Hot Giant Loop Holography

Gianluca Grignani\(^1\), Joanna L. Karczmarek\(^2\) and Gordon W. Semenoff\(^2\)

\(^1\)Dipartimento di Fisica, Università di Perugia, INFN Sezione di Perugia, Via A. Pascoli, 06123 Perugia, Italia
\(^2\)Department of Physics and Astronomy, University of British Columbia, Vancouver, British Columbia V6T 1Z1

We argue that there is a phase transition in the expectation value of the Polyakov loop operator in the large \(N\) limit of the high temperature deconfined phase of \(\mathcal{N} = 4\) Yang-Mills theory on a spatial \(S^3\). It occurs for large completely symmetric representation of the \(SU(N)\) symmetry group.

We speculate that this transition is reflected in the D-branes which are the string theory duals of giant loops.

The Hawking-Page phase transition \(^[1]\) is the collapse of hot anti-de Sitter space to an anti-de Sitter-Schwarzschild black hole. A beautiful picture of the holographic dual of this transition has emerged in the context of AdS/CFT duality \(^[2] - [3]\). On the type IIB supergravity side is black hole formation with asymptotic \(AdS_5 \times S^5\) geometry. On the gauge theory side, we have the deconfinement phase transition of large \(N\) \(\mathcal{N} = 4\) supersymmetric Yang-Mills theory defined on the spatial three-sphere \(S^3\). The confined phase is dual to hot \(AdS_5 \times S^5\) whereas the deconfined phase is dual to the black hole geometry.

Confining behavior of \(\mathcal{N} = 4\) Yang-Mills theory is governed by the representation of its center symmetry and characterized by the expectation value of the Polyakov loop, which is the trace of gauge theory holonomy in periodic Euclidean time \(^[4] [5]\).

\[ \text{Tr} U(\vec{x}) = \text{Tr} \mathcal{P} e^{i \int_0^\beta d\tau A_0(\tau, \vec{x})} . \]  

The loop transforms as \(\text{Tr} U(\vec{x}) \rightarrow c \text{Tr} U(\vec{x})\), where \(c = e^{2\pi i/ N}\) is the generator of the \(Z_N\) center of the \(SU(N)\) gauge group. Its expectation value vanishes in the low temperature confining phase where center symmetry is good, and it can be non-zero in the high temperature deconfined phase where the center symmetry is broken.

We shall show in the following that \(^[2]\) can have interesting behavior which depends on the size and nature of the representation. We will consider completely symmetric representations \(S_k\) whose Young tableau is a single row with \(k\) boxes and completely antisymmetric representations \(A_k\) whose Young tableau is a single column with \(k\) boxes. We shall consider large values of \(k\) so that \(\frac{k}{N}\) remains finite as \(N \rightarrow \infty\). Both representations \(S_k\) and \(A_k\) can be non-zero in the confined phase when this charge is non-zero. The characters can be non-zero in the deconfined phase.

The study of the Polyakov loop using effective field theory has a long history \(^[6]\); however, it was applied to \(\mathcal{N} = 4\) Yang-Mills theory on a spatial \(S^3\) only relatively recently \(^[7] - [8]\). In the weak coupling limit, all of the fields in \(\mathcal{N} = 4\) Yang-Mills theory have a mass gap arising from their conformal coupling to the curvature of the \(S^3\). Integrating them out to find an effective field theory in which one can compute the expectation value of the Polyakov loop is a well-defined procedure. The effective theory is a unitary one-matrix model with effective action \(S_{\text{eff}}[U]\). Gauge symmetry implies \(S_{\text{eff}}[U] = S_{\text{eff}}[W U W^\dagger]\), where \(W\) is a unitary matrix, while center symmetry implies \(S_{\text{eff}}[U] = S_{\text{eff}}[c U]\). Further, the action is of order \(N^2\), \(S_{\text{eff}}[U = 1] \sim N^2\). Since \(\mathcal{N} = 4\) Yang-Mills theory has conformal symmetry, the effective action depends on the temperature \(T\) and the \(S^3\) radius \(R\) through the product \(T R\). It also depends on the ’t Hooft coupling \(\lambda = g_{YM}^2 N\) and on \(N\) which we assume is taken to infinity holding \(\lambda\) fixed. At weak coupling, where \(\lambda \rightarrow 0\), the phase transition is found by tuning \(T R\) to a critical value. When the coupling \(\lambda\) is turned on, this phase transition is thought to persist and at large \(\lambda\) to coincide with the Hawking-Page transition of the gravity dual. The effective field theory description should be reliable when \(T R < 1\); however, it is thought to have a broader applicability. We will assume that it can be used to discuss the deconfined phase, at least in the vicinity of the phase transition that occurs when \(T R \sim 1\).

The unitary matrix model can be used to calculate the expectation value of the Polyakov loop operator in any irreducible representation \(R\) of the \(SU(N)\) gauge group,

\[ \langle \text{Tr}_R U(x) \rangle = \frac{\int [dU] e^{-S_{\text{eff}}[U]} \text{Tr}_R U \int [dU] e^{-S_{\text{eff}}[U]} } {\int [dU] e^{-S_{\text{eff}}[U]} } . \]  

We shall show in the following that \(^[2]\) can have interesting behavior which depends on the size and nature of the representation. We will consider completely symmetric representations \(S_k\) whose Young tableau is a single row with \(k\) boxes and completely antisymmetric representations \(A_k\) whose Young tableau is a single column with \(k\) boxes. We shall consider large values of \(k\) so that \(\frac{k}{N}\) remains finite as \(N \rightarrow \infty\). Both representations \(S_k\) and \(A_k\) have center charge \(k\) mod \(N\) so that \(^[2]\) vanishes in the confined phase when this charge is non-zero. The characters can be non-zero in the deconfined phase.

In the duality between gauge fields and strings, the expectation value of the Wilson loop is dual to an open fundamental string amplitude. This has been made precise for the Maldacena-Wilson loop \(^[9]\) which differs from the Polyakov loop \(^[3]\) in that it contains the scalar fields of the \(\mathcal{N} = 4\) theory as well as the gauge field. In that case, the boundary of the fundamental string worldsheet is located on the loop contour placed at the asymptotic boundary of \(AdS_5\). In the zero temperature Yang-Mills theory defined on a spatial \(R^3\), an interesting phenomenon occurs for loops in representations where the number of boxes \(k\) in the Young tableau is large so that \(\frac{k}{N}\) is finite in the large \(N\) limit. The dual fundamental string worldsheet is replaced by a D-brane with world-volume electric flux \(^[10] - [12]\). This was found by studying highly supersymmetric \(k\)-BPS loops, where some results are known for all values of the coupling constant \(^[13]\). For the anti-symmetric representation, the dual is a D5-brane whose world volume is a direct product of \(AdS_2 \subset AdS_5\) and \(S^3 \subset S^5\). For a symmetric representation, it is a D3-brane with world volume \(AdS_2 \times S^2 \subset AdS_5\). It is interesting to ask whether these D-branes exist in the...
finite temperature geometry where they would be dual to a gauge theory loop linking periodic Euclidean time. This question has already been studied by Hartnoll and Kumar [12] who looked for solutions of the appropriate Born-Infeld actions on the black hole background. For the D5-brane wrapped on $S^4 \subset S^5$ which corresponds to a totally antisymmetric representation on the gauge theory side, there seem to be solutions for any $\frac{k}{N}$ with the usual cutoff at $k = N$ dictated by the maximum size of an antisymmetric representation on the gauge theory side and a maximum radius for embedding $S^4$ in $S^5$ on the supergravity side. However, in the case of the D3-brane which should correspond to a totally symmetric representation, Hartnoll and Kumar could not find any solutions at all. This fundamental difference between the two cases is what motivated our work on the gauge theory which we shall now summarize. Afterward, we will revisit the question of the supergravity side. We emphasize that in supergravity we are studying the dual of the Maldacena-Wilson loop whereas in gauge theory our analysis is limited to the Polyakov loop. Both are governed by the center symmetry and both can become non-zero at the deconfinement transition. At high temperature, due to decoupling of the scalar fields, they should become similar.

To study large totally symmetric or totally antisymmetric representations, it is convenient to obtain the eigenvalue density, $e^{-NT\hat{S}_k/A_k} \equiv \langle Tr S_k/A_k \ U \rangle = \int dt \frac{e^{\frac{T}{2}N\ln(1+\hat{t}U)}}{2\pi i t^k+1}$, where the upper/lower sign is for symmetric/antisymmetric representations $(S_k/A_k)$ respectively and the contour in the integral over $t$ encircles the origin. In the large $N$ limit, these integrals can be computed using two saddle point approximations. The first occurs while integrating over unitary matrices in [2]. Because of the gauge symmetry, this is an eigenvalue model – the gauge symmetry can be used to diagonalize $U = \text{diag}[e^{i\phi_1}, ..., e^{i\phi_N}]$. At large $N$, the eigenvalues become classical variables and their distribution is found by minimizing $S_{\text{eff}}$ plus a Jacobian from the unitary integral measure. As long as $k << N^2$, the loop operators in [3] do not modify the eigenvalue distribution which is given by a density $\rho(\phi)$. $\rho(\phi)d\phi = \frac{1}{N}$ times the number of eigenvalues between $\phi$ and $\phi + d\phi$ and is normalized, $\int_{-\pi}^{\pi} d\phi \rho(\phi) = 1$. Center symmetry is now an invariance under a simultaneous translation of all eigenvalues, $\phi_k \rightarrow \phi_k + \text{constant}$. In the center-symmetric confined phase, the distribution is translation invariant, eigenvalues are uniformly distributed on the unit circle and $\rho_{\text{conf}} = \frac{1}{2\pi}$. In the de-confined phase, the eigenvalues are clumped. We will assume their distribution is symmetric about zero so that $\rho(\phi) = \rho(-\phi)$. In the large $N$ limit the expectation values in Eq. [3] are computed using the eigenvalue density,

$$e^{-NT\hat{S}_k/A_k} = \int dt \frac{e^{\frac{T}{2}N\int_{-\pi}^{\pi} d\phi \rho(\phi) \ln(1+\hat{t}e^{i\phi})}}{2\pi i t^k+1}.$$  (4)

The second use of a saddle-point approximation is to evaluate the integral over $t$ in [4]. Let $\hat{t}$ satisfy the saddle-point equation

$$R_{S_k/A_k}(\hat{t}) = \int_{-\pi}^{\pi} d\phi \rho(\phi) \ln(1+\hat{t}e^{i\phi}) = \frac{k}{N}. \quad (5)$$

Then, the free energy is given by

$$\Gamma_{S_k/A_k} = \pm \int_{-\pi}^{\pi} d\phi \rho(\phi) \ln(1+\hat{t}e^{i\phi}) + \frac{k}{N} \ln \hat{t}. \quad (6)$$

The functions $R_{S_k/A_k}(t)$ in [3] are related to the resolvent of the matrix model and are holomorphic functions of $t$ with cut singularities on the unit circle determined by the support of $\rho(\phi)$.

Let us begin with the symmetric representation $S_k$. We shall consider three examples of eigenvalue distributions. First, the confining phase has $\rho_{\text{conf}} = \frac{1}{\pi}$, $R_{S_k}(t)$ vanishes if $|t| < 1$ and is $-1$ if $|t| > 1$. This is the expected discontinuity at the unit circle. Eq. [5] has solutions only when $\frac{k}{N} = 0$, consistent with confinement.

As a second example consider $\rho(\phi) = \frac{1}{\pi} (1 + 2p \cos \phi)$. $p = \frac{k}{N}$ (Tr $U$) $= \int d\phi \rho(\phi)e^{i\phi}$ is the fundamental representation loop. Positivity of the density requires $0 \leq p \leq \frac{1}{2}$. This distribution depends on $\phi$ and therefore is deconfined. While it is not realistic for $\mathcal{N} = 4$ Yang-Mills theory, it does occur in the strong-coupling phase of large $\mathcal{N}$ 2-dimensional lattice Yang-Mills theory [16].

There is one solution of $R_{S_k}(t) = \frac{k}{N}$ in the region $|t| < 1$ at $t = \frac{1}{2\pi}/p$. (If $\text{frac} \ k N$ and $p$ are such that $|t| > 1$, both $R_{S_k}$ and $\Gamma_{S_k}$ should be extended there by analytic continuation.) The free energy is

$$\Gamma_{S_k} = \frac{k}{N} \ln \left( \frac{k/N}{ep} \right). \quad (7)$$

where $e = 2.718, ..., \Gamma_{S_k}$ has the interesting feature that, as $\frac{k}{N}$ is increased, it changes sign from negative to positive. This results in a phase transition, which occurs when $\frac{k}{N} = (\frac{p}{\pi})_{\text{crit}} = ep$. When $\frac{k}{N} < (\frac{p}{\pi})_{\text{crit}}$, $\Gamma_{S_k}$ is negative and the loop expectation value, $e^{-NT\hat{S}_k}$, is exponentially large. When $\frac{k}{N} > (\frac{p}{\pi})_{\text{crit}}$, $\Gamma_{S_k}$ is positive and the loop vanishes for $N \rightarrow \infty$. This phase transition implies that, even in the deconfined phase, sufficiently large symmetric representations are still confined.

At this point, the reader might wonder how the expectation value of a unitary matrix can grow exponentially. The exponential comes from the traces needed to get the large representation and which give $k$ and $N$ dependent factors. Note that we did not normalize the loop (which would divide by a $k$ and $N$-dependent factor). In Ref.-[12], it was shown that it is the un-normalized loop that...
should be compared with supergravity, which is our eventual aim.

As a check of the saddle-point approximation to the \( t \)-integral in this simple example, observe that, if for the moment we assume that \( k \) and \( N \) are finite, we can integrate (4) explicitly to get \( e^{-NF_S_k} = \frac{N^k}{k!} t^k \). Using the Stirling formula and taking \( k \sim N \rightarrow \infty \) reproduces (7).

We note that the presence of the phase transition is a universal property of the confining phase. From (5) and (6), we see that \( \frac{dF_S}{d(k/N)} = \ln t \) where \( t \) solves (4). Further, by inspecting (6) we see that \( \Gamma_{S_k} \) is real and negative when \( t \) is real and \( t < 1 \). As \( t \) increases, \( \Gamma_{S_k} \) decreases to a minimum at \( t = 1 \), then begins increasing in the region \( t > 1 \) and eventually becomes positive. \( t \) increases with \( k/N \) throughout this region.

To see this behavior in another example, consider the semi-circle distribution which, for \( \theta < 2 \arcsin \sqrt{2 - 2t} \), is

\[
\rho(\phi) = \frac{\cos \frac{\phi}{2}}{\pi(2 - 2t)} \sqrt{2 - 2t - \sin^2 \frac{\phi}{2}}
\]

and which vanishes in the gap \( 2 \arcsin \sqrt{2 - 2t} \leq |\phi| \leq \pi \). We still use the fundamental loop, \( p \), as a parameter and now \( \frac{1}{2} \leq p \leq 1 \). This is the distribution in the weak coupling phase of 2-dimensional lattice Yang-Mills theory (11). It is also an approximation to the deconfined distribution for weakly coupled \( \mathcal{N} = 4 \) Yang-Mills theory (13) (17). For sufficiently weak coupling, it could be accurate near the phase transition where \( p = \frac{1}{2} \). The saddle point computation can be done explicitly near \( t = 0 \) and analytically continued. The free energy is

\[
\Gamma_{S_k} = (2\theta \cosh \theta - \sinh \theta) \frac{\sinh \theta + \sqrt{\sinh^2 \theta + 2 - 2p}}{2 - 2p}
\]

\[ - \frac{1}{2} \ln \left[ \frac{\sinh \theta + \sqrt{\sinh^2 \theta + 2 - 2p}}{2 - 2p} \right], \] (9)

where \( \theta \) is defined by \( \hat{t} = e^{2\theta} \) and is determined by the saddle-point equation

\[
\frac{k}{N} + \frac{1}{2} = \cosh \theta \left[ \frac{\sinh \theta + \sqrt{\sinh^2 \theta + 2 - 2p}}{2 - 2p} \right],
\] (10)

which can be solved for \( \sinh(\theta) \). The free energy is zero when \( k = 0 \), negative for small \( k \), goes to zero at a critical \( \frac{k}{N} \) and is positive thereafter. This is so for any value of \( p \) in the allowed range. A graph of \( \Gamma_{S_k} \) versus \( \frac{k}{N} \) for \( p = 0.51 \) is plotted in Fig. 1. With this value of \( p \), the free energy becomes positive at \( \theta \approx 0.50 \) which corresponds to \( \frac{k}{N} \text{crit.} \approx 1.3 \).

Now, consider the antisymmetric representation. For a large class of distributions gapped around \( \phi = \pi \) and with \( \frac{d\Gamma_{A_k}}{dt} > 0 \), which includes the semi-circle distribution (5), we can argue that \( \Gamma_{A_k} \) is always negative and the phase transition that we are discussing does not occur. To begin, by changing variables in (6), we observe that \( \Gamma_{A_k} = \Gamma_{A_{N-k}} \). This symmetry is reflected in the saddle-point equation (5) which, using our assumption that \( \rho(\phi) = \rho(-\phi) \), can be re-written as

\[
\frac{1}{2} \int_{-\pi}^{\pi} d\phi \rho(\phi) \left( \hat{t}^2 e^{2\phi} - \hat{t}^{-1} e^{-i\phi} \right) = k/N - \frac{1}{2}
\] (11)

and implies \( \hat{t}(k/N) = 1/\hat{t}(1 - k/N) \). The free energy,

\[
\Gamma_{A_k} = -\int_{-\pi}^{\pi} d\phi \rho(\phi) \ln \left( \hat{t}^2 e^{2\phi} + \hat{t}^{-1} e^{-i\phi} \right) + \left( k/N - \frac{1}{2} \right) \ln \hat{t}
\] (12)

is symmetric under \( \frac{k}{N} \rightarrow 1 - \frac{k}{N} \). Moreover, with a gapped distribution, \( \rho(\phi) = 0 \), and the integral in (11) is continuous at \( \hat{t} = 1 \). From \( \frac{d\Gamma_{A_k}}{dt} > 0 \), \( \hat{t}(k) \) is monotone, and one can see in (11) that \( \hat{t} = 0 \) corresponds to \( k/N = 0 \), \( \hat{t} = \infty \) to \( k/N = 1 \) and \( \hat{t} = 1 \) to \( k/N = \frac{1}{2} \). Furthermore, since \( \frac{d^2\Gamma_{A_k}}{dt^2} = \ln \hat{t}(k) \), \( \frac{d^2\Gamma_{A_k}}{dt^2} > 0 \), thus \( \Gamma_{A_k} \) is a convex function which decreases from 0 to a negative minimum as \( \frac{k}{N} \) goes from 0 to \( \frac{1}{2} \) and then increases back to zero when \( \frac{k}{N} \) goes from \( \frac{1}{2} \) to 1. \( \Gamma_{A_k} \) does not become positive and there is no phase transition of the kind that we found for symmetric representations. When the distribution is ungapped, or when \( \frac{d\Gamma_{A_k}}{dt} \) becomes negative (for example when \( p < 0 \)), interesting behavior can occur. We put off a discussion of it to a future investigation.

We have found a difference between the symmetric and antisymmetric representation Polyakov loops which is qualitatively similar to the one found by Hartnoll and Kumar (13) for the dual objects in supergravity: the antisymmetric loop is non-zero in the deconfined phase for all allowed \( \frac{k}{N} \) and the dual D5-brane exists whereas the gauge theory symmetric representation loop has a phase transition. The numerical search for the dual D3-brane in (13) combined with analytic arguments at large \( \kappa = \frac{k}{N} \) found no solution. If we take this to mean that the expectation value of the gauge theory quantity vanishes, it suggests that, at strong coupling, the critical value \( k/N \) goes to zero faster than \( \frac{1}{\sqrt{\kappa}} \).

We can also examine the alternative that the phase transition occurs for a value of \( \kappa \) so small that solutions
where substituted into the equation of motion for \( F \) and 

\[
\frac{\partial L}{\partial A_t} = \Pi_t = 0.
\]

The first term is 

\[ L = r^2 \sin^2 \chi \left[ \sqrt{1 + r^2 f(r)(r')^2} - \frac{4\pi^2}{\lambda} F^2 - r^2 r' \right]. \]  

(15)

The last term is the Chern-Simons term. Consistent with symmetries and equations of motion, one can take \( F_{ab} \) with one nonzero component \( F_{tt} = A'(r) \). The canonical momentum \( \Pi = \partial L / \partial A'_t(r) \) is a constant equal to the number of units of electric flux, \( k \). Solving for \( F \) and substituting into the equation of motion for \( \chi \) yields

\[ 2r^4 \sin^3 \chi \cos \chi \left( \sqrt{1 + r^2 f(r)(r')^2} \right) - 4r^3 \sin^2 \chi
\]

(16)

We fix the boundary condition at \( r \rightarrow \infty \) to match the zero temperature \( \frac{1}{2} \)-BPS D3-brane. It is a solution of the same equation with \( f(r) = r^2 \) and where, to get the Poincaré coordinates, \( (\sin \chi, \cos \chi) \) are replaced by \( (\chi, 1) \). Then \( \chi(r) = \frac{\pi}{2} \) is an exact solution of \( (10) \) and the brane geometry is a simple direct product of AdS5 with radius \( \sqrt{1 + \kappa^2} \) and \( S^5 \) with radius \( \kappa \). We seek solutions with the asymptotic behavior \( \chi(r) \sim \frac{\pi}{2} \) for large \( r \). By studying the large \( r \) regime, it is easy to see that there is no solution of \( (10) \) for \( \chi \) which goes to zero at least as fast as \( r^{-1} \) unless \( \kappa \) is non-zero. By studying the region near the horizon, we can see that there is no solution in the large \( \kappa \) limit. So, if there is a solution at all, it will only exist if kappa is non-zero but not large. We have attempted to solve \( (10) \) numerically with small values of \( \kappa \). We have positive evidence for a solution in a corner of the parameter space obtained by taking the infinite temperature limit (replacing \( r \) with \( r_L, \) \( b = \frac{\kappa^2}{L} \) with \( \kappa^2 \)). The resulting differential equation has an exact solution for \( r_+ = 0, \chi = \kappa / (r + b) + O(r^2) \) where \( b \) is an integration constant. Restoring \( r_+ > 0 \), we employed a shooting technique to look for solutions which asymptote to \( \tilde{\chi} \). With \( \kappa = 0.001 \), there appears to be a solution at \( b \sim 10^1 \). Our work is on-going and we shall present the details elsewhere.

The authors acknowledge hospitality of the Galileo Galilei Institute, Aspen Center for Physics and Perimeter Institute. This work is supported in part by NSERC of Canada and the INFN of Italy.

[1] S. W. Hawking and D. N. Page, Commun. Math. Phys. 87, 577 (1983).

[2] E. Witten, Adv. Theor. Math. Phys. 2, 505 (1998) [arXiv:hep-th/980331].

[3] B. Sundborg, Nucl. Phys. B 573, 349 (2000) [arXiv:hep-th/9908001].

[4] A. M. Polyakov, Int. J. Mod. Phys. A 1751, 119 (2002) [arXiv:hep-th/0110196].

[5] O. Aharony, J. Marsano, S. Minwalla, K. Papadodimas and M. Van Raamsdonk, Adv. Theor. Math. Phys. 8, 603 (2004) [arXiv:hep-th/0310285].

[6] A. M. Polyakov, Phys. Lett. B 72, 477 (1978).

[7] L. Susskind, Phys. Rev. D 20, 2610 (1979).

[8] L. D. McLerran and B. Svetitsky, Phys. Lett. B 98, 195 (1981); B. Svetitsky and L. G. Yaffe, Nucl. Phys. B 210, 423 (1982); L. G. Yaffe and B. Svetitsky, Phys. Rev. D 26, 963 (1982); A. Dumitru, Y. Hatta, J. Lenaghan, K. Orginos and R. D. Pisarski, Phys. Rev. D 70, 034511 (2004) [arXiv:hep-th/0311223].

[9] J. M. Maldacena, Phys. Rev. Lett. 80, 4859 (1998) [arXiv:hep-th/9803002].

[10] N. Drukker and B. Fiol, JHEP 0502, 010 (2005) [arXiv:hep-th/0501109].

[11] S. Yamaguchi, Int. J. Mod. Phys. A 22, 1353 (2007) [arXiv:hep-th/0601089]; S. Yamaguchi, JHEP 0605, 037 (2006) [arXiv:hep-th/0605208].

[12] J. Gomis and F. Passerini, JHEP 0608, 074 (2006) [arXiv:hep-th/0604007]; J. Gomis and F. Passerini, JHEP 0701, 097 (2007) [arXiv:hep-th/0612022]; J. Gomis, S. Matsura, T. Okuda and D. Trancanelli, JHEP 0808, 068 (2008) [arXiv:0807.3330 [hep-th]].

[13] D. Rodriguez-Gomez, Nucl. Phys. B 752, 316 (2006) [arXiv:hep-th/0604031]; K. Okuyama and G. W. Semenoff, JHEP 0606, 057 (2006) [arXiv:hep-th/0604209]; S. Giombi, R. Ricci and D. Trancanelli, JHEP 0610, 045 (2006) [arXiv:hep-th/0608077].

[14] J. K. Erickson, G. W. Semenoff and K. Zarembo, Nucl. Phys. B 582, 155 (2000) [arXiv:hep-th/0003055]; V. Pestun, arXiv:0712.2824 [hep-th].

[15] S. A. Hartnoll and S. Prem Kumar, Phys. Rev. D 74, 026001 (2006) [arXiv:hep-th/0603190].

[16] D. J. Gross and E. Witten, Phys. Rev. D 21 (1980) 446.

[17] J. Jurkiewicz and K. Zalewski, Nucl. Phys. B 220, 167 (1983).