Ising spin-glass transition in magnetic field out of mean-field: Numerical simulations and experiments

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Different Theories and Models (droplet, TNT and RSB).

Spin Glasses with Long Range Interactions.

The one dimensional diluted spin glass with long range interactions.
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What are Spin glasses

- Materials with disorder and frustration.
- Quenched disorder.
- Canonical Spin Glass: Metallic host (Cu) with magnetic impurities (Mn). RKKY interaction between magnetic moments:
  \[ J(r) \sim \frac{\cos(2k_F r)}{r^3}. \]
Some Definitions

- The typical Spin Glass Hamiltonian:

$$\mathcal{H} = - \sum_{i,j} J_{ij} \sigma_i \sigma_j$$

- The order parameter is:

$$q_{EA} = \langle \sigma_i \rangle^2$$

Using two real replicas:

$$\mathcal{H} = - \sum_{i,j} J_{ij} (\sigma_i \sigma_j + \tau_i \tau_j)$$

Let $$q_i = \sigma_i \tau_j$$ be the normal overlap, then: $$q_{EA} = \langle \sigma_i \tau_i \rangle.$$ We also define the link overlap: $$q_{i,\mu}^{\text{link}} = q_i q_{i+\mu}.$$
The Droplet Model

- Based on the Migdal-Kadanoff implementation (approximate) of the Renormalization Group (exact in $D = 1$).
- **Disguished Ferromagnet**: Only two pure states with order parameter $\pm q_{EA}$ (related by spin flip).
- Compact Excitations of fractal dimension $d_f$. The energy of a excitation of linear size $L$ grows as $L^\theta$.
- Any amount of magnetic field destroys the spin glass phase (even for Heisenberg spin glasses).
- Trivial probability distributions of the overlaps (both the normal overlap and the link one).
The Trivial Non Trivial (TNT) Model

- *Disguished Ferromagnet with Anti Periodic Boundary conditions.*
- Trivial probability distributions for the link overlap (the interface has no effect) but Non Trivial probability distribution for the normal one (induced by the interface).
Replica Symmetry Breaking (RSB) Theory

- Exact in $D = \infty$.
- Infinite number of phases (pure states) not related by any kind of symmetry.
- These (pure) states are organized in an ultrametric fashion.
- The spin glass phase is stable under (small) magnetic field.
- The excitations of the ground state are space filling.
Long Range Interactions

- Hamiltonian (Action) for the long range model ($J(r) \sim r^{-\rho/2}$):
  \[
  S_n = H_n \propto \int d^d k \left( k^{\rho-d} + \tau \right) \text{Tr} Q^2 + \int d^D x \left[ g_3 \text{Tr}(Q^3) + \lambda \sum Q^4_{ab} \right]
  \]

- $\text{dim}_p(g_3) = d - \frac{3}{4} \rho$. In MF: $\eta = d + 2 - \rho$ (holds in IRD!) and $1/\nu = \rho - d$.

- Hence, the Mean Field and Infrared region are ($d = 1$):

| $\rho$ | $D(\rho)$ | transition type |
|--------|------------|-----------------|
| $\leq 1$ | $\infty$ | Bethe lattice like |
| $(1, 4/3]$ | $[6, \infty)$ | $2^{nd}$ order, MF |
| $(4/3, 2]$ | $[2.5, 6)$ | $2^{nd}$ order, non-MF |
| 2 | 2.5 | Kosterlitz-Thouless or $T = 0$-like |
| $> 2$ | $< 2.5$ | none |

- It is possible to show (equivalence $D$-SR and $1d$-LR):
  \[
  \frac{2 - \eta(D)}{D} = \rho - 1 ; \ \rho = 1.8 \rightarrow D = 3
  \]
Numerical Simulations

- The spins live on a finite connectivity network ($z = 6$) with periodic boundary conditions: $J_{ij} = 0, \pm 1$ with $P(J_{ij} \neq 0) \propto r_{ij}^{-\rho}$. With this choice one has $J_{ij}^2 \propto r_{ij}^{-\rho}$.
- We have implemented the Parallel Tempering Method.
- We have used multispin coding (64 bits) on a C++ code.
- We have simulated a Gaussian magnetic field and only two replicas.
- We have run on PC’s Clusters.
Some Observables

- The spin glass correlation function:

\[ C(x) = \sum_{i=1}^{L} \left( \langle \sigma_i \sigma_{i+x} \rangle - \langle \sigma_i \rangle \langle \sigma_{i+x} \rangle \right)^2 \]

- The associated spin glass correlation length:

\[ \xi \equiv \frac{1}{2} \sin \left( \frac{\pi}{L} \right) \left[ \tilde{C}(0) \tilde{C}(\frac{2\pi}{L}) - 1 \right]^{\frac{1}{\rho - 1}} \]
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- FSSA in the MF regime (1 < \rho < 4/3):

\[ \frac{\chi_{sg}}{L^{1/3}} = \tilde{\chi} \left( L^{1/3} (T - T_c) \right), \quad \frac{\xi}{L^{\nu/3}} = \tilde{\xi} \left( L^{1/3} (T - T_c) \right) \]

with \( \nu = 1/(\rho - 1) \),
Some Observables

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- FSSA in the MF regime (1 < \rho \leq 4/3):
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  with \( \nu = 1 / (\rho - 1) \),

- FSSA in the IRD regime (\rho > 4/3):
  \[ \frac{\chi_{\text{sg}}}{L^{2-\eta}} = \tilde{\chi} \left( L^{1/\nu} (T - T_c) \right), \quad \frac{\xi}{L} = \tilde{\xi} \left( L^{1/\nu} (T - T_c) \right) \].
The negative overlap problem

- $P(q)$ in a magnetic field: SK results and numerical ones.

![Graph showing $P(q)$ vs $q$ with a peak at $q=0$.]
The negative overlap problem

- $P(q)$ in a magnetic field: SK results and numerical ones.

![Graph showing $P(q)$ as a function of $q$ with a negative overlap region inducing large corrections in $\tilde{C}(0)$](image)
The negative overlap problem

- $P(q)$ in a magnetic field: SK results and numerical ones.

- The negative overlap region induces large corrections in $\tilde{C}(0)$!!

\[ P(q) \]

\[ q=0 \quad q \]

\[ \rho = 1.2 \text{ (MF)}, \ h = 0.3 \]

\[ N = 2^6 \]

\[ N = 2^{13} \]
Numerical Analysis of the Correlation function $(h = 0)$

- $h = 0$ and $\rho = 1.8$. 

![Graph showing the relationship between $T$ and $\chi_{SG} L^{\eta 2}$ for different values of $\rho$.](image)
Numerical Analysis of the Correlation function ($h = 0$)

- $h = 0$ and $\rho = 1.8$. 
Numerical Analysis of the Correlation function ($h = 0$)

- $h = 0$ and $\rho = 1.8$. 

![Graphs showing correlation function analysis](image)
We will avoid the $k = 0$ value by fitting ($k > 0$): 

\[
\left( \frac{1}{\tilde{C}_4(k)} \right)^{\text{fit}} = A(L, T) + B(L, T) \left[ \frac{\sin(k/2)}{\pi} \right]^{\rho-1}
\]
Numerical Analysis of the Correlation function

- We will avoid the $k = 0$ value by fitting ($k > 0$):

$$
\left( \frac{1}{\tilde{C}_4(k)} \right)^{\text{fit}} = A(L, T) + B(L, T) [\sin(k/2)/\pi]^\rho-1
$$

- We can analyze the $L$ and $T$ dependence of

$$A(L, T) \equiv \lim_{k \to 0} \frac{1}{\tilde{C}_4(k)}$$
Numerical Analysis of the Correlation function

- We will avoid the \( k = 0 \) value by fitting \( (k > 0) \):
  \[
  \left( \frac{1}{\bar{C}_4(k)} \right)^{\text{fit}} = A(L, T) + B(L, T) \left[ \sin \frac{k}{2} / \pi \right]^{\rho - 1}
  \]

- We can analyze the \( L \) and \( T \) dependence of
  
  \[
  A(L, T) \equiv \lim_{k \to 0} \frac{1}{\bar{C}_4(k)}
  \]

- We fix the \( L \)-dependent critical temperature by means:
  
  \[
  A(L, T_{c}(L)) = 0
  \]
Numerical Analysis of the Correlation function

\[ T_c(L) \]

vs.

\[ 1/L \]
$h = 0.1$ and $\rho = 1.5$. 
Numerical Analysis of the Correlation function ($h \neq 0$)

- $h = 0.1$ and $\rho = 1.5$. 
Numerical Analysis of the Correlation function ($h \neq 0$)

- $h = 0.1$ and $\rho = 1.5$. 

![Image of correlation function graphs]

- Various diagrams showing the correlation function $C(k)$ as a function of $|\sin(k/2)/\pi|^{1/2}$ for different values of $\kappa = 6, 7, 8, 9, 10, 11, 12$.

- Plots of $A(L,T)$ vs. $T$ for different values of $\rho = 1.5$ and $h = 0.1$.

- Data fits for $C^{-1}(k)$ for $T = 2.1$, $\rho = 1.5$, $h = 0.1$.
### Characterization of the de Almeida-Thouless line

|     | $\rho$ | “$D$” | $\hbar$ | $T_c$ from $\tilde{C}(0)$ | $T_c$ from $A(L, T)$ |
|-----|--------|--------|---------|--------------------------|---------------------|
| MF  | 1.2    | 10     | 0.0     | 2.24(1)                  | 2.34(3)             |
|     | 1.2    | 10     | 0.1     | 2.02(2)                  | 1.9(2)              |
|     | 1.2    | 10     | 0.2     | 1.67(3)                  | 1.4(2)              |
|     | 1.2    | 10     | 0.3     | 1.46(3)                  | 1.5(4)              |
|     | 1.25   | 8      | 0.0     | 2.191(5)                 | 2.23(2)             |
| IRD | 1.4    | 5      | 0.0     | 1.954(3)                 | 1.970(2)            |
|     | 1.4    | 5      | 0.1     | $\sim$ 1.5               | 1.67(7)             |
|     | 1.4    | 5      | 0.2     | $\sim$ 1.1               | 1.2(2)              |
|     | 1.5    | 4      | 0.0     | 1.758(4)                 | 1.770(5)            |
|     | 1.5    | 4      | 0.1     | —                         | 1.46(3)             |
|     | 1.5    | 4      | 0.15    | —                         | 1.20(7)             |
Experiments

Relative decrease of $T_c(h)/T_c(0)$ with increase field for $\rho = 1.5$ and $h = 0, 0.1, 0.15$ and $0.2$ versus the relative decrease of $\chi^*$ (ZFC susceptibility). Experimental data from Fe$_{0.5}$Mn$_{0.5}$TiO$_3$ (Jönsson et al.).

Hence, the critical field should be $H_c < 1000$ Oe.
Experimental data from Fe$_{0.5}$Mn$_{0.5}$TiO$_3$ (Jönsson et al.).
More on Experiments

- Experimental data from Fe$_{0.5}$Mn$_{0.5}$TiO$_3$ (Jönsson et al.).

\[ \omega/2\pi = 0.51 \text{ Hz} \]

\[ h = 3 \text{ Oe} \]

\[ [\chi_{eq} - \chi'(\tau)] / \chi_{eq} \]

\[ H (\text{Oe}) \]

\[ T_f (\text{K}) \]
More on Experiments

- Experimental data from Fe$_{0.5}$Mn$_{0.5}$TiO$_3$ (Jönsson et al.).

- $q(t) \sim 1/t^x$ clear signature of a Spin Glass Phase (Ogileski).
Conclusions

- We have introduced a new analysis method to bypass the bias which induces the large plateau (at negative overlap) in $\tilde{C}(0)$.

- Experimental studies in Heisenberg spin glass find Phase Transition (Campbell et al.).

- Experimental studies in Ising spin glass find NO Phase Transition (Jönsson et al.) for fields $H > 1000\text{Oe}$.

- We suggest to reanalyze the experimental data for $H < 1000\text{Oe}$ on Fe$_{0.5}$Mn$_{0.5}$TiO$_3$.

- Recent experiments find spin glass order in a magnetic field for small external fields ($H \simeq 500\text{Oe}$) in RKKY Spin Glasses.
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