Determining influence of four-wave mixing effect on quantum key distribution

D N Vavulin, V I Egorov, A V Gleim, S A Chivilikhin
Saint Petersburg National Research University of Information Technologies, Mechanics and Optics
dima-vavulin@mail.ru

Abstract. We consider the possibility of multiplexing the classical and quantum signals in a quantum cryptography system with optical fiber used as a transmission medium. If the quantum signal is located at a frequency close to the frequency of classical signals, a set of nonlinear effects such as FWM (four-wave mixing) and Raman scattering is observed. The impact of four-wave mixing (FWM) effect on error level is described and analyzed in this work in case of large frequency diversity between classical and quantum signals. It is shown that the influence of FWM is negligible for convenient quantum key distribution.

1. Introduction
Modern quantum cryptography systems [1] that perform secret key distribution usually operate with not only a weak quantum signal in the secure channel, but also a classical (reference) signal used for clock synchronization in transmitter and receiver modules, and data transfer. However, in most experiments two different fibers are currently used for this purpose: one for the quantum and another for the classical channel. At the same time, a necessity of installing a separate optical fiber for each quantum channel may lead to significant costs and make quantum cryptography economically ineffective.

If the classical signal and the quantum signal, which is at least eight orders of magnitude less intense, are transmitted at close frequencies, a number of nonlinear effects that dramatically decrease the signal-noise ratio of the quantum channel would appear. Two effects that are most important in case of quantum communications are four-wave mixing (FWM) and Raman scattering. Several recent papers investigate the possibility of using a single fiber for quantum key distribution (QKD) and duplex information channel, with most attention paid to Raman scattering [2-10].

Quantum bit error rate (QBER), an important parameter that limits secure key generation rate and transmission distance in quantum cryptography systems, is determined by the following formula:

\[ QBER = QBER_{opt} + QBER_{det} + QBER_{int} + QBER_{FWM} + QBER_{SRS}. \]

The first term, \( QBER_{opt} \), is due to the polarization extinction ratio resulting mainly from passive optical components within the optical system. This is a measure of the optical quality of the setup and is independent of transmission distance. The second term, \( QBER_{det} \), arises from the detector dark counts. The third term, \( QBER_{int} \), is the intersymbol interference, where errors are caused by counts from a given clock period that leak into the adjacent period due to source jitter, detector jitter, and dispersion in the transmission medium. The third term is responsible for the FWM. The fourth term is responsible for the spontaneous Raman scattering. Typically, researchers consider only the first three
terms, because the FWM and spontaneous Raman scattering only appear when multiplexing of classical and quantum signals. In this work we analyzed the influence of FWM for different classical signal power levels. In classical telecommunication systems FWM is the key parasitic effect in DWDM [1]. Due to this effect, the quantum key transmission at a frequency close to the frequency of the classical signal becomes greatly difficult. Therefore, in our calculations we used 1310 nm wavelength for quantum channel and 1550 nm for classical channel. In this case of distant bands not only FWM, but also Raman effects are minimized. However, quantum signal adsorption would be significantly higher, thus limiting QKD bitrate and distance.

2. Initial equations, parameters and conditions

In our calculations we used 1310 nm wavelength for quantum channel and 1550 nm for classical channel. In this case of distant bands not only FWM, but also Raman effects are minimized. However, quantum signal adsorption would be significantly higher, thus limiting QKD bitrate and distance. Initial equations for solving the stated problem were taken from [11]. These equations describe alterations of x and y components of complex amplitudes of signals passing through an optical fiber: pump field (p – classic channel, synchronization pulse), signal field (s – quantum channel) and idler wave (i – parasite wave that appears due to FWM). We used equations only for x components pump, idler and signal waves, while the y components were assumed to be zero, i.e. all three waves had the same polarization.

Since we are using only x-polarization component for the signal wavelength and pump wavelength without y-polarization component and excluding the longitudinal field components in the optical fiber (z-components), in our case the effect of FWM process is the greatest. We have shown that the effect of FWM is not critical even in this case; therefore, there is no need to examine any other wave polarization state.

Birefringence was not considered in our model. Then, we added terms that describe attenuation in the fiber, which leads us to final system:

\[ \dot{\phi}_p = 0 \]  \hspace{1cm} (1)
\[ \dot{\phi}_s = 0 \]  \hspace{1cm} (2)
\[ \dot{\phi}_i = 0 \]  \hspace{1cm} (3)
\[ i \frac{\partial \phi_p}{\partial z} + \frac{8}{9} (2\phi_p \phi_i^* + \phi_i \phi_p^* + 2\phi_p \phi_s \phi_i^* e^{i\Delta k} + 2\phi_p \phi_i \phi_s^* e^{-i\Delta k}) + i\alpha(\omega_p)\phi_p = 0 \]  \hspace{1cm} (4)
\[ i \frac{\partial \phi_s}{\partial z} + \frac{8}{9} (\phi_p^2 e^{-i\Delta k} + 2\phi_i \phi_s \phi_i^* + 2\phi_i \phi_s \phi_p^* + \phi_s^2 \phi_i^*) + i\alpha(\omega_s)\phi_s = 0 \]  \hspace{1cm} (5)
\[ i \frac{\partial \phi_i}{\partial z} + \frac{8}{9} (\phi_p^2 e^{i\Delta k} + 2\phi_i \phi_s \phi_i^* + 2\phi_i \phi_s \phi_p^* + \phi_s^2 \phi_i^*) + i\alpha(\omega_i)\phi_i = 0 \]  \hspace{1cm} (6)

Fields \( \phi_p, \phi_s, \phi_i \) here are complex amplitudes of x polarization components of pump (p), signal (s) and idler fields. Correspondingly, \( \phi_{p,x}, \phi_{s,x}, \phi_{i,x} \) are complex amplitudes of y polarization components of pump (p), signal (s) and idler fields. Amplitudes here are measured in W^{1/2}, so \( |\phi_{p,x}|^2 \) is peak power of pump filed x component in Watts. In these equations we are using the following normalization: longitudinal coordinate z here is a product of physical fiber length Z, measured in km and Kerr nonlinearity coefficient \( \gamma \), measured in (W/km)^{-1} [11].

In this work relatively small signal intensities were studied, so Kerr nonlinearity is not observed, and \( \gamma \) value was 1 (W/km)^{-1}, that is typical for optical fibers. Therefore, longitudinal coordinate z is numerically equal to physical fiber length Z. \( \Delta k \) is a variable known from FWM theory defined below. Pump, signal and idler wave frequencies do not depend on wave polarization: \( \omega_{p} = \omega_{2p} = \omega_{p}, \omega_{s} = \omega_{2s} = \omega_{s}, \omega_{i} = \omega_{2i} = \omega_{i} \). Values of \( \alpha_{p} (\omega_{p}) \), \( \alpha_{s} (\omega_{s}) \) and \( \alpha_{i} (\omega_{i}) \) are decay coefficients for pump, signal and idler fields respectively.
Value of $\Delta k$ is calculated as follows: $\Delta k = k_i + k_s - 2k_p$. Here $k_i = n_i \omega_i / c$ is propagation constant of idler wave at frequency $\omega_i$, $c$ – speed of light in vacuum, $n_i$ – fiber refraction index at $\omega_i$ (similar for $k_s$ and $k_p$). Wave frequencies are bounded by condition: $2\omega_p = \omega_s + \omega_i$. Here $\omega_p$ corresponds to wavelength $\lambda_p = 1.55$ µm, and $\omega_s$ – to $\lambda_s = 1.3$ µm. Thus, idler wavelength was found as:

$$\lambda_i = (\lambda_p \ast \lambda_s) / (2\lambda_p - \lambda_s) = 1.92$$ µm

Indices of refraction for all wavelengths: $\lambda_p$, $\lambda_s$, $\lambda_i$ were calculated using Sellmeier equation. Propagation constants $k_s$, $k_p$, $k_i$ were calculated (with corresponding $n$ and $\lambda$) by definition:

$$k = 2\pi n / \lambda .$$

In our calculations we used standard decoy coefficients for telecommunication fibers (SMF-28) at target frequencies: $\alpha_s (1.3 ~\mu m) = 0.34$ dB/km, $\alpha_p (1.55 ~\mu m) = 0.2$ dB/km, $\alpha_i (1.92 ~\mu m) = 6.5$ dB/km.

Classical (p) and quantum (s) channels are initially ($z = 0$) defined by their complex amplitudes, which corresponds to the following powers:

$$P_{1p} = 10^{-3} ~W, P_{2p} = 0 ~W$$
$$P_{1s} = 10^{-11} ~W, P_{2s} = 0 ~W$$

Idler wave is not excited: $P_{1i} = 0 ~W, P_{2i} = 0 ~W$

These values of complex amplitudes for classic and quantum signals were chosen keeping in mind that power of a common laser source in fiber is around 1 uW, and power of attenuated laser used as a single photon (coherent states) source is 10 pW.

It must be noted that although they describe classic (intense) light field, they can still be applied to quantum cryptography. The reason for it is that even in latest experiments dedicated to creating QKD nets [8, 9] a strongly attenuated laser radiation is used as a light source, and coherent state field with mean photon number $< 1$ is treated as quantum bits. For this assumption, experimental data demonstrates good compliance with theory.

The calculations were carried out for the case when the field in a quantum channel is assumed continuous. This is due to the fact that in the quantized mode we consider each photon of secret key as a separate wave which propagates together with the classical wave and generates FWM. In this case, the FWM efficiency is equal to the average QBER at long-term transfer of the key. It is responsible for the applicability of these equations to assess the impact of FWM effect.

3. Numerical calculation

Figure 1 shows the results of calculation for pumping wave (p, classical channel), signal wave (s, quantum channel) and idler wave (i) at 100km distance. For illustration the values for quantum channel and idler waves were increased in $10^5$ and in $10^{17}$ times correspondingly.

**Figure 1.** Calculation for pumping wave (p, classical channel), signal wave (s, quantum channel) and idler wave (i) at 100km distance.
Figure 2 illustrates the difference between intensities of signal waves ($\Delta P_s$) in presence of pumping (1 mW) and without, compared to signal wave intensity (with pumping). The initial intensity of signal wave is about $10^{-11}$ W, and, maximal difference between the signal intensities is around $10^{-21}$ W, i.e. 10 orders smaller. Consequently, the presence of classical signal has almost no affect at quantum signal.

![Figure 2](image)

**Figure 2.** The difference between intensities of signal waves in presence of pumping and without it compared to signal wave intensity: 1 mW pumping

Calculation of FWM efficiency dependence on fiber length was performed for 1 mW pumping. FWM efficiency was defined as the difference between signal waves intensities with pumping and without it, divided by the intensity of signal wave in presence of pumping (figure 3). We can see that FWM efficiency is limited by some value at large fiber length.

![Figure 3](image)

**Figure 3.** Difference between signal waves with pumping (1 mW) and without it, divided by signal wave intensity in presence of pumping.
4. Conclusion
It was shown that four-wave mixing effects on quantum key distribution are negligible if the classical and quantum channels are transmitted in different optical fiber transparency windows, because mean intensity of idler photons is 17-18 orders of magnitude lower than that of the quantum signal. This statement is also true in case of using DWDM in information (pump) channel. According to literature data, in this case the effects of Raman scattering can also be minimized at certain conditions. At the same time, at 1310 nm wavelength the quantum key generation rate related to losses in optical fiber would decrease about 10 times for large distances. Therefore, large frequency diversity of quantum and classical channels allows finding an optimal balance between nonlinear effect influence and raw key rate for small and medium distances (up to 100 km).

5. Acknowledgements
We would like to thank V.V. Kozlov for his participation in this work. The work was supported by the Dynasty foundation Grant for students and by the St. Petersburg government grant for students. The work was also supported by program of SPIE Scholarships.
This work was financially supported by Government of Russian Federation, Grant 074-U01.

References
[1] Scarani V, Bechmann-Pasquinucci H, Cerf N et al. 2009 The security of practical quantum key distribution. Rev. Mod. Phys., 81, 1301-1350.
[2] Patel K A, Dynes J F, Choi I, Sharpe A W, Dixon A R, Yuan Z L, Penty R V, and Shields A J 2012 Coexistence of High-Bit-Rate Quantum Key Distribution and Data on Optical Fiber. PHYSICAL REVIEW X, 2, 041010
[3] Lin Q, Yaman F, and Agrawal G P 2007 Photon-pair generation in optical fibers through four-wave mixing: Role of Raman scattering and pump polarization. PHYSICAL REVIEW A 75, 023803
[4] Chapuran T E, Toliver P, Peters N A, Jackel J, Goodman M S, Runser R J, McNown S R, Dallmann N, Hughes R J, McCabe K P, Nordholt J E, Peterson C G, Tyagi K T, Mercer L, and Dardy H 2009 Optical networking for quantum key distribution and quantum communications, New J. Phys. 11, 105001
[5] Townsend P D 1997 Simultaneous quantum cryptographic key distribution and conventional data transmission over installed fibre using wavelength-division multiplexing, Electron. Lett. 33, 188–190.
[6] Choi, Young R J, and Townsend P D 2011 Quantum information to the home, New. J. Phys. 13, 063039
[7] Eraerds P, Walenta N, LeGr’e M, Gisin N, and Zbinden H 2010 Quantum key distribution and 1 Gbps data encryption over a single fibre, New J. Phys. 12, 063027
[8] Lancho D, Martinez J, Elkouss D, Soto M, and Martin V 2010 QKD in standard optical telecommunication networks, Lect. Notes Inst. Comput. Sci., Soc. Inf. Telecom. Eng. 36, 142–149
[9] Townsend P D 1998 Experimental Investigation of the Performance Limits for First Telecommunications-Window Quantum Cryptography Systems, IEEE Photonics Technology Letters Vol. 10, No. 7, 1048-1050
[10] Qi B, Zhu W, Qian L, and Lo H K 2010 Feasibility of quantum key distribution through a dense wavelength division multiplexing network, New J. Phys. 12, 103042
[11] Guasoni M, Kozlov V V, Wabnitz S 2012 Theory of polarization attraction in parametric amplifiers based on telecommunication fibers J. Opt. Soc. Am. B, 29 №10, 2710-2720
[12] Egorov V I, Vavulin D N, Latypov I Z, Gleim A V, Rupasov A V 2013 Analysis of a sidebands based quantum cryptography system with different detector types NANOSYSTEMS: PHYSICS, CHEMISTRY, MATHEMATICS, 4 (2), pp. 190–195

5
Ivanova A E, Egorov V I, Chivilikhin S A, Gleim A V 2013 Investigation of quantum random number generation based on space-time division of photons *NANOSYSTEMS: PHYSICS, CHEMISTRY, MATHEMATICS*, V. 4(4), pp. 505-554