Cosmic string loops and large-scale structure

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We investigate the contribution made by small loops from a cosmic string network as seeds for large-scale structure formation. We show that cosmic string loops are highly correlated with the long-string network on large scales and therefore contribute significantly to the power spectrum of density perturbations if the average loop lifetime is comparable to or above one Hubble time. This effect further improves the large-scale bias problem previously identified in earlier studies of cosmic string models.

A. Introduction

Quantitative predictions for the large-scale structure induced by cosmic strings have taken some time to crystallise as the understanding of cosmic string physics has improved. In particular, the role of small loops produced by the string network has evolved from a potential one-to-one correspondence between loops and cosmological objects through to a completely subsidiary role relative to the wakes swept out by long strings. This dethronement of loops was a result of numerical studies which showed that the average loop size \( \ell \approx \alpha t \) was much smaller than the horizon, \( \ell \ll \ell_H \): they might even be as small as the lengthscale set by gravitational backreaction \( \alpha \sim 10^{-4} \), a value appropriate for GUT-scale strings. Add the high ballistic loop velocities observed \( \bar{v} \approx c/\sqrt{2} \) and it was not surprising that these tiny loops have been assumed to be more or less uniformly distributed and hence a negligible source relative to the long string network. Nevertheless, small loops always make up a significant fraction of the total string energy density at any one time and, as we demonstrate here, loop-induced inhomogeneities are considerable if their lifetime is not much smaller than the Hubble time. By properly incorporating these loop perturbations, we show that their contribution relative to the long string wakes is almost comparable and also highly correlated with these wakes.

The context for this work is a major programme of structure formation simulations seeded by high resolution cosmic string networks with very large dynamic ranges. This work demonstrated that for open or \( \Lambda \) models with \( \Gamma = \Omega_h = 0.1-0.2 \) and a cold dark matter (CDM) background, the linear density fluctuation power spectrum has both an amplitude at \( 8h^{-1}\text{Mpc} \), \( \sigma_8 \), and an overall shape which are consistent within uncertainties with those currently inferred from galaxy surveys. This result has also been confirmed using semi-analytical phenomenological models which incorporated some of the main features of long string networks.

In this letter we investigate the contribution of cosmic string loops to the linear power spectrum of cosmic string induced density perturbations. This component has been ignored and excluded in previous work, due to both the computational difficulties and assumptions about the homogeneity of the loop distribution. To this end we first perform very high resolution numerical simulations of a cosmic string network with a dynamic range extending from well before the radiation-matter transition through to deep into the matter era. We then use this network as a source for density perturbations (as described in ref. \cite{1}) taking into account the large-scale power contributed from cosmic string loops. This is done by modelling cosmic string loops smaller than a fixed fraction of the horizon size as relativistic point masses. The effects of the evaporation of these loops into gravitational waves and the damping of loop motion due to expansion are also included. Note, however, that this should be clearly distinguished from recent work \cite{2}, which attempts to incorporate network decay products in the power spectrum of an additional fluid (with a variety of possible equations of state). This does not appear to properly account for the phase correlation between long strings and moving loops.

Unless otherwise indicated, we use \( h = 0.7, \Omega_m = 1 \) and \( \Omega_\Lambda = 0 \) in the results presented here. A verified accurate rescaling scheme for the resulting power spectrum with different choices of \( h, \Omega_m \) and \( \Omega_\Lambda \) is straightforward and described in ref. \cite{3}.

B. Cosmic string and loop evolution

The Nambu equations of motion for cosmic strings in an expanding universe can be averaged to yield:

\[
\frac{d\rho_\infty}{dt} + 2H(1 + \langle v^2 \rangle)\rho_\infty = -X_L, \tag{1}
\]

where \( \rho_\infty \) is the long string energy, \( t \) the physical time, \( H = \dot{a}/a \) the Hubble parameter, \( a(t) \) the scale factor, \( \langle v^2 \rangle \) the mean square velocity of strings, and \( X_L \) the transfer rate of energy density from long strings into loops. In the scaling regime the long string energy density should scale with the background energy density evolving as...
\[ \frac{d\rho_\infty}{dt} = -2\frac{\rho_\infty}{t}. \]  

Substituting this into (1) to eliminate \( d\rho_\infty/dt \) gives

\[ \frac{tX_L}{\rho_\infty} = \begin{cases} (1 - \langle v_r^2 \rangle) \sim 0.6 & \text{in radiation era}, \\ \frac{3}{4}(1 - 2\langle v_m^2 \rangle) \sim 0.2 & \text{in matter era}, \end{cases} \tag{3} \]

where \( \langle v_r^2 \rangle \gtrsim \langle v_m^2 \rangle \sim 0.6 \). Both (2) and (3) provide a check for the scaling behavior of long strings and loops in the cosmic string network simulations.

We know that the loops produced by a cosmic string network will decay into gravitational radiation, with a roughly constant decay rate \( \Gamma G \mu^2 \), where \( \mu \) is the string linear energy density. Typically \( \Gamma = 50 - 100 \) with an average \( \langle \Gamma \rangle \sim 65 \). Now, if we assume the loop production to be 'monochromatic' so that all loops formed at the same time will have the same mass, we can write the initial rest mass of a loop formed at time \( t_* \) as

\[ M_L(t_*) = \alpha \mu t_* \equiv f \Gamma G \mu^2 t_* \tag{4} \]

Here, the parameter \( f = \alpha / \Gamma G \mu \) is expected to be of order unity if the size of the radiation or matter era is determined by gravitational radiation back-reaction, which smooths scales on smaller than \( \Gamma G \mu \). With the decay rate introduced earlier, we have the rest mass of a loop formed at time \( t_* \) evolving as

\[ M_L(t_* t) = M_L(t_*) \rho(t_* t) \tag{5} \]

where

\[ W(t_* t) = \begin{cases} 1 - \frac{t - t_*}{\tau(t_*)} & \text{for } t_* \leq t \leq t_* + \tau(t_*) \\ 0 & \text{otherwise} \end{cases} \tag{6} \]

Here \( \tau(t_*) \approx ft_* \) is the lifetime of loops produced at time \( t_* \) (\( f = 2, 3 \) implies the decay occurs in one horizon time in the radiation and matter era respectively). The evolution of the loop energy density is then given by:

\[ \rho_L(t) = \frac{\int_0^t X_L(t') \left[ \frac{a(t')}{a(t)} \right]^3 W(t',t) dt'}{f/t^2} \]

\[ \propto \begin{cases} f/t^2 & \text{for } f \ll 1 \; (\text{radiation era}), \\ \sqrt{f}/t^2 & \text{for } f \gg 1 \; (\text{matter era}), \end{cases} \tag{7} \]

where we have used the scaling behaviour of (2) and (3).

Consequently, the scaling of the power spectrum induced by loops in \( f \) should interpolate between \( f^2 \) and \( f \) (radiation era) or \( \ln(f)/t^2 \) (matter era). Notice in (7) that we have ignored the effect of loop velocity redshifting due to the expansion of the Universe, which causes a change in the effective mass. Because loops are formed with relativistic velocities, we expect this damping mechanism to have the strongest effect for \( f \gg 1 \), but to be negligible for \( f \ll 1 \).

\[ \frac{t}{t_*} \gtrsim 1 + \frac{f}{1 + \Gamma_p / (\Gamma \gamma v)} \tag{10} \]

which affects only the final stages of the loop lifetime as long as \( \Gamma_p / (\Gamma \gamma v) < 1 \), or equivalently \( v \gtrsim 0.15c \). For a typical \( v \sim c/\sqrt{2} \), one requires a loop lifetime \( \gtrsim 43t_* \) in the radiation era and \( \gtrsim 17t_* \) in the matter era to redshift down to this critical velocity according to (7).

Since the values of \( v \) we explore here are of order unity, it is a reasonable approximation to neglect the transfer of momentum due to gravitational radiation.

### C. Results and discussion

We first perform string simulations with a string sampling spacing 1/1000 of the simulation box sizes. The dynamic range cover from 0.05 to 300 \( \eta_{eq} \), where \( \eta_{eq} \) is the conformal time at radiation-matter energy density equality. We then perform the structure formation simulations with box sizes ranging from 20–120\( h^{-1}\)Mpc, and a
FIG. 2. Small dynamic range power spectra of density perturbations seeded by long strings (thick solid), by loops with initial velocities $v_\ast$ switched to zero (dot-dashed), and by loops with $v_\ast$ determined by string network evolution (dashed and thin solid). The thin solid line includes the effect of gravitational decay of the loop energy, while the other two loop lines don’t but with loops removed after a period of time $\tau_\ast = t_\ast$.

The resolution of $128^3 \sim 512^3$. Figure 2 shows the evolution of $X_L$. We can see that the expected amount of energy was converted into loops in our simulations so that $X_L$ has the correct asymptotic behavior given by (7). However, the typical loop-size (and consequently their lifetime) does not approach scaling so rapidly and is therefore larger than physically expected for most of the duration in the simulations (solid). To overcome this problem we rescale the loop lifetime according to equation (4). Thus the uncertainty in the average mass and therefore the lifetime of loops formed at a given time is quantified by the choice of the parameter $f$. The initial rms velocity of loops observed from the simulations is $\langle v_\ast^2 \rangle^{1/2} \approx 0.7c$ throughout all the regimes.

Figure 2 shows the power spectrum of density perturbations induced by long strings and by cosmic string loops for $f = 1$ for a small dynamic range from 2.5 to $5\sigma_{eq}$. We can see that when compared with the spectrum induced by static loops (dot dashed), the amplitude of small-scale perturbations induced by moving loops (dashed) is clearly reduced by their motion. However, their large-scale power is higher because of the dependence of the gravitational interaction on the loop velocities, especially when they are relativistic. We also see that the gravitational decay of loop energy (thin solid) damps the overall amplitude of the power spectrum (dashed) by about a factor of 3. We notice that between the long-string correlation scale $k_c \approx 20/v_\ast$ and the scale $k_L \approx 10k_c$, the slope of the long-string spectrum $n \approx -2.25$ is exactly the same as that of the long-string spectrum $n \approx -2.25$. We believe that this close correspondence is due to copious loop production being strongly correlated with long string intercommuting events and the collapse of highly curved long string regions (1), that is, near the strongest long string perturbations. Moreover, these correlations persist in time with the subsequent motion of loops and long strings lying preferentially in the same directions, a phenomenon which has been verified by observing animations of string network evolution. These correlations between loops and long strings, however, have a lower cutoff represented by the mean loop spacing $d_L \sim k_L^{-1}$. Below $d_L$, the effects of individual filaments swept out by moving loops can be identified. In terms of the power spectrum, for $k < k_L$ the loops are strongly correlated with the long strings and therefore reinforce the wake-like perturbations, while for $k > k_L$ their filamentary perturbations increase the spectral index by about one to $n \approx -1.25$; this change is expected on geometrical grounds.

In figure 3 we plot the power spectra of density perturbations seeded by long strings $P_\infty(k)$, by small loops

FIG. 3. The lower set of 3 lines are $P_L(k)$ for $f = 0.5$ (dotted), 1 (dot-dashed) and 2 (dashed). $P_\infty(k)$ is plotted as a solid line. The upper set of lines are $P_{tot}(k)$ with corresponding line styles and $f$ values to the lower set of lines.

FIG. 4. The correlation coefficient between the long-string and loop induced perturbations, with $f = 0.5, 1, 2, 4, 6$ (downwards).
F. Conclusion

In this Letter we have described the results of high-resolution numerical simulations of structure formation seeded by a cosmic string network with a large dynamic range, taking into account for the first time the loops produced by the network. We show that on large scales the loops behave like part of the long-string network and can therefore contribute significantly to the total power spectrum of density perturbations, provided their lifetime is not much smaller than one Hubble time. At present, the typical size and lifetime of loops formed by a string network remains to be studied in more detail; the problem is both computationally and analytically challenging. However, within the scale range of interest further developments in this area have the potential to affect the overall amplitude of the spectrum, while
leaving the shape largely unchanged. The results presented here provide further encouragement for more detailed work on both the nature of cosmic string evolution and the large-scale structures they induce in cosmologies with $\Gamma = \Omega h = 0.1$–0.2.

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[1] For a review see A. Vilenkin and E. P. S. Shellard, Cosmic strings and other topological defects (Cambridge University Press, 1994).
[2] A. Vilenkin, Phys. Rev. Lett. 46, 1169 (1981)
[3] J. Silk and A. Vilenkin, Phys. Rev. Lett. 53, 1700 (1984); Ya. B. Zel’dovich, Mon. Not. R. Astron. Soc. 192, 663 (1980).
[4] B. Allen and E. P. S. Shellard, Phys. Rev. Lett. 64, 119 (1990), E. P. S. Shellard and B. Allen, “On the Evolution of Cosmic Strings” in The Formation and Evolution of Cosmic Strings (Cambridge University Press: Cambridge) (1990).
[5] D. P. Bennett and F. R. Bouchet, Phys. Rev. D 41, 2408 (1990).
[6] A. Stebbins, “A New Picture for Cosmic String seeded structure formation” in The Formation and Evolution of Cosmic Strings (Cambridge University Press: Cambridge) (1990).
[7] P. P. Avelino, E. P. S. Shellard, J. H. P. Wu, B. Allen, Phys. Rev. Lett. 81, 2008 (1998)
[8] P. P. Avelino, E. P. S. Shellard, J. H. P. Wu, B. Allen, astro-ph/9803124 (Ap. J. Lett. in press)
[9] P. P. Avelino, E. P. S. Shellard, J. H. P. Wu, B. Allen, (in preparation).
[10] P. P. Avelino, R. R. Caldwell, and C. J. A. P. Martins Phys. Rev. D 56, 4568 (1997).
[11] R. A. Battye, J. Robinson and A. Albrecht, Phys. Rev. Lett. 80, 4847 (1998).
[12] C. Contaldi, M. Hindmarsh & J. Magueijo, astro-ph/9808209.
[13] P. P. Avelino, J. P. de Carvalho, astro-ph/9810362.
[14] R. J. Scerrer, J. M. Quashnock, D. N. Spergel and W. H. Press Phys. Rev. D 39, 371 (1989).
[15] B. Allen, E. P. S. Shellard Phys. Rev. D 45, 1898 (1992).
[16] T. Vachaspati, A. Vilenkin Phys. Rev. D 31, 3052 (1985).
[17] J. A. Peacock and S. J. Dodds, Mon. Not. R. Astron. Soc. 267, 1020 (1994).
[18] B. Allen, R. R. Caldwell, S. Dodelson, L. Knox, E. P. S. Shellard, and A. Stebbins, Phys. Rev. Lett. 79, 2624 (1997).
[19] B. Allen, R. R. Caldwell, E. P. S. Shellard, A. Stebbins and S. Veeraraghavan, Phys. Rev. Lett. 77, 3061 (1996).
[20] A. Albrecht, R. A. Battye, and J. Robinson, Phys. Rev. Lett. 79, 4736 (1997).
[21] P. P. Avelino, R. R. Caldwell Phys. Rev. D 53, 5339 (1996).