Sum-Rate Maximization of RIS-Aided Multi-User MIMO Systems With Statistical CSI

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Abstract—This paper investigates a reconfigurable intelligent surface (RIS)-aided multi-user multiple-input multiple-output (MIMO) system by considering only the statistical channel state information (CSI) at the base station (BS). We aim to maximize its sum-rate via the joint optimization of precoding matrix at the BS and phase shifts vector at the RIS. However, the multi-user MIMO transmissions and the spatial correlations make the optimization cumbersome. For tractability, an asymptotic sum-rate is derived under a large number of the reflecting elements. By adopting the asymptotic sum-rate as the objective function, optimal designs of the transmit precoding matrix and the phase shifts vector can be decoupled and solved individually. More specifically, a high-quality suboptimal solution of the transmit precoding matrix and phase shifts vectors can be obtained by utilizing the water-filling algorithm and the projected gradient ascent (PGA) algorithm, respectively. Comparing to the case of the instantaneous CSI assumed at the BS, the proposed algorithm based on the statistical CSI can achieve comparable performance but with much lower channel estimation overhead, information feedback overhead, and computational complexity, which is more affordable and appealing for practical applications. Moreover, the impact of spatial correlation on the asymptotic sum-rate is examined by using majorization theory.

Index Terms—Reconfigurable intelligent surface (RIS), multi-user MIMO, spatial correlations, phase shifts.

I. INTRODUCTION

A. Background

MASSIVE multiple-input multiple-output (MIMO), millimeter wave (mmWave) communications, ultrasound networks (UDN), and non-orthogonal multiple access (NOMA) have been recognized as the key enabling techniques to achieve higher spectral efficiency and massive machine-type communications (mMTCs) for the next generation mobile cellular systems [1], [2], [3], [4]. Particularly, by deploying a massive number of antennas over 30-300 GHz bands, the mmWave massive MIMO communication paradigm has shown its great potentials for providing ultra-high data-rate transmission as well as massive device connectivity in the upcoming 5G systems [5], [6]. Although the mmWave massive MIMO has attracted numerous interests from both the academic and industry communities, the confronted challenges span the broad fields of communication engineering and allied disciplines, such as escalating signal processing complexity, short-range communications, increasing hardware costs, high power consumption, etc. Moreover, the UDN and NOMA techniques also bring the additional deployment costs and imperfect successive interference cancellation (SIC) bottleneck, respectively. To meet the need for economic and sustainable future cellular networks, more efficient techniques have been proposed to address the spectrum shortage problem while improving the system performance.

With the rapid development of radio frequency (RF) micro electromechanical systems (MEMS) and metamaterial (e.g., metasurface), reconfigurable intelligent surfaces (RIS) is a band-new technology which has gained significant momentum [7], [8]. Specifically, RIS is an artificial metamaterial, which is composed of a large number of passive reflecting elements with low-cost. It is capable of digitally manipulating electromagnetic waves by inducing a certain phase shift on it to achieve certain communication objectives, e.g., enhancing the received signal energy, expanding the coverage region, alleviating interference, etc. [9]. Comparing with conventional
cooperative communication systems, i.e., amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF) relaying schemes, the RIS assisted wireless communications can obtain preferable electromagnetic propagation environment with limited power consumption.

B. Related Works

By properly adjusting the phase shifts to improve the electromagnetic propagation environment with a low cost and energy consumption, the RIS has attracted tremendous attention from both industry and academics, recently [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31]. The authors in [10] proposed RIS-aided multiple-input single-output (MISO) system communications by jointly optimizing the transmit precoding matrix at the base station (BS) and phase shifts vector at the RIS to minimize the total transmit power consumption. Numerical results showed that the RIS not only achieves the comparable performance to the conventional massive MIMO or multi-antenna AF relay, but also is more cost efficient. Similar conclusions were drawn in [11] by extending the continuous phase shifts to discrete phase shifts due to the hardware limitation at reflecting elements. In [12] and [13], the authors considered the sum-rate maximization problem for RIS-assisted multi-group multi-cast MISO and multi-user MISO systems, respectively, in which the optimal solutions of the transmit precoding matrix and the phase shifts vector were obtained by considering the transmit power constraint. In order to reap the benefit of diversity gain at user sides, the symbol error rate (SER) minimization problem for the RIS-assisted MIMO systems and the sum-rate maximization problem for the RIS-assisted multi-user MIMO systems also have been investigated in [14] and [15], respectively. Furthermore, the RIS-assisted user cooperation has been extended to the wireless powered communications, NOMA network, backscatter communications, physical layer security, unmanned aerial vehicle (UAV) communications, mmWave networks, etc. Reference [17], [18], [19], [20], [21], [22], [23].

Although the wireless communication systems can be significantly improved with the help of the RIS, the aforementioned works for the RIS-aided MIMO systems design, i.e., [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], are all based on instantaneous channel state information (CSI) available at the BS. As opposed to the traditional wireless systems where channel acquisition is a straightforward matter, due to the passive feature of RIS, the low-cost reflecting elements can not possess any active RF chains to facilitate channel estimation. In [24], the authors estimated the reflecting cascaded channel via element-wise on-off operation at each reflecting element. Another alternative approach to estimate the channel coefficients is leveraging compressive sensing and deep learning tools by proposing a novel RIS architecture where sparse channel sensors are assumed and a few number of elements are connected to the baseband of the RIS controller [25]. However, the channel training overhead becomes excessively high as the number of RIS reflecting elements and/or RIS-aided users increases. To overcome this challenge, in [26], [27], and [28], the authors proposed statistical CSI-based schemes to maximize the mutual information of RIS-aided MIMO systems with a single user. Comparing with the instantaneous CSI, the statistical CSI can be easily acquired at the BS and commonly stable over many consecutive coherence intervals. This advantage can significantly reduce the channel estimation overhead. Furthermore, another difficulty for RIS-aided MIMO systems is the joint design of the transmit precoding matrix at the BS and the phase shifts vector at the RIS. Since the optimization problems in [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], and [30] is non-convex with coupled optimization variables, the alternating algorithms have been widely adopted with high computational complexity.

The spatial correlation between transceiver antennas or reflecting elements for RIS-aided system always exist in realistic propagation environments due to space limitations for adjacent antenna or reflecting elements, non-rich scattering environments, etc. [28], [29], [30], [31], [32]. However, for analytical simplicity, the RIS-aided MISO/MIMO systems for multi-user case in existing works, i.e., [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], and [23], assumed the independence among antenna and reflecting elements. Even though the spatial correlation has been taken into account for RIS in [29], [31], and [30], those works only considered single-side spatial correlation on RIS (from RIS to users link) for single-input single-output (SISO) or MISO communication systems.

C. Motivation and Contributions

Motivated by the above challenges and issues, we focus on the maximization of the sum-rate of an RIS-assisted multi-user MIMO systems by leveraging only statistical CSI, in which the spatially correlated channels are considered. The main contributions of this paper can be summarized as follows.

- To reap the benefit of diversity gain from MIMO and RIS, we consider an RIS-aided multi-user MIMO system. Unlike the aforementioned works, the spatial correlations at the BS, RIS and users are considered in the paper. Based on this practical system model, the random matrix theory is invoked to derive the asymptotic sum-rate.

- Thanks to the asymptotic sum-rate, the impacts of the spatial correlation at each terminal are investigated with the help of majorization theory. We find that the spatial correlation at users have a negative impact on the asymptotic sum-rate. Moreover, the asymptotic sum-rate can benefit from the spatial correlation at the RIS through proper settings of the phase shifts at the RIS.

- In contrast to conventional instantaneous CSI-based schemes in [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], and [23], in this paper we propose a statistical CSI-based scheme to maximize the sum-rate, in which the statistical CSI includes the path-loss coefficients, the first order and the second-order statistics of small-scale fading. Comparing to the assumption of the instantaneous CSI, the proposed statistical CSI-based scheme in this paper can significantly reduce
the channel estimation overhead and phase shift information feedback overhead. Hence, our proposed scheme could be more suitable for RIS-aided multi-user MIMO systems in practical communications especially for the case of time-varying channels, where the overhead of instantaneous CSI acquisition may be prohibitively high given a large number of passive reflecting elements.

- As opposed to statistical CSI-based schemes for single user systems in [26] and [28], where the joint design of the transmit precoding matrix and the phase shifts vector are highly coupled and the solutions was obtained by using the alternative algorithm, in this paper, we resort to the asymptotic sum-rate to decouple the joint optimization problem into two sub-problems. These two sub-problems can be solved with two efficient algorithms, i.e., water-filling based algorithm and projected gradient ascent (PGA) algorithm, with a low computational complexity as compared to the alternative algorithm.

D. Organization of the Paper

The rest of the paper is organized as follows. In Section II, the system model of RIS aided multi-user MIMO systems under spatial correlation is presented. Section III derives the asymptotic sum-rate, with which the impact of spatial correlation is thoroughly investigated. In section IV, the asymptotic sum-rate is maximized by jointly optimizing the transmit precoding matrix and phase shifts vector. In Section V, the numerical results and discussion are presented and followed by Monte-Carlo simulations which demonstrated the effectiveness of our proposed scheme. Finally, Section VI concludes the paper.

Notations: The following notations are used throughout this paper. We use lower-case and upper-case letters to indicate column vectors and matrices, respectively. \( \mathbf{d} \) denotes an \( N \times 1 \) vector, \( \mathbf{A} \) denotes an \( N \times N \) identity matrix. The superscripts \((\cdot)^T\) and \((\cdot)^H\) denote the transpose and the conjugate-transpose operations, respectively. The notations \( (\cdot)^* \), \( \text{Tr} (\cdot) \) and \( \det (\cdot) \) represent the matrix principal square root, trace and determinant of matrices, respectively. \( (\cdot)^* \) denotes complex conjugate. \( \mathbb{C} \) stands for the set of complex numbers. \( \mathbf{A}(i,j) \) refers to the \((i,j)\)-th element of matrix \( \mathbf{A} \). We denote \( \mathbf{A} \succeq \mathbf{0} \) if \( \mathbf{A} \) is positive semidefinite. \( \mathbb{E} (\cdot) \) is the statistical expectation operator. The Kronecker product and the Hadamard product between two matrices \( \mathbf{A} \) and \( \mathbf{B} \) are denoted by \( \mathbf{A} \otimes \mathbf{B} \) and \( \mathbf{A} \circ \mathbf{B} \), respectively.

II. System Model and Problem Formulation

A. Transmission Model

This paper investigates an RIS-aided multi-user MIMO communication systems as shown in Fig. 1, where a multi-antenna BS serves \( K \) users with the help of a multi-reflecting element RIS. We assume that the BS and users are equipped with \( M \) and \( L \) antennas with uniform linear array (ULA) topology, respectively, and the BS located in the vicinity of the RIS. To enhance the system performance, a uniform planar array (UPA) topology RIS composed of a large number of passive and low-cost reflecting elements is mounted on the wall of a surrounding high-rise building to assist the BS in communicating with the users. The number of reflecting elements at the RIS is \( N \equiv N_r \times N_h \), where \( N \gg M \geq L \), \( N_r \) and \( N_h \) are the numbers of RIS elements at each row and each column, respectively. Similar to [28] and [31], we assume that the direct links from the BS to users are unavailable due to the obstacles, such as buildings. Moreover, each reflecting element of RIS can adjust the phase shift of the incident electromagnetic wave through the RIS controller, where the RIS controller receives the tune information from the BS via wired or wireless backhaul/control links. The channel matrices from the BS to the BS and from the RIS to the \( k \)-th user are denoted by \( \mathbf{H}_{Bi} \in \mathbb{C}^{N \times M} \) and \( \mathbf{H}_{Iu_k} \in \mathbb{C}^{L \times N} \), \( k \in \{1, \cdots, K\} \), respectively. All the channels are assumed to be Rayleigh fading. We denote the diagonal reflection matrix of RIS as \( \mathbf{\Theta} = \text{diag}(e^{j\theta_1}, \cdots, e^{j\theta_N}) \) and the phase shifts vector by \( \mathbf{\theta} = [\theta_1, \cdots, \theta_N]^T \), where \( \theta_n \in [0, 2\pi) \) represents the phase shift induced by the \( n \)-th element and can be adjusted to collaboratively achieve passive beamforming [10]. Accordingly, the received signal at user \( k \) is given by

\[
\mathbf{y}_k = \mathbf{H}_{Iu_k} \mathbf{\Theta} \mathbf{H}_{Bi} \mathbf{W}_k \mathbf{s} + \mathbf{n}_k,
\]

where

- \( \mathbf{s} = [\mathbf{s}_1^T, \cdots, \mathbf{s}_K^T] \in \mathbb{C}^{KD \times 1} \), \( \mathbf{s}_k \in \mathbb{C}^{D \times 1} \) denotes the transmit symbol vector of user \( k \) with \( D \) data streams satisfying \( 1 \leq KD \leq \min(M, KL) \). Furthermore, it is assumed that \( \mathbb{E} (\mathbf{s}_i \mathbf{s}_j^H) = \mathbf{I}_D \), and \( \mathbb{E} (\mathbf{r}_i \mathbf{r}_j^H) = \mathbf{0}, \) for \( i \neq j \).
- \( \mathbf{W} = [\mathbf{W}_1, \cdots, \mathbf{W}_K] \) is the \( M \times KD \) transmit precoding matrix at the BS, in which \( \mathbf{W}_k \in \mathbb{C}^{M \times D} \) is the linear precoding matrix for the \( k \)-th user and with total transmit power constraint \( \text{Tr}(\mathbf{W} \mathbf{W}^H) = K \sum_{k=1}^{K} \text{Tr}(\mathbf{W}_k \mathbf{W}_k^H) \leq P_{\text{max}} \).
- \( \mathbf{n}_k \) represents the additive white Gaussian noise at the \( k \)-th user with zero mean and variance matrix of \( \sigma_k^2 \mathbf{I}_L \). i.e., \( \mathbf{n}_k \sim \mathcal{CN}(\mathbf{0}, \sigma_k^2 \mathbf{I}_L) \).

When the perfect CSI for each link and phase shifts at the RIS are available at the \( k \)-th user, the downlink instantaneous
achievable rate for the $k$-th user can be expressed as
\[
R_k = \log_2 \det \left( I_L + \Psi_k \left( \sum_{i=1, i \neq k}^K \Psi_i + \sigma_L^2 I_L \right)^{-1} \right),
\] (2)
where $\Psi_j = \mathbf{H}_{IU_k} \otimes \mathbf{H}_{BI} \mathbf{W}_j \mathbf{H}_{IU_k}^H \mathbf{H}_{BI}^H$, $j \in \{k, i\}$.

**B. Spatially Correlated Channels**

Due to multiple antennas and multiple reflecting elements mounted at the users, BS and RIS, the close spacing among antennas/reflecting elements will result in the spatial correlation in realistic propagation environments. The spatial correlation significantly affects the achievable rate, which cannot be ignored especially for limited size of RIS and terminals [31], [32], [33]. To account for the impact of spatial correlation, the channel matrices $\mathbf{H}_{BI}$ and $\mathbf{H}_{IU_k}$ are modeled by the widely used Kronecker correlation channel model as [32]
\[
\mathbf{H}_{BI} = \beta_{BI} \mathbf{R}_B^{\frac{1}{2}} \mathbf{H}_{BI} \mathbf{T}_B^{\frac{1}{2}},
\]
\[
\mathbf{H}_{IU_k} = \beta_{IU_k} \mathbf{R}_{IU_k}^{\frac{1}{2}} \mathbf{H}_{IU_k} \mathbf{T}_I^{\frac{1}{2}}, k \in \{1, \cdots, K\}. \tag{3}
\]
It can be seen from (3) and (4) that the channel matrices are composed of four independent components. Specifically, $\beta_{BI}$ and $\beta_{IU_k}$ denote the (long-term) path-loss effect. $\mathbf{H}_{BI} \in \mathbb{C}^{N \times M}$, and $\mathbf{H}_{IU_k} \in \mathbb{C}^{L \times N}$ are random matrices whose entries are independent and identically distributed (i.i.d.) complex circularly symmetric Gaussian random variables, i.e., $\text{vec}(\mathbf{H}_{BI}) \sim \mathcal{CN}(0_{MN}, \mathbf{I}_M \otimes \mathbf{I}_N)$ and $\text{vec}(\mathbf{H}_{IU_k}) \sim \mathcal{CN}(0_{NL}, \mathbf{I}_N \otimes \mathbf{I}_L)$. $\mathbf{T}_B \in \mathbb{C}^{M \times M}$, $\mathbf{R}_B \in \mathbb{C}^{L \times L}$, $\mathbf{T}_I \in \mathbb{C}^{N \times N}$, and $\mathbf{R}_I \in \mathbb{C}^{N \times N}$ are the deterministic covariance matrices that characterize the spatial correlation among the channels of the transmit antennas, the receive antennas and the RIS elements, respectively. In addition, all the correlation matrices are positive semi-definite Hermitian matrices. For analytical simplicity, we assume those correlation matrices are normalized such that $\text{Tr}(\mathbf{T}_B) = M$, $\text{Tr}(\mathbf{T}_I) = \text{Tr}(\mathbf{R}_B) = N$ and $\text{Tr}(\mathbf{R}_I) = L$. Moreover, it is worth noting that all the mentioned spatially correlated random matrices can be assumed to be time-invariant, because the spatial correlation matrices commonly keep constant over a fairly long time, which depends mainly on the configurations of MIMO antennas and reflecting elements, and the operating frequency [34], [35].

**C. Problem Formulation**

For the multi-user communications systems, the sum-rate is the most significant and essential performance metric to measure the spectral efficiency. Hence, we aim at maximizing the sum-rate by jointly designing the transmit precoding matrix $\mathbf{W}$ at the BS and the phase shifts vector $\theta$ at the RIS subject to the transmit power constraint at the BS. The sum-rate maximization problem can be formulated as:
\[
\mathcal{P} : \max_{\mathbf{W}, \theta} \sum_{k=1}^{K} R_k, \tag{5}
\]
where (C1) and (C2) represent the transmit power constraint and phase shift range constraint, respectively.

It is obviously found that the problem $\mathcal{P}$ is difficult to solve, due to the coupling relationship between the preceding matrix $\mathbf{W}$ and phase shifts vector $\theta$. To tackle this challenging problem, most of works resort to an alternating way by iteratively optimizing the transmit precoder and phase shifts until the convergence is reached, e.g. [10], [11], [12], [13], [14], [15], [16], [17]. Although the optimization variables can be decoupled with the help of alternating optimization and the subproblem at the $t$-th iteration can be efficiently solved, the objective functions in [10], [11], [12], [13], [14], [15], [16], and [17] are derived under the requirement of instantaneous CSI at both the BS and the RIS. However, this assumption is impractical for RIS assisted wireless systems with large number of reflecting elements. In practice, since the CSI is usually estimated with the aid of pilot sequences, the passive RIS entails prohibitively high channel estimation accuracy, where “passive” can only change the reflecting angles of incident signals without the capability of amplifying the signal and frequent signal exchange between the BS and the RIS [24], [25]. Hence, the prior alternating optimization algorithms relying on instantaneous CSI have to face the critical issue of channel estimation and high phase adjustment costs for RIS-assisted systems. Besides, comparing to RIS-assisted systems with single antenna receiver or single user in [10], [11], [12], [13], [14], [16], [17], [29], [31], and [30], it is more intractable to maximize the instantaneous achievable sum-rate in (5), due to the involvement of multi-data streams, multi-antenna transceivers and multiple users.

To overcome the above challenges, the benefit brought by a large number of RIS can be utilized by exploiting the channel hardening property. Thanks to the low energy cost features of RIS-assisted system, hundreds of reflecting elements for RIS are more affordable by comparing to hundreds of antennas equipped in massive MIMO systems in practice [36]. In such circumstance, we derive the asymptotic sum-rate by applying random matrix theory tools. The analytical result enables us to formulate a new optimization problem by exploiting statistical CSI as well as gain some meaningful insights.

**III. ANALYSIS OF THE SUM-RATE**

**A. Asymptotic Analysis**

Since the multi-user MIMO, multi-data streams and RIS cascaded channel are considered in the system model, the objective function for instantaneous sum-rate in (5) involves complicated matrix operations such as matrix inversion, determinant, and matrix chain products. These operations significantly challenges the joint optimization design. For tractability,
we first derive the asymptotic sum-rate for the RIS-assisted multi-user MIMO systems in Theorem 1.

Theorem 1: As the number of reflecting elements at the RIS increases, i.e., $N \to \infty$, the asymptotic sum-rate is given by

$$R \to_{a.s.} \frac{1}{N} \log_2 \det \left[ I_L + \frac{\Phi_k}{N} \sum_{i=1}^{K} \Phi_i + \frac{\sigma_r^2}{\varphi_k} \text{Tr}(T_\text{f} \Theta R_{\text{f}} \Theta^H) I_L \right]^{-1}$$

where $\Phi_{k,j} = \frac{1}{N} \text{Tr} \left( T_\text{f} W_j W_k^H \right) R_{U_k,j} j \in \{k,i\}$ and $\varphi_k = \sum_{i=1}^{N} \sum_{i \neq k} r_{k,i}$. 

Proof: See Appendix A.

By considering a large number of reflecting elements in RIS-assisted multi-user MIMO systems, it is worth noting that the instantaneous sum-rate converges to the asymptotic sum-rate which only depends on the statistical CSI, including path-loss coefficients and the spatial correlation matrices. Similar to the massive MIMO systems, the deterministic property for the large scale RIS-assisted wireless communication systems is due to the channel hardening effect, in which the channel variations become small and the fading channels turn to be deterministic with increasing number of reflecting elements on the RIS [35], [37].

With the help of random matrix theory, the sum-rate for RIS-aided multi-user MIMO scenarios can be represented by tractable expression as (6). Section V will verify that the asymptotic result matches well with Monte Carlo simulations when the RIS equips with more than 400 reflecting elements. However, when a large number of reflecting elements are deployed at the RIS, the distance between any two adjacent elements is very short. This leads to more severe spatial correlation at the RIS. Hence, it is necessary and interesting to investigate the impact of spatial correlation for RIS-aided MIMO systems, and some meaningful insights will be provided in the following.

B. Impact of Spatial Correlation

Based on Eq. (6), we can find that the spatial correlation at the BS, RIS and user sides influence the asymptotic sum-rate through $\text{Tr}()$ and $\det()$, respectively. For the spatial correlation matrix at the BS, it is involved in the $\text{Tr}(T_B W_k W_k^H)$ and $\text{Tr}(T_B W_j W_j^H)$ terms. Hence, the design of the transmit precoding matrix highly depends on the spatial correlation matrix at the BS $T_B$, which will be demonstrated in next section. To provide qualitative conclusion on the impact of the spatial correlation at the RIS and user sides, some basic concepts are introduced to facilitate our analysis, including majorization, Schur-convexity, and Schur-concavity [41].

Definition 1: (Majorization [42]) For two positive semi-definite matrices $A_1$ and $A_2$ with identical dimension

where $N \times N$, the descending order vector $\lambda_1 = (\lambda_{1,1}, \cdots, \lambda_{1,N})^T$ and $\lambda_2 = (\lambda_{2,1}, \cdots, \lambda_{2,N})^T$ represent the vectors of the eigenvalues for $A_1$ and $A_2$, respectively. If

$$\begin{align*}
\sum_{i=1}^{k} \lambda_{1,i} \geq \sum_{i=1}^{k} \lambda_{2,i}, \quad k = 1, 2, \ldots, N - 1 \\
\sum_{i=1}^{k} \lambda_{1,i} = \sum_{i=1}^{k} \lambda_{2,i}, \quad k = 1, 2, \ldots, N - 1
\end{align*}$$

we say $\lambda_1 \succeq \lambda_2$ and the matrix $A_1$ is more correlated than the matrix $A_2$ [43]. Therefore, the eigenvalue vector $[0, 0, \cdots, 0]^T$ and $[1, 1, \cdots, 1]^T$ correspond to full- and independent-correlated channel models w.r.t. matrix $A_1$.

Definition 2: (Schur-Convexity and Schur-Concavity [42])

For any two arbitrary vectors $x, y \in \mathbb{R}^n$ and real valued function $f : \mathbb{R}^n \to \mathbb{R}$, if $x \succeq y$, $f$ is said to be Schur-convexity for $f(x) \geq f(y)$. Conversely, $f$ is said to be Schur-concavity for $f(x) \leq f(y)$.

In what follows, the impacts of spatial correlations at the $k$-th user and the RIS on the asymptotic sum-rate will be examined individually.

1) Impact of Spatial Correlation at Users: By capitalizing on eigenvalue decomposition (EVD) of $R_{U_k} = U_{U_k} A_{U_k} U_{U_k}^H$ into (6), where $U_{U_k} \in \mathbb{C}^{L \times L}$ is a unitary matrix and $A_{U_k} = \text{diag} (\lambda_{U_{k,1}}, \cdots, \lambda_{U_{k,L}})$ denotes a diagonal matrix and its diagonal elements are the eigenvalues of $R_{U_k}$. Then, for deterministic matrices $T_B$, $T_f$ and $R_f$, the asymptotic sum-rate w.r.t. $R_{U_k}$ is re-expressed as

$${\bar R}^{\text{asy}}(R_{U_k}) = \sum_{k=1}^{K} \log_2 \det \left[ I_L + \frac{\epsilon_k U_{U_k} A_{U_k} U_{U_k}^H}{N} + \frac{\zeta_k}{\epsilon_k} I_L \right]^{-1}$$

$$= \sum_{k=1}^{K} \log_2 \left[ \text{det} \left( 1 + \frac{\epsilon_k}{\epsilon_k} + \frac{\zeta_k}{\epsilon_k} I_L \right)^{-1} \right]$$

$$= \sum_{k=1}^{K} \sum_{l=1}^{L} \log_2 \left[ 1 + \frac{\epsilon_k}{\epsilon_k} + \frac{\zeta_k}{\epsilon_k} I_L \right]^{-1},$$

where $\epsilon_k = \frac{1}{N} \text{Tr} \left( T_B W_k W_k^H \right)$, $\zeta_k = \frac{1}{N} \text{Tr} \left( T_B W_l W_l^H \right)$, $\varphi_k = \frac{\sigma_r^2}{\varphi_k} \text{Tr}(T_\text{f} \Theta R_{\text{f}} \Theta^H)$ and $\varphi > 0$. Based on Definitions 1 and 2, we have the following theorem.

Theorem 2: The asymptotic sum-rate $\bar R^{\text{asy}}$ is Schur-concave with respect to the eigenvalue vector of $R_{U_k}$. If $R_{U_{k_1}}$ is more correlated than $R_{U_{k_2}}$, i.e., $\lambda_{U_{k_1}} \succeq \lambda_{U_{k_2}}$, we have

$$\bar R^{\text{asy}}(R_{U_{k_1}}) \leq \bar R^{\text{asy}}(R_{U_{k_2}}).$$

Proof: By defining a function $f(\lambda_{U_k}) = -\log_2 \left[ 1 + (\frac{\epsilon_k}{\epsilon_k} + \frac{\zeta_k}{\epsilon_k} I_L)^{-1} \right]$, $\lambda_{U_{k_1}} \in (0, L)$, we can find that $f(\lambda_{U_{k_1}})$ is convex and twice differentiable with respect to $f(\lambda_{U_{k_1}})$. By using [42], Proposition

\[ \text{Proposition} \]
3.C.1] and [44, Proposition 2.7], it can be proved that the negative value of deterministic sum-rate
\[-R^{arsy} = \sum_{k=1}^{K} \sum_{l=1}^{L_f} f(\lambda_{U_k})\]
is Schur-convex w.r.t. the eigenvalue vector \(\lambda_{U_k} = [\lambda_{U_k,1}, \ldots, \lambda_{U_k,L_f}]^T\) of \(R_{U_k}\).
According to [42, A.I. Definition], \(R^{arsy}\) is Schur-concave w.r.t. \(\lambda_{U_k}\) if and only if \(-R^{arsy}\) is Schur-convex w.r.t. \(\lambda_{U_k}\).

Finally, we arrive at Theorem 2.

With Theorem 2, it can be found that the spatial correlation at the \(k\)-th user has a negative impact on the asymptotic sum-rate.

2) Impact of Spatial Correlation at the RIS: From (6), both \(T_I\) and \(R_I\) are in trace operation, i.e., \(\text{Tr}(T_I\Theta R_I\Theta^H)\), and it is hard to investigate the impact of \(T_I\) or \(R_I\) even when the corresponding matrix \((R_I\) or \(T_I\)) is deterministic, since the trace function \(\text{Tr}(\cdot)\) is both Schur-concave and Schur-convex w.r.t. the eigenvalue vector of \(T_I\) or \(R_I\).

Therefore, the trace of \(\text{Tr}(T_I\Theta R_I\Theta^H)\) can be written as \(\text{Tr}(T_I\Theta R_I\Theta^H) = \text{Tr}(T_I^2) = \sum_{n=1}^{N} g(\lambda_{T_I,n}), \lambda_{T_I,n} \in (0, N),\)
where \(g(x) = x^2\).

Since \(g(x)\) is convex and twice differentiable w.r.t. \(x\), with the help of [42, Proposition 3.C.1] and [44, Proposition 2.7],
we can easily prove that \(\text{Tr}(T_I^2)\) is convex w.r.t. \(\lambda_{T_I}\).

By using the monotonically increasing property of \(R^{arsy}(T_I)\) w.r.t. \(\text{Tr}(T_I^2)\), the impact of reflect-correlated matrix \(T_I\) on the asymptotic sum-rate can be revealed in the following theorem.

Theorem 3: Under the identity reflection matrix at the RIS and \(T_I = R_I\) assumptions, if \(T_I\), is more correlated than \(T_{I_2}\), i.e., \(\lambda_{T_I,n} \geq \lambda_{T_{I_2},n}\).

\[R^{arsy}(T_{I_1}) \geq R^{arsy}(T_{I_2}).\] (10)

Hence, it can be found that the spatial correlation at the RIS has a positive impact on the asymptotic sum-rate under the \(\Theta = I_N\) and \(R_I = T_I\) assumptions. Besides, based on [42, H.1.g Theorem], we can prove that \(\text{Tr}(T_I\Theta R_I\Theta^H) \leq \prod_{n=1}^{N} \lambda_{T_I,n}^L \lambda_{R_I,n}^R\),
where \(\lambda_{T_I}^L\) and \(\lambda_{R_I}^R\) denote the \(n\)-th eigenvalue of \(T_I\) and \(R_I\) with descending order, respectively.

By defining a function \(f(\lambda_{T_I}) = \prod_{n=1}^{N} \lambda_{T_I,n}^L \lambda_{R_I,n}^R\),
where \(\lambda_{T_I} = [\lambda_{T_I,1}, \ldots, \lambda_{T_I,L_I}]^T\),
we can easily obtain that the function \(f(\lambda_{T_I})\) is Schur-convex w.r.t. \(\lambda_{T_I}\).

Hence, if the upper bound of \(\text{Tr}(T_I\Theta R_I\Theta^H)\) can be achieved, i.e., \(\text{Tr}(T_I\Theta R_I\Theta^H) = \prod_{n=1}^{N} \lambda_{T_I,n}^L \lambda_{R_I,n}^R\),
we can also reach the conclusion that the spatial correlation at the RIS has a positive impact on the asymptotic sum-rate.

With the help of the asymptotic result in (6) and specific assumptions, we show that the spatial correlations at the RIS and users have opposite impacts on the asymptotic sum-rate.

More specifically, on the one hand, increasing spatial correlation on the user side reduces the effective dimensionality of the receiver. On the other hand, increasing spatial correlation on the RIS side also enables focusing power. Those results for spatial correlation impacts are consistent with [45]. In the next section, we will formulate an optimization problem by adopting the asymptotic sum-rate as the objective function.

IV. SUM-RATE MAXIMIZATION

Although jointly optimization of transmit precoding matrix and phase shifts vector for the RIS-aided multi-user MIMO system has been proposed in [15], the high signaling overhead of instantaneous CSI acquisition and computational complexity of alternating optimization algorithm hinder its application in practice. To address these issues, we resort to maximize the asymptotic sum-rate instead. By comparing with previous works that maximized the sum-rate based on instantaneous CSI [12], [13], [15], the asymptotic sum-rate in (6) only relies on statistical CSI. Accordingly, the reformulated problem is given as

\[
\mathcal{P} : \max_{W, \theta} \sum_{k=1}^{K} \log_2 \det \left( I_L + \Phi_k \left( \sum_{i=1, i\neq k}^{K} \Phi_i + \bar{\sigma}^2 \right)^{\frac{1}{2}} \right) 
\]

s.t. \(\text{Tr}(WW^H) \leq P_{max}\),

\[C1) : \theta_n \leq 2\pi, n = 1, \ldots, M.\] (11)

Observing the objective function in (11), we can find that the transmit precoding matrix \(W\) and phase shifts vector \(\theta\) are only related to \(\phi_{j}, j \in \{1, \ldots, K\}\) and \(\text{Tr}(T_I\Theta R_I\Theta^H)\), respectively. Besides, \(C1)\) and \(C2)\) are the constraints for two optimization variables, respectively.

Unlike the previous works [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [28], [29], [31], [46], [47], and [30], it can be found that the designs of transmit precoding matrix \(W\) and RIS phase shifts vector \(\theta\) can be decoupled in the reformulated problem. In what follows, the optimizations of \(W\) and \(\theta\) are tackled one by one.3

A. Optimization of Phase Shifts

Noticing that \(\theta\) is only related to the trace term \(\text{Tr}(T_I\Theta R_I\Theta^H)\) in the denominator of the inverse matrix in (11), and the objective function is monotonically increasing w.r.t. \(\text{Tr}(T_I\Theta R_I\Theta^H)\).

The optimal phase shifts design can be transformed to the maximization of the trace term \(\text{Tr}(T_I\Theta R_I\Theta^H)\) under the constraint of (C2). Then, the corresponding optimization problem w.r.t. \(\theta\) can be casted as

\[
P_1 : \max_{\theta} \text{Tr}(T_I\Theta R_I\Theta^H),
\]

s.t. \(C2) : 0 \leq \theta_n \leq 2\pi, n = 1, \ldots, M.\) (12)

Let \(v = [e^{j\theta_1}, \ldots, e^{j\theta_n}, \ldots, e^{j\theta_N}]^T\), together with the matrix identity of [48, Lemma 7.5.2], problem (12) can be reduced to

\[
\mathcal{P}_1 : \max_{V} \text{tr}(EV),
\]

s.t. \(V_{n,n} = 1, n = 1, \ldots, N, V \succeq 0 \& \text{rank}(V) = 1,\) (13)

where \(E = T_I \odot R_I^T\). Since both \(T_I\) and \(R_I\) are positive semi-definite Hermitian matrices, \(E\) is also a positive semi-definite Hermitian matrix [48, P477].

It is well known that (13) is a NP-hard quadratically constrained quadratic programs (QCQP) problem due to the

3It is worth mentioning that the optimal design to be carried out at the BS, and the optimal phase shifts are sent from the BS to the RIS through a RIS controller for the configuration of reflecting elements.
rank-one constraint. There are some popular tools that can deal with this problem, such as semi-definite relaxation (SDR) algorithm [10], majorization-minimization (MM) algorithm [15], iterative algorithm for continuous phase case (IA- CPC) [49], wideband beampattern formation via iterative techniques (WBFIT) [50], etc. Although, the SDR algorithm has been widely adopted to solve the NP-hard QCQP problem with rank one constraint, it relies heavily on the convex optimization solvers such as CVX, and requires a sufficiently large number of randomizations to guarantee the approximation with high computational complexity [51].

In order to reduce the computational complexity, we apply the PGA algorithm to obtain a sub-optimal solution $\theta^*$ in this work. The main idea of PGA algorithm is similar to the concept of gradient descent technique by employing gradient ascent to monotonically increase the objective function of (12). However, at each iteration, the solution should project onto the closest feasible point that satisfies the unit-modulus constraint. The main steps of PGA algorithm in each iteration $(t)$ is given as following.

First we compute the gradient ascent direction by taking the first-order derivative of $\psi = \text{Tr} (T_\theta \Theta R \Theta^H)$ w.r.t. $v_n = e^{j\theta_n}$ as

$$\frac{\partial \psi}{\partial v_n} = \text{Tr}(T_\theta(E_{nn} R \Theta^H - v_n^{-1} \Theta R \Theta E_{nn})), \quad (14)$$

where $E_{nn}$ denotes the all-zero $N \times N$ matrix except that the $(n, n)$-th element is 1.

We denote $v^{(t)} = [v_1^{(t)}, \ldots, v_N^{(t)}]$, where $|v_n^{(t)}| = 1, n \in \{1, \ldots, N\}$. Then, the new vector $\tilde{v}^{(t+1)}$ is updated by setting $p^{(t)} = \left[ \frac{\partial \psi}{\partial v_1}, \ldots, \frac{\partial \psi}{\partial v_N} \right]$ as ascent direction,

$$\tilde{v}^{(t+1)} = v^{(t)} + \alpha p^{(t)}, \quad (15)$$

where $\alpha$ is the step size of gradient ascent.

It can be readily found that the update vector $\tilde{v}^{(t+1)}$ violates the unit-modulus constraint in (13). Hence, it has to be projected onto the closest feasible point that satisfies the constraint, and the corresponding phase shifts vector is given as

$$v^{(t+1)} = \exp(j \text{arg}(\tilde{v}^{(t+1)})). \quad (16)$$

Since gradient ascent is along the monotonically increasing direction of $\psi$ at each iteration and $\text{tr} (E v^{(t)} v^{(t)H})$ is upper bounded, the convergence of the PGA algorithm is therefore guaranteed. When the algorithm converges, we can obtain the sub-optimal phase shifts vector as $\theta^* = \text{arg}(v^{(t)})$. It is worth emphasizing that global optimality of the phase shifts cannot be guaranteed unless the problem (12) is not convex w.r.t. $\theta$.

B. Optimization of Transmit Precoding Matrix $W$

Next, the transmit precoding matrix $W$ at the BS will be optimally designed to maximize the asymptotic sum-rate of RIS-aided multi-user systems. Since the phase shifts are independent of $W$ and have been designed in Section IV-A, the optimization problem for $W$ can be simplified as

$$\mathcal{P}_2 : \max_W \sum_{k=1}^{K} \log_2 \det \left[ I_L + \text{Tr}(T_B W_k W_k^H) \tilde{R}_{U_k} \right] \times \left[ (1 - \eta_k) \tilde{P} \tilde{R}_{U_k} + \epsilon_k I_L \right]^{-1},$$

s.t. (C1) : $\text{Tr}(WW^H) \leq P_{max},$ \quad (C2) : $\tilde{R}_{U_k} = R_{U_k}/N$ and $\epsilon_k = \frac{\sigma_k^2}{\text{tr}(\tilde{R}_{U_k} \Theta \Theta^H)}$ is constant for a given optimal phase shift vector $\theta^*$, as shown in Section IV-A.

This problem is also non-convex since the optimization variables are highly coupled, which appears in both the numerator and the denominator of the objective function. To solve this problem, we provide an efficient low-complexity algorithm to obtain high-quality suboptimal transmit precoding matrix in the following. At first, by introducing auxiliary variables $\tilde{P}$ and $\eta = [\eta_1, \ldots, \eta_K]^T, (0 \leq \eta_k \leq 1, \sum_{k=1}^{K} \eta_k = 1)$, where $\tilde{P} = \text{Tr}(T_B WW^H)$, and $\eta$ denotes the power weight vector for each user precoder after multiplying by the spatial correlation matrix $T_B$, i.e., $\eta_k \tilde{P} = \text{Tr}(T_B W_k W_k^H)$, the optimization problem $\mathcal{P}_2$ can be relaxed as

$$\max_{\tilde{P}, \eta, W} \sum_{k=1}^{K} \log_2 \det \left[ I_L + \eta_k \tilde{P} \tilde{R}_{U_k} \right] \times \left[ (1 - \eta_k) \tilde{P} \tilde{R}_{U_k} + \epsilon_k I_L \right]^{-1},$$

s.t. (C1) : $\text{Tr}(WW^H) \leq P_{max},$ \quad (C2) : $\tilde{P} = \text{Tr}(T_B WW^H),$ \quad (C4) : $\sum_{k=1}^{K} \eta_k = 1, \eta_k \geq 0.$ \quad (18)

The objective function in (18) can be rewritten as

$$f(\tilde{P}, \eta) = \sum_{k=1}^{K} \log_2 \det \left[ \left( \epsilon_k I_L + \tilde{P} \tilde{R}_{U_k} \right) \left( (1 - \eta_k) \tilde{P} \til{R}_{U_k} + \epsilon_k I_L \right)^{-1} \right],$$

$$= \sum_{k=1}^{K} \sum_{l=1}^{L} \log_2 \frac{\epsilon_k + r_{k,l} \tilde{P}}{\epsilon_k + (1 - \eta_k) r_{k,l} \tilde{P}} \quad (19)$$

where $r_{k,l} = \lambda U_{k,l}/N$, $L$ is the l-th eigenvalue of matrix $\tilde{R}_{U_k}$. It is readily found from (19) that $f(\tilde{P}, \eta)$ is a monotonically increasing function of $\tilde{P}$ since the first derivative of $\log_2 \frac{\epsilon_k + r_{k,l} \tilde{P}}{\epsilon_k + (1 - \eta_k) r_{k,l} \til{P}}$ w.r.t. $\tilde{P}$ is always greater than 0. Therefore, $W$ should be optimally chosen to maximize $\tilde{P}$, such that

$$\mathcal{P}_3 : \max_W \text{Tr}(T_B WW^H),$$

s.t. (C1) : $\text{Tr}(WW^H) \leq P_{max},$ \quad (C5) : $\text{Rank}(W) = KD.$ \quad (20)

where the constraint (C5) denotes the $KD$ data streams at the BS. It can be identified that the above problem is equivalent to Rayleigh quotient problem [52]. As such, we present the
optimal structure of $\mathbf{W}$ to maximize $\bar{P}$ in the following theorem.

**Theorem 4:** Assuming that the EVD of $\mathbf{T}_B$ is $\mathbf{T}_B = \mathbf{U}_T \Sigma_T \mathbf{U}_T^H$, where $\{\mathbf{U}_T, \Sigma_T\} \in \mathbb{C}^{M \times M}$, $\Sigma_T$ denotes a diagonal matrix and its diagonal elements are the eigenvalues of $\mathbf{T}_B$ with descending order, i.e., $\Sigma_T(1, 1) \geq \Sigma_T(2, 2) \geq \cdots \geq \Sigma_T(M, M)$. The singular value decomposition (SVD) of $W$ is $\mathbf{W} = \mathbf{U}_W \Sigma_W \mathbf{V}_W^H$, where $\mathbf{U}_W \in \mathbb{C}^{M \times M}$, $\Sigma_W \in \mathbb{C}^{M \times K_d}$, $\mathbf{V}_W^H \in \mathbb{C}^{K_d \times K_d}$. We can obtain that the optimal solution for $\mathbf{W}$ is the one which satisfies

$$\mathbf{W}^* = \mathbf{U}_T \Sigma_W \mathbf{V}_W^H,$$  \hspace{1cm} (21)

where $\Sigma_W$ denotes the all-zero $M \times K_d$ matrix except that the entry of the $(1, 1)$-th element is $P_{max}$, $\mathbf{V}_W^H$ is the arbitrary unitary matrix.

By substituting (21) into (20), we obtain the maximum $\bar{P}$ as $\bar{P}^* = t_{max} P_{max}$, where $t_{max}$ is the maximum eigenvalue of $\mathbf{T}_B$.

Now we put our focus on optimizing the power weight vector $\eta$. By substituting $\mathbf{P}^*$ into (18) and ignoring the constant terms, the asymptotic sum-rate maximization w.r.t. $\eta$ is reformulated as

$$\min_{\eta} \sum_{k=1}^{K} \ln \det \left[ \epsilon_k \mathbf{I}_L + (1 - \eta_k) \mathbf{P}^* \mathbf{R}_{uk} \right],$$  \hspace{1cm} s.t. (C4) : $\sum_{k=1}^{K} \eta_k = 1$, $\eta_k \geq 0$.  \hspace{1cm} (22)

We can find that (22) is a water-filling problem, and the corresponding Lagrangian function is given by

$$\mathcal{L}(\eta, \tau) = \sum_{k=1}^{K} \sum_{i=1}^{L} \ln \left[ \epsilon_k + (1 - \eta_k) r_{k,i} \mathbf{P}^* \right] + \tau \left( \sum_{k=1}^{K} \eta_k - 1 \right),$$  \hspace{1cm} (23)

where $\tau$ is the lagrange multipliers associated with constraint $\sum_{k=1}^{K} \eta_k = 1$. Due to complementary slackness, the optimal lagrange multipliers are zeros in constraints $\eta_k \geq 0$.

By taking the derivative of $\mathcal{L}(\eta, \tau)$ w.r.t. $\eta_k$, we have

$$\frac{\partial \mathcal{L}(\eta, \tau)}{\partial \eta_k} = \tau - \sum_{l=1}^{L} \frac{1}{1 - \eta_k + \epsilon_k / (r_{k,i} \mathbf{P}^*)}.  \hspace{1cm} (24)$$

Note that by setting $\frac{\partial \mathcal{L}(\eta, \tau)}{\partial \eta_k} = 0$, we can obtain the optimal solution of $\eta$. However, according to [53, Corollary 17.5.4], there is no closed-from solution for the equation $\frac{\partial \mathcal{L}(\eta, \tau)}{\partial \eta_k} = 0$ w.r.t. $\eta_k$ if $L \geq 5$. To deal with this issue, we have one proposition for the optimal solution $\eta^*$ as follows.

**Proposition 1:** By setting (24) equal to zero, the optimal solution $\eta^*$ can be obtained by using a nested bisection method, with which the globally optimal solution can be obtained.

**Proof:** Since $\eta_k \in [0, 1]$, we can easily obtain the upper and lower bounds of $\tau$, which are respectively given by

$$\tau < \max_{k=1, \cdots, K} \{ \sum_{i=1}^{L} \frac{1}{1 + \epsilon_k / (r_{k,i} \mathbf{P}^*)} \} = \tau_{up},  \hspace{1cm} (25)$$

and

$$\tau_{lb} \triangleq \min_{k=1, \cdots, K} \sum_{i=1}^{L} \frac{1}{1 + \epsilon_k / (r_{k,i} \mathbf{P}^*)} < \tau.  \hspace{1cm} (26)$$

Given an initial value $\tau^{(0)}$, we can always obtain a solution for each $\eta_k^{(i)}$ by using nested bisection method, where $i$ is the number of the iterations and $k \in \{1, \cdots, K\}$. We set $\tau^{lb} = \tau^{i}$ if $\sum_{k=1}^{K} \eta_k^{(i)} < 1$, and $\tau^{ub} = \tau^{i}$ otherwise. Then $\tau^{(i)}$ is updated as $\tau^{(i+1)} = \frac{\tau^{lb} + \tau^{ub}}{2}$. With the updated $\tau$, $\eta_k^{(i+1)}$ is recalculated. This process is repeated until $\sum_{k=1}^{K} \eta_k^{(i+1)} - 1$ less than the error tolerance $\epsilon$. Since the first-order derivative function $\frac{\partial \mathcal{L}(\eta, \tau)}{\partial \eta_k}$ is monotone decreasing and increasing w.r.t. $\eta_k$ and $\tau$, respectively, we can guarantee that the nested bisection method converges to the unique globally optimal solution. The nested bisection method is summarized in Algorithm 1 in next subsection.

Based on the optimal solution $\bar{P}^* = t_{max} P_{max}$ and the optimal power weight vector $\eta^*$, we can obtain the optimal solution for $\mathbf{W}_k, k \in \{1, \cdots, K\}$ as

$$\mathbf{W}_k = \mathbf{U}_T \Sigma_{W_k} \mathbf{V}_{W_k}^H,$$  \hspace{1cm} (27)

where $\Sigma_{W_k}$ denotes the all-zero $M \times d$ matrix except that the $(1, 1)$-th element is $\eta_k P_{max}$, $\mathbf{V}_{W_k} \in \mathbb{C}^{d \times d}$ is an arbitrary unitary matrix.

Finally, the optimal design method for $\mathbf{W}_k$ can be summarized as follows. On the one hand, the left-singular vectors of $\mathbf{W}_k$ should be consistent with eigenvectors of $\mathbf{T}_B$, and the maximum singular value of $\mathbf{W}_k$ corresponds to the dominant eigenvalue of $\mathbf{T}_B$ to maximize $\text{Tr}(\mathbf{T}_B \mathbf{W}_k \mathbf{W}_k^H)$. On the other hand, the power allocation among $\{\mathbf{W}_1, \cdots, \mathbf{W}_K\}$ for each user should be designed on the basis of the statistical CSI, including path-loss for cascaded channel, the second-order statistics of small-scale fading for each channel, and the spatial correlation matrix at each user, which can be solved by applying the bisection search.

### C. Summary and Discussions

In Sections IV-A and IV-B, we solve the optimization problem by decoupling it into two sub-problems. Clearly from (27), the direction of the optimal transmit precoding matrix $\mathbf{W}_k$ is designed to be aligned with the transmit correlation $\mathbf{T}_B$ at the BS, while the power allocation for each user is determined by the receive correlation at the user itself. Meanwhile, it is shown in (12) that the phase shifts $\theta$ are designed purely by the spatial correlations at the RIS. The pseudo-code of the joint optimization of the precoding matrix $\mathbf{W}$ and RIS phase shifts vector $\theta$ is provided in Algorithm 1.

Next, we investigate the computational complexity of Algorithm 1, which includes the optimal phase shifts and transmit precoder design in Section IV-A and IV-B. At the beginning of the optimal phase design, we have to calculate $\mathbf{Z} = \mathbf{T}_r \odot \mathbf{R}_T^T$, the computational complexity of which is $\mathcal{O}(N^2)$. For the iteration of the PGA algorithm, the computational complexity mainly depends on the gradient calculation, which is also $\mathcal{O}(N^2)$. By denoting the converged numbers of iterations for
the PGA algorithm as $t_p$, the complexity of PGA algorithm from Step 1 to 8 is given by $O(t_pN^2)$. For the transmit precoding matrix design, i.e., Step 9 to 20, the mainly complexity comes from the EVD operation and nested bisection method. Firstly, we have to obtain the eigenvectors $U_k$ in Step 9 by EVD operation, whose computational complexity is given by $O(M^3)$. For the nested bisection method, it includes two nested binary searching loops. The outer loop varies the Lagrange multiplier $\tau$ to meet the weighting factor constraint. The inner loop searches the water level for each hop at a given value of $\tau$ to satisfy (24). The outer loop involves $\log_2\left(\frac{N_{\text{out}} - N_{\text{in}}}{\varepsilon_{\text{out}}} \right)$ iterations where $\varepsilon_{\text{out}}$ represents outer loop accuracy [54]. The inner loop has $K$ binary searches, and each involves $\log_2\left(\frac{N_{\text{in}}}{\varepsilon_{\text{in}}} \right)$ iterations, where $\varepsilon_{\text{in}}$ is the inter loop accuracy. Therefore, the overall computational complexity of Algorithm 1 is given by $O(t_pN^2 + M^3 + K \log_2\left(\frac{N_{\text{out}} - N_{\text{in}}}{\varepsilon_{\text{out}}} \right) \cdot \log_2\left(\frac{N_{\text{in}}}{\varepsilon_{\text{in}}} \right))$. By employing the BCD algorithm with instantaneous CSI in [15], the total computational complexity for the sum-rate maximization based on instantaneous CSI is $O(t_{\text{BCD}}(\max\{K,M,\rho\}^3, N^3 + t_{\text{MM}}N^2))$, where $t_{\text{BCD}}$ and $t_{\text{MM}}$ represent the numbers of iterations required until convergence for the BCD algorithm and Majorization-Minimization (MM) algorithm for phase shift design, respectively. The computational complexities of the proposed Algorithm 1 and BCD algorithm in [15] are summarized in Table I. Since the number of RIS elements $N$ is usually much larger than the number of BS antennas $M$, it can be observed that the complexity of the proposed algorithm is much lower than that of the BCD algorithm in [15].

Moreover, the channel estimation and the information feedback overhead from the BS to the RIS in this paper is lower than the instantaneous CSI-based scheme in [15] as compared in Table I. Specifically, the instantaneous CSI-based scheme has to estimate the instantaneous cascaded channel information, calculate the precoding matrix and phase shifts vector, and feed the information of the optimal phase shifts back to RIS within coherence time, as shown in Fig. 2. However, for the statistical CSI-based scheme in this paper, the statistical CSI can be acquired by averaging over the channel realizations across several coherence intervals [55]. This is due to the fact that the statistical CSI is rather static and stable over a fairly long time, which have been verified by measurements in [56]. Therefore, our proposed scheme are more attractive in practice due to the avoidance of frequent channel estimations and CSI feedbacks.

V. SIMULATION RESULTS

In this section, numerical results are presented to validate our foregoing analysis and examine the sum-rate performance achieved by our proposed algorithm. The simulated scenario involves a BS communicating to two users with RIS assisted through the Rayleigh fading channel suffering from severe path loss. We assume the antennas at the BS and each user in the form of ULA topology, while the RIS is configured with $N = N_v \times N_h$ reflecting elements in the form of UPA topology. $N_v$ and $N_h$ are the numbers of elements to per row and per column, respectively. Both the inter-antenna separation at transceiver and the inter-element separation at the RIS are half wavelength. Moreover, we adopt the exponential correlation model to characterize the spatial correlation among antennas, which have been widely utilized in the literatures [38], [39]. Specifically, for the ULA topology at the BS and the users, the $(i,j)$-th element of correlation matrices $T_B$ and $R_{U_k}$ are given as $T_B(i,j) = \rho_{T_B}^{i-j}$ and $R_{U_k}(i,j) = \rho_{R_{U_k}}^{i-j}$, respectively, where $k \in \{1,2\}$. Regarding the spatial correlation matrices $R_I$ and $T_I$ at the RIS, we assume $R_I = T_I$, and apply the Kronecker product of the vertical correlation matrix $R_v \in \mathbb{C}^{N_v \times N_v}$ and the horizontal correlation matrix $R_h \in \mathbb{C}^{N_h \times N_h}$ to the correlation model, i.e., $R_I = T_I = R_v \otimes R_h$ [28], [57]. The elements in $R_v$ and $R_h$ are modeled as $R_v(i,j) = \rho_{R_v}^{i-j}$ and $R_h(i,j) = \rho_{R_h}^{i-j}$, respectively, where $0 \leq \{\rho_{T_B}, \rho_{R_{U_1}}, \rho_{R_{U_2}}, \rho_{R_h}\} \leq 1$ denote the correlation coefficients between any two adjacent antennas or reflecting elements. Compared to the complicated spatial correlation models for ULA topology in [26] and for UPA topology in [31], the exponential correlation model adopted in this paper is relatively simple, since it is controlled by only one parameter. The large-scale path loss is modeled as $\beta_{BI} = (d_{BI}/d_0)^{-\alpha_{BI}}$ and $\beta_{IU_k} = (d_{IU_k}/d_0)^{-\alpha_{IU_k}}$, where $\{d_{BI}, d_{IU_k}\}$ and $\{\alpha_{BI}, \alpha_{IU_k}\}$ denote the communication distance and path loss exponent of corresponding channel, respectively. $d_0 = 10\text{ m}$ stands for a reference distance. Unless otherwise mentioned, other system parameters are set $M = 10$, $L = 3$, $N_v = 20$, $d_{BI} = 10\text{ m}$, $d_{IU_k} = d_{IU_2} = 60\text{ m}$, $\alpha_{BI} = 2.2$, $\alpha_{IU_k} = \alpha_{IU_2} = 2.8$, $D = 2$, and the signal-to-noise ratio (SNR) for each user is $\frac{P_{\text{out}}}{\sigma_i^2} = 15\text{ dB}$. The simulated result are averaged over $10^2$ channel realizations.

A. Verifications

Fig. 3 depicts the simulated result (labeled as “Sim.”) and the asymptotic sum-rate (labeled as “Asy.”) under different number of user antennas $L$ as the reflecting elements number $N$ increases, wherein $N = N_v \times N_h$ by increasing the column number of reflecting element at the RIS from $N_h = 1$ to $N_h = 25$. Moreover, the equal power allocation (EPA) scheme $W_kW_k^H = \frac{P_{\text{out}}}{K}I_M$ is employed at the BS, the phase shift for each element is set as $\theta_{n} = \frac{\pi}{3}$ and the correlation coefficients of each equipments are set as $\rho_{T_B} = \rho_{R_{U_1}} = \rho_{R_{U_2}} = \rho_{R_h}$.
The feasible solution \( \theta^{(1)} \); 
- The iteration number \( t = 1 \), and the accuracy \( \varepsilon \); 
3. \textbf{repeat} 
4. Calculate the gradient \( p^{(t)} = \left[ \frac{\partial \phi}{\partial \theta_1}, \cdots, \frac{\partial \phi}{\partial \theta_N} \right] \), where \( \frac{\partial \phi}{\partial \theta_n} \) is given in (14); 
5. Calculate \( \hat{v}^{(t+1)} = v^{(t)} + \alpha p^{(t)} \) for gradient ascent direction; 
6. Update \( v^{(t+1)} = \text{exp}(j \text{arg}(\hat{v}^{(t+1)})) \); 
7. Calculate the value of the OF in (13) as \( \phi(v^{(t+1)}) \); 
8. until \( |\phi(v^{(t+1)}) - \phi(v^{(t)})| \leq \varepsilon \); 
9. Calculate \( U_T \) according to eigenvalue descending order, given \( T \); 
10. Initialize 
   - The upper and lower bounds on \( \tau, \tau^{ub} \) in (25) and \( \tau^{lb} \) in (26); 
   - The upper and lower bounds on \( \eta_k, \eta_k^{lb} = 0 \) and \( \eta_k^{ub} = 1 \) \( (k \in \{0, \ldots, K\}) \); 
   - The user index \( k = 0 \); 
11. \textbf{repeat} 
12. \( \tau = (\tau^{lb} + \tau^{ub})/2 \); 
13. \textbf{repeat} 
14. \( k = k + 1 \); 
15. Calculate \( \eta_k^* \) with bisection search method based on (24); 
16. until \( k = K \) 
17. if \( \sum_{k=1}^{K} \eta_k - 1 \leq 0 \) then \( \tau^{ub} = \tau \); 
18. else \( \tau^{lb} = \tau \); 
19. until \( \sum_{k=1}^{K} \eta_k - 1 \leq \varepsilon \); 
20. Calculate \( \mathbf{W}_k^* = U_T \Sigma_k^{1/2} \mathbf{V}_k^\dagger \Sigma_k^{1/2} \); 
\textbf{Ensure:} The optimal phase shifts vector \( \theta^* = \text{arg}(v^{(t)}) \) and the precoder \( \mathbf{W}^* = [\mathbf{W}_1^*, \cdots, \mathbf{W}_K^*] \).

\( \rho_{R_k} = 0.3 \). It can be seen from Fig. 3 that the simulation result approaches to the asymptotic sum-rate as the number of reflecting element increases, and coincides well with the asymptotic result for over \( 20 \times 20 \) reflecting elements when \( L = 3 \). This confirms the correctness of the asymptotic analysis in Section III. Fig. 3 also shows that the asymptotic sum-rate increases with \( L \) due to the benefit of spatial diversity gain for multi-antenna receiver. In the meantime, it can be observed from this figure that the asymptotic sum-rate in (6) can be regarded as an upper bound of the achievable sum-rate.

In Fig. 4 (a), we plot the asymptotic sum-rate versus the number of reflecting elements \( N \) for different values of exponential correlation coefficient \( \rho_{R_k} \) by setting \( \rho_T = \rho_{R_k} = \rho_{R_u} = 0.1 \). While in Fig. 4 (b), we plot the asymptotic sum-rate versus the number of reflecting elements \( N \) for different values of the exponential correlation coefficient \( \rho_{R_u} \) together with \( \rho_T = \rho_{R_k} = 0.1 \). It can be observed from Figs. 4(a)-(b) that the increase of the number of reflecting elements causes the enhancement of the asymptotic sum-rate. However, the spatial correlation at user \( R_{U_k} \) negatively influences the asymptotic sum-rate as observed in Fig. 4(a). In contrast to the spatial correlation at user side, it is readily seen in Fig. 4(b) that the spatial correlation at the RIS has a positive effect on the asymptotic sum-rate.

### B. Optimal System Design

In this subsection, we present numerical results to examine the sum-rate achieved by our proposed algorithm. We compare the performance of the proposed algorithm based on statistical CSI with other schemes. Fig. 5 investigates the sum-rate of different transmit schemes by utilizing the “random phase shifts and EPA” as the benchmarking scheme, where the phase shifts vector \( \theta^* \) and transmit precoding matrix \( \mathbf{W}^* \) are calculated based on our proposed algorithm in Section IV. We can find that the jointly optimized scheme (“\( \theta^* \) & \( \mathbf{W}^* \)”) performs better than the individually optimized schemes (“\( \theta^* \) & EPA” and “random phase shift & \( \mathbf{W}^* \)”), and the “random phase shifts and EPA” scheme shows the worst performance, which justifies the significance of joint phase shift and precoding design. Besides, it can be seen from Fig. 5 that the “random phase shift & \( \mathbf{W}^* \)” scheme provides superior performance.
Fig. 4. Sum-rate comparison between the proposed scheme and four baseline schemes with $N = 80$ and $\rho_{T_B} = \rho_{R_{U_k}} = \rho_{R_{v}} = \rho_{R_{h}} = 0.3$.

Fig. 5. Sum-rate versus the number of reflecting elements $N$ for various schemes.

This is due to the fact that the optimization improvement by (12) and (17) is limited by comparing with transmit power increasing.

For comparison purpose, the performance of the block coordinate descent (BCD) algorithm in [15] is also plotted in Fig. 5. It is readily found that the BCD scheme in [15] achieves the best performance among all the schemes because of the utilization of instantaneous CSI. However, the instantaneous CSI acquisition is considerably challenging in RIS-assisted wireless systems. In order to show the superiority of our proposed algorithm, other three benchmarking CSI-based schemes will be used for comparison in Fig. 5. In Fig. 6, we plot the sum-rate of RIS-aided multi-user MIMO systems versus the number of reflecting elements $N$ considering three different CSI conditions at the BS, including perfect CSI, statistical CSI, and without CSI. For the perfect CSI, we employ the block coordinate descent (BCD) algorithm as [15], where the precoding matrices and phase shifts are alternately optimized based on the instantaneous CSI, and the sum-rate is obtained by averaging over 200 channel realizations. In the case of statistical CSI, the proposed algorithm in Section IV is adopted. In the case of without CSI, the EPA precoder and random phase shifts scheme are employed as in [26]. It can be observed from Fig. 6 that the perfect CSI performs the best among the three schemes at the cost of frequent CSI reportings and high signaling overhead as analyzed in Section IV-C. However, As the number of the reflecting elements increases, the gap between the perfect CSI-based scheme and the statistical CSI-based scheme decreases due to the channel hardening effect. Although the scheme without CSI is the simplest one for implementation, it shows the worst performance. To strike a balanced trade-off between computational complexity and performance, the proposed algorithm is very appealing particularly for the large number of reflecting elements at the RIS assisted wireless systems with a comparable performance gain.

VI. CONCLUSION

This paper aimed to enhance the RIS-aided multi-user MIMO system via the joint design of transmit precoder and phase shifts. In contrast to aforementioned literature, the
spatial correlation was considered in this paper, which significantly distinguishes our analysis and optimal design from the prior ones. To address the above issues, the asymptotic sum-rate was derived by capitalizing on random matrix theory. Thanks to the tractable expression of the asymptotic sum-rate, some meaningful insights considering the impacts of the spatial correlation on RIS-aided multi-user MIMO systems were gained with the aid of majorization theory. Besides, the asymptotic result not only enabled the accurate evaluation of the sum-rate under a large number of reflecting elements, but also decoupled the transmit precoding matrix and RIS phase shift vector. This motivated us to formulate a new optimization problem by adopting it as the objective function, which only depends on the statistical CSI. Moreover, an efficient algorithm was proposed based on water-filling and PGA method to solve this problem. Comparing with the designs requiring instantaneous CSI in the literature, our proposed algorithm is more attractive with comparable performance but much lower complexity and communication overhead.

APPENDIX A PROOF OF THEOREM 1

In order to derive the asymptotic rate of user $k$, the following lemma regarding the deterministic equivalent of the random matrix chain products is given below.

**Lemma 1:** Assume two random matrix with the form $H = R_1^{1/2}HT_1^{1/2}$ and $G = R_2^{1/2}GT_2^{1/2}$, where $T_1 \in \mathbb{C}^{M \times N}$, $R_1 \in \mathbb{C}^{N \times N}$, $T_2 \in \mathbb{C}^{N \times N}$, and $R_2 \in \mathbb{C}^{L \times L}$ are Hermitian positive deterministic matrices with bounded spectral norm, i.e., $\max \{ \| T_1 \|, \| R_1 \|, \| T_2 \|, \| R_2 \| \} < \infty$ and diagonal elements being one. $H \in \mathbb{C}^{N \times M}$, $G \in \mathbb{C}^{L \times N}$ are two mutually independent random matrices each having i.i.d. entries such that $\text{vec}(H) \sim CN(0_{N \times M}, \sigma_1^2 I_M \otimes I_N)$ and $\text{vec}(G) \sim CN(0_{L \times N}, \sigma_2^2 I_N \otimes I_L)$. Given a deterministic matrix $W$ with compatible dimension, then we have the matrix chain products deterministically approaches to

$$
\frac{1}{N} \text{GHWH}^HHG^{\frac{1}{2}} \xrightarrow{a.s.} \frac{\sigma_1^2 \sigma_2^2}{N^2} \text{Tr}(T_2R_1) \text{Tr}(T_2W)R_2.
$$

**(8)**

**Proof:** Lemma 1 can be obtained by directly replacing $H$ and $G$ in [40, Theorem 1], the detailed proof is omitted here to avoid redundancy.

By dividing both the numerator and denominator terms of (2) by $1/N$, the method of deterministic equivalent is then applied to derive compact expressions for $\Psi_k/N$ and $\Psi_t/N$. Specifically, by using Lemma 1, it yields a deterministic approximation of $\Psi_k/N$ as

$$
\Psi_k = \frac{1}{N} H_{IU_k}^H \Theta B_I W_k^H (H_{IU_k} \Theta B_I W_k) \xrightarrow{a.s.} \frac{\sigma_1^2 \sigma_2^2}{N^2} \text{Tr}(T_2R_1) \text{Tr}(T_2W)R_2.
$$

Similarly, the derivation of $\Psi_t/N$ can be obtained by following the same method. If the number of the reflecting elements is very large, i.e., $N \rightarrow \infty$, the achievable sum-rate approaches to a deterministic equivalent as (6) by substituting approximation of $\Psi_k/N$ and $\Psi_t/N$ into (2). We thus complete the proof.

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