Performance analysis of dual-function multiple-input multiple-output radar-communications using frequency hopping waveforms and phase shift keying signalling

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Abstract
A signal embedding scheme into the emission of multiple-input multiple-output (MIMO) frequency hopping (FH) radar is analysed. In this scheme, the phase shift keying (PSK) communication symbols are multiplied with the frequency hops in fast-time, resulting in dual-function radar-communication (DFRC) waveforms. The authors examine the impact of PSK symbol embedding on the ambiguity function (AF) and power spectral density (PSD) and show that modulation of the radar pulse can benefit the radar functionality. The effect of PSK symbol embedding on the radar target detection performance of the DFRC system is delineated. The authors’ analyses show that the use of the DFRC waveforms enhances target detection performance. The authors also provide the symbol error rates (SERs) corresponding to different phase constellations at the communication receiver and address the practical data transfer rates that can be achieved with the underlying embedding scheme.

1 | INTRODUCTION

Radar and communication systems, which have been for long designed and developed independently, are now emerging as compatible or a unified system. This development has been, in major part, a response to the congestion of the radio frequency (RF) spectrum caused by various applications such as wearable devices, Internet of things (IOT), vehicular communications [1–3]. The two radar and communication systems can share the spectrum following two different approaches, namely, co-habitation and co-design [4–8]. In the co-habitation approach, radar and communications co-exist as separate systems, adapting and responding to each other’s existence. The co-design approach, in principle, avoids any frequency contentions by signal design, dynamic frequency allocation, or through a dual functional system approach. In the latter, both services are launched from the same system, sharing platform resources such as waveform, power, bandwidth, and aperture [9–15]. Radar operations, however, should not be compromised when sharing resources with communications under the auspices of dual-function radar-communication (DFRC) system [16–25]. On the other hand, communications should strive to benefit from the DFRC platform, specifically fine quality hardware and high transmitting power.

DFRC systems can implement different embedding strategies, including the use of time modulated arrays [16], sidelobe control [26,27], casting the radar waveforms as communication symbols [24,28], code diversity [29–31] and signal phase modulations [20,32–34]. In time-modulated array DFRC, phases of the transmit array are modified on a pulse-to-pulse basis to introduce variations in the sidelobe levels while leaving the mainlobe unaltered. Although the waveform remains the same from one pulse to another, the optimization criteria to design multiple transmit power distribution patterns with the same mainlobe are quite complex. In the sidelobe embedding DFRC systems, communications take place only in the side-lobes region and hence cannot proceed or continue when the mainlobe is steered towards the communication receiver. Code diversity defines the code-shift keying embedding strategy and typically suffers from low data rates [29,30]. Information embedding associated with multiple-input multiple-output (MIMO) radar has been considered in [35–37]. It is noted that alternative names used in the literature for the DFRC system are ‘Intentional Modulation on a Pulse’ and ‘Co-Radar’ [38].

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Towards enabling communications in the mainlobe and to achieve high data rates, we consider a phase modulation symbol embedding strategy for data transmission. The platform considered is frequency hopping (FH) MIMO radar [20]. FH waveforms have constant modulus and are easy to generate. These waveforms are also immune to interference and meet the MIMO radar requirements [21,39]. In the embedding scheme considered, phase shift keying (PSK) communication symbols are multiplied with the FH waveforms in fast time. A single antenna communication receiver, which is phase and time synchronized with the DFRC transmitter, is assumed. The PSK symbols are demodulated by applying matched filter (MF) to the FH waveforms and estimating the phase of the resulting MF outputs.

We examine the impact of embedding PSK communication symbols on the MIMO radar functionality by analysing the ambiguity function (AF) characteristics and the power spectral density (PSD) pre- and post-signal embedding. For a single function FH-based MIMO radar system, large AF sidelobe levels (SLLs) can be exhibited due to re-using FH coefficients within the same pulse [32]. This would, in turn, limit the number of orthogonal FH waveforms that can be synthesized. We show that, in the dual function system counterpart, a new paradigm regarding re-use of the FH coefficient emerges. Unlike other DFRC schemes discussed in [13,24], FH code re-use has the advantage of increasing the number of FH waveforms that can be synthesized, thus, enabling high data rate communications to be achieved. From the radar perspective, the SLLs remain low since the randomness of the PSK symbols breaks coherence between two similar hops.

From the spectrum perspective, an important issue in DFRC system is the need to adhere to the radar specified frequency bandwidth. Owing to signal embedding, the DFRC system waveforms exhibit phase rotations during each hop. This has the adverse effect of disrupting the continuous phase of the FH waveforms, resulting in spectral leakage into adjacent frequency bands. In this regard, different phase constellations would provide different trade-offs between the range sidelobe levels (RSLs) and spectral sidelobe levels (SSLs).

In our preliminary work in [32,33], initial results of the AF and PSD were discussed. In this article, detailed and complete analysis with the proof of concept is presented. The paper contributions are summarized as follows:

1. We derive the DFRC AF expression and provide proof that information embedding reduces AF SLLs.
2. We analyse the radar receiver under DFRC to assess the respective changes in radar target detection performance.
3. We analyse the communication system performance for different phase constellations in terms of symbol error rates (SERs).
4. We provide expressions for the achievable data rate with and without FH code re-use and show the data rate gain of the FH-DFRC system as compared to existing techniques.

The rest of the article is organized as follows. Details of the DFRC system design, the FH waveforms, PSK symbol embedding, bandwidth requirements and the MIMO radar signal model are presented in Section 2. The AF, PSD, and data rate of the proposed waveform design are analysed in Section 3. Radar detection performance is discussed in detail in Section 4. Simulation results are given in Section 5, and conclusions are drawn in Section 6.

2 | MIMO RADAR SIGNAL MODEL

We consider a dual-function system consisting of $M_T$ omni-directional co-located transmit antennas. The MIMO radar receive array comprises $N_R$ antennas arranged in a linear shape. For transmission through the MIMO platform, the radar waveforms $\phi_m(t)$, $m = 1\ldots M_T$, should satisfy the orthogonality condition

$$\int_{T_p} \phi_m(t)\phi_{m'}^*(t + \tau) e^{2\pi \nu t} dt = \begin{cases} \delta(\tau)\delta(\nu), & m = m', \\ 0, & \text{otherwise}, \end{cases}$$

(1)

where $t$ is the fast-time index, $T_p$ is the pulse duration, $(\cdot)^*$ denotes the conjugate of a complex number, $\tau$ and $\nu$ denote time delay and the Doppler shift, respectively and $\delta(\cdot)$ is the Kronecker delta function. Orthogonal waveforms obeying Equation (1) are difficult to realize. However, waveforms with low auto- and cross-correlation SLLs can be efficiently synthesized in practice (see [35]; and references therein).

2.1 Frequency-hopping waveforms

FH waveforms meet the MIMO radar requirements, like high transmit power efficiency, high range and Doppler resolution properties. The use of FH waveforms for MIMO radar is reported in a number of articles [21,39]. The FH waveform, transmitted from the m-th antenna during n-th pulse duration, can be expressed as

$$\phi_m(t; n) = \sum_{q=1}^{Q} e^{2\pi n\nu q \Delta_f t} \text{rect}(t - q \Delta_f - n T_0),$$

(2)

where $\nu_{mq}$, $m = 1, \ldots, M_T$, $q = 1, \ldots, Q$ denotes the FH coefficients, $n = 1, \ldots, N_p$ is the pulse number, $T_0$ is the pulse repetition interval (PRI), $Q$ is the number of sub-pulses derived from $K$ available frequencies ($K \geq Q$), $\Delta_f$ and $\Delta_r$ are the frequency step size and the sub-pulse duration, respectively, and

$$\text{rect}(t) \triangleq \begin{cases} 1, & 0 < t < \Delta_r, \\ 0, & \text{otherwise}, \end{cases}$$

(3)

is a rectangular pulse of duration $\Delta_r$. From Equation (2), the pulse duration can be inferred as $T_p = Q \Delta_r$. It is also assumed
that \( \Delta \Delta f \) is an integer. Selection of FH code matrix is important for designing the FH waveforms and was investigated in \([32, 39]\).

The frequency hopping pattern during a PRI can be observed from the associated time-frequency plot, such as the one shown in Figure 1. It is important to note that in the FH scheme considered, the FH code remains the same from one PRI to another within the coherent processing interval (CPI).

## 2.2 | PSK symbol embedding

This section provides a brief overview of PSK symbol embedding into MIMO radar emission using FH waveforms. Let \( \{ \Omega^{(m)}_{(m,q)} \in D_{PSK}, m = 1, \ldots, M_f, q = 1, \ldots, Q \} \) be a set of PSK symbols to be embedded into the MIMO radar emission during the \( n \)-th pulse, where the PSK dictionary of size \( J \) is defined as \( D_{PSK} = \{ 0, \frac{2\pi}{J}, \ldots, \frac{2\pi (J-1)}{J} \} \). The PSK symbols are positioned on a uniform grid between 0 and \( 2\pi \). Each PSK symbol comprises \( N_b = \log_2 J \) bits. The PSK-modulated FH radar waveforms can be expressed as,

\[
\psi_{m}(t; n) = \sum_{q=1}^{Q} e^{j2\pi n q} h_{m,q}(t) \text{rect}(t - q\Delta t - nT_0), \quad (4)
\]

where \( h_{m,q}(t) \triangleq e^{j2\pi \theta_{m,q} t} \) is the FH signal associated with the \( m \)-th antenna during the \( q \)-th sub-pulse.

It is worth noting that the FH waveforms (Equation 2) and the PSK-modulated FH waveforms in Equation (4) have the same structure and the exact FH code. The only difference between Equations (2) and (4) is that the \( q \)-th sub-pulse of the \( m \)-th waveform is phase rotated by \( \Omega^{(m,q)} \). Such rotation can be clearly seen in the real and imaginary parts of the complex FH waveforms with and without PSK symbol embedding as depicted in Figures 2 and 3. The impact of this PSK symbol embedding on the FH waveform properties is analysed in Section 3.

## 2.3 | Bandwidth requirements

Let \( BW \) be the bandwidth assigned to the DFRC system. The FH code, \( \Omega_{(m,q)} \), should be selected from the set of integers \( \{1, \ldots, K\} \). We assume that the value of \( K \) is properly selected such that the condition \( BW_{\text{eff}} \leq BW \) is satisfied, where

\[
BW_{\text{eff}} \approx (K - 1)\Delta f + \frac{1}{\Delta f}, \quad (5)
\]

approximates the effective bandwidth. This ensures that the spectral contents of the orthogonal FH waveforms are confined to the available bandwidth. From the above equation, the time-bandwidth product of the DFRC system is given as

\[
BW_T = \left( (K - 1)\Delta f + \frac{1}{\Delta f} \right) Q\Delta f \approx KQ. \quad (6)
\]

It is worth noting that, in general, the maximum number of orthogonal waveforms which can be synthesized equals to the time bandwidth product. Therefore, the total number of PSK-modulated waveforms which can be synthesized is less than or equal to \( KQ \). However, synthesizing such a large number of FH waveforms requires using the FH code values multiple times, that is, FH code repetition. This leads to increase in the AF SLLs. It will be shown in Section 3 that embedding of inherently random PSK symbols results in reducing the AF SLLs.

## 2.4 | MIMO radar receiver signal model

We assume that there are \( L \) targets of interest located in the far-field of the DFRC platform. The signal reflected by the \( \ell \)-th target impinges on the MIMO radar receiver from direction \( \theta_{\ell} \). The signal model, expressing the \( N_T \times 1 \) complex-valued vector of the received baseband signals, is

\[
x(t; n) = \sum_{\ell=1}^{L} x_{\ell}(t; n) + x_m(t; n) + x_{\text{noise}}(t; n), \quad (7)
\]

where \( x_{\ell}(t; n) \) is the \( N_T \times 1 \) vector of the desired target signal, \( x_m(t; n) \) is the \( N_T \times 1 \) vector of interference which summarize all interference terms from the spatial sidelobes as well as the range sidelobes during the \( n \)-th pulse, \( x_{\text{noise}}(t; n) \) represents the \( N_T \times 1 \) vector of additive Gaussian noise with zero mean and co-variance \( \sigma^2 I_{N_T} \), and \( I_{N_T} \) denotes an identity matrix of size \( N_T \times N_T \). In Equation (7), the target signal can be expressed as

\[
x_{\ell}(t; n) = \beta_{\ell}(n) \left( a^T(\theta_{\ell}) \psi(t - t; n) e^{j2\pi \nu t} \right) b(\theta_{\ell}), \quad (8)
\]

where \( \nu \) is the Doppler shift associated with the target of interest, \( \beta_{\ell}(n) \) is the target reflection coefficient during the \( n \)-th pulse, \( \psi(t; n) \triangleq [\psi_1(t; n), \ldots, \psi_{M_f}(t; n)]^T \) is the vector of PSK modulated FH waveforms, \( a(\theta_{\ell}) \) and \( b(\theta_{\ell}) \) are the steering vectors of the transmit and receive arrays, respectively, and \( (\cdot)^T \) stands for the transpose. The received signal components associated with the individual transmitted waveforms can be obtained by matched filtering (Equation 7) to the PSK-modulated waveforms (Equation 4). Hence, the signals observed at the output of the MF are the \( M_f N_T \times 1 \) extended vector of virtual data,

\[
y_{\text{rad}}(n) = \text{vec} \left( \int_{t} x(t; n) \psi^H(t; n) dt \right), \quad (9)
\]

where \( \text{vec} (\cdot) \) denotes the vectorization operator that stacks the columns of a matrix into one long column vector, \( (\cdot)^H \) stands for the Hermitian transpose.
2.5 | Communication receiver signal model

Consider a single-antenna communication receiver located in the spatial direction $\theta_{\text{com}}$ with respect to the MIMO radar. The signal at the output of the communication receiver can be expressed as

$$r(t; n) = a_{ch} a^T(\theta_{\text{com}}) \psi(t; n) + w(t; n),$$

(10)

where $a_{ch}$ is the channel coefficient which summarizes the propagation environment between the MIMO radar transmit array and the communication receiver, and $w(t; n)$ represents the additive white Gaussian noise with zero mean and variance $\sigma_w^2$. In order to maintain the orthogonality between the FH waveforms, the condition on code selection,

$$c_{m,q} \neq c_{m',q}, \quad \forall q, m \neq m',$$

(11)

should be satisfied. This condition also enables symbol detection at the communication receiver.

Assume that time and phase synchronizations between the MIMO radar and the communication receiver are achieved. Then, matched filtering $r(t; n)$ to the FH sub-pulses yields

$$y_{m,q}(n) = \int_D r(t, n) b_{m,q}^*(t) \text{rect}(t - q\Delta_c - nT_0) \, dt$$

$$= a_{ch} \hat{\Omega}_{(m,q)} - 2\pi d_{m}\sin(\theta_{\text{com}}) + w_{m,q}(n),$$

(12)

where $d_m$ is the displacement between the first and the $m$-th elements of the transmit array measured in wavelength, and $w_{m,q}(n) \triangleq \int_D w(t, n) b_{m,q}^*(t) \text{rect}(t - q\Delta_c - nT_0) \, dt$ is the additive noise term at the output of the $(m,q)$-th MF with zero mean and variance $\sigma_w^2$. The embedded PSK symbols can be estimated using [20] as

$$\hat{\Omega}_{(m,q)} = \angle y_{m,q}(n) - \varphi_{ch} + 2\pi d_m\sin(\theta_{\text{com}}),$$

(13)

where $\angle(\cdot)$ stands for the angle of a complex number and $\varphi_{ch} \triangleq \angle(a_{ch})$ is the phase of the channel coefficient.

3 | ANALYSIS OF THE DFRC WAVEFORMS

In this section, we discuss in details the AF, PSD and the data rate that can be achieved by the DFRC system. The AF is analysed for the case of a single pulse as well as for the case of a pulse train. In both cases, we draw comparisons between the AF for FH waveforms and PSK-modulated FH waveforms. The PSD for both types of waveforms is also analysed. In all cases, we assume that the FH code used is pre-designed, for example, using a slightly modified version of the method reported in [39] by incorporating the additional orthogonality condition (Equation 11).

3.1 | Ambiguity function analysis

Without loss of generality, assume that the DFRC platform is equipped with uniform linear arrays (ULAs). The inter-element spacings associated with the transmit and receive arrays are denoted as $d_T$ and $d_R$, respectively. The spatial frequency of a hypothetical target located in direction $\theta$ is defined as $\hat{f} = 2\pi d_{g}\sin(\theta)$, where $d_g$ is measured in wavelength. Adopting the AF definition from [21], the AF expression for the MIMO radar can be written as
\[
\chi(\tau, \nu, f, f') \triangleq \frac{1}{MTQ} \sum_{m=1}^{M_T} \sum_{m'=1}^{M_T} \chi_{m,m'}(\tau, \nu) e^{j2\pi(f_m-f_{m'})\gamma}, \tag{14}
\]

where, \(f'\) denotes the spatial frequency shift, \(\gamma = \frac{d_T}{d_R}\), and

\[
\chi_{m,m'}(\tau, \nu) \triangleq \int_0^T \phi_m(t)\phi_{m'}^*(t+\tau) e^{j2\pi\nu t} dt, \tag{15}
\]
is the cross-ambiguity between the waveforms from antennas \(m\) and \(m'\).

### 3.1.1 Without information embedding

The AF for MIMO radar using FH waveforms is readily given by

\[
|\chi_{\text{rad}}(\tau, \nu, f, f')| = \frac{1}{MTQ} \left| \sum_{m=1}^{M_T} \sum_{m'=1}^{M_T} \sum_{q=1}^{Q} \sum_{q'=1}^{Q} \chi_{m,m'}^{(\Delta)}(\tau, \nu) e^{j2\pi(f_m-f_{m'})\gamma} \right|. \tag{16}
\]

**FIGURE 2** Real part of FH waveforms without (top) and with (bottom) PSK symbol embedding. FH, frequency hopping; PSK, phase shift keying

**FIGURE 3** Imaginary part of FH waveforms without (top) and with (bottom) PSK symbol embedding. FH, frequency hopping; PSK, phase shift keying
The term $\chi_{\nu}^{(\Delta)}(\tau, \nu)$ in Equation (16) is the cross AF of any two FH sub-pulses computed over the duration $2\Delta t$. It has a peak at the middle of this interval and zero values at the edges (zeroes everywhere else), at zero Doppler shift. This term is given by [21] as

$$\chi_{m,m'}^{(\Delta)}(\tau, \nu) = \chi_{m,m'}^{\text{rect}}(\tau, \nu) e^{j2\pi c_{m,q} / \Delta t}$$

where

$$\chi_{m,m'}^{\text{rect}}(\tau, \nu) = \int_{0}^{\Delta t} \text{rect}(t) \text{rect}(t + \tau) e^{j2\pi \tau t} dt,$$

$$= \frac{\Delta t - |\tau|}{\Delta t} \text{sinc}(\nu (\Delta t - |\tau|)), \quad \frac{\Delta t - |\tau|}{\Delta t}$$

In Equation (17), $\nu = (c_{m,q} - c_{m',q'}) \Delta t / \nu$ and $\tau = (q - q') \Delta t$ are auxiliary Doppler-shift and time-delay variables, respectively. The behaviour of the overall AF (Equation (16)) depends on the behaviour of the cross-AF of the sub-pulses (Equation (17)). The peak of Equation (17) occurs when the Doppler shift is one and the two FH sub-pulses are perfectly overlapped, that is, when $r = p \Delta t$, where $p$ is an integer. Specifically, the peak of Equation (17) can be mathematically expressed as

$$\chi_{m,m'}^{(\Delta)}(p \Delta t, 0) \triangleq \begin{cases} 1, & \nu = 0, \\ 0, & \nu \neq 0, \\ \text{sinc}(\nu (\Delta t - |\tau|)), & \nu \neq 0, \end{cases}$$

$$= \text{sinc}(\nu (\Delta t - |\tau|)).$$

This property shows that when the FH code is used more than once within the same pulse (i.e., when $K < M_f Q$), AF sidelobe peaks are exhibited. In what follows, we gain insights into the AF behaviour by considering specific values of the parameters involved.

The MIMO radar AF in Equation (16) exhibits spike-like SLL at the delays (assuming zero Doppler shift) $|r| = p \Delta t$, $p = 0, \ldots, Q - 1$, where every pair of sub-pulses separated by $\tau = \Delta t - (q - q') \Delta t = i \Delta t$, $i = 0$ or 1, becomes fully overlapped. In this case, the cross AF in Equation (17) of each pair of entirely overlapped sub-pulses at zero Doppler shift with $\nu = (c_{m,q} - c_{m',q'}) \Delta t / \nu$, $\nu = 0$, and $\Delta t / \nu$ can be represented as follows

$$\chi_{m,m'}^{(\Delta)}(r, 0) = (1 - i) e^{j2\pi (c_{m,q} - c_{m',q'}) / \nu} \text{sinc}((c_{m,q} - c_{m',q'}) / \nu).$$

The above sinc(·) function attains its maximum value when $c_{m,q} = c_{m',q'}$. The term $e^{j2\pi c_{m,q} / \nu}$ is an integer, and hence the term $\chi_{m,m'}^{(\Delta)}(r, 0) = 1 - i$. In the case of $c_{m,q} \neq c_{m',q'}$, the sinc(·) function takes zero value theoretically and hence, $\chi_{m,m'}^{(\Delta)}(r, 0) = 0$. Therefore, at a given spatial frequency $f = f'$, and $\nu = 0$, Equation (16) can be re-written using Equation (20) at delays of $|r| = i \Delta t$ as

$$|\chi_{\nu}(r, 0)| = \frac{1}{M_f Q} \left| \sum_{m=1}^{M_f} \sum_{m'=1}^{M_f} \sum_{q=1}^{Q} \sum_{q'=1}^{Q} \chi_{m,m'}^{(\Delta)}(r, 0) \right|,$$

$$= \frac{1}{M_f Q} \left| \sum_{m=1}^{M_f} \sum_{m'=1}^{M_f} \sum_{q=1}^{Q} \sum_{q'=1}^{Q} (1 - i) \right|,$$

where the terms in the summation of Equation (22) can take either 1 or 0 values.

3.1.2 With PSK symbol embedding

Symbol embedding is represented by a multiplicative term inside the summation of Equation (16) as

$$|\chi_{\nu}(r, f, f')| = \frac{1}{M_f Q} \left| \sum_{m=1}^{M_f} \sum_{m'=1}^{M_f} \sum_{q=1}^{Q} \sum_{q'=1}^{Q} \chi_{m,m'}^{(\Delta)}(r, \nu) \right| \left| e^{j2\pi f (m - f') q} \right|.$$}

In this case, Equation (21) becomes

$$|\chi_{\nu}(r, 0)| = \frac{1}{M_f Q} \left| \sum_{m=1}^{M_f} \sum_{m'=1}^{M_f} \sum_{q=1}^{Q} \sum_{q'=1}^{Q} \chi_{m,m'}^{(\Delta)}(r, 0) \right|$$

$$\left| e^{j2\pi (\nu - \Omega_{m,q}) q} \right|.$$}

The SLLs of the AF of the DFRC system, given in Equation (24), depend on the embedded PSK symbols $\Omega_{m,q}$ that are randomly generated for $c_{n,q} = c_{n',q'}$, we recognize two different cases of Equation (24).

1. $\Omega_{m,q} \neq \Omega_{m',q'}$: The term $e^{j2\pi (\nu - \Omega_{m,q}) q} = (k_1 + jk_2)$, where $k_1$ and $k_2$ take real values between $-1$ and 1 and $|e^{j2\pi (\nu - \Omega_{m,q}) q}| = 1$. Therefore, the terms in the summations which correspond to the delay $|r - (q - q') \Delta t| = i \Delta t$, tend to cancel each other, resulting in lower values of $|\chi_{\nu}(r, 0)|$. This argument is applicable along both the delay and Doppler axes.

2. $\Omega_{m,q} = \Omega_{m',q'}$: $e^{j2\pi (\nu - \Omega_{m,q}) q} = 1$. Therefore, the terms in Equation (24) under this case are the same as those of Equation (21).

Since the summations in Equation (24) contain terms from both case 1 and case 2, the resulting value is smaller than or equal to the values at the respective delays in Equation (21). Accordingly, the spike-like SLLs are reduced as a result of symbol embedding. This reduction is more pronounced when the FH code matrix includes more repetitive values.
There is a situation where reductions are less of a problem. This occurs when the code \( c_{m,q} \) is designed in such a way that all the sub-pulses are nearly orthogonal to each other, that is, the correlation between the sub-pulses is minimum. To further explain this case and to examine the importance of designing \( c_{m,q} \) for FH waveforms, we also consider the FH code design with the following constraint,

\[
c_{m,q} \neq c_{m',q'}, \quad m \neq m', \quad q \neq q'.
\]

In this case the theoretical value of \( \chi_{m,m'}^{(\Delta)}(\tau, 0) = 0 \) at \( \tau = i \Delta \), and hence the AF does not profess spike-like SLLs. In turn, PSK symbol embedding would have minimum effect in terms of the AF SLL reduction. Also, if the phase values \( \Omega_{m,q} \) are relatively low, that is, for higher phase constellations \( (\bar{f}) \), the phase shift between the FH sub-pulses introduced by the PSK symbols is distributed on the unit circle on a uniform grid of phase shift between the FH sub-pulses. In such a case, the statistical average of Equation (24) is still zero, that is, \( \mathbb{E}\{\chi_{DF}(i \Delta, 0)\} = 0 \), for any value of \( \bar{f} \). However, for a sample FH-DSRC waveform realization with large constellation size \( \bar{f} \), the reduction in RSLs becomes more noticeable when the FH code matrix includes a large number of repetitive values, that is, when \( QM_T \gg K \).

### 3.2 Ambiguity function for a train of pulses

The AF for MIMO radar using FH waveforms for \( N_p \) pulses becomes [21],

\[
\chi_{rad}^{(PT)}(\tau, \nu) = \frac{1}{N_p M_T Q} \left| \sum_{n=0}^{N_p-1} \sum_{n'=0}^{N_p-1} \sum_{m=1}^{M_T} \sum_{m'=1}^{M_T} \sum_{q=1}^{Q} \sum_{q'=1}^{Q} \chi_{m,m'}^{(\Delta)}(\tau - (n' - n) T_0, \nu) e^{j2\pi q n T_0} \right|.
\]

Since the radar pulses have identical hopping patterns, then strong correlations arise when these pulses overlap, that is, when the delay \( \tau = n_p T_0 \) with \( n_p \) assumes integer numbers, 0, 1, 2, ..., \( N_p - 1 \). It is clear from Equation (26) that the number of pulses overlapping goes down linearly with \( n_p \), causing SLLs to go down accordingly. To analyse the DFRC waveforms over \( N_p \) pulses, we compute the AF of the PSK modulated waveform (Equation 4) as

\[
\chi_{DF}^{(PT)}(\tau, \nu) = \frac{1}{N_p M_T Q} \left| \sum_{n=0}^{Q} \chi_{m,m'}^{(\Delta)}(\tau - (n' - n) T_0, \nu) e^{j2\pi q n T_0} \right|
\]

At delay \( n_p T_0 \), the above equation can be simplified as

\[
\left| \chi_{DF}^{(PT)}(\tau, \nu) \right| = \frac{1}{N_p M_T Q} \left| \sum_{n=0}^{N_p-1} N_p-1 \sum_{n'=0}^{N_p-1} M_T \sum_{m=1}^{M_T} \sum_{q=1}^{Q} \sum_{q'=1}^{Q} \chi_{m,m'}^{(\Delta)}(n_p T_0 - (n' - n) T_0, \nu) e^{j(\Omega_{m,q} - \Omega_{m',q'})} e^{j2\pi q n T_0} \right|
\]

Since each pulse has the same value of \( M_T Q \) hops, then for \( N_p - n_p \) pulses, there will be \( (N_p - n_p)M_T Q \) PSK symbols embedded, computing Equation (28), results in significant reductions in the SLLs of the AF. If the number of pulses is sufficiently large, the SLLs reduction would be more pronounced due to random nature of the PSK symbols and AF will be close to the thumbtack shape. The simulation results supporting the above analysis are presented in Section 5.

### 3.3 Data rate

The achievable communication data rate (DR) for the DFRC system can be readily shown using [20] to be proportional to the pulse repetition frequency (PRF), the number of transmit elements \( M_T \), the length of the FH code \( Q \), and the size of the PSK constellation \( \bar{f} \). We consider three different cases to analyse the data rate of the FH DFRC system in detail.

1. \( M_T \leq K \): By implementing condition (Equation 25), that is, no repeated hopping code values, the designed hopping code would be restricted to a maximum of \( M_T Q = K \) values. The maximum data rate in this case becomes \( DR = PRF.K \log_2(\bar{f}) \).

2. \( M_T = K \): Here, a total number of \( K = M_T \) orthogonal waveforms can be designed and hence, the data rate becomes, \( DR = PRF.K.Q \log_2(\bar{f}) \).

3. \( K < M_T \leq KQ \): The FH code repetitions are allowed and the data rate in this case is also equal to, \( DR = PRF.K.Q \log_2(\bar{f}) \).

It is important to note that the data rate for cases 2 and 3 is equal. However, case 3 enables the MIMO radar function to utilize more waveforms as needed by allowing repetitions in the FH code. Therefore, with minimal effort in designing the FH code matrix, we can utilize PSK symbol embedding to reduce the RSLs, thereby improving the data rate of the system.

### 3.4 Power spectral density analysis

In this subsection, we analyse the PSD of both the FH MIMO radar and FH-PSK MIMO DFRC waveforms. Since the hopping frequencies are integer multiples of \( \Delta f \) and \( \Delta f \) is an integer, then
the FH waveforms have continuous phase at sub-pulse boundaries as depicted in Figures 2 and 3. The PSD of the FH radar waveforms from m-th antenna can be adopted from [33] as

$$S_{rad}(f) = |\Delta_t \sum_{q=1}^{Q} \text{sinc}(2\pi \Delta_f (f - c_m q \Delta_f)) e^{-j2\pi q \phi_n}|^2. \quad (29)$$

For \( f = c_m q \Delta_f \) the \( \text{sinc}(\cdot) \) function attains its maximum value. If the FH coefficient \( c_{m,q} \) is used \( n_f \) times in the FH code matrix, then the PSD at this frequency increases to \( |n_f \Delta_f|^2 \).

Similarly, the PSD of the FH-PSK MIMO DFRC waveform can be expressed as

$$S_{PSK}(f) = |\Delta_t \sum_{q=1}^{Q} \text{sinc}(2\pi \Delta_f (f - c_m q \Delta_f)) e^{-j2\pi q \phi_n}|^2. \quad (30)$$

The phase change introduced by \( e^{2\pi \omega_n \phi} \) in Equation (30) disrupts the continuous phase property of the FH sub-pulses. The amount of this phase change depends on the size of the phase constellation used, that is, number of bits transmitted. The abrupt phase transitions over consecutive hops lead to spectral leakage into the side bands and an increase in the SSLs. It is noted that the phase change associated with BPSK symbols is greater than those of other PSK constellations, and as such, causes high out-of-band power leakage [40].

When \( f = c_m q \Delta_f \) and in particular when the hopping frequencies are repeated, due to the presence of the random phasor term \( e^{2\pi \omega_n \phi} \), fluctuations in PSD amplitudes will occur. It is noted that the variation in amplitude and phase due to this information embedding makes it difficult for the eavesdropper to intercept the FH pattern [41].

In order to minimize the spectral leakage and confine the spectral contents of the DFRC waveforms to the pre-allocated bandwidth, we seek a modulation scheme in which the higher and lower hopping frequency bands in the bandwidth are left unmodulated. This can be accomplished by forcing the phase of the PSK symbols, \( \Omega_{(m,q)} = 0^0 \) at the lower and higher frequencies of the spectrum. The spectral sidelobes therefore decrease, and the frequencies will adhere to the pre-allocated bandwidth. However, a consequence of such an action is that if \( \Omega_{(m,q)} = 0^0 \), the waveforms at those sub-pulses will be similar to the FH waveforms without symbol embedding. Accordingly, the randomness of the PSK symbols decreases causing the AF’s SSLs to slightly increase. This trade-off between the AF’s SSLs and spectral leakage should be examined when incorporating PSK information into FH sub-pulses.

### 4 | RADAR DETECTION PERFORMANCE

In this section, we examine the detection performance of the FH MIMO waveforms with and without symbol embedding.

We consider that the data at the radar receiver (Equation 9) has a target signal with a signal-to-noise ratio (SNR), a stationary interference signal at interference-to-noise ratio (INR) which summarizes all range sidelobes and spatial sidelobes in the presence of AWGN. Applying beamforming to the radar received signal given in Equation (9) yields a scalar output comprising the target signal from the range cell under test, in addition to interference plus noise, that is,

$$y_{rad}(n) = \sum_{\ell=1}^{L} w^H y_{rad}(n)$$

$$= \sum_{\ell=1}^{L} w^H (\beta_{\ell}(n) \chi(\tau_{\ell}, \nu_{\ell}) [a(\theta_{\ell}) \otimes b(\theta_{\ell})]) + y_{in}(n) + y_{noise}(n), \quad (31)$$

where subscript \( \ell = 1, ..., L \) refers to the \( \ell \)-th target located in the spatial direction \( \theta_{\ell} \), \( \chi(\tau_{\ell}, \nu_{\ell}) \) is the AF computed at time delay \( \tau_{\ell} \) and Doppler shift \( \nu_{\ell} \), respectively, \( y_{in}(n) \triangleq w^H y_{in}(n) \) and \( y_{noise}(n) \triangleq w^H y_{noise}(n) \), denote the interference and noise terms at the beamformer output, respectively, and \( w \) is the \( M \times N \times 1 \) unit-norm receive beamforming weight vector corresponding to the beam pattern of the target located at the spatial direction \( \theta_{\ell} \). Both non-adaptive and adaptive beamforming techniques can be adopted in Equation (31). We focus on AF analysis when conventional beamforming is used. In this case,

$$w = \frac{1}{\sqrt{M \times N \times T}} a(\theta_{\ell}) \otimes b(\theta_{\ell}), \quad (32)$$

and, therefore, the beamformer output becomes

$$y_{rad}(n) = \sqrt{M \times N \times T} \beta_{\ell}(n) \chi(\tau_{\ell}, \nu_{\ell}) + y_{in}(n) + y_{noise}(n)$$

$$+ \sqrt{M \times N \times T} \sum_{\ell'=1, \ell' \neq \ell}^{L} \beta_{\ell'}(n) \gamma_{\ell}(\theta_{\ell'}) \chi(\tau_{\ell'}, \nu_{\ell'})$$

$$\approx \sqrt{M \times N \times T} \beta_{\ell}(n) \chi(\tau_{\ell}, \nu_{\ell}) + y_{in}(n) + y_{noise}(n), \quad (33)$$

where

$$|\gamma_{\ell}(\theta_{\ell'})| \triangleq \frac{\left| [a^H(\theta_{\ell'}) \otimes b^H(\theta_{\ell'})]^H [a(\theta_{\ell}) \otimes b(\theta_{\ell})] \right|}{\|a(\theta_{\ell})\| \cdot \|b(\theta_{\ell})\|} \approx 0.$$ 

Note that the noise term \( y_{noise}(n) \) has zero mean and variance \( \sigma_n^2 \).

For a range cell under test, the target detection problem under the target present/absent \( (H_1/H_0) \) hypotheses can be formulated as follows:

$$H_0: \quad y_{rad}(n) = y_{in}(n) + y_{noise}(n), \quad (34)$$

$$H_1: \quad y_{rad}(n) = y_{rad}(n),$$

$$H_2: \quad y_{rad}(n) = y_{rad}(n).$$
\[ y_{rad}(n) = \sqrt{M_T} N_T \beta_r(n) (\nu_r, \nu_r) + y_{in}(n) + y_{noise}(n). \]  

(35)

The detection decision can be performed using the standard square law detector, where a magnitude squaring operation is performed on the beamformer output, that is,

\[ |y_{rad}(n)|^2 \leq \zeta, \]  

(36)

where \( \zeta \) is an appropriately selected threshold for a given probability of false alarm \( (P_f) \). The data processing steps for detection are depicted in Figure 4.

The detection performance and the corresponding receiver operating characteristics (ROC) depend on the statistics of the interference plus noise. To simplify the analysis, we consider the interference scatters located in the same spatial angle as that of the desired targets. \(^4\) The interference source is assumed to be stationary and its reflection is delayed by \( \tau_d \) with respect to the target reflection such that \( \tau_{in} = \tau_r + \tau_d \). Accordingly, \( M_T N_T \times 1 \) vector of interference for the antenna array steered towards the target at the spatial angle \( \theta_r \) in Equation (31) can be represented as

\[ y_{in}(n) = \text{vec} \left[ \int_{\theta_r} \beta_{in}(\theta_r) \left( a^T(\theta_r) \psi(t-\tau_{in}) \right) b(\theta_r) \psi^H(t) dt \right], \]  

(37)

where \( \beta_{in}(n) \) is the interference reflection coefficient. Considering the Doppler shift to be negligible and using Equations (26) and (28), the interference term at the output of the beamformer can be expressed as

\[ y_{in}(n) = M_T N_T \beta_{in}(n) \chi(\tau_{in}, 0), \]  

(38)

where \( \chi(\tau_{in}, 0) \) is the interference AF RSL at delay \( \tau_{in} \) located in the spatial direction \( \theta_r \). Therefore, the beamformer output (Equation 31) for the target of interest, can be simplified to

\[ y_{rad}(n) = \tilde{\beta}_r(n) \chi(\tau_r, \nu_r) + \tilde{\beta}_{in}(n) \chi(\tau_{in}, 0) + y_{noise}(n), \]  

(39)

where, \( \tilde{\beta}_r(n) \triangleq \sqrt{M_T N_T} \beta_r(n) \) and \( \tilde{\beta}_{in}(n) \triangleq \sqrt{M_T N_T} \beta_{in}(n) \) are signal amplitudes of the target and interference at the beamformer output, respectively. The term \( \chi(\tau_r, \nu_r) \) can assume either \( \chi_{rad}^{PT}(\tau_r, \nu_r) \) or \( \chi_{DF}(\tau_r, \nu_r) \) and the interference AF SLLs \( \chi(\tau_{in}, 0) \), can be either \( \chi_{in}^{PT}(\tau_{in}, 0) \) or \( \chi_{DF}(\tau_{in}, 0) \), depending on whether the PSK symbols are embedded into the FH waveforms. Assuming the target is present in the range cell under test, the target detection problem simplifies to the following hypothesis test:

\[ H_0 : \tilde{\beta}_r(n) \chi(\tau_r, \nu_r) + y_{noise}(n), \]  

(40)

\[ H_1 : \tilde{\beta}_r(n) \chi(\tau_r, \nu_r) + \tilde{\beta}_{in}(n) \chi(\tau_{in}, 0) + y_{noise}(n). \]  

(41)

The ROC is determined by the statistics of \( \tilde{\beta}_r(n) \) and \( \tilde{\beta}_{in}(n) \), and \( \chi(\tau_{in}, 0) \). We assume that \( \tilde{\beta}_r(n) \) and \( \tilde{\beta}_{in}(n) \) are unknown deterministic complex scalars. The statistics of \( \chi(\tau_{in}, 0) \) depends on whether PSK symbol embedding is applied. It is worth noting that the worst-case RSL corresponds to \( \tau_{in} = \pm \Delta \). In what follows, we restrict our analysis to this specific case and discuss the receiver ROC for both the radar alone, that is, no signal embedding, and the DFRC system.

4.1 Without embedding

The data under hypothesis \( H_0 \) in Equation (40) has AWGN with variance \( \sigma_n^2 \). The interfering signal has the signal amplitude \( \tilde{\beta}_{in}(n) \) and AF SLL \( \chi(\tau_{in}, 0) \), described by Equation (26). In the range bin of interest, at integer delays \( \tau_{in} = \pm \Delta \), the detection problem becomes detection of the target signal in the presence of spike-like AF SLLs and AWGN. The hypothesis \( H_0 \) for the output of square law detector can be defined as

\[ H_0 : \tilde{\beta}_r(n) \chi(\tau_r, \nu_r) + y_{noise}(n), \]  

(42)

where, the noise at the output of the beamformer is modelled as \( y_{noise}(n) \sim N(0, \sigma_n^2) \), and \( \chi(\tau_{in}, 0) \sim N(\mu_{\text{rad}}, \sigma_n^2) \) represents the RSLs which has an unknown deterministic value. From Equation (42), it can be inferred that the probability density function (PDF) of \( H_0 \) is a Non-Central Chi-square distribution with the mean power of the interfering signal RSLs, \( \mu_{\text{H}_0}^{\text{rad}} = \text{Re}(\mu_{\text{H}_0}^{\text{rad}}) + j\text{Im}(\mu_{\text{H}_0}^{\text{rad}}) \). Here, \( \text{Re}(\cdot) \) and \( \text{Im}(\cdot) \) denote the real and imaginary parts of a complex number, respectively. On the other hand, with the presence of target, \( H_1 \) Equation (41) can be further simplified as

\[ H_1 : \tilde{\beta}_r(n) + \tilde{\beta}_{in}(n) \chi(\tau_{in}, 0) + y_{noise}(n), \]  

(43)

where \( \tilde{\beta}_{in}(n) \chi(\tau_{in}, 0) \sim N(\mu_{\text{H}_1}^{\text{rad}}, \sigma_n^2) \), and \( y_{noise}(n) \sim N(0, \sigma_n^2) \). From Equation (43), it is evident that the PDF of \( H_1 \) is also a Non-Central Chi-square distribution which clusters around the

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\( ^4 \) It is assumed that spatial side-lobe interference is effectively mitigated by the receive beamforming step in Equation (31).
sum and difference of the values of \( \tilde{\mu}_f(n) \) and \( \tilde{\mu}_m(n) \) with mean value \( \mu_{\tilde{\mu}_f}^{\text{rad}} = \text{Re}(\mu_{\tilde{\mu}_f}^{\text{rad}}) + j\text{Im}(\mu_{\tilde{\mu}_f}^{\text{rad}}) \) and variance \( \sigma_{\tilde{\mu}_f}^{\text{rad}} \).

For detection of the signals, the threshold \( \xi^{\text{FH}} \) is calculated for a given fixed value of \( P_{fa} \) using the hypothesis \( \mathcal{H}_0 \). Clearly, \( \xi^{\text{FH}} \) increases with increased INR.

### 4.2 With PSK symbol embedding

When PSK symbols are embedded into the FH sub-pulses, the randomness of the PSK symbols break the correlation exhibited by the FH waveforms, leading to lower AF SLs. It is also worth noting that the maximum of the RSLs which can be achieved with embedding are always less than the RSLs exhibited by the FH MIMO system without symbol embedding and can be represented as \( |x^{\text{pt}}_{\text{DF}}(\tau_m, 0)| \leq |x^{\text{pr}}_{\text{rad}}(\tau_m, 0)| \).

The hypothesis \( \mathcal{H}_0 \) for the DFRC system is similar to Equation (40), with the random variable \( |x^{\text{pt}}_{\text{DF}}(\tau_m, 0)| = |x^{\text{pr}}_{\text{DF}}(\tau_m, 0)| \), drawn from a truncated Rayleigh distribution, where \( x^{\text{pr}}_{\text{df}}(\tau_m, 0) \sim N(0, \sigma_{\text{psk}}^2) \), and \( y_{\text{noise}}(n) \sim N(0, \sigma_{\text{n}}^2) \). Therefore, from Equation (42), and the PDFs of \( x^{\text{pr}}_{\text{df}}(\tau_m, 0) \) and \( y_{\text{noise}}(n) \), we assume that the PDF of \( \mathcal{H}_0 \) of the DFRC system is a Central Chi-square distribution with zero mean and variance \( \sigma_{\text{n}}^2 \).

Hypothesis \( \mathcal{H}_1 \) has the target signal present along with the AWGN and reduced RSLs of the interfering signal. This hypothesis is similar to Equation (43), with \( y_{\text{noise}}(n) \sim N(0, \sigma_{\text{n}}^2) \), \( \tilde{\mu}_m(n)x^{\text{pr}}_{\text{df}}(\tau_m, 0) \sim N(0, \sigma_{\text{psk}}^2) \), and \( \tilde{\mu}_f(n) \sim N\left(\mu_{\tilde{\mu}_f}^{\text{df}}, \sigma_{\tilde{\mu}_f}^2\right) \). \( \mu_{\tilde{\mu}_f}^{\text{df}} = (\text{Re}(\mu_{\tilde{\mu}_f}^{\text{df}}) + j\text{Im}(\mu_{\tilde{\mu}_f}^{\text{df}})) \). Therefore, the PDF of \( \mathcal{H}_1 \) is a Non-Central Chi-square distribution. For a given \( P_{fa} \), the corresponding threshold \( \xi^{\text{DF}} \) is calculated from the PDF of \( \mathcal{H}_0 \). From the description of hypotheses \( \mathcal{H}_0, \mathcal{H}_1 \), in Sections 4.1 and 4.2, it can be observed that the threshold value \( \xi^{\text{FH}} > \xi^{\text{DF}} \). This is due to the presence of a strong unknown deterministic component \( x^{\text{pr}}_{\text{df}}(\tau_m, 0) \) in the MF output of the FH MIMO radar system without embedding.

The effect of SNR, INR, RSLs and \( P_{fa} \) on the probability of detection \( (P_D) \) is presented in Section 5.

### 5 SIMULATION RESULTS AND DISCUSSION

We consider a MIMO radar system operating at X-band with carrier frequency \( f_c = 8.2 \) GHz and bandwidth 115 MHz. The sampling frequency is taken as the Nyquist rate, that is, \( \frac{f_c}{2} = 1.15 \times 10^8 \) samples/s. The PRI is \( T_0 = 10 \) μs, that is, the PRF is 100 KHz. The transmit array is considered to be an ULA comprising \( M_r = 16 \) omni-directional transmit antennas spaced half a wavelength apart. We generate a set of 16 FH waveforms. The parameter \( K = 16 \) is chosen such that the FH step is \( \Delta_f = 6.4 \) MHz. The FH code length \( Q = 16 \) is assumed and the FH interval duration \( \Delta_f = 0.156 \) μs is used. For communications, \( J = \{2, 4, 8, 16\} \) and \( N_0 = \{1, 2, 3, 4\} \) are considered. We demonstrate the effect of PSK symbol embedding on the FH MIMO radar operation using different simulations. We also consider different values of \( M_T \) and \( Q \) to demonstrate the effectiveness of the DFRC scheme considered.

#### 5.1 Range sidelobe levels

From Figure 5, it can be found that, with repeated hopping code values, there is a substantial reduction of 20 dB in the RSLs of the FH MIMO radar system resulting from the PSK symbol embedding. When a large communication phase constellation, that is, \( J = 16 \), 16–PSK is considered, the phase change introduced between the FH sub-pulses is relatively small. Therefore, the RSLs will slightly increase compared to the case when \( J = 2 \). This can be seen in Figure 5, as there is 2 dB improvement in the RSLs of the FH DFRC system employing BPSK, compared to 16–PSK. It is noted that when repetitions in the hopping code are disallowed, the PSK symbol embedding does not lend itself to tangible changes in RSLs, as shown in Figure 6.

Though the FH code remains invariant with PRI within the CPI, the communication symbol changes from one pulse to another. This leads to significant reductions in the AF RSLs computed at delays \( n_p T_0 \). Figure 7 shows 25 dB reduction for \( N_p = 10 \).

#### 5.2 Power spectral density

Once the PSK symbols are embedded in the FH radar waveforms, the phase shift caused by the PSK symbols results in variations in the PSD amplitude of the FH MIMO radar system, as shown in Figure 8. It can also be observed from the zoomed in region of Figure 8 that, the spectral broadening is proportional to the degree of phase change induced by the PSK symbols, where BPSK exhibits more spectral broadening and higher SSLs compared to other PSK constellations.

#### 5.3 Data rate

For \( K = 16 \), \( M_T = 16 \) and \( Q = 16 \), the achievable communication data rates for the DFRC system considered in this paper are 25.6, 51.2, 76.8, and 102.4 Mbps for \( J = \{2, 4, 8, 16\} \), respectively. We compare the DRs corresponding to the proposed FH DFRC system with the FH DFRC system in [24]. In [24], the system uses FH code selection to represent the communication symbols without any re-modulation of the radar waveforms. In this particular DFRC system configuration, the FH code sequences are selected in a combinatorial manner and the maximum data rate occurs when \( M_T = K/2 \). Consequently, the DFRC framework in [24] suffers from low data rates compared to the DFRC system in this article with QPSK and 16–PSK symbol embedding. The DFRC system in [24] also suffers from high RSLs due to the use of repetitive hopping code values that are used to define the communication symbols. In current embedding scheme, when the repetition of 16 coding values is disallowed in the FH code design, the number of distinct FH waveforms that can be designed is reduced to \( M_T Q = K \). This, in turn, reduces the DR of the FH DFRC system by a factor \( \frac{1}{Q} \). For example, when \( K = 16 \), \( M_T = 4 \), and \( Q = 4 \), that is, \( K = M_T Q \), the attainable
communication data rate of the DFRC system with BPSK embedding will be 1.6 Mbps. The variation of DR with the number of antenna elements is shown in Figure 9.

The above simulations show that FH DFRC framework considered takes advantage of PSK symbol embedding to achieve low RSLs and high communication data rates by enabling the re-use of hopping code values in the FH code design.

5.4 | Radar detection performance

To examine the detection performance of FH MIMO and DFRC MIMO systems, we considered a scenario where the target signal of 10 d SNR is present with an interference of INR equals to 12 dB, plus AWGN. The MF output of the signals computed for 128 pulses along both the delay and the Doppler axes, without and with PSK symbol embedding is
shown in Figure 10a,b respectively. It can be clearly observed from Figure 10a that, in the range cell of interest, the desired signal is masked by the AF SLLs of the interference signal. On the other hand, the target signal is clearly distinguishable with PSK embedding, as shown in Figure 10b. The strong spike like AF SLLs along delay and the Doppler axes would render target detection difficult. In another example, we consider both SNR = 0 dB and 15 dB, with INR = 15 dB. The reflection coefficients are modelled as $\beta_{\text{in}}, \beta_x \in e^{j\theta_p}$, where $|e^{j\theta_p}| = 1$ and $\theta_p$ is a random phase. The output of beamformer is considered at the range cell under test where the target of interest is present. We plot the data of the output of the beamformer under hypothesis $H_0$ and $H_1$ without and with PSK symbol embedding in Figures 11 and 12, respectively. The data under $H_1$ for FH MIMO system without embedding cluster around the sum and the difference of the target and interference signal amplitudes, as displayed in Figure 11. For a fixed value of $P_{fa} = 10^{-4}$, and INR = 15 dB, the threshold values for detection are computed as $\zeta_{\text{FH}} = 1.8951$ and $\zeta_{\text{DF}} = 0.8232$. For a given value of INR, the threshold value for the FH MIMO system without embedding is higher compared to that of the DFRC system due to the presence of spike like AF SLLs. As a
result, the data under $\mathcal{H}_1$ of the radar system without embedding do not completely cross the threshold value as shown in Figure 11, for SNR = 15 dB. With PSK information embedding, the data under $\mathcal{H}_1$ of the DFRC system completely cross the threshold value owing to the weaker SLLs, and hence higher probability of detection. This behaviour is evident in Figure 12. It can also be noted from Figures 11 and 12 that, higher SNR is required to increase the distance between the PDFs of $\mathcal{H}_0$ and $\mathcal{H}_1$ to improve the probability of detection for a given false alarm rate.

The variation of $P_f$ for variable INR and SNR is shown in Figure 13. As shown in Figure 13a, for a broad range of SNR values, the FH-DFRC system exhibits better detection performance compared to the radar system without embedding. This is due to the presence of a strong RSL component in the beamformer output of the data of the FH MIMO radar system,
which raises the threshold $\xi_{FH}$ for detection compared to that of the DFRC system. Also, as INR values increase, due to the presence of RSL component, the threshold value increases and hence $P_d$ decreases for the FH system without embedding compared to the FH-DFRC system considered. This characteristic is shown in Figure 13b. Also, in Figure 14, as $P_{FA}$ decreases, the threshold values $\xi_{FH}$ and $\xi_{DF}$ increases, resulting in the decrease of $P_d$ values [42]. Therefore, using the above discussion, it is inferred that the FH-DFRC system shows considerably better performance compared to the FH MIMO radar system operating alone.

5.5 | Performance at the communication receiver

The communication channel coefficient $\alpha_{ch}$ is selected such that $|\alpha_{ch}| = 1$ and its phase is randomly drawn from $\angle \alpha_{ch} \in [0, 2\pi]$. The SERs of the DFRC system considered in this paper depends on the size of the phase constellation. As the value of $J$ increases, the communication system requires more SNR to achieve the desirable communication performance. In [29], the authors constructed a code-shift keying dictionary to represent the communication symbols and multiplied the communication symbol waveforms with the FH waveforms in fast-time. The DFRC scheme in [29], also referred to as scheme-1 in this article, and uses a series of phase values to represent the communication symbols. For scheme-1 to achieve the same bit rate as the DFRC method considered in this article, the system needs to implement higher phase constellation along with a large number of frequency hops. Since each communication symbol is defined by a sequence of phases, higher SNR is required for the communication system to achieve the desirable performance. It can be seen in Figure 15, that the binary phase constellation of scheme-1 and the DFRC system considered in this
paper exhibit almost the same SER performance, whereas for $J = 8$, scheme-1 requires 3 dB more SNR to achieve the same performance as the DFRC system considered in this paper.

6 | CONCLUSION

We examined the effects of PSK information symbol embedding on FH radar performance. The random nature of the PSK communication symbols breaks the correlation between repetitive hops of the FH sub-pulses, leading to substantial reduction in the radar AF SLLs along both range and the Doppler axes. The effect of phase discontinuity induced by the PSK communication symbols on the bandwidth was analysed and the trade-off between the spectral leakage and the AF SLLs was presented. The detection performance of the DFRC system using PSK communication symbols was analysed and compared with that of the radar system alone without information embedding. Our analyses and results showed that the DFRC MIMO system has lower RSLs and shows better detection performance compared to that of the nominal FH MIMO radar system without embedding.
CONFLICT OF INTEREST
We do not have any conflict of interests.

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