Self-consistent single-nucleon and single-Λ potentials in strange nuclear matter with the Dirac-Brueckner-Hartree-Fock approach

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Abstract

We calculate self-consistent single-nucleon and single-Λ potentials in hyperonic matter for different Λ concentrations. The predictions include relativistic Dirac effects as reported previously in a calculation of the Λ binding energy in symmetric nuclear matter. We discuss the dependence on momentum, density, and Λ fraction.

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1 Introduction

Studies of hyperon energies in nuclear matter are a useful starting point for calculations of bulk properties of hypernuclei. Furthermore, inclusion of strangeness in the equation of state is important towards a better understanding of the properties of matter inside neutron stars. This work is another step in our systematic exploring of nuclear and neutron matter under diverse conditions.

Previously [1] we reported predictions of the Λ binding energy in nuclear matter using the most recent meson-exchange nucleon-hyperon (NY) potential from the Jülich group [2] (hereafter referred to as “NY05”) and including relativistic “Dirac” effects on the $^{1}N\Lambda$ potential. In this paper, we will explore the behaviour of the self-consistently calculated single-Λ and single-nucleon potentials with changing Λ fraction.

In our previous paper on the subject [1], we discussed how some major quantitative differences between NY05 and the earlier version of the Jülich NY meson-exchange potential [7] impact predictions in dense matter. Accordingly, we expect large quantitative differences (beyond relativistic effects) when comparing with earlier microscopic calculations conducted within the Brueckner-Hartree-Fock framework (see, for instance, Refs. [3, 4]), which used the Nijmegen [5] and/or the Jülich [6, 7] NY meson-exchange potentials.

2 The framework

For matter with non-vanishing hyperonic densities, the nucleon, Λ, and Σ single-particle potentials are the solution of a coupled self-consistency problem:

$$U_N = \int_{k<k_F^N} G_{NN} + \int_{k<k_F^\Lambda} G_{N\Lambda} \int_{k<k_F^\Sigma} G_{N\Sigma}$$

(1)
$$U_\Lambda = \int_{k<k_F^\Lambda} G_{\Lambda N} + \int_{k<k_F^\Lambda} G_{\Lambda \Lambda} + \int_{k<k_F^\Lambda} G_{\Lambda \Sigma}$$
$$U_\Sigma = \int_{k<k_F^\Sigma} G_{\Sigma N} + \int_{k<k_F^\Sigma} G_{\Sigma \Lambda} + \int_{k<k_F^\Sigma} G_{\Sigma \Sigma}$$

In the above equations, $G_{NN}$, $G_{NY}$, and $G_{YY'}$, $(Y,Y' = \Lambda, \Sigma)$, are the nucleon-nucleon, nucleon-hyperon, and hyperon-hyperon $G$-matrices at some nucleon and hyperon densities defined by the Fermi momenta $k_F^N$ and $k_F^\Lambda$.

In the present calculation we consider a non-vanishing density of $\Lambda$’s but ignore the presence of real $\Sigma$’s in the medium (although both $\Lambda$ and $\Sigma$ are included in the coupled-channel calculation of the $NY$ $G$-matrix, with free-space energies used for the latter). This scenario can be justified noticing that the (strong) reaction $N + \Sigma \rightarrow N + \Lambda$ is energetically always allowed, although it could be prevented by the Pauli principle at large hyperon concentrations. Thus, over a time scale which is long relative to strong processes, and for small hyperon concentrations, equilibrated matter can be reasonably assumed to contain mostly nucleons and $\Lambda$’s [3]. Also, we neglect the $YY'$ interaction, as very little information is available about it. For these reasons, we keep the $\Lambda$ concentration relatively low.

We start with symmetric nuclear matter at some Fermi momentum $k_F^N$ in the presence of a “$\Lambda$ impurity”, i.e. $k_F^\Lambda \approx 0$ and then increase the $\Lambda$ concentration. The parameters of both the nucleon and the $\Lambda$ potentials are calculated self-consistently with the $G_{NY}$ and the $G_{NN}$ interactions, which are the solution of the Bethe-Goldstone equation with $NY$ and $NN$ potentials, respectively. We solve the coupled self-consistency problem above using the same techniques as described previously for isospin-asymmetric matter [8]. In the Brueckner calculation, density-dependent effects come in through angle-averaged Pauli blocking and dispersion. Most of the details for the nucleon-nucleon sector are given in Ref. [8]. For two particles with masses $M_N$ and $M_Y$, $(Y = \Lambda, \Sigma)$, and Fermi momenta $k_F^N$ and $k_F^Y$, Pauli blocking requires

$$\frac{(\frac{M_N}{M} P)^2 + k^2 - (k_F^N)^2}{2P\frac{k}{M}} > \cos \theta > -\frac{(\frac{M_Y}{M} P)^2 + k^2 - (k_F^Y)^2}{2P\frac{k}{M}}.$$  \tag{2}$$

where $\theta$ is the angle between the total ($\vec{P}$) and the relative ($\vec{k}$) momenta of the two particles, and $M = M_Y + M_N$. Angle-averaging is then applied in the usual way.

Dirac effects are applied in the $NN$ as well as the $NA$ potentials. This entails involving the $\Lambda$ single-particle Dirac wave function in the self-consistent calculation through the $\Lambda$ effective mass [1]. For the nucleon-nucleon sector, we use the Bonn B potential [9], which is a relativistic one-boson exchange potential developed within the context of the Thompson equation and uses the pseudovector coupling for pseudoscalar mesons. The problem associated with the DBHF approach and the use of the pseudoscalar coupling for the interactions of pions and kaons with nucleons and hyperons, (which occurs for the Jülich NY potential), was discussed in Ref. [1].
Figure 1: Single-nucleon and single-Λ potentials as a function of the momentum at constant total density and for varying Λ concentrations. The dotted and the dashed lines are DBHF predictions for the Λ and the nucleon potentials, respectively, while the dash-dotted and the solid curves are the corresponding results without the Dirac effects. See text for more details.
Figure 2: The nucleon (dashed line) and the Λ (dotted) effective masses versus the Λ fraction at fixed density. The predictions correspond to the dashed and dotted lines in Fig. 1.

3 The single-particle potentials

We show in Fig. 1 the single-nucleon and the single-Λ potentials as a function of the momentum. The first panel corresponds to a nucleon Fermi momentum of $1.35 \text{fm}^{-1}$ and zero density of Λ particles, that is, the situation of a Λ impurity in nuclear matter. Notice that this implies

$$k_F^N = k_F(1 + \alpha)^{1/3} = 1.35 \text{fm}^{-1},$$

and

$$k_F^\Lambda = k_F(1 - \alpha)^{1/3} = 0,$$

where $k_F$ is the average Fermi momentum and $\alpha = \frac{\rho_N - \rho_\Lambda}{\rho_N + \rho_\Lambda} = 1$. The other three cases correspond to $\alpha = 0.8$, 0.7, and 0.6, keeping the average Fermi momentum constant. In terms of Λ fraction, $Y_\Lambda$, the above $\alpha$ values correspond to $Y_\Lambda = 0$, 0.1, 0.15, and 0.2, respectively. In all panels, the two lower curves (solid and dash) are the results of a conventional Brueckner calculation (solid) and of a DBHF calculation (dash) for the nucleon potential, whereas the higher curves have the same meaning for the Λ single-particle potentials, with the dotted lines being the Dirac predictions. In the first panel of Fig.1, the value of $U_\Lambda$ at $k=0$ is of course the Λ binding energy we calculated in Ref. [1]. The Dirac effect is small at low momentum but increases considerably with increasing momentum, and even more so for the nucleon.

With increasing Λ concentration, the total density remaining constant, both potentials become less attractive, the effect being more pronounced for the Λ. Consistent with Fig. 1, the Λ effective mass becomes larger with increasing $Y_\Lambda$ (although very slowly), while the nucleon effective mass is considerably less sensitive to these (small) variations, see Fig. 2.
Figure 3: DBHF predictions for the single-nucleon and the single-Λ potentials as a function of the momentum at two different densities and fixed Λ concentrations. The dotted(dash-dotted) and the dashed(solid) lines refer to the Λ and the nucleon potentials, respectively, at the larger(smaller) density. See text for more details.

The results shown in Fig. 1 can be understood observing that, as the Λ fraction increases, $U_\Lambda$ will receive contribution from a smaller number of nucleons (the ΛΛ interaction is neglected, see second line of Eq. (1)), whereas $U_N$ will have a slightly reduced contribution from $G_{NN}$ and a slightly enhanced contribution from $G_{NA}$, see first line of Eq. (1). Also, the Dirac effect becomes smaller with increasing Λ concentration, which is also plausible since this effect is larger for nucleons.

Comparing with the microscopic predictions from Ref. [3], we see that the qualitative features are similar, but there are very large quantitative differences due to the use of different NY potentials, the depth of $U_\Lambda$ at zero Λ density being about -30 MeV in Ref. [3].

In Fig. 3 we compare the single-nucleon and the single-Λ potentials (DBHF predictions only) at two different densities, corresponding to a nucleon Fermi momentum equal to 1.25 fm$^{-1}$ and 1.4 fm$^{-1}$, respectively, with the Λ fraction kept fixed at 20%. Both potentials start deeper at the larger density. As already clear from Fig. 1, $U_N$ grows repulsive (as a function of the momentum) at a much faster rate than $U_\Lambda$. Figure 3 shows that this is more so the larger the density, most likely due to the (very density-dependent) Dirac effect, which impacts nucleons more strongly.

4 Conclusions

We have reported non-relativistic and Dirac-Brueckner-Hartree-Fock predictions of the Λ and nucleon single-particle potentials in nuclear matter at moderately high densities. We examined the significance of Dirac effects as a function of momentum.
and Λ concentration. Dirac effects on hyperons may be important at the higher densities, where high momenta are probed.

We have neglected the ΛΛ interaction, which is not a serious problem at low Λ concentrations. The hyperon fraction in β-equilibrated matter is of course density dependent, but we expect that it will remain relatively small, especially if the NY interaction is fully taken into account (as compared to a model of non-interacting hyperons). This is because the (repulsive) short-range NY interaction will become effective at high density and, to a certain degree, may prevent the growth of the hyperon fraction (at the expenses of the lepton population) which has been observed in models of non-interacting hyperons [10].

Hypernuclei can be produced in large number using relativistic heavy ion collisions [11]. Calculations such as the present one can be useful as a foundation for studies of hypernuclei based on generalized mass formulas [12] and thus predictions of stability of systems where one or more nucleons are replaced with Λ’s. Since the energy of a single Λ impurity in symmetric matter (at normal nuclear matter density) is about -50 MeV (or about -30 MeV, depending on the NY potential being used), whereas the average energy of a nucleon in nuclear matter is approximately -16 MeV, one may conclude that it is energetically favorable to replace a single nucleon with a single Λ. Of course this may no longer be true with increasing Λ fraction, as the Λ particles will have to acquire kinetic energy to obey the Pauli principle. This and other issues will be explored further in forthcoming work.

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