Analysis of the Finite Element Strength Reduction Method to Slope Stability with a thin weak layer

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Abstract. The strength of the weak layer not only controls the instability mechanism of the slope, but also the factor of safety to the slope. Studying the stability of the weak layer slope has important theoretical significance and practical engineering value. The instability mechanism and the factor of safety to slope stability with a thin weak layer were studied by finite element strength reduction method and limit equilibrium method. The results show that factor of safety to slope stability decreases with the strength of the weak layer decreases. When the strength of the weak layer is less than 0.6 times the strength of the soil, the instability mechanism of the slope changes from circular arc sliding to polygonal sliding, and its stability is controlled by the strength of the weak layer. The slope stability analysis by limit equilibrium method must be based on the analysis of the instability mechanism to accurately evaluate the slope stability. However, the finite element strength reduction method can be applied to calculate the factor of safety to slope stability and analyze the instability mechanism, and to previous limit equilibrium method.

1. Introduction

Slope stability analysis is an important field in geotechnical engineering. Most textbooks on soil mechanics include several methods of slope stability analysis. A detailed review of equilibrium methods for slope stability analysis is presented by Duncan[1]. These methods include the ordinary method of slices, Bishop’s modified method, force equilibrium methods, Janbu’s generalized procedure of Slices, Morgenstern and Price’s method and Spencer’s method. These methods, in general, require the soil mass to be divided into slices. The directions of the forces acting on each slice in the slope are assumed. This assumption is a key role in distinguishing one limit equilibrium method from another. Limit equilibrium methods require a continuous surface passes the soil mass. This surface is essential in calculating the minimum factor of safety (FOS) against sliding or shear failure. Before the calculation of slope stability in these methods, some assumptions, for example, the side forces and their directions, have to be given out artificially in order to build the equations of equilibrium. With the development of cheaper personal computer, finite element method has been increasingly used in slope stability analysis. The advantage of a finite element approach in the analysis of slope stability problems over traditional limit equilibrium methods is that no assumption needs to be made in advance about the shape or location of the failure surface, slice side forces and their directions. The method can be applied with complex slope configurations and soil deposits in two or three dimensions to model virtually all types of mechanisms. General soil material models that include Mohr-Coulomb and numerous others can be employed. The equilibrium stresses, strains, and the associated shear strengths
in the soil mass can be computed very accurately. The critical failure mechanism developed can be extremely general and need not be simple circular or logarithmic spiral arcs. The method can be extended to account for seepage induced failures, brittle soil behaviors, random field soil properties, and engineering interventions such as geo-textiles, soil nailing, drains and retaining walls[3]. This method can give information about the deformations at working stress levels and is able to monitor progressive failure including overall shear failure[4].

In practical engineering slope stability analysis, limit equilibrium method is mostly applied in the area of slope stability analysis, and few applications of finite element strength reduction method in slope stability analysis have been reported. Obviously, its potential applications in the most of areas in geotechnical engineering will be shown with time passing and the accumulation of users’ experience. So far, the advantages of finite element strength reduction method to the complexity of engineering problems appear in some aspects, which is widely used in practical engineering and often used in combination with limit equilibrium analysis method. This manuscript is demonstrated that the instability mechanism and stability of slope with weak interlayer by finite element strength reduction method and rigid body limit equilibrium method. The factor of safety to slope stability obtained by finite element strength reduction method and limit equilibrium method is compared and analysed, and the applicability of using finite element strength reduction method in slope stability analysis is validated, which is presented and compared to previous FEM work and limit equilibrium methods.

2. Finite element strength reduction analysis of slope stability

2.1. Finite element strength reduction method
Slope fails because of its material shear strength on the sliding surface is insufficient to resist the actual shear stresses. Factor of safety is a value that is used to examine the stability state of slopes. For $FOS$ values greater than 1 means the slope is stable, while values lower that 1 means slope is instable. In accordance to the shear failure, the factor of safety against slope failure is simply calculated as:

$$FOS = \frac{\tau}{\tau_f}$$  \hspace{1cm} (1)

Where $\tau$ is the shear strength of the slope material, which is calculated through Mohr-Coulomb criterion as:

$$\tau = C + \sigma_n \tan \phi$$  \hspace{1cm} (2)

And $\tau_f$ is the shear stress on the sliding surface. It can be calculated as:

$$\tau_f = C_f + \sigma_n \tan \phi_f$$  \hspace{1cm} (3)

Where the factored shear strength parameters $C_f$ and $\phi_f$ are:

$$C_f = \frac{C}{SRF}$$  \hspace{1cm} (4)

$$\phi_f = \tan^{-1}\left(\frac{\tan \phi}{SRF}\right)$$  \hspace{1cm} (5)

Where $SRF$ is strength reduction factor. This method has been referred to as the ‘shear strength reduction method’. To achieve the correct $SRF$, it is essential to trace the value of $FOS$ that will just cause the slope to fail. This work applied only for two-dimensional plain-strain problems. The Mohr-Coulomb constitutive model used to describe the soil (or rock) material properties. The Mohr-Coulomb criterion relates the shear strength of the material to the cohesion, normal stress and angle of internal friction of the material. The failure surface of the Mohr-Coulomb model can be presented as:

$$f = \frac{1}{3} \sin \phi + \sqrt{J_2} \left[ \cos \Theta - \frac{1}{3} \sin \Theta \sin \phi \right] - C \cos \phi$$  \hspace{1cm} (6)
where $\phi$ is the angle of internal friction, $C$ is cohesion and

$$I_1 = (\sigma_1 + \sigma_2 + \sigma_3) = 3\sigma_m$$  \hspace{1cm} (7)

$$J_2 = \left( \frac{1}{2} (S_x^2 + S_y^2 + S_z^2) + \tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2 \right)$$  \hspace{1cm} (8)

$$\Theta = \frac{1}{3} \sin^{-1} \left[ \frac{3\sqrt{3} J_3}{2 J_2^{3/2}} \right]$$  \hspace{1cm} (9)

Where, $J_3 = S_y^2 S_z^2 - S_y^2 S_x^2 - S_y^2 S_z^2 - S_z^2 S_x^2 - S_x^2 S_z^2$, and $S_x = \sigma_x - \sigma_m$, $S_y = \sigma_y - \sigma_m$, $S_z = \sigma_z - \sigma_m$.

For Mohr-Coulomb material model, six material properties are required. These properties are the friction angle $\phi$, cohesion $C$, dilation angle $\psi$, Young’s modulus $E$, Poisson’s ratio $\mu$ and unit weight of soil $\gamma$. Young’s modulus and Poisson’s ratio have a profound influence on the computed deformations prior to slope failure, but they have little influence on the predicted factor of safety in slope stability analysis. Dilation angle, $\psi$ affects directly the volume change during soil yielding. If $\psi = \phi$, the plasticity flow rule is known as “associated”, the influence of soil dilatancy on slope stability is considered too much. If $\psi \neq \phi$, the plasticity flow rule is considered as “no-associated”, it is considered that the volume of slope does not change when yielding. A great many studies show that with the increase of dilatancy angle, the safety factor of slope increases gradually, which shows that the increase of dilatancy angle increases the resistance of soil, and they concluded that applying the associated flow rule($\psi' = \phi'$) yields higher and relatively unsafe FoSs because of overestimated soil strength. However, the change in the volume during the failure is not considered in this study and therefore the dilation angle is taken as 0. Therefore, only three parameters (friction angle, cohesion and unit weight of material) of the model material are considered in the modeling of slope failure.

2.2. Criterion of slope failure

In the finite element strength reduction analysis, Mohr-Coulomb yield criterion is usually used to analyse factor of safety to slope stability. Generally, three methods are used to judge the instability of slope, as follows:

(1) Non-convergence of finite element numerical iteration is taken as a sign of slope instability.
(2) The abrupt change of displacement (including horizontal displacement, vertical displacement or total displacement) in characteristic parts (such as the top of slope and the maximum displacement in slope, etc.) is taken as a sign of slope failure.
(3) The penetration of generalized plastic strain or equivalent plastic strain from the foot of slope to the top of slope is a sign of slope instability.

Non-convergence within a user-specified number of iterations in finite element program is taken as a suitable indicator of slope failure. This actually means that no stress distribution can be achieved to satisfy both the Mohr-Coulomb criterion and global equilibrium. Slope failure and numerical non-convergence take place at the same time and are joined by an increase in the displacements. Usually, value of the maximum nodal displacement just after slope failure has a big jump compared to the one before failure.

3. Stability analysis of slope with a weak layer

3.1. Establishment of analytical model

The slope model consists of a thin layer of week material. The weak layer runs parallel to the slope and then turns to be horizontal in the toe zone. The geometry of the slope model is presented in Figure 1.
The presence of this thin weak layer in the slope influences the stability of slope. In this example, different values of $C_{U2}/C_{U1}$ was considered. The slope height is 10 meters and $C_u/\gamma H$ ratio is taken as 0.25. Table 1 presents the material properties for the slope model.

### Table 1. Material properties of slope

| $\gamma$ (kN/m$^3$) | $\phi$ (°) | $C_{U1}$ (kN/m$^2$) | $C_{U2}$ | $C_{U2}$ | $C_{U2}$ |
|---|---|---|---|---|---|
| 20 | 0.0 | 50 | 50 | 30 | 10 |

3.2. Analysis results of Finite Element Strength Reduction Method

In this example, the soil was modeled as six-node triangular elements, bounded material with Mohr–Coulomb failure criterion. The meshes of slope are discretized with 2467 elements. Figure 2 shows the meshes of slope. The base was fixed in all degrees of freedom and all sidewalls were constrained in the normal direction corresponding to smooth rigid boundaries.

### Table 2. Factor of slope safety under different weak layer parameters by Finite Element Strength Reduction Method

| Parameters of the weak layer | $C_{U2}$=50kPa $(C_{U2}/C_{U1}=1)$ | $C_{U2}$=30kPa $(C_{U2}/C_{U1}=0.6)$ | $C_{U2}$=10kPa $(C_{U2}/C_{U1}=0.2)$ |
|---|---|---|---|
| Factor of Safety (FOS) | 1.46 | 1.37 | 0.575 |

According to the results of strength reduction analysis by finite element method, the stability of slope decreases with the decrease of strength of weak layer. When the strength of weak interlayer is reduced to a specific value, the sliding surface shape of slope gradually changes from circular sliding surface to polygonal sliding surface, and the stability of its slope will also be controlled by the strength of weak layer.

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Fig. 3 Deformed meshes for strength reduction \( \left( \frac{C_{U2}}{C_{U1}} = 1 \right) \)

Fig. 4 Contours of total displacements for strength reduction \( \left( \frac{C_{U2}}{C_{U1}} = 1 \right) \)

Fig. 5 Deformed meshes for strength reduction \( \left( \frac{C_{U2}}{C_{U1}} = 0.6 \right) \)

Fig. 6 Contours of total displacements for strength reduction \( \left( \frac{C_{U2}}{C_{U1}} = 0.6 \right) \)

Fig. 7 Deformed meshes for strength reduction \( \left( \frac{C_{U2}}{C_{U1}} = 0.2 \right) \)
3.3. Analysis results of Finite Element Strength Reduction Method
The safety factors of the slope layer under different strength conditions are calculated by rigid body limit equilibrium method, and the safety factors of the slope under different layer parameters are obtained as shown in Table 3, and the stability calculation results of the slope obtained by different sliding search methods are shown in Figures 9-11. Figure 9-11 shows the failure mechanisms of $C_{u2}/C_{u1}=0.2$, $C_{u2}/C_{u1}=0.6$ and $C_{u2}/C_{u1}=1.0$.
According to the calculation results of rigid body limit equilibrium, the stability of slope gradually decreases with the decrease of strength of weak layer. However, the results of slope stability obtained by different search methods are quite different, and the possible sliding surfaces of slopes are also quite different.

Table 3. Factor of slope safety under different weak layer parameters by Rigid Body Limit Equilibrium Method

| Parameters of the weak layer | $C_{u2}=50kPa$ | $C_{u2}=30kPa$ | $C_{u2}=10kPa$ |
|-----------------------------|----------------|----------------|----------------|
|                            | $C_{u2}/C_{u1}=1$ | $C_{u2}/C_{u1}=0.6$ | $C_{u2}/C_{u1}=0.2$ |
| Search method of sliding surface | Circular | Polygonal | Composite | Circular | Polygonal | Composite | Circular | Polygonal | Composite |
| Factor of safety (FOS) | 1.481 | 1.888 | 1.362 | 1.395 | 1.455 | 1.297 | 1.267 | 0.758 | 0.652 |

(a) Circular sliding surface (FOS=1.493)

(b) Polygonal sliding surface (FOS=1.888)
(c) Composite sliding surface  (FOS=1.362)

Fig.9 Position and shape of slope sliding surface by Limit Equilibrium Method  \( \frac{C_{U2}}{C_{U1}}=1 \)

(a) Circular sliding surface  (FOS=1.395)

(b) Polygonal sliding surface  (FOS=1.455)

(c) Composite sliding surface  (FOS=1.297)

Fig.10 Position and shape of slope sliding surface by Limit Equilibrium Method  \( \frac{C_{U2}}{C_{U1}}=0.6 \)

(a) Circular sliding surface  (FOS=1.267)

(b) Polygonal sliding surface  (FOS=0.758)
Composite sliding surface (FOS=0.652)

**图 11 Position and shape of slope sliding surface by Limit Equilibrium Method (CU2/CU1=0.2)**

For the homogeneous slope model (CU1/CU2=1.0), and factor of safety to slope stability obtained by circular sliding surface method and composite sliding surface method are similar, while the factor of safety obtained by polygonal sliding surface method are obviously higher. For the case of CU1/CU2≈0.6, the sliding surface of the slope changes obviously whether it is circular sliding surface, composite sliding surface or polygonal sliding surface. The factor of safety obtained by circular sliding surface is in the middle, and polygonal sliding surface is the highest, while the composite sliding surface is the smallest. For the case of CU1/CU2≈0.2, the factor of safety obtained by circular sliding surface is the highest, which is greater than 1.2, while the factor of safety obtained by composite sliding surface and polygonal sliding surface is similar, both of which are less than 0.8, which is obviously lower than that obtained by circular sliding surface. Therefore, when CU1/CU2 is greater than 0.6, the instability mode of slope is circular sliding, and the factor of safety obtained by circular and compound sliding surface is similar, so the factor of safety should not be calculated by polygonal sliding surface. However, when CU1/CU2 is less than 0.6, the instability mode of slope will change from circular sliding to polygonal sliding along the thin weak layer, and the factor of safety obtained by composite sliding surface and polygonal sliding surface is similar, so the factor of safety calculated by circular sliding surface should not be adopted. Based on the above analysis, when rigid body limit equilibrium is used to analyse the stability of the slope with weak layer, it is necessary to understand the deformation and instability mode of the slope in order to obtain the factor of slope safety which is consistent with the actual situation.

4. Discussion on Stability of Slope with a Weak layer

The calculation results obtained by the finite element method and the rigid body limit equilibrium method are shown in Figure 12. The results obtained in this paper are compared well with Griffiths’ FE results as it is shown in Figure 11, and there failure mechanisms are same as those obtained by Griffiths. For a homogeneous slope (CU1/CU2=1.0), the factor of safety was close to the Taylor solution (Taylor, 1937) of FOS 1.47 and gave the expected circular base failure mechanism. As the strength of the thin layer was gradually reduced, a distinct change in the nature of the results was observed when CU1/CU2=0.6. Also shown on this figure are rigid body limit equilibrium solutions obtained using Janbu’s method assuming circular sliding surface, polygonal sliding surface and composite sliding surface mechanism following the path of weak layer. When the strength of weak layer decreases gradually, the factor of safety also decreases gradually. However, the factor of safety decreases the least by circular sliding surface, the factor of safety obtained using the circular sliding method is least affected by the strength of weak layer, and the factor of safety with other sliding modes are greatly affected by the strength of weak layer.
Fig.12 FOS for different values of $C_{u2}/C_{u1}$ from Finite Element Strength Reduction Method and Rigid Body Limit Equilibrium Method

For the homogeneous slope model ($C_{u1}/C_{u2}=1.0$), the factors of safety obtained by circular sliding method, finite element strength reduction method and composite sliding surface are close. The failure mechanism showed a circular sliding which confirms the expectation. While the factor of safety obtained by polygonal sliding surface is high, which does not conform to the deformation and instability mode of slope. For the case of $C_{u2}/C_{u1}$<0.6, a distinct change is observed, and the potential sliding surface of the slope occurs circular failure along the weak layer. It shows that for $C_{u2}/C_{u1}$>0.6, the safety factors obtained by circular sliding surface, finite element strength reduction method and composite sliding surface are close, the failure mechanism governs the slope behaviour and the weak layer doesn’t influence the factor of safety. For $C_{u2}/C_{u1}$<0.6, the slope is a non-circular failure sliding along the weak layer, and the factors of safety obtained by finite element strength reduction method, polygonal sliding surface and composite sliding surface are close, and the factor of safety falls linearly with the strength reduction of the weak layer. Therefore, when the weak layer of slope is reduced to a specific value, the thin weak layer mechanism controls the slope behaviour, and the deformation and failure mode of slope will change from circular sliding to polygonal sliding. If the circular sliding method is used to analyse the slope stability, the factor of safety will be much higher other methods, which is contrary to the actual instability mode of the slope.

Based on the above analysis, using finite element strength reduction method to calculate the factor of safety can not ony reveal the instability mechanism of slope obviously, but also obtain the factor of safety. However, rigid body limit equilibrium method is used to analyse the stability of the slope with weak layer, the instability mechanism of the slope must be analysed, then the calculation method consistent with the sliding mode can be selected, so that the factor of safety can be accurately obtained and the stability of the slope can be reasonably evaluated.

5. Conclusion
The instability mechanism and stability of slope with weak layer are studied by finite element strength reduction method and rigid body limit equilibrium method, and the factors of safety to slope stability by finite element strength reduction method and limit equilibrium method are compared and analysed. The conclusions are as follows.
(1) Slope stability decreases with the decrease of weak layer strength. When the slope is homogeneous, the factor of safety to slope stability obtained by circular arc method, finite element strength reduction
method and composite sliding search method are close, which is the instability mechanism of circular sliding surface. When the strength of weak interlayer is greater than 0.6 times that of soil, the potential sliding surface of slope is not affected by weak layer. When the strength of weak layer is less than 0.6 times that of soil, the instability mechanism of slope changes from circular arc sliding to polygonal sliding along weak layer, and its stability is controlled by the strength of weak layer.

(2) When rigid body limit equilibrium is used to analyse the stability of the slope with weak layer, the instability mechanism of the slope must be analysed, and then the appropriate calculation method can be selected, so that the safety of factor to slope stability can be accurately obtained and the stability of the slope can be reasonably evaluated.

(3) Compared with the rigid body limit equilibrium method, the finite element strength reduction method can be used to calculate the factor of safety to slope stability without analysing the instability mechanism of the slope in advance, and only the finite element strength reduction analysis is needed. The analysis results that the instability mechanism of the slope is not only revealed obviously, but also the factor of safety to slope stability is obtained.

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