Hadron mass generation and the strong interaction

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Abstract

Based on a Lagrangian with a coupling of two gluons to $J^z = 0^+$ (the quantum numbers of the vacuum) which decay to $q\bar{q}$ pairs, a model is presented, in which hadrons couple directly to the absolute vacuum of fluctuating gluon fields. By self-consistency requirements the confinement potential as well as $q\bar{q}$ densities and masses are obtained, which are in good agreement with experimental data on scalar and vector mesons. In comparison with potential models additional states are predicted, which can explain the large continuum of scalar mesons in the low mass spectrum and new states detected recently in the charm region.

The presented model is consistent with the concept, that the hadron masses can be understood by binding effects of the quarks.

PACS/keywords: 12.38.Aw, 12.38.Lg, 12.39.Mk/ Gluon-gluon coupling to $q\bar{q}$-pairs, generation of stable $q\bar{q}$-systems assuming massless quarks, confinement potential, masses of $0^{++}$ and $1^{--}$ mesons.

In the hierarchy of quantum systems hadrons represent the smallest complex substructures known inside of atoms and nuclei. This is supported by the property of asymptotic freedom [1] of the strong interaction. Therefore, hadrons can be related directly to the absolute vacuum of fluctuating gluon fields (with average energy $E_{vac} = 0$) if the quark masses are zero. In quantum chromodynamics (QCD) the vacuum is more complex with a finite mass of the quarks, which may be coupled to scalar Higgs particles, subject of extensive searches with the available and planned high energy experiments at Fermilab and CERN.

To investigate hadron mass generation we start from a Lagrangian, in which two gluons couple to scalar fields, from which $q\bar{q}$ pairs are emitted. The possibility that two gluons coupled to $0^+$ may produce colour singlet bound states has been mentioned already by Cornwall [2], but in context to the structure of QCD. Since the overlap of gluon fields is
non-local, such a model could serve also as an effective theory to investigate the strong scalar fields observed in hadron excitations \cite{3} and scattering \cite{4}–\cite{7}, which may be difficult to extract from QCD.

Assuming a scalar coupling of gluon fields of the form $gg \rightarrow (q\bar{q})^n$ we write the Lagrangian in the form

$$\mathcal{L}_{SI} = \bar{\Psi} i\gamma_{\mu}D^{\mu}\Psi - \frac{1}{4}(F_{\mu\nu}F^{\mu\nu} - G_{\mu}G^{\mu}), \quad (1)$$

with $D^{\mu}$ being the covariant derivative $D^{\mu} = \partial^{\mu} - ig_s A^{\mu}$ and $F^{\mu\nu}$ the Abelian field strength tensor $F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$, with $A^{\mu}$ being the gluon fields. $G_{\mu}G^{\mu}$ couples two gluon fields with $J^\pi = 0^+$ to $q\bar{q}$-pairs with

$$G^{\mu} = -g_s A^{\mu} \left[ b_1(\bar{\Psi}V_{1g}\Psi) + b_2(\bar{\Psi}V_{12g}\Psi)^2 + b_3(\bar{\Psi}V_{1g}\Psi)^3 + \ldots \right], \quad (2)$$

where $(\bar{\Psi}V_{1g}\Psi)$ stands for the creation of $q\bar{q}$-pairs, which interact by gluon exchange. Corresponding Feynman diagrams are shown in the upper part of fig. 1.

The structure of $\mathcal{L}_s = \frac{1}{4}G_{\mu}G^{\mu}$ implies a colour neutral coupling of two gluon fields. Hence, the symmetry is simply isospin SU(2) (two quarks with different charge and one gluon) without colour and (because of massless quarks as seen below) without flavour degree of freedom. By the coupling of two gluons to $J^\pi = 0^+$ the Lagrangian has no chiral symmetry, leading naturally to the sequence of hadronic states as observed experimentally.

A two-gluon field is produced only, if there is spacial overlap of two gluon fields. We write the radial part of $\mathcal{L}_s$ by a matrix element

$$\Phi(r_1 - r_2) = \alpha_s < A_1(r_1) | b_1(a_q^\dagger V_{1g}a_q) + b_2(a_q^\dagger V_{12g}a_q)^2 + b_3(\bar{\Psi}V_{1g}\Psi)^3 + \ldots | A_2(r_2) >, \quad (3)$$

where $A_i(r)$ are radial gluon fields, $a_q^\dagger$ and $a_q$ quark and antiquark creation operators, and $V_{1g}$ an interaction dominated by 1-gluon exchange between these quarks.\footnote{More-gluon exchange as well as spin-spin and spin-orbit effects have not been considered.} For the following discussion only the first term in eq. (3) is needed, contributions due to $(q\bar{q})^2$ contributions will be discussed briefly at the end of the paper.

Due to the non-local 2-gluon field, given by a 2-gluon density $\rho_2(r)$, the recoiling local quark fields are also smeared out and $a_q^\dagger V_{1g}a_q$ can be written as a q-q potential $V_{qq}(r)$ described by folding a $q\bar{q}$-density with the gluon-exchange interaction. The fact, that
$V_{qq}(r)$ is a scalar potential but the created $q\bar{q}$-pair has negative parity, requires a p-wave $q\bar{q}$-density $\rho_{qq}^p(\vec{r}) = \rho_{qq}(r) Y_{1,m}(\theta, \phi)$ leading to

$$V_{qq}(r) = \int dr' \rho_{qq}^p(r') Y_{1,m}(\theta', \phi') V_{1g}(r - r') ,$$

(4)

where $V_{1g}(r)$ can act only within the density $\rho_\Phi(r)$. Therefore the interaction has to be cut towards large radii and we use the following form

$$V_{1g}(r) = (-\alpha_s/r) e^{-cr}$$

(5)

with cut-off parameter $c$ determined in a self-consistent fit of the 2-gluon density. Note, that eq. (3) indicates a departure from a purely relativistic description with a lifetime of the created system $\Delta t$. Causality is fulfilled, if $\Delta t > (r - r')/c$. The shape of the folding potential (4) has been calculated by multiplying the density and potential in momentum space and retransforming the product to r-space.

The p-wave character of the $q\bar{q}$-density gives rise to the constraint $< r_{q\bar{q}} > = \int d\tau r \rho_{qq}(r) = 0$ and thus

$$\rho_{qq}(r) = \sqrt{3} (1 + \beta \cdot d/dr) \rho_\Phi(r) ,$$

(6)

where $\beta$ is determined from the condition $< r_{q\bar{q}} > = 0$. A consequence of eq. (3) is, that the radial dependence of $V_{qq}(r)$ should be the same as $\rho_\Phi(r)$

$$\rho_\Phi(r) = V_{qq}(r) .$$

(7)

From the different relations between densities and potential in eqs. (4), (6) and (7) the two-gluon density is determined. Self-consistent solutions of eq. (7) are obtained assuming a form

$$\rho_\Phi(r) = \rho_o \left[ exp\{-(r/a)\kappa}\right]^2 \text{ with } \kappa \sim 1.5$$

(8)

and interaction cut-off parameter $c$, which yields a mean square radius of the effective interaction between 20 and 80 % larger than $< r_\Phi^2 >$. The self-consistency condition is rather strict (see the lower parts of fig. 1): using a pure exponential form ($\kappa=1$) a very steep rise of $\rho_\Phi(r)$ is obtained for $r \to 0$, but an almost negligible and flat potential, which cannot satisfy eq. (7). Also for a Gaussian form ($\kappa=2$) no consistency is obtained: normalised to the inner part of $\rho_\Phi(r)$, the deduced potential falls off more rapidly towards larger radii than the density. Only by a density with $\kappa \sim 1.5$ a satisfactory solution is
obtained. A self-consistent solution is also shown (in r- and momentum (Q)-space) in fig. 2 with a quantitative agreement for radii > 0.1 fm and momenta < 2.5 GeV/c. The localisation in space indicates, that a stationary (mesonic) system is formed. Interestingly, the emerging quarks can couple again to two gluons giving rise to stable glueballs as well.

In the Fourier transformation to Q-space the process $gg \rightarrow q\bar{q}$ in question is elastic and consequently the created $q\bar{q}$-pair has no mass. However, if we take a finite mass of the created quarks of 1.4 GeV (such a mass has been assumed in potential models [8] for systems of similar size), the dashed line in the lower part of fig. 2 is obtained and no self-consistent solution is possible. Thus, our solutions require massless quarks and consequently the deduced hadronic systems can be related directly to the absolute vacuum of fluctuating gluon fields.

For a quark-gluon system with finite mean square radius as shown in fig. 2 the corresponding binding potential can be obtained from a three-dimensional reduction of the Bethe-Salpeter equation in form of a (relativistic) Schrödinger equation

$$
-\left(\frac{\hbar^2}{2\mu} \left[\frac{d^2}{dr^2} + 2 \frac{d}{r} \frac{d}{dr}\right] - V \right) \psi(r) = E \psi(r),
$$

(9)

where $\mu$ is the mass parameter, which is related to the mass $m$ of the $q\bar{q}$ system by $\mu = \frac{1}{2} m - \delta m$ with a correction $\delta m \neq 0$ only for very light systems. Further, $\psi(r)$ is the wave function, which is given for a system of uncorrelated gluons by $|\psi(r)|^2 = \rho(r)$.

The dependence of the potential as a function of $\psi(r)$ is then given by

$$
V(r) = \frac{\hbar^2}{2\mu} \left[\frac{d^2\psi}{dr^2} + 2 \frac{d\psi}{r} \frac{d}{dr}\right] \frac{1}{\psi} + V_o.
$$

(10)

Inserting the form of the density in eq. (8) yields explicitly

$$
V(r) = \frac{\hbar^2}{2\mu} \left[\frac{\kappa}{a^2} \left(\frac{r}{a}\right)^{\kappa-2} \left[\kappa(\frac{r}{a})^\kappa - (\kappa + 1)\right]\right] + V_o.
$$

(11)

Since the $(2g - q\bar{q})$ system couples to the vacuum, $V_o$ is assumed to be zero. With the values of $\kappa$ and $a$ from the self-consistent solutions (8) and $\mu \sim 1.5$ GeV (which is determined below also self-consistently) the deduced densities (shown in figs. 1 and 2) yield a binding potential given in the upper part of fig. 3. This has the same form as the known confinement potential $V_{conf} = -\alpha_s/r + br$ deduced from potential models [8] and the lattice data of Bali et al. [9]. Further, the potential (11) is the same for other solutions of smaller
or larger radii and can therefore be identified with the ‘universal’ confinement potential. It is important to note, that the self-induced confinement potential (11) reproduces the $1/r +$ linear form without any assumption on its distance behavior; this is entirely a consequence of the deduced radial form (8) of the two-gluon density. Interestingly, because of the rather simple confinement mechanism in our approach, a direct connection to strings (which have been related to the linear part of confinement potential) appears possible.

To determine bound state energies of basic scalar and vector $q\bar{q}$ states (with $J^{PC} = 0^{++}$ and $1^{--}$) binding in the self-induced confinement potential (11) but also in the $q\bar{q}$ potential has to be considered, which (different from the confinement potential) depends strongly on the radius of the density. For smaller radii a deepening of this potential is observed, which leads to more strongly bound states.

We define the mass of the system by the energy to balance binding (this is also the way hadron masses are observed). This yields

$$M_i = -E_{qq} + E_i ,$$

(12)

where $E_{qq}$ and $E_i$ are the binding energies in $V_{qq}(r)$ and $V_{\Phi}(r)$, respectively (note that the eigenstates of $V_{qq}(r)$ have negative energy).

We apply another constraint (generation of mass by binding), requiring that the mass $M_o$ of the lowest bound state is equal to $m_\Phi$ (appearing in the mass parameter $\mu_\Phi = \frac{1}{2}m_\Phi - \delta m$ with $\delta m \sim 0$). With this condition we find $m_\Phi \sim 1/\langle r^2_\Phi \rangle^{1.5}$ for scalar states with a coupling constant $\alpha_s$ decreasing for smaller systems. For the five systems in table 1 we find values of $\alpha_s$ of about 1.3, 0.5, 0.4, 0.32 and 0.23. Together with other solutions we find $\alpha_s(M_o) \approx 1.5 \alpha_s^{QCD}(Q)$ for $M_o = Q$ up to large masses (with values of $\alpha_s^{QCD}(Q)$ from the systematics in ref. [10]). This yields evidence for asymptotic freedom also in our model. More details will be discussed elsewhere.

So far we have no constraint on the radial extent of the $(2g - q\bar{q})$ system, giving rise to continuous mass spectra. Discretisation is provided by a vacuum potential sum rule $\Phi_{vac}(r) = \sum_n \Phi^n(r)$, requiring that the sum of two-gluon matrix elements (3) (where $\Phi^n(r) = \alpha^n_s \rho_{\Phi_n}(r)$) is equal to the total 1-gluon exchange force in the vacuum. For this we take the simplest form, demanding that the total cut-off by the finite size of the overlapping gluon fields shows the same $1/r$ dependence as the 1-gluon exchange force.
Table 1: Deduced masses (in GeV) of scalar and vector $q\bar{q}$ states in comparison with known $0^{++}$ and $1^{--}$ mesons [10]. Only the 1s states in the q-q potential are given. The radii of the known states of mass $M_o$ are fine-tuned to agree with experimental data.

| Solution | (meson) | $M_0$ | $M_1$ | $M_2$ | $M_0^{exp}$ | $M_1^{exp}$ | $M_2^{exp}$ |
|----------|---------|-------|-------|-------|-------------|-------------|-------------|
| 1        | $0^{++}$ | $\sigma$ | 0.60  | 1.60±0.2 | 2.30±0.3 | 0.60±0.1   |             |
| 2        | $0^{++}$ | $f_o$  | 1.30  | 2.15±0.2 | 2.80±0.2 | 1.30±0.2   |             |
|          | $1^{--}$ | $\omega$ | 0.78  | 1.66±0.2 | 2.28±0.3 | 0.78       | 1.65±0.02  |
| 3        | $0^{++}$ | $f_o$  | 2.30  | 3.00±0.2 | 3.55±0.2 | 2.30±0.2   |             |
|          | $1^{--}$ | $\Phi$ | 1.00  | 1.69     | 2.27     | 1.02       | 1.68±0.02  |
| 4        | $0^{++}$ | not seen | 4.5±1.0 | $M_o$+0.5 | $M_o$+1.0 |             | —           |
|          | $1^{--}$ | $J/\Psi$ | 3.10  | 3.68     | 4.19     | 3.097      | 3.686 (4.160) |
| 5        | $0^{++}$ | not seen | 22.0±8 | $M_o$+0.4 | $M_o$+0.7 |             | —           |
|          | $1^{--}$ | $\Upsilon$ | 9.46  | 9.99     | 10.36    | 9.46       | 10.023      | 10.355      |

From this we obtain

$$\Phi_{vac}(r) = -\frac{\tilde{\alpha}_s}{r^2} = -\sum_n \alpha^n_s \rho_n(r) .$$

(13)

Five self-consistent solutions for scalar states below 50 GeV have been extracted with mean square radii $<r^2_\Phi>$ of about 0.49, 0.26, 0.12, 0.06, and 0.02 fm$^2$, respectively, and masses $M_o$ given in table 1. In the lower part of fig. 3 the different solutions $\Phi^n(r)$ for $n=1-5$ are shown together with their sum (solid line). This is compared to the sum rule (13) with $\tilde{\alpha}_s=0.35$ (lower dot-dashed line). Good agreement is obtained, indicating that the sum rule is fulfilled. Only for radii <$0.15$ fm a deviation is observed, which is explained by contributions from solutions with masses significantly larger than 50 GeV.

Between the scalar states discussed so far and corresponding vector states there is a direct relation: the p-wave $q\bar{q}$ density has to be replaced by the corresponding s-wave density. This leads to momentum distributions of the vector states shifted to smaller momenta (and corresponding smaller masses) than the scalar states. We obtain mean square radii $<r^2_\Phi>$ of about 0.38, 0.24, 0.10, and 0.04 fm$^2$ and masses given in table 1, which are in good agreement with the masses of the strong $1^{--}$ mesons (together with their radial excitations) of the (isoscalar) “flavour families” $\omega$, $\Phi$, $J/\Psi$ and $\Upsilon$. The masses of the
known $0^{++}$ states are also in general agreement with experiment: the lowest $0^{++}$ state corresponds to the $\sigma(600)$ meson, which has been clearly identified \cite{11} as a broad meson resonance in $J/\psi$-decay. The next $0^{++}$ states at 1.3 and 2.3 GeV may correspond to the scalar resonances $f_0(1300)$, see also ref. \cite{12}, and $f_0(2300)$, whereas the higher $0^{++}$ states (not found so far) may not be observable in $e^+e^-$ collision experiments.

As compared to potential models with finite quark masses, as e.g. in ref. \cite{8}, we obtain significantly more states, bound states in the confinement and in the q-q potential. The solutions in table 1 correspond only to the 1s levels in the q-q potential, in addition we have calculated Ns levels for N=2, 3, and 4. Most of these states, however, have a relatively small mass below 3 GeV. As the q-q potential is Coulomb like, it creates a continuum of Ns levels which ranges down in mass to the threshold region. This continuum should mix with the states in the confinement potential, giving rise to large scalar phase shifts at low energies, which are observed experimentally but so far not well understood in other models.

Concerning masses above 3 GeV, solution 5 of table 1 yields additional $0^{++}$ 2s and 3s states in the q-q potential at masses of about 12 and 8.8 GeV, respectively, whereas an extra $1^{--}$ 2s state is obtained (between the most likely $\Psi(3s)$ and $\Psi(4s)$ states at 4.160 GeV and 4.415 GeV) at a mass of about 4.2 GeV. This state may be identified with the recently discovered X(4260), see ref. \cite{10}. Corresponding excited states in the confinement potential \cite{11} should be found at masses of 4.9, 5.3 and 5.5 GeV with uncertainties of 0.2-0.3 GeV.

By identifying the deduced $q\bar{q}$ states with known mesons we can check the overall consistency of our model by the observed widths of these states. For the lifetime we assume $<\Delta t> = <r_\phi^2>^{1/2} /c$ and get for the heavier mesons in table 1 values which are small compared to the lifetime deduced from the relatively small experimental widths. Differently, for the lightest meson in table 1 we obtain a value of $<\Delta t>$ of 2.4 $10^{-24}$ s corresponding to a width of about 300 MeV, which is less than the width ($\sim$ 500-600 MeV) extracted \cite{11} for $\sigma(600)$. However, as the Coulomb like q-q potential gives rise to a low energy continuum of Ns states, the width of the lowest bound state in table 1 has to be much smaller, indicating that also in this case our approach is valid.

Finally, possible $(q\bar{q})^2$ contributions corresponding to the second term in eq. (3) are ad-
dressed. In the results shown in fig. 2 and the upper part of fig. 4 the folding potential deviates from $\rho_\Phi(r)$ only at small radii. This difference could be filled by a small $(q\bar{q})^2$ component. This is supported by Monte Carlo simulations [13], in which good agreement between $\rho_\Phi(r)$ and $V_{qq}(r)$ in eq. (7) is obtained in the entire $r$ and $Q$ region by the inclusion of a $(q\bar{q})^2$ contribution.

In conclusion, a model has been presented, in which hadron masses are described as bound states of quarks with a very simple structure of the vacuum. This leads to a good description of the confinement potential and hadron masses. Other results, including a discussion of asymptotic freedom and the stability of baryons will be discussed later.

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Figure 1: Upper part: Relevant Feynman diagrams $2g \rightarrow q\bar{q}$ and $2g \rightarrow (q\bar{q})^2$ ($q$ denotes quark or antiquark). Lower parts: density $\rho_\Phi(r)$ with $<r^2>=0.06$ fm$^2$ (dot-dashed lines) and folding potential $\Pi$ (solid lines) for $\kappa=1$, 1.5 and 2, respectively.
Figure 2: Upper part: Self-consistent solution of eq. (4) for a scalar state with two-gluon density and q-q potential given by dot-dashed and solid lines, respectively. Lower part: Same as in the upper part but transformed to $Q$-space (multiplied by $Q^2$). The dashed line corresponds to a calculation assuming quark masses of 1.4 GeV.
Figure 3: Upper part: Confinement potential from lattice calculations [9] in comparison with the $q\bar{q}$ binding potential (11) given by solid line. Lower part: $q$-$q$ potentials for the different solutions in table 1 with their sum given by solid line compared to the vacuum sum rule (13), lower dot-dashed line.