_j$ has spin $j$, we get a multiplet with spins $(j, j + \frac{1}{2})$. Similarly for $N = 2$ we get multiplets with spins $(j, j + \frac{1}{2}, j + \frac{1}{2}, j + 1)$. The multiplet structures are given in tab. 2. One sees that the massive $D = 3$ multiplets are for arbitrary $N$ identical to the massless $D = 4$ multiplets, as expected.

| Spin | $\Omega_0$ | $\Omega_{1/2}$ | $\Omega_1$ | $\Omega_{3/2}$ | Spin | $\Omega_0$ | $\Omega_{1/2}$ | $\Omega_1$ |
|------|---------|----------|---------|----------|------|---------|----------|---------|
| 0    | 1       | 1        | 1       | 1        | 0    | 1       | 1        | 1       |
| 1    | $\frac{1}{2}$ | 1        | 1        | 1        | $\frac{1}{2}$ | 2        | 1        | 1        |
| 1    | 1       | 1        | 1       | 1        | 1    | 1       | 1        | 2       |
| 2    | $\frac{3}{2}$ | 1        | 1        | 1        | $\frac{3}{2}$ | 1        | 2       |

Table 2: Massive Multiplets for $N = 1$ and $N = 2$

References

[1] S. Cecotti, L. Girardello and M. Porrati, “Two Into One Won’t Go,” Phys. Lett. B 145 (1984) 61; “Constraints On Partial Superhiggs,” Nucl. Phys. B 268 (1986) 295.

[2] I. Antoniadis, H. Partouche and T. R. Taylor, “Spontaneous Breaking of N=2 Global Supersymmetry”, Phys. Lett. B372 (1996) 83 [arXiv:hep-th/9512006].
[3] J. Bagger and A. Galperin, “Matter couplings in partially broken extended supersymmetry”, Phys. Lett. B336 (1994) 25 [arXiv:hep-th/9406217]; “A new Goldstone multiplet for partially broken supersymmetry”, Phys. Rev. D55 (1997) 1091 [arXiv:hep-th/9608177]; “Linear and nonlinear supersymmetries”, arXiv:hep-th/9810109; R. Altendorfer and J. Bagger, “Dual supersymmetry algebras from partial supersymmetry breaking,” Phys. Lett. B460 (1999) 127 [arXiv:hep-th/9904213]; S. Bellucci, E. Ivanov and S. Krivonos, “Superworldvolume dynamics of superbranes from non-linear realizations”, Phys. Lett. B482 (2000) 233 [arXiv:hep-th/0003273].

[4] S. Ferrara, L. Girardello and M. Porrati, “Minimal Higgs Branch for the Breaking of Half of the Supersymmetries in N=2 Supergravity”, Phys. Lett. B366 (1996) 155 [arXiv:hep-th/9510074]; “Spontaneous Breaking of N=2 to N=1 in Rigid and Local Supersymmetric Theories”, Phys. Lett. B376 (1996) 275 [arXiv:hep-th/9512180].

[5] J. Hughes and J. Polchinski, “Partially Broken Global Supersymmetry And The Superstring”, Nucl. Phys. B278 (1986) 147; J. Hughes, J. Liu and J. Polchinski, “Supermembranes”, Phys. Lett. B180 (1986) 370.

[6] E. Kiritsis and C. Kounnas, “Perturbative and non-perturbative partial supersymmetry breaking: N = 4 → N = 2 → N = 1”, Nucl. Phys. B503 (1997) 117 [arXiv:hep-th/9703059].

[7] T. Taylor and C. Vafa, “RR Flux on Calabi-Yau and Partial Supersymmetry Breaking”, Phys. Lett. B474 (2000) 130 [arXiv:hep-th/9912152].

[8] P. Mayr, “On supersymmetry breaking in string theory and its realization in brane worlds”, Nucl. Phys. B593 (2001) 99 [arXiv:hep-th/0003198].

[9] G. Curio, A. Klemm, D. Lüst and S. Theisen, “On the vacuum structure of type II string compactifications on Calabi-Yau spaces with H-fluxes”, Nucl. Phys. B609 (2001) 3 [arXiv:hep-th/0012213].

[10] P. Fre, L. Girardello, I. Pesando and M. Trigiante, “Spontaneous N = 2 → N = 1 local supersymmetry breaking with surviving compact gauge groups”, Nucl. Phys. B493 (1997) 231 [arXiv:hep-th/9607032].

[11] J. Louis, “Aspects of Spontaneous N = 2 → N = 1 Breaking in Supergravity,” arXiv:hep-th/0203138.

[12] L. Andrianopoli, R. D'Auria, S. Ferrara and M. A. Lledo, “Super Higgs effect in extended supergravity”, Nucl. Phys. B 640 (2002) 46 [arXiv:hep-th/0202116]; “Super Higgs effect in extended supergravity,” Nucl. Phys. B 640 (2002) 46 [arXiv:hep-th/0202116]; “Gauging of flat groups in four dimensional supergravity,” JHEP 0207 (2002) 010 [arXiv:hep-th/0203206]; “Gauged extended supergravity without cosmological constant: No-scale structure and supersymmetry breaking,” Mod. Phys. Lett. A 18 (2003) 1001 [arXiv:hep-th/0212141]; “N = 2 super-Higgs, N = 1 Poincare vacua and quaternionic geometry,” JHEP 0301 (2003) 045 [arXiv:hep-th/0212236].
L. Andrianopoli, S. Ferrara and M. Trigiante, "Fluxes, supersymmetry breaking and gauged supergravity," arXiv:hep-th/0307139.

[13] E. Ivanov and S. Krivonos, "N = 1 D = 4 supermembrane in the coset approach," Phys. Lett. B453 (1999) 237 arXiv:hep-th/9901003.

[14] K. Becker and M. Becker, "M-Theory on Eight-Manifolds," Nucl. Phys. B477 (1996) 155 arXiv:hep-th/9605053; "Supersymmetry breaking, M-theory and fluxes," JHEP 0107 (2001) 038 arXiv:hep-th/0107044.

[15] S. Gukov, C. Vafa and E. Witten, "CFT’s from Calabi-Yau four-folds," Nucl. Phys. B 584 (2000) 69 [Erratum-ibid. B 608 (2001) 477] arXiv:hep-th/9906070.

[16] K. Becker, G. Rajesh and S. Sethi, "M theory, orientifolds and G-flux," JHEP 9908 (1999) 023 arXiv:hep-th/9908088.

[17] M. Haack and J. Louis, "Duality in heterotic vacua with four supercharges," Nucl. Phys. B 575 (2000) 107 arXiv:hep-th/9912181; "M-theory compactified on Calabi-Yau fourfolds with background flux," Phys. Lett. B 507 (2001) 296 arXiv:hep-th/0103068.

[18] B. R. Greene, K. Schalm and G. Shiu, "Warped compactifications in M and F theory," Nucl. Phys. B 584 (2000) 480 arXiv:hep-th/0004103.

[19] K. Becker, M. Becker, M. Haack and J. Louis, "Supersymmetry breaking and alpha' -corrections to flux induced potentials," JHEP 0206 (2002) 060 arXiv:hep-th/0204254.

[20] M. Berg, M. Haack and H. Samtleben, "Calabi-Yau fourfolds with flux and supersymmetry breaking," JHEP 0304 (2003) 046 arXiv:hep-th/0212255.

[21] J. Schonfeld, "A Mass Term for Three-dimensional Gauge Fields," Nucl. Phys. B185 (1981) 157.

[22] S. Deser, R. Jackiw and S. Templeton, "Topologically Massive Gauge Theories," Ann. of Phys. 140 (1982) 372.

[23] S. Deser and R. Jackiw, "Self-Duality of Topological Massive Gauge Theories," Phys. Lett. B139 (1984) 371.

[24] P.K. Townsend, K.Pilch and P.Van Nieuwenhuizen, "Self-Duality in Odd Dimensions," Physics Letters 136B (1984) 38.

[25] S. Deser and Z. Yang, "A Remark on the Higgs Effect in Presence of Chern-Simons Terms," Mod. Phys. Lett. A4 (1989) 2123.

[26] B. de Wit, A. K. Tollsten and H. Nicolai, "Locally supersymmetric D = 3 nonlinear sigma models," Nucl. Phys. B392 (1993) 3 arXiv:hep-th/9208074.

[27] B. de Wit, I. Herger and H. Samtleben, "Gauged locally supersymmetric D = 3 nonlinear sigma models," Nucl. Phys. B671 (2003) 175 arXiv:hep-th/0307006.
[28] B. de Wit, H. Nicolai and H. Samtleben, “Gauged Supergravities in Three Dimensions: A Panoramic Overview,” arXiv:hep-th/0403014.

[29] H. Nicolai and H. Samtleben, “Chern-Simons vs. Yang-Mills gaugings in three dimensions,” Nucl. Phys. B 668 (2003) 167 arXiv:hep-th/0303213.

[30] For a review see, for example, J. Wess and J. Bagger, “Supersymmetry and Supergravity,” (2 ed.), Princeton University Press (1992).

[31] B. Gunara, “Spontaneous N=2 to N=1 Supersymmetry Breaking and the Super-Higgs Effect in Supergravity,” PhD-thesis, University of Halle (2003), http://www.desy.de/uni-th/stringth/Works/arbeiten.html.

[32] S. Deser, “Massive Spin 3/2 Theories in 3 Dimensions,” Phys. Lett. B140 (1984) 321.

[33] T. Fischbacher, H. Nicolai and H. Samtleben, “Vacua of Maximal Gauged $D = 3$ Supergravities,” Class. Quant. Grav. 19 (2002) 5297 arXiv:hep-th/0207206.

[34] H. Nicolai and H. Samtleben, “Kaluza-Klein supergravity on AdS$_3 \times S^3$,” JHEP 0309 (2003) 036 arXiv:hep-th/0306202.

[35] S. Deser and J.H. Kay, “Topologically Massive Supergravity,” Phys. Lett. B120 (1983) 97.

[36] I.V. Tyutin and M.A. Vasiliev, “Lagrangian formulation of irreducible massive fields of arbitrary spin in 2+1 dimensions,” Theor. Math. Phys. 113 (1997) 1244 arXiv:hep-th/9704132.

[37] B. Binegar, “Relativistic field theories in three dimensions”, J. Math. Phys. 23 (1982) 1511.