Research Article

On the Study of Reverse Degree-Based Topological Properties for the Third Type of $p$th Chain Hex-Derived Network

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Vertices and edges are made from a network, with the degree of a vertex referring to the number of connected edges. The chance of every vertex possessing a given degree is represented by a network’s degree appropriation, which reveals important global network characteristics. Many fields, including sociology, public health, business, medicine, engineering, computer science, and basic sciences, use network theory. Logistical networks, gene regulatory networks, metabolic networks, social networks, and driven networks are some of the most significant networks. In physical, theoretical, and environmental chemistry, a topological index is a numerical value assigned to a molecular structure/network that is used for correlation analysis. Hexagonal networks of dimension $t$ are used to build hex-derived networks, which have a wide range of applications in computer science, medicine, and engineering. For the third type of hex-derived networks, topological indices of reverse degree based are discussed in this study.

1. Introduction

A topological descriptor is a numerical value that represents the complete structure of a graph. In the study of topological descriptors, graph theory has shown to be a fruitful field of study. The primary elements of topological indices link the many chemical and physical characteristics of fundamental chemical substances. Vertex-edge-based topological indices are employed in the research of QSAR/QSPR for the prediction of bio-activity of different chemical compounds. With the dimension $p$, hexagonal networks create hex-derived networks, which have a wide range of implementations in engineering, computer science, and also medicine. In [1], researchers created a new form of graph known as “third type of hex-derived networks” [2, 3] and continued this work by calculating degree-based topological descriptors for these networks, in which they computed exact values of some vertex-edge named topological indices for this network.

Researchers have used graph theory to develop a range of helpful tools, including graph labeling, topological indices, and finding numbers. The subject of graph theory has several applications and implementations in various fields of study, including chemistry, medicine, and engineering. A polynomial, a series of integers, a numeric value, or a matrix can all be used to identify a graph. A chemical compound can be represented as a graph (or a diagram) or usually denoted as a molecular graph, nodes played a role of atoms, and the bonding between atoms is usually labeled as edges in the molecular graph theory. Recently, a new topic called cheminformatics was established, which is a mix of chemistry, information science, and mathematics, in which the QSPR/QSAR connection, bio-activity, and characterization of chemical compounds are investigated and reported in [4].

The topological descriptor is a numerical number associated with chemical compositions that maintain the relationship between chemical structures and a variety of physico-chemical characteristics, biological activity, and chemical reactivity. To describe the topology of a chemical network, it translated into a number, which is further used to create topological indices. Distance-based topological indices, degree-based topological indices, and counting-related topological indices are some of the most common
forms of topological indices for graphs. Many academics have recently discovered topological indices for studying basic features of molecular graphs or networks. In [5–12], these networks have extremely compelling topological qualities that have been examined in distinct characteristics.

Let $p$ and $q$ represent the number of rows and number of triangles in each row of third-type $p$th chain hex-derived networks $G_{p,q}^3$, respectively, shown in Figure 1. Let $G$ be a simple connected network, with a set of vertex and edges denoted by $V$ and $E$, respectively. $|V|$ represents the order of $G$ and $|E|$ represents the size of $G$. Let $d_{\theta}$ be the degree of a vertex $\theta \in V$ in $G$ and $R_{\theta}$ be its reverse degree that was introduced by Kulli [13] and defined as $\Delta = d_{\theta} = \max\{d_{\theta} | \theta \in V\}$, where $\Delta$ denotes the maximum degree of the given graph. Let $E_{R_{\theta},R_{\theta}}$ represent the edge partition of the given graph based on reverse degree of end vertices of an edge $\theta \theta \in E$ and $|E_{R_{\theta},R_{\theta}}|$ represent its cardinality.

We define general reverse degree topological invariant $T(G)$ as follows:

\[
T(G) = \sum_{\theta \theta \in E(G)} \lambda(R_{\theta}, R_{\theta}),
\]

If $\lambda(R_{\theta}, R_{\theta}) = (R_{\theta} \times R_{\theta})^\alpha$, then $T(G)$ represents the general reverse Randić index $(T_{R_{\alpha}}(G))$ for $\alpha = 1/2, -1/2, 1, -1$, $\alpha = 0$ then it is known as the second reverse Zagreb index $(R_{\theta}, R_{\theta})(G)$,

If $\lambda(R_{\theta}, R_{\theta}) = \sqrt{R_{\theta} + R_{\theta} - 2}$, then $T(G)$ represents the reverse atom – bond connectivity index $(ABC_{\theta})(G)$,

If $\lambda(R_{\theta}, R_{\theta}) = 2\sqrt{R_{\theta} \times R_{\theta}}$, then $T(G)$ represents the reverse geometric – arithmetic index $(R_{\theta})(G)$,

If $\lambda(R_{\theta}, R_{\theta}) = R_{\theta} + R_{\theta}$, then $T(G)$ represents the first reverse Zagreb index $(R_{\theta})(G)$,

If $\lambda(R_{\theta}, R_{\theta}) = (R_{\theta} + R_{\theta})^2$, then $T(G)$ represents the reverse hyper Zagreb index $(R_{H}(G))$,

If $\lambda(R_{\theta}, R_{\theta}) = (R_{\theta}^2 + R_{\theta}^2)$, then $T(G)$ represents the reverse forgotten index $(R_{F}(G))$,

If $\lambda(R_{\theta}, R_{\theta}) = (R_{\theta} + R_{\theta})^\alpha$, then $T(G)$ represents the first reverse redefined index $(R_{\theta}Z_{\alpha}(G))$ for $\alpha = 1, \beta = -1$,

the second reverse redefined index $(R_{\theta}Z_{\beta}(G))$ for $\alpha = -1, \beta = 1$,

the third reverse redefined index $(R_{\theta}Z_{\gamma}(G))$ for $\alpha = 1, \beta = 1$.

For latest results on topological descriptors for different chemical and computer networks and for general graphs, we refer to see [14–25]. In this current research work, we determine the exact values of all the above reverse indices.
2. Structure of Third-Type Hex-Derived Networks

With the help of complete graphs of order 3 (K₃), Chen et al. [26] assembled a hexagonal mesh. In terms of chemistry, these K₃ graphs are also called oxide graphs. Figure 1 is obtained by joining these K₃ graphs. Two-dimensional mesh graph HX(2) (see Figure 2(a)) is obtained by joining six K₃ graphs and three-dimensional mesh graph HX(3) (see Figure 2(b)) is obtained by putting K₃ graphs around all sides of HX(2) [27]. Furthermore, repeating the same process by putting the tK₃ graph around each hexagon, we obtained the t-th hexagonal mesh. We have to note that the one-dimensional hexagonal mesh graph does not exist.

The novel network, labeled the third category of hex-derived networks, was developed in [1]. In [2, 3], they defined the graphically construction algorithm for the third type of hexagonal hex-derived network HHDN3(t). Huo et al. [28] explained the graphical construction algorithm for m-th chain hex-derived networks of third type. In this paper, we denote it by Gₐpq, and different priorities of p and q the chain hex-derived networks are shown in Figure 3. In [29–33], you may find related research that utilizes this idea and that may benefit from the new research’s visions.

3. Main Results

In this section, we study the third-type p-th chain hex-derived networks Gₐpq in the following three cases.

(i) Case 1: for p = q, (p, q) ≥ 1.

(ii) Case 2: for p < q, p is odd and q is a natural number. For p > q, p is odd and q is a natural number. For p < q, p and q both are even. For p > q, p and q both are even.

(iii) Case 3: for p < q, p is even and q is odd. For p > q, p is even and q is odd.

3.1. Results for Case 1. We provide a formula that would be used to calculate any reverse degree topological descriptors of Case 1 for Gₐpq.

Lemma 1. Let Gₐpq be a third-type p-th chain hex-derived networks. Then,

\[ T_1(Gₐpq) = 3pq(\lambda(1, 1) + 2\lambda(1, 5) + \lambda(5, 5)) + 3(p + q)(\lambda(5, 5) - \lambda(1, 1)) + 2(\lambda(1, 1) - 2\lambda(1, 5) + \lambda(5, 5)). \]

Proof. The graph Gₐpq contains 12pq edges and maximum degree in Gₐpq graph is 8. There are two types of reverse degree vertices in Gₐpq that are 1 and 5. Let us partition the edges of Gₐpq according to its reverse degrees according to Case 1 as

\[ E_{1,1} = \{\theta \in E(Gₐpq); R_\theta = 1, R_\theta = 1\}, \]

\[ E_{1,5} = \{\theta \in E(Gₐpq); R_\theta = 1, R_\theta = 5\}, \]

\[ E_{5,5} = \{\theta \in E(Gₐpq); R_\theta = 5, R_\theta = 5\}. \]

Note that \( E(Gₐpq) = E_{1,1} \cup E_{1,5} \cup E_{5,5} \) and \( |E_{1,1}| = 3pq - 3p - 3q + 2 \), \( |E_{1,5}| = 6pq - 4 \), and \( |E_{5,5}| = 3pq + 3p + 3q + 2 \). Hence,

\[ T_1(Gₐpq) = \sum_{\theta \in E(Gₐpq)} \lambda(R_\theta) \]

\[ = \sum_{\theta \in E_{1,1}} (p + q) + \sum_{\theta \in E_{1,5}} 2\lambda(1, 5) + \sum_{\theta \in E_{5,5}} 108pq \]

\[ + 3(p + q)(\lambda(5, 5) - \lambda(1, 1)) + 2(\lambda(1, 1) - 2\lambda(1, 5) + \lambda(5, 5)). \]

After simplification, we obtain

\[ T_1(Gₐpq) = 3pq(\lambda(1, 1) + 2\lambda(1, 5) + \lambda(5, 5)) + 3(p + q)(\lambda(5, 5) - \lambda(1, 1)) + 2(\lambda(1, 1) - 2\lambda(1, 5) + \lambda(5, 5)). \]

Theorem 1. The general reverse Randić index of Gₐpq is equal to

\[ \mathcal{R}_{\alpha} Gₐpq = \begin{cases} 108pq + 72(p + q) + 32, & \text{for } \alpha = 1, \\ (18 + 6\sqrt{5})pq + 12(p + q) + 12 - 4\sqrt{5}, & \text{for } \alpha = \frac{1}{2}, \\ (\frac{18}{5} + \frac{6\sqrt{5}}{5})pq - \frac{12}{5}(p + q) + \frac{12}{5} \cdot \frac{4\sqrt{5}}{5}, & \text{for } \alpha = -\frac{1}{2}, \\ 108 \frac{pq}{25} - \frac{72}{25}(p + q) + \frac{32}{25}, & \text{for } \alpha = -1. \end{cases} \]
Proof. For $\mathcal{R}_{\alpha}(G_{p,q}^3)$ which is the general reverse Randić index of $G_{p,q}^3$, from equation (2), we have

$$\lambda(\mathcal{R}_{\theta}, \mathcal{R}_{\vartheta}) = (\mathcal{R}_{\theta} \times \mathcal{R}_{\vartheta})^\alpha;$$

therefore,

$$\lambda(1, 1) = 1,$$

$$\lambda(1, 5) = (5)^\alpha,$$

and

$$\lambda(5, 5) = (25)^\alpha.$$ Vˇ hus, by Lemma 1,

$$\mathcal{R}_{\alpha}(G_{p,q}^3) = \frac{3pq(1 + 2(5)^n) + 3(p + q)((25)^n - 1) + 2(1 - 2(5)^n + (25)^n)}{R_{\theta} \times R_{\vartheta}}.$$ (14)

Put $\alpha = 1$, and we have

$$\mathcal{R}_{1}(G_{p,q}^3) = 108pq + 72(p + q) + 32.$$ (15)

Put $\alpha = (1/2)$, and we have

$$\mathcal{R}_{1/2}(G_{p,q}^3) = (18 + 6\sqrt{5})pq + 12(p + q) + 12 - 4\sqrt{5}.$$ (16)

Put $\alpha = (-1/2)$, and we have

$$\mathcal{R}_{-1/2}(G_{p,q}^3) = \left(\frac{18}{5} + 6\sqrt{5}\right)pq - \frac{12}{5}(p + q) + \frac{12}{5} - \frac{4\sqrt{5}}{5}.$$ (17)

Put $\alpha = -1$, and we have

$$\mathcal{R}_{-1}(G_{p,q}^3) = \frac{108}{25}pq - \frac{72}{25}(p + q) + \frac{32}{25}.$$ (18)

Theorem 2. Let $G_{p,q}^3$ be a third-type $p$th chain hex-derived networks. Then, the reverse atom-bond connectivity index is

$$\mathcal{R}_{\alpha}(G_{p,q}^3) = \left(\frac{12\sqrt{5}}{5} + \frac{6\sqrt{5}}{5}\right)pq - \frac{8\sqrt{5}}{5} + 2(3p + 3q - 2\sqrt{5})$$ (19)

The reverse geometric-arithmetic index is

$$\mathcal{R}_{G}(G_{p,q}^3) = (6 + 2\sqrt{5})pq + 4 - \frac{4\sqrt{5}}{3}.$$ (20)

The first reverse Zagreb index is

$$\mathcal{R}_{M_1}(G_{p,q}^3) = 72pq + 24p + 24q.$$ (21)

The reverse hyper-Zagreb index is

$$\mathcal{R}_{HM}(G_{p,q}^3) = 528pq + 288p + 288q + 64.$$ (22)

The reverse forgotten index is

$$\mathcal{R}_{F}(G_{p,q}^3) = 312pq + 144p + 144q.$$ (23)

Proof. For $\mathcal{R}_{\alpha}(G_{p,q}^3)$ which is the reverse atom-bond connectivity index of $G_{p,q}^3$ from equation (3), we have

$$\lambda(\mathcal{R}_{\theta}, \mathcal{R}_{\vartheta}) = (\mathcal{R}_{\theta} + \mathcal{R}_{\vartheta} - 2)/\mathcal{R}_{\theta} \times \mathcal{R}_{\vartheta};$$

therefore,

$$\lambda(1, 1) = 1,$$

$$\lambda(1, 5) = (5)^\alpha,$$

and

$$\lambda(5, 5) = (25)^\alpha.$$ Vˇ hus, by Lemma 1,
\(= 0, \lambda(1, 5) = \sqrt{4/5}, \) and \(\lambda(5, 5) = (\sqrt{8}/5). \) Thus, by Lemma 1 and after simplification,

\[
\mathcal{RA}(G_{p,q}^3) = \left( \frac{12\sqrt{5}}{5} + \frac{6\sqrt{5}}{5} \right) p q - \frac{8\sqrt{5}}{5} + 2\left(3p + 3q + 2\right)\sqrt{5}
\] (24)

For \(\mathcal{RGA}(G_{p,q}^3)\) which is the reverse geometric-arithmetic index of \(G_{p,q}^3\) from equation (4), we have \(\lambda(\mathcal{R}_p, \mathcal{R}_q) = (2\sqrt{\mathcal{R}_p \times \mathcal{R}_q}/(\mathcal{R}_p + \mathcal{R}_q))\); therefore, \((1, 1) = 1, \lambda(1, 5) = (\sqrt{5}/3),\) and \(\lambda(5, 5) = 1.\) Thus, by Lemma 1 and after simplification,

\[
\mathcal{RGA}(G_{p,q}^3) = (6 + 2\sqrt{5})pq + 4 - \frac{4\sqrt{5}}{3}.
\] (25)

For \(\mathcal{RM}_1(G_{p,q}^3)\) which is the first reverse Zagreb index of \(G_{p,q}^3\) from equation (5), we have \(\lambda(\mathcal{R}_p, \mathcal{R}_q) = (\mathcal{R}_p + \mathcal{R}_q)^2;\) therefore, \((1, 1) = 2, \lambda(1, 5) = 6,\) and \(\lambda(5, 5) = 10.\) Thus, by Lemma 1 and after simplification,

\[
\mathcal{RM}_1(G_{p,q}^3) = 72pq + 24p + 24q.
\] (26)

For \(\mathcal{RM}_2(G_{p,q}^3)\) which is the first reverse hyper-Zagreb index of \(G_{p,q}^3\) from equation (6), we have \(\lambda(\mathcal{R}_p, \mathcal{R}_q) = (\mathcal{R}_p^2 + \mathcal{R}_q^2);\) therefore, \((1, 1) = 4, \lambda(1, 5) = 36,\) and \(\lambda(5, 5) = 100.\) Thus, by Lemma 1 and after simplification,

\[
\mathcal{RM}_2(G_{p,q}^3) = 528pq + 288p + 288q + 64.
\] (27)

For \(\mathcal{RF}(G_{p,q}^3)\) which is the reverse forgotten index of \(G_{p,q}^3\) from equation (7), we have \(\lambda(\mathcal{R}_p, \mathcal{R}_q) = ((\mathcal{R}_p)^2 + (\mathcal{R}_q)^2);\) therefore, \((1, 1) = 2, \lambda(1, 5) = 26,\) and \(\lambda(5, 5) = 50.\) Thus, by Lemma 1 and after simplification,

\[
\mathcal{RF}(G_{p,q}^3) = 312pq + 144p + 144q.
\] (28)

Theorem 3. Let \(G_{p,q}^3\) be a third-type \(p\)th chain hex-derived networks. Then, the first reverse redefined index is

\[
\mathcal{RRZ}_1(G_{p,q}^3) = \frac{72pq}{5} - \frac{24p}{5} - \frac{24q}{5}
\] (29)

The second reverse redefined index is

\[
\mathcal{RRZ}_2(G_{p,q}^3) = 14pq + 6p + 6q + \frac{8}{3}
\] (30)

The third reverse redefined index is

\[
\mathcal{RRZ}_3(G_{p,q}^3) = 936pq + 744p + 744q + 384.
\] (31)

Proof. For \(\mathcal{RRZ}_1(G_{p,q}^3)\) which is the first reverse redefined index of \(G_{p,q}^3\) from equation (8), we have \(\lambda(\mathcal{R}_p, \mathcal{R}_q) = \frac{(\mathcal{R}_p + \mathcal{R}_q)^3}{(\mathcal{R}_p \times \mathcal{R}_q)};\) therefore, \((1, 1) = 2, \lambda(1, 5) = (6/5),\) and \(\lambda(5, 5) = (2/5).\) Thus, by Lemma 1 and after simplification,

\[
\mathcal{RRZ}_1(G_{p,q}^3) = \frac{72pq}{5} - \frac{24p}{5} - \frac{24q}{5}
\] (32)

For \(\mathcal{RRZ}_2(G_{p,q}^3)\) which is the second reverse redefined index of \(G_{p,q}^3\) from equation (8), we have \(\lambda(\mathcal{R}_p, \mathcal{R}_q) = \frac{(\mathcal{R}_p + \mathcal{R}_q)^4}{(\mathcal{R}_p \times \mathcal{R}_q)^2};\) therefore, \((1, 1) = (1/2), \lambda(1, 5) = (5/6),\) and \(\lambda(5, 5) = (5/2).\) Thus, by Lemma 1 and after simplification,

\[
\mathcal{RRZ}_2(G_{p,q}^3) = 14pq + 6p + 6q + \frac{8}{3}
\] (33)

For \(\mathcal{RRZ}_3(G_{p,q}^3)\) which is the third reverse redefined index of \(G_{p,q}^3\) from equation (8), we have \(\lambda(\mathcal{R}_p, \mathcal{R}_q) = \frac{(\mathcal{R}_p + \mathcal{R}_q)^5}{(\mathcal{R}_p \times \mathcal{R}_q)^3};\) therefore, \((1, 1) = 2, \lambda(1, 5) = 30,\) and \(\lambda(5, 5) = 250.\) Thus, by Lemma 1 and after simplification,

\[
\mathcal{RRZ}_3(G_{p,q}^3) = 936pq + 744p + 744q + 384.
\] (34)

3.2. Results for Case 2. We provide a formula that would be used to calculate any reverse degree topological descriptors of Case 2 for \(G_{p,q}^3\).

Lemma 2. Let \(G_{p,q}^3\) be a third-type \(p\)th chain hex-derived networks. Then,

\[
\mathcal{T}_2(G_{p,q}^3) = 3pq(\lambda(1, 1) + 2\lambda(1, 5) + \lambda(5, 5))
\]
\[
+ 2(2p + q)(\lambda(5, 5) - \lambda(1, 1)) + 2(\lambda(1, 1) - 2\lambda(1, 5) + \lambda(5, 5)).
\] (35)

Proof. The graph \(G_{p,q}^3\) contains \(12pq\) edges and maximum degree in \(G_{p,q}^3\) graph is 8. There are two types of reverse degree vertices in \(G_{p,q}^3\) that are 1 and 5. Let us partition the edges of \(G_{p,q}^3\) according to its reverse degrees according to Case 2 as

\[
E_{1,1} = \{p \in E(G_{p,q}^3) : \mathcal{R}_1 = 1, \mathcal{R}_8 = 1\},
\]
\[
E_{1,5} = \{p \in E(G_{p,q}^3) : \mathcal{R}_1 = 1, \mathcal{R}_8 = 5\},
\]
\[
E_{5,5} = \{p \in E(G_{p,q}^3) : \mathcal{R}_1 = 5, \mathcal{R}_8 = 5\}.
\]

Note that \(E(G_{p,q}^3) = E_{1,1} \cup E_{1,5} \cup E_{5,5}\) and \(|E_{1,1}| = 3pq - 4p - 2q + 2, |E_{1,5}| = 6pq - 4,\) and \(|E_{5,5}| = 3pq + 4p + 2q + 2.\) Hence,

\[
\mathcal{T}_2(G_{p,q}^3) = \sum_{p \in E(G_{p,q}^3)} \lambda(\mathcal{R}_1, \mathcal{R}_8)
\]
\[
= \sum_{p \in E_{1,1}} \lambda(1, 1) + \sum_{p \in E_{1,5}} \lambda(1, 5) + \sum_{p \in E_{5,5}} \lambda(5, 5)
\]
\[
= (3pq - 4p - 2q + 2)\lambda(1, 1) + (6pq - 4)\lambda(1, 5)
\]
\[
+ (3pq + 4p + 2q + 2)\lambda(5, 5).
\] (37)

After simplification, we obtain
\[ T_2(G_{p,q}^3) = 3pq(\lambda(1,1) + 2\lambda(1,5) + \lambda(5,5)) + 2(2p + q)(\lambda(5,5) - \lambda(1,1)) + 2(\lambda(1,1) - 2\lambda(1,5) + \lambda(5,5)). \]

**Theorem 4.** The general reverse Randić index of \( G_{p,q}^3 \) is equal to

\[
\mathcal{R}_{\alpha}(G_{p,q}^3) = \begin{cases} 
108pq + 96p + 48q + 32, & \text{for } \alpha = 1, \\
(18 + 6\sqrt{5})pq + 16p + 8q + 12 - 4\sqrt{5}, & \text{for } \alpha = \frac{1}{2}, \\
\left(\frac{18}{5} + \frac{6\sqrt{5}}{5}\right)pq - \frac{16p}{5} - \frac{8q}{5} + \frac{12}{5} - \frac{4\sqrt{5}}{5}, & \text{for } \alpha = -\frac{1}{2}, \\
\frac{108}{25}pq - \frac{96}{25}p - \frac{48}{25}q + \frac{32}{25}, & \text{for } \alpha = -1.
\end{cases}
\]

**Proof.** For \( \mathcal{R}_{\alpha}(G_{p,q}^3) \) which is the general reverse Randić index of \( G_{p,q}^3 \), from equation (2), we have \( \lambda(\mathcal{R}_{a}, \mathcal{R}_{d}) = (\mathcal{R}_{a} \times \mathcal{R}_{d})^\alpha \); therefore, \( \lambda(1,1) = 1 \), \( \lambda(1,5) = (5)^\alpha \), and \( \lambda(5,5) = (25)^\alpha \). Thus, by Lemma 2,

\[
\mathcal{R}_{\alpha}(G_{p,q}^3) = 3pq(1 + 2(5)^\alpha + (25)^\alpha)
\]

\[ + 2(2p + q)((25)^\alpha - 1) + 2(1 - 2(5)^\alpha + (25)^\alpha). \]

Put \( \alpha = 1 \), and we have

\[
\mathcal{R}_{1}(G_{p,q}^3) = 108pq + 96p + 48q + 32.
\]

Put \( \alpha = \frac{1}{2} \), and we have

\[ \mathcal{R}_{\frac{1}{2}}(G_{p,q}^3) = (18 + 6\sqrt{5})pq + 16p + 8q + 12 - 4\sqrt{5}. \]

Put \( \alpha = -\frac{1}{2} \), and we have

\[ \mathcal{R}_{-\frac{1}{2}}(G_{p,q}^3) = \left(\frac{18}{5} + \frac{6\sqrt{5}}{5}\right)pq - \frac{16p}{5} - \frac{8q}{5} + \frac{12}{5} - \frac{4\sqrt{5}}{5}. \]

Put \( \alpha = -1 \), and we have

\[ \mathcal{R}_{-1}(G_{p,q}^3) = \frac{108}{25}pq - \frac{96}{25}p - \frac{48}{25}q + \frac{32}{25}. \]

\[ \square \]

**Theorem 5.** Let \( G_{p,q}^3 \) be a third-type \( p \)th chain hex-derived networks. Then, the reverse atom-bond connectivity index is

\[
\mathcal{R}_{ABC}(G_{p,q}^3) = \left(\frac{12\sqrt{5}}{5} + \frac{6\sqrt{2}}{5}\right)pq - \frac{8\sqrt{5}}{5} + \frac{2(4p + 2q + 2)\sqrt{2}}{5}.
\]

The reverse geometric-arithmetic index is

\[ \mathcal{R}_{GA}(G_{p,q}^3) = (6 + 2\sqrt{5})pq + 4 - \frac{4\sqrt{5}}{3}. \]

The first reverse Zagreb index is

\[ \mathcal{R}_{M_1}(G_{p,q}^3) = 72pq + 32p + 16q \]

The reverse hyper-Zagreb index is

\[ \mathcal{R}_{H}(G_{p,q}^3) = 528pq + 384p + 192q + 64. \]

The reverse forgotten index is

\[ \mathcal{R}_{F}(G_{p,q}^3) = 312pq + 192p + 96q. \]

**Proof.** For \( \mathcal{R}_{ABC}(G_{p,q}^3) \) which is the reverse atom-bond connectivity index of \( G_{p,q}^3 \), from equation (3), we have

\[ \lambda(\mathcal{R}_{a}, \mathcal{R}_{d}) = \sqrt{(\mathcal{R}_{a} + \mathcal{R}_{d}) - 2}/\mathcal{R}_{a} \times \mathcal{R}_{d}; \] therefore, \( \lambda(1,1) = 0 \), \( \lambda(1,5) = \sqrt{4/5} \), and \( \lambda(5,5) = (\sqrt{8}/5) \). Thus, by Lemma 2 and after simplification,

\[ \mathcal{R}_{ABC}(G_{p,q}^3) = \left(\frac{12\sqrt{5}}{5} + \frac{6\sqrt{2}}{5}\right)pq - \frac{8\sqrt{5}}{5} + \frac{2(4p + 2q + 2)\sqrt{2}}{5}. \]

\[ \square \]
For $\mathcal{R}_H(G_{p,q})$, which is the first reverse hyper-Zagreb index of $G_{p,q}$, from equation (6), we have $\lambda(\mathcal{R}_H, \mathcal{R}_H) = \left(\mathcal{R}_H + \mathcal{R}_H\right)^2$; therefore, $\lambda(1,1) = 4$, $\lambda(1,5) = 36$, and $\lambda(5,5) = 100$. Thus, by Lemma 2 and after simplification,

$$\mathcal{R}_H(G_{p,q}) = 528pq + 384p + 192q + 64. \quad (53)$$

For $\mathcal{R}_F(G_{p,q})$, which is the reverse forgotten index of $G_{p,q}$, from equation (7), we have $\lambda(\mathcal{R}_F, \mathcal{R}_F) = \left(\mathcal{R}_F \times \mathcal{R}_F\right)^2$; therefore, $\lambda(1,1) = 2$, $\lambda(1,5) = 26$, and $\lambda(5,5) = 50$. Thus, by Lemma 2 and after simplification,

$$\mathcal{R}_F(G_{p,q}) = 312pq + 192p + 96q. \quad (54)$$

Theorem 6. Let $G_{p,q}$ be a third-type $p$th chain hex-derived networks. Then, the first reverse redefined index is

$$\mathcal{R}_RZ_1(G_{p,q}) = \frac{72pq + 32p + 16q}{5}. \quad (55)$$

The second reverse redefined index is

$$\mathcal{R}_RZ_2(G_{p,q}) = 14pq + 8p + 4q + \frac{8}{3}. \quad (56)$$

The third reverse redefined index is

$$\mathcal{R}_RZ_3(G_{p,q}) = 936pq + 992p + 496q + 384. \quad (57)$$

Proof. For $\mathcal{R}_RZ_1(G_{p,q})$, which is the first reverse redefined index of $G_{p,q}$, from equation (8), we have $\lambda(\mathcal{R}_R, \mathcal{R}_R) = \left(\mathcal{R}_R \times \mathcal{R}_R\right)^{-1}$; therefore, $\lambda(1,1) = 2$, $\lambda(1,5) = (6/5)$, and $\lambda(5,5) = (2/5)$. Thus, by Lemma 2 and after simplification,

$$\mathcal{R}_RZ_1(G_{p,q}) = \frac{72pq + 32p + 16q}{5}. \quad (58)$$

For $\mathcal{R}_RZ_2(G_{p,q})$ which is the second reverse redefined index of $G_{p,q}$, from equation (8), we have $\lambda(\mathcal{R}_R, \mathcal{R}_R) = \left(\mathcal{R}_R \times \mathcal{R}_R\right)^{-1}$; therefore, $\lambda(1,1) = (1/2)$, $\lambda(1,5) = (5/6)$, and $\lambda(5,5) = (5/2)$. Thus, by Lemma 2 and after simplification,

$$\mathcal{R}_RZ_2(G_{p,q}) = 14pq + 8p + 4q + \frac{8}{3}. \quad (59)$$

For $\mathcal{R}_RZ_3(G_{p,q})$ which is the third reverse redefined index of $G_{p,q}$, from equation (8), we have $\lambda(\mathcal{R}_R, \mathcal{R}_R) = \left(\mathcal{R}_R \times \mathcal{R}_R\right)$; therefore, $\lambda(1,1) = 2$, $\lambda(1,5) = 30$, and $\lambda(5,5) = 250$. Thus, by Lemma 2 and after simplification,

$$\mathcal{R}_RZ_3(G_{p,q}) = 936pq + 992p + 496q + 384. \quad (60)$$

3.3 Results for Case 3. We provide a formula that would be used to calculate any reverse degree topological descriptors of Case 3 for $G_{p,q}$.

Lemma 3. Let $G_{p,q}$ be a third-type $p$th chain hex-derived networks. Then,

$$\mathcal{F}_3(G_{p,q}) = 3pq(\lambda(1,1) + 2\lambda(1,5) + \lambda(5,5)) + 2(\lambda(5,5) - \lambda(1,1))(2p + q) + 2(\lambda(1,1) - 4\lambda(1,5) + 3\lambda(5,5)). \quad (61)$$

Proof. The graph $G_{p,q}$ contains 12pq edges, and maximum degree in $G_{p,q}$ graph is 8. There are two types of reverse degree vertices in $G_{p,q}$ that are 1 and 5. Let us partition the edges of $G_{p,q}$ according to its reverse degrees according to Case 3 as

$$E_1 = \{\emptyset \in E(G_{p,q}) : \mathcal{R}_H = 1, \mathcal{R}_H = 1\},$$

$$E_5 = \{\emptyset \in E(G_{p,q}) : \mathcal{R}_H = 5, \mathcal{R}_H = 5\}. \quad (62)$$

Hence,

$$\mathcal{F}_3(G_{p,q}) = \sum_{\emptyset \in E_1} \lambda(\mathcal{R}_H, \mathcal{R}_H) = \sum_{\emptyset \in E_1} \lambda(1,1) + \sum_{\emptyset \in E_5} \lambda(1,5) + \sum_{\emptyset \in E_5} \lambda(5,5) = (3pq - 4p - 2q + 2)\lambda(1,1) + (6pq - 8)\lambda(1,5) + (3pq + 4p + 2q + 6)\lambda(5,5). \quad (63)$$

After simplification, we obtain

$$\mathcal{F}_3(G_{p,q}) = 3pq(\lambda(1,1) + 2\lambda(1,5) + \lambda(5,5)) + 2(\lambda(5,5) - \lambda(1,1))(2p + q) + 2(\lambda(1,1) - 4\lambda(1,5) + 3\lambda(5,5)). \quad (64)$$

Theorem 7. The general reverse Randić index of $G_{p,q}$ is equal to
Proof. For $\mathcal{R}_\alpha(G^3_{p,q})$ which is the general reverse Randić index of $G^3_{p,q}$, from equation (2), we have

$$\mathcal{R}_\alpha(G^3_{p,q}) = \begin{cases} 
108pq + 96p + 48q + 112, & \text{for } \alpha = 1, \\
(18 + 6\sqrt{5})pq + 16p + 8q + 32 - 8\sqrt{5}, & \text{for } \alpha = \frac{1}{2}, \\
\left(\frac{18}{5} + \frac{6\sqrt{5}}{5}\right)pq - \frac{16p}{5} + \frac{8q}{5} + \frac{16}{5} - \frac{8\sqrt{5}}{5}, & \text{for } \alpha = -\frac{1}{2}, \\
108pq - 25 - \frac{96p}{25} - \frac{48q}{25} + \frac{16}{25} & \text{for } \alpha = -1.
\end{cases}$$

(65)

The reverse geometric-arithmetic index is

$$\mathcal{R}_{GA}(G^3_{p,q}) = (6 + 2\sqrt{5})pq + 8 - \frac{8\sqrt{5}}{3},$$

(72)

The first reverse Zagreb index is

$$\mathcal{R}_{M}(G^3_{p,q}) = 72pq + 32p + 16q + 16.$$

(73)

The reverse hyper-Zagreb index is

$$\mathcal{R}_{HM}(G^3_{p,q}) = 528pq + 384p + 192q + 320.$$

(74)

The reverse forgotten index is

$$\mathcal{R}_{F}(G^3_{p,q}) = 312pq + 192p + 96q + 96.$$

(75)

Proof. For $\mathcal{R}_{ABC}(G^3_{p,q})$ which is the reverse atom-bond connectivity index of $G^3_{p,q}$, from equation (3), we have

$$\mathcal{R}_{ABC}(G^3_{p,q}) = 3pq(1 + 2(5^\alpha + 25^\alpha) + 2((25^\alpha - 1)(2p + q) + 2(1 - 4(5^\alpha + 3(25^\alpha)).$$

(66)

Put $\alpha = 1$, and we have

$$\mathcal{R}_{1}(G^3_{p,q}) = 108pq + 96p + 48q + 112.$$

(67)

Put $\alpha = (1/2)$, and we have

$$\mathcal{R}_{(1/2)}(G^3_{p,q}) = (18 + 6\sqrt{5})pq + 16p + 8q + 32 - 8\sqrt{5}.$$  

(68)

Put $\alpha = (-1/2)$, and we have

$$\mathcal{R}_{(-1/2)}(G^3_{p,q}) = \left(\frac{18}{5} + \frac{6\sqrt{5}}{5}\right)pq - \frac{16p}{5} + \frac{8q}{5} + \frac{16}{5} - \frac{8\sqrt{5}}{5}.$$  

(69)

Theorem 8. Let $G^3_{p,q}$ be a third-type $p$th chain hex-derived networks. Then, the reverse atom-bond connectivity index is

$$\mathcal{R}_{ABC}(G^3_{p,q}) = \left(\frac{12\sqrt{5}}{5} + \frac{6\sqrt{2}}{5}\right)pq - \frac{16\sqrt{5}}{5} + \frac{2(4p + 2q + 6\sqrt{2})}{5},$$

(71)

For $\mathcal{R}_{GA}(G^3_{p,q})$ which is the reverse geometric-arithmetic index of $G^3_{p,q}$, from equation (4), we have

$$\mathcal{R}_{GA}(G^3_{p,q}) = 3pq(1 + 2(5^\alpha + 25^\alpha) + 2((25^\alpha - 1)(2p + q) + 2(1 - 4(5^\alpha + 3(25^\alpha).$$

(66)

Put $\alpha = 1$, and we have

$$\mathcal{R}_{GA}(G^3_{p,q}) = (6 + 2\sqrt{5})pq + 8 - \frac{8\sqrt{5}}{3}. $$

(77)
Lemma 3 and after simplification, from equation (7), we have \( \lambda \), from equation (5), we have \( \lambda \), which is the reverse forgotten index of \( \lambda \), which is the first reverse hyper-Zagreb index of \( \lambda \), which is the second reverse redefined index of \( \lambda \), and \( \lambda \), therefore, \( \lambda (1, 1) = 2 \), \( \lambda (1, 5) = 6 \), and \( \lambda (5, 5) = 10 \). Thus, by Lemma 3 and after simplification,

\[
\mathcal{R}_{1}\left(G_{p,q}^{3}\right) = 72pq + 32p + 16q + 16. \tag{78}
\]

For \( \mathcal{R}_{1}\left(G_{p,q}^{3}\right) \) which is the first reverse Zagreb index of \( G_{p,q}^{3} \), from equation (5), we have \( \lambda (\mathcal{R}_{1}, \mathcal{R}_{1}) = (\mathcal{R}_{1} + \mathcal{R}_{1}) \); therefore, \( \lambda (1, 1) = 2 \), \( \lambda (1, 5) = 6 \), and \( \lambda (5, 5) = 10 \). Thus, by Lemma 3 and after simplification,

\[
\mathcal{R}_{1}\left(G_{p,q}^{3}\right) = 528pq + 384p + 192q + 320. \tag{79}
\]

For \( \mathcal{R}_{2}\left(G_{p,q}^{3}\right) \) which is the reverse forgotten index of \( G_{p,q}^{3} \), from equation (7), we have \( \lambda (\mathcal{R}_{1}, \mathcal{R}_{1}) = ((\mathcal{R}_{1}^{2})^{2}) + ((\mathcal{R}_{1}^{2})^{2}) \); therefore, \( \lambda (1, 1) = 2 \), \( \lambda (1, 1) = 6 \), and \( \lambda (5, 5) = 10 \). Thus, by Lemma 3 and after simplification,

\[
\mathcal{R}_{2}\left(G_{p,q}^{3}\right) = 312pq + 192p + 96q + 96. \tag{80}
\]

Theorem 9. Let \( G_{p,q}^{3} \) be a third-type \( p \)th chain hex-derived networks. Then, the first reverse redefined index is

\[
\mathcal{R}_{1}\left(G_{p,q}^{3}\right) = \frac{72pq}{5} - \frac{32p}{5} - \frac{16q}{5} + \frac{16}{5}. \tag{81}
\]

The second reverse redefined index is

\[
\mathcal{R}_{2}\left(G_{p,q}^{3}\right) = 14pq + 8p + 4q + \frac{28}{3}. \tag{82}
\]

The third reverse redefined index is
\[ \mathcal{R} \mathcal{R} Z_3(G_3^{p,q}) = 936pq + 992p + 496q + 1264. \]  \hfill (83)

Proof. For \( \mathcal{R} \mathcal{R} Z_1(G_3^{p,q}) \), which is the first reverse redefined index of \( G_3^{p,q} \), from equation (8), we have \( \lambda(\mathcal{R}_p, \mathcal{R}_q) = (\mathcal{R}_p + \mathcal{R}_q)^{-1}(\mathcal{R}_p \times \mathcal{R}_q)^{-1} \); therefore, \( \lambda(1,1) = 2, \lambda(1,5) = 6/5, \) and \( \lambda(5,5) = 5/2 \). Thus, by Lemma 3 and after simplification,

\[ \mathcal{R} \mathcal{R} Z_1(G_3^{p,q}) = \frac{72pq}{5} - \frac{32p}{5} - \frac{16q}{5} - \frac{16}{5}. \]  \hfill (84)

For \( \mathcal{R} \mathcal{R} Z_2(G_3^{p,q}) \), which is the second reverse redefined index of \( G_3^{p,q} \), from equation (8), we have \( \lambda(\mathcal{R}_p, \mathcal{R}_q) = (\mathcal{R}_p + \mathcal{R}_q)^{-1}(\mathcal{R}_p \times \mathcal{R}_q)^{-1} \); therefore, \( \lambda(1,1) = 1/2, \lambda(1,5) = 5/6, \) and \( \lambda(5,5) = 5/2 \). Thus, by Lemma 3 and after simplification,

\[ \mathcal{R} \mathcal{R} Z_2(G_3^{p,q}) = 14pq + 8p + 4q + \frac{28}{3}. \]  \hfill (85)

For \( \mathcal{R} \mathcal{R} Z_3(G_3^{p,q}) \), which is the third reverse redefined index of \( G_3^{p,q} \), from equation (8), we have \( \lambda(\mathcal{R}_p, \mathcal{R}_q) = (\mathcal{R}_p + \mathcal{R}_q)(\mathcal{R}_p \times \mathcal{R}_q) \); therefore, \( \lambda(1,1) = 2, \lambda(1,5) = 30, \) and \( \lambda(5,5) = 350 \). Thus, by Lemma 3 and after simplification,

\[ \mathcal{R} \mathcal{R} Z_3(G_3^{p,q}) = 936pq + 992p + 496q + 1264. \]  \hfill (86)
4. Numerical and Graphical Representation

In this section, we determine the numerical values of $R_{ABC}$, $R_{GA}$, $R_{M1}$, $R_{HM}$, and $R_{F}$ in Tables 1–3, for Case 1, Case 2, and Case 3, respectively. We represent these results graphically in Figures 4–6.

Data Availability

No data were used to support the findings of the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] F. Simonraj and A. George, "On the metric dimension of HDN$_3$ and PHDN$_3$," in Proceedings of the IEEE International Conference on Power, Control, Signals and Instrumentation Engineering (ICPCSI), pp. 1333–1336, Chennai, India, September 2017.

[2] H. Ali, M. A. Binyamin, M. K. Shafiq, and W. Gao, "On the degree-based topological indices of some derived networks," Mathematics, vol. 7, no. 7, p. 612, 2019.

[3] C.-C. Wei, H. Ali, M. A. Binyamin, M. N. Naem, and J.-B. Liu, "Computing degree based topological properties of third type of hex-derived networks," Mathematics, vol. 7, no. 4, p. 368, 2019.

[4] M. Karelson, Molecular Descriptors in QSAR/QSPR, Wiley, New York, NY, USA, 2000.

[5] A. Aslam, Y. Bashir, S. Ahmad, and W. Gao, "On topological indices of certain dendrimer structures," Zeitschrift für Naturforschung, vol. 72, no. 6, pp. 559–566, 2017.

[6] M. Bača, J. Horváthová, M. Mokrišová, and A. Suháňiová, "On topological indices of fullerenes," Applied Mathematics and Computation, vol. 251, pp. 154–161, 2015.

[7] A. Q. Baig, M. Imran, and H. Ali, "On topological indices of poly oxide, poly silicate, DOX, and DSL networks," Canadian Journal of Chemistry, vol. 93, no. 7, pp. 730–739, 2015.

[8] S. Hayat and M. Imran, "Computation of topological indices of certain networks," Applied Mathematics and Computation, vol. 240, pp. 213–228, 2014.

[9] A. Koam and A. Ahmad, "Polynomials of degree-based indices for three-dimensional mesh network," Computers, Materials & Continua, vol. 65, no. 2, pp. 1271–1282, 2020.

[10] C. Rui, G. Ali, G. Rahmat, M. Y. Khan, A. Semanicova-Fenovcikova, and J.-B. Liu, "Investigation of general power sum-connectivity index for some classes of extremal graphs," Complexity, vol. 2021, Article ID 6623277, 6 pages, 2021.

[11] G. Hong, Z. Gu, M. Javaid, H. M. Awais, and M. K. Siddiqui, "Degree-based topological invariants of metal-organic networks," IEEE Access, vol. 8, pp. 68288–68300, 2020.

[12] Z. Iqbal, A. Aslam, M. Ishaq, and M. Aamir, "Characteristic study of irregularity measures of some nanotubes," Canadian Journal of Physics, vol. 97, no. 10, pp. 1125–1132, 2019.

[13] V. R. Kulli, "Reverse Zagreb and reverse hyper-Zagreb indices and their polynomials of rhombus silicate networks," Annals of Pure and Applied Mathematics, vol. 16, no. 1, pp. 47–51, 2018.

[14] A. Ahmad, "On the degree based topological indices of benzene ring embedded in P-type-surface in 2D network," Hacettepe Journal of Mathematics and Statistics, vol. 47, no. 1, pp. 9–18, 2018.

[15] M. Imran, A. Q. Baig, and H. Ali, "On molecular topological properties of hex-derived networks," Journal of Chemistry, vol. 30, no. 3, pp. 121–129, 2016.

[16] D. Zhao, Y.-M. Chu, M. K. Siddiqui et al., "On reverse degree based topological indices of polycyclic metal organic network," Polycyclic Aromatic Compounds, vol. 5, 2021.

[17] A. Ahmad, "Comparative study of ve-degree and ev-degree topological descriptors for benzene ring embedded in p-type-surface in 2d network," Polycyclic Aromatic Compounds, vol. 5, 2020.

[18] J. Zhang, M. K. Siddiqui, A. Rauf, and M. Ishfaq, "On ve-degree and ev-degree based topological properties of single walled titanium dioxide nanotube," Journal of Cluster Science, 2020.

[19] N. Zahra, M. Ibrahim, and M. K. Siddiqui, "On topological indices for swapped networks modeled by optical transpose
interconnection system,” *IEEE Access*, vol. 8, pp. 200091–200099, 2020.

[20] Z. Raza, “Leap Zagreb connection numbers for some networks models,” *Indonesian Journal of Chemistry*, vol. 20, no. 6, pp. 1407–1413, 2020.

[21] M. K. Siddiqui, M. Imran, and A. Ahmad, “On Zagreb indices, Zagreb polynomials of some nanostar dendrimers,” *Applied Mathematics and Computation*, vol. 280, pp. 132–139, 2016.

[22] A. Aslam, S. Ahmad, and W. Gao, “On certain topological indices of boron triangular nanotubes,” *Zeitschrift für Naturforschung A*, vol. 72, no. 8, pp. 711–716, 2017.

[23] M. Javaid, H. Zafar, Q. Zhu, and A. M. Alanazi, “Improved lower bound of LFMD with applications of prism-related networks,” *Mathematical Problems in Engineering*, vol. 2021, Article ID 9950310, 9 pages, 2021.

[24] J.-B. Liu, Z. Raza, and M. Javaid, “Zagreb connection numbers for cellular neural networks,” *Discrete Dynamics in Nature and Society*, vol. 2020, Article ID 8038304, 8 pages, 2020.

[25] E. A. Refae and A. Ahmad, “A study of hexagon star network with vertex-edge-based topological descriptors,” *Complexity*, vol. 2021, Article ID 9911308, 7 pages, 2021.

[26] M.-S. Chen, K. G. Shin, and D. D. Kandlur, “Addressing, routing, and broadcasting in hexagonal mesh multiprocessors,” *IEEE Transactions on Computers*, vol. 39, no. 1, pp. 10–18, 1990.

[27] A. N. A. Koam, A. Ahmad, A. Ahmad, and Y. Ahmad, “Computation of reverse degree-based topological indices of hex-derived networks,” *AIMS Mathematics*, vol. 6, no. 10, pp. 11330–11345, 2021.

[28] Y. Huo, H. Ali, M. A. Binyamin, S. S. Asghar, U. Babar, and J.-B. Liu, “On topological indices of mth chain hex-derived network of third type,” *Frontiers in Physics*, vol. 8, Article ID 593275, 2020.

[29] A. N. A. Koam, A. Ahmad, and M. F. Nadeem, “Comparative study of valency-based topological descriptor for hexagon star network,” *Computer Systems Science and Engineering*, vol. 36, no. 2, pp. 293–306, 2021.

[30] H. Ali, A. Sajjad, and A. Sajjad, “On further results of hex derived networks,” *Open Journal of Discrete Applied Mathematics*, vol. 2, no. 1, pp. 32–40, 2019.

[31] M. Imran, A. Q. Baig, S. U. Rehman, H. Ali, and R. Hasni, “Computing topological polynomials of mesh-derived networks,” *Discrete Mathematics, Algorithms and Applications*, vol. 10, no. 6, Article ID 1850077, 2018.

[32] M. Imran, A. Q. Baig, and H. Ali, “On topological properties of dominating David derived networks,” *Canadian Journal of Chemistry*, vol. 94, pp. 137–148, 2015.

[33] A. Aslam, S. Ahmad, and W. Gao, “On certain topological indices of boron triangular nanotubes,” *Zeitschrift für Naturforschung*, vol. 72, no. 8, pp. 711–716, 2017.