Analysis of the vertices $\rho NN$, $\rho \Sigma \Sigma$ and $\rho \Xi \Xi$ with light-cone QCD sum rules

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Abstract

In this article, we calculate the strong coupling constants of the $\rho NN$, $\rho \Sigma \Sigma$ and $\rho \Xi \Xi$ in the framework of the light-cone QCD sum rules approach. The strong coupling constants of the meson-baryon-baryon are the fundamental parameters in the one-boson exchange model which describes the baryon-baryon interactions successfully. The numerical values are in agreement with the existing calculations in part. The electric and magnetic $F/(F + D)$ ratios deviate from the prediction of the vector meson dominance theory, the $SU(3)$ symmetry breaking effects are very large.

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1 Introduction

One of the successful approaches in describing the two-baryon interactions is the one-boson exchange (or the Nijmegen soft-core potential) model [1, 2]. In this model, the baryon-baryon interactions are mediated by the intermediate mesons, such as the pseudoscalar octet mesons $\pi$, $K$, $\eta$, the vector octet mesons $\rho$, $K^*$, $\omega$ and the scalar octet mesons $\sigma$, $a_0$, $f_0$, etc. The strong coupling constants of the meson-baryon-baryon are the fundamental parameters, they have been empirically determined (or fitted) to reproduce the data of the nucleon-nucleon, hyperon-nucleon and hyperon-hyperon interactions. The strong coupling constants of the vector mesons with the octet baryons (thereafter we will denote them by $g_{VNN}$) can be written in term of the $\rho NN$ couplings and the electric (and magnetic) $F/(F + D)$ ratios. The vector meson dominance theory indicates that the electric $F/(F + D)$ ratio $\alpha_e$ be $\alpha_e = 1$ via the universal coupling of the $\rho$ meson to the isospin current [3].

It is important to determine those fundamental quantities directly from the quantum chromodynamics. Based on the assumption of the strong couplings between the quarks and vector mesons, the $g_{VNN}$ have been calculated in the external field QCD sum rules approach, while the coupling constants of the quark-meson were determined in some phenomenological models [4]. The strong coupling constants of the scalar mesons with the octet baryons have also been calculated in the external field QCD sum rules [5]. In the external field QCD sum rules, the operator product expansion is used to expand the time-ordered currents into a series of quark condensates, gluon condensates and vacuum susceptibilities which parameterize the...
long distance properties of the QCD vacuum and the non-perturbative interactions of the quarks and gluons with the external field \[6\].

In this article, we calculate the strong coupling constants of the \(\rho NN\), \(\rho \Sigma \Sigma\) and \(\rho \Xi \Xi\) in the framework of the light-cone QCD sum rules approach, and determine the electric (and magnetic) \(F/(F+D)\) ratios \(\alpha_e\) (and \(\alpha_m\)). The strong coupling constants of the \(\rho NN\) have been calculated with the light-cone QCD sum rules approach \[7\], we revisit this subject and obtain different predictions. Furthermore, the strong coupling constants of the pseudoscalar mesons with the octet baryons have also been calculated with the light-cone QCD sum rules \[8\]. The light-cone QCD sum rules approach carries out the operator product expansion near the light-cone \(x^2 \approx 0\) instead of the short distance \(x \approx 0\) while the non-perturbative hadronic matrix elements are parameterized by the light-cone distribution amplitudes which classified according to their twists instead of the vacuum condensates \[9\] \[10\]. The non-perturbative parameters in the light-cone distribution amplitudes are calculated by the conventional QCD sum rules and the values are universal \[11\].

The article is arranged as: in Section 2, we derive the strong coupling constants \(\rho NN\), \(\rho \Sigma \Sigma\) and \(\rho \Xi \Xi\) in the light-cone QCD sum rules approach; in Section 3, the numerical result and discussion; and in Section 4, conclusion.

## 2 Strong coupling constants \(\rho NN\), \(\rho \Sigma \Sigma\) and \(\rho \Xi \Xi\) with light-cone QCD sum rules

In the following, we write down the two-point correlation functions \(\Pi_i(p, q)\),

\[
\Pi_N(p, q) = i \int d^4 x e^{-i q \cdot x} \langle 0 | T \{ J_N(0) \bar{J}_N(x) \} | \rho(p) \rangle, \\
\Pi_\Sigma(p, q) = i \int d^4 x e^{-i q \cdot x} \langle 0 | T \{ J_\Sigma(0) \bar{J}_\Sigma(x) \} | \rho(p) \rangle, \\
\Pi_\Xi(p, q) = i \int d^4 x e^{-i q \cdot x} \langle 0 | T \{ J_\Xi(0) \bar{J}_\Xi(x) \} | \rho(p) \rangle,
\]

\[
J_N(x) = \epsilon^{abc} u_a^T(x) C \gamma_\mu u_b(x) \gamma_5 \gamma_\mu d_c(x), \\
J_\Sigma(x) = \epsilon^{abc} u_a^T(x) C \gamma_\mu u_b(x) \gamma_5 \gamma_\mu s_c(x), \\
J_\Xi(x) = \epsilon^{abc} s_a^T(x) C \gamma_\mu s_b(x) \gamma_5 \gamma_\mu u_c(x),
\]

where the baryon currents \(J_N(x)\), \(J_\Sigma(x)\) and \(J_\Xi(x)\) interpolate the octet baryons \(p\), \(\Sigma\) and \(\Xi\), respectively \[12\], the external state \(\rho_0\) has the four momentum \(p_\mu\) with \(p^2 = m_\rho^2\).

The vector meson \(V_\mu\) can couple with the vector current \(J_\mu\) with the following Lagrangian,

\[
\mathcal{L} = -g J_\mu(x) V^\mu(x),
\]
where the $g$ denotes the coupling constant. The form factors of the vector current between two octet baryons can be written as

$$
\langle N(p_1)|J_\mu(0)|N(p_2)\rangle = \overline{N}(p_1) \left\{ g_V \gamma_\mu + ig_T \frac{\sigma_{\mu\nu} p^\nu}{2m} + g_P \frac{p_\mu}{2m} \right\} N(p_2),
$$

$$
\int dx \langle N(p_1)|i\mathcal{L}(x)|N(p_2)\rho(p)\rangle = -i\overline{N}(p_1) \left\{ g_V \not\!q + ig_T \frac{\epsilon_\mu \sigma_{\mu\nu} p^\nu}{2m} + g_P \frac{p \cdot \epsilon}{2m} \right\} N(p_2),
$$

where the $m$ is the average value of the masses of the two octet baryons. In the limit $p^2 = m_\rho^2$, $\epsilon \cdot p = 0$, the form factors $g_V(p^2 = m_\rho^2)$ and $g_T(p^2 = m_\rho^2)$ are reduced to the strong coupling constants of the phenomenological Lagrangian,

$$
\mathcal{L} = -g_V \bar{\psi} \gamma_\mu \psi V^\mu + \frac{g_T}{4m} \bar{\psi} \sigma^{\mu\nu} \psi (\partial_\mu V_\nu - \partial_\nu V_\mu).
$$

According to the basic assumption of current-hadron duality in the QCD sum rules approach \[11\], we can insert a complete series of intermediate states with the same quantum numbers as the current operators $J_N(x)$, $J_\Sigma(x)$ and $J_\Xi(x)$ into the correlation functions $\Pi_i(p,q)$ to obtain the hadronic representation. After isolating the ground state contributions from the pole terms of the baryons $p$, $\Sigma$ and $\Xi$, we get the following results,

$$
\Pi_i(p,q) = \frac{\lambda_i^2}{[m_i^2 - (q + p)^2][m_i^2 - q^2]} \left\{ - [g_V + g_T] \frac{m_\rho^2}{2} \not\!q - g_V \epsilon \cdot q [2 \not\!q + \not\!p] \right\} + \cdots,
$$

$$
= \Pi_i^1(p,q) \not\!q + \Pi_i^2(p,q) \epsilon \cdot q \not\!q + \cdots,
$$

where the following definitions have been used,

$$
\langle 0|J_i(0)|N(p)\rangle = \lambda_i U(p),
$$

$$
U(p)\overline{U}(p) = \not\!p + m_N,
$$

here we use the notation $N$ to represent the octet baryons $p$, $\Sigma$ and $\Xi$. We have not shown the contributions from the single pole terms in Eq.(8) explicitly, they can be deleted completely after the double Borel transformation. In the original QCD sum rules analysis of the nucleon magnetic moments \[6\], the interval of dimensions (of the condensates) for the chiral odd structures is larger than the interval of dimensions for the chiral even structures, one may expect a better accuracy of the results obtained from the sum rules with the chiral odd structures. In this article, we choose the tensor structures $\not\!q$ and $\epsilon \cdot q \not\!q$ for analysis.

In the following, we briefly outline the operator product expansion for the correlation functions $\Pi_i(p,q)$ in perturbative QCD theory. The calculations are performed at the large space-like momentum regions $(q + p)^2 \ll 0$ and $q^2 \ll 0$, which correspond to the small light-cone distance $x^2 \approx 0$ required by the validity of the
distribution amplitudes of the \( \rho \) meson in the coordinate space.

The following Fierz re-ordering for one of the quark propagators in the correlation functions in Eqs. (11-13), perform

\[
\langle 0 | T[q_\alpha(x)\bar{q}_\beta(0)] | 0 \rangle = \frac{i\delta_{ab}}{2\pi^2x^4} - \frac{\delta_{ab}m_q}{4\pi^2x^2} - \frac{\delta_{ab}q} {12} \frac{m_q}{48} \bar{q}q + \frac{i\delta_{ab}x^2}{192} \langle \bar{q}g_s\sigma Gq \rangle + \frac{i\delta_{ab}}{1152} \frac{m_q}{32\pi^2x^2} \bar{q}g_s\sigma Gq \bar{q}q - \frac{i}{32\pi^2x^2} \frac{\lambda_{ab}^A}{2} G_{\mu\nu}(\not{\sigma}^{\mu\nu} + \sigma^{\mu\nu} \not{\not{x}}) + \cdots ,
\]

then contract the quark fields in the correlation functions \( \Pi_i(p,q) \) with the Wick theorem, and obtain the results

\[
\Pi_N(p,q) = 2\epsilon_{abc}\epsilon_{ijk} \int d^4x e^{-i\not{q}x} Tr \left[ \gamma_5\gamma^\mu D_{ck}(-x)\gamma^\nu \gamma_5 \right],
\]

\[
\Pi_S(p,q) = 2\epsilon_{abc}\epsilon_{ijk} \int d^4x e^{-i\not{q}x} Tr \left[ \gamma_5\gamma^\mu S_{ck}(-x)\gamma^\nu \gamma_5 \right],
\]

\[
\Pi_{\Xi}(p,q) = 2\epsilon_{abc}\epsilon_{ijk} \int d^4x e^{-i\not{q}x} Tr \left[ \gamma_5\gamma^\mu U_{ck}(-x)\gamma^\nu \gamma_5 \right],
\]

where the \( U \), \( D \) and \( S \) stand for the full propagators of the \( u \), \( d \) and \( s \) quarks respectively. Take the replacement

\[
U_{\alpha\beta}(-x) \rightarrow \langle 0 | u_\alpha^a(x)\bar{u}_\beta^b(x) | \rho(p) \rangle ,
\]

\[
D_{\alpha\beta}(-x) \rightarrow \langle 0 | d_\alpha^a(x)\bar{d}_\beta^b(x) | \rho(p) \rangle ,
\]

for one of the quark propagators in the correlation functions in Eqs.(11-13), perform the following Fierz re-ordering

\[
q_\alpha^a(0)\bar{q}_\beta^b(x) = -\frac{1}{12} \delta_{ab}\delta_{\alpha\beta}\bar{q}(x)q(0) - \frac{1}{12} \delta_{ab}(\gamma^\mu)_{\alpha\beta}\bar{q}(x)\gamma_\mu q(0) - \frac{1}{24} \delta_{ab}(\sigma^{\mu\nu})_{\alpha\beta}\bar{q}(x)\sigma_{\mu\nu}q(0) + \frac{1}{12} \delta_{ab}(i\gamma_5)_{\alpha\beta}\bar{q}(x)i\gamma_5 q(0) ,
\]

and substitute the hadronic matrix elements ( such as the \( \langle 0 | \bar{u}(x)\gamma_\mu u(0) | \rho(p) \rangle , \langle 0 | \bar{u}(x)u(0) | \rho(p) \rangle , \langle 0 | \bar{u}(x)\sigma_{\mu\nu}u(0) | \rho(p) \rangle , \) etc. ) with the corresponding light-cone distribution amplitudes of the \( \rho \) meson, finally we obtain the spectral densities at the coordinate space.

\(^2\)One can consult the second article of Ref.[11] for the technical details in deriving the full propagator.
In calculation, we can encounter some terms like \( x^\mu \langle 0| \bar{u}(x) \gamma_\mu d(0)|\rho(p)\rangle \), there are two approaches to deal with them,

\[
\langle 0| \bar{u}(x) \not{d}(0)|\rho(p)\rangle = f_\rho m_\rho \epsilon \cdot x \int_0^1 du \int_0^{\epsilon x} d\lambda C(\lambda),
\]

and

\[
\langle 0| \bar{u}(x) \not{d}(0)|\rho(p)\rangle = i f_\rho m_\rho \epsilon \cdot x \int_0^1 du \int_0^{\epsilon x} d\lambda C(\lambda)
\]

The two approaches can lead to different results, the analytical expressions of the second approach are more cumbersome, after the double Borel transformation and some technical details, we can prove that the two approaches are equal. In the following, we will present the analytical results of the first approach only. Once the spectral densities in the coordinate space are obtained, we can translate them to the momentum space with the \( D = 4 + 2\epsilon \) dimensional Fourier transformation,

\[
\sqrt{2}\Pi^1_N = -\frac{1}{2\pi^2} f_\rho m_\rho \int_0^1 du g_\perp^{(v)}(u) \frac{\Gamma(\epsilon - 1)}{(-Q^2)^{\epsilon - 1}}
\]

\[
+ \frac{1}{2\pi^2} f_\rho m_\rho^3 \int_0^1 du \int_0^u dt \int_0^t d\lambda C(\lambda) \frac{\Gamma(\epsilon)}{(-Q^2)^\epsilon}
\]

\[
+ \frac{2}{3} \langle \bar{q}q \rangle f_\rho^T m_\rho^2 \int_0^1 du h_\parallel^{(s)}(u) \frac{\Gamma(1)}{(-Q^2)^1}
\]

\[
- \frac{1}{12} \langle \alpha_s GG \rangle f_\rho m_\rho \int_0^1 du g_\perp^{(v)}(u) \frac{\Gamma(1)}{(-Q^2)^1}
\]

\[
- \frac{1}{6} \langle \bar{q}g_\sigma Gq \rangle f_\rho^T m_\rho^2 \int_0^1 du h_\parallel^{(s)}(u) \frac{\Gamma(2)}{(-Q^2)^2}
\]

\[
+ \frac{1}{12} \langle \alpha_s GG \rangle f_\rho m_\rho^3 \int_0^1 du \int_0^u dt \int_0^t d\lambda C(\lambda) \frac{\Gamma(2)}{(-Q^2)^2},
\]

\( ^3 \)Here we use the \( \rho^- \) meson to illustrate the calculation.
\[
\sqrt{2}\Pi_N^2 = \frac{1}{\pi^2} f_{\rho m_\rho}^3 \int_0^1 du \int_0^u dt \int_0^t d\lambda C(\lambda) \left( \frac{\Gamma(1)}{(-Q^2)^{1/3}} \right) \\
+ \frac{4}{3} \langle \bar{q} q \rangle f_{\rho}^T m_\rho^2 \int_0^1 du \left( h_{||}^{(s)}(u) \right) \frac{\Gamma(2)}{(-Q^2)^2} \\
- \frac{1}{3} \langle \bar{q} g_\sigma G q \rangle f_{\rho}^T m_\rho^2 \int_0^1 du \left( h_{||}^{(s)}(u) \right) \frac{\Gamma(3)}{(-Q^2)^3} \\
+ \frac{1}{6} \left( \frac{\alpha_s G}{\pi} \right) f_{\rho m_\rho}^3 \int_0^1 du \int_0^u dt \int_0^t d\lambda C(\lambda) \left( \frac{\Gamma(3)}{(-Q^2)^{1/3}} \right), \quad (20)
\]

\[
\sqrt{2}\Pi_S^1 = -\frac{1}{12\pi^2} f_{\rho m_\rho} \int_0^1 du \left[ \phi_{||}(u) + 6 g_{\perp}^{(v)}(u) \right] \frac{\Gamma(\epsilon - 1)}{(-Q^2)^{\epsilon - 1}} \\
+ \frac{1}{\pi^2} f_{\rho m_\rho}^3 \int_0^1 du \int_0^u dt \int_0^t d\lambda C(\lambda) \frac{\Gamma(\epsilon)}{(-Q^2)^\epsilon} \\
+ \frac{1}{16\pi^2} f_{\rho m_\rho}^3 \int_0^1 du A(u) \frac{\Gamma(\epsilon)}{(-Q^2)^\epsilon} \\
- \frac{1}{3} m_s \langle \bar{s} s \rangle f_{\rho m_\rho} \int_0^1 du \left[ \phi_{||}(u) + 2 g_{\perp}^{(v)}(u) \right] \frac{\Gamma(1)}{(-Q^2)^{1/3}} \\
+ \frac{2}{3} \langle \bar{q} q \rangle f_{\rho}^T m_\rho^2 \int_0^1 du \left( h_{||}^{(s)}(u) \right) \frac{\Gamma(1)}{(-Q^2)^{1/3}} \\
- \frac{1}{12} \left( \frac{\alpha_s G}{\pi} \right) f_{\rho m_\rho} \int_0^1 du \left[ 4 g_{\perp}^{(v)}(u) + \phi_{||}(u) \right] \frac{\Gamma(1)}{(-Q^2)^{1/3}} \\
+ \frac{1}{18} m_s \langle \bar{s} g_\sigma G s \rangle f_{\rho m_\rho} \int_0^1 du \phi_{||}(u) \frac{\Gamma(2)}{(-Q^2)^2} \\
+ \frac{1}{12} m_s \langle \bar{s} s \rangle f_{\rho m_\rho}^3 \int_0^1 du A(u) \frac{\Gamma(2)}{(-Q^2)^2} \\
- \frac{1}{6} \langle \bar{q} g_\sigma G q \rangle f_{\rho}^T m_\rho^2 \int_0^1 du \left( h_{||}^{(s)}(u) \right) \frac{\Gamma(2)}{(-Q^2)^2} \\
+ \frac{4}{3} m_s \langle \bar{s} s \rangle f_{\rho m_\rho}^3 \int_0^1 du \int_0^u dt \int_0^t d\lambda C(\lambda) \frac{\Gamma(2)}{(-Q^2)^2} \\
+ \frac{1}{12} \left( \frac{\alpha_s G}{\pi} \right) f_{\rho m_\rho}^3 \int_0^1 du \int_0^u dt \int_0^t d\lambda C(\lambda) \frac{\Gamma(2)}{(-Q^2)^2} \\
+ \frac{1}{288} \left( \frac{\alpha_s G}{\pi} \right) f_{\rho m_\rho}^3 \int_0^1 du A(u) \frac{\Gamma(2)}{(-Q^2)^2}, \quad (21)
\]
\[
\sqrt{2}\Pi_E^2 = -\frac{1}{6\pi^2} f_\rho m_\rho \int_0^1 du \phi_{\|}(u) \frac{\Gamma(\epsilon)}{(-Q^2)^\epsilon} \\
+ \frac{1}{8\pi^2} f_\rho m_\rho^3 \int_0^1 du A(u) \frac{\Gamma(1)}{(-Q^2)^1} \\
+ \frac{2}{\pi^2} f_\rho m_\rho^3 \int_0^1 du \int_0^u dt \int_0^t d\lambda C(\lambda) \frac{\Gamma(1)}{(-Q^2)^1} \\
- \frac{2}{5} m_s \langle \bar{s}s \rangle f_\rho m_\rho \int_0^1 du \phi_{\||}(u) \frac{\Gamma(2)}{(-Q^2)^2} \\
+ \frac{4}{3} \langle \bar{q}q \rangle f^T_\rho m_\rho^2 \int_0^1 du h_{\|}(s)(u) \frac{\Gamma(2)}{(-Q^2)^2} \\
- \frac{1}{36} \langle \frac{\alpha_sGG}{\pi} \rangle f_\rho m_\rho \int_0^1 du \phi_{\|}(u) \frac{\Gamma(2)}{(-Q^2)^2} \\
+ \frac{1}{9} m_s \langle s g_s \sigma G s \rangle f_\rho m_\rho \int_0^1 du \phi_{\|}(u) \frac{\Gamma(3)}{(-Q^2)^3} \\
+ \frac{1}{6} m_s \langle \bar{s}s \rangle f_\rho m_\rho^3 \int_0^1 du A(u) \frac{\Gamma(3)}{(-Q^2)^3} \\
- \frac{1}{3} \langle \bar{q}g_s \sigma G q \rangle f^T_\rho m_\rho^2 \int_0^1 du h_{\|}(s)(u) \frac{\Gamma(3)}{(-Q^2)^3} \\
+ \frac{8}{3} m_s \langle \bar{s}s \rangle f_\rho m_\rho^3 \int_0^1 du \int_0^u dt \int_0^t d\lambda C(\lambda) \frac{\Gamma(3)}{(-Q^2)^3} \\
+ \frac{1}{6} \langle \frac{\alpha_sGG}{\pi} \rangle f_\rho m_\rho^3 \int_0^1 du \int_0^u dt \int_0^t d\lambda C(\lambda) \frac{\Gamma(3)}{(-Q^2)^3} \\
+ \frac{1}{144} \langle \frac{\alpha_sGG}{\pi} \rangle f_\rho m_\rho^3 \int_0^1 du A(u) \frac{\Gamma(3)}{(-Q^2)^3}, \quad (22)
\]
\[
\sqrt{2} \Pi^1_\Xi = -\frac{1}{12 \pi^2} f_\rho m_\rho \int_0^1 du \phi_{\parallel}(u) \frac{\Gamma(\epsilon - 1)}{(-Q^2)^{\epsilon - 1}}
\]
\[
+ \frac{1}{16 \pi^2} f_\rho m_\rho^3 \int_0^1 du A(u) \frac{\Gamma(\epsilon)}{(-Q^2)^\epsilon}
\]
\[
+ \frac{1}{2 \pi^2} f_\rho m_\rho^3 \int_0^1 du \int_0^t dt \int_0^\lambda d\lambda C(\lambda) \frac{\Gamma(\epsilon)}{(-Q^2)^\epsilon}
\]
\[
- \frac{2}{3} m_s \langle \bar{s}s\rangle f_\rho m_\rho \int_0^1 du \left[ \phi_{\parallel}(u) - 2 g_{\perp}(u) \right] \frac{\Gamma(1)}{(-Q^2)^1}
\]
\[
+ \frac{1}{72} \left( \frac{\alpha_s GG}{\pi} \right) f_\rho m_\rho \int_0^1 du \left[ 2 g_{\perp}(u) - \phi_{\parallel}(u) \right] \frac{\Gamma(1)}{(-Q^2)^1}
\]
\[
+ \frac{1}{9} m_s \langle \bar{s}g_s Gs\rangle f_\rho m_\rho \int_0^1 du \phi_{\parallel}(u) \frac{\Gamma(2)}{(-Q^2)^2}
\]
\[
+ \frac{1}{6} m_s \langle \bar{s}s\rangle f_\rho m_\rho^3 \int_0^1 du A(u) \frac{\Gamma(2)}{(-Q^2)^2}
\]
\[
+ \frac{1}{288} \left( \frac{\alpha_s GG}{\pi} \right) f_\rho m_\rho^3 \int_0^1 du A(u) \frac{\Gamma(2)}{(-Q^2)^2}.
\]

where

\[
Q_\mu = q_\mu + up_\mu,
\]

the \( \epsilon \) is a small positive quantity, after taking the double Borel transformation, we can take the limit \( \epsilon \to 0 \). The light-cone distribution amplitudes \( \phi_{\parallel}(u), g_{\perp}(u), h_{\parallel}^{(s)}(u), C(u) \) and \( A(u) \) of the \( \rho \) meson are presented in the appendix.
the non-perturbative parameters in the light-cone distribution amplitudes are scale dependent, in this article, the energy scale is taken to be $\mu = 1 \text{GeV}$. Here we have neglected the contributions from the gluons $G_{\mu\nu}$, the contributions proportional to the $G_{\mu\nu}$ can give rise to three-particle (and four-particle) meson distribution amplitudes with a gluon (and quark-antiquark pair) in addition to the two valence quarks, their corrections are usually not expected to play any significant role.

Matching the hadronic representations in Eq.(8) with the corresponding ones in Eqs.(19-24) below the threshold $s_0^i$, then perform the double Borel transformation with respect to the variables $Q_1 = -q_2$ and $Q_2 = -(p + q)^2$ respectively, subtract the contributions from the continuum states,

$$B_{M_1}B_{M_2}\frac{\Gamma[n]}{[u(1-u)m_B^2 + (1-u)Q_1^2 + uQ_2^2]^n} = \frac{M^{2(2-n)}M_1M_2}{M_1^2M_2^2}e^{-\frac{u(1-u)m_B^2}{M_2^2}}\delta(u-u_0),$$

$$\frac{1}{M^2} = \frac{1}{M_1^2} + \frac{1}{M_2^2},$$

$$u_0 = \frac{M_1^2}{M_1^2 + M_2^2},$$

$$M^{2n} \to \frac{1}{\Gamma[n]}\int_0^{\infty} ds s^{n-1}e^{-\frac{s}{M^2}},$$

finally we obtain the following six sum rules for the strong coupling constants,

$$g_N^v = -\frac{1}{2\sqrt{2}\lambda_N^2}\frac{m_N^2 - m_Q^2}{M^2}e^{\frac{m_N^2 - m_Q^2}{M^2}}\left\{\frac{1}{\pi^2}M^2E_0(x)f_\rho m_\rho^3\int_0^{u_0} dt \int_0^\ell d\lambda C(\lambda) \right.$$  

$$+\frac{4}{3}\langle q\bar{q}\rangle f_\rho^T m_\rho^2 h^{(s)}(u_0) - \frac{1}{3M^2}\langle qg_\sigma Gq\rangle f_\rho^T m_\rho^2 h^{(s)}(u_0)$$

$$+\frac{1}{6M^2}\left(\frac{\alpha_sGG}{\pi}\right)f_\rho m_\rho^3\int_0^{u_0} dt \int_0^\ell d\lambda C(\lambda) \right\},$$

For examples, in the decay $B \to \chi_{c0}K$, the factorizable contribution is zero and the non-factorizable contributions from the soft hadronic matrix elements are too small to accommodate the experimental data \cite{17}; the net contributions from the three-valence particle light-cone distribution amplitudes to the strong coupling constant $g_{D_{1,2}D^*K}$ are rather small, about 20\% \cite{18}. The contributions of the three-particle (quark-antiquark-gluon) distribution amplitudes of the mesons are always of minor importance comparing with the two-particle (quark-antiquark) distribution amplitudes in the light-cone QCD sum rules. In our previous work, we study the four form-factors $f_1(Q^2)$, $f_2(Q^2)$, $g_1(Q^2)$ and $g_2(Q^2)$ of the $\Sigma \to n$ in the framework of the light-cone QCD sum rules approach up to twist-6 three-quark light-cone distribution amplitudes and obtain satisfactory results \cite{19}. In the light-cone QCD sum rules, we can neglect the contributions from the valence gluons and make relatively rough estimations.
\[ g^V_\Sigma = -\frac{1}{2\sqrt{2}\lambda_\Sigma^2} e^{\frac{m_\Sigma^2 - u_0(1 - u_0)m_\Sigma^2}{M^2}} \left\{ -\frac{1}{6\pi^2} M^4 E_1(x) f_\rho m_\rho \phi ||(u_0) \\
+ \frac{1}{8\pi^2} M^2 E_0(x) f_\rho m_\rho^3 A(u_0) \\
+ \frac{2}{\pi^2} M^2 E_0(x) f_\rho m_\rho^3 \int_0^{u_0} dt \int_0^t d\lambda C(\lambda) \\
- \frac{2}{3} m_\Sigma \langle \bar{s}s \rangle f_\rho m_\rho \phi ||(u_0) + \frac{4}{3} \langle \bar{q}q \rangle f_\rho^T m_\rho^2 h^{(s)}(u_0) \\
- \frac{1}{36} \langle \alpha_s G G \rangle f_\rho m_\rho \phi ||(u_0) \\
+ \frac{1}{9M^2} m_\Sigma \langle \bar{s}g_s \sigma G s \rangle f_\rho m_\rho \phi ||(u_0) \\
+ \frac{1}{6M^2} m_\Sigma \langle \bar{s}s \rangle f_\rho m_\rho^3 A(u_0) \\
- \frac{1}{3M^2} \langle \bar{q}g_s \sigma G q \rangle f_\rho^T m_\rho^2 h^{(s)}(u_0) \\
+ \frac{8}{3M^2} m_\Sigma \langle \bar{s}s \rangle f_\rho m_\rho^3 \int_0^{u_0} dt \int_0^t d\lambda C(\lambda) \\
+ \frac{1}{6M^2} \langle \alpha_s G G \rangle f_\rho m_\rho^3 \int_0^{u_0} dt \int_0^t d\lambda C(\lambda) \\
+ \frac{1}{144M^2} \langle \alpha_s G G \rangle f_\rho m_\rho^3 A(u_0) \right\}, \tag{27} \]

\[ g^V_\Xi = -\frac{1}{2\sqrt{2}\lambda_\Xi^2} e^{\frac{m_\Xi^2 - u_0(1 - u_0)m_\Xi^2}{M^2}} \left\{ -\frac{1}{6\pi^2} M^4 E_1(x) f_\rho m_\rho \phi ||(u_0) \\
+ \frac{1}{8\pi^2} M^2 E_0(x) f_\rho m_\rho^3 A(u_0) \\
+ \frac{1}{\pi^2} M^2 E_0(x) f_\rho m_\rho^3 \int_0^{u_0} dt \int_0^t d\lambda C(\lambda) \\
- \frac{4}{3} m_\Xi \langle \bar{s}s \rangle f_\rho m_\rho \phi ||(u_0) - \frac{1}{36} \langle \alpha_s G G \rangle f_\rho m_\rho \phi ||(u_0) \\
+ \frac{2}{9M^2} m_\Xi \langle \bar{s}g_s \sigma G s \rangle f_\rho m_\rho \phi ||(u_0) \\
+ \frac{1}{3M^2} m_\Xi \langle \bar{s}s \rangle f_\rho m_\rho^3 A(u_0) \\
+ \frac{1}{144M^2} \langle \alpha_s G G \rangle f_\rho m_\rho^3 A(u_0) \right\}, \tag{28} \]
\[ g^V_N + g^T_N = -\frac{\sqrt{2}}{\lambda_N^2 m_\rho^4} e^{\frac{m_N^2 - m_\rho^2 (1 - u_0)^2}{2 m_\rho^2} \frac{u_0}{t}} \left\{ -\frac{1}{2\pi^2} M^6 E_2(x) f_\rho m_\rho g_\perp^{(v)}(u_0) \\
+ \frac{1}{2\pi^2} M^4 E_1(x) f_\rho m_\rho \int_0^{u_0} dt \int_0^t d\lambda C(\lambda) \\
+ \frac{2}{3} M^2 E_0(x) (\bar{q} q) f_\rho m_\rho^2 h^{(s)}(u_0) \\
- \frac{1}{12} M^2 E_0(x) (\alpha_s G G) f_\rho m_\rho g_\perp^{(v)}(u_0) \\
- \frac{1}{6} (\bar{q} g_s \sigma G q) f_\rho m_\rho^2 h^{(s)}(u_0) \\
+ \frac{1}{12} (\alpha_s G G) f_\rho m_\rho^3 \int_0^{u_0} dt \int_0^t d\lambda C(\lambda) \right\} , \tag{29} \]

\[ g^V_\Sigma + g^T_\Sigma = -\frac{\sqrt{2}}{\lambda_\Sigma^2 m_\rho^4} e^{\frac{m_\Sigma^2 - m_\rho^2 (1 - u_0)^2}{2 m_\rho^2} \frac{u_0}{t}} \left\{ -\frac{1}{12\pi^2} M^6 E_2(x) f_\rho m_\rho \left[ \phi_\parallel(u_0) + 6 g_\perp^{(v)}(u_0) \right] \\
+ \frac{1}{16\pi^2} M^4 E_1(x) f_\rho m_\rho^3 A(u_0) \\
+ \frac{1}{\pi^2} M^4 E_1(x) f_\rho m_\rho^3 \int_0^{u_0} dt \int_0^t d\lambda C(\lambda) \\
- \frac{1}{3} M^2 E_0(x) m_s (\bar{s} s) f_\rho m_\rho \left[ \phi_\parallel(u_0) + 2 g_\perp^{(v)}(u_0) \right] \\
+ \frac{2}{3} M^2 E_0(x) (\bar{q} q) f_\rho m_\rho^2 h^{(s)}(u_0) \\
- \frac{1}{72} M^2 E_0(x) (\alpha_s G G) f_\rho m_\rho^3 \left[ 4 g_\perp^{(v)}(u_0) + \phi_\parallel(u_0) \right] \\
+ \frac{1}{18} m_s (\bar{s} g_s \sigma G s) f_\rho m_\rho \phi_\parallel(u_0) + \frac{1}{12} m_s (\bar{s} s) f_\rho m_\rho^3 A(u_0) \\
- \frac{1}{6} (\bar{q} g_s \sigma G q) f_\rho m_\rho^2 h^{(s)}(u_0) \\
+ \frac{4}{3} m_s (\bar{s} s) f_\rho m_\rho^3 \int_0^{u_0} dt \int_0^t d\lambda C(\lambda) \\
+ \frac{1}{12} (\alpha_s G G) f_\rho m_\rho^3 \int_0^{u_0} dt \int_0^t d\lambda C(\lambda) \\
+ \frac{1}{288} (\alpha_s G G) f_\rho m_\rho^3 A(u_0) \right\} , \tag{30} \]
\[ g_\perp + g_\parallel = \frac{\sqrt{2}}{\lambda_2 m_\rho^2} e^{m_\perp^2 - u_0 (1-u_0)m_\rho^2} \left\{ - \frac{1}{12\pi^2} M^6 E_2(x) f_\rho m_\rho \phi_\parallel(u_0) \right. \\
+ \frac{1}{16\pi^2} M^4 E_1(x) f_\rho m_\rho^3 A(u_0) \right. \\
+ \frac{1}{2\pi^2} M^4 E_1(x) f_\rho m_\rho^3 \int_0^{u_0} dt \int_0^t d\lambda C(\lambda) \right. \\
- \frac{2}{3} M^2 E_0(x) m_\perp \rho G \rho \left[ \phi_\parallel(u_0) - 2g_\perp(u_0) \right] \\
+ \frac{1}{72} M^2 E_0(x) \left( \frac{\alpha_s GG}{\pi} \right) f_\rho m_\rho \left[ 2g_\perp(u_0) - \phi_\parallel(u_0) \right] \\
+ \frac{1}{9} m_\perp sG_\rho G \rho f_\rho m_\rho \phi_\parallel(u_0) + \frac{1}{6} m_\perp sG \rho f_\rho m_\rho^3 A(u_0) \\
+ \frac{1}{288} \left( \frac{\alpha_s GG}{\pi} \right) f_\rho m_\rho^3 A(u_0) \right\}, \tag{31} \]

where

\[
E_n(x) = 1 - (1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!}) e^{-x},
\]

\[ x = \frac{s_0^0}{M^2}, \]

the \( s_0^0 \) are the threshold parameters.

## 3 Numerical result and discussion

The input parameters are taken as (\( \bar{q}q \)) = -(0.24 \pm 0.01 GeV)^3, (\( \bar{s}s \)) = (0.8 \pm 0.1)(\( \bar{q}q \)), (\( \bar{q}gGq \)) = m_0^2(\( \bar{q}q \)), (\( \bar{s}gG \rho G \rho \)) = m_0^2(\( \bar{s}s \)), m_0^2 = (0.8 \pm 0.1) GeV^2, (\( \frac{\alpha GG}{\pi} \)) = (0.33 GeV)^4, m_u = 0, m_d = 0, m_s = (140 \pm 10) MeV [9 10 11], \( f_\rho = (0.198 \pm 0.007) \text{GeV} \), \( f_\perp = (0.160 \pm 0.010) \text{GeV} \), \( a_1^2 = 0.20 \pm 0.10 \), \( a_2^2 = 0.18 \pm 0.10 \), \( s_3^3 = 0.032 \pm 0.010 \), \( s_4^4 = 0.15 \pm 0.10 \), \( s_5^5 = 0.10 \pm 0.05 \), \( s_6^6 = -0.10 \pm 0.05 \), \( \omega_3^3 = -2.1 \pm 1.0 \), \( \omega_5^5 = 3.8 \pm 1.8 \), \( \omega_7^7 = 7.0 \pm 7.0 \) [13], \( m_\rho = 0.77 \text{GeV} \), \( m_\rho = 0.938 \text{GeV} \), \( m_\Sigma = 1.189 \text{GeV} \), \( M_\Xi = 1.315 \text{GeV} \), \( \lambda_\rho = (2.4 \pm 0.2) \times 10^{-2} \text{GeV}^3 \), \( \lambda_\Sigma = (3.2 \pm 0.2) \times 10^{-2} \text{GeV}^3 \), \( \lambda_\Xi = (3.8 \pm 0.2) \times 10^{-2} \text{GeV}^3 \), \( s_0^0 = 2.3 \text{GeV}^2 \), \( s_0^0 = 3.2 \text{GeV}^2 \) and \( s_0^0 = 3.6 \text{GeV}^2 \) [15]. The values of the vacuum condensates have been updated with the experimental data for the \( \tau \) decays, the QCD sum rules for the baryon masses and analysis of the charmonium spectrum [20 21 22], in this article, we choose the standard (or old) values to keep in consistent with the sum rules used in determining the non-perturbative parameters in the light-cone distribution amplitudes. The Borel parameters are chosen as \( M_1^2 = M_2^2 = (2-4) \text{GeV}^2 \) and \( M_2^2 = (1-2) \text{GeV}^2 \), in those regions, the values of the strong coupling constants \( g_\perp, g_\parallel, g_\perp \) are rather stable with the variation of the Borel parameter \( M^2 \) from the sum rules in Eqs.(26-28), while the
sum rules for the $g_N^V + g_N^T$, $g_S^V + g_S^T$ and $g_Z^V + g_Z^T$ in Eqs.(29-31) are not as stable as the corresponding ones for the $g_N^V$, $g_S^V$ and $g_Z^V$, which are shown in the Figs.(1-3).

Taking into account all the uncertainties, finally we obtain the numerical results of the strong coupling constants $g_N^V$, $g_S^V$, $g_Z^V$, $g_N^T$, $g_S^T$ and $g_Z^T$, which are shown in the Figs.1-3,

\[
\begin{align*}
g_N^V &= 3.2 \pm 0.9, \\
g_S^V &= 4.0 \pm 1.0, \\
g_Z^V &= 1.5 \pm 1.1, \\
g_N^T &= 36.8 \pm 13.0, \\
g_S^T &= 53.5 \pm 19.0, \\
g_Z^T &= -5.3 \pm 3.3. 
\end{align*}
\]

The numerical values are in agreement with the existing calculations in part, for examples, the values of the external field QCD sum rules ($g_N^V = 2.4 \pm 0.6$, $g_S^V = 4.8 \pm 1.2$ $g_Z^V = 2.4 \pm 0.6$, $g_N^T = 7.7 \pm 1.9$, $g_S^T = 2.3 \pm 0.4$, $g_Z^T = -5.0 \pm 1.0$) [4] and the phenomenologically fitted values of the one-boson exchange model ($g_N^V = 3.0$, $g_S^V = 5.9$ $g_Z^V = 3.0$, $g_N^T = 12.5$, $g_S^T = 9.1$, $g_Z^T = -3.4$) [2]. In calculation, we observe that the main contributions come from the perturbative terms and the quark condensates terms (\(\langle \bar{q}q \rangle\) and \(\langle \bar{s}s \rangle\)), the contributions of the mixed condensates and gluon condensates are of minor importance. It is not un-expected. From the Table.1, we can see that the contributions from the terms of quark condensates are comparable with the perturbative terms in the region $M^2 = (1-2)GeV^2$. The presence of the chiral symmetry breaking quark condensates and the existence of the massive baryons are closely related to each other, $m_p \simeq -\frac{8\pi^2\langle \bar{q}q \rangle}{M^2}$ [12], the contributions from the quark condensates (of order $O(M^2)$ or $O(M^0)$) in the sum rules in Eqs.(26-31) are of great importance, the subtractions of the continuum states can be implemented by the simple replacement $M^{2n} \to M^{2n} E_{n-1}(x)$ for $n \geq 1$, we have to introduce some unknown parameters $C_A$ and $C_B$ to subtract the contributions from the continuum states of order $O(M^0)$ with $g_V \to g_V + C_A$ and $g_V + g_T \to g_V + g_T + C_B$, however, it is difficult to obtain the values of the $C_A$ and $C_B$. In the region $M^2 = (1-2)GeV^2$, $\frac{\alpha_s(M)}{\pi} \sim 0.14 - 0.18$ [22], if the radiative $\alpha_s$ corrections to the perturbative terms are companied with large numerical factors, just like in the case of the QCD sum rules for the masses of the proton [21], the contributions of order $O(\alpha_s)$ may be large, neglecting them can impair the predictive power. The contributions of the three-particle (quark-quark-gluon) light-cone distribution amplitudes may be of the same order as the mixed condensates (\(\langle \bar{q}g_s\sigma Gq \rangle\) and \(\langle \bar{s}g_s\sigma Gs \rangle\)), neglecting them would not impair the predictive power much. The consistent and complete light-cone QCD sum rules analysis should include the contributions from the perturbative $\alpha_s$ corrections, the distribution amplitudes with additional valence gluons and quark-antiquark pairs, the reasonable subtractions of the continuum states of order $O(M^0)$, and improve the parameters which enter in the light-cone QCD sum rules, that may be our next work.
Figure 1: The $g_N^V$ and $g_N^V + g_T^N$ with the Borel parameter $M^2$.

Figure 2: The $g_{\Sigma}^V$ and $g_{\Sigma}^V + g_{\Sigma}^T$ with the Borel parameter $M^2$. 
\[ g_\Sigma = 2\alpha g_N, \]
\[ g_\Xi = (2\alpha - 1)g_N, \]

where we use the same notation \( \alpha \) to present the electric and magnetic ratios \( \alpha_e \) and \( \alpha_m \). Our numerical values \( \alpha_e \approx 0.6 - 0.7 \) and \( \alpha_m \approx 0.4 - 0.7 \) deviate greatly from the prediction of the vector meson dominance theory, \( \alpha_e = 1 \) [3]. The numerical values of the \( g_\Sigma, g_\Xi, g_{\Sigma}, g_{\Xi}, g_T, g_T \) do not obey the simple relation obtained from the group theory in Eq.(33), the \( SU(3) \) symmetry breaking effects are very large.

### 4 Conclusion

In this article, we calculate the strong coupling constants \( g_\Sigma, g_\Xi, g_\Sigma, g_{\Sigma}, g_{\Xi}, g_T, g_T \) of the \( \rho NN, \rho \Sigma \Sigma \) and \( \rho \Xi \Xi \) in the framework of the light-cone QCD sum rules approach. The strong coupling constants of the meson-baryon-baryon are the fundamental
parameters in the one-boson exchange model which describes the baryon-baryon interactions successfully. The numerical results are in agreement with the existing calculations in part, for examples, the predictions of the external field QCD sum rules and the phenomenologically fitted values of the one-boson exchange model. The electric and magnetic $F/(F + D)$ ratios deviate greatly from the prediction of the vector meson dominance theory, the $SU(3)$ symmetry breaking effects are very large. Those deviations may be due to the failure to take into account the perturbative $\alpha_s$ corrections, the un-subtracted contributions from the continuum states of order $\mathcal{O}(M^0)$, and the neglected distribution amplitudes with additional valence gluons and quark-antiquark pairs. The consistent and complete light-cone QCD sum rules analysis should include the contributions from the perturbative $\alpha_s$ corrections, the distribution amplitudes with additional valence gluons and quark-antiquark pairs, the reasonable subtractions of continuum states of order $\mathcal{O}(M^0)$, and improve the parameters which enter in the light-cone QCD sum rules.

Appenidix

The light-cone distribution amplitudes of the $\rho$ meson are defined by

\[
\langle 0 | \bar{u}(x) \gamma_\mu d(0) | \rho(p) \rangle = p_\mu f_\rho m_\rho \frac{\epsilon \cdot x}{p \cdot x} \int_0^1 du e^{-i u p \cdot x} \left\{ \phi_\parallel(u) + \frac{m_\rho^2 x^2}{16} A(u) \right\} \\
+ \left[ \epsilon_\mu - p_\mu \frac{\epsilon \cdot x}{p \cdot x} \right] f_\rho m_\rho \int_0^1 du e^{-i u p \cdot x} g_\perp^{(v)}(u) \\
- \frac{1}{2} x_\mu \frac{\epsilon \cdot x}{(p \cdot x)^2} f_\rho m_\rho^3 \int_0^1 du e^{-i u p \cdot x} C(u), \quad (34)
\]

\[
\langle 0 | \bar{u}(x)d(0) | \rho(p) \rangle = -\frac{i}{2} \left[ f_\rho^T - f_\rho \frac{m_u + m_d}{m_\rho} \right] m_\rho^2 \epsilon \cdot x \int_0^1 du e^{-i u p \cdot x} h_\parallel^{(s)}(u). \quad (35)
\]
The light-cone distribution amplitudes are parameterized as \[13\]

\[
\begin{align*}
\phi_{\parallel}(u, \mu) &= 6u(1-u) \left\{ 1 + a_2^{\parallel} \frac{3}{2} (5\xi^2 - 1) \right\}, \\
g_{\perp}^{(v)}(u, \mu) &= \frac{3}{4} (1 + \xi^2) + \left\{ \frac{3}{7} a_2^{\parallel} + 5 \varsigma_3 \right\} (3\xi^2 - 1) \\
&\quad + \left\{ \frac{9}{112} a_2^{\parallel} + \frac{15}{64} \varsigma_3 (3\omega^V_3 - \omega^A_3) \right\} (3 - 30\xi^2 + 35\xi^4), \\
g_3(u, \mu) &= 1 + \left\{ -1 - \frac{2}{7} a_2^{\parallel} + \frac{40}{3} \varsigma_3 - \frac{20}{3} \varsigma_4 \right\} C_2^1(\xi) \\
&\quad + \left\{ -\frac{27}{28} a_2^{\parallel} + \frac{5}{4} \varsigma_3 - \frac{15}{16} \varsigma_3 (\omega^A_3 + 3\omega^V_3) \right\} C_4^1(\xi), \\
h_{\parallel}^{(s)}(u, \mu) &= 6u(1-u) \left\{ 1 + \left( \frac{1}{4} a_2^{\parallel} + \frac{5}{8} \varsigma_3 \omega^T_3 \right) (5\xi^2 - 1) \right\}, \\
A(u, \mu) &= 30u^2(1-u)^2 \left\{ \frac{4}{5} + \frac{4}{105} a_2^{\parallel} + \frac{20}{9} \varsigma_4 + \frac{8}{9} \varsigma_3 \right\}, \\
C(u, \mu) &= g_3(u, \mu) + \phi_{\parallel}(u, \mu) - 2g_{\perp}^{(v)}(u, \mu),
\end{align*}
\]

where the $\xi = 2u - 1$, and the $C_2^1$ and $C_4^1$ are Gegenbauer polynomials \[13\].

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