Robust anomalous metallic states and vestiges of self-duality in two-dimensional granular In composites

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Many experiments investigating magnetic field tuned superconductor-insulator transition (H-SIT), often exhibit low-temperature resistance saturation, which is interpreted as an anomalous metallic phase emerging from a “failed superconductor,” thus challenging conventional theory. Here we use a random granular array of indium islands grown on a gateable layer of indium-oxide. Tuning the intergrain couplings, we reveal a wide range of magnetic fields where resistance saturation is observed, under conditions of careful electromagnetic filtering and within a wide range of linear response. Exposure to external broadband noise or microwave radiation is shown to strengthen the tendency of superconductivity, where at low field a global superconducting phase is restored. Increasing magnetic field unveils an “avoided H-SIT,” exhibiting granularity-induced logarithmic divergence of the resistance/conductance above/below that transition respectively, pointing to possible vestiges of the original emergent duality observed in a true H-SIT. We conclude that anomalous metallic phase is intimately associated with inherent inhomogeneities, exhibiting robust behavior at attainable temperatures for strongly granular two-dimensional systems.

I. INTRODUCTION

An increasing number of recent experiments have been pointing to the possibility of zero temperature transition from a superconducting state to an “anomalous metallic regime” with $T \to 0$ electronic properties that cannot be understood on the basis of conventional Fermi liquid/Drude theory (for a recent review see [1]). In particular it has been argued that the anomalous metal behaves as a “failed superconductor,” a state in which there are significant superconducting correlations, yet the system fails to globally condense even as $T \to 0$, settling at a finite conductivity that can be orders of magnitude larger than the Drude conductivity. Among its striking features, current in the anomalous metal regime is carried by bosonic quantum fluctuations of the superconducting order parameter, while exhibiting giant positive magneto-resistance, a much suppressed Hall response [2] and absence of cyclotron resonance [3].

Anomalous metallic phases often emerge in searches for a zero-temperature superconductor-insulator transition (SIT) in disordered superconducting films, yielding instead a quantum superconductor to metal transition (QSMT), typically triggered by varying external parameters such as magnetic field, gate voltage, and degree of disorder [2,4,12]. Focusing on the magnetic-field tuned SIT (H-SIT), an “avoided” transition (that is, a higher temperature signature of H-SIT that gives way to resistance saturation at lower temperatures) is often found above the QSMT, accompanied by low-temperature resistance saturation that may persist on both sides of this transition. While much of the current focus is on anomalous metallic phases proximate to a QSMT, an anomalous metallic regime has also been ubiquitously identified on the insulating side of a putative SIT. Such a behavior was identified for example in amorphous superconductors tuned by magnetic field [6], or disorder [13], and assumed to be originating from superconducting fluctuations that persist to the high resistance state. This scenario can be rationalized in the presence of inherent inhomogeneities, where a study of the effect of a ground plane next to the sample further suggested the importance of local superconducting phase coherence [14].

From the very nature of the phenomenon, it is clear that the failed superconductor is extremely fragile, primarily due to the inhomogeneous nature of the superconducting state. This can be a result of a granular morphology, or a result of microscopic disorder that is “amplified” in the superconducting state to yield an effective inhomogeneous microstructure [15,17] or unstable nonequilibrium state due to fluctuations [18]. Indeed, recent experiments probed the stability of magnetic-field tuned superconducting amorphous indium-oxide [19] and MoGe [20] films, concluding that an observed metallic state can be largely eliminated by adequately filtering external radiation. Since in practice any such statement is subject to the limited-sensitivity of the experiment, the ultimate question arises whether in the absence of any external perturbation, the film’s resistance will saturate to a finite resistance, may it be smaller than the experimental limit, as $T \to 0$. This challenging question arises following many experiments where attempts to eliminate the metallic phase failed, and by theoretical solution of a model of superconducting grains embedded in a metallic matrix, where such anomalous metallic behavior can occur in the neighborhood of a QSMT [11].

With the reasonable assumption that a superconducting transition in 2D disordered metallic films is domi-
nated by phase fluctuations [21,23], and thus can be modeled as superconducting grains embedded in a metallic matrix, we may search for a system where phase fluctuations are enhanced, and material parameters can be tuned to allow interrogation of an observed anomalous metallic phase within the sensitivity of the experimental system. Since quantum fluctuations of the phase of an isolated superconducting grain are associated with the charging energy, which is further controlled by the dielectric response of the surrounding matrix, a properly designed granular system should be our starting point.

In this paper we examine the robustness of an observed metallic phase in 2D InOx/In composite system, where a thin layer of amorphous indium-oxide (InOx), tuned to be “barely metallic,” provides the Josephson-coupling for pure indium (In) islands grown on top of it. The unique microstructure of the films, allowing for fine control of the fragility of the zero-field superconducting state, yield a highly tunable yet robust anomalous metallic state. While this metallic phase is observed under conditions of careful electromagnetic filtering and within a wide range of linear response, exposing it to external broad band noise or microwave radiation is shown to strengthen the tendency of superconductivity. Furthermore, in a wide range of parameters the external radiation restore a true superconducting phase (within the sensitivity of the measurements,) which we argue below is a direct consequence of enhanced phase coherence in the film. Close to the QSMT, the metallic phase saturates to a resistance value that is much smaller than the normal state “Drude value,” and depends on magnetic field as a power law. Increasing the magnetic field, a logarithmic divergence of the conductance is observed, with isotherms merge into an “avoided” H-SIT (that is, a higher temperature signature of H-SIT that gives way to resistance saturation as the temperature is lower). Further increase of the magnetic field yields an anomalous logarithmic divergence of the resistance with a large coefficient [24], which mimics the conductance behavior at lower magnetic fields, thus suggest some form of duality between the low-filed and high-field sides of the crossing point. The origin of the logarithmic behavior can be attributed to the granular nature of the system [25]. In fact, the relatively large In grains ensures thermal equilibrium in the superconducting state, where global superconductivity is achieved via phase coherence among the InOx coupled grains. Indeed, this versatile system was recently shown to undergo a true magnetic field tuned SIT (H-SIT) [26], similar to that of uniform InOx, exhibiting a “giant” magneto-resistance above the H-SIT [27,28] and critical behavior, that manifests the duality between Cooper pairs and vortices [29].

II. EXPERIMENT

Extreme granular inhomogeneity is achieved by depositing poorly wetting In metal onto uniform amorphous InOx thin film. The underlying layer of 300-Å InOx was electron-beam evaporated onto a commercial lithium-ion conductive glass ceramics (Li+ -ICGC) substrate (MTI corporation) that enables depletion of electrons up to a 2d carrier density of $\sim 10^{14}$ cm$^{-2}$ in back-gating configuration, as was also found in a recent study [31]. A deposition rate of 0.32 Å/s in an oxygen partial pressure of $9.5 \times 10^{-6}$ Torr yielded a weakly insulating film. Without interrupting the vacuum, at a base pressure of $1 \times 10^{-7}$ Torr, indium was evaporated in situ at a rate of 5 Å/s for

![Graph and Table]

**FIG. 1. Sample characterization.** (a) SEM micrograph of the InOx/In sample. Bright grains on the foreground are metallic indium, while dark background that lies uniformly underneath is weakly-insulating amorphous InOx. Note the existence of In grains of all scales including the interstitial ones. (b) Histogram of logarithm of grain area, extracted from (a) using a clustering algorithm (see supplemental material [30]), shows a broad distribution of grain size. The top axis shows equivalent diameter of a grain. Limitation of image resolution cuts off the distribution below ~0.01 μm. (c) Schematics showing resistance measurement and coupling of RF signal to the sample. See experiment section for filter specifications. The RF signal generator generates at output power 0 dBm (1 mW) unless otherwise noted. “Breakout” is a room-temperature breakout box for measurement wires, whose shielding serves as a common ground for the entire measurement system. (d) Zero-field sheet resistivity as a function of temperature. Data have been smoothed. Superconducting transition temperature ($T_c \approx 3.4$ K) of indium is indicated by the arrow where resistivity drops by around 10%. As sample anneals, resistivity decreases and a global superconducting ground state emerges.
100 seconds, yielding a nominal 500-Å layer of granular indium. The granular structure was confirmed by scanning electron microscopy (SEM) as shown in Fig. 1(a) along with a histogram showing grain size distribution in Fig. 1(b). The composite film was then patterned in Hall bar geometry (200 μm×100 μm), and titanium/gold contacts were subsequently patterned onto the sample. Great care was taken to keep the sample below 50 °C at all times to preserve the amorphous nature of the underlying InOx. Effective tuning of disorder, manifested by slow decrease in resistivity, is achieved by annealing the sample in vacuum at room temperature.

Resistance was measured using standard four-point lock-in technique at 3–13 Hz using 0.1–1 nA excitation current. Linear response was verified at various temperatures and magnetic fields. Measurement and filtering schematics can be found in Fig. 1(c).

Extensive filtering minimizes electron heating caused by external radiation and improves electron thermalization. Twisted-pair signal lines are filtered at mixing chamber plate using a commercial QFilter RC/RF filter (QDevil ApS), offering over -8 dB attenuation above 100 kHz and -50 dB above 300 MHz. Additionally, all signal lines are filtered at room temperature by a commercial in-line π-filter (API technologies corporation), providing an extra -50 dB attenuation above 200 MHz. Gate voltage was applied through a room-temperature-filtered twisted-pair well thermally-anchored at mixing chamber plate. Sample phonon temperature is measured by a calibrated on-chip Ruthenium-Oxide (RuO$_2$) thermometer positioned close to the sample.

III. RESULTS AND DISCUSSION

As made InOx/In samples were prepared intentionally to be initially non-superconducting. Subsequent room-temperature annealing in vacuum with minimal air contact reduced sheet resistance as demonstrated for 4 sample anneal stages in Fig. 4(d): S0 - as prepared sample, S1 - the sample after 1st anneal, S2 - the sample after a subsequent anneal, and S2G - the second-annal sample with a -50 V back gate applied. For every anneal stage including the back-gated, sample resistivity increases when cooled from 300 K to 3.2 K similar to plain InOx. Around 3.2 K, however, resistivity drops by \( \sim 10\% \) marking the onset of superconductivity in In grains. Upon further cooling, S0 resistivity fails to exhibit global superconductivity, but instead saturates at \( T \rightarrow 0 \) [32]. As the sample anneals, zero-resistance is almost achieved at base temperature for S1, and can be clearly identified at 0.5 K and 0.35 K for S2 and S2G respectively. Hall measurements on the annealed samples confirmed a high carrier density of \( 4 \times 10^{15} \) cm$^{-2}$, dominated by the In grains. Thus, while the -50 V back gating effectively reduced carrier concentration leading to a reduction in \( T_c \), it was not enough to turn S2 through the QSMT. We henceforth focus on S2 and S2G to study the transition between a superconducting and its proximate ground states in a perpendicular magnetic field.

A. General trends

With the application of perpendicular magnetic field, the fragile superconducting state in the In/InOx system is rapidly terminated at a magnetic-field-tuned QSMT, and an anomalous metallic phase is identified for \( 0 \lesssim \mu_0 H < 0.04 \) T. Fig. 2(a) shows resistivity that saturates as \( T \rightarrow 0 \) at a level orders of magnitude lower than the Drude value. As magnetic field increases, resistivity initially rises as a power-law that is essentially temperature-independent below 300 mK for both S2 and S2G (see Fig. 2(b)). Then, at \( H \approx 0.04 \) T, resistivity quickly picks up, leading to a giant magneto-resistance (MR) peak. The transition between saturating and diverging temperature dependence of resistivity is marked in the inset of Fig. 2(b) at 0.052 T and 0.043 T, for S2 and S2G respectively. This is reminiscent of a true H-SIT, with a transition to boson-dominated insulating ground state [27,28]. At \( H \approx 0.08 \) T, resistivity peaks at 40 \( k\Omega/\square \) at the highest. Beyond the peak, resistivity slowly decreases up to 8 T, but remains orders of magnitude larger than the normal state value, indicating the persistence of superconducting correlation well into the insulating state [28,33]. Qualitatively, S0 and S1 behave similarly, but have much higher resistivity up to 0.5\( M\Omega/\square \) at the peak (See supplemental material [30]).

B. Low-field regime: a failed superconductor

Starting with the low-field regime, where a failed superconducting phase yields a saturated resistance, the temperature-independent power-law MR before the rapid upturn, \( \rho(H) \propto H^{0.66} \), fully overlaps with the anomalous metallic regime. Assuming the resistivity can be written as an activation law using an effective temperature \( T_{\text{eff}} \), then \( \rho(H) \propto \exp(-U(H)/k_B T_{\text{eff}}) \). Combining the two expressions for \( \rho(H) \) gives \( U(H) \propto k_B T_{\text{eff}} \ln(1/H) \), a result widely observed [33,34], and attributed to the activation of free dislocations in a vortex lattice with short spatial correlations [33]. Since this mechanism should strongly depend on the morphology of the film, it is no surprise that the magneto-resistance curves in the anomalous metallic phase for both S2 and S2G films collapse on the same curve. Slight differences between the two samples are observed as the resistance increases above saturation towards a putative crossing point, which we will dub as “avoided true H-SIT.” We will come back to analyze the form of the resistance increase.
FIG. 2. **Anomalous metallic phase and the avoided H-SIT.** (a) Arrhenius plot of resistivity in various magnetic fields for sample S2. Robust resistivity saturation can be found in low magnetic field at as high as 300 mK, indicated by dashed horizontal lines. (b) Log-log plot of magneto-resistance (MR) at different temperatures for S2 and S2G. In anomalous metallic phase between 0 and 0.04 T, resistivity is essentially temperature-independent below 300 mK. The data collapse into a single power law \( \rho \propto H^{0.66} \), indicated by the dashed line. (Inset) Expanded view of the same data between 0.035 T and 0.065 T shows isotherm crossing points separating metallic from insulating behavior. Arrow indicates resistivity of S2 at 4 T. (c) Scaling function for resistivity \( \rho(\mid H - H_c\mid T^{-1/z\nu}) \) of S2 at 100, 200, 300, and 500 mK. The best data collapse occurs for \( z\nu \approx 1.5 \).

### C. Avoided H-SIT

Beyond the resistivity upturn, isotherm crossing points that separate increasing resistance on the low-field regime from increasing resistance on the high-field regime are observed (marked by full circles) in the inset of Fig. 2(b). This type of behavior would describe a true H-SIT if indeed a true superconducting phase would be attained, accompanied by divergence of the resistance on the insulating side. Since resistance saturation is observed in the low temperature limit of both regimes, we may consider this avoided criticality as a crossover and attempt scaling at higher temperatures (here \( T > 100\text{mK} \)), before the system is dominated by the anomalous metallic phase. Similar approach was demonstrated previously \([8, 36]\) to lead to scaling exponents of order \( z\nu \approx 1.5 \) when we scale the data according to \( \rho(T, H) = \rho_c F(\mid H - H_c\mid T^{-1/z\nu}) \) in Fig. 2(c). It is also interesting to note that such a behavior with a similar critical exponent was previously observed in a 2D system of aluminum islands coupled by 2DEG \([11]\), thus can be added to the list of common features between systems of induced superconducting granularity in otherwise presumed homogeneous materials such as amorphous-MoGe \([5, 6]\) and systems with extreme granularity in ordered \([11]\) and disordered (this manuscript) films.

### D. Behavior proximate to the “H-SIT”

Examination of the resistance trends on both sides of the crossing point reveals an intriguing behavior which may indicate the existence of vestiges of duality despite it being an “avoided transition.”

Starting with the “insulating regime,” again S2 and S2G show similar behavior, exhibiting anomalous logarithmic divergence of resistivity that is much weaker than the commonly observed activation or variable-range hopping trends. As shown in Figs. 3(a) and 3(b) where resistivity is plotted versus logarithm of temperature, within a broad range of temperature, the data can be fit by a straight line in the form \( \Delta R/R(1K) = A \cdot R(1K) \ln(1/T) \), where the slope \( A = (8.44 \pm 0.12) \times 10^{-5} \Omega^{-1} \) for S2 at 4 T. Similar magnitude slopes are found for other magnetic fields, and for the different anneal stages. The data and particularly the slopes of the log-\( T \) behavior are neither compatible with Kondo-effect behavior, nor are they compatible with weak localization corrections. Similar logarithmic divergence has been observed in the high-field insulating state of underdoped \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \) \([37]\), and in amorphous InOx \([24]\). Invoking inhomogeneous microstructure for these otherwise considered homogeneous materials, a possible connection between granularity and logarithmic divergence of the temperature-dependent resistivity has been previously discussed (see e.g. Ref. \([25]\)). We further note that for magnetic fields close to the crossing point, the logarithmic divergence tends to saturation, while increasing the magnetic field beyond the magnetoresistance peak seem to recover a “normal” insulating behavior commensurate with the initial 2D resistivity of the film before superconductivity sets in. Since the high field resistivity is much smaller than the peak in magnetoresistance, we interpret the saturation on the insulating side as an increase in local phase coherence due to remnant superconductivity.

Turning to the transition to the “anomalous metal regime” below the putative crossing point, we find an anomalous logarithmic divergence of the conductivity (calculated as the inverse resistivity since the Hall contribution is negligible), that is much weaker than the commonly observed activation or variable-range hopping commonly found where a true H-SIT is observed \([2, 26]\). As shown in Figs. 3(c) and 3(d) where conductivity is
to “large grains” — a fraction of a \( \mu \)m. Hence, in the insulating state, the system consists of a wide range of tunneling barriers between the metallic (or superconducting) islands. These barriers are determined by a competition between normal or Josephson tunneling and charging energy mediated by the dielectric background environment. Assuming a log-uniform distribution of barrier’s energy \( \Delta \), \( p(\Delta) = 1/\Delta \) and summing parallel conductances, we show that the resultant resistance can exhibit weak power law or logarithmic divergence. At much lower temperature, however, the resistivity will diverge much faster in an activated fashion. A more detailed account of our numerical analysis is given in the supplemental material \[30\]. This emerging behavior further supports our intuition that transport is determined by a collective effect of the In grains coupled within the InOx matrix.

**E. Phase diagram**

It was recently demonstrated that where true H-SIT is observed, the quantum phase transition exhibits the emergence of self duality with an exponential diverging variable range hoping behavior of the resistivity above, and the conductivity below, the transition \[26\]–\[29\]. The insulating phase was then identified as a Hall insulator, while the superconducting phase would be an equivalent phase for vortices. Moreover, the assembly of the critical behavior effects were found to be similar to that of quantum Hall to insulator transition \[35\], particularly the critical exponents and the emergence of self duality \[26\]–\[29\]. In fact, in attempting to summarize the above results into a phase diagram we observe further similarity between the two phenomena.

Fig. 4 is a phase diagram for the In/InOx system based on the experiments of Hen et al. \[26\] observing a true H-SIT and the present data showing an intervening metallic phase. The vertical axis represents an external parameter that controls the nature of the critical behavior. While in uniform films it could be the disorder, in the presence of strong granularity it may represent the distribution of the intergrain couplings. Where material parameters are tuned to observe an anomalous metal (e.g. weak intergrain couplings), a putative crossing point is observed as well with a non-universal critical resistance and different critical exponents, typically \( \nu \rightarrow \frac{1}{2} \). The fact that the resistance above the crossing resembles the conductance below the crossing, where both lead to a regime of saturated resistance suggest that vestiges of the self-duality line that emerges at the true H-SIT remain also in the regime where a metallic phase emerges, hence marks two dual regimes of anomalous metallicity. This observation is marked with the dash-dotted line in the phase diagram. Thus, while for true H-SIT a \( \sigma_{xx} \sim e^{(\Delta_e/T)\delta} \) on the superconducting side transforms to \( \rho_{xx} \sim e^{(\Delta_i/T)\delta} \) in the insulating side \[29\] (here \( \Delta_e \) and \( \Delta_i \) are activation gap scales for the superconducting and insulating regimes respectively). At weaker control parameter a
**FIG. 4. Phase Diagram of Superconductor-Metal-Insulator in 2D.** Phase diagram is drawn as a compilation of the data presented in Hen et al. [26] where true H-SIT is observed, together with the results discussed in this manuscript. The thick solid line represents a true H-SIT, with its associated critical behavior and emerging self duality [29]. At low field and small control parameter, an intervening metallic phase emerges from a quantum superconductor-to-metal transition (solid line), showing Bose character until it fades at a crossover to a stronger, Bose-dominated insulating phase (dashed line). At higher magnetic field pairing is quenched and Fermi-dominated insulating behavior is recovered (thin dashed line). Dash-dotted line represents a line that separates the two regimes of metallic phase, where vestiges of self duality are observed (see text).

\[ \sigma_{xx} \sim \ln(\Delta_s/T) \] that appears following the resistance saturation crosses over to \[ \rho_{xx} \sim \ln(\Delta_I/T) \] with increasing magnetic field. The insulating side then exhibits further increase of the resistance before pairing is quenched and Fermi-dominated insulating behavior is recovered.

**F. Possible non-equilibrium effects and response to external radiation**

Despite extensive filtering at different stages of our dilution refrigerator, the anomalous metallic phase appears robust. Nonetheless, fragility of the underlying superconducting state can cause this inhomogeneous system to be extremely sensitive to environmental perturbation. In a given magnetic field, barriers associated with collective vortex effects are established throughout the sample, leading to a saturated resistance commensurate with that field. However, it is important to explore whether the observed resistance is due to non-equilibrium effects where, for example, electrons fail to thermalize with the lattice due to poor electron-phonon coupling, or external noise heats the electrons, thus obstruct coherence in weaker Josephson couplings and cause disruption of global superconductivity. We already discussed the unique microstructure of our samples, where the large In grains ensure good thermalization of the local pair amplitudes. Indeed, Fig. 5(a) shows linear response of the resistance measurements in the anomalous metallic regime over a wide range of currents used. Non-linearities (Fig. 5(b)) are only observed at higher currents, and are presumably associated with junctions locally exceeding their critical current.

**FIG. 5. Differential resistivity in perpendicular magnetic field or subject to RF injection.** (a) Differential resistivity \( dV/dI \) versus direct current bias for S2G at 60 mK, showing a superconducting critical field around 1 µA. Injection of RF signal of 2 GHz (dark purple) and 0.2 GHz (light purple) leads to a barely visible increase in critical current. Linear response below at least 20 nA is demonstrated in the anomalous metallic phase. Dark gray curve is \( dV/dI \) at 4 K, marking the normal state value. (b) Same data shown in full linear scale. Zero-bias features evolve from a shallow dip in low fields to a sharp peak in higher fields, corresponding to the anomalous metallic phase and insulating phase, respectively.

Establishing linear response, we further need to explore the possible influence of external radiation on the occurrence of anomalous metallicity. To test for this effect, we may introduce external noise by removing successive layers of filtering in a controlled fashion, or by injecting radio-frequency (RF) signal into the sample and measure its response in resistivity. In Fig. 6, we show Arrhenius plot of resistivity under different experimental conditions. By either introducing 2 GHz signal or bypassing the room temperature low-pass filter (see Fig. 1(c),
FIG. 6. Radio-frequency (RF) signal and broad-band noise enhancing superconductivity. Enhanced conductivity by coupling RF signal to the sample S2G, at 0.03 T and 0.0002 T. In the latter case, a zero-resistance state emerges as a result of injecting 2 GHz signal. “No RF” means no output power at RF signal generator. “Pin 19” denotes a voltage lead connected to the sample in the middle along the Hall bar (See Fig. 1(c)). Other wires are checked for de-coupling from the sample at RF, see supplemental material [30]. “Bypass RT filter” means RT filter is bypassed and broad-band noise enters signal line. (No RF signal is injected in this case; data smoothed for higher visibility.) See supplemental material [30] for an estimate of RF power transmitted to the sample. Lower panel shows 0.0002 T data in linear scale, further demonstrating the effect of RF injection.

Resistivity is lowered by as much as ~ 25% at 0.03 T. This is contrary to the electron heating scenario, where an elevated electron temperature would raise saturation resistance. In an extreme case, at a field strength of 0.0002 T or 2 Gauss, the anomalous metallic behavior is suppressed completely and true superconductivity is recovered, where resistivity drops below our measurement sensitivity (See the lower curves in Fig. 6). It is interesting to note that in a completely different system of nanopore-modulated YBCO thin films, where anomalous metallic state has been observed, the removal of filtering resulted in a similar reduction of the saturated resistance at low temperatures [39]. This result was taken as a proof of true cooling of electrons in that highly inhomogeneous system. However, the introduction of deliberate microwave radiation in addition to removal of parts of the filtering system can further illuminate the nature of the robustness of the metallic phase and in particular the effect of re-entrant superconductivity.

Discovered over fifty years ago, microwave-enhanced superconductivity has been studied experimentally in constraint-type [40, 41] and superconductor-normal-superconductor (SNS) Josephson junctions [42, 43]. Dubbed as “microwave-stimulated superconductivity,” [44] this phenomenon was found to be ubiquitous in studies of SNS weak links. Initial theoretical understanding of this phenomenon invoked a non-equilibrium gap-enhancement proposed by Eliashberg [45]. However, more recent analyses, especially taking into account proximity-effect under external radiation, concluded that much of the enhancement of critical current in such SNS junctions arises from enhanced phase coherence (for a recent review see Klapwijk and de Visser [44]).

As demonstrated above, global superconductivity in the In/InOx system is established via phase coherence among the In grains, coupled through the underlying InOx layer. In the presence of magnetic field, pinned vs. mobile collective vortex effects reflect the restoration or loss of local phase coherence respectively. Thus, a metallic state appears over a zero-resistance superconducting state when a vortex-induced global phase slip appears — marking the loss of global phase coherence. However, the emerging metallic phase retains much of the character of the superconducting phase, where significant superconducting correlations are present, thus establishing a resistance much lower than the respective “Drude resistance.” The microwave enhanced conductivity of this metallic state, and the restoration of the superconducting state at very low fields are a manifestation of the robustness of this failed superconducting state.

We return to Fig. 5 which shows non-linear differential resistance $dV/dI$ in presence of a direct current bias. In zero magnetic field, superconductivity is quenched by a critical current of 1 $\mu$A, which is very slightly enhanced by application of RF signal. In an external magnetic field, $dV/dI$ yields a zero-bias minimum value in low field, and a zero-bias peak in relatively large field, more clearly shown in full linear scale in Fig. 5(b).

The evolution of the differential resistance behavior displayed in Fig. 5 has been observed previously in a 2D system of aluminum islands, coupled with a gated 2D electron gas [11]. Such a system may be considered an ordered array of islands, as shown by the oscillations in the array’s resistance at integer and certain fraction of flux quanta entering their periodic sample. While the overall behavior of the two systems is the same, the random distribution of the In islands in our system blurs any possible ordered oscillation in the magneto-resistance, although it may explain the non-smooth MR data to be discussed in the next section.
G. Flux effects

Fig. 7 shows the effect of RF radiation on the magneto-resistance. When no RF signal is present, as a result of the sample’s extreme sensitivity to external magnetic field, we cannot clearly identify a lower critical field of a QSMT. Beyond the limitations in measurement sensitivity and magnet field resolution, the main observation is the large fluctuations in MR, up to 30% of the averaged behavior. Such repeatable fluctuations are likely a consequence of flux commensuration effect allowing flux quanta into the granular structure. While such an effect shows precise periodicity in ordered array of coupled grains, such as in the coupled array of Al islands [11], or in studies of square aluminum wire networks in a magnetic field [10], here the area enclosed by loops connecting indium islands through intergrain coupling is random, and the smeared oscillations we observe are a consequence of the mesoscopic nature of the effect, which in turn is also responsible for the observed low critical field. For example, the deep minimum at ∼15 G corresponds to a flux quantum fitting in a loop of average diameter ∼1.3 μm, which in inspecting Figs. 1(a) and 1(b) would correspond to a fundamental loop that includes the larger grains. Indeed, similar effects have been observed in granular cuprates [17], inhomogeneous nanowires [18] and granular lead-film bridges [19]. The fact that the MR fluctuations are related to flux penetration through loops of grains connected by junctions is further evidenced from the effect of radiation. As we demonstrated above, the intergrain coupling in the In/InOx/In junctions in our films is enhanced in the presence of microwave radiation, which in turn should make the films effectively more homogeneous and thus reduce the MR fluctuations. Indeed, Fig. 7 shows the suppression of the MR fluctuations, accompanied by enhancement in critical field in the presence of 0.2 GHz or 2 GHz RF radiation.

IV. SUMMARY

Many experiments focused on the study of magnetic-field tuned superconductor to insulator transition (H-SIT) in two-dimensions end up exhibiting low-temperature saturation of the resistance, which is interpreted as an anomalous metallic phase emerging from a “failed superconductor.” Challenged by possible non-equilibrium effects and sensitivity to external perturbations, it has become of utmost importance to demonstrate the robustness of such a metallic phase when observed. Here we use a random granular array of indium islands grown on a gateable layer of indium-oxide. Tuning the intergrain couplings by a combination of oxygen annealing and back-gating, we are able reveal a wide range of magnetic fields where resistance saturation is observed. While these observations are obtained under conditions of careful electromagnetic filtering and within a wide range of linear response, exposure to external broad band noise or microwave radiation is shown to strengthen the tendency of superconductivity, where at low field a global superconducting phase is restored. Increasing the magnetic field exposes an “avoided H-SIT,” which above that transition is characterized by an anomalous logarithmic divergence of the resistance, while below the transition a logarithmic divergence of the conductivity is observed. This further highlights the granular nature of the system, and points to possible vestiges of the original duality observed in a true H-SIT. We are then led to conclude that anomalous metallic phase is intimately associated with inherent inhomogeneities, exhibiting robust behavior at attainable temperatures for strongly granular two-dimensional systems.

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[1] A. Kapitulnik, S. A. Kivelson, and B. Spivak, Colloquium: Anomalous metals: Failed superconductors, Rev. Mod. Phys. 91, 011002 (2019).
[2] N. P. Breznay and A. Kapitulnik, Particle-hole symmetry reveals failed superconductivity in the metallic phase of two-dimensional superconducting films, Science Advances 3, e1700612 (2017).
[3] Y. Wang, I. Tamir, D. Shahar, and N. P. Armitage, Absence of cyclotron resonance in the anomalous metallic phase in InO, Phys. Rev. Lett. 120, 167002 (2018).
[4] A. Yazdani and A. Kapitulnik, Superconducting-insulating transition in two-dimensional a-MoGe thin films, Phys. Rev. Lett. 74, 3037 (1995).
[5] D. Epifron, A. Yazdani, A. Kapitulnik, and M. R. Beasley, Observation of quantum dissipation in the vortex state of a highly disordered superconducting thin film, Phys. Rev. Lett. 76, 1529 (1996).
[6] N. Mason and A. Kapitulnik, Dissipation effects on the superconductor-insulator transition in 2d superconductors, Phys. Rev. Lett. 82, 5341 (1999).
[7] N. Mason and A. Kapitulnik, True superconductivity in a two-dimensional superconducting-insulating system, Phys. Rev. B 64, 060504(R) (2001).
[8] Y. Qin, C. L. Vicente, and J. Yoon, Magnetically induced metallic phase in superconducting tantalum films, Phys. Rev. B 73, 100505(R) (2006).
[9] S. Eley, S. Gopalakrishnan, P. M. Goldbart, and N. Mason, Approaching zero-temperature metallic states in mesoscopic superconductor-normal-superconductor arrays, Nature Physics 8, 59 (2012).
[10] Y. Saito, Y. Kasahara, J. Ye, Y. Iwasa, and T. Nojima, Metallic ground state in an ion-gated two-dimensional superconductor, Science 350, 409 (2015).
[11] C. G. L. Bottcher, F. Nichele, M. Kjaergaard, H. J. Suominen, J. Shabani, C. J. Palmstrom, and C. M. Marcus, Superconducting, insulating and anomalous metallic regimes in a gated two-dimensional semiconductor-superconductor array, Nature Physics 14, 1138 (2018).
[12] Z. Chen, A. G. Swartz, H. Yoon, H. Inoue, T. A. Merz, D. Lu, Y. Xie, H. Yuan, Y. Hikita, S. Raghu, and H. Y. Hwang, Carrier density and disorder tuned superconductor-metal transition in a two-dimensional electron system, Nature Communications 9, 4008 (2018).
[13] F. Couedo, O. Crauste, A. A. Drillien, V. Humbert, L. Bergé, C. A. Marrache-Kikuchi, and L. Dumoulin, Dissipative phases across the superconductor-to-insulator transition, Scientific Reports 6, 35834 (2016).
[14] N. Mason and A. Kapitulnik, Superconductor-insulator transition in a capacitively coupled dissipative environment, Phys. Rev. B 65, 220505(R) (2002).
[15] E. Shimshoni, A. Auerbach, and A. Kapitulnik, Transport through quantum melts, Phys. Rev. Lett. 80, 3352 (1998).
[16] A. Ghosal, M. Randeria, and N. Trivedi, Role of spatial amplitude fluctuations in highly disordered s-wave superconductors, Phys. Rev. Lett. 81, 3940 (1998).
[17] Y. Dubi, Y. Meir, and Y. Avishai, Nature of the superconductor-insulator transition in disordered superconductors, Nature 449, 876 (2007).
[18] A. G. Aronov and B. Z. Spivak, Stability of nonequilibrium states of superconductors with respect to finite fluctuations, Physics Letters A 78, 391 (1980).
[19] I. Tamir, A. Benyamini, E. J. Telford, F. Gorniaczyk, A. Doron, T. Levinson, D. Wang, F. Gay, B. Sæcø, J. Hone, K. Watanabe, T. Taniguchi, C. R. Dean, A. N. Pasupathy, and D. Shahar, Sensitivity of the superconducting state in thin films 5, 10.1126/sciadv.aau3826 (2019).
[20] S. Dutta, I. Roy, S. Mandal, J. Jesudasan, V. Bagwe, and P. Raychaudhuri, Extreme sensitivity of the vortex state in a-MoGe films to radio-frequency electromagnetic perturbation, Phys. Rev. B 100, 214518 (2019).
[21] M. V. Feigel’man and A. I. Larkin, Quantum superconductor-metal transition in a 2d proximity-coupled array, Chemical Physics 235, 107 (1998).
[22] B. Spivak, A. Zyzum, and M. Hruska, Quantum superconductor-metal transition, Phys. Rev. B 64, 132502 (2001).
[23] B. Spivak, P. Oretto, and S. A. Kivelson, Theory of quantum metal to superconductor transitions in highly conducting systems, Phys. Rev. B 77, 214523 (2008).
[24] M. A. Steiner, G. Boebinger, and A. Kapitulnik, Possible field-tuned superconductor-insulator transition in high-Tc superconductors: Implications for pairing at high magnetic fields, Phys. Rev. Lett. 94, 107008 (2005).
[25] I. S. Beloborodov, A. V. Lopatin, V. M. Vinokur, and K. B. Efetov, Granular electronic systems, Rev. Mod. Phys. 79, 469 (2007).
[26] B. Hen, X. Zhang, V. Shelykhin, A. Kapitulnik, and A. Palevski, Superconductor-insulator transition and the crossover to non equilibrium in two-dimensional indium - indium-oxide composite (2020), arXiv:2003.05723 [cond-mat.supr-con].
[27] G. Sambandamurthy, L. W. Engel, A. Johansson, and D. Shahar, Superconductivity-related insulating behavior, Phys. Rev. Lett. 92, 107005 (2004).
[28] M. Steiner and A. Kapitulnik, Superconductivity in the insulating phase above the field-tuned superconductor-insulator transition in disordered indium oxide films, Physica C: Superconductivity 422, 16 (2005).
[29] N. P. Breznay, M. A. Steiner, S. A. Kivelson, and A. Kapitulnik, Self-duality and a hall-insulator phase near the superconductor-to-insulator transition in indium-oxide films, Proceedings of the National Academy of Sciences of the United States of America 113, 280 (2016).
[30] See Supplemental Material at [URL will be inserted by publisher] for details on sample characterization, full MR data, logarithmic divergence numerical analysis, RF injection experiment and further results.
[31] M. Philipp, I. Gutierrez-Lezama, N. Ubrig, and A. F. Morpurgo, Lithium-ion conducting glass ceramics for electrostatic gating, Appl. Phys. Lett. 113, 033502 (2018).
[32] Such samples, which do not show a zero-field global superconductivity, will be discussed in a different publication.
[33] T. I. Baturina, C. Strunk, M. R. Baklanov, and A. Satta, Quantum metallicity on the high-field side of the superconductor-insulator transition, Phys. Rev. Lett. 98, 127003 (2007).
[34] W. R. White, A. Kapitulnik, and M. R. Beasley, Collective vortex motion in a-MoGe superconducting thin
films. [Phys. Rev. Lett. 70, 670 (1993)]

[35] M. V. Feigel’man, V. B. Geshkenbein, and A. I. Larkin, Pinning and creep in layered superconductors, Physica C: Superconductivity 167, 177 (1990)

[36] A. Kapitulnik, N. Mason, S. A. Kivelson, and S. Chakravarty, Effects of dissipation on quantum phase transitions, Phys. Rev. B 63, 125322 (2001)

[37] Y. Ando, G. S. Boebinger, A. Passner, T. Kimura, and K. Kishio, Logarithmic divergence of both in-plane and out-of-plane normal-state resistivities of superconducting La$_{2-x}$Sr$_x$CuO$_4$ in the zero-temperature limit, Phys. Rev. Lett. 75, 4662 (1995)

[38] M. Mulligan and S. Raghu, Composite fermions and the field-tuned superconductor-insulator transition, Phys. Rev. B 93, 205116 (2016).

[39] C. Yang, Y. Liu, Y. Wang, L. Feng, Q. He, J. Sun, Y. Tang, C. Wu, J. Xiong, W. Zhang, X. Lin, H. Yao, H. Liu, G. Fernandes, J. Xu, J. M. Valles, J. Wang, and Y. Li, Intermediate bosonic metallic state in the superconductor-insulator transition 10.1126/science.aax5798 (2019).

[40] A. F. G. Wyatt, V. M. Dmitriev, W. S. Moore, and F. W. Sheard, Microwave-enhanced critical supercurrents in constricted tin films, Phys. Rev. Lett. 16, 1166 (1966).

[41] A. H. Dayem and J. J. Wiegand, Behavior of thin-film superconducting bridges in a microwave field, Physical Review 155, 419 (1967).

[42] H. A. Notarys, M. L. Yu, and J. E. Mercereau, Josephson effects at high current density, Phys. Rev. Lett. 30, 743 (1973).

[43] J. M. Warlaumont, J. C. Brown, T. Foxe, and R. A. Buhrman, Microwave-enhanced proximity effect in superconductor-normal-metal-superconductor microjunctions, Phys. Rev. Lett. 43, 169 (1979).

[44] T. M. Klapwijk and P. J. de Visser, The discovery, disappearance and re-emergence of radiation-stimulated superconductivity, ANNALS OF PHYSICS 417, 10.1016/j.aop.2020.168104 (2020).

[45] G. Eliashberg, Film superconductivity stimulated by a high-frequency field, Sov. Phys. JETP-Lett. 11, 114 (1970).

[46] C. W. Wilks, R. Bojko, and P. M. Chaikin, Field dependence of the resistive transition for a square wire network, Phys. Rev. B 43, 2721 (1991).

[47] R. Steinmann, L. P., J. Chaussy, and B. Pannetier, Magnetoresistance oscillations in bulk high-$T_c$ superconductor, PHYSICA C 153, 1487 (1988).

[48] U. Patel, Z. L. Xiao, A. Gurevich, S. Avci, J. Hua, R. Diyan, U. Welp, and W. K. Kwok, Magnetoresistance oscillations in superconducting granular niobium nitride nanowires, Phys. Rev. B 80, 012504 (2009).

[49] J. Wang, X. Ma, S. Ji, Y. Qi, Y. Fu, A. Jin, L. Lu, C. Gu, X. C. Xie, M. Tian, J. Jia, and Q. Xue, Magnetoresistance oscillations of ultrathin Pb bridges, NANO RESEARCH 2, 671 (2009).
Supplemental Material for:
Robust anomalous metallic states and vestiges of self duality in two-dimensional granular In composites

S-I. GRAIN SIZE DISTRIBUTION FROM CLUSTERING ANALYSIS ON SEM IMAGES

Fig. S1. Grain clustering analysis on SEM image. Identification of grain clusters in the raw image presented in Fig. [1(a)] of the main manuscript using open-source scipy and opencv packages in Python. Grain size histogram (Fig. [1(b)] of the main manuscript) was calculated based on this result. The axes are shown in a unit of pixel, where 1 px = 0.006 μm. Each color represents a distinct grain.

Fig. S1 shows the result of grain clustering analysis on SEM image shown in Fig. [1(a)]. The raw image was preprocessed using opencv package in Python. To find the contour of grain boundary, we performed threshold the gray-scale image to create a binary bitmap. A series of morphology transformation (2-dilate/1-erode) was performed to highlight the contours. Then, we took the binary bitmap as a mask, overlaid with the original image, and perfected the grain boundary on the mask according to the raw image. After such image processing procedure, we have created a binary bitmap (mask) illustrating correct grain morphology (white: within a grain; black: grain boundary). Subsequently, a standard clustering analysis was performed on the mask and grain statistics was generated by scipy package in Python, rendering the histogram shown in Fig. [1(b)]. The size distribution of these well-separated grains spans 2 orders of magnitude in length from as large as ~1 μm to as small as ~0.01 μm (1 or 2 pixels wide), where the distribution is cut-off by limitations in the image resolution.

S-II. MAGNETORESISTANCE OF SAMPLE S0 AND S1

Fig. S2. Magneto-resistance and temperature dependence of resistivity of S0 and S1. (a–b) Magneto-resistance in log-log plot for S0 and S1 which are not shown in the main text. Data smoothed for higher visibility. Similarly, in the saturation regime MR behaves as a power-law. MR peak and the trailing tail are both similar to S2/S2G but at a higher resistivity level. Arrows indicate resistivity at 8 T.

(c–d) Resistivity versus logarithm of temperature for S0 and S1. Logarithmic divergence similar to those in S2 and S2G can be seen in S1.

S-III. NUMERICAL ANALYSIS SUGGESTING CONNECTION BETWEEN LOGARITHMIC RESISTIVITY DIVERGENCE AND EXTREME GRANULAR INHOMOGENEITY

Fig. S3 shows the result of our numerical analysis on logarithmic resistivity divergence (or other weak divergence), plotted as fractional change in resistivity compared to resistivity at T = 1. This analysis is inspired by the experimental result (a weak resistivity di-
where $\Delta$. The resulting expression for resistivity is

$$R = (E1\Delta_r/T)^{-1} \approx (\gamma - \ln(\Delta_r/T) + \Delta_r/T)^{-1}$$

(S.2)

where $\gamma \approx 0.577$ is the Euler-Mascheroni constant and $\Delta_r$ is a lower energy cut-off for barrier height $\Delta \equiv 10^{-6}$.

S-IV. INEFFECTIVE RF INJECTION AND ANALYSIS OF RF TRANSMISSION

Fig. 4 of the main manuscript shows RF injection experiments that have no effect on sample resistivity. The colored data are reference data with no RF injection, and is the same data as shown in Fig. 3 of the main manuscript. All RF injection used a minimum output power of 0 dBm (1 mW) from a PTS 6400 signal generator.

S-V. RF INJECTION IN THE INSULATING REGIME

Fig. S5 shows resistivity versus logarithm of temperature for S2G in insulating regime, where an anomalous
FIG. S5. RF injection experiments in the insulating regime. Resistivity versus logarithm of temperature for S2G at 0.07 T. The resistivity in the absence of RF signal diverges logarithmically, indicated by the dashed line. Red trace shows effect of 2 GHz signal coupled to the sample.

logarithmic resistivity divergence is found ubiquitously on the insulating side of the avoided H-SIT. Here we demonstrate that by applying 2 GHz signal, using the same method as in Fig. 4, the resistivity is also reduced by $\sim 20\%$. For the same reason discussed in text, such enhancement to the conductivity indicates that the insulating regime is still dominated by local superconductivity.