Euler Root Mean Square (ERMS): A New Modified Euler Method to Improve Accuracy of Resistor-Inductor (RL) Circuit Equation

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Abstract

Numerical method is a technique of obtaining the nearest approximation for the solution of various problems that can be described in the form of derivative equations. Engineering issues cannot be simply overcome using analytical concepts. This study aims to propose a new scheme from an enhanced Euler method for testing the resistor-inductor (RL) circuit equation. For this purpose, the paper proposes the Euler Root Mean Square (ERMS), a modified Euler method with improved accuracy. It will justify that the new scheme can be as accurate as possible in providing the exact solution by applying an average concept of using root mean square. It will focus on this accuracy by comparing the exact solution and actual solution between the new ERMS scheme and a modified Euler known as Euler Arithmetic. This study has demonstrated that the ERMS provided solutions that are similar to the exact solutions at $t=0.5$. It proves that an enhanced Euler method can be applied in various fields, especially in electrical engineering. In conclusion, the ERMS can be used as an alternative algorithm to solve RL circuit problems.

1.0 INTRODUCTION

Engineering issues cannot be easily overcome using analytical concepts. Numerical approaches are most commonly used as analytical solutions to various problems. Numerical methods are usually iterative in nature, requiring a number of intermediate steps in order to arrive at a solution. Electrical circuit is defined as the interconnection of electrical elements or electrical devices [1] and can be written mathematically using differential equations of first, second, and upper order. An RL circuit (also known as a resistor-inductor circuit) is an electrical circuit consisting of a resistor (R) and an inductor (L) connected in series or in parallel circuitry. An RL circuit is called a first-order circuit as any voltage or current in the circuit can be described by a first-order differential equation for circuit analysis. An RL circuit, as shown in Figure 1, is an electric circuit consisting of a resistor and an inductor driven by voltage or source of current. A first-order RL circuit with one resistor and one inductor and is the simplest type of an RL circuit.
The Euler method is known as a straight-line method and the easiest numerical method to solve initial value problems in ODE. In practice, the Euler method is used to solve ordinary differentiation equation (ODE) problems. In a modified Euler method, the average of two points on the slope of $x_i$ and $x_{i+1}$ is taken. This average value is then applied to $y_{i+1}$. An alternative solution was discovered to have the ability to match the performance accuracy of the Euler method as a more stable scheme [2]. The scheme is able to obtain an exact solution by using the original Euler concept and mean.

This paper presents a newly modified Euler scheme derived from the original Euler method and an enhanced Euler method known as the Euler Arithmetic (EA) scheme [2]. The EA scheme has been proven to attain a better accuracy compared to the Euler method. The Euler Arithmetic (EA), which was developed by [2], is a well-known technique for improving the Euler method. This paper uses the Euler methods demonstrated in [2] and [3] as the basis for developing the new scheme. This new scheme will combine Euler, EA, and mean concept. The mean used in this research is root mean square (RMS). The new scheme is named Euler Root Mean Square (ERMS). RMS has been selected because it is able to provide a higher accuracy than the original Euler scheme. A study showed that using the root mean square and the fourth stage of Runge-Kutta provide more accuracy than using the Euler method and the second method of Runge-Kutta [4]. Hence, this study selects the root mean square as the mean concept to solve the equation in the modified Euler scheme.

The new scheme will be tested with three RL circuit equations. The purpose is to compare the accuracy of the enhanced Euler method in the RL circuit. A study by [3] used Scilab as a programming language to test the Euler method in solving electrical and electronic circuit equations. The Euler scheme has been proven as one of the easiest methods to programme in completing a differentiation using a digital computer [5]. The modified Euler scheme only resulted in a small error for every calculation in the electrical equation [6]. The Euler scheme has also been proven as the best alternative in enhancing the Euler method [7].

2.0 METHODOLOGY

The new scheme was developed by combining Euler (R) and mean (T) schemes, as shown in Figure 2 below. The original Euler scheme with the general formula $n + 1 = y_n + hf(x_n, y_n)$ was selected as the basis for developing the new scheme. Root mean square (RMS), marked as (T1), was selected for developing this proposed scheme. The combination of Euler (R) and RMS (T1) schemes produced the proposed scheme known as Euler Arithmetic Root Mean Square (ERMS) $(R_1,T_1)$. Figure 2 shows how the proposed scheme had been developed.
Figure 3.0 shows how the new scheme was derived using the root mean square. The concept of mean was proposed in a study by [8].

\[
R_{1i}T_1 = y_n + h f(x_n, y_n) + \frac{h}{2} \left[ f(x_n, y_n) + f(x_n + h, y_n + f(x_n + y_n)) \right], \sqrt{\left(x^2 + y^2\right)/2}
\]

Figure 3

For the analysis in this work, the ideal linear circuit elements are focused on what is considered the following constitutive laws for each element, as depicted in equation (1)

\[
L, \frac{dl}{dt} + R \cdot I = V(t)
\]

Ohm’s law shows the voltage drop caused by a resistor as \( R \times I \), and the voltage drop caused by an inductor as \( L \times \frac{dl}{dt} \). One of Kirchhoff’s Laws states that the sum of the voltage drops is equal to the supplied voltage \( E(t) \). From that, equation (2) is established:

\[
\Delta V_I + \Delta V_R = V(t)
\]

Table 1 lists the RL circuit equations derived from using equation (1). These equations were derived by computing the value of \( V \), \( I \), and \( R \) into equation (1). All values are inserted into the RL circuit, as shown in Figure 1. Time constant was computed from 0.1s to 0.5s for better accuracy.
Table 1: List of RL circuit equations derived from given V, I, and R

| Equation | V(t)  | R    | L    |
|----------|-------|------|------|
| 15-3I    | 60 v  | 12 H | 4 Ω  |
| 15-I     | 15 v  | 1 H  | 1 Ω  |
| 1-2I     | 4 v   | 8 H  | 4 Ω  |

The derived RL circuit equations shown in Table 1 have used a basic circuit, as in Figure 1. Table 2 below shows circuits with the derived value for each V, L, and R to provide an understanding of the association between a circuit and its respective equation (2).

### Table 2: Relation between Circuit and Equation

| $\frac{dI}{dt} = \frac{V(t) - RI}{L}$ | RL Circuit Diagram |
|--------------------------------------|--------------------|
| $\frac{dI}{dt} = \frac{60 - 12I}{4}$ | ![Circuit Diagram](image) |
| $\frac{dI}{dt} = \frac{15 - 1I}{1}$  | ![Circuit Diagram](image) |
| $\frac{dI}{dt} = \frac{4 - 8I}{4}$  | ![Circuit Diagram](image) |

3.0 RESULTS AND DISCUSSION

Experiments were conducted on three equations derived from Table 3. All these equations were tested to the two schemes, namely the Euler Arithmetic (EA), i.e. the original enhanced Euler method; and
the new enhanced Euler method, i.e. ERMS. The results were compared with the exact solutions to ascertain accuracy. The exact solution for each LR circuit equation is calculated using equation (3) below:

\[
i(t) = \frac{V}{R} \left(1 - e^{-\frac{t}{\tau}}\right)
\]  

(3)

To obtain the maximum error, the exact value and the Euler value will be deducted, as shown in equation (4).

\[
Max\_Error = \frac{|Exact\_Value - Euler\_Value|}{Exact\_Value}.
\]  

(4)

Problem 1 shows that the ERMS provided better accuracy compared to that of EA. All equations were run for the same duration, \(t=0.5s\). The equation \(\frac{dl}{dt} = 15 - 3I\) provided an exact solution of 3.884349 after being calculated using equation (3). After having run using both schemes, the ERMS demonstrated the best accuracy with a maximum error of 0.016233 compared to the EA’s error of 0.021069. Figure 4 shows the plotted maximum error values for both schemes.

Problem 2, with its equation \(\frac{dl}{dt} = 15 - I\), provided the exact solution of 5.902 after being calculated using equation (2). After having run using both schemes, ERMS still provided the best accuracy with a maximum error of 0.004064 even if the difference was not much; the EA showed an error of 0.004394. Figure 5 shows how the maximum error values are plotted close to each other.
Finally, Problem 3, with its equation \( \frac{dl}{dt} = 1 - 2I \), resulted in an exact solution of 0.31606 after the value had been calculated using equation (2). After having run using both schemes, the ERMS demonstrated the best accuracy with a maximum error of 0.000695. This is by far the best accuracy when compared to the EA with an error of 0.000819, as plotted in Figure 6.
CONCLUSION

In this contribution, this paper presented a new scheme for a modified Euler method to model the electric circuit. This paper started with the development of the modified Euler method using root mean square and modelled in the RL circuit. The feasibility of ERMS was compared with the EA scheme and the exact solution by being tested in an RL circuit equation. This study has demonstrated that the ERMS provides solutions that are similar to the exact solutions at t=0.5. It has proven that the enhanced Euler method can be applied in various fields, especially in electrical engineering. In conclusion, the ERMS can be used as an alternative algorithm to solve RL circuit problems.

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