Cosmic Strings and Structure Formation

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Abstract. We discuss structure formation in cosmic string theories. Emphasis is laid on the properties of the peculiar velocity field, non–gaussian features and clusters of galaxies. It is found that the predicted peculiar velocity field is quiet ($v < 150 \text{ km/s}$ for $L > 30h^{-1}\text{ Mpc}$) for models with $(\Omega_0 h \leq 0.2)$ which is in disagreement with the results from POTENT at large scales ($> 30h^{-1}\text{ Mpc}$), consistent, however, with the analysis of the peculiar velocity field of spiral galaxies by Giovannelli et al. [16]. Using the Press–Schechter formalism we calculate the abundances of X–ray clusters. It is found that in CDM models a mass fluctuation $\sigma_8 = 0.6...0.7$ is needed with $\Omega_0 h = 0.1...0.2$ in order to explain the current data.

1. Introduction

An outstanding problem in cosmology is to explain the observed structures in the universe [6]. Two paradigms have been proposed: the inflationary cosmology and topological defects, which are well motivated by particle physics [13, 28, 7, 12]. Both could, at least in principle, be tested with observations of the microwave background anisotropies and with deep redshift surveys, because they make different predictions of the matter distribution and of the spectrum of the anisotropies of the cosmic microwave background radiation (CMBR).

We discuss here the cosmic string theory, its prediction of the properties of the peculiar velocity field, the non-gaussian features in cosmic string models and the implications of non–gaussianity on the abundances of clusters of galaxies. We do not discuss power spectra and CMBR anisotropies.
Table 1. The four representative cosmological models. $k$ is the curvature parameter, $\Omega_0 = 8\pi G \rho / (3H_0^2)$ is the matter density parameter, $\lambda_0 = \Lambda / (3H_0^2)$ is the cosmological term, $H_0$ is the Hubble parameter (in km s$^{-1}$Mpc$^{-1}$). $R_0$ and $R_{eq}$ are the scale factor at the present time and at matter–radiation equality, respectively.

| Model | $k$ | $\Omega_0$ | $\lambda_0$ | $H_0$ | $\log(R_0/R_{eq})$ |
|-------|-----|------------|-------------|-------|------------------|
| 1     | +1  | 0.014      | 1.08        | 90    | 2.66             |
| 2     | 0   | 1.0        | 0.0         | 60    | 4.16             |
| 3     | -1  | 0.1        | 0.0         | 60    | 3.16             |
| 4     | 0   | 0.1        | 0.9         | 60    | 3.16             |

and refer instead to the contribution by Richard Battye (these proceedings and references therein).

The paper is organized as follows: in section 2 we describe the cosmic string network evolution on large scales, which is relevant for structure formation. In section 3 we discuss the properties of the peculiar velocity field and in section 4 the non–gaussian features in cosmic string theory. Our conclusions are given in section 5.

We take the opportunity to include some points which could not be covered in the talk.

2. Network evolution

A network of cosmic strings, which are line–like objects, will originate in a phase transition if the vacuum manifold is non–simply connected. To know how cosmic strings could form structures, we need to understand the evolution of such a network. Therefore we discuss this in more detail in this section, where we focus on the case of local cosmic strings.

Our approach is based on the modified “one–scale” model by Martins & Shellard [15]. In this (phenomenological) model, the string network is characterized by a length scale $L$ and the RMS velocity $v_{\text{RMS}}$ of the strings. The length scale $L$ is defined by

$$\rho_\infty = \frac{\mu}{L^2},$$

where $\rho_\infty$ is the energy density contained in the long strings (i.e. the strings with curvature radius larger than the Hubble radius) and $\mu$ the energy per unit length on the string. The model can not only be applied to cosmic

$^1$ $\mu$ is related to the energy scale of the phase transition where the string network was created.
Figure 1. Behaviour of $L/R_H$ as a function of $\log(R/R(t_{eq}))$, where $R(t_{eq})$ is the scale factor at the time of matter–radiation equality and $R_H$ is the (time–dependent) Hubble radius, adapted from [25].

...strings but also to line defects in the condensed matter context. The reader is referred to the paper by Martins & Shellard, where further material and justifications of the equations are given [15]. We note that the equations are only exact in flat space–times. However, curvature effects can easily been incorporated. Our results presented below do not depend strongly on curvature effects. Curvature leads only to a faster release of energy of the network. Our results are in agreement with [1].

The equations, which determine the parameter $L$ and $v_{\text{RMS}}$, are given by (we set the speed of light $c = 1$):

$$\frac{dL}{dt} = HL(1 + v_{\text{RMS}}^2) + \frac{\tilde{c}v_{\text{RMS}}}{2}. \quad (2)$$

$$\frac{dv_{\text{RMS}}}{dt} = (1 - v_{\text{RMS}}^2)\left(\frac{\kappa}{\gamma} - 2Hv_{\text{RMS}}\right), \quad (3)$$

where $\kappa$ is a parameter related to the small scale structure on the strings and $\tilde{c}$ is a parameter describing the energy loss due to loop formation. The expansion rate $H(t)$ of the universe obeys the Friedmann equation:

$$H^2(t) = \left(\frac{\dot{R}(t)}{R(t)}\right)^2 = \frac{8\pi G}{3} \rho(t) + \frac{\Lambda}{3} - \frac{k}{R(t)^2}, \quad (4)$$
where $\rho = \rho_{\text{matter}} + \rho_{\text{radiaton}}$ is sum of the total matter density and radiation density, $\Lambda$ the cosmological constant, $k$ the curvature parameter and $R$ the scale factor.

We solve the network equations for four cosmological models summarized in the table 1 [25]. The models include an open model, a flat model with a cosmological constant, a model with a loitering phase (see the article by van de Bruck and Priester, these proceedings) and the Einstein–de Sitter model for comparison. The behaviour of the length scale $L$ is plotted in Fig.1 and Fig.2.

It is usually argued that the cosmic string network evolves according to the scaling solution, which states that the ratio of the length $L$ to the Hubble radius $R_H(t) = 1/H(t)$ is constant: scaling solution $L/R_H = \text{const}$. We found that the scaling behaviour is always found in the radiation dominated epoch, but only reached in the matter dominated epoch in the Einstein–de Sitter model. In all other models, the scaling behaviour can not be found after the radiation dominated epoch [25]. One can see, that even in the matter dominated epoch the scaling behaviour can not be found. This result can

2 A epoch is called radiation dominated if the radiation density dominates all other terms in the Friedmann equation. The terms “matter dominated epoch” or “$\Lambda$–dominated” epoch are explained in the same manner.
easily be understood: As emphasized by Martins & Shellard, the transition from the scaling behaviour in the radiation dominated epoch to the scaling in the matter dominated epoch is long. In the low density models 1, 3 and 4, the matter dominated epoch is too short for the string network to settle down towards the scaling behaviour, before the curvature term or the $\Lambda$–term dominates the expansion of the universe.

Of course, in models with $\Omega_0 \approx 0.9$ the string network reaches scaling in the matter dominated epoch. However, such a model is indistinguishable from an Einstein–de Sitter model from the viewpoint of structure formation.

Very important for structure formation is the transition–regime between the radiation to the matter dominated epoch. We refer to the contribution by Richard Battye in these proceedings.

In the cosmic string theory three mechanism for structure formation are possible: accretion on loops, wake formation due to fast moving strings and filamentary accretion onto slow moving strings, see e.g. [28, 7, 8]. We us restrict here only on one of these possibilities: the formation of wakes, which is suggested by detailed simulations of the network evolution, see the discussion in [28] and references therein.

3. The peculiar velocity field in cosmic string models

Every model of structure formation is confronted with the observed properties of the peculiar velocity field of the galaxies. An astonishing observed feature is that very large volumes ($R \geq 20$ Mpc) flow coherently through space (bulk flows) with large velocities ($v_{\text{bulk}} \geq 100$ km/s, depending on the scale), suggesting the existence of structures on very large scales, see e.g. [10] and references therein.

Here, we calculate the bulk flows predicted from the cosmic string theory. In cosmic string theory, the peculiar velocity field of matter originates due to the conical space–time structure of a cosmic string. If a cosmic string passes a particle it will get a velocity kick in the direction swept out by the string. The velocity kick is given by

$$v_{\text{kick}} = 4\pi G\mu\gamma_s v_s f = 3.8(G\mu)\gamma_s v_s f \text{ km/s,} \quad (5)$$

with

$$f = 1 + \frac{1}{2\gamma_s^2} \left( 1 - \frac{T}{\mu} \right). \quad (6)$$

The term $f$ corresponds to the small scale structure along the string, where $\mu$ is the effective mass per unit length along the string and $T$ is the string tension. $v_s$ is the string velocity and $\gamma_s$ is the corresponding Lorentz–factor.
On this basis it is easy to calculate the peculiar velocity field: the peculiar velocity \( \vec{v} \) of a given volume is the sum of the influence of many strings, which could influence the volume:

\[
\vec{v} = v_{\text{kick}} \sum A_i \vec{k}_i,
\]

where \( v_{\text{kick}} \) is the value of the kick velocity due to a string, \( A_i \) is the amplification factor, which describes the amplification of the velocity kick due to gravitational instability and \( \vec{k}_i \) is an unit vector pointing into the direction of the area swept out by string \( i \). The numbers of strings which could influence the volume are calculated with the modified one-scale model by Martins & Shellard [15]. The amplification factors can be calculated with the Zeldovich–approximation and we assume that the strings are uncorrelated and therefore \( \langle \vec{k}_i \cdot \vec{k}_j \rangle = \delta_{ij} \), where \( \delta_{ij} \) is the Kronecker symbol [26].

![Figure 3](image-url)

**Figure 3.** Probability function of the peculiar–velocity field \( v_{\text{rms}} \) (in units of the kick velocity \( v_{\text{kick}} \)) in the flat \( \Lambda \)–dominated model (model 4). After 50000 realisations the simulations produce an Maxwellian distribution, which shows the Gaussian character of each component of the peculiar velocity field, adapted from [26]. The dotted line is a gaussian curve to calculate the expected bulk flow (eq.10).

We note that the peculiar velocity field can be described by a gaussian distribution (i.e. each component of \( \vec{v} \) is a random variable with a gaussian
distribution). This is consistent with the results by other authors (e.g. [19]). The probability distribution of the rms velocity \(v_{\text{rms}}\) is plotted in Fig.3 for the flat \(\Lambda\)-dominated model (model 4) and in Fig.4 for the open model (model 3). For the other models we observe the same behaviour.

Figure 4. The same as in Fig.3 but here for the open model. The solid line is a gaussian curve to calculate the expected bulk flow (eq.11).

We calculate \(v_{\text{rms}}\) from the probability distribution by fitting a gaussian (see Fig.3 and Fig.4). In the models (Table 1) we obtain a peculiar velocity at a scale corresponding to \(L(t_{eq})\) given by (the length scales are calculated with \(H_0 = 100\text{km}/(\text{s}\cdot\text{Mpc})\)) [26]:

\[
v_{\text{pec}}(L_{eq} \approx 70\text{ Mpc})_{\text{closed}} = (460 \pm 200)(G\mu)_{6}(\gamma_s v_s)f\text{ km/s}, \tag{8}
\]

\[
v_{\text{pec}}(L_{eq} \approx 1\text{ Mpc})_{\text{EdS}} = (1740 \pm 760)(G\mu)_{6}(\gamma_s v_s)f\text{ km/s}, \tag{9}
\]

\[
v_{\text{pec}}(L_{eq} \approx 10\text{ Mpc})_{\Lambda,\text{flat}} = (280 \pm 120)(G\mu)_{6}(\gamma_s v_s)f\text{ km/s}, \tag{10}
\]

\[
v_{\text{pec}}(L_{eq} \approx 10\text{ Mpc})_{\text{open}} = (80 \pm 35)(G\mu)_{6}(\gamma_s v_s)f\text{ km/s.} \tag{11}
\]
The length scale corresponding to the time $t_{eq}$ is set to be $0.1H^{-1}(t_{eq})$: 

$$L_{eq} \approx 1.1 \frac{1}{\Omega_0} h_0^{-2} \text{Mpc}$$  

(12)

Figure 5. Expected bulk flows $v_{bulk}$ in cosmic string theory as a function of redshift in a flat, $\Lambda$–dominated model (model 4) and an open model (model 3). Here we assume $(G\mu)_6(\gamma_s v_s) f \approx 1$. The curves would shift if this quantity differs from unity (see text for details). One can see that cosmic strings in open or flat cosmological models predict a “quiet” peculiar velocity field at large scales. Note that in the closed model 1 this is not the case: the peculiar velocities are constant over a scale of $\leq 10000 \text{ km/s}$ and of the order $450 \pm 200 \text{ km/s}$.

Because the primordial spectrum of the density fluctuations in cosmic string theories is nearly of a Harrison–Zeldovich type, the peculiar velocity field is roughly proportional to $L^{-1}$ (see Fig.5). From this follows, for example in the open model (model 3) with $\Omega_0 = 0.1$ at a scale of $L = 50h^{-1} \text{ Mpc}$, a bulk velocity around 20 km/s, whereas in the flat $\Lambda$–dominated model (model 4) the velocity is around 70 km/s. This is inconsistent with the POTENT results (see e.g. [10]), as far as $(G\mu)_6(\gamma_s v_s) f$ is of the order 1, and with the Lauer–Postman result [14], which suggest bulk flows on large scales. These results are, however, consistent with the observed values obtained by Giovanelli et al. [16], which suggest a much more quiescent peculiar velocity field. The closed model fits the POTENT results well at each length scales.
The fact, that many cosmic strings influence the matter in the universe and the peculiar velocity field led us to ask the question if we can then estimate the density parameter from the comparison between the density field and the peculiar velocities. Such investigations are based on the equation
\[ \nabla \cdot \mathbf{v} = -\beta H \delta, \]
with \( \beta = \Omega_0^{0.6} \), which holds in linear perturbation theory for an irrotational fluid\(^3\). However, even if there is a relation between velocity and density field, eq.(13) is derived from the continuity equation and one can give examples in which an overestimated \( \Omega_0 \) could be derived from eq.(13) (see the discussion for the case of the explosion scenario in \[\textcite{3}\]). In the case of strings one can easily show that there is an connection between density and peculiar velocity field of the form
\[ \nabla \cdot \mathbf{v} = -\beta_{\text{eff}} H \delta, \]
with
\[ \beta_{\text{eff}} = \frac{\sum_{i,j} \delta_{i,j} \beta_{\text{eff},i,j}}{\sum_{i,j} \delta_{i,j}}. \]

Here, \( \delta_{i,j} \) is the density fluctuation created by the string \( i,j \) and \( \beta_{\text{eff},i,j} \) is an “effective” value which connects the density and the velocity field of the string \( i,j \), see \[\textcite{26}\]. In general, \( \beta_{\text{eff}} \) should be different from \( \Omega_0^{0.6} \). This let us to conclude that in general on can not obtain \( \Omega_0 \) from a comparison of the density and the velocity field if cosmic strings where responsible for structure formation \[\textcite{26}\]. However, important for this conclusion is the knowledge of the length scale at which non–gaussianity becomes important\(^4\). One has to find this length scale with numerical simulations.

4. Non–gaussianity in cosmic string models

Avelino et al. investigated this question for CDM and HDM models in detail \[\textcite{2}\]. They calculated the non–gaussian scale \( R_{\text{ng}} \), below which the non–gaussian statistics of the density field is very important. For scales larger than \( R_{\text{ng}} \), the statistics of the density field can be described by a gaussian distribution to high accuracy. We discuss here the case of CDM.

In order to find \( R_{\text{ng}} \), Avelino et al. calculated the skewness and kurtosis of the smoothed density field (i.e. of the probability density function) \[\textcite{2}\]. Because the kurtosis and the skewness are zero for a gaussian distribution, these statistics give a hint for non–gaussianity if they are non–zero. Avelino et al. also calculated the genus curves of the density field, which is also a good indicator for the topology of the density field.

\(^3\) Cosmic strings might generate vorticity also at large scales. However, these quickly decay by cosmic expansion (\( \propto R^{-2} \)) so that we can neglect it at the current epoch.

\(^4\) I am grateful to Pedro Avelino for pointing out this to me.
Figure 6. The tails of the effective probability function $fP(y)$ for gaussian and cosmic string models on the scale $R_{ng}$. The cosmic string data are taken from \cite{2}. $y = \delta/\sigma_R$, where $\delta$ is the density contrast and $\sigma_R$ is the mass fluctuation at the scale $R_{ng}$, see the text for details.

As a result they found $R_{ng} = 1.5(\Omega_0 h^2)^{-1} \text{ Mpc}$. For models with $\Omega_0 \approx 1$ this length scale is very small compared to 50 Mpc at which large scale flows are observed. Therefore, for these models the density and peculiar velocity field are related by equation (13). In fact, only for models with $\Omega_0 h^2 \leq 0.1$ the scale is large enough that a significant departure from eq. (13) is important (for example model 1). Then eq. (14) holds. Cosmic strings in such very low density models might be ruled out by the correlation analysis in Fourier space, see Stirling & Peacock \cite{21}. They found from the combined QDOT and 1.2–Jy IRAS galaxy survey that the modes with wavelength $\lambda > 30h^{-1} \text{ Mpc}$ are in good agreement with a gaussian distribution. While this is a provisional conclusion, with future observations this test will place limits on the degree of non–gaussianity in the density fluctuation field on small and large scales. For another discussion on non–gaussianity see \cite{18}.

In low–density models with $\Omega_0 h = 0.1...0.2$ the length scale $R_{ng}$ is about 10 Mpc. From this length scale clusters collapse. Therefore, different predictions of the number density of cluster of galaxies are expected. In fact, clusters are used to test cosmological models, see e.g. \cite{5} and references therein. Here we use the Press–Schechter method to calculate the abundances of clusters in cosmic string theory. Because of non–gaussianity we have to use a modified approach, see e.g. \cite{9}.
We consider the smoothed density field $\delta$. The probability density function $p(\delta)$ can be described by a rescaled function $P(y)$ with $y = \delta/\sigma_R$, where $\sigma_R$ is the mass fluctuation at the scale $R$. We assume $p(\delta) = P(y)/\sigma_R$ which should hold in the theories considered here and on the relevant length scales. The probability density function $P(y)$ is shown for cosmic string models and gaussian models in Fig.6.

In the Press–Schechter formalism the number density $n(R)$ of collapsed objects (collapsed from a radius $R$) is given by

$$n(R)dR = \frac{3}{4\pi R^3} \left| \int_{y_c}^{\infty} f P(y)dy \right| dR,$$

where $f = 1/\int_{0}^{\infty} P(y)dy$ and $y_c = \delta_c/\sigma_R$. Here $\delta_c$ is the critical density contrast for collapse. We assume the spherical collapse model with $\delta_c = 1.68$, because this quantity depends only very weakly on the cosmological model [11]. From the power spectrum we calculated $\sigma_R$. It can be shown that $\sigma_R \propto R^{-\alpha}$, where $\alpha \approx 1$. Our results for the range 1 keV to 10 keV depend not very strongly on the slope of the power spectrum if $\alpha = 0.75...1.25$, which was also observed by others [11]. We give details of our calculations elsewhere [27].

Because the length scale $R$ from which a cluster collapsed is not an observable, we have to relate $R$ with the cluster mass $M$ or the X–ray temperature of the cluster gas. Here we calculate the temperature function and assume a spherical cluster collapse. The resulting cluster is described by an isothermal sphere with a radius of virialisation equal to half its maximum expansion radius (e.g. [30]). Then, for a hydrogen mass fraction of 0.76,

$$kT = 8.6 \text{ keV } \Omega_0^{2/3} \left( \frac{R}{10h^{-1}\text{Mpc}} \right)^2 \left( \frac{\Omega_0}{\Omega(z_f)} \right)^{1/3} \left[ \frac{\Delta_c}{178} \right]^{1/3},$$

where $z_f$ is the redshift of collapse, which we calculate with the spherical collapse model. We calculate the number density $n(> kT)$ of clusters which X–ray temperatures greater than $kT$ (temperature function).

Because high density peaks are much more common in cosmic string models as in gaussian models, for a given density parameter, cosmological constant, Hubble parameter and mass fluctuation at 8 $h^{-1}$ Mpc, cosmic strings predict more clusters with high X–ray temperatures as compared to gaussian models. This can be seen in Fig.7. If we change $\sigma_8$ the temperature function changes (Fig.8). Also shown are the data by Henry & Arnaud (1991) (the vertical bars with the range of uncertainty). We found that $\sigma_8$ should lie in the range 0.6 – 0.7 to fit the data (for a flat model with $\Omega_0 = 0.3$). The reader is referred to our forthcoming work, where more data will be used and where the errors, which could enter the calculations, are discussed in detail [27].
Figure 7. Number densities of clusters of galaxies as a function of the temperature $kT$ in a gaussian and a cosmic string model. The cosmological parameters are $\Omega_0 = 0.3$, $\lambda_0 = 0.7$, $h = 0.7$ and $\sigma_8 = 0.75$.

Similar to gaussian models we find that the temperature function (and also the mass function) are good discriminators for non–gaussian models. We refer to our future work, where these functions and cluster evolution will be discussed in some depth [27].

We are not able to discuss cluster abundances in the baryonic model 1 of Tab.1 because we don’t know the probability function on the scale relevant for cluster formation.

In conclusion we point out, that several uncertainties enter the analysis:

- We concentrate here only on the wakes. How important are cosmic string loops for structure formation? Is filament formation due to slow moving strings also important? If these mechanism turn out to be important, the probability density function of the density field will certainly change.

- How does the cluster gas evolve and what is its impact on the temperature function (e.g. [22])? Here, an detailed analysis of the formation and evolution of wakes seems necessary, see [23, 29].
Figure 8. Number density of clusters of galaxies in cosmic string models with $\Omega_0 = 0.3$, $\lambda_0 = 0.7$, $h = 0.7$ and different $\sigma_8$ as a function of the X–ray temperature. Shown are also the temperature function according to Henry & Arnaud [17].

- How good is the spherical collapse model? What is the typical formation redshift of a cluster?

- Is the Press–Schechter formalism good enough to describe the statistics of high–density peaks in non–gaussian models?

At the present time we are far from conclusive answers for the questions above.

The case for HDM models + cosmic strings is not discussed here. But the simulations suggest that on cluster scales the probability density function is gaussian in these models and cosmic strings may have the same problems as in “usual” gaussian models [5]. Note, however, that it is important to know which kind of perturbations are important in cosmic string theory: wakes, filaments and/or loops are possible, for a discussion see e.g. [8, 31].

5 However, see [20] for a discussion of the formation of high redshift objects in cosmic strings + HDM.
5. Discussion

We have discussed some aspects of the cosmic string theory of structure formation. Our results suggest that low density models are very encouraging for cosmic string theory. Problems might arise if the peculiar velocity field is observed to be of the order of 300 km/s at large scales ($cz \geq 3000$ km/s). Then only models with $\Omega_0 h \leq 0.1$ and a cosmological constant could produce such large flows at these scales (if $(G\mu_0(\gamma_s v_s)f \approx 1$). Such models predict non–gaussian features at these scales and could be constrained with topological statistics, see e.g. [18] or with correlation analysis in Fourier space, see e.g. [21].

Further observational efforts will help to decide if cosmic strings can be responsible for structure formation. In particular if the peculiar velocity field is better constrained from observations at large scales and if we understand cluster formation and evolution more thoroughly, we should able to test these theories apart from CMB observations and matter distribution.

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