Scaling in the structure of directory trees in a computer cluster

Konstantin Klemm,1 Víctor M. Eguíluz,2 and Maxi San Miguel2

1Interdisciplinary Centre for Bioinformatics / Bioinformatics Group of the Institute for Computer Science, University Leipzig, Krenzstr. 76, 04103 Leipzig, Germany
2Instituto Mediterráneo de Estudios Avanzados IMEDEA (CSIC-UIB), E07122 Palma de Mallorca (Spain)

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We describe the topological structure and the underlying organization principles of the directories created by users of a computer cluster when storing his/her own files. We analyze degree distributions, average distance between files, distribution of communities and allometric scaling exponents of the directory trees. We find that users create trees with a broad, scale-free degree distribution. The structure of the directories is well captured by a growth model with a single parameter. The degree distribution of the different trees has a non-universal exponent associated with different values of the parameter of the model. However, the distribution of community sizes has a universal exponent analytically obtained from our model.

The processes of storing and retrieving information are rapidly gaining importance in science as well as society as a whole. A considerable effort is being undertaken, firstly to characterize and describe how publicly available information, for example in the world wide web, is actually organized, and secondly, to design efficient methods to access this information. It seems clear that to design methods for accessing information we first need to know how information is actually stored or organized as it is being produced.

Within this general framework a crucial step in building general knowledge on these processes, is the understanding of how each of us organizes knowledge and information produced by ourselves. To be specific, we pose the question of general organizational principles in the managing of our own electronic files. To answer this question we analyze the structure and organization of the files stored in a computer cluster by the users of the computer facilities at a research institute. Within the general study of complex networks, we are here looking at trees and we report a first observation of the scale free property in trees. It is important to point out that we are not studying a single large tree, but rather we are considering a forest of many trees, each of them being the result of an individual construction. We are then able to consider samples of organizational schemes of many different sizes, since each user has created a structure with a different number of directories. This allows the study of different samples of the same reality. We also note that contrary to other networks like the WWW or food webs, the structures considered here are not the outcome of a collective action but the creation of a single individual. Our research gives information about the management of information at the individual level.

Two a priori possible answers to the question posed are that we follow a random process of file storing or that, on the contrary, we implement a careful planned structure as we do when organizing the sections and chapters of a PhD thesis or a scientific paper. What we find is the signature of a complex system halfway between these two possibilities, but still with well defined patterns of organization. In this paper we report an extensive characterization of individual user computer directory trees, calculating a number of quantitative measures. These include degree distributions, average distance between directories, distribution of community sizes in the tree, and allometric scaling exponents. Our data turns out to be well described by a directory attachment model for constructing the tree. The model depends on a single parameter $q$ that interpolates between random placement of new directories and the agglomeration into a star structure. The trees of the different users are described by different values of the parameter $q$: diversity in individual behavior here boils down to a different value of a parameter.

Data analysis. – The data material under consideration is taken from the computer facilities of the Cross-disciplinary Physics Department of IMEDEA (Mediterranean Institute for Advanced Studies). The personal accounts of the 63 users running Linux and UNIX have been considered. The users include academic staff, postdocs, graduate students and long-time visitors. Each user is able to choose freely his/her own organizational scheme without specific software. The nodes in the directory tree of a given user are all directories (file folders) stored in the user’s computer account. There is a direct link between nodes $i$ and $j$ if directory $i$ is a subdirectory of directory $j$ or vice versa. We consider the trees as rooted with the home directory as the root. In the following, we analyze the trees in terms of the distributions of degree and of community sizes as well as the allometric scaling.

A local measure of the importance of a given node $i$ is the nodal degree $k_i$ counting the number of nodes directly connected to $i$. In a tree of $N$ nodes the average degree is always $\langle k \rangle = 2 - 2/N$. The distribution of the degree, however, varies strongly across different types of structures. The distribution is narrow in simple chains and binary trees while it is broadest for a star (having $N - 1$ nodes with degree $k = 1$ and one center node with degree $k = N - 1$). The degree distributions of
the observed directory trees (Fig. 1(a)) lie in between these two extremes. Directory trees are scale-free. The probability of finding a node with degree $k$ decays as a power law $k^{-\gamma}$ with a cut-off at the maximum degree $k_{\text{max}}$ due to finite size. There is no indication of an upper bound on the degree that would limit the scaling at large $k$. Given trees generated by different users, the observed values of $\gamma$ do not coincide in general. The degree exponent is not universal.

An alternative characterization of the trees is obtained by iterative decomposition into subtrees rather than single nodes. Here we consider the community structure of the trees. For each node $i$, a community $S_i$ is the subtree rooted at the node $i$ and all nodes below $i$. In the directory trees, a community $S_i$ is the tree formed by a directory $i$, all its subdirectories, the subdirectories of these and so forth. A community $S_i$ is again a rooted tree with node $i$ as the root. Calculating the sizes $A_i = |S_i|$ of all communities for each tree, we find the statistics in Fig. 1(b). The distribution of community sizes decays as a power law $A^{-\tau}$. The exponent $\tau = 2$ appears to be universal. The scaling of community size $A$ is a property independent of the scaling of the degree $k$. When the trees are randomized under conserving degrees of all nodes, the functional form of the community size distribution changes and obtains a scaling region with a larger exponent $\tau > 2$.

In order to capture also the correlations between community sizes we perform allometric scaling analysis. For each community $S_i$ we calculate the quantity $C_i = \sum_{j \in S_i} A_j$, i.e. we sum up all the sizes of all communities contained in $S_i$, including $S_i$ itself. Figure 1(c) shows the data point $(A_i, C_i)$ for each community $i$ in the 63 trees. We find that the growth of $C$ with $A$ is superlinear.

Modeling.—Let us now consider a stochastic model for the construction of a directory tree. We assume that users build their trees by iteratively adding nodes, i.e. creating new directories. Then for each possible tree the model assigns an attachment probability to each of the nodes. The attachment probability $\Pi_i$ of a node $i$ is the probability that $i$ becomes the parent of the next added node. In the simplest case, the structure of the tree is irrelevant for the attachment process. Then we have homogeneous attachment. Each directory has the same probability to become the parent of a new directory. Another conceivable rule is copying of directories. If directories are chosen for duplication with equal probability, a directory obtains a new subdirectory with a probability proportional to the number of subdirectories it already has. Here we formulate a model comprising both these mechanisms at tunable ratio. In a tree with $N$ nodes, a node with degree $k$ becomes the parent of the next added node with probability

$$\Pi(k) = q \frac{k - 1}{N} + (1 - q) \frac{1}{N}.$$
The tunable parameter \( q \in [0, 1] \) is the probability that duplication of a node is performed. With probability \( 1 - q \) a randomly chosen node is the parent of the added directory. Qualitatively \( q \) measures how often the individual creating the tree likes to subdivide a directory. Note that the pure rules \((q = 0, q = 1)\) cannot produce trees as in Fig. 1(a). Homogeneous attachment \((q = 0)\) leads to trees with an exponential degree distribution. The pure duplication mechanism \((q = 1)\) can only generate stars because it cannot turn a leaf into an inner node. By rewriting Eq. 1 as \( \Pi(k) \propto k^q + a \) with the number of links \( k^\text{in} = k - 1 \) received after creation of the node and the "initial attractiveness" \( a = 1/q - 1 \) we see the equivalence of our model with the network growth model by Dorogovtsev et al. [13], restricted to a single new link added per node. The case \( q = 1/2 \), giving \( \gamma = 3 \), is the scale-free model by Barabasi and Albert [14]. For general \( q \in [0, 1] \), the model produces scale-free trees with degree exponent \( \gamma = 2 + a = 1 + 1/q > 2 \).

The evolution of community sizes is described by the probability

\[
\Pi(A) = q \frac{A - 1}{N} + (1 - q) \frac{A}{N} = \frac{A - q}{N}.
\]

that the next node is attached to one of the nodes of a given community of size \( A \), thereby incrementing \( A \).

From a continuous rate equation approach [14] we obtain \( A_i(N) = (1 - q)N/i + q \) as the expected size of community \( S_i \) in a tree of size \( N \). The index \( i \) is the time step of creation of the community as a single node with \( A = 1 \). The linear growth of \( A \) with \( N \) implies that the community size distribution of the model decays asymptotically as \( A^{-\tau} \) with universal (\( q \)-independent) exponent \( \tau = 2 \), in agreement with the data.

For an estimate of the allometric scaling, first note the general property \( C_i = A_i + \sum_{j \in S_i} d_{ij} \) where the chemical distance \( d_{ij} \) is the number of nodes contained in the direct path between nodes \( i \) and \( j \). Adding a new node \( j^* \) to community \( S_i \), the expected distance \( \langle d_{ij^*} \rangle \) from node \( i \) is \( C_i/A - 1 \) for copying and \( C_i/A \) for homogeneous attachment. Thus on average \( C \) grows as \( dC/dA = 1 + C/A - q \), where the finite difference has been approximated by the derivative and the index \( i \) is suppressed. For the initial condition \( C(1) = 1 \) we obtain the solution \( C(A) = A[(1 - q) \ln A + 1] \). The allometric scaling of the model trees is linear with logarithmic correction. In order to compare with the observed trees we replotted the binned data as \((A_i, C_i/A_i)\) in the inset of Fig. 1(c). The data are captured well by a logarithmic dependence (best fit \( C/A = 0.59 \ln A + 0.99 \), correlation coefficient \( r = 0.997 \)) in good agreement with the prediction of the model.

In order to provide a more stringent check of the validity of the model (Eq. 1) we first project the trees into a space of four observables, namely the second, third and fourth moments of the degree distribution and the average chemical distance between nodes. For a given value \( x \) of an observable and given tree size \( N \) we estimate the most likely parameter value \( q_x \) by weighting all possible values \( q \in [0, 1] \) with the probability that they produce \( x \) up to a small error. Figure 2 shows the results and gives details of the method in the caption. For almost all trees there is excellent agreement between the four parameter estimates based on different observables. Thus after choice of a single parameter the model accurately reproduces the projection of the trees into a four-dimensional space. The projection takes into account the distribution of the degree as a local property, and the average distance \( \langle d_{ij} \rangle \) between nodes as a global property. This is strong evidence that the proposed growth mechanism
produces statistically the same structures as seen in the directory trees.

Discussion.—The structure of directory trees has been characterized from a statistical point of view. Our main result is the striking structural similarity between trees created by independent users in the absence of common constraints. Users create trees with a broad, scale-free degree distribution with a non-universal exponent. The distribution of community sizes, however, scales with a universal exponent $\tau \approx 2$. The allometric scaling is linear with a logarithmic correction. Community structure and allometric scaling are significantly different in random surrogate trees with the same degree distribution.

The statistical properties of the empirical trees are reproduced by a model that generates trees by adding nodes iteratively. The model has a single parameter $q$ controlling the tendency to accumulate many subdirectories in the same parent directory. By varying $q$, the degree exponent can be tuned in the empirically observed range $\gamma$. The exponent $\tau \approx 2$ and the allometric scaling $C \sim A \ln A$ have been derived analytically and are independent of the parameter $q$. The validity of the model has been evidenced further by determining the most likely value of the parameter $q$. For a given tree, estimates based on different moments of the degree distribution as well as the diameter coincide, while estimates vary across trees. Consequently, directory trees can be distinguished by their specific value of the growth parameter $q$.

A generally interesting question is to decide about universal properties and universality classes of different natural and artificial or man-made complex networks. The community distribution exponent $\tau \approx 2$ that we find for our directory trees is in agreement with the one reported for the Internet \[15\] and for the communities of scientific collaborations \[17, 18\]. However, a different class is formed by river networks \[14, 20, 21\], informal networks in organizations \[4\] and jazz musician networks \[18\], where the corresponding exponent gives a value $\tau \approx 1.45$ \[17\]. These examples seem to belong to the class of efficient networks obtained from an optimization principle in which transportation costs are minimized \[10\]. For the class of efficient networks one can prove \[10, 22, 23\] that allometric scaling is given by a power law dependence $C \sim A^\eta$, with a universal exponent $\eta = (D + 1)/D$, where $D$ is the embedding dimension. At difference with the prediction from efficiency, we find $C \sim A \ln A$ for the directory trees as reproduced by our growth model. This result is also compatible with effective (apparent) exponents observed in food webs \[11\].

We have shown that directory trees as individually man-made but not designed objects are an interesting direction of further research into hierarchical networks. Analyzing the wealth of readily available tree data on computers around the world offers improved insight into how people naturally structure information.

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