Numerical study of MHD supersonic flow control

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Abstract. Supersonic MHD flow around a blunted body with a constant external magnetic field has been simulated for a number of geometries as well as a range of the flow parameters. Solvers based on Balbas-Tadmor MHD schemes and HLLC-Roe Godunov-type method have been developed within the OpenFOAM framework. The stability of the solution varies depending on the intensity of magnetic interaction. The obtained solutions show the potential of MHD flow control and provide insights into the development of the flow control system. The analysis of the results proves the applicability of numerical schemes, that are being used in the solvers. A number of ways to improve both the mathematical model of the process and the developed solvers are proposed.

1. Introduction
The recent challenges in the aerospace industry have caused the resurgence of research in flow control concepts. One of such concepts is the magnetohydrodynamic (MHD) flow control. It is based on the idea of altering the shock wave structure by using the interaction between charged particles in the flow and a magnetic field. The field, that is generated by either a coil with a current or a magnet, affects the charged particles in an incoming gas flow through the ponderomotive force and joule heating. This effect can be used to reduce aerodynamic heating on an atmospheric entry capsule or even to change its descent trajectory [1]. Another possible application is controlling the pressure position jump within the engines of hypersonic scramjet during the changing of altitude or regime of the flight. The development of such a system will require both theoretical and experimental research into the aspects of this interaction. Computational fluid dynamics (CFD) can be an invaluable instrument to the research supplementing limited experimental capabilities. In the recent decades, there have been several studies concerning MHD flow control for different regimes [2],[3]. Most of them deal with the hypersonic flow, since it is characteristic for both descent capsules and scramjets. We aim to develop a comprehensive model of the phenomenon based on a free CFD software to aid the design of MHD flow control system. In this paper, we deal primarily with supersonic flow. The applications developed and the results obtained in this study will serve as a springboard towards a comprehensive open-source simulation tool for MHD hypersonic flow.

2. Physical description
We consider a supersonic flow around a blunted body of a shape characteristic for a re-entry capsule. Three different cases are considered. Case geometries are cylindrically symmetrical in order to reduce required computational power. First case geometry simulates the shape of “Soyuz” spaceship’s...
atmospheric return capsule, while the second one is shaped similar to the analogous capsule of NASA’s Phoenix spacecraft.

**Figure 1.** Sample of mesh for case geometries 1 and 2

In the first two cases magnetic field is generated by a coil placed inside the model of each craft at the conjunction between the forebody and middle body. Typical configuration of magnetic field is shown on the picture.

**Figure 2.** Magnetic field configuration for the case 2

Finally, third case represents an experiment conducted in Ioffe Institute [2]. The setup consists of a cylinder with a cone on top. It has an electrode core, surrounded by a magnetic core, as well as a ring electrode at the base of its conical top. The electrodes are plugged to a power source, so that a discharge between them ionizes the gas in front of the body. By raising the conductivity of the gas the intensity of interaction between the incoming flow and generated magnetic field is therefore increased.

**Figure 3.** Scheme (left) and a photo of an experimental setup that is represented by case 3.

Hydrodynamic parameters of the flow are listed in the table below. The gas in consideration is air. The intensity of MHD interaction is characterized by Stuart number which can range from 0 to 0.5.
Table 1. The values of characteristic dimensionless quantities of the studied flow

| Number          | Value          |
|-----------------|----------------|
| M (Mach)        | 2-6            |
| Re (Reynolds)   | ~10^5          |
| Re_m (magnetic  | ~0.1           |
| Reynolds)       |                 |
| Kn (Knudsen)    | 10^3 – 10^5    |
| S (Stuart)      | 0 - 0.5        |
| Ha (Hartmann)   | ~100           |

3. Mathematical model

Our simulation uses magnetohydrodynamic model, which is widely used in similar problems [3]. Almost all of the flow’s parameters are well within the range of applicability of the continuous representation. It should, however be noted, that in a real case Knudsen number can locally fall below the threshold of model’s applicability. We have tried to numerically determine where such a breakdown can occur.

The system of MHD equations consists of a set of conservation laws as well as magnetic field equation. However, in this study we only consider cases with a constant magnetic field so the latter can be dropped. The effect of magnetic interaction therefore is described by adding ponderomotive force term into the equation of motion and joule heating term into energy conservation equation. Also, due to low magnetic Reynolds number we can disregard self-induced electrical field. Using these assumptions and the generalized Ohm law we can rewrite the governing system of equations as follows.

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,
\]

\[
\frac{\partial (\rho \mathbf{v})}{\partial t} + \mathbf{v} \cdot \nabla P + \mathbf{j} \times \mathbf{B},
\]

\[
\frac{\partial}{\partial t} \left( \frac{1}{2} \rho \mathbf{v}^2 + \rho e \right) = \mathbf{v} \cdot \left[ P \mathbf{v} + \lambda \nabla T \right] + \mathbf{j} \cdot \mathbf{E},
\]

\[
\mathbf{j} = \sigma (\mathbf{E} + \mathbf{v} \times \mathbf{B}), \quad \mathbf{B} = \mathbf{B}_0 (x, y, z)
\]

4. Numerical method

The studied flow is highly compressible, which gives a density-based approach a significant advantage in shock resolution accuracy. We use rhoCentralFoam, a density-based solver from OpenFOAM standard toolkit, as a starting point for our MHD solver. rhoCentralFoam implements Kurganov-Tadmor difference scheme [4], that is reliable at resolving steep pressure gradients, associated with shock wave formation. In addition Kurganov-Tadmor scheme has a MHD modification, proposed by Balbas and Tadmor [5]. This scheme for the system of MHD equations provides a guideline into how to modify rhoCentralFoam solver according to our needs.

Another solver for MHD supersonic flow has been developed based on dbnsTurbFoam from foam-extend fork of OpenFOAM. It is a Godunov-type coupled density-based solver utilizing Rusanov flux. We used a positive conservative method based on HLL and Roe methods [6] to add MHD functionality to dbnsTurbFoam.

To achieve the convergence of the method we start with a constant distribution for the temperature, pressure and velocity field and solve the problem without MHD interaction. After the solution settles to a stationary regime, we start iterations with a magnetic field and continue them until another stationary
flow regime is achieved. To keep the computations stable, time step had to be limited by a local cell Courant number threshold of 0.9.

To resolve the flow separation that can occur behind the body we have used Spalart-Allmaras IDDES turbulence model. We can take liberty with choosing the turbulence parameters since the turbulence effect is negligible in front of the body and the region, where it is more substantial, is not of a particular interest to us in the scope of this study.

5. Analysis of the Results
Numerical results for supersonic flow around blunted body without MHD interaction can be validated using theoretical values for the shock wave pressure jump [7]:

\[ P_2 = \frac{P_1}{\gamma + 1} \left[ 2\gamma M^2 - (\gamma - 1) \right] \]  

where \( P_1 \) is the free-stream flow pressure and \( P_2 \) is the pressure behind the shockwave.

![Figure 4. Numerically obtained density distribution for case 1](image)

**Table 2.** Comparison between free-stream flow pressure numerically obtained pressure behind the shockwave

| Mach | \( P_1 (10^3 \text{ Pa}) \) | \( P_2 (10^3 \text{ Pa}) \) |
|------|----------------|----------------|
| 2    | 135            |
| 4    | 555            |
| 6    | 1255           |

The comparison of experimental and numerical shock fronts show agreement despite the fact that conductivity in numerical model was set to be constant, which isn’t the case in the real experiment.

![Figure 5. Comparison between numerical and experimental shock from (left) and a shadowgram of experimental shock front (right)](image)

Graphs below present numerically obtained profiles of temperature on the generatrix of case 2 body with and without magnetic field for Mach 2, 4 and 6 flows. Characteristic magnetic field induction in these cases is \( B_c = 1.5T \), which makes Stuart number 0.5, 0.3, 0.1 respectively. The zone around 0.6m
mark on the generatrix corresponds to the maximum of magnetic field. Numerically obtained temperature graphs show significant dip in this region. The source of this dip is obviously the magnetic interaction between the generated field and the incoming ionized gas flow. The dip becomes extreme with lower Mach/high Stuart flows. This can signify a breakdown of continuous model for the extremely low density flow. This fact presents a challenge that will require significant reworking of the model to fully overcome.

![Figure 6](image_url)

**Figure 6.** Comparison between temperature profiles on the generatrix of the body with (solid) and without (dashed) MHD interaction for Mach 2(a), 4(b) and 6(c) flows ($B_c = 1.5T$).

Both central difference and Godunov-type solver proved stable for the flows with Stuart number ranging from 0 to 0.5. Instabilities for higher Stuart number can be attributed to low-density effects.

6. Conclusion
The aforementioned results show the potential of MHD flow control to significantly reduce the surface temperature of supersonic vehicle if the magnetic field is strong enough. However, our simulation lacks proper models for such important aspects as radiational heat transfer and ionization. It also should be
modified to include multi-species capabilities integral for the modeling of hypersonic flows, for which the concept of MHD control is more relevant. Such a modification can also bring resolution to the problem of low-density effects.

The developed solvers have shown the applicability of both approaches to our problem. Both solvers can be used to resolve MHD supersonic flows with Stuart number ranging from 0 to 0.5.

Other question that remain open include determining necessary energy input for ionizing the gas to sufficient degree as well as choosing the optimal configuration of the magnetic field. Answering these question is going to require enhancing the model as well as additional numerical experimentation.

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