Matrix model and string field theory

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Abstract: In this short note we would like to show the relation between the cubic open string field theory for $N$ D-instantons and the string field theory in the presence of the background B field.

Keywords: D-branes, string field theory, noncommutative geometry
1. Introduction

Tachyon condensation has been one of the most studied problems in string theory in the past two years [1, 2, 3], for review see [4, 8, 9] and for the recent discussion the relation between the tachyon condensation and K-theory, see [5, 6, 7]. Evidence for this proposal was given from the analysis of CFT description of this system [1], for review of this approach, see [8, 9]. It was also shown on many examples that string field theory approach to this problem is very effective in the calculation of the tachyon potential [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 32, 33, 34, 35], for review see [31]. This problem was recently studied from the point of view of the Witten's background independent open string field theory [36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49] as well. Success of the string field theory in the analysis of the tachyon condensation indicates that the string field theory could play more fundamental role in the nonperturbative formulation of string theory.

The second approach to the problem of tachyon condensation is based on the idea of the noncommutative geometry [51]. This analysis has been inspired with the seminal paper [52]. Application of this approach to the problem of the tachyon condensation was pioneered in [53, 54]. This research was then developed in other papers [55, 56, 57, 58, 59, 60, 61, 62, 63]. The tachyon condensation has been also studied from the point of view of the matrix model [64, 65, 66]. Success of these two approaches immediately leads to the question whether there is any relation between the matrix model and consequently noncommutative theory [55] and the string field theory. The similar problem was analysed in the recent paper [62] in the framework of the effective string field theory.
In this paper we would like to address this problem in the context of the string field theory for general configuration of $N$ D(-1)-branes (D-instantons) in bosonic string theory. We propose the generalised form of the open string field theory that allows description of any configuration of D-instantons. We will make many consistent checks justifying our approach, in particular, we will show that the new matrix valued BRST operator is nilpotent on condition that the background configuration of $N$ D-instantons obeys the equation of motion familiar from the matrix models. We will show that this theory obeys generalised form of the string field theory axioms that were recently discussed in the papers. We will also show that the emergence of lower dimensional D-branes from D25-brane is very natural process from the point of view of matrix string field theory. In fact, the efficiency of the matrix theory description of the tachyon condensation was recently stressed in.

In section (2) we review the basis facts about cubic string field theory. We will be very brief, more information can be found in the nice papers.

In section (3) we will discuss the string field theory for $N$ D(-1)-branes (D-instantons). We propose the modification of the BRST operator for $N$ D-instantons in such a way that we will be able to take into account their general space-time positions. We will show that when we consider the configuration of $N$ separate D-instantons then off-diagonal modes of string field become massive according to the nonzero string winding charge. Then we will study the noncommutative background of D-instantons and we show that the string field action for $N$ D-instantons in the limit $N \to \infty$ becomes the string field theory action for D-brane with the non-commutative world-volume. In this way we will show that string field theory of $N$ D-instantons is able to describe all even dimensional D-branes in the same way as D-branes emerge in the matrix theory.

In section (4) we will discuss the tachyon condensation on the world-volume of the D25-brane in the presence of the background B field. Using the result given in the section (3) we present a simple solution describing the tachyon condensation into $k$ D2p-branes.

In the conclusion (5) we will discuss some open problems and implication of our results.

2. Brief review of string field theory

In this section we will briefly review the Witten’s open string field theory. The Witten’s formulation is in noncommutative extension of differential geometry, where string fields, BRST operator $Q$ and the integration over string configuration $\int$ in string theory are analogies of differential forms, the exterior derivative $d$ and the integration over the manifold $M$ in the differential geometry, respectively. The ghost number assigned to the string field corresponds to the degree of differential form. Also
the noncommutative products between string fields $\star$ is interpreted as an analogy of the wedge product $\wedge$ between differential forms.

The axioms obeyed the system of $f, \star$ and $Q$, are

\[ \int QA = 0 \,, \]

\[ Q(A \star B) = (QA) \star B + (-1)^A A \star (QB) \,, \]

\[ (A \star B) \star C = A \star (B \star C) \,, \]

\[ \int A \star B = (-1)^{AB} \int B \star A \,, \]

(2.1)

where $A, B, C$ are arbitrary string fields. As was shown in [10] in order to describe a gauge invariant string field theory we must include the full Hilbert space of states of the first quantized open string theory including the $b$ and $c$ ghost fields, subject to the condition that the state must carry ghost number 1. Here we are using the convention that $b$ carries ghost number $-1$ and $c$ carries ghost number 1 and $SL(2, R)$ invariant vacuum $|0\rangle$ carries ghost number 0. In the previous expression $Q$ is BRST operator of the first quantized open string. The string field theory action for Dp-brane is

\[ S = \frac{2\pi^2 T_p}{g_s} \int \left( \frac{1}{2} \langle \Psi | Q | \Psi \rangle + \frac{1}{3} \langle f_1 \circ \Phi(0) f_2 \circ \Phi(0) f_3 \circ \Phi(0) \rangle \right) , \]

(2.2)

where $T_p = 2\pi/(4\pi^2\alpha')^{(p+1)/2}$ is a D-brane tension and $g_s$ is a string coupling constant. This abstract form of the string field action can be written in the other form appropriate for the calculation. It is useful to write it in terms of the conformal field theory (CFT) [39]. Let $|\Phi\rangle$ be an arbitrary state in $\mathcal{H}$ the full Hilbert space of states of the first quantized open string theory and let $\Phi(x)$ be a local field (vertex operator) in conformal field theory which creates this state $|\Phi\rangle$ from out of $SL(2, R)$ invariant vacuum

\[ |\Phi\rangle = \Phi(0) |0\rangle . \]

(2.3)

In the CFT language, the string field action is given

\[ S = \frac{2\pi^2 T_p}{g_s} \left( \frac{1}{2} \langle \Psi | Q | \Psi \rangle + \frac{1}{3} \langle f_1 \circ \Phi(0) f_2 \circ \Phi(0) f_3 \circ \Phi(0) \rangle \right) , \]

(2.4)

where $f_i$ are known conformal maps reviewed in [11] and $f \circ \Phi(0)$ denotes conformal transformation of the vertex operator $\Phi$ by $f$.

We must also mention that there is a formulation of the string field theory in terms of the operator formalism [72, 73]. The operator formalism was used in [70, 71] where the string field theory in the constant B field background was studied.

In the following we will work mainly with the abstract definition of the string field theory given in [10]. In the next section we will discuss the string field theory for $N$ D-instantons.

\footnote{We use normalisation given in [11] and we work in Euclidean signature.}
3. String field theory for $N$ D-instantons

In this section we propose the action for $N$ D-instantons. This can be done very easily in such a way that all string fields will carry the indeces corresponding to the adjoint representation of the gauge group $U(N)$. Then the string field action for $N$ D-instantons has a form

$$S = \frac{2\pi^2 T_{-1}}{g_s} \int \left( \frac{1}{2} \Psi_{ij} \star \tilde{Q}_{\text{inst}}^{ij} \Psi_{ki} + \frac{1}{3} \Psi_{ij} \star \Psi_{jk} \star \Psi_{ki} \right),$$

(3.1)

where

$$\tilde{Q}_{\text{inst}}^{ij} = \tilde{Q}_{\text{inst}}^{ij} \delta_{ij},$$

(3.2)

with the BRST operator $\tilde{Q}_{\text{inst}}^{ij}$ of the string living on one single D-instanton. However, there is a one important issue with this action. This action describes the fluctuations around the background corresponding to $N$ D-instantons in the same place in the space-time so that the $U(N)$ symmetry of the action is unbroken. Under this symmetry the string field transform as $\Psi' = U \Psi U^{-1}, U \in U(N)$. In order to include the more general background configuration of $N$ D-instantons, we propose the new BRST operator for $N$ D-instantons in the form

$$Q_{ij} = Q_{ij}^{\text{inst}} + Q_{ij}^{0},$$

(3.3)

where $Q_{ij}^{\text{inst}} = Q_{ij}^{\text{inst}} \delta_{ij}$ is the instanton BRST operator without zero mode part and $Q_{ij}^{0}$ is a generalised zero mode part of the BRST operator for $N$ instantons in the form

$$Q_{ij}^{0} = \sum_{n=-\infty}^{\infty} c_n L_{-n}^{0} = \frac{1}{2} c_0 g_{IJ}(\alpha_{0}^I \alpha_{0}^J)_{ij} + \sum_{n=-\infty, n \neq 0}^{\infty} c_n g_{IJ}(\alpha_{-n}^I \alpha_{0}^J)_{ij},$$

$$\frac{1}{2} c_0 g_{IJ}(\alpha_{0}^I \alpha_{0}^J)_{ij} = \frac{1}{4 \pi^2 \alpha'} c_0 g_{IJ} [X^I, [X^J, \Psi]]_{ij}, c_n g_{IJ}(\alpha_{-n}^I \alpha_{0}^J)_{ij} = \frac{\sqrt{2}}{2 \pi \sqrt{\alpha'}} c_n g_{IJ}(\alpha_{-n}^I [X^J, \Psi])_{ij},$$

(3.4)

where the action of this operator on any string field $\Psi$ is defined as

$$Q(\Psi)_{ij} = Q^{\text{inst}} \Psi_{ij} + c_0 \frac{1}{4 \pi^2 \alpha'} g_{IJ} [X^I, [X^J, \Psi]]_{ij} + \frac{\sqrt{2}}{2 \pi \sqrt{\alpha'}} \sum_{n=-\infty, n \neq 0}^{\infty} c_n g_{IJ} \alpha_{-n}^I [X^J, \Psi]_{ij}.$$

(3.5)

and where $X^I, I = 1, \ldots, 26$ are $N \times N$ matrices describing the background configuration of $N$ D-instantons. And finally, the normalisation of the various terms given above will be clear from next discussion.

Since (3.3) differs from the ordinary BRST operator, in particular, it is matrix valued, we should prove that it is nilpotent and that the string field theory defined in this way obeys all string field theory axioms (2.1).
We start with the proof of the nilpotence of $Q$ whose obvious generalisation is \[ Q_{ij}Q_{jk} = 0 \] (3.6)

Since we know that all new properties are included in the zero mode part of $\alpha_0$, in particular, the ghost part is the same as in the abelian case, it is sufficient for our purposes to show that $L_n$ obey the correct Virasoro algebra. In order to do that we define oscillator modes $\alpha_m$ as follows

\[(\alpha^I_m)_{ij} = \alpha^I_m \otimes \delta_{ij}, \ m \neq 0, \ [(\alpha^I_m), (\alpha^J_n)]_{ij} = m\delta_{m+n}\delta^I_j\delta_{ij}, \ m, n \neq 0 , \] (3.7)

where the matrix multiplication is understood. It is also clear that $\alpha^I_0$ commutes with $\alpha^I_m$ since $\alpha^I_m, m \neq 0$ are proportional to the identity matrix in the space of the Chan-Paton factors and $X_{ij}$ commutes with $\alpha_m$ from the basis definition of the commutation relations. The only not trivial task is to compute the commutator $[\alpha^I_0, \alpha^J_0]$. Firstly we define Virasoro generators

\[(L_m)_{ij} = \frac{1}{2} \sum_{n=-\infty}^{\infty} g_{IJ}(\alpha^I_{m-n}\alpha^J_n)_{ij}, (L_0)_{ij} = \frac{1}{2} g_{IJ}(\alpha^I_0\alpha^J_0)_{ij} + g_{IJ} \sum_{n=1}^{\infty} (\alpha^I_{-n}\alpha^J_n)_{ij} \] (3.8)

with

\[(\alpha^I_0)_{ij} = \frac{1}{\pi \sqrt{2\alpha'}} [X^I, \cdot]_{ij} . \] (3.9)

Now the commutator of two $\alpha^I_0$ is (when acts on any matrix $M$) equal to

\[ [\alpha^I_0, \alpha^J_0] M = \frac{1}{\sqrt{2\alpha'}\pi}(\alpha^I_0[X^J, M] - \alpha^J_0[X^I, M]) = \] \[ = \frac{1}{2\pi^2\alpha'}([X^I, [X^J, M]] - [X^J, [X^I, M]]) = \frac{1}{2\pi^2\alpha'}[[X^I, X^J], M] . \] (3.10)

Now we are ready to calculate the commutator $[L_m, L_n]$. In fact, the calculation of this commutator is well known for a long time, see for example [84]. The novelty in our approach is in the presence of the matrix valued zero mode operators $\alpha_0$. To illustrate this issue let us work out the commutator that is present in the calculation of the commutator of two Virasoro generators with $m + n \neq 0$

\[ [g_{IJ} \alpha^I_m \alpha^J_0, g_{KL}\alpha^K_n \alpha^L_0] = g_{IJ}g_{KL}(\alpha^I_m\alpha^K_n\alpha^J_0\alpha^L_0 - \alpha^K_n\alpha^I_m\alpha^L_0\alpha^J_0) = \] \[ = g_{IJ}g_{KL}(g^{IK} m\delta_{m+n}\alpha^J_0\alpha^L_0 + \alpha^K_m \alpha^I_m [\alpha^J_0, \alpha^L_0]) = g_{IJ}g_{KL} m \alpha^K_m \alpha^I_m [\alpha^J_0, \alpha^L_0] , \] (3.11)

where we have used the fact that $m + n \neq 0$. It can be shown that with nonzero upper result we cannot obtain the correct form of the Virasoro algebra. For that
reason we must demand the vanishing of the commutator $[\alpha^I_0, \alpha^J_0]$ that leads to the condition

$$[X^I, X^J] = i\theta^{IJ}1_{N\times N}$$

(3.12)

as we can see from (3.10). It is clear that the commutator can be nonzero only in the case of infinite dimensional matrices. Note that this expression has a form of the solution of the equations of motion obtained from the matrix model [74, 75, 76, 77].

$$[X^I, [X^I, X^J]] = 0.$$  

(3.13)

This result is an analogue of the case of the string propagating in the general background when the requirement of the conformal invariance leads to the condition that the background fields should be solutions of the equation of motion obtained from the space-time effective action. In our case this effective action is D-instanton effective action [76, 77]

$$S \sim \text{Tr}g_{IK}g_{JL}[X^I, X^J][X^K, X^L].$$

(3.14)

Using (3.12) it is now straightforward to prove that the Virasoro algebra has a correct form. We do not repeat the standard analysis here, more details can be found in [84] where it was also shown how we can determine the central charge of the Virasoro algebra so that we obtain the result

$$[L_m, L_n]_{ij} = (m - n)L_{ij} + A(m)\delta_{m+n} \delta_{ij}, \quad i, j = 1, \ldots, N,$$

(3.15)

where $A(m) = \frac{1}{12}c(m^3 - m)$ is a central charge of the Virasoro algebra. In the previous expression we have written explicitly the matrix indeces $i, j$ to stress the matrix nature of the generators $L_m$. Some comments about the previous result. Since the central charge is not affected by matrix valued nature of the Virasoro generators it is proportional to the unit matrix. In the same way we can argue that the ghost part of the action does not depend on the matrix notation. Then it immediately follows that the generalised BRST operator is nilpotent

$$Q^2_{ij} = \frac{1}{2}\{Q, Q\}_{ij} = 0,$$

(3.16)

in case of the critical bosonic string theory $D = 26$ [84]. Of course, for the existence of the nilpotent BRST generator is crucial the condition (3.12) which is nothing else than the requirement that the background configuration of $N$ D-instantons must be solution of the equations of motion of the low energy action.

Now we are ready to prove that the string field theory with the generalised BRST operator (3.3) obeys all axioms given in (2.1). In fact, we should consider the more general form of the axiomatic formulation which is appropriate for the general configuration of D-branes. In fact, this has been done in the abstract form in [85, 86] and more recently in the series of papers [87, 88, 89]. We do not mean to discuss these general constructions, see for example [88] for very nice explanation.
For our purposes it is sufficient to know that now the BRST operator $Q$ depends on the background configuration of D-branes. For that reason we should generalise the axioms given in (2.1) in order to include these properties. Intuitively, this can be seen as follows. Let us presume that we have a configuration of $N$ D-instantons described with the matrices

$$X^I = \begin{pmatrix} x^I_1 & 0 & \ldots & 0 \\ 0 & x^I_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \ldots & \ldots & x^I_N \end{pmatrix}, \quad I = 1, \ldots, 26,$$ (3.17)

then the second term in (3.5) acting on any string field $\Psi_{ij}$ gives

$$g_{II}[X^I, [X^J, \Psi]]_{ij} = g_{II}[X^I_{im}[X^J, \Psi]_{mj} - [X^J, \Psi]_{im}X^I_{mj}] =$$

$$= [x^I_i \delta_{im}[x^J_m \delta_{mk} \Psi_{kj} - \Psi_{mk} x^J_k \delta_{kj}]] - [x^I_i \delta_{ik} \Psi_{km} - \Psi_{ik} \delta_{km} x^J_k] x^I_m \delta_{mj} g_{II} =$$

$$= [x^I_i \delta_{im}[x^J_m - x^J_j]] \Psi_{mj} - [x^I_i - x^I_m] \Psi_{im} x^J_m \delta_{mj} g_{II} =$$

$$= g_{II}[x^I_i - x^I_j][x^I_i - x^J_j] \Psi_{ij},$$ (3.18)

which, after multiplication with $\frac{1}{2(2\pi\alpha')^2}$, gives precisely the value $\frac{1}{2}(M^2_{ij})$ where $M^2_{ij} = \frac{(\Delta x)^2}{(2\pi\alpha')^2}$ is a minimal mass for the string stretched between i-th and j-th D-instanton. Of course, it is slightly awkward to speak about the mass of the string with all Dirichlet conditions but as we will see in the moment the D-instanton configuration allows also existence of higher dimensional D-brane and then the previous calculation could be applied to the string stretched between different D-branes so that this analysis is correct. In the same way we can analyse the third term in (3.5). Finally, the emergence of the second factor $\alpha'$ in the previous result comes from $\alpha'p^2$ in the BRST operator. This momentum operator naturally arises from particular configuration of $N$ instantons as we will show in a moment. The previous result suggests that for D-instantons background configuration (3.17) the generalised BRST operator $Q$ can be written as a sum of the BRST operators

$$Q_N = \sum_{i,j} Q_{ij},$$ (3.19)

where $Q_{ij}$ is the BRST operator for the string connecting i-th and j-th D-instanton. It is understood in the previous expression that $Q_{ij}$ acts on the string field sector corresponding to the string going from i-th to j-th D-instanton only and consequently the BRST operator $Q_{ij}$ does not act on string states from different string sectors. It is well known that for each $i, j$ string sector the string field theory is correctly defined. Then we can very easily generalise the axioms (2.1) for any D-instanton background in the same way as in [51, 58, 59]. Firstly, it is clear that the generalisation of the
expression $\int \Psi$ which is linear in CP indeces is given as $\int \text{Tr}\Psi$. With this definiton it is natural the generalise the expression $\int Q\Psi$ as follows

$$\text{Tr} \int Q\Psi = 0$$

(3.20)

which for (3.17) gives

$$\int Q_{ij}\Psi_{ji} = \sum_{ij} \int Q_{ij}\Psi_{ij} = 0 .$$

(3.21)

We then see that the matrix valued BRST operator obeys the generalised first axiom in (2.1) for the background (3.17). Since we know that $Q$ is the correct BRST operator for any configuration of D-instantons (obeying (3.12)) and as is well known from the matrix theory proposal there is not any fundamental difference between (3.17) and more general configurations given in (3.12) we can claim that the generalised $Q$ operator obeys (3.20) as well.

As a next thing we turn to the second axiom in (2.1). As was shown in [88] this axiom should be modified in the presence of D-branes as follows. Let $A_{ij}$ corresponds to some string field for the string stretching between i-th and j-th D-instanton. Then it is clear that this string can be glued with strings ending or starting on i-th or j-th D-instantons. We see that the gluing operation $\star$ is naturally generalised to the matrix valued multiplication between matrix valued string fields $A, B$. We then obtain the string field $(A \star B)_{ij} = A_{ik} \star B_{kj}$ which corresponds to the string going from i-th to j-th D-brane that arises from the string stretching between i-th D-instanton and k-th D-instanton where it glues with the string stretching between k-th D-instanton and j-th instanton. Since there is no preferred D-instanton we should sum over all D-instantons which corresponds to the sum over $k$ in the upper expression. It is then natural to expect that the appropriate BRST operator acting on this string (going from i-th to j-th D-instanton) is $Q_{ij}$. Then we immediatelly obtain the generalisation of the second axiom in (2.1) in the form [88] (No summation over $i, j$ and we explicitly write the sum over $k$.)

$$Q_{ij}(A \star B)_{ij} = \sum_k (Q_{ik}A_{ik}) \star B_{kj} + (-1)^{A} \sum_k A_{ik} \star (Q_{kj}B_{kj}) .$$

(3.22)

We propose more general form of the second axiom in (2.1) that reduces to (3.22) in case of D-instanton background (3.17). This new form has an advantage that holds for any configuration of D-instantons on condition of validity (3.12). Simply, we propose that the second axiom in (2.1) is

$$Q(A \star B)_{ij} = (QA)_{ik} \star B_{kj} + (-1)^{A}A_{ik} \star (QB_{kj}) ,$$

(3.23)

where the acting of $Q$ on string fields is defined in (3.5). For (3.17), the left hand side of (3.23) is equal to

$$Q(A \star B)_{ij} = Q_{ij}(A \star B)_{ij}$$

(3.24)
and the right hand side

$$(Q(A) \ast B)_{ij} + (-1)^A (A \ast Q(B))_{ij} = \sum_k Q_{ik}(A)_{ik} B_{kj} + \sum_k (-1)^A A_{ik} (Q B)_{kj} .$$

(3.25)

so that we obtained from (3.23) the generalised second axiom (3.22). Using these results we can claim that the matrix valued BRST operator $Q$ obeys the first two generalised axioms (3.20),(3.23). To see this more precisely, we can argue as follows.

It is natural to expect that general configuration of D-instantons (when the background obeys the matrix theory equation of motion) arises from (3.17) as its solution of equation of motion. In other words, let us presume that general BRST operator can be written as

$$QA = Q_N A + \Phi_0 \ast A - (-1)^A A \ast \Phi_0 ,$$

(3.26)

where $A$ is any string field and where the matrix multiplication is understood (We will say more about this approach in the next section). From the fact that $Q^2 = Q_N^2 = 0$ we get from the upper expression (No summation over $i, j$)

$$(Q^2 A)_{ij} = 0 = [Q_{ik}(\Phi_0)_{ik} + (\Phi_0)_{il} \ast (\Phi_0)_{lk}] \ast A_{kj} +$$

$$+ (-1)^{2A+1} A_{ik} \ast [Q_{kj}(\Phi_0)_{kj} + (\Phi_0)_{kl} \ast (\Phi_0)_{lj}]$$

(3.27)

We see that the general BRST operator will be nilpotent in case when the string field $\Phi_0$ obeys the string field equation of motion for D-instantons background (3.17) (No summation over $i, k$)

$$Q_{ik}(\Phi_0)_{ik} + (\Phi_0)_{il} \ast (\Phi_0)_{lk} = 0 .$$

(3.28)

Then it is easy to see that the generalised BRST operator obeys all axioms given in (2.1). Firstly, we have

$$\text{Tr} \int QA = \int \left( \sum_{i,j} Q_{ij} A_{ij} + (\Phi_0)_{ij} \ast A_{ji} - (-1)^A A_{ij} \ast (\Phi_0)_{ji} \right) = 0 ,$$

(3.29)

where we have used the fact that $Q_{ij}$ obeys the first axiom in (2.1) and also we have used the fourth axiom in (2.1) together with the fact that $\Phi_0$ has a ghost number one. We can also show that (No summation over $i, j$)

$$Q(A \ast B)_{ij} = Q_{ij}(A \ast B)_{ij} + (\Phi_0)_{ik} \ast (A \ast B)_{kj} - (-1)^{A+B} (A \ast B)_{ik} \ast (\Phi_0)_{kj} =$$

$$= (QA)_{ik} \ast B_{kj} + (-1)^A A_{ik} \ast (QB)_{kj} .$$

(3.30)

So we see that the the generalised BRST operator obeys the second axiom (3.23).
As a further support of our proposal we will consider the background configuration of D-instantons in the form

\[ [X^a, X^b] = i\theta^{ab}_1 \eta_{N_N}, \quad a, b = 1, \ldots, 2p, \quad X^i = 0, \quad i = 2p + 1, \ldots, 26. \] (3.31)

We see that (3.31) belong to the class of the background configuration (3.12) hence BRST operator (3.3) is nilpotent and defines correct string field theory. Then we get

\[ \frac{1}{4\pi^2\alpha'} g_{ab} [X^a, [X^b, \Psi]] = \frac{1}{4\pi^2\alpha'} g_{ac} \theta^{cd} \theta_{ef} [X^e, [X^f, \Psi]] = -\alpha' G^{ab}[C_a, [C_b, \Psi]], \] (3.32)

where we have used

\[ G^{ab} = -(2\pi\alpha')^{-2} \theta^{ac} g_{cd} \theta^{db}, \quad C_a = -i \theta_{ab} X^b, \] (3.33)

In the same way we obtain

\[ \frac{\sqrt{2}}{2\pi\sqrt{\alpha'}} c_n g_{ab} \alpha_{-n}^a [X^b, \Psi] = \frac{i\sqrt{2}}{2\pi\sqrt{\alpha'}} c_n (2\pi\alpha')^2 G^{ce} \theta_{cd} \alpha_{-n}^d [C_e, \Psi] = -i\sqrt{2}\alpha' c_n G^{ab} \tilde{\alpha}_{a,-n} [C_b, \Psi], \quad \tilde{\alpha}_{a,-n} = -(2\pi\alpha') \theta_{ab} \alpha_{-n}^b. \] (3.34)

We see that the expression \( [C_a, \Psi]_{ij} \) is the derivation in the operator formalism of the noncommutative theory \([52, 57]\). Using the correspondence between operators and functions on noncommutative space-time we obtain

\[ [C_a, \Psi] \leftrightarrow \partial_a \Psi(x), \] (3.35)

where now \( \Psi \) looses all the gauge group indeces and becomes function on the noncommutative space-time with coordinates \( x^a, a = 1, \ldots, 2p. \)

This can be seen more precisely as follows. The general string field \([10]\) should have a ghost number 1. For that reason we can write any string field as

\[ |\Psi\rangle_{ij} = \sum_{n,m,l} A^{nm} |n, m, l\rangle, \] (3.36)

where

\[ |n, m, l\rangle = \alpha_{-n}^{\mu_1} \ldots \alpha_{-n_i}^{\mu_i} b_{-m_1} \ldots b_{-m_j} c_{-l_1} \ldots c_{-l_k} |\Omega\rangle, \quad |\Omega\rangle = c_1 |0\rangle, \quad n > 0, m > 0, l \geq 0 \] (3.37)

form the basis of the of the Hilbert space of the first quantised open string restricted to the states with the ghost number 1 and obeying Dirichlet boundary conditions.

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3We work with the metric in the diagonal form \( g_{IJ} = g_{II} \delta_{IJ}. \)
Note that $A_{nm}$ are $N \times N$ matrices describing various string fields. In the following we will presume the limit $N \to \infty$.

From (3.31) we can also anticipate that the correct open string quantities are

$$G_{ab} = -(2\pi \alpha')^2 (Bg^{-1} B)_{ab},$$
$$G_s = g_s \det(2\pi \alpha' Bg^{-1})^{1/2},$$
$$B_{ab} = \left(\frac{1}{\theta}\right)_{ab},$$

and consequently

$$\frac{T_{-1}}{g_s} = \frac{T_{2p-1}}{G_s} \sqrt{\det G(2\pi)^p} \sqrt{\det \theta}. \quad (3.38)$$

Then we can write

$$\frac{2\pi^2 T_{-1}}{g_s} \text{Tr} \langle \Psi | -\alpha' G^{ab} c_0[C_a, [C_b, |\Psi\rangle]] = -\frac{2\pi^2 T_{2p-1}}{G_s} \sqrt{\det G(2\pi)^p} \sqrt{\det \theta} \times$$

$$\times \text{Tr} \sum_{m,n,l} \langle n, m, l | A^{nm} \sum_{m',n',l'} \frac{\alpha'}{2} G^{ab} c_0[C_a, [C_b, A^{m'n'l'}]] |n', m', l'\rangle =$$

$$= -\frac{2\pi^2 T_{2p-1}}{G_s} \int \sqrt{\det G} d^{2p} x \sum_{m,n,l} \langle n, m, l | A^{nm}(x) \sum_{m',n',l'} \frac{\alpha'}{2} G^{ab} c_0 \partial_a \partial_b A^{m'n'l'}(x) |n', m', l'\rangle. \quad (3.40)$$

Using

$$A(x)^{nm} = \int d\tilde{k} e^{i\tilde{k}x} A(k)^{nm}, \quad (3.41)$$

where $\tilde{k}$ is equal to $k(4\pi^2 \alpha')^{1/2}$, (3.40) is equal to

$$\frac{2\pi^2 T_{2p-1}}{G_s} \sum_{m,n,l,m',n',l'} \int d\tilde{k} d\tilde{k}' \langle m, n, l | A^{nm}(k') (4\pi^2 \alpha')^p \delta(k + k') \frac{\alpha'}{2} \times$$

$$\times c_0 G^{ab} k_a k_b A(k)^{m'n'l'}, |m', n', l'\rangle, \quad (3.42)$$

where we have used $\int \sqrt{\det G} d^{2p} x e^{ix(k + k')} = (4\pi^2 \alpha')^p \delta(k + k')$. From this definition of the delta function we immediately see that $\delta(0) = \int \sqrt{\det G} d^{2p} x = V_{26}$.

We see that (3.42) corresponds to the kinetic term in the string field theory action for D(2p-1)-brane. More precisely, after identification $p^2 \sqrt{2\alpha'} = \alpha_0^a$ this expression arises from the part of the BRST operator for D(2p-1)-brane proportional to $\sim c_0 g_{IJ} \alpha_I^a \alpha_0^a$ that acts on the string field in the form

$$|\Psi\rangle = (4\pi^2 \alpha')^{p/2} \sum_{m,n,l} \int d^{2p} \tilde{k} A(k)^{nm} |m, n, l, k\rangle, \quad (3.43)$$
Using (3.34) we can write the remaining contribution to the zero mode part of the BRST operator as

$$
\frac{2\pi^2 T_{-1}}{g_s} \text{Tr} \langle \Psi \mid \sqrt{\frac{2}{\pi}} \sqrt{\alpha}^t \sum_{N = -\infty, N \neq 0}^{\infty} c_N g_{ab} \alpha_{-N}^a X^b, |\Psi\rangle =
$$

$$
= \frac{2\pi^2 T_{2p-1}}{G_s} \sum_{n,m,l,n',m',l',N} d^{p} k d^{p} k' \langle n, m, l \mid \sqrt{2\alpha'} c_N A(k)^{nm} (4\pi^2 \alpha')^p \delta(k + k') \times
$$

$$
\times G_{ab} \bar{\alpha}_{-N,a} k'_b A(k')^{n'm'l'} |n', m', l'\rangle = \frac{2\pi^2 T_{2p-1}}{G_s} \int \langle \Psi \mid \sum_{N = -\infty, N \neq 0}^{\infty} c_N G_{ab} \alpha_{-N}^a \alpha_b^0 |\Psi\rangle ,
$$

where we have used (3.43) and we have omitted tilde on $\alpha$. We see that (3.42) and (3.44) give the correct contribution to the zero-mode part of the BRST operator

$$
\frac{1}{2} G_{ab} \alpha_0^a \alpha_b^0 + \sum_{n = -\infty, n \neq 0}^{\infty} c_n G_{ab} \alpha_{-n}^a \alpha_0^b ,
$$

so that we can claim that $Q^{\text{inst}}$ (after appropriate rescaling $\alpha_n^a$) with the zero mode part given above leads to the BRST operator $Q_{2p-1}$ for D(2p-1)-brane with the noncommutative world-volume and with the open string parameters given in (3.33).

In order to finish this identification we must also discuss the interaction part in (2.2) which has a form

$$
\frac{2\pi^2 T_{-1}}{3g_s} \int \text{Tr} \Psi \star \Psi \star \Psi = \frac{2\pi^2 T_{2p-1}}{3G_s} \sqrt{\det G} (2\pi)^p \sqrt{\det \theta} \int \text{Tr} \Psi \star \Psi \star \Psi =
$$

$$
= \frac{2\pi^2 T_{2p-1}}{3G_s} \int \Psi \star \Psi \star \Psi ,
$$

where $\star$ is a modified star product that includes the Moyal star product arising from the noncommutative nature of the theory. More precisely, the explicit form of the star product was given in [69, 72, 73] in terms of some overlap vertices in the string field theory operator formalism. The conditions which these vertices must obey are completely universal for any background and are completely determined from the form of the BRST operator. Since we have shown that the resulting BRST operator corresponds to the noncommutative background, we could proceed in the same way as in [70, 71] to construct corresponding overlap vertices resulting to the modification of the start product cited above. It would be certainly nice to construct overlap vertices for any D-instanton background. We hope to return to this question in the future. For our purposes in this paper the abstract definition of the star product [10] is sufficient.
As a result, we obtain the string field theory for D(2p-1)-brane in the presence of the background B field

\[
S = \frac{2\pi^2 T_{2p-1}}{G_s} \int \left( \frac{1}{2} \hat{\Psi} \hat{Q} \hat{\Psi} + \frac{1}{3} \hat{\Psi} \hat{\Psi} \hat{\Psi} \right). \tag{3.47}
\]

In this section we have seen that all D-branes of even dimensions arise from the single D-instanton string field theory with the modified BRST operator. We have seen that this operator is correct BRST operator for any background configuration of D-instantons. In fact, we can regard this BRST operator as a particular solution of the pregeometrical string field theory [80] which will be seen more precisely in the next section.

We can also generalise this construction to the configuration of \(k\) D(2p-1)-branes. In this case we take the background configuration of D-instantons

\[
[X^a, X^b] = 1_{k \times k} \otimes i \theta^{ab} 1_{N \times N}, \quad a, b = 1, \ldots, 2p, \quad X^m = 0, \quad m = 2p + 1, \ldots, 26. \tag{3.48}
\]

It is easy to see that this configuration leads to the non-abelian \(U(k)\) string field theory describing \(k\) coincident D(2p-1)-branes. This simply follows from the decomposition of the string field as

\[
\Psi_{IJ} = (\psi_{ab})_{mn}, \quad I = m \times N + a, \quad J = n \times N + b, \quad m, n = 0, \ldots, k-1, \quad a, b = 1, \ldots, N, \quad N \to \infty. \tag{3.49}
\]

We can easily generalised this solution to solution describing \(k\) D(2p-1)-branes with general transverse positions. We replace the solution \(X^m = 0\) in (3.48) with the more general one

\[
X^m = \begin{pmatrix}
x_1^m \otimes 1_{N \times N} & 0 & \ldots & 0 \\
0 & x_1^m \otimes 1_{N \times N} & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & \ldots & 0 & x_k^m \otimes 1_{N \times N}
\end{pmatrix}, \quad m = 2p + 1, \ldots, 26, \quad N \to \infty. \tag{3.50}
\]

4. Tachyon condensation and noncommutative string field theory

In this section we would like to study the problem of the emergence of lower dimensional D-branes from the string field theory describing space-time filling D25-brane in the background B-field. As was shown in [71], and for the case of string field theory in [70, 71], the resulting theory is a noncommutative one. The precise analysis of the string field theory was given in the beautiful papers [70, 71] where it was shown that the string field theory in the presence of the background B field differs from the string field theory in the trivial background in the modification of the string field
start product which now also incorporates the Moyal star product of the noncommutative theory. In fact, we have obtained this result in the previous section from slightly different point of view. The next difference is that all quantities in the string field theory are calculated with the open string quantities \[51\]. In other words, the string field action for D25-brane in the presence of the background B field has a form

\[
S = \frac{2\pi^2 T_{25}}{G_s} \int \left( \frac{1}{2} \Psi \hat{\star} Q \Psi + \frac{1}{3} \Psi \hat{\star} \Psi \hat{\star} \Psi \right).
\]

(4.1)

We will show that with using this action we will be able to obtain all lower dimensional D-branes of even codimensions in the same way as in the case of the tachyon condensation in the effective field theory \[52, 53, 54, 57, 62\]. In order to show this we must say a few words about the star product in string field theory. In the original Witten’s string field theory \[10\], the star product was defined as an abstract operation describing joining two strings which formally does not depend on the background. On the other hand, the modified star product depends on the background B field through the noncommutative parameter. It follows that in the process of the tachyon condensation the modified star product \(\hat{\star}\) changes since the lower dimensional D-brane has different noncommutative parameter, whereas the formal string field star product \(\star\) does not change. For that reason the best thing how to study the tachyon condensation is in such a way where we can replace the Moyal star product with the other formulation of the noncommutative geometry of the world-volume. For that reason we will transform the string field action into the operator formalism \[52, 57\]. Since we will work with the open string parameters \[51\] given in \(3.38\) we can use the results presented in the previous section and express the noncommutative D25-brane in terms of D-instanton matrix model.

Now we would like to claim that the emergence of any configuration of even dimensional D-branes from D25-brane is rather straightforward procedure which does not need to carry about mysterious nothing state \[30, 33\]. For simplicity, let us study the emergence of \(k\) D2p-branes from D25-brane. For that reason we propose the string field \(\Phi_0\) that leads to this configuration as follows

\[
Q_{2p}^k A = Q_{25} A + \Phi_0 \star A - (-1)^A A \star \Phi_0, \forall A.
\]

(4.2)

The index \(k\) in \(Q_{2p}^k\) indices that this is the BRST operator for any configuration of \(k\) D2p-branes that corresponds to the D-instanton background \(3.48\) and \(3.50\). It is also understood the matrix multiplication in \(1.2\). And finally, we express all quantities in terms of D-instanton matrix model and in terms of closed string metric and coupling constant which is a reflection of the background independence of the noncommutative theory \[51, 53\].

Since we know that both \(Q_{2p}^k, Q_{25}\) are nilpotent operators, \(1.2\) leads to the condition that \(\Phi_0\) must be a solution of the string field theory equation of motion

\[
(Q_{25} \Phi_0)_{ij} + (\Phi_0)_{ik} \star (\Phi_0)_{kj} = 0.
\]

(4.3)
We do not give the explicit form of this solution however it is clear that such a solution should exist from the existence of the correct BRST operators $Q_{2p}, Q_{25}$. In fact, we can follow very elegant approach presented in [80] and extend it to the non-abelian case in the form

$$S = \frac{4\pi^3}{3g_s} \text{Tr} \int \Phi \Phi \Phi , \ 2\pi^2 T_{-1} = 4\pi^3 ,$$

with the equation of motion

$$\Phi_{ij} \Phi_{jk} = 0 .$$

Following [80] we can construct for any solution of the equation of motion (4.5) a (matrix valued) operator $D_{\Phi_0}$

$$(D_{\Phi_0}B)_{ik} = (\Phi_0)_{ij} B_{jk} - (-1)^B B_{ij} \Phi (\Phi_0)_{jk} .$$

Then we can see (using axioms (2.1) and their generalised form (3.20), (3.23)) that

$$\text{Tr} \int D_{\Phi_0}B = \int \left[ (\Phi_0)_{ij} B_{ji} - (-1)^2 B (\Phi_0)_{ji} B_{ij} \right] = 0 ,$$

and finally

$$(D_{\Phi_0}^2 B)_{im} = (D_{\Phi_0})_{ij} \left[ (\Phi_0)_{jk} B_{km} - (-1)^B B_{jk} \Phi (\Phi_0)_{km} \right] =$$

$$= ((-1)^B \Phi_0 B \Phi_0 - (-1)^B \Phi_0 B \Phi_0)_{im} = 0 ,$$

where we have used $(\Phi_0)_{ik} \Phi (\Phi_0)_{kj} = 0$. These results imply that $D_{\Phi_0}$ is a derivative.

We will argue that in this way we can construct a BRST operator in noncommutative theory or equivalently the BRST operator for infinite number of D-instantons. In fact, we have implicitly used this construction in the previous section and the approach given here can serve as further support of our proposal. We write the BRST operator as follows

$$(Q_{25}A)_{ij} = Q_{\text{inst}} A_{ij} + (Q_{25}^0 A)_{ij} = \phi_0 A_{ij} - (-1)^A A_{ij} \phi_0 + (\phi_{25})_{ik} A_{kj} - (-1)^A A_{ij} (\phi_{25})_{kj} .$$

Since we know that $Q_{\text{inst}}$ is a correct BRST operator for the background of $N$ D-instantons sitting in the points $x^I = 0$ then can be written as

$$Q_{\text{inst}} A_{ij} = \phi_0 A_{ij} - (-1)^A A_{ij} \phi_0 ,$$
where \( \phi_0 \) can be found as in [80]. Using the explicit form of \( Q_{25}^0 \) (3.3) expressed in the D-instanton form with the classical configuration given in (3.31) we see that it can be written as follows (The matrix multiplication is understood)

\[
Q_{25}^L A = \frac{1}{4\pi^2\alpha'} c_{0IJ}[X^IX^J A - X^IAX^J] + \frac{\sqrt{2}}{2\pi\sqrt{\alpha'}} \sum_{n=-\infty}^{\infty} c_n g_{IJ} \alpha^{-n}_{-n} X^J A,
\]

\[
Q_{25}^R A = \frac{1}{4\pi^2\alpha'} c_{0IJ}[AX^IX^I - X^JAX^I] - \frac{\sqrt{2}}{2\pi\sqrt{\alpha'}} \sum_{n=-\infty}^{\infty} c_n g_{IJ} \alpha^{-n}_{-n} AX^J.
\]

(4.12)

Now we would like to argue that the string field \( \phi_{25} \) acts on any string field \( A \) as

\[
(\phi_{25})_{ik} \star A_{kj} = (Q_{25}^L A)_{ij},
\]

(4.13)

which allows us to express the string field in terms of the zero mode part of the BRST operator \( Q_L \). To support this idea, let us write

\[
\int (\phi_{25})_{ij} \star A_{ji} = (-1)^A \int A_{ji} \star (\phi_{25})_{ij} = \text{Tr} \int Q_{25}^L A = \text{Tr} \int \left( \frac{1}{4\pi^2\alpha'} c_{0IJ}[X^IX^J A - X^IAX^J] + \frac{\sqrt{2}}{2\pi\sqrt{\alpha'}} \sum_{n=-\infty}^{\infty} c_n g_{IJ} \alpha^{-n}_{-n} X^J A \right) =
\]

\[
= -\text{Tr} \int \left( \frac{1}{4\pi^2\alpha'} c_{0IJ}[AX^IX^I - X^JAX^I] - \frac{\sqrt{2}}{2\pi\sqrt{\alpha'}} \sum_{n=-\infty}^{\infty} c_n g_{IJ} \alpha^{-n}_{-n} AX^J \right) \Rightarrow (-1)^A \text{Tr} \int A \star \phi_{25} = -\text{Tr} \int Q_{25}^R A \Rightarrow (-1)^A A_{ij} \star (\phi_{25})_{jk} = -(Q_{25}^R A)_{ij}
\]

(4.14)

and consequently

\[
(Q_{25}^0 A)_{ij} = (\phi_{25})_{ik} \star A_{kj} - (-1)^A A_{ik} \star (\phi_{25})_{kj} = (Q_{25}^R A)_{ij} + (Q_{25}^L A)_{ij} = (Q_{25}^0 A)_{ij}
\]

(4.15)

which we wanted to prove. Finally we must also show that \( \Phi_0 = \phi_0 + \phi_{25} \) is a solution of the equation of motion for the cubic string field theory \( \Phi \star \Phi = 0 \). This equation leads to

\[
(\Phi_0)_{ij} \star (\Phi_0)_{jk} = Q_{\text{inst}}(\phi_{25})_{ij} + (\phi_{25})_{ik} \star (\phi_{25})_{kj} = 0
\]

(4.16)

using (1.14). In other words, \( \phi_0 \) should be a solution of the equation of motion for D-instanton string field theory. Again, this can be seen from the fact that \( Q_{25} \) is nilpotent operator (As we have proven in the previous section) so we have

\[
Q_{25}^2 = 0 = Q_{\text{inst}}(\phi_{25})_{ij} + (\phi_{25})_{ik} \star (\phi_{25})_{kj} = 0
\]

(4.17)

so that \( \phi_{25} \) is a solution of the equation of motion.
Using these results it is easy to find string field describing the tachyon condensation in the noncommutative version of the string field theory from D25-brane to any lower dimensional configurations of D2p-branes. We have (in matrix notation)

\[ Q_{25}A = \Phi_{25} \ast A - (-1)^{\Lambda} A \ast \Phi_{25}, \]
\[ Q_{2p}^k A = \Phi_{2p} \ast A - (-1)^{\Lambda} A \ast \Phi_{2p}, \]
\[ Q_{2p}^k A = Q_{25}A + \Phi_0 \ast A - (-1)^{\Lambda} A \ast \Phi_0 \Rightarrow \Phi_0 = \phi^0_{2p} - \phi_{25}^0, \]

(4.18)

where \( \phi_{25}, \phi^0_{2p} \) are given in (4.13). When we rewrite the action for non-commutative D25-brane in terms of the matrix model and then we use the upper relation between the BRST operator and string field \( \Phi_{25} \) we can write the action for non-commutative D25-brane as follows

\[ S = \frac{4\pi^3}{3g_s} \text{Tr} \int (\Phi_{25} + \Psi) \ast (\Phi_{25} + \Psi) \ast (\Phi_{25} + \Psi). \]

(4.19)

When we expand around the solution \( \Phi_0 \) we obtain precisely the string field action for the configuration of \( k \) D2p-branes. In other words, when we write the string field \( \Psi \) in (4.19) as

\[ \Psi = \Phi_0 + \phi \]

(4.20)

and insert it into (4.19) we get

\[ S = \frac{2\pi^2 T_{-1}}{3g_s} \text{Tr} \int (\Phi_{2p} + \phi) \ast (\Phi_{2p} + \phi) \ast (\Phi_{2p} + \phi) = \frac{2\pi^2 T_{-1}}{g_s} \text{Tr} \int \left( \frac{1}{2} \phi \ast Q_{2p}^k \phi + \frac{1}{3} \phi \ast \phi \ast \phi \right). \]

(4.21)

which is precisely the string field action for D2p-brane written in the matrix model formalism.

In this section we have shown that the description of the string field theory in the noncommutative background in terms of the generalised matrix string field theory can very easily describe the emergence of the lower dimensional D-branes from D25-brane.

5. Conclusion

In this short paper we have tried to present an alternative description of the Witten’s string field theory \([10]\) in the presence of the background B field. We have argued for the existence of more general string field action for \( N \) D-instantons which would have many properties of the matrix models \([74, 75, 76, 77, 78, 79]\). We have seen that matrix description of the string field theory allows naturally to describe the tachyon condensation to D-branes of even dimensions. We have made many calculations which should support our proposal. In particular, we have shown that the
reuirament of the nilpotence of the BRST operator leads to the conclusion that the background configuration of D-instantions should obey the equations of motion of the low energy effective theory.

We believe that the matrix string field description can give more accurate description of the tachyon condensation. In fact, the importance of the matrix theory analysis of this problem has been suggested previously in [65, 66]. Of course, in this approach we cannot describe odd dimensional D-branes which is the same problem as their description in terms of noncommutative theory. We also cannot much to say about the tachyon condensation to the closed string vacuum that is very difficult problem. However, there is now considerable progress in its solution [30, 33].

It would be also very interesting to try to extend this analysis to the case of the supersymmetric string field theory.

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