Planck Constraints on Holographic Dark Energy

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Abstract

We perform a detailed investigation on the cosmological constraints on the holographic dark energy (HDE) model by using the Planck data. We find that HDE can provide a good fit to the Planck high-$\ell$ ($\ell \gtrsim 40$) temperature power spectrum, while the discrepancy at $\ell \approx 20 - 40$ found in the $\Lambda$CDM model remains unsolved in the HDE model. The Planck data alone can lead to strong and reliable constraint on the HDE parameter $c$. At the 68% confidence level (CL), we obtain $c = 0.508 \pm 0.207$ with Planck+WP+lensing, favoring the present phantom behavior of HDE at the more than 2$\sigma$ CL. By combining Planck+WP with the external astrophysical data sets, i.e. the BAO measurements from 6dFGS+SDSS DR7(R)+BOSS DR9, the direct Hubble constant measurement result ($H_0 = 73.8 \pm 2.4$ km s$^{-1}$ Mpc$^{-1}$) from the HST, the SNLS3 supernovae data set, and Union2.1 supernovae data set, we get the 68% CL constraint results $c = 0.484 \pm 0.070$, $0.474 \pm 0.049$, $0.594 \pm 0.051$, and $0.642 \pm 0.066$, respectively. The constraints can be improved by 2%–15% if we further add the Planck lensing data into the analysis. Compared with the WMAP-9 results, the Planck results reduce the error by 30%–60%, and prefer a phantom-like HDE at higher significant level. We also investigate the tension between different data sets. We find no evident tension when we combine Planck data with BAO and HST. Especially, we find that the strong correlation between $\Omega_m h^3$ and dark energy parameters is helpful in relieving the tension between the Planck and HST measurements. The residual value of $\chi^2_{\text{Planck+WP+HST}} - \chi^2_{\text{Planck+WP}}$ is 7.8 in the $\Lambda$CDM model, and is reduced to 1.0 or 0.3 if we switch the dark energy to $w$ model or the holographic model. When we introduce supernovae data sets into the analysis, some tension appears. We find that the SNLS3 data set is in tension with all other data sets; for example, for the Planck+WP, WMAP-9 and BAO+HST, the corresponding $\Delta \chi^2$ is equal to 6.4, 3.5 and 4.1, respectively. As a comparison, the Union2.1 data set is consistent with all three data sets, but the combination Union2.1+BAO+HST is in tension with Planck+WP+lensing, corresponding to a large $\Delta \chi^2$ that is equal to 8.6 (1.4% probability). Thus, combining internal inconsistent data sets (SNIa+BAO+HST with Planck+WP+lensing) can lead to ambiguous results, and it is necessary to perform the HDE data analysis for each independent data sets. Our tightest self-consistent constraint is $c = 0.495 \pm 0.039$ obtained from Planck+WP+BAO+HST+lensing.

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I. INTRODUCTION

Since the discovery of the cosmic acceleration [1], dark energy has become one of the most important research areas in modern cosmology [2]. From the last decade, although a variety of dark energy models have been proposed to explain the reason of cosmic acceleration, the physical nature of dark energy is still a mystery.

The dark energy problem may be in essence an issue of quantum gravity [3]. It is commonly believed that the holographic principle is a fundamental principle of quantum gravity [4]. Based on the effective quantum field theory, Cohen et al. [5] suggested that quantum zero-energy energy of a system with size $L$ should not exceed the mass of a black hole with the same size, i.e., $L^3 \Lambda^4 \leq L M_{\text{Pl}}^2$ (here $M_{\text{Pl}} \equiv 1/\sqrt{8\pi G}$ is the reduced Planck mass, and $\Lambda$ is the ultraviolet (UV) cutoff of the system). In this way, the UV cutoff of a system is related to its infrared (IR) cutoff. When we consider the whole universe, the vacuum energy related to this holographic principle can be viewed as dark energy, and therefore the holographic dark energy density becomes

$$\rho_{\text{de}} = 3c^2 M_{\text{Pl}}^2 L^{-2},$$

where $c$ is a dimensionless model parameter which modulates the dark energy density [6]. In [6], Li suggested that the IR length-scale cutoff should be chosen as the size of the future event horizon of the universe, i.e.,

$$L = a \int_0^{+\infty} \frac{dt}{a},$$

This leads to such an equation of state of dark energy

$$w_{\text{de}}(z) = \frac{1}{3} - \frac{2}{3c} \sqrt{\Omega_{\text{de}}(z)},$$

which satisfies $w_{\text{de}} \approx -0.9$ for $\Omega_{\text{de}} = 0.7$ and $c = 1$. Thus, an accelerated expanding universe can be realized in this model. In Eq. (3), the function $\Omega_{\text{de}}(z)$ is determined by the following coupled differential equation system

$$\frac{1}{E(z)} \frac{dE(z)}{dz} = -\frac{\Omega_{\text{de}}(z)}{1 + z} \left( \frac{1}{c} \sqrt{\Omega_{\text{de}}(z)} + \frac{1}{2} - \frac{\Omega_r(z) + 3}{2\Omega_{\text{de}}(z)} \right),$$

$$\frac{d\Omega_{\text{de}}(z)}{dz} = -\frac{2\Omega_{\text{de}}(z)(1 - \Omega_{\text{de}}(z))}{1 + z} \left( \frac{1}{c} \sqrt{\Omega_{\text{de}}(z)} + \frac{1}{2} + \frac{\Omega_r(z)}{2(1 - \Omega_{\text{de}}(z))} \right),$$

where $E(z) \equiv H(z)/H_0$ is the dimensionless Hubble expansion rate, and $\Omega_r(z) = \Omega_r(1 + z)^4/E(z)^2$. Note that in this paper we only consider a spatially flat universe. The initial conditions are $E(0) = 1$ and $\Omega_{\text{de}}(0) = 1 - \Omega_c - \Omega_b - \Omega_r$. 

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The holographic dark energy (HDE) model described above is a viable and physically plausible dark energy candidate, as an alternative to the standard cosmological constant model ($\Lambda$). The model has been widely studied both theoretically [7] and observationally [8]. The data used in these works mainly include the type Ia supernovae (SNIa), baryon acoustic oscillations (BAO), the direct measurement of Hubble constant, and the Cosmic Microwave Background (CMB) data from the Wilkinson Microwave Anisotropy Probe (WMAP). These works show that the HDE model can provide a good fit to the data, and $c < 1$ is favored by the data. For example, a recent analysis reports the 68% confidence level (CL) constraint $c = 0.680^{+0.064}_{-0.066}$ from WMAP-7+SNIa+BAO+HST [9].

In this March, the European Space Agency (ESA) and the Planck Collaboration publicly released the CMB data based on the first 15.5 months of Planck operations, along with a lot of scientific results [10]. They show that the standard six-parameter $\Lambda$CDM model provides an extremely good fit to the Planck spectra at high multipoles, while there are some discrepancy at $\ell \approx 20-40$. Some cosmological parameters, e.g., $n_s$, $\Omega_k$, and $N_{\text{eff}}$, are measured with unprecedented precision. Interestingly, the Planck values for some $\Lambda$CDM parameters are significantly different from those previously measured. For the matter density parameter, the Planck data give $\Omega_m = 0.315 \pm 0.017$ (68% CL) [11]. This value is higher than the WMAP-7 result $\Omega_m = 0.273 \pm 0.030$ [12] and the WMAP-9 result $\Omega_m = 0.279 \pm 0.025$ [13], and is in tension with the SNLS3 result $\Omega_m = 0.211 \pm 0.069$ [14]. For the Hubble constant, Planck gives a low value $H_0 = 67.3 \pm 1.2$ km s$^{-1}$ Mpc$^{-1}$, which is in tension with the results of the direct measurements of $H_0$, i.e., $H_0 = 73.8 \pm 2.4$ km s$^{-1}$ Mpc$^{-1}$ reported by Riess et al. [15], and $H_0 = 74.3 \pm 1.5$ (statistical) $\pm 2.1$ (systematic) km s$^{-1}$ Mpc$^{-1}$ reported by Freedman et al. [16]. The discrepancy is at about the 2.5$\sigma$ level. They also show that, the Planck constraints of $\Omega_m$ and $H_0$, although are in tension with SNLS3 and HST observations, are in agreement with the geometrical constraints from BAO surveys [11].

The Planck data also improve the constraints on dark energy [11]. Actually, the results can be significantly different if the Planck data are combined with different astrophysical data sets. For a constant $w$ model (hereafter, $w$CDM model), the Planck results give $w = -1.13^{+0.13}_{-0.10}$ and $w = -1.09 \pm 0.17$ (95% CL) by using CMB combined with BAO and Union2.1 [17] data, respectively, which are consistent with the cosmological constant. However, when combined with SNLS3 data and $H_0$ measurement, the results are $w = -1.13^{+0.13}_{-0.14}$ and $w = -1.24^{+0.18}_{-0.19}$ (2$\sigma$ CL), respectively, favoring $w < -1$ at the 2$\sigma$ level. For a dynamical equation of state $w = w_0 + w_a(1 - a)$, the results from the Planck+WP+BAO and Planck+WP+Union2.1 data combination are in agreement with a cosmological constant, while the Planck+WP+$H_0$ and Planck+WP+SNLS3 (here, WP represents the WMAP-9 polarization data) results are in tension with $w = -1$ at the more than 2$\sigma$ level.

Based on the arrival of a bunch of new data sets, it is very important to re-analyze the HDE model
in light of *Planck* and *WMAP* 9-year data. This will enable us to answer a lot of interesting questions: What are the constraint results of the cosmological parameters in the HDE model from the *Planck* data? What is the difference between the fitting results of *Planck* and *WMAP*? What are the results if we combine the *Planck* data with the BAO, SNIa, and *HST* data? Whether are they consistent or in tension with each other? Since the Hubble constant $H_0$ is correlated with the HDE parameter $c$, can HDE help us to relax the tension between the *Planck* data and the direct measurements of $H_0$? Since a phantom dark energy can reduce the TT power spectrum amplitude at large scales, can HDE help us to relieve the mismatches between theoretical and observational power spectra at $\ell \approx 20 - 40$? To give firm and reliable answers to these stimulating questions is the main aim of this paper.

This paper is organized as follows. In Sec. II, we give a brief introduction to the data used in this work and our method of data analysis. In Sec. III, we present and compare the fitting results of HDE by using the CMB-only data of *Planck* and *WMAP*-9. In Sec. IV, we combine the CMB data with the external astrophysical data sets including BAO, SNLS3, Union2.1 and *HST*, and discuss the fitting results and the tensions. Some concluding remarks are given in Sec. V. In this work, we assume today’s scale factor $a_0 = 1$, so the redshift $z$ satisfies $z = 1/a - 1$. We use negative redshifts to represent the future; in this way, $z = -1$ corresponds to the infinite future when $a \rightarrow \infty$. The subscript “0” indicates the present value of the corresponding quantity unless otherwise specified.

II. DATA ANALYSIS METHODOLOGY

To analyze the HDE, we modify the *CAMB* package [18] to incorporate the background equations of the HDE model. Furthermore, to investigate the dark energy perturbations, we apply the “parameterized post-Friedmann” (PPF) approach [19]. This method of dealing with dark energy perturbation has been widely used by *WMAP* [12, 13] and *Planck* team [11]. In our previous work of HDE data analysis [9], we have already employed this method into our pipeline.

The same as [11], we sample cosmological parameter space with Markov Chain Monte Carlo (MCMC) method with the publicly available code *COSMOMC* [20]. For each analysis, we execute about 8–16 chains until they are converged, satisfying the standard Gelman and Rubin criterion $R - 1 < 0.01$ [21]. To make sure that the tails of the distribution are well enough explored, we also check the convergence of confidence limits with the setting MPILimitConverge = 0.025 in *COSMOMC*.

The base $\Lambda$CDM model has the standard “six-parameter” as

$$P = \{\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \ln(10^{10}A_s)\},$$

(6)
where $\Omega_b h^2$ and $\Omega_c h^2$ are the current density of baryon and cold dark matter, respectively, $100\theta_{MC}$ is 100 times the approximation to $r_s/D_A$ in COSMOMC ($r_s = r_s(z_{\text{drag}})$ is the comoving size of sound horizon at baryon-drag epoch, and $D_A$ is the angular diameter distance), $\tau$ is the Thomson scattering optical depth due to reionization, $n_s$ is the scalar spectrum index at the pivot scale $k_0 = 0.05$ Mpc$^{-1}$, $\ln(10^{10} A_s)$ is the log power of the primordial curvature perturbations at $k_0$.

In the following, we will also discuss the holographic dark energy model and the $w$CDM model, each of which has an extra parameter to describe the dynamic evolution of dark energy. For HDE model, the extra parameter is $c$, as described in Eq. (1), and for $w$CDM model, the extra parameter is $w$. Therefore, when we compare $\Lambda$CDM model with $w$CDM and HDE model, we should bear in mind that we are comparing a model with 6 parameters with models with 7 parameters.

To make our results comparable with the results of the Planck Collaboration, baselines and priors for the parameters in our analysis are adopted same as [11]. In our MCMC chains, these parameters are varied with uniform priors, within the ranges listed in Table 1 of [11]. The range of $c$ is [0.001, 3.5], which is wide enough for covering the physically interesting region. Additionally, a “hard” prior [20, 100] km s$^{-1}$ Mpc$^{-1}$ is imposed to the Hubble constant $^1$. The same as [11], we assume a minimal-mass normal hierarchy for the neutrino masses by setting a single massive eigenstate $m_\nu = 0.06$ eV.

Cosmological data used in this work fall into two parts: the CMB data from Planck and WMAP, and the other data sets including BAO, SNIa and $H_0$. We introduce them in the following two subsections.

### A. CMB data

The CMB data based on the first 15.5 months of Planck operations are publicly released by the ESA and Planck Collaboration in March 2013 [10]. At the same time, the Planck likelihood softwares are also made publicly downloadable. $^2$ The likelihood software provided by the Planck Collaboration includes the following four parts:

- The high-$\ell$ temperature likelihood CamSpec. At $\ell = 50 – 2500$, a correlated Gaussian approximation is employed to obtain the likelihood, based on a fine-grained set of angular cross-spectra derived from multiple detector combinations between the 100, 143, and 217 GHz frequency channels.

- The low-$\ell$ temperature likelihood. At $\ell < 50$, the likelihood exploits all Planck frequency channels from 30-353 GHz, separating the CMB signal from the diffuse Galactic foregrounds through a

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$^1$ In the MCMC, samples with $H_0$ out of this range are rejected.

$^2$ http://pla.esac.esa.int/pla/aio/planckProducts.html
physically motivated Bayesian component separation technique.

- The low-$$\ell$$ polarization likelihood. The present \textit{Planck} data release includes only temperature data, and the \textit{Planck} Collaboration supplements the \textit{Planck} likelihood with the 9-year WMAP (\textit{WMAP}-9) polarization likelihood derived from the \textit{WMAP} polarization maps at 33, 41, and 61 GHz (K, Q, and V bands).

- The \textit{Planck} lensing likelihood. Lensing is detected independently in \textit{Planck} 100, 143, and 217 GHz channels with an overall significance of greater than 25$$\sigma$$ [22]. The gravitational lensing data are good at constraining dark energy through the lensing effect coming from the distortion of the large scale structure that emerged after $$z = 10$$ (at this stage, the universe is dark energy dominated).

In the following context, we will use “\textit{Planck}” to represent the \textit{Planck} temperature likelihood (including both the low-$$\ell$$ and high-$$\ell$$ parts), “\textit{WP}” to represent the \textit{WMAP} polarization likelihood as a supplement of \textit{Planck}, and “\textit{lensing}” to represent the likelihood of \textit{Planck} lensing data.

To study the difference between the fitting results by using \textit{Planck} and \textit{WMAP} data, in this work we also perform the analysis of HDE by using \textit{WMAP}-9 data. The data and likelihood software are downloadable at the Legacy Archive for Microwave Background Data Analysis (LAMBDA). ³ We will not use the high-resolution CMB data of the Atacama Cosmology Telescope and the South Pole Telescope [23]. They are not publicly available in the current version of \textit{COSMOMC} package, and only marginally affect the fitting results compared with \textit{Planck} or \textit{WMAP}-9.

\section*{B. External astrophysical data sets}

The CMB data alone are not powerful in constraining dark energy parameters, since dark energy affects the late time cosmic evolution. When combined with the external astrophysical data sets (hereafter, “\textit{Ext}” or “\textit{Exts}”), CMB data are helpful in breaking the degeneracies between parameters and improving the constraints on dark energy parameters [24]. In our analysis, we will consider the following four Exts:

- The BAO data can provide effective constraints on dark energy from the angular diameter distance–redshift relation. In our analysis, similar to [11], we use the following data sets, the 6dF Galaxy Survey $$D_V(0.106) = (457 \pm 27)$$Mpc [25] ($$D_V$$ is a distance indicator similar to angular diameter distance $$D_A$$, see Eq. (46) in [11]), the reanalyzed SDSS DR7 BAO measurement $$D_V(0.33)/r_s = 8.88 \pm 0.17$$ [26], and the BOSS DR9 measurement $$D_V(0.57)/r_s = 13.67 \pm 0.22$$ [27]. SDSS DR7 and

³ \url{http://lambda.gsfc.nasa.gov}
BOSS DR9 are the two most accurate BAO measurements, and the correlation between the surveys is a marginal effect to the parameter estimation.

- The direct measurement of the Hubble constant, \( H_0 = 73.8 \pm 2.4 \text{ km s}^{-1} \text{ Mpc}^{-1} \) (1\(\sigma\) CL) [15], from the supernova magnitude–redshift relation calibrated by the HST observations of Cepheid variables in the host galaxies of eight SNe Ia. Here the uncertainty is 1\(\sigma\) and includes known sources of systematic errors.

- The Union2.1 compilation [17], consisting of 580 SNe, calibrated by the SALT2 light-curve fitting model [28].

- The SNLS3 “combined” sample [14], consisting of 472 SNe, calibrated by both SiFTO [29] and SALT2 [28]. For simplicity, we do not consider the SNLS3 compilation calibrated separately by SiFTO or SALT2.

In the following context, we will use “BAO”, “HST”, “Union2.1” and “SNLS3” to represent these four ExtS. We will also use “SNIa” to represent a supernovae data set, either Union2.1 or SNLS3.

III. CMB-ONLY RESULTS

| Data            | \( \Omega_m \)    | \( c \)             | \( H_0 \)            | \(- \ln L_{\text{max}}\) |
|-----------------|------------------|---------------------|----------------------|--------------------------|
| Planck          | 0.142            | 0.261 ± 0.097       | 0.301                | 0.587 ± 0.449            | 100.00 | 77.34 ± 12.82 | 3894.4 |
| Planck+lensing  | 0.150            | 0.248 ± 0.084       | 0.317                | 0.531 ± 0.296            | 96.80  | 78.51 ± 12.09 | 3899.8 |
| Planck+WP       | 0.157            | 0.268 ± 0.100       | 0.317                | 0.612 ± 0.433            | 95.57  | 75.60 ± 12.83 | 4902.6 |
| Planck+WP+lensing | 0.180          | 0.248 ± 0.079       | 0.354                | 0.508 ± 0.207            | 88.65  | 78.36 ± 11.36 | 4907.3 |
| WMAP–9          | 0.350            | 0.401 ± 0.082       | 0.965                | 1.88^{+0.79}_{-1.20}    | 62.37  | 59.57 ± 8.15  | 3779.0 |

In this section we present the CMB-only fitting results of the HDE model. The CMB+Ext fitting results are discussed in the next section.

In Table 1, we list the fitting results of the HDE model from the CMB data alone. Best-fit values as well as the 68% CL limits for \( \Omega_m \), \( c \) and \( H_0 \) are listed in columns 2–7. The minus log-maximal likelihood is listed in the last column. The first 4 rows list the results of Planck, Planck+lensing, Planck+WP, and Planck+WP+lensing. For comparison, the WMAP–9 results are listed in the last row.
In the following two subsections, we firstly introduce the temperature power spectra with the best-fit parameters, and then discuss the constraints on cosmological parameters.

A. Temperature power spectra

In the upper panel of Fig. 1 we show the temperature power spectrum of the best-fit HDE model (green dotted) by using the Planck+WP data. As comparisons, best-fit spectra of the ΛCDM model and the wCDM model from Planck+WP are also plotted in black solid and red dashed lines. To see the difference between the three spectra, the residuals compared with the best-fit six-parameter ΛCDM model are shown in the lower panel. We find that all the three models can provide a good fit to the Planck high-ℓ power spectrum, while at ℓ ≃ 20 – 40 there are some mismatches, as reported by Planck [11]. The HDE model is not helpful in relieving this discrepancy. The main difference among the power spectra of the three models lie in the ℓ ≲ 20 region, where we find that amplitudes of HDE and wCDM spectra are lower than the ΛCDM spectrum. This phenomenon is consistent with the result of [9], where it is shown that a phantom-like dark energy component leads to smaller $C_{ℓ}^{TT}$ at low-ℓ region.

![Figure 1](image_url)

**FIG. 1:** Upper panel: CMB TT power spectrum plotted with the best-fit parameters of ΛCDM model (black solid), wCDM model (red dashed), and HDE model (green dotted), from the Planck+WP data. The ordinate axis shows $D_{ℓ} ≡ ℓ(ℓ + 1)C_{ℓ}/2π$ in units of $μK^2$. The Planck binned temperature spectrum is shown in black dots with error bars. Lower panel: Residuals with respect to the temperature power spectrum of the best-fit six-parameter ΛCDM model.
It is also of interest to compare the WMAP and Planck spectra in the HDE model. The Appendix A of [11] shows some inconsistency between the Planck and WMAP spectra. It is found that the WMAP power spectrum re-scaled by a multiplicative factor of 0.975 agree to remarkable precision with the Planck spectrum [11]. Thus, in Fig. 2 we plot the WMAP-9 and Planck+WP spectra for the ΛCDM (upper panel), wCDM (middle panel) and HDE (lower panel) models. As expected, in all these three models, we find that the WMAP-9 power spectrum (with a multiplicative factor 0.975) matches well with the Planck power spectrum. The best-fit power spectra of the three models are similar to each other. More interestingly, in all models we find that at ℓ ~ 1600 ~ 2000 the theoretical power spectra of Planck and WMAP-9 have higher amplitudes than the Planck data. This scale corresponds to ~10 times the scale of galaxy clusters, and this discrepancy may be due to some unclear physics on this scale.
FIG. 2: The WMAP-9 and Planck+WP best-fit power spectra for the ΛCDM (upper panel), wCDM (middle panel) and HDE (lower panel) models. To see the difference between the theoretical power spectra and the observational data at the high-\(\ell\) region, we choose to plot the \(\ell^2D_\ell\) (in units of mK\(^2\)) rather than \(D_\ell\). The Planck+WP best-fit power spectra are plotted in green lines, and the WMAP-9 best-fit power spectra multiplied by 0.975 are plotted in red lines. The black points with error bars mark the Planck temperature power spectrum data.

B. Constraints on cosmological parameters

In this subsection we discuss the constraints on cosmological parameters in the HDE model.
The likelihood distributions of $c$ are shown in the left panel of Fig. 3. We find that Planck data lead to small values of best-fit $c$, i.e., $0.30–0.35$. The 68% CL errors of $c$ are about 0.45 for Planck and Planck+WP, and are reduced to 0.3 and 0.21 when the lensing data are added. Compared with the WMAP-9 alone constraint, $c = 1.786 \pm 0.880$, the Planck results reduce the error bar by about 45%–75%. There are clear discrepancies between the mean (see the 68% limits listed in Table I) and the best-fit values of $c$, implying that the likelihood distribution of $c$ is highly deviated from symmetric form.

The right panel of Fig. 3 shows the $\Omega_m$–$c$ contours of the CMB-only constraints. Results of WMAP-9, Planck+WP and Planck+WP+lensing are plotted. To see the behavior of HDE under the constraints, we also plot the “crossing $w = -1$ redshift” in dashed lines: e.g., parameter space above/bellow the dashed blue line corresponds to a quintessence/phantom behavior of holographic dark energy at $z = 1.0$. We see that the WMAP-9 data alone does not lead to any interesting constraint on $c$, while the Planck+WP results show the preference for $c < 1$ at the 1$\sigma$ CL. Adding the lensing data tightens the constraint, and the present phantom behavior of holographic dark energy is preferred at the more than 1$\sigma$ CL. Besides, we find that in the HDE model $\Omega_m$ is constrained to be $0.26–0.28$ (68% CL) by the Planck data, which is smaller than the result in the $\Lambda$CDM model. The WMAP data alone cannot lead to effective constraint on $\Omega_m$ in the HDE model.

The CMB-only constraints on $H_0$ in the HDE model are listed in the 5th and 6th columns of Table I. Compared with the $\Lambda$CDM result, $H_0 = 67.4 \pm 1.4$ (68% CL; Planck) [11], the error bars are significantly larger. $^4$ Similar phenomenon appears in the WMAP-9 results, where we find $H_0 = 59.57 \pm 8.15$ in the HDE

$^4$ Since we impose a prior [20, 100] on $H_0$ in the analysis, the error bars are, actually, under-estimated.
model.

FIG. 4: CMB-only fitting results of the HDE model. Left panel: Marginalized likelihood distributions of $H_0$. Right panel: Marginalized 68% and 95% CL contours in the $c$–$H_0$ plane.

To make a comparison, in the left panel of Fig. 4 we plot the likelihood distributions of $H_0$ in the $\Lambda$CDM, $w$CDM and HDE models, constrained by Planck+WP and WMAP-9 data. We find that, in the $\Lambda$CDM model, $H_0$ is tightly constrained, while in the HDE and $w$CDM models it cannot be effectively constrained. The right panel shows the $c$–$H_0$ contours constrained by WMAP-9, Planck+WP, and Planck+WP+lensing. We see that $c$ and $H_0$ are strongly anti-correlated with each other. This explains why in the HDE model $H_0$ cannot be well constrained by CMB-only data.
FIG. 5: Upper panel: Marginalized likelihood distributions of \( \Omega_m h^2 \) and \( \Omega_m h^3 \), for the \( \Lambda \)CDM (black solid), \( w \)CDM (red dashed) and HDE (green dotted) models. Lower panel: Marginalized 68\% and 95\% CL contours in the \( \Omega_m h^3 - w \) and \( \Omega_m h^3 - c \) planes. The gray band marks the constraint \( \Omega_m h^3 = 0.059 \pm 0.0006 \) (68\% CL; \textit{Planck}) in the \( \Lambda \)CDM model [11].

Furthermore, in order to understand why the likelihood distribution of \( H_0 \) is greatly widened in the \( w \)CDM and HDE models, in the upper panels of Fig. 5 we plot the likelihood distributions of \( \Omega_m h^2 \) and \( \Omega_m h^3 \) in the three models, constrained by \textit{Planck}+WP+lensing. Interestingly, we find similar likelihood distributions of \( \Omega_m h^2 \) in the three models, but that \( \Omega_m h^3 \) has much broader distribution in the \( w \)CDM and HDE models than in the \( \Lambda \)CDM model. The lower panels show that the above phenomenon is due to the strong anti-correlation between \( \Omega_m h^3 \) and dark energy parameters. In the \( \Lambda \)CDM model, the precise measurement of acoustic scale in \textit{Planck} leads to a strong constraint on \( \Omega_m h^3 \), i.e., \( \Omega_m h^3 = 0.059 \pm 0.0006 \) (68\% CL) [11], (shown as the gray band in the lower panels), so together with the constraint on \( \Omega_m h^2 \) we expect a strong constraint on \( H_0 \). However, when we add dark energy parameters like \( w \) or \( c \) into the analysis, the strong correlation between the parameters makes \( \Omega_m h^3 \) unconstrained, and so \( H_0 \) also becomes unconstrained.
It is expected that the widened $H_0$ distribution is helpful in relieving the tension between Planck and HST observations; see [30] for a related work. We will discuss this topic in the next section.

IV. CMB COMBINED WITH ASTROPHYSICAL DATA SET RESULTS

The CMB+Ext fitting results of the HDE model are listed in Table II. Best-fit values as well as 68% CL limits for $\Omega_m$, $c$ and $H_0$ are listed in the columns 2–7. The maximal likelihood values are listed in the 7th column, and the residual $\chi^2$ values, defined as $\Delta\chi^2_{CMB+Ext} \equiv \chi^2_{CMB+Ext} - \chi^2_{CMB} - \chi^2_{Ext}$, are listed in the last column. The results from Planck+WP combined with external astrophysical data sets are listed in the first nine rows, while the WMAP-9 results are listed in the following seven rows. As a comparison, in the last six rows we also list the results from the astrophysical data sets only, including the fitting results of BAO, BAO+HST, SNLS3, Union2.1, SNLS3+BAO+HST, and Union2.1+BAO+HST.

![Fitting results of the HDE model, from Planck+WP combined with external astrophysical data sets of BAO (red), HST (green), SNLS3 (black) and Union2.1 (blue). Left panel: Marginalized likelihood distributions of $c$. Right panel: Marginalized 68% and 95% CL contours in the $\Omega_m$–$c$ plane.](image)

We find that adding external astrophysical dataset reduces the error of $c$ to 0.05–0.07. The likelihood distributions of $c$ and the $\Omega_m$–$c$ contours for Planck+WP and Planck+WP+Ext are plotted in Fig. 6. The best-fit values of $c$ for Planck+WP+BAO and Planck+WP+HST constraints are around 0.5, while the values for Planck+WP+SNLS3 and Planck+WP+Union2.1 constraints are around 0.6. As a comparison, the best-fit value of $c$ from WMAP-9 combined one Ext is larger, i.e., 0.55–0.77, and the error is also larger, i.e., 0.08–0.17.

We find that the Planck lensing data are helpful in improving the constraint on $c$. By adding the lensing data into the analysis of Planck+WP combined with the Ext, such as BAO, HST, BAO+HST, SNLS3 and
TABLE II: CMB+Ext fitting results of the HDE model.

| Data | $\Omega_m$ | $c$ | $H_0$ | $-\ln \mathcal{L}_{\text{max}}$ | $\Delta \chi^2$ | $a$ |
|------|------------|-----|-------|-------------------------------|----------------|-----|
|      | Best fit  | 68% limits | Best fit  | 68% limits | Best fit  | 68% limits |
| Planck+WP + BAO | 0.270 | 0.254 ± 0.024 | 0.506 | 0.484 ± 0.070 | 72.63 | 75.06 ± 3.82 | 4903.6 | 1.7 |
| Planck+WP + BAO + lensing | 0.262 | 0.256 ± 0.022 | 0.498 | 0.494 ± 0.062 | 73.62 | 74.65 ± 3.39 | 4908.4 | 1.9 |
| Planck+WP + HST | 0.266 | 0.257 ± 0.019 | 0.463 | 0.474 ± 0.049 | 73.78 | 74.77 ± 2.68 | 4902.8 | 0.3 $^c$ |
| Planck+WP + HST + lensing | 0.260 | 0.256 ± 0.019 | 0.498 | 0.489 ± 0.048 | 73.81 | 74.62 ± 2.69 | 4907.9 | 1.1 $^c$ |
| Planck+WP + BAO + HST | 0.252 | 0.255 ± 0.014 | 0.470 | 0.481 ± 0.046 | 75.22 | 74.75 ± 2.19 | 4906.6 | 0.9 |
| Planck+WP + BAO + HST + lensing | 0.245 | 0.255 ± 0.013 | 0.481 | 0.495 ± 0.039 | 75.83 | 74.5 ± 2.0 | 4908.5 | 1.3 |
| Planck+WP + SNLS3 | 0.300 | 0.305 ± 0.019 | 0.584 | 0.594 ± 0.051 | 68.81 | 68.46 ± 1.93 | 5115.8 | 6.4 |
| Planck+WP + SNLS3 + lensing | 0.310 | 0.301 ± 0.019 | 0.610 | 0.603 ± 0.049 | 67.73 | 67.66 ± 1.92 | 5120.8 | 7.3 |
| Planck+WP + Union2.1 | 0.327 | 0.324 ± 0.021 | 0.618 | 0.642 ± 0.066 | 66.35 | 66.74 ± 1.94 | 5176.1 | 1.6 |
| Planck+WP + Union2.1 + lensing | 0.321 | 0.321 ± 0.021 | 0.617 | 0.645 ± 0.063 | 66.72 | 66.68 ± 2.03 | 5181.6 | 3.4 |
| Planck+WP + SNLS3 + BAO + HST | 0.269 | 0.275 ± 0.011 | 0.583 | 0.563 ± 0.035 | 72.41 | 71.46 ± 1.37 | 5123.2 | 10.9 $^d$ |
| Planck+WP + Union2.1 + BAO + HST + lensing | 0.276 | 0.281 ± 0.012 | 0.551 | 0.577 ± 0.039 | 71.49 | 70.68 ± 1.40 | 5185.3 | 9.6 $^d$ |
| WMAP-9 + BAO | 0.274 | 0.284 ± 0.021 | 0.623 | 0.746 ± 0.165 | 70.41 | 68.93 ± 3.18 | 3779.6 | 0.9 |
| WMAP-9 + BAO + HST | 0.251 | 0.250 ± 0.020 | 0.552 | 0.569 ± 0.086 | 73.98 | 73.99 ± 2.71 | 3779.1 | 0.2 $^c$ |
| WMAP-9 + SNLS3 | 0.255 | 0.259 ± 0.015 | 0.534 | 0.567 ± 0.081 | 73.65 | 72.96 ± 2.37 | 3779.7 | 0.3 $^c$ |
| WMAP-9 + Union2.1 | 0.277 | 0.280 ± 0.022 | 0.664 | 0.696 ± 0.078 | 69.79 | 69.44 ± 2.23 | 3990.6 | 3.5 |
| WMAP-9 + SNLS3 + BAO + HST | 0.304 | 0.299 ± 0.023 | 0.767 | 0.782 ± 0.105 | 66.76 | 67.24 ± 2.18 | 4051.6 | 0.1 |
| WMAP-9 + SNLS3 + BAO + HST + lensing | 0.269 | 0.270 ± 0.011 | 0.626 | 0.645 ± 0.060 | 70.89 | 70.89 ± 1.46 | 3992.2 | 5.6 $^d$ |
| WMAP-9 + Union2.1 + BAO + HST | 0.276 | 0.276 ± 0.011 | 0.659 | 0.711 ± 0.074 | 70.17 | 69.64 ± 1.37 | 4054.3 | 4.3 $^d$ |
| BAO | 0.227 | 0.215 ± 0.124 | 2.391 | 1.579 ± 0.772 | × $^e$ | × | 0.1 | -- |
| BAO + HST | 0.289 | 0.332 ± 0.974 | 0.552 | 0.666 ± 0.241 | 73.56 | 73.49 ± 2.38 | 0.6 | -- |
| SNLS3 | 0.129 | 0.118 ± 0.072 | 1.294 | 1.519 ± 0.514 | × | × | 209.8 | -- |
| Union2.1 | 0.256 | 0.173 ± 0.099 | 0.024 | 1.68 ± 0.78 | × | × | 272.5 | -- |
| SNLS3 + BAO + HST | 0.294 | 0.295 ± 0.029 | 0.612 | 0.622 ± 0.071 | 72.27 | 72.37 ± 2.36 | 212.5 | 4.1 $^b$ |
| Union2.1 + BAO + HST | 0.323 | 0.326 ± 0.030 | 0.608 | 0.633 ± 0.086 | 73.42 | 73.09 ± 2.36 | 273.6 | 1.0 $^b$ |

$^a$ $\Delta \chi^2_{\text{CMB+Ext}} \equiv \chi^2_{\text{CMB+Ext}} - \chi^2_{\text{CMB}} - \chi^2_{\text{Ext}}$

$^b$ $\Delta \chi^2_{\text{SNLS3+BAO+HST}} \equiv \chi^2_{\text{SNLS3+BAO+HST}} - \chi^2_{\text{SNLS3}} - \chi^2_{\text{BAO+HST}}$

$^c$ $\Delta \chi^2_{\text{Bao+HST}} \equiv \chi^2_{\text{Bao+HST}} - \chi^2_{\text{CMB}}$

$^d$ $\Delta \chi^2_{\text{CMB+SNLS3+BAO+HST}} \equiv \chi^2_{\text{CMB+SNLS3+BAO+HST}} - \chi^2_{\text{CMB}} - \chi^2_{\text{SNLS3}} - \chi^2_{\text{BAO+HST}}$

$^e$ The cross “×” indicates that the parameter is unconstrained by the chosen data sets.

Union2.1, the constraint results are improved by 11%, 2%, 15%, 4% and 5%, respectively.
FIG. 7: Marginalized likelihood distributions of dark energy equation of state at $z = 0$ (upper left and lower panels) and $z = -1$ (upper right panel). In the two upper panels, the results of Planck+WP combined with lensing (gray solid) and BAO (red solid) are shown. In the lower panel, we also plot the Planck+WP+SNLS3 results (blue). The black thick line marks $w = -1$. As comparisons, the WMAP-9 (gray dashed) and WMAP-9+BAO (red dashed) results are plotted in the two upper panels, and the $w$CDM results (dashed lines) are plotted in the lower panel.

To see the dynamical behavior of HDE, in Fig. 7 we plot the likelihood distributions of the dark energy equation of state at $z = 0$ (upper left and lower panels) and $z = -1$ (upper right panel). We find that, by using the Planck data, a phantom-like holographic dark energy is favored at high confidence level in both current and future epochs. The result of $w_0 < -1$ can be obtained at more than 2$\sigma$ level by using Planck+WP+lensing, even without any Ext combined. These are different from that of the $w$CDM results (dashed lines in the lower panel), where $w = -1$ is still consistent with the fitting results at a relatively high confidence level.

Furthermore, to investigate the tension between CMB and Ext, in the last column of Table II we list the $\Delta \chi^2$ values for the different combinations. In most combinations we find a small $\Delta \chi^2$, except for
the CMB+SNLS3 results, where $\chi^2_{\text{CMB+SNLS3}} - \chi^2_{\text{CMB}} - \chi^2_{\text{SNLS3}} = 6.4, 7.3, \text{ and } 3.5$ for Planck+WP, Planck+WP+lensing and WMAP-9, implying an evident tension. For no-CMB constraints, the result $\chi^2_{\text{SNLS3+BAO+HST}} - \chi^2_{\text{SNLS3}} - \chi^2_{\text{BAO+HST}} = 4.1$ means that SNLS3 is also in tension with BAO+HST. The HST combined results lead to $\chi^2_{\text{CMB+HST}} - \chi^2_{\text{CMB}} = 1.7, 1.1 \text{ and } 0.2$ for Planck+WP, Planck+WP+lensing and WMAP-9, implying that there is no severe tension between HST and CMB in the HDE model.

In the following, we will discuss the fitting results in detail. We will discuss the fitting results of CMB combined with BAO and HST in the first subsection, and the fitting results of CMB combined with SNLS3 and Union2.1 in the second subsection.

A. Combined with BAO and HST

FIG. 8: Fitting results of the HDE model, from CMB combined with BAO and HST. The upper-left panel shows the marginalized distributions of $c$. The other three panels show the marginalized 68% and 95% CL contours in the $\Omega_m$–$c$ plane, including the CMB+BAO results for Planck and WMAP-9 (upper-right), CMB+HST results for Planck and WMAP-9 (lower-left), and the CMB+HST/BAO results for Planck (lower-right).
At the 68% CL, we obtain $c = 0.484 \pm 0.070$ (Planck+WP+BAO), $c = 0.474 \pm 0.049$ (Planck+WP+HST), $c = 0.746 \pm 0.165$ (WMAP-9+BAO) and $c = 0.569 \pm 0.086$ (WMAP-9+HST). Compared with the WMAP-9 results, the best-fit values of $c$ from the Planck data are smaller by 0.1–0.3, and the error bars are reduced by 40%–60%. These results can be seen clearly in the likelihood distributions plotted in the upper-left panel of Fig. 8.

The other three panels of Fig. 8 show the 68% and 95% CL contours in the $\Omega_m$–$c$ plane, including the CMB+BAO results for Planck and WMAP-9 (upper-right), CMB+HST results for Planck and WMAP-9 (lower-left), and the Planck+HST/BAO results (lower-right). Interestingly, in the lower-right panel we see that the Planck+WP+BAO (red solid) and Planck+WP+HST (green solid) contours lie in the same position, showing that Planck+WP+BAO and Planck+WP+HST lead to consistent fitting results. This figure also shows a consistent overlap of Planck+WP (gray solid) and BAO+HST (purple dotted). The combined Planck+WP+BAO+HST (blue filled region) data lead to a self-consistent constraint, $c = 0.481 \pm 0.046$. Moreover, we can further tighten the constraints by adding the lensing data into the analysis. Table II shows that, by adding the lensing data, the Planck+WP+BAO constraint on $c$ is improved from $0.484 \pm 0.070$ to $0.494 \pm 0.062$, and the Planck+WP+BAO+HST result is improved from $0.481 \pm 0.046$ to $c = 0.495 \pm 0.039$. The error bars are reduced by 12%–15%. The $\Delta \chi^2$ values for the two lensing combined results are 1.9 and 1.3, respectively, showing a good consistency. Actually, the constraint result, $c = 0.495 \pm 0.039$, from Planck+WP+BAO+HST+lensing is our tightest self-consistent constraint on $c$. If we further add the supernova data set into the analysis, the error bars can be slightly reduced, but a significant inconsistency among the data sets appears. This result also has 35%–50% smaller error bars compared with the WMAP-9 all-combined results, where the constraints on $c$ are $c = 0.645 \pm 0.060$ and $c = 0.711 \pm 0.074$ for WMAP-9+BAO+HST combined with SNLS3 and Union2.1, respectively.
In the last section, we find that in the wCDM and HDE models the CMB-only constraints allow a wide range of $H_0$ (see Fig. 4). Now, let us see if the tension between CMB and HST can be relieved in these two models when the external astrophysical data are added in the analysis. In the upper panels of Fig. 9 we plot the likelihood distributions of $H_0$ in the ΛCDM (left), wCDM (middle) and HDE (right) models, obtained by using Planck+WP (green solid), Planck+WP+lensing (green dashed), Planck+WP+BAO (red solid), Planck+WP+HST (blue solid), and Planck+WP+BAO+HST (black solid), respectively. In all plottings, the HST measurement result, $H_0 = 73.8 \pm 2.4$ [15], is shown in the gray filled region. In the upper-left panel, we see that the constraints on $H_0$ are fairly tight in the ΛCDM model, and the results of various combinations involving Planck+WP are all in tension with the HST measurement. However, for the wCDM and HDE models, all the CMB combined constraints overlap well with the gray region, showing that the
tension between CMB and HST is effectively relieved in these two models if the Planck data are combined with BAO or HST. This phenomenon can be seen more clearly in the \( \Omega_m - H_0 \) plane for the three models (the lower-left panel). We see that the allowed parameter space of the \( \Lambda \)CDM model is tightly confined by the CMB data, and the positions of Planck+WP+lensing (green solid) and Planck+WP+HST (black solid) contours evidently deviate from the HST measurement (the gray band). On the other hand, the CMB data alone cannot effectively constrain the \( \Omega_m - H_0 \) parameter space for HDE (dark blue dashed). The positions of Planck+WP+HST contours for the HDE (light blue solid) and \( w \)CDM (red solid) models are all well consistent with the HST measurement.

### TABLE III: Residual \( \chi^2 \) values in the \( \Lambda \)CDM, \( w \)CDM and HDE models

| Model | \( \chi^2_{\text{Planck+WP+BAO}} - \chi^2_{\text{Planck+WP}} \) | \( \chi^2_{\text{Planck+WP+HST}} - \chi^2_{\text{Planck+WP}} \) | \( \chi^2_{\text{Planck+WP+BAO+HST}} - \chi^2_{\text{Planck+WP}} \) |
|-------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|
| \( \Lambda \)CDM | 2.5 | 7.8 | 9.1 |
| \( w \)CDM | 2.6 | 1.0 | 3.7 |
| HDE | 1.9 | 0.3 | 1.9 |

The tension between CMB and the external data sets (e.g., BAO and HST) in the HDE model can be characterized by the \( \Delta \chi^2 \) values, as listed in the last column of Table II. The results are \( \Delta \chi^2 = 1.7, 0.3, 0.9, 0.9 \), and 0.2 for Planck+WP+BAO, Planck+WP+HST, Planck+WP+BAO+HST, WMAP-9+BAO, and WMAP-9+HST, respectively. These values are small, showing that there is no severe tension between the data sets in the HDE model. As a comparison with the \( w \)CDM and \( \Lambda \)CDM models, in Table III we show the residuals \( \chi^2 \) values of Planck+WP+BAO, Planck+WP+HST and Planck+WP+BAO+HST with respect to Planck+WP in the three models. For the \( \Lambda \)CDM model, adding HST and BAO+HST significantly increases the \( \chi^2 \) value by 7.8 and 9.1. The increments are 1.0 and 3.7 for the \( w \)CDM model, and only 0.3 and 1.9 for the HDE model. Thus, the tension with HST measurement is effectively relieved in the two dynamic dark energy models.

Moreover, Fig. 9 and Table III show that there is a better consistency among data sets in the HDE model than in the \( w \)CDM model. The best-fit values of \( H_0 \) from Planck+WP+BAO are 67.63, 69.68 and 72.63 for the \( \Lambda \)CDM, \( w \)CDM and HDE models, among which the HDE result is the most close to the HST measurement. To understand why Planck+WP+BAO gives a higher \( H_0 \) in the HDE model than in the \( w \)CDM model, in the lower-right panel of Fig. 9 we plot the \( H_0-r_s/D_V(0.57) \) contours for the three models. In this figure, we also show the joint 1 and 2\( \sigma \) likelihood region for BOSS DR9 + HST measurements in the dark and light gray shaded contours. We see that, for the \( \Lambda \)CDM model, the Planck+WP contours (green solid) are consistent with the BOSS DR9 measurement, but are in tension with the HST measurement. In the \( w \)CDM
and HDE models, the allowed parameter spaces are greatly broadened, and their Planck+WP contours (dashed lines) overlap with the gray contours. Interestingly, the positions of the wCDM and HDE contours are different: the HDE contours lie in the smaller $r_\chi/D_V(0.57)$ region, below the wCDM contours, so they overlap with the gray contours at higher $H_0$ region. This helps us to understand why the Planck+WP+BAO (black filled region) result of the HDE model has higher values of $H_0$ than the Planck+WP+BAO (blue filled region) result of the wCDM model.

Besides, it should be mentioned that, due to the anti-correlation between $w$ (or $c$) and $H_0$, the Planck+WP+HST leads to phantom results in the wCDM and HDE models. In [11], the Planck Collaboration reported a result $w = -1.24^{+0.18}_{-0.19}$ (95% CL, Planck+WP+highL+BAO+HST) for the wCDM model, which is in tension with $w = -1$ at the more than 2$\sigma$ level. For the HDE model, the lower-left panel of Fig. 8 shows that the 95% CL contour from Planck+WP+HST (red filled region) lies below the $z=0.5$ phantom divide line (red dashed).

### B. Combined with SNIa

In this subsection, we discuss the SNIa combined fitting results.

The CMB+SNIa fitting results are plotted in Fig. 10. The likelihood distributions of $c$ are shown in the upper-left panel. At the 68% CL, we get $c = 0.594 \pm 0.051$, $c = 0.642 \pm 0.066$, $c = 0.696 \pm 0.078$ and $c = 0.782 \pm 0.105$ for Planck+WP+SNLS3, Planck+WP+Union2.1, WMAP-9+SNLS3 and WMAP-9+Union2.1, respectively. Similar as the above results, compared with the WMAP-9 results, the Planck results have smaller best-fit values and error bars. Adding lensing into the analysis effectively tightens the constraint, yielding $c = 0.583 \pm 0.042$ and $c = 0.645 \pm 0.063$ for Planck+WP+lensing combined with SNLS3 and Union2.1. Compared with CMB+Union2.1, we find that CMB+SNLS3 yields more phantom-like result.

In [11], the Planck Collaboration reported that there exists some tension between Planck and supernovae data sets, and the tension between Planck and SNLS3 is more severe than that between Planck and Union2.1. To investigate the tension between CMB and SNIa data sets in the HDE model, in the lower panels we plot the 68% and 95% CL contours in the $\Omega_m$–$c$ plane from Planck+WP (orange), WMAP-9 (gray), SNIa (blue), Planck+WP+SNIa (red filled) and WMAP-9+SNIa (green filled). The SNLS3 plottings are shown in the lower-left panel, and the Union2.1 plottings are shown in the lower-right panel. From the positions of the contours, we see that the CMB data are consistent with Union2.1, but in tension with SNLS3 (the 1$\sigma$ contours of CMB and SNIa do not overlap). Table II shows that $\Delta\chi^2_{Planck+WP+SNIa}$, $\Delta\chi^2_{Planck+WP+lensing+SNIa}$ and $\Delta\chi^2_{WMAP-9+WP+SNIa}$ are 6.4, 7.3 and 3.5 for SNLS3, respectively, while only 1.6, 3.4 and 0.1 for Union2.1,
respectively. Besides, as mentioned above, the results in Table II also show some tension between SNLS3 and BAO+HST: for SNLS3 we have \( \chi^2_{\text{SNIa+BAO+HST}} - \chi^2_{\text{SNIa}} = 4.1 \), while for Union2.1 the value is only 1.0. So, it is fairly remarkable that for the HDE model the SNLS3 data set is in weak tension with all other data sets.

Another interesting phenomenon is that, although there is no severe tension when we combine Union2.1 with BAO+HST or Planck+WP, evident tension appears when we combine all these data sets together. Table II shows that \( \Delta \chi^2_{\text{Planck+BAO+HST}} = 9.6 \), as large as \( \Delta \chi^2_{\text{Planck+SNLS3+BAO+HST}} \) (that is equal to 10.9). This tension mainly comes from the discrepancy between the results of Planck+WP+lensing and Union2.1+BAO+HST: we find that \( \chi^2_{\text{Planck+BAO+HST+lensing}} - \chi^2_{\text{Planck+SNLS3+BAO+HST+lensing}} = 8.6 \). The fitting results of Union2.1+BAO+HST are \( \Omega_m = 0.326 \pm 0.030, c = 0.633 \pm 0.086 \) and \( H_0 = 73.09 \pm 2.36 \), while for Planck+WP+lensing the results are
\[ \Omega_m = 0.248 \pm 0.079 \text{ and } c = 0.508 \pm 0.207. \] When we combine them, we get \[ \Omega_m = 0.281 \pm 0.012, \]
\[ c = 0.577 \pm 0.039 \text{ and } H_0 = 70.68 \pm 1.40. \] These three sets of results do not match with each other. Especially, the constraint result of \( H_0 \) in the all-combined analysis is in tension with the \( HST \) measurement.

For \( WMAP-9 \) we find that \[ \chi^2_{\text{WMAP-9+SNIa+BAO+HST}} = 5.6 \] and 4.3 for SNLS3 and Union2.1, respectively, which also implies some tension, but not so severe as the \( Planck \) case. Thus, it is no longer viable to do a all-combined analysis by combining \( Planck \) data with all the external data sets of SNIa, BAO and \( HST \). Our tightest self-consistent constraint is \[ c = 0.495 \pm 0.039 \] obtained from \( Planck+WP+BAO+HST+lensing \).

V. CONCLUDING REMARKS

In this paper we perform detailed investigation on the constraints on the HDE model by using the \( Planck \) data. We find the following results:

- HDE provides a good fit to the \( Planck \) high-\( \ell \) temperature power spectrum. The discrepancy at \( \ell \lesssim 20 \) – 40 found in the \( \Lambda \)CDM model remains unsolved in the HDE model. The best-fit power spectra of the \( \Lambda \)CDM, \( w \)CDM and HDE models are similar to each other at \( \ell \gtrsim 25 \). In the \( \ell \lesssim 25 \) region, the \( w \)CDM and HDE spectra have slightly lower amplitudes than the \( \Lambda \)CDM spectrum.

- \( Planck \) data alone can lead to interesting constraint on \( c \). By using \( Planck+WP+lensing \), we get \[ c = 0.508 \pm 0.207 \text{ (68\% CL)}, \] favoring the present phantom behavior of HDE at the more than \( 2\sigma \) CL. Comparably, by using \( WMAP-9 \) data alone we cannot get valuable constraint on \( c \).

- At the 68\% CL, the results are \[ c = 0.484 \pm 0.070, \]
\[ c = 0.474 \pm 0.049, \]
\[ c = 0.594 \pm 0.051, \] and \[ c = 0.642 \pm 0.066 \] from \( Planck+WP \) combined with BAO, \( HST \), SNLS3 and Union2.1, respectively. The constraints can be improved by 2\%–15\% if we further add the \( Planck \) lensing data into the analysis. The results from \( WMAP-9 \) combined with each Ext are \[ c = 0.746 \pm 0.165, c = 0.569 \pm 0.086, \]
\[ c = 0.696 \pm 0.078 \] and \[ c = 0.782 \pm 0.105. \] Compared with \( WMAP-9+\text{Ext} \) results, we find that \( Planck+WP+\text{Ext} \) results reduce the error by 30\%–60\%, and prefer a more phantom-like HDE.

- Non-standard dark energy models are helpful in relieving the tension between CMB and \( HST \) measurements. In the CMB-only analysis, the strong correlation between \( c \) (\( w \)) and \( \Omega_m h^3 \) in the HDE (\( w \)CDM) model makes \( H_0 \) unconstrained. We find that \[ \chi^2_{\text{Planck+WP+HST}} - \chi^2_{\text{Planck+WP}} = 7.8, 1.0 \text{ and } 0.3 \] for the \( \Lambda \)CDM, \( w \)CDM and HDE models, respectively.

- There is no evident tension when we combine \( Planck+WP \) with BAO, \( HST \) or Union2.1: values of \[ \Delta \chi^2 \equiv \chi^2_{\text{Planck+WP+Ext}} - \chi^2_{\text{Planck+WP}} - \chi^2_{\text{Ext}} \text{ for them are } 1.7, 0.3 \text{ and } 1.6, \] respectively. The SNLS3
data set is in weak tension with the other data sets. When SNLS3 is combined with Planck+WP, Planck+WP+lensing, WMAP-9 and BAO+HST, we obtain large values of $\Delta \chi^2$, equal to 6.4, 7.3, 3.5 and 4.1, respectively.

- The Planck+WP+BAO and Planck+WP+HST results are in good agreement with each other. The best-fit and 68% CL constraints on $H_0$ in the Planck+WP+BAO analysis are $H_0 = 72.63$ and $H_0 = 75.06 \pm 3.82$, close to the HST measurement result, $H_0 = 73.8 \pm 2.4$.

- Although Union2.1 is not in tension with CMB or BAO+HST, the combination Union2.1+BAO+HST is in tension with the combination Planck+WP+lensing. When we combine the two together, we find $\Delta \chi^2 = 8.6$. So it is not viable to do an all-combined analysis for HDE by using the Planck data combined with all the Exts. Our tightest self-consistent constraint is $c = 0.495 \pm 0.039$ obtained from Planck+WP+BAO+HST+lensing.

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