Passive harmonic filters for high voltage transmission systems

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Abstract. This paper presents a new passive filter configurations that can reduce the harmonic penetration into high-voltage transmission system. Proposed configurations utilize the secondary winding of a step-down transformer as a part of the filter. A general method for harmonic filters design, based on the theory of passive circuit synthesis, is considered. The simulation results show that the proposed filter is an efficient tool for attenuating harmonics penetrating high voltage transmission systems.

1. Introduction
The problem of ensuring the power quality caused by the wide use of nonlinear loads that create currents of nonsinusoidal form has gained importance in recent years. The proliferation of non-linear loads is growing steadily due to the increasing use of power electronics in industry and in everyday life [1 – 5].

The widespread application of nonlinear loads causes distortion of currents and voltages not only in distribution networks, but also in high voltage transmission systems, which leads to serious problems such as capacitor overloading, resonant overvoltages, negative impact on electrical equipment [1, 2, 6].

The most popular and effective mitigation method for harmonic problems in the power systems are passive harmonic filters (PHF) [7 – 11]. Main advantage of passive filters are simplicity, high reliability, low cost and ease of operation. However, their application is rather expensive in case of 110–220kV systems. Low-voltage operation can contribute to reducing costs and simplifying maintenance.

Many substation transformers have a tertiary winding. In [12, 13], the variant is considered when the PHF is connected to the low voltage tertiary winding of a three-winding transformer. However, the analysis shows that in such cases, the frequency characteristics of the traditional passive filters are distorted due to the influence of transformer windings inductances. Thus, different configurations of passive filters capable of taking into account the inductance of the transformer and the grid are required.

The paper describes a general method for passive filters design, based on the theory of passive circuit synthesis. The proposed method allows us to consider the inductance of the transformer winding as a component of the filter. The advantage of the proposed approach is that the filter configuration can be varied during the implementation process by choosing the appropriate canonical form of the reactive one-port.
2. Proposed mitigation schemes
An equivalent circuit diagram of the transmission system with the passive filter connected to the tertiary winding at the frequency of the k-th harmonic is shown in Figure 1. The current source represents the nonlinear load.

The following notations are used in Figure 1:
- \( Z_{T_1} = R_{T_1} + jX_{T_1} \) – the impedance of the transformer primary winding;
- \( Z_{T_2} = R_{T_2} + jX_{T_2} \) – the impedance of transformer secondary winding;
- \( Z_S = R_S + jX_S \) – the impedance of transmission system;
- \( Z_{PF} = R_{PF} + jX_{PF} \) – the impedance of the passive filter;

The current of the k-th harmonic transmitted to the transmission system is:

\[
I_S = \frac{Z_{T_3}(j\omega_k) + Z_{PF}(j\omega_k)}{Z_S(j\omega_k) + Z_{T_1}(j\omega_k) + Z_{T_3}(j\omega_k) + Z_{PF}(j\omega_k)} J_k
\]  

(1)

Figure 2 shows the equivalent circuit corresponding to the variant when the passive filter is connected via a step-down transformer.

Then, the current of the transmission system is:

\[
I_S = \frac{Z_{T_2}(j\omega_k) + Z_{PF}(j\omega_k)}{Z_S(j\omega_k) + Z_{T_2}(j\omega_k) + Z_{PF}(j\omega_k)} J_k
\]  

(2)

According to formulas (1) and (2), the minimum value of the current transmission coefficient to the transmission system is reached, when the numerator reactance is equal to zero:

\[
X_T + X_{PF} = 0
\]

Thus, it is crucial to consider the inductance of the transformer when choosing the configuration of the filter and designing the components.

3. Canonical forms of the passive one-ports
We consider the main principles involved concerning the theory of passive circuit synthesis. They are necessary for obtaining new passive filter configurations. It is known [15] that the input impedance and admittance of LC one-port is the ratio of two polynomials with simple zeros and poles bounded by the axis.

The input impedance of an LC one-port is determined by the equation:

\[
Z(s) = H \frac{\prod_{i=1}^{n} (s^2 + \omega_i^2)}{\prod_{j=1}^{n} (s^2 + \omega_j^2)} = H \frac{N(s)}{D(s)}
\]
Here, $s$ is a complex variable, $\omega_{zi}$ and $\omega_{pj}$ denote the zeros and poles of the input resistance, respectively.

The procedure for passive one-ports synthesis is based on function decomposition, either into the sum of elementary terms or a continued fraction. Each term is implemented by the simplest series or parallel circuit form. These passive one-port configurations are known as Foster and Cauer canonical forms.

The first canonical Foster form corresponds to decomposition of the LC one-port input resistance into the sum of simple fractions:

$$Z(s) = \frac{k_0}{s} + k_\infty s + \sum_{j=1}^{n} \frac{k_j s}{s^2 + \omega^2_{pj}}$$

(3)

where $k_\infty = \frac{Z(s)}{s} \bigg|_{s \to \infty}$ is the residue, corresponding to the pole at infinity and $k_0$ is the residue, corresponding to the pole at the origin, when $s = 0$.

The values of residues at the poles $\omega_{pj}$ are obtained using the formula:

$$k_j = \frac{\left(s^2 + \omega^2_{pj}\right)Z(s)}{s^2} \bigg|_{s^2 = -\omega^2_{pj}}$$

Formula (3) corresponds to the series connection of elementary one-ports (Figure 3). The inductive element $L_1$ implements the pole of the input resistance at infinity, and the capacitor element $C_1$ implements the pole $Z(s)$ at the origin. The parallel circuits correspond to the second order terms. Parallel contour parameters are calculated using the formulas:

$$C_j = \frac{1}{k_j}; \quad L_j = \frac{k_j}{\omega^2_{pj}}$$

![Figure 3. The first Foster canonical form.](image)

The second Foster canonical form is based on decomposing the input admittance function of a reactive one-port:

$$Y(s) = \frac{k_0}{s} + k_\infty s + \sum_{i=1}^{n} \frac{k_i s}{s^2 + \omega^2_{zi}}$$

(4)

A single inductive element implements the pole at the origin, and a single capacitor implements the pole at infinity. The value of residue $k_i$ is determined using the formula:
Formula (4) corresponds to the second Foster canonical form, created by the parallel connection of series second order circuits having resonant frequencies $\omega_s$ (Figure 4).

$$k_i = \left. \frac{\left(s^2 + \omega_s^2\right)Y(s)}{s} \right|_{s^2 = -\omega_s^2}$$  

(5)

The values of the $i$-th series circuit components are found using the formulas:

$$L_i = \frac{1}{k_i} \quad \text{and} \quad C_i = \frac{k_i}{\omega_s^2}$$  

(6)

The first and second Cauer canonical forms are obtained by decomposing the input function into a continued fraction. This decomposition is implemented in the ladder one-ports (Figure 5 and 6).

Figure 4. The second Foster canonical form.

Figure 5. The first Cauer canonical form

Figure 6. The second Cauer canonical form

One should note that the traditionally structured passive filter is the implementation of the second Foster form. New variants of passive filters can be designed by choosing the other canonical implementation of reactive one-ports or their combinations. In this case, it is preferable to use the first ones of the Foster and Cauer forms (Figure 3 and 5, respectively). It will allow using the transformer winding as a part of the filter.

4. Passive filter design procedure

The proposed filter design procedure includes the following stages.

- In the early stage, we analyze spectral decomposition of non-sinusoidal voltages and currents generated by non-linear loads and then create a system model “transmission system - passive filter”.
- Based on the data obtained, we develop the requirements for the filter impedance. To simplify calculations, it is advisable to use the transfer function, normalized to the fundamental frequency.
- Next, we synthesize reactive two-port implementing resistance $Z_{PF}(s)$, and calculate the normalized filter elements values $C_{rs}$, $L_{rs}$.
At a later stage, we the elements values with respect to the frequency \( \omega_0 \): \( C_i = C_i^* / \omega_0 \), \( L_i = L_i^* / \omega_0 \). The denormalized frequency \( \omega_0 \) should be 4-5% lower than the fundamental frequency. It could help to take into account the effect of reducing the capacitance due to insulation ageing.

And finally, filter elements are denormalized with according to impedance level.

The design procedure proposed implies the possibility of taking into account the inductance of the transformer winding and provides the required filter setting.

### 5. Passive filter design example

To illustrate the proposed method, we consider the example of passive filters design. It is required to design a filter which provides attenuation of the 5th and 7th harmonics generated by a three-phase non-linear load.

The function of the filter input resistance, normalized to the fundamental frequency, can be expressed in the form:

\[
Z(s) = \frac{s^2 + 25}{s(s^2 + 36)}
\]

Figure 7 shows the filter configuration in the first Cauer form.

**Figure 7.** The filter configuration.

The normalized element values are: \( L_1 = 1 \) H, \( C_2 = 0.026 \) F, \( L_3 = 10.11 \) H, \( C_4 = 0.0031 \) F. After the frequency and impedance denormalization we obtain the following element values: \( L_1 = 1.61 \) mH, \( C_2 = 168 \) \( \mu \)F, \( L_3 = 16.3 \) mH, \( C_4 = 20 \) \( \mu \)F.

Figure 8 shows the frequency response of the current transfer coefficient after the filter has been installed.
Figure 8.

The values of harmonic voltages in the transmission system (% of fundamental) after the filter installation are presented in Table 1.

Table 1. The values of harmonic voltages in the transmission system (% of fundamental) after the filter installation.

| Variant      | $I_5$ | $I_7$ | $I_{11}$ | $I_{13}$ |
|--------------|-------|-------|----------|----------|
| No filter    | 17    | 12    | 8        | 5        |
| With filter  | 1.7   | 0.85  | 2.8      | 1.9      |

6. Conclusions

The paper describes the procedure for designing passive filters, which provide attenuation of harmonics penetrating into high-voltage transmission systems. To reduce costs and simplify maintenance, the filter is connected to the low voltage winding of the substation transformer. We proposed a new filter configuration including the inductance of the transformer winding as a filter component. Simulation has shown that the proposed filter is an efficient tool for attenuating harmonics penetrating high voltage transmission systems.

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