Regge trajectories of Excited Baryons, quark-diquark models and quark-hadron duality

Pere Masjuan\*,\*

1Grup de Física Teòrica, Departament de Física, Universitat Autònoma de Barcelona and Institut de Física d’Altes Energies (IFAE), The Barcelona Institute of Science and Technology (BIST), Campus UAB, E-08193 Bellaterra (Barcelona), Spain

2Departamento de Física Atómica, Molecular y Nuclear and Instituto Carlos I de Física Teórica y Computacional Universidad de Granada, E-18071 Granada, Spain.

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The Particle Data Group (PDG) booklet [1] compiles the agreed upon result of the many existing phenomenological analyses and identification of states, mostly resonances, determined from hadronic reactions and, till now, the classification and listings faithfully feature the quark model classification of states. This scheme records and rates single states one-by-one with growing number of stars, , , , , , , depending on the increasing estimated confidence level on their existence [2]. The rating of states changes with time depending on the detailed features of the analysis [3].

One should remind that within a Hamiltonian viewpoint, resonances in the continuum are identified as the so-called Gamow states which are not normalizable in the usual Hilbert space sense, as they are not conventional irreducible representations of the Poincaré’s group [4]. Even in the simplest potential scattering situation the completeness relation involves bound states and the continuum, which can be rewritten as a discrete sum of the Gamow states and a remainder which is generally non-vanishing [5]. This circumstance adds more difficulties to a practical definition of completeness [6].

The completeness issue becomes particularly severe in the case of baryons; the exceedingly many baryonic excited states predicted by the quark model (where baryons are \(qqq\) states) [7] have apparently not yet been found. This persistent puzzle is referred to as the missing resonance problem [8] (see also [9] and [10] for reviews and references therein). The two commonly accepted, not necessarily incompatible, possible explanations to the puzzle assume i) Weak coupling of the predicted states to the particular production process (photoproduction, \(\pi N\) scattering etc.) or ii) Dynamical reduction of degrees of freedom due to diquark clustering (for a review see [11]). Both possibilities have triggered a great deal of experimental as well as theoretical activity, but again based on individual one-to-one mapping of resonance states which have a mass spectrum and which are produced with different backgrounds. More recently, a revision of the resonance ratings problem suggests that the missing resonances might indeed be found in the intricacies of partial wave analyses to a large database [3]. Under these circumstances independent sources of information are most welcome.

In the opposite extreme to the individual resonance approach, a thermodynamic approach to the completeness problem is more global as it concerns all states as a whole and can be verified from the study of the equation of state; at not too high temperatures most resonances behave as narrow and they can effectively be regarded as particles in the partition function [12] [13]. Some separation of quantum numbers can be imposed with the study of susceptibilities of conserved charges, where a combination of degeneracy and level density is involved (see e.g. Ref. [14] and references included).

In the present work we return to an intermediate possi-

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1 Gamow states belong instead to the rigged Hilbert space where completeness has a well defined meaning, see e.g. Ref. [6] for a pedagogical exposition and references therein. For the present paper we stay within the conventional Hilbert space interpretation.
bility to address completeness for baryons inspired by the old notion of Quark-Hadron duality, namely the coupling of QCD quark bilinear currents to hadrons close to the Bjorken limit. Quark-hadron duality was first established for mesons. Building on previous works \[19\] a complete analysis of qq states, the situation is much simpler and quark-hadron duality provides a sort of closure (completeness) relation for narrow resonant states.

In the meson case, where currents connect the vacuum with bar states \[20\], in the present paper we follow a similar strategy to analyze excited baryons. The paper is organized as follows. In Section II we review the asymptotic uniformly distributed mass squared spectrum (spinor details are lengthy and handled in Section V. Finally, in Section VI we draw our main conclusions.

II. BJORKEN SCALING AND BARYON SPECTRUM

We start by reviewing here in a simplified way the main argument \[17\] leading to the asymptotic uniformly distributed mass squared spectrum (spinor details are lengthy and handled in that work). For unpolarized nucleon targets the forward Compton scattering in the gauge invariant form reads\[2\]

\[
W_{\mu\nu}(p,q,s) = \frac{1}{4\pi} \int d^4x e^{i\mathbf{x}\cdot\mathbf{q}} \left\langle p, \lambda | J_{\mu}(x), J_{\nu}(0) | p, \lambda \right\rangle
\]

\[
= \left( -g_{\mu\nu} + \frac{q_{\mu} q_{\nu}}{q^2} \right) W_1(v, Q^2)
\]

\[
+ \left( p_{\mu} - q_{\mu} \frac{p \cdot q}{q^2} \right) \left( p_{\nu} - q_{\nu} \frac{p \cdot q}{q^2} \right) \frac{1}{M^2} W_2(v, Q^2)
\]

with \( J_{\mu} = \bar{q} \hat{Q} q \) the e.m. current and \( \hat{Q} = (2/3, -1/3, -1/3) \) the charge operator for (u,d,s) quarks, \( p^2 = M^2 \) with \( M \) the nucleon mass, \( \lambda \) the nucleon helicity, \( v = p \cdot q / M \) and \( Q^2 = -q^2 \). For the parton model in the Bjorken limit, \( Q^2 \to \infty \) with \( x = Q^2/(2p \cdot q) \) fixed, one obtains both scaling

\[
W_1(x, Q^2) \to F_1(x), \quad \frac{p \cdot q}{M^2} W_2(x, Q^2) \to F_2(x),
\]

and the Callan-Gross relation \( F_2(x) = 2xF_1(x) \) featuring the Spin 1/2 nature of partons, so that we may focus on just one, say \( F_1(x) \). Defining \( J_x = J(0) \cdot \epsilon \) with \( \epsilon \) a four-vector such that \( \epsilon \cdot x = -1 \) and \( \epsilon \cdot q = \epsilon \cdot p = 0 \) we get \( W_1 = \epsilon_{\alpha} W_{\mu\nu} \). Inserting a complete set of eigenstates \( |\alpha\rangle \), we get

\[
W_1 = (2\pi)^3 \frac{1}{2} \sum_{\alpha, \lambda} \left| \langle p, \lambda | J_x | \alpha \rangle \right|^2 \delta(p + q - P_\alpha).
\]

Taking \( |\alpha\rangle \equiv |p + q, J_m v n\rangle \) where \( J, m, v, n \) specify the angular momentum, magnetic, normality and radial quantum numbers respectively for the resonance \( R \) with mass \( M_R = M_{Jvn} \), in our normalization we have

\[
\sum_{\alpha} \to \sum_{J_m v n} \int \frac{d^3 P_R}{(2\pi)^3} \frac{M_R}{E_R}
\]

\[
= \sum_{J_m v n} \int \frac{d^3 P_R}{(2\pi)^3} 2M_R \delta(P_R^2 - M_R^2).
\]

In order to see how scaling arises in the Bjorken limit at the hadronic level consider the invariant \( N + \gamma^* \) squared mass

\[
s = (p + q)^2 = M^2 + Q^2(1/x - 1),
\]

and assume that \( N \to R \) transition form factors fulfills a scaling relation in terms of the resonance mass, \( M_R \), of the form

\[
\sum_{J_m v n} |\langle p, \lambda | J_x | p + q, J_m v n \rangle|^2 = \frac{M_R M^2}{Q^2} \left( G_{Jv} \left( -\frac{Q^2}{M_R^2} \right) \right)^2.
\]

We are left with \( \sum_{J_m v n} \) and replacing the sum in \( n \) by an integral we can evaluate the Dirac delta function and get a factor \( d\nu / dM^2 = 1/(dM_{Jvn}^2 / d\nu) \).

\[
W_1 = \sum_{J_m v n} \left[ G_{Jv}(-Q^2/M_{Jvn}^2) \right] \frac{M_{Jvn}^2}{Q^2} \delta(s - M_{Jvn}^2)
\]

\[
\to F_1(x) = \frac{1}{x} \frac{1}{x} \sum_{J_m v n} \left[ G_{Jv} \left( \frac{x}{x-1} \right) \right]^2 \frac{M^2}{dM_{Jvn}^2 / d\nu}.
\]

Scaling follows from a constant mass squared level density

\[
dM_{Jvn}^2 / d\nu = \frac{\mu_{Jv}^2}{4\pi}. \]

In Ref. \[17\] it was found that this also implies the Drell-Yan-West asymptotic relation between the form factor \( G(-Q^2) \to Q^{-n} \) at large \( Q \) and the structure function \( F_1(x) \to (1 - x)^{n-1} \) for \( x \to 1 \). While the analysis of Ref. \[17\] assumed the completeness relation for narrow resonances, the authors reinstated the finite width by replacing the Dirac-\( \delta \) of Eq. \[4\] into a Breit-Wigner form and by making use of the relation \( M_{Jvn} \Gamma_{Jvn} = \gamma (M_{Jvn}^2 - M_{J,0}^2) \) with \( \gamma = 0.13 \) empirically noted by Suranyi in 1967\[3\] (see below for an upgrade) accounting...
for the existing SLAC data at the time. Based on this duality picture best empirical fits to precision inclusive electron-neutron cross-sections data in the resonance region have been presented (see e.g. Ref. [21, 22] for a more recent analysis).

The previous discussion was restricted to vector currents. On more general grounds this argument holds also for any composite bilinear current \( J = \bar{q} \Phi q \) with \( \Phi \) a Dirac spinor operator with any quantum number connecting the nucleon with a baryon resonance. Thus, we expect any excited baryon to asymptotically follow the equal spacing mass squared formula, Eq. (8).

### III. QUARK-DIQUARK MODELS

In the conventional quark model, baryons are made of three quarks. However, there has traditionally been some mounting evidence that the baryonic spectrum can be understood in terms of quark-diquark degrees of freedom (see e.g. Ref. [11] for a review but also Ref. [3] for evidence against it). Within the non-relativistic quark model, diquark clustering has been investigated [23]. There exist analyses at the Non-relativistic [24] and Relativistic [25, 26] levels where scalar and axial-vector diquarks have a mass of about 600 MeV and 400 MeV respectively (the diquark mass difference seems quite model independent and about 200 MeV). While diquarks do not resolve the missing resonance problem, they ameliorate it since many states predicted by the quark model do not appear [7]. Actually, the relativistic diquark model [25, 26] does not predict missing states below 2 GeV, whereas Isgur and Capstick have 5 [7]. On the lattice, some evidence on diquarks correlations in the nucleon [27] and the dominance of the scalar diquark channel [28] have been reported. More recently, within the framework of Dyson-Schwinger and Faddeev equations, the diquark-approximation has been found to work well [29]. Radial Regge behavior in the relativistic quark-diquark picture has been found from a numerical analysis of the spectrum [30].

The scaling requirement at the hadronic level implies an equidistant mass squared spectrum for the intermediate baryonic states/resonances. We will first argue that this condition may be interpreted as a quark-diquark (qD) spectrum and adopt the same argument as in the \( \bar{q}q \) case based on the Salpeter equation [31]. The scaling of Dirac fermions with a heavy elementary diquark with a confining potential has also been studied exactly and in the WKB approximation [32, 33]. If, for simplicity, we assume scalar particles and a confining qD potential, massive diquark with mass \( m_D \) the Hamiltonian in the CM frame reads

\[
M = \sqrt{p^2 + m_D^2} + p + \sigma_{qD},
\]

where \( \sigma_{qD} \) is a qD “string tension”. This dynamical setup is common to qD models (see e.g. [24, 26, 30, 34]) where, in addition, Coulomb, spin-spin, spin-orbit splitting terms are added. Besides some phenomenological success in predicting the baryonic spectrum and the familiarity with the \( \bar{q}q \) case, we do not know of any theoretical justification of the qD linear potential term. Good spectra are obtained for \((u, s, d)\) with \( \sigma_{qD} = 2.15 \text{fm}^{-2} \) [25], for \((u, d)\) \( \sigma_{qD} = 1.57 \text{fm}^{-2} \) [34] and for \((c, b)\) with \( \sigma_{qD} = 4.5 \text{fm}^{-2} \) [30].

For excited states we can work at the semi-classical level where just the long-distance and high-momentum configurations dominate. Indeed, the Bohr-Sommerfeld quantization condition for \( L = 0 \) yields (see e.g. Ref. [31])

\[
\oint p_{\alpha} dr = 2\pi (n + \alpha) \rightarrow \frac{dn}{dM^2} = \frac{1}{4\pi \sigma_{qD}} \left( 1 - \frac{m_D^2}{M^2} \right) \quad (10)
\]

with \( \alpha \) a constant of order unity \( ^4 \). For \( M \gg m_D \) implies a Radial Regge spectrum for large \( n \) exactly as Eq. (8) with

\[
\mu_{qD}^2 = 4\pi \sigma_{qD}. \quad (11)
\]

Thus, quark-diquark dynamics with a linearly confining interaction generates excited Baryon states whose masses are asymptotically consistent with Quark-Hadron duality in DIS. The corresponding slopes found in qD models are flavour dependent, \( \mu_{qD}^2 = 0.76 \text{GeV}^2 \) for \((u, d)\) [34], \( \mu_{qD}^2 = 1.05 \text{GeV}^2 \) for \((u, d, s)\) [25] and \( \mu_{qD}^2 = 2.20 \text{GeV}^2 \) for \((c, b)\) [30].

The role played by diquarks in high-energy processes has been repeatedly described in the past [11] but the connection between this linearly confining qD interaction and DIS has been overlooked. In our view, this connection also provides an a posteriori reason-of-being for recent quark-diquark models so that their phenomenological success appears more natural [24, 26]. Nonetheless, while the absence of missing states below 2 GeV in (relativistic) quark-diquark models [25, 26] speaks in favor of the diquarks as effective degrees of freedom, it is unclear how the agreement should be quantified taking into account that they are resonances.

### IV. PHENOMENOLOGICAL REGGE ANALYSIS

Based on the previous discussion, for fixed \( J^{PC} \) baryon quantum numbers we propose the simple fitting formula

\[
M_{vn}^2 = \mu_{vn}^2 n + M_{0,vn}^2. \quad (12)
\]

In order to verify the accuracy of the radial Regge formula, Eq. (12), based on the PDG listings [11] we should provide an educated guess for the error, taking into account that almost all states are in fact resonances having a mass \( M \) and a width \( \Gamma \). Contrary to what it is often stated, the hadron resonances listed in the PDG are, in average, narrow since the width/mass ratio becomes \( \Gamma/M = 0.12(8) \) both for mesons and baryons [35, 36] a figure which upgrades Suranyi’s result [12] and, actually, agrees remarkably well with a salient feature of the large-\( N_c \) limit of QCD where \( \Gamma/M = \theta(N_c^{-1}) \) [38] which for \( N_c = 3 \) would numerically give a natural value \( \Gamma/M \sim 0.33 \).
In order to carry out our analysis based on the 2016 PDG compilation \( [1] \) we invoke the half-width rule (HWR) motivated previously \( [20] [39] \). That way we side-step possible channel-dependent and model-dependent extractions of the resonance parameters assuming process-dependent backgrounds and take into account their natural width. Moreover, we are then able to undertake an error analysis. The method complies to the fact that the pole position of the resonance is typically shifted from channel-dependent extractions by an amount \( \sim \pm \Gamma/2 \). Specifically, to incorporate the half-width rule in practice, we take along with Refs. \( [20] [36] [37] [40] \) the \( \sim \pm \) amount is typically shifted from channel-dependent extractions by an amount \( \sim \pm \Gamma/2 \).

In our analysis we use \( n_N = 22 \) and \( n_\Delta = 6 \) states taken from the 2016 PDG compilation \( [1] \). Our results are presented in Figs. 1 and tables \( \text{II} \) and \( \text{I} \) corresponding to fitting all states and just the first three states in the radial Regge trajectories respectively. As we see, all \( N \)-states and \( \Delta \)-states slopes are very much alike alike, but there is some difference between the \( N \) and \( \Delta \) slopes when fitted separately, \( \mu_N^2 \sim 0.62 \text{GeV}^{-2} \) and \( \mu_\Delta^2 \sim 1 \text{GeV}^{-2} \). A combined \( N \) and \( \Delta \) fit with the same radial slope using \( \chi_{tot}^2 = \chi_N^2 + \chi_\Delta^2 \) produces

\[
\mu^2 = 0.651(40) \text{GeV}^2, \quad \chi_{tot}^2/\text{DOF} = 0.49. \tag{14}
\]

The number comes so much closer to the \( N \)-case since we have about four times more \( N \)-states than \( \Delta \)-states. If we weight as to give the same relative importance for both \( N \) and

\[
\begin{array}{lll}
N, j^P & \mu^2 \, [\text{GeV}^2] & \chi^2/\text{DOF} \quad j^P & \Delta, j^P & \mu^2 \, [\text{GeV}^2] & \chi^2/\text{DOF} \\
\hline 1^+ & 0.764(177) & 0.06 & 3^+ & 1.061(203) & 0.01 \\
2^+ & 0.657(36) & 0.01 & 5^+ & 0.719(127) & 0.15 \\
5^+ & 0.569(19) & 0.02 & \quad & \quad & \quad \\
1^- & 0.700(126) & 0.64 & 7^- & 0.757(21) & 0.02 \\
3^- & 0.640(69) & 0.16 & \quad & \quad & \quad \\
5^- & 0.757(21) & 0.02 & \quad & \quad & \quad \\
\hline
\text{Weighted Av.} & 0.657(13) & 1.011(153) & \quad & \quad & \quad \\
\text{Combined fit} & 0.617(42) & 0.25 & \quad & \quad & \quad \\
\end{array}
\]

\[
\begin{array}{lll}
N, j^P & \mu^2 \, [\text{GeV}^2] & \chi^2/\text{DOF} \quad j^P & \Delta, j^P & \mu^2 \, [\text{GeV}^2] & \chi^2/\text{DOF} \\
\hline 1^+ & 0.764(177) & 0.06 & 3^+ & 1.061(203) & 0.01 \\
2^+ & 0.657(36) & 0.01 & 5^+ & 0.719(127) & 0.15 \\
5^+ & 0.569(19) & 0.02 & \quad & \quad & \quad \\
1^- & 0.700(126) & 0.64 & 7^- & 0.757(21) & 0.02 \\
3^- & 0.640(69) & 0.16 & \quad & \quad & \quad \\
5^- & 0.757(21) & 0.02 & \quad & \quad & \quad \\
\hline
\text{Weighted Av.} & 0.647(12) & 1.011(153) & \quad & \quad & \quad \\
\text{Combined fit} & 0.594(46) & 0.20 & \quad & \quad & \quad \\
\end{array}
\]

FIG. 1. Radial Regge trajectories for the excited \( N \) baryons (top) and excited \( \Delta \) baryons (bottom). The \( \pm \) sign indicates the Parity of the state: \( 1/2^+ \) is shown as solid black, \( 3/2^- \) as red dashed, \( 3/2^+ \) as dotted blue, \( 5/2^- \) as long dashed brown, \( 5/2^+ \) as dot-dashed orange, \( 5/2^- \) as very long dashed green.

\[
\Delta \text{-states as follows}
\]

\[
\chi^2 = (n_N + n_\Delta) \left[ \frac{1}{n_N} \chi_N^2 + \frac{1}{n_\Delta} \chi_\Delta^2 \right], \tag{15}
\]

we get

\[
\tilde{\mu}^2 = 0.750(32) \text{GeV}^2, \quad \tilde{\chi}^2/\text{DOF} = 1.28. \tag{16}
\]

These values should be compared with the \( \tilde{\mu}_{qD}^2 = 0.76 \text{GeV}^2 \) for \((u,d)\) flavours \( [34] \), whereas \( \tilde{\mu}_{qD}^2 = 1.05 \text{GeV}^2 \) for

\[
\begin{array}{lll}
N, j^P & \mu^2 \, [\text{GeV}^2] & \chi^2/\text{DOF} \quad j^P & \Delta, j^P & \mu^2 \, [\text{GeV}^2] & \chi^2/\text{DOF} \\
\hline 1^+ & 0.764(177) & 0.06 & 3^+ & 1.061(203) & 0.01 \\
2^+ & 0.657(36) & 0.01 & 5^+ & 0.719(127) & 0.15 \\
5^+ & 0.569(19) & 0.02 & \quad & \quad & \quad \\
1^- & 0.700(126) & 0.64 & 7^- & 0.757(21) & 0.02 \\
3^- & 0.640(69) & 0.16 & \quad & \quad & \quad \\
5^- & 0.757(21) & 0.02 & \quad & \quad & \quad \\
\hline
\text{Weighted Av.} & 0.647(12) & 1.011(153) & \quad & \quad & \quad \\
\text{Combined fit} & 0.594(46) & 0.20 & \quad & \quad & \quad \\
\end{array}
\]
(u,d,s) \[25\] and $\mu_{qD}^2 = 2.20 \text{GeV}^2$ for (c,b) \[30\]. Unfortunately none of these works provides uncertainties nor a value of the $\chi^2$, but the agreement is nonetheless encouraging as we only have (u,d) quarks. A hybrid quark-diquark baryon model provides directly a mass formula with splitting terms including or not both qD as well as qq terms \[43\] finding $\mu^2 = 1.24(5), 1.48(5) \text{GeV}^2$ and $\mu^2 = 1.50(13) \text{GeV}^2$ in different scenarios.

Motivated by quark-hadron duality, Eq. \[1\], assuming instead $G_{vn}$, when the role of $n$ and $J$ in the double summation are interchanged we should get $dM^2_{J\nu}/dJ = \beta_{\nu n}$. This suggests the angular-momentum Regge trajectory

$$M^2_{J\nu} = a_{\nu n} + \beta^2_{\nu n} J = a_{\nu n} + \beta^2_{\nu n} (L + S),$$

with $S = 1/2$ for excited N and $S = 3/2$ for excited $\Delta$. Results for the fit are given in Table \[III\] and are plotted in Fig. \[2\]. Note that if $G_{J\nu} = G_{vn} = G_v$ we should have from Eq. \[1\] that $\beta^2_{J\nu} = \beta^2_{vn}$, which does not hold since we have $\mu^2 = 0.750(30) \text{GeV}^2$ and $\beta^2 = 1.128(56) \text{GeV}^2$.

**VI. CONCLUSIONS**

We summarize our results. Quark-Hadron duality for bilinear quark currents connecting the nucleon with Baryon resonances provides in the Bjorken limit a restriction on the mass spectrum of excited baryonic states and, more specifically, on the asymptotic level density . While the study of the Baryonic spectrum is generally and notoriously more difficult than the Mesonic spectrum some features become remarkably similar on the light of quark-hadron duality. The uniform mass squared spectrum distribution befits a radial Regge-like spectrum in both cases, and points to an effective two-body $q\bar{q}$ and qD dynamics for mesons and baryons respectively with a long distance linearly growing potential. Besides, the phenomenological description of the PDG listings of these radial formula,

$$M^2_{J\nu} = 4\lambda^2 (n + L + 3/2) - 2(6\lambda^2 - M^2_N)\kappa,$$

with $M_N = 0.940 \text{GeV}$ and $\kappa$ a good diquark fraction for each baryon. (In particular, $\kappa = 0$ for all $\Delta$ and spin–3/2 nucleon resonances, $\kappa = 1/2$ for nucleons in the ground state and $\kappa = 1/4$ for the spin–1/2 negative-parity nucleon excitations.) From our previous results of the combined $N$ and $\Delta$ fit on Eq. (12), $\lambda = 0.40(5) \text{GeV}$. However, Forkel and Klempt, used $\lambda = 0.52 \text{GeV}$ for all of them. The reason for such a difference is basically our more restricted set of states: we considered trajectories with at least three states which immediately rejects those with $J > 5/2$. If they were included a larger slope would be found as it can be inferred from the result in table \[III\]. Beyond that, some states have changed since 2009 thanks to improvements in spectroscopy experiments, including those that simply disappeared since then \[2\].

Still within the same classification scheme \[44\], one can also consider a non-universal radial and angular-momentum trajectories and fit the following function:

$$M^2_{J\nu} = 4\lambda^2 (n + 3/2) + 4\lambda^2 L - 2(6\lambda^2 - M^2_N)\kappa,$$

with $\lambda_n = 0.507(10)$ and $\lambda_L = 0.514(11) \text{GeV}$, well compatible within errors with $\chi^2$/DOF = 0.26. Fitting the nucleon sector exclusively, we find $\lambda_n = 0.520(14)$, and $\lambda_L = 0.512(14) \text{GeV}$, with $\chi^2$/DOF = 0.23.

Instead of fitting the spectrum using their ansatz for the mass square, we can use a generic $M_{J\nu}^2 = a + bL + cJ$. We will obtain $a = 1.45(9) \text{GeV}^2$, $b = 1.00(6) \text{GeV}^2$, and $c = 0.83(9) \text{GeV}^2$, which shows certain tension, with $\chi^2$/DOF = 0.82.

A similar exercise can be performed with the spectrum proposed by Ferretti, Vassallo, and Santopinto based on solving a quark-diquark model with a linearly confining term \[25\] for the $\Delta$ baryons. Again, a fit to $M_{J\nu}^2 = a + bL + cJ$ yields $a = 1.53(12) \text{GeV}^2$, $b = 1.16(34) \text{GeV}^2$, and $c = 1.11(13) \text{GeV}^2$, which shows nice agreement between slopes, with $\chi^2$/DOF = 1.36. This is in harmony with our semi-classical argument in Section \[III\].

**V. COMPARISON WITH OTHER STUDIES**

The fitting equations discussed above should be compared with other mass formulas to the PDG listings. Forkel and Klempt based on the holographic approach \[44\] take the mass...
Regge trajectories is satisfactory if the finite width of the resonances is included in the error budget. Of course, the previous arguments invoke only consistency conditions based on high energy completeness of the baryon resonance spectrum. More work would be needed to provide a microscopic and dynamical justification for a dominating quark-diquark picture for excited baryons. Conversely, we also expect some quantitative guiding information on the slope of the radial Regge trajectories from the analysis of structure functions with different conserved currents as they correspond to fluctuations inside the nucleon, probing the relative three quarks vs quark-diquark content in the excited baryon spectrum.

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