NUCLEAR EFFECTS IN THE $F_3$ STRUCTURE FUNCTION

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We discuss nuclear effects in the structure function $F_3$ measured in neutrino deep-inelastic scattering on heavy nuclear targets.

Experiments on deep-inelastic scattering (DIS) of charged leptons and neutrino remains an important source of information about the nucleon as well as about the nuclear structure. DIS experiments with neutrino beams has reached now an accuracy comparable to that of experiments with charged leptons. Note in this respect that because of statistics reasons neutrino data are collected for heavy nuclei like iron.

Nuclear effects have been extensively discussed for the spin independent structure functions (SF) $F_{1,2}$ as well as for the spin SF $g_{1,2}$ which are measured in the charged lepton DIS (for a recent reviews of experimental situation and theoretical approaches to nuclear effects in DIS see refs. [3], [4], [5], [6]), however until now only a little attention was payed to nuclear effects in neutrino DIS. It is usually assumed in analysing neutrino data that nuclear corrections are the same as in charged lepton DIS. At large $Q^2$ this assumption can be motivated by the parton model where the charged lepton and neutrino SF are expressed in terms of universal parton distributions. However the similarity between the charge lepton and neutrino $F_2$ fails at small $x$ where they are different in the strange and charm quarks content as well as when one studies effects due to finite $Q^2$. In this respect the situation is even more uncertain with the SF $F_3$, which does not have its analog in the charged lepton DIS.

In the present contribution we report on the results of our studies of nuclear effects in the SF $F_3$ within an approach applicable at finite $Q^2$. More detailed discussion on this matter can be found in ref.

Nuclear effects in DIS are usually discussed within the context of the impulse approximation for the nucleons, where the nuclear Compton scattering amplitude is approximated by the uncoherent sum of the scattering amplitudes from bound nucleons neglecting final state interactions. An argument to support this approximation comes from the analysis of characteristic space-time scales involved in deep-inelastic scattering. In the laboratory frame a characteristic time for DIS is $1/Mx$, where $M$ is the nucleon mass (see, e.g., ref. [11]). For $x > 0.2$ this time is smaller than the typical average distance between bound nucleons in the nucleus and the scattering proceeds uncoherently.
Major nuclear effects in this region are due to Fermi-motion and nuclear binding, and possible off-shell modification of nucleon structure functions (for more discussion and references see ref. 4).

On the other hand at small $x$ the coherent multiple scattering effects should be important. It is known that for the SF $F_2$ these effects cause the nuclear shadowing phenomenon (for a review of models for the nuclear shadowing see refs. 5, 3, 6). We note however that not much is known yet about the shadowing effect in the SF $F_3$ (the preliminary results of ref. 18 show that the nuclear shadowing effect in $F_3$ is even more prominent than the one in $F_2$).

We comment also that correction due to meson exchange currents in $F_3$ vanishes, because light mesons, such as $\pi$ and $\rho$, have vanishing $F_3$ structure functions (if one neglects a small contributions due to $s-c$ and $\bar{s}-c$).

Based on these observations we consider nuclear Compton amplitude in the impulse approximation. In order to find a connection between the nuclear and nucleon SF one has to take the imaginary part of the Compton amplitude and then extract the desired SF. We note that there are few subtle points in this procedure, which are usually neglected in many calculations based on convolution model. First of all we observe that bound nucleons are off-mass-shell ($p^2 \neq M^2$) and nuclear SF are not determined by those of on-mass-shell nucleon but are sensitive to their off-shell behavior. As it is discussed in ref. 14, the Lorentz-Dirac structure of the nucleon hadronic tensor in off-shell region is more complicated than that of the on-shell one. A direct reason for this is that one can not use the Dirac equation for the off-shell nucleon which greatly reduces the number of independent amplitudes in the hadronic tensor. In particular we found that the SF $F_3$ splits into four independent inelastic form-factors for the nucleon off-shell. Similar observations have been already done for the SF $F_2$ as well as for $g_1$ and $g_2$. One should conclude therefore that even in the impulse approximation there is no simple factorization between the nuclear and nucleon SF, that is assumed in convolution model calculations.

One can show however that things simplify considerably in the limit of weak nuclear binding. We have done a systematic $1/M$-expansion of nuclear matrix elements which enter the nuclear Compton amplitude and found that the factorization for the SF is remarkably recovered if one keeps only terms to order $1/M^2$ (including the latter). In this approximation the nuclear $F_3$ can be expressed in terms of the generalized convolution of the (non-relativistic) nuclear spectral function $P(\varepsilon, \mathbf{p})$ and a combination of the four off-shell nucleon inelastic form factors which we take for the definition of the “off-shell nucleon structure function” $F_3(x, Q^2; p^2)$ (notice the dependence on...
the nucleon “virtuality” $p^2$ as an additional variable). The final result reads:

$$x F_A^3(x, Q^2) = \sum_{\tau=p,n} \int \frac{d\varepsilon dp}{(2\pi)^4} \mathcal{P}_\tau(\varepsilon, p) \left( 1 + \frac{p_z}{\gamma M} \right) x' F_3^\tau(x', Q^2; p^2), \quad (1)$$

where the integration is done over the nucleon four-momentum, $p = (M + \varepsilon, p)$, $x' = Q^2/2p \cdot q$ is the Bjorken variable of the bound nucleon and $\gamma = |q|/q_0$ is the ratio of the space to time components of the momentum transfer. We note that no approximation is done with respect to $Q^2$, so that Eq.(1) is valid, in general, for any $Q^2$.

We use Eq.(1) to separate $Q^2$ dependence of the nuclear SF due to nuclear effects. A useful observation is that these effects come through dependence on $\gamma$ of the variable $x'$ as well as a “flux” factor in Eq.(1). Making use of this observation we expand Eq.(1) in nuclear $Q^{-2}$ series:

$$x F_A^3(x, Q^2)/A \approx \left\langle \left( 1 + \frac{p_z}{M} \right) x' F_3(x', Q^2; p^2) \right\rangle - \frac{2M^2x^2}{Q^2} \left\langle \frac{p_z}{M} \frac{\partial}{\partial x'} \left( x'^2 F_3(x', Q^2; p^2) \right) \right\rangle + \cdots \quad (2)$$

Here to simplify the notations the brackets denote the averaging over the nuclear spectral function. Note that $x'$ in Eq.(2) corresponds to the light-cone kinematics when $\gamma = 1$. The dots denote terms of order $Q^{-4}$ and higher which are not written here explicitly (for more details see ref.

Few comments on Eq.(2) are in order. First we note that Eq.(2) makes it possible to separate the effects due to $Q^2$ dependence of the nucleon SF itself and those which are due to nuclear effects. It was argued in ref.

that an effective parameter in this expansion is $p_{\text{char}}^2 x^2/Q^2$ with $p_{\text{char}}$ being the characteristic momentum of the bound nucleon. This allows us to apply the $Q^{-2}$ expansion in Eq.(2) down to small $Q^2 \sim 0.1 \text{ GeV}^2$ and keep only $Q^{-2}$ correction at practically interesting $Q^2$. Fig.1 shows the $x$- and $Q^2$-dependence of the ratio $R_3 = \frac{1}{A} F_A^3(x, Q^2)/F_N^3(x, Q^2)$ calculated for the $^{56}\text{Fe}$ nucleus assuming the latter to be an isoscalar target with $N = Z$.

Eq.(2) is particularly useful in calculating nuclear corrections to the Gross-Llewellyn-Smith sum rule [17]. Nuclear corrections to the GLS sum rule cancel out in the leading order, which is due to the baryon charge conservation in strong interaction. The $Q^{-2}$ correction to the GLS sum rule turned out to be negative and small. For the iron and deuterium nuclei we have the following estimates for the corrections: $\delta S_{\text{GLS}}^{\text{Fe}} = -1.2 \cdot 10^{-2}/Q^2$, $\delta S_{\text{GLS}}^{\text{D}} = -1.9 \cdot 10^{-3}/Q^2$, where the coefficients are taken in the units of GeV$^2$.

Notice however that nuclear corrections are sizable for the GLS integrals truncated from below, $S_{\text{GLS}}(x, Q^2) = \int_x^1 dx' F_3(x', Q^2)$, as it is illustrated in
Fig. 2, that may have an impact on extraction of the value of the nucleon GLS sum rule from the data.

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Figure 1: Shown is the $Q^2$ dependence of the ratio $R_3$ for the iron nucleus. The dashed-dotted line corresponds to $Q^2 = 15$ GeV$^2$, the dashed line — $Q^2 = 5$ GeV$^2$, while the solid line — $Q^2 = 3$ GeV$^2$.

Figure 2: Shown is the $x$ dependence of the nucleus/nucleon ratio of the GLS integrals calculated at $Q^2 = 5$ GeV$^2$. The dashed line corresponds to the deuteron nucleus, while the solid line to the iron nucleus.