Weak axial exchange currents for the Bethe-Salpeter equation

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Abstract

We construct weak axial one-boson exchange currents for the Bethe-Salpeter equation, starting from chiral Lagrangians of the NΔ(1236)πρ a_1 ω system. The currents fulfill the Ward-Takahashi identities and the matrix element of the full current between the two-body solutions of the Bethe-Salpeter equation satisfies the PCAC constraint exactly.

I. INTRODUCTION

The quantitative studies of the (anti)neutrino interaction with nuclei have now become reality, thanks to the existence of several neutrino detectors. Actually, the main goal of all these studies is a firm empirical evidence for the neutrino oscillations \cite{1}, \cite{2}, \cite{3}.

Here we deal with the (anti)neutrino-deuteron reactions,
\begin{align}
\nu + d & \rightarrow \nu + n + p, \\
\nu_e + d & \rightarrow e^- + p + p, \\
\bar{\nu} + d & \rightarrow \bar{\nu} + n + p, \\
\bar{\nu}_e + d & \rightarrow e^+ + n + n.
\end{align}

In the recent experiment \cite{4} on the electron antineutrino-induced deuteron disintegration done at the Bugey reactor, the neutral current (NCA) reaction (1.3) and the charge current (CCA) deuteron disintegration (1.4) were probed at low energies (\sim 1 MeV). While the measured cross section for the reaction NCA is in agreement with the previous measurements \cite{5,6} and calculations \cite{7,8}, the new CCA cross section differs considerably from the previous results.

Reactions (1.1) (NCN) and (1.2) (CCN) are important for studying the solar electron neutrino oscillations. They are the main object of the SNO detector \cite{9} and should be measured soon. While the charge current effect will come presumably from the solar $^8$B neutrinos ($E_\nu < 15$ MeV), the neutral current effect should reflect the interaction of all the neutrino species. If the ratio of the observed rates for reactions (1.2) and (1.1) would be measured significantly less than the standard model prediction, then the mixing of the neutrino species would be clearly demonstrated.
High energy neutrinos (E_ν > 50 MeV) can be expected from solar bursts and from reactions of cosmic rays in the atmosphere (atmospheric neutrinos).

The cross sections for reactions (1.1)-(1.4) up to energies 170 MeV are presented in [10]. They were calculated within the frame of the standard nuclear physics calculations using the nuclear wave functions obtained by solving the Schroedinger equation with the commonly used nucleon-nucleon (NN) realistic potentials [11]. Besides the one-nucleon currents, static weak one-pion meson exchange currents (MECs) were also taken into account.

At high energies, relativistic effects both in the potentials and currents (consistency of calculations) should be considered. In the standard non-relativistic approach, relativistic corrections are calculated in the 1/M expansion scheme (M is the nucleon mass). Actually, some of the components of the MECs (the time component of the vector MEC and the space component of the axial MEC) exist only as relativistic corrections of order 1/M^2 which makes the calculations difficult. On the other hand, relativistic calculations can be carried out from the beginning within the framework of relativistic equations. This approach has recently been applied to the electron-deuteron interaction at high energies [12], where the general discussion was done for the Bethe-Salpeter (BS) [13] equation and furthermore the electromagnetic interaction has been constructed in accord with the spectator (Gross) quasipotential equation. Here we apply the approach of the above mentioned paper to study the structure of the weak interaction in conjunction with the BS equation, having in mind to perform later a similar study for the Gross equation and to use the weak MECs in calculations of the neutrino-deuteron cross sections at high energies.

The search for the MEC effects at low and intermediate energies has a long and successful history [14–19]. The one-pion range MEC effects were firmly established in the reactions governed by the space part of the electromagnetic current and by the time component of the weak axial current which are of the same order ∼ 1/M as the corresponding components of the one-nucleon current. Actually, the recent precise measurement [20] of the transition rate for the reaction

\[ \mu^- + ^3\text{He} \rightarrow \nu_\mu + ^3\text{H} \]  

and its comparison with the calculations [21] demonstrates clearly also the presence of the effect of the space component of the weak axial MEC. At higher energies, other meson exchanges [22,23] and relativistic effects start to play an important role.

At low energies, the starting point for constructing the MEC operators are the low-energy theorems. In order to pass to higher energies, one employs chiral Lagrangians relevant to a nuclear system under investigation. In constructing the Lagrangians, symmetries play an essential role and dynamical symmetries imbedded in the Lagrangian reflect dynamical principles governing the system under investigation. Such a basic principle governing the physics of electroweak processes in nuclei at intermediate energies is the vector dominance model [24]. The vector meson fields are introduced into Lagrangians first as massless Yang-Mills (YM) gauge fields [25] belonging to the linear realizations of the chiral symmetry. Introduction of the mass terms for the vector meson fields by hand makes this concept internally inconsistent. Also the interaction of the particles with the external fields has to be introduced by hand since there are no other charges available which could be related to the external fields. These defects of the concept of the massive YM fields are later removed in the hidden local symmetries (HLS) approach [26,27].
In this approach, a given global symmetry group \( G_g \) of a system Lagrangian is extended to a larger one by a local group \( H_l \) and the Higgs mechanism generates the mass terms for gauge fields of the local group in such a way that the local symmetry is conserved. For the chiral group \( G_g \equiv [SU(2)_L \times SU(2)_R]_g \) and \( H_l \equiv [SU(2)_L \times SU(2)_R]_l \), the gauge particles are identified [26,27] with the \( \rho \) - and \( a_1 \) mesons. An additional extension by the group \( U(1)_l \) allows one to include the isoscalar \( \omega \) meson as well [28]. Moreover, external gauge fields, which are related to the electroweak interactions of the Standard Model, are included by gauging the global chiral symmetry group \( G_g \).

Let us note that the YM approach and the HLS concept with the vector meson fields belonging to the linear realization of the chiral symmetry are formally equivalent [26,27]. The linear and non-linear realizations of the HLS are related by the Stueckelberg transformation [29,27]. Lagrangians written in terms of fields of any realization are equivalent: the physical content of the model cannot depend on the parametrization of fields.

Our starting point for constructing the weak axial MECs are chiral Lagrangians of the \( N\Delta(1236)\pi\rho a_1\omega \) system. The first one is that with the YM vector meson fields belonging to the linear realization of the \( SU(2) \times SU(2) \) chiral symmetry [30], [31]. It is a minimal Lagrangian and contains no more than two derivatives of fields in each term. Another suitable Lagrangian constructed already within the concept of the HLS can be found in [32]. These two Lagrangians differ by the vertex \( \pi\rho a_1 \) which brings into the MECs a model dependence. The model dependence of the electromagnetic isovector one-pion MEC upon this difference was studied in the backward deuteron electrodisintegration in [32]. It influences considerably the cross sections at energies about 800 MeV.

The chosen Lagrangian has already been used in constructing the weak axial MEC operator of the one-pion range [33] within the extended S-matrix method [34]. In this method, the two-nucleon transition amplitude is represented by a set of tree Feynman graphs of the one-boson exchange type with the external nucleon lines on-shell. It is gauge- and Lorentz-invariant and it satisfies the PCAC equation. It is also assumed that the nuclear states are described by the Schroedinger equation solved with a one-boson exchange potential. The nuclear weak axial MEC operator is obtained after subtracting the first iterated Born approximation from the two-nucleon transition amplitude. Let us stress that the first iterated Born approximation generally differs from the positive-frequency part of the nucleon Born term entering the two-nucleon transition amplitude in higher order in \( \frac{1}{M} \). As mentioned above, this MEC operator has been employed in [21] to calculate the weak axial MEC effects in reaction (1.5).

Here we construct the weak axial MEC operator for the case when the two-nucleon nuclear states are described by the BS relativistic equation. In Sect. II, we present the Lagrangians. The transition operator is constructed in Sect. III starting from the YM Lagrangian and its model dependence is studied in Sect. IV. For the construction, the same Feynman diagram technique is employed as in [33], but the nucleons are off-shell. In order to get the correct MEC operator, one should omit the nucleon Born terms from the transition operator, which are already accounted for when calculating the matrix element of the one-nucleon current between the nuclear wave functions. We show that the full BS current satisfies the correct Ward-Takahashi identity (WTI) and that the resulting matrix element of the MEC operator between the two-body solutions of the BS equation satisfies the PCAC constraint exactly.
Similar program for the electromagnetic interaction has been done earlier in Ref. [35]. However, Gross and Riska use the minimal substitution in the kernel of the relativistic equation for generating the potential exchange currents, while the non-potential currents are derived from vertices. We derive both kinds of the currents from chiral Lagrangians. Generally, some difference can appear, since the currents derived from Lagrangians might be non-minimal.

Our results and conclusions are given in Sect. V.

The investigation made here is based on the normal Lagrangians. The anomalous Lagrangians describe processes with the change of the internal parity of particles and they are related to the phenomenon of chiral anomaly [36]. In our case, the vertices $\pi\rho\omega$ and $\rho\omega a_1$ would play a role. They are characterized by the presence of the tensor $\varepsilon^{\alpha\beta\gamma\delta}$ causing the change of the internal parity. The relevant Lagrangians can be found in [28], [37]. However, the construction of these currents is beyond the scope of this paper.

The weak axial MECs based on the Lagrangian approach have recently been studied in other models in [38, 39].

We use the metrics defined by $\left( + - - - \right)$.

## II. CHIRAL LAGRANGIANS

We construct the MEC operators from a chiral Lagrangian of the $N\pi\rho a_1\omega$ system. Its essential part is the Lagrangian of the subsystem $\pi\rho a_1$. The conventional Lagrangians of the $\pi\rho a_1$ system contain trilinear and quadrilinear derivatives of the fields. They were studied already in sixties within the framework of the YM massless compensating fields. As it was shown in [40], a minimal effective Lagrangian $L_{\pi\rho a_1}^M$ of the $\pi\rho a_1$ system can be constructed so that it contains no more than two field derivatives in each term. This restriction determines the Lagrangian completely and it describes reasonably the related elementary processes up to 1 GeV. This construction was extended in [30], [31] to the $N\Delta\pi\rho a_1$ system.

However, the approach of the massless YM fields is internally inconsistent, as the masses of the heavy mesons should be put into the theory by hands and they violate the underlying symmetry. It was realized later [26], [27] that the internal inconsistency of the YM approach can be removed in the theory of HLS. In this theory, the masses of the heavy mesons are generated by the Higgs mechanism and they do not violate the symmetry any more. The HLS approach was applied also to the construction of the Lagrangian of the $\pi\rho a_1$ system [26], where the high energy behaviour of the amplitudes was corrected by a different choice of correction Lagrangians in comparison with [40], thus introducing a model dependence. However, the construction [26] was done for the heavy mesons on–shell and the Lagrangian cannot be used for constructing the MEC operators. Moreover, it also provides for the anomalous magnetic moment of the $a_1$ meson $\delta_{an}=+3$, in contradiction to its commonly accepted value $\delta_{an}=-1$ [10]. Both gaps in construction have recently been removed [32].

It has been also shown in [26], [27] that the Lagrangians of the YM approach are formally equivalent to those constructed from the fields of the linear realizations of the HLS and that they are a sub–manifold of the manifold of the BKY Lagrangians. We first present the $N\Delta\pi\rho a_1$ Lagrangian $L^M$ of the YM type [30], [31], [10]. It can be easily extended to contain also the $\omega$ meson field. Then we introduce the HLS Lagrangian $L^H$ [32], [11] and
discuss possible source of the model dependence of our MEC operators.

The YM Lagrangian of the $N\Delta\pi\rho a_1$ system can be written with an arbitrary mixing of the pseudoscalar and pseudovector $\pi NN$ couplings \cite{31} but we restrict ourselves to the pseudoscalar one only. With the $\omega$ meson included, it is of the form

$$\mathcal{L}^M = \mathcal{L}^{M}_{N\pi\rho a_1\omega} + \mathcal{L}^{M}_{N\Delta\pi\rho a_1} + \mathcal{L}^{M}_{\pi\rho a_1},$$ (2.1)

$$\mathcal{L}^{M(0)}_{N\pi\rho a_1\omega} = \mathcal{L}^{M(0)}_{N\pi\rho a_1\omega} + \frac{g_\rho}{2f_\pi}(1 - 2g_A^2) \bar{\Psi} \gamma^\mu (\vec{\tau} \cdot \vec{a}_\mu) \Psi$$

$$+ \frac{g_\rho}{2} \bar{\Psi} \left( \gamma^\mu \bar{\rho}_\mu + \frac{\kappa^V}{4M} \sigma^{\mu\nu} \bar{\rho}_\mu \right) \cdot \vec{\tau} \Psi + \frac{i}{4} g_\rho \frac{\kappa^V}{2M} \bar{\Psi} \gamma_5 \sigma^{\mu\nu} \left( \vec{z}'_\mu \times \vec{a}_\mu \right) \Psi$$

$$+ \mathcal{O}(\pi^2),$$ (2.2)

$$\mathcal{L}^{M(0)}_{N\pi\rho a_1\omega} = \bar{\Psi} (i \rho - M) \Psi - ig\bar{\Psi} \vec{\pi} \cdot \vec{\tau} \gamma_5 \Psi - g_A g_\rho \bar{\Psi} \gamma^\mu \gamma_5 (\vec{\tau} \cdot \vec{a}_\mu) \Psi$$

$$+ \frac{M}{2} \left( \frac{g_A}{f_\pi} \right)^2 \bar{\Psi} \vec{\pi}^2 \Psi + \left( \frac{g_A}{2f_\pi} \right)^2 \bar{\Psi} \gamma^\mu (\vec{\tau} \cdot \vec{a}_\mu) \Psi$$

$$+ \frac{g_\omega}{2} \bar{\Psi} \left( \gamma^\mu \omega^\mu + \frac{\kappa^S}{4M} \sigma^{\mu\nu} \omega_{\mu\nu} \right) \Psi + \frac{i}{4} g_\omega \frac{\kappa^S}{2M} \bar{\Psi} \gamma_5 \sigma^{\mu\nu} \left( \vec{\tau} \cdot \vec{a}_\mu \right) \Psi$$

$$+ \mathcal{O}(\pi^2).$$ (2.3)

Here we define

$$\vec{z}'_\mu = \vec{\rho}_\mu + g_\rho \vec{a}_\mu - \frac{1}{f_\pi} \left[ \vec{a}_\mu \times \vec{\tau} \vec{\pi} - \vec{\tau} \vec{\pi} \times \vec{a}_\mu \right] + \mathcal{O}(\pi^2),$$

and

$$\vec{\rho}_\mu = \partial_\mu \vec{a}_\mu - \partial_\nu \vec{a}_\mu, \quad \vec{a}_\mu = \partial_\mu \vec{a}_\mu - \partial_\nu \vec{a}_\mu, \quad \omega_{\mu\nu} = \partial_\mu \omega_{\nu} - \partial_\nu \omega_{\mu}. $$

Furthermore

$$\mathcal{L}^M_{N\Delta\pi\rho a_1} = \frac{2f_{\pi\Delta}}{m_\pi} \bar{\Psi} \gamma^\mu \vec{T} \Psi \cdot \nabla_\mu \vec{\pi} + i g_\rho \frac{G_1}{M} \bar{\Psi} \gamma_5 \gamma^\nu \vec{T} \Psi \cdot \vec{\rho}_\mu + h. c.$$ (2.4)

Here the transition isospin operator $\vec{T}$ is defined \cite{12} as

$$\left( \vec{T} \right)_{MTm_t} = \sum_l \left( \frac{3}{2} M_T \mid 1 l \frac{1}{2} m_t \right) t^{*l} \left( \pm i \sqrt{2} \right) = \left( \begin{array}{c} \frac{1}{\sqrt{2}} \pm i \frac{1}{\sqrt{2}} \\ \pm i \frac{1}{\sqrt{2}} \pm i \frac{1}{\sqrt{2}} \end{array} \right), \quad \vec{t}^0 = \left( \begin{array}{c} 0 \\ 0 \\ 1 \end{array} \right).$$ (2.5)

Further in Eq. (2.4)

$$\nabla_\mu \vec{\pi} = \frac{1}{2} \partial_\mu \vec{\pi} - f_\pi g_\rho \vec{a}_\mu + \mathcal{O}(\pi^2).$$ (2.6)

As it was discussed in \cite{30}, $G_1 \approx 2.6$. Actually, another term $\sim G_2$ present in the original Lagrangian was neglected in (2.4) because of one additional factor $\sim 1/M_\Delta$. 


The Lagrangian of the $\pi \rho a_1$ system is of the form
\[
L_{\pi \rho a_1}^M = g_\rho \bar{\rho}^\mu \cdot \vec{\pi} \times \partial_\mu \vec{\pi} + g_\rho \bar{\rho}^\mu \times \vec{\partial}_\mu \bar{\rho} - g_\rho (\bar{\rho}^\mu \times \vec{a}^\nu).
\]

In this approach, the hadron currents are mediated by mesons,
\[
J_M^S = -\frac{m_\omega^2}{g_\omega} \omega^\mu, \quad \vec{J}_M^\mu = -\frac{m_\rho^2}{g_\rho} \bar{\rho}^\mu, \quad \vec{J}_M^5 = -\frac{m_\rho^2}{g_\rho} \bar{a}^\mu - f_\pi \partial^\mu \vec{\pi} + \frac{1}{4} \vec{\pi} \times \vec{a}^\mu.
\]

(2.7)

The Lagrangian $L_M$ (2.1) together with the currents (2.8) consist of all the vertices necessary for constructing the operators of the isovector vector and axial one–nucleon and $\pi$, $\rho$, $a_1$ and $\omega$ MECs in the tree approximation.

The HLS Lagrangian $L^H$ consists of three terms like the Lagrangian $L^M$, but now the heavy meson fields represent the nonlinear realization of the HLS for the groups $H_l \equiv SU(2)_L \times SU(2)_R \times U(1)_I$. In comparison with (2.2) we have
\[
L_{\pi \rho a_1}^H = L_{\pi \rho a_1}^{H(0)} - g_\rho \frac{g_A}{2 f_\pi} \bar{\Psi} \gamma_\mu \gamma_5 (\vec{\tau} \cdot \bar{\rho}_\mu \times \vec{\Pi}) \Psi + \frac{g_\rho^2}{f_\pi} \bar{\Psi} \gamma_\mu (\vec{\tau} \cdot \bar{\rho}_\mu \times \vec{\Pi}) \Psi
\]
\[
+ \frac{g_\rho}{2} \bar{\Psi} \left( \gamma_\mu \frac{\kappa^\rho}{4 M} \sigma^{\mu\nu} \vec{F}^\rho_{\mu\nu} \right) \cdot \vec{\tau} \Psi + \frac{i}{4} g_\rho \frac{g_A}{M} \bar{\Psi} \gamma_5 \sigma^{\mu\nu} \left( \vec{F}^\rho_{\mu\nu} \cdot \vec{\Pi} \right) \Psi
\]
\[
+ O(\pi^2),
\]

(2.9)

with
\[
\vec{F}^\rho_{\mu\nu} = \bar{\rho}_{\mu\nu} + \bar{\rho}_{\rho\nu} \times \vec{\rho} \,
\]

(2.10)

and the Lagrangian $L_{\pi \rho a_1}^{H(0)}$ formally coincides with $L_{\pi \rho a_1}^{M(0)}$. The same holds also for $L_{\Delta \pi \rho a_1}^H$. As to the Lagrangian $L_{\pi \rho a_1}^H$, it is of the same form as given in Eq. (2.7) with the last two terms changed to
\[
\Delta L_{\pi \rho a_1}^H = \frac{1}{f_\pi} \left( \bar{\rho}_{\nu}^\mu \cdot \bar{\rho}_\mu \times \vec{\Pi} + \frac{1}{2} \bar{\rho}_\mu \cdot \vec{\partial} \bar{\rho}_\mu \times \vec{\Pi} \right).
\]

(2.11)

The currents-analogues of Eqs. (2.8) are given by the following set of equations
\[
J_H^S = -\frac{m_\omega^2}{g_\omega} \omega^\mu, \quad \vec{J}_H^\mu = -\frac{m_\rho^2}{g_\rho} \bar{\rho}_\mu \times \vec{\Pi} + O(\pi^2),
\]
\[
\vec{J}_H^5 = -\frac{m_\rho^2}{g_\rho} \bar{a}_\mu - f_\pi \partial_\mu \vec{\Pi} - 2 f_\pi g_\rho \bar{\rho}_\mu \times \vec{\Pi}
\]
\[
+ \frac{1}{g_\rho} \left( -\frac{1}{2 f_\pi} \vec{\partial} \vec{\Pi} - g_\rho \bar{\rho}_\nu + eA'_\nu \right) \times \bar{\rho}_\nu + O(\pi^2).
\]

(2.13)

In contrast to the YM currents Eq. (2.8), the HLS currents contain also the external weak axial field $A'_\nu$.

Checking the Lagrangians and currents for the possible source of difference between the MECs one finds that it can arise from the $\pi \rho a_1$ and NN$\pi a_1$ vertices.
III. THE AXIAL MEC OPERATOR FOR THE BETHE-SALPETER EQUATION

We first write the operator of the weak axial current for the $i$th nucleon ($i = 1, 2$)

$$\hat{J}^{a \mu}(1, i) = \frac{g_A}{2} m_{a_i}^2 \Delta^{\mu \nu}_a(q) \hat{\Gamma}^{a \nu}_i + i f_a \Delta_F^\pi(q^2) q^\mu \hat{\Gamma}^a_i. \quad (3.1)$$

Here $g_A$ is the weak axial coupling (the nucleon axial charge), $g_A = -1.256$, $q_i = p'_i - p_i$, the vector-meson propagator is generally designed as

$$\Delta^{\mu \nu}_B(q) = \left( g^{\mu \nu} - \frac{q^\mu q^\nu}{m_B^2} \right) \Delta_B(q^2), \quad \Delta_B(q^2) = \frac{1}{m_B^2 - q^2}, \quad (3.2)$$

and the pseudovector and pseudoscalar vertices are defined as

$$\hat{\Gamma}^{a \nu}_i = (\gamma_\nu \gamma_5 \tau^a)\, \hat{\Gamma}^a_i \equiv i g (\gamma_5 \tau^a)\, \hat{\Gamma}^a_i. \quad (3.3)$$

The divergence of the current (3.1) is

$$q_{\mu} \hat{J}^{a \mu}(1, i) = \left[ G^{-1}(p'_i) \hat{\epsilon}_A(i) + \hat{\epsilon}_A(i) G^{-1}(p_i) \right] + i f_a m^2 \Delta_F^\pi(q^2) \hat{\Gamma}^a_i,$$

$$\equiv \left[ \hat{\epsilon}_A(i), G^{-1}_i \right] + i f_a m^2 \Delta_F^\pi(q^2) \hat{\Gamma}^a_i. \quad (3.4)$$

Here the operator of the nucleon axial charge $\hat{\epsilon}_A(i)$ and the inverse of the nucleon propagator $G^{-1}(p)$ are defined as

$$\hat{\epsilon}_A(i) = g_A \left( \gamma_5 \frac{\tau^a}{2} \right)_i, \quad G^{-1}(p) = \hat{p} - \hat{M}. \quad (3.5)$$

The general structure of the weak axial MEC operators is given in Fig. 1. The weak axial interaction is mediated by the meson $B$ which is either $\pi$ or $a_1$ meson. The range of the current is given by the meson $B_2$ which is here $\pi, \rho, a_1$ or $\omega$ meson. In the graphs 1a and 1b, $N$ is either for the nucleon $N$ (in this case the graphs are the standard nucleon Born terms) or for the $\Delta(1236)$ isobar. We shall write the corresponding currents as $\hat{J}^{a \mu}_{B_2}(N, B)$. In the case of the BS equation, the nucleon Born terms do not enter the nuclear MECs because they are fully taken into account when calculating the matrix element of the one-nucleon current with the solutions of this equation. The graph 1c represents one type of contact terms which we shall denote as $\hat{J}^{a \mu}_{c B_2}(B)$. It is connected with the weak production amplitude of the $B_2$ meson on the nucleon. Another type of the contact terms is given by the graph 1d where the weak axial current interacts directly with the mesons $B_1$ and $B_2$. We shall write it as $\hat{J}^{a \mu}_{c B_1 B_2}$. The graph 1e is for a mesonic current which we shall write in the form $\hat{J}^{a \mu}_{c B_1 B_2}(B)$. The associated pion absorption amplitudes will be written as $\hat{M}^{a}_{B_2}(N)$, $\hat{M}^{a}_{c B_2}$ and $\hat{M}^{a}_{B_1 B_2}$. For convenience, we shall modify the notations in some cases.

A. The axial MEC operator of the pion range

This operator is closely related to the one derived in [33] from the Lagrangians $\mathcal{L}^{M}_{N, \pi \rho a_1, \omega}$ and $\mathcal{L}^{M}_{\pi \rho a_1}$. Essentially, it has potential and non-potential parts, with the nucleon Born
terms in the potential part omitted. Let us note that the Lagrangian used for the derivation of the \( \pi \) MEC operator corresponds to the non-linear chiral model with the pseudoscalar \( \pi \)NN coupling which is connected to another model with any mixing of the pseudoscalar and pseudovector couplings by the unitary Foldy-Dyson transformation which makes all the models physically equivalent.

In the given case, the potential amplitude consists of several contact terms where the weak interaction is mediated either by the \( a_1 \)- or \( \pi \) meson. The \( a_1 \)-pole contact terms of the type \( \hat{J}^{a,\mu}_{c_1 \pi}(a_1) \) are

\[
\hat{J}^{a,\mu}_{c_1 \pi}(a_1) = -\frac{1}{4f_\pi} m^2_{a_1} \Delta^{\mu \nu}_{a_1}(q) \varepsilon^{amn} \hat{\Gamma}_{1\nu}(q_1) \Delta^\pi_F(q_2^2) \hat{\Gamma}_2^n + (1 \leftrightarrow 2),
\]

with

\[
\hat{\Gamma}^m_{1\nu}(q_j) = \left( \gamma_\nu + i \frac{K^V_\rho}{2M} \sigma_\nu \delta q_j^\delta \right) \tau_i^m,
\]

and

\[
\hat{J}^{a,\mu}_{c_2 \pi}(a_1) = \frac{g^2_\pi}{2f_\pi} m^2_{a_1} \Delta^{\mu \nu}_{a_1}(q) \varepsilon^{amn} (\gamma_\nu \tau^m)_{1\Delta^\pi_F(q_2^2) \hat{\Gamma}_2^n + (1 \leftrightarrow 2).}
\]

The \( \pi \) meson contact terms of the type \( \hat{J}^{a,\mu}_{c \pi}(\pi) \) are of the form

\[
\hat{J}^{a,\mu}_{c_1 \pi}(\pi) \equiv i f_\pi q^\mu \Delta^\pi_F(q^2) \hat{M}^a_{c_1 \pi},
\]

\[
\Delta \hat{J}^{a,\mu}_{c_1 \pi}(\pi) \equiv i f_\pi q^\mu \Delta^\pi_F(q^2) \Delta \hat{M}^a_{c_1 \pi},
\]

where the pion absorption amplitudes are

\[
\hat{M}^a_{c_1 \pi} = -\frac{i}{2} \left( \frac{g}{M} \right)^2 \varepsilon^{amn} F_{1A} \Delta^\pi_F(q_2^2) \hat{\Gamma}_2^n + (1 \leftrightarrow 2),
\]

\[
\Delta \hat{M}^a_{c_1 \pi} = -\frac{i}{4} \left( \frac{g}{M} \right)^2 \varepsilon^{amn} F_{1A} \Delta^\pi_F(q_2^2) \hat{\Gamma}_2^n + (1 \leftrightarrow 2),
\]

and

\[
\hat{J}^{a,\mu}_{c_2 \pi}(\pi) = i f_\pi q^\mu \Delta^\pi_F(q^2) \frac{g^2_\pi}{M} \delta_{an} \Delta^\pi_F(q_2^2) \hat{\Gamma}_2^n + (1 \leftrightarrow 2) \equiv i f_\pi q^\mu \Delta^\pi_F(q^2) \hat{M}^a_{c_2 \pi}.
\]

In contrast to ref. [33], besides the current (3.9) we have an additional term (3.10) which disappears for the on-shell nucleons.

A direct calculation of the divergence of the currents (3.8), (3.9), (3.10) and (3.13) yields

\[
q_\mu \left[ \hat{J}^{a,\mu}_{c_2 \pi}(a_1) + \hat{J}^{a,\mu}_{c_1 \pi}(\pi) + \Delta \hat{J}^{a,\mu}_{c_1 \pi}(\pi) + \hat{J}^{a,\mu}_{c_2 \pi}(\pi) \right] = i f_\pi m^2_{a_1} \Delta^\pi_F(q^2) \left[ \hat{M}^a_{c_1 \pi} + 2\Delta \hat{M}^a_{c_1 \pi} + \hat{M}^a_{c_2 \pi} \right] - i f_\pi q^2 \Delta^\pi_F(q^2) \Delta \hat{M}^a_{c_1 \pi} - i f_\pi \hat{M}^a_{c_2 \pi}.
\]

It can be shown that

\[
- i f_\pi \hat{M}^a_{c_2 \pi} = \left[ \hat{e}^A(1) + \hat{e}^A(2), \hat{V}_\pi \right]_+ \left[ \hat{e}^A(1) + \hat{e}^A(2), \hat{V}_\pi \right]_+ \left[ \hat{e}^A(1) + \hat{e}^A(2), \hat{V}_\pi \right]_+.
\]
where the one-pion exchange potential $\hat{V}_\pi$ is defined as

$$\hat{V}_\pi = \Gamma^n_1 \Delta_F(q_2^2) \hat{\Gamma}_2^n. \quad (3.16)$$

Taking into account Eqs. (3.10) and (3.15), we can write Eq. (3.14) in the form

$$q_\mu \left[ \hat{j}^a_\mu(c_1 \pi(a_1)) + \hat{j}^a_\mu(c_2 \pi) + 2\Delta \hat{J}^a_\mu(c_1 \pi) + \hat{j}^a_\mu(c_2 \pi) \right] = i f_\pi m_\pi^2 \Delta_F(q^2) \left[ \hat{M}^{a}_1 \pi \right. + 2\Delta \hat{M}^{a}_1 \pi + \hat{M}^{a}_2 \pi] + \left[ \hat{e}_A(1) + \hat{e}_A(2), \hat{V}_\pi \right]. \quad (3.17)$$

This continuity equation allows us to define the potential axial $\pi$ MEC operator related to the BSE as

$$\hat{j}^{a}_\nu(BS \pi(p)) = \hat{j}^a_\mu(c_1 \pi(a_1)) + \hat{j}^a_\mu(c_2 \pi) + 2\Delta \hat{J}^a_\mu(c_1 \pi) + \hat{j}^a_\mu(c_2 \pi). \quad (3.18)$$

Let us note that the current (3.6) does not enter the resulting potential current (3.18). As in the on-shell case [13], it satisfies the continuity equation together with the non-potential currents (see below) which are constructed from the $\pi\rho a_1$ vertices. So we find its natural place among these non-potential currents, which we will consider now. They are analogues of the amplitudes given in Sect. 4 of Ref. [13]. The explicit form of the currents is

$$\hat{j}^{a\mu}_\rho(\pi(a_1)) = \frac{m_\rho^2}{8 f_\pi} \Delta^{a\nu}_\rho(q) \varepsilon^{ann} \Gamma^{m}_1(q_1) \Delta^\rho_F(q_1^2) \left[ (q_1 \cdot q_2 - q_1^2) q^{\nu m} + q^{\mu n} (q_1^2 - q_2^2) \right] \times$$

$$\Delta^\rho_F(q_2^2) \hat{\Gamma}_2^n + (1 \leftrightarrow 2), \quad (3.19)$$

$$\hat{j}^{a\mu}_\rho(\pi) = i f_\pi q^{\mu} \Delta^\pi_F(q^2) \left[ \hat{M}^{a\rho}_\pi + \Delta \hat{M}^{\rho\pi}_\pi \right], \quad (3.20)$$

$$\hat{M}^{a\rho}_\pi = i \frac{m_\rho^2}{2 f_\pi} \varepsilon^{ann} q_2 \varepsilon^{\nu m} \hat{\Gamma}^{\nu}_1(q_1) \Delta^\rho_F(q_1^2) \Delta^\pi_F(q_2^2) \hat{\Gamma}_2^n + (1 \leftrightarrow 2), \quad (3.21)$$

$$\Delta \hat{M}^{a\rho}_\pi = i \frac{m_\rho^2}{4 f_\pi} \varepsilon^{ann} q_1 \varepsilon^{\mu n} \hat{\Gamma}^{\mu}_1(q_1) \Delta^\rho_F(q_1^2) \Delta^\pi_F(q_2^2) \hat{\Gamma}_2^n + (1 \leftrightarrow 2), \quad (3.22)$$

$$\hat{j}^{a}_\rho = -\frac{m_\rho^2}{4 f_\pi} \varepsilon^{ann} \Delta^{\rho\mu}_\rho(q_1) \hat{\Gamma}^{\mu}_1(q_1) \Delta^\rho_F(q_1^2) \Delta^\pi_F(q_2^2) \hat{\Gamma}_2^n + (1 \leftrightarrow 2). \quad (3.23)$$

Straightforward calculation yields for the divergence of the currents (3.3) and (3.19) - (3.23)

$$q_\mu \left[ \hat{j}^a_\mu(c_1 \pi(a_1)) + \hat{j}^a_\mu(c_2 \pi) + \hat{j}^{a\mu}_\rho(\pi) + \Delta \hat{j}^{a\mu}_\rho(n \pi) \right] = i f_\pi m_\pi^2 \Delta_F(q^2) \left[ \hat{M}^a_\pi + \hat{\Delta}^a_\pi \right], \quad (3.24)$$

where an additional current $\Delta \hat{j}^{a\mu}_\rho(n \pi)$ defined as

$$\Delta \hat{j}^{a\mu}_\rho(n \pi) = -\frac{1}{4 f_\pi} q^{\mu} \Delta_F(q^2) \varepsilon^{ann} \hat{\eta}_1 \hat{\eta}^{mn}_1 \Delta^\rho_F(q_1^2) \hat{\Gamma}_2^n + (1 \leftrightarrow 2), \quad (3.25)$$

and a new pion absorption amplitude

$$\hat{\Delta}^a = \frac{i}{2 f_\pi} \varepsilon^{ann} \hat{\eta}_1 \hat{\eta}^{mn}_1 \left[ (q_1 \cdot q_2) \Delta^\rho_F(q_1^2) \right] \Delta^\pi_F(q_2^2) \hat{\Gamma}_2^n + (1 \leftrightarrow 2), \quad (3.26)$$
appear for the off-shell nucleons.

It can be verified that the sum of the currents $\hat{J}_{\rho;\pi}^a(\pi)$ and $\Delta\hat{J}_{\rho;\pi}^a(np)$ is

$$\hat{J}_{\rho;\pi}^a(\pi) + \Delta\hat{J}_{\rho;\pi}^a(np) = if_\pi q^\mu \Delta_{F}(q^2) \left[ \hat{M}_{\rho;\pi}^a + \hat{M}_{\rho;\pi}^a \right],$$

(3.27)

whereas the other contact terms satisfy

$$q_\mu \left[ \hat{J}_{c_1;\pi}^a(a_1) + \hat{J}_{\rho;\pi}^a(a_1) + \hat{J}_{\rho;\pi}^a(\pi) \right] = if_\pi \left[ \hat{M}_{\rho;\pi}^a + \hat{M}_{\rho;\pi}^a \right],$$

(3.28)

as it should be, if the continuity equation (3.24) should hold also for the off-shell nucleons.

Evidently, the non-potential axial $\pi$ MEC operator is defined by Eq. (3.24) as

$$\hat{J}_{\text{BS};\pi}^a(np) = \hat{J}_{c_1;\pi}^a(a_1) + \hat{J}_{\rho;\pi}^a(a_1) + \hat{J}_{\rho;\pi}^a(\pi) + \Delta\hat{J}_{\rho;\pi}^a(np).$$

(3.29)

As noted above, the natural place for the current $\hat{J}_{c_1;\pi}^a(\pi)$ is among the non-potential ones.

We now consider another contribution to the non-potential currents of the pion range coming from the $\Delta(1236)$ excitation currents. These currents appear because of vertices $a_1 N\Delta$ and $\pi N\Delta$ and they are of the form

$$\hat{J}_{\pi}^a(\Delta, a_1) = if_\pi \left( \frac{f_\pi N\Delta}{m_\pi} \right)^2 m_{a_1}^2 \Delta_{a_1,\nu}(q) \hat{O}_{\pi}^{a\nu}(\Delta),$$

(3.30)

$$\hat{J}_{\pi}^a(\Delta, \pi) = if_\pi q^\mu \Delta_{F}(q) \hat{M}_{\pi}^a(\Delta),$$

(3.31)

$$\hat{M}_{\pi}^a(\Delta) = \left( \frac{f_\pi N\Delta}{m_\pi} \right)^2 q_\mu \hat{O}_{\pi}^{a\mu}(\Delta),$$

(3.32)

$$\hat{O}_{\pi}^{a\nu}(\Delta) = \left[ (T^+)^a T^0 S_F^{\lambda\nu}(P) + (T^+)^a T^b S_F^{\nu\lambda}(Q) \right] \left[ q_2 \chi \Delta_{F}(q_2) \hat{\Gamma}_2 + (1 \leftrightarrow 2) \right].$$

(3.33)

Here the propagator of the $\Delta$ isobar is defined as

$$S_F^{\alpha\beta}(p) = \frac{1}{p - M_\Delta + i\varepsilon} \left[ g^{\alpha\beta} - \frac{1}{3} \gamma^\alpha \gamma^\beta - \frac{2}{3M_\Delta} p^\alpha p^\beta + \frac{1}{3M_\Delta} (p^\alpha \gamma^\beta - p^\beta \gamma^\alpha) \right],$$

(3.34)

and the transition isospin operator $\hat{T}$ is defined accord with Eq. (2.3). The total $\Delta$ isobar current of the pion range is

$$\hat{J}_{\pi}^a(\Delta) = \hat{J}_{\pi}^a(\Delta, a_1) + \hat{J}_{\pi}^a(\Delta, \pi),$$

(3.35)

with the divergence

$$q_\mu \hat{J}_{\pi}^a(\Delta) = if_\pi m_\pi^2 \Delta_{F}(q) \hat{M}_{\pi}^a(\Delta).$$

(3.36)

We define the BS weak axial $\pi$ MEC operator as

$$\hat{J}_{\text{BS};\pi}(ex) = \hat{J}_{\text{BS};\pi}(p) + \hat{J}_{\text{BS};\pi}(np) + \hat{J}_{\pi}^a(\Delta),$$

(3.37)

with $\hat{J}_{\text{BS};\pi}(p)$, $\hat{J}_{\text{BS};\pi}(np)$ and $\hat{J}_{\pi}^a(\Delta)$ given in Eqs. (3.13), (3.29) and (3.33), respectively. Using Eqs. (3.17), (3.24) and (3.38) we obtain the WTI for the $\pi$ MEC operator.
\[ q_\mu \hat{J}^{a\mu}_{E_\pi}(ex) = \left[ \hat{e}_A(1) + \hat{e}_A(2), \hat{V}_\pi \right]_+ + if_\pi m_\pi^2 \Delta^\pi_F(q^2) \hat{\mathcal{M}}^a_\pi(2), \] (3.38)

where the pion absorption amplitude \( \hat{\mathcal{M}}^a_\pi(2) \) is

\[ \hat{\mathcal{M}}^a_\pi(2) = \hat{\mathcal{M}}^a_{c_1}\pi + 2\Delta \hat{\mathcal{M}}^a_{c_2}\pi + \hat{\mathcal{M}}^a_{c_1}\pi + \hat{\mathcal{M}}^a_{\rho}\pi + \hat{\mathcal{M}}^a_{\rho}\pi(\Delta). \] (3.39)

At low energies, the weak axial potential \( \pi \) MECs play little role. The non-potential current (3.23) dominates the time component of the weak axial \( \pi \) and \( \rho \) MECs and its effect can be clearly seen in light nuclei [14], [16] - [19]. On the other side, the current (3.30) contributes significantly to the space component of this current, as observed in the weak reactions in lightest nuclei, particularly in the reaction (1.5) [20, 21].

At higher energies, as in the case of the electromagnetic interaction, heavier mesons are expected to play a non-negligible role. Now we consider the weak axial MECs of the \( \rho, a_1 \) and \( \omega \) range.

### B. The axial MEC operator of the \( \rho \) and \( a_1 \) meson range

As it will become clear soon the \( \rho \) and \( a_1 \) exchanges should be considered in the given model simultaneously. The weak axial MECs of the \( \rho \) meson range derived from our model Lagrangian \( \mathcal{L}^{Y,M} \) are as follows. The only contact term is

\[ \hat{J}_{c\rho}^{a,\mu}(\pi) = if_\pi q^\mu \Delta^\pi_F(q^2) \hat{\mathcal{M}}^a_{c\rho}, \] (3.40)

where

\[ \hat{\mathcal{M}}^a_{c\rho} = \frac{g_A}{f_\pi} \left( \frac{g_\rho}{2} \right)^2 \frac{K^V}{2M} \delta_{a_n}(\gamma_5 \sigma_\eta) q_{2\delta} \Delta^\eta_\rho(q_2) \hat{\Gamma}_{2\delta}(q_2) + (1 \leftrightarrow 2). \] (3.41)

The mesonic currents are

\[ \hat{J}_{a_1\rho}^{a,\mu}(a_1) = i \frac{g_A}{2} g_\rho^2 m_\rho^2 \left[ (q_\rho + q_1\nu) \Delta^\mu_\rho(q_1) \Delta^\nu_\rho(q_1) - q_\zeta \Delta^\mu_\rho(q_1) \Delta^\nu_\rho(q_1) - \right. \]

\[ \left. - q_\zeta \Delta^\mu_\rho(q_1) \Delta^\nu_\rho(q_1) \right] \varepsilon^{amn} \hat{\Gamma}_{1\lambda} q_{2\delta} \Delta^\nu_\rho(q_2) \hat{\Gamma}_{2\delta}(q_2) + (1 \leftrightarrow 2), \] (3.42)

and

\[ \hat{J}_{a_1\rho}^{a,\mu}(\pi) = if_\pi q^\mu \Delta^\pi_F(q^2) \hat{\mathcal{M}}^a_{a_1\rho}(\pi), \] (3.43)

with

\[ \hat{\mathcal{M}}^a_{a_1\rho}(\pi) = \frac{g_A}{f_\pi} \left( \frac{g_\rho}{2} \right)^2 \left[ (2q_{2\zeta} + q_1\zeta) \Delta^\lambda_\rho(q_1) - 2q_{2\zeta} + q_1\zeta \right] \Delta^\lambda_\rho(q_1) \times \]

\[ \varepsilon^{amn} \hat{\Gamma}_{1\lambda} q_{2\delta} \Delta^\nu_\rho(q_2) \hat{\Gamma}_{2\delta}(q_2) + (1 \leftrightarrow 2). \] (3.44)

Here we modify the notation for the pion absorption amplitude from \( \hat{\mathcal{M}}^a_{a_1\rho} \) to \( \hat{\mathcal{M}}^a_{a_1\rho}(\pi) \).

The operator for the \( \rho \) meson exchange potential is
For the current of Eq. (3.40), we can write the divergence in the form
\[ q_\mu \tilde{J}^\mu_{c,\rho} (\pi) = i f_\pi m_\pi^2 \Delta_\pi (q^2) \tilde{\mathcal{M}}^a_{c,\rho} - i f_\pi \tilde{\mathcal{M}}^a_{c,\rho}. \] (3.46)

Using the explicit form of the amplitude \( \tilde{\mathcal{M}}^a_{c,\rho} \) from Eq. (3.41), the second term in Eq. (3.46) can be cast into the form
\[- i f_\pi \tilde{\mathcal{M}}^a_{c,\rho} = \left[ \hat{e}_A (1) + \hat{e}_A (2), \hat{V}_\rho^{an} \right]_+, \] (3.47)

where \( \hat{V}_\rho^{an} \) is the part of the potential \( \hat{V}_\rho \) given by the anomalous part of the \( \rho \)NN vertex entering the anticommutator of the potential with the nucleon axial charge. We denote the missing part of the anticommutator by \( -\hat{\Delta}_\rho \). It is of the form
\[- \hat{\Delta}_\rho \equiv \left[ \hat{e}_A (1) + \hat{e}_A (2), \hat{V}_\rho^a \right]_+ = i g_A \left( \frac{g_\rho}{2} \right)^2 \varepsilon^{amn} \hat{\Gamma}_{1,\mu}^\pi (q_1^2) \hat{\Gamma}_{2,\mu}^\pi (q_2^2) + (1 \leftrightarrow 2). \] (3.48)

In the next step of deriving the WTI for the \( \rho \) meson MECS we calculate
\[ q_\mu \tilde{J}_{a,\rho}^\mu (a_1) + \hat{\Delta}_\rho = i g_A \left( \frac{g_\rho}{2} \right)^2 \varepsilon^{amn} \hat{\Gamma}_{1,\mu}^\pi (q_1^2) \left[ q_1 \cdot q_2 + m_\rho^2 \right] \varepsilon^{\nu\lambda} (1 \leftrightarrow 2) \equiv i f_\pi \tilde{\mathcal{M}}^a_{a_1,\rho} (x). \] (3.49)

However, the amplitude \( \tilde{\mathcal{M}}^a_{a_1,\rho} (x) \) does not coincide with the amplitude \( \tilde{\mathcal{M}}^a_{a_1,\rho} (\pi) \) from Eq. (3.44) and we get
\[ q_\mu \left( \hat{J}_{a,\rho}^\mu (a_1) + \hat{J}_{a,\rho}^\mu (\pi) \right) = i f_\pi m_\pi^2 \Delta_\pi (q^2) \tilde{\mathcal{M}}^a_{a_1,\rho} (\pi) + \left[ \hat{e}_A (1) + \hat{e}_A (2), \hat{V}_\rho^a \right]_+ + i f_\pi \Delta_\pi (q^2) \left( \tilde{\mathcal{M}}^a_{a_1,\rho} (x) - \tilde{\mathcal{M}}^a_{a_1,\rho} (\pi) \right), \] (3.50)

which together with Eqs. (3.46) and (3.47) provides for the exchange current \( \tilde{J}^\mu_{BS,\rho} (ex) \), defined as
\[ \tilde{J}^\mu_{BS,\rho} (ex) = \hat{J}^\mu_{c,\rho} (\pi) + \hat{J}^\mu_{a_1,\rho} (a_1) + \hat{J}^\mu_{a_1,\rho} (\pi), \] (3.51)

with the WTI in the form
\[ q_\mu \tilde{J}^\mu_{BS,\rho} (ex) = \left[ \hat{e}_A (1) + \hat{e}_A (2), \hat{V}_\rho \right]_+ + i f_\pi m_\pi^2 \Delta_\pi (q^2) \tilde{\mathcal{M}}^a_{\rho} (2) + i f_\pi \left( \tilde{\mathcal{M}}^a_{a_1,\rho} (x) - \tilde{\mathcal{M}}^a_{a_1,\rho} (\pi) \right), \] (3.52)

where the pion absorption amplitude is given as
\[ \tilde{\mathcal{M}}^a_{\rho} (2) = \tilde{\mathcal{M}}^a_{c,\rho} + \tilde{\mathcal{M}}^a_{a_1,\rho} (\pi). \] (3.53)
Using Eqs. (3.44) and (3.49), we derive for the difference of the amplitudes $\hat{M}_{a_1,\rho}^a(x)$ and $\hat{M}_{a_1,\rho}^a(\pi)$ the following equation

$$i f_\pi \left( \hat{M}_{a_1,\rho}^a(x) - \hat{M}_{a_1,\rho}^a(\pi) \right) = -i g_A \frac{g_\rho^2}{2} \varepsilon^{amn} (\gamma_\mu \tau^m)_1 \Delta_{a_1}^{\mu\nu}(q_2) \hat{\Gamma}_n^{2\nu} + (1 \leftrightarrow 2).$$

(3.54)

Evidently, this difference is an operator of the $a_1$ meson range. As we shall see below, it will be compensated by the contribution from the weak axial exchange currents of the $a_1$ meson range.

Considered together with the $\rho$ meson MECs, the $a_1$ meson MECs contain only 3 contact terms $\hat{J}_{c_{i,1}}^{a\mu}(\pi)$ $(i=1,2,3)$

$$\hat{J}_{c_{i,1}}^{a\mu}(\pi) = i f_\pi g^\mu \Delta_F^\pi(q^2) \hat{M}_{c_{i,1}}^a.$$

(3.55)

The associated pion absorption amplitudes are

$$\hat{M}_{c_1,1}^a = -\frac{g_A}{f_\pi} (g_A g_\rho)^2 \varepsilon^{amn} (\gamma_\mu \tau^m)_1 \Delta_{a_1}^{\mu\nu}(q_2) \hat{\Gamma}_n^{2\nu} + (1 \leftrightarrow 2),$$

(3.56)

$$\hat{M}_{c_2,1}^a = \frac{g_A g_\rho^2}{f_\pi} \varepsilon^{amn} (\gamma_\mu \tau^m)_1 \Delta_{a_1}^{\mu\nu}(q_2) \hat{\Gamma}_n^{2\nu} + (1 \leftrightarrow 2),$$

(3.57)

$$\hat{M}_{c_3,1}^a = -i \frac{g_A g_\rho^2}{f_\pi} \frac{\kappa_V^2}{2} \varepsilon^{amn} q_{1\mu} (\sigma_\nu \tau^m)_1 \Delta_{a_1}^{\nu\lambda}(q_2) \hat{\Gamma}_n^{2\lambda} + (1 \leftrightarrow 2).$$

(3.58)

Let us note that the currents (3.55) are derived from the same NN$\pi a_1$ vertices as the $\pi$ MECs (3.6)–(3.8), only the role of the weakly and strongly interacting mesons is exchanged.

The $a_1$ meson exchange potential is

$$\hat{V}_{a_1} = (g_A g_\rho)^2 \hat{\Gamma}_{1\mu}^{5n} \Delta_{a_1}^{\mu\nu}(q_2) \hat{\Gamma}_n^{2\nu}.$$

(3.59)

Let us now calculate the divergence of the currents $\hat{J}_{c_{i,1}}^{a\mu}(\pi)$. In the first step we compute

$$q_\mu \hat{J}_{c_{i,1}}^{a\mu}(\pi) = i f_\pi m_\pi^2 \Delta_F^\pi(q^2) \hat{M}_{c_{i,1}}^{a} - i f_\pi \hat{M}_{c_{i,1}}^{a}.$$

(3.60)

Using the explicit form (3.56) of $\hat{M}_{c_{i,1}}^{a}$ it follows that

$$-i f_\pi \hat{M}_{c_{1,1}}^{a} = \left[ \hat{\epsilon}_{\pi,1}(1) + \hat{\epsilon}_{\pi,2}, \hat{V}_{a_1} \right].$$

(3.61)

We next calculate

$$q_\mu \left( \hat{J}_{c_{2,1}}^{a\mu}(\pi) + \hat{J}_{c_{3,1}}^{a\mu}(\pi) \right) = i f_\pi m_\pi^2 \Delta_F^\pi(q^2) \left( \hat{M}_{c_{2,1}}^{a} + \hat{M}_{c_{3,1}}^{a} \right) - i f_\pi \left( \hat{M}_{c_{2,1}}^{a} + \hat{M}_{c_{3,1}}^{a} \right).$$

(3.62)

Employing Eqs. (3.60)–(3.62), we have the following WTI for the weak axial $a_1$ MEC derived so far

$$q_\mu \hat{J}_{BS,1}^{a\mu}(ex) = \left[ \hat{\epsilon}_{\pi,1} + \hat{\epsilon}_{\pi,2}, \hat{V}_{a_1} \right] + i f_\pi m_\pi^2 \Delta_F^\pi(q^2) \hat{M}_{a_1}^{a}(2) - i f_\pi \left( \hat{M}_{c_{2,1}}^{a} + \hat{M}_{c_{3,1}}^{a} \right).$$

(3.63)
with the current $\hat{J}_{BS a_1}^{a, \mu}(ex)$ defined as

$$\hat{J}_{BS a_1}^{a, \mu}(ex) = \sum_{i=1}^{3} \hat{j}_{c_i a_1}^{a, \mu}(\pi),$$

(3.64)

and with the pion absorption amplitude $\hat{M}_{a_1}^{a}(2)$ given by

$$\hat{M}_{a_1}^{a}(2) = \sum_{i=1}^{3} \hat{M}_{c_i a_1}^{a}.$$

(3.65)

The sum of the last two terms at the right hand side of Eq. (3.63) yield

$$-if_{\pi} \left( \hat{M}_{c_2 a_1}^{a} + \hat{M}_{c_3 a_1}^{a} \right) = ig_{A} \frac{g_{\rho}^{2}}{2} \varepsilon_{\alpha \mu \nu \rho \tau}^{\alpha m n} \hat{\Gamma}_{1 m}^{5 \mu} \Delta_{a_1}^{\mu \nu}(q_{1}) \hat{\Gamma}_{2 \nu}^{n}(q_{2}) + (1 \leftrightarrow 2).$$

(3.66)

Observing that the right hand sides of Eqs. (3.54) and (3.66) are of the same form but with the opposite sign we derive the WTI for the sum of the weak axial $\rho$ and $a_1$ MECs

$$q_{\mu} \left( \hat{J}_{BS \rho}(ex) + \hat{J}_{BS a_1}(ex) \right) = \left[ \hat{e}_{A}(1) + \hat{e}_{A}(2), \hat{V}_{\rho} + \hat{V}_{a_1} \right]_{+}$$

$$+if_{\pi} m_{\pi}^{2} \Delta_{F}^{\rho}(q^{2}) \left( \hat{M}_{\rho}^{a}(2) + \hat{M}_{a_1}^{a}(2) \right).$$

(3.67)

Using Eqs. (3.54) and (3.66) once more, we get from Eqs. (3.53) and (3.65)

$$\hat{M}_{\rho}^{a}(2) + \hat{M}_{a_1}^{a}(2) = \hat{M}_{c_2}^{a} + \hat{M}_{c_3}^{a} + \hat{M}_{c_1 a_1}^{a}.$$

(3.68)

We now derive the $\Delta$ excitation currents of the $\rho$ and $a_1$ range. In analogy with Eqs. (3.30)–(3.36) we have for the $\rho$ MEC the following set of equations

$$\hat{j}_{\rho}^{a, \mu}(a_1, \pi) = -e_{\pi} \frac{f_{\pi N A}}{m_{\pi}} \frac{G_{1}}{M} (g_{\rho} m_{\rho})^{2} \Delta_{a_1}^{a} \hat{\rho}_{\rho}^{a, \mu}(\Delta),$$

(3.69)

$$\hat{j}_{\rho}^{a, \mu}(a_1, \rho) = if_{\pi} q_{\mu} \Delta_{F}^{\rho}(q) \hat{M}_{\rho}^{a}(\Delta),$$

(3.70)

$$\hat{M}_{\rho}^{a}(\Delta) = i \frac{f_{\pi N A}}{m_{\pi}} \frac{G_{1}}{M} \frac{g_{\rho}^{2}}{2} q_{\nu} \hat{\rho}_{\rho}^{a, \nu}(\Delta),$$

(3.71)

$$\hat{\rho}_{\rho}^{a, \nu}(\Delta) = \left[ (T^{+})^{n} T^{n} \gamma_{5} \gamma_{5} S_{F}^{\lambda}(P) + (T^{+})^{n} T^{n} S_{F}^{\lambda}(Q) \gamma_{5} \gamma_{5} \right]_{1} \left( q_{2} \gamma_{5} g_{\rho}^{c} \gamma_{5} \right. \left. - q_{2} \gamma_{5} \gamma_{5} \right) \Delta_{F}^{\rho}(q_{2}) \hat{\Gamma}_{2 \rho}^{n}(q_{2}) + (1 \leftrightarrow 2).$$

(3.72)

The total $\Delta$ isobar current of the $\rho$ meson range is

$$\hat{j}_{\rho}^{a, \mu}(\Delta) = \hat{j}_{\rho}^{a, \mu}(a_1, \pi) + \hat{j}_{\rho}^{a, \mu}(a_1, \rho),$$

(3.73)

with the divergence

$$q_{\mu} \hat{j}_{\rho}^{a, \mu}(\Delta) = if_{\pi} m_{\pi}^{2} \Delta_{F}^{\rho}(q^{2}) \hat{M}_{\rho}^{a}(\Delta).$$

(3.74)

Similarly we have for the $\Delta$ excitation currents of the $a_1$ meson range
\[ \hat{J}_{a_1}^{\mu}(\Delta, a_1) = g_A \left(2f_\pi g_\rho m_\rho\right)^2 \left(\frac{f_\pi N}{m_\pi}\right)^2 \Delta^{\mu}_{a_1, \nu}(q) \hat{O}_{a_1}^{\alpha\nu}(\Delta), \]  
(3.75)

\[ \hat{J}_{a_1}^{\mu}(\Delta, \pi) = if_\pi q^\mu \frac{\Delta^{\pi}_{q}(q)}{m_\pi} \hat{\mathcal{M}}_{a_1}^{\alpha}(\Delta), \]  
(3.76)

\[ \hat{\mathcal{M}}_{a_1}^{\alpha}(\Delta) = -2ig_A f_\pi g_\rho^2 \left(\frac{f_\pi N}{m_\pi}\right)^2 q_\nu \hat{O}_{a_1}^{\alpha\nu}(\Delta), \]  
(3.77)

\[ \hat{O}_{a_1}^{\alpha\nu}(\Delta) = \left[(T^+)^n T^m S_F^\nu(q) + (T^+)^m T^n S_F^\nu(\lambda, Q)\right] \Delta^{\eta}_{a_1, \chi}(q_2) \hat{\Gamma}_{2\eta}^{\gamma n}(1 \leftrightarrow 2). \]  
(3.78)

The total \( \Delta \) isobar current of the \( a_1 \) meson range is

\[ \hat{J}_{a_1}^{\alpha\mu}(\Delta) = \hat{J}_{a_1}^{\alpha\mu}(\Delta, a_1) + \hat{J}_{a_1}^{\alpha\mu}(\Delta, \pi), \]  
(3.79)

with the divergence

\[ q_\mu \hat{J}_{a_1}^{\alpha\mu}(\Delta) = if_\pi m_\pi^2 \Delta^{\pi}_{q}(q^2) \hat{\mathcal{M}}_{a_1}^{\alpha}(\Delta). \]  
(3.80)

The total BS weak axial \( \rho + a_1 \) MEC is defined as

\[ \hat{J}_{BS, \rho + a_1}^{\alpha\mu}(ex) = \hat{J}_{BS, \rho}^{\alpha\mu}(ex) + \hat{J}_{BS, a_1}^{\alpha\mu}(ex), \]  
(3.81)

and it satisfies the WTI

\[ q_\mu \hat{J}_{BS, \rho + a_1}^{\alpha\mu}(ex) = [\hat{e}_A(1) + \hat{e}_A(2), \hat{V}_\rho + \hat{V}_{\alpha_1}]_+ + if_\pi m_\pi^2 \Delta^{\pi}_{q}(q^2) \hat{\mathcal{M}}_{\rho + a_1}^{\alpha}(2), \]  
(3.82)

with the pion absorption amplitude \( \hat{\mathcal{M}}_{\rho + a_1}^{\alpha}(2) \) given by

\[ \hat{\mathcal{M}}_{\rho + a_1}^{\alpha}(2) = \hat{\mathcal{M}}_{\rho}^{\alpha} + \hat{\mathcal{M}}_{a_1}^{\alpha}(x) + \hat{\mathcal{M}}_{c_{\alpha_1}}^{\alpha} + \hat{\mathcal{M}}_{\rho}(\Delta) + \hat{\mathcal{M}}_{a_1}(\Delta). \]  
(3.83)

The last MEC operator we will now construct is that of the \( \omega \) meson range.

### C. The axial MEC operator of the \( \omega \) meson range

The \( \omega \) meson is a vector meson with the isospin zero and the only MEC operator which appears is the contact current \( \hat{J}_{c_{\omega}}^{\alpha\mu}(\pi) \)

\[ \hat{J}_{c_{\omega}}^{\alpha\mu}(\pi) = if_\pi q_\mu \Delta^{\pi}_{F}(q) \hat{\mathcal{M}}_{c_{\omega}}^{\alpha}, \]  
(3.84)

\[ \hat{\mathcal{M}}_{c_{\omega}}^{\alpha} = g_\omega \left(\frac{2f_\omega}{g_\rho^2}\right)^2 \frac{\kappa S}{2M} \left(\gamma_5 \sigma^\nu_{q_2} \tau^\mu_{q_2}\right) \Delta_{T_{\omega}}^{\mu\nu}(q_2) \hat{\Gamma}_{2\eta}(q_2) + (1 \leftrightarrow 2). \]  
(3.85)

Here \( \hat{\Gamma}_{2\eta}(q_2) \) is given by Eq. (3.7) with the isospin operator \( \tau^m \) omitted and with the change \( \kappa_\rho^V \rightarrow \kappa_\omega^S \).

The \( \omega \) exchange potential reads

\[ \hat{V}_\omega = \left(\frac{g_\omega}{2}\right)^2 \hat{\Gamma}_{1\mu}(-q_2) \Delta_{T_{\omega}}^{\mu\nu}(q_2) \hat{\Gamma}_{2\nu}(q_2). \]  
(3.86)

Calculating the divergence of the current (3.84) yields
\[ q_\mu \hat{J}_{c_\omega}^{a\mu}(\pi) = if_\pi m^2_\pi \Delta_F^\pi(q^2) \hat{M}^a_{c_\omega} - if_\pi \hat{M}^a_{c_\omega}. \]  

(3.87)

Using Eq. (3.85) for the amplitude \( \hat{M}^a_{c_\omega} \) and Eq. (3.86) for the \( \omega \) exchange potential \( \hat{V}_\omega \) we verify that the second term on the right hand side of Eq. (3.87) provides the needed anticommutator of the nucleon axial charge and the potential. Finally we have the WTI for the BS weak axial \( \omega \) MEC

\[ q_\mu \hat{J}_{c_\omega}^{a\mu}(\pi) = if_\pi m^2_\pi \Delta_F^\pi(q^2) \hat{M}^a_{c_\omega} + \left[ \hat{e}_A(1) + \hat{e}_A(2), \hat{V}_\omega \right]. \]  

(3.88)

We have completed the derivation of the BS weak axial one-body operator and of the \( \pi, \rho, a_1 \) and \( \omega \) MEC operators in the tree approximation. These operators satisfy separately the WTI (3.4), (3.38), (3.82) and (3.88). Starting from them, we verify in the next step that the matrix element of the total current satisfies the standard PCAC equation.

D. The continuity equation for the current matrix element

We define, in accord with Sect. 2 of Ref. [12] the full BS weak axial current as

\[ \hat{J}_{BS}^{a\mu} = i\hat{J}^{a\mu}(1,1)G_2^{-1} + i\hat{J}^{a\mu}(1,2)G_1^{-1} + \hat{J}_{BS}^{a\mu}(ex) = \hat{J}_{IA}^{a\mu} + \hat{J}_{BS}^{a\mu}(ex), \]  

(3.89)

where

\[ \hat{J}_{BS}^{a\mu}(ex) = \hat{J}_{BS\pi}^{a\mu}(ex) + \hat{J}_{BS\rho+a_1}^{a\mu}(ex) + \hat{J}_{c_\omega}^{a\mu}(\pi). \]  

(3.90)

Using the WTIs for the one- and two-nucleon currents yields for the divergence of the full BS current

\[ q_\mu \hat{J}_{BS}^{a\mu} = \left[ \hat{e}_A(1) + \hat{e}_A(2), \mathcal{G}^{-1} \right] + if_\pi m^2_\pi \Delta_F^\pi(q^2) \hat{M}^a, \]  

(3.91)

where the inverse Green function is

\[ \mathcal{G}^{-1} = G_{BS}^{-1} + \hat{V}, \]  

(3.92)

with

\[ \hat{V} = \hat{V}_\pi + \hat{V}_\rho + \hat{V}_{a_1} + \hat{V}_\omega. \]  

(3.93)

The BS propagator in term of the single-particle propagators reads \( G_{BS} = -iG_1G_2 \) and

\[ \hat{M}^a = i\hat{\Gamma}_1G_2^{-1} + i\hat{\Gamma}_2G_1^{-1} + \hat{M}_\pi^a(2) + \hat{M}_{\rho+a_1}^a(2) + \hat{M}_{c_\omega}^a. \]  

(3.94)

Because the two-body BS wave functions for both bound and scattering states satisfy the equation

\[ \mathcal{G}^{-1}\psi = \psi|\mathcal{G}^{-1} = 0, \]  

(3.95)

the matrix element of the divergence of the BS current (3.91) fulfill the standard PCAC constraint

\[ q_\mu \psi|\hat{J}_{BS}^{a\mu}\psi> = if_\pi m^2_\pi \Delta_F^\pi(q^2) \psi|\hat{M}^a\psi>. \]  

(3.96)
E. The strong form factors

We have dealt with the point BNN vertices so far. The usual way to introduce the strong interaction effects in the BNN vertices is to introduce the form factors $F_{BNN}(q_i^2)$ with the normalization $F_{BNN}(m_B^2)=1$. Then the vertex $\hat{\Gamma}_i^a$ defined in (3.3) becomes

$$\hat{\Gamma}_i^a \rightarrow \hat{\Gamma}_i^a F_{\pi NN}(q_i^2).$$

(3.97)

It is a simple matter to verify that the one-nucleon current (3.1) should be redefined similarly

$$\hat{J}_a^\mu(1,i) \rightarrow \hat{J}_a^\mu(1,i) F_{\pi NN}(q_i^2).$$

(3.98)

Then for the divergence of this current we have the same Eq. (3.4) but with the vertex $\hat{\Gamma}_i^a$ from (3.97) and with the nucleon axial charge $\hat{e}_A(i)$ redefined according to the prescription (3.97)

$$\hat{e}_A(i) \rightarrow \hat{e}_A(i) F_{\pi NN}(q_i^2).$$

(3.99)

We could also leave the axial charge unchanged but then we should redefine the propagator in a similar manner

$$G^{-1}(p) \rightarrow G^{-1}(p) F_{\pi NN}(q_i^2) \equiv \hat{p} - M - \Sigma,$$

(3.100)

with

$$\Sigma = (\hat{p} - M)[1 - F_{\pi NN}(q_i^2)].$$

(3.101)

Introducing the strong form factors into the potential currents (3.8) and (3.9)-(3.13) and to the $\Delta$ excitation currents (3.30)-(3.31) follows the standard prescription (3.8) and the pion propagator connecting the baryon lines is modified as

$$\Delta_{\pi F}(q_i^2) \rightarrow \Delta_{\pi F}(q_i^2) F_{\pi NN}(q_i^2).$$

(3.102)

This prescription is not suitable for the non-potential currents (3.19)-(3.23). In this case the correct procedure is

$$\Delta_{F}(q_i^2) \rightarrow \Delta_{F}(q_i^2) F_{BNN}(q_i^2), \quad B = \pi, \rho.$$ 

(3.103)

In order that the WTI (3.24) would be satisfied in the presence of the strong form factors, the pion propagator of the current (3.6) should be modified as

$$\Delta_{\pi F}(q_i^2) \rightarrow \Delta_{\pi F}(q_i^2) F_{\pi NN}(q_i^2) F_{\rho NN}(q_j^2).$$

(3.104)

Checking the WTI for operators of the $\rho$ and $a_1$ meson range we conclude that it would be valid also with the strong form factors included provided

1. In all operators of the $\rho$ range

$$\Delta_{F}(q_i^2) \rightarrow \Delta_{F}(q_i^2) F_{\rho NN}(q_i^2).$$

(3.105)
2. In the amplitudes $\hat{M}_{c_2 a_1}^a$ and $\hat{M}_{c_3 a_1}^a$
\[\Delta F^a(q_i^2) \rightarrow \Delta F^a(q_i^2) F^2_{\rho NN}(q_i^2). \quad (3.106)\]

3. In the amplitude $\hat{M}_{c_1 a_1}^a$ and in the potential $\hat{V}_{a_1}$ and in the currents $\hat{J}_{a_1}^a(\Delta, \pi)$
\[\Delta F^a(q_i^2) \rightarrow \Delta F^a(q_i^2) F^2_{a_1 NN}(q_i^2). \quad (3.107)\]

Finally it holds for the operators of the $\omega$ meson range that the necessary modification is
\[\Delta F^a(q_i^2) \rightarrow \Delta F^a(q_i^2) F^2_{\omega NN}(q_i^2). \quad (3.108)\]

In the next section, we investigate the model dependence of our MECs appearing due to the difference in the physical content of the YM type minimal Lagrangian $L^M$ and of the HLS Lagrangian $L^H$.

**IV. MODEL DEPENDENCE OF THE BS MECS**

The derivation of the MECs from the HLS Lagrangians and currents (2.9) - (2.13) is analogous to that of the previous sections. Therefore, we present here briefly the results and emphasize the difference leading to the model dependence of the currents. The currents of this section will be labeled by an additional label H in order to distinguish them from the currents derived from the minimal Lagrangian $L^M$.

A. The axial MEC operator of the pion range

It is straightforward to check that the potential axial $\pi$ MEC operator $\hat{J}^{a\mu}_{BS \pi}(p, H)$ coincides with the current $\hat{J}^{a\mu}_{BS \pi}(p)$ of Eq. (3.18). But generally, the non-potential currents differ. It holds only for the new $\rho\pi\pi$ current $\hat{J}^{a\mu}_{\rho \pi}(\pi, H)$ that
\[\hat{J}^{a\mu}_{\rho \pi}(\pi, H) = \hat{J}^{a\mu}_{\rho \pi}(\pi), \quad (4.1)\]
where $\hat{J}^{a\mu}_{\rho \pi}(\pi)$ is given in Eq. (3.20). Checking further the currents $\hat{J}^{5\mu}_{M}(\pi)$ of Eq. (2.8) and $\hat{J}^{5\mu}_{H}$ of Eq. (2.13) we find that the new $\rho\pi$ contact term is now
\[\hat{J}^{a\mu}_{\rho \pi}(1, H) = 2 \hat{J}^{a\mu}_{\rho \pi}, \quad (4.2)\]
where the current $\hat{J}^{a\mu}_{\rho \pi}$ is given in Eq. (3.23). Here we label the current (4.2) additionally, because another $\rho\pi$ term appears due to the presence of the vertex $\sim \partial^\nu \Pi \times \tilde{\rho}_\nu$ in the current $\hat{J}^{5\mu}_{H}$,
\[\hat{J}^{a\mu}_{\rho \pi}(2, H) = \frac{1}{4f_\pi} \varepsilon^{a mn} \Gamma^m_{1 \nu}(q_1) [q_1^\mu q_2^\nu - (q_1 \cdot q_2) g^\mu\nu] \Delta F_F(q_2^2) \hat{\Gamma}^n_2 + (1 \leftrightarrow 2). \quad (4.3)\]
The new current \( \hat{J}_{\rho \pi}^a (a_1, H) \), obtained using the Lagrangian \( \Delta \mathcal{L}_{\rho \pi}^H \) of Eq. (2.11) is

\[
\hat{J}_{\rho \pi}^a (a_1, H) = \frac{m_{a_1}^2}{4 f_\pi} \Delta_{\rho \pi}^a \varepsilon^{\mu\nu\eta} \hat{g}_m \eta (q_1) \Delta \gamma_{F}^2 (q_2^2) [g_\nu^\mu g_\eta^\eta - (q_1 \cdot g_\nu^\pi) g_\eta^\eta] \Delta \gamma_{F}^2 (q_2^2) \hat{\Gamma}_2 + m_{a_1}^2 \varepsilon^{\mu\nu\eta} \hat{g}_m \eta (q_1) \Delta \gamma_{F}^2 (q_2^2) [g_\nu^\mu g_\eta^\eta - (q \cdot g_\nu^\pi) g_\eta^\eta] \Delta \gamma_{F}^2 (q_2^2) \hat{\Gamma}_2 + (1 \leftrightarrow 2),
\]

(4.4)

The new non-potential currents satisfy the following WTIs

\[
q_\mu \left[ \hat{J}_{\rho \pi}^a (a_1, H) + \hat{J}_{\rho \pi}^a (2, H) \right] = 0,
\]

(4.5)

\[
q_\mu \left[ \hat{J}_{\rho \pi}^a (1, H) + \hat{J}_{\rho \pi}^a (\pi, H) + \Delta \hat{J}_{\rho \pi}^a (n\rho) \right] = i f_\pi m_{a_1}^2 \Delta \gamma_{F}^2 (q_2^2) \left[ \hat{\mathcal{M}}_{\rho \pi}^a + \hat{\mathcal{M}}_{\rho \pi}^a \right],
\]

(4.6)

where the current \( \Delta \hat{J}_{\rho \pi}^a (n\rho) \) is given in Eq. (3.23) and the pion absorption amplitudes \( \hat{\mathcal{M}}_{\rho \pi}^a \) and \( \hat{\mathcal{M}}_{\rho \pi}^a \) are from Eqs. (3.21) and (3.26), respectively. In analogy with the non-potential currents of the Sect. III A, in addition to Eq. (3.27) we have now

\[
q_\mu \hat{J}_{\rho \pi}^a (1, H) = i f_\pi \left[ \hat{\mathcal{M}}_{\rho \pi}^a + \hat{\mathcal{M}}_{\rho \pi}^a \right],
\]

(4.7)

instead of Eq. (3.28).

**B. The axial MEC operator of the \( \rho \) and \( a_1 \) meson range**

In this model, the \( \rho \) and \( a_1 \) axial MECS satisfy the WTI separately. We first consider the \( \rho \) meson axial MECS.

Instead of the current \( \hat{J}_{c_{1\rho}}^a (\pi) \) of Eq. (3.40), we have now two contact terms \( \hat{J}_{c_{1\rho}}^a (\pi, H) \) and \( \hat{J}_{c_{2\rho}}^a (\pi, H) \)

\[
\hat{J}_{c_{1\rho}}^a (\pi, H) = \hat{J}_{c_{1\rho}}^a (\pi),
\]

(4.8)

and

\[
\hat{J}_{c_{2\rho}}^a (\pi, H) = i f_\pi q^\mu \Delta \gamma_{F}^2 (q_2^2) \hat{\mathcal{M}}_{c_{2\rho}}^a (H),
\]

(4.9)

where the pion absorption amplitude \( \hat{\mathcal{M}}_{c_{2\rho}}^a (H) \) is

\[
\hat{\mathcal{M}}_{c_{2\rho}}^a (H) = - \frac{g_A}{f_\pi} \left( \frac{q_\rho}{2} \right)^2 \varepsilon^{\mu\nu\eta} \hat{g}_m \eta (q_1) \Delta \gamma_{F}^2 (q_2^2) \hat{\Gamma}_2 \eta (q_2) + (1 \leftrightarrow 2).
\]

(4.10)

The first contact term appears due to the piece \( \sim \hat{\pi}^\mu \times \hat{P} \) in the current (2.13), while the second one comes from the second term on the right hand side of the Lagrangian (2.9). It also holds that

\[
\hat{J}_{a_{1\rho}}^a (a_1, H) = \hat{J}_{a_{1\rho}}^a (a_1),
\]

(4.11)
with the current \( \hat{J}_{a_1 \rho}(a_1) \) given in Eq. (3.42). An additional contact current is now generated from the vertex \( \sim \vec{a}_\nu \times \vec{p}_\mu \) in the current (2.13)

\[
\hat{j}^{a_1 \rho}(H) = ig_A \frac{g_2}{2} \varepsilon^{a m n} \Delta^\lambda_{a_1} (q_1) \hat{\Gamma}_{1 \lambda}^{5 m} \left[ q_2^\rho \Delta^\eta_{\mu}(q_2) - q_2^\nu \Delta^\eta_{\mu}(q_2) \right] \hat{\Gamma}_{2 \eta}^{n}(q_2) + (1 \leftrightarrow 2) . \tag{4.12}
\]

The last current we consider is the mesonic current

\[
\hat{j}^{a_1 \rho}(\pi, H) = i f_\pi q^\rho \Delta^\pi_{\mu}(q_2) \hat{\mathcal{M}}^{a_1 \rho}(H) , \tag{4.13}
\]

with

\[
\hat{\mathcal{M}}^{a_1 \rho}(H) = -g_A \left( \frac{g_2}{2} \right)^2 \varepsilon^{a m n} [2q_2 \zeta q_2^\nu + q_2 \zeta q_1^\nu - q_1 \zeta q_1^\nu] \Delta^\lambda_{a_1} (q_1) + (-2q_2 - q_1 \cdot q_2 + q_1^\rho) \Delta^\lambda_{a_1} (q_1) \Delta^\eta_{\mu}(q_2) + (1 \leftrightarrow 2) . \tag{4.14}
\]

It is straightforward to find that the rho meson axial MECS of this section

\[
\hat{j}^{a_1 \rho}(ex, H) = \hat{j}^{a_1 \rho}(\pi, H) + \hat{j}_{c_2 \rho}(\pi, H) + \hat{j}_{c_1 \rho}(a_1, H) + \hat{j}_{a_1 \rho}(H) + \hat{j}_{a_1 \rho}(\pi, H) , \tag{4.15}
\]

satisfies the WTI

\[
q^\rho \hat{J}_{BS \rho}(ex, H) = [\hat{e}_A(1) + \hat{e}_A(2), \hat{\mathcal{M}}_{\rho}(2, H) , \tag{4.16}
\]

with the pion absorption amplitude given by

\[
\hat{\mathcal{M}}_{\rho}(2, H) = \hat{\mathcal{M}}_{c_1 \rho}(H) + \hat{\mathcal{M}}_{c_2 \rho}(H) + \hat{\mathcal{M}}_{a_1 \rho}(H) . \tag{4.17}
\]

As to the \( a_1 \) meson axial MECS, we have now only the current

\[
\hat{j}_{BS a_1}(ex, H) = \hat{j}_{c_1 a_1}(\pi) , \tag{4.18}
\]

of the currents (3.55), for which the WTI (3.60) is valid.

C. The continuity equation for the current matrix element

In analogy with Eq. (3.89), the full BS weak axial current is now

\[
\hat{j}^{a_1 \rho}(H) = \hat{j}_{IA}^{a_1 \rho} + \hat{j}^{a_1 \rho}(ex, H) , \tag{4.19}
\]

where

\[
\hat{j}^{a_1 \rho}(ex, H) = \hat{j}_{BS \rho}^{a_1 \rho}(ex, H) + \hat{j}_{BS \pi}^{a_1 \rho}(ex, H) + \hat{j}_{BS a_1}^{a_1 \rho}(ex, H) + \hat{j}_{e \omega}^{a_1 \rho}(\pi) , \tag{4.20}
\]

\[
\hat{j}_{BS \pi}^{a_1 \rho}(ex, H) = \hat{j}_{BS \pi}^{a_1 \rho}(\pi) + \hat{j}_{BS \pi}^{a_1 \rho}(np, H) + \hat{j}_{BS \pi}^{a_1 \rho}(\Delta) , \tag{4.21}
\]

\[
\hat{j}_{BS \rho}^{a_1 \rho}(ex, H) = \hat{j}_{BS \rho}^{a_1 \rho}(ex, H) + \hat{j}_{BS \rho}^{a_1 \rho}(\Delta) , \tag{4.22}
\]

\[
\hat{j}_{BS a_1}^{a_1 \rho}(ex, H) = \hat{j}_{BS a_1}^{a_1 \rho}(ex, H) + \hat{j}_{BS a_1}^{a_1 \rho}(\Delta) . \tag{4.23}
\]
The associated pion absorption amplitudes read
\[
\hat{\mathcal{M}}^a(H) = i\hat{\Gamma}_1^a G_2^{-1} + i\hat{\Gamma}_2^a G_1^{-1} + \hat{\mathcal{M}}^a_\pi(2) + \hat{\mathcal{M}}^a_\rho(2, H) + \hat{\mathcal{M}}^a_{a_1}(2, H) + \hat{\mathcal{M}}^a_{c\omega},
\]
(4.24)
\[
\hat{\mathcal{M}}^a_\rho(2, H) = \hat{\mathcal{M}}^a_\rho(2, H) + \hat{\mathcal{M}}^a_\rho(\Delta),
\]
(4.25)
\[
\hat{\mathcal{M}}^a_{a_1}(2, H) = \hat{\mathcal{M}}^a_{c_1 a_1} + \hat{\mathcal{M}}^a_{a_1}(\Delta).
\]
(4.26)

An analogue of Eq. (3.91) is then
\[
q_\mu \hat{J}^{a\mu}_{\text{BS}}(H) = [\hat{e}_A(1) + \hat{e}_A(2), G^{-1}]_+ + if_\pi m_\pi^2 \Delta_F^\pi(q^2) \hat{\mathcal{M}}^a(H),
\]
(4.27)
which again leads to the standard PCAC constraint
\[
q_\mu < \psi| \hat{J}^{a\mu}_{\text{BS}}(H)|\psi > = if_\pi m_\pi^2 \Delta_F^\pi(q^2) < \psi| \hat{\mathcal{M}}^a(H)|\psi > .
\]
(4.28)

The possible model dependence is given by effects due to the difference between the currents \( \hat{J}^{a\mu}_{\text{BS}}(e x) \) and \( \hat{J}^{a\mu}_{\text{BS}}(e x, H) \), which reduces to investigating the difference between the currents
\[
\hat{J}^{a\mu}(M) = \hat{J}^{a\mu}_{c_1\pi}(a_1) + \hat{J}^{a\mu}_{\rho\pi}(a_1) + \hat{J}^{a\mu}_{a_1\rho}(\pi) + \hat{J}^{a\mu}_{c_2 a_1}(\pi) + \hat{J}^{a\mu}_{c_3 a_1}(\pi),
\]
(4.29)
and
\[
\hat{J}^{a\mu}(H) = \frac{1}{2} \hat{J}^{a\mu}_{\rho\pi}(1, H) + \hat{J}^{a\mu}_{\rho\pi}(2, H) + \hat{J}^{a\mu}_{\rho\pi}(a_1, H) + \hat{J}^{a\mu}_{a_1\rho}(H) + \hat{J}^{a\mu}_{a_1\rho}(\pi, H) + \hat{J}^{a\mu}_{c_2 \rho}(\pi, H).
\]
(4.30)

The presence of the \( \pi \) meson axial MECs entering these two equations may indicate that the model dependence can appear already at relatively low energies. However, inspecting the structure of the currents \( \hat{J}^{a\mu}_{c_1\pi}(a_1) \) and \( \frac{1}{2} \hat{J}^{a\mu}_{\rho\pi}(1, H) \) shows that the leading terms cancel each other and only the momentum dependent terms survive. So the model dependence should be expected presumably of the short range nature and can be expected at high energies.

V. RESULTS AND CONCLUSIONS

In this paper, we have investigated the structure of the axial MECs for the \( N\Delta\pi\rho a_1\omega \) system in the conjunction with the BS equation. Our starting points are Lagrangians and one-body currents of this system (Sect. II). One of the Lagrangians, \( \mathcal{L}^M \), is the minimal one and it is approximately invariant under the local \([SU(2)_L \times SU(2)_R] \times U(1)\) symmetry. The heavy meson fields are introduced as massless Yang-Mills compensating fields belonging to the linear realization of the chiral symmetry. The symmetry is violated by the heavy meson masses, which are introduced, together with the external electroweak interaction, by hands.

Another choice considered here is the Lagrangian \( \mathcal{L}^H \), which reflects the \([SU(2)_L \times SU(2)_R] \times U(1)\) hidden local symmetry. In this case, the symmetry is not violated by the heavy meson mass terms and the external electroweak interaction enters naturally into the scheme as gauge fields of the initial global chiral group \([SU(2)_L \times SU(2)_R]_g\). We take the heavy meson fields belonging to the non-linear realization of the HLS, having in mind that
the Lagrangian expressed in terms of the fields belonging to the linear realization of the HLS is physically equivalent. Let us remind that the transition between the representations is done using the Stueckelberg transformation.

Essentially, both Lagrangians differ by the choice of correction Lagrangians needed to obtain correct amplitudes of elementary processes up to the energies $\sim 1$ GeV.

In Sect. III, (a) the $\pi$, $\rho$, $a_1$ and $\omega$ meson axial MECs from the minimal Lagrangian $L^M$ are derived. These currents satisfy the WTI's separately but the case of the $\rho$ and $a_1$ meson axial MECs, which should be considered together, (b) the WTI for the full BS current and the divergence of its matrix element between the two-body BS wave functions is given and it is shown that it satisfies the standard PCAC constraint and (c) the introduction of the strong form factors into the BNN vertices is discussed.

The derivation of the analogous currents for the Lagrangian $L^H$ is presented in Sect. IV, where also the model dependence is indicated.

The derived currents, with the nucleon Born terms added and sandwiched between the Dirac spinors, can be used in the standard nuclear physics calculations. To our knowledge, more complete set of the axial MECs has not yet been published.

In comparison with earlier works [38], [39], we can say that the set of the axial MECs derived here is more realistic. Indeed, besides the $a_1$ meson exchange, the considered $\pi$, $\rho$ and $\omega$ meson exchanges enter the realistic NN potentials of the OBE type [13]. Let us note that the $a_1$ meson exchange was included into OBEPQB in [14]. From the meson exchanges which are still not included, only the $\sigma$ meson exchange is of importance for OBEPs in describing the NN attraction at medium distances. This particle is controversial and it is difficult to include it consistently. It enters the linear $\sigma$ model naturally as a partner of the $\pi$ meson. However, the mass of the resonance [45] having the quantum numbers of the $\sigma$ meson is by a factor $\sim 2$ larger than its mass extracted from the fit of OBEPs to the data [43]. On the other hand, in the non-linear $\sigma$ models, to which our models belong, this particle does not enter by construction and can be put in by hands only.

Besides, Ref. [38] contains an error [46] in sections dealing with the non-linear $\sigma$ meson model. Indeed, it should be $(p'_1 - p_1 - 2q)_\nu$ in Eq. (40) instead of $(p'_1 - p_1 - q)_\nu$, which removes (a) the confusion about the axial current nonconservation even in the chiral limit given in the paragraph after this equation and (b) the $q$ dependence on the right hand side of Eq. (60). It is seen then that the correct current can be simply redefined so that it satisfies the WTI and the matrix element of its divergence PCAC.

As the next step would be calculations of the cross sections for the weak reactions like Eqs. (1.1)-(1.4). In order to do them consistently, one has to use solutions of the BS equation for the nuclear wave functions using a OBEP constructed in the same model. Such calculations would give more confidence in conclusions about the charged and neutral processes in neutrino disintegration of the deuteron and consequently, about the neutrino oscillations. These are important considerations for understanding the solar neutrino experiments at SNO and other possible experiments at higher energies.
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Fig. 1. The general structure of the weak axial MEC operators considered in this paper. The weak axial interaction is mediated by the meson B which is either $\pi$ or $a_1$ meson. The range of the current is given by the meson $B_2$ which is here $\pi$, $\rho$, $a_1$ or $\omega$ meson. The graphs a, b, represent the current $\hat{J}_{B_2}^{a,\mu}(N, B)$ with $N$ either for the nucleon $N$ or for the $\Delta(1236)$ isobar. The graph c represents a contact current $\hat{J}_{c_{B_2}}^{a,\mu}(B)$. It is connected with the weak production amplitude of the $B_2$ meson on the nucleon. Another type of the contact terms is given by the graph d, $\hat{J}_{B_1 B_2}^{a,\mu}$, where the weak axial current interacts directly with the mesons $B_1$ and $B_2$. The graph e is for a mesonic current $\hat{J}_{B_1 B_2}^{a,\mu}(B)$. The associated pion absorption amplitudes correspond to the graphs where the weak axial interaction is mediated by the pion, but with the weak interaction wavy line removed. There are three types of these amplitudes in our models: $\hat{M}_{B_2}^{a}(N)$, $\hat{M}_{c_{B_2}}^{a}$ and $\hat{M}_{B_1 B_2}^{a}$. 