$^4\Lambda\Lambda$H in halo effective field theory

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I. INTRODUCTION

Light double-$\Lambda$ hypernuclei are exotic few-body systems that provide opportunities to investigate the flavor SU(3) structure of baryon-baryon interactions in the strangeness $S = -2$ channel. They are also expected to have a key role to resolve the long-standing strangeness $\Lambda\Lambda$ problem. In the KEK-E373 experiment the $\Lambda\Lambda$ interaction energy is not determined [7, 8]. However, there are only a few reports on the observation of double-$\Lambda$ hypernuclei, and our understanding on these systems is still very poor. Theoretical studies which provide opportunities to investigate the flavor SU(3) structure of baryon-baryon interactions in the strangeness $S = -2$ channel are expected to have a key role to resolve the long-standing strangeness $\Lambda\Lambda$ problem. In the KEK-E373 experiment the $\Lambda\Lambda$ interaction energy is not determined [7, 8]. However, there are only a few reports on the observation of double-$\Lambda$ hypernuclei, and our understanding on these systems is still very poor. Theoretical studies which provide opportunities to investigate the flavor SU(3) structure of baryon-baryon interactions in the strangeness $S = -2$ channel are expected to have a key role to resolve the long-standing strangeness $\Lambda\Lambda$ problem.

The $^4\Lambda\Lambda$H bound state and the $S$-wave hypertriton-$^3\Lambda$H-$\Lambda$ scattering in spin singlet and triplet channels below the hypertriton breakup momentum scale are studied in halo/cluster effective field theory at leading order by treating the $^4\Lambda\Lambda$H system as a three-cluster ($\Lambda$-$\Lambda$-deuteron) system. In the spin singlet channel, the amplitude is insensitive to the cutoff parameter $\Lambda_c$ introduced in the integral equation, and we find that there is no bound state. In this case, the scattering length of the hypertriton-$\Lambda$ scattering is found to be $a_{\text{LO}} = 16.0 \pm 3.0$ fm. In the spin triplet channel, however, the amplitude obtained by the coupled integral equations is sensitive to $\Lambda_c$, and we introduce the three-body contact interaction $g_1(\Lambda_c)$. After phenomenologically fixing $g_1(\Lambda_c)$, we find that the correlation between the two-$\Lambda$ separation energy $B_{\text{LO}}$ from the $^4\Lambda\Lambda$H bound state and the scattering length $a_{\Lambda\Lambda}$ of the $S$-wave $\Lambda\Lambda$ scattering is significantly sensitive to the value of $\Lambda_c$.

The methods of EFT nowadays become popular in many fields. (For a review, see, e.g., Refs. 21, 22.) In this scheme, a theory is constructed based on a scale which separates low energy and high energy degrees of freedom, and the theory constructed in such a way provides a systematic perturbative expansion in powers of $Q/\Lambda_H$, where $Q$ is the typical scale of the reaction in question and $\Lambda_H$ is the large (or high energy) scale. High energy degrees of freedom above $\Lambda_H$ are integrated out and their effects are accounted for through the coefficients of contact interactions, so-called low energy constants, in higher order.

In this work, we investigate the relation between the $^4\Lambda\Lambda$H bound state and the $S$-wave hypertriton-$\Lambda$ scattering below the hypertriton breakup momentum for spin singlet and triplet channels by employing Halo/Cluster EFT at leading order (LO). In particular, we treat the $^4\Lambda\Lambda$H hypernucleus as a three-body $\Lambda\Lambda d$ system, where $d$ stands for a deuteron. Although the scattering experiment with double-$\Lambda$ systems is not feasible in near future, the scattering length of the hypertriton-$\Lambda$ scattering is obtained as $a_{\Lambda\Lambda} \geq 12.5$ fm in Ref. 19. These values are consistent with those extracted from the leading order calculations for the $S = -2$ baryon-baryon interactions in chiral effective theory [2] and in the Nijmegen ESC04d phenomenological potential model [20]. On the other hand, other phenomenological potential model predictions are scattered in values from $-0.27$ fm to $-3.80$ fm even though such models could explain the existence of the $^6\Lambda\Lambda$H bound state. The present situation is summarized, for example, in Table I of Ref. 17. This may imply that the parameter space of potential models would be too large to determine unambiguously the parameter values from the currently available experimental data. In such a situation, it would be worth studying the structure of hypernuclei by employing a very low energy effective field theory (EFT) which has a low separation scale, a well-defined expansion scheme, and a few parameters to determine.
ture, a qualitative/theoretical information from the scattering results can be possibly connected to the bound state problem, which makes the main motivation of the present work.

Below the hypertriton breakup momentum, we can choose the typical momentum \( Q \) of the reaction as the \( \Lambda \) particle separation momentum from the hypertriton, which is defined by \( \gamma_\Lambda = \sqrt{2m_\Lambda B_\Lambda} \sim 13.5 \pm 2.6 \text{ MeV} \), where \( m_\Lambda \) is the mass of the \( \Lambda \) system and \( B_\Lambda \) is the \( \Lambda \) particle separation energy from the hypertriton. On the other hand, there is no cutoff dependence in the effective range parameters, however, there are no experimental data to constrain \( g_1(\Lambda_c) \) for \( \Lambda_\Lambda \). This suggests that the system is insensitive to the short range mechanism \[24\]. This is treated as a higher order term.

The \( \Lambda\Lambda\) channel of spin-1 and the other is the deuteron and double-\( \Lambda \) system, where we assume that the double-\( \Lambda \) state problem on the existence of bound states we investigate the effect of the contact term in the \( \Lambda_\Lambda \) system. For this purpose we consider two cases. In the first case, we do not include the contact interaction by setting \( g_1 = 0 \). Then the system is found to have a large negative scattering length at \( \Lambda_c \simeq \Lambda_H \), which may imply the formation of a quasi-bound state. Furthermore, if \( \Lambda \) is sent to the asymptotic limit, \( \Lambda_c \to \infty \), we find that a bound state arises in the system.

In the second case, we turn on the contact interaction. To constrain the value of \( g_1(\Lambda_c) \), we employ the results of the potential model calculations of Refs. \[11\] \[12\] and determine \( g_1(\Lambda_c) \) by using the computed double-\( \Lambda \) separation energy \( B_{\Lambda\Lambda} \) for given values of \( \sigma_{\Lambda\Lambda} \). Then we find that the renormalized \( g_1(\Lambda_c) \) exhibits so-called the limit-cycle when \( \Lambda_c \) is sent to the asymptotic limit. In the present work, we also calculate \( B_{\Lambda\Lambda} \) as a function of \( \sigma_{\Lambda\Lambda} \) for a fixed \( g_1(\Lambda_c) \) and a correlation between \( B_{\Lambda\Lambda} \) and \( 1/a_1 \) as well, where \( a_1 \) is the scattering length of the S-wave hypertriton-\( \Lambda \) scattering in the spin triplet channel at \( \Lambda_H \). We find that the \( \sigma_{\Lambda\Lambda} \)-dependence of \( B_{\Lambda\Lambda} \) is quite sensitive to the value of \( \Lambda_c \). For example, \( B_{\Lambda\Lambda} \) is found to be almost insensitive to \( \sigma_{\Lambda\Lambda} \) when \( \Lambda_c \simeq \Lambda_H \). On the other hand, the reported \( \sigma_{\Lambda\Lambda} \)-dependence of \( B_{\Lambda\Lambda} \) in the potential model calculations of Refs. \[11\] \[12\] is recovered when \( \Lambda_c \simeq 6 \Lambda_H \). In the present work, we will investigate the implications of the choice on the cutoff \( \Lambda_c \) and the \( \sigma_{\Lambda\Lambda} \)-dependence of the properties of \( \Lambda_\Lambda \) system in the cluster theory.

This paper is organized as follows. We start with the relevant effective Lagrangian in the next section, which defines notations and our basic tools for studying hypernuclei. In Sec. \[III\] the two-body parts of the \( \Lambda\Lambda \) system, i.e., the dressed \( \Lambda\Lambda \) dibaryon propagator in \( ^1S_0 \) channel and the dressed hypertriton propagator (as a \( \Lambda\Lambda \) system), are constructed. In Sec. \[IV\] the integral equations of the \( \Lambda\Lambda \) three-body system for the S-wave hypertriton-\( \Lambda \) scattering are constructed in the spin singlet and triplet states. The numerical results are presented in Sec. \[V\] and Section \[VI\] contains a summary and conclusions of this work.

II. EFFECTIVE LAGRANGIAN

In EFT, effective Lagrangian is constructed on the symmetry requirement with relevant degrees of freedom...
at low energies being expanded in terms of the number of derivatives order by order \[27\]. The effective Lagrangian at LO for this work can be written as

\[ \mathcal{L} = \mathcal{L}_\Lambda + \mathcal{L}_d + \mathcal{L}_s + \mathcal{L}_t + \mathcal{L}_M. \]  

(1)

Here, \( \mathcal{L}_\Lambda \) and \( \mathcal{L}_d \) are the standard one-body \( \Lambda \) and (elementary) deuteron Lagrangian in the heavy-baryon formalism \[28\], which read

\[ \mathcal{L}_\Lambda = B_\Lambda^I \left[ iv \cdot \partial + \frac{(v \cdot \partial)^2 - \partial^2}{2m_\Lambda} \right] B_\Lambda + \cdots, \]  

(2)

\[ \mathcal{L}_d = d_i^t \left[ iv \cdot \partial + \frac{(v \cdot \partial)^2 - \partial^2}{2m_d} \right] d_i + \cdots, \]  

(3)

where \( B_\Lambda \) is the \( \Lambda \)-baryon field of spin-1/2, \( d_i \) is the deuteron (vector) field of spin-1, and \( v^\mu \) is a velocity vector with \( v^\mu = (1, 0) \) in our case. The \( \Lambda \) and deuteron masses are represented by \( m_\Lambda \) and \( m_d \), respectively. The dots denote the higher order terms that are irrelevant for the LO calculations.

Equation (1) also contains the Lagrangian for the composites containing strangeness. For this purpose, we introduce \( s \) and \( t \) fields to denote the \( \Lambda \Lambda \) dibaryon in the \( ^1S_0 \) state and the \( \Lambda d \) composite in the \( ^2S_1/2 \) state. Then \( \mathcal{L}_s \) and \( \mathcal{L}_t \) are the Lagrangians for these fields including \( s \leftrightarrow \Lambda \Lambda \) and \( t \leftrightarrow \Lambda d \) interactions, which read \[29\] \[31\]

\[ \mathcal{L}_s = \sigma_s s^t \left[ iv \cdot \partial + \frac{(v \cdot \partial)^2 - \partial^2}{4m_\Lambda} + \Delta_s \right] s - y_s \left[ s^t \left( B_\Lambda^t P^{(1S_0)} B_\Lambda \right) + \text{H.c.} \right] + \cdots, \]  

(4)

\[ \mathcal{L}_t = \sigma_t t^t \left[ iv \cdot \partial + \frac{(v \cdot \partial)^2 - \partial^2}{2(m_d + m_\Lambda)} + \Delta_t \right] t + \frac{y_t}{\sqrt{3}} \left[ t^t \sigma \cdot d B_\Lambda + \text{H.c.} \right] + \cdots, \]  

(5)

where \( \sigma_s \) and \( \sigma_t \) are sign factors, \( \Delta_s \) and \( \Delta_t \) are the mass differences between the composite states and their constituents, and \( y_s \) and \( y_t \) are coupling constants. The spin projection operator of the \( \Lambda \Lambda \) composite onto the \(^1S_0 \) state is

\[ P^{(1S_0)} = -\frac{i}{2} \sigma_2, \]  

(6)

The three-body contact interaction is given by the Lagrangian \( \mathcal{L}_{At} \), where \( t \) and \( \Lambda \) fields are in the \(^3S_1 \) channel, which reads

\[ \mathcal{L}_{At} = -\frac{g_1(A_c)}{\Lambda_c^2} \left( B_\Lambda^T P_i^{(3S_1)} t \right)^\dagger \left( B_\Lambda T P_i^{(3S_1)} t \right) + \cdots, \]  

(7)

with the spin projection operator onto the \(^3S_1 \) state,

\[ P_i^{(3S_1)} = -\frac{i}{2} \sigma_2 \sigma_i. \]  

(8)

The coupling constant of the three-body contact interaction is given by \( g_1(A_c) \) as a function of the cutoff \( \Lambda_c \), which will be introduced in the integral equations below.

### III. TWO-BODY PART

#### A. S-wave \( \Lambda\Lambda \) scattering in \(^1S_0 \) channel

At low energies, we assume that the dominant partial wave of \( \Lambda \Lambda \) scattering is the \(^1S_0 \) state and the scattering process can be described by the effective range parameters. Therefore, this is similar to the low-energy nucleon-nucleon scattering in the \(^1S_0 \) channel studied, for example, in Ref. \[30\]. Diagrams for the dressed dibaryon field and for the scattering amplitude are shown in Figs. 1 and 2 respectively.

Referring the details to Ref. \[30\], we can obtain the scattering amplitude in the center-of-mass (CM) frame as

\[ A(E) = \frac{4\pi}{m_\Lambda} \left( -\frac{1}{a_{\Lambda\Lambda}} + \frac{1}{2} r_{\Lambda\Lambda} k^2 - ik \right)^{-1}, \]  

(9)

where \( a_{\Lambda\Lambda} \) and \( r_{\Lambda\Lambda} \) are the scattering length and effective range of \( \Lambda \Lambda \) scattering in the \(^1S_0 \) channel. The on-shell total energy is \( E = k^2 / m_\Lambda \) with \( k = |k| \).

Thus the renormalized dressed dibaryon propagator can be written as

\[ D_s(p_0, p) = \frac{4\pi}{m_\Lambda y_s^2} \left[ \frac{1}{a_{\Lambda\Lambda}} + \frac{1}{2} r_{\Lambda\Lambda} \left( -m_\Lambda p_0 + \frac{1}{4} p^2 - i\epsilon \right) \right]^{-1} - \sqrt{-m_\Lambda p_0 + \frac{1}{4} p^2 - i\epsilon} \]  

(10)

parameters from the bubble diagrams. We use the same cutoff value for renormalizing \( a_{\Lambda\Lambda} \) and \( r_{\Lambda\Lambda} \) in the three-body part, which will be discussed in Sec. IV.
its A separation energy is $B_\Lambda = 0.13 \pm 0.05$ MeV. We refer the readers to Ref. [32] for a study on this state within pionless EFT.

**B. S-wave $\Lambda d$ system in hypertriton channel**

The hypertriton ($^3\Lambda$H) has the quantum numbers of $J^P = 1/2^+$ and $T = 0$, where $T$ stands for isospin, and its $\Lambda$ separation energy is $B_\Lambda = 0.13 \pm 0.05$ MeV. We refer the readers to Ref. [32] for a study on this state within pionless EFT.

Shown in Fig. 4 are the diagrams for the dressed hypertriton ($t$ field) propagator as a $\Lambda d$ composite state. Then the renormalized dressed hypertriton propagator is obtained as

\[
D_t(p_0, p) = \frac{2\pi}{\mu_{\Lambda d} y_t} \left\{ \frac{1}{a_{\Lambda d}} + \frac{1}{2} r_{\Lambda d} \left[ -2\mu_{\Lambda d} \left( p_0 - \frac{1}{2(m_\Lambda + m_d)} p^2 + i\epsilon \right) \right] \right\}^{-1}
\]

with

\[
y_t = -\frac{1}{\mu_{\Lambda d}} \sqrt{\frac{2\pi}{r_{\Lambda d}}},
\]

where $\mu_{\Lambda d}$ is the reduced mass of the $\Lambda d$ system, i.e., $\mu_{\Lambda d} = m_\Lambda m_d / (m_\Lambda + m_d)$, and $a_{\Lambda d}$ and $r_{\Lambda d}$ are the effective range parameters of the $S$-wave $\Lambda - d$ scattering in the hypertriton channel. In Ref. [32], these effective range parameters are estimated as $a_{\Lambda d} = 16.8^{+4.4}_{-2.4}$ fm, and $r_{\Lambda d} = 2.3 \pm 0.3$ fm, which leads to $\gamma_{\Lambda d} = 1/a_{\Lambda d} + r_{\Lambda d}^2/2 \approx 12.8$ MeV when we use the central values of the parameters. This value is consistent with the one given in Sec. 4 within error.

Since there exists a bound state for hypertriton, the propagator should have a pole at $k = i\gamma_{\Lambda d}$ and we may rewrite the on-energy-shell dressed propagator as

\[
D_t(E) = \frac{2\pi}{\mu_{\Lambda d} y_t} \left[ \gamma_{\Lambda d} - \frac{1}{2} r_{\Lambda d} (k^2 + \gamma_{\Lambda d}^2) + ik \right]^{-1}
\]

where $E = k^2 / (2\mu_{\Lambda d})$. Furthermore, near the pole, the propagator can be further simplified as

\[
D_t(E) \approx \frac{Z_{\Lambda d}}{E + B_\Lambda} \quad \text{with} \quad Z_{\Lambda d} = \frac{\gamma_{\Lambda d} r_{\Lambda d}}{1 - \gamma_{\Lambda d} r_{\Lambda d}},
\]

where $Z_{\Lambda d}$ is the wave function normalization factor of the hypertriton as a $\Lambda d$ system. Since the inverse of the effective range has a large scale, $r_{\Lambda d}^{-1} \approx 86$ MeV, one can see that the KSW counting rules, where the propagator and $Z_{\Lambda d}$ are expanded in terms of $r_{\Lambda d}^{-1}$, would be a good approximation, which can be seen from the fact that $\gamma_{\Lambda d} r_{\Lambda d} \approx 0.16 < 1/3$.

**IV. THREE-BODY PART**

In this Section, we construct the integral equations for $S$-wave scattering of hypertriton and $\Lambda$, which has two spin channels, $S = 0$ and $1$, because both the hypertriton and $\Lambda$ have spin-1/2. For $S = 0$ channel, the amplitude $t(p, k; E)$ consists of hypertriton-$\Lambda$ channel only. In Fig. 3 diagrams of the integral equation for the scattering amplitude are shown, which lead to

\[
t(p, k; E) = -3K_{(a)}(p, k; E) + \frac{1}{2\pi^2} \int_0^{\Lambda_c} d\ell \ell^2 3K_{(a)}(p, \ell; E)D_t \left( E - \frac{\ell^2}{2m_\Lambda}, \ell \right) t(\ell, k; E)
\]
with the one-deuteron-exchange interaction $K_{(a)}(p, ℓ; E)$,
\[
K_{(a)}(p, ℓ; E) = \frac{1}{3} \frac{m_d y_t^2}{2p\ell} \ln \left( \frac{m_d y_t^2 (p^2 + \ell^2) + p\ell - m_d E}{m_d y_t^2 (p^2 + \ell^2) - p\ell - m_d E} \right),
\]
(17)
where $p$ and $k$ are relative off-shell and on-shell momenta of hypertriton-$\Lambda$ scattering in the CM frame, respectively, and $E$ is the total energy,
\[
E = -\frac{\gamma_{\Lambda d}^2}{2\mu_{\Lambda d}} + \frac{1}{2\mu_{\Lambda(\Lambda d)}} k^2,
\]
with $\mu_{\Lambda(\Lambda d)}$ being the reduced mass of the $\Lambda$-$\Lambda d$ system so that $\mu_{\Lambda(\Lambda d)} = m_\Lambda (m_\Lambda + m_d)/(2m_\Lambda + m_d)$. A sharp cutoff momentum $\Lambda_c$ was introduced as before in the integral equation. However, as we shall see below, the integral equation is insensitive to the value of $\Lambda_c$, which weakens the necessity of three-body contact interactions.

For $S = 1$ channel, however, we have two scattering amplitudes, namely, $a(p, k; E)$ for the spin triplet $\Lambda(\Lambda d)$ (hypertriton) channel and $b(p, k; E)$ that connects the $\Lambda(\Lambda d)$ to the $ds$ (deuteron and hypertriton) channel. In Fig. 4 diagrams of coupled integral equations are presented, from which we obtain
\[
a(p, k; E) = K_{(a)}(p, k; E) - \frac{g_1(\Lambda_c)}{\Lambda_c^2} \frac{1}{2\pi^2} \int_0^{\Lambda_c} d\ell \ell^2 \left[ K_{(a)}(p, \ell; E) - \frac{g_1(\Lambda_c)}{\Lambda_c^2} \right] D_t \left( E - \frac{\ell^2}{2\mu_{\Lambda d}}, \ell \right) a(\ell, k; E)
\]
\[
- \frac{1}{2\pi^2} \int_0^{\Lambda_c} d\ell \ell^2 K_{(b1)}(p, \ell; E) D_s \left( E - \frac{\ell^2}{2\mu_{\Lambda d}}, \ell \right) b(\ell, k; E),
\]
\[
b(p, k; E) = K_{(b2)}(p, k; E) - \frac{1}{2\pi^2} \int_0^{\Lambda_c} d\ell \ell^2 K_{(b2)}(p, \ell; E) D_t \left( E - \frac{\ell^2}{2\mu_{\Lambda d}}, \ell \right) a(\ell, k; E),
\]
(19)
with one-$\Lambda$-exchange interactions $K_{(b1)}(p, \ell; E)$ and $K_{(b2)}(p, \ell; E)$, which read
\[
K_{(b1)}(p, \ell; E) = -\sqrt{2} \frac{m_\Lambda y_t s_t}{3} \frac{1}{2p\ell} \ln \left[ \frac{p^2 + \frac{m_\Lambda}{2p_{\Lambda d}} \ell^2 + p\ell - m_\Lambda E}{p^2 + \frac{m_\Lambda}{2p_{\Lambda d}} \ell^2 - p\ell - m_\Lambda E} \right],
\]
(20)
\[
K_{(b2)}(p, \ell; E) = -\sqrt{2} \frac{m_\Lambda y_t s_t}{3} \frac{1}{2p\ell} \ln \left[ \frac{m_\Lambda p^2 + \ell^2 + p\ell - m_\Lambda E}{m_\Lambda p^2 + \ell^2 - p\ell - m_\Lambda E} \right],
\]
(21)
In Eq. (19), we have introduced the three-body contact interaction that contains the coupling constant $g_1(\Lambda_c)$\(^2\)

\[^2\text{The coupling constant } g_1(\Lambda_c)\text{ is a dimensionless quantity.}\]
FIG. 5. Diagrams of coupled integral equations for $S$-wave scattering of hypertriton and $\Lambda$ for spin triplet ($S = 1$) channel. See the captions of Figs. 1 and 3 as well.

V. NUMERICAL RESULTS

A. $S$-wave scattering of hypertriton and $\Lambda$ in $S = 0$ channel

In the dressed hypertriton propagator $D_t$ given in Eq. (12), there are two singularities at $\ell \simeq 13$ MeV and $\ell \simeq 172$ MeV when $E = 0$ in Eq. (19). The first one corresponds to the binding momentum of the hypertriton in the $\Lambda$-d system and the second one to an unphysical deeply-bound state. We avoid the effect from the unphysical deeply bound state by expanding the effective range correction, as mentioned above, employing the KSW counting rules.

The on-shell scattering matrix is given by

$$T(p, k) = -3 \gamma_{\Lambda d} r_{\Lambda d} K_{(0)}(p, k; E)$$

$$- \frac{3}{2\pi^2 \mu_{\Lambda (d)}} \int_0^{\Lambda_c} d\ell K_{(0)}(p, \ell; E) \left\{ \gamma_{\Lambda d} + \sqrt{\frac{\gamma_{\Lambda d}^2 + \frac{\mu_{\Lambda d}}{\mu_{\Lambda (d)}} (\ell^2 - k^2)}{\ell^2 - k^2 - i\epsilon}} \right\} \frac{\ell^2 T(\ell, k)}{\ell^2 - k^2 - i\epsilon},$$

so that it reduces to the scattering length as $a_0 = a(0, 0)$.

We numerically calculate the off-diagonal part of the scattering length $a_0(p, 0)$ with $\Lambda_c \sim 170$ MeV to find that the off-diagonal part of the scattering length becomes indeed very small when the off-shell momentum $p$ is larger than the large scale $\Lambda_H \sim \gamma \simeq 45.7$ MeV. We also calculate the scattering length $a_0(0, 0)$ as a function of the cutoff $\Lambda_c$ to find that $a(0, 0)$ is nearly independent of the cutoff if it is relatively small such as $\Lambda_c \simeq 20$ MeV. Therefore, the $S$-wave hypertriton-$\Lambda$ scattering in spin singlet channel would be well described by considering the cutoff region of $\Lambda_c \simeq \Lambda_H$. From this procedure we obtain

$$a_0 = 16.0 \pm 3.0 \text{ fm},$$

This shows that the integral equation is expressed in terms of two parameters, namely, $\gamma_{\Lambda d}$ and $\Lambda_c$, in addition to the deuteron and $\Lambda$ masses. As mentioned before, this integral equation is insensitive to the value of $\Lambda_c$ and thus the scattering in the $S = 0$ channel is well controlled by one effective range parameter, $\gamma_{\Lambda d}$.

The scattering length $a_0$ of the $S$-wave hypertriton-$\Lambda$ scattering in the $S = 0$ channel is then computed by taking the limit for the on-shell momentum $k \to 0$, which leads to $T(0, 0) = -\frac{2\pi}{\rho_{\Lambda (d)}} a_0$. Here, we introduce the half-off-shell scattering length $a(p, 0)$ as

$$a(p, 0) = -\frac{\mu_{\Lambda (d)}}{2\pi} T(p, 0),$$

FIG. 6. Phase shift $\delta_0$ (in degrees) of the $S$-wave hypertriton-$\Lambda$ scattering in the spin singlet channel as a function of momentum $k$ (in MeV).

so that it reduces to the scattering length as $a_0 = a(0, 0)$.
which is our prediction on the scattering length, where the error was estimated from the uncertainties in $\gamma_{\Lambda d}$.

In Fig. 6, the calculated phase shift $\delta_0$ of the $S$-wave hypertriton-$\Lambda$ scattering in the spin singlet channel is presented as a function of $k$. The form of the calculated phase shift $\delta_0$ determines the two effective range parameters as $a_0 \approx 16.0$ fm and $r_0 \approx 2$ fm. In addition, we find no limit-cycle in the numerical calculation of the integral equation within the range up to $\Lambda_c \sim 10^8$ MeV.

### B. $\Lambda^3\Lambda H$ bound state and $S$-wave scattering of hypertriton-$\Lambda$ in $S = 1$ channel

For the spin triplet channel, the coupled integral equations can be rewritten in terms of the half-off-shell scattering amplitudes $a_1(p, k)$ and $b_1(p, k)$ which are defined by

\begin{align}
a_1(p, k) &= -\frac{Z_{\Lambda d}}{2\pi} \mu_{\Lambda(\Lambda d)} \left[ K_{(a)}(p, k; E) - \frac{g_1(\Lambda_c)}{\Lambda_c^2} \right] - \frac{1}{2\pi^2} \int_0^{\Lambda_c} d\ell \int_0^{\Lambda_c} d\ell' \left[ K_{(a)}(p, \ell; E) - \frac{g_1(\Lambda_c)}{\Lambda_c^2} \right] D_t \left( E - \frac{\ell^2}{2m_\Lambda}, \ell \right) a_1(\ell, k),

b_1(p, k) &= -\frac{Z_{\Lambda d}}{2\pi} \mu_{\Lambda(\Lambda d)} K_{(b2)}(p, k; E) - \frac{1}{2\pi^2} \int_0^{\Lambda_c} d\ell \int_0^{\Lambda_c} d\ell' K_{(b2)}(p, \ell; E) D_t \left( E - \frac{\ell^2}{2m_\Lambda}, \ell \right) a_1(\ell, k),
\end{align}

with the normalizations

\begin{align}
a_1(k, k) &= \sqrt{Z_{\Lambda d}} a(k, k) \sqrt{Z_{\Lambda d}},

b_1(k, k) &= \sqrt{Z_{\Lambda d}} b(k, k) \sqrt{Z_{\Lambda d}}.
\end{align}

The scattering length $a_1$ is then defined as

\begin{align}
a_1 = -\frac{\mu_{\Lambda(\Lambda d)}}{2\pi} a_1(0, 0).
\end{align}

Because the effect from the unphysical singularities in the dressed dibaryon and hypertriton propagators ($D_s$ and $D_t$) to the scattering length $a_1$ is significant, we employ the KSW counting rules and expand the propagators and the wave function normalization factor $Z_{\Lambda d}$ in terms of the effective ranges $r_{\Lambda d}$ and $r_{d(\Lambda \Lambda)}$, as discussed in Sec. \[\text{II}\]. Therefore, at LO, the propagators $D_s$ and $D_t$ and the wave function normalization factor $Z_{\Lambda d}$ are written as

\begin{align}
& Z_{\Lambda d}^{\text{LO}} = \gamma_{\Lambda d} r_{\Lambda d},

& D_t^{\text{LO}} \left( E - \frac{\ell^2}{2m_\Lambda}, \ell \right) = -\frac{2\pi \mu_{\Lambda(\Lambda d)}}{\mu_{d(\Lambda \Lambda)} \gamma_{\Lambda d}} \left[ \gamma_{\Lambda d} + \sqrt{\gamma_{\Lambda d}^2 + \frac{\mu_{\Lambda(\Lambda d)}}{\gamma_{\Lambda d}} (\ell^2 - k^2)} \right] \frac{1}{\ell^2 - k^2 - i\epsilon},

& D_s^{\text{LO}} \left( E - \frac{\ell^2}{2m_\Lambda}, \ell \right) = \frac{4\pi}{m_\Lambda y_\Lambda^2} \left[ \frac{1}{a_{\Lambda \Lambda}} - \sqrt{\frac{m_\Lambda}{2\mu_{d(\Lambda \Lambda)}} \gamma_{\Lambda d}^2 - \frac{3}{2} \left( \frac{\ell^2}{\mu_{d(\Lambda \Lambda)}} \right)} \right]^{-1},
\end{align}

where $\mu_{d(\Lambda \Lambda)}$ is the reduced mass of the $d$-(\Lambda \Lambda) system, $\mu_{d(\Lambda \Lambda)} = 2m_\Lambda m_d / (2m_\Lambda + m_d)$.

In addition to the masses, therefore, we have four parameters, namely, $\gamma_{\Lambda d}$, $a_{\Lambda \Lambda}$, $g_1(\Lambda_c)$, and $\Lambda_c$. In the present work, we fix $\gamma_{\Lambda d}$ by the hypertriton binding energy. The parameter $a_{\Lambda \Lambda}$ may be determined from other available empirical information. However, there exists no available information from the three-body system to con-
strain the value of $g_1(\Lambda_c)$. In the present work, therefore, instead of studying the energy levels of the $ΛΛ^4H$ hypernucleus, we examine the effect of the coupling $g_1(\Lambda_c)$ in this system.

1. Scattering length $a_1$ without three-body contact interaction

We first consider the case when $g_1(\Lambda_c) = 0$ and calculate the two-$Λ$ separation energy $B_{ΛΛ}$ in the $ΛΛ^4H$ bound state and the scattering length $a_1$ of the $S$-wave hypertriton-$Λ$ scattering for the spin triplet channel at LO. In this case we find that there is no bound state formed with the cutoff value in the range of $\Lambda_c = 50 \sim 300$ MeV.

In Fig. 7, we present our results for the LO scattering length $a_1$ with several values of $a_{ΛΛ}$, namely, $a_{ΛΛ} = -0.5$, $-1.0$, $-1.5$, $-2.0$ fm, as a function of the momentum cutoff $\Lambda_c$. This shows that the calculated $a_1$ curves show a significant dependence on $\Lambda_c$ as well as on $a_{ΛΛ}$. The $a_{ΛΛ}$-dependence of $a_1$ becomes more significant when $\Lambda_c$ is larger than $Λ_H$ as shown in Fig. 7. When the cutoff parameter $\Lambda_c$ is down close to the large scale of the theory, i.e., $\Lambda_c \simeq Λ_H \sim 45.7$ MeV, such a dependence becomes mild. We then obtain negative values for the scattering length, namely, $a_1 \approx -21.7$, $-22.7$, $-23.8$, $-24.8$ fm for $a_{ΛΛ} = -0.5$, $-1.0$, $-1.5$, $-2.0$ fm, respectively, with $\Lambda_c = 45.7$ MeV. Since $a_1$ is negative and its magnitude is large, it may imply a formation of a quasi-bound state.

As $\Lambda_c$ increases, $a_1$ decreases until it shows a pole-structure at around $\Lambda_c \sim 80$, $33$, $17$, $10$ GeV depending on the value of $a_{ΛΛ}$. After passing the pole, $a_1$ changes the sign as shown in Fig. 7. This corresponds to a formation of a bound state with zero binding energy at such a huge cutoff. In other words, the one-deuteron-exchange interaction has a sensitivity to $\Lambda_c$ and it becomes attractive enough to make a bound state at the asymptotic limit of the cutoff.

To make the result cutoff-independent, however, one needs to promote the three-body contact interaction at LO so that the cutoff dependence is controlled by the additional coupling constant $g_1$. We work on in this scheme below.

2. $ΛΛ^4H$ bound state with three-body contact interaction

We now consider the case with $g_1(\Lambda_c) \neq 0$ to investigate its role in the $ΛΛ^4H$ hypernucleus. Since there is no experimental information to constrain the value of $g_1(\Lambda_c)$, we adopt the values of this coupling constant determined as follows. We first assume a formation of the $ΛΛ^4H$ bound state due to the three-body-contact interaction and fit $g_1(\Lambda_c)$ to reproduce the potential model results of Refs. [11, 12]. To be specific, we choose the following three sets for $B_{ΛΛ}$ and $a_{ΛΛ}$:

$$
\text{(I)} \quad B_{ΛΛ} = 0.2 \text{ MeV and } a_{ΛΛ} = -0.5 \text{ fm,} \\
\text{(II)} \quad B_{ΛΛ} = 0.6 \text{ MeV and } a_{ΛΛ} = -1.5 \text{ fm,} \\
\text{(III)} \quad B_{ΛΛ} = 1.0 \text{ MeV and } a_{ΛΛ} = -2.5 \text{ fm.} 
$$

In Fig. 8 we show the calculated strength of the three-body contact interaction $g_1(\Lambda_c)$ as a function of the cutoff $\Lambda_c$, which can reproduce the three parameter sets of Eq. (33).

One can see that the curves of $g_1(\Lambda_c)$ are rather mildly varying at $\Lambda_c = 10 \sim 10^4$ MeV, and each curve has a singularity at $\Lambda_c \sim 10^5$ MeV indicating the possibility of the first cycle of the limit-cycle. This implies that the one-deuteron-exchange interaction for the $S = 1$ channel contains an attractive (singular) interaction at very high momentum, say, $\Lambda_c \sim 10^5$ MeV. This property has also been observed in the calculation of $a_1$ as shown in Fig. 7.
At such a very high momentum, however, the applicability of the present theory, a very low energy EFT, cannot be guaranteed and thus the mechanisms of the formation of a bound state must have different origins. We note, on the other hand, that, if we choose $\Lambda_c$ smaller at $\Lambda_c = 50, 150, 300$ MeV. Here, the coupling $g_4(\Lambda_c)$ is in a natural size and may be generated from the mechanisms of high energy such as $\sigma$-meson exchange or two-pion exchange near the intermediate range of nuclear force, i.e., $\Lambda_c = 300 \sim 600$ MeV.

In order to study the correlation between $B_{\Lambda\Lambda}$ and $a_{\Lambda\Lambda}$, we calculate $B_{\Lambda\Lambda}$ as a function of $a_{\Lambda\Lambda}$ and show the results in Fig. 9 for various cutoff values, i.e., $\Lambda_c = 50, 150, 300$ MeV. Here, the coupling $g_4(\Lambda_c)$ is fixed by using the parameter set (I), i.e., $B_{\Lambda\Lambda} = 0.2$ MeV and $a_{\Lambda\Lambda} = -0.5$ fm, which is marked by a filled square in Fig. 9. This is achieved with $g_4(\Lambda_c) \simeq -2.48, -2.83, -2.96$ for $\Lambda_c = 50, 150, 300$ MeV, respectively. Once the starting values are fixed, we vary the value of $a_{\Lambda\Lambda}$ for a fixed value of $\Lambda_c$, which changes the values of $B_{\Lambda\Lambda}$. We then find that the behaviors of the $B_{\Lambda\Lambda}$ curves as functions of $a_{\Lambda\Lambda}$ are quite sensitive to the values of the cutoff $\Lambda_c$. For example, when we choose $\Lambda_c = 50$, i.e., $\Lambda_c = 50$ MeV, $B_{\Lambda\Lambda}$ is insensitive to the value of $a_{\Lambda\Lambda}$ and makes a nearly flat curve as shown by the dotted line in Fig. 9. However, with a larger cutoff value, $\Lambda_c = 300$ MeV, $B_{\Lambda\Lambda}$ strongly depends on $a_{\Lambda\Lambda}$ and we can fairly well reproduce the $a_{\Lambda\Lambda}$-dependence of $B_{\Lambda\Lambda}$ obtained by Filikhin and Gal [11] or Nemura et al. [12].

This may imply that the main part of the correlation between $B_{\Lambda\Lambda}$ and $a_{\Lambda\Lambda}$ in potential model calculations is related to the high momentum part and, when we choose the cutoff $\Lambda_c \simeq \Lambda_H$, the mechanisms with high momentum are integrated out and their effects are absorbed by the renormalized three-body contact interaction $g_4(\Lambda_c)$. Thus we do not have the dynamics that is sensitive to the high momentum regime and this leads to the cutoff-insensitive results. Therefore, when we choose $\Lambda_c = 50$ MeV in our cluster EFT, the theory does not to adequately probe the $\Lambda$-$\Lambda$ interactions, but, when we choose $\Lambda_c = 300$ MeV, we can fairly well reproduce the results obtained in the potential model calculations. However, in the latter case, the theory becomes inconsistent because of neglecting other mechanisms relevant in the high momentum region, such as the channels of deuteron break-up into two nucleons and of meson-exchanges among baryons.

In Fig. 10 we present our results on the correlation between $B_{\Lambda\Lambda}$ and $1/a_1$ with four values of $a_{\Lambda\Lambda}$ where $g_4(\Lambda_c)$ is fixed by using the condition that $B_{\Lambda\Lambda} = 0.2$ MeV at $\Lambda_c = 50$ MeV. Thus with $\Lambda_c = 50$ MeV, we have $g_4 = -2.48, -2.45, -2.43, -2.40$ for $a_{\Lambda\Lambda} = -0.5, -1.0, -1.5, -2.0$ fm, respectively. Then the curves are obtained by varying $\Lambda_c$ from 50 MeV to 300 MeV with the fixed values of $g_4$ determined at $\Lambda_c = 50$ MeV. We find that, at $\Lambda_c = 50$ MeV, which gives the starting points of the curves at the top right corner (marked by open squares), the calculated scattering length $a_1$ at LO turned out to be positive due to the existence of $^4\Lambda_\Lambda$ bound state and the positions of these points are not sensitive to the value of $a_{\Lambda\Lambda}$, as was seen in Fig. 9 for the case of $B_{\Lambda\Lambda}$ with $\Lambda_c = 50$ MeV. Thus we have $a_1 \sim 5.7$ fm corresponding to $B_{\Lambda\Lambda} \simeq 0.2$ MeV. By increasing the cutoff values, we obtain the lower values of $B_{\Lambda\Lambda}$. When $a_{\Lambda\Lambda} = -0.5$ fm, the $^4\Lambda_\Lambda$ bound state eventually becomes unbound, and when $a_{\Lambda\Lambda} = -2.0$ fm, $B_{\Lambda\Lambda}$ has a minimum.

Fig. 9. (Color online) Calculated two-$\Lambda$ separation energy $B_{\Lambda\Lambda}$ from $^4\Lambda_\Lambda$ bound state as a function of the scattering length $a_{\Lambda\Lambda}$ of the S-wave $\Lambda\Lambda$ scattering for the $^1S_0$ channel with the cutoff values $\Lambda_c = 50, 150, 300$ MeV. The value of $g_4(\Lambda_c)$ of all three curves is fitted at the point (I): $B_{\Lambda\Lambda} = 0.2$ MeV and $a_{\Lambda\Lambda} = -0.5$ fm, marked by a filled square. The points (II) and (III) are also included as blank squares in the figure.

Fig. 10. (Color online) Correlations between $B_{\Lambda\Lambda}$ and $1/a_1$ with four values of $a_{\Lambda\Lambda}$ where $g_4(\Lambda_c)$ is fixed by using the condition that $B_{\Lambda\Lambda} = 0.2$ MeV at $\Lambda_c = 50$ MeV. Thus with $\Lambda_c = 50$ MeV, we have $g_4 = -2.48, -2.45, -2.43, -2.40$ for $a_{\Lambda\Lambda} = -0.5, -1.0, -1.5, -2.0$ fm, respectively. Then the curves are obtained by varying $\Lambda_c$ from 50 MeV to 300 MeV with the fixed values of $g_4$ determined at $\Lambda_c = 50$ MeV. We find that, at $\Lambda_c = 50$ MeV, which gives the starting points of the curves at the top right corner (marked by open squares), the calculated scattering length $a_1$ at LO turned out to be positive due to the existence of $^4\Lambda_\Lambda$ bound state and the positions of these points are not sensitive to the value of $a_{\Lambda\Lambda}$, as was seen in Fig. 9 for the case of $B_{\Lambda\Lambda}$ with $\Lambda_c = 50$ MeV. Thus we have $a_1 \sim 5.7$ fm corresponding to $B_{\Lambda\Lambda} \simeq 0.2$ MeV. By increasing the cutoff values, we obtain the lower values of $B_{\Lambda\Lambda}$. When $a_{\Lambda\Lambda} = -0.5$ fm, the $^4\Lambda_\Lambda$ bound state eventually becomes unbound, and when $a_{\Lambda\Lambda} = -2.0$ fm, $B_{\Lambda\Lambda}$ has a minimum.
and then starts to increase with increasing cutoff. We also find that the correlations do not show the sensitivity to $a_{\Lambda\Lambda}$.

VI. SUMMARY AND DISCUSSION

In the present work, we studied the $^4\Lambda\Lambda H$ bound state and S-wave hypertriton-Λ scattering for spin singlet and triplet channels below the hypertriton breakup momentum in Halo EFT at LO by treating the $^4\Lambda\Lambda H$ system as a three-body $\Lambda\Lambda d$ cluster system. In this approach, the hypertriton breakup momentum $\gamma_{\Lambda H} \simeq 13.4$ MeV is chosen to be the typical scale $Q$ of the theory, whereas the deuteron binding momentum $\gamma \simeq 45.7$ MeV to be the high momentum scale $\Lambda_H$. Thus, in such a small typical momentum scale, the deuteron is not broken into two nucleons, which justifies the treatment of the deuteron field as a cluster (elementary) field. Furthermore, our expansion parameter is $Q/\Lambda_H \sim \gamma_{\Lambda d}/\gamma \sim 1/3$.

For the spin singlet channel of the S-wave hypertriton-Λ scattering, the amplitude is nearly independent of the cutoff, thus there is no need to introduce the three-body contact interaction at LO. Consequently, the integral equation at LO is well described by one effective range parameter, $\gamma_{\Lambda d}$. This leads to the value of the scattering length $a_0$ of the S-wave hypertriton-Λ scattering for the spin singlet channel as $a_0 = 16.0 \pm 3.0$ fm. We also found no bound state in this channel at LO.

For the spin triplet channel of the S-wave scattering of hypertriton and Λ, the scattering amplitudes are obtained through two coupled integral equations. We find that when the cutoff parameter $\Lambda_c$ is close to the asymptotic limit, the coupling of the three-body contact interaction, i.e., $g_1(\Lambda_c)$, exhibits the limit-cycle, and thus the three-body contact interaction should be included in the spin triplet channel. Consequently, the coupled integral equations are represented by four parameters, $\gamma_{\Lambda d}$, $a_{\Lambda\Lambda}$, $g_1(\Lambda_c)$, and $\Lambda_c$. The value of $\gamma_{\Lambda d}$ can be fixed from the Λ separation energy of the hypertriton and that of $a_{\Lambda\Lambda}$ may be fixed from other experiments or possibly lattice QCD simulations. However, there is no available experimental data to constrain the value of $g_1(\Lambda_c)$.

When we do not introduce $g_1(\Lambda_c)$ in the theory, we obtain $a_1 \simeq -25 \sim -22$ fm with $\Lambda_c \simeq \Lambda_H$. This may imply that the hypertriton-Λ interaction is attractive but it is not strong enough to form a bound state. Thus, if the $^4\Lambda\Lambda H$ bound state is formed, the main binding mechanism should stem from the mechanisms of high momentum region, which is represented by the coupling $g_1(\Lambda_c)$ in the present approach. Therefore, to take into account this effect, we assume a formation of the $^4\Lambda\Lambda H$ bound state and employ the results of the potential model calculations for the two-Λ separation energy $B_{\Lambda\Lambda}$ for several values of $a_{\Lambda\Lambda}$ to constrain the value of $g_1(\Lambda_c)$. Using the fixed $g_1(\Lambda_c)$, we then calculate $B_{\Lambda\Lambda}$ as a function of $a_{\Lambda\Lambda}$. We also calculate the correlations between $B_{\Lambda\Lambda}$ and $1/a_1$, where $a_1$ is the scattering length of the S-wave hypertriton-Λ scattering for spin triplet channel.

As can be notably seen in the numerical results for the correlation between $B_{\Lambda\Lambda}$ and $a_{\Lambda\Lambda}$ as given in Fig. 9, when the cutoff is chosen to be the large scale of the theory, i.e., $\Lambda_c \simeq \Lambda_H$, $B_{\Lambda\Lambda}$ is insensitive to the value of $a_{\Lambda\Lambda}$. But, when $\Lambda_c$ is larger than $\Lambda_H$, say $\Lambda_c \simeq 6\Lambda_H$, $B_{\Lambda\Lambda}$ is sensitive to $a_{\Lambda\Lambda}$, which gives results similar to the potential model predictions. This would be a natural consequence because $a_{\Lambda\Lambda}$ is a quantity of a large scale, $|a_{\Lambda\Lambda}| \simeq 100 \sim 400$ MeV, compared to the typical scale of the system, $Q \sim \gamma_{\Lambda d} \simeq 13.4$ MeV. In addition, the dynamics that exhibits the sensitivity to the $\Lambda-\Lambda$ interaction above $\Lambda_c \simeq \Lambda_H$ is integrated out and its effect in high momentum is embedded in the contact interaction $g_1(\Lambda_c)$. Meanwhile, although the deuteron cluster theory with a large cutoff value such as $\Lambda_c \simeq 6\Lambda_H$ can reproduce the $a_{\Lambda\Lambda}$-dependence of $B_{\Lambda\Lambda}$ similar to the potential model predictions, this would be inconsistent with the construction principles of EFT and it will miss the important dynamic mechanisms as discussed before. Therefore, the $a_{\Lambda\Lambda}$-sensitivities in the physical observables for the $^4\Lambda\Lambda H$ hypernucleus, such as $B_{\Lambda\Lambda}$, inevitably depend on the scale of the theory. Investigating the $a_{\Lambda\Lambda}$-sensitivity in more detail at another scale in the $^4\Lambda\Lambda H$ system requires to work with a non-cluster theory such as the pionless theory for four-body systems.

Experimentally, we still do not have enough information to judge whether the $^4\Lambda\Lambda H$ system is bound or not. This causes the difficulty for studying the energy levels of the $^4\Lambda\Lambda H$ hypernucleus within EFT since the value of the contact interaction $g_1(\Lambda_c)$ cannot be constrained by other information. Therefore, it would be interesting to apply this approach to other double-Λ hypernuclei, where some empirical data are available such as the $^4\Lambda\Lambda H$ system. The $^6\Lambda\Lambda H$ hypernucleus as a $\Lambda\Lambda\alpha$ three-body cluster system can be investigated in the scheme of EFT. Because the binding energy, or equivalently the two-Λ separation energy, of $^6\Lambda\Lambda H$ is experimentally known, it can be used to determine the strength of the three-body contact interaction in the $\Lambda\Lambda\alpha$ system. Moreover, because the $\alpha$ particle is more tightly bound than the deuteron, the high momentum scale of the cluster theory becomes larger than $\Lambda_H$ of the present work. Therefore, the study of $^6\Lambda\Lambda H$ in Halo/Cluster EFT can provide another tool to study $a_{\Lambda\Lambda}$ in the exotic systems and shed light on our understanding of strong interactions in the strangeness sector. Work in this direction is under progress and will be reported elsewhere.

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