Cosmic Microwave Background Tests of Inflation

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Inflation provides a unified paradigm for understanding the isotropy of the cosmic microwave background (CMB), the flatness problem, and the origin of large-scale structure. Although the physics responsible for inflation is not yet well understood, slow-roll inflation generically makes several predictions: a flat Universe, primordial adiabatic density perturbations, and a stochastic gravity-wave background. Inflation further predicts specific relations between the amplitudes and shapes of the spectrum of density perturbations and gravity waves. There are now excellent prospects for testing precisely these predictions with forthcoming CMB temperature and polarization maps. Here I discuss these new CMB tests of inflation.

1. INTRODUCTION

Despite its major triumphs (the expansion, nucleosynthesis, and the cosmic microwave background), the big-bang theory for the origin of the Universe leaves several questions unanswered. Chief amongst these is the horizon problem: When cosmic microwave background (CMB) photons last scattered, the age of the Universe was roughly 100,000 years, much smaller than its current age of roughly 10 billion years. After taking into account the expansion of the Universe, one finds that the angle subtended by a causally connected region at the surface of last scatter is roughly $1\degree$. However, there are 40,000 square degrees on the surface of the sky. Therefore, when we look at the CMB over the entire sky, we are looking at 40,000 disconnected regions of the Universe. But quite remarkably, each has the same temperature to roughly one part in $10^{5}$!

The most satisfying (only?) explanation for this is slow-roll inflation, a period of accelerated expansion in the early Universe driven by the vacuum energy most likely associated with a symmetric phase of a GUT Higgs field (or perhaps Planck-scale physics or Peccei-Quinn symmetry breaking). Although the physics responsible for inflation is still not well understood, inflation generically predicts (1) a flat Universe; (2) that primordial adiabatic (i.e., curvature) perturbations are responsible for the large-scale structure (LSS) in the Universe today; and (3) a stochastic gravity-wave background. More precisely, inflation predicts a spectrum $P_s = A_s k^{n_s}$ (with $n_s$ near unity) of primordial density (scalar metric) perturbations, and a stochastic gravity-wave background (tensor metric perturbations) with spectrum $P_t = A_t k^{n_t}$ (with $n_t$ small compared with unity). Inflation further uniquely predicts (4) specific relations between the “inflationary observables,” the amplitudes $A_s$ and $A_t$ and spectral indices $n_s$ and $n_t$ of the scalar and tensor perturbations. The amplitude of the gravity-wave background is proportional to the height of the inflaton potential, and the spectral indices depend on the shape of the inflaton potential. Therefore, determination of these parameters would illuminate the physics responsible for inflation.

Until recently, none of these predictions could be tested with precision. Measured values for the density of the Universe span almost an order of magnitude. Furthermore, most do not probe the possible contribution of a cosmological constant (or some other diffuse matter component), so they do not address the geometry of the Universe. The only observable effects of a stochastic gravity-wave background are in the CMB. COBE observations do in fact provide an upper limit to the tensor amplitude, and therefore an inflaton-potential height near the GUT scale. However,
there is no way to disentangle the scalar and tensor contributions to the COBE anisotropy.

In recent years, it has become increasingly likely that adiabatic perturbations are responsible for the origin of structure. Before COBE, there were numerous plausible models for structure formation: e.g., isocurvature perturbations both with and without cold dark matter, late-time or slow phase transitions, topological defects (cosmic strings or textures), superconducting cosmic strings, explosive or seed models, a “loitering” Universe, etc. However, after COBE, only primordial adiabatic perturbations and topological defects were still considered seriously. And in the past few months, some leading proponents of topological defects have conceded that these models have difficulty accounting for the origin of large-scale structure [5].

We are now entering an exciting new era, driven by new theoretical ideas and developments in detector technology, in which the predictions of inflation will be tested with unprecedented precision. It is even conceivable that early in the next century, we will move from verification of inflation to direct investigation of the high-energy physics responsible for inflation.

The purpose of this talk is to review how forthcoming CMB experiments will test several of these predictions. I will first review the predictions of inflation for density perturbations and gravity waves. I will then discuss how a CMB polarization map may be used to isolate the gravity waves and briefly review how detection of these tensor modes may be used to learn about the physics responsible for inflation. I close with some brief remarks about further testable consequences of inflation.

2. INFLATIONARY OBSERVABLES

Inflation occurs when the energy density of the Universe is dominated by the vacuum energy $V(\phi)$ associated with some scalar field $\phi$ (the “inflaton”). During this time, the quantum fluctuations in $\phi$ produce classical scalar perturbations, and quantum fluctuations in the spacetime metric produce gravitational waves. If the inflaton potential $V(\phi)$ is given in units of $m_{pl}^2$, and the inflaton $\phi$ is in units of $m_{pl}$, then the scalar and tensor spectral indices are

$$1 - n_s = \frac{1}{8\pi} \left( \frac{V'}{V} \right)^2 - \frac{1}{4\pi} \left( \frac{V'}{V} \right)' ,$$

$$n_t = \frac{1}{8\pi} \left( \frac{V'}{V} \right)^2 . \quad (1)$$

The amplitudes can be fixed by their contribution to $C_2^{TT}$, the quadrupole moment of the CMB temperature,

$$S \equiv 6C_2^{TT,\text{scalar}} = 33.2 \left[ \frac{V^3}{(V')^2} \right] ,$$

$$T \equiv 6C_2^{TT,\text{tensor}} = 9.2V . \quad (2)$$

For the slow-roll conditions to be satisfied, we must have

$$\left( 1/16\pi \right) (V'/V)^2 \ll 1 , \quad (3)$$

$$\left( 1/8\pi \right) (V''/V) \ll 1 , \quad (4)$$

which guarantee that inflation lasts long enough to make the Universe flat and to solve the horizon problem.

When combined with COBE results, current degree-scale–anisotropy and large-scale-structure observations suggest that $T/S$ is less than order unity in inflationary models, which restricts $V \lesssim 5 \times 10^{-12}$. If the consistency relation $T/S \simeq -7n_t$ [implied by Eqs. (1) and (2)] holds, the tensor spectrum must be nearly scale invariant ($n_t \simeq 0$).

3. TEMPERATURE ANISOTROPIES

The primary goal of CMB experiments that map the temperature as a function of position on the sky is recovery of the temperature autocorrelation function or angular power spectrum of the CMB. The fractional temperature perturbation $\Delta T(\hat{n})/T$ in a given direction $\hat{n}$ can be expanded in spherical harmonics,

$$\frac{\Delta T(\hat{n})}{T} = \sum_{lm} a_{lm}^T Y_{lm}(\hat{n}) , \quad (5)$$

where the multipole coefficients are given by

$$a_{lm}^T = \int d\hat{n} Y_{lm}^*(\hat{n}) \frac{\Delta T(\hat{n})}{T} . \quad (6)$$
Statistical isotropy and homogeneity of the Universe imply that these coefficients have expectation values \( \langle a_{lmm'}^T a_{lm'm}^T \rangle = C_{l}^{TT} \delta_{ll'} \delta_{m'm} \), when averaged over the sky. Roughly speaking, the multipole moments \( C_{l}^{TT} \) measure the mean-square temperature difference between two points separated by an angle \((\theta/\text{1}^\circ) \sim 200/l \).

Predictions for the \( C_l \)'s can be made given a theory for structure formation and the values of several cosmological parameters. Fig. 1 shows predictions for models with primordial adiabatic perturbations. The wriggles come from oscillations in the photon-baryon fluid at the surface of last scatter. Each panel shows the effect of independent variation of one of the cosmological parameters. As illustrated, the height, width, and spacing of the acoustic peaks in the angular spectrum depend on these (and other) cosmological parameters.

In Ref. 6, we argued that these small-angle CMB anisotropies can be used to determine the geometry of the Universe. The angle subtended by the horizon at the surface of last scatter is \( \theta_H \sim \Omega^{1/2} \text{1}^\circ \), and the peaks in the CMB spectrum are due to causal processes at the surface of last scatter. Therefore, the angles (or values of \( l \)) at which the peaks occur determine the geometry of the Universe. This is illustrated in Fig. 1(a) where the CMB spectra for several values of \( \Omega \) are shown. As illustrated in the other panels, the angular position of the first peak is relatively insensitive to the values of other undetermined (or still imprecisely determined) cosmological parameters such as the baryon density, the Hubble constant, and the cosmological constant (as well as several others not shown such as the spectral indices and amplitudes of the scalar and tensor spectra and the ionization history of the Universe). Therefore, determination of the location of this first acoustic peak should provide a robust measure of the geometry of the Universe.

The precision attainable is ultimately limited by cosmic variance and practically by the finite angular resolution, instrumental noise, and partial sky coverage in a realistic CMB mapping experiment. Taking these considerations into account, my collaborators and I showed that future satellite missions may potentially determine \( \Omega \) to better than 10% after marginalizing over all other undetermined parameters (we considered 7 more parameters in addition to the 4 shown in Fig. 1), and better than 1% if the other parameters can be fixed by independent observations or assumption. This would be far more accurate than any traditional determinations of the geometry.

We also found that the CMB should provide determinations of the cosmological constant and baryon density far more precise than those from traditional observations. If there is more nonrelativistic matter in the Universe than baryons can account for—as suggested by current observations—it will become increasingly clear with future CMB measurements. Subsequent analyses have confirmed these estimates with more precise numerical calculations.

Although these forecasts relied on the assumptions that adiabatic perturbations were responsible for structure formation and that reionization would not erase CMB anisotropies, these assumptions have become increasingly justifiable in the past few years. As discussed above, the leading alternative theories for structure formation now appear to be in trouble, and recent detections of CMB anisotropy at degree angular separations show that the effects of reionization are small.

NASA has recently approved the flight of a satellite mission, the Microwave Anisotropy Probe (MAP) in the year 2000 to carry out these measurements, and ESA has approved the flight of a subsequent more precise experiment, the Planck Surveyor. Therefore, it appears increasingly likely that the inflationary prediction of a flat Universe will be carried out precisely in the near future.

The predictions of a nearly scale-free spectrum of primordial adiabatic perturbations will also be further tested with measurements of small-angle CMB anisotropies. The existence and structure of the acoustic peaks shown in Fig. 1 will provide an unmistakable signature of adiabatic perturbations and the spectral index \( n_s \) can be determined from fitting the theoretical curves to the data in the same way that the density, cosmological constant, baryon density, and Hubble constant are also fit.
Figure 1. Theoretical predictions for CMB spectra as a function of multipole moment $l$ for models with primordial adiabatic perturbations. In each case, the heavy curve is that for the canonical standard-CDM values, a total density $\Omega = 1$, cosmological constant $\Lambda = 0$, baryon density $\Omega_b = 0.06$, and Hubble parameter $h = 0.5$. Each graph shows the effect of variation of one of these parameters. In (d), $\Omega + \Lambda = 1$. 
Temperature anisotropies produced by a stochastic gravity-wave background would affect the shape of the angular CMB spectrum, but there is no way to disentangle the scalar and tensor contributions to the CMB anisotropy in a model-independent way. Unless the tensor signal is large, the cosmic variance from the dominant scalar modes will provide an irreducible limit to the sensitivity of a temperature map to a tensor signal.

4. CMB POLARIZATION AND GRAVITY WAVES

Although a CMB temperature map cannot unambiguously distinguish between the density-perturbation and gravity-wave contributions to the CMB, the two can be decomposed in a model-independent fashion with a map of the CMB polarization.\(^4\) Suppose we measure the linear-polarization “vector” \(\vec{P}(\mathbf{n})\) at every point \(\mathbf{n}\) on the sky. Such a vector field can be written as the gradient of a scalar function \(\nabla\) plus the curl of a vector field \(\mathbf{B}\),

\[
\vec{P}(\mathbf{n}) = \nabla A + \nabla \times \mathbf{B}.
\] (7)

The gradient (i.e., curl-free) and curl components can be decomposed by taking the divergence or curl of \(\vec{P}(\mathbf{n})\) respectively. Density perturbations are scalar metric perturbations, so they have no handedness. They can therefore produce no curl. On the other hand, gravitational waves do have a handedness so they can (and we have shown that they do) produce a curl. This therefore provides a way to detect the inflationary stochastic gravity-wave background and thereby test the relations between the inflationary observables. It should also allow one to determine (or at least constrain in the case of a nondetection) the height of the inflaton potential.

More precisely, the Stokes parameters \(Q(\mathbf{n})\) and \(U(\mathbf{n})\) (where \(Q\) and \(U\) are measured with respect to the polar \(\hat{\theta}\) and azimuthal \(\phi\) axes) which specify the linear polarization in direction \(\mathbf{n}\) are

\[
\left[\begin{array}{c}
Q(\mathbf{n}) \\
U(\mathbf{n})
\end{array}\right] = \frac{1}{2} \left[
\begin{array}{cc}
Q(\mathbf{n}) & -U(\mathbf{n}) \\
U(\mathbf{n}) & Q(\mathbf{n})
\end{array}\right] \sin \theta
\]

\[
\left[\begin{array}{c}
Q(\mathbf{n}) \\
U(\mathbf{n})
\end{array}\right] = \frac{1}{2} \left[
\begin{array}{cc}
Q(\mathbf{n}) & -U(\mathbf{n}) \\
U(\mathbf{n}) & Q(\mathbf{n})
\end{array}\right] \sin \theta
\]

where the subscripts \(ab\) are tensor indices. Just as the temperature is expanded in terms of spherical harmonics, the polarization tensor can be expanded,

\[
\frac{P_{ab}(\mathbf{n})}{T_0} = \sum_{lm} \left[ G_{(im)ab} Y^{G}_{(lm)}(\mathbf{n}) \right] + C_{(im)ab} Y^{C}_{(lm)}(\mathbf{n}),
\] (9)

in terms of the tensor spherical harmonics \(Y^{G}_{(lm)ab}\) and \(Y^{C}_{(lm)ab}\), which are a complete basis for the “gradient” (i.e., curl-free) and “curl” components of the tensor field, respectively. The mode amplitudes are given by

\[
a_{(lm)ab}^{G} = \frac{1}{T_0} \int d\mathbf{n} P_{ab}(\mathbf{n}) Y^{G*}_{(lm)}(\mathbf{n}),
\]

\[
a_{(lm)ab}^{C} = \frac{1}{T_0} \int d\mathbf{n} P_{ab}(\mathbf{n}) Y^{C*}_{(lm)}(\mathbf{n}),
\]

which can be derived from the orthonormality properties,

\[
\int d\mathbf{n} Y^{G*}_{(lm)ab}(\mathbf{n}) Y^{G}_{(l'm')}(\mathbf{n}) = \delta_{ll'} \delta_{mm'},
\]

\[
\int d\mathbf{n} Y^{C*}_{(lm)ab}(\mathbf{n}) Y^{C}_{(l'm')}(\mathbf{n}) = \delta_{ll'} \delta_{mm'},
\]

\[
\int d\mathbf{n} Y^{G*}_{(lm)ab}(\mathbf{n}) Y^{C}_{(l'm')}(\mathbf{n}) = 0.
\]

Here \(T_0\) is the cosmological mean CMB temperature and \(Q\) and \(U\) are given in brightness temperature units rather than flux units. Scalar perturbations have no handedness. Therefore, they can produce no curl, so \(a_{(lm)}^{C} = 0\) for scalar modes. On the other hand tensor modes do have a handedness, so they produce a non-zero curl, \(a_{(lm)}^{C} \neq 0\).

A given inflationary model predicts that the \(a_{(lm)}^{X}\) are gaussian random variables with zero mean, \(\langle a_{(lm)}^{X} \rangle = 0\) (for \(X, X' = \{T, G, C\}\) and covariance \(\langle a_{(l'm')}^{X'} a_{(lm)}^{X} \rangle = C_{ll'}^{XX'} \delta_{mm'}\).
Parity demands that \( C_l^{TC} = C_l^{GC} = 0 \). Therefore the statistics of the CMB temperature-polarization map are completely specified by the four sets of moments, \( C_l^{TT}, C_l^{TG}, C_l^{CG}, \) and \( C_l^{CC} \). Also, as stated above, only tensor modes will produce nonzero \( C_l^{CC} \).

To illustrate, Fig. 2 shows the four temperature-polarization power spectra. The dotted curves correspond to a COBE-normalized inflationary model with cold dark matter and no cosmological constant \((\Lambda = 0)\), Hubble constant (in units of 100 km sec\(^{-1}\) Mpc\(^{-1}\)) \( h = 0.65 \), baryon density \( \Omega_b h^2 = 0.024 \), scalar spectral index \( n_s = 1 \), no reionization, and no gravitational waves. The solid curves show the spectra for a COBE-normalized stochastic gravity-wave background with a flat scale-invariant spectrum \((h = 0.65, \Omega_b h^2 = 0.024, \text{and } \Lambda = 0)\) in a critical-density Universe. Note that the panel for \( C_l^{CC} \) contains no dotted curve since scalar perturbations produce no C polarization component. The dashed curve in the CC panel shows the tensor spectrum for a reionized model with optical depth \( \tau = 0.1 \) to the surface of last scatter.

As with a temperature map, the sensitivity of a polarization map to gravity waves will be determined by the instrumental noise and fraction of sky covered, and by the angular resolution. Suppose the detector sensitivity is \( s \) and the experiment lasts for \( t_{yr} \) years with an angular resolution better than 1°. Suppose further that we consider only the CC component of the polarization in our analysis. Then the smallest tensor amplitude \( T_{min} \) to which the experiment will be sensitive at 1σ is [16]

\[
\frac{T_{min}}{6 C_l^{TT}} \simeq 5 \times 10^{-4} \left( \frac{s}{\mu K \sqrt{sec}} \right)^2 t_{yr}^{-1}.
\]

(12)

Thus, the curl component of a full-sky polarization map is sensitive to inflaton potentials \( V \gtrsim 5 \times 10^{-15} t_{yr}^{-1} (s/\mu K \sqrt{sec})^2 \). Improvement on current constraints with only the curl polarization component requires a detector sensitivity \( s \lesssim 40^{1/2} \mu K \sqrt{sec} \). For comparison, the detector sensitivity of MAP will be \( s = O(100) \mu K \sqrt{sec} \). However, Planck may conceivably get sensitivities around \( s = 25 \mu K \sqrt{sec} \).

Even a small amount of reionization will significantly increase the polarization signal at low \( l \) [17], as shown in the CC panel of Fig. 2 for \( \tau = 0.1 \). With such a level of reionization, the sensitivity to the tensor amplitude is increased by more than a factor of 5 over that in Eq. (12). This level of reionization (if not more) is expected in cold dark matter models [18, 19], so if anything, Eq. (12) provides a conservative estimate.

Furthermore, the estimate in Eq. (12) takes into account only the information provided by the CC polarization moments. A complete likelihood analysis will fit the temperature-polarization map to the four complete sets of moments shown in Fig. 4, and this will improve the sensitivity significantly over that given in Eq. (12) [16].

5. DISCUSSION

If MAP and Planck find a CMB temperature-\( \Delta \)isotropy spectrum consistent with a flat Universe and nearly–scale-free primordial adiabatic perturbations, then the next step will be to isolate the gravity waves with the polarization of the CMB. If inflation has something to do with grand unification, then it is possible that Planck’s polarization sensitivity will be sufficient to see the polarization signature of gravity waves. However, it is also quite plausible that the height of the inflaton potential may be low enough to elude detection by Planck. If so, then a subsequent experiment with better sensitivity to polarization will need to be done.

Inflation also predicts that the distribution of primordial density perturbations is gaussian, and this can be tested with CMB temperature maps and with the study of the large-scale distribution of galaxies. Since big-bang nucleosynthesis predicts that the baryon density is \( \Omega_b \lesssim 0.1 \) and inflation predicts \( \Omega = 1 \), another prediction of inflation is a significant component of nonbaryonic dark matter. This can be either in the form of vacuum energy (i.e., a cosmological constant), and/or some new elementary particle. Therefore, discovery of particle dark matter could be interpreted as evidence for inflation.
Figure 2. Theoretical predictions for the four nonzero CMB temperature-polarization spectra as a function of multipole moment $l$. The dotted curves are from scalar perturbations in a COBE-normalized standard-CDM model. The solid curves are for a COBE-normalized scale-invariant spectrum of tensor modes. The dashed curve in the CC panel shows the tensor spectrum for a reionized model with optical depth $\tau = 0.1$ to the surface of last scatter.
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