A calculation method for seepage of vertical wells with concentric rings in horizontal rock layers

Ma Xuqiang, Huang Shuling*, Ding Xiuli, He Jun, Zhang Yuting

Key Laboratory of Geotechnical Mechanics and Engineering of Ministry of Water Resources, Changjiang River Scientific Research Institute, Wuhan, Hubei 430010, China

*Corresponding author, Huang Shuling, E-mail address, huangsl@mail.crsri.cn

Abstract. Some vertical wells are excavated in the horizontal rock layers with different thicknesses and permeability coefficients, and have concentric rings such as lining and grouting circle. The exiting formulas, which are derived for the unlined wells in homogeneous body, are inapplicable to such complex wells. Therefore, a calculation method is developed. Based on the Dupuit assumption, the vertical velocity of groundwater is ignored, and only the horizontal flow is considered. Consequently, each layer is assumed to be independent of the other during seepage. The water inflow and the water pressure of each layer are obtained according to the Darcy law. The total water inflow is the sum of those of all layers. Then, the seepage analysis of a vertical well is carried out by using this method and numerical simulation. The two results are in good agreement. It is found that the water pressure on the lining or that on the grouting ring generally increases with depth and also changes with the permeability coefficient of each layer. The impact of grouting circle parameters on water inflow is discussed. The research in this paper can provide useful references for well seepage analysis and design of grouting circle.

1. Introduction

The vertical well sometimes is a transportation facility between the ground and the underground tunnel in the mine and hydraulic engineering. To evaluate the well’s safety, it is necessary to predict the water inflow and the water pressure on lining.

In the early days, Dupuit derived the equations of water inflow into a well in a phreatic or confined aquifer, but the well is unlined and in a homogeneous formation. Until recently, the water inflow from each aquifer was predicted by the Dupuit equations and then added up to obtain the total water inflow, but the lining and its water pressure were not considered[1-3]. On the contrary, a tunnel with lining and grouting circle has been fully studied. For a tunnel, the equations of the water inflow...
and the water pressure on lining have been obtained\cite{4,5} and verified by the numerical simulation\cite{6,7} and the model test\cite{8}. However, there are significant differences between the well and the tunnel.

Seepage analysis of a well with complex structures such as lining, grouting circle and horizontal rock layers is presented in this paper. The structure features and the boundary conditions of the well are described. The research on the tunnel seepage is used as a reference. A calculation method is proposed for such wells and compared with the numerical software FLAC3D\cite{6,9}. Besides, the sensitivity of water inflow into a well to grouting circle parameters is discussed.

2. Method for the problem
As shown in Figure 1, a well with radius $r_0$ passes through horizontal rock layers vertically. The well has a lining with outer radius $r_c$ and permeability coefficient $k_c$, and has a grouting circle with outer radius $r_g$ and permeability coefficient $k_g$.

![Figure 1. Schematic of a well with lining and grouting circle in horizontal rock layers.](image)
Each layer is regarded as a porous medium. Flow is governed by Darcy’s law. The permeability coefficient of each layer is written as

$$k_i = k_{i'} \quad (z_{i-1} < z < z_i)$$  \hfill (1)

where $i$ is the number of layer, and ranges from 1 to $n$; $z_i$ is the bottom depth of layer $i$.

The far field ($r = R$) water head at depth $z$ is defined as

$$H_z = z - z_0 \quad (z_0 < z \leq z_n)$$  \hfill (2)

where $R$ is the radius of influence; $z_0$ is the depth of initial water level; $z_n$ is the depth of well bottom.

The inner ($r = r_0$) boundary is that the water pressure in the well always equals zero.

The well bottom is an impervious boundary.

A method is proposed in the next to solve the water inflow into the well ($Q_w$), the water pressure on lining at depth $z$ ($\gamma h_w$) and the water pressure on grouting circle at depth $z$ ($\gamma h_{gw}$).

### 2.1 Water inflow into a well

According to Dupuit assumption, horizontal flow around a well is dominating compared to vertical flow. To simply the analysis, only horizontal flow is considered for a well.

The cross section of tunnel is like that of the well shown in Figure 1(b). The water inflow into a tunnel has been derived as$^{[4,5]}$

$$Q = 2\pi k_i H \left( \ln \frac{R}{r_g} + \frac{k_i}{k_g} \ln \frac{r_g}{r_0} + \frac{k_i}{k_c} \ln \frac{r_c}{r_0} \right)^{-1}$$  \hfill (3)

However, tunnel seepage is studied as a plane problem. The permeability coefficient of surrounding rock $k_i$ and the far field water head $H$ remain constant along the tunnel axis.

Because of these differences, $k_i$ is replaced with $k_i$ according to Equation (1), and $H$ is replaced with and $H_z$ according to Equation (2). Thus, the water inflow into a well from depth $z$ can be given as

$$Q_z = 2\pi k_i (z - z_0) \left( \ln \frac{R}{r_g} + \frac{k_i}{k_g} \ln \frac{r_g}{r_0} + \frac{k_i}{k_c} \ln \frac{r_c}{r_0} \right)^{-1}$$  \hfill (4)

After integration, the water inflow from layer $i$ can be described by

$$Q_i = \int_{z_{i-1}}^{z_i} Q_z \, dz = \pi k_i (z_i + z_{i-1} - 2z_0)(z_i - z_{i-1}) \left( \ln \frac{R}{r_g} + \frac{k_i}{k_g} \ln \frac{r_g}{r_0} + \frac{k_i}{k_c} \ln \frac{r_c}{r_0} \right)^{-1}$$  \hfill (5)

Then, the total water inflow into a well is obtained as

$$Q_w = \sum_{i=1}^{n} Q_i$$  \hfill (6)
If all the permeability coefficients are the same \((k = k_i = k_g = k_c)\), the total water inflow into a well is simplified as

\[
Q_w = \frac{\pi k (z_a - z_0)^2}{\ln \frac{R}{r_0}} = \frac{\pi k H^2}{\ln \frac{R}{r_0}} \tag{7}
\]

This equation has been put forward by Dupuit, and applies to a fully penetrating bare well in a phreatic aquifer.

2.2. Water pressures on lining and grouting circle of well

For a tunnel, the water pressures on lining and grouting circle have been derived as\[4,5]\n
\[
P_c = \gamma h_c = \gamma H \ln \frac{r_c}{r_0} \left( \frac{k_i}{k_c} \ln \frac{R}{r_g} + \frac{k_c}{k_i} \ln \frac{r_i}{r_c} \right)^{-1} \tag{8}
\]

\[
P_g = \gamma h_g = \gamma H - \gamma H \ln \frac{R}{r_g} \left( \frac{k_i}{k_c} \ln \frac{r_i}{r_c} + \frac{k_c}{k_i} \ln \frac{r_c}{r_g} \right)^{-1} \tag{9}
\]

where \(\gamma\) is the unit weight of groundwater; \(h_c\) is the water head on lining; \(h_g\) is the water head on grouting circle.

Similarly, \(k_c\) and \(H\) are replaced with \(k_i\) and \(H_z\), respectively. The water pressures on lining and grouting circle at depth \(z\) can be obtained for a well

\[
P_{ci} = \gamma h_{ci} = \gamma (z - z_0) \ln \frac{r_c}{r_0} \left( \frac{k_i}{k_c} \ln \frac{R}{r_g} + \frac{k_c}{k_i} \ln \frac{r_i}{r_c} \right)^{-1} \quad (z_{i-1} < z < z_i) \tag{10}
\]

\[
P_{gi} = \gamma h_{gi} = \gamma (z - z_0) - \gamma (z - z_0) \ln \frac{R}{r_g} \left( \frac{k_i}{k_c} \ln \frac{r_i}{r_c} + \frac{k_c}{k_i} \ln \frac{r_c}{r_g} \right)^{-1} \quad (z_{i-1} < z < z_i) \tag{11}
\]

3. Results and discussion

3.1. Validity of the method

An example of well is provided in this section. The proposed analytical method is used to calculate the water inflow and the water pressure of the well. The numerical software FLAC3D is also applied. To validate the method, the analytical results are compared with the numerical results.

A 3D model of the well is built with FLAC3D and the dimensions are shown in Figure 2. Owing to the symmetry, only a quarter of the entire geological structure is modeled. The model has a height of 200 m. From top to bottom, it is divided into 5 layers with bottom depths of 40 m, 90 m, 130 m, 180 m and 200 m. The model width is 150 m and used as the value of the radius of influence. It is large in
comparison with the well radius which is 5 m. The outer radiiuses of lining and grouting circle are 6 m and 14 m, respectively.

The boundary conditions are as follows: The top surface of the model is the ground where the initial water level is located; the water pressure increases linearly from 0 to 2 MPa on the lateral surfaces at \( x = 150 \) and \( y = 150 \); the impermeable boundaries are the bottom surface and the lateral surfaces at \( x = 0 \) and \( y = 0 \); the water pressure on the well wall remains at zero.

![3D model of a well](image)

**Figure 2.** 3D model of a well.

All the permeability coefficients are listed in Table 1.

**Table 1.** Permeability coefficients.

| Permeability coefficient \((10^{-5} \text{ cm s}^{-1})\) | Lining | Grouting circle | Layer 1 | Layer 2 | Layer 3 | Layer 4 | Layer 5 |
|-----------------------------------------------|--------|----------------|--------|--------|--------|--------|--------|
| 6                                             | 8      | 20             | 40     | 100    | 30     | 50     |

The model has 16000 zones and 18491 grid points. It took 212127 steps to complete the numerical calculation in FLAC3D. Figure 3 shows the distributions of water pressure and flow vector around the well. Water pressures decrease from far to near in the radial direction. The dip angles of flow vectors are mostly small, which proves that the assumption of horizontal flow is acceptable for the well. The lengths of flow vectors are proportionate to specific discharges at the grid points, and increase with depth which has also been revealed by Equation (4).
Based on the Equation (5), the water inflow from layer $i$ can be predicted. In FLAC3D, water inflow can be obtained by adding up the flow discharges at grid points. Figure 4 shows the two methods have similar results for water inflow. As mentioned earlier, the specific discharge increases with depth. The thicknesses of the layer 1, 2, 3 and 4 are close to each other. Therefore, the water inflow increases layer by layer. In addition, the water inflow from the layer 5 is less than that from the layer 4, because the thickness of the layer 5 is about half of the layer 4.

![Water pressure distribution](image)

**Figure 3.** Distributions of water pressure and flow vector around the well obtained using FLAC3D.

Based on the Equations (10) and (11), the water pressures on lining and grouting circle at depth $z$ can be obtained. In FLAC3D, water pressures at grid points can be printed. Figure 5 shows the two kinds of results are in good agreement with each other. As the water pressure decreases from far to near in the radial direction, the water pressure on lining is lower than that on grouting circle. Generally, the water pressures increase linearly with depth. Besides, the water pressures fluctuate with the permeability coefficients of layers. The abrupt change of water pressure is caused by the difference between two neighboring layers in permeability coefficients.

![Water pressure versus depth](image)

**Figure 4.** Calculated water inflow from each layer based on Equation (5) and FLAC3D.
Figure 5. Calculated water pressures on lining and grouting circle at different depths based on Equations (10), (11) and FLAC3D.

3.2. Sensitivity analysis

According to Equation (5), the water inflow from layer \( i \) is dependent on the following factors: average water head of the layer \( (z_i + z_{i-1} - z_0)/2 \); layer thickness \( (z_i - z_{i-1}) \); radii such as \( r_0, r_c, r_g \) and \( R \); permeability coefficients such as \( k_c, k_g \) and \( k_i \). Some factors belong to natural factors, including water head, thickness and permeability coefficient of the layer, which are certain for a specific project. Other factors are human factors which can be adjusted. Due to space limitations, only the sensitivity of the water inflow to the grouting circle parameters will now be discussed. The discussion can provide a basis for controlling water inflow into a well.

Take the well mentioned earlier as an example. Figure 6 shows the relationship between water inflow and grouting circle parameters including its thickness and permeability coefficient.

Figure 6. Relationship between water inflow and grouting circle parameters.
Figure 6(a) shows that the water inflow decreases with the increasing thickness of grouting circle. However, the difference between neighboring water inflows decreases gradually. When the difference becomes small enough, it is unnecessary to further increase the thickness of grouting circle. It can be assumed that the grouting circle has an economical thickness when the difference between neighboring water inflows is less than 5%. In this case, when the thicknesses of grouting circle are 5, 6 and 7 m, the water inflows are 6497, 6159 and 5878 m³/d, and the differences are 5.2% and 4.6%. Therefore, the economical thickness is about 7 m.

In addition, Figure 6(b) shows that the water inflow increases rapidly when the permeability coefficient of grouting circle increases from 0 to $10 \times 10^{-5}$ cm/s. However, it is difficult to further lower the permeability coefficient when it has been small. Taken together, it is prefer to control the water inflow by selecting a reasonable thickness of grouting circle.

4. Conclusions

A method is proposed to predict the water inflow and the water pressure of a well with concentric rings in horizontal rock layers. It has been proved that the method have a good prediction accuracy by comparing with numerical simulation.

It is found that the water inflow into a well can be reduced by increasing the thickness or lowering the permeability coefficient of grouting circle. The water pressure on lining generally increases with depth, and fluctuates with the permeability coefficient of each layer.

Acknowledgements

This work is supported by the National Nature Science Foundation of China (Grant Nos. 51979008; 51779018; 41807249; 51809014) and the Basic Research Fund for Central Research Institutes of Public Causes (Nos. CKSF2021715/YT; CKSF2021457/YT; CKSF2021458/YT).

References

[1] Lin ZB, Li YH, Gui CL, Liu JQ and Qin XL 2013 Chin. J. Geotech. Eng. 35 2290–2297
[2] He Z, Wan HD, Hu KG and Xu T 2018 Miner. Eng. Res. 33 31–35
[3] Li YF and Guo TZ 2007 Geol. Anhui. 17 275–277
[4] Wang JY 2003 Mod. Tunn. Tech. 40 5–10
[5] Wang XY, Wang MS and Zhang M 2004 J. Northern. Jiaotong. Univ. 28 8–10
[6] Hua FC 2013 Rock. Soil. Mech. 34 299–304
[7] Zheng B, Wang JY and Wu J 2012 Mod. Tunn. Tech. 49 60–65
[8] Yang SZ, He C, Li Z, Yang WB and Luo YW 2017 Chin. J. Univ. Mining. Tech. 46 546–553
[9] Zhang XT, Wang HL, Zhou HM and Lu LB 2008 Rock. Soil. Mech. 29 258–262