Quenching of Spin Operators in the Calculation of Radiative Corrections for Nuclear Beta Decay

I. S. Towner

AECL Research, Chalk River Laboratories, Chalk River
Ontario K0J 1J0, Canada

August 1, 2018

Abstract

Calculations of the axial-vector component to the radiative correction for superallowed Fermi $0^+ \rightarrow 0^+$ nuclear beta decay are here modified with quenched rather than free-nucleon coupling constants for the axial-vector and electromagnetic interactions with nucleons. The result increases the deduced value of $V_{ud}$ but does not restore unitarity in the CKM matrix.
Superallowed Fermi $0^+ \rightarrow 0^+$ nuclear beta decays [1] provide both the best test of the Conserved Vector Current (CVC) hypothesis in weak interactions and, together with the muon lifetime, the most accurate value for the up-down quark-mixing matrix element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, $V_{ud}$. Recent developments [2, 3], however, indicate a deterioration in the quality of the CVC test and a lowering of the $V_{ud}$ value such that, with standard values [4] of the other elements of the CKM matrix, the unitarity test from the sum of the squares of the elements in the first row fail to meet unity by more than twice the estimated error.

Much of this deterioration is a consequence of the recent improvements [5, 6, 7] in the calculation of the nuclear-structure dependent part of the radiative correction. If the CVC hypothesis were correct, then the $\mathcal{F}t$ values derived by correcting measured $ft$ values for the effects of isospin-symmetry breaking and radiative corrections, should be the same for all superallowed Fermi transitions in all nuclei. In the 1990 analysis [1] the $^{26}\text{m} \alpha \ell$ data point had the lowest $\mathcal{F}t$ value; in particular it was lower than the $^{14}\text{O}$ data point. However the revised radiative correction calculation reverses this situation, leaving the $^{14}\text{O}$ data point with the smallest $\mathcal{F}t$ value. Taken with the other seven precision data from $^{26}\text{m} \alpha \ell$ to $^{54}\text{Co}$ the new analysis suggests a $Z$-dependence in the set of $\mathcal{F}t$ values, where $Z$ is the charge number of the daughter nucleus in the beta decay. Such a $Z$-dependence indicates either an electromagnetic correction is still not accounted for or that the CVC hypothesis is false.

Although we are discussing a purely vector interaction between spin $0^+$ states, the axial-vector interaction does play a role in the radiative corrections. An axial-vector interaction may flip the nucleon spin and then be followed by an electromagnetic interaction that may flip it back again. This axial contribution to the radiative correction was considered by Marciano and Sirlin [8], who cast the result into the following expression and estimated its value:

$$\frac{\alpha}{2\pi} \left[ \ln(m_p/m_A) + 2C \right] = (0.12 \pm 0.18)\%.$$  \hspace{1cm} (1)

Here $\alpha$ is the fine structure constant, $m_p$ the proton mass, $m_A$ a mass of order 1 GeV that provides a short-distance cut-off, and $C$ represents the nonasymptotic long-range correction. We write $C$ as

$$C = C_{\text{Born}} + C_{\text{NS}},$$  \hspace{1cm} (2)

where $C_{\text{Born}}$ refers to the Born graph in which the axial-vector and electromagnetic interactions occur on the same nucleon and $C_{\text{NS}}$ is a nuclear-structure dependent correction in which the interactions occur on different nucleons.
In the calculations [3, 8, 7] for C, the axial-vector and electromagnetic vertices are evaluated with free-nucleon coupling constants. Yet there is ample evidence in nuclear physics that coupling constants for spin-flip processes are quenched in the nuclear medium [9, 10]. Thus the purpose of this Letter is to repeat the calculations of [7] with quenched coupling constants and investigate to what extent this ameliorates the deterioration in the CVC test.

We assume that the axial-vector and electromagnetic vertices can be described by on-shell form factors, even though in the diagrams in question they are off-shell. Further we will use nonrelativistic reductions and consider the form factors in the zero-momentum limit and characterised by coupling constants \( g(k^2 \to 0) \). These coupling constants can then be equated with well-known coupling constants of nuclear physics as deduced from electromagnetic \( \gamma \)-transitions (and magnetic moments) and Gamow-Teller \( \beta \)-decay transitions [3].

We follow the notation of [10, 11] and write the magnetic-moment operator as

\[
\mu_{\text{eff}}^{(I)} = g_{L,\text{eff}}^{(I)}L + g_{S,\text{eff}}^{(I)}S + g_{T,\text{eff}}^{(I)}[Y_2, S],
\]

where \([Y_2, S]\) represents a spherical harmonic of rank 2, vector coupled to the spin operator, \( S \), to form a spherical tensor of rank 1. Here the superscript, \( I \), denotes the isospin structure: \( I = 0 \) being isoscalar and \( I = 1 \) isovector. The effective coupling constants are written as

\[
g_{\text{eff}} = g + \delta g,
\]

where \( g \) is the free-nucleon value and \( \delta g \) a nuclear-medium correction. The free-nucleon values are: \( g_{L}^{(0)} = 0.5, g_{L}^{(1)} = 0.5, g_{S}^{(0)} = 0.88, g_{S}^{(1)} = 4.706 \), \( g_{P}^{(0)} = 0.0 \) and \( g_{P}^{(1)} = 0.0 \). The nuclear-medium correction can also be expressed in terms of a quenching factor

\[
q = g_{\text{eff}}/g.
\]

Calculations of the nuclear-medium correction \( \delta g \) are given in [10, 11] for closed-shell-plus (or minus)-one nuclei \( A = 5, 15, 17, 39, 41 \). They are based on corrections to the single-particle wavefunction for these nuclei being evaluated through to second order in core polarisation, and on corrections for meson-exchange currents and isobars. Here we will use the values from Table 26 of [11] and extrapolate or interpolate for other mass values, \( A \). These values are in good agreement with the empirical values deduced by Brown and Wildenthal [8] in fits of shell-model calculations to experimental data on magnetic moments and M1 \( \gamma \)-transition rates in \( sd \)-shell nuclei.
Similarly we write the Gamow-Teller $\beta$-decay operator as

$$(\text{GT})_{\text{eff}} = g_{L_A,\text{eff}} L + g_{A,\text{eff}} \sigma + g_{P_A,\text{eff}} [Y_2, \sigma],$$

and note the traditional use of the Pauli matrix $\sigma$ rather than $S$. The free-nucleon values are $g_{L_A} = 0.0$, $g_A = 1.26$ and $g_{P_A} = 0.0$. Calculations of $\delta g$ can be found in [10, 11]. Here we will take values from Table 27 of [10] and again extrapolate or interpolate as required. Here the results are not in such good accord with the empirical values of Brown and Wildenthal [9] obtained in fits of shell-model calculations to experimental Gamow-Teller $\beta$-decay rates. For example, the quenching factors $q_A$ from [9] are 0.761, 0.737 and 0.727 for $A = 26$, 34 and 38 respectively showing greater quenching than the values we propose to use, as listed in Table 1, and obtained from the calculations of [10, 11]. However the use of these stronger quenching factors for $sd$-shell nuclei in the present analysis would not alter significantly the conclusions to be drawn here.

In Table 1, we list the quenching factors to be used here. For $p$-shell nuclei, they represent linear interpolations between $A = 5$ and $A = 15$; for the $sd$-shell linear interpolations between $A = 17$ and $A = 39$; while for the $pf$-shell they are extrapolated from the $A = 41$ values using a scaling factor of $(A/41)^{0.35}$ applied to $\delta g$.

For the case in which the axial-vector and electromagnetic interactions occur at the same nucleon, the radiative correction is universal, i.e. the same for all nuclei, and has the value [8, 4]

$$C_{\text{Born}}(\text{free}) = 3 g_A g_s^{(0)} I,$$

where $I$ is a loop integral. With the replacements $g_A \rightarrow q_A g_A$ and $g_s^{(0)} \rightarrow q_s^{(0)} g_s^{(0)}$ the universality is now broken, and the contribution from 1-body graphs is written

$$C_{\text{Born}} = 3 g_A g_s^{(0)} I + (q_A q_s^{(0)} - 1) 3 g_A g_s^{(0)} I = C_{\text{Born}}(\text{free}) + (q_A q_s^{(0)} - 1) C_{\text{Born}}(\text{free}),$$

where the second term becomes part of the nuclear-structure dependence of the radiative correction. For $C_{\text{Born}}(\text{free})$, we use the value $0.881 \pm 0.030$ [4].

For the two-nucleon graphs the operator is complicated and comprises 12 terms as listed in Table 2 of [6]. Different terms originate in different pieces of the electromagnetic couplings. Terms 1 and 2 are proportional to the isoscalar spin coupling.

---

#1There is one typographical error in Table 27. The entry for $\delta g_A$ for $A = 40$ 0$d_{5/2}$ should read $-0.255$. 

Table 1: Quenching factors used in the present study

|          | Electromagnetic | Weak |
|----------|-----------------|------|
|          | $q_L^{(0)}$ | $q_s^{(0)}$ | $q_L^{(1)}$ | $q_s^{(1)}$ | $q_A$ |
| $A = 10$ | 1.042 | 0.897 | 1.173 | 0.927 | 0.878 |
| $A = 14$ | 1.044 | 0.873 | 1.201 | 0.934 | 0.858 |
| $A = 26$ | 1.023 | 0.869 | 1.146 | 0.877 | 0.835 |
| $A = 34$ | 1.026 | 0.850 | 1.155 | 0.870 | 0.812 |
| $A = 38$ | 1.028 | 0.840 | 1.159 | 0.866 | 0.801 |
| $A = 42$ | 1.010 | 0.862 | 1.133 | 0.866 | 0.824 |
| $A = 46$ | 1.010 | 0.857 | 1.137 | 0.862 | 0.818 |
| $A = 50$ | 1.011 | 0.853 | 1.141 | 0.857 | 0.812 |
| $A = 54$ | 1.011 | 0.849 | 1.145 | 0.854 | 0.807 |

Table 2: Revised values for the nuclear-structure part of $C$ obtained through the introduction of quenching factors

|          | Unquenched | Quenched |
|----------|------------|----------|
|          | $C_{NS}$  | $(q_Aq_s^{(0)} - 1)C_{Born}(\text{free})$ |
| $A = 10$ | $-1.67 \pm 0.20$ | $-1.35 \pm 0.16$ |
| $A = 14$ | $-1.15 \pm 0.30$ | $-0.88 \pm 0.23$ |
| $A = 26$ | $0.25 \pm 0.05$  | $0.20 \pm 0.04$  |
| $A = 34$ | $-0.17 \pm 0.06$ | $-0.13 \pm 0.05$ |
| $A = 38$ | $-0.10 \pm 0.10$ | $-0.09 \pm 0.09$ |
| $A = 42$ | $0.50 \pm 0.10$  | $0.40 \pm 0.07$  |
| $A = 46$ | $0.16 \pm 0.03$  | $0.14 \pm 0.03$  |
| $A = 50$ | $0.16 \pm 0.03$  | $0.14 \pm 0.03$  |
| $A = 54$ | $0.20 \pm 0.03$  | $0.17 \pm 0.03$  |
$g_s^{(0)}$; terms 3 and 4 proportional to $g_s^{(1)}$; terms 5, 6, 9 and 10 proportional to $g_L^{(0)}$; and terms 7, 8, 11 and 12 proportional to $g_L^{(1)}$. The prescription then is to modify the 2-body operator by its appropriate electromagnetic quenching factor and by the weak quenching factor, $q_A$. Numerical results are given in Table 2.

The data base for superallowed $\beta$-transitions produced by the Chalk River group [1] in 1990 is here updated to include four new lifetimes [12] and four new $Q$-values [12, 13, 14]. The methodology for handling the data and the theoretical corrections remains the same as that used in [1, 7] except for the introduction of quenching factors in the radiative corrections. The corrected $f_t$-values, $F_t$, for the eight precision data cases are fitted by a one-parameter function

$$F_t = F_t(0) = \text{constant} \quad (9)$$

or a two-parameter function

$$F_t = F_t(0) [1 + a_1 Z] \quad (10)$$

The results are given in Table 3, both with and without the quenching factors. In both cases the introduction of quenching factors improves the fit (reduces the $\chi^2$). In the 1-parameter fit there is a reduction of 2.0s in $F_t(0)$ and a concomitant increase in $V_{ud}$, but not enough to restore unitarity. In the 2-parameter fit, the intercept $F_t(0)$ is essentially unchanged and hence there is little change in $V_{ud}$, but the slope $a_1$ is reduced. This indicates that the quenching factors are responsible for about 20% of the putative $Z$-dependence in the current data base. Thus, in spite of the considerations given here, the deterioration in the CVC test in the precision superallowed Fermi decay data persists.

The author acknowledges a conversation at the WEIN92 conference with Prof. Yu. V. Gaponov, who insisted a calculation such as this should be performed.

References

[1] J. C. Hardy, I. S. Towner, V. T. Koslowsky, E. Hagberg and H. Schmeing, Nucl. Phys. A509 (1990) 429

[2] D. H. Wilkinson, Nucl. Phys. A511 (1990) 301; D. H. Wilkinson, Nucl. Inst. and Meth. 335 (1993) 172, 182, 201; D. H. Wilkinson, preprint TRI-PP-93-85 ‘$G_V$, CKM unitarity, neutron decay; $W_R$’ (1993).
Table 3: Fitted values of $\mathcal{F}t(0)$ and $a_1$ for the eight precision superallowed Fermi $\beta$-decay data, and the deduced values of $V_{ud}$ and the unitarity sum

|                     | Unquenched  | Quenched   |
|---------------------|-------------|------------|
|                     | $C_{NS}$    | $C_{NS}$   |
| 1-parameter fit     |             |            |
| $\mathcal{F}t(0)$   | 3075.0 ± 3.5 s | 3073.0 ± 3.3 s |
| $\chi^2/N$          | 1.91        | 1.40       |
| $V_{ud}$            | 0.9733 ± 0.0007 | 0.9736 ± 0.0007 |
| $V_{ud}^2 + V_{us}^2 + V_{ub}^2$ | 0.9959 ± 0.0016 | 0.9965 ± 0.0015 |
| 2-parameter fit     |             |            |
| $\mathcal{F}t(0)$   | 3067.4 ± 3.5 s | 3067.1 ± 3.3 s |
| $a_1$               | $(1.4 \pm 0.5) \times 10^{-4}$ | $(1.1 \pm 0.5) \times 10^{-4}$ |
| $\chi^2/N$          | 1.17        | 0.94       |
| $V_{ud}$            | 0.9745 ± 0.0007 | 0.9745 ± 0.0007 |
| $V_{ud}^2 + V_{us}^2 + V_{ub}^2$ | 0.9982 ± 0.0016 | 0.9983 ± 0.0015 |
[3] P. Quin, Nucl. Phys. A553 (1993) 319c; J. Deutsch and P. Quin, *Symmetry tests in semileptonic weak interactions: A search for new physics* to be published in *Precision tests of the standard electroweak model*, ed. P. Langacker (World Scientific, Singapore, 1993)

[4] Particle Data Group, Phys. Rev. D11 (1992) S1

[5] W. Jaus and G. Rasche, Phys. Rev. D41 (1990) 166

[6] F. C. Barker, B. A. Brown, W. Jaus and G. Rasche, Nucl. Phys. A540 (1992) 501

[7] I. S. Towner, Nucl. Phys. A540 (1992) 478

[8] W. J. Marciano and A. Sirlin, Phys. Rev. Lett. 56 (1986) 22

[9] B. A. Brown and B. H. Wildenthal, Phys. Rev. C28 (1983) 2397; B. A. Brown and B. H. Wildenthal, At. Data Nucl. Data Tables 33 (1985) 347; B. A. Brown and B. H. Wildenthal, Nucl. Phys. A474 (1987) 290

[10] I. S. Towner, Phys. Reports 155 (1987) 263

[11] A. Arima, K. Shimizu, W. Bentz and H. Hyuga, Adv. in Nucl. Phys. 18 (1987) 1

[12] V. T. Koslowsky *et al.*, to be published

[13] S. W. Kikstra *et al.*, Nucl. Phys. A529 (1991) 39

[14] S. A. Brindhaban and P. H. Barker, to be published; S. Lin, S. A. Brindhaban and P. H. Barker, to be published