Pressure dependence of the quality factor of a micromachined cantilever in rarefied gases

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Abstract. We present a study of the damping behavior of monocrystalline silicon cantilevers in different rarefied gas regimes. Mechanical quality factors $Q$ were analyzed at controlled ambient pressures in the range of 0.01 Pa to 100 Pa. Emphasis was laid on the investigation of the fundamental vibration mode. Hence, the test structures were harmonically excited by the Lorentz force acting on the current carrying lead attached to the top surface of the cantilever. The micromachined clamped-free cantilevers featuring a length of 2 mm, a width of 1.5 mm and a thickness of 20 $\mu$m, were manufactured in SOI technology. The experimental results were compared with existing theories revealing an underestimate of the damping parameter for the Knudsen range $Kn = 0.1$ to 10. So far, squeeze-film damping by free molecular flow and kinetic damping were taken into account in damping models for the quasi-molecular regime. However, our measurements indicate that also the ongoing molecular flow around the test structures has to be considered. Hence the damping coefficient has to be calculated with methods of the free molecular aerodynamics. Thus, we used an algorithm based on the random walk model that allows the usage of already available knowledge in the field of Direct Simulation Monte Carlo. With this approach the quality factor of a squeezed-film damped cantilever in the quasi-molecular regime was derived. The results were compared with the most recent stochastic model, where the theoretical predictions and the experimental investigations indicate significant squeezing up to a Knudsen number of 10. In a superposition of both damping mechanisms, kinetic and squeeze-film damping, a satisfactory characterization of the damping behavior of an oscillating cantilever in the quasi-molecular regime with Knudsen numbers in the range of 10 down to 0.02 was achieved.

1. Introduction
For many years, micromechanical structures with all kinds of geometries have been used in various ways in microelectromechanical systems (MEMS-devices). Single-crystalline silicon has superior mechanical properties with regard to elastic parameters which permit reproducibility and long-term stability better than at other materials [1]. Furthermore, silicon-based production enables an integration of microelectronic and micromechanical components.

The resolution of systems with resonant vibrating structures is determined by the mechanical quality factor. With increasing quality factor, the resonance amplification becomes more and more pronounced. As a result, the resolution of a sensor that evaluates the resonant frequency is improved, too.
If the mechanical quality factor is derived theoretically, several kinds of damping mechanisms, like thermo-elastic damping (TED), gas damping or structural damping have to be considered which are adequately described in literature [2–9].

This work emphasizes the dependency of the quality factor on the ambient pressure, since the related extrinsic damping mechanism dominates for typical MEMS structures in the pressure range of 1 up to 100 Pa. Due to the gas-solid interactions this range is further divided into molecular and transition regime. In these regimes, a part of the extrinsic damping is caused by collisions of impinging molecules with the structure. The regime names indicate that the conventional equations of continuum theory are inappropriate to describe rarefied gas flows. Continuum (gas) damping requires a sufficient particle density in the flow, which is not guaranteed in the transition regime. The calculation of transition flows can only be carried out on the basis of the kinetic theory, where the characteristic equation is the Boltzmann-equation. This equation ensures not only a complete description of the free molecular flow, but implies also continuum flows as an approximation. There are three dimensionless parameters which determine the flow regime: the Mach number $M$, the Knudsen number $Kn$ and the Reynolds number $Re$ [10]. For typical MEMS applications two of them, $M$ and $Re \ll 1$, are negligible because of subsonic vibration velocities. This leaves the Knudsen number as the most important fundamental parameter,

$$Kn = \frac{\lambda}{b} = \frac{k_B T}{\sqrt{2 \pi p \sigma^2 b}},$$

where $\lambda$ is the mean free path, $k_B$ the Boltzmann constant, $T$ the ambient temperature, $p$ the ambient pressure, $\sigma = 4.19 \cdot 10^{-10}$ m the diameter of air molecules and $b$ the width of the cantilever [11]. Both in the theoretical and experimental description, the overall pressure range is divided into an intrinsic (0.01 Pa to 1 Pa) and a quasi-molecular (1 Pa to 100 Pa) regime, depending on the dominant damping mechanism [2]. For cantilevers with a characteristic width $b$ of 1.5 mm the Knudsen number in the intrinsic regime is always greater than or equal to 10. This is therefore a sufficient condition to separate the molecular (Knudsen) from the transition flow.

The dominant damping mechanism in the intrinsic regime is structural damping caused by the mounting of the vibrating structure to the chip carrier [4, 8]. Another damping mechanism in the intrinsic regime is the thermo-elastic damping (TED), where the flexural oscillations are modeled as acoustic modes interacting with the thermally excited phonons of the vibrating structure. Through the interaction between the field of the mechanical stress and strain and the temperature field of thermal phonons, energy can be dissipated.

The quasi-molecular regime coincides with Knudsen numbers in the range of 0.1 to 10, whereas at the lower boundary the molecular flow enters into the slip flow. The dominant damping mechanisms in this regime are structure-molecular and intermolecular collisions. Analytical solutions of the Boltzmann equation in the quasi-molecular regime do not exist, but several approximations like the Burnett equation are available [12]. One possibility to get a better insight into this transition regime is to use the assumptions of the free molecular flow and to compare the systematic deviations of the measurement results with the analytical solutions of the free molecular flow. As an important precondition for the experimental evaluation surrounding surfaces from the bulk material, the mounting and the measurement chamber are in sufficiently large distance to the cantilever so that their influence on the damping of the fundamental mode of the device can be neglected.

2. Device Fabrication

The fabrication process for the device is based on the silicon-on-insulator (SOI) technology. The handle layer of the SOI wafer is passivated with silicon dioxide ($\text{SiO}_2$). First of all, metal lines are evaporated
and patterned with lift-off technique using an image reversal photoresist (AZ 5214). Both the current-carrying leads and the bond pads consists of a 300 nm thick gold layer and a 70 nm chromium layer acting as bonding agent. In the resulting multi-layered structure the stresses of the chromium and the gold layer compensate each other and the structure remains undeflected. In a second step, the silicon wafer is etched from the back side anisotropically with 30 wt % KOH solution at 75 °C. Afterwards, the membrane is dry etched from the top side using a DRIE (Deep-Reactive-Ion-Etching) process, which finally forms the cantilever.

The structure, schematically shown in figure 1, consists of a clamped-free cantilever with a length $l$ of 2 mm, a width $b$ of 1.5 mm, and a thickness $h$ of the silicon device layer of 20 µm. The thickness of the handle wafer is 350 µm.

![Diagram of cantilever](image)

Figure 1: Schematic top view and cross-section of the test device with the Au-lead on the top of the Lorentz-force actuated cantilever.

### 3. Experimental Setup

A scanning laser Doppler-vibrometer (Polytec MSA400 Micromotion Analyzer, shown in figure 2) comprising displacement and velocity decoders is used to characterize the out-of-plane vibrations of the cantilever structures. Because of its considerable broadband noise, the vibrometer output signal is measured with a lock-in amplifier (SR830, Stanford Research Instruments). A waveform generator excites the vibrating structure around its natural frequency in a static magnetic field. The sinusoidal current in the lead on the top of the cantilever causes a harmonic oscillation due to the Lorentz force. The waveform generator sweeps stepwise ($\Delta f = 0.1$ Hz) around the natural frequency and synchronizes the vibrometer DSP and the Lock-in amplifier measuring the vibrometer’s output signal. Owing to high intrinsic quality factors, the cantilever vibration may take several seconds to settle. The amplitude of the cantilever vibration due to the excitation is always much smaller than its length $l$ to ensure linearity.
The test device shown in figure 3 is placed in a vacuum chamber surrounded by a Helmholtz coil that generates the static magnetic field. The nitrogen pressure in the vacuum-chamber is measured with a high vacuum gauge (Pfeiffer IMR 265) and maintained by a flow controller (MKS 50 sccm) at the high vacuum port of a turbomolecular pump, enabling a dynamic equilibrium in the pressure range of 20 mPa to 200 Pa. As a result a fully automated control of the nitrogen atmosphere is established for interactions with the vibrating test structure.

4. Mechanical Quality Factor of Vibrating Cantilevers

One possible and widely used definition of the quality factor of a damped mechanical system is based on energy considerations

\[ Q = \frac{2\pi E_i}{E_d}, \]  

where \( E_i \) is the stored vibrational energy and \( E_d \) is the dissipated energy per oscillation period. In the case of a weakly damped system it is feasible to determine the quality factor using the linewidth of the resonance. At the lower boundary of the investigated pressure range intrinsic and quasi-molecular damping governs the quality factor. According to equation (2) the resulting quality factor

\[ Q = \left( \frac{1}{Q_{\text{intrinsic}}} + \frac{1}{Q_{\text{quasi-molecular}}} \right)^{-1} \]  

is the reciprocal sum of all inverse individual quality factors. The intrinsic quality factor is independent of the ambient pressure and can therefore be found experimentally at the ultimate pressure of the vacuum system (20 mPa). In 1992 Blom et al. first published the quasi-molecular quality factor of a cantilever,
\[ Q_{\text{Cantilever}} = \frac{\rho A \omega}{f_1}, \quad (4) \]

where \( \rho \) is the density of the beam material, \( A \) is the cross-sectional area, \( \omega \) and \( f_1 \) denote the resonance angular frequency and the dissipative drag parameter per unit length [2]. The drag parameter

\[ f_1 = k_m b \rho \quad (5) \]

was first published by Drawin in 1962, where \( k_m \) is the damping coefficient, \( b \) is the width of the cantilever and \( \rho \) the ambient pressure [13]. The idea of his derivation is based on a net difference of the particle density between the top and the bottom side of the vibrating cantilever. In consequence, a nonzero molecular pressure difference acts on the cantilever, which counteracts the propulsive forces. Several publications have laid emphasis on the determination of the damping coefficient, but Martin et al. in 2008 has given an upper limit

\[ k_m = C_D \frac{c}{\epsilon} = 3.9008/c, \quad (6) \]

with full momentum accommodation, where the damping coefficient \( k_m \) is expressed in terms of the most probable speed \( c \) (= thermal speed) and a damping parameter \( C_D \) [2, 14, 15]. Thermal effects are completely neglected, but the result of equation (6) is valid for a wide range of MEMS cantilevers, where the height \( h \) is much smaller than the width \( b \) of the beam. The minimum quality factor of a cantilever can be retrieved by combining equations (4), (5) and (6)

\[ Q_{\text{kinetic}} = \frac{\rho h \omega \ c}{p \ C_D}, \quad (7) \]

with a dominant damping mechanism due to structure-molecular collisions in the quasi-molecular regime.

5. Squeeze-film Damping in the Quasi-Molecular Regime

In the range of Knudsen numbers \( Kn \ll 1 \) viscous damping mechanisms like squeeze-film damping dominate the gas-surface interactions. Viscous squeeze-film damping is well known and exactly analyzed [16, 17]. Due to flexural vibrations of the cantilever perpendicular to a fixed wall the gas in between is compressed [16]. The gas can partially escape through air slits that surround the moving cantilever opposite to the chip carrier, which induces friction losses since compression work is not fully recovered. Rarefaction effects were considered by Veijola et al. into the linearized Reynolds equation using an effective viscosity model, which permits the prediction of the flow in rarefied gases [18]. The Reynolds equation is a simplification of the Navier-Stokes equation. Hence, it stands for a continuum approach, that is adapted to the transition between the macroscopic and microscopic description of rarefied gas flows [19]. But the Navier-Stokes equation in turn applies only to a subset of problems covered by the Boltzmann equation.
A numerical solution of the latter based on the DSMC method was chosen by Alexeenko et al., and a promising analytical approach was introduced by Suijlen et al. in 2009 [20, 21]. This work presents a model of squeezed-film damped beams in the molecular regime. The basic idea of this derivation is a variation of the particle density between the vibrating cantilever and the fixed wall. The dynamic behavior of the beam counteracts the random walk of the gas molecules which are forced to sustain a steady-state particle density in the gap. For harmonic motion, this results in a complex-valued force, where the diffusion time

\[ \tau = \frac{8}{\pi^3} \cdot \frac{A}{(d\bar{c})} \]  

(8)

of the gas molecules is calculated from the Brownian motion. In equation (8) \( A \) is the surface area of the cantilever, \( d \) is the averaged distance between the moving cantilever and the fixed wall and \( \bar{c} \) denotes the mean speed of the gas molecules (Maxwell–Boltzmann distribution). In an earlier publication by Legtenberg et al. in 1995 it was already speculated that in the quasi-molecular regime not only kinetic, but also squeeze-film damping lowers the quality factor \( Q \) [22]. According to the calculations of [2, 21, 23] the quality factor of a squeeze-film damped cantilever in the free molecular regime was derived

\[ Q_{\text{squeeze}} = \frac{\rho h \omega}{p} \cdot \frac{1 + (\omega \tau)^2}{\tau}. \]  

(9)

Adding the losses is equal to summing up all contributions to the inverse of the quality factor, and the pressure-dependent quality factors can be summarized to

\[ Q_{\text{quasi–molecular}} = \left( \frac{1}{Q_{\text{kinetic}}} + \frac{1}{Q_{\text{squeeze}}} \right)^{-1}. \]  

(10)

6. Results and Discussion

In order to obtain data with significant as well as negligible squeeze-film damping several identical structures like the micrograph depicted in figure 4 are produced. In the case of testing exclusively kinetic damping the PCB underneath the vibrating structure is partially removed (see figure 5).

Otherwise a fixed wall is placed 350 \( \mu \)m (which conforms to the thickness of the handle wafer) below the test-structure. Usually the lateral air slit is 100 \( \mu \)m, whereat the influence of different slit widths on the quality factor is tested in the transition regime. The experimental data shown in figure 6 are compared with analytical solutions of the equations (3) and (7) on the one hand and with the most recent stochastic model by Okada et al. on the other hand [24].

From figure 6 it is obvious that the assumptions of the free molecular flow are no longer valid for Knudsen numbers in the range of 0.1 or smaller. The intermolecular collision frequency increases with decreasing Knudsen numbers, where in figure 6(a) the Knudsen flow evolves into the pressure-independent viscous flow [2]. Compared to that, there is a stronger structure-molecular dependence observable in the results in figure 6(b) due to the smaller air slit.
Figure 4: Micrograph of the top view of a cantilever with a typical air slit width of 100 µm.

Figure 5: Micrograph of a bottom view with removed fixed wall below the test structure.

Figure 6: Experimental and theoretical results of the pressure dependence of the quality factor due to intrinsic and kinetic damping mechanisms of a cantilever with a length of 2 mm and a width of 1.5 mm, and with two different widths of the air slit.

This structure-molecular dependence increases with a fixed wall below the vibrating structure. The experimental data shown in figure 7 are similarly compared with analytical solutions of the equations (3) and (7), but additionally with equations (9) and (10), because of the fixed wall below the cantilever. In figure 7 the uncertainty is mainly determined by the lack of knowledge of the equilibrium distance $d$ between the vibrating structure and the fixed wall. It is assumed that this distance corresponds to the thickness of the handle wafer, which is 350 µm with an estimated uncertainty of 10%. The red dotted curve depicted in figure 7 represents exclusively kinetic damping. Evidently, squeeze-film damping according to equations (9) is the second dominant damping mechanism in the transition regime, where the modeled quality factor $Q$ perfectly matches the experimental data.
7. Conclusion and Outlook

A comprehensive study of the damping behavior of monocrystalline silicon cantilevers in different rarefied gas regimes is presented and experimentally verified with the use of a micromotion analyzer. It was demonstrated that two damping mechanisms, namely kinetic and squeeze-film damping, dominate in the pressure-dependent transition regime with low Knudsen numbers in the range of 0.02 to 10. Both damping mechanisms are derived from the assumptions of a free molecular flow, avoiding a priori phenomenological presumptions.

Further analytical models in the transition regime have to be carried out and analyzed with MEMS devices in order to test which kinetic equation would describe the interaction of diatomic gases in a non-thermal equilibrium with the structure. Similarly, the dependency of squeeze-film damping on different gaps $d$ between vibrating structure and fixed wall has to be studied for a thorough verification of the models. These considerations are topics of further examinations.

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