Goldstone Superfield Actions for Partially Broken AdS\(_5\) Supersymmetry

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Abstract

We explicitly construct \(N = 1\) worldvolume supersymmetric minimal off-shell Goldstone superfield actions for two options of 1/2 partial spontaneous breaking of AdS\(_5\) supersymmetry \(SU(2,2|1)\) corresponding to its nonlinear realizations in the supercosets with the AdS\(_5\) and AdS\(_5\) × S\(_1\) bosonic parts. The relevant Goldstone supermultiplets are comprised, respectively, by improved tensor and chiral \(N = 1\) superfields. The second action is obtained from the first one by duality transformation. In the bosonic sectors they yield static-gauge Nambu-Goto actions for L3-brane on AdS\(_5\) and scalar 3-brane on AdS\(_5\) × S\(_1\).

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1. Introduction. The concept of partial breaking of global supersymmetry (PBGS) [1] provides a manifestly worldvolume supersymmetric description of various superbranes in terms of Goldstone superfields [2].

Most of the PBGS theories known to date correspond to superbranes on flat super Minkowski backgrounds (see [3, 4] and refs. therein). On the other hand, keeping in mind the renowned AdS/CFT correspondence [5], it is the AdSₙ × Sᵐ and PP-wave type [6] superbackgrounds which are of primary interest. However, not too many explicit examples of the worldvolume superfield PBGS actions on such backgrounds were constructed so far. Such actions were given only for N = 1 supermembrane in AdS₄ [7] and some its dimensional reductions [8, 9].

It is tempting to construct PBGS versions of superstring and D3-brane on the AdS₅ × S⁵ background which is in the heart of the original AdS/CFT conjecture. These systems should be associated with the partial breaking of N = 4, d = 4 superconformal group SU(2, 2|4) which determines the corresponding superisometries. ¹ It is natural to firstly study some truncations of these models based on simpler N = 1 and N = 2, d = 4 superconformal groups SU(2, 2|1) and SU(2, 2|2). An attempt to construct a PBGS model for SU(2, 2|1) which would generalize that of [12] was undertaken in [13]. This model involves Goldstone N = 1 chiral superfield as the basic one and is expected to describe a scalar 3-brane on AdS₅ × S¹. However, no proper Goldstone superfield action in the explicit form was given.

The aim of this letter is to present AdS₅ generalizations of the two versions of the off-shell minimal Goldstone superfield actions of partially broken N = 2, d = 4 Poincaré supersymmetry: the one with the N = 1 Goldstone tensor multiplet [12, 14, 15] and the one with the chiral Goldstone N = 1 supermultiplet [16, 12]. Instead of dealing with a nonlinear realization of SU(2, 2|1) in the standard approach [17] like this has been done in [13], we prefer to follow the line of refs. [12, 14, 15, 7, 18]. As a first step, we construct a nonlinear realization of SU(2, 2|1) on the set of three N = 1 superfields: an improved N = 1 tensor superfield L and mutually conjugated chiral superfields F, F. This set is subjected to some nonlinear covariant constraints which leave us with the single superfield L as the only Goldstone one. Its SU(2, 2|1) invariant action describes N = 1 L3-brane on AdS₅.² The bosonic core of this action is a static-gauge Nambu-Goto action of L3-brane in AdS₅, with one scalar physical field of L being a transverse brane coordinate and another (on-shell) bosonic degree of freedom being carried out by the notoph field strength. Then we dualize L into a pair of mutually conjugated chiral N = 1 superfields and obtain an analog of the action of ref. [16, 12]. It describes a scalar super 3-brane on AdS₅ × S¹. This action corresponds to the PBGS option studied in [13] and in the bosonic sector precisely yields the S⁵ → S¹ reduction of the scalar part of D3-brane action on AdS₅ × S⁵ [5, 11].

²See e.g. [19] for the relevant nomenclature.

2. Goldstone tensor N = 1 multiplet in a flat background. The idea to utilize N = 1 tensor multiplet as the one for describing the partial breaking of the global N = 2, d = 4 Poincaré supersymmetry down to N = 1 has been worked out in [12, 14, 15].

One starts with N = 2, d = 4 Poincaré superalgebra extended by a real central charge D

\[
\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2P_{a\dot{a}}, \{S_\alpha, \bar{S}_{\dot{\alpha}}\} = 2P_{a\dot{a}}, \{Q_\alpha, S_\beta\} = -\varepsilon_{\alpha\dot{\alpha}D}, \{\bar{Q}_{\dot{\alpha}}, \bar{S}_{\dot{\beta}}\} = -\varepsilon_{\dot{\alpha}\dot{\beta}D}. \quad (1)
\]

Here Q_\alpha, \bar{Q}_{\dot{\alpha}} and S_\alpha, \bar{S}_{\dot{\alpha}} are generators of the unbroken and broken N = 1 supersymmetries,
respectively. These generators and the 4-translation generator $P_{a\dot{a}}$ possess the standard
commutation relations with the Lorentz so(1, 3) generators $(M_{\alpha\beta}, \bar{M}_{\dot{\alpha}\dot{\beta}})$:

\[
i [M_{\alpha\beta}, M_{\rho\sigma}] = \varepsilon_{\alpha\rho}M_{\beta\sigma} + \varepsilon_{\alpha\sigma}M_{\beta\rho} + \varepsilon_{\beta\rho}M_{\alpha\sigma} + \varepsilon_{\beta\sigma}M_{\alpha\rho} \equiv (M)_{\alpha\beta, \rho\sigma},
\]
\[
i [\bar{M}_{\dot{\alpha}\dot{\beta}}, \bar{M}_{\dot{\rho}\dot{\sigma}}] = (\bar{M})_{\dot{\alpha}\dot{\beta}, \dot{\rho}\dot{\sigma}},
\]
\[
i [M_{\alpha\beta}, P_{\rho\dot{\rho}}] = \varepsilon_{\alpha\rho}P_{\beta\dot{\rho}} + \varepsilon_{\beta\rho}P_{\alpha\dot{\rho}}.,
\]
\[
i [M_{\alpha\beta}, Q_{\gamma\dot{\gamma}}] = \varepsilon_{\alpha\gamma}Q_{\beta\dot{\gamma}} + \varepsilon_{\beta\gamma}Q_{\alpha\dot{\gamma}} \equiv (Q)_{\alpha\beta, \gamma\dot{\gamma}},
\]
\[
i [M_{\alpha\beta}, S_{\gamma\dot{\gamma}}] = (S)_{\alpha\beta, \gamma\dot{\gamma}}.
\]

(2)

Then one introduces two $N = 1$ superfields: a real one $L(x, \theta)$ subjected to the constraint

\[
D^2 L = \bar{D}^2 L = 0 ,
\]

(3)

and so describing a linear (or tensor) $N = 1$ supermultiplet, and a complex chiral $N = 1$
superfield $F, \bar{F}$,

\[
D_{\alpha} F = \bar{D}_{\dot{\alpha}} \bar{F} = 0 .
\]

(4)

Here

\[
D_{\alpha} = \frac{\partial}{\partial \theta^\alpha} + i \theta^\alpha \partial_{\alpha \dot{\alpha}},
\]

\[
\bar{D}_{\dot{\alpha}} = - \frac{\partial}{\partial \bar{\theta}^\dot{\alpha}} - i \bar{\theta}^\dot{\alpha} \partial_{\dot{\alpha} \alpha},
\]

\[
D^2 = D^\alpha D_{\alpha}, \bar{D}^2 = \bar{D}_{\dot{\alpha}} \bar{D}^\dot{\alpha} .
\]

(5)

On these $N = 1$ superfields one implements [12] the following off-shell representation of the full
$N = 2$ supersymmetry (1):

\[
\delta L = - i \left( \eta^\alpha \theta_{\alpha} - \bar{\eta}_{\dot{\alpha}} \bar{\theta}^\dot{\alpha} \right) + \eta^\alpha D_{\alpha} \bar{F} - \bar{\eta}^\dot{\alpha} \bar{D}_{\dot{\alpha}} F, \delta F = - \eta^\alpha D_{\alpha} L, \delta \bar{F} = \bar{\eta}^\dot{\alpha} \bar{D}_{\dot{\alpha}} L .
\]

(6)

where $\eta_{\alpha}, \bar{\eta}_{\dot{\alpha}}$ are the infinitesimal transformation parameters associated with the generators $S_{\alpha}, \bar{S}_{\dot{\alpha}}$. It is a modification of the transformation law of $N = 2$ tensor multiplet [20] written in
terms of its $N = 1$ superfield components. This modification is such that we are in fact facing
the Goldstone $N = 2$ tensor multiplet: the spinor derivatives $D_{\alpha} L, \bar{D}_{\dot{\alpha}} L$ are shifted by $\eta_{\alpha}, \bar{\eta}_{\dot{\alpha}}$ and so are Goldstone fermions for the partial spontaneous breaking $N = 2 \rightarrow N = 1$, while $L$
is shifted by a constant under the action of the generator $D$ and so is the relevant Goldstone
field ($\mid$ means restriction to the $\theta, \bar{\theta}$ independent parts).

One can construct the simplest invariant ‘action’ as follows

\[
S = \frac{1}{4} \int d^4 x d^2 \bar{\theta} F + \frac{1}{4} \int d^4 x d^2 \theta \bar{F} .
\]

(7)

To make it meaningful one should express the chiral supermultiplet $F, \bar{F}$ in terms of the Gold-
stone tensor multiplet $L$ by imposing proper covariant constraints. These additional constraints
were simply guessed in [12] and later re-derived in [14] from the nilpotency conditions imposed
on the appropriate superfields. They read

\[
F = - \frac{D^\alpha L}{2 - D^2} D_{\alpha} L, \bar{F} = - \frac{\bar{D}_{\dot{\alpha}} L}{2 - \bar{D}^2} \bar{D}^\dot{\alpha} L .
\]

(8)

and can be easily solved [12, 14]

\[
F = - \psi^2 + \frac{1}{2} D^2 \left[ \frac{\psi^2 \bar{\psi}^2}{1 + \frac{1}{2} A + \sqrt{1 + A + \frac{1}{4} B^2}} \right],
\]

(9)
Finally, the action (7) becomes

\[ S = -\frac{1}{4} \int d^4 x d^2 \theta \psi^2 - \frac{1}{4} \int d^4 x d^2 \theta \theta^2 + \frac{1}{4} \int d^4 x d^2 \theta \frac{\psi^2 \overline{\psi}^2}{1 + \frac{1}{2} A + \sqrt{1 + A + \frac{1}{4} B^2}}. \]  

(11)

It is a nonlinear extension of the standard \( N = 1 \) tensor multiplet action. In the bosonic sector it gives rise to the static-gauge Nambu-Goto action for L3-brane in \( d = 5 \) Minkowski space, with one physical scalar of \( L \) being the transverse brane coordinate and another one represented by the notoph field strength. After dualizing \( L \) into a pair of conjugated chiral and antichiral \( N = 1 \) superfields (the notoph strength is dualized into a scalar field) the PBGS form of the worldvolume action of super 3-brane in \( d = 6 \) is reproduced [12].

We would like to point out that the constraints (8) which play the central role in deriving the action (11) are intimately related to the 5-dimensional nature of the brane under consideration. They guarantee 5-dimensional Lorentz covariance.

Indeed, the generator \( D \) in (1) can be treated as the generator of translations in 5th dimension and the full automorphism algebra of (1) can be checked to be \( so(1, 4) \) (we ignore the \( R \)-symmetry \( SU(2) \) automorphisms which are explicitly broken in (11)). The 5D Lorentz algebra \( so(1, 4) \) includes, besides 4D Lorentz generators \( M_{\alpha \beta}, \bar{M}_{\dot{\alpha} \dot{\beta}} \), an additional 4D vector \( K_{\alpha \dot{\alpha}} \) belonging to the coset \( SO(1, 4)/SO(1, 3) \). The full set of additional commutation relations is as follows:

\[ i [M_{\alpha \beta}, K_{\rho \dot{\rho}}] = \varepsilon_{\alpha \rho} K_{\beta \dot{\rho}} + \varepsilon_{\beta \rho} K_{\alpha \dot{\rho}}; \quad i [K_{\alpha \dot{\alpha}}, K_{\beta \dot{\beta}}] = -\varepsilon_{\alpha \beta} M_{\alpha \dot{\beta}} - \varepsilon_{\dot{\alpha} \dot{\beta}} M_{\alpha \beta}, \]

\[ i [D, K_{\alpha \dot{\alpha}}] = 2P_{\alpha \dot{\alpha}}; \quad i [P_{\alpha \dot{\alpha}}, K_{\beta \dot{\beta}}] = \varepsilon_{\alpha \beta} \varepsilon_{\dot{\alpha} \dot{\beta}} D, \]

\[ i [K_{\alpha \dot{\alpha}}, Q_\beta] = -\varepsilon_{\alpha \beta} S_{\dot{\alpha}}; \quad i [K_{\alpha \dot{\alpha}}, \bar{S}_\beta] = \varepsilon_{\dot{\alpha} \dot{\beta}} Q_\alpha. \]  

(12)

Now one can check that the following nonlinear transformations

\[ \delta^* L = a_{\alpha \dot{\alpha}} x^{\alpha \dot{\alpha}} - a^{\alpha \dot{\alpha}} \partial_{\alpha \dot{\alpha}} (L^2 - 2FF) + ia^{\alpha \dot{\alpha}} \theta_\alpha D_{\dot{\alpha}} F - ia^{\alpha \dot{\alpha}} \bar{\theta}_{\dot{\alpha}} D_\alpha F, \]

\[ \delta^* F = -2a^{\alpha \dot{\alpha}} \partial_{\alpha \dot{\alpha}} (FL) + ia^{\alpha \dot{\alpha}} \bar{\theta}_{\dot{\alpha}} D_\alpha L, \quad \delta^* \bar{F} = -2a^{\alpha \dot{\alpha}} \partial_{\alpha \dot{\alpha}} (FL) - ia^{\alpha \dot{\alpha}} \theta_\alpha \bar{D}_{\dot{\alpha}} L \]  

(13)

are just the \( SO(1, 4)/SO(1, 3) \) ones, with \( a^{\alpha \dot{\alpha}} \) being a transformation parameter related to the additional generator \( K_{\alpha \dot{\alpha}} \). They have a correct closure on \( SO(1, 3) \) and are compatible with the defining constraints (3), (4) only provided the nonlinear constraints (8) are imposed. The action (11) is invariant under these transformations.

3. \textbf{AdS}_5 background. \textit{Now we wish to generalize the flat superspace construction described in the previous Section to the case of partial spontaneous breaking of the simplest AdS}_5 super-symmetry which is \( SU(2, 2|1) \), that is \( N = 1 \) superconformal group in \( d = 4 \).

The superalgebra \( su(2, 2|1) \) contains \( so(2, 4) \times u(1) \) bosonic subalgebra with the generators \( \{ P_{\alpha \dot{\alpha}}, M_{\alpha \beta}, \bar{M}_{\dot{\alpha} \dot{\beta}}, K_{\alpha \dot{\alpha}}, D \} \) and \{ \( J \) \} and eight supercharges \( \{ Q_\alpha, Q_{\dot{\alpha}}, S_\alpha, S_{\dot{\alpha}} \} \). We choose the basis in a such way, that the generators \( K_{\alpha \dot{\alpha}} \) form \( so(1, 4) \) subalgebra together with the
$d = 4$ Lorentz generators $\{M_{\alpha\beta}, \bar{M}_{\dot{\alpha}\dot{\beta}}\}$, as in the first line of (12). The rest of non-trivial (anti)commutators reads

\[
i [D, P_{a\dot{a}}] = mp_{a\dot{a}}, \quad i [D, K_{a\dot{a}}] = 2p_{a\dot{a}} - mK_{a\dot{a}},
\]
\[
i [P_{a\dot{a}}, K_{\beta\dot{\beta}}] = \varepsilon_{a\beta}\varepsilon_{\dot{a}\dot{\beta}}D - \frac{m}{2} \left( \varepsilon_{a\beta}\bar{M}_{\dot{a}\dot{\beta}} + \varepsilon_{\dot{a}\dot{\beta}}M_{a\beta} \right),
\]
\[
\{Q_{\alpha}, S_{\beta}\} = -\varepsilon_{a\beta} (D + imJ) + mM_{a\beta}, \quad \{Q_{\alpha}, \bar{Q}_{\dot{\alpha}}\} = 2p_{a\dot{a}}, \quad \{S_{\alpha}, \bar{S}_{\dot{\alpha}}\} = 2p_{a\dot{a}} - 2mK_{a\dot{a}},
\]
\[
i [D, Q_{\alpha}] = \frac{m}{2} Q_{\alpha}, \quad i [D, S_{\alpha}] = -\frac{m}{2} S_{\alpha}, \quad [J, Q_{\alpha}] = \frac{3}{2} Q_{\alpha}, \quad [J, S_{\alpha}] = -\frac{3}{2} S_{\alpha},
\]
\[
i [K_{a\dot{a}}, Q_{\beta}] = -\varepsilon_{a\beta} \bar{S}_{\dot{a}}, \quad i [K_{a\dot{a}}, S_{\beta}] = \varepsilon_{a\beta} \bar{Q}_{\dot{a}}, \quad i [P_{a\dot{a}}, S_{\beta}] = m\varepsilon_{a\beta} \bar{Q}_{\dot{a}}.
\]

(14)

This basis is an example of the ‘AdS basis’ of conformal superalgebras [21, 22, 7, 23] which perfectly suits their interpretation as the superisometry groups of the appropriate AdS superspaces. Indeed, the generators $P_{a\dot{a}}, D, J$ form a maximal solvable bosonic subgroup in $su(2, 2|1)$ and span the coset $SO(2, 4)/SO(1, 4) \times U(1) \sim AdS_5 \times S^1$. The parameter $m$ has the meaning of the inverse AdS radius, $m = \ell^{-1}$. In the limit $m = 0$ ($\ell = \infty$) one recovers from (14) the $N = 1, d = 5$ Poincaré superalgebra, with $D$ becoming the 5th component of momenta. The generators $J$ and $K_{a\dot{a}}, M_{a\beta}, \bar{M}_{\dot{a}\dot{\beta}}$ decouple and generate outer $u(1) \oplus so(1, 4)$ automorphisms.

Our goal is to construct an AdS$_5$ version of the nonlinear realization (6), (8). The main hints which allowed us to do this are as follows. Firstly, we assert that this realization involves some modification of $N = 1$ tensor multiplet $L$ and, as before, a pair of mutually conjugated $N = 1$ chiral and anti-chiral superfields $F, \bar{F}$ subjected to some generalization of (8). Second, in a close analogy with the flat case we require that the following ‘action’

\[
S \sim \int d^4xd^2\theta F + \int d^4xd^2\bar{\theta}\bar{F}
\]

(15)

is an invariant of the AdS$_5$ supersymmetry. Since the right-chiral integration measure $d^4xd^2\bar{\theta}$ has the $D$ weight $-3m$ and, with our normalization of $J$, the $U(1)$ charge $-3$, the superfield $F$ should carry the $D$ and $J$ weights equal to $3m$ and 3 ($F$ has the same $D$ weight and the $J$ charge equal to $-3$). Third, in the limit $m = 0$ our construction should reproduce the flat case outlined in Sec. 2. At last, it is sufficient to find the realization of conformal $S$ supersymmetry, since the rest of $SU(2, 2|1)$ transformations appears in the closure of these $S$ transformations with themselves and with those of $N = 1$ Poincaré supersymmetry.

It turns out that this reasoning almost uniquely fixes the sought transformation laws and constraints (more details of the derivation are given in [24]). These are

\[
\delta^a \bar{F} = 6im\theta^\alpha \eta_{\alpha} \bar{F} - \Delta x^{a\dot{\alpha}} \partial_{a\dot{\alpha}} \bar{F} + \Delta \theta^a D_\alpha \bar{F} + ie^{-2mL} \bar{\eta} \bar{D}_\dot{a} L,
\]
\[
\delta^a F = -6im\bar{\theta}_{\dot{\alpha}} \bar{\eta}^\dot{\alpha} F - \Delta x^{a\dot{\alpha}} \partial_{a\dot{\alpha}} F - \Delta \bar{\theta}^{\dot{\alpha}} \bar{D}_{\dot{\alpha}} F + ie^{-2mL} \eta^\alpha D_\alpha L,
\]
\[
\delta^a L = -i(\theta^\alpha \eta_{\alpha} - \bar{\theta}^{\dot{\alpha}} \bar{\eta}^\dot{\alpha}) - \Delta x^{a\dot{\alpha}} \partial_{a\dot{\alpha}} L + \Delta \theta^a D_\alpha L - \Delta \bar{\theta}^{\dot{\alpha}} \bar{D}_{\dot{\alpha}} L - ie^{-2mL} \left[ \bar{\eta}^\alpha D_\alpha \left( e^{2mL} \bar{F} \right) + \bar{\eta}^{\dot{\alpha}} \bar{D}_{\dot{\alpha}} \left( e^{2mL} F \right) \right],
\]
\[
\frac{1}{m} D^2 e^{-2mL} = \frac{1}{m} D^2 e^{-2mL} = 0, \quad D_\alpha F = D_{\dot{\alpha}} F = 0,
\]
\[
F = \frac{e^{-2mL} D^a L D_{\alpha} L}{2 - e^{4mL} D^2 F}, \quad \bar{F} = \frac{e^{-2mL} \bar{D}_{\dot{\alpha}} \bar{L} \bar{D}^{\dot{\alpha}} L}{2 - e^{4mL} D^2 F}.
\]

(16)
Here

$$\Delta x^{\alpha \dot{\alpha}} = 2im \left( \eta_\beta x^{\beta \dot{\alpha}} \theta^\alpha + \bar{\eta}_\beta x^{\alpha \dot{\beta}} \bar{\theta}^\dot{\alpha} \right) - m \left( \theta^2 \eta^\alpha \bar{\theta}^\dot{\alpha} - \bar{\theta}^2 \eta^\dot{\alpha} \theta^\alpha \right),$$

$$\Delta \theta^\alpha = m \bar{\eta}_\alpha x^{\alpha \dot{\alpha}} + im \left( \bar{\theta} \eta^\alpha - \bar{\eta} \theta^\dot{\alpha} \right), \quad \Delta \bar{\theta}^\dot{\alpha} = m \eta_\alpha x^{\alpha \dot{\beta}} - im \left( \theta \bar{\eta}^\dot{\alpha} - \eta \bar{\theta}^\alpha \right)$$

(19)

are the standard transformations of the $N = 1$ superspace coordinates with respect to the conformal supersymmetry.

In the limit $m = 0$ eqs. (16), (17) and (18) go, respectively, into (6), (3), (4) and (8). We have checked that, on the surface of the nonlinear constraints (18), the off-shell transformations (16) are, first, compatible with the differential constraints (17) and, second, produce the whole $SU(2,2|1)$ symmetry when commuted among themselves and with $N = 1$ Poincaré supersymmetry. Had we neglected the last nonlinear terms in (16), we would recover the standard linear $N = 1$ superconformal transformation laws of the improved tensor superfield $e^{-2mL}$ and chiral superfields $F, \bar{F}$ which close without any need in the nonlinear constraints (18). It is just due to the presence of these extra mixed terms the transformations (16) constitute a realization of $SU(2,2|1)$ as the superisometry group of super $AdS_5$ background and correctly generalize the flat superspace realization (6). A striking difference between (6) and (16) lies, however, in the fact that (6) close on $N = 2$ Poincaré superalgebra before imposing the constraints (8), while (16) define a closed supergroup structure only provided the corresponding constraints (18) are imposed from the very beginning. In this sense the situation is similar to the implementation of the $SO(1,4)$ transformations (13) in the flat case, which are closed (together with the $SO(1,3)$ ones) only on the surface of (8). Since in the case of the supergroup $SU(2,2|1)$ these $SO(1,4)$ transformations appear in the anticommutators of the $Q$ and $S$ supersymmetry generators, it is quite natural that the constraints (18) should enter the game already at the stage of defining $S$ supersymmetry transformations. It is straightforward to check that (18) by themselves are covariant under the transformations (16).

Inspecting (16), one can be convinced that this realization just corresponds to a half-breaking of the $SU(2,2|1)$ supersymmetry: the spinor derivatives of $L$ are shifted by spinor parameters under the action of $S$ supersymmetry, thus signaling that the latter is spontaneously broken. Broken are also $D$ transformations (with $L$ as the Goldstone field) and the $SO(1,4)/SO(1,3)$ transformations generated by $K_{\alpha \dot{\alpha}}$ (with $\partial_{\alpha \dot{\alpha}} L$ as the relevant ‘Goldstone field’).

Like their flat counterparts, the constraint (18) can be easily solved

$$F = -e^{-2mL} \psi^2 + \frac{1}{2} D^2 \left[ \frac{\psi^2 \bar{\psi}^2}{1 + \frac{1}{2} A + \sqrt{1 + A + \frac{1}{4} B^2}} \right],$$

(20)

where

$$\psi_\alpha \equiv D_\alpha L, \quad \bar{\psi}_{\dot{\alpha}} \equiv \bar{D}_{\dot{\alpha}} L, \quad A = \frac{1}{2} e^{2mL} \left( D^2 \bar{\psi}^2 + \bar{D}^2 \psi^2 \right), \quad B = \frac{1}{2} e^{2mL} \left( D^2 \bar{\psi}^2 - \bar{D}^2 \psi^2 \right).$$

(21)

Finally, the action (15) can be written in the form

$$S = -\frac{1}{4} \int d^4 x d^2 \theta e^{-2mL} \bar{\psi}^2 - \frac{1}{4} \int d^4 x d^2 \bar{\theta} e^{-2mL} \psi^2 + \frac{1}{4} \int d^4 x d^4 \theta \frac{\psi^2 \bar{\psi}^2}{1 + \frac{1}{2} A + \sqrt{1 + A + \frac{1}{4} B^2}}.$$

(22)
The first two terms in (22) are recognized as the action of the improved tensor \( N = 1 \) superfield [25]. In the limit \( m = 0 \) (22) converts into the flat superspace Goldstone superfield action (11).

Defining the bosonic components as

\[
\phi = L|_{\theta = 0}, \quad [D_\alpha, \bar{D}_{\dot{\alpha}}] e^{-2mL}|_{\theta = 0} = -2mV_{\alpha\dot{\alpha}},
\]

(23)

where in virtue of (18)

\[
\partial_{\alpha\dot{\alpha}} V^{\alpha\dot{\alpha}} = 0,
\]

(24)

the bosonic part of (22) proves to be

\[
S_B = \int d^4 x e^{-4m\phi} \left[ 1 - \sqrt{1 + \frac{1}{2} e^{6m\phi} V^2 - 2e^{2m\phi}(\partial\phi)^2 - e^{8m\phi}(V^{\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}}\phi)^2} \right].
\]

(25)

It is a conformally-invariant extension of the static gauge Nambu-Goto action for L3-brane in \( d = 5 \); the dilaton \( \phi \) can be interpreted as a radial brane coordinate, while \( V^{\alpha\dot{\alpha}} \) is the field strength of notoph which contributes one more scalar degree of freedom on shell. As is well known, \( V^{\alpha\dot{\alpha}} \) can be dualized in an off-shell scalar by introducing the constraint (24) into the action with a Lagrange scalar multiplier and then eliminating \( V^{\alpha\dot{\alpha}} \) by its algebraic equation of motion. Extending (25) as

\[
S_B \Rightarrow S_{\text{dual}}^B = S_B + \int d^4 x \lambda \partial_{\alpha\dot{\alpha}} V^{\alpha\dot{\alpha}}
\]

(26)

and eliminating \( V^{\alpha\dot{\alpha}} \), after some algebra we get

\[
S_{\text{dual}}^B = \int d^4 x |Z|^4 \left[ 1 - \sqrt{1 - \det \left( \frac{2}{m^2} \eta_{\mu\nu} - \frac{\partial_{\mu} Z^n \partial_{\nu} Z^n}{|Z|^4} \right)} \right],
\]

(27)

where

\[
Z^1 = r \cos \vartheta, \quad Z^2 = r \sin \vartheta, \quad r \equiv e^{-m\phi}, \quad \vartheta \equiv m\lambda, \quad \eta_{\mu\nu} = \text{diag}(+---).
\]

(28)

The action (27) is recognized as the \( S^5 \to S^1 \) reduction of the scalar part of the D3-brane action on AdS\(_5 \times S^5 \) [5], that is the static-gauge Nambu-Goto action of scalar 3-brane on AdS\(_5 \times S^5 \).

4. AdS\(_5 \times S^1 \) Goldstone superfield action. Here we repeat the above duality transformation at the full superfield level and obtain in this way an \( SU(2, 2|1) \) invariant action of Goldstone chiral \( N = 1 \) superfield which generalizes the action of [16, 12, 14, 15] and describes a super 3-brane on AdS\(_5 \times S^1 \) superbackground. We shall be sketchy about details which can be found in [24]. In its basic steps this dualization procedure is similar to the flat superspace one of [15].

We start with the superfield action (22) and relax the constraints for \( L \) in (17) by adding a Lagrange multiplier term to the superfield Lagrangian

\[
S_{\text{dual}} = \frac{1}{4} \int d^4 x d^2 \theta d^2 \bar{\theta} \left[ -\frac{1}{2m^2} Y (\ln Y - 1) + \frac{Y^{-4}}{(2m)^4} (DY)^2 (\bar{D}Y)^2 f + \frac{Y}{2m} (\varphi + \bar{\varphi}) \right].
\]

(29)

Here

\[
Y \equiv e^{-2mL}, \quad \bar{D}_{\dot{\alpha}} \varphi = D_\alpha \bar{\varphi} = 0, \quad f = \frac{1}{1 + \frac{1}{2} A + \frac{1}{4} B^2}.
\]

(30)
Next we vary the action (29) with respect to $Y$ in order to obtain an algebraic equation that would allow us to trade $Y$ for $\phi, \bar{\phi}$. Though the expression for $Y$ is rather complicated [24], the calculations are greatly simplified due to the property that only terms bilinear in fermions really contribute to the dualized action after substitution of this expression back into (29). Also, the terms $\sim D^2Y, D^3Y$ can be reabsorbed into a redefinition of chiral Lagrange multiplier like in the flat case [15]. Skipping details, the dual action turns out to be as follows

$$S_{\text{dual}} = \frac{1}{8} \int d^4x d^4\theta \left( \frac{e^{m(\phi + \bar{\phi})}}{m^2} \right.$$

$$\left. + \frac{\frac{1}{8}(D\phi)^2(D\bar{\phi})^2}{1 - e^{-m(\phi + \bar{\phi})}\partial\phi\partial\bar{\phi}} + \sqrt{(1 - e^{-m(\phi + \bar{\phi})}\partial\phi\partial\bar{\phi})^2 - e^{-2m(\phi + \bar{\phi})}(\partial\phi)^2(\partial\bar{\phi})^2} \right).$$

(31)

This action goes into the flat $N = 2 \rightarrow N = 1$ chiral Goldstone superfield action of [12, 14, 15] in the limit $m = 0$ and is obviously $SU(2, 2|1)$ invariant as it was obtained by dualizing the $SU(2, 2|1)$ invariant action (22). We do not give the precise form of the $SU(2, 2|1)$ transformations of the chiral superfields $\phi, \bar{\phi}$ because they look not too illuminating. However, it is noteworthy that the standard $U(1)$ isometry associated with the duality transformation, viz. $\delta\phi = i\alpha, \delta\bar{\phi} = -i\alpha$, now appears in the closure of the $Q$ and $S$ transformations on these Goldstone superfields, with the imaginary part of $\phi$ being the related extra Goldstone field. It is just the $J$ (or $\gamma_5$) symmetry of $SU(2, 2|1)$, i.e. the duality transformation brings this symmetry from the stability subgroup into the coset. A similar phenomenon was observed in [26] in the context of the duality between real and complex forms of $N = 2$ superconformal mechanics. The bosonic core of the action (31) coincides with (27) after the identification

$$\phi = -\frac{1}{2}(\phi + \bar{\phi}), \quad \lambda = \frac{i}{2}(\phi - \bar{\phi}).$$

(32)

Thus we conclude that the Goldstone superfield action (31) describes the option when the internal $U(1)$ R-symmetry with the generator $J$ is also broken in addition to the (super)isometries broken in the action (22). The bosonic coset is basically $\text{AdS}_5 \times S^1 \propto \{x^{a\dot{a}}, \phi\} \times \{\lambda\}$ and the bosonic part of the action (31) is just the static-gauge Nambu-Goto action of a 3-brane on this manifold. This solves the problem of constructing an invariant Goldstone superfield action for such PBGS option, as it was posed in [13]. Note that both the Goldstone superfield actions (22), (31) are uniquely restored from the $SU(2, 2|1)$ invariance and do not involve any free parameters, like their flat superspace counterparts. It is also worth mentioning that the corresponding Lagrangian densities, once again in tight analogy with the Goldstone superfield Lagrangians on the Minkowski superbackgrounds, are invariant under $SU(2, 2|1)$ only up to full derivatives and are similar in this respect to WZW or CS Lagrangians.

5. Concluding remarks. In this note we have presented new nonlinear realizations of the simplest $\text{AdS}_5$ supersymmetry group $SU(2, 2|1)$ in terms of $N = 1$ tensor and chiral Goldstone superfields. We have explicitly given the corresponding minimal Goldstone superfield actions, for the second option by dualizing the action for the first one, and shown that they provide a manifestly $N = 1$ supersymmetric off-shell superfield form of worldvolume actions of L3-superbrane on $\text{AdS}_5$ and scalar super 3-brane on $\text{AdS}_5 \times S^1$. The latter is a truncation of the $\text{AdS}_5 \times S^5$ D3-action. In the limit of infinite $\text{AdS}_5$ radius these new actions go into their flat
superspace counterparts describing the partial breaking of \( N = 2, d = 4 \) supersymmetry down to \( N = 1 \) supersymmetry [12, 14, 15, 16].

This study can be considered as a first step towards finding out Goldstone superfield actions for various patterns of partial breaking of AdS\(_5\) supersymmetries. As was already mentioned in [13], it is interesting to look for the action corresponding to the half-breaking of \( N = 2 \) AdS\(_5\) supergroup \( SU(2,2|2) \) in a supercoset with the AdS\(_5 \times S^1\) bosonic part. The basic Goldstone superfield which we can expect to encounter in this case should be the appropriate generalization of the \( N = 2 \) Maxwell superfield strength. This action should be a superconformal version of Dirac-Born-Infeld action describing the \( N = 4 \rightarrow N = 2 \) partial breaking in the flat superspace [27, 28]. In this connection, let us recall that in the flat case there exists one more \( N = 2 \rightarrow N = 1 \) PBGS option associated with the choice of vector \( N = 1, d = 4 \) multiplet as the Goldstone one and corresponding to the space-filling \( N = 1 \) D3-brane [29]. No AdS\(_5\) analog of this realization can be defined. The reason is that for achieving \( SU(2,2|1) \) invariance one always needs a dilaton among the worldvolume Goldstone fields and hence within the relevant \( N = 1 \) Goldstone superfield. In the vector Goldstone \( N = 2 \) supermultiplet there are two scalar fields and, therefore, the above objection is evaded. An interesting related problem is to construct PBGS actions for the PP-wave type superbackgrounds via proper contractions of AdS supersymmetries and their Goldstone superfield actions.

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