Near-horizon geometry with torsion

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Abstract

We investigate near-horizon geometry of the rotating BTZ black hole with torsion. Our main motivation is to gain insight into the role of torsion in the near-horizon geometry, which is well understood in the Riemannian case. We consequently obtain that near-horizon geometry represents generalization of AdS self-dual orbifold with non-trivial torsion. We analyze its asymptotic structure and derive the corresponding algebra of asymptotic symmetries, which consists of chiral Virasoro and \( u(1) \) Kac-Moody algebra.

1 Introduction

Black holes represent some of the most fascinating objects in our Universe which, since their discovery by Schwarzschild, don’t stop to puzzle and inspire us. After Hawking’s discovery of black hole radiation we stumbled into quite a few problems. First of them is an information paradox. Namely, if black hole radiates thermally, after its evaporation the information about the matter it was constituted of is inevitably lost. The second one is the problem of black hole micro-states which are responsible for the black hole entropy and crucial for the information paradox resolving.

There are numerous approaches to the previously mentioned problems, in this article we are particularly interested in Kerr/CFT [1]. Basic idea of Kerr/CFT is that near horizon geometry encodes many important information about the full geometry itself if not them all. The only drawback is that near-horizon geometry is well-defined only for the extremal black holes. Nevertheless, keeping in mind the importance of the subject, gaining divergent insights is valuable.

It is worth noting that charges that generate asymptotic symmetry of near-horizon geometry are by construction soft. This is interesting because there are propositions that soft hairs are black hole micro-states [2] [3]. So, one might hope to gain a small insight into importance of torsion for this approach as well.

Although Kerr/CFT is, already, known for a decade there is no analogous analysis for the gravity with torsion. Our intention is to fill this gap and initiate investigation of near-horizon
geometries with torsion. Before proceeding to the subject of our work it is instructive to give a short note on the role of torsion in gravity.

Einstein’s general relativity (GR), prototype of all modern gravitational theories, postulates that connection is Christoffel or, equivalently, that there is no torsion. Although successful in explaining many of the existing macroscopic phenomena it still lacks the status of the fundamental theory on the microscopic level, due its non-renormalizability. One also needs to include dark matter and dark energy into GR to accomplish the agreement with observations.

Instead of including mysterious new form of matter one may turn to alternative theories of gravity. If we allow the presence of torsion in gravitational theories we are exposed to a vast number of new possibilities. One of them is that dark matter represents one of the manifestations of non-trivial torsion, see [4, 5, 6]. In cosmology there are also considerations in which both dark matter and energy are replaced by torsion [7]. Additionally, there are proposals that torsion plays a crucial role in inflation. Namely in [8] a simple model of inflation is proposed which, among other issues, also solves the problem of cosmological singularity.

However, there is an approach to gravity based on localization of Poincaré group in which basic dynamical variables are vielbein and spin connection. For the comprehensive overview of the subject see [9]. This formulation of gravity naturally incorporates presence of torsion and metric postulate is a consequence of an antisymmetry of spin connection. Also coupling of fermions to gravity is natural, which makes this approach more appealing formulation than standard metric one.

The paper is organized as follows. In the next section we collect main results of Mielke-Baekler (MB) model: action, equations of motion, BTZ black hole with torsion, canonical structure and generator of gauge symmetries. Third section is devoted to the main results of this paper. We construct near-horizon geometry of BTZ black hole with torsion which represents a new solution of MB model, self-dual orbifold with torsion and give its basic properties. After introducing suitable asymptotic conditions for vielbein and spin connection we derive the kinematical algebra of asymptotic symmetries. Using the results of the second section we construct the corresponding charges that generate this symmetry and derive commutation relations between generators.

Our conventions are as follows. The Latin indices (i, j, k, ...) refer to the local Lorentz frame with the signature of the metric (+, −, −), Levi-Civita symbol εijk is normalized to ε012 = 1. We use Greek indices (µ, ν, ρ, ...) for the coordinate frame. The orthonormal triad (coframe 1-form) is denoted with ei, while ωij is the spin connection (1-form). The field strengths are torsion Ti = dei + ωij ∧ em and the curvature Ri = dωij + ωik ∧ ωkj (2-forms). The Lie dual of an antisymmetric form Xij is Xi := −εijkXjk/2 and the exterior product of forms is implicit.

2 Mielke-Baekler model

2.1 Mielke-Baekler model in a nutshell

Basic dynamical variables in the first order formulation of gravity are vielbein eiµ and spin connection ωijµ. Very common is to use differential forms which, often, simplify notation
and calculation, so we introduce vielbein and spin connection 1-forms
\[
e^i = e^i_{\mu} dx^{\mu}, \quad \omega^{ij} = \omega^{ij}_{\mu} dx^{\mu}
\]
(2.1)

In three dimensions (3D) it is useful to pass on dual spin connection \( \omega^i = -\frac{1}{2} \varepsilon_{ijk} \omega^j \), and in completely same manner for every other two-index Lorentz tensors. In this notation torsion and curvature 2-forms are given by
\[
T^i = de^i + \varepsilon_{ijk} \omega^j e^k, \quad R^i = d\omega^i + \varepsilon_{ijk} \omega^j \omega^k.
\]

MB topological model \([10]\) of 3D gravity is described by the action
\[
I = aI_1 + \Lambda I_2 + \alpha_3 I_3 + \alpha_4 I_4 + I_M,
\]
\[
I_1 = 2 \int e^i R_i, \quad I_2 = -\frac{1}{3} \int \varepsilon_{ijk} e^i e^j e^k, \quad I_3 = \int \omega_i d\omega^i + \frac{1}{3} \varepsilon_{ijk} \omega^i \omega^j \omega^k, \quad I_4 = \int e^i T_i,
\]
(2.2)

where \( I_M \) is the matter field contribution. The first term with \( a = 1/16\pi G \) is the Einstein Cartan action, the second term is the cosmological one, the third terms is Chern-Simons action for Lorentz connection, while the fourth term explicitly depends on torsion. In the previous expressions and in what follows wedge product between forms is implicit.

We are particularly interested in the non-degenerate sector of the theory in which the following relation holds \( a^2 - \alpha_3 \alpha_4 \neq 0 \). If the previous relation is true then equations of motion can be cast in the simple form \([11]\)
\[
2T^i = p \varepsilon_{ijk} e^j e^k, \quad p = \frac{\alpha_3 A + \alpha_4 a}{\alpha_3 \alpha_4 - a^2},
\]
(2.3a)
\[
2R^i = q \varepsilon_{ijk} e^j e^k, \quad q = -\frac{\alpha_3^2 + a A}{\alpha_3 \alpha_4 - a^2}.
\]
(2.3b)

The vacuum configuration is characterized by constant torsion and constant curvature. For \( p = 0 \), or \( q = 0 \) the vacuum geometry is Riemannian \( (T^i = 0) \) or teleparallel \( (R^i = 0) \).

From the torsion equation (2.3a) it follows that contorsion one form is particular simple, \( K^i = \frac{p}{2} e^i \), so that connection reads
\[
\omega^i = \bar{\omega}^i + \frac{p}{2} e^i,
\]
(2.4)

where \( \bar{\omega}^i \) is Levi-Chivita (Riemannian) connection. The curvature equation of motion (2.3b) now implies that Riemannian piece of curvature is
\[
2\bar{R}^i = \Lambda_{\text{eff}} \varepsilon_{ijk} e^j e^k, \quad \Lambda_{\text{eff}} = q - \frac{p^2}{4},
\]
(2.5a)

where \( \Lambda_{\text{eff}} \) is an effective cosmological constant. In what follows we shall restrict ourselves to the AdS sector of the theory with negative effective cosmological constant
\[
\Lambda_{\text{eff}} := -\frac{1}{\ell^2},
\]
(2.5b)

where \( \ell \) is an AdS radius.
2.2 BTZ black hole with torsion

MB model has an important and interesting solution which is generalization of BTZ black hole which has non-trivial torsion [12].

The metric of this solution parametrized by $m$ and $j$ is given by

$$ds^2 = N^2 dt^2 - \frac{dr^2}{N^2} - r^2 (N_\varphi dt + d\varphi)^2,$$

where

$$N^2 = -8Gm + \frac{r^2}{\ell^2} + \frac{16G^2j^2}{r^2}, \quad N_\varphi = \frac{4Gj}{r^2}.$$

Vielbein can be chosen in the simple diagonal form

$$e^0 = N dt, \quad e^1 = \frac{dr}{N}, \quad e^2 = r (N_\varphi dt + d\varphi),$$

Spin connection of the solution is

$$\omega^i = \tilde{\omega}^i + \frac{p}{2} e^i$$

where $\tilde{\omega}^i$ is Levi-Chivita spin connection.

Conserved charges of the black hole are energy

$$E = m \left( a + \frac{\alpha_3 p}{2} \right) - \frac{\alpha_3 j}{\ell^2},$$

and angular momentum

$$J = j \left( a + \frac{\alpha_3 p}{2} \right) - \alpha_3 m.$$ 

The entropy of the solution contains a contribution stemming from torsion. For more details see [13].

2.3 Canonical generator of gauge symmetries

First, we review some results about canonical structure of MB model, detailed analysis can be found in [11].

There are six first class primary constraints in the theory:

$$\pi_i^0 \approx 0, \quad \Pi_i^0 \approx 0,$$

where $\pi_i^\mu$ and $\Pi_i^\mu$ are momenta conjugate to basic dynamical variables $e^i_\mu$ and $\omega^i_\mu$. Secondary first class constraints in the reduced phase space, obtained from the original phase space after elimination of second class constraints read

$$\mathcal{H}_i = -\varepsilon^{0\alpha\beta} \left( a R_i_{\alpha\beta} + \alpha_3 T_i_{\alpha\beta} + \Lambda \varepsilon_{ijk} e^j_\alpha e^k_\beta \right),$$

$$\mathcal{K}_i = -\varepsilon^{0\alpha\beta} \left( a T_i_{\alpha\beta} + \alpha_3 R_i_{\alpha\beta} + \alpha_4 \varepsilon_{ijk} e^k_\alpha e^i_\beta \right).$$

(2.11b)
Now we can state the result of the form of the generator of gauge symmetry, which will be used in later section for determining asymptotic symmetries. The general procedure for its construction is given in [14]. Generator of gauge symmetries $G$ consists of two parts

$$G = -G_1 - G_2.$$ 

The first part generates diffeomorphisms and has the following form

$$G_1 = \xi^\mu \left( e^i_\rho \pi^0_i + \omega^i_\rho \Pi_i^0 \right) + \xi^\rho \left( e^j_\rho \mathcal{H}_i + \omega^j_\rho \mathcal{K}_i + \left( \partial_\rho e^i_0 \right) \pi^0_i + \left( \partial_\rho \omega^j_0 \right) \Pi_i^0 \right), \quad (2.12a)$$

while the second generates local Lorentz transformations and is given by

$$G_2 = \theta^i \Pi_i^0 + \theta^i \left( \mathcal{K}_i - \varepsilon_{ijk} \left( e^j_0 \pi^k_0 + \omega^j_0 \Pi^k_0 \right) \right). \quad (2.12b)$$

## 3 AdS self-dual orbifold with torsion

### 3.1 Near-horizon of the BTZ with torsion

We study the near-horizon limit of the extremal BTZ black hole with torsion. Extremal BTZ with torsion is characterized by the following relation between the parameters

$$j = \pm m\ell, \quad (3.1)$$

from which we deduce the value of the radius of the event horizon

$$r_0 = 2\ell \sqrt{Gm}, \quad (3.2a)$$

and angular velocity

$$\Omega = N_\varphi(r_0) = \frac{4Gj}{r_0^2} = \frac{1}{\ell}. \quad (3.2b)$$

To arrive at the the near-horizon region we need to change the variables in the following manner

$$\varphi \rightarrow \varphi - \Omega t/\epsilon, \quad r \rightarrow r_0 + r\epsilon, \quad t \rightarrow t/\epsilon. \quad (3.3)$$

and after taking the limit $\epsilon \rightarrow 0$, we derive the near-horizon geometry of the BTZ with torsion, which is characterized by

$$N^2 dt^2 \rightarrow \frac{4r^2}{\ell^2} dt^2, \quad \frac{dr^2}{N^2} \rightarrow \frac{\ell^2}{4r^2} dr^2, \quad d\varphi + N_\varphi dt \rightarrow d\varphi - \frac{2r}{r_0 \ell} dt,$$

so we arrive at the metric of the near-horizon

$$ds^2 = \frac{4r_0 r}{\ell} dt d\varphi - \frac{\ell^2}{4r^2} dr^2 - r_0^2 d\varphi^2. \quad (3.4)$$

The triad fields can be easily derived from the metric

$$e^0 = \frac{2r}{\ell} dt, \quad e^1 = \frac{\ell}{2r} dr, \quad e^2 = -r_0 d\varphi + \frac{2r}{\ell} dt. \quad (3.5)$$
The Levi-Chivita connection is given by
\[ \tilde{\omega}^0 = -\frac{e^0}{\ell}, \quad \tilde{\omega}^1 = -\frac{e^1}{\ell}, \quad \tilde{\omega}^2 = -2\frac{e^0}{\ell} + \frac{e^2}{\ell}, \] (3.6a)
and the Cartan spin connection reads
\[ \omega^i = \tilde{\omega}^i + \frac{p}{2} e^i. \] (3.6b)

In this way we constructed the new solution of the MB model which represents the generalization of $AdS_3$ self-dual orbifold [15], with non-trivial torsion.

### 3.2 Asymptotic conditions

To further explore properties of the AdS orbifold with torsion we analyze asymptotic structure of this solution. First, we introduce asymptotic conditions of the metric, which are
\[ g_{\mu\nu} \sim \begin{pmatrix} O_0 & O_3 & O_{-1} \\ O_3 & -\frac{\ell^2}{4r^2} + O_4 & O_1 \\ O_{-1} & O_1 & O_0 \end{pmatrix}, \] (3.7a)

where $O_n$ stands for that at infinity behaves as $r^{-n}$. The asymptotic behavior of the vielbein is given by
\[ e^i_\mu \sim \begin{pmatrix} 2r/\ell + O_1 & O_4 & O_0 \\ O_2 & 2r/2r + O_3 & O_0 \\ 2r/\ell + O_1 & O_0 & O_0 \end{pmatrix}. \] (3.7b)

Asymptotic form of the Levi-Chivita connection reads
\[ \tilde{\omega}^i_\mu \sim \begin{pmatrix} -\frac{2r}{\ell^2} + O_1 & O_2 & O_0 \\ O_1 & -\frac{1}{2r} + O_2 & O_0 \\ -\frac{2r}{\ell^2} + O_1 & O_2 & O_0 \end{pmatrix}. \] (3.8a)

By using $\omega^i = \tilde{\omega}^i + \frac{p}{2} e^i$ we conclude that asymptotic form of Cartan connection is
\[ \omega^i_\mu \sim \begin{pmatrix} -\frac{2r}{\ell^2} + \frac{pr}{\ell} + O_1 & O_2 & O_0 \\ O_1 & -\frac{1}{2r} + \frac{p\ell}{4r} + O_2 & O_0 \\ -\frac{2r}{\ell^2} + \frac{pr}{\ell} + O_1 & O_2 & O_0 \end{pmatrix}. \] (3.8b)

From the adopted asymptotic behavior of the fields we can derive kinematical algebra of asymptotic symmetries. The most simple way of deriving sub-algebra of the algebra of diffeomorphisms under which the adopted asymptotic forms of fields are invariant is by looking at the invariance of metric. Under diffeomorphisms metric transforms in the following way
\[ \delta_0 g_{\mu\nu} = -\xi^\rho \partial_\rho g_{\mu\nu} - \partial_\mu \xi^\rho g_{\rho\nu} - \partial_\nu \xi^\rho g_{\mu\rho}. \] (3.9)
Note that metric does not transform under local Lorentz rotations. Invariance of vielbein gives as preliminary result the sub-algebra of local Lorentz transformations that respect asymptotic conditions, while invariance of spin connection may and will give further restrictions. Transformation of vielbein under diffeomorphisms and local Lorentz rotations is of the form

\[ \delta_0 e^i_\mu = -\epsilon^{ijk} e_{j\mu} \theta_k - (\partial_\mu \xi^\rho) e^i_\rho - \xi^\mu \partial_\rho e^i_\mu. \]  

(3.10)

The local symmetries that preserve the asymptotic form of vielbein are parametrized by

\[ \xi^t = T(t) + O_2, \quad \xi^\varphi = rU(\varphi) + O_1, \]
\[ \theta^0 = \frac{r^2}{\ell^2} \partial_r \xi^t + O_3, \quad \theta^1 = -\frac{\xi^\varphi}{r} + \partial_t \xi^t + O_2, \]
\[ \theta^2 = \frac{r^2}{\ell^2} \partial_r \xi^t + O_3. \]  

(3.11)

(3.12)

By inspecting the invariance of the asymptotic form of spin connection

\[ \delta_0 \omega^i_\mu = -\nabla_\mu \theta^i - (\partial_\mu \xi^\rho) \omega^i_\rho - \xi^\rho \partial_\rho \omega^i_\mu, \]  

(3.13)

we conclude that only invariance of \( \omega^1_\varphi \) gives further restriction \( \partial_\varphi^2 \xi^t = O_2 \) with the solution

\[ \xi^t = at + b + O_2. \]  

(3.14)

### 3.3 Charges

The previous derivation is purely kinematical and is valid in any theory of gravity that has AdS orbifold with torsion as a solution. To conclude whether these symmetries are true ones or pure gauge we need to chose specific theory and calculate the charges that generate them. We decided to use canonical approach to asymptotic symmetries \[16\].

Canonical charges are obtained by requiring that generator of gauge symmetry has well-defined functional derivatives for the given asymptotic behavior of fields. Generally, this leads to adding of a surface term \( \Gamma \) to a generator of symmetry \( G \)

\[ \hat{G} = G + \Gamma. \]  

(3.15)

In this way we obtain the improved generator \( \hat{G} \) \[16\].

After a shorter calculation we obtain that surface term needed that has to be added to a time translations generator is

\[ \Gamma[\xi^t] = 2 \int d\varphi \xi^t \left[ e^2_t (a\omega^2_\varphi + \alpha_4 e^2_\varphi) + \omega^2_t (ae^2_\varphi + \alpha_3 \omega^2_\varphi) \right. \]
\[ - e^0_t (a\omega^0_\varphi + \alpha_4 e^0_\varphi) - \omega^0_t (ae^0_\varphi + \alpha_3 \omega^0_\varphi) \right]. \]  

(3.16)

In the same manner we obtain that surface term for symmetry generated by \( \xi^\varphi \) is

\[ \Gamma[\xi^\varphi] = - \int d\varphi S(\varphi) \left[ 2ae^i_\varphi \omega^i_\varphi + \alpha_4 e^i_\varphi e^i_\varphi + \alpha_3 \omega^i_\varphi \omega^i_\varphi \right]. \]  

(3.17)
For $\xi^r$ we obtain the following surface term

$$\Gamma[\xi^r] = \int_0^{2\pi} d\varphi U(\varphi) \left[ e^1_\varphi \left( \ell\alpha_4 + a \left( p\ell^2 - 3 \right) \right) + \omega^1_\varphi \left( a\ell + \alpha_3 \left( p\ell^2 - 3 \right) \right) \right]. \quad (3.18)$$

To actually see which symmetry this generators correspond to we need to go to Fourier mods in the following way

$$\tilde{G}[\xi^\varphi] = \sum_n e^{in\varphi} L_n, \quad (3.19)$$

$$\tilde{G}[\xi^r] = \sum_n e^{-in\varphi} J_n. \quad (3.20)$$

The commutation relations now, easily, follow from the previous definition of the generators and explicit form of surface terms [17]. Generators $L_n$ satisfy Virasoro algebra commutation relations

$$i[L_n, L_m] = (n - m)L_{n+m}, \quad (3.21)$$

without the central extension.

While, the generators $J_n$ obey $u(1)$ Kac-Moody algebra commutation relations

$$i[J_n, J_m] = n\delta_{n+m,0}\kappa, \quad (3.22)$$

with central extension $\kappa$ whose value is

$$\kappa = 4\pi \left[ \alpha_3 \left( \frac{p\ell^2}{2} - 1 \right) + a\ell \right]. \quad (3.23)$$

The values of the zero mode generators on the orbifold background are given by

$$L_0^{on-shell} = \frac{4\pi r_0^2}{\ell^2} \left[ \alpha_3 \left( \frac{p\ell^2}{2} + 1 \right) + a\ell \right], \quad (3.24)$$

$$J_0^{on-shell} = 0.$$

### 3.4 Alternative coordinates

We can, alternatively, introduce change of coordinates $r = \rho^2$ in which metric of the orbifold is

$$ds^2 = \frac{2r_0\rho^2}{\ell} dtd\varphi - \frac{\ell^2}{\rho^2} d\rho^2 - r_0^2 d\varphi^2. \quad (3.25)$$

Vielbeins are chosen in the following form

$$e^0 = \frac{\rho^2}{\ell^2} dt, \quad e^1 = \frac{\ell}{\rho} d\rho, \quad e^2 = \frac{\rho^2}{\ell^2} dt - r_0 d\varphi. \quad (3.26)$$

In this coordinates metric takes the same form as that of near-horizon of rotating OTT black hole [18], so we can introduce the same asymptotic conditions on the metric

$$g_{\mu\nu} \sim \begin{pmatrix} O_{-1} & O_3 & O_{-2} \\ O_3 & -\frac{\ell^2}{r^2} + O_4 & O_1 \\ O_{-2} & O_1 & O_0 \end{pmatrix}. \quad (3.27)$$
The asymptotic behavior of the triad fields is also the same as for the rotating OTT
\[ e_i^\mu \sim \begin{pmatrix} \frac{\rho^2}{\ell^2} + O_1 & O_5 & O_0 \\ O_1 & \frac{\ell}{\rho} + O_3 & O_0 \\ \frac{\rho^2}{\ell^2} + O_1 & O_5 & O_0 \end{pmatrix} \]  
(3.28)

Asymptotic form of the spin connection is given by
\[ \omega_i^\mu \sim \begin{pmatrix} -\frac{\rho^2}{\ell^3} + \frac{\rho^2 p}{2\ell^2} + O_1 & O_2 & O_0 \\ O_1 & -\frac{1}{\rho} + \frac{p\ell}{2\rho} + O_2 & O_0 \\ -\frac{\rho^2}{\ell^3} + \frac{\rho^2 p}{2\ell^2} + O_1 & O_2 & O_0 \end{pmatrix} \]  
(3.29)

and differs from the rotating OTT in the falloff of the \( \omega^t_1 \) component and in presence of torsional part parametrized with \( p \). Asymptotic symmetries for this falloff conditions are

**Diffeomorphisms**

\[
\begin{align*}
\xi^t &= at + b + O_3, \\
\xi^r &= rU(\varphi) + O_1, \\
\xi^\varphi &= S(\varphi) + O_4,
\end{align*}
\]  
(3.30)

**Lorentz transformations**

\[
\begin{align*}
\theta^0 &= \partial_r \xi^t e^2_t e^1_r + O_2 = O_1, \\
\theta^1 &= -\frac{2\xi^r}{r} + \partial_t \xi^t + O_4, \\
\theta^2 &= \frac{e^0_t}{e^1_r} \partial_r \xi^t + O_2 = O_1.
\end{align*}
\]  
(3.31)

We obtained the same algebra as before, the only difference is that \( \theta^1 \) is now two times larger, which will lead to a change of central extension.

Surface terms for generators corresponding to \( \xi^t \) and \( \xi^\varphi \) are the same as for the previous asymptotic conditions, while the surface term for the generator \( G[\xi^r] \) takes form
\[\Gamma[\xi^r] = \int_0^{2\pi} d\varphi U(\varphi) [e^1_\varphi (2\ell \alpha_1 + a(p\ell - 6)) + \omega^1_\varphi (2a\ell + \alpha_3(p\ell - 6))].\]  
(3.33)

If we expand in Fourier modes, in the same manner as before, we obtain
\[i[J_n, J_m] = n\delta_{n+m,0} \kappa,\]  
(3.34)
where the central extension $\kappa$ is given by

$$\kappa = 16\pi \left[ \alpha_3 \left( \frac{p\ell}{2} - 1 \right) + a\ell \right]$$

(3.35)

The values of the zero mode generators on the orbifold background are given by

$$L_0^{\text{on-shell}} = \frac{4\pi r_0^2}{\ell^2} \left[ \alpha_3 \left( \frac{p\ell}{2} + 1 \right) + a\ell \right]$$

$$J_0^{\text{on-shell}} = 0.$$ 

(3.36)

4 Concluding remarks

We analyzed near-horizon geometry with non-trivial torsion with desire to understand influence of later on applicability of Kerr/CFT. For simplicity, we undertook the investigation of simplest case of extremal BTZ black hole with torsion. After deriving the corresponding near-horizon geometry which is the generalization of AdS self-dual orbifold which represents the near-horizon of BTZ black hole. After introducing the suitable asymptotic conditions we derived the algebra of asymptotic symmetries, which consists of two Killing vectors of the orbifold and direct sum of chiral Virasoro and $u(1)$ Kac-Moody algebra, known also as Warped CFT symmetry. Virasoro algebra is not centrally extended, while Kac-Moody algebra possesses non-zero central extension $\kappa$.

It is worth noting that investigation of asymptotic structure of BTZ black hole with torsion was done in [11] with the result that asymptotic symmetry is Virasoro algebra with central charges $c$ and $\bar{c}$. Central extension of Kac-Moody algebra, that we obtained, is proportional to $c$, and on-shell value of Virasoro zero mode generator is proportional to $\bar{c}$.

There is the procedure, known as Sugawara-Sommerfeld construction [20], for constructing Virasoro algebra from bilinear combinations of Kac-Moody generators. In this way it is possible to obtain Virasoro algebra which allows application of Cardy formula. Drawback of this approach is that one central charge is proportional to arbitrary constant which is fixed by demand that Cardy formula correctly reproduces black hole entropy.

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A $\Gamma[\xi^t]$ is finite

Because the surface term is the same for both asymptotic conditions it is enough to show that it is finite in one of them.

For $\rho$ coordinates the following relations between Riemann part of connection and viel-
\begin{align}
\frac{e^0}{\ell} - \frac{e^2}{\ell} + \bar{\omega}^2 - \bar{\omega}^0 &= \mathcal{O}_2, \quad \text{(A.1a)} \\
\bar{\omega}^2 &= -\frac{e^2}{\ell} + \mathcal{O}_1 = -\frac{\rho^2}{\ell^3} + \mathcal{O}_1, \quad \text{(A.1b)} \\
\bar{\omega}^0 &= -\frac{e^0}{\ell} + \mathcal{O}_1 = -\frac{\rho^2}{\ell^3} + \mathcal{O}_1. \quad \text{(A.1c)}
\end{align}

The first identity is consequence of \( T^i = 0 \), the other two are obvious from asymptotic behavior of fields.

If we expand spin connection as \( \omega_i = \bar{\omega}^i + \frac{p}{2} e^i \), use relation between Lagrangian parameters \( \alpha_4 = \frac{\alpha_3}{\ell^2} - ap - \frac{\alpha_3 p^2}{5} \) and last two equalities, after short calculation, we derive

\[ \Gamma[\xi^t] = \int d\varphi \frac{\beta^2}{\ell^2}(2a + \alpha_3 p - 2\alpha_3)\left(\frac{e^0}{\ell} - \frac{e^2}{\ell} + \bar{\omega}^2 - \bar{\omega}^0\right) + \mathcal{O}_1. \] (A.2)

Now, from the first equality follows that \( \Gamma[\xi^t] = \mathcal{O}_0 \), i.e. it is finite.

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