Rare kaon decay $K^+ \rightarrow \pi^- \mu^+ \mu^+$ as the key event for the right-handed weak interaction effects

Yoshio Koide

$^a$ Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan
E-mail address: koide@epp.phys.sci.osaka-u.ac.jp

We discuss on the search for the right-handed weak interaction effects in the SU(2)$_L \times$SU(2)$_R$ model with lepton doublets $(\nu_\ell, \ell^-)_L$ and $(N_\ell, \ell^-)_R$ ($\ell = e, \mu, \tau$). We will point out that only the chance of the observation of the right-handed weak interaction effect will be in the rare decay $K^+ \rightarrow \pi^- \mu^+ \mu^+$.

1. Introduction

We are interested in the right-handed weak interaction effect in the SU(2)$_L \times$SU(2)$_R \times U(1)_{B-L}$ model [1]. Let us denote the lepton doublets as

$$
\begin{pmatrix}
\nu_\ell \\
\ell^-
\end{pmatrix}_L,
\begin{pmatrix}
N_\ell \\
\ell^-
\end{pmatrix}_R,
(\ell = e, \mu, \tau). \tag{1.1}
$$

Here, the neutral leptons $\nu_\ell$ and $N_\ell$ have Majorana masses separately.

Of course, the right-handed lepton doublets $(N_\ell, \ell^-)_L$ interact with the right-handed weak boson $W_R$. At present, it is known [2] that the mass of $W_R$ is experimentally $M(W_R) > 3 \text{ TeV}$. \tag{1.2}

Therefore, events induced by the right-handed weak interaction are highly suppressed by the factor

$$
\left( \frac{M(W_L)}{M(W_R)} \right)^4 < \left( \frac{0.1 \text{ TeV}}{4 \text{ TeV}} \right)^4 \approx 10^{-6}. \tag{1.3}
$$

Here and hereafter, for simplicity, we take $g_L = g_R$ for the gauge coupling constants with the weak bosons $W_L$ and $W_R$.

For example, we know that the $D_s$ meson (mass is 1.9684 GeV) decays into muon and neutrino with the branching ratio $Br(D_s \rightarrow \mu + \nu_\mu) = 5.50 \pm 0.23 \%$ [3]. If there is the right-handed weak boson $W_R$, the leptonically decay is also possible by mediating $W_R$.

In general, the momenta $p_1$ and $p_2$ in the final state at the rest frame of parent particle are given by

$$
p_1 = p_2 = \frac{1}{M} \left[ M^4 - 2M^2(m_1^2 + m_2^2) + m_1^4 + m_2^4 - 2m_1^2m_2^2 \right]^{1/2}. \tag{1.4}
$$
Here, $M$ is a mass of the ps-meson in the initial state, and $m_1$ and $m_2$ are lepton masses of the final state. Therefore, the momenta of the final state muon are quite different from each other:

**Event via $W_L$**

$$\left(p_{\mu}\right)_L \simeq \frac{1}{2M} \left( M^4 - 2M^2m_\mu^2 + m_\mu^4 \right)^{1/2} = \frac{M}{2} \left( 1 - \frac{m_\mu^2}{M^2} \right),$$  \hspace{1cm} (1.5)

**Event via $W_R$**

$$\left(p_{\mu}\right)_R \simeq \frac{1}{2M} \left( M^4 - 2M^2m_N^2 + m_N^4 \right)^{1/2} = \frac{M}{2} \left( 1 - \frac{m_N^2}{M^2} \right),$$  \hspace{1cm} (1.6)

where $M$ is the mass of $Ds$ meson. Thus, by measuring the momentum of the muon, i.e. (1.5) or (1.6), we can know that the case is due to $W_L$ or $W_R$.

However, it is noted that the events (1.6) are highly suppressed by the factor (1.3), so that we cannot observe the events substantially.

In this paper, we would like point out that only the visible effect induced by $W_R$ is a rare decay $K^+ \rightarrow \pi^- \mu^+\mu^+$.  

2. Rare decay $K^+ \rightarrow \pi^- \mu^+\mu^+$

The rare decay $K^+ \rightarrow \pi^- \mu^+\mu^+$ is only possible event which we can observe under the right-handed weak interaction. In this section, we will discuss the detail.

For convenience, let us give only the leptonic part of the matrix element. As we show in Appendix A, the matrix element of the lepton part due to the exchange of $W_L$ for $K^+ \rightarrow \pi^- \mu^+\mu^+$ is given by

$$\mathcal{M}_L = m_\nu \overline{\nu}(k_1) i C^{-1} \frac{1 - \gamma_5}{2} \not{q} \not{v}(k_2)$$  \hspace{1cm} (2.1)

and that due to the exchange of $W_R$ is given by

$$\mathcal{M}_R = M_N \overline{\nu}(k_1) i C^{-1} \frac{1 + \gamma_5}{2} \not{q} \not{v}(k_2).$$  \hspace{1cm} (2.2)

Then, for the case $M_N^2 \ll k_2$, we get

[Case A]

$$R_{R/L}^A \equiv \frac{\mathcal{M}_R}{\mathcal{M}_L} \sim \frac{M_N/k_2}{m_\nu/k_2} = \frac{M_N}{m_\nu}.$$  \hspace{1cm} (2.3)
On the other case, for the case $M_N^2 \gg k^2$, we get

[Case B]

$$R_{R/L}^B = \frac{M_R}{M_L} \sim \frac{1/M_N}{m_\nu/k^2} = \frac{k^2}{m_\nu M_N}. \quad (2.4)$$

Note that the case $M_N^2 \gg k^2$ does not always give a large enhancement of $R_{R/L}$ as we see in Eq.(2.4).

Let us see the numerical value of the ratio $R_{R/L}$. For example, we take $m_\nu \sim 10$ meV $\sim 10^{-11}$ GeV, and $k^2 \sim 0.2$ GeV$^2$.

Since we want as possible as large value of $R_{R/L}$, for the case A, we take $M_N \sim 10^{-1}$ GeV.

Then, we obtain

[Case A]

$$R_{R/L}^A \sim \frac{M_N}{m_\nu} \sim \frac{10^{-1}}{10^{-11}} \sim 10^{10}. \quad (2.5)$$

On the other hand, for the case B, for the $M_N$ mass $M_N \sim 10$ GeV, we obtain

[Case B]

$$R_{R/L}^B \sim \frac{k^2}{m_\nu M_N} \sim \frac{10 \text{ GeV}^2}{10^{-11} \text{ GeV} \times 10 \text{ GeV}} \sim 10^{11}. \quad (2.6)$$

Of course, since the ratio $R_{R/L}$ is the ratio for the lepton part, so that we must multiply the weak boson mass ratio $(M_{W_L}/M_{W_R})^4 \sim 10^{-6}$.

$$\left(\frac{M(W_L)}{M(W_R)}\right)^4 R_{R/L}^A \sim 10^4, \quad \left(\frac{M(W_L)}{M(W_R)}\right)^4 R_{R/L}^B \sim 10^5. \quad (2.7)$$

(Note that this diagram has two weak boson exchanges. But the ratios (2.3) and (2.4) are ones for the amplitude.) Therefore, our result (2.7) shows that the right-handed weak interaction effect is enhanced in the rare decay $K^- \to \pi^- \mu^+\mu^+$. We can confirm by seeing whether the muons in the final state are $\mu_R$ not $\mu_L$.

4. Conclusion

In conclusion, only the chance of the search for the right-handed weak interaction will be in the rare decay $K^+ \to \pi^- \mu^+\mu^+$. If we observe the decay $K^+ \to \pi^- \mu^+\mu^+$ in a near future, it will be caused by the exchange of the right-handed weak boson. This will be confirmed by the helicity of muons in the final state, i.e. $\langle \mu^+ \rangle_R$.

Acknowledgements
The author is grateful to Prof. M. Tanaka for his enjoyable discussion and his helpful comments. And, also, the author is grateful to Prof. T. Yamashita for his helpful assistance in preparing this manuscript. And he is also grateful to Prof. A. Sato for his valuable advice on the experimental behavior of muon. This work is supported by JPS KAKENHI Grant number JP1903826.

**Appendix A – Decay $K^+ \rightarrow \pi^- \mu^+ \mu^+$**

There are two diagrams for the $K^+ \rightarrow \pi^- \mu^+ \mu^+$ decay as shown in Fig.1 and Fig.2.

![Box-type diagram](image1)

**Fig.1** Box-type diagram for $K^+ (q) \rightarrow \pi^- (p) + \mu^+ (k_1) + \mu^+ (k_2)$ decay

![Tree-type diagram](image2)

**Fig.2** Tree-type diagram for $K^+ (q) \rightarrow \pi^- (p) + \mu^+ (k_1) + \mu^+ (k_2)$ decay

However, our interest is only in the lepton part whose matrix element is given by the following:

$$
\mathcal{M}_L \equiv q_{\nu} p_{\mu} \bar{v}(k_1) \gamma^\nu \frac{1 - \gamma^5}{2} \left( \frac{iC^{-1}(k' + m_{\mu})}{k'^2 - m_\mu^2 + i\varepsilon} \right) \gamma^\mu \frac{1 + \gamma^5}{2} v(k_2)
$$
\[ = \bar{v}(k_1)q' \frac{1 - \gamma_5}{2} \frac{iC^{-1}}{k^2 - m_\nu^2} (k/ + m_\nu) \frac{1 + \gamma_5}{2} v(k_2) \]

\[ = \bar{v}(k_1)q' \frac{1 - \gamma_5}{2} \frac{iC^{-1}}{k^2 - m_\nu^2} v(k_2) \]

\[ = m_\nu \bar{v}(k_1)q' \frac{iC^{-1}}{k^2 - m_\nu^2} \frac{1 - \gamma_5}{2} v(k_2) \]

(A.1)

Here, we have used the following formulae

\[ \Psi^c \equiv C \bar{\Psi}^T, \]  
\[ \bar{\Psi}^c = (\Psi^c)^\dagger \gamma^0 = (C \bar{\Psi}^T)^\dagger \gamma^0 = \Psi^T C (\gamma^0)^2 = -\Psi^T C, \]

(A.2)

(A.3)

\[ C^{-1} \gamma^\mu C = -(\gamma^\mu)^T, \]

\[ C^{-1} \gamma^5 C = \gamma^5, \]

\[ C = C^{-1} \quad (C^2 = 1). \]

(A.4)

(A.5)

(A.6)

The propagator of Majorana neutrino was quoted from the Haber and Kane’s paper \[5\].

Similarly to the case of the \[W_L\] exchange, (A.1), the result in the \[W_R\] exchange is given as follows:

\[ \mathcal{M}_R = M_N \bar{v}(k_1)q' \frac{iC^{-1}}{k^2 - M_N^2} \frac{1 + \gamma_5}{2} v(k_2). \]

(A.7)

References

[1] J. C. Pati and A. Salam, Phys. Rev. D 10, 275 (1974), R. N. Mohapatra and J. C. Pati, Phys. Rev. D 11 566 (1975), R. N. Mohapatra G. Senjanovic and R. N. Mohapatra, Phys. Rev. D 12 1502 (1975).

[2] M. Frank, Ö. Özdal and P. Foulose, Phys. Rev. D99, 035001 (2019).

[3] P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020).

[4] P. Minkowski, Phys. Lett. B67 421, (1977); M. Gell-Mann, P. Ramond, R. Slansky,. in “Supergravity”. Amsterdam, North Holland, 315-321. ISBN 044485438X. (1979). T. Yanagida, Prog. Thor. Phys. 64, 1103, (1980); S. L. Glashow, M. Levy, J. L. Basdevant, (1980); R. N. Mohapatra, G. Senjanovic, Phys. Rev. Lett. 44 912, (1980); J. Schechter, J. Valle, Phys. Rev. D22, 2227, (1980).
[5] H. E. Haber and G. L. Kane, Phys. Rep. 117, 95 (1985).