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A Note on Moment Inequality for Quadratic Forms

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Abstract

Moment inequality for quadratic forms of random vectors is of particular interest in covariance matrix testing and estimation problems. In this paper, we prove a Rosenthal-type inequality, which exhibits new features and certain improvement beyond the unstructured Rosenthal inequality of quadratic forms when dimension of the vectors increases without bound. Applications to test the block diagonal structures and detect the sparsity in the high-dimensional covariance matrix are presented.

Keywords:
Quadratic forms, Rosenthal’s inequality, high-dimensional covariance matrix.

1. Introduction

Covariance matrix plays a central role in multivariate analysis, spatial statistics, pattern recognition and array signal processing. Let $\mathbf{x} = (X_1, \cdots, X_p)^T$ be a $p$-variate random vector with mean zero and covariance matrix $\Sigma = \mathbb{E}(\mathbf{x}\mathbf{x}^T)$. Let $\mathbf{x}_i = (X_{i1}, \cdots, X_{ip})^T$ be independent and identically distributed (iid) copies of $\mathbf{x}$. To test and estimate the covariance matrix $\Sigma$ based on the observed values of $\mathbf{x}_i$, there are $p(p+1)/2$ parameters. When the dimension $p$ is large relatively to the sample size $n$, the parameter space grows quadratically in $p$, thus making the covariance testing and estimation problems challenging. Leveraging low-dimensional structures in $\Sigma$, recent literature focuses on structures detection and regularized estimation of the covariance matrix; see e.g. Chen et al. (2010); Bickel and Levina (2008a,b); Chen et al. (2013) among many others. A key step of the success of those regularized estimates relies on sharp large deviation and moment inequalities of the second-order partial sum process involving the quadratic form $\sum_{i=1}^{n}(X_{ji}X_{ki} - \sigma_{jk})$. Large
