REVISITING THE CONFRONTATION OF THE ENERGY CONDITIONS WITH SUPERNOVAE DATA

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In the standard Friedmann–Lemaître–Robertson–Walker (FLRW) approach to model the Universe the violation of the so-called energy conditions is related to some important properties of the Universe as, for example, the current and the inflationary accelerating expansion phases. The energy conditions are also necessary in the formulation and proofs of Hawking-Penrose singularity theorems. In two recent articles we have derived bounds from energy conditions and made confrontations of these bounds with supernovae data. Here, we extend these results in following way: first, by using our most recent statistical procedure for calculating new $q(z)$ estimates from the gold and combined type Ia supernovae samples; second, we use these estimates to obtain a new picture of the energy conditions fulfillment and violation for the recent past ($z \leq 1$) in the context of the standard cosmology.

Keywords: Energy conditions; energy condition confrontation with supernovae data.

1. Introduction

In the absence of constraints on the energy-momentum tensor $T_{\mu\nu}$ any metric satisfies Einstein’s equations since they can be regarded as a definition of $T_{\mu\nu}$, i.e., a set of equations determining $T_{\mu\nu}$ for any given metric $g_{\mu\nu}$. However, if one wishes to explore general properties that hold for a variety of different physical sources it is convenient to impose the so-called energy conditions that limit the arbitrariness of $T_{\mu\nu}$ on physical grounds.

On scales relevant for cosmology, an important point in the study of the energy conditions is the confrontation of their predictions with the observational data. By using model-independent energy-condition integrated bounds on the cosmological observables as, for example, the distance modulus and lookback time, this confrontation has been made in some recent articles (see also the pioneering Refs. [9] by Visser). In Ref. [10] however, it was shown that the fulfillment (or the violation) of these integrated bounds at a specific redshift $z$ is not a sufficient (nor a necessary)
local condition for the fulfillment (or respectively the violation) of the associated energy condition at $z$. In this way, the confrontation between the prediction of these integrated bounds and observational data cannot be used to draw conclusions on the fulfillment (or violation) of the energy conditions at $z$. In Ref. [10] this problem was overcome by deriving new non-integrated energy-condition bounds, and confrontations between the new bounds with type Ia supernovae (SNe Ia) data of the gold [14] and combined [15] samples were performed by using the upper and lower limits of confidence regions on $E(z) - q(z)$ plane. More recently, in Ref. [16] a new statistical way for estimating the deceleration parameter $q(z)$ was carried out, and a new picture of the energy conditions fulfillment and violation for recent past ($z \leq 1$) was calculated by using the recently compiled Union sample [17].

In this work, we use the most recent statistical procedure introduced in Ref. [16] along with the gold [14] and combined [15] samples to obtain estimates of $q(z)$ in order to build up a new picture of the confrontation between the energy condition integrated bounds and these SNe Ia data sets, completing therefore the cycle of this type of analysis which involves these three samples and the two statistical procedures to $q(z)$ estimates of Refs. [10] and [16].

2. Preliminaries

It is known that the energy conditions can be stated in a coordinate-invariant way in terms of $T_{\mu\nu}$ and vector fields of fixed character (timelike, null and spacelike). However, within the framework of the standard Friedmann-Lemaître-Robertson-Walker (FLRW) model, we only need to consider the energy-momentum tensor of a perfect fluid with density $\rho$ and pressure $p$, i.e., $T_{\mu\nu} = (\rho + p) u_\mu u_\nu - p g_{\mu\nu}$. In this context, the energy conditions take the following forms:

\begin{align}
\text{NEC : } & \quad \rho + p \geq 0 , \\
\text{WEC : } & \quad \rho \geq 0 \quad \text{and} \quad \rho + p \geq 0 , \\
\text{SEC : } & \quad \rho + 3p \geq 0 \quad \text{and} \quad \rho + p \geq 0 , \\
\text{DEC : } & \quad \rho \geq 0 \quad \text{and} \quad -\rho \leq p \leq \rho ,
\end{align}

where NEC, WEC, SEC and DEC correspond, respectively, to the null, weak, strong, and dominant energy conditions. For a FLRW metric with a scale factor $a(t)$, the density $\rho$ and pressure $p$ of the cosmological fluid are given by

\begin{align}
\rho &= \frac{3}{8\pi G} \left[ \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right] \quad \text{and} \quad p = -\frac{1}{8\pi G} \left[ 2 \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right],
\end{align}

where overdots denote the derivative with respect to the time $t$ and $G$ is Newton’s gravitational constant.

\text{Energy conditions constraints on the so-called } f(R) \text{-gravity have also been investigated in Ref. [11] and more recently in Refs. [12] and [13].}
The non-integrated bounds from energy conditions derived in Ref. 10 can be obtained in terms of the deceleration parameter $q(z) = -\ddot{a}/aH^2$, the normalized Hubble function $E(z) = H(z)/H_0$, and the curvature density parameter $\Omega_{k0} = -k/(a_0H_0)^2$, simply by substituting Eqs. (2) into Eqs. (1). This gives

\begin{align*}
\text{NEC} & \iff q(z) - \Omega_{k0} (1 + z)^2 \geq -1, \\
\text{WEC} & \iff \frac{E^2(z)}{(1 + z)^2} \geq \Omega_{k0}, \\
\text{SEC} & \iff q(z) \geq 0, \\
\text{DEC} & \iff q(z) + 2\Omega_{k0} (1 + z)^2 \frac{E^2(z)}{E^2(z)} \leq 2,
\end{align*}

where $z = (a_0/a) - 1$ is the redshift, $H(z) = \dot{a}/a$, and the subscript 0 stands for present-day quantities.

In this work, we focus on the FLRW flat ($\Omega_{k0} = 0$) universe. In this case the NEC, SEC and DEC bounds reduce, respectively, to $q(z) \geq -1$, $q(z) \geq 0$ and $q(z) \leq 2$, while the WEC bound is fulfilled identically. Thus, having estimates of $q(z_\star)$ for different redshifts $z_\star$, one can test the fulfillment or violation of the energy conditions at each $z_\star$.

Now, the $q(z_\star)$ estimates are obtained by using a SNe Ia data set, by approximating the deceleration parameter $q(z)$ function as the following linear piecewise continuous function:

$$q(z) = q_l + q'_l \Delta z_l, \quad z \in (z_l, z_{l+1}),$$

where the subscript $l$ means that the quantity is taken at $z_l$, $\Delta z_l \equiv (z - z_l)$, and the prime denotes the derivative with respect to $z$. The supernovae observations provide the redshifts and distance modulus

$$\mu(z) = 5 \log_{10} \left[ \frac{c(1 + z)}{H_0 1\text{Mpc}} \int_0^z \frac{dz'}{E(z')} \right] + 25.$$

Then, by using the following well known relation between $q(z)$ and $E(z)$:

$$E(z) = \exp \int_0^z \frac{1 + q(z')}{1 + z'} \, dz',$$

along with Eq. (8), we fitted the parameters of the $q(z)$, as given by (7), by using the SNe Ia redshift–distance modulus data from the gold[13] and combined[15] samples.

3. Results and Conclusions

Since in the flat case the energy condition bounds given by Eqs. (3), (5) and (6) depend only on $q(z)$, we have obtained the $q(z_\star)$ estimates at 1σ – 3σ confidence

\footnote{Through out this paper we use the notation of Ref. 10 in which NEC, WEC, SEC and DEC correspond, respectively, to $\rho + p \geq 0$, $\rho \geq 0$, $\rho + 3p \geq 0$ and $\rho - p \geq 0$.}
levels from gold and combined SNe Ia samples by marginalizing over $E(z_\star)$ and the other parameters ($q'_l$’s) of the $q(z)$ function [Eq.(7)].

A global picture of the breakdown and fulfillment of the energy conditions in the recent past has been built up with the $q(z_\star)$ estimates at 200 equally spaced redshifts in the interval $(0,1]$. Fig. 1(a) shows the NEC, SEC, and DEC bounds along with the best-fit values and the $1\sigma$, $2\sigma$ and $3\sigma$ limits of $q(z_\star)$ in the $q(z) - z$ plane. We recall that WEC bound [$E(z) \geq 0$] is fulfilled identically.

![Fig. 1](image_url)

**Fig. 1.** The best-fit, the upper and lower $1\sigma$, $2\sigma$ and $3\sigma$ limits of $q(z_\star)$ estimates, obtained with the gold [panel (a)] and the combined [panel (b)] samples, for 200 equally spaced redshifts. The NEC and SEC lower bounds, and also the DEC upper bound are shown. This figure shows that the SEC is violated with $1\sigma$ confidence level from $\approx 0$ until $z \approx 0.31$ [gold sample, panel (a)], and until $z \approx 0.52$ [combined sample, panel (b)]. It also shows that the DEC and NEC is violated within $3\sigma$ confidence level for high redshifts for both supernovae samples, and that the NEC is violated for $z \lesssim 0.105$ [panel (a)] and $z \lesssim 0.085$ [panel (b)].

In Fig. 1 it is showed that the SEC bound is violated with $1\sigma$ confidence level until $z = 0.31$ for gold and $z = 0.52$ for combined sample, while in the redshift intervals $(0.09,0.17)$ [panel (a)] and $(0.08,0.18)$ [panel (b)] this violation occurs with more than $3\sigma$ confidence level, where the highest evidence is found at $z = 0.135$ with $\approx 3.86\sigma$ [gold, panel (a)] and $\approx 4.28\sigma$ [combined, panel (b)]. Unlike the result of Ref. 10 wherein these analyses have been performed by computing the confidence regions on the $E(z_\star) - q(z_\star)$ plane, revealing no redshift value for the SEC fulfillment with at least $1\sigma$, we note here the SEC is fulfilled with more than $1\sigma$ for $z \gtrsim 0.615$ [gold, panel (a)] and $z \gtrsim 0.855$ [combined, panel (b)]. According to the present SEC analysis, with $1\sigma$ confidence level, the universe crosses over from a decelerated expansion phase to an accelerated expansion during the redshift interval $(\approx 0.31, \approx 0.615)$ for gold and $(\approx 0.52, \approx 0.855)$ for combined sample.

\(^{d}\)We note that this statistical approach has been previously used in Ref. 16 but for the SNe Ia Union sample.17

\(^{d}\)We recall that in a similar SEC analysis of Ref. 10 performed by using the Union sample, the
Regarding the NEC, Fig. 1 indicates its breakdown within 3σ confidence level for low redshift, \( z \lesssim 0.105 \) [panel (a)] and \( z \lesssim 0.085 \) [panel (b)]. For higher values of redshift, NEC is violated within 3σ at \( z \gtrsim 0.94 \) for gold and \( z \gtrsim 0.96 \) for combined sample.

Concerning the DEC, Fig. 1 indicates that it is violated within 3σ for \( z \gtrsim 0.795 \) [gold, panel (a)] and \( z \gtrsim 0.83 \) [combined, panel (b)], which are intervals where the error in the estimates of \( q(z) \) grow significantly, though. Finally, we note that the DEC violation of the present analysis is weaker than that obtained in Ref. 10 in the sense that, differently from that analysis, now the DEC is fulfilled with 1σ confidence level in the entire redshift interval for both samples.

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Deceleration to acceleration transition expansion phase of the universe took place in the redshift interval \( (\simeq 0.4, \simeq 0.64) \).