Possible scaling behaviour of the multiplicities ratio in leptoproduction of charged pions in nuclear medium

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In this paper it is demonstrated that based on two-dimensional distributions in semi-inclusive deep inelastic scattering (SIDIS) data, obtained recently by the HERMES experiment at DESY on different nuclei, which contains data for charged pions produced in \( z \) slices as a function of \( \nu \), and in a \( \nu \) slices as a function of \( \nu \), it is possible to parametrise a ratio of multiplicities on nucleus and deuterium (per nucleon) \( R_M^{\nu} \) in a form of a function of a single variable \( \tau \), which has the physical meaning of the formation time of hadron. We call this effect \( \tau \) scaling. \( \tau \) is a function of two variables \( \nu \) and \( z \). It is also shown that \( R_M^{\nu} \) can be presented in a form of a linear polynomial of \( \tau \), \( a_{11} + \tau a_{12} \), where parameters \( a_{11} \) and \( a_{12} \) do not depend on \( \nu \) and \( z \).

I. INTRODUCTION

Hadronization is the process through which partons, created in an elementary interaction, turn into the hadrons. Experimentally the hadronization process in free space (vacuum) has been studied extensively in \( e^+e^- \) annihilation and in semi-inclusive lepton-proton deep inelastic scattering (DIS). As a result, the spectra of hadrons produced and their kinematical dependences are rather well known. However, little is known about the space-time evolution of the process, because the products of this process can only be observed in a detector that is separated from the reaction point by a macroscopic distance. It is worth to mention that according to theoretical estimates the hadronization process occurs over length scales that vary from less than a femtometer to several tens of femtometers. The nuclear medium can serve as a detector located directly at the place where microscopic interaction happens. Consequently, leptoproduction of hadrons on atomic nuclei provides a way to investigate the space-time picture of the hadronization process. The semi-inclusive DIS of leptons on nuclear targets is widely used for the study of this process [3, 4]. It is most effective to observe at moderate energies of the virtual photon, when the formation time of the hadron is comparable with the nuclear radius. Such possibility provides HERMES experiment at DESY, which uses electron (positron) beam with energy 27.5 GeV and fixed nuclear targets.

The most convenient observable measured experimentally for this process is the nuclear attenuation ratio, which is a ratio of multiplicities on nucleus and deuterium (per nucleon) for a given hadron. We shall denote it as \( R_M^{\nu} \). For a more profound understanding of the hadronization mechanism, it is important to find a variable which allows to present this observable in the most simple functional form.

Usually it is supposed, that \( R_M^{\nu} \) is a function of two variables \( \nu \) and \( z \), which are the energy of photon and the fraction of this energy carried by the final hadron with energy \( E_h (z = E_h/\nu) \) respectively\(^1\).

In our preceding work [5] we performed a fit for the evidence that the formation time \( \tau \) is the best variable for \( R_M^{\nu} \), i.e. that it can be parametrized as a function of a single variable \( \tau \). Three widely known representations for \( \tau \) were used for the fit. The experimental data for pions on nitrogen and for identified hadrons on krypton nuclei obtained by the HERMES experiment [5, 6] were used for this fit. We have demonstrated that the nuclear attenuation ratio can be presented, with good precision, as a function of a single variable \( \tau \) instead of a function of two variables \( \nu \) and \( z \). Moreover, \( R_M^{\nu} \) is a linear function of \( \tau \). We named \( \tau \) a scaling variable, because it contains all \( \nu \) and \( z \) dependencies of \( R_M^{\nu} \). For the fit we have obtained \( R_M^{\nu} \) as a function of \( \tau \) from experimental data [3, 4], where it was measured as a function of \( \nu \) with the integration over \( z \), and as a function of \( z \) with the integration over \( \nu \). This means, that the data were taken at an unequal binning over \( \nu \) and \( z \). In case of \( \nu \)-dependence the detailed bins over the variable \( \nu \) were taken, and for each value of \( \nu \) the value of \( z \) averaged over whole range of measured \( z ( < z > ) \) was taken. In case of \( z \)-dependence we also performed the above mentioned procedure.

Recent work published by the HERMES experiment [6] allows to escape this difficulty, because the data published contains, among others, the so called two-dimensional data, i.e. multiplicity ratio \( R_M^{\nu} \) for charged pions produced in a \( z \) slices as a function of \( \nu \), and in a \( \nu \) slices as a function of \( \nu \). The data were obtained for four nuclear targets and used for a new fit of \( R_M^{\nu} \) as a function of \( \tau \).

The main aim of this work is to show, that \( R_M^{\nu} \) is a

\(^1\) In fact, \( R_M^{\nu} \) also depends on the photon virtuality \( Q^2 \) and on the square of the hadron transverse momentum in respect to the virtual photon direction, \( p_t^2 \). However, from the experimental data, it is known that \( R_M^{\nu} \) is a much sensitive function of \( \nu \) and \( z \) in comparison with \( Q^2 \) and \( p_t^2 \).
function of single variable \( \tau \), rather than a function of two variables \( \nu \) and \( z \), using the new set of \( R^h_M \), given by the two-dimensional analysis, where the data is split into more regular \( \nu \) and \( z \) bins than in case of traditional presentation in form of \( \nu \) - and \( z \) - dependences. This will allow to verify the results of our preceding work \([5]\) in more favourable conditions, i.e. to confirm that in electroproduction of hadrons in nuclear medium we indeed observe scaling, where \( \tau \) takes over the role of the scaling variable.

This paper is organized as follows. Nuclear attenuation in an absorption model is presented in the next section. In Section 3 we discuss the choice of an appropriate form for the variable \( \tau \). Section 4 presents results of the fit. Conclusions are given in Section 5.

II. NUCLEAR ATTENUATION IN ABSORPTION MODEL

The semi-inclusive DIS of lepton on nucleus of atomic mass number \( A \) is

\[
l_i + A \rightarrow l_f + h + X, \tag{1}
\]

where \( l_i(l_f) \) and \( h \) denote the initial (final) leptons and the hadron observed in the final state. This process is usually investigated in terms of \( R^h_M \):

\[
R^h_M(\nu, z) = 2d\sigma_A(\nu, z)/A d\sigma_D(\nu, z). \tag{2}
\]

Experimental data are usually presented at precise values of one variable and average values of another\(^2\). In this work we adopt a model according to which the origin of the nuclear attenuation is the absorption of the prehadron (string, dipole) and final hadron in the nuclear medium. In that case \( R^h_M \) has the following form:

\[
R^h_M = \int d^2 b \int_{-\infty}^{\infty} \rho(b, x) W(b, x) |(A-1)dx, \tag{3}
\]

where \( b \) is the impact parameter and \( x \) the longitudinal coordinate of the DIS point. \( \rho \) is the nuclear density function with a normalization condition \( \int \rho(r)d^3 r = 1 \). \( W(b, x) \) is the probability that neither the prehadron nor the final hadron \( h \) are absorbed by a nucleon located anywhere in the nucleus. For \( W(b, x) \) we use the one-scale model proposed in Ref. \([7]\):

\[
W(b, x) = 1 - \sigma_q \int_x^{\infty} P_q(x' - x) \rho(b, x') dx' - \sigma_h \int_x^{\infty} P_h(x' - x) \rho(b, x') dx', \tag{4}
\]

where \( \sigma_q \) and \( \sigma_h \) are the inelastic cross sections for prehadron-nucleon and hadron-nucleon interactions, respectively. Generally speaking \( \sigma_q \) is a function of the distance \( x' - x \), \( \nu \) and \( z \).\(^3\) However a comparison of simple theoretical models containing \( \sigma_q \) as a parameter \([2, 8]\), with the experimental data obtained in different kinematical regions, (in particular in different domains of \( \nu \)), shows that the approximation \( \sigma_q = \text{const} \) leads to a very acceptable agreement with the data. Further, taking into account the qualitative character of this work, we shall use this approximation. In the region of moderate energies, \( \sigma_h \) are approximately constant for all \( h \). \( P_q(x' - x) \) is the probability that at distance \( x' - x \) from the DIS point, the particle is a prehadron and \( P_h(x' - x) \) is the probability that the particle is a hadron. The abovementioned probabilities are related via a condition

\[
P_h(x' - x) = 1 - P_q(x' - x). \tag{5}
\]

In analogy with the survival probability for a particle having lifetime \( \tau \) in a system where it travels a distance \( x' - x \) before decaying, \( P_q(x' - x) \) can be expressed in the form

\[
P_q(x' - x) = \exp[-(x' - x)/\tau], \tag{6}
\]

where \( \tau \) is the formation time. Substituting expressions for \( P_q(x' - x) \) and \( P_h(x' - x) \) in eq.\,(4) one obtains

\[
W(b, x) \approx 1 - \sigma_h \int_x^{\infty} \rho(b, x') dx' + \tau(\sigma_h - \sigma_q) \rho(b, x) \approx w_1(b, x) + \tau(\nu, z) w_2(b, x). \tag{7}
\]

In the framework of our suppositions about \( \sigma_q \) and \( \sigma_h \), \( W \) depends on \( \nu \) and \( z \) only by means of \( \tau(\nu, z) \).

In more detail, the formation time in the string models can be divided in two parts (see, for instance, the two scale model presented in Refs. \([2, 8]\)). The first part is the constituent formation time \( \tau_{\text{c}} \), which defines the time elapsed from the moment of the DIS until the production of the first constituent of the final hadron. The second time interval begins with the production of the first constituent until the second one, which coincides with the yo-yo\(^4\) or final hadron production. Comparison with the experimental data shows that in the second interval, the prehadron-nucleon cross section has values close to the hadron-nucleon cross section \( \sigma_h \). If the difference between these cross-sections is neglected, the model is reduced to one scale model with \( \tau = \tau_{\text{c}} \). Substituting \( W(b, x) \) in \( R^h_M \) we obtain

\[
R^h_M \approx \int d^2 b \int_{-\infty}^{\infty} \rho(b, x)(w_1 + \tau w_2)(A-1)dx \approx a_1 + \tau a_2 + \tau^2 a_3 + \cdots, \tag{8}
\]

\(^2\) In case where the \( \nu \) - dependence is studied, we have \( R^h_M(\nu, z) \), where \( z \) are the average values of \( z \) for each \( \nu \) bin. For \( \nu \) - dependence \( R^h_M(\nu, z) \), where \( \nu \) are the average values of \( \nu \) for each \( z \) bin.

\(^3\) \( \sigma_q \) is a function of formation time \( \tau \) rather than a function of variables \( \nu \) and \( z \) separately.

\(^4\) The yo-yo formation means that a colorless system with valence contents and quantum numbers of the final hadron is formed, but without its “sea” partons.
where $i$ is the maximal power of $\tau$ with which we are limited. Although $R^h_M$ is a polynomial of $\tau$ with maximal power $A-1$, it is expected that $a_{i1} > a_{i2} > a_{i3} > \cdots$. The coefficients $a_{ij}$ depend on $A$, $\sigma_0$, $\sigma_h$, and nuclear density. For fit we use three expressions for $R^h_M$ as first, second and third order polynomials of $\tau$:

\begin{align*}
R^h_M[P_1] &= a_{11} + \tau a_{32}, \\
R^h_M[P_2] &= a_{21} + \tau a_{22} + \tau^2 a_{23}, \\
R^h_M[P_3] &= a_{31} + \tau a_{32} + \tau^2 a_{33} + \tau^3 a_{34}.
\end{align*}

(9) (10) (11)

In order to get the information on the influence of highest order polynomial forms for $R^h_M$, $R^h_M[P_3]$ expression also was checked (see section 4).

### III. FORMATION TIME

Equation (8) shows that within our approximation, $R^h_M$ depends on $\nu$ and $z$ only by means of $\tau(\nu, z)$. This is the reason why we call $\tau$ a scaling variable. In this section we shall discuss the physical meaning and possible expressions of the formation time $\tau$. There are different definitions for the formation time. We define it as a time scale which is necessary for the prehadron-nucleon cross section to reach the value of the hadron-nucleon cross-section. In the literature there are three qualitatively different definitions for $\tau$. In the first extreme case it is assumed that $\tau = 0$ (Glauber approach). In the second extreme case $\tau \gg r_A$, where $r_A$ is the nuclear radius (energy loss model [9]). And at last, and in our opinion more realistic definition of the formation time, as a function of $\nu$ and $z$ which can change from zero up to values larger than $r_A$. Experimental data seem to confirm that for moderate values of $\nu$ (on the order of 10GeV) the formation time is comparable with the nuclear size, i.e. the hadronization mostly takes place within the nucleus. This follows from the comparison of the experimental data for $R^h_M$ obtained in the region of moderate [4] and high [2] energies. At moderate energies $R^h_M$ significantly differs from unity and is a sensitive function of $\nu$ and $z$, at high energies $R^h_M \approx 1$ and weakly depends on $\nu$ and $z$. For the formation time we shall use expressions which do not contradict the third definition mentioned above. The following expressions are used:

1. Formation time for the leading hadron [10], which follows from the energy-momentum conservation law

\[ \tau_{\text{lead.}} = (1 - z)\nu/\kappa, \]

(12)

where $\kappa$ is the string tension (string constant) with numerical value $\kappa = 1\text{GeV}/fm$

2. Formation time for the fast hadron, which is composed of characteristic formation time of the hadron $h$

\[ \tau_{\text{fast}} = \tau_{\text{lead.}} + \Delta \tau, \]

where \( \Delta \tau \) is the additional time required for the fast hadron to be formed.

3. Formation time for the reduced hadron $h'$

\[ \tau_{\text{reduced}} = \tau_{\text{lead.}} + \Delta \tau_{\text{reduced}}, \]

where \( \Delta \tau_{\text{reduced}} \) is the additional time required for the reduced hadron to be formed.


\[ \tau_{\text{Lund}} = \frac{\ln(1/z^2) - 1 + z^2}{1 - z^2} \frac{\nu}{\kappa}. \]

(14)

One should note that all three types of formation time have similar behavior with $\nu$, but different behavior with $z$. At the values of $z$ typical for the HERMES kinematics ($z \geq 0.2$) the behavior of $\tau$ defined as in eqs.(12) and (14) with $z$ is similar, i.e. they are decreasing with the increase of $z$, while $\tau$ defined as in eq.(13) is increasing with the increase of $z$.

### IV. RESULTS

The two-dimensional data from [6], i.e. the multiplicity ratio $R^h_M$ for charged pions produced on helium, neon, krypton and xenon nuclei, in a $z$ slices as a function of $\nu$, and in a $\nu$ slices as a function of $z$ were used to perform the fit. The independent fit including 47 experimental points was performed for each nucleus. As it is clear from eq.(2), experimental points corresponding to $\nu$-dependence $R^h_M(\nu, z)$, and $z$-dependence $R^h_M(\nu, z)$ enter in fit on equal basis, as a values of function $R^h_M(\nu, z)$ at values of variables $(\nu, z)$ equal $(\nu, z)$ and $(\nu, z)$, respectively. For the fit $R^h_M$ has been taken in polynomial forms $R^h_M[P_{1,2,3}]$ [see eqs.(9)-(11)], and formation times (lengths) as in eqs.(12)-(14). The results for the reduced $\chi^2$ denoted as $\chi^2/d.o.f.$, are close for the polynomial approximations $R^h_M[P_1]$, $R^h_M[P_2]$, and $R^h_M[P_3]$, which means that the inclusion in consideration of the higher order polynomials of $\tau$ does not essentially improve the description of the data.

\[ \tau_{\text{Lund}} = \tau_0 - \frac{E_h}{m_h} = \tau_0 - \frac{z\nu}{m_h}, \]

(13)

where $E_h$ and $m_h$ are the energy and mass of the hadron $h$, respectively. In Ref. [5] we have discussed formation time $\tau_{\text{Lund}}$ in detail. In particular we considered the possibility of that $\tau_0$ being proportional to $m_h$. In the present paper we have to deal with hadrons of one type only - charged pions - and looking at interested kinematics ($z \geq 0.2$) the behavior of $\tau$ defined as in eq.(9) is not similar to $\tau_0$ being proportional to $m_h$. In Ref. [7] we have discussed formation time $\tau_{\text{Lund}}$ in detail. In particular we considered the possibility of that $\tau_0$ being proportional to $m_h$. In the present paper we have to deal with hadrons of one type only - charged pions - and looking at interested kinematics ($z \geq 0.2$) the behavior of $\tau$ defined as in eq.(9) is not similar to $\tau_0$ being proportional to $m_h$.
In order to test this, we have also calculated the $R_{M}^{h}[P_{3}]$ polynomial form and obtained the values of $\chi^{2}/d.o.f.$ close to the ones in case of $R_{M}^{h}[P_{3}]$. From Table I one can see that the fit gives unexpectedly good values for $\chi^{2}/d.o.f.$ in case of $\tau_{\text{lead}}$, and $\tau_{\text{Lund}}$, for $\tau_{\text{Lor}}$, the agreement is much worse. As it is known from experiment [3, 4, 6], $R_{M}^{h}(\nu, z)$ increases with increasing of $\nu$, and $R_{M}^{h}(\nu, z)$ decreases with increasing of $z$ for all nuclei. Our assumption is that these functions indeed present different representations of the same function, which depends on variable $\tau$ only. In turn $\tau$ is a function of $\nu$ and $z$. Now let us discuss the figure. Using eqs.(12)-(14), we present $R_{M}^{h}(\nu, z)$ and $R_{M}^{h}(\nu, z)$ as functions of $\tau$. Experimental points and results of the fit are presented in Fig. I. Solid points correspond to the $R_{M}^{h}(\nu, z)$ obtained from the experimental data for $\nu$-dependence, open points to the $R_{M}^{h}(\nu, z)$ from $z$-dependence. From the figure one can easily note that experimental points for $R_{M}^{h}(\nu, z)$ and $R_{M}^{h}(\nu, z)$ as functions of $\tau$ have the same behavior and approximately coincide when $\tau_{\text{lead}}$ and $\tau_{\text{Lund}}$ serve as the variables. The reason for this is that these variables are approximately proportional to $\nu$ and $1 - z$. In contrary, variable $\tau_{\text{Lor}}$, is proportional to $\nu$ and $z$, and as a consequence $R_{M}^{h}(\nu, z)$ and $R_{M}^{h}(\nu, z)$ have opposite behavior as functions of $\tau_{\text{Lor}}$. This means, that without any calculations one can state that $\tau_{\text{lead}}$ and $\tau_{\text{Lund}}$ can serve as a scaling variables, but $\tau_{\text{Lor}}$ cannot. For the sake of convenience we have renormalized $\tau$ to $x = \tau/\tau(\text{max})$, where $\tau(\text{max})$ are the maximum values of $\tau$ for each set of data and each choice of the $\tau$ expression.

The range of variation of $\tau$ and the numerical values for $\tau(\text{max})$ in all scenarios for instance on krypton are: $2.46 \leq \tau_{\text{lead}} \leq 14.6 fm$, $0.37 \leq \tau_{\text{lead}} \leq 15.5 fm$, $0.36 \leq \tau_{\text{Lund}} \leq 8.28 fm$. Presentation of $R_{M}^{h}$ as a function of $x$ allows us to place all data in an interval $(0, 1)$. This choice does not influence the results of the fit and the values of $R_{M}^{h}$. On the figure the linear polynomial is presented $a_{11} + x a_{12}$ with values $a_{11}$ and $a_{12}$ corresponding to the best fit. Solid, dashed and dotted curves represent the $R_{M}^{h}[P_{1}], R_{M}^{h}[P_{2}]$, and $R_{M}^{h}[P_{3}]$ polynomial fit, respectively. One can easily see that the difference between the curves corresponding to $R_{M}^{h}[P_{1}], R_{M}^{h}[P_{2}]$, and $R_{M}^{h}[P_{3}]$ is small. The vertical positions of the experimental points are the same in all scenarios. The experimental points can be closer together (or not) depending on the type of the formation time definition. When looking at the $\tau$ dependencies, the points corresponding to $\nu$-dependence preserve their order in all scenarios. In case of $z$-dependence, the order is the same in the scenario with $\tau_{\text{Lor}}$, but changes to opposite in other scenarios.

As a last remark, one should note that results of this analysis do not depend from the values of the parameters, in particular from the values of $\kappa$ and $\tau_{0}$.

V. CONCLUSIONS.

The two-dimensional nuclear attenuation data for charged pions on helium, neon, krypton and xenon nuclei obtained recently by the HERMES experiment [6] were used to perform the fit. So far it has been supposed that experimentally measured function $R_{M}^{h}(\nu, z)$ depends on variable $\nu$ only, and $R_{M}^{h}(\nu, z)$ from $z$ only. In our preceding work [3], we have assumed that these functions indeed present different representations of the same function, which depends on only one variable $\tau$. In turn $\tau$ is a function of $\nu$ and $z$. In this work based on two-dimensional distributions (more suitable for this investigation), the results of our preceding work [3] are confirmed. We demonstrate (see Table I and figure) that $R_{M}^{h}(\nu, z)$ and $R_{M}^{h}(\nu, z)$ as functions of $\tau$ have the same behavior and approximately coincide, when $\tau$ is used in form of $\tau_{\text{lead}}$ and $\tau_{\text{Lund}}$. In contrary, they have opposite behavior as a functions of $\tau_{\text{Lor}}$. This indicates that $\tau_{\text{lead}}$ and $\tau_{\text{Lund}}$ can serve as scaling variables, but $\tau_{\text{Lor}}$ cannot. We also show that $R_{M}^{h}(\tau)$, with a good precision, can be parametrized in a form of a linear polynomial $a_{11} + \tau a_{12}$, where the fitting parameters $a_{11}$ and $a_{12}$
TABLE I: The $\chi^2 / d.o.f.$ values obtained from polynomial fit. $P_{1,2,3}$ denote the expressions $R^k_M[P_{1,2,3}]$ used as fitting functions. The necessary details concerning the data sets used for the fit are given in the text.

|       | $\tau_{\text{lead.}}$ | $\tau_{\text{Lor.}}$ | $\tau_{\text{Lund}}$ |
|-------|------------------------|-----------------------|----------------------|
|       | A | $N_{\text{exp}}$ | $\text{Had.}$ | $P_1$ | $P_2$ | $P_3$ | $P_1$ | $P_2$ | $P_3$ | $P_1$ | $P_2$ | $P_3$ |
| $^4\text{He}$ | 47 & $< \pi >$ & 0.43 | 0.42 | 0.39 | 0.37 | 0.37 | 0.38 | 0.42 | 0.39 | 0.36 |
| $^{20}\text{Ne}$ | 47 & $< \pi >$ & 0.62 | 0.55 | 0.50 | 0.98 | 0.98 | 0.96 | 0.45 | 0.38 | 0.35 |
| $^{84}\text{Kr}$ | 47 & $< \pi >$ & 1.27 | 0.83 | 0.76 | 7.18 | 7.31 | 7.47 | 0.73 | 0.57 | 0.50 |
| $^{131}\text{Xe}$ | 47 & $< \pi >$ & 0.83 | 0.55 | 0.50 | 7.68 | 7.85 | 7.98 | 0.54 | 0.45 | 0.42 |

do not depend on $\nu$ and $z$. We conclude that experimentally measured function $R^k_M$ is a function of single variable $\tau$, and that $\tau$ scaling follows naturally from the absorption model, but we do not confirm that scaling is a property of an absorption model only. There could exist other mechanisms which can also lead to the scaling behavior - this is an open question, which requires further investigation.

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