Perspective

Future directions for active matter on ordered substrates

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Abstract – Active matter is a term encompassing particle-based assemblies with some form of self-propulsion, including certain biological systems as well as synthetic systems such as artificial colloidal swimmers, all of which can exhibit a remarkable variety of new kinds of nonequilibrium phenomena. A wealth of non-active condensed matter systems can be described in terms of a collection of particles coupled to periodic substrates, leading to the emergence of commensurate-incommensurate effects, Mott phases, tribology effects, and pattern formation. It is natural to ask how such phases are modified when the system is active. Here we provide an overview and future directions for studying individual and collectively interacting active matter particles coupled to periodic substrates, where new types of commensuration effects, directional locking, and active phases can occur. Further directions for exploration include directional locking effects, the realization of active solitons or active defects in incommensurate phases, active Mott phases, active artificial spin ice, active doping transitions, active floating phases, active surface physics, active matter time crystals, and active tribology.

Introduction. – Active matter systems, composed of particles or components that have some form of self-propulsion [1,2], are relevant to a wide variety of areas including biological systems [1,2], synthetic systems such as artificial colloidal swimmers undergoing chemical catalysis [1,2], crowds [3], and robotic systems [4]. There are also continuum versions of active matter such as cell motion and active nematics [1,5,6]. In the case of particle-based active matter systems, the individual particles show anomalous diffusion due to the finite persistence or run length of their motion. For collectively interacting active particle systems, other types of behaviors arise, with one of the best known examples being motility-induced phase separation where the particles form a high-density solid phase coexisting with a low-density gas when the activity is large enough [7–12]. When active particles are placed in confinement, they can accumulate along walls or boundaries [13–15], and the resulting active pressure forces can show marked differences from forces observed in equilibrium systems [16–19]. With asymmetric boundaries, active ratchet effects can arise [20,21]. There have also been a variety of studies of active matter systems in disordered media where the disorder can break up the active matter clusters or lead to anomalous diffusion [22–25].

A class of nonactive systems that has been extensively studied on the individual and collectively interacting particle level involves the coupling of particles to a periodic substrate [26], which may take the form of a periodic array of obstacles or trapping sites created optically or with nanostructures. A series of commensuration effects occur when the number of particles or the spacing between particles matches the substrate periodicity. At commensuration, the system can be highly ordered and the motility is strongly reduced. Such effects have been observed for ions in periodic substrates [27], cold atoms in optical arrays [28–30], colloidal particles [31,32], surface physics [33,34], and superconducting vortex systems [35,36]. Other effects including novel crystalline ordering, melting, and Mott transitions, where there is one active particle per potential minimum and the mobility is suppressed by particle-particle interactions, have also been studied as a function of the substrate strength. In more complex periodic substrate geometries, frustration effects can arise even under commensurate conditions, leading to the formation of excitations such as monopoles of the type found in two-dimensional artificial spin ice systems [37]. Under an applied drive, particles on periodic substrates exhibit numerous types of depinning, sliding, and frictional behavior, including directional locking where the motion becomes locked to symmetry directions.
of the substrate lattice, and in incommensurate states, depinning transitions can occur via the nucleation and motion of solitons [26,38,39].

Here we give a perspective on active matter systems interacting with periodic or ordered substrates, highlighting what has been done so far and emphasizing the new directions to pursue in this area. There have been a variety of works showing how the motion of active matter over two-dimensional periodic arrays differs from that of Brownian particles, and describing the appearance of anomalous diffusion, scattering, and directional locking [40–48], as well as trapping and sorting [49,50], guiding of the motion along certain directions [51–53], and the organization of large scale dynamic structures [54,55]. Possible future directions include studying active matter Mott phases at commensurate states, or the enhancement of localization by activity. At incommensurate states, there could be motion or depinning of solitons due to the activity, leading to the emergence of new types of active solitons. In chiral active matter systems, it could be possible to realize active versions of spin systems and spin ordered states, and there has been some work examining periodic vortex formation in systems with periodic obstacle arrays [54]. It may also be possible to create dynamical states that are periodic in time in order to realize an active matter version of time crystals. Other directions include studying the impact of activity on tribology effects such as friction, lubrication, and wear by exploring the sliding of active matter over periodic sites or by enclosing active matter particles between sliding plates. It would also be interesting to add activity to well-known models for particles on periodic substrates in both classical and quantum contexts to examine active matter versions of spin ice, active Mott phases, active insulator to fluid transitions, active doping, and so forth. Other directions include studying active continuum models such as active charged systems, active nematics and active polymers.

Modeling different types of substrates. – The simplest representation of particle-based active matter employs self motile disks with a finite short range repulsion and a motor force. This approach has been used extensively to study active matter behaviors such as active clustering [2,7,8,12]. Other more complicated interactions can also be included such as elongated particles, hydrodynamic effects, and even charged particles. Typically, active matter particle systems are modeled as overdamped systems with a motor force for propulsion, interaction forces between the particles, and a substrate interaction:

\[ \eta \frac{dr_i}{dt} = F_{\text{inter}} + F_{\text{sub}} + F_m. \]  

(1)

The particle position is \( r_i \), and its velocity is \( v_i = \frac{dr_i}{dt} \), while the damping constant \( \eta \) in the examples shown here is set to \( \eta = 1 \). If the interactions are assumed to be of hard disk type, then for a sample containing \( N_a \) particles, the disk-disk interaction is given by

![Fig. 1: Illustration of two types of substrates. (a) An egg-carton substrate for active particles, where there are well-defined trapping locations that can capture one or more particles (green). (b) An array of obstacles or posts (red) where active particles (green) can move freely between obstacles but collide with or are guided by the obstacles. These different substrate types produce different effects.](Image)

In two-dimensional systems, there are two general types of attractive interaction potential. The egg-carton potential illustrated in fig. 1(a) is composed of a periodic array with lattice constant \( a_s \) of traps that can capture one or more particles. Alternatively, an array of obstacles can be used as shown in fig. 1(b). Other types of substrates are possible, including the muffin tin substrate in which the well-defined pinning sites have flat spaces between them, quasi-one-dimensional substrates in the form of aligned troughs, and quasi-periodic substrates. For the substrate in fig. 1(a), the minima act as trapping sites with a maximum trapping force of \( F_p \) such that a single particle becomes trapped in a site when \( F_m < F_p \). Even when \( F_m > F_p \), if the persistence length \( l_a \) is very small, a particle can remain trapped by a site for long periods of time. In principle, when \( l_a > a_s \) and \( F_m > F_p \), a particle should...
be continuously hopping from one site to the next. In contrast, for the obstacle substrate shown in fig. 1(b), where the obstacles have radius $r_{\text{obs}}$, single particles are generally not trapped but move through the flat space between obstacles. Encounters with the obstacles can impede or guide the particle motion. This shows that different types of substrate produce distinct effects.

Even in a system containing only a single active particle, the presence of a periodic substrate alters the particle motion. In fig. 2(a), (b) we illustrate an array of posts along with the trajectory of a single active disk for the system studied in ref. [57] where the obstacle lattice constant is $a_s = 4.0$, the active disk radius is $r_a = 0.5$, and the obstacle radius is $r_{\text{obs}} = 0.75$. As shown in fig. 2(a), when the run length $l_a = 0.8$ is very short, the system behaves in a Brownian manner and the particle trajectory fills space uniformly over time. In fig. 2(b), the run length $l_a = 20a_s$, where $a_s$ is the spacing between obstacles and $r_{\text{obs}} = 0.5$, and the motion is constrained to follow symmetry directions of the lattice at angles $\theta = 0^\circ$, $45^\circ$, and $90^\circ$ to the $x$-axis. The distribution $P(v_x)$ of instantaneous $x$-direction velocities for the small run length particle from fig. 2(a) appears in fig. 2(c) and has a Gaussian form, while in fig. 2(d), $P(v_x)$ for the $l_a = 20a_s$ particle from fig. 2(b) has a series of spikes corresponding to the different locking directions.

For a system with an egg carton potential, different kinds of single particle dynamical effects could arise for $F_m/F_p > 1.0$ whenever there is matching between the persistence length and the substrate lattice spacing such that $l_a/a_s = n$ with $n$ integer, leading to an oscillating mobility as a function of varying $F_m$.

**Commensuration effects.** – In fig. 1(a), it is clear that commensuration effects appear when there is a matching density of one particle per substrate trap. It would be interesting to explore the effective mobility of the system as a function of the ratio of the number of active particles $N_a$ to the number of substrate minima $N_s$. These effects will also depend on the values of the motor force and the persistence length. When $F_m/F_p > 1.0$, in the single particle limit the particle can hop to the next trap; however, for $N_a/N_s = 1.0$, all of the wells are occupied and there is no free trap available to accommodate hopping without particle-particle collisions, causing a strong reduction of the mobility and producing an effectively insulating or Mott state. One question is whether there is an optimal activity that would enhance the mobility and produce an insulating state.

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![Fig. 2: (a) An array of posts (red) and trajectories (green) of a single active particle in the Brownian limit. Here the lattice spacing $a_s = 4.0$, the active particle radius is $r_a = 0.5$, and the obstacle radius is $r_{\text{obs}} = 0.75$. (b) The same but for a system of run-and-tumble particles with a run length $l_a = 20a_s$, where the obstacle radius is $r_{\text{obs}} = 0.5$. The motion locks to specific symmetry directions of the substrate. (c) The distribution $P(v_x)$ of instantaneous $x$-direction velocities from the system in panel (a) has Gaussian features. (d) $P(v_x)$ from the system in panel (b) shows several peaks that correspond to the locking of the particle motion to certain substrate symmetry directions. Results are from ref. [57]. Reprinted with permission from Reichhardt C. et al., Phys. Rev. E, 102 (2020) 042616. Copyright 2020 by the American Physical Society.]
Commensuration effects can occur even when $N_o/N_s < 1$. This phenomenon has already been studied for active run-and-tumble particles interacting with a square array of obstacles [58]. Figure 3(a) shows a system of mobile disks in a square obstacle array with lattice constant $a_s = 3.0$ in the Brownian limit of small $l_o$. The active particle radius is fixed to $r_a = 0.5$ and the obstacle radius $r_{\text{obs}}$ is varied. In this case, the area covered by the disks is $\phi = \pi r_a^2/L^2 = 0.32$, where $L$ is the length of one side of the sample. This is well below the jamming density of $\phi = 0.9$, and as the figure illustrates, the system forms a uniform fluid. In fig. 3(b), the same system with a long running length or large $l_o$ forms a motility-induced phase separated state in the absence of a substrate. In the presence of the substrate, local areas of high density form a commensurate triangular ordering that coexists with a low-density gas. The triangular ordering occurs when the obstacle diameter is chosen such that an integer number of disks can fit between the obstacles. For other obstacle diameters, a cluster can still form when the activity is large but it is a lower-density disordered cluster. The active commensurate states can be detected by measuring the ordering as a function of obstacle diameter $d = 2r_{\text{obs}}$, as shown in fig. 3(d) where the fraction of sixfold coordinated particles $P_6$ is plotted vs. $d$. There is a peak at $d = 1.0$ where the crystalline state shown in fig. 3(b) occurs. For smaller $d$, the obstacles break up the clusters. Figure 3(c) shows the corresponding mobility $M = N_o^{-1} \sum_i N_s v_x$ vs. $d$ for the same system. The peak in $P_6$ corresponds to the lowest mobility, indicating that the commensurate cluster is pinned. There are also a series of other bumps and dips corresponding to different kinds of commensurate-incommensurate ordering. The peak in the mobility near $d = 1.35$ appears when the system forms a loosely packed disordered solid. There are many future directions to study. For example, one could look at mixtures of active and passive particles to see if they would mix or demix. Also of interest would be a mixture of elongated active particles such as bacteria or anisotropic colloids where one length scale matches the substrate lattice constant and the other does not. Such a system could form a commensurate cluster state for only one of the two types of particles, producing a much richer variety of ordering.

It would also be interesting to explore more complex periodic substrates. For example, there are geometries known as artificial spin ice that can be modeled as particles coupled to a specific type of periodic substrate that gives rise to a frustrated ground state and supports excitations such as monopoles [37]. In active systems, the monopoles could behave like active emergent particles, and active order to disorder transitions could be studied as a function of activity.

Another form of active matter is chiral swimmers or particles that move in circular paths [59–61]. The direction of the chirality can be viewed as an effective spin degree of freedom for active particles, allowing spinners on a periodic substrate to be used as active matter versions of various spin models. For example, in a system of clockwise and counterclockwise swimmers placed on a square periodic lattice, if long range interactions couple different plaquettes of the sample, an ordered checkerboard state could form as shown in fig. 4(a). On the other hand, if the swimmers are placed on a triangular lattice, they will generally be frustrated and disordered as shown in fig. 4(b), although other orderings could arise such as stripe phases. Another interesting possibility is that chiral or nonchiral active matter on a periodic substrate could synchronize to form dynamical states that repeat as a function of time, producing an active matter version of classical time crystals [62–64]. Other effects to explore include phase locking, chaos, or intermittent states similar to those found for coupled oscillators.

**Incommensurate phases.** – In systems with periodic substrates, a rich variety of effects appear when the num-
Fig. 4: (a) A square array of obstacles (red) interacting with a binary assembly of chiral swimmers (blue, clockwise; green, counterclockwise) at $N_a/N_s = 1$ matching where the swimmers can form a checkerboard ordered state. (b) The same but for a triangular substrate array where the system forms a disordered state.

Fig. 5: (a) Schematics of an active matter system on a periodic substrate where there is a kink or soliton present. The soliton could show active motion while the other active particles remain pinned. Top: a vacancy soliton. Upper middle: a commensurate state. Lower middle: an interstitial soliton. Bottom: a motility-induced phase separated (MIPS) state. (b) Schematic showing possible behavior of the mobility $M$ of an incommensurate active matter system as a function of increasing motor force $F_M$ showing a pinned phase, an active soliton phase, an active fluid, and MIPS states.

Fig. 6: An example of active matter tribology showing two periodic plates sliding past one another with active particles (yellow) in the space between the plates. Arrows indicate the direction of motion of each plate. (a) Low density of active particles. (b) Higher density of active particles.
could match the periodicity of the substrate. Since many of the systems that have previously been studied involve charged particles or particle-particle interactions that are longer range than hard spheres, a natural future direction is to study active charged systems or active particles with Yukawa interactions. It could also be possible to make particles with a Bessel function interaction active. This type of interaction arises for superconducting vortices, skyrmions, and screened charged systems. There are some efforts underway currently to create such systems experimentally, allowing more direct comparisons to the commensurate states and dynamics found in colloidal and superconducting vortex systems without activity. Another approach would be simply to take many of the known models for systems that have been studied on periodic lattices and add an activity term. This could be done for both classical and even quantum systems, where correlated noise or a persistence length could be added. There may be a way to create such systems experimentally using active optical feedback traps.

An additional area is to consider systems with a combination of periodic pinning and an asymmetry in order to create active ratchet effects that can be impacted by commensuration. Some studies along these lines have been performed in nonactive systems [66–68] as well as for active matter on periodic asymmetric substrates [21,69–71]. Other directions are to study chiral active matter on periodic asymmetric substrates [72,73] where Hall effects and odd viscosity can arise. Finally, the flocking of swarming models of self-propelled particles could be examined in the presence of a periodic substrate [74,75].

Summary. – Active systems have opened a new field of physics and materials and represent a new state of matter. In nonactive systems there is an extensive amount of phenomena that has been observed under coupling to a periodic substrate, such as commensuration effects, formation of kinks and anti-kinks, diffusion, Mott and fluid-like phases, tribology, and pattern formation for both hard and soft matter systems. Examining active matter on periodic substrates is a natural direction for further work where new types of commensurate effects, directional locking, ordering and frustrated order could be observed. We highlighted several of the directions that could be studied for different types of periodic substrates, including commensuration effects for increasing activity and motor forces, dynamical commensuration effects when the persistence length is a multiple of the substrate periodicity, active solitons or active excitations in incommensurate phases, new ways to control friction, and new types of pattern formation. It would also be interesting to incorporate activity into already existing models in both classical and quantum systems coupled to periodic substrates.

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