Fatigue Life Calculation of Shaft System Based on Bend-Torsion Coupling

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Abstract. The increase of the propulsion power of the ship and the increase of the distance between the two adjacent bearings lead to the reduction of the fatigue life of the shaft system. The research on the fatigue life of the ship propulsion shaft system has important theoretical significance and practical engineering application value for the ship design. Based on the ANSYS Workbench platform, this paper builds the physical model of the coupling part of the bending and torsion, obtains the transient analysis result under the working speed, and introduces it into ANSYS Ncode for fatigue analysis under flexural and torsional coupling.

1. Introduction

The ship propulsion shafting is an important part of the ship's power plant, and its structural performance affects the ship's operational reliability and safety. When the ship is operating, the ship's propulsion shafting is subjected to complex forces, including the output torque of the diesel engine, the self-weight of the shaft, the bearing support force, the weight of the propeller and the reaction force of the water. Under the continuous action of the above forces, the concentrated stress and random load will cause the normal deformation and slip zone of the shafting material, and the crack will sprout. After the expansion stage, the ship propulsion shafting will be unstable and fractured. Since the shafting structure is subjected to stress that is less than the strength limit of the material but exceeds the fatigue limit of the material, the damage caused by accumulating to a certain extent is called fatigue failure of the ship propulsion shafting. Fatigue damage is one of the main reasons for the failure of the ship's propulsion shafting, which affects the normal operation of the propulsion shafting and the navigation safety of the ship.\textsuperscript{[1]}

The existing shaft torsional vibration calculation models are divided into three categories: spring mass equivalent systems, solid models and partial solid models. Some models of the bending and torsion coupling of the propulsion shaft system have not been studied in this paper. In this case, based on ANSYS Workbench, the finite element analysis and subsequent fatigue analysis of the propulsion shaft bending and torsion coupling model are proposed.

2. Fatigue life calculation

2.1. Fatigue calculation research

In 1942, CepeHceH C.B. published the calculation formula of conventional fatigue and proposed the infinite life design based on the horizontal section of S-N curve and the finite life design based on the oblique section of S-N curve. In order to consider the influence of mean stress on life span, Miner proposed a linear cumulative damage rule based on a lot of experimental studies on fatigue cumulative damage. After the 1950s, the damage caused by fatigue gradually attracted people's attention, and a
large number of fatigue test studies were carried out. During this period, the fatigue theory was also promoted more rapidly. In low cycle fatigue, Manson-Coffin equations for plastic strain and fatigue life were published in the 1950s.

Manson put forward the method of estimating material fatigue S-N curve in the 1960s, which saved a lot of money and time cost of fatigue test and had certain prediction accuracy, and is still used today. In the early 1980s, when Martindale and Stahl were doing fatigue estimation, S-N curve was considered in sections to improve the calculation accuracy of torsion fatigue life. [2-5]

2.2. Fatigue calculation method
Existing fatigue calculation methods are divided into three categories: quasi-static, dynamic and stochastic. Based on the results of static analysis, the maximum stress is extracted, and then the simple fatigue property is assigned to the load. Dynamic fatigue calculation is based on the transient calculation results. The cycle number under different loads is calculated by rain-flow counting method, and then the irregular load is transformed into variable amplitude load, and then into cyclic symmetric load. The S-N curve measured by the experiment is used for fatigue calculation. The fatigue calculation method for random load is proposed.

2.3. Stress fatigue and strain fatigue
Fatigue analysis can be divided into stress fatigue and strain fatigue, which is determined by the properties of the material itself. The strain and stress curves of ideal elastic-plastic materials are shown in FIG. 1. Before the inflection point 1, the stress changes significantly with the shape, so the stress is a relatively sensitive index. However, after the inflection point 1, large deformation can only cause small stress changes, that is, the stress is not a more sensitive index at this time, so for this kind of load condition, it is more appropriate to describe the fatigue life of the material with strain, because it is a more sensitive index.

According to the above analysis, stress fatigue generally refers to high cycle fatigue when the stress is small. On the contrary, strain fatigue belongs to low cycle fatigue. If the stress amplitude at the resonance speed of the shafting or outside the restricted zone of the rotation speed does not exceed the allowable stress, the number of cycles of the propulsion shafting is generally higher than 10^5 times, so it belongs to high cycle fatigue. [6]

\[
R = \frac{S_{\text{min}}}{S_{\text{max}}} = \frac{S - S_a}{S_a + S_a} 
\]

(1)

2.4. S-N curve
The S-N curve of the material refers to the fatigue life of the standard specimen made of the raw material into a circular bar under a single force of tension, compression, bending or torsion. Therefore, the S-N curve of different parts of the same material is different due to their different surface quality, shape and size.

The S-N curve takes the stress S of the material standard specimen as the ordinate and the number of cycles N as the abscissa. FIG.2 is a typical stress fatigue curve. [7]

The stress level S is usually described by the stress ratio R and the stress amplitude S_a.

\[
R = \frac{S_{\text{min}}}{S_{\text{max}}} = \frac{S - S_a}{S_a + S_a} 
\]
\[ S' = \frac{S_{\text{max}} - S_{\text{min}}}{2} \]  

(2)

S\text{max} is the maximum value of the load spectrum, S\text{min} is the minimum value of the load spectrum, and Sm is the average stress.

\[ S' = \frac{S_{\text{max}} - S_{\text{min}}}{2} \]  

(3)

The figure is illustrated as follows:

1. S-N curve characterizes the relationship between stress amplitude and failure cycle number;
2. If a component bears cyclic load, after a certain number of cycles, the component will generate cracks or crack propagation, and may lead to failure of the component;
3. The greater the stress, the smaller the failure cycle;
4. There are many factors influencing S-N curve, such as Young's modulus, Poisson's ratio, density and surface quality.

The mathematical expression of S-N curve can be written as \( S_m \cdot N = C \), where \( m \) and \( C \) are related to material and stress ratio loading mode.

2.5. Prediction method for fatigue life of shafting

Fatigue can be divided into high cycle fatigue and low cycle fatigue according to failure cycle. If the shaft system resonates near the working speed, the energy rapidly accumulates, and the shaft system breaks quickly, then the cycle time is less than \( 10^4 \), which belongs to the category of low cycle fatigue. In this case, no further fatigue life calculation is needed for the shaft system.

Under the action of cyclic stress which is lower than its yield strength, the failure occurring after more than \( 10^4 \sim 10^5 \) cycles is called high cycle fatigue. If the stress amplitude under the resonance speed of the shaft or the stress amplitude outside the restricted zone of the speed does not exceed the allowable stress, the fatigue life of the propulsion shaft system will be greatly extended, and its cycle times are generally higher than \( 10^5 \) times, so it belongs to high cycle fatigue. [8]

When a part undergoes an alternating load, its interior produces an alternating stress varying with time. Even if the maximum stress amplitude is lower than the yield limit strength of the material, it will eventually generate a crack until the fracture. Such failure is called fatigue failure.

3. Transient analysis of shafting

Under the operating speed, the stress is less than the yield strength, and the shaft system will not have a static fracture. Therefore, the transient analysis and subsequent fatigue calculation are of certain significance. In the transient analysis, part of the solid model of the shafting was established. Different from the harmonic response analysis, the model considered the influence of bending moment and torque at the same time, and obtained the stress history of the shafting in several cycles through the transient analysis, which was then imported into Ncode DesignLife for fatigue analysis.
3.1. Entity model creation

In this paper, beam188 beam element, combin14 torsion spring element and mass21 mass element are used to simulate the equivalent crankshaft model.

First, the torsional stiffness of the crankshaft segment is satisfied. Beam188 unit is adopted and the diameter is set as 500 mm. Then use the formula of equivalent beam length Internal damping is simulated by the combin14 torsional spring unit. To sum up, through joint simulation of beam188 beam element, combin14 torsional spring element and mass21 concentrated mass element, the crankshaft model is simulated, which not only has torsional characteristics (torsional stiffness and damping), but also retains the mass attribute reflecting bending moment. In addition, the number of units in this equivalent model is very small, less than 100 units, so the computational efficiency can be greatly improved.

3.2. Bearing simulation

For the simulation of crankshaft bearing of the rotating damping to consider here, regardless of bearing stiffness, therefore the crankshaft bearing can use Cylindrical Support in a way that is loaded. The cylinder support is essentially a combination of CONTA174 and the Target unit, which can be used to define the rotational damping at the bearing.

Because the middle bearing and the tail bearing only consider the bearing stiffness and do not consider the rotary damping, the bearing model of Combin214 is selected.

The bearing element can consider not only the stiffness and damping coefficients in the horizontal and vertical directions, but also the influence of the coupling stiffness and damping coefficients in the horizontal and vertical directions, and can apply the damping coefficient varying with the speed. The unit is a powerful 2D support unit. FIG. 3 shows the detailed properties of the element. K11, K22, K12 and K21 are stiffness coefficients, while C11, C12, C21 and C22 are damping coefficients.

![Figure 3. Unit Combin214 attributes](image)

3.3. Mesh generation

The mesh was divided according to the mesh size when calculating the stress concentration coefficient. In order to improve the calculation accuracy and reduce the calculation time, hexahedral grid was used to divide as far as possible.

3.4. Cylinder excitation force

Though consulting literature. The Graph of tangential force of gas and the Angle of crankshaft under 100% load conditions was obtained. Origin and other software are used to read the points on the curve, and the Fourier transform principle is used to program in Matlab to obtain the coefficients av and bv of each order.
\[
a_i = \frac{1}{n} \sum_{k=0}^{2n-1} x(k \Delta t) \cos \left( \frac{\pi k \Delta t}{n} \right), \\
b_i = \frac{1}{n} \sum_{k=0}^{2n-1} x(k \Delta t) \sin \left( \frac{\pi k \Delta t}{n} \right), \\
a_0 = \frac{1}{n} \sum_{k=0}^{2n-1} x(k \Delta t);
\]

(4)

Type, \(a_0\) is initial value, \(\Delta t\) is the interval.

That is, the numerical expression of tangential force of diesel cylinder is:

\[
p_{\theta} = a_0 + \sum (a \cos(i\omega t) + b \sin(i\omega t))
\]

(5)

3.5. Crankshaft driving torque

According to the knowledge of diesel engine dynamics, the tangential gas pressure \(F_t\) acting on the crank can be obtained:

\[
F_t = P_{\theta} \left( \frac{\pi}{4} D^2 \right) \sin(\alpha + \beta) \cos \beta
\]

(6)

The torque \(T\) transferred to the crankshaft through the connecting rod is:

\[
T = P_{\theta} \left( \frac{\pi}{4} D^2 \right) R \sin(\alpha + \beta) \cos \beta
\]

(7)

Where, \(D\) -- piston diameter; \(R\) -- crank radius.

This expression indicates that the torque received by the crankshaft is also a torque that changes periodically with time, so torque \(T\) can be expressed as:

\[
T = \frac{\pi}{4} RD^2 (a \cos(i\omega t) + b \sin(i\omega t))
\]

(8)

3.6. Transient analysis Settings and results

The line time is set to 5 seconds and the load step is set to 2000 steps. There are two reasons for setting the load step in this way: one is not to miss the peak valley value; the other is to avoid the difficulty of convergence caused by setting the sub-step too large. According to the results, the maximum stress is generated at the tail bearing, flange hole, shaft and flange joint.

4. Fatigue life analysis under bending - torsion coupling

The transient calculation result file rst based on ANSYS contains the displacement and stress of the entire shaft system. In Ansys Ncode DesignLife, Ncode reads each unit under each time step (or Node) stress results and strain results, etc., that is, as long as the data stored in the RST result file, Ncode can be read.

Since Ncode DesignLife has been integrated into ANSYS Workbench, it is very convenient to transfer the results of transient analysis to Ncode for fatigue analysis.

The roughness is set as 6.3 micron, and other correction parameters can also be set in this field, such as material proportion factor, surface treatment factor, etc.

The S-N calculation method is selected as the standard S-N algorithm, and the material properties should also correspond to it. The stress combination mode is the maximum principal stress, and the correction mode of the average stress is Goodman curve.

According to the above analysis results, under normal operation of the shaft system, the whole shaft system is designed with infinite life. This is also consistent with the actual design. As the propulsion shaft system is the heart of the propulsion system, it is quite dangerous to lose power.
during the voyage, so the detailed design is usually conservative. The fatigue life of shaft system at rated speed is analyzed in this paper. In actual use, the start-up and parking of the main engine, as well as other impact loads, such as propellers hitting ice cubes or anchor chains, and the breaking of teeth in the gearbox, will reduce the actual life of the shaft system.

5. Conclusion
In this paper, the fatigue life of shafts under bending and torsion coupling is analyzed, and the basic principle and flow of fatigue calculation are discussed in detail. Based on ANSYS Workbench, a part of the solid model of shaft bending and torsion coupling was established. To reduce the calculation time, a method of simplifying the model was described in detail. The equivalent crankshaft model and the connection form of the motion pair were used to calculate the transient analysis results at the working speed, which were imported into ANSYS Ncode for the fatigue analysis under the coupling of bending and torsion.

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