How Rare is the Bullet Cluster?

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20 March 2022

ABSTRACT
The galaxy cluster 1E 0657-56 has a bullet-like subcluster that is moving away from the centre of the main cluster at high speed. Markevitch et al. (2004) recently estimated a relative velocity of \( V_{\text{bullet}} = 4500^{+1100}_{-800} \) km s\(^{-1}\), based on observations of the bow shock in front of the subcluster. The weak lensing analysis of Clowe et al. (2004) indicates that a substantial secondary mass peak is associated with this subcluster. We estimate the likelihood of such a configuration by examining the distribution of subhalo velocities for clusters in the Millennium Run, a large ΛCDM cosmological simulation. We find that the most massive subhalo has a velocity as high as that of the bullet subcluster in only about 1 out of every 100 cluster-sized halos. This estimate is strongly dependent on the precise velocity adopted for the bullet. One of the ten most massive subhalos has such a high velocity about 40% of the time. We conclude that the velocity of the bullet subcluster is not exceptionally high for a cluster substructure, and can be accommodated within the currently favoured ΛCDM cosmogony.

Key words: galaxies: clusters: individual (1E0657-56) - cosmology: dark matter - galaxies: formation

1 INTRODUCTION
Galaxy cluster 1E 0657-56 is a singular example of a merging system. X-ray observations of the cluster have revealed a bow shock propagating in front of a bullet-like gas cloud moving away from the core of the main cluster. Based on the gas density jump across the shock front, Markevitch et al. (2002) first derived an estimate of velocity of the bullet subcluster, \( V_{\text{bullet}} \approx 4000 \) km s\(^{-1}\). The unique geometry of the system was recently exploited by Clowe et al. (2004), who combined the X-ray observations with weak lensing analysis to show that the mass distribution does not follow that of the hot cluster gas, but could plausibly be associated with a collisionless dark matter component. These authors argue that the data are not easily reconciled with modified Newtonian Dynamics (MOND) in which the mass budget would be dominated by the gas. Markevitch et al. (2004) also use 1E 0657-56 to derive constraints on the cross-section for self-interaction of the dark matter based on the spatial offset between the peaks in the X-ray and mass distributions.

In this paper we estimate the probability of finding such a high velocity subcluster in a ΛCDM cosmology by examining the statistics of dark matter substructure halos (subhalos) in a very large cosmological N-body simulation. We calculate the distribution of subhalo velocities relative to their host halos for a large sample of cluster-sized hosts and use this to determine the fraction of clusters which contain a high velocity subcluster.

2 PROPERTIES OF THE BULLET CLUSTER
The cluster 1E 0657-56 was discovered as an extended source in the Einstein imaging proportional counter (IPC) database and was identified as a rich cluster of galaxies by Tucker et al. (1995). Tucker et al. (1998) established the cluster redshift as \( z = 0.296 \) and identified the bullet subcomponent in a ROSAT X-ray image of the cluster. They also measured the temperature of the hot cluster gas as \( kT \sim 17 \) keV, making 1E 0657-56 one of the hottest known clusters. Markevitch et al. (2004) subsequently revised this to an average cluster temperature of 14-15 keV, albeit with large spatial variations, based on Chandra observations of the system.

The transverse separation between the bullet subcluster and the centre of the main cluster is \( \sim 0.48 \ h^{-1}\text{Mpc} \) (Clowe et al. 2004). Markevitch et al. (2004) estimate the velocity of the bullet subcluster, \( V_{\text{bullet}} = 4500^{+1100}_{-800} \) km s\(^{-1}\); here and elsewhere we adopt a Hubble constant of \( H_0 = 73 \) km s\(^{-1}\) Mpc. The line-of-sight velocity of the subcluster is \( \sim 600 \) km s\(^{-1}\) relative...
to the cluster centre (Barrena et al. 2002), so the direction of the bullet’s motion is very nearly in the plane of the sky.

Clowe et al. (2004) use weak lensing to model the mass distribution of the main cluster and the bullet subcluster. They find that the main cluster is well fit by a NFW (Navarro et al. 1996) profile with concentration $c_{200} = 3.0$ and virial radius $r_{200} = 1.64 \ h^{-1}\text{Mpc}$, corresponding to a virial mass and velocity of $M_{200} = 2.16 \times 10^{15} \ h^{-1}M_\odot$ and $V_{200} = (GM_{200}/r_{200})^{1/2} = 2380 \ \text{km s}^{-1}$, respectively. They also find a secondary peak in the mass distribution that is clearly associated with the bullet subcluster, and they measure a mass of $(5.3 \pm 1.5) \times 10^{14} \ h^{-1}M_\odot$ for the subcluster within a cylinder of radius 0.11 $h^{-1}\text{Mpc}$. We adopt these values for the mass of the main cluster and the bullet rather than estimates based on the cluster galaxy velocity dispersion or the gas temperature since the latter rely on assumptions of isotropy and hydrostatic equilibrium which may not be valid for a merging cluster system.

We note that the virial mass we adopt corresponds to an overall X-ray temperature of $kT = 15 - 16 \ \text{keV}$ according to the mass-temperature relations of Arnaud et al. (2005), in good agreement with the observed X-ray temperature.

To summarize, the main properties of this system relevant to this study are as follows:

- Cluster 1E 0657-56 is a very massive cluster, as evidenced by its high temperature and confirmed by the high mass detected by weak lensing analysis.
- The bullet subcluster is the most massive substructure in the cluster, and represents a mass $\sim 2\%$ that of the main cluster.
- The bullet subcluster is moving away from the main cluster at velocity $V_{\text{bullet}} \approx 4500 \ \text{km s}^{-1} = 1.9 \ V_{200}$ and is at least 0.48 $h^{-1}\text{Mpc} \approx 0.3 \ r_{200}$ from the cluster centre.

In the following section we assess the likelihood of such an object by examining the statistics of dark matter halos in a large cosmological simulation.

### 3 SEARCHING FOR THE BULLET

This study makes use of the Millennium Run (Springel et al. 2005), a very large cosmological N-body simulation carried out by the Virgo Consortium.\footnote{1 http://www.virgo.dur.ac.uk/} In this simulation a flat ΛCDM cosmology is adopted, with $\Omega_{\text{dm}} = 0.205$ and $\Omega_b = 0.045$ for the current densities in cold dark matter and baryons, $h = 0.73$ for the present dimensionless value of the Hubble constant, $\sigma_8 = 0.9$ for the rms linear mass fluctuation in a sphere of radius 8 $h^{-1}\text{Mpc}$ extrapolated to $z = 0$, and $n = 1$ for the slope of the primordial fluctuation spectrum. The simulation follows 2160\footnote{2 http://www.mpa-garching.mpg.de/galform/virgo/millennium/} dark matter particles from $z = 127$ to $z = 0$ within a cubic region 500 $h^{-1}\text{Mpc}$ on a side. The individual particle mass is thus $8.6 \times 10^8 h^{-1}M_\odot$, and the gravitational force is softened with a Plummer-equivalent comoving softening of 5 $h^{-1}\text{kpc}$. Initial conditions were generated using the Boltzmann code CMBFAST (Seljak and Zaldarriaga 1996) to generate a realization of the desired power spectrum which was then imposed on a glass-like uniform particle load (White 1993). A modified version of the Tree-PM N-body code GADGET2 (Springel et al. 2001a, 2005) was used to carry out the simulation and full particle data are stored at 64 output times approximately equally spaced in the logarithm of the expansion factor.

In each output of the simulation, halos are identified using a friends-of-friends (FoF) groupfinder with a linking length of $b = 0.2$ (Davis et al. 1985). The virial radius, $r_{200}$, is defined for each FoF halo by calculating the radius of a sphere, centered on the particle with the minimum potential, that encompasses a mean density 200 times the critical value. Each FoF halo is decomposed into a collection of locally overdense, self-bound substructures (or subhalos) using the SUBFIND algorithm of Springel et al. (2001a). Of these subhalos, one is typically much larger than the others and contains most of the mass of the halo. We identify this as the main halo and subtract its centre of mass velocity from that of the remaining subhalos to compute the relative velocity between subhalos and their host halo. Hereafter, we refer to this as the subhalo velocity, not to be confused with the internal circular velocity of the subhalo.

We search the Millennium simulation for halos whose most massive subhalo has a velocity relative to the main halo comparable to that of the bullet subcluster, i.e., $V_{\text{sub}} > 1.9 \ V_{200}$. We focus our search on cluster- and group-sized halos in the $z = 0.28$ output of the Millennium simulation, the output closest in redshift to 1E 0657-56.

Figure 1 shows the cumulative distribution of velocities for the most massive subhalo in host halos with $M_{200} > 10^{14} h^{-1}M_\odot$, $3 \times 10^{14} h^{-1}M_\odot$, and $10^{15} h^{-1}M_\odot$. The number of hosts in each of these mass ranges is $N_{\text{hosts}} = 1491, 157, \text{ and } 5$, respectively. We note that 1E 0657-56 is one of the hottest known clusters and that very few clusters in the Millennium Run have masses comparable to that of 1E 0657-56 due to the limited volume of the simulation. According to the cluster temperature function of Henry (2004), one expects less than 0.1 clusters as hot as 1E 0657-56 in a volume similar to that of the Millennium simulation. In fact, at $z = 0.28$, the Millennium simulation contains one cluster halo with $M_{200} > 2 \times 10^{15} h^{-1}M_\odot$.

The shape of the velocity distribution appears relatively insensitive to host halo mass and the median subhalo velocity is $1.1 \ V_{200}$. We note that all subhalos have velocities much less than the maximum escape velocity, $V_{\text{esc}} \approx 3.3 \ V_{200}$ for an NFW potential with $r_{200} \approx 6$.

The fraction of halos with $M_{200} > 10^{14} h^{-1}M_\odot$ whose most massive subhalo has a velocity greater than that of the bullet subcluster (scaled to the virial velocity of the host halo) is 16 out of 1491 or approximately 1%. However the velocity distribution drops steeply at high velocities, and this percentage increases (decreases)
to 10% (≪ 0.1%) if the lower (upper) limit is adopted for the bullet velocity. We note that this is in agreement with the simulation results of Gill et al. (2005) who also find a small but significant fraction of high velocity subhalos in cluster-sized host halos.

We find that the velocity distribution is well fit by a function of the form shown in the solid curve in Figure 1.

\[
\log \frac{N_i(> V_{\text{sub}})}{N_{\text{hosts}}} = - \left( \frac{V_{\text{sub}}}{V_{200}} \right) \alpha^\frac{1}{\alpha},
\]

(1)

where \(v_{10\%}\) is the velocity in units of \(V_{200}\) at which the fraction of halos drops to 0.1. Fitting this function to the \(M_{200} > 10^{14} h^{-1} M_\odot\) distribution at \(z = 0.28\) yields best fit values of \(v_{10\%} = 1.55\) and \(\alpha = 3.3\). We note that the best fit value of \(v_{10\%}\) tends to decrease with increasing redshift: \(v_{10\%} = 1.64, 1.52,\) and \(1.32\) at \(z = 0.5, 0.11,\) and \(0,\) respectively. The value of \(\alpha\) shows no significant change with redshift and is typically consistent with \(\alpha ≃ 3.0 - 3.1.\) We attribute this to the increase in the virial velocity of the halo with respect to the velocities of subhalos at the time of infall. Indeed, the mean \(V_{200}\) increases from 780 km s\(^{-1}\) at \(z = 0.5\) to 920 km s\(^{-1}\) at \(z = 0.0,\) whereas the mean subhalo velocity decreases by only a few percent, from 1190 km s\(^{-1}\) to 1129 km s\(^{-1}\), over the same period.

Having quantified the likelihood that a halo has a most massive subhalo with a velocity comparable to that of the bullet subcluster, we now estimate the probability of finding a high velocity subhalo amongst a halo’s \(n\) most massive subhalos. The probability of drawing at least one subhalo with velocity \(V_{\text{sub}}\) from the \(n\) most massive subhalos is given by

\[
P(> V_{\text{sub}}) = 1 - \prod_{i=1}^{n} \left(1 - \frac{N_i(> V_{\text{sub}})}{N_{\text{hosts}}}\right).
\]

(2)

We investigate whether subhalo velocity is independent of subhalo mass in Figure 2 by comparing the velocity distributions for the 1st, 2nd, 3rd and 10th most massive subhalos in halos of mass \(M_{200} > 10^{14} h^{-1} M_\odot\). We find that more massive subhalos are slightly biased toward lower velocities. The best fit values of \(v_{10\%}\) are 1.55, 1.64, 1.71, and 1.79 for the velocity distributions of the 1st, 2nd, 3rd and 10th most massive subhalos, and \(v_{10\%} ≃ 1.8\) for subhalos of higher rank. We find that the following formula accurately describes the trend:

\[
v_{10\%}(i) = 1.8 - 0.25 \exp(-0.45 (i - 1)).
\]

(3)

Note that the value of \(\alpha\) decreases slightly with increasing subhalo rank, but we find that a constant value of \(\alpha = 3.1\) provides an adequate fit to the distributions for all subhalo ranks.

We combine eqs. (1) and (2) in order to predict the probability of at least one subhalo with \(V_{\text{sub}} > 1.9 V_{200}\). This gives a probability of 39.7% which agrees well with the actual fraction, 40.8%, found for halos in the Millennium simulation with \(M_{200} > 10^{14} h^{-1} M_\odot\). In comparison, if we adopt a constant value for \(v_{10\%}\) of 1.55 (1.8) we predict a probability of 12.4% (49.3%).

In order to convert the subhalo mass rank into a fraction of the host halo mass, we compute the subhalo mass function for our halo sample. We find that...
the differential subhalo mass function for halos with $M_{200} > 10^{14}$ $h^{-1} M_\odot$ is well fit by a power law with slope $-0.88$, in agreement with the mass functions of cluster halos presented by De Lucia et al. (2004). Integrating the differential mass function yields the total number of subhalos with mass greater than $M_{\text{sub}}$:

$$\log N(> M_{\text{sub}}) = -0.88 \log \frac{M_{\text{sub}}}{M_{200}} - 1.67. \quad (4)$$

Solving this equation for $M_{\text{sub}}$ gives the subhalo mass corresponding to subhalo mass rank $N$. For example, for $N = 10$ we find $M_{\text{sub}} = 0.001 M_{200}$, corresponding to subhalos of mass $10^{12} h^{-1} M_\odot$ for cluster-sized host halos of mass $M_{200} \approx 10^{15} h^{-1} M_\odot$. Our previous result therefore implies that 40% of massive cluster halos have at least one subhalo of mass greater than $10^{12} h^{-1} M_\odot$ with a velocity comparable to that of the bullet subcluster.

We now return to the bias in the velocity distribution of the most massive subhalos (see Figure 1). This is related to the fact that more massive substructures are preferentially located in the external regions of their host halos, as noted by De Lucia et al. (2004). These subhalos are closer to the apocentre of their orbits, and therefore have lower velocities compared to subhalos near pericentre. Conversely, we expect to find high velocity subhalos near the centre of their host halos. Indeed, of the 16 high velocity subhalos that are bullet subcluster candidates, all but one are located at $r < 0.6 r_{200}$, whereas only 15% of total sample of most massive subhalos are found within this radius. This correlation between subhalo velocity and clustercentric distance was also noted by Gill et al. (2006).

In Figure 3 we compare the velocity distributions of subhalos in the inner and outer regions. Subhalos within the central 0.6 $r_{200}$ of the host halo (about half of the total within $r_{200}$) are indeed biased to higher velocities. The best fit values of $v_{10\%}$ are 1.74, 2.16, and 1.87 for the velocity distributions of subhalos with $r \leq 0.6 r_{200}$, $r > 0.6 r_{200}$ and the combined sample, respectively. We also note that velocity distribution of outer subhalos is not as well fit by eq. (4) as the other distributions, however the deviations are typically $\lesssim 0.1$ dex.

Finally, we note that the direction of the bullet subcluster’s velocity is an additional constraint that we have not so far considered. As stated in Figure 1, the bullet subcluster is moving away from the cluster centre, i.e., has a positive radial velocity. Of the 16 bullet subcluster candidates, five have positive radial velocities. Overall, we find that a smaller fraction of massive subhalos tend to have positive radial velocities compared to less massive subhalos; this fraction is 20%, 30% and 40% for the most massive, second most massive, and tenth most massive subhalo samples.

We attribute this to the mass loss and disruption of subhalos by tidal stripping, which occurs near orbital pericentre. This can decrease the mass of a subhalo by as much as 90% (e.g., Hayashi et al. 2003), potentially downgrading the mass rank of more massive subhalos after they pass through pericentre. Less massive subhalos can be disrupted altogether resulting in the depletion of subhalos with positive radial velocities. However, the velocity distributions of subhalos with positive and negative radial velocities are very similar; subhalos with positive radial velocities are slightly biased toward lower velocities, but the difference in the best fit values of $v_{10\%}$ and $\alpha$ are typically $\sim 0.05$ and $\sim 0.5$, respectively. We therefore conclude that if the direction of the bullet subcluster is considered, the probabilities we have estimated are reduced by about 60 – 70%, the fraction of subhalos with negative radial velocities.

4 CONCLUSIONS

The bullet subcluster is a massive substructure in galaxy cluster 1E 0657-56 moving at a high velocity relative to the centre of the main cluster. In units of the virial velocity of the cluster, the velocity of the bullet subcluster is $V_{\text{bullet}} \approx 1.9 V_{200}$. We have examined the distribution of subhalo velocities relative to their host halos in a large cosmological simulation in order to assess the likelihood of a system like 1E 0657-56.

We calculate the velocity distribution of the most massive subhalos in 1491 host halos with virial masses $M_{200} > 10^{14} h^{-1} M_\odot$ and find that about 1 in 100 have velocities comparable to that of the bullet subcluster. If the best estimate of the velocity of the bullet subcluster from the work of Markevitch et al. (2004) is adopted for the bullet velocity, however, this fraction depends strongly on the velocity of the bullet and ranges from $\ll 0.1\%$ to $10\%$ for the upper and lower limits on the bullet velocity, respectively. We find that more massive subhalos are biased towards lower velocities, as are subhalos in the outer regions of halos. Taking this into account, we find that at least one of the ten most massive subhalos has a velocity as high as that of the bullet sub-
cluster in 40% of all host halos. We also find that subhalos are preferentially found to be moving toward the centres of halos, most likely a result of tidal depletion of subhalos at pericentre. With this additional constraint, the likelihood of the bullet cluster drops to about 1 in 500. We conclude that the best estimate for the velocity of the bullet subcluster is high but not extraordinary considering the mass of its host cluster. It is a rare but not an impossible event within the currently favoured ΛCDM cosmogony.

ACKNOWLEDGMENTS

This work was conceived after discussions with M. Bradáč. We thank I. McCarthy, M. Balogh, and S. McQueen for useful discussions.

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