Updates of the nuclear equation of state for core-collapse supernovae and neutron stars: effects of 3-body forces, QCD, and magnetic fields

G J Mathews¹, M Meixner¹, J P Olson¹, I-S Suh¹, T Kajino², T Maruyama³, J Hidaka², C-Y Ryu⁴, M-K Cheoun⁵ and N Q Lan⁶

¹Department of Physics/Center for Astrophysics, University of Notre Dame, Notre Dame, United States
²Theory Division, National Astronomical Observatory of Japan, Mitaka, Tokyo, Japan
³College of Bioresource Sciences, Nihon University, Fujisawa 252-8510, Japan
⁴General Education Curriculum Center, Hanyang University, Seoul 133-791, Korea
⁵Department of Physics, Soongsil University, Seoul, South Korea
⁶Hanoi National University of Education, 136 Xuan Thuy, Hanoi, Vietnam

E-mail: gmathews@nd.edu

Abstract. We summarize several new developments in the nuclear equation of state for supernova simulations and neutron stars. We discuss an updated and improved Notre-Dame-Livermore Equation of State (NDL EoS) for use in supernova simulations. This Eos contains many updates. Among them are the effects of 3-body nuclear forces at high densities and the possible transition to a QCD chiral and/or superconducting color phase at densities. We also consider the neutron star equation of state and neutrino transport in the presence of strong magnetic fields. We study a new quantum hadrodynamic (QHD) equation of state for neutron stars (with and without hyperons) in the presence of strong magnetic fields. The parameters are constrained by deduced masses and radii. The calculated adiabatic index for these magnetized neutron stars exhibit rapid changes with density. This may provide a mechanism for star-quakes and flares in magnetars. We also investigate the strong magnetic field effects on the moments of inertia and spin down of neutron stars. The change of the moment of inertia associated with emitted magnetic flares is shown to match well with observed glitches in some magnetars. We also discuss a perturbative calculation of neutrino scattering and absorption in hot and dense hyperonic neutron-star matter in the presence of a strong magnetic field. The absorption cross-sections show a remarkable angular dependence in that the neutrino absorption strength is reduced in a direction parallel to the magnetic field and enhanced in the opposite direction. The pulsar kick velocities associated with this asymmetry comparable to observed pulsar velocities and may affect the early spin down rate of proto-neutron star magnetars with a toroidal field configuration.

1. Introduction

To describe the structure and hydrodynamics of compact matter; an equation of state (EoS) is needed to relate the physics of the state variables [1]. In supernovae the EoS determines the dynamics of the collapse and the outgoing shock, and determines whether the remnant ends up as a neutron star or a black hole. In a neutron star, it determines the mass-radius relationship, stellar composition, cool-down time and dynamics of neutron star spin down and mergers. In this...
paper we summarize some progress in the development of Equations of state for supernova and neutron star simulations. In particular we highlight the role of 3-body forces, QCD, magnetic fields and neutrino transport.

2. Three body forces and the EoS

At present, only a few hadronic EoSs are commonly employed that cover large enough ranges in density, temperature and electron fraction to be of use in core-collapse supernova simulations. The two most employed in astrophysical simulations are the EoS of Lattimer & Swesty (LS91) [2] and that of H. Shen et. al. (Shen98) [3, 4]. The former utilizes a non relativistic parameterization of nuclear interactions in which nuclei are treated as a compressible liquid drop including surface effects. The latter is based upon a Relativistic Mean Field (RMF) model using the TMI parameter set in which nuclei are calculated in a Thomas-Fermi approximation. Baryonic matter was parameterized with a new RMF model that treated nuclei and non-uniform matter with the statistical model of Hempel et. al. [5].

Here, we discuss a new Notre Dame-Livermore (NDL) EoS [6]. This EoS evolves from the original Livermore formulation [7, 8], but unlike the previous version this NDL EoS, is consistent with known experimental nuclear matter constraints and recent [14] mass and radii measurements of neutron stars. Below nuclear matter density, the conditions for nuclear statistical equilibrium (NSE) are achieved at a temperature of $T \approx 0.5$ MeV. Below this temperature the nuclear matter is a approximated by a nine element reaction network which must be evolved dynamically. Above this temperature, the nuclear constituents are represented by free nucleons, alphas and a single “representative” heavy nucleus. Among the new features in the NDL EoS is that high density phase of the EoS is treated with a parameterized Skyrme energy density functional that utilizes a modified zero range 3-body interaction. The effects of pions on the state variables at high densities is also included as well as the consequences of a

\[ P = n^2 \frac{\partial}{\partial n} \left( \frac{E}{A} \right). \]

The volume compressibility of symmetric nuclear matter is calculated from the derivative of the pressure: $K = 9 \left( \frac{\partial P}{\partial n} \right) = 18 \frac{P}{E} + 9n^2 \frac{\partial^2}{\partial n^2} \left( \frac{E}{A} \right)$. The skewness coefficient, is from the third derivative of the free energy per nucleon $Q_0 = 2$.
\[
27n^3 \frac{\partial^3}{\partial n^3} \left( \frac{E}{A} \right) .
\]
Applying the saturation condition \(P(n = n_0) = n^2 \frac{\partial^3}{\partial n^3} \left( \frac{E}{A} \right) |_{n=n_0} = 0\), one obtains [6] a system of four equations in terms of \(t_0\), \((3t_1 + 5t_2)\), \(t_3\), and \(\sigma\). Solving this system for \(\sigma\) yields

\[
\sigma = \frac{9}{5} T_{F_0} - 2 K_0 + Q_0 - 45E_0
\]
\[3K_0 + 45E_0 - \frac{22}{9} T_{F_0}, \tag{4}\]
where the subscript zero denotes values at the saturation density.

For our purposes, we adopt inferred values of \(n_0, E_0, K_0, Q_0\) from the literature and use these to determine the Skyrme model parameters. We also demand that these parameters allow neutron star masses \(\geq 1.97 \pm 0.04 \, M_\odot\) [14]. The saturation density \(n_0 \approx 0.16 \, \text{fm}^{-3}\) and the binding energy per nucleon \(E_0 = -16 \, \text{MeV}\) are reasonably well established [15]. The determination of the compressibility parameter from experimental data on the giant monopole resonance on finite nuclei has been a long standing conundrum. For our purposes we adopt the median value and uncertainty from Ref. [16], i.e. \(K_0 = 240 \pm 10 \, \text{MeV}\) as this is appropriate for the Skyrme force approach employed here. Solving the saturation conditions self consistently, we therefore determine the best range for the nuclear compressibility consistent with the results of [16].

There is even more uncertainty in the skewness parameter \(Q_0\). Breathing mode data [17] implies \(Q_0 = -700 \pm 500 \, \text{MeV}\). Using the range for \(K_0\) given by [16] and solving the saturation conditions, we find [6] a skewness coefficient of \(Q_0 = -390 \pm 90 \, \text{MeV}\) consistent within the range given in Ref. [16]. The fiducial NDL EoS is then constructed [6] using the The Skyrme coefficients \(t_0 = -1718 \, \text{MeVfm}^3\), \((3t_1 + 5t_2) = -102 \, \text{MeVfm}^3\), \(t_3 = 13226 \, \text{MeVfm}^3\), and \(\sigma = 0.369\).

The density dependence of the symmetry energy beyond saturation is highly uncertain. For many Skyrme models the symmetry energy either saturates at high densities, or in the worst case becomes negative. This results in a negative pressure deep inside the neutron star core. We implemented [6] a linearly increasing function of density. The symmetry energy at saturation was determined by the difference between the energy per particle for pure neutron matter and that of symmetric matter at \(T = 0 \, \text{MeV}\). For all relevant parameter sets the NDL EoS symmetry energy at saturation is \(S_0 = 30.5 \, \text{MeV}\) [6].

3. QCD and the EoS
For sufficiently high densities and/or temperature a transition from hadronic matter to quark-gluon plasma (QGP) can occur [18]. Progress [19] in lattice gauge theory (LGT) has shown that at high temperature and low density a deconfinement and chiral symmetry restoration occur simultaneously. In particular, it has been found [19] that the order parameters for deconfinement and chiral symmetry restoration changes abruptly for temperatures of \(T = 145 - 170 \, \text{MeV}\) [20, 21] as a smooth crossover.

At low density the hadron phase can be approximated as a pion-nucleon gas, while the QGP phase can be approximated in a bag model as a non-interacting relativistic gas of quarks and gluons [22]. The LGT results then imply a range for the QCD vacuum energy of \(165 \leq B^{1/4} \leq 225 \, \text{MeV}\). Also requiring that the maximum mass of a neutron star exceed \(1.97 \pm 0.04 \, M_\odot\) [14] implies a value for \(B^{1/4}\) near the top end of that range.

For the description of quark matter we utilize a bag model with 2-loop corrections, and construct the EoS from a phase-space integral representation over scattering amplitudes. We allow for the possibility of a coexistence mixed phase in a 1st order transition or a simple cross over transition. It is convenient to compute the QGP in terms of the grand potential, \(\Omega(T, V, \mu)\), where the grand potential for the quark-gluon plasma takes the form:

\[
\Omega = \sum_i (\Omega_{q0}^i + \Omega_{q2}^i) + \Omega_{g0} + \Omega_{g2} + BV. \tag{5}\]
Where $q_0$ and $g_0$ denote the 0th-order bag model thermodynamic potentials for quarks and gluons, respectively, and $q_2$ and $g_2$ denote the 2-loop corrections. In most calculations sufficient accuracy is obtained by using fixed current algebra masses (e.g. $m_u \sim m_d \sim 0$ GeV, $m_s \sim 0.1 \sim 0.3$ GeV). For this work we chose the strange quark mass to be $m_s = 150$ MeV and a bag constant $B^{1/4} = 165 - 220$ MeV. The quark contribution to the thermodynamic potential is given [18] in terms of a sum of the ideal gas contribution plus a two loop correction from phase-space integrals over Feynman amplitudes [23].

Fig. 1 compares [6] the neutron star mass radius relation for the NDL EoS for: 1) a hadronic EoS with 3-body forces (solid line); 2) a first order QCD transition with $B^{1/4} = 220$ MeV; and 3) a simple QCD cross over transition. Also, shown for comparison are results from the LS180 EoS, Shen EoS and the original Bowers & Wilson EoS. Note, that all three versions of the NDL EoS easily accommodate a maximum neutron star mass $\geq 1.97 \pm 0.04 \, M_\odot$, however, the hadronic version must have the 3-body forces at high baryon density. A first order phase transition to a QGP is consistent with the high maximum neutron star mass constraint [14] for a bag constant $B^{1/4} > 220$ MeV. This imposes a low baryon density transition temperature of $T_c = 158$ MeV [22] which is consistent with the current range of crossover temperatures determined from LGT [19].

**Figure 1.** Mass-radius relations for the EoS of Shen [3], Lattimer & Swesty (LS180) [2], Bowers & Wilson [7], compared with the new NDL EoS [6] with and without a mixed phase transition or a simple crossover transition to a QGP. Note that the soft LS180 hadronic EoS and the previous Bowers & Wilson EoS without 3-body forces cannot satisfy the astrophysical constraint of a maximum neutron star mass $\geq 1.97 \pm 0.04 \, M_\odot$ [14] as shown.
4. Magnetic Fields and The Eos

We have also considered the neutron star equation of state and neutrino transport in the presence of strong magnetic fields [25]-[30]. Indeed, magnetic fields are everywhere in Nature and frequently play a role in astrophysical phenomena. In particular the existence of magnetars and magnetar flares [31, 32, 33], along with the observed asymmetry in supernova explosions and the observed pulsar kick velocities all suggest the strong magnetic fields play an important role in supernova explosions and the formation of proto-neutron stars [34, 35, 36]. In view of this we have undertaken studies of a variety of phenomena.

In [25] we considered the role that strong interior magnetic fields ($B \sim 10^{17}$ G) would have on neutron star structure and stability. We considered the nuclear equation of state for an ideal $npe$ gas in a strong magnetic field. In particular, we calculated the proton concentration, the threshold densities for neutron, muon, and pion production and pion condensation in a strong magnetic field both without and with the effect of the nucleon anomalous magnetic moments. It was shown [25] that the higher Landau levels are significant at high density in spite of the existence of a very strong magnetic field. In particular, at high density, the proton concentration approaches the nonmagnetic limit. In particular, we have obtained the neutron appearance threshold density in a magnetic field when the nucleon anomalous magnetic moment is included. We also have shown [25] that the muon and pion threshold densities are not affected by magnetic fields for $B < 10^{12}$ G. We also obtained an equation of state for a pion condensate in strong magnetic fields. We found [25] that magnetic fields reduce the amount of pion condensation. However, we still could find distinguishable effects from a pion condensate in strongly magnetized neutron stars. In addition, we demonstrated an oscillatory behavior of the adiabatic index in both strongly magnetized $n, p, e$ and $n, p, e, \mu, \pi$ gases at high density. Here we speculated that this behavior might lead to an interior pulsational instability.

In [26] we investigated the possibility that soft gamma-ray repeaters (SGRs) and anomalous X-ray pulsars (AXPs) might be observational evidence for a magnetic phase separation in magnetars. We studied such magnetic domain formation as a new mechanism for SGRs and AXPs in which magnetar matter separates into phases containing different flux densities. We identified the parameter space in matter density and magnetic field strength at which there is an instability for magnetic domain formation. We showed that such instabilities will likely occur in the deep outer crust for the magnetic Baym, Pethick, and Sutherland (BPS) model and in the inner crust and core for magnetars described in relativistic Hartree theory. Moreover, we estimated that the energy released by the onset of this instability is comparable with the energy emitted by SGRs.

In [27] this has recently been extend to a study a new quantum hadrodynamic (QHD) equation of state for neutron stars (with and without hyperons) in the presence of strong magnetic fields. The parameters were constrained by the condition that deduced neutron star masses and radii that must be consistent with the recent observations [14] of a high mass neutron star. The calculated adiabatic index for these magnetized neutron stars exhibited the same rapid changes with density. This was hypothesized to provide possible insight into the mechanism of starquakes and flares in magnetars. We also investigated the strong magnetic field effects on the moments of inertia of neutron stars. The change of the moments of inertia associated with emitted magnetic flares was shown to match well with observed glitches in some magnetars.

In [29, 30] we explored a perturbative calculation of neutrino scattering and absorption in hot and dense hyperonic neutron-star matter in the presence of a strong magnetic field. We found that the absorption cross-sections show a remarkable angular dependence in that the neutrino absorption strength is reduced in a direction parallel to the magnetic field and enhanced in the opposite direction. This asymmetry in the neutrino absorption can be as much as 2 % of the entire neutrino momentum for an large interior magnetic field. We estimate the associated pulsar kick velocities associated with this asymmetry in a fully relativistic mean-field theory.
formulation and show that the kick velocities are comparable to observed pulsar velocities. In [30] we have extended this calculation to include a toroidal magnetic field configuration. In this case, there can be an asymmetric emission of neutrino momentum along the magnetic field lines that are in the direction of the neutron star spin. This can substantially accelerate the spin down of a neutron star in the early cooling phase, ~ 10 sec after core bounce. This is to be compared with [28] in which we considered the spin down of a neutron star purely from the outflow of neutrinos without a magnetic field.

Acknowledgments

Work at the University of Notre Dame is supported by the U.S. Department of Energy under Nuclear Theory Grant DE-FG02-95-ER40934. One of the authors (N.Q.L.) was supported in part by the National Science Foundation through the Joint Institute for Nuclear Theory (JINA)

References

[1] Lattimer J M 2012 Ann. Rev. Nucl. and Part. Sci. 62 485
[2] Lattimer J M and Swesty F D 1991 Nuclear Physics A 535 331
[3] Shen H, Toki H, Oyamatsu K and Sumiyoshi K 1998 Nuclear Physics A 637 435
[4] Shen H, Toki H, Oyamatsu K and Sumiyoshi K 1998 Progress of Theoretical Physics 100 1013
[5] Hempel M and Schaner-Bielich J 2010 Nuclear Physics A 837 210
[6] Meixner M, Olson J P, Mathews G J, Lan N Q and Dalhed H E 2013 Submitted to Phys. Rev. C
[7] Bowers R L and Wilson J R 1982 Phys. Rev. C 50 115
[8] Wilson J R and Mathews G J 2003 Relativistic Numerical Hydrodynamics (Cambridge University Press)
[9] Vautherin D and Brink D M 1972 Phys. Rev. C 5 626
[10] Ring P and Schuck P 2000 The nuclear many-body problem (Springer)
[11] Mansour H M M 1990 Acta Physica Polonica B 21 741
[12] Kohler H S 1965 Phys. Rev. 138 831
[13] Krivine H, Treiner J and Bohigas O 1980 Nuclear Physics A 336 155
[14] Demorest P B, Pennucci T, Ransom S M, Roberts M S E and Hessels J W T 2010 Nature 467 1081
[15] Li BA and Ko C M 1997 Nuclear Physics A 618 498
[16] Colò G, van Giai N, Meyer J, Bennaceur K and Bonche P 2004 Phys. Rev. C 70 024307
[17] Farine M, Pearson J M and Tondeur F 1997 Nuclear Physics A 615 135
[18] McLerran L 1986 Reviews of Modern Physics 58 1021
[19] Kronfeld A S 2012 Ann. Rev. Nucl. and Particle Sci. 62 265
[20] Borsanyi S, et al 2012 Journal of High Energy Physics 8 126
[21] Bazavov A, et al 2012 Phys. Rev. D 86 094503
[22] Fuller G M, Mathews G J and Alcock C R 1988 Phys. Rev. D 37 1380
[23] Kapusta J J 1978 High temperature matter and heavy ion collisions Ph.D. thesis, Univ. California , Berkeley
[24] Braithwaite J and Spruit H C 2004 Phys. Rev. C 70 024307
[25] Suh I-S and Mathews G J 2003 Astrophys. J. 586 127
[26] Suh I-S and Mathews G J 2010 Astrophys. J. 717 843
[27] Ryu C-Y, Cheoun M-K, Maruyama T and Mathews G J 2012 Astroparticle Physics 38 25-30
[28] Ryu C-Y, Maruyama T, Kajino T, Mathews G J and Cheoun M K 2012 Phys. Rev. C 85 045803
[29] Maruyama T, Yasutake T, Cheoun M-K, Hidaka J, Kajino T and Mathews G J 2012 Phys. Rev. D 86 123003
[30] Maruyama T, Hidaka J, Kajino T, Yasutake T, Cheoun M-K, Ryu C Y and Mathews G J 2013 Submitted to Phys. Rev. Lett. (Preprint arXiv:1301.7495 [astro-ph])
[31] Duncan R C and Thompson C 1992 Astrophys. J. Lett. 392 L19
[32] Kunihiro K and Thompson C 1992 Phys. Rev. D 45 1270
[33] Hurley K, et al 1999 Astrophys. J. 510 L111
[34] Lyne A G and Lorimer D R 1994 Nature 369 127
[35] Paczynski B 1992 Acta. Astron. 41 145
[36] Chanmugam G 1992 Ann. Rev. Astron. Astrophys. 30 143