Cooperative Cognitive Relaying Protocol for an Energy Harvesting Cognitive Radio User

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Abstract—In this paper, we investigate the maximum secondary throughput of a saturated rechargeable secondary terminal sharing the spectrum with a primary terminal. The secondary transmitter (ST) harvests energy packets from the environment with a certain harvesting rate. All transmitters are assumed to have data buffers. In addition to its own traffic buffer, the ST has a buffer for storing the admitted primary packets for relaying; and a buffer for storing the energy packets harvested from the environment. We propose a new cooperative cognitive relaying protocol that allows the ST to relay a fraction of the undelivered primary packets. We consider an interference channel model (or a multipacket reception (MPR) channel model), where concurrent transmissions can survive with certain probability characterized by the complement of channel outages. The proposed protocol exploits the primary queue burstiness and receivers' MPR capabilities. In addition, it efficiently expends the secondary energy packets. We derive formulas of misdetection and false alarm probabilities for the proposed cognitive setting. Our numerical results show the benefits of the cooperation, the receivers’ MPR capabilities, and the secondary energy queue arrival rate on the system performance from network layer standpoint.

Index Terms—Cognitive radio, relaying, protocol design, cooperation, throughput analysis, queue stability.

I. INTRODUCTION

Secondary utilization of a licensed system can efficiently enhance the electromagnetic spectrum usage. Secondary users (SUs) can use the spectrum under certain quality of service for the primary users (PUs). High performance wireless communication networks relies, among other technologies, on cooperative communications.

In many practical situations and applications, the secondary transmitter (ST) is a rechargeable device. The secondary operation, which involves spectrum sensing and access, is accompanied by energy consumption. Consequently, an energy-aware ST must optimize its sensing and access decisions to efficiently invest the available energy.

Energy harvesting technology is an emerging technology for energy-constrained terminals which allows the transmitter to collect (harvest) energy from its environment. For a comprehensive overview of the different energy harvesting technologies, the reader is referred to [1] and the references therein.

Data transmission by an energy harvester with a rechargeable battery has got a lot of attention recently [2]–[10]. In [2], the optimal online policy for controlling admissions into the data buffer is derived using a dynamic programming framework. In [3], energy management policies which stabilize the data queue are proposed for single-user communication and some delay-optimal properties are derived. In [4], the optimality of a variant of the back-pressure algorithm using energy queues is shown.

In a cognitive setting, the authors of [5] considered the scenario two different priority nodes share a common channel. The higher priority user (PU) has a rechargeable battery, whereas the lower priority user (SU) is plugged to a reliable power supply and therefore has energy each time slot without limitations. In [6], the authors investigated a cognitive setting with one PU and one rechargeable SU. The ST randomly accesses and senses the primary channel and can possibly leverage primary feedback. Receivers are capable of decoding under interference as they have multipacket reception (MPR) capabilities. The authors investigated the maximum secondary throughput under stability and delay constraints on the primary queue. In [7], the ST randomly accesses the channel at the beginning of the time slot to exploit the MPR capability of receivers. The ST aims at maximizing its throughput under stability and queueing delay constraints on the primary queue. In [8], El Shafie et al. investigated the maximum stable throughput of an energy harvesting ST under stability of an energy harvesting primary transmitter (PT). The SU selects a sensing duration each time slot from a predefined set such that its stable throughput is maximized under the stability of the primary queue.

Cooperative cognitive relaying, where cooperation among primary and secondary nodes, has got extensive attention recently [9]–[14]. In [11], Sadek et al. proposed cognitive protocols for a multiple access system with a single relay aids the transmitting nodes transmission. The proposed cooperative protocols enable the relaying node to aid the transmitters operating in a time-division multiple access network in their silent periods due to source burstiness. The secondary throughput of the proposed protocol as well as the delay of symmetric nodes were investigated. The authors of [12] investigated a network composing of one primary transmitter-receiver pair and one secondary transmitter-receiver pair. The cognitive radio transmitter aims at maximizing its throughput via optimizing its transmit power such that the primary and the relaying queues are kept stable. In [13], the authors considered a cognitive setting with one PU and one SU. The relaying process is managed using an admitting coefficient which controls the flow of arrivals to the relaying queue. A priority of transmission is given to the relaying packets over secondary own packets. The authors of [14] proposed a cluster
of SUs helping the PT with a single relaying queue accessible by all the SUs.

Emerging cooperative communications and energy harvesting technologies has been considered in [9], [10]. In [9], the authors investigate the effects of network layer cooperation in a wireless three-node network with energy harvesting nodes and bursty data traffic. The authors derived the maximum stable throughput of the source as well as the required transmitted power for both a non-cooperative and an orthogonal decode-and-forward cooperative schemes. In [10], the authors studied the impact of the energy queue on the maximum stable throughput of an energy harvesting SU utilizes the spectrum whenever the primary queue being empty and capable of relaying the undelivered primary packets. The authors assumed an energy packet consumption in data decoding and data transmission. Inner and outer bounds on the secondary throughput were proposed.

In this work, we investigate the maximum secondary throughput for an energy harvesting ST in presence of a PT. In contrast to [9], [10], in this paper, we consider a generalized MPR channel model and propose a new cooperative protocol which exploit the MPR capabilities of the receivers. In addition, we assume a finite capacity energy queue. In the proposed cooperative cognitive relaying protocol, the ST cooperatively relays a certain fraction of undelivered primary packets when primary destination fails in decoding them. The flow of the primary packet at the ST is controlled using some designable parameters which depend on the channels quality and other queues state. The proposed cooperative cognitive relaying protocol allows the ST to transmit simultaneously with the PT at a fraction of the time slots to exploit the MPR capabilities of the receiving nodes. If the ST has energy packets, it may access the channel at the beginning of the time slot or decides to receive the primary packet. After $\tau$ seconds relative to the beginning of the time slot, which we refer to as decision duration, the ST uses the gathered primary samples to declare the state of the PT based on their energy and then the ST decides whether to resume decoding the primary packet or access the channel. The decision duration is designed to render the detection errors probabilities of the PT negligible. If the PT is inactive, the ST accesses with probability one, whereas if the PT is active, the ST may access simultaneously with the PT with a certain probability. If in a given time slot the secondary energy queue is empty at the beginning of the time slot, the operation of the ST becomes a decision on receiving the primary packet or not. In all cases (with and without the availability of secondary energy packets), when the PT is active, if the primary destination fails in decoding it, the ST decides at the end of the time slot whether to accept or reject the correctly decoded primary packet. The proposed protocol is simple and doesn’t require continuous estimation of the channel state information (CSI) at the transmitting terminals.

The contributions of this paper can be summarized as follows:

- We assume a finite buffer energy queue, and investigate the impact of buffer size on the secondary throughput.
- We investigate the impact of the MPR capabilities of the receivers and the secondary energy queue on the secondary throughput. To make the characterization of the secondary throughput feasible, we consider three approximated systems: two of them are inner bounds for the original system, whereas the third is an outer bound.

This paper is structured as follows: Next we describe the system model adopted in this paper. We explain the proposed cooperative cognitive relaying protocol and provide the analysis of the queues rates and the problem formulation in Section III. In Section IV we provide some numerical results. The conclusions are drawn in Section V.

II. SYSTEM MODEL

We consider a simple configuration comprising of one rechargeable secondary transmitter ‘s’, one secondary destination ‘$d_s$’, one primary transmitter ‘$p$’ and one primary destination ‘$d_p$’. The primary transmitter-receiver pair operates over slotted channels. Time is slotted and a slot is $T$ seconds in length. Each transmitter has an infinite buffer (queue) to store its own incoming fixed-length data packets, denoted by $Q_p$, $Q_e$, $Q_s$, and an infinite capacity relaying queue to store the accepted primary packets for relaying. Let $Q_1$ denote the secondary relaying queue and $Q_e$ denote the secondary energy queue with mean arrival rate $0 \leq \lambda_e \leq 1$ packets/slot and $0 \leq \lambda_e \leq 1$ energy packets/slot, respectively. The secondary queue is assumed to be saturated (always backlogged). Arrivals at queues $Q_p$ and $Q_e$ are assumed to be Bernoulli random variables [15], [16], with means $\lambda_p \in [0,1]$ packets/slot and $\lambda_e \in [0,1]$ energy packets/slot, respectively. The arrivals at each queue are assumed to be independent and identically distributed (i.i.d.). The Bernoulli model is simple, but it captures the random availability of ambient energy sources. More importantly, in the analysis of discrete-time queues, Bernoulli arrivals see time averages (BASTA), which is equivalent to the Poisson arrivals see time averages (PASTA) property in continuous-time systems [17]. Arrivals are also independent from queue to queue. All data packets are of size $B$ bits. The energy queue has energy packets each of $e$ energy units. The primary and secondary queues and links are shown in Fig. 2. For similar assumptions of infinite size of data buffers and modeling the arrivals of data and energy queues as Bernoulli arrivals, the reader is referred to [15]-[17] and the references therein.

The proposed cooperative cognitive relaying protocol and the theoretical development in this work can be readily generalized to networks with more than one PU and more than one cognitive radio user, where several PUs may choose one or more cognitive radio users or the best cognitive radio user for cooperation.\footnote{The considered network can be seen as part of a larger network with multiple primary nodes assigned to orthogonal frequency bands or different time slots via employing frequency division multiple-access or time division multiple-access, respectively.}
All wireless links exhibit a stationary non-selective Rayleigh block fading, which means that the instantaneous gain of link \( j \to k \), connecting nodes \( j \) and \( k \), remains constant during a given time slot \( T \in \{1, 2, 3, \ldots, T\} \) with value \( \zeta_{jk}^2 \), but changes independently from one slot to another according to a circularly symmetric complex Gaussian distribution with zero mean and variance \( \sigma_{jk}^2 \). Received signals are corrupted by additive white Gaussian noise (AWGN) with zero mean with variance \( N_0 \) Watts. Hereinafter, we omit the time superscript of the symbols. Let \( \zeta_{jk} = |\zeta_{jk}|^2 \) denote the fading coefficient of link \( j \to k \). We do not assume the availability of CSI at the transmitters. Since the PT transmits at the beginning of the time slot over the whole slot duration if its queue is nonempty, the spectral efficiency of the primary terminal is \( R_{p} = B/(TW) \) bits/sec/Hz, where \( W \) is the channel bandwidth. The cognitive radio user may transmit either at the beginning of the time slot or after \( \tau \) seconds relative to the beginning of the time slot. Hence, the secondary transmission time is \( T_s^{(i)} = T - i\tau \), where \( i = 0 \) if the ST transmits at \( t = 0 \), and \( i = 1 \) if the ST transmits at \( t = \tau \). The spectral efficiency of the secondary transmission is either \( R_s^{(0)} = B/(TW) \) bits/sec/Hz or \( R_s^{(1)} = B/((T - \tau)W) \) bits/sec/Hz for \( i = 0 \) and \( i = 1 \), respectively. Note that the decision duration, \( \tau \), should be long enough to justify the perfect detection of the primary state assumption, as discussed in the next Section. The PT transmits data with a fixed power \( P_p \) Watts, whereas the ST transmits with power \( P_s^{(i)} = e/T_i \) Watts, \( i \in \{0, 1\} \). The secondary transmit power is a function of the time instant in which the ST starts data transmission within the time slot. Outage of a link occurs when the instantaneous capacity of that link is lower than the transmitted spectral efficiency rate \( 6, 7, 9 \).

Assume that node \( j \) transmits a packet to node \( k \) and at the same time node \( \nu \) transmits to its respective receiver. Due to the broadcast nature of the wireless communication channel, the signal transmitted by node \( \nu \) arrives at node \( k \) and causes interference with the signal transmitted by node \( j \). Let us assume that node \( j \) starts transmission at \( t = i\tau \), whereas node \( \nu \) starts transmission at \( t = n\tau \), where \( i, n \in \{0, 1\} \). Under this setting, the probability that a transmitted packet by node \( j \) being successfully received at node \( k \) is \( P_{jk, \text{in}}^{(c)} = 1 - P_{jk, \text{in}}^{(c)} \) (see Appendix A for the exact expression). If transmitter \( j \) sends its packet alone (without interference) to node \( k \), and starts transmission at \( t = i\tau \), the probability of that packet being successfully decoded at \( k \) is \( P_{jk, \text{in}}^{(c)} \). The physical layer is explained with details in Appendix A.

At the far end of each time slot, a feedback acknowledgement/negative-acknowledgement (ACK/NACK) signal is sent from the destination to inform the respective transmitter about the decodability status of its packet. The feedback message is overheard by all nodes in the network due to the wireless channel broadcast nature. Decoding error of the feedback messages at the transmitters is negligible, which is reasonable for short length packets as low rate and strong codes can be employed in the feedback channel [11]. If a packet is received correctly at its destination, it is then removed from the respective transmitter’s queue.

For the primary packets, if the primary destination can decode the transmitted packet, it sends an ACK and both the PT and the ST drops that packet. If the ST can decode or decides to reject the primary packet and the primary destination fails in decoding the packet, the ST retransmits that packet at the following time slot. We note that the feedback signals sent by the ST and the primary destination are separated either in time or frequency. A fundamental performance measure of a communication network is the stability of the queues. Stability can be defined rigorously as follows: For every queue represented by an irreducible and aperiodic Markov chain with countable number of states, the chain and its associated queue are called stable if and only if there is a positive probability for the queue, represented by the chain, to become empty. Denote by \( Q^{(i)} \) the length of queue \( Q \) at the beginning of time slot \( t \). Queue \( Q^{(i)} \) is said to be stable if \( \lim_{k \to \infty} \lim_{t \to \infty} \Pr\{Q^{(i)} < \kappa\} = 1 \) [11], where \( \Pr\{\cdot\} \) denotes the probability of the argument. In a multiqueue system, the system is stable when all queues are stable. We can apply the following theorem to check the stability of a queue [11]. Loynes theorem: if the arrival process and the service process of a queue are strictly stationary, and the average arrival rate is greater than the average service rate
of the queue, then the queue is unstable. If the average arrival rate is lower than the average service rate, then the queue is stable \( \Phi \).

III. PROPOSED COOPERATIVE COGNITIVE RELAYING PROTOCOL

In this section, we analyze the proposed cooperative cognitive relaying protocol, denoted by \( S \). The time slot structure is shown in Fig. 1. The operation of the ST can be summarized as follows. At the beginning of the time slot, if the secondary energy queue is nonempty, the ST accesses using its own packets with probability \( f \) or decide to access the channel using one of its queues with probability \( 1-f \), where \( \phi = 1-\phi \). Accessing the channel at the beginning of the time slot is motivated by the following facts. First of all, it may be the case that using the whole time in data transmission provides better throughput than wasting \( \tau \) seconds for channel sensing, specially at low primary arrival rate as the PT will be inactive most of the time slots. Recall that the probability of outage of certain link decreases with the total time used in data transmission. This fact is discussed and proved in Appendix A. Secondly, the presence of MPR capability at the receiving nodes allows packets decoding under interference with nonzero probability, which can be exploited by the ST to boost its throughput. Thirdly, as will be explained in details later, due to the fixed energy transmission property of the energy harvesting ST, secondary delays of channel access may increase the interference at the primary destination due to increasing of the secondary transmitted power, which in turn reduces the probability of successful decoding of the primary packets at the primary destination. Based on these observations, channel accessing at the beginning of the time slot may be useful for certain scenarios and under specific system and channels parameters. If the ST decides to receive the primary packet in a time slot, it takes another action/decision after \( \tau \) seconds of primary packet reception. The decision duration \( \tau \) is designed such that the probability of detecting the state of the primary activity is one. This is important for designing an efficient access protocol on the basis of the actual state of the time slot, i.e., busy/free.

Based on the gathered samples of the primary transmission, the ST perfectly detects the state of the PT\(^2\) if the PT is active, the ST decides whether to resume primary packet reception, which occurs with probability \( \omega \); or to access the channel concurrently with the PU using one of its queues, which occurs with probability \( \omega \). In this case, accessing the channel simultaneously with the PT is motivated by the presence of the MPR capabilities at receivers. If the PT is inactive and the secondary energy queue is nonempty, the ST accesses with probability \( \beta \) using one of its queues.

If at the beginning of the time slot the ST has no energy packets in its energy queue, it decides whether to receive the primary packet, which occurs with probability \( \alpha \), or not. Note that since there is no energy in the secondary energy queue, there is no need to take another decision at \( t = \tau \) seconds.

This is because the ST is incapable of establishing any data transmission due to the lack of energy. In such cases, the probability of receiving the primary packet is \( \alpha \), whereas the probability of remaining silent till the end of the current time slot is \( \pi \). We would like to emphasize here the importance of having different parameters associated with the different state of the queues in the system. Having such parameters enhance the system performance and help in achieving the optimal performance of the network under investigation.

At the far end of the time slot, the ST decides, on the basis of its ability to decode the primary packet and the status of primary packet decoding at the primary destination, whether to accept or reject the admission of the primary packet to the relaying queue. The acceptance probability of a primary packet is \( \beta \), whereas the rejection probability is \( \beta = 1-\beta \).

If the relaying queue is nonempty, the ST selects one of its packets for transmission with probability \( \Gamma = 1-\Gamma \); or selects one of the relaying packets with probability \( \Gamma \). If the relaying queue is empty, the ST accesses using its own packets with probability \( \beta \). The selection probability \( \Gamma \) represents the relative importance of the primary relaying packets and is used for controlling the throughput of the relaying queue. Choosing \( \Gamma = 1 \) gives full priority to the relaying packets over the secondary packets, while \( \Gamma = 0 \) favors the secondary packets (i.e., no selection for the relaying packets). By varying \( \Gamma \) between 0 and 1, we can maximize the secondary throughput under stability of the other queues.

It should be noted that the probability of outage of a certain link depends on the time available for data transmission. Hence, the probability of outage when the ST transmits at the beginning of the time slot is less than the outage probability when it starts the transmission at \( t = \tau \). Although using lower transmission time raises the secondary transmitted power, \( e/(T-\tau) \), the channel outage raises as well [6], [3] (see Appendix A for proof). We should note that the interference caused by the ST on the PT’s transmission increases with delaying in secondary transmission. This happens because the secondary transmit power increases as mentioned earlier. The reader is referred to Appendix A for the proof.

A. Queues Service and Arrival Processes

Let us first consider the packets of the primary queue, \( Q_p \). A packet at the head of the primary queue is served in either one of the following events. If the link \( p \rightarrow d_p \) is not in outage; or if the link \( p \rightarrow d_p \) is in outage, the link \( p \rightarrow s \) is not in outage, and the ST decides to admit the packet to the relaying queue. A successfully received packet by either the primary destination or the ST will be dropped form the primary queue. The mean service rate of the primary queue is then given by

\[
\mu_p = P_{ps,0} \left[ \frac{Pr\{Q_e \neq 0\} + fPr\{Q_e \neq 0\}}{1 - \phi} \right] + Pr\{Q_e \neq 0\} \left( \delta_{pd_p,00} \bar{f} + \delta_{pd_p,01}/\omega \right)
\]

\[
+ P_{ps,0} P_{ps,0} (\alpha Pr\{Q_e \neq 0\} + fPr\{Q_e \neq 0\}) \beta
\]  (1)

\(^2\)Perfect declaration of the PT due to the fact that \( \tau \) is designed for negligible detection errors as will be explained and justified later.
where $\delta_{p_{d},00}$ and $\delta_{p_{d},0}$ denote the reduction in $P_{p_{d},0}$ due to concurrent transmission when the ST accesses at $t = 0$ and $t = \tau$, respectively. The definition and derivation of $P_{j,k}, i$ and $\delta_{j,k, in}$ are provided in Appendix A. It should be pointed out here that without cooperation the maximum mean service rate for the primary queue is $P_{p_{d},0}$, whereas with cooperation the maximum achievable primary mean service rate is $P_{p_{d},0} + P_{p_{d},0} P_{p_{s},0}$, which attained when the ST sets $\beta = \alpha = f = \omega = 1$. Thus, the maximum achievable throughput of the ST is increased by $P_{p_{d},0} P_{p_{s},0}$ packets per time slot.

A packet from $Q_{e}$ is served if the secondary energy queue is nonempty, the ST decides to access the channel using $Q_{e}$, and the link $s \rightarrow d_{s}$ is not in outage. The mean service rate of $Q_{e}$ is given by

$$\mu_{e} = \frac{P_{s_{d},0}}{Q_{s_{d},0}} \left[ \Pr\{Q_{p} \neq 0, Q_{e} \neq 0\} \delta_{s_{d},00} + \Pr\{Q_{p} = 0, Q_{e} \neq 0\} \delta_{s_{d},10} \right] + \Pr\{Q_{p} = 0, Q_{e} \neq 0\} \]$$

(2)

where $\delta_{j,k} = \frac{P_{s_{d},0}}{P_{s_{d},0} + P_{p_{d},0}}$ is defined in Appendix B.

Similarly, the mean service rate of $Q_{e}$ is given by

$$\mu_{e} = \frac{P_{s_{d},0}}{Q_{s_{d},0}} \left[ \Pr\{Q_{p} \neq 0, Q_{e} \neq 0\} \delta_{s_{d},00} + \Pr\{Q_{p} = 0, Q_{e} \neq 0\} \delta_{s_{d},10} \right] + \Pr\{Q_{p} = 0, Q_{e} \neq 0\} \]$$

(3)

The mean arrival rate of the relaying queue is obtained directly from (1). That is,

$$\lambda_{e} = P_{p_{d},0} P_{p_{s},0} \Pr\{Q_{e} \neq 0\} + f \Pr\{Q_{e} \neq 0\} \beta \Pr\{Q_{p} \neq 0\}$$

(4)

where $\Pr\{Q_{p} \neq 0\}$ in (4) means that the arrival of a primary packet at $Q_{e}$ occurs when the primary queue is nonempty.

An energy packet is consumed from the secondary energy queue in a time slot if the ST decides to transmit a data packet from one of its queues. The mean service rate of $Q_{e}$ is then given by

$$\mu_{e} = f \Pr\{Q_{p} \neq 0\} f \omega + f \Pr\{Q_{p} \neq 0\} = 1 - \Pr\{Q_{p} \neq 0\} f \omega$$

(5)

In (5), $f$ means that the ST accesses the channel at $t = 0$; $\Pr\{Q_{p} \neq 0\} f \omega$ means that the ST decides to access the channel at $t = \tau$ seconds, which occurs with probability $\omega$ when $\{Q_{p} \neq 0\}$; and $f \Pr\{Q_{p} \neq 0\}$ means that the ST decides to access the channel after $\tau$ seconds with probability one when $\{Q_{p} \neq 0\}$.

Relaying the primary packets by the ST may seem to waste the time slots that could be used for its own packets. However, it turns out that the ST is indeed gaining since opportunistic relaying of primary packets results in emptying (servicing) the primary queue faster as the service process of the primary queue increases; in return, more network resources can be utilized for delivering the secondary packets. As a result, all users simultaneously achieve performance gains.

B. Radio Sensing and Design of $\tau$

We note that the choice of $\tau$ depends on the required values of misdetection and false alarm probabilities. In this paper, we assume perfect detection of the state of PT’s activity. This can be obtained via adjusting the decision duration, $\tau$, such that the detection errors are kept below certain threshold $\mathcal{T}$. The detection problem at slot $\mathcal{T} \in \{1, 2, 3, \ldots\}$ (assuming that $\tau F_{s}$ is an integer, where $F_{s}$ is the sampling frequency of spectrum sensing [17]) is described as follows:

$$\mathcal{H}_{1} : s(k) = \zeta_{ps} x(k) + \sigma(k)$$

$$\mathcal{H}_{0} : s(k) = \sigma(k)$$

(6)

$$\mathcal{T}(s) = \frac{1}{F_{s} \tau} \sum_{k=1}^{F_{s} \tau} |s(k)|^{2}$$

(7)

where $|\zeta_{ps}|^{2} = \zeta_{ps}$ is channel gain of link $p \rightarrow s$, hypotheses $\mathcal{H}_{1}$ and $\mathcal{H}_{0}$ denote the cases where the PT is active and inactive, respectively, $\tau F_{s}$ is the total number of used samples for primary activity detection, $\sigma$ is the noise instantaneous value at time slot $\mathcal{T}$, $x$ is the primary transmitted signal at slot $\mathcal{T}$ with variance $P_{p}$, $x(k)$ is the $k$th sample of the primary transmit signal, $s(k)$ is the $k$th received sample of the primary signal at the ST and $\mathcal{T}(.)$ is the test statistic of the energy detector.

The quality of the sensing process outcome is determined by the probability of detection, $P_{D}$, and the probability of false alarm, $P_{FA}$, which are defined as the probabilities that the sensing algorithm (technique) detects a PT under hypotheses $\mathcal{H}_{1}$ and $\mathcal{H}_{0}$, respectively. Obviously, for a good detection algorithm, the probabilities of misdetection (complement of detection) and false alarm should be as low as possible. The lower the probability of misdetection, the better protection the PT receives specially when the MPR capability of the primary receiver is low. Hence, secondary decisions on accessing the channel under the constraint on stability of primary and relaying queues can be more efficient when the ST knows the exact state of the PT. The exact knowledge of the primary state at a slot would, on the average, increase both the primary and the secondary throughput as the primary queue gets emptied faster due to the increasing of its queue service rate. This may also allow more interference-free time slots for the ST to be used for its own packets transmission and the relaying packets transmission as well. In addition, the lower the probability of false alarm, there are more chances for the ST to use almost all the free time slots alone, which provide a better throughput for the secondary queues relative to case of transmissions under interference. From the above, we can conclude that both detection errors probabilities affect the primary and the secondary rates. Hence, designing and controlling such probabilities can make the users exploit the channel resources in a better way.
Using the central limit theorem (CLT), the test statistic $T$ for hypothesis $H_0$, $\theta \in \{0, 1\}$, can be approximated by Gaussian distributions $\text{CLT}$ with parameters

$$\Lambda_\theta = \theta\zeta_{ps}\mathbb{P}_p + N_0, \quad \sigma_\theta = \frac{(\theta\zeta_{ps}\mathbb{P}_p + N_0)^2}{F_s\tau}$$

(8)

where $\Lambda_\theta$ and $\sigma_\theta$ denote the mean and the variance of the Gaussian distribution for the hypothesis $H_\theta$, where $\theta \in \{0, 1\}$. Since $\zeta_{ps}$ is Exponentially distributed random variable with parameter $1/\sigma_{\text{ps}}$, the probabilities $P_{FA}$ and $P_D$ can be written as (for proof see Appendix C)

$$P_D = \Pr\{T(s) > \epsilon | H_1\} = \frac{\exp\left(\frac{-N_0}{\sigma_{\text{ps}}\mathbb{P}_p}\right)}{\sigma_{\text{ps}}\mathbb{P}_p} \int_{N_0}^{\infty} Q\left(\sqrt{\frac{F_s\tau}{\sigma_{\text{ps}}\mathbb{P}_p}}\left(\frac{\epsilon}{N_0} - 1\right)\right) \exp\left(-\frac{Z}{\sigma_{\text{ps}}\mathbb{P}_p}\right) dZ$$

(9)

$$P_{FA} = \Pr\{T(s) > \epsilon | H_0\} = Q\left(\sqrt{\frac{F_s\tau}{\sigma_{\text{ps}}\mathbb{P}_p}}\left(\frac{\epsilon}{N_0} - 1\right)\right)$$

(10)

where $\exp(.)$ denotes the exponential function, $\epsilon$ is the energy threshold and $Q(\cdot) = \frac{1}{\sqrt{2\pi}} \int_{\epsilon}^{\infty} \exp(-z^2/2)dz$ is the $Q$-function.

For a targeted false alarm probability, $\hat{P}_{FA}$, the value of the threshold $\epsilon$ is given by

$$\epsilon = N_0 \left(\frac{Q^{-1}(\hat{P}_{FA})}{\sqrt{F_s\tau}} + 1\right)$$

(11)

Thus, for a targeted false alarm probability, $\hat{P}_{FA}$, the probability of misdetection is given by substituting Eqn. (11) into Eqn. (9).

$$P_{MD} = 1 - \frac{1}{\sigma_{\text{ps}}\mathbb{P}_p} \exp\left(-\frac{1}{\sigma_{\text{ps}}\mathbb{P}_p}N_0\right) \left[\frac{N_0}{\sqrt{2\pi}} \int_{N_0}^{\infty} Q\left(\sqrt{\frac{F_s\tau}{\sigma_{\text{ps}}\mathbb{P}_p}}\left(\frac{Q^{-1}(\hat{P}_{FA})}{\sqrt{F_s\tau}} + 1\right)\right) \exp\left(-\frac{Z}{\sigma_{\text{ps}}\mathbb{P}_p}\right) dZ\right]$$

(12)

where $Q^{-1}(\cdot)$ is the inverse of $Q$-function. The argument of $Q$-function in (12) can be rewritten as

$$\frac{N_0}{\sqrt{2\pi}} \sqrt{\frac{F_s\tau}{\sigma_{\text{ps}}\mathbb{P}_p}}\left(\frac{Q^{-1}(\hat{P}_{FA})}{\sqrt{F_s\tau}} + 1\right)$$

(13)

Since $Z \geq N_0$, $\left(\frac{N_0}{\sqrt{2\pi}} - 1\right)$ is definitely a non-positive value. Hence, $\sqrt{F_s\tau}\left(\frac{N_0}{\sqrt{2\pi}} - 1\right)$ is monotonically decreasing with $\tau$. Since the $Q$-function is decreasing with increasing of the argument, increasing $\tau$ decreases the probability of misdetection for a targeted false alarm.

The minimum decision period $\tau$ which results in a negligible detection errors probabilities is obtained via solving the following optimization problem:

$$\min_{0 < \tau < T} \tau, \quad \text{s.t.} \quad P_{FA}(\tau), P_{MD}(\tau) \leq \mathcal{L}$$

(14)

where $\mathcal{L}$ is chosen such that the detection probabilities defined as being sufficiently negligible. It is assumed here to be in order of $10^{-3}$. Substituting (12) into (14) and putting $\hat{P}_{FA} = \mathcal{L}$, we get the formula (15) at the top of the following page. Note that as we mentioned earlier, as $\tau$ increases, the primary actual state detection probabilities decrease, hence the minimum value of the decision duration, $\tau$, over all feasible values is attained when the inequality of the constraints in (14) and (15) hold to equality. Furthermore, as the primary transmit power increases, $\mathbb{P}_p$, $\Lambda_1$ increases as well. Consequently, from (5-1) in Appendix C, the detection probability, $P_D$, increases due to decreasing of the argument of the $Q$-function. This reduction would make the required $P_D$ achievable with lower decision duration $\tau$. The problem in (15) can be solved numerically or via grid search over $\tau$. The optimal $\tau$ is taken as the lowest value of $0 < \tau < T$ which satisfies the constraint $P_D \geq 1 - \mathcal{L}$ for a targeted $\hat{P}_{FA} = \mathcal{L}$.

As discussed above, detection reliability depends on the decision duration, $\tau$. That is, as $\tau$ increases, primary detection at a slot becomes more reliable at the expense of reducing the time available for secondary transmission either for transmitting its own packets or retransmitting the primary relaying packets which, in turn, increase the outage probabilities of the links $s \rightarrow \text{sd}$ and $s \rightarrow \text{pd}$ as shown in Appendix A. This is the essence of the sensing-throughput tradeoff in cognitive radio systems $[17]$.

### C. Approximated Systems

The service processes of the primary data queue and the secondary energy queue are coupled, i.e., interacting queues. This means that the departure of a packet at any of them depends on the state of the other. Hence, we cannot analyze the system performance or compute the service process of each queue directly. For this reasoning, we study three approximated systems. Two of them are inner bounds for the actual performance of the original system and the third is an outer bound.

In the first approximated system, we assume that the PT will transmit dummy packets when its queue is empty. These packets may interfere with the ST in case of concurrent transmission, but do not contribute the throughput of the PT. The essence of such assumption is to cause a constant interference with the ST to decouple the queues interaction and to render the computation of nodes’ service rates feasible. Under such assumption, the probability of the primary queue being empty is zero; that is, $\Pr\{Q_p = 0\} = 0$ and $\Pr\{Q_p \neq 0\} = 1$.

Since the PT is always backlogged (has at least one packet at its queue each time slot), the probability of the ST finds a free time slot is zero. Thus, all time slots that the ST decides to access in are occupied by the a primary transmission. Hence, the service rates of the secondary queues, $Q_s$ and $Q_r$, are reduced relative to the original system in which the PT’s queue may be emptied in some time slots and the ST can access the channel alone.

Accordingly, this system is an inner bound for the original system.

---

3 This is actually the stochastic dominance approach extensively investigated in the literature, see for example [6, 7, 8, 12, 13, 18]

4 Accessing the channel alone (without interference) provides a successful packet decoding at the relevant receiver higher than the case of concurrent transmission as is obvious. The reader is referred to Appendix A for proofs and further details.
In the second approximated system, we assume an energy packet consumption each time slot, which implies that \( \mu_e = 1 \) energy packets per time slot. Under such assumption, the probability of the energy queue being empty is increased relative to the original system. Consequently, the secondary packets get service less frequently. Furthermore, the relaying packets get service in a lower rate, hence the event of primary queue being empty decreases. Thus, the possibility of having a free time slot or an interference-free time slot for the ST is reduced as well. Accordingly, this system is an inner bound for the original system.

In the third approximated system, we assume that the departure of the energy queue is almost zero, or equivalently, the probability of having an energy packet stored in the secondary energy queue in any time slot is one. This system is an outer bound for the original system as the ST will always be able to access the channel for transmitting its own packets or retransmitting the relayed primary packets each time slot, if there is a chance for the SU to access the channel. Hence, all data queues service rates will be increased simultaneously.

1) First Approximated System: Inner Bound: In this case, denoted by \( S_1 \), the PT is always backlogged. If the ST decides not to access the channel at the beginning of the time slot, it will not access later at \( t = \tau \). This is because the PT is always active and wasting \( \tau \) seconds for knowing the activity state of the PT will not lead to any gains in terms of secondary queues throughput. Therefore, the optimal \( \omega = \omega^* = 1 \). Moreover, the decision on accessing the channel or receiving of the primary packet is taken at the early beginning of the time slot, specifically at \( t = 0 \). If the secondary energy queue is nonempty, the ST decides to access the channel by one of its queue with probability \( \ell \) or decides to receive the possible primary transmission with probability \( f \). If the secondary energy queue is empty, the ST cannot transmit data and its decision becomes whether to receive of the possible primary transmission with probability \( \alpha \) or remain idle with probability \( 1 - \alpha \). At the end of the time slot, the ST decides whether to admit the primary packet or to reject it, as explained earlier. Under the first approximated system, the mean service rate of the energy queue is given by

\[
\mu_e = 1 - f
\]

Using the results provided in Appendix D (setting \( \mu = \mu_e = 1 - f \)), the probability of the energy queue being empty is given by

\[
\nu_o = \frac{\eta}{\eta - f/\lambda_e - \eta^K} = \frac{\ell/\lambda_e - 1}{\ell/\lambda_e - \eta^K}
\]

The probability of nonempty energy queue is given by

\[
\nu_o = \frac{1 - \eta^K}{f/\lambda_e - \eta^K}
\]

Based on this, the relaying queue departure and arrival mean rates are given by

\[
\mu_r = P_{sdp,0} \Gamma f/\nu_o \delta_{sdp,0}
\]

\[
\lambda_r = P_{psd,0} P_{ps,0} (\alpha \nu_o + f/\nu_o) \beta
\]

The probability of the relaying queue being nonempty is given by

\[
Pr[Q_r \neq 0] = \pi_r = \frac{\lambda_r}{\mu_r}
\]

The mean service rate of \( Q_s \) becomes

\[
\mu_s = P_{sdp,0} \Gamma f/\nu_o \delta_{sdp,0} \left( \Gamma \nu_o + \pi_r \right)
\]

The primary queue mean service rate is given by

\[
\mu_p = P_{psd,0} \left[ (1 - \nu_o) \ell + \delta_{psdp,0} \nu_o \right] + P_{psd,0} P_{ps,0} (\alpha \nu_o + f/\nu_o) \beta
\]

We note that the queues are not interacting anymore. Hence, we can apply Loynes theorem to check the stability of the queues and obtain the maximum stable throughput based on the first approximated system via solving the following constrained optimization problem.

\[
\max_{\beta, \alpha, \Gamma} \mu_s, \quad \text{s.t.} \quad \lambda_r \leq \mu_p, \lambda_r \leq \mu_p
\]

where \( \mu_r, \lambda_r, \mu_p \) and \( \mu_p \) are in (19), (20), (22) and (23), respectively.

For a given \( f \) and \( \beta \), we can get a closed-form expressions for \( \Gamma \) and \( \alpha \), then we solve a family of convex optimization problems parameterized by \( \beta \) and \( f \). Specifically, the optimal solutions of \( \Gamma \) and \( \alpha \) are a set of points which satisfies the stability constraint of the primary and relaying queues stability, respectively. Using (23), the optimal \( \alpha \) for a fixed \( f \) and \( \beta \) is given by

\[
\alpha^* \geq \frac{\lambda_r - P_{psdp,0} (1 - \nu_o) \ell \delta_{psdp,0} \nu_o}{P_{psdp,0} P_{ps,0} \delta_{ps,0} \beta} - P_{ps,0} \nu_o
\]

Using the constraint on the stability of the relaying queue,

\[
\nu_o \geq \frac{\lambda_r - P_{psdp,0} (1 - \nu_o) \ell \delta_{psdp,0} \nu_o}{P_{psdp,0} P_{ps,0} \delta_{ps,0} \beta} - f/\nu_o
\]

The expression in (21) is obtained via solving the Markov chain modeling the relaying queue when its arrival and service processes are decoupled of the other queue and become computable. This formula is exactly \( 1 - \nu_o \) in Appendix D with the relevant mean service and arrival rates of the relaying queue and with setting the buffer size \( K \) to infinity.
the optimal $\Gamma$ is given by
\begin{equation}
\Gamma^* \geq \frac{P_{pd_p,0}F_{ps,0}(\alpha^* \nu_e + f \nu_e)\beta}{P_{sd_p,0}F_{\nu_0}d_{sd_p,0} \nu_0}
\end{equation}
where $\alpha^*$ is given in (25). The optimal $\beta$ and $f$ are obtained via grid search and are selected as the pair of parameters that yields the highest objective function in (24). From (26), we note that the optimal selection probability of the relaying queue for transmission, $\Gamma^*$, increases with increasing the acceptance probability of the primary undelivered packets, $\beta$, and the flow rate to the relaying queue $P_{pd_p,0}F_{ps,0}(\alpha^* \nu_e + f \nu_e)\beta$. This is because the ST should increase the selection of $Q_e$ for transmission to maintain the relaying queue stability. In addition, $\Gamma^*$ increases with decreasing of $P_{sd_p,0}F_{\nu_0}d_{sd_p,0} \nu_0$. This is because $P_{sd_p,0}F_{\nu_0}d_{sd_p,0} \nu_0$ determines the probability of certain transmitted packet from the relaying queue being correctly received at the primary destination and therefore if this term is high, the ST will not need several transmission for the same packet each time slot. Hence, the ST can reduce the probability of choosing the relaying queue for transmission at a time slot and rather it could use that time slot for the transmission of its own packets.

2) Second Approximated System: Inner Bound: In this approximated system, denoted by $S_2$, we assume that an energy packet is consumed per time slot. That is, $\mu_e$ = 1 energy packets per time slot. Using the results in Appendix D, if $\mu = \mu_e = 1$, $\eta = 0$ and the probability of the energy queue being empty is given by
\begin{equation}
\nu_0 = 1 - \lambda_e
\end{equation}
We can interpret the probability $\lambda_e$ as the fraction of time slots that can be used by the ST for data transmission. It should be pointed out here that the buffer size does not change the state probabilities. Hence, does not have any impact on the queues’ rates. The Markov chain modeling the energy queue in this case is composed of two states only: state 0 where the energy queue has no packets, and state 1 where the energy queue has only one packet. The probability of the energy queue having more than one packet, $\nu_k$, $k \geq 2$, is zero. This case is discussed in Appendix D. The primary throughput is given by
\begin{equation}
\mu_p = P_{pd_p,0} \left[ (\lambda_e + f \lambda_e \omega) + \nu_0 (\delta_{pd_p,0} + f \delta_{pd_p,0} f \omega) \right]
\end{equation}
The probability of the primary queue being nonempty is given by
\begin{equation}
\Pr\{Q_p \neq 0\} = \pi_p = \frac{\lambda_p}{\mu_p}
\end{equation}
The relaying queue mean service and arrival rates are given by
\begin{equation}
\mu_r = \lambda_e P_{sd_p,0} \left[ (\lambda_e + f \lambda_e \omega) + f \delta_{sd_p,10 \omega} \pi_p + \pi_r \right]
\end{equation}
\begin{equation}
\lambda_r = P_{pd_p,0}F_{ps,0}(\alpha^* \nu_e + f \nu_e)\beta \pi_p
\end{equation}

The mean service rate of $Q_e$ is then given by
\begin{equation}
\mu_s = P_{sd_p,0} \left[ (\pi_p \delta_{sd_p,00} + \pi_p) + \delta_{sd_p,0} f (\omega \pi_p \delta_{sd_p,10} + \pi_p) \right]
\end{equation}
\begin{equation}
\times \left( \Gamma \pi_r + \pi_r \right)
\end{equation}
Since the queues are decoupled in the second approximated system, the maximum secondary throughput is given by solving the following problem.
\begin{equation}
\max_{\beta, f, \omega, \Gamma} \mu_s, \text{ s.t. } \lambda_r \leq \mu_r, \lambda_p \leq \mu_p
\end{equation}
where $\mu_p$, $\mu_r$, $\lambda_r$ and $\mu_s$ are in (28), (30), (31) and (32), respectively.

3) Third Approximated System: Outer Bound: In this case, denoted by $S_3$, we consider a backlogged energy queue. This means that there exists at least one energy packet each time slot in $Q_e$. This case can happen if the energy arrival rate is greater than or equal to the energy departure rate or when $\lambda_e = 1$ regardless of the value of $\mu_e$. In this case, the probability of the energy queue being nonempty approaches the unity. The mean service and arrival rates of the queues are then given by
\begin{equation}
\mu_p = P_{pd_p,0} \left[ f \omega + (\delta_{pd_p,0} + f \delta_{pd_p,0} f \omega) \right] + P_{pd_p,0}F_{ps,0} f \omega \beta,
\end{equation}
\begin{equation}
\mu_s = P_{sd_p,0} \left[ f (\pi_p \delta_{sd_p,00} + \pi_p) + f \delta_{sd_p,0} (\omega \pi_p \delta_{sd_p,10} + \pi_p) \right]
\end{equation}
\begin{equation}
\lambda_r = P_{pd_p,0}F_{ps,0} f \omega \beta \pi_p,
\end{equation}
where $\pi_p$ in (35) and (36) follows (29) with $\mu_p$ in (34), and
\begin{equation}
\lambda_s = P_{sd_p,0} \left[ f (\pi_p \delta_{sd_p,00} + \pi_p) + f \delta_{sd_p,0} (\omega \pi_p \delta_{sd_p,10} + \pi_p) \right]
\end{equation}
\begin{equation}
\times \left( \Gamma \pi_r + \pi_r \right)
\end{equation}
where $\pi_r$ follows (21) with $\mu_r$ and $\lambda_r$ in (35) and (36), respectively.

The outer bound can be computed by solving the following problem
\begin{equation}
\max_{\beta, f, \omega, \Gamma} \mu_s, \text{ s.t. } \lambda_r \leq \mu_r, \lambda_p \leq \mu_p
\end{equation}
with $\mu_p$, $\mu_r$, $\lambda_r$ and $\mu_s$ are in (14), (35), (36), and (37), respectively. The optimization problems (24), (33) and (38) are solved numerically at the ST for a given channels and system parameters. Specifically, for a given parameters, the ST solves the optimization problem and use the optimal parameters for the system’s operation.

D. Some Important Remarks
Following are some important remarks.
1) **First Remark:** Using the results in Appendix A, the complement of outage probability of link \( p \rightarrow d_p \) when the ST starts transmission at the beginning of the time slot is given by

\[
P_{c}(c)_{pd,0,0} = \frac{1}{1 + \left( \frac{2^{\frac{a}{\gamma}} - 1}{\gamma_{pd,0,0}} \right) \exp \left( - \frac{2^{\frac{a}{\gamma}} - 1}{\gamma_{pd,0,0}} \right)}
\]  

while the probability of that link being not in outage when the ST starts transmission at \( t = \tau \) is given by

\[
P_{c}(c)_{pd,0,1} = \frac{1}{1 + \left( \frac{2^{\frac{a}{\gamma}} - 1}{\gamma_{pd,0,0}} \right) \exp \left( - \frac{2^{\frac{a}{\gamma}} - 1}{\gamma_{pd,0,0}} \right)}
\]  

The ratio of (39) to (40) is given by

\[
\rho = \frac{P_{c}(c)_{pd,0,1}}{P_{c}(c)_{pd,0,0}} = 1 + \frac{1}{1 + \frac{a}{1 - \tau/\gamma}}
\]

We note that \( \gamma_{sd,0,1} = \gamma_{sd,0}/(1 - \tau/T) \) and \( a = \left( \frac{2^{\frac{a}{\gamma}} - 1}{\gamma_{pd,0,0}} \right) \gamma_{pd,0,0}. \)

If \( a \gg 1 \), the reduction of the primary packet correct reception probability due to secondary access delay (when the ST accesses at \( t = \tau \)) is \( \rho \approx 1 - \tau/T \). Therefore, if the secondary decides to access after \( \tau \) seconds of primary packet reception based on the primary activity, the probability of the primary packet decoding reduces by a factor \( 1 - \tau/T \) relative to the case when the ST accesses at the beginning of the slot. The reduction of the primary packet correct reception is a linear function of \( \tau \). If the decision time, \( \tau \), is high, the primary packet decoding will be reduced significantly.

Assume that the primary transmits with a very low power. This makes \( a \) much greater than 1. Thus, we can approximate the reduction, due to secondary access delay, of the probability of the primary channel not being in outage by \( \rho \approx 1 - \tau/T \). At the same time, since the primary transmit power is low, the required \( \tau \) for perfect primary detection is high, as discussed beneath (14). This means that the reduction of the primary packet decoding at the primary destination due to concurrent transmissions is significantly high. In this case, the secondary access probability at \( t = 0 \) is definitely higher than the access probability at \( t = \tau \) when the PT is detected to be active and the ST decides to access the channel. Moreover, it may be better for the ST to access the channel at \( t = 0 \) to use the whole slot time in data transmission; and at \( t = \tau \) if the PT is declared to be inactive, if the PT is declared to be active, it may be better to resume receiving the primary packet because concurrent transmission would be harmful for the PT as explained earlier.

2) **Second Remark:** Assume that the current primary arrival rate is \( \lambda_p = \lambda_p^* \). Increasing the primary arrival rate to \( \lambda_p + \Delta \lambda_p \geq 0 \), increases the probability of the primary queue being nonempty. This is because the probability of having an arrival at a certain time slot is increased. Consequently, the number of empty time slots that the ST can detect or access alone decreases as well. In addition, the probability of relaying queue selection, \( \Gamma \), must be increased to maintain the stability of the relaying queue as the arrival rate of the relaying queue is increased due to the increasing of \( \lambda_p \). These two observations lead to the fact that the achievable secondary rate is increased relative to the case of \( \lambda_p = \lambda_p^* \). This means that the secondary service rate, \( \mu_s \), is a non-increasing function of the primary arrival rate \( \lambda_p \).

3) **Third Remark:** From the expressions of the service rates of the queues, the service processes are functions of channel outages probabilities. Based on the formulas of the channel outage in Appendix A, the outage probability of a certain link is a decreasing function of \( R_p = B/(TW) \). Therefore, increasing the targeted primary spectral efficiency rate, \( R_p \), decreases all queues service rates. This leads to a reduction in the maximum achievable secondary throughput, \( \mu_s \). This means that the secondary service rate, \( \mu_s \), is a non-increasing function of the primary targeted spectral efficiency rate \( R_p = B/(TW) \).

The following proposition summarize the main observations in the second and the third remarks.

**Proposition 1:** For a given channel and system parameters, let \( \mu_s^*(\lambda_p, R_p) \) be the maximum secondary throughput at the pair \( (\lambda_p, R_p) \). The optimal secondary throughput satisfies the following properties:

- \( \mu_s^*(\lambda_p, R_p) \geq \mu_s^*(\lambda_p + \Delta \lambda_p, R_p) \), \( \Delta \lambda_p \geq 0 \).
- \( \mu_s^*(\lambda_p, R_p) \geq \mu_s^*(\lambda_p, R_p + \Delta R_p) \), \( \Delta R_p \geq 0 \).

**IV. Numerical Results**

In this section, we provide some numerical results for the optimization problems presented in this paper. The decision duration \( \tau \) is chosen such that \( P_{MD}, P_{PA} \leq 10^{-3} \). We define here the conventional scheme, denoted by \( S_c \), where the ST senses the channel for \( \tau \) seconds and if the primary data queue and the secondary energy queue are simultaneously empty and nonempty, respectively, the ST accesses with probability 1 using one of its queues probabilistically if the relaying queue is nonempty. In addition, if the PT is transmitting a packet to its destination, the ST accepts with probability one to relay and admit the transmitted packet if the primary destination fails in decoding that packet. The secondary throughput of the conventional system is obviously a subset of the proposed cooperative system, \( S \), and can be obtained from \( S \) via setting \( \beta = 1, \omega = 1, f = 1 \) and \( \alpha = 0 \). The other parameters are optimized over their domain to achieve the maximum secondary throughput.

Figs. 3 and 4 represent the maximum secondary throughput of the approximated systems of system \( S \). The figures are generated using the following common parameters: \( P_{sd,0} = 0.08, \quad \delta_{sd,00} = 0.3, \quad P_{sd,0} = 0.7, \quad P_{ps,0} = 0.8, \quad \delta_{sd,00} = 0.3, \quad K = 60, \quad P_{pd,0} = 0, \quad P_{pd,0} = P_{pd,0} = 0, \quad \delta_{sd} = 0.7, \quad \delta_{sd,10} = 0.2, \quad \delta_{sd,10} = 0.2 \). In Fig. 3 the maximum secondary throughput under the approximated systems is plotted versus \( \lambda_p \). This figure is plotted with \( \lambda_e = 0.9 \) energy packets per time slot. The figure shows that the second approximated system provides throughput higher than the first approximated system, hence the union, which represents an inner bound on the actual performance of system \( S \), is the second approximated system. The outer bound which represents the case of backlogged energy queue is close to the inner bound.
Fig. 4 reveals two important observations. First, the figure reveals the impact of the arrival rate of the secondary energy queue on the system’s inner bound. Precisely, as the energy arrival rate increases, the inner and the outer bound become close to each other and they overlap for all $\lambda_p$ when $\lambda_c = 1$ energy packets/slot. Second, the figure reveals that the inner bound of the proposed system can outperform the outer bound of the conventional cooperation protocol with reliable energy source plugged to the ST, system $S_e$ is plotted with $\lambda_c = 1$ energy packets per time slot (outer bound on $S_e$).

We note that for Figs. 3 and 4, without cooperation the primary packets outage probability is $1 - \frac{P_{sd,p,0}}{P_{pd,p,0}} = 1$ which implies that the probability of a primary packet being served at an arbitrary time slot is zero. Hence, the primary queue is always backlogged and will never be emptied. On the other hand, with cooperation the maximum feasible primary arrival rate is 0.3 packets per time slot.

Fig. 5 demonstrates the impact of buffer size, $K$, on the first approximated system performance. As expected, increasing the buffer size boosts the secondary throughput. This is because when the buffer size is high, the ST can store more packets to be used in future for transmitting its own packets or retransmitting the primary packets. The case of finite buffer capacity is obviously a subset of the case of infinite or large buffer capacity. The figure is generated with $\lambda_e = 0.8$ energy packets per time slot, $P_{sd,p,0} = 0.5$, $\delta_{sd,00} = 0.5$, $P_{sd,e,0} = 0.5$, $P_{ps,e,0} = 0.8$, $\delta_{sd,e,0} = 0.1$, $P_{pd,p,0} = 0$, $P_{pd,e,0}^{(c)} = 0$, $\delta_{sd,e,10} = 0.1$, $\delta_{sd,10} = 0.1$, $\delta_{sd,10} = 0.1$.

Finally, the impact of MPR capabilities is shown in Fig. 6. The figure reveals the gains of the MPR capability on achieving higher throughput for both users. The parameters are chosen to be: $\lambda_e = 0.8$, $P_{sd,p,0} = 0.8$, $P_{sd,e,0} = 0.7$, $P_{ps,e,0} = 0.8$, $P_{pd,p,0}^{(c)} = 0.6$, $\delta_{sd,e,0} = 0.5$, $\delta_{sd,e,0} = 0.5$, and $P_{pd,e,0}^{(c)} = \delta_{sd,e,0} = \delta_{sd,e,0} = \delta_{sd,10} = \delta_{sd,10} = e$, which represents the MPR strength. At strong MPR, we can achieve orthogonal channels for terminals over most $\lambda_p$ range. The plot also shows that the inner and the outer bounds coincide for high $\lambda_p$. This happens because the energy queue is backlogged under the used parameters.

From the figures, it is noted that cooperation boosts both primary and secondary throughput. Furthermore, the energy arrival rate increases the probabilities of the secondary packets and the relayed primary packets being served which, in turn, boost both primary and secondary throughput. The figures also show that the increasing of $\lambda_p$ decreases the maximum achievable secondary throughput.

V. CONCLUSION

In this paper, we have proposed a new cooperative cognitive relaying protocol, where the ST relays some of the undelivered primary packets. We have considered a generalized MPR channel model, and investigated the impact of the receivers’ MPR capabilities on the users throughout. We also have investigated the impact of the secondary energy queue on the system performance. We have provided two inner bounds and an outer bound on the secondary throughput, and showed that the bounds are coincide when the secondary energy queue is
The outage probability can be written as

\[ P_j^{(c)} = 1 - \frac{1}{1 + \left( \frac{2R_j^{(i)}}{\gamma_{jk,n}\sigma_{jk}} - 1 \right)} \exp \left( -\frac{\gamma_{jk,i}\sigma_{jk}}{\gamma_{jk,n}\sigma_{nk}} \right) \]  

(43)

We note that from the outage probability (43), the numerator is increasing function of \( R_j^{(i)} \) and the denominator is a decreasing function of \( R_j^{(i)} \). Hence, the outage probability \( P_j^{(c)} \) increases with \( R_j^{(i)} \). The probability of correct reception \( P_j^{(c)} = 1 - P_j^{(c)} \) is thus given by

\[ P_j^{(c)} = 1 - \frac{\delta_{jk,in}^{(c)} - \delta_{jk,in}}{\delta_{jk,in}^{(c)}} \]  

(44)

where \( \delta_{jk,in} = \exp \left( -\frac{\gamma_{jk,i}\sigma_{jk}}{\gamma_{jk,n}\sigma_{nk}} \right) \) is the probability of correct packet reception when node \( j \) transmits alone (without interference) and \( \delta_{jk,in} \leq 1 \). As is obvious, the probability of correct reception is lowered in the case of interference. Based on (44), we note that

\[ \frac{P_j^{(c)}}{\delta_{jk,in}} = \frac{\delta_{jk,in}^{(c)}}{\delta_{jk,in}} \]  

(45)

Following are some important notes. First, note that if the PT’s queue is nonempty, the PT transmits the packet at the head of its queue at the beginning of the time slot (at \( t = 0 \)) with a fixed transmit power \( P_p \) and data transmission time \( T_p = T \). Accordingly, the superscript \( i \) which represents the instant that a transmitting node starts transmission in is removed in case of PU.

Second, for the ST, the formula of probability of complement outage of link \( s \rightarrow k \) when the PT is active is given by

\[ P_{sk,0}^{(c)} = \frac{\exp \left( -\frac{\gamma_{sk,n}\sigma_{sk}}{\gamma_{sk,i}\sigma_{sk}} \right) - 1}{\delta_{sk,0}^{(c)}} \]  

(46)

where \( n = 0 \) because the PT always transmits at \( t = 0 \) and \( \gamma_{sk,i} = e/\left(T(1-i\tau/T) \right) = \gamma_{sk,0}/(1-i\tau/T) \). The denominator of (46) is proportional to \( \left( \frac{2\gamma_{sk,0}}{\gamma_{sk,i}\sigma_{sk}} \right) \), which in turn monotonically decreasing with \( i\tau \). The first derivative with respect to \( i\tau \), the numerator of (46),

\[ T_{sk,i} = \exp \left( -\frac{\gamma_{sk,n}\sigma_{sk}}{\gamma_{sk,i}\sigma_{sk}} \right) \]  

(47)

can be easily shown to be decreasing with \( i\tau \) as in [6, 8]. Since the numerator of (46) is monotonically decreasing with \( i\tau \) and the denominator is monotonically increasing with \( i \), \( P_{sk,0}^{(c)} \) is monotonically decreasing with \( i\tau \). Therefore, the delay in the secondary access causes reduction in the probabilities of the secondary packets correct reception and the primary relayed packets correct reception at their destinations.
APPENDIX B

In this Appendix, we compute the ratio \( \frac{P_{jk,1}}{P_{jk,0}} \). Let

\[
\hat{\delta}_{jk} = \frac{P_{jk,1}}{P_{jk,0}}
\]

From (44), we have

\[
P_{jk,1} = \frac{P_{jk,1}^{(e)}}{\delta_{jk,1n}}
\]

Thus,

\[
\hat{\delta}_{jk} = \frac{P_{jk,1}}{P_{jk,0}} = \frac{P_{jk,1}^{(e)}}{\delta_{jk,1n}}
\]

After some mathematical manipulations, the ratio \( \frac{P_{jk,1}^{(e)}}{P_{jk,0}} \) is given by

\[
\frac{P_{jk,1}^{(e)}}{P_{jk,0}} = \delta_{jk,1n}\hat{\delta}_{jk}
\]

APPENDIX C

In this Appendix, we prove the misdetection and false alarm probabilities provided in (9) and (10). Using the mean and variance of each hypothesis provided in (8), we have

\[
\begin{align*}
\Lambda_0 &= N_0, \\
\sigma_0 &= N_0^2/F_s \tau, \\
\lambda_1 &= \zeta_p \sigma_p + N_0, \\
\sigma_1 &= (\zeta_p \sigma_p + N_0)^2/F_s \tau
\end{align*}
\]

(51)

Given hypothesis \( \mathcal{H}_1 \), a correct detection of the primary activity occurs when the test function, \( T(s) \), exceeds the threshold \( \epsilon \). That is,

\[
P_D = \operatorname{Pr}\{T(s) > \epsilon | \mathcal{H}_1\} = \int_0^{\infty} \frac{1}{\sqrt{2\pi \Lambda_1^2/F_s \tau}} \exp\left(-\frac{2}{\Lambda_1^2/F_s \tau}\right) G(\zeta_{ps}) d\zeta_{ps}
\]

(52)

where \( G(\zeta_{ps}) \) is the probability density function of \( \zeta_{ps} \), which is Exponential with parameter \( 1/\sigma_{ps} \) in case Rayleigh fading. Making the change of variable \( Y = \sqrt{F_s \tau} (\frac{2}{\Lambda_1}) \) and substituting into (52), we get

\[
P_D = \operatorname{Pr}\{T(s) > \epsilon | \mathcal{H}_1\} = \int_0^{\infty} \frac{1}{\sqrt{2\pi \Lambda_1^2/F_s \tau}} \exp\left(-\frac{2}{\Lambda_1^2/F_s \tau}\right) G(\zeta_{ps}) d\zeta_{ps}
\]

(53)

The probability of detection can be rewritten as

\[
P_D = \operatorname{Pr}\{T(s) > \epsilon | \mathcal{H}_1\} = \int_0^{\infty} Q(\sqrt{F_s \tau} (\frac{\epsilon - \Lambda_1}{\Lambda_1})) G(\zeta_{ps}) d\zeta_{ps}
\]

(54)

where \( Q(Y) = \frac{1}{\sqrt{2\pi}} \int_0^{\infty} \exp(-z^2/2) \, dz \) is the \( Q \)-function.

Rewriting (54) in terms of \( \Lambda_1 \), we get

\[
P_D = \operatorname{Pr}\{T(s) > \epsilon | \mathcal{H}_1\} = \frac{1}{\sigma_{ps} \sigma_p} \int_0^{\infty} Q(\sqrt{F_s \tau} (\frac{\epsilon - \Lambda_1}{\Lambda_1})) \exp(-\frac{\Lambda_1}{\sigma_{ps} \sigma_p}) d\Lambda_1
\]

(55)

In a similar fashion, using the hypothesis \( \mathcal{H}_0 \), the false alarm probability is given by

\[
P_{FA} = \operatorname{Pr}\{T(s) > \epsilon | \mathcal{H}_0\} = Q(\sqrt{F_s \tau} (\frac{\epsilon - 1}{\Lambda_o}))
\]

(56)

APPENDIX D

When the arrival and departure of the secondary energy queue become decoupled from all other queues in the network as in the approximated systems, we can construct and solve its Markov chain. The Markov chain is shown in Fig. 7, where the mean arrival rate is \( \lambda_o \) and the mean service rate is \( \mu \). Solving the state balance equations of the Markov chain modeling the secondary energy queue, it is straightforward to show that the probability that the energy queue has 1 \( \leq \theta \leq K \) packets, \( \nu_{\theta} \), is

\[
\nu_{\theta} = \nu_0 \frac{1}{\mu} \left[ \frac{\lambda_o \mu}{\lambda_o \mu} \right]_{\theta=1}^{\theta=K} = \nu_0 \frac{\eta^\theta}{\mu^\theta}, \quad \theta = 1, 2, \ldots, K
\]

(57)

where \( \eta = \frac{\lambda_o}{\mu} \). Since the sum of all states’ probabilities is the unity, \( \sum_{\theta=0}^{K} \nu_{\theta} = 1 \). The probability of the secondary energy being empty is obtained via solving the following linear equation:

\[
\nu_o + \sum_{\theta=1}^{K} \nu_{\theta} = \nu_o + \nu_o \sum_{\theta=1}^{K} \frac{1}{\mu} \eta^\theta = 1
\]

(58)

After some mathematical manipulations, \( \nu_o \) is given by

\[
\nu_o = \frac{\mu - 1}{\mu - \eta^K}
\]

(59)

with \( \lambda_o < \mu \). If \( \lambda_o \geq \mu \), the energy queue saturates, i.e., always backlogged. Thus, \( \nu_o = 0 \), which boosts the secondary rate.

If the buffer size is large, i.e., \( K \rightarrow \infty \), the probability of the primary energy queue being empty is \( 1 - \lambda_o / \mu \). The blocking probability of an arrived packet to the secondary energy queue is equal to the probability that the queue is full, i.e., storing \( K \) packets in its buffer. Thus,

\[
P_B = \nu_K = \frac{\mu - 1}{\mu - \eta^K}
\]

(60)

The blocking probability represents the amount of energy packets that will be rejected at the energy queue due to storage limitations. As the buffer size, \( K \), increases, the blocking probability vanishes.

If \( \mu = 1 \), \( \eta = 0 \) and the probability that the energy queue having more than one packet is zero. The states probabilities in such case are given by

\[
\nu_0 = 1 - \lambda_o, \quad \nu_1 = \lambda_o, \quad \nu_\theta = 0, \quad \theta = 2, \ldots, K
\]

(61)
Fig. 7. The Markov chain modeling the secondary energy queue when its service rate is independent of the other queues and has a mean service rate $0 \leq \mu \leq 1$.

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