Gravitational Waves from Long Gamma-Ray Bursts and Supernovae

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ABSTRACT

Gamma-ray bursts (GRBs) are produced during the propagation of ultra-relativistic jets. While our understanding of these jets have improved notably during the last decades, it is currently impossible to study directly the jet close to the central source, due to the high opacity of the medium. In this paper, we present numerical simulations of relativistic jets propagating through a massive, stripped envelope star associated to long GRBs, breaking out of the star and accelerating into the circumstellar medium. We compute the resulting gravitational wave (GW) signal, showing that several key parameters of the jet propagation can be directly determined by the associated GW signal. The signal presents two peaks, the first one corresponding to the jet duration, while the second one corresponding to the end of the acceleration phase. Depending on the observer location (with respect to the jet axis) this peak corresponds to the break-out time for observer located close to the jet axis (which in turn depends on the stellar size), or to much larger times (corresponding to the end of the acceleration phase) for off-axis observers. We also show that the slope of the GW signal before and around the first peak tracks the jet luminosity history and the structure of the progenitor star. The amplitude of the GW signal is $h_D \sim$ hundreds to several thousands. Although this signal, for extragalactic sources, is outside the range of detectability of current GW detectors, it can be detected by future detectors as BBO, DECIGO and ALIA. Our results illustrate that future detections of GW associated to GRB jets will represent a revolution in our understanding of this phenomenon.

Key words: relativistic processes – methods: numerical – gamma-ray burst: general – stars: jets gravitational waves: Burst, LIGO, Noise modeling, Power Spectrum, Wavelets

1 INTRODUCTION

Gamma-ray bursts (GRBs) are extremely luminous pulses of gamma-rays (with an isotropic energy of $10^{51} - 10^{54}$ ergs) lasting typically from $\sim$ a fraction of a second to $\sim$ hundreds of seconds. GRBs are classified based on their duration. Short GRB (SGRBs), typically lasting $\lesssim 2$ s, are produced during the coalescence of neutron stars (NS), while long GRBs (LGRBs), lasting $\gtrsim 2$ s, are associated to the collapse of massive stars and their explosion as type Ic supernovae (SNe) (for a review, see, e.g., Kumar & Zhang 2015).

The gamma-ray emission observed in these events is produced by highly relativistic jet, moving with Lorentz factors $\Gamma_j \sim 100 - 1000$. These jets are ejected from a black hole or a magnetar (the so-called “central-engine”) formed during the collapse of a massive star (in the case of LGRBs, see, e.g., Hjorth & Bloom 2012; Cano et al. 2017) or as a result of the coalescence of a binary NS system (for SGRBs, see, e.g., Berger 2014).

Once the jet is ejected from the central engine, it propagates through the dense, optically thick surrounding medium formed by the progenitor star (in LGRBs) or the debris of the binary NS system (in SGRBs), before breaking out at distances of $\sim 10^{10} - 10^{11}$ cm. Theoretical studies show that, during this phase, the jet moves with sub-relativistic velocities ($\sim 0.1 - 0.5c$), being $c$ the light speed (e.g., Bromberg et al. 2011b; Nakar & Piran 2016; De Colle et al. 2018). When the jet breaks out from the dense environment, it accelerates to large jet Lorentz factors $\Gamma_j (\sim E_j/M_j c^2$ where $E_j$ and $M_j$ are the jet energy and mass), before emitting the observed gamma-ray emission at larger distances ($\gtrsim 10^{13} - 10^{15}$ cm), once the hot plasma becomes optically thin to gamma-ray radiation.

The prompt gamma-ray emission is followed by a multi-wavelength afterglow emission covering the full electromagnetic spectrum, from radio to X-rays, and lasting from minutes to several years. Thus, the late phases of evolution of the relativistic jets (from $\sim 10^{13}$ cm to $\gtrsim 10^{18}$ cm) can be studied by analyzing these rich electromagnetic signatures (see, e.g., Kumar & Zhang 2015 and references therein). On the other hand, it is much more difficult to study the early phases of evolution of the jet, corresponding to distances $\lesssim 10^{10} - 10^{13}$ cm, as the high densities make the jet plasma optically thick to electromagnetic radiation. In particular, only neutrinos (e.g., Kimura 2022) and gravitational waves (GWs) could probe directly the behaviour of the jet while it is crossing the dense environment.
In addition to oscillating GW signals associated to the coalescence of compact objects (Abbott et al. 2017b), the possibility of detecting non-oscillating, low frequency signals (the so-called "memory" signal produced by unbound material over timescales $\gtrsim 1$ s), has been proposed long time ago (Braginskii & Thorne 1987). These “memory” signals have been studied extensively, e.g., in the context of supernovae (SNe) explosions (e.g. Kotake et al. 2006; Murphy et al. 2009; Müller et al. 2012; Müller et al. 2013; Wongwathanarat, A. et al. 2015; Yakunin et al. 2015; Powell & Müller 2019; Hübner et al. 2020; Mezzacappa et al. 2020; Richardson et al. 2022a). Richardson et al. (2022a), in particular, also discuss the frequency range related to the memory production, with respect to other mechanisms like the resonances of the f/g modes of the proto-neutron stars.

The focus of these studies was to discuss under which circumstances (in terms of specific instrument and signal morphology) the memory component of the signal spectral density is above the interferometric noise spectral density. This is a semiquantitative measure of the detectability of the memory (in the sense that it is an important metric but it is not related to a specific algorithm). It is also worth stressing that for detectability the whole spectrum of the memory development over time matters, not just the zero frequency component produced by the asymptotic value.

Previous studies of the GWs produced by GRB jets have focused on the propagation of the jet through the dense envelope, or to the acceleration of the jet after the break-out (Segalis & Ori 2001; Sago et al. 2004a; Sun et al. 2012; Akiba et al. 2013; Birnholtz & Piran 2013; Du et al. 2018; Leiderschneider & Piran 2021a). These studies have shown that the amplitude of the GW increases with time due to the continuous injection of energy into the jet from the central engine, or due to the jet acceleration once it expands through the environment.

Previous studies estimating the GW memory from GRB jets were based on simple analytical and/or semi-analytical estimations. Although these calculations provide a qualitative understanding of the GW memory, quantitative estimations can be obtained only by detailed numerical calculations. In this work, we study the propagation of relativistic jets associated to LGRBs through the progenitor star, and its propagation through the wind of the progenitor star up to large distances ($10^{13} \text{ cm}$). We compute the resulting GW signal as a function of time and observer angle (with respect to the main axis of the jet). We also consider the possible presence of a supernova (SN) component, and how its GW signal is affected by the presence of the jet. As we will discuss below, although the simulations presented refer to the LGRB case (in which the jet is propagating through a massive progenitor star), the expected GW signal will be qualitatively similar in short GRBs.

The paper is structured as follows: in Section 2 we discuss the initial conditions of the hydrodynamic simulations, and the methods used to compute the GW directly from the simulations. Section 3 presents the results of the calculations, in particular, the jet dynamics as the jets propagate through the progenitor and its environment, and the calculation of the resulting GW. In section 4 we discuss our results, in the context of present and future GW detectors. Our conclusions are presented in section 5.

### 2 METHODS

#### 2.1 Numerical simulations

We study the first 300 s of evolution of relativistic jets associated to gamma-ray bursts by running a series of numerical simulations. The simulations employ the adaptive mesh refinement code Mezzal (De Colle et al. 2012), which integrates the special relativistic, hydrodynamics equations by employing a second-order (both in space and time), shock-capturing scheme.

We run five numerical simulations (see Table 1) in two dimensional (2D), cylindrical (axisymmetric) coordinates. In all the models, the computational box extends from $(r, z) = 0 \text{ cm}$ to $(r_{\text{max}}, z_{\text{max}}) = 10^{13} \text{ cm}$, and is resolved by employing $40 \times 40$ cells at the coarsest level of refinement and 17 levels of refinement, corresponding to a maximum resolution of $\Delta r_{\text{fin}} \approx \Delta z_{\text{fin}} = 3.8 \times 10^9 \text{ cm}$. Long GRBs are associated to broad-line, type Ic SNe, which are produced during the collapse of massive, compact Wolf-Rayet stars. We set the density in the computational box by considering the pre-collapse stellar models 12TH and 16TH taken from Woosley & Heger (2006). These models correspond to stripped-envelope progenitor stars with stellar masses $M_* = 9.23 M_\odot$ and 11.45 $M_\odot$ and stellar radii $R_* = 4.5 \times 10^{10} \text{ cm}$ and $9 \times 10^{10} \text{ cm}$ for the 12TH and the 16TH models respectively. For radial distances $r > R_*$, we consider a medium shaped by the wind of the Wolf-Rayet progenitor, i.e. with a density $\rho(r) = M_\odot/(4\pi r^2 v_w)$, being $M_w = 10^{-5} M_\odot \text{ yr}^{-1}$ and $v_w = 10^4 \text{ km s}^{-1}$ typical values for the mass-loss rate and the velocity of the wind from a Wolf-Rayet star (e.g., Vink 2011). The pressure in both the star and the wind is negligible (as in strong shock it does not affect the shock dynamics) and it is set as $p = 10^{-5} \text{ pc}^2$.

The relativistic jet is injected from an inner boundary located at $r_{\text{in}} = 5 \times 10^2 \text{ cm}$, with a jet Lorentz factor $\Gamma_j = 10$. The jet energy is largely dominated by thermal energy, with the jet pressure given as $P_j = \rho_j c^2 (\Gamma_j^{-1} - 1)/4$, being $\rho_j$ the jet mass density and $\Gamma_j = 100$ the final jet velocity, eventually achieved once the jet breaks out of the star and accelerates by converting its thermal to kinetic energy. A more detailed description of the initial conditions can be found in Urrutia et al. (2022). In two of the simulations (differing by the presence of a SN), we inject the jet during $t_j = 10 \text{ s}$, such that its total energy is $E_j = 10^{51} \text{ erg}$ and its luminosity is $L_j = 10^{49} \text{ erg s}^{-1}$, while in one model we inject the jet during $t_j = 2.5 \text{ s}$ with a total energy of $E_j = 10^{52} \text{ erg}$, corresponding to a much larger luminosity $L_j = 4 \times 10^{51} \text{ erg s}^{-1}$. In all these cases the jet opening angle is $\theta_j = 0.1 \text{ rad}$ and, as we will discuss in detail below, the jet successfully breaks out of the star and accelerates to highly relativistic speeds through the progenitor wind. We also consider a simulation in which the jet also lasts for $t_j = 10 \text{ s}$, with a total energy $E_j = 10^{51} \text{ erg}$, but with a larger jet opening angle $\theta_j = 0.2 \text{ rad}$. In this case, the jet will not be able to break out successfully from the star. We refer to this case as the choked or failed GRB case.

To study how the GW memory signal is affected by the presence of both a SN and a GRB, we also inject, in two of the five simulations (see Table 1), a supernova shock front from the same inner boundary

| Scenario            | $t_{\text{inj}}$ (s) | Energy (erg) | Label           | Progenitor |
|---------------------|----------------------|--------------|-----------------|------------|
| Successful jet      | 10                   | $10^{51}$    | JET-S1          | 12TH       |
| Successful jet      | 2.5                  | $10^{52}$    | JET-S2          | 16TH       |
| Failed jet          | 10                   | $10^{51}$    | JET-F           | 12TH       |
| Supernova           | 1                    | $10^{52}$    | SN              | 12TH       |
| Jet + Supernova     | 10                   | $10^{51}$    | JETSN-S         | 12TH       |

Table 1. Numerical simulations presented in this paper. The columns refer to: the scenarios considered, the time during which the jet/SN is injected into the computational box, its energy, and the label of each model (see the main text for a detailed description of each model). The progenitors 12TH and 16TH correspond to 12 $M_\odot$ and 16 $M_\odot$ initial masses, respectively.
at \( t = 0 \) s. Following De Colle et al. (2022) and Urrutia et al. (2022), we inject, from \( r_{\text{in}} \), a SN shock front during \( t_{\text{sh}} = 0.1 \) s, with a total energy of \( E_{\text{sh}} = 4 \times 10^{51} \) erg and a mass \( M_{\text{sh}} = 0.1 M_\odot \). We assume that 10\% of the SN energy is thermal, while 90\% is kinetic. Type Ic, broad-line SNe associated to Long GRBs present a certain degree of asymmetry (as inferred from polarization measurements, see, e.g., Maund et al. 2007; Tanaka et al. 2017, or by the analysis of line emission during the nebular phase, see, e.g., Taubenberger et al. 2009). To qualitatively reproduce this asymmetry, we set an angular dependence for the energy injected in the SN as \( E_{\text{SN}}(\theta) \propto \cos^2 \theta \), being \( \theta \) the angle polar measured with respect to the \( x \)-axis.

In the models in which both SN and jet are present, the jet is injected with a delay of 1 s with respect to the SN. The origin of the SN associated to GRBs is debated. The models proposed include a wind from a collapsar disk (MacFadyen & Woosley 1999), energy ejection from a magnetar (e.g., Metzger et al. 2015), or the jittering wind from a collapsar disk (MacFadyen & Woosley 1999), energy injected with a delay of 1 s with respect to the SN. The origin of the fluid element has been fixed as vertical of radial, in this paper we leave it completely general, and determined directly from the numerical simulations.

Bragsinskii & Thorne 1987; Segalis & Ori 2001 obtained explicit expressions for the GW memory polarization components \( h_+ \) and \( h_\times \) in the transverse-traceless (TT) gauge. The explicit expressions for \( h_+ \) and \( h_\times \) are:

\[
\begin{align*}
  h_+ &\equiv h_{TT} = -h_{yy} = \frac{2 G E}{c^4} \frac{\beta^2 \sin^2 \theta_\nu}{1 - \beta \cos \theta_\nu} \cos 2\Phi, \\
  h_\times &\equiv h_{xy} = h_{yx} = \frac{2 G E}{c^4} \frac{\beta^2 \sin^2 \theta_\nu}{1 - \beta \cos \theta_\nu} \sin 2\Phi,
\end{align*}
\]

where \( G \) is the gravitational constant, \( D \) the distance between the object and the observer, \( \beta = v/c \) is the velocity normalized with respect to the speed of light, \( \theta_\nu \) is the angle between the direction of the observer and the direction of the velocity vector, i.e.

\[
\cos \theta_\nu = \hat{n} \cdot \hat{\nu} = (\beta R \sin \theta_\nu \cos \phi + \beta_s \cos \theta_\nu)/\beta,
\]

\( E = (\rho H \gamma^2 c^2 - p) \Delta V \) is the energy of the fluid element, being \( \rho \) the mass density, \( \gamma \) the Lorentz factor, \( p \) the pressure, \( H = 1 + 4p/(\rho c^2) \) the specific enthalpy (by considering a hot plasma with an adiabatic index \( \Gamma_{\text{ad}} = 4/3 \)), \( \Delta V \) the volume of the fluid element which induces the metric perturbation, and \( \Phi \) is the polar coordinate, measured in the observer frame.

To find the value of \( \Phi \), we consider the following geometrical relations between the angles evaluated in the observer frames (indicated by the capital Greek letters \( \Phi \) and \( \Theta \) for the azimuthal and polar directions respectively) and those in the laboratory frame (e.g., the frame centered on the central engine; see, Akiba et al. 2013):

\[
\begin{align*}
  \cos \Theta &\equiv \hat{n} \cdot \hat{r} = \sin \theta \cos \phi \sin \theta_{\text{obs}} + \cos \theta \cos \theta_{\text{obs}}, \\
  \sin \theta \sin \phi &\equiv \sin \Theta \sin \Phi, \\
  \sin \theta \cos \phi &\equiv \sin \Theta \cos \Phi \cos \theta_{\text{obs}} + \cos \Theta \sin \theta_{\text{obs}},
\end{align*}
\]

which lead to

\[
\begin{align*}
  \cos(2\Phi) &= \frac{\sin(2\Phi)}{\sin^2 \Theta}, \\
  \cos(2\Phi) &= \frac{(\sin \theta \cos \phi \cos \theta_{\text{obs}} - \cos \theta \sin \theta_{\text{obs}})^2 - \sin^2 \phi}{\sin^2 \Theta}.
\end{align*}
\]

In the case of an on-axis observer, i.e. located along the \( z \)-axis, \( \theta_{\text{obs}} = 0 \), and we recover the obvious result \( \Phi = \phi \). In this case, for the symmetry of the problem, we get \( h_+ = h_\times = 0 \).

On the other hand, in the case of a particle moving along the \( z \) axis, we have \( \theta = 0 \), which implies \( \sin(2\Phi) = 0 \), \( \cos(2\Phi) = 1 \), and \( h_\times = 0 \). Also, being \( \beta = \beta_s \) in this case, we get \( \cos \theta_\nu = \cos \theta_{\text{obs}} \), and

\[
\beta^2 \sin^2 \theta_\nu = \beta^2 (1 - \cos^2 \theta_{\text{obs}}),
\]

This function has a maximum \((\gamma = 2)/\gamma \) at \( \cos \theta_{\text{obs}} = \beta \gamma / (\gamma + 1) \). In particular, for an ultra-relativistic flow, \( \gamma \gg 1 \), and the maximum \((\gamma = 2)\) is at \( \theta_{\text{obs}}^2 \sim 2/\gamma \). Thus, the GW signal determined from equation (1) is weakly boosted along the direction of the observer, except for observers located nearly along the jet axis (in which case \( h_+ = 0 \) as shown above).
In practice, the calculation of the GW signals proceeds as follows. We save a large number of snapshots of our two-dimensional, axisymmetric simulations at \( t = t_i \), with \( i = 1, \ldots, 600 \) (i.e., 600 outputs, spaced by 0.5 s, during the total integration time of 300 s).

The data files include the positions \( R, Z \) and the size \( \Delta V \) of each cell, in addition to the thermal pressure, mass density and the velocity vector. Then, we remap each cell along the azimuthal \( \phi \) direction. We compute the values of \( h_+ \) and \( h_\times \) (to verify that it remains \( \sim 0 \) at all times). Then, we compute the arrival time of the GW signal generated by that particular cell, that is,

\[
t_{\text{obs}} = t_i - (R/c) \cos \phi \sin \theta_{\text{obs}} - (z/c) \cos \theta_{\text{obs}}.
\]

We divide the time-space in the observer frame in \( N_{\text{obs}} \) equally-spaced time-bins. Then, we add the contribution of a certain cell to the corresponding time bin to determine \( h_+ \) as a function of the observer time.

3 RESULTS

3.1 Jet dynamics

In this section, we describe the dynamics of the system for the different numerical simulations. Figure 2 shows three different evolutionary times (at 7 s, 14 s and 300 s from the top to the bottom panels) for the successful jet without and with an associated SN, for the choked jet and for a SN-like explosion (from the left to the right panels). The model JET-S2 is qualitatively similar to the model JET-S1 (although the jet breaks out on a shorter timescale, as we will discuss below) and it is not shown in the figure.

As shown in figure 2 (top panels), the jet expands through the stellar material. At the shock front, the stellar material is heated and accelerated by the forward shock, while (in the lab frame) the jet material, launched from the central engine and propagating through the jet channel, is heated and decelerated by the reverse shock. The hot, entropy rich post-shock material expands sideways into the progenitor star, producing an extended cocoon (see, e.g., Bromberg et al. 2011b; Gottlieb et al. 2018), which helps collimating the jet. Despite this extra collimation, the jet velocity is sub-relativistic while the jet moves through the star (see figures 2 and 3).

Once the jet breaks out from the stellar surface (figure 2, left-middle panel), the cocoon expands laterally quickly engulfing the low density region surrounding the progenitor star, while the entropy rich material, close to the jet axis, accelerates converting thermal to kinetic energy. The cocoon material remains strongly stratified both along the radial and the polar direction, moving at mildly relativistic speeds (close to the jet axis) and sub-relativistic speeds close to the equatorial plane.

Once the jet expands to larger distances (figure 2, left-bottom panel), the fast moving material remains confined into a thin shell with size \( \gtrsim t_j c (\sim 3 \times 10^7) \) in the successful jet simulations shown.

Figure 2. Two-dimensional plots of \( \Gamma^2 \rho H c^2 \). Left to right panels: successful jet, jet associated to a supernova, choked jet and SN explosion, respectively. Top to bottom panels: different evolutionary phases of the system, corresponding to 7 s, 14 s and 300 s.
in the figure), where $t_j$ is the jet during which the jet is injected by the central engine. On the other hand, the cocoon begins to decelerate, specially close to the equatorial plane where the cocoon energy is lower, as indicated by the presence of Rayleigh-Taylor instabilities visible in figure 2.

The simulation of the jet associated to a SN is qualitatively similar to the one without the SN. In this simulation, the jet is launched with a delay of 1 s with respect to the SN. After a few seconds, the jet head reaches the SN shock front, breaking out of it and expanding through the progenitor star. The late phases are also similar to the case of a jet without a SN discussed above, except that, at large times, the SN shock front breaks out from the progenitor star into the jet cocoon.

We notice that the general outcome of the system depends on the time when the jet breaks out from the SN. If, for instance, the jet energy, opening angle and duration are such that the SN shock front breaks out first from the stellar surface, then the jet will remain trapped inside the expanding SN, depositing its energy in the deep layers of the SN ejecta. The result of the interaction between the SN, the jet and its cocoon leads to a rich landscape of scenarios which have not been studied in detail yet (see De Colle et al. 2022, for a qualitative description).

The third column of figure 2 shows the case of a choked jet. In this case, the jet opening angle is larger by a factor of $\sim 2$, so that the luminosity per unit solid angle drops by a factor of $\sim 4$. Then, the jet duration (10 s) is not large enough for the jet to break through the progenitor star. Once the jet power is switched off, the relativistic moving material crosses the jet channel in a time $R_\beta/c \sim \beta_h t_j$, being $R_\beta$ and $\beta_h \sim 0.1 - 0.3$ c the head position and velocity, and $t_j$ the jet injection time. Once all the jet material arrives to the head of the jet, the jet quickly expands laterally and decelerate. Then, it can break out from the stellar surface into a more spherical explosion (see the bottom panel of the figure).

The last column of figure 2 shows a nearly spherical explosion, qualitatively representing a SN explosion. In this case, the shock breakout is also nearly spherical. Nevertheless, we notice that realistic 3D simulations of SN explosions show a much more asymmetric, turbulent behaviour not captured in these 2D simulations.

Figure 3 shows the evolution of the head of the jet ($z_{sh}$ hereafter) and its average velocity, as a function of time, for the different models. As discussed above, the velocity of the shock front is sub-relativistic inside the progenitor star. Once the shock front approaches the stellar surface, it quickly accelerates due to the large density gradients. This is visible both in the top panel of figure 3, where the slope of the curves showing $z_{sh}$ vs $t$ becomes steeper just after the breakout (represented by the vertical dotted lines), and in the bottom panel, where the average velocity increases quickly after the breakout. Then, the SN and the choked jet cases achieve a velocity of $\sim 0.2$ c, while the successful jets (with or without SN associated) continue accelerating until the end of the simulation. As mentioned before, the acceleration process is related to the conversion of thermal to kinetic energy. At the end of the process, the jet head will arrive to a terminal Lorentz factor $\Gamma_j \sim E_j/M_j c^2 \gg 1$.

Finally, we notice that the high luminosity model is qualitatively similar to the successful jet model, with the main difference being the timescales for the different phases to occur. As the luminosity is larger, the jet duration is shorter, and the progenitor star is smaller, the jet will break out from the stellar surface in a much smaller time, and it will accelerate faster to its final velocity (see figure 3).

### 3.2 GW emission

To understand where the GW signal originates, we show in figure 4 the amplitude of the GW signal $h_+$ as a function of $z$, at different times, i.e., integrating over the radial and azimuthal directions. During the first 10 s, the jet is continuously injected into the computational box, and the jet energy increases along the jet channel (see figure 3). As evident in the top panel of the figure, the GW signal is produced along most of the jet channel. The small fluctuations correspond to the presence of recollimation shocks. As the jet pressure is larger than the cocoon pressure, the jet expands laterally into the cocoon, until when both pressures are approximately equal. Then, a recollimation shock is created, pinching the jet onto the jet axis. This produces strong fluctuations in the jet velocity and energies, which lead to the observed fluctuations in the GW signal seen in figure 3.

Once the jet breaks out from the star, the energy and velocity into the emitting region becomes more uniform. As discussed above, the jet velocity increases strongly achieving a Lorentz factor close to the terminal value (set to 100 in the simulation, see the methods section). While a fraction of the total energy is stored in the cocoon, the cocoon does not contribute significantly to the GW signal, as it moves at most at mildly relativistic speeds. This can be seen in the bottom panel of figure 3, in which it is evident that the region emitting the GW signal is limited to the fast moving jet material.
As soon as the jet luminosity starts dropping at $t = 9\,\text{s}$, the GW amplitude quickly drops with time. At larger distances, the GW amplitude increases again due to the acceleration of the jet material. Once the jet achieves its terminal velocity, that is, after transforming most of its thermal to kinetic energy, the GW amplitude achieves a second peak before dropping again with time. Unfortunately, the second peak is not completely resolved in our simulations, as it happens (in the lab frame) at times larger than the simulated 300 s. Then, the value of the GW signal at the second peak should then be taken as a lower limit to the real value. In the lab frame, the dependence on the observing angle is weak. Except for observer located exactly on the jet axis, for which $h\gamma = 0$, there is a difference $\lesssim 2$ between the values of $h\gamma$ computed at different observer angles.

The central and right panels of figure 5 show the same calculations, but in the observer frame. A qualitative understanding of the behaviour of $h\gamma$ in this case can be attained by assuming that all GW signal is coming from a region very close to the jet axis. In this case, $R = 0$, and equation (9) reduces to

$$t_{\text{obs}} = t_{\text{n}} - (z/c) \cos \theta_{\text{obs}}.$$  

Then, assuming that the emission comes from a single point source moving with constant velocity $\beta$, we get

$$t_{\text{obs}} = t \left(1 - \beta \cos \theta_{\text{obs}}\right).$$  

For observers located at large observing angles, $\theta_{\text{obs}} \gg 0$, $t_{\text{obs}} \sim t$ and the GW arrival time is the same as the time when the signal is produced (except of course for the time $D/c$ needed for the signal to propagate from the source to the Earth). On the other hand, for observers located at small observing angles, $\cos \theta_{\text{obs}} \sim 1 - \theta_{\text{obs}}^2/2$, and $t_{\text{obs}} \sim t(1 - \beta + \theta_{\text{obs}}^2/2) \sim t/(2\Gamma^2)(1 + \Gamma^2\theta_{\text{obs}}^2)$. Then, for $\theta_{\text{obs}} \ll 1/\Gamma \sim 6^{\circ}(\Gamma/10)^{-1}$, we have $t_{\text{obs}} \sim t/2\Gamma^2$ and the GW signal arrival time is reduced by a factor of a few hundred with respect to the GW signal as seen in the lab frame, while for $\theta_{\text{obs}} \gg 1/\Gamma$, we have $t_{\text{obs}} \sim t\theta_{\text{obs}}^2/2$.

As shown in figure 5, the GW signal is very different in the observer frame with respect to the lab frame. Consistently with the discussion above, the second peak moves to increasingly smaller observer times for smaller observer angles. So, at $\theta_{\text{obs}} = 5^{\circ}$, the second peak drops substantially, overlapping the first peak. As the simulations output files are saved every 0.5 s, this implies that, for this observer angle, the two peaks are separated by less than 0.5 s, while, e.g., the second peak moves at $\sim 12$ s, $\sim 22$ s for observers located at $\theta_{\text{obs}} = 10^{\circ}$, $20^{\circ}$ respectively. As more GW radiation arrives during a shorter time, the amplitude of the two peaks increase substantially, specially for observer located at small observer angles.

The right panel shows that the maximum in the GW signal is obtained when $\theta_{\text{obs}} = 5^{\circ}$, i.e., for observer located at the edge of the jet. Although it is barely visible due to the size of the bins in time (0.5 s as mentioned before), the break-out from the progenitor star produces a small change in the slope of the curves.

Figure 6 shows the GW amplitude $h\gamma$ for the other models considered. The successful jet models with and without an associated SN are very similar (compare the upper left of figure 6 with the middle panel of figure 5). The GW produced by the luminous, shorter jet shown in the second panel from the top also presents a similar behaviour, but with peaks located at shorter times, and a much larger

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1 The jet injection time is $t_j = 10$ s, but, to avoid numerical problems related with the strong rarefaction wave produced once the jet is switched off, we set a jet luminosity dropping linearly between 9 s and 10 s.
amplitude at peak $\sim 13000 \text{ cm vs } \sim 650 \text{ cm}$. In the case of the failed jet, $h_+^\text{obs}$ increases for $t \leq t_j$, to then drop on a short timescale $\lesssim 0.5 \text{ s}$. The peak achieved for this model is $\sim 2-3$ order of magnitude smaller than in the other cases. Finally, the GW signal produced by a SN is several orders of magnitude smaller, as the velocity of the SN shock front remains always sub-relativistic. Anyway, we notice that our simulations do not capture the initial, larger GW signal produced by the early propagation of the SN shock front.

4 DISCUSSION

In this paper, we have presented numerical simulations of the propagation of relativistic jets through a massive, progenitor star, the break-out and the expansion of the jet up to distances $\sim 10^{13} \text{ cm}$, and computed the resulting GW signal as a function of the observer angle. Previous studies of GW memory from GRB jets have focused on the the neutrinos produced by the central engine during the jet formation (Hiramatsu et al. 2005a; Suwa & Murase 2009; Kotake et al. 2012), on internal shocks (Akiba et al. 2013) and on the jet acceleration (Akiba et al. 2013; Birnholtz & Piran 2013; Yu 2020; Leiderschneider & Piran 2021b). These studies have used a simple analytic description of the jet, often taken as an accelerating point mass. Although our results qualitatively confirm previous findings, our numerical simulations allow us to give a quantitative prediction of the expected GW signal.

The GW signal is “anti-beamed” (Segalis & Ori 2001; Sago et al. 2004a; Birnholtz & Piran 2013; Leiderschneider & Piran 2021b). Nevertheless, we notice that the GW signal is strongly suppressed only for observer located at $\theta_\text{obs} \approx 0^\circ$. As shown in the right panel of figure 5, it increases for larger observer angles, peaking at $\theta_\text{obs} \sim \approx 2^\circ$ (e.g., the GW signal is $\sim 1/2$ of the peak at $\theta_\text{obs} = \theta_j/2$). In contrast with the prediction obtained by considering simple accelerating point masses, then, we expect to see GWs associated to GRBs seen nearly on-axis.

The other clear feature resulting from our models is the presence of a double peak structure in the GW signal, due to two characteristic acceleration phases: a) inside the progenitor star, as the jet move through a lower density medium as it approaches the stellar surface; and b) after the breakout, as the jet accelerates converting thermal to kinetic energy. The timescales of the two peaks reflect directly the duration of the jet $t_j$ (the first peak) and the observer angle (with larger timescales corresponding to larger $\theta_\text{obs}$, see figures 5 and 6).

As discusses above, the slope of the GW signal before and after the first peak (see, e.g., figure 6) depends on the stellar structure and on the jet luminosity. For instance, we can expect a shallower increase for a jet with a luminosity decreasing with time. Thus, observations of $h_+$ by future detectors will provide direct information on the central engine activity (e.g., jet duration and luminosity history), the stellar structure, the observer angle and the acceleration process after breakout.

Figure 7 shows the Amplitude Spectral Density (ASD) of the GW from GRB jets compared with the instrumental limits. In agreement with previous estimates (Sago et al. 2004b; Hiramatsu et al. 2005b; Suwa & Murase 2009; Kotake et al. 2012; Sun et al. 2012; Akiba et al. 2013; Birnholtz & Piran 2013; Du et al. 2018; Yu 2020; Leiderschneider & Piran 2021b), GRBs are expected to be undetectable with LIGO and KAGRA. Given the (uncertain) expected GRB rate of $100-1000 \text{ Gpc}^{-3} \text{ yr}^{-1}$ (see, e.g., Fryer et al. 2002; Wanderman & Piran 2010; Cao et al. 2011; Abbott et al. 2017a), galactic GRBs, eventually detectable with the Einstein Telescope and/or eLISA, are very rare. Future low-frequency instruments, as DECIGO and BBO, will easily detect GRBs located at $\sim 40 \text{ Mpc}$ ($\sim 1 \text{ yr}^{-1}$).

The typical interesting range of distances for memory detectability in the case of SNe is considered to be the galactic radius. The range of possible amplitudes of the memory production from type IIb CCSNe is still under study (Richardson et al. 2022b). The rate of CCSNe in our galaxy was estimated to be up to a few each century. In this paper we discuss a different scenario where the amplitude of the memory component of the GW emission is in the range $h_+ D$ up to thousands. This means that much larger distances could be considered for future interferometers. As shown in table 2, on-axis GRBs (observed at $\theta_\text{obs} \sim 5^\circ$) could be seen up to distances $\lesssim 1.2 \text{ Gpc}$ by DECIGO and BBO, while off-axis GRBs can be detected up to distances $\sim 40 \text{ Mpc}$. Then, the GWs of $\sim 60$ on-axis GRBs per year can be detected by these instruments, $\sim 20 \text{ yr}^{-1}$ at larger angles ($10^\circ \lesssim \theta_\text{obs} \lesssim 40^\circ$), and $\sim 1 \text{ yr}^{-1}$ completely off-axis.

While the GW memory generated by GRBs is unlikely to be detected by network of detectors using LIGO (Aasi et al. 2015), VIRGO (Acernese et al. 2015) and KAGRA (Aso et al. 2013), technology improvements will notably increase the sensitivity of the future generation of GW detectors such as DECIGO and BBO during the next decades (see, e.g., Moore et al. 2014; Bailes et al. 2021).

2 These values were estimated by assuming a GRB rate of $10^3 \text{ Gpc}^{-3} \text{ yr}^{-1}$, and scale linearly with the (uncertain) GRB rates.
In addition to successful jets, producing the observed gamma-ray emission, other high energy transients are likely associated to a central engine activity and to the propagation of a relativistic jets, including low-luminosity GRBs (Campana et al. 2006; Soderberg et al. 2006; Starling et al. 2011; Margutti et al. 2013), relativistic SNe (Soderberg et al. 2010; Margutti et al. 2014; Milisavljevic et al. 2015), and X-ray flashes (Pian et al. 2006; Bromberg et al. 2011a; Nakar & Sari 2012). In addition, it has been suggested that SNe (in particular, broad-line type Ic) could be produced by the propagation of a choked jet (e.g., Piran et al. 2019; Soker 2022).

These events could be detectable at shorter distances. Our results show that the GW strain $h_+$ depends mainly on the jet luminosity and the jet velocity. Jets choked while deep inside the progenitor stars, as the one simulated in this paper, will have a very low signal (see figure 6, third panel from the top) as their velocity is only mildly relativistic when the jet is switched-off from the central engine. Nevertheless, jets lasting for longer times, i.e. arriving closer to the stellar surface before being choked, will accelerate to relativistic speeds producing signals similar to those of successful jets.

Finally, we notice that, while we have simulated relativistic jets leading to LGRBs (i.e., associated to the collapse of massive stars), a similar outcome is expected for SGRBs, associated to the coalescence of massive stars. These jets are expected to last for shorter times, to have smaller total energies and can move through smaller density media, so than they could achieves relativistic velocities on shorter timescales. Detailed numerical simulations are needed to understand whereas the expected signal would be larger for jets associated to LGRBs or SGRBs.

5 CONCLUSIONS

In this paper, we have presented numerical simulations of relativistic jets associated to long GRB. We have computed the resulting GW signal for successful jets, choked jets, and jets associated to a SN. In successful jets (accompanied or not by a SN), the GW signal is characterized by a double peak structure, with amplitudes $h_+D$ ranging from hundreds to several thousand. The first peak corresponds to the jet injection from the central engine, while the second peak corresponds to the jet acceleration while it breaks out from the star. In addition, the slope of the GW signals track directly the luminosity history of the GRB jets, and the structure of the progenitor star.

As GRBs are the product of collimated jets seen nearly on-axis, given the detected GRB rate, the volumetric rate depends on the jet angle and on the jet structure. Thus, the GRB volumetric rate is highly uncertain ($\sim 100-1000 \text{ Gpc}^{-5} \text{ yr}^{-1}$). As illustrated in figures 5 and 6, the GW signal presents a second peak which strongly depend on the observer angle. Thus, the observer angle can be determined precisely by observing the GW signal. In addition, by observing the associated multi-wavelength afterglows, the jet structure can be determined. Thus, observations of the GW signal will provide us with a precise estimate of the volumetric rate of GRBs.

The predicted GW signal is below the detection limits of LIGO, KAGRA and similar Earth-based detectors, and is expected to be seen by lower-frequency space-based detectors as BBO and DECIGO. Future detections of GWs from GRBs will provide information on optically thick regions impossible to explore by electromagnetic radiation, clarifying the jet duration, the structure of the progenitor star and the jet acceleration process. It is also worth pointing out that the GW detectability can be improved with a network of interferometers with the rough rule that the signal-to-noise ratio (SNR) achievable with a network of identical interferometers is the single
Figure 7. Comparison between the amplitude spectral density (ASD) GW signal computed from the successful jet (Jet-S2) launched during $t_j = 2.5$ s at $D = 1$ Mpc, and the detection limit of Ligo O4, Kagra, the Einstein Telescope, eLISA, DECIGO, the Big-bang Observatory (BBO) and the Advanced Laser Interferometer Antenna ALIA. The detection limits were taken from Moore et al. (2015). The GRB GW signal will be detectable by BBO up to $\sim 100$ Mpc at all observer angles, and by LISA up to distances $\lesssim 200$ kpc.

Table 2. The columns refer to: the observatories considered (see figure 7), the signal-to-noise ratio (SNR) for an observer observing the jet at $\theta_{\text{obs}} = 5^\circ$, $70^\circ$ and at a distance of 40 Mpc, the distance where SNR = 5, and the number of events detected per year along different solid angles. The values refer to the model JET-S2.

| Detector | SNR ($\theta_{\text{obs}}$) | Distance [Mpc] | $0^\circ - 10^\circ$ | Rate [$\text{yr}^{-1}$] |
|----------|---------------------|----------------|-----------------|-----------------|
| LIGO O4  | 0.0038              | 0.0304         | 0.1021          | $1.18 \times 10^{-14}$ | $1.53 \times 10^{-9}$ | $3.40 \times 10^{-9}$ |
| eLISA    | 0.0214              | 0.1710         | 0.0311          | $4.37 \times 10^{-15}$ | $2.97 \times 10^{-9}$ | $3.69 \times 10^{-9}$ |
| ALIA     | 1.6084              | 12.8676        | 0.7489          | $1.04 \times 10^{-4}$ | $9.73 \times 10^{-5}$ | $3.58 \times 10^{-2}$ |
| DECIGO   | 150.2016            | 1201.6127      | 37.6217         | 60.12            | 17.28             | 0.80              |
| BBO      | 152.0613            | 1216.4902      | 43.8227         | 63.43            | 20.41             | 1.02              |

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DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding author.

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