PERIODIC AND ALMOST PERIODIC OSCILLATIONS IN A DELAY DIFFERENTIAL EQUATION SYSTEM WITH TIME-VARYING COEFFICIENTS

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Abstract. It is extremely difficult to establish the existence of almost periodic solutions for delay differential equations via methods that need the compactness conditions such as Schauder’s fixed point theorem. To overcome this difficulty, in this paper, we employ a novel technique to construct a contraction mapping, which enables us to establish the existence of almost periodic solution for a delay differential equation system with time-varying coefficients. When the system’s coefficients are periodic, coincide degree theory is used to establish the existence of periodic solutions. Global stability results are also obtained by the method of Liapunov functionals.

1. Introduction. Recently there has been increasing attention to migrant workers. The number of migrant workers in many countries has been increasing dramatically. This is particularly true in China, where millions of individuals from the rural area temporarily leave their home seeking for employment in urban cities every year. The number of migrant workers in China was as high as 281.71 million in 2016 according to the national migrant workers monitoring report [25]. Studies have shown that migrant workers are much more vulnerable to infectious diseases such as tuberculosis, HIV and other sexually transmitted infections [18, 24]. To study the influence of temporary migration on the transmission of infectious diseases in migrant workers’ home residence, Wang and Wang proposed and studied the following model in [27]

\[
\begin{align*}
S'(t) &= \Lambda - \mu_SS(t) - \beta S(t)I(t) - m_S S(t) + m_S (1 - p) e^{-\delta \tau} S(t - \tau), \\
I'(t) &= \beta S(t)I(t) - \gamma I(t) - \mu_I I(t) + m_I S e^{-\delta \tau} S(t - \tau), \\
R'(t) &= \gamma I(t) - \mu_R R(t) - m_R R(t) + m_R e^{-\delta \tau} R(t - \tau).
\end{align*}
\] (1.1)

Here \(S(t), I(t)\) and \(R(t)\) denote the population sizes at time \(t\) of three disjoint compartments, namely, susceptible, infectious and recovered, respectively. The constant \(\Lambda > 0\) is the recruitment rate of the susceptibles, \(\beta > 0\) is the infection transmission coefficient, \(\mu_S, \mu_I, \mu_R\) are the natural death rates of susceptible, infectious and
recovered individuals, respectively, and $\gamma$ is the rate at which infectious individuals recover. $m_S$ and $m_R$ denote the rates at which susceptible and recovered individuals migrate, respectively. The time delay $\tau$ measures the average time that a migrant worker spends working away from his/her home residence. It is assumed in (1.1) that a migrant worker has a probability (without taking death into account) of $p(\tau)$ to return home with an infection.

One notable phenomenon is that many migrant workers go back to their home residence for traditional holidays. In China, traditional holidays such as the Spring Festival are determined by the Chinese calendar. Compared to the Western calendar, the Chinese calendar’s years and months are irregular. In addition, some holidays in China are determined according to the Western calendar. For example, May 1st is the Labor’s Day, and October 1st is the National Day and there is a 7-day break for each of these holidays. Figure 1 shows how the major Chinese holidays vary according to the Western calendar during 2012-2016. Clearly, the distributions of holidays in China are not precisely periodic, but rather almost periodic. For the definition of an almost periodic function, we refer the reader to [9, 10].

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{holidays.png}
\caption{Distributions of holidays in China for the period of 2012-2016.}
\end{figure}

The special mobility pattern of migrant workers motivates us to generalize model (1.1) to the case with periodic/almost periodic coefficients. More precisely, in this paper, we are concerned with the following delay differential equation system with time-varying coefficients:

\begin{equation}
\begin{aligned}
S'(t) &= \Lambda(t) - \mu_S S(t) - \beta(t) S(t) I(t) - m_S(t) S(t) + m_S(t)(1 - p)e^{-\delta\tau} S(t - \tau), \\
I'(t) &= \beta(t) S(t) I(t) - \gamma I(t) - \mu_I I(t) + m_S(t)p e^{-\delta\tau} S(t - \tau), \\
R'(t) &= \gamma I(t) - \mu_R R(t) - m_R(t) R(t) + m_R(t)e^{-\delta\tau} R(t - \tau),
\end{aligned}
\end{equation}

where $\Lambda(t)$, $\beta(t)$, $m_S(t)$ and $m_R(t)$ are almost periodic (including periodic case as a special case).

Mathematically, the analysis of (1.2) is much more challenging, since traditional tools such as Schauder’s fixed point theorem require compactness of the family of almost periodic functions, which indeed is extremely difficult to obtain. To overcome this difficulty, we employ a novel technique to construct a contraction mapping and establish the existence result by means of the contraction mapping theorem.

We point out that the history of mathematical modeling on infectious diseases could be traced back to Bernoulli’s work on the spread of smallpox [21]. The classical foundation work is due to Kermack and McKendrick [19]. The SARS outbreak in 2003 [24], the 2009 H1N1 pandemic [31] and the 2014 Ebola virus epidemic in West Africa [2, 11] promoted further interest and efforts in disease modeling. Many aspects have been incorporated into disease modeling in order to accurately
Preliminaries. For convenience, we first introduce some notations. Set \( AP(\mathbb{R}, \mathbb{R}_+) = \{ x(t) : x(t) \in C(\mathbb{R}, \mathbb{R}_+), x(t) \) is an almost periodic function \}. Define \( \| x(t) \| = \sup_{t \in \mathbb{R}} |x(t)|, \forall x \in AP(\mathbb{R}, \mathbb{R}_+) \) or \( \forall x \in C(\mathbb{R}, \mathbb{R}_+) \), then \( AP(\mathbb{R}, \mathbb{R}_+) \) and \( C(\mathbb{R}, \mathbb{R}_+) \) are Banach spaces with the norm \( \| \cdot \| \).

Since continuous periodic or almost periodic functions are bounded and for a continuous periodic or almost periodic function \( f \), we define

\[
 f^* = \max_{\mathbb{R}} f(t) \quad \text{and} \quad f_* = \min_{\mathbb{R}} f(t).
\]

Lemma 2.1. For any positive continuous function \( f \) in \( \mathbb{R} \), the following inequality holds

\[
 \int_{-\infty}^{t} e^{-\int_{r}^{t} f(r)dr} f(s)ds \leq 1.
\]

Proof. Since the positive function \( f \) is continuous on \( \mathbb{R} \), one can get \( e^{-\int_{-\infty}^{t} f(r)dr} \geq 0 \). We further have

\[
 \int_{-\infty}^{t} e^{-\int_{r}^{t} f(r)dr} f(s)ds = \int_{-\infty}^{t} d\left( e^{-\int_{r}^{t} f(r)dr} \right) = 1 - e^{-\int_{-\infty}^{t} f(r)dr} \leq 1.
\]

Theorem 2.2. Consider \( (1.2) \) with initial conditions \( S(\theta) = \varphi(\theta) \geq 0, I(0) > 0 \) and \( R(\theta) \geq 0 \) for \( \theta \in [-\tau, 0] \), where \( \varphi \) is continuous on \( [-\tau, 0] \) with \( \varphi(0) > 0 \). Then system \( (1.2) \) has a unique solution \( (S(t), I(t), R(t)) \) which is positive for \( t > 0 \).

Proof. By the method of steps [15], for any given initial condition satisfying \( S(\theta) = \varphi(\theta) \geq 0, I(0) > 0 \) and \( R(\theta) \geq 0 \) for \( \theta \in [-\tau, 0] \) with \( \varphi(0) > 0 \), system \( (1.2) \) has a unique solution on \( (0, +\infty) \).

First we prove \( S(t) \) and \( I(t) \) are positive for \( t \in (0, \tau] \). Suppose on the contrary, there is \( t_1 \in (0, \tau] \) such that \( S(t_1)I(t_1) = 0 \), \( S(t) > 0 \) and \( I(t) > 0 \) for \( t \geq 0 \). Then we have either (i): \( I(t_1) = 0 \) and \( S(t) \geq 0 \) for \( t \in [0, t_1] \) or (ii): \( S(t_1) = 0 \) with \( S'(t_1) \leq 0 \) and \( I(t) \geq 0 \) for \( t \in [0, t_1] \). For case (i), it follows from \( (1.2) \) that \( I'(t) \geq -(\gamma + \mu_I)I(t) \) for \( t \in [0, t_1] \). Thus, \( I(t_1) \geq I(0)e^{-\int_{0}^{t_1} (\gamma + \mu_I)dr} > 0 \). A contradiction. For case (ii), it follows from the first equation of \( (1.2) \) that \( S'(t_1) \geq \Lambda(t_1) - \mu_S(t_1)S(t_1) - \beta(t_1)S(t_1)I(t_1) - \mu_S(t_1)S(t_1) = \Lambda(t_1) > 0 \), which is a contradiction. Thus, \( S(t) \) and \( I(t) \) are positive for \( t \in (0, \tau] \). If there is a \( t_2 \in (0, \tau] \) such that \( R(t_2) = 0 \) and \( R(t) \geq 0 \) for \( t \in [0, t_2] \), then by the third equation of system \( (1.2) \) and note that \( I(t) > 0 \) for \( t \in [0, \tau] \), one can easily get

\[
 R(t_2) > R(0)e^{-\int_{0}^{t_2} (\mu_R + m_R(r))dr} \geq 0,
\]

a contradiction. Hence, \( S(t) > 0, I(t) > 0 \) and \( R(t) > 0 \) for \( t \in (0, \tau] \). Applying this argument repeatedly, one can show that \( S(t) > 0, I(t) > 0 \) and \( R(t) > 0 \) for \( t \in (0, \infty) \). The proof is complete. \( \square \)
Theorem 2.3. Let
\[ M_1 = \frac{\Lambda^*}{(\mu_s + m_s^*)(1 - (1 - p)e^{-\delta \tau})}. \] (2.1)
If \( \frac{\beta^* M_1}{\mu I + \gamma} < 1 \), then any positive almost periodic solution \((S(t), I(t), R(t))\) of system (1.2) satisfies that
\[ \|S\| \leq M_1 \quad \text{and} \quad \|I\| \leq M_2, \] (2.2)
where
\[ M_2 = \frac{m_s^2 p e^{-\delta \tau} M_1}{\mu I + \gamma} / \left(1 - \frac{\beta^* M_1}{\mu I + \gamma}\right). \] (2.3)

Proof. It follows from the theory of dichotomies [2] that if \((S(t), I(t), R(t))\) is a positive almost periodic solution of (1.2), then
\[ \begin{aligned}
S(t) &= \int_{-\infty}^{t} h(t,s) \left(\Lambda(s) + m_S(s)(1-p)e^{-\delta \tau} S(s-\tau)\right) ds, \\
I(t) &= \int_{-\infty}^{t} e^{-\int_{t-s}^{t} (\mu_I + \gamma) dr} \left(\beta(r) I(r) S(r) + m_S(s)e^{-\delta \tau} S(s-\tau)\right) ds, \\
R(t) &= \int_{-\infty}^{t} e^{-\int_{t-s}^{t} (\mu_R + m_R(r)) dr} \left(\gamma I(s) + m_R(s)e^{-\delta \tau} R(s-\tau)\right) ds
\end{aligned} \] (2.4)
where
\[ h(t,s) = e^{-\int_{t}^{s} (\mu_s + \beta(r) I(r) + m_S(r)) dr}. \] (2.5)

From [10], continuous almost periodic function \((S(t), I(t), R(t))\) is uniformly bounded and it follows from the first equation of (2.4), \(I(t) > 0\) and Lemma 2.1 that
\[ \begin{aligned}
S(t) &= \int_{-\infty}^{t} h(t,s) \left(\Lambda(s) + m_S(s)(1-p)e^{-\delta \tau} S(s-\tau)\right) ds \\
&\leq \int_{-\infty}^{t} e^{-\int_{t-s}^{t} (\mu_s + m_S) dr} \left(\Lambda^* + m_S(s)(1-p)e^{-\delta \tau} S(s-\tau)\right) ds \\
&\leq \int_{-\infty}^{t} e^{-\int_{t-s}^{t} \Lambda^* (1-p)e^{-\delta \tau} S(s-\tau) ds} \\
&\quad + \int_{-\infty}^{t} e^{-\int_{t-s}^{t} m_S dr} m_S(s)(1-p)e^{-\delta \tau} S(s-\tau) ds \\
&\leq \frac{\Lambda^*}{\mu_s + m_s^*} + (1-p)e^{-\delta \tau} \|S\| \int_{-\infty}^{t} e^{-\int_{t-s}^{t} m_S dr} m_S(s) ds \\
&\leq \frac{\Lambda^*}{\mu_s + m_s^*} + (1-p)e^{-\delta \tau} \|S\|.
\end{aligned} \]

This implies that
\[ \|S\| \leq \frac{\Lambda^*}{(\mu_s + m_s^*)(1 - (1 - p)e^{-\delta \tau})} = M_1. \]

Note that \(M_1\) is independent of initial conditions.

The uniform boundedness of \(S(t)\), together with the second equation of (2.4), yields
\[ \begin{aligned}
I(t) &\leq \int_{-\infty}^{t} e^{-\int_{t-s}^{t} (\mu_I + \gamma) dr} \left(\beta^* I(t) M_1 + m_s^2 p e^{-\delta \tau} M_1\right) ds \\
&\leq \frac{\beta^* M_1}{\mu_I + \gamma} \|I\| + \frac{m_s^2 p e^{-\delta \tau} M_1}{\mu_I + \gamma},
\end{aligned} \]
which means
\[ \| I \| \leq \frac{m_2p e^{-\delta T} M_1}{\mu_1 + \gamma} \left( 1 - \frac{\beta^* M_1}{\mu_1 + \gamma} \right) = M_2. \]

This completes the proof. \( \square \)

For an almost periodic function \( f \in AP(\mathbb{R}, \mathbb{R}_+) \), let
\[ \alpha(f, \lambda) = \lim_{T \to +\infty} \frac{1}{T} \int_0^T f(t) e^{-i\lambda t} dt, \quad \Lambda = \{ \lambda \in \mathbb{R} \mid \alpha(f, \lambda) \neq 0 \}. \]

Then according to \[10\], the module of \( f \) is defined as
\[ \text{mod}(f) = \left\{ \mu \mid \mu = \sum_{i=1}^{N} n_i \lambda_i, \ n_i, N \text{ are integers, } N \geq 1, \lambda_i \in \Lambda \right\}. \]

The following lemma taken from \[10\] Theorem 4.5 will be used to prove our mains results on existence of almost periodic solutions of system \[1.2\].

**Lemma 2.4.** The following statements are equivalent for any \( f \) and \( g \in AP(\mathbb{R}, \mathbb{R}_+) \):

(i) \( \text{mod}(f) \supset \text{mod}(g) \);

(ii) For any sequence \( \{ t'_n \} \subset \mathbb{R} \), if \( \lim_{n \to +\infty} f(t + t'_n) = f(t) \) uniformly in \( t \in \mathbb{R} \), then there is a subsequence \( \{ t_n \} \subset \{ t'_n \} \) such that \( \lim_{n \to +\infty} g(t + t_n) = g(t) \) uniformly in \( t \in \mathbb{R} \).

3. **Almost periodic case.** In this section, we consider system \[1.2\] in which the parameters \( \Lambda(t), \beta(t), m_S(t) \) and \( m_R(t) \) are almost periodic. We first establish the existence and uniqueness of almost periodic solution for \[1.2\] in the following theorem.

**Theorem 3.1.** System \[1.2\] admits a unique positive almost periodic solution \( (S(t), I(t), R(t)) \) such that \( \text{mod}(S, I, R) \subset \text{mod}(\Lambda, \beta, m_S, m_R) \) provided that
\[ 0 \leq q \overset{\Delta}{=} (1 - p)e^{-\delta T} + \frac{\beta^* M_2}{\mu_S + m_S^*} < 1 \]

and
\[ 0 \leq w \overset{\Delta}{=} \frac{\beta^* M_1}{\mu_1 + \gamma} + \frac{\beta^* M_1(m_2 p e^{-\delta T} + \beta^* M_2)}{(1 - q)(\mu_S + m_{S^*})(\mu_1 + \gamma)} < 1, \]

where \( M_1 \) and \( M_2 \) are constants by defined in \[2.1\] and \[2.3\], respectively.

**Proof.** It follows from the theory of dichotomies \[7\] that for any given \( I(t) \in AP(\mathbb{R}, \mathbb{R}_+) \), \( S(t) \) is a positive almost periodic solution to the first equation of system \[1.2\] if and only if
\[ S(t) = \int_{-\infty}^t h(t, s) \left( \Lambda(s) + m_S(s)(1 - p)e^{-\delta T}S(s - \tau) \right) ds, \]

where \( h(t, s) \) is defined in \[2.3\]. For any given \( I(t) \in AP(\mathbb{R}, \mathbb{R}_+) \), define an operator \( F : AP(\mathbb{R}, \mathbb{R}_+) \to AP(\mathbb{R}, \mathbb{R}_+) \) as follows
\[ [FS](t) = \int_{-\infty}^t h(t, s) \left( \Lambda(s) + m_S(s)(1 - p)e^{-\delta T}S(s - \tau) \right) ds. \]

Then for any $S_1(t), S_2(t) \in AP(\mathbb{R}, \mathbb{R}_+)$, we have

$$
\| [FS_1](t) - [FS_2](t) \|
= \left| (1 - p)e^{-\delta t} \int_{-\infty}^t h(t, s) m_S(s) (S_1(s - \tau) - S_2(s - \tau)) ds \right|
\leq (1 - p)e^{-\delta t} \int_{-\infty}^t e^{-\int_s^t \beta(r)dr} m_S(s) ds \| S_1 - S_2 \|
\leq (1 - p)e^{-\delta \tau} \| S_1 - S_2 \|.
$$

This shows that $F$ is a contraction mapping. By the contraction mapping principle, for any given $I(t) \in AP(\mathbb{R}, \mathbb{R}_+)$, the first equation of (1.2) has a unique positive almost periodic solution $S(t) = S_1(t)$. From the proof of Theorem 2.3, we have $S_1(t) \in (0, M_1]$.

Next we define an operator $G : AP(\mathbb{R}, \mathbb{R}_+) \to AP(\mathbb{R}, \mathbb{R}_+)$ as

$$
[GI](t) = \int_{-\infty}^t e^{-\int_s^t (\mu_1 + \gamma)dr} (\beta(s)S_1(s)I(s) + m_S(s)p)e^{-\delta \tau} S_1(s - \tau) ds.
$$

Then for any $I_1(t), I_2(t) \in AP(\mathbb{R}, \mathbb{R}_+)$, we have

$$
\| [GI_1](t) - [GI_2](t) \|
\leq \int_{-\infty}^t e^{-\int_s^t (\mu_1 + \gamma)dr} \beta(s) (S_{I_1}(s)I_1(s) - S_{I_2}(s)I_2(s)) ds
+ \int_{-\infty}^t e^{-\int_s^t (\mu_1 + \gamma)dr} m_S(s)p e^{-\delta \tau} (S_{I_1}(s - \tau) - S_{I_2}(s - \tau)) ds.
$$

Thus, we have

$$
\| [GI_1](t) - [GI_2](t) \|
\leq \int_{-\infty}^t e^{-\int_s^t (\mu_1 + \gamma)dr} \beta(s) (S_{I_1}(s)I_1(s) - I_2(s)) + I_2(s)(S_{I_1}(s) - S_{I_2}(s)) ds
+ m_S \cdot p \cdot e^{-\delta \tau} \cdot \| S_{I_1} - S_{I_2} \|
\leq \frac{\delta \cdot M_1}{\mu_1 + \gamma} \| I_1 - I_2 \| + \frac{m_S \cdot p \cdot e^{-\delta \tau} + \beta \cdot M_2}{\mu_1 + \gamma} \| S_{I_1} - S_{I_2} \|.
$$

On the other hand, $S(t)$ is a positive almost periodic solution of the first equation of system (1.2) if and only if

$$
S(t) = \int_{-\infty}^t e^{-\int_s^t (\mu_2 + m_S(r))dr} (\Lambda(s) - \beta(s)S(s)I(s)) ds
+ \int_{-\infty}^t e^{-\int_s^t (\mu_2 + m_S(r))dr} m_S(s)(1 - p) e^{-\delta \tau} S(s - \tau) ds.
$$

Then for any $I_1(t), I_2(t) \in AP(\mathbb{R}_+, (0, M_2])$, there are corresponding positive almost periodic solutions $S_{I_1}(t)$ and $S_{I_2}(t)$ to the first equation of system (1.2), satisfying

$$
\| S_{I_1}(t) - S_{I_2}(t) \|
= \left| \int_{-\infty}^t e^{-\int_s^t (\mu_2 + m_S(r))dr} \beta(s) (S_{I_1}(s)I_1(s) - S_{I_2}(s)I_2(s)) ds
+ \int_{-\infty}^t e^{-\int_s^t (\mu_2 + m_S(r))dr} m_S(s)(1 - p) e^{-\delta \tau} (S_{I_1}(s - \tau) - S_{I_2}(s - \tau)) ds \right|.
This implies that

\[ \text{and hence} \]

Therefore, the operator system (1.2) has a unique positive almost periodic solution \( \tilde{S} \), the corresponding positive almost periodic solution to the first equation of (1.2).

\[ \text{Define an operator} \]

Then for any positive almost periodic solution \( \tilde{S} \), it is not difficult to find that \( (\tilde{S} + t_0) \) and \( \tilde{S} \) are almost periodic in the first equation of (1.2).

For any positive almost periodic solution \( \tilde{I}(t) \) of the first and second equation in system (1.2), \( R(t) \) is a positive almost periodic solution of the third equation in system (1.2) if and only if it satisfies

\[ R(t) = \int_{-\infty}^{t} e^{-\int_{0}^{\tau} (\mu R + mR(\tau))d\tau} \left( \gamma \tilde{I}(s) + mR(s)e^{-\delta\tau} R(s - \tau) \right) ds. \]

Define an operator \( H : AP(\mathbb{R}, \mathbb{R}^+) \rightarrow AP(\mathbb{R}, \mathbb{R}^+) \) as

\[ [HR](t) = \int_{-\infty}^{t} e^{-\int_{0}^{\tau} (\mu R + mR(\tau))d\tau} \left( \gamma \tilde{I}(s) + mR(s)e^{-\delta\tau} R(s - \tau) \right) ds. \]

Then for any \( R_1(t), R_2(t) \in AP(\mathbb{R}, \mathbb{R}^+) \), it follows from Lemma 2.1 that

\[ \left| [HR_1](t) - [HR_2](t) \right| \]

Since \( pe^{-\delta\tau} < 1 \), the operator \( H \) is a contraction mapping and there is a unique positive almost periodic solution \( \tilde{R}(t) =: \tilde{R}(t) \) to the third equation of system (1.2).

Finally, we show that \( mod(\tilde{S}, \tilde{I}, \tilde{R}) \subset mod(\Lambda, \beta, mS, mR) \). For any sequence \( \{t'_n\} \subset \mathbb{R}, \lim_{n \to +\infty} \Lambda(t + t'_n) = \Lambda(t), \lim_{n \to +\infty} \beta(t + t'_n) = \beta(t), \lim_{n \to +\infty} mS(t + t'_n) = mS(t) \)

and \( \lim_{n \to +\infty} mR(t + t'_n) = mR(t) \) uniformly in \( t \in \mathbb{R} \), then there is a subsequence \( \{t_n\} \subset \{t'_n\} \) such that \( \lim_{n \to +\infty} \tilde{S}(t + t_n) = S(t), \lim_{n \to +\infty} \tilde{I}(t + t_n) = I(t) \)

and \( \lim_{n \to +\infty} \tilde{R}(t + t_n) = R(t) \) uniformly in \( t \in \mathbb{R} \) by almost periodicity of \( \tilde{S}(t), \tilde{I}(t) \) and \( \tilde{R}(t) \). It is not difficult to find that \( (S(t), I(t), R(t)) \) is also a positive almost periodic solution of system (1.2). It follows from the uniqueness of almost periodic solution that \( \tilde{S}(t) = S(t), \tilde{I}(t) = I(t) \) and \( \tilde{R}(t) = R(t) \). Since
Theorem 3.2. The positive almost periodic solution \((\hat{S}(t), \hat{I}(t), \hat{R}(t))\) of system \((1.2)\) attracts all positive solutions if \(\mu_S + m_{S*} > m_{S} e^{-\delta r} \) and \(\gamma + \mu_I > 2\beta^* M_1\), where \(M_1\) is defined in \((2.1)\).

Proof. Suppose \((S(t), I(t), R(t))\) is a positive solution of system \((1.2)\). Let \(x(t) = S(t) - \hat{S}(t), y(t) = I(t) - \hat{I}(t)\) and \(R(t) = R(t) - \hat{R}(t)\), then \(x(t), y(t)\) and \(z(t)\) satisfy

\[
x'(t) = -(\mu_S + m_{S}(t))x(t) - \beta(t)I(t)x(t) - \beta(t)S(t)y(t) + m_{S}(t)(1 - p)e^{-\delta x}x(t - \tau) + m_{S}(t)\gamma y(t) - \mu_R + m_{R}(t)z(t) + m_{R}(t)e^{-\delta r}z(t - \tau),
\]

\[
y'(t) = -(\gamma + \mu_I)I(t)x(t) + \beta(t)I(t)y(t) + \beta(t)S(t)y(t) + m_{S}(t)pe^{-\delta r}x(t - \tau) + e^{-\delta r}m_{S}(t + \tau)|x(t)| - e^{-\delta r}m_{S}(t)|x(t - \tau)|
\]

\[
z'(t) = \gamma y(t) - (\mu_R + m_{R}(t))z(t) + m_{R}(t)e^{-\delta r}z(t - \tau).
\]

Then it suffices to show that \(\lim_{t \to \infty} x(t) = 0, \lim_{t \to \infty} y(t) = 0\) and \(\lim_{t \to \infty} z(t) = 0\). Motivated by \((11)\), we define a Liapunov functional \(V(t) = V[x, y](t)\) as follows:

\[
V(t) = |x(t)| + |y(t)| + e^{-\delta t} \int_{t-\tau}^{t} m_{S}(s + \tau)|x(s)|ds.
\]

Calculating the upper right derivative of \(V\) along the solutions of \((3.6)\) yields

\[
\frac{dV(t)}{dt} = \text{sgn}[x(t)] \left\{ - (\mu_S + m_{S}(t))x(t) - \beta(t)I(t)x(t) - \beta(t)S(t)y(t) + m_{S}(t)(1 - p)e^{-\delta x}x(t - \tau) + m_{S}(t)\gamma y(t) - (\gamma + \mu_I)I(t)x(t) + \beta(t)I(t)y(t) + \beta(t)S(t)y(t) + m_{S}(t)pe^{-\delta r}x(t - \tau) + e^{-\delta r}m_{S}(t + \tau)|x(t)| - e^{-\delta r}m_{S}(t)|x(t - \tau)|\right\} \leq - (\mu_S + m_{S}(t) - m_{S}(t + \tau)e^{-\delta r}) \cdot |x(t)| - \gamma y(t) - (\gamma + \mu_I - 2\beta^* M_1) \cdot |y(t)| = -\sigma \{ |x(t)| + |y(t)| \},
\]

where \(\sigma = \min\{\mu_S + m_{S*} - m_{S} e^{-\delta r}, \gamma + \mu_I - 2\beta^* M_1\} > 0\) provided \(\mu_S + m_{S*} > m_{S} e^{-\delta r} \) and \(\gamma + \mu_I > 2\beta^* M_1\). Here, for a continuous and differentiable function \(u(t)\), we define \(\text{sgn}[u(t)]\) as that

\[
\text{sgn}[u(t)] = \begin{cases} 
1 & \text{if } u(t) > 0 \text{ or } u(t) = 0 \text{ and } u'(t) > 0, \\
0 & \text{if } u(t) = 0 \text{ or } u'(t) = 0, \\
-1 & \text{if } u(t) < 0 \text{ or } u(t) = 0 \text{ and } u'(t) < 0.
\end{cases}
\]

For any \(t > 0\), \((3.8)\) implies that

\[
V(t) + \sigma \int_{0}^{t} (|x(s)| + |y(s)|) ds \leq V(0).
\]

It follows from Theorem 2.2 that \(x(t)\) and \(y(t)\) are bounded on \((0, +\infty)\). This implies that \(V\) is bounded on \((0, +\infty)\). Note from the first two equations in \((3.6)\)
that \( x'(t) \) and \( y'(t) \) are bounded on \((0, +\infty)\) and hence \( x(t) \) and \( y(t) \) are uniformly continuous on \((0, +\infty)\). Consequently
\[
|x(t)| + |y(t)| \in L_1((0, \infty)). \tag{3.10}
\]
By Barbalat’s Lemma [13], we know that \( (|x(t)| + |y(t)|) \to 0 \) as \( t \to +\infty \). Thus \( \lim_{t \to \infty} y(t) = 0 \) and \( \lim_{t \to \infty} x(t) = 0 \). \( \lim_{t \to \infty} z(t) = 0 \) follows directly from \( \lim_{t \to \infty} y(t) = 0 \) and [4] Lemma 1.

4. Periodic case. In this section, we deal with the periodic case. To apply the coincide degree theory to establish the existence of positive periodic solutions of system (1.2), we need some standard notations. Let \( X, Z \) be real Banach spaces, \( L : \text{Dom}L \subset X \to Z \) be a linear Fredholm mapping of index 0, and \( N : X \to Z \) be continuous. Let \( P : X \to X, Q : Z \to Z \) be continuous projections such that \( \text{Im}P = \text{Ker}L, \text{Ker}Q = \text{Im}L \) and \( X = \text{Ker}L \oplus \text{Ker}P, Z = \text{Im}L \oplus \text{Im}Q \). Then by Gaines and Mawhin[12], \( L : \text{Dom}L \cap \text{Ker}P \to \text{Im}P \) is one to one, so its inverse \( K_P : \text{Im}L \to \text{Dom}L \cap \text{Ker}P \) exists. \( J : \text{Im}Q \to \text{Ker}L \) is an isomorphism of \( \text{Im}Q \) onto \( \text{Ker}L \).

We need the following continuation theorem to prove our existence result.

**Lemma 4.1.** (Gaines and Mawhin[12], Theorem IV.1) Let \( \Omega \subset X \) be an open bounded set and let \( N : X \to Z \) be a continuous operator which is \( L \)-compact on \( \Omega \) (i.e., \( QN \) and \( K_P(\bar{I} - Q)N \) are relatively compact on \( \Omega \)). Assume that

(H1): for each \( \lambda \in (0, 1), x \in \text{Dom}L \cap \partial \Omega \), \( Lx \neq \lambda Nx \);

(H2): for each \( \lambda \in (0, 1) \), \( x \in \text{Ker}L \cap \partial \Omega, QNx \neq 0 \) and \( \deg\{JQN, \text{Ker}L \cap \Omega, 0\} \neq 0 \).

Then the equation \( Lx = Nx \) has at least one solution in \( \text{Dom}L \cap \Omega \), where, \( \bar{I} \) is an identity operator.

In this section, we denote
\[
C_{\omega} = \{u : u(t) = (x(t), y(t))^T \in C[0, \infty), u(t) \equiv u(t + \omega), \forall t \in \mathbb{R}\}
\]
and define \( ||u|| = \max\{|x_0|, |y_0|\} \), where \( |x_0| = \max_{t \in [0, \omega]} |x(t)| \) and \( |y_0| = \max_{t \in [0, \omega]} |y(t)| \). Then \( C_{\omega} \) is a Banach space with the norm \( || \cdot || \). Denote
\[
\bar{\tau} = \frac{1}{\omega} \int_{0}^{\omega} f(t) dt, \quad f \text{ is a continuous } \omega \text{-periodic function in } \mathbb{R}.
\]

Assume that \( S(t) \) and \( I(t) \) are positive periodic solutions of the first two equations of system (1.2), let \( x(t) = \ln S(t) \) and \( y(t) = \ln I(t) \), then \( x(t) \) and \( y(t) \) satisfy
\[
\begin{align*}
x'(t) &= \Lambda(t)e^{-\tau(t)} - (\mu_S + \bar{m}_S(t)) - \beta(t)e^y(t) \\
&\quad + \bar{m}_S(t)(1 - p)e^{-\delta \tau}e^{-x(t) + x(t - \tau)}.
\end{align*}
\tag{4.1}
\]
\[
\begin{align*}
y'(t) &= \beta(t)e^x(t) - (\gamma + \mu_I) \bar{m}_S(t)pe^{-\delta \tau}e^{-y(t) + y(t - \tau)}.
\end{align*}
\]
By some algebra, we can show the following system
\[
\begin{align*}
\bar{\Lambda}e^{-x} - (\mu_S + \bar{m}_S) - \beta e^y + \bar{m}_S(1 - p)e^{-\delta \tau} = 0, \\
\bar{\beta}e^{-x} - (\gamma + \mu_I) + \bar{m}_Spe^{-\delta \tau}e^{-y + x} = 0
\end{align*}
\tag{4.2}
\]
has a unique solution \((x^*, y^*)\).

Choose \( M > \max\{|\ln M_1| + 1, |\ln M_2| + 1\} \) large enough such that \( \max\{|x^*|, |y^*|\} < M \), where \( M_1 \) and \( M_2 \) are defined in (2.1) and (2.3).
Set $X = Z = C_{\bar{\Omega}}$. Define $L : \text{Dom} L \cap X \to X$ as $Lu = \frac{du}{dt} = \left( \frac{dx(t)}{dt}, \frac{dy(t)}{dt} \right)$ and $N : X \to X$ as

$$Nu = \begin{pmatrix}
A(t)e^{-z(t)} - (\mu_S + m_S(t)) - \beta(t)e^y(t) + m_S(t)(1-p)e^{-\delta}\tau e^{-z(t)-z(t-\tau)} \\
\beta(t)e^z(t) - (\gamma + \mu_I) + m_S(t)pe^{-\delta}\tau e^{-x(t)-x(t-\tau)}
\end{pmatrix}.$$

Denote the two continuous projection operators $P$ and $Q$ by

$$Pu = \frac{1}{\omega} \int_0^\omega u(t)dt = \left( \frac{1}{\omega} \int_0^\omega x(t)dt, \frac{1}{\omega} \int_0^\omega y(t)dt \right), \quad u \in X,$$

$$Qz = \frac{1}{\omega} \int_0^\omega z(t)dt = \left( \frac{1}{\omega} \int_0^\omega z_1(t)dt, \frac{1}{\omega} \int_0^\omega z_2(t)dt \right), \quad z \in Z.$$

Then

$$\text{Ker}L = \{ u : u \in X, u = c, c \in \mathbb{R}^2 \}, \quad \text{Im}L = \{ z : z \in Z, \int_0^\omega z(t)dt = 0 \}$$

and $L$ is a Fredholm mapping of index 0. Thus $P$ and $Q$ satisfy

$$\text{Im}P = \text{Ker}L, \text{Ker}Q = \text{Im}L = \text{Im}(I - Q)$$

and $L$ admits an inverse $K_P : \text{Im}L \to \text{Dom}L \cap \text{Ker}P$ with

$$K_P(z)(t) = \int_0^t z(s)ds - \frac{1}{\omega} \int_0^\omega \int_0^t z(s)ds dt.$$

By a direct calculation, we obtain

$$QN(u)(t) = \omega \int_0^\omega N(u)(t)dt \quad (4.3)$$

$$K_P(I - Q)N(u)(t)$$

$$= \int_0^t N(u)(s)ds - \frac{1}{\omega} \int_0^\omega \int_0^t N(u)(s)ds - \left( \frac{t}{\omega} - t \right) \int_0^\omega N(u)(s)ds. \quad (4.4)$$

**Lemma 4.2.** Let $\Omega = \{ u : u \in X, \| u \| \leq M \}$, then $N : X \to X$ is $L$-compact on $\bar{\Omega}$.

**Proof.** We only need to show that $QN$ and $K_P(I - Q)N$ are relatively compact on $\bar{\Omega}$. $QN$ is relatively compact on $\bar{\Omega}$ following directly from (4.3). Note that $K_P(I - Q)N$ is uniformly bounded on $\bar{\Omega}$, by the Ascoli-Arzela Theorem, we then only need to show the function family $K_P(I - Q)N(\bar{\Omega})$ is equi-continuous. From (4.4), for any $u \in \bar{\Omega}$, we have

$$\frac{d}{dt}[K_P(I - Q)N(x)(t)] = N(u)(t) - \left( \frac{1}{\omega} - 1 \right) \int_0^\omega N(u)(s)ds. \quad (4.5)$$

Then there is a positive constant $\tilde{M}$ such that $\| \frac{d}{dt}[K_P(I - Q)N(u)(t)] \| \leq \tilde{M}, \forall u \in \bar{\Omega}$. This implies that the function family $K_P(I - Q)N(\bar{\Omega})$ is equi-continuous. Therefore, $K_P(I - Q)N$ is relatively compact on $\bar{\Omega}$ and $N$ is $L$-compact on $\bar{\Omega}$.

**Lemma 4.3.** For any $\lambda \in (0, 1)$, every periodic solution $u$ of the operator $Lu = \lambda Nu$ satisfies that $\| u \| < M$. 

Proof. In fact, for any \( \lambda \in (0, 1) \), if \( u(t) \) is an arbitrary periodic solution of \( Lu = \lambda Nu \), then \( u(t) \) satisfies
\[
\begin{cases}
\dot{x}'(t) = \lambda \left( \Lambda(t)e^{-x(t)} - (\mu_S + m_S(t)) \right) - \beta(t)e^{y(t)} \\
+ \lambda (m_S(t)(1 - p)e^{-\delta t}e^{-x(t)} - x(t)) ,
\end{cases}
\]
\[
\begin{cases}
y'(t) = \lambda \left( \beta(t)e^{x(t)} - (\gamma + \mu_I) + m_S(t)pe^{-\delta t}e^{-x(t)} - x(t) \right) .
\end{cases}
\]
Let \( z_1 = e^{x(t)} \), \( z_2 = e^{y(t)} \), then the above system can be rewritten as
\[
\begin{cases}
z_1(t) = \lambda (\Lambda(t) - (\mu_S + m_S(t))z_1(t) - \beta(t)z_1(t)z_2(t)) \\
+ \lambda (m_S(t)(1 - p)e^{-\delta t}z_1(t - \tau)) ,
\end{cases}
\]
\[
\begin{cases}
z_2(t) = \lambda (\beta(t)z_1(t)z_2(t) - (\gamma + \mu_I)z_2 + m_S(t)pe^{-\delta t}z_1(t - \tau)) .
\end{cases}
\]
By using the same argument as in the proofs of Theorems 2.2 and 2.3, we can obtain \( ||z_1|| < M_1 \) and \( ||z_2|| < M_2 \). Thus the proof is complete.

**Theorem 4.4.** If \( \Lambda(t), \beta(t), m_S(t) \) and \( m_R(t) \) are positive \( \omega \)-periodic, then system (1.2) admits at least one positive \( \omega \)-periodic solution \( (\tilde{S}(t), \tilde{I}(t), \tilde{R}(t)) \).

**Proof.** Define \( \Omega \) as in Lemma 4.2, then by Lemma 4.3, the first assumption of Lemma 4.1 holds true. For \( u \in \text{Ker}L \cap \partial \Omega \), then \( u = (x, y)^T \in \mathbb{R}^2 \) and \( ||u|| = M \), we have
\[
QN_x = \left( \begin{array}{c}
\lambda e^{-x} - (\mu_S + \bar{m}_S)e^{y} - \bar{\beta}e^{y} - \bar{\beta}e^{y} \\
\bar{\beta}e^{x} - (\gamma + \mu_I) + \bar{m}_S e^{-\delta t}e^{-x} - \bar{m}_S e^{-\delta t}e^{-x}
\end{array} \right) \neq \left( \begin{array}{c}
0 \\
0
\end{array} \right).
\]
Noting that \( \text{Im}Q = \text{Ker}L \), therefore \( J \) is an identity mapping from \( \text{Im}Q \) to \( \text{Ker}L \). Further
\[
deq{\text{deg}(JQN, \text{Ker}L \cap \Omega, 0)} = \text{sign} \left\{ \det \left( \begin{array}{cc}
-\lambda e^{-x} & -\bar{\beta}e^{y} \\
\bar{\beta}e^{x} & -\bar{\beta}e^{y}
\end{array} \right) \right\} = 1.
\]
From Lemma 4.1 and thus system (1.1) has at least one \( \omega \)-periodic solution \( \hat{u}(t) = (\hat{x}(t), \hat{y}(t))^T \). That is, system (1.2) has at least one positive \( \omega \)-periodic solution \( (\hat{S}(t), \hat{I}(t), \hat{R}(t))^T = (e^{\hat{x}(t)}, e^{\hat{y}(t)})^T \).

A similar argument can be applied to show the following equation
\[
R'(t) = \gamma \tilde{I}(t) - (\mu_R + m_R(t))R(t) + m_R(t)e^{-\delta t}R(t - \tau)
\]
has a positive periodic solution \( \hat{R}(t) \).

Theorems 3.2 and 4.4 immediately give the following result.

**Theorem 4.5.** System (1.2) has a unique positive periodic solution \( (\tilde{S}(t), \tilde{I}(t), \tilde{R}(t)) \), which is globally asymptotically stable provided that \( \mu_S + m_{S*} > m_S^* e^{-\delta t} \) and \( \gamma + \mu_I > 2\beta^* M_1 \), where \( M_1 \) is defined as in (2.1).

5. Numerical simulations. In this section, we carry out numerical simulations to illustrate our theoretical results. Throughout this section, we assume \( \mu_S = \mu_R \) and \( \mu_I = \mu_S = 0.0001 \) and take \( \Lambda(t) = \lambda_0(1 - \lambda_1(\cos(\omega_1 t) + \sin(\omega_2 t))), \beta(t) = \beta_0(1 - \beta_1(\cos(\omega_1 t) + \sin(\omega_2 t))), m_S(t) = m_R(t) = m_0(1 - m_1(\cos(\omega_1 t) + \sin(\omega_2 t))). \)
Remark 1. If $\frac{\omega_1}{\omega_2}$ is irrational, then the functions $\Lambda(t)$, $\beta(t)$, $m_S(t)$ and $m_R(t)$ are almost periodic.

5.1. Almost periodic case. We take parameter values as $\lambda_0 = 0.5$, $\lambda_1 = 0.2$, $\beta_0 = 0.0003$, $\beta_1 = 0.01$, $m_0 = 0.4$, $m_1 = 0.01$, $\omega_1 = \frac{\pi}{77}$, $\omega_2 = 1$, $\mu_S = \mu_R = \frac{1}{77}$, $\gamma = 0.05$, $\delta = \frac{1}{7}$, $\tau = 0.78$, $p = \frac{2\pi}{100}$. This gives $q \approx 0.97 < 1$, $w \approx 0.17 < 1$, $\mu_S + m_{S-} - m_{S+}e^{-\delta \tau} \approx 0.004 > 0$ and $\gamma + \mu_1 - \beta^* M_1 \approx 0.05 > 0$. All conditions of Theorem 3.1 and Theorem 3.2 are satisfied and system (1.2) admits a unique positive almost periodic solution, which is globally stable. Numerical solutions are plotted in Figure 2, where the following three sets of initial conditions are used:

$$\text{IV1} \triangleq \begin{cases} S(\theta) = 5(1 + 0.1(\cos(\omega_1 \theta) + \sin(\omega_2 \theta))), & \theta \in [-\tau, 0], \\ I(0) = 1.9, & \theta \in [-\tau, 0], \\ R(\theta) = 0.9, & \theta \in [-\tau, 0], \end{cases} \quad (5.1)$$

$$\text{IV2} \triangleq \begin{cases} S(\theta) = 5.5(1 + 0.1(\cos(\omega_1 \theta) + \sin(\omega_2 \theta))), & \theta \in [-\tau, 0], \\ I(0) = 2.1, & \theta \in [-\tau, 0], \\ R(\theta) = 1.1, & \theta \in [-\tau, 0], \end{cases} \quad (5.2)$$

and

$$\text{IV3} \triangleq \begin{cases} S(\theta) = 6(1 + 0.1(\cos(\omega_1 \theta) + \sin(\omega_2 \theta))), & \theta \in [-\tau, 0], \\ I(0) = 3, & \theta \in [-\tau, 0], \\ R(\theta) = 2.2, & \theta \in [-\tau, 0]. \end{cases} \quad (5.3)$$

![Fig. 2. Numerical solutions of (1.2) with $\lambda_0 = 0.5$, $\lambda_1 = 0.2$, $\beta_0 = 0.0003$, $\beta_1 = 0.01$, $m_0 = 0.4$, $m_1 = 0.01$, $\omega_1 = \frac{\pi}{77}$, $\omega_2 = 1$, $\mu_S = \mu_R = \frac{1}{77}$, $\gamma = 0.05$, $\delta = \frac{1}{7}$, $\tau = 0.78$, $p = \frac{2\pi}{100}$. Three sets of initial conditions IV1, IV2 and IV3 are used.](image)

We should point out that the conditions given in Theorem 3.1 and Theorem 3.2 are only sufficient and not necessary. If we keep the same parameter values except that we take $\lambda_0 = 1.5$ and $m_1 = 0.2$, then in this case we find $q \approx 1.01 > 1$, $w \approx -3.23 < 0$, $\mu_S + m_{S-} - m_{S+}e^{-\delta \tau} \approx -0.30 < 0$ and $\gamma + \mu_1 - \beta^* M_1 \approx -0.007 < 0$. Theorem 3.1 and Theorem 3.2 do not apply. However, numerical simulations presented in Figure 3 indicate that system (1.2) still admits a globally stable positive almost periodic solution.
5.2. Periodic case. Theorem 4.4 implies that system (1.2) admits at least one positive periodic solution as long as the system parameters are positive and periodic. Take parameter values $\lambda_0 = 0.5$, $\lambda_1 = 0.2$, $\beta_0 = 0.001$, $\beta_1 = 0.01$, $m_0 = 0.4$, $m_1 = 0.2$, $\omega_1 = 1$, $\omega_2 = 0$, $\mu_S = \mu_R = \frac{1}{12}$, $\gamma = 0.05$, $\delta = \frac{1}{50}$, $\tau = 0.78$ and $p = \frac{2\tau}{160+2\tau}$. Then $\Lambda(t)$, $\beta(t)$ and $m_S(t) = m_R(t)$ are all periodic. Choose the initial conditions as $I(0) = 2$, $S(\theta) = 5(1 + 0.1 \cos(\omega_1 \theta))$ and $R(\theta) = 1$ for $\theta \in [-\tau, 0]$, as shown in Figure 4, there is a positive periodic solution. If the conditions of Theorem 4.5 are satisfied, then system (1.2) admits a unique periodic solution, which is globally stable. Next we explore the situation when one of the conditions in Theorem 4.5 is not valid. To this end, we take $\lambda_0 = 0.5$, $\lambda_1 = 0.2$, $\beta_0 = 0.001$, $\beta_1 = 0.01$, $m_0 = 0.3$, $m_1 = 0.3$, $\omega_1 = 1$, $\omega_2 = 0$, $\mu_S = \mu_R = \frac{1}{12}$, $\gamma = 0.05$, $\delta = \frac{1}{50}$, $\tau = 0.78$ and $p = \frac{2\tau}{160+2\tau}$. Then we have $\mu_S + m_S - m_0 e^{-\delta \tau} \approx -0.34 < 0$ and $\gamma + \mu_I - \beta^* M_1 \approx 0.02$. Thus Theorem 4.5 does not apply. As demonstrated in Figure 5, system (1.2) still admits a unique periodic solution which is globally stable.
6. Summary and discussion. In this paper, motivated by the mobility patterns of migrant workers in China, we have considered a delay differential equation system with time-vary parameters. More specifically we have considered the almost periodic case and periodic case. For the almost periodic case, by a novel technique, we applied the the contraction mapping theorem to establish the existence and uniqueness of almost periodic solution. In addition, for the periodic case, the standard coincidence degree theory has been employed to establish the existence of periodic solutions. For both cases, global stability results are obtained via the method of Liapunov functional. Since there is not standard procedure to construct feasible Liapunov functionals, the stability conditions we have obtained clearly are not the sharpest. Numerical simulations suggest that system (1.2) may still admit a globally stable (almost) periodic solution even if the conditions given in Theorems 3.1 and 4.5 are not satisfied. Based on our extensive simulations, we propose the following conjecture:

**Conjecture** If the time-varying coefficients in (1.2) are positive almost periodic (periodic), then (1.2) has a unique positive almost periodic (periodic), which is globally stable with respect to the nonnegative initial conditions.

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