A novel finite-time optimal output regulator for discrete-time linear systems via an information fusion estimation method

Huihui Yang

Henan College of Industry and Information Technology, Jiaozuo, People's Republic of China

ABSTRACT
The finite-time optimal output regulation problem is investigated in this paper. In order to simplify the design process of linear quadratic finite-time optimal output regulation control, a new information fusion estimation-based optimal output regulation control scheme is proposed. The information fusion estimation-based optimal output regulator converts all information into measurement equations for control input and co-state, and then obtains the optimal estimation of control input by the information fusion estimation scheme. Finally, the proposed control scheme is employed for position and velocity regulator design of a brushless linear DC motor system, and the simulation experiment illustrates the effectiveness of the proposed control strategy.

ARTICLE HISTORY
Received 28 June 2019
Accepted 30 August 2019

KEYWORDS
Information fusion estimation; optimal output regulation; finite time

1. Introduction
Output regulation control aims at designing a feedback control strategy that guarantees asymptotic tracking and disturbance rejection (Bhawal & Pal, 2019; Byrnes, Delli Priscoli, Isidori, & Kang, 1997; Davison, 1972; Francis, 1977; Marconi & Praly, 2008), which has a wide range of applications in practical engineering, such as unmanned aerial vehicle attitude control (Tran, Sakamoto, Sato, & Muraoka, 2017), robotic manipulators control (Lucibello, Panzieri, & Pascucci, 2003), and motor control (Xie, Tang, Song, Zhou, & Guo, 2019). Generally, the output regulation control is converted into an LQ optimal regulation problem, and the optimal control law is obtained by solving the LQ optimal performance index.

Plentiful researches have been conducted to address the problems. In Gao and Jiang (2019), a measurement feedback-based adaptive optimal output regulation solution was proposed for the regulation problems of discrete-time linear systems with input time delay. Based on reinforcement learning and adaptive dynamic programming, an approximate optimal control policy was obtained via recursive numerical algorithms using online information. Linear quadratic (LQ) control has been extensively employed to solve the output regulation problems (Bernhard, 2017; Hashikura & Kojima, 2017; Wen, Tao, & Yang, 2007, 2016). An LQ-based adaptive actuator failure compensation control approach was developed to guarantee the system stability and output regulation performances in the presence of plant and actuator failure uncertainties (Wen, Tao, Yang, & Jiang, 2019). In Wada, Sugiyama, and Saeki (2005), a feedback control law that can minimize the LQ type cost function was proposed for the output regulation in the presence of actuator saturation. LQ control is an effective and optimal control method and is widely used in practical engineering. However, the traditional LQ optimal regulation control is used generally for optimal zero-state regulation. For the optimal output regulation problem, the control problem is usually transformed into a zero-state regulation problem, which increases the matrix dimensions and complexity of the controller design process.

There are many different approaches to the solution of LQ optimal control problems, such as the minimization method using Lagrange multipliers, dynamic programming, and Lyapunov function. Information fusion is used to obtain more accurate and definite inferences from the data provided by any single information source by integrating multiple information sources. The most important application area of information fusion is track fusion or track-to-track fusion in a target tracking system, which has been investigated for more than two decades. In Luo, Wang, Wei, and Alsaaedi (2018), the non-fragile sensor fault estimation problem for a class of two-dimensional (2-D) nonlinear systems was addressed. In Li, Zhu, Wang, and Han (2003), three estimation fusion architectures including centralized, distributed, and hybrid were proposed. Moreover, a unified linear data model and
two optimal fusion rules based on the best linear unbiased estimation (BLUE) and the weighted least squares (WLS) were also developed. It is much more general and flexible than the previous approach of matching centralized and distributed fusion rules based on a Kalman filter. In Sun and Deng (2004), a new multi-sensor optimal information fusion criterion weighted by matrices in the linear minimum variance (LMV) sense was presented, which is equivalent to the maximum likelihood fusion criterion under the assumption of normal distribution. The concept of the information weight was proposed in Wang, Wang, and Zhen (2007), which represents the contribution of the measurement on the estimated variable. Besides, a unified information fusion rule based on a least-square estimation criterion for linear and nonlinear measurements was presented. The information weight is essentially equal to the Fisher information matrix in case of Gaussian white noise. Information fusion is an ideology which universally exists in decision-making problems. Both estimation problem and control problem belong to the decision-making problem. In Wang et al. (2007), an information fusion estimation-based control theory is proposed for the first time to solve the optimal control problem of a discrete-time system. Subsequently, the information fusion estimation-based control method has been extended to solve various optimal control problems. In Zhen, Jiang, Wang, and Chen (2015), an information fusion state regulator was deduced for a large civil aircraft system to keep incline attitude in the presence of external disturbance. In Zhen, Jiang, Wang, and Wang (2015), a decoupling control scheme based on information fusion estimation technique was proposed for a multivariable nonlinear system. In Zhen, Xu, and Jiang (2018), an information fusion estimation-based control method was proposed for the flight control of a nonlinear jumbo jet to restrain wind disturbance. The information fusion estimation-based control method regards all information, including the desired trajectory, system dynamic equation and ideal control strategy, as the measurement information of the control input and obtain the optimal estimation of control input via the information fusion estimation scheme.

Motivated by the above discussions, the finite-time optimal output regulation problem for a linear system is investigated in this paper. In order to simplify the design complexity of traditional LQ output regulation control, a new information fusion estimation-based finite-time optimal output regulation control scheme is proposed. The control method proposed in this work converts all information into measurement equations for control input and co-state, and then obtains the optimal estimation of control input by the information fusion estimation scheme.

The main contributions of this paper are summarized as follows:

(1) Different from the information fusion estimation-based control approaches proposed in Wang et al. (2007), Zhen et al. (2015), Zhen et al. (2015), and Zhen et al. (2018), this paper investigates the optimal output regulation problem of discrete-time linear systems via the information fusion estimation method. By the traditional optimal output regulator, the original output regulation problem is transformed into the zero-state regulation problem, which increases the complexity of controller design and is not convenient for practical applications. In our work, all the information related to the control law, including the system state equation and the performance index function, are converted into measurement equations, and the control law for the optimal output regulation problem is directly derived based on the information fusion estimation method and without any transformation of the control problem, which has an intuitive design idea and can significantly reduce design complexity.

(2) The information fusion estimation-based output regulation control scheme proposed in this paper is equivalent to the traditional LQ output regulation method. The control sequence of the optimal output regulator obtained by the proposed method is consistent with the optimal control sequence of the traditional solution. The recursive equation for co-state information weight given by the proposed method is equivalent to discrete Riccati equations in traditional optimal control. Finally, a simulation experiment for a finite-time optimal output regulation problem is given to verify the effectiveness of the proposed control algorithm.

The remainder of this paper is organized as follows. The finite-time optimal output regulation problem of a linear system is presented in Section 2. The proposed control strategy is exposed in Section 3. Some simulation results are shown in Section 4. Finally, the last section gives a brief summary of the obtained results and the conclusion.

2. Problem formulation

Consider that the controlled discrete linear system is described as

\[
\begin{align*}
x_{k+1} &= A_k x_k + B_k u_k, \\
y_k &= C_k x_k, & k = 0, 1, \ldots, N
\end{align*}
\]  

(1)
where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^r$, $y_k \in \mathbb{R}^m$ represent the state vector, the control input vector and the output vector, respectively, $A_k \in \mathbb{R}^{n \times n}$, $B_k \in \mathbb{R}^{n \times r}$, $C_k \in \mathbb{R}^{m \times n}$ are the system matrices with appropriate dimensions, and $N$ is the regulation horizon.

Assuming that the system described above is controllable and observable, the finite-time optimal output regulation law can be obtained by minimizing the following LQ performance index:

$$
J = \frac{1}{2} y_N^T F y_N + \frac{1}{2} \sum_{k=0}^{N-1} \{ y_k^T Q_k y_k + u_k^T R_k u_k \},
$$

(2)

where $F$ is a symmetric non-negative definite matrix, and $Q_k, R_k$ are non-negative and positive definite symmetric weighting matrices, respectively.

3. Controller design

The control problem considered in this research is an LQ finite-time optimal output regulation problem. For this kind of a problem, the traditional approach requires to transform the original control into a zero-state regulation problem using the information fusion estimation theory, we get 2 virtual measurement equations expressed as follows:

$$
0 = C_k x_k + q_k
$$

and

$$
l(0 \mid x_k) = C_k^T Q_k C_k,
$$

(7)

$$
0 = u_k + f_k
$$

and

$$
l(0 \mid u_k) = R_k,
$$

(8)

where $q_k$ is the random error between the desired output $y_k^* = 0$ and measuring output $y_k$ with $E[q_k] = 0, E[q_k q_k^T] = Q_k^{-1}$, $f_k$ is the random error between the desired control input $u_k^* = 0$ and optimal estimation control signal $\hat{u}_k$ with $E[f_k] = 0, E[f_k f_k^T] = R_k^{-1}$, $l(0 \mid x_k)$ represents the information weight of the system desired state about its actual state, and $l(0 \mid u_k)$ represents the information weight of the system desired state about its control input.

Step 2. Assuming that the co-state fusion estimation $\hat{x}_{k+1}$ and its information weight $P_{k+1}$ have been obtained, then we get the information equation

$$
\hat{x}_{k+1} = x_{k+1} + P_{k+1}
$$

and

$$
l(\hat{x}_{k+1} \mid x_{k+1}) = P_{k+1},
$$

(9)

where $E[P_{k+1}] = 0, E[P_{k+1} P_{k+1}^T] = P_{k+1}^{-1}$, $l(\hat{x}_{k+1} \mid x_{k+1})$ represents the information weight of the system estimated state about its actual state.

Since the objective of this paper is to regulate the output of the controlled system to origin, the desired co-state is always set as $\hat{x}_{k+1} = 0$. Then, one can obtain that

$$
0 = x_{k+1} + P_{k+1},
$$

(10)

Associating system dynamic equation (1), we get

$$
0 = A_k x_k + B_k \hat{u}_k + P_{k+1},
$$

(11)

where $\hat{x}_{k+1} = 0$ is taken as a virtual measurement of $x_{k+1}$ and $l(\hat{x}_{k+1} \mid x_k)$ represents the information weight of the system co-state $\hat{x}_{k+1}$ about its actual state $x_k$.

Fusing all measurement equations for the system status $x_k$ given in (7) and (11), the recursive equation for the information weight can be obtained via the information fusion estimation technique

$$
l(\hat{x}_k \mid x_k) = P_k = l(0 \mid x_k) + l(\hat{x}_{k+1} \mid x_k)
$$

= $A_k^T [B_k^T P_{k+1}^{-1} B_k + P_{k+1}^{-1}]^{-1} A_k + C_k^T Q_k C_k$.

(12)

Considering the performance function given in (2), the measurement equation for the terminal boundary status
can be written as
\[
0 = C_N x_N + f_N,
\]
\[
l[0 \mid x_N] = P_N = C_N^T F C_N,
\]
where \(f_N\) is the random error between the desired output \(y^*_N = 0\) and measuring output \(y_N\) with \(E[f_N] = 0\), \(E[f_N^T f_N] = F^{-1}\), and \(l[0 \mid x_N]\) represents the information weight of the system desired state about its actual state at the terminal horizon \(N\).

Step 3. Measurement equation (11) can be represented as
\[
-A_k x_k = B_k \hat{u}_k + p_{k+1},
\]
\[
l[\hat{x}_{k+1} \mid \hat{u}_k] = B_k^T P_{k+1} B_k,
\]
where \(l[\hat{x}_{k+1} \mid \hat{u}_k]\) represents the information weight of the system co-state about its estimated control input at the time \(k\).

According to the information fusion estimation theory and the two measurement equations for control input \(\hat{u}_k\) given as (8) and (13), the information fusion estimation-based optimal control law can be obtained as
\[
\hat{u}_k = -l[\hat{u}_k]^{-1} B_k^T P_{k+1} A_k x_k,
\]
\[
l[\hat{u}_k] = R_k + B_k^T P_{k+1} B_k,
\]
where \(l[\hat{u}_k]\) represents the information weight of the estimated control input about itself.

Based on the control law given in (15), the recursive equation expressed as (12) and terminal boundary measurement equation (13), these three equations are equivalent to the optimal regulator, the discrete Riccati equation as well as its boundary condition in a traditional finite-time optimal output regulator. Thus, optimal performance index (4) can be obtained.

The proof ends.

**Remark 3.1:** The information weight refers to the contribution of the related information to the decision amount. The larger the information weight, the greater is the contribution of the related information to the decision amount.

**Remark 3.2:** From the design process given above, it can been seen that the proposed control scheme converts all information into measurement equations for control input and co-state, and then obtains the optimal estimation of control input by the information fusion estimation scheme. Compared with the traditional approach, the proposed control strategy has the same result but a simpler and more intuitive design process, which is very important for practical applications. The detailed design of the finite-time information fusion estimation-based optimal output regulation control scheme is described as follows.

**Algorithm 1:** Finite-time information fusion estimation-based optimal output regulator

1. Set \(k = 0, \hat{u}_0, x_0, y_0\), terminal time \(N\);
2. Calculate the co-state information weight \(P_k\) based on Equation (5)(6) and save, \(k = N, N - 1, \ldots, 1\);
3. While \(k \leq N\) do
   - **Step 1:** Set \(x_{k+1} = 0\) and take \(P_{k+1}\) from work space;
   - **Step 2:** Calculate the optimal control signal \(\hat{u}_k\) according to (3);
   - **Step 3:** Calculate system status \(x_{k+1}\);
   - **Step 4:** Set \(x_k = x_{k+1}, k = k + 1\);

4. **Simulation results**

In this section, a simulation experiment for a finite-time optimal output regulation problem of a linear brushless DC motor is conducted to illustrate the effectiveness of the proposed control scheme. The input current \(i\) of the linear brushless DC motor drive circuit is selected as the control input signal. The position \(p\) and speed \(v\) of the motor are used as the state variables of the control system, and the system position \(p\) is used as the output. The system model can be described as
\[
\begin{bmatrix}
\dot{p}(t) \\
\dot{v}(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & -D
\end{bmatrix}
\begin{bmatrix}
p(t) \\
v(t)
\end{bmatrix} +
\begin{bmatrix}
0 \\
K_f
\end{bmatrix} i(t).
\]

The parameters of the DC motor are given as mass of movable part \(M = 1.82\) kg, coefficient of friction \(D = 3.48\) Ns/m and thrust coefficient \(K_f = 3.8\) N/A. Assume that the sampling time \(T = 0.001\) s; the discrete system model can be expressed as
\[
\begin{bmatrix}
p_{k+1} \\
v_{k+1}
\end{bmatrix} =
\begin{bmatrix}
1 & 9.96 \times 10^{-4} \\
0 & 9.91 \times 10^{-1}
\end{bmatrix}
\begin{bmatrix}
p_k \\
v_k
\end{bmatrix} +
\begin{bmatrix}
4.74 \times 10^{-6} \\
9.46 \times 10^{-3}
\end{bmatrix} i_k,
\]
\[
y_k = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix}
p_k \\
v_k
\end{bmatrix}.
\]

Set the terminal time \(N = 1000\), and the weight matrices contained in the performance index function are set to constant values as
\[
F = Q = \begin{bmatrix} 10^4 & 0 \\ 0 & 0 \end{bmatrix}, \quad R = 1.
\]

The output regulation results under the control scheme proposed in this paper are shown in Figure 1.
Figure 1(a,b) shows the response curves of output regulation and the control input signal, respectively. From these two figures, it can be clearly seen that the position output rapidly converges to the origin and is maintained at the origin. As shown in Figure 1(b), the control input is rapidly reduced during the adjustment process, and the control input remains at 0 when the system reaches the origin. According to these simulation results, the effectiveness of the proposed control method has been verified.

5. Conclusion

This paper investigates the finite-time optimal output regulation problem for discrete linear systems. In order to simplify the design process of LQ optimal output regulation control, an information fusion estimation-based control scheme is proposed. The optimal output regulation control law is obtained by fusing the reference output, the desired control input and the system dynamics via the information fusion estimation technique. Finally, the simulation experiment verifies the effectiveness of the proposed control algorithm.

Compared with the traditional finite-time LQ optimal output regulation control, the control scheme proposed in this work has the advantage of an intuitive design process and can effectively reduce the design complexity. However, lots of matrix inversions exist in the control process, which is not conducive to practical applications. As a future work, it would be interesting to study how to avoid matrix inversion, which will significantly improve the performance of the proposed control approach.

Disclosure statement

No potential conflict of interest was reported by the author.

References

Bernhard, S. (2017). Time-invariant control in LQ optimal tracking: An alternative to output regulation. *IFAC Papersonline, 50*(1), 4912–4919. doi:10.1016/j.ifacol.2017.08.746

Bhawal, C., & Pal, D. (2019). Almost every single-input LQR optimal control problem admits a PD feedback solution. *IEEE Control Systems Letters, 3*, 452–457. doi:10.1109/LCSYS.2019.2898388

Byrnes, C., Delli Priscoli, F., Isidori, A., & Kang, W. (1997). Structurally stable output regulation of nonlinear systems. *Automatica, 33*, 369–385. doi:10.1016/S0005-1098(96)00184-7

Davison, E. (1972). The output control of linear time-invariant multivariable systems with unmeasurable arbitrary disturbances. *IEEE Transactions on Automatic Control, 17*, 621–630. doi:10.1109/TAC.1972.1100084

Francis, B. (1977). The linear multivariable regulator problem. *SIAM Journal on Control and Optimization, 15*, 486–505. doi:10.1137/0315033

Gao, W., & Jiang, Z.-P. (2019). Adaptive optimal output regulation of time-delay systems via measurement feedback. *IEEE Transactions on Neural Networks and Learning Systems, 30*(3), 938–945. doi:10.1109/TNNLS.2018.2850520

Hashikura, K., & Kojima, A. (2017). LQ preview state feedback with output regulation constraint. In: 2017 11TH Asian Control Conference (ASCC) (pp. 2760–2765). Gold Coast, Australia.

Li, X., Zhu, Y., Wang, J., & Han, C. (2003). Optimal linear estimation fusion - Part I: Unified fusion rules. *IEEE Transactions on Information Theory, 49*(9). doi:10.1109/TIT.2003.815774

Lucibello, P., Panzieri, S., & Pascucci, F. (2003). Suboptimal output regulation of robotic manipulators by iterative learning. In: Proceedings of the 11th International Conference on Advanced Robotics (Vol. 1–3, pp. 441–446). Coimbra, Portugal.

Luo, Y., Wang, Z., Wei, G., & Alsaadi, F. (2018). Nonfragile l2-linfinity fault estimation for Markovian jump 2-d systems with specified power bounds. *IEEE Transactions on Systems, Man, and Cybernetics: Systems, 1–12*. doi:10.1109/TSMC.2018.2794414

Marconi, L., & Praly, L. (2008). Uniform practical nonlinear output regulation. *IEEE Transactions on Automatic Control, 53*, 1184–1202. doi:10.1109/TAC.2008.923674
Sun, S., & Deng, Z. (2004). Multi-sensor optimal information fusion Kalman filter. *Automatica, 40*(6). doi:10.1016/j.automatica.2004.01.014

Tran, A. T., Sakamoto, N., Sato, M., & Muraoka, K. (2017). Control augmentation system design for quad-tilt-wing unmanned aerial vehicle via robust output regulation method. *IEEE Transactions on Aerospace and Electronic Systems, 53*, 357–369. doi:10.1109/TAES.2017.2650618

Wada, N., Sugiyama, D., & Saeki, M. (2005). Optimal design of a feedback controller that achieves output regulation in the presence of actuator saturation. In 2005 44TH IEEE conference on decision and control & European control conference (Vol. 1–8, pp. 7576–7580). Seville, Spain.

Wang, Z., Wang, D., & Zhen, Z. (2007). Primary exploration of nonlinear information fusion control theory. *Science in China Series F Information Sciences, 50*(5), 686–696. doi:10.1007/s11432-007-0049-y

Wen, L. Y., Tao, G., Yang, H., & Jiang, B. (2019). Adaptive actuator failure compensation for possibly nonminimum-phase systems using control separation based LQ design. *IEEE Transactions on Automatic Control, 64*(1), 143–158. doi:10.1109/TAC.2018.2809719

Wen, L. Y., Tao, G., & Yang, H. (2007). Optimal LQ-feedback regulation of a nonisothermal plug flow reactor model by spectral factorization. *IEEE Transactions on Automatic Control, 52*(7), 1179–1193. doi:10.1109/TAC.2007.900823

Wen, L. Y., Tao, G., & Yang, H. (2016). LQ control based actuator failure compensation. *Optimal Control Applications & Methods, 37*(2), 227–247. doi:10.1002/oca.2162

Xie, Y., Tang, X., Song, B., Zhou, X., & Guo, Y. (2019). Model-free tuning strategy of fractional-order pi controller for speed regulation of permanent magnet synchronous motor. *Transactions of the Institute of Measurement and Control, 41*(1), 23–35. doi:10.1177/0142331217751040

Zhen, Z., Jiang, J., Wang, X., & Chen, G. (2015). Information fusion based optimal control for large civil aircraft system. *ISA Transactions, 55*, 81–91. doi:10.1016/j.isatra.2014.09.017

Zhen, Z., Jiang, J., Wang, Z., & Wang, X. (2015). Information fusion based decoupling control for multivariable nonlinear system. *Mathematical Problems in Engineering, 9*. doi:10.1155/2015/361581

Zhen, Z., Xu, Y., & Jiang, J. (2018). Information fusion estimation-based flight control of a nonlinear jumbo jet under wind disturbance. *Optimal Control Applications & Methods, 39*(2,SI). doi:10.1002/oca.2357