Resonant transmission of normal electrons through Andreev states in ferromagnets

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Giant oscillations of the conductance of a superconductor - ferromagnet - superconductor Andreev interferometer are predicted. The effect is due to the resonant transmission of normal electrons through Andreev levels when the voltage $V$ applied to the ferromagnet is close to $2h_0/e$ ($h_0$ is the spin-dependant part of the electron energy). The effect of bias voltage and phase difference between the superconductors on the current and the differential conductance is presented. These effects allow a direct spectroscopy of Andreev levels in the ferromagnet.

Recently a high sensitivity of the conductance of mesoscopic systems to the superconductor phase difference has been observed and theoretically considered in superconductor - normal conductor - superconductor heterostructures (S/N/S heterostructures) (see, e.g., the review paper by Lambert and Raimondi [1]). This effect arises because of a quantum interference of quasiparticles due to Andreev scattering at two (or more) N-S interfaces. This is caused by the fact that the phase of the superconducting condensate is imposed on the quasiparticle wave function in the normal metal. One of the manifestations of the quantum interference is giant oscillations of the conductance of the normal metal as a function of the phase difference between the superconductor predicted in [2].

A single electron in a normal metal with energy below the superconductor energy gap, $\Delta$, can not penetrate into the superconductor. However, under Andreev reflection at an N-S interface two electrons with nearly opposite momenta and spins leave the normal metal to create a Cooper pair in the superconductor; hence the incident electron is transformed into a hole with the opposite direction of the spin. The spin flip does not affect the interference pattern of the non-magnetic normal metal because all energy levels are doubly degenerate with respect to spin. In ferromagnets, however, this degeneracy is lifted due to the interaction of the electron spin with the ferromagnet’s spontaneous moment ($h_0$, usually $h_0 > 0$). The Josephson current in a superconductor-ferromagnet-superconductor (S/F/S) structure was investigated in Refs. [3,4]; transport properties of F/S junctions were investigated in [4,5]; experiments on the boundary resistance of an S/F/S system was reported in [6], and phase coherent effects in the conductance of a ferromagnet contacting a superconductor were observed in [7].

In this Letter we predict giant oscillations in the conductance of an S/F/S heterostructure, of the Andreev interferometer type, in which the ferromagnet part is separated from the reservoirs of normal electrons with potential barriers ("beam splitters") of low transparency, $t_r \ll 1$, (see the insert in Fig. 2).

In the case of Andreev reflections, the paramagnetic effect essentially modifies the interference pattern in the ferromagnetic region. The momentum of an electron with spin up/down, $p_{\uparrow}^e / p_{\downarrow}^e$, and the momentum of the reflected hole with the spin down/up, $p_{\downarrow}^h / p_{\uparrow}^h$ are (see [8]):

$$p_{\uparrow}^{(e)} = \sqrt{p_F^2 + 2m(E \pm h_0)}; \quad p_{\downarrow}^{(h)} = \sqrt{p_F^2 - 2m(E \pm h_0)}$$

where $E$ is the energy of the incident electron measured from the Fermi level $\epsilon_F$, $p_F$ is the Fermi momentum, and $m$ is the electron mass.

From Eq. (8) it follows that in contrast to the non-magnetic case, near the Fermi level ($E \approx 0$) the electron and the hole momenta in the ferromagnet are different, and for large enough $h_0$ (usually $h_0 > 0$) the interference effects are absent due
to the destructive interference. This fact demonstrates the conflict between superconductivity and magnetic ordering in S/F/S structures.

FIG. 2. Schematic representation of resonant transmission of an incident electron which tunnels through potential barrier I, moves along a 1D disordered chain of scatterers (dots) where Andreev scattering (back-scattering) + normal scattering (forward scattering) takes place, and is reflected back through the first barrier I as an electron and transmitted through the second barrier II as a hole. Semi-classical electron and hole paths are shown with full and dashed lines, respectively. Trajectory sections connecting successive scattering events at different N/S-interfaces have lengths $L_i$; $i = 0, \pm 1, \ldots$. The insertion schematically shows the geometry of the system under consideration and a classical trajectory contributing to the resonant part of the conductance. Thick lines indicate potential barriers.

However, interference effects in the ferromagnet can exist albeit at some finite voltage $V$ applied between the reservoirs. If the energy $|E| \approx h_0 < |\Delta|$ the change of the quasi-particle momentum under Andreev reflection is small (see Eq. [1]), while the velocity changes its sign, and an essential cancellation of the phase gain along trajectories including electron-hole transformations at the superconducting boundaries takes place. At $E = h_0$ any such a classical trajectory is closed (in this case, under Andreev reflection the electron and hole momenta are equal and hence the reflected quasi-particle is sent exactly back along the classical path of the incident quasiparticle), and this cancellation is complete at $\phi = \pi(2l + 1)$, $l = 0, \pm 1, \ldots$ irrespective of the geometry and the length of the trajectory [1]. From here it follows that at $|E - h_0| \ll E_{Th}$ and $\phi$ close to odd numbers of $\pi$, such paths take part in the constructive interference resulting in resonant transmission through Andreev levels. In our calculations of the probability amplitude of the electron-hole reflection back to the reservoir of the electron injection [13] we use the approach developed by us in Ref. [13] assuming the motion of quasi-particles inside the ferromagnet to be semi-classical (this assumption is valid if the de-Broglie wave length $\lambda_F$ of electrons is the shortest length in the ferromagnet). Within this approach one can find the wave function of the scattered quasiparticles by mapping the incident wave along classical paths determining the phase of rapid oscillations $\Theta = S/\hbar$ as a classical action $S = \int pdl$ along the path. A typical classical trajectory of this kind for an incident electron which undergoes a number of Andreev and normal reflections at F-S boundaries is shown in the insert of Fig. 2 (solid and dashed lines are for electronic and hole paths, respectively). The electron-hole transmission along this trajectory is similar to the resonant transmission of an electron through a two-barriers system (schematically shown in Fig. 2) in which the incident electron tunnels through a potential barrier I (solid line 1), moves along a one-dimensional chain of scatterers (black dots in Fig. 2 representing Andreev and normal reflections at F/S interfaces), and then is reflected back as an electron through potential barrier I and transmitted through potential barrier II as a hole. $L_i$ ($i = 0, \pm 1, \ldots$) is the length of the quasi-particle path between two successive scatterings at F/S interfaces which is the distance between the neighboring scatterers for the 1D chain of Fig. 2. The paths $L_i$ are uncorrelated and hence the chain of Fig. 2 is a 1D system with disordered distances between the scatterers. In the same way as in Ref. [19], it can be shown that due to the above-mentioned phase compensation the motion of the quasi-particle in this chain is reduced to the conventional quantum motion of an electron with energy $E - h_0$ (but having the Fermi velocity $v_F$) in the 1D disordered chain of centers of back-scatterings where the back-scattering amplitude is the probability amplitude of the Andreev reflection $r_A^{1,2}$ and the amplitude to pass to the next section of the chain is the probability amplitude of the normal reflection $r_N^{1,2}$ at F/S interfaces 1 and 2 (the probability amplitudes $r_A$ and $r_N$ are given in [22]). In this situation, for $E \neq h_0$ the phase gains between successive back-scatterings are random, and quasi-particle localization takes place. For $|r_N^{1,2}| \ll 1$ and $t_r \ll 1$ a sharp resonant transmission between points I and II through discrete energy levels (of the Andreev type) which correspond to the quasiparticle states localized around the section of the electron injection, occurs. Matching amplitudes $a_i^{e,h}$ at the centers of scattering (dots in Fig. 2) and taking into account the phase gains along the paths between them show the probability of electron-hole resonant transmission through an energy level $E_\alpha$ [23] to be of the Breit-Wigner form, $T(E, \alpha) \propto t_r^2/((E - E_\alpha)^2/h^2 + t_r^2 \times \text{const.})$, where $\tau_0$ is the time of motion in the section of injection.

The total electron-hole transmission probability $T_{eh}(E)$ is a sum of $T(E, \alpha)$ with respect to the starting points of the semi-classical trajectories inside the reservoir separated by the distance of the order of $\lambda_F$. These trajectories meet different ”random” sets of impurities, and hence their path lengths and the times of quasiparticle propagation along them are randomly distributed. Therefore, the summation over the starting points is equivalent to averaging the transmission probability with respect to realizations of times $\tau_i$ ($\tau_i$ is the
time of propagation along section $i$ (see [19]). It seems reasonable to assume the propagation times $\tau_i$ to be uncorrelated. Under this assumption, as is shown in [13], the total transmission probability $T_{\text{eh}}(E)$ is proportional to the density of localized states in the 1D disordered chain of Fig. 2, and using the Lambert formula [24] one gets the transport current for temperature $T = 0$ as

$$I = (t_r N_r e/\hbar) E T_{\text{eh}} \sum_{\tau_+} \int_{-eV/2}^{eV/2} <\nu_{\text{rand}}^{(1\tau)}(E)> dE \quad (2)$$

(here and below we assume $t_r \ll (|r_N^{(1)}| + |r_N^{(2)}|)/2 \ll 1$).

In Eq. (2) $N_r = S/\lambda_F^2$, $S$ is the F/S contact area, $\lambda_F$ is the electron wave length, $<\nu_{\text{rand}}^{(1\tau)}(E)>$ is the density of states for a quasi-particle with the spin up ($\uparrow$) or down ($\downarrow$) averaged with respect to the configurations of $\tau_r$.

Now we assume the distribution $P(\tau)$ for the propagation times to be of the Lorentzian form

$$P(\tau) = \gamma/\pi[(\tau - \bar{\tau})^2 + \gamma^2]$$

($\bar{\tau} = L_S^2/D$ and $L_S$ is the distance between the superconductors) that, for the configuration of Fig. 2, permits to find the density of state exactly. Using Eq. (2) one finds the resonant phase-sensitive part of the differential conductance of the system $G = dI/dV$ to be

$$G = \frac{-\pi e^2}{h} N_r t_r |\bar{V}| \times \left\{ \frac{\sqrt{4\epsilon^4 + \epsilon_1^4}}{\sqrt{4\epsilon^4 + \epsilon_1^4}} \right\}^{1/2} \quad (3)$$

where $\bar{V} = (eV/2 - h_0)/E T_{\text{eh}}$, $\epsilon_{a,b} = \delta \phi^2 + |r_N^{(1)}| \pm |r_N^{(2)}|^2/2$ is the dimensionless applied voltage measured from $h_0$, $\delta \phi$ is the minimal value of $|\phi - \pi(2l + 1)|$, $l = 0, \pm 1, \pm 2, \ldots$. While writing Eq. (3) we took $\gamma = \bar{\tau}$, and assumed $|h_0 - eV/2| \ll E T_{\text{eh}}$. Eqs. (2) and (3) describe the the current and differential conductance at $T = 0$ for both the magnetic $h_0 \neq 0$ and non-magnetic $h_0 = 0$ cases.

Numerical results for the conductance and the current based on Eq. (3) are shown in Figs. 3 and 4. They demonstrate a high sensitivity of the conductance and the non-linear current-voltage characteristics to both the superconductor phase difference $\phi$ and the applied voltage $V$.

At odd multiples of $\pi$ and $|r_N^{(1)}| = |r_N^{(2)}|$ there is a symmetry between the clock- and counter-clockwise motions of electron-hole pairs in the ferromagnet, and the energy level $E = h_0$ is degenerate (see above) [23]. Under this condition the maximum of the resonant transmission through Andreev levels is at $eV/2 = h_0$, and a resonant peak in the conductance is observed (Fig. 3a). Even a small deviation of $\phi$ from an odd multiple of $\pi$, will repel Andreev levels from $h_0$, and the conductance peak splits in two peaks.

In a more realistic experimental case when $|r_N^{(1)}| \neq |r_N^{(2)}|$ the symmetry is broken, and Andreev levels are repelled from the level $h_0$ (see [27]), the shift being proportional to $\delta r_N = |r_N^{(1)}| - |r_N^{(2)}|$. As a result the resonant peaks of the conductance are split as shown in Fig. 3b.

![FIG. 3. Normalized differential conductance $G = dI/dV$ of the S/F/S structure for $|r_N^{(1)}| = |r_N^{(2)}| = 0.1$ and $|r_N^{(1)}| = 0.05$, $|r_N^{(2)}| = 0.1$ shown in Fig. 3a and Fig3b, respectively, at phase differences $\phi = \pi$ (full line), $\phi = 1.1\pi$ (dotted line) and $\phi = 1.2\pi$ (dashed line); $G_0 = (\sqrt{2e^2}/h) N_r t_r$.](image)

Current-voltage characteristics for different $\phi$ are presented in Fig. 4.

![FIG. 4. Normalized current-voltage characteristics for phase differences $\phi = \pi$ (full line), $\phi = 1.1\pi$ (dotted line) and $\phi = 1.2\pi$ (dashed line) shown for $|r_N^{(1)}| = 0.05$, $|r_N^{(2)}| = 0.1$ and $h_0 = E T_{\text{eh}}$; $I_0 = (\sqrt{2e^2}/2\pi h) N_r t_r (2h_0/e)$.](image)
At low voltages, far from $2h_0/e$, we have a resonant tunneling of quasi-particles through separate Andreev levels, and the current level is low. When $eV/2 \approx h_0$ and $\phi = \pi$ Andreev levels concentrate near $h_0$, and we have simultaneous resonant transport through the whole number of $N_\perp$ states resulting in a jump of the current $\Delta I = |r^{(1)}_N| + |r^{(2)}_N|G_{\text{max}}h_0/2e$ ($G_{\text{max}}$ is the maximal value of the conductance). When $\phi$ deviates from $\pi$ the number of Andreev levels concentrated near $h_0$ is decreasing that results in a decrease of the sensitivity of the current to the voltage.

We note here that the curve for the differential conductance $G$ as a function of $eV$ repeats the density of Andreev states in the diffusive ferromagnet permitting a direct spectroscopy of the Andreev levels by conductance and current measurements.

In conclusion we have demonstrated a pronounced possibility for spectroscopy of Andreev states in ferromagnets at energies even greater than the Thouless energy. The paramagnetic effect determines sharp peaks for the conductance as a function of the superconductor phase difference $\phi$ and the applied voltage $V$ near $\phi = \pi(2l + 1)$, $l = 0, \pm 1, \pm 2, \ldots$ and $V = 2h_0/e$, respectively. This phenomenon is a convenient tool for the Andreev level spectroscopy, and enables applications, e.g. as a double-gate ferromagnet transistor and an AND logic element as described in [23].

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