New Insight into the stellar mass function of Galactic globular clusters

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ABSTRACT

We present the results of the analysis of deep photometric data of 32 Galactic globular clusters. We analysed 69 parallel field images observed with the Wide Field Channel of the Advanced Camera for Surveys of the Hubble Space Telescope which complemented the already available photometry from the globular cluster treasury project covering the central regions of these clusters. This unprecedented data set has been used to calculate the relative fraction of stars at different masses (i.e. the present-day mass function) in these clusters by comparing the observed distribution of stars along the cluster main sequence and across the analysed field of view with the prediction of multimass dynamical models. For a subsample of 31 clusters, we were able to obtain also the half-mass radii, mass-to-light ratios and the mass fraction of dark remnants using available radial velocity information. We found that the majority of globular clusters have single power law mass functions $F(m) \propto m^\alpha$ with slopes $\alpha > -1$ in the mass range $0.2 < m/M_\odot < 0.8$. By exploring the correlations between the structural/dynamical and orbital parameters, we confirm the tight anticorrelation between the mass function slopes and the half-mass relaxation times already reported in previous works, and possible second-order dependence on the cluster metallicity. This might indicate the relative importance of both initial conditions and evolutionary effects on the present-day shape of the mass function.

Key words: methods: numerical - techniques: photometric - techniques: radial velocities - stars: kinematics and dynamics - stars: luminosity function, mass function - globular clusters: general

1 INTRODUCTION

Globular clusters (GCs) are among the most useful objects to study stellar astrophysics. Consisting of a large number of stars with similar ages and chemical compositions they are valuable to test stellar evolution models. Beside their importance in stellar evolution studies, GCs are the oldest collisional systems such that dynamical drivers (like two-body relaxation) and external tidal field have changed their structures. By comparing observational coordinates in phase-space (projected positions and three-dimensional motions) with theoretical models, one can derive many important parameters of GCs like their masses, mass functions and degree of mass segregation.

The present-day structure of GCs depends directly on the mass distribution of stars that extends from the faintest stars at the hydrogen burning limit (with masses around $0.1 M_\odot$) to massive black holes (with masses larger than $15 M_\odot$). The shape of the present-day mass function (MF) is the result of the effect of complex mechanisms of dynamical and stellar evolution from the initial mass function (IMF). The universality of the IMF is a highly debated topic in astrophysics (Bastian, Covey & Meyer 2010; Kroupa et al. 2013). The first attempt to introduce a parameterized IMF as a single power-law function was made by Salpeter (1955) and followed by a log-normal (Miller & Scalo 1979; Chabrier 2003), a multi-segment power-law (Kroupa, Tout & Gilmore 1993; Kroupa 2001) and a tapered power-law (Kroupa, Tout & Gilmore 1993; Kroupa 2001) and a tapered power-law IMF (de Marchi, Paresce & Portegies Zwart 2005). In recent decades, many studies have tried to fit the above functions to the observed MF of various unevolved groups of stars, e.g. field stars (Czekaj et al. 2014; Rybizki & Just 2015;
Mor et al. 2019; Sollima 2019), young and embedded clusters (Weights et al. 2009; Weisz et al. 2013), OB associations (Massey 2003; Da Rio et al. 2012), open clusters (Moraux et al. 2003; Shekhi et al. 2016), and dwarf galaxies (Cappellari et al. 2006; Gennaro et al. 2018). Most studies of resolved stellar populations in the disk of the Milky Way showed that stars form following an IMF that has a universal form (Kroupa 2001, 2002) which is referred to as the “canonical” IMF. This poses a problem for star formation theories which predict a dependence on the environment where star formation takes place (Kroupa et al. 2013; Chabrier, Hennahelle & Charlot 2014). In spite of no evidence of significant variations of the IMF in these studies, the universality of the IMF is still a matter of debate. In particular, recent integrated light spectroscopic studies in the centres of giant ellipticals seem to favour a bottom-heavy IMF (Couron, van Dokkum & Villaume 2017; van Dokkum et al. 2017), while some extragalactic super starburst regions indicate a top-heavy IMF (Zhang et al. 2018).

From the IMF to the present-day MF, the evolution of GC MFs depends on many processes. At early stages, stellar evolution leads to the disruption of the most massive stars depriving the high-mass tail of the IMF. On long timescales, gravitational encounters among the stars and the interaction of the GC with the external tidal field also alter the shape of the MF. The tendency toward kinetic energy equipartition leads high-mass stars to sink into the core and low-mass stars to migrate toward the outskirts (Spitzer 1987). Low-mass stars can gain enough energy to escape from the GC more efficiently than high-mass ones, with a consequent depletion of the low-mass end of the IMF (Lamers, Baumgardt & Gieles 2013). As a result, as the GC loses a significant fraction of its low-mass stars, the faint end of its IMF evolves from its initial toward a flatter shape (Baumgardt & Makino 2003). This process is enhanced by the tidal force exerted by the host galaxy which accelerates the process of mass loss (Gieles, Heggie & Zhao 2011). This so-called “dynamical” mass segregation might be accompanied by a “primordial” mass segregation. The evidence and effect of this latter kind of mass segregation has been widely discussed on the basis of both observational (Frank, Grebel & Kupper 2014) and theoretical grounds (Haghi et al. 2014; 2015). For instance, Zonoozi et al. (2011, 2014, 2017) have shown that to explain the MF flattening in the centres of Pal 4 and Pal 14, the presence of both types of mass segregation is necessary. The effect of the various drivers on the evolution of GC MFs have been considered by many studies including many further ingredients, e.g. Galactic disc/bulge shocking (Ostriker, Spitzer, & Chevalier 1972; Aguilar, Hut, & Ostriker 1988), binary star evolution and interaction (Schneider et al. 2015) and orbital eccentricity (Madrid, Hurley, & Martig 2014; Webb et al. 2014).

The proper modelling of mass segregation as a function of radius is crucial to estimate the global MF of a star cluster. Indeed, observations are often localized in a restricted portion of the GC, and suitable corrections are necessary to account for such MF variations (Paust et al. 2010).

From a theoretical perspective, there are a few methods to investigate the dynamical evolution of gravitational systems: N-body simulations (Baumgardt & Makino 2003; Webb & Leigh 2015), Fokker-Planck models (Takahashi & Lee 2000; Murphy, Cohn & Lugger 2011), Monte Carlo models (Giersz 2001; Joshi, Nave & Rasio 2001) and through analytic methods (e.g. like the EMACSS code Alexander et al. 2014). All these models have been used to study the mechanisms driving the evolution from the IMF to the present-day MF.

During the last decades, the Hubble Space Telescope (HST) has allowed us an unprecedented insight into the GC stellar populations. Within the HST Treasury program, Sarajedini et al. (2007) performed uniform photometry of stars in the central region of 65 GCs sampling stars as faint as 0.2 M⊙ with a S/N ≳ 10. This program has served many studies aimed at testing stellar evolution, isochrones, luminosity functions and synthetic horizontal-branch models (Dotter et al. 2007), also deriving the ages of ∼ 60 GCs (Marín-Franch et al. 2009, Dotter et al. 2010) and the MF of a sample of 17 GCs (Paust et al. 2010). Moreover, the HST Ultraviolet Legacy Survey of Galactic GCs program designed to find multiple stellar populations extended the observational datasets to ultraviolet wavelengths (Piotto et al. 2015; Milone et al. 2017). In the context of this project, Simioni et al. (2018) presented a photometric catalogues of 110 parallel fields in the outskirts of 48 Galactic GCs. By combining the catalogues of the two above mentioned programs, the photometric catalogues of 56 Galactic GCs have been made public (Nardiello et al. 2018). The second data release (DR2) of the Gaia mission has also improved the observational scenario of GCs. Among the recent works based on Gaia DR2, Baumgardt et al. (2019) have derived the mean proper motions and space velocities of 154 Galactic GCs and the velocity dispersion profiles of 141 GCs.

By comparing observational data and simple dynamical models, several studies have attempted to calculate GC MFs and evaluate possible correlations between GC parameters. Among the most comprehensive studies, but based on heterogeneous measurements, Capaccioli, Pietto & Stiavelli (1993) and Djorgovski, Pietto, & Capaccioli (1993) reported a dependence of the MF slopes on the Galacticcentric distances and on the heights above the Galactic plane. Pietto & Zoccali (1999) analysed deep HST images taken near the half-mass radii of seven GCs and found that the MF slopes correlate with the orbital destruction rates of the clusters and anticorrelate with their half-mass relaxation times, although their small sample hampered any firm conclusion on the significance of these correlations. de Marchi, Paresce, & Pulone (2007), derived the MF of 20 GCs using HST and Very Large Telescope (VLT) data, reporting a well-defined correlation between the slope of their MFs and their King model concentration parameter c. Paust et al. (2010) provided the MF for 17 GCs comparing the luminosity function derived from the HST Treasury program data and multimass models and found that the MF slope correlates with central density, but with neither metallicity nor Galactic location. Using to the same data set and a similar technique, Sollima & Baumgardt (2017; hereafter SB17) expanded the sample to 35 GCs. They determined the structural and dynamical parameters of 29 GCs with available radial velocity information and revealed a tight anticorrelation between MF slopes and half-mass relaxation times and correlation with the dark remnant fractions. They concluded that the internal dynamical evolution is the main responsible in shaping the present-day MFs. These last works, while representing the most complete census of MF based on the deepest pho-
Figure 1. Maps of the pointings around the 6 GCs analysed here and not present in the Simioni et al. (2018) sample. The squares in the centre show the regions covered by the HST Treasury program. The squares labelled by ‘P’ mark the location of the parallel pointings. The half-mass radii (from the present work) are marked with circles. The maps of the other GCs of our sample are available in Simioni et al. (2018).

This paper is organized as follows: in section 2 the observational material is presented and the data reduction technique is described. The multimass dynamical model, the algorithm of MF determination and fitting technique are described in section 3 and 4. In section 5 we present the derived GCs MF and look for the possible correlations with various parameters. We summarize our results in Section 6.

2 PHOTOMETRY AND DATA REDUCTION

Images have been obtained with the Advanced Camera for Surveys (ACS) on board HST using the single channel camera WFC. The detector includes two similar chips of 2048 × 4096 pixels each with a pixel-scale of 0.05 arcsec per pixel. Therefore the entire channel covers an effective field of view of 202 arcsec × 202 arcsec. The photometric catalogues in the F606W and F814W filters of the region around the centers of 65 GCs have been published as part of the HST Treasury program (Sarajedini et al. 2007). The results of artificial star experiment performed on this data set have been also provided by Anderson et al. (2008).

Parallel pointings for all the GCs included in the HST Treasury project are also available as ancillary products of the HST UV legacy survey of Galactic GCs program. In a recent work, Simioni et al. (2018) provided accurate photom-
Table 1. Observing logs.

| NGC | Pointing | Filter | Exposure time (s) |
|-----|----------|--------|------------------|
| 104 | 1        | F475W  | 1050; 2×986; 947; 2×880 |
|     |          |        | F814W 877; 870; 806; 800; 767; 760 |
|     | 2        | F606W  | 1498; 1457; 1443 |
|     |          |        | 142; 136; 137 |
|     |          | F814W 1457; 1358; 1357 |
|     |          |        | 1303; 1118; 100 |
| 288 | 1        | F606W  | 3×200; 15 |
|     |          |        | F814W 3×150; 10 |
| 362 | 1        | F606W  | 172; 86; 39 |
|     |          |        | F814W 60; 25 |
| 1261| 1        | F475W  | 770 |
|     |          |        | F814W 694 |
|     | 2        | F475W  | 745 |
|     |          |        | F814W 669 |
|     | 3        | F475W  | 766 |
|     |          |        | F814W 690 |
|     | 4        | F475W  | 745 |
|     |          |        | F814W 669 |
|     | 5        | F475W  | 829 |
|     |          |        | F814W 753 |
| 1851| 1        | F475W  | 2×1277; 1237; 2×40 |
|     |          |        | F814W 6×488; 40 |
|     | 2        | F475W  | 2×1277; 1237; 2×40 |
|     |          |        | F814W 8×488; 2×40 |
| 2298| 1        | F475W  | 2×785 |
|     |          |        | F814W 2×683 |
|     | 2        | F475W  | 887; 885 |
|     |          |        | F814W 816; 815 |
| 3201| 1        | F475W  | 685 |
|     |          |        | F814W 612 |
|     | 2        | F475W  | 689 |
|     |          |        | F814W 616 |
| 4590| 1        | F475W  | 627 |
|     |          |        | F814W 554 |
|     | 2        | F475W  | 627 |
|     |          |        | F814W 554 |
| 5024| 1        | F475W  | 4×725; 2×723 |
|     |          |        | F814W 3×370 |
|     | 2        | F475W  | 4×775; 2×774 |
|     |          |        | F814W 3×375 |
| 5053| 1        | F475W  | 740 |
|     |          |        | F814W 664 |
|     | 2        | F475W  | 740 |
|     |          |        | F814W 664 |
|     | 3        | F475W  | 790 |
|     |          |        | F814W 714 |
|     | 4        | F475W  | 790 |
|     |          |        | F814W 714 |
|     | 5        | F475W  | 765 |
|     |          |        | F814W 689 |
| 5272| 1        | F475W  | 3×800 |
|     |          |        | F814W 3×760 |
| 5286| 1        | F475W  | 728 |
|     |          |        | F814W 655 |
|     | 2        | F475W  | 603 |
|     |          |        | F814W 559 |

Table 1 – continued

| NGC | Pointing | Filter | Exposure time (s) |
|-----|----------|--------|------------------|
| 5466| 1        | F475W  | 835; 834 |
|     |          |        | F814W 765; 763 |
|     | 2        | F475W  | 2×776 |
|     |          |        | F814W 2×700 |
| 5897| 1        | F475W  | 833; 830 |
|     |          |        | F814W 2×761 |
|     | 2        | F475W  | 781; 779 |
|     |          |        | F814W 710; 709 |
| 5904| 1        | F475W  | 620 |
|     |          |        | F814W 559 |
|     | 2        | F475W  | 621 |
|     |          |        | F814W 559 |
| 5986| 1        | F475W  | 676 |
|     |          |        | F814W 603 |
|     | 2        | F475W  | 2×603 |
|     |          |        | F814W 2×559 |
| 6093| 1        | F475W  | 5×845; 5×760 |
|     |          |        | F814W 5×539 |
| 6101| 1        | F475W  | 762 |
|     |          |        | F814W 686 |
|     | 2        | F475W  | 762 |
|     |          |        | F814W 686 |
|     | 3        | F475W  | 800 |
|     |          |        | F814W 724 |
|     | 4        | F475W  | 851 |
|     |          |        | F814W 775 |
|     | 5        | F475W  | 800 |
|     |          |        | F814W 724 |
| 6144| 1        | F475W  | 769 |
|     |          |        | F814W 606 |
|     | 2        | F475W  | 679 |
|     |          |        | F814W 606 |
| 6218| 1        | F475W  | 721 |
|     |          |        | F814W 648 |
|     | 2        | F475W  | 645 |
|     |          |        | F814W 572 |
| 6254| 1        | F475W  | 721 |
|     |          |        | F814W 648 |
|     | 2        | F475W  | 644 |
|     |          |        | F814W 571 |
| 6341| 1        | F475W  | 638 |
|     |          |        | F814W 565 |
|     | 2        | F475W  | 750 |
|     |          |        | F814W 677 |
| 6362| 1        | F475W  | 651 |
|     |          |        | F814W 578 |
|     | 2        | F475W  | 760 |
|     |          |        | F814W 687 |
| 6541| 1        | F475W  | 689 |
|     |          |        | F814W 616 |
|     | 2        | F475W  | 639 |
|     |          |        | F814W 566 |
| 6584| 1        | F475W  | 640 |
|     |          |        | F814W 567 |
|     | 2        | F475W  | 726 |
|     |          |        | F814W 653 |

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Table 1 – continued

| NGC  | Pointing | Filter | Exposure time (s) |
|------|----------|--------|-------------------|
| 6723 | 1        | F475W  | 666               |
|      |          | F814W  | 592               |
|      | 2        | F475W  | 626               |
|      |          | F814W  | 551               |
|      | 3        | F475W  | 624               |
|      |          | F814W  | 551               |
| 6752 | 1        | F606W  | 500               |
|      |          | F814W  | 200; 15           |
| 6779 | 1        | F475W  | 731               |
|      |          | F814W  | 658               |
|      | 2        | F475W  | 637               |
|      |          | F814W  | 564               |
| 6809 | 1        | F475W  | 753               |
|      |          | F814W  | 680               |
|      | 2        | F475W  | 677               |
|      |          | F814W  | 604               |
| 7078 | 1        | F475W  | 4×702             |
|      |          | F814W  | 4×121             |
| 7089 | 1        | F475W  | 717               |
|      |          | F814W  | 643               |
|      | 2        | F475W  | 668               |
|      |          | F814W  | 593               |
|      | 3        | F475W  | 611               |
|      |          | F814W  | 534               |
| 7099 | 1        | F475W  | 656               |
|      |          | F814W  | 583               |
|      | 2        | F475W  | 656               |
|      |          | F814W  | 583               |

We retrieved images of 69 parallel fields (pointings) in the outer region of these 32 GCs provided by the HST/ACS/WFC public archive and released before 2018. These pointings are centered approximately from 6 arcmin to 12 arcmin from the center of the target GCs. In Fig 1, the maps of the region sampled by those observations are shown. Each pointing consists of several images differing from each other because of their different filters and exposure times. We selected them in order to ensure at least two images observed through two different filters (including F814W) and with an exposure time long enough to allow to construct a deep colour-magnitude diagram (CMD) sampling stars with masses down to ~0.15 M⊙. In our collected samples, each parallel pointing has only one filter pair: (F606W, F814W) or (F475W, F814W). All GCs with multiple parallel pointings (with the exception of NGC 104) have been observed with the same filter pair. For simplicity, in spite of their different throughput, we will name the F475W/F606W magnitudes as V magnitude and the F814W magnitude as I magnitude. The logs of the observations are listed in Table 1.

We performed the photometric analysis on the fielded HST/ACS/WFC images corrected for CTE (f1c) while the drizzled (drc) images were used for source detection. We employed the stellar photometry package DOLPHOT v2 optimized in the HSTphot extention (Dolphin 2000) to analyse ACS images (Dolphin 2016). A high-signal to noise image has been created by aligning and stacking all available drc images of each pointing and used to create a list of detected sources. This reference image has then been used as input by the DOLPHOT point spread function (PSF)-fitting routine which has been run on individual images. Aperture corrections have been computed using the most isolated and bright sources and applied to the output catalogue. In this catalog, the average magnitude of each detected star and those measured in all the individual f1c images were listed together with quality flags, S/N, roundness, sharpness and star positions in pixels. To select an optimal set of stars, we set the following criteria: (i) at least two reasonable magnitudes should exist per filter for each star in each pointing, (ii) an absolute value of sharpness smaller than 0.1,2 (iii) an object type flag equal to 1, which corresponds to good stars; (iv) a photometry quality flag less than 4, which includes stars with negligible photometric errors. We cross-correlated our catalogues with those provided by Simioni et al. (2018) to convert (x, y) coordinates into the standard astrometric reference system (RA, Dec). A star-by-star comparison between our photometry and that of Simioni et al. (2018) indicates average differences between the two datasets of ∆V, ∆I (Present work - Simioni et al.) = 0.04 in V and I passbands for all the GCs of the sample, indicating a small shift in the calibration process.

To perform artificial star experiments for a pointing in each GC, an input set of fake stars has been simulated. The positions of fake stars have been chosen by defining a regular grid of 190×190 cells separated by 20 pixels in each direction, which cover homogeneously the ACS/WFC field of view. Artificial stars have been placed at a random position within a 16 × 16 pixel square centered around each grid knot (one

1 The CMDs of all the GCs are available as supplementary material in the online version of the paper.

2 This criterion came from the definition of sharpness (Dolphin 2000): the sharpness of completely flat, sharp and perfectly-fit stars are -1, 1 and 0, respectively.
star per cell). Given the typical size of the ACS/WFC PSF (FWHM $\sim 2$ px) this ensures that artificial stars cannot blend each other (self-crowding). The magnitudes of stars have been extracted from the theoretical isochrone provided by Dotter et al. (2007) with the suitable age, metallicity and alpha-element abundance of each GC (Dotter et al. 2010). The absolute magnitudes of stars have been converted into the apparent ones by adopting the distance moduli and reddening determined by Dotter et al. (2010). The masses of fake stars were extracted from a Kroupa IMF (Kroupa 2001) denoting determined by Dotter et al. (2010). The masses of the apparent ones by adopting the distance moduli and red-
al- element abundance of each GC (Dotter et al. 2010).

Blend each other (self-crowding). The magnitudes of fake stars have been covering the full range of masses covered by the isochrone fake stars were extracted from a Kroupa IMF (Kroupa 2001) denoting determined by Dotter et al. (2010). The masses of fake stars were extracted from a Kroupa IMF (Kroupa 2001) covering the full range of masses covered by the isochrone from the hydrogen burning limit up to the red giant branch tip. The corresponding magnitudes of fake stars have been used by DOLPHOT to normalize the fluxes of the PSF which have been placed into the original science images and the photometric analysis has then been performed on synthetic images with the same prescriptions adopted for the original frames. With the above procedure, 36,100 fake stars have been simulated per run. Moreover, several (from 1 to 5, de-
pending on the number of GC pointings) independent runs have been performed in each GC and the artificial star cat-
alogues of different runs have been stacked together in a unique list containing from 36,100 to 180,500 artificial stars. An artificial star has been considered as recovered if the difference between its input and output magnitudes is smaller than 2.5 log(2) $\sim$ 0.752 mag in both V and I magnitudes.

For a subsample of 31 GCs we were able to derive the dynamical parameters through the comparison with the set of available radial velocities. For this purpose we used the sample of radial velocities derived by BH18 which have been obtained from a combination more than 45,000 high-resolution spectra observed with FLAMES@VLT and DEIMOS@Keck stars with literature data. For the 31 GCs in our sample, the number of stars with available radial velocities ranges from 19 (for the lowest mass GC NGC 6144) to 2867 (for the most massive GC NGC 104).

3 MODELS

To convert the relative distribution of stars at different radii into the global MF, a suitable dynamical model is needed. The structure of multimass stellar systems can be obtained from the phase-space distribution of stars proposed by Michie (1963a) and King (1966). The isotropic form of this distribution is (Gunn & Griffin 1979):

$$f(E) = \prod_{i=1}^{H} f(m, r, v) = \prod_{i=1}^{H} k_i \left[ \exp \left( -\frac{A_i E}{\sigma_K^2} \right) - 1 \right],$$

where $H$ is the number of mass components, $m_i$ is the mass of the $i$th component, $v$ and $r$ are the 3D velocity and distance from the GC centre, $k_i$ are coefficients determining the number of stars in each mass bin, $A_i$ are coefficients regulating the equilibrium among the stars with different masses and $\sigma_K^2$ is a constant determining the normalization in energy. $E = v^2/2 + \psi(r)$ is the energy per unit mass, where $\psi$ is the effective potential defined as the difference between the potential at an arbitrary radius and the potential at the GC tidal radius, $r_t$. Following Gunn & Griffin (1979), we adopted $A_i \propto m_i$. A degree of radial anisotropy can be accounted for by multiplying the above distribution function by a term depending on the angular momentum. Watkins et al. (2015) analysed the HST proper motions of a sample of 22 nearby GCs and showed that deviations from isotropy are negligible within GC cores and only a mild anisotropic velocity distributions near the half-mass radius is detectable. Thus, to limit the number of free parameters, we adopted the isotropic form of the distribution functions (eq. 1).

By integrating the distribution function over $v$ and $r$, the 3D number density, velocity dispersion, number of stars, projected number density and line-of-sight velocity dispersion can be derived for each mass group, respectively, as:

$$n_i(r) = \int_{0}^{\sqrt{-2\psi(r)}} 4\pi v^2 k_i f(m_i, r, v) \, dv,$$

$$\sigma^2_{v,i}(r) = \frac{1}{n_i(r)} \int_{0}^{\sqrt{-2\psi(r)}} 4\pi v^4 k_i f(m_i, r, v) \, dv,$$

Figure 2. CMD of NGC 104 (left panels) NGC 288 (right panels). The same plot for the entire sample of GCs is available as supplementary material in the online version of the paper.

\[3\] https://people.smp.uq.edu.au/HolgerBaumgardt/globular/
\[ N_i = \int_0^\infty 4\pi r^2 n_i(r) \, dr, \]
\[ \Gamma(m_i, R) = 2 \int_R^\infty \frac{r n_i(r)}{\sqrt{r^2 - R^2}} \, dr, \]
\[ \sigma^2_{\text{los},i}(R) = \frac{2}{3} \int_R^\infty \frac{r n_i(r) \sigma^2_{\nu}(r)}{\sqrt{r^2 - R^2}} \, dr, \]
while the mean mass is given by
\[ \bar{m} = \frac{\sum_{i=1}^H N_i m_i}{\sum_{i=1}^H N_i}. \]
The radial gradient of \( \psi(r) \) is given by the Possion equation as:
\[ \nabla^2 \psi(r) = 4\pi G \rho(r), \]
while the global 3D density is determined by the following expression:
\[ \rho(r) = \sum_{i=1}^H m_i n_i(r). \]
To integrate the above equations, appropriate boundary conditions need to be set. At the center the value of the potential \( W_0 \equiv -\psi_0/\sigma^2_k \) has been left as a free parameter while its derivative has been set to \( d\psi/dr(0) = 0 \). At the tidal radius, defined as the distance where both density and potential vanish, we set \( \psi(r_t) = 0 \).
Along with \( k_i \), two additional free parameters complete the definition of the models: \( r_c \equiv \sqrt{9\sigma^2_k/4\pi G \rho_0} \) and \( \sigma^2_k \), which determine the size and mass of the system. The total mass, luminosity and surface brightness profile can be obtained respectively as:
\[ M = \sum_{i=1}^H N_i m_i, \]
\[ L_V = \sum_{i=1}^H N_i F_i, \]
\[ \mu = -2.5 \log \left( \frac{1}{3} \sum_{i=1}^H \Gamma_i F_i \right), \]
where \( F_i \) is the average V-band flux of stars in the \( i \)-th component.

4 METHOD

Sollima, Bellazzini & Lee (2012), Sollima et al. (2017) and SB17 have described a method to determine the global GCs MF and the structural/dynamical parameters of GCs and we adopt their method in this paper.

Briefly, an iterative procedure has been implemented. At each iteration, a guess of the MF and of the model parameters is chosen and a synthetic population of particles with the corresponding masses (magnitudes) and radial distributions has been simulated. The effect of completeness and photometric errors are included using the artificial star experiments described in Sect. 2 and a mock catalog of masses and distances is created. The corrections to the guess MF are calculated by comparing the distribution of synthetic and observed stars in the mass-distance plane, and the updated MF is used as input for the next iteration. This algorithm converges after a few iterations providing the output global MF.

Specifically, the algorithm can be schematically summarized as follows:

(i) An initial guess of the MF (defined by the coefficients \( k_i \)) is made. In the present analysis the coefficients corresponding to a Kroupa (2001) IMF have been adopted as first guess.

(ii) A synthetic stellar population has been created by extracting \( 10^5 \) stars from the adopted MF with masses between 0.1 and 8 M\(_\odot\). It has been assumed that the mass loss of stars with masses \( < M_{\text{tip}} \) (the mass at the tip of the red giant branch, RGB) is negligible while those with masses \( > M_{\text{tip}} \) have been evolved into white dwarfs (WDs) following the initial-final mass relation of Kalliari et al. (2009).

\[ m_{\text{WD}} = 0.109 m + 0.428. \]

According to the upper limit of the IMF, the white dwarfs are all retained by the GC unlike the other types of compact remnants. This implicitly assumes that the neutron stars and the black holes are ejected during the GC evolution by natal kicks and/or the Spitzer instability (Spitzer 1987). The fraction of mass in remnants in each bin (\( \nu_i \)) has also been calculated. No binaries have been simulated. These objects constitute only a small (\(< 5\%\)) fraction of objects in GCs (Milone et al. 2012) and their presence is not expected to significantly affect the final MF.

(iii) The corresponding V and I magnitudes of the synthetic stars have been derived by interpolating the masses of visible stars through the mass-luminosity relation of the adopted isochrone from the Dotter et al. (2007) database, adopting the best fit metallicity, age, distance modulus and reddening provided by Dotter et al. (2010) (see section 2). Zero luminosity in both bands has been assumed for the population of remnants. Note that we used the same isochrone, distance modulus and reddening for each GC, both in photometric data reduction and in MF analysis processes.

(iv) A synthetic population of field stars has been derived from the Galactic model provided by Robin et al. (2003) covering an area of 1 sq. deg around each GC centre and the V and I magnitudes have been transformed into the HST/ACS photometric system using the transformation provided by Sirianni et al. (2005).

(v) We defined eight evenly-spaced mass groups ranging from 0.1 M\(_\odot\) to \( M_{\text{tip}} \) and one additional bin for stars more massive than \( M_{\text{tip}} \) including massive WDs. Eight I-band magnitude intervals containing the above defined mass bins (excluding the one related to remnants) have been also defined accordingly. Real, field and synthetic stars with colours within three times the mean locus of MS stars, have been binned in these groups. The projected density of field stars in each bin \( \Gamma_{\text{field}}(i) \) is calculated by dividing the number of field stars contained in each mass bin by the area of the extracted field catalog (1 sq. deg).
where $N_{\text{tot}}^{\text{obs}}$ is the total number of observed stars, $m_j$ and $R_j$ are the mass and projected distance of the $j$-th observed star and $P(m_j, R_j)$ is the total probability density function to find a star with mass $m_j$ at a projected distance from the centre $R_j$. This last function is calculated as the model density in the $m-R$ plane, after correcting for completeness and azimuthal coverage

$$P(m_j, R_j) = \left[\frac{N_{\text{tot}}^{\text{obs}} - N_{\text{tot}}^{\text{field}}}{\sum_{i=1}^H N_i} \Gamma(m_j, R_j)(1 - t_j) + \Gamma_{t_j}^{\text{field}}\right] \times C(m_j, R_j) A_z(R_j),$$

(15)

where $N_i$ and $\Gamma(m_j, R_j)$ are calculated from eq.s 4 and 5, $\Gamma_{t_j}^{\text{field}}$ is the projected number density of field star in the same mass bin of the $j$th star, $N_{\text{tot}}^{\text{obs}}$ is the total number of stars in the observed catalog and

$$N_{\text{tot}}^{\text{field}} = \sum_{i=1}^H \Gamma_{t_j}^{\text{field}} \int_0^\rho 2\pi R C(m_i, R) A_z(R) dR.$$

(ix) For the guess choice of $k_i$, the $W_0$, $r_c$ space has been searched to find the values maximizing the above merit function using a Powell’s direction set algorithm (Brent 1973).

(x) For the best fit pair of $(W_0, r_c)$ values, the corresponding multimass model has been computed. Synthetic stars have been distributed according to the density profile of their corresponding mass bin across the field of view.

(xi) For each synthetic star, a particle from the artificial star library with I magnitude within 0.25 mag and the distance from the GC centre within 0.1 arcmin, with respect to
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5 RESULTS

5.1 An Overview of GCs Mass Function and Their Parameters

The derived global MFs and the structural/dynamical parameters of 32 GCs are listed in Table 2 and the present-day number of stars in the eight mass bins in the last iteration (eq. 4).

The best fit of the projected density profiles for two GCs and of the distribution of stars in the $m - R$ plane for NGC 5286 are shown in Figs. 3 and 4, as an example. According to Fig. 4, the density contours of the best-fitting model are generally in good agreement with the distribution of stars in the $m - R$ plane. As an alternative view, in Fig. 3, the surface density predicted by the best-fitting model for three mass groups is overplotted to the observational density profile for the same GCs. Note that, while the model captures the general behaviour of mass-segregation, some discrepancies are apparent inside the core of these GCs.

The stellar mass of each GC has been derived by normalizing the total number of stars in the best-fitting model to those counted in the observed catalogue

$$M_{\text{um}} = \frac{N_{\text{obs}}}{N_{\text{mock}}^{1/2}} \sum_{i=1}^{N} m_i (1 - \nu_i),$$

(17)

The dynamical mass has been estimated by comparing the observed velocity dispersion profile with the prediction of the best-fit model. In particular, the mass of the model has been chosen as the one minimizing the penalty function:

$$L = \sum_{j=1}^{N} \left\{ \frac{(v_j - \bar{v})^2}{\sigma_{\text{LOS},j}^2(R_j)} + \ln \left[ \sigma_{\text{LOS},j}^2(R_j) + \epsilon_j^2 \right] \right\},$$

(18)

where $N$ is the number of radial velocities, $v_j$ is the radial velocity of the $j$th star, $\bar{v}$ is the mean velocity of the sample, $\epsilon_j$ is the uncertainty of the radial velocity and $\sigma_{\text{LOS},j}(R_j)$ is the line-of-sight velocity dispersion predicted by the best-fitting model at the projected distance $R_j$ of the $j$th star for the eighth mass group (see eq. 6). The choice of the 8-th bin is justified by the fact that the considered radial velocity catalogues contain stars along the RGB covering a limited range of masses.

The total V-band luminosity has been calculated using equation 11, allowing to derive the mass-to-light ratio, $M_{\text{dyn}}/L_V$.

The fraction of dark remnants has been estimated using the following relation:

$$f_{\text{rem}} = 1 - \frac{M_{\text{um}}}{M_{\text{dyn}}}$$

(19)

The half-radius, $r_h$, has been evaluated as the radius including half of the GC mass while the half-mass relaxation time as (Spitzer 1987)

$$t_{r_h} = 0.138 \frac{M_{\text{dyn}}^{1/2} \gamma^{3/2}}{G^{1/2} \langle \gamma \rangle M_{\text{dyn}}/\bar{m}}$$

(20)

where $\gamma = 0.11$ (Giersz & Heggie 1996) and $\bar{m}$ is the mean mass of stars (eq. 7).

The entire procedure has been repeated iteratively until convergence. The global MF is determined by the relative quantities of the given star has been selected and, if recovered (see section 2), its output-input magnitude and colour shift have been added to those of the corresponding star. The same procedure has been applied to field stars.

(xii) For each synthetic star located at a distance $R_j$ from the centre, a random number $\eta$ uniformly distributed between 0 and 1 has been extracted and the star is rejected if $\eta > A\zeta(R_j)$. The same procedure has been applied to field stars. At the end of this step a mock catalog of synthetic stars corresponding to the guess choice of $k_i$, $W_0$, and $r_c$ is obtained, accounting for the photometric errors, incompleteness and azimuthal coverage.

(xiii) The number of stars in the observed ($N_{\text{obs}}$), field ($N_{\text{field}}$) and synthetic ($N_{\text{mock}}$) catalogues contained in the eight I magnitude bins have been counted and the $k_i$ coefficients have been updated using the relation

$$k'_i = k_i \frac{N_{\text{mock}}^{1/2} (N_{\text{obs}} - N_{\text{field}})}{(N_{\text{tot}} - N_{\text{field}})} N_{\text{mock}}^{1/2}$$

(16)

The same quantities of the given star have been added to those of the corresponding star. The same procedure has been applied to field stars.

(xiv) The number of stars in the observed ($N_{\text{obs}}$), field ($N_{\text{field}}$) and synthetic ($N_{\text{mock}}$) catalogues contained in the eight mass bins have been counted and the $k_i$ coefficients have been updated using the relation

$$k'_i = k_i \frac{N_{\text{mock}}^{1/2} (N_{\text{obs}} - N_{\text{field}})}{(N_{\text{tot}} - N_{\text{field}})} N_{\text{mock}}^{1/2}$$

(16)
MFs of these GCs are shown in Fig 5. The MF slopes $\alpha$ calculated by fitting a single power-law, $F(m) \propto m^{\alpha}$, in the mass range $0.2 < m/M_\odot < 0.8$ have also been calculated.

In the considered sample, the derived parameters cover wide ranges in MF slope ($-1.26 < \alpha < 0.02$), mass ($2.2 \times 10^4 < M/M_\odot < 10^6$), mass-to-light ratio ($0.4 < M/L_V < 2.9$) and half-mass radius ($4 < r_h/pc < 17.6$). For reference, a Kroupa (2001) IMF in this mass range has an average slope of $-1.567$. As already found in SB17, GCs with a steeper MF slope tend to have a convex shape with a systematic depletion of stars at $M < 0.2 M_\odot$, while the MF of GCs with a flatter MF are much better fitted by single power-laws.

5.2 Dependence on assumptions: NGC 104 as test case

The MF of the GC NGC 104 has been studied by de Marchi & Paresce (1995) who used HST/WFPC2 observations at 4.6 arcmin from the centre through F606W and F814W filters i.e. similar to one of the ACS pointing used in the present analysis (P2 in Fig. 1; at ~6′), adopting a different distance and mass-luminosity relation. The MF slope calculated from this work in the range $-0.60 < \log(m/M_\odot) < -0.15$ turns out to be $\alpha = -1.15$, flatter than what is estimated in the present work ($\alpha = -1.26$). This gives the opportunity to test the effect of the various assumptions on the derived MF providing also a validation of the photometric analysis and completeness correction with a completely independent study. Note that our analyzed region is located on the opposite side of the de Marchi & Paresce (1995) field with respect to the cluster centre and is larger. In the top panel of Fig. 6 the luminosity functions of de Marchi & Paresce (1995) and our "P2" region are compared, with and without applying the completeness correction. A good agreement between the two works is apparent, with only random fluctuations of < 0.1 dex amplitude and no systematic trend. This is particularly apparent when completeness correction are applied in both works, accounting for the different depth of the two different observations. This agreement indicates that the difference in the MF slope between these two works does not depend on either the photometry or on the completeness, but on the different assumptions on distance and/or mass-luminosity relation.

de Marchi & Paresce (1995) derived the MF of stars within their region using a Bergbusch & VandenBerg (1992) isochrone with [Fe/H]=−0.65 and age of 14 Gyr for NGC 104 and adopt a distance of 4.6 kpc (Webbink 1985), while in the present work we use a Dotter et al. (2007) isochrone with [Fe/H]=−0.7 and age of 12.75 Gyr and adopt a distance of 4.5 kpc.

To check the effect of such assumptions we derived the MF of region P2 using all the combinations of isochrones (Dotter et al. 2007; Bergbusch & VandenBerg 1992) and distance moduli (Dotter et al. 2007; Webbink 1985), and compared these four resulting MFs with the MF derived by de Marchi & Paresce (1995) in their analysed field. In the bottom panel of Fig. 6 this comparison is shown. It is clear that the small difference in the adopted distance produces a negligible effect across the entire mass range. A larger effect is produced by the different mass-luminosity relation, with the Bergbusch & VandenBerg (1992) isochrone providing a flatter MF than that derived using Dotter et al. (2007) models ($\Delta \alpha \sim 0.1$). So, this effect can entirely explain the difference in the measured slope.

Summarizing, at least for the case of NGC 104, the impact of assumptions on distance and adopted models should not produce an effect exceeding $\Delta \alpha \sim 0.15$ in the MF slope.
Figure 5. Present-day mass function of the 32 GCs in our sample. A vertical shift has been added to each cluster in each panel for clarity. From left to right panel and top to the bottom panel, GCs have been sorted according to their MF slope, in descending order.
5.4 Mass segregation

The MFs derived here rely on the assumption that King-Michie multimass models (computed according to the prescriptions of Gunn & Griffin 1979) provide an adequate description of the mass segregation occurring in GCs. It is interesting to check this hypothesis by comparing the predicted and observed variation of the MF slope at different distances from the cluster center.

For this purpose, we considered for each GC five regions at different distances from the cluster center: four annular regions of 0.4′′ width in the central field and a single bin containing the parallel pointings. Then we computed the completeness-corrected MF slopes at these radial bins. The radial behaviour of MF slopes measured in two GCs (NGC 5286 and NGC 5466) are compared with those predicted by the best-fitting model in Fig. 9. It is clear that the model reproduces well the observed radial trend of the MF slope for NGC 5286 while it does not match the MF slope variation in NGC 5466. In particular, in this last GC the observed MF slope varies much more quickly (by $\Delta \alpha \sim 6$) with radius than what is predicted by the best fit model ($\Delta \alpha \sim 1.2$). In other words, NGC 5286 appears to be over-segregated than what is predicted by the best fit model ($\Delta \alpha \sim 1.2$).

In NGC 5466, while it does not match the MF slope variation predicted and observed variation of the MF slope at different distances from the cluster center.

In Fig 8, the half-mass radii of 32 GCs, masses and mass-to-light ratios of the 31 GCs in common between the present work and BH18 are compared. These authors determined the masses and structural/dynamical parameters of GCs by fitting a large set of N-body simulations to the velocity dispersion profile from the same set of radial velocities adopted in this work. The mean differences in the estimated masses and $M/L$ ratios indicate good agreement within the errors in spite of the difference between our adopted multimass models and N-body simulation. The estimated half-mass radii are instead slightly larger in this work. This difference arises mainly from the more compact GCs, where the constraint provided by the external field photometry tends to favour a slightly larger mass at large radii.

5.3 Comparison with previous works

In this section we compare the results obtained in this analysis with those obtained by SB17 and Paust et al. (2010) (who applied a similar technique using only data in the central cluster region) and with those by BH18. The statistics of the differences between our work with those mentioned above have been summarized in Table 3. In Fig 7, the MF slopes and half-mass radii of 28 GCs and the dynamical masses and the mass-to-light ratios of 23 GCs in common between this work and SB17 are compared. The largest differences in the estimated MF slopes ($\Delta \alpha > 0.2$) are related to the two GCs which have the steepest MF among 28 GCs in SB17 (e.g. NGC 5466 and NGC 6101) while they have flatter MF in the present work. The mean differences between the two studies indicate good agreements in estimated MF slopes, dynamical masses and mass-to-light ratios. The difference between two studies in estimated half-mass radii shows that the three GCs with the largest difference between the half-mass radii estimated in these works i.e. NGC 6101, NGC 5466, NGC 5024, are those with the largest difference in the estimated MF slope. This indicates the importance of the MF constraint outside the ACS field of view for these extended clusters.

Fifteen GCs are in common with the work by Paust et al. (2010). While the difference between these two works is not statistically significant, our MFs are on average flatter than those measured by Paust et al. (2010). These authors used the same HST/ACS photometric data in the central region and the same multimass model fitted in an annular region close to the half-light radius.

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For this purpose, we considered for each GC five regions at different distances from the cluster center: four annular regions of 0.4′′ width in the central field and a single bin containing the parallel pointings. Then we computed the completeness-corrected MF slopes at these radial bins. The radial behaviour of MF slopes measured in two GCs (NGC 5286 and NGC 5466) are compared with those predicted by the best-fitting model in Fig. 9. It is clear that the model reproduces well the observed radial trend of the MF slope for NGC 5466, while it does not match the MF slope variation in NGC 5286. In particular, in this last GC the observed MF slope varies much more quickly (by $\Delta \alpha \sim 6$) with radius than what is predicted by the best fit model ($\Delta \alpha \sim 1.2$). In other words, NGC 5286 appears to be over-segregated with respect to the prediction of the King-Michie model.

To quantify the discrepancy between models and observations, for each GC we performed a least-squares fit to the residuals of the best fit MF slope ($\Delta \alpha$) as a function of the distance from the center (in units of $r_h$) and adopted the slope of this linear fit $\xi \equiv d\Delta \alpha/d\log R$ as an indicator of the
Table 3. The mean (µ) and standard deviation (σ) of differences between this work and the others in estimated MF slopes (third column), dynamical masses (fourth column), half-mass radii (fifth column), and mass-to-light ratios (sixth column).

| Statistic          | ∆α    | ∆log(M/M⊙) | ∆log(rh/pc) | ∆[(M/M⊙)/(L/L⊙)] |
|--------------------|-------|------------|-------------|-------------------|
| This Work - Paust et al. (2010) | µ 0.23 ± 0.12 | | | |
|                     | σ 0.43 | | | |
| This Work - BH18   | µ 0.04 ± 0.02 | 0.10 ± 0.01 | -0.27 ± 0.08 | |
|                     | σ 0.11 | 0.08 | 0.46 | |
| This Work - SB17   | µ 0.03 ± 0.03 | 0.005 ± 0.020 | 0.02 ± 0.02 | -0.10 ± 0.06 |
|                     | σ 0.16 | 0.10 | 0.10 | 0.31 |

Figure 7. Comparison between the MF slopes (top-left panel), half-mass radii (top-right panel), dynamical masses (bottom-left panel) and mass-to-light ratios (bottom-right) derived in the present work and those determined by SB17. The one-to-one relation is marked by the dashed line.

observed over-segregation. The over-segregation parameter measured in the whole GCs sample is plotted as a function of the projected central density and half-mass density in Fig. 10. Both densities clearly correlate with ξ such that, while at relatively small densities models correctly reproduce the observed degree of mass segregation (ξ ∼ 0), denser GCs show significantly steeper gradients of α (ξ < 0). Note that ξ does not seem to significantly correlate with other quantities. This evidence can be interpreted in two alternative ways: i) the estimated completeness is spuriously high in the extremely crowded conditions occurring in dense GCs, or ii) This is a real effect such that the densest GCs with shorter central relaxation time reach a degree of mass segregation higher than what is predicted by multimass models. In this context, it is interesting to note that the correlation with the central projected density (an indicator of the maximum crowding) has a larger spread than that with the half-mass density (which is an intrinsic parameter of the GC).

Note that the global MF slopes (listed in Table 2) are strongly constrained by the radial interval close to the half-mass radius containing the largest fraction of stars. So, while any inadequacy of the adopted models reflects in the accuracy of the estimated MF slopes, no significant systematic errors are expected. This fact is supported by the simulations of Baumgardt & Makino (2003) who found that the global and local MFs at around 60 per cent of the half-light radius are approximately the same.
Figure 8. Comparison between the dynamical masses (left panel), half-mass radii (middle panel) and mass-to-light ratios (right panel) derived in the present work and those determined by BH18. The one-to-one relation is marked by the dashed line.

Figure 9. Radial variation of the MF slopes of two GCs: NGC 5286 (top panel) and NGC 5466 (bottom panel). The slope predicted by the corresponding best-fit model are shown by solid lines.

5.5 Tidal Radii of GCs

We calculated the present-day Jacobi radius ($r_J$; defined as the distance of the inner Lagrangian point from the cluster center) of each GC in our sample using the same formulas described in Sollima & Mastrobuono Battisti (2014) (see their appendix A2). For this purpose, we used the dynamical masses estimated here and the velocities from Gaia DR2 determined by Baumgardt et al. (2019). We have assumed a three-component Galactic potential given by the superposition of (i) a Hernquist bulge with $c = 0.7$ kpc and $M_b = 1.3 \times 10^{10} M_\odot$, (ii) a Miyamoto & Nagai disk with $a = 6.5$ kpc, $b = 0.26$ kpc and $M_d = 1.085 \times 10^{11} M_\odot$, (iii) a logarithmic halo with $v_0 = 165$ km/s and $d = 12$ kpc (in analogy with the potential adopted by Johnston, Spergel & Hernquist (1995), with a slightly different normalization of the components to best-fit the post-Gaia Galactic kinematics of GCs; Sollima et al., in prep.).

In Fig 11, the ratio of the half-mass radius to the Jacobi radius as a function of Galactocentric distance derived from Harris (1996, 2010 edition) is shown. Among the innermost GCs with $R_{GC} < 8$ kpc, two extreme cases are notable: (i) NGC 6144 has the highest ratio ($r_h/r_J = 0.26$) because it is the lightest and one of the nearest GCs from the centre of the Galaxy, (ii) The lowest ratio belongs to NGC 104 with $r_h/r_J = 0.03$ which is the most massive GC in our sample and the most distant GC from the centre of the Galaxy within the mentioned interval. Generally, there is no clear relationship between $r_h/r_J$ and $R_{GC}$ in this interval. For outer GCs with $R_{GC} > 8$ kpc, there are two recognizable groups: (i) 5 GCs with $r_h/r_J > 0.08$, i.e. NGC 3201, NGC 4590, NGC 5053, NGC 5466 and NGC 6101 whose mean mass and mean half-mass radii are $M_{dyn} \approx 10^5 M_\odot$ and $r_h \approx 13$ pc respectively, so they are the extended and tidally filling GCs. (ii) 12 GCs with $r_h/r_J \lesssim 0.08$. Their mean mass and mean half-mass radii are $M_{dyn} \approx 3.2 \times 10^5 M_\odot$ and $r_h \approx 7$ pc, respectively. Most of the GCs in this group are compact and tidally underfilling.

The GCs in the former group experience a stronger tidal field and are expected to dissolve faster than the ones in the latter group. The existence of these two distinct groups confirms the previous findings by Baumgardt et al. (2010).
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5.6 Analysis of Correlations

Our sample of GCs constitutes a large enough data set to investigate the correlations between MF slope and the dynamical, structural and orbital parameters. The following parameters have been considered: the Galactocentric distance ($R_{GC}$), distance from Galactic plane ($Z$), the average apogalactic ($R_{apo}$) and perigalactic ($R_{per}$) distance and the average dissolution time ($t_{diss}$) derived by Baumgardt et al. (2019), the concentration ($c$) derived from McLaughlin & van der Marel (2005), the age ($t_{age}$) and metallicity ([Fe/H]) derived by Dotter et al. (2010), the luminous mass ($M_{lum}$), dynamical mass ($M_{dyn}$), remnant mass fraction ($f_{rem}$), V-band magnitude ($M_V$), mass-to-light ratio ($M_{dyn}/L_V$), half-mass radius ($r_h$), half-mass relaxation time ($t_{rh}$) and half-mass density ($\rho_h \equiv 3M_{dyn}/(8\pi r_h^3)$), all derived from our adopted best-fitting model.

We have employed a permutation test to calculate the significance of the univariate correlations between MF slope and the other parameters. For each parameter, an error-weighted least-squares fit has been performed and the corresponding $\chi^2$ value has been estimated. The same process has been performed on $10^4$ realizations simulated by randomly swapping the values of the independent variable. The significance of the correlation is defined by the fraction of realizations with a $\chi^2$ larger than the one calculated in the observed sample. The complete set of correlations and their related probabilities are shown in Figs 12. We found a tight anticorrelation with $P > 99.7\%$ between the present-day MF slope and the following parameters: $t_{rh}$, $r_h$, $R_{GC}$, $\log|Z|$, $\log R_{apo}$, $\log R_{per}$ and $\log t_{diss}$, and marginal ($95\% < P < 99.7\%$) correlations with $\log \rho_h$ and $f_{rem}$. Note that most of these parameters are correlated to each other so that it is not easy to distinguish the leading parameter determining the correlation.

The anticorrelation with half-mass relaxation time confirms what was found in SB17. The left panel of Fig. 13 shows another view of the correlation between MF slope and the ratio between the GC age and present-day half-mass relaxation time. The tight correlation is clear: less-evolved GCs have the steeper MFs. It is apparent in this plot that an increase of $\alpha$ by a factor of 2 between $\alpha \approx -1.2$ and $\alpha \approx -0.6$ implies an increase of $t_{age}/t_{rh}$ by a factor of 5. Only a marginal correlation is instead found with the remnant fraction, as claimed in SB17 (2017).

Moreover, the correlations with the present-day Galactic position (found by Djorgovski, Piotto, & Capaccioli 1993; Capaccioli, Piotto & Stiavelli 1993; Piotto & Zoccali 1999) are also confirmed.

Instead, the correlation with concentration, previously reported by de Marchi, Paresce, & Pulone (2007), is found to be not significant.

As explained above, there are a lot of significant correlations between individual parameters and the MF slope. Some of these parameters are expressible in terms of each...
Figure 12. Univariate correlations between the global MF slope $\alpha$ and various dynamical, structural and orbital parameters. The statistical significance ($P$) of each correlation is indicated.

other so that the significance of their univariate correlation with the MF slope cannot be interpreted as a proof of the physical dependence of the MF on these parameters. For instance, both the Galactocentric distance and half-mass relaxation time separately show significant correlations with the MF slope. However, GCs at large Galactocentric distances have on average large half-mass radii (van den Bergh, Morbey & Pazder 1991) and thus long relaxation times. So, the correlation with Galactocentric distance can be driven by the effect of the relaxation time rather than by the current location in the Galaxy.

To avoid this problem, we also considered the bivariate correlations of all the possible pairs of parameters listed above. In this case, for any pair of variables a bilinear fit is performed providing a $\chi^2_{biv}$. A permutation test by randomly swapping the second independent variable is then performed providing $10^4$ individual $\chi^2_{perm}$. The second independent variable is considered an independent correlator if the fraction of random realizations with a $\chi^2_{perm} > \chi^2_{biv}$ is larger than 99.7%. This test allows to identify the strongest among two correlating variables. Indeed, if we adopt $t_{age}/t_{rh}$ as the first independent variable, the only significant bivariate correlator would be metallicity. Instead, if we assumed any other parameter as the first independent variable, we would find the only significant bivariate correlation using $t_{rh}$ as the second independent variable. This indicates that the driving parameter in determining the MF slope is the half-mass relaxation time, while all the other correlations are...
only consequences of implicit covariances. In other word, the residuals of a fit in the $\alpha - \log(t_{\text{age}}/t_{\text{rh}})$ plane correlate with metallicity only.

In the right panel of Fig 13, the residual of the least-square fit in the $\alpha - \log(t_{\text{age}}/t_{\text{rh}})$ plane

$$\alpha = (1.17 \pm 0.12) \log(t_{\text{age}}/t_{\text{rh}}) - (1.35 \pm 0.07)$$

are shown as a function of metallicity. The slope of the apparent trend is $d\alpha/d\text{[Fe/H]} \sim 0.37$. We note that most of this correlation is driven by two GCs at the metal-poor end (NGC 4590 and NGC 7099) and by the most metal-rich GC of our sample (NGC 104).

6 DISCUSSION

In this paper, we performed the deepest photometry of HST/ACS/WFC parallel fields for 32 Galactic GCs, combined them with HST data available for the central regions of these GCs from the ACS Treasury Program (Sarajedini et al. 2007), and compared this dataset with multimass dynamical models to derive the present-day global MFs. Additionally, the masses, mass-to-light ratios, half-mass radii and fraction of remnants have been estimated for 31 GCs with available radial velocity information.

This represents one of the largest data sets for the GC present-day MFs constraining its variation with measurements in the outer regions of GCs. The MF slopes within our sample vary in the range $-1.2 \lesssim \alpha \lesssim 0$ and are comparable with the MFs found by SB17, who estimated MFs by focusing only on the GCs central region. This work represents an improvement with respect to the work by SB17 since most GCs of their sample extend far outside the field of view of the ACS, possibly under/over-estimating the degree of mass segregation (see Sollima et al. 2015). However, our result showed that the average difference between the two works is $\Delta \alpha = 0.03$ which is even smaller than the formal errors of the MF slope for each GC. Nevertheless, we have estimated flatter MFs for a few GCs (actually the most extended of the SB17 sample).

The estimated masses and half-mass radii are in good agreement with those found by SB17 and BH18, although significant differences are apparent for some extended GCs. By analyzing the radial variation of the MF slopes for each GC, we found that King-Michie multimass models adequately reproduce the observations for most GCs, but they underestimate the degree of mass segregation in dense GCs. While we cannot exclude that this evidence is an artifact due to an over-estimation of the completeness in the crowded central regions of denser GCs, this evidence could be real and indicate that GCs with denser cores evolve faster and reach a larger degree of mass segregation than what is predicted by King-Michie models. In this respect, a similar result has been found by Sollima et al. (2017) comparing various snapshots of N-body simulations selected at different stages of evolution with the same family of multimass models adopted here and found that over a long time interval, as the GC approaches the core-collapse phase, simulations reach a degree of mass segregation much larger than that indicated by the best fit analytical models.

We evaluated the present-day Jacobi radii for our GCs and estimated that, while around 60% of them are tidally underfilling with $r_h/r_J \lesssim 0.08$, in the outer Galactic region (at $R_{GC} > 8$ kpc) two distinct groups are distinguishable: the tidally underfilling and compact GCs with mean half-

Figure 13. Left panel: Slope of GC MFs as a function of the ratio of lifetime to present-day half-mass relaxation time. The least-square fit is marked by the dashed line. Right panel: Residual of fit in the $\alpha - \log(t_{\text{age}}/t_{\text{rh}})$ plane against metallicity.
mass density $\rho_{h,m} \approx 110 M_\odot pc^{-3}$ and the tidally filling and extended GCs with $\rho_{h,m} \approx 6 M_\odot pc^{-3}$. The GCs in the second group have small masses and are closer to dissolution. The existence of these two groups of GCs was put forward by Baumgardt et al. (2010) on the basis of a sample of half-mass and Jacobi radii calculated using single-mass fit to the projected density profile and assuming a simplified Galactic potential. They interpreted this evidence as a dichotomy in the primordial size of proto-GCs, similarly to what is observed in dwarf galaxies and young clusters in the Milky Way (Da Costa et al. 2009; Pfalzner 2009). Unfortunately, only a few GCs of our sample populate the region at large Galactocentric distances, where most of the tidally-filling GCs reside in the Baumgardt et al. (2010) sample. In particular, only 3 GCs classified by Baumgardt et al. (2010) as tidally filling (NGC 288, NGC 5053 and NGC 5466) are included in our sample, two of them have $r_0/r_J > 0.08$.

By investigating possible correlations between various parameters, we found that MF slope correlates significantly with half-mass relaxation time as previously suggested by Paust et al. (2010) and SB17. Other correlations have been found to be less significant and mainly driven by implicit correlations with $r_{18}$ (like the ones with the present-day positions of GCs in the Galaxy, $R_{GC}$ and $Z$, claimed by Djorgovski, Piotto, & Capaccioli 1993; Capaccioli, Piotto & Staivel 1993; Piotto & Zoccali 1999, and those with the average apo-/perigalactic distances). This result confirms the theoretical predictions provided by several studies (Gieles, Heggie & Zhao 2011; Baumgardt & Makino 2003), indicating that the MF slope is controlled by two-body relaxation and evaporation across the cluster tidal boundary. In particular, two-body relaxation leads to an efficient exchange of kinetic energy between GC stars, with the less massive stars gaining orbital energy. These stars reach the border of the GC potential well and escape in a time-scale proportional to the inverse of the squared energy (Fukushige & Heggie 2000). Hence, the short relaxation time causes a fast and efficient depletion of low-mass stars in the MF (see left panel of Fig. 13).

We investigate the significance of bivariate correlations between all the possible pairs of parameters. We found that, adopting the relaxation time as a first correlator, a significant bivariate correlation is found with metallicity. This second-order correlation strongly depends on a few GCs lying at the extremes of the metallicity range covered by our sample. Its significance is therefore not clear. However, if true, this correlation would indicate a flatter MF for more metal-rich GCs, in agreement with the predictions of star formation theories on the dependence of the IMF slope on metal content (Silk 1977; Marks et al. 2012; Chabrier, Hennebelle & Charlot 2014). We do not find any correlation between $\alpha$ and concentration, as claimed by de Marchi, Paresce, & Pulone (2007). Such a correlation has not been found in any of the studies based on HST datasets, i.e. Paust et al. (2010) and SB17.

The analysis presented here represents a further step forward in the determination of the dynamical properties of GCs, in particular their MF. While improvements on the observational side (like a better photometric coverage of the cluster extent, a larger sample of kinematics information, etc.) would be valuable to further reduce uncertainties and the possible bias, the next important improvement is likely linked to the theoretical modelling of these stellar systems. In particular, the results presented here indicate that analytical models provide only a first-order approximation of the mass segregation occurring in GCs. This might lead to significant systematics in the determined structural/dynamical quantities. Sophisticated techniques like direct N-body fitting (BH18) including a realistic treatment of tidal interaction, the development of velocity anisotropy and the effect of binary interactions are becoming feasible in the near future and represent the next step-forward in the determination of structural/dynamical parameters of GCs.

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