Motion of a falling object

Kalimullah, Madhur Garg, P. Arun
Department of Physics & Electronics, S.G.T.B. Khalsa College,
University of Delhi, Delhi - 110 007, India

&
F.M.S. Lima
Instituto de Fisica
Universidade de Brasilia
P.O. Box 04455
70919-970 Brasilia-DF
Brazil.

Abstract

A simple setup was assembled to study the motion of an object while it falls. The setup was used to determine the instantaneous velocity, terminal velocity and acceleration due to gravity. Also, since the whole project was done within $\leq 20 it can easily be popularized.

1 Introduction

One of the first topics students of physics learn is the concept of bodies falling towards the center of the earth with an acceleration (g). This acceleration due to gravity is usually determined in undergraduate laboratories by measuring the oscillation time period of a simple pendulum. Some students do wonder as to why acceleration due to gravity is not just determined by measuring the time taken by a freely falling body to fall through a given height. Trivial calculation using the equation

\[ s = ut + \frac{1}{2}gt^2 \]  

shows that the body has to drop across an appreciable height to increase the body’s time of flight. This is required to minimize the error in measurement of the fall’s time. Historically this would explain why Galileo did his experiment from the Tower of Pisa. Over the last three decades various contraptions have been designed to measure the time of flight in laboratory conditions i.e. drop through limited height [1]-[9]. This has been made possible by the advent of cheap and easily available electronic circuitry capable of measuring accurately in milliseconds. In most of these experiments, the body is released by either a lever or electromagnet. The position of the release mechanism would be taken as s=0 and the initial velocity on release would obviously be zero. A second sensor kept at a distance one meter below the release point (s=0) would stop the timing circuit when the body crosses it. The acceleration due to gravity is calculated using eqn(1). However, this scheme introduces an error in time ‘t’ due to the uncertainty in the time of release. In Lindemuth’s [1] work, error was introduced due to the body having an initial velocity
by the time it arrived to the first sensor that initiated the clock. In all schemes discussed, there was an error either in the measurement of time of fall or knowledge of initial velocity. Also, these experiments rely on the a prior knowledge/validity of eqn(1). To overcome this problem, a method devised in use is to allow a metallic ball to drop in between two parallel wires in which high tension current flows in pulses. Sparks generated by the traveling ball is recorded on a photographic strip. In another method two test masses to required to determine acceleration due to gravity [1]. These techniques either require high (unsafe) voltages or the contraptions require a certain amount of skill in designing which would not be easily available in standard under-graduate colleges of developing countries.

We have designed a simple setup to measure the acceleration due to gravity. It was designed keeping in mind minimum skill should be required to set it up and use it as also the validity of eqn(1) would not be taken for granted. In the following sections we elaborate our setup designed by using LEDs, photodiodes, op amps and microprocessor and comment on the results we obtained.

## 2 Apparatus

A hollow tube (which we shall hence refer to as the "drop tower") of length 1 meter housed eight LEDs (Light Emitting Diodes) and eight LDRs (Light Dependent Resistances). They were arranged along the length of the drop tower facing each other (see fig 1). The distance between two neighboring LEDs (and in turn LDRs) was 10cm. The first pair of LED-LDR was kept at a distance 10cm from the top edge of the drop tower. This was to make sure that stray light from the room did not lead to false triggering of the circuitry.

As the name suggests, LDRs are light light sensitive devices and hence are popularly used as optical sensors. Their resistance depends on the amount of light falling on it. When light is incident on the LDR, its resistance is low. Thus, when a falling object cuts the lights path, no light falls on the LDR and its resistance increases. This variation in resistance when converted into voltage can be (wave-)shaped into pulses which then can be fed to a computer or microprocessor for identifying and storing the information as to the instant the object passed the LDR. The required circuitry used for wave-shaping is shown in fig(2). The LDR resistance is converted into a corresponding voltage using a voltage divider circuit. The voltage across the LDR, $R_1$, is high ($V_H$) when light falls on the LDR since its resistance diminishes. Conversely, if the LDR is in shadow region its resistance is high and the voltage across $R_1$ goes low ($V_L$). This voltage is given to the op amp

![Figure 1: Setup used to measure the time of fall of a falling object.](image)
comparator circuit. The variable voltage is set at a value between $V_L$ and $V_H$. This ensures that the output of the LM741 op amp swings to $+12\text{v}$ when light falls on the LDR and to $-12\text{v}$ when shadow falls on it. This voltage is made TTL/microprocessor compatible (vary between 0 and $+5\text{v}$) using a 4.7v zener diode.

Figure 2: Wave-shaping circuit used to convert variation of LDR’s resistance into square pulses. The DC supplies used were $V_1 = 5\text{v}$ and $V_2 = 12\text{v}$ and the resistances in the circuit were $R_1 = 3.3K\Omega$, $R_2 = 10K\Omega$ and $R_3 = 1K\Omega$. Diodes used were the usual 1N4001 rectification diodes.

Now we explain the final block of our setup which is a IC8085 microprocessor training kit. To follow in detail this section requires an understanding of IC8085 and can be acquired from Gaonkar’s book. A standard 8085 microprocessor training kit comes with a programmable input/output device (IC8155) which has as many as twenty two input pins (divided into Port A with 8 pins, Port B with 8 pins and Port C with 6 pins). The output of each wave-shaping circuit can be fed to one of these pins. We have only used eight pins of Port A. This was done keeping the following in mind

(i) Optimize the amount of hardware (wave-shaping circuits),

(ii) eight data points would be reasonably enough to fit a curve reliability and

(iii) the circuit then could easily be extended for interfacing with a computer since the computers parallel port can only input eight bits.
When no object cuts the light path, all the pins of Port A (of IC8155) are high and informs the microprocessor by sending the word \( FF_H \) (the subscript 'H' means that it is the hexa representation of binary number 1111 1111). This word changes when a object cuts the light’s path. For example on cutting the first LED-light’s path the word sent to the microprocessor is 1111 1110 \( (FE_H) \) and when the object cuts the second LED-light’s path the microprocessor receives the word 1111 1101 \( (FD_H) \) and so forth as the object continuous to fall. The microprocessor is programmed to run a counter (clock) from the negative edge of one input signal to that of the second (see fig 3). On receipt of the second input signal’s negative edge, the microprocessor stores the counter’s count in it’s memory and restarts the counter. This contributes in an error (time measured would be lower than actual) in the measurement of the time of flight between the next two pair of LED-LDR. The counts stored in the memory is converted to time in seconds using the formula

\[
\begin{align*}
t_1 &= (30n + 34) \times 0.326\mu s \\
t_2 \text{ to } t_8 &= (30n + 41) \times 0.326\mu s
\end{align*}
\]

Notice both the wave-shaping circuit and the software are designed to minimize any delay in response by the cadmium sulphide LDR used. The assembly language instruction set used for the project is listed at the end.

### 3 Results and Discussions

The drop tower was kept vertical and attempt was made to make sure that the objects would fall in a straight line. A plumb line was used to ascertain the alignment. For conducting this experiment we used four different balls whose physical features, dimensions and mass are listed in Table 1.

The balls were dropped from the top edge of the drop tower. We made sure that the ball in use would cut all eight light beams. As soon as the ball crosses the first LED, the microprocessor starts it’s clock. This point would act as the co-ordinates \((0,0)\). The microprocessor stores the counter’s count as the ball passes the second LED (the count in turn would be used to calculate time). This gives the co-ordinates \((t_{21}, 0.1)\) where the subscript ‘21’ indicates this as the time...
taken for the fall between the first and second LED. Since, the LEDs are 10cm apart, in meters we have $s=0.1\text{cm}$. In this manner, eight data points are generated and stored in the microprocessor. The experiment was repeated as many as 200 times with each ball.

Fig. 4 shows two curves each for the results obtained with Ball 1 and Ball 2. As can be seen the data lie on a parabola. However, before fitting a second polynomial that would represent eq. (1), one has to remember that eq. (1) represents a object falling freely under the action of gravity. No other forces should be acting on it. However, there are two possible forces acting in the opposite direction to gravity when the object is dropped, namely forces of buoyancy and air resistance. The contribution due to air resistance is proportional to the square of the velocity with which the object is moving. The general equation of motion in this case is given as

$$m \frac{d^2s}{dt^2} = (m - m_{\text{air}})g - kv^2$$

where $m_{\text{air}}$ is the mass of the air displaced by the ball and ‘$k$’ is the proportionality constant. Since, the dimensions of the balls are small, the volume and in turn the mass of air displaced is very small. Hence, contributions of the buoyancy force can be neglected. Hence, the equation of motion reduces to

$$m \frac{d^2s}{dt^2} = mg - kv^2 \quad (2)$$

| Ball Name | Description       | Mass (grams) | Radius (cm) |
|-----------|-------------------|--------------|-------------|
| Ball 1    | Smooth Rubber Ball| 19.00        | 1.4         |
| Ball 2    | Ping Pong Ball    | 1.75         | 1.9         |
| Ball 3    | Nylon Ball        | 5.50         | 1.9         |
Before proceeding, for completeness it should be stated here that the force acting as a resistance on the falling body can either be proportional to its velocity (given by Navier-Stokes equation, $F = 6\pi \mu rv$) or the square of the velocity. This depends if motion of the viscous medium around the falling body remains laminar or not and is denoted by the Reynolds number $(R)$

$$R = \frac{d \rho v}{\mu}$$

where this number characterises the media (viscosity, $\mu$ and density, $\rho$) through which the body (of diameter, $d$) falls with velocity, $v$. In a very viscous media ($\mu$ large), the body would face large resistance and would fall slowly ($v$ small). Thus the Reynolds number would be less than one. Under such conditions the Navier-Stokes equation should be considered. However, as in our case, the body is falling through (dry) air, whose viscosity is small (around $1.9 \times 10^{-5}$kg/m.sec) with density $1.225$kg/m$^3$. The Reynolds number works out be $\approx 3000$, i.e. $R \gg 1$. The drag force acting on the body is then given as

$$F = \frac{1}{2} \rho AC_D v^2$$

or $k = \frac{1}{2}C_D \rho A$, where 'A' is the area of cross-section and $C_D$ is the drag coefficient. The drag coefficient depends on the shape, surface characteristics and the Reynolds number $[10]$. The value of $C_D$ for smooth spheres as a function of Reynolds number has been listed in Landau and Lifshitz book $[11]$. For $R=3000$, $C_D \approx 0.5$. For the radii of the balls used in our experiment (see Table 1), $k$ works out to be $\approx 3.8 \times 10^{-4}$(Ns$^2$/m$^2$). However, we shall not use this value immediately but at the end compare our experimentally determined values with this theoretical value.

The solution of eqn(2) and its power approximation has been detailed by Lindemuth and is given as

$$s = \left(\frac{v_T^2}{g}\right) \ln \left[ \cosh \frac{gt}{v_T} + \frac{u}{v_T} \sinh \frac{gt}{v_T} \right]$$

$$\approx ut + \frac{1}{2} g \left(1 - \frac{u^2}{v_T^2}\right) t^2 - \frac{g^2 u}{3v_T} \left(1 - \frac{u^2}{v_T^2}\right) t^3$$

where

$$v_T = \sqrt{\frac{mg}{k}}$$

is the terminal velocity the falling body would achieve when it moves with no acceleration, i.e. net force acting on it is zero in the case of free fall else when all other higher terms cancel out. The coefficient of $t^3$ represents the rate of change in acceleration (dg/dt).

Thus, under the conditions in which this experiment was done, the data points were fit with a third order polynomial (eqn 3). Curves were fit to these data points using standard and freely available software "Curxpt v3.0".
All the curves for the 200 odd experiments done with each ball lie within the two curves shown in fig. 4. The coefficient of $t$, $t^2$ and $t^3$ as returned by the software was used to determine the ball’s velocity ($u_1$) as it crosses the first sensor where the ball has traveled through 10+cm (10cm from the drop tower’s top edge to the first sensor and the unknown height from the edge from where the ball was dropped) and it’s acceleration ($g$). Results for the graphs shown in fig. 4 are listed in Table 2. The error in ‘$g$’ is 1 in part of $10^3$ with as small as $+7\mu$sec (max) error in the measurement of time of flight. The acceleration due to gravity in Delhi can be calculated using the equation

$$g = 9.780327[1 + 0.0053024\sin^2L - 0.000058\sin^22L]$$

(4)

where $0.499455 (28^o37'N)$ is the latitude (L) of Delhi in radian. This gives the acceleration due to gravity in Delhi as $9.7918\text{m/s}^2$. Our experimental value of ‘$g$’ is in good agreement with the theoretical value. Notice that the velocity ($u_1$) is different in each case (see Table 2). This is expected since even with the best of care taken, the balls would have been dropped from different heights (position from the drop tower’s edge).

The power series of eqn. 3 suggests that a knowledge of the coefficients should also enable us to estimate the objects terminal velocity. However, the power approximation leads to the situation that if the body’s initial velocity is zero then the terms associated with terminal velocity also collapses. Also, curve fitting the original solution relating distance to time proves tedious due to the existence of logarithm and cosine hyperbolic terms. However, the functional relation between velocity and time for a body falling through a viscous medium is given as [4]

$$v = v_T\tanh\left[\frac{g}{v_T}t + \tanh^{-1}\left(\frac{u}{v_T}\right)\right]$$

When the body’s initial velocity is zero, the above expression reduces to

$$v = v_T\tanh\left[\frac{g}{v_T}t\right]$$

(5)

One can either use the coefficients ‘$g$’ and ‘$dg/dt$’ in

$$v = gt - 3\frac{dg}{dt}t^2$$
Table 3.

| Ball Name | $v_T$ (m/s) | t (sec) | s (m) | $k = \frac{mg}{v_T^2} \times 10^{-4} (\text{Ns}^2/\text{m}^2)$ |
|-----------|-------------|---------|-------|--------------------------------------------------|
| Ball 1    | 35.11       | 3.58    | 62.9  | 1.27                                             |
| II        | 35.11       | 3.58    | 62.9  | 1.27                                             |
| Ball 2    | 6.32        | 0.58    | 1.77  | 4.3                                              |
| II        | 5.89        | 0.53    | 1.52  | 4.92                                             |
| Ball 3    | 7.97        | 0.74    | 2.87  | 8.47                                             |
| II        | 9.97        | 0.94    | 4.6   | 5.4                                              |

To generate data for curve fitting eqn (5) or find the gradient along the curves shown in fig(4). Both methods would introduce an error abid small since the fitting as represented by co-relation factor (see Table 2) was very good. The terminal velocities thus evaluated is listed in Table 3. Before concluding, it would be interesting to compute the time a body would take to attain terminal velocity and the distance through which it would fall to attain this velocity. For this one would have to solve the quadratic equation

$$v_T = gt - \frac{3}{2} \frac{dg}{dt} t^2$$

and substitute the time taken thus obtained in the equation

$$s = \frac{1}{2} gt^2 - \frac{dg}{dt} t^3$$

The data of Table 3 indicates that the terminal velocity, the time in which the object attains this velocity and the distance through which it has to travel for attaining the same all strongly depends on the objects mass. The drag coefficient also depends on the surface area (and hence diameter) hence comparison was sort to be done on balls with similar diameter and different mass. Obtaining balls of different mass and similar dimensions was not easy. However, the designed experimental setup proved handy in estimating the acceleration due to gravity. Above which, it also proved to be a rich source of information on the motion of an object while falling. Providing ample scope of and experience in data analysis, the setup gives an idea of the terminal velocity, time taken to achieve it and the distance through which the body has to travel before attaining it. All this is possible even though the length of the drop tower is less than the distance through which object has to fall to attain $v_T$. On a final note it must be mentioned that this project was done in a graduate college laboratory, with a small budget of $20.

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Program required.

| Mem. add. | Hex Code | Instruction          |
|-----------|----------|----------------------|
| C100      | 31       | LXI SP               |
| C101      | 00       |                      |
| C102      | C6       | C6                   |
| C103      | 21       | LXI H                |
| C104      | 00       | 00                   |
| C105      | C0       | C0                   |
| C106      | 3E       | MVI A                |
| C107      | 00       | 00                   |
| C108      | D3       | OUT 08 (CR)          |
| C109      | 08       | 08                   |
| C10A      | DB       | IN 09 (PA)           |
| C10B      | 09       | 09                   |
| C10C      | A6       | ANA M                |
| C10D      | C2       | C2                   |
| C10E      | 0A       | 0A                   |
| C10F      | C1       | C1                   |
| C110      | 23       | INX H                |
| C111      | 11       | LXI D (initiate counter) |
| C112      | 00       | 00                   |
| C113      | 00       | 00                   |
| C114      | 13       | INX D                |
| C115      | DB       | IN 09 (PA)           |
| C116      | 09       | 09                   |
| C117      | A6       | ANA M                |
| C118      | C2       | JNZ                  |
| C119      | 11       | 11                   |
| C11A      | C1       | C1                   |
| C11B      | 23       | PUSH D               |
| C11C      | D5       | INX H                |
| C11D      | C3       | JMP                  |
| C11E      | 11       | 11                   |
| C11F      | C1       | C1                   |
Data used by main program.

| Mem. add. | Data |
|-----------|------|
| C000      | 01   |
| C001      | 02   |
| C002      | 04   |
| C003      | 08   |
| C004      | 10   |
| C005      | 20   |
| C006      | 30   |
| C007      | 80   |