Seismically Determined Acoustic Gruneisen Parameter in the Earth’s core

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Abstract. The determination of the thermodynamic properties of the Earth’s core requires the computation of acoustic Grüneisen parameter of solids at the prevailing pressure and temperature of the Earth’s core. The acoustic Grüneisen parameter, γa, of the Earth’s core were determined seismically from the velocity and density profiles. In this paper, the seismic data from the Preliminary Reference Earth Model (PREM) were used in computing the acoustic Grüneisen parameter γa at each depth of the Earth’s core. The thermal derivatives \(\frac{d (\ln V_s)}{d (\ln \rho)}\) and \(\frac{d (\ln V_p)}{d (\ln \rho)}\) which defines the modes from the velocity and density profiles were used to determine the values of the acoustic Grüneneisen parameter. The result for the outer core showed that the average \(\gamma_a\) is 1.53 which is found to be consistent with result obtained from previous studies.

Keywords: Velocity profiles, Thermodynamic properties, Earth’s core, Density profiles, Earth’s model

1. Introduction

The Earth’s interior is inaccessible. Consequently, human understanding of the Earth’s interior is limited. Detail information about the properties in the Earth’s interior can be obtained from seismological investigation of seismic waves [1]. An understanding of the Physics of the Earth’s interior requires information about the thermodynamic properties of its constituent minerals. However, the determination of these thermodynamic properties has been an age long challenge in Solid Earth Physics [2]. The thermodynamic properties of interest includes the Gruneisen parameter which is pertinent in understanding the physical properties of the Earth’s core and materials subjected to...
conditions of high temperatures and pressure obtained in the core. The core occupies the central portion of the Earth and it consists of the outer core and the inner core. The radius of the outer core is 3480 km while that of the inner core is 1216 km. Reviews of some of the properties of the Earth’s core have been published by [3-10]. The Gruneisen parameter can be determined from seismic profiles such as primary wave (V_p), shear wave (V_s) and density at each depth which are all contained in seismic earth model. The existing seismic data obtained from seismic earth models such as Parametric Earth Model (PEM) [11], thermal model of the earth [12] and Debye model [13] had been employed in the determination of Gruneisen parameter in the deep earth’s interior. However, [14] suggested the use of improved seismic models to accurately determine Gruneisen parameter which serves as a direct link between the thermal and seismic properties of the earth’s interior. In an attempt to accurately determine the Gruneisen parameter in the earth’s core, a globally accepted parameterized earth model called the Preliminary Reference Earth Model (PREM)[15] was employed.

The purpose of this paper is to determine the values of Gruneisen parameter in the Earth’s core using seismic data obtained from the PREM.

2. Method of Analysis

The thermal Gruneisen parameter is an important thermodynamic property of the Earth’s interior and is defined as:

$$\gamma = \frac{\alpha K_s}{\rho C_p}$$  \hspace{1cm} (1)

where $\alpha$ is the thermal expansivity, $K_s$ is the adiabatic incompressibility, $\rho$ is the density and $C_p$ is the specific heat at constant pressure.

Another definition of Gruneisen parameter which establishes the relationship between the thermal and elastic properties of a solid material is:

$$\gamma = V \left( \frac{\partial P}{\partial U} \right)_V$$ \hspace{1cm} (2)

where $P, V, U$ are pressure, volume and energy density, respectively. Equation (2) shows that Gruneisen parameter can be considered as the measure of the change in pressure resulting from the increase in energy density at constant volume. For the purpose of this study which involves the use of seismic data, high temperature acoustic Gruneisen parameter defined below is used:

$$\gamma_a = \frac{1}{3} \left( 2\gamma_s + \gamma_p \right)$$ \hspace{1cm} (3)

where

$$\gamma_s = \frac{1}{3} + \frac{d(\ln V_s)}{d(\ln \rho)}$$ \hspace{1cm} (4)

and
Equation (3) shows that $\gamma_a$ is the average of the P wave mode $\gamma_p$ and the two shear wave modes $\gamma_s$. The detail derivation of equations (1-5) is given by [16]. All the seismic parameters $V_s, V_p, \rho$ at every depth in the Earth’s core are contained in the PREM. The acoustic Gruneisen parameter is significant because it uses all the seismic data, $V_s, V_p, \rho$ at every depth.

3. Results and Discussion

Previous studies such as [12] used the classical point of view to determine the values of Gruneisen parameter in the lower mantle. In this present study, we have adopted the atomic point of view by using equation (3) to compute the values of the acoustic Gruneisen parameter in the earth’s inner core and outer core. This method is preferable because it uses all the seismic data contained in PREM seismic earth model at every depth. Seismic data such as the $V_p, V_s, \rho$ are in four significant figures and this allowed the computation of $\gamma_a$ to be in three significant figures. The computed values of $\gamma_a$ for the outer core are shown in Table 1 while the values of $\gamma_a$ for the inner core are displayed in Table 2. The corresponding values of $\gamma_a$ for the outer core presented by [14] are shown in the last column of Table 3 and this is compared to the values obtained in this present study using the same depth. The value of $\gamma_a$ decreased with increasing depth except at 4971 km depth where phase changes exist and corresponds to inner-outer ore boundary (ICB). From Table 1, the average value of $\gamma_a$ is 1.53. Computed values of $\gamma_a$ for the inner core using PREM data for this present study along with Anderson’s values of $\gamma_a$ are reported in Table 4. The detailed calculations of $\gamma_a$ of hexagonal close-packed (hcp) iron by [16] are reported in Table 5. It is seen from Table 5, that $\gamma_a$ for the core mantle boundary (CMB) at pressure 135 GPa is 1.62. This correlate well with the value of 1.64 obtained from this study at 2891 km depth. [14] reported $\gamma_a = 1.8$ for $\alpha$-iron at pressure 330 Gpa which compares favourably with $\gamma_a = 1.82$ at depth 5171 km in Table 1. Thus, the results obtained from this present study are contained within a narrow range determined by experimentalist for iron which is useful for further geophysical studies.

4. Conclusion

The values of Gruneisen parameter in the earth’s core had been determined using the seismic data from the Preliminary Reference Earth Model. The value of the Gruneisen parameter for the outer core is 1.53 which is consistent with the results obtained from the previous studies. The value
of Gruneisen parameter in the inner core decreases slowly with the depth from its value at the inner–outer core boundary. The value of Gruneisen parameter decreases from 1.82 at the inner–outer core boundary to 1.64 at the core–mantle boundary. The results obtained could lead to further research on the properties of iron at the prevailing conditions of high temperature and pressure in the earth’s core.

Table 1: Gruneisen parameter in the Outer core

| Pressure/GPa | Depth/Km | $\gamma_S$ | $\gamma_P$ | $\gamma_a$ |
|--------------|----------|------------|------------|------------|
| 135.8        | 2891.0   | 0          | 1.64       | 1.64       |
| 144.2        | 2971.0   | 0          | 1.62       | 1.62       |
| 154.7        | 3071.0   | 0          | 1.61       | 1.61       |
| 165.1        | 3171.0   | 0          | 1.59       | 1.59       |
| 175.4        | 3271.0   | 0          | 1.57       | 1.57       |
| 185.6        | 3371.0   | 0          | 1.56       | 1.56       |
| 195.7        | 3471.0   | 0          | 1.54       | 1.54       |
| 205.6        | 3571.0   | 0          | 1.53       | 1.53       |
| 215.3        | 3671.0   | 0          | 1.52       | 1.52       |
| 224.8        | 3771.0   | 0          | 1.51       | 1.51       |
| 234.2        | 3871.0   | 0          | 1.50       | 1.50       |
| 243.2        | 3971.0   | 0          | 1.49       | 1.49       |
| 252.1        | 4071.0   | 0          | 1.48       | 1.48       |
| 260.7        | 4171.0   | 0          | 1.48       | 1.48       |
| 269.0        | 4271.0   | 0          | 1.48       | 1.48       |
| 277.0        | 4371.0   | 0          | 1.49       | 1.49       |
| 284.8        | 4471.0   | 0          | 1.49       | 1.49       |
| 292.2        | 4571.0   | 0          | 1.50       | 1.50       |
| 299.3        | 4671.0   | 0          | 1.52       | 1.52       |
| 306.1        | 4771.0   | 0          | 1.46       | 1.46       |
| 312.6        | 4871.0   | 0          | 1.57       | 1.57       |
| 318.7        | 4971.0   | 0          | 1.60       | 1.60       |
| 324.5        | 5071.0   | 0          | 1.64       | 1.64       |

Table 2: Gruneisen parameter in the Inner core

| Pressure/GPa | Depth/Km | $\gamma_S$ | $\gamma_P$ | $\gamma_a$ |
|--------------|----------|------------|------------|------------|
| 328.9        | 5149.5   | 2.11       | 1.22       | 1.81       |
| 330.0        | 5171.0   | 2.15       | 1.15       | 1.82       |
### Table 3: Comparison of computed values of Gruneisen parameter for the outer core using data from PREM with other reported data

| Depth/km | V<sub>S</sub>/Kms<sup>-1</sup> | V<sub>P</sub>/Kms<sup>-1</sup> | Density ρ/gcm<sup>-3</sup> | γ<sub>a</sub> = γ<sub>ρ</sub> Present | γ<sub>a</sub> [14] |
|----------|------------------|------------------|------------------|------------------|------------------|
| 2971.0   | 0                | 8.199            | 10.029           | 1.62             | 1.50             |
| 3171.0   | 0                | 8.513            | 10.327           | 1.59             | 1.68             |
| 3371.0   | 0                | 8.796            | 10.602           | 1.56             | 1.56             |
| 3571.0   | 0                | 9.050            | 10.853           | 1.53             | 1.66             |
| 3771.0   | 0                | 9.279            | 11.083           | 1.51             | 1.40             |
| 3971.0   | 0                | 9.484            | 11.293           | 1.49             | 1.61             |
| 4171.0   | 0                | 9.669            | 11.483           | 1.48             | 1.56             |
| 4371.0   | 0                | 9.835            | 11.655           | 1.49             | 1.50             |
| 4571.0   | 0                | 9.986            | 11.809           | 1.50             | 1.49             |
| 4771.0   | 0                | 10.123           | 11.947           | 1.46             | 1.25             |
| 4971.0   | 0                | 10.250           | 12.069           | 1.60             | 1.50             |

### Table 4: Comparison of computed values of Gruneisen parameter for the inner core using data from PREM with other reported data

| Depth/km | V<sub>S</sub>/Kms<sup>-1</sup> | V<sub>P</sub>/Kms<sup>-1</sup> | Density ρ/gcm<sup>-3</sup> | γ<sub>a</sub> (present) | γ<sub>a</sub> [14] |
|----------|------------------|------------------|------------------|------------------|------------------|
| 5171.0   | 3.510            | 11.036           | 12.775           | 1.82             | 1.54             |
| 5271.0   | 3.535            | 11.072           | 12.825           | 1.82             | 1.47             |
| 5371.0   | 3.558            | 11.105           | 12.871           | 1.85             | 1.56             |
| 5471.0   | 3.579            | 11.135           | 12.912           | 1.83             | 1.50             |
| 5571.0   | 3.598            | 11.162           | 12.949           | 1.84             | 1.54             |
| 5671.0   | 3.614            | 11.185           | 12.982           | 1.79             | 1.53             |
Table 5: Gruneisen parameter along the calculated solidus of hcp iron[16]

| Pressure/GPa | $\gamma_a$ |
|--------------|------------|
| 55.0         | 1.74       |
| 58.1         | 1.73       |
| 71.6         | 1.70       |
| 88.0         | 1.68       |
| 97.5         | 1.67       |
| 119.5        | 1.64       |
| 135.0        | 1.62       |
| 146.5        | 1.62       |
| 162.2        | 1.61       |
| 179.7        | 1.60       |
| 199.2        | 1.59       |
| 220.8        | 1.58       |
| 240.0        | 1.56       |
| 244.9        | 1.56       |
| 271.7        | 1.55       |
| 301.7        | 1.54       |
| 330.0        | 1.53       |
| 335.3        | 1.53       |
| 372.8        | 1.52       |
| 414.9        | 1.50       |

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