Effects of the Cosmological Constant as the Origin of the Cosmic-Ray Paradox

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Abstract

We show that a tiny but non-zero positive cosmological constant, which has been strongly suggested by the recent astronomical observations on supernovae and CMBR, may change notably the behaviors of the ultrahigh energy cosmic ray interacting with soft photons. The threshold anomaly of the ultrahigh energy cosmic ray disappears naturally after the effects of the cosmological constant are taken into account.

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Recently there is great interest in the study of the ultrahigh energy cosmic ray (UHECR) and the TeV-photon paradoxes. Hundreds of events with energies above $10^{19}$eV and about 20 events above $10^{20}$eV have been observed \cite{1}-\cite{5}. All these observed events exceed the Greisen-Zatsepin-Kuzmin (GZK) \cite{6} threshold. In principle, the photopion production process by the cosmic microwave background radiation (CMBR) should decrease the energies of these protons to the level which is below the corresponding threshold. The second paradox \cite{7} comes from the detected 20TeV photons from the Mrk 501 (a BL Lac object at a distance of 150Mpc). Similar to the case

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of UHECR, due to interaction with the IR background photons, the 20TeV photons should have disappeared before arriving at the ground-based detectors. The two puzzles have a common feature that both of them can be considered to be some threshold anomalies: energy of an expected threshold is reached but the threshold is not observed. There are yet numerous published suggestions [8]-[10] for possible solution of the UHECR and the TeV-γ paradoxes. In particular, authors [11]-[18] have suggested that Planck scale physics and the violation of the Lorentz invariance can be the origin of these anomalies. Along this way, noncommutative geometry [19] and doubly special relativity approaches [20]-[23] for the cosmic-ray paradox have been set up and many interesting results have been obtained. However, all of these investigations are far beyond the standard cosmology theory and the standard model of particle physics.

The most important progress made in cosmology in recent years is that the astronomical observations on supernovae [24, 25] and CMBR [26] show that about two third of the whole energy in the universe is contributed by a small positive cosmological constant. An asymptotic de Sitter (dS) spacetime is promised naturally. The physics in an asymptotic dS spacetime has been discussed extensively [27]-[29].

In this Letter, starting from solving equations of motion for a free particle in the dS sapcetime, we try to give a reasonable solution to the cosmic ray paradox by taking the effects of the cosmological constant into account.

The dS spacetime can be described as a submanifold of a five dimensional pseudo-Euclidean space

\[(\xi^0)^2 - (\xi^1)^2 - (\xi^2)^2 - (\xi^3)^2 - (\xi^5)^2 = -\frac{1}{\lambda}, \quad (1)\]

\[ds^2 = (d\xi^0)^2 - (d\xi^1)^2 - (d\xi^2)^2 - (d\xi^3)^2 - (d\xi^5)^2, \quad (2)\]

where \(\lambda\) is the curvature of the dS spacetime. This realization of dS spacetime is invariant obviously under the action of the dS group \(SO(4, 1)\).

It is convenient to study the kinematics in dS spacetime by introducing the Beltrami coordinates \(x^i\),

\[x^i \equiv \frac{\xi^i}{\sqrt{\lambda} \xi^5}, \quad (i = 0, 1, 2, 3). \quad (3)\]
In the Beltrami coordinate system, the dS spacetime can be rewritten into the form

\[ \sigma \equiv \sigma(x, x) = 1 - \lambda \eta_{ij} x^i x^j > 0, \]

\[ ds^2 = \left( \eta_{ij} + \frac{\lambda \eta_{kl} x^l x^k}{\sigma^2} \right) dx^i dx^j, \]

where \( \eta_{ij} = \text{diag}(1, -1, -1, -1) \). It is easy to check that Eqs. (4) and (5) are invariant under transformations of the \( SO(4, 1) \)

\[ x^i \rightarrow \tilde{x}^i = \sigma(a, a)^{1/2} \sigma(a, x)^{-1} (x^j - a^j) D_j^i, \]

\[ D_j^i = L_j^i + \lambda \left( \sigma(a, a) + \sigma(a, a)^{1/2} \right)^{-1} \eta_k l a^l L_j^k, \]

\[ L \equiv (L_j^i) \in SO(3,1), \]

\[ \sigma(a, a) > 0. \]

We notice that there is a subgroup \( SO(4) \) of the de Sitter one \( SO(4,1) \), which consists of spatial transformations among \( x^\alpha (\alpha = 1, 2, 3) \). It is not difficult to show that \( \xi^0 (\equiv \sigma(x,x)^{-1/2}x^0) \) is invariant under the spatial transformations. Thus, we can say that two spacelike events are simultaneous if they satisfy

\[ \sigma(x, x)^{-1/2} x^0 = \xi^0 = \text{constant}. \]

Therefore, it is convenient to discuss physics of the dS spacetime in the coordinate \((\xi^0, x^\alpha)\). In this coordinate, the metric can be rewritten into the form

\[ ds^2 = \frac{d\xi^0 d\xi^0}{1 + \lambda \xi^0 \xi^0} - \left( 1 + \lambda \xi^0 \xi^0 \right) \left[ \frac{d\rho^2}{(1 + \lambda \rho^2)^2} + \frac{\rho^2}{1 + \lambda \rho^2} d\Omega^2 \right], \]

where \( \rho^2 \equiv \Sigma x^\alpha x^\alpha \) and \( d\Omega^2 \) denotes the metric on the sphere \( S^2 \).

If a proper time \( \tau \) is introduced as

\[ \tau \equiv \frac{1}{\sqrt{\lambda}} \sinh^{-1}(\sqrt{\lambda} \xi^0), \]

one can get a Robertson-Walker-like metric

\[ ds^2 = d\tau^2 - \cosh^2(\sqrt{\lambda} \tau) \left[ \frac{d\rho^2}{(1 + \lambda \rho^2)^2} + \frac{\rho^2}{1 + \lambda \rho^2} d\Omega^2 \right]. \]
The Casimir operator of the de Sitter group can be used to express the one-particle states

\[ \left( \frac{1}{\sqrt{-g}} \partial_i (\sqrt{-g} g^{ij} \partial_j) + m_0^2 \right) \Phi(\xi^0, x^\alpha) = 0 , \tag{11} \]

where \( \Phi(\xi^0, x^\alpha) \) denotes a scalar field or a component of vector field for a particle with given spin \( s \).

Making use of the diagonal metric (8), we can rewrite the de Sitter invariant operator in the following form

\[
\frac{1}{\sqrt{-g}} \partial_i (\sqrt{-g} g^{ij} \partial_j) = \left( 1 + \lambda \xi^0 \xi^0 \right) \partial_{\xi^0}^2 + 4 \lambda \xi^0 \partial_{\xi^0} \\
- \left( 1 + \lambda \xi^0 \xi^0 \right)^{-1} \left[ (1 + \lambda \rho^2)^2 \partial_\rho^2 + 2 \rho^{-1} (1 + \lambda \rho^2) \partial_\rho \right] \\
+ \left( 1 + \lambda \xi^0 \xi^0 \right)^{-1} (1 + \lambda \rho^2) \rho^{-2} \left[ -\partial_u^2 + s(s+1) \right] ,
\]

where \( \partial_u^2 \) denotes the Laplace operator on \( S^2 \).

To solve the equation of motion, we write the field \( \Phi(\xi^0, x^\alpha) \) into the form

\[
\Phi(\xi^0, \rho, u) = T(\xi^0)U(\rho)Y_{lm}(u) .
\]

This form of the field transforms the equation of motion into [29]

\[
[ \left( 1 + \lambda \xi^0 \xi^0 \right)^2 \partial_{\xi^0}^2 + 4 \lambda \xi^0 (1 + \lambda \xi^0 \xi^0) \partial_{\xi^0} + m_0^2 (1 + \lambda \xi^0 \xi^0) + (\varepsilon^2 - m_0^2) ] \, T(\xi^0) = 0 ,
\]

\[
\left[ \partial_\rho^2 + 2 \frac{\rho}{\rho} \partial_\rho - \frac{m_0^2 - \varepsilon^2}{(1 + \lambda \rho^2)^2} \frac{l(l+1) + s(s+1)}{\rho^2(1 + \lambda \rho^2)} \right] U(\rho) = 0 ,
\]

\[
[ \partial_u^2 + l(l+1) ] Y_{lm}(u) = 0 , \tag{13}
\]

where \( Y_{lm}(u) \) is the spherical harmonic function and \( \varepsilon \) is a constant.

Solutions of timelike part of the field are of the forms

\[
T(\xi^0) \sim (1 + \lambda \xi^0 \xi^0)^{-1/2} \cdot \begin{cases} P^\mu_\nu(i \sqrt{\lambda} \xi^0) , \\ Q^\mu_\nu(i \sqrt{\lambda} \xi^0) , \end{cases}
\]
where $\mu, \nu$ satisfy
\[
\nu(\nu + 1) = 2 - \lambda^{-1}m_0^2 ,
\]
\[
\mu^2 = 1 + \lambda^{-1}(\epsilon^2 - m_0^2) .
\]
For the radial equation of the field, we can write the solutions as the form
\[
U(\rho) \sim \rho^l(1 + \lambda\rho^2)^{k/2}F\left(\frac{1}{2}(l + s + k + 1), \frac{1}{2}(l + s + k), l + s + \frac{3}{2}; -\lambda\rho^2\right) ,
\]
where $k$ denotes the radial quantum number
\[
k^2 - 2k - \lambda^{-1}(\epsilon^2 - m_0^2) = 0 .
\]
To be normalizable, the hypergeometric function in the radial part of the field has to break off, leading to the quantum condition
\[
\frac{l + s + k}{2} = -n , \quad (n \in \mathbb{N}) .
\]
Then, we get the dispersion relation for a free particle in the dS spacetime
\[
E^2 = m_0^2 + \epsilon'^2 + \lambda(2n + l + s)(2n + l + s + 2) .
\]
The dispersion relation (17) combined with the conservation laws forms a powerful and elegant means of treating the kinematics in the collision and decay processes in the spacetime with a positive cosmological constant.

We first consider the head-on collision between a soft photon of energy $\epsilon$, momentum $q$ and a high energy particle of energy $E_1$, momentum $p_1$, which leads to the production of two particles with energies $E_2, E_3$ and momenta $p_2, p_3$, respectively. From the laws of conservation of energy and momentum, we have
\[
E_1 + \epsilon = E_2 + E_3 ,
\]
\[
p_1 - q = p_2 + p_3 .
\]
In the C. M. frame, $m_2$ and $m_3$ are at rest at threshold, so that they have the same velocity in the lab frame. It’s easy to give the following relation

$$\frac{p_2}{p_3} = \frac{m_2}{m_3}. \tag{20}$$

For the process of the UHECR interacting with the CMBR photons,

$$p + \gamma \rightarrow p + \pi,$$

we obtain the threshold

$$E_{\text{UHECR}}^{\text{th}, \lambda} \simeq \frac{(m_N + m_{\pi})^2 - m_N^2 + \lambda^* \left(1 + \frac{m_N}{m_{\pi}} + \frac{m_{\pi}}{m_N}\right)}{2 \left(\epsilon + \sqrt{\epsilon^2 - \lambda^*}\right)}, \tag{21}$$

where $\lambda^* = \lambda(2n + l + s)(2n + l + s + 2)$, and we have used the dispersion relation (17) and popular approximated relations for relativistic particles

$$\epsilon^2 = q^2 + \lambda^* = q^2 + \lambda(2n + l + s)(2n + l + s + 2), \tag{22}$$

$$E_i = \sqrt{m_i^2 + p_i^2 + \lambda_i^*} \simeq p_i + \frac{m_i^2}{2p_i} + \frac{\lambda_i^*}{2p_i}, \quad (i = 1, 2, 3). \tag{23}$$

It should be noticed that the $\lambda$ dependent term in the expression of the threshold (21) can not be omitted in spite of the tiny value of the observed cosmological constant $\lambda (\approx 10^{-85}\text{GeV}^2)$. The reason is that the angular momentum, which appears in the dispersion relation of a free particle in dS spacetime, is a cosmological quantity. The distance between the particle and the coordinate origin is at the level of $\frac{1}{\sqrt{\lambda}}$.

In FIG.1, we give a plot of the dependence of the threshold $E_{\text{th}, \lambda}^{\text{UHECR}}$ on the value of the curvature ($\lambda$) of the dS spacetime.
FIG. 1  The cosmological constant \((3\lambda)\) dependence of the threshold \(E_{th,\lambda}^{UHECR}\) in the interaction process between the UHECR protons and the CMBR photons \((10^{-3}\text{eV})\).

From FIG. 1, we know clearly that a tiny but non-zero positive cosmological constant increases the threshold sharply in the photopion production process of the CMBR photons with the ultrahigh energy cosmic ray. For the observed cosmological constant, if the CMBR takes the quantum number \((2n + l)\) to be about \(10^{30}\), the energies of all the observed UHECR events are below the theoretical threshold and the threshold anomaly disappears.

In the interaction process between the TeV-\(\gamma\) ray and the IR background photons,

\[
\gamma + \gamma \rightarrow e^+ + e^- ,
\]

we have similar dispersion relations as

\[
e^2 = q^2 + \lambda^* = q^2 + \lambda(2n + l + s)(2n + l + s + 2) , \tag{24}
\]

\[
E_i = \sqrt{m_i^2 + p_i^2 + \lambda_i^*} \simeq p_i + \frac{m_i^2}{2p_i} + \frac{\lambda_i^*}{2p_i} , \quad (m_1 = 0) . \tag{25}
\]

The threshold we obtained is of the form

\[
E_{th,\lambda}^{\gamma} \simeq \frac{2m_e^2 + 3\lambda^*}{\epsilon + \sqrt{\epsilon^2 - \lambda^*}} . \tag{26}
\]

We present a plot of the curvature \((\lambda)\) dependence of the threshold \(E_{th,\lambda}^{\gamma}\) in FIG. 2.
From **FIG.2**, one can see that the threshold $E_{\gamma, \lambda}^{\gamma}$ is also very sensitive to the varying of the cosmological constant if the IR background also has a quantum number $(2n + l)$ about $10^{31}$. Similar to UHECR, we now have a new theoretical threshold of 40 TeV for this process and the threshold anomaly doesn’t confuse us any more.

In this Letter, we have investigated kinematics in the de Sitter spacetime and obtained a deformed dispersion relation for free particles. In particular, the CMBR and IR background interacting with extremely high energy cosmic rays have been presented in the framework. We noticed that the familiar GZK cutoff might be deviated from if the effects of the cosmological constant were taken into account. Therefore, the cosmological constant may be the origin of the threshold anomaly of the cosmic ray. Of course, the origin of the UHECR is one of the outstanding puzzles of modern astrophysics[30]–[32]. Today’s understanding of the phenomena responsible for the production of UHECR is still limited. Furthermore, it is well-known that to solve the so-called flatness problem, the horizon problem and magnetic monopoles in the standard big-bang theory, we should introduce the inflation models[33]. In fact, recent observations of WMAP show strong evidence for inflation[34]. After all of these factors
have been dealt with carefully, a more reliable scenario of the threshold anomaly of the cosmic ray can be obtained.

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