On the Value of LES Models for Evaluating Spatio-Temporal Tropospheric Variability in Multitemporal SAR Interferograms

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Abstract—Atmospheric delay has a profound impact on synthetic aperture radar (SAR) interferometry, inducing a spatial signal that significantly devaluates interferometric products. While the wide-scale variability of the atmosphere can be adequately modeled with global or regional weather models, it is especially the turbulent and convective part of the atmosphere that at smaller spatial and temporal scales is typically poorly captured. Due to the high resolution and precision of InSAR there is a need for a realistic modeling of the 3-D distribution of turbulent refractivity in the boundary layer. This would enable assessment of the impact of a temporal or spatial model misalignment on the interferometric products, and contribute to studying the impact for future SAR missions. Here we test the feasibility of an advanced large eddy simulation (LES) model to simulate a time-series refractivity distribution with a high spatio-temporal resolution to show the spatio-temporal variability of the troposphere on short time scales. We found for a fair-weather situation that the LES model produces realistic atmospheric simulations that match stochastically with results found in interferometric studies and that tropospheric delay variation leads to significant phase gradients within several minutes. This implies that even when using an (unrealistic) perfect weather model with 

resolutions similar to the SAR image, realizations that are several minutes apart from the time of the SAR acquisition will lead to significant phase errors. We propose the use of LES models as a realistic instrument to perform InSAR quality-assessments and for the development and simulation for future missions.

Index Terms—Large-eddy simulation (LES), multitemporal interferometric synthetic aperture radar (InSAR), tropospheric delay.

I. INTRODUCTION

SYNTHETIC aperture radar (SAR) plays an important role in remote sensing, by observing the Earth’s surface with a large coverage and acquiring information about its physical properties. Using interferometric (In)SAR, the interferometric phase denotes the path delay between two SAR observations along the line of sight, reflecting the topographic height difference, surface motion, and atmospheric delay [1], [2], [3], [4], [5]. Depending on the purpose of the interferometric study, the atmospheric delay is either a nuisance signal that needs to be mitigated, or the meteorological signal of interest that reveals information on the integrated refractivity and water vapor distribution [6], [7]. For both objectives, atmospheric dynamics at meso-gamma-scales (2–20 km) are most challenging, as the synoptic, meso-alpha, and meso-beta scales are better recoverable with global and regional numerical weather prediction (NWP) models [8], [9].

As a nuisance signal, we need to better understand the nature of the variability of the atmospheric delay signal at meso-gamma scales. For these scales, temporal and spatial resolvability are intimately entwined, which implies that a misalignment, either temporal or spatial, has a significant impact. Thus, there is a limit to the use of weather models for atmospheric correction of SAR interferograms, below which a correction is likely to result in an adverse negative effect.

As a meteorological signal of interest, the precision and spatial resolution of InSAR holds significant potential for assimilation into NWP models, e.g., for identifying extreme rain fall [10], [11]. Moreover, future geosynchronous (GEO) SAR missions potentially have a short revisit time and large coverage of potentially more than 1000 km [12], leading to an opportunity to obtain real-time surface observation for fast disaster warning [13]. Such short temporal baselines enable the frequent and continuous production of integrated refractivity maps with high resolution. Near real-time integrated refractivity can improve weather forecasting and consequently provide early warnings for severe weather or increased risks for landslides [14].

Large eddy simulation (LES) models use computational fluid dynamics to model turbulence. Simulations of moist convection are run at resolutions high enough to resolve convection explicitly [8], making them very well suited for studying meso-gamma scale atmospheric signal in InSAR. Yet, the use of LES models for InSAR has not been investigated in depth thus far. To investigate the value of LES models for this purpose, we conduct a proof-of-concept study of a simulated fair-weather day that depicts typical weather conditions over mid-latitude land. We introduce a standard approach to generate atmospheric interferograms, i.e., synthetic interferograms only sensitive to
atmospheric delay variability, constructed from a hypothetical ideal SAR mission, showing the time-varying characteristics of the troposphere. We refer to them as “synthetic interferograms” in this study. First, the Dutch atmospheric large eddy simulation (DALES) model and its output atmospheric parameters are briefly reviewed, including details of the transformation from model parameters to refractivity. Slant delays are calculated with variable orbit parameters. Finally, time series of atmospheric interferograms are generated and the impacts of their spatio-temporal characteristics are shown.

II. Interferometric Delay

In this part, we introduce the standard approach to generate the synthetic interferograms using the atmospheric parameters simulated by the DALES model. It contains four parts, i.e., parameters conversion, refractivity calculation, ray tracing the slant radar path, and interferogram generation.

A. Definition of the Slant Delay

The two-way slant delay difference $2\delta_{p,t}$ for a location $p$ at acquisition $t$ expressed as phase delay ($\psi_{p,t}$) is given by

$$\psi_{p,t} = \frac{4\pi}{\lambda} \delta_{p,t}$$

where $\lambda$ is the radar wavelength. The observed phase difference $\phi_p$ in the interferogram between acquisitions $t_1$ and $t_2$ is

$$\phi_p = \frac{4\pi}{\lambda} (\delta_{p,t_2} - \delta_{p,t_1}).$$

Given the antenna position $a$, the delay at the image pixel $p$ is obtained by integrating along the path $l$ of the line of sight using the refractivity $N$, which is defined as \cite{17}, \cite{15}

$$\delta_{p,t} = 10^{-6} \int_{l=p}^{a} N(x(l), y(l), z(l), t) dl.$$  \hfill (3)

Considering all types of atmospheric delay, the original expressions of the refractivity can be written as \cite{16}

$$N = k_1 \frac{P_d}{T} + \left( k_2 \frac{e}{T} + k_3 \frac{e}{T^2} \right) + k_4 \frac{n_e}{T} + k_5 W$$  \hfill (4)

where $P_d$ is partial pressure of dry air, $e$ denotes partial pressure of water vapor, $T$ is temperature, $n_e$ is the electron density per cubic meter, $f$ is the radar frequency, and $W$ is the liquid water content. The constants used in this article are $k_1 = 77.6 \text{ K hPa}^{-1}$, $k_2 = 70.4 \text{ K hPa}^{-1}$, $k_3 = 3.739 \times 10^5 \text{ K}^2 \text{ hPa}^{-1}$, $k_4 = 4.028 \times 10^7 \text{ m}^{-3}$, and $k_5 = 1.45 \text{ m}^3 \text{ g}^{-1}$. The last two terms in (4) are the ionospheric term and the liquid term, which are not considered in this study. The other three terms are referred to as the tropospheric terms.

Assuming that the total atmospheric pressure is $P = P_d + e$, the tropospheric refractivity $N_{\text{mo}}$ is the summation of a hydrostatic and a “wet” component

$$N_{\text{mo}} = k_1 \frac{P}{T} + \left( k_2 \frac{e}{T} + k_3 \frac{e}{T^2} \right)$$  \hfill (5)

where $k_2 = k_2 - k_1 R_d R_c^{-1}$, with $R_d = 287.053 \text{ J K}^{-1} \text{ kg}^{-1}$ and $R_c = 461.524 \text{ J K}^{-1} \text{ kg}^{-1}$.

B. Parameters in the DALES Model

The numerical atmospheric model used in this research is DALES, the Dutch Atmospheric Large Eddy Simulation model \cite{17}, which can provide reliable results for a multitude of atmospheric conditions. The LES uses liquid water potential temperature $T_L$ and specific humidity $q_r$, as prognostic variables and a diagnostic hydrostatic total pressure $p$ and liquid water specific humidity $q_l$. Since the tropospheric refractivity depends on partial pressure of water vapor, total pressure and temperature, see (5), we need to convert the DALES parameters for $e$ and $T$. The definition of the specific humidity is

$$q_v = \frac{m_v}{m_t}$$  \hfill (6)

where $m_v$ denotes the specific mass of water vapor and $m_t = m_d + m_v$ denotes the sum of the dry air specific mass and the specific mass of water vapor. Using the gas law and Dalton’s law we can rewrite $q_v$ in terms of the vapor pressure as \cite{17}

$$q_v = e \frac{e}{p + e(e - 1)}$$  \hfill (7)

where $e = R_d/R_c$. Assuming that $q_v$ is much smaller than 1, inverting this gives the desired relation

$$e = \frac{p}{e} 1 + (1/e - 1)q_v \approx \frac{p}{e} q_v.$$  \hfill (8)

To compute the temperature $T$, we use the linearized form of the liquid water potential temperature $T_L$ in DALES \cite{15}, \cite{18}

$$\theta_L = 1 \frac{1}{\Pi} T - \frac{L_v}{c_p \Pi} q_l$$  \hfill (9)

where the latent heat of vaporization $L_v = 2.25 \times 10^6 \text{ J} \cdot \text{kg}^{-1}$, and the specific heat of dry air $c_p = 1004 \text{ J} \cdot \text{K}^{-1} \cdot \text{kg}^{-1}$. We use the Exner function defined as $\Pi = (\frac{p}{p_0})^{R_d/c_p}$ and the standard pressure $p_0 = 1000 \text{ hPa}$. Inverting this allows to write the temperature in terms of $\theta_L$, $q_l$, and pressure $p$

$$T = \Pi \theta_L + \frac{L_v}{c_p} q_l.$$  \hfill (10)

Finally, with (8) and (10), the 4-D refractivity distribution can be obtained using (5) with all derived parameters.

We compute the tropospheric delay along the line of sight using ray-tracing \cite{19} based on the SAR geometry. In Fig. 1, the radar signal passes the troposphere from point $B$ to illuminate target $A$. Although the path of the signal is not a straight line, the contribution of the refractivity change due to the bending error is negligible for typical SAR incidence angles \cite{2}, \cite{20}. Thus, the total tropospheric delay for one acquisition is obtained by integrating the refractivity from the elevation of the target $A$ to the total height of the troposphere, cf. (3). Subsequently, the tropospheric delay is calculated using (1).

C. Interferogram Generation

Considering $m$ SAR acquisitions, $m - 1$ independent interferograms are generated and the combination of the interferograms can be one of the following two types. In the single master approach, only one SAR image is chosen as the master image,
and all interferograms are obtained by

$$\phi^i_p = \text{wrap}\{\psi^{i+1}_p - \psi^1_p\}$$  (11)

where \(\text{wrap}\{\ldots\}\) denotes the wrapping operator [2]. In the daisy chain approach, all interferograms are generated using two subsequent SAR acquisitions, as given by

$$\phi^i_p = \text{wrap}\{\psi^{i+1}_p - \psi^i_p\}.$$  (12)

Time series interferograms only show the relative changes of the interferometric phase over time. Retrieving the undifferenced integrated refractivity distribution corresponding to each SAR acquisition can be accomplished by adding a constraint using external data [21], such as MODIS [22] and GNSS data [7] to fix the observation in one of the epochs.

III. SIMULATION

We conduct large-eddy simulations using DALES, version 4.2. The simulation setup is based on radiosonde and ground observations on 21 June 1997 at the Southern Great Plains site, an extensive atmospheric measurement site in Oklahoma and Kansas, USA [23]. The simulation covers an area of 49.3 \(\times\) 49.3 \(\text{km}^2\) and is characterized by shallow cumulus convection with a cloud cover between 20% and 30%. Shallow cumulus clouds are typical fair-weather clouds and a common cloud type over continental mid-latitudes [24]. They can be simulated very well in LES [23] and are thus ideally suited for the present proof-of-concept study. After 6 h of simulation time (when sufficient atmospheric turbulence and clouds have developed), we produce 3-D output at a temporal resolution of 10 s for 15 min of simulation time to yield the 4-D refractivity distribution. The relevant parameters are shown in Table I.

The most spatio-temporally variable parameters in the simulated refractivity distribution are the specific humidity \(q_v\) and the liquid water potential temperature \(\theta_L\). Fig. 2 shows the vertical profile of these parameters, averaged over the entire spatial domain per elevation level. The average of the specific humidity, see Fig. 2(a), decreases significantly at higher altitudes, especially in the range from 1000 to 2500 m. This implies that most of the wet delay occurs lower than 2500 m. The liquid water potential temperature, see Fig. 2(b), stays nearly constant until 1000 m and increases significantly above 1000 m. Finally, the difference in the specific humidity between the first (\(t_0\)) and the last epoch (\(t_0 + 15\) min) as a function of height is shown in Fig. 2(c), indicating that the water vapor is increasing during the simulated period. Although the difference of the specific humidity within 15 min is small (\(\sim 0.3\) g/kg), it will lead to a significant change in partial pressure of water vapor, see (8). Additionally, the maximum value of the difference is situated at a height of 1340 m, which demonstrates that the total water vapor varies most significantly between 1000 and 2000 m, which is the height where clouds are present.

In (3) and (5), the tropospheric delay is divided into two parts, the hydrostatic and the wet delay. Fig. 3 shows the vertical delay for both components at \(t_0\) for a subset of 6.7 \(\times\) 6.7 km of the original dataset. While the absolute value of the hydrostatic delay is \(\sim 914\) mm, which is larger than that of the wet delay, \(\sim 218\) mm, the spatial variability range of the wet delay is dominant over that of both temperature and pressure: the spatial range of the hydrostatic delay (\(\sim 1\) mm), is much smaller than that of wet delay (\(\sim 16\) mm), see Fig. 3(a) and (b).

Note that the DALES model is evaluated up to a maximum elevation of 4.5 km. As a consequence the total hydrostatic delay is less than typically experienced when dealing with the actual atmosphere in space geodetic methods.
The tropospheric delay time series of the hydrostatic and the wet delay, evaluated at one position, is shown in Fig. 4(a) and (b), respectively. This confirms that also the temporal variation of the hydrostatic delay is smaller than that of wet delay. The hydrostatic delay varies over ~0.4 mm, while the wet delay varies over ~4 mm during the 15’ of the evaluation. Thus, the effects of the hydrostatic delay between subsequent acquisitions are extremely small (i.e., negligible over the synthetic aperture integration of a GEO InSAR mission). To investigate the sensitivity of the observed signal delays to the atmospheric parameters, we compared $T^{-1}$ and $e$, cf. (5), with the delay, for a horizontal cross section of the delay at $t_0$. Fig. 5(a) and (b) show the horizontal variation of tropospheric delay, here evaluated at a height of 900 m, which means that the tropospheric delay is integrated until 900 m and the parameters including temperature and water vapor are computed at 900 m. It shows that the spatial variation of the hydrostatic delay is highly correlated with the inverse temperature (0.9996) while that of the wet delay is very correlated with water vapor pressure (0.9998). Additionally, Fig. 5(c) and (d) show that the vertical range of variability of the tropospheric delay is much more than the horizontal variability. The vertical variability of hydrostatic delay is less correlated with the temperature while the vertical variability of the wet delay shows a strong correlation with water vapor pressure.

IV. ANALYSIS OF SYNTHETIC INTERFEROMETERS

A. Statistical Characteristics

Using a C-band radar wavelength of 0.056 m, two “tropospheric” synthetic interferograms are simulated, assuming vertical (zenith) delays, i.e., not projected to the line of sight. Based on the assumption that the refractivity distribution can be considered frozen within a 10-s window, a 15-min (15’) synthetic interferogram (combining epoch 1 and epoch 90) with a color
Fig. 6. Tropospheric synthetic interferogram over (a) 15 min (epoch 1–90) and (d) 10 s (epoch 89–90). Note the difference in the colorbar range. (b) and (c) are turbulence-only and frozen flow synthetic interferograms of 15 min synthetic interferograms, respectively. (e) shows the corresponding power spectra of (a), (b), and (c). (a) Tropospheric delay. (b) Turbulence. (c) Frozen flow. (d) Daisy chain. (e) Power spectra.

The short-term tropospheric synthetic interferogram signal can now be disentangled into two temporal effects: 1) the “frozen flow” drift of the entire refractivity distribution due to the prevailing wind (Taylor’s hypothesis [25]), combined with 2) turbulent advection during this time period. To investigate which of the two is dominant, we shift a copy of the tropospheric delay signal at \( t_0 \) to the position that maximizes the correlation with the observed synthetic interferogram between \( t_0 \) and \( (t_0 + 15 \text{ min}) \), yielding the frozen flow synthetic interferogram, see Fig. 6(c). The difference between the observed synthetic interferogram and the frozen flow synthetic interferogram, i.e., Fig. 6(a) and (d), respectively, demonstrating the significant influence of a longer temporal separation.

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The correlation coefficient time series can be an indicator to express the impact of temporal variations. We compute correlation coefficient time series by comparing the total vertical delay per epoch with the first one, showing the effect of the cumulative total vertical delay, as shown by the dashed line in Fig. 7. This shows that the correlation coefficient drops to less than 0.1 in 5 min, i.e., near-decorrelation. Alternatively, using a correction with the estimated velocity based on Taylor’s hypothesis, the corrected correlation coefficient time series are obtained: the solid line in Fig. 7. Compared with the result without velocity correction, the corrected one shows less loss of correlation, suggesting that the decorrelation is dominantly a result of the translation of the entire refractivity distribution as a result of the prevailing wind direction. Yet, after about 9 min the correlation half-time value is reached, which indicates that the use of Taylor’s hypothesis is only valid for short time spans.

This result has significant impact for strategies using numerical weather models to correct for atmospheric delay in SAR interferograms. The impact of a temporal, hence spatial, misalignment is very significant, requiring unrealistic temporal alignment constraints on the realization of the model.

Additionally, we compare the tropospheric power spectra between the unwrapped 15’ synthetic interferogram and the corresponding single epoch delays, see Fig. 8(a). It shows that the atmospheric delay signals of one epoch and an epoch 15’ later are similar in terms of their power and scaling behavior, and that the construction of a synthetic interferogram leads to a differential atmospheric signal that has more power over all of its spatial wavelengths. Fig. 8(b) shows that the scaling behavior is a distinct function of the wavelengths, with the typical \(-8/3\) scaling law for scales between 0.7 and 5 km. The less steep power-law slopes for the other wavelengths indicate a more “rough,” or noise-like, signal for these domains. The fact that this matches very well suggests that the LES model indeed provides a physically realistic refractivity distribution. As the DALES model is evaluated up to a maximum elevation
Fig. 10. Spatio-temporal variograms of the total delay for each epoch, showing a sill at a range of $\sim 1.5$ km, and temporal and spatial periodicity.

Fig. 11. Total slant delay time series for one pixel for two viewing angles ($-23^\circ$ and $23^\circ$, angular difference $46^\circ$) for (a) the most changing pixel and (b) the least changing pixel in the domain. Note the differences in range on the vertical axis.

Fig. 12. 15 min tropospheric interferogram (epoch 1–90) with different heading of (a) $300^\circ$ and (b) $120^\circ$ and their corresponding magnified views (c) and (d) (Red box in (a) and (b) denotes the test area).

Fig. 9 demonstrates how the power in a synthetic interferogram, for different spatial scales, will increase with increasing temporal separation of the acquisitions. With increasing temporal separation, i.e., the color in Fig. 9(a), we converge to a maximum. For example, at 200-m scales, the colors orange, yellow, and green are obscured by the (longest) $15^\circ$ temporal separation, while we do see lower power levels for temporal separations up to three minutes. This demonstrates a temporal ceiling of $\sim 3$ min in the energy for scales around 200 m, and of $\sim 10$ min for scales around 10 km. Again, we note that this understanding cannot be obtained from real-life interferograms, as such repeat frequencies are not available for study. Fig. 9(b) demonstrates the amount of power we may encounter in computing so-called “daisy-chain” synthetic interferograms, which have a $10$-s separation. These interferograms exhibit a similarly low absolute power, corresponding with very limited spatial variation between subsequent acquisitions, see Fig. 9(b).

Finally, Fig. 10 visualizes the spatio-temporal variograms for each epoch, showing that the semivariance increases as time goes by, a sill at scales of 1500 m, and periodicity in the variance in space and time.

B. Slant Delay Interferogram

Given the satellite orbit parameters, slant delays and (synthetic) interferograms can be obtained using ray-tracing. It is a general analysis for both LEO-SAR and GEO-SAR. Only the viewing (incidence) angle, $\theta$, should be adapted to consider the variety of orbits. In particular, the $(\cos \theta)^{-1}$ ramp is a common term in each single acquisition, which can be neglected since
the interferogram is only sensitive to the relative phase change between different acquisitions.

To assess the influence of the temporally changing atmosphere, we show the total slant delay time series of the most and the least changing pixel in the domain, see Fig. 11(a) and (b). During the 15' of the evaluation, the total slant delay varies over $\sim 40$ mm for the most variable pixel (equivalent to 1.4 cycles for C-band), while it varies only within $\sim 3$ mm for the least variable pixel. The latter is negligible during the radar imaging, while the former may have a significant effect in the SAR focusing in case of a GEOSAR imaging geometry.$^2$

Obviously, the significantly longer integration times of GEOSAR configurations, in combination with a changing atmospheric refractivity distribution, will influence the focusing, which is not covered in this study. The synthetic interferograms correspond to the APSs (atmospheric phase screens) in a system with a short integration time and a spatial resolution higher than the troposphere model. This couldn’t be applied to short time GeoSAR acquisitions, whose resolution is in the same order of magnitude of the synthetic interferogram spatial variations. In that case, the APS cannot be assumed smooth compared to the system resolution.
A change in the viewing geometry during the SAR integration time results in a different path through the atmosphere, and will, therefore, also have an effect on the phase. This may influence the SAR focusing quality, which normally assumes a “frozen” atmosphere during the aperture time [25]. We simulated extreme cases of a difference in viewing angles from $-23^\circ$ to $+23^\circ$, see the solid and dashed lines in Fig. 11(a) and (b). Note that this difference is comparable for LEOsar satellites with opposing headings (ascending and descending). For these extreme geometric differences, we see that the different path through the atmosphere may lead to more than 10 mm in delay difference over time, for the fair-weather circumstances in this simulation. 15-min tropospheric interferograms with heading angles of $300^\circ$ and $120^\circ$, showing the interferograms of ascending and descending geometry with an angular difference of $46^\circ$ are shown in Fig. 12(a) and (b). While one would expect to see a mere “shifted” version of the same signal, zoomed-in subsets shown in Fig. 12(c) and (d) show quite a significant delay difference due to the different viewing geometries. This is due to the complex 3-D distribution of tropospheric refractivity, resulting in different integration paths. Obviously, the viewing geometry differences are extreme for current SAR constellations, but the advent of (combinations of) bistatic and geosynchronous missions will force us to consider such differences.

V. IMPLICATIONS FOR GEO SAR ORBITS

Since the illumination time for GEOSAR is much longer, the space-time variability of the atmosphere may be significant. Variations of the atmospheric state during the SAR illumination time should be considered in the slant delay interferogram, inducing additional space decorrelation. We used the simulated 4-D refractivity distribution to analyze interferograms from a GeoSAR orbit. Table II shows the main parameters of the GeoSAR orbit we used in this simulation. As the time period of the simulation is 15 min, the maximal integration time is 900 s in the following analysis.

A. Analysis of Wind Direction

Based on the analysis of the vertical delay interferogram, the frozen flow plays an important role in the decorrelation of the short term interferograms. Considering the slant delay interferogram, the direction and velocity of the wind with respect to the satellite will directly affect the correlation.

The wind speed of the simulated LES scenario is 3.3 m/s with an azimuth angle of $147^\circ$. Then, setting the velocity of the satellite to this value, the slant delay interferograms with the heading angles of $147^\circ$, $327^\circ$, and $57^\circ$ are obtained. 210-s tropospheric interferograms with heading angles of $147^\circ$, $327^\circ$, and $57^\circ$ are shown in Fig. 13(a)–(c). It shows quite a significant delay difference due to the different viewing geometries. The 210-s tropospheric interferogram with a heading angle of $147^\circ$ has the least spatial variation since the space decorrelation induced by the frozen flow can be neglected, see Fig. 13(a).

The corresponding correlation coefficient time series relative to the first epoch with different heading angles are shown in Fig. 14. It shows that if the wind moves along the same direction as the satellite, the interferometric phase suffers from less decorrelation, which is similar to the result with velocity correlation. In contrast, if the wind direction is opposite to the satellite, it will induce large phase change, i.e., stronger space decorrelation. Fig. 15 shows the correlation coefficient time series with or without space decorrelation, indicating that the space variability leads to faster decorrelation and the correlation coefficient drops to less than 0.1 in two minutes.

Additionally, space decorrelation also changes the one-dimension power spectra of the interferograms with increasing temporal separation, which are shown in Fig. 16. It shows that the lower power levels of the $327^\circ$ interferograms for temporal separations is up to two minutes while that of the $147^\circ$ interferogram is 10 min.

B. Interferograms With Varying Spatial Resolution

During GEOSAR radar imaging, the tropospheric delay may lead to decorrelation in case of long integration times required for high-resolution imaging. To investigate the effect as a function of the spatial resolution, the total delay boxplots with different integration times are shown in Fig. 17, indicating that the variation of the total slant delay during the original integration time (900 s) exceeds 30 mm. If the integration time is limited to 100 s, the change in the total delay ranges from $-10$ to 10 mm, which is smaller than one cycle. Thus, interferograms with shorter integration times will suffer less from tropospheric delay. Note that the total delays in Fig. 17 are not zero mean since the actual delay is known. In GEOSAR radar imaging, high resolution requires a long integration time, suffering more from the troposphere dynamics, while it has a less influence

![Fig. 17. Variation of the total delay for one location with different integration times.](image_url)
Fig. 18. Effect of integrated refractivity resolution on the phase residual of the compensated interferogram (15 minute single master interferogram). (a)–(c) are compensated 30-m interferograms by 60, 120, and 250-m integrated refractivity. (d) and (e) are compensated 60 m interferograms by 120 and 250-m integrated refractivity. (f) is compensated 120-m interferograms by 250-m integrated refractivity. (a) 30–60 m. (b) 30–120 m. (c) 30–250 m. (d) 60–120 m. (e) 60–250 m. (f) 120–250 m.

on lower resolution images due to the shorter integration time. As a result, the refractivity distribution can be estimated in first order from a low resolution image, which is subsequently used for atmospheric correction of the high resolution image. Based on interferograms with the four resolutions (900, 450, 225, and 100 s there are six potential combinations for multiscale atmospheric correction of the 15’ single master interferogram.

Supposing that there are \( u \) low-resolution interferograms during the integration time of the high-resolution interferogram. The atmospheric correction part can be expressed using the sum of these low-resolution interferograms, as follows:

\[
\phi_L = \sum_{i=1}^{u} e^{-j\phi_i L}.
\]

Then the \( \phi_L \) will be unwrapped, interpolated to the same grid as the high-resolution interferogram and rewrapped. Finally, the rewrapped phase can be used to correct the atmospheric delay in the high-resolution interferogram.

The effectiveness of the atmospheric correction can be evaluated using the phase residuals of the 15’ compensated interferogram, which is shown in Fig. 18. Comparing Fig. 18(a)–(c) shows that the larger the difference is between the original and the corrected interferogram resolution, the higher phase SD we get. Phase residuals of the 30-m interferogram corrected by the 60-m interferogram are less than 2 mm, while a correction by the 250-m interferogram leads to phase residuals exceeding 5 mm. Thus, the effects of the troposphere dynamics during

radar imaging with long integration times can be well eliminated using the integrated refractivity estimated by interferograms with shorter integration times.

The convenience of the 30–60- or the 30–250-m solutions depends on the spatial and temporal resolution of the DTD. 30–60 is more convenient for fine spatial and low temporal scale variations, while the 30–250 configuration is preferable with fast temporal fluctuations.

VI. Conclusion

Using an advanced large eddy simulation model (DALES) for a fair-weather situation with shallow cumulus convection, typical of continental mid-latitudes, we demonstrated the high value of high-resolution LES models for the realistic spatio-temporal evolution of refractivity in relation to InSAR studies. The results quantify to what extent the turbulent part of the atmosphere contributes to the spatio-temporal variation of the tropospheric delay, and show that the stochastic properties of the LES simulations match with results found in real InSAR studies. We find that the 2-D tropospheric delay signal changes rapidly in time, decorrelating to a half-time value within 3 min. By applying a frozen-flow correction, using the main wind direction as proxy, we can delay this decorrelation to about 9 min. For longer time intervals turbulent advection becomes the dominant part of the signal.

For a particular location in the image, the maximum delay variation can be more than 30 mm within 15 min. Thus, a
temporal misalignment with even a perfect numerical weather model realization may lead to significant interpretation errors in the interferometric phase, for example in deformation studies. This suggests an intrinsic limitation in the ability of weather models to correct InSAR products for tropospheric turbulent delay signals. For future bistatic or GEO SAR missions, with more extreme variability in the viewing geometries and/or longer integration times, our results show that the LES models are effective to quantify the impact of tropospheric delay variability in the design phase.

While the proposed methodology and metrics is demonstrated for a fair-weather situation, it can be easily repeated for cases with more significant turbulence conditions. A DALES-InSAR toolbox is developed to simulate atmospheric phase screens from the parameters of DALES and to generate both vertical and slant delay interferograms with specific orbital and viewing parameters, available via GitHub.3

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