Dynamical Phase Transitions in the Quantum Dimer Model on a Square Lattice

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We consider the quench dynamics of a two-dimensional (2D) constrained quantum dimer model and determine its rich dynamical phase diagram. By means of exact diagonalization on systems of sizes up to $8 \times 8$, we show that order parameters generically relax to their thermal expectation values. This allows us to interpret the far-from-equilibrium dynamics in terms of the underlying equilibrium phase transitions consisting of a BKT-transition between a columnar ordered valence bond solid (VBS) and a valence bond liquid (VBL), as well as a first order transition between a staggered VBS and the VBL. For quenches across the BKT transition, the Loschmidt rate develops non-analyticities at the zero-crossings of the order parameter, fixed by microscopic model parameters. By contrast, the relaxation time across the first order transition scales linearly with the correlation length of the initial state, preventing the formation of sharp kinks for infinitely long-ranged correlations.

Within the staggered VBS, some local observables even fail to thermalize as a result of the kinematic constraints of the quantum dimer model.

**Introduction.**— Recent experimental progress on realizing and controlling synthetic quantum matter has granted new access to the rich nature of many-body dynamics and permits the exploration of foundational aspects of non-equilibrium statistical physics [1, 2]. Far-from equilibrium states of strongly interacting and non-integrable quantum many-body systems are generally believed to quickly relax to a local thermal equilibrium at infinite temperature. By contrast, a recent experiment with one-dimensional ultracold atoms in the Rydberg blockade regime has found that certain highly-excited states feature long-lived coherent oscillations of local observables [3]. Due to the Rydberg blockade [3–10], a constrained quantum many-body system is realized in which two excitations are forbidden to occupy neighboring lattice sites. Theoretical works have proposed a set of exceptional eigenstates, entitled “quantum many-body scars”, and nearby integrable points to be responsible for the exotic quantum dynamics [11–17].

The 1D Rydberg chain admits a direct mapping to yet another constrained model: the close-packed dimer coverings on a two-rung ladder [18]. Here, we consider the far-from-equilibrium quantum dynamics of a quantum dimer model (QDM) in two spatial dimensions, where previous work has conjectured a connection to the slow dynamics of glassy systems and failure of ergodicity [19]. Our main focus is to determine the rich dynamical phase diagram upon quenching the model far from thermal equilibrium. The restricted Hilbert space of the 2D QDM allows us to study new types of dynamical phase transitions (DPTs) [20–25], that thus far are mainly confined to numerical simulations in one-dimensional systems (see however Ref. 26). The QDM on a square lattice possesses particularly exotic properties including a continuous (BKT) as well as a discontinuous 1st-order finite temperature phase transition [27–33]. In this paper, we therefore seek to provide direct interpretation to the multifaceted dynamics of the QDM following a quench by connecting it to its phase diagram, see Fig. 1 for an overview.

**Model.**— We start by introducing the QDM on the square lattice, where a hard-core constraint enforces each site to be occupied by exactly one singlet dimer, see insets in Fig. 1 for the illustration of a few dimer configurations. The dynamics is generated by the following Hamiltonian, originally introduced by Rokhsar and Kivelson (RK) [34],

$$
\hat{H} = \hat{H}_V + \hat{H}_J \\
\hat{H}_V = V \sum_{\text{plaq.}} \left( \left| \begin{array}{c} 1 \end{array} \right\rangle \left\langle 1 \right| + \left| \begin{array}{c} 2 \end{array} \right\rangle \left\langle 2 \right| \right) \\
\hat{H}_J = -J \sum_{\text{plaq.}} \left( \left| \begin{array}{c} 1 \end{array} \right\rangle \left\langle 1 \right| + \left| \begin{array}{c} 2 \end{array} \right\rangle \left\langle 2 \right| \right)
$$

Here, $\hat{H}_V$ gives a constant energy-offset to each pair of parallel dimers, while the off-diagonal kinetic term $\hat{H}_J$ flips a pair of resonant singlets. Importantly, the dimer model features non-local conservation laws represented by winding numbers $W_x$ and $W_y$, which provide a staggered count of the number of
dimers intersecting a given straight line through the sample \[35\]. For the remainder of this work, we restrict to the zero-winding sector \(W_c = W_r = 0\), which constitutes the largest part of the full Hilbert space.

The nature of the equilibrium phase diagram of the square-lattice QDM remains a matter of high interest, but seems to converge to a framework similar to the one depicted in Fig.1 [27–33]. In particular, for \(T = 0\), the model possesses a RK-point at \(V = J\), where the exact ground state wave function can be constructed as an equal weight superposition of all dimer coverings within each winding number sector \[34\]. The RK-point separates two crystalline VBS phases, that show columnar order for \(V < J\) and staggered order for \(V > J\). Both VBS phases are expected to extend up to certain finite temperatures, before the transition to a disordered VBL phase, which is conjectured to extend all the way to the RK-point at \(T = 0\) [32, 36]. Most notably for our purposes, while the columnar-VBL - transition is expected to be of BKT-form, and thus continuous, the staggered-VBL - transition is of 1st-order [28, 30].

The BKT-transition can be captured by introducing an order parameter that detects the spontaneous breaking of \(C_4\)-rotational lattice symmetry,

\[
\hat{\phi}_c = \frac{2}{L^2} \sum_{\text{plaq.}} \left( | \hat{\psi}_{\uparrow\uparrow} \rangle \langle \hat{\psi}_{\downarrow\downarrow} | - | \hat{\psi}_{\downarrow\uparrow} \rangle \langle \hat{\psi}_{\uparrow\downarrow} | \right),
\]

which counts the imbalance between horizontal and vertical plaquettes on an \(L \times L\) square lattice. Restricting to the zero momentum sector on periodic boundary conditions (PBCs), \(\hat{\phi}_c\) distinguishes the two existing, translational invariant columnar ground states \(|c_A\rangle\) and \(|c_B\rangle\), related by a \(\pi/2\)-rotation and \(\hat{\phi}_c \langle c_A/B \rangle = \pm \langle c_A/B \rangle\). In contrast, concerning the 1st order transition, the fully staggered state is not part of the zero-winding sector and is dynamically completely frozen. We thus construct the exact groundstate in the limit \(V/J \rightarrow \infty\) within the zero-winding sector, which yields a state of “pyramid”-like shape, where the tip of the pyramid serves as a dynamically active domain wall between extended areas of staggered configurations, see Fig.1 inset. In the thermodynamic limit on PBCs, the staggered ground state contains four such pyramidal domain walls [33]. For a quench from the staggered phase to the disordered regime however, the dynamics will mainly be governed by the behavior of individual pyramid states, which we can effectively capture by considering a single pyramid on open boundary conditions (OBCs). This choice allows us to double the length scale \(\xi\) of the initial state, given our finite size limitations. On an OBC-geometry, there exist two pyramidal ground states \(|p_A\rangle\) and \(|p_B\rangle\), from which we construct a \(\mathbb{Z}_2\) order parameter to distinguish between them:

\[
\hat{\phi}_s := \frac{2}{L^2} \left\{ \sum_{l_A} \hat{n}_{l_A} - \sum_{l_B} \hat{n}_{l_B} \right\}.
\]

Here, the indices \(l_A\) denote the bonds occupied in the \(|p_A\rangle\)-state, \(\hat{n}_{l_A} = |p_A\rangle\langle p_A|\), and correspondingly for \(l_B\), such that again \(\hat{\phi}_s \langle p_{A/B} \rangle = \pm \langle p_{A/B} \rangle\). In a given quench protocol starting from e.g. \(|p_A\rangle\), the computation of \(\langle \hat{\phi}_s(t) \rangle\) thus involves the memory-function of the initial state, and (non-)relaxation of the same is intuitively connected to spontaneous symmetry breaking. Even though the Hilbert space is somewhat restricted on OBCs, we still expect the dynamics of the 1st-order transition to be well captured in our approach, as quenches within the VBS phase should induce dynamics only within a pyramid. However, once the complete pyramid can be melted by quenching across the phase boundary, the system will disorder even on OBCs.

**Thermalization.** – To study thermalization in our system, we compute the relaxation of order parameters in a quench protocol to their corresponding thermal expectation values. We thus consider the long-time averaged values

\[
\langle \hat{O} \rangle_q := \lim_{t \to \infty} \frac{1}{T} \int_0^T dt \langle \psi(t) | \hat{O} | \psi(t) \rangle
\]

of a given observable \(\hat{O}\) following a quench from an initial state \(|\psi(t = 0)\rangle = |\psi_0\rangle\) on systems of linear size \(L \in \{4, 6, 8\}\) and adopt the following convention regarding the choice of \(|\psi_0\rangle\): For quenches to final values of \(V \leq 0\), the initial state is chosen as a columnar state on PBCs, \(|\psi_0\rangle = |c_A\rangle\), while quenches to \(V \geq 0\) start from a maximally staggered state on OBCs, \(|\psi_0\rangle = |p_A\rangle\). The averages of Eq.(4) are to be com-

![Figure 2. Thermalization and phase transitions in the QDM.](image-url)
pared with the corresponding thermal expectation values

$$\langle \hat{O} \rangle_\beta = \text{Tr} \left\{ e^{-\beta \hat{H}} \hat{O} \right\}, \quad \langle \hat{H} \rangle_\beta = \langle \psi_0 | \hat{H} | \psi_0 \rangle,$$

(5)

where the effective inverse temperature $\beta$ is chosen to match the energy of the initial state. For $L = 8$, the system cannot be diagonalized fully, so we use the typicality approach [37] where the expectation values of Eq. (5) are drawn from random (infinite temperature) initial states, which are subsequently evolved in imaginary time up to $\tau = \beta$ such that Eq. (5) is fulfilled.

As displayed in Fig. 2 (a+b), both columnar and staggered order parameters show efficient relaxation to thermal values, less accurate only in the vicinity of the arising phase transitions. The thermalization of order parameters allows for the determination of finite-temperature phase transitions via finite size scaling arguments, as has been done for ground state phases of the model [27]. Even though the exact transition point between columnar and disordered phase may turn out quite inaccurate in such small systems [27, 32], the qualitative picture is expected to hold, nonetheless. On the left hand part of the phase diagram, $V < 0$, the Binder cumulant $U_B \propto 1 - \langle \hat{\phi}_c^4 \rangle_\beta / 3 \langle \hat{\phi}_c^2 \rangle_\beta^2$ shows a crossing at $V \approx -3.1$, which signals the transition to the $C_4$-symmetry-breaking columnar phase. For $V > 0$, order parameter dynamics shows signals of a first-order transition very close to the RK-point at $V = J$, with a sharp rise of $\langle \hat{\phi}_c^2 \rangle_\beta$ emerging for larger systems. Time-averages were computed up to $tJ = 300$ on $L = 8$, which supports a quick order parameter thermalization for both initial states in a wide range of model parameters. Notably, while in Ref. 19 memory functions of certain initial states at $V > J$ showed relaxation at late times accessible on $L = 6$, here the squared order parameter $\langle \hat{\phi}_c^2(t) \rangle$, connected to such memory functions via Eq. (3), goes to finite values quickly. Examining $\langle \hat{\phi}_c(t) \rangle$ at $V > J$, we find oscillations between positive and negative extremal values of $\hat{\phi}_c$ on large timescales, the system thus oscillates slowly between the two $C_4$-related staggered ground states, as the symmetry is not broken explicitly.

While both order parameters $\hat{\phi}_c, \hat{\phi}_s$ relax to thermal values fast even deep within the VBS phases, a more diverse picture emerges for the relaxation of local observables. For this purpose, we define the dimer-dimer correlation functions $C(r)$ as

$$C(r) = \langle \hat{n}_i \hat{n}_j \rangle - \langle \hat{n}_i \rangle \langle \hat{n}_j \rangle,$$

(6)

where $\hat{n}_i$ is the dimer occupation number at bond $r$. Quenches from $|c_A\rangle$ even deep inside the columnar phase are accompanied by fast relaxation of $C(r)$, save for spontaneous symmetry breaking. The latter can be accounted for by choosing a rotationally invariant initial state $|\psi_0\rangle = \frac{1}{\sqrt{2}}(|c_A\rangle + |c_B\rangle)$, which leads to the results of Fig. 2 (b). Inside the staggered phase, absence of thermalization of local observables can be demonstrated most illustratively by mapping out the potential energy landscape, i.e., the plaquette density, following a quench. Here, we start from a symmetrized staggered state $|\psi_0\rangle = \frac{1}{\sqrt{2}}(|p_A\rangle + |p_B\rangle)$, quenching to $V = 3$. We find the thermal potential energy landscape to include delocalized plaquettes along the system diagonals, Fig. 3 (b). In sharp contrast, the distribution of plaquettes following the quench, averaged up to $tJ = 300$ on $L = 8$, remains localized around the center, see Fig. 3 (c). An analogous analysis on $L = 6$, where the system may be diagonalized fully and arbitrarily long timescales become accessible, also shows no signs of thermalization at late times. We are thus led to the conclusion that the dynamics of local observables is “frozen” inside the staggered VBS.

To understand this property, we can consider the large-$V/J$ dynamics around, say, $|p_A\rangle$ as an effective single-particle problem on a finite, 1D lattice in a potential $U_i$, where $U_0 = V, U_1 = 3V$, and $U_{i\geq 2} = 4V$. Here, $i = 0$ corresponds to $|p_A\rangle, i = 1$ labels the second state of Fig. 3 (a) with three flippable plaquettes, and all states with $i \geq 2$ have four plaquettes. It can then be shown that for sufficiently large $V/J$, the large overlap of the initial state $|\psi(0)\rangle = |i = 0\rangle$ with the bound states of the effective potential well located around $i = 0$ defies global eigenstate thermalization and leads to an enhanced localization of the long-time averaged state $|\psi(t)\rangle$, in good agreement with the results of Fig. 3. Similar effects can be observed in the dynamics of columnar states containing string-like excitations, composed of finite columns aligned perpendicular to the background state. These states are strongly repulsively bound, leading to large relaxation time scales. In either case, the arising slow/non-thermalizing dynamics due to effective trapping potentials may signal a connection to the confining nature of the emergent $U(1)$-gauge field on both sides of the RK point.

**Dynamical Phase Transitions.** With the presence of the two transitions being established, we proceed to characterize...
and compare them. A hallmark in the study of DPTs is the emergence of non-analytic, kink-like structures in the return amplitude to the original ground state manifold following a quench, the Loschmidt rate, defined as

$$\lambda(t) = -\frac{2}{L^2} \log \left( \sum_{n \in [g]} \left| \langle n | \phi(t) \rangle \right|^2 \right). \tag{7}$$

Here, the manifold \([g]\) consists of \(|c_A\rangle, |c_B\rangle\) for the BKT, and \(|p_A\rangle, |p_B\rangle\) for the 1\(^{\text{st}}\)-order transition. The behavior of \(\lambda(t)\) upon quenching across the BKT-transition is shown in Fig. 4(a), where an initial columnar state is taken to the VBL phase at \(V = 0\). The time evolution of \(\phi_s(t)\) has already converged reasonably well for \(L = 6, 8\), while \(\lambda(t)\) exhibits significant finite size fluctuations which dominate the Loschmidt rate at late times. This is evidenced by the suppressed magnitude of oscillations at late times on \(L = 8\) as compared to \(L = 6\). Nonetheless, there exist two systematic crossings of the individual ground state weights \(\lambda_{A/B}(t) = -2/L^2 \log \left| \langle c_{A/B} | \phi(t) \rangle \right|^2\), at which sharp kinks are expected to form for \(L \rightarrow \infty\). The critical times of the kinks are in rough agreement with the zeros of the order parameter \(\phi_s(t)\), Fig. 4(b).

An analogous analysis carried out for the 1\(^{\text{st}}\) order DPT yields vastly different results, shown in Fig. 4(c), where the rate \(\lambda(t)\) is given for \(L = 4, 6, 8\), in a quench of \(|p_A\rangle\) to \(V = 0\). Here, we can identify a number of noteworthy observations: First, the sharp features visible in \(\lambda(t)\), marked by black stars in Fig. 4(c), correspond to resonances specific to the point \(V = 0\). They can be understood in a simplified picture of the pyramid tip as a single plaquette embedded in a staggered, and thus anisotropic, mean field background. The dynamics of the central plaquette is then described by an effective two-level Hamiltonian \(H_{\text{eff}} \propto -J \sigma_x + V \sigma_z\), which, for \(V \neq 0\), sustains a finite population of the initial ground state at any time. For \(V = 0\), the background becomes isotropic and the central plaquette undergoes coherent Rabi-oscillations, which depopulate the ground state with period \(\pi/J\), roughly corresponding to the separation of the black stars in Fig. 4(c). The temporal positions of the resonances are similar on all system sizes \(L\), which indicates that the short-time dynamics following the quench is dominated by only small regions around the central plaquette. This characterizes the melting dynamics proliferating around the dynamical center of the initial state, which successively has to work its way to the outside.

The scaling of this process is revealed by the dynamical order parameter \(\phi_s(t)\) in Fig. 4(d), which shows an approximate collapse of the zero-crossings and extrema of \(\phi_s(t)\) upon rescaling \(t \rightarrow t/L\). Therefore, considering an initial state not composed of a single pyramid covering the lattice, but rather multiple pyramids of average size \(\xi\), corresponding to the staggered correlation length, the timescale \(\tau_{eq}\) of thermalization is effectively set by \(\xi\). Relating to the phase diagram of Fig. 1, this corresponds to a quench across the transition not from the ground state at \(V/J \rightarrow \infty\), but an initial state with energy corresponding to a finite-\(T\) state at a finite ratio \(V/J\).

Finally, letting \(L \rightarrow \infty\) and \(\xi \rightarrow \infty\), the \(\propto L\)-scaling of the relaxation dynamics also shifts the crossing of \(\lambda_{A/B}(t)\) to increasingly late times. Hence, while \(L \rightarrow \infty\) can be shown to yield \(\lambda(t) = \min(\lambda_A(t), \lambda_B(t))\), thus developing sharp features, the angle at which \(\lambda_{A/B}(t)\) cross becomes increasingly flat. Assuming a corresponding \(\propto 1/L\)-scaling of this angle, a simple scaling analysis for \(\lambda(t)\) shows the cancellation of both effects and no discontinuity develops in \(\partial_t \lambda(t)\) for \(\xi \sim L \rightarrow \infty\).

**Conclusion & Outlook.** In this work, we have examined the dynamical phase diagram of the square lattice quantum dimer model and established the presence and nature of the

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**Figure 4. Dynamical Phase Transitions.** (a) Loschmidt rate \(\lambda(t)\) for \(L = 6, 8\) following a quench from of a columnar product state to \(V = 0\). The light blue dashed lines indicate the weights \(\lambda_{A/B}\) of the individual ground states \(|c_{A/B}\rangle\). (b) Columnar order parameter dynamics for the same quench. The dashed lines mark the zero-crossings of \(\phi_s(t)\) and kinks of \(\lambda(t)\), respectively. (c) shows \(\lambda(t)\) for \(L = 4, 6, 8\) starting from a staggered initial state, with light dashed lines corresponding to the weights \(\lambda_{A/B}(t)\) of \(|p_{A/B}\rangle\) in the vicinity of their crossing points, marked by arrows. Black stars mark resonances. (d) shows \(t/L\)-scaling of the staggered order parameter dynamics.
arising continuous and 1\textsuperscript{st}-order dynamical phase transitions. Underlying these observations, we found a qualitatively distinct melting of crystalline order for quenches from either columnar or staggered valence bond solid phases to a disordered regime. The associated emergence of a scale \( \xi \) in the time evolution across the 1\textsuperscript{st}-order transition has a drastic impact on the dynamics of order parameter and Loschmidt-echo as signatures of dynamical phase transitions.

Future lines of investigation may address the existence of quantum-many body scars in two dimensions. In particular, determining the dimensional crossover from the two-leg ladder to the fully two-dimensional quantum dimer model may provide further insights. Moreover, additional studies on the effects of excitations on top of the initial states could yield further insights on the dynamical phase diagram of Fig. 1. Interesting questions on the dynamical nature of dimerized systems also naturally extend to the various manifestations of the quantum dimer model on different lattice geometries. In particular, the presence of local constraints permits to study various types of emergent dynamical gauge fields besides the \( U(1) \)-liquid of the square lattice. Direct connections between constrained models in dimensions larger than one and the controllable dynamics of Rydberg-blockaded atoms would thus be much desirable, as they could possibly open up new roads to the experimental investigation of fundamental matters in nonequilibrium quantum physics.

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