Digital meter of frequency instability and phase noise of high stable oscillators

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Abstract This paper discusses the measurement of frequency instability and phase noise of real oscillators. As part of this work, a device was developed in LabVIEW, which allows implementing the multiplier-conversion method (MCM), in the form of a subprogram for oscillations with a low level of frequency instability. Another device was developed, which allows the simulation of a signal with a given phase noise grade, for applying MCM to increase standard deviation as many times as is needed. The use of these devices confirmed the operability and efficiency of the MCM for increasing the oscillation instability by a given number of times. The research showed that in the case of oscillation with very high frequency stability, it is possible to increase this small instability by a concise number of times, using MCM, and thereby provide the possibility of measuring frequency and phase instability, by devices with a larger error than is required for the measurements of very small instability.

1. Introduction
Wherever radio engineering and telecommunications are used, the requirements towards the frequency stability of generated oscillations are increasing yearly [1-15]. This leads to the need of improving methods for measuring and monitoring the frequency and phase parameters of oscillations generated by highly stable oscillators. This paper is a logical continuation of publications [16, 17] and is devoted to the both numeric and graphic representation of the results obtained during the development of a digital meter of frequency and phase instability of real oscillators based upon the multiplier-conversion method (MCM) [18-20], thank to which the reference devices are useless. The modeling has been performed in LabVIEW environment using National Instruments technologies and MCM.

2. Instantaneous frequency of SFG-2110 oscillator
The realization of the instantaneous frequency of the SFG-2110 oscillator, with the frequency set at 10 MHz, which was measured by Ch3-85 frequency meter, is shown in figure 1(a). The data show, that in the observation interval of 45 seconds length the measurement of 450 instantaneous frequency values is performed and each value is obtained within 100 ms

The realization of the instantaneous frequency contains outstanding and fast fluctuations which represent Jitter and considerably slower and more systematic (less chaotic) changes, which characterize Wander. Each frequency value, e.g. f = 9999865.037543956 Hz, is measured with an accuracy of 1 nHz. Thus, the achievable high relative accuracy of the oscillation frequency is equal to $10^{-9}/10^7 = 10^{-16}$.

This is formally obtainable using the Ch3-85 frequency meter in the combined method of discrete
counting, which consists about dividing two values with the possibility of obtaining any number of decimals, which is limited only by the computer capacity.

In order to obtain the Allan deviation, the systematic component (Wander) is extracted from the instantaneous frequency signal (figure 1(a)) then, basing on the centered dependency of the components of frequency fluctuations (figure 1(b)), the Allan deviation (figure 1(c)) is calculated and the distribution histogram (figure 1(d)) is depicted.

![Figure 1](image)

**Figure 1.** The results of the oscillation frequency measurement of SFG-2110 oscillator

3. **Jitter measurement by instantaneous frequency of the signal and by power spectral density of the instantaneous frequency**

Focusing on the available data given by the mentioned combined discrete count method, and willing to simulate multiple instantaneous frequency oscillation signal of a real SFG-2110 oscillator, it was necessary to develop a LabVIEW virtual instrument (vi). Short-term frequency instability or Jitter is being quantitatively estimated basing on the realization of the instantaneous frequency, in other words, there should be the calculation of the standard deviation (SD) of frequency fluctuations $\sigma_f$ within 2 sec. The figure 2(a) shows one of the realizations, obtained during the simulation of the instantaneous frequency of a signal provided by SFG-2110 oscillator. And the figure 2(b) shows the fluctuation component of the frequency $df(t)$. It should be pointed out, that during the simulation; the initial frequency $f_0$, the initial SD $\sigma_0$, the maximum spectrum frequency $f_m$ and the initial frequency relative instability $\delta_0$ of the instantaneous frequency oscillation signal were set to: 1 kHz, 1 mHz, 25 Hz and $10^{-6}$ respectively. After averaging the instantaneous frequency oscillation signal, the SD of the selected frequency fluctuation component $\sigma_f$ resulted equal to 0.993 mHz almost equal to the input value 1 mHz. Thus, the high accuracy of the calculus is confirmed.
The following power spectral density (PSD), (figure 3) is the result of having applied the Fourier transform to the fluctuation component of the frequency $df(t)$.

The averaging of the accumulated PSD of instantaneous frequency took place and the standard deviation of the average process happened to be $0.995 \text{ mHz}$.

Despite the strict interrelation between frequency instability and phase noise, this paper shows, that evaluating instability might be, in some cases done with higher efficacy using phase noise.

4. The measurement of phase noise using realization and PSD of the phase

The integration of the fluctuation component of the instantaneous frequency allows to switch to the fluctuation component of the instantaneous phase, and after that, it is possible to determine the phase noise from the timing diagram (figure 4(a)) or from the PSD of the instantaneous phase (figure 4(b)). There are respective standard deviations of the phase noise on the upper right corner of each of these figures. As well as the upper case; the displayed standard deviations correspond to the average phase fluctuation process, $\sigma_{\phi,t} = 62.45 \mu \text{rad}$ and the average PSD process, $\sigma_{\phi,f} = 62.33 \mu \text{rad}$ respectively.

As you can see, the obtained values are quite close and fully correspond to the theory.
5. The measurement of SD of the phase noise via the PSD of the oscillation

The formation of the linear part of the argument of the oscillation, that simulates a harmonic signal from the SFG-2110 oscillator allows moving to the full instantaneous phase of the harmonic and form an s(t) oscillation with a given amplitude, frequency instability and the corresponding phase noise. After obtaining this oscillation, we proceed to solve two important tasks, that are related to obtaining an estimation of the phase noise, which is in the s(t) oscillation.

The first task is as follows. Using the Hilbert transform, switching from the s(t) oscillation to the analytical signal is possible. Now, the complex envelope is extracted from the analytical signal and, from complex envelope the instantaneous phase is extracted. A fragment of the block diagram for solving this task is shown in figure 5(a). The realization of the instantaneous phase, which is embedded in the model of the signal, is shown in the figure 5(b). This phase completely coincides with the phase noise (figure 5(c)), which is extracted from the signal argument s(t).

![Diagram](image)

Figure 5. Extraction of the instantaneous phase from the s(t) oscillation

The second task is to find the PSD of the oscillation with the already known phase noise. Then once again evaluate this phase noise, finding SD of the phase noise $\sigma_{\phi,s}$ using the PSD of the signal. PSD of the s(t) oscillation with a given phase noise is obtained by applying the corresponding Fourier transform. The result of this transformation is shown in figure 6.

To obtain an estimation SD of the phase noise is recommended to follow these steps. The $P_0$ harmonic on the carrier frequency is excluded from the PSD of the signal. All other $P_n$ harmonics are summed and the sum is multiplied by two, after that the result is affected by the square root and the result is divided by the oscillation amplitude $A$, as it’s shown in the equation:

$$\sigma_{\phi,s} = \frac{1}{A} \sqrt{2 \sum_{n=0}^{N} P_n} ,$$  \hspace{1cm} (1)

where: $n = 0,1,2, ..., N$; $P_n$ – all harmonics in the PSD, except the carrier frequency harmonic.

Eventually, the obtained value of SD of the phase noise ($\sigma_{\phi,s}$) using the PSD of the signal is equal to $6.233 \times 10^{-5}$ rad (figure 6) and this value coincides with the value of SD of the phase noise using the
realization of the instantaneous phase \( \sigma_{\phi,t} = 6.245 \cdot 10^{-5}\) rad. And this value coincides with the value of SD of the phase noise, using the PSD of the phase of the signal \( \sigma_{\phi,f} = 6.233 \cdot 10^{-5} \) rad.

Figure 6. The power spectral density of the oscillation of SFG-2110 oscillator

6. Estimation of jitter of highly stable generators using MCM
It was earlier [18-20] shown, that in case of very insignificant frequency fluctuations, MCM can be efficiently used for measuring instability, relying on its property of transforming the oscillation, so that the frequency instability increases by several orders of magnitude and, then its measurement can be performed rather easily by known methods.

As part of this work, a device was developed in LabVIEW in the form of a subprogram, which allows implementing the MCM for oscillations with a low level of frequency instability.

Another device was also developed, which allows the simulation of a signal with a given phase noise grade and then use the MCM to increase of frequency instability as many times as is needed. The use of these devices fully confirms the operability and efficiency of the MCM for increasing the oscillation instability by a given number of times.

7. Conclusion
The paper deals with the measurement of frequency instability and phase noise of real oscillators. It is shown, that in the case of an oscillation with a very high frequency stability, it is possible to increase this small instability by a certain number of times, using the multiplier-conversion method and, thereby provide the possibility of obtaining an estimation of frequency instability and phase noise by devices that have a greater error than is required for direct measurements.

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References
[1] Ochkov D, Silaev E and Formalnov I 2006 Methodology for estimating the coherence intervals of radar radio paths. M.: Radio engineering (Journal in journal) V 4 pp 36-38 https://rastr-radio.ru/pdf/coherence.pdf
[2] Ochkov D, Silaev E and Formalnov I 2007 The influence of the value of the frequency of the reference signal on the magnitude of the parasitic incursion of the phase of the quartz-based heterodyne. M.: Radar location and communication V 10 pp 94-99 https://rastr-radio.ru/pdf/coherence_2.pdf
[3] Belchikov S 2009 Phase Noise: How to Get Below \(-120\) dBc / Hz at 10 kHz Offset in the Frequency Range up to 14 GHz, or The Decibel Struggle Components and Technologies V 5 pp 139-46
[4] Molchanov E, Ochkov D, Silaev E and Formalnov I 2007 Allan variation and its modifications in assessing the short-term instability of the heterodyne frequency of a coherent radar rastradio M.: Radiotechnique V 10 pp 108-110 rastr-radio.ru/pdf/allan.pdf

[5] Belloni M, Gioia M and Beretta S 2011 Space Mini Passive Hydrogen Masers - A Compact Passive Hydrogen Maser for Space Applications Proc. IEEE Intern. Frequency Control Symposium and European Frequency and Time Forum, San Fransisco, USA pp 906-910

[6] Vasilyev V and Kozlov S 2013 Figure of merit and limit of short-term stability in passive hydrogen maser Proc. of the joint conference IFCS-EFTF, Prague, Czech Republic, July 21-25 pp 768-770

[7] Vasilyev V 2014 Time and frequency measurements M.: Measurement Techniques V 2 pp 37-40 https://www.elibrary.ru/item.asp?id=21394529

[8] Vasilyev V 2016 Investigation of the limiting short-term instability of the output signal of a passive hydrogen frequency standard M.: Measurement Techniques V 9 pp 25-29 https://www.elibrary.ru/item.asp?id=27195360&

[9] Akulov V and Pashev G 2019 Analysis of the digital linear systems of automatic frequency adjustment of high-stability generators by radio signals of the global positioning system M.: Measurement Techniques V 3 pp 254-258 https://www.elibrary.ru/item.asp?id=37247987

[10] Verveyko A, Lappo I, Arkushenko P and Yusukhno S 2019 Frequency instability measurement device based on the pulse coincidence principle Bulletin of National Technical University of Ukraine. Series Radiotechnique Radioapparatus building V 76 pp 29-36 https://www.elibrary.ru/item.asp?id=37199649

[11] Kychak V. and Gavrasienko P. 2014 Device of Radio Frequency Instability Measurements Visnyk NTUU KPI Seriia - Radiotekhnika Radioapparatobuduvannia V 58 pp 83-89 https://doi.org/10.20535/RADAP.2014.58.83-89

[12] Mrachkovskiy O and Vishnevyy S 2008 Modelling of the multifunctional generator of video and radio signals in program application LabVIEW National Technical University of Ukraine, Kyiv Politechnic Institute Kiev V 36 pp 34-37 https://www.elibrary.ru/item.asp?id=2933878

[13] Goncharov A, Bonert A, Baraulya V, Tropnikov M, Kuznetsov S, Taichenachev A and Bagayev S 2018 Laser frequency stabilisation on narrow resonances of cold magnesium atoms at the $^{1s_0}_0-^{3p_1}$ transition Quantum electronics V 5 pp 410-414 https://www.elibrary.ru/item.asp?id=34940655

[14] Epikhin V, Baryshev V, Sylyusarev S, Aprelev A and Blinov I 2019 Acousto-optic modulators for a controlled frequency shift of light beams in optical and microwave cold-atom frequency standards Quantum electronics V 9 pp 857-862 https://www.elibrary.ru/item.asp?id=41701544

[15] Goncharov A, Baraulya V, Bonert A and Tropnikov M 2020 457-nm radiation source based on a diode laser for precision spectroscopy of magnesium atoms Quantum Electronics V 3 pp 272-276 https://www.elibrary.ru/item.asp?id=42649746

[16] Nsue J, Fedosov V and Tereshkov V 2016 Evaluation of frequency instability using indicators in the time domain Rostov Scientific Journal: Network Journal V 4 16 pp 5-15

[17] Nsue J, Fedosov V and Tereshkov V 2016 Measurement of frequency instability of highly stable generators using indicators in the time domain Rostov Scientific Journal: Network Journal V 4 17 pp 63-70

[18] Nsue J and Fedosov V 2016 A digital algorithm for measuring the short-term frequency instability of highly stable generators by the multiply-converting method Trends in the development of science and education: collection of articles scientific works, based on the materials of the XV International scientific conf. June 25, - The publishing house of the Research Center "L-Journal" P 3 pp 16-18
[19] Nsue J, Fedosov V and Kucheryavenko S 2018 Measurement of short-term frequency instability of ultra-stable quasi-harmonic signals Electronic scientific journal “Engineering Bulletin of the Don” V 1 pp 24-28

[20] Nsue J, Fedosov V and Kucheryavenko S 2019 Experimental measurement of relative instability of frequency of oscillators by digital multiplier-converting method. Izvestiya SFedU. Engineering Sciences V 1 pp 69-70