Natural frequency analysis of fluid-conveying pipes in the ADINA system

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Abstract. In this paper we present analysis and numerical simulations for the natural frequency of pipes conveying fluid in the ADINA system. The slender pipe structures are modelled as shells and the fluid flows are supposed to be three-dimensional (3D) and incompressible. The fluid and structural models are mechanically coupled on their interface through slip and displacement conditions. Several numerical examples for pipes shaped as different configurations show that the natural frequencies obtained based on ADINA codes are generally lower than those predicted by analytical or dynamic stiffness methods. In particular, for curved pipes conveying fluid with relatively high flow velocity, it is found that the evolution trend of natural frequencies with increasing flow velocity is similar as that predicted by the inextensible theory for curved pipes conveying fluid.

1. Introduction
The dynamical system of slender pipes conveying fluid is capable of displaying rich dynamical behaviour. As it is well known that, van der Pol’s oscillator and the Lorenz equations are examples of systems the study of which has been instrumental in the development of modern dynamical theory [1, 2]. Now the model problem of pipes conveying fluid has established itself as another new paradigm in dynamics. This is a physically simple system, easily modelled by simple equation, and is a fairly easily realizable system with the possibility of theoretical and experimental investigation in parallel. Moreover, the model problem of pipes conveying fluid may be considered as the simplest fluid-structure interaction system [2]. The knowledge gained from the canonical problem of fluid-conveying pipes may radiate into other dynamic problems across applied mechanics [3]. Indeed, the dynamics of a pipe conveying fluid has been used in the concept of novel methods of marine propulsion, in the study of pulmonary collapse, and as medium for explaining the nature of turbulence and the dynamics of solar wind [2]. It is not surprising, therefore, that the literature on this topic is extensive and constantly expanding.

Certainly, some theoretical aspects of the problem have been studied extensively in the past decades. For instance, the vibration characteristics of straight pipes conveying fluid have been analyzed by many researchers. It would seem an easy task to extend the analysis of straight pipes conveying fluid to that of curved pipes. This is not so, however. In fact, in contrast to the systems discussed so far, there remained considerable uncertainty as to the vibration characteristics of curved pipes conveying fluid. The curved pipes shaped as circular arcs may be studied analytically. More
complex L- and S-shaped configurations (or arbitrary ones), however, should be analysed by numerical methods such as finite-element method (FEM) [2].

In this paper, based on the FEM, numerical simulations are performed for the vibration characteristics of fluid-conveying pipes in the ADINA system. The ADINA system is a general-purpose finite element and finite volume code for the analysis of structures, fluid flows and fluid-structure interaction [4]. The objective of the numerical simulations is to evaluate the capabilities of the ADINA code for analyzing the vibration characteristics of fluid-conveying pipes shaped with various configurations.

2. Modelling of fluid-conveying pipes in the ADINA system

2.1. Geometric model
In this paper, several typical configurations of fluid-conveying pipes with both ends simply supported will be considered. The systems of pipes shaped as straight, slightly curved and circular arcs will be studied. As a great deal of work has been done on the dynamics of straight pipes conveying fluid over the past 60 years or so, numerical simulations based on ADINA codes will be performed first, to demonstrate the validity of the solution procedure. In general, pipes may be curved and twisted into complex spatial forms. In this paper, however, reflecting the state of the art, curved pipes which initially lie within a given plane will be considered. For comparison convenience, only the in-plane vibration will be analyzed in the numerical results.

2.2. Assumptions
In the interest of analysing the vibration of fluid-conveying pipes in the transverse direction, only slender pipes will be studied. Thus, even for pipes with relatively thin wall, the lowest natural frequency will be only related to the beam-type mode. The internal fluid flow is assumed to be laminar and incompressible. Moreover, the friction between the fluid and pipe wall will be neglected according to the discussion by Paidoussis [5]. Thus, slip conditions between the internal fluid flow and the pipe wall may be used in the ADINA simulations.

2.3. Mechanical coupling on interface
The fluid and pipe structure in the ADINA system can be coupled through their interface. At fluid-structure interfaces, besides the slip conditions between the fluid flow and the pipe wall mentioned above, the compatibility of displacements/velocities must be satisfied. Since the internal fluid is flowing along the pipe length, the displacement compatibility may be viewed as a kinematic condition that can be realized along the normal direction of the interface.

2.4. Solution procedure
In the ADINA system, the presence of powerful analysis features for both solid and fluid models is essential for the accurate analysis of couple problems [4]. However, the coupling scheme is not readily available in either fluid or solid models. Thus, the well-organized core FSI coupling scheme should be used to link the two models. In order to handle the moving interface between fluids and solids, an arbitrary Lagrangian-Eulerian (ALE) formulation can be applied to the fluid. In the solution procedure, the direct computing method is used to solve the fully coupled equations of solid and fluid.

3. Results and discussion
In this section, some numerical examples that illustrate the capabilities of the ADINA system for predicting the vibration characteristics of fluid-conveying pipes will be presented.

The problems of fluid-conveying pipes, of course, have been extensively studied in both theoretical and experimental investigations. However, our purpose is to analyze the natural frequencies of such FSI systems by numerical simulations in the ADINA system. Here, it is of interest to discuss the effect on the natural frequency of the internal fluid flow. In the ADINA-FSI core scheme, the natural
frequencies of the coupled system could not be calculated directly. If, however, an additional harmonic force is applied at the middle of the pipe, the frequency response curve may be calculated for each fluid velocity. From the frequency response curve, the natural frequency can be readily recognized.

For calculating convenience, the pipe systems considered are assumed to be slender and the dimensional parameters are chosen to be: pipe outer diameter $D=52\text{mm}$, inner diameter $d=48\text{mm}$, Young’s modulus $E=210\text{GPa}$, Possion ratio $\nu=0.3$, pipe mass density $\rho_p=7800\text{kg/m}^3$ and fluid mass density $\rho_f=1000\text{kg/m}^3$.

In the following simulations, the pipe structures are all modeled using nine-node shell elements while the fluid domain is modelled using 3D four-node elements. The initial fluid velocity at the inlet of the pipe is prescribed at $0\text{m/s}$. The increase of this fluid velocity is set to $0.1\text{m/s}$. A transient analysis will be performed with a time step length of $0.1\text{s}$.

3.1. Straight pipe conveying fluid

Let us analyse the vibrations of a straight pipe conveying fluid with simply supported boundary conditions first. The schematic view of the straight pipe is shown in Figure 1. For this geometrical pipe system, it has been verified theoretically and experimentally that the natural frequency would decrease with increasing fluid velocity.
In Figure 2, the calculations have been carried out for the first mode of a straight pipe with length $L=1.2m$. It can be seen that the natural frequencies decrease with increasing flow velocity. The results obtained using the ADINA code show reasonable agreement with those predicted by dynamic stiffness method (DSM) [6] and analytical method [5]. It is obvious that, moreover, the natural frequency predicted by ADINA system is lower than that obtained by dynamic stiffness method or analytical method. For sufficiently high flow velocity close to the critical value, the difference between the results predicted by the ADINA code and the other two methods becomes remarkable. In the case of sufficiently high flow velocity, it is suggested that maintaining numerical stability is not easy in the ADINA system.

3.2. Slightly curved pipe conveying fluid

Next, a slightly curved pipe shaped as circular arc will be analyzed. The slightly curved pipe model is shown in Figure 3. The length of the pipe is chosen to be $L=1.2m$. The curvature radius of the pipe is $R=7.0m$.

![Figure 3. Schematic view of a slightly curved pipe conveying fluid.](image)

![Figure 4. Natural frequencies as functions of the flow velocity for a slightly curved pipe with pinned-pinned boundary conditions.](image)

In this case, the pipe is slightly curved; hence one expects the natural frequency would be higher than that of a straight pipe conveying fluid. The natural frequency of the system is shown in Figure 4. Here, comparing the ADINA results to the results predicted by dynamic stiffness method based on an “inextensible theory” [7, 8], it is noted the natural frequency predicted by ADINA system is, once
again, lower than that predicted by the dynamic stiffness method. It should be pointed out that the inextensible theory for curved pipes conveying fluid presumes that the centreline of the pipe is essentially inextensible and all steady-state stress resultants are zero.

For higher flow velocity, similarly as before, numerical stability in the ADINA system could not maintain; and hence we have not obtained the corresponding natural frequencies.

3.3. Semi-circular curved pipe conveying fluid

It is then of interest to see how a semi-circular curved pipe behaves. For comparison purpose, the analytical method [7] and the dynamic stiffness method will be used to calculate the natural frequencies of the system. It should be mentioned that, for the dynamic stiffness method and analytical method, the equations of motion used are based on the inextensible theory initiated by Chen [7].

Figure 5. Schematic view of a semi-circular curved pipe conveying fluid.

Figure 6. In-plane natural frequencies as functions of the flow velocity for a semi-circular curved pipe with pinned-pinned boundary conditions.
For a semi-circular curved pipe with both ends simply supported and curvature radius \( R = 0.5 \text{m} \) (see Figure 5), the evolution of the in-plane natural frequency with increasing dimensionless flow velocity is shown in Figure 6. As seen in this figure, the natural frequency is much lower than that predicted by the dynamic stiffness method or analytical method. For high flow velocity, similarly as before, the natural frequency could not be calculated by the ADINA code since the numerical stability is not easy to maintain.

### 3.4. Discussion

Clearly there are two significant characteristics from the ADINA system for predicting the vibrations of fluid-conveying pipes. The first is associated with the quantitative results of natural frequencies. The natural frequencies predicted by the ADINA system are much lower than those obtained by analytical method or dynamic stiffness method. For high flow velocity close to the critical value, the natural frequency could not be calculated correctly due to the numerical instability. The second is associated with the evolution trend of the natural frequencies with increasing flow velocity for curved pipes. In the results shown in Figures 4 and 6, the natural frequencies for curved pipes are shown to decrease with increasing flow velocity in the ADINA system. This trend agrees with that predicted by the analytical method and dynamic stiffness method. In these two figures, however, the analytical solutions or DSM solutions were all based on the inextensible theory. As discussed by Paidoussis [5] and Misra and Paidoussis [9], the inextensible theory and its conclusions regarding evolution trend of natural frequencies with increasing flow velocity may be not reliable.

A more reliable theory for curved pipe conveying fluid is the so-called “extensible theory” [9]. The extensible theory, as the name implies, do not make the assumption of that the centreline of the pipe is inextensible, and generally take into account the changes in form with increasing flow velocity, as well as the force generated thereby [5]. According to the results predicted by extensible theory [5, 9], the natural frequency of curved pipe may change slightly with increasing flow velocity. The curved pipe is stiffened by the flow-induced steady-state stresses (mainly centrifugally induced tension) at a rate. Thus, the flow-induced steady-state stresses may result in a stable system, no matter how high is the flow velocity.

### 4. Conclusions

This paper has initiated to model the vibration problem of fluid-conveying pipes in the ADINA system. By investigating the natural frequencies of pipes shaped as straight, slightly curved and semi-circular curved configurations, it is found that the natural frequencies of ADINA solutions are generally lower than those predicted by the analytical method and dynamic stiffness method. For curved pipes containing internal fluid flow, particularly the case of curved pipes conveying relatively high flow velocity, the evolution trend of natural frequencies with increasing fluid velocity is similar as that predicted by theoretical models based on flow velocity.

Further studies require more comprehensive numerical simulations for a wider range of pipe configurations. Especially, fluid-conveying pipes shaped as various three-dimensional configurations need to be analyzed based on the ADINA code. Moreover, there exist some possible measures to improve numerical stability in the ADINA system. These and many more are some of the aspects of the problem should be considered in the near future.

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