Unsteady flow damping force prediction of MR dampers subjected to sinusoidal loading

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Abstract. So far quasi-steady models are usually used to design magnetorheological (MR) dampers, but these models are not sufficient to describe the MR damper behavior under unsteady dynamic loading, for fluid inertia is neglected in quasi-steady models, which will bring more error between computer simulation and experimental results. Under unsteady flow model, the fluid inertia terms will bring error calculated up to 10\%, so it is necessary to be considered in the governing equation. In this paper, force-stroke behavior of MR damper with flow mode due to sinusoidal loading excitation is mainly investigated, to simplify the analysis, the one-dimensional axisymmetric annular duct geometry of MR dampers is approximated as a rectangular duct. The rectangular duct can be divided into 3 regions for the velocity profile of the incompressible MR fluid flow, in each region, a partial differential equation is composed of by Navier-Stokes equations, boundary conditions and initial conditions to determine the velocity solution. In addition, in this work, not only Bingham plastic model but the Herschel—Bulkley model is adopted to analyze the MR damper performance. The damping force resulting from the pressure drop of unsteady MR dampers can be obtained and used to design or size MR dampers. Compared with the quasi-steady flow damping force, the damping force of unsteady MR dampers is more close to practice, particularly for the high-speed unsteady movement of MR dampers.

1. Introduction

It is known that magnetorheological (MR) and electrorheological (ER) fluids are a type of non-Newtonian fluids, and the fluid flow obey the governing Navier–Stokes (NS) equations. Due to its analysis simplicity, Quasi-static analysis model is usually used to provide the required force-stroke profile as an analytical method for the design of MR /ER fluids device, the quasi-static model neglects the effect of fluid inertia on the MR fluid flow [1-3]. However, fluid inertia term acts on the fluid flow in practical application and should be incorporated in the governing NS equation for oscillatory or unsteady fluid flow [4]. For example, under the sinusoidal excitation, it is not significant for the fluid inertial effect on the unsteady flow of MR fluid in the very low range of the oscillatory frequency, but it is obvious for the higher oscillatory frequency [5]. Chen et al. proposed and solved the velocity profile and pressure gradient of the unsteady state unidirectional flow of Bingham fluid between parallel plates by the Laplace transform method [6]. Based on the unsteady flow analysis of an ER fluid...
fluid, Nguyen and Choi developed a dynamic modeling of the ER damper without experimental results for parameter identification considering the unsteady behavior of ER fluid in a high frequency and large stroke damper. In addition, experiments were conducted to validate the proposed model and showed the effect of unsteady behavior is significant and should be taken into account at high frequency [7].

It is necessary to study dynamic damping force of the MR/ER dampers for its geometric parameter configuration or control modeling parameter identification without experimental results. The main objective of this paper is to introduce a theoretically analyze method of unsteady flow of MR fluids, based on Bingham-plastic (BP) [8] and Herschel–Bulkley (HB) model [2], the MR unsteady fluid flow within flow mode MR dampers is described using partial differential equations (PDE). According to the PDE, numerical velocity solution of MR fluid unsteady flow is solved, and then the dynamic damping force of MR dampers can be predicted.

2. Simplified mathematical formulations

2.1. Constitutive equations

In this paper, both BP model and HB model are used to describe the incompressible MR fluid behavior. The constitutive equations of the two models are described as

BP model

\[
\tau = \tau_y \text{sgn}(\dot{\gamma}) + \eta \dot{\gamma}, |\tau| > \tau_y \\
\dot{\gamma}_y = 0, |\tau| < \tau_y
\]

(1)

where \(\tau\) is shear stress, \(\tau_y\) is the critical yield stress, \(\eta\) is zero field viscosity of MR fluids, \(\dot{\gamma}\) is the rate of stress strain.

HB model

\[
\tau = \tau_y \text{sgn}(\dot{\gamma}) + \kappa (\dot{\gamma})^m, |\tau| > \tau_y \\
\dot{\gamma}_y = 0, |\tau| < \tau_y
\]

(2)

where \(m\), \(\kappa\) are the power-law exponent and the consistency index, respectively. The HB model accounts for shear-thinning/thickening behavior displayed by MR fluids at higher strain rates. The BP model can be seen as a generalization of the HB model that takes into account shear-thinning/thickening behavior of MR fluid by changing in the effective viscosity with the applied shear rate through a power-law behavior. There is a substitution relationship between BP and HB models, namely:

\[
\eta = \kappa (\dot{\gamma})^{m-1}
\]

(3)

2.2. Governing equations

In this section, a flow mode MR damper with an approximate parallel-plate geometry model instead of the actual annular damping channel is investigated, for the damping channel width \(d\) is smaller than damping channel inner radius \(R_i\) and effective length \(L\). The pressure gradient between piston head sides \(\Delta P(t)\) is assumed to evenly distribute along the damping channel length \(L\). In addition, the Cartesian coordinate system \((x, y, z)\) is used, \(x\)-axis is taken as the centre line direction in the two parallel plates, \(y\) is the coordinate normal to the plate, \(z\) presents the perimeter of the annular damping channel. According to NS equations and constitutive equation of MR fluid, here, a one-dimensional governing equation of the parallel-plate model can be confirmed and written as

\[
\rho \frac{\partial u(y,t)}{\partial t} = -\frac{\Delta P(t)}{L} + \frac{\partial \tau(y,t)}{\partial y}
\]

(4)
where $\rho$ is the fluid density, $t$ is the time variable. $u(y,t)$ is the fluid velocity, $\tau(y,t)$ is the shear stress.

### 2.3 Boundary conditions

Figure 1 shows a typical velocity profile of MR fluids flowing through annular damping channel of flow mode MR dampers. According to velocity profile, boundary of parallel-plate gap, inner boundary of the plug region ($y_{pi}$) and outer boundary of the plug region ($y_{po}$) divide parallel-plate gap into three regions, region1 (post-yield) ($0 \leq y \leq y_{pi}$), region2 (pre-yield) ($y_{pi} \leq y \leq y_{po}$) and region3 (post-yield) ($y_{po} \leq y \leq d$). Each region has different boundary conditions, which are listed in table 1.

![Figure 1. Typical MR fluids velocity profile in the annular damping channel of flow mode MR dampers.](image)

| Regions    | Region1 | Region2 | Region3 |
|------------|---------|---------|---------|
| Boundary conditions | $u(0,t) = 0$, $\tau(y_{pi},t) = \tau_y$, $u_y(y_{po},t) = 0$, $\nu_y = \varphi(y)$ | $u_y(y_{pi},t) = 0$, $\tau(y_{po},t) = -\tau_y$, $u(d,t) = 0$ |

#### 2.4 Mathematical equation

The question of MR fluids flow through damping channel can be described by the partial differential equation (PDE) incorporating the governing equation, boundary conditions, initial condition in each region. Velocities of every regions $u_i(y,t)$ satisfies a independent PDE, namely:

In the region 1,

$$
\begin{align*}
\begin{cases}
\dot{u}_i(y,t) &= \nu \cdot u_{iy}(y,t) + \Delta P(t)/L \\
\dot{u}_i(0,t) &= 0, u_{iy}(y_{pi},t) = 0 \\
\dot{u}_i(y,0) &= \varphi(y)
\end{cases}
\end{align*}
$$

(5)

In the region 2,

$$
\begin{align*}
&u_2\left(y_{pi} \leq y \leq y_{po}, t\right) = u_i(y_{pi}, t) = u_i(y_{po}, t)
\end{align*}
$$

(6)
In the region 3,
\[
\begin{align*}
  u_3(i,y,t) &= \mathbf{v} \cdot u_{3yy}(y,t) + \Delta P(t)/L \\
  u_3(d,t) &= 0, u_{3y}(y_{po},t) = 0 \\
  u_3(y,0) &= \phi(y)
\end{align*}
\]

(7)

where subscript \(i\) is a representative of numbers 1, 2 and 3, \( \nu = \eta/\rho \), \( u_y(y,t) = \partial u_i(y,t)/\partial t \), \( u_{3y}(y,t) = \partial u_i(y,t)/\partial y \). \( u_{3yy}(y,t) = \partial u_i(y,t)/\partial y^2 \).

3. Methodology of solution

3.1. Velocity solutions

To solve the PDE, the pressure gradient between piston head sides \(\Delta P(t)\) should be given by a known functional form. The nonlinear constitutive law for the MR fluid dampers is assumed to be

\[
F(t) = c \left| \frac{dX(t)}{dt} \right|^\alpha \text{sgn}\left(\frac{dX(t)}{dt}\right)
\]

(8)

where \(F(t)\) represents the damping force on the piston, \(X(t)\) is the displacement of the piston, \(c\) is the damping coefficient and \(\alpha\) is a fractional exponent that accounts for the nonlinearity inherent in a MR fluid damper with the range \(0 \leq \alpha \leq 1.5\) \[9\]. It can be seen from equation (8) that there is a determinate \(\Delta P(t)\) for an excitation velocity at the certain time \(t\). In this paper, to simply the analysis, assume the pressure gradient \(\Delta P(t)\) is an approximate linear function of load velocity on the MR damper, namely

\[
\Delta P(t) = \varepsilon V_{p}(t)
\]

(9)

where \(\varepsilon\) is the undetermined constant, \(V_{p}(t)\) is the velocity of the piston motion. The PDE can be solved with variable separation method, and the velocity of MR fluid through the annular damping channel \(u(y,t)\) can be obtained. In addition, according to equation (3), the velocity solutions of BP model \(u_b(y,t)\) is also used to calculate the effective viscosity of HB model.

Assume \(\Delta P(t)/(\rho L) = f(t)\), the velocity solutions of BP model are:

\[
\begin{align*}
  u_{b1}(y,t) &= \sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi} \int f(t)e^{K_{b1}t}dt + C[e^{-K_{b1}t} \sin \left(\frac{(2n-1)\pi y}{2y_{pi}} \right) \\
  u_{b2}(y,t) &= -\sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi} \int f(t)e^{K_{b2}t}dt + C[e^{-K_{b2}t}] \\
  u_{b3}(y,t) &= \sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi} \int f(t)e^{K_{b3}t}dt + C[e^{-K_{b3}t} \sin \left(\frac{(2n-1)\pi(y-d)}{2(d-y_{po})} \right)]
\end{align*}
\]

(10)

where \(K_{b1} = \left[\frac{(2n-1)\pi}{2y_{pi}}\right]^2 \nu\), \(K_{b2} = \left[\frac{(2n-1)\pi}{2(d-y_{po})}\right]^2 \nu\).

The velocity solutions of HB model are
where

\[ u_{H1}(y,t) = -\sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi} \int f(t) e^{K_{\text{H1}} y dt} + C e^{-K_{\text{H1}} y} \sin \left( \frac{(2n-1)\pi y}{2y_{\text{pi}}} \right) \]

\[ u_{H2}(y,t) = -\sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi} \int f(t) e^{K_{\text{H2}} y dt} + C e^{-K_{\text{H2}} y} \sin \left( \frac{(2n-1)\pi y}{2y_{\text{pi}}} \right) \]

\[ u_{H3}(y,t) = \sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi} \int f(t) e^{K_{\text{H3}} y dt} + C e^{-K_{\text{H3}} y} \sin \left( \frac{(2n-1)\pi y}{2(d-y_{\text{po}})} \right) \]  

(11)

where \( K_{\text{H1}} = \frac{(2n-1)\pi}{2y_{\text{pi}}} \kappa \frac{du_{\text{B1}}(y,t)}{dy} \) \( w^{-1} \), \( K_{\text{H2}} = \frac{(2n-1)\pi}{2(d-y_{\text{po}})} \kappa \frac{du_{\text{B3}}(y,t)}{dy} \) \( w^{-1} \), the arbitrary constant \( C \) can be confirmed based on the initial conditions.

Based on the above velocity solutions, it can be seen that there is a comprehensive characteristic function affected by the piston excitation velocities for the velocity solution, that is

\[ \phi(t) = \frac{4}{(2n-1)\pi} \int f(t) e^{K_{\text{H1}} y dt} + C e^{-K_{\text{H1}} y} \sin \left( \frac{(2n-1)\pi y}{2y_{\text{pi}}} \right) \]

(12)

It is a function of the time variable and concerned with the input excitation mode (piston motion).

3.2. Dynamic damping force

To calculate \( \Delta P(t) \), there are at least three equations for three unknown quantities \( \Delta P(t) \), \( y_{\text{pi}} \) and \( y_{\text{po}} \).

One equation is about volume flow. For incompressible continuous MR fluids, it is known that the total volume flux through the damping channel \( Q_{c} \) equals the volume flux displaced by the piston \( Q_{p} \), namely

\[ Q_{c} = Q_{p} = S_{p} V_{p}(t) \]  

(13)

where \( S_{p} \) is the active cross-section of piston head. The total volume flux composes of the volume fluxes in each region obtained by velocity-annular gap width integral, namely

\[ Q_{c} = [Q_{1} + Q_{2} + Q_{3}] = b \int_{0}^{y_{\text{pi}}} u_{1}(y,t)dy + b \int_{y_{\text{pi}}}^{y_{\text{po}}} u_{2}(y,t)dy + b \int_{y_{\text{po}}}^{y_{\text{po}}} u_{3}(y,t)dy \]  

(14)

where \( b \) is the average perimeter of the annular damping channel, \( u_{1}(y,t) \) is general term for \( u_{\text{B1}}(y,t) \) and \( u_{\text{H1}}(y,t) \), \( u_{2}(y,t) \) is general term for \( u_{\text{B2}}(y,t) \) and \( u_{\text{H2}}(y,t) \),  \( u_{3}(y,t) \) is general term for \( u_{\text{B3}}(y,t) \) and \( u_{\text{H3}}(y,t) \).

The plug flow region 2 is rigid and the total yield stress \( \tau(y_{\text{po}}) = 2\tau_{r} \) linearly varies along the plug thickness. Based on force equilibrium of plug region 2, the plug thickness can be obtained and can rewritten as

\[ y_{\text{po}} - y_{\text{pi}} = 2\tau_{r} \left| \frac{\Delta P(t)}{L} - \rho \frac{\partial u_{2}(y,t)}{\partial t} \right|^{-1} \]  

(15)

Another equation, according to equations (6), (10) or (11), the system of equations can be obtained, that is

\[ y_{\text{po}} + y_{\text{pi}} = d \]  

(16)

Incorporating equations (14), (15) and (16), the numerical solution of pressure drop \( \Delta P(t) \) can be worked out, and the damping force can be confirmed by the equation

\[ F(t) = \Delta P(t) S_{p} \]  

(17)
4. Illustration of examples
For the unsteady analysis of MR dampers, four piston excitation motion cases are discussed. The flow due to linearly accelerated piston motion and suddenly piston motion proposed by Das and Arakeri are analyzed [10], in addition, the piston velocity $V_p(t)$ moves with an exponential velocity, and the oscillatory piston motion are also considered. For a certain excitation load, if the comprehensive function of equation (12) is confirmed, its damping force can be calculated. Hence, only the comprehensive functions of four piston excitation motion is solved.

4.1. Constant acceleration piston motion
For a piston with constant acceleration ($a_p$, where $V_p(t) = a_p t$), the corresponding characteristic function is

$$\phi(t) = \left[ \frac{4B a_p (K_{Bi} t - 1) e^{K_{Bi} t}}{(2n-1)\pi K_{Bi}^2} + C \right] e^{-K_{Bi} t}$$

where $B$ is a constant.

4.2. Suddenly started piston motion
For a piston motion with a suddenly started velocity, the motion equation is

$$V_p(t) = 0, (t \leq 0)$$
$$V_p(t) = V_0, (t > 0)$$

where $V_0$ is the constant velocity. The corresponding characteristic function is

$$\phi(t) = \left[ \frac{4B V_0 e^{K_{Bi} t}}{(2n-1)\pi K_{Bi}^2} + C \right] e^{-K_{Bi} t}$$

4.3. Exponential velocity piston motion
The piston motion with exponential velocity can be described as

$$V_p(t) = V_1 e^{-\lambda t}$$

where $\lambda$ is the attenuation coefficient of velocity, $V_1$ is the initial velocity. The corresponding characteristic function is

$$\phi(t) = \left[ \frac{4B V_1 e^{(K_{Bi} - \lambda) t}}{(2n-1)\pi (K_{Bi} - \lambda)} + C \right] e^{-K_{Bi} t}$$

4.4. Oscillatory piston motion
The sinusoidal displacement excitation is usually applied on MR dampers to characterize their properties, such as force-stroke and force-velocity profiles. Here, the sinusoidal displacement excitation is

$$X(t) = A \sin(\omega t)$$
where $A$, $\omega$ are the sinusoidal excitation displacement amplitude and angular velocity, respectively. The corresponding characteristic function is

$$\phi(t) = \frac{4B}{(2n-1)\pi(K_{B_1}^2 + \omega^2)} \left[ (K_{B_1} \cos \omega t + \omega \sin \omega t)e^{K_{B_1}t} + C \right] e^{-K_{B_1}t}$$

(24)

According to the solution analysis procedure in section 3, damping forces for each piston motions can be numerically solved. In this work, force-stroke and force-velocity profiles of flow mode MR dampers are investigated, in addition, assuming the initial condition is satisfied the following equation

$$u(y, 0) = -u(y, T/2)$$

(25)

Assume there is an MR damper with the configuration parameters listed in table 2 referring to design methods of ER or MR dampers in the literatures [11, 12].

### Table 2. Simulation parameters.

| Parameters                              | Value               |
|-----------------------------------------|---------------------|
| Sinusoidal excitation displacement amplitude | 0.005m              |
| MR fluid yield stress                   | 20kPa               |
| Sinusoidal excitation frequency         | 40Hz                |
| Damping channel length                  | 0.02 m              |
| Damping channel inner radius            | 0.0185 m            |
| Damping channel outer radius            | 0.02 m              |
| MR dampers piston rod radius            | 0.01 m              |
| Average perimeter of the damping channel | 0.1209 m            |
| damping channel width                   | 0.0015 m            |
| MR fluids mass density                  | $2.650 \times 103$ kgm$^{-3}$ |
| MR fluid zero field viscosity           | 1.000 Pa s          |

The relationships of the MR damper response between force versus displacement and force versus velocity with yield stress 20 kPa and frequency 40 Hz are shown in Figure 2(a) and 2(b), respectively. The consistency index $\kappa$ in equations (2) and (3) is assumed to equal zero field viscosity of BP model $\eta$, in addition, the power-law exponent of HB model $m$ of 0.9 and 1.0 are discussed, since most of the MR fluids show the shear thinning behaviours $(m < 1)$. For the non-stationary motion of MR damper piston, the dynamic damping force of MR damper can be predicted by the numerical analysis solutions of MR fluid unsteady flow. Compared with the damping force of MR damper for quasi-static analysis, the dynamic damping force can well present the effect of fluid inertia on the unsteady fluid flow with high velocity rates, shown in figure 2.

### 5. Conclusions

In this paper, a numerical solution procedure of dynamic damping force is given for unsteady flow of MR fluid within flow mode MR damper for the controlled non-stationary piston motion. According to the governing NS equations, the boundary and initial conditions, the analytical solutions of velocity profile in the annular damping channel can be derived and used to confirm the dynamic damping force. The numerical solution method can be applied to arbitrary non-stationary piston of MR damper.
Figure 2. Relationships of (a) force versus displacement (b) force versus velocity for different power-law exponent of HB (0.9, 1.0).

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