Implementation of holonomic quantum computation through engineering and manipulating environment

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We consider an atom-field coupled system, in which two pairs of four-level atoms are respectively
driven by laser fields and trapped in two distant cavities that are connected by an optical fiber. First,
we show that an effective squeezing reservoir can be engineered under appropriate conditions. Then,
we show that a two-qubit geometric CPHASE gate between the atoms in the two cavities can be
implemented through adiabatically manipulating the engineered reservoir along a closed loop. This
scheme that combines engineering environment with decoherence-free space and geometric phase
quantum computation together has the remarkable feature: a CPHASE gate with arbitrary phase
shift is implemented by simply changing the strength and relative phase of the driving fields.

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I. INTRODUCTION

Quantum computation, attracting much current interest since Shor’s algorithm [1] was proposed, depends on
two key factors: quantum entanglement and precision control of quantum systems. Unfortunately, quantum
systems are inevitably coupled to their environment so that entanglement is too fragile to be retained. This
makes the realization of quantum computation extremely difficult in the real world. In order to overcome this
difficulty, one proposed the decoherence-free space concept [2, 3]. It is found that when qubits involved in quantum
computation collectively interact with a same environment there exists a “protected” subspace in the entire
Hilbert space, in which the qubits are immune to the decoherence effects induced by the environment. This
subspace is called decoherence-free space (DFS). To perform quantum computation in a DFS, one has to design
the specific Hamiltonian containing controlling parameters, which eigenspace is spanned by DFS states and the
state-unitary manipulation related to quantum computation goal is implemented by changing the controlling
parameters [4].

As well known, instantaneous eigenstates of a quantum system with the time-dependent Hamiltonian may
acquire a geometric phase when the time-dependent parameters adiabatically undergo a closed loop in the parameter
space [5]. The phase depends only on the swept solid angle by the parameter vector in the parameter space. This
feature can be utilized to implement geometric quantum computation (GQC) which is resilient to stochastic con-
trol errors [6, 7, 8]. On combining the DFS approach with the GQC scheme, one may build quantum gates
which may be immune to both the environment-induced decoherence effects and the control-led errors [9]. In the
scheme, quantum logical bits are represented by degenerate eigenstates of the parameterized Hamiltonian. These
states have the features: they belong to DFS, and unitarily evolve in time and acquire a geometric phase when the
controlling parameters adiabatically vary and undergo a closed loop.

In the recent paper [10], Carollo and coworkers showed that a cascade three-level atom interacting with a broad-
band squeezed vacuum bosonic bath can be prepared in a state which is decoupled to the environment. This state
depends on the reservoir parameters such as squeezing degree and phase angle. As the squeezing parameters
smoothly vary, the atomic state can unitarily evolve in time and always be in the manifold of the DFS. More-
over, after a cyclic evolution of the squeezing parameters, the state acquires a geometric phase. This investigation
has been generalized to cases where both quantum systems and manipulated reservoir under consideration are
not restricted to cascade three-level atoms and squeezed vacuum [11]. These results strongly inspire us that in-
stead of engineering Hamiltonian one may implement the decoherence-free GQC by engineering and manipulating
reservoir.

In this paper, we propose a scheme in which the quantum-reservoir engineering [12, 13, 14] is combined
with DFS and Berry phase together to realize a two-qubit CPHASE gate [15]. We show that atomic states
can unitarily evolve in time in a DFS if the change rate of reservoir parameters is much smaller than the charac-
teristic relaxation time of an atom-reservoir coupled system. Moreover, we find that as the reservoir parameters
adiabatically change in time along an appropriate closed loop, the atomic state in the DFS acquires a Berry phase
and a CPHASE gate with arbitrary phase shift can be realized. To our knowledge, it is the first proposal for the
realization of quantum gates by engineering and steering the environment.

This paper is organized as follows. In Sec. II, we introduce a cavity-atom coupling model in which two pairs
of four-level atoms are respectively trapped in two distant cavities that are connected by an optical fiber. In
the model, each of pairs of the atoms are simultaneously driven by laser fields and coupled to the local cavity

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modes through the double Raman transition configuration. Under large detuning and bad cavity limits, we investigate to engineer an effective broadband squeezing reservoir for the atoms. In Sec. III, we analyze how to realize controlling gates between the atoms trapped in the two cavities by steering the squeezing reservoir. Section IV contains conclusions of our investigations.

![Diagram of atom-field coupling scheme](image)

**FIG. 1.** Atom-field coupling scheme.

![Diagram of atomic level configuration](image)

**FIG. 2.** Atomic level configuration for atom $j$ in cavity $n$.

**II. ENGINEERING A SQUEEZING ENVIRONMENT AND GENERATING A DECOHERENCE-FREE SUBSPACE**

Our scheme is shown in Fig. 1. A pair of four-level atoms are trapped in each of two distant cavities, respectively, which are connected through an optical fiber. In the short fiber limit $[16, 17, 18]$, only one fiber mode $b$ is excited and coupled to cavity modes $a_1$ and $a_2$ with strength $\nu_1$ [19]. We assume that the cavity modes and the fiber mode have the same frequency $\omega$. The level scheme of atoms is shown in Fig. 2. Atom $j$ in cavity $n$ is labeled by the index $jn$ with $j, n = 1, 2$. The distance between the atoms in the same cavity is assumed to be large enough that there is no direct interaction between the atoms. The levels $|g_{jn}\rangle$ and $|e_{jn}\rangle$ of atom $j$ in cavity $n$, with $j, n = 1, 2$, are stable with a long lifetime. The energy of the level $|g_{jn}\rangle$ is taken to be zero as the energy reference point. The lower lying level $|e_{jn}\rangle$, and upper levels $|r_{jn}\rangle$ and $|s_{jn}\rangle$ have the energy $\delta_{jn}$ and $\omega_{jn}$, respectively, in the unit with $\hbar = 1$. Transitions $|g_{jn}\rangle \leftrightarrow |s_{jn}\rangle$ and $|e_{jn}\rangle \leftrightarrow |r_{jn}\rangle$ are driven by laser fields of frequencies $\omega_{jn}^g$ and $\omega_{jn}^s$ with Rabi frequencies $\Omega_{jn}^g$ and $\Omega_{jn}^s$, and relative phase $\varphi$, respectively. Transitions $|g_{jn}\rangle \leftrightarrow |r_{jn}\rangle$ and $|e_{jn}\rangle \leftrightarrow |s_{jn}\rangle$ are coupled to the cavity mode $a_n$ with the strengths $g_{jn}^g$ and $g_{jn}^s$, respectively.

Here, we set $\Delta_{jn}^g = \omega_{jn}^g - \omega - \delta_{jn} = \omega_{jn}^g - \omega^L_{jn}$, and $\Delta_{jn}^s = \omega_{jn}^s - \omega - \delta_{jn} = \omega_{jn}^s - \omega^L_{jn}$.

Under the Markovian approximation, the master equation of the density matrix for the whole system under consideration can be written as [14]

$$\dot{\rho}T = -i[H, \rho T] + L_{\text{cav}}\rho T + L_{\text{cav}}\rho T + L_{\text{fiber}}\rho T, \quad (1)$$

where $H = H_0 + H_d + H_{ac} + H_{ef}$ with

$$H_0 = \sum_{j,n=1}^2 \left( \omega_{jn}^g |g_{jn}\rangle \langle g_{jn}| + \omega_{jn}^s |s_{jn}\rangle \langle s_{jn}| + \delta_{jn} |e_{jn}\rangle \langle e_{jn}| \right) + \omega \left( \sum_{n=1}^2 a_n^\dagger a_n + b^\dagger b \right),$$

$$H_d = \sum_{j,n=1}^2 \left( \frac{\Omega_{jn}^g}{2} e^{-i\omega_{jn}^g t} |g_{jn}\rangle \langle g_{jn}| + \frac{\Omega_{jn}^s}{2} e^{-i(\omega_{jn}^s t + \varphi)} |e_{jn}\rangle \langle e_{jn}| + 2 \omega^2 \beta \right),$$

$$H_{ac} = \sum_{j,n=1}^2 (g_{jn}^r |r_{jn}\rangle \langle g_{jn}| a_n + g_{jn}^s |s_{jn}\rangle \langle e_{jn}| a_n + \text{H.c.}),$$

$$H_{ef} = \nu \left[ b(a_n^\dagger + a_n) + \text{H.c.} \right]. \quad (2)$$

Here, $H_0$ is the free energy of atoms and cavity fields, $H_d$ is the interaction energy between the atoms and laser fields, $H_{ac}$ is the interaction energy between the atoms and the cavity fields, and $H_{ef}$ describes the interaction between the cavity modes and the fiber mode. The last three terms in (1) describe the relaxation processes of the cavity and fiber modes in the usual vacuum reservoir, taking the forms

$$L_{\text{cav}}\rho T = \kappa_n (2a_n^\dagger \rho T a_n - a_n^\dagger a_n \rho T - \rho T a_n^\dagger a_n),$$

$$L_{\text{fiber}}\rho T = \kappa_f (2b^\dagger \rho T b - b^\dagger b \rho T - \rho T b^\dagger b), \quad (3)$$

where $\kappa_n$ is the leakage rate of photons from cavity $n$, and $\kappa_f$ is the decay rate of the fiber mode.

Let’s introduce collective basis: $|a_n\rangle = (|g_{1n}\rangle |e_{2n}\rangle - |e_{1n}\rangle |g_{2n}\rangle) / \sqrt{2}$, $|1_n\rangle = |g_{1n}\rangle |g_{2n}\rangle$, $|0\rangle_n = (|g_{1n}\rangle |e_{2n}\rangle + |e_{1n}\rangle |g_{2n}\rangle) / \sqrt{2}$, $|1_n\rangle = |e_{1n}\rangle |g_{2n}\rangle$. The states $|a_n\rangle$ and $|1_n\rangle$ are taken as a qubit $n$ for quantum computation. In the large detuning limit, adiabatically eliminating the excited states and setting $\Omega_{jn}^g \frac{g_{jn}^g}{2}\frac{\omega_{jn}^g}{\Delta_{jn}^g} = \Omega_{jn}^s \frac{g_{jn}^s}{2}\frac{\omega_{jn}^s}{\Delta_{jn}^s} = \beta_n^g$, from (2), we obtain the effective interaction Hamiltonian

$$H_{ef} = \sum_n \sqrt{2} \left[ a_n (\beta_n^g e^{i\varepsilon S_n^+ + \beta_n^s S_n} + \text{H.c.}) + H_{ef} \right], \quad (4)$$

where $S_n^+ = |0\rangle_n \langle -1| + |1\rangle_n \langle 0|$. In the derivation of (4), we have assumed the resonant condition $\frac{g_{jn}^2}{\Delta_{jn}^g} (a_n^\dagger a_n + b^\dagger b)$.
\[ \frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2 \rho}{\partial \varphi^2} + \frac{\partial^2 \rho}{\partial \theta^2}, \]  
\[ \Omega_{\varphi} = \frac{1}{2} \left( \omega \varphi + \omega \varphi - \sqrt{2} \varphi \right) \langle a_n a_n \rangle + \beta_{\varphi}^+ \]  
In order to satisfy the condition with the flexible choice of \( \Omega_{\varphi}^+, \Omega_{\varphi}^-, \Delta_{\varphi}^+, \) and \( \Delta_{\varphi}^-, \) we have introduced additional ac-Stark shifts \( \beta_{\varphi} \) to states \( |g_{\varphi} \rangle, \) which can be generated by using a laser field to couple the level \( |g_{\varphi} \rangle \) to an ancillary level.

We now introduce three normal modes \( c \) and \( c_s \) with frequencies \( \omega \) and \( \omega \pm \sqrt{2} \nu \) by use of the unitary transformation \( a_1 = \frac{1}{2} (c_s + c_r + c_r + c_s - \sqrt{2} c_{\varphi}) , \) \( a_2 = \frac{1}{2} (c_s + c_r - c_r + c_s + \sqrt{2} c_{\varphi}) , \) \( b = \frac{1}{2 \sqrt{2}}(c_r - c_r) \) \( \mathbf{11, 12}. \) In the limit \( \nu \gg |\beta_{\varphi}^+|, |\beta_{\varphi}^-| \), neglecting the far off-resonant modes \( c_s \) and setting \( \beta_{\varphi}^+ = -\beta_{\varphi}^- = \beta_s \) with \( \nu = r, s, \) we can approximately write the effective Hamiltonian \[ \mathbf{13} \]

\[ H_{\text{eff}} = (\beta_s e^{i \varphi} S^+ + \beta_s S^c) + H_{\text{c.c.}}, \]

where \( S^+ = S^T + S^S \).

Since the modes \( c_s \) are nearly not excited and decoupled with the resonant mode \( c \), the fiber mode \( b \) is mostly in the vacuum state, therefore, \( L_{\text{fiber}} \rho \) can be neglected, and \( L_{\text{cav}1, \text{PT}} + L_{\text{cav}2, \text{PT}} \) can be approximated as

\[ L_{\text{cav}1, \text{PT}} = \kappa (2 c_r c_T^c - c^c c_T - \rho_T c^c c), \]

with \( \kappa = (\kappa_1 + \kappa_2)/2 \).

In the bad cavity limit, \( \kappa \gg \beta \), adiabatically eliminating the mode \( c \) \( \mathbf{12, 14} \), from Eq. \( \mathbf{11} \) with the replacement of the Hamiltonian \( \mathbf{2} \) and the relaxation terms \( \mathbf{3} \) by the effective Hamiltonian \( \mathbf{11} \) and the relaxation term \( \mathbf{13} \), respectively, we can obtain the master equation for the density matrix of the atoms

\[ \dot{\rho} = -\frac{\Gamma}{2} (R^R \rho R + \rho R^R R - 2 \hbar \rho R^R), \]

where \( \rho = \text{Tr} \rho, \) \( R = S \cosh \varphi + e^{i \varphi} S^1 \sinh \varphi, \) \( r = \cosh^{-1}(\beta_s / \sqrt{\beta_s^2 - \beta_s^2}) \) and \( \Gamma = 2(\beta_s^2 - \beta_s^2) / \kappa \)

\[ \mathbf{10} \]

describes the collective interaction of two cascade three-level atoms with the effective squeezed vacuum reservoir \( \mathbf{10} \). The parameters \( \beta_s, \beta_s^+ \) and \( \varphi \) are easily changed and controlled at will by varying the strength and phase of the driving lasers \( \mathbf{8} \). We will show that a geometric phase gate can be realized through changing these parameters.

The DFS of the atomic system is spanned by the states which satisfy the equation \( R(\varphi, \varphi) \left| \psi_{\text{DFS}}(\varphi, \varphi) \right\rangle = 0 \) \( \mathbf{10} \). In terms of basis states \( |e_1 \rangle = |a_1 a_2 \rangle, \) \( |e_2 \rangle = |a_1 a_1, a_2 \rangle, \) \( |e_3 \rangle = |a_1 a_2, a_2 \rangle, \) \( |e_4 \rangle = |a_1 a_2 a_2 \rangle, \) \( |e_5 \rangle = |0_1 0_2, a_2 \rangle, \) \( |e_6 \rangle = |a_1 a_2, a_2 \rangle, \) \( |e_7 \rangle = |a_1 a_2 a_2 \rangle, |e_8 \rangle = |1_1 1_2, a_2 \rangle, |e_9 \rangle = \sqrt{\lambda_5} (1_1 0_2 + 0_1 1_2) , |e_{10} \rangle = -1_1 1_2 + 1_1 0_2 , |e_{11} \rangle = \frac{\sqrt{\lambda_5}}{2} (0_1 - 1_1 - 1_2 + -1_1 0_2) , |e_{12} \rangle = \frac{\sqrt{\lambda_5}}{2} (0_1 - 1_1 0_2 + 1_1 0_2) , \)

Therefore, in the frame dragged adiabatically by the reservoir, the state of the atoms in the DFS unitarily evolves in time.

\textbf{III. REALIZING CONTROLLING PHASE GATES THROUGH MANIPULATING THE SQUEEZING ENVIRONMENT}

In this section, we investigate how to realize a CPHASE gate through manipulating the engineered reservoir. Suppose that at the initial time the laser field driving the transition \( |g \rangle \leftrightarrow |s \rangle \) is switched off but the laser field driving the transition \( |r \rangle \leftrightarrow |e \rangle \) is switched on and the atoms are in the DFS state \( \left| \psi(0) \right\rangle = \frac{1}{2} \left( |a_1 a_1 a_2 + a_1 a_2 a_2 + -1_1 a_1 + -1_1 a_2 + 1_1 a_2 + -1_1 a_2 + -1_1 a_2 + -1_1 a_2 \right) \) \( \mathbf{14} \). To generate a geometric phase for the atomic state, we smoothly change the parameters of the engineered reservoir along a closed loop, which is divided into the following three steps: (1) From time 0 to \( T_1 \), hold on \( \varphi = 0 \), and adiabatically increase the parameter \( r \) from 0 to \( r_0 \); (2) From time \( T_1 \) to \( T_2 \), hold on \( r = r_0 \), and adiabatically change the phase \( \varphi \) from 0 to \( \varphi_0 \); (3) From time \( T_2 \) to \( T_3 \), hold on \( \varphi = \varphi_0 \), and adiabatically decrease \( r \) from \( r_0 \) to 0. When the cyclic evolution ends, the atomic state becomes

\[ \left| \psi(T_3) \right\rangle = \frac{1}{2} \left( |e_1 \rangle + e^{i \chi_1} |e_2 \rangle + e^{i \chi_2} |e_3 \rangle + e^{i \chi_2} |e_{10} \rangle \right), \]

where geometric phases \( \chi_1 = -\nu_1 \varphi_0, \chi_2 = -\nu_1 \varphi_0 \) with \( \nu_1 = \frac{\sinh^2 r_0}{\cosh^2 r_0 + \sinh^2 r_0} \), \( \nu_1 = \frac{2 \tanh^2 r_0 + \frac{\tanh^2 r_0}{\cosh^2 r_0}}{2 \tanh^2 r_0 + \frac{\tanh^2 r_0}{\cosh^2 r_0} + 1} \) \( \mathbf{15} \). By performing local transformations \( U_1 = e^{-i \chi_1} |1 \rangle_{111} (-1 \rangle_{111} \) and
If $r_0 = \text{atanh}(\sqrt{4/3} - 1) \approx 0.4157$, $|\nu_{12}| = |\nu_f|$. Under this condition with $\varphi_0 = \pi/\nu_f$, the state of the atoms at the time $T_3$ is $|\Psi^\alpha(T_3)\rangle_a = \frac{1}{\sqrt{3}}(-|a_1|a_2 + |a_1|1 - 1) + |1\rangle_1|a_2 + e^{i\Delta}|-1\rangle_1 - 1\rangle_2)$. In this case, the Controlled-Z gate between the two qubits is realized without local transformations.

**FIG. 3.** Fidelity $F_r$ of the atomic state.

**FIG. 4.** Fidelity $F_p$ of the atomic state.

The above results depend on the adiabatical approximation. To check the adiabatical condition, we numerically simulate the following two examples. In the first example, we suppose that at the initial time the atoms are in the state $|\Psi_1\rangle_a = (|a_1|a_2 + |\psi_{DF}(0,0)|_{12})/\sqrt{2}$ and the laser field driving the transition $|e\rangle \leftrightarrow |r\rangle$ are turned on. Then, by slowly switching the laser field driving the transition $\langle g| \rightarrow |s\rangle$, we increase the parameter $r$ from $0$ to $r_0$ according to the linear function $r(t) = r_0/T$. In the adiabatical limit ($T \gg 1$), the atomic state becomes $|\Psi_1\rangle_a = (|a_1|a_2 + |\psi_{DF}(r_0,0)|_{12})/\sqrt{2}$ at the time $T$. On the other hand, in the Hilbert space spanned by the basis states $\{|e_i\rangle\}$ for $i = 1, 2, \cdots, 12$, we can numerically solve Eq. (7) and obtain the density matrix $\rho(T)$ of the atoms. Let’s define $F_r = a(|\Psi(T)|^2)/\langle S\rangle$ as the fidelity for this process. As shown in Fig. 3, if $T > 100/\Gamma$, $F_r$ is always bigger than 0.997 if $r \in (0, 0.8)$, corresponding to the almost perfect evolution.

In the second example, we suppose that the atoms are initially in the state $|\Psi_2\rangle_a = (|a_1|a_2 + |\psi_{DF}(r,0)|_{12})/\sqrt{2}$ and all the driving fields are turned on to hold the parameters $r = r_0$ and $\varphi = 0$. By adiabatically changing the phase $\varphi$ from $0$ to $2\pi$ at the rate $\dot{\varphi} = 2\pi/T$, the atomic state at the time $T$ becomes $|\Psi_2\rangle_a = (|a_1|a_2 + e^{i\Delta} |\psi_{DF}(r,2\pi)|_{12})/\sqrt{2}$. Let’s define the fidelity for this example as $F_p = a(|\Psi(T)|^2)/\langle S\rangle$, where $\rho(T)$ is the numerical solution of Eq. (7). As shown in Fig. 4, $F_p$ increases as $T$ increases but decreases as the parameter $r_0$ increases. If $T > 1000/T$, $F_p$ is larger than 0.992 for $0 < r_0 < 0.8$. From these two examples, we find that to fulfill the adiabatical condition the time used in the steps 2 should be much longer than in the steps 1 and 3.

A controlled-Z gate has been numerically simulated by directly solving Eq. (7) with $r_0 = 0.5$, and $\varphi_0 = \pi/(2\nu_1 - \nu_{12})$. In the simulation, we set $r = r_0/T_1$ in the steps 1 and 3, and $\dot{\varphi} = \varphi_0/(T_2 - T_1)$ in the step 2 with $T_1 = 0.05 T_2$ and $T_2 - T_1 = 0.90 T_3$. If $T_3 > 1100/T$, we find that the fidelity $F_p = a(|\Psi(T)|^2)/\langle S\rangle$ is larger than 0.95. For an almost perfect controlled-Z gate with $F > 0.99$, we find that $T_3$ must be longer than 6000/T.

Now let’s briefly discuss the effects of the atomic spontaneous emission, the fiber mode decay and cavity photon leakage. For simplicity but without the loss of generality, we suppose that atomic spontaneous emission rates of the excited levels are equal to $\gamma$. In the large detuning limit, the characteristic spontaneous emission rate of the atoms is $\gamma_{eff} = \gamma(\Omega^2/2 \Delta^2)$ and the effective decay rate of the fiber mode is $\kappa_{eff} = \kappa_f \Omega^2 g^2/(4 \Delta^2 \nu^2)$. If $\kappa_f \ll \gamma$ and $g^2 \ll \nu^2$, $\kappa_{eff}$ can be much smaller than $\gamma_{eff}$. Under this condition, the present scheme is feasible if $\Gamma \gg \gamma_{eff}$. In the current cavity quantum dynamic (CQED) experiment, the parameters ($g, \kappa, \gamma$) = (2000, 10, 10) MHz could be available. If setting $\Omega/(2\Delta) = 10^3 \times 10^{-9}$, we have $\Gamma \simeq 4 \times 10^4 \gamma_{eff}$. The condition is held. In the present scheme, the large cavity decay rate is required to ensure that the cavity modes are in a broadband squeezed vacuum reservoir and then the atoms always “see” the broadband squeezed vacuum reservoir during the dynamic evolution. For an arbitrary small but nonzero value of the squeezing degree of the reservoir, a CPHASE gate with arbitrary high fidelity can always be realized in the represent scheme. The cavity decay does...
not directly affect the fidelity of the realized CPHASE gates. However, the larger the decay rate is, the longer the operation time of the CPHASE gates is. Thus, we have the condition $\kappa >> \beta, \gamma_{\text{eff}}$ for realizing the reliable CPHASE gates. Based on the parameters quoted above, this condition can be well satisfied. With the parameters of the current CQED experiment, we find that the operation time of the controlled-Z gate, with fidelity larger than 0.95, is about 2.8 ms. It is much shorter than both $1/\gamma_{\text{eff}}$ and the single-atom trapping time in cavity [21]. On the other hand, the present scheme needs a strong coupling between the cavity and the fiber. This could be realized at the current experiment [22]. Therefore, the requirement for the realization of the present scheme can be satisfied with the current technology.

### IV. CONCLUSIONS

We propose a cavity-atom coupled scheme for the realization of quantum controlling gates, in which each of two pairs of four-level atoms in two distant cavities connected by a short optical fibre are simultaneously driven by laser fields and coupled to the local cavity modes through the double Raman transition configuration. We show that an effective squeezing reservoir coupled to the multilevel atoms can be engineered under appropriate driving condition and bad cavity limit. We find that in the scheme a CPHASE gate with arbitrary phase shift can be implemented through adiabatically changing the strength and phase of driving fields along a closed loop. It is also noticed that the larger the effective coupling strength between the environment and the atoms is, the more reliable the realized CPHASE gate is.

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