Dipole-dipole scattering in CGC/saturation approach at high energy: summing Pomeron loops

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Abstract: In this paper we demonstrate that the dense system of partons (gluons) can be produced in dilute-dilute system scattering, using the example of dipole-dipole collisions. This increase in density stems from the intensive gluon cascades that can be described by the enhanced BFKL Pomeron diagrams (Pomeron loops). For the first time we found the analytical solution to the equation for diffraction production in the dipole-dense parton system scattering, using the simplified BFKL kernel. Having this solution as well as the solution to Balitsky-Kovchegov equation we developed technique that allowed us to calculate the total cross section, cross sections for single and double diffractions in the MPSI approximation. Calculating inclusive production and two gluon correlations we see that the dense and strongly correlated system of gluons can be produced at high energy in the dipole-dipole scattering.

Keywords: Color Glass Condensate, gluon saturation, BFKL Pomeron, calculus, non-linear evolution, geometric scaling behavior.

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1. Introduction

LHC data support the assumption that the dense system of partons is produced in the proton-proton collisions at high energy. Such dense system of partons naturally appears in the CGC/saturation approach to high energy QCD [1–6]. The success in description of both the general properties of the bias event [7] and the long range rapidity angular correlations in the framework of CGC/saturation approach [8] makes this assumption a working hypothesis which allows us to look at hadron-hadron, hadron-nucleus and nucleus-nucleus interactions from the unique point of view. We wish to single out two sets of the experimental data which confirm the hypothesis. The first one is the measurement of the long range rapidity correlations in the azimuthal angle between two produced hadrons [9] which have the same pattern as such correlations in proton-nucleus [10] and nucleus-nucleus collisions [11]. The second set of the data is the measurement of the double parton interaction (DPI) [12]. In these experiments the double inclusive cross sections of two pair of back-to-back jets with momenta $p_{T,1}$ and $p_{T,2}$, were measured with rapidities of two pairs ($y_1$ and $y_2$) which are close to each other ($y_1 \approx y_2$). These pairs can be produced only from two different parton showers. The data were parameterized in the form

\[
\frac{d\sigma}{dy_1d^2p_{T,1}dy_2d^2p_{T,2}} = \frac{m}{2\sigma_{eff}} \frac{d\sigma}{dy_1d^2p_{T,1}} \frac{d\sigma}{dy_2d^2p_{T,2}}
\]  

(1.1)

where $m = 2$ for different pairs of jet and $m = 1$ for identical pairs. One can calculate the rapidity correlation function using Eq. (1.1)

\[
R(y_1,y_2) = \frac{1}{\sigma_{in}} \frac{d\sigma}{dy_1d^2p_{T,1}dy_2d^2p_{T,2}} - 1 = \frac{\sigma_{in}}{\sigma_{eff}} - 1 \approx 2
\]  

(1.2)

For the above the estimates we use $\sigma_{eff} = 12 - 15$ mb (see Refs. [12]) and $\sigma_{in} = \sigma_{tot} - \sigma_{el} - \sigma_{sd} - \sigma_{dd} \approx 50$ mb for the energy $W = 7$ TeV (see Ref. [13] and references therein). Using that $y_1 \approx y_2 \approx 4 \div 5$ in ATLAS experiment at $W = 7$ TeV (see Ref. [12]) and the estimates that one gluon jet decays in two hadrons [7] we can evaluate the density of parton in rapidity, namely, $dN_{parton}/dy \approx 1$ from the inclusive cross sections measured at the LHC [14]. Therefore, we can conclude that at $W = 7$ TeV the dense system of parton produced and these parton strongly interact with each other.

Since at low energy the proton consists of a moderate number of partons and can be considered as a dilute parton system, the only way, how the proton could become a dense system of partons, is due to intensive decay of partons inside the parton cascade (see Fig. 1). In other words, we need to sum the enhanced BFKL Pomeron [15,16] diagrams to create the dense system of partons. The first such diagram is shown in Fig. 1. Therefore, we face the difficult problem of summing enhanced diagrams in the framework of the BFKL Pomeron calculus [17]. The goal of this paper is to sum all enhanced diagrams for dipole-dipole scattering at high energy. For solution of this problem we are going to exploit Mueller-Patel-Salam-Iancu (MPSI) approximation [18] shown in Fig. 2 following the procedure suggested in Ref. [19]. One can see that the MPSI approximation is based on two key features of high density QCD: on $t$-channel unitarity and on the simple dipole cascade generated by dilute system (one upper or lower dipole in our case). This cascade
can be described by the simple generating functional \[20,21\] which is equivalent to Balitsky-Kovchegov \[4\] and JIMWLK(KLWMIG) \[5\] evolutions. In the BFKL Pomeron calculus this cascade corresponds to summation of fan Pomeron diagrams in the region of \(Y - Y' \gg 1\) and \(Y' - 0 \gg 1\) in Fig. 2). Rapidity \(Y'\) is artificial rapidity which does not enter the final answer. Indeed, the BFKL Pomeron has the following property from the \(t\)-channel unitarity \[1,22\] at any value of \(Y'\):

\[
N_P = \int d^2r_1 d^2r_2 d^2b_1 d^2b_2 G_P \left( r, r_1, \vec{b} - \vec{b}_1 \middle| Y - Y' \right) \gamma \left( r_1, r_2, \vec{b}_1 - \vec{b}_2 \right) G_P \left( r_2, R, \vec{b}_2 \middle| Y' \right)
\]

In Eq. (1.3) \( N_P \) describes the dipole-dipoles scattering amplitude due to the single BFKL Pomeron exchange, \( G_P \) denotes the Green function of the BFKL Pomeron. \( \gamma \left( r_1, r_2, \vec{b}_1 - \vec{b}_2 \right) \) is the amplitude of interaction of two dipoles with sizes: \( r_1 \) and \( r_2 \) at the impact parameter \( \vec{b}_1 - \vec{b}_2 \) in the Born Approximation of perturbative QCD in which two dipoles interact due to exchange of two gluons. \( r \) and \( R \) are the sizes of the scattering dipoles.

In the next section we will find the amplitude for dipole-dipole scattering (or the resulting Green function of the BFKL Pomeron) at high energy using the approach proposed in Ref. [23].

2. Parton cascade of the fast dipole

Most of the material of this section is not new and has already appeared in [23, 24]. We include it here for completeness in order to present a coherent picture of the approach.
Figure 2: MPSI approximation: all notations are shown in the insertion. Wavy lines denote the BFKL Pomerons, the blob stands for the scattering amplitude of two dipoles with sizes $r_1$ and $r_2$ in the Born approximation (due to exchange of two gluons). $G_{3P}$ is the triple BFKL Pomerons vertex.

2.1 Simplified BFKL kernel

BFKL kernel is rather complicated and the analytical solution of the non-linear equation with this kernel has not been found. In Ref. [23] it was suggested to simplify the kernel by taking into account only log contributions. From formal point of view this simplification means that we consider only leading twist contribution to the BFKL kernel. Note that the full BFKL kernel includes all twists contributions. Actually we have two kinds of logs: $(\bar{\alpha}_S \ln \left( \frac{r^2 \Lambda_{QCD}^2}{Q_s^2(Y,b)} \right))^{n}$ outside of the saturation region ($r^2 Q_s^2(Y,b) \equiv \tau \ll 1$); and $(\bar{\alpha}_S \ln \left( \frac{r^2 Q_s^2(Y,b)}{Q_s^2(Y,b)} \right))^{n}$ inside the saturation domain ($\tau \gg 1$). To sum all logs for $\tau \ll 1$ we can simplify the BFKL kernel $K(r; r')$ in the following way [23], since $r' \gg r$ and $|\vec{r} - \vec{r'}| > r$

$$
\int d^2r' K(r; r') \equiv \int d^2r' \frac{r^2}{r^2 (\vec{r} - \vec{r'})^2} \to \pi r^2 \int_1^{r^2} \frac{dr'^2}{r'^4} \quad (2.1)
$$

Inside of the saturation region where $\tau > 1$ the logs are originated from the decay of the large size dipole into one small size dipole and one large size dipole. However, the size of the small dipole is still larger than $1/Q_s$. This observation can be translated in the following form of the kernel

$$
\int d^2r' K(r; r') \to \pi \int_{1/Q_s^2(Y,b)}^{r^2} \frac{dr'^2}{r'^2} + \pi \int_{1/Q_s^2(Y,b)}^{r^2} \frac{d|\vec{r} - \vec{r'}|^2}{|\vec{r} - \vec{r'}|^2} \quad (2.2)
$$

The Mellin transform of the full BFKL kernel has the form

$$
\chi(\gamma) = \int \frac{d\xi}{2\pi i} e^{-\gamma \xi} K(r; r') = 2\psi(1) - \psi(\gamma) - \psi(1 - \gamma) \quad (2.3)
$$
where $\xi = \ln(r^2/r'^2)$ and $\psi(z) = d\ln \Gamma(z)/dz$ with $\Gamma(z)$ equal to Euler gamma function. The simplified kernel replaces Eq. (2.3) by the following expression

$$
\chi(\gamma) = \begin{cases} 
\frac{1}{\gamma} & \text{for } \tau \geq 1; \\
\frac{1}{1-\gamma} & \text{for } \tau \leq 1;
\end{cases}
$$

(2.4)

One can see that the advantage of the simplified kernel of Eq. (2.4) is that it provides a matching with the DGLAP evolution equation [25] in Double Log Approximation (DLA) for $\tau < 1$.

The non-linear BK equation takes two different forms outside and inside the saturation region. For $\tau < 1$ it can be written as

$$
\frac{\partial^2 n(r,Y;b)}{\partial Y \partial \ln(1/(r^2\Lambda_{QCD}^2))} = \frac{\bar{\alpha}_S}{2} \left( 2n(r,Y;t=0) - n^2(r,Y;b) \right)
$$

(2.5)

for $n(r,Y;b) = N(r,Y;b)/r^2$ where $N(r,Y;b)$ is the dipole scattering amplitude.

Inside the saturation region where $\tau > 1$ the BK equation takes the form

$$
\frac{\partial^2 \tilde{N}(r,Y;b)}{\partial Y \partial \ln r^2} = \bar{\alpha}_S \left\{ 1 - \frac{\partial \tilde{N}(r,Y;b)}{\partial \ln r^2} \right\} \tilde{N}(r,Y;b)
$$

(2.6)

where $\tilde{N}(r,Y;b) = \int r^2 dr'^2 N(r',Y;b)/r'^2$.

### 2.2 Solution to BK equation

Outside the saturation region the non-linear corrections in Eq. (2.5) affect the behaviour of the solution to the linear BFKL equation only in the vicinity of the saturation scale ($\tau \rightarrow 1$) where the solution takes the following form [26]

$$
N(Y;r,b) \propto (r^2Q_s^2(Y_0))^{1-\gamma_{cr}}
$$

(2.7)

where the critical anomalous dimension $\gamma_{cr}$ given by

$$
-\frac{\partial \omega(\gamma_{cr})}{\partial \gamma_{cr}} = \frac{\omega(\gamma_{cr})}{1-\gamma_{cr}}
$$

(2.8)

One can see that Eq. (2.7) shows the geometric scaling behaviour [27] and it takes the form

$$
N(Y;r,b) = N_0 e^{\frac{z}{2}} \quad \text{where} \quad z = \ln \tau = 4 \bar{\alpha}_S (Y - Y_0) + \ln(r^2Q_s^2(Y = Y_0;b)) = \xi_s + \xi
$$

(2.9)

since for the kernel of Eq. (2.4) $\gamma_{cr}=1/2$. In Eq. (2.3) $\xi_s = 4 \bar{\alpha}_S (Y - Y_0)$ and $\xi = \ln(r^2Q_s^2(Y = Y_0;b))$.

In the entire kinematic region $\tau < 1$ the solution takes the following form for the simplified kernel of Eq. (2.5)

$$
N(Y;r,b) = N_0 \exp\left(\sqrt{-\xi_s \xi} + \xi\right) \xrightarrow{\tau \rightarrow 1} N_0 e^{\frac{z}{2}} \exp\left(-\frac{z^2}{8\xi_s}\right)
$$

(2.10)
Recall that $N_0$ is the value of the dipole amplitude at $z = 0$ and $\xi < 0$ for $\tau < 1$. One can see that the geometric scaling behaviour holds at $z \ll 8 \xi_s$.

Solution of Eq. (2.9) provides the boundary condition for the solution inside the saturation region:

$$N(Y; \xi = -\xi_s, b) = N_0(b); \quad \frac{\partial \ln N(Y; \xi = -\xi_s, b)}{\partial z} = \frac{1}{2}; \quad (2.11)$$

Inside the saturation region ($z > 0$) we are looking for the solution in the form

$$\tilde{N} = \int_{\xi_s}^{\xi} d\xi' \left( 1 - e^{-\phi(Y, \xi')} \right) \quad (2.12)$$

Substituting Eq. (2.12) into Eq. (2.6) we obtain

$$\phi_Y e^{-\phi} = \bar{\alpha} S \tilde{N} e^{-\phi} \quad (2.13)$$

Canceling $e^{-\phi}$ and differentiating with respect to $\xi$ we obtain the equation in the form:

$$\frac{\partial^2 \phi}{\partial Y \partial \xi} = \bar{\alpha} S \left( 1 - e^{-\phi(Y, \xi)} \right) \quad (2.14)$$

Using variable $\xi_s$ and $\xi$ we can rewrite Eq. (2.13) in the form

$$\frac{\partial^2 \phi}{\partial Y \partial \xi_s} \frac{\partial^2 \phi}{\partial \xi \partial \xi} = \frac{1}{4} \left( 1 - e^{-\phi(Y, \xi)} \right) \quad \text{or in the form} \quad \frac{\partial^2 \phi}{\partial z^2} - \frac{\partial^2 \phi}{\partial x^2} = \frac{1}{4} \left( 1 - e^{-\phi(Y, \xi)} \right) \quad (2.15)$$

with $z$ defined in Eq. (2.9) and $x = \xi_s - \xi$.

Eq. (2.13) has general traveling wave solution (see Ref. [28] formula 3.4.1)

$$\int_{\phi_0}^{\phi} \frac{d\phi'}{\sqrt{c + \frac{1}{2(\lambda^2 - \kappa^2)} (\phi' - 1 + e^{-\phi'})}} = \kappa x + \lambda z \quad (2.16)$$

where $c, \phi_0, \lambda$ and $\kappa$ are arbitrary constants that should be found from the initial and boundary conditions.

From the matching with the perturbative QCD region (see Eq. (2.11)) we have the following initial conditions for small values of $\phi_0$:

$$\phi(t \equiv z = 0, x) = \phi_0(b); \quad \phi_z(t \equiv z = 0, x) = \frac{1}{2} \phi_0(b) \quad (2.17)$$

These conditions allow us to find that $\kappa = 0$ and $c = 0$ for $\phi_0 \ll 1$. Therefore, solution of Eq. (2.16) leads to the geometric scaling since it depends only on one variable: $z$. It takes the form [23, 28] for small values of $\phi_0$

$$\sqrt{2} \int_{\phi_0}^{\phi} \frac{d\phi'}{\sqrt{\phi' - 1 + e^{-\phi'}}} = z \quad (2.18)$$
Figure 3: The scattering amplitude of two dipoles with sizes $r$ and $R$. Fig. 3-a shows the Green function of the ‘bare’ Pomeron while in Fig. 3-b the dressed Pomeron Green function is shown.

2.3 Generating functional for dilute system

Eq. (2.18) we can re-write in a different form, namely,

$$\frac{1}{\sqrt{2}} \int_{\phi_0}^{\phi} \left\{ \frac{1}{\sqrt{\phi' - 1 + e^{-\phi'}}} - \frac{\sqrt{\phi'}}{\phi'} \right\} = \ln N_P (\phi_0, z)$$

(2.19)

where

$$N_P (\phi_0, z) = \frac{\alpha_s^2}{16\pi^2} G_P (\phi_0, z) = \phi_0 (b) e^{\frac{1}{2} z}$$

(2.20)

where $G_P$ is the contribution of one BFKL Pomeron in the saturation region (see Fig. 3-a). The solution to Eq. (2.19) takes a general form

$$N (G_P (\phi_0, z)) = 1 - e^{-\phi(z)} = \sum_{n=1}^{\infty} (-1)^{n+1} C_n (\phi_0) N^n_P (\phi_0, z)$$

(2.21)

where $C_n (\phi_0)$ gives the probability to have $n$ BFKL Pomerons at $Y = Y_0$ from the dipole with size $r$ at high energy ($Y$). The sizes of all dipoles delivered by the BFKL Pomerons are equal and about $1/Q_s$ ($Y = Y_0, b$). $C_n (\phi_0)$ are independent of the size of the initial dipole due to the geometric scaling behaviour.

In general the scattering amplitude that we need for the MPSI approximation (see Fig. 4), looks as follows [19]:

$$N (Y - Y', r, \{r_i, b_i\}) = \sum_{n=1}^{\infty} (-1)^{n+1} \bar{C}_n (\phi_0, r) \prod_{i=1}^{n} N_P (Y - Y'; r, r_i, b_i)$$

(2.22)

Comparing Eq. (2.22) with Eq. (2.21) we see that

$$\bar{C}_n (\phi_0, r) = C_n (\phi_0)$$

(2.23)

and they do not depend on the size of the initial dipole due to the geometric scaling behaviour of the scattering amplitude. Actually, Eq. (2.23) is the master equation of this paper which will allow us to approach the dipole-dipole scattering amplitude in the MPSI approximation. Proof-as well as a more detailed discussion of this equation, is given in Ref. [19].
Solving Eq. (2.19) numerically we find \( N(G) \) (see \( N_{\text{ext}}(G) \) in Fig. 4-a). We find that simple function

\[
N_{\text{apr}}(N) = \kappa (1 - \exp(-N)) + (1 - \kappa) \frac{N}{1 + N}
\]

with \( \kappa = 0.65 \) describes the exact solution within accuracy less that 2.5% (see Fig. 4-b).

### 3. Dressed BFKL Pomeron

Eq. (2.22) we can find in MPSI approximation [18,19,29] the scattering amplitude of two dipoles with sizes \( r \) and \( R \) and impact parameter \( b \) which gives us the Green function of the dressed (final) BFKL Pomeron (see Fig. 3-b). It takes the form (see Fig. 3)

\[
\frac{\alpha_s^2}{16\pi^2} G_F(Y - Y_0, r, R, b) \equiv N_F(Y - Y_0, r, R, b) = \sum_{n=1}^{\infty} n! (-1)^n \left( \frac{\alpha_s}{4\pi} \right)^{2n}
\]

\[
C_n \prod_{i=1}^{n} \int d^2 r_i d^2 r_i' d^2 b_i d^2 b_i' G_{\text{bare}}^\#(r_i r_i' - b_i - b_i'|Y - Y') \gamma(r_i r_i', b_i - b_i') G_{\text{bare}}^\#(r_i' R, b_i'|Y')
\]

Eq. (3.1) can be re-written, using Eq. (1.3) in the form (see Fig. 3 for notations):

\[
N_{\text{dipole-dipole}}(Y - Y_0, r, R, b) = N_F(Y - Y_0, r, R, b) = \frac{\alpha_s^2}{16\pi^2} G_{\text{dress}}^\#(r, R, b|Y - Y_0)^n
\]
From Eq. (2.24) we derive the approximate simple formula for $N (Y - Y_0, r, R, b)$ which looks as follows

$$
N_{\text{dipole-dipole}}^{\text{appr}} (Y - Y_0, r, R, b) = N_{f}^{\text{appr}} (Y - Y_0, r, R, b)
$$

$$
\kappa^2 \left\{ 1 - \exp \left( -T (Y - Y_0, r, R, b) \right) \right\} + 2\kappa (1 - \kappa) \frac{T (Y - Y_0, r, R, b)}{1 + T (Y - Y_0, r, R, b)}
$$

$$
+ (1 - \kappa)^2 \left\{ 1 - \exp \left( \frac{1}{T (Y - Y_0, r, R, b)} \right) \right\} \frac{1}{T (Y - Y_0, r, R, b)} \Gamma \left( 0, \frac{1}{T (Y - Y_0, r, R, b)} \right)
$$

where $\Gamma (0, T)$ is incomplete Euler gamma-function (see Ref. [36] formulae 8.35) and $T (Y - Y_0, r, R, b) = \frac{\alpha_5^2}{16\pi^2} G_{fS}^{\text{bare}} (r, R, b | Y - Y_0) = N_{f} (Y - Y'; r, R, b)$

Let us discuss the most difficult problem of CGC/saturation approach [30]: the impact parameter dependence of the Pomeron Green Function. For our simplified kernel which coincide with the DGLAP kernel outside of the saturation region, we can factorize out the non-perturbative large $b$ behaviour writing for the scattering amplitude in the form:

$$
N_{f} (b, Y, r, R) = S (b) \int d^2 b' N_{f}^{\text{DGLAP}} (b', Y, r) = N_0 r R S (b) e^{b_{\parallel}}
$$

where $N_0$ is a constant (see Ref. [31]). In our estimates we choose $r = R = 1/m$.

Indeed, considering the scattering amplitude at fixed transferred momentum $q$ (which is Fourier conjugated to $b$), one can see that for $q < \mu_{\text{soft}}$ the evolutions in $\ln(1/r)$ do not depend on $q$. However,
for $q > \mu_{soft}$ the logs take the form $\ln(1/(rq))$ and the $q$ dependence cannot be absorbed in $S(b)$ in Eq. (3.5) [31]. Using Eq. (3.5) we can absorbed the non-perturbative corrections at large $b$ in the definition of the saturation scale $Q_s(Y;b)$ [17,23,32–34]. $S(b)$ is a non-perturbative form factor which we parameterize in the form

$$S(b) = \frac{m^2}{2\pi} e^{-mb}; \quad \int d^2b \ S(b) = 1;$$  

(3.6)

introducing mass $m$. Using the experimental data on cross section off the double parton interaction (see Eq. (1.1)) we can write that (see Fig. 6)

$$2C^2 \frac{m^2}{8\pi} = \frac{1}{\sigma_{eff}}$$  

(3.7)

Eq. (3.7) is written assuming that only Pomeron loops contribute to the double parton interaction (see Fig. 6). This assumption looks natural in CGC/saturation approach for the proton-proton scattering as has been discussed in the introduction but, being phenomenological, it should be re-check in the future description of proton-proton date based on the result of this paper. Plugging the experimental value $\sigma_{eff} = 12 \div 15 \text{mb}$ we obtain $m = 0.86 \div 1 \text{GeV}$. This value of $m$ is in a good agreement with other indications of the second dimensional scale in the proton [35]. The variable $z$ in Eq. (3.5) is defined as $z = 4\alpha_s(Y - Y_0) + \ln(r^2/R^2)$. Collecting Eq. (3.5) and Eq. (3.6) the Pomeron Green function takes the following form

$$N_P(b,Y,r,R) = rR N_0 S(b) e^{2\alpha_s(Y - Y_0) + 2\ln(r^2/R^2)}$$  

(3.8)

In all our numerical estimates we take $r = R = 1/m$ and $N_0 = 1/2$. It should be mentioned that any other choice will lead only to redefinition of the value for $N_0$ in Eq. (3.8). With our choice we have

$$N_P(b,Y,r,R) = 0.08 e^{-mb} e^{2\alpha_s(Y - Y_0) + 2\ln(r^2/R^2)}$$  

(3.9)

Certainly $N_P(b = 0, z = 0) = 0.08 \ll 1$ satisfies the condition, that we have used obtaining the solution to the non-linear equation.

4. Diffractive production in dipole-dense target interaction: solution to the equation

The equation for the diffractive production in dipole-target scattering (see Fig. 7) has been known for more than decade [37] (see also Ref. [6]). It has the same form as BK equation but for the following amplitude

$$G(Y;Y_0,r,b) = 2N(Y,r,b) - N^D(Y,Y_0,r,b)$$  

(4.1)

The cross section of diffractive production with the rapidity gap $(Y - \ln M^2$ in Fig. 8) larger than $Y_0$ can be written through $N^D(Y,Y_0,r,b) :$

$$\sigma_{diff} = \int d^2b \ N^D(Y,Y_0,r,b)$$  

(4.2)
From Eq. (4.2) and unitarity constraint follows that $G(Y; Y_0, r, b)$ is the inelastic cross section cross section of all processes except the diffractive production with the rapidity gap $\geq Y_0$. At $Y = Y_0$

$$G(Y_0; Y_0, r, b) = 2N(Y_0, r, b) - N^2(Y, Y_0, r, b)$$

(4.3)

where $N^2$ is the elastic cross section. Eq. (4.3) determines the initial condition for the equation for $G(Y; Y_0, r, b)$. Since the equation for $G$ is the same as for $N$, therefore, the solution in the saturation region is equal to $G(z, z_0) = 1 - \exp(-\phi(z))$ where $\phi$ is given by Eq. (2.16). The difference is the initial condition is given by Eq. (4.3) at $Y = Y_0$. Here, we consider the case when both $Y$ and $Y_0$ are so large that we have the geometric scaling behaviour for the amplitude both at $Y_0$ and $Y$. We introduce two variables

$$z = 4\bar{\alpha}_s Y + \ln(r^2/R^2); \quad z_0 = 4\bar{\alpha}_s Y_0 + \ln(r^2/R^2)$$

(4.4)

where $r$ and $r'$ are the sizes of the dipoles with rapidities $Y$ and $Y_0$, respectively.

Rewriting the initial conditions in the variables of Eq. (4.4) we have

$$G(z, z_0; b) = 1 - \exp(-\phi(\Delta z)) = 1 - \exp(-2\phi(z_0; b)); \quad \phi(\Delta z(z_0; b)) = 2\phi(z_0; b)$$

(4.5)

Final solution takes the form

$$G(z, z_0; b) = 1 - \exp\left(-\phi\left(z - z_0 + \Delta (z_0; b)\right)\right)$$

(4.6)

and

$$N^D(z, z_0; b) = 1 - 2 \exp\left(-\phi(z; b)\right) + \exp\left(-\phi\left(z - z_0 + \Delta (z_0; b)\right)\right)$$

(4.7)

In Fig. 8 we plot the total and diffraction cross sections. We fix $N_{prom}$ in Eq. (3.8) considering $N_0 = 0.5$. Using this form of $N_{\mathcal{P}}$ and solution of Eq. (2.14) we can calculate

$$\sigma_{tot} = 2 \int d^2b\left(1 - \exp\left(-\phi\left(N_{\mathcal{P}}(z, b)\right)\right)\right);$$

$$\sigma_{diff} = \int d^2b\left(1 - 2 \exp\left(-\phi\left(N_{\mathcal{P}}(z, b)\right)\right) + \exp\left(-\phi\left(N_{\mathcal{P}}(z - z_0 + \Delta (z_0, b); b)\right)\right)\right)$$

(4.8)
Figure 7: Example of the diagrams for the diffractive production: single diffraction (Fig. 7-a) and double diffraction (Fig. 7-b). Helix lines denote gluons, wavy lines describe the BFKL Pomeron.

Figure 8: The total cross section and the cross section of the diffraction production with $z_0 = 3$. In Eq. (3.8) $\tilde{\alpha}_S = 0.2, N_0 = 0.5$

In Fig. 8 we plot $\Delta (z_0, b) - z_0$ versus $z_0$ and $b$. One can see that at small $z_0$ Eq. (4.5) can be re-written in the form

$$\phi (\Delta z(0)) = 2 \phi (N\mathcal{P} (z_0, b)) \overset{N_0\ll 1}{\longrightarrow} \phi (2N\mathcal{P} (z_0, b))$$

leading to $\Delta (z_0) - z_0 = 2 \ln 2$ for $\tilde{\alpha}_S = 0.25$. From Eq. (4.8) we can calculate $M^2 d\sigma_{diff}/d\ln M^2$ (see Fig. 7) which takes the form

$$\frac{d\sigma_{diff}}{d\ln M^2} = - \frac{dN^D}{dY_0} = - 4 \tilde{\alpha}_S \frac{dN^D}{dz_0} = 2\tilde{\alpha}_S \frac{d\phi (N\mathcal{P})}{d\ln N\mathcal{P}} \frac{d (\Delta (z_0, b) - z_0)}{dz_0} \ e^{-\phi (N\mathcal{P}(z-z_0+\Delta (z_0, b), b))}$$

(4.10)
Figure 9: Behaviour of $\Delta(z_0, b)$ versus $z_0$ (Fig. 9-a) and versus $b$ (Fig. 9-b).

Figure 10: Fig. 10-a shows the impact parameter dependence of the single diffraction at different values of $z_0$ for fixed $z = 17.75$. The curves are multiplied by factors: at $z_0 = 6$ by 0.1 and at $z_0 = 14$ by 0.2. For elastic cross section the factor is 0.1. Both $\frac{d\sigma_{diff}}{d\ln M^2 db}$ and $b$ are shown in 1/GeV. In Fig. 10-b it is plotted the dependence of the diffraction cross section at fixed mass ($d\sigma_{diff}/d\ln M$) at $z = 17.75$ which corresponds to the LHC energy for $\bar{\alpha}_S = 0.25$.

In Fig. 10 we plot $\frac{d\sigma_{diff}}{d\ln M^2 db}$ at fixed $z = 17.75$ which for $\bar{\alpha}_S = 0.25$ corresponds to the LHC energy. One can see that the largest contribution stems from the large mass kinematic region and the $b$-dependence shows a peripheral - type of behaviour versus $b$ with maximum at $b \approx 6$. We plot in this figure the elastic cross section at fixed $b$ ($A^2_{el}(b)$) to illustrate the peripheral character of the diffraction production.
5. Single diffractive production for dipole-dipole interaction in the MPSI approximation

The pattern of calculation of diffractive production in the dipole-dipole scattering is shown in Fig. 11. This picture illustrates the main difference between calculation of the scattering amplitude and the cross section of the diffractive production: for the latter we need to introduce the difference between BFKL Pomeron in the scattering amplitude (in black in Fig. 11) and in the complex conjugated amplitude shown in blue in Fig. 11 (see Ref. [6, 38] and references therein). The general equation for the cross section of the single diffractive production takes the form

\[
N^D_{\text{dipole-dipole}} = N^D (z - z', z_0, \hat{b} - \hat{b}') \bigotimes N (z', b') N^* (z', b') = \sum_{n=1}^{\infty} \sum_{k=1}^{n-1} n! \left( \frac{(n-k)!k!}{n!} \right)^2 (-1)^n \times \left( \frac{\alpha_s}{4\pi} \right)^{2n} C_{n-k,k} C_{n-k} C_k \left( G_{\text{bare}}^D (z + \Delta (z_0,b) - z_0,b) \right)^{n-k} \left( \tilde{G}_{\text{bare}}^D (z + \Delta (z_0,b) - z_0,b) \right)^k
\]

where \( \tilde{G} \) denotes the Pomeron Green’s function in the complex conjugated amplitude \( (N^*) \) and \( C_{n-k,k} \) and \( C_n \) are the coefficients in the series:

\[
N^D (z - z', z_0, b) = \sum_{n=1}^{\infty} \sum_{k=1}^{n-1} C_{n-k,k} (G_{\text{bare}}^D (z - z' - z_0 + \Delta (z_0,b)); b)^{n-k} (\tilde{G}_{\text{bare}}^D (z - z' - z_0 + \Delta (z_0,b)); b)^k
\]

\[
N (z') = \sum_{n=1}^{\infty} C_n (G_{\text{bare}}^D (z'; b))^n
\] (5.2)

As has been shown in Ref. [38] we need to re-write Eq. (5.3) and Eq. (4.7) in the following form to obtain coefficients \( C_{n-k,k}^D \)

\[
G \left( G_{\text{bare}}^D (z - z' ; b), \tilde{G}_{\text{bare}}^D (z - z' ; b) \right) = \left( 1 - \exp \left( - \phi (e^{\Delta (z_0,b)} - \ln 2 \left\{ G_{\text{bare}}^D (z - z' - z_0; b) + \tilde{G}_{\text{bare}}^D (z - z' - z_0; b) \right\} \right) \right)
\]

\[
N^D (z - z', z_0; b) = \left( N \left( G_{\text{bare}}^D (z - z' ; b) \right) + N \left( \tilde{G}_{\text{bare}}^D (z - z' ; b) \right) - G \left( G_{\text{bare}}^D (z - z' ; b), \tilde{G}_{\text{bare}}^D (z - z' ; b) \right) \right)
\]

Using Eq. (5.3) and Eq. (5.4) we can simplify Eq. (5.2) reducing it to the form:

\[
N^D (z - z', z_0 ; b) = \left( \sum_{n=1}^{\infty} \sum_{k=1}^{n-1} \frac{n!}{(n-k)!k!} C_n^D (G_{\text{bare}}^D (z - z' - z_0 + \Delta (z_0,b)); b)^{n-k} (\tilde{G}_{\text{bare}}^D (z - z' - z_0 + \Delta (z_0,b)); b)^k \right)
\]

(5.5)
Figure 11: MPSI approximation for diffraction production: all notations are shown in the insertion. Wavy lines denote the BFKL Pomeron, the blob stands for the scattering amplitude of two dipoles with sizes $r_1$ and $r_2$ in the Born approximation (due to exchange of two gluons). $G_{3P}$ is the triple BFKL Pomeron vertex. $\mathcal{P}$ denotes the cut Pomeron shown in the insertion.

leading to $C_{n-k,k}^D = \frac{n!}{(n-k)k!} C_n^D$. Note, that two first terms in Eq. (5.4) do not contribute to Eq. (5.1).

Performing summation over $n$ and $k$ in Eq. (5.2) we use for $N$ the approximate expression of Eq. (2.24) which we re-write in the following form

$$N_{\text{appr}} (T_{\mathcal{P}}^{\text{bare}}) = \int_0^\infty dt e^{-t} \left( 1 - \kappa e^{-T_{\mathcal{P}}^{\text{bare}}} - (1 - \kappa) e^{-t T_{\mathcal{P}}^{\text{bare}}} \right)$$

(5.6)

The result of lengthy but simple calculation takes the form

$$N_{\text{dipole-dipole}}^D (z, z_0, b) = \kappa \left( \kappa (1 - \exp (-T)) + (1 - \kappa) \frac{T}{1 + T} \right)^2 + \kappa^2 (1 - \kappa) \frac{2T^2}{(1 + T)(1 + 2T)}$$

$$+ \kappa (1 - \kappa)^2 \left( \frac{T}{1 + T} + \exp \left( 1 + \frac{1}{T} \right) \Gamma \left( 0, 1 + \frac{1}{T} \right) - \exp \left( \frac{1}{T} \right) \Gamma \left( 0, \frac{1}{T} \right) \right)$$

$$+ (1 - \kappa)^3 \frac{1}{T^2} \left( T (1 + T) - \exp \left( \frac{1}{T} \right) (1 + 2T) \Gamma \left( 0, \frac{1}{T} \right) \right)$$

(5.7)

where

$$T \equiv \frac{\alpha_s^2}{16\pi^2} e^{2\alpha_s (\Delta(z_0,b) - 2 \ln 2)} G_{\mathcal{P}}^{\text{bare}} (z, b)$$

(5.8)
6. Double diffractive production for dipole-dipole interaction in the MPSI approximation

We can calculate in the framework of the MPSI approximation the cross section of the double diffractive production (see Fig. 7-b and Fig. 13) using the following equations

\[
N^{DD}_{\text{dipole-dipole}} = \sum_{n=1}^{\infty} \sum_{k=1}^{n-1} n! \left( \frac{(n-k)!}{n!} \right)^2 (-1)^n \left( \frac{\alpha_S}{4\pi} \right)^{2n} \left( C_{n-k,k}^{D} \right)^2 \times \left( G_{P}^{\text{bare}}(z + \Delta(z_0,b) + \Delta(z'_0,b) - z_0 - z'_0;b) \right)^{n-k} \left( \tilde{G}_{P}^{\text{bare}}(z + \Delta(z_0,b) + \Delta(z'_0,b) - z_0 - z'_0;b) \right)^{k}
\]

Substituting in Eq. (6.1) Eq. (5.3) and Eq. (5.4) we obtain a very economic expression for \(N^{DD}_{\text{dipole-dipole}}\), viz.

\[
N^{DD}_{\text{dipole-dipole}}(z,z_0,z'_0;b) = N_{\text{dipole-dipole}}(z + \Delta(z_0,b) + \Delta(z'_0,b) - 2\ln 2 - z_0 - z'_0) \quad (6.2)
\]

Fig. 12 demonstrates that the cross section for the double diffractive production is close to the cross sections of the single diffractive production and its value can be rather large.
Figure 13: MPSI approximation for double diffraction production: all notations are shown in the insertion. Wavy lines denote the BFKL Pomerons, the blob stands for the scattering amplitude of two dipoles with sizes $r_1$ and $r_2$ in the Born approximation (due to exchange of two gluons). $G_{3P}$ is the triple BFKL Pomerons vertex. $\bar{P}$ denotes the cut Pomeron shown in the insertion.

7. Density of the produced gluons

7.1 Single inclusive production

Armed with the knowledge of the dipole-dipole scattering amplitude we are ready to answer what kind of parton system is produced at high energy. First, we will find the density of the produced gluons per unit of rapidity ($dN_G/dY_1$) which is equal to [39] (see also Refs. [6, 7] for details).

$$\frac{dN_G}{dY_1} = \int d^2p_T \frac{1}{\sigma_{in}} \frac{d\sigma}{dy d^2p_T} = 2C_F \alpha_s(2\pi)^4 \int d^2p_T \frac{1}{p_T^2} \int d^2b d^2b' d^2r_\perp e^{i\vec{p_T} \cdot \vec{r}_\perp} \nabla^2 N_{\text{dipole-dipole}}^G(Y_1; r_\perp; b) \nabla^2 N_{\text{dipole-dipole}}^G(Y_1; r_\perp; b')$$

(7.1)

where [39]

$$N_{\text{dipole-dipole}}^G(Y_1; r_\perp; b) = 2N_{\text{dipole-dipole}}(Y_1; r_\perp; b) - N_{\text{dipole-dipole}}^2(Y_1; r_\perp; b)$$

(7.2)

Integration over $r_\perp$ in Eq. (7.1) spans over saturation region ($z > 0$) as well as over perturbative QCD domain ($z < 0$). Since we calculated $N_{\text{dipole-dipole}}$ in the saturation region we can claim that

$$\frac{dN_G}{dY_1} \geq \frac{1}{\sigma_{in}} \frac{2C_F}{\alpha_s(2\pi)^4} \int d^2p_T \frac{1}{p_T^2} \int d^2b d^2b' \int d^2r_\perp e^{i\vec{p_T} \cdot \vec{r}_\perp} \nabla^2 N_{\text{dipole-dipole}}^G(Y_1; r_\perp; b) \nabla^2 N_{\text{dipole-dipole}}^G(Y_1; r_\perp; b')$$

(7.3)
which in the notation of Eq. (2.3) can be re-written in the form

$$
\frac{dN_G}{dY_1} \geq \frac{1}{\sigma_{in}} \frac{2C_F}{\bar{\alpha}_S\pi^2} \int dl \int d^2b d^2b' \int_{-\xi_s}^{\infty} d\xi e^{-\xi} J_0 \left( e^{\frac{l^2}{2}+\frac{i}{2} \xi} \right) \frac{d^2}{d\xi^2} N^G_{\text{dipole-dipole}} (\xi_s + \xi, b) N^G_{\text{dipole-dipole}} (\xi_s + \xi, b')
$$

(7.4)

where $l = \ln p_T^2$.

Performing integration over $p_T$ for $p_T \geq 0.25 GeV$ we find that the density of gluons $dN_G/dY = 2 \div 2.7$ for energies $W \geq 1.8 TeV$ for $\bar{\alpha}_S = 0.2 \div 0.25$. Therefore, at high energy the dense system of partons is produced.

### 7.2 Correlations and multiparton interactions (MPI)

The value of the correlation function $R(y_1, y_2)$ we have discussed in the introduction (see Eq. (1.2)). Noticing that $\sigma_{in} = 58mb$ for $W = 7 TeV$ in our approach, we obtain that $R(y_1, y_2) \approx 3$ for $\sigma_{eff} = 15mb$. These estimates depend on the value of $N_0$ in Eq. (3.3). For example, changing $N_0$ from $N_0 = 0.5$ for which all previous estimates were done, to $N_0 = 0.1$ does not change the density of gluons but the value of $\sigma_{in}$ becomes $\sigma_{in} = 37mb$ leading to $R(y_1, y_2) \approx 1.5 \div 2$.

Hence we can claim that in the extreme case of dilute-dilute system scattering: dipole-dipole interaction at high energy, the dense and strongly correlated system of gluon is produced.

In this system gluons are mostly originated from many parton showers. For example, the large ratio $\sigma_{in}/\sigma_{eff} \approx 2$ indicates that the probability of two parton showers production is larger than one parton shower. We can calculate the inclusive production of $n$-pair of jet using our approach. Parameterizing the inclusive cross section of $n$-pair production* in the spirit of Eq. (1.1), namely,

$$
\frac{d^n \sigma}{\prod_{i=1}^n dy_i d^2p_{T,i}} = \frac{1}{(\sigma_{eff})^n} \prod_{i=1}^n \frac{d\sigma}{dy_i d^2p_{T,i}}
$$

(7.5)

*For simplicity we consider all $n$ pairs of jets being non identical.
Using Eq. (3.3) one can see that
\[
\frac{1}{\sigma_{\text{eff}}^{(n)}} = \left\{ \frac{1}{n!} (a + (1-a)n!)^2 \right\} \int d^2 b S^n(b) = 2\pi \left( \frac{m^2}{2\pi} \right)^n \frac{1}{(nm)^2} \left\{ \frac{1}{n!} (a + (1-a)n!)^2 \right\}
\]

(7.6)

Using \( m = 0.86 \text{GeV} \) and \( a = 0.65 \) we estimate the values of \( \sigma^{(n)} \) obtaining \( \sigma_{\text{eff}}^{(2)} = 15 \text{mb}, \sigma_{\text{eff}}^{(3)} = 9 \text{mb}, \sigma_{\text{eff}}^{(4)} = 5.7 \text{mb} \). These numbers illustrate that the production of large number of parton cascade gives the main contribution at high energy. Therefore, the large gluon density in dipole-dipole high energy scattering stems from the production of numerous parton showers.

8. Elastic slope

Looking at Fig. 12, we cannot escape the feeling that the dipole is similar to the proton, having total and diffractive cross sections qualitatively similar to the cross sections of the proton-proton interaction at high energies. However, we face a problem with the shrinkage of the diffraction peak which was observed experimentally in proton-proton interaction and, at first sight, which is not expected in dipole-dipole scattering. Indeed, the BFKL Pomeron is not moving singularity (it is a standing branch point) and we do not expect the shrinkage of the diffraction peak for the single Pomeron exchange. On the other hand, the multi Pomeron exchanges and interactions induce the effective shrinkage. These exchanges and interactions started to slow down the increase of the scattering amplitude due to BFKL Pomeron exchange at

\[
N_P(z,b) \propto e^{-mb_0 + \frac{1}{2}z} \approx N_0 < 1
\]

(8.1)

leading to

\[
b_0 = \frac{1}{2m} z = \frac{2\tilde{\alpha}_S}{m} Y
\]

(8.2)

In Fig. 13 we plot the value of the elastic slope which is equal to

\[
B(z) = \frac{1}{2} \frac{\int b^2 d^2 b N_{\text{dipole-dipole}}}{\int d^2 b N_{\text{dipole-dipole}}}
\]

(8.3)

One can see that the slope is rather large and increases with the energy. Fig. 13 as well as Fig. 12 encourage us develop the description of proton-proton interaction a high energy based on CGC/saturation approach. However, such an approach could be only phenomenological at the moment since we do not have theory of the confinement. We are going to develop such approach in the nearest future.

9. Conclusions

The main physical result of this paper is to demonstrate that the dense system of partons (gluons) can be produced in dilute-dilute system scattering. We illustrated this using the extreme case of dipole-dipole...
scattering. This increase in density is originated by the intensive gluon cascades that can be described by the enhanced BFKL Pomeron diagrams (Pomeron loops).

For the first time we found the analytical solution to the equation for diffraction production proposed in Ref. [29] using the simplified BFKL kernel. Having this solution as well as the solution to Balitsky-Kovchegov equation we developed technique that allowed us to calculate the total cross section, cross sections for single and double diffraction in the MPSI approximation [18]. Hence we can discuss physics of the dilute-dilute parton system at high energy. Calculating inclusive production and two gluon correlations we see that the dense and strongly correlated system of gluons can be produced at high energy in the dipole-dipole scattering.

It should be stressed that using the BFKL Pomeron calculus and the MPSI approximation we satisfy the $t$-channel unitarity constraint at every stage of our calculations and demonstrate that the resulting scattering amplitude does not contradict the $s$-channel unitarity. Generally speaking we have, at the moment, two approaches for the high parton density QCD: Colour Glass Condensate (CGC) approach and the BFKL Pomeron calculus. However, in the frame of the MPSI approximation these approaches give the same amplitude since, as it was shown in Ref. [40], the gluon cascade initiated by one dipole is the same in both.

However, at very large values of $Y$ we cannot trust the MPSI approximation. Indeed, when density of dipoles at rapidity $Y'$ become large not only one dipole from the upper Pomeron can interact with the dipole in low Pomeron in Fig. 15, but the interaction two and more dipoles can be essential (see Fig. 16). In terms of the BFKL Pomeron calculus it means that not only triple Pomeron interaction should be taken into account but also multiPomeron vertices have to be included.

Choosing $Y' = Y/2$ we estimate the first four Pomeron interaction (see Fig. 16), which has not been taken into account, to demonstrate the region of validity for the MPSI approximation. We do not know the

![Figure 15: The elastic slope $B(z)$ versus $z$. $\bar{\alpha}_s$ is taken to be equal to 0.2. The vertical line shows the LHC energy range $W = 7 TeV$.](image)
four Pomeron vertex but it is proportional to $\bar{\alpha}_S^4$, or in other words, it has the same order of magnitude as $\gamma^2$ (see Eq. (1.3)). In our estimates we replace the four Pomeron vertex by $\gamma^2$ and introduce the parameter $R(z, b)$ which characterizes the strength of the contribution of the four Pomeron term in the scattering amplitude and which takes the form

$$R(z, b) = \frac{1}{T^2(0, 0)} \int d^2b' T^2 \left( \frac{z}{2}, \vec{b}' - \vec{b} \right) T^2 \left( \frac{z}{2}, \vec{b}' \right) S^2(z, b) = R_1(z, b) R_2(z, b) \quad (9.1)$$

The factor in front is need for a correct normalization for four Pomeron vertex being equal to $\gamma^2$.

The survival probability $S^2$ is equal to

$$S^2(z, b) = \left. \frac{1}{2} d^2 N_{\text{dipole-dipole}}^{\text{appr}}(T) \right|_{T=T(z, b)} \quad (9.2)$$

This parameter is plotted in Fig. 17. One can see that it falls down at large $z$. It happens because $R_1 \propto T^2$ while $S^2 \propto 1/T^3$ at large $T$.

Fig. 17-c illustrates the $b$ dependence of $R(z, b)$. One can see that $R$ has a maximum at fixed $b$ but in spite of this maximum the accuracy of our calculation us less than 4% even at large values of $z(Y)$. Although it should be noted, the maximum of $R(z, b)$ increases with the energy and it could reach the value of several percents. Since the maximum of $R$ increases with $Y$ we see that at high energies the corrections to the MSPI approximation become essential but our estimates show that it would happen for higher energies than the LHC one. We can safely use the MPSI approximation for the entire region of accessible energies ($W \leq 57$ TeV).

Bearing in mind all the assumptions that have been made: simplified BFKL kernel, MPSI approximation, phenomenological input for impact parameter dependence, we consider this paper as the first try to approach dilute-dilute scattering theoretically.
Figure 17: $R(z, b =)$ versus $z$ (see Fig. 17-a and Fig. 17-b) at $b = 0$ and versus $b$ at different $z$ (see Fig. 17-c). $\bar{\alpha}_S$ is taken to be equal to 0.25. The vertical line shows the LHC energy range $W = 7$ TeV.

Using the simplified BFKL kernel we were able to introduce the non-perturbative corrections at large impact parameters and the only phenomenological parameter which describes the large $b$ behaviour of the scattering amplitude, we extracted from the experimental data on double parton interaction as we discussed in the introduction.

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