How can one detect the rotation of the Earth “around the Moon”?
Part 1: With a Foucault pendulum

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Abstract  It will be shown that the rotation of the Earth in the Earth-Moon system can be detected by comparing the deflection of a Foucault pendulum at noon on the one hand and at midnight on the other hand. More precisely, on 21 June the midnight experiment would give a deflection about 4% larger than at noon. In other words, with a Foucault pendulum having an accuracy of the order of 1% one should be able to identify this effect through a single measurement. Moreover, if the experiment is repeated on \(N\) successive days, the reduction of the error bar by a factor \(1/\sqrt{N}\) which comes with the averaging process will allow identification of the Moon effect even with a pendulum of poorer accuracy, say of the order of a few percent. In spite of the fact that this effect appears fairly easy to detect, it does not seem that its observation has attracted much attention so far. We hope that this paper will encourage some new observations.

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Introduction

The Foucault pendulum seen as a gyrometer

Improving the accuracy of experiments and measurement devices has been a major means of progress in physics.

A device for detecting movements of rotation is called a gyrometer or gyrosensor or angular accelerometer. The Foucault pendulum is a highly sensitive gyrometer. In this paper we examine how its accuracy can be further improved.

As is well known, the Foucault pendulum was introduced in 1851 by the physicist Léon Foucault. At that time, in spite of a length of 67 meters, the accuracy that was achieved was not better than a few percent.

In his PhD thesis of 1879 the Dutch physicist Heike Kamerlingh-Onnes showed that it was possible to build a short and nevertheless fairly accurate Foucault pendulum. His pendulum had a length of about 1.50m and an accuracy of about 1%. In recent years as many as 18 short Foucault pendulums based on different techniques were designed and built by Marcel Bétrisey. Surprisingly, however, it seems that the physical applications of the Foucault pendulum have been restricted to showing the movement of the Earth around its axis. This was not really something new even in 1851.

For what observation can the Foucault pendulum be used?

The question raised in this paper is whether one can improve the accuracy of the Foucault pendulum and then use it to make meaningful physical observations. The Earth is subject to many movements of rotation, some of which are summarized in Table 1.

Can we use the Foucault pendulum as a “rotation observatory” in order to detect some of these movements?

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1There are many commercial models of gyrometers. However, the sensitivity of most of them is too low for the kind of measurements considered here. A possible exception may be the gyroimeters used for the inertial navigation of nuclear submarines, but (for obvious reasons) it is almost impossible to get detailed information about the performances of such devices.

2The rationale for using a long pendulum is related to the so-called Puiseux effect which can be stated as follows. A spherical pendulum whose trajectory is an ellipse of major axis $2a$ and minor axis $2b$ undergoes a rotation of its major axis with an angular velocity $\Omega_P$ given by: $\Omega_P = \frac{3}{8} \left( \frac{ab}{L^{5/2}} \right) \sqrt{g}$ where $L$ is the length of the pendulum and $g$ the acceleration of gravity. The Puiseux effect has exactly the same appearance as the Foucault effect, namely a rotation of the major axis. Therefore, if $ab/L^{5/2}$ is too large it will completely hide and obliterate the Foucault effect.

3The following website provides additional information. It gives useful descriptions of several of the pendulums and of some of the problems raised by their design and construction. Moreover, broader information about pendulums can be found in a recent book by Leslie Pook (2011)

4In this connection let us recall that when Galileo (allegedly) said “E pur si muove!” (“And yet it moves”) he was speaking of the rotation of the Earth around the Sun for which by the way, he had no scientific justification whatsoever. His belief was chiefly based on the observation of Jupiter and its satellites that he saw as a small scale model of the solar system.
### Table 1 Movements of rotation of the Earth

| Rotation                                      | Period [day] | Angular Velocity [degree/24 h] | Precision required [%] |
|-----------------------------------------------|--------------|-------------------------------|------------------------|
| 1 Rotation of the Earth on its axis           | 1            | 360                           |                        |
| 2 Rotation of the Earth “around the Moon”    | 27           | 13                            | 1                      |
| 3 Rotation of the Earth around the Sun       | 365          | 0.98                          | 0.20                   |
| 4 Precession of the equinoxes                |              | 1.4 $10^{-3}$                 | 4 $10^{-4}$            |
| 5 Rotation of the solar system around the center of the galaxy | 73 $10^9$ | 1.3 $10^{-11}$ | 4 $10^{-12}$ |

Notes: The angular velocity column gives the angular deviation in 24 hours of the plane of a Foucault pendulum located at the North pole as observed by a terrestrial observer. The last column gives the precision with which one must measure the deviation in order to be able to detect the rotation mentioned in the same line of the table. The accuracy of standard Foucault pendulum experiments is comprised between 0.5% and 1% and is therefore too low for the effects number 3, 4 and 5 to be observable. The so-called precession of the equinoxes is a slow change in the direction of the axis of rotation of the Earth, an effect which is similar to the phenomenon by which the axis of a spinning top “wobbles” when a torque is applied to it.

In addition of the movements listed in the table our galaxy is also moving toward the Andromeda galaxy with a relative velocity equal to 220 km/s (which is about 7 times more than the speed of the Earth on its orbit around the Sun). However, at the time of writing, it is not clear whether this movement is also a rotation around some (still unknown) center. *Source: Roehner (2007, chapter 5)*

After the rotation of the Earth around its axis the first possible candidate is the movement with respect to the Moon. We use to say that the Moon rotates around the Earth and that the Earth revolves around the Sun, but in fact in both cases the rotation is around the center of gravity of the two-body systems. In the case of the Earth-Moon system the center of gravity is in fact inside the Earth as shown in Fig. 1.

### Two interpretations of the Foucault effect

There are two ways to consider the movement of a Foucault pendulum depending on which frame of reference one uses.

- In the first approach one uses a geocentric frame of reference $GC$ whose axis are stationary with respect to distant stars. At the North Pole a Foucault pendulum is expected\(^5\) to experience an angular deflection of 360 degrees in 24 hours\(^6\).

In the $GC$ frame of reference this observation can be interpreted by saying that the plane of oscillation of the pendulum is motionless with respect to the “stars” and that the Earth rotates below it.

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\(^5\)In fact, we do not know if the Foucault pendulum experiment has ever been made right at the North Pole, but this conclusion can be extrapolated from numerous observations made at various latitudes.

\(^6\)In many textbooks one reads that this deflection corresponds to a *sideral day* of 23.944 hours, rather than to the normal 24-hour day. However the difference between the two is so small (namely 0.27%) that the distinction is beyond observation.
• The second approach is to work in a frame of reference located on the Earth and rotating with it. Therefore, in order to be able to use Newton’s law one must add to the external forces the so-called Coriolis force $\vec{F}_c = 2m \vec{v} \times \vec{\Omega}$, where $\vec{\Omega}$ is the vector of angular velocity which is parallel to the axis of rotation of the Earth, and $\vec{v}$ is the velocity of the pendulum’s mass. At the North pole (Fig. 2a) $\vec{\Omega}$ is vertical and upward; therefore any horizontal movement will be deflected toward the right. As a result the trajectory of the pendulum will be a kind of marguerite and the plane of oscillation will turn clock-wise. This conclusion is in agreement with the first perspective because the rotation is counter-clockwise when watched from the pendulum.

At another location the vector $\vec{\Omega}$ will have both a vertical and a horizontal component (Fig. 2b). As the movement of the pendulum is almost horizontal the horizontal component will generate a vertical force which will be absorbed into the tension of the wire and which plays no role therefore. It is the vertical component $\omega_v = \Omega \sin \lambda$, where $\lambda$ is the latitude of the location, which will deflect the pendulum. As $\omega_v$ is smaller than $\Omega$ the period of the Foucault pendulum will be longer than 24 hours and given by: $T = \frac{2\pi}{\omega_v} = \frac{24}{\sin \lambda}$. On the Equator $\omega_v$ will be zero (Fig. 2c) and there will therefore be no Foucault effect if one restricts oneself to the rotation of the
Detection of the movement of the Earth around the Moon

In this section, we also take into account the two other rotations of the Earth (numbered as 2 and 3 in Table 1). Basically, we need to find the vertical projection, $\omega_v$, of the total angular velocity:

$$\vec{\omega} = \vec{\Omega} + \vec{\omega}_1 + \vec{\omega}_2$$

where $\vec{\omega}_1$ and $\vec{\omega}_2$ are the rotation vectors with respect to the Moon and Sun respectively.

Let us again begin by the simple case of the North Pole (Fig. 2a).

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**Fig. 2a: Vectors of angular velocity at the North Pole on 21 June (at noon).** For the clarity of the diagram these vectors are represented with same length. In fact, the green vector (Moon) should be 27 times shorter than the blue vector (Earth around its axis) and the red vector (Sun) should be 365 times shorter. Similarly, for the sake of clarity the figure corresponds to the situation on 21 June. On this day the plane containing the center of the Sun and the axis of rotation of the Earth is perpendicular to the orbit of the Earth. For 21 June the diagram is easier to draw but the Foucault effect is the same on other days. For instance, 6 months later, that is to say on 21 December, the only difference is that the North Pole is in the night during the whole day.

As the movement of the Foucault pendulum is almost horizontal, only the vertical components of the vectors play a role. Without the green and red rotation-vectors, the period of the Foucault pendulum would be one day. These additional rotations will accelerate the Foucault deflection and make its period shorter. However, at the North Pole (in contrast to other latitudes) there will be no change in the Foucault effect during the course of one day. As will be seen in Fig. 2c, the most drastic change of the Foucault effect in the course of one day occurs on the Equator.

- In this situation the angle between $\vec{\omega}_2$ and $\vec{\Omega}$ is equal to the angle between the Equator of the Earth and the ecliptic, that is to say $\epsilon = 24.5$ degrees. We denote by
the angle between the orbit of the Moon and the ecliptic: \( m = 5 \) degrees. Then the angle between the vertical and \( \vec{\omega}_1 \) is \( \epsilon + m \). In this case there is no change of the total vertical projection of \( \vec{\omega} \) in the course of one day.

- At another location of latitude \( \lambda \) the vertical projection of \( \vec{\omega}_1 + \vec{\omega}_2 \) will change in the course of one day (Fig. 2b). Indeed, whereas the vertical projection of \( \vec{\Omega} \)

\[
\omega_v^{(12)} = \Omega \sin \lambda + \left( \frac{\Omega}{27} \right) \sin(\lambda - m - \epsilon) + \left( \frac{\Omega}{365} \right) \sin(\lambda - \epsilon) \quad (1a)
\]

\[
\omega_v^{(24)} = \Omega \sin \lambda + \left( \frac{\Omega}{27} \right) \sin(\lambda + m + \epsilon) + \left( \frac{\Omega}{365} \right) \sin(\lambda + \epsilon) \quad (1b)
\]

For a different latitude these expressions are slightly modified depending on the specific position of the vertical with respect to the vectors \( \vec{\omega}_1 \) and \( \vec{\omega}_2 \).
For $\lambda = 40$ degrees, these formulas lead to:

$$\frac{\omega_v^{(24)} - \omega_v^{(12)}}{\omega_v^{(12)}} = 4.4\%$$

In other words, there will be a difference of 4.4% between measurements carried out at midnight and noon respectively.

- The difference between noon and midnight is much more drastic at the Equator (Fig. 2c) because in this case $\vec{\Omega}$ does not contribute to the vertical projection. The

![Angular velocity vectors for the Earth rotations in 1, 27, 365 days](image)

**Fig. 2c: Comparison of the Coriolis force at noon and midnight on the Equator (on 21 June).** This case is interesting because the behavior of the Foucault pendulum is drastically different at noon and midnight. This comes from the fact that on the Equator the effect due to the rotation of the Earth is inexistant (the vertical component of the blue vector is zero). Therefore, the Foucault effect is determined solely by the projections of the green and red vectors. The fact that these projections are of opposite sign (upward at midnight and downward at noon) means that the corresponding deflections of the Foucault pendulum are in opposite directions.

On 21 December the diagram is the same with the only difference that noon and midnight are interchanged.

The figure shows that the vertical projections of $\vec{\omega}_1$ and $\vec{\omega}_2$ is upward at midnight and downward at noon which means that the Foucault pendulum will be deflected clock-wise at midnight and counter-clockwise at noon. Of course, the angular velocity is much slower. A whole rotation will take $27/\sin(23.5 + 5) = 56$ days.

In the next section we examine how the previous predictions can be tested experimentally.

**How to set up experimental tests**

It seems that in recent decades most of the Foucault pendulums which have been constructed were more destined to decoration purposes than to making physical
measurements. Because they must be able to move permanently these pendulums included an electromagnetic device for maintaining the oscillations. Of course, this device introduces external forces which may slightly change the behavior of the pendulum. This leads us to suggest that for physical measurements such devices should be disconnected. As the tests described below require only fairly short measurements over time intervals of a few minutes, such devices will not be necessary anyway.

**Measurement of the angular deflection**

At the latitude of 40 degrees, the angular deflection of an ideal Foucault pendulum will be 0.16 degree per minute. In order to make such an angle measurable with reasonable accuracy, it must be amplified. Several methods can be proposed. A simple method is to take pictures which will then be magnified on a computer screen. One can also design an optical method. Such a method should work even when the mass of the pendulum rotates around the direction of the wire because this is what will indeed occur except for pendulums which have a rigid suspension. Moreover, the measurement should not be affected by the damping of the oscillations. The following method satisfies these two conditions.

1. One uses a cylindrical mass in order to be able to fix a circular mirror onto the upper surface of the cylinder.
2. In the position where the pendulum is at the end of its trajectory a well-focused laser beam is directed toward the mirror. This beam comes from above with an angle of about 45 degree.
3. After the beam has been reflected upward a reflexion on a second mirror makes the direction of the beam almost horizontal so that it can be sent toward a screen located at a distance $D$ of the order of 10 to 20 meters.
4. What is the amplification power of such a detection system? By conducting static trials we found that on the screen the deviation of the laser spot is about 5mm per degree of rotation of the plane of oscillation of the pendulum and per meter of distance between the second mirror and the screen.

As an illustration, we consider an experiment which lasts 2mn and produces a deflection of $2 \times 0.16 = 0.32$ degree. With $D = 10m$ the horizontal deviation of the spot on the screen will be about $5 \times 0.32 \times 10 = 15mm$.

When, due to damping, the amplitude of the pendulum is reduced, the angle of the circular mirror with respect to the horizontal will decrease. As a result the spot on the screen will become lower but its horizontal position should not be affected.

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**It may be argued that a “passive” (that is to say without any source of current) Charron ring should not change the behavior of the pendulum. Indeed, its purpose is to prevent the trajectory from becoming more elliptic, thus keeping the behavior of the pendulum closer to the theoretical model. This can be beneficial but on the other hand may not be necessary for within a few minutes the “ovalisation” will remain limited. So, one should probably try the two options, with and without Charron ring, in order to see what difference it makes.**
Improving the accuracy through averaging

In physics averaging is a standard procedure for reducing the error-bar of measurements. This procedure is based on the fact that the standard deviation of an average of \( N \) uncorrelated measurements \( X_k \) each of which has a standard deviation \( \sigma \) is \( \sigma / \sqrt{N} \).

\[
X_m = \frac{X_1 + X_2 + \ldots + X_N}{N}, \quad \sigma(X_m) = \frac{\sigma}{\sqrt{N}}
\]

Usually, one must distinguish between systematic and random errors. Random errors may for instance be due to vibrations or changes in friction forces. Clearly, the reduction by \( \sqrt{N} \) applies only to random errors.

However, for the comparison of noon and midnight deflections one does not have to care about systematic errors provided they are identical at noon and at midnight. On the contrary, if the measurement is modified by a change in temperature and if (as is likely) the midnight temperature is systematically lower than the noon temperature, then the comparison will be affected. The same can be said for vibrations for they are also likely to be lower at midnight than at noon.

Another question which must be addressed is the following.

Is it better to make 10 separate measurements over 2-minute time intervals or rather one measurement over a 20-minute time interval?

To get a better insight into this question let us consider a simpler example.

When one measures the period of a pendulum with a chronometer it is more accurate to count 20 oscillations rather than to make 10 measurements of only 2 oscillations. Why?

In this case the main source of error is the uncertainty \( \Delta \) when starting and stopping the chronometer. While the measurement of 20 oscillations comprise an uncertainty of \( 2\Delta \), the 10 measurements of two oscillations comprise an uncertainty of \( 20\Delta \); due to the averaging process, this uncertainty will be reduced to \( (20/\sqrt{10})\Delta \), but this is still larger (by a factor \( \sqrt{10} \)) than the \( 2\Delta \) uncertainty.

In other words, the answer to the previous question depends on the level of uncertainty in launching the pendulum (at the end of the 2-minute run there is no special uncertainty). Clearly the answer will also depend upon the magnitude and effects of the “ovalisation” of the trajectory.

Prospects

It is difficult to know in advance which level of accuracy can be achieved. The mea-

\[^9\text{At the time of writing (December 2011) this method is used at the CERN Laboratory in Geneva for deciding whether the Higgs particle exists or not. The method simply requires that one accumulates a sufficiently large number of events so that the error bar can be reduced accordingly. To record such a large number of events the LHC accelerator must run for months.}\]
measurements can be repeated 25 times, 100 times or (once the procedure has been automatized) 1,000 times. As there is in principle no limit to the number of repetitions, thanks to $1/\sqrt{N}$ factor of the averaging process, the accuracy of the experiment can become fairly high. With a Foucault pendulum which has an intrinsic accuracy of 1%, $N = 1,000$ would lead to an accuracy of $1/\sqrt{1000} = 0.03\%$. Such an accuracy would be sufficient to observe the effect of $\omega_2^2$, that is to say the rotation of the Earth around the Sun.

However, the best chance to identify the effect of the rotation of the Earth around the Sun is by a dual measurement done at noon and midnight on 21 June and 21 December respectively. The noon and midnight results should be interverted (see above Fig. 2c). For this measurement one needs only the 1% accuracy that is required for observing the rotation around the Moon.

**Conclusion**

In the second of this series of two papers we will introduce a different kind of gyrometer. However, before introducing a new method, it is natural to try those who already exist. As the Foucault pendulum has been studied (and possibly improved) for over one and a half century, it is an obvious candidate. We have described how its accuracy can be improved, firstly through amplification of the angular deflection, secondly through an appropriate number of repetitions. We have also shown what kind of observations must be done in order to identify specific movements of rotation of the Earth.

It appears that the accuracy required for detecting the movement in the Earth-Moon system can be achieved fairly easily. By automatising the measurement process one would be able to carry out a great number of repetitions and in this way one may get an accuracy sufficient for detecting the rotation of the Earth around the Sun. That was the dream of Galileo some four centuries ago and the realization of this dream would be an appropriate tribute to the founding father of modern physics.

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