Meson spectrum in Regge phenomenology

De-Min Li $^{1,2}$, Bing Ma $^1$, Yu-Xiao Li $^1$, Qian-Kai Yao $^1$, Hong Yu $^2$

$^1$ Department of Physics, Zhengzhou University, Zhengzhou, Henan 450052, P. R. China
$^2$ Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100039, P. R. China

March 26, 2022

Abstract

Under the assumption that both light and heavy quarkonia populate approximately linear Regge trajectories with the requirements of additivity of intercepts and inverse slopes, the masses of different meson multiplets are estimated. The predictions derived from the quasi-linear Regge trajectories are in reasonable agreement with those given by many other references.
1 Introduction

The investigation of the meson spectrum is of great importance for better understanding the dynamics of the strong interactions, since the mesons are the ideal laboratory for the study of strong interactions in the strongly coupled non-perturbative regime[1]. According to the recent issue of Review of Particle Physics[2], there are many mesons so far absent from the Meson Summary Table, therefore, for the sake of the completeness of meson spectrum, especially for the heavy meson spectrum, there are still a lot of work to be done both theoretically and experimentally.

Regge theory is concerned with the particle spectrum, the forces between particles, and the high energy behavior of scattering amplitudes[3]. One of the most distinctive features of Regge theory is the Regge trajectory by which the mass and the spin of a hadron are related. Knowledge of the Regge trajectories is useful not only for spectral purpose, but also for many non-spectral purpose. The intercepts and slopes of the Regge trajectories are of fundamental importance in hadron physics[4].

To a large extent, our knowledge of meson spectrum is based on some phenomenological QCD motivated models. A series of recent papers by Anisovich et al.[5] show that meson states fit to the quasi-linear Regge trajectories with sufficiently good accuracy, although some suggestions that the realistic Regge trajectories could be nonlinear exist[6, 7].

In the analysis of Ref.[5], the Regge trajectories for heavy mesons were not concerned. In the analysis of Refs.[8, 9], the mass relation for heavy mesons was investigated in the quasi-linear Regge trajectory ansatz with the simplification that the Regge slopes in the light quark sector are the same for all the meson multiplets. With the help of the rich available experimental data, in the present work, we shall extract the parameters of the quasi-linear Regge trajectories for both light and heavy meson states, and estimate the masses of the states lying on these Regge trajectories. In our consideration, we don’t constrain that the Regge slope is flavor-independent, and we shall adopt the argument adopted by Ref.[5] that the state with spin $J$ and its partners with the same quantum numbers but spin $J + 2$, $J + 4$, ..., rather than both $J^{PC}$ and $(J + 1)^{-P,-C}$ states, populate a common quasi-linear Regge trajectory. The suggestion[9]
that the Regge trajectories are not linear but rather have curvatures in the region of lower spin
may be relevant to the usual assignment that both $J^{PC}$ and $(J + 1)^{-P-C}$ states belong to a
common linear Regge trajectory. In fact, based on this assignment, if one tries to fit the $\pi$ ($0^{-+}$)
to the linear trajectory on which the $b_1(1235)$ ($1^{-+}$) and $\pi_2(1670)$ ($2^{-+}$) lie, one can obtain that
the mass of $\pi$ is about 0.696 GeV, which is much higher than the experimental value of 0.135
GeV[2]. The argument that both $J^{PC}$ and $(J + 1)^{-P-C}$ states populate a common linear Regge
trajectory is in fact based on the hypothesis of exchange-degeneracy of Regge trajectories[3].
However, it has already been pointed out by Desgrolard et al.[10] that the hypothesis of exact
exchange-degeneracy, even in its weak formulation, is not supported by the present data.

The paper is organized as follows. The parameters of different Regge trajectories and the
masses of the meson states lying on different Regge trajectories are given in Section 2. The
discussions of our results appear in section 3. The summary and conclusion are presented in
Section 4.

2 The parameters of Regge trajectories and spectroscopy

By assuming the existence of the quasi-linear Regge trajectories for a meson multiplet, one
can have

$$J = \alpha_{i\bar{i}}(0) + \alpha'_{i\bar{i}} M^2_{i\bar{i}},$$

where $i$ ($\bar{i}$) refers to the quark (antiquark) flavor, $J$ and $M_{i\bar{i}}$ are respectively the spin and mass
of the $i\bar{i}$ meson, $\alpha_{i\bar{i}}(0)$ and $\alpha'_{i\bar{i}}$ are respectively the intercept and slope of the trajectory on
which the $i\bar{i}$ meson lies. For a meson multiplet, the parameters for different flavors can be
related by the following relations proposed in the literature:

(i) additivity of intercepts,

$$\alpha_{i\bar{i}}(0) + \alpha_{j\bar{j}}(0) = 2\alpha_{ji}(0),$$

(ii) additivity of inverse slopes,

$$\frac{1}{\alpha'_{i\bar{i}}} + \frac{1}{\alpha'_{j\bar{j}}} = \frac{2}{\alpha'_{ji}}.$$
(iii) factorization of slopes,

\[ \alpha'_i \beta'_j = (\alpha'_j)^2. \]

The relation (2) is first derived for $u(d)$- and $s$- quarks in the dual-resonance model[11], and it is satisfied in two-dimensional QCD[12], the dual-analytic model[13], and the quark bremsstrahlung model[14]. Also, it saturates inequalities for Regge trajectories[15] which follow from the $s$-channel unitarity condition. The relation (3) is derived based on topological expansion and the $q\bar{q}$-string picture of hadrons[17], and the relation (4) follows from the factorization of residues of the $t$-channel poles[18]. The paper by Burakovsky et al.[16] shows that only additivity of inverse Regge slopes is consistent with the formal chiral and heavy quark limits for both mesons and baryons, and factorization of Regge slopes, although consistent in the formal chiral limit, fails in the heavy quark limit. In our present work, we shall assume that the relations (2) and (3) are valid for the quasi-linear Regge trajectory.

Based on the quasi-linear Regge trajectory (1), together with the relations (2) and (3), we can construct the Regge trajectories. The starting point for constructing a meson Regge trajectory is the meson assignment. According to the argument that the state with spin $J$ and its partners with the same quantum numbers but spin $J + 2, J + 4, \ldots$ populate a common linear Regge trajectory[10], the meson assignment is shown in Table 1. In the following, $n$ denotes $u$- or $d$-quark.

| Trajectories | Meson states |
|--------------|--------------|
| $1^3S_0$    | $1^1S_0 (0^-)$, $1^1D_2 (2^-)$, $1^3G_4 (4^-)$, \ldots |
| $2^3S_0$    | $2^1S_0 (0^-)$, $2^1D_2 (2^-)$, $2^3G_4 (4^-)$, \ldots |
| $1^3S_1$    | $1^1S_1 (1^-)$, $1^3D_3 (3^-)$, $1^3G_5 (5^-)$, \ldots |
| $2^3S_1$    | $2^1S_1 (1^-)$, $2^3D_3 (3^-)$, $2^3G_5 (5^-)$, \ldots |
| $1^3P_0$    | $1^3P_0 (0^+)$, $1^3P_2 (2^+)$, $1^3H_4 (4^+)$, \ldots |
| $1^1P_1$    | $1^1P_1 (1^+)$, $1^3P_3 (3^+)$, $1^3H_5 (5^+)$, \ldots |
| $1^3P_1$    | $1^3P_1 (1^+)$, $1^3P_3 (3^+)$, $1^3H_5 (5^+)$, \ldots |
| $1^3P_2$    | $1^3P_2 (2^+)$, $1^3P_4 (4^+)$, $1^3H_6 (6^+)$, \ldots |

Table 1. The assignment for the meson states lying on different linear Regge trajectories.

### 2.1 The $1^1S_0$ trajectories

For the $1^1S_0$ trajectories, inserting the masses of $\pi$, $\pi_2(1670)$, $K$, $K_2(1770)$, $\eta_c(1S)$, $D$, \ldots
and $B^1$ into the following equations

\begin{align}
0 &= \alpha_{n\bar{n}}(0) + \alpha'_{n\bar{n}} M_{K}^2, \\
2 &= \alpha_{n\bar{n}}(0) + \alpha'_{n\bar{n}} M_{\rho_{2}(1570)}^2, \\
0 &= \alpha_{n\bar{s}}(0) + \alpha'_{n\bar{s}} M_{K}^2, \\
2 &= \alpha_{n\bar{s}}(0) + \alpha'_{n\bar{s}} M_{\rho_{2}(1770)}^2, \\
0 &= \alpha_{c\bar{c}}(0) + \alpha'_{c\bar{c}} M_{\eta_c(1S)}^2, \\
0 &= \alpha_{c\bar{c}}(0) + \alpha'_{c\bar{c}} M_{D}^2, \\
0 &= \alpha_{b\bar{b}}(0) + \alpha'_{b\bar{b}} M_{\eta_b(1S)}^2, \\
0 &= \alpha_{n\bar{b}}(0) + \alpha'_{n\bar{b}} M_{B}^2,
\end{align}

and with the help of the relations (2) and (3), one can extract the parameters of the $1\ ^1S_0$ trajectories shown in Table 2.

|        | $n\bar{n}$ | $s\bar{s}$ | $n\bar{s}$ | $c\bar{c}$ | $b\bar{b}$ |
|--------|------------|------------|------------|------------|------------|
| $\alpha(0)$ | -0.01316  | -0.3260   | -0.1696   | -3.6178   | -1.8155    |
| $\alpha'(\text{GeV}^{-2})$ | 0.7218     | 0.6613    | 0.6902    | 0.4075    | 0.5209     |
| $\alpha(0)$ | -1.9719   | -17.9790  | -8.9960   | -9.1525   | -10.7980   |
| $\alpha'(\text{GeV}^{-2})$ | 0.5043    | 0.2079    | 0.3228    | 0.3164    | 0.2753     |

**Table 2.** Parameters of the $1\ ^1S_0$ trajectories of the form (1).

Based on these parameters, the masses of the $J = 0$, $J = 2$ and $J = 4$ states lying on the $1\ ^1S_0$ trajectories can be estimated. Comparison of our predictions with those given by other references is shown in Tables 3-I $\sim$ 3-III. Hereafter, the masses used as input for our calculation are shown in boldface.

### 2.2 The $1\ ^1P_1$ and $1\ ^3P_1$ trajectories

In our calculation of the masses of the states lying on the $1\ ^1P_1$ ($1\ ^3P_1$) trajectories, we adopt the assumption presented by Ref.[7] that the slopes of the parity partners trajectories coincide, and further, that the slopes do not depend on charge conjugation in accordance with

\[ M_K = (M_{K^0} + M_{K^\pm})/2, \quad M_D = (M_{D^0} + M_{D^\pm})/2, \quad M_B = (M_{B^0} + M_{B^\pm})/2. \]

Here and below, all the masses used as input for our calculation are taken from PDG2002[2].
Table 3-I. The masses of the $n\bar{n}$, $s\bar{s}$ and $n\bar{s}$ states lying on the $1^1S_0$ trajectories.

| Reference | $M_{n\bar{n}}$ (GeV) | $M_{s\bar{s}}$ (GeV) | $M_{n\bar{s}}$ (GeV) |
|-----------|----------------------|----------------------|----------------------|
| Present work | 0.135 1.67 | 0.702 1.875 | 0.4957 1.773 2.458 |
| Exp.[2] | 0.135 1.67 | 0.4957 1.773 |
| Ref.[7] | 0.135 1.677 2.237 | 0.689 1.869 2.429 | 0.493 1.773 2.333 |
| Ref.[19] | 0.15 1.68 2.33 | 0.96 1.89 2.51 | 0.47 1.78 2.41 |

Table 3-II. The masses of the $c\bar{n}$, $c\bar{s}$, $n\bar{b}$ and $s\bar{b}$ states lying on the $1^1S_0$ trajectories.

| Reference | $M_{c\bar{n}}$ (GeV) | $M_{c\bar{s}}$ (GeV) | $M_{n\bar{b}}$ (GeV) | $M_{s\bar{b}}$ (GeV) |
|-----------|----------------------|----------------------|----------------------|----------------------|
| Present work | 1.8669 2.706 3.341 | 1.977 2.806 3.441 | 5.2792 5.837 6.345 | 5.378 5.937 6.447 |
| Exp.[2] | 1.8669 1.9685 | 5.2792 |
| Ref.[7] | 1.8641 2.692 3.228 | 1.971 2.786 3.323 | 5.2798 5.83 6.286 | 5.3696 5.92 6.376 |
| Ref.[19] | 1.88 1.98 | 5.31 | 5.39 |
| Ref.[20] | 1.875 1.981 | 5.285 | 5.375 |
| Ref.[21] | 1.865 1.969 | 5.279 | 5.383 |
| Ref.[22] | 1.868 2.775 | 1.965 2.900 | 5.279 5.925 | 5.373 6.095 |
| Ref.[23] | 1.85 2.74 3.24 | 1.94 2.86 3.37 | 5.28 5.96 6.36 | 5.37 6.07 6.47 |

the C-invariance of QCD. Under this assumption, the corresponding slopes of the $1^1P_1$ ($1^3P_1$) are the same as those of the $1^1S_0$ trajectories. Therefore, for the $1^1P_1$ trajectories, inserting the masses of $b_1(1235)$, $D_1(2420)$ and $D_{s1}(2536)$ as well as the values of $\alpha'_{n\bar{n}}$, $\alpha'_{c\bar{c}}$ and $\alpha'_{c\bar{s}}$ shown in Table 2 into the following equations

$$1 = \alpha_{n\bar{n}}(0) + \alpha'_{n\bar{n}}M^2_{b_1(1235)},$$

$$1 = \alpha_{c\bar{n}}(0) + \alpha'_{c\bar{n}}M^2_{D_1(2420)},$$

$$1 = \alpha_{c\bar{s}}(0) + \alpha'_{c\bar{s}}M^2_{D_{s1}(2536)},$$

together with the relations (2) and (3), one can extract the parameters of the $1^1P_1$ trajectories summarized in Table 4. With these parameters, the masses of the $J = 1$, $J = 3$ and $J = 5$ states lying on the $1^1P_1$ trajectories can be obtained. Comparison of our calculations with those performed by other references is given in Tables 5-I ~ 5-II.

For the $1^3P_1$ trajectories, in order to extract the value of $\alpha_{n\bar{s}}(0)$, we should know the mass of $K_{1A}$ since the states containing the s-quarks of the $1^3P_1$ multiplet are not established experimentally except for $K_{1A}$. It is well-known that the physical states $K_1(1400)$ and $K_1(1270)$ are the mixtures of $K_{1A}$ and $K_{1B}$, the strange members of $1^3P_1$ and $1^1P_1$, therefore, the masses
Table 3-III. The masses of the $c\bar{c}$, $b\bar{b}$ and $c\bar{b}$ states lying on the $1^1S_0$ trajectories.

| Reference          | $M_{c\bar{c}}$ (GeV) | $M_{b\bar{b}}$ (GeV) | $M_{c\bar{b}}$ (GeV) |
|-------------------|----------------------|----------------------|----------------------|
| Present work      | 2.9797               | 3.713                | 9.3                  |
| Exp. [2]          | 2.9797               | 9.3                  |                      |
| Ref. [7]          | 2.9798               | 3.692                | 9.424                |
| Ref. [19]         | 2.97                 | 3.84                 | 9.40                 |
| Ref. [23]         | 3.60                 | 3.82                 | 9.41                 |
| Ref. [24]         | 2.979                | 3.811                | 9.4                  |
| Ref. [25]         | 2.993                | 3.778                | 9.435                |
| Ref. [26]         | 2.98                 | 9.477                | 10.127               |
| Ref. [27]         | 2.98                 | 9.403                | 10.155               |
| Ref. [28]         | 2.98                 | 9.406                | 6.286                |
| Ref. [29]         | 2.979                | 3.796                | 9.359                |
| Ref. [30]         | 2.979                | 9.408                |                      |
| Ref. [31]         | 2.921                | 3.867                | 9.369                |
| Ref. [32]         | 2.987                | 3.872                | 9.413                |
| Ref. [33]         |                      |                      | 6.253                |
| Ref. [34]         |                      |                      | 6.310                |
| Ref. [35]         |                      |                      | 6.255                |
| Ref. [36]         |                      |                      | 6.290                |
| Ref. [37]         |                      |                      | 6.255                |
| Ref. [38]         |                      |                      | 6.326                |
| Ref. [39]         |                      |                      | 6.386                |
| Ref. [40]         |                      |                      | 6.194-6.292          |
| Ref. [41]         |                      |                      | $\geq 6.220$         |

Table 4. Parameters of the $1^1P_1$ trajectories of the form (1).

\[
M_{K_{1A}}^2 + M_{K_{1B}}^2 = M_{K_1(1400)}^2 + M_{K_1(1270)}^2,
\]

then using $M_{K_1(1400)} = 1.402$ GeV, $M_{K_1(1270)} = 1.273$ GeV[2], and $M_{K_{1B}} = 1.36$ GeV shown in Table 5-I, one can have $M_{K_{1A}} = 1.318$ GeV. Based on the values of the slopes shown in Table 2, inserting $M_{K_{1A}} = 1.318$ GeV as well as the masses of $a_1(1260)$, $\chi_{c1}(1P)$ and $\chi_{b1}(1P)$ into the following equations

\[
1 = \alpha_{n\bar{n}}(0) + \alpha'_{n\bar{n}} M_{a_1(1260)}^2,
\]

\[
1 = \alpha_{n\bar{s}}(0) + \alpha'_{n\bar{s}} M_{K_{1A}}^2,
\]

\[
1 = \alpha_{c\bar{c}}(0) + \alpha'_{c\bar{c}} M_{\chi_{c1}(1P)}^2,
\]
Table 5-I. The masses of the $n\bar{n}$, $s\bar{s}$ and $n\bar{s}$ states lying on the $1^3P_1$ trajectories.

$$1 = \alpha_{bb}(0) + \alpha'_{bb} M_{\chi_{bb}}^2(1P), \quad (20)$$

and using the relations (2) and (3), one can extract the parameters of the $1^3P_1$ trajectories summarized in Table 6. In terms of these parameters, the masses of the $J = 1$, $J = 3$ and $J = 5$ states lying on the $1^3P_1$ trajectories can be given. Comparison of our calculations with those performed by other references is shown in Tables 7-I ~ 7-III.
The masses of the $n\bar{n}$, $s\bar{s}$ and $n\bar{s}$ states lying on the $1^3P_1$ trajectories.

Table 7-I.

| Reference | $M_{n\bar{n}}$ (GeV) | $M_{s\bar{s}}$ (GeV) | $M_{n\bar{s}}$ (GeV) |
|-----------|----------------------|----------------------|----------------------|
| Present work | 1.23 | 2.070 | 2.656 |
| Exp. [2] | 1.23 | | |
| Ref. [7] | 1.230 | 2.000 | 2.427 |
| Ref. [19] | | | |

Table 7-II.

| Reference | $M_{c\bar{n}}$ (GeV) | $M_{c\bar{s}}$ (GeV) | $M_{n\bar{b}}$ (GeV) | $M_{s\bar{b}}$ (GeV) |
|-----------|----------------------|----------------------|----------------------|----------------------|
| Present work | 2.423 | 3.116 | 3.681 | 5.849 |
| Exp. [2] | 2.418 | 3.042 | 3.473 | 5.796 |
| Ref. [7] | | | 2.505 | 6.367 |
| Ref. [19] | | | | 6.845 |

2.3 The $1^3S_1$ trajectories

Inserting the masses of $\rho$, $\rho_3(1690)$, $K^*(892)$, $K_3^*(1780)$, $J/\psi$, $D^*(2010)$, $\Upsilon$ and $B^*$ into the following equations

\[
1 = \alpha_{n\bar{n}}(0) + \alpha'_{n\bar{n}} M^2_{\rho},
\]
\[
3 = \alpha_{n\bar{n}}(0) + \alpha'_{n\bar{n}} M^2_{\rho_3(1690)},
\]
\[
1 = \alpha_{n\bar{s}}(0) + \alpha'_{n\bar{s}} M^2_{K^*(892)},
\]
\[
3 = \alpha_{n\bar{s}}(0) + \alpha'_{n\bar{s}} M^2_{K_3^*(1780)},
\]
\[
1 = \alpha_{c\bar{b}}(0) + \alpha'_{c\bar{b}} M^2_{J/\psi},
\]
\[
1 = \alpha_{c\bar{n}}(0) + \alpha'_{c\bar{n}} M^2_{D^*(2010)},
\]
\[
1 = \alpha_{b\bar{b}}(0) + \alpha'_{b\bar{b}} M^2_{\Upsilon(1S)},
\]
\[
1 = \alpha_{n\bar{b}}(0) + \alpha'_{n\bar{b}} M^2_{B^*},
\]

and by means of the relations (2) and (3), one can extract the parameters of the $1^3S_1$ trajectories as shown in Table 8. For the masses of the $J = 1$, $J = 3$ and $J = 5$ states lying on the $1^3S_1$ trajectories, we have

\[
M_{K^*(892)} = (M_{K^*(892)} + M_{K^*(892)^*}) / 2, \quad M_{D^*(2010)} = (M_{D^*(2010)} + M_{K^*(2007)^*}) / 2.
\]
The masses of the $c\bar{c}$, $b\bar{b}$ and $c\bar{b}$ states lying on the $1^{3}P_{1}$ trajectories.

Table 7-III. The masses of the $c\bar{c}$, $b\bar{b}$ and $c\bar{b}$ states lying on the $1^{3}P_{1}$ trajectories.

| Reference  | $M_{c\bar{c}}$ (GeV) | $M_{b\bar{b}}$ (GeV) | $M_{c\bar{b}}$ (GeV) |
|------------|----------------------|----------------------|----------------------|
| Present work | 3.51051 | 4.151 | 4.705 | 9.8927 | 10.368 | 10.822 | 6.788 | 7.303 | 7.785 |
| Exp.[2]     | 3.51051 |             | 9.8927 |             |             |             |             |             |             |
| Ref.[7]     | 3.5105 | 4.08 | 4.513 | 9.8919 | 10.33 | 10.727 | 6.74 | 7.215 | 7.62 |
| Ref.[19]    | 3.51 | 4.10 | 9.88 | 10.35 | 6.742 |             |             |             |             |
| Ref.[23]    | 3.50 | 4.06 | 9.87 | 10.36 | 6.74 | 7.25 |             |             |             |
| Ref.[24]    | 3.510 | 9.892 |             |             |             |             |             |             |             |
| Ref.[25]    | 3.455 | 9.885 |             |             |             |             |             |             |
| Ref.[26]    | 3.486 | 9.864 |             |             |             |             |             |             |
| Ref.[27]    | 3.502 | 9.891 | 10.347 |             |             |             |             |             |
| Ref.[28]    | 3.482 | 9.891 |             |             |             |             |             |             |
| Ref.[29]    | 3.482 | 9.895 |             |             |             |             |             |             |
| Ref.[30]    | 3.511 | 9.893 |             |             |             |             |             |             |
| Ref.[31]    | 3.506 | 9.893 |             |             |             |             |             |             |
| Ref.[32]    | 3.513 | 9.893 |             |             |             |             |             |             |
| Ref.[33]    |             |             |             |             |             |             |             | 6.718 |
| Ref.[34]    |             |             |             |             |             |             |             | 6.760 |
| Ref.[35]    |             |             |             |             |             |             |             | 6.730 |
| Ref.[36]    |             |             |             |             |             |             |             | 6.750 |

Table 8. Parameters of the $1^{3}S_{1}$ trajectories of the form (1).

| $\alpha(0)$ | $\alpha'(0)$ | $\alpha'(1)$ | $\alpha'(2)$ | $\alpha'(3)$ | $\alpha'(4)$ |
|-------------|--------------|--------------|--------------|--------------|--------------|
| $n\bar{n}$  | 0.4749       | 0.1675       | 0.3212       | -3.1851      | -1.3551      |
| $s\bar{s}$  | 0.8830       | 0.8181       | 0.8493       | 0.4364       | 0.5841       |
| $c\bar{c}$  | -1.5088      | -17.338      | -8.4316      | -8.5853      | -10.2616     |
| $b\bar{b}$  | 0.5092       | 0.2049       | 0.3326       | 0.3277       | 0.2789       |

2.4 The $1^{3}P_{2}$ trajectories

Because the tensor meson trajectories are the parity partners of the vector meson trajectories, according to the assumption mentioned in Section 2.2, the corresponding slopes of the tensor meson trajectories are the same as those of the vector meson trajectories. So, according to the values of the slopes shown in Table 8, in the presence of the relations (2) and (3), inserting the
masses of $a_2(1320)$, $K_2^*(1430)^3$, $\chi_{c2}(1P)$ and $\chi_{b2}(1P)$ into the following equations

\begin{align}
2 &= \alpha_{n\bar{n}}(0) + \alpha'_{n\bar{n}} M_{a_2(1320)}^2, \quad (29) \\
2 &= \alpha_{n\bar{s}}(0) + \alpha'_{n\bar{s}} M_{K_2^*(1430)}^2, \quad (30) \\
2 &= \alpha_{c\bar{c}}(0) + \alpha'_{c\bar{c}} M_{\chi_{c2}(1P)}^2, \quad (31) \\
2 &= \alpha_{b\bar{b}}(0) + \alpha'_{b\bar{b}} M_{\chi_{b2}(1P)}^2, \quad (32)
\end{align}

one can extract the parameters of the tensor meson trajectories as summarized in Table 10.

Based on these parameters, the masses of the $J = 2$, $J = 4$ and $J = 6$ states lying on the tensor meson trajectories can be obtained. Comparison of our predictions with those given by other references is shown in Tables 11-I ∼ 11-III.

2.5 The $2 \ ^3S_1$ trajectories

Finally, we wish to discuss the $2 \ ^3S_1$ trajectories. According to Ref.[5], the corresponding slopes of the $2 \ ^3S_1$ and $1 \ ^3S_1$ trajectories coincide. Therefore, based on the values of the slopes

\begin{equation}
3 M_{K_2^*(1430)} = (M_{K_2^*(1430)}^2 + M_{K_2^*(1430)}^2)/2.
\end{equation}
### Table 9-III. The masses of the $c\bar{c}$, $b\bar{b}$ and $c\bar{b}$ states lying on the $1^3S_1$ trajectories.

| Reference | $M_{c\bar{c}}$ | | | $M_{b\bar{b}}$ | | | $M_{c\bar{b}}$ | |
|-----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Present work | 3.09687 | 3.705 | 4.331 | 9.4603 | 9.963 | 10.44 | 6.354 | 6.896 | 7.397 |
| Exp.[2] | 3.09687 | | | 9.4603 | | | | | |
| Ref.[7] | 3.0969 | 3.753 | 4.24 | 9.4604 | 9.906 | 10.30 | 6.356 | 6.854 | 7.276 |
| Ref.[19] | 3.1 | 3.85 | | 9.46 | 10.16 | | 6.34 | 7.04 | |
| Ref.[23] | 3.19 | 3.83 | | 9.46 | 10.15 | | 6.34 | 7.04 | |
| Ref.[24] | 3.096 | 3.815 | | 9.460 | 10.162 | | 6.332 | 7.081 | |
| Ref.[25] | 3.093 | 3.913 | | 9.451 | 10.165 | | 6.347 | 7.086 | |
| Ref.[26] | 3.097 | | | 9.464 | 10.130 | | 6.337 | 7.005 | |
| Ref.[27] | 3.097 | | | 9.460 | 10.163 | | 6.349 | 7.049 | |
| Ref.[28] | 3.098 | | | 9.461 | | | 6.341 | 7.032 | |
| Ref.[29] | 3.118 | | | 9.462 | 10.149 | | 6.372 | | |
| Ref.[30] | 3.097 | | | 9.460 | | | 6.308 | | |
| Ref.[31] | 3.125 | 3.867 | | 9.461 | 10.172 | | | | |
| Ref.[32] | 3.104 | 3.884 | | 9.459 | 10.172 | | | | |
| Ref.[33] | | | | | | | 6.317 | | |
| Ref.[34] | | | | | | | 6.355 | | |
| Ref.[35] | | | | | | | 6.320 | | |
| Ref.[36] | | | | | | | 6.321 | | |
| Ref.[37] | | | | | | | 6.333 | | |
| Ref.[40] | | | | | | | 6.284-6.357 | | |
| Ref.[41] | | | | | | | $\geq 6.279$ | | |

### Table 10. Parameters of the $1^3P_2$ trajectories of the form (1).

| | $n\bar{n}$ | $s\bar{s}$ | $n\bar{s}$ | $c\bar{c}$ | $c\bar{b}$ |
|----------------|--------------|--------------|--------------|--------------|--------------|
| $\alpha(0)$ | 0.4661 | 0.0653 | 0.2657 | -3.5189 | -1.5264 |
| $\alpha'(\text{GeV}^{-2})$ | 0.8830 | 0.8181 | 0.8493 | 0.4364 | 0.5841 |
| $c\bar{s}$ | $b\bar{s}$ | $a\bar{b}$ | $a\bar{s}$ | $c\bar{b}$ |
| $\alpha(0)$ | -1.7266 | -18.1354 | -8.8337 | -9.0341 | -10.8261 |
| $\alpha'(\text{GeV}^{-2})$ | 0.5692 | 0.3049 | 0.3326 | 0.3277 | 0.2789 |

shown in Table 8, inserting the masses of $\rho(1450)$, $\psi(2S)$ and $\Upsilon(2S)$ into the following equations,

$$
1 = \alpha_{n\bar{n}}(0) + \alpha'_{n\bar{n}} M^2_{\rho(1450)},
$$

$$
1 = \alpha_{c\bar{c}}(0) + \alpha'_{c\bar{c}} M^2_{\psi(2S)},
$$

$$
1 = \alpha_{b\bar{b}}(0) + \alpha'_{b\bar{b}} M^2_{\Upsilon(2S)},
$$

one can extract the values of $\alpha_{n\bar{n}}(0)$, $\alpha_{c\bar{c}}(0)$ and $\alpha_{b\bar{b}}(0)$. In order to obtain the parameters of the trajectories on which the states containing $s$-quarks lie, we should have the masses of the states containing $s$-quarks of the $2^3S_1$ multiplet. According to Ref.[2], only the mass of the state $\phi(1680)$ is well established experimentally\textsuperscript{4}. Also, the physical state $\phi(1680)$ is usually believed

\textsuperscript{4}The assignment that $K^{*}(1410)$ belongs to the $2^3S_1$ multiplet is problematic, which will be discussed below.
Table 11-I. The masses of the \( n\bar{n}, s\bar{s} \) and \( n\bar{s} \) states lying on the \( 1^3P_2 \) trajectories.

| Reference | \( M_{n\bar{n}} \) | \( M_{s\bar{s}} \) | \( M_{n\bar{s}} \) |
|-----------|-----------------|-----------------|-----------------|
| Present work | 1.318 | 2.001 | 2.530 |
| Exp.[2] | 1.318 | 2.011 | 2.045 |
| Ref.[7] | 1.3181 | 1.927 | 2.256 |
| Ref.[19] | 1.31 | 2.01 | 1.54 |

Table 11-II. The masses of the \( c\bar{n}, c\bar{s}, n\bar{b} \) and \( s\bar{b} \) states lying on the \( 1^3P_2 \) trajectories.

| Reference | \( M_{c\bar{n}} \) | \( M_{c\bar{s}} \) | \( M_{n\bar{b}} \) | \( M_{s\bar{b}} \) |
|-----------|-----------------|-----------------|-----------------|-----------------|
| Present work | 2.457 | 3.076 | 3.590 | 2.559 |
| Exp.[2] | 2.458 | 3.01 | 3.19 | 5.707 |
| Ref.[7] | 2.454 | 3.11 | 5.698 | 5.797 |
| Ref.[19] | 2.459 | 2.560 | 5.733 | 5.844 |
| Ref.[20] | 2.465 | 2.573 | 5.759 | 5.875 |
| Ref.[21] | 2.460 | 3.091 | 5.714 | 5.820 |
| Ref.[22] | 2.46 | 3.03 | 5.71 | 5.82 |

Based on the value of \( \alpha_{n\bar{n}}(0), \alpha_{c\bar{n}}(0), \alpha_{n\bar{b}}(0) \) and \( \alpha_{s\bar{b}}(0) \), from the relations (2) and (3), all the parameters of the \( 2^3S_1 \) trajectories are presented in Table 12. The masses of the \( J = 1, J = 3 \) and \( J = 5 \) states lying on the \( 2^3S_1 \) trajectories given by the present work as well as those
| Reference   | \( J = 2 \) | \( J = 4 \) | \( J = 6 \) | \( J = 2 \) | \( J = 4 \) | \( J = 6 \) | \( J = 2 \) | \( J = 4 \) | \( J = 6 \) |
|------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Present work | 3.55618     | 4.151       | 4.670       | 9.9126      | 10.393      | 10.854      | 6.781       | 7.291       | 7.767       |
| Exp.[2]     | 3.55618     |             |             |             |             |             |             |             |             |
| Ref.[7]     | 3.5562      | 4.092       | 4.498       | 9.9132      | 10.310      | 10.655      | 6.781       | 7.213       | 7.582       |
| Ref.[19]    | 3.55        | 4.06        |             | 9.89        | 10.36       |             | 6.77        | 7.27        |             |
| Ref.[23]    | 3.54        | 4.09        |             | 9.90        | 10.36       |             | 6.76        | 7.25        |             |
| Ref.[24]    | 3.556       |             |             | 9.913       |             |             | 6.762       |             |             |
| Ref.[25]    | 3.589       |             |             | 9.921       |             |             | 6.890       |             |             |
| Ref.[26]    | 3.507       |             |             | 9.886       |             |             | 6.747       |             |             |
| Ref.[27]    | 3.556       |             |             | 9.913       | 10.353      |             | 6.787       |             |             |
| Ref.[28]    | 3.530       |             |             | 9.910       |             |             | 6.772       |             |             |
| Ref.[29]    | 3.527       |             |             | 9.917       |             |             |             |             |             |
| Ref.[30]    | 3.557       |             |             | 9.914       |             |             | 6.773       |             |             |
| Ref.[31]    | 3.561       |             |             | 9.912       |             |             |             |             |             |
| Ref.[32]    | 3.557       |             |             | 9.911       |             |             |             |             |             |
| Ref.[33]    |             |             |             |             |             |             |             |             |             |
| Ref.[34]    |             |             |             |             |             |             |             |             |             |
| Ref.[35]    |             |             |             |             |             |             |             |             |             |
| Ref.[36]    |             |             |             |             |             |             |             |             |             |

Table 11-III. The masses of the \( c\bar{c}, b\bar{b} \) and \( c\bar{b} \) states lying on the \( 1\,^3P_2 \) trajectories.

This table predicted by other references are shown in Tables 13-I ~ 13-III.

### Table 12. Parameters of the \( 2\,^3S_1 \) trajectories of the form (1).

| \( \alpha(0) \) | \( n\bar{n} \) | \( s\bar{s} \) | \( n\bar{s} \) | \( c\bar{c} \) | \( c\bar{n} \) | \( c\bar{b} \) | \( s\bar{b} \) | \( n\bar{b} \) | \( a'_{n\bar{b}} \) | \( a'_{s\bar{s}} \) | \( a'_{s\bar{b}} \) | \( a'_{c\bar{c}} \) | \( a'_{c\bar{s}} \) | \( a'_{c\bar{b}} \) | \( a'_{n\bar{b}} \) | \( a'_{s\bar{b}} \) |
|----------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( a'_{n\bar{b}} \) | -0.9591 | -1.3090 | -1.1021 | -4.9291 | -2.9121 | | | | | | | | | | |
| \( a'_{s\bar{s}} \) | 0.8830 | 0.8181 | 0.8493 | 0.4364 | 0.5841 | | | | | | | | | | |
| \( a'_{s\bar{b}} \) | -0.1219 | -0.5854 | -0.2403 | -10.4472 | -12.2572 | | | | | | | | | | |
| \( a'_{c\bar{c}} \) | 0.5692 | 0.2049 | 0.3126 | 0.3277 | 0.2789 | | | | | | | | | | |

3 Discussions of our results

Comparison of the predictions given by the present work with those given by other references clearly illustrates that the agreement between our results and those of many other approaches is generally satisfactory, which indicates that in the presence of the relations (2) and (3), the quasi-linear Regge trajectory can give a reasonable description for the spectrum of both light and heavy mesons.

From the parameters shown in Tables 2 and 8, we find that the slopes of Regge trajectories are flavor-dependent and approximately satisfy \( a'_{n\bar{b}} > a'_{s\bar{s}} > a'_{s\bar{b}} > a'_{c\bar{c}} > a'_{c\bar{s}} > a'_{c\bar{b}} > a'_{n\bar{b}} > a'_{s\bar{b}} \).
With the help of the relation (36) and the parameters shown in Tables 2 and 8, one can have

\[ \alpha'_i M_{in}^2 + \alpha'_{jj} M_{jj}^2 = 2 \alpha'_{j1} M_{j1}^2. \]  

(36)

With the help of the relation (36) and the parameters shown in Tables 2 and 8, one can have

\[ \begin{align*}
8.13 M_{n\bar{s}}^2 + 4.80 M_{c\bar{c}}^2 &= 6.14 M_{c\bar{b}}^2 + 5.94 M_{c\bar{s}}^2 \\
17.1 M_{n\bar{s}}^2 + 5.15 M_{b\bar{b}}^2 &= 8.0 M_{n\bar{b}}^2 + 7.84 M_{b\bar{s}}^2
\end{align*} \]

for the \(1^1S_0\)-like trajectories,  

(37)

\[ \begin{align*}
8.13 M_{n\bar{s}}^2 + 4.18 M_{c\bar{c}}^2 &= 5.59 M_{c\bar{b}}^2 + 5.45 M_{c\bar{s}}^2 \\
17.1 M_{n\bar{s}}^2 + 4.13 M_{b\bar{b}}^2 &= 6.70 M_{n\bar{b}}^2 + 6.60 M_{b\bar{s}}^2
\end{align*} \]

for the \(1^3S_1\)-like trajectories,  

(38)

here and below, the \(1^1S_0\)-like trajectories (\(1^3S_1\)-like trajectories) denote the trajectories whose slopes coincide with those of the \(1^1S_0\) trajectories (\(1^3S_1\) trajectories). The similar relations

\[ \begin{align*}
8.13 M_{n\bar{s}}^2 + 4.75 M_{c\bar{c}}^2 &= 6 M_{c\bar{b}}^2 + 6 M_{c\bar{s}}^2 \\
17.1 M_{n\bar{s}}^2 + 3.64 M_{b\bar{b}}^2 &= 6 M_{n\bar{b}}^2 + 6 M_{b\bar{s}}^2
\end{align*} \]

for all the trajectories,  

(39)

have been proposed in Ref.[8] based on the simplification that the Regge slopes in the light quark sector are the same for all the meson multiplets. The non-integer coefficients in (37), (38)
Reference $J = 1$ $J = 3$ $J = 5$ $J = 1$ $J = 3$ $J = 5$ $J = 1$ $J = 3$ $J = 5$

Present work 3.68596 4.263 4.770 10.02326 10.499 10.954 6.894 7.396 7.866

Exp.[2] 3.68596 10.02326

Ref.[19] 3.68 4.22 10.00 10.45 6.89

Ref.[23] 3.73 4.24 10.02 10.47 6.90 7.37

Ref.[24] 3.686 10.023 6.89

Ref.[25] 3.719 4.332 10.023 10.47 6.90 7.459

Ref.[26] 3.686 10.007 6.881

Ref.[27] 3.719 4.332 10.023 10.465 6.935 7.459

Ref.[28] 3.693 10.027 6.914

Ref.[29] 3.716 10.004 10.44

Ref.[30] 3.686 10.016 6.886

Ref.[31] 3.685 10.019

Ref.[32] 3.670 10.015

Ref.[33] 6.902

Ref.[34] 6.917

Ref.[35] 6.900

Ref.[36] 6.990

Table 13-III. The masses of the $c\bar{c}$, $b\bar{b}$ and $c\bar{b}$ states lying on the $2^3S_1$ trajectories.

and (39) reflect the uncertainty in fitting the values of the Regge slopes. Note that the accuracy of our results, (37) and (38), is better than that of (39). For example, for the $1^3S_1$ multiplet, $17M_{n\bar{s}}^2 + 4.13M_{b\bar{b}}^2 = 6.70M_{n\bar{b}}^2 + 6.60M_{s\bar{b}}^2$ gives 383.29 GeV$^2$ on the l.h.s. vs. 383.58 GeV$^2$ on the r.h.s., with an accuracy of $\sim 0.08\%$, while $17M_{n\bar{s}}^2 + 3.64M_{b\bar{b}}^2 = 6M_{n\bar{b}}^2 + 6M_{s\bar{b}}^2$ gives 339.44 GeV$^2$ on the l.h.s. vs. 346.13 GeV$^2$ on the r.h.s., with an accuracy of $\sim 2\%$.

Also, based on the predicted masses shown in section 2, and comparing $M_{n\bar{n}}^2 + M_{s\bar{s}}^2$ with $2M_{n\bar{s}}^2$, one can find that the relation $M_{n\bar{n}}^2 + M_{s\bar{s}}^2 = 2M_{n\bar{s}}^2$ holds with an accuracy of $\sim 4\%$ for the $1^1S_0$ multiplet, and with an accuracy of $\leq 1\%$ for the remaining multiplets considered in our present work. In fact, from the parameters shown in Tables 2 and 8, together with the formula (36), we can have

\[
1.09M_{n\bar{n}}^2 + M_{s\bar{s}}^2 = 2 \times 1.04M_{n\bar{s}}^2 \quad \text{for } 1^1S_0\text{-like trajectories,}
\]

\[
1.08M_{n\bar{n}}^2 + M_{s\bar{s}}^2 = 2 \times 1.04M_{n\bar{s}}^2 \quad \text{for } 1^3S_1\text{-like trajectories,}
\]

from which, one can naturally expect that the Gell-Mann-Okubo mass relation $M_{n\bar{n}}^2 + M_{s\bar{s}}^2 = 2M_{n\bar{s}}^2$[44] can hold for all the multiplets with a good accuracy.

In our present work, the masses of the strange members of $1^1P_1$ and $1^3P_1$, $M_{K_{1B}}$ and $M_{K_{1A}}$, are determined to be the values of 1.36 GeV and 1.318 GeV, respectively. Inserting $M_{K_{1B}}$ and
one can extract $\theta_K$, the mixing angle of $K_{1A}$ and $K_{1B}$, is about 54.5°. Such result is inconsistent with $\theta_K \sim 33°$ suggested in Refs.[19, 42], however, it should be noted that the masses of $K_{1B}$ and $K_{1A}$ predicted by us are in excellent agreement with the results that and $M_{K_{1A}} = 1322$ MeV suggested by Burakovsky and Goldman in a nonrelativistic constituent quark model[45], also with the results that $M_{K_{1B}} \simeq 1.368$ MeV and $M_{K_{1A}} \simeq 1.31$ GeV given in Ref.[46]. Further, inputting $M_{s\bar{s}} = 1.405$ GeV which is obtained in the presence of $M_{K_{1A}} = 1.318$ GeV, and repeating the calculation of our previous paper[47], one can have $|f_1(1410)\rangle = \cos \theta |8\rangle - \sin \theta |1\rangle$ and $|f_1(1285)\rangle = \sin \theta |8\rangle + \cos \theta |1\rangle$ with $\theta = 47.3°$, which is in agreement with the value of $\theta \sim 50°$ suggested by Close and Kirk[48].

Particle Data Group state that the $K^*(1410)$ could be replaced by the $K^*(1680)$ as the $2\,^3S_1$ state[2]. The problem with the $K^*(1410)$ is that it is much too light to be the $2\,^3S_1$ state, even if one takes into account the $2\,^3S_1 - 1\,^3D_1$ mixing, therefore it is suggested by Törnqvist[49] that one can well doubt the existence of the $K^*(1410)$. In Table 13-I, the mass of the strange member of the $2\,^3S_1$ multiplet is determined to be the value of 1.573 GeV, which is in excellent agreement with the value of 1.58 GeV predicted by Godfrey and Isgur[19] in a relativistic quark model. Comparison of 1573 MeV and $1414 \pm 15$ MeV, the mass of the state $K^*(1410)$[2], would challenge the assignment that the state $K^*(1410)$ is the strange member of the $2\,^3S_1$ multiplet. It has been suggested[49] that the state $K^*(1680)$ should be resolved into two separate states of normal widths ($\Gamma \approx 150$ MeV) fitting well the $1\,^3D_1$ ($\approx 1784$ MeV) and $2\,^3S_1$ ($\approx 1608$ MeV) states. Our results support the state $K^*(1573)$, rather than the $K^*(1410)$, being the member of the $2\,^3S_1$ multiplet, which also agrees with the conclusion given by Ref.[50].

The masses of the pure $s\bar{s}$ states predicted by us can not be directly measured experimentally, since the pure isoscalar $n\bar{n}$ and $s\bar{s}$ states usually can mix. However, comparison of the mass of the pure $s\bar{s}$ state with that of the physical states (mainly SU(3) singlet) can help us to understand the mixing of the two isoscalar physical states of a meson nonet. For example, for
the $1^1S_0$ nonet, $M_{\eta'} \approx 9.578$ GeV\cite{2} and $M_{s\bar{s}} \approx 0.702$ GeV shown in Table 3-I, which implies that the $\eta$ and $\eta'$ must be non-ideally mixing. However, for the $1^3S_1$ nonet, $M_\phi \approx 1.02$ GeV\cite{2} and $M_{s\bar{s}} \approx 1.01$ GeV shown in Table 9-I, which implies that the $\omega$ and $\phi$ are almost ideally mixing. The above deduction is consistent with the usual understanding of the mixing picture about $\eta - \eta'$ ($\omega - \phi$).

For these heavy-quark states such as the $\eta_b(1S)$, $B_s^*$ and $h_c(1P)$ which are not included in the Meson Summary Table but appear in the Table 13.2 of Ref.\cite{2}, we want to give some comments. In our analysis of the $1^1S_0$ trajectories, we input the mass of the state $\eta_b(1S)$, 9.3 GeV, as the mass of the $b\bar{b}$ state, although the $b\bar{b}$ member of the $1^1S_0$ is not well established experimentally. Note that, as shown in Table 3-III, many theoretical predictions on the mass of the $b\bar{b}$ state of the $1^1S_0$ multiplet are in good agreement with the ALEPH measurement that $M_{\eta_b(1S)} = 9300 \pm 20 \pm 20$ MeV\cite{51}. Also, our predicted masses of $B_s$ and $B_c$, which are obtained with the help of $M_{\eta_b(1S)} = 9.3$ GeV, are in good agreement with the experimental data and the predictions given by many other references listed in Tables 3-II and 3-III. Therefore, if the state $\eta_b(1S)$ is confirmed experimentally, it would be a good candidate for the $b\bar{b}$ member of the $1^1S_0$ multiplet. From Table 5-II, it is clear that the agreement between the theoretical result on the mass of the $c\bar{c}$ state of the $1^1P_1$ multiplet and the experimental result that $M_{h_c(1P)} = 3526.14 \pm 0.24$ MeV\cite{2} is good. Also, Table 9-II indicates that many predicted results on the mass of the $s\bar{b}$ state of the $1^3S_1$ multiplet are in good agreement with the measured result that $M_{B_s^*} = 5416.6 \pm 3.5$ MeV\cite{2}. Therefore, if the $h_c(1P)$ and $B_s^*$ are confirmed experimentally, the suggestion that the $h_c(1P)$ is the $c\bar{c}$ member of the $1^1P_1$ multiplet and the $B_s^*$ is the $s\bar{b}$ member of the $1^3S_1$ multiplet seems satisfactory.

Recently, the quantum numbers of the $D_{sJ}(2457)$ was measured by the Belle Collaboration\cite{52}, the mass of the $D_{sJ}(2457)$ is determined to be the value of $2456.5 \pm 1.3 \pm 1.3$ MeV, and the measured results on the branching fractions are consistent with the spin-parity assignment for the $D_{sJ}(2457)$ of $1^+$. In the 2004 edition of Review of Particle Physics\cite{53}, this state has been included in the Meson Summary Table\textsuperscript{5}. Such experimental results support our prediction that

\textsuperscript{5}In Ref.\cite{53}, Particle Data Group use the $D_{sJ}(2460)$ with mass of $2459.3 \pm 1.3$ MeV for the $c\bar{s}$ state of the $1^3P_1$ multiplet.
the mass of the $c\bar{s}$ state of the $1^3P_1$ multiplet is about 2.5 GeV.

4 Summary and conclusion

In the quasi-linear Regge trajectory ansatz, with the help of additivity of inverse slopes and intercepts, the parameters of the $1^1S_0$, $1^1P_1$, $1^3P_1$, $1^3S_1$, $1^3P_2$ and $2^3S_1$ trajectories are extracted. Based on these parameters, the masses of the states lying on these Regge trajectories mentioned above are estimated. Our predictions are in reasonable agreement with those suggested by many other different approaches. We therefore conclude that the quasi-linear Regge trajectory can, at least at present, give a reasonable description for the meson spectroscopy, and its predictions may be useful for the discovery of the meson states which have not yet been observed.

Acknowledgments. This work is supported in part by National Natural Science Foundation of China under Contract No. 10205012, Henan Provincial Science Foundation for Outstanding Young Scholar under Contract No. 0412000300, Henan Provincial Natural Science Foundation under Contract No. 0311010800, and Foundation of the Education Department of Henan Province under Contract No. 2003140025.

References

[1] S. Godfrey and J. Napolitano, Rev. Mod. Phys. 71, 1411 (1999)
[2] K. Hagiwara et al., Phys. Rev. D 66, 1 (2002)
[3] P. D. Collins, An introduction to Regge theory and high energy physics, Cambridge University Press, 1977
[4] L. Basdevant, S. Boukraa, Z. Phys. C 28, 413 (1985)
[5] A. V. Anisovich, V. V. Anisovich and A. V. Sarantsev, Phys. Rev. D 62, 051502 (2000); V. V. Anisovich, hep-ph/0110326; ibid. hep-ph/0208123; ibid. hep-ph/0310165
[6] W. K. Tang, Phys. Rev. D 48, 2019 (1993); A. Brandat et al., Nucl. Phys. B 514, 3 (1999); M. M. Brisudova, L. Burakovsky and T. Goldmann, Phys. Lett. B 460, 1 (1999); A. Tang, J. W. Norbury, Phys. Rev. D 62, 016006 (2000); L. Pando Zayas, J. Sonnenschein, D. Vaman, hep-th/0311190; S. S. Afonin et al., hep-ph/0403268
[7] M. M. Brisudova, L. Burakovsky and T. Goldmann, Phys. Rev. D 61, 054013 (2000)

[8] L. Burakovsky, T. Goldman and L. P. Horwitz, Phys. Rev. D 56, 7119 (1997); ibid. J. Phys. G 24, 771 (1998)

[9] L. Burakovsky, T. Goldman and L. P. Horwitz, hep-ph/9708468

[10] P. Desgrolard, M. Giffon, E. Martynov and E. Predazzi, Eur. Phys. J. C 18, 555 (2001)

[11] K. Kawarabayashi, S. Kitakado and H. Yabuki, Phys. Lett. B 28, 423 (1969)

[12] R. Brower, J. Ellis, M. Schmidt and J. Weis, Nucl. Phys. B 128, 175 (1977)

[13] N. Kobylinsky, E. Martynov and A. Prognimak, Ukr. Phys. Zh. 24, 969 (1979)

[14] V. Dixit and L. Balazs, Phys. Rev. D 20, 816 (1979)

[15] K. Igi and s. Yazaki, Phys. Lett. B 71, 158 (1977)

[16] L. Burakovsky, T. Goldman, Phys. Lett. B 434, 251 (1998)

[17] A. B. Kaidalov, Z. Phys. C 12, 63 (1982)

[18] J. Pasupathy, Phys. Rev. Lett. 37, 1336 (1976); K. Igi, Phys. Lett. B 66, 276 (1977); Phys. Rev. D 16, 196 (1977); M. Kuroda, B. L. Young, Phys. Rev. D 16, 204 (1977)

[19] S. Godfrey, N. Isgur, Phys. Rev. D 32, 189 (1985); S. Godfrey, hep-ph/0406228

[20] D. Ebert, V. O. Galkin, R. N. Faustov, Phys. Rev. D 57, 5663 (1998)

[21] S. N. Gupta, J. M. Johnson, Phys. Rev. D 51, 168 (1995)

[22] M. Di Pierro, E. Eichten, Phys. Rev. D 64, 114004(2001)

[23] J. Zeng, J. W. Van orden and W. Roberts, Phys. Rev. D 52, 5229 (1995)

[24] D. Ebert, R. N. Faustov, V. O. Galkin, Phys. Rev. D 67, 014027 (2003)

[25] S. M. Ikhdair, R. Sever, hep-ph/0403280; ibid. hep-ph/0406005

[26] E. J. Eichten, C. Quigg, Phys. Rev. D 49, 5845 (1994)

[27] L. Motyka, K. Zalewski, Eur. Phys. J. C 4, 107 (1998)

[28] L. P. Fulcher, Phys. Rev. D 60, 074006 (1999)

[29] M. Baker, J. S. Ball, F. Zachariasen, Phys. Rev. D 45, 910 (1992)

[30] S. N. Gupta, J. Johnson, Phys. Rev. D 53, 312 (1996)
[31] L. P. Fulcher, Phys. Rev. D 42, 2337 (1990)
[32] L. P. Fulcher, Phys. Rev. D 44, 2097 (1991)
[33] S. S. Gershtein, V. V. Kiselev, A. K. Likhoded, A. V. Tkabladze, Phys. Rev. D 51, 3613 (1995)
[34] Y. Q. Chen, Y. P. Kuang, Phys. Rev. D 46, 1165 (1992)
[35] R. Roncaglia, A. Dzierba, D. B. Lichtenberg, E. Prdazzi, Phys. Rev. D 51, 1248 (1995)
[36] C. T. H. Davies et al., Phys. Lett. B 382, 131 (1996)
[37] E. Bagan, H. G. Dosch, P. Gospodinsky, S. Narison and J. M. Richard, Z. Phys. C 64, 57 (1994)
[38] N. Brambilla and A. Vairo, Phys. Rev. D 62, 094019 (2000)
[39] H. P. Shanahan, P. Boyle, C. Davies, H. Newton, Phys. Lett. B 453 (1999) 289.
[40] W. Kwong, J. L. Rosner, Phys. Rev. D 44, 212 (1991)
[41] S. Nussinov, M. A. Lampert, Phys. Rep. 363, 193 (2002)
[42] M. Suzuki, Phys. Rev. D 47, 1252 (1993)
[43] L. Kope and N. Wermes, Phys. Rep. 174, 67 (1989)
[44] S. Okubo, Prog. Theor. Phys. 27, 949 (1062)
[45] L. Burakovsky and T. Goldman, Phys. Rev. D 57, 2879 (1998)
[46] De-Min Li et al., Mod. Phys. Lett. A 18, 2775 (2003)
[47] De-Min Li, Hong Yu, Qi-Xing Shen, Chin. Phys. Lett. 17, 558 (2000)
[48] F. E. Close and A. Kirk, Z. Phys. C 76, 469 (1997)
[49] N. A. Törnqvist, Nucl. Phys. B 21 (Proc. Suppl.), 196 (1991)
[50] L. Burakovsky and L. P. Horwitz, Nucl. Phys. A 609, 585 (1996) L. Burakovsky and T. Goldman, Nucl. Phys. A 625, 220 (1997)
[51] ALEPH Collaboration, Phys. Lett. B 530, 56 (2002)
[52] Belle Collaboration, Phys. Rev. Lett. 92, 012002 (2004)
[53] S. Eidelman et al., Phys. Lett. B 592, 1 (2004)