Application of complex transition density to nuclear reaction and effect of phase factor

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Abstract

Complex transition density can be constructed by a nuclear structure model with a complex basis and/or complex coefficient. In general, the complex transition density is converted to the real one with phase factor. In this study, we apply the complex transition density directly to the microscopic reaction model. We compare with scattering cross sections calculated with the real and complex transition densities in the frameworks of the optical model, the distorted wave Born approximation, and the coupled-channel (CC) calculation, respectively. In addition, we investigate the dependence of the phase factor for the transition density in the elastic and inelastic cross sections. The effect of the phase factor on the elastic and inelastic cross sections can be seen in the CC calculation. Finally, we found an important role of the phase factor in the nuclear elastic and inelastic scatterings.

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Introduction. In nuclear physics, the microscopic description has been developed to describe the nuclear structure and reaction with the development of science and technology. For the microscopic description of nuclear reactions, the folding model is one of the powerful and reliable models. By folding the effective nucleon-nucleon (NN) interaction with the nucleon density of the projectile and/or target nuclei, the interacting potential between the projectile and target particles is obtained. The folding models, which are called as the single folding model for nucleon-nucleus system and the double folding model for nucleus-nucleus system, are widely used to make clear exotic properties of nuclear structure, nuclear force, and nuclear reaction [1–16].

To describe the nuclear reaction with the single and double folding models, the nucleon density and the effective NN interaction have important role. Because the properties of the nucleus and the nuclear force are described by the nucleon density and the effective NN interaction, respectively. They are not independent, as they are intertwined with each other. Therefore, their properties are increasingly understood to be complementary. In such a situation, it is remarkable that the effective NN interaction has been developed based on the realistic NN interaction. For the nucleon-nucleus system, the complex G-matrix interaction based on the realistic NN interaction is applied to describe the nuclear reaction [10, 17–20]. On the other hand, the phenomenological approach is advanced to reproduce the experimental data and to understand the medium effect in colliding two nuclei [2, 21–23]. Recently, the complex effective NN interaction based on the realistic NN interaction has been applied to the nucleus-nucleus systems [12, 24, 25].

The construction of the nucleon density is also important to describe the nuclear reaction. At the first stage, the diagonal and transition densities are phenomenologically constructed by assuming the Woods-Saxon and its derivative forms, respectively [3]. With the development of science and technology, the nuclear structure model gives more realistic diagonal and transition densities [8, 26–32]. Here, the transition density can be complex when the complex basis and/or the complex coefficient are used in the microscopic nuclear structure model, but the diagonal density is real. In general, the complex transition density is converted to the real one with phase factor. It is well known that the phase factor for a state does not affect expectation value. Therefore, the phase is real and arbitrary. However, it is reported that the difference in the phase factor (the phase difference) between two quantum states can be observed as summarized in Ref. [33].

In this study, we apply the complex transition density directly to the microscopic reaction model. The microscopic reaction model is concretely applied to ${}^{10}\text{Be} + \text{p}$ elastic and inelastic cross sections at $E/A = 59.4$ MeV. The inelastic cross section is obtained by the complex transi-
tion (coupling) potential in the frameworks of the distorted wave Born approximation (DWBA) and the coupled-channel (CC) calculation. The complex transition (coupling) potentials are constructed from the real and complex transition densities with the complex effective $NN$ interaction. The calculated elastic and inelastic cross sections with the real and complex transition densities are compared with each other. In addition, we investigate the effect of the phase factor for the conversion of the transition density. To understand the effect of the phase factor on the cross sections, the simplified model is introduced. Finally, we find an important role of the phase factor in nuclear scatterings.

**Formalism.** To obtain the elastic and inelastic cross sections, we perform the optical model (OM), the DWBA and the CC calculation. In the calculations, the microscopic folding model potential is applied. Here, we briefly introduce the derivation of the folding model potential. The detail is introduced in Refs. [7, 10, 16, 34]. The folding model potential for the proton-nucleus system is described as

$$U_{\beta\alpha}(R) = \int \rho_{\beta\alpha}(r)u(s,\rho)dr,$$

(1)

where $s = r - R$. Here, we define the complex transition (coupling) potential, $U_{\beta\alpha}$, between the entrance (initial) channel $\alpha$ and the exit (final) channel $\beta$. When $\alpha$ and $\beta = 0$, the potential for the elastic channel is obtained. $\rho_{\beta\alpha}$ is the transition density between the initial state $\alpha$ and the final state $\beta$. If $\alpha = \beta$, the diagonal density is obtained. $u$ is the density-dependent $NN$ interaction. In this study, the complex $G$-matrix interaction is applied to obtain the complex potential between proton and nucleus. $\rho$ in $u$ is the local density to evaluate the medium effect. Eq. (1) is so-called the direct part of the central potential. We mention that the exchange part of the central potential and the spin-orbit potential are also obtained by the microscopic procedure in the same manner as Refs. [16, 34], but the transition density is replaced by the complex type. The Coulomb potential is obtained by the microscopic procedure with the proton density and the proton-proton Coulomb interaction. The transition part of the Coulomb potential is also complex when the folded transition density is complex.

Here, we introduce the phase factor for the transition density. The transition density obtained by the microscopic structure model can be complex. Almost complex transition densities can be converted to the real ones with reasonable phase factor while the values of the phase difference between two states are different for each [35]. With a phase difference $\theta_{\beta\alpha}$, the complex transition
density is rewritten as

\[ \rho_{\beta\alpha} \rightarrow e^{i\theta_{\beta\alpha}} \rho_{\beta\alpha}, \]

(2)

where \( e^{i\theta_{\beta\alpha}} \rho_{\beta\alpha} \) is real. The effect of the ambiguity of the sign of the real transition density on the elastic and inelastic cross sections will be discussed later. Namely, it means the difference of \( e^{i\theta_{\beta\alpha}} \rho_{\beta\alpha} \) and \( e^{i(\theta_{\beta\alpha}+\pi)} \rho_{\beta\alpha} \). Again, we mention that the diagonal density is real independent of the phase factor.

Results and discussion. In this paper, we apply the present microscopic folding model potential with real and complex transition densities to the \(^{10}\text{Be} + p\) elastic and inelastic scatterings at \( E/A = 59.4 \text{ MeV}\) in the frameworks of the OM, the DWBA, and the CC calculation. The relativistic kinematics is used to compute the cross sections. We apply the complex \( G\)-matrix interaction MPa \([36, 37]\) derived from the realistic \( NN\) interaction, ESC08 \([38]\) to the density-dependent \( NN\) interaction. With complex \( G\)-matrix interaction, the renormalization factor is often introduced to reproduce the experimental data. In this paper, we take the incident-energy-dependent renormalization factor, \( N_w = 0.5 + (E/A)/1000 \) \([39]\), in the folding calculation as

\[ u = v + iw \rightarrow v + iN_w w, \]

(3)

where \( v \) and \( w \) are the real and imaginary parts of the complex \( G\)-matrix interaction MPa, respectively. This means that we have no additional parameter to calculate the elastic and inelastic cross section in this paper. The elastic and inelastic cross sections are obtained in the standard way \([3]\).

The transition density of \(^{10}\text{Be}\) is taken from the version of \( V_{LS} = 2000 \text{ MeV}\) in Ref. \([16]\). This \(^{10}\text{Be}\) nucleus is constructed in the microscopic nuclear structure model based on the four-body (\( \alpha + \alpha + n + n \)) clusters. The ground (\( 0^+_1 \)), \( 2^+_1, 2^+_2 \), and \( 0^+_2 \) states are included in the present calculation. The complex transition density is obtained with the complex coefficient. In Ref. \([16]\), the real transition density converted from the complex one is applied to the nuclear reaction. However, we use both the real and complex transition densities in this paper.

Here, we show the results of the application of the complex transition density to the microscopic nuclear reaction model. The elastic cross sections are obtained by the OM and the CC calculation. The inelastic cross sections are obtained by the DWBA and the CC calculation. We focus on the comparison with the results of the real and complex transition densities. Figures \([1]\) and \([2]\) show the calculated elastic and inelastic cross sections with the real and complex transition densities for the \(^{10}\text{Be} + p\) system at \( E/A = 59.4 \text{ MeV}\). The experimental data for the elastic cross section and
FIG. 1: (a) Elastic cross section and inelastic cross sections of (b) \(2^+_1\), (c) \(2^+_2\), and (d) \(0^+_2\) states obtained by the OM and DWBA with the real and complex transition densities. The dotted and solid curves are obtained with the real and complex transition densities, respectively. The experimental data are taken from Refs. [40–42].

the inelastic cross section of the \(2^+_1\) state are well reproduced. The real and complex transition densities give consistent results as shown in Fig. 1. In the CC calculation, they also give consistent results, except for the inelastic cross section of the \(0^+_2\) state as shown in Fig. 2. We mention that the result of the \(0^+_2\) state by the CC calculation is not a numerical error. This minor discrepancy indicates an important nuclear reaction mechanism in the CC calculation. Because only the phase factor gives the discrepancy between the real and complex transition densities.

To make clear the mechanism of the CC result, we here introduce the constant phase difference \(\theta_c\) as,

\[
\rho_{\beta\alpha} \rightarrow e^{i\theta_c} \rho_{\beta\alpha}.
\]

By fixing the \(\theta_c\) value to be 0 (original complex transition density), \(\pi/3\), \(2\pi/3\), and \(\pi\), we investigate the effect of phase factor on the nuclear scatterings. Figures 3 and 4 show the calculated elastic and inelastic cross sections of the \(^{10}\)Be + p system at \(E/A = 59.4\) MeV. Again, the experimental data are well reproduced. In the frameworks of the OM and DWBA, the complex transition densities
with constant phase differences give consistent results as shown in Fig 3. It is obvious that the elastic cross sections are consistent because the OM potential independents on the phase factor. The inelastic cross section by the DWBA calculation is well-known to be described as

$$
\frac{d\sigma}{d\Omega} \propto | < \psi^{(-)}_\beta | U_{\beta0} | \psi^{(+)}_0 |^2,
$$

where $\psi^{(+)}_0$ and $\psi^{(-)}_\beta$ are the distorted waves for the entrance (elastic) and exit channels, respectively.

In this paper, we apply the complex transition density with the constant phase difference. Namely, the complex coupling potential is also affected by the constant phase difference $\theta_c$. However, the effect of the phase factor vanishes for the cross section even if $U_{\beta0}$ is replaced by $e^{i\theta_c} U_{\beta0}$ in Eq. (5).

On the other hand, the transition densities with the constant phase difference $\theta_c$ in the CC calculation give inconsistent results as shown in Fig 4. Not only the inelastic cross section but also the elastic cross section is affected by the phase factor. Here, we introduce a simplified model adopted in Refs. [5, 6, 14] to understand the result of the CC calculation. In this model, one assumes the same radial form factor to the real and imaginary parts of the complex coupling potential as

$$
U_{\beta0}(R) = (N_V + iN_W)f_{\beta0}(R),
$$

FIG. 2: Same as Fig. 1 but by the CC calculation.
FIG. 3: (a) Elastic cross section and inelastic cross sections of (b) $2^+_1$, (c) $2^+_2$, and (d) $0^+_2$ states obtained by the OM and DWBA with complex transition density including the constant phase $\theta_c$. The solid, dotted, dashed, dot-dashed curves are obtained with $\theta_c = 0$, $\pi/3$, $2\pi/3$, and $\pi$, respectively. The experimental data are taken from Refs. [40–42].

where $f_{\beta 0}(R)$ denotes the real radial form factor of the coupling potential. $N_V$ and $N_W$ represent the strength parameters for the real and imaginary parts of the complex coupling potential, respectively. The dynamical polarization potential (DPP) generated by the complex coupling between the elastic channel and various excited channels in the second-order perturbation theory will be written symbolically as

$$\Delta U_{\text{DPP}} = \sum_{\beta} U_{0\beta} \hat{G}^{(+)}_{\beta} U_{\beta 0}$$

$$= (N_V + iN_W)^2 \sum_{\beta} f_{0\beta} \hat{G}_{\beta} f_{\beta 0}$$

$$\equiv (N_V + iN_W)^2 (\Delta v + i\Delta w),$$

where $\hat{G}^{(+)}_{\beta}$ represents the Green function operator (propagator) in the $\beta$ channel. $\Delta v + i\Delta w$ is the complex DPP generated by the real coupling potential $f_{\beta 0}(R)$ with a unit strength. Following Refs. [3, 43, 44], the DPP generated by the real coupling potential having a moderate strength corresponding to the normal collective excitation is dominated by the imaginary part of an absorptive
nature ($\Delta w < 0$) and the real part of the DPP is rather small or even negligible, $|\Delta w| \gg |\Delta v| \approx 0$. Finally, the simplified complex coupling potential $U_{\beta 0}(R)$ generates the real and imaginary parts of the complex DPP as

$$\Delta V_{\text{DPP}} \simeq -2N_VN_W\Delta w,$$

$$\Delta W_{\text{DPP}} \simeq (N_V^2 - N_W^2)\Delta w,$$

in good approximation. This complex DPP tells us the effect of the coupling potential on the elastic cross section.

Again, we apply the complex transition density with the constant phase difference in this paper. Therefore, the constant phase difference $\theta_c$ replaces Eq. (6) by

$$e^{i\theta_c}U_{\beta 0}(R) = (N_V + iN_W)e^{i\theta_c}f_{\beta 0}(R).$$

FIG. 4: Same as Fig. 3 but by the CC calculation.
As a result, the constant phase difference for the transition density (potential) gives

\[ \Delta V_{\text{DPP}} \approx \left[ -2 N_V N_W \cos (2 \theta_c) + (N_V^2 - N_W^2) \sin (2 \theta_c) \right] \Delta w, \]

\[ \Delta W_{\text{DPP}} \approx \left[ (N_V^2 - N_W^2) \cos (2 \theta_c) - 2 N_V N_W \sin (2 \theta_c) \right] \Delta w, \]

for the complex DPP. Eqs. (13) and (14) clearly show the effect of the phase factor on the elastic cross section in the CC calculation. In addition, the effect of the elastic channel appears in the inelastic cross section. Here, we note that the DPP with \( \theta_c = \pi \) is consistent with the original DPP (\( \theta_c = 0 \)). It implies that the sign of the transition potential (density) does not affect the elastic and inelastic cross sections. In fact, the calculated cross sections with \( \theta_c = 0 \) and \( \pi \) are consistent with each other as shown in Fig. [4].

As the above, the effect of the constant phase difference on the elastic cross section is estimated through the DPP. However, the values of the phase difference between two states can be different. In fact, the values of the phase difference between two states in the conversion from the complex transition density to the real one are different for \(^{10}\)Be used in this paper. Therefore, the above estimation in the elastic cross section will be more complicated. It is considered that the complex coupling potential has a decisive role in the elastic cross section through the channel coupling. However, the origin of the imaginary part of the complex coupling potential is unclear. In Ref. [14], the author and his collaborator discussed the role of the imaginary part of the complex coupling potential in the inelastic cross section. In this paper, the effect of the phase factor, which can give the imaginary part of the complex coupling potential, is indicated on the cross sections. In addition, we mention the nucleon-nucleon interaction. In this paper, we use the complex \( G \)-matrix interaction MPa to investigate the effect of the phase factor on the nuclear scatterings. We confirm that the JLM interaction [17] also shows the effect of the phase factor on the elastic and inelastic cross sections. We also confirm the effect of the phase factor on the nucleus-nucleus scatterings based on the double folding model [14, 29, 45]. Here, we would like to discuss the imaginary part of the complex coupling potential from the viewpoint of the \( NN \) interaction. In the present calculation, we applied the complex \( G \)-matrix interaction to construct the diagonal and coupling potentials. Namely, the complex coupling potential is derived from the complex \( G \)-matrix interaction when the transition density is real. However, it is not clear that the origin of the imaginary part of the complex coupling potential is the same as that of the complex \( G \)-matrix interaction.
Namely, it is a possibility that the effect of the phase factor has an important role to obtain the complex coupling potential. Nuclear elastic and inelastic scatterings are useful to understand the role of the phase factor and the property of the $NN$ interaction for the transition.

**Summary.** We have constructed a microscopic folding model potential with the real and complex transition densities. The real transition densities were obtained by the complex one with the phase factor. The elastic and inelastic cross sections were obtained with the folding model potentials in the frameworks of the OM, the DWBA, and the CC calculation. In the frameworks of the OM and DWBA, the results with the real and complex transition density were consistent with each other. However, the real and complex transition densities in the CC calculation showed inconsistent results. To understand the situation, the constant phase difference was introduced to the transition density. The effect of the constant phase difference was demonstrated in the elastic and inelastic cross section. The mechanism of the effect of the phase factor on the elastic scattering was understood by introducing the simplified model. Finally, we found an important role of the phase factor in the nuclear elastic and inelastic scatterings.

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