Large power dissipation of hot Dirac fermions in twisted bilayer graphene

S S Kubakaddi

Department of Physics, K L E Technological University, Hubballi-580031, Karnataka, India

E-mail: sskubakaddi@gmail.com

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Abstract

We have carried out a theoretical investigation of hot electron power loss \( P \), involving electron–acoustic phonon interaction, as a function of twist angle \( \theta \), electron temperature \( T_e \) and electron density \( n_s \) in twisted bilayer graphene. It is found that as \( \theta \) decreases closer to magic angle \( \theta_m \), \( P \) enhances strongly and \( \theta \) acts as an important tunable parameter, apart from \( T_e \) and \( n_s \). In the range of \( T_e = 1–50 \) K, this enhancement is \( \sim 250–450 \) times the \( P \) in monolayer graphene (MLG), which is manifestation of the great suppression of Fermi velocity \( v_F \). As \( \theta \) increases away from \( \theta_m \), the impact of \( \theta \) on \( P \) decreases, tending to that of MLG at \( \theta \sim 3^\circ \). In the Bloch–Grüneisen (BG) regime, \( P \sim T_e^{-1/2} n_s^{-1/2} \) and \( v_F^{-2} \). In the higher temperature region (\( \sim 10–50 \) K), \( P \sim T_e^{-2} \), with \( \delta \sim 2.0 \), and the behavior is still super linear in \( T_e \), unlike the phonon limited linear-in-\( T \) (lattice temperature) resistivity \( \rho_p \). \( P \) is weakly, decreasing (increasing) with increasing \( n_s \) at lower (higher) \( T_e \) as found in MLG. The energy relaxation time \( \tau_e \) is also discussed as a function of \( \theta \) and \( T_e \). Expressing the power loss \( P = F_e(T_e) - F_e(T) \), in the BG regime, we have obtained a simple and useful relation \( F_e(T)\mu_p(T) = (e\nu_s^2/2)T_e\mu_p \) where \( \mu_p \) is the acoustic phonon limited mobility and \( \nu_s \) is the acoustic phonon velocity. The \( \mu_p \) estimated from this relation using our calculated \( F_e(T) \) is nearly agreeing with the \( \mu_p \) of Wu et al (2019 Phys. Rev. B 99 165112).

Keywords: twisted bilayer graphene, Dirac fermions, electron–phonon interaction, hot electron power loss

(Some figures may appear in colour only in the online journal)

1. Introduction

Recent pioneering experimental discoveries in twisted bilayer graphene (tBLG) by Cao et al [1, 2], have created great interest in the study of their electronic properties and has ushered in a new era in the condensed matter physics [3–13]. Among the discoveries, the existence of correlated insulating phases and superconductivity at low temperatures and a highly resistive linear-in-temperature \( T \) resistivity \( \rho \) at high temperature, are remarkable and exciting [1, 2, 4]. Very recently, the observation of a quantum anomalous Hall effect in tBLG aligned to hexagonal boron nitride has been reported in tBLG [9]. In twisted tBLG a small twist angle \( \theta \), near the magic angle \( \theta_m \), between the two layers plays the most significant role and acts as one of the tunable parameters, similar to the carrier density \( n_s \) and temperature \( T \), of the samples in limiting their electronic properties [1, 2, 4, 12, 13]. The transport results of Cao et al [11] establish magic angle bilayer graphene (BLG) as a highly tunable platform to investigate ‘strange metal’ behavior. Because of the twist between the layers the band structure is a moiré flat band with the twist angle dependent suppressed Fermi velocity \( v_F^*(\theta) < v_F \), the bare Fermi velocity in monolayer graphene (MLG), and the large density of states \( D(E_k) \) near \( \theta_m \) at which \( v_F^*(\theta) = 0 \) [12–14]. The strongly enhanced electrical resistivity \( \rho \), near \( \theta_m \), with linear-in-temperature behavior has been observed for \( T > \sim 5 \) K [10, 11].

Theoretically, the electrical resistivity has been investigated in tBLG, at higher temperature and away from the moiré miniband edge, by considering the effect of electron–acoustic...
phonon (el–ap) interaction [10, 12, 13]. It is shown that the phonon limited resistivity $\rho_P \equiv \rho(T, \theta)$ is strongly enhanced in magnitude, twist-angle dependent and linear-in-$T$ occurring for $T > T_{BLG}$, where $T_{BLG}$ (on the order of few kelvins) is the temperature above which linearity in $\rho(T, \theta)$ develops. This linear-in-$T$ is observed for $T_{BLG} \approx T_{BG}/\sqrt{5}$ [13], where $T_{BG} = 2\hbar \nu_s k_B$ is the Bloch–Grüneisen (BG) temperature, $\nu_s$ is the acoustic phonon velocity, and $k_F = \sqrt{\pi n_e/2}$ is the Fermi wave vector in tBLG. The enhancement in $\rho(T, \theta)$, about three orders of magnitude greater than that in MLG at $T \approx 10$ K, is shown to arise from the large increase in the effective el–ap scattering in tBLG due to the suppression of $\nu_P$ induced by the moiré flat band. In the metallic regime i.e. for $T > T_{m}(< T_{BLG})$, where $T_{m}$ is the metallic temperature, $d\rho(T, \theta)/dT > 0$, and it is $n_e$ and $\theta$ dependent. The $\rho(T, \theta)$ is found to increase with increasing $T$ as the twist angle $\theta$ approaches $\theta_m$. The linear dispersion taken for the Dirac fermions in tBLG is an approximation that is valid for Fermi energy near the Dirac point and hence its transport study is limited to the $n_e \leq 10^{12}$ cm$^{-2}$. Interestingly, it is also shown that the same enhanced el–ap interaction can also produce superconductivity with $T_c \sim 1$ K in s, p, d and f orbital pairing channels [3, 12].

The theory of Wu et al. [12, 13], explains the available experimental data of $\rho$ in tBLG [10, 11] well for $T > 5$ K, taking the Fermi energy slightly away from the Dirac point. The resistivity of a sample arises from the momentum relaxation of the charge carriers (Dirac fermions in tBLG) due to their interaction with the impurities, disorders and phonons. In their theory of resistivity in tBLG, Wu and co-workers [12, 13] ignore the effects of impurities and disorder assuming that the system is extremely clean and $\rho$ is limited by only phonons. Moreover, $\rho$ obtained from the momentum relaxation time is investigated in the low electric field (i.e. Ohmic) region. However, hot electron/Dirac fermions power loss $P$, which is the average energy loss rate by the hot carriers to the phonons, is an important transport property governed by only electron–phonon interaction and independent of disorders and impurities. This property, unlike the resistivity, involves the energy relaxation of the hot electrons/Dirac fermions in large electric field (non-Ohmic) region as given below.

The electron (Dirac fermion) system in samples (tBLG) subject to large electric fields or photoexcitation establishes its internal thermal equilibrium at an electron temperature $T_e$ greater than the lattice temperature $T$ because electron–electron interaction occurs at the time scale of several femtoseconds which is much smaller than the electron–phonon scattering time. Consequently, the electron system is driven out of equilibrium with the lattice. In steady state, these electrons will relax toward equilibrium with the lattice by dissipating energy with phonons as the cooling channels. The hot electron power loss $P$ is an important phenomenon as it affects thermal dissipation and heat management which are key issues in nanoscale electronics device. Moreover, it is crucial for applications in variety of devices such as calorimeters, bolometers, infrared detectors, ultrafast electronics and high speed communications. Hot electron cooling has been extensively studied theoretically and experimentally in MLG [15–24] and conventional BLG [18, 25–27].

In the present work, we investigate the effect of enhanced el–ap coupling on the power dissipation $P$ of the hot Dirac fermions in moiré flat band in tBLG. It is studied as a function of twist angle, electron temperature and electron density. We show that the twist angle $\theta$ acts as one of the strong tunable parameters of $P$. Additionally, a relation between power loss and phonon limited mobility $\mu_P$ is brought out in BG regime.

2. Theoretical model

Wu et al. [12] have used the Dirac Hamiltonian with a renormalized velocity for electron energy spectrum, in order to obtain their analytical results. In moiré flat band, the electron energy spectrum is assumed to be Dirac dispersion $E_k = \hbar v_F |k|$, which is an approximation that is valid for near Dirac point, with an effective Fermi velocity $v_F^* = v_F(\theta)$. Because of this approximation our theory will be limited to the carrier density $n_e \leq 10^{12}$ cm$^{-2}$. The density of states is $D(E_k) = (g(E_k)/[2\pi(\hbar v_F^*)^2])$ with the degeneracy $g = g_s, g_f$, where $g_s, g_f$ are, respectively, spin, valley and layer degeneracy each with the value of 2. We consider electron–acoustic phonon interaction within the deformation potential approximation with the longitudinal acoustic (LA) phonons of energy $\hbar \omega_P$ and wave vector $\mathbf{q}$ interacting with the tBLG Dirac electrons in the moiré miniband. The LA phonons in tBLG are assumed to be unaffected by the tBLG structure and are taken to be the same as the MLG phonons. In MLG the experimental observations of electrical conductivity [28] and power loss [19, 20, 22] are very well explained by the electron interaction with only LA phonons, without screening. Wu et al. [12, 13] have explained the linear-in-$T$ resistivity data in tBLG with only electron–LA phonon interaction.

The hot Dirac fermions power loss is obtained by calculating the energy gained by the phonons from the electrons and dividing by the total number of electrons [15, 29]. It is given by $P = (1/N_e) \sum \hbar \omega_P (dN_q/dt)_{el-phon}$, where $N_e$ is the total number of electrons, $dN_q/dt)_{el-phon} = \hbar \omega_P [\sum |M(q)|^2 [(N_q + 1)/f(E_k + \hbar \omega_P) - N_q f(E_k)]/(1 - f(E_k + \hbar \omega_P))] \delta(E_k - \hbar \omega_P)$ is the rate of change of phonon distribution $N_q$ due to the electron–phonon (el–ph) coupling and $|M(q)|^2$ is the electron–Dirac fermion interaction matrix element. The Dirac fermion distribution function is assumed to be given by the heated Fermi–Dirac distribution $f(E_k) = \{\exp[(E_k - \mu)/\hbar k_B T_e] + 1\}^{-1}$, where $\mu$ is the chemical potential of the Dirac fermions determined by the 2D electron density $n_e = \int f(E_k) D(E_k) dE_k$.

We use the modified ordinary el–ap matrix element [12] $|M(q)|^2 = (D^2 \hbar q F(\theta)/[2 \Delta \rho_P \nu_s]) (1 - q^2/4k^2)$, where $D$ is the first-order acoustic deformation potential coupling constant, $A$ is the area of the tBLG, $\rho_P$ is the areal mass density and $\nu_s$ is the LA phonon velocity. The detailed tBLG moiré wave function gives rise to the form factor function $F(\theta)$ which modifies the el–ap interaction matrix element in tBLG as compared with the MLG [12]. It is shown to be between 0.5 and 1.0 and being nearly parabolic for $1^\circ < \theta < 2^\circ$ in the neighborhood of a minimum at $\theta = 1.3^\circ$ [13]. Following the references [15, 29, 30], and taking care of additional layer degeneracy, we obtain an expression for the electron power loss in tBLG...
and it is given by

\[ P = -\frac{gD^2F(\theta)}{4\pi^2n_p\hbar^2v_F^3} \int_0^\infty d(\hbar\omega_q) \left( \hbar\omega_q \right)^2 \]

\[ \times \int_\gamma dE_k \frac{(E_k + \hbar\omega_q)}{[1 - (\gamma/E_k)^2]^{1/2}} \times G(E_q, E_k) \left[ (N_q(T_e) - N_q(T)) \right] [f(E_k) - f(E_k + \hbar\omega_q)], \tag{1} \]

where \( n_p \) is the electron density, \( \gamma = (E_q/2) \), \( E_q = \hbar v_F q \), \( N_q(T) \) = [exp(\hbar\omega_q/k_BT) - 1]^{-1} is the Bose–Einstein distribution at lattice temperature \( T \) and \( G(E_q, E_k) = [1 - (\gamma/E_k)^2] \), is due to the spinor wave function of the electron in the electron–phonon matrix element, in the quasi-elastic approximation [15]. By setting \( F(\theta) = 1 \), \( g_1 = 1 \) and \( v_F^*(\theta) = v_F \) in equation (1), we regain the equation that is applicable to MLG [15] and silicene [31], similar to the acoustic phonon induced resistivity in tBLG [10, 12].

The twist angle dependence of \( v_F^* \equiv v_F^*(\theta) \) is shown to be very well approximated by [10, 13]

\[ v_F^*(\theta) \approx 0.5 |\theta - \theta_m| v_F, \tag{2} \]

which clearly indicates that twist angle effect is very large for \( \theta \) closer to \( \theta_m \). We use this relation while computing \( P \) for different twist angles.

In the Bloch–Grüneisen (BG) regime \( T, T_e \ll T_{BG}, q \ll 2v_F \), the power loss is given by

\[ P = \Sigma(T_e^4 - T^4)/n_s^{1/2}, \tag{3} \]

where \( \Sigma = \Sigma_0(D^2/v_F^3) \) and \( \Sigma_0 = (g\pi^{5/2}k_B^4F(\theta))/(60\sqrt{2}/\rho_m \hbar^4v_F^2) \). Hence, in BG regime \( P \sim T_e^4 \), \( n_s^{-1/2} \) and \( v_F^{-2} \).

3. Results and discussion

We obtain the following numerical results of the power loss in tBLG using the parameters [12, 13]: \( \rho_m = 7.6 \times 10^{-8} \text{ gm cm}^{-2} \), \( \theta_m = 1.02^\circ \), \( v_t = 2 \times 10^3 \text{ cm s}^{-1} \), \( v_F = 1 \times 10^6 \text{ cm s}^{-1} \) and \( D = 20 \text{ eV} \) [15, 17, 20, 28, 32, 33], noting that Polshyn et al [10] and Wu et al [12] have used \( D = 25 \pm 5 \text{ eV} \). In order to bring out the angular dependence of the power loss, we confine our illustrations for \( \theta = 1.1^\circ, 1.2^\circ \) and \( 1.3^\circ \) which are closer to magic angle \( \theta_m = 1.02^\circ \). For these angles, the effective Fermi velocity \( v_F^* = 4, 9 \) and \( 14 \times 10^5 \text{ cm s}^{-1} (> 1.5v_t \) [12]), respectively, which are much smaller than the bare \( v_F \) and the effect of \( v_F^* \) on the transport coefficients will be very large. For further increase of \( \theta \), \( v_F^* \) tends to \( v_F \) at about 3.0°. The value of the function \( F(\theta) \) for different \( \theta \) are taken from figure 3 of Das Sarma et al [13], and because of its value between 0.5 and 1, it will have smaller influence on \( P \) than \( v_F^* \). We have presented the calculations for lattice temperature \( T = 0.1 \text{ K} \) and \( n_e = 0.1 - 1n_0 \), with \( n_0 = 1 \times 10^{15} \text{ cm}^{-2} \), which keeps us slightly away from the Dirac point and within the linear region of moiré flat band. For \( n_e = n_0, T_{BG} = 38.3\sqrt{N} \) which is smaller by a factor of \( \sqrt{2} \) compared to MLG.

First we explore the dependence of power loss \( P \) on electron temperature \( T_e \) for twist angles \( \theta = 1.1^\circ, 1.2^\circ \) and \( 1.3^\circ \). In figure 1(a), \( P \) is presented as a function of \( T_e(1-50 \text{ K}) \) for \( n_e = n_0 \). For all the \( \theta \), we observe the generic nature of the behavior, where in at very low \( T_e \) the power loss increases rapidly then slows down at higher temperature. At higher \( T_e \) the phonon energy \( \hbar\omega_q \) becomes smaller than \( k_BT_e \), and hence the increase in the number of phonons \( N_q(T_e) \) is slower resulting into slowing down of the power loss. For the temperatures \( T_e \ll T_{BG}, \) the rapid increase may be attributed to the increasing number of phonons as their wave vector \( q \approx k_BT_e/\hbar v_t \) increases linearly with \( T_e \). For \( \theta = 1.1^\circ \), the power law \( P \sim T_e^{5/2} \)

![Figure 1](image-url)

Figure 1. Electron temperature \( T_e \) dependence of the power loss \( P \) in tBLG for twist angle \( \theta = 1.1^\circ, 1.2^\circ \) and \( 1.3^\circ \). (a) \( P \) vs \( T_e \) for \( n_e = n_0 \), (b) \( P/T_e^3 \) vs vs \( T_e \) for \( n_e = n_0 \) and (c) \( P \) vs \( T_e \) for \( n_e = 0.5n_0 \).
is found to be obeyed for $T_e < \sim 2.5$ K, which is about $T_{BG}/15$. The exponent 4 of $T_e$ is manifestation of two-dimensional phonons with unscreened electron–phonon coupling.

In order to see the effect of $\theta$ on the range of validity of the power law, we have plotted $P/T_e^4$ vs $T_e$ in figure 1(b). It is found that, as $\theta$ increases the range of $T_e$ in which power law is obeyed marginally increases. For example, for $\theta = 1.2^\circ$ and $1.3^\circ$, power law is found to be satisfied for $T_e$ up to about 3 and 3.5 K, respectively, although $T_{BG}$ is same. This happens because as $\theta$ increases $\nu_F^4$ also increases and tends toward $\nu_F$.

In the BG regime, in which $P \sim \nu_F^{-2}$, we find $\Sigma = 2.66 \times 10^{-15}/\sqrt{N} \ W \ K^{-4} \ cm$ and $5.13 \times 10^{-16}/\sqrt{N} \ W \ K^{-4} \ cm$ and $2.1 \times 10^{-16}/\sqrt{N} \ W \ K^{-4} \ cm$, for $\theta = 1.1^\circ$, $1.2^\circ$ and $1.3^\circ$, respectively, as compared to $5.23 \times 10^{-18}/\sqrt{N} \ W \ K^{-4} \ cm$ in MLG. In the higher temperature region of $T_e = 10–50$ K ($30–50$ K), $P \sim T_e^\delta$ with $\delta \sim 2.0–2.2(\sim 1.7–2.0)$, for all $\theta$s, as compared to the resistivity which is found to show linear-in-temperature for temperature $\geq T_{BG}/8$ [13].

In figure 1(c), $T_e$ dependence of $P$ is shown for $n_s = 0.5 n_0$ and the same behavior is observed as in figure 1(a). However, for the same $\theta$, in the low temperature region $P$ is found to be marginally larger (smaller) at lower (higher) $T_e$ than that for $n_s = n_0$. We observe that the temperature below which the power law $P \sim T_e^\delta$ is obeyed shifts to lower side for smaller $n_s$. For example, for $\theta = 1.1^\circ$ power law is obeyed for $T_e < \sim 2$ K, which may be due to the lower $T_{BG} = 27.0$ K, for $n_s = 0.5 n_0$.

More importantly, from the figures 1(a)–(c) (and figures 2–4) we find the strong influence of $\theta$ on $P$. As $\theta$ decreases $P$ is found to be very significantly enhanced and it is consistent with the resistivity calculations [12, 13]. This enhancement of $P$ is attributed to the strong moiré flatband induced enhancement of the electron–phonon interaction as evinced by the presence of $\nu_F^{-4}$ in the denominator of equation (1). Thus, $\theta$ is emerging as an important tunable parameter of the power loss, in addition to $T_e$ and $n_s$. The influence of $n_s$ on $P$ is very much small compared to that of $\theta$. For $\theta = 1.1^\circ$ and $1.3^\circ$, for the $T_e$ range considered, $P$ is in the range $\sim 10^{1–10^6} \ eV \ s^{-1}$ and $\sim 10^{1–10^9} \ eV \ s^{-1}$, respectively. These values are comparable to those in monolayer MoS$_2$ [30] but about three and four orders of magnitude greater than those in GaAs heterojunction [34] and Si-inversion layer [35], respectively.

In order to compare the power loss in tBLG with that in MLG and conventional BLG, $P$ dependence on $T_e$ is depicted in figure 2, for $n_s = n_0$ with $P$(BLG) taken for $\theta = 1.1^\circ$. We find that the power loss in tBLG is very large ($\sim 2 \times 10^6$–$2 \times 10^9 \ eV \ s^{-1}$) compared to that in MLG ($\sim 4 \times 10^4$–$6 \times 10^6 \ eV \ s^{-1}$) and BLG ($\sim 9 \times 10^3$–$3 \times 10^7 \ eV \ s^{-1}$) [26]. Defining a ratio $R_p = P$(tBLG)/$P$(MLG), it is found that $R_p = \sim 450$, 260 and 300, respectively, at 1, 10 and 50 K, and $R_p$ is expected to be smaller for larger $\theta$. This enhancement is attributed to the significantly reduced $\nu_F^4$. This may be compared with the $\rho$ enhancement in tBLG, which is of three orders of magnitude greater than that in MLG at $\sim 10 \ K$ [12, 13]. It is also noticed that the range of $T_e$ in which power law is obeyed is much larger in MLG than in tBLG.

We have presented in figure 3 the electron density $(n_s = 0.1–1.0 n_0)$ dependence of the power dissipation for two electron temperatures $T_e = 1 \ K$ (figures 3(a)) and $5 \ K$ (figure 3(b)). For $n_s = 0.1 (1.0) n_0$ the $T_{BG} = 12.1 (38.3)$ K. From figure 3(a), we see that $P$ is found to decrease with
increasing $n_s$, as found in MLG [15, 22], with power law $P \sim n_e^{-1/2}$ being followed at larger $n_e$ and small deviation occurring at lower $n_e$. This is due to the fact that $T_{BG}$ goes on decreasing with decreasing $n_s$. The $P \sim n_e^{-1/2}$ dependence in tBLG is in contrast to the $P \sim n_e^{-3/2}$ dependence in conventional BLG [26, 27]. On the other hand for $T_e = 5$ K (figure 3(b)), $P$ increases (flattens) with increasing $n_s$ in the low (high) $n_s$ region, because we are moving away from the $T_e \ll T_{BG}$ region.

The energy relaxation time $\tau_\varepsilon$ is another important quantity studied in the hot electron relaxation process, as it determines the samples suitability for its applications in optical detectors (bolometer, calorimeter and infrared detectors) and high speed devices. For a degenerate electron gas it is given by $\tau_\varepsilon = [(p + 1)(\pi k_B)^2(T_e^2 - T^2)/(6e^2P)]$, where $p$ is the exponent of energy in density of states and $E_F$ is the Fermi energy [20, 36]. In BG regime, since $P \sim T_e^4$ and $n_e^{-1/2}$, we find $\tau_\varepsilon \sim T_e^{-2}$ and independent of $n_e$ (as $E_F \sim n_e^{1/2}$). In figure 4, $\tau_\varepsilon$ is presented as a function of $T_e$, for $\theta = 1.1^\circ$, $1.2^\circ$ and $1.3^\circ$, in tBLG along with the $\tau_\varepsilon$ in MLG for $n_s = n_0$. In both tBLG and MLG, $\tau_\varepsilon$ is found to decrease with increasing $T_e$ and the decrease is rapid at lower temperature ($\ll 10$ K). It is found that $\tau_\varepsilon$ in MLG, for $\theta = 1.1^\circ$, is an order of magnitude smaller than that in MLG and this difference decreases with increasing $\theta$. The ratio $\tau_\varepsilon$ (MLG)/$\tau_\varepsilon$(tBLG), for $\theta = 1.1^\circ$, is found to be 10.0, 6.6 and 6.9, respectively, for $T_e = 5, 10$ and 20 K. This ratio is not as large as the ratio of $P$ s, because of the product $E_FP$ in the denominator of the expression for $\tau_\varepsilon$, noting that for the same $n_s$, the $E_F$ in tBLG is much smaller than that in MLG. By increasing $\theta$ the $\tau_\varepsilon$ increases significantly, indicating that twist angle is an important tunable parameter for $\tau_\varepsilon$ also. It may be noted that samples with faster energy relaxation (i.e. smaller $\tau_\varepsilon$) find applications in ultrafast electronics and high speed communications. On the other hand, samples with longer energy relaxation time are preferred in photodetectors and energy harvesting devices like hot carrier solar cells.

Finally, in BG regime, we bring out a simple relation of $P$ with phonon limited mobility $\mu_\rho$ and resistivity $\rho_\rho$ in tBLG. In this regime, $P$, $\mu_\rho$ and $\rho_\rho$ are sensitive measures of the el–ap coupling. While $P$ is determined by the energy relaxation through el–ap interaction, $\mu_\rho$ and $\rho_\rho$ involve momentum relaxation through the same mechanism. A relation between these measurable properties is expected because of the same underlying mechanism. This kind of relation between $P$ and $\mu_\rho$ is listed for different electron systems in Ref [37]. In BLG, the equation $\rho_\rho(T) = A T^p$ for the phonon limited resistivity is obtained from Min et al [38] with the suitable replacement of $g_{eg}$ by $g_{g_s}g_{g_s}$, $k_B = (\pi n_s/2)$ and inserting $F(\theta)$ in the numerator in their Ref [38] for $A$. There by, using the relation $\mu_\rho(T) = 1/(n_s \epsilon_\rho(T))$, the phonon limited mobility is found to be $\mu_\rho(T) = [(15\hbar e^2/\epsilon_\rho)^{4/5}n_s^{1/2}T^{-4}]/[16\sqrt{2}\pi^2D^2 k_B^4 F(\theta)]$, where $e$ is the electron charge. Expressing equation (3) as $P = F_\varepsilon(T_e) - F_\varepsilon(T)$ [15, 34], where $F_\varepsilon(T) = \Sigma T^4/n_s^{1/2}$ and $P = F_\varepsilon(T_e)$ for $T_e \gg T$, we obtain a very simple relation $F_\varepsilon(T_e)$ $\mu_\rho(T) = (e v_F^2/2)$, which is exactly same as that of MLG [37]. This relation is analogous to Herring’s law [39], which relates phonon-drag thermopower $S'$ and $\rho_\rho$. Alternatively, power loss can be related to $\rho_\rho$ by the formula $F_\varepsilon(T_e) = (n_e e^2 v_F^2/2) \rho_\rho(T)$.

The advantage of these relations is, if $F_\varepsilon(T)$ is measured then $\mu_\rho(T)$ and $\rho_\rho(T)$ can be determined or the vice-versa, and the measurements of power loss may be preferred as it is independent of lattice disorder and impurities. From our calculated value of $P = F_\varepsilon(T_e) = 4.45 \times 10^{-14}$ W at 2 K for $\theta = 1.1^\circ$ and $n_s = n_0$, we estimate $\rho_\rho(T) = 0.8 \Omega$, which is nearly agreeing with the value obtained by Wu et al (see figure 4(a) of [12]), and $\mu_\rho(T) = 7.5 \times 10^6$ cm$^2$ V$^{-1}$s$^{-1}$.

We would like to make the following remarks. In the literature the values of $\theta_m$ given are varying between 1.02$^\circ$ to 1.1$^\circ$ [1, 8, 12, 13]. However, we believe that our findings and analysis with $\theta_m = 1.02^\circ$ [12, 13] hold good for the $\theta$ values closer to any chosen $\theta_m$. We want to emphasize that, our analytical results will be of great help to experimental researchers and secondly can be used to determine $D$ as the measurements of $P$ are independent of lattice disorder and impurities, unlike resistivity.

4. Conclusion

We have studied the hot electron power loss $P$ due to the simple acoustic phonon interaction, via deformation potential coupling, in tBLG of low electron density $n_s \leq 10^{12}$ cm$^{-2}$ for small twist angles $\theta$ and for $T_e \geq 1$ K. For $\theta$ closer to the magic angle, $P$ is enhanced by a few 100 times that in MLG due to the strong suppression of the Fermi velocity $v_F$ leading to the strong el–ap scattering. Consequently, twist angle emerges as an additional important tunable parameter of $P$. Although BG regime power law $P \sim T_e^4$ is obeyed in low $T_e$ region, $P$ vs $T_e$ behavior still remains super linear at higher $T_e$ where acoustic phonon limited resistivity $\rho_\rho$ is linear-in-temperature. For a given $n_s$, although $T_{BG}$ is independent of $\theta$, the range of $T_e$ in which $P \sim T_e^4$ is obeyed increases marginally with increasing $\theta$. The energy relaxation time $\tau_\varepsilon$ is found to be smaller by an order of magnitude than in MLG and decreasing with increasing $T_e$. As $\theta$ approaches $\theta_m$ the $\tau_\varepsilon$ decreases significantly indicating that $\theta$ can be used as an important parameter to tune $\tau_\varepsilon$ also. Finally, simple and useful relations of $P$ with
μ_p and ρ_p are obtained in the BG regime. From the relation between P and ρ_p, using our calculated P, the estimated value is closer to the ρ_p of Wu et al [12]. Experimental observations may test the validity of our predictions.

ORCID iDs
S S Kubakaddi https://orcid.org/0000-0002-4702-868X

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