Online Residential Demand Response via Contextual Multi-Armed Bandits

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Abstract—Residential load demands have huge potential to be exploited to enhance the efficiency and reliability of power system operation through demand response (DR) programs. This paper studies the strategies to select right customers for residential DR from the perspective of load service entities (LSEs). One of the main challenges to implement residential DR is that customer responses to the incentives are uncertain and unknown, which are influenced by various personal and environmental factors. To address this challenge, this paper employs the contextual multi-armed bandit (CMAB) method to model the optimal customer selection problem with uncertainty. Based on Thompson sampling framework, an online learning and decision-making algorithm is proposed to learn customer behaviors and select appropriate customers for load reduction. This algorithm takes the contextual information into consideration and is applicable to complicated DR settings. Numerical simulations are performed to demonstrate the efficiency and learning effectiveness of the proposed algorithm.

Index Terms—Residential demand response, online learning, multi-armed bandits, uncertainty.

I. INTRODUCTION

With the deepening penetration of renewable generation and growing peak load demands, power systems are inclined to confront a deficiency of reserve capacity for power balance. Instead of deploying additional generators, demand response (DR) is an economical and environmentally friendly alternative solution that motivates the change of flexible load demands to fit the needs of power supply. Most of the existing DR programs focus on industrial and large commercial consumers through the direct load control and interruptible loads [1], [2]. While residential loads actually take up a significant share of the total electricity usage (e.g. about 38% in the U.S. [3]), which have a huge potential to be exploited to facilitate power system operation. Moreover, the proliferation of smart meters, smart sensors, and automatic devices, enables the remote monitoring and control of home electric appliances with a two-way communication between load service entities (LSEs) and households. As a consequence, residential DR has attracted a great deal of recent interest from both academia and industry.

The mechanisms for residential DR are mainly categorized as price-based and incentive-based [4]. The price-based DR programs [5–7] use various pricing schemes, such as time-of-use pricing, critical peak pricing, and real-time pricing, to influence and guide the residential electricity usage. While the reactions of customers to the price change signals are highly uncontrollable and may lead to undesired load dynamics like rebound peak [5]. In incentive-based DR programs [8–10], the LSEs recruit customers to participate in an upcoming DR event with financial incentives, e.g. cash, coupon, raffle, rebate, etc. During the DR event, customers reduce their electricity consumption to earn the revenue but are allowed to opt out at any time. For the LSEs, it is significant to target appropriate customers among the large population, since each recruitment comes with a cost. Accordingly, this paper focuses on the incentive-based residential DR programs, and studies the strategies to select right customers for load reduction from the perspective of LSEs.

In practical DR implementation, the LSEs confront a major challenge that the customer behaviors to the incentives are uncertain and unknown. According to the investigations in [11–13], the user acceptances of DR load control are influenced by individual preference and environmental factors. The former relates to customers’ intrinsic socio-demographic characteristics, e.g., income, age, education, household size, attitude to energy saving, etc. The latter refers to immediate externalities such as indoor temperature, offered revenue, electricity price, fatigue effect, weather conditions, etc. However, LSEs barely have the access to customers’ individual preferences or the knowledge how environmental factors affect their opt-out behaviors. Without considering the actual willingness, a blind customer selection scheme for DR participation may lead to high opt-out rate and inefficient load shedding.

To address this challenge, a natural idea is to learn unknown customer behaviors through interaction and observation. In particular, the multi-armed bandit (MAB) framework [14] can be used to model the residential DR as an online learning and decision-making problem. MAB deals with the uncertain optimization problems where an agent selects actions (arms) sequentially and upon each action observes a reward, while the agent is uncertain about how his actions influence the rewards. After each action-reward pair is observed, the agent learns about the rewarding system, and this allows him to improve the action strategies. In addition, when the reward (or outcome) to each action is not fixed but affected by some contexts, e.g. environmental factors and agent profiles, such problems are referred as contextual MAB (CMAB) [14]. Due to its useful structure, (C)MAB has been successfully applied in many fields, such as recommendation system [15], clinical trial [16], web advertisement [17], and etc.

To achieve high performance in MAB problems, it necessitates an effective balance between exploration and exploita-
tion, i.e., whether taking a risk to explore poorly understood actions or exploiting current knowledge for decision-making. To this end, upper confidence bound (UCB) and Thompson sampling (TS) are two prominent algorithms that solve MAB problems. UCB algorithms \cite{18, 19} employ the upper confidence bound as an optimistic estimate of the mean reward for each action and encourage the exploration with optimism. In a Bayesian style, TS algorithms \cite{20} form a prior distribution over the unknown parameters and select actions based on a random sample from the posterior distribution, which motivates the exploration via sampling randomness. Without the necessity of sophisticated design for UCB functions, TS has a simple and flexible solution structure that can be generalized to complex online optimization problems, and generally achieves better empirical performance than UCB algorithms \cite{21}.

For the residential DR, most existing literature does not consider or overly simplify the uncertainty of customer behaviors, which causes a disconnection between theory and practice. A number of studies \cite{22, 23, 24, 25} use online learning techniques to deal with the unknown information in DR problems. In terms of the price-based DR, references \cite{22, 23} use reinforcement learning to learn the customer dissatisfaction on job delay, and automatically schedule the household electricity usage under the time-varying price. From the perspective of LSEs, references \cite{24, 25} design the dynamical pricing schemes based on the MAB method and an online linear regression algorithm respectively, considering the uncertain user responses to the price change. For the incentive-based DR, in \cite{26, 27, 28, 29}, MAB and its variations like adversarial MAB, restless MAB, are utilized to learn customer behaviors and unknown load parameters and select right customers to send DR signals, where UCB algorithms, policy index, and other heuristic algorithms are applied for solution. In these researches, simplified DR problem formulations are employed, and the effects of time-varying environmental factors on customer behaviors are mostly neglected.

In this paper, to deal with the uncertain customer behaviors and the environmental influence, the CMAB method is used to model the customer selection issue in incentive-based DR programs as an online learning and decision-making problem. Specifically, the logistic regression approach \cite{30} is employed to predict the customer opt-out behaviors under given contextual conditions. Based on the TS framework, we develop the online learning and selection (OLS) algorithm for practical DR implementation. There are two main contributions of the proposed OLS algorithm:

1) The contextual information, including individual diversity and time-varying environmental factors, are considered when learning and predicting customer behaviors.
2) With the decomposition structure of learning and optimization, the proposed OLS algorithm is applicable to complex DR settings and applications.

Besides, the theoretical performance guarantee on the proposed algorithm is provided, which shows that a sublinear Bayesian regret can be achieved.

The remainder of this paper is organized as follows: Section \textbf{II} introduces the residential DR problem formulation with uncertainty. Section \textbf{III} develops the online learning and customer selection algorithm. Section \textbf{IV} analyzes the performance of the proposed OLS algorithm. Numerical tests are carried out in Section \textbf{V} and conclusions are drawn in Section \textbf{VI}.

\section*{II. Problem Formulation}

\subsection*{A. Residential DR Model}

Consider the residential DR program with a system aggregator (SA) and \(N\) customers over a time horizon \([T] := \{1, \cdots, T\}\), where each time \(t \in [T]\) corresponds to one DR event. As illustrated in Figure \ref{fig:1}, there are two phases in a typical DR event \cite{31}. Phase 0 denotes the preparation period, when the SA calls upon customers for load reduction with incentives and selects a subset of participating customers under a certain budget. In phase 1, the selected customers decrease their electric usage, e.g., shut down the air conditioner or increase the setting temperature, while they can choose to opt out if unsatisfied. In the end, the SA pays the selected customers according to their contributions to the load reduction.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig1}
\caption{Two phases of a residential DR event.}
\end{figure}

For customer \(i \in [N] := \{1, \cdots, N\}\), let \(d_{i,t}\) be the load demand that can be shut off at \(t\)-th DR event, and \(r_{i,t}\) be the revenue (or the bidding price) for such load reduction. Denote \(z_{i,t} \in \{0, 1\}\) as the binary variable indicating whether customer \(i\) stays in (equal 1) or opts out (equal 0) during \(t\)-th DR event if selected. Assume that \(z_{i,t}\) is a random variable following Bernoulli distribution with \(z_{i,t} \sim \text{Bern}(p_{i,t})\), which is independent across times and customers. Without loss of generality, assume that all customers decide to participate in every DR event at phase 0, otherwise let \(r_{i,t}\) be sufficiently large for the unpaticipated customers.

This paper studies the customer selection strategies at phase 0 from the perspective of the SA. At \(t\)-th DR event, based on the reported \((d_{i,t}, r_{i,t})_{i \in [N]}\), the SA aims to select a subset of customers to achieve certain DR goals under the given budget \(b_t\). Accordingly, the optimal customer selection (OCS) problem can be formulated as

\begin{equation}
\text{Obj.} \max_{S_t \subseteq [N]} g_t \left( (z_{i,t})_{i \in [N]}, (d_{i,t})_{i \in [N]}, S_t \right)
\end{equation}

\begin{equation}
s.t. \ h_t \left( (z_{i,t})_{i \in [N]}, (r_{i,t})_{i \in [N]}, S_t \right) \leq b_t, \ S_t \in \Pi_t
\end{equation}

where \(g_t\) and \(h_t\) represent the objective function and the cost function respectively, \(S_t\) is the decision variable denoting the set of selected customers, \(\Pi_t\) is the feasible set of \(S_t\) that describes other physical constraints. For example, the network and power flow constraints can be captured by \(\Pi_t\), which is elaborated in Appendix \textbf{A}.

The OCS model \textbf{(1)} is a general formulation. Depending on practical DR applications, functions \(g_t\) and \(h_t\) can take different forms. Two concrete examples are provided as follows.
Example 1. Model \( \text{OLS} \) maximizes the total expected load reduction, and constraint \( \text{OLS} \) ensures that the revenue payment does not exceed the given budget.

\[
\begin{align*}
\text{Obj.} & \quad \max_{\mathcal{S}_t \subseteq [N]} \mathbb{E}(\sum_{i \in \mathcal{S}_t} d_{i,t}z_{i,t}) = \sum_{i \in \mathcal{S}_t} d_{i,t}p_{i,t} \\
\text{s.t.} & \quad \sum_{i \in \mathcal{S}_t} r_{i,t} \leq b_t
\end{align*}
\]

Example 2. Model \( \text{OLS} \) aims to track a load reduction target \( D_t \), where the objective \( \text{OLS} \) minimizes the expected squared deviation. Equation \( \text{OLS} \) is a cardinality constraint on \( S_t \), which limits the number of selected customers by \( b_t \).

\[
\begin{align*}
\text{Obj.} & \quad \min_{\mathcal{S}_t \subseteq [N]} \mathbb{E}(\sum_{i \in \mathcal{S}_t} d_{i,t}z_{i,t} - D_t)^2 \\
\text{s.t.} & \quad |S_t| = \sum_{i \in \mathcal{S}_t} 1 \leq b_t
\end{align*}
\]

Since \( z_{i,t} \sim \text{Bern}(p_{i,t}) \) is assumed to be independent across customers, the objective function in \( \text{OLS} \) is equivalent to

\[
\min_{\mathcal{S}_t \subseteq [N]} (\sum_{i \in \mathcal{S}_t} d_{i,t}p_{i,t} - D_t)^2 + \sum_{i \in \mathcal{S}_t} d_{i,t}^2p_{i,t}(1-p_{i,t})
\]

In the follows, Example 1 with the OCS model \( \text{OLS} \) is used for the illustration of algorithm design, while the proposed framework is clearly applicable to other application cases.

B. Contextual MAB Modelling

If the probability profiles \( p_i := (p_{i,t})_{t \in [N]} \) of customers are known, the OCS model \( \text{OLS} \) is purely an optimization problem. However, the SA barely has access to \( p_i \) in practice, which actually depict the customer opt-out behaviors. Moreover, the probability profiles \( p_i \) are time-varying and influenced by various environmental factors. To address this uncertainty issue, the CMAB framework is leveraged to model the residential DR program as an online learning and decision-making problem.

In the CMAB language, each customer \( i \in [N] \) is treated as an independent arm. A set of multiple arms, called a superarm, constitutes a possible action that the agent takes. At each time \( t \in [T] \), the SA plays the agent role and takes the action \( S_t \). Then the SA observes the outcomes \( (z_{i,t})_{i \in S_t} \) that are generated from the distributions \( \text{Bern}(p_{i,t}) \), and enjoys the reward of \( \sum_{i \in S_t} d_{i,t}z_{i,t} \). Since the customer opt-out outcome \( z_{i,t} \) is binary, the widely-used logistic regression method \( \text{OLS} \) is utilized to learn the unknown \( p_{i,t} \):

\[
p_{i,t} = \frac{\exp(\alpha_i + x_{i,t}^\top\beta_i)}{1 + \exp(\alpha_i + x_{i,t}^\top\beta_i)} \quad \forall i \in [N], t \in [T]
\]

\[x_{i,t} \in \mathbb{R}^m \] is the feature vector that captures the environmental factors for customer \( i \) at \( t \)-th DR event. Each entry of \( x_{i,t} \) corresponds to a quantified factor, such as the offered revenue, indoor temperature, real-time electricity price, the fatigue effect of being repeatedly selected, etc. \( \beta_i \in \mathbb{R}^m \) is the weight vector describing how customer \( i \) reacts to those factors, and \( \alpha_i \) denotes its individual preference. Denote \( x_{i,t} := (1, x_{i,t}) \) as the context vector and \( \theta_i := (\alpha_i, \beta_i) \), then the linear term in \( \text{OLS} \) becomes \( x_{i,t}^\top\theta_i \), and the unknown parameter of each customer \( i \) is summarized by \( \theta_i \).

As a result, the sequential customer selection in residential DR is modelled as a CMAB problem. The SA aims to learn the unknown \( (\theta_i)_{i \in [N]} \) and improves the customer selection strategies. To this end, we propose the online learning and selection (OLS) algorithm to solve this CMAB problem efficiently, which is elaborated in the next section.

III. ALGORITHM DESIGN

In this section, the TS algorithm is introduced, and the offline optimization method and Bayesian inference method are presented. Then we assemble these methods and develop the OLS algorithm for residential DR.

A. Thompson Sampling Algorithm

Consider a general \( T \)-times MAB problem where an agent selects an action \( a_t \) from the action set \( A_t \) at each time \( t \). After applying action \( a_t \), the agent observes an outcome \( u_t \), which is randomly generated from a conditional probability distribution \( P(\cdot | a_t) \), and then obtains a reward \( R_t = R(u_t) \) with known deterministic function \( R(\cdot) \). The agent intends to maximize the total expected reward but is initially uncertain about the value of \( \theta \). Thompson sampling (TS) algorithm \([20]\) is a Bayesian learning framework that solves such MAB problems with effective balance between exploration and exploitation.

Algorithm 1 Thompson Sampling Algorithm

1: \textbf{Input:} Prior distribution \( P \) on \( \theta \).
2: \textbf{for} \( t = 1 \) to \( T \) \textbf{do}
3: \hspace{1em} Sample \( \theta \sim P \).
4: \hspace{1em} \( a_t \leftarrow \arg \max_{a \in A_t} \mathbb{E}_{P(\cdot | a)}[R(u_t) | a_t = a] \).
5: \hspace{1em} Apply \( a_t \) and observe \( u_t \).
6: \textbf{end for}

As illustrated in Algorithm 1, TS algorithm represents the initial belief on \( \theta \) using a prior distribution \( P \). At each time \( t \), TS draws a random sample \( \theta \) from \( P \), then takes the optimal action based on the sample \( \theta \). After the outcome \( u_t \) is observed, the Bayesian rule is applied to update the belief and obtain the posterior distribution of \( \theta \). There are three key observations about the TS algorithm:

1) As outcome data accumulate, the predefined prior distribution will be washed out and the posterior converges to the true distribution or true value of \( \theta \).
2) The TS algorithm encourages exploration by the random sampling. As the posterior distribution gradually concentrates, less exploration and more exploitation will be performed, which strikes an effective balance.
3) The crucial advantage of TS algorithm is that the complex online problem is decomposed into a Bayesian learning task and a deterministic optimization task. In particular,
the optimization problem remains the original model formulation, which enables efficient solution methods.

In the follows, the specific methods for the optimization task and the Bayesian learning task are presented respectively.

B. Offline Optimization Method

At each DR event, given the customer probability profiles \( p_i \), the OCS model \([2]\) can be equivalently reformulated as the binary optimization problem \([5]\):

\[
\text{Obj.} \max_{y_{i,t} \in \{0, 1\}} \sum_{i=1}^{N} d_{i,t} p_{i,t} y_{i,t} \quad (5a)
\]

\[
\text{s.t.} \sum_{i=1}^{N} r_{i,t} y_{i,t} \leq b_t \quad (5b)
\]

where binary variable \( y_{i,t} \in \{0, 1\} \) is introduced to indicate whether the SA selects customer \( i \) or not at \( t \)-th DR event.

The binary optimization model \([5]\) can be solved efficiently using many available optimizers such as IBM CPLEX and Gurobi, which are employed as the offline solution tools. Let \( \hat{y}_i^t := (y_{i,t})_{i \in [N]} \) be the optimal solution of model \([5]\). For concise expression, the optimizer tools are denoted as an offline oracle \( O : p_i \rightarrow \hat{y}_i^t \). It is worth mentioning that when taking the power flow constraints \([19]\) \([20]\) in Appendix A into consideration, the OCS model becomes a mixed integer linear programming, which can be solved efficiently as well.

C. Bayesian Inference for Logistic Model

For clear expression, we abuse notations a bit and discard subscripts \( i \) and \( t \) in this part. Under the TS framework, a prior distribution \( P(\theta) \) on the unknown parameter \( \theta \) is constructed. After the outcome \((\hat{x}, \hat{z})\) is observed, the posterior distribution is calculated by the Bayesian law:

\[
P(\theta|\hat{x}, \hat{z}) = \frac{P(\theta)P(z|\theta, \hat{x})}{\int_{\theta} P(\theta)P(z|\theta, \hat{x})d\theta} \quad (6)
\]

and the logistic likelihood function \( P(z|\theta, \hat{x}) \) is given by

\[
P(z|\theta, \hat{x}) = \phi \left( (2\hat{z} - 1)\hat{x}^\top \theta \right) \quad (7)
\]

where \( \phi(x) := 1/(1 + e^{-x}) \) and \((7)\) is equivalent to \((4)\).

Due to the analytically inconvenient form of the likelihood function, Bayesian inference for the logistic regression model is recognized as an intrinsically hard problem \([32]\), and thus the exact posterior \( P(\theta|\hat{x}, \hat{z}) \) \([6]\) is intractable to compute. To address this issue, the variational Bayesian inference approach \([33]\) is employed to develop an analytical approximation of the posterior. The fundamental tool at the heart of this approach is a lower bound approximation of the likelihood function \((7)\):

\[
P(z|\theta, \hat{x}) \geq \phi(\xi) \exp \left[ \frac{s - \xi}{2} + \ell(\xi)(s^2 - \xi^2) \right] := P(z|\theta, \hat{x}, \xi) \quad (8)
\]

where \( s := (2\hat{z} - 1)\hat{x}^\top \theta \), \( \ell(\xi) := (1/2 - \phi(\xi))/2\xi \), and \( \xi \) is the variational parameter.

The variational distribution \( P(z|\theta, \hat{x}, \xi) \) in \((8)\) has a convenient property that it depends on \( \theta \) only quadratically in the exponent. As the prior is a Gaussian distribution with \( P(\theta) \sim \mathcal{N}(\mu, \Sigma) \), we use the Gaussian-like variational distribution \( P(z|\theta, \hat{x}, \xi) \) to approximate the logistic likelihood function \( P(z|\theta, \hat{x}) \) in the Bayesian inference \([6]\). As a result, the posterior is also a Gaussian distribution \( P(\theta|\hat{x}, z) \sim \mathcal{N}(\hat{\mu}, \Sigma) \) with the closed-form update rule \([9]\):

\[
\hat{\Sigma}^{-1} = \Sigma^{-1} + 2[\ell(\xi)|\hat{x}\hat{x}^\top] \quad (9a)
\]

\[
\hat{\mu} = \hat{\Sigma} \left[ \Sigma^{-1} \mu + (z - 1/2)|\hat{x} \right] \quad (9b)
\]

See \([33]\) for the detailed derivation.

Since the posterior covariance matrix \( \hat{\Sigma} \) depends on the variational parameter \( \xi \), its value needs to be specified such that the lower bound approximation in \((8)\) is optimized. The optimal \( \xi \) is achieved by maximizing the expected complete log-likelihood function \( \mathbb{E}[\log P(\theta)P(z|\theta, \hat{x}, \xi)] \), where the expectation is taken over \( P(\theta|\hat{x}, z, \xi^\Theta) \), and this leads to a closed form solution:

\[
\xi = \sqrt{\hat{x}^\top \hat{\Sigma} \hat{x} + (\hat{x}^\top \hat{\mu})^2} \quad (10)
\]

Alternating between the posterior update \((9)\) and the \( \xi \) update \((10)\) monotonically improves the posterior approximation \([33]\). Generally, by two or three iterations, an accurate approximation can be achieved.

D. Online Learning and Selection Algorithm

For each customer \( i \in [N] \), we construct a Gaussian prior \( \mathcal{N}(\mu_i, \Sigma_i) \) on the unknown \( \theta_i \) based on historical information. Using the TS framework, the online learning and selection (OLS) algorithm for the residential DR problem is developed as Algorithm \([2]\). From statement \([3]\) to \([7]\) it generates a random sample of \( \theta_{i,t} \) and then calculates the probability \( p_{i,t} \) with the contextual information \( \hat{x}_{i,t} \) for each customer. With the obtained probability profiles \( p_i \), the SA determines the optimal selection of customers for load reduction by solving the OCS model \([5]\), when available optimizers can be used. After observing the behavior outcome \( z_{i,t} \) of each selected customer, statements \([9]\) \([13]\) update the posterior on \( \theta_i \) using the variational Bayesian inference approach in section \([III-C]\).

On the other hand, the OLS algorithm inherits the merits of TS, and decomposes the online DR problem into learning and optimization two separate parts. Since the optimization problem remains the original formulation without being corrupted by the learning task, the OLS algorithm can be applied to the practical DR problems with complicated objectives and constraints. On the other hand, the closed form formula for posterior update enables very convenient Bayesian inference on the unknown parameter, which leads to efficient implementation of the OLS algorithm.

IV. PERFORMANCE ANALYSIS

In this section, we provide the performance analysis for the proposed online algorithm and prove that it achieves a \( O(\sqrt{T\log T}) \) Bayesian regret bound when exact Bayesian inference is used.
bounds on Bayesian regret are essentially asymptotic bounds. Which is the expectation of \( \text{Regret}(T) \) since \( \text{Regret}(T) = \sum_{t=1}^{T} (\hat{R}_t - R(t)) \), which indicates that the online algorithm can eventually learn the optimal solution, since \( \text{Regret}(T) \) decreases as the number of rounds increases. The expectation in (12) is taken over the randomness of the customer selection decision made by the online algorithm.

Theoretical results in [21, Section 7] are applied to prove Theorem 1 in the next subsection.

**Proof Sketch of Theorem 2**

We first show that our problem formulation satisfies the two assumptions imposed in [21, Section 7]. For [21, Assumption 1], the reward function (11) is bounded as

\[
0 \leq f_t^\Theta(S_t) \leq \sup_{t \in [T]} \sum_{i \in S_t} d_{i,t} := \bar{D}, \quad \forall t \in [T]
\]

In terms of [21, Assumption 2], we have the following lemma, whose proof is provided in Appendix B-A.

**Lemma 1.** For all \( t \in [T] \), \( f_t^\Theta(S_t) \) conditioned on \( (\Theta, S_t) \) is \( 1/2 \)-\( D \)-sub-Gaussian.

Under Assumption 2, define the reward function class as

\[
\mathcal{F}_t := \{ f_t^\Theta | \Theta \in \Psi \}, \quad t \in [T]
\]

where \( \Psi := \{ \Theta \in \mathbb{R}^{Nm} | ||\Theta||_{\infty} \leq L \} \). The reward function class \( \mathcal{F}_t \) is time dependent due to the time-variant \( d_{i,t} \) and \( \hat{x}_{i,t} \). This is a bit different from the setting in [21, Section 7], but it can be checked the regret analysis still holds since \( d_{i,t} \) and \( \hat{x}_{i,t} \) are given parameters and the task is to learn the unknown \( \Theta \). By applying the result in [21, Proposition 10], we have the following Bayesian regret bound.
Lemma 2. Under Assumption 1 and 2, the Bayesian regret of the OLS algorithm is bounded by

\[
\text{BayRegret}(T) \leq \sup_{t \in [T]} \left\{ 1 + \left[ \dim_E(F_t, T^{-1}) + 1 \right] D + 8 \bar{D} \sqrt{\dim_E(F_t, T^{-1})(1 + o(1) + \dim(K(F_t))T \log T) \right\}
\]

(16)

In (16), \(\dim_K(F_t)\) is the Kolmogorov dimension defined by [21, Definition 1] and \(\dim_E(F_t, T^{-1})\) denotes the \(1/T\) eluder dimension defined by [21, Definition 3] of function class \(F_t\). Intuitively, the Kolmogorov dimension is related to the measure of complexity to learn a function class, while the eluder dimension captures how effectively the unknown values can be inferred from the observed samples. To achieve the final result, we further bound \(\dim_E(F_t, T^{-1})\) and \(\dim_K(F_t)\) in (16) by the following two lemmas, whose proofs are provided in Appendix B-B and B-C respectively.

Lemma 3. Under Assumption 2, we have

\[
\sup_{t \in [T]} \dim_E(F_t, T^{-1}) \leq C_3 \log \left[ C_2(1 + C_1 T^2) \right] + 1 \tag{17}
\]

where the definitions of constants \(C_1, C_2, C_3\) are given as (24).

Lemma 4. Under Assumption 2 we have

\[
\sup_{t \in [T]} \dim_K(F_t) \leq N m m \tag{18}
\]

Combing the results in Lemma 2, we obtain Theorem 1.

V. Numerical Simulations

A. Simulation Setting

Consider the residential DR with \(N = 1000\) customers. The reduced load \(d_{i,t}\) and the offered revenue \(r_{i,t}\) are randomly and independently generated from the uniform distribution \(\text{Unif}[0, 1]\) for each customer, which are fixed for different time steps. At each time \(t\), the SA randomly samples a budget \(b_t\) from \(\text{Unif}[300, 400]\) to recruit customers for DR participation. Set the number of contextual features as \(m = 9\), and let all customers share a same feature vector \(x_i \in \mathbb{R}^9\) at each time \(t\), whose elements are independently generated from \(\text{Unif}[0, 2]\). Assume that there exists an underlying ground truth \(\theta^*_i \in \mathbb{R}^{10}\) associated with each customer, which is generated from \(\text{Unif}[-0.4, 0.6]\) to simulate the customer behaviors. In the OLS algorithm, we assign a Gaussian prior distribution \(N(\theta^*_i + \delta u_i, \sigma^2 I)\) for each customer, where each element of \(u_i\) is randomly generated from \(\text{Unif}[-1, 1]\) and \(I\) denotes the identity matrix. Parameters \(\delta\) and \(\sigma\) denote the mean error and standard deviation level of the prior distribution respectively.

B. Algorithm Performance

We compare the proposed OLS algorithm with the UCB-based online learning DR algorithm proposed in [26], which does not consider the contextual influence on customer behaviors. We set \(\delta = 0.3, \sigma = 0.3\) for the prior distributions in the OLS algorithm, and let the used UCB function be

\[
p_{i,t} = \bar{p}_{i,t} + \sqrt{\frac{3 \log t}{2 T_i(t)}}
\]

where \(\bar{p}_{i,t}\) is the sample average of the historically realized \(z_{i,t}\) by time \(t\), and \(T_i(t)\) denotes the number of times selecting customer \(i\) by time \(t\).

The simulation results are shown as Figure 2. It is observed that the OLS algorithm exhibits a sublinear cumulative regret curve, and its regret at each time gradually decreases to zero value. In contrast, without considering the contextual factors, the UCB-based DR algorithm does not learn the customer behaviors well and maintains high regret at each time.

C. Effects of Prior Distribution

This part studies the effects of the prior distributions and tests the proposed OLS algorithm with different parameters \(\delta\) and \(\sigma\). The simulated regret results are illustrated as Figure 3. It is observed that when the mean error \(\delta\) is fixed, the cumulative regret is generally higher with a larger standard deviation \(\sigma\), because a larger \(\sigma\) indicates greater uncertainty about the true value and leads to more explorations. However, if the standard deviation \(\sigma\) is too small, e.g. the case with \(\sigma = 0.1\), a high cumulative regret occurs, because it sticks to the erroneous prior belief and does not perform sufficient explorations. As for the case with fixed standard deviation \(\sigma = 0.4\), it is seen that the cumulative regret becomes lower as the mean error \(\delta\) decreases, which is consistent with the intuition that a more accurate prior belief leads to better performance.

VI. Conclusion

In this paper, the CMAB method is employed to model the customer selection problem in residential DR, considering the uncertain customer behaviors and the influence of contextual
factors. Based on TS framework, the OLS algorithm is proposed to learn customer behaviors and select appropriate customers for load reduction with the balance between exploration and exploitation. The simulation results validate the efficiency and learning effectiveness of the OLS algorithm. Future work will study the detailed control strategies for each DR event with the consideration of internal system dynamics.

APPENDIX A
POWER FLOW CONSTRAINTS
Consider an underlying power distribution network delineated by the graph $G(V, E)$, where $V$ denotes the set of buses and $E \subset V \times V$ denotes the set of distribution lines. For line $jk \in E$, denote $P_{jk,t}$ and $Q_{jk,t}$ as the active and reactive power flow from bus $j$ to bus $k$ at time $t$; and denote $R_{jk}$ and $X_{jk}$ as the line resistance and reactance. For bus $k \in V$, denote $P_{kn,t}$ and $Q_{kn,t}$ as the net active and reactive power injection right before the $t$-th DR event, and let $U_{k,t}$ be the squared voltage magnitude of bus $k$. Using the linearized Distflow model [34], the power flow equations are formulated as [19]:

\begin{align}
\sum_{k \rightarrow l} P_{kl,t} - \sum_{j \rightarrow k} P_{j,k,t} &= P_{k,n,t} + \sum_{i \in C(k)} y_{i,t} d_{i,t}, \quad (19a) \\
\sum_{k \rightarrow l} Q_{kl,t} - \sum_{j \rightarrow k} Q_{j,k,t} &= Q_{k,n,t} + \sum_{i \in C(k)} y_{i,t} \eta d_{i,t}, \quad (19b) \\
U_{j,t} - U_{k,t} &= 2(R_{jk} P_{jk,t} + X_{jk} Q_{jk,t}), \quad (19c)
\end{align}

where $d_{i,t}$ is the active load power of customer $i$ that can be reduced at time $t$. Binary variable $y_{i,t} \in \{0, 1\}$ denotes whether customer $i$ is selected for load reduction, and $C(k)$ denotes the set of customers whose loads are attached to bus $k$. In [19], a constant load power factor is assumed with the constant $\eta$. Then the line thermal constraints and voltage limits are formulated as [20]:

\begin{align}
P_{jk,t}^2 + Q_{jk,t}^2 &\leq S_{jk}^2, \quad \forall jk \in E \quad (20a) \\
U_k - \bar{U}_k &\leq U_{k,t} \leq \bar{U}_k, \quad \forall k \in V \quad (20b)
\end{align}

where $S_{jk}$ is the apparent power capacity of line $jk$; $\bar{U}_k$ and $\bar{U}_k$ are the lower and upper limits of the squared voltage magnitude respectively.

As a result, the feasible set $\Pi_t$ that captures the network and power flow constraints is constructed as [21]

$$\Pi_t := \{s_{yi} \subseteq [N] \mid \text{Equation } (19a)(20)\} \quad (21)$$

where $s_{yi}$ denotes the subset of customers with

$$\{i \in s_{yi}, \text{ if } y_{i,t} = 1 \text{ } \quad \forall i \in [N].$$

$$\text{if } y_{i,t} = 0$$

APPENDIX B
PROOF OF LEMMATA
A. Proof of Lemma [7]
By definition, $R_t - f_t^\Theta(S_t) = \sum_{i \in S_t} d_{i,t}(z_{i,t} - p_{i,t})$. As $z_{i,t}$ is a Bernoulli random variable with $z_{i,t} \sim \text{Bern}(p_{i,t})$, $\frac{1}{2}$-sub-Gaussian by Hoeffding’s lemma. Then for any $\lambda$,

$$\mathbb{E} \left[ \exp(\lambda \sum_{i \in S_t} d_{i,t}(z_{i,t} - p_{i,t})) \right] \leq \prod_{i \in S_t} \exp \left(\frac{1}{8} \lambda^2 d_{i,t}^2 \right) \leq \exp \left(\frac{1}{8} \sum_{i \in S_t} d_{i,t}^2 \right) \leq \exp \left(\frac{1}{8} \sum_{i \in S_t} (z_{i,t} - p_{i,t})^2 \right) \leq \exp \left(\frac{1}{8} \sum_{i \in S_t} d_{i,t}^2 \right)$$

where the first inequality is because $z_{i,t} - p_{i,t}$ is $\frac{1}{2}$-sub-Gaussian, and the second inequality is due to [14]. Hence, $R_t - f_t^\Theta(S_t)$ conditioned on $(\Theta, S_t)$ is $\frac{1}{2}D$-sub-Gaussian. □

B. Proof of Lemma [3]
For any $t \in [T]$ and $S_t$, define the $N\hat{m}$-dimension vector $x_t(S_t) = (x_{1,t}^T, 1 \in S_1, \ldots, x_{N,t}^T, 1 \{N \in S_t\})^T$ where $1\{\cdot\}$ is the indicator function. To facilitate the proof, we abuse the notation a bit and adjust $\Theta$ to the matrix form

$$\Theta = \text{blockdiag}(\theta_1, \theta_2, \ldots, \theta_N) \in \mathbb{R}^{N\hat{m} \times N}$$

Define function $g(y) = \sum_{i=1}^{N} d_{i,t} \phi(y_i)$ for $y \in \mathbb{R}^N$ with $\phi(\cdot)$ defined in [7]. Define a new reward function class by

$$\tilde{f}_t := \{f_t^\Theta(S_t) = g(\Theta^T x_t(S_t)) \mid \Theta \in \Psi\} \quad (22)$$

Denote $d$ and $\bar{d}$ as the lower and upper bound of $d_{i,t}$ for all $i \in [N]$ and $t \in [T]$ respectively with $0 < d \leq d_{i,t} \leq \bar{d}$. Let $y_i := \Theta^T x_{i,t}$, then the partial derivative of $g(y_i)$ in (22) is lower and upper bounded by

$$\frac{d}{4eL\hat{m}} \leq \frac{\partial g}{\partial y_i} = \frac{d_{i,t}}{e^{y_i} - e^{y_i} + 1} + 2 \leq \frac{\bar{d}}{4} \quad \forall i \in [N]$$

Note that $||\Theta||_2 \leq L\sqrt{N\hat{m}}$ and $||x||_2 \leq \sqrt{N\hat{m}}$. Then by [35 Proposition 4], we have

$$\text{dim}_E(\tilde{F}_t, T^{-1}) \leq C_3 \log \left( C_2(1 + C_1 T^2) \right) + 1 \quad (23)$$

where the constants are given by

$$C_1 := 4L^2N^2\hat{m}^2 \quad (24a)$$
$$C_2 := \left(\frac{\bar{d}}{d}\right)^2(4N - 2)e^{2L\hat{m}} + 1 \quad (24b)$$
$$C_3 := N\hat{m} \frac{e}{e - 1} C_2 \quad (24c)$$

Since the relation between the new reward function in (22) and the reward function [11] is $f_t^\Theta(S_t) = f_t^{\Theta^1}(S_t) + \frac{1}{2} \sum_{i \notin S_t} d_{i,t}$, for any $\Theta^1, \Theta^2 \in \Psi$, $t \in [T]$ and action $S_t$, we have

$$f_t^{\Theta^1}(S_t) - f_t^{\Theta^2}(S_t) = f_t^{\Theta^1}(S_t) - f_t^{\Theta^2}(S_t)$$

Hence, by the definition of eluder dimension in [21] Definition 3, we obtain $\text{dim}_E(\tilde{F}_t, 1/T) = \text{dim}_E(F_t, 1/T)$ for all $t \in [T]$, which leads to Lemma [3]. □
C. Proof of lemma [24]

For any \( t \in [T] \) and \( \Theta^1, \Theta^2 \in \Psi \), with function \( \phi(\cdot) \) defined in [24], we have

\[
\| f_{t}^{\Theta^1} - f_{t}^{\Theta^2} \|_\infty \leq \frac{\hat{m} \bar{D}}{4} \| \Theta^1 - \Theta^2 \|_\infty
\]

where the first equality is due to the Lagrange's mean value theorem for some appropriate \( \xi_{i,t} \), and the second inequality uses the property \( |\phi(\cdot)| \leq \frac{\hat{m}}{2} \). Hence, for any \( f_{t}^{\Theta^1}, f_{t}^{\Theta^2} \in \mathcal{F}_t \), we have

\[
\| f_{t}^{\Theta^1} - f_{t}^{\Theta^2} \|_\infty \leq \frac{\hat{m} \bar{D}}{4} \| \Theta^1 - \Theta^2 \|_\infty
\]

Thus an \( \alpha \)-covering of \( \mathcal{F}_t \) can be achieved through a \( \left( \frac{4 \alpha}{\bar{D} m} \right) \)-covering of the set \( \Psi \). Denote \( n_t(\alpha) \) as the \( \alpha \)-covering number of \( \mathcal{F}_t \) in the \( \alpha \)-norm. By the definition of Kolmogorov dimension [21] Definition 1), it obtains

\[
dim_K(\mathcal{F}_t) = \limsup_{\alpha \downarrow 0} \frac{\log n_t(\alpha)}{\log(1/\alpha)} \leq \limsup_{\alpha \downarrow 0} \frac{\log \left( \frac{\bar{D} m}{\alpha} \right) N \hat{m}}{\log(1/\alpha)} = N \hat{m}
\]

where the inequality is because an \( \varepsilon \)-covering of the set \( \Psi \) requires at most \( \left( \frac{\bar{D} m}{\alpha} \right) N \hat{m} \) elements [36]. Since the above upper bound is independent of time \( t \), Lemma [24] is proved. □

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