Corrigendum: Navigation and control of an UAV quadrotor in search and surveillance missions

L A Frederico¹, L S Martins-Filho¹ and A L Da Silva²

¹Universidade Federal do ABC, Centro de Engenharia, Modelagem e Ciências Sociais Aplicadas, Santo André, São Paulo, Brazil
²Universidade Federal de Santa Maria, Cachoeira do Sul, Rio Grande do Sul, Brazil

The initially published version of the article included some text errors and the changes in this new version are:

- the title was corrected (there was a typing mistake);
- the equations 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29 and 30 were corrected in terms of wrong characters caused by PDF production problems;
- the figure 2 was updated (the previous version contained texts in Portuguese).
Corrigendum: Navigation and control of a UVA quadrotor in search and surveillance missions

L A Frederico¹, L S Martins-Filho¹ and A L Da Silva²
¹Universidade Federal do ABC, Centro de Engenharia, Modelagem e Ciências Sociais Aplicadas, Santo André, São Paulo, Brazil
²Universidade Federal de Santa Maria, Cachoeira do Sul, Rio Grande do Sul, Brazil

The published version of the article includes some text errors and the corrections are:

- the correct title is Navigation and control of an UAV quadrotor in search and surveillance missions;
- the following equations contain wrong characters and the correct ones are:

\[ \mathbf{\tau} = \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} I_{xx} \ddot{\phi} + (I_{zz} - I_{yy}) \dot{\theta} \dot{\psi} \\ I_{yy} \ddot{\theta} + (I_{xx} - I_{zz}) \phi \dot{\psi} \\ I_{zz} \ddot{\psi} + (I_{yy} - I_{xx}) \phi \dot{\theta} \end{bmatrix} \]  \hspace{1cm} (5)

\[ \begin{bmatrix} \Omega_1^2 \\ \Omega_2^2 \\ \Omega_3^2 \\ \Omega_4^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \frac{1}{2k_f L_c} & \frac{1}{4k_m} \\ \frac{4k_f}{2k_f L_c} & 1 & -1 & 0 \\ \frac{4k_f}{2k_f L_c} & 0 & 1 & -1 \\ \frac{4k_f}{2k_f L_c} & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} T \\ L \\ M \\ N \end{bmatrix} \] \hspace{1cm} (6)

\[ m \begin{bmatrix} \dot{v}_n \\ \dot{v}_d \\ \dot{v}_e \end{bmatrix} = mg \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - C_0 \begin{bmatrix} 0 \\ 0 \\ k_f(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \end{bmatrix} \] \hspace{1cm} (7)

\[ \ddot{\phi} = \frac{L + (I_{yy} - I_{zz}) \theta \dot{\psi}}{I_{xx}} \quad \dot{\theta} = \frac{M + (I_{xx} - I_{zz}) \phi \dot{\psi}}{I_{yy}} \quad \dot{\psi} = \frac{N + (I_{xx} - I_{yy}) \phi \dot{\theta}}{I_{zz}} \] \hspace{1cm} (8)
\begin{align*}
X &= [\phi \ \dot{\phi} \ \theta \ \dot{\theta} \ \psi \ \dot{\psi} \ z \ \dot{z} \ x \ \dot{x} \ y \ \dot{y}]^T \\
X &= [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7 \ x_8 \ x_9 \ x_{10} \ x_{11} \ x_{12}]^T \\
\dot{X} &= [\dot{x}_1 \ \dot{x}_2 \ \dot{x}_3 \ \dot{x}_4 \ \dot{x}_5 \ \dot{x}_6 \ \dot{x}_7 \ \dot{x}_8 \ \dot{x}_9 \ \dot{x}_{10} \ \dot{x}_{11} \ \dot{x}_{12}]^T \\
X &= \begin{bmatrix}
    x_4x_6a_1 + (L/I_{xx}) \\
x_2x_6a_2 + (M/I_{yy}) \\
x_2x_4a_3 + (N/I_{zz}) \\
g - (1/m)c_{x1}c_{x3}T \\
-x(1/m)(u_x)T \\
-x(1/m)(u_y)T \\
\end{bmatrix} \\
a_1 &= \frac{(l_{yy} - l_{zz})\dot{\psi}}{l_{xx}}, \quad a_2 = \frac{(l_{zz} - l_{xx})\dot{\phi}\dot{\psi}}{l_{yy}}, \quad a_3 = \frac{(l_{xx} - l_{yy})\dot{\phi}\dot{\theta}}{l_{zz}} \\
u_x &= c_\phi s_\theta c_\psi + s_\phi s_\psi, \quad u_y = c_\phi s_\psi s_\theta - s_\phi c_\psi \\
z_{2k-1} &= x_{2k-1}^{\text{ref}} - x_{2k-1} \\
z_{2k} &= x_{2k} - x_{2k-1}^{\text{ref}} - \alpha_{2k-1}z_{2k-1} \\
V(z_{2k-1}, z_{2k}) &= \frac{(z_{2k-1})^2 + (z_{2k})^2}{2} \\
\dot{x}_{2k} &= z_{2k} - \alpha_{2k}x_{2k} - \alpha_{2k-1}(z_{2k} + \alpha_{2k-1}z_{2k-1}) \\
x_{2k-1} &= [x_{2k-1}^{\text{ref}} - x_{2k-1}(0)](1 - e^{-\omega_n t}[1 + \omega_n t]) + x_{2k-1}(0) \\
x_{2k} &= [x_{2k-1}^{\text{ref}} - x_{2k-1}(0)]\omega_n^2 t e^{-\omega_n t} \\
T &= \begin{bmatrix}
m\{g + [x_7^{\text{ref}} - x_7(0)]\omega_n^2 e^{-\omega_n t} (\omega_n t - 1)\} \\
l_{xx}[x_4^{\text{ref}} - x_1(0)]\omega_n^2 e^{-\omega_n t}(1 - \omega_n t) \\
l_{yy}[x_3^{\text{ref}} - x_3(0)]\omega_n^2 e^{-\omega_n t}(1 - \omega_n t) \\
l_{zz}[x_5^{\text{ref}} - x_5(0)]\omega_n^2 e^{-\omega_n t}(1 - \omega_n t) \\
c_{x1} \{[(a_9 + \alpha_{19})(x_{10}) + (1 + \alpha_9 \alpha_{19})(x_9 - x_9^{\text{ref}})] \\
c_{x2} \{[(a_1 + \alpha_{12})(x_{12}) + (1 + \alpha_1 \alpha_{12})(x_{11} - x_{11}^{\text{ref}})] \\
L &= \begin{bmatrix}
M = \begin{bmatrix}
\end{bmatrix} \\
N = \begin{bmatrix}
\end{bmatrix} \\
u_x = \begin{bmatrix}
\end{bmatrix} \\
u_y = \begin{bmatrix}
\end{bmatrix} \\
\end{align*}
\[
\alpha_{2k-1} - \alpha_{2k} = \pm 2
\]
\[
\omega_n = \frac{\alpha_{2k-1} + \alpha_{2k}}{2}
\]
\[
0 \leq \Omega_i^2 \leq \Omega_{\text{max}}^2
\]
\[
\Omega_{\text{max}}^2 \geq \frac{T}{4k_f} + \frac{M}{2k_f l_c} \quad 0 \leq \frac{T}{4k_f} + \frac{M}{2k_f l_c} \quad \Omega_{\text{max}}^2 \geq \frac{T}{4k_f} - \frac{M}{2k_f l_c} \quad 0 \leq \frac{T}{4k_f} - \frac{M}{2k_f l_c}
\]
\[
\theta(t = 0) = \pm 45^\circ \quad \theta^{\text{ref}} = \mp 45^\circ \quad \dot{\theta} = 0
\]
\[
1 < \alpha_1 \leq 8.153 \quad -1 < \alpha_2 \leq 6.153 \quad 1 < \alpha_3 \leq 8.153 \quad -1 < \alpha_4 \leq 6.153
\]
\[
\Omega_{\text{max}}^2 \geq \frac{T}{4k_f} + \frac{N}{4k_m} \quad 0 \leq \frac{T}{4k_f} + \frac{N}{4k_m} \quad \Omega_{\text{max}}^2 \geq \frac{T}{4k_f} - \frac{N}{4k_m} \quad 0 \leq \frac{T}{4k_f} - \frac{N}{4k_m}
\]
\[
1 < \alpha_5 \leq 2.382 \quad -1 < \alpha_6 \leq 0.382
\]
\[
-7 \leq [x_7^{\text{ref}} - x_7(0)]\omega_n^2 t e^{-\omega_n t} \quad 4 \geq [x_7^{\text{ref}} - x_7(0)]\omega_n^2 t e^{-\omega_n t}
\]
\[
x_7^{\text{ref}} = -60 \quad x_7(0) = 0
\]
\[
x_7^{\text{ref}} = 0 \quad x_7(0) = -60
\]
\[
1 < \omega_7 \leq 1.181 \quad -1 < \omega_8 \leq -0.819
\]
\[
\ddot{x}_{2k} = \frac{1}{5\left(\frac{\omega_n}{\omega_h}\right)} \int_0^{5\frac{\omega_n}{\omega_h}} x_{2k} dt \quad \ddot{x}_{2k} \approx 0.192 \omega_n [x_{2k-1}^{\text{ref}} - x_{2k-1}(0)]
\]
\[
\bar{v}_h = \sqrt{x_{10}^2 + x_{12}^2} = 0.192 \omega_n l
\]
\[
\alpha_9 = \alpha_{11} = 1 + \frac{10.2}{l} \quad \alpha_{10} = \alpha_{12} = -1 + \frac{10.2}{l}
\]
the figure 2 contains texts in Portuguese, the correct version is:

Figure 2. Control Diagram.
Navigation and control of an UAV quadrotor in search and surveillance missions

L A Frederico¹, L S Martins-Filho¹ and A L Da Silva²
¹Universidade Federal do ABC, Centro de Engenharia, Modelagem e Ciências Sociais Aplicadas, Santo André, São Paulo, Brazil
²Universidade Federal de Santa Maria, Cachoeira do Sul, Rio Grande do Sul, Brazil
E-mail: leo_de_avellar@hotmail.com

Abstract. This study addresses the problem of exploration of areas for surveillance and searching using an Unmanned Aerial Vehicle quadrotor (UAV). Their agility provides them operational flexibility, as required by such applications. Other desirable characteristics are unpredictable motion path (from the point of view of intruders) and fast scan of the area. Random trajectories have been studied in these cases. Here, we study trajectories based on a specific random motions, known as Levy flights. In addition, this work concerns the study of the flight dynamics and the control to achieve the mission objectives. The backstepping control developed was implemented in simulation model, and tested together the Levy flight path generator. A procedure to determine the control gains in order to satisfy the Levy flight requirements and quadrotor constraints was proposed. Results show this control is adequate for the execution of the Levy flight respecting the operational and dynamic constraints.

1. Introduction
This work deals with the application of Lévy flight in an UAV for searching and surveillance missions. The Lévy flight has been observed in some animal species as an optimal solution to the problem of finding food, shelter or other target [1-2]. Fast area scan and unpredictability of the trajectory are desirable characteristics, becoming the Lévy flights an interesting solution. The Lévy function has the following form [1-2]:

\[ p(l) = l^{-\mu}, \quad l \geq l_0 \]  \hspace{1cm} (1)

In equation (1), \( l \) is the trajectory length in each step of the mission, \( \mu \) is an exponent and \( l_0 \) is the minimum trajectory length possible. For each value of the trajectory length will be there a probability value of that trajectory length be chosen. The approximation in equation (1) shows it depends on a maximum value; so that its distribution is equal the unit between these values. Levy flight is a super diffusive motion, so that tends to stay in the same direction any longer. The exponent \( \mu \) must be: 1 < \( \mu \) < 3. In this gap the motion is super diffusive, or, in other words, a Lévy flight motion type.

1.1 Model of Mission
The task model is given as follows: the flight is defined by waypoints. These waypoints are determined by the length of trajectory (randomly, according to Lévy function) and the yaw angle (according to a uniform density function). In this work, \( \mu = 1.125 \). The operational altitude is 60 m, and the operating speed is 20 km/h. Besides, the maximum range of the radio link and weather conditions are disregarded. The monitoring is accomplished by imaging, with the camera's viewing...
range $\alpha = 40^\circ$. The UAV autonomy is 20 min and the overflown region is square, 180 m long. The surveillance area size, the camera's viewing angle and autonomy of quadrotor UAV are related: the area is divided into square subareas whose side length is 30 m long [5]. Each area is a square subscribed in the circumference of specified radius. Once the UAV has imaged all subareas, the search is considered complete. It was observed that the mean total length for the mission is approximately 5.8 km, which, given the values of autonomy and maximum speed considered valid [5]. The steps of the mission are: 1-Takeoff; 2-Choice of the random variables (trajectory length and yaw angle); 3-Implementation of yaw motion; 4-Implementation of horizontal motion. If the full scan of the area has not been reached, the process returns to step 2, otherwise the UAV comes back to the launch point. If the next waypoint is specified outside the surveillance area, the UAV moves to the edge of the area in the predetermined direction, covering the remaining distance at a mirrored angle.

1.2. Dynamic Model of Quadrotor Flight

The adopted definitions are basically the same as other studies [3-7]. The following assumptions are adopted: the vehicle is considered to be a rigid body; its structure is symmetrical; the upper quadrotor is one where the propellers are located; the four propellers are contained in the same plane (propellers plan); the distance between the center of mass and the plane of the propellers is negligible; the engines are numbered 1-4, clockwise, looking up from top to bottom; engines 1 and 3 rotate in a counterclockwise direction; the motors 2 and 4, clockwise; the propellers cause tensile forces parallel and perpendicular to the plane of propellers; the four motors are identical; the drag force is negligible. Two coordinate systems are used: an inertial system and a mobile system. Figure 1 shows them:

![Figure 1. Inertial coordinate and UAV body systems. Adapted from [4].](image)

The mobile system is fixed on quadrotor mass center. Such as [3], [5] and [6], for quadrotor system, the Euler-Lagrange formalism states the generalized coordinates are $q = \left[ x_n \ y_e \ z_d \ \phi \ \theta \ \psi \right]^T$. The origin of the inertial system is taken to be the place of the take-off. The variables, $x_n$, $y_e$ and $z_d$ are the translations in the X, Y and Z, respectively. The variables $\phi$, $\theta$ and $\psi$ represent the orientation in relation to the inertial system through successive rotations about the axes x, y and z. The sequence of rotations that takes the orientation of the body system to the orientation of the inertial one is [3-7]:

$$C_0 \ R = \begin{pmatrix}
    c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\
    c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\
    -s_\theta & s_\phi c_\theta & c_\phi c_\theta
\end{pmatrix}$$

(2)

And the Lagrangian function $\Lambda$ of the quadrotor system is:

$$\Lambda = (1/2)[m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + I_{xx}(\dot{\phi} - \dot{\psi} s_\theta)^2 + I_{yy}(\dot{\theta} c_\phi + \dot{\phi} s_\phi)^2 + I_{zz}(\dot{\psi} c_\phi c_\theta - \dot{\theta} s_\phi)^2] + m g z$$

(3)

From equation (3), considering the first three variables of the vector $q$, the forces related to these coordinates are respectively:
For the rotational case it is considered the Euler angles are near zero. This simplifies the control, wherein each angle: \( \sin(\alpha) \approx \alpha \approx 0 \) and \( \cos(\alpha) \approx 1 \) (where \( \alpha \) is \( \phi \), \( \theta \)). Such approach is also considered valid in [3-7]. From equation (3) again, and after considering \( \alpha \approx 0 \) [3-7]:

\[
\mathbf{\tau} = \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} I_{xx}\dot{\phi} + (I_{zz} - I_{yy})\dot{\theta}\dot{\psi} \\ I_{yy}\dot{\theta} + (I_{xx} - I_{zz})\dot{\phi}\dot{\psi} \\ I_{zz}\dot{\psi} + (I_{yy} - I_{xx})\dot{\phi}\dot{\theta} \end{bmatrix}
\] (5)

Equation (5) is generic. The forces generated by the propellers make the equations particular. The four efforts those governing the motion are: the lift force and the three moments associated with the angular motions. The relationship between effort and rotations is given by [3-7]:

\[
\begin{bmatrix} \Omega_x^2 \\ \Omega_y^2 \\ \Omega_z^2 \\ \Omega_\omega^2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2k_f l_c \\ 2k_f l_c \\ 2k_f l_c \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 4k_m \end{bmatrix} \begin{bmatrix} \Omega_x^2 \\ \Omega_y^2 \\ \Omega_z^2 \\ \Omega_\omega^2 \end{bmatrix}
\] (6)

In which \( k_f \) is the thrust factor, \( k_m \) is the drag factor and \( l_c \) is the quadrotor arm length. Each of the motors will produce a force along the normal direction of the propellers plane, so that:

\[
m\begin{bmatrix} \dot{v}_n \\ \dot{v}_d \end{bmatrix} = mg \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - C_0 \begin{bmatrix} 0 \\ 0 \\ k_f(\Omega_x^2 + \Omega_y^2 + \Omega_z^2 + \Omega_\omega^2) \end{bmatrix}
\] (7)

Once \( \phi \approx 0 \) e \( \theta \approx 0 \), it implies \( \tau_\phi \approx L \), \( \tau_\theta \approx M \) e \( \tau_\psi \approx N \). So:

\[
\dot{\phi} = \frac{L + (I_{yy} - I_{zz})\dot{\psi}}{I_{xx}} \quad \dot{\theta} = \frac{M + (I_{xx} - I_{zz})\dot{\phi}\dot{\psi}}{I_{yy}} \quad \dot{\psi} = \frac{N + (I_{yy} - I_{xx})\dot{\phi}\dot{\theta}}{I_{zz}}
\] (8)

1.3. Flight Control

Given the nonlinear nature of quadrotor dynamics, a nonlinear control technique is implemented: the backstepping control. The use of this control is widespread in literature [3-7]. The state vector is:
\[
X = \begin{bmatrix}
\phi & \dot{\phi} & \theta & \dot{\theta} & \psi & \dot{\psi} & z & \dot{z} & x & \dot{x} & y & \dot{y}
\end{bmatrix}^T
\]
\[
\dot{X} = \begin{bmatrix}
x_4 x_6 a_1 + (L/1_{xx}) & x_4 & x_2 x_6 a_2 + (M/1_{yy}) & x_6 & x_2 x_4 a_3 + (N/1_{zz}) & x_8 & g - (1/m)c_x c_{x3} T & x_{10} & -(1/m)(u_x)T & x_{12} & -(1/m)(u_y)T
\end{bmatrix}
\]

Wherein \(c_x = \cos(x)\) e \(s_x = \sin(x)\). And:

\[
a_1 = \frac{(I_{yy} - I_{xz})\dot{\theta}\dot{\psi}}{I_{xx}}, \quad a_2 = \frac{(I_{zz} - I_{xx})\phi\dot{\psi}}{I_{yy}}, \quad a_3 = \frac{(I_{xx} - I_{yy})\phi\dot{\theta}}{I_{zz}}
\]

\[
u_x = c_\phi s_\theta c_{\psi} + s_\phi s_\psi, \quad \nu_y = c_\phi s_\psi s_\theta - s_\phi c_{\psi}
\]

For \(x_{2k-1} = 1, 2, 3, 4, 5, 6\), the following variable change are adopted:

\[
z_{2k-1} = x_{2k-1}^{ref} - x_{2k-1}, \quad z_{2k} = x_{2k} - x_{2k-1}^{ref} - \alpha_{2k-1} z_{2k-1}
\]

The candidate Lyapunov function that stabilizes the system is:

\[
V(z_{2k-1}, z_{2k}) = \frac{(z_{2k-1})^2 + (z_{2k})^2}{2}
\]

From its derivative (considering the derivatives of \(x_{2k-1}^{ref}\), in time, is zero) the solution is [3, 5, 6]:

\[
\dot{z}_{2k} = z_{2k-1} - \alpha_{2k} z_{2k} - \alpha_{2k-1} (z_{2k} + \alpha_{2k-1} z_{2k-1})
\]

For each of \(k\) values, it is possible to write an expression for each of efforts. The intention is that the control of the motion does not ask for input values that exceed the capacity of the engines. During each stage, only the motion in question is executed. For the mission it is considered that the motions are critically damped. In this way, from the last assumptions and from equation (14) is possible write:

\[
x_{2k-1} = [x_{2k-1}^{ref} - x_{2k-1}(0)] [1 - e^{-\omega_n t}] [1 + \omega_n t] + x_{2k-1}(0)
\]

\[
x_{2k} = [x_{2k-1}^{ref} - x_{2k-1}(0)] \omega_n^2 t e^{-\omega_n t}
\]

From equation (15) and the assumption during a motion that one will be the unique, efforts will be:
Wherein:

\[
\begin{bmatrix}
T \\
L \\
M \\
N \\
ux \\
uy
\end{bmatrix} = \\
\begin{bmatrix}
m\{g + [x^r_e - x_7(0)]\omega_n^2 e^{-\omega_n t}(\omega_n t - 1)} \\
l_{xx}[x^r_e - x_1(0)]\omega_n^2 e^{-\omega_n t}(1 - \omega_n t)} \\
l_{yy}[x^r_e - x_3(0)]\omega_n^2 e^{-\omega_n t}(1 - \omega_n t)} \\
l_{zz}[x^r_e - x_5(0)]\omega_n^2 e^{-\omega_n t}(1 - \omega_n t)} \\
\frac{c_{x3}}{g}[(\alpha_9 + \alpha_{10})(x_{10}) + (1 + \alpha_9\alpha_{10})(x_9 - x^r_e)] \\
\frac{c_{x3}}{g}[(\alpha_{11} + \alpha_{12})(x_{12}) + (1 + \alpha_{11}\alpha_{12})(x_{11} - x^r_e)]
\end{bmatrix}
\]

\begin{align}
\alpha_{2k-1} - \alpha_{2k} &= \pm 2 \\
\omega_n &= \frac{(\alpha_{2k-1} + \alpha_{2k})}{2}
\end{align}

One criterion adopted here is: at no time, mission control must request values for the efforts that exceed the limits of the actuators. So, the following inequality must be satisfied:

\[0 \leq \Omega^2 \leq \Omega_{\text{max}}^2\] (19)

1.3.1. Roll and Pitch Control

The first four values of gains, \(\alpha_1\) to \(\alpha_4\), are related. The pair \(\alpha_1\) and \(\alpha_3\) follow the same gap, as well as the pair \(\alpha_2\) and \(\alpha_4\). It occurs because the conditions under the pitch motion that are extended to roll motion, due to the symmetry. From equations (6) and (19), the constraints to the pitch motion are:

\[
\Omega_{\text{max}}^2 \geq \frac{T}{4k_f} + \frac{M}{2k_f l_c} \\
\Omega_{\text{max}}^2 \geq \frac{T}{4k_f} - \frac{M}{2k_f l_c}
\]

Now, from the equation (6) is possible explicit the frequency associate with this motion, or in other words, the gains \(\alpha_3\) and \(\alpha_4\). In this way, more one consideration is made: \(|\theta| \leq 45^\circ\). For calculation of these gains, a limit case motion is considered:

\[
\theta(t = 0) = \pm 45^\circ \\
\theta_{\text{ref}} = \mp 45^\circ \\
\dot{\theta} = 0
\]

The limit case would be a state where the quadrotor would be in a position in \(\theta = + 45^\circ\) at the initial instant of the pitch motion, trying to reach the opposite state. Now two instants are considered: the initial instant of the motion and the instant after \(2/\omega_n\) seconds. The first is the instant in which the effort's modulus is maximum, and the second one, the minimum modulus [5]. So, considering an instant in that \(\theta(t=0) = -45^\circ\) and \(\theta_{\text{ref}} = + 45^\circ\), taking the instant \(t = 0\) for the first and the last equations from (20), and the instant \(t = 2/\omega_n\) for the other two ones, the most restrictive gap is given by the last equation of (20):

\[1 < \alpha_1 \leq 8.153 \quad -1 < \alpha_2 \leq 6.153 \quad 1 < \alpha_3 \leq 8.153 \quad -1 < \alpha_4 \leq 6.153\] (22)

In fact, the consideration of the instant \(t = 2/\omega_n\) have not taken to restrict conditions [5]. The taking into account of this instant in these equations was made only to ensure that the constraints would be respected yet.

1.3.2. Yaw Control

Again, four constraints are considered to ensure satisfactory values of the efforts:
\[ \Omega_{\text{max}}^2 \geq \frac{T}{4k_f} + \frac{N}{4k_m}, \quad 0 \leq \frac{T}{4k_f} + \frac{N}{4k_m} \quad \Omega_{\text{max}}^2 \geq \frac{T}{4k_f} - \frac{N}{4k_m}, \quad 0 \leq \frac{T}{4k_f} - \frac{N}{4k_m} \]  

(23)

It occurs not only because equation (6), but because the duality of motion of the propeller pairs. When a pair rotates in clockwise another one rotates in counterclockwise. So, the considerations for only one case ensure the validity of the motion. The maximum amplitude considered for the yaw motion is \( x_{5}^{\text{ref}} - x_{5}(0) = \pi \), a half total rotation. Now, taking equation for N, the instant \( t = 0 \) for the first and the last equations from equation (23), and the instant \( t = 2/\omega_n \) (as in the previous case) for the other two ones, the most restrictive gap is given by the last equation of (23) [5]:

\[ 1 < \alpha_5 \leq 2.382 \quad -1 < \alpha_6 \leq 0.382 \]  

(24)

1.3.3. Altitude Control

In this case, the constraints from the equation (19) were supplanted for other two conditions: during takeoff and landing, the values of the rates of climb and descent should not exceed certain values. The values of maximum rate of climb and descent, respectively, are 7 m/s and 4 m/s, from the platform considered [5]:

\[-7 \leq [x_7^{\text{ref}} - x_7(0)]\omega_n^2 t e^{-\omega_n t} \quad 4 \geq [x_7^{\text{ref}} - x_7(0)]\omega_n^2 t e^{-\omega_n t} \]  

And:

\[
\begin{align*}
x_7^{\text{ref}} &= -60 \quad x_7(0) = 0 \\
x_7^{\text{ref}} &= 0 \quad x_7(0) = -60
\end{align*}
\]  

(26)

The first line of (26) represents the takeoff. The second one does the landing. The maximum amplitude for both occurs at \( t = 1/\omega_n \) [5]. So, from these, the second equation from (25) gives the following gap:

\[ 1 < \omega_7 \leq 1.181 \quad -1 < \omega_8 \leq -0.819 \]  

(27)

1.3.4. Horizontal Motion Control

For the horizontal motion, is considered the average horizontal velocity. The average horizontal velocity is considered approximately constant, being the same in each step:

\[ \ddot{x}_{2k} = \frac{1}{5/\omega_n} \int_0^{5/\omega_n} x_{2k} dt \quad \ddot{x}_{2k} \approx 0.192 \omega_n [x_{2k-1}^{\text{ref}} - x_{2k-1}(0)] \]  

(28)

In this case, is considered the same gain pair \( (\alpha_5 = \alpha_{11}, \alpha_{10} = \alpha_{12}) \). In these cases, \( k = 5 \) e \( k = 6 \). So:

\[ \beta_h = \sqrt{x_{10}^2 + x_{12}^2} = 0.192 \omega_n l \]  

(29)

\[ \alpha_9 = \alpha_{11} = 1 + \frac{10.2}{l} \quad \alpha_{10} = \alpha_{12} = -1 + \frac{10.2}{l} \]  

(30)

The average horizontal velocity has supplanted other considered conditions, showing itself the best one [5]. The four last values of the gains are dependent on the trajectory length, as shows the equation (29). The average speed adopted for the steps is 2m/s [5]. The control diagram is shown in Figure 2:
2. Methodology
The methodology is divided in two parts. The first one is about Levy flight simulations, around the mean total length needed to the full scan of the area. The simulations considered a squared area, 180 m long. During the mission, the quadrotor motion should be kept inside the area. During the implementation of the horizontal motion, if the quadrotor tried to leave the area, a change of direction (a reflexive motion in the area border) is made. The area was divided into squared subareas, 30 m long. In this way, a mission was considered completed when all subareas were visited by the quadrotor. 200 simulations about the vigilance mission were done. The second simulations cases concerned the proposed control laws. Firstly, each control law was tested separately, and after all of them were tested together.

3. Results and Discussions
Figures 3 to 7 show the UAV flight for trajectories generated from the design based on Levy flights. The tests were conducted using numerical simulations, from the simulation model developed in [6] and adapted in this work. The gains are: $\alpha_1 = \alpha_3 = 8$, $\alpha_2 = \alpha_4 = 6$, $\alpha_5 = 2.3$, $\alpha_6 = 0.3$, $\alpha_7 = 1.18$ and $\alpha_8 = -0.82$. The other gains depend on the length path step. The values of the gains $\alpha_{2k}$ can assume negative values. This is true, since the stability is given by $\omega_n$ ($\omega_n > 0$).
Figures 3 to 6 show the performance of the UAV. Throughout the mission, the quadrotor managed to reach the points determined by the Lévy flight. Figure 3 shows the yaw angle and the other two ones. As can be seen, at no instant roll and pitch modules are bigger than the value proposed. Yaw angle reach the chosen values determined. Green lines represent the references. Figure 4 shows the horizontal and vertical displacements. As can be seen, each waypoint is reached. The maximum rates of climb and descent values did not exceed 4 m/s -4 m/s respectively, as can be seen in Figure 5. In Figure 6, at no instant the horizontal speed exceeded the value of 20 km/h (approximately 5.56 m/s). The success of the mission shows the constraint represented by equation (19) was satisfied.

4. Conclusion

In this study, we addressed the control of a vehicle quadrotor for search and surveillance missions, through a random trajectories generator biologically inspired by the Lévy flight. For this, it agreed on a mission model and a backstepping to control the motion of the UAV. Given the control law, were imposed certain restrictions on the motion of quadrotor, leading to gains value ranges. Thus, it was possible to implement the Lévy flight for a search mission and surveillance restricted to a certain area.

References
[1] Viswanathan G M, Afanasyev V, Buldyrev S V, Murphy E J, Prince P A and Stanley, H E 1996 Lévy flight search patterns of wandering albatrosses. Nature 381, p. 413–415
[2] Viswanathan G M, Raposo E P and Da Luz M G E 2008 Lévy flights and superdiffusion in the context of biological encounters and random searches. PLR, 5, p. 133–150
[3] Bouabdallah S 2007. Design and Control of Quadrotors with Application to Autonomous Flying. PhD Thesis - Ecole Polytechnique Federale de Lausanne. 154
[4] Silva A L 2012. Voo Autônomo de Veículo Aéreo Não Tripulado Tipo quadrotor. Relatório Final de Pós Doutorado – Universidade de São Paulo. 71
[5] Frederico L A 2015. Planejamento de Trajetórias e Controle de Voo em Missões de Busca Vigilância de um Veículo Aéreo Não Tripulado. Dissertação – Universidade Federal do ABC. 203
[6] Santana P H R Q A and Borges G A 2009. Modelagem e controle de quadrotores. Brasília. In: SBA (Ed.), IX Simpósio Brasileiro de Automação Inteligente - SBAI 2009.
[7] Mellinger D, Michael N and Kumar V 2010. Trajectory generation and control for precise aggressive maneuvers. The International Journal of Robotics Research, p. 11