Estimation of propagation losses for infrared laser 
beam in turbulent atmosphere

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Abstract. In present work, the radiation propagation in atmosphere from laser source to the 
receiver is considered by taking into account deviations of optical beam due to turbulence. The 
photon flux density on the receiver has been evaluated.

1. Introduction

Laser technologies are actively used in aviation and space equipment today. For example, laser 
is used in distance Earth probing and monitoring of near-Earth space systems, network of 
specialized laser stations is created, which is the part of International Laser Ranging Service 
(IRLS). The main aim of IRLS is to control orbits of near-Earth spacecraft and objects of near-
Earth space garbage. The power of lasers, which are used in these systems, is in the range 
from watts to kilowatts. Powerful lasers, which can be used in these systems, can interact with 
different aerial vehicles and devices on their boards. It can be harmful for some devices. That 
is why the task of estimating losses of laser radiation (LR) when passing the atmosphere, a 
significant impact on the value of which has turbulence, is of the day. Method [1] of estimating 
the loss of near infrared (IR) LR of when passing the atmosphere was improved to account 
of the impact of the turbulence. Improved method includes number of steps, which are describes 
below.

2. Calculation of the route, which the laser beam passes

Distance between laser source and receiver is determined using the approximate values of 
spherical coordinates by the formula:

\[ L = \sqrt{\left(\Delta \delta R_3\right)^2 + \left(h_p - h_c\right)^2}, \]

where \( \Delta \delta = 2 \arcsin \left(\sqrt{\sin^2((\phi_2 - \phi_1)/2) + \cos \phi_2 \cos \phi_1 \sin^2((\Lambda_2 - \Lambda_1)/2)}\right) \) —angular distance, 
radians; \( R_3 \) —radius of the Earth, km; \( \phi \) —latitude, radians; \( \Lambda \) —longitude, radians; \( h_p \) —the 
height of the source of LR, km; \( h_c \) —the height of the receiver of LR, km.

Angles of the approach of LR to the receiver are also calculated.
3. Calculation of laser attenuation due to atmosphere scattering, absorption by water vapor and carbon dioxide

Attenuation of LR, which passes from the source to the receiver, by atmosphere, is calculated on the basis of the initial data of the parameters of the route. For this, all route splits into elementary sections with vertical drop of each section not more than 10 m. Next, transmission coefficients of the scattering layer, water vapor and carbon dioxide in the atmosphere for each elementary section are calculated for evaluating attenuation of LR on the route. Integral transmission coefficient is calculated by multiplying transmission coefficients of all elementary sites:

\[ K_\Sigma = \prod_{i=1}^{n} K_i^P K_i^K Y_i. \]

The scattering coefficient of LR by atmosphere on each segment is calculated according to the formula [1]:

\[ K_i^P = e^{-k_{\lambda i} l_i}, \quad k_{\lambda i} = \frac{8\pi^3(n^2 - 1)^2 6 + 3\chi}{3N\lambda^4 6 - 7\chi}, \]

where \( k_{\lambda i} \)—scattering coefficient per unit layer thickness, \( \text{km}^{-1} \); \( l_i \)—layer thickness, \( \text{km} \); \( n \)—the refractive index of the optical environment; \( N \)—the number of molecules per unit volume, \( \text{km}^{-3} \); \( \lambda \)—wavelength, \( \text{km} \); \( \chi \)—depolarization factor, in this paper is accepted \( \chi = 0 \).

The refractive index of the optical environment is calculated by the formula [2]:

\[ n = (n_0 - 1) \frac{P}{P_0} \left(\frac{1 + \gamma P}{1 + \alpha T}\right), \quad P = P_0 \exp\left(\frac{Mg h_0 - h}{RT}\right), \]

where \( P \)—ambient pressure at the altitude, \( \text{Pa} \); \( \gamma = 7 \times 10^{-7} \) (for air); \( \alpha = 3.67 \times 10^{-3}, \text{K}^{-1} \) (for air); \( n_0 \)—the refractive index of index of optical environment under normal conditions; \( P_0 \)—ambient pressure at sea level, \( \text{Pa} \); \( T \)—ambient temperature at the height, \( \text{K} \); \( M \)—molar mass, \( \text{g/mol} \); \( g \)—acceleration of gravity, \( \text{m/s}^2 \); \( h \)—height, \( \text{m} \); \( h_0 \)—height above sea level, \( \text{m} \); \( R \)—the universal gas constant.

The number of molecules per unit volume as a function of the height is determined by the formula:

\[ N = N_0 \exp\left(\frac{mg h_0 - h}{kT}\right), \]

where \( N_0 \)—the number of molecules per unit volume at sea level, \( \text{m}^{-3} \); \( m \)—molecular mass, \( \text{g} \); \( k \)—Boltzmann constant.

The number of molecules per unit volume as a function of the height is determined by the formula:

\[ K_i^K = -0.183 \ln(W) + 0.296, \]

where \( W = \sum_{i=1}^{n} W_i \)—height of besieged water column of the atmosphere of this section, \( \text{mm} \);

\[ W_i = \frac{N}{N_0} \frac{R_0(h)}{T(h)} E_{00}(h) l_i, \quad E_{00}(h) = 1.12 \times 10^{-9} \exp(0.081T(h)), \]

\( R_0(h) \)—relative air humidity, \( \% \) (\( R_0(h) = 100\% \)); \( T(h) \)—temperature at a height, \( \text{K} \); \( E_{00}(h) \)—elasticity of saturated vapor, \( \text{mbar} \); \( l_i \)—length of \( i \)-th section, \( \text{km} \).

Transmission coefficient of the atmosphere for carbon dioxide is calculated on the basis of the data [1] by the approximate formula:

\[ K_i^Y = -0.002 \ln(L_{\text{eff}}) + 0.9996, \quad L_{\text{eff}} = \frac{N}{N_0} l_i, \]

where \( L_{\text{eff}} \)—effective range by absorption of carbon dioxide, \( \text{km} \).
Attenuation coefficient of flux density of LR \( K_{LR} \) calculates by divide integral transmission coefficient at sinuses of angles of approach of LR to the receiver:

\[
K_{LR} = \frac{K_\Sigma}{\sin(\phi_1)\sin(\phi_2)},
\]

where \( \phi_1 \) and \( \phi_2 \)—angles of approach of LR to the receiver.

4. Calculation of the divergence and the minimum radius of the spot of laser radiation on the receiver taking into account the atmospheric turbulence on the track

Mathematical model of turbulence of Kolmogorov was used for determination of divergence of LR on the route taking into account the turbulence. It can be assumed [4], value \( C_n^2(h) \) is constant along the optical routes, which are locally parallel to the Earth’s surface. Corresponding values of all the elementary sections, on which the optical route was splitted earlier, depending on the height (from 5 to 20 km) is calculated by the following formula:

\[
C_n^2(h) = 8.2 \times 10^{-5} U^2 h^{-10} e^{-\tilde{h}_i/1000} + 2.7 \times 10^{-16} e^{-\tilde{h}_i/1500},
\]

where \( \tilde{h}_i \)—height above sea level of \( i \)-th elementary section, m; \( U \)—RMS wind speed, obtained by averaging over the height from 5 to 20 km [5] (for the deterministic model it can be considered equal 27 m/s). For a probabilistic model of the atmosphere, it is considered to be a Gaussian random variable, oscillating each day with an average value of 27 m/s and RMS deviation 9 m/s. At altitudes greater than 20 km \( C_n^2 = 0 \) [4].

Usage of the adaptive optics can reduce \( C_n^2 \) tenfold [6].

Divergence of LR on elementary section of the route, that passes LR, is calculated by the following formula [6]:

\[
\Theta_i(\lambda) = \left( 4.42 \left( \frac{K\lambda}{d} \right)^{5/3} + 37.1 \lambda^{-1/3} C_n^2(h) l_i \right)^{6/5} + 33 D^{-1/3} C_n^2(h) l_i \right)^{1/2},
\]

where \( D \)—diameter of the aperture, m; \( K \)—optical quality of the laser beam, equal to the ratio of real radiation divergence to its diffraction limit.

The minimum radius of the spot of LR at the approach to the surface of the receiver is calculated by summing the minimum radiuses of the spot of LR at all elementary sections:

\[
r_{LR} = \Sigma_{i=1}^n l_i \sin(\Theta_i(\lambda)).
\]

5. Calculating of the flux density of laser radiation on the receiver

Flux density of LR on the surface of the receiver is calculated by the following formula:

\[
q = \frac{Q K_{LR}}{\pi r_{LR}},
\]

where \( Q \)—output power of the LR source, W.

6. Calculation results

The results of the calculations by the method, taking into account the atmospheric turbulence and without it, in estimating path loss of near infrared laser radiation are shown in table 1. Position and height of the LR receiver were changed in initial data of calculations. Position and height of the LR receiver were changed in initial data of calculations. Position and height of the LR source in calculations were \( \phi_1 = 59.3978 \) rad; \( \Lambda_1 = 27.7486 \) rad; \( h_1 = 14000 \) m.
7. Conclusion

Thus, problem of turbulence accounting in estimating path loss of near infrared LR was set and solved at this article. With this purpose, method of estimating the loss near infrared LR during the passage of the atmosphere, differs from the known [1], that turbulence is further considered, is offered. Test calculations were carried out in accordance with the proposed improved method. Analysis of the values in the last two columns of table 1 suggests that the neglect of turbulence increases the flux density of LR on the receiver in about 6 times. Increasing the angle of the route as well as reducing the distance between the source and receiver leads to an increase of flux density of LR on the receiver.

References

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