FLEET OPTIMIZATION BASED ON THE MONTE CARLO ALGORITHM

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Keywords: management science, optimisation, simulation, vehicle fleet capacity, Monte Carlo algorithm

Abstract: With the development of computers and software products, there is now greater use of quantitative methods in industrial enterprises when making managerial decisions. One of the most applicable solutions to computer simulation algorithms is the Monte Carlo method. The application of the Monte Carlo algorithm lies in finding a relation between the individual variables, which are the solutions to the problem and represent the characteristics of random processes reproducible on computers. The aim of this article is to show the application of simulations from the Monte Carlo algorithm using the example of optimising vehicle fleet capacity so that the total daily costs spent on transporting goods are minimal.

1 Introduction

Over the last two decades, there have been great advances in computer technology and these advances have also affected managerial decision-making [1]. Mathematical methods with the use of computer technologies have been increasingly used in most problems of managerial decision-making [2,3]. One of the most applicable methods of computer implementation is the Monte Carlo method [4]. A static, stochastic method uses random (pseudo-random) numbers in the course of the calculation [5]. In this article, its use will be shown while optimising vehicle fleet capacity so that the daily costs spent on transporting goods are minimal.

2. Monte Carlo simulation

The term Monte Carlo is widely used to denote a wide class of computing methods that use random sampling to obtain numerical solutions. Monte Carlo methods are ubiquitous in science and engineering; they are preferred due to their simplicity [6].

Monte Carlo methods are used to simulate the behaviour of physical or mathematical systems, especially when analytical solutions are difficult to obtain [7]. These methods are nondeterministic or stochastic. The applications of Monte Carlo methods are very diverse: these include physics, computer science, engineering, environmental sciences, finance, fleet management etc. And systems with uncertainties other than pure mathematical systems that require no uncertainty [8].

A random value of $x$ is known as a random number. In practice, $x$ values can be determined deterministically, and the random numbers so generated are known as pseudo-random numbers: such a pseudo-random number contains a limited number of digits, which means that the continuous uniform distribution is approximated by the discrete. Pseudorandom numbers are often used in simulation studies [9]. Monte Carlo methods require many pseudo-random numbers for computer processing.

Simulation models are usually used to decide on something that involves a risk, i.e. a model in which the behaviour of one or more components is not known with certainty. Bank lines or the number of trucks that arrive between specific time intervals are cases where a component that is not listed with confidence is considered a random variable and a probability distribution is used to simulate the behaviour of the random variable [10].

In the practical example used, the simulation is applied to the number of customer orders per day, operator intensity, and the simulated expected overtime operation.

3. The task

A company provides the transportation of goods based on customer orders according to the following rule:

| Ordered on | Transported on |
|------------|----------------|
| Monday     | Tuesday        |
| Tuesday    | Wednesday      |
| Wednesday  | Thursday       |
| Thursday   | Friday         |
| Friday     | Monday         |

Orders are not received on Saturdays or Sundays and there is no transportation on these days. Transportation is carried out by using trucks. The daily cost of operating one truck is 600 € – provided the trucks operate 8 hours each day.
If an order requires overtime operation, i.e., an operation that exceeds the set 8-hour operational time, the costs for each overtime hour are 250 €.

Through statistical investigation for the assumption of a normal distribution of probability, the following data were determined:
- Data on the transport capacity of cars – on average, during an 8-hour running time, 1 car transports 80 orders with a standard deviation of 15 orders,
- Data on customer demand.

### Table 2 Individual customer demands per respective days

| Days    | The average number of orders \( x \) | Standard deviation \( \sigma \) |
|---------|-------------------------------------|-------------------------------|
| Monday  | 7,800                               | 800                           |
| Tuesday | 6,400                               | 700                           |
| Wednesday | 5,500                             | 600                           |
| Thursday | 7,000                               | 750                           |
| Friday  | 5,000                               | 500                           |

The business unit must set an optimal vehicle fleet capacity so that the total daily costs spent on transportation are minimal.

### Analysis of the solution

From the substantive point of view, this is a task that belongs to the methodology of the queuing theory [11]. It is a simple task to determine the optimal dimension (size, capacity) of the operating system:
- Operating system = vehicle fleet that provides transportation services,
- Two main variables are assumed (variables crucial to solution of the task)
  - operation intensity = transport capacity of the vehicle,
  - the intensity of the requirements entering the operating system = the number of orders.
- Number of the contracts has the character of random variables with a normal distribution of probability.

Two extreme situations (two extreme variations in the capacity of the operating system) that lead to relatively high costs of the activity may occur in terms of optimization:
- A heavily oversized operating system (too many vehicles):
  - high costs of the vehicle fleet,
  - costs of overtime work converge to zero; the requests do not queue.
- A heavily undersized operating system (the number of vehicles is too small):
  - low costs of the vehicle fleet,
  - high costs for overtime work; formation of request queues (possible loss of profit due to queues).

Between the extreme variants, there is a number of possible variants of dimensioning the vehicle fleet that are associated with the different daily costs of transportation [12,13]. Of the possible variants, it is necessary to choose the optimal variant - the one with the lowest daily costs for transportation.

The following applies (1) for the \( i \)-th variation of the operating system dimension:

\[
N = N_V + N_M
\]

\( N \) – daily costs for transportation,
\( N_V \) – daily costs for operating the vehicle fleet,
\( N_M \) – daily overtime cost.

### 5. The procedure of solving the task using a simulation of the Monte Carlo algorithm

For the assumption of normal probability distribution of both main variables:

1. The simulations (predictions) of the number of orders are performed according to the intensity of the requirements entering the operation. The simulation itself is in Table 3.

### Table 3 Simulation of the number of orders

| Day     | Simulated value of the distribution function \( x \) | Determinant variable \( t \) | Prediction of the number of contracts \( x = \bar{x} + t \cdot \sigma \) |
|---------|--------------------------------------------------|----------------------------|------------------------------------------------------------------------------------------------|
| Mon     | 0.88147                                          | 1.1                       | 8,680                                                                                           |
| Tue     | 0.9970                                           | 2.7                       | 8,290                                                                                           |
| Wed     | 0.11941                                          | -1.1                      | 4,840                                                                                           |
| Thu     | 0.78817                                          | 0.8                       | 7,600                                                                                           |
| Fri     | 0.17061                                          | -0.9                      | 4,550                                                                                           |

\( \bar{x} \) from the table of random numbers, we assume five-digit values for the distribution function of the normal probability distribution.

\( x = \bar{x} + t \cdot \sigma \) Prediction of the number of contracts \( x \) (2):

\[
x = \bar{x} + t \cdot \sigma 
\]

For Monday: \( x = 7,800 + 1.1 \times 800 \) \( x = 8,680 \)
For Tuesday: \( x = 6,400 + 2.7 \times 700 \) \( x = 8,290 \)
For Wednesday: \( x = 5,500 – 1.1 \times 600 \) \( x = 4,840 \)
For Thursday: \( x = 7,000 + 0.8 \times 750 \) \( x = 7,600 \)
For Friday: \( x = 5,000 – 0.9 \times 500 \) \( x = 4,550 \)

\( x \) – the average number of orders,
\( \sigma \) – determinant deviation.

2. A simulation of the operational intensity of the 8-hour transportation capacity for the chosen number of vehicles is carried out: 70, 80, ..., 110
Determining the number of vehicles for the highest number of orders:
The highest number of orders = 7,800.
Determinant deviation = 800.
One vehicle transports an average of 80 orders per hour.
The total number of vehicles for the highest number of orders = \((7,800 + 800)/80 = 110\).

Determining the number of vehicles for the lowest number of orders:
The lowest number of orders = 5,000.
Determinant deviation = 500.
One vehicle transports 80 orders per hour.
The total number of vehicles for the lowest number of orders = \((5,000 + 500)/80 = 70\).

In the following step (Table 4), a simulation of the operational intensity is performed.

### Table 4 Simulation of operational intensity

| The Number of Vehicles | Day | The simulated value of the distribution function of normal distribution | Determinant variable | \( \bar{x} = n \cdot \bar{x} \) | \( \sigma = \sqrt{n} \cdot \sigma \) | Prediction of 8-hour transport capacity |
|------------------------|-----|------------------------------------------------|---------------------|-----------------|-----------------|---------------------------------------|
| 70                     | Mon | 0.33166 -0.4 | 5,600 | 126 | 5,550 |
|                        | Tue | 0.87094 1.1 | 5,600 | 126 | 5,739 |
|                        | Wed | 0.11120 -1.2 | 5,600 | 126 | 5,449 |
|                        | Thu | 0.22254 -0.7 | 5,600 | 126 | 5,512 |
|                        | Fri | 0.96023 1.7 | 5,600 | 126 | 5,814 |
| 80                     | Mon | 0.76869 0.7 | 6,400 | 134 | 6,494 |
|                        | Tue | 0.39300 -0.2 | 6,400 | 134 | 6,373 |
|                        | Wed | 0.02982 -1.8 | 6,400 | 134 | 6,159 |
|                        | Thu | 0.57991 0.2 | 6,400 | 134 | 6,427 |
|                        | Fri | 0.94479 1.6 | 6,400 | 134 | 6,614 |
| 90                     | Mon | 0.96023 1.7 | 7,200 | 142 | 7,441 |
|                        | Tue | 0.88936 1.2 | 7,200 | 142 | 7,370 |
|                        | Wed | 0.88936 -0.5 | 7,200 | 142 | 7,129 |
|                        | Thu | 0.55013 0.1 | 7,200 | 142 | 7,214 |
|                        | Fri | 0.10920 -1.2 | 7,200 | 142 | 7,030 |
| 100                    | Mon | 0.26299 -0.6 | 8,000 | 150 | 7,910 |
|                        | Tue | 0.77806 0.7 | 8,000 | 150 | 8,105 |
|                        | Wed | 0.12446 -1.1 | 8,000 | 150 | 7,835 |
|                        | Thu | 0.23510 -0.7 | 8,000 | 150 | 7,895 |
|                        | Fri | 0.68774 0.4 | 8,000 | 150 | 8,060 |
| 110                    | Mon | 0.48454 0 | 8,800 | 157 | 8,800 |
|                        | Tue | 0.65269 0.4 | 8,800 | 157 | 8,863 |
|                        | Wed | 0.18167 -0.9 | 8,800 | 157 | 8,659 |
|                        | Thu | 0.84631 1.0 | 8,800 | 157 | 8,957 |
|                        | Fri | 0.74108 0.6 | 8,800 | 157 | 8,894 |

Calculation example for \( n = 70 \)
\[
\begin{align*}
\bar{x} &= 70 \times 80 = 5,600 \\
\sigma &= \sqrt{70} \cdot 15 = 126
\end{align*}
\]

3. The expected overtime is simulated (from the simulation of the number of orders and the eight-hour capacity of the fleet) in Table 5.
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Table 5 Simulation of expected overtime operation

| n   | Day | The number of orders | 8-hour capacity of the vehicle fleet | The number of pending orders | The number of overtime hours | The total number of overtime hours | The average number of daily overtime hours |
|-----|-----|----------------------|-------------------------------------|-----------------------------|---------------------------|-----------------------------------|-------------------------------------------|
| 70  | Mon | 7,800                | 5,550                               | 2,250                       | 225                       | 445                               | 89                                        |
|     | Tue | 6,400                | 5,739                               | 661                         | 66                        |                                   |                                           |
|     | Wed | 5,500                | 5,449                               | 51                          | 5                         |                                   |                                           |
|     | Thu | 7,000                | 5,512                               | 1,488                       | 149                       |                                   |                                           |
|     | Fri | 5,000                | 5,814                               | 0                           | 0                         |                                   |                                           |
| 80  | Mon | 7,800                | 6,494                               | 1,306                       | 131                       | 191                               | 38.2                                     |
|     | Tue | 6,400                | 6,373                               | 27                          | 3                         |                                   |                                           |
|     | Wed | 5,500                | 6,159                               | 0                           | 0                         |                                   |                                           |
|     | Thu | 7,000                | 6,427                               | 573                         | 57                        |                                   |                                           |
|     | Fri | 5,000                | 6,614                               | 0                           | 0                         |                                   |                                           |
| 90  | Mon | 7,800                | 7,441                               | 359                         | 36                        |                                   |                                           |
|     | Tue | 6,400                | 7,370                               | 0                           | 0                         |                                   |                                           |
|     | Wed | 5,500                | 7,129                               | 0                           | 0                         |                                   |                                           |
|     | Thu | 7,000                | 7,214                               | 0                           | 0                         |                                   |                                           |
|     | Fri | 5,000                | 7,030                               | 0                           | 0                         |                                   |                                           |
| 100 | Mon | 7,800                | 7,910                               | 0                           | 0                         | 0                                 | 0                                         |
|     | Tue | 6,400                | 8,105                               | 0                           | 0                         | 0                                 | 0                                         |
|     | Wed | 5,500                | 7,835                               | 0                           | 0                         |                                   |                                           |
|     | Thu | 7,000                | 7,895                               | 0                           | 0                         |                                   |                                           |
|     | Fri | 5,000                | 8,060                               | 0                           | 0                         |                                   |                                           |
| 110 | Mon | 7,800                | 8,800                               | 0                           | 0                         | 0                                 | 0                                         |
|     | Tue | 6,400                | 8,863                               | 0                           | 0                         | 0                                 | 0                                         |
|     | Wed | 5,500                | 8,659                               | 0                           | 0                         |                                   |                                           |
|     | Thu | 7,000                | 8,957                               | 0                           | 0                         |                                   |                                           |
|     | Fri | 5,000                | 8,893                               | 0                           | 0                         | 0                                 | 0                                         |

6. The result of the task - recommended by the simulation calculations
A calculation of the total daily costs for transportation with the individual numbers of vehicles is shown below:

\[ N = N_0 + N_m \]

\[ N_70 = 70 \times 600 + 89 \times 250 = 64,250 \text{ €} \]

\[ N_{80} = 80 \times 600 + 38.2 \times 250 = 57,500 \text{ €} \]

\[ N_{90} = 90 \times 600 + 7.2 \times 250 = 58,500 \text{ €} \]

\[ N_{100} = 100 \times 600 + 0 \times 250 = 60,000 \text{ €} \]

\[ N_{110} = 110 \times 600 + 0 \times 250 = 66,000 \text{ €} \]

Under the given conditions, the total daily transport costs will be minimised when operating 80 vehicles or 80 to 90 vehicles.

7. Conclusions
As seen from the article, in this case, the use of the Monte Carlo algorithm is very appropriate when dealing with the problem of optimal vehicle fleet capacity. It was found that the overall lowest daily transport costs are 57,500 € when using 80 trucks. In this case, MS Excel was used to generate pseudorandom numbers. Computers play an indispensable role at the application of simulations and not using them in today’s field of quantitative methods of managerial decision-making is unimaginable.

Further research into fleet issues should focus on the use of telematics, which presents new possibilities to accelerate the processing of information throughout the process and increase customer satisfaction. Research trends consist in developing new efficient fleet management models, optimization and simulation models, and integrating them into the entire decision support system. Computational efficiency of methods can be increased by integrating precise algorithms and developing mechanisms based on mathematical programming principles. The use of artificial intelligence principles and methods is also the right direction.

Acknowledgements
The work was supported by the specific university research of Ministry of Education, Youth and Sports of the Czech Republic No. SP2019/42.
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Review process
Single-blind peer review process.