Method for experimental determination of the coefficients of stiffness and damping of rubber insulators

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Abstract. Rubber insulators are used in many dynamic systems to reduce the impact of vibrations. They are located in the mount system of the engine, gearbox, suspension, et cetera. Two of their characteristics are the dynamic coefficients of stiffness and damping. These characteristics are influenced by the frequency of vibration, the preload in the mounts and the hardness of the rubber. The characteristics of the rubber insulators in the suspension elements also influence the handling of the vehicle. In the work are investigated rubber insulators from different suspensions in the frequency range 70 - 220 Hz. This frequency range contains the resonant frequencies of the radial type pneumatic tyre due its interference with the road surface. The purpose of the work is to develop a method for obtaining the dynamic coefficients of stiffness and damping of the rubber insulators.

1. Introduction and overview of the problem

Rubber insulators used in the mounting of the suspension influence the magnitude and frequency of high frequency oscillations [1]. In the modeling and optimization of suspension behavior, the values of the stiffness and damping coefficients of the insulators are essential. The dynamic coefficients of stiffness and damping of the rubber mounts have non-linear characteristics and they depend on the frequency of the oscillations, the preload on them and the hardness of the rubber. These coefficients are different from those in static measurements. In the modeling of the behaviour of the dynamic system operating under a fixed operating mode (constant frequency), it is possible to use constant values of the coefficients of stiffness and damping [2]. In the case of road disturbance to the tyre and the suspension elements, the frequency of excitation depends on the speed of motion. The dynamic system operates in a wide frequency range and it is necessary to study the values of these coefficients in each resonance of the tyre and the suspension elements. The resonant frequencies of the radial type pneumatic tyre, due to its design characteristics, occur in the 80 - 250 Hz frequency range [3, 4]. The absorption of these oscillations in this frequency range is mainly carried out by the rubber insulators located at the mounts of the suspension. Above 50 Hz, regardless of its type, the shock absorber does not affect the oscillation absorption [5]. In modeling the dynamic behaviour of the suspension, the coefficients of stiffness and damping of the rubber mounts have to be set and predetermined for the respective resonance frequencies of the tyre and the suspension elements [6]. In [7], the commonly used models of rubber insulators are considered and the authors offer their own model that describes its non-linear dynamic behavior. In [8], the authors investigate the structure-borne vibrations generated by the engine and their impact on the...
vibro-acoustic comfort in the vehicle cabin. For the numerical method, it is used the coefficient of stiffness, which is experimentally obtained. The results show that the value of stiffness coefficient increases with the increase in frequency. In work [9] are obtained the forces before and after the rubber insulators of the diesel generator. The forces are presented as functions of the frequency of oscillation, the angle of rotation of the crankshaft and the load on the diesel unit. Suspension elements are responsible for the vehicle handling. Because of that, the optimization of the stiffness values of the rubber insulators, in order to improve comfort, is made within narrow limits [10].

This work presents a method for obtaining the dynamic coefficients of stiffness and damping of rubber insulators. Object of the study are rubber mounts for different types of suspension: the rubber insulators in the lower arm (position 2 in figure 1(a)) and in the strut rod (position 1 in figure 1(a)) for an independent front torsion spring suspension; the two rubber insulators in the lower control arm (position 3 and 4 in figure 1(b)) for front MacPherson strut suspension; the rubber insulators in the shock absorber mounts (position 6 in figure 1(c)) and that in the trailing arm (position 5 in figure 1(c)) for twist-beam rear suspension.

Figure 1. Rubber mounts for different types of suspension: (a) - torsion spring suspension; (b) - MacPherson strut suspension; (c) - twist-beam rear suspension [11].

2. Description of the method

The method for experimentally determining the coefficients of stiffness and damping of rubber insulators is based on a comparative method of two dynamic systems presented in figure 2. In the first dynamic system (A), the mass \( m_b \) and coefficients of stiffness \( c_b \) and damping \( \beta_b \) are known. To create the second system (B), three new parameters are added, from which the mass \( m_v \) is known and the coefficients of damping \( \beta_v \) and stiffness \( c_v \) are unknown. The first system (A), which is called “spring – mass”, is a simplified model of the free end of a cantilever beam. The second system (B), which is called “spring – mass – insulator”, is created by installing fixed rubber insulator to the free end of the beam. The unknown coefficients \( \beta_v \) and \( c_v \) are respectively the dynamic coefficients of damping and stiffness of the rubber insulator.

Figure 2. Dynamic system (A) “spring – mass” and dynamic system (B) “spring – mass – insulator”.

By using the relationship between the described parameters in the case of free damped oscillations, the unknown coefficients \( \beta_v \) and \( c_v \) can be determined. For free damped oscillations, the following can be written [12]:

\[
\tau = \frac{2\pi}{\Omega} = \frac{2\pi}{\sqrt{\omega^2 - \bar{n}^2}},
\] (1)
where $\tau$ is the period of the free damped oscillations;
$\Omega$ – the frequency of the free damped oscillations;
$$\omega = \sqrt{\frac{c}{m}}$$ – the natural frequency of the dynamic system;
$$n = \frac{\beta}{2m}$$ – coefficient that parameterizes the strength of the damping.

For the specific dynamic systems (A) and (B) $n<\omega$, so in this case the damped oscillations are periodic. Figure 3 is the time-domain response of the dynamic systems after initial perturbation. The continuous line record (figure 3) corresponds to the free damped oscillations of system (A) and the dashed line record corresponds to the free damped oscillations of system (B). Changing the masses of the systems also changes their frequency of oscillation.

![Figure 3. Time-domain response of dynamic systems (A) and (B).](image)

For each system, the logarithmic decrement is determined. The logarithmic decrement is defined as the natural logarithm of the ratio of any two successive amplitudes:
$$\delta = \ln \frac{A_i}{A_{i+1}} = \frac{n\tau}{2} = \frac{\pi n}{\sqrt{\omega^2 - n^2}}$$  \hspace{1cm} (2)

The logarithmic decrement characterizes damping in the dynamic systems. Possessing the time-domain responses of the two systems (A) and (B), the dynamic coefficients of damping and stiffness of the rubber insulator can be determined from equations (1) and (2). The coefficients of damping $\beta_v$ and stiffness $c_v$ are respectively:
$$\beta_v = \frac{4m_S\delta_S}{\tau_S} - \frac{4m_b\delta_b}{\tau_b}$$ \hspace{1cm} (3)
$$c_v = \frac{4m_S}{\tau_S^2} (\delta_S^2 + \pi^2) - \frac{4m_b}{\tau_b^2} (\delta_b^2 + \pi^2)$$ \hspace{1cm} (4)

where $m_b$ and $m_S$ are respectively the mass of the system (A) “spring – mass” and the mass of the system (B) “spring – mass – insulator” (figure 2);
$\tau_b$ and $\tau_S$ – respectively the periods of oscillation of system (A) and system (B);
$\delta_b$ and $\delta_S$ – respectively the logarithmic decrement of system (A) and system (B).

3. Method of conducting the experiment

3.1. Description of the experimental setup

A flexible steel beam 6 is welded to the upper metal plate of housing 9 (figure 4). The housing is a metal cylinder, the volume of which is filled with reinforced concrete. The natural frequency of the housing must be at least five times higher than that of the steel beam. Together with the correcting weights 5, the beam provides the possibility of varying the frequency of the oscillation of the test object in the frequency range 70 – 220 Hz. The test object (rubber insulator) 4 is attached to the bracket 3, which can be moved vertically by the loading device 2 in the support 1. The loading device 2 presses the rubber insulator to the free end of the beam with the required preload force.
Figure 4. Experimental setup for obtaining oscillograms of free damped vibrations (1 - support, 2 - loading device, 3 - bracket, 4 - test object, 5 - correcting weights, 6 - steel beam, 7 - displacement transducer, 8 - strain gage, 9 - housing.

The excitation of the free damped oscillations in the system is performed by a single impact on the beam. The vibrations of the beam are measured by a strain gage 8 or an accelerometer (figure 4). The deflection of the free end of the beam is measured by a displacement transducer 7. In the layout of the measuring system are also included analog-to-digital converters for voltage inputs and for measuring with strain gages (positions 4 and 5 in figure 5). The obtained oscillograms are recorded with a computer. The requirements for conducting the experiment are: room temperature 20±5 °C; accuracy of the measuring equipment less than 1%; the mass (housing), to which the elastic beam is mounted, needs to has at least five times higher natural frequency than that of the beam; the elastic beam at different loads needs to has frequency of oscillation in the range of 70-220 Hz.

Figure 5. Layout of the measuring system (1 - power supply, 2 - read out (PC), 3 - active USB HUB, 4 - analog-to-digital converter DQ401, 5 - analog-to-digital converter DQ430, 6 - inductive displacement transducer, 7 - half bridge strain gage).

3.2. Sequence of conducting the measurements
Firstly are conducted the measurements for free damped oscillations of the cantilever steel beam – the system (A) “spring – mass” (figure 2). To change the frequency of the oscillations, different weights are mounted at the free end of the beam. The beam is excited by initial impact. The recording device saves the time-domain responses.

Secondly are conducted the measurements for free damped oscillations of the steel beam with rubber insulator mounted at its free end – the system (B) “spring - mass - insulator” (figure 2). The investigated rubber insulator is pressed against the beam with varying preload force. To change the frequency of the oscillations, different weights are mounted at the end of the beam. The beam is excited by initial impact. The recording device saves the time-domain responses.

By conducting series of experiments in the desired frequency range 70 - 220 Hz, a set of oscillograms are obtained both for the “spring – mass” system and the “spring - mass - insulator” system.
3.3. **Sequence of processing the experimentally obtained oscillograms**

Comparisons are made between the two systems with the same period of oscillation $\tau_C = \tau_b = \tau$. From the oscillograms for the both systems are obtained the period $\tau$ and the values of two successive amplitudes $A_i$ and $A_{i+1}$ (the peak stresses in the beam). The logarithmic decrements $\delta_C$ and $\delta_b$ of the two dynamic systems are calculated. At last the coefficients of damping $\beta_v$ and stiffness $c_v$ of the rubber insulator are determined.

4. **Results**

The results of the study show that the dynamic characteristics of the rubber insulators are non-linear. Figures 6 and 7 show the coefficients of damping $\beta_v$ and stiffness $c_v$ for the front and rear rubber insulators in a lower control arm (position 3 and 4 in figure 1(b)) of MacPherson strut suspension. The dynamic coefficients are presented as functions of the frequency and the preload.

![Figure 6](image_url)

**Figure 6.** Coefficients of damping and stiffness for front rubber insulator in a lower control arm (position 4 in figure 1(b)) of a MacPherson strut suspension.

![Figure 7](image_url)

**Figure 7.** Coefficients of damping and stiffness for rear rubber insulator in a lower control arm (position 3 in figure 1(b)) of a MacPherson strut suspension.

![Figure 8](image_url)

**Figure 8.** Coefficients of damping and stiffness for rubber insulator in a strut rod (position 1 in figure 1(a)) of a torsion spring suspension.
From the results it is noticed that the coefficient of stiffness increases with increasing the frequency and the preload force. It is observed that the coefficient of damping decreases with increasing the frequency. Also there is slight increase of the damping coefficient with increasing the preload force. Figure 8 shows the coefficients of damping $\beta$ and stiffness $c$ for the rubber insulator in a strut rod (position 2 in figure 1(a)) of a torsion spring suspension. Figure 9 shows the damping and stiffness coefficients for the rubber insulator in a trailing arm (position 5 in figure 1(c)) for twist-beam rear suspension. Since these dynamic systems have limited power, the increase in frequency itself leads to a reduction in the deformation of the rubber, which is one reason for reducing the damping coefficient.

**Figure 8.** Coefficients of damping and stiffness for rubber insulator in a strut rod (position 2 in figure 1(a)) of a torsion spring suspension.

The tested rubber elements have different hardness, and those with harder rubber have a higher damping and stiffness coefficients. For measuring the hardness, it is used Shore durometer Type A. The damping coefficient is also influenced by the volume of the rubber layer. Increasing the volume of the rubber material leads to higher damping coefficient (figure 10). For suspension components that are responsible for the handling of the vehicle, the rubber insulators have higher hardness (Shore A 70) than the others (Shore A 50).

**Figure 9.** Coefficients of damping and stiffness for rubber insulator in a trailing arm (position 5 in figure 1(c)) for twist-beam rear suspension.

**Figure 10.** Coefficient of damping as a function of the volume of the rubber layer in the insulators for frequency 100 Hz and preload force 50 kg.

5. Conclusions
The proposed method provides the following opportunities:
- Possibility to obtain the damping coefficient of the rubber insulators as a function of the frequency and the preload force;
- Possibility to obtain the stiffness coefficient of the rubber insulators as a function of the frequency and the preload force;
- Possibility to estimate the damping coefficient of rubber insulators depending on the volume of the rubber layer and the hardness of the rubber;
- Possibility to study the dynamic coefficients of stiffness and damping for other rubber insulators such as the mounts of the engine, gearbox, et cetera.
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