The role of nuclear form factors in Dark Matter calculations

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Abstract

The momentum transfer dependence of the total cross section for elastic scattering of cold dark matter candidates, i.e. lightest superymmetric particle (LSP), with nuclei is examined. We find that even though the energy transfer is small ($\leq 100K\epsilon V$) the momentum transfer can be quite big for large mass of the LSP and heavy nuclei. The total cross section can in such instances be reduced by a factor of about five.
There is ample evidence that about 90% of the matter in the universe is non-luminous and non-baryonic of unknown nature [1-3]. Furthermore, in order to accommodate large scale structure of the universe one is forced to assume the existence of two kinds of dark matter [3]. One kind is composed of particles which were relativistic at the time of the structure formation. This is called Hot Dark Matter (HDM). The other kind is composed of particles which were non-relativistic at the time of structure formation. These constitute the Cold Dark Matter (CDM) component of the universe. The COBE data [4] by examining the anisotropy on background radiation suggest that the ratio of CDM to HDM is 2:1. Since about 10% of the matter of the universe is known to be baryonic, we know that we have 60% CDM, 30% HDM and 10% baryonic matter.

The most natural candidates for HDM are the neutrinos provided that they have a mass greater than $1eV/c^2$. The situation is less clear in the case of CDM. The most appealing possibility, linked closely with Supersymmetry (SUSY), is the LSP i.e. the Lightest Supersymmetric Particle.

In recent years the phenomenological implications of Supersymmetry are being taken very seriously [5-7]. Pretty accurate predictions at low energies are now feasible in terms of few input parameters in the context of SUSY models without any commitment to specific gauge groups. More or less such predictions do not appear to depend on arbitrary choices of the relevant parameters or untested assumptions.

In such theories derived from Supergravity the LSP is expected to be a neutral fermion with mass in the $10 - 100GeV/c^2$ region travelling with non-relativistic velocities ($\beta \simeq 10^{-3}$) i.e. with energies in the KeV region. In the absence of R-parity violation this particle is absolutely stable. But, even in the presence of R-parity violation, it may live long enough to be a CDM candidate.

The detection of the LSP, which is going to be denoted by $\chi_1$, is extremely difficult, since this particle interacts with matter extremely weakly. One possibility is the detection of high energy neutrinos which are produced by pair annihilation in the sun where this particle is trapped i.e. via the reaction

$$\chi_1 + \chi_1 \rightarrow \nu + \bar{\nu}$$

(1)
The above reaction is possible since the LSP is a Majorana particle, i.e. its own antiparticle (à la $\pi^0$). Such high energy neutrinos can be detected via neutrino telescopes.

The other possibility, to be examined in the present work, is the detection of the energy of the recoiling nucleus in the reaction

$$\chi_1 + (A, Z) \rightarrow \chi_1 + (A, Z)$$

This energy can be converted into phonon energy and detected by a temperature rise in cryostatic detector with sufficiently high Debye temperature \cite{3,8,9}. The detector should be large enough to allow a sufficient number of counts but not too large to permit anticoincidence shielding to reduce background. A compromise of about 1Kg is achieved. Another possibility is the use of superconducting granules suspended in a magnetic field. The heat produced will destroy the superconductor and one can detect the resulting magnetic flux. Again a target of about 1Kg is favored.

There are many targets which can be employed. The most popular ones contain the nuclei $^{3}He$, $^{19}F$, $^{23}Na$, $^{40}Ca$, $^{72,76}Ge$, $^{75}As$, $^{127}I$, $^{134}Xe$, and $^{207}Pb$.

It has recently been shown that process (2) can be described by a four fermion interaction \cite{10-16} of the type \cite{17}

$$L_{eff} = -\frac{G_F}{\sqrt{2}} [J_\lambda \bar{\chi}_1 \gamma^\lambda \gamma^5 \chi_1 + J \bar{\chi}_1 \chi_1]$$

where

$$J_\lambda = \bar{N}\gamma_\lambda [f_V^0 + f_V^1 \tau_3 + (f_A^0 + f_A^1 \tau_3) \gamma_5] N$$

and

$$J = \bar{N}(f_S^0 + f_S^1 \tau_3) N$$
where we have neglected the uninteresting pseudoscalar and tensor currents. Note that, due to the majorana nature of the LSP, $\bar{\chi}_1 \gamma^\lambda \chi_1 = 0$ (identically). The vector and axial vector form factors can arise out of Z-exchange and s-quark exchange [10-15] (s-quarks are the SUSY partners of quarks with spin zero). They have uncertainties in them (see ref. [15] for three choices in the allowed parameter space of ref. [5]). The transition from the quark to the nucleon level is pretty straightforward in this case. We will see later that, due to the majorana nature of the LSP, the contribution of the vector current, which can lead to a coherent effect of all nucleons, is suppressed [10-15]. Thus, the axial current, especially in the case of light and intermediate mass nuclei, cannot be ignored. The scalar form factors arise out of the Higgs exchange or via S-quark exchange when there is mixing between s-quarks $\tilde{q}_L$ and $\tilde{q}_R$ [10-12] (the partners of the left-handed and right-handed quarks). They have two types of uncertainties in them [18]. One, which is the most important, at the quark level due to the uncertainties in the Higgs sector. The other in going from the quark to the nucleon level [16-17]. Such couplings are proportional to the quark masses and hence sensitive to the small admixtures of $q \bar{q}$ (q other than u and d) present in the nucleon. Again values of $f_0^S$ and $f_1^S$ in the allowed SUSY parameter space can be found in ref. [15].

The invariant amplitude in the case of non relativistic LSP takes the form [15]

$$|m|^2 = \frac{E_f E_i - m_1^2 + \mathbf{p}_i \cdot \mathbf{p}_f}{m_1^2} |J_0|^2 + |J|^2 + |J|^2 \simeq \beta^2 |J_0|^2 + |J|^2 + |J|^2$$

where $|J_0|$ and $|\mathbf{J}|$ indicate the matrix elements of the time component and space component of the current $|J_3|$ of eq. (4) and $\mathbf{J}$ the matrix element of the scalar current $\mathbf{J}$ of eq. (5). Notice that $|J_0|^2$ is multiplied by $\beta^2$ (the suppression due to the majorana nature of LSP mentioned above). It is straightforward to show that

$$|J_0|^2 = A^2 |F(q^2)|^2 \left( f_0^V - f_1^V \frac{N - Z}{A} \right)^2$$

(7)
\[ J^2 = A^2 |F(q^2)|^2 \left( f_S^0 - f_S^1 \frac{N-Z}{A} \right)^2 \]  
\[ |J|^2 = \frac{1}{2J_i + 1} | < J_i || f_A^0 \Omega_0(q) + f_A^1 \Omega_1(q) || J_i > |^2 \]  
with

\[ \Omega_0(q) = \sum_{j=1}^{A} \sigma(j)e^{-i\mathbf{q} \cdot \mathbf{x}_j}, \quad \Omega_1(q) = \sum_{j=1}^{A} \sigma(j)\tau_3(j)e^{-i\mathbf{q} \cdot \mathbf{x}_j} \]

where \( \sigma(j), \tau_3(j), \mathbf{x}_j \) are the spin, third component of isospin \((\tau_3|p > = |p >)\) and coordinate of the \( j \)-th nucleon and \( \mathbf{q} \) is the momentum transferred to the nucleus.

The differential cross section in the laboratory frame takes the form [15]

\[ \frac{d\sigma}{d\Omega} = \frac{\sigma_0}{\pi} \frac{m_1^2}{m_p} \frac{1}{(1+\eta)^2} \xi \{ \beta^2 |J_0|^2 [1 - \frac{2\eta + 1}{(1+\eta)^2} \xi^2] + |J|^2 + |J|^2 \} \]

where \( \eta = m_1/m_p A \) (\( m_p \) = proton mass), \( \beta = v/c \) (\( v \) is the velocity of LSP), \( m_1 \) is the mass of LSP, \( \xi = \hat{p}_i \cdot \hat{q} \geq 0 \) (forward scattering) and

\[ \sigma_0 = \frac{1}{2\pi} (G_F m_p)^2 \simeq 0.77 \times 10^{-38} \text{cm}^2 \]

\( |J_0|^2, |J|^2 \) and \( |J|^2 \) are given by eqs. (7)-(9). The momentum transfer \( \mathbf{q} \) is given by

\[ |\mathbf{q}| = q_0 \xi, \quad q_0 = \frac{\beta m_1 c}{1 + \eta} \]

Some values of \( q_0 \) (forward momentum transfer) for some characteristic values of \( m_1 \) and representative nuclear systems (light, intermediate and
heavy) are given in table 1. It is clear that the momentum transfer can be stable for large \(m_1\) and heavy nuclei.

The total cross section can be cast in the form

\[
\sigma = \sigma_0 \frac{m_1^2}{m_p} \frac{1}{(1 + \eta)^2} \{ A^2 [\beta^2 (f_0^N - f_1^N - Z)^2 } + (f_S^0 - f_S^1 N - Z/A)^2 I_0(q_0^2) - \frac{2\eta + 1}{2(1 + \eta)^2} (f_1^V - f_1^V - N - Z)^2 I_1(q_0^2) \right] + (f_A^0 \Omega_0(0))^2 I_{00}(q_0^2) - 2f_A^0 f_A^1 \Omega_0(0) \Omega_1(0) I_{01}(q_0^2) + (f_A^1 \Omega_1(0))^2 I_{11}(q_0^2) \right) \}
\]

(14)

where

\[
I_\rho(q_0^2) = 2(\rho + 1) \int_0^1 \xi^{1+2\rho} |F(q_0^2 \xi^2)|^2 d\xi, \quad \rho = 0, 1
\]

(15)

\[
\Omega_\rho(q) = (2J_i + 1)^{-\frac{Z}{2}} < J_i ||\Omega_\rho(q)|| J_i >, \quad \rho = 0, 1
\]

(16)

(see eq. (10) for the definition of \(\Omega_\rho\)) and

\[
I_{\rho\rho'}(q_0^2) = 2 \int_0^1 \xi \frac{\Omega_\rho(q_0^2 \xi^2)}{\Omega_\rho(0)} \frac{\Omega_{\rho'}(q_0^2 \xi^2)}{\Omega_{\rho'}(0)} d\xi, \quad \rho, \rho' = 0, 1
\]

(17)

In a previous paper [16] we have shown that the nuclear form factor can be adequately described within the harmonic oscillator model as follows

\[
F(q^2) = \left[ Z \Phi(qb, Z) + N \Phi(qb, N) \right] e^{-q^2 b^2/4}
\]

(18)

where \(\Phi\) is a polynomial of the form [18]

\[
\Phi(qb, \alpha) = \sum_{\lambda=0}^{N_{\text{max}}(\alpha)} \theta_\lambda^{(\alpha)}(qb)^{2\lambda}, \quad \alpha = Z, N
\]

(19)
$N_{\text{max}}(Z)$ and $N_{\text{max}}(N)$ depend on the major harmonic oscillator shell occupied by protons and neutrons [16], respectively. The integral $I_{\rho}(q_0^2)$ can be written as

$$I_{\rho}(q_0^2) \rightarrow I_{\rho}(u) = \int_0^u x^{1+\rho} |F(2x/b^2)|^2 \, dx,$$  

where

$$u = q_0^2b^2/2, \quad b = 1.0A^{1/3} \, fm \quad (21)$$

With the use of eqs. (18), (19) we obtain

$$I_{\rho}(u) = \frac{1}{A^2} \{ Z^2 I_{ZZ}^{(\rho)}(u) + 2NZ I_{NZ}^{(\rho)}(u) + N^2 I_{NN}^{(\rho)}(u) \} \quad (22)$$

where

$$I_{\alpha\beta}^{(\rho)}(u) = \sum_{\lambda=0}^{N_{\text{max}}(\alpha)} \sum_{\nu=0}^{N_{\text{max}}(\beta)} \frac{\theta_\lambda^{(\alpha)} \theta_\nu^{(\beta)}}{\alpha \beta} \frac{2^{\lambda+\nu+\rho}(\lambda+\nu+\rho)!}{u^{\lambda+\rho}} \left[ 1 - e^{-u} \sum_{\kappa=0}^{\lambda+\nu+\rho} \frac{u^\kappa}{\kappa!} \right] \quad (23)$$

with $\alpha, \beta = N, Z$

The coefficients $\theta_\lambda^{(\alpha)}$ for light and medium nuclei have been computed in ref. [16]. In table 2 we present them by including in addition those for heavy nuclei. The integrals $I_{\rho}(u)$ for three typical nuclei (${}^{40}_{20}\text{Ca}$, ${}^{72}_{32}\text{Ge}$ and ${}^{208}_{82}\text{Pb}$) are presented in fig. 1 as a function of $m_1$. We see that for light nuclei the modification of the cross section by the inclusion of the form factor is small. For heavy nuclei and massive $m_1$ the form factor has a dramatic effect on the cross section and may decrease it by a factor of about 5. The integral $I_1(u)$ is even more suppressed but it is not very important.

The spin matrix elements depend on the details of the structure of the nucleus considered. So is the spin form factor. The spin matrix element, since it does not show coherence, is expected to be more important in the case of odd light and intermediate nuclei. In the present work we will examine
the $q^2$ dependence of the spin matrix element in the cases of $^{207}\text{Pb}$ and $^{19}\text{F}$ whose structure is believed to be simple.

To a good approximation [15,17] the ground state of the $^{207}\text{Pb}$ nucleus can be described as a $2s_1/2$ neutron hole in the $^{208}\text{Pb}$ closed shell. One then finds

$$\Omega_0(q) = (1/\sqrt{3})F_{2s}(q^2), \quad \Omega_1(q) = -(1/\sqrt{3})F_{2s}(q^2)$$

(24)

and

$$I_{00} = I_{01} = I_{11} = 2 \int_0^1 \xi [F_{2s}(q^2)]^2 d\xi$$

(25)

Even though the probability of finding a pure $2s_1/2$ neutron hole in the $^{1/2}^-$ ground state of $^{207}\text{Pb}$ is greater than 95%, the ground state magnetic moment is quenched due to the $1^+$ p-h excitation involving the spin orbit partners. Hence we expect a similar suppression of the isovector spin matrix elements. Thus we write

$$|(1/2)^- > _{gs} = C_0(2s1/2)^{-1} \left[ 1 + C_1[0i_{11/2}(n)(0i_{13/2})^{-1}(n)]1^+ > + C_2[0h_{9/2}(p)(0h_{11/2})^{-1}(p)]1^+ > + ... \right]$$

(26)

Retaining terms which are most linear in the coefficients $C_1, C_2$ we obtain

$$\Omega_0(q) = C_0^2 \left\{ F_{2s}(q^2)/\sqrt{3} - 8 [(7/13)^{1/2}C_1F_{0i}(q^2) + (5/11)^{1/2}C_2F_{0h}(q^2)] \right\}$$

(27)

$$\Omega_1(q) = C_0^2 \left\{ F_{2s}(q^2)/\sqrt{3} - 8 [(7/13)^{1/2}C_1F_{0i}(q^2) - (5/11)^{1/2}C_2F_{0h}(q^2)] \right\}$$

(28)

where

$$F_{nl}(q^2) = e^{-q^2b^2/4} \sum_{\lambda=0}^{N_{max}} \gamma_{\lambda}^{(nl)}(qb)^{2\lambda}$$

(29)
The coefficients $\gamma^{(nl)}_\lambda$ are given in table 3.

The coefficients $C_0, C_1$ and $C_2$ were obtained by diagonalizing the Kuo-Brown G-matrix [18,19] in a model space of $2h-1p$ configurations. Thus we find

$$C_0 = 0.973350, \quad C_1 = 0.005295, \quad C_2 = -0.006984$$

We also find

$$\Omega_0(0) = -(1/\sqrt{3})(0.95659), \quad (small\ retardation) \quad (30)$$

$$\Omega_1(0) = -(1/\sqrt{3})(0.83296), \quad (sizable\ retardation) \quad (31)$$

The amount of retardation of the total matrix element depends on the values of $f_0^A$ and $f_1^A$. Using eqs. (25) and (26) we can evaluate the integrals $I_{00}, I_{01}$ and $I_{00}$. The results are presented in fig. 2. We see that for a heavy nucleus and high LSP mass the momentum transfer dependence of the spin matrix elements cannot be ignored.

In the second example we examine the spin matrix elements of the light nucleus $^19_F$. Assuming that the ground state wave function is a pure $SU(3)$ state with the largest symmetry i.e $f = [3], (\lambda\mu) = (60)$, we obtain [20,21] the expression

$$\frac{\Omega_1(q)}{\Omega_1(0)} = \frac{4}{9} F_{2s}(q^2) + \frac{5}{9} F_{0d}(q^2) \quad (32)$$

There is only isovector component contribution (the isoscalar matrix element vanishes). The results for the $I_{11}(u)$ integral are shown in fig. 3. We see that the effect of the nuclear form factor on the cross section for light nuclei is insignificant.

In the present paper we have examined the momentum transfer dependence of the nuclear matrix elements entering the elastic scattering of cold...
dark matter candidates (LSP) with nuclei. We have found that such a mo-
mentum transfer dependence is very pronounced for heavy nuclear targets
and mass of the LSP in the 100 GeV region.
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Figure Captions

Fig. 1. The integral $I_0(u)$, which describes the main coherent contribution to the total cross section as a function of the LSP mass ($m_1$), for three typical nuclei: $^{40}_{20}Ca$, $^{72}_{32}Ge$ and $^{208}_{82}Pb$.

Fig. 2. The integral $I_1(u)$, entering the total coherent cross section as a function of the LSP mass ($m_1$), for three typical nuclei: $^{40}_{20}Ca$, $^{72}_{32}Ge$ and $^{208}_{82}Pb$. For its definition see eqs (11) and (15) of the text.

Fig. 3. The integral $I_{11}$, associated with the spin isovector - isovector matrix elements for $^{207}_{82}Pb$ and $^{19}_{9}F$ as a function of the LSP mass ($m_1$). The other two integrals $I_{00}$ and $I_{01}$ are almost identical and are not shown.
Table 1: The quantity $q_0$ (forward momentum transfer) in units of $fm^{-1}$ for three values of $m_1$ and three typical nuclei.

| Nucleus | $m_1 = 30 GeV$ | $m_1 = 100 GeV$ | $m_1 = 150 GeV$ |
|---------|----------------|-----------------|-----------------|
| $^{40}$Ca | .174 | .290 | .321 |
| $^{72}$Ge | .215 | .425 | .494 |
| $^{208}$Pb | .267 | .685 | .885 |

Table 2. The coefficients $\theta_\lambda$ determining the proton and neutron form factors for all closed (sub)shell nuclei. In a harmonic oscillator basis they are rational numbers. The coefficients for $\lambda = 0$ are equal to $Z$ (or $N$).

| nlj-level | $\lambda = 0$ | $\lambda = 1$ | $\lambda = 2$ | $\lambda = 3$ | $\lambda = 4$ | $\lambda = 5$ | $\lambda = 6$ |
|-----------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| 0s$_{1/2}$ | 2 | -2/3 | | | | | |
| 0p$_{3/2}$ | 6 | -1 | | | | | |
| 0p$_{1/2}$ | 8 | 1/10 | | | | | |
| 0d$_{5/2}$ | 14 | -3 | 1/4 | | | | |
| 1s$_{1/2}$ | 16 | -3/10 | 11/60 | | | | |
| 0d$_{3/2}$ | 20 | -5 | 1/4 | | | | |
| 0f$_{7/2}$ | 28 | -9 | 13/20 | -1/105 | | | |
| 1p$_{3/2}$ | 32 | -3/10 | 61/60 | -11/420 | | | |
| 0f$_{5/2}$ | 38 | -14 | 79/60 | -1/30 | | | |
| 1p$_{1/2}$ | 40 | -15 | 3/2 | -1/24 | | | |
| 0g$_{9/2}$ | 50 | -5/65 | 5/2 | -5/56 | 1/1512 | | |
| 0g$_{7/2}$ | 58 | -27 | 33/10 | -107/840 | 1/840 | | |
| 1d$_{3/2}$ | 64 | -31 | 17/4 | -173/840 | 1/336 | | |
| 1d$_{3/2}$ | 68 | -101/3 | 293/60 | -31/120 | 1/240 | | |
| 2s$_{1/2}$ | 70 | -35 | 21/4 | -7/24 | 1/192 | | |
| 0h$_{11/2}$ | 82 | -45 | 29/4 | -73/168 | 37/4032 | -1/27720 | | |
| 0h$_{9/2}$ | 92 | -160/3 | 107/12 | -31/56 | 151/12096 | -1/15120 | | |
| 1f$_{7/2}$ | 100 | -60 | 217/20 | -653/840 | 449/20160 | -1/5040 | | |
| 1f$_{5/2}$ | 106 | -65 | 123/10 | -397/420 | 199/6720 | -1/3360 | | |
| 2p$_{3/2}$ | 110 | -205/3 | 403/30 | -153/140 | 253/6720 | -1/2240 | | |
| 2p$_{1/2}$ | 112 | -70 | 14 | -7/6 | 1/24 | -1/1920 | | |
| 0i$_{13/2}$ | 126 | -84 | 35/2 | -3/2 | 1/18 | -49/63360 | 1/617760 | |
Table 3. The coefficients $\gamma^{(nl)}_\lambda$, entering the polynomial describing the form factor (see eq. (29)) of a single particle harmonic oscillator wave function up to $6\hbar\omega$, i.e. throughout the periodic table.

| $n\,l$ | $\lambda = 0$ | $\lambda = 1$ | $\lambda = 2$ | $\lambda = 3$ | $\lambda = 4$ | $\lambda = 5$ | $\lambda = 6$ |
|--------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 0 0    | 1              |                |                |                |                |                |                |
| 0 1    | 1              | -1/6           |                |                |                |                |                |
| 1 0    | 1              | -1/3           | 1/24           |                |                |                |                |
| 0 2    | 1              | -1/3           | 1/60           |                |                |                |                |
| 1 1    | 1              | -1/2           | 11/120         | -1/240         |                |                |                |
| 0 3    | 1              | -1/2           | 1/20           | -1/840         |                |                |                |
| 2 0    | 1              | -2/3           | 11/60          | -1/60          | 1/1920         |                |                |
| 1 2    | 1              | -2/3           | 19/120         | -11/840        | 1/3360         |                |                |
| 0 4    | 1              | -2/3           | 1/10           | -1/210         | 1/15120        |                |                |
| 2 1    | 1              | -5/6           | 17/60          | -31/840        | 9/4480         | -1/26880       |                |
| 1 3    | 1              | -5/6           | 29/120         | -47/1680       | 37/30240       | -1/60480       |                |
| 0 5    | 1              | -5/6           | 1/6            | -1/84          | 1/3024         | -1/332640      |                |
| 3 0    | 1              | -1             | 17/40          | -31/420        | 27/4480        | -1/4480        | 1/322560       |
| 2 2    | 1              | -1             | 2/5            | -1/15          | 41/8064        | -1/5760        | 1/483840       |
| 1 4    | 1              | -1             | 41/120         | -1/20          | 1/315          | -1/11880       | 1/1330560      |
| 0 6    | 1              | -1             | 1/4            | -1/42          | 1/1008         | -1/55440       | 1/8648640      |
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