Modelling the impact of a time-varying wave angle on the nonlinear evolution of sand bars in the surf zone

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ABSTRACT: Sandy beaches are often characterized by the presence of sand bars, whose characteristics (growth, migration speed, etc.) strongly depend on offshore wave conditions, such as wave height and angle of wave incidence. This study addresses the impact of a sinusoidally time-varying wave angle of incidence with different time-means on the saturation height, migration speed and longshore spacing of sand bars. Model results show that shore-transverse sand bars (so-called TBR bars) eventually develop under a time-varying wave angle. Depending on the time-mean, amplitude and period of the varying angle of wave incidence, the mean heights and mean migration speeds of the bars can be larger or smaller than their corresponding values in the case of time-invariant angles. Bars might not even form when the wave angle varies around a too large oblique mean value, whereas bars exist in the case of a time-invariant wave angle. The oscillations in both bar height and migration speed are large if the period of the time-varying wave angle is close to the adjustment timescale of the system and if large differences in the local growth and migration rates of the bars occur during one oscillation period. The oscillations in bar height are a combination of harmonics with the principal period and half the period of the time-varying wave angle, whereas those of migration speed contain only the principal period. Bars that are subject to time-varying wave angles have larger longshore crest-to-crest spacings than those which form under fixed wave angles. Physical explanations for these findings are given. © 2020 The Authors. Earth Surface Processes and Landforms published by John Wiley & Sons Ltd

KEYWORDS: time-varying wave angle; sand bars; morphodynamics; beach states; height and migration; resonance; oblique wave incidence

Introduction

The surf zone of many sandy uninterrupted beaches often features the presence of different types of sand bars, such as shore-parallel bars, crescentic bars and transverse bars (e.g. Wright and Short, 1984; Komar, 1998; van Enckevort et al., 2004). Transverse bars often develop from the shallowest sections of crescentic bars that progress shoreward until they attach to the shoreline. These bars, which are called TBR bars (transverse bar and rip), are aligned perpendicularly or obliquely to the coast (Figure 1). The presence of sand bars has a direct impact on the shoreline and beach safety by, for example, creating areas of erosion and accretion and rip currents (Lippmann and Holman, 1990; Komar, 1998). Therefore, increasing our knowledge of the dynamics of these bedforms is important, for example, to enhance our capacity to accurately simulate the evolution of coastal systems.

Fair weather conditions favour the growth of longshore-rhythmic sand bars, of which the growth and migration rates strongly depend on the wave conditions (e.g. Wright and Short, 1984; Lippmann and Holman, 1990). During severe storms, shore-parallel bars can form from sand that is eroded from the beach, while meandering crescentic bars might be reshaped into shore-parallel bars (bar straightening) without longshore variability (Lippmann and Holman, 1990; van Enckevort et al., 2004). Bar straightening might also occur for highly oblique wave angles of incidence (with respect to the shore-normal Price and Ruessink, 2011).

Various models have been developed to study the initial formation and long-term evolution of sand bars. These models simulate waves, currents, sand transport and bed evolution in the surf zone, with incoming waves being the main driver (see the review by Ribas et al., 2015). The saturation of growth of sand bars results from competition between two processes: (1) accumulation of sand in bar areas due to correlation between currents and the gradient of the depth-averaged sediment concentration and (2) erosion of bars due to downslope sediment transport (Garnier et al., 2006). A major limitation of nonlinear models that describe the evolution of sand bars is that wave conditions (wave height, angle of wave incidence, etc.) have been assumed to be time invariant, whereas in reality these conditions change continuously with time. An exception is the study by Castelle and Ruessink (2011), who studied the impact of a time-varying wave forcing on the evolution of finite-amplitude crescentic bars. They demonstrated that especially periodic variations in the wave angle of incidence were
crucial for the development of these bars in terms of their height, migration speed and alongshore spacing. A limitation in the study by Castelle and Ruessink (2011) was that only variations of wave angles around a zero time-mean (normal incidence) were considered. Obviously, in many beaches featuring bars (e.g. Gold Coast, Australia), the mean angle in the study by Castelle and Ruessink (2011) was that only variations of wave angles around a zero time-mean (normal incidence) were considered. Obviously, in many beaches featuring bars (e.g. Gold Coast, Australia), the mean angle (in 18 m depth) might reach values up to 15°-30° (Price and Ruessink, 2013).

This study, which is motivated by the work of Castelle and Ruessink (2011), focuses on sand bars that develop under a time-varying wave forcing. Specifically, the following question is addressed: what is the impact of a sinusoidally time-varying wave angle of incidence on the height, migration speed and alongshore spacing of sand bars? Angle variations around a zero mean (normal incidence) as well as around oblique means will be considered. To this end, simulations will be performed with an available numerical model (morfo55), developed by Caballeria et al. (2002) and Garnier et al. (2008), who used this model to simulate the evolution of finite-amplitude sand bars in a single-barred beach system in the case of time-invariant wave forcing. Here, this model will be extended such that it includes a time-varying wave angle of incidence.

The next section gives a brief outline of the model and of the methodology applied in this study. Model results are then presented, followed by a discussion and the conclusions.

Material and Methods

Model description and setting

The model setting of Garnier et al. (2008) is used in this study. The study area is schematized as a rectangular domain with a cross-shore extent of 250 m and a longshore extent of 2000 m, which is bounded by a straight coast (Figure 2). A Cartesian coordinate system is used, where the x-axis points in the offshore direction, the y-axis in the longshore direction and the z-axis in the vertical direction. The undisturbed water level is at z = 0, while the location of the coastline is at x = 0.

Morfo55 solves the phase-averaged equations for waves that are characterized by a narrow frequency–direction spectrum, together with the depth-averaged shallow water equations for currents, sediment transport due to the joint action of waves and currents (the formulation of Soulsby and van Rijn is used; see Soulsby, 1997) and the Exner equation that describes the changes in the bed due to convergence and divergence of sediment transport. The morfo55 model thus includes wave-topography feedbacks, wave shoaling and refraction, wave breaking and wave radiation stresses. Further details are given in Garnier et al. (2008).

Periodic boundary conditions are applied at the lateral boundaries (y = 0 and y = Ly) for each variable in the model (e.g. water level, velocity, bed level), as well as for their y-derivatives. At the shore boundary (x = 0), a vanishing cross-shore (u) and longshore (v) flow are assumed. At the offshore boundary (x = Lx), a constant root-mean-square-wave height (\(H_{rms}\)), wave period (\(T_p\)) and a time-varying angle of wave incidence \(\theta\) are imposed. The latter varies in time according to

\[
\theta(t) = \theta_0 + \theta \sin \left( \frac{2\pi t}{T} \right)
\]

with \(\theta_0\) the time-mean angle, \(\theta\) the amplitude of the variation in the angle and T the period of this variation. Non-cohesive sediment is assumed with a single size \(d_{50} = 250\mu m\).
Model simulations start from a single-barred beach profile:

\[
Z_b(x) = -a_0 - a_1 \left( 1 - \frac{\beta_2}{\beta_1} \tanh \left( \frac{\beta_1 x}{a_1} \right) \right) - \beta_2 x + a_2 \exp \left( -5 \left( \frac{x - x_c}{x_c} \right)^2 \right) 
\]

which is depicted in Figures 3a and 3b. This profile is based on measurements at Duck, North Carolina (Yu and Slinn, 2003). In Equation (2), \( x = x_c \) is the bar location, \( a_2 \) is the bar amplitude, and \( a_0 \) and \( a_1 \) are the water depths at the swash and surf zone boundaries, respectively. Coefficients \( \beta_1 \) and \( \beta_2 \) are the shoreline and offshore bottom slopes, respectively. To trigger sand bar development, random bottom perturbations \( h \) with a given root mean square amplitude are added to the initial bottom profile \( Z_b(0)(x) \).

The equations are solved on a computational staggered grid with grid sizes \( \Delta x \) and \( \Delta y \) in the \( x \)- and \( y \)-direction, respectively. Spatial derivatives in the model equations are approximated by the centred finite-difference numerical scheme, while temporal derivatives are computed following the Adams–Bashforth method (Caballeria et al. 2002). The hydrodynamic time step is \( \Delta t \). Following Garnier et al. (2008), morphodynamic processes are artificially accelerated by a morphological amplification factor \( (Ma_c) \).

### Table 1. Overview of model parameters, which are based on observations at Duck, North Carolina (Yu and Slinn, 2003). These parameters are adopted from Garnier et al. (2008)

| Parameter | Value |
|-----------|-------|
| Cross-shore extent \( L_x \) | 250 m |
| Longshore extent \( L_y \) | 2000 m |
| Root-mean-square wave height \( H_{\text{rms}} \) | 1 m |
| Wave period \( T_p \) | 6 s |
| Sediment size \( d_{10} \) | 250 \( \mu \)m |
| Bar location/amplitude \( x_c/a_2 \) | 80/1.5 m |
| Coefficients bottom profile \( \{a_0,a_1,\beta_1,\beta_2\} \) | [0.25 m, 2.97 m, 0.075, 0.0064] |
| Amplitude initial bottom perturbations \( h \) | 2 cm |
| Grid size (cross-shore) \( \Delta x \) | 5 m |
| Grid size (alongshore) \( \Delta y \) | 10 m |
| Time step \( \Delta t \) | 0.05 s |
| Amplification factor \( Ma_c \) | 90 |

### Model experiments

Values of the model parameters are representative of the sandy beach Duck in North Carolina (see Table 1). These values are adopted from Garnier et al. (2008). The values of the numerical parameters \( \{\Delta x, \Delta y, \Delta t\} \) were chosen such that model results did not change if smaller values were chosen. To reduce the...
Influence of the mean angle $\theta_0$ and period of variation $T$

Figure 5a shows differences $\Delta \hat{h}$ between the mean heights of the saturated bars of the cases of time-varying and

Table 2. Overview model experiments

| Model Type                  | $\theta$          |
|-----------------------------|-------------------|
| ConstantAngle               | $\theta = [0, 2, 4, 6, 7]^\circ$ |
| TimeVaryingAngle            | $\theta_0 = 4^\circ$, $T = 28$ d, $\theta = 2^\circ$ |
| SensToPeriodMeanAngle       | $\theta_0 = [0, 2, 4, 6, 7]^\circ$, $T = [1, 2, 5, 7, 28, 56, 112]$ |
| SensToAmplitude             | $\theta_0 = [0, 2, 4]^\circ$, $T = 28$ d, $\theta = [0.5, 1, 4, 8]^\circ$ |

computation time, the morphodynamic processes have been artificially accelerated by a Moac factor of 90. Results from a test run (Figure S1 in the Supporting Information) demonstrate that the use of a smaller value (Moac = 20) does not affect the height and migration of the saturated bars. The only quantititative difference is that bars saturate more rapidly if Moac is increased.

To assess the impact of a time-varying angle on the growth and migration speed of bars, a run was conducted (’TimeVaryingAngle’ in Table 2), whereby a sinusoidally time-varying angle of wave incidence ($\theta(t)$; Equation (1)) with a time-mean angle $\theta_0 = 4^\circ$ and amplitude $\theta = 2^\circ$ was imposed at the offshore boundary. The period with which $\theta(t)$ varied was $T = 28$ days, which is of the order of the adjustment timescale of the bars. Smaller and larger periods $T$ were also considered. Results from this run were compared with those of the case of a time-invariant angle, i.e. $\theta = 4^\circ$ (from run series ’ConstantAngle’ in Table 2). To examine the sensitivity of model results to the mean angle $\theta_0$ and period $T$, additional runs were conducted (’SensToPeriodMeanAngle’), in which values of $\theta_0$ and periods $T$ are in the range $\theta_0 = [0, 2, 4, 6, 7]^\circ$ and $T = [1, 2, 5, 7, 28, 56, 112]$ d. To quantify the influence of the amplitude $\theta$ of the variation in angle $\theta(t)$ on the height, migration speed and longshore spacings of the bars, another series of runs was performed (’SensToAmplitude’) for values of $\theta$ ranging between $0.5^\circ$ and $8^\circ$. This influence was studied for different $\theta_0 = [0, 2, 4]^\circ$. To deduce the relative impact of a time-varying wave angle on bar evolution with respect to the case of a time-invariant wave angle, additional simulations (run series ’ConstantAngle’) were conducted without time variation in $\theta$, i.e. $\theta = \theta_0$. Finally, note that a maximum angle $\theta_0 = 7^\circ$ is considered in this study because preliminary model results reveal that bars do not form for higher angles $\theta_0$ (see the Discussion). The experiments were run for a maximum simulation period of 300 days.

Analysis of model results

The analysis of model results focuses on the height, migration speed and longshore dominant spacing of the sand bars. The height and migration speed of these bars are expressed by, respectively, their mean bar height ($\Sigma h$) and global migration speed ($c$) as follows (Vis-Star et al. 2008):

$$||h|| = \left( \frac{h}{\Delta} \right)^{1/2},$$

and

$$c = \frac{1}{\Sigma h} \frac{\partial \Delta}{\partial t}.$$  

In these expressions, the overbar indicates averaging over the entire model domain, i.e. $\frac{1}{L_x L_y} \int_0^{\Delta} \int_0^{\Sigma h} \cdot dx dy$. Discrete Fourier transform was used to retrieve the periods of the harmonic signals that might arise in the time response of the bar height and migration speed to the imposed sinusoidally time-varying wave angle.

The longshore dominant spacing of the sand bars ($\lambda_d$) is computed using the discrete Fourier transform of the bottom perturbations $h$ in the longshore section $x = 50$ m. This dominant spacing is defined as the longshore spacing between bars for which the modulus of the Fourier coefficients is maximal (Garnier et al. 2006).

Results

Time-varying versus time-invariant wave angle

Model simulations start from an alongshore uniform beach with a shore-parallel bar located 80 m from the shoreline, which is depicted in Figure 3a. A cross-shore profile of the beach is provided in Figure 3b. Snapshots of the simulated bed levels at $t = 25$ days and $t = 300$ days of the cases of the time-invariant angle $\theta = 4^\circ$ and the time-varying angle around the mean value $\theta_0 = 4^\circ$ are shown in Figures 3c–3f. It appears that, in both cases, crescentic bar patterns initially appear (Figures 3c and 3d) from the deformation of the shore-parallel bar. In the course of time, the shoals of the crescentic bar pattern migrate onshore to eventually form transverse bars (TBR bars; Figures 3e and 3f). The TBR bars, which are somewhat downcurrent-oriented, are alongshore periodic with a dominant longshore spacing $\lambda_d$ of about 250 m, their heights (crest-to-trough distance) are $\sim 1$ m and their average migration speeds are $\sim 20$ md$^{-1}$. A moderate rip current system forms with maximum cross-shore currents of 0.37 ms$^{-1}$ (Figures 3g and 3h). These currents are directed offshore in the channels and onshore over the bars. The modelled characteristics of the bars compare well with observations (e.g. Hunter et al. 1979; Konicki and Holman, 2000; Ribas et al. 2015).

Figure 4a displays the time evolution of the longshore profile of bottom perturbations $h$ at $x = 50$ m of the case of the time-invariant angle. After the first emergence of bars around $t \sim 30$ days, a complex spatial–temporal behaviour occurs, with the tendency of the bars to merge and to split in the course of time. In the case of a time-varying wave angle, oscillating bars develop, which alternately appear and disappear (Figure 4b) with the period of the time-varying angle ($T = 28$ days). This oscillating behaviour of the bars is also seen in the periodic time response of the root mean square height ($\Sigma h$) and global migration speed ($c$) of the bars (Figures 4c and 4d, red lines). The saturated bars have root mean square heights $\Sigma h$ ranging between 0.10 and 0.17 m (range $||h||_{\text{max}} - ||h||_{\text{min}} = 0.07$ m, equivalent to a variation in the bar height of about 20 cm) and migration speeds $c$ that vary between 9 and 35 md$^{-1}$ (range $c_{\text{max}} - c_{\text{min}} = 29$ md$^{-1}$). Next, the mean bar height $\bar{h}$ and mean migration speed $\bar{c}$ were calculated from an average over the last 100 days of the simulation period. These were compared with the bar height $h_t$ and migration speed $c_t$ in the case of time-invariant forcing. It was found that $\Delta h = \bar{h} - h_t \sim 0.02$ m, so mean bar height is smaller in the case of time-varying forcing. On average, the migration speed of the bars does not significantly differ from that in the case of a time-invariant angle ($\bar{c} = \bar{c}_t$).
Figure 4. (a-d) Time evolution of (a-b) the longshore profile of bottom perturbations $h$ at $x=50$ m (indicated by the dashed horizontal black lines in Fig. 3), root-mean-square height of bars $\|h\|$ (c) and of their migration speed $c$ (d) in the two cases. [Colour figure can be viewed at wileyonlinelibrary.com]

Figure 5. Results from run series ‘SensToPeriodMeanAngle’. (a) Difference $\Delta \bar{h}$ between the mean heights of the saturated bars of the cases of time-varying ($\bar{h}$) and time-invariant angles ($\bar{h}_s$). Difference $\Delta \bar{h}$ is scaled with the total mean heights of the two cases ($\bar{h} + \bar{h}_s$). The black line indicates the 0-contour level. Bars do not form in the grey area. (b,c) Time series of the longshore profile of bottom perturbations $h$ at $x=50$ m for the cases of time-varying angles around $\theta_0=6^\circ$ and which have periods $T=1$ d (b) and $T=112$ d (c). Note that the colour bar in panel (c) has a different scale. (d,e) As in panel (a) but for ranges $\|h\|_{\text{max}} - \|h\|_{\text{min}}$ and $c_{\text{max}} - c_{\text{min}}$ of the variations in, respectively, height $\|h\|$ (d) and migration speeds $c$ (e) of the saturated bars. [Colour figure can be viewed at wileyonlinelibrary.com]
time-invariant angles in the \([T,\theta_0]\) parameter space (run series ‘SensToPeriodMeanAngle’). This figure reveals that, compared with the cases of time-invariant angles, bars have larger (mean) heights in the cases of time-varying wave angles around 0-means and smaller heights in the cases of varying angles around non-zero means \(\theta_0\). This difference between the height of the bars of the two cases occurs especially for periods \(T\) that are of the order of the adjustment timescale of the bars or longer. The height of the bars in the cases of smaller periods \(T\) (order of days) is almost identical to that of the bars of the cases of time-invariant angles. Bars strongly decay or they do not form at all (grey area in Figures 5a, 5d and 5e) when angle \(\theta\) varies around large means \(\theta_0\) (> 4°) and has periods \(T\) that are long compared to the adjustment timescale of the bars. This bar decay can also be seen in Figure S2 (Supporting Information), which shows the bed levels of the saturated bars for different periods \(T\) and mean angles \(\theta_0\). Model results (not shown) indicate that the mean migration speeds (\(\bar{c}\)) of bars of the cases of time-varying angles with different periods \(T\) do not significantly differ from those of the cases of time-invariant angles (\(\bar{c}_0\)). The longshore spacing \(\lambda_d\) between bars is mostly affected by the angle of wave incidence; namely, the bars are more spaced apart for larger mean angles \(\theta_0\) (Figures S2 and S3a). In the cases of time-varying wave angles around small mean angles \(\theta_0\), the bars generally have larger longshore spacings (up to 20 m) compared with those in the cases of time-invariant angles (Figure S3b).

Model results further show that in all the cases with different mean angles \(\theta_0\) and periods \(T\), the response of the heights of bars to the presence of waves with sinusoidally varying angles \(\theta(t)\) is not purely sinusoidal. Especially for large periods \(T\) and oblique mean angles \(\theta_0\), oscillations in \(\|h\|\) are highly asymmetric (see Figures S4a–S4c, Supporting Information). The oscillations in migration speed \(c\) are less distorted compared with those in height \(\|h\|\) (Figures S4b–S4d). If \(\theta(t)\) varies around a 0-mean, the period with which the bar height oscillates \(T_h\) is always twice as small as the period \(T\) \((T_h = T/2)\). This feature does not occur in the migration speed of the bars, whose period of oscillation \(T_c\) equals that of the wave angle \(\theta(t)\). The range of the oscillations in \(\|h\|\) \((\|h\|_{\text{max}} - |\|h\|_{\text{min}}|\) increases when mean angle \(\theta_0\) is in the range [0,4]° and it decreases for \(\theta_0 > 4°\). Range \(\|h\|_{\text{max}} - |\|h\|_{\text{min}}|\) attains a maximum value when \(\theta(t)\) varies with a period of \(T = 28\) d and around the mean angle \(\theta_0 = 4°\). The range of variations in the migration speed of the bars \((c_{\text{max}} - c_{\text{min}})\) exhibits a similar behaviour to that of the bar height (Figure 5e); i.e. range \(c_{\text{max}} - c_{\text{min}}\) increases when mean angle \(\theta_0\) is in the range [0,4]°, attains a maximum around \(\theta_0 = 4°\) and decreases for \(\theta_0 > 4°\).

Influence of the amplitude of variation \(\hat{\theta}\)

Figure 6a demonstrates that, with respect to the cases of time-invariant angles, the mean height of the saturated sand bars \(\bar{h}\) of the cases of time-varying wave angles \(\theta(t)\) with non-zero mean angles \(\theta_0\) becomes smaller with increasing amplitude of the variation in the wave angle \(\hat{\theta}\). The larger is \(\theta_0\), the stronger the decrease in the height of bars (compare also

![Figure 6](https://image-url.com)

**Figure 6.** Results from run series ‘SensToAmplitude’. (a) As in Figure 5a, but in the \([\theta_0, \hat{\theta}]\) parameter space. (b,c) Time series of the longshore profile of bottom perturbations \(h\) at \(x = 50\) m for the cases of time-varying angles around \(\theta_0 = 4°\) and which have amplitudes \(\hat{\theta} = 0.5°\) (b) and \(\hat{\theta} = 6°\) (c). (d,e) As in Figures 5d and 5e, but in the parameter space \([\theta_0, \hat{\theta}]\). [Colour figure can be viewed at wileyonlinelibrary.com]
Figures 6b and 6c). When angle \( \theta(t) \) varies around \( \theta_0 = 0^\circ \), the mean height \( \bar{h} \) of the bars is larger for \( \bar{\theta} \) ranging between 0\(^{\circ}\) and 4\(^{\circ}\) and smaller for \( \bar{\theta} > 4^\circ \), compared with those of the time-invariant angle \( \bar{\theta} = 0^\circ \). Also in the cases of different amplitudes of \( \bar{\theta} \), there are no significant differences between mean migration speeds for time-varying \( \bar{\theta} \) and a constant \( \bar{\theta} \).

Overall, the increase in longshore spacing between bars when introducing time variations in the wave angle appears for almost all the amplitudes of variations \( \bar{\theta} \) (Figures S3c and S5, Supporting Information).

Model results further show that (Figure S6a, Supporting Information) higher amplitudes of the oscillation in the wave angle (\( \bar{\theta} \)) enhance the asymmetry of the oscillations in the bar height. The oscillations in migration speed \( c \) are less distorted compared with those in the height (Figure S6b). In all the cases with different amplitudes \( \bar{\theta} \), when \( \theta(t) \) varies around a 0-mean, the bar height oscillates with a period that is twice as small as that of the wave angle \( (T_0 = T/2) \). In the case of large amplitudes \( \theta = 8^\circ \), the oscillation in height \( ||h|| \) also contains modes that have the period of angle \( \theta(t) \). Finally, as can be seen from Figures 6d and 6e, the ranges of oscillations in height \( ||h|| \) and migration speed \( (||h||_{\text{max}} - ||h||_{\text{min}} \) and \( c_{\text{max}} - c_{\text{min}} \)) increase with increasing mean angle \( \theta_0 \) and amplitude \( \bar{\theta} \)

reaching maximum values (red area), after which they decrease to 0 (grey area).

**Discussion**

**Physical interpretation**

Mean bar height

To explain the model results, it is useful to know how the mean height of the bars \( \bar{h}_i \) (averaged over the last 100 days) depends on wave angle \( \bar{\theta} \) in the case of a time-invariant forcing. As is shown in Figure 7a, bars attain maximum heights for certain angles \( \theta_m \) which are approximately equal to \( \theta_m = [\pm 1.5, \pm 3] \) (indicated by the dashed red lines). Bars strongly decay when \( \theta_m > \max(|\theta_m|) \). No bars form beyond a critical wave angle \( \theta_c \) (\( \approx \pm 7^\circ \)). Note that the height of the bars is symmetric with respect to angle \( \theta = 0^\circ \).

**Case 1:** \( |\theta(t)| \leq \max(|\theta_m|) \). Model results (see ‘Influence of the mean angle \( \theta_0 \) and period of variation \( T \) above shown that, when the wave angle \( \theta(t) \) varies inside the region of maximum angles \( (|\theta(t)| \leq \max(|\theta_m|)) \), the mean height of the bars \( \bar{h} \) might be larger than their corresponding height in the case of time-invariant angles \( \bar{h}_i \), depending on the wave period \( T \). For periods \( T \) that are small (order of days) compared with the adjustment timescale of the bars (order of weeks), the mean height of the bars is almost identical to that of the case of a time-invariant angle; i.e. \( \bar{h} \approx \bar{h}_i(\theta_0) \). Due to inertia, the bar system does not have sufficient time to effectively respond to the time-varying wave forcing. For large periods \( T \) (order of months or longer), the bar system is in quasi-equilibrium with the varying wave forcing; i.e. the bars are able to fully adapt to the changing wave angle \( (|\theta(t)| \approx \bar{h}_i(\theta(t))) \). This means that, for these large periods, a time-varying wave angle \( \theta(t) \) leads to (on average) higher bars, i.e. \( \bar{h} \approx \bar{h}_i(\theta_0) \), particularly when \( \theta(t) \) varies around 0\(^{\circ}\) (Figure 7a). For moderate periods \( T \) (of the order of the adjustment timescale of the bars), model results showed that the maximum height the bars reach during one oscillation period strongly increases, suggesting that the bar height exhibits a resonance behaviour. This behaviour might explain why, for moderate periods, the bars are on average higher than those in the case of a time-invariant angle \( \bar{h} > \bar{h}_i(\theta_0) \). The physical mechanisms underlying this resonant behaviour are still unclear and are a topic of further research.

**Case 2:** \( |\theta(t)| > \max(|\theta_m|) \). Similar to the previous case, the response of the height of bars to a wave angle that varies outside the region of maximum angles \( \theta_m \) \( (|\theta(t)| > \max(|\theta_m|)) \) can be explained based on Figure 7a. For moderate or large periods \( T \), the bars are on average lower than those in the case of time-invariant angles \( \theta_m \), particularly when time-mean \( \theta_0 \) and/or amplitude \( \bar{\theta} \) have values of the order of the critical angle \( \theta_c \) (\( \approx \pm 7^\circ \)). The same critical angle was reported in the studies by Garnier et al. (2008, 2013), which is not surprising because their parameter setting is the same as the setting used in the present work. According to Garnier et al. (2013), a weakening of the cross-shore current and a decreasing correlation between the cross-shore current and the bottom perturbations are responsible for the existence of this critical angle. At many beaches where sand bars are present (e.g. Gold Coast, Australia), wave angles ranging between 15\(^{\circ}\) and 30\(^{\circ}\) (in 18 m) were measured (Price and Ruessink, 2013). In the present model, those waves would have angles between 10\(^{\circ}\) and 20\(^{\circ}\) at the offshore boundary. Such angles

![Figure 7](https://wileyonlinelibrary.com)
are larger than \( \theta_c \) found in the present study. However, for an initially longshore uniform beach profile without the presence of a longshore-parallel bar, Garnier et al. (2006) could simulate bars up to a wave angle of 25°. This means that, even though the precise value of \( \theta_c \) might be different in the case of other beach systems, the response of bars to time-varying wave angles close to this critical angle in these systems is expected to be qualitatively the same as that in the present study.

Longshore current reversal
The behaviour of the bars during an oscillation period of \( \theta \) can be understood by the application of the concepts that are discussed in Ribas et al. (2015). They demonstrate that bars can increase in height if the flow and the gradient of the depth-averaged sediment concentration \( C \) in the bar areas have opposite directions. Analysis of model results revealed that in the present model the bar growth is mainly due to the cross-shore flow component and the cross-shore gradient of \( C \). As is shown in Figure S7 of the Supporting Information, for both positive \( \theta \) and negative \( \theta \) the cross-shore current is onshore over the bars, while \( C \) in the bar areas increases in the seaward direction. This demonstrates that the bars maintain themselves against damping by slope-induced diffusive sediment transport. However, during the oscillation period the net growth varies, as can be seen in the figure from the opposite orientations of the bars with respect to the shore-normal. This indicates that longshore currents for positive \( \theta \) contribute to build-up of bars of which the crests are shifted downstream with respect to their shore attachments. However, when \( \theta \) (and thus the longshore current) changes sign, the deflection of the longshore current over that bar causes divergence of sediment transport at the offshore end of the crest and convergence of sediment transport further downstream. As a result, the orientation of the bar changes.

Period and range of the response
The fact that the height of the bars is symmetric with respect to angle \( \theta = 0^\circ \) means that bars that are subject to a time-varying wave angle \( \theta(t) \) around a zero-mean will grow during two phases within one oscillation period \( T \) -- namely, either when \( \theta(t) \) increases from 0° to \( \theta \) or when it decreases from 0° to \( -\theta \). This means that changes in the height of the bars occur twice as often as those in the wave angle. Thus the height of the bars oscillates with a period that is half of that in \( \theta(t) \). Note that this halving of the period is also expected to occur when \( \theta(t) \) would vary either in the range \([-3,-1.5]^\circ \) or \([1.5,3]^\circ \). The fact that the migration speed for time-invariant angles is anti-symmetric in \( \theta \) (Figure 7b) means that in the cases of time-varying wave angles the periods of the oscillations in the migration speed and the wave angle are identical. Bars that are subject to a time-varying \( \theta(t) \) around an oblique mean angle will decay if \( |\theta(t)| > \max(|\theta|) \). For instance, if \( \theta(t) \) has a time mean \( \theta_0 = 4^\circ \) and amplitude \( \theta = 2^\circ \), the bar will experience a strong decay and growth during one oscillation period (Figure 7a), leading to large differences (range) in their height during this oscillation. These differences are more pronounced when period \( T \) is of the order of the adjustment timescale of the bars. Similar arguments apply for the response of the migration speed of the bars to the time-varying wave angle.

Comparison with earlier works
The results of this study show that a time-varying wave angle might become so oblique during one oscillation period that the bars that were previously formed are wiped away. These outcomes are comparable with the results of Garnier et al. (2013), who found that meandering crescentic bars become shore-parallel bars (so-called sandbar straightening) with increasing wave obliquity. This implies that transitions between different bar states are not limited only to high-energetic wave events, as is often stated in the existing literature (e.g. Lippmann and Holman, 1990), but these transitions might also be triggered by variations in the angle of wave incidence within the same wave energy, especially when the waves are on average obliquely incident to the coast. The outcomes of this study support the statement by Price and Ruessink (2011) that it is important to include the wave angle of incidence in relating state transitions of bars to the amount of wave energy.

The findings of this study are in many respects similar to those of Castelle and Ruessink (2011) on crescentic sand bars. A noticeable difference is that crescentic bars, which initially form in the present model, migrate onshore to form shore-attached transverse bars (so-called TBR bars). This transition from crescentic bars to TBR bars was extensively described in the work by Garnier et al. (2008). Another difference is that the mean height of crescentic sand bars in Castelle and Ruessink (2011) for the case of a time-varying angle of wave incidence \( \theta \) (with zero-mean) is always smaller than that in the case of constant \( \theta \). Such behaviour does not always appear in the case of the TBR bars that form in the present model. These bars might become higher than those of the case of time-invariant angle, which occurs when angle \( \theta(t) \) varies in the range \([-\theta_m,\theta_m]\). This difference is attributed to the different configurations applied in the two model studies. One of them is that morfo55 uses a fixed coastline, whereas the model of Castelle and Ruessink (2011) uses a dynamic coastline (its location changes in time). Another difference is that the two models use different wave modules; i.e. morfo55 assumes waves to have narrow spectrum, regarding both frequency and direction, whereas Castelle and Ruessink (2011) use the spectral wave SWAN (Holthuijsen, 2007, and references therein), which accounts for a broad spectrum with multiple frequencies and directions. Finally, Castelle and Ruessink (2011) computed the sediment transport based on Ballard (1981) formulations, whereas the formula of Soulsby (1997) is applied in the present work. All of these aspects need further study.

Other initial bottom perturbations
The simulations of this study start from random bottom perturbations (with an amplitude of 2 cm) that are superimposed on the initial bathymetry. Additional simulations were conducted to quantify the sensitivity of model results to a higher amplitude (4 cm) of the random bottom perturbations and to other types of initial bottom perturbations (a hump of sand and longshore periodic bars). Results from these simulations (Figure S8, Supporting Information) show that only the transient behaviour is different; i.e. after an adjustment time of several weeks, the system reaches the same non-transient (saturated) state.

Conclusions
From this study, the following four main conclusions are drawn. The first is that bars, which are subject to angles of wave incidence \( \theta(t) \) that vary sinusoidally around a non-zero time-mean \( \theta_0 \) have generally a mean height that is smaller than that in the case of time-invariant angles, especially for amplitudes \( \theta > \theta_m \) (with \( \theta_m \) the angle where bars attain maximum heights in the case of a forcing with a time-invariant angle) and oscillation periods \( T \) of the order of the adjustment timescale of the system (in the order of weeks). For angle variations around oblique
means that are close to the critical angle $\theta_c$ (~7°, which is the time-invariant angle beyond which bars do not form), bars decay or they do not even form. When the wave angle $\theta(t)$ varies in the range $[-\theta_{min}, \theta_{max}]$, bars are on average higher than those of the case of time-invariant angles. Overall, the average migration speeds of bars are not significantly different from those in the cases of time-invariant angles.

The second is that bars, which experience large changes in their growth and migration speed during one oscillation period of the wave angle, might exhibit strong oscillations of their heights and migration speeds if the period $T$ is close to the adjustment time scale of the bars. For the setting in this study, these strong oscillations occurred for $\theta_0 = 4^\circ$, $\theta = 6^\circ$ and $T = 28$ d.

The third conclusion is that the oscillations in the bar height and migration speed might contain harmonics with either the period $T$ of the time-varying wave angle, half the period or a combination of the two. In this study, the half-period response dominates the oscillations in bar height if the angle varies around $\theta_0 = 0^\circ$, while the $T$-periodic response dominates for variations around large oblique means ($\theta_0 > 3^\circ$) and a combination of the two modes for mean angles in between. These differences in the time response of the bar height are determined by the degree of asymmetry in the growth rate of the bars to the left ($\theta(t) < \theta_0$) and to the right ($\theta(t) > \theta_0$) of the mean angle. The oscillations in the migration speed of the bars always have the period of the forcing.

The fourth conclusion is that bars that form in the cases of time-varying wave angles have larger longshore spacings than those in the cases of time-invariant angles.

The results of this study confirm previous work (e.g. Price and Ruessink, 2011; Garnier et al. 2013) that large angles of oblique wave incidence can trigger transitions between different beach states. It is thus important that in the description and modelling of beach states, account is made of the wave angle of incidence.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflict of Interest Statement

The authors declare that there is no conflict of interest.

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Supporting Information

Additional supporting information may be found online in the Supporting Information section at the end of the article.

Figure S1. a) Height $h$ and migration speed $c$ (b) of the bars for two values of moac (90 and 20) versus time.

Figure S2. Results of run series ‘SensToPeriodMeanAngle’ (Table 2 in the main manuscript). Contour plots of bed level $z_b$ at $t = 300$ days in the cases of time-varying wave angles with increasing period $T$ ($T = [5; 7; 28; 56; 112]$ days; top→bottom panels) and mean wave angle $\theta_0$ ($\theta_0 = [0; 6^\circ]$); left→right panels). Note that the top panels (a and f) show the cases of time-invariant angles. In all the other cases, the amplitude of angle variation is $\theta = 2^\circ$.

Figure S3. a) The dominant longshore spacing $\lambda_d$ of the saturated bars at $t = 300$ days and $\Delta \lambda_d$ between the longshore spacings of the cases of time-varying and time-invariant wave angles in the parameter space $[\theta_0; \theta_c]$. c) As in a, but in the $[\theta'; \theta_0]$ parameter space. The grey box indicates the area where no bars form.

Figure S4. Time evolution of the root-mean-square height $h$ (a) and global migration speed $c$ (b) of the bars for different time-mean values $\theta_0$ and periods $T$ of the time-varying wave angle (see legend). Zoom-ins of this time evolution are shown in panels c and d.

Figure S5. Results of run series ‘SensToAmplitude’ (Table 2 in the main manuscript). As in Fig. S2, but for the cases with
increasing amplitude of variation $\theta' = [0; 1; 4; 6; 8]^\circ$) and mean wave angle $\theta_0$ ($\theta^0 = [0; 4]^\circ$).

**Figure S6.** As in Fig. S4, but for different values $\theta_0$ and amplitudes of variation $\theta'$.

**Figure S7.** a–b) Snapshots of the depth-averaged sediment concentration $C$ (m$^3$/m$^3$; colours) with superimposed the flow field (arrows) at two different time points ($t = 286; 299$ days; a and b, respectively) during one oscillation of the time-varying wave angle $\theta(t)$ (c). Shoals (troughs) are indicated by solid (dashed) contour lines. The vertical black dashed lines in panel c correspond to the times at which the snapshots shown in the top panels were taken.

**Figure S8.** Time evolution of the root-mean-square height $||h||$ (a) and global migration speed $c$ (b) of the bars for different initial bottom perturbations. The case when starting from random perturbations with an amplitude of 2 cm (the default case applied in all the simulations of this study) is represented by the black lines; the case where this amplitude is increased by a factor of 2 (4 cm) is represented by the blue lines; the case where starting from a sand hump (with amplitude of 2 cm; at location (x, y) = (50; 1000) m) is represented by the red lines; and the case when starting from longshore periodic bars (with amplitude of 2 cm and wavelength of 200 m) is represented by the green lines. All these cases are conducted for a time-varying wave angle $\theta(t)$, having a mean $\theta_0 = 0^\circ$, amplitude $\theta^\circ = 2^\circ$ and period $T = 28$ days.