Asteroid (469219) 2016 HO₃, the smallest and closest Earth quasi-satellite

C. de la Fuente Marcos* and R. de la Fuente Marcos
Apartado de Correos 3413, E-28080 Madrid, Spain

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ABSTRACT
A number of Earth co-orbital asteroids experience repeated transitions between the quasi-satellite and horseshoe dynamical states. Asteroids 2001 GO₂, 2002 AA₂⁹, 2003 YN₁₀⁷ and 2015 SO₂ are well-documented cases of such a dynamical behaviour. These transitions depend on the gravitational influence of other planets, owing to the overlapping of a multiplicity of secular resonances. Here, we show that the recently discovered asteroid (469219) 2016 HO₃ is a quasi-satellite of our planet – the fifth one, joining the ranks of (164207) 2004 GU₉, (277810) 2006 FV₃₅, 2013 LX₂₈, and 2014 OL₃₅⁹. This new Earth co-orbital also switches repeatedly between the quasi-satellite and horseshoe configurations. Its current quasi-satellite episode started nearly 100 yr ago and it will end in about 300 yr from now. The orbital solution currently available for this object is very robust and our full N-body calculations show that it may be a long-term companion (time-scale of Myr) to our planet. Among the known Earth quasi-satellites, it is the closest to our planet and as such, a potentially accessible target for future in situ study. Due to its presumably lengthy dynamical relationship with the Earth and given the fact that at present and for many decades this transient object remains well positioned with respect to our planet, the results of spectroscopic studies of this small body, 26–115 m, may be particularly useful to improve our understanding of the origins – local or captured – of Earth’s co-orbital asteroid population. The non-negligible effect of the uncertainty in the value of the mass of Jupiter on the stability of this type of co-orbitals is also briefly explored.

Key words: methods: numerical – celestial mechanics – minor planets, asteroids: general – minor minor planets, asteroids: individual: (469219) 2016 HO₃ – planets and satellites: individual: Earth.

1 INTRODUCTION
The study of the orbital dynamics of small-body populations in the Solar system has multiple practical applications ranging from the computation of impact and cratering rates on to planetary bodies (see e.g. Michel & Morbidelli 2007) to selection of spacecraft accessible targets (see e.g. Adamo et al. 2010). In the particular case of the near-Earth objects (NEOs), the analysis of their present-day orbital architecture, their past orbital evolution, and their future scattering events has made possible to improve steadily our understanding of the dynamical properties of the immediate neighbourhood of our planet.

In general and for a given planetary host, the small-body populations subjected to direct gravitational interaction with the host are either primordial or transient. Transient small-body populations consistently dominate, but primordial dynamical groups are known to exist. Primordial dynamical groups in direct gravitational interaction with a planetary host are captured permanently in resonance and they may have formed near their present location. Within this context, perhaps the most relevant resonance is the 1:1 mean motion resonance that makes objects go around a central star in almost exactly one planetary orbital period. These interesting bodies are termed co-orbitals even if their orbits only resemble that of the host planet in terms of orbital period but they are very eccentric and/or inclined (Morais & Morbidelli 2002). Frozen in orbital parameter space, primordial co-orbital small bodies are the remnants of the physical and dynamical processes that molded their planetary host and this fact makes them particularly interesting when attempting to reconstruct the conditions reigning at that moment. Well-known examples of these populations are the Trojans of Jupiter (see e.g. Di Sisto, Ramos & Beaugé 2014; Wong, Brown & Emery 2014), Mars (see e.g. Marzari et al. 2002; Connors et al. 2005; Scholl, Marzari & Tricarico 2005; Cuk, Christou & Hamilton 2015) or Neptune (see e.g. Parker 2015; Gerdes et al. 2016). In addition to primordial co-orbitals, objects from the planet-crossing small-body populations can undergo temporary co-orbital capture giving rise to transient co-orbital populations. In a multi-planet environment, the orbital

*E-mail: carlosdlfmarcos@gmail.com

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evolution of transient co-orbitals (including capture and ejection) is mostly controlled by secular resonances induced by planets other than the host.

Small bodies are classified dynamically (for instance as Apollos, Apollos, Centaurs, etc.) based on their present-day orbital solutions. The case of co-orbitals, both primordial and transient, is however more complicated. Having a particular set of orbital elements at present time is not enough to claim a co-orbital relationship with a host; a representative set of statistically compatible orbits must be integrated forward and backwards in time to show that the dynamical evolution of the object over a reasonable amount of time is also consistent with being locked in a 1:1 mean motion resonance with the host. Therefore, co-orbital objects are identified indirectly after investigating numerically their orbital evolution.

Minor bodies are classified as co-orbitals of a given host after studying the behaviour of a critical angle. The variable of interest here is the relative mean longitude, $\lambda_r$, or difference between the mean longitude of the object and that of its host. In celestial mechanics, the mean longitude of an object – planet or minor body – is given by $\lambda = M + \Omega + \omega$, where $M$ is the mean anomaly, $\Omega$ is the longitude of the ascending node, and $\omega$ is the argument of perihelion (see e.g. Murray & Dermott 1999). The relative mean longitude of a non-resonant object with respect to a second body circulates; i.e. $\lambda_r \in (0, 360^\circ)$ with all the values being equally probable. However, if an object is locked in a 1:1 mean motion resonance with a host, the critical or resonant angle $\lambda_r$ librates or oscillates around a certain, well-defined value. Although the critical value is a function of the orbital eccentricity and inclination of the objects involved (Namouni, Christou & Murray 1999; Namouni & Murray 2000), $0^\circ$, $\pm 60^\circ$ or $180^\circ$ are often cited in the literature as the signposts of co-orbital behaviour (see e.g. Murray & Dermott 1999).

There are three elementary or primary co-orbital configurations: quasi-satellite or retrograde satellite, Trojan or tadpole, and horseshoe. However, in a multi-planet environment, the co-orbital dynamics experienced by an object may show a surprisingly high level of complexity as compound states (see e.g. Morais & Morbidelli 2002) and recurrent transitions between the three elementary co-orbital dynamical states (Namouni et al. 1999; Namouni & Murray 2000) are feasible. Out of the three primary co-orbital configurations, the quasi-satellite dynamical state is the rarest and it is characterized by the libration of $\lambda_r$ about $0^\circ$. This value of the resonant angle is independent from the nature of the orbital eccentricity and inclination of the objects involved. Although a bona fide quasi-satellite appears to travel around its host when its motion is viewed in a frame of reference rotating with the host (see e.g. Fig. 1), the object is not gravitationally bound to it and describes complex, drifting loops as observed from the host (see e.g. Fig. 2).

The term ‘quasi-satellite’ was brought before and popularized among the astronomical community by Mikkola & Innanen (1997). However, the associated concept had been first studied by Jackson (1913) and its energy balance was initially explored by Hénon (1969), who coined the term ‘retrograde satellites’ to refer to them, although this term is seldom used nowadays. The details of this interesting co-orbital configuration were further studied by Szebehely (1967), Broucke (1968), Benest (1976, 1977), Dermott & Murray (1981), Kogan (1989) and Lidov & Vashkov’yak (1993, 1994a,b); the stability of the quasi-satellite dynamical state has been studied by Mikkola et al. (2006), Siderenko et al. (2014) and Pousse, Robutel & Vienne (2016). Most of the results presented in the works cited have been obtained within the simplified framework of the restricted elliptic three-body problem. However, the interest in this orbital configuration is far from strictly theoretical. Quasi-satellite configurations may have played a role in the origin of the Earth–Moon system (Kortenkamp & Hartmann 2016); they have also been found numerically (de la Fuente Marcos, de la Fuente Marcos & Aarseth 2016) within the context of the Planet Nine hypothesis (Batygin & Brown 2016).

Although the existence of quasi-satellites was predicted over a century ago, the first minor body to be confirmed to follow a quasi-satellite trajectory, in this case with respect to Venus, was 2002 VE69 in 2004 (Mikkola et al. 2004; de la Fuente Marcos & de la Fuente Marcos 2012a). Asteroid 2002 VE69 is so far the only known quasi-satellite of Venus. Among the inner planets, the Earth hosts four confirmed quasi-satellite companions: (164207) 2004 GU9 and (469219) 2016 HO10 trace figure-eight paths.

Figure 1. The motion of (469219) 2016 HO10 over the time interval (−100, 200) yr according to the nominal orbit in Table 1 is displayed projected on to the ecliptic plane in a heliocentric frame of reference that rotates with the Earth. The orbit and position of our planet are also indicated. All the investigated control orbits exhibit the same behaviour within this time interval.

Figure 2. Apparent motion in geocentric equatorial coordinates of the known quasi-satellites, 2010 TK7, a Trojan, and 2015 SO2, a horseshoe libator, over the time range (0, 10) yr. Earth’s quasi-satellites describe complex, drifting loops as observed from our planet. Both (164207) 2004 GU9 and (469219) 2016 HO10 trace figure-eight paths.
The recurrent dynamics of 469219

2 Numerical Model

As pointed out in the previous section, the identification of co-orbital objects is not based on the present-day values of their orbital parameters alone, but on the statistical analysis of the results of large sets of numerical integrations. Aiming at studying the orbital evolution of (469219) 2016 HO₃ and following the steps outlined in de la Fuente Marcos & de la Fuente Marcos (2012a, 2015b), extensive N-body calculations have been performed. These

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1 http://neo.jpl.nasa.gov/nhats/
numerical integrations have been carried out using the Hermite scheme (Makino 1991; Aarseth 2003). The standard version of the N-body code used in this study is publicly available from S. J. Aarseth’s website. As explained in detail in de la Fuente Marcos & de la Fuente Marcos (2012a), results from this code compare well with those from Laskar et al. (2011) among others. At the end of the simulations, the relative errors in the total energy are <1 \times 10^{-14} and those in the total angular momentum are several orders of magnitude smaller. Our integrations have two main ingredients: initial conditions and physical model.

In the case of a nominal orbit, our initial conditions (positions and velocities in the barycentre of the Solar system) have been provided directly by the JPL horizons system (Giorgini et al. 1996; Giorgini & Yeomans 1999; Giorgini, Chodas & Yeomans 2001). They are relative to the JD TDB (Julian Date, Barycentric Dynamical Time) epoch 2457600.5 (2016-July-31.0), which is the \( t = 0 \) instant in the figures. These initial conditions, for both planets and minor bodies, are based on the DE405 planetary orbital ephemerides (Standish 1998). The initial conditions for all the control orbits of 469219 are based on the available orbital solution (see Section 3.5 for details).

The physical model includes the perturbations from the eight major planets, the Moon, the barycentre of the Pluto–Charon system, and the three largest asteroids. An example of a typical input file with details can be found in the appendix (table 9) of de la Fuente Marcos, de la Fuente Marcos & Aarseth (2015). Our physical model does not include non-gravitational forces, relativistic or relativistic effects in the integrated equations of motion. We have neglected the effect of the Yarkovsky and Yarkovsky–O’Keefe–Radzievskii–Paddack (YORP) effects (see e.g. Bottke et al. 2006), but ignoring these effects has not relevant impact on the evaluation of the present-day dynamical status of 469219. Not including these non-gravitational forces in the calculations may affect both the reconstruction of the dynamical past of this object and any predictions made regarding its future orbital evolution. However, accurate modelling of the Yarkovsky force requires relatively precise knowledge of the physical properties – such as rotation rate, albedo, bulk density, surface conductivity or emissivity – of the objects under study, which is not the case here (see Section 3.1). On the other hand, the effects derived from these forces may be unimportant when objects are tumbling or in chaotic rotation, and NEOs this small often are. Effects resulting from the theory of general relativity are insignificant for objects following orbits like that of 469219 (see e.g. Benitez & Gallardo 2008). The role of the oblateness of the Earth can be neglected for cases like the one studied here – see the analysis in Dmitriev, Lupovka & Gritsevich (2015) for the Chelyabinsk superbolide.

3 ASTEROID (469219) 2016 HO\(_3\), AN APOLLO QUASI-SATURN

Here, we show the data available for this recently discovered NEO and study both its short- and long-term orbital evolution. Emphasis is made on the statistical robustness of our results.

3.1 Data

Asteroid (469219) 2016 HO\(_3\) was discovered on 2016 April 27 by B. Gibson, T. Goggia, N. Primak, A. Schultz and M. 2

Table 1. Heliocentric Keplerian orbital elements of (469219) 2016 HO\(_3\) used in this study. The orbital solution is based on 80 observations spanning a data-arc of 4468 d or 12.23 yr, from 2004 March 17 to 2016 June 10. Values include the 1σ uncertainty. The orbit has been computed at epoch JD 2457600.5 that corresponds to 00:00:00.000 TDB on 2016 July 31 (J2000.0 ecliptic and equinox. Source: JPL Small-Body Database.)

| Element          | Value          |
|------------------|----------------|
| Semimajor axis, \(a\) (au) | \(1.001229935 \pm 0.000000003\) |
| Eccentricity, \(e\)       | \(0.1041429 \pm 0.00000005\) |
| Inclination, \(i\) (°)     | \(7.77140 \pm 0.000004\) |
| Longitude of the ascending node, \(\Omega\) (°) | \(66.51326 \pm 0.000004\) |
| Argument of perihelion, \(\omega\) (°) | \(307.22767 \pm 0.000007\) |
| Mean anomaly, \(M\) (°)    | \(297.53122 \pm 0.000010\) |
| Perihelion, \(q\) (au)     | \(0.8969589 \pm 0.000000005\) |
| Aphelion, \(Q\) (au)       | \(1.105500933 \pm 0.000000003\) |
| Absolute magnitude, \(H\) (mag) | \(24.1 \pm 0.5\) |

Willman observing with the 1.8-m Ritchey-Chretien telescope of the Pan-STARRS Project (Kaiser et al. 2004) from Haleakala at an apparent magnitude \(m\) of 21.5 (Mastaler et al. 2016). Its absolute magnitude, \(H = 24.1\) (assumed \(G = 0.15\)), suggested a diameter \(<\)7 m. Our integrations have two main ingredients: initial conditions and physical model.

In the case of a nominal orbit, our initial conditions (positions and velocities in the barycentre of the Solar system) have been provided directly by the JPL horizons system (Giorgini et al. 1996; Giorgini & Yeomans 1999; Giorgini, Chodas & Yeomans 2001). They are relative to the JD TDB (Julian Date, Barycentric Dynamical Time) epoch 2457600.5 (2016-July-31.0), which is the \(t = 0\) instant in the figures. These initial conditions, for both planets and minor bodies, are based on the DE405 planetary orbital ephemerides (Standish 1998). The initial conditions for all the control orbits of 469219 are based on the available orbital solution (see Section 3.5 for details).

The physical model includes the perturbations from the eight major planets, the Moon, the barycentre of the Pluto–Charon system, and the three largest asteroids. An example of a typical input file with details can be found in the appendix (table 9) of de la Fuente Marcos, de la Fuente Marcos & Aarseth (2015). Our physical model does not include non-gravitational forces, relativistic or relativistic effects in the integrated equations of motion. We have neglected the effect of the Yarkovsky and Yarkovsky–O’Keefe–Radzievskii–Paddack (YORP) effects (see e.g. Bottke et al. 2006), but ignoring these effects has not relevant impact on the evaluation of the present-day dynamical status of 469219. Not including these non-gravitational forces in the calculations may affect both the reconstruction of the dynamical past of this object and any predictions made regarding its future orbital evolution. However, accurate modelling of the Yarkovsky force requires relatively precise knowledge of the physical properties – such as rotation rate, albedo, bulk density, surface conductivity or emissivity – of the objects under study, which is not the case here (see Section 3.1). On the other hand, the effects derived from these forces may be unimportant when objects are tumbling or in chaotic rotation, and NEOs this small often are. Effects resulting from the theory of general relativity are insignificant for objects following orbits like that of 469219 (see e.g. Benitez & Gallardo 2008). The role of the oblateness of the Earth can be neglected for cases like the one studied here – see the analysis in Dmitriev, Lupovka & Gritsevich (2015) for the Chelyabinsk superbolide.

3.2 Quasi-satellite of the Earth

Fig. 1 shows the motion of (469219) 2016 HO\(_3\) (in red, nominal orbit in Table 1) over the time interval (−100, 200) yr projected on to the ecliptic plane as seen in a frame of reference centred at the Sun and rotating with the Earth. All the investigated control orbits of this small body exhibit the same resonant behaviour within this time interval; the observed evolution is consistent with the one described in Mikkola et al. (2006), Sidorenko et al. (2014) or Pousse et al. (2016). This result shows that 469219 is a co-orbital that currently follows a quasi-satellite orbit around our planet. It joins the ranks of (164207) 2004 GU\(_9\), (277810) 2006 FV\(_{35}\), 2013 L\(_{X}\), and 2014 OL\(_{339}\) as the fifth quasi-satellite of the Earth. It is also the smallest

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2 http://www.ast.cam.ac.uk/~sverre/web/pages/nbody.htm
3 http://sde.jpl.nasa.gov/?horizons
4 http://www.minorplanetcenter.net/mpec/K16/K16H63.html
5 http://www.minorplanetcenter.net/mpec/K16/K16K07.html
6 http://www.minorplanetcenter.net/mpec/K16/K16L96.html
7 http://www.minorplanetcenter.net/mpec/K16/K16L07.html
8 http://www.jpl.nasa.gov/news/news.php?feature=6537
one by 1.5 mag in $H$; the second smallest Earth quasi-satellite is 2014 OL$_{339}$ with $H = 22.6$ mag.

Fig. 2 shows that 469219 describes a shifting figure-eight retrograde path – as 164207 does – when viewed from our planet over the course of a sidereal year. When compared with other Earth co-orbitals (2010 TK$_{11}$ and 2015 SO$_{2}$), quasi-satellites delineate very conspicuous paths in the sky, very different from those of gravitationally bound satellites like the Moon or other unbound co-orbitals like 2010 TK$_{11}$ and 2015 SO$_{2}$. When the motion of the known quasi-satellites is plotted as a function of their distance from the Earth (see Fig. 3), it becomes clear why 469219 has a better orbital solution than several other co-orbitals in spite of its small size: its average distance from the Earth is $\sim0.2$ au and never goes beyond 0.3 au during its current quasi-satellite episode. Only 164207 exhibits a comparable behaviour in terms of distance from the Earth. Most known quasi-satellites have a wide difference between their perigee and apogee distances; asteroid 2014 OL$_{339}$ is the most extreme example (see Fig. 3). The behaviour observed in Fig. 3 makes both 164207 and 469219 attractive targets regarding affordable accessibility from our planet in terms of scheduling.

3.3 Recurrent co-orbital dynamics

Fig. 4 shows the time evolution of various parameters for the nominal orbit of (469219) 2016 HO$_{3}$ in Table 1 during the time interval ($-4$, 5) kyr. The evolution of the critical angle around $t = 0$ displayed in panel C confirms that the value of $\lambda_{h}$ librates about $0^\circ$, which is the condition for being engaged in quasi-satellite behaviour. Panel C also shows that 469219 switches repeatedly between the quasi-satellite and horseshoe configurations. Its current quasi-satellite episode started nearly 100 yr ago and it will end in about 300 yr from now. The object experiences three quasi-satellite episodes and their associated transitions to and from the horseshoe librator dynamical state during the displayed timeframe. These transitions take place at quasi-regular intervals as seen in panel C. During the quasi-satellite episodes, the value of $e$ is the highest (see panel E) and that of $i$, the lowest (see panel F); the value of the argument of perihelion decreases (see panel G) as predicted by Namouni (1999).

The orbital behaviour observed in Fig. 4 is very similar to the one displayed by Earth co-orbital 2015 SO$_{2}$ in Fig. 2 of de la Fuente Marcos & de la Fuente Marcos (2016a). The short-term dynamical evolution of various parameters is virtually identical for these two objects although they are out of phase by a few centuries and the average duration of the quasi-satellite episodes of 469219 is longer, 400 yr versus 150 yr for 2015 SO$_{2}$. The mechanism that controls the transitions is also analogous (see below). Fig. 4 is a detailed view of the entire integration displayed in Fig. 5, central panels. The behaviour of the various parameters is very similar along the entire simulated time; the evolution of the nominal orbit of 469219 is unusually stable. About 24 (a maximum of 35 has been found for other control orbits) quasi-satellite episodes take place in 100 kyr of simulated time. Among similarly behaved co-orbitals like 2015 SO$_{2}$, the recurrent co-orbital dynamics displayed by 469219 is the lengthiest.

Asteroid 469219 becomes a quasi-satellite of the Earth whenever its descending node is farthest from the Sun and its ascending node is closest to the Sun (see Fig. 4, panel H). The object becomes a horseshoe when the nodal positions are inverted with respect to the previous situation (see Fig. 4, panel H). We pointed out above that the current value of the MOID of this object is 0.0345 au, which is not particularly small when compared with the value of the radius

Figure 4. The various panels show the time evolution of relevant parameters for the nominal orbit of (469219) 2016 HO$_{3}$ in Table 1 during the time interval ($-4$, 5) kyr. Panel A shows how the distance from the Earth changes over time and the geocentric distance equivalent to the value of the radius of the Hill sphere of the Earth, 0.0098 au, is plotted as a dashed line. Panel B displays the evolution of the Kozai–Lidov parameter $\sqrt{1-e^{2}}\cos i$. Panel C shows how the value of the resonant angle, $\lambda_{h}$, changes during the displayed time interval. Panel D displays the behaviour of the semimajor axis, $a$. Panel E shows how the orbital eccentricity, $e$, changes over time. Panel F plots the value of the orbital inclination, $i$. Panel G displays how the value of the argument of perihelion, $\omega$, changes over time. Panel H shows the values of the distances to the descending (thick line) and ascending nodes (dotted line) and those of Earth’s aphelion and perihelion distances. The nodal distances have been computed using Equation (1).
of the Hill sphere of the Earth, 0.0098 au. Therefore, no truly close approaches are expected and this is what is seen in Fig. 5, panel A; this behaviour has been observed for all the integrated control orbits. Close encounters with the Earth–Moon system are only possible in the vicinity of the nodes. For a prograde orbit, the distance between the Sun and the nodes is given by the expression:

$$r = a(1 - e^2)/(1 \pm e \cos \omega),$$

where the ‘+’ sign is for the ascending node (where the orbit crosses the Ecliptic from South to North) and the ‘−’ sign is for the descending node. Both distances appear in Figs 4 and 5, panel H.

Fig. 4, panels C and H, shows that transitions between co-orbital states occur when the nodes of the orbit of this minor body are farthest from the Earth and the average gravitational influence of the Earth–Moon system on the asteroid is the weakest. Therefore, at those moments, the secular influence of more distant planets must be at its peak.

Jupiter is the dominant secular perturber for this object (see the discussion in the following section). During the quasi-satellite phase both nodes cross the path of the Earth; then, distant encounters with the Earth–Moon system are possible at both nodes (see Fig. 4, panel H) and the sustained action of these encounters slowly...
increases the asteroid’s orbital energy making the transition to a horseshoe trajectory possible. These relatively distant close approaches are well beyond the radius of the Hill sphere (see Fig. 5, A-panels) and cannot perturb the orbit dramatically. The described mechanism (see also de la Fuente Marcos & de la Fuente Marcos 2016a) is also affecting other Earth co-orbitals. In addition to 2015 SO2 (and now 469219), asteroids 2001 GO2, 2002 AA39 and 2003 YN107 are well-documented cases of such a dynamical behaviour (Brasser et al. 2004). The existence of these transitions had been previously predicted and explained by Namouni (1999) and Christou (2000a).

3.4 Kozai–Lidov resonance? Not quite

Fig. 4 shows coupled oscillations in eccentricity (panel E), inclination (panel F) and argument of perihelion (panel G). These are widely considered as the signposts of the Kozai–Lidov mechanism (Kozai 1962; Lidov 1962); also, the value of the Kozai–Lidov parameter (panel B) remains fairly constant. In our case and in principle, the Kozai–Lidov scenario is defined by the presence of a primary (the Sun in our case), a perturbed body (an asteroid), and a massive outer perturber (Jupiter in our case) such as the ratio of semimajor axes (perturbed versus perturber) tends to zero. Under this orbital architecture the libration of the argument of perihelion occurs at $\omega = 90^\circ$ or $270^\circ$. This is observed in Fig. 4, panel G, where $\omega$ librates about $270^\circ$ (or $-90^\circ$). Under these conditions, aphelion always occurs away from the orbital plane of the perturber. A classical example of a minor body subjected to the Kozai–Lidov effect induced by an outer perturber is the asteroid (3040) Kozai that is perturbed by Jupiter.

Another variant of the Kozai–Lidov scenario is found when the ratio of semimajor axes (perturbed versus perturber) is close to one which is typical of asteroids trapped in a 1:1 mean motion resonance with a host planet. In this case, the libration occurs at argument of perihelion equal to 0 or $180^\circ$. In this alternative scenario, the nodes are located at perihelion and at aphelion, i.e. away from the massive perturber (see e.g. Milani et al. 1989; Michel & Thomas 1996). If $\omega$ librates about 0 then the object subjected to the Kozai–Lidov secular resonance reaches perihelion at nearly the same time it crosses the ecliptic plane from South to North (ascending node); conversely, if $\omega$ oscillates about $180^\circ$ then the Kozai–Lidov librator reaches perihelion while approaching the descending node. Objects trapped at either version of the Kozai–Lidov resonance ($\omega \sim 90^\circ$, $270^\circ$ or $\omega \sim 0^\circ$, $180^\circ$) exhibit a precession rate of $\omega$ compatible with zero.

In principle, Fig. 5, central panel G, shows that (469219) 2016 HO₃ may have been locked in a Kozai–Lidov resonance with $\omega$ librating about $270^\circ$ for nearly 100 kyr and probably more. Because of the Kozai–Lidov resonance, both $e$ (central panel E) and $i$ (central panel F) oscillate with the same frequency but out of phase (for a more detailed view, see Fig. 4, panels E and F); when the value of $e$ reaches its maximum the value of $i$ is the lowest and vice versa ($\sqrt{1-e^2}\cos i \sim \text{constant}$, see Fig. 4, panel B). During the simulated time and for the nominal orbit, 469219 reaches perihelion and aphelion the farthest possible from the ecliptic plane (South to North, ascending node; conversely, if $\omega$ oscillates about $180^\circ$ then the Kozai–Lidov librator reaches perihelion while approaching the descending node. Objects trapped at either version of the Kozai–Lidov resonance ($\omega \sim 90^\circ$, $270^\circ$ or $\omega \sim 0^\circ$, $180^\circ$) exhibit a precession rate of $\omega$ compatible with zero.

Figure 6. The $e_\omega$-portrait relative to Venus, the Earth, and Jupiter for (469219) 2016 HO₃.
3.5 The impact of errors

In contrast with other Earth co-orbitals or Kozai librators (see e.g. de la Fuente Marcos & de la Fuente Marcos 2015c, 2016b), (469219) 2016 HO$_3$ follows a rather stable orbit that is only subjected to the direct perturbation of the Earth–Moon system. Its current orbital solution (see Table 1) has very small associated uncertainties, but moving embedded in a web of secular resonances and switching regularly between co-orbital states, can induce unexpected instabilities.

In addition to the integrations that make use of the nominal orbital elements in Table 1 as initial conditions, we have performed 50 control simulations with series of orbital parameters drawn from the nominal ones within the quoted uncertainties and assuming Gaussian distributions for them. In the computation of these additional sets of orbital elements, the Box–Muller method (Box & Muller 1958; Press et al. 2007) has been applied to produce random numbers according to the standard normal distribution with mean 0 and standard deviation 1. When computers are used to generate a uniform random variable – in our case to seed the Box–Muller method – it will inevitably have some inaccuracies because, numerically, there is a lower bound on how close numbers can be to 0.

For a 64 bits computer the smallest non-zero number is 2$^{-64}$ which means that the Box–Muller method will not produce random variables more than 9.42 standard deviations from the mean (Press et al. 2007). Representative results from these calculations are displayed in Figs 5 and 8. Here, when an orbit is labelled ‘±3σ’ (Fig. 5), it has been obtained by adding (+) or subtracting (−) three times the uncertainty from the orbital parameters (the six elements) in Table 1; an equivalent approach has been followed for orbits labelled ‘±6σ’ (Fig. 8).

Figs 5 and 8 show that the orbital evolution of this object is similar within ±15 kyr of $t = 0$. Beyond this time interval and according to the results of the multiplicity of simulations performed, the object may start being subjected to the Kozai–Lidov resonance but it will remain confined inside Earth’s co-orbital region, in particular, evolving dynamically as a horseshoe libration. This object never approaches the Earth under 0.04 au within ±50 kyr of $t = 0$. Recurrent transitions between the horseshoe and quasi-satellite dynamical states are observed for all the simulated control orbits.

A more rigorous and detailed evaluation of the role of errors on our short-term results has been carried out by studying how the changing values of the orbital parameters of the test orbits at $t = 0$ influence the evolution of the osculating orbital elements as the simulation progresses. Three additional sets of 100 shorter control simulations (±2 kyr of $t = 0$) are analysed here. The first set (see Fig. 9, left-hand panels), has been generated as previously described with the initial orbital elements of each control orbit varying randomly, within the ranges defined by their mean values and standard deviations. For instance, new values of the semimajor axis have been found using the expression $a_i = (a) + n \sigma_a, r_i$ where $a_i$ is the semimajor axis of the control orbit, $(a)$ is the value of the semimajor axis from the nominal orbit (Table 1), $n$ is an integer (in our case 1, left-hand panels, or 6, central panels), $\sigma_a$ is the standard deviation of $a$ supplied with the nominal orbit (Table 1), and $r_i$ is a (pseudo) random number with standard normal distribution (see above). For this object, the errors are so small and the orbital evolution so smooth that nominal and error-based results fully overlap within ±2 kyr of $t = 0$. Artificially increasing the values of the standard deviations – up to 6 times in the simulations displayed in Fig. 9, right-hand panels – gives nearly the same results.

Sitarski (1998, 1999, 2006) has shown that the somewhat classical approach used above is equivalent to considering a set of distinct
virtual minor bodies following similar orbits, but not a sample of test orbits resulting from an actual set of observations associated with a single object. The orbit in Table 1 matches all the available observations for 469219 within certain, very strict tolerances. In this context, the statistically correct procedure to compute control orbits is to consider how the elements influence each other and their associated uncertainties, applying the Monte Carlo using the Covariance Matrix (MCCM) approach (Bordovitsyna, Avdyushev & Chernitsov 2001; Avdyushev & Banschikova 2007).

Fig. 9, right-hand panels, shows the results of a third set of short simulations whose initial conditions have been generated using the implementation of the MCCM approach discussed in de la Fuente Marcos & de la Fuente Marcos (2015b). The control orbits studied here have initial parameters drawn from the nominal orbit (Table 1) adding random noise on each initial orbital element as described by the covariance matrix. The covariance matrix used here was provided by the JPL Small-Body Database9 and it has been computed at epoch 2457232.5 TDB. Our results show that, for this particular object, the MCCM and the classical approaches produce consistent results; this is often the case when precise orbital solutions of stable orbits are used (see e.g. de la Fuente Marcos & de la Fuente Marcos 2015b, 2016a). We can confirm that 469219 is a present-day quasi-satellite of our planet and the probability of this assessment being

9 http://ssd.jpl.nasa.gov/sbdb.cgi
3.6 Long-term stability

The stability of quasi-satellite orbits has been explored in the theoretical works of Mikkola et al. (2006), Sidorenko et al. (2014) and Pousse et al. (2016). Arguably, the most stable known Earth’s co-orbital is 2010 SO\(_{16}\). This object stays as horseshoe libraror for at least 120 kyr and possibly for up to 1 Myr (Christou & Asher 2011). This is quite remarkable because it remains in the same co-orbital state during this time span with no transitions.

Fig. 10 is an extension of the data displayed in Figs 5 and 8, central panels and it shows the long-term dynamical evolution of the nominal orbit of (469219) 2016 HO\(_{3}\) (Table 1). Asteroid 469219 could be as stable as 2010 SO\(_{16}\), perhaps even more stable, but it experiences transitions between co-orbital states. However, if we assess the overall stability of an Earth co-orbital in terms of how much time the object stays confined within Earth’s co-orbital zone – that, based on the results of the integrations performed, currently goes from \(\sim0.994\) au to \(\sim1.006\) au – then the object discussed here is, with little doubt, the most stable known Earth co-orbital. A quantitative measure of the level of dynamical stability associated with the orbital solution currently available for 469219 is in the value of its Lyapunov time – or time-scale for exponential divergence of integrated orbits starting arbitrarily close to each other – that is \(\sim7500\) yr; this time-scale is nearly the same for both forward and backwards integrations.

3.7 Objects in similar orbits

Minor bodies as small as 469219 2016 HO\(_{3}\) are not expected to be primordial, but fragments of larger objects. In this case, it is logical to assume that other small bodies could be moving along paths similar to that of 469219 if they are trapped in some web of secular resonances like the one described in the previous sections. In order to test this reasonable hypothesis, here we use the D-criteria of Southworth & Hawkins (1963), \(D_{\text{SH}}\), Lindblad & Southworth (1971), \(D_{\text{LS}}\) (in the form of equation 1 in Lindblad 1994 or equation 1 in Foglia & Masi 2004), Drummond (1981), \(D_{\text{R}}\), and the \(D_{\text{R}}\) from Valsecchi, Jopek & Froeschl (1999) to search for a possible dynamical link between 469219 and other known minor bodies.

An exploration of all the NEOs currently catalogued (as of 2016 July 7) by the JPL Small-Body Database\(^{3}\) using these criteria produced the list in Table 2. In this list, objects are sorted by ascending \(D_{\text{LS}}\) and only those with \(D_{\text{LS}}\) and \(D_{\text{R}} < 0.05\) are shown. The list includes 2015 SO\(_{2}\) that is a previously documented horseshoe libraror (de la Fuente Marcos & de la Fuente Marcos 2016a). It has already been pointed out that the orbital evolution of 2015 SO\(_{2}\) resembles, in many aspects, that of 469219 but, without proper integrations, it is not possible to draw any conclusions about the other objects in Table 2. Given the high degree of dynamical chaos that characterizes the neighbourhood of the orbit of our planet, finding two or more similar present-day orbits is not enough to claim a relationship, dynamical or otherwise, among them; a representative set of test orbits must be integrated to show that the dynamical evolution of the candidate related objects over a reasonable time interval is also similar (see e.g. Jopek & Williams 2013).

Fig. 11 shows the comparative orbital evolution of 2009 SH\(_{2}\), 2009 DA\(_{\text{A3}}\) and 2012 VU\(_{16}\). With the notable exception of 2009 DA\(_{\text{A13}}\), the orbital evolution of the other two objects is rather unstable as expected from the low values of their respective MOIDs (see Table 2). None of the three objects is a present-day Earth co-orbital and they are located well outside Earth’s co-orbital zone. Asteroid 2009 DA\(_{\text{A3}}\) shows a very peculiar and stable orbital evolution controlled by the Kozai–Lidov resonance; in this case, the value of its \(\omega\) librates about 180°, a variant of the Kozai–Lidov mechanism that arises when the ratio of semimajor axes of both perturbed and perturber bodies is close to one. For 2009 DA\(_{\text{A3}}\) the dominant secular perturbation is that of the nearly co-orbital perturber (Earth).

4 THE ROLE OF THE UNCERTAINTIES IN THE MASSES OF THE JOVIAN PLANETS

Figs 6 and 7 strongly suggest that Jupiter plays a major role on the secular evolution of (469219) 2016 HO\(_{3}\). Our physical model
that of the value of the mass of the Earth (0.0006 $\times 10^{24}$ kg) and it is nearly 0.022 $M_\oplus$ ($M_\oplus = 5.972 19 \times 10^{24}$ kg). This value of the mass of Jupiter corresponds to the ephemeris JUP230 and it has been used to obtain Fig. 10 and the other figures with the exception of Fig. 12.

If instead of using the ephemeris JUP230, we make use of the most updated ephemeris JUP310, the value of the mass of Jupiter is 1.898 26 $\pm 0.000 23 \times 10^{27}$ kg. Fig. 12 shows the comparative long-term evolution of the values of various parameters (as in Fig. 4) for the nominal orbit of 469219 (Table 1). Three representative values of the mass of Jupiter from the ephemeris JUP310 are considered, nominal value minus 1σ (left-hand panels), nominal value (central panels), and nominal value plus 1σ (right-hand panels). Changing the value of the mass of the main secular perturber has visible effects on the long-term orbital evolution of 469219. Small differences already appear beyond $\pm 10$ kyr of integrated time. In principle, a heavier Jupiter may further stabilize the dynamical evolution of this object, but more calculations are needed to provide a statistically robust answer.

Our results here should be understood as a cautionary note regarding long-term predictions of the orbital evolution of NEOs based on the currently available values of the masses of the Jovian planets. It is unlikely that the masses of Uranus or Neptune will be improved within the next decade or so, but the ongoing Juno mission (Bolton et al. 2010) should be able to reduce the degree of uncertainty in the value of the mass of Jupiter considerably (Le Maistre et al. 2016). Such a strong improvement in the precision of Jupiter’s mass parameter determination will indirectly improve our assessment of the stability of Earth co-orbitals.

Figure 10. As in Fig. 4 but displaying an extended integration.


5ASTEROID (469219) 2016 HO: A SUITABLE PROBE INTO THE ORIGINS OF EARTH’S CO-ORBITAL ASTEROID POPULATION

Among known Earth co-orbitals, the long-term evolution of (469219) 2016 HO\textsubscript{3} clearly stands out. The results of our calculations strongly suggest that it may have been a companion to our planet for at least 1 Myr and perhaps more. Fig. 10 indicates that this object may have arrived to Earth’s co-orbital zone as early as 600 kyr ago, but probably even 1 Myr ago (not shown). It may have been captured from the general NEO population but, being such a long-term companion to our planet, other scenarios regarding its putative origin may also be plausible. Unlikely but not impossible is an artificial origin for this object. Albeit improbable, 469219 could be an artificial interloper, a relatively large piece of hardware from previous space missions returning to the neighbourhood of the Earth–Moon system (see the discussion in section 9 of de la Fuente Marcos & de la Fuente Marcos 2015c). In contrast with previous sections, our discussion here is significantly more speculative because the influence of orbital chaos severely limits our chances of determining with confidence the sources of the NEOs (Connors et al. 2004). It is possible that some NEOs may have been produced in the Earth–Moon system (see e.g. Margot & Nicholson 2003); however, this scenario is disfavoured by other authors who argue that sources...
Table 2. Orbital elements, orbital periods ($P$), perihelia ($q = a(1 - e)$), aphelia ($Q = a(1 + e)$), number of observations ($n$), data-arc, absolute magnitudes ($H$) and MOID of minor bodies with orbits similar to that of (469219) 2016 HO$_{3}$ (see Table 1). The various $D$-criteria ($D_{SH}$, $D_{LS}$, $D_{D}$ and $D_{R}$) are also shown. The objects are sorted by ascending $D_{LS}$ (equation 1 in Lindblad 1994 or equation 1 in Foglia & Masi 2004). Only objects with $D_{LS}$ and $D_{R} < 0.05$ are shown. The orbits are referred to the Epoch 2457600.5 (2016-July-31.0) TDB. Data as of 2016 July 7.

| Asteroid | $a$ (au) | $e$ | $i$ (°) | $\Omega$ (°) | $\omega$ (°) | $P$ (yr) | $q$ (au) | $Q$ (au) | $n$ | arc (d) | $H$ (mag) | MOID (au) | $D_{SH}$ | $D_{LS}$ | $D_{D}$ | $D_{R}$ |
|----------|---------|-----|--------|-------------|-------------|---------|---------|---------|-----|--------|-----------|-----------|---------|--------|-------|-------|-------|
| 2009 SH$_{2}$ | 0.991 | 0.994 | 6.811 | 6.690 | 12 | 101.646 | 0.99 | 0.8979 | 1.0849 | 109 | 14 | 24.90 | 0.900 | 0.1942 | 0.0195 | 0.0829 | 0.0299 |
| 2009 DA$_{43}$ | 1.016 | 0.118 | 6.700 | 57.875 | 71 | 215.440 | 31 | 0.8965 | 1.1372 | 21 | 14 | 24.60 | 0.0826 | 0.1813 | 0.0235 | 0.0861 | 0.0486 |
| 2015 SO$_{3}$ | 0.998 | 0.108 | 9.186 | 182.931 | 94 | 290.047 | 19 | 0.8909 | 1.1070 | 84 | 9 | 23.90 | 0.019 | 0.2983 | 0.0257 | 0.1007 | 0.0281 |
| 2012 VU$_{76}$ | 1.018 | 0.128 | 6.758 | 52.371 | 81 | 88.189 | 82 | 0.8875 | 1.1489 | 47 | 96 | 25.70 | 0.0037 | 0.2127 | 0.0314 | 0.1331 | 0.0462 |
| 2016 CO$_{246}$ | 1.000 | 0.124 | 6.422 | 137.159 | 57 | 118.360 | 21 | 0.8766 | 1.1251 | 21 | 27 | 26.00 | 0.0378 | 0.2452 | 0.0370 | 0.1248 | 0.0037 |

Figure 11. Analogous to Fig. 5 for the nominal orbits of 2009 SH$_{2}$, 2009 DA$_{43}$ and 2012 VU$_{76}$. 

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in the main asteroid belt are probably more likely (see e.g. Morais & Morbidelli 2002). The initial hypothesis suggesting an origin in the Earth–Moon system for some of these objects assumes that they could be the result of impacts on the Moon (see e.g. Warren 1994; Gladman et al. 1995; Bottke et al. 1996; Gladman 1996); i.e. within the framework of this hypothesis a fraction of NEOs are nothing but Lunar debris. However, an origin in the Earth–Moon system does not require a physical impact on our satellite.

Objects like 469219 may be relatively recent fragments of an already co-orbital and older parent body. In the Earth’s neighbourhood, at least two processes can dislodge large rocks from a weakly bound minor body. Resurfacing events can be triggered when an asteroid encounters our planet during flybys at relatively large planetary distances, in the range 5–20 planetary radii (see e.g. Keane & Matsuyama 2015); even more likely, fragments can also be released after the spin rate of an asteroid is dramatically altered during a close encounter with our planet (Scheeres et al. 2005), such a sudden change may induce subsequent structural failure that results in large rocky blocks being ejected from the larger, crumbling asteroid (see e.g. Denneau et al. 2015). In the framework of this hypothetical – but certainly not impossible – scenario, the relatively large fragments released during one of these partial disruption events may

Figure 12. Comparative long-term dynamical evolution of various parameters (as in Fig. 4) for the nominal orbit of (469219) 2016 HO₃ as in Table 1 and three representative values of the mass of Jupiter, nominal value–1σ (left-hand panels), nominal value (central panels), and nominal value+1σ (JUP310 ephemeris, see the text for details).
remain within the orbital neighbourhood of our planet confined inside a dynamical mesh where resonances interweave in a complex pattern that pervades the entire region (see Section 3).

Due to the singular orbital evolution of this small body, it will remain observable from the ground under relatively favourable conditions for many decades to come (see Figs 2–4). Its apparent visual magnitude at perigee could be as low as 22 mag or slightly lower in April. These facts could turn this object into an eventual ‘Rosetta stone’ whose study could lead to uncovering the true origin of Earth’s co-orbitals or at least of some of them. Spectroscopic observations during its future close approaches to our planet should be able to confirm a possible genesis in the Earth–Moon system as Lunar debris or even a physical connection with 2015 SO$_2$ which is already a close dynamical relative, and also discard a putative artificial origin for this object. Such studies may be particularly helpful in improving our understanding of the origins – local versus captured – of Earth’s co-orbital asteroid population.

6 DISCUSSION

The subject of NEOs currently engaged in quasi-satellite behaviour with our planet has been revisited in de la Fuente Marcos & de la Fuente Marcos (2014, 2016c). The conclusion of this study was that the known quasi-satellites form a very heterogeneous, dynamically speaking, transient group. This result suggests that they have been temporarily captured from the Earth-crossing small-body populations. If we compare the results obtained here with those in de la Fuente Marcos & de la Fuente Marcos (2014, 2016c), that conclusion is only confirmed although the orbital evolution of (162407) 2004 GU$_3$ clearly resembles that of (469219) 2016 HO$_3$, even if their orbital inclinations are somewhat different. Asteroid 469219 is also significantly more stable than 162407 as it remains for a longer period of time inside Earth’s co-orbital zone. The quality of the orbital solutions of both objects is comparable, therefore their different level of dynamical stability is a robust, distinctive feature. In principle, it may appear somewhat surprising being able to find so much orbital diversity among transient Earth quasi-satellites however, as pointed out above (see Section 3), the overlapping of a multiplicity of secular resonances makes this region particularly chaotic.

The effect of secular resonances on the dynamics of minor bodies following orbits with $a$ smaller than 2 au and relatively low values of $e$ was first studied by Michel & Froeschlé (1997). These early results were further extended by Michel (1997, 1998). Michel & Froeschlé (1997) concluded that minor bodies with $0.9 < a < 1.1$ au are subjected to the Kozai–Lidov resonance that, at low inclination, induces oscillation of the argument of perihelion around the values $0^\circ$ or $180^\circ$, the second variant of the Kozai–Lidov mechanism (see e.g. Michel & Thomas 1996). This behaviour has been found for 2009 DA$_{13}$ (see Section 3.7), but 469219 and several other co-orbitals are not currently affected by the Kozai–Lidov mechanism.

Asteroid 469219 exhibits a number of dynamical features that make it unusual. In addition, it is not often that an Earth co-orbital as small as this one – that encounters our planet at relatively large distances – sports an orbital determination as precise as the one in Table 1. This is possible because, in terms of average geocentric distance, it is one of the closest known co-orbitals if not the closest. Earth quasi-satellites have not been the target subject of specific surveys. For instance, the discovery of 2014 OL$_{139}$ (Vaduvescu et al. 2014, 2015) was the by-product of a standard NEO survey, EURONEAR (Vaduvescu et al. 2008); the others were serendipitous, not planned, findings. Therefore, the identification by chance in less than a year of two objects moving in similar orbits, 2015 SO$_2$ and 469219, that also exhibit matching orbital evolution within several thousand years backwards and forward in time, strongly suggests that these two and other – yet to be discovered – NEOs may be part of a group of dynamical origin as the ones described in e.g. de la Fuente Marcos & de la Fuente Marcos (2016d).

Asteroid 469219 cannot be considered part of the group of NEOs moving in Earth-like orbits or Arjunas (de la Fuente Marcos & de la Fuente Marcos 2013, 2015a,c) because its eccentricity is perhaps too high. However, it shares a number of dynamical features with them, in particular those derived from switching between co-orbital states.

Our calculations show that 469219 is unlikely to collide with our planet. Its orbital properties are such that it always remains at a safe distance from the Earth but still close enough to make it an attractive target for future in situ study. Its relative velocity at encounter remains in the range $\sim3$ to $5 \text{ km s}^{-1}$ for approaches in the near future.

7 CONCLUSIONS

In this paper, we have explored the orbital evolution of the recently discovered NEO (469219) 2016 HO$_3$. This study has been performed using $N$-body simulations. In addition, a number of dynamical issues regarding the stability of Earth’s co-orbital asteroid population have been re-examined. Our conclusions can be summarized as follows.

(i) Asteroid 469219 is an Earth co-orbital, the fifth known quasi-satellite of our planet and the smallest. Its present quasi-satellite dynamical state started nearly 100 yr ago and it will end in about 300 yr from now, transitioning to a horseshoe state. It is the closest known Earth quasi-satellite, in terms of average distance from our planet.

(ii) Extensive $N$-body simulations show that it can be counted among the most stable known Earth co-orbitals, with a Lyapunov time close to 7500 yr. This object may stay within Earth’s co-orbital zone for a time interval well in excess of 1 Myr.

(iii) Like a few other known Earth co-orbitals, 469219 experiences repeated transitions between the quasi-satellite and horseshoe dynamical states. Jupiter plays a major role in the operation of the dynamical mechanism responsible for these transitions.

(iv) Although a few other NEOs move in orbits similar to that of 469219, only one of them – 2015 SO$_2$ – exhibits a dynamical evolution that closely resembles that of the object discussed here.

(v) Our orbital analysis singles out 469219 as a very suitable candidate for spectroscopic studies as it will remain well positioned with respect to our planet for many decades. Its location within the NEO orbital parameter space makes it also an attractive target for future in situ study.

(vi) Exploratory calculations show that the assessment of the stability of Earth co-orbitals following paths similar to those of 2015 SO$_2$ and 469219 depends on the uncertainty in the value of the mass of Jupiter. In general, an improved determination of the value of the Jovian mass will translate into a more robust statistical evaluation of the stability of Earth co-orbitals.

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