TUNE DETERMINATION OF STRONGLY COUPLED BETATRON OSCILLATIONS IN A FAST-RAMPING SYNCHROTRON*

Y. Alexahin#, E. Gianfelice-Wendt, W. Marsh, K. Triplett, FNAL, Batavia, IL 60510, U.S.A.

Abstract
Tune identification - i.e. attribution of the spectral peak to a particular normal mode of oscillations - can present a significant difficulty in the presence of strong transverse coupling when the normal mode with a lower damping rate dominates spectra of Turn-by-Turn oscillations in both planes. The introduced earlier phased sum algorithm [1] helped to recover the weaker normal mode signal from the noise, but by itself proved to be insufficient for automatic peak identification in the case of close phase advance distribution in both planes. To resolve this difficulty we modified the algorithm by taking and analyzing Turn-by-Turn data for two different ramps with the beam oscillation excited in each plane in turn. Comparison of relative amplitudes of Fourier components allows for correct automatic tune identification. The proposed algorithm was implemented in the Fermilab Booster B38 console application and successfully used for tune, coupling and chromaticity measurements.

INTRODUCTION
Turn-by-turn (TBT) position measurements of oscillating beam can provide a wealth of information on the machine and beam parameters, such as tunes and optics functions, coupling and chromaticity. The starting point is finding the oscillation tunes.

There is a number of reasons making the betatron tune evaluation from the TBT Beam Position Monitor (BPM) data particularly difficult in fast ramping synchrotrons like the Fermilab Booster: rapid changes of the tunes (sometimes in the course of a few dozen turns), high level of noise, complicate spectra. In the following Section we discuss methods specifically developed for tune evaluation in such conditions: first we recall the phased sum algorithm [1] that made it possible to recover a weak signal from noise, and then introduce new algorithm – which we called a "two-ramps" algorithm – for automatic spectral peak identification.

Next we discuss implementation of these algorithms in the Fermilab Booster B38 console application and finally give examples of its usage for tune, coupling and chromaticity measurements.

BASIC THEORY
Continuous Fourier Transform
With a small number of turns $N$ available the resolution of the standard FFT ($1/N$) may be insufficient.

Much better precision can be achieved with the so-called Continuous Fourier Transform (CFT) method [2] which consists in evaluation of the sum

$$X(\nu) = \frac{1}{N} \sum_{n=1}^{N} e^{-2\pi i \nu x_n^k}$$

as a function of continuous variable $\nu$ and finding the maximum of $|X(\nu)|$.

In absence of random noise CFT provides precision $\sim 1/N^2$, while in its presence the precision is $\sim 1/N^{3/2}$ [1].

An alternative method – the Least Square Fit (LSF) of data with a number of sinusoids (two in our case) has the same tune error dependence on the number of turns [3].

BPM Phasing
If the betatron phase advance between BPMs does not differ too much from theoretical values $\phi^{(k)}_{x,y}$, $k$ being the BPM index, we may try to use information from all available BPMs constructing a phased sum

$$\bar{x}_n = \sum_k w_k x^{(k)}_n \exp[-i\phi^{(k)}_{x,y}]$$

for each turn $n$, where $w_k$ are some weights, and perform the CFT analysis using $\bar{x}_n$ [1]. Weights $w_k$ may reflect the quality of individual BPM data, here we set $w_k = 1$.

It is easy to verify that the proper part of the signal propagating with expected phase advance is amplified by a factor of $N_{BPM}$ whereas the alien modes and random noise are amplified only as $\sqrt{N_{BPM}}$ so that the signal to noise ratio is improved by a factor of $\sqrt{N_{BPM}}$ [1]. A more rigorous tune error estimate is:

$$\sigma_\nu \approx \frac{\sqrt{6\sigma}}{\pi a N^{1/2} N_{BPM}^{1/2}},$$

where $\sigma$ is the r.m.s. value of BPM errors and $a$ is the betatron oscillations amplitude.

Two-ramps Algorithm
In the presence of coupling both transverse mode peaks are present in the spectra of oscillations. The mode with lowest damping rate (e.g. due to low chromaticity) may dominate the spectra of oscillations in both planes making it difficult to attribute spectral peaks to particular modes.

To resolve this problem we propose the following algorithm:

- Take data for two ramps, one with horizontal pings on the beam, the other with vertical pings.
- Looking at the horizontal BPM data from a ramp with h-pings:
  - Find the highest spectral peak, determine the corresponding tune $Q_{11}$ and the damping constant, subtract the peak from the data (see Appendix).
  - Find the next highest peak tune $Q_{12}$.

* Work supported by Fermi Research Alliance, LLC under Contract DE-AC02-07CH11359 with the U.S. DOE.
# alexahin@fnal.gov
• Do the same for the vertical BPM data from v-pings to find tunes of the highest and next highest peaks $Q_{y1}, Q_{y2}$.
• If the main peaks are well separated, $|Q_{y1} - Q_{y2}| > \varepsilon$, where $\varepsilon \sim 1/N$, then simply: $Q_y = Q_{y1}$. $Q_y = Q_{y2}$.
• If they coincide, $|Q_{y1} - Q_{y2}| < \varepsilon$, then compare the ratios of secondary to primary peak heights in both spectra:
  - If $|X(Q_{y2})/X(Q_{y1})| > |Y(Q_{y2})/Y(Q_{y1})|$ then $Q_y = Q_{y2}$, $Q_y \approx Q_{y1}$.
  - If $|X(Q_{y2})/X(Q_{y1})| < |Y(Q_{y2})/Y(Q_{y1})|$ then $Q_y = Q_{y1} \approx Q_{y2}$.

To achieve a better precision in determination of secondary peaks the primary peaks can be subtracted from the spectra as discussed in Appendix. Alternatively, all peaks can be found with a simultaneous least-square fit which gives even better precision but is more difficult to implement in a console application.

Figure 1 gives an example of the spectrum of coupled oscillations of the Fermilab Booster beam which required the application of the described algorithm.

For each turn the horizontal and vertical BPM readings are separately combined according to equation (2) and for each ping horizontal and vertical CFT’s are computed. The tunes are calculated using the above described two-ramps algorithm. Thus the beam momentum is varied by shifting the closed orbit through the radial RF phase feedback. Another set of horizontal and vertical pinged data is acquired. Resorting to the theoretical dispersion the beam momentum relative variation, $\delta = \Delta p/p$, is computed from the closed orbit differences measured at the BPM’s and chromaticity, $\xi$, is obtained by the usual expression $\epsilon_{x,y} = \Delta Q_{x,y}/\delta$.

RECENT BOOSTER PROGRESS

The new version of B38 application made it possible to calibrate the skew quadrupole and sextupole circuits and correct coupling and chromaticity.

As concerns transverse coupling our goal was to correct the global coupling coefficient $C$. [4] through the whole ramp. The absolute value of this coefficient determines the closest tune approach which should be minimized for chromaticity measurements.

The Booster has 24 long and 24 short straights with large vertical and horizontal $\beta$-functions respectively. Correctors in the long and short straights can be used as two families of each kind – quadrupole, skew- quadrupole and sextupole – for global correction of the tunes, coupling and chromaticity respectively. The response to the skew-quad and sextupole families was found to be:

$$\Delta ReC_s = (0.0645 - 0.0660 - 0.0600) \cdot 3.183/Bp$$
$$\Delta ImC_s = ( -0.0137 - 0.0037 - 0.0037) \cdot 3.183/Bp$$

$$\Delta Q_x = (8.0 - 0.9 - 0.9 - 0.9) \cdot 3.183/Bp$$

$$\Delta Q_y = -(0.1 - 2.6 - 2.6 - 2.6) \cdot 3.183/Bp$$

with known calibration constants it was possible to reduce coupling to insignificant level in just two iterations. Figure 3 shows the values of coupling coefficient before and after correction.
It is interesting to note that after coupling correction the beam became prone to coherent instability which could have been suppressed by either moving tunes closer to each other (as shown in Fig.1) or increasing chromaticity. More information on the observed instability is presented at this Conference in a different paper [5].

REFERENCES
[1] Y. Alexahin, E. Gianfelice-Wendt, W. Marsh, Proc. IPAC’10, Kyoto, May 2010, p. 1179.
[2] R. Bartolini et al., Particle Accelerators, 56, 167-199 (1996).
[3] V. Lebedev, private communication.
[4] W. Marsh, Y. Alexahin, E. Gianfelice-Wendt, Proc. PAC’11 New York, March 2011, p. 516.
[5] Y. Alexahin, N. Eddy, E. Gianfelice-Wendt et al., this Conference, WEPPR085.

APPENDIX
To subtract the main peak from the spectrum we must determine its tune, amplitude, phase and damping constant.

Consider a model signal

\[ u_n = e^{2\pi i n \nu_0}, \quad n = 1, \ldots, N, \]  

(A.1)

with tune \( \nu_0 \) possibly complex. Its CFT is

\[ U(\nu) = \frac{1}{\sqrt{N}} \sin \frac{N\pi}{\sin \pi \nu} \]

(A.2)

For complex \( \chi = \chi' + i\chi'' \), \( |\chi| < 1 \), \( |F(\chi)| \) has a maximum at \( \chi' = 0 \) (or \( \nu = \text{Re} \nu_0 \)). Information on \( \text{Im} \nu_0 \) can be obtained from the derivative

\[ \left. \frac{\partial F}{\partial \chi' \left|_{\chi'} \right.} \right|_{\chi'} = -i(N+1)(N+1/3 \chi^2) + \ldots \]  

(A.3)

Now, given the (combined) signal \( \tilde{X}_n \), we can find the tune as the position of the maximum of \( \tilde{X}(\nu) \) and the damping constant, i.e., the imaginary part of \( \nu_0 \), from

\[ \text{Im} \nu_0 = \frac{3}{\pi (N-1)} \left[ 1 + \frac{1}{\pi (N+1)} \right] \text{Im} \left. \frac{\partial \log \tilde{X}}{\partial \nu} \right|_{\nu = \text{Re} \nu_0} \]  

(A.4)

Subtraction of the found main peak is performed as

\[ \tilde{X} = \tilde{X} - \frac{\langle u, \tilde{X} \rangle}{\langle u, u \rangle} \leq \frac{1}{N} \sum_{n=1}^{N} w_n u_n, \]  

(A.5)

and does not explicitly involve the amplitude and phase of TBT oscillations.

Figure 4 (color): Chromaticities as found 11/17/2011 (top) and after 02/08/2012 studies (bottom) with the central part of the ramp excluded around transition energy excluded. The spike in \( Q'_y \) in the lower plot is an artefact of measurement.