GAUGE AND YUKAWA UNIFICATION IN MODELS WITH GAUGE-MEDIATED SUPERSYMMETRY BREAKING

JONATHAN A. BAGGER, a
KONSTANTIN T. MATCHEV, a
DAMIEN M. PIERCE b and
REN-JIE ZHANG a

a Department of Physics and Astronomy
Johns Hopkins University
Baltimore, Maryland 21218

b Stanford Linear Accelerator Center
Stanford University
Stanford, California 94309

Abstract

We examine gauge and Yukawa coupling unification in models with gauge-mediated supersymmetry breaking. We work consistently to two-loop order, and include all weak, messenger and unification-scale threshold corrections. We find that successful unification requires unification-scale threshold corrections that are in conflict with the minimal SU(5) model, but are consistent with the modified missing doublet SU(5) model for small tan β, and large tan β with μ > 0.

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The apparent unification of the gauge couplings in the minimal supersymmetric standard model (MSSM) [1] has sparked much interest in supersymmetric extensions to the standard model. In their present form, most phenomenologically viable models have two sectors: a hidden sector, in which supersymmetry is broken, and a visible sector, which contains the standard-model particles and their supersymmetric partners. Supersymmetry breaking is transmitted to the visible sector by gravitational interactions (as in supergravity-inspired models) or by standard-model gauge interactions (as in models with gauge-mediated dynamical supersymmetry breaking).

Models with gauge-mediated supersymmetry breaking are usually constructed to preserve gauge coupling unification to one-loop order. In this letter we will report on a closer look at unification in gauge-mediated models. We will present the results of a complete two-loop analysis for gauge and Yukawa coupling unification. Our computation takes all one-loop thresholds into account, including those at the weak, messenger and unification scales. The thresholds include finite terms which turn out to be very important for our precision analysis.

We will present our results in terms of the model-independent unification-scale threshold corrections $\epsilon_g$ and $\epsilon_b$ [2]. These parameters describe conditions that must be satisfied by any viable unification model. We will illustrate the range of these parameters for the minimal [3] and (modified) missing-doublet [4, 5] SU(5) models. We will see that present precision measurements exclude the minimal model, but are consistent with gauge and Yukawa unification in the modified missing-doublet case.

In the simplest models of gauge-mediated supersymmetry breaking [6], the messenger sector contains a set of vector-like fields which couple only to a standard-model singlet spurion through trilinear terms in the superpotential. The vector-like messenger fields are chosen to transform in $5 + \overline{5}$ or $10 + \overline{10}$ representations of SU(5). Requiring the gauge couplings to remain perturbative restricts attention to at most four $5 + \overline{5}$ or one $10 + \overline{10}$ plus one $5 + \overline{5}$ pair of fields. (An additional $5 + \overline{5}$ pair can be accommodated if the messenger particles are sufficiently heavy.)

We assume that the lowest ($S$) and highest ($F$) components of the spurion acquire vevs through their interactions with the hidden sector. These interactions remove the mass degeneracy of the messenger superfields and transmit supersymmetry breaking from the hidden to the visible sector through loop diagrams which contain spurion insertions. At the messenger scale, gaugino and soft scalar masses are induced by one-loop and two-loop diagrams, respectively. The flavor-blind nature of the gauge interactions ensures that flavor-changing neutral currents are suppressed. To this order, the soft supersymmetry-breaking $A$-parameter is not generated.

The supersymmetric Higgs mass parameter $\mu$ and the soft supersymmetry-breaking $B$-parameter violate a Peccei-Quinn symmetry and cannot be generated by standard-model gauge interactions. We will assume that they are generated by some minimal mechanism. The region where $B = 0$ is theoretically appealing [7] because it gives rise to a large ratio of vevs ($\tan \beta$) without fine tuning. In this region, all CP-violating phases are generated only radiatively, so CP violation is naturally small.

Our approach is as follows. We start with the Fermi constant, $G_F$, the electromagnetic coupling, $\alpha_{em}$, the Z-boson mass, $M_Z$, the $\overline{\text{MS}}$ strong coupling constant, $\alpha_s(M_Z)$, and the top-, bottom-quark and tau-lepton pole masses, $m_t$, $m_b$ and $m_\tau$ (for details, see
We then assume a supersymmetric spectrum and use the full one-loop corrections to calculate the DR couplings $g_1, g_2, g_3, \lambda_t, \lambda_b$ and $\lambda_\tau$ for a given value of tan $\beta$. We run these couplings to the messenger scale, $M$, using the two-loop MSSM renormalization group equations. At $M$ we fix the gaugino and soft scalar masses [9]. We then run the soft parameters back to the squark mass scale, where we impose electroweak symmetry breaking and calculate the supersymmetric spectrum. We iterate the procedure several times to achieve a consistent solution.

Our calculations of the one-loop threshold corrections include the finite and logarithmic terms. The finite corrections, which are often neglected in the literature, allow a precise determination of the gauge couplings $g_1$ and $g_2$ at the scale $M_Z$ [2, 10]. The finite corrections to the bottom and tau Yukawa couplings also play an important role in our analysis.

Once we determine the gauge and Yukawa couplings at the messenger scale, we extrapolate them to the unification scale, $M_{GUT}$, which we define to be the scale where $g_1$ and $g_2$ meet. We use the usual two-loop beta functions to compute the evolution of the gauge and Yukawa couplings. We also include the messenger contributions, those listed in Ref. [11, 12], and [13]

$$\frac{\mu dg_i}{d\mu} = - \frac{g_i^3}{16\pi^2} \left( \sum_f D_{if} y_f^2 \right) + \ldots$$ (1)

$$\frac{\mu d\lambda_a}{d\mu} = (n_5 + 3n_{10}) \frac{\lambda_a}{16\pi^2} \left( \sum_{i=1}^3 C_{ai} g_i^4 \right) + \ldots$$ (2)

The sum over $f$ runs over all messenger multiplets, $n_5$ and $n_{10}$ are the number of 5 + $\overline{5}$ and 10 + $\overline{10}$ messenger fields, and

$$D_{if} = \begin{bmatrix} \frac{4}{5} & \frac{6}{5} & \frac{2}{5} & \frac{16}{5} & \frac{12}{5} \\ 0 & 2 & 6 & 0 & 0 \\ 2 & 0 & 4 & 2 & 0 \end{bmatrix}, \quad f = d, \ell, q, u, e,$$ (3)

$$C_{ai} = \begin{bmatrix} \frac{13}{5} & 3 & \frac{16}{3} \\ \frac{7}{5} & 3 & \frac{16}{3} \\ \frac{9}{5} & 3 & 0 \end{bmatrix}, \quad a = t, b, \tau.$$ (4)

At $M_{GUT}$ we set the messenger Yukawas to a common value, $y_m$. We run the messenger Yukawas back to the messenger scale according to their one-loop evolution equations,

$$\frac{\mu dy_f}{d\mu} = \frac{y_f}{16\pi^2} \left( 2y_f^2 + T - 4 \sum_{i=1}^3 g_i^2 C_i(f) \right),$$ (5)

where $T = n_5(3y_d^2 + 2y_{\ell}^2) + n_{10}(6y_q^2 + 3y_u^2 + y_e^2)$ and the $C_i$’s are the quadratic Casimirs, $3Y^2/5$, $3/4$, and $4/3$ for fundamental representations. (The messenger-Yukawa evolution equations can receive additional model-dependent contributions from the hidden-sector particles. The extra terms do not affect the messenger mass splittings, so we can ignore them in our analysis. Note that the one-loop equations suffice because the
messenger-sector Yukawas enter our calculation only through the messenger threshold corrections.

From the set of $y_f(M)$, we determine the messenger-particle mass spectrum and compute the messenger-scale threshold corrections to the gauge couplings,

$$\Delta \alpha_i^{-1}(M) = \sum_f \frac{D_{ij}}{8\pi} \left[ \ln \frac{M_i^2}{M_j^2} + \frac{1}{6} \ln \left( 1 - \frac{\Lambda^2}{M_j^2} \right) \right],$$

(6)

where $\Lambda = F/S$ is the supersymmetry-breaking scale and $M_j$ is the messenger fermion mass. The second term in the brackets is small for $\Lambda/M_j \ll 1$, in which case there is a near degeneracy among the masses in the vector-like supermultiplets. Note that there are no messenger-scale Yukawa thresholds to this order.

We iterate this procedure to find a consistent solution in the region between $M$ and $M_{GUT}$. At $M_{GUT}$ we define the threshold corrections for the gauge and Yukawa couplings, $\epsilon_g$ and $\epsilon_b$, as follows,

$$g_3(M_{GUT}) = g_3(M_{GUT})(1 + \epsilon_g),$$

$$\lambda_b(M_{GUT}) = \lambda_b(M_{GUT})(1 + \epsilon_b).$$

(7)

The parameters $\epsilon_g$ and $\epsilon_b$ describe the unification-scale threshold corrections that are necessary to achieve unification in any particular model. In what follows, we will indicate the allowed ranges of $\epsilon_g$ and $\epsilon_b$ for two of the simplest unification models, the minimal and the modified missing-doublet SU(5) models.

In the minimal SU(5) model, the unification-scale gauge threshold correction is

$$\epsilon_g = \frac{3 g_3^2}{40\pi^2} \ln \left( \frac{M_H}{M_{GUT}} \right),$$

(8)

where $M_H$ is the mass of the color-triplet Higgs multiplet that mediates nucleon decay. Generally, $M_H$ is bounded from below by the proton decay limits [14], which imply $M_H \gtrsim M_{GUT}$, so $\epsilon_g \gtrsim 0$.

The missing-doublet model is an alternative SU(5) theory in which the heavy color-triplet Higgs particles are split naturally from the light Higgs doublets [4]. This requires large SU(5) representations, such as the 75 and 50 + $\overline{50}$, so the SU(5) coupling $g_5$ diverges below the Planck scale. The modified missing-doublet (MMD) model solves this problem for $n_5 \leq 1$ by lifting the mass of the 50 + $\overline{50}$ to the Planck scale and suppressing the nucleon decay rate through an extra Peccei-Quinn symmetry [5]. In this way the modified missing doublet model can accommodate two color-triplet Higgs particles with masses between $10^{13} - 10^{15}$ GeV.

In the modified missing-doublet model, the unification-scale gauge threshold can be written as

$$\epsilon_g = \frac{3 g_3^2}{40\pi^2} \left\{ \ln \left( \frac{M_{H}^{\text{eff}}}{M_{GUT}} \right) - 9.72 \right\},$$

(9)

where $M_{H}^{\text{eff}}$ is the effective mass that enters the proton decay amplitude, so the previous lower bounds on $M_H$ apply here as well. In the MMD case, the effective mass is also bounded from above, $M_{H}^{\text{eff}} \lesssim 10^{20}$ GeV [3].
The Yukawa threshold in minimal SU(5) can be written as follows \[2, 14\]:

\[
\epsilon_b = \frac{1}{16\pi^2} \left\{ 4g_{GUT}^2 \left[ \ln \left( \frac{M_V}{M_{GUT}} \right) - \frac{1}{2} \right] - \lambda_t^2(M_{GUT}) \left[ \ln \left( \frac{M_H}{M_{GUT}} \right) - \frac{1}{2} \right] \right\}, \tag{10}
\]

where \( M_V \) is the mass of a superheavy SU(5) gauge boson. For the minimal SU(5) model, the most stringent lower limit on \( M_V \) comes from requiring that the 5 + \( \overline{5} \) Higgs coupling remain perturbative to the Planck scale. This implies \( M_V \gtrsim 0.5 M_H \) \([15]\). We take the upper limit on \( M_V \) to be the Planck scale, \( M_V \lesssim 10^{19} \text{ GeV} \).

For the modified missing-doublet model, the Yukawa threshold has the same form as eq. (10), with the color-triplet Higgs mass, \( M_H \), replaced by the effective mass, \( M_{H}^{\text{eff}} \). In this case, the lower limit on \( M_V \) comes from proton decay experiments, which imply \( M_V/g_{GUT} \gtrsim 3.8 \times 10^{15} \text{ GeV} \) \([18]\). As before, we impose \( M_V \leq 10^{19} \text{ GeV} \). Hence, both models have the same upper limit on \( \epsilon_b \), but the lower limit in the MMD model is lower, by virtue of the fact that \( M_V \) can be smaller and \( M_H \) larger.

In what follows, we present our results for gauge-mediated models. In particular, we calculate \( \epsilon_g, \epsilon_b, \alpha_{GUT} \) and \( M_{GUT} \) as functions of the input parameters, which we take to be \( \tan \beta \), the numbers \( n_5 \) and \( n_{10} \), the supersymmetry-breaking scale \( \Lambda \), the messenger scale \( M \), and the messenger Yukawa at the unification scale, \( y_m \). To examine bottom-tau unification, we fix the sign of \( \mu \) to be positive.

We find the range of \( \alpha_{GUT} \) and \( M_{GUT} \) by scanning over the parameter space, with \( m_t = 175 \text{ GeV}, m_b = 4.9 \text{ GeV} \), \( \Lambda \leq 300 \text{ GeV}, 1.03 \leq M/\Lambda \leq 10^4 \), \( 0.03 \leq y_m \leq 3.0 \) and \( \tan \beta \) in the allowed range. For the case \( n_5 = 1 \), we determine \( \alpha_{GUT} \simeq (0.044 - 0.054) \) and \( M_{GUT} \simeq (1.5 - 5.0) \times 10^{16} \text{ GeV} \). For \( n_5 = 10 \), we find \( \alpha_{GUT} \simeq (0.062 - 0.28) \) and \( M_{GUT} \simeq (1.2 - 7.0) \times 10^{16} \text{ GeV} \).

In Fig. 4 we plot \( \epsilon_g \) and \( \epsilon_b \) for \( n_5 = 1 \), \( M/\Lambda = 2 \), \( m_b = 4.9 \text{ GeV} \) and \( y_m = 1 \), versus \( \Lambda \) and \( \tan \beta \), respectively. In (a) we choose \( \tan \beta = 20 \), while in (b) we take \( \Lambda = 100 \text{ TeV} \). In each case the short-dashed (long-dashed) lines correspond to \( \alpha_s(M_Z) = 0.124 \) (0.112). The black bands correspond to \( \alpha_s(M_Z) = 0.118 \) with \( m_t \) varying from 170 to 180 GeV. The uncertainty in \( \epsilon_b \) from varying \( m_b = 4.9 \pm 0.3 \text{ GeV} \) is almost the same as that from changing \( \alpha_s(M_Z) = 0.118 \pm 0.006 \).

In Fig. 4(a) we also show the allowed values for \( \epsilon_g \) in the minimal and modified missing-doublet SU(5) models. The region of allowed \( \epsilon_g \) in the modified missing-doublet model almost completely overlaps the region with \( \alpha_s(M_Z) = 0.118 \pm 0.006 \). In contrast, we see that minimal SU(5) is inconsistent with \( \alpha_s(M_Z) \) by more than 2\( \sigma \).

For \( n_5 = 1 \) we find that the messenger sector corrections decrease \( \epsilon_g \). The change is induced by the messenger thresholds and the differences in the two-loop gauge coupling evolution. Both of these effects are of approximately equal importance.

From Fig. 4(a) we see that raising the supersymmetry-breaking scale \( \Lambda \) decreases the size of the gauge-coupling unification-scale threshold. This is because the superpartner masses scale with \( \Lambda \), and larger masses decrease the size of the required thresholds \([2, 14]\).

Figure 4(b) illustrates the well-known fact that bottom-tau unification can only be achieved for very small (\( \lesssim 1.8 \)) or rather large (\( \gtrsim 35 \)) \( \tan \beta \). (Very large values of \( \tan \beta \) are excluded by the requirement of proper electroweak symmetry breaking.) Figure
Figure 1: The unification-scale threshold corrections with \( n_5 = 1, \mu > 0, \ M/\Lambda = 2, \) and \( y_m = 1. \) (a) The gauge coupling unification-scale threshold correction \( \epsilon_g \) versus \( \Lambda, \) for \( \tan \beta = 20, \) and \( \alpha_s(M_Z) = 0.118 \) (black band), 0.124 (short-dashed) and 0.112 (long-dashed). (b) The Yukawa coupling unification-scale threshold correction, \( \epsilon_b, \) versus \( \tan \beta, \) for \( \Lambda = 100 \text{ TeV} \) and the same values for \( \alpha_s(M_Z) \) as in (a). In each case, the black band is obtained by varying the top mass from 170 to 180 GeV. The shaded regions indicate the allowed range for (a) \( \epsilon_g \) and (b) \( \epsilon_b \) in the minimal and modified missing-doublet SU(5) models.

(b) also shows the allowed bands for \( \epsilon_b \) in the minimal and modified missing-doublet SU(5) models.

As above, we can compare this plot to the case with no messengers. There, one typically finds that the bottom and tau Yukawa couplings meet much earlier than the scale \( M_{\text{GUT}}, \) so a rather large and negative threshold correction \( \epsilon_b \) is required. For the case at hand, the extra messenger multiplets change the Yukawa evolution equations at two loops. More importantly, however, they also increase the gauge couplings, which feed into the Yukawa evolution equations and cause the bottom and tau couplings to meet even earlier. This makes \( \epsilon_b \) even more negative.

Fortunately, at large \( \tan \beta \) there are significant finite threshold corrections to the bottom (and to a smaller extent, tau) Yukawa couplings \[8]. These corrections, which are proportional to \( \mu \tan \beta, \) are sufficiently important to permit bottom-tau unification at large \( \tan \beta \) for \( \mu > 0. \) (The case \( \mu < 0 \) is completely excluded at large \( \tan \beta. \)) These finite corrections were omitted in the analysis of Ref. \[12\], which came to a different conclusion.

For \( n_5 = 1, \) the value of \( \epsilon_g \) is not significantly affected by changes in \( \tan \beta \) or \( M/\Lambda. \) At the smallest values of \( \tan \beta, \) \( \epsilon_g \) increases by about 0.5%, while for \( M/\Lambda = 10^4, \) \( \epsilon_g \) increases by about 0.2%. The parameter \( \epsilon_b \) is more sensitive to changes in \( M/\Lambda. \) For
$M/\Lambda = 10^4$, the $\epsilon_b$ curve is 2.5 to 3% higher at intermediate $\tan\beta$, and rises to $+20\%$ at $\tan\beta \simeq 40$.

In Fig. 2 we plot $\epsilon_g$ and $\epsilon_b$ for the case of $n_5 = n_{10} = 1$, versus $\Lambda$ and $\tan\beta$ respectively. The other parameters are fixed as in Fig. 1, except that in Fig. 2(b), $\Lambda = 50$ TeV (to keep the scalar masses unchanged) and $M/\Lambda = 100$. (Two $M/\Lambda = 2$ curves are shown in dotted lines.) Figure 2(a) shows that everything shifts because of the larger $\alpha_{\text{GUT}}$, but the overlap between the band from the MMD model and the allowed region for $\alpha_s(M_Z)$ is still almost complete. In this case, increasing $M/\Lambda$ to $10^4$ significantly changes Fig. 2(a). The central value of $\epsilon_g$ runs from $-4\%$ for $\Lambda = 20$ TeV to $-1.5\%$ for $\Lambda = 100$ TeV. The band for the MMD model is such that the required value of $\epsilon_g$ lies entirely within the band. (Note, however, that $n_5 = n_{10} = 1$ gives rise to nonperturbative couplings above $M_{\text{GUT}}$ in the MMD case.)

The change in Fig. 2(b) as compared to Fig. 1(b) is more dramatic. Because the gauge couplings are even larger than in the previous case, bottom-tau unification turns out to be barely possible for $M/\Lambda = 2$ (dot-dashed lines). Note, however, that there is still a significant region for unification in the missing doublet model with $M/\Lambda = 100$. In Fig. 2(b) we also show the necessary threshold, $\epsilon_t$, for top-tau Yukawa unification. We see from Fig. 2(b) that the top, bottom and tau couplings unify at the largest values of $\tan\beta$ (in the region where $B \simeq 0$). Such a unification is expected in SO(10) models. However, the thresholds in any particular SO(10) model must be calculated to be sure the model is consistent with data.
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