QCD RELATIONS BETWEEN STRUCTURE FUNCTIONS AT SMALL X

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We present very simple non-integral relations between deep inelastic structure functions $F_L$, $F_2$ and the gluon distribution at small $x$ based on perturbative QCD which are useful for the phenomenological analysis of data at low $x$. As an application we extract the deep inelastic scattering cross-sections ratio $R = \sigma_L/\sigma_T$ in the range $10^{-4} \leq x \leq 10^{-2}$ from $F_2$ HERA data.

1 Introduction

For experimental studies of high energy hadron-hadron and lepton-hadron processes it is necessary to know in detail the values of the parton (quark and gluon) distributions (PD) of nucleons, especially at small values of $x$. Of great relevance is the determination of the gluon density at low $x$, where gluons are expected to be dominant.

The basic information on the gluon structure of nucleons is extracted from the measurement of the deep inelastic structure function $F_2$ in lepton-hadron scattering (DIS). One of the usual procedures compares experimental data with the theoretical prediction for $F_2$ obtained from the solution of a system of complicated coupled integro-differential quark and gluon evolution equations. It is also possible to extract the gluon distribution more directly from $F_2$ scaling violations using a very simple relation with the $Q^2$ derivative of $F_2$.\footnote{At present there are only preliminary measurements by H1.}

By other part, $F_L$ or the ratio $R = F_L/(F_2 - F_L)$, is also an interesting quantity, because it is a very sensitive QCD characteristic. For example, future $F_L$ measurements\footnote{At present there are only preliminary measurements by H1.} will be used as a signal of the gluon structure at low $x$.

In perturbative QCD, there is the possibility to connect $F_L$ with $F_2$ due to the fact that at small $x$ the DIS structure functions depend really on only two \emph{independent} functions, the gluon and the singlet quark distribution (the non-
singlet quark density is negligible at small $x$), which in turn can be expressed in terms of $F_2$ and its derivative $dF_2/dlnQ^2$.

In this article we present very simple linear relations between the gluon density and $F_L(x, Q^2)$ with $F_2(x, Q^2)$ and $dF_2(x, Q^2)/dlnQ^2$ at small $x$. Using these formulas we exploit the possibility of extracting information about the gluon distribution and $F_L$ at small $x$, directly from the measurement of the $F_2$ scaling violations. This method complement the standard analysis where quarks and gluons, determined from complex fits to data, are integrated for the calculation of $F_L$. With our formulas it is possible to take into account the experimental uncertainty in the theoretical calculation more directly.

The standard initial form for the singlet quark $s(x, Q^2_0)$ and gluon $g(x, Q^2_0)$ distributions at some $Q^2_0$ are parameterized by:

$$p(x, Q^2_0) = A_p x^{-\delta_p}(1 - x)^{\nu_p} (1 + \epsilon_p \sqrt{x} + \gamma_p x) \quad (p = S, g) \quad (1)$$

Until the recent time the value of $\delta_p$ was a matter of discussion, but the new HERA data start to overcome this controversy. From the theoretical side, the type of evolution of the PD in Eq. 1 depends on the value and form of $\delta_p$. For example, a $Q^2$-independent $\delta_S = \delta_g$ obey the DGLAP equation when $x^{-\delta_p} \gg 1$. However, if $\delta_p(Q^2_0) = 0$ in some point $Q^2_0 \geq 1 GeV^2$, the behaviour $p(x, Q^2) \sim Const$ is not compatible with DGLAP evolution and a more singular behaviour is generated. These cases have been recently evolved to a common picture where partons are really a combination of two solutions (at $x^{-\delta_p} \gg 1$ and at $\delta_p \sim 0$) linked at some $Q^2$ point. For a Regge-like form of structure functions, one obtains $p(x, Q^2) \sim x^{-\delta_p}(Q^2)$ with next-to-leading order (NLO) intercept trajectories. Without any restrictions, it generates the double-logarithmical behaviour,

$$p(x, Q^2) \sim \exp \left(2 \sqrt{\delta_p(Q^2) \ln \frac{1}{x}} \right) \quad (2)$$

where at NLO and for $f = 4$ active quarks one has:

$$\delta_g(Q^2) = \frac{36}{25} t - \frac{91096}{5625} r, \quad \delta_S(Q^2) = \delta_g(Q^2) - 20r$$

being $t = \ln(\alpha(Q^2_0)/\alpha(Q^2))$ and $r = \alpha(Q^2_0) - \alpha(Q^2)$.

\[b\] Some of them have been already published.

\[c\] We use PD multiplied by $x$ and neglect the nonsinglet quark distribution at small $x$.

\[d\] Hereafter we use $\alpha(Q^2) = \alpha_s(Q^2)/4\pi$. Sometimes it is denoted as $\alpha$ to shorten long formulae.
2 The gluon and $F_L$ as a function of $F_2$ and $dF_2/d\ln Q^2$

By lack of space we will only present the final formulas of the calculation while the details will be given elsewhere. Assuming the Regge-like behaviour of Eq. (1) for $x^{-\delta_p} \gg 1$, one can replace at small $x$ the convolution integrals by ordinary products in the $Q^2$ evolution equations and in the formulas relating $F_2$ and $F_L$ to PD. From the $F_2$ equation one can extract the singlet quark combination $S(x, Q^2)$ as a function of $F_2(x, Q^2)$ and substitute it into the equations for $F_L$ and for $dF_2/d\ln Q^2$.

The case of the non-Regge type behaviour has to be treated independently, but it is possible to combine both cases (Regge and non-Regge types) in a single formula valid for any value of $\delta_p$:

$$g(x, Q^2) = -\frac{2f}{ae^2} \int \frac{dy}{y} \frac{g(y, Q^2)}{g(x, Q^2)} \left[ \frac{dF_2(x, Q^2)}{d\ln Q^2} + \frac{\alpha_s}{\alpha} \right],$$

$$F_L(x, Q^2) = \alpha \frac{B_L^{g,1+\delta_g}}{\gamma_S^{g}(1, 1+\delta_g)} \left[ \frac{dF_2(x, Q^2)}{d\ln Q^2} + \frac{\alpha_s}{\alpha} \right],$$

where $e = \sum_i e_i^2$ is the sum of squares of $f$ quark charges. The variables $B_k^{\eta,\eta}$ ($k = 2, L$) and $\gamma_k^{(p, l)}$ ($p, l = g, S$) are respectively the one loop parts of the Wilson coefficients and anomalous dimensions of the operators. The second order quantities are related to the two loop Wilson coefficients and anomalous dimensions by:

$$\overline{\Gamma}_L^{g} = R_L^{g,1+\delta_g} - B_2^{g,1+\delta_g} B_L^{S,1+\delta_s} / B_L^{g,1+\delta_g},$$

$$\overline{\Gamma}_S^{g} = \gamma_S^{g}(1, 1+\delta_g) + B_2^{g,1+\delta_g} \gamma_S^{g}(0, 1+\delta_g) + B_2^{g,1+\delta_g} \left( 2\beta_0 + \gamma_S^{g}(0, 1+\delta_g) \right) - \gamma_S^{g}(0, 1+\delta_g).$$

All these coefficients have to be continued analytically from the well known integer values $n$ to the non-integer ones $\eta$.

The values of $\tilde{\gamma}_S^{(1)}$ and $\tilde{R}_L^{g}$ coincide with $\gamma_S^{(1)}$ and $\Gamma_L^{g}$, respectively, with the replacement:

$$\frac{1}{\delta_g} \to \int_x^1 \frac{dy}{y} \frac{g(y, Q^2)}{g(x, Q^2)}$$

(5)
In the cases $x^{-\delta_S} \gg \text{Const}$ and $\delta_S \to 0$, the r.h.s. of Eq. 5 leads to $1/\delta_g$ and $1/\delta_p$, respectively, where

$$
\frac{1}{\delta_p} = \sqrt{\frac{\ln(1/x)}{\delta_p(Q^2)}} - \frac{1}{4\delta_p(Q^2)} \left[ 1 + \frac{1}{8\sqrt{4\delta_p(Q^2)\ln(1/x)}} + O \left( \frac{1}{4\delta_p \ln(1/x)} \right) \right] \tag{6}
$$

The comparison of our results with others is performed elsewhere.

3 The extraction of the ratio $R$ at low $x$

As an application we have extracted the ratio $R(x, Q^2)$ from H1 1994 data, determining the slopes $dF_2/d\ln Q^2$ from straight line fits. The systematic errors have been taken from an early analysis of H1. The running coupling constant $\alpha_s(Q^2)$ has been calculated at two loops using $\Lambda^{(4)}(\overline{\text{MS}}) = 225 MeV$.

Figure 1 shows the extracted ratio $R$ at $Q^2 = 20 \text{ GeV}^2$ for $\delta_S = 0.3$ and two different exponent for the gluon, $\delta_g = \delta_S$ and $\delta_g = \delta_S + 0.05$. These values are very close to those obtained by various groups from QCD fits to H1 data. Fig. 1 also shows some experimental data points at high $x$. For comparison...
we have also plotted various predictions for $R$ using QCD formulae at $O(\alpha_s^2)$ and parton densities extracted from fits to HERA data. The large difference between the result from MRS(G) and the latest set MRS(R1) shows the sensitivity of $R$ to the update of these parton densities to new HERA data. One can notice that there is good agreement between MRS(R1) prediction and our points when $\delta_g = \delta_S + 0.05 = 0.35$, even though it is not surprising because these values were taken from the MRS analysis. However our result gives extra information about the uncertainty. A more precise future measurements at low $x$ should lie within the error bars of the results presented in Fig. 1.

By other part recent theoretical predictions on $R$ based on conventional NLO DGLAP evolution analysis of HERA data (LBY) and on the dipole picture of BFKL dynamics (NPRW), both finding values $\delta_S \equiv \delta_g \approx 0.3$, lie between the two of our above cases.

Acknowledgments

This work was partially supported by CICYT (AEN96-1773), by Xunta de Galicia (XUGA-20604A96), and by RFFR (95-02-04314-a).

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