MODELING THE STAR FORMATION IN GALAXIES USING THE CHEMO - DYNAMICAL SPH CODE

Abstract.
A new Chemo - Dynamical Smoothed Particle Hydrodynamic (CD - SPH) code is presented. The disk galaxy is described as a multi - fragmented gas and star system, embedded in a cold dark matter halo. The star formation (SF) process, SNII, SNIa and PN events as well as chemical enrichment of gas have been considered within the framework of the standard SPH model. Using this model we describe the dynamical and chemical evolution of triaxial disk - like galaxies. It is found that such approach provides a realistic description of the process of formation, chemical and dynamical evolution of disk galaxies over a cosmological timescale.

1. Introduction
The dynamical and chemical evolution of galaxies is one of the most interesting and complex problems. Naturally, galaxy formation is tightly connected with the process of large - scale structure formation in the Universe. The main role in the scenario of large - scale structure formation seems to be played by the dark matter. It is believed that the Universe was seeded at some early epoch with small density fluctuations of dark non - baryonic matter, and the evolving distribution of these dark halos provides the arena for galaxy formation. Galaxy formation itself involves collapse of baryons within potential wells of dark halos (White & Rees, 1978). The properties of forming galaxies depend on the amount of baryonic matter, that can be accumulated in such halos, and the efficiency of star formation. The observational support for this scenario of galaxy formation comes from the recent
COBE detection of fluctuations in the microwave background e.g. (Bennett et al., 1993).

The investigation of the process of galaxy formation is a highly complex subject requiring many different approaches. The formation of self-gravitating inhomogeneities of protogalactic size, the ratio of baryonic and non-baryonic matter (Bardeen et al., 1986; White & Silk, 1979; Peebles, 1993; Dar, 1995) the origin of the protogalaxy’s initial angular momentum (Voglis & Hiotelis, 1989; Zurek et al., 1988; Eisenstein & Loeb, 1995; Steinmetz & Bartelmann, 1995), the protogalaxy’s collapse and its subsequent evolution are usually considered as separated problems. The recent advances in computer technology and numerical methods have made possible the detailed modeling of baryon matter dynamics in the universe dominated by collisionless dark matter and, therefore, the detailed gravitational and hydrodynamical description of the formation and evolution of galaxies. The most sophisticated models additionally include radiative processes, star formation and supernova feedback e.g. (Katz, 1992; Steinmetz & Muller, 1994; Friedli & Benz, 1995).

However, results of numerical simulations are essentially affected by the star formation algorithm incorporated into modeling techniques. The star formation and accompanied processes are still not well understood on either small or large spatial scales. The star formation algorithm by which the gas material is converted into stars can be based only on simple theoretical assumptions or on results of empirical observations of nearby galaxies. The other most important effect of star formation on the global evolution of a galaxy is caused by a large amount of energy released in supernova explosions and stellar winds.

Among numerous methods developed for the modeling of complex three-dimensional hydrodynamic phenomena Smoothed Particle Hydrodynamics (SPH) is one of the most popular (Monaghan, 1992). Its Lagrangian nature allows to be combined easily with fast N-body algorithms, making it very suitable for simultaneous description of complex dynamics of gas–stellar system (Friedli & Benz, 1995). As an example of such combination, the TREE - SPH code (Hernquist & Katz, 1989; Navarro & White, 1993) can be mentioned, which was successfully applied to the detailed modelling of disk galaxy mergers (Mihos & Hernquist, 1996) and galaxy formation and evolution (Katz, 1992). The second good example is a GRAPE - SPH code (Steinmetz & Muller, 1994; Steinmetz & Muller, 1995) which was successfully used to model of the evolution of disk galaxy structure and kinematics.

In recent few years, we had an high number of excellent papers concerning the complex SPH modeling of galaxy formation and evolution (Raiteri et al., 1996; Carraro et al., 1997). In our code, we propose new ”energetic”
criteria for SF, and make a more realistic account of returned chemically enriched gas fraction via SNII, SNIa and PN events.

The simplicity and numerical efficiency of the SPH method were the main reasons why we chose this technique for the modeling of the evolution of complex, triaxial protogalactic systems. We used our own modification of the hybrid N - body/SPH method (Berczik & Kravchuk, 1996; Berczik, 1998), which we call the chemodynamical SPH (CD - SPH) code.

The "stars" are included into the standard SPH algorithm as the N - body collisionless system of particles, which can interact with the gas component only through the gravitation (Katz, 1992). The star formation process and supernova explosions are included into the scheme in the manner proposed by Raiteri et al. (1996), but with our own modifications.

2. The CD - SPH code

2.1. THE SPH CODE

Continuous hydrodynamic fields in SPH are described by the interpolation functions constructed from the known values of these functions at randomly positioned particles. In this case the mean value of a physical field \( f(\mathbf{r}) \) at the point \( \mathbf{r} \) can be written as (Monaghan, 1992):

\[
<f(\mathbf{r})> = \int f(\mathbf{r}') \cdot W(r - r'; h) dr',
\]  

where \( W(\mathbf{r} - \mathbf{r}'; h) \) is the kernel function and \( h \) is the softening constant.

The function \( W(\mathbf{r} - \mathbf{r}'; h) \) is strongly peaked at \(|\mathbf{r} - \mathbf{r}'| = 0\), and we can consider it without any loss of generality as belonging to the class of even functions. In this case it is not difficult to demonstrate (Hernquist & Katz, 1989) that the average value \(<f(\mathbf{r})>\) represents the real function \( f(\mathbf{r}) \), with an error not higher than \( O(h^2) \). Consider a fluid with the density \( \rho(\mathbf{r}) \), then we can rewrite (1) in the form:

\[
<f(\mathbf{r})> = \int \frac{f(\mathbf{r}')}{\rho(\mathbf{r}')} \cdot W(\mathbf{r} - \mathbf{r}'; h) \cdot \rho(\mathbf{r}') d\mathbf{r}'.
\]  

(2)

Let us imagine that \( f(\mathbf{r}) \) and \( \rho(\mathbf{r}) \) are known only at \( N \) discrete points \( \mathbf{r}_i \). Then, equation (2) gives:

\[
f_i = \sum_{j=1}^{N} m_j \cdot \frac{f_j}{\rho_j} \cdot W_{ij}.
\]  

(3)

Here \( f_i \equiv f(\mathbf{r}_i) \), \( \rho_i \equiv \rho(\mathbf{r}_i) \), \( W_{ij} \equiv W(\mathbf{r}_i - \mathbf{r}_j; h) \) and \( m_i \) is the mass of particle \( i \). Using (3), we approximate the hydrodynamic field by an analytical function which is differentiable as many times as the kernel \( W_{ij} \).
Following Monaghan & Lattanzio (1985) we use for the kernel function \( W_{ij} \) the spline expression in the form:

\[
W_{ij} = \frac{1}{\pi h^3} \begin{cases} 
1 - \frac{3}{2} u_{ij}^2 + \frac{3}{4} u_{ij}^3, & \text{if } 0 \leq u_{ij} < 1, \\
\frac{1}{4}(2 - u_{ij})^3, & \text{if } 1 \leq u_{ij} < 2, \\
0, & \text{otherwise}.
\end{cases}
\]

Here \( u_{ij} = r_{ij}/h \).

The local resolution in SPH is defined by the chosen smoothing length \( h \). If it is the same for all points, the hydrodynamic field will be evidently approximated more smoothly in the regions where the points lie more closely, i.e. where the density is higher.

To achieve the same level of accuracy for all points in the fluid it is necessary to use a spatially variable smoothing length. In this case each particle has its individual value of \( h \). Following Hernquist & Katz (1989), we write instead of (1):

\[
<f(r)> = \int f(r') \cdot \frac{1}{2} \cdot [W(\delta r; h) + W(\delta r; h')] dr'.
\]

Here \( \delta r \equiv r - r' \), \( h \equiv h(r) \) and \( h' \equiv h(r') \). For the density \( \rho(r) \) it gives

\[
<\rho(r_i)> = \sum_{j=1}^{N} m_j \cdot \frac{1}{2} \cdot [W(r_{ij}; h_i) + W(r_{ij}; h_j)].
\]

Other hydrodynamic functions are written in the same manner.

In our calculations the values of \( h_i \) were determined from the condition that the number of particles \( N_B \) in the neighborhood of each particle within the \( 2 \cdot h_i \) remains constant (Mihos & Hernquist, 1996). The value of \( N_B \) is chosen such that a certain fraction from the total number of "gas" particles \( N \) affects the local flow characteristics (Hiotelis & Voglis, 1991). If the defined \( h_i \) become smaller than the minimal smoothing length \( h_{min} \), we set the value \( h_i = h_{min} \). For "star" particles (with Plummer density profiles) we use, accordingly, the fixed gravitational smoothing lengths \( h_{star} \).

If the density is computed according to equation (6), then the continuity equation is satisfied automatically. Equations of motion for particle \( i \) are

\[
\frac{dr_i}{dt} = v_i, \\
\frac{dv_i}{dt} = -\frac{\nabla_i P_i}{\rho_i} + a^{\text{vis}}_i - \nabla_i \Phi_i - \nabla_i \Phi_i^{\text{ext}},
\]
where $P_i$ is the pressure, $\Phi_i$ is the self gravitational potential, $\Phi_i^{ext}$ is a gravitational potential of possible external halo, $a_i^{vis}$ is an artificial viscosity term. Using equations (5) and (6) we can write:

$$\frac{\nabla_i P_i}{p_i} = \frac{1}{2} \sum_{j=1}^{N} \frac{m_j (P_i/\rho_i^2 + P_j/\rho_j^2)}{p_j} \nabla_i [W(r_{ij}; h_i) + W(r_{ij}; h_j)].$$

(9)

The artificial viscosity term $a_i^{vis}$ is introduced so as to describe the flows with shock waves. In present calculations the viscous acceleration is introduced by replacing $(P_i/\rho_i^2 + P_j/\rho_j^2)$ in equation (9) by $(P_i/\rho_i^2 + P_j/\rho_j^2) \cdot (1 + \pi_{ij})$. The expression for $\pi_{ij}$ has the form (Hiotelis et al., 1991):

$$\pi_{ij} = -\alpha \cdot \mu_{ij} + \beta \cdot \mu_{ij}^2,$$

(10)

where $\alpha$ and $\beta$ are constants, and $\mu_{ij}$ is defined by the relation:

$$\mu_{ij} = \begin{cases} 
\frac{h_{ij}}{c_{ij}} \cdot (v_{ij} \cdot r_{ij})/(r_{ij}^2 + n^2 h_{ij}^2), & \text{if } (v_{ij} \cdot r_{ij}) < 0, \\
0, & \text{otherwise}. 
\end{cases}$$

(11)

Here $c_{ij} = (c_i + c_j)/2$ is the sound speed averaged for points $i$ and $j$, $h_{ij} = (h_i + h_j)/2$, $r_{ij} = (r_i - r_j)$ and $v_{ij} = (v_i - v_j)$ is the relative velocity vector for the points $i$ and $j$. The term $n^2 \cdot h_{ij}^2$ is inserted to prevent divergences when $r_{ij} = 0$ and it should be small enough. The constant $n$ was set equal to 0.1. For the constant $\alpha$ and $\beta$ in (10) the values $\alpha = 1$ and $\beta = 2$ give good results in a wide range of the Mach numbers (Monaghan, 1992).

By using equation (4), one gets for the gravitational acceleration (Hiotelis & Voglis, 1991):

$$-\nabla_i \Phi_i = \frac{1}{2} \sum_{j=1}^{N} G \cdot m_j \cdot \frac{r_{ij}}{r_{ij}^2} \cdot [g_i + g_j],$$

(12)

where:

$$g_k = \begin{cases} 
\frac{4}{3} u_k^3 - \frac{6}{5} u_k^5 + \frac{4}{5} u_k^6, & 0 \leq u_k < 1, \\
-\frac{1}{15} + \frac{8}{3} u_k^3 - 3 u_k^4 + \frac{8}{5} u_k^5 - \frac{4}{5} u_k^6, & 1 \leq u_k < 2, \\
1, & \text{otherwise},
\end{cases}$$

(13)

and $u_k = r_{ij}/h_k$. 

When isothermal flows are considered, the system of equations is closed by adding the equation of state:

\[ P_i = \rho_i \cdot c_i^2, \quad (14) \]

where \( c_i \) is the isothermal speed of sound.

In the particle representation for complex hydrodynamic flows the energy equation has the form:

\[
\frac{du_i}{dt} = \sum_{j=1}^{N} \frac{m_j}{4} \left( \frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} \right) (1 + \pi_{ij}) \times \\
\times \nabla_i [W(r_{ij}; h_i) + W(r_{ij}; h_j)] \nu_{ij} + \frac{\Gamma_i - \Lambda_i}{\rho_i}. \quad (15)
\]

Here \( u_i \) is the specific internal energy of particle \( i \). The term \((\Gamma_i - \Lambda_i)/\rho_i\) accounts for nonadiabatic processes not associated with the artificial viscosity (in our calculations \( \Gamma_i \equiv 0 \)). We present the radiative cooling in the form:

\[
\Lambda_i = \Lambda_i(u_i, \rho_i) = \Lambda^*_i(T_i) \cdot n_i^2, \quad (16)
\]

where \( n_i \) is the hydrogen number density and \( T_i \) is the temperature. To follow its subsequent thermal behaviour in numerical simulations we use an analytical approximation of the standard cooling function \( \Lambda^*(T) \) for an optically thin primordial plasma in ionization equilibrium (Dalgarno & McCray, 1972; Katz & Gunn, 1991). Its absolute cutoff temperature is set equal to \( 10^4 \) K.

If \( \log T \leq 6.2 \):

\[
\Lambda^*(T) = 10^{-28} \cdot \left[ 10^{-0.1 \cdot 1.88(5.23 - \log T)}^4 + \\
+ 10^{-1.7 \cdot 0.2(6.2 - \log T)}^4 \right] \text{J/s}, \quad (17)
\]

else

\[
\Lambda^*(T) = 10^{-28} \cdot \left[ 10^{-0.1 \cdot 1.88(5.23 - \log T)}^4 + \\
+ 10^{-1.7} \right] \text{J/s}, \quad (18)
\]

The equation of state must be added to close the system:

\[ P_i = \rho_i \cdot (\gamma - 1) \cdot u_i, \quad (19) \]
where $\gamma = 5/3$ is the adiabatic index.

2.2. TIME INTEGRATION

To solve the system of equations \((7), (8)\) and \((15)\) we use the leapfrog integrator (Hernquist & Katz, 1989). This system of equations has the form:

\[
\begin{align*}
\frac{dr_i}{dt} &= v_i, \\
\frac{dv_i}{dt} &= a_i(h, \rho, T, c, P, r, v), \\
\frac{du_i}{dt} &= \dot{u}_i(h, \rho, T, c, P, r, v).
\end{align*}
\] (20)

The time step $\delta t_i$ for each particle depends on the particle’s acceleration $a_i$ and velocity $v_i$, as well as on viscous forces. To define $\delta t_i$ we use the relation (Hiotelis & Voglis, 1991):

\[
\delta t_i = C_n \min \left( \sqrt{\frac{h_i}{a_i}}, \frac{h_i}{|v_i|}, \frac{h_i}{s_i} \right),
\] (21)

where $s_i = c_i \cdot (1 + \alpha + 0.68 \cdot \beta \cdot \max_j |\mu_{ij}|)$ and $C_n$ is the Courant’s number. We adopt $C_n = 0.1$. The minimum time step value is set equal to:

\[
\Delta t = \min_i (\delta t_i),
\] (22)

for all particles. The integrator has a second order accuracy in the time step $\Delta t$. As the first step for particle $i$ we define:

\[
\begin{align*}
r_i^{n+\frac{1}{2}} &= r_i^n + 0.5 \cdot \Delta t \cdot v_i^n, \\
v_i^{n+\frac{1}{2}} &= v_i^n + 0.5 \cdot \Delta t \cdot a_i^n, \\
u_i^{n+\frac{1}{2}} &= u_i^n + 0.5 \cdot \Delta t \cdot \dot{u}_i^n.
\end{align*}
\] (23)

After that:

\[
\begin{align*}
r_i^{n+1} &= r_i^n + \Delta t \cdot v_i^{n+\frac{1}{2}} + O(\Delta t^3), \\
v_i^{n+1} &= v_i^n + \Delta t \cdot a_i^{n+\frac{1}{2}} + O(\Delta t^3), \\
u_i^{n+1} &= u_i^n + \Delta t \cdot u_i^{n+\frac{1}{2}} + O(\Delta t^3).
\end{align*}
\] (24)
We have carried out (Berczik & Kolesnik, 1993) a large series of test calculations to check that the code is correct, the conservation laws are obeyed and the hydrodynamic fields are represented adequately. These tests have shown very good results.

2.3. THE STAR FORMATION ALGORITHM

It is well known that star formation (SF) regions are tightly connected to giant molecular complexes, especially to regions, which reach some threshold for dynamical instabilities. The overall picture of star formation seems to be understood, but the detailed physics of star formation and accompanying processes on either small or large scales remains sketchy (Larson, 1969; Silk, 1987; Leitherer et al., 1992).

For describing of the process of conversion of gas material into stars we modify the standard SPH star formation algorithm (Katz, 1992; Navarro & White, 1993), taking into account the presence of chaotic motions in the gaseous environment and the time lag between the initial development of suitable conditions for star formation and star formation itself (Berczik & Kravchuk, 1996; Berczik, 1998). The first reasonable requirement incorporated into this algorithm allows selecting "gas" particles that are potentially eligible to form stars. It states that in the separate "gas" particle the SF can start if the absolute value of the "gas" particle’s gravitational energy exceeds the sum of its thermal energy and energy of chaotic motions:

\[ |E_{gr}^i| > E_{th}^i + E_{ch}^i. \] (25)

Gravitational and thermal energies and energy of chaotic motions for the "gas" particle \(i\) in model simulation are defined as:

\[
\begin{aligned}
E_{gr}^i &= -\frac{2}{3} \cdot G \cdot m_i^2 / h_i, \\
E_{th}^i &= \frac{3}{2} \cdot m_i \cdot c_i^2, \\
E_{ch}^i &= \frac{1}{2} \cdot m_i \cdot \Delta v_i^2.
\end{aligned}
\] (26)

Where \(c_i = \sqrt{\mathcal{R} \cdot T_i / \mu}\) is the isothermal sound speed of particle \(i\). We set \(\mu = 1.3\) and define the chaotic or "turbulent" square velocities near particle \(i\) as:

\[
\Delta v_i^2 = \sum_{j=1}^{N_B} m_j \cdot (v_j - v_c)^2 / \sum_{j=1}^{N_B} m_j,
\] (27)

where:
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\[ \mathbf{v}_c = \frac{\sum_{j=1}^{N_B} m_j \cdot \mathbf{v}_j}{\sum_{j=1}^{N_B} m_j}. \] (28)

For practical reasons, it is useful to define a critical temperature for SF onset in particle \( i \) as:

\[ T_i^{\text{crit}} = \frac{\mu 3}{3R} \cdot \left( \frac{8}{5} \cdot \pi \cdot G \cdot \rho_i \cdot h_i^2 - \Delta v_i^2 \right). \] (29)

Then if the temperature of the "gas" particle \( i \), drops below the critical one, SF can proceed:

\[ T_i < T_i^{\text{crit}}. \] (30)

We think that requirement (25), or in another form (30), is only a necessary one. It seems reasonable that the chosen "gas" particle will produce stars only if the above condition will hold over the time interval exceeding its free-fall time \( t_{\text{ff}} = \sqrt{\frac{3}{\pi} \cdot \frac{G \cdot \rho}{32}} \). This condition is based on the well known fact that due to gravitational instability all substructures of collapsing system are formed on such timescale. Using it we exclude from the consideration transient structures, that are destroyed by the tidal action of surrounding matter. This last condition we assume to be a sufficient one.

We also define which "gas" particles remain cool, i.e. \( t_{\text{cool}} < t_{\text{ff}} \). We rewrite this condition in the manner presented in Navarro & White (1993): \( \rho_i > \rho_{\text{crit}} \). Here we use the value of \( \rho_{\text{crit}} = 0.03 \text{ cm}^{-3} \).

When the collapsing particle \( i \) is defined, we create the new "star" particle with mass \( m^{\text{star}} \) and update the "gas" particle \( m_i \) using these simple equations:

\[
\begin{align*}
  m^{\text{star}} &= \epsilon \cdot m_i, \\
  m_i &= (1 - \epsilon) \cdot m_i.
\end{align*}
\] (31)

Here \( \epsilon \), defined as the global efficiency of star formation, is the fraction of gas converted into stars according to the appropriate initial mass function (IMF). The typical values for SF efficiency in our Galaxy on the scale of giant molecular clouds are in the range \( \epsilon \approx 0.01 - 0.4 \) (Duerr et al., 1982; Wilking & Lada, 1983). But it is still a poorly known quantity. In numerical simulation it is the model parameter which has to be checked by comparison of numerical simulation results with available observational data. Here we define \( \epsilon \) as:
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\( \epsilon = 1 - (E_i^{th} + E_i^{ch})/ |E_i^{gr}|, \)

(32)

with the requirement that all excess mass of the gas component in star forming particle, which provide the inequality \(|E_i^{gr}| > E_i^{th} + E_i^{ch}\), is transformed into the star component. In the code we set the absolute maximum value of the mass of such a "star" particle \(m_{\text{star}}^{\text{max}} = 2.5 \cdot 10^6 \, M_\odot\), i.e. \(\approx 5\%\) of the initial particle mass \(m_i\).

At the moment of the birth, the position and velocities of new "star" particles are equal to those of parent "gas" particles. Thereafter these "star" particles interact with other "gas" and "star" particles only by gravitation. The gravitational smoothing length for these (Plummer like) particles is set equal to \(h_{\text{star}}\).

2.4. THE THERMAL SNII FEED-BACK

We try to include in our code the events of SNII, SNIa and PN into the complex gasdynamic picture of galaxy evolution. But, for the thermal budget of the ISM, only SNII play the main role. Following Katz (1992), we assume that the explosion energy is converted totally to the thermal energy. The stellar wind action seems not to be essential in the energy budget (Ferriere, 1995). The total energy released by SNII explosions \((10^{44} \, \text{J per SNII})\) within a "star" particles is calculated at each time step and distributed uniformly among the surrounding (i.e. \(r_{ij} < h_{\text{star}}\)) "gas" particles (Raiteri et al., 1996).

2.5. THE CHEMICAL ENRICHMENT OF GAS

Every new "star" particle in our SF scheme represents a separate, gravitationally closed star formation macro region (like a globular cluster). The "star" particle is characterized its own time of birth \(t_{\text{begSF}}\) which is set equal to the moment of particle formation. After the formation, these particles return the chemically enriched gas to surrounding "gas" particles due to SNII, SNIa and PN events. For the description of this process we use the approximation proposed by Raiteri et al. (1996). We consider only the production of \(^{16}\text{O}\) and \(^{56}\text{Fe}\), and try to describe the full galactic time evolution of these elements, from the beginning to present time (i.e. \(t_{\text{eval}} \approx 13.0 \, \text{Gyr}\)).

We use the multi-power IMF law defined by Kroupa et al. (1993). The distribution of stellar masses within a "star" particle of mass \(m_{\text{star}}\) is then:
\[
\Psi(m) = \psi \cdot A \cdot \begin{cases} 
2^{0.9m^{-1.3}}, & \text{if } 0.1 \leq m < 0.5, \\
2^{2.2m}, & \text{if } 0.5 \leq m < 1.0, \\
2^{2.7m}, & \text{if } 1.0 \leq m.
\end{cases}
\]  

(33)

Here \( m \) is the star mass in solar units and the normalization constant is \( A = 0.310146 \). The value of \( A \) is different from Raiteri et al. (1996), because we set the integration limits \( m_{\text{low}} = 0.1 M_\odot \) and \( m_{\text{upp}} = 100 M_\odot \).

For the definition of stellar lifetimes we use the equation (Raiteri et al., 1996):

\[
\log t_{\text{dead}} = a_0(Z) - a_1(Z) \cdot \log m + a_2(Z) \cdot (\log m)^2,
\]

(34)

where \( t_{\text{dead}} \) is expressed in years and \( m \) is in solar unit and coefficients are defined as:

\[
\begin{align*}
    a_0(Z) &= 10.130 + 0.0755 \cdot \log Z - 0.0081 \cdot (\log Z)^2, \\
    a_1(Z) &= 4.4240 + 0.7939 \cdot \log Z + 0.1187 \cdot (\log Z)^2, \\
    a_2(Z) &= 1.2620 + 0.3385 \cdot \log Z + 0.0542 \cdot (\log Z)^2.
\end{align*}
\]

(35)

These relations are based on the calculations of the Padova group (Alongi et al., 1993; Bressan et al., 1993; Bertelli et al., 1994) and give a reasonable approximation to stellar lifetimes in the mass range from 0.6 \( M_\odot \) to 120 \( M_\odot \) and metallicities \( Z = 7 \cdot 10^{-5} - 0.03 \) (defined as a mass of all elements heavier than He). In our calculation, we assume following, Raiteri et al. (1996), that \( Z \) scales with the oxygen abundance as \( Z/Z_\odot = ^{16}O/^{16}O_\odot \). For lower or higher metallicities we take the value corresponding to the extremes of this interval.

We can define the number of SNII explosions inside a given "star" particle during the time from \( t \) to \( t + \Delta t \) using a simple equation:

\[
\Delta N_{\text{SNII}} = \int_{m_{\text{dead}(t + \Delta t)}}^{m_{\text{dead}(t)}} \Psi(m) \, dm.
\]

(36)

Here \( m_{\text{dead}(t)} \) and \( m_{\text{dead}(t + \Delta t)} \) are masses of stars that end their lifetimes at the beginning and at the end of the time step respectively. Contrary to Raiteri et al. (1996), we assume that all stars with masses
between $8 \, M_\odot$ and $100 \, M_\odot$ end their lives as SNII. For SNII we use the yields from Woosley & Weaver (1995). We use the approximation formulae from Raiteri et al. (1996) for defining the total ejected mass by one SNII $m_{\text{ej}}^{\text{tot}}$, as well as for the ejected mass of iron $m_{\text{ej}}^{\text{Fe}}$ and oxygen $m_{\text{ej}}^{\text{O}}$ as a function of stellar mass (in solar unit).

\[\begin{align*}
    m_{\text{ej}}^{\text{tot}} &= 7.682 \cdot 10^{-1} \cdot m^{1.056}, \\
    m_{\text{ej}}^{\text{Fe}} &= 2.802 \cdot 10^{-4} \cdot m^{1.864}, \\
    m_{\text{ej}}^{\text{O}} &= 4.586 \cdot 10^{-4} \cdot m^{2.721}.
\end{align*}\] (37)

To take into account PN events inside the "star" particle we use the equation, like (36):

\[\Delta N_{\text{PN}} = \int_{m_{\text{dead}}(t+\Delta t)}^{m_{\text{dead}}(t)} \Psi(m) dm.\] (38)

Following van den Hoek & Groenewegen (1997), Samland (1997) and Samland et al. (1997), we assume that all stars with masses between $1 \, M_\odot$ and $8 \, M_\odot$ end their lives as PN. We define the average ejected masses (in solar unit) of one PN event as (Renzini & Voli, 1981; van den Hoek & Groenewegen, 1997):

\[\begin{align*}
    m_{\text{ej}}^{\text{tot}} &= 1.63, \\
    m_{\text{ej}}^{\text{Fe}} &= 0.00, \\
    m_{\text{ej}}^{\text{O}} &= 0.00.
\end{align*}\] (39)

The method described in Raiteri et al. (1996) and proposed in Greggio & Renzini (1983) and Matteucci & Greggio (1986) is used to account of SNIa. In simulations, the number of SNIa exploding inside a selected "star" particle during each time step is given by:

\[\Delta N_{\text{SNIa}} = \int_{m_{\text{dead}}(t+\Delta t)}^{m_{\text{dead}}(t)} \Psi_2(m_2) dm_2.\] (40)

The quantity $\Psi_2(m_2)$ represents the initial mass function of the secondary component and includes the distribution function of the secondary’s mass relative to the total mass of the binary system, $m_B$. 
\[
\Psi_2(m_2) = m_{\text{star}} \cdot A_2 \cdot \int_{m_{\text{inf}}}^{m_{\text{sup}}} \left( \frac{m_2}{m_B} \right)^2 \cdot m_B^{-2.7} dm_B,
\]

(41)

where \( m_{\text{inf}} = \max(2 \cdot m_2, \ 3 \ 10^4 M_\odot) \) and \( m_{\text{sup}} = m_2 + 8 \ 10^4 M_\odot \). The value of normalization constant we set, following the van den Berg & McClure (1994), equal to \( A_2 = 0.16 \cdot A \).

Then the total ejected mass (in solar unit) is (Thielemann et al., 1986; Nomoto et al., 1984):

\[
\begin{aligned}
&\begin{cases}
  m_{e_j}^{\text{tot}} = 1.41, \\
  m_{e_j}^{\text{Fe}} = 0.63, \\
  m_{e_j}^{\text{O}} = 0.13.
\end{cases}
\end{aligned}
\]

(42)

In summary, a new "star" particle (with metalicity \( Z = 10^{-4} \)) with mass \( 10^4 M_\odot \), during the total time of evolution \( t_{\text{evol}} \) produces:

\[
\Delta N_{\text{SNII}} \approx 52.5, \ \Delta N_{\text{PN}} \approx 1770.0, \ \Delta N_{\text{SNIa}} \approx 8.48.
\]
Figure 2. The returned mass of $^{56}$Fe

Figure 3. The returned mass of $^{16}$O
We present in Fig. 1. the number of SNII, SNIa and PN events for this "star" particle.

In Fig. 2. and Fig. 3. we present the returned masses of $^{56}\text{Fe}$ and $^{16}\text{O}$, respectively. We can estimate the total masses ($\text{H, He, } ^{56}\text{Fe, } ^{16}\text{O}$) (in solar masses) returned to the surrounding "gas" particles, due to these processes as:

$$\begin{align*}
\Delta m_{\text{SNII}}^{\text{H}} &= 477, & \Delta m_{\text{PN}}^{\text{H}} &= 2164, & \Delta m_{\text{SNIa}}^{\text{H}} &= 4.14, \\
\Delta m_{\text{SNII}}^{\text{He}} &= 159, & \Delta m_{\text{PN}}^{\text{He}} &= 721.3, & \Delta m_{\text{SNIa}}^{\text{He}} &= 1.38, \\
\Delta m_{\text{SNII}}^{\text{Fe}} &= 3.5, & \Delta m_{\text{PN}}^{\text{Fe}} &= 0.000, & \Delta m_{\text{SNIa}}^{\text{Fe}} &= 5.35, \\
\Delta m_{\text{SNII}}^{\text{O}} &= 119, & \Delta m_{\text{PN}}^{\text{O}} &= 0.000, & \Delta m_{\text{SNIa}}^{\text{O}} &= 1.10.
\end{align*}$$

(43)

2.6. THE COLD DARK MATTER HALO

In the literature we have found some, sometimes controversial, profiles for the galaxies Cold Dark Matter Haloes (CDMH) (Burkert, 1995; Navarro, 1998). For resolved structures of CDMH: $\rho_{\text{halo}}(r) \sim r^{-1.4}$ (Moore et al., 1997). The structure of CDMH high-resolution N-body simulations show can be described by: $\rho_{\text{halo}}(r) \sim r^{-1}$ (Navarro et al., 1996; Navarro et al., 1997). Finally, in Kravtsov et al. (1997), we find that the cores of DM dominated galaxies may have a central profile: $\rho_{\text{halo}}(r) \sim r^{-0.2}$.

Because we concentrate our attention mainly on the problem of the formation and evolution (dynamical and chemical) of the disk structure in galaxy we using the simplest dark matter halo model. In our calculations, as a first approximation, it is assumed that the model galaxy halo contains the CDMH component with Plummer - type density profiles (Douphole & Colin, 1995):

$$\rho_{\text{halo}}(r) = \frac{M_{\text{halo}}}{\frac{4}{3} \pi b_{\text{halo}}^5} \cdot \frac{b_{\text{halo}}^5}{(r^2 + b_{\text{halo}}^2)^{\frac{3}{2}}}. \quad (44)$$

Therefore we can write for the external force acting on the "gas" and "star" particles:

$$-\nabla_i \Phi_i^{\text{ext}} = -G \cdot \frac{M_{\text{halo}}}{(r_i^2 + b_{\text{halo}}^2)^{\frac{3}{2}}} \cdot r_i. \quad (45)$$

The inclusion of the dynamically evolved halo model is a next step in our calculations.
3. Results and discussion

3.1. INITIAL CONDITIONS

We test our code and demonstrate that simple assumptions lead to a reasonable galaxy model. The SPH calculations were carried out for \( N_{\text{gas}} = 2109 \) "gas" particles. According to Navarro & White (1993) and Raiteri et al. (1996), such number seems to be quite enough to provide a qualitatively correct description of the system behaviour. Even such small number of "gas" particles produces \( N_{\text{star}} = 31631 \) "star" particles at the end of the calculation. We also check the dependency of our model with the number of "gas" particles. We calculate the first \( \sim 5 \) Gyr evolution of the systems with identical initial data but with different numbers of \( N_{\text{gas}} = 2109; 4169; 11513 \) with correspondent \( N_B = 21; 41; 115 \). All three models show the practically identical star formation rate and chemical evolution history.

The value of the smoothing length \( h_i \) was chosen requiring that each "gas" particle had \( N_B = 21 \) neighbors within \( 2 \cdot h_i \). Minimal \( h_{\text{min}} \) was set equal to 1 kpc. This value is defined as a largest characteristic dimensions of instability in galactic disk. This \( h_{\text{min}} \) is enough only for a crude description of the galaxy disk structure but is quite enough for a dimensions of SSP (Single Stellar Population) in chemical evolution modeling. Any dynamical processes below this scale we can't describe correctly in our SPH code.

We use also the fixed gravitational smoothing length \( h_{\text{star}} = 1 \) kpc for the "star" particles. Our results show that such a value of \( N_B \approx 1\% \ N_{\text{gas}} \) provides qualitatively correct treatment of the system’s large scale evolution.

As the initial model (relevant for CDM - scenario) we took constant - density homogeneous gaseous triaxial configuration \( (M_{\text{gas}} = 10^{11} \ M_\odot) \) within the dark matter halo \( (M_{\text{halo}} = 10^{12} \ M_\odot) \). We set \( A = 100 \) kpc, \( B = 75 \) kpc and \( C = 50 \) kpc for semi-axes of system. Such a triaxial configurations are reported in cosmological simulations of the dark matter halo formation (Eisenstein & Loeb, 1995; Frenk et al., 1988; Warren et al., 1992). Initially, the centers of all particles were placed on a homogeneous grid inside this triaxial configuration. We set the smoothing parameter of CDMH: \( b_{\text{halo}} = 25 \) kpc. Such values of \( M_{\text{halo}} \) and \( b_{\text{halo}} \) are typical for CDMH in disk galaxies (Navarro et al., 1996; Navarro et al., 1997; Burkert, 1995).

The gas component was assumed to be initially cold, \( T_0 = 10^4 \) K. As we see in our calculations, the influence of chaotic motions essentially reduces the dependence of model parameters on the adopted temperature cutoff and, therefore, on the adopted form of cooling function itself.

The gas was assumed to be involved into the Hubble flow \( (H_0 = 65 \) km/s/Mpc) and into the solid - body rotation around \( z \) - axis. We added the small random components of velocities \( (\Delta |v| = 10 \) km/s) to account
for the chaotic motions of fragments. The initial velocity field was defined as:

\[ \mathbf{v}(x, y, z) = [\mathbf{\Omega}(x, y, z) \times \mathbf{r}] + H_0 \cdot \mathbf{r} + \Delta \mathbf{v}(x, y, z), \]  

(46)

where \( \mathbf{\Omega}(x, y, z) \) is the angular velocity of an initially rigidly rotating system.

The spin parameter in our numerical simulations is \( \lambda \approx 0.08 \). This parameter is defined in Peebles (1969) as:

\[ \lambda = \frac{| \mathbf{L}_0 | \cdot \sqrt{ | E_{gr}^0 | }}{G \cdot (M_{\text{gas}} + M_{\text{halo}})^{5/2}}, \]

(47)

\( \mathbf{L}_0 \) is the total initial angular momentum and \( E_{gr}^0 \) is the total initial gravitational energy of a protogalaxy. It is to be noted that for a system, in which angular momentum is acquired through the tidal torque of the surrounding matter, the standard spin parameter does not exceed \( \lambda \approx 0.11 \) (Steinmetz & Bartelmann, 1995). Moreover, its typical values range between \( \lambda \approx 0.07^{+0.04}_{-0.05} \), e.g. \( 0.02 \leq \lambda \leq 0.11 \).

3.2. DYNAMICAL MODEL

At the final time step \((t_{\text{evol}} \approx 13.0 \text{ Gyr})\) the "star" and "gas" particle distributions have dimensions typical for a disk galaxy. The radial extension is approximately 25–30 kpc. The disk height is around 1–2 kpc. The "gas" particles are located mainly within the central 5–10 kpc.

We present in Fig. 4. the density distribution of gas \( \rho(r) \). Except for the central region \((< 2 \text{ kpc})\), the gas distribution has an exponential form with radial scale length \( \approx 2.8 \text{ kpc} \).

The column density distributions of gas \( \sigma_{\text{gas}}(r) \) and stars \( \sigma_{\ast}(r) \) are presented in Fig. 5. The total column density is defined as: \( \sigma_{\text{tot}}(r) = \sigma_{\text{gas}}(r) + \sigma_{\ast}(r) \). The total column density distribution \( \sigma_{\text{tot}}(r) \) is well approximated (in the interval from 5 kpc to 15 kpc) with an exponential profile characterised with a \( \approx 3.5 \text{ kpc} \) radial scale length.

In the literature we found a large scatter of values for this radial scale length obtained for our Galaxy:

- 5.5 kpc (van der Kruit, 1986),
- 3.0 kpc (Eaton et al., 1984),
- 2.5 kpc (Lewis & Freeman, 1989),
- 2.0 kpc (Jones et al., 1981),
- 1.0 kpc (Mikami & Ishida, 1981),
- 2.1 ± 0.3 kpc (Porcel et al., 1998),
- 2.3 ± 0.1 kpc (Ruphy et al., 1996),
Our value is very close with a recently reported value: 3.5 kpc (Mera et al., 1998b).

The value of $\sigma_{\text{tot}} \approx 55 \, M_\odot/\text{pc}^2$ near the location of the Sun ($r \approx 9 \, \text{kpc}$) is very close to a recent determination of the total density $52 \pm 13 \, M_\odot/\text{pc}^2$ (Mera et al., 1998a).

We present in Fig. 6. the rotational velocity distribution of gas $V_{\text{rot}}(r)$, at the end of calculation. In the figure we show the rotational curve for our Galaxy (Vallee, 1994). As we can see from the figure, the modelled disk galaxy rotation velocities are very close to our Galaxy rotation curve.

The gaseous radial $V_{\text{rad}}(r)$ and normal $V_z(r)$ velocity distributions we present in Fig. 7. and Fig. 8. The radial velocity dispersion has a maximum value $\approx 60 \, \text{km/s}$ in the center (this high value is mainly caused by the central strong bar structure). Near the Sun this dispersion drops down to $\approx 20 \, \text{km/s}$. Such radial dispersion is reported in the kinematics study of the stellar motions in the solar neighborhood (Bienayme, 1998). The normal dispersion is near $\approx 20 \, \text{km/s}$ in the whole disk. This value also coincides with the vertical dispersion velocity near the Sun (Bienayme, 1998).

We present in Fig. 9. the temperature distribution of gas $T(r)$. As we can see from the figure, the distribution of $T(r)$ has a very large scatter from $10^4 \, \text{K}$ to $10^6 \, \text{K}$. Because, in our calculation we set the cutoff temperature for the cooling function at $10^4 \, \text{K}$, the gas can’t cool to lower temperatures. The most ($\approx 90 \, \%$) of the gaseous mass in the galaxy have a temperatures $\approx 10^4 \, \text{K}$. The modeled process of SNII explosions injects to the gas a great amount of thermal energy and generates a very large temperature scatter. Such a temperature scatter is typical for our Galaxy’s ISM.

Therefore, as we can see, even our crude numerical approximation gives a good fit for all dynamical and thermal distributions of gas and stars in a typical disk galaxy like our Galaxy.

3.3. CHEMICAL CHARACTERISTICS

We present in Fig. 10. the time evolution of the SFR in galaxy $SFR(t) = dM_*(t)/dt$. About $\approx 90\%$ of gas is converted into stars at the end of calculation. The most intensive SF burst happened in the first $\approx 1 \, \text{Gyr}$, with a maximal SFR $\approx 35 \, M_\odot/\text{yr}$. After $\approx 1.5 - 2 \, \text{Gyr}$ the SFR is coming down like an “exponential function”. The SFR has a value $\approx 1 \, M_\odot/\text{yr}$ at the end of the simulation.

To check the SF and chemical enrichment algorithm in our SPH code, we use the chemical characteristics of the disk in the "solar" cylinder ($8 \, \text{kpc} < r < 10 \, \text{kpc}$).
Figure 4. \( \rho(r) \). The density distribution of gas in the final step

Figure 5. \( \sigma_{\text{gas}}(r) \), \( \sigma_*(r) \) and \( \sigma_{\text{tot}}(r) = \sigma_{\text{gas}}(r) + \sigma_*(r) \). The column density distribution in the disk of gas and stars in the final step
Figure 6. $V_{\text{rot}}(r)$. The rotational velocity distribution of gas in the final step.

Figure 7. $V_{\text{rad}}(r)$. The radial velocity distribution of gas in the final step.
Figure 8. $V_z(r)$. The perpendicular to disk normal velocity distribution of gas in the final step.

Figure 9. $T(r)$. The temperature distribution of gas in the final step.
The age - metalicity relation of the "star" particles in the "solar" cylinder, \([\text{Fe}/\text{H}](t)\), we present in Fig. 11. The observational data in this figure are taken from Meusinger et al. (1991) and Edvardsson et al. (1993).

We presented in Fig. 12. the metalicity distribution of the "star" particles in the "solar" cylinder \(N_\ast([\text{Fe}/\text{H}])\). The model data are scaled to the observed number of stars (Edvardsson et al., 1993).

The \([\text{O}/\text{Fe}]\) vs. \([\text{Fe}/\text{H}]\) distribution of the "star" particles in the "solar" cylinder we presented in Fig. 13. In this figure we also present the observational data from Edvardsson et al. (1993) and Tomkin et al. (1992).

All these model distributions are in good agreement, not only with presented observational data, but also with other data collected from Portinari et al. (1997).

The \([\text{O}/\text{H}]\) radial distribution \([\text{O}/\text{H}](r)\) we present in Fig. 14. The approximation presented in the figure is obtained by a least - squares linear fit. At distances \(5 \text{ kpc} < r < 11 \text{ kpc}\) the model radial abundance gradient is \(-0.06 \text{ dex/kpc}\). In the literature we found different values of this gradient defined in objects of different types. From observations of HII regions (Peimbert, 1979; Shaver et al., 1983) we obtained that the oxygen radial gradient is \(-0.07 \text{ dex/kpc}\). From observations of PN of different types (Maciel & Koppen, 1994) we obtained the values: \(-0.03 \text{ dex/kpc}\) for PNI, \(-0.069 \text{ dex/kpc}\) for PNII, \(-0.058 \text{ dex/kpc}\) for PNIII, \(-0.062 \text{ dex/kpc}\) for PNIIa, \(-0.057 \text{ dex/kpc}\) for PNIIb. Therefore our model agrees well with the oxygen radial gradient in the our Galaxy.

3.4. CONCLUSION

This simple model provides a reasonable, self - consistent picture of the process of galaxy formation and star formation in the galaxy. Our calculations show that even a relatively small number of "gas" particles with physically motivated star formation criteria can reproduce the most of observed dynamical and chemical characteristic in the galactic disks. The dynamical and chemical evolution of the modeled disk - like galaxy is coincident with the results of observations for our own Galaxy. The results of our modeling give a good base for a wide use of the proposed SF and chemical enrichment algorithm in other SPH simulations.

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Figure 10. \( SFR(t) = \frac{dM_*(t)}{dt} \). The time evolution of the SFR in galaxy

Figure 11. \([\text{Fe/H}] (t)\). The age metalicity relation of the "star" particles in the "solar" cylinder (8 kpc < \( r < 10 \) kpc)
Figure 12. $N([\text{Fe}/\text{H}])$. The metallicity distribution of the "star" particles in the "solar" cylinder (8 kpc < $r$ < 10 kpc)

Figure 13. The $[\text{O}/\text{Fe}]$ vs. $[\text{Fe}/\text{H}]$ distribution of the "star" particles in the "solar" cylinder (8 kpc < $r$ < 10 kpc)
Figure 14. [O/H](r). The [O/H] radial distribution

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