CONSTRAINTS ON LOW ENERGY SUPERSYMMETRY

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We review the available constraints on the low energy supersymmetry. The bulk of the electroweak data is well screened from supersymmetric loop effects, due to the structure of the theory, even with superpartners generically light, $O(M_Z)$. The only exception are the left-handed squarks of the third generation which have to be $\gtrsim O(300 \text{ GeV})$ to maintain the success of the SM in describing the precision data. The other superpartners can still be light, at their present experimental mass limits. As an application of the derived constraints (supplemented by the requirement of "naturalness") we discuss the predictions for the mass of the lighter MSSM Higgs boson in specific scanarios of supersymmetry breaking.

1 Introduction

Supersymmetry offers an interesting solution to the well known hierarchy puzzle of the SM and, moreover, has several other theoretical and phenomenological (gauge coupling unification) virtues. The supersymmetry breaking scale (often it can be defined only in some average sense) i.e. the scale of the mass spectrum of the superpartners provides the necessary cut-off to the SM. Two immediate and most important remarks about the superpartner spectrum are the following ones: if supersymmetry is to cure the hierarchy problem that scale is expected to be not much above the electroweak scale. On the other hand, it is totally unknown in detail, as we do not have at present any realistic model of supersymmetry breaking. Therefore, the Minimal Supersymmetric Standard Model (MSSM) is a very well defined theoretical framework but contains many free parameters: superpartner soft masses and their dimensionful couplings.

It is, therefore, very interesting to discuss the question to what extent the superpartner spectrum can manifest itself through virtual (loop) effects on the electroweak observables. Do very high precision measurements of the electroweak observables provide us with a tool to see supersymmetric effects.

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indirectly or, at least, to put stronger limits on its spectrum? We remember the important rôle played by precision measurements in seeing, indirectly, some evidence for the top quark long ago its direct discovery and with the mass quite close to its measured mass. Also, the present level of precision makes the electroweak measurements to some extent sensitive even to the Higgs boson mass, although the dependence is only logarithmic. With supersymmetric corrections the situation is different. The dependence on the top quark (and Higgs) mass in the SM is due to nondecoupling of heavy particles which get their masses through the mechanism of spontaneous symmetry breaking. The soft SUSY breaking is explicit and the Appelquist-Carazzone theorem applies to the superpartner spectrum. Thus, supersymmetric virtual effects disappear at least as $O(1/M_{SUSY})$. Nevertheless, several interesting questions can be discussed.

2 Constraints on general MSSM

The bulk of the electroweak precision measurements ($M_W$, $Z^0$-pole observables, $\nu e$, $ep$ scattering data, etc.) shows that the global comparison of the SM predictions with the data is impressive (for more details see e.g.\cite{2}). The simplest interpretation of the success of the SM within the MSSM is that the superpartners are heavy enough to decouple from the electroweak observables. Explicit calculations show that this happens if the common supersymmetry breaking scale is $\geq O(300 - 400)$ GeV\cite{3,4}. This is very important as such a scale of supersymmetry breaking is still low enough for supersymmetry to cure the hierarchy problem. However, in this case the only supersymmetric signature at the electroweak scale and just above it is the Higgs sector with a light, $M_h \leq O(150)$ GeV, Higgs boson. This prediction is consistent with global fits in the SM which give $M_h \approx 130^{+130}_{-70}$ GeV (the 95\% C.L. upper bound is around 470 GeV\cite{5}). We can, therefore, conclude at this point that the supersymmetric extension of the SM, with all superpartners $\geq O(300)$ GeV, is phenomenologically as successful as the SM itself and has the virtue of solving the hierarchy problem. Discovery of a light Higgs boson is the crucial test for such an extension.

The relatively heavy superpartners discussed in the previous paragraph are sufficient for explaining the success of the SM. But is there a room for some light superpartners with masses $O(M_Z)$ or even below? This question is of great importance for LEP2. Indeed, a closer look at the electroweak observables shows that the answer to this question is positive. The dominant quantum corrections to the electroweak observables are the so-called "oblique" corrections to the gauge boson self-energies. They are economically summa-
rized in terms of the $S, T, U$ parameters

$$\alpha S \sim \Pi'_{3Y}(0) = \Pi'_{L3,R3} + \Pi'_{L3,B-L}$$

(1)

(the last decomposition is labelled by the $SU_L(2) \times SU_R(2) \times U_{B-L}(1)$ quantum numbers).

$$\alpha T \equiv \Delta \rho \sim \Pi_{11}(0) - \Pi_{33}(0)$$

(2)

$$\alpha U \sim \Pi'_{11}(0) - \Pi'_{33}(0)$$

(3)

where $\Pi_{ij}(0)$ ($\Pi'_{ij}(0)$) are the $(i,j)$ gauge boson self-energies at the zero momentum (their derivatives) and the self-energy contribution to the $S$ parameter originates from mixing between $W_3^\mu$ and $B_\mu$ gauge bosons. The parameters $S, T, U$ have important symmetry properties: $T$ and $U$ vanish when quantum corrections to the gauge boson self-energies leave unbroken "custodial" $SU_V(2)$ symmetry. The parameter $S$ vanishes if $SU_L(2)$ remains an exact symmetry.

In terms of the parameters $S, T$ and $U$ the “new physics” contribution to the basic electroweak observables can be approximately written e.g. as

$$\delta M_W = M_W \frac{\alpha}{c_W^2 - s_W^2} \left( c_W^2 T_{new} - \frac{1}{2} s_{new}^2 + \frac{c_W^2 - s_W^2}{4 s_W^2} U_{new} \right)$$

(4)

$$\delta \sin^2 \theta_{eff} = - \frac{s_W^2 c_W}{c_W^2 - s_W^2} \left( \alpha T_{new} - \frac{\alpha}{4 s_W^2 c_W^2} S_{new} \right)$$

(5)

where the parameters $M_W$, $c_W \equiv M_W/M_Z$, $s_W$, etc. are computed in the SM (taking into account loop corrections) with some reference values of $m_t$ and $M_h$. $S_{new}$, $T_{new}$, $U_{new}$ contain only the contributions from physics beyond the SM.

The success of the SM means that it has just the right amount of the $SU_V(2)$ breaking (and of the $SU_L(2)$ breaking), encoded mainly in the top quark-bottom quark mass splitting. Any extension of the SM, to be consistent with the precision data, should not introduce additional sources of large $SU_V(2)$ breaking.

In the MSSM, there are potentially two new $SU_V(2)$ breaking effects. Firstly, masses of the superpartners of the up and down-type left-handed fermions are split through the $D-$term contribution.

$$M_{\tilde{f}_{down}}^2 - M_{\tilde{f}_{up}}^2 \approx - \cos 2\beta M_W^2$$

(6)
For example for light sleptons the splitting between the left-handed slepton and sneutrino masses becomes non-negligible. Assuming masses of all three generation of sleptons to be the same and requiring $\Delta\chi^2 < 4$ for the MSSM fit to the electroweak data one gets the lower bound on the mass of the left-handed charged slepton shown in Fig. 1. This limit, for $\tan\beta \sim 2$ is still better than the limit from the direct search (also marked on the plot) of sleptons at LEP2.

Twice as big (because of two generations of squarks instead of three species of sleptons but times the color factor 3) effects of the $SU(2)$ violation are present also for light left-handed squarks from the first two generations. However in this case no limits similar to those for sleptons can be obtained from the fit. The reason for that is a different behaviour of the $S$ parameter. Indeed, for the contribution of a doublet of left-handed sfermions, we find:

$$S = \frac{N_c}{12\pi} \log \frac{M_{\tilde{f}_{\text{down}}}}{M_{\tilde{f}_{\text{up}}}}$$

(7)

where $N_c = 1(3)$, $\lambda = -1(+1/3)$ for sleptons (squarks). Since for the sfermions under consideration $M_{\tilde{f}_{\text{down}}} > M_{\tilde{f}_{\text{up}}}$ $S$ is negative for sleptons and therefore adds up to the positive contribution of $T$ in eqs (4,5). On the contrary, for squarks $S$ is positive and compensates the effects of $T$. In principle, $T$ itself is

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$^6$The direct Tevatron bounds, $M_{\tilde{q}} \gtrsim 150$ GeV, for $\tilde{q} \neq \tilde{t}$, may not apply e.g. if $R$–parity is broken or if the gluino is heavier than $\mathcal{O}(350)$ GeV.
Figure 2: Lower bounds on the heavier stop mass $M_{\tilde{t}_2}$, as a function of $M_{\tilde{t}_1}$ for $\tan\beta = 1.6$ and $\tan\beta = 10$ from the fit (solid lines) and from the assumed experimental limit on $M_h$ (dashed lines). A scan over the top quark mass and the top squarks mixing angle $\theta_{\tilde{t}}$ has been performed. In this plot, $M_{\tilde{t}_1} < M_{\tilde{t}_2}$ by definition.

closely related to an observable $\Delta \rho$ but the existing result $\rho = 1.006 \pm 0.034$ is not accurate enough to constrain masses of the left-handed squarks (if heavier than 45 GeV).

Another, and far more important, source of $SU_V(2)$ violation is due to the splitting between the left-handed stop and sbottom masses:

$$M_{\tilde{t}_L}^2 - M_{\tilde{b}_L}^2 = m_{\tilde{t}}^2 - m_{\tilde{b}}^2 + \cos 2\beta M_W^2$$

This source of the $SU_V(2)$ violation cannot, however, be treated independently of the similar effects originating from the left-right mixing of stops. Thus, in this case the fit depends on the physical masses $M_{\tilde{t}_1}^2$, $M_{\tilde{t}_2}^2$ of both stops and on their mixing angle $\theta_{\tilde{t}}$:

$$\hat{t}_L = \cos \theta_{\tilde{t}} \hat{t}_1 - \sin \theta_{\tilde{t}} \hat{t}_2, \quad \hat{t}_R = \sin \theta_{\tilde{t}} \hat{t}_1 + \cos \theta_{\tilde{t}} \hat{t}_2$$

Absolute (i.e. optimized with respect to $\theta_{\tilde{t}}$) lower limit derived from the fit to precision data on the mass of the heavier stop as a function of the mass of the lighter one is shown by solid lines for low and large values of $\tan \beta$.

Another interesting observation is that the upper limit on the mass of the lighter MSSM Higgs boson $h^0$ depends on the same set of parameters ($M_{\tilde{t}_1}^2$, $M_{\tilde{t}_2}^2$).
\[ M_{i_2}^2, \theta_i \]

\[ M_{h^0}^2 < M_{Z^0}^2 \cos^2 2\beta + \frac{3\alpha}{4\pi s_W M_W^2} \frac{m_t^4}{m_t^2} \left[ \log \frac{M_{ii_1}^2 M_{ii_2}^2}{m_t^2} + f \left( M_{i_1}^2, M_{i_2}^2, \theta_i \right) \right] \quad (9) \]

Therefore, similar absolute bound on \( M_{i_2}^2 \) as a function of \( M_{ii}^2 \) can be derived from the limits on \( M_h \) from direct experimental searches. They are shown in Fig. 2 by dashed lines for various assumed experimental limits on \( M_h \). For low \( \tan \beta \) where the MSSM lighter Higgs boson is SM-like the present experimental limit \( M_{h^0}^{SM} > 77 \text{ GeV} \) can be used and gives the bound on \( M_{i_2}^2 \) which is already stronger than the one obtained from the precision data. For large \( \tan \beta \) the limit from the fit is lower compared to the low \( \tan \beta \) case (due to cancellation of \( m_t^2 \) and \( M_{ii}^2 \) terms in eq. (8)) but, nevertheless, still stronger than the limits coming from the Higgs searches.

For fixed mass \( M_{i_2}^2 \) of one of the stops the precision data (and the direct Higgs boson mass limit) determine the allowed region in the \( (M_{i_1}^2, \theta_i) \) plane shown in Fig. 3 for two different values of \( \tan \beta \) and two values of \( M_{i_2}^2 \). The possibility of the existence of light, mostly left-handed stop (for \( \theta_i \gtrsim 90^\circ \) the stop, whose mass is fixed on the Fig. 3 to 100 or 200 GeV becomes almost purely left-handed), which seems to contradict the standard argument that it should be heavy, can be understood from the formula for their contribution to the \( T \) parameter [14] which (neglecting the mixing of sbottoms) reads:

\[ T = \cos^2 \theta_i f \left( M_{i_1}^2, M_{ii}^2 \right) + \sin^2 \theta_i f \left( M_{i_2}^2, M_{ii}^2 \right) - \sin^2 \theta_i \cos^2 \theta_i f \left( M_{i_1}^2, M_{i_2}^2 \right) \]

where \( f(x, y) = (3/16\pi s_W^2 M_W^2) \left( x + y - \frac{2xy}{x+y} \log \frac{x}{y} \right) \) and \( M_{ii} \) is related to the physical stop masses through the relations

\[ M_{i_1}^2 = \cos^2 \theta_i M_{i_1}^2 + \sin^2 \theta_i M_{i_2}^2, \quad M_{i_2}^2 = \sin^2 \theta_i M_{i_1}^2 + \cos^2 \theta_i M_{i_2}^2 \]

and eq. (8). For \( \theta_i \lesssim 90^\circ \) (but \( \neq 90^\circ \)) and \( M_{ii} \) large enough to make \( M_{i_2} \approx M_{ii} \), the negative third term in the expression for \( T \) can cancel the first one and the second can also remain small. From the above formulae it is clear, however, that those solutions require \( M_{i_2} \gg M_{ii} \) which leads to very large inverse hierarchy \( m_{\tilde{t}_2}^2 \gg m_{\tilde{t}_1}^2 \) of the corresponding soft SUSY breaking mass parameters. This is very unnatural from the point of view of the GUT boundary conditions which usually lead to \( m_{\tilde{t}_2}^2 \lesssim m_Q^2 \). It is also interesting to note that precisely this configuration is exploited in one of the most interesting attempts [13] to explain the anomalous HERA events [14]. The mechanism proposed
Figure 3: Allowed by $\Delta \chi^2 < 4$ and $M_h > 70$ GeV regions in the plane $(M_{t_2}, \theta_t)$ for two different values of $M_{t_1}$ and two different values of $\tan \beta$. In the region between the two solid lines $170$ GeV $< M_{b_1} < 180$ GeV.
in [3] requires mostly left-handed stop with mass \( \sim 200 \) GeV and (mostly left-handed) \( \tilde{b} \) with \( 170 < M_{\tilde{b}} < 180 \) GeV. From Fig. 3 where this \( \tilde{b} \) mass range is marked by solid lines one can judge how likely this (otherwise very appealing) solution to the HERA puzzle is.

Another process which can further constrain the MSSM parameter space is the \( b \to s\gamma \) decay. Due to the recent progress the theoretical prediction for \( BR(b \to s\gamma) \) is now available with reduced error of \( \approx 15\% \). The existing measurement of \( BR(b \to X_s\gamma) \) implies therefore the important correlation of the charged (or, by the relation \( M_{H^+}^2 = M_{\tilde{A}}^2 + M_{W}^2 \), CP-odd) Higgs boson mass and masses and compositions of the lighter stop and chargino. To understand them, it is important to remember that the charged Higgs contribution to the \( b \to s\gamma \) amplitude has always the same sign as the SM one whereas the chargino-stop contribution to this amplitude may have opposite sign. Since the actually measured value of \( BR(b \to s\gamma) \) is close to the SM prediction, SUSY and charged Higgs contributions must either be small by themselves or cancel each other to a large extent.

**Figure 4:**

a) Lower limits on \( M_{\tilde{A}} \) from \( b \to s\gamma \) as a function of \( \tan\beta \). Solid line correspond to very heavy, \( > \mathcal{O}(1 \) TeV) sparticles. Dashed (dotted) line show the limit for \( m_{\tilde{C}^1} = M_{\tilde{t}_1} = 250 \) (500) GeV. b) Lower limits on \( M_{\tilde{A}} \) as a function of \( M_2/|\mu| \), based on CLEO \( BR(B \to X_s\gamma) \) measurement. Thick lines show limits for \( \mu > 0 \), thin lines for \( \mu < 0 \). Solid, dashed and dotted lines show limits for lighter stop and chargino masses \( M_{\tilde{t}_1} = m_{\tilde{C}^1} = 90, 150 \) and 300 GeV, respectively.

Fig. 4a shows the lower limits on \( M_{\tilde{A}} \) from \( b \to s\gamma \) as a function of \( \tan\beta \). Solid line correspond to very heavy, \( > \mathcal{O}(1 \) TeV) sparticles. Dashed (dotted) line show the limit for \( m_{\tilde{C}^1} = M_{\tilde{t}_1} = 250 \) (500) GeV. b) Lower limits on \( M_{\tilde{A}} \) as a function of \( M_2/|\mu| \), based on CLEO \( BR(B \to X_s\gamma) \) measurement. Thick lines show limits for \( \mu > 0 \), thin lines for \( \mu < 0 \). Solid, dashed and dotted lines show limits for lighter stop and chargino masses \( M_{\tilde{t}_1} = m_{\tilde{C}^1} = 90, 150 \) and 300 GeV, respectively.
all the superpartner masses are very large (above 1 TeV). Dashed (dotted) lines in Fig. 3a show the same limits in the presence of chargino and stop with masses $m_{C_1} = M_{\tilde{t}_1} = 500$ (250) GeV (with all other sparticles heavy) obtained by scanning over the values of $r = M_2/|\mu|$ and $\theta_{\tilde{t}}$. In the presence of light stop and chargino limits on $M_A$ are significantly weaker and totally disappear for large values of $\tan\beta$. Another important observation is that, large chargino-stop contribution to $b \to s\gamma$ amplitude arise when the chargino is higgsino-like rather then gaugino-like i.e. when $M_2/|\mu| > 1$. In addition, it depends also on the stop mixing angle $\theta_{\tilde{t}}$. Fig. 3b (taken from ref. 18) shows the lower limit on the allowed pseudoscalar Higgs boson mass $M_A$ as a function of $r = M_2/|\mu|$ for three different values of the lighter chargino and lighter stop masses. For small $M_2/|\mu|$, i.e. for gaugino-like lighter chargino (when the chargino-stop contribution to $BR(b \to s\gamma)$ is suppressed) the resulting limits on $M_A$ are quite strong even for very light chargino and stop e.g. $M_A \geq \mathcal{O}(200 \text{ GeV})$ for $M_{\tilde{t}_1} = m_{C_1^\pm} = 90 \text{ GeV}$. The limits decrease when $M_2/|\mu|$ increases and approximately saturate for $M_2/|\mu| \geq 1$.

Other rare processes like e.g. neutral meson mixing ($K^0 - \bar{K}^0$ or $B^0 - \bar{B}^0$) etc. give much weaker constraints 18.

3 Constraints on specific supersymmetry breaking scenarios

Up to now we have been discussing bounds and correlations imposed by precision measurements on the parameter space of the unconstrained MSSM. These bounds were “absolute” in the sense that e.g. to obtain lower limits on slepton masses all other sparticles were kept heavy in order to reduce their impact on the fit. In the following we shall impose the constraints discussed in the preceding section on the specific models of supersymmetry breaking, such as minimal SUGRA model or very popular recently gauge mediated models. In those models the resulting at the electroweak scale MSSM parameters are strongly correlated by specific boundary conditions assumed at the GUT or intermediate scales (for example, sleptons are always lighter than third generation squarks). For this reason, low energy precision measurements constrain such models very efficiently. We will illustrate those effects below by discussing the predictions of these models for the mass of the lighter MSSM Higgs boson as this issue is of direct interest for physics at LEP2. More detailed discussion can be found in 20. In this context there has also been often addressed the question of fine-tuning in the Higgs potential in models with the soft terms generated at large scales 19. Indeed, if supersymmetry is to be the solution to the hierarchy problem in the SM, it should not introduce another fine-tuning in the Higgs
potential. “Naturalness” of a given parameter set can be quantified e.g. by calculating the derivatives of $M_2^2$ with respect to the GUT (or intermediate) scale parameters of the model:

$$
\Delta_i \equiv \left| a_i \frac{\partial M_2^2}{\partial a_i} \right|
$$

In the global analysis, which combines the electroweak breaking with experimental constraints, it is interesting to check the “naturalness” of different parameter regions i.e. to check the values $\Delta_i$ for each parameter set. Before presenting the results for the specific models, one has to remember that the “naturalness” is only a qualitative criterion. Compared to the fine-tuning of many orders of magnitude in the SM, cancellations of two or even three orders of magnitude are still very small. Secondly, the as yet unknown fundamental theory of SUSY breaking may predict soft supersymmetry breaking terms correlated to each other, thus “explaining” the cancellations between them. We now present the results of our global analysis, for the three scenarios considered and for several values of $\tan \beta$. In each case the lightest Higgs boson mass is shown as a function of the heavier stop.

In the simplest, the so-called minimal supergravity model (Ansatz), all superpartner masses are given in terms of five parameters: $m_0^2$, $M_{1/2}$, $\mu$, $A_0$ and $B_0$. Two of them can be traded for $M_2^0$ and $\tan \beta$ (for analytical solution of the RG equations see 21). Thus, we get strongly correlated superpartner spectrum and correlated with the Higgs boson masses. It is now particularly simple to follow our global analysis and to determine the allowed range of the lightest Higgs boson mass as a function of the heavier stop mass. In Fig. 5 we show the results for $\tan \beta = 1.65$ (corresponding to the infrared fixed point for the top quark Yukawa coupling) and 2.5. We see that, in this model, requiring the proper breaking of the electroweak symmetry and with the imposed experimental constraints the lightest Higgs boson mass is bounded from below: $M_{h^0} > \sim 75(85) \text{ GeV}$ for $\tan \beta = 1.65(2.5)$ (for $\tan \beta = 10$ the lower bound is around 105 GeV).

The model also gives lower bound on $M_{A^0}$ of about 500 GeV at $\tan \beta = 1.65$ and decreasing to 300 GeV at $\tan \beta = 10$. The heavier stop is bounded from below at $\sim 450$ GeV. Of course, the crucial role in obtaining those bounds is played by the universality Ansatz combined with the existing experimental constraints. The mass $M_{h^0}$ is bounded from above at 95, 105 and 120 GeV for $\tan \beta = 1.65$, 2.5 and 10, respectively. Thus the general upper bound on $M_{h^0}$ can be reached even in this constrained model.

Turning now to the fine-tuning problem we observe first that the model does not admit at all solutions with all $\Delta_i < 10$. This is mainly because of
the imposed experimental limit \( m_{CZ} > 90 \text{ GeV} \) which pushes \( M_{1/2} \) into the region with \( \Delta_{M_{1/2}} \gtrsim 10 \) for all \( \tan\beta \) values. Moreover, close to the IR fixed point (for \( \tan\beta \approx 1.65 \)), there do not even exist solutions with all \( \Delta_i < 100 \).

As expected from the general arguments, cancellations become weaker with increasing \( \tan\beta \). In Fig. 6a (b) we show the results for \( \tan\beta = 2.5(10) \) with the cut \( \Delta_{\text{max}} < 100 \). We note that in this case such a cut leaves a non-empty parameter region but gives stronger upper bounds on the Higgs boson mass for the same values of \( M_{1/2} \). They result mainly from the bound on \( A_0 \) (i.e. on the left-right mixing) obtained due to increasing \( \Delta_{M_{1/2}} \) with increasing \( A_0 \). Moreover, the cut \( \Delta_{\text{max}} < 100 \) gives also an upper bound on \( M_{1/2} \). Finally we note one more interesting effect: a cut on \( \Delta \)'s gives almost flat (instead of logarithmic) dependence of \( M_{h^0} \) on \( M_{1/2} \). An increase in \( M_{1/2} \) is balanced by a decrease in \( A_0 \) (i.e. in the left-right mixing) to keep \( \Delta \)'s below the imposed bound.

In the next step one can study a less restrictive model, with the pattern of the soft terms consistent with the \( SO(10) \) unification, i.e. with the universal sfermion masses and the two Higgs boson masses taken as independent parameters. It turns out that the predictions the \( M_{h^0}, M_{A^0} \) and \( M_{1/2} \) (as well as the degree of fine-tuning) are very similar to those in the universal case and need not be independently shown here. This can be partly understood in terms of the important role played by the limit \( m_{CZ} > 90 \text{ GeV} \) and by the constraints
Figure 6: $M_{h^0}$ versus the mass of the heavier stop ($M_{\tilde{t}_2}$) obtained from universal boundary conditions at the scale $2 \times 10^{16}$ GeV and requiring $\Delta_i < 100$

(a) for $\tan\beta = 2.5$ ($y \approx 0.80$)

(b) for $\tan\beta = 10$ ($y \approx 0.71$).

from $b \rightarrow s\gamma$ and precision data, which are not sensitive to the assumed non-universality in the Higgs boson mass parameters. Moreover, there is no change in the values of $\Delta_i$'s since their expected decrease with the increasing number of free parameters is now reduced by the disappearence of certain cancellations in $\Delta_i$'s which are present in the universal case. Thus, the results presented in Figs. 5 and 6 are representative also for the partial breakdown of universality.

Finally, it is interesting to compare the supergravity scenario with models in which supersymmetry breaking is transmitted to the observable sector through ordinary $SU(3) \times SU(2) \times U(1)$ gauge interactions of the so-called messenger fields at scales $M \ll M_{GUT}$ (see eq. [22]). In general, these gauge-mediated models of SUSY breaking are characterized by two scales: the scale $M$, which is of the order of the average messenger mass and the scale $\sqrt{F}$ ($\sqrt{F} < M$) of supersymmetry breaking. Messenger fields are assumed to form complete $5 + \overline{5}$ (or $10 + \overline{10}$) $SU(5)$ representations. Their number $n$ is restricted to $n \leq 4$ by perturbativity of the gauge couplings up to the GUT scale.

In terms of $M$ and $x \equiv F/M^2$ the soft supersymmetry breaking parameters of the MSSM at the scale $\sim M$ are given by:

$$M_i = \frac{\alpha_i(M)}{4\pi} M \ n \ x \ g(x) = \frac{\alpha_i(M)}{4\pi} M \ y$$ (11)
\[ m^2_f = 2M^2 \, n \, x^2 \, f(x) \sum_i \left( \frac{\alpha_i(M)}{4\pi} \right) C_i = 2M^2 \, y^2 \, z \sum_i \left( \frac{\alpha_i(M)}{4\pi} \right) C_i \quad (12) \]

where \( C_3 = 4/3, C_2 = 3/4, C_1 = (3/5)Y^2 \) (\( Y \) being the hypercharge of the scalar \( \tilde{f} \)), the functions \( g(x) \) and \( f(x) \) (\( g(0) = f(0) = 1, g(1) \approx 1.4, f(1) \approx 0.70 \)) can be found in ref. 24 and the factor \( z \equiv f(x)/ny^2(x) \). Thus, for fixed messenger sector (i.e. fixed \( n \)) and fixed scale \( M \) all soft supersymmetry breaking masses are predicted in terms of \( y \) (\( 0 < y < n_{\text{max}} \, g(1) \approx 5.6 \)) in those models we also have \( A_0 \approx 0 \) as the \( A_0 \) parameter can be generated at two loop only. However, the values of the soft masses \( m_{H^\pm}^2 \) may differ significantly from their values given by eq. (12) since they can be modified by physics involved in generation of \( B_0 \) and \( \mu_0 \) parameters 2. Therefore, in our scans we take \( y, m_{H_1}, m_{H_2}, \mu_0 \) and \( B_0 \) as free parameters (the last two are fixed by \( M_{2\tilde{b}}^2 \) and \( \tan \beta \)). To be general, the factor \( z \) in eq. (12) is scanned between \( z_{\text{min}} = f(1)/n_{\text{max}}g^2(1) \approx 0.15 \) and \( z_{\text{max}} = 1 \). For definiteness we will consider \( M = 10^5 \) GeV only.

We follow the same simple approach we used for the supergravity models. On the parameter space consistent with the electroweak symmetry breaking we impose the discussed earlier experimental constraints (now we require \( m_{C^\pm} > 120 \) GeV, \( M_{\tilde{t}_1} > 140 \) GeV). Very important rôle is played by \( b \to s\gamma \). The requirement of good \( b \to s\gamma \) rate reduces otherwise rather widely spread out \( h^0 \) and \( A^0 \) Higgs boson masses (for \( \tan \beta = 2.5, M_{h^0} < 100 \) GeV) to a narrow band (\( 80 < M_{h^0} < 100 \) GeV, \( M_{A^0} > 200 \) GeV). This effect can be easily understood (see Fig. 3a) because in the model considered squarks and charginos are rather heavy so a light \( A^0 \) is not allowed by \( b \to s\gamma \) and light \( h^0 \) is always associated with light \( A^0 \). Moreover, surviving small values of \( M_{h^0} \) (\( \sim 80 \) GeV for \( \tan \beta = 2.5 \) are associated with lowest values of \( M_{\tilde{t}_2} \) (\( \lesssim 500 \) GeV) which are eliminated by imposing the \( \Delta \chi^2 < 4 \) cut. Finally, if we also require “naturalness” e.g. by demanding \( \Delta_{\text{max}} < 100 \) we constrain the heavier stop mass \( M_{\tilde{t}_2} \) and \( C^\pm \)-odd higgs boson mass \( M_{A^0} \) from above to \( \lesssim 700 \) GeV.

Final results are shown in Fig. 3a as a plot of \( M_{h^0} \) versus the mass of the heavier stop \( M_{\tilde{t}_2} \) predicted in models of gauge mediated supersymmetry breaking with \( M = 10^5 \) GeV for \( \tan \beta = 2.5 \) and 10. As in the case of supergravity models, the restriction of the chargino and stop masses eliminates solutions with \( \Delta_{\text{max}} < 10 \). With all constraints imposed, \( M_{A^0} \) turns out to be surprisingly tightly constrained. For \( \tan \beta = 2.5(10) \) values of the lightest scalar Higgs

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6In addition, because \( \mu \) values required by electroweak symmetry breaking are large, the lighter chargino turns out to have only small higgsino component and hence its \( b\tilde{t}C^- \) coupling is weaker than that of the pure higgsino chargino which is responsible for the limit shown in Fig. 3a.
Figure 7: Results for gauge mediated supersymmetry breaking models with $M = 10^5$ GeV for $\tan \beta = 2.5$: a) $M_{h^0}$ versus the mass of the heavier stop ($M_{\tilde{t}_2}$) b) versus the mass of the CP-odd Higgs boson ($M_{A^0}$). Condition $\Delta_i < 100$ is imposed.

boson are bounded by $90(108) \text{ GeV} \lesssim M_{h^0} \lesssim 97(115) \text{ GeV}$. Masses of the CP-odd Higgs boson are bounded to $280(200) \text{ GeV} \lesssim M_{A^0} \lesssim 700(850) \text{ GeV}$. These upper bounds should be compared to the ones obtained in [27] in the restricted model of gauge mediated supersymmetry breaking (with $x = 1, n = 1$ and with $m_{H_{1,2}}$ as given by eq. (12)) without imposing any additional constraints. It is interesting that in the much more general scenario described above, after imposing experimental and naturalness cuts, one gets upper bounds on $M_{h^0}$ not higher than those obtained in [27].

4 Summary

There is an apparent contradiction between the hierarchy problem (which suggest new physics to be close to the electroweak scale) and the striking success of the Standard Model in describing the electroweak data. The supersymmetric extension of the SM offers an interesting solution to this puzzle. The bulk of the electroweak data is well screened from supersymmetric loop effects, due to the structure of the theory, even with superpartners light, $O(M_Z)$. In order to maintain the overall success of the SM, only left-handed squarks from the third generation have to be $\gtrsim O(300 \text{ GeV})$. Some, rather weak limits do exist also for left-handed sleptons. The other superpartners can still be light, at their present experimental mass limits, and would manifest themselves through vir-
tual corrections to the small number of observables such as $b \to s\gamma$, $K^0-\bar{K}^0$, and $B^0-\bar{B}^0$ mixing and a few more for large $\tan \beta$ (not discussed in this talk).

In unconstrained minimal supersymmetric extension of the Standard Model there exists the well known upper bound on the mass of the lightest supersymmetric Higgs. The available parameter space can be efficiently reduced by additional theoretical assumptions, related to the extrapolation of the model to very large energy scales. When these assumptions are supplemented by the low energy experimental constraints and qualitative criterion of “naturalness” more more definite predictions for $M_{h^0}$ are obtained. In this context, it is interesting to note that in the models considered $M_{h^0} > 75$ GeV (roughly the current experimental direct bound). Moreover, several arguments point toward $M_{h^0} < 100$ GeV. Both, the discovery or the absence of the Higgs boson in this mass range will have strong implications for the supersymmetric extension of the Standard Model.

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