What is the Magnetic Moment of the Electron?

Othmar Steinmann

Fakultät für Physik, Universität Bielefeld, 33501 Bielefeld, Germany

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Dedicated to Rudolf Haag on the occasion of his 80th birthday

Abstract: Because of infrared effects the charged sectors of QED contain no eigenstates of the mass operator. The electron is therefore not definable as a Wigner particle. There exists no sharp, unambiguous, definition of the notion of a 1-electron state. The assignment of a fixed value of the magnetic moment – or similar quantities – to the electron is therefore at first problematic. It is not clear a priori that such a notion is meaningful. Conventionally this problem is solved by first calculating the desired quantity in an IR-regularized theory and then removing the regularization. If this method yields a finite value, that is considered sufficient proof of its soundness. This is clearly less than satisfactory. Here we propose a more convincing way of defining the intrinsic magnetic moment of the electron, which does not use any regularizations and is not based on an interaction with external fields. A pseudostatic 1-electron state is defined in a phenomenological way. Its magnetic moment, as defined here, does not depend on the unavoidable ambiguities inherent in this definition. The method leads to the same analytic expression as the conventional approach, thus preserving the excellent agreement between theory and experiment.

1. Introduction

The spectacular accuracy with which the theoretical QED-values of the anomalous magnetic moments of the electron and the muon agree with measurement\(^1\) is one of the major triumphs of relativistic quantum field theory. It must be said, however, that from the point of view of a rigorous formulation of field theory, the theoretical derivation of these numbers leaves much to be desired. The method typically used, as briefly described in [3], can be epitomized by the following quote from this reference: “The magnetic property of an electron can be studied most conveniently by examining its scattering by a static magnetic field”. But the standard scattering formalism used in carrying out this program

\(^1\) A fairly recent review is [1]. The latest measurement in the muon case is reported in [2].
is not really applicable to QED even in the absence of an external field, due to the notorious infrared (IR) problems. Taking these problems seriously it is found that for charged particles a 1-particle state is a much more complex object than usually assumed. In particular it is not definable as an eigenstate of the mass operator \( M^2 = P_\mu P^\mu \) \((P_\mu \text{ the 4-momentum operator})\). Green’s functions and the like cannot be meaningfully restricted to the mass shell. A “1-particle state” can therefore not be specified by a 3-dimensional wave function. Customarily this fact is described by stating that a charged particle is necessarily accompanied by a cloud of soft photons, the exact composition of this cloud not being derivable from first principles. What is fixed is, crudely speaking, only the form of the singularity of \( n(\omega) \) for \( \omega \to 0 \), when \( n(\omega) \) is the number of soft photons of energy \( \omega \).\(^2\) Besides invalidating the conventional scattering formalism, this unavoidable vagueness of the 1-particle states creates an obvious problem with the definition of their magnetic moment and similar quantities. Might they not be indeterminate because depending on the shape of the photon cloud?

The problem becomes even more serious if the system is acted upon by an external electromagnetic field. This destroys the Poincaré invariance of the theory, in particular the translation invariance which is a powerful tool in the ordinary treatment. This raises, for instance, the important question of how to define the vacuum state, which seems to be a prerequisite for a meaningful definition of a 1-particle state. Also, it is not clear that the quantum fields can still be expected to satisfy asymptotic conditions like the LSZ condition, that is, to converge in a suitable sense to free fields for large positive or negative times, unless the external fields tend to zero fast enough in this limit, which is clearly not the case for static external fields.

As a result, the scattering amplitude underlying the conventional formalism does not actually exist. In a perturbative treatment this non-existence manifests itself most prominently by the IR divergences of the formal expression of the amplitude. The traditional way of handling this problem consists in starting from an IR-regularized theory, typically by introducing a positive photon mass \( \mu \), and letting \( \mu \) tend to zero in the final expression for whatever physical quantity one is interested in. But the fact that this derivation yields a finite (i.e. divergence-free) value of the magnetic moment does not make it any less suspect. It has hardly more than a heuristic value. Indeed, it has been shown in [6] that in the case of scattering cross sections this method very likely produces erroneous results.

In view of the undeniable success in describing observation, the theoretical formula for the magnetic moment thus deviously obtained is, however, undoubtedly correct. But because of the importance of this result a more convincing derivation is desirable. Such a derivation will be proposed in the present paper.

2. Outline of the Method

The method to be presented is based on the particle notion introduced in [6].\(^3\) Particles play no fundamental role in the theory. They are secondary phenomenological objects which are useful for the description of observations. We are especially interested in the magnetic moment of the electron due to its spin. The major ingredients of the formalism are the notion of an approximate 1-electron state and an intrinsic definition of the magnetic moment not relying on its response to external fields.

\(^2\) This description uses the language of the interaction representation, which is mathematically unsound because of Haag’s theorem [4, 5], a fact that is unfortunately still largely ignored in the literature.

\(^3\) This reference will henceforth be quoted as BK.