The strange quark mass from scalar sum rules updated

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The talk discusses preliminary results of an updated analysis of the strange quark mass from the scalar current QCD sum rules [1]. In particular the role of the scalar form factor which is a main ingredient in the analysis is especially emphasised. The sources of the uncertainties in the sum rule determination are briefly reviewed.

1. Introduction

A precise determination of the strange quark mass, being one of the fundamental parameters in the Standard Model (SM), is of paramount interest in several areas of present day particle phenomenology. Until today two main methods have been employed to achieve this task. QCD sum rules [2] have been applied to various channels containing strange quantum numbers, in particular the scalar channel that will be the subject of this talk [3–5], and more recently also lattice QCD simulations have been used to extract the strange quark mass [6,7].

The basic object which is investigated in the simplest version of QCD sum rules is the two-point function $\Psi(q^2)$ of two hadronic currents

$$ \Psi(q^2) \equiv i \int dx e^{iqx} \langle 0 | T \{ j(x) j(0) \} | 0 \rangle, \quad (1) $$

where in our case $j(x)$ will be the divergence of the vector current,

$$ j(x) = \partial^\mu (\bar{s} \gamma_\mu u)(x) = i (m_s - m_u)(\bar{s} u)(x). \quad (2) $$

$\Psi(q^2)$ is thus approximately given by $m_s^2$ times the two-point function of the scalar current.

After taking two derivatives of $\Psi(q^2)$ with respect to $q^2$, $\Psi''(q^2)$ vanishes for large $q^2$, and satisfies a dispersion relation without subtractions,

$$ \Psi''(q^2) = 2 \int_0^{\infty} \frac{\rho(s)}{(s-q^2-i\varepsilon)^3} ds, \quad (3) $$

where $\rho(s)$ is defined to be the spectral function corresponding to $\Psi(s)$,

$$ \rho(s) \equiv \frac{1}{\pi} \text{Im} \Psi(s + i\varepsilon). \quad (4) $$

To suppress contributions in the dispersion integral coming from higher excited states, it is further convenient to apply a Borel transformation to eq. (3). The left-hand side of the resulting equation is calculable in QCD, whereas under the assumption of duality, the right-hand side can be evaluated in a hadron-based picture, thereby relating hadronic quantities like masses and decay widths to the fundamental SM parameters.

Generally, however, from experiments the phenomenological spectral function $\rho_{\text{ph}}(s)$ is only known from threshold up to some energy $s_0$. Above this value, we shall use the perturbative expression $\rho_{\text{th}}(s)$ also for the right-hand side. This is legitimate if $s_0$ is large enough so that perturbation theory is applicable. The central equation of our sum-rule analysis is then:

$$ u^3 \tilde{\Psi}_{\text{th}}''(u) = \int_0^{s_0} e^{-s/u} \rho_{\text{ph}}(s) ds + \int_{s_0}^{\infty} e^{-s/u} \rho_{\text{th}}(s) ds. \quad (5) $$

In addition, in the analysis one can also use the first derivative of this equation with respect to $u$ the so called “first-moment sum rule”.

The main ingredients in these equations, namely the theoretical expression for the two-point function as well as its phenomenological parameterisation, will be discussed below.
2. Theoretical two-point function

In the framework of the operator product expansion the Borel transformed two-point function \( \hat{\Psi}(u) \) can be expanded in inverse powers of the Borel variable \( u \):

\[
\hat{\Psi}(u) = (m_s - m_u)^2 u \left\{ \frac{\Psi_0(u) + \Psi_2(u)}{u} + \frac{\Psi_4(u)}{u^2} + \frac{\Psi_6(u)}{u^3} + \ldots \right\}. \tag{6}
\]

The \( \Psi_n \) contain operators of dimension \( n \), and their remaining \( u \) dependence is only logarithmic. The purely perturbative contribution \( \Psi_0(u) \) is presently known up to \( O(\alpha_s^2) \), with the last term having been calculated only very recently \( \hat{3} \). Numerically, the expansion reads

\[
\Psi_0(u) = \frac{3}{8 \pi^2} \left[ 1 + 1.53 \alpha_s + 2.23 \alpha_s^2 + 1.71 \alpha_s^3 \right]. \tag{7}
\]

In this expression the strong coupling constant \( \alpha_s(u) \) should be evaluated at the scale \( u \). Therefore, even for \( \alpha_s(1\text{ GeV}) \approx 0.5 \) the last term is roughly 20% and the perturbative expansion displays a reasonable convergence. Because the two-point function scales as \( m_s^2 \), the resulting uncertainty for \( m_s \) from higher orders is at most 10%. In practice it is somewhat smaller since the average scale at which the sum rule is evaluated lies around 1.5 GeV.

The next term in the operator product expansion \( \Psi_2(u)/u \) only receives contributions proportional to the quark masses squared. Already at a scale of \( u = 1 \text{ GeV}^2 \) its size is approximately 2%, decreasing like 1/\( u \) for higher scales. Although it has been included in the phenomenological analysis, for the error estimates on the strange quark mass it can be safely neglected.

The same holds true for the dimension-four operators. Here there are contributions from the quark and gluon condensates as well as explicit mass corrections \( \sim m_s^4 \). Again, at a scale of \( u = 1 \text{ GeV}^2 \) the size of \( \Psi_4(u) \) is below 1% of the full two-point function, hence being negligible for the \( m_s \) analysis. Nevertheless, the \( \Psi_4 \) and in addition the \( \Psi_6 \) contributions have been included for the numerical investigations.

3. Hadronic spectral function

Generally, all intermediate states with the correct quantum numbers contribute to the hadronic spectral function. In the case of the scalar two-point function the lowest lying state is the \( K\pi \) system in an s-wave isospin 1/2 state. The contribution of this intermediate state yields the inequality \( \hat{3} \)

\[
\rho(s) \geq \frac{3 s - s_+}{32 \pi^2 s} \sqrt{(s - s_+)(s - s_-)} |d(s)|^2 \tag{8}
\]

where

\[
s_+ = (M_K + M_\pi)^2, \quad s_- = (M_K - M_\pi)^2, \tag{9}
\]

and \( d(s) \) is the strangeness-changing scalar form factor

\[
d(s) = -i \sqrt{2} \langle \pi^0 K^-|\bar{\psi}\gamma_i u|0\rangle(0) \tag{10}
\]

also appearing in \( K\pi \) decays. The scalar form factor \( d(s) \) admits an Omnès representation which can be found in \( \hat{3} \) and depends on the \( K\pi \) s-wave, \( I = 1/2 \) phase shift \( \delta_0^{1/2} \). Similarly to an analysis of the pion form factor \( \hat{3} \) the Omnès representation can be improved by using knowledge on effective hadronic theories and the 1/\( N_c \) expansion thereby fixing a polynomial ambiguity which exists in the Omnès representation \( \hat{3} \):

\[
d(s) = \frac{d(0) M_K^2}{(M_K^2 - s - i M_R \Gamma_R(s))},
\]

\[
\exp \left\{ \frac{s}{\pi} \int_{s_+}^{\infty} \frac{\delta_0^{1/2}(t)}{t(t - s - i\varepsilon)} dt \right\}, \tag{11}
\]

with

\[
d(0) = 0.977 \cdot (M_K^2 - M_\pi^2), \tag{12}
\]

\( M_R \) and \( \Gamma_R(s) \) are the mass and energy dependent width of the lowest lying resonance, the \( K_0^*(1430) \) in this case, and \( \delta_0^{1/2} \) is the background contribution to the s-wave phase shift which can be extracted from experimental data on \( K\pi \) scattering.

Further details on this representation of the scalar form factor and a discussion of the deficiencies of the representation used in ref. \( \hat{3} \) can be found in \( \hat{3} \).
4. Phenomenological analysis

Evaluating the sum rule of eq. (5) and the corresponding first moment sum rule with the theoretical two-point function of section 2 and the hadronic spectral function of section 3, the resulting values for the running strange quark mass $m_s(1\text{ GeV})$ in the $\overline{MS}$ scheme as a function of $\sqrt{u}$ are displayed in figure 1 (solid and dashed lines respectively). The continuum threshold $s_0$ has been determined to be approximately $s_0 = 3.4\text{ GeV}^2$ by requiring duality, namely equality of the strange masses obtained from the zeroth and first moment sum rules. A value of this size is also expected from the fact that in this region the second resonance, the $K^*_0(1950)$, which has not been included in the analysis, is found.

On the other hand, in an interval for $u$ where we expect the sum rules to be valid, the sum rules should be stable and the extracted strange quark mass should be independent of $u$. However, the stability of the curves shown in figure 1 is not very good. This is due to the fact that the region where duality holds overlaps with the region of the second resonance which should thus be included. The contribution of multi-particle intermediate states like $K\pi\pi\pi$ is of higher order in the chiral expansion and should be suppressed.

Estimating the error from the variation of the strange mass in the range $1\text{ GeV}^2 < u < 9\text{ GeV}^2$, as a preliminary result from our analysis we find $m_s(1\text{ GeV}) = 160 \pm 30\text{ MeV}$. This indicates that the dominant error in the determination of the strange quark mass stems from the parameterisation of the hadronic spectral function. Compared to this error the uncertainties from higher order $\alpha_s$ corrections are small. A detailed discussion of the phenomenological spectral function as well as the inclusion of the second resonance can be found in ref. [1].

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Discussions

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*What is the theoretical uncertainty in the final result corresponding to the two-parameter freedom in the choice of the renormalization scheme for the next-next-to-leading order perturbative QCD correction?*

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*The scale uncertainty has been estimated to be about 5% by varying the renormalization scale. An error for the dependence of the quark mass on the renormalization scheme has not been included in the analysis.*