The Spatial Correlation Function From An X–ray Selected Sample of Abell Clusters

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Abstract

We present here the Spatial Two–Point Correlation Function for a complete sample of 67 X–ray selected Abell clusters of galaxies. We find a correlation length of $16.1 \pm 3.4 h^{-1} \text{Mpc}$ with no significant clustering beyond $\approx 40 h^{-1} \text{Mpc}$. This is the lowest uncorrected value for the correlation length ever derived from the Abell catalogue of clusters. In addition, we have investigated the anisotropy of the correlation function between the radial and transverse directions. This can be characterised by the magnitude of pair–wise cluster peculiar velocities such anisotropy predicts, which we find to be $\approx 800 \text{km s}^{-1}$ for our sample. Again, this is the lowest uncorrected value ever seen for the Abell catalogue. We therefore, no longer need to invoke high cluster peculiar velocities or line of sight clustering to understand the correlation function as derived from an Abell sample of clusters. Furthermore, our result is consistent with recently published correlation functions computed from automated selections of optical and X–ray clusters. Therefore, we are now approaching a coherent picture for the form of the cluster spatial correlation function which will be used to place confident constraints on theories of galaxy formation.

1 Introduction

Clusters of galaxies are key tracers of the large–scale structure in the universe, since their typical separation is $\sim 10 h^{-1} \text{Mpc}$. The most popular statistic used to quantify the distribution of clusters has been the Spatial Two–Point Correlation Function ($\xi_{cc}(r)$), whose observed shape and amplitude have been the centre of much debate in the astronomical literature over the last 10 years.

The bench–mark in this area of study has been the $\xi_{cc}(r)$ derived by Bahcall & Soneira (BS83, 1983). For a sample of 104 $R \geq 1$ Abell clusters (Abell 1958), they found that the correlation function had the form; $\xi_{cc}(r) = (r/r_o)^{-1.8}$ with a correlation length of $r_o = 25 h^{-1} \text{Mpc}$ and a positive tail out to $150 h^{-1} \text{Mpc}$. One of the most severe consequences of this result was it suggested that clusters of galaxies were over 15 times more clustered than galaxies, indicating that both galaxies and clusters can not simultaneously be fair

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²Throughout this paper, we use $H_o = 100 \text{km s}^{-1} \text{Mpc}^{-1}$ and $q_o = \frac{1}{2}$.
tracers of the underlying mass distribution. Furthermore, the form of the BS83 result has
been a strong constraint on models of galaxy formation. For example, the popular theory
of standard Cold Dark Matter (CDM, Davis et al. 1985) can not reproduce the high level of
clustering seen in the BS83 correlation function, or its positive tail out to high separations
(White et al. 1987). More recently, Lilje (1990) has also shown that the BS83 result can
not be accounted for in a Hot Dark Matter dominated universe.

Over the past few years, the original BS83 $\xi_{cc}(r)$ has been confirmed, to some degree,
with much larger redshift surveys of Abell clusters. Table 1 highlights all published correla-
tion analyses carried out on the Abell catalogue and the correlation functions derived from
these studies. The largest surveys presented in this table, those of Huchra et al. (1990),
Postman et al. (1992) and Peacock & West (1992), all find a correlation length for $\xi_{cc}(r)$
of $r_o \simeq 21 \pm 3h^{-1}\text{Mpc}$ with a positive tail out beyond $r > 50h^{-1}\text{Mpc}$. Again, all these
results are in conflict with CDM predictions for $\xi_{cc}(r)$.

Table 1: (a) Published determinations of $\xi_{cc}(r)$. The table includes the original reference,
the number of clusters used, the published value of the correlation length ($r_o$), the number
density of clusters and finally, the slope used for the analysis.

| Survey                  | No. | $r_o$ ($h^{-1}\text{Mpc}$) | $n_c$ ($10^{-5}h^3\text{Mpc}^{-3}$) | $\gamma$ |
|-------------------------|-----|---------------------------|------------------------------------|---------|
| Abell determinations of $\xi_{cc}(r)$ |
| Bahcall & Soneira 1983  | 104 | 25.0                      | 0.6                                | 1.8     |
| Ling et al. 1986        | 104 | 21.9$^{+7.1}_{-5.1}$      | 0.6                                | 1.7$^{\pm0.17}$ |
| Postman et al. 1986     | 136 | 20.0$^{+4.6}_{-2.3}$      | 1.8                                |
| Postman et al. 1986     | 1207| 24.0$^{+3.0}_{-3.0}$      | 1.8                                |
| Postman et al. 1986     | 370 | 42.0$^{+8.0}_{-9.1}$      | 1.8                                |
| Huchra et al. 1990      | 145 | 20.3$^{+8.5}_{-5.1}$      | $\sim$ 1.0                         | 1.8     |
| Huchra et al. 1990      | 92  | 20.9$^{+6.7}_{-6.9}$      | $\sim$ 0.6                         | 1.8     |
| West & van der Bergh 1991 | 64  | 22.1$\pm$ 6.8             | 1.7$^{\pm0.5}$                     |
| Postman et al. 1992     | 351 | 20.0$^{+4.6}_{-4.0}$      | 1.2                                | 2.5$^{\pm0.2}$ |
| Postman et al. 1992     | 156 | 23.7$^{+7.9}_{-9.0}$      | $\sim$ 1.0                         | 1.8$^{\pm0.2}$ |
| Peacock & West 1992      | 195 | 21.1$\pm$ 1.3             | 0.7                                | 2.0$^{\pm0.2}$ |
| Peacock & West 1992      | 232 | 20.6$\pm$ 1.5             | 0.9                                | 1.5$^{\pm0.2}$ |
| Projection Corrected Abell determinations of $\xi_{cc}(r)$ |
| Sutherland 1988         | 533 | 14.0$^{+4.0}_{-3.0}$      | 0.6                                | 1.8     |
| Dekel et al. 1989       | 102 | $\sim$ 15.0               | 1.8                                |
| Sutherland & Efstathiou 1991 | 113 | 9.0                       | $\sim$ 3.0                         | 1.8     |
| Sutherland & Efstathiou 1991 | 145 | 14.0                      | 0.7                                | 1.8     |
| Efstathiou et al. 1992  | 298 | $\sim$ 13                 | 1.4                                | $\sim$ 2.0 |
| Non–Abell determinations of $\xi_{cc}(r)$ |
| Lahav et al. 1989       | 53  | 21.0                      | 1.8                                |
| Dalton et al. 1992      | 220 | 12.9$\pm$ 1.4             | 2.4                                | 1.9$^{\pm0.3}$ |
| Dalton et al. 1992      | 93  | 14.4$\pm$ 4.0             | 1.1                                | 2.0     |
| Nichol et al. 1992      | 79  | 16.0$\pm$ 4.0             | 1.0                                | 2.1$^{\pm0.3}$ |
| Romer et al. 1993       | 129 | 13.7$\pm$ 2.3             | 1.9                                | 1.9$^{\pm0.4}$ |
| Romer et al. 1993       | 129 | 15.6$\pm$ 2.4             | 1.4                                | 1.4$^{\pm0.4}$ |
| This Paper              |     |                           |                                    |
| Nichol et al. 1993      | 67  | 16.1$\pm$ 3.4             | 0.8                                | 1.9$^{\pm0.3}$ |
Table 1: (b) Published determinations of \( \xi_{cc}(r) \). This table details the cluster samples used by the authors in Table 1a. \( RC \) is Richness Class and \( D \) is Distance Class as originally defined by Abell (1958).

| Survey                        | Comments                                                                 |
|-------------------------------|--------------------------------------------------------------------------|
| Bahcall & Soneira 1983        | \( RC \geq 1, D \leq 4 \) Abell clusters.                               |
| Ling et al. 1986              | same data as above.                                                     |
| Postman et al. 1986           | Abell statistical sample \( z \leq 0.1 \).                              |
| Postman et al. 1986           | All Abell \( RC \geq 1 \) clusters (80% estimated redshifts).         |
| Postman et al. 1986           | All Abell \( RC \geq 2 \) clusters (75% estimated redshifts).         |
| Huchra et al. 1990            | Deep Abell survey.                                                      |
| West & van der Bergh 1991     | cD Abell clusters.                                                      |
| Postman et al. 1992           | All Abell clusters \( m_{10} \geq 16.5 \).                            |
| Postman et al. 1992           | \( RC \geq 1 \) Abell clusters \( m_{10} \geq 16.5 \).                |
| Peacock & West 1992           | Volume limited sample of \( RC \geq 1 \) Abell clusters.               |
| Peacock & West 1992           | Volume limited sample of \( RC \geq 0 \) Abell clusters.               |
| Sutherland 1988               | Abell catalogue + projection correction.                                |
| Dekel et al. 1989             | \( RC \geq 1, D \leq 4 \) Abell clusters + projection correction.      |
| Sutherland & Efstathiou 1991  | Shectman (1985) clusters + projection correction.                      |
| Sutherland & Efstathiou 1991  | Huchra et al. (1990) survey + projection correction.                   |
| Efstathiou et al. 1992        | Postman et al. (1992) survey + projection correction.                   |
| Lahav et al. 1989             | EXOSAT X-ray clusters.                                                  |
| Dalton et al. 1992            | APM survey, \( R \geq 20 \).                                           |
| Dalton et al. 1992            | APM survey, \( R \geq 35 \).                                           |
| Nichol et al. 1992            | EM survey, \( R \geq 22 \).                                            |
| Romer et al. 1993             | ROSAT All-Sky Survey.                                                  |
| Romer et al. 1993             | ROSAT All-Sky Survey. Fit \( r < 35h^{-1}\) Mpc                       |
| Nichol et al. 1993            | ROSAT detections of the Huchra et al. (1992) sample.                    |

However, several authors (Sutherland 1988, Sutherland & Efstathiou 1991 & Efstathiou et al. 1992) have claimed that the high correlation length of \( \xi_{cc}(r) \) derived from the Abell catalogue, and its southern counterpart (Abell, Corwin & Olowin 1989), is due to systematic biases introduced by the subjective manner in which these catalogues were constructed. Sutherland argues that the catalogues are plagued by projection effects, which he defines as; ‘angular correlations that are not due to genuine clustering in redshift space’. These projection effects would therefore, result in the correlation function being artificially elongated in the redshift direction since there would be an excess of cluster pairs angularly close on the sky but with very different redshifts. When Sutherland corrected the BS83 result for these projection effects, he found the correlation length of the \( \xi_{cc}(r) \) decreased to \( 14^{+4 \, -3}h^{-1}\) Mpc (Table 1); a result far less discrepant with models of structure formation.

The elongation of \( \xi_{cc}(r) \) in the redshift direction was originally noted by BS83 themselves, however, they claimed the effect was due to large cluster peculiar velocities (\( \simeq \)
\( \simeq \))
\( \xi_{cc}(r) \) for X-ray selected Abell clusters

2000 \( \text{km s}^{-1} \), Bahcall, Soneira & Burgett 1986) which would have the effect of smoothing \( \xi_{cc}(r) \) in the line of sight direction. More recently, Jing, Plionis & Valdarnini (1992) have simulated the effect and claim the true cause of these redshift elongations is real line of sight clustering. Clearly, there is little consensus within the astronomical literature over the true nature of these redshift elongations.

This debate has increased dramatically over the past two years with the publication of \( \xi_{cc}(r) \) from new, fully automated selections of clusters. For example, Nichol et al. (1992) have presented \( \xi_{cc}(r) \) for 79 rich clusters selected objectively for the Edinburgh/Durham Cluster Catalogue (Lumsden et al. 1992) which constitutes the Edinburgh/Milano cluster redshift survey (Guzzo et al. 1992). Each of these clusters has an average of 10 galaxy redshift measurements, thus removing the problems of phantom clusters and spurious cluster redshifts. For this unique database, they found a correlation length of \( 16 \pm 4h^{-1}\text{Mpc} \) with no positive tail beyond \( \simeq 40h^{-1}\text{Mpc} \). Furthermore, they showed that \( \xi_{cc}(r) \) was isotropic with little indication of extensive redshift elongations as seen in the BS83 result. The best fit from this result for the magnitude of pair-wise cluster peculiar velocities was \( 442^{+398}_{-40} \text{km s}^{-1} \), which ruled out the value given by Bahcall et al. (1986) at the \( > 3\sigma \) level.

Finally, in addition to the Sutherland effect, several authors have argued that the true frequency of phantom clusters is also a serious problem in the Abell catalogue. This is the chance alignment of groups/galaxies along the line of sight that give the impression of a rich cluster, as seen in 2–D. Simulations of this phenomenon have claimed that as many as 50\% of clusters, seen in 2–D, are spurious (Lucey 1983, Frenk et al. 1990). However, recent results from Briel & Henry (1993) have shown that over 80\% of Abell clusters are X–ray emitters above a flux limit of \( 10^{-11} \text{erg s}^{-1} \), thus strongly suggesting that this is not as severe a problem as predicted and therefore, has a much lower significance than the proposed projection effects.

In this paper, we present \( \xi_{cc}(r) \) derived from an X–ray selected sample of Abell clusters. The data are the combination of the deep Abell sample of Huchra et al. (1990) and the corresponding X–ray detections as given by Briel & Henry (1993). The motivation behind this work was a hope that the X–ray data would provide a more robust sample of Abell clusters and minimise any projection effects that might be present. In the next section, we discuss the exact sample of clusters used in the analysis. In Section 3, we derive the correlation function for this sample and in Section 4, compute the spatial number density of our clusters. We investigate the degree of anisotropy for our sample in Section 5 and finally, end the paper with a discussion of our result in the light of previous correlation functions.

2 Sample of Abell Clusters

The results given in this paper are primarily based on the data published by Huchra et al. (1990). They presented the redshift measurements for all 145 Abell clusters in the region \( 10^h \leq \text{Right Ascension} \leq 15^h \) and \( 58^\circ \leq \text{Declination} \leq 78^\circ \), which is at high galactic latitude and has an effective volume similar to that used by BS83. The median redshift of the sample is \( z = 0.17 \) with a maximum redshift of \( z = 0.35 \) and an estimated redshift completeness limit of \( z = 0.24 \). Finally, only \( \simeq 25\% \) of the cluster redshifts were derived
from a single galaxy redshift measurement thus reducing the probability of assigning clusters a spurious redshift.

Recently, Briel & Henry (1993) presented the X–ray details, obtained from analysis of the ROSAT All–Sky survey, for the clusters from the Huchra et al. deep sample. They detected 66 of the 145 clusters at the $3\sigma$ level above the background and for these clusters, they presented the X–ray flux and luminosity of the cluster. For the remaining clusters an upper limit on the flux and luminosity were given in their paper. However, all of these remaining clusters do have a measured flux and luminosity, just at a lower significance level. We refer the reader to Briel & Henry (1993) for a complete description of the X–ray data used here.

We used all the measured luminosities and X–ray detection levels to create a sub-sample from the Huchra et al. (1990) clusters for our correlation analysis. In total, we selected 67 clusters with the criteria of $z \leq 0.24$, an X–ray luminosity of $\geq 10^{43}$ erg s$^{-1}$ and a detection significance of $\sigma \geq 2$ above the background. The redshift cut corresponded to the completeness limit of clusters given by Huchra et al. which they derived from the observed space density of the clusters as a function of redshift. The two other criteria were derived empirically to obtain a balance between selecting legitimate massive systems, while still having enough clusters to carry out a confident correlation analysis. Hence, we selected the clusters for our analysis from the Abell sample of Huchra et al. based only on the X–ray characteristics of these clusters (Briel & Henry 1993). We did not introduce any optical selection criteria (other than those already intrinsic to the Abell catalogue) since, at some level, these are the basis for supposed projection effects i.e. the optical richness of distant clusters being artificially boosted because of the presence of a nearby rich cluster.

3 The Spatial Two–Point Correlation Function

The Spatial Two–Point Correlation Function is usually estimated by comparing the observed distribution of cluster pairs with that obtained from a random catalogue of clusters distributed within the same survey boundaries as the data. The advantage of this technique is that it minimises the problems of edge effects, as well as allowing any selection biases in the data to be incorporated into the random catalogue. The function is therefore, represented by;

$$\xi_{cc}(r) = \frac{2N_r}{N_d} \frac{n_{dd}}{n_{dr}} - 1,$$

where $N_d$ and $N_r$ are the number of data and random clusters respectively and $n_{dd}$ and $n_{dr}$ are the number of data–data pairs and data–random pairs within the separation interval $r \pm \Delta r/2$ ($\Delta r$ is the binsize). Comoving separations ($r$) between the clusters were calculated using the standard Friedmann cosmology. The reader is referred to Davis & Peebles (1983) and/or Peebles (1980) for a full discussion of the merits of this estimator over other possible forms. In all estimations of $\xi_{cc}(r)$ discussed below, we used 3 random catalogues ($N_r = 100 N_d$) which were then averaged together to obtain a single $\xi_{cc}(r)$ (see Nichol 1992).

\footnote{We computed our luminosities using $H_0 = 100$ km s$^{-1}$ Mpc$^{-1}$. This is in contrast to the value used by Briel & Henry. However, we were able to reproduce their luminosities when using the same value of $H_0$.}
The redshift distributions for these random catalogues were constrained to have the same distribution as the data, after it had been smoothed with a Gaussian of width 3000 km s\(^{-1}\). This removed the need for detailed modelling of the redshift selection function of the data and was the same procedure as originally used by Huchra et al. and many other authors (Postman et al. 1992, Nichol et al. 1992). The overall form of \(\xi_{cc}(r)\) was insensitive to the exact value of the smoothing width. The correlation function was derived using logarithmic binning with a binsize of \(\Delta\log r = 0.1\) and a maximum separation of 300\(h^{-1}\)Mpc. Again, this was the same procedure as used by Huchra et al. and therefore, allowed for an exact comparison between these results. Finally, bootstrap resampling errors on all correlation functions were computed using the method described by Mo, Jing & Börner (1992).

The correlation function derived from our X–ray sample of 67 Abell clusters is shown in Figure 1. Included in this plot are the best fit forms of \(\xi_{cc}(r)\) as computed by Huchra et al. (1990) and BS83. It is worth noting here, that we re–computed \(\xi_{cc}(r)\) for all 145 clusters in the deep Huchra et al. sample and obtained exactly the same result as published by them. We fitted our \(\xi_{cc}(r)\) using the model \(\xi_{cc}(r) = (r/r_o)^\gamma\) and obtained a best fit for the correlation length and slope of \(r_o = 16.1 \pm 3.4h^{-1}\)Mpc and \(\gamma = 1.9 \pm 0.3\) for the range \(5 \leq r \leq 35h^{-1}\)Mpc (\(\chi^2 = 2.1\) for 3 degrees of freedom). In addition, our \(\xi_{cc}(r)\) passed through zero at \(\approx 40h^{-1}\)Mpc with no significant clustering beyond this.

4 The Number Density of Clusters

It is vital to accompany any discussion of the correlation function with an accurate measure of the number density of clusters being analysed. This is due to claims that the correlation length of \(\xi_{cc}(r)\) is a strong function of the mean separation between the clusters in question (see Bahcall & West 1992). The basis for such claims can be seen in Table 1 where richer clusters (\(i.e.\) RC \(\geq 2\)) have a larger correlation length than other estimates of \(\xi_{cc}(r)\). Therefore, it is important to derive the number density of our sample so that we can confidently compare our results with the correlation estimates in the literature.

The number density, \(n_c\), of any survey can be computed using the following formulae:

\[
\begin{align*}
n_{tot} &= n_c \int \frac{dV}{dz} S(z) E(z) \, dz, \\
\frac{dV}{dz} &= 4d\Omega \left( \frac{c}{H_o} \right)^3 \frac{(z - \sqrt{z + 1} + 1)^2}{(1 + z)^2},
\end{align*}
\]

(Kolb & Turner 1990, \(q_o = \frac{1}{2}\)) where \(n_{tot}\) is the total number of clusters observed, \(S(z)\) is the selection function, \(E(z)\) is the evolution of \(n_c\) and \(d\Omega\) is the solid angle subtended by the survey region.

In Figure 2, we have plotted the product of \(n_c S(z) E(z)\) as a function of redshift for our sample of 67 clusters. This was computed by dividing the observed redshift histogram of our 67 clusters (after it had been smoothed with a Gaussian of width 3000 km s\(^{-1}\)) by Equation 3. The error bars shown are \(\sqrt{dN}\), where \(dN\) is the number of clusters in each redshift bin. From this figure, our sample is consistent with a constant number density out to a redshift of \(z = 0.15\), which strongly suggests that neither the selection function or
cluster density evolution are significant. Beyond this redshift, the observed number density of our sample drops by a factor of three. This would indicate that we are approaching the limit of the Abell catalogue, which is consistent with previous authors’ estimations for the completeness depth of the Abell catalogue (BS83, Scaramella et al. 1991) and/or, we are seeing X-ray evolution in our deep sample of X-ray clusters (i.e. Henry et al. 1992). Therefore, from Figure 2, the underlying number density of our X-ray selected sample is \( \sim 0.8 \times 10^{-5} h^3 \text{Mpc}^{-3} \), which is consistent with the number densities of other Abell samples as shown in Table 1. More specifically, our \( n_c \) agrees well with the mean number density quoted by Huchra et al. (1990) and appears to be midway between the observed \( n_c \) for \( R \geq 0 \) and \( R \geq 1 \) Abell clusters (Peacock & West 1992).

As a further check, we compared the mean richness of our individual clusters with that of other Abell samples. This was achieved using the individual cluster galaxy counts given by Abell et al. (1989) for the northern clusters. We found our mean richness to be \( R_{\text{mean}} = 75 \pm 31 \) which is in good agreement with the mean richness of the Abell \( RC \geq 1 \) clusters; \( R_{\text{mean}} = 74 \) (Bahcall & West 1992). The scatter on our mean reflects the fact that our X-ray selection criteria have selected clusters over a wide range of Abell richnesses which can be seen in the poor correlation between X-ray luminosity and galaxy richness (see Briel & Henry 1993). Furthermore, this scatter is consistent with the findings of Lumsden et al. (1992) who showed that the true external error on the quoted richness of individual Abell clusters was \( \approx 35 \) irrespective of richness and distant class; this is a factor of two greater than the error quoted by Abell. This agreement would suggest that overall, we would expect to see similar clustering characteristics as \( RC \geq 1 \) Abell clusters.

5 Anisotropy of \( \xi_{cc}(r) \)

At the centre of the debate over the true form of \( \xi_{cc}(r) \) are the observed elongations in the redshift direction for functions derived from the Abell catalogue. As discussed in the Introduction, Sutherland (1988) and others claim that these elongations are due to projection effects, while Bahcall et al. (1986) claim they are the result of large cluster peculiar velocities. Therefore, we investigated our \( \xi_{cc}(r) \) as both a function of transverse separation \( (r_p) \) and radial separation \( (r_z) \). Figure 3 shows the contours of our \( \xi_{cc}(r_z, r_p) \), as well as those for the whole Huchra et al. sample (132 clusters) and the \( RC \geq 1 \) Huchra et al. clusters (92 clusters) both with \( z \leq 0.24 \).

The best method of quantifying the amount of anisotropy seen in these contour plots is via the value of cluster peculiar velocities they predict. This can be achieved using the function,

\[
\xi_{cc}(r_z, r_p) = \frac{r_p^{\gamma}}{\sqrt{2\pi} \sigma_v} \times \\
\int_{-\infty}^{+\infty} (r_p^2 + (r_z - x)^2)^{-\gamma/2} e^{-x^2/2\sigma_v^2} \, dx,
\]

where \( \sigma_v \) is the value of the pair–wise cluster peculiar velocities. This expression is the product of convolving the power–law form of \( \xi_{cc}(r) \) with a Gaussian dispersion. Using the
values of $r_o$ and $\gamma$ derived above, we fitted our $\xi_{cc}(r_z, r_p)$ with the transverse direction constrained to $0 \leq r_p \leq 10h^{-1}$ Mpc and obtained a best fit value of $\sigma_v = 789^{+432}_{-407}$ km s$^{-1}$ (70% confidence limit). We also performed the same analysis on the full sample of deep clusters and the $RC \geq 1$ clusters, both with $z \leq 0.24$, as shown in Figure 3. Using the Huchra et al. values of $r_o$ and $\gamma$, we obtained $\sigma_v = 1241^{+342}_{-320}$ km s$^{-1}$ and $\sigma_v = 1178^{+278}_{-252}$ km s$^{-1}$ for the full and $RC \geq 1$ Huchra et al. datasets respectively. These values are higher than those quoted by Huchra et al., but are still substantially less than that advocated by Bahcall et al. (1986). Due to the large statistical errors on these data, all the results are within the 70% confidence limits ($\approx 1\sigma$) of each other. It should be noted that all $\xi_{cc}(r_z, r_p)$ derived in this paper did have a local minima in the fitted chi–squared at zero peculiar velocities, but these minima were all larger than the eventual best fitted peculiar velocity values.

6 Discussion

The value of $r_o$ computed from our sample of 67 X–ray selected Abell clusters is the lowest uncorrected value of the correlation length ever derived for the Abell catalogue. Although other authors have derived lower values for $r_o$ from the Abell catalogue (Table 1) this has been achieved by correcting the correlation function for projection effects which remains controversial and leads to large uncertainties on the correlation length (Dekel et al. 1990). In Figure 1, our $\xi_{cc}(r)$ is systematically below both the Huchra et al. and BS83 results on all scales. If we constrain the comparison of our result to samples with a similar number density (i.e. $RC \geq 1$), we still find we have a lower value of $r_o$ and a lower degree of anisotropy. For example, we can compare our anisotropy plots with that observed in similar samples of Abell clusters without X–ray selection (Nichol et al. 1992, Peacock & West 1992 and the $RC \geq 1$ Huchra et al. sample presented in Figure 3c). The $\xi_{cc}(r_p, r_z) = 1$ contour in these latter samples is clearly extended by more than a factor of 2:1 in the redshift direction for $RC \geq 1$ clusters. The same contour in our anisotropy plot is isotropic with little sign of extension (Figure 3a). This difference is quantified by the drop in the predicted cluster peculiar velocities we see between our sample and the aforementioned works. Furthermore, our result is in good agreement with the $\xi_{cc}(r)$ calculated by Nichol et al. (1992) for an automated selection of galaxy clusters, which has a similar number density to the sample used here, and other non–Abell determinations (Table 1).

It should be stressed here, that the statistical errors on all the measurements of $r_o$, either from Abell samples or not, make it difficult to conclusively argue the difference seen in Table 1 or in our result. No result is more than $2\sigma$ away from any other determination of $r_o$. However, we feel that the combination of a lower $r_o$ and a smaller degree of anisotropy seen in our sample, suggests that this is a real change in the clustering characteristics of our Abell sample. Moreover, this debate should not be restricted to an exclusive discussion of the correlation length; we also see different clustering characteristics for our clusters on scales $> r_o$.

At face value, it is no longer necessary to invoke high cluster peculiar velocities (Bahcall et al. 1986) or line of sight clustering (Jing et al. 1992) to explain our result. If these physical effects were real, it is hard to understand why we no longer see them in our sample.
Our lower value for the anisotropy is probably due to the combination of two effects: (i) we are selecting clusters based solely on their X–rays luminosity which is much less prone to error than the projected 2–D galaxy counts; (ii) a large majority of the Huchra *et al.* clusters have multiple redshift measurements, thus reducing the problems of assigning a cluster a spurious redshift which would have the effect of smoothing the correlation function preferentially in the redshift direction.

We can compare our result with other X–ray determinations of $\xi_{cc}(r)$ in the astronomical literature (see Table 1). Lahav *et al.* (1989) have published an $\xi_{cc}(r)$ for a similarly sized sample of X–ray bright nearby clusters of galaxies (53 clusters in total). They found a correlation length of $r_o = 21h^{-1}$Mpc which is in agreement with the standard Abell value (Table 1) and therefore, appears to be in conflict with other non–Abell samples of clusters and the findings of this paper. However, the authors themselves comment that their sample may be incomplete near the galactic plane and when they curtail their sample to high galactic latitudes, they find an $r_o$ of $17h^{-1}$Mpc thus removing the apparent disagreement. Furthermore, they force their correlation function to have a predetermined slope of $\gamma = 1.8$, which will have the effect of increasing $r_o$ compared to that determined from a steeper slope like $\gamma = 2$ ($\log r_o \propto \gamma^{-1}$). More recently, Romer *et al.* (1993) have presented the long awaited determination of $\xi_{cc}(r)$ from a large objective sample of ROSAT clusters. They find a correlation length of $r_o = 15.6 \pm 2.4 h^{-1}$Mpc, over the same fitting range, with no elongations in the redshift direction. This is in good agreement with the results presented here and results derived from automated optical cluster catalogues (Table 1).

Bahcall & West (1992) have argued for the existence of a universal correlation function whose $r_o$ is directly related to the number density of the clusters being analysed ($r_o = 0.4 n^{-\frac{1}{3}}$ where $n$ is the observed number density). Their argument is analogous to the idea that richer clusters have a higher correlation length, since they are intrinsically rarer objects and thus have a larger mean separation. From the Bahcall & West paper, the observed number density presented in Section 4 for our sample of 67 clusters would imply an observed $r_o$ of $20 h^{-1}$Mpc for this sample. Our result would therefore, appear to contradict this hypothesis since we observe a lower correction length than that predicted. However, the large error on both our observed $r_o$ and the number density of our sample makes it hard to conclusively argue this point. As a matter of interest, the correlation length for the remaining 78 clusters from the 145 Huchra *et al.* sample not selected by our X–ray selection criteria is $10.8 \pm 4.4 h^{-1}$Mpc with a slope of $2.2 \pm 0.8$. This lower value of $r_o$ is consistent with the idea that the correlation length is dependent on the mass of the clusters analysed as we would expect these 78 clusters to be, on a whole, less massive systems than those in our main sample based on their X–ray luminosity.

Our result removes the tentative disagreement seen in Table 1 between Non–Abell and Abell samples of clusters, thus suggesting that we are now approaching a coherent picture for the distribution of clusters and their peculiar velocities. Therefore, this allows us to use the correlation function of clusters to confidently place tight constraints on theories of galaxy formation. This has recently been carried out by several authors (*i.e.* Olivier *et al.* 1993, Dalton *et al.* 1992) all of which still find an excess of power on large scales compared to predictions from the popular standard CDM model. However, the discrepancy is far less severe than that seen with the original BS83 result. In addition, there are now several
alternative CDM models available that might be able to reconcile this theory with these new cluster correlation observations (see Bahcall & Cen 1992 & Mann, Heavens & Peacock 1993 and Croft & Efstathiou 1993).

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