HOW TO MEASURE $m_t$: A BRIEF OVERVIEW

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ABSTRACT

This talk is an overview of prospects for measuring the top mass $m_t$ at present and future colliders. Methods for extracting $m_t$ appropriate to experiments at the Tevatron, Large Hadron Collider, and Next Linear Collider are discussed, with examples given for each. Sources of systematic uncertainties specific to each method and experiment are identified.

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1. Introduction

Discovery of the top quark will be followed immediately by attempts to measure its mass $m_t$. What we know so far is suggestive but not conclusive. As of this conference, direct searches have been unsuccessful, leading to lower limits on $m_t$ of 113 GeV and 131 GeV from CDF$^{1)}$ and D0$^{2)}$, respectively. Global fits to precision electroweak measurements at the $Z^0$ from LEP and SLC imply$^{3)}$ $m_t = 174^{+11+17}_{-12-19}$ GeV. The top mass is of interest not only as a parameter of the Standard Model (SM). Since top is heavier by far than the other quarks, it may be uniquely able to shed light on the origin of mass, and many quantities both within and beyond the SM depend on $m_t$, some quite sensitively. Hence even our ability to make predictions to test extensions or alternatives to the SM depend on knowing the top mass.

Measuring the mass of the top quark is not so straightforward as for the other quarks. Because top is so heavy, it has a large width ($\Gamma_t \approx 1$ GeV for $m_t = 150$ GeV; $\Gamma_t$ increases as $m_t^3$) and it decays before it can form hadrons. The absence of a sharp $t\bar{t}$ resonance makes $m_t$ hard to measure. In this talk we discuss methods for measuring $m_t$ that we do have at our disposal at present and future hadron and $e^+e^-$ colliders. This is not meant to be a comprehensive review; rather we present a brief overview with examples of available methods.

Methods for measuring $m_t$ fall into three categories: (i.) measuring the production cross section and comparing observed event rates with those expected; (ii.) reconstructing the top quark’s four-momentum from the momenta of its decay products; and (iii.) measuring other distributions that are sensitive to $m_t$. We will see examples of each in the discussions below. Exactly how well we can do will depend, among other things, on the value of $m_t$ itself. The heavier top is, the harder it is to produce, and the larger is its intrinsic width.

Before discussing specific experiments, some comments about top detection modes are in order. The $t$ quark decays to a real $W$ boson and a $b$ quark nearly 100% of the time. Since top is predominately produced in pairs, its detection modes are characterized by the $W$ decays.

- In the dilepton mode, each of the $W$’s decays to an $e$ or a $\mu$, and such events contain two isolated charged leptons, two $b$ jets, and missing energy from two undetected neutrinos. This is the cleanest channel, with the lowest backgrounds. Unfortunately it has a low branching ratio ($\sim 5\%$), and the neutrinos prohibit top momentum reconstruction.
In the *all jets* mode, both $W$’s decay hadronically. These events contain six jets, and so all of the decay products from both top quarks are, in principle, detectable. With a branching ratio of 45%, this mode occurs at the largest rate. However, at hadron colliders, large QCD backgrounds can overwhelm the signal.

A compromise is struck in the *lepton+jets* mode, with hadronic and one leptonic $W$ decay. The branching ratio is appreciable (15%) and the events are characterized by one isolated lepton, four jets (two from $b$’s), and missing energy from a single neutrino. One of the top momenta can be fully reconstructed from jets.

We will see below how the detection channels and methods of measurement are best combined in particular experiments, and what this implies about sources of systematic uncertainties.

2. **Top mass measurement at the Tevatron**

For a top quark mass on the order of 150 GeV, the Tevatron $p\bar{p}$ collider will produce about $10^4 t\bar{t}$ pairs in $10^3$ pb$^{-1}$ of integrated luminosity. The challenge will be to make the most of the relatively modest numbers of events left over after branching ratios and cuts to reduce background are taken into account. Of the mass determination methods mentioned above, reconstructing the top momentum has the best prospects here. Measuring the production cross section is useful for setting lower limits on $m_t$ in the absence of a significant signal, but theoretical uncertainties are too large to make it of much use for measuring $m_t$ once a signal has been obtained. And the event rate will be insufficient for measuring distributions other than that of the reconstructed mass.

Reconstructing the top momentum at the Tevatron will be best achieved in the lepton+jets channel. Requiring a single lepton cuts down on QCD backgrounds while allowing full reconstruction of one of the top momenta. Furthermore, the other top momentum can be reconstructed (with a two-fold ambiguity) by attributing missing transverse energy to the undetected neutrino. This method is discussed, *e.g.*, in Ref. [4], from which we show some results for $m_t = 150$ GeV in Figure 1. Kinematic cuts and energy smearing have been applied, but no hadronization or detector effects are included. At least one $b$ tag is required. Figs. 1(a) and 1(b) show, respectively, the reconstructed $bl\nu$ mass (using the tagged $b$) and the reconstructed 3-jet mass (with a $W$ mass constraint on two of the jets). Combinatorial backgrounds from wrong jet combinations are included in the $t\bar{t}$ curves and the $W$+jets background is shown as the dashed line. A narrower signal peak can be obtained by combining the two methods, as shown in Fig. 1(c). It shows
the distribution of the mean of the reconstructed–mass pair which gives the closest two values of \( m_t \), with their difference constrained to be < 50 GeV.

The sources of systematic uncertainties include, from the theory, higher order QCD effects in predictions for both signal and backgrounds, and on the experimental side, jet definition and energy scale, detector efficiencies and acceptance, and effects related to \( b \)-tagging. In addition, extra gluons can be radiated before and after top decays and can lead to further ambiguities in mass reconstruction.\(^5\) From such analyses we can expect eventually to obtain a top mass measurement at the Tevatron to about 10–15 GeV, depending on \( m_t \).

3. Top mass measurement at the Large Hadron Collider

The LHC will be a top factory: with a yearly integrated luminosity of \( 10^4 \) pb\(^{-1} \), it will produce on the order of \( 10^7 \) \( t \bar{t} \) pairs each year. Unfortunately, with high luminosity we get multiple interactions, which leads to problems for any measurement involving jets because of extra hadronic activity. There are two ways to get around this for purposes of measuring \( m_t \):\(^6\) we can run at reduced luminosity, and we can run at design luminosity, but avoid using jets.
3.1 Mass reconstruction at low luminosity

The cross section for top production is large enough at LHC energies that we can afford to reduce the luminosity by a factor of ten to ameliorate some of the problems associated with high luminosity, and still have an appreciable event rate. We can then use the lepton+jets channel to reconstruct the top momentum as at the Tevatron. Figure 2 shows the three-jet invariant mass distribution obtained in Ref. [6] for top events at the LHC, for $m_t = 130$ GeV and $m_t = 200$ GeV. The distributions show clear peaks at the top mass. In addition to kinematic cuts and a $W$ mass constraint, $b$ fragmentation and detector effects are included. Backgrounds can be reduced further by using $b$ tagging, though some signal events are lost in the process as well. This measurement will have larger systematic than statistical uncertainties (the large fluctuations shown are the result of limited Monte Carlo statistics), and the dominant sources of systematic uncertainties will be those associated with measuring jet energies and possibly $b$ tagging. These include initial state radiation and hadronic activity from the underlying event, the jet cone algorithm, and especially $b$ fragmentation and calorimeter response.

3.2 Sequential $l\ell'$ mass distribution at design luminosity

At design luminosity at the LHC, dealing with jets in top events will be difficult. However, the $t\bar{t}$ event rate is so large that we can hope to find an easily measured distribution that is sensitive to $m_t$ but does not get spoiled by multiple interactions. Such a distribution can be obtained in the clean dilepton channel if we require that one of the
Figure 3: Sequential $ll'$ mass distribution at LHC using the dilepton channel with semileptonic $b$ decay. (a) $m_t = 130$ GeV. (b) $m_t = 200$ GeV. From Ref. [6].

$b$'s also decays leptonically, e.g., $t \rightarrow W^+ (\rightarrow l^+ \nu) b (\rightarrow c l^- \bar{\nu})$ with $\bar{t} \rightarrow W^- (\rightarrow l^- \bar{\nu}) b$. The distribution of the invariant mass of the opposite-sign pair of leptons that come from the same top (e.g., the $l^+$ and $l^-$ from the $t$ in the above example) has the desired properties. Hence one searches for two isolated leptons and one additional non-isolated lepton from the $b$ decay. In Figure 3 (from Ref. [6]), which shows $m_{ll'}$ distributions for $m_t = 130$ and 200 GeV, we see that the peak and mean values increase with $m_t$. (Again, fluctuations are due to limited Monte Carlo statistics.) Here the systematic effects are dominated by the lepton energy scales and details of the $b$ fragmentation, and there may be contributions from dependence of $m_{ll'}$ on the transverse momentum of the top. Ultimately, using such distributions, we expect the sensitivity of top mass measurements that may be achieved at the LHC to be a few GeV, and perhaps as low as 2 GeV.\textsuperscript{7} Note that the expected sensitivity here begins to be of the same order of magnitude as the intrinsic width of the top quark.

4. Top mass measurement at the Next Linear Collider

A precision measurement of $m_t$ requires the clean environment of a high energy $e^+e^-$ collider. All three methods discussed in the introduction can be used, and a measurement of the top mass to about a few hundred MeV can be obtained. We can study the top production cross section and top momentum distribution near $t\bar{t}$ threshold and, at higher energies, reconstruct the top mass, to obtain independent measurements of $m_t$. (In practice, it is likely that we will use the latter measurement to tell us at what energies to
Figure 4: Cross section and top momentum distribution near $t\bar{t}$ threshold at NLC. (a) Production cross section as a function of collision energy. From Ref. [8]. (b) Top 3-momentum distribution for $m_t = 149, 150, 151$ GeV. From Ref. [9].

perform the threshold studies.)

4.1 Production cross section at $t\bar{t}$ threshold

As mentioned above, the top width is too large for a narrow toponium resonance peak to appear in the production cross section. Nonetheless, there is some structure in the threshold region due to the attractive Coulomb-like QCD potential between the $t$ and $\bar{t}$ quarks. Near the $t\bar{t}$ threshold, the cross section as a function of collision energy exhibits a bump which spreads out and eventually disappears as $m_t$ increases. The height and position of the bump behave similarly when $\alpha_s$ decreases (see Figure 4(a), from Ref. [8]); this is because $\alpha_s$ determines the depth of the potential. Thus one obtains correlated measurements of $m_t$ and $\alpha_s$. Here the all jets channel can be used to identify the top signal. Systematic uncertainties in this case arise from effects due to initial state radiation, beam energy spread, and beamstrahlung, and are related to understanding the energy of the hard collision.

Another handle available near the $t\bar{t}$ threshold is the 3-momentum distribution $d\sigma/d|\vec{p}_t|$ of the top quarks, which have some Fermi motion. Here one can use the lepton+jets channel (to reduce combinatorial backgrounds) and measure $d\sigma/d|\vec{p}_t|$ at fixed collision energy, as shown in Fig. 4(b) from Ref. [9]. The magnitude of the 3-momentum at which $d\sigma/d|\vec{p}_t|$ peaks (typically of order 10–20 GeV) is very sensitive to $m_t$ and can be relatively insensitive to $\alpha_s$. The systematic uncertainties here are related to energy
and momentum measurement and arise from hadronic effects and undetected sources of momentum imbalance.

Because of the insensitivity of the $|\vec{p}_t|$ distribution to $\alpha_s$, the best prospects for measuring $m_t$ come from combining a threshold energy scan with a measurement of $d\sigma/d|\vec{p}_t|$ at fixed energy near threshold. Such a combined measurement is expected to give $m_t$ to a few hundred MeV, depending on the value of $m_t$.

4.2 Mass reconstruction at high energy

At energies well above $t\bar{t}$ threshold at the NLC, it will be straightforward to reconstruct the top 4-momentum from its decay products. Using either the all jets or lepton+jets channel, three-jet invariant masses can be reconstructed to obtain $m_t$. Figure 5, from Ref. [10], shows $m_{jjj}$ distributions for $m_t = 150$ GeV and $\sqrt{s} = 500$ GeV. We see that the lepton+jets channel gives a narrower distribution, but the all jets channel has a larger overall rate. Systematic effects here arise from measurement of missing energy, detector acceptances, jet resolution, and QCD-related effects, such as ambiguities in assigning gluon jets in reconstructed momenta.\(^5\)

Note that the systematic effects involved in the high energy measurement are completely different from those in the threshold measurement, so that these methods give independent measurements of $m_t$. (The threshold measurement gives a smaller overall uncertainty.)
5. Summary

We expect top mass measurements to improve markedly with each new machine at which top is produced. How well we can do depends on the actual value of $m_t$, but the following numbers are reasonable estimates. Mass reconstruction at the Tevatron can be expected to give $m_t$ to about 10–15 GeV. At the LHC, methods such as mass reconstruction at low luminosity or measurement of the sequential $ll'$ distribution at design luminosity should reduce uncertainties in $m_t$ to a few, perhaps 2, GeV. Finally, precision measurements will be possible at the NLC via cross section and top 3-momentum measurements at $t\bar{t}$ threshold, and mass reconstruction at higher energies, resulting in an $m_t$ measurement to a few hundred MeV.

Note added: As of this writing (May 1994), CDF has reported\(^\text{11}\) seeing evidence for top quark production. Assuming the excess they see is indeed due to top, they obtain $m_t = 174 \pm 10^{+13}_{-12}$ GeV from fits to their data. Their method for obtaining $m_t$ is similar in spirit to that discussed in section 2 above; see Ref. [11] for details.

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