A Near Horizon CFT Dual for Kerr-Newman-AdS

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We show that the near horizon regime of a Kerr-Newman-AdS (KNAdS) black hole, given by its two dimensional analogue a la Robinson and Wilczek (2005 Phys. Rev. Lett. 95 011303), is asymptotically AdS$_2$ and dual to a one dimensional quantum conformal field theory (CFT). The s-wave contribution of the resulting CFT’s energy-momentum-tensor together with the asymptotic symmetries, generate a centrally extended Virasoro algebra, whose central charge reproduces the Bekenstein-Hawking entropy via Cardy’s Formula. Our derived central charge also agrees with the near extremal Kerr/CFT Correspondence (2009 Phys. Rev. D 80, 124008) in the appropriate limits. We also compute the Hawking temperature of the KNAdS black hole by coupling its Robinson and Wilczek two dimensional analogue (RW2DA) to conformal matter.

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1. Introduction

Black holes provide a unique scenario for studying quantum gravitational phenomena, since near a black hole, horizon degrees of freedom tend to redshift away reducing physics to two dimensions. This convenient fact was first utilized by Christensen and Fulling 1 by studying the s-wave contribution of a semi-classical scalar in a Schwarzschild spacetime. Christensen and Fulling showed that the trace anomaly of the resulting effective action’s energy-momentum-tensor contained the respective spacetime’s Hawking temperature:

$$T_H = \frac{\hbar \kappa}{2 \pi},$$

where $\kappa$ is the surface gravity of the black hole horizon. The derivation of $T_H$ via effective actions and their associated energy-momentum-tensors of semiclassical scalars has been explored in various settings. The general idea follows from studying the effective action of the functional:

$$Z(\varphi, g) = \int D\varphi e^{-iS[\varphi, g]},$$

where $S_D[\varphi, g]$ is the action of a free scalar field in $D$ dimensions on the background spacetime $g^{(D)}_{\mu\nu}$. For the case where $D = 2$ the effective action is given by the
Polyakov Action \textsuperscript{9,10}

\[ \Gamma_{\text{Polyakov}} = \frac{1}{96\pi} \int d^2 x \sqrt{-g^{(2)}} R^{(2)} \frac{1}{\Box_{g^{(2)}}} R^{(2)}, \]  

which has shown to play an important roll for computing quantum gravitational quantities near black hole horizons \textsuperscript{11,12,13,14} and exhibits a unique relationship to conformal algebras \textsuperscript{15,16}.

Motivated by the above arguments Robinson and Wilczek (RW) initiated the study of two-dimensional chiral scalars in the near horizon regime of black holes \textsuperscript{17}. Two-dimensional chiral theories are known to be anomalous. RW showed that to ensure a unitary quantum field theory in this regime requires black hole radiation at temperature $T_H$. In other words, quantum gravitational phenomena cancels chiral/gravitational anomalies \textsuperscript{18}. This method has been applied to various types of black holes with various gauge, gravitational and covariant anomalies \textsuperscript{12,13,19,20,21,22,23,24,25,26,27,28,29,30,31,32} and has provided two dimensional analogues for various types of black holes beyond Schwarzschild. These two dimensional analogues are the remnant metric degrees of freedom which dominate $S^{(D)}[\varphi, g]$ in the near horizon regime, where all potential terms vanish exponentially fast upon transformation to tortoise coordinates \textsuperscript{33}.

Combining two dimensional near horizon physics with holography has provided a unique scenario for studying black hole entropy by asserting that quantum gravity in two dimensions is dual to a conformal field theory of equal or lesser dimension. This duality is richly exemplified in the well known $AdS/CFT$ correspondence of string theory \textsuperscript{34}. In fact the seminal work of Brown and Henneaux showed the algebra of the asymptotic symmetry group of three dimensional gravity with a negative cosmological constant is Virasoro with calculable central charge \textsuperscript{35}. This is widely considered to be the first example of an $AdS_3/CFT_2$ correspondence. Applying this to the three dimensional $BTZ$-black hole \textsuperscript{36}, Strominger \textsuperscript{37} reproduced the correct Hawking-Bekenstein Entropy \textsuperscript{38}

\[ S_{BH} = \frac{A}{4\hbar G} \]  

via Cardy’s Formula \textsuperscript{39,40}. This idea has been generalized and applied to various black holes in near horizon regimes and at asymptotic infinity by Carlip and others \textsuperscript{11,41,14,42,43,44,45,46,47,48}, where the general idea is summarized as follows. Given a set of consistent metric boundary or fall-off conditions, there exists an associated asymptotic symmetry group (ASG). This ASG is generated by a finite set of diffeomorphisms parametrized by some discrete $\xi_n$ for all $n \in \mathbb{Z}$ satisfying a $\text{Diff}(S^1)$ subalgebra:

\[ i \{ \xi_m, \xi_n \} = (m - n)\xi_{m+n} \]  

Consistency necessitates that these generators, $\xi_n$, be finite and well behaved at the respective boundary. Upon quantization, $\xi_n \rightarrow Q_n$ via Hamiltonian or Covariant
Lagrangian techniques, Brown and Henneaux showed\(^{35}\)

\[
[Q_m, Q_n] = (m - n)Q_{m+n} + \frac{c}{12}m(m^2 - 1)\delta_{m+n,0}
\]  

(6)

where \(c\) is a calculable central extension. We should note that (6) assumes a fixed normalization of the lowest Virasoro mode due to the linear term in the center. This ambiguity was first addressed by string theory in\(^{49,50,37,51}\), where it was shown that the massive BTZ black hole is a solution to low energy superstring theory lying in the Neveu-Schwarz sector (antiperiodic BC). This implies a mass shift \(Q_0 = \frac{c}{24}\) and thus fixes the normalization such that:

\[
\left( Q_0 - \frac{c}{24} \right) |0\rangle = 0
\]

(7)

In the case for non supersymmetric theories the requirement for the generators of the ASG to include a proper \(SL(2,\mathbb{R})\) subgroup, i.e. \(\{Q_{-1}, Q_0, Q_1\}\) form a proper \(\mathfrak{sl}(2,\mathbb{R})\) subalgebra, is synonymous to the requirement that the vacuum be annihilated according to (7). The Bekenstein-Hawking entropy is then obtained from Cardy’s Formula\(^ {39,40}\) in terms of \(c\) and the proper normalized lowest eigen-mode via:

\[
S = 2\pi \sqrt{\frac{c \cdot Q_0}{6}}
\]

(8)

Applying a similar program, as outlined above, Guica, Hartman, Song and Strominger (GHSS)\(^ {52}\) proposed that the near horizon geometry of an extremal Kerr black hole is holographically dual to a 2-dimensional chiral CFT\(^a\) with non vanishing left central extension \(c_L\). In this approach the Bekenstein-Hawking entropy is recovered by the Thermal Cardy Formula\(^ {53}\)

\[
S_{BH} = \frac{\pi^2}{3}c_LT_L
\]

(9)

where \(T_L\) is the Frolov-Thorne vacuum temperature for generic Kerr geometry\(^ {54}\). This correspondence has been extended to various classical and exotic black holes in string theory, higher dimensional theories and gauged supergravities to name a few\(^{55,56,57,58,59,60,61,62,53,63,64}\).

In addition to demonstrating a duality between extremal black holes and CFT, GHSS\(^ {52}\) apply the affluent principle of holographic duality to an astrophysical object/black-hole GRS 1915+105. This is a binary black hole system located 11000pc away in the constelation Aquila\(^ {65}\). GHSS showed that the companion black hole is holographically dual to a 2-dimensional chiral CFT with \(c_L = (2\pm1) \times 10^{39}\). For an extremal Kerr black hole, the inner most stable circular orbit corresponds to the horizon. Thus, GHSS conclude that any radiation emanating from the inner most circular orbit should be well described by the 2-dimensional chiral CFT,

\(^a\)Distinct from the 2-dimensional chiral-scalar field theory employed by RW.
making the Kerr/CFT correspondence a possible theoretical tool in a future astrophysical observation.

The aim of this manuscript is to apply the rich ideas of holography to the near horizon two dimensional black hole analogue of the \textit{KNAdS} black hole. This Robinson and Wilczek 2-dimensional analogue (RW2DA) is shown to be asymptotically \textit{AdS} for a suitable choice of BC. Using a covariant Lagrangian technique and a Liouville type action resulting from the $s$-channel of a minimally coupled scalar, we compute the asymptotic quantum generator (charge) algebra, which is a centrally extended Virasoro algebra. This central extension together with the lowest normalized eigen-mode reproduce the correct form of the \textit{KNAdS} Bekenstein-Hawking entropy inside Cardy’s Formula. We also show that the RW2DA reproduces the Hawking temperature of the \textit{KNAdS} black hole by coupling it to conformal matter and computing the resulting anomalous quantum energy momentum tensor. The manuscript is outlined as follows: In Section 2.1 we briefly review the ingredients of the main calculation and talk about various techniques for arriving at a dimensionally reduced gravitational theory. Section 2.2 contains the main results of the paper namely the asymptotic symmetry generators, both classical and quantum, of the RW2DA field theory and their respective generator algebras. In Section 2.4 we apply the main results to compute the Bekenstein-Hawking entropy of the \textit{KNAdS} black hole. In Section 2.5 we couple the RW2DA to conformal matter and derive the Hawking temperature of the \textit{KNAdS} spacetime and finally in Section 3 we discuss our results and possible future directions.

2. Kerr-Newman-\textit{AdS} and Near Horizon Field Theory

2.1. \textit{Near Horizon Geometry}

The Kerr-Newman-\textit{AdS} metric is a solution to the Einstein-Hilbert Action with negative cosmological constant coupled to a Maxwell field given by the line element \ref{eq:KNAdS_metric}.

\begin{align}
\nonumber ds^2 &= g^{(4)\mu\nu}dx^\mu dx^\nu \\
&= -\frac{\Delta(r)}{\rho^2}\left(dt - a \frac{\sin^2 \theta}{\Xi} d\phi\right)^2 + \frac{\rho^2}{\Delta(r)} dr^2 + \frac{\rho^2}{\Delta} d\theta^2 \\
&\quad + \frac{\Delta \sin^2 \theta}{\rho^2}\left(adt - r^2 + a^2 \frac{\sin^2 \theta}{\Xi} d\phi\right)^2,
\end{align}

(10)
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where

\[ \Delta(r) = (r^2 + a^2) \left( 1 + \frac{r^2}{l^2} \right) - 2GMr + GQ^2, \]

\[ \Delta_\theta = 1 - \frac{a^2}{l^2} \cos^2 \theta, \]

\[ \rho^2 = r^2 + a^2 \cos^2 \theta \]

and \( M \) is the mass, \( a \) is the angular momentum per unit mass, \( Q \) is the charge, \( G \) is Newton’s constant and \( l \) the de Sitter radius. In general there are four horizon radii for which \( \Delta(r) \) vanishes, but only two are physical. Of these two, only one, \( r^+ \), reduces to Kerr-Newman, Kerr, Reissner-Nordstrom and Schwarzschild black hole horizons in the appropriate limits. Thus, given we choose this respective horizon radius, any closed forms for entropy and temperature will hold in general for all sub-leading black holes in their respective limits.

The RW2DA is found by examining the functional

\[ S^{(4)}(\varphi, g) = \frac{1}{2} \int dx^4 \sqrt{-g} \nabla_\mu \varphi \nabla^\mu \varphi \]

in the regime where \( r \) is close to \( r^+ \). Expanding \( \varphi \) in terms of spherical harmonics, transforming to tortoise coordinates and integrating out the angular degrees of freedom, we obtain the near horizon theory\(^{21,33}\):

\[ S^{(4)}(\varphi, g) \xrightarrow{r \to r^+} S^{(2)}(\varphi_{lm}, g^{(2)}) = \frac{(r^2 + a^2)}{2\Xi} \int dt dr \varphi^*_{lm} \left[ \frac{1}{f(r)} (\partial t - iA_t)^2 - \partial_r f(r) \partial_r \right] \varphi_{lm} \]

where

\[ f(r) = \frac{\Delta(r)}{r^2 + a^2} \]

and RW2DA\(^b\)

\[ g^{(2)}_{\mu \nu} = \begin{pmatrix} \Delta(r) & 0 \\ 0 & \frac{1}{f(r)} \end{pmatrix} \]

and a gauge field containing the contributions of the \( U(1) \) charge and angular momentum

\[ A_t = -\frac{eQr}{r^2 + a^2} - \frac{\Xi am}{r^2 + a^2}, \]

\(^b\)We should note that for any two dimensional Riemannian-Levi-Civita connection 2-form \( \omega_{\alpha \beta} \), \( d\omega_{12} = K \text{vol}^2 \), where \( K = \frac{1}{(2\pi)^2} R^{(2)} \) is the Gauss curvature. This implies that the curvature of any Riemann-Surface is completely determined by its scalar variant. Thus classically, in 2-dimensions, there are no general relativistic dynamics and any gravitational effects that are present must have quantum gravitational implication/origin with semi-classical metric \( g^{(2)}_{\mu \nu} \).
The above dimensional reduction suggests that in the near horizon regime the KNAdS metric has the form:

\[ ds^2 = g^{(2)}_{\mu\nu} dx^\mu dx^\nu + B(\varphi) (d\varphi - A_t dt)^2 + C(\varphi) d\theta^2 \]  

assuming we consider \( \varphi \) as a component of the gravitational field. Similar approaches, for axisymmetric spacetimes, have been considered in \( 67,68,69,70 \), with the aim of arriving at an effective two dimensional gravitational theory in the near horizon regime. For spherically symmetric black holes this is readily straightforward given the metric ansatz

\[ ds^2 = g^{(2)}_{\mu\nu} dx^\mu dx^\nu + \frac{1}{\lambda^2} e^{-2\varphi} d\Omega_{(2)}^2 \]  

and substituting into the Einstein-Hilbert action. Then integrating out the angular degrees of freedom we are left with two dimensional dilaton gravity:

\[ S_{DG} = \frac{1}{2\pi} \int d^2 x \sqrt{-g^{(2)}(r)} \left\{ R^{(2)} + 2 (\nabla \varphi)^2 + \lambda^2 e^{2\varphi} \right\}, \]  

with a dimensionless coupling of \( e^{-2\varphi(r)} = \frac{4\pi^2}{16\pi G} \) and \( \lambda^2 = \frac{\pi}{2 G} \). The above functional is in general not a conformal field theory, yet for a specific choice of conformal transformation and field redefinition \( 19 \) becomes Liouville with central extension proportional to \( \frac{4\pi r_+^2}{6G} \). In fact, we know from the c-Theorem \( 41,72 \) that \( 19 \) must flow, under the renormalization group, to a CFT. There exists strong evidence \( 11,14,41,73,74,71 \), that in the near horizon this CFT takes the form of Liouville Theory

\[ S_{Liouville} \sim \int d^2 x \sqrt{-g^{(2)}} \left\{ -\Phi \Box g^{(2)} \Phi + 2\Phi R^{(2)} \right\} \]  

with effective dimensionless coupling proportional to:

\[ \frac{A}{16\pi G}, \]  

where the numerator originates from the dimensional reduction procedure and the denominator is a remnant of the parent classical gravitational theory (general relativity).

For a large class of non extremal weakly isolated horizons, including cosmological and of non spherical spacetimes, a recent analysis by Chung \( 69,70 \) showed by considering near horizon Gauss Null Coordinates, given by the line element

\[ ds^2 = \tilde{r} F(\tilde{r}) du^2 + 2 dud\tilde{r} + 2 \tilde{r} h_{ij} du dx^i + g_{ij} dx^i dx^j \]  

which takes the form

\[ ds^2 = 2g_{++} (dx^+ dx^- + h_{++} dx^+ dx^+) + g_{ij} dx^i dx^j \]  

on the light cone,\(^c\) that in the near horizon regime general relativity reduces to a two dimensional Liouville type conformal field theory to \( O(\tilde{r}) \). This was done by

\(^c\)For the line element \( 22 \) the horizon is located at \( \tilde{r} = 0 \) and \( x^\pm \) are defined in terms of \( (u, \tilde{r}) \).
considering diffeomorphisms $\xi^\pm$ preserving specific metric boundary conditions on the isolated horizon, then evaluating the Einstein-Hilbert Action for $g'_{\mu\nu} = g_{\mu\nu} + \mathcal{L}_\xi g_{\mu\nu}$ and integrating out the angular degrees of freedom. This near horizon theory again exhibited the same pre-factor $\frac{A}{16\pi G}$ and a center proportional to this coupling.

Motivated by these approaches, we will consider a slightly different one, by elevating $\varphi_{lm}$ to a gravitational field via the field redefinition

$$\varphi_{lm} = \sqrt{\frac{6}{G}} \psi_{lm},$$  \hspace{1cm}  \text{(24)}$$

where $\psi_{lm}$ is now unit less and the $\sqrt{6}$ was chosen to recover the Einstein coupling $\frac{1}{16\pi G}$ in the quantum gravitational effective action of (13) within the $s$-wave approximation.

2.2. Effective Action and Asymptotic Symmetries

Applying the field redefinition (24) to (13) yields:

$$S^{(2)}[\psi, g] = \frac{3(r_+^2 + a^2)}{G\Xi} \int d^2x \sqrt{-g^{(2)}} \psi^*_{lm} \left[D_\mu \left(\sqrt{-g} g^{(2)}_{\mu\nu} D_\nu \right)\right] \psi_{lm},$$  \hspace{1cm}  \text{(25)}$$

where $D_\mu$ is the gauge covariant derivative. The effective action of each partial wave is given by the sum of two functionals

$$\Gamma_{(lm)} = \Gamma_{\text{grav}} + \Gamma_{U(1)},$$  \hspace{1cm}  \text{(26)}$$

where

$$\Gamma_{\text{grav}} = \frac{(r_+^2 + a^2)}{16\pi G\Xi} \int d^2x \sqrt{-g^{(2)}} R^{(2)} \frac{1}{\Box g^{(2)}} R^{(2)}$$

and

$$\Gamma_{U(1)} = \frac{3e^2(r_+^2 + a^2)}{\pi G\Xi} \int F \frac{1}{\Box g^{(2)}} F.$$  \hspace{1cm}  \text{(27)}$$

We will discuss the $s$-wave contribution of (26) shortly and instead turn our attention to computing the ASG of (25). The asymptotic or large $r$ behavior of (15) and (16) are given by

$$g^{(0)}_{\mu\nu} = \begin{pmatrix}
-\frac{r^2}{r^2} - 1 + \frac{2GM}{r} - \frac{GQ^2}{r^2} + \mathcal{O} \left(\left(\frac{1}{r}\right)^3\right) & 0 \\
0 & \frac{l^2}{r^2} + \mathcal{O} \left(\left(\frac{1}{r}\right)^3\right)
\end{pmatrix},$$  \hspace{1cm}  \text{(28)}$$

and define an asymptotically $AdS_2$ configuration with Ricci Scalar, $R = -\frac{2}{r^2} + \mathcal{O} \left(\left(\frac{1}{r}\right)^3\right)$. We also impose the following metric and gauge field boundary or fall-off conditions:

$$\delta g_{\mu\nu} = \begin{pmatrix}
\mathcal{O} (r) & \mathcal{O} \left(\left(\frac{1}{r}\right)^0\right) \\
\mathcal{O} \left(\left(\frac{1}{r}\right)^0\right) & \mathcal{O} \left(\left(\frac{1}{r}\right)^3\right)
\end{pmatrix}$$

and $\delta A = \mathcal{O} \left(\left(\frac{1}{r}\right)^3\right)$.
A set of diffeomorphisms preserving the asymptotic metric structure is given by
\[ \xi_n = \xi_1(r)e^{i\kappa(t \pm r^*)} \frac{\partial_t}{\kappa} + \xi_2(r)e^{i\kappa(t \pm r^*)} \frac{\partial_r}{\kappa}, \] (31)
where \( r^* \) is the tortoise coordinate defined by \( dr^* = \frac{1}{f(r)} dr \),
\[ \xi_1 = \frac{i A r^4 e^{i\kappa r^*}}{n\kappa (-2G\ell^2 Mr + G\ell^2 Q^2 + \ell^2 r^2 + r^4)}, \quad \xi_2 = A r e^{i\kappa r^*}, \] (32)
A is an arbitrary normalization constant and \( \kappa \) is the surface gravity of the KN AdS black hole. Applying this set of diffeomorphisms to the gauge field we find
\[ \delta \xi A_\mu = \left( -\frac{3 (eQ^2 n e^{i\kappa t} \kappa)}{r^2} + \mathcal{O} \left( \frac{1}{r} \right)^3, \mathcal{O} \left( \frac{1}{r} \right)^4 \right). \] (33)
Thus to satisfy all the imposed fall of conditions we must consider total symmetries of the action, which implies
\[ \delta \xi \to \delta \xi + \Lambda, \] (34)
where
\[ \Lambda = -\frac{3ieQ^2 n e^{i\kappa t} \kappa}{r^2}. \] (35)
Evaluating the gauge field under this total symmetry we find
\[ \delta \xi + \Lambda A = \mathcal{O} \left( \frac{1}{r} \right)^3 \] (36)
in accordance with (30). Finally switching to light cone coordinates \( x^\pm = t \pm r^* \),
the set \( \xi_n^\pm \) is well behaved on the \( r \to \infty \) boundary and obey the centerless Virasoro or \( Diff(S^1) \) subalgebra
\[ i \{ \xi_m^+, \xi_n^- \} = (m - n) \xi_{m+n}^\pm. \] (37)
Evaluating the wave equation
\[ D_\mu \left( \sqrt{-g} g^{(2)\mu\nu} D_\nu \right) \psi_{lm} = 0 \] (38)
in this asymptotic behavior we find a product solution for \( \psi_{lm} \), which is complex hypergeometrical in \( r \), but decays exponentially fast in \( t \) for higher orders in \( m \).
Thus we only consider the \( s \)-wave contribution to (26), \( \Gamma_{00} = \Gamma \), leaving us with a near horizon effective action:
\[ \Gamma = \frac{(r^2_+ + a^2)}{16\pi G \Xi} \int d^2x \sqrt{-g^{(2)}(2) R^{(2)}} \frac{1}{\square_{g^{(2)}}} R^{(2)} \] (39)
\[ + \frac{3e^2 (r^2_+ + a^2)}{\pi G \Xi} \int F \frac{1}{\square_{g^{(2)}}} F. \]
\[ ^d \text{Large } r \text{ behavior will be synonymous with large } x^+ \text{ behavior.} \]
The above functional may be recast in the familiar form of a Liouville type CFT by introducing auxiliary scalars $\Phi$ and $B$ satisfying

$$\Box g^{(2)} \Phi = R^{(2)}$$
$$\Box g^{(2)} B = \epsilon^{\mu \nu} \partial_\mu A_\nu.$$  

In terms of these new fields our near horizon CFT takes its final form:

$$S_{NH\text{CFT}} = \frac{(r_+^2 + a^2)}{16\pi G \Xi} \int d^2 x \sqrt{-g^{(2)}} \left\{ -\Phi \Box g^{(2)} \Phi + 2 \Phi R^{(2)} \right\} + \frac{3e^2 (r_+^2 + a^2)}{\pi G \Xi} \int d^2 x \sqrt{-g^{(2)}} \left\{ -B \Box g^{(2)} B + 2 B \left( \frac{\epsilon^{\mu \nu}}{\sqrt{-g^{(2)}}} \partial_\mu A_\nu \right) \right\}.$$  

2.3. Energy Momentum and The Virasoro algebra

The energy momentum tensor and $U(1)$ current of (41) are defined as:

$$\langle T_{\mu \nu} \rangle = \frac{2}{\sqrt{-g^{(2)}}} \frac{\delta S_{NH\text{CFT}}}{\delta g^{(2)}{\mu \nu}}$$

$$= \frac{r_+^2 + a^2}{8\pi G \Xi} \left\{ \partial_\mu \Phi \partial_\nu \Phi - 2 \nabla_\mu \partial_\nu \Phi + g^{(2)}{\mu \nu} \left[ 2R^{(2)} - \frac{1}{2} \nabla_\alpha \Phi \nabla^\alpha \Phi \right] \right\}$$
$$+ \frac{6e^2 (r_+^2 + a^2)}{\pi G \Xi} \left\{ \partial_\mu B \partial_\nu B - \frac{1}{2} g^{\mu \nu} \partial_\alpha B \partial^\alpha B \right\}$$

$$\langle J^\mu \rangle = \frac{1}{\sqrt{-g^{(2)}}} \frac{\delta S_{NH\text{CFT}}}{\delta A^\mu} = \frac{6e^2 (r_+^2 + a^2)}{4\pi G \Xi} \frac{1}{\sqrt{-g^{(2)}}} \epsilon^{\mu \nu} \partial_\nu B$$

and the equation of motions for the auxiliary fields are:

$$\Box g^{(2)} \Phi = R^{(2)}$$
$$\Box g^{(2)} B = \epsilon^{\mu \nu} \partial_\mu A_\nu.$$  

Thus, given the metric (15) and gauge field (16) and adopting modified Unruh Vacuum boundary conditions (MUBC) 76

$$\begin{align*}
\langle T_{++} \rangle &= \langle J_+ \rangle = 0 \quad r \to \infty, \ l \to \infty, \\
\langle T_{--} \rangle &= \langle J_- \rangle = 0 \quad r \to r_+ \end{align*}$$

where the modification takes the $AdS$ radius into account, all relevant integration constants of (42) and (43) are determined. for large $r$ and to $O(\frac{1}{r^2})$, the resulting energy momentum tensor is dominated by one holomorphic component, $\langle T_{--} \rangle$. Expanding this component and the $U(1)$ current in terms of the boundary fields (28) and (29), we compute their responses to the total symmetry $\delta \xi^{+\Lambda}$, yielding:

$$\begin{align*}
\delta \xi_{n+\Lambda} \langle T_{--} \rangle &= \xi_{n} \langle T_{--} \rangle' + 2 \langle T_{--} \rangle \langle \xi_{n} \rangle' + \frac{r_+^2 + a^2}{4\pi G \Xi} \langle \xi_{n} \rangle'' + O \left( \frac{1}{r^2} \right) \\
\delta \xi_{n+\Lambda} \langle J_- \rangle &= O \left( \frac{1}{r^2} \right)
\end{align*}$$
This shows that \( \langle T_{\mu \nu} \rangle \) transforms asymptotically as the energy momentum tensor of a one dimensional CFT with center:

\[
\frac{c}{24\pi} = \frac{r_+^2 + a^2}{4\pi G \Xi} \Rightarrow c = \frac{3A}{2\pi G},
\]

where \( A = \frac{4\pi (r_+^2 + a^2)}{\Xi} \) is the horizon area of the KNAdS black hole. It is well known that a 2-dimensional CFT exhibits a conformal/trace anomaly of the form

\[
\langle T_{\mu \mu} \rangle = -\frac{c}{24\pi} R^{(2)}
\]

and evaluating the trace of (42) agrees with the above equation yielding the same center as in (46).

The entropy of our near horizon CFT will be determined by counting the microstates of the total quantum asymptotic symmetry generators on the \( r \to \infty \) boundary via the Cardy formula (8). The quantum generators are defined via the charge:

\[
Q_n = \lim_{r \to \infty} \int dx^- \langle T_{-} \rangle \xi_n^-,
\]

Computing its response to a total symmetry and compactifying the \( x^- \) coordinate to a circle from \( 0 \to 2\pi/\kappa \) yields the charge algebra:

\[
\delta_{\xi_n \Lambda} Q_n = [Q_m, Q_n] = (m - n) Q_n + \frac{c}{12} m (m^2 - 1) \delta_{m+n,0},
\]

which takes the familiar form of a centrally extended Virasoro algebra.

### 2.4. Entropy

Summarizing our results from (46) through (49) we have:

\[
c = \frac{3A}{2\pi G}, \quad Q_0 = \frac{A}{16\pi G},
\]

Substituting this into the Cardy Formula (8) we obtain:

\[
S = 2\pi \sqrt{\frac{c Q_0}{6}} = \frac{A}{4G},
\]

which is in agreement with the Bekenstein-Hawking entropy of the 4-dimensional KNAdS black hole. Taking the limit of (46) to Kerr and to extremality yields

\[
\lim_{l \to \infty, Q \to 0, M \to a} c = 12J,
\]

which is the same value of the left central charge obtained in the Kerr/CFT correspondence, further strengthening the proposal of GHSS.
2.5. Temperature
To compute the black hole temperature, we will couple the metric (15) to a single quantum conformal field $\Phi$ with Liouville functional

$$S_{\text{Liouville}} = \frac{1}{96\pi} \int d^2x \sqrt{-g^{(2)}} \left\{ -\Phi \Box_{g^{(2)}} \Phi + 2\Phi R^{(2)} \right\}$$

(53)

and following the steps (42) through (44), we obtain an energy momentum tensor which is dominated by one holomorphic component in the limit $r = r^+$ given by:

$$\langle T_{++} \rangle = -\frac{f'(r^+)^2}{192\pi}$$

(54)

This is the value of the Hawking Flux of the KNAdS black hole, from which we obtain the known Hawking temperature

$$HF = -\frac{\pi}{12} (T_H)^2 \Rightarrow T_H = \frac{f'(r^+)}{4\pi}$$

(55)

3. Conclusion
To conclude, we have analyzed quantum black hole properties in the near horizon regime via CFT techniques, extending the analysis of 11 to the more general KNAdS spacetime. In this regime the KNAdS black hole is dual to a two dimensional Liouville type quantum CFT whose conformal symmetry is generated by the centrally extended Virasoro algebra. The central charge and lowest Virasoro eigen-mode (50) together reproduce the correct form of the Bekenstein-Hawking entropy and analysis of the RW2DA (15) coupled to a single quantum conformal field reproduce the known form of the Hawking temperature.

It is interesting to note that the lowest Virasoro eigen-mode satisfies

$$Q_0 = GM_{irr}^2$$

(56)

where $M_{irr}^2$ is the irreducible mass of the KNAdS black hole, i.e. the final mass state after radiating away its angular momentum via a Penrose type process. This suggests that the eigen value of a CFT’s Hamiltonian is proportional to the irreducible mass of its black hole dual.

In (24) we elevated the scalar field to a gravitational one. This was first suggested and outlined by Solodukhin in 71 and extended to compute Hawking radiation by RW in their seminal work 17. Yet, in this approach the scalar field is still treated mathematically as a matter field. It is also unclear the exact details of the four dimensional gravitational theory, perhaps an ultraviolet complete extension of general relativity that dimensionally reduces to (41) except that it has the same coupling as standard Einstein gravity.

It still remains an open question to generalize the methods of this note to more exotic, higher dimensional black holes. In 23,24,27 the authors showed that the RW method for computing Hawking radiation via gauge and gravitational anomalies holds for their respective exotic black holes in arbitrary topologies and thus we
believe our construction for a near horizon CFT dual should extend to these cases as well.

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