RAPID TeV FLARING IN MARKARIAN 501

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ABSTRACT

We investigate rapid TeV flaring in Markarian 501 in the frame of a time-dependent one-zone synchrotron self-Compton model. In this model, electrons are accelerated to extra-relativistic energy through the stochastic particle acceleration and evolve with time, and non-thermal photons are produced by both synchrotron and inverse Compton scattering off synchrotron photons. Moreover, non-thermal photons during a pre-flare are produced by the relativistic electrons in the steady state and those during a flare are produced by the electrons whose injection rate is changed during some time interval. We apply the model to the rapid flare of Markarian 501 on 2005 July 9, and obtain the multi-wavelength spectra during the pre-flare and during the flare. Our results show that the time-dependent properties of flares can be reproduced by varying the injection rate of electrons and a clear canonical counterclockwise loop can be given.

Key words: acceleration of particles – BL Lacertae objects: individual (Markarian 501) – radiation mechanisms: non-thermal

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1. INTRODUCTION

Variability, which is found from radio to TeV γ-ray bands, is one of the major characteristics of blazars. The variability timescales from a few minutes to days in the optical band have been extensively investigated (e.g., Sillanpää et al. 1991; Wagner & Witzel 1996; Lainela et al. 1999). Particularly in the X-ray and TeV regimes in which photons are produced by radiation of ultra-relativistic electrons close to their maximum energy, the observations of variability timescales constrain the particle acceleration mechanism in TeV blazars. For example, Kataoka et al. (2001) reported the observations of the X-ray flares with timescales from hours to days for three TeV blazars (Markarian 421, Markarian 501, and PKS 2155-304), and Albert et al. (2007) obtained a rapid TeV variability of several tens of minutes for Markarian 501 by MAGIC. The observed short timescales indicate that the variability is associated with small regions in the relativistic jet, which is located at a distance in excess of 100 Schwarzschild radii ($r_s$) with a central black hole mass $M = 10^9 M_\odot$, rather than the center region (Begelman et al. 2008). Relativistic particles may be responsible for the emission flare. These particles are ejected from the central region alone with the subsistent jet structure, and radiate away their energy at 100$r_s$ quickly, or the particles are accelerated within the jet, close to the emission region.

Generally, soft lags can be interpreted as being due to electron cooling (Kirk et al. 1998; Kirk & Mastichiadis 1999; Kusunose et al. 2000). However, with the fast TeV γ-ray flare in Markarian 501 on 2005 July 9, the evidence that hard γ-rays lagged the soft ones by 4 ± 1 minutes was discovered (Albert et al. 2007). To explain these abnormal phenomena, Bednarek & Wagner (2008) proposed that the radiating blob accelerated during the flare, but in their model the particles would only undergo cooling processes without any acceleration around the high blob Lorentz factor plasma flow. Mastichiadis & Moraitis (2008) showed that allowing the particles to accelerate gradually can explain the observed feature, and reach energies to $\gamma \sim 10^6$; the acceleration timescales is of the order of hours. Following their model, Tammi & Duffy (2009) compared four different acceleration mechanisms, and pointed out that the timescale may be too long for first-order Fermi acceleration, so the stochastic acceleration may be a promising candidate for the energy-dependent time delays.

Motivated by the above arguments, we study the time-dependent one-zone synchrotron self-Compton (SSC) model in the presence of stochastic particle acceleration, and then apply the model to Markarian 501 to explain its flare and time delay properties, especially the rapid flare of Markarian 501 on 2005 July 9. Throughout the paper, we assume the Hubble constant $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$, the matter energy density $\Omega_M = 0.27$, the radiation energy density $\Omega_r = 0$, and the dimensionless cosmological constant $\Omega_\Lambda = 0.73$.

2. THE MODEL

Assuming that the accelerated particles have an isotropic diffusion in momentum space, the evolution of the energetic particle distribution can be described by the momentum diffusion equation (Tverskoi 1967)

$$\frac{\partial f(p, t)}{\partial t} = \frac{1}{p^2} \frac{\partial}{\partial p} \left[ p^2 D(p, t) \frac{\partial f(p, t)}{\partial p} \right].$$

(1)

where $f(p, t)$ is the isotropic, homogeneous phase space density, $p$ is the dimensionless particle momentum, $\beta = \beta \gamma$, $D(p, t)$ is the momentum–diffusion coefficient due to interactions with magnetohydrodynamic waves, $\gamma$ is the particle Lorentz factor, and $\beta$ is the particle velocity in units of light velocity $c$. The particle number density $N(p, t) = 4\pi p^2 f(p, t)$ is directly related to the phase space density.

For a specific source, after including injection, radiation, and escape of the particles, Equation (1) can be rewritten as (Katarzynski et al. 2006)

$$\frac{\partial N(\gamma, t)}{\partial t} = \frac{\partial}{\partial \gamma} \left\{ [C(\gamma, t) - A(\gamma, t)]N(\gamma, t) + D(\gamma, t)\frac{\partial N(\gamma, t)}{\partial \gamma} \right\} + Q(\gamma, t) - E(\gamma, t).$$

(2)
where we have assumed that the particles are ultra-relativistic, $\beta \approx 1$, so the momentum becomes equivalent to the Lorentz factor of the particle ($p = \gamma$). In Equation (2), $C(\gamma, t) = (d\gamma/dt)_\text{syn} + (d\gamma/dt)_\text{IC}$ is the radiative cooling parameter that describes the synchrotron and inverse-Compton (IC) cooling of the particles at time $t$. For the synchrotron cooling, $(d\gamma/dt)_\text{syn} = (4/3)\sigma_T c/m_e c^2 U_B(t)\gamma^2$ is the rate of the synchrotron loss, $U_B$ is the energy densities of the magnetic field, $m_e$ is the electron rest mass, and $\sigma_T$ is the Thomson cross section. For the IC cooling, the Klein–Nishina (KN) effects at high energy will be important and will modify the electron distribution and the IC spectrum (e.g., Moderski et al. 2005; Nakar et al. 2009). The rate of IC energy losses in which the KN corrections is included is given by (Moderski et al. 2005)

$$\frac{d\gamma}{dt}_\text{IC} = \frac{4\sigma_T c}{3m_e c^2} U_{\text{rad}}(\gamma, t)\gamma^2 F_{\text{KN}},$$

where $U_{\text{rad}}(\gamma, t) = \int_{E_{\text{min}}}^{E_{\text{max}}} U(\epsilon_0) d\epsilon_0$ is the total energy density of the radiation field, $U(\epsilon_0)$ is the energy distribution of the soft photons, $\epsilon_0$ is the soft photon energy of the synchrotron radiation, and $F_{\text{KN}} = [1/U_{\text{rad}}(\gamma, t)] \int_{E_{\text{min}}}^{E_{\text{max}}} \frac{K}{\gamma U(\epsilon_0)} d\epsilon_0$. Here, $\chi = 4\gamma\epsilon_0$, and the function $f_{\text{KN}}$ can be approximated as (Moderski et al. 2005) follows:

$$f_{\text{KN}} \simeq \frac{1}{2}\left\{\ln \chi - \frac{\chi}{\chi + 1}\right\} \chi \ll 1 \text{ (Thomson limit)}$$

When $\chi \lesssim 10^4$, $f_{\text{KN}} \simeq 1/(1 + \chi)^{3/2}$. Therefore, the radiative cooling parameter is given by

$$C(\gamma, t) = \frac{4}{3} \frac{\sigma_T c}{m_e c^2} \left[ U_B(t) + U_{\text{rad}}(\gamma, t) F_{\text{KN}} \right] \gamma^2.$$

Other terms on the left-hand side of Equation (2) are as follows. $A(\gamma, t) = \gamma/\tau_{\text{esc}}$ is the acceleration term that describes the particle energy gain per unit time, which is given by

$$A(\gamma, t) = \frac{\gamma}{\tau_{\text{esc}}} = \frac{2D(\gamma, t)}{\gamma},$$

where the acceleration time $\tau_{\text{esc}} = \gamma^2/2D(\gamma, t)$ is used. $E(\gamma, t)$ represents the escape term, which is

$$E(\gamma, t) = \frac{N(\gamma, t)}{\tau_{\text{esc}}} = \frac{c}{R} N(\gamma, t),$$

where the escape timescale $\tau_{\text{esc}} = R/c$ depends on the emission region size $R$. $Q(\gamma, t)$ is the source term. Here, we consider the continuous injection case, i.e., the particles are continuously injected at the lower energy ($1 \leq \gamma \leq 2$) and systematically accelerated up to the equilibrium energy ($\gamma_e$), where the acceleration process is fully compensated for by the cooling, i.e., $\tau_{\text{cool}}(\gamma_e) = \tau_{\text{esc}}$.

In the time-dependent one-zone SSC model, Equation (2) needs to be solved by a numerical method because of the nonlinearity process involved. We adopt an implicit difference scheme given by Chang & Copper (1970). In our calculations, we adopt the forward differentiation in time and the centered differentiation in the energy (see Press et al. 1996 for a detail discussion). The merits of the implicit difference scheme are as follows: (1) the solution is always positive; (2) the particle number is always conserved; and (3) we can significantly reduce the number of mesh points in the calculation with no loss of accuracy. We define the energy mesh points of electrons with logarithmic steps:

$$\gamma_j = \gamma_{\text{min}} \left( \frac{\gamma_{\text{min}}}{\gamma_{\text{max}}} \right)^{(j-1)/(n_{\text{max}}-1)}; \quad j = 1, 2, 3, \ldots, n_{\text{max}}, \quad \text{(8)}$$

where $j_{\text{max}}$ is the number of the mesh points, and $\gamma_{\text{min}}$ and $\gamma_{\text{max}}$ are the minimum and the maximum Lorentz factors of electrons to be used in the calculation, respectively. In our calculation, a grid of 200 points has been used both for particle energy and photon frequency. Since we assume an exponential cutoff at $\gamma = \gamma_{\text{max}}$ for the particle distribution $N(\gamma, t)$, $\gamma_{\text{max}}$ is taken to be much larger than $\gamma_{\text{max}}$. Here, we adopt $\gamma_{\text{max}} = 10^7$. By defining $r = (\gamma_{\text{inf}}/\gamma_{\text{min}})^{1/(j-1)}$, the energy intervals can be expressed as $\Delta \gamma_j = (r-1)\gamma_j$, $\Delta \gamma_{j+1/2} = (r-1)\gamma_{j+1/2} = (1/r)\gamma_{j+1/2} + (\gamma_{j+1/2} + \gamma_j)$, and $\Delta \gamma_{j-1/2} = (r-1)\gamma_{j-1/2} = (1/r)\gamma_{j-1/2} - (\gamma_{j-1/2} + \gamma_j)$, e.g., see Park & Petrosian 1996. Quantities with the subscript $j+1/2$ are calculated at half grid points. In order to discretize the continuity equation, we define

$$G(\gamma, t) = [C(\gamma, t) - A(\gamma, t)]N(\gamma, t) + D(\gamma, t) \frac{\partial N(\gamma, t)}{\partial \gamma}, \quad \text{(9)}$$

and $N_j^n = N(\gamma_j, n\Delta \gamma)$. Therefore, Equation (2) can be written as

$$N_j^{n+1} - N_j^n = \frac{G_{j+1/2}^{n+1} - G_{j-1/2}^{n+1}}{\Delta \gamma_j} + Q_j^n - \frac{N_j^{n+1}}{\tau_{\text{esc}}}, \quad \text{(10)}$$

where

$$G_{j+1/2}^{n+1} = \left(C_{j+1/2}^{n+1} - A_{j+1/2}^{n+1}\right)N_j^{n+1} + D_{j+1/2}^{n+1} \frac{N_j^{n+1} - N_{j+1}^{n+1}}{\Delta \gamma_{j+1/2}}$$

and

$$G_{j-1/2}^{n+1} = \left(C_{j-1/2}^{n+1} - A_{j-1/2}^{n+1}\right)N_j^{n+1} + D_{j-1/2}^{n+1} \frac{N_j^{n+1} - N_{j-1}^{n+1}}{\Delta \gamma_{j-1/2}}.$$  

(12)

In this case, we have $N_j^{n+1} = (1/2)(N_j^n + N_{j-1}^n)$, $A_{j+1/2}^{n+1} = (1/2)(A_j + A_{j+1})$, $C_{j+1/2}^{n+1} = (1/2)(C_j + C_{j+1})$, $D_{j+1/2}^{n+1} = (1/2)(D_j + D_{j+1})$, and $N_{j+1/2} = (1/2)(N_j + N_{j-1})$, $A_{j+1/2} = (1/2)(A_j + A_{j-1})$, $C_{j+1/2} = (1/2)(C_j + C_{j-1})$, $D_{j+1/2} = (1/2)(D_j + D_{j-1})$. With the energy interval $\Delta \gamma_j$ and the time interval $\Delta t$, using the no-flux boundary condition (Park & Petrosian 1995), Equation (2) can be written in a tri-diagonal matrix and can be solved by a numerical approach (e.g., Press 1989). If the electron number density $N(\gamma, t)$ at time $t = 0$ is given, then the number density $N(\gamma, t)$ can be calculated at time $t = \Delta t$. The iteration of the above prescription gives the electron number density at an arbitrary time $t$ (e.g., Chaiberge & Ghisellini 1999).

After calculating the electron number density $N(\gamma, t)$ at a time $t$, we can use the formulas given by Katarzynski et al. (2001) to calculate the synchrotron intensity $I_s(v, t)$ and the intensity of self-Compton radiation $I_c(v, t)$, and then calculate the flux density observed at the Earth as follows:

$$F_{\text{tot}}(v, t) = \frac{\pi R^2}{d^2} \delta^3 (1 + z)[I_s(v, t) + I_c(v, t)]. \quad \text{(13)}$$

Here, $d$ is the luminosity distance, $z$ is the redshift, and $\delta = [\Gamma(1 - \beta \cos \theta)]^{-1}$ is the Doppler factor where $\Gamma$ is the blob Lorentz factor, $\theta$ is the angle of the blob vector velocity to the line of sight, and $\beta = v/c$. Since at high energies the
Compton photons may produce pairs by interacting with the synchrotron photons, this process may decrease the observed high energy radiation (Coppi & Blandford 1990; Finke et al. 2009). Katarzynski et al. (2001) analyze the absorption effect due to pair production inside the source; they found that this process is almost negligible. On the other hand, very high energy (VHE) photons from the source are attenuated by photons from the extragalactic background light (EBL). Therefore, after taking the absorption effect, the flux density observed at the Earth becomes

\[ F(\nu) = F_{\infty}(\nu, t) \exp[-\tau(\nu, z)], \]  

where \( \tau(\nu, z) \) is the absorption optical depth due to interactions with the EBL (Kneiske et al. 2004; Dwek & Krennrich 2005). In our calculation, we use the absorption optical depth which is deduced by the average EBL model in Dwek & Krennrich (2005).

3. VALIDATION OF THE NUMERICAL CODE

In order to validate our numerical code, we compare the time evolution of the electron spectrum calculated in our code with the analytic solutions given by Chang & Cooper (1970). In our calculation, we assume that electrons lose energy by synchrotron and IC cooling, where the IC cooling is assumed to occur in the Thomson regime. Since the loss rates of both synchrotron radiation and IC scattering satisfy \( d\gamma/dt \propto \gamma^2 \), we can write the characteristic cooling time as \( t_{\text{cool}}(\gamma') = 1/C_0\gamma' \) with a cooling coefficient \( C_0 \). Otherwise, the system has no injection and particles escape during the evolution process (i.e., \( Q(\gamma', t) = 0 \) and \( E(\gamma', t) = 0 \)). Under the above assumptions, Equation (2) can be written as in the steady state \( (\partial N(\gamma)/\partial t = 0) \):

\[ \frac{\partial}{\partial \gamma} \left[ (C(\gamma) - A(\gamma))N(\gamma) + D(\gamma) \frac{\partial N(\gamma)}{\partial \gamma} \right] = 0. \]  

Chang & Cooper (1970) gave the general solution of the above equation as

\[ N(\gamma) = x \exp \left( -\int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} \frac{C(\gamma') - A(\gamma')}{D(\gamma')} d\gamma' \right), \]  

where \( x \) is an integration constant. Assuming \( N_{\text{init}}(\gamma) \) is the initial electron distribution between \( \gamma_{\text{min}} \) and \( \gamma_{\text{max}} \), the total number of the particles in the system is given by \( N_{\text{total}} = \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} N_{\text{init}}(\gamma) d\gamma' \), and then the integration constant can be determined by

\[ x = \frac{N_{\text{total}}}{\exp \left( -\int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} \frac{C(\gamma') - A(\gamma')}{D(\gamma')} d\gamma' \right)}. \]  

Using \( C(\gamma) = C_0\gamma'^2 \) and \( D(\gamma) = \gamma'^2/2t_{\text{acc}} \), we can obtain

\[ N(\gamma) = x\gamma^2 \exp(-2C_0\gamma_{\text{acc}}(\gamma - 1)). \]  

At the equilibrium energy \( \gamma_e \), the acceleration process is fully compensated for by the cooling and then the acceleration time can be given by \( t_{\text{acc}} = 1/C_0\gamma_e \). When the electron energy \( \gamma \) is equal to \( \gamma_e \), Equation (11) has a maximum value. In our tests, we adopt the following values: \( \gamma_e = 10^{43}, N_{\text{init}} = 1 \, \text{cm}^{-3}, \) and \( C_0 = 3.48 \times 10^{-11} \, \text{s}^{-1} \).

In Figure 1, we show the results of the particle number density \( N(\gamma, t) \) at different evolution timescales calculated in our numerical code for three cases. For comparison, the analytic solutions given by Equation (18) are shown with black lines. The top panel (a) shows the initial electron distribution between \( \gamma_{\text{min}} = 1 \) and \( \gamma_{\text{max}} = 2 \) (case 1), the middle panel (b) shows the initial electron distribution between \( \gamma_{\text{min}} = 10^4 \) and \( \gamma_{\text{max}} = 10^5 \) (case 2), and the bottom panel (c) shows the initial electron distribution between \( \gamma_{\text{min}} = 10 \) and \( \gamma_{\text{max}} = 10^6 \) (case 3). Marks near color lines represent the evolution timescales in units of \( t_{\text{acc}} \).

(A color version of this figure is available in the online journal.)
number density of the system decreases in the initial energy range and simultaneously increases around the equilibrium as the evolution timescale increases. When the evolution timescale is increased to be more than $25 t_{\text{acc}}$, the system reaches the stationary Maxwellian distribution given by Equation (18). (See the middle panel of Figure 1.) In the second case, we assume the initial electron distribution to be between $0_{\text{min}}$ and $0_{\text{max}} = 10^6$, which indicates that the electron energy is about the equilibrium energy, and electrons cooling should dominate over all evolution processes. When the evolution timescale increases to $10 t_{\text{acc}}$, the system reaches the stationary Maxwellian distribution given by Equation (18). (See the middle panel of Figure 1.) In the third case, we use the initial electron distribution to be between $0_{\text{min}} = 10$ and $0_{\text{max}} = 10^6$. In this case, the evolution of the electron energy distribution with time depends on both electron cooling and acceleration processes and the system reaches the stationary Maxwellian distribution when the evolution timescale is about $10 t_{\text{acc}}$. (See the bottom panel of Figure 1.) In a nutshell, the electron spectra with larger evolution timescales ($t = 20 t_{\text{acc}}$ for the first case and $t \approx 10 t_{\text{acc}}$ for the second and third cases) calculated in our code are in agreement with the analytic solution (i.e., Equation (18)) given by Chang & Cooper (1970).

4. APPLICATION TO THE FLARING IN MARKARIAN 501

Using the time-dependent one-zone SSC solution for spherical geometry, we can calculate X-ray/TeV $\gamma$-ray spectra in the steady (pre-burst) and variable (in-burst) state. In order to do so, first we search for the steady-state solution for electron and photon spectra. Assuming a constant initial electron distribution $N_{\text{inj}}(\gamma, 0) = 2.1 \, \text{cm}^{-3}$ for $1 \leq \gamma \leq 2$, we calculate the time evolution of the spectra to $t = 15 t_{\text{acc}}$, where the injection rate of the electron population is $Q(\gamma) = 2.1 \, \text{cm}^{-3} \, \text{s}^{-1}$ for $1 \leq \gamma \leq 2$ and a constant escape for all evolution processes is assumed. The parameters are used as follows. The minimum and the maximum Lorentz factors of electrons are $\gamma_{\text{min}} = 1$ and $\gamma_{\text{max}} = 10^5$, respectively, the magnetic field strength is $B = 0.71 \, \text{G}$, the emission region size is $R = 0.205 \times 10^{15} \, \text{cm}$, the Doppler factor is $\delta = 22.5$, and the acceleration timescale $t_{\text{acc}} = t_{\text{esc}} = R/c$. In Figure 2, we show the changes when calculating the energy flux $\nu F_{\nu}$ at the four energy bands of 0.15–0.25 TeV, 0.25–0.6 TeV, 0.6–1.2 TeV, and 1.2–10 TeV with the evolution time normalized to the acceleration time. It can be seen from Figure 2 that the steady states for all TeV energy bands can be reached when the evolution time $t \geq 10 t_{\text{acc}}$.

We assume that relativistic electrons are in the steady state during the pre-burst of X-rays and TeV $\gamma$-rays. Therefore, we can calculate the pre-burst X-ray/TeV $\gamma$-ray spectrum in the one-zone SSC model using the steady-state electron spectrum. In Figure 3, we show predicted pre-burst spectrum from the X-ray to TeV $\gamma$-ray bands (the solid curve). For comparison, the observed data of Markarian 501 at the X-ray band and the TeV band on 2005 July 9 (Albert et al. 2007) are also shown, where black solid circles with error bars represent the observed values at the pre-burst. It can be seen that the observed data in the pre-burst state can be reproduced in the SSC model.

We now consider the properties of the TeV $\gamma$-ray flare of Markarian 501 in 2005 July. In order to do it, we use the physical parameters selected above and consider the resulting steady-state spectrum as an initial condition, but we change the injection rate of the electron population to

$$Q(\gamma) = \begin{cases} 5.88 \, \text{cm}^{-3} \, \text{s}^{-1} & \text{for } 1 \leq \gamma \leq 2 \\ 2.1 \, \text{cm}^{-3} \, \text{s}^{-1} & \text{otherwise} \end{cases}$$

(19)

Under the above assumptions, we reproduce the observed TeV photon spectrum (the dashed curve) of Markarian 501 on 2005 July 9 in Figure 3. Furthermore we simulated the light curves at energy bands of 0.15–0.25 TeV, 0.25–0.6 TeV, 0.6–1.2 TeV, and 1.2–10 TeV, respectively, and show the results in Figure 4, where the fluxes are normalized to the pre-burst state. It can be seen that (1) the quasi-symmetric light curve during the flare is reproduced quite well; (2) the peaking time of the flare at higher energies lags relative to that at lower energies; and (3) the amplitude of the flare becomes smaller as the photon energy increases.

In order to compare the simulated light curves with the observations, we show the comparisons of the predicted light curves with the observations by MAGIC (Albert et al. 2008) in Figure 5. The integrated fluxes on the left side of the vertical dashed lines of this figure are estimated using the differential spectra showed in Figure 3. It can be seen from this figure that our model can reproduce the flare at the energy bands of 0.15–0.25 TeV, 0.25–0.6 TeV, 0.6–1.2 TeV, and 1.2–10 TeV.
Figure 4. Simulated light curves at energy bands of 0.15–0.25 TeV, 0.25–0.6 TeV, 0.6–1.2 TeV, and 1.2–10 TeV for the rapid flare of Markarian 501 on 2005 July 9. The fluxes at different wavelengths are normalized to the pre-burst state value. The quasi-symmetric shape of the light curve and decreasing time lag of the peak with increasing energy are clearly seen.

Figure 5. Comparisons of simulated light curves (solid lines) with observational light curves (data points) from the Albert et al. (2008) for the night of 2005 July 9. The vertical dashed lines divide the light curves into steady (i.e., pre-burst) and variable (i.e., in-burst) states.

Finally, we calculate the time lag between 0.15–0.25 TeV and 1.2–10 TeV using the Gaussian fit of the simulated light curves, and find out that the light curve at higher energies (1.2–10 TeV) lags relative to that at lower energies (0.15–0.25 TeV) by a factor of about 0.9\(t_{\text{acc}}\); this timescale corresponds to about 4.7 minutes in the observer’s frame. We also calculate the evolution of the hardness ratio which is defined as the ratio \(\frac{F(1.2–10 \text{ TeV})}{F(0.6–1.2 \text{ TeV})}\). The evolution of the hardness ratio with the emitted flux above 1.2 TeV is shown in Figure 6. It can be seen from Figure 6 that the evolution of the flare points shows a clear canonical counterclockwise loop.

5. DISCUSSION AND CONCLUSIONS

In this paper, we have tried to explain the TeV \(\gamma\)-ray flare of Markarian 501 observed by MAGIC telescope on 2005 July 9, in the context of the time-dependent one-zone SSC model which includes stochastic particle acceleration. In this model, particles with low energy are assumed to be injected and then are accelerated to higher energies by the second-order Fermi acceleration mechanism (Fermi 1949); the most important photon targets for IC scattering by relativistic electrons are the synchrotron photons. We have studied time-dependent properties of flares by reproducing the pre-burst spectrum of the source and varying the injection rate. In our results, the behavior of the mean multi-frequencies spectra before and during the flare is a little different. The peaks of both synchrotron and IC emissions move to lower frequencies. We argue that this can be explained by the energy loss of the electrons during the outburst. In this scenario, the hard lag flare can be obtained and during the flare, it shows a clear canonical counterclockwise loop.

It should be noted that hard lags require some sort of particle acceleration. If the variability timescale is faster than the cooling timescale, the radiation from accelerated particles would show a hard lag (Albert et al. 2008). Kirk et al. (1998) argued that the hard lag from the acceleration process induces to the counterclockwise-loop pattern. In this view, Albert et al. (2007) concluded that the acceleration process of low energy particles probably dominate over the TeV \(\gamma\)-ray flare. Assuming low energy electron injection and stochastic acceleration, our calculations predicted a hard lag-dependent flaring activity and showed a counterclockwise-loop evolution of the hardness ratio with the flux. These are in agreement with the observational results on 2005 July 9, and imply that, during the flare, the dynamics of the system is dominated by the acceleration, rather than by the cooling processes. However, a detailed investigation
of electron acceleration in the presence of losses has so far been performed only by a few investigators (e.g., Mastichiadis & Moraits 2008). Given the complexity of the flaring activity of high energy radiation, this requires more detailed observations and that the issue remain open.

The magnetohydrodynamic turbulence will be generated if standing shocks form in the neighborhood of the central object, which amplify any incoming upstream turbulence in the downstream accretion shock magnetosheath (Campeanu & Schlickeiser 1992). These magnetohydrodynamic plasma waves are free energy and lead to stochastic acceleration of charged particles. Actually, stochastic acceleration occurs wherever there are turbulent magnetic fields and can spread to an extended region; the size is determined by the turbulence generation and decay rates. Virtanen & Vainio (2005) simulated the stochastic acceleration in relativistic shocks and showed, when the particles were accelerated behind the discontinuity, a gradual shift of the whole particle spectrum toward higher energy. Some recent observations of particle spectra with hard power-law spectral indices, \( N(y) \propto y^{-n} \) with \( n < 2 \), suggests that the stochastic acceleration is seen in the observations (Katarzynski et al. 2006; Böttcher et al. 2008). The model presented here contains the stochastic acceleration process. For simplicity, we introduced a constant acceleration term, which is associated with the momentum diffusion coefficient \( D(p, t) \). The form of the diffusion coefficient due to interactions with magnetohydrodynamic waves has been discussed in detail (e.g., Kulsrud & Ferrari 1971; Schlickeiser 2002). In our model, both constant acceleration and escape times are assumed, leading to \( D(y, t) = y^2/2\tau_{acc} \propto y^2 \). The form of the diffusion coefficient corresponds to the hard-sphere approximation, in which the free path for article–wave interaction is independent of particle energy, and probably induces a complicated spectrum. Furthermore, since the basic shock acceleration models postulate that \( \tau_{esc} \propto \tau_{acc} \) (e.g., Katarzynski et al. 2006), we adopt shorter acceleration and escape timescales \( \tau_{esc} = \tau_{acc} = \tau_{acc} = \tau_{acc} \) than other investigators (generally, \( \tau_{esc} > \tau_{esc} > \tau_{esc} > \tau_{esc} \); see, e.g., Kirk et al. 1998; Mastichiadis & Moraits 2008). These assumptions can lead to a higher acceleration rate and a lower escape rate, and make more particles accelerate to high energy rapidly.

There are two scenarios for explaining the intrinsic variability. The first scenario assumes that the observed variations originate from the geometry of emitting sources (e.g., Camenzind & Krockenberger 1992; Gopal-Krishna & Wiita 1992). The second scenario assumes that the variability is generated by the change of the emission condition. A typical example is that fresh particles are injected into the acceleration region and then are accelerated (e.g., Blandford & Konigl 1979; Marscher & Gear 1985; Celotti et al. 1991; Kirk et al. 1998). In order to reproduce both high energy radiation and variability of Markarian 501, we change the injection rate of the low energy particles. It should be noted that when the shock front over-runs a region in the jet in which the local plasma density is enhanced, the number of particles increase as an avalanche occurs in the jet, and the injection rate can be expected to change.

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