Deformed Intersecting D6-Brane GUTS II

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ABSTRACT

By employing D6-branes intersecting at angles in $D = 4$ type I strings, we construct five stack string GUT models (PS-II class), that contain at low energy exactly the Standard model with no extra matter and/or extra gauge group factors. These classes of models are based on the Pati-Salam (PS) gauge group $SU(4)_C \times SU(2)_L \times SU(2)_R$. They represent deformations around the quark and lepton basic intersection number structure.

The models possess the same phenomenological characteristics of some recently discussed examples (PS-A and PS-I class) of four stack PS GUTS. Namely, there are no colour triplet couplings to mediate proton decay and proton is stable as baryon number is a gauged symmetry. Neutrinos get masses of the correct sizes. Also the mass relation $m_e = m_d$ at the GUT scale is recovered. The conditions for the non-anomalous $U(1)$’s to survive massless the Green-Schwarz mechanism are equivalent, to the conditions, coming from the presence of $N=1$ supersymmetry, in sectors involving the presence of extra branes and also required to guarantee the existence of the Majorana mass term for the right handed neutrinos. These conditions are independent from the number of extra $U(1)$ branes. We also discuss the relative size of the leading worldsheet instanton correction to the trilinear Yukawa couplings in a general GUT model.

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1 Introduction

The purpose of this paper is to present three generation five stack string GUT models that break at low energy exactly to the Standard Model (SM), $SU(3)_C \times SU(2)_L \times U(1)_Y$, without any extra chiral fermions and/or extra gauge group factors. The four-dimensional models are non-supersymmetric and are based on the Pati-Salam (PS) $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge group. The basic structure behind the models includes D6-branes intersecting each other at non-trivial angles, in an orientifolded compactification of IIA theory on a factorized six-torus, where O$_6$ orientifold planes are on top of D6-branes [1, 2].

The proposed classes of models have some distinctive features:

- The models, from now on characterized as belonging to the PS-II class, are being built with a gauge group $U(4) \times U(2) \times U(2) \times U(1) \times U(1)$ at the string scale. At the scale of symmetry breaking of the left-right symmetry $M_{GUT}$, the initial symmetry group breaks to the the standard model $SU(3)_C \times SU(2)_L \times U(1)_Y$ augmented with two extra anomaly free $U(1)$ symmetries. The additional $U(1)$'s may break by the vev of some charged singlet scalars, e.g. $s^1_B$, $s^2_B$ at a scale set by their vevs, leaving at low energies the SM itself. The fermions get charged under the broken $U(1)$ symmetry, acquiring a flavour symmetry. The singlets responsible for breaking the $U(1)$ symmetries are obtained by demanding certain open sectors to respect $N = 1$ supersymmetry, e.g. $dd^*$, $ee^*$.

- A numbers of extra U(1)'s added to cancel the RR tadpoles results in scalar singlet generation in combination with preserving N=1 SUSY on intersections.

- Neutrinos gets a mass of the right order, consistent with the LSND oscillation experiments, from a see-saw mechanism, where the Dirac and Majorana terms are, of the Frogatt-Nielsen type.

- Proton is stable due to the fact that baryon number is an unbroken gauged global symmetry surviving at low energies and no colour triplet couplings that could mediate proton decay exist. Thus a gauged baryon number provides a natural explanation for proton stability. As in the other D6-models [3, 4, 5, 6], on the same background with just the SM at low energy [1], the baryon number

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1 could be as high as the string scale

2 We note that there are also intersecting D5-brane constructions [7] [8] with exactly the SM at low energy. They will be mentioned later in this section.
associated $U(1)$ gauge boson becomes massive through its couplings to Green-Schwarz mechanism. That has an an immediate effect that baryon number is surviving as a global symmetry to low energies providing for a natural explanation for proton stability in general brane-world scenarios.

- The model uses small Higgs representations in the adjoint to break the PS symmetry, instead of using large Higgs representations \(^3\).

Some of the major problems of string theory is the hierarchy of scale and particle masses after supersymmetry breaking. These phenomenological issues have by far been explored in the context of construction of semirealistic supersymmetric models of weakly coupled $N = 1$ (orbifold) compactifications of the heterotic string theories. In these theories many problems remain unsolved, we mention briefly one of them, namely that the string scale which is of the order of $10^{18}$ GeV is in clear disagreement with the observed unification of gauge coupling constants in the MSSM of $10^{16}$ GeV. The latter problem remained unclear even though the observed discrepancy between the two high scales was attributed \(^4\) to the presence of the $N = 1$ string threshold corrections to the gauge coupling constants \(^5\). Also semirealistic model building has by far been explored in the context of orientifold models \(^6\).

On the contrary in type I models, the string scale, which is a free parameter, can be lowered in the TeV range \(^7\) thus suggesting that non-SUSY models with a string scale in the TeV region is a viable possibility. In this spirit, recently some new constructions have appeared in a type I string vacuum background which use intersecting branes \(^8\) and give four dimensional non-supersymmetric models.

Hence, by using background fluxes in a D9 brane type I background \(^9\) it was possible for open string modes to be formulated \(^10\) that break supersymmetry on the brane and give chiral fermions with an even number of generations \(^11\). In these models the fermions get localized in the intersections between branes \(^12\). With the introduction of a quantized background NS-NS B field \(^13\), \(^14\), \(^15\), that makes the tori tilted, it was then possible to give rise to semirealistic models with three generations \(^16\). It should be noted that these backgrounds are T-dual to models with magnetic deformations \(^17\).

\(^3\) E.g. 126 like in the standard $SO(10)$ models.

\(^4\) among other options,

\(^5\) In the T-dual language these backgrounds are represented by D6 branes wrapping 3-cycles on a dual torus and intersecting each other at certain angles.
Furthermore, an important step was taken in [3], by showing how to construct the standard model (SM) spectrum together with right handed neutrinos in a systematic way. The authors considered, as a starting point, IIA theory compactified on $T^6$ assigned with an orientifold product $\Omega \times R$, where $\Omega$ is the worldsheet parity operator and $R$ is the reflection operator with respect to one of the axis of each tori. In this case, the four stack D6-branes contain Minkowski space and each of the three remaining dimensions is wrapped up on a different $T^2$ torus. In this construction the proton is stable since the baryon number is a gauged $U(1)$ global symmetry. A special feature of these models is that the neutrinos can only get Dirac mass. These models have been generalized to models with five stacks [4] and six stacks of D6-branes at the string scale [5]. The models of [4, 5] are build as deformations of the QCD intersection numbers, namely they are build around the left and right handed quarks intersection numbers. Also, they hold exactly the same phenomenological properties of [3]. They also, have a special feature since by demanding the presence of $N = 1$ supersymmetric sectors, we are able to break the extra, beyond the SM gauge group, $U(1)'s$, and thus predicting the unique existence of one supersymmetric partner of the right neutrino or two supersymmetric partners of the right neutrinos in the five and the six stack SM’s respectively.

In addition, in [6] we presented the first examples of GUT models in a string theory context, and in the context of intersecting branes, that break completely to the SM at low energies. The models predict uniquely the existence of light weak fermion doublets with energy between the range 90 - 246 GeV, that is they can be directly tested at present of future accelerators. Deformations of these models will be pursued in this work.

We note that apart from D6 models with exactly the SM at low energy just mentioned, there are studies using intersecting D5-branes and only the SM at low energy [7, 8]. In the latter models [8] there are special classes of theories, again appearing as deformations of the QCD intersection structure, which have not only the SM at low energy but exactly the same low energy effective theory including fermion and scalar spectrum.

Also non-SUSY and SUSY constructions in the context of intersecting branes with the SM plus additional massless exotic matter were considered in [17, 18]. For constructions with intersecting branes on compact Calabi-Yau spaces see [19] and for intersecting branes on non-compact Calabi-Yau spaces see [20]. For some other work in the context of intersecting branes see [21, 22, 23, 24, 25, 26].
The paper is organized as follows. In section two we describe the general rules for building chiral GUT models in orientifolded $T^6$ compactifications and the possible open string sectors. In section 3, we discuss the basic fermion and scalar structure of the PS-II class of models that will mainly focus in this work. In section 4, we discuss the parametrization of the solutions to the RR tadpole cancellation conditions. In section 5 we discuss the cancellation of $U(1)$ anomalies in the presence of a generalized Green-Schwarz (GS) mechanism and extra $U(1)$ branes. In section 6, we discuss the conditions for the absence of tachyons as well describing the Higgs sector of the models including the low energy and high energy Higgs sector of the classes of GUTS presented. In subsection 7.1 we discuss the importance of creating sectors preserving N=1 SUSY for the realization of the see-saw mechanism. In subsection 7.2 we discuss in which way by adding extra $U(1)$ branes we create scalar singlets and satisfy RR tadpole conditions. In subsection 7.3 we discuss the breaking of the surviving the Green-Schwarz mechanism massless $U(1)$’s with the use of singlets coming the non-trivial intersections of the extra branes and leptonic branes as well from sectors in the form $j^j$. In subsection 8.1 we examine the structure of GUT Yukawa couplings in intersecting braneworlds and compute the leading worldsheet corrections for the present models. In subsection 8.2 we examine the problem of neutrino masses. In subsection 8.3 we show that all additional exotic fermions beyond those of SM present in the models become massive and disappear from the low energy spectrum. Section 9 contains our conclusions. Finally, Appendix I, includes the conditions for the absence of tachyonic modes in the spectrum of the PS-II class of models presented, In Appendix II, we discuss the importance of choosing appropriate locations for the extra branes across the three-cycles, such that the models realize the presence of Higgsses, needed for electroweak symmetry breaking.

2 Tadpole structure and spectrum rules

Next, we describe the construction of the PS classes of models. It is based on type I string with D9-branes compactified on a six-dimensional orientifolded torus $T^6$, where internal background gauge fluxes on the branes are turned on \[,\]. By performing a T-duality transformation on the $x^4, x^5, x^6$, directions the D9-branes with fluxes are translated into D6-branes intersecting at angles. The branes are not parallel to the orientifold planes. We assume that the D6$_a$-branes are wrapping 1-cycles ($n^i_a, m^i_a$) along each of the $T^2$ torus of the factorized $T^6$ torus, namely $T^6 = T^2 \times T^2 \times T^2$. 
In order to build a PS model with minimal Higgs structure we consider four stacks of D6-branes giving rise to their world-volume to an initial gauge group $U(4) \times U(2) \times U(2) \times U(1) \times U(1)$ at the string scale. In addition, we consider the addition of NS B-flux, such that the tori are not orthogonal, avoiding in this way an even number of families, and leading to effective tilted wrapping numbers,

$$(n^i, m = \tilde{m}^i + n^i/2); \ n, \tilde{m} \in \mathbb{Z},$$  \hspace{1cm} (2.1)\

that allows semi-integer values for the m-numbers.

In the presence of $\Omega R$ symmetry, where $\Omega$ is the worldvolume parity and $R$ is the reflection on the T-dualized coordinates,

$$T(\Omega R)T^{-1} = \Omega R,$$  \hspace{1cm} (2.2)\

and thus each D6$_a$-brane 1-cycle, must have its $\Omega R$ partner $(n^i_a, -m^i_a)$.

Chiral fermions are obtained by stretched open strings between intersecting D6-branes [12]. Also the chiral spectrum of the models may be obtained after solving simultaneously the intersection constraints coming from the existence of the different sectors together with the RR tadpole cancellation conditions.

There are a number of different sectors, which should be taken into account when computing the chiral spectrum. We will denote the action of $\Omega R$ on a sector $\alpha, \beta$, by $\alpha^\star, \beta^\star$, respectively. The possible sectors are:

- The $\alpha\beta + \beta\alpha$ sector: involves open strings stretching between the D6$_\alpha$ and D6$_\beta$ branes. Under the $\Omega R$ symmetry this sector is mapped to its image, $\alpha^\star \beta^\star + \beta^\star \alpha^\star$ sector. The number, $I_{\alpha\beta}$, of chiral fermions in this sector, transforms in the bifundamental representation $(N_\alpha, \bar{N}_\beta)$ of $U(N_\alpha) \times U(N_\beta)$, and reads

$$I_{\alpha\beta} = (n^1_\alpha m^1_\beta - m^1_\alpha n^1_\beta)(n^2_\alpha m^2_\beta - m^2_\alpha n^2_\beta)(n^3_\alpha m^3_\beta - m^3_\alpha n^3_\beta),$$  \hspace{1cm} (2.3)\

where $I_{\alpha\beta}$ is the intersection number of the wrapped cycles. Note that the sign of $I_{\alpha\beta}$ denotes the chirality of the fermion and with $I_{\alpha\beta} > 0$ we denote left handed fermions. Negative multiplicity denotes opposite chirality.

- The $\alpha\alpha$ sector: it involves open strings stretching on a single stack of D6$_\alpha$ branes. Under the $\Omega R$ symmetry this sector is mapped to its image $\alpha^\star \alpha^\star$. This sector contain $N = 4$ super Yang-Mills and if it exists SO(N), SP(N) groups appear. This sector is of no importance to us as we will be dealing with unitary groups.
The $\alpha\beta^* + \beta^*\alpha$ sector: It involves chiral fermions transforming into the $(N_\alpha, N_\beta)$ representation with multiplicity given by

$$I_{\alpha\beta^*} = -(m_1^1 m_2^1 + m_2^1 n_3^1)(n_2^2 m_3^2 + m_2^2 n_3^2)(n_3^3 m_3^3 + m_3^3 n_3^3).$$

(2.4)

Under the $\Omega R$ symmetry transforms to itself.

- the $\alpha\alpha^*$ sector: under the $\Omega R$ symmetry is transformed to itself. From this sector the invariant intersections will give $8m_1^1 m_2^2 m_3^3$ fermions in the antisymmetric representation and the non-invariant intersections that come in pairs provide us with $4m_1^1 m_2^2 m_3^3 (n_1^1 n_2^2 n_3^3 - 1)$ additional fermions in the symmetric and antisymmetric representation of the $U(N_\alpha)$ gauge group.

Also any vacuum derived from the previous intersection number constraints of the chiral spectrum is subject to constraints coming from RR tadpole cancellation conditions \[1\]. That requires cancellation of D6-branes charges \[4\] wrapping on three cycles with homology $[\Pi_a]$ and O6-plane 7-form charges wrapping on 3-cycles with homology $[\Pi_{O6}]$. In formal terms, the RR tadpole cancellation conditions in terms of cancellations of RR charges in homology, read:

$$\sum_a N_a[\Pi_a] + \sum_{\alpha^*} N_{\alpha^*}[\Pi_{\alpha^*}] - 32[\Pi_{O6}] = 0.$$  \hspace{1cm} (2.5)

Explicitly, the RR tadpole conditions read:

$$\sum_a N_a n_1^a n_2^a n_3^a = 16,$$
$$\sum_a N_a m_1^a m_2^a m_3^a = 0,$$
$$\sum_a N_a m_1^a n_2^a m_3^a = 0,$$
$$\sum_a N_a n_1^a m_2^a m_3^a = 0.$$  \hspace{1cm} (2.6)

That ensures absence of non-abelian gauge anomalies. A comment is in order. It is important to notice that the RR tadpole cancellation condition can be understood as a constraint that demands that for each gauge group the number of fundamentals to be equal to the number of bifundamentals. As a general rule to D-brane model building, by considering $a$ stacks of D-brane configurations with $N_a, a = 1, \ldots, N$, paralled branes, the gauge group that appears is in the form $U(N_1) \times U(N_2) \times \cdots \times U(N_a)$. Thus,

\[6\] Taken together with their orientifold images $(n_1^a, -m_1^a)$ wrapping on three cycles of homology class $[\Pi_{\alpha^*}]$.
effectively each $U(N_i)$ factor will give rise to an $SU(N_i)$, charged under the associated $U(1_i)$ gauge group factor, that appears in the decomposition $SU(N_a) \times U(1_a)$. A brane configuration with the unique minimal PS particle content such that intersection numbers, tadpole conditions and various phenomenological requirements including the presence of exotic representations are accommodated, can be obtained by considering five stacks of branes yielding an initial $U(4)_a \times U(2)_b \times U(1)_c \times U(1)_d \times U(1)_e$. In this case the equivalent gauge group is an $SU(4)_a \times SU(2)_b \times SU(2)_b \times U(1)_a \times U(1)_b \times U(1)_c \times U(1)_d \times U(1)_e$. Thus, in the first instance, we can identify, without loss of generality, $SU(4)_a$ as the $SU(4)_c$ colour group that its breaking could induce the $SU(3)$ colour group of strong interactions, the $SU(2)_b$ with $SU(2)_L$ of weak interactions and $SU(2)_c$ with $SU(2)_R$ of left-right symmetric PS models.

3 The basic fermion structure

The basic PS-II class of models that we will center our attention in this work, will be a three family non-supersymmetric GUT model with the left-right symmetric Pati-Salam model structure $SU(4)_C \times SU(2)_L \times SU(2)_R$. The open string background on which the models will be build will be intersecting D6-branes wrapping on 3-cycles of decomposable toroidal ($T^6$) orientifolds of type IIA in four dimensions [1, 2].

The three generations of quark and lepton fields are accommodated into the following representations:

\[
F_L = (4, \bar{2}, 1) = (q(3, \bar{2}, \frac{1}{6}) + l(1, \bar{2}, -\frac{1}{2}) \equiv (u, d, l),
\]

\[
\bar{F}_R = (\bar{4}, 1, 2) = (\bar{u}^c(3, 1, -\frac{2}{3}) + \bar{d}^c(3, 1, \frac{1}{3}) + e^c(1, 1, 1) + N_c(1, 1, 0) \equiv (\bar{u}^c, \bar{d}^c, \bar{e}^c),
\]

where the quantum numbers on the right hand side of (3.1) are with respect to the decomposition of the $SU(4)_C \times SU(2)_L \times SU(2)_R$ under the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group and $l = (\nu, e)$ is the standard left handed lepton doublet, $l^c = (N^c, e^c)$ are the right handed leptons. Also the assignment of the accommodation of the quarks and leptons into the representations $F_L + \bar{F}_R$ is the one appearing in the spinorial decomposition of the 16 representation of $SO(10)$ under the PS gauge group.

A set of useful fermions appear also in the model

\[
\chi_L = (1, \bar{2}, 1), \chi_R = (1, 1, \bar{2}).
\]

These fermions are a general prediction of left-right symmetric theories as the existence of these representations follows from RR tadpole cancellation conditions.
The symmetry breaking of the left-right PS symmetry at the $M_{GUT}$ scale proceeds through the representations of the set of Higgs fields,

$$H_1 = (\bar{4}, 1, 2), \quad H_2 = (4, 1, 2),$$  \hspace{1cm} (3.3)

where,

$$H_1 = (\bar{4}, 1, 2) = u_H(3, 1, \frac{2}{3}) + d_H(\bar{3}, 1, -\frac{1}{3}) + e_H(1, 1, -1) + \nu_H(1, 1, 0).$$  \hspace{1cm} (3.4)

The electroweak symmetry breaking is delivered through bi-doublet Higgs fields $h_i$ for $i = 3, 4$, field in the representations

$$h_3 = (1, 2, 2), \quad h_4 = (1, 2, 2).$$  \hspace{1cm} (3.5)

Because of the imposition of N=1 SUSY on some open string sectors, there are also present the massless scalar superpartners of the quarks, leptons and antiparticles

$$\tilde{F}_R^H = (\bar{4}, 1, 2) = u_H^c(3, 1, -\frac{4}{6}) + d_H^c(\bar{3}, 1, \frac{1}{3}) + e_H^c(1, 1, 1) + N_H^c(1, 1, 0) \equiv (u_H^c, d_H^c, l_H^c).$$  \hspace{1cm} (3.6)

The latter fields characterize all vacua coming from these type IIA orientifolded tori constructions is the replication of massless fermion spectrum by an equal number of massive particles in the same representations and with the same quantum numbers. This is the basic fermionic structure appearing in the PS models that we have considered in [6] and will be appearing later in this work. Also, a number of charged exotic fermion fields, which receive a string scale mass, appear

$$6(6, 1, 1), \quad 6(\bar{10}, 1, 1).$$  \hspace{1cm} (3.7)

The complete accommodation of the fermion structure of the PS-II classes of models under study in this work can be seen in table (1).

### 4 Tadpole cancellation for PS-II classes of GUTS

To understand the solution of the RR tadpole cancellation condition, that it will be given in parametric form, we should make the following comments:

a) The need to realize certain couplings will force us to demand that some intersections will preserve some supersymmetry. Thus some massive fields will be “pulled out” from

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7In principle this scale could be as high as the string scale.
8 are replicas of the fermion fields appearing in the intersection ac and they receive a vev
Table 1: Fermionic spectrum of the $SU(4)_{C} \times SU(2)_{L} \times SU(2)_{R}$, PS-II class of models together with $U(1)$ charges. We note that at energies of order $M_z$ only the Standard model survives.

The massive spectrum and become massless. For example, in order to realize a Majorana mass term for the right handed neutrinos we will demand that the sector $ac$ preserves $N = 1$ SUSY. That will have as an immediate effect to "pull out" from the massive mode spectrum the $\bar{F}H_R$ particles.

b) The intersection numbers, in table (1), of the fermions $F_L + \bar{F}_R$ are chosen such that $I_{ac} = -3$, $I_{ab^r} = 3$. Here, $-3$ denotes opposite chirality to that of a left handed fermion. The choice of additional fermion representations $(1, \bar{2}, 1), (1, 1, \bar{2})$ is imposed to us by the RR tadpole cancellation conditions that are equivalent to $SU(N_a)$ gauge anomaly cancellation, in this case of $SU(2)_L, SU(2)_R$ gauge anomalies,

$$\sum I_{ia} N_a = 0, \ a = L, R. \hspace{1cm} (4.1)$$

c) The PS-II class of models don’t accommodate representations of scalar sextets $(6, 1, 1)$ fields, that appear in attempts to construct realistic 4D $N = 1$ PS heterotic models from the fermionic formulation [31], even through heterotic fermionic models where those representations are lacking exist [30]. Those representations were imposed earlier in attempts to produce a realistic PS model as a recipe for saving the models from proton decay. Fast proton decay was avoided by making the mediating $d_H$ triplets
of (3,4) superheavy and of order of the $SU(2)_R$ breaking scale via their couplings to the sextets. In the models we examine in this work, baryon number is a gauged global symmetry, so that proton is stable. Thus there is no need to introduce sextets to save the models from fact proton decay as proton is stable.

Also in the present PS-II GUTS, there is no problem of having $d_H$ becoming light enough and causing catastrophic proton decay, as the only way this could happen, is through the existence of the $d_H$ coupling to sextets to quarks and leptons. However, this coupling is forbidden by the symmetries of the models.

The theory breaks just to the standard model $SU(3) \times SU(2) \times U(1)_Y$ at low energies. The tadpole solutions of PS-II models are presented in table (2).

d) The mixed anomalies $A_{ij}$ of the seven surplus $U(1)$'s with the non-abelian gauge groups $SU(N_a)$ of the theory cancel through a generalized GS mechanism \cite{31, 32, 3, 4, 5}, involving close string modes couplings to worldsheet gauge fields. Two combinations of the $U(1)$'s are anomalous and become massive through their couplings to RR fields, their orthogonal non-anomalous combinations survives, combining to a single $U(1)$ that remains massless. Crucial for achieving the RR tadpole cancellation is the presence of $N_h$ extra branes. Contrary, of what is happening in D6-brane models \cite{10}, with exactly the SM at low energy, and a Standard-like structure at the string scale \cite{3, 4, 5} where the extra branes have no intersection with the branes, in the intersecting GUT models there is a non-vanishing intersection of the extra branes with the rest of the branes. As a consequence, this becomes a singlet generation mechanism after imposing $N = 1$ SUSY between $U(1)$ leptonic (the $d, e$ branes) and the $U(1)$ extra branes. Also, contrary to the SM's of \cite{3, 4, 5, 7, 8} the extra branes do not form a $U(N_h)$ gauge group but rather a $U(1)^{N_1} \times U(1)^{N_2} \cdots U(1)^{N_h}$ one.

e) The constraint

$$\Pi_{i=1}^3 m^i = 0. \quad (4.2)$$

is not imposed and thus leads to the appearance of the non-trivial chiral fermion content from the $a\bar{a}^* , d\bar{d}^* , e\bar{e}^*$ sector with corresponding fermions $\omega_L, z_R, s_1^R, s_2^R$.

f) After breaking the PS left-right symmetry at $M_{GUT}$, the surviving gauge symmetry is that of the SM augmented by four anomaly free $U(1)$ symmetries, including the added extra $U(1)$ branes, surviving the Green-Schwarz mechanism. To break the latter

\footnote{We examine for convenience the case of two added extra U(1)’s.}

\footnote{Also happening in intersecting D5-brane models, with exactly the SM at low energy and a Standard-like structure at the string scale \cite{3, 8}.}
$U(1)$ symmetries we will impose that the $dd^*, ee^*dh$, $dh^*$ sectors \[\] respects $N = 1$ SUSY. Thus singlets scalars will appear, that are superpartners of the corresponding fermions.

f) Demanding $I_{ab^*} = 3$, $I_{ac} = -3$, it implies that the third tori should be tilted. By looking at the intersection numbers of table one, we conclude that the b-brane should be parallel to the c-brane and the a-brane should be parallel to the d, e branes as there is an absence of intersection numbers for those branes. The cancellation of the RR crosscap tadpole constraints is solved from multiparametric sets of solutions which are given in table (2).

| $N_i$ | $(n_1^i, m_1^i)$ | $(n_2^i, m_2^i)$ | $(n_3^i, m_3^i)$ |
|-------|-----------------|-----------------|-----------------|
| $N_a = 4$ | $(0, \epsilon)$ | $(n_a^2, 3\epsilon\tilde{\epsilon} \beta_2)$ | $(1, \tilde{\epsilon}/2)$ |
| $N_b = 2$ | $(-1, \epsilon m_b^1)$ | $(1/\beta_2, 0)$ | $(1, \tilde{\epsilon}/2)$ |
| $N_c = 2$ | $(1, \epsilon m_c^1)$ | $(1/\beta_2, 0)$ | $(1, -\tilde{\epsilon}/2)$ |
| $N_d = 1$ | $(0, \epsilon)$ | $(n_d^2, -4\epsilon\tilde{\epsilon} \beta_2)$ | $(2, \tilde{\epsilon})$ |
| $N_e = 1$ | $(0, \epsilon)$ | $(n_e^2, -2\epsilon\tilde{\epsilon} \beta_2)$ | $(2, -\tilde{\epsilon})$ |
| 1 | $(1/\beta_1, 0)$ | $(1/\beta_2, 0)$ | $(2, 0)$ |
| \vdots | \vdots | \vdots | \vdots |
| $N_h$ | $(1/\beta_1, 0)$ | $(1/\beta_2, 0)$ | $(2, 0)$ |

Table 2: Tadpole solutions for PS-II type models where the five stack of D6-branes wrapping numbers giving rise to the fermionic spectrum and the SM, $SU(3)_C \times SU(2)_L \times U(1)_Y$, gauge group at low energies. The wrappings depend on two integer parameters, $n_a^2$, $n_d^2$, $n_e^2$, the NS-background $\beta_i$ and the phase parameters $\epsilon = \tilde{\epsilon} = \pm 1$. Also there is an additional dependence on the two wrapping numbers, integer of half integer, $m_b^1$, $m_c^1$. Note also that the presence of the $N_h$ extra U(1) branes.

A comment is in order. The location of extra branes needed to satisfy the RR tadpoles is particularly important. Choosing e.g. $\beta_1 = \beta_2 = 1/2$ and their location to \[\]

\[\]

\[\]

WE denoted by $h$ the presence of extra branes.
be at
\[(1/\beta^1,0)(1/\beta^1,0)(1,m/2), \ m \in 2Z + 1\] (4.3)
it results in classes of models with no electroweak bidoublets \(h_1, h_2, h_3, h_4\). The relevant analysis can be seen in Appendix II. Thus we choose to add \(N_h\) extra branes located at
\[(1/\beta_1,0)(1/\beta_2,0)(2,0)\] (4.4)
In principle the extra branes could have a different from but great care should be taken, as the non-zero intersections of the extra branes with the rest of the branes could create massless exotic fermions that cannot be made massive and disappear from the low energy spectrum.

The first tadpole condition in (2.6) depends on the number of extra branes that it is added. Thus it becomes
\[N_h \frac{2}{\beta_1 \beta_2} = 16.\]
(4.5)
The third tadpole condition becomes
\[(2n_a^2 + n_d^2 - n_e^2) + \frac{1}{\beta_2}(m_b^1 - m_c^1) = 0.\]
(4.6)
To see clearly the cancellation of tadpoles, we have to choose a consistent numerical set of wrapping numbers, e.g.
\[\epsilon = \bar{\epsilon} = 1, \ n_a^2 = 1, \ m_b^1 = -3/2, \ m_c^1 = 1/2, \ n_d^2 = 1, \ n_e^2 = 1, \ \beta_1 = 1/2, \ \beta_2 = 1.\]
(4.7)
The latter can be satisfied with the addition of four extra U(1) D6-branes.

We note that in the model described by the wrapping numbers of table (3) we cannot get the SM at low energy as all the fermions are charged under the U(1)’s (The U(1)’s can be seen in (5.4) and at this stage they do not remain massless as they have non-zero couplings to RR fields.).

f) the hypercharge operator is defined as usual in this classes of GUT models (see also [5]) as a linear combination of the three diagonal generators of the \(SU(4), SU(2)_L, SU(2)_R\) groups:
\[Y = \frac{1}{2}T_{3R} + \frac{1}{2}T_{B-L}, \ T_{3R} = \text{diag}(1,-1), \ T_{B-L} = \text{diag} \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1\right).\]
(4.8)
Also,
\[Q = Y + \frac{1}{2}T_{3L}.\]
(4.9)
Table 3: Wrapping number set consistent with the tadpole constraint (4.6). We have not include the extra U(1) branes.

5 Cancellation of U(1) Anomalies

The mixed anomalies $A_{ij}$ of the four $U(1)$’s with the non-Abelian gauge groups are given by

$$A_{ij} = \frac{1}{2}(I_{ij} - I_{ij'})N_i. \quad (5.1)$$

Moreover, analyzing the mixed anomalies of the extra $U(1)$’s with the non-abelian gauge groups $SU(4)_c$, $SU(2)_R$, $SU(2)_L$ we can see that there are three anomaly free combinations $Q_b - Q_c$, $Q_a + Q_d - Q_e$ and $Q_a + 4Q_d + 5Q_e$. Note that gravitational anomalies cancel since D6-branes never intersect O6-planes. In the orientifolded type I torus models gauge anomaly cancellation [32] proceeds through a generalized GS mechanism [3] that makes use of the 10-dimensional RR gauge fields $C_2$ and $C_6$ and gives at four dimensions the couplings to gauge fields

$$N_a m_a^1 m_a^2 m_a^3 \int_{M_4} B_2^o \wedge F_a; \quad n_b^1 n_b^2 n_b^3 \int_{M_4} C^o \wedge F_b \wedge F_b, \quad (5.2)$$

$$N_a n_a^1 n_a^2 n_a^3 \int_{M_4} B_2^l \wedge F_a; \quad n_b^1 m_b^1 m_b^K \int_{M_4} C^I \wedge F_b \wedge F_b, \quad (5.3)$$

where $C_2 \equiv B_2^o$ and $B_2^l \equiv \int_{(T^2)^1 \times (T^2)^2 \times (T^2)^3} C_6$ with $I = 1, 2, 3$ and $I \neq J \neq K$. Notice the four dimensional duals of $B_2^o$, $B_2^l$:

$$C^o \equiv \int_{(T^2)^1 \times (T^2)^2 \times (T^2)^3} C_6; \quad C^I \equiv \int_{(T^2)^1 \times (T^2)^2 \times (T^2)^3} C_2, \quad (5.4)$$

where $dC^o = -\ast dB_2^o$, $dC^I = -\ast dB_2^l$. 

| $N_i$ | $(n_i^1, m_i^1)$ | $(n_i^2, m_i^2)$ | $(n_i^3, m_i^3)$ |
|-------|----------------|----------------|----------------|
| $N_a = 4$ | $(0, 1)$ | $(1, 3)$ | $(1, 1/2)$ |
| $N_b = 2$ | $(-1, -3/2)$ | $(1, 0)$ | $(1, 1/2)$ |
| $N_c = 2$ | $(1, 1/2)$ | $(1, 0)$ | $(1, -1/2)$ |
| $N_d = 1$ | $(0, 1)$ | $(1, -4)$ | $(2, 1)$ |
| $N_e = 1$ | $(0, 1)$ | $(1, -2)$ | $(2, -1)$ |
The triangle anomalies (5.1) cancel from the existence of the string amplitude involved in the GS mechanism [31] in four dimensions [32]. The latter amplitude, where the $U(1)\alpha$ gauge field couples to one of the propagating $B_2$ fields, coupled to dual scalars, that couple in turn to two $SU(N)$ gauge bosons, is proportional [3] to

$$- N_a m_a^1 m_a^2 m_a^3 n_b^1 n_b^2 n_b^3 - N_a \sum_I n_a^I n_a^J n_b^K m_a^I m_b^J m_b^K , I \neq J, K$$

(5.5)

We make the minimal choice

$$\beta_1 = \beta_2 = 1/2$$

(5.6)

that requires two extra D6 branes.

In this case the structure of $U(1)$ couplings reads:

$$B_2^3 \wedge [2\tilde{\epsilon}][-(F^b + F^c)],$$

$$B_2^1 \wedge [\epsilon][4n_a^2 F^a + 4m_b^1 F^b + 4m_c^1 F^c + 2n_d^2 F^d + 2n_e^2 F^e],$$

$$B_2^0 \wedge (3F^a - 2F^d + F^e).$$

(5.7)

As can be seen from (5.7) two anomalous combinations of $U(1)$'s, e.g. $3F^a - 2F^d + F^e$, $-(F^b + F^c)$ become massive through their couplings to RR fields $B_2^0$, $B_2^3$. Also there is an anomaly free model dependent $U(1)$ which is getting massive from its coupling to the RR field $B_2^1$. In addition, there are four non-anomalous $U(1)$'s which also are getting broken by vevs of singlet scalars generated by imposing N=1 SUSY on certain sectors. They are:

$$U(1)^{(4)} = (Q^b - Q^c) + (Q^a + Q^d - Q^e),$$

$$U(1)^{(5)} = F^{b1}, \quad U(1)^{(6)} = F^{b2},$$

$$U(1)^{(7)} = Q^a + 4Q^d + 5Q^e.$$  

(5.8)

The choice of $U(1)$'s (5.8) have no couplings to RR fields, and thus survive massless the presence of the generalized Green-Shwarz mechanism, if

$$2n_a^2 + 4n_b^2 + 5n_e^2 = 0$$

(5.9)

At this point we should list the couplings of the dual scalars $C^d$ of $B_2^I$ required to cancel the mixed anomalies of the $U(1)$'s with the non-abelian gauge groups $SU(N_a)$. They are given by

$$C^o \wedge 2[-(F^b \wedge F^b) + (F^c \wedge F^c)],$$

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\[ C^2 \wedge [\xi \bar{\xi}][2n_a^2(F^a \wedge F^a) + 2m_b^1(F^b \wedge F^b) - 2m_c^1(F^c \wedge F^c) + n_d^2(F^d \wedge F^d) - n_e^2(F^e \wedge F^e)], \]
\[ C^3 \wedge \left[ \frac{\bar{\xi}}{2} \right][3(F^a \wedge F^a) - 8(F^d \wedge F^d) - 4(F^e \wedge F^e)], \tag{5.10} \]

As it will be shown later, the conditions for demanding that some sectors respect \( N=1 \) SUSY, that in turn guarantee the existence of a Majorana coupling for right handed neutrinos as well creating singlets necessary to break the \( U(1)'s \) (5.8), solve the condition (5.9). We note that if we had chosen \( \beta_1 = 1, \beta_2 = 1/2 \) that is \( N_h = 4 \) extra branes, we simply would have two more \( U(1) \) generators surviving massless that is \( U(1)^{(8)} = F^{h3}, U(1)^{(9)} = F^{h4} \). In a similar way we can treat the case \( \beta_1 = \beta_2 = 1 \).

6 Higgs sector, global symmetries, proton stability, \( N = 1 \) SUSY on intersections and neutrino masses

6.1 Stability of the configurations and Higgs sector

We have so far seen the appearance in the R-sector of \( I_{ab} \) massless fermions in the D-brane intersections transforming under bifundamental representations \( N_a, \bar{N}_b \). In intersecting brane words, besides the actual presence of massless fermions at each intersection, we have evident the presence of an equal number of massive bosons, in the NS-sector, in the same representations as the massless fermions \( [17] \). Their mass is of order of the string scale and it should be taken into account when examining phenomenological applications related to the renormalization group equations. However, it is possible that some of those massive bosons may become tachyonic \( [12] \), especially when their mass, that depends on the angles between the branes, is such that is decreases the world volume of the 3-cycles involved in the recombination process of joining the two branes into a single one \( [33] \). Denoting the twist vector by \( (\vartheta_1, \vartheta_2, \vartheta_3, 0) \), in the NS open string sector the lowest lying states are given by \( [13] \)

| State                     | Mass                      |
|---------------------------|---------------------------|
| \((-1 + \vartheta_1, \vartheta_2, \vartheta_3, 0)\) | \( \alpha' M^2 = \frac{1}{2}(-\vartheta_1 + \vartheta_2 + \vartheta_3) \) |
| \((\vartheta_1, -1 + \vartheta_2, \vartheta_3, 0)\) | \( \alpha' M^2 = \frac{1}{2}(\vartheta_1 - \vartheta_2 + \vartheta_3) \) |
| \((\vartheta_1, \vartheta_2, -1 + \vartheta_3, 0)\) | \( \alpha' M^2 = \frac{1}{2}(\vartheta_1 + \vartheta_2 - \vartheta_3) \) |
| \((-1 + \vartheta_1, -1 + \vartheta_2, -1 + \vartheta_3, 0)\) | \( \alpha' M^2 = 1 - \frac{1}{2}(\vartheta_1 + \vartheta_2 + \vartheta_3) \) |

\( [12] \) For consequences when these set of fields may become massless see \( [22] \).

\( [13] \) we assume \( 0 \leq \vartheta_i \leq 1 \).

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Exactly at the point, where one of these masses may become massless we have preservation of $\mathcal{N} = 1$ SUSY. We note that the angles at the four different intersections can be expressed in terms of the parameters of the tadpole solutions.

- **Angle structure and Higgs fields for PS-II classes of models**

  The angles at the different intersections can be expressed in terms of the tadpole solution parameters. We define the angles:

  \[
  \theta_1 = \frac{1}{\pi} \cot^{-1} \frac{R_{11}}{\epsilon m_b R_{21}}; \quad \theta_2 = \frac{1}{\pi} \cot^{-1} \frac{\epsilon \tilde{e} \tilde{n}_a^2 R_{11}^{(2)}}{3 \epsilon \tilde{\beta}_2 R_{22}^{(2)}}; \quad \theta_3 = \frac{1}{\pi} \cot^{-1} \frac{2 R_{11}^{(3)}}{R_{22}^{(3)}}; \\
  \bar{\theta}_1 = \frac{1}{\pi} \cot^{-1} \frac{R_{11}}{\epsilon m_c R_{22}^{(1)}}; \quad \bar{\theta}_2 = \frac{1}{\pi} \cot^{-1} \frac{\epsilon \tilde{e} \tilde{n}_d^2 R_{11}^{(1)}}{4 \beta_2 R_{22}^{(1)}}; \quad \bar{\theta}_2 = \frac{1}{\pi} \cot^{-1} \frac{\epsilon \tilde{e} \tilde{n}_e^2 R_{11}^{(1)}}{2 \beta_2 R_{22}^{(1)}} \tag{6.2}
  \]

  where we consider $\epsilon \tilde{e} > 0$, $\epsilon m_b > 0$, $\epsilon m_c > 0$ and $R_{i}^{(j)}$, $i = 1, 2$ are the compactification radii for the three $j = 1, 2, 3$ tori, namely projections of the radii onto the cartesian axis $X^{(i)}$ directions when the NS flux B field, $b^k$, $k = 1, 2$ is turned on.

  At each of the six non-trivial intersections we have the presence of four states $t_i$, $i = 1, \ldots, 4$, that could become massless, associated to the states (6.1). Hence we have a total of twenty four different scalars in the model. The setup is seen clearly if we look at figure one. These scalars are generally massive but for some values of their angles could become tachyonic (or massless).

  Also, if we demand that the scalars associated with (3.1) and PS-II models may not be tachyonic, we obtain a total of eighteen conditions for the PS-II type models with a D6-brane at angles configuration to be stable. They are given in Appendix I. We don’t consider the scalars from the $aa^*, dd^*, ee^*$ intersections. For these sectors we will require later that they preserve $N = 1$ SUSY. As a result all scalars in these sectors may become massive or receive vevs and becoming eventually massive.

  Lets us now turn our discussion to the Higgs sector of PS-II models. In general there are two different Higgs fields that may be used to break the PS symmetry. We remind that they were given in (3.3). The question is if $H_1, H_2$ are present in the spectrum of PS-II models. In general, tachyonic scalars stretching between two different branes $\tilde{a}, \tilde{b}$, can be used as Higgs scalars as they can become non-tachyonic by varying the distance between the branes. Looking at the $I_{ac^*}$ intersection we can confirm that the scalar doublets $H^\pm$ get localized. They come from open strings stretching between the $U(4)$ $a$-brane and $U(2)_R c^*$-brane.
Figure 1: Assignment of angles between D6-branes on the PS-II class of models based on the initial gauge group $U(4)_C \times U(2)_L \times U(2)_R$. The angles between branes are shown on a product of $T^2 \times T^2 \times T^2$. We have chosen $m_b^1, m_e^1, n_d^2, n_e^2 > 0, \epsilon = \bar{\epsilon} = 1$. These models break to low energies to exactly the SM.
Table 4: Higgs fields responsible for the breaking of $SU(4) \times SU(2)_R$ symmetry of the $SU(4)_C \times SU(2)_L \times SU(2)_R$ with D6-branes intersecting at angles. These Higgs are responsible for giving masses to the right handed neutrinos in a single family.

| Intersection | PS breaking Higgs | $Q_a$ | $Q_b$ | $Q_c$ | $Q_d$ |
|--------------|-------------------|------|------|------|------|
| $ac^*$       | $H_1$             | 1    | 0    | 1    | 0    |
| $ac^*$       | $H_2$             | -1   | 0    | -1   | 0    |

The $H^\pm$'s come from the NS sector and correspond to the states

\[
\begin{align*}
\text{State} & & \text{Mass}^2 \\
(-1 + \vartheta_1, \vartheta_2, 0, 0) & & \alpha' (\text{Mass})^2_{H^+} = \frac{Z_3}{4\pi^2} + \frac{1}{2}(\vartheta_2 - \vartheta_1) \\
(\vartheta_1, -1 + \vartheta_2, 0, 0) & & \alpha' (\text{Mass})^2_{H^-} = \frac{Z_3}{4\pi^2} + \frac{1}{2}(\vartheta_1 - \vartheta_2)
\end{align*}
\]

where $Z_3$ is the distance$^2$ in transverse space along the third torus, $\vartheta_1$, $\vartheta_2$ are the (relative) angles between the $a$-, $c^*$-branes in the first and second complex planes respectively. The presence of scalar doublets $H^\pm$ can be seen as coming from the field theory mass matrix

\[
(H_1^* H_2^m) \begin{pmatrix} M^2 & H_1 \\ H_2^* & \end{pmatrix} + \text{h.c.}
\]

where

\[
M^2 = M_s^2 \begin{pmatrix} Z_3^{(ac^*)} (4\pi^2)^{-1} & \frac{1}{2}\vartheta_1^{(ac^*)} - \vartheta_2^{(ac^*)} \\ \frac{1}{2}\vartheta_1^{(ac^*)} - \vartheta_2^{(ac^*)} & Z_3^{(ac^*)} (4\pi^2)^{-1} \end{pmatrix},
\]

The fields $H_1$ and $H_2$ are thus defined as

\[
H^\pm = \frac{1}{2}(H_1^* \pm H_2)
\]

where their charges are given in table (4). Hence the effective potential which corresponds to the spectrum of the PS symmetry breaking Higgs scalars is given by

\[
V_{Higgs} = m_H^2 (|H_1|^2 + |H_2|^2) + (m_B^2 H_1 H_2 + \text{h.c})
\]

where

\[
m_H^2 = \frac{Z_3^{(ac^*)}}{4\pi^2 \alpha'}; \quad m_B^2 = \frac{1}{2\alpha'} |\vartheta_1^{(ac^*)} - \vartheta_2^{(ac^*)}|
\]

\footnote{a similar set of states was used in \cite{3} to provide the model with electroweak Higgs scalars.}
| Intersection | Higgs | $Q_a$ | $Q_b$ | $Q_c$ | $Q_d$ |
|--------------|-------|-------|-------|-------|-------|
| $bc^*$       | $h_1 = (1, 2, 2)$ | 0     | 1     | 1     | 0     |
| $bc^*$       | $h_2 = (1, 2, 2)$ | 0     | -1    | -1    | 0     |

Table 5: Higgs fields present in the intersection $bc^*$ of the $SU(4)_C \times SU(2)_L \times SU(2)_R$ classes of models with D6-branes intersecting at angles. These Higgs give masses to the quarks and leptons in a single family and could have been responsible for electroweak symmetry breaking if their net number was not zero.

The precise values of $m_H^2$, $m_B^2$, for PS-II classes of models are given by

$$m_H^2 \overset{PS-II}{=} \frac{(\xi'_a + \xi'_c)^2}{\alpha'}, \quad m_B^2 \overset{PS-II}{=} \frac{1}{2\alpha'} \left| \frac{1}{2} + \tilde{\theta}_1 - \tilde{\theta}_2 \right|, \quad (6.9)$$

where $\xi'_a(\xi'_c)$ is the distance between the orientifold plane and the $a(c)$ branes and $\tilde{\theta}_1$, $\tilde{\theta}_2$ were defined in (6.2). Thus

$$m_B^2 \overset{PS-II}{=} \frac{1}{2} \left| m_{h_R}^2(t_2) + m_{h_R}^2(t_3) - m_{h_L}^2(t_1) - m_{h_L}^2(t_3) \right|$$

$$= \frac{1}{2} \left| m_{h_R}^2(t_2) + m_{h_R}^2(t_3) - m_{F_R}^2(t_1) - m_{F_R}^2(t_3) \right| \quad (6.10)$$

For PS-II models the number of Higgs present is equal to the intersection number product between the $a(c)$, $c^*$-branes in the first and second complex planes,

$$n_{Hz} \overset{PS-II}{=} I_{ac^*} = 3. \quad (6.11)$$

A comment is in order. For PS-II models the number of PS Higgs is three. That means that we have three intersections and to each one we have a Higgs particle which is a linear combination of the Higgs $H_1$ and $H_2$.

The electroweak symmetry breaking could be delivered through the bidoublets Higgs present in the $bc^*$ intersection (seen in table (5)). In principle these can be used to give mass to quarks and leptons. In the present models their number is given by the intersection number of the $b$, $c^*$ branes in the first tori

$$n_{h_1, h_2}^{bc^*} \overset{PS-II}{=} |\epsilon(m_c^1 - m_b^1)| = |\beta^2(2n_a^2 + n_d^1 - n_c^2)| \quad (6.12)$$

A comment is in order. Because the number of the electroweak bidoublets in the PS-II models depends on the difference $|m_b^1 - m_c^1|$, given the conditions for N=1 SUSY in
Table 6: Higgs fields present in the intersection $bc$ of the $SU(4)_C \times SU(2)_L \times SU(2)_R$ classes of models with D6-branes intersecting at angles. These Higgs give masses to the quarks and leptons in a single family and are responsible for electroweak symmetry breaking.

| Intersection | Higgs      | $Q_a$ | $Q_b$ | $Q_c$ | $Q_d$ |
|--------------|------------|-------|-------|-------|-------|
| $bc$         | $h_3 = (1, 2, 2)$ | 0     | -1    | 1     | 0     |
| $bc$         | $h_4 = (1, 2, 2)$ | 0     | 1     | -1    | 0     |

some sectors in the models (see (7.13), (7.14) at next section), we get $n_{h^\pm} = 0$ and thus $m_b^1 = m_c^1$. However, this is not a problem for electroweak symmetry breaking as (see section 8) a different term is used to provide Dirac masses to quarks, leptons and neutrinos. In the present models is it important that

$$I_{bc} = |m_c^1 + m_b^1| = 2|m_b^1|$$

(6.13)

may be chosen different from zero. Thus an alternative set of electroweak Higgs may be provided from the the NS sector where the lightest scalar states $h^\pm$ originate from open strings stretching between the $bc$ branes, e.g. named as $h_3$, $h_4$.

$$\begin{align*}
\text{State} & \quad \alpha'(\text{Mass})^2 \quad \text{Mass}^2 \\
(-1 + \vartheta_1, 0, \vartheta_3, 0) & \quad = \frac{Z_{bc}^2}{4\pi^2} + \frac{i}{2}(\vartheta_3 - \vartheta_1) \\
(\vartheta_1, 0, -1 + \vartheta_3, 0) & \quad = \frac{Z_{bc}^2}{4\pi^2} + \frac{i}{2}(\vartheta_1 - \vartheta_3)
\end{align*}$$

(6.14)

where $Z_{bc}^2$ is the relative distance in transverse space along the second torus from the orientifold plane, $\vartheta_1$, $\vartheta_3$, are the (relative) angle between the $b$-, $c$-branes in the first and third complex planes.

Hence the presence of scalar doublets $h^\pm$ defined as

$$h^\pm = \frac{1}{2}(h_3^* \pm h_4^*)$$

(6.15)

can be seen as coming from the field theory mass matrix

$$\begin{pmatrix} h_3^* \\ h_4^* \end{pmatrix} \begin{pmatrix} M^2 \end{pmatrix} \begin{pmatrix} h_3 \\ h_4 \end{pmatrix} + h.c.$$ 

(6.16)

where

$$M^2 = M_s^2 \begin{pmatrix} 2Z_{23}^{(bc)}(4\pi^2)^{-1} & \frac{1}{2}|\vartheta_1^{(bc)} - \vartheta_3^{(bc)}| \\ \frac{1}{2}|\vartheta_1^{(bc)} - \vartheta_3^{(bc)}| & Z_{23}^{(bc)}(4\pi^2)^{-1} \end{pmatrix}$$

(6.17)
The effective potential which corresponds to the spectrum of electroweak Higgs $h_3$, $h_4$ may be written as

$$V_{	ext{Higgs}}^{bc} = m_H^2(|h_3|^2 + |h_4|^2) + (m_B^2 h_3 h_4 + h.c) \quad (6.18)$$

where

$$m_H^2 = \frac{Z_2^{(bc)}}{4\pi^2\alpha'} \, \, ; \, \, m_B^2 = \frac{1}{2\alpha'}|\vartheta_1^{(bc)} - \vartheta_3^{(bc)}| \quad (6.19)$$

The precise values for PS-II classes of models $m_H^2, m_B^2$ are

$$\bar{m}_H^{PS-II} = \left( \tilde{\vartheta}_B^{(2)} + \tilde{\vartheta}_C^{(2)} \right)^2 \, \, ; \, \, \bar{m}_B^{PS-II} = \frac{1}{2\alpha'}|\theta_1 - \tilde{\theta}_1 - 2\theta_3| \quad (6.20)$$

where $\theta_1, \tilde{\theta}_1, \theta_3$ were defined in (6.2). Also $\tilde{\vartheta}_B, \tilde{\vartheta}_C$ are the distances of the $b$, $c$ branes from the orientifold plane in the second tori. The values of the angles $\vartheta_1, \tilde{\vartheta}_1, \tilde{\vartheta}_2$, can be expressed in terms of the scalar masses in the various intersections. We list them for convenience

$$\frac{1}{\pi} \theta_1 = \frac{1}{2} |1 - m^2_{ab}(t_2) + m^2_{ab'}(t_3)|$$
$$= \frac{1}{2} |1 + m^2_{be}(t_2) + m^2_{be}(t_3)|$$
$$= \frac{1}{2} |1 - m^2_{bd}(t_2) + m^2_{bd}(t_3)| \quad (6.21)$$

$$\frac{1}{\pi} \tilde{\theta}_1 = \frac{1}{2} |1 - m^2_{ac}(t_2) - m^2_{ac}(t_3)|$$
$$= \frac{1}{2} |1 - m^2_{cd}(t_2) + m^2_{cd}(t_3)|$$
$$= \frac{1}{2} |1 + m^2_{ce}(t_2) + m^2_{ce}(t_3)| \quad (6.22)$$

$$\frac{1}{\pi} \theta_2 = \frac{1}{2} |m^2_{ab}(t_1) + m^2_{ab'}(t_3)|$$
$$= \frac{1}{2} |m^2_{ac}(t_1) + m^2_{ac}(t_3)| \quad (6.23)$$

$$\frac{1}{\pi} \theta_3 = \frac{1}{4} |m^2_{\chi_L}(t_1) + m^2_{\chi_L}(t_2)| = \frac{1}{4} |m^2_{\chi_R}(t_1) + m^2_{\chi_R}(t_2)|$$
$$= \frac{1}{4} |m^2_{\chi_L}(t_1) + m^2_{\chi_L}(t_2)| = \frac{1}{4} |m^2_{\chi_R}(t_1) + m^2_{\chi_R}(t_2)|$$
$$= \frac{1}{4} |m^2_{\chi_L}(t_1) + m^2_{\chi_L}(t_2)| = \frac{1}{4} |m^2_{\chi_R}(t_1) + m^2_{\chi_R}(t_2)| \quad (6.24)$$
7 Singlet scalar generation - $N = 1$ SUSY on Intersections

In this section, we intend to demand that certain open string sectors respect $N = 1$ supersymmetry. In particular we will focus in showing how we can create sectors which preserve $N=1$ supersymmetry in non-supersymmetric intersecting D6-brane models. This is most interesting for the good phenomenology of the models, as SUSY sectors guarantee the generation of singlets. The singlet scalars will be necessary for giving masses to the extra $U(1)$ gauge bosons which they don’t have any couplings to the RR fields and also realizing a majotana mass term for the right handed neutrinos. We note that the spectrum of PS-II classes of models described in table (I) is massless at this point. Thus supersymmetry will create singlet scalars which receive vevs and generate masses for the otherwise massless fermions $\chi_1^L, \chi_2^L, \chi_1^R, \chi_2^R, \omega_L, y_R, s_1^R, s_2^R$. For the status of vevs in the context of intersecting branes see a relevant comment on the concluding section.

Before presenting the analysis, let us note that a Majorana mass term for right neutrinos appears only once we impose $N = 1$ SUSY on an intersection. That will have as an effect the appearance of the massless scalar superpartners of the $\bar{F}_R^H$ fermions, the $\bar{F}_R^H$’s, thus allowing a dimension 5 Majorana mass term for $\nu_R, F_R F_R \bar{F}_R^H \bar{F}_R^H$. The see-saw mechanism for the Majorana neutrinos will be discussed in section 8.

7.1 PS-II models with $N=1$ SUSY

In this part we will show that model dependent conditions, obtained by demanding that the extra $U(1)$’s do not have non-zero couplings to the RR fields, are necessary conditions in order to have scalar singlet generation that could effectively break the extra $U(1)$’s. These conditions will be alternatively obtained by demanding that certain string sectors respect $N = 1$ supersymmetry.

In general, for $N = 1$ supersymmetry to be preserved at some intersection between two branes $L, M$, we need to satisfy $\pm \vartheta_{ab}^1 \pm \vartheta_{ab}^2 \pm \vartheta_{ab}^3$ for some choice of signs, where $\vartheta_{i, \alpha \beta}, i = 1, 2, 3$ are the relative angles of the branes $L, M$ across the three 2-tori. The latter rule will be our main tool in getting $N=1$ SUSY on intersections.

- The $ac$ sector respects $\mathcal{N} = 1$ supersymmetry.

\[15\text{Without } N=1 \text{ SUSY this coupling would have been absent and thus the models useless for good phenomenology as in the base case the right handed neutrinos would survive massless to low energy. See also } \footnote{4} \text{ and section (8.2).} \]
The condition for $N = 1$ SUSY on the $ac$-sector is

$$\pm \left( \frac{\pi}{2} + \tilde{\vartheta}_1 \right) \pm \vartheta_2 \pm 2\vartheta_3 = 0, \quad (7.1)$$

This condition can be solved by choosing:

$$ac \rightarrow \left( \frac{\pi}{2} + \tilde{\vartheta}_1 \right) + \vartheta_2 - 2\vartheta_3 = 0, \quad (7.2)$$

and thus may be solved by the choice

$$- \tilde{\vartheta}_1 = \vartheta_2 = \vartheta_3 = \frac{\pi}{4}, \quad (7.3)$$

effectively giving us

$$- \frac{1}{\epsilon m_c^1 U^{(1)}} = \frac{(\epsilon \tilde{\epsilon}) n_d^2}{3\beta_2 U^{(2)}} = \frac{2\tilde{\epsilon}}{U^{(3)}} = \frac{\pi}{4}. \quad (7.4)$$

By imposing $N = 1$ SUSY on an intersection $ac$ the massless scalar superpartner of $\tilde{F}_R$ appears, the $\tilde{F}_R^B$. Note that in (7.4) the imposition of N=1 SUSY connects the complex structure moduli $U^i$ in the different tori and thus reduces the moduli degeneracy of the theory.

- The $dd^*$ sector preserves $\mathcal{N} = 1$ supersymmetry

As we noted in the appendix the presence of N=1 supersymmetry in the sectors $dd^*, ee^*$ is equivalent to the absence of tachyons in those sectors.

The general form of the $\mathcal{N} = 1$ supersymmetry condition on this sector is

$$\pm \pi \pm 2\tilde{\vartheta}_2 \pm 2\vartheta_3 = 0, \quad (7.5)$$

which may be solved by the choice

$$\pi + 2\tilde{\vartheta}_2 - 2\vartheta_3 = 0, \quad (7.6)$$

Hence

$$- \tilde{\vartheta}_2 = \vartheta_3 = \frac{\pi}{4}, \quad (7.7)$$

that is

$$- \frac{\epsilon \tilde{\epsilon} n_d^2}{4\beta_2 U^{(2)}} = \frac{2}{\tilde{\epsilon}} U^{(3)} = \frac{\pi}{4}. \quad (7.8)$$

$^{16}$We have chosen $m_c^1 < 0$.

$^{17}$We have set $U^{(i)} = n_d^{(i)} \tilde{\xi}^{(i)}$, $i = 1, 2, 3$. 

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• The ee* sector preserves $\mathcal{N} = 1$ supersymmetry

The general form of the $\mathcal{N} = 1$ supersymmetry condition on this sector is

$$\pm \pi \pm 2\vartheta_2 \pm 2\vartheta_3 = 0, \quad (7.9)$$

which we may recast in the form

$$- \pi + 2\vartheta_2 + 2\vartheta_3 = 0, \quad (7.10)$$

be solved by the choice

$$\vartheta_2 = \vartheta_3 = \frac{\pi}{4}, \quad (7.11)$$

that is

$$\frac{(\epsilon\bar{\epsilon})n_e^2}{2\beta_2 U^{(2)}} = \frac{2}{\epsilon} U^{(3)} = \frac{\pi}{4}. \quad (7.12)$$

From (7.12), (7.8), (7.4), we derive the conditions

$$2n_a^2 = 3n_e^2, \quad (7.13)$$

$$n_d^2 = -2n_e^2. \quad (7.14)$$

An important comment is in order. The presence of N=1 supersymmetry in $dd^*$, $ee^*$ sectors signals the presence of the scalar superpartners of $s^1_R$, $s^2_R$, namely the $s^1_B$, $s^2_B$ respectively. The latter scalars may receive vevs. Thus imposing N=1 SUSY in leptonic sectors guarantee the presence of gauge singlets in the models.

Also what is is evident by looking at conditions (4.6), (7.13), (7.14) is that the conditions of orthogonality for the extra $U(1)$’s to survive massless the generalized Green-Schwarz mechanism is equivalent to the conditions for N=1 supersymmetry in the leptonic sectors $dd^*$, $ee^*$. The latter condition is equivalent to the absence of tachyons in the sectors $dd^*$, $ee^*$ as someone might check.

A numerical set of wrapping numbers consistent with the RR tadpole constraints (4.6), (7.13) and (7.14) is

$$\epsilon = \bar{\epsilon} = 1, \quad n_a^2 = 15, \quad m_b^1 = 1, \quad m_c^1 = 1, \quad n_d^2 = -20, \quad n_e^2 = 10, \quad \beta_1 = 1, \quad \beta_2 = 1/2. \quad (7.15)$$

The latter can be satisfied with the addition of four extra U(1) D6-branes.

We note that the constraint (4.6) is independent from the number of extra $U(1)$’s added to satisfy the RR tadpole conditions. Thus we may safely choose $\beta = 1, \beta = 1/2$ that is $N_h = 4$ in table (4) positioned at (1.4). We note that the entries of the d-brane in the second tori is such that it appears to be corresponding to a U(2) brane wrapping
Table 7: Wrapping number set consistent with the tadpole constraint (4.6) and the \( N = 1 \) SUSY preserving conditions (7.13), (7.14). The SUSY conditions can be shown to be equivalent to the model dependent conditions for \( U(1)'s \) surviving massless the generalized Green-Schwarz mechanism (see section 5). We have not included the extra \( U(1) \) branes.

| \( N_i \) | \( (n_i^1, m_i^1) \) | \( (n_i^2, m_i^2) \) | \( (n_i^3, m_i^3) \) |
|---------|-----------------|-----------------|-----------------|
| \( N_a = 4 \) | \( (0, 1) \) | \( (15, 3/2) \) | \( (1, 1/2) \) |
| \( N_b = 2 \) | \( (-1, 1) \) | \( (2, 0) \) | \( (1, 1/2) \) |
| \( N_c = 2 \) | \( (1, 1) \) | \( (2, 0) \) | \( (1, -1/2) \) |
| \( N_d = 1 \) | \( (0, 1) \) | \( (-20, -2) \) | \( (2, 1) \) |
| \( N_e = 1 \) | \( (0, 1) \) | \( (10, -1) \) | \( (2, -1) \) |

Once around or a \( U(1) \) brane wrapping twice along the one-cycle on the 2nd tori. However, as has been noted before this is not a problem since a multiwrapping of this form can be absorbed in a \( U(1)_d \) field redefinition in e.g. the anomaly free \( U(1)^{(4)} \), that survives massless the generalized Green-Schwarz mechanism in section 5.

### 7.2 Gauge singlet generation from the extra \( U(1) \) branes

In this section, we will present an alternative mechanism for generating singlet scalars. We had already seen that in leptonic sectors involving \( U(1) \) branes, e.g. \( dd^*, ee^* \), brane imposing \( N=1 \) SUSY creates singlet scalars. This is reflected in the fact that in \( U(1) \) \( j \)-branes, sectors in the form \( jj^* \) had localized in their intersection gauge singlet fermions. Thus imposing \( N=1 \) SUSY on those sectors help us to get rid of these massless fermions, by making them massive through their couplings to their superpartner gauge singlet scalars.

What we will become clear in this sector is that the presence of supersymmetry in particular sectors involving the extra branes creates singlet scalars that provide the couplings that make massive some non-SM fermions.

In order to show the creation of gauge singlets from sectors involving extra branes we will make our points by using only one of the extra \( N_h U(1) \) branes, e.g. the \( N_{h_1} \)

\(^{18}\)see the first reference of [8]
one. The following discussion can be identically repeated for the other extra branes.

Thus due to the non-zero intersection numbers of the $N_{h_1}$ U(1) brane with $a,d$ branes the following sectors are present: $ah, ah^*, dh, dh^*$.

- **$ah$-sector**

  Because $I_{ah} = -\frac{3}{\beta_1}$ we have present $|I_{ah}|$ massless fermions $\kappa_1^f$ in the representations

  \[ \kappa_1^f \rightarrow (\bar{4}, 1, 1)_{(-1, 0, 0, 0, 1)} \]  

  (7.16)

  where the subscript last entry denotes the U(1) charge of the ‘sixth’ U(1) extra brane.

- **$ah^*$-sector**

  Because $I_{ah^*} = -\frac{3}{\beta_1} < 0$, there are present $|I_{ah^*}|$ fermions $\kappa_2^f$ appearing as a linear combination of the representations

  \[ \kappa_2^f \rightarrow (\bar{4}, 1, 1)_{(-1, 0, 0, 0, -1)} dh^2 \]  

  (7.17)

- **$dh$-sector**

  Because $I_{dh} = \frac{8}{\beta_1}$, there are present $|I_{dh}|$ fermions $\kappa_3^f$ transforming in the representations

  \[ \kappa_3^f \rightarrow (1, 1, 1)_{(0, 0, 1, 0, -1)} \]  

  (7.18)

  We further require that this sector respects $N = 1$ supersymmetry. In this case we have also present the massless scalar fields $\kappa_3^B$, \n
  \[ \kappa_3^B \rightarrow (1, 1, 1)_{(0, 0, 1, 0, -1)}, laradh^5 \]  

  (7.19)

  The latter scalars receive a vev which we assume to be of order of the string scale. The condition for $N = 1$ supersymmetry in this sector is exactly

  \[ -\frac{\pi}{2} + \tilde{\vartheta}_2 + (\vartheta_3) = 0 \]  

  (7.20)

  which is satisfied when $\tilde{\vartheta}_2, \vartheta_3$ take the value $\pi/4$ in consistency with (7.10) and subsequently (7.10).  

\footnote{We don’t exhibit the beyond the sixth entry of the rest of the extra branes as they are obviously zero for the present discussion.}
• \( dh^\ast\)-sector

Because \( I_{dh^\ast} = \frac{8}{\beta_1} \neq 0 \), there are present \( |I_{ah^\ast}| \) fermions \( \kappa_4^f \) in the representations

\[
\kappa_4^f \rightarrow (1, 1, 1)_{(0,0,0,1,0,1)} l dh4
\]  

(7.21)

The condition that this sector respects N=1 SUSY is equivalent to the one is the \( dh\)-sector.

• \( eh\)-sector

Because \( I_{eh} = -\frac{4}{\beta_1} \), there are present \( |I_{eh}| \) fermions \( \kappa_5^f \) transforming in the representations

\[
\kappa_5^f \rightarrow (1, 1, 1)_{(0,0,0,0,-1,1)}
\]  

(7.22)

Also we require that this sector preserves N=1 SUSY. Because of N=1 SUSY and \( I_{eh} = -\frac{4}{\beta_1} \), there are present \( |I_{eh}| \) bosons \( \kappa_5^B \) transforming in the representations

\[
\kappa_5^B \rightarrow (1, 1, 1)_{(0,0,0,0,-1,1)}
\]  

(7.23)

The condition for N=1 SUSY is

\[
\pm \frac{\pi}{2} \pm \bar{\vartheta}_2 \pm \vartheta_3 = 0
\]  

(7.24)

which is exactly ‘half’ of the supersymmetry condition (7.34). When it is rearranged into the form

\[
\frac{\pi}{2} + \bar{\vartheta}_2 - \vartheta_3 = 0,
\]  

(7.25)

it is solved by the choice (7.11).

Summarizing we have found that the conditions (7.13, 7.14) derived as the model dependent conditions of the U(1)’s that survive the generalized Green-Schwarz mechanism, are equivalent:

• to have the leptonic branes, d, e, preserve N=1 SUSY on the sectors \( dd^\ast, ee^\ast \).

• to have the sectors made of a mixture of the extra and leptonic branes preserve N=1 SUSY. The presence of these conditions is independent from the number of extra U(1) branes present.

• \( eh^\ast\)-sector In this sector, \( I_{eh^\ast} = -\frac{4}{\beta_1} \). Thus there are present \( |I_{eh^\ast}| \) fermions \( \kappa_6^f \) transforming in the representations

\[
\kappa_6^f \rightarrow (1, 1, 1)_{(0,0,0,0,0,-1,-1)}
\]  

(7.26)
The condition for N=1 SUSY to be preserved by this section is exactly (7.25). Thus we have present $|L_{eh}|$ bosons $\kappa_6^B$ transforming in the representations

$$\kappa_6^B \rightarrow (1, 1, 1)(0, 0, 0, -1; -1)$$

(7.27)

We will now show that all fermions, appearing from the non-zero intersections of the extra brane $U(N_h)$ with the branes $a, d, e$, receive string scale mass and disappear from the low energy spectrum (see also a related discussion in the concluding section).

• The mass term for the $\kappa_1^f$ fermion reads:

$$(4, 1, 1)_{(1, 0, 0, 0, 0; -1)} \times \langle \bar{\kappa}_1^f \rangle \langle F_R^H \rangle \langle \kappa_5^B \rangle \sim \bar{\kappa}_1^f \kappa_1^f M_s$$

(7.29)

• The mass term for the $\kappa_2^f$ fermion reads:

$$(4, 1, 1)_{(1, 0, 0, 0, 0; 1)} \times \langle \bar{\kappa}_2^f \rangle \langle F_R^H \rangle \langle \kappa_6^B \rangle \sim \bar{\kappa}_2^f \kappa_2^f M_s$$

(7.31)

• The mass term for the $\kappa_3^f$ fermion reads:

$$(1, 1, 1)_{(0, 0, 0, -1, 0; 1)} \times \langle \bar{\kappa}_3^f \rangle \langle \kappa_3^B \rangle \sim \bar{\kappa}_3^f \kappa_3^f M_s$$

(7.33)

• The mass term for the $\kappa_4^f$ fermion reads:

$$(1, 1, 1)_{(0, 0, 0, -1, 0; -1)} \times \langle \bar{\kappa}_4^f \rangle \langle \kappa_4^B \rangle \sim \bar{\kappa}_4^f \kappa_4^f M_s$$

(7.35)
• The mass term for the \( \kappa_f^5 \) fermion reads:

\[
(1, 1, 1)_{(0, 0, 0, 0, -1)} \langle (1, 1, 1)_{(0, 0, -1, 0, 0)} \rangle \langle (1, 1, 1)_{(0, 0, -1, 0, 1)} \rangle \tag{7.36}
\]

or

\[
\bar{\kappa}_f^5 \kappa_f^5 \langle \kappa_f^B \rangle \langle \kappa_f^B \rangle \sim M_s \bar{\kappa}_f^5 \kappa_f^5 \tag{7.37}
\]

• The mass term for the \( \kappa_f^6 \) fermion reads:

\[
(1, 1, 1)_{(0, 0, 0, 1)} \langle (1, 1, 1)_{(0, 0, 0, -1)} \rangle \langle (1, 1, 1)_{(0, 0, 0, -1, 0, 1)} \rangle \tag{7.38}
\]

or

\[
\bar{\kappa}_f^6 \kappa_f^6 \langle \kappa_f^B \rangle \langle \kappa_f^B \rangle \sim M_s \bar{\kappa}_f^6 \kappa_f^6 \tag{7.39}
\]

### 7.3 Breaking the anomaly free massless \( U(1) \)'s

As in the standard version of a left-right Pati-Salam \( SU(4) \times SU(2)_L \times SU(2)_R \) model, if the neutral component of \( H_1 \) (resp. \( H_2 \), \( \nu_H \), acquires a vev, e.g. \( \langle \nu_H \rangle \), then the initial gauge symmetry, \( SU(4) \times SU(2)_L \times SU(2)_R \times U(1)_a \times U(1)_b \times U(1)_c \times U(1)_d \), can break to the standard model gauge group \( SU(3) \times U(2) \times U(1)_Y \) augmented by the extra, non-anomalous, \( U(1) \)'s, \( Q^{(4)}, Q^{(5)}, Q^{(6)}, Q^{(7)} \). Note that we have considered for simplicity the possibility of two extra \( U(1) \) branes. For convenience the following discussion will focus on one of the extra branes, as identical results hold for the other \( U(1) \) brane.

The extra \( U(1) \)'s may be broken by appropriate Higgsing. In the PS-A, PS-I models, by imposing SUSY on the sectors \( dd^*, dh, dh^* \) we made it possible to generate the appearance of the scalar superpartners of the fermions appearing on the respecting intersections. In the present models the sectors by preserving \( N = 1 \) SUSY on sectors \( dd^*, ee^*, dh, dh^* \) we have available the singlets responsible for breaking the \( U(1) \)'s that survive massless the Green-Schwarz mechanism. Thus, looking at (5.8), \( U(1)^{(4)} \) may break if \( s_B^1 \) gets a vev, \( U(1)^{(7)} \) may break if \( s_B^2 \) gets a vev, \( U(1)^{(5)} \) may break if \( \kappa_f^B \) (or one of the \( \kappa_f^B, \kappa_f^B, \kappa_f^B \)) receives a vev. Also the extra \( U(1)^{(6)} \) brane is treated in the same way as \( U(1)^{(5)} \).

Note that in this case the extra non-anomalous \( U(1) \)'s have some important phenomenological properties. In particular they do not charge the PS symmetry breaking Higgs scalars \( H_1, H_2 \) thus avoiding the appearance of axions.
We note that up to this point the only issue remaining is how we can give non-zero masses to all exotic fermions of table (1) beyond those that accommodate the quarks and leptons of the SM.

8 Geometrical Yukawa couplings and lepton masses

In this section, we will examine the mechanism of generating neutrino masses in the $SU(4) \times S(2)_L \times SU(2)_R$ classes of PS-II GUTS. Also we examine some aspects of the geometry of the Yukawa couplings. Particular emphasis is given to the exhibition of the couplings giving masses to all the fermions appearing in table 1, beyond those making the quarks and lepton structure.

8.1 Yukawa couplings

Proton decay is one of the most important problems of grand unifies theories. In the standard versions of left-right symmetric PS models this problem is avoided as B-L is a gauged symmetry but the problem persists in baryon number violating operators of sixth order, contributing to proton decay. In the PS-I models proton decay is absent as baryon number survives as a global symmetry to low energies. That provides for an explanation for the origin of proton stability in general brane-world scenarios. Clearly $Q_a = 3B + L$ and the baryon $B$ is given by

$$B = \frac{Q_a + Q_{B-L}}{4}. \quad (8.1)$$

In intersecting brane worlds the usual tree level SM fermion mass generating trilinear Yukawa couplings between the fermion states $F^i_L$, $\bar{F}^j_R$ and the Higgs fields $H^k$ arise from the stretching of the worldsheet between the three D6-branes which cross at those intersections. In the Pati-Salam GUTS we examine, the trilinear Yukawa is

$$Y^{ijk} F^i_L \bar{F}^j_R H^k \quad (8.2)$$

Its general form for a six dimensional torus is in the leading order [7],

$$Y^{ijk} = e^{-\tilde{A}_{ijk}}, \quad (8.3)$$

where $\tilde{A}_{ijk}$ is the worldsheet area connecting the three vertices. The areas of each of the two dimensional torus involved in this interaction is typically of order one in string
units. In [3] we have assumed that the areas of the second and third tori are close to zero. In this case, the area of the full Yukawa coupling (8.3) was given in the form

$$Y^{ijk} = e^{-\frac{R_1 R_2}{a'} A_{ijk}},$$

(8.4)

where $R_1$, $R_2$ the radii and $A_{ijk}$ the area of the two dimensional tori in the first complex plane. Here we exhibit the leading worldsheet correction coming from the first tori, as it holds for any PS GUT model constructed as a deformation of the quark and lepton intersection numbers, e.g. PS-A, PS-I classes in [3] and the present PS-II classes of models.

Let us analyze a bit further the relation (8.4). The area of the interaction (8.2) in the graphic representation seen in figure 2 is depicted, in the first tori, by the triangle AABBCCC, with sides a, b, c named as the branes lying on them. A comment is in order at this point. The classes of GUT models we have been considering recently [3] and at the present work, have as their low energy theory in energies of order $M_z$ the Standard model. Their common characteristic is that they represent deformations around the basic intersection structure of the Quark and Lepton structure,

$$I_{ab} = 3, \quad I_{ac} = -3.$$  

(8.6)

Thus they all share the same intersection numbers along the ‘baryonic’ a and the left and right ‘weak’ b and c, D6 branes. As we will show the area of the trilinear Yukawa couplings can be reexpressed in a simple form in terms of only intersection numbers of the b, c branes along which the bidoublets $h_1$, $h_2$, $h_3$, $h_4$ responsible in general for electroweak symmetry breaking, are localized. This provides us with a quantitative relation that may be useful is showing the hierarchies among neutrino masses in the general left-right PS GUT models. Assuming that the triangle areas in the 2nd, 3rd tori are close to zero, for the interaction (8.2) the worldsheet areas for (8.2) are given by

$$|A^{(T_2^{(1)})}| = |A^{(T_2^{(2)})}| = 0,$$

(8.7)

$$|A^{(T_2^{(3)})}| = |A^{(T_2^{(3)})}| = 0,$$

$$|A^{(T_2^{(3)})}| = |A^{(T_2^{(3)})}| = 0.$$  

(8.5)

The universal relations (8.7) describe the triangle area for all classes of models based on the present PS-II classes of GUTS and the PS-A, PS-I of [3].

\[20\] This area in simple Euclidean geometry terms is given by

$$A = \sqrt{(a - b)^2(a - c)^2 - ((a - b) \cdot (a - c))^2}.$$
Figure 2: Assignment of branes a, b, c based on the initial gauge group $U(4)_C \times U(2)_L \times U(2)_R$. The angles between branes shown are localized on the first tori.

Hence we have found that the worldsheet area entering the trilinear Yukawa couplings is parametrized in terms of the parameters describing the RR tadpole conditions. In the leading worldsheet instanton expansion the trilinear Yukawa couplings may be given by

$$Y_{ijk} F_L \bar{F}_R h^k \sim e^{-\frac{h_1^{(1)} h_2^{(1)} - |m_b^1 - m_c^1|}{2\pi} F_L \bar{F}_R h^k}$$  \hspace{1cm} (8.8)

In the present models $m_b^1 - m_c^1 = 0$.

### 8.2 Neutrino masses

In the present class of GUTS the electroweak bidoublets (8.2) are absent at tree level. Fortunately for us, there is another coupling, which is non-renormalizable, of the same order as the tree level one. It is given by

$$F_L \bar{F}_R \langle h_3 \rangle \langle F_R^B \rangle \langle H_2 \rangle \sim v F_L \bar{F}_R$$  \hspace{1cm} (8.9)

For a dimension five interaction term, like those involved in the Majorana mass term for the right handed neutrinos the interaction term is in the form

$$Y_{lmi} = e^{-\tilde{A}_{lmi}},$$  \hspace{1cm} (8.10)

where $\tilde{A}_{lmi}$ the worldsheet area connecting the four interaction vertices. Assuming that the areas of the second and third ‘tetragonal’ are close to zero, the four term coupling can be approximated as

$$Y^{ijk} = e^{-\frac{h_1^{(1)} h_2^{(1)}}{2\pi} \tilde{A}_{lmi}},$$  \hspace{1cm} (8.11)
where the area of the $A_{tiny}$ may be of order one in string units.

Thus the full Yukawa interaction for the chiral spectrum of the PS-II models

$$\lambda_1 F_L \bar{F}_R \langle h_3 \rangle \langle F_R^B \rangle \langle H_2 \rangle + \lambda_2 \frac{F_RF_R\langle F_R^H \rangle \langle F_R^H \rangle}{M_s}, \quad (8.12)$$

where

$$\lambda_1 \equiv e^{-\frac{B_1 B_2 A_1}{\alpha}}, \quad \lambda_2 \equiv e^{-\frac{B_1 B_2 A_2}{\alpha}}. \quad (8.13)$$

and the Majorana coupling involves the massless scalar $^{21}$ superpartners $\bar{F}_R^H$ of the antiparticles $\bar{F}_R$. This coupling is unconventional, in the sense that the $\bar{F}_R^H$ is generated by imposing SUSY on a sector of a non-SUSY model. We note the presence of $N = 1$ SUSY at the sector $ac$. As can be seen by comparison with (3.6) the $\bar{F}_R^H$ has a neutral direction that receives the vev $< H >$. There is no restriction on the vev of $\bar{F}_R^H$ from first principles and its vev can be anywhere between the scale of electroweak symmetry breaking and $M_s$.

The Yukawa term

$$F_L \bar{F}_R \langle h_3 \rangle \langle F_R^B \rangle \langle H_2 \rangle \sim \nu F_L \bar{F}_R \quad (8.14)$$

is responsible for the electroweak symmetry breaking. This term generates Dirac masses to up quarks and neutrinos. Thus, we get

$$\lambda_1 F_L \bar{F}_R \langle h_3 \rangle \langle F_R^B \rangle \langle H_2 \rangle \to (\lambda_1 \nu)(u_i u^c_i + \nu_i N^c_i) + (\lambda_1 \bar{\nu}) \cdot (d_i d^c_i + e_i e^c_i), \quad (8.15)$$

where we have assumed that

$$\langle h_3 \rangle \langle F_R^B \rangle \langle H_2 \rangle = \begin{pmatrix} v & 0 \\ 0 & \bar{v} \end{pmatrix} \quad (8.16)$$

We observe that the model gives non-zero tree level masses to the fields present. These mass relations may be retained at tree level only, since as the model has a non-supersymmetric fermion spectrum, it will receive higher order corrections. It is interesting that from (8.10) we derive the GUT relation $^{33}$

$$m_d = m_e. \quad (8.17)$$

as well the unwanted

$$m_u = m_{N^c \nu}. \quad (8.18)$$

$^{21}$Of order of the string scale.

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In the case of neutrino masses, the “unwanted” \((8.18)\), associated to the \(\nu - N^c\) mixing, is modified due to the presence of the Majorana term in \((8.12)\) leading to the see-saw mixing type neutrino mass matrix

\[
\left( \begin{array}{cc} \nu & N^c \end{array} \right) \times \left( \begin{array}{cc} 0 & m \\ m & M \end{array} \right) \times \left( \begin{array}{c} \nu \\ N^c \end{array} \right),
\]

where

\[m = \lambda_1 \nu. \tag{8.20}\]

After diagonalization the neutrino mass matrix gives us two eigenvalues, the heavy eigenvalue

\[m_{\text{heavy}} \approx M = \frac{\lambda_2 \langle H \rangle^2}{M_s}, \tag{8.21}\]

corresponding to the right handed neutrino and the light eigenvalue

\[m_{\text{light}} \approx \frac{m^2}{M} = \frac{\lambda_2}{\lambda_1} \times \frac{\nu^2 M_s}{\langle H \rangle^2} \tag{8.22}\]

corresponding to the left handed neutrino. In the present models, \(\lambda_1 = 1\).

Values of the parameters giving us values for neutrino masses between 0.1-10 eV, consistent with the observed neutrino mixing in neutrino oscillation measurements, will not be presented here, as they have already been discussed in [6]. The analysis remain the same, as the mass scales of the theory do not change. We note that the hierarchy of neutrino masses has been investigated by examining several different scenario associated with a light \(\nu_L\) mass including the cases \(\langle H \rangle = |M_s|, \langle H \rangle < |M_s|\). In both cases the hierarchy of neutrino masses is easily obtained.

### 8.3 Exotic fermion couplings

Our main focus in this part is to show that all additional particles, appearing in table (1), beyond those of SM get a heavy mass and disappear from the low energy spectrum. The only exception will be the light masses of \(\chi_L^1, \chi_L^2\), weak fermion doublets which are of order of the electroweak symmetry breaking scale, e.g. 246 GeV.

Lets us discuss the latter issue in more detail.

The left handed fermions \(\chi_L^1\) receive a mass from the coupling

\[
(1, 2, 1)(1, 2, 1)e^{-4\langle h_2 \rangle \langle h_2 \rangle \langle H \rangle \langle H \rangle} \frac{\langle h_2 \rangle \langle h_2 \rangle \langle H \rangle \langle H \rangle}{M_s^4} A \sim \eta^2 \eta (1, 2, 1)(1, 2, 1) \tag{8.23}
\]

\(^{22}\) The neutrino mass matrix is of the type of an extended Frogatt- Nielsen mechanism mixing light with heavy states.
that is in representation form

\[
(1, 2, 1)_{(0,1,0,1,0)} (1, 2, 1)_{(0,1,0,1,0)} \langle (1, \bar{2}, \bar{2})_{(0,-1,-1,0,0)} \rangle \langle (1, \bar{2}, \bar{2})_{(0,-1,-1,0,0)} \rangle \\
\times \langle (\bar{4}, 1, 2)_{(-1,0,1,0,0)} \rangle \langle (4, 1, 2)_{(1,0,1,0,0)} \rangle \langle (1, 1, 1)_{(0,0,0,-2,0)} \rangle \tag{8.24}
\]

In [8.23] we have included the leading contribution of the worksheet area connecting the seven vertices. In the following for simplicity reasons we will set the leading contribution of the different couplings to one e.g. area tends to zero.

The left handed fermions $\chi^2_L$ receive an order $M_s$ mass from the coupling

\[
(1, 2, 1)(1, 2, 1) \frac{\langle h_2 \rangle \langle \bar{F}^H_R \rangle \langle H_1 \rangle \langle s_B^2 \rangle}{M_s^4} \overset{A \to 0}{\sim} \frac{v^2}{M_s} (1, 2, 1)(1, 2, 1) \tag{8.25}
\]

that is in representation form:

\[
(1, 2, 1)_{(0,1,0,0,-1)} (1, 2, 1)_{(0,1,0,0,-1)} \langle (1, \bar{2}, \bar{2})_{(0,-1,-1,0,0)} \rangle \langle (1, \bar{2}, \bar{2})_{(0,-1,-1,0,0)} \rangle \\
\times \langle (\bar{4}, 1, 2)_{(-1,0,1,0,0)} \rangle \langle (4, 1, 2)_{(1,0,1,0,0)} \rangle \langle (1, 1, 1)_{(0,0,0,0,2)} \rangle \tag{8.26}
\]

Altogether, $\chi^1_L$, $\chi^2_L$, receive of order $v^2/M_s$. Thus the Pati-Salam models predict light weak doublets with mass between 90 and $v = 246$ GeV. This is a general prediction of all classes of models based on intersecting D6-branes.

The $\chi^1_R$ doublet fermions receive heavy masses of order $M_s$ in the following way.

The mass term

\[
(1, 1, 2)(1, 1, 2) \frac{\langle H_2 \rangle \langle \bar{F}^H_R \rangle \langle s_B^1 \rangle}{M_s^2} \tag{8.27}
\]

can be realized. In explicit representation form

\[
(1, 1, 2)_{(0,0,1,-1,0)} (1, 1, 2)_{(0,0,1,-1,0)} \langle (\bar{4}, 1, 2)_{(-1,0,-1,0,0)} \rangle \langle (4, 1, 2)_{(1,0,-1,0,0)} \rangle \\
\times \langle (1, 1, 1)_{(0,0,0,2,0)} \rangle \tag{8.28}
\]

With vevs $< H_2 > \sim < F^H_R > \sim M_s$, the mass of $\chi^1_R$ is of order $M_s$.

The $\chi^2_R$ doublet fermions receive heavy masses of order $M_s$ in the following way:

\[
(1, 1, 2)(1, 1, 2) \frac{\langle H_2 \rangle \langle \bar{F}^H_R \rangle \langle s_B^2 \rangle}{M_s^2} \tag{8.29}
\]

In explicit representation form

\[
(1, 1, 2)_{(0,0,1,0,1)} (1, 1, 2)_{(0,0,1,0,1)} \langle (\bar{4}, 1, 2)_{(-1,0,-1,0,0)} \rangle \langle (4, 1, 2)_{(1,0,-1,0,0)} \rangle \\
\times \langle (1, 1, 1)_{(0,0,0,2,0)} \rangle \tag{8.30}
\]

With vevs $< H_2 > \sim < F^H_R > \sim M_s$, the mass of $\chi^2_R$ is of order $M_s$. 

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The 6-plet fermions, \( \omega_L \), receive a mass term of order \( M_s \) from the coupling,

\[
\frac{\langle H_1 \rangle \langle F^H_R \rangle \langle H_1 \rangle \langle F^H_R \rangle}{M^3_s} \tag{8.31}
\]

where we have made use of the \( SU(4) \) tensor products \( 6 \otimes 6 = 1 + 15 + 20, \ 4 \otimes 4 = 6 + 10 \).

Explicitly, in representation form,

\[
(6, 1, 1)(-2,0,0,0,0) \langle (4, 1, 2)_{1,0,1,0,0} \rangle \langle (4, 1, 2)_{1,0,1,0,0} \rangle \\
\times \langle (4, 1, \bar{2})_{1,0,-1,0,0} \rangle \langle (4, 1, \bar{2})_{1,0,-1,0,0} \rangle \tag{8.32}
\]

The 10-plet fermions \( z_R \) receive a heavy mass of order \( M_s \) from the coupling

\[
\frac{\langle \bar{F}^H_R \rangle \langle \bar{F}^H_R \rangle \langle H_2 \rangle \langle H_2 \rangle}{M^3_s} \tag{8.33}
\]

where we have used the tensor product representations for \( SU(4) \), \( 10 \otimes 10 = 20 + 35 + 45, \ 20 \otimes \bar{4} = 15 + 20, \ 20 \otimes \bar{4} = 6 + 10, \ 10 \otimes \bar{4} = 4 + 36, \ 4 \otimes \bar{4} = 1 + 15 \). Explicitly, in representation form,

\[
(10, 1, 1)(10, 1, 1) \langle (\bar{4}, 1, 2)_{-1,0,1,0,0} \rangle \langle (\bar{4}, 1, 2)_{-1,0,1,0,0} \rangle \\
\times \langle (\bar{4}, 1, \bar{2})_{-1,0,-1,0,0} \rangle \langle (\bar{4}, 1, \bar{2})_{-1,0,-1,0,0} \rangle \tag{8.34}
\]

9 Conclusions

Recently the first examples of three generation string GUT models that break exactly to the SM at low energies were constructed [6]. These models were based on the \( SU(4)_C \times SU(2)_L \times SU(2)_R \) structure at the string scale. They are build on a background of intersecting D6-branes wrapping on 1-cycles across each of the three \( T^2 \)-tori appearing in the decomposition \( T^6 = T^2 \times T^2 \times T^2 \) in IIA orientifolds [1]. In this work, we extended the four stack constructions of [6] to five stacks. The different classes of GUT models are constructed as deformations around the basic intersection structure of the accommodated quark and lepton representations. Thus the massless structure of the quark-lepton and the Higgs sector is being shared by the present PS-II GUTS and also by the Pati-Salam PS-A, PS-I [6] classes of \( SU(4) \times SU(2)_L \times SU(2)_R \) GUT models.

The new classes of models preserve several features of the original models [6]. Among them we mention that the proton is stable as the baryon number is a gauged symmetry, the corresponding gauge boson become massive through its BFF couplings, and thus baryon number survives as a global symmetry to low energies. Particularly
important in the satisfaction of the RR tadpole cancellation conditions is the addition of extra $U(1)$ branes. This is to be contrasted with models with just the SM at low energy from an extended Standard model structure at the string scale \cite{3,4,5,7,8}. In those cases the presence of extra branes has no intersection with the rest of the branes \footnote{Also in this case the extra branes can be characterized as hidden one's as they don't charge the chiral fermion context of the models.}. In the present constructions the extra branes are handled in such a way that their presence has non-trivial intersection numbers with the colour $a$- and the leptonic $U(1)$ $d$, $e$- branes. The presence of extra branes creates scalar singlet scalars that may be used to break the additional extra $U(1)$'s that survive massless the Green-Schwarz mechanism.

Also in the construction of the models we allow exotic, antisymmetric and symmetric, fermionic representations of the colour, and $U(1)$ degrees of freedom arising from brane-orientifold image brane sectors. In this way we engineer the models such that they have the capacity to accommodate couplings that give a mass of order $M_*$ to all these exotic fermions, and also create singlets that may be used to break the extra $U(1)$'s surviving massless the presence of the generalized Green-Schwarz mechanism.

Small neutrino masses, of order $0.1-10$ eV, in consistency with neutrino oscillation experiments, can be easily accommodated as the worldsheet area, involved in the Yukawa couplings, between the intersecting branes works practically as a moduli parameter. Moreover, colour triplet Higgs couplings that could couple to quarks and leptons and cause a problem to proton decay are absent in all classes of models.

The present non-supersymmetric model constructions if the angle stabilization conditions of Appendix I hold, are free of tachyons. However, this is not enough as there will always be closed string NSNS tadpoles that cannot all be removed at once. Some ways that these tadpoles may be removed have been suggested in \cite{35} by freezing the complex moduli to discrete values, or by background redefinition in terms of wrapped metrics \cite{36}. However, a dilaton tadpole always remain that could in principle reintroduce tadpoles in the next leading order. Forcing us to rethink a solution in terms of the Fischer-Susskind mechanism \cite{37}. We also note that the complex structure moduli \footnote{As was noted in \cite{8} the Kähler moduli could be fixed from its value at the string scale, using relations involving the product radii (see (7.4) ) but in this way we could use a large fine tuning which seems unnatural in a string theory context, where moduli should be assigned values dynamically.} can be fixed to discrete values using the supersymmetry conditions, e.g. see (7.4), and in this way it is possible that some if not all, of the NS tadpoles can be removed. We leave this task for a future investigation. Also related is the fact that we have
tacitly assumed that the values of the scalar singlets present in the models are of the order of the string scale. In principle, whether these scalars, appearing in $N = 1$ supermultiplets, really receive a vev is highly non-trivial dynamical question. A full solution to the problem involves a) a solution to the stability of the present non-SUSY backgrounds and also b) a determination by a string calculation of the full effective potential of the scalar fields. Both problems are open problems as involve non-trivial dynamics and are beyond our calculational ability at present.

One point that we want to emphasize is that until recently, in orientifolded six-torus compactifications there was not any obvious explanation for keeping the string scale low [11], e.g. to the 1-100 TeV region. Thus the usual explanation of explaining the hierarchy by making the Planck scale large, while keeping the string scale low, by varying the radii of the transverse directions [11] could not be applied [25]. However, as was noted in [6] there is an alternative mechanism that keeps the string scale $M_s$ low. In particular the existence of the light weak doublets $\chi^1_L, \chi^2_L$ with mass of order $v^2/M_S$ and up to 246 GeV, makes a definite prediction for a low string scale in the energy range less than 650 GeV if a fermion weak doublet has to be detected in $e^+e^-$ experiments, in the energy range over 90 GeV. If the lightest weak fermion doublet is over 100 GeV as being favoured by the experiments at present the $M_s < 600 GeV$. That effectively, makes the PS-II class of D6-brane models (also the PS-A, PS-I classes) directly testable to present or feature accelerators. Also it would be interesting if we could analyze the low energy implications for the PS-I, PS-A, PS-II GUT models in terms of a variant of the analysis performed in [38].

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25 As there are no simultaneously transverse torus directions to all D6-branes [1].
10 Appendix I

In the appendix we list the conditions, mentioned in section 5, under which the PS-II model D6-brane configurations of tadpole solutions of table (2), are tachyon free. Note that the conditions are expressed in terms of the angles defined in (6.2). A comment is in order. Note that we have included the contributions from the sectors $ab^*$, $ac$, $bd^*$, $cd$, $be$, $ce^*$. We have not present the tachyon free conditions from the sectors $dd^*$, $ee^*$, as these conditions will be shown to be equivalent to the presence of N=1 supersymmetry in these sectors.

\begin{align}
-\left(\frac{\pi}{2} + \vartheta_1\right) &+ \vartheta_2 + 2\vartheta_3 \geq 0 \\
-\left(\frac{\pi}{2} - \tilde{\vartheta}_1\right) &+ \vartheta_2 + 2\vartheta_3 \geq 0 \\
-\left(\frac{\pi}{2} + \vartheta_1\right) &+ \tilde{\vartheta}_2 + 2\vartheta_3 \geq 0 \\
-\left(\frac{\pi}{2} - \tilde{\vartheta}_1\right) &+ \tilde{\vartheta}_2 + 2\vartheta_3 \geq 0 \\
-\left(-\frac{\pi}{2} + \vartheta_1\right) &+ \vartheta_2 + 2\vartheta_3 \geq 0 \\
-\left(-\frac{\pi}{2} + \tilde{\vartheta}_1\right) &+ \tilde{\vartheta}_2 + 2\vartheta_3 \geq 0 \tag{10.1}
\end{align}

\begin{align}
\left(-\frac{\pi}{2} + \vartheta_1\right) &- \vartheta_2 + 2\vartheta_3 \geq 0 \\
\left(-\frac{\pi}{2} - \tilde{\vartheta}_1\right) &- \vartheta_2 + 2\vartheta_3 \geq 0 \\
\left(-\frac{\pi}{2} + \vartheta_1\right) &- \tilde{\vartheta}_2 + 2\vartheta_3 \geq 0 \\
\left(-\frac{\pi}{2} - \tilde{\vartheta}_1\right) &- \tilde{\vartheta}_2 + 2\vartheta_3 \geq 0 \\
\left(-\frac{\pi}{2} + \vartheta_1\right) &- \vartheta_2 + 2\vartheta_3 \geq 0 \\
\left(-\frac{\pi}{2} + \tilde{\vartheta}_1\right) &- \tilde{\vartheta}_2 + 2\vartheta_3 \geq 0 \tag{10.2}
\end{align}
11 Appendix II

In this Appendix, following a comment in (4.3) we emphasize the importance of choosing an appropriate location for the presence of extra branes needed to satisfy the RR tadpole cancellation conditions. By choosing the location of the extra branes, in a general point like \((1/\beta_1, 0)(1/\beta_1, 0)(1, m/2)\) and e.g. \(\beta_1 = \beta_2 = 1/2, m = 1\), we are getting a GUT class of models where there are no electroweak bidoublets \(h_1, h_2, h_3, h_4\) and thus there is no Dirac term allowed to give mass to quarks and leptons.

The precise arguments have as follows: In this case the number of extra branes required is \(N_h = 4\). The structure of \(U(1)\) anomalies gives us the following couplings of the RR fields to the \(U(1)\)'s of the new classes of models:

\[
\begin{align*}
B_2^3 \wedge [2\tilde{c}] & \left[-(F^b + F^c) + (F^{h_1} + F^{h_2} + F^{h_3} + F^{h_4})]\right], \\
B_2^1 \wedge [\tilde{c}] & \left[4n_a^2 F^a + 4m_b^1 F^b + 4m_c^1 F^c + 2n_4^2 F^d + 2n_4^2 F^e\right], \\
B_2^0 \wedge \left(3F^a - 2F^d + F^e\right). \quad (11.1)
\end{align*}
\]

The couplings of the dual scalars \(C^l\) of \(B_2^l\), required to cancel the mixed anomalies of the \(U(1)\)'s with the non-abelian gauge groups \(SU(N_a)\), are given by

\[
\begin{align*}
C^0 & \wedge 2[-(F^b \wedge F^b) + (F^c \wedge F^c) + 2(F^{h_1} \wedge F^{h_1} + F^{h_2} \wedge F^{h_2} + F^{h_3} \wedge F^{h_3} + F^{h_4} \wedge F^{h_4})], \\
C^2 & \wedge [\tilde{c}\tilde{\epsilon}]\left[2n_a^2 (F^a \wedge F^a) + 2m_b^1 (F^b \wedge F^b) - 2m_c^1 (F^c \wedge F^c) + n_4^2 (F^d \wedge F^d) - n_4^2 (F^e \wedge F^e)\right], \\
C^0 & \wedge [\tilde{c}\tilde{\epsilon}]\left[2n_a^2 (F^a \wedge F^a) - 4F^d \wedge F^d - 4(F^e \wedge F^e)\right]. \quad (11.2)
\end{align*}
\]

As can be seen two anomalous combinations of \(U(1)\)'s, e.g. \(3F^a - 2F + F^c, -(F^b + F^c) + F^{h_1} + F^{h_2} + F^{h_3} + F^{h_4}\) become massive through their couplings to RR fields \(B_2^0, B_2^1\). Also there is an anomaly free model dependent \(U(1)\) which is getting massive from its coupling to the RR field \(B_2^2\). In addition, there are six non-an anomalous \(U(1)\)'s which also are getting broken by vevs of singlet scalars generated by imposing \(N=1\) SUSY on certain sectors. They are:

\[
\begin{align*}
U(1)^{(4)} &= \tilde{c}(F^{h_1} - F^{h_2} + F^{h_3} - F^{h_4}), \\
U(1)^{(5)} &= \tilde{c}(F^{h_1} - F^{h_2} - F^{h_3} + F^{h_4}), \\
U(1)^{(6)} &= (Q_b - Q_c) + (Q_a + Q_d - Q_e) + \tilde{c}(F^{h_1} + F^{h_2} - F^{h_3} - F^{h_4}), \\
U(1)^{(7)} &= \frac{1}{5}(Q_b - Q_a + (Q_a + Q_d - Q_e)) + \frac{1}{4}(-F^{h_1} - F^{h_2} + F^{h_3} + F^{h_4}).
\end{align*}
\]
\[ U(1)^{(8)} = (Q_b + Q_c) + \frac{1}{2}(F^{h_1} + F^{h_2} + F^{h_3} + F^{h_4}) \]
\[ U(1)^{(9)} = Q_a + 4Q_d + 5Q_e \] (11.3)

The choice of U(1)'s (11.3) gives the constraints

\[ m^1_b = -m^1_c \] (11.4)
\[ 2n^2_a + 4n^2_d + 5n^2_e = 0 \] (11.5)

The number of electroweak bidoublets \( h_1, h_2 \) taking into account the constraint (11.5) is zero. We note that this number depends on the difference \( |m^1_b - m^1_c| \). Also the number of electroweak bidoublets \( h_3, h_4 \) is zero, as it depends on the difference \( |m^1_b + m^1_c| \).
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