The effect of thermal feedback and statistical technological dispersion of microcircuits parameters on their thermal modes

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Abstract. During functioning of integrated circuit, the consumed power converts into heat, which makes high temperatures appear in it. Due to the strong dependence of the power on the temperature, its value changes as a result of self-heating, and the changed power, in turn, makes the microcircuit’s temperature change, thus closing the thermal feedback loop. Neglecting the thermal feedback in a microcircuit results in a significant discrepancy between the real temperature and its prognosticated values. The statistical technological dispersion of microcircuits electric and thermal parameters is the second most important factor that must be taken into account for adequately modeling thermal processes therein. Due to this dispersion, microcircuits’ real temperatures are not exactly known and determinate; on the contrary, they are stochastic, and their values are vary within some intervals. The statistical dispersion of a microcircuit’s parameters is conditioned by the technology of manufacturing and mounting the microcircuit in an electronic device, as well as by the stochastic parameters of ambient, such as the temperature, velocity and direction of flow of environment. In this article examines mathematical modeling the separate and simultaneous influence of the thermal feedback and statistical dispersion of the parameters on a microcircuit chip’s temperature. It has been shown that neglecting the said factors leads to the inadequate modeling of thermal and electrical regimes of microcircuits and to errors of design of electronic devices based on them.

1. Introduction

While functioning, integrated semiconductor microcircuits (ICs) are exposed to a plethora of destabilizing factors, thermal factors being the most critical to ensuring the operability of ICs and electronic devices (EDs) based on them. This is conditioned by the fact that the energy consumed by a IC is dispersed mostly as heat [1, 2], and only a small part of it is expended to create useful electric signals. This circumstance makes the IC self-heat significantly and establishes high temperatures in it. The IC operation practice shows that in an IC as it functions, mainly two factors, namely, thermal feedback (TF) and the stochastic character of the thermal and electric processes, essentially influence thermal processes in the IC and temperature values on the chip and casing.

The TF phenomenon is conditioned both by an IC’s self-heating, the appearance of high temperature
values on the chip, and the significant dependence of the electric parameters, including the IC’s consumed power, on the temperature [3 – 5]. As a result, in the IC, the change in the temperature entails a change in the IC’s electric parameters and consumed power, and the change in the consumed powers leads, in turn, to a change in the temperature, thus closing the feedback loop. The mutually conditioned influence of the IC’s temperature on its consumed power and of the consumed power on the temperature is called the TF phenomenon.

The TF phenomenon is inherent in any ICs and EDs built on their basis; it cannot be eliminated. TF can be both positive and negative. If TF is positive, in an IC the consumed power value and temperature levels increase like an avalanche and lead to breakdowns, the IC’s overheating, the IC’s electric and thermal regime parameters’ overstepping the limits of the permissible values. A negative TF manifests itself in the form of a decrease in the consumed power and IC’s temperature, thus favoring the achievement of steady thermal and electric regimes and reduced levels of temperature.

As a result, when creating competitive ICs, it is necessary to take into account that the negative TF is a useful phenomenon, while the positive TF is a harmful one. Despite the substantial influence of the TF on the electric and thermal parameters of ICs, at present the electric and thermal regimes of ICs and EDs are calculated, as a rule, with the assumed independence of the temperature and heat release power, which is significantly conditioned by complexity of the mathematical modeling of TF.

Research into real ICs under various operation conditions demonstrates that the IC’s thermal and electric regimes, as shown in [2, 6], are indeterminate, intervally stochastic values. The indeterminate, intervally stochastic character of IC’s thermal and electric regimes is conditioned, firstly, by the statistical technological dispersion of the electric and thermal parameters both when manufacturing the ICs and mounting the ICs into real EDs, secondly, by the stochastic character of the IC’s thermal and electric parameters that appears while the ICs operate, and thirdly, by the stochastic nature of the environment parameters (temperature, velocity and direction of the flow).

Let us note that due to the statistical technological dispersion during the manufacture, the values of the parameters of separate samples of ICs in a batch of similar ones differ from each other and vary from a sample to a sample. At that, in a batch, the values of the IC’s parameters fall within some intervals, the boundaries of which are determined by the general technological spread when manufacturing the whole batch of ICs. Study [5] established that a random value uniformly distributed inside the variation interval is the adequate mathematical model of the indeterminate parameters of an IC’s electric and thermal regimes subject to statistical technological dispersion.

The TF phenomenon and the intervally stochastic character of the parameters of the thermal and
electric regimes exert a significant influence on thermal processes and temperature levels in real ICs and condition their inter Valley stochastic nature. Both factors mentioned are inherent in all ICs and cannot be eliminated through any manufacturing and mounting technologies, in any operation modes.

Based on mathematical modelling, this article researches the influence of TF and stochastic indeterminacy of the parameters on the temperature of an IC chip confined in a casing installed on a printed circuit board.

The modelled structure includes the IC in the casing (figure 1) installed on a multilayered printed circuit board (PCB) and its thermal model. During heat exchange, the power \( P \) consumed by the IC chip converts into heat, which spreads across the structure of the casing (thermal resistance \( R_{jc} \)), partly goes from the surface of the IC casing to the environment (thermal resistance \( R_{ca} \)), partly goes to the PCB through the casing outputs and the clearance between the IC casing and the PCB (thermal resistance \( R_{cp} \)), disperses across the board and is further entrained to the environment with the temperature \( T_a \) (thermal resistance \( R_{pa} \)) through heat exchange.

At first, let us consider the influence of TF on the IC chip’s temperature without taking into account the statistical technological dispersion of the IC’s parameters, and then we will analyze the joint influence of both factors.

2. The Microcircuit’s Thermal Mode with Thermal Feedback

The mathematical model of the thermal process in the IC structure (figure 1), if all the IC’s parameters are determinate:

Equation as normal text:

\[
C \frac{dT_j}{dt} + \frac{T_j-T_a}{R_{ja}} = P(T_j),
\]

(1)

where \( T_j(0) = T_a \); \( R_{ja} = R_{jc} + R_{ca}(R_{cp} + R_{pa})/(R_{ca} + R_{cp} + R_{pa}T_j) \) is the IC chip’s temperature; \( T_c \) is the IC casing’s temperature; \( T_p \) is the PCB’s temperature; \( T_a \) is the environment temperature; \( P(T_j) \) is the IC’s consumed power; \( R_{jc} \) is the thermal chip-casing resistance; \( R_{ca} \) is the thermal casing-environment resistance; \( R_{cp} \) is the thermal resistance between the IC casing and the board; \( R_{pa} \) is the thermal resistance from the board to the environment; \( C \) is the volumetric heat capacity of the encapsulated IC (in casing).

For the simplicity of the analysis, let us accept that the temperature dependence of the consumed power is described by the linear function \( P(T_j) = P_0 + \alpha \cdot T_j \), where \( \alpha \) is the TF coefficient, and at \( \alpha > 0 \) TF is positive, at \( \alpha < 0 \) TF is negative and at \( \alpha = 0 \) there is no TF.

The solution of equation (1) equals \( (g_{ja} = 1/R_{jc}) \)

\[
T_j(t) = \frac{P_0+g_{ja}T_a}{g_{ja}-\alpha} - \frac{P_0+kT_a}{g_{ja}-\alpha} \exp \left(-\frac{g_{ja}-\alpha}{c}t\right).
\]

(2)

The results of the calculations of the chip temperature \( T_j(t) \) are given for the environment temperature \( T_a = 20°C \) both for positive and negative TF (figure 2). For comparison, figure 2 also shows the results of the calculations, if the TF effect is neglected. The analysis of the results shows that at positive TF the IC chip’s temperature significantly (by more than 42%) exceeds the IC chip’s temperature at negative TF, and this exceedance will only grow with the passage of time. And neglecting the influence of the TF phenomenon on the IC’s thermal regime results in the IC chip’s temperature calculation accuracy that exceeds 25%.

Thus, the results of the IC chip’s temperature calculations both with and without taking the TF into account make it possible to conclude that the adequate modeling of the IC’s electric and thermal regimes can be achieved only with taking TF into account; but if one neglects the TF, this can lead to significant inaccuracies when designing ICs and EDs built on them.
3. The Microcircuit’s Thermal Mode with Thermal Feedback and the statistical dispersion of the parameters

Under real operation conditions, IC’s thermal and electric regime parameters and, along with them, the IC temperature levels and thermal processes are influenced by both the TF phenomenon modelled by the dependence of the consumed power $P(T_j)$ on the chip’s temperature $T_j$ and the statistical dispersion of the parameters that determine the IC’s thermal regime. They include the following parameters, which are, in general, interval stochastic: thermal resistances $R_{jc}(\omega)$ and $R_{cp}(\omega)$, environment temperature $T_e(\omega)$, consumed power $P(T_j, \omega)$ and TF coefficient $\alpha(\omega)$, where $\omega$ are elementary events from the sample space $\Omega$. The thermal resistances $R_{ca}$ and $R_{pa}$ are determinate, which corresponds with the reality.

The IC’s mathematical stochastic model that describes the IC chip’s stationary interval stochastic temperature $T_j(\omega)$ with taking the TF effect into account has the appearance ($\omega \in \Omega$):

$$T_j(\omega) = T_a(\omega) + \left( R_{jc}(\omega) + R_{c-p-a}(\omega) \right) \cdot P(T_j, \omega), \quad (3)$$

where $P(T_j, \omega)$ is the interval stochastic power linearly dependent on the IC chip’s temperature $P(T_j, \omega) = P_0(\omega) + \alpha(\omega) \cdot T_j(\omega)$; $P_0(\omega)$ is the interval stochastic power value at the chip’s temperature $T_{j0}$; $\alpha(\omega)$ is the interval stochastic value of the TF coefficient; $R_{c-p-a}(\omega)$ is the interval stochastic value of thermal resistance between the IC casing and environment equal to

$$R_{c-p-a}(\omega) = R_{ca} \left( R_{cp}(\omega) + R_{pa} \right)/(R_{ca} + R_{cp}(\omega) + R_{pa}). \quad (4)$$

The interval stochastic character of the thermal resistance $R_{cp}(\omega)$ is conditioned by the statistical technological dispersion of the thickness of the clearance between the IC casing and the printed circuit board (figure 1); and for the purpose of revealing the influence of TF more clearly, this article accepts that the thickness of the clearance between the IC casing and the printed circuit board is determinate, so the thermal resistance $R_{cp}$ of the clearance will also be determinate. In general, the TF coefficient $\alpha(\omega)$
is an intervally stochastic value conditioned by the statistical technological dispersion of manufacturing the IC; however, like in the case of the thermal resistance $R_{jc}(\omega)$, we will consider it determinate for the simplicity of analysis. In the mathematical model (3), (4), the other random values, namely, the IC casing’s thermal resistance $R_{jc}(\omega)$, consumed power $P(T_j, \omega)$ and environment temperature $T_a(\omega)$ are intervally stochastic and statistically independent, which corresponds with the practice.

Taking into account the given comments and introducing a new variable $\Delta T_j(\omega) = T_j(\omega) - T_a(\omega)$ equal to the IC chip’s temperature exceedance $T_j(\omega)$ over the environment temperature $T_a(\omega)$, from the correlations (3) and (4) we will obtain an expression for the intervally stochastic temperature $\Delta T_j(\omega)$ that takes into account both the TF phenomenon and the statistical technological dispersion when manufacturing the IC:

$$
\Delta T_j(\omega) = \left( R_{jc}(\omega) + R_{c-p-a} \right) P_0(\omega)/(1 - \alpha \cdot \left( R_{jc}(\omega) + R_{c-p-a} \right))
$$

(5)

From the last expression (5), it follows that the intervally stochastic character of the IC chip’s temperature $T_j(\omega)$ is conditioned both by the statistical technological dispersion when manufacturing the IC casings, which manifests itself in the form of stochasticity of the casing’s thermal resistance $R_{jc}(\omega)$, and by the power $P_0(\omega)$ consumed by the IC. The intervally stochastic values of the thermal resistance $R_{jc}(\omega) \in [\bar{R}_{jc}, \overline{R}_{jc}]$ and consumed power $P_0(\omega) \in [\bar{P}_0, \overline{P}_0]$ are modelled by uniformly distributed random values with the probability densities $p_{R_{jc}} = 1/\Delta R_{jc}$ and $p_{P_0} = 1/\Delta P_0$ determined within intervals $\Delta R_{jc} = \overline{R}_{jc} - \bar{R}_{jc}$ and $\Delta P_0 = \overline{P}_0 - \bar{P}_0$ long respectively, where $\bar{R}_{jc}, \overline{R}_{jc}$ and $\bar{P}_0, \overline{P}_0$ are the lower and upper boundaries of the thermal resistance $R_{jc}(\omega)$ and consumed power $P_0(\omega)$ intervals. The statistical measures of the IC chip’s intervally stochastic temperature $\Delta T_j(\omega)$ (5), namely, the mathematical expectation $E\{\Delta T_j(\omega)\}$ and variance $D_{\Delta T_j} = E\{(\Delta T_j(\omega) - E\{\Delta T_j(\omega)\})^2\}$, where $E\{\bullet\}$ is the operator of the mathematical expectation, will be determined by the following expressions:

$$
E\{\Delta T_j(\omega)\} = \frac{1}{\Delta R_{jc}} \left( -\frac{\Delta R_{jc}}{\alpha} + \frac{1}{\alpha^2} \ln \frac{A}{B} \right) \cdot \overline{P}_0
$$

(6)

$$
D_{\Delta T_j} = \frac{1}{\alpha^2 \Delta R_{jc}} \left( \alpha \cdot \Delta R_{jc} - 2 \ln \frac{A}{B} + \frac{\alpha \Delta R_{jc}}{A \cdot B} \right) \cdot \left( D_{P_0} + (E\{P_0(\omega)\})^2 \right) - (E\{\Delta T_j(\omega)\})^2,
$$

(7)

$$
A = 1 - \alpha \cdot (\bar{R}_{jc} + R_{c-p-a}), \quad B = 1 - \alpha \cdot (\overline{R}_{jc} + R_{c-p-a}),
$$

(8)

where $E\{P_0(\omega)\}$ and $D_{P_0}$ are the mathematical expectation and variance of the IC’s stochastic consumed power $P_0(\omega)$.

The calculated values of the mathematical expectation $E\{\Delta T_j(\omega)\}$ and variance $D_{\Delta T_j}$ determine the interval $\Delta T_j(\omega) \in [\Delta T_j', \overline{\Delta T}_j]$, within which the real values of the IC chip’s temperature $\Delta T_j(\omega)$ would vary and which can be encountered in practice during the operation of the IC. Let us note that the said interval takes into account both the TF effect and the intervally stochastic indeterminacy of the IC parameters, namely, the consumed power and thermal resistance of the IC casing. The lower and upper boundaries of the IC chip’s temperature value interval are determined according to the formulas: $\Delta T_j = E\{\Delta T_j(\omega)\} - 3 \sigma_{\Delta T_j}$, $\overline{\Delta T}_j = E\{\Delta T_j(\omega)\} + 3 \sigma_{\Delta T_j}$, where $\sigma_{\Delta T_j} = \sqrt{D_{\Delta T_j}}$ is the root-mean-square deviation of the exceedance of the IC chip’s stochastic temperature $T_j(\omega)$ over the environment temperature $T_a(\omega)$.

The calculated dependences of the mathematical expectation $E\{\Delta T_j(\omega)\}$ and the boundaries of the
interval \( [\Delta T_j, \overline{\Delta T_j}] \) of the IC chip’s temperature \( \Delta T_j(\omega) \) on the TF coefficient \( \alpha \) (is determinate) under the joint effect of TF and intervally stochastic indeterminacy of the casing’s thermal resistance value are given in figure 3.

The influence of the spread of the interval-stochastic TF coefficient \( \alpha(\omega) \) and the power of the IC on the value of the mathematical expectation \( E\{\Delta T_j(\omega)\} \) and the variance \( D_{\Delta T_j} \) of the temperature of the crystal of the IC \( \Delta T_j(\omega) \) is determined by the formulas

\[
E\{\Delta T_j(\omega)\} = \frac{E_0}{\Delta_\alpha} \ln \frac{1-a R_{ja}}{1-\alpha R_{ja}},
\]

\[
D_{\Delta T_j} = \frac{R_{ja}^2 (p_{ja} + E_0^2)}{(1-\alpha R_{ja})^2 (1-\alpha R_{ja})} - \Delta T_j^2,
\]

where \( R_{ja} = R_{jc} + R_{c-p-a} \) is the thermal resistance of junction-environment; \( \underline{a}, \overline{a} \) are the lower and upper bound of the interval \( [\underline{a}, \overline{a}] \) of variation of the interval-stochastic TF coefficient \( \alpha(\omega) \) uniformly distributed within the interval.

Calculation results of the dependence of the relative magnitude of the temperature dispersion of the IC crystal \( (\Delta_{\alpha T}/E\{\Delta T_j(\omega)\}) \) on the value of the relative dispersion of the interval-stochastic coefficient of TF \( (\Delta_{\alpha}/E\{\alpha(\omega)\}) \), in the absence of the spread of the power consumption of the IC, and the various values of the mathematical expectation of the TF coefficient \( E\{\alpha(\omega)\} \), are shown in figure 4, where \( \Delta_{\alpha T} = \overline{\Delta T_j} - \Delta T_j \).

An analysis of the results shows the following:

- the spread of the interval-stochastic value of the TF coefficient \( \alpha(\omega) \) has a significant effect on the spread of the temperature of the IC crystal \( \Delta T_j(\omega) \). So, a change in the relative spread of the TF coefficient by 20% entails a change in the relative spread in the temperature of the IC crystal by 10% (for given initial data);
- the magnitude of the temperature spread of the IC crystal substantially depends on the mathematical expectation of the TF coefficient \( E\{\alpha(\omega)\} \) and grows with an increase in the latter. Thus, an increase in TF \( E\{\alpha(\omega)\} \) from 0,01 to 0,015 W/K leads to an increase in the...
relative dispersion of the temperature of the IC crystal by 22%; but an increase in $E\{\alpha(\omega)\}$ from 0.015 to 0.02 W/K leads to an increase in the relative temperature spread of the IC crystal by 30%.

Figure 4. The value of the relative spread of the temperature of the IC crystal depending on the value of the relative spread of the TF coefficient for various values of the mathematical expectation of the TF coefficient $E\{\alpha(\omega)\}$. The spread of the power consumption of the IC is absent.

The analysis of the obtained dependences shows that the joint effect of TF and interval indeterminacy of parameters of IC on the IC chip’s temperature is significant and neglecting them leads to significant errors in prognosticating the IC’s thermal regimes. For example, the mathematical expectation of the IC chip’s temperature $E\{\Delta T_i(\omega)\}$ calculated with taking into account the TF ($\alpha = 0.016$) rises in comparison to the temperature $E\{\Delta T_i(\omega)\}$ calculated with neglecting the TF ($\alpha = 0$) by 46%. As to the interval within which will fall the real values of the IC chip’s temperature $\Delta T_i(\omega)$, its span increases, as the TF intensity coefficient $\alpha$ rises by 47% relative to the temperature interval calculated with neglecting the influence of TF ($\alpha = 0$). The spread of the interval-stochastic coefficient of TF makes an additional and significant contribution to the temperature spread of the IC crystal. Namely, an increase in the spread of the TF coefficient entails an increase in the temperature spread of the IC crystal in excess of 20% of the mathematical expectation of the temperature of the IC crystal.

4. Conclusion
The thermal regime of an IC in a casing installed on a printed circuit board has been modelled in nonstationary mode with the TF effect only and in stationary mode under the conditions of the simultaneous TF effect and statistical dispersion of the IC’s electric and thermal parameters.

The comparison of the results of modelling the IC chip’s nonstationary temperature shows a substantial discrepancy between the temperature levels for a positive TF and a negative TF (by 42%), which evidences the advisability of modelling ICs with a negative TF. When modelling thermal processes in ICs, neglecting the TF leads to inadequate results that differ from the TF-including results by more than 35%, and this inaccuracy will only grow with the passage of time.

The analysis of the joint influence of the TF and statistical dispersion of the IC’s electric and thermal parameters on the IC chip’s temperature shows the following. If there is no TF effect (TF coefficient $\alpha = 0$), neglecting the spread of the IC’s parameters leads to a point value of $\Delta T_i \approx 80^\circ C$, which would hardly be encountered in the real practice of IC operation. At the same time, the real spread of the IC’s parameters entails the interval of the chip’s possible temperature values $\Delta T_i$, which would fall within
the interval [60, 100],°C. If the TF effect (for example, \( \alpha = 0.016 \)) is also taken into account together with the statistical dispersion of the IC’s parameters, the interval of the IC chip’s possible real temperature values would be as large as [96, 145],°C, which substantially exceeds the IC’s temperature value interval without taking TF into account. This is why neglecting the factor of the statistical technological dispersion of the IC’s parameters leads to the significant inaccuracy of modeling its thermal regime.

Thus, the joint effect of the TF and statistical technological dispersion of the IC’s parameters exerts a significant influence on the IC’s thermal regime, and neglecting them leads to inadequately modelling ICs’ thermal regimes, to significant inaccuracy of prognosticating the temperature levels in ICs and, as a result, to errors in designing ICs and EDs built on their basis. Let us note that taking into account the stochastic nature of the environment temperature \( T_a(\omega) \) contributes additionally to the span of the interval of variation of an IC’s stochastic temperature.

It is necessary to note that this article accepts that the IC’s consumed power changes linearly depending on the IC chip’s temperature, while the real dependence of the power consumed by an IC on the chip’s temperature can significantly differ from a linear one and have rather a complex non-linear shape. In the article, the following assumption is connected with taking into account some intervally stochastic parameters of an IC. For example, the article has accepted that only the IC casing’s thermal resistance \( R_{ic}(\omega) \) is intervally stochastic, while the thermal resistance of the clearance between the IC and printed circuit board \( R_{cp}(\omega) \) is determinate. Along with that, the assumptions accepted by the article are rather common, significantly simplify the mathematical modeling and, at the same time, permit discovering the main regularities of the influence of the TF and statistical technological dispersion of the IC’s parameters on thermal processes and temperature levels in real ICs.

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