Astronomical bounds on future big freeze singularity

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Recently it was found that dark energy in the form of phantom generalized Chaplygin gas may lead to a new form of the cosmic doomsday, the big freeze singularity. Like the big rip singularity, the big freeze singularity would also take place at a finite future cosmic time, but unlike the big rip singularity it happens for a finite scale factor. Our goal is to test if a universe filled with phantom generalized Chaplygin gas can conform to the data of astronomical observations. We shall see that if the universe is only filled with generalized phantom Chaplygin gas with equation of state \( p = -c^2 s^2 / \rho^\alpha \) with \( \alpha < -1 \), then such a model cannot be matched to the data of astronomical observations. To construct matched models one actually need to add dark matter. This procedure results in cosmological scenarios which do not contradict the data of astronomical observations and allows one to estimate how long we are now from the future big freeze doomsday.

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I. INTRODUCTION

Till recently there were only two types of singularities in cosmology: The big bang and the big crunch. The latter singularity appeared in the universe if it would go through a contracting stage which is possible both in closed universes (filled with baryon matter and radiation) and in open universes filled with scalar field with negative potential [1].

This picture changed dramatically with the coming of the dark energy revolution. Now we know about many new scenarios for the end of the universe among which there is the big rip singularity [2] and the sudden future singularity [3]. Another way to obtain a cosmological doomsday was suggested in [4]. According to this scenario our vacuum should be rather unstable and should decay within 20 Gyr (which is possible if the gravitino is superheavy).

A new type of future singularity, the so-called big freeze singularity, has been quite recently considered [5]. It is a type III singularity in the notation of Ref. [6]. Such a type of doomsday is possible if the universe is filled with phantom generalized Chaplygin gas (which was originally introduced in [7], [8], [9]). Like the big rip singularity, this singularity would also take place at a finite future cosmic time, but unlike the big rip singularity, it happened for a finite scale factor. This feature results in very interesting conclusions some of which are discussed in this paper.

The aim of this article is to find what bounds on the future lifetime of the universe filled with phantom generalized Chaplygin gas can be placed by current cosmological observations. As we shall see in Sec.2 and Sec.3, if the universe is only filled with generalized phantom Chaplygin gas with equation of state \( p = -c^2 s^2 / \rho^\alpha \) with \( \alpha < -1 \), then such a model cannot match the data of astronomical observations. To construct models compatible with observations one need to add dark matter which allows one to obtain a class of good models where the total lifetime of the universe can be calculated. It is interesting to note that whereas the present age of the universe \( t_0 \) is weakly depending on the parameter of the equation of state \( w \) and the Chaplygin parameter \( \alpha \), the total lifetime of the universe \( t_f \) and the time to be elapsed from now to the final big freeze singularity \( (t_f - t_0) \) are very sensitive to these parameters. For example, if \( \alpha = -2 \) then for the flat universe with \( w = -1.01 \) we have \( t_f - t_0 = 10.3 \) Gyr, \( t_0 = 14.4 \) Gyr, whereas for the case with \( w = -1.05 \) we have \( t_f - t_0 = 6.4 \) Gyr and \( t_0 = 13.8 \) Gyr.

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II. THE MODEL

Let us consider a universe filled with dark matter \( p = 0 \) and a generalized Chaplygin gas with equation of state \( p = -\frac{c^2}{\alpha} \beta \rho^{3/\epsilon} \). The Friedmann equations would read

\[
\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \left( \rho + \frac{c^2}{a^3} \right) - \frac{kc^2}{a^2}, \hspace{1cm} (1)
\]

\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} + \frac{c^2}{a^3} \right), \hspace{1cm} (2)
\]

where we have denoted \( \rho = \beta^{-6/\epsilon}(a_f - a)^{-3/\epsilon} \) and \( p = -\frac{c^2}{\alpha} \beta \rho^{3/\epsilon} \) for, respectively, the energy density and pressure of the generalized Chaplygin gas.

Observations demand that

\[
\frac{\ddot{a}a}{a} = k_0 H_0^2, \hspace{1cm} (3)
\]

where \( k_0 \approx 0.7 \). Using (1,2,3) we get

\[
p_0 = -\frac{c^2}{8\pi G} \left( (2k_0 + 1)H_0^2 + \frac{kc^2}{a_0^3} \right). \hspace{1cm} (4)
\]

However, \( p_0 = -\frac{c^2}{\alpha} \beta^{-6/\epsilon}(a_f - a_0)^{-1-3/\epsilon} \), therefore we can find the unknown constant \( \beta \) given by

\[
\beta^{-6/\epsilon} = \frac{a_0^3(N_0 - 1)^{1+3/\epsilon}}{8\pi GN_0} \left( (2k_0 + 1)H_0^2 + \frac{kc^2}{a_0^3} \right), \hspace{1cm} (5)
\]

where \( N_0 = \frac{a_f}{a_0}. \) The current parameter of the equation of state is \( w_0 = \frac{p_0}{\rho_0} \), so that

\[
w_0 = \frac{\rho_0}{c^2 \rho_0} = -\beta^2 N_0 a_0^3 \frac{\epsilon}{\beta} = -\frac{N_0}{N_0 - 1} \hspace{1cm} (6)
\]

Finally, we have for the current vacuum energy density

\[
\rho_0 = \frac{N_0 - 1}{8\pi GN_0} \left( \frac{kc^2}{a_0^3} + (2k_0 + 1)H_0^2 \right) \hspace{1cm} (7)
\]

Using (1) and (7) we can get \( c_d^2 \):

\[
c_d^2 = \frac{H_0^2 a_0^3}{4\pi Gw_0} \left( \frac{3w_0 + 1}{2} \left( \frac{kc^2}{H_0^2 a_0^3} + 1 \right) + k_0 \right) \hspace{1cm} (8)
\]

and

\[
\Omega_0 \equiv \frac{\rho_0}{\rho_c} = \frac{N_0 - 1}{3N_0} \left( \frac{kc^2}{a_0^3 H_0^2} + 2k_0 + 1 \right). \hspace{1cm} (9)
\]

Using (8) we can derive from (9) the second formula for \( \Omega_0 \)

\[
\Omega_0 = -\frac{1}{3w_0} \left( \frac{kc^2}{a_0^3 H_0^2} + 2k_0 + 1 \right) \hspace{1cm} (10)
\]

We introduce now the parameter \( \Omega_{d,0} = \frac{\rho_d}{\rho_c} \). It is obvious that

\[
\Omega_{d,0} = 1 + k_0 \frac{c^2}{H_0 a_0^3} - \Omega_0.
\]
Because \( w_0 = -\frac{N}{N - 1} < 0 \) the condition \( c_d^2 > 0 \) will hold for

\[
    w_0 < -\frac{kc^2 + H_0^2 a_0^2 (2k_0 + 1)}{3(kc^2 + H_0^2 a_0^2)}
\]

Let us put \( k_0 = 0.7 \) and consider the value of \( \Omega_0 \) derived from (10) for various values of \( w_0 \) and \( \frac{k c^2}{a_0 H_0^2} \). The results attained by such a calculation are given in the following tables.

1. \( k = 0 \)

| \( w_0 \) | \( \Omega_0 \) |
|-------|-------|
| -1.01 | 0.792 |
| -1.02 | 0.784 |
| -1.03 | 0.777 |
| -1.04 | 0.769 |
| -1.05 | 0.762 |
| -1.1  | 0.727 |

2. \( k = +1 \)

| \( w_0 \) | \( \Omega_0 \) for \( H_0^2 a_0^2 = c^2 \) | \( \Omega_0 \) for \( H_0^2 a_0^2 = 5c^2 \) | \( \Omega_0 \) for \( H_0^2 a_0^2 = 10c^2 \) |
|-------|----------------|----------------|----------------|
| -1.01 | 1.122          | 0.858          | 0.825          |
| -1.02 | 1.111          | 0.850          | 0.817          |
| -1.03 | 1.100          | 0.841          | 0.809          |
| -1.04 | 1.090          | 0.833          | 0.801          |
| -1.05 | 1.079          | 0.825          | 0.794          |
| -1.1  | 1.030          | 0.788          | 0.758          |

3. \( k = -1 \)

| \( w_0 \) | \( \Omega_0 \) for \( H_0^2 a_0^2 = c^2 \) | \( \Omega_0 \) for \( H_0^2 a_0^2 = 5c^2 \) | \( \Omega_0 \) for \( H_0^2 a_0^2 = 10c^2 \) |
|-------|----------------|----------------|----------------|
| -1.01 | 0.462          | 0.726          | 0.759          |
| -1.02 | 0.458          | 0.719          | 0.752          |
| -1.03 | 0.453          | 0.712          | 0.744          |
| -1.04 | 0.449          | 0.705          | 0.737          |
| -1.05 | 0.444          | 0.698          | 0.730          |
| -1.1  | 0.424          | 0.667          | 0.697          |

**III. INTEGRATION PROCEDURE**

We have

\[
    \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \left( \frac{\beta^{-6/\epsilon}}{(\alpha^\epsilon - \alpha^\epsilon)^{3/\epsilon}} + c_d^2 \frac{a}{a^3} - \frac{3kc^2}{8\pi Ga^2} \right),
\]

Let us consider the change of variables \( a^\epsilon = \xi f \sin^2 \eta \), where \( \xi_f = a^\epsilon_f, \eta = \eta(t) \). Then, because

\[
    \beta^{-6/\epsilon} = \frac{a^3 H_0^2}{8\pi G} |w_0| + 1|3/\epsilon| \left( 2k_0 + 1 + \frac{kc^2}{H_0^2 a_0^2} \right),
\]

\[
    c_d^2 = \frac{a^3 H_0^2}{8\pi G} \left( \frac{(3w_0 + 1)}{w_0} \left( 1 + \frac{kc^2}{H_0^2 a_0^2} \right) + \frac{2k_0}{w_0} \right),
\]

\[
    a_0 = \xi_f^{1/\epsilon} \sin^{2/\epsilon} \eta_0,
\]
\[
\sin^2 \eta_0 = 1/N_0 = \frac{w_0 + 1}{w_0},
\]

Eq. (11) after simplifications results in

\[
\frac{2\sqrt{3}}{c H_0} \left( \frac{w_0}{w_0 + 1} \right)^{3/\epsilon} \int \frac{(\cos \eta)^{1+3/\epsilon} (\sin \eta)^{-1+3/\epsilon} \, d\eta}{\sqrt{\mu(w_0, a_0, \epsilon) \sin^6 \eta + \nu(w_0, a_0, \epsilon) \cos^6 \eta + \gamma(w_0, a_0, \epsilon) \sin^2 \eta \cos^6 \eta}} = \int dt,
\]

where we have introduced the following three parametric coefficients

\[
\mu(w_0, a_0, \epsilon) = \left| \frac{w_0}{w_0 + 1} \right|^{3/\epsilon} \left( 2k_0 + 1 + \frac{kc^2}{H_0^2 a_0^2} \right),
\]

\[
\nu(w_0, a_0, \epsilon) = \frac{3w_0 + 1}{w_0} \left( 1 + \frac{kc^2}{H_0^2 a_0^2} \right) + \frac{2k_0}{w_0},
\]

\[
\gamma(w_0, a_0, \epsilon) = -\frac{3kc^2}{H_0^2 a_0^2} \left( \frac{w_0}{w_0 + 1} \right)^{1/\epsilon}.
\]

Thus, integration of Eq. (12) from \( \eta = 0 \) to \( \eta = \eta_0 = \arcsin \left( \frac{1 + 1}{w_0} \right) \) gives the age of the universe. The lifetime of the universe can be found by integrating Eq. (12) from \( \eta = 0 \) to \( \eta = \pi/2 \). We shall in what follows consider various possible cases.

1. Flat spacetime. The coefficients \( \mu \) and \( \nu \) in this case depend only on \( w_0 \) and \( \epsilon \). Coefficient \( \gamma = 0 \). The numerical calculations of the age of the universe and the difference between time of final singularity \( t_f \) and \( t_0 \) for various values of \( w_0 \) and \( \epsilon \) are given in the following table. The used Hubble parameter for that calculation is \( H_0 = 72 \text{ km/s/Mpc} \).

| \( w_0 = -1.01 \) | \( w_0 = -1.05 \) | \( w_0 = -1.1 \) |
|---|---|---|
| \( \epsilon \) | \( t_f - t_0 \), Gyr | \( t_f - t_0 \), Gyr | \( t_f - t_0 \), Gyr | \( t_f - t_0 \), Gyr | \( t_f - t_0 \), Gyr |
| 0.1 | 252.116 | 14.443 | 103.026 | 13.842 | 61.938 | 13.245 |
| 0.2 | 164.752 | 14.442 | 76.156 | 13.838 | 48.557 | 13.838 |
| 0.5 | 86.958 | 14.440 | 45.763 | 13.827 | 45.763 | 13.827 |
| 1 | 50.798 | 14.436 | 28.691 | 13.813 | 28.691 | 13.813 |
| 3 | 19.726 | 14.429 | 11.916 | 13.771 | 11.916 | 13.771 |
| 6 | 10.286 | 14.423 | 6.353 | 13.756 | 6.353 | 13.756 |
| 9 | 6.929 | 14.420 | 4.319 | 13.745 | 4.319 | 13.745 |

2. Spacetime with positive spatial curvature. The coefficients \( \mu, \nu, \gamma \) depend on \( w_0, \epsilon \) and \( a_0 \). We use the values 1, 5 and 10 for the relation \( a_0^2 H_0^2 / c^2 \). We obtain in this case

a) \( a_0^2 H_0^2 / c^2 = 1 \).

| \( w_0 = -1.01 \) | \( w_0 = -1.05 \) | \( w_0 = -1.1 \) |
|---|---|---|
| \( \epsilon \) | \( t_f - t_0 \), Gyr | \( t_f - t_0 \), Gyr | \( t_f - t_0 \), Gyr | \( t_f - t_0 \), Gyr | \( t_f - t_0 \), Gyr |
| 0.5 | 74.752 | 11.335 | 39.815 | 13.842 | 61.938 | 13.245 |
| 1 | 44.344 | 11.333 | 25.400 | 13.838 | 48.557 | 13.838 |
| 3 | 18.017 | 11.327 | 11.916 | 13.771 | 45.763 | 13.827 |
| 6 | 9.734 | 11.323 | 6.353 | 13.756 | 28.691 | 13.813 |
| 9 | 6.671 | 11.300 | 4.319 | 13.745 | 11.916 | 13.771 |

b) \( a_0^2 H_0^2 / c^2 = 5 \).
Putting and with growing the values 5 and 10 for relation

a) $a_0^2 H_0^2/c^2 = 5$.  

| $w_0$ = -1.01 | $w_0$ = -1.05 | $w_0$ = -1.1 |
|----------------|----------------|----------------|
| $\epsilon$ | $t_f - t_0$, Gyr | $t_f - t_0$, Gyr | $t_f - t_0$, Gyr |
| 0.5 | 83.422 | 12.335 | 43.897 | 11.932 | 29.912 | 11.518 |
| 1 | 48.680 | 12.333 | 27.454 | 11.923 | 19.471 | 11.501 |
| 3 | 18.825 | 12.327 | 11.341 | 11.899 | 8.359 | 11.459 |
| 6 | 9.763 | 12.324 | 6.013 | 11.883 | 4.491 | 11.429 |
| 9 | 6.550 | 12.288 | 4.073 | 11.873 | 3.060 | 11.413 |

b) $a_0^2 H_0^2/c^2 = 10$.  

| $w_0$ = -1.01 | $w_0$ = -1.05 | $w_0$ = -1.1 |
|----------------|----------------|----------------|
| $\epsilon$ | $t_f - t_0$, Gyr | $t_f - t_0$, Gyr | $t_f - t_0$, Gyr |
| 0.5 | 85.997 | 13.177 | 44.749 | 12.700 | 30.527 | 12.216 |
| 1 | 49.668 | 13.174 | 28.022 | 12.689 | 19.881 | 12.197 |
| 3 | 19.227 | 13.168 | 11.594 | 12.622 | 8.550 | 12.148 |
| 6 | 9.989 | 13.164 | 6.159 | 12.643 | 4.603 | 12.114 |
| 9 | 6.711 | 13.125 | 4.177 | 12.620 | 3.139 | 12.086 |

c) $a_0^2 H_0^2/c^2 = 10$.  

| $w_0$ = -1.01 | $w_0$ = -1.05 | $w_0$ = -1.1 |
|----------------|----------------|----------------|
| $\epsilon$ | $t_f - t_0$, Gyr | $t_f - t_0$, Gyr | $t_f - t_0$, Gyr |
| 0.5 | 90.991 | 18.500 | 47.949 | 17.462 | 32.756 | 16.261 |
| 1 | 53.224 | 18.842 | 30.118 | 17.435 | 21.405 | 16.219 |
| 3 | 20.775 | 18.826 | 12.592 | 17.373 | 9.305 | 16.119 |
| 6 | 10.907 | 18.817 | 6.762 | 17.334 | 5.060 | 16.054 |
| 9 | 7.385 | 18.800 | 4.617 | 17.241 | 3.471 | 15.943 |

It can be checked that the above numerical estimates provide realistic results for an age of the universe which weekly depends on $\epsilon$ and $w_0$. The lifetime of universe, which on the contrary strongly depends on $\epsilon$ and $w_0$, decreases with growing $\epsilon$ and $w_0$.

IV. NEW SOLUTIONS WITH BIG FREEZE SINGULARITY

We shall consider in this section a flat universe which shows a big freeze singularity (BFS) in its future, i.e.

$$\rho = \beta^{-6/\epsilon} (a_f^\epsilon - a^\epsilon)^{-3/\epsilon},$$

and

$$p = -c^2 a_f^\epsilon \beta^{-6/\epsilon} (a_f^\epsilon - a^\epsilon)^{1-3/\epsilon}.$$ 

Putting $\epsilon = 3$, we have

$$p = -c^2 \beta^2 a_f^3 \rho.$$
In this case, one can solve the above equations to find the scale factor. We obtain it in parametric form

\[ a = a_f (\sin \eta)^{2/3}, \]
\[ t = t_f + \frac{1}{\kappa} \left( \ln |\tan \frac{\eta}{2}| + \cos \eta \right), \quad (13) \]

where

\[ \kappa = \frac{1}{\beta} \sqrt{\frac{6\pi G}{a_f^3}}, \quad dt = \frac{\cos^2 \eta}{\kappa \sin \eta} d\eta. \]

We set now \( t_f = 0 \) and therefore \( \eta = 0 \) corresponds to \( t = -\infty \), \( \eta = \pi/2 \) to \( t = 0 \) (BFS) and \( \eta = \pi \) to \( t = +\infty \).

Now one can see that if \( \psi = a^3 \) then the following equation holds

\[ \frac{d^2 \psi}{dt^2} = (v(t) - \lambda)\psi, \quad (14) \]

in which

\[ v = \frac{2\kappa^2 \sin^2 \eta(2\cos^2 \eta + 1)}{\cos^4 \eta}, \quad \lambda = -4\kappa^2. \]

This is very interesting point for the spectral theory of the Schrödinger equation. The potential \( v(t) \to +0 \) at \( t \to \pm \infty \) and \( v(0) = +\infty \). But we have a bounded state (\( \psi \in L^2 \) and no zeros at \( t \in (-\infty; +\infty) \)), and this is the case notwithstanding for which the potential has a singularity at \( t = 0 \) (\( \eta = \pi/2 \)). One can check that

\[ \int_{-\infty}^{\infty} \psi^2 dt = 1, \]

if \( \psi = \sqrt{15\kappa/4}\sin^2 \eta \).

Now we can use Eq. (14) to find the second solution \( \hat{\psi} \) with the same potential \( v \) and the same value of the spectral parameter \( \lambda \), i.e.

\[ \hat{\psi} = \psi \int \frac{dt}{\psi^2}. \]

We get

\[ \hat{\psi} = -2 \cos \eta + \frac{4 \cos \eta}{\sin^2 \eta} - 2 \sin^2 \eta \ln \left| \cot \frac{\eta}{2} \right|. \quad (15) \]

Therefore we have a new solution for the same expression

\[ v - \lambda = 12\pi G \left( \rho - \frac{p}{c^2} \right). \]

This solution describes two universes: the first one begins at \( t = -\infty \) and then progressively contracts until a big crunch singularity at \( t = 0 \) (or \( \eta = \pi/2 \)). The second solution begins at \( t = 0 \) (big bang) and then starts expanding. One can check that the Hubble root \( \hat{H} = d\ln \hat{\psi}^{1/3}/dt \) has the asymptotic behavior given by

\[ \lim_{t \to \pm \infty} \hat{H} = \pm 2\kappa, \]

and \( \hat{H}(0) = \infty \). Thus we have dS universe at \( t \to \pm \infty \).

A most interesting solution is the superposition of \( \psi = \sin^2 \eta \) and \( \hat{\psi} \). We can see that \( \Psi = c_1\psi + c_2\hat{\psi} \) results in a new solution \( a_{\text{general}} = \Psi^{1/3} \), such that

\[ \frac{d^2 \Psi}{dt^2} = (v(t) - \lambda)\Psi, \quad (16) \]

for the same \( v \) and \( \lambda \). This solution describes three distinct kinds of universes. If \( c_1 = 1 \) and \( c_2 = -0.1 \), then we get Universe I. This universe begins at \( t = -\infty \) and then progressively contracts until a big crunch singularity at \( t = t_i: -\infty < t_i < 0 \) (for the case \( c_1 = 1, c_2 = -0.1 \) we have \( t_i \sim 0.72 \)).
Universe II. This universe begins at $t = t_i$ (big bang at $a_{\text{general}}(t_i) = 0$) after which it starts expanding until a BFS which takes place at $t = 0$. It can be shown that this type of universe cannot be fitted to the data of astronomical observations and that it does not lead to a phase of accelerating expansion.

Universe III. This universe begins at $t = 0$ as a BFS (with $a_{\text{general}} \neq 0$) and then starts expanding until it finally behaves like De Sitter for large $t$

$$\lim_{t \to \infty} H_{\text{general}} = \frac{2\kappa}{3}.$$ 

We find the latter two kinds of cosmological evolution to be most interesting.

V. SOLUTIONS WITH COSMOLOGICAL CONSTANT

Let’s consider the universe filled with generalized Chaplygin gas and nonzero vacuum energy. The equation for the scale factor

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G_f}{3} \left( \frac{\beta^{-6/\epsilon}}{(a_f^{-3} - a^{-3})^{3/\epsilon}} + \Lambda \right)$$  

(17)

can be conveniently rewritten in terms of the rescaled variables $A = a/a_f$ and $T = (8\pi G|\Lambda|/3)^{1/2}t$. If, without loss of generality we set $\beta^{-6/\epsilon}a_f^{-3}|\Lambda|^{-1/2} = 1$, we finally derive from (17) an equation for $A$,

$$\frac{A'}{A^2} = \frac{1}{(1 + \Lambda)^{3/\epsilon}} + \frac{|\Lambda|}{\Lambda},$$

with the prime denoting derivation with respect to $T$. The solution for this equation can be now given in parametric form:

a) For a universe starting at a big bang

$$A = (\sin \eta)^{2/\epsilon},$$
$$T = T_f - 2\epsilon^{-1} \int_{\eta}^{\pi/2} d\eta \cot(\cos \eta)^{3/\epsilon}(1 \pm (\cos \eta)^{6/\epsilon})^{-1/2},$$

(18)

where the signs “+” and “−” correspond to positive and negative cosmological constant, respectively. We put $T_f = 0$ and therefore $\eta = 0$ corresponds to $T = -\infty$ (big band) and $\eta = \pi/2$ to $T = 0$ (BFS).

b) For a universe starting at a BFS

$$A = (\sin \eta)^{2/\epsilon},$$
$$T = T_f + 2\epsilon^{-1} \int_{\pi/2}^{\eta} d\eta \cot(\cos \eta)^{3/\epsilon}(1 \pm (\cos \eta)^{6/\epsilon})^{-1/2},$$

(19)

Setting $T_f = 0$ and therefore $\eta = \pi/2$ corresponds to $T = 0$ (BFS) and $\eta = \pi$ to $T = -\infty$ (big chrunch).

VI. SUMMARY AND CONCLUSION

This paper deals with several new cosmic solutions, which show or do not show a future big freeze singularity, derived from a universe filled with generalized Chaplygin in the cases where: (i) dark energy is the sole component of the universal energy, (ii) the universe contains in addition some amount of dark matter, and (iii) the universe is also equipped with a cosmological constant. We discuss the observational feasibility of such theoretical solutions, reaching the conclusion that some nonzero amount of dark matter should be present in order for the given model with future big freeze singularity to be compatible with current observations.

It is worth noticing that the above conclusion and the alluded results are only valid in a classical framework. It could well be expected that our classical treatment would break down in the neighborhood of the singularity where the energy density tends to infinity. Actually, a quantum-gravity consideration which contemplated the Planck length as the ultimate resolution limit would smooth out such a singularity and allowed for a further evolution for the universe.

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