On the hidden charm state at 3872 MeV

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Abstract

We discuss some puzzling aspects of the narrow hidden charm resonance that was recently discovered by the Belle Collaboration at mass 3872 MeV. In order to determine its quantum numbers, a crucial piece of information is the spin of the dipion in the decay final state $\pi^+\pi^- J/\psi$. We give the angular distributions and correlations of the final particles in the decay which will provide this information about the nature of this resonance.

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I. INTRODUCTION

A new narrow resonance has been discovered at mass 3872 MeV by the Belle Collaboration [1] through the process,

\[ B^\pm \rightarrow K^\pm X(3872) \rightarrow K^{\pm} \pi^+ \pi^- J/\psi. \]  

(1)

Following Belle [2], we denote this resonance tentatively by \( X(3872) \) in this paper. The invariant mass of \( \pi^+ \pi^- \) extends to the upper end of its kinematical boundary (\( \approx 775 \) MeV), but it is not known at present whether the dipion is in \( s \)-wave, \( p \)-wave, or even \( d \)-wave. Experiment does not exclude the possibility that the dipion in the \( \rho \) mass region is actually in \( s \)-wave. We should keep in mind that the \( s \)-wave \( \pi \pi \) scattering cross section rises rapidly starting just below the \( \rho \) mass. In fact, this partial-wave or spin of the dipion provides the most important clue to the quantum numbers of this resonance. Search of the radiative decays \( X(3872) \rightarrow \gamma \chi_{c1} \) has so far not produced a positive result [2]. As the upper bound set on the radiative decay branching becomes more severe, it will impose a strong constraint on the nature of the resonance.

The most likely candidate for \( X(3872) \) is an excited charmonium state. In fact, the experimental study of the \( \pi^+ \pi^- J/\psi \) final state in \( B \) decay was motivated with search of the excited charmonia [3]. Meanwhile, closeness of the \( DD^* \) threshold to 3872 MeV suggests another explanation, a loosely bound molecular state of \( D\bar{D}^* \) and \( D\bar{D}^* \) [4]. However, both the charmonium and the molecule interpretation may encounter some problems. In this paper we are not rigidly constrained by the potential model predictions on charmonia since uncertainties are large for the excited charmonium states near the open charm thresholds. We will be open minded about dynamics of the molecules. Instead we narrow down the possible quantum numbers of \( X(3872) \) with the information extracted from the Belle experiment and then focus on possibility of determining the quantum numbers purely experimentally by the decay angular distributions and correlations.

II. CURRENT EXPERIMENTAL INFORMATION

The Belle experiment [1] has so far imposed the following constraints on this resonance. The width is narrow (< 2.3 MeV [2]) despite the ample phase space (\( p_{cm} \approx 500 \) MeV) for the decay \( X(3872) \rightarrow D\bar{D} \). For comparison, the width of \( \psi^\prime(3770) \rightarrow D\bar{D} \) is 24 MeV with \( p_{cm} = 242 \) MeV. Provided that the decay into \( D\bar{D} \) is forbidden by selection rules of quantum numbers rather than by some unknown dynamical suppression, we expect that \( X(3872) \) should have an unnatural spin-parity,

\[ J^P = 0^-, 1^+, 2^- \cdots, \]  

(2)

or an unnatural spin-charge-parity,

\[ J^{PC} = 0^{+-}, 1^{-+}, 2^{+-} \cdots. \]  

(3)

To select a right \( J^{PC} \) out of these choices, the dipion quantum numbers provide the most important clue. If \( \pi^+ \pi^- \) forms a scalar dipion of \( J^{PC}_{\pi\pi} = 0^{++} \) or a tensor dipion of \( J^{PC}_{\pi\pi} = 2^{++} \),
charge parity $C$ of $\pi^+\pi^- J/\psi$ is negative and isospin is $I = 0$ or 2. We do not consider $I = 2$ further in view of lack of any candidate. Combining $\pi^+\pi^-$ with $J/\psi$ in relative orbital angular momentum $L$, one can make the unnatural spin-parity and charge-parity states of $0^{+-}$, $1^{+-}$, $2^{+-}$ \ldots with a scalar or a tensor dipion. If $\pi^+\pi^-$ forms a vector dipion of $J^{PC}_{\pi\pi} = 1^{--}$, charge parity of $\pi^+\pi^- J/\psi$ is positive and isospin is $I = 1$. The most relevant unnatural spin-parity state is $1^{++}$ with $L = 0$ in this case. These unnatural quantum number states are listed in Table I. Since only a limited phase space is available for the $\pi^+\pi^- J/\psi$ decay, the Table includes only the cases of $L \leq 2$ for the scalar dipion, and $L \leq 1$ for the vector and tensor dipion.

### III. CHARMONIA

Since $I = 0$ for the charmonia, the following quantum numbers are selected for charmonium candidates:

$$J^{PC} = \begin{cases} 2^{--}, & (L = 0, 2) \\ 1^{+-}, & (L = 1). \end{cases}$$

The charmonia carrying these quantum numbers are $^3D_2(2^{--})$ and $^1P_1(1^{+-})$.

The mass spectrum calculations by potential models [5,6] suggest that the $^3D_2$ state should be much closer to the $^1D_1$ state, which is believed to be $\psi'(3770)$. The mass of the $^1D_2$ state was predicted at 30~\text{MeV} lower than 3872~\text{MeV} [5]. However, the charmonium levels near the $D\overline{D}$ and $D\overline{D}^*$ thresholds are subject to large uncertainties due to the open charm channel coupling and the large relativistic corrections ($\sqrt{<v^2/c^2>} \simeq 0.54$ in one estimate [6]). Therefore we are not too concerned with the discrepancy between experiment and the potential model expectation for the masses in the case of those higher excited charmonia.

The radiative decay into $\gamma\chi_{cJ}$ is allowed for both $2^{--}$ and $1^{+-}$ by quantum numbers. The decay $^3D_2(2^{--}) \rightarrow \gamma\chi_{cJ}$ is the $E1$ transitions and should occur without suppression. Let us compare it with a similar radiative decay of $\psi(2S)$. We know $\Gamma(\psi(2S) \rightarrow \chi_{c1}) \simeq 26$~\text{keV} and $\Gamma(\psi(2S) \rightarrow \pi^+\pi^- J/\psi) \simeq 93$~\text{MeV} from experiment. The strong decay $\psi(2S) \rightarrow \pi^+\pi^- J/\psi$ occurs into a scalar dipion with $s$-wave ($L = 0$) relative to $J/\psi$. Apart from difference in the dipole strength $\langle r \rangle$ between $\psi(2S) \rightarrow \chi_{cJ}$ and $^3D_2 \rightarrow \chi_{cJ}$, the spin factors and phase space corrections combined enhances $\Gamma(^3D_2(3872) \rightarrow \gamma\chi_{c1})$ by a factor of $\approx 5$ relative to $\Gamma(\psi(2S) \rightarrow \gamma\chi_{c1})$. The dipole matrix element is sensitive to the coupling to the $p$-wave $D\overline{D}^*$ channel. Comparison of the strong decay is at least as sensitive or even more uncertain since the decay $X(3872) \rightarrow \pi^+\pi^- J/\psi$ involves $d$ wave either in the dipion or in the relative orbital angular momentum $L$. The rescaling factor of the three-body phase space can be as large as $\approx 10$, which is severely compensated by the $d$-wave suppression factor $|p_{\pi\pi}/E_{\pi\pi}|^4$ for a scalar dipion and $|p_{\pi}/E_{\pi}|^4$ for a tensor dipion. It depends sensitively on the dipion mass distribution. Despite the larger phase space for $X(3872) \rightarrow \pi^+\pi^- J/\psi$ than for $\psi(2S) \rightarrow \pi^+\pi^- J/\psi$, the net result is most likely that $\Gamma(X(3872) \rightarrow \pi^+\pi^- J/\psi)$ is smaller than $\Gamma(\psi(2S) \rightarrow \pi^+\pi^- J/\psi)$. With this order of magnitude consideration we expect $B(X(3872) \rightarrow \gamma\chi_{c1})/B(X(3872) \rightarrow \pi^+\pi^- J/\psi) > 1$. Then the non-observation of the radiative decay $X(3872) \rightarrow \gamma\chi_{c1}$ [2],
\[ B(X(3872) \rightarrow \gamma \chi_{c1})/B(X(3872) \rightarrow \pi^+\pi^-J/\psi) < 0.89 \ (90\%\ C.L.), \tag{5} \]

poses a problem on the \(^3D_2\) assignment. The Belle Collaboration \[2\] compares the upper bound of Eq.\(5\) above with the potential model prediction of \(5\) \[3\]. Considering the large uncertainties involved in the potential model calculations, however, it is prudent to take a "wait and see" attitude.

The other possibility of the radially excited \(^1P_1(1^+\!-\!-\!\!\!-)\) encounters a larger discrepancy with the mass prediction of the potential models. The mass 3872 MeV is roughly 100 MeV lower than the potential model. Nonetheless we do not reject \(^1P_1\) at present for the same reason stated above for \(^1S_0\). The radiative decay \(^2P_1\!\rightarrow\!\gamma\chi_{cJ}\) is an \(M1\) transition that is caused by spin flip. Since the spatial wave functions are orthogonal between \(\chi_{cJ}(1^3P_J)\) and \(^2P_1\) in the nonrelativistic limit, the \(M1\) transition is considerably weaker than the allowed \(E1\) transitions. The strong decay \(^2P_1\!\rightarrow\!\pi^+\pi^-J/\psi\) occurs into a scalar dipion with \(L=1\) instead of \(L=2\) for \(^1S_0\!\rightarrow\!\pi^+\pi^-J/\psi\). Therefore the radiative branching fraction is much smaller, relative to the \(\pi^+\pi^-J/\psi\) decay branching, for \(^2P_1\) than for \(^1S_0\). It can be easily consistent with the current upper bound. If the non-observation of the radiative decays into \(\gamma\chi_{cJ}\) continues to a higher precision, we should consider \(^2P_1\) as a better candidate than \(^1S_0\). We should keep in mind, however, that even the lowest \(^1P_1\) charmonium \(h_c\) has not yet been reported in \(B\) decay despite continuing searches by both the Belle and the BaBar Collaboration.

To wit, if \(X(3872)\) is a charmonium, it should be either the \(^1S_0\!\rightarrow\!\pi^+\pi^-J/\psi\) state or the \(^2P_1(1^+\!-\!-\!\!\!-)\) state.

### IV. MOLECULES

The idea of loosely bound molecule states of two hadrons has been entertained for a long time \[4\], but no meson has so far been positively identified as such a state. The close proximity of the mass 3872 MeV to the \(DD^*\) thresholds \((3971.2 \pm 0.7\ \text{MeV for} \ D^0D^{*0} \ \text{and} \ 3979.3 \pm 0.7\ \text{MeV for} \ D^+D^{*-}\) has prompted reconsideration of this possibility for \(X(3872)\) \[7\]:

\[(1/\sqrt{2})(DD^* \pm DD^*)\tag{6}\]

Hereafter we refer to the charge-parity eigenstates of the molecule states simply by \(DD^*\). Then the following unnatural spin-parity and charge-parity states can be formed as a molecule:

\[J^{PC} = \begin{cases} 
1^{\pm\pm}, 0^{\pm\pm}, 1^{++}, 2^{\pm\pm} & (L_{DD^*} = 0), \\
\ & (L_{DD^*} = 1). 
\end{cases} \tag{7}\]

Since charge parity of \(\pi^+\pi^-J/\psi\) is correlated to isospin by \(C = -(-1)^I\), the molecule should be in \(I = 0\) for \(C = -\) and in \(I = 1\) for \(C = +\). For the relative orbital angular momentum of \(DD^*\), the closeness of 3872 MeV to the \(DD^*\) threshold strongly favors \(L_{DD^*} = 0\) over \(L_{DD^*} = 1\). A qualitative argument of dynamics based on the one-pion exchange favors formation of the \(I = 0\) molecules over the \(I = 1\) ones, in particular, \(0^{++}\) and \(1^{++}\) \[8\]. However, the states of \(0^{++}\) and \(1^{++}\) with \(I = 0\) are not found in Table I since they are..
inconsistent with the decay into $\pi^+\pi^- J/\psi$. Is it possible that the states of $0^{-+}$ and $1^{++}$ with $I = 0$ decay into the $I = 1$ channels of $\pi^+\pi^- J/\psi$ with isospin symmetry breaking. The answer is affirmative, but such a state cannot feed a scalar dipion in $\pi^+\pi^- J/\psi$ because of $C$ invariance. In this case, the dipions observed with mass below the $\rho$ region would be entirely due to the vector dipion produced by isospin breaking.

We do expect a large isospin violation in a molecular bound state of $D\bar{D}^*$ at 3872 MeV, since the mass 3872 MeV almost coincides with the $D^0\bar{D}^{*0}$ threshold but as much as 7 MeV below the $D^{+}\bar{D}^-$ threshold. No matter how large isospin violation is, however, no single resonance can produce both a scalar dipion and a vector dipion in $\pi^+\pi^- J/\psi$ by $C$ invariance: Any strong interaction resonance of zero net flavor must be an eigenstate of charge conjugation even when isospin is broken. Since the states of $I = 0$ and $I = 1$ have opposite $C$, a single resonance cannot be responsible for both the $I = 0$ final state (a scalar dipion) and the $I = 1$ final state (a vector dipion) of $\pi^+\pi^- J/\psi$. In order to feed both the scalar/tensor and the vector dipion, there must be two (almost perfectly degenerate) resonances of opposite charge parities, one with $I = 0$ and odd $C$ and the other with $I = 1$ and even $C$. Forming two such molecules in degeneracy in hidden flavor channels would be a highly unlikely accident. With these observations, we should consider only a molecule of either $J^{PC} = 1^{-+}$ with $I = 0$ or $J^{PC} = 1^{++}$ with $I = 1$ as the molecule candidates among the entries in Eq. (7).

It is likely that the branching fractions of the radiative transitions to $\gamma \chi_{cJ}$ are naturally small for the $D\bar{D}^*$ molecules in general. For $J^{PC} = 1^{++}$, the radiative transitions into $\gamma \chi_{cJ}$ is completely forbidden by $C$ invariance. Even for the molecules of $1^{-+}$, the radiative decay width should be rather small. Hence a small or negligible radiative decay width poses no problem for the interpretation in terms of a molecule.

If $J^{PC}$ of the most attractive channel of $D\bar{D}^*$ coincides with those of a charmonium, a substantial mixing can occur between $D\bar{D}^*$ and the charmonium [9]. This is a real possibility for our most favorite charmonium candidates with $J^{PC} = 2^{--}$ and $1^{++}$. If $X(3872)$ is such a mixed state of a molecule and a charmonium, its radiative decay width is suppressed relative to that of a pure charmonium by the fraction of the molecule mixing.

In fact, this kind of mixing is needed for a very loosely bound molecule state to be produced in $B$ decay with a significant rate [10]. The production amplitude of a bound state is proportional to the “wave function at origin” $|\Psi(0)|$ for the same reason that the produciton amplitudes of $\pi$ and $K$ are proportional to $f_\pi$ and $f_K$, respectively, which are interpreted as the wave functons at the origin of quark-antiquark. For a loosely bound state with binding energy $\Delta E$, $|\Psi(0)|^2$ is a small quantity proportional to $(m\Delta E)^{3/2}$. For the $D\bar{D}^*$ molecule, this is much smaller than $|\Psi(0)|^2$ of the charmonia for which the charm quark mass is not sharply defined. If one accepts this argument, production of a pure $D\bar{D}^*$ molecule is minuscule relative to charmonium produciton in $B$ decay. Physically speaking, $D$ and $\bar{D}^*$ must fly in parallel with practically zero relative velocities in order to form a molecule in $B$ decay. Such a phase space is a tiny, almost negligible fraction of the final-state phase space of $B$ decay. The only way to enhance the molecule production in $B$ decay is through a substantial mixing with a charmonium.\(^1\)

\(^1\)For production of a pure $D\bar{D}^*$ molecule, a more favorable environment is near the $D\bar{D}^*$ threshold.
V. DIPION AND ANGULAR DISTRIBUTIONS

A. Scalar dipion

The most important information in determining the quantum numbers of $X(3872)$ is the spin of the dipion in $\pi^+\pi^- J/\psi$. Let us first consider the case that the dipions of $I = 0$ are entirely in $s$-wave, $J^{PC} = 0^{++}$. If one recalls that the $d$-wave $\pi\pi$ cross section is negligibly small in this region, this is a reasonable possibility. However, it is experiment that should eventually determine whether $J^{\pi^+\pi^-} = 0$ or not. If $\pi^+\pi^-$ is really a scalar dipion, the $\pi^+$ (or $\pi^-$) momentum in dipion rest frame should show no angular correlation with other vectors:

$$\frac{d\Gamma}{d\Omega_{\pi\pi}} = \frac{1}{4\pi} \Gamma_0,$$

(8)

where $\Omega_{\pi\pi}$ is the solid angle of $p_{\pi^+} - p_{\pi^-}$ in the dipion rest frame, as measured with respect to the direction of any momentum, e.g., the $J/\psi$ momentum in the $X$ rest frame. If this test shows that the dipion is indeed a scalar, the dipion angular distribution in the rest frame of $X(3872)$ happens to be independent of dynamics and unique for $J^P = 1^{++}$ and $2^{-}$ since the zero-helicity amplitude of $J/\psi$ is forbidden for them. We elaborate on this below.

The $X(3872)$ state is produced in the zero-helicity state in the $B$ rest frame when it is produced in $B^\pm \rightarrow K^\mp X(3872)$. In the $X$ rest frame, $X(3872)$ is in $|J,0\rangle$ when the quantization axis (call it the $z$-axis) of $J$ is chosen along $p_X$ in the $B$ rest frame. According to the parity constraint of the helicity formalism [11], the helicity amplitudes $\langle s_b, \lambda_b; s_c, \lambda_c | J_a, M \rangle$ for $a \rightarrow b + c$ in the $a$-rest frame obey

$$\langle s_b, \lambda_b; s_c, \lambda_c | J, 0 \rangle = \eta_a \eta_b \eta_c (-1)^{J+s_b-\lambda_b+s_c-\lambda_c} \langle s_b, -\lambda_b; s_c, -\lambda_c | J, 0 \rangle,$$

(9)

where $\eta_{a,b,c}$ are the intrinsic parities of $a(= X), b(= \pi^+\pi^-), c(= J/\psi)$. In the present case, $\eta_b = (-1)^{J_{\pi\pi}}$ and $\eta_c(= \eta_{J/\psi}) = -1$, and $s_c = J_{J/\psi} = 1$. We apply this relation to the case of a scalar dipion, $s_b = \lambda_b = 0$. Denoting the helicity of $J/\psi$ by $h$ instead of $\lambda_c$, we obtain from the parity constraint

$$\langle 0, 0; 1, h | J, 0 \rangle = \eta_a (-1)^{J-h} \langle 0, 0; 1, -h | J, 0 \rangle.$$

(10)

The factor $\eta_a (-1)^J$ is equal to $-1$ for $J^P = 2^-$ and $1^+$. Therefore the $h = 0$ amplitudes of $J/\psi$ must vanish:

$$\langle 0, 0; 1, 0 | J, 0 \rangle = 0, \ (J^{PC} = 2^- \text{ and } 1^+).$$

(11)

Let us suppose that one measures the angular distribution of the dipion momentum $p_{\pi\pi} = p_{\pi^+} + p_{\pi^-}$ in the $X$ rest frame, choosing the $z$-axis along the $X$ momentum in the $B^\pm$ rest frame. Following the standard formulae [11], we obtain with Eq. (11)

$$\frac{d\Gamma}{d\cos \theta_{\pi\pi}} \propto \begin{cases} \cos^2 \theta_{\pi\pi} \sin^2 \theta_{\pi\pi}, & (J^{PC} = 2^-) \\ \sin^2 \theta_{\pi\pi}, & (J^{PC} = 1^+), \end{cases}$$

(12)

where $\theta_{\pi\pi}$ is the polar angle of $p_{\pi\pi}$. (See Fig.1.)

in $e^+e^-$ annihilation where the relative motion of $D$ and $\overline{D}$ is restricted to be small.
FIG. 1. The dipion scattering angle $\theta_{\pi\pi}$, which is defined in the $X$ rest frame.

For the positive parity states other than $J^{PC} = 1^{-+}$, the longitudinal polarization decay amplitude enters the angular distribution:

$$\frac{d\Gamma}{d\cos\theta_{\pi\pi}} \propto \left\{ \begin{array}{ll}
1 & (J^{PC} = 0^{+-}), \\
\cos^2\theta_{\pi\pi} \sin^2\theta_{\pi\pi} + \frac{1}{6}\kappa (3 \cos^2\theta_{\pi\pi} - 1)^2 & (J^{PC} = 2^{+-}),
\end{array} \right.$$  \hspace{1cm} (13)

where $\kappa = |A_0|^2/(|A_1|^2 + |A_{-1}|^2)$ with $|A_1| = |A_{-1}|$. The angular distribution for $J^{PC} = 2^{+-}$ can mimic that of $2^{--}$ when $\kappa$ is small.

B. Tensor dipion

The zero total helicity state $\lambda_{\pi\pi} - h = 0$ can be produced as well as the nonzero helicity states for the tensor dipion. That is, more than one value of $L$ is allowed for a given $J^P$ of $X(3872)$ in the case of the tensor dipion. Consequently the angular distributioins and correlations are dependent on dynamics. However, kinematics of the $X(3872)$ decay allow us to make a special approximation. Since the tensor dipion mass is produced with the invariant mass close to the upper end of the kinematical boundary ($\simeq 775$ MeV), the dipion and $J/\psi$ are most often produced nearly at rest in the $X$ rest frame. Let us select those fat dipions. We then expect that the relative orbital angular momentum between the dipion and $J/\psi$ is in s-wave ($L = 0$) and that the nonrelativistic approximation should be good. Therefore, first of all, the angular distribution of the tensor dipion momentum should be flat in the $X$ rest frame:

$$d\Gamma/d\cos\theta_{\pi\pi} \simeq (1/2)\Gamma_0,$$  \hspace{1cm} (14)

where the sign of $\simeq$ means “in the nonrelativistic approximation”. This may not be an easy test since the fat dipions do not move much in the $X$ rest frame so that determination of the $p_{\pi\pi}$ direction is subject to large errors.

Eq. (14) only tests $L = 0$, not directly the quantum numbers of the dipion. We shall be able to test the tensor nature of dipion by measuring the angular correlation of $p_\pi = p_{\pi^+} - p_{\pi^-}$ of the dipion rest frame with the $X$ momentum in the $B^{\pm}$ rest frame. Since angular momenta conservation holds among spins alone for $L = 0$, the spin components of $\pi^+\pi^-$ and $J/\psi$ make up $|J, M\rangle = |2, 0\rangle$ of $X(3872)$ as

$$|2, 0\rangle_X = \frac{1}{\sqrt{2}}(|2, +1\rangle_{\pi\pi}|1, -1\rangle_{J/\psi} - |2, -1\rangle_{\pi\pi}|1, +1\rangle_{J/\psi}),$$  \hspace{1cm} (15)
when the spin quantization axis is chosen along the $X$ momentum in the $B^{±}$ rest frame. The $s_z = 0$ states are missing in the right-hand of Eq. (15), which is characteristic of the Clebsch-Gordan composition involving $|j, m⟩ = |1, 0⟩$. This leads to the angular distribution of $p_{π}$ as

$$dΓ/d cosθ_{π} ≃ \frac{15}{4}Γ_0 cos^2θ_{π} sin^2θ_{π}, \quad (J^{PC} = 2^{−−})$$ (16)

where $θ_{π}$ is the polar angle of $p_{π}$ in the dipion rest frame as measured from the $X$ momentum in the $B$ rest frame. Eq. (16) can be obtained by an elementary calculation of the nonrelativistic decay amplitude made of three polarizations, $ε^{jkl} ε^{ij}_X ε^{ik}_{π\pi} ε^{l}_{J/ψ}$. The test will be done most accurately if one selects the dipions of $700 \text{ MeV} < m_{ππ} < 775 \text{ MeV}$. The Belle data show many events in this mass region relative to the region below it [1,2].

**C. Vector dipion**

The spin parity of interest in the case of the vector dipion is $J^{PC} = 1^{++}$ for the molecule. The vector dipion most often forms a $ρ$ meson. Since $ρ$ and $J/ψ$ are nearly at rest in the $X$ rest frame, the relative orbital angular momentum between them should be $L = 0$ just as in the case of the tensor dipion. In this circumstance the same simplification occurs. The angular distribution of $p_{ππ}$ tests $L = 0$ by Eq. (14). The vector nature of the dipion can be tested by the angular correlation,

$$dΓ/d cosθ_{π} ≃ \frac{3}{4}Γ_0 sin^2θ_{π}, \quad (J^{PC} = 1^{++})$$ (17)

where $θ_{π}$ is defined in the same way as in Eq. (16).

**VI. SUMMARY**

The most likely candidate for the narrow resonance discovered by the Belle Collaboration is the $1^3D_2$ charmonium state. Depending on the outcome of the radiative decay measurement, however, the $2^1P_1$ charmonium may become a better alternative. Among the molecules, the $J^{PC} = 1^{+±}$ molecule states of $D\bar{D}^*$ and $D\bar{D}^{*}$ are acceptable as far as quantum numbers are concerned and furthermore would easily satisfy the constraints imposed by the absence of radiative decay mode. On the other hand, the final states of $π^+ π^- J/ψ$ with a scalar dipion and a vector dipion cannot be produced by a single resonance because of charge conjugation invariance no matter how badly isospin symmetry is violated.

The crucial information leading to determination of the quantum numbers of this resonance should come from spin of the dipion. The various angular distributions and correlations that have been presented here will help us in identifying the nature of this resonant state conclusively.

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TABLE I. Unnatural spin-parity and charge-parity state. $L$ stands for the relative orbital angular momentum of $\pi^+\pi^-$ and $J/\psi$.

| $J^{PC}_{\pi\pi}$ | I   | L   | $J^{PC}$          |
|------------------|-----|-----|------------------|
| 0$^{++}$         | 0, 2| 1   | 0$^{+-}, 1^{+-}, 2^{+-}$ |
|                  |     | 2   | 2$^{-}$          |
| 2$^{++}$         | 0, 2| 0   | 2$^{-}$          |
|                  |     | 1   | 0$^{+-}, 1^{+-}, 2^{+-}, 3^{+-}, 4^{+-}$ |
| 1$^{-+}$         | 1   | 0   | 1$^{++}$         |
|                  |     | 1   | 0$^{-+}, 1^{-+}, 2^{-+}, 3^{-+}$ |

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