Determination of the gluon condensate with DIS experiments using holographic approach

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Abstract

We study the deep inelastic scattering (DIS) of a proton-targeted lepton in the presence of gluon condensation using gauge/gravity duality. We use a modified $AdS_5$ background where the modification parameter $c$ corresponds to the gluon condensation in the boundary theory. First, when examining the electromagnetic field, we find that non-zero $c$ can increase the magnitude of the field. In the next step, we calculate wave function equations for baryonic states where the mass of the proton target requires a value contribution of $c$ as $c = 0.0120 \text{ GeV}^4$. Proceeding by electromagnetic field and baryonic states, we derive the holographic interaction action related to the amplitude of the scattering. Finally, we compute the corresponding structure functions numerically as functions of $x$ and $q$, which are Björken variables and the lepton momentum transfers, respectively. Comparing the Jlab Hall C data with our theoretical calculations, our results are acceptable.

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1 Introduction

The Deep inelastic scattering (DIS) process is used to study internal structure of proton which is a puzzle in quantum chromodynamic (QCD). The main difficulty arises from the fact that massless gluons and nearly massless quarks give rise to the mass of the proton $M \sim 1\, \text{GeV}$. One can say that from the point of view of special relativity the missing mass could be considered as initiation by kinetic energy of quarks and gluons inside the proton, but it turns out that the mentioned kinetic energy is not enough to describe the total vanishing mass. To learn more about this topic, breaking the conformal symmetry in QCD is another reason that should be considered. It leads to anomalous dimension contributions in the DIS. Recall that QCD coupling is large at low energies, so perturbation calculations cannot be used to study many properties of hadrons. The alternative way for this purpose, is to use a holographic approach to study DIS.

In AdS/CFT description a strongly coupled field theory at the boundary of the AdS space could be described by a weakly coupled gravitational theory in the bulk of the AdS. In other words a ten dimensional geometry at the corresponding boundary is an exact dual of a supersymmetric $SU(N)$ gauge theory with large $N$ in the bulk. In particular, the string theory in the space $AdS_5 \times S_5$ is a dual of a four dimensional gauge theory. AdS/CFT duality is for conformal theories originally, but fortunately it has been generalized to non-conformal theories like QCD, so in any case of interest one can study a QCD problem by assuming an appropriate AdS background. In the current holographic study a DIS process with a proton target is considered using a modified $AdS_5$ background. In general in a phenomenological holographic approach, AdS/QCD tries to adapt a five-dimensional effective field theory to QCD as much as possible. Therefore mass gap, confinement and supersymmetry breaking are obtained by considering some modifications in gravitational duals. To break the conformal symmetry, one can modify the radial coordinate, the 5th dimension of spacetime, as was done in the reference.
We should adopt a modified AdS that introduces the gluon condensation on the QCD side of the duality. Originally, gluon condensation was a measure of the non-perturbative physics in zero-temperature QCD \[5\]. It was later identified as an order parameter for confinement to study some non-perturbative phenomena \[6–9\].

A well-known modified holographic model introducing gluon condensation in the boundary theory is given by the following background metric in Minkowski spacetime \[10\],

\[
ds^2 = \frac{L^2}{z^2} \left( \sqrt{1 - c^2 z^8} (-dt^2 + \sum_i dx_i^2) + dz^2 \right),
\]

\((1)\)

\[
\varphi(z) = \sqrt{\frac{3}{2}} \ln \frac{1 + cz^4}{1 - cz^4} + \varphi_0.
\]

\((2)\)

In the above dilaton-wall solution, \(i = 1, 2, 3\) are orthogonal spatial boundary coordinates, \(z\) denotes the 5th dimension, radial coordinate and \(z = 0\) sets the boundary. \(\varphi_0\) is a constant, \(c = \frac{1}{z_c^4}\) and \(z_c\) denotes the IR cutoff. It shows that \(z\) is defined from zero to the IR limit as usual. To clarify, parameter \(c\) does not bound upper limit of \(z\) to values less than cutoff, in fact it should be interpreted as \(c < \frac{1}{z_c^4}\). In other words, for a very large value of \(z\), the parameter \(c\) tends to an infinitesimal value based on the range \(1 - c^2 z^8 > 0\). Also we calculate in the unit where \(L = 1\). To investigate how the forth correction of radial coordinate (with the coefficient denoted \(c\)) appears in the metric, let us mention that the dilaton field is dual to a scalar operator and the metric is dual to the energy-momentum tensor of the dual field theory \[11\] (for more discussions see \[12–14\]). Expanding the dilaton profile near \(z = 0\) will give,

\[
\varphi(z) = \varphi_0 + \sqrt{6} cz^4 + ....
\]

\((3)\)

According to the holographic dictionary \(\varphi\) and \(c\) are the source and the parameter associated with the confinement respectively. Obviously \(c\) in the background metric breaks the conformal symmetry so the gluon condensation appears in the boundary theory. The relevant phenomenological information show its value generally lie in the range \(0 < c \leq 0.9\,\text{GeV}^4\) \[15–17\] however we are interested in finding the exact value of \(c\) in a proton-targeted DIS process.

The holographic description of the gluon condensation allows many physical quantities to be studied in this context. Firstly in order to get familiar with its phenomenological aspects notice that the dilaton wall solution represented by \([11, 2]\) is related to the zero temperature case, hence this is suitable for studying DIS and its physics. As it must be, one can readily check that in the limit \(c \rightarrow 0\), \([11]\) reduces to \(\text{AdS}_5\) which does not present mass gap, while modification of radial coordinate would yield more phenomenological results. In fact, it has become an approach to discuss more phenomenological aspects using modified AdS \[18–20\].

In the most related work to our case in \([21]\), such a scattering has been investigated by using a deformed AdS. Since models with anomalous dimension in AdS/QCD lead to mass-scale fermionic field generation, many works have used them to deal with DIS \[21,54\]. In view of all the above
motivations, we will use a holographic model of gluon condensation (1) and (2) in the current work to study DIS with proton target.

This paper is structured as follows, after a brief overview of the DIS properties using holography in section 2 we will study electromagnetic interactions and baryonic states in deep inelastic scattering in sections 3 and 4 respectively. Based on these results, Section 5 provides the interaction action, then we will study structure functions according to the relationship between such action and scattering amplitude. In section 6 we briefly review and discuss our results.

2 DIS parametrization and holography

We begin this section with a brief overview of deep inelastic scattering to clarify our motivation and goals. The main application of DIS in particle physics is the study of internal hadronic structure and strong interactions. Consider a DIS process in which a lepton scattered off a proton target. During this scattering a virtual photon is exchanged. Proton fragmentation creates a lepton and some final hadronic states. It should be noted that production of final hadronic states depends on four momentum the initial lepton transfers. Therefore, the four momentum causes the inner quarks and gluons of proton expelled out, finally in the next step quark anti-quark pairs are hadronized. According to [55] DIS is parametrized by Bjorken dynamical variable which is defined as,

\[ x = -\frac{q^2}{2P \cdot q}, \]  

where \( q \) is the momentum that lepton transfers to the proton target via a virtual photon and \( P \) is the initial momentum of the proton. We adopt the method has been explained in [55] and rederived in [21, 56]. Doing so, the hadronic transition amplitude is given as,

\[ W^{\mu\nu} = F_1(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}) + \frac{2x}{q^2} F_2(P^\mu + \frac{q^\mu}{2x})(P^\nu + \frac{q^\nu}{2x}), \]  

where \( F_{1,2} = F_{1,2}(x, q^2) \) are some structure functions.

Now let’s relate the matrix above to holography. From the AdS/QCD dictionary, elements of (5) on the QCD side are associated with the interaction action on the AdS side as [22],

\[ \eta_{\mu} < P + q, s_X | J^\mu(0) | P, s_i > = K_{eff} S_{int}, \]  

where \( \eta_{\mu} \) is polarization of virtual photon, \( | P, s_i > \) represents a normalizable proton state with spin \( s_i \), \( J^\mu \) is the electromagnetic quark current and \( s_X \) denotes the final state. It is worth to mention that \( K_{eff} \) is an effective factor that adjusts the bulk supergravity quantities to the boundary phenomenologically. This is based on a different perspective in reference [41] that bulk/boundary quantities of (6) are proportional, and not necessarily equal.

The interaction action is written as,

\[ S_{int} = g_V \int d^4y e^{-\phi} \sqrt{-g} \bar{\psi} X \Gamma^\mu \psi_i, \]  

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and $g_V$ is a coupling constant related to the electric charge of the baryon, $\varphi$ is the dilaton field and $\sqrt{-g}$ is given by the metric, $\phi^\mu$ is the electromagnetic gauge field, $\Psi_1$ and $\Psi_X$ are the initial and final state spinors for the baryon, respectively and $\Gamma_\mu$ are Dirac gamma matrices in the curved space. By computing all above quantities according to (1) and (2), we study interaction action of DIS.

3 Electromagnetic interactions in deep inelastic scattering

A photon is exchanged during scattering, so we study the electromagnetic interactions in the bulk. It can be described as the presence of photon in the modified AdS. The action for a five dimensional massless gauge field $\phi^m$ is given by,

$$ S = -\frac{1}{4} \int d^5x e^{-\varphi} \sqrt{-g} F^{mn} F_{mn}, $$  \hfill (8)

where $F^{mn} = \partial^m \phi^n - \partial^n \phi^m$, and $m,n$ refer to the 5-dimensional space includes Minkowski spacetime coordinates, $\mu, \nu$ and $z$, and $\varphi$ is the dilaton field given by (2). Note that $\varphi$ and $\phi$ should be differentiated. In fact (8) is an action showing the gauge field $\phi$ on a background coupled to a dilaton field $\varphi$. From (8) the equation of motion of such an electromagnetic field is derived as,

$$ \partial_m [e^{-\varphi} \sqrt{-g} F^{mn}] = 0. $$  \hfill (9)

Considering $m, n \equiv \mu, \nu, z$ the relation (9) leads to,

$$ \partial_\mu \left[ \frac{1}{z} (1 + cz^4)^{1 - \sqrt{3}/2} (1 - cz^4)^{1 + \sqrt{3}/2} F^{\mu z}] = 0, \right. $$

$$ \partial_z \left[ \frac{1}{z} (1 + cz^4)^{1 - \sqrt{3}/2} (1 - cz^4)^{1 + \sqrt{3}/2} F^{z \mu}] = 0. \right. $$  \hfill (10)

In order to solve the equations of motion of the gauge field in (10), we should first fix the gauge. Suppose there is an electromagnetic field in the bulk defined with the metric (1). This obeys the 5–dimensional Maxwell equation supplemented by a gauge condition which we take to be,

$$ e^{-\varphi} \sqrt{-g} \partial_\mu \phi^\mu + \partial_z (e^{-\varphi} \sqrt{-g} \phi_z) = 0. $$  \hfill (11)

From (11) one can write,

$$ \partial_\mu \phi^\mu + \frac{z}{(1 + cz^4)^{1 - \sqrt{3}/2} (1 - cz^4)^{1 + \sqrt{3}/2}} \partial_z \left[ \frac{(1 + cz^4)^{1 - \sqrt{3}/2} (1 - cz^4)^{1 + \sqrt{3}/2}}{z} \phi_z \right] = 0, $$  \hfill (12)

so,

$$ \Box \phi_\mu + \partial_\mu \partial_z \phi_z - \frac{1 + 4 \sqrt{6} cz^4 + 7 c^2 z^8}{z(1 - c^2 z^8)} \partial_\mu \phi_z = 0. $$  \hfill (13)
Using the gauge (13) together with (10) leads to the following equations,

\[ \square \phi_z - \partial_\mu \partial_z \phi^\mu = 0, \]  

\[ \square \phi_\mu + \partial_z^2 \phi_\mu - \frac{1 + 4\sqrt{6}c^4 + 7c^8z^8}{z(1 - c^2z^8)} \partial_z \phi_\mu = 0. \]  

(14)  

(15)

At this point one could consider a photon with a particular polarization as \( \eta_\mu q^\mu = 0 \) for simplicity, hence only the \( \phi^\mu \) component contributes in the scattering \([22, 25, 56]\). In the latter case we need to solve only (15). This equation cannot be solved analytically and we have to use numerical methods. Let us consider \( \phi_\mu(z, q, y) = \eta_\mu e^{iq.\eta} \phi_1(z, q) \), and the condition \( \phi_\mu(z, q, y) \big|_{z=0} = \eta_\mu e^{iq.\eta} \) in (15). We use reference \([41]\) for the initial condition, because we should get same results at the boundary \( z = 0 \). At the IR limit we take the Neumann boundary condition. So \( \phi_1(z, q) \) should be convergent and the assumption is taken as follows, \( \frac{\partial \phi_1}{\partial z} \big|_{z=z_c} = 0 \). Then we can get \( \phi_1(z, q) \) versus \( z \) that describes the behaviour of the electromagnetic field in the bulk. Figure 1 shows \( \phi_1(z, q) \) for different values of \( q^2 \) and \( c \).

Figure 1: Electromagnetic field in the bulk with a) large value of \( q^2 \) and two different values of \( c \), b) small value of \( q^2 \) and two different values of \( c \), c) two different values of parameter \( q^2 \) at fixed value of \( c \).

4 Baryonic state equations in deep inelastic scattering

In this section, we study the baryonic initial and final states for further requirements of the interaction action (7). The equations of motion of fermionic states are,

\[ (\not{D} - m_5) \Psi = 0, \]  

(16)
where $m_5$ is the baryon bulk mass and the operator $\mathcal{D}$ is defined as,

$$\mathcal{D} = g^{mn} \epsilon^a_n \gamma_a (\partial_m + \frac{1}{2} \omega_m \Sigma_{bc}),$$  \hfill (17)

in which $\gamma_a = (\gamma_\mu, \gamma_5)$, $\{ \gamma_a, \gamma_b \} = 2 \eta_{ab}$ and $\Sigma_{bc} = \frac{1}{4} [\gamma_\mu, \gamma_5]$ \textsuperscript{57,61}. $\gamma_\mu$ are Dirac’s gamma matrices. a, b, c are flat space and, m, n, p, q are AdS space indices respectively. As before $\mu, \nu$ represent the Minkowski space. With the metric (1) Vielbein are computed as,

$$e^a_n = (1 - \frac{c^2 z^8}{z})^{\frac{1}{2}} \delta^a_n, \hfill (18)$$

The above terms give us first term of (17). Now we should calculate the second term of that. Spin connection is given by,

$$\omega_{ab}^m = e^a_n \partial_m e^b_n + e^a_n e^p e^{pm} \Gamma^{m}_{pm}, \hfill (19)$$

where the Christoffel symbols are,

$$\Gamma^{p}_{mn} = \frac{1}{2} g^{pq} (\partial_n g_{mq} + \partial_m g_{nq} - \partial_q g_{mn}). \hfill (20)$$

From the metric (1), one may write, $g_{\mu\nu} = \sqrt{1 - \frac{c^2 z^8}{z}} \eta_{\mu\nu}$ and $g_{zz} = \frac{1}{z^2}$. So the only non vanishing terms are, $\Gamma^{z}_{\mu\nu}, \Gamma^{z}_{zz}, \Gamma^{\mu}_{\nu z}$. After computation they are written as,

$$\Gamma^{z}_{\mu\nu} = - \frac{(1 + \frac{c^2 z^8}{z})}{z (1 - \frac{c^2 z^8}{z})} \eta_{\mu\nu}$$

$$\Gamma^{z}_{zz} = \frac{1}{z}$$

$$\Gamma^{\mu}_{\nu z} = \frac{(1 + \frac{c^2 z^8}{z})}{z (1 - \frac{c^2 z^8}{z})} \delta^{\mu}_{\nu}. \hfill (21)$$

Also from (1) together with (18) and (20) the relation (19) turns to,

$$\omega_{m}^{z\nu} = - \omega_{m}^{\nu z} = - \frac{(1 + \frac{c^2 z^8}{z})}{z (1 - \frac{c^2 z^8}{z})} \delta^{\nu}_{\mu}, \hfill (22)$$

hence other components of $\omega_{m}^{ab}$ are zero. Using these solutions, (17) is given by,

$$\mathcal{D} = z \gamma^5 \partial_z + \frac{z}{(1 - \frac{c^2 z^8}{z})^{\frac{1}{2}}} \gamma^\mu \partial_\mu - 2 \frac{(1 + \frac{c^2 z^8}{z})}{z (1 - \frac{c^2 z^8}{z})^{\frac{1}{2}}} \gamma_5,$$  \hfill (23)

and the EOM (16) is written as,

$$[z \gamma^5 \partial_z + \frac{z}{(1 - \frac{c^2 z^8}{z})^{\frac{1}{2}}} \gamma^\mu \partial_\mu - 2 \frac{(1 + \frac{c^2 z^8}{z})}{z (1 - \frac{c^2 z^8}{z})^{\frac{1}{2}}} \gamma_5 - m_5] \Psi = 0. \hfill (24)$$
According to the fact that spinor is either left-handed or right-handed, and since Kaluza-Klein modes are dual to the chirality spinors we decompose these components and expand as,

$$\Psi_{L/R}(x^\mu, z) = \sum_n f_{L/R}^n(x^\mu) \chi_{L/R}^n(z),$$

(25)

by applying (25) in the equation of motion (24) we find the coupled equations as,

$$\left(\partial_z - 2 \frac{(1 + c^2 z^8)}{z^2(1 - c^2 z^8)^{\frac{3}{4}}} + \frac{m_5}{z}\right)\chi_L(z) = \frac{M_n}{(1 - c^2 z^8)^{\frac{3}{4}}} \chi_R(z),$$

(26)

$$\left(\partial_z - 2 \frac{(1 + c^2 z^8)}{z^2(1 - c^2 z^8)^{\frac{3}{4}}} - \frac{m_5}{z}\right)\chi_R(z) = \frac{-M_n}{(1 - c^2 z^8)^{\frac{3}{4}}} \chi_L(z).$$

(27)

Decoupling (26) and (27) leads to the following equation which describes both left-handed and right-handed sectors as,

$$-(1 - c^2 z^8)^{\frac{3}{4}} \left(\partial_z - 2 \frac{(1 + c^2 z^8)}{z^2(1 - c^2 z^8)^{\frac{3}{4}}} \pm \frac{m_5}{z}\right)\chi_R(z) - 2 \frac{(1 + c^2 z^8)}{z^2(1 - c^2 z^8)^{\frac{3}{4}}} \chi_R(z) = M_n^2 \chi_{R/L}(z).$$

(28)

Below we create a Schrödinger-like equation by applying a transformation like this,

$$\chi_{R/L}(z) = e^{-\frac{2 (1 - c^2 z^8)^{\frac{3}{4}}}{z^2(1 - c^2 z^8)^{\frac{3}{4}}}} \psi_{R/L}(z),$$

(29)

so the equation (28) is written as,

$$\sqrt{1 - c^2 z^8} \left(-\psi''_{R/L}(z) + \frac{m_5 (m_5 \mp 1) - c^2 z^8 (2m_5^2 + 7) + c^4 z^16 m_5 (m_5 \pm 1)}{z^2(1 - c^2 z^8)^2} \psi_{R/L}(z)\right) = M_n^2 \psi_{R/L}(z)$$

(30)

In (30), $m_5$ is a parameter on the AdS side of gauge/ gravity duality and is related to the baryon mass on the gauge side, so the normalizable solutions of the equations above are dual to the states in the boundary theory. In pure AdS space, the bulk mass is related to the canonical conformal dimension $\Delta_{can}$ of a boundary operator as,

$$|m_5^{AdS}| = \Delta_{can} - 2.$$  

(31)

Recall that QCD is not a conformal field theory since it has a mass gap. So the gravity side should be modified somehow and then it is not pure AdS any more. If one modifies AdS, the canonical dimension $\Delta_{can}$ of an operator has an anomalous contribution $\gamma$ implying an effective scaling dimension.

$$|m_5| = \Delta_{can} + \gamma - 2.$$  

(32)
The contribution of the anomaly is related to how one modifies the theory. For example in [56] modification of the scale introduces the mass gap in the theory. Therefore the anomalous contribution represents the energy scale in the theory and leads to the mass spectra. So, the main task is to find the value of the bulk mass in (32). In AdS/CFT dictionary, the bulk mass is related to the dimension, means the energy scale of the boundary theory is holographically related to the localization in the z-coordinate, therefore we have z-dependent mass in the bulk. Let us focus on \( m_5 \). One can fit \( m_5 \) numerically as the equations (30) have normalizable solutions. By fixing \( M \) as proton mass, we should find suitable values for \( c \) and \( m_5 \) which give us well defined answers.

Figures 2 and 3 show initial (n=1) and final (for two excited states as n=2,3) chiral components of the wave function respectively. To solve the equations (30) numerically, we fix proton mass \( M \) as eigenvalue of equation, therefore \( m_5 \) and \( c \) are found as \( c = 0.0120 \) GeV\(^4 \) and \( m_5 = 0.081 \) GeV. Interestingly, the value of parameter \( c \) on the AdS side is very close to the phenomenological GC value of QCD as found \( G_2 = 0.010 \pm 0.0023 \) GeV\(^4 \) in the reference [18]. Another consequence of the presence of \( c \) is that the anomaly \( \gamma \) in (32) affects the bulk mass intensely. After finding both left-handed and right-handed modes from (30) we have,

\[
\Psi_i = \frac{e^{-2(1-c^2z^2)^{1/2}}}{(1-c^2z^2)^{1/2}} e^{ip\cdot y \cdot [(1+\gamma_5/2)\psi^i_L + (1-\gamma_5/2)\psi^i_R]} u_{s_i}(p),
\]

as the initial wave function for the target proton and,

\[
\Psi_X = \frac{e^{-2(1-c^2z^2)^{1/2}}}{(1-c^2z^2)^{1/2}} e^{ip_X\cdot y \cdot [(1+\gamma_5/2)\psi^X_L + (1-\gamma_5/2)\psi^X_R]} u_{s_X}(p),
\]

as the final wave function for the hadronic state. These will be used later.
5 Modified geometry and the action of deep inelastic scattering

According to (6) and (7) we find the interaction action with the electromagnetic field and the baryonic states obtained from (15) and (33)-(34) respectively. The interaction action (7) is written as,

\[ S_{int} = gV \int dz d^{4}y e^{-i \Phi_{X}} \sqrt{-\phi} \phi^\mu \bar{\Psi} \chi_{\mu \nu} \Psi_{i} \]

and from (34) one writes,

\[ \bar{\Psi} = e^{-iP_{X} \cdot y} \bar{u}_{sX}(\hat{p}_{L} \psi_{L} + \hat{p}_{R} \psi_{R}) \gamma_{\mu} \left( \frac{1 + \gamma_{5}}{2} \psi_{X}^{L} + \frac{1 - \gamma_{5}}{2} \psi_{X}^{R} \right). \]

Therefore (35) is given by,

\[ S_{int} = \frac{gV}{2} \int dz d^{4}y e^{-i(P_{X} - P - q) \cdot y} \eta_{\nu} \tilde{\mu} \phi_{1} \frac{1}{z^{2}} (1 - cz^{4})^{\frac{3}{2} + \sqrt{2}} (1 + cz^{4})^{\frac{3}{2} - \sqrt{2}} \]

\[ e^{- \frac{4(1 - c^{2}z^{4})}{z}} \left[ \bar{u}_{sX}(\hat{P}_{L} \psi_{L}^{X} + \hat{P}_{R} \psi_{R}^{X}) \gamma_{\mu}(\hat{P}_{L} \psi_{L}^{i} + \hat{P}_{R} \psi_{R}^{i})u_{s} \right] \]

\[ = \frac{gV}{2} (2\pi)^{4} \delta^{4}(P_{X} - P - q) \eta_{\nu} \int dz \frac{1}{z^{2}} (1 - cz^{4})^{\frac{3}{2} + \sqrt{2}} (1 + cz^{4})^{\frac{3}{2} - \sqrt{2}} \]

\[ e^{- \frac{4(1 - c^{2}z^{4})}{z}} \phi_{1} \left[ \bar{u}_{sX} \gamma_{\mu} \tilde{P}_{R} u_{s} \psi_{L}^{i} + \bar{u}_{sX} \gamma_{\mu} \tilde{P}_{L} u_{s} \psi_{R}^{i} \right]. \]

(37)
By defining the following integral,

\[ B_{R,L} = \int dz \frac{e^{-\frac{(1-c^2z^2)^{1/2}}{2}}}{(1-c^2z^2)^{1/2}} \frac{1}{\pi^2} (1-c^2z^2)^{1/2} \psi_{R,L}(\frac{z}{\sqrt{2}}) \overline{\psi}_{R,L}(\frac{z}{\sqrt{2}}), \]  

(38)

is written as,

\[ S_{int} = \frac{g_{\nu}}{2} (2\pi)^4 \delta^4(P_X - P - q) \eta_\mu [\overline{u}_s \gamma_\mu \hat{P}_{R} u_s B_L + \overline{u}_s \gamma_\mu \hat{P}_L u_s B_R], \]  

(39)

and \( (5) \) is written as,

\[ \eta_\mu < P_X |J_\mu(q)| P_i > = \frac{g_{\mu \nu}}{2} \delta^4(P_X - P - q) \eta_\mu [\overline{u}_s \gamma_\mu \hat{P}_{R} u_s B_L + \overline{u}_s \gamma_\mu \hat{P}_L u_s B_R], \]

\[ \eta_\nu < P_i |J_\mu(q)| P_X > = \frac{g_{\mu \nu}}{2} \delta^4(P_X - P - q) \eta_\nu [\overline{u}_s \gamma_\mu \hat{P}_{R} u_s B_L + \overline{u}_s \gamma_\mu \hat{P}_L u_s B_R], \]  

(40)

where \( g_{\mu \nu}^2 = \kappa_{eff}^2 g_{\nu}^2 (2\pi)^8 \), and \( g_{\nu}^2 = \frac{1}{4\pi^2} \). \( \kappa_{eff}^2 \) should be fitted numerically as shown in table 1. Considering the above equations and after some calculations (for details see [21,22,23]) the relation \( (5) \) is obtained as,

\[ \eta_\mu \eta_\nu W^{\mu \nu} = \eta^2 F_1(q^2, x) + \frac{2x}{q^2} (\eta.P)^2 F_2(q^2, x), \]  

(41)

where \( F_1 \) and \( F_2 \) are,

\[ F_1(q^2, x) = \frac{g_{\mu \nu}^2}{4} \left[ M_0 M_X B_L B_R + (B_L^2 + B_R^2) \frac{q^2}{4x} + \frac{1}{M_X^2} \right], \]  

(42)

and

\[ F_2(q^2, x) = \frac{g_{\mu \nu}^2 q^2}{8x} (B_L^2 + B_R^2) \frac{1}{M_X^2}, \]  

(43)

respectively. Also \( M_0 \) is the mass of the initial hadron and \( M_X \) is the mass of the final hadron as,

\[ M_X = \sqrt{M_0^2 + q^2 \left( \frac{1-x}{x} \right)}. \]  

(44)

**Numerical strategy**

As we mentioned in section 4 in the equation of states the eigenvalue of the ground state should be close to the square of the proton mass. So, we consider the ranges \( 0 < m_5 < 1 \text{GeV} \) and \( 0.001 < c < 1 \text{GeV}^4 \), while the eigenvalue of the ground state equation, is in the range from 0.876 GeV to 1 GeV (\( M_{\text{proton}} = 0.938 \text{GeV} \)). Accordingly we get a set of suitable values of the parameters \( m_5 \) and \( c \). Their approximate ranges are \( 0.001 < m_5 < 0.2 \text{GeV} \) and \( 0.006 < c < 0.02 \text{GeV}^4 \). In the next step we look for the appropriate values of \( \kappa_{eff}^2 \) since \( m_5 \) and \( c \) satisfy their ranges and our theoretical calculations can be fitted with the experimental data for \( F_2 \). We continue by focusing only on 0.01 – 0.04 order of \( x \) and small \( q^2 \). In sweep spectrum form, we determine a set of \( m_5 \) and \( c \) and then for each set, we fit the experimental data for \( F_2 \) to
get $K_{eff}^2$ by Least squares method. What needs to be mentioned here is that for $m_5$, the scanning step is 0.01, and for $c$, the scanning step is 0.001. The optimal parameter values with the smallest uncertainty are $m_5 = 0.081$ GeV, $c = 0.0120$ GeV$^4$, $K_{eff}^2 = 37.3259$. The uncertainty of $c$ comes from the size of scanning step. Using this parameters set, we will get the proton structure function $F_2$ as a function of $q^2$.

| $x$   | $m_5$/GeV | $c$/GeV$^4$ | $K_{eff}^2$ |
|-------|-----------|-------------|-------------|
| 0.015 | 0.081     | 0.012       | 37.3259     |
| 0.025 |           |             |             |
| 0.04  |           |             |             |

Table 1: Adjustment of parameter $K_{eff}^2$ at different $x$ with $c = 0.0120$ GeV$^4$. Remind that the value of parameter $c$ is demanded by phenomenological value of proton mass.

Figure 4: a), b), c) Comparison between Jlab Hall C data [62] and our theoretical results. Dashed lines are theoretical results and square dots are experimental data.

Figure 4 is a comparison between Jlab Hall C data [62] and our theoretical results. In plots a), b), c) our results have good agreements with experimental data at $x = 0.015$, $x = 0.025$ and $x = 0.04$.

6 Conclusions

In a holographic description of DIS we found effects of the parameter $c$ appearing in the background metric representing gluon condensation in the boundary theory. Since there is a proton target in the scattering, the mass of the proton and the value of $c$ parameter both play an important role in this study. One of our main aim was to determine the value of $c$ from experimental data. First, we found the behaviour of the electromagnetic field with respect to $z$ for different
values of $c$. We have shown that the $c$ parameter can increase the magnitude of this field, especially for small values of $q^2$. Then, we solved the equation of baryonic wave function numerically and set the proton mass as the ground state eigenvalue to find the best bulk mass, $c$ parameter and the $k_{eff}^2$ values. Hence, only small values of $c$ lead to a well-defined answer of the equation, or proton target requires a small value of $c$. It could be suggested that since the $c$ parameter breaks the conformal symmetry, its value represents the confinement. So in our study case confinement is not strong.

Based on the above results, we discussed the structure functions in the scattering versus Jlab Hall C data (with order $0.01 - 0.04$ of $x$ and small $q^2$). Numerically, we used an appropriate set of bulk mass and $c$ in the form of a sweep spectrum which were already determined to fit experimental data and the structure function. Our results are useful for understanding QCD and proton structure for small $x$, small $q^2$ and weak confinement.

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