Optimal Cooperative Relaying Schemes for Improving Wireless Physical Layer Security

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Abstract

We consider a cooperative wireless network in the presence of one or more eavesdroppers, and exploit node cooperation for achieving physical (PHY) layer based security. Two different cooperation schemes are considered. In the first scheme, cooperating nodes retransmit a weighted version of the source signal in a decode-and-forward (DF) fashion. In the second scheme, while the source is transmitting, cooperating nodes transmit weighted noise to confound the eavesdropper (cooperative jamming (CJ)). We investigate two objectives, i.e., maximization of achievable secrecy rate subject to a total power constraint, and minimization of total power transmit power under a secrecy rate constraint. For the first design objective with a single eavesdropper we obtain expressions for optimal weights under the DF protocol in closed form, and give an algorithm that converges to the optimal solution for the CJ scheme; while for multiple eavesdroppers we give an algorithm for the solution using the DF protocol that is guaranteed to converge to the optimal solution for two eavesdroppers. For the second design objective, existing works introduced additional constraints in order to reduce the degree of difficulty, thus resulting in suboptimal solutions. In this work, either a closed form solution is obtained, or algorithms to search for the solution are proposed. Numerical results are presented to illustrate the proposed schemes and demonstrate the advantages of cooperation as compared to direct transmission.

Index Terms

Secrecy rate, node cooperation, physical layer based security, semi-definite programming.
I. INTRODUCTION

Privacy and security issues play an important role in wireless networks. Although security is typically addressed via cryptographic approaches, there have been several attempts at addressing security at the physical layer, following the pioneering work of [8]. Recently wireless physical (PHY) layer based security from an information-theoretic point of view has received considerable attention, e.g., [3]-[7]. The wiretap channel, first introduced and studied by Wyner [8], is the most basic physical layer model that captures the problem of communication security. Wyner showed that when an eavesdropper’s channel is a degraded version of the main channel, the source and destination can achieve a positive perfect information rate (secrecy rate). The maximal secrecy rate from the source to the destination is defined as the secrecy capacity and for the degraded wiretap channel is given as the difference between the rate at the legitimate receiver and the rate at the eavesdropper. The Gaussian wiretap channel, in which the outputs at the legitimate receiver and at the eavesdropper are corrupted by additive white Gaussian noise (AWGN), was studied in [9]. Along the same lines, the Gaussian MIMO wiretap channel was investigated and the secrecy capacity of the MIMO wiretap channel was established in terms of an optimization problem over all possible input covariance matrices [10], [13]. There have also been some recent works focusing on secrecy rates based on partial CSI or channel statistics [3], [4], [15]. In [15], the authors derived the ergodic secrecy capacity of Gaussian MIMO wiretap channel and showed that a circularly symmetric Gaussian input is optimal.

The secrecy rate is affected by channel conditions between the source and the destination and also channel conditions between the source and the eavesdroppers. A low cost approach to increase the achievable secrecy rate by exploiting/mitigating channel effects is node cooperation via relays [16]-[21]. A two-stage cooperative approach was recently proposed in [22], [23], and their extended version [24]. In [24], the source first transmits locally to a set of trusted relays, and subsequently, the relays retransmit a weighted version of the signal that they heard (amplify-and-forward (AF)), or a weighted version of the decoded signal (decode-and-forward (DF)). Alternatively, the relays can transmit weighted noise to confound the eavesdropper while the source is transmitting (cooperative jamming (CJ)). In all cases, the objective is to select the weights so as to maximize the secrecy rate under total power constraints, or to minimize the total power under a secrecy rate constraint. The results in [22]-[24] contain sub-optimal weights for both a single eavesdropper and multiple eavesdroppers, due to the difficulty of solving the associated optimization problems. In particular, several criteria for sub-optimal weight design were proposed such as completely nulling out the message signal at all eavesdroppers for DF and AF, and
completely nulling out the jamming signal at the destination for CJ. These sub-optimal weights may yield a reduction in the achievable secrecy rate or minimization of total power.

In this paper, we consider the same scenario and problem as in [22]-[24], but focus on obtaining the optimal solution for the DF and CJ schemes. Obtaining the solution for the AF scheme is a more difficult problem and will be addressed in future work. Exploiting certain properties of the objective functions and the constraints, enables us to either obtain closed form solutions, or, if a closed form solution is not possible, propose algorithms to search for the solution.

The remainder of this paper is organized as follows. The mathematical model is introduced in §II. In §III we derive the optimal relay weights that maximize the achievable secrecy rate subject to a total power constraint in the presence of a single eavesdropper for the DF and CJ protocols, and multiple eavesdroppers for DF protocol. In §IV we study the optimal weights that minimize the total power under a secrecy rate constraint for the DF and CJ protocols. Numerical results in §V are presented to illustrate the proposed solutions. Finally, §VI provides some concluding remarks.

A. Discussion of related work

Our work falls under the general scenario where the source-destination communication is aided by a relay or a helper. Relevant results include the work of [16], where multiple users communicate with a common receiver in the presence of an eavesdropper, and the transmit power allocation policy is determined that maximizes the secrecy sum-rate. In [17], a source, destination, eavesdropper and relay model is considered, in which the relay transmits a noise signal in order to jam the eavesdropper. The rate-equivocation region is derived to show gains and applicable scenarios for cooperation, with the equivocation denoting the uncertainty of the eavesdropper about the source message.

A generalization of [16] and [17] was proposed in [18], in which the helper transmits signals from another source encoder. In [19], inner and outer bounds on the rate-equivocation region were derived for the four-node model for both discrete memoryless and Gaussian channels. In [21], the secrecy rate of orthogonal relay and eavesdropper channels was studied.

Our work in this paper is different from the aforementioned works in the sense that we address the more general case of multiple relays and multiple eavesdroppers. Also, existing works primarily focus on rate-achieving relaying strategies. In our work, we consider pre-defined cooperative schemes without claiming that those schemes are optimal, and determine relay weight and power allocation design that optimize the achievable secrecy rate subject to a power constraint, or minimize the transmit power subject to a secrecy rate constraint.
B. Notation

Upper case and lower case bold symbols denote matrices and vectors, respectively. Superscripts $\ast$, $T$ and $\dagger$ denote respectively conjugate, transposition and conjugate transposition. $\text{Tr}(A)$ denotes the trace of matrix $A$. $A \succeq 0$ means that $A$ is a Hermitian positive semi-definite matrix. $\text{rank}(A)$ denotes the rank of matrix $A$. $\|a\|$ denotes Euclidean norm of vector $a$. $I_n$ denotes the identity matrix of order $n$ (the subscript is dropped when the dimension is obvious). $f'(x)$ and $f''(x)$ denote first- and second- order derivatives of $f(x)$, respectively.

Fig. 1. System model: source $S$ wishes to communicate to destination $D$ in the presence of $J$ eavesdroppers, $E_1, \ldots, E_J$. The $N$ relays, $R_1, \ldots, R_N$ implement decode and forward (DF) or cooperative jamming (CJ), based protocols. In each case the objective is to select the relay weights ($w$) and the source power ($P_s$) to maximize the achievable secrecy rate subject to a total power constraint ($P_0$), or to minimize the total power constraint under a secrecy rate constraint.

II. SYSTEM MODEL AND PROBLEM STATEMENT

We consider a wireless network model depicted in Fig. 1 consisting of one source node $S$, a set of $N$ relay nodes ($R_i$, $i = 1, \ldots, N$), a destination node $D$, and a set of $J$ passive eavesdroppers ($E_j$, $j = 1, \ldots, J$). The symbols used in the paper are listed in Table I.

The source message is uniformly distributed over the message set $\mathcal{W} = \{1, 2, \ldots, 2^{nR}\}$, which is transmitted in $n$ channel uses. Here, $R$ denotes the source rate (unit: bits per channel use) and the message has entropy $nR$ bits. A stochastic encoder at the source maps each message to a codeword from an alphabet of length-$n$. For the purpose of evaluating the achievable secrecy rate, we assume that the codewords used at the source are Gaussian. We consider a time division multiple access system, in which there are $n$ time units in each transmission slot. In a time unit, the average power of an encoded source symbol is normalized to unity. The noise at any node is assumed to be zero-mean white complex Gaussian with variance $\sigma^2$. Each node is equipped with a single omni-directional antenna and operates in a half-duplex mode.
### Table I

**Mathematical notation**

| Symbol | Description |
|--------|-------------|
| $N$    | number of relays |
| $J$    | number of eavesdroppers |
| $P_0$  | total power (source power plus the relays’ power) |
| $P_s$  | transmit power at the source |
| $\sigma^2$ | noise variance |
| $h_0$  | baseband complex channel gain between the source and the destination |
| $h_i$  | baseband complex channel gain between the $i$th relay and the destination |
| $a_i$  | baseband complex channel gain between the source and the $i$th relay |
| $g_{0j}$ | baseband complex channel gain between the source and the $j$th eavesdropper |
| $g_{ij}$ | baseband complex channel gain between the $i$th relay and the $j$th eavesdropper |
| $w$    | weight vector at the relays, $[w_1, \cdots, w_N]^T$ |
| $h$    | $[h_1, \cdots, h_N]^T$ |
| $g_j$  | $[g_{0j}, \cdots, g_{Nj}]^T$ |
| $R_h$  | $hh^\dagger$ |
| $R_j^*$ | $g_jg_j^\dagger$ |

**Notes:** The index $j$ is dropped when $J = 1$.

All channels are assumed to be flat fading. We assume that global channel state information (CSI) is available, including the eavesdroppers’ channels. This corresponds to the cases where the eavesdroppers are active in the network and their transmissions can be monitored [25]. We should note that there have been some recent works focusing on secrecy rates based on partial CSI or channel statistics (e.g., chapter 5 in [7], and [4]). Adapting the proposed work to cooperative schemes that uses partial CSI or channel statistics will be considered in future work.

Similarly as in [20], [24], we assume that the source encoding scheme, the decoding methods at destination and eavesdroppers, and the cooperative protocol, are all public information.

Let us fix the relay weight vector $w$ and the source transmission power $P_s$. Then the following expressions give the rates with the destination and the eavesdroppers as a function of $w$, $P_s$, the noise $\sigma^2$, and the various channel gains. For the DF-based protocol, the rate at the destination and the $j$th
eavesdropper are, respectively
\[ R_d = \frac{1}{2} \log \left( 1 + \frac{P_s|h_0|^2 + w^\dagger R_h w}{\sigma^2} \right), \]  
\[ R^j_e = \frac{1}{2} \log \left( 1 + \frac{P_s|g_{0j}|^2 + w^\dagger R^j_j w}{\sigma^2} \right), \]

where the scalar factor 1/2 is due to the fact that two time units are required in two stages. Here we have assumed an additional constraint, i.e., \( P_s \geq P^\text{min}_0 \) where \( P^\text{min}_0 \) is the minimum source power requirement for cooperative nodes to correctly decode the source message with high probability. We assume \( P^\text{min}_0 \) is known a priori. For the CJ-based protocol, the rate at the destination and the \( j \)th eavesdropper are, respectively
\[ R_d = \log \left( 1 + \frac{P_s|h_0|^2}{w^\dagger R_h w + \sigma^2} \right), \]  
\[ R^j_e = \log \left( 1 + \frac{P_s|g_{0j}|^2}{w^\dagger R^j_j w + \sigma^2} \right). \]

For both the DF and CJ protocols, the achievable secrecy rate in the presence of \( J \) eavesdroppers is given by [26]
\[ R_s = \max\{0, R_d - \max_{1 \leq j \leq J} R^j_e\}. \]  

In particular, when \( J = 1 \), i.e., a single eavesdropper, the secrecy rate in (5) becomes [13], [14]
\[ R_s = \max\{0, R_d - R_e\}. \]  

We consider the practical case in which the system can be designed so that the secrecy rate is positive. In that case, the achievable secrecy rate can be rewritten as
\[ R_s = R_d - \max_{1 \leq j \leq J} R^j_e. \]

Achievability of the above rates can be shown based on existing results for MIMO wire-tap channels, such as [11], [14], [12], for one eavesdropper, and [26] for multiple eavesdroppers.

The CJ scheme can be viewed as a 1×2 SIMO system, so that MIMO results are directly applicable. As discussed in [24], for the DF scheme, MIMO results are applicable if we assume that the received signal at destination/eavesdropper at time \( i \) depends only on the relays’ transmitted encoded signals at time \( i \) (though a relay’s transmitted signal at time \( i \) depends on its received signal before time \( i \)). This is usually referred to as the “memoryless relay channel” [17], [19]. For convenience, as in [24] we focus on case in which the relays use the same codewords as the source to re-encode the signal before transmission; in that
case the rates of source-destination and source-eavesdropper links admit simple closed-form expressions [24].

The problems addressed in this paper are described as follows.

**Problem 1 (maximize secrecy rate under power constraint using DF):** Given total power (source plus relays) $P_0$, select source power $P_s$ and cooperative nodes’ weights $w$ to maximize the secrecy rate

$$\max_{P_s, w} R_s \quad \text{s.t. } P_s + \|w\|^2 = P_0, P_s \geq P_0^{\text{min}}.$$  (8)

where $R_s$ is given by (1), (2) and (7).

The solution of Problem 1 for one eavesdropper is provided in Section [III-A1] and for multiple eavesdroppers in Section [III-B].

**Problem 2 (maximize secrecy rate under power constraint using CJ):** Given total power (source plus relays) $P_0$, select source power $P_s$ and cooperative nodes’ weights $w$ to maximize the secrecy rate

$$\max_{P_s, w} R_s \quad \text{s.t. } P_s + \|w\|^2 = P_0.$$  (9)

where $R_s$ is given by (3), (4) and (7).

The solution of Problem 2 for one eavesdropper is provided in Section [III-A2].

**Problem 3 (minimize transmit power under secrecy rate constraint using DF):** Given the secrecy rate constraint $R_s^0$, select the source power $P_s$ and cooperative nodes’ weights, $w$, to minimize the total power (source plus relays)

$$\min_{P_s, w} \left[ P_0 = P_s + \|w\|^2 \right] \quad \text{s.t. } R_s = R_s^0, P_s \geq P_0^{\text{min}}.$$  (10)

where $R_s$ is given by (1), (2) and (7).

The solution of Problem 3 is provided in Section [IV-A].

**Problem 4 (minimize transmit power under secrecy rate constraint using CJ):** Given the secrecy rate constraint $R_s^0$, select the source power $P_s$ and cooperative nodes’ weights, $w$, to minimize the total power (source plus relays)

$$\min_{P_s, w} \left[ P_0 = P_s + \|w\|^2 \right] \quad \text{s.t. } R_s = R_s^0.$$  (11)

where $R_s$ is given by (3), (4) and (7).

The solution of Problem 4 is provided in Section [IV-B].

Before proceeding, we provide Lemma [1] and [2] which will be the basis of the results to follow. Please see Appendix [A] and [B] for details.
Lemma 1: Let \( r \) and \( s \) be (known) linearly uncorrelated vectors. Let \( \theta \) be the argument of \( r^\dagger s \). Denote \( i = \sqrt{-1} \). The matrix \( rr^\dagger - ss^\dagger \) has only two nonzero eigenvalues, i.e., \( \eta_1 > 0 \) and \( \eta_2 < 0 \), given by
\[
\eta_1 = \|r\|^2 - |c_2|r^\dagger s|, \quad \eta_2 = \|r\|^2 - |c_4|r^\dagger s|.
\] (12)

The corresponding eigenvectors are
\[
e_1 = c_1(r + |c_2|e^{i(\pi - \theta)}s), \quad e_2 = c_3(r + |c_4|e^{i(\pi - \theta)}s)
\] (13)

where \( c_1 = 1/\sqrt{\|r\|^2 + |c_2|^2}\|s\|^2 - 2|c_2||r^\dagger s|, \quad c_3 = 1/\sqrt{\|r\|^2 + |c_4|^2}\|s\|^2 - 2|c_4||r^\dagger s| \), \( 2|c_2||r^\dagger s| = \|r\|^2 - \|s\|^2 - \sqrt{\|r\|^2 + \|s\|^2)^2 - 4|r^\dagger s|^2} \), and \( 2|c_4||r^\dagger s| = \|r\|^2 + \|s\|^2 + \sqrt{\|r\|^2 + \|s\|^2)^2 - 4|r^\dagger s|^2} \).

Lemma 2: Let \( d_1 \) and \( d_2 \) be (known) unit-norm vectors. Let \( \phi \in (-\pi, \pi] \) be the argument of \( d_2^\dagger d_1 \), \( r = |d_1^\dagger d_2| \), and consider \( 0 \leq q \leq 1 \). The solution of
\[
\max_z z^\dagger d_2 d_1^\dagger z \quad \text{s.t.} \quad z^\dagger d_1 d_1^\dagger z = q, \quad \|z\| = 1
\] (14)
is given by \( z^\phi = c_1 d_1 + c_2 d_2 \), where \( c_2 = \sqrt{(1-q)/(1-r^2)} \) and \( c_1 = (rc_2 - \sqrt{q})e^{i(\pi - \phi)} \). The maximum is \( z^\phi d_2 d_1^\phi z^\phi = 1 - (r\sqrt{1-q} - \sqrt{(1-r^2)q})^2 \).

III. Secrecy Rate Maximization Under Power Constraint

In this section, we address Problems [1] and [2].

A. One Eavesdropper (\( J = 1 \))

In this subsection we address Problem [1] for the case of a single eavesdropper. Since \( J = 1 \) we drop the superscript \( j \) from \( R_j^c \).

1) DF-based protocol: Problem [1] can be recast as
\[
\max_{P, w} \frac{1}{2} \log \left( \frac{\sigma^2 + P_s|g_0|^2 + w^\dagger R_h w}{\sigma^2 + P_s|g_0|^2 + w^\dagger R_g w} \right)
\] (15)
s.t. \( P_s \geq P_0^{\min}, w^\dagger w = P_0 - P_s \).

Denote \( u_1 = g/\|g\|, u_2 = h/\|h\| \) and \( \zeta = |u_1^\dagger u_2| \). Let \( \theta \in (-\pi, \pi] \) be the argument of \( u_2^\dagger u_1 \). The solution of Problem [1] is given in the following theorem. The proof is given in Appendix [C].

Theorem 1: The solution of (15) is given by
\[
P_s^\phi = \begin{cases} P_0^{\min} & \text{if } J(P_0^{\min}) > J(P_0); \\ P_0 & \text{else}; \end{cases}
\] (16)
\[ w^\circ = \begin{cases} c_1u_1 + c_2u_2 & \text{if } J(P_{0}^{\min}) > J(P_0); \\ 0 & \text{else} \end{cases} \] (17)

where the function \( J(P_s) \) is defined by

\[
J(P_s) = \frac{\sigma^2 + P_s|h_0|^2 + (P_0 - P_s)\|h\|^2 L(z(P_s))}{\sigma^2 + P_s|g_0|^2 + (P_0 - P_s)\|g\|^2 z(P_s)}
\] (18)

and

\[
L(z) \triangleq 1 - (\zeta\sqrt{1-z} - \sqrt{(1-\zeta^2)z})^2
\] (19)

\[
z(P_s) = (B - \sqrt{B^2 - 4C})/(2bC) - a/b
\] (20)

\[
a = \sigma^2 + P_s|h_0|^2,
\]

\[
b = (P_0 - P_s)\|g\|^2,
\]

\[
c = \sigma^2 + P_s|h_0|^2 + (P_0 - P_s)\|h\|^2(1 - \zeta^2),
\]

\[
d = (P_0 - P_s)\|h\|^2(1 - 2\zeta^2),
\]

\[
f = 2(P_0 - P_s)\|h\|^2\zeta\sqrt{1-\zeta^2},
\]

\[
B = (2a + b)/(a^2 + ab),
\]

\[
C = \frac{f^2(2a + b)^2 + 4(ad + bc)^2}{4f^2(a^2 + ab)^2 + 4(ad + bc)^2(a^2 + ab)}
\]

\[
c_2 = \sqrt{P_0 - P_s}\sqrt{(1 - z(P_s^0))(1 - \zeta^2)}
\]

\[
c_1 = \sqrt{P_0 - P_s^0}(\zeta c_2 - \sqrt{z(P_s^0)})e^{i(\pi - \theta)}.
\]

Remarks: In particular, Theorem 1 states that, depending on the relative values of \( J(P_{0}^{\min}) \) and \( J(P_0) \), the optimal solution is either i) the source uses minimum power and the relay weights are a linear combination of the normalized relay and eavesdropper channel vectors, or ii) the source uses all the power and the relays are unused.

2) CJ-based protocol: Problem (2) is recast as

\[
\max_{P_s, w} \log \left( 1 + \frac{P_s|h_0|^2}{w^\dagger R_h w + \sigma^2} \right) - \log \left( 1 + \frac{P_s|g_0|^2}{w^\dagger R_g w + \sigma^2} \right)
\] (21)

s.t. \( w^\dagger w = P_0 - P_s, P_s \in [0, P_0] \).

By denoting \( w = \sqrt{P_0 - P_s} x, v_1 = h/\|h\|, v_2 = g/\|g\| \), the problem of (21) can be rewritten as

\[
\max_{P_s, x} \log \left( 1 + \frac{P_s|h_0|^2}{(P_0 - P_s)\|h\|^2 x^\dagger v_1 v_1^\dagger x + \sigma^2} \right) - \log \left( 1 + \frac{P_s|g_0|^2}{(P_0 - P_s)\|g\|^2 x^\dagger v_2 v_2^\dagger x + \sigma^2} \right)
\] (22)

s.t. \( x^\dagger x = 1, P_s \in [0, P_0] \).
Let \( x \) be a feasible point. Denote \( x^\dagger v_1^\dagger v_1^\dagger x = z, z \in [0, 1] \). For fixed \( z \), a larger \( x^\dagger v_2^\dagger v_2^\dagger x \) results in a larger objective value. With this and from Lemma 2 we know that the optimal \( x^\dagger v_2^\dagger v_2^\dagger x \) equals \( G(z) \) where \( G(z) \triangleq 1 - (\eta \sqrt{1-z} - \sqrt{(1-\eta^2)z})^2, \eta = |v_1^\dagger v_2| \). With these, we can rewrite the optimization of (22) as

\[
\max_{P_s, z} \log \left( 1 + \frac{P_s}{(P_0 - P_s)\alpha_1 z + \alpha_2} \right) - \log \left( 1 + \frac{P_s}{(P_0 - P_s)\alpha_3 G(z) + \alpha_4} \right) \quad (23)
\]

s.t. \( z \in [0, 1], P_s \in [0, P_0] \)

where \( \alpha_1 = ||h||^2/|h_0|^2, \alpha_2 = \sigma^2/|h_0|^2, \alpha_3 = ||g||^2/|g_0|^2 \) and \( \alpha_4 = \sigma^2/|g_0|^2 \).

The problem of (23) makes sense when the maximum is greater than zero, i.e., when positive secrecy rate is achieved. The conditions under which positive secrecy rate is achieved are given in the following lemma. The proof is given in Appendix D.

**Lemma 3:** The condition under which positive secrecy rate is achieved is:

- \( \alpha_2 < \alpha_4 \) (i.e., \( |h_0|^2 > |g_0|^2 \));
- \( \alpha_2 > \alpha_4, P_0 > (\alpha_2 - \alpha_4)/(\alpha_3 G(z_0) - \alpha_1 z_0) \) where \( z_0 \) is the unique root of \( \alpha_3 G'(z) = \alpha_1 \) given by

\[
z_0 = 1/(1 + u_0^2), u_0 = [\alpha_1/\alpha_3 - (2\eta^2 - 1) + \sqrt{(\alpha_1/\alpha_3)^2 + 1 - 2(\alpha_1/\alpha_3)(2\eta^2 - 1)})]/(2\eta \sqrt{1 - \eta^2})
\]

**Remarks:** The second condition means that if the source-destination channel is weaker than source-eavesdropper channel, direct transmission can not achieve positive secrecy rate, at that time, relays should be used and the total power (source plus relay) should be greater than a threshold.

In the following analysis, we assume the conditions in Lemma 3 hold. Denote the objective in (23) as \( M_1(z) \). It is easy to show that \( M_1'(z) > 0 \). Thus, \( z = 0 \) is not the optimal point. Before proceeding, we give a suboptimal solution which turns out to be the suboptimal solution proposed in [24], which is a special case corresponding to \( z = 0 \) (please see Appendix D for details).

**Lemma 4:** When \( z = 0 \) is fixed, a suboptimal solution is given:

- if \( \alpha_2 > \alpha_4, P_0 < (\alpha_2 - \alpha_4)/(\alpha_3(1 - \eta^2)) \), then \( P_{s, \text{sub}} = 0 \);
- if \( \{ \alpha_2 < \alpha_4 \text{ or } \alpha_2 > \alpha_4 \}, P_0 > (\alpha_2 - \alpha_4)/(\alpha_3(1 - \eta^2)) \) and \( (P_0 + \alpha_4)\alpha_4 > (P_0 + \alpha_2)(P_0\alpha_3(1 - \eta^2) + \alpha_4) \), then \( P_{s, \text{sub}} = P_0 \);
- if \( \{ \alpha_2 < \alpha_4 \text{ or } \alpha_2 > \alpha_4 \}, P_0 > (\alpha_2 - \alpha_4)/(\alpha_3(1 - \eta^2)) \) and \( (P_0 + \alpha_4)\alpha_4 < (P_0 + \alpha_2)(P_0\alpha_3(1 - \eta^2) + \alpha_4) \), then \( P_{s, \text{sub}} = -(c_3d_2 + \sqrt{c_3^2d_2^2 - c_3d_2(1-d_2)(\alpha_2 - \alpha_3)})/(d_2(1 - d_2)), \) where \( d_2 = \alpha_3(1 - \eta^2), c_3 = \alpha_4 + P_0\alpha_3(1 - \eta^2) \).

Now we proceed. The methodology to solve the problem of (23) is: 1) fix \( z \), find the optimal \( P_s \); 2) fix \( P_s \), find the optimal \( z \). Based on this, we propose an algorithm to search for the optimal \( P_s \) and \( z \) as follows.
Algorithm 1: Take a feasible point \( z^{(1)} \) as initial point. Subsequently, find the optimal \( P_s^{(1)} \) and then the optimal \( z^{(2)} \). Then find the optimal \( P_s^{(2)} \), and so on. The procedure converges to the optimal \( P_s^0 \) and \( z^0 \).

The algorithm is not complete without providing the methods to find the optimal \( z \) for fixed \( P_s \) and the optimal \( P_s \) for fixed \( z \). Next, we provide such methods. First, we consider the problem: find the optimal \( P_s \) for fixed \( z \). This corresponds to an optimization problem of a single variable \( P_s \), and the maximum is achieved at either 0, \( P_0 \), or the points with zero derivative. Setting the derivative of the objective to zero leads to the following quadratic equation

\[
A_1 P_s^2 + B_1 P_s + C_1 = 0
\]

where \( A_1 = a_1 d_1 (1 - d_1) - b_1 c_1 (1 - b_1), B_1 = 2a_1 c_1 (d_1 - b_1), C_1 = a_1 c_1 (1 - c_1), a_1 = P_0 \alpha_1 z + \alpha_2, b_1 = \alpha_1 z, c_1 = P_0 \alpha_3 G(z) + \alpha_4 \) and \( d_1 = \alpha_3 G(z) \). We can obtain \( P_s \) explicitly as a function of \( z \). Second, we consider the problem: when \( P_s \) is fixed, find the optimal \( z \). This corresponds to an optimization problem of a single variable \( z \), and the maximum is achieved at one of the following points: 0, 1 and the points with zero derivative. The problem to solve can now be rewritten as

\[
\max_z R_s(z) = \log \frac{\alpha_1 z + b_3}{\alpha_1 z + a_3} - \log \frac{\alpha_3 G(z) + d_3}{\alpha_3 G(z) + c_3}
\]

s.t. \( z \in [0, 1] \)

where \( a_3 = \alpha_2/(P_0 - P_s), b_3 = (\alpha_2 + P_s)/(P_0 - P_s), c_3 = \alpha_4/(P_0 - P_s) \) and \( d_3 = (\alpha_4 + P_s)/(P_0 - P_s) \).

The derivative of \( R_s(z) \) given by

\[
R'_s(z) = \frac{\partial R_s}{\partial z} = \frac{P_s}{P_0 - P_s} \left( \alpha_3 G'(z) \right) = \frac{\alpha_1}{(\alpha_1 z + a_3)(\alpha_1 z + b_3)}.
\]

It is easy to verify that \( R'_s(0) > 0 \), \( R'_s(1) < 0 \). Thus, the optimal \( z \) must be the points with zero derivative. Note that \( R_s(z) > 0 \) holds only when \( \alpha_1 z + a_3 < \alpha_3 G(z) + c_3 \), which determines an interval \( (\underline{z}, \bar{z}) \subset [0, 1] \). Here \( \underline{z} \) and \( \bar{z} \) can be expressed in closed form from the fact: if \( G(z) = \beta_1 z + \beta_2 \) has real roots over \([0,1]\), then its roots can be expressed as \( z = 1/(1 + u_0^2) \) where \( u_0 = (\eta \sqrt{1 - \eta^2} + \sqrt{\eta^2(1 - \eta^2) + (\beta_2 - 1 + \eta^2)(\eta^2 - \beta_1 - \beta_2)})/(\beta_2 - 1 + \eta^2) \). With these, we can restrict our attention to the root of \( (26) \) over \((\underline{z}, \bar{z})\).

To proceed, we need the following result. The proof is given in Appendix F.

Property 1: For \( R_s(z) \) defined in (25), \( \frac{\partial^2 R_s}{\partial z^2} |_{z'} < 0 \) for the stationary point \( z' \in (\underline{z}, \bar{z}) \) (i.e., the point with \( \frac{\partial R_s}{\partial z} |_{z'} = 0 \)).
According to Property 1, we know that the equation (26) has a unique root \( z' \) such that when \( z < z' \), \( \frac{\partial R_s}{\partial z} > 0 \) and when \( z > z' \), \( \frac{\partial R_s}{\partial z} < 0 \). This property ensures that the Newton method would be very effective in searching for \( z' \) and would enjoy quadratic convergence.

B. Multiple Eavesdroppers

We now turn to the case of multiple eavesdroppers \((J > 1)\). We restrict our attention to the DF protocol. The CJ protocol case with multiple eavesdroppers is a more difficult problem and will be addressed in future work.

Problem 1 now becomes

\[
\max_{P_s, w} \min_{j \in I} \frac{1}{2} \log \frac{\sigma^2 + P_s|h_0|^2 + w^\dagger R_{h,w}}{\sigma^2 + P_s|g_0|^2 + w^\dagger R_{g,w}}
\]

\[\text{s.t. } P_s \in [P_{0 \text{min}}, P_0], \quad w^\dagger w = P_0 - P_s\]

where \( I = \{1, \ldots, J\} \), \( P_{0 \text{min}} \) is defined as in the case of a single eavesdropper. By letting \( w = \sqrt{P_0 - P_s} x \), we can rewrite the above problem as

\[
\max_{P_s, x} \min_{j \in I} \frac{1}{2} \log \frac{\sigma^2 + P_s|h_0|^2 + (P_0 - P_s)x^\dagger R_{h,x}}{\sigma^2 + P_s|g_0|^2 + (P_0 - P_s)x^\dagger R_{g,x}}
\]

\[\text{s.t. } P_s \in [P_{0 \text{min}}, P_0], \quad x^\dagger x = 1\]

Before proceeding, we give a suboptimal solution which turns out to be the suboptimal solution in [24]. In [24], if \( N \geq J + 1 \), a suboptimal solution is obtained when an additional constraint is added: \( w^\dagger G = 0 \) where \( G = [g_1, g_2, \ldots, g_J] \). This additional constraint means nulling the energy to the eavesdroppers, and this requires \( N > J \) so that we have enough degrees of freedom to ensure this is possible. The proof is given in Appendix G.

**Lemma 5:** When the constraint \( w^\dagger G = 0 \) is added, a suboptimal solution is obtained as

\[
P_{s,\text{sub}} = \begin{cases} 
P_0 & \text{if } f_2(P_0) > f_2(P_{0 \text{min}}); \\
P_{0 \text{min}} & \text{else}
\end{cases}
\]

where the function \( f_2(P_s) \) is defined by

\[
f_2(P_s) = \frac{\sigma^2 + P_s|h_0|^2 + (P_0 - P_s)||E^\dagger h||^2}{\sigma^2 + P_s \max_{j \in I} \{|g_0|^2\}}
\]

and \( E \) is the null space of \( G^\dagger \) with \( E^\dagger E = I \).

Now we proceed. The methodology to solve the problem of (28) is: 1) fix \( x \), find the optimal \( P_s \); 2) fix \( P_s \), find the optimal \( x \). Based on this, we propose an algorithm to search for the optimal \( P_s \) and \( x \) as follows.
**Algorithm 2:** Take a feasible $P_s^{(1)}$ as an initial point. Subsequently, find the optimal $x^{(1)}$ and then the optimal $P_s^{(2)}$. Then find the optimal $x^{(2)}$, and so on.

**Remarks:** We will see later that: for $J = 2$, the procedure always converges to the optimal $P_s$ and $x$, while for $J > 2$ the procedure does not necessarily converge to the optimal solution. We will discuss this in the sequel.

The algorithm is not complete without providing the methods to find the optimal $x$ for fixed $P_s$ and find the optimal $P_s$ for fixed $x$. Next, we provide such methods.

First, we consider the problem: find the optimal $P_s$ for fixed $x$. We need to solve

$$\begin{align*}
\max_{P_s} & \quad \frac{\sigma^2 + P_s |h_0|^2 + (P_0 - P_s)x^\dagger R_h x}{\sigma^2 + \max_{j \in I} \{P_s |g_{0j}|^2 + (P_0 - P_s)x^\dagger R_{gj} x\}} \\
\text{s.t.} & \quad P_s \in [P_{0\min}, P_0].
\end{align*} \tag{31}$$

Note that $\max_{j \in I} \{P_s |g_{0j}|^2 + (P_0 - P_s)x^\dagger R_{gj} x\}$ denotes a polygonal line $\Gamma$ whose vertices are located at the points $P_{s,k}$, $k = 0, 1, \cdots, M$, $P_{s,0} = P_{0\min}$, $P_{s,M} = P_0$. An example is plotted in Fig. 2. Note that the objective in (31) is a linear fractional function and hence quasi-linear [27] in each line segment $P_{s,k}P_{s,k+1}$, $k = 0, 1, \cdots, M - 1$. Thus, the optimal $P_s$ is one of the vertices of the polygonal, i.e., $P_{s,k}$, $k = 0, 1, \cdots, M - 1$. It is easy to find $P_{s,k}$.

![Fig. 2. Polygonal line for the problem (31).](image)

Second, we consider the problem: find the optimal $x$ for fixed $P_s$. We need to solve

$$\begin{align*}
\max_x & \quad \min_{j \in I} \frac{\sigma^2 + P_s |h_0|^2 + (P_0 - P_s)x^\dagger R_h x}{\sigma^2 + P_s |g_{0j}|^2 + (P_0 - P_s)x^\dagger R_{gj} x} \\
\text{s.t.} & \quad x^\dagger x = 1.
\end{align*} \tag{32}$$

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We can show that the problem of (32) is equivalent to
\[
\max_v \|v\|^2 \quad \text{s.t.} \quad v^\dagger A_j v \leq 1, j \in I
\] (33)
where
\[
A_j = \tilde{R}_h^{-1/2} \tilde{R}_g \tilde{R}_h^{-1/2}, j \in I
\] (34)
\[
\tilde{R}_h = (\sigma^2 + P_s|h_0|^2)I + (P_0 - P_s)R_h,
\] (35)
\[
\tilde{R}_g^j = (\sigma^2 + P_s|g_0|^2)I + (P_0 - P_s)R_g^j, j \in I.
\] (36)

Let the solution of (33) be \(v^\circ\). Then, the solution of (32) is given by \(x^\circ = \tilde{R}_h^{-1/2}v^\circ/\|\tilde{R}_h^{-1/2}v^\circ\|\). The proof is given in Appendix H.

The problem of (33) belongs to the quadratic constrained quadratic programs (QCQP) which is in general difficult [30], [31]. Denote \(Z = vv^\dagger\) which enables us to rewrite it as
\[
\max_Z \text{Tr}(Z) \quad \text{s.t.} \quad Z \succeq 0, \text{rank}(Z) = 1, \text{Tr}(A_j Z) \leq 1, j \in I.
\] (37)
Dropping the constraint \(\text{rank}(Z) = 1\), the semi-definite relaxation (SDR) of the problem of (37) is given by
\[
\max_Z \text{Tr}(Z) \quad \text{s.t.} \quad Z \succeq 0, \text{Tr}(A_j Z) \leq 1, j \in I.
\] (38)
This is a semi-definite program (SDP) and can be effectively solved by CVX [32].

If the problems of (33) and (38) achieve the same objective value, we say the SDR of (38) is tight (its solution \(Z^\circ\) does not necessarily have rank one). Obviously, if the solution \(Z^\circ\) of (38) has rank one, the SDR (38) is tight and the optimal solution \(v^\circ\) of the problem of (33) can be obtained by simply eigen-decomposing \(Z^\circ = vv^\dagger\). If \(Z^\circ\) does not have rank one, the problem for the solution of (33) is in general difficult [30], [31]. But for the special case \(J = 2\), the problem can be solved in polynomial time [30 §2.2], [31 Lemma 2.2.3]. We state it as the following theorem.

**Theorem 2:** When \(J = 2\), the SDR (38) is always tight and the optimal solution for the problem of (33) can be constructed in polynomial time.

For \(J > 2\), if \(Z^\circ\) does not have rank one, we use Gaussian randomization procedure (GRP) to obtain an approximate solution based on the SDR solution [28], [29]. In detail, we calculate the eigen-decomposition of \(Z^\circ = UDU^\dagger\) and generate \(v_l = \mu_l UD^{1/2}\xi_l\) for \(l = 1, \cdots, L\), where \(\xi_l\) is a vector of zero-mean, unit-variance complex circularly symmetric uncorrelated Gaussian random variables, \(\mu_l\) is chosen such that \(\max_{j \in I} v_l^\dagger A_j v_l = 1\) (i.e., the constraint of (33) holds). Suppose
\[
l^* = \arg \max_l \|v_l\|^2.
\] (39)
Then select \( v^{(i^*)} \) as an approximation solution.

IV. TRANSMIT POWER MINIMIZATION UNDER SECRECY RATE CONSTRAINT

In this section, we address Problems 3 and 4.

A. DF-based protocol

Problem 3 is to solve

\[
\begin{align*}
\min_{P_s, w} & \quad P_0 = P_s + \|w\|^2 \\
\text{s.t.} & \quad \frac{\sigma^2 + P_s|h_0|^2 + w^\dagger R_h w}{\sigma^2 + P_s|g_0|^2 + w^\dagger R_g w} = 4^{R_s}, \quad P_s \geq P_0^{\min}.
\end{align*}
\]

We provide a closed form solution to this problem which reveals that the optimal weight vector \( w^* \) is a linear combination of \( h \) and \( g \). We give the main result of the problem as a theorem. The proof is given in Appendix [1].

**Theorem 3:** When \( 4^{R_s}|g_0|^2 - |h_0|^2 \neq 0 \),

\[
(P_s^*, w^*) = \begin{cases} 
(\zeta, 0) & \text{if } \zeta \geq P_0^{\min}, \lambda_2 \geq -1 \\
(P_0^{\min}, \sqrt{\frac{\zeta - P_0^{\min}}{\lambda_2}} u_2) & \text{if } \zeta \geq P_0^{\min}, \lambda_2 < -1 \\
(P_0^{\min}, \sqrt{\frac{\zeta - P_0^{\min}}{\lambda_1}} u_1) & \text{if } \zeta < P_0^{\min}
\end{cases}
\]

where \( \zeta = (4^{R_s} - 1)\sigma^2/(|h_0|^2 - 4^{R_s}|g_0|^2); \lambda_1 > 0, \lambda_2 < 0 \) are the only two nonzero eigenvalues of \( \bar{R} = (R_h - 4^{R_s} R_g)/(4^{R_s}|g_0|^2 - |h_0|^2) \) with associated eigenvectors \( u_1 \) and \( u_2 \) (see Lemma [7]).

When \( 4^{R_s}|g_0|^2 - |h_0|^2 = 0 \), the solution is

\[
P_s^* = P_0^{\min} \quad \text{and} \quad w^* = \sqrt{(4^{R_s} - 1)\sigma^2/\xi_1} v_1
\]

where \( \xi_1 > 0, \xi_2 < 0 \) are the only two nonzero eigenvalues of \( (R_h - 4^{R_s} R_g) \) with associated eigenvectors \( v_1 \) and \( v_2 \) (see Lemma [7]).

Before ending of this subsection, we should point out that the results in [22]-[24] address only the case \( 4^{R_s}|g_0|^2 - |h_0|^2 > 0 \), thus our analysis here is more complete. Further, our analysis reveals that \( (R_h - 4^{R_s} R_g) \) has only two nonzero eigenvalues (one positive, and one negative) and, unlike [22]-[24], provides simple expression for them.
B. CI-based protocol

Problem 4 is to solve

\[
\min_{P_s, w} P_0 = P_s + \|w\|^2
\]
\[
s.t. \quad \frac{|h_0|^2}{w^\dagger R_h w + \sigma^2} - \frac{2R_0^s |g_0|^2}{w^\dagger R_g w + \sigma^2} = \frac{2R_0^s - 1}{P_s}, \quad P_s > 0.
\]

By denoting \(\|w\|^2 = \gamma\), \(x = w/\|w\|\), \(v_1 = h/\|h\|\), \(v_2 = g/\|g\|\), the problem of (44) can be rewritten as

\[
\min_{P_s, x} P_0 = P_s + \gamma
\]
\[
s.t. \quad \frac{|h_0|^2}{\gamma\|h\|^2 x^\dagger v_1 v_1^\dagger x + \sigma^2} - \frac{2R_0^s |g_0|^2}{\gamma\|g\|^2 x^\dagger v_2 v_2^\dagger x + \sigma^2} = \frac{2R_0^s - 1}{P_s}, \quad P_s > 0.
\]

Let \(x\) be a feasible point, and \(x^\dagger v_1 v_1^\dagger x = z, z \in [0, 1]\). For fixed \(z\) and \(\gamma\), a larger \(x^\dagger v_2 v_2^\dagger x\) results in a smaller \(P_s\). With this and from Lemma 2 we know that the optimal \(x^\dagger v_2 v_2^\dagger x\) equals \(F(z)\) where \(F(z) \triangleq 1 - (\rho \sqrt{1 - z} - \sqrt{(1 - \rho^2)z})^2\), \(\rho = |v_1^\dagger v_2|\). With this in mind, we can rewrite (44) as

\[
\min_{P_s, \gamma, z} P_0 = P_s + \gamma
\]
\[
s.t. \quad \frac{1}{\gamma \alpha_1 z + \alpha_2} - \frac{1}{\gamma \alpha_3 F(z) + \alpha_4} = \frac{1}{P_s},
\]
\[
z \in [0, 1], \quad P_s > 0, \quad \gamma \geq 0
\]

where \(\alpha_1 = \|h\|^2(2R_0^s - 1)/|h_0|^2\), \(\alpha_2 = \sigma^2(2R_0^s - 1)/|h_0|^2\), \(\alpha_3 = \|g\|^2(2R_0^s - 1)/(2R_0^s |g_0|^2)\) and \(\alpha_4 = \sigma^2(2R_0^s - 1)/(2R_0^s |g_0|^2)\). Further, we can rewrite the problem of (46) as

\[
\min_{\gamma, z} P_0 = \frac{(\gamma \alpha_1 z + \alpha_2)[\gamma \alpha_3 F(z) + \alpha_4]}{\gamma (\alpha_3 F(z) - \alpha_1 z) + \alpha_4 - \alpha_2} + \gamma
\]
\[
s.t. \quad \gamma (\alpha_3 F(z) - \alpha_1 z) + \alpha_4 - \alpha_2 > 0,
\]
\[
z \in [0, 1], \quad \gamma \geq 0
\]

Before proceeding, we give a suboptimal solution which is turns out to be the same as the the suboptimal solution of [24]. Please see Appendix I for details.

**Lemma 6:** When \(z = 0\) is fixed, a suboptimal solution is obtained as

\[
P_{0,\text{sub}} = P_{s,\text{sub}} + \gamma_{\text{sub}}
\]
where
\[
\gamma_{\text{sub}} = \begin{cases} 
0 & \text{if } \frac{\alpha_3}{\alpha_2} > 1 + \sqrt{\frac{\alpha_3(1 - \rho^2)}{\alpha_2}} \\
\frac{\alpha_2 \sqrt{\alpha_3(1 - \rho^2) + \alpha_2 - \alpha_4}}{\alpha_3(1 - \rho^2)} & \text{else.}
\end{cases}
\] (49)
\[
P_{s,\text{sub}} = \frac{1}{\frac{1}{\alpha_2} - \frac{1}{\gamma_{\text{sub}} \alpha_3(1 - \rho^2) + \alpha_4}}.
\] (50)

Now we proceed. The methodology to solve the problem of (47) is: 1) fix \(z\), find the optimal \(\gamma\); 2) fix \(\gamma\), find the optimal \(z\). Based on this, we propose an algorithm to search for the optimal \(\gamma\) and \(z\) as follows.

**Algorithm 3:** Take a feasible point \(z^{(1)}\) as initial point. Subsequently, find the optimal \(\gamma^{(1)}\) and then the optimal \(z^{(2)}\). Then find the optimal \(\gamma^{(2)}\), and so on. The procedure converges to the optimal \(\gamma^*\) and \(z^*\).

The algorithm is not complete without providing the methods to find the optimal \(z\) for fixed \(\gamma\) and find the optimal \(\gamma\) for fixed \(z\). Next, we provide such methods.

First, we consider the problem: find the optimal \(\gamma\) for fixed \(z\). This corresponds to an optimization problem of a single variable \(\gamma\), and the maximum is achieved at one of the following points: 0 and the points with zero derivative. With this, we obtain
\[
\gamma = \max \left\{ \frac{\sqrt{f_1(z)} - (\alpha_4 - \alpha_2)}{\alpha_3 F(z) - \alpha_1 z}, 0 \right\}
\] (51)
where \(f_1(z) = (\alpha_2 \alpha_3 F(z) - \alpha_1 \alpha_4 z)^2 / [\alpha_3 F(z) - \alpha_1 z + \alpha_1 \alpha_3 z F(z)]\). We can obtain \(\gamma\) explicitly as a function of \(z\) (in this sense, we in fact reduce the original problem to a single variable optimization \(P_0 = P_0(z), z \in [0, 1]\) explicitly).

Second, we consider the problem: when \(\gamma\) is fixed, find the optimal \(z\). Let us denote the left side of the first constraint in (46) by \(f_\gamma(z)\), namely
\[
f_\gamma(z) = \frac{1}{\gamma \alpha_1 z + \alpha_2} - \frac{1}{\gamma \alpha_3 F(z) + \alpha_4}.
\] (52)
We know that the optimal \(z\) must maximize \(f_\gamma(z)\) which results in the minimal \(P_s\) (see 46). The derivative of \(f_\gamma(z)\) given by
\[
f'_\gamma(z) = \frac{\partial f_\gamma}{\partial z} = -\gamma \frac{\alpha_1}{(\gamma \alpha_1 z + \alpha_2)^2} + \frac{\gamma \alpha_3 F'(z)}{(\gamma \alpha_3 F(z) + \alpha_4)^2}.
\] (53)
It is easy to verify that \(f'_\gamma(0) > 0, f'_\gamma(1) < 0\). Thus, \(z = 0\) is not the optimal point, and the optimal \(z\) must be the points with zero derivative. Note that \(P_s > 0\) (i.e., \(f_\gamma(z) > 0\)) holds only when \(\gamma \alpha_1 z + \alpha_2 < \gamma \alpha_3 F(z) + \alpha_4\) which determines an interval \((\underline{z}, \bar{z}) \subset [0, 1]\). Here \(\underline{z}\) and \(\bar{z}\) can be expressed in closed form from the fact: if \(F(z) = \beta_1 z + \beta_2\) has real roots over \([0, 1]\), then its roots can be expressed as
\[ z = 1/(1 + u_0^2) \] where \( u_0 = (\rho \sqrt{1 - \rho^2} \pm \sqrt{\rho^2(1 - \rho^2) + (\beta_2 - 1 + \rho^2)(\rho^2 - \beta_1 - \beta_2)})/((\beta_2 - 1 + \rho^2)). \]

With these, we can restrict our attention to the root of (53) over \((z, \bar{z})\).

To proceed, we need the following result. The proof is given in Appendix K.

**Property 2:** For \( f_\gamma(z) \) defined in (52), \( \frac{\partial^2 f_\gamma}{\partial z^2} \mid_{z'} < 0 \) for the stationary point \( z' \in (z, \bar{z}) \) (i.e., the point with \( \frac{\partial f_\gamma}{\partial z} \mid_{z'} = 0 \)).

According to Property 2, we know that the equation (53) has a unique root \( z' \) such that when \( z < z' \), \( f_\gamma'(z) > 0 \) and when \( z > z' \), \( f_\gamma'(z) < 0 \). This property ensures that the Newton method would be very effective in searching for \( z' \) and would enjoy quadratic convergence.

**V. Numerical Simulations**

In this section we provide some numerical simulations to illustrate the proposed solutions. We use the same system configuration as that in [24], where source, relays, destination and eavesdroppers are placed along a line. Channels between any two nodes are modeled as a line-of-sight (LOS) channel \( \rho_0 d^{-c/2} e^{i\theta} \), where \( d \) is the distance between two nodes, \( \rho_0 \) is a constant, \( c \) is the path loss exponent, and \( \theta \) is the phase uniformly distributed within \([0, 2\pi]\). In our simulations we set \( c = 3.5 \) and \( \rho_0 = 1 \). We assume the distances between relays are much smaller than the distances between relays and source/destination, such that the path loss between different relays and source/destination can be taken as approximately the same. Similarly, the path loss between different eavesdroppers and source/destination/relay are approximately the same as well. The results are obtained using Monte-Carlo simulations consisting of 500 independent trials.

First, we vary the position of the destination so that the source-destination distance changes from 10 m to 100 m, as shown in the upper row of Fig. 3. The source-relay distances are fixed at 5 m, the number of relays is \( N = 10 \), the source-eavesdropper distances are fixed at 50 m, the power constraint is fixed at 30 dBm, the secrecy rate constraint is fixed at 1 bits/s/Hz. The secrecy rate for a single eavesdropper and multiple eavesdroppers is depicted in Figs. 4 and 5 respectively. From these two figures, one can see that when the destination moves past the eavesdropper direct transmission cannot sustain positive secrecy rate. On the other hand, both DF and CJ maintain positive secrecy rate even when the destination is further away from the source than the eavesdropper. The fact that there is a cooperation advantage even when the destination is at the same location as the eavesdropper is because of the phases differences of the corresponding channels. Although the propagation environment would be the same for both destination and eavesdropper in that case, the phases will be different due to different receiver phase offsets. The DF scheme yields the higher secrecy rate, while the optimal and suboptimal CJ schemes produce the same
average rate.

Similar observations can be drawn from Fig. 5 for the case of multiple eavesdropper as far as the advantage of cooperation over direct transmission is concerned.

In Fig. 6 the secrecy rate for a fixed configuration and variable number of eavesdroppers is shown. It can be seen from Fig. 6 that, when the number of eavesdroppers increases, the suboptimal solution for DF in Lemma 5 becomes inferior as compared to the optimal solution. The minimal transmit power is depicted in Fig. 7. For comparison purposes, the suboptimal solution for CJ in Lemma 4 and the direct transmission result are also shown on the same figure.

Second, in Fig. 8 we show the performance when the eavesdroppers’ positions change while the source-destination distance is fixed at 50 m and the source-relay distances are fixed at 5 m. When the source-eavesdropper distance changes from 25 m to 100 m as shown in the lower row of Fig. 3, the minimum transmit power for CJ first increases a little, and then decreases, while the minimum transmit power for DF always decreases. The results show that cooperation can significantly improve the system performance as compared to direct transmission. In particular, when the source-eavesdropper distance is smaller than 65 m, using direct transmission there is no level of transmit power than can meet the secrecy rate constraint. Also, for source-eavesdropper distance greater than 85 m direct transmission and the CJ scheme are equivalent in terms of the minimum required transmit power. The DF approach requires significantly smaller power to meet the secrecy rate constraint. It is interesting to note that in the average sense, the suboptimal solution for CJ in Lemma 4 and Lemma 6 is a very good approximation of the optimal solution.

VI. CONCLUSION

We have given explicit constructions for the optimal relay weights and source transmission power for maximizing the secrecy rate or minimizing the total transmit power (source transmission power plus relay power) under secrecy rate constraint using the DF and CJ protocols in the presence of a single eavesdropper or multiple eavesdroppers. We present numerical results to compare the secrecy rate under our optimal solutions with the secrecy rate under the sub-optimal solutions in [22]-[24]. Numerical results illustrate that cooperation can significantly improve the system performance as compared to direct transmission.
APPENDIX A

PROOF OF LEMMA 1

Let $\lambda \neq 0$ be the eigenvalue of $r r^\dagger - s s^\dagger$ associated with the eigenvector $a$. Thus we have $(r r^\dagger - s s^\dagger)a = \lambda a$ which leads to $a = (1/\lambda)[r(r^\dagger a) - s(s^\dagger a)]$. Thus, $a$ has the form of a linear combination of $r$ and $s$. With this, we let $a = \pi_1 r + \pi_2 s$ where $\pi_1$ and $\pi_2$ will be determined as follows. Since $a$ has unit norm, we have

$$|\pi_1|^2||r||^2 + \pi_1 \pi_2^*(s^\dagger r) + \pi_1^* \pi_2 (r^\dagger s) + |\pi_2|^2||s||^2 = 1. \quad (54)$$

On the other hand, by inserting $a = \pi_1 r + \pi_2 s$ into $(r r^\dagger - s s^\dagger)a = \lambda a$, we get

$$[\pi_1(||r||^2 - \lambda) + \pi_2(r^\dagger s)]r = [\pi_1(s^\dagger r) + \pi_2(||s||^2 + \lambda)]s. \quad (55)$$

Since $r$ and $s$ are linearly uncorrelated, we have

$$\pi_1(||r||^2 - \lambda) + \pi_2(r^\dagger s) = \pi_1(s^\dagger r) + \pi_2(||s||^2 + \lambda) = 0 \quad (56)$$

which leads to

$$\lambda^2 - (||r||^2 - ||s||^2)\lambda - (||r||^2||s||^2 - ||r^\dagger s||^2) = 0 \quad (57)$$

$$\pi_2 = \frac{\lambda - ||r||^2}{r^\dagger s} \pi_1. \quad (58)$$

From Cauchy inequality, we get $||r||^2||s||^2 - ||r^\dagger s||^2 > 0$. Thus, The equation (57) has a positive root and a negative root. As a result, we obtain $\lambda$ and the corresponding $\pi_1$ and $\pi_2$.

APPENDIX B

PROOF OF LEMMA 2

The solution $z^\circ$ of (14) is a linear combination of $d_1$ and $d_2$, which follows from its optimality condition $d_2 d_2^\dagger z - \mu_1 d_1 d_1^\dagger z - \mu_2 z = 0$ or further $\mu_2 z = (d_2^\dagger z)d_2 - (\mu_1 d_1^\dagger z)d_1$ where $\mu_1$ and $\mu_2$ are Lagrange multipliers. Note that $e^{-i\theta_2}z^\circ = e^{-i\theta_2}c_1 d_1 + |c_2|d_2$ is also solution of (14), where $\theta_2$ is the argument of $c_2$. Consequently, we can restrict $c_2 \geq 0$. Inserting $z = c_1 d_1 + c_2 d_2$ into the constraints and objective, results in

$$|c_1|^2 + c_2^2 + c_1^* c_2 d_1^\dagger d_2 + c_1 c_2 d_2^\dagger d_1 = 1 \quad (59)$$

$$|c_1|^2 + c_2^2|d_1^\dagger d_2|^2 + c_1^* c_2 d_1^\dagger d_2 + c_1 c_2 d_2^\dagger d_1 = q \quad (60)$$

$$z^\dagger d_2 d_2^\dagger z = 1 - |c_1|^2(1 - |d_1^\dagger d_2|^2). \quad (61)$$
From (61), we need to minimize \( |c_1|^2 \). From (59) and (60), we get \( c_2^2(1 - |d_1^\dagger d_2|^2) = 1 - q \) which leads to \( c_2 = \sqrt{(1 - q)/(1 - r^2)} \). By denoting \( c_1 = |c_1|e^{i\theta} \) where \( \theta \) is the argument of \( c_1 \), we can rewrite (59) as
\[
|c_1|^2 + |c_1|c_2 r (e^{-i(\phi + \theta)} + e^{i(\phi + \theta)}) + (c_2^2 - 1) = 0.
\] (62)

It is not difficult to show that the optimal \( \theta \) given by
\[
\theta = \begin{cases} 
-\phi & \text{if } c_2^2 - 1 < 0 \\
\pi - \phi & \text{if } c_2^2 - 1 \geq 0
\end{cases}
\] (63)

and the optimal \( |c_1| \) is given by
\[
|c_1| = \begin{cases} 
\sqrt{q} - c_2 r & \text{if } c_2^2 - 1 < 0 \\
c_2 r - \sqrt{q} & \text{if } c_2^2 - 1 \geq 0
\end{cases}.
\] (64)

With these, we obtain the optimal \( c_1 = (c_2 r - \sqrt{q})e^{i(\pi - \phi)} \). Further, from (61), we obtain
\[
z^\dagger d_2 d_2^\dagger z = 1 - (c_2 r - \sqrt{q})^2 (1 - r^2) = 1 - (r \sqrt{1 - q} - \sqrt{(1 - r^2)q})^2.
\] (65)

APPENDIX C

PROOF OF THEOREM 1

First, we derive the optimal weight vector \( w \) for fixed \( P_s \geq P_0^{\text{min}} \). By denoting \( w = \sqrt{P_0 - P_s} \, x \), we can rewrite the problem of (15) as
\[
\max_x \frac{1}{2} \log \left( \frac{\sigma^2 + P_s |h_0|^2 + (P_0 - P_s)\|h\|^2 x^\dagger u_2 u_2^\dagger x}{\sigma^2 + P_s |g_0|^2 + (P_0 - P_s)\|g\|^2 x^\dagger u_1 u_1^\dagger x} \right)
\] (66)
\[\text{s.t. } x^\dagger x = 1.\]

Let \( x \) be a feasible point and \( x^\dagger u_1 u_1^\dagger x = z \), \( z \in [0, 1] \). For fixed \( z \), a larger \( x^\dagger u_2 u_2^\dagger x \) results in a larger objective value in the problem of (66). With this and from Lemma 2, we know that \( x^\dagger u_2 u_2^\dagger x \) equals \( L(z) \). Thus, the problem of (66) can be rewritten as
\[
\max_z M(z) = \frac{1}{2} \log \left( \frac{\sigma^2 + P_s |h_0|^2 + (P_0 - P_s)\|h\|^2 L(z)}{\sigma^2 + P_s |g_0|^2 + (P_0 - P_s)\|g\|^2 z} \right)
\] (67)
\[\text{s.t. } 0 \leq z \leq 1.\]

This is an optimization problem of a single variable \( z \). It is easy to show that \( M'(0) > 0, M'(1) < 0 \). Thus, the optimal \( z \) must be the points with zero derivative, i.e., \( M'(z) = 0 \). As a result, the solution of (67), denoted by \( z(P_s) \) as a function of \( P_s \), can be expressed in closed form (20).
Next we consider the optimal $P_s$ and let it be $P_s^o$. We can state that $P_s^o$ is also the solution of the following associated problem

$$
\max_{P_s} \frac{1}{2} \log \left( \frac{\sigma^2 + P_s |h_0|^2 + (P_0 - P_s) ||h||^2 L(z(P_s^o))}{\sigma^2 + P_s |g_0|^2 + (P_0 - P_s) ||g||^2 L(z(P_s^o))} \right)
$$

s.t. $P_s \in [P_{0\text{min}}, P_0]$.

To see why this is the case, let assume the solution of (68) is $P_s'$ but not $P_s^o$. Denote the objective in (67) as $\frac{1}{2} \log(J_1(z(P_s'), P_s'))$. Recall that for fixed $P_s'$, the solution of (67) is $z(P_s')$, which leads to $\frac{1}{2} \log(J_1(z(P_s'), P_s')) \geq \frac{1}{2} \log(J_1(z(P_s^o), P_s'))$. On the other hand, $\frac{1}{2} \log(J_1(z(P_s^o), P_s^o)) \geq \frac{1}{2} \log(J_1(z(P_s^o), P_s'))$ since $P_s^o$ is the solution of (15). Combining both gives $\frac{1}{2} \log(J_1(z(P_s^o), P_s^o)) \geq \frac{1}{2} \log(J_1(z(P_s^o), P_s'))$ which violates the assumption that the solution of (68) is $P_s'$ but not $P_s^o$.

Further, $J_1(z, P_s)$ is a linear fractional function known to be quasi-linear [27], thus the maximum always occurs at one of the two ends. Thus, the solution of (15) must be $P_{0\text{min}}$ or $P_0$. With this, we can also obtain the optimal $w$ according to Lemma 2.

APPENDIX D

PROOF OF LEMMA 3

From the objective in the problem of (23), it should hold that $(P_0 - P_s) \alpha_1 z + \alpha_2 < (P_0 - P_s) \alpha_3 G(z) + \alpha_4$ for some $P_s \in [0, P_0]$, $z \in [0, 1]$. In other words, it should hold $(P_0 - P_s)(\alpha_3 G(z) - \alpha_1 z) > \alpha_2 - \alpha_4$ for some $P_s \in [0, P_0]$, $z \in [0, 1]$. Denote $K(z) = \alpha_3 G(z) - \alpha_1 z$, $z \in [0, 1]$. It is easy to verify that: 1) $K(0) = \alpha_3 (1 - \eta^2) > 0$; 2) $K''(z) < 0$; 3) $K'(z) \to +\infty$ as $z \to 0$, $K'(0) < 0$. Here $K'(z)$ and $K''(z)$ denote the first- and second- order derivatives, respectively. With these, first, if $\alpha_2 < \alpha_4$, then $(P_0 - P_s)K(z) > \alpha_2 - \alpha_4$ for some $P_s \in [0, P_0]$, $z \in [0, 1]$ holds since $K(0) > 0$; second, if $\alpha_2 > \alpha_4$, we know that $K(z), z \in [0, 1]$ achieves its maximum at $z_0$ with $K'(z_0) = 0$ (i.e., the unique root of the equation $\alpha_3 G'(z) - \alpha_1 = 0$), and $(P_0 - P_s)K(z)$, $P_s \in [0, P_0]$, $z \in [0, 1]$ achieves its maximum $P_0 K(z_0)$, then, the condition should be $P_0 K(z_0) > \alpha_2 - \alpha_4$.

APPENDIX E

PROOF OF LEMMA 4

When $z = 0$ is fixed, the problem of (23) is reduced to

$$
\max_{P_s} \log \left( 1 + \frac{P_s}{\alpha_2} \right) - \log \left( 1 + \frac{P_s}{(P_0 - P_s) \alpha_3 (1 - \eta^2) + \alpha_4} \right)
$$

s.t. $P_s \in [0, P_0]$.
This is an optimization problem of a single variable $P_s$, and the maximum is achieved at one of the following points: 0, $P_0$ and the points with zero derivative. After some calculations, the desired results are obtained.

**APPENDIX F**

**Proof of Property 1**

According to (23), we can write

$$R_s(z) = \log (1 + \frac{1}{q_1 z + q_2}) - \log (1 + \frac{1}{q_3 G(z) + q_4}) \tag{70}$$

where $q_1 = (P_0 - P_s)\alpha_1/P_s$, $q_2 = \alpha_2/P_s$, $q_3 = (P_0 - P_s)\alpha_3/P_s$ and $q_4 = \alpha_4/P_s$. It follows from

$$\left. \frac{\partial R_s}{\partial z} \right|_{z'} = -\frac{q_1}{(q_1 z' + q_2 + 1)(q_1 z' + q_2)} + \frac{q_3 G'(z')}{(q_3 G'(z') + q_4 + 1)(q_3 G(z') + q_4)} = 0 \tag{71}$$

that

$$\frac{(q_3 G(z') + q_4 + 1)(q_3 G(z') + q_4)}{(q_1 z' + q_2 + 1)(q_1 z' + q_2)} = \frac{q_3 G'(z')}{q_1} \tag{72}$$

On the other hand, we know

$$\left. \frac{\partial^2 R_s}{\partial z^2} \right|_{z'} = \frac{q_1^2 (2(q_1 z' + q_2) + 1)}{[(q_1 z' + q_2 + 1)(q_1 z' + q_2)]^2} - \frac{(q_3 G'(z'))^2 (2(q_3 G(z') + q_4) + 1)}{[(q_3 G'(z') + q_4 + 1)(q_3 G(z') + q_4)]^2} + \frac{q_3 G''(z')}{(q_3 G(z') + q_4 + 1)(q_3 G(z') + q_4)} \tag{73}$$

Inserting (72) into (73) and using the fact: $G''(z') < 0$ and $q_3 G(z') + q_4 > q_1 z' + q_2$ (since $z' \in (\underline{z}, \bar{z})$) leads to the desired result.

**APPENDIX G**

**Proof of Lemma 5**

When the constraint $w^\dagger G = 0$ is added, the problem of (27) is reduced to

$$\max_{P_s, w} \frac{1}{2} \log \frac{\sigma^2 + P_s |h_0|^2 + w^\dagger R_h w}{\sigma^2 + P_s \max_{j \in I} \{|g_{0j}|^2\}} \tag{74}$$

s.t. $P_s \in [P_0^{\min}, P_0^\dagger]$, $w^\dagger w = P_0 - P_s$, $w^\dagger G = 0$. 

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From $w^\dagger w = P_0 - P_s$ and $w^\dagger G = 0$, we can obtain $w = \sqrt{P_0 - P_s}E z$ and $z^\dagger z = 1$ which, when inserted into (74), results in

$$\max_{P_s, z} \frac{1}{2} \log \frac{\sigma^2 + P_s|h_0|^2 + (P_0 - P_s)z^\dagger E^\dagger R_h E z}{\sigma^2 + P_s \max_{j \in I} \{|g_{0j}|^2\}}$$  \hspace{1cm} (75)

s.t. $P_s \in [P_0^{\text{min}}, P_0]$, $z^\dagger z = 1$.

By using the fact $z^\dagger E^\dagger R_h E z$ achieves its maximum at $z = E^\dagger h / \|E^\dagger h\|$, we rewrite the problem of (75) as

$$\max_{P_s} \frac{1}{2} \log \frac{\sigma^2 + P_s|h_0|^2 + (P_0 - P_s)\|E^\dagger h\|^2}{\sigma^2 + P_s \max_{j \in I} \{|g_{0j}|^2\}}$$  \hspace{1cm} (76)

s.t. $P_s \in [P_0^{\text{min}}, P_0]$.

The objective in (76) is $\frac{1}{2} \log f_2(P_s)$. Note that $f_2(P_s)$ is quasi-linear [27], thus the maximum occurs at one of the two ends, namely, $P_0^{\text{min}}$ or $P_0$.

**APPENDIX H**

**PROOF OF EQUIVALENT PROBLEM (33)**

From the constraint $x^\dagger x = 1$, we can rewrite $\sigma^2 + P_s|h_0|^2 = (\sigma^2 + P_s|h_0|^2)x^\dagger x$ and $\sigma^2 + P_s|g_{0j}|^2 = (\sigma^2 + P_s|g_{0j}|^2)x^\dagger x$, which enables us to rewrite the problem of (32) as

$$\max_{\|x\|=1} \min_{j \in I} \frac{x^\dagger \tilde{R}_h x}{x^\dagger \tilde{R}_j x}$$  \hspace{1cm} (77)

Further, by denoting $u = \tilde{R}_h^{1/2} x / \|\tilde{R}_h^{1/2} x\|$, we can rewrite the problem of (77) as

$$\max_{u} \min_{j \in I} \frac{1}{u^\dagger A_j u} \text{ s.t. } u^\dagger u = 1.$$  \hspace{1cm} (78)

By introducing the slack variable $y$ to rewrite $\min_{j \in I} 1/(u^\dagger A_j u) = 1/y$, we can rewrite the problem of (78) as

$$\max_{u, y} \frac{1}{y} \text{ s.t. } u^\dagger u = 1, u^\dagger A_j u \leq y, j \in I.$$  \hspace{1cm} (79)

Let $v = u / \sqrt{y}$, then $u^\dagger u = 1$ is equivalent to $\|v\|^2 = 1/y$ and we can rewrite the problem of (79) as the problem of (33). Let the solution of (33) be $v^\circ$, then the solution of (78) is $u^\circ = v^\circ / \|v^\circ\|$, the solution of (77) is $x^\circ = \tilde{R}_h^{-1/2} v^\circ / \|\tilde{R}_h^{-1/2} v^\circ\|$.
APPENDIX I

PROOF OF THEOREM 3

The first constraint of (40) leads to
\[(4R^2 |g_0|^2 - |h_0|^2)P_s = w^\dagger (R_h - 4R^2 R_g)w - (4R^2 - 1)\sigma^2. \tag{80}\]

There are two cases. If \(4R^2 |g_0|^2 - |h_0|^2 \neq 0\), it follows from (80) that \(P_s = w^\dagger \tilde{R}w + \zeta\). With this, we can rewrite the problem of (40) as
\[
\min_w w^\dagger w + w^\dagger \tilde{R}w + \zeta \quad \text{s.t.} \quad w^\dagger \tilde{R}w + \zeta \geq P_0^{\text{min}}. \tag{81}\]

The optimal relay weight vector \(w^\circ\) has the form of \((d_1 u_1 + d_2 u_2)\) which follows from its optimality condition of (81) \[27\]
\[
w + \tilde{R}w - \nu \tilde{R}w = 0. \tag{82}\]

where \(\nu \geq 0\) is the Lagrange multiplier. Indeed, inserting \(\tilde{R} = \lambda_1 u_1 u_1^\dagger + \lambda_2 u_2 u_2^\dagger\) into (82) gives
\[
w = (\nu - 1)\lambda_1 (u_1^\dagger w)u_1 + (\nu - 1)\lambda_2 (u_2^\dagger w)u_2. \tag{83}\]

with this, we can rewrite (81) as
\[
\min_{d_1, d_2} (1 + \lambda_1)|d_1|^2 + (1 + \lambda_2)|d_2|^2 + \zeta \quad \text{s.t.} \quad \lambda_1|d_1|^2 + \lambda_2|d_2|^2 + \zeta \geq P_0^{\text{min}}. \]

There are several case. If \(\zeta \geq P_0^{\text{min}}, \lambda_2 \geq -1\), then the solution is \(|d_1| = 0, |d_2| = 0\); If \(\zeta \geq P_0^{\text{min}}, \lambda_2 < -1\), then the solution is \(|d_1| = 0, |d_2|^2 = (\zeta - P_0^{\text{min}})/|\lambda_2|\); If \(\zeta < P_0^{\text{min}}, \lambda_2 \geq -1\), the solution is \(|d_1|^2 = (P_0^{\text{min}} - \zeta)/\lambda_1, |d_2| = 0\).

Similarly, we can obtain the other results in Theorem 3.

APPENDIX J

PROOF OF LEMMA 6

When \(z = 0\) is fixed, the problem of (47) is reduced to
\[
\min_{\gamma} P_0 = \frac{\alpha_2[\gamma \alpha_3(1 - \rho^2) + \alpha_4]}{\gamma \alpha_3(1 - \rho^2) + \alpha_4 - \alpha_2} + \gamma \tag{84}\]

s.t. \(\gamma > \frac{\alpha_2 - \alpha_4}{\alpha_3(1 - \rho^2)}, \gamma \geq 0\).

This is an optimization problem of a single variable \(\gamma\), and the maximal is achieved at one of the following points: 0 and the points with zero derivative. After some calculations, the desired results are obtained.
APPENDIX K

PROOF OF PROPERTY [2]

It follows from
\[
\frac{\partial f_{72}}{\partial z} \bigg|_{z'} = \frac{-\gamma \alpha_1}{(\gamma \alpha_1 z' + \alpha_2)^2} + \frac{\gamma \alpha_3 F'(z')}{(\gamma \alpha_3 F(z') + \alpha_4)^2} = 0
\]
(85)

that
\[
\frac{(\gamma \alpha_3 F(z') + \alpha_4)^2}{(\gamma \alpha_1 z' + \alpha_2)^2} = \frac{\alpha_3 F'(z')}{\alpha_1}.
\]
(86)

On the other hand, we know
\[
\frac{\partial^2 f_{72}}{\partial z^2} \bigg|_{z'} = \frac{2(\gamma \alpha_1)^2}{(\gamma \alpha_1 z' + \alpha_2)^3} - \frac{2(\gamma \alpha_3 F'(z'))^2}{(\gamma \alpha_3 F(z') + \alpha_4)^3}
\]
\[
+ \frac{\gamma \alpha_3 F''(z')}{(\gamma \alpha_3 F(z') + \alpha_4)^2}.
\]
(87)

Inserting (86) into (87) and using the fact: \( F''(z') < 0 \) and \( \gamma \alpha_3 F(z') + \alpha_4 > \gamma \alpha_1 z' + \alpha_2 \) (since \( z' \in (\bar{z}, \tilde{z}) \)) leads to the desired result.

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Fig. 3. Simulation model: source, relays, destination, eavesdroppers are placed along a line; Upper one for Fig. 4, 5, 7; Lower one for Fig. 8.

Fig. 4. Secrecy rate vs. source-destination distance (power constraint: 30 dBm, source-relay distance: 5 m, number of relays: \( N = 10 \), one eavesdropper \((J = 1)\), source-eavesdropper distance: 50 m).
Fig. 5. Secrecy rate vs. source-destination distance (power constraint: 30 dBm, source-relay distance: 5 m, number of relays: $N = 10$, number of eavesdroppers: $J = 7$, source-eavesdropper distance: 50 m).

Fig. 6. Secrecy rate vs. number of eavesdroppers (power constraint: 30 dBm, source-relay distance: 5 m, number of relays: $N = 10$, source-destination distance: 25 m, source-eavesdropper distance: 50 m).
Fig. 7. Transmit power vs. source-destination distance (secrecy rate constraint: 1 bits/s/Hz, source-relay distance: 5 m, the number of relays: $N = 10$, one eavesdropper ($J = 1$), source-eavesdropper distance: 50 m).

Fig. 8. Transmit power vs. source-eavesdropper distance (secrecy rate constraint: 1 bits/s/Hz, source-relay distance: 5 m, number of relays: $N = 10$, one eavesdropper ($J = 1$), source-destination distance: 50 m).