Neutrino Mixing and CP Phase Correlations

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Abstract

A special form of the $3 \times 3$ Majorana neutrino mass matrix derivable from $\mu - \tau$ interchange symmetry accompanied by a generalized $CP$ transformation was obtained many years ago. It predicts $\theta_{23} = \pi/4$ as well as $\delta_{CP} = \pm \pi/2$, with $\theta_{13} \neq 0$. Whereas this is consistent with present data, we explore a deviation of this result which occurs naturally in a recent proposed model of radiative inverse seesaw neutrino mass.
A special form of the $3 \times 3$ Majorana neutrino mass matrix first appeared in 2002 \cite{1,2}, i.e.

$$
\mathcal{M}_\nu = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix},
$$

where $A, B$ are real. It was shown that $\theta_{13} \neq 0$ and yet both $\theta_{23}$ and the $CP$ nonconserving phase $\delta_{CP}$ are maximal, i.e. $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm \pi/2$. Subsequently, this pattern was shown \cite{3} to be protected by a symmetry, i.e. $e \leftrightarrow e$ and $\mu \leftrightarrow \tau$ exchange with $CP$ conjugation. All three predictions are consistent with present experimental data. Recently, a radiative (scotogenic) model of inverse seesaw neutrino mass has been proposed \cite{4} which naturally obtains

$$
\mathcal{M}_\nu^\lambda = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \lambda \end{pmatrix} \mathcal{M}_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \lambda \end{pmatrix},
$$

where $\lambda = f_\tau/f_\mu$ is the ratio of two real Yukawa couplings.

This model has three real singlet scalars $s_{1,2,3}$ and one Dirac fermion doublet $(E^0, E^-)$ and one Dirac fermion singlet $N$, all of which are odd under an exactly conserved (dark) $Z_2$ symmetry. As a result, the third one-loop radiative mechanism proposed in 1998 \cite{5} for generating neutrino mass is realized, as shown below.

![Figure 1: One-loop generation of inverse seesaw neutrino mass.](image)

The mass matrix linking $(\bar{N}_L, \bar{E}_L^0)$ to $(N_R, E_R^0)$ is given by

$$
\mathcal{M}_{N,E} = \begin{pmatrix} m_N & m_D \\ m_F & m_E \end{pmatrix},
$$
where $m_N, m_E$ are invariant mass terms, and $m_D, m_F$ come from the Higgs vacuum expectation value $\langle \phi^0 \rangle = v/\sqrt{2}$. As a result, $N$ and $E^0$ mix to form two Dirac fermions of masses $m_{1,2}$, with mixing angles

$$m_Dm_E + m_Fm_N = \sin \theta_L \cos \theta_L (m_1^2 - m_2^2), \quad (4)$$

$$m_Dm_N + m_Fm_E = \sin \theta_R \cos \theta_R (m_1^2 - m_2^2). \quad (5)$$

To connect the loop, Majorana mass terms $(m_L/2)N_LN_L$ and $(m_R/2)N_RN_R$ are assumed. Since both $E$ and $N$ may be defined to carry lepton number, these new terms violate lepton number softly and may be naturally small, thus realizing the mechanism of inverse seesaw [6, 7, 8] as explained in Ref. [4]. Using the Yukawa interaction $fsE^0_R\nu_L$, the one-loop Majorana neutrino mass is given by

$$m_\nu = f^2m_R \sin^2 \theta_R \cos^2 \theta_R (m_1^2 - m_2^2) \int \frac{d^4k}{(2\pi)^4 (k^2 - m_s^2) (k^2 - m_1^2)} \frac{1}{(k^2 - m_2^2)} \frac{1}{(k^2 - m_2^2)}$$

$$+ f^2m_Lm_1^2 \sin^2 \theta_L \cos^2 \theta_R \int \frac{d^4k}{(2\pi)^4 (k^2 - m_s^2) (k^2 - m_1^2)^2}$$

$$+ f^2m_Lm_2^2 \sin^2 \theta_R \cos^2 \theta_L \int \frac{d^4k}{(2\pi)^4 (k^2 - m_s^2) (k^2 - m_2^2)^2}$$

$$- 2f^2m_Lm_1m_2 \sin \theta_L \sin \theta_R \cos \theta_L \cos \theta_R \int \frac{d^4k}{(2\pi)^4 (k^2 - m_s^2) (k^2 - m_1^2)} \frac{1}{(k^2 - m_2^2)} \frac{1}{(k^2 - m_2^2)}. \quad (6)$$

It was also shown in Ref. [4] that the implementation of a discrete flavor $Z_3$ symmetry, which is softly broken by the $3 \times 3$ real scalar mass matrix spanning $s_{1,2,3}$, leads to $M_\nu^λ$ of Eq. (2).

To explore how the predictions $θ_{23} = \pi/4$ and $δ_{CP} = ±\pi/2$ are changed for $λ ≠ 1$, consider the general diagonalization of $M_\nu$, i.e.

$$M_\nu = E_\alpha U E_β M_d E_β U^T E_α, \quad (7)$$

where

$$E_α = \begin{pmatrix} e^{iα_1} & 0 & 0 \\ 0 & e^{iα_2} & 0 \\ 0 & 0 & e^{iα_3} \end{pmatrix}, \quad E_β = \begin{pmatrix} e^{iβ_1} & 0 & 0 \\ 0 & e^{iβ_2} & 0 \\ 0 & 0 & e^{iβ_3} \end{pmatrix}, \quad M_d = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \quad (8)$$
Hence

\[ M_{\nu} M_{\nu}^\dagger = E_{\alpha} U M_{d}^2 U^\dagger E_{\alpha}^\dagger. \]  \hfill (9)

We then have

\[ M_{\nu}^\lambda (M_{\nu}^\lambda)^\dagger = E_{\alpha} U [1 + \Delta] M_{\lambda d}^2 [1 + \Delta^\dagger] U^\dagger E_{\alpha}^\dagger, \]  \hfill (10)

where

\[ \Delta = U^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \lambda - 1 \end{pmatrix} U, \quad M_{\lambda d}^2 = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & \lambda^2 m_3^2 \end{pmatrix}. \]  \hfill (11)

We now diagonalize numerically

\[ [1 + \Delta] M_{\lambda d}^2 [1 + \Delta^\dagger] = O M_{\text{new}}^2 O^T, \]  \hfill (12)

where \( O \) is an orthogonal matrix, and \( M_{\text{new}}^2 \) is diagonal with mass eigenvalues equal to the squares of the physical neutrino masses. Let us define

\[ A = (1 + \Delta)^{-1} O, \]  \hfill (13)

then

\[ A M_{\text{new}}^2 A^\dagger = M_{\lambda d}^2. \]  \hfill (14)

Since \( U \) is known with \( \theta_{23} = \pi/4 \) and \( \delta = \pm \pi/2 \), we know \( \Delta \) once \( \lambda \) is chosen. The orthogonal matrix \( O \) has three angles as parameters, so \( A \) has three parameters. In Eq. (14), once the three physical neutrino mass eigenvalues of \( M_{\text{new}}^2 \) are given, the three off-diagonal entries of \( M_{\lambda d}^2 \) are constrained to be zero, thus determining the three unknown parameters of \( O \). Once \( O \) is known, \( UO \) is the new neutrino mixing matrix, from which we can extract the correlation of \( \theta_{23} \) with \( \delta_{CP} \). There is of course an ambiguity in choosing the three physical neutrino masses, since only \( \Delta m_{32}^2 \) and \( \Delta m_{21}^2 \) are known. There are also the two different choices of \( m_1 < m_2 < m_3 \) (normal ordering) and \( m_3 < m_1 < m_2 \) (inverted ordering). We consider each case, and choose a value of either \( m_1 \) or \( m_3 \) starting from zero. We then obtain numerically the values of \( \sin^2(2\theta_{23}) \) and \( \delta_{CP} \) as functions of \( \lambda \neq 1 \). We need also to
adjust the input values of $\theta_{12}$ and $\theta_{13}$, so that their output values for $\lambda \neq 1$ are the preferred experimental values.

We use the 2014 Particle Data Group values \cite{PDG} of neutrino parameters:

\begin{align}
\sin^2(2\theta_{12}) &= 0.846 \pm 0.021, \quad \Delta m^2_{21} = (7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2, \\
\sin^2(2\theta_{23}) &= 0.999 \begin{pmatrix} +0.001 \\ -0.018 \end{pmatrix}, \quad \Delta m^2_{32} = (2.44 \pm 0.06) \times 10^{-3} \text{ eV}^2 \text{ (normal)},
\end{align}
\[
\sin^2(2\theta_{23}) = 1.000 \left( +0.000 \right. -0.017 \left. \right) , \quad \Delta m^2_{32} = (2.52 \pm 0.07) \times 10^{-3} \text{ eV}^2 \text{ (inverted)}, \quad (17)
\]
\[
\sin^2(2\theta_{13}) = (9.3 \pm 0.8) \times 10^{-2} . \quad (18)
\]

We consider first normal ordering, choosing the three representative values \( m_1 = 0, 0.03, 0.06 \) eV. We then vary the value of \( \lambda > 1 \). [The case \( \lambda < 1 \) is equivalent to \( \lambda^{-1} > 1 \) with \( \mu - \tau \) exchange.] Following the algorithm already mentioned, we obtain numerically the values of \( \sin^2(2\theta_{23}) \) and \( \delta_{CP} \) as functions of \( \lambda \). Our solutions are fixed by the central values of \( \Delta m^2_{21} \), \( \Delta m^2_{32} \), \( \sin^2(2\theta_{12}) \), and \( \sin^2(2\theta_{13}) \). In Figs. 2 and 3 we plot \( \sin^2(2\theta_{23}) \) and \( \delta_{CP} \) respectively versus \( \lambda \). We see from Fig. 2 that \( \lambda < 1.15 \) is required for \( \sin^2(2\theta_{23}) > 0.98 \). We also see from Fig. 3 that \( \delta_{CP} \) is not sensitive to \( m_1 \). Note that our scheme does not distinguish \( \delta_{CP} \) from \( -\delta_{CP} \). In Fig. 4 we plot \( \sin^2(2\theta_{23}) \) versus \( \delta_{CP} \). We see that \( \delta_{CP}/(\pi/2) > 0.95 \) is required for \( \sin^2(2\theta_{23}) > 0.98 \).

![Figure 4: \( \sin^2(2\theta_{23}) \) versus \( \delta_{CP} \) in normal ordering.](image)

We then consider inverted ordering, using \( m_3 \) instead of \( m_1 \). We plot in Figs. 5, 6, and 7 the corresponding results. Note that in our scheme, the effective neutrino mass \( m_{ee} \) measured
in neutrinoless double beta decay is very close to $m_1$ in normal ordering and $m_3 + \sqrt{\Delta m_{32}^2}$ in inverted ordering. We see similar constraints on $\sin^2(2\theta_{23})$ and $\delta_{CP}$. In other words, our scheme is insensitive to whether normal or inverted ordering is chosen. Finally, we have checked numerically that $\theta_{23} < \pi/4$ if $\lambda > 1$, and $\theta_{23} > \pi/4$ if $\lambda < 1$. As we already mentioned, the two solutions are related by the mapping $\lambda \rightarrow \lambda^{-1}$.

Figure 5: $\sin^2(2\theta_{23})$ versus $\lambda$ in inverted ordering.

Figure 6: $\delta_{CP}$ versus $\lambda$ in inverted ordering.
In conclusion, we have explored the possible deviation from the prediction of maximal $\theta_{23}$ and maximal $\delta_{CP}$ in a model of radiative inverse seesaw neutrino mass. We find that given the present 1$\sigma$ bound of 0.98 on $\sin^2(2\theta_{23})$, $\delta_{CP}/(\pi/2)$ must be greater than about 0.95.

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