Comment on “Spin light of neutrino in matter: a new type of electromagnetic radiation”

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Abstract

We show that a criticism made in hep-ph/0610294 against our paper Mod. Phys. Lett. A21, 1769 (2006) has no ground. We confirm that all results of investigations of the so-called “spin light of neutrino (SLν)” are incorrect because of the medium influence on photon dispersion. With taking account of this influence, both the SLν radiation rate and the total power are zero for neutrinos in all real astrophysical conditions.

In an extended series of papers [1–11], there was proposed and investigated in detail the so-called “spin light of neutrino (SLν),” the process of the radiative neutrino transition $\nu_L \rightarrow \nu_R \gamma$, caused by the additional energy $W$ acquired in medium by the ultra-relativistic left-handed neutrino [12]. In their analysis the authors neglected the medium influence on the photon dispersion. It looked unnatural, because the neutrino dispersion was defined by the weak interaction while the photon dispersion was defined by the electromagnetic interaction. In the paper [13] we evaluated the effective mass acquired by a photon in the astrophysical conditions used by those authors, and we have shown qualitatively that consideration of the radiative neutrino transition $\nu_L \rightarrow \nu_R \gamma$ without taking account of the photon dispersion in medium should be incorrect. In a short reply [14], the authors wrote that our analysis was wrong because “the momentum conservation law has not been accounted for”. It was declared instead of performing an accurate application of this conservation law to the process.

As we have shown further in the papers [15, 16], just the energy-momentum conservation law with the photon effective mass taken into account, has led to the threshold value for the initial neutrino energy:

$$E > E_0 \simeq \frac{m_\gamma^2}{2W},$$

where $m_\gamma$ is the effective mass of the photon (plasmon) in medium. Evaluation of these threshold energies for different astrophysical situations shows that the value $E_0$ is always greater in orders of magnitude than the typical neutrino energy. Thus, the photon dispersion leaves no room for “spin light of neutrino”, except for a pure theoretical possibility when an ultra-high energy...
neutrino threads, e.g., a neutron star. The threshold neutrino energy for this case is $E_0 \simeq 10 \text{ TeV}$.

In spite of a pure methodical meaning of this problem, we have performed in the paper [16] an accurate calculation of the process $\nu_L \rightarrow \nu_R \gamma$ width, as well as the energy and angular distributions of the photons created. We have evaluated the mean free path $L$ of an ultra-high energy neutrino with respect to the process, and we have obtained

$$L \gtrsim 10^{19} \text{ cm},$$

(2)
to be compared with the neutron star radius $\sim 10^6 \text{ cm}$. This obviously illustrates the extreme weakness of the effect considered.

In a recent preprint [17] the authors have continued a discussion. They have totally ignored the basic point of our kinematical analysis, namely, the conclusion on the threshold behaviour of the process which closes the effect of “spin light of neutrino” in real astrophysical situations. The main objections by the authors [17] to our paper [16] were fixed on our expression (18) for the amplitude squared, which was ill-defined, in their opinion. First, they made a statement that it was not positively-defined, and second, they asked a puzzling question, how this expression could be obtained at all. Now we show, that the amplitude squared was defined well enough, and that its positivity was unquestionable inside the kinematical region of the process. We present also some details of calculation of the amplitude squared, which could be helpful for someone, while it should be a standard exercise for graduate students.

At first sight, the positivity of the amplitude squared is not obvious:

$$|\mathcal{M}|^2 = 4 \mu_\nu^2 E^2 \left[ 2 W^2 \left( 1 - \frac{\omega}{E} \right) - m_\gamma^2 \sin^2 \theta \right].$$

(3)

Here, $E = |\vec{p}|$ and $\omega$ are the neutrino and plasmon energies, $\theta$ is the angle between the initial neutrino momentum $\vec{p}$ and the plasmon momentum $\vec{k}$.

Really, the plasmon mass $m_\gamma$ in the second negative term is much greater than the Wolfenstein energy $W$. However, one should wonder what is the restriction on the $\theta$ angle arising from the above-mentioned energy-momentum conservation law. Similarly, the differential probability for the classic weak process of muon decay contains the expression $(3 m_\mu - 4 E_e)$, however, nobody doubts about its positivity, knowing the range of the electron energies $E_e$.

As for the “problem” of positivity of Eq. (3), one should analyse the spatial distribution of final photons, and see that they are created inside the narrow cone with the opening angle $\theta_0$, see Eq. (21) of [16]:

$$\theta < \theta_0 \simeq \frac{\varepsilon - 1}{\varepsilon} \frac{W}{m_\gamma},$$

(4)
where $\varepsilon = E/E_0$, $E_0$ is the threshold neutrino energy [11].

There could be another hint for an attentive reader about the positivity of Eq. (3). Integration in Eq. (19) of [16] was performed with the $\delta$ functions only, which means just the substitutions, and it led to Eq. (20). May be, it would be easier to check the positivity of the function $f(x, \varepsilon)$. Surely, a detailed quantitative analysis confirms these qualitative arguments.

Now let us turn back to the calculation of the amplitude squared (3). Let $p'$ and $q$ are the four-momenta of the final right-handed neutrino and photon, respectively, with $p'^2 = 0$ (we neglect the neutrino vacuum mass $m_\nu$ in our analysis), and $q^2 = m_\gamma^2$.

The initial left-handed neutrino acquires in medium the additional energy $W$, and its four-momentum can be written as $P^\alpha = p^\alpha + W u^\alpha$, where $P^\alpha$ is the neutrino four-momentum in medium, while $p^\alpha = (E, \vec{p})$ would form the neutrino four-momentum in vacuum, with $p^2 = 0$. The four-vector $u^\alpha$ of plasma velocity in its rest frame is $u^\alpha = (1, 0)$.

From the Lagrangian (17) of [16] one readily obtains the amplitude of the process $\nu_L \rightarrow \nu_R \gamma^{(\lambda)}$ with a creation of the plasmon with the polarization $\lambda$:

$$\mathcal{M}^{(\lambda)} = \mu_\nu \left( \bar{u}_R \gamma_\lambda u_L \right),$$

(5)
where \( u_L \) and \( u'_R \) are the bispinor amplitudes for the initial left-handed and the final right-handed neutrinos. One should take care of the four-momentum which the bispinor amplitude \( u_L \) depends on. To define it, one should write down the Dirac equation

\[
(\not{\hat{P}} - \Sigma) u_L = 0, \tag{6}
\]

where \( \Sigma \) is the neutrino self-energy operator in plasma, \( \Sigma = W \hat{\gamma} (1 - \gamma_5)/2 \). In the plasma rest frame one has \( \Sigma = W \gamma_0 (1 - \gamma_5)/2 \). Substituting the zero component \( P_0 = E + W \) and \( \Sigma \) into Eq. (6), and taking into account that \((1 - \gamma_5)/2\) \( u_L = u_L \), one obtains:

\[
(E \gamma_0 - \not{\hat{p}} \gamma) u_L = 0. \tag{7}
\]

Thus, the bispinor amplitude \( u_L \) depends on \( p^a = (E, \hat{p}) \), \( E = |\hat{p}| \).

The amplitude (5) squared

\[
|\mathcal{M}^{(\lambda)}|^2 = \mu_{\nu}^2 \text{Tr} \left[ \rho_L(p) \bar{\xi}^{(\lambda)} \hat{q} \rho_R(p') \bar{\xi}^{(\lambda)} \right], \tag{8}
\]

with the neutrino density matrices substituted, \( \rho_L(p) = u_L \bar{u}_L = \hat{p} (1 - \gamma_5)/2 \), \( \rho_R(p') = u'_R \bar{u}'_R = \hat{p}' (1 + \gamma_5)/2 \), is:

\[
|\mathcal{M}^{(\lambda)}|^2 = 2 \mu_{\nu}^2 \left[ 2(pq) (p'q) - q^2 (pp') - 2 q^2 \left( p\bar{\xi}^{(\lambda)} \right) \left( p'\bar{\xi}^{(\lambda)} \right) \right]. \tag{9}
\]

Using the energy-momentum conservation law and keeping in mind that \( E > E_0 \gg W \), one obtains:

\[
(pq) = W (E - \omega) + m^2_{\tau}/2, \quad (p'q) = W (E - m^2_{\tau}/2), \quad (pp') = W \omega - m^2_{\tau}/2. \tag{10}
\]

Substituting Eqs. (10) into Eq. (9) and summarizing over the transversal plasmon polarizations:

\[
\sum_{\lambda} \left( p\bar{\xi}^{(\lambda)} \right) \left( p'\bar{\xi}^{(\lambda)} \right) = E^2 \sin^2 \theta, \quad \sum_{\lambda} |\mathcal{M}^{(\lambda)}|^2 = |\mathcal{M}|^2, \tag{11}
\]

one readily obtains the amplitude squared (3).

As is seen from [14, 17], the authors believe that they have shown in Refs. [7, 9] that “for the case of high-energy neutrinos the matter influence on the photon dispersion can be neglected” because “plasma is transparent for electromagnetic radiation on frequencies greater than the plasmon frequency”. It is rather naive consideration. Really, one can see that from the side of kinematics the discussed process \( \nu_L \to \nu_R \gamma \) in plasma is very similar to the process \( \bar{\nu}_e e^- \to \tau^- \bar{\nu}_\tau \), where the high-energy electron anti-neutrino scattered off the electron in rest, creates the \( \tau \) lepton. Neglecting the neutrino masses, one can see that the threshold value for the initial neutrino energy arises:

\[
E > E_0 \simeq \frac{m^2_{\tau}}{2 m_e}, \tag{12}
\]

to be compared with Eq. (11). The similarity is deliberately not accidental. Both inequalities are caused by the minimal value of the Mandelstam \( S \) variable which is equal to the mass squared of the heavy particle in the final state, \( m^2_{\tau} \) (or \( m^2_{\bar{\nu}} \) in our case). At the same time, the mass of the initial electron in rest is kinematically identical to the additional neutrino energy \( W \). Taking the approach of the authors [1–11], one should forget about the threshold (12) and conclude that the process \( \bar{\nu}_e e^- \to \tau^- \bar{\nu}_\tau \) is open if only the medium (vacuum in this case) is transparent for \( \tau \) leptons.

It is interesting to note that it was not the first case when the plasma influence was taken into account for one participant of the physical process while it was not taken for other participant. The history is repeated.
As E. Braaten wrote in Ref. [18]:

“...In Ref. [19], it was argued that their calculation for the emissivities from photon and plasmon decay would break down at temperatures large enough that \( m_\gamma > 2m_e \), since the decay \( \gamma \rightarrow e^+e^- \) is then kinematically allowed. This statement, which has been repeated in subsequent papers [20–23], is simply untrue. The plasma effects which generate the photon mass \( m_\gamma \) also generate corrections to the electron mass such that the decay \( \gamma \rightarrow e^+e^- \) is always kinematically forbidden.”

Thus, the authors [1–11] made the same mistake when they considered the plasma-induced additional neutrino energy \( W \) and ignored the effective photon mass \( m_\gamma \) arising by the same reason.

In the recent papers [24–26] the authors have extended their approach to the so-called “spin light of electron” (\( SLe \)), \( \epsilon_L \rightarrow \epsilon_R^\gamma \). It should be mentioned that in these papers the authors have repeated just the same mistake of ignoring the photon dispersion in plasma.

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