ABSTRACT

“Pay-per-last-\(N\)-shares” (PPLNS) is one of the most common payout strategies used by mining pools in Proof-of-Work (PoW) cryptocurrencies such as Bitcoin. As with any payment scheme, it is imperative to study issues of incentive compatibility of miners within the pool. For PPLNS this question has only been partially answered; we know that reasonably-sized miners within a PPLNS pool prefer following the pool protocol over employing specific deviations. In this paper, we present a novel modification to PPLNS where we randomise the protocol in a natural way. We call our protocol “Randomised pay-per-last-\(N\)-shares” (RPPLNS), and note that the randomised structure of the protocol greatly simplifies the study of its incentive compatibility. We show that RPPLNS maintains the strengths of PPLNS (i.e., fairness, variance reduction, and resistance to pool hopping), while also being robust against a richer class of strategic mining than what has been shown for PPLNS.

KEYWORDS

blockchain; strategic mining; mining pools

1 INTRODUCTION

In Bitcoin, miners expend computational resources to maintain a ledger of transactions for all users in the system. This is done by successfully mining ”blocks” of transactions, for which miners earn a reward. The nature of mining blocks is inherently variable. Hence, miners often form groups and pool their resources so that rather than rarely earning large rewards they earn smaller rewards at a more consistent rate. In order to share rewards, said pools must reliably identify the computational contribution of each miner. This is done by accepting ”near-misses” (called shares) to Bitcoin’s desired hash rate, created at a rate directly proportional to the computational effort spent on extending the blockchain. A pool operator collects shares reported by every miner in the pool, and uses them to distribute payments once an actual block is found. Participating in a pool should have (at least) the following guarantees:

(1) Fairness: miners earn the same block reward in expectation as mining alone.
(2) Variance reduction: miners have lower variance in block reward than when mining alone.
(3) Robustness against pool hopping: at no point of time does the miner benefit from switching between similar pools.
(4) Incentive Compatibility: to maximize their reward, each participating miner should always expend maximum effort and report shares/blocks immediately as they are generated.

One of the most popular pool mining protocols which (partially) satisfies these properties is ”Pay-per-last-\(N\)-shares” (PPLNS). Miners report shares to a pool operator which maintains a queue of the \(N\) most recent shares reported to it, and if a block is found and reported by the pool, the owners of these \(N\) shares are paid proportionally ( times the value of a block for each such share). The structure of PPLNS is such that it satisfies properties 1-3 above [13]. With respect to property 4, [15, 16] demonstrate that miners in PPLNS pools are incentivised to act honestly if they are only permitted specific deviations, hence PPLNS only partially fulfils it.

Our Contributions. We modify PPLNS so that a pool operator maintains a ”bag” of \(N\) shares and when a new share is found, it replaces a random share from the bag. We call our protocol ”Randomised pay-per-last-\(N\)-shares” (RPPLNS), and note that the randomised structure greatly simplifies the study of its incentive compatibility. We show1 that RPPLNS maintains the strengths of PPLNS, and we use experimental evidence to show that RPPLNS is robust to strategic pool miners who may arbitrarily hoard share and blocks to publish at a later time.

Related Work. Strategic mining has been studied since the inception of Bitcoin [10]. In [4], the authors demonstrate that honest mining is not robust to strategic mining in terms of block reward, even when a miner has less than a majority computational stake in the Bitcoin ecosystem. This work is refined in [14], [11] and [6] by generalising selfish mining, pairing selfish mining with network-level attacks, and proving limited incentive compatibility of honest mining at low hash rates. Further incentives at the individual miner level have also been studied in [2, 7, 9]. At the pool level, the authors of [3] and [8] study infiltration attacks pools can wage against

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1The full version of the paper can be found in [5].
each other which lead to an iterated prisoner’s dilemma between operators and [1] study reward sharing for stake pools.

An extensive survey of pool protocols can be found in [13]. In [15], the authors study incentive compatibility in pool protocols that decide how to make payments on the basis of the quantity of shares each miner reported, irrespective of the order in which they are received. They also study PPLNS and show that honest pool mining is robust to specific share/block hoarding strategies whereby miners enter a hoarding phase for shares, and simply publish all secret shares upon finding a new block. The authors of [16] study a similar set of strategic deviations where for a fixed $x \in \mathbb{N}$ a miner hoards at most $x$ shares. Subsequent shares are published immediately, and whenever a block is found, those $x$ shares are published immediately before publishing the block. Their analysis makes the assumption that each strategic miner reaches their threshold $x$, and show when being honest outperforms being strategic in this setting. The authors of [12] find reporting strategies that can be beneficial to strategic miners at high enough hash rates.

2 PROPERTIES OF RPPLNS

To analyse RPPLNS, it suffices to consider a single strategic pool miner, $m_1$ with hash power $\alpha$, a single honest pool miner $m_2$ with hash power $\beta$, and a single honest non-pool miner $m_0$ with hash power $\gamma$. Indeed, $m_2$ and $m_0$ could be composed of multiple miners, but if they are honest, we can model their behaviour as that of a single miner of their aggregate hash power. Pool miners operate within an RPPLNS pool with a bag of size $N$, and such that the block difficulty threshold is $D$ times higher than that of shares. To model revenues, we consider a turn-based process. Every turn, either $m_1$, $m_2$ or $m_0$ find a share with probability $\alpha$, $\beta$ and $\gamma$ respectively, and each share has a further $\frac{1}{D}$ probability of being a full block. $m_0$ finds shares in the sense that it computes a block with a hash that is a near-miss to the target hash (by a factor of $D$), but does not actually report this near miss to the pool since it is not a part of the pool. $m_0$ does publish blocks immediately to all agents of the Bitcoin ecosystem, including the pool operator. $m_1$ may hoard shares/blocks found, but since $m_2$ is an honest pool miner, whenever they find a share/block they communicate this immediately to the RPPLNS pool. The full version can be found in [5].

Fairness, Variance Reduction and Pool Hopping. In the full version of the paper, we show that if $m_1$ is honest, then their expected block reward per turn is precisely $\alpha/D$. Since each share has a $\frac{1}{D}$ probability of being a block, this coincides with the expected $\alpha$ block reward $m_1$ would get (per block mined by the system) by mining solo. In addition, we demonstrate that RPPLNS enjoys similar variance reduction in block reward to that of PPLNS.

\textbf{Theorem 2.1.} Suppose that $m_1$ is honest with hash power $\alpha$. Their expected per-turn block reward in a RPPLNS pool is $\frac{\alpha}{D}$ and the variance of the reward is $\frac{1}{D^2}(\alpha - \alpha^2) + \frac{\alpha}{ND}$. In deterministic PPLNS, block reward variance can be computed in an identical fashion, and it is $\frac{\alpha}{D^2} + \frac{\alpha}{N} - \frac{\alpha}{D^2} = \frac{\alpha}{N}$. Typically, pools have $N = 2D$, in which case the PPLNS variance becomes $\frac{\alpha}{D^2}(\alpha - \alpha^2) - \frac{\alpha}{2D}$. For this difficulty setting, RPPLNS block reward variance becomes $\frac{1}{D^2}(\alpha - \alpha^2) + \frac{\alpha}{2D}$. Though this is more than with standard PPLNS, this still vanishes at the same asymptotic rate of $O(1/N^2)$ when $N = \Theta(D)$. Finally, we show that for a given time horizon, a pool miner’s reward only depends on total effort invested in the pool, and not on when the effort is invested, which leads to:

\textbf{Theorem 2.2 (Informal).} RPPLNS is resistant to pool hopping.

\textbf{When is Honest Mining a Dominant Strategy.} We wish to find conditions such that $m_1$ is honest. To this end, in the full paper, we provide a recurrence that bounds the block reward that $m_1$ can obtain from being optimally strategic. We compute the best possible reward of $m_1$, assuming $N = 1000$, $D = 500$ and a finite horizon of $k = 150$ steps. We compare against expected rewards in the same number of steps if $m_1$ follows the protocol honestly. The following graphs show the best action for $m_1$ given an initial fraction of $F$ shares in the bag. We do not include graphs for $F \in (0.33, 0.7)$ as $m_1$ is only honest in these cases. We witness some strategic behaviour, though it is important to note that this occurs at hash rates of $m_1$ where having the given initial fraction of $F$ shares is improbable. In the full version of the paper, we justify the strategic deviations we see, and prove that PPLNS is susceptible to the same deviations at similarly improbable extremal queue states.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{$F \leq 0.30$ and $0.75 \leq F \leq 0.95$}
\end{figure}

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