The New Minimal Supersymmetric GUT
I: Spectra, RG analysis and fitting
formulae

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ABSTRACT

The Supersymmetric SO(10) GUT based on the $210 \oplus 10 \oplus 120 \oplus 126 \oplus \overline{126}$
Higgs system has 40 real superpotential parameters of which 2 can be fixed by the
Light Higgs doublet fine tuning and one by the determination of the unification scale
by the RG flow. It provides a minimal framework for the emergence of R-parity
exact Supersymmetric Standard Model at low energies and viable supersymmetric
seesaw explanation for the observed neutrino masses and mixing angles. We present
complete formulae for MSSM decomposition of the superpotential invariants, explicit
GUT scale one step spontaneous symmetry breaking down to the MSSM, tree level
light charged fermion effective yukawa couplings, Weinberg neutrino mass generation
operator, and the complete set of effective superpotential coefficients of LLLL and
RRRR $d = 5$ Baryon violating operators(which have contributions from novel $120$-
plet Higgs channels) in terms of GUT superpotential parameters. We survey the gauge
RG flow including threshold corrections due to the calculated super heavy particle
spectra. The formulae given are used in following papers to determine complete
realistic Susy SO(10) GUT fits of all MSSM data.
1 Introduction

The discovery of neutrino mass was both preceded by[1] and itself provoked[2,3,4,5,6,7] intensive investigation of unifying theories that naturally incorporate supersymmetry and the seesaw mechanisms[8]: in particular models with the Left-Right gauge group as a part of the gauge symmetry and $B - L$ broken at a high scale and R/M-parity preserved to low energies[1]. The close contiguity of the seesaw scale and the Grand Unified scale pushed SO(10) GUTs, which are the natural GUT home of both Type I and Type II seesaw mechanisms, but were long relegated as baroque cousins of the -seemingly- more elegant minimal SU(5) GUT, into centre stage. The understanding that the Susy SO(10) GUT based on the $210 \oplus 10 \oplus 126 \oplus \overline{126}$ Higgs system proposed[9,10] long ago was the best candidate for the Minimal Supersymmetric GUT (MSGUT) crystallized after the demonstration of its minimality on parameter counting grounds and an elegant reduction of its spontaneous symmetry breaking problem to a single cubic equation with just one unknown parameter[11]. Careful computations of the symmetry breaking[9,10,11] and mass spectra[3,4,5,6] became available for the MSGUT. These theories naturally maintain a structural distinction between Higgs and matter fields and therefore naturally preserve R-parity down to low energies[12,11,2].

The initial euphoria[7] that the version utilizing only $10, \overline{126}$ Higgs representations might prove sufficient[13] to fit all low energy fermion data in an elegant and predictive way ran aground when the successful generic fits of fermion mass data were shown to be unrealizable[14,15,16] in the context of the actual Seesaw mechanisms(both Type I and Type II) available in the MSGUT. Both types of seesaw yielded neutrino masses that were too small and Type I was shown to generically dominate Type II. Faced with this impasse it is natural to have recourse to the third allowed type of Fermion Mass (FM) Higgs, i.e the $120$-plet of SO(10). The $120$-plet had previously played a relatively minor role in fitting the fermion mass data[17,18]. In particular in [18] the $120$-plet, with "spontaneous CP violation" and Type II seesaw(assumed to be viable), was shown to allow fits with CKM CP-violating phases in the first quadrant: which otherwise required a fine tuning in the MSGUT[19].

In view of our no-go result in the MSGUT, however, we proposed[15] a re-allocation of roles among the three types of FM Higgs representation by suppressing the $\overline{126}$ yukawa couplings relative to those of $10,120$. Since the Type I seesaw neutrino masses are inversely proportional to the $\overline{126}$ yukawa coupling this would enhance the Type I seesaw masses to viable levels(Type II contributions get further suppressed) while perhaps still allowing sufficient freedom to fit all the fermion mass and mixing data. This also has the interesting consequence that Right handed Neutrino masses would be significantly lowered into a range $10^8 - 10^{12}$ GeV compatible with Lepto-genesis.

Related subsequent work[20,21,22] gave mixed signals regarding the viability of
our proposal to use $10 + 120$ Higgs to fit charged fermion and small $126$ couplings to raise Type I neutrino masses by lowering $126$ right handed neutrino masses. However we find accurate NMSGUT specific fits using ultra small $126$ couplings but a somewhat enlarged fitting scenario that takes recourse to the strong influence (at the large $\tan \beta$ values typically favoured by SO(10) Susy GUTs) of threshold corrections on the down type quark masses. This is done in order to lower the yukawa couplings of the down and strange quarks to values that can be accommodated by the NMSGUT specific fitting formulae. These new fits will be described in detail in the second paper of this series. They are manifestly distinct from the accurate generic fits found in [21, 22], which besides being un-utilizable in the NMSGUT[24] also give a distinct picture of right handed neutrino masses. In our fits we find that right handed neutrinos are much lighter than the GUT scale and strongly hierarchical while neither statement applies to the fits of [21, 22].

Thus the GUT based on the $210 \oplus 10 \oplus 120 \oplus 126 \oplus 126$ Higgs system emerges as a New Minimal Supersymmetric GUT (NMSGUT) capable of fitting all the known fermion mass and mixing data. The New MSGUT calls for and deserves the same detailed analysis of its superheavy Renormalization Group(RG) flow threshold effects, fermion mass fit compatibility and exotic effect effective superpotential that we earlier provided[9, 3, 8, 14, 15] for the theory without the $120$ which we previously called the MSGUT[11] but whose claim on that name is now tenuous and faded. In this paper we begin this detailed investigation by presenting the required spontaneous symmetry breaking, spectra, Higgs doublet fine-tuning and “Higgs-fraction” determination leading to matter fermion yukawa expressions in terms of GUT parameters (as well as Weinberg operator coefficients leading to seesaw neutrino masses) and threshold effect formulae in the gauge RG flow from $M_Z$ up to $M_X$. In particular we find that the unification scale is generically raised over most of the viable parameter space. Although the gauge coupling is still perturbative even at the modified unification scale the NMSGUT gauge coupling exhibits [20] a Landau pole at $\Lambda_X$ lying just above the perturbative unification scale $M_X$ due to the huge SO(10) gauge beta functions implied by the large representations used. Thus the raising of the unification scale to near the Planck scale (where in any case gravitational effects become strong) somewhat softens the asymptotic strength problem of such GUTs and points to a common origin with strong gravity with $\Lambda_X \sim M_{Planck}$ serving as a physical cutoff beyond which both SO(10) and gravity are strongly coupled (and exactly supersymmetric). We have speculated[30] that the apparent vice of Asymptotic Strength(AS) of (N)MSGUTs may be turned to good account to construct a theory of dynamical calculable GUT symmetry breaking using an extension of the techniques for dealing with strongly coupled supersymmetric theories[31], and even that AS Susy GUTs(ASSGUTs), which determine by their RG flow their own physical UV cutoffs, escape the objections to the induced gravity program that halted its development in the 80’s[32]. This induced gravity program would gain plausibility
if the Planck scale and $\Lambda_X$ coincided. Such a coincidence is an obvious consequence of raising the perturbative unification scale to just below the Planck scale. We shall see [43] that the requirement of viable unification and $d = 5$ B violation suppression in the presence of high scale threshold corrections in the NMSGUT leads us almost inevitably to regions of the parameter space where $M_X$, together with baryon decay mediating triplet masses, are raised well above $10^{17} GeV$ and $\Lambda_X$ approaches close to $M_{Planck}$.

Even after it fits the fermion data the NMSGUT must still face the challenge posed by the non observation of proton decay. Minimal Susy GUTs generically imply proton decay rates via $d = 5$ operators which are higher than the current experimental upper bounds[25]. The raising of the unification scale does not affect the RG flow of the MSSM in the grand desert but the threshold corrections due to particles with masses of order the one loop unification scale or greater raise the mass of particles that mediate proton decay. Soft susy breaking parameters and in particular the sfermion masses and mixing matrices have a crucial influence on the rates of the $d = 5$ mediated proton decay. Our results in [23, 43] show that the NMSGUT yields consistent ( but not unique ) sets of the Soft Susy parameters to be used in the calculation of the $d = 5$ mediated Baryon decay. In these subsequent papers[23, 43] we show that NMSGUT- Supergravity(SUGRY) soft parameter freedom (with non-universal Higgs masses(NUHM) and inclusion of threshold effects on fermion yukawas due to both Susy thresholds in the TeV range and $120$-plet thresholds near $M_X$ ) allows accurate NMSGUT-mSUGRY-NUHM fits of all the fermion data that also imply perfectly acceptable B decay rates. In this paper, to begin this elaborate demonstration, we derive also the $\Delta B \neq 0$ effective superpotential by integrating out heavy fields to prepare for the explicit evaluation of the proton decay rate using NMSGUT-mSUGRY-NUHM parameters.

In Section 2 we recapitulate our notation and the basics of such models. Detailed accounts of our techniques have already been given earlier[3, 5, 14]. In Section 3 and Appendix A we give the mass spectra and in Appendix B we describe an SU(5) reassembly crosscheck of the spectra we obtain. In Section 4 we discuss how the threshold effects calculated using these spectra determine regions where perturbative unification is viable and Baryon decay mass scales are raised. We shall see how one is generically led towards a raised unification scale. This leads to a potential resolution of the some of the basic problems of Susy GUTs discussed above, without any contrived cancellations. In Section 5 we give the fermion mass formulae in the presence of the $120$—plet using analytic expressions for the null eigenvectors(after fine tuning to keep one pair of doublets light) of the $6 \times 6$ Higgs doublet ([1, 2, ±1]) mass matrix (Appendix C). In Section 6 we integrate out the heavy triplets that mediate Baryon decay via $d = 5$ operators and give the resultant effective superpotential in terms of the matter superfields of the effective MSSM. We conclude, in Section 7, with a brief discussion of issues and a preview of the fermion fits and baryon decay
2 The New Minimal Susy GUT

2.1 MSGUT couplings, vevs and masses

The original MSGUT was the renormalizable globally supersymmetric $SO(10)$ GUT whose Higgs chiral supermultiplets consist of AM(Adjoint Multiplet) type totally antisymmetric tensors: $210(\Phi_{ijkl}), \mathbf{126}(\Sigma_{ijklm}), \mathbf{126}(\Sigma_{ijklm})(i,j=1...10)$ which break the GUT symmetry to the MSSM, together with Fermion mass (FM) Higgs $10$-plet($H_i$). The $\mathbf{126}$ plays a dual or AM-FM role since it also enables the generation of realistic charged fermion and neutrino masses and mixings (via the Type I and/or Type II Seesaw mechanisms); three $16$-plets $\Psi_A(A=1,2,3)$ contain the matter including the three conjugate neutrinos ($\bar{\nu}_L^A$).

The superpotential (see[11, 3, 4, 5] for comprehensive details) contains the mass parameters

$$m: 210^2; \quad M: 126 \cdot \mathbf{126}; \quad M_H: 10^2$$

(1)

and trilinear couplings

$$\lambda: 210^3; \quad \eta: 210 \cdot 126 \cdot \mathbf{126}; \quad \gamma \oplus \bar{\gamma}: 10 \cdot 210 \cdot (126 \oplus \mathbf{126})$$

(2)

In addition one has two complex symmetric matrices $h_{AB}, f_{AB}$ of Yukawa couplings of the $10, \mathbf{126}$ Higgs multiplets to the $16, \mathbf{16}$ matter bilinears. The $U(3)$ ambiguity due to $SO(10)$ ‘flavour’ redefinitions can be used to remove 9 of the 24 real parameters in $f, h$. In addition rephasing of the remaining 4 fields $\Phi, H, \Sigma, \bar{\Sigma}$ removes 4 phases from the 14 parameters in $m, M, M_H, \lambda, \eta, \gamma, \bar{\gamma}$ leaving 25 superpotential parameters to begin with. Strictly speaking, since a fine tuning to keep one pair of doublets light is an intrinsic part of the MSGUT scenario, an additional complex parameter (say $M_H$) may be considered as fixed so that there are actually 23 free superpotential parameters. In addition the electroweak scale vev $v_W$, and $\tan \beta$ are relevant external parameters for the light fermion spectrum determined by the GUT Yukawa structures. The overall superheavy scale(identified with the real parameter $m_{210}$) is fixed by the identification of the unification scale determined by the RG flow with the mass of the gauge $X[3,2,\pm \frac{4}{3}]$ sub-multiplet.

The GUT scale vevs that break the gauge symmetry down to the SM symmetry (in the notation of[3]) are[9, 10]

$$\langle (15, 1, 1) \rangle_{210} : \langle \phi_{abcd} \rangle = \frac{a}{2} \epsilon_{abcdef} \epsilon_{ef}$$

(3)

$$\langle (15, 1, 3) \rangle_{210} : \langle \phi_{a\bar{\alpha} \bar{\beta} \bar{\delta}} \rangle = \omega \epsilon_{ab} \epsilon_{\bar{\alpha} \bar{\beta} \bar{\delta}}$$

(4)

$$\langle (1, 1, 1) \rangle_{210} : \langle \phi_{\bar{\alpha} \bar{\beta} \bar{\gamma} \bar{\delta}} \rangle = p \epsilon_{\bar{\alpha} \bar{\beta} \bar{\gamma} \bar{\delta}}$$

(5)
\begin{align*}
\langle (10, 1, 3) \rangle_{126} & : \langle \Sigma_{13586} \rangle = \bar{\sigma} \\
\langle (10, 1, 3) \rangle_{126} & : \langle \Sigma_{24675} \rangle = \sigma.
\end{align*}

The vanishing of the D-terms of the SO(10) gauge sector potential imposes only the condition $|\sigma| = |\bar{\sigma}|$. Except for the simpler cases corresponding to enhanced unbroken gauge symmetry ($SU(5) \times U(1), SU(5), G_{3,2,2,B-L}, G_{3,2,R,B-L}$ etc) , this system of equations is essentially cubic and can be reduced to a single equation for a variable $x = -\lambda \omega/m$, in terms of which the vevs $a, \omega, p, \sigma, \bar{\sigma}$ are specified :

\begin{equation}
C(x, \xi) = 8x^3 - 15x^2 + 14x - 3 + \xi(1 - x)^2 = 0
\end{equation}

where $\xi = \frac{\lambda M}{\eta m}$. Then the dimensionless vevs in units of $(m/\lambda)$ are $\bar{\omega} = -x$ and

\begin{align*}
\bar{a} &= \frac{x^2 + 2x - 1}{(1 - x)} ; \\
\bar{p} &= \frac{x(5x^2 - 1)}{(1 - x)^2} ; \\
\bar{\sigma} &= \frac{2 \lambda x(1 - 3x)(1 + x^2)}{\eta (1 - x)^2}.
\end{align*}

This exhibits the crucial importance of the parameters $\xi, x$. Note that one can trade the parameter $\xi$ for $x$ with advantage using equation (8) since $\xi$ is uniquely fixed by $x$. By a survey of the behaviour of the theory as a function of the complex parameter $x$ we thus cover the behaviour of the three different solutions possible for each complex value of $\xi$.

2.1.1 Characteristics of the SSB solutions

A knowledge of the 3 solutions $x_i(\xi)(i = 1, 2, 3) |C(x_i(\xi), \xi) = 0)$ of the cubic equation is important for surveying the properties of the theory. In Fig. 1.2 we exhibit plots of $x_i(\xi)$ for real $\xi$. For $\xi < -27.917$ all three solutions are real. Moreover it is clear that for $|x| >> 1, x \approx -\xi/8$ specifies one real branch for real $\xi$ as visible in Fig.1. On the other two (complex conjugate) branches the real and imaginary parts of $x(\xi)$ are bounded above and below (real part $\in (-1.8, 1.0)$ and imaginary part magnitude $\in [0, 1.1]$).

Using the above vevs and the methods of we calculated the complete gauge and chiral multiplet GUT scale spectra and couplings for the 52 different MSSM multiplet sets falling into 26 different MSSM multiplet types (prompting a natural alphabetization of their naming convention) of which 18 are unmixxed while the other 8 types occur in multiple copies which mix. The (full details of these) spectra may be found and equivalent results (with slightly differing conventions) are presented in. A related calculation with very different conventions has been reported in. The initially controversial relation between the overlapping parts of these papers was discussed and resolved in.

Among the mass matrices is the all important $4 \times 4$ Higgs doublet mass matrix $H$ which can be diagonalized by a bi-unitary transformation: from the
Figure 1: Solutions of eqn.[9] which governs GUT ssb: Plot of $Re[x_i(\xi)]$ vs $\xi$ for $i = 1, 2, 3$. The vertical straight line segments are “reconnection artifacts” induced by a switch over between real and complex solutions and vice versa.

4 pairs of Higgs doublets $h^{(i)}, \bar{h}^{(i)}$ arising from the SO(10) fields to a new set $H^{(i)}, \bar{H}^{(i)}$ of fields in terms of which the doublet mass terms are diagonal.

$$U^T H U = \text{Diag}(m_H^{(1)}, m_H^{(2)}, ...)$$

$$h^{(i)} = U_{ij} H^{(j)} ; \quad \bar{h}^{(i)} = \bar{U}_{ij} \bar{H}^{(j)}$$

(10)

To keep one pair of these doublets light one tunes $M_H$ so that $\text{Det} H = 0$. In the effective theory at low energies the GUT Higgs doublets $h^{(i)}, \bar{h}^{(i)}$ are present in the massless doublets $H^{(1)}, \bar{H}^{(1)}$ in a proportion determined by the first columns of the matrices $U, \bar{U}$:

$$E \ll M_X : \quad h^{(i)} \rightarrow \alpha_i H^{(1)} ; \quad \alpha_i = U_{i1}$$

$$\bar{h}^{(i)} \rightarrow \bar{\alpha}_i \bar{H}^{(1)} ; \quad \bar{\alpha}_i = \bar{U}_{i1}$$

(11)

The all important normalized 4-tuples $\alpha, \bar{\alpha}$ can be easily determined[[11, 4, 14, 33, 15]] by solving the zero mode conditions: $\mathcal{H}\alpha = 0 ; \quad \bar{\alpha}^T \mathcal{H} = 0$. 

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2.2 Additional terms introduced by the 120

The introduction of the 120-plet Higgs representation leads to new couplings in the superpotential. The additional terms are

\[
W_{\text{MSGUT}} = \frac{m_o}{2(3!)} O_{ijk} O_{ijk} + \frac{k}{3!} O_{ijk} H_m \Phi_{mijk} + \frac{\rho}{4!} O_{ijk} O_{mnk} \Phi_{ijmn} + \frac{1}{2(3!)} O_{ijklmn} (\Sigma_{lmmij} + \bar{\Sigma}_{lmmij}) + \frac{1}{5!} g_{AB} \Psi_A^T C^{(5)} B_{i1} \gamma_i \gamma_i \gamma_i \Psi_B O_{iiviziz}.
\]

The Yukawa coupling \( g_{AB} \) is a complex antisymmetric \( 3 \times 3 \) matrix. The \( SU(4) \times SU(2)_L \times SU(2)_R \) (Pati-Salam) decomposition of the 120plet is as follows:

\[
O_{ijk}(120) = O_{\mu}^{(s)} (10, 1, 1) + \mathcal{O}_{(s)}^{\mu} (10, 1, 1) + O_{\nu\alpha} (15, 2, 2) + O_{\mu\alpha} (6, 1, 3) + O_{\nu\alpha} (6, 3, 1) + O_{\nu\alpha} (1, 2, 2)
\]

The sub/superscripts "(s), (a)" denote symmetry and antisymmetry in SU(4) indices \( \mu, \nu \). Note that this multiplet contains no SM singlet so that the MSGUT high scale spontaneous symmetry breaking analysis remains the same. The arbitrary
phase of the $120$ reduces the effective number of the extra couplings ($m_0, \rho, \zeta, \bar{\zeta}, k (5_e - 1_r = 9_r)$ and $g_{AB}(3_e = 6_r)$) so they amount to 15 additional parameters. Thus the relative advantage\cite{11, 35} with respect to $SU(5)$ theories using additional fields or higher dimensional operators to correct the fermion mass relations of the simplest $SU(5)$ model seems weakened but is still not lost.

In fact the old MSGUT fails to fit the fermion mass data due to difficulties with the overall neutrino mass scale\cite{14, 15}. An alternative scenario within the NMSGUT which successfully removes the problems of the old MSGUT was proposed and elaborated in\cite{15, 28, 29}. In this scenario the Yukawa couplings ($f_{AB}$) of the $126$ are much smaller than those of the $10, 120$. This boosts the value of the Type I seesaw masses (which were in any case dominant over the Type II seesaw masses but still too small) so that they are generically viable.

The NMSGUT even with only real Yukawas (except the fine tuned complex parameter $M_H$) i.e. with a total of 23 (= 12 fermion Yukawas + 11 AM Yukawas) real parameters (further reduced to 22 by the fine tuning condition) may be first tried to fit the fermion data ( 12 masses + CKM phase + 3 CKM angles + 3 PMNS angles + 3 PMNS phases = 22 parameters). Such a theory will have less parameters than even the (unsuccessful) old MSGUT. However restriction to real values is somewhat arbitrary since it cannot be justified by a CP symmetry in view of the complexity of the fine tuned value of $M_H$. Thus we shall allow all parameters to be complex. In that case number of free NMSGUT superpotential parameters mounts to 37.

The FM Higgs $120$ does not contain any SM singlets and hence the analysis of the GUT scale symmetry breaking in MSGUT carries over unchanged to the NMSGUT. In particular there is still only one complex parameter($x$) whose variation directly affects the vevs and thus the masses in the theory. The additional kinetic terms are given by covariantizing in the standard way the global $SO(10)$ invariant D-terms $\left[ \frac{1}{(3)!} O_{ijk}^a O_{ijkl}^a \right]_D$\cite{3, 5}.

\section{AM Chiral masses via PS}

As in the case of the MSGUT\cite{3, 5} we open up the maze of NMSGUT interactions by decomposing $SO(10)$ invariants in the superpotential first into Pati-Salam invariants and then, after substituting the GUT scale vevs in PS notation we obtain the superpotential in the MSSM vacuum in terms of MSSM invariants. The results for the old MSGUT case were already given in\cite{3, 5} thus we list only the effect of the additional terms in the superpotential. The PS form of $W_{NMSGUT}$ is (we have inserted line numbers for easy reference):

$$\frac{k}{3!}H_i O_{jkl} \Phi_{ijkl} = k \left[ \frac{1}{\sqrt{2}} i (\tilde{H}^{\mu} O^{(a)}_{\mu \lambda} \Phi_{\lambda} + H^{(a)}_{\mu \nu} O_{(s)}^{\mu \lambda} \Phi_{\lambda} \nu) \right] (13)$$

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\[
\frac{m_0}{2(3!)} O_{ijk} O_{ijk} = \frac{m_0}{12} \left[ 6 \Phi^{(a)\mu} O_{\sigma(s)}^{\mu \nu} + 6 O_{\sigma(s)}^{\mu \nu} O_{\lambda \alpha \lambda \sigma} + 3 \bar{O}^{(a)(\mu)} (\bar{O}^{(a)\nu(R)} \cdot \bar{O}^{(a)(\nu)}(L)) \right]
\]

\[
\frac{\rho}{4!} O_{ijm} O_{klm} \Phi_{ijkl} = \frac{\rho}{4!} \left[ 8 i O_{\mu \lambda}(s) O_{\nu \lambda}^{(a)\mu} \Phi_{\nu \lambda}^{(a)} + 8 i O_{\sigma(s)}^{\mu \nu \rho \lambda} O_{\lambda \alpha \lambda \sigma}(s) \Phi_{\rho \lambda}^{(a)} \right]
\]
The purely chiral superheavy supermultiplet masses can be determined from these expressions simply by substituting in the AM Higgs vevs and breaking up the contributions according to MSSM labels.

It is again easiest to keep track of Chiral fermion masses since all others follow using supersymmetry and the organization provided by the gauge super Higgs effect.

There are three types of mass terms involving fermions from chiral supermultiplets in such models:

- Unmixed Chiral
- Mixed pure chiral
- Mixed chiral-gauge. We briefly discuss the notable features of the mass spectrum calculation and give the actual mass formulae in the Appendix.
### 3.1 Unmixed Chiral

A pair of Chiral fermions transforming as $SU(3) \times SU(2)_{L} \times U(1)_{Y}$ conjugates pairs up to form a massive Dirac fermion. For example for the properly normalized fields

$$
\bar{A}[1, 1, -4] = \frac{\Sigma^{44}(R^{-})}{\sqrt{2}} \\
A[1, 1, 4] = \frac{\Sigma^{44}(R^{+})}{\sqrt{2}}
$$

one obtains the mass term

$$2(M + \eta(p + 3a + 6\omega))\bar{A}A = m_{A}\bar{A}A$$

The physical Dirac fermion mass is then $|m_{A}|$ since the phase can be absorbed by a field redefinition. By supersymmetry this mass is shared by a pair of complex scalar fields with the same quantum numbers.

In the MSGUT case there are 19 pairs of chiral multiplets which form Dirac supermultiplets pairwise and two Majorana singletons, none of which mix with others of their ilk. In the NMSGUT 6 of these pairs become mixed with others of the same type leaving 11 Dirac supermultiplets and 2 Majorana supermultiplets (S,Q) which are unmixed. If the representation is real rather than complex one obtains an extra factor of 2 in the masses. The relevant representations, field components and masses are given in Table 1.

### 3.2 Mixed Pure Chiral

For such multiplets there is no mixing with the massive coset gauginos but there is a mixing among several multiplets with the same SM quantum numbers. There were only three such multiplet types in the MSGUT (i.e. $R[8, 1, 0], h[1, 2, \pm 1], t[3, 1, \pm \frac{2}{3}]$) but in the NMSGUT, there are an additional 5 mixed pure chiral types namely the $C[8, 2, \pm 1], D[3, 2, \pm \frac{2}{3}], K[3, 1, \pm \frac{2}{3}], L[6, 1, \pm \frac{2}{3}], P[3, 3, \pm \frac{2}{3}]$. As for the multiplet types which had mixed pure chiral mass terms in the MSGUT, the type $R[8, 1, 0]$ acquires no new partners and has an unchanged mass matrix since the 120 has no such submultiplets. However the other two mixed pure chiral multiplet types of the MSGUT do acquire new contributions:

- $[1, 2, -1](\bar{h}_{1}, \bar{h}_{2}, \bar{h}_{3}, \bar{h}_{4}, \bar{h}_{5}, \bar{h}_{6}) \bigoplus [1, 2, 1](h_{1}, h_{2}, h_{3}, h_{4}, h_{5}, h_{6}) \equiv (H_{2}^{\alpha}, \Sigma_{2}^{(15)\alpha}, \Sigma_{2}^{(15)\alpha}, \frac{\Phi_{2}^{44\alpha}}{\sqrt{2}}, O_{2}^{\alpha}, O_{2}^{(15)\alpha}) \bigoplus (H_{a1}^{\alpha}, \Sigma_{a1}^{(15)}, \Sigma_{a1}^{(15)}, \frac{\Phi_{a1}^{44\alpha}}{\sqrt{2}}, O_{a1}^{\alpha}, O_{a1}^{(15)})$

Here one gets an additional 2 rows and 2 columns relative to the MSGUT since the 120-plet contains two pairs of doublets with MSSM type Higgs doublet quantum numbers so that the mass matrix $\mathcal{H}$ is $6 \times 6$. To keep one pair of light doublets in the low energy effective theory, it is necessary to fine tune one of the parameters of the superpotential (e.g $M_{H}$) so that $\text{Det}\mathcal{H} = 0$. By extracting
the null eigenvectors of $\mathcal{H}'\mathcal{H}$ and $\mathcal{H}\mathcal{H}'$ one can compute the composition of the light doublet pair in terms of the doublet fields in the full SO(10) GUT, and, in particular, we can find the proportions of the doublets coming from the 10, 26, 120 multiplets which couple to the matter sector (see Section 5 and Appendix C). This information is crucial for investigating whether a fit of the fermion data accomplished by using the generic form of the SO(10) fermion mass formulae is compatible with the dictates of the MSGUT. In fact precisely such considerations led [14, 15, 16] to the conclusion that the MSGUT is Type I Seesaw dominated yet gives too small neutrino masses.

- $\{3, 1, \pm \frac{2}{3}\}(\tilde{t}_1, \tilde{t}_2, \tilde{t}_3, \tilde{t}_4, \tilde{t}_5, \tilde{t}_7) \oplus [3, 1, \mp \frac{2}{3}](t_1, t_2, t_3, t_4, t_5, t_6, t_7) \equiv (H_{\mu 4}, \Sigma_{(\alpha)}, \Sigma_{(R0)}, \Phi_{4(R+)}, O_{\mu 4(a)}, O_{\mu 4}^4, (H_{\mu 4}, \Sigma_{\mu 4(a)}, \Sigma_{\mu 4(R0)}, \Phi_{\mu(R+)}, O_{\mu 4(s)}, O_{\mu 4(R0)})$

With the contribution of the 120-plet one gets two additional rows and columns and the dimension of $\{3, 1, \pm \frac{2}{3}\}$ mass matrix $T$ becomes $7 \times 7$. These triplets and antitriplets participate in baryon violating process since the exchange of $(\tilde{t}_1, t_2, t_3, t_6, t_7) \oplus (\tilde{t}_1, \tilde{t}_2, \tilde{t}_6, \tilde{t}_7)$ Higgsinos generates $d = 5$ operators of type QQQL and $\tilde{L} \tilde{u} \tilde{d} \tilde{d}$. The strength of the operator is controlled by the inverse of the $\tilde{t} - t$ mass matrix $T$.

### 3.3 Mixed Chiral-Gauge

Finally we come to the mixing matrices for the chiral modes that mix with the gauge particles as well as among themselves. There is no direct mixing between MSSM fields contained in 120-plet with gauge particles. However mixing is present via other MSSM submultiplets present in MSGUT Higgs fields which further mix with gauge fields. This occurs for all such multiplet types except $G[1, 1, 0]$ and $X[3, 2, \pm \frac{4}{3}]$ which are unchanged, while for $E[3, 2, \pm \frac{1}{3}], F[1, 1, \pm 2], J[3, 1, \pm \frac{4}{3}]$ mass matrices acquire additional rows and columns. Thus

- $\{3, 2, \mp \frac{1}{3}\}(\tilde{E}_1, \tilde{E}_2, \tilde{E}_3, \tilde{E}_4, \tilde{E}_5, \tilde{E}_6) \oplus [3, 2, \pm \frac{1}{3}](E_1, E_2, E_3, E_4, E_5, E_6) \equiv (\Sigma_{44}, \Sigma_{44}, \Phi_{(s)}^{(\alpha)}, \Phi_{(a)}^{(\alpha)}, \lambda_2, O_{44}^4) \oplus (\Sigma_{44}, \Sigma_{44}, \Phi_{4a}, \Phi_{4a}, \lambda_{4a}, O_{44}^4)$

The 6 $\times$ 6 mass matrix $E$ has the usual super-Higgs structure : complex conjugates of the 5th row and column (omitting the diagonal entry) furnish left and right null eigenvectors of the chiral 5 $\times$ 5 submatrix $E$ obtained by omitting the fifth row and column. $E$ has non zero determinant although the determinant of $E$ vanishes.

- $\{1, 1, -2\}(\tilde{F}_1, \tilde{F}_2, \tilde{F}_3, \tilde{F}_4) \oplus [1, 1, 2](F_1, F_2, F_3, F_4) \equiv (\Sigma_{44}, \Phi_{(s)}^{(\alpha)}, \lambda_{(R+)}, O_{44}^4) \oplus (\Sigma_{(R0)}^4, \Phi_{(s)}^{(\alpha)}, \lambda_{(R+)}, O_{44}^4)$

The 4 $\times$ 4 mass matrix $F$ has the usual super-Higgs structure : complex conjugates of the 3rd row and column (omitting the diagonal entry) furnish left and
right null eigenvectors of the chiral $3 \times 3$ submatrix $F$ obtained by omitting the third row and column. $F$ has non zero determinant although the determinant of $F$ vanishes.

- $[3, 1, -\frac{4}{3}] (J_1, J_2, J_3, J_4, J_5) \oplus [3, 1, \frac{4}{3}] (J_1, J_2, J_3, J_4, J_5)$
  $$\equiv (\Sigma^R_{(R-)}, \phi^R_4, \phi^L_{(R+)}), \lambda^R_4, O^R_{(R-)} \oplus (\Sigma^L_{(R+)}, \phi^L_4, \phi^L_{(R-)}), \lambda^L_4, O^L_{(R+)}$$

The $5 \times 5$ mass matrix $J$ also has the super-Higgs structure: complex conjugates of the 4th row and column (omitting the diagonal entry) furnish left and right null eigenvectors of the chiral $4 \times 4$ submatrix $J$ obtained by omitting the fourth row and column. $J$ has non zero determinant although the determinant of $J$ vanishes.

This concludes our description of the superheavy mass spectrum of the NMSGUT, explicit details are given in Appendices A, B.

4 RG Analysis

In [5, 14], for the case of the MSGUT, we exhibited plots of the threshold corrections $(\Delta_G, \Delta_W, \Delta_X)$ to $\alpha_G(M_X)^{-1}, \sin^2 \theta_W(M_S)$ and $\log_{10} M_X$ versus $\xi$. In this paper we shall illustrate the position for the case of the NMSGUT but, following [36] and the standard practice in GUT RG studies, we shall take the now precisely measured value of $\sin^2 \theta_W(M_Z)$ as given and evaluate the threshold corrections $(\Delta_G, \Delta_3, \Delta_X)$ to $\alpha_G(M_X)^{-1}, \alpha_3(M_Z)$ and $\log_{10} M_X$. Note that we follow the approach of [37] in which $M_X$ is taken to be the mass of the lightest gauge multiplet which mediates proton decay and not a scale where the 3 MSSM gauge couplings (should) cross. In [15] we synthesized the three batches of information corresponding to the three roots of the cubic equation by exhibiting contour plots of $\Delta_G, \Delta_W, \Delta_X$ on the $x$--plane at representative values of the other (’slow’) parameters $\lambda, \eta, \gamma, \bar{\gamma}$. In this section we illustrate the $x$ values allowed by imposing plausible ‘realistic’ constraints on the magnitudes of the threshold corrections to the gauge couplings. We pay particular attention to the scenario where $M_X$ and with it dangerous colour triplet masses are pushed above $10^{16}$GeV. To implement the consistency requirements that the SO(10) theory remain perturbative after threshold and two loop corrections and, conversely, that $\alpha_G$ not decrease so much as to to invalidate the neglect of one-loop effects in the chiral couplings we impose an upper limit of 25 and a lower limit of -22 on the change in $\alpha_G^{-1}$. The lower limit corresponds to $\alpha_G = .28$ which is still marginally perturbative. As we have discussed elsewhere [26, 30] this type of unified theory is inevitably strongly coupled in the ultraviolet. Inasmuch as the unification scale $M_X$

\[\text{\textsuperscript{11}}\text{A possible way out is if, in a Robinson-Wilczek type scenario of gravity tempered gauge coupling evolution, the interplay between negative power law corrections from gravity and the strongly growing gauge coupling leads to a non-trivial gauge-gravity fixed point at the Planck scale.}\]
is found to be raised close to $10^{18}$ GeV i.e within an order of magnitude of the scale where both SO(10) and gravity become strongly coupled the requirement that $\alpha_G$ be small is moot. We allow a maximum value of 0.3 which is still perturbative. We expect that the mass of the lightest baryon decay mediating gauge bosons should not be lowered by more than one order of magnitude in order to respect the current bounds on $d=6$ mediated nucleon decay and not be raised by more than 3 orders of magnitude.

Since it is $\alpha_3(M_Z)$ which carries the largest uncertainty while $\alpha_{em}(M_Z)$, $\sin^2 \theta_W(M_Z)$ are quite precisely known (better than 0.01%, 0.1% respectively) it is usual\cite{38,39} to choose to predict $\alpha_3(M_Z)$. Using updated parameter values\cite{40}

\[
M_H = 117 GeV \quad ; \quad M_Z = 91.1876 \pm 0.0021 GeV \\
\alpha(M_Z)^{−1} = 127.918 \pm 0.018 \quad ; \quad \bar{s}_Z^2 = 0.23122 \pm 0.00015 \\
m_{\text{pole}} = 172.7 \pm 2.9 GeV 
\]

we find from the equations of \cite{39}

\[
\alpha_s(M_Z) - \Delta_{\alpha_s} = 0.130 \pm 0.001 + 3.1 \times 10^{-7} GeV^{-2} \times [(m_{\text{pole}})^2 - (172.7 GeV)^2] + H_{\alpha_s}(85)
\]

where $\Delta_{\alpha_s} = \Delta_{\alpha_s}^{GUT} + \Delta_{\alpha_s}^{Susy}$ threshold corrections.

The effect of the two loop Yukawa coupling corrections $H_{\alpha_s}$ was estimated\cite{39} to be bounded : $-0.003 < H_{\alpha_s}(h_t, h_b) < 0$

The Susy thresholds can raise or lower the value of $\alpha_s(M_Z)$. For $250 GeV > M_{SUSY} > 20 GeV$ one find \cite{39} that $0.005 > \Delta_{\alpha_s}^{Susy} > -0.003$. It appears that $\alpha_s(M_Z) - \Delta_{\alpha_s}^{GUT}$ could be as high as 0.135 or as low as 0.124 so that superheavy threshold corrections in the range $-0.004 > \Delta_{\alpha_s}^{GUT} > -0.015$ are required to reconcile with the measured value\cite{40} $\alpha_3(M_Z) = 0.1176 \pm 0.002$

Thus we demand :

\[
-22.0 \leq \Delta_G \equiv \Delta(\alpha_G^{-1}(M_X)) \leq 25 \\
3 \geq \Delta_X \equiv \Delta(\log_{10} M_X) \geq -0.3 \\
-0.017 < \Delta_3 \equiv \alpha_3(M_Z) < -0.004
\]

The threshold correction\cite{5,14} formulae are

\[
\Delta^{(th)}(ln M_X) = \frac{\lambda_1(M_X) - \lambda_2(M_X)}{2(b_1 - b_2)} \\
\Delta_X \equiv \Delta^{(th)}(\log_{10} M_X) = 0.0232 + 0.178(\bar{b}_1 - \bar{b}_2) \log_{10} \frac{M'}{M_X} \\
\Delta_3 \equiv \Delta^{(th)}(\alpha_3(M_Z)) = \frac{100 \pi (b_1 - b_2) \alpha(M_Z)^2}{[(5b_1 + 3b_2 - 8b_3) \sin^2 \theta_w(M_Z) - 3(b_2 - b_3)]^2} \sum_{ijk} \epsilon_{ijk} (b_i - b_j) \lambda_k(M_X) \\
= 0.00155 + 0.008942 \sum_{M'} (4\bar{b}_1' - 9.6\bar{b}_2' + 5.6\bar{b}_3') \log_{10} \frac{M'}{M_X}
\]
\[ \Delta_G \equiv \Delta^{(th)}(\alpha^{-1}_G(M_X)) = \frac{4\pi(b_1\lambda_2(M_X) - b_2\lambda_1(M_X))}{b_1 - b_2} \]
\[ = .1507 + .065 \sum_{M'} (6.6\tilde{b}'_2 - \tilde{b}'_1) \text{Log}_{10} \frac{M'}{M_X} \]  

(87)

Where \( \tilde{b}'_i = 16\pi^2 b'_i \) are 1-loop \( \beta \) function coefficients \( (\beta_i = b_i g_i^3) \) for multiplets with mass \( M' \) and \( \lambda_i \) are the leading contributions of the superheavy thresholds\(^{37, 5}\). These corrections, together with the two loop gauge corrections, modify the one loop values corresponding to the successful gauge unification of the MSSM but inspite of the large number of superheavy fields still give viable unification over extended regions of the GUT parameter space belying early expectations that the unification exercise was futile in SO(10) Susy GUTs\(^{11}\) (see \[3, 14, 15\] for details). Since the development of the NMSGUT was motivated by the need to reconcile the demands of unification and constraints imposed by a fit of the fermion data using the specific fermion mass formulae we do not attempt a survey of RG corrections over the huge parameter space but only illustrate some typical results for values of the slow parameters derived from successful fermion fits. The parameter \( \xi = \lambda M/\eta m \) is the only numerical parameter that enters into the cubic eqn.(8) that determines the parameter \( x \) in terms of which all the superheavy vevs are given. It is thus the most crucial determinant of the mass spectrum. The rest of the coupling parameters divide into “diagonal” \( (\lambda, \eta, \rho) \) and “non-diagonal” \( (\gamma, \bar{\gamma}, \zeta, \bar{\zeta}, k) \) couplings with the latter exerting a weaker influence on the unification parameters. The dependence of the threshold corrections on the “diagonal” couplings is also comparatively mild except when coherent e.g when many masses are lowered together leading to \( \alpha_G \) explosion, \( \text{Log} M_X \) collapse or large changes in \( \alpha_3(M_Z) \). This happens when we lower these couplings too much. In the second paper of this series\(^{23}\) we have found GUT parameter sets consistent with the known fermion data and with unification constraints. A crucial point\(^{14}\) is that the threshold corrections depend only on ratios of masses and are independent of the overall scale parameter which we choose to be \( m \). Since \( M_X = 10^{16.25+\Delta x} \text{GeV} \) it follows that

\[ \Delta_x = \Delta(\text{Log}_{10} \frac{M_X}{1 \text{GeV}}) \]
\[ |m| = 10^{16.25+\Delta x} \frac{|\lambda|}{g\sqrt{|4|\tilde{a} + \tilde{w}|^2 + 2|\tilde{p} + \tilde{\omega}|^2}} \text{GeV} \]

(88)

It is thus clear that this factor will enter every superheavy mass so that they must all rise or fall in tandem with \( M_X \) i.e with \( \Delta_x \). The SO(10) gauge coupling in this formula may be improved by using its threshold corrected value.

In Fig 3. we have given a contour plot over the complex \( x \)-plane obeying constraints\(^{56}\) with the superpotential couplings taken from a typical solution of the type found found in\(^{23}\). A noteworthy feature is that the allowed regions of the \( x \)-plane are dominantly those where the unification scale is raised above \( 10^{17.25} \text{GeV} \)(the
Figure 3: Allowed regions of $x$–plane for slow couplings fixed at values taken from a viable fit at $|\xi| = 2.0925$. Regions of the x-plane compatible with the unification constraints are shaded. The darkest regions have $2 \geq \Delta_X > 1$ (corresponding to $M_X > 10^{17.25}\, GeV$), the next darkest $1 \geq \Delta_X > 0$ the lightest shade $0 \geq \Delta_X > -0.3$ and the white regions are disallowed.

darkest shaded parts of Fig.3). We find that this behaviour is generic when one restricts the slow parameters to values $\simeq O(1)$ and also occurs around the viable parameter sets we have found[23, 43] by fitting the fermion data. Thus the NMSGUT points towards a resolution of the difficulties with $d = 5$ baryon decay and a too low gauge Landau pole by an across the board elevation of GUT scale masses.
A special case which is more easily surveyed is when $\xi$ is real. Then it follows that
the 3 solutions $x_i$ of eqn. (8) form a conjugate pair accompanied by a real solution
or else are all independently real. Due to the presence of a reflection symmetry
that interchanges the complex conjugate pair of solutions of eqn. (8) it is sufficient to
study solutions with positive real imaginary part only. The complex conjugate pair
of solutions exists only for $\xi > -27.916$. Then we may ask for what (real) values
of $\xi$ can we obtain complex $x$ values compatible with the constraints of unification.
As already seen the answer depends on the values of $\lambda, \eta, \rho$. In Fig. 4 we show a
parametric plot (vs $\xi$) of the branch $x_+(\xi)$ of the solution of eqn. (8) with positive
imaginary part. The reflection symmetry in the Re[$x$] axis makes discussion of just
the positive imaginary part branch sufficient.

![Figure 4: Parametric plot of $x(\xi)$ $|Im[x(\xi)] > 0, \xi \in (-27.917, 1000)$. The terminus
point near (2.3, 0) corresponds to $\xi \to -27.917$ and that near (1, 0) to $\xi \to \infty$](image)

As discussed in the introduction, an interesting question is whether there are
viable regions of the parameter space where the theory is still perturbative yet the
masses of the colour triplet Higgsinos that mediate proton decay are sufficiently large
as to remove or mitigate the challenge to GUTs posed by the non-detection of proton
decay. As is well known [42], the $d = 5$ proton decay rates are extremely sensitive
functions of the (so far completely unknown) flavour mixing matrices in the squark
sector. Even large (say $M_{\text{Triplet}} \geq 7 \times 10^{16}\text{GeV}$) masses of the triplet Higgs that
mediate baryon violation may not be sufficient to suppress the rate adequately. In
the NMSGUT (see Section 6) there is not one pair but a plethora of triplets -of three
different MSSM types \((t[3, 1, \pm \frac{2}{3}], P[3, 3, \pm \frac{2}{3}], K[3, 1, \pm \frac{8}{3}])\)- that can mediate Baryon
decay. However it is somewhat reassuring, in view of the tight upper bound on the
masses of baryon decay mediating triplets in the renormalizable \(SU(5)\) theory\[25\],
that in the NMSGUT the scale \(M_X\) (and with it the masses of all baryon decay
mediating triplets) is raised over the viable parameter space. In \[43\] we shall actually
exhibit fits with acceptable B violation rates.

To illustrate the effect of varying \(\xi\), in Figs.5-7 we plot the values of \(\Delta_3, \Delta_X, \Delta_G\)
versus \(\xi\) with the slow couplings fixed at values from a viable solution and with
\(x = x_+(\xi)\). There is a sharp peak in \(m\) (the overall mass scale in all superheavy
masses written in terms of the dimensionless vevs) due to a peak in \(\Delta_X\) at \(\xi = -5\).
(which on the \(x_+(\xi)\) branch corresponds to \(x = 1 + i\)). On this complex branch this
spike is not due to any special gauge symmetry as can be checked from the values of
the \(SO(10)/G_{123}\) coset gaugino (i.e E,X,G,F,J type gauginos ) masses which remain
distinct and non zero. Rather it is because a certain special multiplet , namely the
lowest mass eigenstate in the \(C[8, 2, \pm 1]\) sector becomes light as one approaches close
to to \(\xi = -5\) ( i.e as \(M_X, \alpha_G\) rise).

In Fig.6 we see that \(\Delta_X > 0\) in a wide region around the solution point where
\(M_X \sim 10^{18} GeV\). Further, since \(\Delta_X > 0\) raises the scale of all superheavy thresholds
in tandem so that they are all above the one loop unification scale(see Tables in
\[23\] for typical values) it seems clear that the running of the gauge couplings in the
Grand Desert is unmodified. Fig.5 shows that the condition on the threshold changes
in \(\alpha_3(M_Z)\) can be satisfied for a range of real values of the control parameter \(\xi\) around
the value corresponding to one of the fits we have found. Fig.7 shows that the value of
the grand unified coupling near the unification scale is \(\alpha_G \geq .05\), and thus still perturbative,
except in a narrow region around the point \(\xi = -5\) where there is a singular behaviour due to the low mass of one of the \(C[8, 2, \pm 1]\) multiplets near that
value of \(\xi\). At the singular point \(C[8, 2, \pm 1]\) is a mixture of only the modes \(C_{1,2}, \bar{C}_{1,2}\)
which have their origin in the \((15, 2, 2)_{PS}\) submultiplets of the \(126, \bar{126}\) multiplets.

From Fig.7 we see that the value of \(\alpha_G\) rises as one raises \(M_X\) but only in a very
narrow region around \(\xi = -5\) which is in any case excluded by too large a value of \(M_X\)
and \(\Delta \alpha_3\) constraints. Obviously the \(d = 6\) i.e gauge mediated baryon decay operators
will be suppressed when \(M_X\) is raised towards the Planck scale (say \(10^{18} GeV\) ) which
will raise the \(d = 6\) operator mediated lifetime by 8 orders of magnitude above the
\(10^{36}\) years usually quoted for \(d = 6\) processes in supersymmetric GUTs. The masses
of the lightest proton decay mediating triplets of each of the three independent types
\(t, K, P\) (see Section 6) all rise in tandem with \(M_X\) due to the increase in the overall
scale parameter \(m\) with \(\Delta_X\).

There is however a price to pay for all this mass scale raising and it is enforced
by the other arm of the Baryon violation- Lepton violation seesaw or balance that
operates in \(SO(10)\) theories. Namely the \(\bar{126}\) vev also rises with \(m\). If one wishes
to have right handed neutrinos much lighter than the GUT scale this would indicate that solutions of the fermion mass fitting problem with very small values of the $T_{26}$ Yukawa couplings are preferable to those that rely on inordinately skewed values of the Higgs fractions. The fits to the fermion masses we have obtained[23] are in fact of precisely this type. Small $T_{26}$ couplings give rise to relatively light($10^9 - 10^{12}$GeV) and strongly hierarchical right handed neutrino masses permitting large enough Type
Figure 6: Plot of $\Delta X$ against $\xi$ on the CP violating solution branch $x_+ (\xi)$ with values of the slow parameters fixed at those of a viable fit with $|\xi| = 2.0925$. Vertical lines mark off the $\Delta \alpha_3(M_Z)$ allowed regions: $\xi \in [-1.55, -0.6], [3.8, 4.6]$.

I (but not Type II) neutrino masses as well as large neutrino mixing angles even though the neutrino yukawa couplings take their natural values $\sim y_{q,l}$ near the GUT scale. This arrangement seems yet another instance of the intricate balance of the Fermion mass hierarchy and its ouroborotic link to the structure of the apparently completely remote GUT scale symmetry breaking and threshold structure.

Raising $M_X$ to values near the Planck scale also alleviates an apparent gross difficulty of the MSGUT one loop unification scenario with $210 \oplus 126 \oplus 126$ Higgs
system without threshold effects [26]: the presence of a Landau pole in the gauge coupling evolution at $\Lambda_X \sim 5M_X$. The problem is only worsened by the introduction of the 120. If the Unification scale is raised close to the Planck scale by the threshold effects that we have calculated then the strongly coupled dynamics at $\Lambda_X$ occurs at or close to the Planck scale itself. The Planck scale becomes a physical cut off in both the gauge and gravity sectors. This strengthens the heuristic arguments [30] that envision a UV condensation of coset gauginos in the supersymmetric GUT which

Figure 7: Plot of $\Delta G$ against $\xi$ on the CP violating solution branch $x_+(\xi)$ with values of the slow parameters fixed at those of a viable fit with $|\xi| = 2.0925$. Vertical lines represent the $\Delta \alpha_3(M_Z)$ allowed region: $\xi \in [-1.55, -0.6], [3.8, 4.6]$. 
drives the breaking of the GUT symmetry\cite{30}. Such a scenario may overcome the objections\cite{32} that led to an abandonment of the induced gravity scenarios of the 1980s. Since the cutoff of the perturbative theory is about an order of magnitude less than the scale of UV condensation and the Planck scale - which coincide - it is natural to surmise that the gauge strong coupling dynamics induces gravity characterized by $M_P \sim \Lambda_X$. In this picture the MSSM Grand Desert evolution finds $SO(10)$ completion when it crosses the superheavy mass thresholds and then $SO(10)$ quickly defines its own physical UV cutoff: $\Lambda_X \sim 5M_X$. A supersymmetric theory with a physical cutoff escapes the objections (raised on grounds of ambiguity of cutoff dependent contributions\cite{32}) against gravity induced by gauge theory dynamics. The coincidence of the scale of condensation and the Planck scale is of course the nub of the matter. Previous attempts to construct induced gravity from asymptotically free theories had no plausible reason why a large Planck scale should be induced by gauge theory (e.g. QCD or some variety of Technicolour) without any intrinsic large scale. Here however we are ‘gifted’ with coincident Planck and strong supersymmetric (therefore holomorphically controlled and calculable\cite{30}) condensation scales with no extra assumptions.

A very interesting but still controversial possibility is raised by the proposal\cite{44} that gravitational corrections to the gauge couplings provide negative, quadratic- in-energy scale corrections to the running gauge coupling. The $SO(10)$ gauge-gravity system may have a nontrivial fixed point- arising from the interplay of the quadratic and logarithmic corrections- in the gauge coupling near $M_{Planck}$.

5 Effective fermion yukawas and Weinberg Operator coefficients from the NMSGUT

As in the case of the MSGUT one imposes the fine tuning condition $\text{Det}\mathcal{H} = 0$ to keep a pair of Higgs doublets $H_{(1)}, \tilde{H}_{(1)}$ (left and right null eigenstates of the mass matrix $\mathcal{H}$) light. The composition of these null eigenstates in terms of the GUT scale doublets then specifies how much the different doublets contribute to the low energy EW scale symmetry breaking. In the Dirac mass matrices we can replace $\langle h_i \rangle \rightarrow \alpha_i v_u, \langle \tilde{h}_i \rangle \rightarrow \bar{\alpha}_i v_d$. The fermion Dirac masses may be read off the decomposition of $16 \cdot 16 \cdot (10 \oplus 120 \oplus 126)$ given in \cite{3, 5} and this yields\cite{15} (we have made slight changes in notation relative to\cite{15}).

$$y^u = (\hat{h} + \hat{f} + \hat{g}) \ ; \ \hat{r}_1 = \frac{\bar{\alpha}_1}{\alpha_1} \ ; \ \hat{r}_2 = \frac{\bar{\alpha}_2}{\alpha_2}$$
$$y^v = (\hat{h} - 3\hat{f} + (\hat{r}_5 - 3)\hat{g}) \ ; \ \hat{r}_5 = \frac{4i\sqrt{3}\alpha_5}{\alpha_6 + i\sqrt{3}\alpha_5}$$
$$y^d = (\hat{r}_1 \hat{h} + \hat{r}_2 \hat{f} + \hat{r}_6 \hat{g}) \ ; \ \hat{r}_6 = \frac{\bar{\alpha}_6 + i\sqrt{3}\bar{\alpha}_5}{\alpha_6 + i\sqrt{3}\alpha_5}$$
\[ y' = (\hat{r}_1 \hat{h} - 3 \hat{r}_2 \hat{f} + (\hat{r}_5 - 3 \hat{r}_6) \hat{g}); \quad \hat{r}_5 = \frac{4i\sqrt{3} \alpha_5}{\alpha_6 + i\sqrt{3} \alpha_5} \quad (89) \]

\[ \hat{g} = 2ig\sqrt{\frac{2}{3}}(\alpha_6 + i\sqrt{3} \alpha_5); \quad \hat{h} = 2\sqrt{2} h \alpha_1; \quad \hat{f} = -4\sqrt{\frac{2}{3}}if \alpha_2 \]

The Yukawa couplings of matter fields with 120 Higgs field give no contribution to the Majorana mass matrix of the superheavy neutrinos \( \bar{\nu}_A \) so it remains \( M_{AB}^\nu = 8\sqrt{2} f_{AB} \sigma \). Thus the Type I contribution is obtained by eliminating \( \bar{\nu}_A \)

\[ W = \frac{1}{2} M_{AB}^\nu \bar{\nu}_A \bar{\nu}_B + \bar{\nu}_A m_{AB}^\nu \nu_B + \ldots \rightarrow \frac{1}{2} M_{AB}^{\nu(I)} \nu_A \nu_B + \ldots \]

\[ M_{AB}^{\nu(I)} = -((m^\nu)^T (M^\nu)^{-1} m^\nu)_{AB} \quad (90) \]

As shown in [14, 15] it is likely that the Type II seesaw contribution is subdominant to the Type I seesaw. However the consistency of the assumption that it is negligible must be checked and quantified so we also evaluate the tadpole that gives rise to the Type II seesaw since the 120–plet does contribute new terms.

For computing the vev \( < \bar{O}(10, 3, 1)_{\frac{1}{15}} > \), inspection of the mass spectrum (Appendix A) yields the relevant terms in the superpotential as

\[ W_{\Sigma}^{\Sigma} = M_O \bar{\Sigma} \cdot \bar{\Sigma} - \frac{\gamma}{\sqrt{2}} H^{\alpha \beta} \Phi_{4\alpha 4\beta} \bar{O}_{\alpha \beta} - \frac{\gamma}{\sqrt{2}} H^{\alpha \beta} \Phi_{4\alpha 4\beta} \bar{O}_{\alpha \beta} \]

\[ - 2\sqrt{2} i \eta (\Sigma_4 \Phi_{4\alpha}^\beta \bar{O}_{\alpha \beta} + \Sigma_4 \Phi_{4\alpha}^\beta \bar{O}_{\alpha \beta}) \]

\[ + \zeta \left[ \frac{1}{2} \bar{O}_\alpha \bar{O}_\alpha \Phi_{4\alpha}^\beta + O_4 \Phi_{4\alpha}^\beta \bar{O}_\alpha \right] \]

\[ + \zeta \left[ \frac{1}{2} \bar{O}_\alpha \bar{O}_\alpha \Phi_{4\alpha}^\beta + O_4 \Phi_{4\alpha}^\beta \bar{O}_\alpha \right] \]

\[ = M_O \bar{O}_+ \bar{O}_- (\frac{i \gamma}{\sqrt{2}} \alpha_1 + i \sqrt{6} \eta \alpha_3 + \sqrt{3} \zeta \alpha_6 + i \zeta \alpha_5) \dot{\alpha}_4 v_\nu \sqrt{2} \]

\[ - O_+ (\frac{i \gamma}{\sqrt{2}} \alpha_1 + i \sqrt{6} \eta \alpha_2 - \sqrt{3} \zeta \alpha_6 + i \zeta \alpha_5) \dot{\alpha}_4 v_\nu \sqrt{2} \quad (91) \]

So

\[ < \bar{O}_- > = (\frac{i \gamma}{\sqrt{2}} \alpha_1 + i \sqrt{6} \eta \alpha_2 - \sqrt{3} \zeta \alpha_6 + i \zeta \alpha_5) \dot{\alpha}_4 \frac{v_\nu^2}{M_O} \quad (92) \]

and \( M_O \) can be read off from Table I to be \( M_O = 2(M + \eta(3a - p)) \). The Type II neutrino mass is then simply \( M_{AB}^\nu = 16i f_{AB} < \bar{O}_- > \).

The NMSGUT derived formulae for the matter fermion yukawas given in this section when combined with the explicit formulae for the Higgs fractions given in Appendix C serve as the basis for our investigation of the ability of the NMSGUT to fit all the fermion mass data now available. As mentioned in the introduction we are
forced to involve the soft supersymmetry breaking spectra in the fermion mass fitting process albeit through the natural route of large \( \tan \beta \) driven large supersymmetry threshold corrections to the SM down type fermion yukawas.

In \cite{23, 43} we present NMSGUT-mSUGRY-NUHM parameters at \( M_X \) that fit the 18 fermion mass/mixing parameters of the MSSM both before and after including GUT scale threshold corrections to the fermion yukawa couplings. The formulae collected in this paper are essential for obtaining those fits.

As seen from the fermion mass formulae the coefficients \( \alpha_i, \bar{\alpha}_i \) are quite important for the phenomenology of these models. They are calculated by determining the null left and right eigenvectors of \( \mathcal{H} \) (the \([1, 2, \pm 1]\) mass matrix) and can be used to check the compatibility of the NMSGUT with the realistic generic fits \cite{28, 29, 21}. To this end we give expressions for the \( \alpha_i, \bar{\alpha}_i \) in Appendix C. An immediate application is to check the conditions under which fits of the fermion data like the spontaneous CP violating generic fit of \cite{21} can be realized in the NMSGUT. Complex GUT scale vevs should require complex values of \( x \). One finds that the six independent phases\cite{21} that appear in the generic "spontaneous CP violation" case in \( 10 - 120 - 126 \) are given in terms of our quantities (in the convention where \( \alpha_1 = \bar{\alpha}_1 \) are both real so that the contributions of the \( 10 \rightarrow \)plet to all Dirac masses are real and the phase of \( \bar{\sigma} \) in the Type I seesaw formula has been absorbed by redefining the neutrino fields(as also in \cite{21}) ) by:

\[
\begin{align*}
\zeta_u &= \text{Arg}[\alpha_2] - \frac{\pi}{2} ; \quad \xi_u = \text{Arg}[i\alpha_6 - \sqrt{3}\alpha_5] \\
\zeta_d - \zeta_u &= \text{Arg}\left[\frac{\bar{\alpha}_2}{\alpha_2}\right] ; \quad \xi_d - \xi_u = \text{Arg}\left[\frac{\bar{\alpha}_6 + i\sqrt{3}\bar{\alpha}_5}{\alpha_6 + i\sqrt{3}\alpha_5}\right] \\
\xi_l - \xi_u &= \text{Arg}[r_7] = \text{Arg}\left[\frac{-3\bar{\alpha}_6 + i\sqrt{3}\bar{\alpha}_5}{\alpha_6 + i\sqrt{3}\alpha_5}\right] ; \quad \xi_D - \xi_u = \text{Arg}\left[\frac{-3\alpha_6 + i\sqrt{3}\alpha_5}{\alpha_6 + i\sqrt{3}\alpha_5}\right]
\end{align*}
\]  

From Appendix C one can check that for real superpotential parameters and real values of \( x \) the \( \alpha_i, \bar{\alpha}_i, i = 1...5 \) are real while \( \alpha_6, \bar{\alpha}_6 \) are pure imaginary. Then it immediately follows that except the trivial \( 126 \) phase convention dependent values \( \zeta_u = \zeta_d = -\frac{\pi}{2} \) (essentially from the factor of \( i \) that accompanies \( f_{AB} \) within the parameter \( \hat{f}_{AB} \)) all other phases are zero and there is no CKM CP violation. The only way to get non trivial phases in the CKM matrix while keeping real superpotential parameters is for \( x \) to be complex.

In the second paper of this series we use the formulae for the fermion masses in terms of the fundamental GUT parameters presented here to attempt a fit to the SM mass data extrapolated to \( M_X \) using the two loop MSSM RGE equations. Ignoring Susy threshold corrections we find that except for the yukawa couplings \( y_{d,s} \) of the down and strange quarks the available data (including large angle neutrino mixing, and adequate neutrino masses in spite of \( M_{B-L} \sim M_X \)) can indeed be accurately fit at very low values of the \( 126 \) yukawa couplings. These ultra small couplings reconcile
the large $M_{B-L}$ breaking scale with the relatively large Type I neutrino masses and thus would exactly realize the scenario that originally motivated the NMSGUT \cite{15} were it not for the difficulty with too small $y_{d,s}$. In view of the persisting under-determination of the GUT parameters by the fermion data we take this difficulty as a welcome structural constraint upon the viability of the NMSGUT. We note that precisely the same difficulty had already been noted in the generic fits of (charged, diagonal) fermion data in the $10-120$ data in \cite{20} where an evaluation of our proposal was attempted. In fact these authors later found \cite{21, 22} accurate generic fits in the $10-120-126$ system, which however seemed to rely on a combination of moderately small $126$ couplings and large $10-120$ yukawas. These fits have no direct relevance to fits in the NMSGUT. Indeed their structure turns out to be un-realizable in searches for NMSGUT specific fits \cite{24}.

The apparent cul-de-sac does however have an exit if one uses the freedom to choose soft Susy breaking parameters in the MSSM at large $\tan \beta$. Then one finds that the large threshold corrections to precisely these (i.e $T_{3L} = -1/2$) types of fermions in the large $\tan \beta$ scenario allow one to find excellent fits using the parameter freedom of the soft Susy breaking masses and couplings. The details of this numerical saga are the content of the next papers of this series, where the implications of the parameter values found are also evaluated with regard to their implications for Baryon number violation. The weaker constraints from other exotic processes such as $b \rightarrow s \gamma$ or precision data on $(g-2)_\mu$, $\rho$ parameter etc are also evaluated there. To prepare for that analysis, in the next section, we complete our suite of NMSGUT formulae by extending our analysis of the effective superpotential for $d=5$ operator mediated Baryon decay from the MSGUT to the NMSGUT.

\section{d = 5 Operators for B, L violation}

In \cite{5} we worked out the effective $d=4$ superpotential for $B+L$ violating processes due to exchange of colour triplet superheavy chiral supermultiplets contained in the $10, \mathbf{T}_{26}$ Higgs multiplets. These included a novel channel due to decays mediated by exchange of triplets $t_{(4)}$ contained in the $\mathbf{T}_{26}$ Higgs irrep. Evidently the inclusion of $120$ plet Higgs will lead to additional channels for baryon violation. These can be easily derived using the Pati-Salam decomposition of the $16.16.120$ $\text{SO}(10)$ invariants\cite{3}:

$$W_{FM}^O = 2\sqrt{2} g_{AB}[\bar{h}_5(\bar{d}_A Q_B + \bar{e}_A L_B) - h_5(\bar{u}_A Q_B + \bar{\nu}_A L_B)]$$
\[-2\sqrt{2}g_{AB}[\sqrt{2}L_2Q_AQ_B + F_1L_AL_B + \sqrt{2}t_6Q_AL_B] \\
+2\sqrt{2}L_2\bar{u}_A\bar{d}_B + \sqrt{2}t_6(\bar{u}_A\bar{e}_B - \bar{d}_A\bar{\nu}_B) + 2F_1\bar{\nu}_A\bar{e}_B] \\
-2\sqrt{2}g_{AB}[2\tilde{C}_3\bar{d}_AQ_B - 2C_3\bar{u}_AQ_B + \frac{i}{\sqrt{3}}\bar{h}_6(\bar{d}_AQ_B - 3\bar{e}_AL_B) \\
- \frac{i}{\sqrt{3}}h_6(\bar{u}_AQ_B - 3\bar{\nu}_AL_B) + 2D_3\bar{e}_AQ_B - 2E_6\bar{\nu}_AQ_B \\
+2E_6\bar{d}_AL_B - 2D_3\bar{u}_AL_B] - 2i\sqrt{2}g_{AB}[\epsilon\bar{J}_5\bar{d}_A\bar{d}_B \\
+2K_2\bar{d}_A\bar{e}_B - \epsilon\bar{K}_2\bar{u}_A\bar{u}_B - 2J_5\bar{u}_A\bar{\nu}_B \\
- \sqrt{2}\epsilon\tilde{t}_7\bar{d}_A\bar{u}_B - \sqrt{2}\epsilon\tilde{t}_7(\bar{d}_A\bar{\nu}_B - \bar{e}_A\bar{u}_B)] - 2g_{AB}[\epsilon P_2Q_AQ_B + 2\tilde{P}_2QAQ_AL_B]
\]

We have suppressed $G_{321}$ indices and used a sub-multiplet naming convention specified in Section 2, conversion to fields of unit norm in the terms containing colour sextets ($L_2, L_2$) is explained in the caption to Table 1.

In order that the exchange of a Higgsino that couples to matter with a given $B + L$ lead to a $B + L$ violating $d = 5$ operator in the effective theory at sub GUT energies it is necessary that it have a nonzero contraction with a conjugate (MSSM) representation Higgsino that couples to a matter chiral bilinear with a $B + L$ different from the conjugate of the first $B + L$ value. On inspection one finds that not only the familiar triplet types $[3, 1, \pm \frac{2}{3}] \subset 120$ i.e $\{\tilde{t}(6), \tilde{t}(7)\} [3, 1, \frac{2}{3}]$ and $\{t(6), t(7)\}$ but also the novel exchange modes from the $P[3, 3, \pm \frac{2}{3}]$ and $K[3, 1, \pm \frac{2}{3}]$ multiplet types can contribute to baryon violation. In the case of the $126$ the $\tilde{P}_1, K_1 \subset 126$ multiplets did couple to the fermions but $P_1, K_1 \subset 120$ did not. The $120$ however contains both $P_2, P_2$ and $K_2, K_2$. Since these mix with $P_1, \tilde{P}_1$ and $K_1, \tilde{K}_1$, a number of fresh contributions appear.

The multiplets $P_2[3, 3, -\frac{2}{3}], \bar{P}_2[\bar{3}, 3, \frac{2}{3}], K_2[3, 1, -\frac{2}{3}], \bar{K}_2[\bar{3}, 1, \frac{2}{3}]$ satisfy the requirement regarding $B + L$ quantum numbers of the fields they couple to. Note in particular that these novel exchanges always lead to contributions in which at least one and possibly both pairs of final state family indices are antisymmetrized.

On integrating out the heavy triplet Higgs supermultiplets one obtains the following additional effective $d = 4$ Superpotential for Baryon Number violating processes in the NMSGUT to leading order in $m_W/M_X$. We have taken the opportunity to insert a missing overall sign and correct minor sub/super-script typos in [5]:

\[
W_{eff}^{A\bar{B}\neq 0} = -L_{ABCD}(\frac{1}{2}(Q_AQ_BQ_CL_D) - R_{ABCD}(\epsilon\bar{e}_A\bar{u}_B\bar{u}_C\bar{d}_D)
\] 

where the coefficients are

\[
L_{ABCD} = S_1^1\bar{h}_{AB}\bar{h}_{CD} + S_1^2\bar{h}_{AB}\bar{f}_{CD} + S_2^1\bar{f}_{AB}\bar{h}_{CD} + S_2^2\bar{f}_{AB}\bar{f}_{CD} \\
- S_6^1\bar{h}_{AB}\bar{g}_{CD} - S_6^2\bar{f}_{AB}\bar{g}_{CD} + \sqrt{2}(P^{-1})_2^1\bar{g}_{AC}\bar{f}_{BD} \\
- (P^{-1})_2^2\bar{g}_{AC}\bar{g}_{BD}
\]

\[\text{(96)}\]
\[ R_{ABCD} = S_1^1 \tilde{h}_{AB} \tilde{h}_{CD} - S_1^2 \tilde{h}_{AB} \tilde{f}_{CD} - S_2^1 \tilde{f}_{AB} \tilde{h}_{CD} + S_2^2 \tilde{f}_{AB} \tilde{f}_{CD} - i\sqrt{2} S_4^1 \tilde{f}_{AB} \tilde{h}_{CD} - i\sqrt{2} S_4^2 \tilde{f}_{AB} \tilde{f}_{CD} + S_6^1 \tilde{\tilde{g}}_{AB} \tilde{h}_{CD} - S_6^2 \tilde{\tilde{g}}_{AB} \tilde{f}_{CD} + iS_7^1 \tilde{\tilde{g}}_{AB} \tilde{h}_{CD} - iS_7^2 \tilde{\tilde{g}}_{AB} \tilde{f}_{CD} + iS_8^7 \tilde{\tilde{g}}_{AB} \tilde{g}_{CD} + \sqrt{2} (\mathcal{K}^{-1})_{1}^2 \tilde{f}_{AD} \tilde{g}_{BC} - (\mathcal{K}^{-1})_{2}^2 \tilde{g}_{AD} \tilde{g}_{BC} \]

(97)

Here \( S = \mathcal{T}^{-1} \) and \( \mathcal{T} \) is the mass matrix for \([3, 1, \pm 2/3]\)-sector triplets: \( W = \bar{\nu}_i t_i \bar{t}_j + \ldots \), while

\[ \tilde{h}_{AB} = 2\sqrt{2} h_{AB} \quad \tilde{f}_{AB} = 4\sqrt{2} f_{AB} \quad \tilde{g}_{AB} = 4 g_{AB} \]

(98)

These operators are dressed by sparticles to yield the \( d = 6 \) effective 4-fermi operators for Baryon decay. This dressing requires knowledge of the scalar spectra and mixing angles. This information is partly supplied by the threshold corrections used to fit the down and strange quark masses which assume adequate (diagonal) scalar spectra for the purpose. However, the scalar mixing which is so crucial to the Baryon decay rate is assumed minimal i.e to be determined simply by evolution of the GUT scale (super)CKM mixing. The rates for B violation via the dominant \( d = 5 \) operators are evaluated using the above formulae and the usual dressing by Gaugino/Higgsino exchange in [23].

7 Discussion and Outlook

In this paper, motivated by successful fits of the fermion data[29, 21] which evade the difficulties that forced an abandonment[14, 15] of the hope[13] that the \( 10 , 126 \) FM Higgs system would be sufficient to describe the entire fermion mass spectrum, we specified the ingredients of a New Minimal Supersymmetric GUT based on the gauge group \( SO(10) \) and the \( 210 \oplus 10 \oplus 120 \oplus 126 \oplus \overline{126} \) Higgs System. While inheriting the Higgs system responsible for GUT scale symmetry breaking unchanged from the MSGUT[2, 10, 11] but reassigning the roles of the FM Higgs the NMSGUT is able to describe all the fermion data at \( M_X \) successfully provided recourse is had to relevant threshold corrections at the Susy breaking scale. This alleviates a problem with fitting down type yukawa couplings using only the \( 10, 120 \) couplings to matter fields (since the \( 126 \) couplings are lowered drastically to make the Type I seesaw neutrino masses viable and are thus irrelevant to charged fermion masses).

Using the techniques we developed for the MSGUT[3, 5] we computed the superheavy spectrum for the NMSGUT and used it to compute threshold effects in
the gauge evolution. We found that the Unification scale defined as the mass of
the Baryon number violating gauge fields is raised above the one loop values. This
increase could take \( M_X \) to values as large as \( 10^{19} \text{ GeV} \) while still remaining in
the perturbative domain. Thus gauge mediated Baryon decay is unmeasurably small in
this theory. Together with \( M_X \) all other masses, in particular those of the three
triplet types that mediate \( d = 5 \) baryon decay, also rise and can be taken(effectively)
well above \( 10^{16} \text{GeV} \). Thus, \textit{prima facie}, not only \( d = 6 \) but also \( d = 5 \) proton decay
may be controllable. In practice we find\cite{13} that inclusion of GUT scale threshold
effects due to the \( 120 \)-plet and searches of the parameter space under a constraint to
suppress B-violation is necessary before palatable B-violation rates are reached.

The increase of \( M_X \) provides resolution of a nagging difficulty\cite{26} in the MSGUT:
the Landau pole in the gauge coupling evolution above \( M_X \). Since \( M_X \) is closer to
the Planck scale the presence of the SO(10) Landau pole just above the Planck scale
strengthens our speculation that the UV condensation to be expected in such a super-
symmetric Asymptotically Strong(AS) theory \cite{26,30} acts as a physical cutoff for
the perturbative SO(10) theory and perhaps even as the scale of an induced gravity
that arises from this theory. We made a beginning in\cite{30} by demonstrating, using Sup-
ersymmetric strong coupling heuristics\cite{31}, that in a toy ASSGUT the condensation
actually takes place and breaks the (toy) GUT symmetry, and that the vevs responsi-
bible are \textit{calculable}. It is encouraging that the development of the theory in regard to
apparently unrelated features has naturally brought us to the point where a number
of intractable fundamental features have become pliable to a synthetic interpretation.

We gave complete formulae for the fermion masses and baryon violating effective
superpotential in the NMSGUT, including lengthy analytic expressions for the Higgs
fractions \((\alpha_i, \bar{\alpha}_i)\) which are determined by the GUT parameters(after a fine tuning)
and are crucial ingredients of both the masses and the \( d = 5 \) B-violation. In the next
papers of this series we use the suite of formulae given here to find fits of the fermion
data and calculate the corresponding B-violation rates.

In sum, the NMSGUT having inherited the strengths of its parent is revealing
new virtues as well as new weaknesses and, while threatening still to plunge into the
yawning crevasse of falsification, yet promises to carry the long winding caravan of
Grand Unification not only across the Grand Desert that set its first horizons but
across threshold jungles beyond that first horizon up into the rarefied heights where
gauge forces and gravity meld into their primordial pleromal\cite{30} unity.

8 Acknowledgments

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ment of Science and Technology of the Govt. of India and that of S.K.G by a
### Table 1: Masses of the unmixed states in terms of the superheavy vevs. The $SU(2)_L$ contraction order is always $F_{α}^{a}F_a$. The absolute value of the expressions in the column “Mass” is understood. For sextets of $SU(3)$ the 6 unit norm fields are denoted by a prime: $\Sigma'_{μν} = \Sigma_{μν}, \bar{μ} > \bar{ν}, \Sigma'_{μ\bar{μ}} = \Sigma_{μ\bar{μ}}/\sqrt{2}$ and similarly for $\bar{6}$.

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**Appendix A**: Tables of masses and mixings

Here mixing matrix rows are labelled by barred irreps and columns by unbarred.

(i) The masses of 13 Unmixed cases are given as Table II.

(ii) Mixed states

\[ [8, 2, -1](C_1, C_2, C_3) \oplus [8, 2, 1](C_1, C_2, C_3) \equiv (Σ_{α2}^A, Σ_{α2}^A, O_{α2}^A) \oplus (Σ_{α1}^A, Σ_{α1}^A, O_{α1}^A)(A = 1,...,8) \]
\[\begin{pmatrix}
2(\bar{M} + \eta(a + \omega)) & 0 & -i(\omega - p)\bar{\zeta} \\
0 & 2(\bar{M} + \eta(a + \omega)) & -i(\omega + p)\bar{\zeta} \\
i(\omega - p)\zeta & i(\omega + p)\zeta & -m_0 + \frac{2}{3}a
\end{pmatrix}\]

b) \[\{3, 2, -\frac{7}{3}\}(\bar{D}_1, \bar{D}_2, \bar{D}_3) \oplus [3, 2, \frac{7}{3}](D_1, D_2, D_3) \equiv (\Sigma_{42}, \Sigma_{42}^\prime, O_{42}^\prime) \oplus (\Sigma_{\mu a 1}, \Sigma_{\mu a 1}^\prime, O_{\mu a 1}^\prime)\]

\[\begin{pmatrix}
2(\bar{M} + \eta(a + \omega)) & 0 & (i\omega + ip - 2ia)\bar{\zeta} \\
0 & 2(\bar{M} + \eta(a + 3\omega)) & (-3i\omega - ip - 2ia)\bar{\zeta} \\
(-i\omega + ip - 2ia)\zeta & (3i\omega + ip + 2ia)\zeta & m_0 + \frac{2}{3}(a + 2\omega)
\end{pmatrix}\]

c) \[\{3, 2, -\frac{1}{3}\}(\bar{E}_1, \bar{E}_2, \bar{E}_3, \bar{E}_4, \bar{E}_5, \bar{E}_6) \oplus [3, 2, \frac{1}{3}](E_1, E_2, E_3, E_4, E_5, E_6) \equiv (\Sigma_{41} h_{i a}, \Sigma_{41}^\prime h_{i a}, \phi^{\mu a 4}_{(s) h_{i a}}, \lambda_{41}^h, O_{41}^h) \oplus (\Sigma_{\mu a 2}, \Sigma_{\mu a 2}^\prime, \phi^{\mu a 4}_{(h) l_{i a}}, \lambda_{\mu a 1}, O_{\mu a 1}^h)\]

\[\begin{pmatrix}
-2(\bar{M} + \eta(a - \omega)) & 0 & 0 & 0 & 0 & (i\omega - ip + 2ia)\bar{\zeta} \\
0 & -2(\bar{M} + \eta(a - 3\omega)) & -2i\sqrt{2}\eta\sigma & 2i\eta\sigma & 0 & (-3i\omega + ip + 2ia)\bar{\zeta} \\
0 & 2\sqrt{2}\eta\sigma & -2(\bar{M} + \lambda(a - \omega)) & -2\sqrt{2}\lambda\omega & 2\sqrt{2}\lambda(a^* - \omega^*) & -\sqrt{2}\zeta \\
0 & -2i\sqrt{2}\eta\sigma^* & -2\sqrt{2}\lambda\omega^* & -2(\bar{M} - \lambda\omega) & \sqrt{2}\lambda(a^* - \omega^*) & -\sqrt{2}\zeta^* \\
(-i\omega + ip - 2ia)\zeta & 0 & 2\sqrt{2}\lambda(a^* - \omega^*) & g\sqrt{2}(\omega^* - \rho^*) & \sqrt{2}\zeta & 0 \\
(3i\omega - ip - 2ia)\zeta & -2\sqrt{4}\lambda(a^* - \omega^*) & 0 & \sqrt{2}\zeta & 0 & -(m_0 + \frac{2}{3}a - \frac{4}{3}\rho\omega)
\end{pmatrix}\]

d) \[\{1, 1, -2\}(\bar{F}_1, \bar{F}_2, \bar{F}_3, \bar{F}_4) \oplus [1, 1, 2]\{F_1, F_2, F_3, F_4\} \equiv (\Sigma^{44}_{h(0)}, \phi^{(15)}_{(R^-)}, \lambda_{(R^-)}, O_{44}^{(15)}) \oplus (\Sigma^{44}_{(R^+)}, \phi^{(15)}_{(R^+)}), \lambda_{(R^+)}, O_{44}^{(15)})\]

\[\begin{pmatrix}
2(\bar{M} + \eta(p + 3a)) & -2i\sqrt{3}\eta\sigma & -g\sqrt{2}\sigma^* & -6i\bar{\zeta}\omega \\
2i\sqrt{3}\eta\sigma & 2(\bar{M} + \lambda(p + 2a)) & \sqrt{2}ig\omega^* & \sqrt{3}\zeta\sigma \\
-g\sqrt{2}\sigma^* & \sqrt{2}i\eta\sigma^* & 0 & 0 \\
6i\zeta\omega & \sqrt{3}\zeta\sigma & 0 & m_0 + a\rho
\end{pmatrix}\]

e) \[\{1, 1, 0\}(G_1, G_2, G_3, G_4, G_5, G_6) \equiv (\phi, \phi^{(15)}_{(h)}, \phi^{(15)}_{(l)}, \Sigma^{44}_{h(0)}, \Sigma^{44}_{(R^+)}, \Sigma^{44}_{(R^+) + \bar{R}(1)^{(15)}})\]

\[G = 2\begin{pmatrix}
m & 0 & \sqrt{6}\lambda\omega & \frac{-i\eta\sigma}{\sqrt{3}} & 0 \\
0 & m + 2a & 2\sqrt{2}\lambda\omega & \frac{i\eta\sigma}{\sqrt{3}} & 0 \\
\sqrt{6}\lambda\omega & 2\sqrt{2}\lambda\omega & m + \lambda(p + 2a) & \frac{-i\eta\sigma}{\sqrt{3}} & 0 \\
\frac{i\eta\sigma}{\sqrt{2}} & i\eta\sigma & -i\eta\sqrt{3}\lambda & i\eta\sqrt{3}\lambda & 0 \\
0 & -i\eta\sigma & \frac{-i\eta\sigma}{\sqrt{2}} & \frac{\sqrt{5}\sigma^*}{2} & M + \eta(p + 3a - 6\omega) \\
0 & 0 & \frac{\sqrt{5}\sigma^*}{2} & M + \eta(p + 3a - 6\omega) & \frac{\sqrt{5}\sigma^*}{2} \end{pmatrix}\]

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\[ \begin{pmatrix} -M_{H} & \gamma \sqrt{3}(\omega - a) & -\gamma \sqrt{3}(\omega + a) & -\gamma \sigma & kp & -\sqrt{3} \kappa \omega \\ -\gamma \sqrt{3}(\omega + a) & -(2M + 4\eta(\omega + a)) & 0 & -2\eta \sqrt{3} \kappa \omega & -\eta(p + 2\omega) \kappa & -m_{o} \\ \gamma \sqrt{3}(\omega - a) & -(2M + 4\eta(\omega - a)) & 0 & 2\eta \sqrt{3} \kappa \omega & -\eta(p - 2\omega) \kappa & -m_{o} \\ -\sigma \gamma & -2\eta \sqrt{3} \kappa \omega & -\sigma \kappa \omega & \kappa \omega & -2\kappa \omega & -m_{o} \\ pk & \sqrt{3} \kappa \omega & -\sqrt{3} \kappa \omega & \kappa \omega & \kappa \omega & -m_{o} \\ \sqrt{3} \kappa \omega & i(p - 2\omega) \kappa & -i(p + 2\omega) \kappa & -\sqrt{3} \kappa \omega & -\sqrt{3} \kappa \omega & -m_{o} \\ \end{pmatrix} \]

The above matrix is to be diagonalized after imposing the fine tuning condition
\( \text{Det} \mathcal{H} = 0 \) to keep one pair of doublets light.

\[ \quad \equiv (\Sigma_{(R+)_{1}}, \phi_{11}, \phi_{41}(R0), \lambda_{1a}, O_{(R-)}^{2}) \oplus (\Sigma_{(R+4)}, \phi_{4}^{R}, \phi_{\mu}^{4}(R0), \lambda_{4}, O_{\mu}^{4}) \]

\[ \mathcal{J} = \begin{pmatrix} 2(M + \eta(a + p - 2\omega)) & -2\eta \sigma & 2\sqrt{2}\eta \sigma & -ig \sqrt{2}\sigma a & 2\zeta(a - 2\omega) \\ 2\sqrt{2}\eta \sigma & -(2M + 4\eta) & -2\sqrt{2}\lambda \omega & -2\sqrt{2}\lambda \omega & -2\sqrt{2} \kappa \omega \sigma \\ -2\sqrt{2}\eta \sigma & -2\sqrt{2}\lambda \omega & -(2M + 4\eta) & -2\sqrt{2} \kappa \omega \sigma & -2\sqrt{2} \kappa \omega \sigma \\ -2\sqrt{2}\eta \sigma & -2\sqrt{2}\lambda \omega & -2\sqrt{2} \kappa \omega \sigma & -(2M + 4\eta) & -2\sqrt{2} \kappa \omega \sigma \\ 2\zeta(a - 2\omega) & \sigma \kappa \omega & -\sqrt{2} \kappa \omega \sigma & -\sqrt{2} \kappa \omega \sigma & 0 \\ \end{pmatrix} \]

\[ \quad \equiv (\Sigma_{(R+)_{1}}, \phi_{11}, \phi_{41}(R0), \lambda_{1a}, O_{(R-)}^{2}) \oplus (\Sigma_{(R+4)}, \phi_{4}^{R}, \phi_{\mu}^{4}(R0), \lambda_{4}, O_{\mu}^{4}) \]

\[ \begin{pmatrix} 2(M + \eta(a + p + 2\omega)) & 2\zeta(a + 2\omega) \\ 2\zeta(a + 2\omega) & m_{o} + \frac{p}{3} \right) \]

\[ \quad \equiv (\Sigma_{(R+)_{1}}, \phi_{11}, \phi_{41}(R0), \lambda_{1a}, O_{(R-)}^{2}) \oplus (\Sigma_{(R+4)}, \phi_{4}^{R}, \phi_{\mu}^{4}(R0), \lambda_{4}, O_{\mu}^{4}) \]

\[ \begin{pmatrix} 2(M + \eta(a - p - a)) & -2i \zeta \omega \\ 2i \zeta \omega & m_{o} - \frac{p}{3} \right) \]

\[ \quad \equiv (\Sigma_{(L)}, \phi_{4}^{L}, \phi_{\mu}^{4}(L), \lambda_{2a}, O_{\mu}^{4}(L)) \]

\[ \begin{pmatrix} 2(M + \eta(a - p)) & 2a \zeta \\ 2a \zeta & m_{o} - \frac{2p}{3} \right) \]

\[ \quad \equiv (\Sigma_{(L)}, \phi_{4}^{L}, \phi_{\mu}^{4}(L), \lambda_{2a}, O_{\mu}^{4}(L)) \]

\[ \quad \equiv (\Sigma_{(L)}, \phi_{4}^{L}, \phi_{\mu}^{4}(L), \lambda_{2a}, O_{\mu}^{4}(L)) \]

\[ \quad \equiv (\Sigma_{(L)}, \phi_{4}^{L}, \phi_{\mu}^{4}(L), \lambda_{2a}, O_{\mu}^{4}(L)) \]

\[ \quad \equiv (\Sigma_{(L)}, \phi_{4}^{L}, \phi_{\mu}^{4}(L), \lambda_{2a}, O_{\mu}^{4}(L)) \]

\[ \quad \equiv (\Sigma_{(L)}, \phi_{4}^{L}, \phi_{\mu}^{4}(L), \lambda_{2a}, O_{\mu}^{4}(L)) \]
\[ \mathcal{R} = 2 \begin{pmatrix} (m - \lambda a) & -\sqrt{2}\lambda \omega \\ -\sqrt{2}\lambda \omega & m + \lambda(p - a) \end{pmatrix} \]

1) \[ [3, 1, \frac{2}{3}] (\tilde{t}_1, \tilde{t}_2, \tilde{t}_3, \tilde{t}_4, \tilde{t}_5, \tilde{t}_6, \tilde{t}_7) \oplus [3, 1, -\frac{2}{3}] (t_1, t_2, t_3, t_4, t_5, t_6, t_7) \equiv (H^{\mu 4}, \Sigma_{\mu 4}^{(a)}, \Sigma_{\mu 4}^{(a)}, O^\mu_{(R_0)}, O^\mu_{(R_0)} \Phi_{\mu 4}^{(R_0)}, \Phi_{\mu 4}^{(R_0)}, O^\mu_{(R_0)}, O^\mu_{(R_0)}) \]

\[
\begin{pmatrix}
M_H & \gamma(p + a) & \gamma(p - a) & 2\sqrt{2}\omega \gamma & i\sigma \gamma & \sqrt{2}k_a & \sqrt{2}k_\omega \\
\gamma(p - a) & 0 & 2M & 0 & 0 & \sqrt{2}a_\zeta & \sqrt{2}a_\omega \\
\gamma(p + a) & 2M & 0 & 4\sqrt{2}\omega \eta & 2i\eta \omega & \sqrt{2}a_\zeta & \sqrt{2}a_\omega \\
-2\sqrt{2}\omega \gamma & -4\sqrt{2}\omega \eta & 0 & 2M + 2\eta \rho + 2\eta a & -2\sqrt{2}\eta \rho & 2\omega \zeta & 2\omega \eta \\
i\sigma \gamma & 2\eta \rho & 0 & 2\sqrt{2}\rho \omega & -2m & 2m & 2m \\
\sqrt{2}k_a & -\sqrt{2}a_\zeta & \sqrt{2}a_\omega & -2i\omega \zeta & \sqrt{2}i\zeta \omega & m_0 + \frac{3}{2}a & -\frac{3}{2}\rho \omega \\
-\sqrt{2}k_\omega & -\sqrt{2}a_\zeta & -\sqrt{2}a_\omega & 2\zeta a & \sqrt{2}2 \zeta \omega & \frac{3}{2}\rho \omega & m_0 + \frac{3}{2}p
\end{pmatrix}
\]

m) \[ [3, 2, \frac{5}{3}] (\bar{X}_1, \bar{X}_2, \bar{X}_3) \oplus [3, 2, -\frac{5}{3}] (X_1, X_2, X_3) \equiv (\phi^{(a)\mu 4}, \phi^{(a)\mu 4}, \lambda^{(a)\mu 4}) \oplus (\phi^{(a)\mu 4}, \phi^{(a)\mu 4}, \lambda^{(a)\mu 4}) \]

\[ \mathcal{X} = \begin{pmatrix}
2(m + \lambda(a + \omega)) & -2\sqrt{2}\lambda \omega & -2g(a^* + \omega^*) \\
-2\sqrt{2}\lambda \omega & 2(m + \lambda \omega) & \sqrt{2}g(\omega^* + p^*) \\
-2g(a^* + \omega^*) & \sqrt{2}2g(\omega^* + p^*) & 0
\end{pmatrix} \]

Appendix B : SU(5) \times U(1) Reassembly Crosscheck

The internal consistency of these spectra and couplings can be verified by considering special values of vevs, e.g

\[ p = a = \pm \omega \] (99)

where the unbroken symmetry includes SU(5)[4]. Then we find that the MSSM labelled mass spectra and couplings given in Appendix A do indeed reassemble into SU(5) invariant form. If we insert \( a = -\omega = p \) in the mass matrices of Appendix A we find that, after diagonalizing the mass matrices of the submultiplets that mix, the resultant spectra group precisely as indicated by the decompositions below with all the subreps of a given SU(5) irrep obtaining the same mass and correct phases to permit reassembly. The delicacy of this reassembly is a non-trivial consistency check of our results.
\[ \begin{align*}
H &= 10 = 5_1 + 5_{-1} \\
5_1 &= h_1(1, 2, 1) + t_1(3, 1, -\frac{2}{3}) \\
5_{-1} &= \bar{h}_1(1, 2, -1) + \bar{t}_1(3, 1, \frac{2}{3}) \\
\Sigma &= 126 = 1_{-5}(G_4) + 5_{-1} + 10_{-3} + 15_{-3} + 45_1 + 50_{-1} \\
5_{-1} &= \bar{h}_3(1, 2, -1) + \bar{t}_{3,4}(3, 1, \frac{2}{3}) \\
10_{-3} &= F_1(1, 1, 2) + J_1(3, 1, -\frac{4}{3}) + E_2(3, 2, \frac{1}{3}) \\
15_{-3} &= \bar{O}(1, 3, 2) + \bar{E}_1(3, 2, -\frac{1}{3}) + \bar{N}(6, 1, \frac{4}{3}) \\
45_1 &= h_3(1, 2, 1) + t_3(3, 1, -\frac{2}{3}) + P_1(3, 3, -\frac{2}{3}) + K_1(3, 1, \frac{8}{3}) + D_1(3, 2, -\frac{7}{3}) \\
&+ \bar{L}_1(6, 1, -\frac{2}{3}) + C_1(8, 2, 1) \\
50_{-1} &= A(1, 1, 4) + \bar{t}_{3,4}(3, 1, \frac{2}{3}) + D_2(3, 2, \frac{7}{3}) + W(6, 3, \frac{2}{3}) + \bar{M}(6, 1, -\frac{8}{3}) + \bar{C}_2(8, 2, -1) \\
\Sigma &= 126 = 1_5(G_5) + 5_1 + 10_3 + 15_{-3} + 45_{-1} + 50_1 \\
5_1 &= h_2(1, 2, 1) + t_{2,4}(3, 1, -\frac{2}{3}) \\
10_3 &= \bar{F}_1(1, 1, -2) + J_1(3, 1, \frac{4}{3}) + \bar{E}_2(3, 2, -\frac{1}{3}) \\
15_{-3} &= \bar{O}(1, 3, 2) + E_1(3, 2, \frac{1}{3}) + N(6, 1, -\frac{4}{3}) \\
45_{-1} &= \bar{h}_2(1, 2, -1) + \bar{t}_2(3, 1, \frac{2}{3}) + \bar{P}_1(3, 3, \frac{2}{3}) + K_1(3, 1, -\frac{8}{3}) + D_1(3, 2, \frac{7}{3}) \\
&+ \bar{L}_1(6, 1, \frac{2}{3}) + \bar{C}_1(8, 2, -1) \\
50_1 &= \bar{A}(1, 1, -4) + t_{2,4}(3, 1, -\frac{2}{3}) + \bar{D}_2(3, 2, -\frac{7}{3}) + \bar{W}(6, 3, -\frac{2}{3}) + M(6, 1, \frac{8}{3}) + C_2(8, 2, 1) \\
\Phi &= 210 = 1_0 + 5_{-4} + 5_4 + 10_2 + 100_{-2} + 24_0 + 40_2 + 400_{-2} + 75_0 \\
1_0 &= G_{1,2,3} \\
5_{-4} &= h_4(1, 2, 1) + t_5(3, 1, -\frac{2}{3}) \\
5_4 &= \bar{h}_4(1, 2, -1) + \bar{t}_5(3, 1, \frac{2}{3})
\end{align*} \]
\[10_2 = F_2(1, 1, 2) + J_{2,3}(3, 1, -\frac{4}{3}) + E_{3,4}(3, 2, \frac{1}{3})\]

\[\overline{10}_{-2} = \bar{F}_2(1, 1, -2) + J_{2,3}(3, 1, \frac{4}{3}) + \bar{E}_{3,4}(3, 2, -\frac{1}{3})\]

\[24_0 = (1, 1, 0)G_{1,2,3} + S(1, 3, 0) + X_{1,2}(3, 2, -\frac{5}{3}) + \bar{X}_{1,2}(\bar{3}, 2, \frac{5}{3}) + R_{1,2}(8, 1, 0)\]

\[40_2 = V(1, 2, -3) + E_{3,4}(3, 2, \frac{1}{3}) + J_{2,3}(3, 1, -\frac{4}{3}) + \bar{U}(3, 3, -\frac{4}{3}) + Z(8, 1, 2) + \bar{Y}(6, 2, \frac{1}{3})\]

\[4\overline{10}_{-2} = V(1, 2, 3) + \bar{E}_{3,4}(3, 2, -\frac{1}{3}) + J_{2,3}(3, 1, \frac{4}{3}) + U(3, 3, \frac{4}{3}) + \bar{Z}(8, 1, -2) + Y(6, 2, -\frac{1}{3})\]

\[75 = (1, 1, 0)G_{1,2,3} + I(3, 1, \frac{10}{3}) + \bar{I}(3, 1, -\frac{10}{3}) + X_{1,2}(3, 2, -\frac{5}{3}) + X_{1,2}(\bar{3}, 2, \frac{5}{3})\]

\[+ B(6, 2, \frac{5}{3}) + \bar{B}(6, 2, -\frac{5}{3}) + R_{1,2}(8, 1, 0) + Q(8, 3, 0)\]

\[(103)\]

\[O = 5_1 + 5_{-1} + 10_{-3} + 10_3 + 45_1 + 45_{-1}\]

\[5_1 = h_{5,6}(1, 2, 1) + t_{6,7}(3, 1, -\frac{2}{3})\]

\[5_{-1} = \bar{h}_{5,6}(1, 2, -1) + \bar{t}_{6,7}(3, 1, \frac{2}{3})\]

\[10_{-3} = F_4(1, 1, 2) + J_{5}(3, 1, -\frac{4}{3}) + E_6(3, 2, \frac{1}{3})\]

\[\overline{10}_3 = \bar{F}_4(1, 1, -2) + J_5(3, 1, \frac{4}{3}) + \bar{E}_6(3, 2, -\frac{1}{3})\]

\[45_1 = h_{5,6}(1, 2, 1) + t_{6,7}(3, 1, -\frac{2}{3}) + P_2(3, 3, -\frac{2}{3}) + \bar{K}_2(3, 1, \frac{8}{3}) + \bar{D}_3(\bar{3}, 2, -\frac{7}{3})\]

\[+ L_2(6, 1, -\frac{2}{3}) + C_3(8, 2, 1)\]

\[45_{-1} = \bar{h}_{5,6}(1, 2, -1) + \bar{t}_{6,7}(3, 1, \frac{2}{3}) + \bar{P}_2(3, 3, \frac{2}{3}) + K_2(3, 1, -\frac{8}{3}) + D_3(3, 2, \frac{7}{3})\]

\[+ L_2(6, 1, \frac{2}{3}) + \bar{C}_3(8, 2, -1)\]

\[(104)\]

Due to the \textbf{120}-plet one obtains the additional \textit{SU}(5) invariant mass terms:

\[(m_0 + \rho p)50_5o\bar{O} + (m_0 + \rho p)10_010_0 + (m_0 - \frac{\rho}{3} p)45_045_0\]

\[+ 2kp(5_0\bar{5}_5 + \bar{5}_505_5) - 2\sqrt{3}p(\zeta 5_0\bar{5}_5 + \bar{\zeta}05_5) + 2(\zeta 5_0\bar{5}_5 + \bar{\zeta}05_5)\]

\[+ 6ip(\zeta 10_010_0 + \bar{\zeta}10_010_0) + \sqrt{3}(\zeta 10_010_0 + \bar{\zeta}10_010_0)\]

\[+ 2p(\zeta 45_045_0 + \bar{\zeta}45_045_0)\]

\[(105)\]

Where every \textit{SU}(5) invariant has been normalized so that the individual \textit{G}_{123} sub-rep masses can be read off directly from the coefficient of the invariant for com-
plex SU(5) representations which pair into Dirac supermultiplets and is \(2\) times the coefficient for the real representations which remain unpaired Majorana/Chiral supermultiplets.

**Appendix C: Doublet fraction Coefficients** \(\alpha_i, \tilde{\alpha}_i\)

In this appendix we give the explicit expressions for the coefficients \(\alpha_i, \tilde{\alpha}_i\) obtained by first imposing the condition \(\text{Det} \mathcal{H} = 0\) and then solving the equations to determine the normalized left and right eigenvectors of \(\mathcal{H}\).

\[
m_0 = m_0 \lambda \quad \tilde{\sigma} = \sqrt{\frac{(1 - 3x)x(1 + x^2)}{(1 - 2x + x^2)}}
\]

\[
N = \frac{e^{-i \text{Arg}[\alpha_1]}}{\sqrt{|\hat{\alpha}_1|^2 + |\hat{\alpha}_2|^2 + |\hat{\alpha}_3|^2 + |\hat{\alpha}_4|^2 + |\hat{\alpha}_5|^2 + |\hat{\alpha}_6|^2}}
\]

\[
\tilde{N} = \frac{e^{-i \text{Arg}[\tilde{\alpha}_1]}}{\sqrt{|\hat{\tilde{\alpha}}_1|^2 + |\hat{\tilde{\alpha}}_2|^2 + |\hat{\tilde{\alpha}}_3|^2 + |\hat{\tilde{\alpha}}_4|^2 + |\hat{\tilde{\alpha}}_5|^2 + |\hat{\tilde{\alpha}}_6|^2}}
\]

\[
A = N\{\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \hat{\alpha}_4, \hat{\alpha}_5, \hat{\alpha}_6\} = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\}
\]

\[
\tilde{A} = \tilde{N}\{\hat{\tilde{\alpha}}_1, \hat{\tilde{\alpha}}_2, \hat{\tilde{\alpha}}_3, \hat{\tilde{\alpha}}_4, \hat{\tilde{\alpha}}_5, \hat{\tilde{\alpha}}_6\} = \{\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3, \tilde{\alpha}_4, \tilde{\alpha}_5, \tilde{\alpha}_6\}
\]

\[
\hat{\alpha}_1 = \tilde{\alpha}_1 = (m_o \eta^2 \lambda P_0 + \tilde{\eta}^2 \tilde{\lambda} P_1 + \tilde{m}_o \tilde{\zeta} \eta \lambda P_2 + \tilde{\zeta} \zeta \eta \lambda \rho P_3 + m_o \eta^2 \lambda \rho P_4 + \eta^2 \lambda \rho^2 P_5)
\]

\[
\hat{\alpha}_2 = (m_o \eta \lambda Q_0 + m_o \bar{\zeta} \gamma \zeta \lambda Q_1 + \bar{\gamma} m_o \zeta^2 \lambda Q_2 + k \tilde{\zeta} \zeta^2 \lambda Q_3 + \tilde{\zeta} \gamma \zeta^2 \lambda Q_4 + \bar{\gamma} \tilde{\zeta} \zeta^3 \lambda Q_5
\]

\[
+ k m_o \zeta \lambda Q_6 + \bar{\zeta} \gamma \zeta \lambda \rho Q_7 + \bar{\gamma} \zeta^2 \lambda \rho Q_8 + m_o \gamma \eta \lambda \rho Q_9 + k \zeta \zeta \lambda \rho Q_{10} + \eta \lambda \rho^2 Q_{11})
\]

\[
\hat{\alpha}_2 = (m_o \eta \lambda \bar{Q}_0 + \tilde{m}_o \tilde{\zeta} \gamma \zeta \lambda \bar{Q}_1 + \gamma \tilde{m}_o \zeta^2 \lambda \bar{Q}_2 + k \tilde{\zeta} \zeta^2 \lambda \bar{Q}_3 + \tilde{\zeta} \gamma \zeta^2 \lambda \bar{Q}_4 + \bar{\gamma} \tilde{\zeta} \zeta^3 \lambda \bar{Q}_5
\]

\[
+ k \tilde{m}_o \zeta \lambda \bar{Q}_6 + \bar{\zeta} \gamma \zeta \lambda \rho \bar{Q}_7 + \tilde{\gamma} \zeta^2 \lambda \rho \bar{Q}_8 + \tilde{m}_o \gamma \bar{\eta} \lambda \rho \bar{Q}_9 + k \zeta \gamma \lambda \rho \bar{Q}_{10} + \eta \bar{\lambda} \rho^2 \bar{Q}_{11})
\]

\[
\hat{\alpha}_3 = (\bar{\gamma} \tilde{m}_o \eta \lambda R_0 + \tilde{m}_o \tilde{\zeta} \gamma \lambda R_1 + \bar{\gamma} \tilde{m}_o \zeta^2 \lambda R_2 + k \tilde{\zeta} \zeta^2 \lambda R_3 + \tilde{\zeta} \gamma \zeta \lambda R_4 + \bar{\gamma} \tilde{\zeta} \zeta^3 \lambda R_5
\]

\[
+ k \tilde{m}_o \zeta \lambda R_6 + \tilde{\zeta} \gamma \lambda \rho R_7 + \bar{\gamma} \tilde{\zeta} \zeta \lambda \rho R_8 + \bar{\gamma} \tilde{m}_o \eta \lambda \rho R_9 + k \zeta \gamma \lambda \rho R_{10} + \eta \lambda \rho^2 \lambda R_{11})
\]
\[\hat{\alpha}_3 = (\gamma \bar{m}_0 \eta \lambda \bar{R}_0 + \bar{m}_0 \bar{\zeta}^2 \gamma \lambda \bar{R}_1 + \gamma \bar{m}_0 \bar{\zeta} \zeta \lambda \bar{R}_2 + k \bar{\zeta}^2 \zeta \lambda \bar{R}_3 + \bar{\zeta}^3 \gamma \zeta \lambda \bar{R}_4 + \gamma \bar{\zeta}^2 \zeta^2 \lambda \bar{R}_5 \\
+ k \bar{m}_0 \zeta \eta \lambda \bar{R}_6 + \bar{\zeta}^2 \gamma \lambda \rho \bar{R}_7 + \bar{\gamma} \bar{\zeta} \zeta \lambda \rho \bar{R}_8 + \gamma \bar{m}_0 \eta \lambda \rho \bar{R}_9 + k \zeta \eta \lambda \rho \bar{R}_{10} + \gamma \eta \rho^2 \lambda \bar{R}_{11})\]

\[\hat{\alpha}_4 = \sqrt{\frac{\lambda}{\eta}} (\bar{m}_0 \gamma \eta^2 S_0 + \bar{\zeta}^2 \gamma \zeta^2 S_1 + \gamma \bar{\zeta} \zeta \gamma S_2 + \bar{m}_0 \bar{\zeta} \zeta \gamma \eta S_3 + \bar{\gamma} \bar{m}_0 \zeta^2 \eta S_4 + k \bar{\zeta} \zeta^2 \eta S_5 \\
+ k \bar{m}_0 \zeta \eta^2 S_6 + \bar{\zeta} \gamma \zeta \eta \rho S_7 + \gamma \zeta^2 \eta \rho S_8 + \bar{m}_0 \zeta \eta^2 \rho S_9 + k \zeta \eta^2 \rho S_{10} + \gamma \eta^2 \rho^2 S_{11})\]

\[\hat{\alpha}_4 = \sqrt{\frac{\lambda}{\eta}} (\bar{m}_0 \gamma \eta^2 \bar{S}_0 + \bar{\zeta}^3 \gamma \zeta \bar{S}_1 + \gamma \bar{\zeta} \zeta \zeta \eta \bar{S}_2 + \bar{m}_0 \bar{\zeta} \zeta \eta \bar{S}_3 + \bar{\gamma} \bar{m}_0 \zeta^2 \eta \bar{S}_4 + k \bar{\zeta} \zeta^2 \eta \bar{S}_5 \\
+ k \bar{m}_0 \zeta \eta^2 \bar{S}_6 + \gamma \bar{\zeta} \zeta \eta \rho \bar{S}_7 + \gamma \zeta^2 \eta \rho \bar{S}_8 + \bar{m}_0 \zeta \eta^2 \rho \bar{S}_9 + k \zeta \eta^2 \rho \bar{S}_{10} + \gamma \eta^2 \rho^2 \bar{S}_{11})\]

\[\hat{\alpha}_5 = (\bar{\zeta}^2 \gamma \zeta \lambda T_0 + \gamma \bar{\zeta} \zeta \gamma \eta \lambda T_1 + \bar{m}_0 \bar{\zeta} \zeta \gamma \eta \lambda T_2 + \gamma \bar{m}_0 \zeta \eta \lambda T_3 + k \bar{\zeta} \zeta \zeta \lambda T_4 \\
+ k \bar{m}_0 \eta^2 \lambda T_5 + \bar{\zeta} \gamma \eta \lambda \rho T_6 + \bar{\gamma} \zeta \eta \lambda \rho T_7 + k \eta^2 \lambda \rho T_8)\]

\[\hat{\alpha}_5 = (\bar{\zeta}^2 \gamma \zeta \lambda T_0 + \gamma \bar{\zeta} \zeta \gamma \eta \lambda T_1 + \bar{m}_0 \bar{\zeta} \zeta \gamma \eta \lambda T_2 + \gamma \bar{m}_0 \zeta \eta \lambda T_3 + k \bar{\zeta} \zeta \zeta \lambda T_4 \\
+ k \bar{m}_0 \eta^2 \lambda T_5 + \bar{\zeta} \gamma \eta \lambda \rho T_6 + \gamma \zeta \eta \lambda \rho T_7 + k \eta^2 \lambda \rho T_8)\]

\[\hat{\alpha}_6 = (\bar{\zeta}^2 \gamma \zeta \lambda U_0 + \gamma \bar{\zeta} \zeta \gamma \eta \lambda U_2 + \bar{m}_0 \bar{\zeta} \zeta \gamma \eta \lambda U_3 + k \bar{\zeta} \zeta \zeta \lambda U_4 + k \bar{m}_0 \eta^2 \lambda U_5 \\
+ \bar{\gamma} \gamma \eta \lambda \rho U_6 + \bar{\gamma} \zeta \zeta \gamma \rho U_7 + k \eta^2 \lambda \rho U_8)\]

\[\hat{\alpha}_6 = (\bar{\zeta}^2 \gamma \zeta \lambda U_0 + \gamma \bar{\zeta} \zeta \gamma \eta \lambda U_2 + \bar{m}_0 \bar{\zeta} \zeta \gamma \eta \lambda U_3 + k \bar{\zeta} \zeta \zeta \lambda U_4 + k \bar{m}_0 \eta^2 \lambda U_5 \\
+ \bar{\gamma} \gamma \eta \lambda \rho U_6 + \gamma \zeta \eta \lambda \rho U_7 + k \eta^2 \lambda \rho U_8)\]

\[t_{(1,1)} = -1 + x\]

\[t_{(2,1)} = -3 + 4x + 3x^2\]

\[t_{(2,2)} = -1 + 5x^2\]

\[t_{(2,3)} = -1 + 2x + x^2\]

\[t_{(2,4)} = 1 - 6x + 7x^2\]

\[t_{(2,5)} = 1 - 5x + 2x^2\]

\[t_{(2,6)} = -3 + 3x + 2x^2\]
$$t_{(3,1)} = -3 + x + 5x^2 + 3x^3$$
$$t_{(3,2)} = -(1 + 3x)(1 + x^2)$$
$$t_{(3,3)} = 2 - 9x + 6x^2 + 5x^3$$
$$t_{(3,4)} = -2 + 9x - 9x^2 + 4x^3$$
$$t_{(4,1)} = 3 - 16x + 21x^2 - 6x^3 + 2x^4$$
$$t_{(4,2)} = 1 - x - 5x^2 + 5x^3 + 4x^4$$
$$t_{(4,3)} = 2 - 7x - 3x^2 + 11x^3 + 13x^4$$
$$t_{(4,4)} = 1 - 5x + 6x^2 - 5x^3 + 7x^4$$
$$t_{(5,1)} = -1 + 9x - 25x^2 + 29x^3 - 18x^4 + 14x^5$$
$$t_{(5,2)} = -2 + 8x - 3x^2 - 2x^3 - 8x^4 + x^5$$
$$t_{(5,3)} = 3 - 3x - 15x^2 - x^3 + 30x^4 + 14x^5$$
$$t_{(5,4)} = -1 - 3x + 11x^2 + 8x^3 - 12x^4 + 5x^5$$
$$t_{(5,5)} = -3 + 9x + 5x^2 - 10x^3 - 14x^4 + x^5$$
$$t_{(6,1)} = 1 + 4x - 38x^2 + 47x^3 + 10x^4 - 23x^5 + 7x^6$$
$$t_{(7,1)} = 1 - 8x + 25x^2 - 44x^3 + 64x^4 - 74x^5 + 36x^6 + 12x^7$$
$$t_{(7,2)} = 3 - 21x + 60x^2 - 114x^3 + 181x^4 - 147x^5 + 4x^6 + 18x^7$$
$$t_{(7,3)} = -1 + 5x - 3x^2 - 2x^3 - 39x^4 + 69x^5 - 37x^6 + 40x^7$$
$$t_{(7,4)} = 3 - 22x + 61x^2 - 102x^3 + 157x^4 - 142x^5 + 11x^6 + 18x^7$$
$$t_{(7,5)} = 2 + 11x - 72x^2 + 34x^3 + 123x^4 - 30x^5 - 49x^6 + 13x^7$$
$$t_{(8,1)} = 2 - 35x + 183x^2 - 398x^3 + 474x^4 - 441x^5 + 387x^6 - 170x^7 + 54x^8$$
$$t_{(8,2)} = 1 - 5x + 9x^2 - 10x^3 - 33x^4 + 195x^5 - 245x^6 + 44x^7 + 12x^8$$
$$t_{(8,3)} = 3 - 77x + 528x^2 - 1660x^3 + 2967x^4 - 3417x^5 + 2702x^6 - 1342x^7 + 360x^8$$
$$t_{(8,4)} = -1 + 19x - 115x^2 + 286x^3 - 211x^4 - 341x^5 + 767x^6 - 548x^7 + 128x^8$$
$$t_{(8,5)} = -3 + 23x - 81x^2 + 191x^3 - 307x^4 + 261x^5 - 91x^6 + 21x^7 + 18x^8$$
$$t_{(9,1)} = -1 + 12x - 39x^2 + 30x^3 + 79x^4 - 172x^5 + 63x^6 + 50x^7 - 6x^8 + 48x^9$$
$$t_{(9,2)} = 1 - 16x + 81x^2 - 174x^3 + 118x^4 + 222x^5 - 529x^6 + 50x^7 - 303x^8 + 84x^9$$
$$t_{(9,3)} = -1 + 21x - 135x^2 + 405x^3 - 599x^4 + 337x^5 + 127x^6 - 173x^7 - 128x^8 + 210x^9$$
$$t_{(9,4)} = 1 - 21x + 97x^2 - 155x^3 + 87x^4 - 29x^5 + 3x^6 + 63x^7 - 108x^8 + 30x^9$$
$$t_{(10,1)} = 5 - 51x + 219x^2 - 537x^3 + 883x^4 - 1025x^5 + 757x^6 - 287x^7 + 16x^8 + 12x^9 + 72x^{10}$$
$$t_{(10,2)} = -1 + 21x - 152x^2 + 572x^3 - 1335x^4 + 2176x^5 - 2631x^6 + 2340x^7 - 1380x^8 + 315x^9 + 171x^{10}$$
$$t_{(10,3)} = -3 + 28x - 150x^2 + 486x^3 - 810x^4 + 422x^5 + 428x^6 - 670x^7 + 605x^8 - 394x^9 + 186x^{10}$$
$$t_{(10,4)} = 1 - 16x + 96x^2 - 338x^3 + 864x^4 - 1546x^5 + 1426x^6 + 114x^7$$
\[-1377x^8 + 762x^9 + 270x^{10}\]

\[
t_{(11,1)} = -1 + 18x - 121x^2 + 482x^3 - 1294x^4 + 1958x^5 - 226x^6 - 4462x^7 + 7239x^8 - 4456x^9 + 483x^{10} + 252x^{11}
\]

\[
t_{(12,1)} = 3 - 56x + 383x^2 - 1302x^3 + 2460x^4 - 2908x^5 + 2826x^6 - 2040x^7 - 2121x^8 + 8532x^9 - 9449x^{10} + 2766x^{11} + 1674x^{12}
\]

\[
p_2 = (-1 + 2x)(1 + x)
\]

\[
p_3 = -1 + 10x - 17x^2 + 12x^3
\]

\[
p_4 = (-1 + 3x)(-1 + 5x + x^3)
\]

\[
p_5 = 1 - 7x + 21x^2 - 32x^3 + 20x^4 + 9x^5
\]

\[
P_0 = -12p_3p_5t_{(1,1)}^4
\]

\[
P_1 = 24x^3t_{(1,1)}t_{(10,1)}
\]

\[
P_2 = -24xt_{(10,2)}t_{(1,1)}^2
\]

\[
P_3 = 4xt_{(11,1)}t_{(1,1)}^2
\]

\[
P_4 = 8p_3p_5t_{(2,3)}t_{(1,1)}^3
\]

\[
P_5 = 4x^2p_3p_5t_{(1,1)}^5
\]

\[
Q_0 = 6\sqrt{3}p_3p_4t_{(1,1)}^5
\]

\[
Q_1 = 6\sqrt{3}x(-1 + 3x)t_{(8,1)}t_{(1,1)}^3
\]

\[
Q_2 = 6\sqrt{3}x^3(-1 + 3x)p_2^2t_{(3,1)}t_{(1,1)}^3
\]

\[
Q_3 = -12\sqrt{3}x^3(-1 + 3x)t_{(1,1)}t_{(9,1)}
\]

\[
Q_4 = -6\sqrt{3}x^3(-3 + 5x)t_{(3,2)}t_{(4,1)}t_{(1,1)}^2
\]

\[
Q_5 = -6\sqrt{3}x^3(-3 + 5x)p_2^2t_{(1,1)}^2t_{(3,2)}
\]

\[
Q_6 = -12\sqrt{3}x^2(-1 + 3x)p_3t_{(4,2)}t_{(1,1)}^3
\]

\[
Q_7 = -2\sqrt{3}x(-1 + 3x)t_{(9,2)}t_{(1,1)}^3
\]

\[
Q_8 = -6\sqrt{3}x^3(-1 + 3x)p_2^2t_{(2,6)}t_{(1,1)}^4
\]

\[
Q_9 = -4\sqrt{3}p_3p_4t_{(2,3)}t_{(1,1)}^4
\]

\[
Q_{10} = 4\sqrt{3}x^3(-1 + 3x)p_3t_{(1,1)}^2t_{(5,3)}
\]

\[
Q_{11} = -2\sqrt{3}x^2p_3p_4t_{(1,1)}^5
\]

\[
\bar{Q}_0 = 6\sqrt{3}p_2p_5t_{(1,1)}^5
\]

\[
\bar{Q}_1 = 12\sqrt{3}x p_2 t_{(7,1)} t_{(1,1)}^3
\]

\[
\bar{Q}_2 = -12\sqrt{3}x^2(-1 + 3x)p_2 t_{(5,2)} t_{(1,1)}^3
\]

\[
\bar{Q}_3 = 12\sqrt{3}x^3 p_2 t_{(7,2)} t_{(1,1)}^2
\]

\[
\bar{Q}_4 = -6\sqrt{3} x^3 t_{(3,2)} p_2^3 t_{(1,1)}^3
\]

\[
\bar{Q}_5 = -6\sqrt{3} x^3 p_2 t_{(3,2)} t_{(4,1)} t_{(1,1)}^2
\]

\[
\bar{Q}_6 = 24\sqrt{3}x^2 p_2 p_5 t_{(1,1)}^4
\]

\[
\bar{Q}_7 = -2\sqrt{3}x p_2 t_{(8,1)} t_{(1,1)}^3
\]

\[
\bar{Q}_8 = 2\sqrt{3}x^2(-1 + 3x)p_2 t_{(6,1)} t_{(1,1)}^3
\]

\[
\bar{Q}_9 = -4\sqrt{3}p_2 p_5 t_{(2,3)} t_{(1,1)}^4
\]

\[
\bar{Q}_{10} = -4\sqrt{3}x^2(1 + x)p_2 p_5 t_{(2,4)} t_{(1,1)}^2
\]

\[
\bar{Q}_{11} = -2\sqrt{3}x^2 p_2 p_5 t_{(1,1)}^5
\]

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\[ \begin{align*}
R_0 &= 6 \sqrt{3} p_2 p_5 t_{(1,1)}^5 \\
R_2 &= 12 \sqrt{3} x p_2 t_{(7,1)}^3 t_{(1,1)}^1 \\
R_4 &= \frac{-6 \sqrt{3}}{\eta} x^3 p_2 t_{(3,2)} t_{(4,1)}^1 t_{(1,1)}^2 \\
R_6 &= 24 \sqrt{3} x^2 p_2 p_5 t_{(1,1)}^4 \\
R_8 &= -2 \sqrt{3} x p_2 t_{(8,2)} t_{(1,1)}^3 \\
R_{10} &= -4 \sqrt{3} x^2 p_2 p_5 (1 + x) t_{(2,4)} t_{(1,1)}^2 \\
R_1 &= -12 \sqrt{3} x^2 (-1 + 3 x) p_2 t_{(5,2)} t_{(1,1)}^3 \\
R_3 &= 12 \sqrt{3} x^2 p_2 t_{(7,2)} t_{(1,1)}^2 \\
R_5 &= \frac{-6 \sqrt{3}}{\eta} x^3 p_2^3 t_{(3,2)}^2 t_{(1,1)}^2 \\
R_7 &= 2 \sqrt{3} x^2 (-1 + 3 x) p_2 t_{(6,1)} t_{(1,1)}^3 \\
R_9 &= -4 \sqrt{3} x^2 p_2 t_{(2,3)} t_{(1,1)}^1 \\
R_{11} &= -2 \sqrt{3} x^2 p_2 p_5 t_{(1,1)}^5 \\
\end{align*} \]

\[ \begin{align*}
\bar{R}_0 &= 6 \sqrt{3} p_3 p_4 t_{(1,1)}^5 \\
\bar{R}_2 &= 6 \sqrt{3} x (-1 + 3 x) t_{(8,1)} t_{(1,1)}^3 \\
\bar{R}_4 &= \frac{-6 \sqrt{3}}{\eta} x^3 (-3 + 5 x) p_2^2 t_{(1,1)}^2 t_{(3,2)} \\
\bar{R}_6 &= -12 \sqrt{3} x^2 (-1 + 3 x) p_3 t_{(4,2)} t_{(1,1)}^3 \\
\bar{R}_8 &= -2 \sqrt{3} x (-1 + 3 x) t_{(9,2)} t_{(1,1)}^3 \\
\bar{R}_{10} &= 4 \sqrt{3} x^3 (-1 + 3 x) p_3 t_{(1,1)}^2 t_{(5,3)} \\
\bar{R}_1 &= 6 \sqrt{3} x^2 (-1 + 3 x) p_2^2 t_{(3,1)} t_{(1,1)}^4 \\
\bar{R}_3 &= -12 \sqrt{3} x^3 (-1 + 3 x) t_{(1,1)} t_{(9,1)} \\
\bar{R}_5 &= \frac{-6 \sqrt{3}}{\eta} x^3 (-3 + 5 x) t_{(1,1)}^2 t_{(3,2)} t_{(1,1)} \\
\bar{R}_7 &= -6 \sqrt{3} x^3 (-1 + 3 x) p_2^2 t_{(2,6)} t_{(1,1)}^4 \\
\bar{R}_9 &= -4 \sqrt{3} p_3 p_4 t_{(2,3)} t_{(1,1)}^4 \\
\bar{R}_{11} &= -2 \sqrt{3} x^2 p_3 p_4 t_{(1,1)}^5 \\
\end{align*} \]

\[ \begin{align*}
S_0 &= -6 \sqrt{2} p_3 \hat{\sigma} t_{(3,4)} t_{(1,1)}^5 \\
S_3 &= -3 \sqrt{2} x \hat{\sigma} t_{(8,3)} t_{(1,1)}^3 \\
S_6 &= 6 \sqrt{2} x \hat{\sigma} p_3 t_{(5,1)} t_{(3,4)} t_{(1,1)}^4 \\
S_9 &= 4 \sqrt{2} p_3 \hat{\sigma} t_{(2,3)} t_{(3,4)} t_{(1,1)}^4 \\
S_{10} &= -2 \sqrt{2} x \hat{\sigma} p_3 t_{(1,1)}^2 t_{(7,3)} \\
S_{11} &= 2 \sqrt{2} x^2 \hat{\sigma} p_3 t_{(3,4)} t_{(1,1)}^5 \\
S_1 &= -18 \sqrt{2} x^3 \hat{\sigma} t_{(4,1)} t_{(1,1)}^6 \\
S_4 &= -9 \sqrt{2} x \hat{\sigma} p_2^2 t_{(2,5)} t_{(1,1)}^5 \\
S_7 &= 3 \sqrt{2} x \hat{\sigma} t_{(8,4)} t_{(1,1)}^4 \\
S_{10} &= -2 \sqrt{2} x \hat{\sigma} p_3 t_{(1,1)}^2 t_{(7,3)} \\
S_{11} &= 2 \sqrt{2} x^2 \hat{\sigma} p_3 t_{(3,4)} t_{(1,1)}^5 \\
S_2 &= -18 \sqrt{2} x^3 \hat{\sigma} p_2^2 t_{(1,1)}^6 \\
S_5 &= 12 \sqrt{2} x^3 \hat{\sigma} t_{(9,3)} t_{(1,1)} \\
S_8 &= 3 \sqrt{2} x \hat{\sigma} p_2^2 t_{(4,4)} t_{(1,1)}^4 \\
S_{10} &= -2 \sqrt{2} x \hat{\sigma} p_3 t_{(1,1)}^2 t_{(7,3)} \\
S_{11} &= 2 \sqrt{2} x^2 \hat{\sigma} p_3 t_{(3,4)} t_{(1,1)}^5 \\
\end{align*} \]
\begin{align*}
T_0 &= 6x^2 (-1 + 3x) t_{(1,1)} t_{(10,3)} \\
T_2 &= 6x (-1 + 3x) p_3 t_{(5,4)} t_{(1,1)}^3 \\
T_4 &= -12x^2 t_{(12,1)} \\
T_6 &= -2x (-1 + 3x) p_3 t_{(1,1)}^2 t_{(7,5)} \\
T_8 &= 4xp_3 p_5 t_{(1,1)} t_{(4,3)} \\

T_1 &= 18x^2 p_2 t_{(7,4)} t_{(1,1)}^3 \\
T_3 &= 18xp_2 p_5 t_{(1,1)}^5 \\
T_5 &= -12xp_3 p_5 t_{(1,1)}^2 t_{(2,2)} \\
T_7 &= -6xp_2 p_5 t_{(3,3)} t_{(1,1)}^3 \\
T_{\bar{0}} &= 18x^2 p_2 t_{(7,4)} t_{(1,1)}^3 \\
T_{\bar{2}} &= 18xp_2 p_5 t_{(1,1)}^5 \\
T_{\bar{4}} &= -12x^2 t_{(12,1)} \\
T_{\bar{6}} &= -6xp_2 p_5 t_{(3,3)} t_{(1,1)}^3 \\
T_{\bar{8}} &= 4xp_3 p_5 t_{(1,1)} t_{(4,3)} \\

U_0 &= 6\sqrt{3}ix^2 (-1 + 3x) t_{(9,4)} t_{(1,1)}^2 \\
U_2 &= -6\sqrt{3}ix (-1 + 3x) p_3 t_{(5,5)} t_{(1,1)}^3 \\
U_4 &= -12\sqrt{3}ix^2 t_{(10,4)} t_{(1,1)}^2 \\
U_6 &= 2\sqrt{3}ix^2 (-1 + 3x) p_3 t_{(5,4)} t_{(1,1)}^3 \\
U_8 &= -4\sqrt{3}ix^2 p_3 p_5 t_{(2,2)} t_{(1,1)}^2 \\

U_{\bar{0}} &= -6\sqrt{3}ix^2 p_2 t_{(8,5)} t_{(1,1)}^2 \\
U_{\bar{2}} &= 6\sqrt{3}ix p_2 p_5 t_{(2,1)} t_{(1,1)}^3 \\
U_{\bar{4}} &= 12\sqrt{3}ix^2 t_{(10,4)} t_{(1,1)}^2 \\
U_{\bar{6}} &= -6\sqrt{3}ix^2 p_2 p_5 t_{(1,1)}^5 \\
U_{\bar{8}} &= 4\sqrt{3}ix^2 p_3 p_5 t_{(2,2)} t_{(1,1)}^2 \\

U_1 &= 6\sqrt{3}ix^2 p_2 t_{(8,5)} t_{(1,1)}^2 \\
U_3 &= -6\sqrt{3}ix p_2 p_5 t_{(2,1)} t_{(1,1)}^3 \\
U_5 &= 12\sqrt{3}ix p_3 p_5 t_{(1,1)}^4 \\
U_7 &= 6\sqrt{3}ix^2 p_2 p_5 t_{(1,1)}^5 \\
U_{\bar{1}} &= -6\sqrt{3}ix^2 (-1 + 3x) t_{(9,4)} t_{(1,1)}^2 \\
U_{\bar{3}} &= 6\sqrt{3}ix (-1 + 3x) p_3 t_{(5,5)} t_{(1,1)}^3 \\
U_{\bar{5}} &= -12\sqrt{3}ix p_3 p_5 t_{(1,1)}^4 \\
U_{\bar{7}} &= -2\sqrt{3}ix^2 (-1 + 3x) p_3 t_{(5,4)} t_{(1,1)}^3 \\

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