Simple implementation of discrete quantum Fourier transform via cavity quantum electrodynamics

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Abstract. We propose a scheme for implementing discrete quantum Fourier transform in cavity quantum electrodynamics (QED). The experimental implementation is appealingly simple because the combined effect of the complex controlled-$R_k$ gate and SWAP gate operations required for implementing discrete quantum Fourier transform in the naive quantum circuit is replaced by the one-step CR$_k$S gate, which can be directly implemented via atom–cavity interaction with the assistance of classical fields. We propose the detailed experimental procedure and analyze the experimental feasibility. The experimental implementation of the scheme would show the full power of quantum algorithm and would open wide prospects for more complicated quantum computation with atoms in QED.

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1. Introduction

The existence of quantum algorithms for specific problems shows that quantum computers can in principle provide tremendous speedup compared to classical computers by using quantum parallelism and the interference effect: such as the factoring problem \([1]\), search problem \([2]\), counting solution problem \([3]\), phase estimation problem \([4]\), hidden subgroup problem \([5, 6]\), etc. Discrete quantum Fourier transform, which is a linear unitary transform and is one of the most important computational problems, is the key ingredient for quantum factoring and many other interesting quantum algorithms. Many real-world applications require that the transform be performed as quickly as possible.

For a given state \(|\phi\rangle = \sum_{j=0}^{N-1} \alpha_j |j\rangle\), the discrete quantum Fourier transform on the state \(|\phi\rangle\) maps it into

\[
F|\phi\rangle = \sum_{k=0}^{N-1} \beta_k |k\rangle,
\]

where

\[
\beta_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} \alpha_j e^{2\pi i j k / N}.
\]

Figure 1 gives the quantum circuit for implementing the discrete quantum Fourier transform, where

\[
R_k = \begin{pmatrix} 1 & 0 \\ 0 & e^{i2\pi/2^k} \end{pmatrix}.
\]

The two-qubit controlled-\(R_k\) gate operation in figure 1 produces a phase \(i2\pi/2^k\) if and only if both qubits in the input states are 1. The implementation of such a controlled-\(R_k\) gate requires two two-qubit controlled-NOT (CNOT) gates and three one-qubit rotation gates, and the implementation of a SWAP gate requires three two-qubit CNOT gates, or can be decomposed into six Hadamard gates \((H\) transformation) and three controlled phase flip (CPF) gates, which corresponds to at least 21 pulses in this system. The quantum circuit decomposition
of the controlled-$R_k$ gate and SWAP gate is shown in figure 2, with

$$U_A = \begin{pmatrix} e^{-i\pi/2^{k+2}} & 0 \\ 0 & e^{i\pi/2^{k+2}} \end{pmatrix} = e^{-i(\pi/2^{k+2})\sigma_z},$$

$$U_B = \begin{pmatrix} e^{i\pi/2^{k+2}} & 0 \\ 0 & e^{-i\pi/2^{k+2}} \end{pmatrix} = e^{i(\pi/2^{k+2})\sigma_z},$$

$$U_C = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/2^{k+1}} \end{pmatrix} = e^{i\pi/2^{k+2}} e^{-i(\pi/2^{k+2})\sigma_z}. \quad (4)$$

Until now, only a few physical schemes have been proposed to implement discrete quantum Fourier transform based on cavity quantum electrodynamics (QED) systems [7]–[9], the nuclear magnetic resonance (NMR) system [10] and the optical system [11, 12]. Therefore, how to efficiently implement discrete quantum Fourier transform, especially for a large number of qubits, is still a challenge in real experiment.

In this paper, we propose a simple cavity QED scheme for implementing discrete quantum Fourier transform in a direct way. In the scheme, we design a new rearranged quantum circuit for implementing discrete quantum Fourier transform based on the CR$_k$S gate generated from the atom–cavity interaction associated with the classical field. The CR$_k$S gate is a combination of the controlled-$R_k$ gate and SWAP gate operations and can be directly implemented in one step by choosing atom–cavity interaction time and interaction parameters appropriately. The scheme has the following significant advantages: (i) the gate operations are greatly reduced due to the combinative effect of the CR$_k$S gate; (ii) the scheme is insensitive to cavity decay and there is no transformation of quantum information between the atomic systems and cavity fields because the cavity fields are always in the vacuum state during the whole process of implementing discrete quantum Fourier transform; (iii) it does not need to detect the states of the cavity fields and a readout process of the discrete quantum Fourier transform is unnecessary because the resulting state of the atoms is the discrete quantum Fourier transform of the input state; (iv) cavity QED not only provides a natural setting for distributed quantum information processing, but also provides good insulation against the environment. Meanwhile, atoms are
Figure 2. (a) Decomposition of the controlled-$R_k$ gate from CNOT gates and one-qubit rotation gates on the two qubits. $U_\alpha$ $(\alpha = A, B, C)$ are the one-qubit rotation gates, denoted by equation (4). (b) Decomposition of the SWAP gate from CNOT gates or CPF gates and $H$ gates on the two qubits. $H$ denotes Hadamard gate transformation and $Z$ is normal Pauli-$z$ operator.

not only the source of local entanglement, but also well suited for storing quantum information in long-lived internal states.

2. The fundamental model

We consider two identical three-level atoms, each having two low levels $|0\rangle$, $|1\rangle$, and a high level $|2\rangle$, simultaneously interacting with a single-mode cavity field, as shown in figure 3. The cavity mode is coupled to the $|1\rangle \leftrightarrow |2\rangle$ transition, and the two classical pulses are coupled to the $|0\rangle \leftrightarrow |2\rangle$ and $|1\rangle \leftrightarrow |2\rangle$ transitions, respectively. The interaction Hamiltonian of the system in the interaction picture is

$$H_I = \sum_{j=1}^{2} \left( g_j e^{i\Delta_j t} a |2\rangle_j \langle 1| + \Omega_{j1} e^{i\Delta_{j1} t} |2\rangle_j \langle 1| + \Omega_{j2} e^{i\Delta_{j2} t} |2\rangle_j \langle 0| \right) + \text{H.c.}, \quad (5)$$

where $a$ is the annihilation operator for the cavity mode, $\Delta_i$ $(i = 1, 2, 3)$ denote the detunings for the cavity mode and classical pulses from the respective atomic transitions, $g_j$ denotes the coupling strength of the $j$th atom to the cavity mode, and $\Omega_{j1}$ and $\Omega_{j2}$ are the Rabi frequencies of the classical pulses for the $j$th atom. For simplicity, we assume that $g_j$, $\Omega_{j1}$ and $\Omega_{j2}$ are real. In the case of $\Delta_1 \gg g_j$, $\Delta_2 \gg \Omega_{j1}$ and $\Delta_3 \gg \Omega_{j2}$, the high level $|2\rangle_j$ can be eliminated.
adiabatically to obtain the following Hamiltonian [13]–[15]:

\[ H'_j = \sum_{j=1}^{2} \left[ \frac{g_j^2}{\Delta_1} a \right]_{j} \langle 1 \rangle_{j} \langle 1 \rangle_{j} + \frac{\Omega_{j1}^2}{\Delta_2} \langle 1 \rangle_{j} \langle 1 \rangle_{j} + \frac{\Omega_{j2}^2}{\Delta_3} \langle 0 \rangle_{j} \langle 0 \rangle_{j} + \lambda_{j1} (a S_j^+ e^{-i\delta_1 t} + a^+ S_j^- e^{i\delta_1 t}) 
+ \lambda_{j2} (S_j^+ e^{-i\delta_2 t} + S_j^- e^{i\delta_2 t}) + \lambda_{j3} (a e^{-i\delta_3 t} + a^+ e^{i\delta_3 t}) \langle 1 \rangle_{j} \langle 1 \rangle_{j} \right], \]

(6)

where \( \delta_1 = \Delta_3 - \Delta_1 \), \( \delta_2 = \Delta_2 - \Delta_3 \), \( \delta_3 = \Delta_2 - \Delta_1 \), \( \lambda_{j1} = g_j \Omega_{j2} (1/\Delta_1 + 1/\Delta_2 + 1/\Delta_3) / 2 \), \( \lambda_{j2} = \Omega_{j1} \Omega_{j2} (1/\Delta_1 + 1/\Delta_3 + 1/\Delta_2) / 2 \), \( \lambda_{j3} = g_j \Omega_{j1} (1/\Delta_1 + 1/\Delta_2 + 1/\Delta_3) / 2 \), \( S_j^+ \rangle \|= \langle 0 \rangle_{j} \langle 1 \rangle_{j} \rangle\) and \( S_j^- \rangle \|= \langle 1 \rangle_{j} \langle 0 \rangle_{j} \rangle\); the first three terms in equation (6) are the Stark shifts for the levels \( |0\rangle_j \rangle \) and \( |1\rangle_j \rangle \); the fifth term denotes the movement of the cavity mode and the classical pulse, respectively; the fourth term is the Jaynes–Cummings interaction, describing the Raman coupling of the two states \( |0\rangle_j \rangle \) and \( |1\rangle_j \rangle \); the last term is the coupling between the cavity mode and the classical field assisted by the atoms.

In the case of \( \delta_i \gg g_j^2 / \Delta_i \), \( \Delta_1^2 / \Delta_2 \), \( \Delta_2^2 / \Delta_3 \), and \( \lambda_{jw} (i, \alpha \in \{1, 2, 3\}) \), there is no energy exchange between the atomic system and the cavity mode, and the photon in the cavity is only virtually excited and the two atoms interfere with each other. The energy-conserving transitions are between \( |0, 1, n\rangle \rangle \) and \( |1, 0, n \rangle \rangle \), mediated by \( |1, 1, n \rangle \rangle \), \( |1, 1, n + 1 \rangle \rangle \), \( |0, 0, n \rangle \rangle \) and \( |0, 0, n - 1 \rangle \rangle \) [16, 17], where \( n \rangle \rangle \) is the photon number of the cavity mode. For the sake of convenience, we set \( g_j = g \), \( \Omega_{j1} = \Omega_1 \), \( \Omega_{j2} = \Omega_2 \); thus we have \( \lambda_{j1} = \lambda_1 = g \Omega_2 (1/\Delta_1 + 1/\Delta_3) / 2 \), \( \lambda_{j2} = \lambda_2 = \Omega_1 \Omega_2 (1/\Delta_1 + 1/\Delta_3) / 2 \), and \( \lambda_{j3} = \lambda_3 = g \Omega_1 (1/\Delta_1 + 1/\Delta_2) / 2 \). Then the effective Hamiltonian is given by

\[ H_{\text{eff}} = \sum_{j=1}^{2} \left( \frac{g_j^2}{\Delta_1} a \right]_{j} \langle 1 \rangle_{j} \langle 1 \rangle_{j} + \frac{\Omega_{j1}^2}{\Delta_2} \langle 1 \rangle_{j} \langle 1 \rangle_{j} + \frac{\Omega_{j2}^2}{\Delta_3} \langle 0 \rangle_{j} \langle 0 \rangle_{j} + \frac{\lambda_1^2}{\delta_1} a^+ a \langle 0 \rangle_{j} \langle 0 \rangle_{j} + \frac{\lambda_2^2}{\delta_2} (S_1^+ S_2^- + S_1^- S_2^+) + \frac{\lambda_3^2}{\delta_3} \langle 1 \rangle_{j} \langle 1 \rangle_{j} \langle 1 \rangle_{j} \langle 1 \rangle_{j} \right] \]

(7)

The last term in equation (7) describes the coupling between atoms 1 and 2 mediated by the cavity mode and the classical pulse. If the cavity field is initially in the vacuum state, the

Figure 3. Configuration of the atomic level structure and relevant transitions.
the choice of $\gamma$ based on the CR

\begin{align*}
H'_{e\text{ff}} &= \sum_{j=1}^{2} \left( \frac{\Omega_2^2}{\Delta_2} |1⟩⟨1| + \frac{\gamma^2}{\delta_3} |1⟩⟨1| + \frac{\Omega_3^2}{\Delta_3} |0⟩⟨0| + \frac{\gamma^2}{\delta_1} |0⟩⟨0| \right) \\
&\quad + \left( \frac{\gamma^2}{\delta_1} + \frac{2\gamma^2}{\delta_2} \right) \left( S_1^+ S_2^- + S_1^- S_2^+ \right) + \frac{2\gamma^2}{\delta_3} |1⟩⟨1| \otimes |1⟩⟨1|.
\end{align*}

(8)

3. Quantum circuit and experimental realization of discrete quantum Fourier transform based on the CR$_k$S gate

The time evolution operator, governed by Hamiltonian (8), in the subspace spanned by $|00⟩_{12}$, $|01⟩_{12}$, $|10⟩_{12}$ and $|11⟩_{12}$, is given by

\begin{equation}
U(t) = e^{-iH'_{e\text{ff}}t} = \begin{pmatrix}
e^{-i2\alpha t} & 0 & 0 & 0 \\
0 & e^{-i(\alpha+\beta)t} \cos \gamma t & -ie^{-i(\alpha+\beta)t} \sin \gamma t & 0 \\
0 & -ie^{-i(\alpha+\beta)t} \sin \gamma t & e^{-i(\alpha+\beta)t} \cos \gamma t & 0 \\
0 & 0 & 0 & e^{-i(2\beta+\phi)t}
\end{pmatrix},
\end{equation}

(9)

where $\alpha = \frac{\Omega_2^2}{\Delta_3} + \frac{\gamma^2}{\delta_3}$, $\beta = \frac{\Omega_2^2}{\Delta_2} + \frac{\gamma^2}{\delta_3}$, $\gamma = \frac{\gamma^2}{\delta_1} + \frac{2\gamma^2}{\delta_2}$ and $\phi = \frac{2\gamma^2}{\delta_3}$. After the performances of the one-qubit rotation operations $R_α^j(t)$, $R_β^j(t)$ and $R_{\pi/2}^j$ ($j = 1, 2$), we have

\begin{equation}
\bigotimes_{j=1}^{2} R_α^j(t) R_β^j(t) R_{\pi/2}^j U(t) = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \gamma t & \sin \gamma t & 0 \\
0 & \sin \gamma t & \cos \gamma t & 0 \\
0 & 0 & 0 & e^{i(\pi-\phi)t}
\end{pmatrix},
\end{equation}

(10)

with

\begin{equation}
R_α^j(t) = \begin{pmatrix}
e^{i\alpha t} & 0 \\
0 & 1
\end{pmatrix}, \quad R_β^j(t) = \begin{pmatrix}
1 & 0 \\
0 & e^{i\beta t}
\end{pmatrix}, \quad R_{\pi/2}^j = \begin{pmatrix}
1 & 0 \\
0 & e^{i\pi/2}
\end{pmatrix},
\end{equation}

(11)

which can be easily realized with the classical microwave pulse. Here the superscripts $j$ $(j = 1, 2)$ denote the one-qubit operation performed on the $j$th qubit. In equation (10), with the choice of $\gamma t = \pi/2$, we obtain a two-qubit CR$_k$S gate, defined as

\begin{equation}
U_{\text{CR}_kS} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & e^{2\pi i k^2}
\end{pmatrix},
\end{equation}

(12)

where $k = \log_2(\frac{4\gamma}{\pi-\phi})$. The CR$_k$S gate is a combination of a controlled-$R_k$ gate and a SWAP gate, taking the form

\begin{equation}
U_{\text{CR}_kS} = U_{\text{CR}_k} U_{\text{SWAP}} = U_{\text{SWAP}} U_{\text{CR}_k}.
\end{equation}

(13)

Based on the CR$_k$S gate indicated in equation (12), we design a rearranged quantum circuit for implementing discrete quantum Fourier transform, as shown in figure 4.

In the following, we illustrate how to experimentally implement the quantum circuit shown in figure 4. The schematic setup for implementing multi-bit discrete quantum Fourier transform
Figure 4. The new rearranged quantum circuit for implementing discrete quantum Fourier transform based on the CR$_k$S gate.

Figure 5. Schematic setup for implementing multi-bit discrete quantum Fourier transform. All the cavities are initially prepared in the vacuum state, and $R_i$ denote the Ramsey zones, in which classical fields rotate the atoms along the $z$-axis, resulting in that one-qubit gate operations denoted by equation (11) are achieved. The atoms go across the series of classical fields and cavity fields from left to right with the same velocity $v$ but at different positions.
are prepared in an arbitrary superposition state

$$a_0|0\rangle_1|0\rangle_2 \cdots |0\rangle_n + a_1|0\rangle_1|1\rangle_2 \cdots |0\rangle_n + \cdots + a_{2^n - 1}|1\rangle_1|1\rangle_2 \cdots |1\rangle_n,$$

where \(\sum_{i=0}^{2^n-1} |a_i|^2 = 1\). Then the \(n\) atoms go through the series of Ramsey zones and cavities with identical velocities from left to right. Each time only two nearest-neighbor atoms interact with the cavity field with the interaction time of \(\tau = \pi/2\gamma\). After the one-qubit rotation operations performed by the classical fields, the CR\(_k\)S (\(k = 2, 3, \ldots, n\)) gate operations are achieved. Here by setting the parameters \(\gamma\) and \(\phi\) appropriately in each of the cavities in advance, i.e. \(\frac{4\gamma}{2\gamma - \phi} = 2^2, 2^3, \ldots, 2^n\), we can get different phase shifts to the state \(|1\rangle_j|1\rangle_{j+1}\). In this way the multi-bit discrete quantum Fourier transform is achieved after the atoms pass through all the cavity fields and classical fields shown in figure 5.

### 4. Discussion and conclusion

We now briefly discuss some issues in relation to the experimental realization of our scheme under practical conditions and give a conclusion. Firstly, to implement the discrete quantum Fourier transform, we need to employ several different one-qubit gate operations. Therefore, the Ramsey zones should be long enough to finish all the required consecutive one-qubit gate operations. Secondly, our scheme requires two atoms simultaneously interacting with a cavity; otherwise an error will be introduced. This is still a challenge for us to simultaneously send two Rydberg atoms through a cavity with precise tracks and velocities in experiment. Therefore, it is necessary for us to consider the effect of the deviation of the operation time caused by positioning the atoms at a precise location in cavity mode. Assume that the actual time atoms 1 and 2 stay in the cavity is

$$t = \pi (1 + \epsilon)/2\lambda,$$

Figure 6. The fidelity of the implementation of discrete quantum Fourier transform by using the one-step CR\(_k\)S gate. \(\epsilon\) is the amount deviation and \(n\) is the number of qubits.
that all the conditions mentioned above can be satisfied very well. Therefore, the needs
quantum Fourier transform; thus the cavity decay can be sufficiently suppressed.
cavity-photon population involved during the whole process of the implementation of discrete
scheme simplifies the experimental implementation greatly. Fifthly, in our scheme, there is no
gate, resulting in that only \( n \) gates (here we have assumed that the controlled-
\( m \) numbers 50 and 51 interacting with high-
DQFT. When \( \epsilon = 0.01 \) and \( n = 50 \), \( F_{\text{DQFT}} \approx 0.85 \). Even when \( n = 100 \), the fidelity
still exceeds 0.5. Therefore, the fidelity of our scheme is considerably high. Thirdly,
to implement the different conditional phase shifts, we need to set the parameters satisfying
\( \frac{4\gamma}{2\gamma - \phi} = 2^m \) \((m = 2, 3, \ldots, n)\). Experimentally, for Rydberg atoms with principal quantum
numbers 50 and 51 interacting with high-Q microwave cavities, \( g/2\pi \sim 50 \text{kHz} \) [19]–[21]. For
simplicity, we set \( \Delta_3 > \Delta_2 > \Delta_1 \), obtaining \( \delta_1 > \delta_2 > \delta_3 \). With the choices of \( \Omega_1 = \Omega_2 = g \),
\( \Delta_1 = 10g \), \( \Delta_2 = 11g \) \((the value of \( \Delta_3 \) will be determined later\)), we have \( \delta_3 = \Delta_2 - \Delta_1 = g \sim 10g/\Delta_1 , 11\Omega_2/\Delta_2, 10.5\lambda_3 \), which satisfy the conditions \( \Delta_1 \gg g \), \( \Delta_2 \gg \Omega_1 \), \( \Delta_3 \gg \Omega_2 \) and
\( \delta_i \gg \frac{\Omega_i^2}{\Delta_i}, \frac{\Omega_i^2}{\Delta_i}, \lambda_i \) \((i = 1, 2, 3)\). According to the relation \( \frac{4\gamma}{2\gamma - \phi} = 2^m \), we numerically calculate
the values of \( \Delta_3 \) when \( m \) is chosen to different values, from 2 to 20, as shown in table 1. We can see
from table 1 that all the conditions mentioned above can be satisfied very well. Therefore, the
different phases required for implementing discrete quantum Fourier transform can be obtained
by choosing the experimental parameters appropriately. Fourthly, the gate operations in our
scheme are greatly reduced in contrast to the usual method. Generally, the implementation of the
quantum circuit shown in figure 1 needs \( n(n-1)/2 \) controlled-\( R_k \) gate, \( n \) Hadamard gates and
\( n/2 \) SWAP gates. Therefore, to achieve the discrete quantum Fourier transform, it needs at least
\( n(n+2)/2 \) gates \( (\text{here we have assumed that the controlled-} R_k \text{\ gates and SWAP gates could be}
directly implemented without performing any auxiliary one-qubit or two-qubit gate operations})
if using the conventional gate-decomposition method shown in figure 2, it will need at least
\( n(9n+2)/2 \) gates. In our scheme, we need \( n \) Hadamard gates and \( n(n-1)/2 \) CR\(_k\)S gates, and
each CR\(_k\)S gate represented by equation (10) consists of six one-qubit gates and one two-qubit
gate, resulting in that only \( n(7n-5)/2 \) gates are required, in all, in our scheme. Therefore, our
scheme simplifies the experimental implementation greatly. Fifthly, in our scheme, there is no
cavity-photon population involved during the whole process of the implementation of discrete
quantum Fourier transform; thus the cavity decay can be sufficiently suppressed.

Table 1. The values that the \( \Delta_3 \) can hold corresponding to different values of \( m \)
when setting \( \Omega_1 = \Omega_2 = g \), \( \Delta_1 = 10g \), \( \Delta_2 = 11g \).

| \( m \) | \( \Delta_3 \) | \( m \) | \( \Delta_3 \) | \( m \) | \( \Delta_3 \) | \( m \) | \( \Delta_3 \) |
|-------|--------|-------|--------|-------|--------|-------|--------|
| 2     | 12.065 386 7755 g | 9     | 13.104 355 6001g | 16    | 13.112 104 0215g |
| 3     | 12.600 657 7521 g | 10    | 13.108 261 0125g | 17    | 13.112 134 5161g |
| 4     | 12.859 416 2966 g | 11    | 13.110 213 1883g | 18    | 13.112 149 7634g |
| 5     | 12.986 530 4135 g | 12    | 13.111 189 1436g | 19    | 13.112 157 3870g |
| 6     | 13.049 530 7334 g | 13    | 13.111 677 0882g | 20    | 13.112 161 1988g |
| 7     | 13.080 893 3663 g | 14    | 13.111 921 0522g |       |                   |
| 8     | 13.096 540 5283 g | 15    | 13.112 043 0321g |       |                   |

where \( \epsilon \) is the amount of deviation. We can calculate the fidelity of the discrete quantum
Fourier transform according to the following relation [18]:

\[
F_{\text{DQFT}} = \overline{\langle \Psi_0 | U_{\text{DQFT}}^\dagger \rho_i U_{\text{DQFT}} | \Psi_0 \rangle},
\]

where the overline indicates average over all possible input states \( |\Psi_0\rangle \), \( U_{\text{DQFT}} \) is the ideal
discrete quantum Fourier transform operation, and \( \rho_i = |\Psi_i\rangle \langle \Psi_i| \), with \( |\Psi_i\rangle \) being the final state
after the discrete quantum Fourier transform in our scheme. We drew the three-dimensional
plot for the fidelity \( F_{\text{DQFT}} \) as a function of deviation amount \( \epsilon \) and the number of qubits \( n \), as
shown in figure 6. When \( \epsilon = 0.01 \) and \( n = 50 \), \( F_{\text{DQFT}} \approx 0.85 \). Even when \( n = 100 \), the fidelity
\( F_{\text{DQFT}} \) still exceeds 0.5. Therefore, the fidelity of our scheme is considerably high. Thirdly,
to implement the different conditional phase shifts, we need to set the parameters satisfying
\( \frac{4\gamma}{2\gamma - \phi} = 2^m \) \((m = 2, 3, \ldots, n)\). Experimentally, for Rydberg atoms with principal quantum
numbers 50 and 51 interacting with high-Q microwave cavities, \( g/2\pi \sim 50 \text{kHz} \) [19]–[21]. For
simplicity, we set \( \Delta_3 > \Delta_2 > \Delta_1 \), obtaining \( \delta_1 > \delta_2 > \delta_3 \). With the choices of \( \Omega_1 = \Omega_2 = g \),
\( \Delta_1 = 10g \), \( \Delta_2 = 11g \) \((the value of \( \Delta_3 \) will be determined later\)), we have \( \delta_3 = \Delta_2 - \Delta_1 = g \sim 10g/\Delta_1 , 11\Omega_2/\Delta_2, 10.5\lambda_3 \), which satisfy the conditions \( \Delta_1 \gg g \), \( \Delta_2 \gg \Omega_1 \), \( \Delta_3 \gg \Omega_2 \) and
\( \delta_i \gg \frac{\Omega_i^2}{\Delta_i}, \frac{\Omega_i^2}{\Delta_i}, \lambda_i \) \((i = 1, 2, 3)\). According to the relation \( \frac{4\gamma}{2\gamma - \phi} = 2^m \), we numerically calculate
the values of \( \Delta_3 \) when \( m \) is chosen to different values, from 2 to 20, as shown in table 1. We can see
from table 1 that all the conditions mentioned above can be satisfied very well. Therefore, the
different phases required for implementing discrete quantum Fourier transform can be obtained
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scheme are greatly reduced in contrast to the usual method. Generally, the implementation of the
quantum circuit shown in figure 1 needs \( n(n-1)/2 \) controlled-\( R_k \) gate, \( n \) Hadamard gates and
\( n/2 \) SWAP gates. Therefore, to achieve the discrete quantum Fourier transform, it needs at least
\( n(n+2)/2 \) gates \( (\text{here we have assumed that the controlled-} R_k \text{\ gates and SWAP gates could be}
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quantum Fourier transform; thus the cavity decay can be sufficiently suppressed.

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Next we make a brief comparison of the present scheme with the previously proposed cavity QED schemes in [7]–[9]. In comparison with [7], the present scheme has the following merits: (i) qubit definitions are the same for the atoms, which makes the experiment easier; (ii) a readout process for the discrete quantum Fourier transform is unnecessary because the resulting state of the atoms is the discrete quantum Fourier transform of the input state; and (iii) it is unnecessary to use the photon as the data bus because the photon in the cavity is only virtually excited in the whole process of the implementation of discrete quantum Fourier transform. Compared with [8, 9], (i) the quantum circuit for implementing the discrete quantum Fourier transform in the present scheme is greatly reduced by using the one-step CR$_{s}$S gate; and (ii) fewer cavities and classical fields are used, which reduces the quantum resources and makes the present scheme simpler and more feasible in real experiment.

In conclusion, we have proposed a simple scheme to implement discrete quantum Fourier transform via cavity QED system. The scheme did not use the cavity mode as the data bus, and the resulting state of the atoms was the discrete quantum Fourier transform of the input state. The atomic states $|0\rangle$, $|1\rangle$, and $|2\rangle$ could be represented by using the hyperfine levels $(6S_{1/2}, F = 3, m = +3)$, $(6S_{1/2}, F = 4, m = +4)$, and $(6P_{3/2}, F = 4, m = +4)$, respectively. Many physical systems with the Λ-type three-level configuration, such as ion traps, superconducting quantum-interference devices (SQUIDs) and quantum dots, can also be used to implement the scheme within cavity QED. The implementation of the scheme in the experiment would promote the development of a quantum computer with cavity QED technology.

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