Observational Constraints on Dark Radiation in Brane Cosmology

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We analyze the observational constraints on brane-world cosmology whereby the universe is described as a three-brane embedded in a five-dimensional anti-de Sitter space. In this brane-universe cosmology, the Friedmann equation is modified by the appearance of extra terms which derive from the existence of the extra dimensions. In the present work we concentrate on the “dark radiation” term which diminishes with cosmic scale factor as $a^{-4}$. We show that, although the observational constraints from primordial abundances allow only a small contribution when this term is positive, a much wider range of negative values is allowed. Furthermore, such a negative contribution can reconcile the tension between the observed primordial $^4$He and D abundances. We also discuss the possible constraints on this term from the power spectrum of CMB anisotropies in the limit of negligible cosmological perturbation on the brane world. We show that BBN limits the possible contribution from dark radiation just before the $e^+e^-$ annihilation epoch to lie between $-123\%$ and $+11\%$ of the background photon energy density. Combining this with the CMB constraint reduces this range to between $-41\%$ and $+10.5\%$ at the $2\sigma$ confidence level.

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I. INTRODUCTION

Brane-world cosmology is of considerable current interest. In such scenarios, our universe is a submanifold embedded in a higher-dimensional spacetime. Physical matter fields are confined to this submanifold, while gravity can reside in the higher-dimensional spacetime. This paradigm was first proposed \cite{1,2} as a means to reconcile the mismatch between of the scales of particle physics and gravity. It lowers the scale of gravity to the weak scale. This compactification is an alternative to the standard Kaluza-Klein (KK) scheme. In their model, our universe is described as a three-brane embedded in a five-dimensional anti-de Sitter space $AdS_5$ (the bulk). This guarantees the usual 4-dimensional Newtonian limit in our brane world.

The cosmological evolution of such brane universes has been extensively investigated. Exact solutions have been found by several authors \cite{3,4,5,6,7,8,9}. These solutions reduce to a generalized Friedmann equation on our brane which can be written as

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3} \rho - \frac{K}{a^2} + \frac{\Lambda_4}{3} + \frac{\kappa_5^4}{36} \rho^2 + \frac{\mu}{a^4}. \quad (1)$$

Here, $a(t)$ is the scale factor at cosmic time $t$, and $\rho$ is the total energy density of matter on our brane.

In equation (1), several identifications of cosmological parameters were required in order to recover standard big-bang cosmology. For one, the first term on the right hand side is obtained by relating the four-dimensional gravitational constant $G_N$ to the five-dimensional gravitational constant, $\kappa_5$. Specifically,

$$G_N = \frac{\kappa_5^4}{48\pi}, \quad (2)$$

where $\lambda$ is the intrinsic tension of the brane and $\kappa_5^2 = M_5^{-3}$, where $M_5$ is the five dimensional Planck mass. Secondly, the four-dimensional cosmological constant $\Lambda_4$ is related to its five-dimensional counterpart $\Lambda_5$,

$$\Lambda_4 = \frac{\kappa_5^4 \lambda^2}{12} + 3 \Lambda_5 / 4. \quad (3)$$

$\Lambda_5$ should be negative in order for $\Lambda_4$ to obtain its presently observed small value.

Standard big-bang cosmology does not contain the fourth and fifth terms of Eq. (1). The fourth term arises from the imposition of a junction condition for the scale factor on the surface of the brane. Physically, it derives from a singular behavior in the energy-momentum tensor which originates in the fact that physical matter fields are confined to the brane. This $\rho^2$ term would decay rapidly as $a^{-8}$ in the early radiation dominated universe. Hence, it is not likely to be significant during the later nucleosynthesis and photon decoupling epochs of interest here.

The fifth term, however, is of considerable interest for the present discussion. It scales just like radiation with a constant $\mu$. Hence, it is called the dark radiation. This term derives from the electric (Coulomb) part of the five-dimensional Weyl tensor $\lambda$. The coefficient
\( \mu \) is a constant of integration obtained by integrating the five-dimensional Einstein equations \[1, 2, 3, 4, 5, 6, 7, 8, 9\]. Both positive and negative \( \mu \) are possible mathematically. Its magnitude and sign can depend on the choice of initial conditions when solving the five-dimensional Einstein equation. Hence, even the sign of \( \mu \) remains an open question \[11\].

Dark radiation should strongly affect both big-bang nucleosynthesis (BBN) and the cosmic microwave background (CMB). Hence, such observations can be used to constrain both the magnitude and sign of the dark radiation. A brief analysis of this was made in literature \[5\]. In the present work, we seek to explore these constraints in more detail utilizing the most recent light-element abundance data sets and the latest combined CMB power spectrum data sets.

## II. BBN CONSTRAINT

The observed primordial light-element abundances constrain the conditions during the BBN epoch from the time of weak reaction freezeout \((t \sim 1 \text{ sec}, T \sim 1 \text{ MeV})\) to the freezeout of nuclear reactions \((t \sim 10^4 \text{ sec}, T \sim 10 \text{ keV})\). The present status the observational constraints have been reviewed in a number of papers (cf. \[12, 22\]). The primordial helium abundance is obtained by measuring extragalactic HII regions in low-metallicity irregular galaxies. The primordial helium abundance \(Y_p\) so deduced tends to reside in one of two possible values (a low value \(Y_p \approx 0.230, \[12\]\) and a high value \(Y_p \approx 0.245, \[13\]\). In view of this controversy, we adopt a conservative range for the primordial \(^4\)He abundance:

\[
0.226 \leq Y_p \leq 0.247. \tag{4}
\]

Primordial deuterium is best determined from its absorption line in high redshift Lyman \(\alpha\) clouds along the line of sight to background quasars. For deuterium there is a similar possibility for either a high or low value. For the present discussion, however, we shall adopt the generally accepted low value for \(D/H\) \[24, 21\].

\[
2.9 \times 10^{-5} \leq D/H \leq 4.0 \times 10^{-5} \tag{5}
\]

The primordial lithium abundance is generally inferred from old low-metallicity halo stars. Such stars exhibit an approximately constant (“Spite plateau”) lithium abundance as a function of surface temperature which is taken to be the primordial abundance. There is, however, some controversy \[22\] concerning the depletion of \(^7\)Li on the surface of such halo stars. If destruction has occurred, the true primordial \(^7\)Li abundance is higher than the plateau value. For the present purposes, therefore, we adopt a conservative \(^7\)Li abundance constraint:

\[
1.67 \times 10^{-10} \leq ^7\text{Li}/H \leq 4.75 \times 10^{-10}. \tag{6}
\]

The constraints on positive extra energy density during the BBN epoch based upon primordial light-element abundances have been recently studied by many authors in context of numbers of neutrino families, lepton asymmetry, or dark energy (cf. \[14, 15, 16\]). The main effect of such additional background energy density is to increase the universal expansion rate. This causes the neutron to proton ratio to be larger because the weak reactions freeze out at a higher temperature and because there is less time for neutrons to decay between the time of weak-reaction freezeout and the onset of BBN. Consequently, adding excess energy density during BBN yields a larger \(^4\)He abundance since most of the free neutrons are converted into \(^4\)He nuclei. \(D/H\) also increases largely because the reactions destroying deuterium fall out of nuclear statistical equilibrium while the deuterium abundance is higher \[15\]. Similarly, there is less time for the destructive reaction \(^7\)Li\((p, \alpha)^4\)He. This causes \(^7\)Li to be more abundant for \(\eta \leq 3 \times 10^{-10}\). However, there is also less time for the \(^4\)He\((^3\)He, \(\gamma)^7\)Be reaction to occur. This causes \(^7\)Li to be less abundant for \(\eta \geq 3 \times 10^{-10}\). On the other hand, when the extra energy component is negative (i.e. negative dark radiation), the opposite results occur.

Figure 2 illustrates the dependence of the nucleosynthesis yields with the dark radiation content. In the following, we will quote the dark radiation content as a fraction of the background photon energy density just before and the onset of the \(e^+e^-\) annihilation and BBN epochs. We have included the current 2\(\sigma\) uncertainties \[19\] arising from the input nuclear reaction rates in our present analysis of the BBN model predictions.

For dark radiation in the range of 0 to +11% of the background photon energy density, the cosmological bounds on the baryon-to-photon ratio \(\eta\) come from the \(^4\)He upper bound and the \(D/H\) upper bound. With negative dark radiation the allowed range for \(\eta\) expands because the \(^4\)He mass fraction and \(D/H\) have opposite dependences on \(\eta\). The addition of more than 2% negative dark radiation reduces the expansion rate and the helium abundance sufficiently so that the adopted \(^4\)He constraint is satisfied for all values of \(\eta\) which satisfy the \(D/H\) constraint. For negative dark radiation in the range of 2% − 112% of the background photon energy density, the constraint on \(\eta\) comes only from \(D/H\) upper and lower limits. Between 112 and 123% negative dark radiation, the constraint on \(\eta\) comes from the lower bounds on \(^4\)He and \(D/H\).

Similarly, the conservative \(^7\)Li abundance constraint adopted here does not significantly constrain the dark radiation component. The shaded region in Figure 2 shows allowed values of the dark radiation fraction, \(\rho_{\text{DR}}/\rho_\gamma\), where \(\rho_\gamma\) is the total energy density in background photons just before the BBN epoch at \(T = 1\text{ MeV}\). Note, that only a small (≤ 11%) positive dark radiation contribution is allowed while substantial negative dark radiation (up to 123%) is allowed and even preferred by the BBN constraints.
III. CMB CONSTRAINT

Next we examine the possible imprint of a dark radiation on the CMB angular power spectrum. It is well known that the CMB spectrum is sensitive to many cosmological parameters which have almost no effect on BBN. For simplicity, therefore, we have fixed most cosmological parameters to their optimum values and explore the effects of varying the dark radiation content and baryon to photon ratio $\eta$. In spite of the name dark “radiation” it has no interaction (e.g., Compton scattering) with other matter fluids. Moreover, we make the further simplifying assumption that it has no intrinsic fluctuation. Cosmological perturbation theory in a five dimensional universe is now extensively under consideration [23]-[26]. Ultimately, one must take the five dimensional (geometrical) perturbative effects into account when calculating the CMB angular power spectrum. In this present paper, however, we only address the dominant effect on the background expansion rate.

We have calculated CMB power spectra using the CMBFAST code of [29]. The $\chi^2$ goodness of fit to the combined BOOMERANG [30], DASI [31], and MAXIMA-1 [32] data sets was evaluated using the widely employed offset log-normal procedure of [33]. The available experimental offsets and window functions were utilized. As a benchmark zero dark-radiation model, we have made a search for a global optimal fit to the combined CMB data set for a flat $\Omega_M + \Omega_\Lambda = 1$ cosmology (with ionization parameter $\tau = 0$). We find a best fit for $\Omega_M = 0.233$, $\Omega_\Lambda = 0.767$, $h = 0.726$, $\Omega_b h^2 = 0.0214$ ($\eta_{10} = 5.75$), $n = 0.9334$. We have also marginalized over the experimental calibration uncertainties and the COBE normalization using Gaussian priors based upon published experimental uncertainties. This fit gives a $\chi^2$ of 30.03 for 31 degrees of freedom. For the present purposes, we restrict our consideration to this parameter set as an optimum 4-dimensional standard cosmology. We then study variations in the goodness of fit as a function of the dark radiation fraction at the photon decoupling epoch and $\Omega_b h^2 = \eta_{10}/268$. In addition, the normalization was optimized for each choice of these two parameters.

The most distinguishable effects of the dark radiation is their influence on the location and amplitude of the acoustic peaks in the CMB power spectrum [28]. Adding positive dark radiation moves the epoch of matter radiation equality to a later epoch. It prevents the growth of perturbations inside the horizon and leads to a decay

FIG. 1: Light-element abundances as a function of baryon to photon ratio $\eta$. Shaded Areas or dashed lines denote $\pm 2\sigma$ uncertainties in the BBN model predictions. Plotted are model s with 0% (blue), +10.5% (red) and -41% (green) dark radiation (relative to the background photon energy density just before the $e^+e^-$ annihilation epoch). $^4$He, D and $^7$Li are shown in the top, center and bottom panels, respectively. The $^4$He abundance predictions are well separated for the three dark radiation models, while the models are barely distinguishable for D and $^7$Li. Observational constraints are indicated as horizontal lines as labeled.
in the gravitational potential. This increases the amplitude of the CMB acoustic oscillations by the integrated Sachs-Wolfe effect. The net result of adding positive dark radiation is therefore an enhanced CMB anisotropy. The opposite is true if negative dark radiation is added.

As a second feature, a more rapid expansion rate due to positive dark radiation causes the epoch of photon decoupling to occur earlier so that the horizon size is smaller at the surface of photon last scattering. Therefore the $l$-values for the acoustic peaks are slightly larger or smaller depending upon whether the dark radiation term is positive or negative.

Figure 3 illustrates effects of both positive and negative dark radiation on the CMB angular power spectrum. This shows the benchmark zero dark-radiation model together with $\pm 3\sigma$ components of dark radiation. These limits correspond to a ratio of dark radiation to photon energy density of $+24\%$ and $-35\%$ at the photon decoupling epoch. Correcting for photon heating at the pair annihilation epoch, these limits expand by a factor of $(11/3)^{4/3} = 3.85$ for the dark-radiation fraction just before the BBN epoch. Hence, these limits would be $+92\%$ and $-135\%$ of the photon energy density just before nucleosynthesis.

The final effect on the power-spectrum depends upon the normalization, and is somewhat counter intuitive. From Figure 3 we see, for example, that a fit with a dark radiation fraction of $-35\%$ of the photon energy density at the CMB epoch increases (rather than decreases) the amplitude of the first acoustic peak by $\approx 10\%$ and shifts the location of the first and third peaks to smaller $l$ values. The increase in the acoustic peak amplitudes is a result of having shifted the normalization to optimize the goodness of fit. Note, that the effects of positive or negative dark radiation are not the same as simply adding or subtracting photons. This is because dark radiation does not behave like relic photons or neutrinos. Dark radiation does not interact either gravitationally or via Compton scattering with the other matter fields. Also, in the present analysis, it does not fluctuate. Therefore the effects of dark radiation on the power spectrum are in principle distinguishable from the effects of normal electromagnetic radiation.

Figure 2 shows the contours of the dark radiation fraction and $\Omega_b h^2$ allowed by nucleosynthesis and the CMB. This shows that the combined nucleosynthesis and CMB constraints severely limit the possible sign and amplitude of the dark radiation. The combined $2\sigma$ 95% confidence limit from the concordance of both constraints corresponds to $-41\% \leq \rho_{DR}/\rho_{γ} \leq +10.5\%$ for $4.73 \leq \eta_0 \leq 5.56$ (or $0.0176 \leq \Omega_b h^2 \leq 0.0207$).

IV. DISCUSSION

We have considered the cosmological constraints on the magnitude and sign of the dark radiation term of the brane-world generalized Friedmann equation (1). If the sign of the dark radiation is positive then it behaves like additional relativistic particles and enhances the expansion rate. This kind of effect has been recently well studied in the context, for example, of additional neutrino flavors or degeneracy and is tightly constrained. We have reexamined this effect for both positive and negative dark energy. We include the nuclear reaction uncertainties in the BBN model predictions. For positive dark radiation the observational upper bound for primordial $^4$He and $\mathrm{D}/\mathrm{H}$ allows at most $\rho_{DR}/\rho_B \leq 0.03$ ($\rho_{DR}/\rho_γ \leq 0.16$) at the BBN epoch. This limit is consistent with the estimate (on thermally generated dark radiation due to bulk graviton production described in [11]).

Such extra energy also affects the power spectrum of CMB fluctuations, but it is too small to be constrained by current CMB measurements. We therefore conclude that BBN places the most stringent constraint on positive dark radiation.

On the other hand, a wider range of dark radiation density relative to the background photon energy density is allowed in the case of negative dark radiation. We deduce an absolute BBN upper limit of 123% negative dark radiation. This maximal value is allowed for $\eta \approx 5.09 \times 10^{-10}$ (or $\Omega_b h^2 \approx 0.019$). This $\eta$ value, however, is $1\sigma$ less than the values consistent with the combined BOOMERANG, DASI, and MAXIMA-1 data sets. For the combined CMB and BBN analysis, we deduce that only a maximum of 41% contribution of negative dark radiation is allowed at the 95% confidence level.

We should, however, point out several caveats to the present work. One is that if one wishes to avoid a naked singularity in the bulk dimension, then there is a relation between the curvature $K$ and the dark radiation $\mu$ when the sign of $\mu$ is negative,

$$\mu \geq -\frac{K^2 l^2}{4}, \text{ for } \mu < 0,$$

where $l$ is the five dimensional curvature length scale which relates to the five dimensional cosmological con-
stant $$\Lambda_5 \equiv -4/l^2$$. If one accepts this cosmic censureship hypothesis, then only minuscule quantities of negative dark radiation are allowed if one wishes to maintain the five dimensional Planck mass above the TeV scale. Hence, the present limits on negative dark radiation only apply if one wishes to accept either a much lower value for $$M_5$$ or a naked singularity in the bulk dimension.

We note that there is an independent constraint on the five-dimensional Planck mass from the quadratic term in equation (1). If one applies the same BBN constraint as the five dimensional Planck mass above the TeV scale. Therefore, we have ignored these perturbations and concentrated on the zeroth order effect of “dark radiation” which appears in equation (1).

Finally we note that although the the acoustic peaks are very useful indicators of the dark radiation, they are also sensitive to other cosmological parameters, especially $$\Omega_m$$ and $$\Omega_\Lambda$$. Hence, one should ultimately do a combined likelihood analysis including other constraints on cosmological parameters to test for the significance of the dark radiation component in the CMB.

V. CONCLUSION

We conclude that the constraints on BBN alone allow for $$-1.23 \leq \rho_{DR}/\rho_\gamma \leq 0.11$$ in a dark radiation component. In order to compute the theoretical prediction of CMB anisotropies exactly, one must eventually solve the perturbations including the contribution from the bulk. However, we have shown that the CMB power spectrum can be used to constrain the dominant expansion-rate effect of the dark-radiation term in the generalized Friedmann equation. If the constraint from effects of dark radiation on the expansion rate are included, the allowed concordance range range of dark radiation content reduces to $$-0.41 \leq \rho_{DR}/\rho_\gamma \leq 0.105$$ at the 95% confidence level.

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