ULTIMATE CAPACITY LIMIT OF A MULTI-SPAN LINK WITH PHASE-INSENSITIVE AMPLIFICATION

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Keywords: OPTICAL FIBER COMMUNICATION, OPTICAL AMPLIFIERS, OPTICAL RECEIVERS, CHANNEL CAPACITY, QUANTUM COMMUNICATION

Abstract

The Shannon capacity of a point-to-point link with an optimised configuration of optical amplifiers is compared with general detection strategies permitted by quantum mechanics. Results suggest that the primary application area of receivers based on such strategies may be unamplified short-distance links or free-space optical communication.

1 Introduction

In the ongoing quest to boost the capacity of optical communication links [1, 2], a relatively unexplored option is to replace conventional detection with more elaborate receivers that operate beyond the standard shot-noise limit. While first designs for such receivers were proposed back in 1970s [3, 4], only the past dozen years have witnessed a number of proof-of-principle experimental demonstrations [5-9]. Performance of any type of a receiver is ultimately limited by the Holevo capacity [10] which takes into account the most general measurement strategies for the received optical signal that are permitted by quantum mechanics. In contrast, the well-established Shannon capacity bound specifically assumes conventional coherent detection of both field quadratures [11]. In the case of a loss-only linear optical channel, the Holevo bound for the spectral efficiency is known to exceed the standard Shannon bound by 1.44 bit/(s × Hz) in the bandwidth-limited regime. This figure defines the maximum increase of the information rate that could be attained with unconventional measurement strategies, although designing receivers operating at the Holevo capacity limit remains a dauntingly non-trivial task [12].

The purpose of this work is to assess the possible enhancement of the spectral efficiency beyond the Shannon limit when the propagating signal is regenerated with the help of phase-insensitive optical amplification [13]. The locations and the gains of optical amplifiers are numerically optimised under the constraint on the total optical power at any point along the link, using the Holevo capacity bound for the entire channel as the cost function. Such a power constraint can follow e.g. from the desire to avoid nonlinear effects in the optical medium. This motivation makes the scenario discussed here distinct compared to the optimisation of the overall energy consumption by an optical transmission system considered recently by Antonelli et al. [14]. It is found that the excess noise introduced by the amplification process closes the gap between the Holevo and the Shannon capacity bounds, leaving rather little room for improvement beyond conventional coherent detection in optically amplified links. This result suggests that the primary area of prospective applications for unconventional receivers may be short-range loss-only links, such as optical interconnects, as well as free-space optical communication, in particular satellite links for which shot-noise limited operation has been experimentally verified [15].

2 Capacity limits

The canonical reference for the capacity limit of a narrowband linear optical channel with additive Gaussian noise follows from the Shannon-Hartley theorem and is given by [11]

\[ S_{\text{Shannon}} = \log_2 \left( 1 + \frac{\tau \bar{n}}{1 + \nu} \right), \]

where \( S \) is the spectral efficiency in bits/(s × Hz), \( \tau \) is the power loss of the optical channel and \( \bar{n} \) is the input signal power spectral density in photons/(s × Hz). The noise term appearing in the denominator is written as a sum of the shot noise of the quadrature measurement, equal to one in the photon number units used here, and the power spectral density \( \nu \) in photons/(s × Hz) of the excess noise acquired in the course of propagation.

The above formula relies on the assumption that the readout of field quadratures is performed using conventional shot-noise limited coherent detection. When the most general detection strategies compatible with quantum theory are allowed, the Shannon formula needs to be replaced by the ultimate Holevo capacity limit [16]

\[ S_{\text{Holevo}} = g(\tau \bar{n} + \nu) - g(\nu), \]

where \( g(x) = \log_2(1 + x) + x \log_2(1 + e^{-1}) \). For a loss-only channel with \( \nu = 0 \), the Holevo bound is given by \( g(\tau \bar{n}) \). In the bandwidth-limited regime when \( \tau \bar{n} \gg 1 \), the second term in the definition of \( g(x) \) can be expanded into a power
Fig. 1 A point-to-point communication link including $N$ phase-insensitive amplifiers with gains $G_i$ at relative locations $L_i$.

series in $x = (\tau \bar{n})^{-1} \ll 1$, which yields $S_{\text{Holevo}} \approx S_{\text{Shannon}} + 1.44 \text{ bit}/(s \times \text{Hz})$.

3 Methodology

We consider a point-to-point optical link extending over a total length $L$ with uniform attenuation $\alpha$ per unit length. As shown in Fig. 1, the transmitted signal is regenerated using phase-insensitive amplification at $N$ intermediate nodes labelled with $i = 1, \ldots, N$ and located at distances $L_i$ measured with respect to the preceding node. The transformation of the optical field from the channel input to the output of the $i$th amplifier is described by two parameters: $\tau_i$, which specifies the ratio of the signal power right after the $i$th node to the input value, and $\nu_i$, which characterises the exceed noise added by the amplification up to the $i$th node. The recursive relations for the parameters read:

$$
\tau_i = G_i \exp(-\alpha L_i) \tau_{i-1},
\nu_i = G_i \exp(-\alpha L_i) \nu_{i-1} + G_i - 1.
$$

The initial values are $\tau_0 = 1$ and $\nu_0 = 0$. Here $G_i$ is the gain of the $i$th amplifier assumed to operate at the quantum limit. Following the general theory of optical amplification [17], this results in the additive term $G_i - 1$ in the equation for $\nu_i$, while the excess noise $\nu_{i-1}$ that has built up until the preceding node is multiplied by the same factor $G_i \exp(-\alpha L_i)$ as the signal. Taking into account the attenuation over the final unamplified span $L_{N+1}$ between the last regeneration node and the channel output, the entire optical channel is characterised by the parameters

$$
\tau = \exp(-\alpha L_{N+1}) \tau_N, \quad \nu = \exp(-\alpha L_{N+1}) \nu_N.
$$

These values are inserted into the expressions for the Shannon and the Holevo capacity limits. For a given input power spectral density $\bar{n}$ and a number of amplifiers $N$, the power constraint demands that the locations $L_i$ of the nodes and the respective gains $G_i$ need to satisfy the set of inequalities

$$
\tau_i \bar{n} + \nu_i \leq \bar{n}, \quad i = 1, \ldots, N
$$

which guarantee that the total optical power does not exceed the input value anywhere along the link.

Fig. 2 (a) The spectral efficiency of a point-to-point link with locations and gains of amplifiers optimised with respect to the Shannon (dashed lines) and the Holevo (solid lines) criterion shown for $N = 2, 4, 8, 16, 64$ regeneration nodes using colour-coding. The two extreme cases of a loss-only channel and distributed-amplification cases are depicted with black and grey lines respectively. (b) A close-up for short distances up to $L = 100$ km.

4 Results

4.1 Numerical optimisation

We carried out numerical optimisation of the locations and the gains of amplifiers for a given number $N$ of regeneration nodes under the constraint that the total power spectral density is less or equal $\bar{n}$ photons/(s $\times$ Hz) at any point of the link. The results of optimisation, carried out independently for the Shannon and the Holevo expressions, are shown in Fig. 2 for the input power spectral density $\bar{n} = 100$ photons/(s $\times$ Hz) equivalent to 12.8 $\mu$W/THz at the 1550 nm wavelength, the number of nodes $N = 2, 4, 8, 16, 64$, and the linear attenuation coefficient $\alpha = 0.05$ km$^{-1}$ corresponding to the transmission of the standard SMF-28 fibre at 1550 nm. At $L = 0$ km the 1.44 bit/(s $\times$ Hz) gap in the spectral efficiency is clearly seen.
4.2 Distributed amplification

The numerical results motivate a model for signal regeneration using distributed optical amplification. In this model, the amplifier gain over an infinitesimal span $dl$ is given by $G(l) \approx 1 + \gamma(l)dl$. The signal loss $\tau(l)$ and the excess noise $\nu(l)$ become continuous functions of the distance $l$ from the channel input and are governed by a pair of differential equations

$$\frac{d\tau}{dl} = [\gamma(l) - \alpha]\tau(l),$$

$$\frac{d\nu}{dl} = [\gamma(l) - \alpha]\nu(l) + \gamma(l) .$$

If the amplification restores the total optical power to the input level one has $\gamma(l) = \alpha n/(1 + n)$ and the analytical solution reads

$$\tau(l) = \exp[-\alpha l/(1 + n)], \quad \nu(l) = n[1 - \tau(l)].$$

It is seen that compared to the loss-only channel the signal attenuation is reduced by a factor $1/(1 + n)$ and the lost signal power is replaced by the amplification noise. If the Holevo capacity limit is considered, the optimal strategy is to leave unamplified the last section of the link. The length $L'$ of that section can obtained by optimising the Holevo expression with the output signal power spectral density $\exp(-\alpha L')\tau(L - L')n$ and the output noise $\exp(-\alpha L')\nu(L - L')$. The optimal termination point $L - L'$ of distributed amplification is shown in Fig. 3 with a grey line.

5 Conclusion

The Holevo capacity limit takes into account the most general detection strategies that go beyond conventional coherent detection of both field quadratures with efficiency characterised by the Shannon limit. The distinct form of the Holevo expression necessitates a revisited analysis of amplified optical communication links, carried out here under the constraint on the total optical power. It is found that the 1.44 bit/(s × Hz) enhancement is material for short distances only, otherwise conventional phase-insensitive amplification combined with coherent detection can deliver performance approaching the Holevo limit. Consequently, unconventional detection strategies can be expected to bring prospectively a meaningful benefit to short-haul pure-loss links such as optical interconnects or in scenarios where signal regeneration is fundamentally impossible, such as satellite optical communication.

6 Acknowledgements

We acknowledge insightful discussions with Cristian Antonelli, Saikat Guha, Christoph Marquardt, Antonio Mecozzi, Mark Shatia, and Jaroslaw P. Turkiewicz, as well as the support of the FNP TEAM project “Quantum Optical Communication Systems”.

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