New Scalar Contributions to $h \rightarrow Z\gamma$

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Abstract

We calculate the Higgs decay rate of $h \rightarrow Z\gamma$ by including the contributions from new scalars with arbitrary quantum numbers of the weak isospin ($T$) and hypercharge ($Y$) in the standard model. We find that our general formula for the decay rate of $h \rightarrow Z\gamma$ matches with that for $h \rightarrow \gamma\gamma$ in the limit of $m_Z = 0$, but it is different from those in the literature. To illustrate our result, by taking the current $2\sigma$ excess of the $h \rightarrow \gamma\gamma$ rate measured by the LHC, we examine the corresponding shift for the $Z\gamma$ decay channel due to the new scalar. We show that the enhancement or reduction of the $h \rightarrow Z\gamma$ rate only depends on the relative size of $T$ and the absolute value of $Y$. Explicitly, we predict $0.76 < R_{Z\gamma} \equiv \Gamma(h \rightarrow Z\gamma)/\Gamma_{SM}(h \rightarrow Z\gamma) < 2.05$ by imposing the observed range of $1.5 < R_{\gamma\gamma} \equiv \Gamma(h \rightarrow Z\gamma\gamma)/\Gamma_{SM}(h \rightarrow \gamma\gamma) < 2$, which is independent of the number of multiplets and the couplings to the Higgs particle as long as the scalars are heavier than 200 GeV. This result provides a clear signature for the future LHC measurements to test physics beyond the standard model.
I. INTRODUCTION

The discovery of the Higgs-like particle \( (h) \) by ATLAS \cite{1} and CMS \cite{2} collaborations is a great triumph of particle physics. The following immediate question is whether this newly discovered particle is really the Higgs particle in the Standard Model (SM). To achieve this goal, one has to determine its couplings to all the SM particles. However, recent results from Higgs searches at the LHC have already shown a hint of new physics in the Higgs diphoton decay channel. The experimental data have shown that the diphoton decay rate is about 1.5 – 2.0 times larger than the SM prediction, while the measurements in other channels including \( WW^* \) and \( ZZ^* \) agree with the SM Higgs properties. Due to the fact that the Higgs particle is electric neutral, its coupling to diphoton must be induced by some charged particles running in the loops, while in the SM, the W-boson and top-quark loops give the dominant contributions \cite{3–7}. If the enhancement in the diphoton channel persists, it clearly indicates \cite{8–26} that there must be some additional new charged particles which couple to the Higgs and mediate this decay process. Based on the electroweak (EW) \( SU(2)_L \times U(1)_Y \) gauge symmetry, these new particles should also contribute to the \( h \to Z\gamma \) channel and generically lead to a shift in the \( Z\gamma \) decay width from the SM expectation. As pointed out in Refs. \cite{27–33}, the simultaneous measurements of the \( \gamma\gamma \) and \( Z\gamma \) channels at the LHC will provide us with valuable information about the structure of new physics.

In this paper, we focus on the simple scenario in which extra contributions to the \( \gamma\gamma \) and \( Z\gamma \) decay widths arise from some new scalars beyond the SM particle content. Some similar scenarios have already been explored in Refs. \cite{27–42}. However, the formulae for the scalar contributions to the \( Z\gamma \) decay width used in the literature \cite{27–32} are not consistent with each other. Moreover, they cannot be reduced to the corresponding rates for \( h \to \gamma\gamma \) by taking the limit of \( m_Z = 0 \) and making the charge replacement. As a result, these numerical predictions for the \( Z\gamma \) decay width may not be reliable. Clearly, it is timely important to re-calculate the scalar contributions to the decay.

This paper is organized as follows. In section II we show the detail calculations of the scalar contributions to the decay rate of \( h \to Z\gamma \). In section III we give phenomenological analyses of the correlations between the \( \gamma\gamma \) and \( Z\gamma \) channels in several specific sets as well as a generic one of the scalar multiplets. We summarize our results in section IV.
measurements, that is, the oblique parameters would constrain the mass differences involving $\gamma$ loop-induced contributions of $O$ components of the scalar $S$ the Weinberg mixing angle). We note that the mass splittings at tree level among the scalar multiplet \[44\] to be smaller than a few $g$ where $h$

II. CALCULATIONS OF SCALAR CONTRIBUTIONS TO $h \rightarrow Z\gamma$

For a general scalar particle $S$ with the third weak isospin charge $T_3^{(S)}$ and non-trivial electric charge $Q_S$ the relevant Lagrangian involving $S$ is given by

$$\mathcal{L}_S = (D_\mu S)^\dagger (D^\mu S) - m_S^2 S^\dagger S - \lambda_{HS}(H^\dagger H)(S^\dagger S) - \bar{\lambda}_{HS}(H^\dagger T^a H)(S^\dagger T^a S), \quad (2.1)$$

where $H$ is the SM Higgs doublet; $T^a$ is the SU(2)$_L$ generators, and the covariant derivative involving $\gamma$ and $Z$ gauge fields is defined as

$$D_\mu S = (\partial_\mu + ieQ_A_\mu + ieg_{ZSS}Z_\mu)S, \quad (2.2)$$

where $g_{ZSS} = (T_3^{(S)} - Q_S s_W^2)/(s_W c_W)$ with $s_W = \sin \theta_W$ and $c_W = \cos \theta_W$ ($\theta_W$ being the Weinberg mixing angle). We note that the mass splittings at tree level among the components of the scalar $S$ come from the last term in Eq. (2.1), while they also receive loop-induced contributions of $O(100)$ MeV due to the gauge boson exchange diagrams \[43\], which can be ignored in our discussion. On the other hand, the electroweak precision measurements, that is, the oblique parameters would constrain the mass differences $\delta m$ among the scalar multiplet \[44\] to be smaller than a few $O(10)$ GeV \[41\], \[45\]. As a result, we assume the masses of the scalar multiplet degenerate in this paper for simplicity. The trilinear coupling between Higgs and the scalar particle is $-g_{hSS}hS^\dagger S = -\lambda_{HSv}hS^\dagger S$ after Higgs develops the vacuum expectation value, $\langle H \rangle = v/\sqrt{2}$. The amplitudes for the three Feynman diagrams shown in Fig. \[1\] can be easily written down:

\[
-i\mathcal{M}^{(1)} = e^2 Q g_{ZSS}\lambda_{HSv} \mu^{4-d} \int \frac{d^4l}{(2\pi)^d} \frac{(2l+q)^\mu (2l+2q+p)^\nu}{[(l+q)^2 - m_Z^2][(l+q)^2 - (l+2q+p)^2 - m_Z^2]} \epsilon^*_\mu(q)\epsilon^A_\nu(p),
\]

\[
-i\mathcal{M}^{(2)} = e^2 Q g_{ZSS}\lambda_{HSv} \mu^{4-d} \int \frac{d^4l}{(2\pi)^d} \frac{(2l-q)^\mu (2l-2q-p)}{(l-q)^2 - m_Z^2} \frac{\epsilon^*_\mu(q)\epsilon^A_\nu(p)}{(l-q-p)^2 - m_Z^2},
\]

\[
-i\mathcal{M}^{(3)} = -2e^2 Q g_{ZSS}\lambda_{HSv} \mu^{4-d} \int \frac{d^4l}{(2\pi)^d} \frac{g^\mu\nu}{(l^2 - m_Z^2)} \epsilon^*_\mu(q)\epsilon^A_\nu(p),
\]

1 We use the convention $Q = T_3 + Y/2$ in this paper.
where we have taken the dimensional regularization. Note that by changing the integration variable \( l \rightarrow -l, -iM^{(1)} \) can be transformed into the form that is exactly identical to \(-iM^{(1)}\), so that we only need to calculate \(2(-iM^{(1)}) + (-iM^{(3)})\). With the standard procedures of the Feynman parametrization and the translation of the integral momentum \( l \), we obtain:

\[
-i(2M^{(1)} + M^{(3)})
\]

\[
e^2(2Q)g_{ZSS} \lambda_{HS} v \mu^{4-d} \int \frac{d^d l}{(2\pi)^d} \frac{\{(2l + q)^\mu(2l + 2q + p)^\nu - [(l + q)^2 - m_S^2]g^{\mu\nu}\}}{(l^2 - m_S^2)(l^2 - (l + q)^2 - m_S^2)(l + q + p)^2 - m_S^2} 
\]

\[
e^2(2Q)g_{ZSS} \lambda_{HS} v \mu^{4-d} \Gamma(3) \int dx dy \int \frac{d^d l}{(2\pi)^d} \left[ \frac{4 \lambda^\mu p^\nu - g^{\mu\nu} l^\mu l^\nu}{(l^2 - \Delta^2)^2} \right] \epsilon_\nu^Z(q) \epsilon_\nu^A(p),
\]

(2.4)

where \( \Delta^2 \equiv m_S^2 - x(1 - x - y)m_Z^2 - y(1 - x - y)m_h^2 \). In the second line of Eq. (2.4), we have used the following on-shell identities:

\[
p^2 = 0, \quad q^2 = m_Z^2, \quad (p + q)^2 = m_h^2, \quad p \cdot \epsilon^A(p) = q \cdot \epsilon^Z(q) = 0.
\]

(2.5)

With the help of the dimensional regularization, it can be proved that in the second equation in Eq. (2.4), the first two terms exactly cancel with each other and only the last finite term is left, given by

\[
-i(2M^{(1)} + M^{(3)}) = \frac{i e^2(2Q)g_{ZSS} \lambda_{HS} v}{16\pi^2} \mu^{4-d} \int dx dy \left\{ \frac{4y(1 - x - y)(p^\mu q^\nu - g^{\mu\nu} p \cdot q)}{m_S^2 - x(1 - x - y)m_Z^2 - y(1 - x - y)m_h^2} \right\} \epsilon_\nu^Z(q) \epsilon_\nu^A(p).
\]

(2.6)

From this equation after the integration, the first term leads to

\[
-i(2M^{(1)} + M^{(3)} = \frac{i e^2(2Q)g_{ZSS} \lambda_{HS} v}{16\pi^2} (p^\mu q^\nu - g^{\mu\nu} p \cdot q) \epsilon_\nu^Z(q) \epsilon_\nu^A(p) A_0^{Z\gamma}(\tau_S, \lambda_S),
\]

(2.7)

while the second one vanishes, where \( \tau_S = 4m_S^2/m_h^2, \lambda_S = 4m_S^2/m_Z^2 \) and the loop function \( A_0^{Z\gamma}(x, y) \) is defined in Appendix [A] By combining the SM contributions and integrating the phase space of outgoing particles, we obtain the \( h \rightarrow Z\gamma \) decay width,

\[
\Gamma(h \rightarrow Z\gamma) = \frac{\alpha^2}{512\pi^3} m_h^3 \left( 1 - \frac{m_Z^2}{m_h^2} \right)^3 \left| A_{SM}^{Z\gamma} - \frac{\lambda_{HS} v}{m_Z^2} (2 \sum_{t_S} Q_S \cdot g_{ZSS}) A_0^{Z\gamma}(\tau_S, \lambda_S) \right|^2,
\]

(2.8)
with
\[ A^{Z\gamma}_{SM} = \frac{2}{v} \left[ \cot \theta_W A^\gamma_1(\tau_W, \lambda_W) + N_c \frac{(2Q_t)(T_d^{(i)} - 2Q_ts_W)}{s_Wc_W} A^{Z\gamma}_{1/2}(\tau_i, \lambda_i) \right], \tag{2.9} \]

where \( \tau_i = 4m_i^2/m_h^2 \), \( \lambda_i = 4m_i^2/m_Z^2 \) \((i = W, t)\), and the summation is over the different isospin components in given \( SU(2)_L \) multiplets. We point out that for the non-SM contribution to the decay rate of \( h \to Z\gamma \), there are extra factors of \(-1/2\) in Refs. [29, 31] and \(-1\) in Ref. [32] in comparing with ours in Eq. (2.8). The different signs for the scalar contributions to \( h \to Z\gamma \) clearly lead to different results by taking limits to \( h \to \gamma\gamma \). The modification in the partial decay width of \( h \to Z\gamma \) is then expressed in terms of the enhancement factor, given by
\[ R_{Z\gamma} \equiv \frac{\Gamma(h \to Z\gamma)}{\Gamma_{SM}(h \to Z\gamma)} = \left| 1 - \tilde{N} \lambda_{HS} v^2 m^2 S \left( \sum_{\tau_3} Q_{S} \cdot g_{ZSS} A^{Z\gamma}_{0}(\tau_S, \lambda_S) \right) / v A^{Z\gamma}_{SM} \right|^2, \tag{2.10} \]

where \( \Gamma_{SM}(h \to Z\gamma) \) represents the decay width in the SM and the factor \( \tilde{N} \) represents the degeneracy of the multiplet. For completeness and convenience for the later discussions, we also present the standard formula for the enhancement factor of the diphoton rate,
\[ R_{\gamma\gamma} \equiv \frac{\Gamma(h \to \gamma\gamma)}{\Gamma_{SM}(h \to \gamma\gamma)} = \left| 1 + \tilde{N} \lambda_{HS} v^2 m^2 S \left( \sum_{\tau_3} Q_{S}^2 \right) / 2 \right| \left( A^{\gamma\gamma}_{0}(\tau_S) / A^{\gamma\gamma}_{SM}(\tau_W) + N_c Q_t A^{\gamma\gamma}_{1/2}(\tau_t) \right) ^2, \tag{2.11} \]

where the loop functions \( A^{\gamma\gamma}_j(x) \) \((j = 0, 1/2, 1)\) are defined in Appendix A. It is straightforward to show that our formula in Eq. (2.10) for the \( Z\gamma \) decay can be retrieved to the one in Eq. (2.11) for the \( \gamma\gamma \) mode when taking \( m_Z \to 0 \) and making the replacement \( g_{ZSS} \to Q_S \). If the mass splittings among components of the multiplet are taken into account, the deviations of the contributions in Eqs. (2.10) and (2.11) are approximately proportional to \( \delta m^2/m_Z^2 \).

The modifications from the mass splittings are at percentage level and are negligible due to the electroweak precision measurements. Consequently, we will simply take Eqs. (2.10) and (2.11) to analyze the qualitative behaviors in \( h \to \gamma\gamma \) and \( h \to Z\gamma \) decays.

### III. PHENOMENOLOGICAL ANALYSES OF CORRELATIONS BETWEEN THE \( \gamma\gamma \) AND \( Z\gamma \) CHANNELS

With formulae in Eqs. (2.10) and (2.11) at hand, we are able to study the correlations between \( h \to \gamma\gamma \) and \( h \to Z\gamma \) decay rates. According to the EW gauge symmetry, one expects that the particle responsible for the observed enhancement in the diphoton channel
could also lead to the modification of the $Z\gamma$ rate. Furthermore, it is interesting to notice that the $Z\gamma$ amplitude depends not only on the electric charges of particles running in the loops but also on their isospins, in contrast with the $\gamma\gamma$ case in which only electric charges can be probed. Consequently, a combined analysis of the decay widths in these two modes can provide us with important information on the EW charges of particles contributing to both processes.

A. Singlet and Doublet Scalars

In this subsection, we revisit two simple cases of singlet and doublet scalars under $SU(2)_L$ with the couplings:

(I) : $g^{(1)}_{ZSS} = -\frac{Q_S s_W^2}{s_W c_W}$,  

(II) : $g^{(2)}_{ZSS} = \frac{1}{s_W c_W} \left( \frac{1}{2} - Q_S s_W^2 \right)$,  

where $Q_S = 1$, corresponding to $(T, Y) = (0, 2)$ and $(1/2, 1)$, respectively. The aim for this part of the investigation is twofold: (a) illustrating the features of the isospin and hypercharge dominations and (b) reexamining the results in Ref. [29]. In the two cases in Eq. (3.1), since only charged particles with $Q_S = 1$ run in the loops, the enhancements for the diphoton rate are expected to be the same. However, due to the different isospin representations, those for the $Z\gamma$ mode are distinct. From Fig. 2 we see that the $Z\gamma$ rate in Case I (II) has a small suppression (enhancement) in comparison with the SM value. These features can be understood from the general formula in Eq. (2.10). In order to enhance the $h \to \gamma\gamma$ rate by a constructive interference, it is required that $\lambda_{HS}$ must be negative since $A_0^\gamma(\tau_S)$ and $A_{SM}^\gamma$ possess an opposite sign. In this case, the $h \to Z\gamma$ rate depends only on the sign of $Q_S g_{ZSS}$ as $\lambda_{HS} < 0$ corresponds to both positive values of $A_{SM}^\gamma$ and $A_0^\gamma(\tau_S, \lambda_S)$ in the physically interesting mass regime. As a result, $R_{Z\gamma}$ is enhanced (suppressed) if $Q_S g_{ZSS}$ is positive (negative). Accordingly, it is easy to see that $Q_S g_{ZSS} < 0 (> 0)$ in Case I (II). Note that our result is opposite to that in Ref. [29], in which the scalar contributions to $h \to \gamma\gamma$ and $h \to Z\gamma$ are positive correlated in Case I and negative correlated in Case II due to the extra minus sign in the formula.

The factor $Q_S \cdot g_{ZSS}$ can be examined in a general scalar multiplet. By using the identity $Q = T_3 + Y/2$, we have

$$Q_S \cdot g_{ZSS} = \frac{1}{s_W c_W} \left( T_3 + \frac{Y}{2} \right) \left( T_3 c_W^2 - \frac{Y s_W^2}{2} \right).$$  

(3.2)
When the isoweak charge $T$ of the scalar multiplet is larger than its hypercharge $Y$, we find $Q_S \cdot g_{ZSS} > 0$, which indicates the enhanced behavior in the $Z\gamma$ channel compared with the SM prediction. In contrast, for the multiplet with $Y \gg T$, we get $Q_S \cdot g_{ZSS} < 0$, leading to a suppressed rate of $h \rightarrow Z\gamma$. Precisely, Cases I and II are the two simple but nontrivial archetypes of the two limits, respectively.

B. Triplet Scalars with $T = 1$ and $Y = 0, 2$

It is known that triplet scalars are introduced in many SM extensions, such as the type II seesaw model [46–52]. Current LHC experiments also plan to search for these triplet scalars, especially the doubly charged particle in the $Y = 2$ case due to its interesting signatures [53, 54]. Instead of a special model, we examine the generic feature of these triplets. In particular, we investigate their loop effects in the $\gamma\gamma$ and $Z\gamma$ decay channels of the Higgs particle. For simplicity, we assume the multiplets of scalars possess the same masses and no mixings between the triplet scalars and the SM Higgs. For more detail analysis one can refer to Refs. [38–42]. Our result is shown in Fig. 3. Clearly, the $Z\gamma$ rate is enhanced in both cases, which belong to the isospin domination class discussed in
FIG. 3. Contours of $R_{\gamma\gamma}$ (solid lines) and $R_{Z\gamma}$ (dashed lines) in $m_S-\lambda_{HS}$ plane for the triplet scalars of (a) $(T,Y) = (1,0)$ and (b) $(T,Y) = (1,2)$, where the solid (dashed) lines from top to bottom correspond to the values of (a) 1.25 (1.25), 1.50 (1.53), 1.75 (1.75), and 2.00 (2.00); and (b) 1.25 (1.05), 1.50 (1.10), 1.75 (1.15), and 2.00 (1.20), respectively.

the previous subsection. Furthermore, the enhancement in the $Y = 0$ case is much larger than that in the $Y = 2$ one, which implies that increasing the value of $Y$ will decrease the $h \rightarrow Z\gamma$ decay rate for a given isospin multiplet. Moreover, in comparison with Case I of $(T,Y) = (0,2)$ in Sec. III-A, $\Gamma(h \rightarrow Z\gamma)$ for the case of $(T,Y) = (1,2)$ for a given rate of $h \rightarrow \gamma\gamma$, say, 1.5 times the SM value, is slightly larger, showing that a large $SU(2)_L$ multiplet helps to increase the $h \rightarrow Z\gamma$ rate.

C. 5-plet Scalars with $T = 2$ and $Y = 0, 2$

The weak $SU(2)_L$ 5-plet scalar particles are also of great phenomenological interest. For example, a recent study in Ref. [55] shows that with the 5-plet and $Y = 2$ scalar, the small Majorana neutrino masses can be generated at two-loop level. The $\gamma\gamma$ and $Z\gamma$ results are presented in Fig. 4. Similar to the $SU(2)_L$ triplet cases, the 5-plet scalars give sizable enhancements in the $h \rightarrow Z\gamma$ channel when the diphoton rate is lying in 1.5 to 2.0 times larger than the SM value. Explicitly, the enhancement factor $R_{Z\gamma}$ is also about 1.5 to 2.0 (1.25 to 1.50) for the $Y = 0 (2)$ case. Again, the behaviors of these two cases can be expected by the rules of the isospin domination. We note that a bit less enhancement in the $Y = 2$ case is due to the hypercharge cancellation.
FIG. 4. Contours of $R_{\gamma\gamma}$ (solid lines) and $R_{Z\gamma}$ (dashed lines) in $m_S\lambda_{HS}$ plane for the 5-plet scalars of (a) $(T,Y) = (2,0)$ and (b) $(T,Y) = (2,2)$, where the solid (dashed) lines from top to bottom correspond to the values of (a) 1.25 (1.25), 1.50 (1.53), 1.75 (1.75), and 2.00 (2.00); and (b) 1.25 (1.10), 1.50 (1.15), 1.75 (1.25), and 2.00 (1.50), respectively.

D. General Discussions

In this subsection, we give a more comprehensive overview of how to use the combined analysis of the $\gamma\gamma$ and $Z\gamma$ modes to investigate the nature of the new scalar multiplets. By assuming that the excess of $h \to \gamma\gamma$ is originated from a constructive interference between the SM and charged scalar loops, we obtain

$$\frac{\sqrt{R_{Z\gamma}} - 1}{\sqrt{R_{\gamma\gamma}} - 1} = -\frac{4[A^\gamma_1(t_W) + N_c Q_T^2 A^\gamma_2(t_t)]}{v A^\gamma_{SM}} \frac{A^\gamma_0(t_S, \lambda_S)}{A^\gamma_0(t_S)} \frac{\sum_{T_3} Q_S \cdot g_{ZSS}}{\sum_{T_3} Q_S^2}$$

for a scalar with general charges of $T$ and $Y$. From Eq. (3.3), $R_{Z\gamma}$ increases with a higher value of $T$ since $A^\gamma_0$ and $A^\gamma_0$ are both positive, whereas it decreases with a larger $|Y|$. Furthermore, when $T \gg |Y|$, $R_{Z\gamma}$ is saturated to a maximal value with a fixed value of $R_{\gamma\gamma}$, given by

$$\frac{\sqrt{R_{Z\gamma}^{\text{max}}} - 1}{\sqrt{R_{\gamma\gamma} - 1}} = 2.09 \cdot \frac{A^\gamma_0(t_S, \lambda_S)}{A^\gamma_0(t_S)}.$$  \hfill (3.4)

The renormalization group running will drive the $SU(2)_L$ gauge coupling to violate perturbativity for $2T + 1 > 8$ \cite{43}. The main point in this paper is to show the quantitative behaviors of the scalars with arbitrary EW quantum numbers.
which is definitely positive, implying that the $Z\gamma$ decay width must be enhanced. This limit corresponds to the isospin domination case as discussed in Sec. III-A. In Fig. 5, we show $R_{Z\gamma}$ as a function of the isospin $T$ with $R_{\gamma\gamma} = 1.5$ and the scalar mass $m_S = 300$ GeV. From the figure, we find that the saturated value of $R_{Z\gamma}^{\text{max}}$ is 1.53.

On the other hand, when we take the large hypercharge limit $|Y| \gg |T|$, that is, the hypercharge domination case, we obtain the minimum value for the factor $R_{Z\gamma}$.

\[
\frac{\sqrt{R_{Z\gamma}^{\text{min}}} - 1}{\sqrt{R_{\gamma\gamma} - 1}} = -0.63 \cdot \frac{A_0^{Z\gamma}(\tau_S, \lambda_S)}{A_0^{\gamma\gamma}(\tau_S)}. \tag{3.5}
\]

In Fig. 6, we show $R_{Z\gamma}$ as a function of the hypercharge $Y$ with $R_{\gamma\gamma} = 1.5$ and $m_S = 300$ GeV. In the large $|Y|$ limit, we get the asymptotic value of $R_{Z\gamma}^{\text{min}} = 0.86$.

The above features become more transparent in the large scalar mass limit, $m_S^2 \to \infty$. In this limit, we have that $A_0^{Z\gamma}(\tau_S, \lambda_S) = A_0^{\gamma\gamma}(\tau_S)/2 = 1/6$, and thus Eq. (3.3) can be reduced
FIG. 7. $R_{Z\gamma}$ in the plane of general $SU(2)_L \times U(1)_Y$ quantum numbers by fixing $R_{\gamma\gamma} = 1.5$, where the solid and dashed lines represent the results of $m_S \to \infty$ and $m_S = 200$ GeV, respectively.

$R_{Z\gamma}$ can be approximated by

$$\sqrt{R_{Z\gamma} - 1} = 1.36 \cdot \frac{4T(T + 1)c_W^2 - 3Y^2s_W^2}{4T(T + 1) + 3Y^2}.$$  \hspace{1cm} (3.6)

In most of the physically interesting regime, Eq. (3.6) is a very good approximation with the error of ten percent for the scalar mass above 200 GeV in which the ratio $\frac{A_{Z\gamma}^0(r_S,a_S)}{A_{\gamma\gamma}^0(r_S)}$ is numerically stable around 0.5. A remarkable feature shown in Eq. (3.6) is that the $h \to Z\gamma$ decay rate for the Higgs boson only depends on the EW quantum numbers assigned to the scalar multiplet for a given enhancement in the $\gamma\gamma$ channel. As an illustration, Fig. 7 shows the contours of $R_{Z\gamma}$ in terms of the general $SU(2)_L \times U(1)_Y$ quantum numbers with $R_{\gamma\gamma} = 1.5$ for both $m_S \to \infty$ and $m_S = 200$ GeV, respectively. In the large mass regime, the maximum and minimum values of $R_{Z\gamma}$ can be easily obtained, $R_{Z\gamma}^{\text{max}} = [1 + 1.04(\sqrt{R_{\gamma\gamma} - 1})]^2$ and $R_{Z\gamma}^{\text{min}} = [1 - 0.31(\sqrt{R_{\gamma\gamma} - 1})]^2$, corresponding to the isospin and hypercharge dominations, respectively.

Finally, we can further demonstrate that the above bounds hold when we extend our discussion to arbitrary numbers of scalar multiplets with different quantum numbers, masses $m_i$, Higgs couplings $\lambda_i$, and the degeneracies $N_i$. The formula in Eq. (3.3) can be generalized to:

$$\frac{\sqrt{R_{Z\gamma} - 1}}{\sqrt{R_{\gamma\gamma} - 1}} = 1.36 \cdot \frac{\sum_i N_i \frac{\lambda_i}{m_i}(2T_i + 1)[4T_i(T_i + 1)c_W^2 - 3Y_i^2s_W^2]}{\sum_i N_i \frac{\lambda_i}{m_i}(2T_i + 1)[4T_i(T_i + 1) + 3Y_i^2]}. \hspace{1cm} (3.7)$$

where we have taken $m_i > 200$ GeV. Note that Eq. (3.7) is valid only for the new scalar...
contributions interfering with the SM part constructively. This assumption follows by two indications: (i) $\lambda_i$ should be negative in sign, so that we can replace $\lambda_i$ with its absolute value in Eq. (3.7); and (ii) contributions to the amplitudes of the $h \to \gamma\gamma$ and $Z\gamma$ modes must be smaller than the corresponding SM parts in size to eliminate the sign ambiguity when we take the square roots of $R_{\gamma\gamma}$ and $R_{Z\gamma}$. We now try to derive a generic bound for $R_{Z\gamma}$. From Eq. (3.7) and inequalities

$$-0.31 \left[4T_i(T_i + 1) + 3Y_i^2\right] < 4T_i(T_i + 1)c_W^2 - 3Y_i^2 s_W^2 < 1.04 \left[4T_i(T_i + 1) + 3Y_i^2\right]$$

(3.8)

for a single multiplet, we find

$$-0.31 < \frac{\sqrt{R_{Z\gamma}} - 1}{\sqrt{R_{\gamma\gamma}} - 1} < 1.04.$$  (3.9)

From the general bound in Eq. (3.9), it is clear that one can deduce the allowed range of the $Z\gamma$ decay width for a given value of $R_{\gamma\gamma}$. Since the current ATLAS and CMS data have indicated the range $1.5 < R_{\gamma\gamma} < 2.0$ for the diphoton mode, one predicts that

$$0.76 < R_{Z\gamma} < 2.05$$

(3.10)

for the $Z\gamma$ mode, where both the upper and lower limits arise from $R_{\gamma\gamma} = 2.0$. We emphasize that the result in Eq. (3.10) does not rely on the details of the extra scalars, such as the quantum numbers and their couplings to the Higgs particle. Thus, if the future-measured diphoton rate is still enhanced about $1.5 - 2.0$ times over the SM value, the $Z\gamma$ decay rate should be in the range of 0.76 to 2.05. Otherwise, our simple scenario discussed in this paper will be excluded.

IV. SUMMARY

We have studied the decay width of the $h \to Z\gamma$ mode with extra scalar multiplets of arbitrary weak isospins and hypercharges. Due to the SM gauge symmetry, these new scalars which result in the excess of the $h \to \gamma\gamma$ decay rate would generically shift the $h \to Z\gamma$ decay rate as well. The combined analysis of these two modes can provide us with valuable information about the new physics structure. Regarding the ambiguity of the formulae for $\Gamma(h \to Z\gamma)$ in the literature, we have revisited some simple extensions to the SM in which new scalar multiplets are introduced. We have found that $\Gamma(h \to Z\gamma)$ only depends on the
relative size of the isospin $T$ and the absolute value of the hypercharge $Y$. In particular, we have shown that the enhancement factor $R_{Z\gamma}$ is a monotonically increasing function of $T$ and a monotonically decreasing one of $|Y|$ with the fixed value of $R_{\gamma\gamma}$. This observation enables us to predict that $0.76 < R_{Z\gamma} < 2.05$ by imposing the observed range of $1.5 < R_{\gamma\gamma} < 2$ if the scalars are heavier than 200 GeV. Note that this range is irrelevant to the number of the scalar multiplets, their representations, and the couplings to the Higgs particle. Our results on the $h \rightarrow Z\gamma$ decay clearly can be tested at the LHC.

**Appendix A: Definition of Loop Functions**

The functions related to $h \rightarrow \gamma\gamma$ and $h \rightarrow Z\gamma$ are defined as follows:

\begin{align*}
A_1^{\gamma\gamma}(x) &= -x^2[2x^{-2} + 3x^{-1} + 3(2x^{-1} - 1)f(x^{-1})], \quad (A1a) \\
A_{1/2}^{\gamma\gamma}(x) &= 2x^2[x^{-1} + (x^{-1} - 1)f(x^{-1})], \quad (A1b) \\
A_0^{\gamma\gamma}(x) &= -x^2[x^{-1} - f(x^{-1})], \quad (A1c) \\
A_1^{Z\gamma}(x, y) &= 4(3 - \tan^2 \theta_W)I_2(x, y) + [(1 + 2x^{-1})\tan^2 \theta_W - (5 + 2x^{-1})]I_1(x, y), \quad (A1d) \\
A_{1/2}^{Z\gamma}(x, y) &= I_1(x, y) - I_2(x, y), \quad (A1e) \\
A_0^{Z\gamma}(x, y) &= I_1(x, y), \quad (A1f)
\end{align*}

where

\begin{align*}
I_1(x, y) &= \frac{xy}{2(x - y)} + \frac{x^2y^2}{2(x - y)^2}[f(x^{-1}) - f(y^{-1})] + \frac{x^2y}{(x - y)^2}[g(x^{-1}) - g(y^{-1})], \quad (A2a) \\
I_2(x, y) &= -\frac{xy}{2(x - y)}[f(x^{-1}) - f(y^{-1})]. \quad (A2b)
\end{align*}

For a Higgs mass smaller than twice of that of the loop particle, i.e. $m_h < 2m_{\text{loop}}$, we have

\begin{align*}
f(x) &= \arcsin^2 \sqrt{x}, \quad (A3a) \\
g(x) &= \sqrt{x^{-1} - 1}\arcsin \sqrt{x}. \quad (A3b)
\end{align*}

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