QCD resummation for jet substructures

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Outlines

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Motivation

To propose a theoretical framework for study of jet physics based on perturbative QCD
Boosted heavy particles

- Large Hadron Collider (LHC) provide a chance to search new physics
- New physics involve heavy particles decaying possibly through cascade to SM light particles
- New particles, if not too heavy, may be produced with sufficient boost -> a single jet
- How to differentiate heavy-particle jets from ordinary QCD jets?
- Similar challenge of identifying energetic top quark at LHC
Fat QCD jet fakes top jet at high pT

Thaler & Wang
0806.0023
Pythia 8.108
Jet substructure

- Make use of jet internal structure in addition to standard event selection criteria
- Energy fraction in cone size of $r$, $\Psi(r)$, $\Psi(R) = 1$
- Quark jet is narrower than gluon jet
- Heavy quark jet energy profile should be different
Why resummation?

• Monte Carlo may have ambiguities from tuning scales for coupling constant
• NLO is not reliable at small jet mass
• Predictions from QCD resummation are necessary

Tevatron data vs MC predictions

N. Varelas 2009
Jet factorization

Achieved by eikonalization and Ward identity
Eikonalization
• Jet is dominated by collinear dynamics from loop momentum \( l \) to parallel jet momentum \( P_j^+ \)
• For attachment of collinear gluon, eikonalization holds -> detachment of gluon
• For \( l^- << l^+ \)

\[
\frac{(p-l)_\alpha \gamma^\alpha + m_t}{(p-l)^2 - m_t^2} \gamma^\mu \approx \frac{\xi^\mu}{-\xi \cdot l}
\]
eikonal vertex, eikonal propagator
-> Wilson line, collect collinear gluons

\[
\Phi^{(f)}_\xi(\infty, 0; 0) = \mathcal{P} \left\{ e^{-ig \int_0^\infty d\eta \xi \cdot A(f)(\eta \xi^\mu)} \right\}
\]
Jet definitions  Almeida et al. 08

- **Eikonalization leads to factorization**

- **Quark**
  \[
  J_i^q(m_J^2, p_0, J_i, R) = \frac{(2\pi)^3}{2\sqrt{2}(p_0, J_i)^2} \frac{\xi_\mu}{N_c} \sum_{N_{J_i}} \text{Tr} \left\{ \gamma^\mu \langle 0 | q(0) \Phi^{(q)}_{\xi}(0, 0) | N_{J_i} \rangle \right\} \delta(m_J^2 - \tilde{m}_J^2(N_{J_i}, R)) \]
  \[
  \times \langle N_{J_i} | \Phi^{(\bar{q})}_{\xi}(\infty, 0) \bar{q}(0) | 0 \rangle \delta(p_0, J_i - \omega(N_{J_c}))
  \]

- **Gluon**
  \[
  J_i^g(m_J^2, p_0, J_i, R) = \frac{(2\pi)^3}{2(p_0, J_i)^3} \sum_{N_{J_i}} \langle 0 | \xi_\sigma F^{\sigma\nu}(0) \Phi^{(g)}_{\xi}(0, \infty) | N_{J_i} \rangle \]
  \[
  \times \langle N_{J_i} | \Phi^{(g)}_{\xi}(0, \infty) F^{\rho\nu}(0) \xi_\rho | 0 \rangle \delta(m_J^2 - \tilde{m}_J^2(N_{J_i}, R)) \]
  \[
  \times \delta^{(2)}(\hat{n} - \tilde{n}(N_{J_i})) \delta(p_0, J_i - \omega(N_{J_c}))
  \]

- **LO jet**
  \[
  J^{(0)}(m_{J_i}^2, p_0, J_i, R) = \delta(m_{J_i}^2)
  \]
NLO diagrams

- quark jet
- gluon jet

$R = 0.4$

$p_T = 600\text{GeV}$

$\sqrt{S} = 1.96\text{TeV}$
Resummation

Technical part, ideas only
See alternative approach based on SCET, Ellis et al. 2010
Key idea

- Vary Wilson line to arbitrary vector $n$
- Collinear dynamics is independent of $n$
- Variation effect does not contain collinear dynamics, and can be factorized from jet
- Study derivative $-\frac{n^2}{P_J \cdot n} P_{J\alpha} \frac{d}{dn_\alpha} J$
- Differentiation applies only to Wilson line

$$-\frac{n^2}{P_J \cdot n} P_{J\alpha} \frac{d}{dn_\alpha} \frac{n_\mu}{n \cdot l} = \frac{n^2}{P_J \cdot n} \left( \frac{P_J \cdot l}{n \cdot l} n_\mu - P_{J\mu} \right) \frac{1}{n \cdot l} = \frac{\hat{n}_\mu}{n \cdot l}$$

- Differentiated gluon gives hard or soft contribution
Soft factorization (virtual)

• If differentiated gluon is soft, special vertex locates at outer end of Wilson line
• If it locates inside (see figure), both gluons are soft
  -> NLO soft kernel

LO soft kernel $K_v^{(1)}$
Soft factorization (real)

- Similar argument applies to factorization of differentiated soft real gluon

LO soft kernel $K_r^{(1)}$

Jet invariant mass excluding soft momentum $l$, $(P_J - l)^2$
Hard factorization

- If differentiated gluon is hard, special vertex locates at inner end of Wilson line
- If it locates at outside, both gluons are hard -> NLO hard kernel

LO hard kernel $G^{(1)}$
Resummation equation

• Up to leading logs, resummation equation

\[- \frac{n^2}{P_j \cdot n} P_j^\alpha \frac{d}{dn^\alpha} J = [G^{(1)} + K_v^{(1)} + K_r^{(1)}] \otimes J\]

• For next-to-leading-logarithm accuracy, G and K are evaluated to two loops

• Solve the equation in Mellin N space

\[\tilde{J}_q(N, P_T, \nu^2, R, \mu^2) \equiv \int_0^1 dx (1 - x)^{N-1} J_q(x, P_T, \nu^2, R, \mu^2),\]

\[x \equiv \frac{m_j^2}{(RP_T)^2} \quad \nu^2 \equiv \frac{4(v \cdot n)^2}{|n^2|} \quad \nu = \frac{P_j}{P_j^0}\]
Predictions for jet mass distribution

- NLL in resummation
- NLO in initial condition
- CTEQ6L PDFs
Jet energy profiles
Jet energy function

- Define jet energy function $J^E(r)$ by associating $k^0_i \Theta(r - \theta_i)$ with each final-state particle $i$ within jet cone $r$, $r<R$
- Still vary Wilson direction $n$.
- Separation $\sum_i k^0_i \Theta(r - \theta_i) = \sum_i k^0_i \Theta(r - \theta_i) + l^0 \Theta(r - \theta)$
- First term gives $[G^{(1)} + K_v^{(1)} + K_r^{(1)}] \otimes J^E$

Differentiated soft real gluon negligible in soft region
Soft gluon effect

- Differentiated soft real gluon renders jet axis of other particles inclined by small angle \( l^0 \sin \theta / P^0_j \).
- This jet axis can not go outside of the subcone.

\[
l^0 \sin \theta / P^0_j < r
\]

Jet axis of other particles

Axis of total jet

Inclination angle
Resummation equation

- Resummation equation for jet profile

\[ K_r^{(1)}(1) = g^2 C_F \int \frac{d^4l}{(2\pi)^3} \frac{n^2}{(n \cdot l + i\epsilon)^2} \delta(l^2 - a^2) \Theta \left( r - \frac{|l| \sin \theta}{P^0_j} \right) \]

\[ - \frac{n^2}{v \cdot n} v_\alpha \frac{d}{dn_\alpha} \bar{J}_q^E (1, P_T, \nu^2, R, r) \]

\[ = 2[G^{(1)} + K^{(1)}(1)] \bar{J}_q^E (1, P_T, \nu^2, R, r) \]

- Have considered N=1 here, corresponding to integration over jet mass (insensitive to nonperturbative physics)

- Resum \( \alpha_S \ln^2 r, \alpha_S \ln r \) from phase space constraint for real gluons
Comparison with CDF data

\[ \Psi(r) = \frac{1}{N_{\text{jet}} \sum_{\text{jets}}} \frac{P_T(0, r)}{P_T(0, R)}, \quad 0 \leq r \leq R \]

quark, gluon jets, convoluted with LO hard scattering, PDFs
Compasion with CMS data

- $20 \text{ GeV} < P_T < 30 \text{ GeV}$
- $40 \text{ GeV} < P_T < 50 \text{ GeV}$
- $60 \text{ GeV} < P_T < 80 \text{ GeV}$
- $80 \text{ GeV} < P_T < 100 \text{ GeV}$
Heavy-quark jet

Work in progress
Factorization (semileptonic)

• Factorize heavy quark-quark jet first at jet energy scale $E_Q$, which contains weak decay
Scale hierarchy $E_Q \gg m_Q \gg m_J$

- The two lower scales $m_Q$ and $m_J$ characterize different dynamics, which can be factorized

![Diagram showing light-quark jet and heavy-quark kernel]
Further factorization (HQET)

• Then factorize the light-quark jet from the total jet at leading $1/m_Q$

\[
J_Q = H_Q \otimes J
\]
Summary

• Jet substructure improves jet identification
• Perturbative calculation is not reliable in extreme kinematic region (e.g. small mJ)
• Event generators may have ambiguities (from tuning scales for coupling constant)
• QCD resummation provides reliable prediction, independent check, and alternative approach
• Analyzed jet function and profile for light particles. Results consistent with current data
• Numerical work on heavy-quark jet in progress
Back-up slides
Jet Finding

- **Calorimeter jet (cone)**
  - jet is a collection of energy deposits with a given cone $R$: $R = \sqrt{\Delta \varphi^2 + \Delta \eta^2}$
  - cone direction maximizes the total $E_T$ of the jet
  - various clustering algorithms

- **Particle jet**
  - a spread of particles running roughly in the same direction as the parton after hadronization

→ correct for finite energy resolution
→ subtract underlying event
→ add out of cone energy
Underlying Event & Hadronization Correction

- UE and hadronization effects are in the opposite directions

CDF Run-2

- With $R=0.7$, the UE effect is larger than the hadronization effects.
- ~10% in cross section at low jet $P_T$