Power law cosmology - a viable alternative

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April 23, 2008

Abstract

A power law cosmology is defined by the cosmological scale factor evolving as $t^\alpha$. In this work, we put bounds on $\alpha$ by using the joint test of the SNe Ia data from Supernova Legacy Survey (SNLS) and $H(z)$ data with curvature constant $k = 0, \pm 1$. We observe that the combined analysis with SNLS and $H(z)$ data favours the open power law cosmology with $\alpha = 1.31^{+0.06}_{-0.05}$. It is also interesting to note that an Einstein - de Sitter model ($\alpha = 2/3$) is ruled out at 2$\sigma$ level.

1 Introduction

Our universe is very well explained by the Standard Cosmological Model (SCM) based on the Hot Big-Bang theory and the inflationary scenario. However, there are still some features of the universe which cannot be understood within the SCM. One of the major unsolved problem is the cosmological constant problem. The standard model fails to explain why the energy density of the vacuum is 120 orders of magnitude smaller than its value at the Planck time [1]. The problems in the SCM and the availability of precise data from various observations have encouraged cosmologists to explain the observed universe through alternative cosmological models.

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One of the interesting alternatives is a **Power Law Cosmology**. In such a model there is a power law evolution of the cosmological scale factor, $a(t) \propto t^\alpha$. The power law evolution with $\alpha \geq 1$ has been discussed at length in a series of earlier articles [2, 3, 4]. The motivation for such a scenario comes from the fact that it does not encounter flatness and the horizon problem at all. Another interesting feature of these models is that they easily accommodate high redshift objects and hence alleviate the age problem. These models are also purged of the fine tuning problem [5, 6]. Such a scaling is a generic feature in a class of models that attempt to dynamically solve the cosmological constant problem [3, 4, 5, 6, 7].

A power law evolution of the cosmological scale factor with $\alpha \approx 1$ is surprisingly an excellent fit to a host of cosmological observations. Any model that can support such a coasting presents itself as a falsifiable model as far as classical cosmological tests are concerned as it exhibits distinguishable and verifiable features. An evolution of this nature is supported by classical cosmological tests such as the galaxy number counts as a function of redshift and the data on angular diameter distance as a function of redshift [2]. However, as these tests are marred by evolutionary effects (e.g. mergers), they have fallen into disfavour as reliable tests of a viable model. With the discovery of Supernovae type Ia, SNe Ia, as reliable standard candles, the status of the Hubble test has been elevated to that of a precision measurement. The Hubble plot relates the magnitude of a standard candle to its redshift in an expanding FRW universe. We demonstrated that linear coasting cosmology accommodates the high redshift objects while the standard model could not [8, 9]. Such a model is also comfortably consistent with the gravitational lensing statistics [8] and the primordial nucleosynthesis [10].

The plan of the paper is as follows. In Section 2, we give the basic equations for open, closed and flat power law scenarios. In Section 3, we find the constraints on the cosmological parameter $\alpha$ from a joint test of the Supernova Legacy Survey SNe Ia data set (SNLS) and the H(z) data. The joint test is performed for open, closed and flat power law cosmologies. The results are summarized in Section 4.

## 2 Power Law Cosmology

For a FRW metric, the line element is

$$ds^2 = c^2 dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta \, d\phi^2) \right]. \quad (1)$$

Here $k = \pm 1, 0$, is the curvature constant, $t$ is the cosmic proper time and $a(t)$ is the cosmological scale factor.

The expansion rate of the universe is described by a Hubble parameter, $H(t) = \dot{a}/a$. The present expansion rate of the universe is defined by the *Hubble constant* $H_0$. Here and subsequently the subscript $0$ on a parameter refers to its present value.
In this paper, we study a general power law cosmology with the scale factor given in terms of a dimensionless parameter $\alpha$

$$a(t) = \frac{c}{H_0} \left( \frac{t}{t_0} \right)^{\alpha}.$$  \hspace{1cm} (2)

In this model, $H(t) = \alpha/t$ and $H_0 = \alpha/t_0$. The scale factor and the redshift at time $t$ are related to their present values by

$$\frac{a_0}{a(z)} = \frac{t_0}{t} = 1 + z.$$

The present ‘radius’ of the universe is defined as

$$a_0 = \frac{c}{H_0}.$$ \hspace{1cm} (4)

The age of the universe at redshift $z$ is given as

$$t(z) = \frac{\alpha}{H_0 (1 + z)^{1/\alpha}}.$$ \hspace{1cm} (5)

The dimensionless Hubble parameter is defined as:

$$E(z) \equiv \frac{H(z)}{H_0} = (1 + z)^{1/\alpha}.$$ \hspace{1cm} (6)

For the power law cosmology, the luminosity distance between two redshifts $z_1$ and $z_2$ is

$$d_L(z_1, z_2) = \frac{c(1 + z_2)}{H_0} S \left( \frac{\alpha}{\alpha - 1} \left\{ (1 + z_2)^{\frac{\alpha - 1}{\alpha}} - (1 + z_1)^{\frac{\alpha - 1}{\alpha}} \right\} \right).$$ \hspace{1cm} (7)

In the limiting case, $\alpha \to 1$, we obtain

$$d_L(z_1, z_2) = \frac{c(1 + z_2)}{H_0} S \left[ \ln(1 + z_2) - \ln(1 + z_1) \right].$$ \hspace{1cm} (8)

Here

$$S(x) = \sinh(x) \text{ for } k = -1$$
$$= \sin(x) \text{ for } k = +1$$
$$= x \text{ for } k = 0$$ \hspace{1cm} (9)
3 Observational Tests

3.1 Constraints from the Supernova Legacy Survey SNe Ia data set (SNLS)

Type Ia supernova (SNe Ia) are excellent cosmological standard candles for estimating the apparent magnitude $m(z)$ at peak brightness after accounting for various corrections. In this work we use the SNLS data set of 115 SNe Ia data points with redshift $z < 1$ \[12\].

For a standard candle of absolute magnitude $M$, the apparent magnitude $m(z)$ can be expressed as:

$$ m(z) = M + 5 \log_{10} D_L(z) . $$

(10)

Here $D_L(z)$ is related to the luminosity distance:

$$ D_L(z) = \frac{H_0}{c} d_L(0, z) $$

$$ = (1 + z) \left[ \frac{\alpha}{\alpha - 1} \left\{ (1 + z) \frac{\alpha - 1}{\alpha} - 1 \right\} \right] $$

(11)

and

$$ M = M - 5 \log_{10} h + 42.38 , $$

(12)

is the “zero point” magnitude. We use $H_0 = h 100 \text{Kms}^{-1} \text{Mpc}^{-1}$.

The distance modulus, $\mu(z)$, is defined as

$$ \mu(z) = m(z) - M = 5 \log_{10} D_L(z) - 5 \log_{10} h + 42.38 . $$

(13)

The $\chi$-square function is defined as

$$ \chi^2(h, \alpha) = \sum_{i=1}^{115} \left[ \frac{\mu_{\text{exp}}^i(h, \alpha, z_i) - \mu_{\text{obs}}^i(z_i)}{\sigma_i} \right]^2 , $$

(14)

where $\mu_{\text{exp}}$ is the expected distance modulus for a supernova at a given redshift $z$ and $\sigma_i$ is the error due to intrinsic dispersion of SNe Ia absolute magnitude and observational uncertainties in SNe Ia peak luminosity. These errors are assumed to be Gaussian and uncorrelated. The observed distance modulus, $\mu_{\text{obs}}$, is given by the supernovae data set.
3.2 Constraints from H(z) data

Simon, Verde and Jimenez (2005) used differential ages of passively evolving galaxies to determine the Hubble parameter as a function of redshift, \( H(z) \) \[13\]. They use a sample of absolute ages of 32 galaxies taken from the Gemini Deep Deep Survey (GDDS) and the archival data to obtain 9 data points of \( H(z) \) with \( 0.09 \leq z \leq 1.75 \). The Hubble parameter and the differential age of the universe, \( \frac{dz}{dt} \), are linked by the equation:

\[
H(z) = -\frac{1}{1+z} \frac{dz}{dt}. \tag{15}
\]

The details of the method for calculating the \( \frac{dz}{dt} \) from the absolute age is given by Simon, Verde and Jimenez (2005) \[13\].

The \( H(z) \) for power law cosmology is given by:

\[
E(z) \equiv \frac{H(z)}{H_0} = (1 + z)^{1/\alpha}. \]

In order to put bounds on the model parameter, \( \alpha \), we define the quantity:

\[
\chi^2(h, \alpha) = \sum_{i=1}^{9} \left( \frac{H_{exp}(z_i, \alpha, h) - H_{obs}(z_i)}{\sigma_i} \right)^2 \tag{16}
\]

Where \( H_{exp} \) is the expected value of the Hubble constant in the power law cosmology, \( H_{obs} \) is the observed value and \( \sigma_i \) is the corresponding 1\( \sigma \) uncertainty in the measurement.

3.3 Joint Test: SNe Ia + H(z)

We find the constraints on the cosmological parameter \( \alpha \) from the joint test of SNe Ia and H(z) data sets. In this joint test we define the quantity

\[
\chi^2_{\text{joint}} = \chi^2_{\text{SNe}} + \chi^2_{\text{H(z)}}, \tag{17}
\]

where \( \chi^2_{\text{SNe}} \) is given by Eq.(14) and \( \chi^2_{\text{H(z)}} \) by Eq.(16).

Considering \( h \) to be a nuisance parameter, we marginalize over \( h \) to obtain the probability distribution function defined as:

\[
L(\alpha) = \int e^{-\chi^2_{\text{joint}}(h, \alpha)/2} P(h) \, dh.
\]

Here \( P(h) \) is the prior probability function for \( h \) which we assume to be Gaussian:

\[
P(h) = \frac{1}{\sqrt{2\pi}\sigma_h} \exp\left[ -\frac{1}{2} \frac{(h - h_{\text{obs}})^2}{\sigma_h^2} \right],
\]
where \( h_{\text{obs}} \) is the value of \( h \) (and \( \sigma_h \) is the error in it) as suggested by independent observations. In this paper, we also study the effect of different priors on the result. We use two set of priors:

1. **Set A:** \( h_{\text{obs}} = 0.68 \pm 0.04 \) as obtained from the median statistics analysis of 461 measurements of \( H_0 \) [14].
2. **Set B:** \( h_{\text{obs}} = 0.77 \pm 0.04 \) as suggested by the Chandra X-ray Observatory results [15].

The best fit model parameter is obtained by minimizing the modified \( \chi^2 \) (obtained after marginalization over \( h \)):

\[
\chi^2 = -2 \ln L(\alpha) \tag{18}
\]

We performed the joint analysis for open, closed and flat power law cosmologies. The best fit value of \( \alpha \) and the constraints on it seem to be independent of the choice of prior in all the three models. The result obtained with both the priors are summarised in Table 1. The joint analysis also shows that the best fit scenario is an open model with \( \alpha_{\text{min}} = 1.31^{+0.06}_{-0.05} \).

### 4 Summary

In this paper, we study observational constraints on the power law cosmology, \( a(t) \propto t^\alpha \). This model of the universe has very interesting features which makes it unique when compared to the other models of the universe. Firstly, for \( \alpha \geq 1 \) it does not encounter the horizon,
flatness and age problem [2, 3, 4]. Secondly, such an evolution is a characteristic feature of models that dynamically solve the cosmological constant problem. Statistically this model may be preferred over other models as we have to fit only one parameter, $\alpha$.

In the work presented here, we use the joint test, which uses the SNLS data and H(z) data, to put constraints on the parameter $\alpha$. To begin with, we work with all the three models - closed, flat and open and put constraints on $\alpha$ in these three cosmologies. The results on the cosmological test are summarized in Table 1. Fig. 1-3 show variation of $\chi^2$ with $\alpha$ for the three models (with set B prior). We also mark the parametric space allowed at 90% CL in the figures. We make the following observations:

- For the three models under consideration, the value of $\chi^2_{\nu}$, the best fit value of $\alpha$ and the constraints on it seem to be independent of the choice of prior.
- The joint test favours an open power law cosmology with $\alpha_{\text{min}} = 1.31^{+0.06}_{-0.05}$. As can be seen in Table 1, $\chi^2_{\nu}$ is minimum for an open power law cosmology.
- The joint analysis does not rule out flat and closed power law cosmologies. However, we do observe that the constraints on $\alpha$ are tighter for an open model.
- The joint analysis rules out linear coasting ($\alpha = 1$) in all the three cosmologies even at 90% CL. (see Fig.1, Fig.2 and Fig.3).

These observations match the conclusions of Zhu et al. (2007) [16]. They test the power-law cosmology against the recent measurements of the X-ray gas mass fractions in clusters of galaxies. They conclude that the best fit is an open model (with $\alpha_{\text{min}} = 1.14 \pm 0.05$) though the flat and closed models can not be ruled out.

In the past, various observational tests have been used to put constraints on the parameter $\alpha$ in an open power law cosmology, such as gravitational lensing, Old High Redshift Galaxies (OHRG), SNe Ia and X-ray gas mass fractions in galaxy clusters. Constraints obtained from the other tests along with the constraints obtained from the SNe Ia data and H(z) data are summarized in Table 2. The interest in an open power law cosmology is on account of the fact that a whole class of dilaton gravity models that dynamically solve the cosmological constant rely on a vanishingly small effective gravitational constant in the early universe [7, 17]. This gives an open FRW model for any reasonable equation of state of matter.

We observe that the joint analysis of SNLS and H(z) data favours an open power law cosmology with $\alpha > 1$. We further observe that the joint analysis (SNLS + H(z)) done in this letter rules out the Einstein-de Sitter universe ($\alpha = 2/3$).

Since the joint test of SNLS and H(z) data presented in this work favours an open power law cosmology, for the sake of completeness we find bounds on $\alpha$ in open model separately using the SNLS data and the H(z) data. We once again marginalize over $h$ to find $\chi^2_{\nu}$ and the best fit values using each test. We find that:

1. **Constraints from SNLS data:** For set A prior, we get the best fit value $\alpha = 1.421^{+0.08}_{-0.07}$ with $\chi^2_{\nu} = 1.07$. With set B, we get the same constraints on the parameter $\alpha$ as obtained
from set A but with $\chi^2_\nu = 1.09$. We find that the constraints on $\alpha$ do not depend upon the choice of the prior. We, therefore, conclude that the SNLS data favours $\alpha > 1$ (best fit value being $\alpha = 1.42^{+0.08}_{-0.07}$). This observational data rules out linear coasting cosmology ($\alpha = 1$) even at at $2\sigma$ level.

2. Constraints from H(z) data: We find that the H(z) data provides tight constraints on the model parameter $\alpha$. With set A prior we obtain best fit value $\alpha = 1.02^{+0.09}_{-0.06}$ with $\chi^2_\nu = 0.834$. With set B prior we get the best fit value as $\alpha = 1.07^{+0.11}_{-0.06}$ with $\chi^2_\nu = 1.06$. We observe that for this test the constraints on $\alpha$ weakly depend upon the choice of priors. The H(z) data, however, strongly favours linear coasting cosmology with the best fit value.

We summarize: An open power law cosmology with $\alpha > 1$ is in excellent agreement with the present day observations. This makes it an attractive alternative. In fact, the possibility of an open linear coasting model as a viable model cannot be ruled out (as suggested by the H(z) data). Concordance of the power-law cosmology with CMB anisotropy measurement is a major area to be explored. There are large numbers of surveys that are ongoing or have been proposed. With the flood of new data (and the possibility that the observational techniques will be improved), the task ahead is to find models of the universe that can explain these observations. It will be interesting to investigate how the future observations will change the constraints on $\alpha$.

Acknowledgments

A. Dev & D. Jain thank Amitabha Mukherjee and Shobhit Mahajan for providing the facilities to carry out the research work.

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| Method                        | Reference   | \( \alpha \)     |
|-------------------------------|-------------|-------------------|
| Lensing Statistics            |             |                   |
| (i) \( n_L \)                 | Dev et al. [8] | 1.09 ± 0.3       |
| (ii) Likelihood Analysis      | Dev et al. [8] | 1.13^{+0.4}_{-0.3} |
| OHRG                          | Dev et al. [8] | ≥ 0.8             |
| SNe Ia (Gold sample)          | Sethi et. al [9] | 1.001       |
| Old Quasar                    | Sethi et. al [9] | ≥ 0.85       |
| Galaxy Clusters               | Zhu et. al. [16] | 1.14 ± 0.05       |
| SNe Ia (SNLS)                 | This Letter | 1.42^{+0.08}_{-0.07} |
| H(z) data                     | This Letter | 1.07^{+0.11}_{-0.09} |
| SNe Ia + H(z) data            | This Letter | 1.31^{+0.06}_{-0.05} |

Table 2: Constraints on \( \alpha \) from various cosmological tests in an open power law cosmology model.
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Figure 1: Results with Gaussian prior $h_{obs} = 0.77 \pm 0.04$ in a closed power law cosmology. The vertical lines at $\alpha = 1.98$ and at $\alpha = 2.69$ mark the parametric space allowed at 90% CL. The minimum of $\chi^2$ occurs at $\alpha = 2.28$. 
Figure 2: Results with Gaussian prior $h_{\text{obs}} = 0.77 \pm 0.04$ in a flat model. The vertical lines at $\alpha = 1.48$ and at $\alpha = 1.79$ mark the parametric space allowed at 90% CL. The best fit value occurs at $\alpha = 1.62$. 
Figure 3: Results with Gaussian prior $h_{\text{obs}} = 0.77 \pm 0.04$ in an open model. The vertical lines at $\alpha = 1.23$ and at $\alpha = 1.41$ mark the parametric space allowed at 90% CL. The minimum occurs at $\alpha = 1.31$. 