On Heat Flow and Non-Reciprocity

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Abstract.

Heat transfer properties in non-reciprocal systems are discussed. An ideal experiment employing microwave or optical isolators is considered in order to investigate the possibility for a spontaneous transfer of energy between two black-bodies at common temperatures. Under the validity of the adopted assumptions on the examined physical system, the effect appears to be weak but feasible.
I. INTRODUCTION.

The second law of thermodynamics is one of the basic principles of this discipline \([1]\). Its foundation from the point of view of the more basic branch : Statistical Physics is well established and its validity is recognized for an enormous range of physical systems and phenomena \([2]\). However, it is also admitted that the definition of Entropy for systems staying far away from thermal equilibrium is not obvious \([3]\). Therefore, the theoretical proof of the fact that Entropy is always growing in non-equilibrium processes (occurring in a completely arbitrary isolated physical system) is also not fully established. This situation confers an status of a basic but heuristic principle to the Second Law. In addition, it is also certain that the standard derivations of the Law from Statistical Physics assume conditions which are obeyed by an enormous class of physical systems but which, in spite of it, are by far non general and excludes less common but very important phenomena in Nature. One particular requirement, which eliminates a large class of situations, is the condition that the spectrum of the Gibbs subsystems are statistically independent among them \([2]\). This property is not satisfied when long range forces are present. Such kind of limitations have given ground to the work of C. Tsallis \([4]\) on introducing alternative expressions for the Entropy for long range interacting systems. This approach recently have received an experimental support \([5]\). However, although new expressions for the entropy are under consideration the general expectation is the survival of the Law of increasing Entropy for isolated systems within a more general picture.

In this work we intend to attract the attention about another aspect of the derivation of the 2\(^{nd}\) Law from Statistical Physics. It is related with the physics of the heat transfer among bodies and the fact that such energy flows are expected to always spontaneously occur from higher to lower temperature regions. This principle is obtained form the Law of increasing Entropy and also implies it. However, its theoretical derivation from statistical physics has received very much limited attention. It can be understood that for wide, but even though, particular class of physical systems having reciprocal dynamics, the heat will
always travel from higher to lower temperature regions. When, however, the system has a non-reciprocal dynamical response this same property is less intuitively evident.

Let us examine for example the following situation. Consider the statistical mean value of the Poynting vector operator, reflecting the electromagnetic energy flux in a many body system coupled with the electromagnetic field. It will be assumed also that the system is subject to an homogeneous and constant magnetic field which therefore destroys the exact parity invariances of the photon propagator and then its reciprocity property. The general expression for the mean value of the Poynting vector operator can be shown to vanish under the assumed translation invariance of the system. However, it can be underlined that in the non reciprocal situation the vanishing of the net energy flux follows from a dynamical equilibrium in which the energy flux of the modes propagating in exact opposite directions exactly cancels, but their polarization vectors does not show identical spatial distribution propagating in opposite senses. This property suggests the possibility that a coupling of such systems with bodies showing reciprocal dynamics could lead to an unbalanced flux of the Poynting vector.

This phenomenon effectively happens in devices such as optical and microwave isolators [6], [7], [8]. However, up to our knowledge, the analysis of the implications of such properties for the energy transfer at thermal equilibrium is not a fully investigated and understood matter.

This work intends to discuss this question. For that purpose an analysis of two types of isolators connecting two regions containing blackbody radiation at common temperatures is done. Within the adopted assumptions, the results seem to indicate the possibility for the occurrence of a stationary energy flow between the two blackbody regions. Although the magnitude of the fluxes evaluated are far from being of practical relevance, the correctness of the analysis could support the search of similar effects in which larger energy flows could occur.

In the next Section 2, concrete versions of the isolator devices to be discussed are presented and the power input at entries are evaluated. In Section 3, the energy flow balance
is investigated by comparing the situations in which the non reciprocity is switched on and off.

II. OPTICAL AND MICROWAVE ISOLATORS

Let us consider a system composed by two big reservoirs $S_-$ and $S_+$ which are thermally isolated between them by a wall $W$ having a low value of the thermal conductivity $\sigma$ due to collisions. The wall is also assumed to be highly absorptive for electromagnetic radiation in a wide frequency region containing most of the black-body spectrum at the temperature value $T$ to be considered. The system is illustrated in Figs.1.

The wall $W$ is considered as permeated by a square lattice of special non reciprocal devices called isolators [9], separated one from another by a lattice period $d$. Let us consider in what follows two types of such devices. The so called resonance isolators which are based in the effect of non reciprocal resonance absorption in ferrites; and the other ones which operation is determined by the Faraday rotation in the same kind of materials. For resonance isolators the width $h$ of $W$ is assumed to coincide with the length $2l + l_1$ of the isolators, where $l_1$ is the length of a thin ferrite rod furnishing the non reciprocal propagation properties of the device. The symbol $l$ indicates the length of two unloaded rectangular waveguides serving as the entries for the portion loaded with the ferrite rod. The structure is shown in Fig. 2.

In the case of the Faraday effect isolator, the width of $W$ is also $h = 2l + l_1$ where again $l$ is the length of the rectangular waveguide entries, but $l_1$ is now the length of a cylindrical waveguide at which axis a ferrite rod producing the Faraday rotation of the waves is situated. In addition, the output rectangular waveguides have their corresponding sides rotated in $\frac{\pi}{4}$ radians between them. Also, at one of the entries there is a thin resistive vane with its plane being parallel to the wide side of the rectangular waveguide. The structure is illustrated in Fig. 3. More details on the operation of such devices can be found in [6], [7], [8].

It can be remarked that such isolators have been extensively studied and applied in
microwave technology. The resonance devices have been investigated theoretically by B. Lax [9]. In general they can be developed in a way showing, by example, 60 db of attenuation for propagation in one sense and allowing the power in the other sense to have as low as 0.5 db of losses.

The general aim of the work is to investigate the flux of heat flowing through each isolator in the wall \( W \). The region at the left of \( W \) will be designated by \( S_- \) and the one at the right by \( S_+ \). The black-body radiation filling them will be assumed to have a common value of the temperature \( T \). Calling \( a \) and \( b \) the longer and shorter sides of the rectangular waveguide entries, the dispersion relations for the EM modes of propagation in these input guides are given by

\[
\frac{\omega}{c} = \frac{\epsilon}{\hbar c} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + k^2} = \sqrt{\left(\frac{\epsilon_{mn}^{cc}}{\hbar c}\right)^2 + k^2},
\]

(1)

\[
\frac{\omega_{mn}^{cc}}{c} = \frac{\epsilon_{mn}^{cc}}{\hbar c} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2},
\]

(2)

where \( m \) and \( n \) are any positive integers, \( k = \frac{2\pi}{\lambda} \) is the wavevector associated to the wavelength \( \lambda \) in the guide and the \( \omega_{mn}^{cc} \) and \( \epsilon_{mn}^{cc} \) are the cutoff frequencies or energies for the \( (m, n) \) propagation modes. Relation (1) defines the photon energy \( \epsilon \) within the waveguide. The lowest cutoff frequency is given by

\[
\frac{\omega_{10}^{cc}}{c} = \frac{\pi}{a}.
\]

(3)

Assuming \( a = 2b \) the next cutoff frequency turns to be the double of the lowest one:

\[
\frac{\omega_{01}^{cc}}{c} = \frac{2\pi}{a}.
\]

(4)

Then, the density of states per unit energy \( \rho(\epsilon) \) of the lowest mode vanishes for values of the energy lower than \( \epsilon_{c} \) and above this value it is given by the expression

\[
dN(\epsilon) = \frac{\Delta k}{2\pi L} = \frac{dk}{\epsilon} \frac{d\epsilon}{2\pi L} = \frac{L}{2\pi \hbar c} \frac{\epsilon}{\sqrt{\epsilon^2 - (\epsilon_{c}^{10})^2}} \frac{d\epsilon}{\epsilon} = \rho(\epsilon) \ d\epsilon.
\]

(5)
Therefore, after multiplying by the Bose distribution function and the photon energy $\epsilon$, the amount of thermal energy per unit photon energy propagating in one of the two allowed senses is given by

$$\frac{du(\epsilon)}{d\epsilon} = \rho(\epsilon) \epsilon f(\epsilon),$$  \hspace{1cm} (6)

$$f(\epsilon) = \frac{1}{\exp\left(\frac{\epsilon}{kT}\right) - 1},$$

where $k$ and $T$ are the Boltzmann constant and the temperature respectively. The power per unit photon energy transmitted in any of the senses in the entry guides is then given by

$$dP(\epsilon) = \frac{1}{T_{trans}} \frac{du(\epsilon)}{d\epsilon} d\epsilon,$$  \hspace{1cm} (7)

where the transit time $T_{trans}$ through a length $L$ of the rectangular waveguide for a photon of energy $\epsilon$, can be obtained though the following argument. Let us consider the fact that the propagation of the first mode can be interpreted as the total reflection of two plane waves which are reflected by two parallel metallic planes obtained by extending the shorter sides of the waveguides. These waves have a momentum component along the waveguide given by $k$ and a component along the large side given by $\pm \frac{\pi}{a}$. Hence, the propagation along the waveguide axis has the velocity

$$v_{\parallel} = \frac{c k}{\sqrt{(\pi a)^2 + k^2}}$$

$$= c \frac{\sqrt{\epsilon^2 - (\epsilon_{c_{10}})^2}}{\epsilon}.$$  \hspace{1cm} (8)

Thus, the time of transit through a length $L$ of the guide and the density of power per unit energy in the entry guides are given by

$$T_{trans} = \frac{L}{v_{\parallel}} = \frac{L}{c} \frac{\epsilon}{\sqrt{\epsilon^2 - (\epsilon_{c_{10}})^2}},$$  \hspace{1cm} (9)

$$dP(\epsilon) = \frac{1}{T_{trans}} \frac{du(\epsilon)}{d\epsilon} d\epsilon = \frac{1}{2\pi h} \frac{\epsilon}{\exp\left(\frac{\epsilon}{kT}\right) - 1} = \frac{dP(\epsilon)}{d\epsilon} d\epsilon.$$  \hspace{1cm} (10)
Let us discuss now the connections between the dynamical and thermodynamical properties in the system under consideration. We would like to stress that the above considered power flow at the entry guides should be established dynamically by the incidence of the black body radiation at both sides of the wall $W$. Let us assume that the transmission coefficient of the entry guides are near to unity at the considered frequency region. The same will be assumed to happens for the rectangular or cylindrical waveguides when the ferrite material is absent. This condition is affordable in transmission lines from microwave to optical regions at laboratory values of the waveguide sizes $[11]$. In other word, it will be assumed that the possible skin or radiation losses at the waveguides is low in order that the decay length of the mode power is much greater than the isolator dimensions.

Moreover, in both cases of the resonance and Faraday rotation isolators, it will be assumed that the ferrite kernel is saturated. This will allow to employ the known expressions for the magnetic susceptibilities in such cases in order to estimate the energy flow in that structures.

III. RECIPROCITY AND ENERGY FLOW BALANCE

Let us analyze now the radiation which is transmitted by the isolators between the regions $S_-$ and $S_+$. For this purpose, consider that a control device is used to switch on the ferrite in its active positions for the resonance or Faraday rotation type isolators. Before the switching on, the system is completely reciprocal. Since we are assuming that the emittance and absorptivity of the walls of the waveguides are small (transmission coefficients near to unity), the power at their interior should be generated mainly by the black body radiation incident at both sides of the wall in each isolator (whenever the inputs are not blocked). If the inputs are poorly matched or closed, then the small but non vanishing emittance of the walls should be the main dynamical source of the thermal power spectrum at the inside. However, when the inputs are nearly matched , the energy of that sources should be insufficient to elevate the power spectrum to the level given by Eq.(10). Therefore, within
the adopted conditions, it seems possible to assume that the incident power at the entry waveguides is propagating as coming from the input and not stochastically generated at the interior. This is a main assumption in the present analysis. In the absence of the ferrite, the vanishing of the net power transmitted from $S_-$ to $S_+$ can be easy understood. For example, in the resonance isolator what connects both regions is a piece of rectangular waveguide of total length $h$. Let us consider two imaginary surfaces covering the inputs and being completely symmetrical under a reflection in a plane parallel to $W$ and equidistant from both sides. Also assuming the boundary conditions of the electromagnetic radiation as represented by sources (through the use of the Green Theorem) the reciprocity property assures that, since the statistical distribution of the sources representing the blackbody radiation are identical in both surfaces, the power propagating in both senses should be identical. It should be noticed that this argument also implies that when the temperature of the blackbody radiation is higher at one of the sides, let say $S_-$, the energy (heat) should flow from the high to lower temperature regions because the equivalent sources are stronger at the higher temperature side.

In the case of the Faraday isolator the vanishing of the net transmitted power can also be understood dynamically. This is because the polarization of the waves coming from $S_-$ to $S_+$ have a $\frac{\pi}{4}$ rotation with respect to the small side of the of the external waveguide and then only a fraction of the power will be transmitted after multiple reflections. Equivalently, the wave coming from $S_+$ to $S_-$ will also arrive with a $\frac{\pi}{4}$ angle of polarization with the short side of the entering waveguide at $S_-$. Then, as the solutions of the electromagnetic propagation are symmetrical under reflection in the above mentioned symmetry plane, it follows that the fraction of the power transmitted in this case should be identical as in the former situation. This remarks assure that the net energy flux from one side to another exactly vanishes.

In the presence of the magnetic effects, the exact reflection invariance is absent and the above arguments can’t be applied.

Let us consider below the switching on of the ferrite in either of the both types of isolators.
A. Faraday Isolator.

After putting the ferrite in its working position, the amount of incident power given by Eq. (10) and flowing from the $S_-$ should propagate with relative small losses (as assumed before) up to the other output whenever the Faraday rotation angle is fixed to be near $\frac{\pi}{4}$ in the appropriate sense. This situation can be attained by commuting the sense of the saturation magnetization direction. Then, a large fraction of the power given by expression (10) can be transmitted to the other side of the isolator if the coupling between the cylindrical and rectangular waveguides is near to be matched. In another hand, the radiation propagating from the entry waveguide at $S_+$ in direction of $S_-$ will suffer almost a total reflection at the junction between the cylindrical to rectangular waveguide near $S_-$, since the non reciprocal dielectric action of the ferrite will rotate an additional $\frac{\pi}{4}$ the polarization of the electric field. This fact makes incompatible the symmetry of the wave with the excitation of the lowest mode in the output rectangular waveguide. After the total reflection and the also propagation back the energy of this wave is dissipated at the resistive vane $R$ at the $S_+$ side in Fig. 3.

The dissipation at the resistive plate should be discussed in more detail. The power absorbed by the vane can be partially radiated back and partially thermally conducted to the $S_+$ side by a good thermal matching of the ferrite with the waveguide walls. Let us consider a design of the resistive card in which the excess power dissipated in it can be efficiently transferred to the waveguide walls and from them to the $S_+$. This procedure should allow to reduce the amount of the dissipated power returned back as non polarized radiation to the waveguide. However, even in the case that the full power absorbed is radiated back in the waveguide (a very improbable outcome) only a fraction of it will have the appropriate polarization to emerge at the $S_-$ side. Thus, even in this extreme case, the balance of the energy fluxes is not evidently occurring.

Let us consider now the determination of the amount of power which could be transmitted if the argued mechanism is effectively at work. This quantity can be evaluated in the
following way. Assume that the Faraday angle is adjusted to be $\frac{\pi}{4}$ for a particular value of the frequency, for example $\sqrt{2}w_{\epsilon 10}$ lying below the cutoff of the second propagation mode. Then, also suppose the working frequencies $w$ are so high that the magnetic Faraday susceptibility show its far from resonance asymptotic dependence [7]:

$$\chi_F \sim \frac{w_M}{w},$$

(11)

where $w_M$ is proportional to the ferrite magnetization. As the Faraday rotation is proportional to $\chi_F$, the rotation angle as a function of the energy $\epsilon$ (or frequency $w$) can be evaluated.

Therefore, the fraction of the power within some energy interval $d\epsilon$, which will transmitted from $S_-$ to $S_+$ will be given by the squared cosine of the difference between the Faraday rotation angle and $\frac{\pi}{4}$. The rest of the power is dissipated in the resistive vane $R$ at the entering of the cylindrical guide near $S_+$. In another hand, the wave incident in the cylindrical guide trough $S_+$ is rotated $\frac{\pi}{4}$ plus the Faraday angle with respect to the small axis of the squared waveguide at $S_-$. Then, the power transmitted to the matched rectangular waveguide entering at $S_-$ is the fraction given by the squared cosine of that angle. The rest of this power is reflected back and the fraction able to pass into the rectangular waveguide escapes, and the other portion is dissipated at the resistive load. Finally, as the incoming powers at the rectangular waveguides are equal, the net energy flux (the integral over all the energies) can be estimated to be given by:

$$P = \frac{\epsilon e^{\frac{10}{kT}}}{2\pi h} \int_1^{\infty} dx \frac{1}{\exp\left(\frac{\epsilon_{\epsilon 10}}{kT}\right) - 1} \left(\cos^2\left(\frac{\pi}{4} \left(\sqrt{2} x - 1\right)\right) - \sin^2\left(\frac{\pi}{4} \left(\sqrt{2} x - 1\right)\right)\right).$$

(12)

The integral is only dependent on $\frac{\epsilon_{\epsilon 10}}{kT}$. For the case $\frac{\epsilon_{\epsilon 10}}{kT} = 1$ the expected power has the expression

$$P = 0.7018 \frac{k^2 T^2}{2\pi h},$$

(13)

which at the room temperature $T = 300 K^0$, predicts a net transmitted power of

$$P = 0.18149 \text{ erg/s.}$$
As it could be expected from the radiative nature of the discussed processes, this a small power transfer. But, it also should be underlined, that at room temperature the size of the waveguide satisfying the condition \( \frac{\epsilon}{kT} = 1 \) is as small as \( 24 \mu m \). Then, after assuming that the lattice of isolators has \( d = 50 \mu m \) as the size of the square unit cell, the power per squared meter can rise up to near \( 7 \text{ watt} / m^2 \). Even in this case the considered heat flux becomes of no significance for practical purposes.

However, with respect to the detectability the situation seems to be different. After assuming that the estimated power in a single isolator Eq. (13) is dissipated in a let say standard resistance of \( 75 \Omega \), the voltage produced should be of the order of \( 1 \text{ mV} \). This result indicates the possibility for the detection of the proposed effect under the current experimental conditions.

B. Resonance Isolator.

Next let us consider the switching of the ferrite rod in the resonance isolator. The following conditions will be also assumed:

1) The ferrite will be supposed to have an efficient heat coupling with the metallic sides of the rectangular waveguides, thus assuring that the power dissipated in it is transferred in a large proportion to the cavity walls.

2) At the same time, the cavity walls are assumed to have a low thermal resistance for the conduction towards the \( S_+ \) side and a very much higher one for the conduction into the \( S_- \) region.

I think that these could not be essentially difficult conditions to match. They basically depend on seemingly controllable aspects of design for the devices, at least, whenever an ideal situation with respect to the available values of the heat conductivities is assumed.

After the switching on of the ferrite the wave can excite the ferromagnetic resonance of the material. The resonance frequency will be assumed to have a value between \( w_{c}^{10} \) and \( w_{c}^{01} \). It can be stressed that at points being near the small side of the waveguides a distance \( \frac{a}{4} \),
the microwave magnetic field is circularly polarized. This is a typical position selected for
the ferrite rod.

However, the resonance occurs only for one of the senses of rotation for the polarization
vector. Since the wave going in opposite spatial senses in the first mode have opposite
senses of rotation for their circular polarization, it occurs that one wave can be absorbed
much more than the other. Under the change in sign of the magnetization the behavior of
the two waves are interchanged. Let us assume that the magnetization is polarized in the
sense that produces resonance for the waves going from \(S_+\) to \(S_-\). Then, when the ferrite is
connected, that wave becomes absorbed and the reverse one is less disturbed. Then, if the
absorbed power in the ferrite is transferred efficiently to the waveguide walls and from them
to the \(S_+\) side, the amount of power passing from \(S_-\) to \(S_+\) appears to be higher that the
one flowing in the reverse sense. It should be noticed that for this conclusion to be valid it
is essential to assume that the emittances at thermal equilibrium of the waveguide walls are
very low. Then, it seems improbable that their effect will compensate for the absorption of
the wave coming from \(S_+\).

Let us evaluate below the net transmitted power which could be expected. For this
purpose consider the transmission coefficients for the waves propagating in both senses. As
cited before, the propagation in waveguides loaded with ferrite bars, as depicted in Fig.
2, have been studied theoretically by Lax \[9\], \[10\]. The analysis can be done using the
perturbative approximation for the derivation of the propagation modes starting from the
unperturbed ones \[8\].

The transmission coefficients for a length \(l_1\) loaded with a thin bar of ferrite as in Fig. 2
are given in the above mentioned perturbative approximation, by

\[
T_{-+} = \exp(-2Re[\Gamma_{-+}]\ l_1),
\]

\[
T_{+-} = \exp(-2Re[\Gamma_{+-}]\ l_1),
\]

(14)

where the absorptive parameters \(\Gamma_{-+}\) and \(\Gamma_{+-}\) are defined by the expressions \[8\]
$$\Gamma_{--} = \frac{\Delta S}{S} \frac{1}{k} \left[ \left( k^2 \sin^2 \left( \frac{\pi x}{a} \right) + \left( \frac{k}{a} \right)^2 \cos^2 \left( \frac{\pi x}{a} \right) \right) \chi_{xx} - 2 \chi_{xy} \sin \left( \frac{\pi x}{a} \right) \cos \left( \frac{\pi x}{a} \right) \right], \quad (15)$$

$$\Gamma_{++} = \frac{\Delta S}{S} \frac{1}{k} \left[ \left( k^2 \sin^2 \left( \frac{\pi x}{a} \right) + \left( \frac{k}{a} \right)^2 \cos^2 \left( \frac{\pi x}{a} \right) \right) \chi_{xx} + 2 \chi_{xy} \sin \left( \frac{\pi x}{a} \right) \cos \left( \frac{\pi x}{a} \right) \right], \quad (16)$$

in which it is assumed that the non diagonal component of the magnetic susceptibility satisfies $\chi_{xy} > 0$; $S, \Delta S$ are the areas of the sections of the waveguide and the ferrite bar respectively and $x$ is the distance of the center of bar from the small side of the waveguide.

The diagonal and non diagonal components of the magnetic susceptibility are given by

$$\chi_{xx} = \frac{\omega_M (\omega_0 + j \omega_L)}{(\omega_0 + j \omega_L)^2 - \omega^2}, \quad (17)$$

$$\chi_{xy} = \frac{\omega_M \omega_j}{(\omega_0 + j \omega_L)^2 - \omega^2}.$$  

The various frequency parameters in (17) are defined as

$$\omega_0 = 2\pi \gamma H_0, \quad (18)$$

$$\omega_M = 2\pi \gamma (4\pi M_s) \mu_0,$$

$$\omega_L = \frac{1}{T} = \frac{2\pi \gamma \Delta H}{T} = \frac{2\pi \gamma \Delta H}{2},$$

$4\pi M_s = saturation~magnetization,$

$T = macroscopic~relaxation~time,$

$$\gamma = 2.8 \, MHz/Oe.$$  

Taking the real part of the $\Gamma$ parameters, the transmission coefficients can be written as

$$T_{+-}(\epsilon) = \exp\left( -\frac{2l_1 \Delta S}{S} \frac{\epsilon_m \epsilon_L}{(\epsilon^2 - \epsilon_0^2 + \epsilon_L^2)^2 + 4 \epsilon_L^2 \epsilon_0^2 \sqrt{\epsilon^2 - (\epsilon_c^{10})^2}} \right) \frac{ch}{(\epsilon^2 - (\epsilon_c^{10})^2)^2} \frac{\epsilon_0}{\epsilon_L} \left( \frac{\epsilon^2 - (\epsilon_c^{10})^2}{\epsilon_0^2 \epsilon_L^2} \sin^2 \left( \frac{\pi x}{a} \right) + \left( \frac{\pi}{a} \right)^2 \cos^2 \left( \frac{\pi x}{a} \right) \right) \left( \epsilon_0^2 + \epsilon_L^2 + \epsilon^2 \right) - 4 \sin \left( \frac{\pi x}{a} \right) \cos \left( \frac{\pi x}{a} \right) \frac{\epsilon}{a h c} \sqrt{\epsilon^2 - (\epsilon_c^{10})^2} \epsilon_0 \), \quad (19)$$
\[ T_+ (\epsilon) = \exp \left( \frac{-2l_1 \Delta S}{S} \frac{\epsilon_m \epsilon_L}{(\epsilon^2 - \epsilon_0^2 + \epsilon_L^2)^2 + 4\epsilon_L^2 \epsilon_0^2 \sqrt{\epsilon^2 - (\epsilon_e^{10})^2}} \right) \]

\[
\left( \frac{\epsilon^2 - (\epsilon_e^{10})^2}{c^2 \hbar^2} \sin^2 \left( \frac{\pi x}{a} \right) + \left( \frac{\pi}{a} \right)^2 \cos^2 \left( \frac{\pi x}{a} \right) \right) (\epsilon_0^2 + \epsilon_L^2 + \epsilon^2) + 4 \sin \left( \frac{\pi x}{a} \right) \cos \left( \frac{\pi x}{a} \right) \frac{\pi}{a} \frac{\hbar c}{\sqrt{\epsilon^2 - (\epsilon_e^{10})^2}} (\epsilon_0 \epsilon_L),
\]

where the newly used energy parameters are defined as

\[ \epsilon_0 = \hbar w_0, \]
\[ \epsilon_m = \hbar w_m, \]
\[ \epsilon_L = \hbar w_L. \]

It is possible to write now for the total transmitted power the expression

\[ P = \int_{\epsilon_{cc}}^{\infty} d\epsilon \frac{dP(\epsilon)}{d\epsilon} (T_+ (\epsilon) - T_{++} (\epsilon)) = \int_{\epsilon_{cc}}^{\infty} \frac{1}{2\pi \hbar} \frac{\epsilon}{\epsilon_T} \exp \left( \frac{\epsilon}{\epsilon_T} \right) - 1 (T_+ (\epsilon) - T_{++} (\epsilon)). \]

Let us assume that \( x = \frac{a}{4} \), as the typical value considered for resonance isolators. Then the formula for the power can be rewritten as follows

\[ P = \frac{\epsilon_e^2}{2\pi \hbar} \int_{1}^{\infty} dx \sum_{\sigma = \pm} \frac{\sigma}{\exp \left( \frac{\epsilon \sigma}{\epsilon_T} \right) - 1} \times \]
\[ \left( l_1 \Delta S \frac{\epsilon_m \epsilon_L}{2S \hbar c} \frac{x^2 (x^2 + (\epsilon_0/\epsilon_e)^2 + (\epsilon_L/\epsilon_e)^2) - 4\sigma x \epsilon_0 \sqrt{x^2 - 1}}{\epsilon_e} \right) \]
\[ \exp \left( \frac{l_1 \Delta S \epsilon_m \epsilon_L}{2S \hbar c} \frac{\sqrt{x^2 - 1}}{\left( x^2 - (\epsilon_0/\epsilon_e)^2 + (\epsilon_L/\epsilon_e)^2 \right)^2 + 4\epsilon_L^2 \epsilon_0^2/\epsilon_e^4} \right) \]

where \( \epsilon_e = \epsilon_e^{10} \) is the cutoff energy of the first mode.

Let us select specific values for the parameters in order to get sense of the order of magnitude for the expected powers which could be attained. Suppose also that the ferromagnetic resonance frequency is given by \( w_0 = \sqrt{2} \frac{\epsilon_e}{h} \), which means that the resonance occurs at \( \sqrt{2} \) times the cutoff frequency of the first mode and also that the resonance width takes the value \( w_L = 0.2 \frac{\epsilon_e}{h} \). Moreover, let us fix the other parameters through adjusting the following relation to be valid

\[ \frac{l_1 \Delta S \epsilon_m \epsilon_L}{2S \hbar c} = 0.1. \]
This condition assures a relatively low absorption for the power of the waves travelling from $S_-$ to $S_+$. Finally, let fix again the condition that the dimension of the waveguide is such that the thermal energy satisfies

$$kT = \epsilon_c.$$  \hspace{1cm} (25)

After that, the numerical integration of the relation (23) gives the result

$$P = 0.36320 \frac{(kT)^2}{2\pi\hbar},$$  \hspace{1cm} (26)

$$= 0.36320 \frac{\epsilon^2_c}{2\pi\hbar} = 0.36320 \frac{\pi c/a)^2}{2\pi} \frac{\hbar}{a^2} = \frac{1.47 \times 10^{-7}}{a^2} \text{ erg/s},$$

in which the values for $\hbar = 1.054572 \times 10^{-27} \text{ erg.s}$ and $c = 3 \times 10^{10} \text{ cm/s}$ have been substituted. It can be observed that for waveguides of sizes of the order of a centimeter the output power is very low. At the ambient temperature $T = 300 \text{ K}$, which is equivalent to a waveguide dimension $a$ of the order of 24 $\mu$m, the power transmitted by one isolator should be

$$P = 0.02552 \text{ erg/s},$$

which is of the same order of magnitude of the one estimated for the Faraday rotation situation. Henceforth, it became clear, that in both of the considered cases, the implied powers are low. However, the real existence of such an effect could open the possibility for the search of alternative kinetic processes which would show improved power fluxes. However, in the sense of the possibility of detecting the effect, the results give a positive answer, at least in the cases of infrared or optical isolators. For them, the signals to be detected stay within the voltage ranges of the standard voltage or electric current measurement instruments.

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Figure Captions

Fig 1. Scheme of the system under consideration. The $S_-$ and $S_+$ are regions each of them containing black-body radiation at a common temperature $T$.

Fig 2. Picture of the type of resonance isolator considered in this work.

Fig 3. Picture of the particular type of Faraday rotation isolator discussed in the text.
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