FURTHER ALL-LOOP RESULTS IN SOFTLY-BROKEN SUPERSYMMETRIC
GAUGE THEORIES

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Abstract

It is proven that the recently found, renormalization-group invariant sum rule for the soft scalar masses in softly-broken $N = 1$ supersymmetric gauge-Yukawa unified theories can be extended to all orders in perturbation theory. In the case of finite unified theories, the sum rule ensures the all-loop finiteness in the soft supersymmetry breaking sector. As a byproduct the exact $\beta$ function for the soft scalar masses in the Novikov-Shifman-Vainstein-Zakharov (NSVZ) scheme for softly-broken supersymmetric QCD is obtained. It is also found that the singularity appearing in the sum rule in the NSVZ scheme exactly coincides with that which has been previously found in a certain class of superstring models in which the massive string states are organized into $N = 4$ supermultiplets.

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1 Introduction

The plethora of free parameters of the, very successful otherwise, Standard Model (SM), can be interpreted as signaling the existence of a more fundamental Physics picture in higher scales, whose remnants appear as free parameters in the SM. In fact after several decades of experience in searching for such a fundamental theory, which in principle could explain everything that is observed today in terms of very few parameters, it seems more realistic to expect that only parts of the fundamental theory are uncovered at various higher scales; maybe the full fundamental theory can only be found close to the Planck scale. The usual theoretical strategy to search for new Physics beyond the SM is to construct more symmetric theories, e.g. Grand Unified Theories (GUTs) at higher scales and subsequently test their predictions against the measured low energy parameters. A representative candidate for carrying some of the information of the fundamental theory at intermediate scales is the $N = 1$ globally supersymmetric $SU(5)$ GUT, given its predictive power for certain low energy free parameters of the SM.

In our recent studies [1]–[10], we have developed another complementary strategy in searching for a more fundamental theory possibly at Planck scale and its consequences that could be missing in ordinary GUTs. Our method consists of hunting for renormalization group invariant (RGI) relations among couplings holding below the Planck scale and which therefore are exactly preserved down to the GUT scale. This programme applied in the dimensionless couplings of supersymmetric GUTs such as gauge and Yukawa couplings had already certain success by predicting correctly, among others, the top quark mass in the finite [1, 4] and in the minimal [3, 4] $N = 1$ supersymmetric $SU(5)$-GUTs.

An impressive aspect of the RGI relations is that one can guarantee their validity to all-orders in perturbation theory by studying the uniqueness of the resulting relations at one-loop, as was proven in the early days of the programme of reduction of couplings [8].

Although supersymmetry seems to be an essential feature for a successful realization of the above programme, its breaking has to be understood too in this framework, which has the ambition to supply the SM with predictions for several of its free parameters. Therefore, the

\footnote{For an extended discussion and a complete list of references, see ref. [11].}
search for RGI relations was naturally extended to the soft supersymmetry breaking (SSB) sector of these theories [12, 5], which involve parameters with dimension one and two. In the case of nonfinite theories, the method to prove the existence of reduction of couplings to all-loop [8–10] can be easily extended for the RGI relations among dimensional parameters [5] if use of a mass-independent renormalization scheme (RS) is assumed [5]. In contrast to this, for the case of finite theories the elegant way of ref. [14] to show finiteness (which is based on a consideration of renormalization of certain anomalies) cannot be simply applied; reduction of couplings is merely one of the conditions for finiteness. The proof of the all-order finiteness is certainly less involved to be performed in a particular RS in which various properties of the RG functions are known and can be assumed [15]. Using the recent results [16–19] on the renormalization properties of the SSB sector in the supersymmetric version of the minimal subtraction scheme, Kazakov [20] has pursued that line of the thought and shown the finiteness in the SSB sector [3]. Soon later Jack, Jones and Pickering [23] have generalized Kazakov’s idea [20] so as to find RGI relations among the SSB parameters in the nonfinite case.

Note that in the formulation of references above the SSB parameters are expressed in terms of the unified gauge coupling $g$ and the unified gaugino mass parameter $M$ only, which may appear as a too strong constraint on the SSB sector for a given phenomenological model. Therefore, there has been attempts [4, 7] to relax this constraint without losing RGI. An interesting observation resulting from the independent analysis of the SSB sector of a $N = 1$ supersymmetric gauge-Yukawa unified theory is the existence of a RGI sum rule for the soft scalar- masses in lower orders; in one-loop for the nonfinite case [3] and in two-loop for the finite case [7]. The sum rule appears to have significant phenomenological consequences and in particular manages to overcome the unpleasant predictions of the previously known “universal” finiteness condition for the soft scalar masses [21, 22]. The universal soft scalar masses apart from their simplicity they were appealing for a number of reasons (a) they are part of the constraints that preserve finiteness up to two-loop [21, 22], (b) they appear to be RGI under a certain constraint, known as the $P = 1/3Q$ condition [12], in

\[^2\text{The proof is also possible without any assumption on a particular RS [13].}\]

\[^3\text{Finiteness in this sector in lower orders are shown in refs. [21, 22].}\]
more general supersymmetric gauge theories, and (3) they appear in the dilaton dominated
supersymmetry breaking superstring scenarios \[24\]. In the latter case, since the dilaton cou-
ples in a universal manner to all particles the universality of soft scalar masses appears as
a quite model independent feature. Unfortunately, further studies have exhibited a number
of problems attributed to the universality of soft scalar masses. For instance (1) in finite
unified theories the universality leads to a charged particle, the superpartner of \(\tau\), the s-\(\tau\),
to be the lightest supersymmetric particle \[25, 7\], (2) the MSSM with universal soft scalar
masses is inconsistent with radiative electroweak symmetry breaking \[26\] and (3) worst of all
the dilaton dominated limit leads to charge and/or colour breaking minima deeper than the
standard vacuum \[27\]. Therefore, the sum rule is a welcome possibility. Furthermore, it was
shown that the same sum rule is satisfied in a certain class of 4D orbifold models, at least at
the tree-level for the nonfinite \[8\] and in two-loop order for the finite case \[7\] if the massive
string states are organized into \(N = 4\) supermultiplets so that they do not contribute to the
quantum modification of the gauge kinetic function \[28\].

The purpose of the present paper is to prove the existence of the RGI soft scalar-mass
sum rule to all-orders for the nonfinite as well as for the finite case, based on the recent
developments on the renormalization properties of the SSB sector of the \(N = 1\) super-
symmetric gauge theories. As an interesting byproduct we obtain the exact \(\beta\) function for
the soft scalar masses in the Novikov-Shifman-Vainstein-Zakharov (NSVZ) scheme \[29\] for
softly-broken \(N = 1\) supersymmetric QCD.

2 Recent results on the renormalization of the SSB parameters

Most of the recent interesting progress \[17\]–\[20\], \[23\] on the renormalization properties of the
SSB parameters is based conceptually and technically on the work of ref. \[16\]. In ref. \[16\] the
powerful supergraph method \[30\] for studying supersymmetric theories has been applied to
the softly-broken ones by using the “spurion” external space-time independent superfields
\[31\]. In the latter method a softly-broken supersymmetric gauge theory is considered as
a supersymmetric one in which the various parameters such as couplings and masses have
been promoted to external superfields that acquire "vacuum expectation values". Based on this method the relations among the soft term renormalization and that of an unbroken supersymmetric gauge theory have been derived.

To be more specific, following the notation of ref. [23], in an \( N = 1 \) supersymmetric gauge theory with superpotential

\[
W(\Phi) = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} h^{ij} \Phi_i \Phi_j
\]

(1)

the SSB part \( L_{SSB} \) can be written as [16]:

\[
L(\Phi, W) = -\left( \int d^2 \theta \eta (\frac{1}{6} h^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} b^{ij} \Phi_i \Phi_j + \frac{1}{2} MW_A^\alpha W_A^\alpha) + \text{h.c.} \right) - \int d^4 \tilde{\theta} \tilde{\eta} \overline{W} (m^2)^i_j (e^{2\tilde{V}})^k_i \Phi_k,
\]

(2)

where \( \eta = \theta^2, \tilde{\eta} = \tilde{\theta}^2 \) are the external spurion superfields and \( \theta, \tilde{\theta} \) are the usual Grassmann parameters, and \( M \) is the gaugino mass. The \( \beta \) functions of the \( M, h \) and \( m^2 \) parameters are found to be:

\[
\beta_M = 2 \mathcal{O} \left( \frac{\beta_g}{g} \right),
\]

(3)

\[
\beta_h^{ijk} = \gamma_i^j h^{lijk} + \gamma_j^i h^{iljk} + \gamma_k^i h^{ijlk} - 2 \gamma_i^j Y^{lijk} - 2 \gamma_j^i Y^{ilk} - 2 \gamma_k^i Y^{ijl},
\]

(4)

\[
(\beta_{m^2})^j_i = \left[ \Delta + X \frac{\partial}{\partial g} \right] \gamma^j_i,
\]

(5)

\[
\mathcal{O} = \left( M g^2 \frac{\partial}{\partial g^2} - h^{lmn} \frac{\partial}{\partial Y_{lmn}} \right),
\]

(6)

\[
\Delta = 2 \mathcal{O} \mathcal{O}^* + 2 |M|^2 g^2 \frac{\partial}{\partial g^2} + \tilde{Y}_{lmn} \frac{\partial}{\partial \tilde{Y}_{lmn}} + \tilde{Y}_{lmn} \frac{\partial}{\partial \tilde{Y}_{lmn}},
\]

(7)

where \( (\gamma_i)^j = \mathcal{O} \gamma_i^j, Y_{lmn} = (Y^{lmn})^* \), and

\[
\tilde{Y}^{ijk} = (m^2)^i_l Y^{lijk} + (m^2)^j_i Y^{ilk} + (m^2)^k_i Y^{ijl}.
\]

(8)

Note that the \( X \) term in (5) is explicitly known only in the lowest order [22, 32]:

\[
X^{(2)} = -\frac{S g^3}{8 \pi^2}, S \delta_{AB} = (m^2)^k_i (R_A R_B)^l_i - |M|^2 C(G) \delta_{AB}.
\]

(9)

We do not consider the \( b \) parameters in the following discussions, because they do not enter into the \( \beta \) functions of the other quantities at all. Moreover they are finite if the other quantities are finite.
In order to express the \( h \) and \( m^2 \) parameters in terms of \( g \) and \( M \) in a RG invariant way, we have to solve the set of coupled reduction equations \[8, 9, 10\]. The key point in the strategy of refs. [20, 23] to solve the reduction equations is the assumption that the differential operators \( O \) and \( \Delta \) given in eqs. (6) and (7) become total derivative operators on the RG invariant surface which is defined by the solution of the reduction solutions. Although we consider this assumption as a subtle one and the extent of its validity requiring further clarification, we accept it throughout our analysis.

Observe that the \( \beta \) functions of the SSB parameters are obtained by applying the differential operators, \( O \) and \( \Delta \), on the RG functions, \( \beta_g \) and \( \gamma_{ji} \), of the unbroken theory, and note next that in a finite theory \( Y^{ijk} \) is a power series of \( g \) and that \( \beta_g \) as well as \( \gamma_{ji} \) have to identically vanish. But in general we expect that

\[
\frac{\partial \gamma_{j'i}(g,Y,Y^*)}{\partial Y} \bigg|_{Y=Y(g),Y^*=Y^*(g)} \neq 0 \quad \text{or} \quad \frac{\partial \gamma_{j'i}(g,Y,Y^*)}{\partial g} \bigg|_{Y=Y(g),Y^*=Y^*(g)} \neq 0 ,
\]

even if \( \gamma_{j'i}(g,Y(g),Y^*(g)) \) vanishes. However, one easily sees that

\[
\frac{d\gamma_{j'i}(g,Y=Y(g),Y^*=Y^*(g))}{dg} = \frac{\partial \gamma_{j'i}(g,Y,Y^*)}{\partial g} \bigg|_{Y=Y(g),Y^*=Y^*(g)} + \frac{\partial \gamma_{j'i}(g,Y,Y^*)}{\partial Y} \bigg|_{Y=Y(g),Y^*=Y^*(g)} \frac{dY(g)}{dg} = 0 ,
\]

if \( \gamma_{j'i}(g,Y=Y(g),Y^*=Y^*(g)) = 0 \). Kazakov [20] examining the finite case was searching for a RG invariant surface on which the differential operators \( O \) and \( \Delta \) can be written as total derivative terms.

In ref. [23] the general case has been considered and has been further assumed that

\[
\gamma_{j'i} = \gamma_{i'j'i} ,
\]

\[
(m^2)^{j'i} = m_i^2 \delta_{j'i} ,
\]

\[
Y^{ijk} \frac{\partial}{\partial Y^{ijk}} = Y^{*ijk} \frac{\partial}{\partial Y^{*ijk}} \quad \text{on the space of the RG functions} ,
\]

and has been shown that if

\[
h^{ijk} = -M(Y^{ijk})' \equiv -M \frac{dY^{ijk}(g)}{d\ln g} ,
\]

\[
m_i^2 = |M|^2 \{ (1 + \bar{X}(g))(g/\beta_g)(\gamma_i(g))' + \frac{1}{2}[ (g/\beta_g)\gamma_i(g)]' \} .
\]
are satisfied, then the differential operators $\mathcal{O}$ and $\Delta$ can be written as

$$
\mathcal{O} = \frac{M}{2} \frac{d}{d \ln g},
$$
(17)

$$
\Delta = |M|^2 \left[ \frac{1}{2} \frac{d^2}{d(\ln g)^2} + \left(1 + \bar{X}(g)/g\right) \frac{d}{d \ln g} \right],
$$
(18)

where

$$
g \bar{X}(g) = \frac{1}{|M|^2} X(g, Y(g), Y^*(g), h(M, g), h^*(M, g), m^2(|M|^2, g)).
$$
(19)

Eqs. (17) and (18) can be derived from

$$
\frac{d \ln Y^{ijk}}{d \ln g} = (\ln Y^{ijk})' = (g/\beta_g)[\gamma_i(g) + \gamma_j(g) + \gamma_k(g)],
$$
(20)

which follows assuming the reduction equation

$$
\beta_g \frac{d Y^{ijk}}{d g} = \delta^{ijk} = Y^{ijk}(g)[\gamma_i(g) + \gamma_j(g) + \gamma_k(g)].
$$
(21)

Note that so far eq. (15) is a solution of the reduction equation (i.e. RG invariant), but eq. (16) is not. At the final step, Jack et al. in ref. [23] require that eq. (16), too, is RG invariant, which fixes $\bar{X}(g)$ uniquely up to a term related to an integration constant. This integration constant term is then fixed by comparing it with the lowest order result in eq. (9). They found

$$
\bar{X}(g) = \frac{1}{2} (\ln(\beta_g/g))' - 1.
$$
(22)

Note that there is no perturbative computation of $X$ beyond two-loop. Therefore eq. (22) may be understood as a prediction of perturbative computation of $X$. If one inserts $\bar{X}$ above into eq. (16), one obtains

$$
m^2_i = \frac{1}{2} |M|^2 (g/\beta_g) (\gamma_i(g))'.
$$
(23)

which together with (15) is the final result of ref. [23].

3 New results

Next let us consider the sum rules for soft scalar masses [3, 4]. In turn, we assume neither (16) nor (23). But we assume that $Y^{ijk}$ and $h^{ijk}$ are already reduced, where $h^{ijk}$ is given in
equ. (15), as well as that (12)–(14) hold. Suppose that the sum rule takes the form
\[ m_i^2 + m_j^2 + m_k^2 = |M|^2 F_{ijk}^M(g) + \sum_l m_l^2 F_{ijk}^l(g). \]  
(24)

We require, as in ref. [20, 23], that \( \Delta \) acting on \( \gamma_i \) can be written as a total derivative operator, and we find that
\[ F_{ijk}^M(g) = (1 + \tilde{X}^M(g))(\ln Y_{ijk})' + \frac{1}{2}(\ln Y_{ijk})'' , \quad F_{ijk}^l(g) = \tilde{X}^l(g)(\ln Y_{ijk})' \]  
(25)

have to be satisfied, where
\[ |M|^2 g |M|^2 + \sum_l m_l^2 g \tilde{X}^l(g) = X(g,Y(g),Y^*(g), h(M,g), h^*(M,g), m^2) . \]  
(26)

Then we have
\[ \beta_{m_i^2} = \Delta \gamma_i \]
\[ = \left\{ |M|^2 \left[ \frac{1}{2} \frac{d^2}{d(\ln g)^2} + (1 + \tilde{X}^M(g)) \frac{d}{d \ln g} \right] + \sum_l m_l^2 \tilde{X}^l(g) \frac{d}{d \ln g} \right\} \gamma_i(g) , \]  
(27)

which vanishes if \( \gamma_i(g) = 0 \). Therefore eq. (24) with (25) is the desired sum rule for the finite theories. Since in two-loop order \( (\ln Y_{ijk})' = 1 , (\ln Y_{ijk})'' = 0 \) and \( X \) is given by eq. (9), we reproduce our previous result [7]
\[ m_i^2 + m_j^2 + m_k^2 = |M|^2 + \tilde{X}^{(2)} , \]  
(28)

where \( \tilde{X}^{(2)} \) is given in (3). The general case is more involved. Following ref. [23] we require that the sum rule (24) with \( F^M \) and \( F^l \) given in (25) is RG invariant in the general case, too. That is, the reduction equation of the form [3]
\[ \mathcal{D}[ m_i^2 + m_j^2 + m_k^2 - |M|^2 F_{ijk}^M(g) - \sum_l m_l^2 F_{ijk}^l ] = 0 \]  
(29)

has to be satisfied, where
\[ \mathcal{D} \equiv \beta_g \frac{\partial}{\partial g} + \beta_M \frac{\partial}{\partial M} + \beta_M^s \frac{\partial}{\partial M^s} + \sum_l \beta_{m_l^2} \frac{\partial}{\partial m_l^2} . \]  
(30)

The equation above implies that
\[ \beta_{m_i^2} + \beta_{m_j^2} + \beta_{m_k^2} \]
\[ |M|^2 \left\{ \frac{d^2}{d(\ln g)^2} + (1 + \tilde{X}^M(g)) \frac{d}{d \ln g} \right\} + \sum_l m_l^2 \tilde{X}^l(g) \frac{d}{d \ln g} \right\} [\gamma_i(g) + \gamma_j(g) + \gamma_k(g)] \]

= \[ |M|^2 \left\{ 2(\beta_g/g)' [(1 + \tilde{X}^M)(\ln Y_{ij})' + \frac{1}{2}(\ln Y_{ij})''] \right. \]

+ \frac{(\beta_g/g)}{(1 + \tilde{X}^M)(\ln Y_{ij})'} + (1 + \tilde{X}^M)(\ln Y_{ij})'' + \frac{1}{2}(\ln Y_{ij})'''

= \sum_l \tilde{X}^l(\ln Y_{ij})' \left[ \frac{1}{2}(\gamma_l)' + (1 + \tilde{X}^M)(\gamma_l)'ight]

+ \sum_l m_l^2 \left( \beta_g/g \right) \left( \tilde{X}^l(\ln Y_{ij})' + \tilde{X}^l(\ln Y_{ij})'' \right) + \tilde{X}^l(\ln Y_{ij})' \sum_m (\gamma_m)' \tilde{X}^m \right\} , \quad (31)

where use has been made of eqs. (3), (5), (20), (27) and

\[ O = \frac{1}{2} M \frac{d}{d \ln g} . \quad (32) \]

The eq. (31) is satisfied if

\[ [(\beta_g/g)\tilde{X}^M]' + \sum_l \tilde{X}^l \left[ \frac{1}{2}(\gamma_l)'' + (1 + \tilde{X}^M)(\gamma_l)'ight] = \frac{1}{2}(\beta_g/g)'' - (\beta_g/g)' , \quad (33) \]

\[ \tilde{X}^i(\beta_g/g)' - (\tilde{X}^i)'(\beta_g/g) = \tilde{X}^i \sum_l \tilde{X}^l(\gamma_l)' \quad (34) \]

are satisfied. It seems a highly non trivial task to solve these nonlinear ordinary differential equations. On the other hand, there is another constraint coming from the result of [23], given in eq. (22), for which it is assumed that \( m_l^2 \) are also reduced in favor of \( g \) and \( M \): It reads

\[ \sum_l \tilde{X}^l(\gamma_l)' = -2(1 + \tilde{X}^M)(\beta_g/g) + (\beta_g/g)' . \quad (35) \]

For a given \( \beta_g \), it may be in principle possible to solve eqs. (33), (34) together with the constraint (35) to find \( \tilde{X}^M(g) \) and \( \tilde{X}^l(g) \). We find that this set of non-linear differential equations can be solved for the \( \beta \) function of Novikov et al. [29] which is given by

\[ \beta^\text{NSVZ}_g = \frac{g^2}{16\pi^2} \left[ \sum_l T(R_l)(1 - \gamma_l/2) - 3C(G) \right] , \quad (36) \]

because \( \beta^\text{NSVZ}_g \) has a certain singularity at

\[ g^2 = \frac{8\pi^2}{C(G)} . \quad (37) \]

We assume that \( \tilde{X}^M \) and \( \tilde{X}^l \) have a singularity of the form

\[ \tilde{X}^M \sim (C(G) - 8\pi^2/g^2)^{-a} , \]

\[ \tilde{X}^l \sim (C(G) - 8\pi^2/g^2)^{-a_l} , \quad (38) \]
and that $\gamma_l(g)$ has no singularity at $g^2 = 8\pi^2/C(G)$. To find $a$ and $a_l$ we derive from eqs. (34) and (35)

$$ (\ln \tilde{X}^l)' = \tilde{X}^M + 1 $$

which requires that $a = 1$. From eq. (35) we find that

$$ 1 \leq a_l \leq 2. $$

Further we find from eqs. (33) and (35) that the leading singularity should be canceled without the $\tilde{X}^l$ terms in these equations, which fixes $a_l$ also to be one. It is then straightforward to find the desired solution:

$$ \tilde{X}^M_{\text{NSVZ}} = -\frac{C(G)}{C(G) - 8\pi^2/g^2}, $$

$$ \tilde{X}^l_{\text{NSVZ}} = \frac{T(R_l)}{C(G) - 8\pi^2/g^2}, $$

where we have used

$$ \sum_l \gamma^\text{NSVZ}_l T(R_l) = (\beta^\text{NSVZ}_g / g) (C(G) - \frac{8\pi^2}{g^2}) + \frac{1}{2} [ \sum_l T(R_l) - 3C(G) ]. $$

Therefore, the sum rule (24) in the NSVZ scheme takes form

$$ m_i^2 + m_j^2 + m_k^2 = |M|^2 \left\{ \frac{1}{1 - g^2 C(G)/(8\pi^2)} \frac{d \ln Y^{ijk}}{d \ln g} + \frac{1}{2} \frac{d^2 \ln Y^{ijk}}{d (\ln g)^2} \right\} $$

$$ + \sum_l m_l^2 T(R_l) \frac{d \ln Y^{ijk}}{C(G) - 8\pi^2/g^2} \frac{d \ln g}{d \ln g}. $$

This result should be compared with the superstring inspired result for the finite case (i.e. $3C(G) = T(R) = \sum_i T(R_i)$)

$$ m_i^2 + m_j^2 + m_k^2 = |M|^2 \frac{1}{1 - g^2 C(G)/(8\pi^2)} + \sum_l m_l^2 T(R_l) \frac{C(G)}{C(G) - 8\pi^2/g^2}, $$

which is valid in a certain class of orbifold models in which the massive string states are organized into $N = 4$ supermultiplets, so that they do not contribute to the quantum modification of the kinetic function. So if $(\ln Y^{ijk})' = 1$, the RG invariant expressions (42) and (45) exactly coincide with the corresponding ones in the superstring models in this particular case.
As a byproduct we obtain the exact $\beta$ function for $m^2$ in the NSVZ scheme:

$$\beta_{m^2}^{\text{NSVZ}} = \left[ |M|^2 \left\{ \frac{1}{1 - g^2 C(G)/(8\pi^2)} \frac{d}{d \ln g} + \frac{1}{2} \frac{d^2}{d(\ln g)^2} \right\} + \sum \frac{m_i^2 T(R_i)}{C(G) - 8\pi^2/g^2} \frac{d}{d \ln g} \right] \gamma_i^{\text{NSVZ}},$$

(46)

where we have used eq. (27), (41) and (42). Note that $\beta_{m^2}^{\text{NSVZ}}$ assumes the form given in the r.h.s. of eq. (46) only on the RG invariant surface defined by $Y = Y(g)$ and eq. (15) in the space of parameters. In theories without Yukawa couplings such as supersymmetric QCD, the $\beta$ function above is valid in the unconstrained space of parameters.

4 Conclusions

In the present paper we have shown to all orders in perturbation theory the existence of the RGI sum rule (24) for the soft scalar masses in the SSB sector of $N = 1$ supersymmetric gauge theories exhibiting gauge-Yukawa unification. The all-loop sum rule (24) with (25) substitutes the universal soft scalar masses (which leads to phenomenological problems), while the previously known relation among $h$’s, $Y$’s $M$ and $g$ still hold to all-loop [20, 23]. Particularly interesting is the fact that the finite unified theories, which could be made all-loop finite in the supersymmetric sector [14, 15, 1] can be made completely finite, i.e. including the SSB sector, in terms of the soft scalar-mass sum rule (24), generalizing the recent result of Kazakov [20] and relaxing his finiteness conditions.

This very appealing theoretical result complements nicely the successful earlier prediction of the top quark mass [1, 2, 4] and the recent prediction of the Higgs masses and the s-spectrum [7].

In the NSVZ scheme, the sum rule can be written in a more explicit form (see (44)), exhibiting a definite singularity at $g^2 = 8\pi^2/C(G)$. The same singular behavior in the exact sum rule (45) in a certain class of superstring models has been observed [7]. This result seems to be suggesting a hint for a possible connection among the two kinds of theories.

References
[1] D. Kapetanakis, M. Mondragon and G. Zoupanos, Zeit. f. Phys. C60 (1993) 181; M. Mondragon and G. Zoupanos, Nucl. Phys. B (Proc. Suppl) 37C (1995) 98.

[2] J. Kubo, M. Mondragon and G. Zoupanos, Nucl. Phys. B424 (1994) 291.

[3] J. Kubo, M. Mondragon, N.D. Tracas and G. Zoupanos, Phys. Lett. B342 (1995) 155; J. Kubo, M. Mondragon, S. Shoda and G. Zoupanos, Nucl. Phys. B469 (1996) 3.

[4] J. Kubo, M. Mondragon, M. Olechowski and G. Zoupanos, Nucl. Phys. B479 (1996) 25.

[5] J. Kubo, M. Mondragon and G. Zoupanos, Phys. Lett. B389 (1996) 523.

[6] T. Kawamura, T. Kobayashi and J. Kubo, Phys. Lett. B405 (1997) 64; hep-ph/9710458.

[7] T. Kobayashi, J. Kubo, M. Mondragon and G. Zoupanos, hep-ph/9707425, to be published in Nucl. Phys. B.

[8] W. Zimmermann, Com. Math. Phys. 97 (1985) 211; R. Oehme and W. Zimmermann, Com. Math. Phys. 97 (1985) 569.

[9] R. Oehme, K. Sibold and W. Zimmermann, Phys. Lett. B147 (1984) 117; B153 (1985) 142.

[10] J. Kubo, K. Sibold and W. Zimmermann, Nucl. Phys. B259 (1985) 331; Phys. Lett. B200 (1989) 185.

[11] J. Kubo, M. Mondragon and G. Zoupanos, Acta Phys. Polon. B27 (1997) 3911.

[12] I. Jack and D.R.T. Jones, Phys. Lett. B349 (1995) 294.

[13] W. Zimmermann, to be published elsewhere.

[14] C. Lucchesi, O. Piquet and K. Sibold, Helv. Phys. Acta 61 (1988) 321; O. Piquet and K. Sibold, Int. J. Mod. Phys. A1 (1986) 913; Phys. Lett. B177 (1986) 373.

[15] A.Z. Ermushev, D.I. Kazakov and O.V. Tarasov, Nucl. Phys. B281 (1987) 72; D.I. Kazakov, Mod. Phys. Lett. A9 (1987) 663.

[16] Y. Yamada, Phys. Rev. D50 (1994) 3537.
[17] J. Hisano and M. Shifman, Phys. Rev. D56 (1997) 5475.

[18] I. Jack and D.R.T. Jones, hep-ph/9709364.

[19] L.V. Avdeev, D.I. Kazakov and I.N. Kondrashuk, hep-ph/9709397.

[20] D.I. Kazakov, hep-ph/9709463.

[21] D.R.T. Jones, L. Mezincescu and Y.-P. Yao, Phys. Lett. B148 (1984) 317.

[22] I. Jack and D.R.T. Jones, Phys. Lett. B333 (1994) 372.

[23] I. Jack, D.R.T. Jones and A. Pickering, hep-ph/9712542.

[24] L.E. Ibanez and D. Lüst, Nucl. Phys. B382 (1992) 305; V.S. Kaplunovsky and J. Louis, Phys. Lett. B306 (1993) 269; A. Brignole, L.E. Ibanez and C. Munoz, Nucl. Phys. B422 (1994) 125 [Erratum: B436 (1995) 747].

[25] K. Yoshioka, hep-ph/9705449.

[26] A. Brignole, L.E. Ibanez and C. Munoz, Phys. Lett. B387 (1996) 305.

[27] J.A. Casas, A. Lleyda and C. Munoz, Phys. Lett. B380 (1996) 59.

[28] J. Kubo and B. Milewski, Nucl. Phys. B254 (1985) 367; L.J. Dixon, V.S. Kaplunovsky and J. Louis, Nucl. Phys. B355 (1991) 649; I. Antoniadis, K.S. Narain and T.R. Taylor, Phys. Lett. B267 (1991) 37.

[29] V. Novikov, M. Shifman, A. Vainstein and V. Zakharov, Nucl. Phys. B229 (1983) 381; Phys. Lett. B166 (1986) 329; M. Shifman, Int.J. Mod. Phys.A11 (1996) 5761 and references therein.

[30] R. Delbourgo, Nuovo Cim 25A (1975) 646; A. Salam and J. Strathdee, Nucl. Phys. B86 (1975) 142; K. Fujikawa and W. Lang, Nucl. Phys. B88 (1975) 61; M.T. Grisaru, M. Rocek and W. Siegel, Nucl. Phys. B59 (1979) 429.

[31] L. Girardello and M.T. Grisaru, Nucl. Phys. B194 (1982) 65; J.A. Helayel-Neto, Phys. Lett. B135 (1984) 78; F. Feruglio, J.A. Helayel-Neto and F. Legovini, Nucl. Phys. B249 (1985) 533; M. Scholl, Zeit. f. Phys. C28 (1985)545.

[32] I. Jack, D.R.T. Jones, S.P. Martin, M.T. Vaughn and Y. Yamada, Phys. Rev. D50 (1994) R5481.