CP violation in a multi-Higgs doublet model with flavor changing neutral current

N.G. Deshpande* and Xiao-Gang He†

Institute of Theoretical Science
University of Oregon
Eugene, OR 97403-5203, USA
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Abstract

We study CP violation in a multi-Higgs doublet model based on a $S_3 \times Z_3$ horizontal symmetry where CKM phase is not the principal source of CP violation. We consider two mechanisms for CP violation in this model: a) CP violation due to complex Yukawa couplings; and b) CP violation due to scalar-pseudoscalar Higgs boson mixings. Both mechanisms can explain the observed CP violation in the neutral Kaon system. $\epsilon'/\epsilon$ due to neutral Higgs boson exchange is small in both mechanisms, but charged Higgs boson contributions can be as large as $10^{-3}$ for a), and $10^{-4}$ for b). CP violation in the neutral B system is, however, quite different from the Minimal Standard Model. The neutron Electric Dipole Moment can be as large as the present experimental bound, and can be used to constrain charged Higgs boson masses. The electron EDM is one order of magnitude below the experimental bound.

*e-mail address: desh@oregon.uoregon.edu
†e-mail address: he@bovine.uoregon.edu
in case b) and smaller in case a).
I. INTRODUCTION

The origin of CP violation, fermion masses and mixings are some of the outstanding problems of particle physics today. The Minimal Standard Model (MSM) provides no explanations for the observed fermion masses and mixings. It is believed that one needs to go beyond the MSM to solve these problems. There is no satisfactory solution so far. The best one can do at the present is to reduce the number of free parameters in the theory regarding fermion masses and mixings. The understanding of CP violation is also very poor. So far CP violation has only been observed in the neutral kaon system. In the MSM CP violation is from the phase in the CKM matrix $V_{KM}$. The model is consistent with observations. It is, however, important to study different processes and other models of CP violation to better understand the origin of CP violation. In this paper we will study some details of CP violation in a model proposed by Ma [2]. The model is based on a horizontal $S_3 \times Z_3$ symmetry. In this model there are some interesting relations between the fermion masses and mixings. The free parameters related to the fermion masses and mixings are reduced. This model also has some interesting consequences for CP violation [3,4]. Many of the considerations in this model will have applicability to more general multi-Higgs models.

There are four Higgs doublets in the $S_3 \times Z_3$ model. Their interactions at the tree level mediate neutral flavor changing current. Like any multi-Higgs doublet models with flavor changing neutral current at the tree level, there are different mechanisms for CP violation. CP violation can arise in three places in this type of models: 1) Non-trivial phase in the $V_{KM}$ matrix; 2)Non-trivial phases in the flavor changing Yukawa couplings; and 3) Mixings of scalar and pseudoscalar Higgs bosons. In cases 2) and 3), CP violation can occurs at the tree level by exchanging neutral Higgs bosons [4]. These models have much richer phenomenology for CP violation than the MSM. In the MSM, CP violation in the neutral Kaon system can be explained by exchanging W-bosons at the one loop level (the "box diagram") [3]. There are specific predictions for $\epsilon'/\epsilon$ resulting from the direct CP violation in $K_{L,S} \rightarrow 2\pi$ [7]; and predictions for CP violation in the neutral B system [8]. The Electric
Dipole Moment of neutron is generated at the three loop level, and is less than $10^{-31}$ ecm \[9\]. The electron EDM is even smaller \[10\]. In the $S_3 \times Z_3$ model, the CP violation coming from the phase in the CKM matrix is inadequate to account for the $\bar{\epsilon}$ parameter in the K system. One of the other two has to be invoked. We consider the consequences of either mechanism for: (i) $\epsilon'/\epsilon$; (ii) CP violation in the neutral B system; and (iii) the neutron and electron EDMs. The results are dramatically different from the MSM.

II. YUKAWA COUPLINGS IN THE $S_3 \times Z_3$ MODEL

In the $S_3 \times Z_3$ model, there are four Higgs doublets, $\phi_{1,2,3,4}$. The quarks and Higgs bosons transform under the $S_3 \times Z_3$ symmetry as \[2\]

$q_{3L}, t_R, b_R, \phi_1 : (1, 1)$,

$(q_{1L}, q_{2L}), (\phi_3, \phi_4) : (2, \omega)$,

$(c_R, u_R), (s_R, d_R) : (2, \omega^2)$,

$\phi_2 : (1, \omega^2)$,

(1)

where $\omega \neq 1, \omega^3 = 1$ is the $Z_3$ element. The Yukawa couplings consistent with the $S_3 \times Z_3$ symmetry are given by

$$L_Y = -f_1(\bar{q}_{1L}\phi_3u_R + \bar{q}_{2L}\phi_4c_R) - f_2\bar{q}_{3L}\phi_1t_R$$

$$- f_3(\bar{q}_{1L}\phi_2s_R + \bar{q}_{2L}\phi_2d_R) - f_4(\bar{q}_{1L}\phi_3b_R + \bar{q}_{2L}\phi_4b_R)$$

$$- f_5(\bar{q}_{3L}\phi_3d_R + \bar{q}_{3L}\phi_4s_R) - f_6\bar{q}_{3L}\phi_1b_R + H.C.$$  

(2)

where $\tilde{\phi}_i = (\phi_i^0, -\phi_i^-)^T$. Without loss of generality we work in a basis where all Vacuum Expectation Values (VEV) are real. When the neutral components develop VEVs, $<\phi_i> = v_i$, we obtain the quark mass matrices

$$M^u = \hat{M}^u = \begin{pmatrix} f_1v_3 & 0 & 0 \\ 0 & f_1v_4 & 0 \\ 0 & 0 & f_2v_1 \end{pmatrix},$$

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\[ M^d = \begin{pmatrix} 0 & f_3v_2 & f_4v_3 \\ f_3v_2 & 0 & f_4v_4 \\ f_5v_3 & f_5v_4 & f_6v_1 \end{pmatrix}. \] (3)

The quark phases can be chosen such that
\[ M^d = \begin{pmatrix} 0 & a & \xi b \\ a & 0 & b \\ \xi c & c & d \end{pmatrix}, \] (4)

with \( a, b, c, d \) real and \( \xi = |\xi|e^{i\sigma} \) complex. \( M^d \) can be diagonalized by a bi-unitary transformation
\[ M^d = V_L \hat{M^d} V_R^T. \] (5)

Here \( \hat{M^d} \) is the diagonalized down quark mass matrix. \( V_L \) and \( V_R \) are unitary matrices. Because the up quark mass matrix is already diagonalized, \( V_L \) is the CKM matrix \( V_{KM} \).

It is convenient to work in a basis of the Higgs bosons in which the Goldstone bosons are removed. To this end we define the following
\[ \left( \begin{array}{c} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{array} \right) = \left( \begin{array}{c} \frac{v_1}{\sqrt{2}} \\ \frac{v_2}{\sqrt{2}} \\ \frac{v_1v_2}{\sqrt{2}} \\ \frac{v_1v_2v_3}{\sqrt{2}} \end{array} \right) \begin{pmatrix} 0 & \frac{v_1}{\sqrt{2}} & \frac{v_1v_2}{\sqrt{2}} & \frac{v_1v_2v_3}{\sqrt{2}} \\ \frac{v_1}{\sqrt{2}} & 0 & \frac{v_1v_2}{\sqrt{2}} & \frac{v_1v_2v_3}{\sqrt{2}} \\ \frac{v_1v_2}{\sqrt{2}} & \frac{v_1v_2}{\sqrt{2}} & 0 & \frac{v_1v_2v_3}{\sqrt{2}} \\ \frac{v_1v_2v_3}{\sqrt{2}} & \frac{v_1v_2v_3}{\sqrt{2}} & \frac{v_1v_2v_3}{\sqrt{2}} & 0 \end{pmatrix} \begin{pmatrix} G \\ H_1 \\ H_2 \\ H_3 \end{pmatrix}, \] (6)

where \( v_{12}^2 = v_1^2 + v_2^2, \) \( v_{124}^2 = v_1^2 + v_2^2 + v_4^2, \) and \( v^2 = v_1^2 + v_2^2 + v_3^2 + v_4^2. \) The transformation is the same for both the neutral and charged Higgs bosons. For the neutral Higgs bosons, \( G = h^0 + iG_Z, \) where \( G_Z \) is the Goldstone boson ‘eaten’ by \( Z, \) and \( h^0 \) is a physical field whose couplings are the same as the Higgs boson in the MSM. For the charged Higgs bosons \( G \) is the Goldstone boson ‘eaten’ by \( W. \) In this basis, we have
\[ L_Y = -(D_L \hat{M^d} D_R + \bar{U}_L \hat{M^u} U_R)(1 + \frac{\text{Re}h^0}{\sqrt{2}}) \]
\[ \quad - \bar{D}_L \bar{Y}_i^d D_R \frac{h^0_i}{\sqrt{2}} - \bar{U}_L \bar{Y}_i^u U_R \frac{h^0_i}{\sqrt{2}} \]
\[ \quad - \bar{U}_L V_{KM} \bar{Y}_i^d D_R h_i^- + \bar{D}_L V_{KM}^\dagger \bar{Y}_i^u U_R h_i^+ + H.C., \] (7)
where $h_i$ are the component fields of $H_i$ with $H_i = (h_i^+, h_i^0/\sqrt{2})$. $U_{L,R} = (u, c, t)^T_{L,R}$, and $D_{L,R} = (d, s, b)^T_{L,R}$. The Yukawa couplings are given by

\[
\tilde{Y}_1^u = \text{Diag}(0, 0, \frac{m_{t}v_2}{v_{12}v_1}), \\
\tilde{Y}_2^u = \text{Diag}(0, -\frac{m_{c}v_{12}}{v_4v_{12}v_2}, \frac{m_{t}v}{v_{12}v_1}), \\
\tilde{Y}_3^u = \text{Diag}(\frac{m_{u}v_{124}}{v_3v}, -\frac{m_{c}v_{3}}{v_{124}v_1}, -\frac{m_{t}v_{3}}{v_{124}v_1}),
\]

\[
\tilde{Y}_1^d = \frac{1}{v_{12}}V_{KM}^\dagger \begin{pmatrix}
0 & -a_{\frac{v_4}{v_{12}}} & 0 \\
-a_{\frac{v_4}{v_{12}}} & 0 & 0 \\
0 & 0 & d_{\frac{v_4}{v_{12}}}
\end{pmatrix} V_R, \\
\tilde{Y}_2^d = \frac{1}{v_{124}}V_{KM}^\dagger \begin{pmatrix}
0 & a_{\frac{v_4}{v_3}} & 0 \\
a_{\frac{v_4}{v_3}} & 0 & -b_{\frac{v_4}{v_4}} \\
0 & -c_{\frac{v_4}{v_4}} & d_{\frac{v_4}{v_{124}}}
\end{pmatrix} V_R, \\
\tilde{Y}_3^d = \frac{1}{v}V_{KM}^\dagger \begin{pmatrix}
0 & -a_{\frac{v_4}{v_{124}}} & \frac{\xi b_{\frac{v_4}{v_{124}}}}{v_3} \\
a_{\frac{v_4}{v_{124}}} & 0 & -b_{\frac{v_4}{v_{124}}} \\
\xi c_{\frac{v_4}{v_{124}}} & -c_{\frac{v_4}{v_{124}}} & -d_{\frac{v_4}{v_{124}}}
\end{pmatrix} V_R.
\]  

\[ (9) \]

In general $h_i^{0,+}$ are not the mass eigenstates. They will mix with each other. In particular if CP is violated in the Higgs potential, $Reh_i^{0}$ and $Imh_i^{0}$ will mix. Also the charged Higgs boson mixing matrix will be complex. We can parametrize the mixings as

\[
\begin{pmatrix}
\begin{pmatrix}
h^0 \\
Reh_i^0 \\
Imh_i^0
\end{pmatrix}
\end{pmatrix} = \begin{pmatrix}
\alpha_{00} & \alpha_{0i} & \beta'_{0j} \\
\alpha_{k0} & \alpha_{ki} & \beta'_{kj} \\
\alpha'_{k0} & \alpha'_{ki} & \beta_{kj}
\end{pmatrix} \begin{pmatrix}
R_0 \\
R_i \\
I_j
\end{pmatrix},
\]

\[ (9) \]

\[ h_i^+ = (\gamma_{ij})n_j^+ . \]

where $R_i$, $I_i$ and $\eta_i$ are the mass eigenstates, the matrix $(\alpha \beta)$ is a $7 \times 7$ orthogonal matrix, and $(\gamma)$ is a $3 \times 3$ unitary matrix.
The specific numbers for $\alpha_{ij}$, $\alpha'_{ij}$, $\beta_{ij}$ and $\beta'_{ij}$ depend on the details of the Higgs potential. Unfortunately they are not determined. To simplify the problem, we will discuss two cases: a) CP violation only come from complex Yukawa couplings; and b) CP violation only come from the mixings of real and imaginary $h_0^i$. Case a) can be realised by constraining certain soft symmetry breaking terms in the potential [4]. We further assume, for simplicity, that $Re h_0^i$ are the mass eigenstate $R_i$ and consider their effects. The same analysis can be easily carried out for $Im h_0^i$ in the same way. The source for CP violation is the non-zero value for $\sigma$ which is a free parameter. We will present our results for $\sigma = 80^0$, which is close to the maximum of the allowed phase. Case b) can be realised by requiring spontaneous CP violation. The value of $\sigma$ will be zero and CP violation arises due to scalar-pesudoscalar Higgs boson mixing. For illustration, we consider the effects of a neutral mixed state

$$R = cos \theta Re h_2^0 + sin \theta Im h_3^0,$$

and for the charged Higgs boson we consider mixing

$$\eta^+ = \gamma_{22} h_2^+ + \gamma_{23} h_3^+$$

(11)

where $\gamma_{ij}$ are complex numbers, and $|\gamma_{22}|^2 + |\gamma_{23}|^2 = 1$. We assume that $R$ and $\eta^+$ are mass eigenstates, and all other Higgs bosons are much heavier and their effects can be neglected.

The details of the results depend on the specific rules of $\sigma$ in case a) and how Higgs bosons are mixed in case b). However, the special cases considered here will serve as a guide for a more complete analysis if the details of the mixings are known. The general features will be the same. We will comment on other cases later.

The parameters $a$, $b$, $c$, and $d$ are constrained from the down quark masses and the CKM mixings. We take as input parameters $a = 0.04GeV$, $b = 0.25GeV$, $c = 2.66GeV$, $d = 4GeV$. The mass eigenvalues for the down quarks are quite insensitive to the phase $\sigma$. For both cases, we have $m_b = 4.8GeV$, $m_s = 149MeV$ and $m_d = 9.5MeV$. These values are well within the allowed regions [11]. The CKM matrix for case a) is

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\[ V_{KM} = \begin{pmatrix} 0.975 & -0.222 & 0.00476 \\ 0.221 + i0.0033 & 0.974 + i0.014 & 0.043 - i0.0015 \\ -0.014 + i1.2 \times 10^{-5} & -0.041 - i6.8 \times 10^{-4} & 0.998 - i0.034 \end{pmatrix}, \quad (12) \]

and for case b)

\[ V_{KM} = \begin{pmatrix} 0.975 & 0.22 & 0.0048 \\ -0.219 & 0.975 & -0.0436 \\ -0.014 & 0.0415 & 0.999 \end{pmatrix}. \quad (13) \]

The values for the VEV’s are not fixed, we only know \( v_3/v_4 = m_u/m_c \). We will use the values in Ref \[1\]: \( v_1 = v_2 = 44 \text{GeV}, \ v_3 = 0.9 \) and \( v_4 = 238 \text{GeV} \) for illustration. We shall comment on effects of changing these values later.

**III. CONSTRAINTS ON THE HIGGS BOSON MASSES FROM THE NEUTRAL K AND B MESON SYSTEMS**

The \( S_3 \times Z_3 \) model has very restrictive allowed values for the non-trivial CP violating phase in the CKM matrix. The CP violating measure \( J \) \[12\] is less than \( 2.5 \times 10^{-6} \). If CP violation is purely from the CKM matrix, it is not possible to explain the observed CP violation in the neutral K system which requires \( J \geq 10^{-5} \). Therefore in this model CP violation due to Higgs boson exchange has to be considered. Because the neutral Higgs bosons mediate flavor changing neutral current at the tree level, there are stringent constraints on their masses and their interactions arising from experimental data. Some of the best constraints are from the mass differences in the neutral K and B systems. We must make sure that the observed CP violation in the neutral K system \( \bar{\epsilon} = 2.3 \times 10^{-3} e^{i\pi/4} \) is explained.

The CP violating parameter \( \bar{\epsilon} \) is given by

\[ \bar{\epsilon} = \frac{\text{Im} M_{12}^K}{\sqrt{2} \Delta m_K} e^{i\pi/4}, \]

where \( M_{12}^K \) is the matrix element which mixes \( K^0 \) with \( \bar{K}^0 \), and \( \Delta m_K \) is the mass difference between \( m_{K_L} \) and \( m_{K_S} \). The \( \Delta S = 2 \) Hamiltonian, responsible for \( M_{12}^K \), generated by exchanging neutral Higgs bosons \( R_i \) is given by
\[ H_{\text{eff}} = -\frac{1}{2M_{R_i}^2} \left( \bar{d}(\alpha_{k_i} + i\alpha'_{k_i})\bar{Y}_{k,12}^{d} \left( \frac{1 + \gamma_5}{2} + (\alpha_{k_i} - i\alpha'_{k_i})\bar{Y}_{k,21}^{d*} \right) \right)^2. \] (15)

We obtain

\[ M_{K_{12}}^K = \langle K^0 | H_{\text{eff}} | \bar{K}^0 \rangle > = -\frac{f_K^2 m_K^2}{2M_{R_i}^2} \left( \frac{5}{24} \left( \frac{m_K^2}{m_s + m_d} \right)^2 \left( (\alpha_{k_i} + i\alpha'_{k_i})\bar{Y}_{k,12}^{d} \right)^2 + \left( (\alpha_{k_i} - i\alpha'_{k_i})\bar{Y}_{k,21}^{d*} \right)^2 \right) \]

\[ + \left( \alpha_{k_i} + i\alpha'_{k_i} \right)\bar{Y}_{k,12}^{d} \left( \alpha'_{k_i} - i\alpha_{k_i} \right)\bar{Y}_{k,21}^{d*} \left( \frac{1}{12} + \frac{1}{2} \left( \frac{m_K^2}{m_s + m_d} \right)^2 \right). \] (16)

Here we have used the vacuum saturation and factorization approximation results for the matrix elements \( [13] \)

\[ < K^0 | \bar{d}(1 \pm \gamma_5)s\bar{d}(1 \mp \gamma_5)s | K^0 > = f_K^2 m_K \left( \frac{1}{6} + \frac{m_K^2}{(m_s + m_d)^2} \right), \]

\[ < K^0 | \bar{d}(1 \pm \gamma_5)s\bar{d}(1 \mp \gamma_5)s | \bar{K}^0 > = -\frac{5}{6} f_K^2 m_K \left( \frac{m_K^2}{(m_s + m_d)^2} \right). \] (17)

The contribution to the mass difference \( \Delta m_K \) is given by \( 2ReM_{12} \). Similar formula holds for the neutral B system.

To constrain the Higgs boson masses, we require that the neutral Higgs boson contributions to the mass differences in the neutral K and B systems to be less than the experimental values: \( \Delta m_K/m_K = 7 \times 10^{-15} \), and \( \Delta m_B/m_B = 8 \times 10^{-14} \). We find that for case a) the tightest constraints on the masses of \( Reh_{1,2}^0 \) are from the mass difference \( \Delta M_B \) of the neutral B mesons which gives \( M_{h_1} > 2.9TeV \) and \( M_{h_2} > 3.1TeV \). With these masses, \( Reh_{1,2}^0 \) can not produce large enough \( \bar{\epsilon} \). Similar consideration yields \( M_{h_3} > 3.5TeV \), and we find the experimental value of \( \bar{\epsilon} \) can now be produced if the mass is about \( 5.6TeV \). The mass difference \( \Delta M_K \) of the neutral K mesons gives weaker bounds in all cases. For case b), the experimental value of \( \Delta M_B \) constrains \( M_R > 3TeV \). From the experimental value of \( \bar{\epsilon} \), we obtain \( \sin\theta\cos\theta/M_{R}^2 = 1.1 \times 10^{-8} \) which implies \( M_R < 7TeV \).

Similar analyses for case a) and case b) have been carried out previously in Ref. [4] and Ref. [3] respectively. Our analysis for case a) is similar to that in Ref. [4]. We used different values for the parameters \( a, b, c, \) and \( d \) which obtain smaller mass for \( m_b \) \((4.8GeV)\) compared
with the value 5.4GeV used in Ref. [4]. The analysis for case b) is different than that in Ref. [3]. In Ref. [3], the averaged effects of Higgs boson exchange were considered.

From the above we see that the Higgs boson masses are constrained to be in the multi TeV region. One would wonder if such heavy Higgs bosons may violate the unitarity bound. However this is not the case. It should be noted that the unitarity bound only apply to $h^0$ Higgs boson mass. Its mass is not constrained in the cases we are considering, $h^0$ can be light. In the above discussions we have neglected mixings between $h_i^0$ with $h^0$. If such mixings are large, the unitarity bound can constrain the $h_i^0$ masses. However there is enough freedom in our model to make the mixings with $h_0$ sufficiently small such that the unitarity bounds are always satisfied [3]. This argument applies to many models. A well known example is the minimal supersymmetric standard model. In this model, when the soft supersymmetry breaking parameter goes to infinite, the other heavy Higgs bosons decouple from the theory and cause no problem to the unitarity bound [14].

**IV. PREDICTIONS FOR $\epsilon'/\epsilon$.**

In this section we study the direct CP violation in $K_{L,S} \to 2\pi$ decays. CP violation in these processes is characterized by the value of $\epsilon'/\epsilon$. $\epsilon'/\epsilon$ is defined as

$$
\frac{\epsilon'}{\epsilon} = \frac{\omega \xi - \text{Im}A_2/\text{Re}A_0}{\xi + \text{Im}M_{12}/\Delta M_K},
$$

where $\omega = \text{Re}A_2/\text{Re}A_0 = 1/20$, $\xi = \text{Im}A_0/\text{Re}A_0$. Here $A_0$ and $A_2$ are the $\Delta I = 1/2$, 3/2 decay amplitudes for $K_{L,S} \to 2\pi$.

In our model, the neutral Higgs boson can induce $\text{Im}A_i$ at the tree level. These amplitudes are constrained to be very small due to large Higgs boson masses. They are small compared with the CP conserving amplitudes $\text{Re}A_i$ generated by W-boson exchange at the tree level. The neutral Higgs boson contributions to $\epsilon'/\epsilon$ are very small. However there may be large contributions from the charged Higgs bosons. The charged Higgs boson contributions to $\text{Im}A_i$ can be generated at the one loop level. The dominant one is from the charged Higgs boson mediated gluon penguin. The relevant $\Delta S = 1$ effective Lagrangian is given by
\[ L_{\Delta S=1} = i d \sigma^{\mu \nu} (\tilde{f}_1 \frac{1 + \gamma_5}{2} + \tilde{f}_2 \frac{1 - \gamma_5}{2}) \lambda^a s G_{\mu \nu}^a , \]  

where \( G_{\mu \nu}^a \) is the gluon field strength, and

\[ \tilde{f}_1 = \frac{g_s(\mu)}{32\pi^2} \frac{m_t}{M_{h_1^+}^2} (\frac{3}{2} - \ln \frac{m_t^2}{M_{h_1^+}^2}) Im\{ (V_{K_M} \bar{Y}_i d_{ij})_{l1} (\bar{Y}_{k}^d V_{K_M} \gamma_{kj})_{l2} \} \zeta_f , \]

\[ \tilde{f}_2 = \frac{g_s(\mu)}{32\pi^2} \frac{m_t}{M_{h_2^+}^2} (\frac{3}{2} - \ln \frac{m_t^2}{M_{h_2^+}^2}) Im\{ (V_{K_M} \bar{Y}_i d_{ij})_{l2} (\bar{Y}_{k}^d V_{K_M} \gamma_{kj})_{l1} \} \zeta_f , \]

where \( \zeta_f = (\alpha_s(m_h)/\alpha_s(\mu))^{14/23} \approx 0.17 \) is the QCD correction factor, and \( l \) is summed over \( u, c \) and \( t \). We will use \( \alpha_s(\mu) \approx 4\pi/6 \) for \( \mu = 1 GeV \). The above effective Lagrangian will generate a non-zero value for \( ImA_0 \) [15]. \( L_{\Delta S=1} \) also generates a non-zero value \( \bar{\epsilon}_{LD} \) for CP violation in \( K^0 \) and \( \bar{K}^0 \) mixing due to long distance interactions through \( K^0 \) and \( \pi, \eta, \eta' \) mixings [16]. One obtains [16,17]

\[ \frac{\xi}{\bar{\epsilon}_{LD}} \approx -0.196D , \]

\[ 2m_K Im M_{12,LD}^K \approx 0.8 \times 10^{-7} (\tilde{f}_1 + \tilde{f}_2)(GeV^3) , \]

where \( D \) is a supression factor of order \( O(m_h^2, m_{\pi}^2)/\Lambda^2 \). \( \xi/\bar{\epsilon}_{LD} \) is of order -0.014 to -0.1.

We find that in both a) and b) cases, the dominant contributions are from the top quark in the loop arising from mixing in the charged Higgs boson couplings. For case a), we have

\[ \bar{\epsilon}_{LD}(h_1^+) \approx 18 \frac{GeV^2}{m_{h_1^+}^2} \frac{m_t}{150 GeV} \frac{m_t^2}{ln \frac{m_t^2}{m_{h_1^+}^2}} , \]

\[ \bar{\epsilon}_{LD}(h_2^+) \approx 25 \frac{GeV^2}{m_{h_2^+}^2} \frac{m_t}{150 GeV} \frac{m_t^2}{ln \frac{m_t^2}{m_{h_2^+}^2}} , \]

\[ \bar{\epsilon}_{LD}(h_3^+) \approx -7 \frac{GeV^2}{m_{h_3^+}^2} \frac{m_t}{150 GeV} \frac{m_t^2}{ln \frac{m_t^2}{m_{h_3^+}^2}} . \]  

And for case b), we have

\[ \bar{\epsilon}_{LD} \approx -7.35 \times 10^2 Im(\gamma_{22}^{\gamma \gamma_23}) \frac{GeV^2}{m_{\eta^+}^2} \frac{m_t}{150 GeV} \frac{m_t^2}{ln \frac{m_t^2}{m_{\eta^+}^2}} . \]

The contributions to \( \bar{\epsilon} \) can be significant in both cases depending on the Higgs boson masses and the CP violating parameter \( Im(\gamma_{22}^{\gamma \gamma_23}) \). If the masses of the charged Higgs bosons are
of order a few hundred GeV, $\bar{\epsilon}_{LD}$ can be as large as the experimental value and $\epsilon'/\epsilon$ can be of order $10^{-3}$. However, there are constraints on the charged Higgs boson masses from the experimental upper bound on the neutron EDM. We will study these constraints in Sec.VI. When these constraints are taken into account, $\bar{\epsilon}_{LD}$ is generally constrained to be less than $3 \times 10^{-5}$ for case a). $\epsilon'/\epsilon$ is then constrained to be less than $3 \times 10^{-5}$. However, for case b), $\bar{\epsilon}_{LD}$ can still be as large as $10^{-3}$ and $\epsilon'/\epsilon$ can be $10^{-3}$.

V. CP VIOLATION IN THE NEUTRAL B SYSTEM.

There are many processes which can test CP violation in the neutral B system. Some particularly interesting ones are \[8\]

\[
\begin{align*}
B_d &\rightarrow J/\psi K_S , \\
B_d &\rightarrow \pi^+ \pi^- , \\
B_s &\rightarrow \rho K_S .
\end{align*}
\]

The differences of time variation of decay rates for the above processes and their CP transformed states are given by

\[a_{fCP} = \frac{\Gamma(B^0(t) \rightarrow f_{CP}) - \Gamma(\bar{B}^0(t) \rightarrow f_{CP})}{\Gamma(B^0(t) \rightarrow f_{CP}) + \Gamma(\bar{B}^0(t) \rightarrow f_{CP})} = \frac{(1 - |\lambda|^2)\cos(\Delta M_B t) - 2Im\lambda \sin(\Delta M_B t)}{1 + |\lambda|^2},\]

where $f_{CP}$ indicates the final states. $\lambda$ is defined as

\[\lambda = \left(\frac{q}{p}\right)_B \frac{\bar{A}}{A} S ,\]

where $(q/p)_B = \sqrt{M_{12}^B/M_{12}^B}$, $A$ and $\bar{A}$ are the decay amplitudes. If the final state contains $K_S$, $S = (q/p)_K$ which has a phase of order $10^{-3}$. For other cases $S$ is equal to one.

Non-zero asymmetry $a_{fCP}$ signals CP violation. If $|\lambda|$ is not equal to one, it indicates that CP is violated in the decay amplitudes. In the MSM $|\lambda|$ is equal to one to a very good approximation for the above three processes. The asymmetries are proportional to $Im\lambda$. 

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In the MSM, the processes in Eq.(24) measure three angles related to CP violation in the CKM matrix,

\[ \text{Im}\lambda(B_d \to J/\psi K_S) = -\sin 2\beta , \]
\[ \text{Im}\lambda(B_d \to \pi^+\pi^-) = \sin 2\alpha , \]
\[ \text{Im}\lambda(B_s \to \rho K_S) = -\sin 2\gamma , \]

where

\[ \alpha = \arg \left( \frac{V_{KM,td} V_{KM,tb}^*}{V_{KM,ud} V_{KM,ub}^*} \right) , \]
\[ \beta = \arg \left( \frac{V_{KM,cd} V_{KM,cb}^*}{V_{KM,td} V_{KM,tb}^*} \right) , \]
\[ \gamma = \arg \left( \frac{V_{KM,ud} V_{KM,ub}^*}{V_{KM,cd} V_{KM,cb}^*} \right) . \]

The sum of these three angles is equal to \(180^\circ\).

In the \(S_3 \times Z_3\) model, the situation is very different. In this model CP violation is mainly due to neutral Higgs boson exchange. The CP violating decay amplitudes \(A\) and \(\bar{A}\) are small because the decay amplitudes are dominated by the CP conserving tree level \(W\) interactions. However the phase of \(\sqrt{M_{12}^B/M_{12}^B}\) in the \(B - \bar{B}\) mixing due to neutral Higgs boson exchange can be large. In case a), there is CP violation arising from the phase in Yukawa coupling of Higgs bosons, as well as CKM matrix, but the former is much larger. The three measurements in Eq.(24) do not measure the angles \(\alpha\), \(\beta\) and \(\gamma\) defined in Eq.(28) anymore. The first two processes will mostly measure the phases in \(M_{12}^{B_d}\). We have

\[ \text{Im}\lambda(B_d \to \pi^+\pi^-) \approx \text{Im}\lambda(B_d \to J/\psi K_S) \leq 0.42 , \text{ from } Reh_1^0 , \]
\[ \text{Im}\lambda(B_d \to \pi^+\pi^-) \approx \text{Im}\lambda(B_d \to J/\psi K_S) \leq 0.19 , \text{ from } Reh_2^0 , \]
\[ \text{Im}\lambda(B_d \to \pi^+\pi^-) \approx \text{Im}\lambda(B_d \to J/\psi K_S) \approx 0.19 , \text{ form } Reh_3^0 . \]

For case b), the CP violation is purely from scalar-pseudoscalar mixing in the Higgs sector and we find

\[ \text{Im}\lambda(B_d \to \pi^+\pi^-) \approx \text{Im}\lambda(B_d \to J/\psi K_S) \approx -0.25 . \]
Im\lambda for B_s \to \rho K_S is different for a) and b). For case a), the neutral Higgs boson contributions to the asymmetry are small. However Im\lambda(B_s \to \rho K_S) due to CP violation in the KM-matrix can be about 0.1. For case b), Im\lambda(B_s \to \rho K_S) from neutral Higgs boson exchange is only about 0.02.

If interpreted as in Eq.(27), we find for case a), \( \sin 2\alpha = -\sin 2\beta, \sin \gamma = 0.05, \) and \( \alpha + \beta + \gamma \neq 180^0. \) For case b), we have, \( \sin 2\alpha = -\sin 2\beta, \sin \gamma = 0.01. \) We again find, \( \alpha + \beta + \gamma \neq 180^0. \) These predictions are different than those of the MSM.

VI. THE NEUTRON ELECTRIC DIPOLE MOMENT.

The prediction for the neutron EDM in the \( S_3 \times Z_3 \) model is very different from the MSM. In both a) and b) cases, the neutron EDM can be generated at the one loop level, and the two loop contributions can be significant. The one loop contributions in this model are also different from multi-Higgs models with neutral flavor conservation for tree level neutral Higgs boson exchange \[17\]. In flavor conserving models the fermion EDMs are proportional to the third powers of the external fermion masses. For u- and d- quarks and electron, the EDMs at the one loop level are very small \( < 10^{-30}ecm \). In the models considered by us, the couplings are quite different from the flavor conserving models. Furthermore the off diagonal couplings may contribute significantly. It is possible to have a large neutron EDM at the one loop level. The d quark EDM due to neutral Higgs boson exchange at the one loop level is given by

\[
d_d = \frac{Q_d e}{16\pi^2} Im(a_{i,dl}a_{i,ld}) \frac{m_l}{m^2_{R_i}} (\frac{3}{2} - \ln \left( \frac{m^2_{R_i}}{m^2_l} \right)) \zeta_d ,
\]

where \( a_{i,ql} = (\alpha_{ki} + i\alpha'_{ki})\tilde{Y}_{k,ql} \), \( \zeta_d = (\alpha_s(m_h)/\alpha_s(\mu))^{16/23} \approx 0.12, \) and l is summed over d, s, and b. The quark EDMs are related to the neutron EDM by quark model,

\[
D_n = 4d_d/3 - d_u/3 .
\]

For case a) we have
\[ D_n(d, Reh_1^0) = \frac{4}{3} d(Reh_1^0) \leq 2 \times 10^{-28} \text{ecm}, \]
\[ D_n(d, Reh_2^0) \leq 0.7 \times 10^{-28} \text{ecm}, \]
\[ D_n(d, Reh_3^0) \approx 2 \times 10^{-29} \text{ecm}. \]  

For case b), we have \( D_n(d) \approx 2 \times 10^{-29} \text{ecm}. \) These values are at least three orders of magnitude smaller than the experimental upper bound of \( 1.2 \times 10^{-25} \text{ecm} \) \[21\]. The u quark EDM is zero at the one loop level.

In multi-Higgs models, there may be large contributions to the neutron EDM at the two loop level from the Weinberg operator \[18\] \( D_n(W) \) and from the color dipole moment of gluon due to Bar-Zee type of diagrams \[13,21\] \( D_n(BZ) \). In our model, we have

\[ D_n(W) \approx \frac{c_u}{64\pi^2} ImZ_i^u \frac{m_i^2}{m_{h_0}^2} \ln \frac{m_i^2}{m_{h_0}^2}, \]
\[ D_n(BZ,q) \approx \frac{m_q c_q}{64\pi^2} \zeta_{bz} \left( \frac{\ln \frac{m_i^2}{m_{h_0}^2}}{\zeta_{bw}} \right)^2 ImZ_{iq}^i, \]

where \( \zeta_W \approx 6 \times 10^{-6} \), and \( \zeta_{bz} \approx 10^{-2} \) are the QCD correction factors, \( c_u = 2 \) and \( c_d = 4 \), and \( \Lambda \approx 1\text{GeV} \) is the chiral symmetry breaking scale. The parameters \( ImZ \) are given by

\[ ImZ_{tt}^i = \frac{1}{m_t^2} Re(\bar{Y}_{k,33}^u(\alpha_{ki} - i\alpha'_{ki})) Im(\bar{Y}_{k',33}^u(\alpha_{k'i} - i\alpha'_{k'i})) \],
\[ ImZ_{tu}^i = \frac{1}{m_u m_t} Im(\bar{Y}_{k,33}^u(\alpha_{ki} - i\alpha'_{ki})) \bar{Y}_{k',11}^u(\alpha_{k'i} - i\alpha'_{k'i})) \]
\[ ImZ_{td}^i = \frac{1}{m_d m_t} Im(\bar{Y}_{k,33}^u(\alpha_{ki} - i\alpha'_{ki})) \bar{Y}_{k',11}^d(\alpha_{k'i} + i\alpha'_{k'i})). \]  

(35)

For case a), because there is no CP violation in the up quark sector only down quark loops contribute, \( D_n(W) \) from the Weinberg operator at the two loop level is small. There are non-zero \( D_n(BZ) \) from d-quark due to Bar-Zee mechanism. We find that the contributions from \( Reh_{12}^0 \), is also small \( (< 4 \times 10^{-28} \text{ecm}). \) \( Reh_{3}^0 \) contribution is even smaller \( (< 10^{-29}). \)

For case b), the two loop contributions to the EDM are significantly larger because in this case there is CP violation in the top quark interaction. We have

\[ D_n(BZ,u) \approx (2 \sim 8) \times 10^{-26} \text{ecm}, \]
\[ D_n(BZ,d) \approx (2 \sim 8) \times 10^{-27} \text{ecm} , \]  

(36)
for $m_t$ between 100 GeV to 200 GeV. The neutron EDM can be as large as the experimental upper bound. The contribution from the Weinberg operator is small, $D_n(W) \leq 10^{-30}_{\text{ecm}}$.

The charged Higgs bosons can also contribute to the neutron EDM. At the one loop level, the u and d quark EDM are given by

$$d_u = -\frac{1}{48\pi^2} \frac{m_l}{m^2_{h^+_1}} \ln \frac{m^2_{h^+_1}}{m^2_{h^+_2}} \text{Im}[\gamma_{ji}\gamma^*_{ki}(V_{KM}Y^d_{ji})(V^d_{KM}Y^u_{ki})_{11}],$$

$$d_d = \frac{1}{24\pi^2} \frac{m_l}{m^2_{h^+_1}} \ln \frac{m^2_{h^+_1}}{m^2_{h^+_2}} \text{Im}[\gamma_{ji}\gamma^*_{ki}(V_{KM}Y^d_{ji})(V^d_{KM}Y^u_{ki})_{11}].$$  (37)

For $d_u$, l is summed over d, s, and b; and for $d_d$, l is summed over u, c, and t. At the two loop level, there is a large contribution from the Weinberg operator,

$$D_n(W) \approx e\zeta'_W M \frac{1}{32\pi^2} \text{Im}Z^u_{it} \frac{m^2_l}{m^2_{h^+_1}} \ln \frac{m^2_{h^+_1}}{m^2_{h^+_2}},$$  (38)

where $\zeta'_W = 3 \times 10^{-4}$ is the QCD correction factor, and

$$\text{Im}Z^u_{it} = \frac{1}{m_b m_t} \text{Im}[\gamma_{ji}\gamma^*_{ki}(V_{KM}Y^d_{ji})(V_{KM}Y^u_{ki})_{33}].$$  (39)

We find that in case a) the dominant contributions are from the two loop Weinberg operator. We have

$$D_n(W) \approx 1.6 \times 10^{-19} \frac{\text{GeV}^2}{m^2_{h^+_1}} \ln \frac{m^2_{h^+_1}}{m^2_{h^+_2}} \frac{m^2_t}{(150 \text{GeV})^2} \text{ecm},$$

$$D_n(W) \approx 1.4 \times 10^{-19} \frac{\text{GeV}^2}{m^2_{h^+_2}} \ln \frac{m^2_{h^+_2}}{m^2_{h^+_3}} \frac{m^2_t}{(150 \text{GeV})^2} \text{ecm},$$

$$D_n(W) \approx 1.2 \times 10^{-25} \frac{\text{GeV}^2}{m^2_{h^+_3}} \ln \frac{m^2_{h^+_3}}{m^2_{h^+_4}} \frac{m^2_t}{(150 \text{GeV})^2} \text{ecm}.$$  (40)

Requiring the contributions to be less than the experimental value, we find the masses of $h^+_{1,2}$ have to be larger than 2.5 TeV. There is no constraint on $h^+_3$ mass. Combining this information with those from Eqs.(22) and (23), we find the charged Higgs boson contributions to $\bar{\epsilon}_{LD}$ is less than $3 \times 10^{-5}$, and $\epsilon'/\epsilon$ is less than $3 \times 10^{-5}$.

For case b), we find the dominant contribution is from the one loop d quark EDM. We have
\[ D_n(d) \approx 5.4 \times 10^{-19} Im(\gamma_{22}\gamma_{23}^*) GeV^2 m_{\eta^+}^2 ln\frac{m_t^2}{m_{\eta^+}^2} \frac{m_t}{150 GeV} \text{ ecm} \]  

(41)

Requiring \( D_n(d) \) to be less than the experimental value, \( \bar{\epsilon}_{LD} \) is constrained to be less than \( 10^{-3} \), and \( \epsilon'/\epsilon \) can still be of order \( 10^{-3} \). Assuming maximum mixing, the mass of \( \eta^+ \) is constrained to be larger than 5 TeV.

We also checked cases for different values of \( \sigma \) and different mixings between \( \text{Re} h_i \) and \( \text{Im} h_j \). We find that for case a) if \( 95^0 > |\sigma| > 85^0 \), it is not possible to produce the experimental value for \( \bar{\epsilon} \) because the constraints on the neutral Higgs boson masses from the mass difference of the neutral B mesons is too strong. The model can explain the observed CP violation in the neutral K system even for \( |\sigma| = 10^0 \), and \( |180^0 - \sigma| = 10^0 \). The neutral Higgs boson masses are typically constrained to be larger than a few TeV. For angles close to zero and \( 180^0 \), CP violation for B system become small. \( Im \lambda(B_d \to J/\psi K_S) \approx Im \lambda(B_d \to \pi^+\pi^-) \) are between 0.05 \~ 0.45. \( Im(B_s \to \rho K_S) \) is less than 0.1. The neutron EDM due to the neutral Higgs bosons is typically less than \( 10^{-27} \text{ ecm} \). The contributions to the neutron EDM from the charged Higgs bosons can be close to the experimental upper bound. \( \epsilon'/\epsilon \) are typically less than \( 10^{-4} \). For case b), \( Im \lambda(B_d \to \pi^+\pi^-) \) and \( Im \lambda(B_d \to J/\psi K_S) \) vary between 0.02 to 0.3. \( Im \lambda(B_s \to \rho K_S) \) is less than 0.1. The neutron EDM due to neutral Higgs bosons is larger than \( 10^{-27} \text{edm} \) and can be as large as the experimental upper bound. The charged Higgs bosons contribution to the neutron EDM can be close to the experimental upper bound. \( \epsilon'/\epsilon \) can be of order \( 10^{-3} \).

The predictions also depend on the choices of the VEVs. The general features are, however, the same. For example for \( v_1 = 210 \text{ GeV}, v_2 = 3; GeV, v_3 = 0.5 \text{ GeV} \) and \( v_4 = 130 \text{ GeV} \), we find: 1) The neutral Higgs bosons are constrained to be in the multi TeV region; 2) \( \epsilon'/\epsilon \) is small in case a) and can be \( 10^{-3} \) in case b); 3) the predictions for CP violation B system maintain the same features; and 4) the predictions for the neutral EDM from the neutral Higgs bosons are in the same regions as discussed earlier.
VII. THE ELECTRON ELECTRIC DIPOLE MOMENT

The $S_3 \times Z_3$ model may also have interesting CP violating signatures in the lepton sector. We assume the same $S_3 \times Z_3$ assignments for the left handed and the charged right handed leptons as their quark partners [2]. The mass matrix and Yukawa couplings for the charged leptons are similar to the down quarks. One simply changes the parameters $(a, b, c, d, \text{ and } \xi)$ for quarks to $(a_l, b_l, c_l, d_l, \text{ and } \xi_l = |\xi|e^{i\sigma'})$ for leptons. We use [3]:

\[ a_l = 0.106 GeV, \quad b_l = 0, \quad c_l = 1.781 GeV, \quad d_l = 8.6 \times 10^{-3} GeV. \]

For this set of parameters, we have $m_e = 0.511 MeV$, $m_\mu = 106 MeV$ and $m_\tau = 1784 MeV$ which are in good agreement with experimental data. We again consider two cases: a) CP violation purely due to complex Yukawa coupling with the phase $\sigma' = 80^0$; and case b) CP violation due to $Reh_2^0$ and $Imh_3^0$ mixing. If right handed neutrinos exist, the charged Higgs bosons will also contribute to electron EDM. However due to very small neutrino masses, the contribution to the electron EDM is negligibly small. We will not consider them here.

For case a) we find that the one loop contributions are small ($< 10^{-29} ecm$). However the two loop contribution due to Bar-Zee mechanism [19,22] can be as large as

\[
    d_e(Reh_1^0) \leq 10^{-27} ecm, \\
    d_e(Reh_2^0) \leq 1.5 \times 10^{-27} 
\]

for $m_t < 200 GeV$. $Reh_3^0$ contribution is much smaller. For case b), we find that the one loop and two loop contributions are small ($< 10^{-33} ecm$).

For case a) the predictions depend on the phase $\sigma'$. However as long as the phase is not too close to $0^0$ and $180^0$ ($|s\sigma'| \geq 0.17$), the electron EDM can be a few times of $10^{-27} ecm$. It is below the experimental upper bound [23]. For case b), different mixings may have different values for electron EDM and can be larger than the special case considered here, but is less than $10^{-27} ecm$. For example, the case for $Re_1^0$ and $Imh_2^0$, $d_e$ is about $2 \times 10^{-28} ecm$. Varying VEVs can change the predictions for the electron EDM. Using the set of VEVs discussed at the end of the last section, we find the value for electron EDM is
smaller by one order of magnitude for case a), and for case b) we again obtain small electron EDM.

VIII. CONCLUSIONS

We have studied in detail some effects due to two different CP violating mechanisms in the $S_3 \times Z_3$ model. Both mechanisms discussed in this paper can explain the observed CP violation in the neutral K system. CP violation in the neutral K system and the mass difference in the neutral B system constrain the neutral Higgs boson masses to be in the multi TeV region. The predictions for other CP violations observables are very different from the MSM.

i) $\epsilon'/\epsilon$: In the MSM, depending on the top quark mass, $\epsilon'/\epsilon$ can be as large as $10^{-3}$. In the $S_3 \times Z_3$ model $\epsilon'/\epsilon$ due to neutral Higgs bosons is small. The charged Higgs boson contribution for $\epsilon'/\epsilon$ in case a) is also small. But for case b), $\epsilon'/\epsilon$ can be as large as $10^{-3}$. The measurement of $\epsilon'/\epsilon$ may distinguish the MSM and case b) from case a).

ii) CP violation in B system: In section V, we discussed three asymmetries in the neutral B system. In the MSM model these three asymmetries measure three angles related to the CKM matrix. The sum of these angles is equal to 180$^\circ$. However in the $S_3 \times Z_3$ model, CP violation in the neutral B system comes from different sources. The predictions are very different from the MSM. If we interpret the measurements of the three parameters in Eq. (27) in terms of the three angles, we find their sums are not equal to 180$^\circ$ in both a) and b) cases and more over $\sin 2\alpha = -\sin 2\beta$. These experiments should distinguish the MSM from the $S_3 \times Z_3$ model and the B factory will provide us with very useful information.

iii) The Electric Dipole Moment: The predictions for the neutron and electron EDMs are several orders of magnitude larger than those from the MSM. There are also dramatic differences between the two cases considered. The neutron EDM can be as large as the experimental upper bound for case b). It is smaller in case a). The electron EDM is below the experimental bound, but future experiments will reach the sensitivity necessary to test
case a). The electron EDM is smaller in case b). It is also interesting to note that some of the charged Higgs boson masses are also constrained to be in the TeV region. This is different from flavor conserving multi-Higgs models, where limits are much weaker.

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