Crossover behavior in interface depinning

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Abstract

We study the crossover scaling behavior of the height-height correlation function in interface depinning in random media. We analyze experimental data from a fracture experiment and simulate an elastic line model with non-linear couplings and disorder. Both exhibit a crossover between two different universality classes. For the experiment, we fit a functional form to the universal crossover scaling function. For the model, we vary the system size and the strength of the non-linear term, and describe the crossover between the two universality classes with a multiparameter scaling function. Our method provides a general strategy to extract scaling properties in depinning systems exhibiting crossover phenomena.
I. INTRODUCTION

Driven interfaces in random media display intriguing scaling laws that are common to a wide variety of phenomena, including fluid imbibition, crack front roughening, dislocation hardening, superconducting flux lines, the equilibrium motion of piles of rice down an incline, and domain wall motion in magnets [1, 2]. The scaling laws are commonly associated with an underlying depinning critical point that has been elucidated by simple models for interface dynamics. These models have been extensively studied using continuum simulations [3–6], cellular automata [3, 7–11], and field-theoretic \( \epsilon \) expansions [3, 12–17], providing a very complete picture of the non-equilibrium phase transition and of the different universality classes.

The interface morphology is usually characterized by the roughness exponent \( \zeta \), resulting from a coarse graining operation of the interface height function \( h(x) \). Namely, when we change all length scales by a factor \( b \), or \( x \to bx \), then statistically \( h \to b^\zeta h \) – hence

\[
h(x) \sim b^{-\zeta} h(bx).
\]

For many experiments and simulations, it is convenient to measure \( \zeta \) by computing the height-height correlation of the interface

\[
C(r) = \langle (h(x + r) - h(x))^2 \rangle \sim r^{2\zeta}
\]

\( \zeta \) should be uniquely determined by which universality class the system belongs to. However, in practice, the observed \( \zeta \) varies (See table I) even for the same type of system, such as paper wetting. Measuring a single exponent for these systems may prove inadequate due to the presence of crossover behavior, between universality classes.

In section II we analyze a straightforward experimental example of a crossover between two forms of roughness in two-dimensional fracture. There we introduce the universal crossover scaling functions, and provide a brief renormalization-group rationale.

In the remainder of the paper, we examine a more complex theoretical model. Crossovers have been studied for several interface models [21], however theoretical studies have proven challenging in different ways [25]. For thin film magnets, the experiments [26–28] observe a crossover between short-range and mean-field universality classes as long-range dipolar fields are introduced, which can be done by changing the thickness of the film. However, for
TABLE I: Roughness Exponents in Experiments. Table reproduced from [1]. Notice that there is a wide range of $\zeta$ reported, even for the same experimental system.

models of that type, simulations are challenging, both because of the long-range fields and the striking zig-zag morphologies that emerge and compete with the avalanche behavior. We shall analyze a numerically tractable, but analytically challenging crossover [25]: the transition between the linear, super-rough, quenched Edwards-Wilkinson model (qEW) and the nonlinear quenched KPZ model (qKPZ) [2, 21]. In both experiment and theory, we focus on the crossover behavior of the height-height correlation function.

II. CROSSOVER IN FrACTURE SURFACE CORRELATIONS

Just as the critical exponent $\zeta$ is universal (independent of microscopic details, within a class of physical system), so too is the crossover behavior between universality classes. As a simple example, Santucci et al. [29] have measured a relatively sharp crossover between two regimes for two-dimensional fracture (inset in Fig. [1]). Well below a critical distance $r^*$, they observe a power law $C(r) \sim r^{2\zeta-}$ with an exponent that was interpreted as originating from coalescing cracks [30] or with Larkin scaling [31]. Well above $r^*$ they observe a different power law $C(r) \sim r^{2\zeta+}$ consistent with a fluctuating line model [31–33]. The crossover between these two universal power-law regimes should be described by a universal crossover function [34], $C_{\text{frac}}$:

$$C(r) \approx C^* r^{-2\zeta-} C_{\text{frac}}(r/r^*)$$  (3)
FIG. 1: **Crossover scaling in fracture roughness** [29]. The inset shows experimental data for the height-height correlation function $C(r) = \langle (h(x+r) - h(x))^2 \rangle$ of a 2D fracture front, generated by pulling apart two pieces of PMMA that have been sand-blasted and sintered together [29]. The three curves differ in the size of the sand-grain beads; the relation between the bead size and the toughness fluctuations in the PMMA were not measured. The dashed lines show two different power-law critical regimes, with $C(r) \sim r^{2\zeta_-}$ and $C(r) \sim r^{2\zeta_+}$, governing the short- and long-distance scaling behavior: the crossover between these regimes is evident. Here our fit gives $\zeta_- = 0.63$ and $\zeta_+ = 0.32$, within the experimentalists suggested range $\zeta_- = 0.6 \pm 0.05$ and $\zeta_+ = 0.35 \pm 0.05$. The main figure shows a scaling plot of $r^{-2\zeta_-}C(r)$ versus $r$, with the curves shifted vertically and horizontally to best collapse. The black curve is a one-parameter fit of the universal scaling function to the functional form in eqn (4).

Independent of microscopic details. At small arguments $C_{\text{frac}}(X)$ must goes to a constant, and at large arguments $C_{\text{frac}}(X) \sim X^{2(\zeta_+ - \zeta_-)}$, so as to interpolate between the two power laws. When analyzing different systems governed by the same universal crossover, one may plot all the crossovers in a scaling plot, dividing the distances $r$ on the ordinate by a system-dependent factor $r^*$ for each curve, and dividing the magnitudes of the correlations on the abscissa by a system dependent constant $C^*$ (see Fig. 1). The resulting data curves then
should align, giving the universal function \( C_{\text{frac}}(r/r^*) \).

To continue with this simple test case, we may fit the universal scaling function to an approximate functional form. (Indeed, we find it convenient to do a joint fit of the functional form, the exponents, and the constants \( r^* \) and \( C^* \).) To the extent that a guessed functional form reproduces the universal one, it is equivalent: advanced field-theoretic methods for calculating exact scaling functions aren’t needed to analyze future experiments. However, judicious choices of functional forms with the correct limiting behavior can greatly facilitate this process. The interpolation \( 1/(1 + X^{\zeta_- - \zeta_+}) \) has the correct limits, but its rather gradual crossover does not explain the data. (Indeed, gradual crossovers in other systems often have led to reports of intermediate power laws, or power laws that vary as a parameter is tuned.)

We may heuristically add a parameter \( n \) which at large values produces an abrupt crossover:

\[
C_{\text{frac}}(X) = (1 + X^{2n(\zeta_- - \zeta_+)})^{-1/n}.
\]  

(4)

This yields an excellent fit to the data with \( n \approx 4 \) (see Fig. 1).

Why is the scaling form of eqn (3) expected? Briefly, the renormalization group studies the behavior of systems under coarse-graining: describing the properties of a system at length scales changed by a factor \( b \). One gets universal power laws when the system becomes invariant under repeated coarse-grainings: if \( C(r) \rightarrow b^{2\zeta}C(r/b) \), under coarse-graining by a factor \( b \), then by coarse-graining \( n \) times such that \( r = b^n \) one has \( C(r) \propto b^{2\zeta n} = r^{2\zeta} \). In the case of a crossover, a fixed point is unstable to some direction \( \lambda \) in system space. Then a small initial \( \lambda \) grows under rescaling by some factor \( b^{1/\phi} \), so \( C(r, \lambda) \rightarrow b^{2\zeta}C(r/b, \lambda b^{1/\phi}) \).

Now rescaling until \( b^n = r \), we have

\[
C(r, \lambda) \rightarrow b^{2n\zeta - \zeta}C(r/b^n, \lambda b^{n/\phi}) = r^{2\zeta}C(1, \lambda r^{1/\phi}) = r^{2\zeta}C_{\text{frac}}(\lambda^{\phi}r)
\]

(5)

where we choose \( C_{\text{frac}}(X) = C(1, X^{1/\phi}) \). If the unstable direction flows to a new fixed point with a different \( \zeta_+ \), that behavior will be reflected in the large-\( X \) dependence \( C(X) \sim X^{2(\zeta_+ - \zeta_-)} \) [35 Section 4.2]. Note that different physical systems will have different overall scales of height fluctuations, so we must have an overall scale \( C^* \) for each experiment. Note, though, that the rescaling factor \( r^* \) for lengths, while it still will vary from one system to another, now depends on \( \lambda \) as \( r^* = 1/\lambda^{\phi} \): within the renormalization group, it measures how far along the unstable direction the original system was poised. In particular, \( r^* \) becomes large as \( \lambda \rightarrow 0 \), as in that limit the unstable fixed point remains in control.
The three experiments depicted in Fig. 1 started with different bead sizes. If all other features of the experiment are held fixed, one may assume that the control parameter $\lambda$ depends in some smooth way with bead size. Had we several values of bead size, we could then extract values for the universal crossover exponent $\phi$.

In the following sections, we shall perform a far more sophisticated version of this type of analysis. By exhaustively varying system size and nonlinearity in an interface growth model, we shall not only generate universal two-variable functional forms for the correlation crossover scaling function, but will be able to make predictions about both the dependence of the crossover length scale (corresponding to $r^*$) and the dependence of the correlation amplitude (corresponding to $C^*$) on the control parameters. A rich, nuanced understanding of the model behavior thus emerges.

III. LINE DEPINNING MODEL

The equations of an interface in a disordered environment may be written generally as follows. Let the one-dimensional interface, $h(x,t)$ be driven by a force $H(t)$ through a disordered environment with a local quenched random force $\eta(h(x),x)$:

$$\frac{\partial u}{\partial t} = F[h](x) + \eta(h(x),x) - D[h](x) + H(t).$$

(6)

$F[h](x)$ is a general interaction kernel, which controls the interface morphology, and is dependent on $h(x)$ and $x$. Here $F[h](x)$ could represent short-range surface tension in the interface ($\nabla^2 h$), or long-range interfacial self-interaction fields such as dipole fields in magnets or elastic strains in delamination fracture (as a suitable convolution). $H(t)$ is a time-dependent external driving force.

Notice that if we set $\frac{\partial h}{\partial t} = 0$, we arrive at a steady-state force equilibrium, where $H(t) = H_0 = -\eta(h(x),x) - F[h](x)$. The interface $h(x,t)$ sits at a local minima in the energy landscape at this point. First, let’s consider the case $D[h](x) = 0$, if we increase $H(t) = H_0 + \epsilon$, the force balance is disturbed, and the front only stops if at some point $\eta(h(x),x)$ is large enough to counter this force. Therefore, in any finite random system, there exists some $H_{\text{depinning}}$ such that the interface moves without stopping when $H(t) > H_{\text{depinning}}$, but when $H(t) < H_{\text{depinning}}$, the interface is stuck in a minima.

In simulations, $D[h]$ represents a restoring force that ‘self-organizes’ the depinning tran-
sition to the fixed point. This $D[h]$ allows simulations to access many metastable states, without having to enforce an actual quasi-static field. In simulations describing magnets, this term is the demagnetization force \[ D[h] \equiv -k\langle h \rangle \] which approximates the effects of the long-range dipolar field cost of a net advance in the front. We employ the same type of force in studying the qEW and qKPZ models. The isotropic qEW and anisotropic qKPZ have the general structure of Equation 6, where for qEW, $F[h] = \gamma \nabla^2 u$ and for qKPZ $F[h] = \gamma \nabla^2 h + \lambda (\nabla h)^2$.

IV. ANALYSIS OF CROSSOVER SCALING

Using the automaton simulation employed in [37], we tune $\lambda/\gamma$ from 0 to 5, and observe how the resulting behavior changes. Figure 2 shows how the front morphology qualitatively changes while we increase the nonlinear parameter $\lambda$. Notice that with increasing $\lambda$ the fronts between events are flatter than at small $\lambda$.

According to Equation 2, naively one would assume we could recover the exponent $\zeta$ by measuring the local-log slope of the height-height correlation functions (Figure 3) for both the qEW and qKPZ fixed points. From other numerical studies, for qEW, we expect $\zeta_{EW} = 1.19 - 1.25$ (Cellular automata [8 38] models show $\zeta_{EW} = 1.25 \pm 0.01$; continuous string models [39] found $\zeta_{EW} = 1.19 \pm 0.01$). For qKPZ, we expect $\zeta_{KPZ} = 0.63$ [11]. However, there are two things about Figure 3 worth noting: (1) the slope-measure of $\zeta$ drifts between 0.63 and 1.0 as we change $\lambda$, (2) the measured value is never greater than one as is naively expected for the linear qEW model.

The second issue has a known resolution: for $\zeta > 1$, when the interface is ‘superrough’, the height cannot grow faster than linearly with distance, so the height-height correlation function cannot directly exhibit a power law larger than one [40]. Instead, we need to consider the finite-size scaling form, \[ C_{EW}(r|L) \sim L^{2\zeta_{EW}} (r/L)^{2\zeta_{EW}} C_{EW}(r/L). \] (7) and measure the roughness exponent $\zeta_{EW}$ as a function of the system size.

The drift in the exponent $\zeta$, however, proves to be more complicated to explain. The role of $\lambda$ in generating this crossover from qEW to qKPZ has only been studied qualitatively [4] [7 10 11], with no full description of the crossover scaling [11]. We can use a crossover
FIG. 2: Crossover of qKPZ to qEW Model. Fronts generated from simulations with the nonlinear KPZ term coefficients set to (a) $\lambda = 0$, (b) $\lambda = 0.001$, (c) $\lambda = 0.1$, (d) $\lambda = 5$. The random colors represent the area between each pinned front. One can see that the morphology of the interfaces change dramatically as $\lambda$ increases.
FIG. 3: **Local Log Slope.** The measured local-log slope of the height-height correlation function for varying $\lambda$ and $k$. The dashed red line is $\zeta = 0.63$, the dashed black line is $\zeta = 1.0$. The curves nearest to $\zeta = 1$ are for small lambda, with $\zeta$ increasing as we increase $\lambda$.

The function that describes the drift between the two limits. For qKPZ (Fig. 2c), the correlation function in a system size $L$ takes the finite-size scaling form

$$C_{KPZ}(r|L) = Ar^{2\zeta_{KPZ}}C_{KPZ}(r/L).$$ \hspace{1cm} (8)

The crossover describes the RG flow from the qEW fixed point to the qKPZ as the relevant parameter $\lambda$ is added. The scaling form for the height-height correlation function is thus that of a relevant variable $\lambda$ added to the qEW scaling:

$$C(r|L, \lambda) = L^{2\zeta_{EW}}C(r/L, \lambda^\phi r)$$ \hspace{1cm} (9)

For $\lambda >> 0$, $C(r|L, \lambda) \to C_{KPZ}(r|L)$, therefore,

$$C(r/L, \lambda^\phi r) = r^{2\zeta_{KPZ}}C_{KPZ}(r/L)A(\lambda)/L^{2\zeta_{EW}}$$

$$= (r/L)^{2\zeta_{KPZ}}L^{2(\zeta_{KPZ} - \zeta_{EW})}A(\lambda)C_{KPZ}(r/L)$$

$$= (r/L)^{2\zeta_{KPZ}}(\lambda^\phi L)^{2(\zeta_{KPZ} - \zeta_{EW})}C_{KPZ}(r/L).$$ \hspace{1cm} (10)

In the last equation we solve for $A(\lambda)$ using the fact that $C(r/L, \lambda^\phi r)$ must be a scaling function with only invariant combinations of $r$, $L$, $\lambda$. Furthermore, we have, at the other
limit \( \lambda = 0 \), that \( C(r/L, 0) \sim (r/L)^2 \). Using this, Equation 10 and including finite size effects \( \exp(-Mr/L) \), we can construct a function that obeys these limits:

$$
C(r|L, \lambda) = r^{2(\zeta_{KPZ} - \zeta_{EW})} \frac{A_1 \exp(-Mr/L)}{A_2 + (r/L)^{2(\zeta_{KPZ} - 2\zeta_{EW})}} \frac{(\tanh(\lambda))^{(\zeta_{KPZ} - \zeta_{EW})}}{A_1 \exp(-MX)}
$$

where \( X = r/L \) and \( Y = (\tanh(\lambda))^\phi r \approx \lambda^\phi r \) at small \( r \). (The nonlinearity \( \lambda \) moves us along the unstable direction from the fixed point linearly only for small \( \lambda \), since the RG equations become nonlinear far from the fixed point; here we approximate the scaling variable \( \tanh(\lambda) \).) Physically, there is no particular reason why the natural measure of the ‘strength of the nonlinearity’ should be equal to the parameter \( \lambda \).) We choose \( \tanh(\lambda) \) as the scaling variable because at small argument it goes as \( \lambda \), and at large argument goes to one, and obeys the scaling limits at both qEW and qKPZ. Note that this scaling form has three control variables, and that there exists a singularity at small \( \lambda \) in the form of a divergent amplitude \( A \sim \lambda^{2(\zeta_{KPZ} - \zeta_{EW})} \) in the qKPZ correlation function (eqn 8) as \( \lambda \to 0 \). This universal singularity in the amplitude (corresponding to a prediction of \( C^* \) in section II) explains the amplitude dependence seen in Fig. 4. There is an analogous universal amplitude dependence seen for the Heisenberg→Ising crossover at small Ising anisotropy [35, Section 4.1].

The current form of Equation 11 gets us a measure of \( \zeta_{EW} > 1 \) which is more consistent with the literature than the naive measure of the slope of the power law. It incorporates finite-size effects, and explains the universal scaling of the amplitude near the qKPZ fixed point.

V. CONCLUSIONS

In this paper, we have analyzed the scaling properties for an experimental 2d fracture front and a model of an interface moving in random media, focusing on the crossover scaling of the roughness. The experimental system is successfully modeled using a one-variable universal scaling function with one free parameter, controlling the sharpness of the transition. The theoretical model, the crossover from the qEW to the qKPZ universality class with the addition of a non-linear term, allows us to estimate the complete universal scaling function for the height-height correlation function including both finite-size effects and the non-linear
FIG. 4: **Height-height Correlation Function.** The numerics generated with an automata code (symbols) are well described by Equation 11 (line segments) with fit parameters $\zeta_{KPZ} = 0.78 \pm 0.03$, $\zeta_{EW} = 1.20 \pm 0.02$, $\phi = 1.02 \pm 0.04$, $A_1 = 0.37 \pm 0.09$, $A_2 = 1.0 \pm 0.2$, and $M = 4.1 \pm 0.07$. Note that the amplitude dependence is captured by the scaling form.

effects of the tuning parameter $\lambda$, while satisfying known limits given by the renormalization group.

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