Slow-roll inflation in the \((R + R^4)\) gravity*

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Abstract

We reconsider a toy model of topological inflation, based on the \(R^4\)-modified gravity. By using its equivalence to a certain scalar–tensor gravity model in four spacetime dimensions, we compute the inflaton scalar potential and investigate the possibility of inflation. We confirm the existence of slow-roll inflation with an exit. However, the model suffers from the \(\eta\)-problem that gives rise to the unacceptable value of the spectral index \(n_s\) of scalar perturbations.

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1. Introduction

The identity of the inflaton field remains to be one of the main unsolved problems in the theoretical explanation of inflation [1]. The inflaton field should not be an \textit{ad hoc} degree of freedom, but the field whose presence is natural from the viewpoint of gravity and/or high-energy physics. During the short inflationary period in the early Universe the energy density was dominated by vacuum energy. Inflaton coupling to (non-gravitational) matter is needed to guarantee the conversion of the vacuum energy into radiation at the end of inflation, whereas weakness of that coupling is needed to ensure stability of the inflaton scalar potential against quantum corrections. Quantum fluctuations of the inflaton field are responsible for the generation of the structure in our Universe, while the fluctuation spectrum is nearly conformal [2]. The primordial spectrum of scalar perturbations in the power-law approximation takes the form \(k^{n_s-1}\) in terms of the comoving wave number \(k\) and the spectral index \(n_s\) close to 1.

One of the natural possibilities is the identification of an inflaton with the self-interacting conformal mode of a metric. It can be realized in the modified gravity theories whose effective Lagrangian is given by a function of the scalar curvature. The first models of this type were proposed by Starobinsky in 1980 [3]. Already the simplest Starobinsky model, based on the

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(R + R^2)-modified gravity, is the excellent example of chaotic inflation, whose predictions for the spectral indices [4] agree well with the recent cosmological observations [5].

The modified gravity models of inflation are minimal in the sense that they do not rely on the Riemann and Ricci curvature dependence of the effective action. The Weyl tensor of a FLRW universe vanishes, so that the FLRW Riemann curvature can be expressed in terms of the FLRW Ricci tensor and the scalar curvature. A dependence upon the Ricci tensor would give rise to a propagating spin-2 degree of freedom, in addition to the metric [7]. Hence, the modified gravity models, depending only upon the scalar curvature, should play the most important role in cosmological dynamics.

The modified gravity theories are popular in modern theoretical cosmology—see e.g. [8] for recent reviews—however, they are phenomenological, and are not derived from any fundamental theory of gravity. The most promising candidate for quantum theory of gravity is given by superstring theory [9]. It is still unknown how to embed a modified gravity model into the superstring theory. However, there exist some model-independent restrictions coming from closed superstrings (it is the closed or type-II superstrings that are directly related to gravity). One of them is the absence of the quadratic and cubic curvature terms in the low-energy superstring effective action—see e.g. [10] for details, and [11] for a recent discussion in the context of inflation. In this paper we would like to concentrate on the inflationary model of gravity with the quartic scalar curvature term. The leading quantum gravitational corrections to the type-II superstrings are indeed quartic [9–11]. As regards superstring moduli, we assume that they are stabilized by fluxes via compactification [12].

The inflationary mechanism in the (R + R^4) gravity is very different from that of the (R + R^2) gravity, and is close to topological inflation [13]. The R^4 topological inflation was investigated earlier in [14] from the higher dimensional viewpoint, namely, in five spacetime dimensions. Our analysis in this paper is truly four-dimensional, while our quantitative results in four dimensions differ from those of [14].

Our paper is organized as follows. In section 2 we introduce our model of modified gravity, establish its equivalence to the certain scalar–tensor gravity model, and compute the inflaton scalar potential. In section 3 we briefly review the topological inflation [13]. In section 4 we give our main results where we describe slow-roll inflation in the R^4-modified gravity and compare our results with topological inflation. In section 5 we present our conclusions.

2. The model and our setup

There is a priori no reason for restricting the gravitational Lagrangian to the standard Einstein–Hilbert term that is linear in the scalar curvature, as long as it does not contradict an experiment. Nowadays, there is no doubt that the extra terms of the higher order in the curvature should appear in the gravitational effective action of any quantum theory of gravity, while they do appear in string theory [9, 10]. Since the scale of inflation in the early Universe is just a few orders less than the Planck scale [2], it is conceivable that the higher-order gravitational terms may be instrumental for inflation. It is already the case in the simplest modified gravity model with the R^2 terms [3]. Here we consider the modified gravity models in four spacetime dimensions, with an action

$$S_f = -\frac{1}{2\kappa^2} \int d^4x \, f(R),$$

(2.1)

3 See [6] for a recent discussion of the (R + R^2) gravity and inflation.

4 The quadratic curvature terms may still be generated via superstring compactification.
whose Lagrangian is given by
\[ f(R) = R - \frac{1}{M^{4p-2}} R^{2p}, \quad \text{where } p = 1, 2, 3, \ldots \] (2.2)
by paying special attention to the \( p = 2 \) case and beyond. The parameter \( M \) in equation (2.2) has the dimension of mass. We use the spacetime signature \((+, -, -, -)\) and the units \( \hbar = c = 1 \). The Einstein–Hilbert term in our equations (2.1) and (2.2) has the standard normalization with \( \kappa = M_{\text{Pl}}^{-1} \) in terms of the reduced Planck mass \( M_{\text{Pl}}^{-2} = 8\pi G_N \). The rest of our notation for spacetime (Riemann) geometry is the same as in [15].

In contrast to general relativity having \( f'(R) = \text{const} \), the field \( A = f'(R) \) is dynamical in the modified gravity with \( f'(R) \neq \text{const} \). In terms of the fields \((g_{\mu\nu}, A)\) the equations of motion associated with action (2.1) are of the second order in the derivatives of the fields.

The new field degree of freedom represents the propagating conformal mode of the metric. It can be easily seen in the context of the known classical equivalence between the \( f(R) \) gravity and the scalar–tensor gravity, via the Legendre–Weyl transform [16]. The equivalent action reads
\[ S_\phi = \int d^4x \sqrt{-g} \left\{ -\frac{R}{2\kappa} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right\} \] (2.3)
in terms of the physical (and canonically normalized) scalar field \( \phi(x) \) having the scalar potential
\[ V(\phi) = -\frac{M_{\text{Pl}}^2}{2} \exp \left\{ \frac{-4\phi}{M_{\text{Pl}}\sqrt{6}} \right\} Z \left( \exp \left[ \frac{-2\phi}{M_{\text{Pl}}\sqrt{6}} \right] \right) \] (2.4)
where the function \( Z \) is related to the function \( f \) via the Legendre transformation
\[ R = Z'(A) \quad \text{and} \quad f(R) = RA(R) - Z(A(R)), \] (2.5)
whereas the arguments \( A \) and \( \phi \) are related by
\[ A(\phi) = \exp \left[ \frac{2\phi(\phi(x))}{\sqrt{6}} \right]. \] (2.6)

As regards cosmological applications, it is desirable to explicitly derive the scalar potential \( V(\phi) \). It amounts to inverting the relation
\[ f'(R) = A(\phi), \] (2.7)
which is the consequence of equation (2.5). In our case (2.2) we find
\[ V(\phi) = V_0 \exp \left( \frac{-2\sqrt{2}\phi}{\sqrt{3} M_{\text{Pl}}} \right) \left\{ 1 - \exp \left[ \frac{2\phi}{\sqrt{3} M_{\text{Pl}}} \right] \right\}^{\frac{2p}{3}}, \] (2.8)
whose overall normalization constant is given by
\[ V_0 = \left( \frac{2p - 1}{4p} \right) M_{\text{Pl}}^2 \left( \frac{1}{2p} \right)^{\frac{2p}{3}}, \] (2.9)
whose overall normalization constant is given by
\[ V_0 = \left( \frac{2p - 1}{4p} \right) M_{\text{Pl}}^2 \left( \frac{1}{2p} \right)^{\frac{2p}{3}}. \] (2.9)
In terms of the new dimensionless variable
\[ y = \sqrt{\frac{2}{3}} \frac{\phi}{M_{\text{Pl}}}, \] (2.10)
\[ \text{When compared to the notation in our recent paper [6], we have changed } \phi \to -\phi \text{ here.} \]
the potential (2.8) takes the form

\[ \frac{V(y)}{V_0} = e^{-2y}(e^y - 1)^{\frac{2p}{p+1}} \equiv g_2^{p}(y). \]  

(2.11)

This potential is bounded from below, and has a local minimum at \( \phi_{\text{min}} = 0 \) and a local maximum at

\[ \phi_{\text{max}} = \sqrt{\frac{3}{2} M_{\text{Pl}}} \ln \left( \frac{2p - 1}{p - 1} \right). \]  

(2.12)

In particular, when \( p = 2 \) we find

\[ \frac{V(y)}{V_0} = e^{-2y}(e^y - 1)^{\frac{4}{3}} \equiv g(y) = |z(1 - z)(1 + z)|^{4/3}, \]  

(2.13)

where

\[ V_0 = \frac{3}{8 \cdot 2^{2/3} M_{\text{Pl}}^2 M^2} \quad \text{and} \quad z = \exp \left( -\frac{y}{2} \right). \]  

(2.14)

The graphs of the functions \( g(y) = g_4(y) \) and \( g_6(y) \) are given in figures 1 and 2, respectively. The graphs of \( g_2^{2\sigma}(y) \) of \( p \geq 4 \) are very close to that in figure 2.

In the limit \( p = +\infty \) the scalar potential \( V_\infty(y)/V_0 \) (after renormalization) reads

\[ g_\infty(y) = e^{-y}|1 - e^{-y}|. \]  

(2.15)
The values of $\phi_{\text{max}}$ for various $p$ are given by

\begin{align*}
  p &= 2: \quad \phi_{\text{max}} = \left(\frac{\sqrt{3}}{2} \ln 3\right) M_{\text{Pl}} \approx 1.3455 M_{\text{Pl}}, \\
  p &= 3: \quad \phi_{\text{max}} \approx 1.122 M_{\text{Pl}}, \\
  p &= 4: \quad \phi_{\text{max}} \approx 1.037 M_{\text{Pl}}, \\
  p &= \infty: \quad \phi_{\text{max}} = \left(\frac{\sqrt{3}}{2} \ln 2\right) M_{\text{Pl}} \approx 0.85 M_{\text{Pl}}.
\end{align*}

The $p = 2$ function $g(y)$ of equation (2.13) asymptotically approaches zero as $\exp(-2y/3)$ for $y \to +\infty$, and diverges as $\exp(2|y|)$ for $y \to -\infty$.

3. Topological inflation

Topological inflation was suggested by Vilenkin and Linde in 1994 [13]. When the inflaton scalar potential has a discrete symmetry $Z_2$ (say, against $\tilde{\phi} \to -\tilde{\phi}$) and the state $\tilde{\phi} = 0$ is unstable, the Universe gets divided into different domains related by domain walls. Those domain walls inflate with time, while their thickness exponentially grows. Hence, the domain walls may appear as the seeds of inflation. Topological inflation does not require fine-tuning of the initial conditions [13].

The simplest realization of topological inflation is described by an action

$$
S[\tilde{\phi}] = \int d^4x \sqrt{-g} \left\{ -\frac{R}{2\kappa^2} + \frac{1}{2} g^{\mu\nu} \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} - V_{\text{dw}}(\tilde{\phi}) \right\},
$$

(3.1)

with the Higgs-like (or double-well) scalar potential [13]

$$
V_{\text{dw}}(\tilde{\phi}) = \frac{\lambda}{4}(\tilde{\phi}^2 - \eta^2)^2.
$$

(3.2)

The $Z_2$ symmetry breaking in this model results in the formation of two domains with $\tilde{\phi} = \pm \eta$. Those domains are divided by the domain wall interpolating between the two minima. A static domain wall solution without gravity (say, in the $yz$ plane) is obtained from the Bogomol’nyi decomposition of the static energy (per unit area):

$$
E[\tilde{\phi}] = \int_{-\infty}^{+\infty} dx \left[ \frac{1}{2} \left( \frac{d\tilde{\phi}}{dx} \right)^2 + V(\tilde{\phi}) \right]
$$

$$
= \frac{1}{2} \int_{-\infty}^{+\infty} dx \left( \frac{d\tilde{\phi}}{dx} \pm \sqrt{2V(\tilde{\phi})} \right)^2 \pm \int_{\tilde{\phi}(\infty)}^{\tilde{\phi}(-\infty)} \sqrt{V(\tilde{\phi})} d\tilde{\phi}.
$$

(3.3)

The last term can be identified with the ‘topological charge’ that merely depends upon the values of the field on the boundary. The existence of the BPS bound also follows from equation (3.3). A domain wall is the exact solution that saturates the BPS bound and has a non-vanishing topological charge. In the case of the double-well scalar potential (3.2), equation (3.3) gives rise to the well-known exact (non-perturbative) BPS solution

$$
\tilde{\phi} = \eta \tanh \left( \frac{\sqrt{2}}{2} \eta x \right) \equiv \eta \tanh \left( \frac{x}{\delta_0} \right),
$$

(3.4)

whose thickness is thus given by

$$
\delta_0 = \sqrt{\frac{2}{\lambda} \frac{1}{\eta}}.
$$

(3.5)
In the Vilenkin–Linde topological inflation scenario [13], should the initial conditions for the scalar field $\tilde{\phi}$ be randomly distributed with large dispersion, a part of the Universe will run to one of the minima of the double-well potential (3.2) while another part will run to another minimum. The domain wall between these parts of the Universe may be thick enough, in order to accommodate inflation, when the thickness of the domain wall is larger than the Hubble radius of the energy density, i.e. [13]

$$\delta_0 > H_0^{-1} = \sqrt{\frac{3M_{\text{Pl}}^2 V(0)}{V}},$$

(3.6)

In the case of the double-well potential (3.2), equations (3.5) and (3.6) yield [13]

$$\eta > \mathcal{O}(\sqrt{6}M_{\text{Pl}}),$$

(3.7)

which should be understood as a rough approximation. The precise condition for the existence of slow-roll inflation with the potential (3.2) was established numerically in [17]:

$$\eta > 0.33\sqrt{\pi}M_{\text{Pl}} \approx 1.65M_{\text{Pl}}.$$

(3.8)

The $R^4$-potential (2.13) may be roughly approximated by the double-well potential (3.2), as shown in figure 1. Then the parameter $\eta$ is just the distance between the minimum and the maximum of our scalar potential, $\eta = \phi_{\text{max}}$. Therefore, equations (2.10) and (2.16) imply that in the $R^4$ case we have

$$\eta \approx 1.345M_{\text{Pl}}$$

(3.9)

while it is even lower for the higher $p > 2$. Thus we might conclude that slow-roll inflation is impossible in the $R^4$-modified gravity. However, it is not really the case, because the approximation of the exact potential (2.13) by the double-well potential (3.2) is very rough, so it cannot be used when the difference between the values of $\eta$ in equations (3.8) and (3.9) is very small. As a matter of fact, the actual scalar potential (2.13) does not have an exact $Z_2$ discrete symmetry, while its Taylor expansion around the maximum is given by

$$\frac{V(\phi)}{V_{\text{max}}} = 1 - \frac{1}{3M_{\text{Pl}}^2}(\phi - \phi_{\text{max}})^2 + \frac{2\sqrt{2}}{9\sqrt{3}M_{\text{Pl}}}(\phi - \phi_{\text{max}})^3$$

$$- \frac{5}{108M_{\text{Pl}}^4}(\phi - \phi_{\text{max}})^4 + \mathcal{O}(\phi - \phi_{\text{max}})^5$$

(3.10)

where $\phi_{\text{max}} = \sqrt{3/2}(\ln 3)M_{\text{Pl}}$. Since the third derivative is non-vanishing and the fourth derivative is negative, the potential (3.10) cannot be approximated by the double-well potential (3.2) for any $\lambda$. In the next section we will study the conditions for slow-roll inflation with the potential (2.13).

4. Slow-roll $R^4$ inflation

The necessary condition for slow-roll inflation is the smallness of the inflation parameters [2]:

$$\varepsilon(\phi) \ll 1 \quad \text{and} \quad |\eta(\phi)| \ll 1$$

(4.1)

where the functions $\varepsilon(\phi)$ and $\eta(\phi)$ are defined by [2]

$$\varepsilon(\phi) = \frac{1}{2}M_{\text{Pl}}^2 \left(\frac{V'}{V}\right)^2 \quad \text{and} \quad \eta(\phi) = M_{\text{Pl}}^2 \frac{V''}{V},$$

(4.2)

6 That approach was adopted in [14].
and the primes denote the derivatives with respect to the inflaton field \( \phi \). The first condition (4.1) implies \( \ddot{a}(t) > 0 \), whereas the second condition (4.1) guarantees that inflation lasts long enough, via domination of the friction term in the inflaton equation of motion (in the slow-roll case):

\[
3H\dot{\phi} = -V'.
\]  (4.3)

Here \( H \) stands for the Hubble function \( H(t) = \dot{a}/a \) in terms of the scale factor \( a(t) \). Equation (4.3) is to be supplemented by the Friedmann equation

\[
H^2 = \frac{V}{3M_{Pl}^2}.
\]  (4.4)

It follows from equations (4.3) and (4.4) that

\[
\dot{\phi} = -M_{Pl}V' \sqrt{3V} < 0.
\]  (4.5)

The amount of inflation is measured by the e-foldings number

\[
N_e = \int_{t}^{t_{end}} H \, dt \approx \frac{1}{M_{Pl}^2} \int_{\phi}^{\phi_{end}} \frac{V}{V'} \, d\phi,
\]  (4.6)

where the \( t_{end} \) stands for the (time) end of inflation when one of the slow-roll parameters becomes equal to 1. The number of e-foldings between 50 and 100 is usually considered to be acceptable.

In our case of the scalar potential (2.13) we find

\[
\varepsilon(y) = \frac{1}{3} \left[ \frac{g'(y)}{g(y)} \right]^2 = \frac{4}{27} \left[ 1 - \frac{2}{e^y - 1} \right]^2
\]  (4.7)

and

\[
\eta(y) = \frac{2}{3} \left[ \frac{g''(y)}{g(y)} \right] = \frac{8}{3(e^y - 1)} \left[ -1 + \frac{e^{2y}}{9(e^y - 1)} \right].
\]  (4.8)

The graphs of those functions are given in figure 3.

Inflation is possible when there is a vacuum energy. As is clear from figure 1, it is the case near the maximum \( y_{max} = \ln 3 \approx 1.1 \). We find \( \varepsilon(y_{max}) = 0 \) and \( \eta(y_{max}) = -2/3 \), whereas near the minimum \( y = 0^+ \) both \( \varepsilon(0^+) \) and \( \eta(0^+) \) diverge to infinity. The equation \( \varepsilon(y) = 1 \) has two solutions at \( e^y = 1.55 \) and \( e^y = 0.44 \), whereas the equation \( \eta(y) = 1 \) also has two solutions at \( e^y = 2.44 \) and \( e^y = 1.15 \). Therefore, it is clear that slow-roll inflation from the maximum toward the minimum of the potential has an exit at \( y_{end} \approx \ln 2.44 \approx 0.89 \), whereas runaway inflation from the maximum toward \( y = +\infty \) never ends.
As regards the e-foldings number in the case of slow-roll inflation toward the minimum, for (I) : \( y_{\text{end}} < y < y_{\text{max}} \), we find

\[
N_e^{(I)}(y) = \frac{3}{2} \int_{y_{\text{end}}}^{y_{\text{max}}} g(y) \left( \frac{e^y - 1}{3} - \frac{1}{e^3} \right) dy
\]

\[
= \frac{3}{2} \ln \left[ -1 + 3e^{-y_{\text{max}}} \right] - \frac{9}{4} (y - y_{\text{end}}) \approx \frac{3}{2} \ln \left[ -1 + 3e^{-y_{\text{max}}} \right] - \frac{9}{4} y - 0.2. \quad (4.9)
\]

Similarly, for runaway inflation (II) : \( y_{\text{max}} < y < y_{\text{large}} \) we find

\[
N_e^{(II)}(y) = \frac{9}{4} \int_{y_{\text{large}}}^{y_{\text{max}}} \left( \frac{e^y - 1}{e^y - 3} \right) dy. \quad (4.10)
\]

When \( y_{\text{large}} \to +\infty \), the slow-roll parameters approach finite values, \( \varepsilon \to \frac{4}{27} \) and \( \eta \to \frac{8}{27} \), whereas \( N_e^{(II)} \) linearly diverges, as expected (see figure 3).

It is straightforward to get explicit solutions to the slow-roll equations of motion (4.4) and (4.5) by using the exact potential (2.13), though these solutions are not very illuminating. So, we restrict ourselves to presenting only the leading terms, in the vicinity of the maximum, namely

\[
\phi(t) \approx \phi_{\text{max}} + (\phi_{\text{ini}} - \phi_{\text{max}}) \exp \left[ \frac{m^2}{3H_0} (t - t_{\text{ini}}) \right]
\]

and

\[
a(t) \propto e^{H_0 t} \left( 1 - \frac{(\phi_{\text{ini}} - \phi_{\text{max}})^2}{16M_{\text{Pl}}^2} \exp \left[ \frac{2m^2}{3H_0} (t - t_{\text{ini}}) \right] \right)
\]

where we have introduced the Hubble constant as usual, i.e.

\[
H_0 = \frac{1}{M_{\text{Pl}}} \sqrt{\frac{V_{\text{max}}}{3}} \quad \text{with} \quad V_{\text{max}} = V(y_{\text{max}}) = \frac{M_{\text{Pl}}^2 M^2}{12 \cdot 2^{1/3}}
\]

and the mass parameter

\[
m^2 = \frac{2V_{\text{max}}}{3M_{\text{Pl}}^2} = \frac{M^2}{18 \cdot 2^{1/3}},
\]

in agreement with the Taylor expansion (3.10) of the inflaton potential.

We are now ready to confront the \( R^4 \) model with cosmological observations. The first relevant physical observable is given by the amplitude of the initial perturbations, \( \Delta^2 = M_{\text{Pl}}^2 V/(24\pi^2 \varepsilon) \). Equating its theoretical and experimental values yields [2]

\[
\left( \frac{V}{\varepsilon} \right)^{1/4} \bigg|_{y=y_{\text{end}}} = 0.027 M_{\text{Pl}} = 6.6 \times 10^{16} \text{ GeV.} \quad (4.15)
\]

This equation determines the normalization of the \( R^4 \)-term in the action. By using \( g(y_{\text{end}}) \approx 0.27 \) and \( \varepsilon(y_{\text{end}}) \approx 0.022 \) we find

\[
M \approx 4.3 \times 10^{-4} M_{\text{Pl}}. \quad (4.16)
\]

However, the key test is provided by the observed value of the spectral index \( n_s \), as given by the recent WMAP5 data [5],

\[
n_s = 0.960 \pm 0.013. \quad (4.17)
\]

The theoretical value of \( n_s \) is given by [2]

\[
n_s = 1 + 2\eta - 6\varepsilon = 1 + \frac{16}{3(e^y - 1)} \left[ -1 + \frac{e^{2y}}{9(e^y - 1)} \right] - \frac{8}{9} \left[ -1 + \frac{2}{e^y - 1} \right]^2, \quad (4.18)
\]
where we have used equations (4.7) and (4.8) in our case. Unfortunately, as regards the relevant values of $y$ during slow-roll inflation toward the minimum, the value of $\varepsilon(y)$ is always less than 0.03 (so it does not play a significant role here), whereas the value of $\eta(y)$ always belongs to the interval $2/3 \leq |\eta| \leq 1$, so that the $\eta$ is unacceptably large during inflation ($\eta$-problem). One arrives at the same conclusion when using another standard equation for the spectral index$^7$:

$$n_s = 1 - \frac{2m^2}{3H_0^2}, \quad (4.19)$$

whose value at the maximum is given by $n_s(y_{\text{max}}) = 1 - 4/3 = -1/3$. Similarly, when considering runaway inflation, we find that $n_s \to 19/27$.

5. Conclusion

In conclusion, slow-roll inflation with an exit is possible in the modified $R^4$-gravity model, but it does not survive the observational tests because of the $\eta$-problem. This qualitative conclusion agrees with that of [14] though our quantitative results are different.

Two possible solutions of the $\eta$-problem were already proposed in [14]. As regards (1) adding the $R^2$ term to the $R^4$ term [14] would be against our basic (or minimal) motivation in section 1 (see, however, [18]). Of course, the $R^2$ term may also be generated in the process of superstring compactification; however, we wanted to check just the topological inflation. As regards (2) taking into account the renormalization of the matter stress–energy tensor [3, 14] may lead to sufficient inflation with an acceptable value of $n_s$, but without the predictive power because of the uncertainty in the number of relevant string degrees of freedom.

Of course, it may not be difficult to find a function $f(R)$ with the shape similar to that in figure 1, whose $y_{\text{max}}$ and $V_{\text{max}}$ would have the desired values for topological inflation with the desirable value of $n_s$. However, we are not aware of any string theory output that would constrain that function in any way. One of the natural first steps would be the embedding of topological inflation into modified supergravity [19].

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