Exploring students’ critical thinking skills in a geometry lesson

D Hidayat and R Rosnawati
Universitas Negeri Yogyakarta, Jl. Colombo No.1, Sleman, Yogyakarta, Indonesia
E-mail: dedenhidayat.2018@student.uny.ac.id

Abstract. The ability of critical thinking in mathematics is an important ability that should be mastered by students. The goal of this research is to describe students’ critical thinking in a geometry lesson. This research is a survey study in which the data are taken by the documentation. Students’ answers used in this data based on critical thinking questions in which 17 students in 8th grade on one of the junior high schools in Yogyakarta are involved in this research. Students’ critical thinking instruments consist of 6 questions which have three aspects, namely analyzing, evaluating, and concluding in which each is with a picture and without the picture. The result of this research shows that analyzing aspect with picture questions has a better result compare to the question without picture. Similarly, in evaluating and concluding aspects, the data show that students’ answers are better with the picture rather than the question without the picture. Furthermore, from the results of the sample students’ exploration answers, it is expected that learning in future geometry can facilitate students' critical thinking skills then, the learning of geometry is better presented by using the pictures.

1. Introduction
The development of education nowadays requires students to actively build their knowledge. Based on the Indonesian 2013 curriculum, learning must place students at the center of learning in the classroom. Mathematics learning in the classroom should focus not only on improving student’s numeracy skills but also on developing students' abilities or skills in thinking. So that developing students' thinking skills will be able to help students in making the right decisions and gaining new knowledge quickly.

The skill of making the right decisions and gaining new knowledge quickly are basically the ability to think critically. Critical thinking is the ability used in everyday life to solve a problem that involves logical reasoning, interpreting, analyzing, and evaluating information that allows making reliable and valid decisions [1]. The ability to think critically enables students to be able to process information logically, determine what information is important and what is not important. Furthermore, students who have the ability to think critically can respond to the problems or make decisions correctly [2]. The same thing also expressed by Carroll [3] that critical thinkers will become better problem solvers and decision-makers.

The importance of critical thinking skills is also contained in the goals of national education, one of which is to be able to develop critical thinking skills [4]. The same thing also expressed by the National Education Association [5] that abilities are considered important and need to be possessed by a student of all types of abilities that includes critical thinking, communication, collaboration, and creativity. Moon [6] stated that an important goal of Education is to develop critical thinking skills. Furthermore, Larson [7] also stated the development of critical thinking skills regarding current educational goals.
According to Ennis [8], critical thinking is rational and reflective thinking that focuses on deciding what to believe and do. While Liu, He, and Le [9] stated that critical thinking is defined as a complex form of a higher process in which there is evaluating available information and testing one’s own understanding as a form of process components, the other expert defined critical thinking as an intellectual concept that encourages an individual to analyze, evaluate and explain through interpreting ideas from a broader perspective [10]. Interpretation of ideas from a broader perspective will make someone able to organize, adjust, change and improve their thinking, so that they can make effective decisions from an issue.

Wolfolk [11] stated that the ability to think critically is an ability to carry out the reasoning in response to a situation based on evidence and other supporters to be able to provide conclusions at the end. Meanwhile, Tittle [12] argued that critical thinking is an evaluation of opinions which have a reason in which is the basis of critical thinking itself is not only in the form of emotions, intuitions, or beliefs but also rational thinking. Based on these opinions, a student’s critical thinking will be guided the students to be able to draw an appropriate conclusion with logical reasons. The standard of critical thinking itself includes analysis which involves determining the relevant information, evaluation which involves assessing the truth of an argument and concluding which involves the activity can make the right conclusion from an existing argument [9, 11-14].

The development of critical thinking skills is an effective way to make students understand the material in mathematics. The ability to think critically also gives students the opportunity to share ideas, strategies, and ways to solve a problem [15]. In line with this, Samanci [16] stated that the ability to think critically influences the achievements of students. Furthermore, the results of Chukwuyenum’s research [1] are students who are able to think critically will be able to predict common mistakes and be able to solve problems systematically.

Mathematics taught in schools consists of various fields namely algebra, arithmetic, geometry, statistics, and others [17]. The importance of critical thinking skills makes the teacher must associate critical thinking skills with every subject taught to their students. Geometry is an important branch in mathematics because it is used by short people and is related to everyday life. Geometry is not only an important part of mathematics but also an important part of daily life [18]. Geometry is an important mathematical material in the school curriculum [19, 20].

According to Gunham [21], geometry is an important branch of mathematics, which makes it possible for people to understand the world by comparing shapes, objects and their relationships. Sahin [22] further stated that geometry is one of the important branches in mathematics education because the purpose of geometry is to teach students to think critically, problem-solving and good understanding. Geometry is not just a series of pure theorems used in education, but it is also very close to our lives where geometry occupies social, intellectual, and institutional contexts. Therefore, to master the geometry is important to be owned by students.

Cambers [23] stated that the nature of visual geometry will make geometry an interesting material to be studied and has the potential to explore a lot of knowledge from mathematics. Clements and Battista [24] found that the students preferred to solve geometrical problems by presented visually rather than verbally. In this study, the researchers describe the students’ critical thinking skills in geometry. By describing the critical thinking skills possessed by students, researchers are expected to be able to understand the strengths and weaknesses of students in critical thinking so that they can be used as a basis for developing critical thinking skills in geometry.

2. Method

This research is a survey research using data collection through documentation study. The data used students’ answers from the critical thinking skills test conducted by 17 students of class VIII at one of the junior high schools in Yogyakarta, Indonesia. The instrument of students’ critical thinking skills consists of six items containing aspects of analyzing, evaluating, and concluding. The three aspects of critical thinking consist of two questions namely questions that are accompanied by pictures and without pictures. The aim is to describe students’ critical thinking skills in geometry.
3. Results and Discussion
In this paper, we explored the students’ answers from tests of critical thinking skills on geometry material. Students’ answers are obtained from six problems (P1-P6), each of which measures indicators of critical thinking skills that are tested on students. Each indicator of critical thinking ability consists of two questions, namely questions without pictures and questions accompanied by pictures. Student answers can be divided into three forms, namely student answers correct, some true or correct but not complete and incorrect or not answer. The result of the critical thinking test is presented in Table 1.

| Answer type     | The number of students |
|-----------------|------------------------|
|                 | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 |
| Correct         | f  | 1  | 3  | 4  | 5  | 4  | 5  |
|                 | %  | 6  | 18 | 24 | 29 | 24 | 29 |
| Partially correct| f  | 14 | 13 | 5  | 4  | 9  | 7  |
|                 | %  | 82 | 76 | 29 | 24 | 53 | 41 |
| Incorrect       | f  | 2  | 1  | 8  | 8  | 4  | 5  |
|                 | %  | 12 | 6  | 47 | 47 | 24 | 29 |

The following is the result of exploratory student answers for each indicator of thinking ability seen from the geometry problem without drawing and accompanied by pictures.

3.1. Problem 1 (P1): Analyzing aspect for geometry problem without picture
In this section, students are expected to be able to determine relevant information from the problems given. The problem given in this section requires students to analyze the amount of plasterboard in the form of a square or rectangle that can be purchased with a certain size and price. After determining the amount of plasterboard purchased, the plasterboard is then designed on the roof of the room provided whether or not it is enough. This question might be considered the easiest for students because students are only told to determine the amount of plasterboard needed. But the results obtained show the opposite. Most students cannot analyze the given problem. Only one student can give the correct answer. The results of these student answers can be seen in Figure 1.

Translation:
The amount of each plasterboard:

a) $100 \text{ cm} \times 100 \text{ cm} = 14 \text{ pieces with price } Rp \ 231.000$

b) $30 \text{ cm} \times 50 \text{ cm} = 16 \text{ pieces with price } Rp \ 184.000$

c) $25 \text{ cm} \times 100 \text{ cm} = 8 \text{ pieces with price } Rp \ 72.000$

Total = Rp 487.000

Design illustration

Figure 1. The correct students' answer for the analyzing aspect of geometry problem without picture
the money provided enough or not to buy the plasterboard. Furthermore, students also lack the understanding that at least three plasterboards must be purchased. Finally, there are many students who are unable to determine the relevant information from the problem given. One form of incorrect as described above is illustrated in Figure 2.

Translation:

**Type A =**

\[
\begin{align*}
\text{500 cm} & \times \text{400 cm} = 20 \text{ pieces} \\
\text{100 cm} & \times \text{100 cm} = 20 \text{ pieces} \\
\end{align*}
\]

\[
\begin{align*}
\times 16.500 = 330,000 \\
\end{align*}
\]

**Type B =**

\[
\begin{align*}
\text{50 cm} & \times \text{50 cm} = 80 \text{ pieces} \\
\text{500 cm} & \times \text{400 cm} = 80 \text{ pieces} \\
\end{align*}
\]

\[
\begin{align*}
\times 14.000 = 1,120,000 \\
\end{align*}
\]

**Type C =**

\[
\begin{align*}
\text{50 cm} & \times \text{50 cm} = 80 \text{ pieces} \\
\text{500 cm} & \times \text{400 cm} = 80 \text{ pieces} \\
\end{align*}
\]

\[
\begin{align*}
\times 11.500 = 920,000 \\
\end{align*}
\]

**Type D =**

\[
\begin{align*}
\text{25 cm} & \times \text{25 cm} = 320 \text{ pieces} \\
\text{500 cm} & \times \text{400 cm} = 320 \text{ pieces} \\
\end{align*}
\]

\[
\begin{align*}
\times 9.000 = 2,280,000 \\
\end{align*}
\]

Money = 550,000

**Type A =**

\[
\begin{align*}
\text{550,000} \times 20 = 33,333 \rightarrow 34 \text{ pieces} \\
\end{align*}
\]

**Type B =**

\[
\begin{align*}
\text{39,28} \rightarrow 40 \text{ pieces} \\
\end{align*}
\]

**Type C =**

\[
\begin{align*}
\text{47,8} \rightarrow 48 \text{ pieces} \\
\end{align*}
\]

**Type D =**

\[
\begin{align*}
\text{61,1} \rightarrow 62 \text{ pieces} \\
\end{align*}
\]

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**Figure 2.** Incorrect student’s answer for the analyzing aspect of geometry problem without picture

3.2. **Problem 2 (P2): Analyzing aspect for geometry problem with picture**

In this section, the problem is presented by displaying the limestone mounting design as a volleyball court line on one side where provided two rectangular lime stones with different sizes. Furthermore, the students are asked to determine whether the design of lime stone installation is appropriate or not. If it is appropriate, it shows how much exactly the limestone needed. The success rate for this question is higher when compared to Problem 1. A correct student answer is presented in Figure 3.

Translation:

Yes, limestone installation is done right
white limestone = 10 cm × 50 cm
red limestone = 10 cm × 40 cm
the size of a volleyball court = 1900 cm × 800 cm
→ 1900:2 = \[
\frac{950}{40 \text{ cm}+50 \text{ cm}} = 10
\]
white limestone = 10 × 50 = 500
red limestone = 10 × 40 = 400
Total limestone = 900
→ 950 − 900 = 50 (red limestone +1)
\[
\frac{800}{90}
\]
white limestone = 8 × 50 = 400
red limestone = 10 × 40 = 400
Total limestone = 800
Total :
white limestone = (10 × 4) + (3 × 2) = 40 + 12 = 56 pieces
red limestone = (11 × 4) + (10 × 2) = 44 + 20 = 64 pieces

**Figure 3.** The correct students’ answer for the analyzing aspect of geometry problem with picture
In Figure 3, the students begin the analysis step by determining the design of limestone mounting on one of the right or incorrect sides. Then after determining the exact or not, the students determine the amount of each limestone needed to be able to line the volleyball field as a whole. Furthermore, the incorrect students answer is also presented in Figure 4.

Translation:

white limestone = \(10 \times 50\) 
red limestone = \(10 \times 40\)

\[
\text{white limestone} = \frac{(1900+800)}{50} \text{ cm} = \frac{2700}{50} = 54 \text{ pieces}
\]
\[
\text{red limestone} = \frac{(1900+800)}{40} = \frac{2700}{40} = 67.5 \approx 68 \text{ pieces}
\]

Figure 4. Incorrect student’s answer for the analyzing aspect of geometry problem with picture

Meanwhile, in Figure 4, the students are not able to do an analysis of the questions given to the problems. This is because students immediately divide half the length of the field by the length of one side of the limestone without regard to the design of the limestone installation. This makes the analysis carried out by students incorrect. The same thing happened to several other students.

3.3 Problem 3 (P3): Evaluating aspect for geometry problem without picture

In this section, the students are asked to be able to assess the truth of the information provided. The problem is presented by giving some coordinate points if connected will form two planar figures with a certain area. The next students are asked to evaluate by assessing the truth of the information provided. The following are examples of correct answers given by students.

Translation: There are two planar figures, rhombus and rectangle.

\[
\text{Rhombus area} = \frac{d_1 \times d_2}{2} = \frac{6 \times 8}{2} = 24 \text{ unit}
\]
\[
\text{Rectangle area} = \frac{a \times t}{2} = \frac{3 \times 4}{2} = 6 \text{ unit}
\]
\[
\text{Total area} = 24 + 6 = 30 \text{ unit.}
\]

Figure 5. Correct student’s answer for the evaluating aspect of geometry problem without picture

In Figure 5, the students are able to evaluate in assessing an argument or information provided by connecting between the concepts. Students are able to distinguish square and rhombus based on correct concepts. Many students are also able to evaluate the type of planar figure, but experience errors for the next stage. Figure 6, shows students are still mistaken in determining the type of planar figure.
Translation:
There are two planar figures, square and rectangle.
Total of the planar figures area is 24 unit.

Figure 6. Incorrect students’ answers for the evaluating aspect of geometry problem without picture

In Figure 6, the students have been able to draw the points in the coordinate plane correctly. However, then students do not see in terms of angles and diagonals of a planar figure so they are doing wrong in evaluating the type of planar figure. Furthermore, students are also wrong in terms of determining the area of the planar figure where students use the unit square area to determine the actual area.

3.4. Problem 4 (P4): Evaluating aspect for geometry problem with pictures

In this section, the problem is presented by displaying the garden design and fertilizer needed for each part of the garden. Next, students were asked to evaluate whether available fertilizer is sufficient for the entire park area. The success rate for Problem 4 is higher than evaluating for Problem 3. One correct student answer is presented in Figure 7. Meanwhile, the incorrect answer is presented in Figure 8.

Translation:
Total area small rectangle
\[= 6 \times \text{area rectangle} = 6 \times \frac{1}{2} \times a \times t = 6 \times \frac{3}{2} \times 4 \times 3 = 36 \text{ cm}^2\]

Total area big rectangle
\[= 2 \times \text{area rectangle} = 2 \times \frac{1}{2} \times 20 \times 15 = 300 \text{ cm}^2\]

Total of garden area \[= 36 \text{ cm}^2 + 300 \text{ cm}^2 = 336 \text{ cm}^2\]

The need of fertilizer \[\frac{1 \text{ kg}}{25 \text{ m}^2}\]
The need of fertilizer \[= \frac{336 \text{ cm}^2 \times 1 \text{ kg}}{2500 \text{ cm}^2} = 15.44 \text{ kg}\]
Not enough, the need of fertilizer for garden area is 15.44 kg while the fertilizer purchased is 14 kg.

Figure 7. The correct student's answers for the evaluating aspect of geometry problem with picture

In Figure 7, the students begin the evaluation step by calculating the area of six congruent triangles. Furthermore, the next students determine the length of one side of a large triangle by first determining
the length of the hypotenuse of a congruent triangle. In the end, students are able to determine the area of the park by adding up the area of a small triangle and the area of a large triangle. Next students evaluate whether the available fertilizer is enough by comparing it with the total area of the park.

Figure 8. Students' incorrect answer for the evaluating aspect of geometry problem with picture

Meanwhile, in Figure 8, the students can’t evaluate the truth of the information provided. This is because students cannot determine the figure of a flat figure correctly formed. As a result, students can’t determine the truth of the information provided on the problem. The same thing happened with several other students.

3.5. Problem 5 (P5): Concluding aspect for geometry problems without picture

In this section, the students are asked to make a conclusion from the given problem. The problem given is about how to divide traditional food whose surface is in the form of a parallelogram in order to obtain the greatest benefit. More than half of the students were able to determine how to divide the surface of the parallelogram, but many students cannot determine the final conclusion. One of the correct student answers is presented in Figure 9.

Figure 9. Students’ correct answer for the concluding aspect of geometry problem without picture
Meanwhile, the incorrect answer of one of the students is presented in Figure 10.

3.6 Problem 6 (P6): Concluding aspect for geometry problems with picture

In this section, like Problem 5, the students were asked to make appropriate conclusions from a given problem. The problem given is whether two colors of paint are provided to be able to paint the existing parallelogram or not. In the first step, the students determine the area of one of the parallelograms provided. After knowing the area, students can determine the amount of paint needed for each parallelogram. Next, the students count how many parallelograms can be painted. Whether the paint that is available is sufficient or not for all existing parallelogram. One of the correct student answers is presented in Figure 11.

Meanwhile, the answer to one of the students who experienced an incorrect answer is presented in Figure 12.
Based on the aforementioned results, through an exploration of each indicator of critical thinking, it was found that in general, the questions with pictures had a greater proportion of students answering correctly than the questions without pictures. The difference in results shows that students more easily solve geometry problems accompanied by pictures. Research conducted by Burger and Shaughnessy [25] found that the level of geometrical thinking of middle school students is still largely at the visualization stage. Based on this, students will find it easier to understand geometry material if the material is presented with pictures. Furthermore, the ease of analyzing the form of picture compared to words or without picture is also supported by research conducted by Clemern and Battista [24]. The results of research conducted by Clements & Battista [24] produce that students prefer to solve geometrical problems presented visually rather than verbally.

Furthermore, White [26] stated whether students can’t find keywords in the problem, or students can read the problem can’t interpret the problem, then students will find it difficult to proceed to the process of solving. Walle [27] stated that junior high school students may be in the visualization stage. Ulfiana [28] also stated that in junior high school results, visual design learning can maximize students’ critical thinking skills. Based on this, the geometry material if presented with a visual form or accompanied by a picture will make it easier for students to complete it. The true learning success in class requires consistent learning in order to develop students’ thinking abilities.

Exploration activities on other abilities need to be carried out by a teacher to be used as a basis for implementing learning. According to Fahmi & Masouleh, teachers are expected to continue to find ways to make students want to think critically [3]. It is intended that students can increase intelligence owned by a higher level.

4. Conclusion
Based on the results of the study, it can be concluded that based on the results of the exploration of students’ answers from each indicator of critical thinking. Where each indicator is viewed from geometry problems without pictures and questions with pictures, it is obtained that in general the questions with pictures viewed from the aspect of analyzing, evaluating, and concluding have a greater proportion of students answering correctly than questions without pictures. Based on these results, it is expected that learning in the future geometry material in order to facilitate students’ critical thinking skills, the learning that is carried out would be better if it is presented with pictures. Next, due to the limitations of the sample of this study, this finding only applies to students in the sample and cannot be considered to represent broader Indonesian students. However, the findings in this study are of interest to teachers and educators in developing learning in the classroom, especially in the geometry material.

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References
[1] Chukwuyenum A N 2013 Impact of critical thinking on performance in mathematics among senior secondary school students in Lagos State J. Res. Method Educ. 3 18–25
[2] Su H F H, Ricci F A and Mnatsakanian M 2016 Mathematical teaching strategies: Pathways to critical thinking and metacognition Int. J. Res. Educ. Sci. 2 190–200
[3] Fahim M and Masouleh N S 2012 Critical thinking in higher education: A pedagogical look Theory Pract. Lang. Stud. 2 1370-5
[4] MOEC 2016 Permendikbud Nomor 22 Tahun 2016 Tentang Standar Proses Pendidikan Dasar dan Menengah (Jakarta: Indonesian Ministry of Education and Culture)
[5] Robinson S P and Kay K 2010 Preparing 21st Century Students for a Global Society (Washington DC: NEA)
[6] Moon J 2008 Critical thinking: An exploration on theory and practice (New York: Routledge)
[7] Larsson K 2017 Understanding and teaching critical thinking: A new approach Int. J. Educ. Res. 84 32–42
[8] Ennis R H 2011 The nature of critical thinking: Outlines of general critical thinking disposition and abilities Sixth Int. Conf. Think. 1–8
[9] Liu Z K, He J and Li B 2015 Critical and creative thinking as learning processes at top-ranking Chinese middle schools: Possibilities and required improvements High Abil. Stud. 26 139–52
[10] Dil Y, Öğretildiği O and Smiflaranda D 2015 Developing critical thinking skills in English language teaching classes through novels Int. J. Lang. Acad. 3 76–90
[11] Wolfolk A 2016 Educational Psychology (Boston: Pearson)
[12] Peg T 2011 Critical thinking: An appeal to reason (New York: Routledge)
[13] Fong C J, Kim Y, Davis C W, Hoang T and Won Y 2017 A meta-analysis on critical thinking and community college student achievement Think. Ski. Creat. 26 71–83
[14] Kereluik K, Fahnoe C and Karr J A 2013 What knowledge is of most worth: Teacher knowledge for 21 J. Digit. Learn. Teach. Educ. 29 127–40
[15] Sultra W S R Y, Usodo B and Pramudya I 2019 Analysis of students’ critical thinking in solving geometry problem based on learning archivment J. Phys. Conf. Ser. 1321 032002
[16] Samancı N K 2015 A study on the link between moral judgment competences and critical thinking skills Int. J. Environ. Sci. Educ. 10 135–43
[17] Sutiarso S and Coesamin M 2018 The effect of various media scaffolding in increasing understanding of students’ geometry concepts J. Math. Educ. 9 95–102
[18] Cherif A H, Gialamas S and Stamati A 2017 Developing mathematical knowledge and skills through the awareness approach of teaching and learning J. Educ. Pract. 8 108–32
[19] Hock T T, Tarmizi R A, Yunus A S and Ayub A F 2015 Understanding the primary school students’ van Hiele levels of geometry thinking in learning shapes and spaces: A q-methodology Eurasia J. Math. Sci. Technol. Educ. 11 793–802
[20] Watan S and Sugiman S 2018 Exploring the relationship between teachers’ instructional and students’ geometrical thinking levels based on van Hiele theory J. Phys. Conf. Ser. 1097 012122
[21] Gunhan B C 2014 A case study on the investigation of reasoning skills in geometry South African J. Educ. 34 1–19
[22] Aydogdu M Z and Kesan C 2014 A research on geometry problem solving strategies used by elementary mathematics teacher candidates J. Educ. ans Instr. Stud. World 4 53–62
[23] Chambers P 2008 Teaching mathematics: developing as a reflective secondary teacher (London: SAGE)
[24] Clements D H and Battista M T 1992 Geometry and spatial reasoning Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics ed D A Grouws (New York: Macmillan Publishing)
[25] Burger W F and Shaughnessy J M 1986 Characterizing the van Hiele levels of development in geometry J. Res. Math. Educ. 17 31–48
[26] White A L 2005 Active mathematics in classrooms: Finding out why children make mistakes and then doing something to help them Sq. One 15 5–9
[27] Walle V 2010 Elementary and Middle School Mathematics: Teaching Developmentally 7th ed (London: Pearson)
[28] Ulfiana E, Mardiyana M and Triyanto T 2019 Determining ways to improve critical thinking skills in the math mathematics in student style J. Phys. Conf. Ser. 1321 022098