A GRAPH CELLULAR AUTOMATON WITH RELATION-BASED
NEIGHBOURHOOD DESCRIBING THE IMPACT OF PEER
INFLUENCE ON THE CONSUMPTION OF MARIJUANA
AMONG COLLEGE-AGED YOUTHS

YUSRA BIBI RUHOMALLY
Department of Mathematics, Faculty of Science
University Of Mauritius, Reduit, Mauritius

MUHAMMAD ZAID DAUHOO*
Department of Mathematics, Faculty of Science
University Of Mauritius, Reduit, Mauritius

LAURENT DUMAS
Laboratoire de Mathématiques de Versailles, UVSQ, CNRS
Université Paris-Saclay, 78035 Versailles, France

(Communicated by Pierre-Emmanuel Jabin)

Abstract. A novel approach depicting the dynamics of marijuana usage to
gauge the effects of peer influence in a school population, is the site of invest-
tigation. Consumption of drug is considered as a contagious social epidemic
which is spread mainly by peer influences. A relation-based graph-CA (r-
GCA) model consisting of 4 states namely, Nonusers (N), Experimental users
(E), Recreational users (R) and Addicts (A), is formulated in order to represent
the prevalence of the epidemic on a campus. The r-GCA model is set up by
local transition rules which delineates the proliferation of marijuana use. Data
available in [4] is opted to verify and validate the r-GCA. Simulations of the
r-GCA system are presented and discussed. The numerical results agree quite
accurately with the observed data. Using the model, the enactment of cam-
paigns of prevention targeting N, E and R states respectively were conducted
and analysed. The results indicate a significant decline in marijuana consump-
tion on the campus when a campaign of prevention targeting the latter three
states simultaneously, is enacted.

1. Introduction. Around 269 million people used drugs worldwide in 2018, which
is approximately 30 per cent more than in 2009, while over 35 million people suffer
from drug use disorders, according to the World Drug Report 2020. It appears
that the Covid-19 pandemic has already caused shortages of drugs on the street,
leading to increased prices and reduced purity [27]. Several states and countries
have legalised marijuana in some jurisdictions [19]. However, to date, the impact
of illicit drug consumption is still blurred. On the other hand, it must be pointed
out that frequent use of marijuana has increased in all of these jurisdictions after

2020 Mathematics Subject Classification. Primary: 37N99.
Key words and phrases. Marijuana, peer influence, relation-based, graph cellular automaton,
targeted campaigns of prevention.

* Corresponding author: Muhammad Zaid Dauhoo.
legalisation. In some of these jurisdictions, numerous potent marijuana products are also seen in the market. In the World Drug Report 2020, it is emphasised that the adverse health consequences of drug use are more widespread than previously thought [27].

According to [13], marijuana has been the most widely used illicit drug among college students. A survey in 2019 showed that past-year use of marijuana is similar for college students and their non-college peers at about 43%, representing an approximate of 7% five-year increase for college students with rates for both groups at historic highs over the past 35 years [24]. As pointed in [13], one in 17 college students uses marijuana daily or near-daily and 28.6% used marijuana in the past three months.

According to the reports [13, 24, 27], it seems that the dynamics of illicit drug use and marijuana among teens and young adults in particular, is taking an increasing trend. By analysing and quantifying its evolution on college campuses, further insights on how to curb this phenomena can be obtained. Studies have highlighted that illicit drug use driven mainly by peer influences on college campuses plays a crucial role in initiation of illicit drug use and is very common nowadays [27], [24], [18]. Additionally, social interactions within a group influences individual decision making [5, 6]. In this sense, such interactions must be integrated in the modelling of illicit drug use proliferation. Social network analysis have revealed that one common feature of adolescence is the clique, which is a small group of at least three adolescents whose primary friendships are with each other [8]. Peer pressure of cliques play an important role in the eventual drug habits of its members, and can provide the social context for using drugs [18].

2. Mathematical modelling of illicit drug consumption. The dynamical study of this social issue can be found in [1], [3], [7], [11], [12], [21], [25] and [23]. However, it must be pointed out that in all these works, the dynamics of the evolution of consumption of illicit drug, under the influence of the social interactions, which are responsible for the drug use spread within drug users community has not been taken into account. These factors can be integrated in the modelling of the proliferation of illicit drug consumption using the cellular automata (CA) approach naturally via the concept of neighbourhood as is next explained.

Whilst the above-mentioned models are based on systems of coupled differential equations and give the global temporal evolution of each category involved, cellular automata have the advantage of giving the evolution of each individual, \( x \) through the simulations, based on prescribed rules. Because these rules take into account the state and evolution of the neighbours of \( x \), this represents one way that CA can be used to consider the peer influence due to social interactions. CA models can thus be used to study the effects of social interactions at the individual level, on the evolution and proliferation of illicit drug consumption in a given group of individuals. Various scenarios can thus be tested using CA [2].

There exists few works dealing with the use of cellular automata to simulate the prevalence of illicit drug use within a given population. The work of [22] is a notable example whereby a drug model based on cellular automata, endowed with the traditional topology using the Von Neumann neighbourhood, was presented. As mentioned in [16], the usual topologies of CAs are chains and regular lattices. Nonetheless, the 2D lattice space exhibit certain drawbacks. The connection topology among the cells is limited to predetermined homogeneous lattice. As a result,
other topologies must be considered when the phenomenon to simulate requires additional features to take into account.

In the classical CA, the most usual types of neighborhood employed are the Von Neumann and Moore neighborhood, which consider some or all of the cells adjacent to a given cell [14]. These neighborhoods can also be extended using a radius around the central cell. In the context of the proliferation of drug consumption, the problem of neighborhood becomes more complex due to the effect of peer influence. It is imperative to highlight that peer influence is not necessarily a function of proximity in considering close neighbors only. Nowadays, teens spend considerable time on social media accounts and this can enhance their social standing.

In this sense, it is worth taking up the idea of the extension of the neighborhood of a particular individual. An interesting approach is to use the concept of a relation-based graph-CA (r-GCA) neighborhood [16]. The r-GCA approach is based on a graph whereby the nodes of the graph stand for the cells of the CA and the neighborhood of a particular cell is comprised by nodes that are related to it. Compared to the classical CA, such neighborhood is dynamic and differs from cell to cell. The graph-CA based approach is mainly used in the analysis of social networks and connections between users who are far away but close in the relationship of common friends, common interests and cliques amongst others. The graph-CA concept has been applied to numerous fields namely, traffic modelling [16], flow of water [20] and in the modelling of infectious diseases [17]. However, to date, no such study has been conducted in the field of illicit drug use.

The present work aims at modelling illicit drug usage within a campus, using the relation-based graph-cellular automata (r-GCA) approach. The model proposed in this study can be considered as a continuation and improvement of the system in [22], whereby this study enables the modelling in the relational space where the neighborhood corresponds to the relationships established between the users. A relation-based graph-cellular automata (r-GCA) is set up by local transition rules that are capable of describing the social interactions and influences among both nonusers and drug users. In [5], it is highlighted that it is difficult to acquire data which is related to crime on drug populations and hence, the cellular automaton developed could not be verified and validated. This limitation is addressed in the current work, using data available in [4]. More details are given in Section 6.1.

3. Overview of cellular automata. Formally, a cellular automaton can be defined as a collection of cells that evolve through a finite number of discrete time steps on a grid of specified shape [14]. A Cartesian mesh is the simplest grid that can be considered in 2D. The spatio-temporal evolution of each cell is according to a set of rules which are based on the states of its neighbouring cells. Thus, the neighborhood over which the cells affect one another must also be set. The rules are then applied iteratively for a number of time steps. These rules can either be deterministic or stochastic, that is, probability-based [9]. More details of the theory and applications of cellular automata can be found in [15].

Each cell of the lattice can be in one state at a time. The next state of each cell depends on the state of the current cell and states of its neighbourhood. Typically, in the Von Neumann or Moore’s neighbourhood, the state of the cell is influenced by a list of adjacent cells that is, neighbouring cells in its environment. In a r-GCA neighbourhood, the cell’s state is determined on the basis of its state and the states and relationships of neighbouring cells that are present in its neighbourhood.
More precisely, the neighbourhood belongs to a set of relations with other cells and are not dependent on the position of the cells in the cell grid [16]. The 2D cellular automaton is pictured as a regular lattice or grid and Figure 1 presents the neighbourhood of cell \(C_{i,j}\) (Red cell).

![Von Neumann neighbourhood.](image1)

![Moore neighbourhood.](image2)

![Relation-based graph cellular automaton (r–GCA) neighbourhood.](image3)

Figure 1. Types of neighbourhood in cellular automata.

Table 1 specifies the neighbouring cells for cell, \(C_{i,j}\) in the Von Neumann, Moore and relation-based graph cellular automaton (r–GCA) neighbourhoods.

| Neighbourhood | Neighbouring cells for \(C_{i,j}\) |
|---------------|----------------------------------|
| Von Neumann   | \(C_{i+1,j}, C_{i-1,j}, C_{i,j+1}, C_{i,j-1}\) |
| Moore         | \(C_{i+1,j}, C_{i-1,j}, C_{i,j+1}, C_{i,j-1}\), \(C_{i+1,j+1}, C_{i-1,j-1}, C_{i-1,j+1}, C_{i+1,j-1}\) |
| r-GCA         | \(C_{i,j-1}, C_{i-2,j-1}, C_{i-2,j+1}, C_{i+1,j+2}, C_{i+2,j-1}\) |

Table 1. Neighbourhood specification.

In this study, the r-GCA approach is opted to simulate the dynamics of illicit drug use within a given population. Such concept is more realistic since the neighbourhood of a given individual is not limited to four or eight neighbours unlike the classical CA. Additionally, the neighbourhood for each cell is inhomogeneous, making the process dynamic and realistic. In the next section, the basic theory about CA on graphs, is presented.
4. Cellular automata on graphs. Some notation for graphs are introduced in this section. A graph $G$ consists of two sets $V(G)$ and $E(G)$. $V(G)$ is a set of vertices (or nodes) \( \{v_1, v_2, \ldots, v_n\} \), where $n$ is the size of the graph. $E(G)$ is a set of edges (or links) between the vertices, where each edge is an unordered pair of vertices \( \{v_i, v_j\} \).

In the current context, a directed graph is used and loops are not admitted so that $v_i v_j \neq v_j v_i$ and $v_i v_i$ is not a member of $E(G)$.

If $V(G) = \{v_1, \ldots, v_n\}$, the adjacency matrix of $G$ is the $n \times n$ matrix, $A = (a_{ij})$ where

$$a_{ij} = \begin{cases} 1, & \text{if } (v_i, v_j) \in E(G) \\ 0, & \text{if } (v_i, v_j) \notin E(G) \end{cases}$$

The neighbourhood of a vertex $v_i \in V(G)$ is the set of vertices $N(v_i)$ which are joined to $v_i$ by edges in $E(G)$:

$$N(v_i) = \{v_j | v_j v_i \in E(G)\}.$$

A cellular automaton for a directed graph $G = (V(G), E(G))$ is a 4-tuple $A = (V, S, N, f)$. The set $V$ defines the cellular space of the CA such that each node stands for a cell of the cellular automaton. $S$ denotes the finite set of states that can be assumed by the nodes at each time step, $t$. The state of the node $v$ at time step $t$ is represented by $s_t^v \in S$ which alters accordingly to the local transition function $f$. $N$ is the neighbourhood function which assigns the respective neighbourhood for each node, that is:

$$N : V \rightarrow 2^V$$

$$v_i \mapsto N(v_i) = N_{v_i} = \{v_{i_1}, v_{i_2}, \ldots, v_{i_{d_v}}\},$$

where $d_v$ represents the degree of the node, that is the number of neighbours in the neighbourhood of node $v_i$.

The local transition function, $f$ determines the state of every node at a particular time step, $t + 1$ from the states of its neighbours at the previous time step $t$, that is:

$$s_{t+1}^{v_i} = f \left(s_t^{v_{i_1}}, s_t^{v_{i_2}}, \ldots, s_t^{v_{i_{d_v}}}\right) \in S,$$

where $N_v = \{v_{i_1}, v_{i_2}, \ldots, v_{i_{d_v}}\}$.

The r-GCA that describes the evolution of illicit drug dynamics is developed in the following section.

5. The relation-based Graph Cellular Automaton (r-GCA). The relation-based Graph Cellular Automaton (r-GCA) model is defined on a 2D lattice that stands for the space where the social epidemic of drug consumption is proliferating. The r-GCA is a discrete model comprising of a grid of $X \times X$ dimensions which is divided into identical square areas, each of them representing a cell of the CA. Each site of the lattice is occupied by only one user that can be in one of the following four states: nonuser (N), experimental user (E), recreational user (R) and addict user (A). The four states that are used in the model are described in Table 2.

Figure 2 pictures the schematic representation of drug dynamics in a given population. The r-GCA system has 12 parameters and the physical interpretation of each parameter involved is given in Table 3. In this model, the r-GCA neighbourhood
| Type of user     | State | Colour |
|-----------------|-------|--------|
| Nonuser - N     | 0     | Green  |
| Experimental user - E | 1     | Blue   |
| Recreational user - R   | 2     | Yellow |
| Addict user - A    | 3     | Red    |

Table 2. Definition of states and colour code of cells.

Figure 2. Schematic representation of the r-GCA model.

| Parameter | Physical Meaning |
|-----------|------------------|
| $\alpha_1(t)$ | Influence rate of $E(t)$ on $N(t)$ |
| $\alpha_2(t)$ | Influence rate of $R(t)$ on $N(t)$ |
| $\alpha_3(t)$ | Influence rate of $R(t)$ on $E(t)$ |
| $\alpha_4(t)$ | Rate at which recreational users change to addicts |
| $\gamma_1(t)$ | Rate at which experimental users quit drugs |
| $\gamma_2(t)$ | Rate at which recreational users quit drugs |
| $\gamma_3(t)$ | Rate at which addicts quit drugs |
| $\beta$ | Proportion of nonusers moving into the population |
| $\omega_N(t)\beta$ | Proportion of nonusers moving out of the population |
| $\omega_E(t)\beta$ | Proportion of experimental users moving out of population |
| $\omega_R(t)\beta$ | Proportion of recreational users moving out of population |
| $\omega_A(t)\beta$ | Proportion of addicts moving out of the population |

Table 3. Interpretation of the parameters involved in the r-GCA model.

is considered to simulate the cellular automaton. The model is run for a certain number of time steps, with the grid being updated at every time step $t$. Each cell "interacts" with cells in its neighbourhood and is updated according to a set of predefined rules [22]. Within this model, the stochastic rules for the transitions in the r-GCA are set up according to the following assumption:
• The allowed transitions between the different states are only the ones that obey the schematic representation of the r-GCA model.

The transition probabilities of the r-GCA [22] are delineated as follows:

1. If a site is occupied by a nonuser and there is at least an experimental user in its neighbourhood, then the site has a certain probability of being occupied by an experimental user in the next time step. This transition is proportional to the parameter $\alpha_1$ and the number of neighbouring experimental sites. This dynamic consists of the following process: $N + E \rightarrow E + E$, by a catalytic reaction which is equivalent to peer pressure exerted by an experimental user on a nonuser.

2. If a site is occupied by a nonuser and there is at least a recreational user in its neighbourhood, then the site has a certain probability of being occupied by an experimental user in the next time step. This transition is proportional to the parameter $\alpha_2$ and the number of neighbouring recreational sites. This dynamic consists of the following process: $N + R \rightarrow E + R$, by a catalytic reaction which is equivalent to peer pressure exerted by a recreational user on a nonuser.

3. If a site is in state $E$ and there exists a recreational user in its neighbourhood, then the probability that the site is in state $R$ in the next instant of time is proportional to the parameter $\alpha_3$ and the number of neighbouring recreational sites, that is, $E + R \rightarrow R + R$.

4. If a site is in the state $R$, it becomes $A$ spontaneously with probability $\alpha_4$ in the next time step. In this process, we note that a recreational user is not influenced by neighbouring addict user sites in order to change state, that is, $R \rightarrow A$ spontaneously.

5. If a site is occupied by an experimental user or a recreational user and there is at least a nonuser in the neighbourhood of that site, then the site has a certain probability of being occupied by a new nonuser in the next time step. These transition probabilities are proportional to the parameters $\gamma_1$ and $\gamma_2$ respectively and the number of neighbouring nonuser sites. The processes are as follows: $E + N \rightarrow N + N$ and $R + N \rightarrow N + N$, by a catalytic reaction, which depicts a positive peer influence of nonusers on experimental and recreational users respectively.

6. If a site is in the state $A$, it becomes $N$ spontaneously with probability $\gamma_3$. In this process, an addict user does not require the presence of a nonuser in order to change its state and it occurs as follows: $A \rightarrow N$ spontaneously.

In contrast to the model of [22], in the present r-GCA system, with each site of the lattice being occupied by only one user that can be in one of the following four states: nonuser (0), experimental user (1), recreational user (2) and addict user (3), an indicator variable is created to represent the age of each cell which is coded as 12, 13, $\cdots$, 18. The age of each cell is updated after every 365 time steps (1 year). When a given cell reaches the age value of 18, the cell is removed from the system and is replaced by a new nonuser. This explains the coefficients $\omega_N(t), \omega_E(t), \omega_R(t)$ and $\omega_A(t)$ that are present in the schematic representation of the model. In the present work, college dropouts have not been considered and thus, outflow of individuals are only for those who are 18 years old.
Moreover, \( \omega_N(t) + \omega_E(t) + \omega_R(t) + \omega_A(t) = 1 \). Therefore, in this model, a constant population is considered. Hence, each cell in the lattice is assigned to 2 values namely, the state and the corresponding age of the user. Consequently, taking into account the age of each cell and the periodic exits of users that occur after every 365 time steps, bring the r-GCA model closer to the reality of a given school population.

Figure 3 illustrates the neighbourhood of a given individual.

![Figure 3](image_url)

**Figure 3.** The neighbourhood of a given individual within a population comprising of 900 individuals. Double arrows denote mutual influences (two-way relationship) and single arrows represent a one-way relationship with the individual. Four mutual influences are present in the neighbourhood of the individual.

Furthermore, the ground where the epidemic is spreading is modelled as a weighted graph where each node stands for an individual and the edge between two nodes denotes the relationship between the corresponding individuals. In this sense, the distance factor between the nodes \( u_{i,j} \) and \( v_{k,l} \) is the weight associated to the edge \( (u, v) \in E(G) \) and it is denoted by \( w_{u,v} \). Generally speaking, the greater the distance, the smaller the influential factor. Thus, the probability of being influenced is inversely proportional to the distance between \( u_{i,j} \) and \( v_{k,l} \). Let

\[
\text{dist}_{u_{i,j}, v_{k,l}} = \frac{1}{\sqrt{(i - k)^2 + (j - l)^2}}. \tag{1}
\]

Then,

\[
\Gamma_{u_{i,j}} = \sum_{v_{k,l}, \neq u_{i,j}} \frac{1}{\sqrt{(i - k)^2 + (j - l)^2}}. \tag{2}
\]

The weight factor, \( w_{u,v} \) of the cells \( u_{i,j} \) and \( v_{k,l} \) is defined as

\[
w_{u,v} = \frac{\text{dist}_{u_{i,j}, v_{k,l}}}{\Gamma_{u_{i,j}}}. \tag{3}
\]

The weight factors for each cell of the CA form the weight matrix, \( W(G) \).
Therefore, for a given r-GCA system with a population of \( n \) individuals, the neighbourhood matrix, \( A(G) \) and the weight matrix, \( W(G) \) are represented as

\[
A(G) = \begin{bmatrix}
a_{11} & \cdots & a_{n1} \\
\vdots & \ddots & \vdots \\
a_{n1} & \cdots & a_{nn}
\end{bmatrix},
\]

where

\[
a_{ij} = \begin{cases} 
1, & \text{if } (v_i, v_j) \in E(G) \\
0, & \text{if } (v_i, v_j) \notin E(G)
\end{cases}
\]

and

\[
W(G) = \begin{bmatrix}
w_{11} & \cdots & w_{n1} \\
\vdots & \ddots & \vdots \\
w_{n1} & \cdots & w_{nn}
\end{bmatrix},
\]

where \( w_{ij} \) is computed using Eq. (3) for \( 1 \leq i \leq n \) and \( 1 \leq j \leq n \).

The weighted neighbourhood matrix, \( W_N(G) \) for the given graph includes weight coefficients of individual edges of the directed graph only and is obtained by performing an Hadamard product as follows:

\[
W_N(G) = \begin{bmatrix}
a_{11} & \cdots & a_{n1} \\
\vdots & \ddots & \vdots \\
a_{n1} & \cdots & a_{nn}
\end{bmatrix} \odot \begin{bmatrix}
w_{11} & \cdots & w_{n1} \\
\vdots & \ddots & \vdots \\
w_{n1} & \cdots & w_{nn}
\end{bmatrix} = \begin{bmatrix}
a_{11}w_{11} & \cdots & a_{n1}w_{n1} \\
\vdots & \ddots & \vdots \\
a_{n1}w_{1n} & \cdots & a_{nn}w_{nn}
\end{bmatrix}.
\]

An Hadamard product is used so that other influences which are not found in the neighbourhood of a given individual is not taken into consideration.

Moreover, for the present r-GCA system, a domain matrix, \( D_{n,4} \) is built to determine the type of each user in the lattice. The four columns of the matrix \( D_{n,4} \) denote the nonuser domain, experimental domain, recreational domain and addict domain respectively. \( D_{n,4} \) is represented as follows:

\[
D_{n,4} = \begin{bmatrix}
d_{11} & d_{12} & d_{13} & d_{14} \\
\vdots & \vdots & \vdots & \vdots \\
d_{n1} & d_{n2} & d_{n3} & d_{n4}
\end{bmatrix},
\]

where

\[
d_{ij} = \begin{cases} 
1, & \text{if the status of the } i^{th} \text{ individual is of } j^{th} \text{ type} \\
0, & \text{otherwise for } 1 \leq i \leq n \text{ and } 1 \leq j \leq 4
\end{cases}
\]

The sum of each row of matrix \( D_{n,4} \) is one since an individual can be in only one state at a given time \( t \).

Matrices \( W_N(G) \) and \( D_{n,4} \) are then multiplied as follows:

\[
M(i, j) = \sum_{k=1}^{n} W_N(i, k) \times D(k, j).
\]

The \((i, k)\) entry of \( W_N \) is the weighted influence of individual \( k \) which is present in the social network of individual \( i \). The \((k, j)\) entry of \( D \) is the status of the \( k^{th} \) individual of \( j^{th} \) type. Thus, the \((i, j)\) entry of \( M \) represents the total influence exerted by \( j^{th} \) type users which is present in the social network of \( i \), on individual \( i \).

The transition probabilities as defined in Table 3 are then multiplied by the matrix \( M \) in order to determine the state of a particular cell in the next time step, \( t \).
In order to use the r-GCA model developed in this section, some numerical simulations are conducted in the section that tails.

6. Numerical experiments.

6.1. Verification and validation of the r-GCA model. One of the primary concerns in designing a mathematical model is the ability to produce adequate results. This implies that the verification of a system is of utmost importance. Therefore, data available in [4] is opted to verify and validate the r-GCA model established in the previous section.

Data is drawn from repeated cross-sectional Ontario Student Drug Use and Health Survey (OSDUHS) of students in grades 7-12 over 18 academic years (1999-2017). This survey is carried out every 2 years with the aim of analysing the epidemiological trends in student drug use [4]. In the present study, the prevalence of marijuana use is of particular concern. Participants were asked: “In the last 4 weeks, how often (if ever) did you use marijuana?” The response options were: not used in the past month, 1-2 times, 1-2 times each week, 3-6 times each week and daily use.

Within the framework of the r-GCA model, the participants were partitioned as follows: not used in the past month responses are coded as nonusers, 1-2 times responses are coded as experimental users, 1-2 times each week and 3-6 times each week responses are classified as recreational users and daily use responses are considered as addicts. Figure 4 pictures the trends in marijuana use for the four different categories of marijuana users for the period 1999-2017, according to the data gathered in Table 3.5.3 in [4].

Figure 4. Trends of the 4 categories of marijuana users for the period 1999-2017 in grades 7-12 according to [4].

In order to verify the r-GCA model, it is required to evaluate the set of parameters given in Table 3. Data available in [4] is used to verify the system. The verification is conducted for the years 1999 to 2006. The model is calibrated using genetic algorithm in order to determine the parameters involved in the system, as explained
The fitness function, $F(x)$ which is the sum of the squared differences between the collected data [4] and the model prediction, takes the following form:

$$F(x) = \sum_{i=1}^{4} (D_i - P_i)^2,$$

(5)

where

- $i = 1, 2, 3, 4$ correspond to the 4 categories: $N, E, R$ and $A$ respectively,
- $D_i$ : the exact value obtained from [4] for each category,
- $P_i$ : the value obtained for each category from the r-GCA simulations and
- $x$ : the vector representing the variables present in the r-GCA which needs to be determined.

Matlab Optintool is used and the fitness function is inserted in the genetic algorithm. Table 4 shows the values of the coefficients obtained after applying genetic algorithm, which result in the square error between the actual data and the simulated one.

| Parameter | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | $\alpha_4$ | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ |
|-----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Value     | 0.101       | 0.109       | 0.116       | 0.117       | 0.126       | 0.318       | 0.114       |

Table 4. Parameter values obtained for the consumption of marijuana for the period 1999-2006, using genetic algorithm.

The following initial condition is incorporated in the r-GCA model in order to conduct the numerical simulations. The initial condition is obtained from Figure 4.

$$(N(0), E(0), R(0), A(0)) = (79.1, 10.2, 8.1, 2.5).$$

The r-GCA model is then run with the set of parameter values given in Table 4. The system is verified for 7 years (up to 2006 whereby each year is run for 365 time steps) and is then validated for the next 11 years, as depicted in Figure 5. The left side of the vertical black line, drawn at the year 2006 represents the verification part while the right side represents the validation part of the r-GCA model.

Figure 5 shows the results of the r-GCA model (dotted lines) superimposed on the data from [4] (solid lines) for the period 1999-2017. The results show good agreement with the observed data. A significant difference is noted in the proportion of recreational users and addicts as compared to the exact trend at the final time step. Nevertheless, it is observed that the model guarantees a good accuracy in the prediction of the proportion of nonusers and experimental users respectively, thereby giving an accurate idea of the proportion of regular marijuana consumers in the population. This is evident in the sense that the present system captures the different characteristics underlying the proliferation of experimental, recreational and abusive marijuana consumption. As a result, the r-GCA model can be a precious tool in forecasting illicit drug consumption in any given population.

Since cellular automata modelling can serve as virtual laboratories, various scenarios can be effectuated. Hence, in the following section, the r-GCA model is applied to a marijuana consuming prone environment in order to examine scrupulously the effects of peer influence on nonusers and marijuana users. A school population is considered and tight social connections are assumed to exist within
the environment, whereby both the positive and negative influences on illicit drug use occur.

6.2. Simulations of the r-GCA model within a school environment. In this subsection, the case for a common place like a school is considered and it is assumed that the total population of the school is, $T = 900^{1}$. The cellular space in the simulations is formed by a 2D array of $30 \times 30$. The simulation starts with an initial configuration of nonusers, experimental users and a group of regular drug users comprising of recreational and addict individuals respectively. These sites corresponding to these individuals are scattered randomly in the lattice.

It is noted that in general, the minimum time for a student to be in the secondary school cycle is 7 years$^{1}$. In the present model, each time step, $t$, represents 1 day. Thus, the r-GCA model is simulated for 2555 time steps and the evolution of each category is studied. The cellular space of the cellular automata is made up of 67.89% of nonusers, 12.22% of experimental users, 10.89% of recreational users and 9% of addicts. The initial configuration used is given in Table 5 and the parameter values given in Table 6 is used to conduct the simulations.

$^{1}$Statistics Mauritius, Of Education Statistics, 2018. Available from: https://statsmauritius.govmu.org/Documents/Statistics/Digests/Education/Digest_Edu_Yr18.pdf (Accessed on 07/02/21)

Statistics Mauritius is the official organisation responsible for collection, compilation, analysis and dissemination of the official statistical data relating to the economic and social activities of the country. Information about the secondary school cycle in Mauritius is sought in Statistics Mauritius.

| State | N  | E  | R  | A  |
|-------|----|----|----|----|
| Value | 611| 110| 98 | 81 |

Table 5. Initial number of individuals that represent each state.

Figure 5. Superimposition of the evolution of the four categories of marijuana users (dotted lines) on the data collected from [4] (solid lines).
| Parameter | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | $\alpha_4$ | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Value     | 0.17      | 0.13      | 0.012     | 0.08      | 0.08      | 0.0101    | 0.0124    |

Table 6. Parameter values used for the scenario.

Figure 6 depicts the spread pattern of illicit drug proliferation at various time steps.

Figure 7 shows the evolution of the 4 categories of users over time. The mean proportion of each category for each year (365 time steps) is plotted over time. A growing tendency towards the population of the experimental users is illustrated in Figure 7. This indicates that a large number of individuals quit the nonuser category to join the experimental category. The remarkable decrease in the number of nonusers in the first 500 time steps clearly evidences for the rise in the experimental, recreational and addict categories. It is also observed that the population of the experimental users has remained almost steady after 1000 time steps.

The above observations indicate that delaying or preventing the nonusers to initiate illicit drug use is crucial. Effective measures are required to remedy this social issue. At the school level, suggested improvements namely, the implementation of targeted campaigns of prevention can be adequate in order to lessen the number of illicit drug users in the school premises.

Hence, in the following section, an investigation is made on the impact of enacting targeted campaigns of prevention directed to the different categories of users, within the school population.

7. The effect of implementing targeted campaigns of prevention. In this section, the effect of 3 types of targeted campaigns of prevention subjected to the population, is analysed. By varying the different rates of influences that drug users exert on each other, many factors that have a preponderant role in illicit drug consumption can be examined. The efficiency of a campaign of prevention can be gauged from the outcomes of numerous scenario analysis. 3 scenarios are considered and the impact on the population is perused thoroughly.

7.1. Scenario 1: Targeting the nonusers. In this case, the effect of enacting campaigns of prevention on the nonusers is examined. It is noted that the experimental and recreational users exert an influence of $\alpha_1$ and $\alpha_2$ on the nonusers respectively. Therefore, a possible campaign of prevention with the aim of targeting the nonusers in order to curb their transition to the drug user’s category is one that reduces both influences.

For instance, campaigns of prevention with the aim of discouraging the first use of illicit drugs and disseminating its consequences can be appropriate\(^2\).

\(^2\)Drug Aware, Real facts, 2018. Available from: https://drugaware.com.au/about-us/previous-campaigns/real-facts-about-drugs/. Accessed on 07/02/21.

Drug Aware was established in 1996 and is an on-going government program on illicit drug education undertaken in Australia. The program is managed by the Alcohol and Other Drug Prevention Services Division of the Mental Health Commission in collaboration with Cancer Council Western Australia. Drug Aware program contributes to behaviour change by implementing campaigns using targeted mass reach education strategies and develops strategies to create safer environments to reduce the incidence and risk of harm from illicit drug use.

Since 2018, the campaign ‘Real Facts’ has been running with the objective of targeting youngsters aged 15-25 years old. The campaign aims at preventing and delaying the early uptake of alcohol and other drugs.
Figure 6. These figures show the snapshots of the r-GCA dynamics over 2555 time steps on a $30 \times 30$ grid. These figures illustrate the temporal evolution of the users. Figure 3 (a) represents the initial configuration with 611 N, 110 E, 98 R and 81 A. The parameters used are $\alpha_1 = 0.17, \alpha_2 = 0.13, \alpha_3 = 0.012, \gamma_1 = 0.08, \gamma_2 = 0.0101$ and $\gamma_3 = 0.0124$. 
CONSUMPTION OF MARIJUANA AMONG COLLEGE-AGED YOUTHS

Within the present r-GCA model, limiting the first use of marijuana is equivalent to the reduction of both $\alpha_1$ and $\alpha_2$. Data given in Table 6, except that $\alpha_1$ and $\alpha_2$ have been decreased by 4% and 3% respectively, in order to simulate campaigns of prevention which target nonusers.

Figure 8 depicts the evolution of the 4 categories of users with campaigns of prevention targeting the nonusers (dotted lines) superimposed on the evolution of the 4 categories of users without any campaign of prevention (solid lines). From

In the model, this is related to the transition of nonusers to the experimental category. Such a campaign of prevention if effective, can decrease the coefficients $\alpha_1$ and $\alpha_2$ respectively.
Figure 8, it is observed that the number of nonusers with campaigns of prevention is much higher as compared to the proportion of nonusers without any campaign of prevention. Additionally, a notable decrease in the number of recreational users and addicts is noted, when campaigns of prevention are directed to the nonuser category.

7.2. Scenario 2: Targeting the experimental and recreational users. A campaign of prevention targeting both the experimental users and recreational users is one that reduces the influence of $R(t)$ on $E(t)$. Moreover, the influence of nonusers on the experimental and recreational users are increased in order to encourage them to join the nonuser category. Data given in Table 6, except that $\alpha_3$ has been decreased by 20%. Moreover, $\gamma_1$ and $\gamma_2$ have been increased by 10% and 12.5% respectively, in order to simulate campaigns of prevention directed to the experimental and recreational categories specifically.

From this scenario, a remarkable difference in the number of recreational users is observed as compared to the previous case. It should be highlighted that in the present scenario, a higher number of nonusers is observed.

These observations indicate that it is essential that campaigns of prevention be set up to discourage recreational use of illicit drugs. From the above scenario, the r-GCA model confirms the importance of setting up such campaigns of prevention in order to thwart the number of illicit drug users on college campuses.

3Drug Aware, Party smarter - safer events and festivals campaign, 2020. Available from: https://drugaware.com.au/about-us/current-campaigns/party-smarter/. Accessed on 07/02/21.

In November 2020, the Drug Aware Safer Events and Venues, ‘Party Smarter’ campaign has been set up in Western Australia. Since college campuses have numerous social events, festivals and night venues, the latter campaign aims to assist in reducing illicit drug related harm at these events. Besides, the campaign has a pre, during and post-event phase respectively whereby each phase has a different evidence-based objective relating to harm minimisation of ecstasy use at music events.
7.3. Scenario 3: Targeting the nonusers, experimental and recreational users. In this scenario, a combination of the measures considered in scenarios 1 and 2 are incorporated in the model simultaneously. The modified parameter values used in scenarios 1 and 2 are used to simulate campaigns of prevention which target the non, experimental and recreational users concurrently.

Figure 10 demonstrates an eventual decline in the 3 categories of drug users which accounts for the significant rise in the population of nonusers. However, it is seen that during the first 500 time steps, the population of experimental users is higher when subjected with campaigns of prevention. A similar observation is also seen for the population of addict users between 750 and 1000 time steps. Nevertheless, at the final time step $t = 2555$, a considerable drop in the number of experimental and recreational users is accounted. Moreover it is remarkable that with the inclusion of both measures in the system, a much higher proportion of nonusers is noted which evidences for the large drop in the 3 categories of drug users, unlike the previous 2 cases.

Thus, this highlights the fact that campaigns of prevention focusing on the nonuser, experimental and recreational categories is crucial in order to curb the transition of nonusers to the drug users’ category.

For example, campaigns$^4$ of prevention with the aim of discouraging the first use of illicit drugs and encouraging current drug users to quit consuming drugs can be

\[\text{Figure 10. Superimposition of the temporal evolution of the users with a campaign of prevention subjected to the non, experimental and recreational users (dotted lines) on the evolution of the users without any campaign of prevention (solid lines).}\]

In the model, this is related to the transition of experimental users to the recreational category and the movement of experimental and recreational users to quit drug use and join the nonuser class. Such a campaign of prevention is meant to decrease the coefficient $\alpha_3$ and increase the coefficients $\gamma_1$ and $\gamma_2$.

$^4$Drug Aware, 78% Don’t Use, 2019. Available from: https://drugaware.com.au/about-us/current-campaigns/78-dont-use/. Accessed on 07/02/21.

In August 2019, the new Drug Aware Generic Drugs ‘78% Don’t Use’ campaign has been established which is ran on social media, radio and out-of-home media in order to target youngsters aged 16 - 22 years old. Studies conducted in Western Australia showed that young people tend to overestimate the number of other young people who consume drugs, assuming that such behaviour
very important. Figure 11 clearly shows the notable differences between the initial proportion of the 4 categories of users with the final proportion for the 3 scenarios, with and without campaigns of prevention. When the effect of both measures in scenarios 1 and 2 are incorporated in the r-GCA model simultaneously, the population size of the experimental, recreational and addict population categories declines to 24.33%, 4.82% and 0.95% respectively, as depicted in Figure 11 (c). It is evident in Figure 11 that the final population size of the nonusers is much higher as compared to that of scenarios 1 and 2 when it is subjected to campaigns of prevention directed to non, experimental and recreational users simultaneously. Consequently, Figure 11 highlights the importance and effectiveness of the type of campaign of preventions to be enacted, in curbing drug consumption.

It should be stressed that the r-GCA model established in this work captures a multitude of influences that are responsible for the pervasiveness of illicit drug consumption in an illicit drug prone environment. The simulations presented highlight the importance of modelling this social epidemic with the relation-based graph-cellular automata approach since the users can be individually followed in time. Additionally, a significant feature of using r-GCA over ordinary differential equations and partial differential equations is that the change in social state of a typical individual can be tracked scrupulously during the latter’s life course.

8. Limitations of the model. In this study, it is assumed that the influence rates $\alpha_1$, $\alpha_2$ and $\alpha_3$ are homogenous for this particular age group. However, prior studies have shown that peer pressure is influenced by many factors. In [10], it is revealed that age is an influential factor for peer pressure among secondary school adolescents. Furthermore, a study [26] has shown that resistance to peer influence increased linearly during adolescence and particularly between ages 14 and 18. In this sense, it is essential to take age into account and vary the probabilities of influence namely, $\alpha_1$, $\alpha_2$ and $\alpha_3$ in the model, with respect to age. Such feature has to be taken into consideration in a future study.

9. Conclusion. In this work, a novel approach that describes the dynamics of marijuana usage to assess the effects of social interactions between drug consumers in a school population, was presented. A relation-based graph-cellular automata (r-GCA) model representing this social epidemic, consisting of 4 states was formulated. The r-GCA model was set up by local transition rules which delineated the proliferation of illicit drug use. The system was verified and validated using data available in [4]. Simulations of the r-GCA system were illustrated and the numerical results agreed quite accurately with the observed data. Additionally, an investigation was made on the enactment of targeted campaigns of prevention whereby 3 scenarios are conducted and analysed. The numerical results indicated a remarkable decline in the number of drug users when a targeted campaign of prevention is made to act on each of the category N, E, R and A respectively, in the r-GCA model simultaneously. To date, we do not know how to measure the effectiveness of is acceptable and normal. The goal of the campaign is to challenge such misconceptions among young people in order to aid them in influencing their decisions to not initiate or continue drug usage just because they think ‘everyone else is doing it’.

In the model, this is related to the transition of nonusers to the experimental class, experimental users to the recreational category and the movement of experimental and recreational users to quit drug use and join the nonuser class. Such a campaign of prevention has an influence on the coefficients $\alpha_1$, $\alpha_2$, $\alpha_3$, $\gamma_1$ and $\gamma_2$. 
Figure 11. A comparison of the proportion of the 4 categories of users when $t = 1$ (Initial) and when $t = 2555$ (Final) for the 3 scenarios, with and without the enactment of campaigns of prevention.

(a) Scenario 1: Targeting the Nonusers.

(b) Scenario 2: Targeting the Experimental and Recreational users.

(c) Scenario 3: Targeting Nonusers, Experimental and Recreational users.
a campaign of prevention and thus, it is a very interesting issue to be addressed in a future work. However, the present study gives insight on the type of campaign of prevention to be implemented. Furthermore, the r–GCA presented in this study can be considered as a mechanism that enables us to obtain a different level of realism in modelling the global phenomenon of illicit drug use, compared to the classical CA.

Acknowledgments. We would like to acknowledge the assistance of Mr Maheshsingh Mungur from the University of Mauritius in the preparation of this work. This work has been conducted under the HEC (Higher Education Commission Mauritius) MPhil/PhD scholarship. The authors would also like to thank the reviewers for their constructive and valuable comments, which led to the improvement of the paper.

REFERENCES

[1] N.-R. Badurally Adam, M. Z. Dauhoo and O. Kavian, An analysis of the dynamical evolution of experimental, recreative and abusive marijuana consumption in the states of Colorado and Washington beyond the implementation of I–502, J. Math. Sociol., 39 (2015), 257–279.
[2] A. Bakhtiari, Social Influences Among Drug Users and Mean Field Approximation of Cellular Automata, Ph.D thesis, Simon Fraser University, 2009.
[3] D. A. Behrens and G. Tragler, The dynamic process of dynamic modelling: The cocaine epidemic in the United States, Bulletin on Narcotics, 53 (2001), 65–78.
[4] A. Boak, H. A. Hamilton, E. M. Adlaf and R. E. Mann, Drug use among Ontario students, 1977–2017: Detailed findings from the Ontario Student and Drug Use Health Survey (OS-DUHS), Centre for Addiction and Mental Health.
[5] V. Dabbaghian, V. Spicer, S. K. Singh, P. Borwein and P. Brantingham, The social impact in a high-risk community: A cellular automata model, J. Comput. Sci., 2 (2011), 238–246.
[6] S. Y. Del Valle, J. M. Hyman, H. W. Hethcote and S. G. Eubank, Mixing patterns between age groups in social networks, Social Networks, 29 (2007), 539–554.
[7] M. Z. Dauhoo, B. S. N. Korimboccus and S. B. Issack, On the dynamics of illicit drug consumption in a given population, IMA J. Appl. Math., 78 (2013), 432–448.
[8] S. T. Ennett and K. E. Bauman, Adolescent Social Networks: Friendship Cliques, Social Isolates, and Drug Use Risk, University of North Carolina at Chapel Hill, 2000.
[9] P. Ghosh, A. Mukhopadhyay, A. Chanda, P. Mondal and A. Akhand, et al., Application of Cellular automata and Markov-chain model in geospatial environmental modeling - A review, Remote Sensing Appl.: Soc. Environ., 5 (2017), 64–77.
[10] R. Gikonyo and K. Ngugi, The influence of demographic factors on peer pressure among secondary school adolescents in Nyahururu Laikipia county, Res. Humanities Soc. Sci., 6 (2016), 2224–5766.
[11] A. Gragnani, S. Rinaldi and G. Feichtinger, Dynamics of drug consumption: A theoretical model, Socio-Economic Planning Sci., 31 (1997), 127–137.
[12] D. Grass, J. P. Caulkins, G. Feichtinger, G. Tragler and D. A. Behrens, Optimal Control of Nonlinear Processes, Springer-Verlag, Berlin, 2008.
[13] L. Johnston, R. Miech, P. O’Malley, J. Bachman, J. Schulenberg and M. Patrick, Monitoring the future national survey results on drug use, 1975-2019: Overview, key findings on adolescent drug use.
[14] J. J. Kari, Basic concepts of cellular automata, in Handbook of Natural Computing, Springer, Berlin, Heidelberg, (2012), 3–24.
[15] P.-Y. Louis and F. R. Nardi, Probabilistic Cellular Automata. Theory, Applications and Future Perspectives, Emergence, Complexity and Computation, 27, Springer, Cham, 2018.
[16] K. Malecki, Graph cellular automata with relation-based neighbourhoods of cells for complex systems modelling: A case of traffic simulation, Symmetry, 9 (2017), 322.
[17] M. J. F. Martínez, E. G. Merino, E. G. Sánchez, J. E. G. Sánchez, A. M. del Rey and G. R. Sánchez, A graph cellular automata model to study the spreading of an infectious disease, in Advances in Artificial Intelligence, Lecture Notes in Computer Science, 7629, Springer, Berlin, Heidelberg, (2012), 458–468.
[18] E. R. Oetting and F. Beauvais, Peer cluster theory: Drugs and the adolescent, *J. Counsel. Develop.*, 65 (1986), 17–22.

[19] R. L. Pacula and R. Smart, Medical marijuana and marijuana legalization, *Annual Review of Clinical Psychology*, 13 (2017), 397–419.

[20] P. Rinaldi, D. Dalponte, M. Venere, A. Clausse et al., Graph-based cellular automata for simulation of surface flows in large plains, *Asian J. Appl. Sci.*, 5 (2012), 224–231.

[21] Y. B. Ruhomally, N. Banon Jahmeerbaccus and M. Z. Dauhoo, The deterministic evolution of illicit drug consumption within a given population, in *CIMPA School on Mathematical Models in Biology and Medicine*, ESAIM Proc. Surveys, 62, EDP Sci., Les Ulis, (2018), 139–157.

[22] Y. B. Ruhomally and M. Z. Dauhoo, The NERA model incorporating cellular automata approach and the analysis of the resulting induced stochastic mean field, *Comput. Appl. Math.*, 39 (2020), 327–356.

[23] Y. B. Ruhomally, M. Z. Dauhoo and L. Dumas, An analysis of the recreational use of marijuana amongst the 21+ population of the state of Washington in the context of I-502 and its aftermath, *Neural, Parallel and Scientific Computations*, 28 (2020), 273–294.

[24] J. Schulenberg, L. Johnston, P. O’Malley, J. Bachman, R. Miech and M. Patrick, Monitoring the Future National Survey Results on Drug Use, 1975-2018: Volume II, college students and adults ages 19-60, 2019.

[25] B. Song, M. Castillo-Garsow, K. R. Rios-Soto, M. Mejran, L. Henso and C. Castillo-Chavez, Raves, clubs and ecstasy: The impact of peer pressure, *Math. Biosci. Eng.*, 3 (2006), 249–266.

[26] L. Steinberg and K. C. Monahan, Age differences in resistance to peer influence, *Develop. Psych.*, 43 (2007), 1531–1543.

[27] UN Office on Drugs and Crime, *World Drug Report 2020*, 2020. Available from: https://wdr.unodc.org/wdr2020/index.html.

Received February 2021; revised February 2021.

E-mail address: bibi.ruhomally@uomail.uom.ac.mu
E-mail address: m.dauhoo@uom.ac.mu
E-mail address: laurent.dumas@uvsq.fr