Robust $H_\infty$ control of time delayed power systems

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Power system is the backbone of our society. The purpose of this work is to design a stable, robust and efficient controller for a power-generation system with time delays, model uncertainties and disturbances. Based on the practical dynamics of generator, prime mover, exciter and automatic voltage regulator, a mathematical power-generation system model is developed with state space dynamical equations involving time delays in the feedback. A novel robust $H_\infty$ control framework based on linear matrix inequalities is proposed in the paper, which controls the energy system effectively. Computer simulations are used to show the efficacy of the proposed control algorithm.

Keywords: time delay; linear matrix inequalities; power system; $H_\infty$

Nomenclature

| Symbol | Description |
|--------|-------------|
| $\delta$ | rotor angle |
| $\omega_s$ | synchronous speed |
| $v$ | normalized frequency $v = \omega/\omega_s$ |
| $T_{do}'$ | equivalent transient rotor time constant |
| $P$ | total number of poles |
| $S_B$ | rated three phase voltage ampere |
| $\omega_B$ | rated speed in electrical radians per second, $\omega_B = \omega_s$ |
| $H$ | the shaft inertia constant is scaled by defining $H = \frac{1}{2}J(\omega_B(2/P))^2/S_B$ |
| $K_d$ | the damping factor |
| $I_d$ | direct axis current |
| $I_q$ | quadrature axis current |
| $X_d$ | direct axis reactance |
| $X_q$ | quadrature axis reactance |
| $X_d'$ | direct axis transient reactance |
| $X_q'$ | quadrature axis transient reactance |
| $E_q$ | quadrature axis voltage |
| $E_q'$ | quadrature axis transient voltage |
| $E_{fd}$ | excitation voltage |
| $E_{fd}'$ | transient excitation voltage |
| $T_{mech}$ | mechanical torque |
| $R_s$ | stator resistance |
| $V_T$ | terminal bus voltage |
| $V_{ref}$ | reference voltage |
| $R_e$ | transmission line resistance |
| $X_e$ | transmission linear reactance |
| $z_1, z_2, z_3$ | state variables of power system stabilizer |
| $U_{pss}$ | power system stabilizer control signal |
| $\ast$ | transposed value of the corresponding element |

1. Introduction

Power systems are the basic infrastructure of modern civilization in our society. Stability analysis and control system development of the smart power grid are becoming more and more important, due to the rapid deployment of the distributed energy resources. In practical engineering applications, time delay plays a significant role in performance and stability of the overall power systems. Severe delays can even lead to catastrophic breakdown of the entire energy system due to instability (Alrifai, Zribi, Rayan, & Mahmoud, 2013; Bayrak & Tatlicioglu, 2012; Mahmoud, 2000; Okuno & Nakabayashi, 2006; Scorletti & Fromion, 1998; Wu, Ni, & Heydt, 2002; Zribi, Mahmoud, Karkoub, & Lie, 2000). For this reason, extensive research of transient and steady-state stability analysis and controller design have been conducted during the past decade (Jiang, 2007). System design engineers should consider time delays in designing and implementing practical power systems due to their significance (Alrifai et al., 2013; Jiang, Cai, Dorsey, & Qu, 1997; Mahmoud & Zribi, 1999; Wu et al., 2002; Yu, Jia, & Zhao, 2008).

Many different control approaches have been studied in the literature for effectively controlling the time delayed power systems Zhang et al. (2012). Based on an optimal control approach, the effect of time delays on the region of stability for small signals variation is studied in Jia, Yu, Yu, and Wang (2008). Zhang, Jiang, Wu, and Wu (2013) present a robust control method to design a PID type load frequency control of power systems considering time delays. Snyder, Ivanescu, Hadjised, Georges,
and Margolin (2000) introduce a robust controller for a wide-area power system involving input time delays. The controller is developed based on the model reduction and linear matrix inequalities (LMIs). An adaptive wide-area damping controller based on generalized predictive control and model identification for time delayed power system is proposed in Yao, Jiang, Wen, Cheng, and Wu (2009). Yu, Zhang, Xie, and Wang (2007) propose a nonlinear robust control algorithm for power system considering signal delays and measurement incompleteness. Yu et al. (2008) discuss the maximal allowable time delay margin for a stable power systems based on the Lyapunov method involving three generators and nine buses. Chowdhury, Kulhare, and Raina (2011) present the nonlinear limit cycle effect of time delays on local stability of the single machine infinite bus system. In Liu, Zhu, and Jiang (2008), the cluster treatment of eigenvalues is introduced to analyze the stability of a power system with time delays in the feedback loop.

This paper presents a general robust control framework based on linear matrix inequality for time delayed power systems. The mathematical dynamics is modeled as a seventh-order nonlinear system with bounded model uncertainties and external disturbances. By formulating the nonlinear control as a convex optimization problem, the linear matrix inequality can provide the optimal and robust solution satisfying the Lyapunov stability and the robust $H_{\infty}$ performance objective.

This paper is organized as follows: Section 2 discusses the mathematical modeling of the power-generation system. Section 3 presents the novel $H_{\infty}$ controller design with linear matrix inequality. Computer simulations conducted with MATLAB is given in Section 4. In Section 5, the conclusion is reached and future work is discussed.

2. Mathematical model of power-generation system

In this section, the mathematical model of an infinite bus power system involving a synchronous generator is developed.

2.1. Synchronous generator model

A model for the synchronous generator is given as follows (Wang & Gu, 2014):

$$\dot{E}_{q}^t = -1 \frac{1}{T_{do}} (E_{q}^t - (X_d - X_q)I_d - E_{id}),$$

(1)

$$\dot{\delta} = \omega - \omega_s,$$

(2)

$$\dot{\omega} = \frac{\omega_s}{2H} [T_{mech} - (E_{q}^t I_q + (X_q - X_d)I_d I_q + K_d(\omega - \omega_s))].$$

(3)

The stator algebraic equations are

$$V_T \sin(\delta - \theta) + R_d I_d - X_q I_q = 0,$$

(4)

$$E_{q}^t - V_T \cos(\delta - \theta) - R_d I_d - X_d I_d = 0.$$  

(5)

Neglecting stator resistance by assuming $R_s = 0$, we can write the stator dynamical equations as

$$V_T \sin(\delta - \theta) - X_q I_q = 0,$$

(6)

$$E_{q}^t - V_T \cos(\delta - \theta) - X_d I_d = 0.$$  

(7)

Since we have

$$V_d = V_T \sin(\delta - \theta),$$

(9)

$$V_q = V_T \cos(\delta - \theta).$$

(10)

Substituting in Equations (6) and (7), we get

$$V_d - X_q I_q = 0,$$

(11)

$$E_{q}^t - V_q - X_d I_d = 0.$$  

(12)

Assuming zero degree phase angle for the infinite bus voltage, we have

$$(I_d + j I_q) \exp(j(\delta - \pi/2)) = \frac{(V_d + j V_q) \exp(j(\delta - \pi/2)) - V_{\infty} \angle 0^\circ}{R_e + j X_e},$$

(13)

By separating the imaginary and real parts of Equation (13), we get

$$R_d I_d - X_q I_q = V_d - V_{\infty} \sin(\delta),$$

(14)

$$X_d I_d - R_d I_q = V_q - V_{\infty} \cos(\delta).$$

By linearizing Equations (11) and (12), we get

$$\begin{bmatrix} \Delta V_d \\ \Delta V_q \end{bmatrix} = \begin{bmatrix} 0 & X_q \\ -X_d & 0 \end{bmatrix} \begin{bmatrix} \Delta I_d \\ \Delta I_q \end{bmatrix} + \begin{bmatrix} 0 \\ \Delta E_{q}^t \end{bmatrix}.$$  

(15)

And by linearizing Equation (14), we get

$$\begin{bmatrix} \Delta V_d \\ \Delta V_q \end{bmatrix} = \begin{bmatrix} R_e & -X_d \\ X_e & R_e \end{bmatrix} \begin{bmatrix} \Delta I_d \\ \Delta I_q \end{bmatrix} + \begin{bmatrix} V_{\infty} \sin(\delta) \\ -V_{\infty} \cos(\delta) \end{bmatrix} \Delta \delta.$$  

(16)

By equating the right-hand sides of Equations (15) and (16), we have

$$\begin{bmatrix} R_e & -(X_e + X_q) \\ (X_e + X_q) & R_e \end{bmatrix} \begin{bmatrix} \Delta I_d \\ \Delta I_q \end{bmatrix} = \begin{bmatrix} 0 \\ \Delta E_{q}^t \end{bmatrix} + \begin{bmatrix} V_{\infty} \cos(\delta) \\ V_{\infty} \sin(\delta) \end{bmatrix} \Delta \delta.$$  

(17)
\[ \frac{\Delta I_d}{\Delta I_q} = \frac{1}{\Delta} \begin{bmatrix} (X_c + X_q) & -R_e V_\infty \cos \delta + V_\infty \sin \delta (X_c + X_q) \\ R_e V_\infty \sin \delta + V_\infty \cos \delta (X_d + X_c) \end{bmatrix} \times \frac{\Delta E_q}{\Delta \delta}, \]  

(18)

where

\[ \Delta = R_c^2 + (X_c + X_q)(X_c + X_q). \]  

(19)

Denote the normalized frequency \( \nu = \omega / \omega_s \). The linearized synchronous generator model of Equations (1)–(3) is given as follows:

\[ \frac{\Delta E_q^*}{\Delta \nu} = \begin{bmatrix} -\frac{1}{T_{do}} & 0 & 0 \\ 0 & 0 & \omega_s \\ -\frac{1}{2H} & 0 & -\frac{K_d}{2H} \end{bmatrix} \begin{bmatrix} \Delta E_q^* \\ \Delta \delta \\ \Delta \nu \end{bmatrix} + \begin{bmatrix} \frac{1}{T_{do}}(X_c - X_q) \\ \frac{1}{2H} (X_d - X_q) I_q^* - \frac{1}{2H} E_q^* \\ \frac{1}{2H} \end{bmatrix}, \]  

(20)

Substitute for \( \Delta I_d, \Delta I_q \), we have

\[ \Delta E_q^* = -\frac{1}{K_3 T_{do}} \Delta E_q^* - \frac{K_4}{T_{do}} \Delta \delta + \frac{1}{T_{do}} \Delta E_{fd}, \]  

(21)

\[ \Delta \dot{\delta} = \omega_s \Delta \nu, \]  

(22)

\[ \Delta \dot{\nu} = \frac{K_2}{2H} \Delta E_q^* - \frac{K_1}{2H} \Delta \delta - \frac{K_d \omega_s}{2H} \Delta \nu + \frac{1}{2H} \Delta T_{\text{mech}}, \]  

(23)

where

\[ \frac{1}{K_3} = 1 + \frac{(X_d - X_q')(X_c + X_c)}{\Delta}, \]  

(24)

\[ K_4 = \frac{V_\infty (X_c + X_q)}{\Delta} [ (X_q + X_c) \sin \delta - R_e \cos \delta], \]  

(25)

\[ K_2 = \frac{1}{\Delta} \left[ I_q^* \Delta - I_q'(X_c' - X_q)(X_q + X_c) \right] \]  

\[ - R_e (X_d' - X_q') I_d' + R_e E_q^*, \]  

(26)

\[ K_1 = -\frac{1}{\Delta} \left[ I_q^* V_\infty (X_c' - X_q)(X_q + X_c) \sin \delta - R_e \cos \delta \right] \]  

\[ + V_\infty \{ (X_d' - X_q') I_d' - E_q^* \} (X_c' + X_c) \cos \delta \]  

\[ + R_e \sin \delta \}. \]  

Since

\[ V_T^2 = V_q^2 + V_q^2, \]  

the differential terms is given as follows:

\[ \Delta V_T = \frac{V_q}{V_T} \Delta V_d + \frac{V_q^q}{V_T} \Delta V_q, \]  

(28)

Substituting Equation (18) into Equation (15), we obtain

\[ \begin{bmatrix} \Delta V_d \\ \Delta V_q \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} 0 \\ X_q R_e \sin \delta + V_\infty \cos \delta (X_c' + X_c) \end{bmatrix} \]  

\[ - \begin{bmatrix} -X_d'(X_q + X_c) \\ -X_d'(R_e V_\infty \cos \delta + V_\infty (X_c + X_c) \sin \delta) \end{bmatrix} \left( \frac{\Delta E_q^*}{\Delta \delta} \right) + \begin{bmatrix} 0 \\ \Delta E_q^* \end{bmatrix}, \]  

(29)

Based on Equations (28) and (29), we get

\[ \Delta V_T = K_5 \Delta \delta + K_6 \Delta E_q^*, \]  

(30)

where

\[ K_5 = \frac{1}{\Delta} \left\{ \frac{V_q}{V_T} \left( X_q R_e \sin \delta + V_\infty \cos \delta (X_c' + X_c) \right) \right\} \]  

\[ + \frac{V_q^q}{V_T} \left( X_q (R_e V_\infty \cos \delta - V_\infty (X_q + X_c) \sin \delta) \right). \]  

(31)

\[ K_6 = \frac{1}{\Delta} \left\{ \frac{V_q}{V_T} X_q R_e \sin \delta - \frac{V_q^q}{V_T} X_q (X_q + X_c) \right\} + \frac{V_q^q}{V_T}. \]  

(32)

### 2.2. Automatic voltage regulator (AVR) and exciter circuit dynamics

The following dynamical equations for AVR and excitation control system are adopted:

\[ \dot{E}_{\text{fd}} = \frac{K_A}{T_A} \left( V_{\text{ref}} - V_T + U_{\text{ps}} - \frac{E_{\text{fd}}}{T_A} \right). \]  

(33)

By linearizing Equation (33), we get

\[ \Delta \dot{E}_{\text{fd}} = \frac{K_A}{T_A} (\Delta V_{\text{ref}} - \Delta V_T + \Delta U_{\text{ps}}) - \frac{\Delta E_{\text{fd}}}{T_A}. \]  

(34)

Based on (30), (34) can be rewritten as

\[ \Delta \dot{E}_{\text{fd}} = -\frac{\Delta E_{\text{fd}}}{T_A} - \frac{K_A K_5}{T_A} \Delta \delta - \frac{K_A K_6}{T_A} \Delta E_q^* \]  

\[ + \frac{K_A}{T_A} \Delta U_{\text{ps}} + \frac{K_A}{T_A} \Delta V_{\text{ref}}. \]  

(35)

A typical power system stabilizer (PSS) control scheme include a washout filter and two lead-lag blocks. The
Denote $x = [\Delta \delta, \Delta \nu, \Delta E'_q, \Delta \delta_\text{fd}, \Delta \delta_1, \Delta \zeta_2, \Delta \zeta_3]^T$ and $u = [\Delta T_\text{mech}, \Delta V_\text{ref}]^T$, the linearized model becomes

$$\dot{x} = Ax(t) + A_d x(t-\tau) + B u(t),$$

where

$$A = \begin{bmatrix}
0 & \omega_s & 0 & 0 \\
-K_1 & -K_d\omega_s & -K_2 & 0 \\
2H & 2H & 2H & 1 \\
-1 & -K_1 T_{do} & 0 & 0 \\
-K_4 T_{do} & 0 & -1 & K_1 T_{do} \\
-K_5 T_A & K_2 T_A & 0 & -1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix},$$

$$A_d = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

$$B = \begin{bmatrix}
0 \\
1/T_{2} \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}.$$
disturbances is considered. The system is of the form:
\[
\dot{x}(t) = (A + \delta A)x(t) + (A_d + \delta A_d)x(t - \tau_1) + (B + \delta B)u(t) + (B_d + \delta B_d)u(t - \tau_2) + D\varpi(t),
\]
(48)
the performance output is chosen as
\[
z(t) = Ex(t)
\]
(49)
and
\[
x(t) = \phi(t) \quad \text{for} \ t \in [-\tau_1, 0].
\]
(50)
Assume that the state variables are available for feedback. Otherwise, estimators can be developed for state estimation purposes. Then, we have state feedback control input as
\[
u(t) = Kx(t)
\]
Therefore, the closed-loop system becomes:
\[
\dot{x}(t) = (A + \delta A + BK + \delta B)x(t) + (A_d + \delta A_d)x(t - \tau_1) + (B_d + \delta B_d)Kx(t - \tau_2) + D\varpi(t),
\]
(51)
Rearranging Equation (51), we get
\[
\dot{x}(t) = A_c x(t) + \delta A_c x(t) + (A_d + \delta A_d)x(t - \tau_1) + (B_d + \delta B_d)Kx(t - \tau_2) + D\varpi(t),
\]
(52)
where
\[
A_c = A + BK,
\]
\[
\delta A_c = \delta A + BK.
\]
Before proceeding to the theorem derivation, Assumption 1 and Lemma 1 are introduced (Wang, Yaz, & Yaz, 2010).

**ASSUMPTION 1** The general form of unstructured $L_2$ bounded uncertainties is used in this work:
\[
\delta A \delta A' \leq \gamma_A I,
\]
\[
\delta A_d \delta A_d' \leq \gamma_A I,
\]
\[
\delta B \delta B' \leq \gamma_B I,
\]
\[
\delta B_d \delta B_d' \leq \gamma_B I.
\]
**LEMMA 1**
\[
AB' + BA' \leq \alpha A A' + \alpha^{-1} B B'.
\]

To prove this inequality, we can consider the following equivalent inequality which always holds, given arbitrary $\alpha > 0$:
\[
(\alpha^{1/2}A - \alpha^{-1/2}B)(\alpha^{1/2}A - \alpha^{-1/2}B)' \geq 0.
\]
Furthermore, if $A$ and $B$ are chosen to be $[a' \ 0]$ and $[0 \ b']$, respectively, we get
\[
\begin{bmatrix}
0 & a' \\
b' & 0
\end{bmatrix} \leq \begin{bmatrix}
\zeta & a' \\
a & 0
\end{bmatrix} \leq \begin{bmatrix}
\zeta d' & 0 \\
0 & \zeta^{-1} b' b
\end{bmatrix},
\]
Based on Assumption 1 and Lemma 1, the main theorem of the paper is summarized as follows:

**THEOREM 1** Under the feedback control law $u(t) = Kx(t)$, the system of Equation (52) is asymptotically stable for all delays satisfying $\tau_1, \tau_2 \geq 0$. And the $H_\infty$ performance objective $\|T_{wy}\|_\infty \leq \gamma, \gamma > 0$ can be satisfied. If there exist matrices $Y, Q^i, Q^i_t > 0$ and $Q^i = Q^i_t > 0$ satisfying the following LMI:
\[
\begin{bmatrix}
m_1 & A_d X & B_d Y & D & X & Y \\
A_d^t & m_2 & 0 & 0 & 0 & 0 \\
Y B_d^t & 0 & m_3 & 0 & 0 & 0 \\
D^t & 0 & 0 & m_4 & 0 & 0 \\
X & 0 & 0 & m_5 & 0 & 0 \\
Y & 0 & 0 & 0 & m_6 & 0
\end{bmatrix} < 0,
\]
(53)
where
\[
m_1 = AX + BY + XA' + Y'B + Q_i + \alpha_1 (\gamma_A + \gamma_B)I,
\]
m_2 = $\alpha_2^{-1} I - Q_i$, 
\[
m_3 = \alpha_3^{-1} I - Q_i,
\]
m_4 = $-\gamma^2 I$, 
\[
m_5 = -[\alpha_1^{-1} I + \alpha_2 \gamma A I + E' E]^{-1},
\]
m_6 = $-\alpha_1^{-1} I + \alpha_3 \gamma_B I]^{-1}$.

**Proof** A Lyapunov–Krasovskii function is chosen as follows:
\[
V(x, t) = x'(t)P x(t) + \int_{t-\tau}^{t} x'(v)Q_1 x(v) \, dv
\]
\[
+ \int_{t-\tau}^{t} x'(v)Q_2 x(v) \, dv
\]
(55)
where $V(x, t)$ is a positive semi-definite functional and the matrices $P, Q_1, Q_2$ are all positive definite.

By taking derivative, we have
\[
\dot{V}(x, t) = x'(t) P x(t) + x'(t) P x(t) + x'(t) Q_1 x(t)
\]
\[
- x'(t - \tau_1) Q_1 x(t - \tau_1) + x'(t) Q_2 x(t)
\]
\[
- x'(t - \tau_2) Q_2 x(t - \tau_2). 
\]
(56)

Based on LaSalle’s theorem, in order to achieve the asymptotic stability, the conditions $V > 0$ and $\dot{V} < 0$ need to be satisfied.

In order to satisfy $H_\infty$ performance objective, the following $H_\infty$ performance inequality needs be employed.
\[
J = \int_0^\infty (z'^2 - \gamma^2 \omega^2 \omega) \, dt < 0.
\]
(57)
The sufficient condition to achieve both the asymptotic stability and $H_\infty$ performance objective is
\[
J = \int_0^\infty (z'^2 - \gamma^2 \omega^2 \omega + \dot{V}) \, dt < 0.
\]
(58)
Condition (58) implies

\[ z'z - \gamma^2 \omega'\omega + \dot{V} < 0. \]  

(59)

By substituting Equations (52) into (56), we get

\[
V(x, t) = [A_c x(t) + \delta A_c x(t) + (A_d + \delta A_d)x(t - d) + (B_d + \delta B_d)K x(t - \tau_t) + D w(t)]^T P x
+ x^T P [A_c x(t) + \delta A_c x(t) + (A_d + \delta A_d)x(t - d) + (B_d + \delta B_d)K x(t - \tau_t) + D w(t)]x(t)
- x'(t - \tau_t)Q_1 x(t - \tau_t) + x'(t)Q_2 x(t)
- x'(t - \tau_t)Q_2 x(t - \tau_t) < 0.
\]

(60)

Based on condition (58), we have

\[
[A_c x(t) + \delta A_c x(t) + (A_d + \delta A_d)x(t - d) + (B_d + \delta B_d)K x(t - \tau_t) + D w(t)]^T P x
+ x^T P [A_c x(t) + \delta A_c x(t) + (A_d + \delta A_d)x(t - d) + (B_d + \delta B_d)K x(t - \tau_t) + D w(t)]x(t)
- x'(t - \tau_t)Q_1 x(t - \tau_t) + x'(t)Q_2 x(t)
- x'(t - \tau_t)Q_2 x(t - \tau_t) + z'z - \gamma^2 I < 0.
\]

(61)

Denote \( \zeta(t) = [x'(t) \quad x'(t - \tau_d) \quad x'(t - \tau_i) \quad \omega'(t)]^T \), then Equation (61) can be written as

\[
\dot{V}(x, t) = \zeta'(t) W_\alpha \zeta(t),
\]

where

\[
W_\alpha = \begin{bmatrix}
P(A_c + \delta A_c) + (A_c + \delta A_c)P + Q_1 + Q_2 + E' E
(A_d + \delta A_d)P
K'(B_d + \delta B_d)P
\end{bmatrix}
\begin{bmatrix}
P(A_d + \delta A_d)
P(B_d + \delta B_d)K
PD
\end{bmatrix}
\begin{bmatrix}
-Q_1
0
0
\end{bmatrix} < 0.
\]

(62)

By pre- and post-multiplying Equation (62) with the diagonal matrix \( \text{diag}(X, I, I, I) \) and denote

\[
X = P^{-1}, \quad Y = K X, \quad Q_1 = X Q_1 X, \quad Q_2 = X Q_2 X,
\]

we get

\[
\begin{bmatrix}
\phi_{11} & \phi_{12} & \phi_{13} & D \\
\star & \phi_{22} & 0 & 0 \\
\star & 0 & \phi_{33} & 0 \\
\star & 0 & 0 & \phi_{44}
\end{bmatrix} < 0,
\]

(63)

where

\[
\phi_{11} = (A + BK + \delta A + \delta BK)X + X (A + BK + \delta A + \delta BK)^T + Q_1 + Q_2 + X E' E X,
\]

\[
\phi_{12} = (A_d + \delta A_d)X,
\]

\[
\phi_{13} = (B_d + \delta B_d)Y,
\]

\[
\phi_{22} = -Q_t,
\]

\[
\phi_{33} = -Q_t,
\]

\[
\phi_{44} = -\gamma^2 I.
\]

Applying Lemma 1, we get

\[
(\delta A + \delta BK)X + X (\delta A + \delta BK)^T < X [I \quad K]^T [\begin{bmatrix} \delta A' \\ \delta B' \end{bmatrix} + [\delta A \quad \delta B] \begin{bmatrix} I \\ K \end{bmatrix}] X,
\]

\[
\leq \alpha_1 [\delta A \quad \delta B] \begin{bmatrix} \delta A' \\ \delta B' \end{bmatrix} + \alpha_1^{-1} X [I \quad K]^T [\begin{bmatrix} I \\ K \end{bmatrix}] X.
\]

(64)

By applying Assumption 1, we have

\[
(\delta A + \delta BK)X + X (\delta A + \delta BK)^T \leq \alpha_1 (\gamma_t I + \gamma_t I)
+ \alpha_1^{-1} X [I \quad K]^T [\begin{bmatrix} I \\ K \end{bmatrix}] X.
\]

Based on Lemma 1 and Assumption 1, the following matrix inequality is reached:

\[
\begin{bmatrix}
0 & \delta A_d X & \delta B_d Y & 0 \\
X' \delta A_d' & 0 & 0 & 0 \\
Y' \delta B_d' & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\alpha_2 \gamma_{h_d} X' X & \alpha_3 \gamma_{h_d} Y Y & 0 & 0 \\
0 & \alpha_4^{-1} I & 0 & 0 \\
0 & 0 & \alpha_5^{-1} I & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \leq 0.
\]

(65)

Now, by substituting Equations (64) and (65) into Equation (63) and applying Schur complement, we obtain the following LMI result:

\[
\begin{bmatrix}
\zeta_{11} & \zeta_{12} & \zeta_{13} & D & X & Y' \\
\star & \zeta_{22} & 0 & 0 & 0 & 0 \\
\star & 0 & \zeta_{33} & 0 & 0 & 0 \\
\star & 0 & 0 & \zeta_{44} & 0 & 0 \\
\star & 0 & 0 & 0 & \zeta_{55} & 0 \\
\star & 0 & 0 & 0 & 0 & \zeta_{66}
\end{bmatrix} < 0.
\]

(66)
where

\[
\begin{align*}
\zeta_{11} &= AX + BY + XA' + Y'B + Q_r + Q_s + \alpha_1 (\gamma_A + \gamma_B)I, \\
\zeta_{12} &= A_dX, \\
\zeta_{13} &= B_dY, \\
\zeta_{12} &= \alpha_{21}^{-1}I - Q_r, \\
\zeta_{13} &= \alpha_{31}^{-1}I - Q_s, \\
\zeta_{22} &= \gamma^2 I, \\
\zeta_{55} &= -[\alpha_1^{-1}I + \alpha_2 \gamma_A I + E'E]^{-1}, \\
\zeta_{66} &= -[\alpha_1^{-1}I + \alpha_3 \gamma_B I]^{-1}.
\end{align*}
\]

\section{Simulation and Results}

The following parameters are used for simulations. Assuming that \( R_e = 0 \), \( X_e = 0.5 \text{pu} \), \( V_T \angle \theta = 1 \angle 15^\circ \text{pu} \), and \( V_\infty \angle 0^\circ = 1.05 \angle 0^\circ \text{pu} \).

Based on Equations (24)–(27), (31) and (32), we can calculate the values:

\[
\begin{align*}
K_1 &= 0.9224, \\
K_2 &= 1.0739, \\
K_3 &= 0.296667, \\
K_4 &= 2.26555, \\
K_5 &= 0.005, \\
K_6 &= 0.3572.
\end{align*}
\]

\( \mathcal{L}_2 \) of disturbance is chosen as \( w(t) = 5 \times 0.9^t \), notice that the disturbance energy is finite.

MATLAB robust control toolbox provide the capability to design the optimal control feedback with LMI. Computer simulation shows that our proposed controller effectively stabilizes the time response of rotor angle in Figure 1, normalized frequency in Figure 2, quadrature axis transient voltage in Figure 3 and excitation voltage in Figure 4. The control input is shown in Figure 5. Simulation results have demonstrated the effectiveness and robustness of our proposed approach (Figure 5).
5. Conclusion

A general robust $H_\infty$ control approach is proposed in this paper for a power-generation system with state and input delays, disturbances and model uncertainties. The uncertainties are assumed to be bounded and unstructured. A novel seventh-order state space model for the power-generation system is developed. Computer simulation studies conducted through the use of MATLAB show the robustness and effectiveness of the novel approach. Notice that our LMI control solution only applies for delay independent power system control applications. In the future development, we would investigate the delay-dependent cases based on the Discretized Lyapunov–Krasovskii functional method.

Disclosure statement

No potential conflict of interest was reported by the authors.

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