Ratio of strange to non-strange quark condensates in QCD

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Abstract

Laplace transform QCD sum rules for two-point functions related to the strangeness-changing scalar and pseudoscalar Green’s functions $\psi(Q^2)$ and $\psi_5(Q^2)$, are used to determine the subtraction constants $\psi(0)$ and $\psi_5(0)$, which fix the ratio $R_{su} \equiv \frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle}$. Our results are $\psi(0) = -(1.06 \pm 0.21) \times 10^{-3}$ GeV$^4$, $\psi_5(0) = (3.35 \pm 0.25) \times 10^{-3}$ GeV$^4$, and $R_{su} = \frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} = 0.5 \pm 0.1$. This implies corrections to kaon-PCAC at the level of 50%, which although large, are not inconsistent with the size of the corrections to Goldberger-Treiman relations in $SU(3) \otimes SU(3)$.

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The quark vacuum condensate ratio \( R_{su} \equiv \frac{<\bar{s}s>}{<\bar{u}u>} \), with \(<\bar{u}u> \sim <\bar{d}d>\), is an important quantity which measures the size of flavour SU(3) symmetry breaking in the QCD vacuum \( [1] \). It also enters in a variety of QCD sum rules \( [2] \), particularly in those used to determine baryon masses \( [3] \). In addition, it provides a measure of the corrections to kaon PCAC (Partial Conservation of the Axial-Vector Current), provided the quark masses are known and neglecting corrections to pion PCAC \( [1],[4] \). In fact, let us consider the two-point function involving the flavour-changing axial-vector current divergences

\[
\psi_5(q^2) = i \int d^4x \ e^{iqx} \left\langle 0 \left| T(\partial^\mu A_\mu(x) \ \partial^\nu A^\dagger_\nu(0)) \right| 0 \right\rangle ,
\]

where flavour indices have been omitted for simplicity, and

\[
\partial^\mu A_\mu(x)|^j_i = (m_i + m_j) : \bar{\psi}_j(x) \ i\gamma_5 \ \psi_i(x) :,
\]

and \((i,j)\) are flavour indices. A well known Ward identity fixes the value of \( \psi_5(q^2) \) at zero momentum \( [1],[4] \), viz.

\[
\psi_5(0)|^j_i = -(m_i + m_j) \langle \bar{\psi}_j \psi_j + \bar{\psi}_i \psi_i \rangle ,
\]

which is a renormalization group invariant quantity. Saturation of Eq.(1) with the lowest lying pseudoscalar meson then leads to the Gell-Mann, Oakes, Renner relations \( [1] \)

\[
-(m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle = 2f_\pi^2 \mu_\pi^2 ,
\]

for \((i,j)=(u,d)\), and

\[
-(m_s + m_u) \langle \bar{s}s + \bar{u}u \rangle = 2f_K^2 \mu_K^2 ,
\]

where now \((i,j)=(u,s)\), \( f_\pi = 92.4 \pm 0.4 \) MeV, and \( f_K = 113.0 \pm 1.5 \) MeV \( [5] \). Considering Eq.(3) for up- and strange-quark flavours and then for up- and down-quark flavours and taking the ratio leads to

\[
R_{AA} \equiv \frac{\psi_5(0)|^s_u}{\psi_5(0)|^d_u} = \frac{1}{2} \frac{m_s + m_u}{m_u + m_d} \left( 1 + \frac{<\bar{s}s>}{<\bar{u}u>} \right) ,
\]

where \(<\bar{u}u> \sim <\bar{d}d>\) is a very good approximation in this case \( [6] \). In principle, this relation could be used to determine \( R_{su} \), if the ratio of the subtraction constants \( \psi_5(0) \), i.e.
\( R_{AA} \), is known independently. Using current values of the quark masses \([5]\) in Eq.(6) leads to

\[
R_{su} \equiv \frac{<\bar{s}s>}{<\bar{u}u>} \simeq 0.15 \ R_{AA} - 1 .
\]  

Since \( R_{AA} \) is expected to be of order \( \mathcal{O}(10) \) (from using PCAC), this procedure would lead to very large uncertainties, unless \( \psi_5(0) \) were to be determined with extreme accuracy. An alternative method not affected by this problem was first proposed in \([7]\), and it is based on examining \( \psi_5(0) \) together with \( \psi(0) \), where

\[
\psi(q^2) = i \int d^4x \ e^{i q x} \langle 0 | T(\partial^\mu V_\mu(x) \partial^\nu V_\nu^\dagger(0)) | 0 \rangle ,
\]  

with

\[
\partial^\mu V_\mu(x) |^j_i = (m_j - m_i) : \bar{\psi}_j(x) \ i \ \psi_i(x) : ,
\]  

being the flavour-changing vector current divergence. The Ward identity analogous to Eq.(3) is now

\[
\psi(0)^j_i = -(m_j - m_i) \langle \bar{\psi}_j \psi_j - \bar{\psi}_i \psi_i \rangle .
\]  

Defining the ratio

\[
R_{VA} \equiv \frac{\psi(0)^s_i}{\psi_5(0)^s_u} ,
\]  

it follows that

\[
R_{su} \equiv \frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} \simeq \frac{1 + R_{VA}}{1 - R_{VA}} ,
\]  

where the reasonable approximation \( (m_s - m_u) \simeq (m_s + m_u) \) has been made. This procedure can provide a more accurate determination of \( R_{su} \), as discussed in \([7]\). Since then, considerable progress has been made on the QCD calculation of the two-point functions Eqs.(1) and (8), viz. the perturbative expressions are now known up to four loops \([8]\), quark-mass corrections have been calculated up to order \( \mathcal{O}(m_q^6) \), and the issue of quark-mass logarithmic singularities has been satisfactorily resolved \([9]-[10]\). On the other hand, there is now better experimental information on the hadronic spectral functions in both the scalar \([11]\) and the pseudoscalar channels \([3]\). In view of this, we discuss here a new determination of the subtraction constants \( \psi_{(5)}(0) \) which are then used to determine the
ratio $R_{su}$ through Eq.(12).

In order to relate the constants $\psi(5)(0)$ to the two-point functions $\psi(5)(q^2)$, Eqs.(1) and (8), one defines the auxiliary functions

$$q^2 D(5)(q^2) = \psi(5)(q^2) - \psi(5)(0).$$

The derivatives of these functions, i.e.

$$\xi(q^2) = \frac{\partial}{\partial q^2} D(q^2),$$

$$\phi(q^2) = \frac{\partial}{\partial q^2} D_5(q^2),$$

have the same imaginary part as $\psi(5)(q^2)$, and satisfy the following dispersion relations

$$\left\{ \begin{array}{l}
\xi(Q^2) \\
\phi(Q^2)
\end{array} \right\} = \frac{1}{\pi} \int_0^{\infty} \frac{ds}{s} \frac{Im \psi(5)(s)}{(s + Q^2)^2},$$

where $Q^2 \equiv -q^2 \geq 0$. After Laplace improvement, these dispersion relations become

$$\left\{ \begin{array}{l}
\xi(M^2_L) \\
\phi(M^2_L)
\end{array} \right\} = \frac{1}{\pi} \int_0^{\infty} \frac{ds}{s} e^{-s/M^2_L} \frac{Im \psi(5)(s)}{s},$$

According to quark-hadron duality, the left hand side of Eq.(17) is computed in QCD using the Operator Product Expansion to organize the perturbative series and the power corrections, and the right hand side is saturated with the experimental data. Using the available QCD expressions for $\psi(5)(q^2)$ \cite{3,4}, performing the derivatives, Laplace transforming, and after Renormalization Group improvement, we obtain the following QCD representations for the left hand sides of Eq.(17) for three quark flavours.
\[
\xi (M^2_L) \phi (M^2_L) \right\} = \frac{3}{8 \pi^2} \frac{m_s^2 (M^2_L)}{M^2_L} \left\{ 1 + \frac{\alpha_s (M^2_L)}{\pi} \left( \frac{17}{3} - 2 \Psi (1) \right) \\
+ \left( \frac{\alpha_s (M^2_L)}{\pi} \right)^2 \left[ \frac{9631}{144} - \frac{35}{2} \zeta (3) - \frac{95}{3} \Psi (1) \right] \\
+ \frac{17}{4} \left( \Psi^2 (1) - \Psi' (1) \right) \right\} \\
+ \left( \frac{\alpha_s (M^2_L)}{\pi} \right)^3 \left\{ 2 - \Psi (2) + \frac{\alpha_s (M^2_L)}{\pi} \left[ \frac{41}{3} - 4 \zeta (3) \right] \\
- \frac{28}{3} \Psi (2) + 2 \left( \Psi^2 (2) - \Psi' (2) \right) \right\} \\
- \frac{1}{2} \frac{m_s^4 (M^2_L)}{M^6_L} \left\{ \langle m_s \bar{s}s \rangle \left[ 1 + \frac{\alpha_s (M^2_L)}{\pi} \left( \frac{14}{3} - 2 \Psi (3) \right) \right] \\
\pm 2 \langle m_u \bar{u}u \rangle \left[ 1 + \frac{\alpha_s (M^2_L)}{\pi} \left( \frac{17}{3} - 2 \Psi (3) \right) \right] \right\} \\
- \frac{1}{8} \frac{m_s^4 (M^2_L)}{M^6_L} \left\{ \langle \alpha_s \pi G^2 \rangle - \frac{3}{8 \pi^2} \frac{m_s^2 (M^2_L)}{M^6_L} \Psi (3) + \frac{\psi_5 (0)}{M^6_L} \right\}, \tag{18}
\]

where \(\Psi(z)\) is the di-gamma function, primes stand for its derivatives, and \(\zeta(z)\) is Riemann’s zeta function. The four-quark vacuum condensate term of dimension-six has been omitted above as it makes a negligible contribution, and the up-quark mass has been neglected in comparison with \(m_s\). The four-loop expressions for the running coupling and quark mass in the \(\overline{\text{MS}}\) renormalization scheme, and for three flavours, are \[3\]

\[
\frac{\alpha_s (Q^2)}{\pi} = \frac{4}{9} \frac{1}{L^2} - \frac{256}{729} \frac{L^2}{L^2} \\
+ \left[ 6794 - 16384 \left( LL - LL^2 \right) \right] \frac{1}{59049} \frac{1}{L^3} + \mathcal{O} \left( \frac{1}{L^4} \right), \tag{19}
\]

\]
\[
\bar{m}_j(Q^2) = \left(\frac{4}{L}\right)^3 \left\{ 1 + (290 - 256LL) \frac{1}{729L} \right. \\
+ \frac{550435}{1062882} - \frac{80}{729} \zeta(3) \\
- \left(388736LL - 106496LL^2\right) \frac{1}{531441} \right\} \frac{1}{L^2} \\
+ \left[ -\frac{126940037}{1162261467} - \frac{256}{177147} \beta_4 + \frac{128}{19683} \gamma_4 + \frac{7520}{531441} \zeta(3) \\
+ \left( -\frac{611418176}{387420489} + \frac{112640}{531441} \zeta(3) \right) LL + \frac{335011840}{387420489} LL^2 \\
- \frac{149946368}{1162261467} LL^3 \right\} \frac{1}{L^3} + \mathcal{O}\left(\frac{1}{L^4}\right) \right). 
\]

where \(L = \ln\left(Q^2/\Lambda^2_{QCD}\right), LL = \ln L,\) and

\[
\beta_4 = -\frac{281198}{4608} - \frac{890}{32} \zeta(3), 
\]

with \(\gamma_4 = 88.5258\) [12], and \(\bar{m}_j\) is the invariant quark-mass. The terms of order \(\mathcal{O}\left(\frac{1}{L^4}\right)\) above are known up to a constant not determined by the renormalization group. This constant can be estimated e.g. using Padè approximants [13]. However, we have checked that our final results are essentially insensitive to terms of this order.

Before we proceed to discuss the hadronic parametrization of the scalar and pseudoscalar spectral functions, it should be stressed that these spectral functions are also used to determine the strange-quark mass from QCD sum rules for \(\psi_5(q^2)\) [14] and \(\psi(q^2)\) [1-13]. The fact that \(m_s\) is one of the most important parameters in Eq.(18), and given the wide range of values obtained from QCD sum rules [10], it is imperative to achieve consistency between the results for \(m_s\) from the scalar and the pseudoscalar channels. Fully consistent
results are obtained as explained below. Another very important parameter in Eq.(18) is $\Lambda_{QCD}$. The value extracted from a variety of experimental data has been increasing steadily over the years, from $\Lambda_{QCD} \simeq 100 - 200 \text{ MeV}$ at the birth of the QCD sum rules [2], to something as high as $\Lambda_{QCD}(N_f = 3) \simeq 300 - 450 \text{ MeV}$ lately [5]. Such high values make radiative corrections to Green’s functions the overwhelming terms in the Operator Product Expansion, thus reducing the numerical importance of the power corrections as parametrized by quark and gluon vacuum condensates. In fact, it has been argued in [17] that if $\Lambda_{QCD}(N_f = 3)$ exceeds $\simeq 330 \text{ MeV}$ the QCD sum rule program may break down, and it becomes extremely difficult to extract numerical values for the condensates from the data (see also [18]). As for the other QCD parameters in Eq.(18), the gluon condensate has the value [19] $\langle \frac{G^2}{\pi} \rangle \simeq 0.01 - 0.04 \text{ GeV}^4$. The light-quark condensate (at 1 GeV) is $\langle \bar{u}u \rangle \simeq -10^{-2} \text{ GeV}^3$. For the strange-quark condensate, since it is the object of this determination, an iteration procedure of fast convergence has been adopted, starting from $\langle \bar{s}s \rangle \simeq \langle \bar{u}u \rangle$.

Turning to the hadronic sector, the spectral function in the pseudoscalar channel is parametrized as in [14], viz.

\[
\frac{1}{\pi} \text{Im} \psi_5(s)|_{HAD} = 2f_K^2M_K^4\delta\left(s-M_K^2\right) \\
+ \frac{1}{\pi} \text{Im} \psi_5(s)|_{K\pi\pi}\left[\frac{BW_1(s)+\lambda BW_2(s)}{(1+\lambda)}\right],
\]

(22)

where the threshold behaviour of the resonant piece, determined using chiral perturbation theory, is given by

\[
\frac{1}{\pi} \text{Im} \psi_5(s)|_{K\pi\pi} = \frac{M_K^4}{2f_{\pi}^2} \frac{3}{2^8\pi^4} \frac{I(s)}{s(M_K^2-s)^2} \theta\left(s-M_K^2\right),
\]

(23)

with

\[
I(s) = \int_{M_K^2}^{s} \frac{du}{u} (u-M_K^2)(s-u) \times \left\{ (M_K^2-s) \left( u - \frac{(s+M_K^2)}{2} \right) \right. \\
- \frac{1}{8u} \left( u^2 - M_K^4 \right)(s-u) + \frac{3}{4} \left( u - M_K^2 \right)^2 |F_{K\pi}(u)|^2 \right\},
\]

(24)
where

$$|F_{K^*}(u)|^2 = \frac{(M_{K^*}^2 - M_K^2)^2 + M_K^2 \Gamma_{K^*}^2}{(M_{K^*}^2 - u)^2 + M_K^2 \Gamma_{K^*}^2}, \quad (25)$$

is the contribution from the resonant sub-channel $K^*(892) - \pi$. The parameter $\lambda$ above controls the importance of the second radial excitation ($K(1830)$) relative to the first ($K(1460)$) ($\lambda \simeq 1$), with both resonances being parametrized by Breit-Wigner forms $BW_{1,2}(s)$. Due to the approximation $m_u = 0$, the pion mass has been neglected. There is another resonant sub-channel, the $\rho(770)K$, which turns out to be numerically negligible [14]. As usual, this resonant hadronic parametrization is used up to some threshold energy, $s_{0,A}$, the continuum threshold, after which the spectral function is assumed to be given by perturbative QCD.

In the case of the scalar channel, there is experimental data on $K\pi$ phase shifts [11] which can be used to reconstruct the spectral function given by

$$\frac{1}{\pi} Im \psi(s) = \frac{3}{32\pi^2} \sqrt{(s-s_+)(s-s_-)} |d(s)|^2, \quad (26)$$

where $s_{\pm} = (\mu_K \pm \mu_\pi)^2$, and $d(s)$ is the scalar form factor. We have used the method of [15], based on the Omnès representation, to relate $d(s)$ to the experimental phase shifts. In this way, the results for $m_s$ from the pseudoscalar channel are fully compatible with those from the scalar channel, giving an invariant strange-quark mass

$$\hat{m}_s = 140 \pm 10 \text{ MeV}, \quad (27)$$

for $\Lambda_{QCD}$ in the range $\Lambda_{QCD} \simeq 300 - 350$ MeV. As mentioned earlier, higher values of $\Lambda_{QCD}$ may invalidate the QCD sum rule program in general, and they do so in this particular application, as they lead to serious instabilities in the results for $\psi(5)(0)$. Therefore, we restrict $\Lambda_{QCD}$ to the above range. In Figs. 1 and 2 we show typical results for $\psi_5(0)$ and $\psi(0)$, respectively, obtained using the values $\Lambda_{QCD} = 300$ MeV, $\hat{m}_s = 145$ MeV, $\langle \frac{a_s G^2}{\pi} \rangle = 0.01 \text{ GeV}^4$, and the asymptotic freedom thresholds $s_{0A} = 6 \text{ GeV}^2$, and $s_{0V} = 4 \text{ GeV}^2$. These results lead to the ratio $R_{su}$ shown in Fig.3. As it may be appreciated from these figures, the results are nicely insensitive to changes in the Laplace variable $M_L^2$; they are also reasonably stable against changes in the continuum thresholds, $s_{0V,A}$. Increasing $\Lambda_{QCD}$ leads to smaller values of $\hat{m}_s$, and higher values of the ratio $R_{su}$. After allowing for
variations in all the relevant parameters, within the ranges indicated above, we obtain

\[ \psi_5(0) = (3.35 \pm 0.25) \times 10^{-3} \text{ GeV}^4, \tag{28} \]

\[ \psi(0) = -(1.06 \pm 0.21) \times 10^{-3} \text{ GeV}^4, \tag{29} \]

\[ R_{su} \equiv \frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} = 0.5 \pm 0.1. \tag{30} \]

A comparison of these results with previous determinations based on similar methods [7], [20] is not very enlightening, as most of the old analyses were done at the two-loop level in perturbative QCD, they were affected by logarithmic quark-mass singularities, and did not make use of the latest experimental data to reconstruct the hadronic spectral functions. However, these problems do not affect determinations based on different methods. For instance, from QCD sum rules for baryon masses [3] \( R_{su} = 0.6 \pm 0.1 \) has been obtained, while an estimate based on non-perturbative quark-mass generation [21] gives \( R_{su} \simeq 0.7 \) (with no error estimates). Our result, Eq.(30) is consistent with these values. Furthermore, Eq.(29) leads to a correction to kaon-PCAC, \( \delta_K \), defined as

\[ \psi_5(0) = 2f_K^2 \mu_K^2 (1 - \delta_K), \tag{31} \]

where using Eq.(29) one finds

\[ \delta_K \simeq 0.5. \tag{32} \]

This result points to a rather large correction to kaon-PCAC, although it is not inconsistent with the expected size of the corrections to Goldberger-Treiman relations in \( SU(3) \otimes SU(3) \) [22].

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Figure Captions

Figure 1. The subtraction constant $\psi_5(0)$ as a function of the Laplace variable $M_L^2$, for $\Lambda_{QCD} = 300$ MeV, $\hat{m}_s = 145$ MeV, $\langle \frac{u^2}{\pi} G^2 \rangle = 0.01$ GeV$^4$, and the continuum thresholds $s_{0A} = 6$ GeV$^2$, and $s_{0V} = 4$ GeV$^2$.

Figure 2. The subtraction constant $\psi(0)$ as a function of the Laplace variable $M_L^2$, for $\Lambda_{QCD} = 300$ MeV, $\hat{m}_s = 145$ MeV, $\langle \frac{u^2}{\pi} G^2 \rangle = 0.01$ GeV$^4$, and the continuum thresholds $s_{0A} = 6$ GeV$^2$, and $s_{0V} = 4$ GeV$^2$.

Figure 3. The ratio $R_{su} \equiv \frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle}$ as a function of the Laplace variable $M_L^2$, for $\Lambda_{QCD} = 300$ MeV, $\hat{m}_s = 145$ MeV, $\langle \frac{u^2}{\pi} G^2 \rangle = 0.01$ GeV$^4$, and the continuum thresholds $s_{0A} = 6$ GeV$^2$, and $s_{0V} = 4$ GeV$^2$. 

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Figure 2:
Figure 3: