Entanglement Swapping with Photons Generated on Demand by a Quantum Dot

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Photonic entanglement swapping, the procedure of entangling photons without any direct interaction, is a fundamental test of quantum mechanics and an essential resource to the realization of quantum networks. Probabilistic sources of nonclassical light were used for seminal demonstration of entanglement swapping, but applications in quantum technologies demand push-button operation requiring single quantum emitters. This, however, turned out to be an extraordinary challenge due to the stringent prerequisites on the efficiency and purity of the generation of entangled states. Here we show a proof-of-concept demonstration of all-photonic entanglement swapping with pairs of polarization-entangled photons generated on demand by a GaAs quantum dot without spectral and temporal filtering. Moreover, we develop a theoretical model that quantitatively reproduces the experimental data and provides insights on the critical figures of merit for the performance of the swapping operation. Our theoretical analysis also indicates how to improve state-of-the-art entangled-photon sources to meet the requirements needed for implementation of quantum dots in long-distance quantum communication protocols.

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Entanglement swapping has been observed in a few different systems, from the original all-photonic scheme that employs a spontaneous parametric down-conversion (SPDC) source [1] to hybrid protocols in which the interference of two photons is used to entangle spins [2] or atoms [3] at a distance. The swapping procedure between pairs of photons is especially relevant to the development of future quantum networks, because it provides a way to overcome the absence of optical communication amplifier for photonic qubits due to the no-cloning theorem and to create entanglement over distances beyond the reach of direct transmission [4,5].

Developing quantum light sources able to operate on demand is an important step towards this goal. Despite the impressive technological achievements up to date [6], SPDC sources are limited by the probabilistic nature of the photon generation process [7]. Quantum emitters, such as atoms, nitrogen vacancies in diamonds, and semiconductor quantum dots (QDs), overcome this hurdle and hold strong promise for deterministic operation. Among these, the latter are receiving attention after recent reports of QD-based single-photon sources overtaking SPDC in terms of brightness, single photon purity, and indistinguishability [8,9]. In addition, they are closing the performance gap concerning the generation of polarization-entangled photons as well [10–14], leading to the recent demonstration of three-photon quantum teleportation [15], even under deterministic photon generation [16].

However, to the best of our knowledge, there is no report on the use of QDs—and more generally of any solid-state-based quantum emitter—for entanglement swapping using photon pairs. This absence of experimental results is likely linked to the outstanding challenges set by the implementation of four-photon swapping protocols involving high quality entangled-photon pairs. Below, we detail the specific challenges and explain how they can be successfully overcome.

Using a single QD we design an entanglement swapping experiment which follows the seminal work of Pan et al. [1], as illustrated in the inset of Fig. 1. Photon pairs in the maximally entangled state $|\Phi^+\rangle = 1/\sqrt{2}(|HH\rangle + |VV\rangle)$ are deterministically generated from a QD with exciton fine structure splitting (FSS) well below the radiative-limited excitonic linewidth [17] by exciting the radiative biexciton-exciton (XX-X) cascade via a two-photon resonant scheme [18–20]. Two XX-X entangled-pairs ($|\Phi_E^+\rangle$ and $|\Phi_L^+\rangle$), linked to the early and late generation pulse) are independently triggered by two subsequent laser pulses, and the photons emitted by the X transition ($X_E$ and $X_L$) are brought to interfere at a beam splitter using a delay line. When these two photons are perfectly indistinguishable, a joint detection at the two output ports of the beam splitter...
demand operation with a preparation fidelity of approximately 90% [32], excellent suppression of multiphoton emission [33] and good indistinguishability [30]. To improve light collection and enhance the brightness of the source, we integrate the QDs into a monolithic planar cavity composed of two asymmetric distributed Bragg reflectors. This convenient approach maintains the QD optical quality and guarantees a broad bandwidth to deal with the wavelength difference between the X and XX transitions. A light extraction efficiency of 7% is estimated in the relevant spectral region of emission (see Sec. SVIII for how this figure was calculated).

We selected a QD with optimal trade-off among the relevant figures of merit (details about how the values listed below are measured are in the Secs. SII–SIV [23,25,26]). A low FSS $S$ of 0.6(5) μeV and an exciton lifetime $\tau_x$ of 270(10) ps ensure a high degree of XX-X entanglement, as supported by the measured value of Bell state fidelity of 0.88(2). The Hong-Ou-Mandel (HOM) visibility $V$ of the X line is 0.63(2). Most importantly, a detection rate of approximately 0.5 MHz is achieved on the detectors recording the BSM, which results in a rate of fourfold coincidences of approximately 3 mHz, in agreement with our predictions on the throughput of the swapping protocol (whose derivation is described in more depth in Sec. SVIII).

The reported count rates are measured in the setup sketched in Fig. 1. With a repetition rate of 160 MHz, a couple of laser pulses separated by a time delay of 1.8 ns excite the QD and trigger the emission of two pairs of entangled photons, which we label $XXE-XE$ and $XXL-XL$. Volume Bragg gratings are used to separate the photons from the two transitions of the cascade, thus ensuring minimal losses. The photons originating from the exciton to ground state transition, $XE$ and $XL$, are sent to an unbalanced Mach-Zehnder interferometer with an internal delay also set to 1.8 ns) to let them interfere at its second beam splitter. The BSM is thus performed by recording joint detection events between $XE$ and $XL$ within a time window of 0.6 ns. Because of the short radiative lifetime this choice does not postselect the emitted photons by applying a temporal filter on the two-photon interference profile. While this technique is known to increase the HOM visibility [34], it is not suitable for applications as it comes at the cost of decreasing the source brightness. The photons from the biexciton to exciton transition, $XXE$ and $XXL$, are instead sent to a nonpolarizing 50:50 beam splitter and identified at the two output ports based on the arrival time. The polarization optics performs different projective measurements on $XXE$ and $XXL$ so to acquire coincidences in the set of polarization bases required for quantum state tomography. Only the detection events within a temporal range of 100 ns from a BSM are recorded.

The fourfold coincidences are recorded as a function of the delays between the BSM and the XX detection events on the two tomography channels, as shown in Fig. 2(a) for a
pair of co- and cross-polarized $XX$ bases. The comparison between the two peaks near zero delay—which contain the fourfold coincidences of photons coming from two subsequent $XX$-$X$ cascades—highlights the presence of polarization correlation. To estimate the correlation visibility, the coincidence counts are normalized with respect to the side peaks stemming from $XX$ photons uncorrelated with the BSM, as discussed more in depth in the Sec. SV. In Fig. 2(b) the data are windowed and binned to obtain second-order intensity correlation histograms for the linear, diagonal, and circular bases. The observed bunching and antibunching behaviors clearly show the presence of a swapping process and are consistent with a projection to a state with a dominant $\Psi^-$ character.

In order to gain complete insight on the result of the swapping operation, we perform the full tomography of the two-photon state and collect $XX_E$-$XX_L$ correlations in the 36 possible combinations of linear, diagonal, and circular polarization bases [35]. Note that $XX_E$ and $XX_L$ are defined by their time of arrival and not by the detector that registers them, therefore permuted pairs of bases are acquired at the same time, and the total number of measurements is reduced to 21 (see Sec. SVI for a list of the performed tomography measurements). The density matrix is reconstructed using a maximum likelihood estimation [36] and is presented in Fig. 3(a).

The raw value of fidelity to the expected Bell state $|\Psi^\mp\rangle$ is calculated from the density matrix (using the standard formula also reported in Sec. SVI) to be 0.58(4), which indicates a strong correlation between photons that are uncorrelated without the information from the BSM (0.25), surpassing the classical threshold [37] of 0.5 by 2 standard deviations. A consistent evidence of the presence of entanglement is offered by the above-zero raw value of the concurrence, 0.15(8). Therefore, our results experimentally demonstrate entanglement swapping between single pairs of entangled photons generated on demand by a quantum emitter. In addition to that, the swapping procedure generates entangled pairs of photons with the same energy and different time bins. These are qualitatively different features with respect to cascaded photons usually observed in QDs.

It is worth emphasizing that the measured level of entanglement between the swapped photons does not consider imperfections stemming from the experimental setup, such as background light and non-ideal beam splitters. Taking these imperfections into account the fidelity would be expected to increase to 0.64(4) and the concurrence to 0.28(8), see below.

While our result offers a valid proof of concept, higher levels of entanglement will be needed for real-life quantum communication, which demands a degree of entanglement...
of the swapped photons large enough to violate Bell’s inequality (since our outcome of the swapping operation closely resembles a Werner state, a concurrence approximately larger than 0.58 would be needed [38]), and to implement error-correction protocols for secure QKD (fidelity larger than 0.8 [39]). As the temporal [34] and spectral [40,41] filtering techniques usually employed to improve indistinguishability and entanglement come at a cost of source brightness, it is unclear whether QDs can be really used as the on-demand entangled photon source needed for long-distance quantum communication.

Below, we argue that the fidelity of the swapping operation can be instead pushed to the required values with future developments of a state of the art QD-photon source. Before explaining how to accomplish this task, we now present a novel theoretical model that not only accounts quantitatively for the experimental observations, but also explains the sources of entanglement degradation and, as a consequence, pinpoints the next steps ahead.

The success of entanglement swapping critically depends on two main parameters: the initial degree of entanglement of the photon pairs and the indistinguishability of the photons involved in the BSM. In contrast with the simple approximation presented in Ref. [42], we consider the specific role of the relevant properties of the quantum emitter. The initial degree of entanglement is known to be limited by finite FSS, spurious photons from background light or multiphoton emission, and decoherence mechanisms during the intermediate step of the cascade [11]. From a theoretical point of view, it is possible [43] to introduce all these contributions in the density matrices of the initial $|\Phi^+_X\rangle$ and $|\Phi^+_L\rangle$ states (that are $\rho_{X,X}^{\Phi^+_X}$ and $\rho_{X,X}^{\Phi^+_L}$) and project the two $X$ polarization modes onto $|\Psi^-\rangle$ [44] to describe the density matrix resulting from the swapping operation

$$\rho_{XXE;XXL}^{\Psi^-} = \text{Tr}_{X_{E};X_{L}} \left( \Pi_{X_{E};X_{L}}^{\Psi^-} \rho_{X_{E};X_{X}E}^{\Phi^+_X}(t_E,t_L) \right) = \sum_{i=\phi^+,\Psi^+} \rho_{XXE;XXL}^{\psi} p_i(BSM) \rho_{XXE;XXL}^{\phi^+_i}.$$ 

Eventually (see Sec. SIX [23,27–29] for the step-by-step procedure), we can derive an analytic expression for the fidelity to $|\Psi^-\rangle$, which reads

$$f_{XXE;XXL}^{\Psi^-} = \frac{1}{4} \left[ 1 + \frac{V}{2} - k² \left( g_{H,V}^{(1)} \right)^2 \left[ 1 + \left( \frac{g_{H,V}^{(1)}}{g_{H,V}^{(1)}} \right)^2 \right] \right],$$

where $k$ is the fraction of uncorrelated photons collected from the $XX-X$ cascade, $g_{H,V}^{(1)} = 1/(1 + \tau_X/\tau_{SS})$, $g_{H,V}^{(2)} = 1/(1 + \tau_X/\tau_{SS} + \tau_X/\tau_{HV})$, and $g_{\text{dep}}^{(1)} = 1/(1 + 2\tau_X/T_2^*)$ with $\tau_{SS}$, $\tau_{HV}$, and $T_2^*$, respectively, defined as the spin-scattering, cross-dephasing, and pure-dephasing characteristic times.

By measuring $g_{XX,XXX}(0)$, $\rho_{XXE;XXE}$, $S$, $\tau_X$, and $V$, and taking the value of the decoherence times from the literature [11,32,43] it is possible to experimentally estimate all the quantities appearing in Eq. (1) and, therefore, predict $f_{XXE;XXL}^{\Psi^-}$ with no fitting parameters. The model returns a swapping fidelity of 0.56 (0.64 in absence of background light and considering beam splitter imperfections), in excellent agreement with the experimental result. As a further proof of our theoretical model, we repeat the experiments and intentionally decrease either the degree of entanglement of our source, selecting a QD with a larger FSS of 5.9(5) μeV, or the indistinguishability of the photons, using an emitter with a HOM visibility of 0.51(2). The comparison between these data and the model, summarized in Fig. 3(b) and discussed in more detail in the Sec. SIX [23], shows once again good agreement between experiment and theory.

Our theoretical model also shows that the swapping fidelity depends in a sublinear fashion on the photon indistinguishability and that our current setup and QD sample cannot be used to perform a Bell test without a postselection technique such as narrow time gating (not suitable for applications, as mentioned above). However, our model can estimate the swapping fidelity for any QD photon source and, considering the best values of entanglement and indistinguishability available from the literature, we can predict whether QDs can be suitable for quantum communication. Discarding approaches using postselection we focus on the following works: (i) Huber et al. [11], who have demonstrated that QDs can deliver entangled photons with fidelities up to 98% using micromachined piezoelectro-
challenge, but our theoretical and experimental work on entanglement swapping anticipates that the strive to reach this goal is certainly worth the efforts, as the realization of the ideal “on-demand entangled photon source” could be revolutionary for quantum communication science and technology.

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\textit{Note added.---}After submission we became aware of a related work \cite{Somaschi2017} that uses temporal postselection techniques and a different experimental setup for the BSM.

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