The overlapping generations model pioneered by Paul Samuelson is used to address an issue in Social Security. In the 1983 Social Security reform, Congress chose to build a substantial trust fund, with principal and interest both to be used for later benefits. That is, Congress chose payroll tax rates higher than pay-as-you-go levels while the baby-boomers were in the labor force in order to have payroll tax rates lower than pay-as-you-go while the baby-boomers were retired. The impact on national capital of these higher...
payroll taxes, with the implied trust fund buildup, has been controversial. The impact depends on the response of the rest of the government budget as well as the responses of individuals to these government actions. It also depends on the effects of future tax changes as well as initial tax changes. This paper explores a simple model distinguishing two types in each cohort - life-cycle savers and nonsavers, and allowing an income tax change to offset a fraction of the additional revenue from any payroll tax change. Analyzing a permanent trust fund increase, even if the unified budget is always balanced, the trust fund buildup increases national capital initially when payroll taxpayers have a lower propensity to save out of payroll taxes than income taxpayers do out of the income tax, as is plausible. The long run impact on capital depends on the fraction of the payroll tax revenue increase that is offset by an income tax decrease.
1 Introduction

Paul Samuelson has had an enormous impact on the public economics of the 20th century. In models with heterogeneity in the population, Pareto improvements are not possible from policies that are restricted to even vaguely resemble realism. Thus, the Bergson-Samuelson social welfare function plays a key role in the evaluation of alternative policies. Since this key contribution was discussed by Kenneth Arrow in the 1983 festschrift for Paul (Brown and Solow, 1983), I will merely say that I foresee no diminution in the importance of this contribution in the 21st century. Similarly, the formulation of public goods by Paul was discussed by Richard Musgrave in the same volume and I foresee no diminution in its importance either. Alas, the use of fiscal policy as part of the stabilization of an economy has moved out of contemporary public economics (and apparently macroeconomics as well). Thus his writings, celebrated by Tobin in the festschrift, must await a revival in this key topic for a future impact. As explained in the introduction to the 1983

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2 Arrow wrote: "The analysis of concepts that lie so close to the roots of the social essence of humanity can never be definitive, but certainly the formulation of Bergson and Samuelson profoundly affected the direction of all future thinking, at least by economists." Page 15.

3 Musgrave wrote: "Never have three pages had so great an impact on the theory of public finance." Page 141.

4 Tobin wrote: "In this appreciation of Paul Samuelson as macroeconomist I shall concentrate on his contributions to the methodology and substance of macromodel building.
festschrift, Paul’s development of the overlapping generations (OLG) model and applications to social security fell between the cracks of chapter assignments for that volume. Thus it is fitting that I provide an example of the use of that extraordinarily fruitful model in analysis of a social security question for this volume.5 This list of Paul’s direct contributions to public economics leaves out Paul’s enormous impact through his writings on both individual choice and equilibrium, which have affected all of economic theory, but that would take me astray.

But first, I want to say a few words about how 21st century public economics may differ from that of the 20th, and how that might affect the Samuelsonian legacy. Looking at recent research trends that I expect to continue, two developments stand out. One is rapid growth of interest in behavioral economics, while the other is the use of computers for calculations of far more complex examples, both deterministic and stochastic, than could

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5Samuelson’s role in the revival and extension of Ramsey pricing was not so central, but that topic will last as long as there are linear taxes. And his analysis of tax deductibility of economic depreciation (1964) while a big help in my analysis of adjusting income taxes for inflation (1975), has not generated much response that I am aware of. And his introduction of the Le Chatelier principle into economics (1947) helped my optimal tax paper with Mirrlees.
have been contemplated before. Computerized examples rely on theory in
the same way that simpler examples do. And while some bottom up interac-
tive simulations might make little use of the economic theory that Paul has
helped to build, overwhelmingly, to date, the calculations and simulations
have relied on the same basic theory. I do not expect that to change.

There are two complementary developments in theoretical behavioral eco-
nomics. One is the development of new models of individual choice that rely
heavily on empirical input from psychology. The second is the development
of equilibrium models that make use of simplified models of behavior that
are informed by both empirical findings on behavior and the first strand of
new individual behavioral modeling. Thus these models are not consistent
with the standard model. In Social Security, such modeling has gone on for
a considerable time. In particular there are a number of analyses employing
the OLG model while assuming nonstandard behavior in savings (and some-
times in labor supply as well). These still rely on the Samuelsonian OLG
model as before - changing the model of individual choice does not remove
the legacy. This model is an example of such modeling, offered as a tribute
to Paul.
In the 1983 Social Security reform, Congress chose to build a substantial trust fund, with principal and interest both to be used for later benefits. That is, Congress chose payroll tax rates higher than pay-as-you-go levels while the baby-boomers were in the labor force in order to have payroll tax rates lower than pay-as-you-go while the baby-boomers were retired. The impact on national capital of these higher payroll taxes, with the implied trust fund buildup, has been controversial. The impact depends on the response of the rest of the government budget as well as the responses of individuals to these government actions.\(^6\)

In the absence of an empirically supported, widely-accepted connection between Social Security and non-Social Security budgets, research has naturally considered the implications of alternative ways of modeling this connection. In particular, Elmendorf and Liebman (2000) analyzed the impact of Social Security savings on national savings under different assumptions as to the response of the rest of the budget to a Social Security surplus. Implicitly, they assumed a representative taxpayer and so did not distinguish between a payroll tax increase and an income tax cut that might be induced by the

\(^{6}\)I ignore any possible impact on employer-provided pensions.
payroll tax increase. Yet the distribution of payroll and income tax burdens by income level are very different and propensities to save by income level are also very different. The top quintile in earners in 1995 paid 71 percent of the individual income tax and 37 percent of the Social Security payroll tax.7

The 1983 legislation can be viewed as a commitment to finance the additional debt in the Trust Fund out of future income taxes insofar as it resulted in offsetting expenditure increases or tax decreases. That is, prefunding through the payroll tax should be seen as a commitment to workers, whether the government and the country save more overall or not. If tax changes are proportional to taxes and if income tax changes fully offset payroll tax changes (a balanced unified budget on the margin), then the legislation could be viewed in part (roughly one-third say, reflecting the differences in shares of the two taxes paid by low earners) as a transfer from current payroll tax

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7In 1995, the individuals and families in the top quintile of people in cash income have incomes above $71,510 (CBO, 1998). These are estimated to pay 71 percent of the individual income tax and 41 percent of social insurance taxes (Table 5). (They also pay 66 percent of the corporate income tax.) Social insurance taxes include the uncapped Medicare tax, and perhaps the unemployment insurance tax, as well as the capped Social Security tax. Ignoring unemployment insurance and using SSA data (2002) to convert the percentage of total payroll taxes into the percentage of Social Security payroll taxes (since almost all of Medicare taxes due to earnings above the Social Security taxable maximum of $61,200 are paid by the top quintile in cash incomes, we calculate as if all of it were), we estimate that those in the top quintile pay 37% of the relevant payroll tax. Thus approximately one-third of exactly offsetting income and payroll tax changes would be a redistribution between the top quintile and the other four quintiles.
payers to current income tax payers with an exactly offsetting (in present discounted value terms) future transfer from income tax payers to payroll tax payers. I believe that very little of the trust fund buildup of the 80’s and early 90’s resulted in offsetting budgetary changes. Since this belief is not held by all analysts, it is helpful to consider the model with a parametric level of offset.

This note contributes to evaluating the impact of the 1983 reform on national savings by considering a one-period rise in the payroll tax to permanently increase the trust fund, with the increased interest income used to finance a decrease in all future payroll tax rates. Since future social security budgets are balanced, it is assumed that the income tax rate decreases to partially offset the social security surplus in the first period and thereafter the income tax rate rises by enough to pay the increase in the interest owing on the national debt. That is, after the initial period it is assumed that both the Social Security budget and the unified budget are always balanced. The impact of these government actions on national capital is solved for the end of the initial period and for the asymptotic steady state (reached after all those alive in the initial period have died since the production technology

8For an interpretation of the historic record, see Diamond and Orszag (2004), pages 47-54.
is assumed to be linear). Capital is larger at the end of the initial period. Whether it is higher or lower in the asymptotic steady state depends on the fraction of the payroll tax revenue change that is offset by the income tax. A simple calculation suggests that the break-even point for long-run capital is with an 80 percent offset. In a setting of a temporary trust fund buildup, eventually the only effects are those of the income tax increase, thereby lowering national capital, before the model eventually returns to the same long-run capital as would have happened without the temporary trust fund buildup.\footnote{A longer working paper version of this paper includes examination of a temporary trust fund buildup, to follow more closely the plan for addressing the retirement of the baby boomers.}

These effects are derived in a two-types model (as in Diamond and Geanakoplos, 2003), where one group, called workers, does no savings while the other group, called savers, are standard life-cycle optimizers.\footnote{Two-types models can have very different results than representative agent models. For example, see Diamond and Geanakoplos (2003) on diversifying Social Security assets and Saez (2000) on taxing interest income. The contrast between types is stark and overstates the differences between them - many people doing little saving may have a small response to a tax change rather than zero and many people doing considerable savings may not be so responsive to future taxes as is assumed in the model.} For convenience the savers are assumed to plan constant consumption as they would with a discount rate equal to the interest rate. Reflecting the empirical observation that those with higher earnings have higher propensities to save (Dynan,
Skinner and Zeldes, 2004, Saez, 2002), and the patterns of payroll and income tax liabilities by earnings level, the model assumes that workers pay only payroll taxes and savers pay only income taxes. Thus the model should be interpreted in terms of the differences between the two types of taxes at the margin in response to a payroll tax excess over the level needed for pay-as-you-go.

The logic behind the effects in the initial period in both settings is clear. While the initial payroll tax increase comes fully out of worker consumption, the initial income tax cut is partially saved. This savings comes from a forward-looking spreading of a one-period income increase over consumption in all the remaining periods of life and from the assumed awareness that future income tax rates will be increased. While solved in a specific model, this result would follow more generally when payroll taxpayers have a lower propensity to save out of payroll taxes than income taxpayers do out of the income tax, as is plausible.

The model structure is presented in Section 3. Analysis of a permanent trust fund buildup is in Section 4, with concluding remarks in Section 5.
3 Model Structure

There is great diversity in earnings, savings propensities and in the ratios of payroll to income taxes paid. The starting place for this model is the diversity in savings. Assume two-types, so that in each cohort there are $n$ "workers" who do no saving whatsoever and $N$ "savers" who are standard life-cycle savers. We use lower-case letters to refer to workers and upper-case letters to refer to savers. For notational simplicity, assume no population growth. Given the positive correlation between savings propensities and earnings (Dynan, Skinner and Zeldes, 2000, Saez, 2002), the ratio of payroll taxes to income taxes is higher for workers than savers. For simplicity, we model this by assuming that the workers pay no income taxes at all - a simplification which calls for interpreting the model relative to tax differences. While most savers are also covered by Social Security, for notational simplicity we assume that they are not. Again, this calls for interpretation in terms of the difference between types of taxes paid.

The workers rely on social security for retirement consumption, while the savers do their own retirement savings. In recognition of the tax advantages of retirement savings, we model the income tax as falling only on the earnings of savers. We also assume that labor is inelastically supplied - with work for $L$
periods (length of career) and retirement for $D - L$. We assume that careers are longer than retirements, $L > D - L$. We do not consider differences in career length or life expectancy between the two types.

For simplicity, assume a linear technology, with each worker earning $w$ per period, each saver earning $W$ per period, and capital earning a gross return $R = 1 + r > 1$.\footnote{Without this linearity assumption we would need to track the changes in wages and interest rates and their impact on taxes and savings.} With constant payroll taxes, $t$, each worker would consume $w(1 - t)$ while working and a social security benefit of $b$ while retired. For simplicity, assume that savers equalize the consumption each period over their entire lives or their remaining lives when there is a policy change. This would follow from the standard model if the savers have additive lifetime utility functions with the same period utility functions in each period and a utility discount rate that equals the interest rate. With savers choosing the same level of consumption, $C$, in each of the $D$ periods of life, the present discounted value (PDV) of period consumption for a saver newly entering the labor force is equal to the PDV of $L$ periods of net-of-tax earnings, $W (1 - T)$. With unanticipated changes in income taxes, $T$, we will have to pay attention to the timing of tax changes.

The social security system is partially funded, with a fund of size $F$. With
equally sized cohorts, the social security budget constraint if the fund is held constant is

\[ n(D - L)b = nLtw + rF \]  

(1)

That is, benefits of \( b \) are paid to each of the \( n(D - L) \) retirees alive in each period. Financing comes from payroll tax revenues and the interest on the trust fund.

For simplicity, we assume no government expenditures other than interest on the outstanding public debt, denoted \( G \). If the debt is constant, per period non-social security budget balance implies

\[ NLTW = rG \]  

(2)

That is, the interest on the total debt outstanding is paid from the income tax on the earnings of the \( NL \) savers in the labor force in each period. Thus the income tax on savers finances the non-social security budget while the payroll tax on workers finances the social security budget. The public debt held by the savers is \( G - F \), the rest of their savings being in physical capital.
4 Permanent Fund Increase

Assume that the government increases the payroll tax rate by $\Delta t$ for one period, using the revenue to permanently increase the trust fund, with the additional interest earnings used to lower the payroll tax rate thereafter. Assume that the government decreases the income tax in the initial period by an amount chosen to offset the fraction $\alpha$ ($0 \leq \alpha \leq 1$) of the additional payroll tax revenue, with no changes in either public consumption or government investment.

$$\alpha nLw\Delta t = -NLW\Delta T$$ (3)

We assume unified budget balance in all later periods. That is, we are assuming that the deviations from budget balance for the non-social security budget are $-\alpha$ times the deviations in the social security budget. The analysis would be different if the non-social security budget responded to the social security payroll tax revenue less benefit payments, thereby ignoring the interest on the trust fund. Initially this policy change decreases the debt held by the public by $(1 - \alpha)$ times the increase in the trust fund. That is, $G$ increases by $\alpha$ times the increase in $F$. The trust fund increases in the initial period by $nLw\Delta t$. Thereafter, neither the trust fund nor the debt held by
the public make further changes.

With benefits and cohort size unchanged, the payroll tax rate can be reduced because of the interest on the increased revenue from the initial tax increase. Thus, the payroll tax rate after the initial period, $t'$, satisfies

$$t' = t - r \Delta t$$

(4)

Similarly, the income tax rate thereafter, $T'$, is increased to pay the increase in interest from the increase in the public debt

$$T' = T - r \Delta T = T + \alpha r \frac{nw}{NW} \Delta t$$

(5)

That is, there is an intertemporal trade between payroll taxpayers and income taxpayers, which is balanced in PDV. This also involves changes in the timing of tax payments by each agent and redistribution across cohorts of each type.

The central question is what happens to the time shape of national capital.
4.1 National savings in the initial period

To analyze the impact of the changes in payroll and income taxes on national savings in the initial period we can examine the changes in consumption of workers and savers. In the period of the initial tax change, the aggregate consumption of workers falls by their tax increase: \( nLw\Delta t \). In all later periods, the aggregate consumption of workers is higher by \( rnLw\Delta t \). This is equal to the return on the increase in the trust fund.\(^{12}\) Thus, if the trust fund increase were fully an increase in national savings, there would be no impact on national savings after the initial period as the increase in consumption by workers would match the increase in national income. That is, national capital would increase in the initial period and remain at the higher level thereafter. This would be the case in this model if the government did not alter the income tax \((\alpha = 0)\). But we have assumed that the income tax may change, so we must examine the response of savers to the income tax changes, which is more complicated.

The change in income tax in the initial period for a saver who is still working is \( W\Delta T \), equal to \(-\alpha nw\Delta t/N\). Thereafter there is a tax change of \(-rW\Delta T \), equal to \(\alpha rnw\Delta t/N\) in each of the remaining \(L - z - 1\) periods.

\(^{12}\)The lack of growth of the labor force simplifies the calculation.
until retirement for a saver of age $z$ in the initial period. In PDV terms, the tax change for an age $z$ saver is $(anw\Delta t/N)\left(-1 + r\sum_{s=z+1}^{L} R^{z-s}\right)$. With a discount rate equal to the interest rate, each saver preserves equal consumption in each remaining period of life. With an unexpected change in taxes starting at age $z$, the change in consumption each remaining period of life that preserves equality of consumption for the rest of life, is

$$\Delta C_z \sum_{s=z}^{D} R^{z-s} = (anw\Delta t/N)\left(1 - r\sum_{s=z+1}^{L} R^{z-s}\right)$$

(6)

The change in consumption in the initial period is less than the tax cut for two reasons - anticipation of future tax increases and the spreading of consumption over the rest of life.

Summing over working savers, we get an initial consumption increase for savers of

$$\Delta C = N \sum_{z=1}^{L} \Delta C_z = anw\Delta t \sum_{z=1}^{L} \frac{1 - r\sum_{s=z+1}^{L} R^{z-s}}{\sum_{s=2}^{D} R^{z-s}}$$

(7)

The change in national savings, $\Delta NS$, in the initial period is equal to minus
the change in aggregate consumption:

\[
\Delta NS = nw\Delta t \left( L - \alpha \sum_{z=1}^{L} \frac{1 - r \sum_{s=z+1}^{L} R^{s-z}}{\sum_{s=z}^{D} R^{s-z}} \right)
\]

\[
= nw\Delta t \sum_{z=1}^{L} \left( 1 - \frac{\alpha}{\sum_{s=z}^{D} R^{s-z}} + \alpha r \frac{\sum_{s=z+1}^{L} R^{s-z}}{\sum_{s=z}^{D} R^{s-z}} \right) > 0 \quad (8)
\]

If savers were to consume all of their income tax cut in the initial period and the tax cut balanced the unified budget \((\alpha = 1)\), there would be no increase in national savings. But they do not consume all of the income tax cut. They save part of their initial tax cut to finance later consumption and part of it to finance higher tax payments in their remaining working years. Thus national savings increase in the initial period even if \(\alpha = 1\). Insofar as \(\alpha\) is less than one there is a further increase in savings in the initial period.

### 4.2 National capital in the steady state

In later years the pattern changes. Once the savers who received the initial tax cut have all died off, all savers have the same net earnings and so the same consumption and we are in a steady state. To examine the impact on steady-state capital, we can consider the impact on consumption since in a steady state with no growth, aggregate consumption equals aggregate output.
With a linear technology, the change in output is equal to the interest rate times the change in capital. For convenience we now switch from discrete time to continuous time.

In aggregate, workers have consumption which is higher by the amount of their tax decrease: $rnLw\Delta t$. For savers we need to consider lifetime planning, which determines the constant level of consumption that they choose. We need to calculate how their consumption changes in response to an income tax rate increase of $\alpha rnw\Delta t/NW$.

The lifetime budget constraint for a saver is

$$C' \int_0^D e^{-rs} ds = W (1 - T') \int_0^L e^{-rs} ds \quad (9)$$

or

$$C' = W (1 - T') \frac{1 - e^{-rL}}{1 - e^{-rD}} \quad (10)$$

The change in aggregate consumption by savers is $ND\Delta C' = -W \Delta T' \frac{1 - e^{-rL}}{1 - e^{-rD}}$, which equals $-D \frac{1 - e^{-rL}}{1 - e^{-rD}} \alpha rnw\Delta t$.

Adding the two pieces, aggregate consumption changes by $rnLw\Delta t - D \frac{1 - e^{-rL}}{1 - e^{-rD}} \alpha rnw\Delta t$, which equals $rnw\Delta t \left( L - \alpha D \frac{1 - e^{-rL}}{1 - e^{-rD}} \right)$. Thus aggregate capital changes by $nw\Delta t \left( L - \alpha D \frac{1 - e^{-rL}}{1 - e^{-rD}} \right)$. 

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Note that if $D$ were equal to $L$ and $\alpha$ equal to 1, this expression would be zero. Moreover, the expression $\frac{D}{1 - e^{-rD}}$ is increasing in $D$, implying that the expression is everywhere negative for $\alpha = 1$ and $D > L$. Thus if $\alpha = 1$, capital is decreased in the steady state. The result follows from the same logic as above, run in reverse. Workers have lower taxes, all of which flows into higher consumption. Savers have higher taxes, but the induced consumption decrease is spread over their entire lives. With positive interest, there is a smaller consumption decrease for savers than the increase in consumption of workers.

Thus national capital increases in the steady state if $\alpha = 0$ and decreases in the steady state if $\alpha = 1$. For intermediate values of $\alpha$ we get an increase or decrease depending on whether $\alpha$ is below or above a critical value, denoted $\alpha^*$. To find this critical value, we set the change in aggregate capital to zero:

$$\alpha^* = \frac{L(1 - e^{-rL})}{D(1 - e^{-rL})} \quad (11)$$

For example, with $L = 40$, $D = 60$, and $r = .03$, we have $\alpha^* = .8$. That is, long-run capital is increased if the income tax cut uses up no more than 80 percent of the revenue raised by the payroll tax increase.
5 Concluding Remarks

The distributions of income and social security taxes are very different. Assuming proportional cuts in income taxes in response to a surplus generated by the payroll tax, there are winners and losers from the two tax changes. Also, there are future winners and losers insofar as current tax changes result in future tax changes. It was convenient to model the economy with savers paying only the income tax and workers paying only the payroll tax. In order to interpret the results, we need to consider the net change in taxes for each group that comes about from the policy change. We have no simple way of distinguishing savers from workers, but approximating this by assuming that top quintile of individuals in terms of family income are savers and the rest are workers, roughly one-third of the payroll tax change can be modeled as above (see footnote 7). For the other two-thirds, an offsetting income tax change results in no net effect.

In the formal model, legislated decisions about income taxes and expenditures are influenced by the deficit but not the level of debt. This simplification is missing an effect that is plausible (at least for high levels of debt) and has been found by Bohn (1998). While the debt level influences the

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13 This assumes that the income tax change is proportional to average taxes.
deficit level through interest expenditures, it is plausible that there is also a
direct influence. (Also missing in the model is any effect of debt levels on
government interest rates.)

Given my view that very little of the Social Security surplus showed up in changes in the rest of the budget during the 1980s and 1990s, I think that most of the surplus has represented an increase in national savings. It is not clear what impact the surplus has had on the Bush tax cuts - the first was sold as not touching the Social Security surplus and the second happened despite touching the surplus. Taking a view that the political process was more responsive to this change in the unified budget balance than I believe to be the case, one still has a short run increase in savings, while in different years after the baby-boomers retire and the accumulated trust fund is used to help finance their benefits, there are increases and then decreases in capital.

The development of the OLG model and its application to analyzing social security represents an achievement of Paul Samuelson that will influence at least the next century.
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