Bioeconomic two predator-prey model of harvesting fishery

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Abstract. In this article, the research conducted is a comparison of the Leslie-Gower model with logistical growth in the harvesting of predatory and prey populations. This study discusses reconstructing a modified logistic growth model by using the Holling III response function as well as voting on predator and prey populations, analyzing equilibrium points, determining and bionomic equilibrium. The bionomic equilibrium from harvesting was carried out on the Leslie-Gower modification model

1. Introduction

The predator-prey system is a model that has long been studied because it deals directly with nature and the existence of creatures in it. This model is so popular that many researchers then study, develop and modify the model to conform to the actual conditions in nature[1]. This model discusses two species that interact with each other, where one of them preys on the other to survive. Predator and prey populations with economic values can be harvested. Predator and prey growth rates are influenced by a large number of populations harvested. If the results of harvesting do benefit, harvesters tend to harvest as much as possible to get maximum profit[2]. Continuous harvesting of populations without considering population numbers can lead to scarcity or extinction and even sustainable harvesting cannot be carried out.

Predator and prey populations with economic values can be harvested. Predator and prey growth rates are influenced by a large number of populations harvested. If the results of harvesting do benefit, harvesters tend to harvest as much as possible to get maximum profit[3]. Continuous harvesting of a population without considering population numbers can lead to scarcity or extinction and even sustainable harvesting cannot be carried out.

[4] analyzes the harvesting dynamics of both species. The growth rate model of the population is logically based on Pontryagin’s maximum principle for determining optimal policies for maximum yield and harvesting can be carried out sustainably. Another study of ecological models was also carried out by [5] who discussed the dynamics of the Leslie-Gower model with the Holling II and Toaha response functions [6] using the Leslie Gower model with the Holling II response function.
2. Model

The Leslie-Gower model assumes that carrying capacity predators are influenced by pre-density, i.e.
\[
\frac{dy}{dt} = r_2y - \frac{y^2}{c}
\]
with \(L = l_1x\) is carrying capacity from of predators which is proportional to the number of prey and \(l\) is the prey conversion coefficient to become a predator, so equation (1) becomes:
\[
\frac{dy}{dt} = r_2y - \frac{y^2}{lx}
\]
and \(\frac{y}{lx}\) the form is assumed to be a loss of predator because of its predominant scarcity[7]. Scarcity in prey can stimulate predators to look for alternative alternatives, but its growth has a limit (carrying capacity), so a positive constant is added \(d\) because predators do not depend on a single prey, so it becomes[6]:
\[
\frac{dy}{dt} = r_2y - \frac{y^2}{lx+d}
\]
With \(u_2 = \frac{1}{l} k_2 = \frac{q_2}{r}\) and Holling III response function as a predation response function, then model becomes:
\[
\frac{dx}{dt} = r_1x \left(1 - \frac{x}{K}\right) - \frac{mxy}{a+xy} - q_1Ex
\]
\[
\frac{dy}{dt} = r_2y \left(1 - \frac{y}{K}\right) - \frac{mbyy}{a+yy} - q_2Ey
\]
there is an effort to harvest \(E(t)\) on predators and prey that have economic value so that it is done in both populations so that they are obtained,
\[
\frac{dx}{dt} = r_1x \left(1 - \frac{x}{K}\right) - \frac{mxy}{a+xy}
\]
\[
\frac{dy}{dt} = r_2y \left(1 - \frac{y}{K}\right) - \frac{mbyy}{a+yy}
\]
Parameter \(r_1, r_2, m, a, \) and \(K\) positive with \(r_1\) and \(r_2\) is the intrinsic growth rate of prey and predator, \(m\) is the rate of interaction between the prey, \(\beta\) predation rate of a predator, \(a\) is the rate of interaction between predators, \(K\) is caring capacity and \(E\) is harvesting done on predators and prey[8]. From the model (6) and (7), two non-feasible equilibrium points are obtained \((0,0), \left(0, \frac{r_1E}{a}\right)\), equilibrium point is unstable \(T_1(x_1^*, 0)\) and an equilibrium point that is asymptotically stable \(T_2(x_2^*, y_2^*)\).

3. Result and Discussion

3.1 Bionomic equilibrium

Bionomic equilibrium is a concept that integrates equilibrium in biology and economic balance. Suppose there are two populations reviewed, i.e. \(x\) and \(y\). Ecological balance is obtained when the population is in a condition \(\frac{dx}{dt} = 0\) dan \(\frac{dy}{dt} = 0\). This balance point is obtained by completing the system of equations for \(x\) and \(y[9]\). This equilibrium point states a condition where both populations are in equilibrium, that is, the two populations do not change because the growth rate is zero[10]. Economic equilibrium is said to be achieved when the total income \((TR)\) obtained from the sale of stock \((population)\) harvested is equal to the total cost \((TC)\) of the effort made in harvesting the population [6]. The general form of harvesting uses the Schaefer harvesting equation (1954) harvesting is formulated [11]:
\[
h = qEx
\]
with \(h\) is the rate of harvesting with total net revenue per unit time. The profit function of a population harvesting business is:
\[
\pi = TR - TC
\]
Total income \((TR)\) is obtained from the biomass unit price \(p\) multiplied by the rate of harvest \(h\). Whereas the total financing \((TC)\) is proportional to the cost of harvesting per business unit \(c\) multiplied by the effort \(E\) formulated [12]:
\[ \pi = ph - cE \]

Thus, the benefit function equation with harvesting carried out in the predator-prey population is as follows:

\[ \pi(E) = (p_1q_1x + p_2q_2y - c)E \]  

With \( p_1 \) the price is prey per unit of biomass, \( p_2 \) the price of predators per unit of biomass and \( c \) is the financing spent in harvesting the population [13].

Bioeconomic balance is \( E_* = (x_*, y_*, E_*) \) with \( x_*, y_*, E_* \) is a positive solution from the following equation:

\[ \frac{dx}{dt} = r_1x \left( 1 - \frac{x}{K} \right) - \frac{mx_1}{a + x} - q_1Ey = 0 \]  
\[ \frac{dy}{dt} = r_2y \left( 1 - \frac{y}{K} \right) - \frac{mby_2}{a + y} - q_2Ey = 0 \]  

\[ \pi(E) = (p_1q_1x + p_2q_2y - c)E = 0 \]

From equations (14),(15) and (16) are obtained:

\[ E = \frac{1}{q_2} \left( r_1 \left( 1 - \frac{x}{K} \right) \frac{a + y}{a + y} - \frac{q_1}{q_2} \left( r_2 \left( 1 - \frac{y}{K} \right) \frac{a + x}{a + x} \right) = 0 \]

If the costs incurred for harvesting are greater than the income from the sale of the harvest \((E > p_1q_1x + p_2q_2y)\), then the profit function will be negative and cause harvesting to stop and no bionomic equilibrium. To have a bionomic equilibrium, it is assumed that the costs incurred for harvesting are smaller than the income obtained from the sale of the harvest[14] \((E < p_1q_1x + p_2q_2y)\). Through equations (14) and (15) are obtained:

\[ y_* = E \]  
\[ b_* = \frac{C}{q_2} \frac{a + x}{a} \frac{a + y}{a + y} \]

With \( \frac{a_2E - k_2p_2q_2}{q_1p_1q_1 + q_1p_1q_2} < y_* < \frac{C}{p_1q_1} \) and \( 0 < y_* < -\frac{1}{p_1q_1} \left( (a_2E - k_2p_2q_2)/(p_1q_1) - C \right) \)

\[ (a_2E - k_2p_2q_2 + Ca_2a_2q_1 - p_2q_2t_2 + k_2p_2q_2/2 + Ca_2 + r_2p_2q_2/(a_2p_2q_1 + p_2q_2t_2)/(a_2p_2q_1 + p_2q_2t_2)(k_2p_2q_2/(a_2p_2q_1 + p_2q_2t_2)) - (k_2p_2q_2 + Ca_2p_2q_2^2 < b_* < (E - a_2p_2q_1 - p_2q_2t_2 + k_2p_2q_2/(a_2p_2q_1 + p_2q_2t_2)) \]

If \( E > b_* \), then the total financing from harvesting is greater than the total income from harvesting, so the harvest will suffer losses, so it cannot be continued. If \( E < b_* \), then harvesting benefits, harvesting can continue.

3.2 Numeric Simulation

Parameter values are based on previous studies and assumptions in this study. The parameter values used in the simulation are \( r_1 = 0.47, r_2 = 0.2, m = 0.8, a = 200, k = 100, p_1 = 0.26, p_2 = 0.32, q_1 = 0.4, q_2 = 0.2, c = 0.18 \).

The equilibrium point in the Leslie-Gower model of the holling III response function without harvesting is obtained[15] \( T_2 = (5.5203, 2.9215) \) with Jacobian Matrix \( J(T_2) = \begin{pmatrix} -0.4552 & -0.1058 \\ 0.0301 & -0.1600 \end{pmatrix} \)

The characteristic equation of the matrix \( J(T_2) \) is \( \lambda^2 + 0.1718 \lambda + 0.0012 \). The eigenvalue obtained from the characteristic equation is \( \lambda_1 = -0.1645 \) dan \( \lambda_2 = -0.0073 \), so that the \( T_2 \) s equilibrium point is asymptotically stable. The trajectory around the equilibrium point \( T_2 \) with some initial values is as follows:
Figure 1. A trajectory around the equilibrium point $T_2$

Bionomic equilibrium occurs when the total income from the sale of harvest equals the financing of harvesting or the profit function equals 0 so that the value obtained $E = 3.602274535$. Value $E = 3.602274535$ then it is substituted for equilibrium point $H_2$ so that the equilibrium point is obtained $H_2(2.1822, 0.1043)$ with the Jacobian matrix $J(H_2) = \begin{pmatrix} -0.1645 & -0.0186 \\ 0.0001 & -0.0073 \end{pmatrix}$. The characteristic equation of the matrix $J(H_2)$ is $\lambda^2 + 0.1718\lambda + 0.0012$. The eigenvalue obtained from the characteristic equation is $\lambda_1 = -0.1645$ dan $\lambda_2 = -0.0073$, so it's an equilibrium point $H_2$ asymptotic stable. Trajectory around equilibrium points $H_2$ with some initial values as follows:

Figure 2. A directory around the equilibrium point $H_2$

Bionomic equilibrium occurs when harvesting is done by harvesting limits for predatory and prey populations $P(x, y, E)$ is $P(2.1822, 0.1043, 3.602274535)$. 


4. Conclusion
The modification of the Leslie-Gower model with the logistical growth of the function of response III and binomic harvesting in two predatory and prey populations is:

\[
\begin{align*}
\frac{dx}{dt} &= r_1x\left(1 - \frac{x}{K}\right) - \frac{mxy}{a + x} - q_1Ex \\
\frac{dy}{dt} &= r_2y\left(1 - \frac{y}{K}\right) - \frac{mxy}{a + x} - q_1Ex
\end{align*}
\]

Produce feasible \((0,0)\), \((\frac{K}{u_1}, 0)\), an unstable equilibrium point \(T_1(x_1^*, 0)\) and an asymptotically stable equilibrium point \(T_2(x_2^*, y_2^*)\). The binomic equilibrium from harvesting was carried out in the modification of the two predator-prey models \(F(x_{x0}, y_{x0}, E_{x0})\) with

\[
0 < y_{x0} < \frac{1}{\alpha_1q_1}\left(\frac{(\alpha_2L - k_2q_2)p_1q_1}{u_2q_1 + p_1q_2} - L\right)
\]

\[
\left((-k_2q_2q_2 + \alpha_2L(u_2q_1 - p_1q_2q_2 + (k_2q_2q_2q_2 - u_2q_1 + p_1q_2q_2q_2)/(\alpha_1p_1q_1 + p_2q_2q_2)(\alpha_2p_1q_1 + p_2q_2q_2q_2))/(u_2q_1 + p_1q_2) - \frac{k_2}{u_2q_1 + p_1q_2q_2} - \frac{L}{u_2q_1p_1q_2q_2} - \frac{E_{x0}}{L}\right)
\]

\[
0 < y_{x0} < \frac{1}{\alpha_1q_1}\left(\frac{(\alpha_2L - k_2q_2)p_1q_1}{u_2q_1 + p_1q_2} - L\right)
\]

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