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System analysis of formation and perception processes of three-dimensional images in volumetric displays

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Abstract. One of the promising devices is currently a volumetric display. Volumetric displays capable to visualize complex three-dimensional information as nearly as possible to its natural – volume form without the use of special glasses. The invention and implementation of volumetric display technology will expand opportunities of information visualization in various spheres of human activity. The article attempts to structure and describe the interrelation of the essential characteristics of objects in the area of volumetric visualization. Also there is proposed a method of calculation of estimate total number of voxels perceived by observers during the 3D demonstration, generated using a volumetric display with a rotating screen. In the future, it is planned to expand the described technique and implement a system for estimation the quality of generated images, depending on the types of biplanes and their initial characteristics.

1. Introduction
One of the development trend of three-dimensional visualization is volumetric display systems. They create volumetric image by voxels (volumetric pixels) or vectors that fill the working space of the display. Working space is determined by the structural features of the device.

Important factors that significantly affect the widespread use of volumetric displays are: the cost, image quality, the possibility of simultaneous unlimited viewing without the using of special tools (various glasses, etc.). Due to the additional opportunities for physical realization that have appeared recently, the volumetric technology is on trend now. However, there are a number of problems need to be solved for the successful use of color-intensive dynamic visualization and eliminate existing contradictions.

The first, and the most significant of them is the contradiction between the required image quality and the data flow necessary to provide it. The color and size of the resulting volume image is substantially limited, in spite of use of high-technology projection devices and computing resources. It is still difficult or cost intensive to create real time volumetric image.

The second, there are the technical, mathematical and software development of tools for quality control of a generated three-dimensional visualization. The task is quite difficult and it includes an assessment of the prospects for the technical implementation of displays of a given type, as well as the rate of perception of the generated image.

Volumetric display technology is based on the principle of scattering or emission of a light from a certain point in the working space of the display. «Inertia» of a human vision creates an illusion of the object which volume fills by volumetric pixels, so-termed voxels [1–4]. The
implementation of the working surface of the display can vary. There are layered arrangement of screens, rotating screen, screen with forward-backward motion, etc.). Thus, it is relevant to consider the process of forming volumetric images from the perspective of system analysis for evaluation the quality of observed images. It could help in developing of promising solutions in the design and use of volumetric displays. The work is carried out as part of the volumetric display development project [5,6] with the use of the method described in [7].

2. Problem definition
There are a number of tasks that must be solved to construct volumetric display and also analyze quality of formed volumetric image. There are:

- Development of a semantic network, which describes relations between volumetric displays and tasks of volumetric dynamic visualization;
- Characteristics of variables describing the process of volumetric image formation;
- Determination of the observed resolving power of volumetric displays with a moving flat screen;

Now, we are move to the consideration of the results of solving above-described points.

3. The development of the semantic network for describing the object domain related with volumetric visualization
Analysis of various types of displays allowed to offer a semantic network describing the subject area associated with the formation of three-dimensional images by volumetric displays. Its fragment is shown in Fig.1.

![Figure 1. Fragment of semantic network](image-url)

According to this scheme, the key concept is a 3D-display, which creates a three-dimensional image that has sets of properties. Among this set, we can split it into a subset of the monocular
and binocular types. The monocular type has the following characteristics: parallax motion and object rotation. The binocular type, in turn, has the following characteristics: convergence and binocular disparity. If the image is formed from two-dimensional, then according to the full version of the semantic network, it has the subset, which is described by the following characteristics: color depth, resolution, size, brightness, contrast, etc.

In the future, the resulting semantic network is used to construct a mathematical model of the process of volumetric visualization, which makes it possible to automate the formation of a description of a specific version of the technical implementation of a volumetric display and the characteristics of its intermediate states obtained at various stages of the process.

As a result of the analysis, the main stages of the transformation of the source images into the final volumetric image were determined.

Source images $X$, sufficient to reproduce volumetric virtual object, are fed to the processing and the formation unit of primary images $P$. Source images include objects that are obtained using three-dimensional models, two-dimensional images from different angles or specially prepared "slices", and also video information for creating an animated visualization.

One of the most important stages in the structural sequence of the formation of the volumetric image $P$ is the stage of creating the primary images, i.e. a cluster of two-dimensional images of the object, which forms a three-dimensional image (Fig.2).

![Figure 2. A generalized scheme of transformations of the three-dimensional image formation](image)

As a result of the transfer of the primary images $P$ using the output device, a composite image $Q$ is obtained, and the synchronization of the component parts of the images is required to correctly combine them. As a result, the image $Q$ is synchronized in time and space, which is then transmitted to the optical system to generate the final volumetric image $Y$, which is directly perceived by the observer (user), taking into account his psychophysiological features $Z$. As the result we will get the final volumetric image $Y$.

4. Characteristics of variables describing the process of building 3D images

It is necessary to identify many characteristics of two-dimensional images that affect the final result. These characteristics determine the scales for measuring the values of the input and output variables of the system. It is necessary to take into account that the various technical means used impose certain restrictions on the values being formed. These restrictions also affect the final three-dimensional image and determine the quality of the perception of volume by a person.

Initial images are primary two-dimensional images, which are usually characterized by such parameters as color depth, resolution, size and brightness of the image.
The main ones are the characteristics of the images, which, in turn, depend on the parameters of the output devices: monitors, projectors or graphics cards of the primary image forming device. The quality of the output volumetric image directly depends on the degree of technical realizability of these parameters. The main criteria for the transfer of graphic information are speed, resolution, frequency, color rendition and viewing angle. In our case, these criteria are the input signals of the system.

The above characteristics affect the quality of flat images, therefore monocular and binocular characteristics should be considered as criteria for the perception of volume (Table 1). To estimate the degree of influence of image characteristics on the final volume perception, weights are introduced for image characteristics. Values of the coefficients are determined on the basis of expert estimates of the degree of influence of the system parameters on the final result. Playback with the specified values of the characteristics will allow to form a three-dimensional image to a certain extent.

We assign to the input variables the depth of the image, i.e. the ability of a person to perceive three-dimensionality of the image. Monocular attributes are taken into account when creating content. They are aimed at enhancing the perception of volume. Examples of similar monocular properties are: relative size of objects, texture gradient, relative positioning, linear and atmospheric perspectives, parallax motion; as well as binocular: binocular convergence, normal binocular disparity, occlusion.

The perception of the object is influenced by psychological factors and physiological characteristics of an individual. Therefore, one more aspect of the development of the method for the formation of a three-dimensional image is the need for its approbation at various groups of recipients.

Several output devices participate in the formation of the volume, which, when projecting the primary images onto the working plane, form a virtual volumetric object. When displaying a volumetric image through multiple devices, one of the main tasks is to synchronize them, while it is advisable to use the time of out-of-frame synchronization as the measured value.

The quality of the image formed by the autostereoscopic image system is estimated subjectively by the observer, the sensations of volume, clarity, color of the image are important at the same time. Therefore, at the first stage of the research, it is proposed to use a relatively rough ordinal scale with possible values: satisfactory, unsatisfactory, later on, more detailed estimates of the species may be used to improve accuracy: excellent, good, satisfactory, unsatisfactory.

As a unified mathematical apparatus for describing the proposed criteria and characteristics of the visualization process, it is proposed to use fuzzy models, which will allow to take into account the heterogeneity of the variables in different scales: from absolute to nominal.

5. The problem of the observed resolving power of a 3D display with a rotating flat screen

Using the above classification, it is planned to develop a mathematical model that will allow to connect the input variables of the volumetric displays with their output characteristics and find the most suitable ones. When constructing the model, a type of volumetric display with a rotating screen is selected for the subsequent analysis. It was widely used in various variations.

An example of such a display is Seelinder [8], which uses a rotating array of LEDs and a cylindrical parallax barrier. This achieves a viewing angle of 1 degree and a resolution of $1254 \times 256$ pixels.

Scientists from the University of Southern California report on the development of the Light Field Display [9], creating a three-dimensional structure (in a horizontal plane), which can be viewed from either side. The display includes a rapidly rotating mirror, with an anisotropic
Table 1. Characteristics and weights

| Monocular factors                                           | Coefficient |
|-------------------------------------------------------------|-------------|
| Motion parallax                                             | 0.05        |
| Object rotating                                             | 0.05        |
| Shadows                                                     | 0.07        |
| Perspective (direct, reverse, panoramic, spherical, air, perceptual) | 0.1         |
| Mutual overlapping of objects (occlusion)                   | 0.03        |
| Inhomogeneity of the shape                                  | 0.08        |
| Gradient texture                                            | 0.07        |

| Biniocular factors                                          |             |
|-------------------------------------------------------------|-------------|
| Convergence                                                 | 0.06        |
| Binocular disparity                                         | 0.15        |

| 2-Dimensional factors                                       |             |
|-------------------------------------------------------------|-------------|
| Size of the image area                                      | 0.07        |
| Viewing Angle                                               | 0.05        |
| Resolution                                                  | 0.06        |
| Lineage                                                     | 0.04        |
| Brightness                                                  | 0.03        |
| Contrast                                                    | 0.03        |
| Color bitness                                               | 0.04        |
| Refresh rate                                                | 0.02        |

| Total value                                                 | 1           |

diffuser applied to it; microcontroller, engaged in decoding video signal, which is transmitted before the standard display; high-speed video projector, as well as a personal computer with the usual characteristics. The creators of the display argue that the resulting solution allows you to see a three-dimensional picture from anywhere around the display point and an unlimited number of viewers.

The resulting image has the following characteristics: refresh rate 15-20 Hz; the diameter of the image is 0.13 m; the rotation speed of the screen is 5400-7200 r/s; angular resolution - 1.25°; two colors are displayed, the resolution is 768 x 768 pixels. Getting a volumetric image provides a video card that creates more than 5000 slices of a three-dimensional object per second. Another similar display is RayModeler [10]. This is a full-color volumetric cylindrical display with a resolution of 96x128 pixels, developed by Sony Corporation and presented at the SIGGRAPH 2010 conference. It allows viewing a 3D image created using a rotating substrate with LEDs attached to it, from any angle and simultaneously from several points.

Consider a generalized scheme of displays of this type (Fig.2) and on its basis we will solve the problem of determining the number of human units perceived by a person depending on the radius of the working area, the resolution of the display and the distance to the observer.

Here it is necessary to make a number of remarks: from different distances and at different angles of vision, the person perceives the same image differently. The image on the display screen of 4K from a distance of 3 meters is practically the same as the image on the display of a standard notebook located at a distance that is optimal for perception of vision. If the display screen is turned azimuthally, the perceived horizontal resolution of the screen will decrease sharply. The screen of the volumetric 3D display rotates around its own vertical axis and, therefore, if the
Figure 3. The scheme of viewing of volumetric display screen with a radius $R$ from a distance $L$ at a rotation angle $\alpha$, an observation angle $\beta$, and a angle of image resolution unit $\theta$.

The actual resolution of the supplied image is constant at different screen rotation angles, its perceived component will be substantially different, which must be taken into account when evaluating the quality of the 3D image being formed.

Proceeding from the geometric relations, we find $\Delta r$ - the interval that is minimally discernable by the observer on the surface of the rotating screen of radius $R$. Let us assume that the angle at which the eye, at a distance $L$ from the screen axis $O$, distinguishes two close vertical strokes is $\theta$.

Define the interval that cuts this angle at the current radius $r$. We get:

$$\Delta r = \frac{2L\cos(\alpha)\sin(\theta)}{\cos(2\alpha - 2\beta + \theta) + \cos(\theta)},$$

(1)

where $\Delta r$ is the visible interval of the image unit at the current angular position of the screen. Let the number of pixels that are placed horizontally on the screen (horizontal resolution) is...
The number of visible image units within this interval is equal to the ratio:

\[ \Delta p = \Delta r, \text{ i.e.} \]

\[ \frac{2R}{Ng} = \frac{2L\cos(\alpha)\sin(\theta)}{\cos(2\alpha - 2\beta + \theta) + \cos(\theta)}. \quad (2) \]

Expressing the angle of observation \( \beta \) and solving this equation, we obtain three roots, which determine the zone where these units pass into each other, i.e., boundaries of calculated areas:

\[ \beta_1 = \frac{4\alpha + 2\theta + 3\pi}{4} - \frac{\arcsin\left( \frac{LN\sin(\alpha + \theta) - LN\sin(\alpha + \theta) + 2R\cos(\theta)}{2R} \right)}{\cos(2\alpha - 2\theta + \beta) + \cos(\theta)}, \]

\[ \beta_2 = \frac{4\alpha + 2\theta - 3\pi}{4} - \frac{\arcsin\left( \frac{LN\sin(\alpha + \theta) - LN\sin(\alpha + \theta) + 2R\cos(\theta)}{2R} \right)}{\cos(2\alpha - 2\theta + \beta) + \cos(\theta)}, \quad (3) \]

\[ \beta_3 = \frac{4\alpha + 2\theta + 3\pi}{4} + \frac{\arcsin\left( \frac{LN\sin(\alpha + \theta) - LN\sin(\alpha + \theta) + 2R\cos(\theta)}{2R} \right)}{\cos(2\alpha - 2\theta + \beta) + \cos(\theta)}. \]

Without reaching the boundary, at \( \Delta p > \Delta r \), it is necessary to calculate the pixels directly perceived by the observer, i.e. The width of this part of the screen area should be divided by the width of the pixels. Passing through the boundary, with \( \Delta p < \Delta r \), we should use the integral characteristic, i.e. determine the number of intervals \( \Delta r \) that fit into the remaining part of the screen.

Here is an example of the screen position in which are formed three different zones: two pixel and one integral. The behavior of the zone boundaries is depend on the angle \( \alpha \) and angle \( \beta \) conduct based on the following data: \( L = 250, R = 100, Ng = 800, \alpha = -60^\circ, \theta = 1^\circ \):

\[ \frac{2 \cdot 100}{800} > \frac{2 \cdot 250 \cos(-60^\circ) \sin(1/60)}{\cos(-2 \cdot 60 - 2\beta + 1/60) + \cos(1/60)}. \quad (4) \]

Solving for the angle \( \beta \), we obtain: \( \beta_1 = 0.13246492, \beta_2 = 0.91502353, \beta_3 = -2.2265691 \).

Thus, it can be argued that the condition \( \Delta r > \Delta p \) is satisfied in the range from 0 to the first transition and from the second transition to \( \pi/2 \).

The length of the portion of the screen radius \( \Delta R \) corresponding to a certain increment of the angle \( \beta \) is equal to:

\[ \Delta R = \Delta R_2 - \Delta R_1 = \frac{L\sin(\beta + \Delta \beta)}{\cos(\alpha - \beta - \Delta \beta)} - \frac{L\sin(\beta)}{\cos(\alpha - \beta)} \quad (5) \]

For \( \Delta R \) can be written as:

\[ \Delta R = \frac{L\cos(\beta)}{\cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)} d\beta. \quad (6) \]

The number of visible image units within this interval is equal to the ratio: \( \Delta n = \Delta R/\Delta r \). Substituting the previously obtained expression for \( \Delta r \), we have:

\[ \Delta n = \frac{\cos(2\alpha - 2\beta + \theta) + \cos(\theta)}{2L\cos(\alpha)\sin(\theta)} \cdot \frac{L\cos(\beta)}{\cos(\alpha - \beta)} d\beta. \quad (7) \]
Expanding the expressions obtained into three separate relations and forming integrals of them, we obtain:

\[
\begin{align*}
    n_1 &= \frac{\cos(2\alpha + \theta)}{4\cos(\alpha)\sin(\theta)} \int_{\beta_3}^{\beta_2} \frac{4\cos(\beta)^3 - 2\cos(\beta)}{\cos(\alpha - \beta)} d\beta, \\
    n_2 &= \frac{4\sin(2\alpha + \theta)}{4\cos(\alpha)\sin(\theta)} \int_{\beta_3}^{\beta_2} \frac{\sin(\beta)\cos(\beta)^2}{\cos(\alpha - \beta)} d\beta, \\
    n_3 &= \frac{\cos(\theta)}{2\cos(\alpha)\sin(\theta)} \int_{\beta_3}^{\beta_2} \frac{\cos(\beta)}{\cos(\alpha - \beta)} d\beta.
\end{align*}
\]

(8)

The number of observed units (pixels) of the image along the horizontal direction of the screen with the given parameters of the system (display and observer) is equal to the sum of the solutions

**Figure 4.** Graph of changing the edges of zones depending on the angle of the screen rotation
of the integral expressions after substituting the found limits:

\[ n_1 = \frac{\cos(2\alpha + \theta)}{4\cos(\alpha)\sin(\theta)} \left( (\sin(3\alpha) - \sin(\alpha))(\ln(\cos(\alpha - \beta_1)) - \ln(\cos(\alpha - \beta_2))) + (\beta_1 - \beta_2) \times \right. \]
\[ \times (\cos(3\alpha) - \cos(\alpha)) + \sin(\alpha + 2\beta_2) - \sin(\alpha + 2\beta_1)), \]
\[ n_2 = \frac{4\sin(2\alpha + \theta)}{4\cos(\alpha)\sin(\theta)} \left( (\ln(\cos(\alpha - \beta_2)) - \ln(\cos(\alpha - \beta_3)))(\cos(3\alpha) - \cos(\alpha)) + (\beta_1 - \beta_2) \times \right. \]
\[ \times (\sin(3\alpha) - \sin(\alpha)) + (\cos(\alpha + 2\beta_3) - \cos(\alpha + 2\beta_2))), \]
\[ n_3 = \frac{\cos(\theta)}{2\cos(\alpha)\sin(\theta)} \left( (\ln(\cos(\alpha)\cos(\beta_1) + \sin(\alpha)\sin(\beta_1)) - \ln(\cos(\alpha)\cos(\beta_3) + \sin(\alpha)\sin(\beta_3))) - (\beta_3 - \beta_2)\cos(\alpha)) \right). \]

Determine the number of image units, depending on the rotation angle of the screen \( \alpha^\circ \). For the initial values we take: \( L = 250, R = 100, \alpha = 0^\circ - 90^\circ, \beta = 0^\circ - 90^\circ, \theta = 1', N_g = 800 \). We define the behavior of the limits of the transition zones \( \beta_1, \beta_2, \) and \( \beta_3 \) and, by results, construct a graph of these functions (Fig.3).

We calculate the change in the total number of observed image units, including unmodified pixels, depending on the angle of rotation of the screen and the results of calculations, we construct the corresponding graph (Fig.4).

**Figure 5.** Graph of the dependence of the number of pixels on the angle of the screen rotation

Multiplying the resulting virtual horizontal screen resolution by its vertical component, which is determined in a similar way, taking into account the absence of azimuthal variations, we determine the screen resolution perceived by the observer for each of its azimuthal positions.
Knowing the screen update time, it is possible to calculate the actual number of voxels visible by the observer during the demonstration of the volumetric image, and also to estimate this characteristic for a particular display and determine the optimal values according to the conditions of formation, data transmission and observation.

Conclusion
Thus, the article attempts to structure and describe the interrelation of the essential characteristics of objects in the area of volumetric visualization. An example of an estimate of the total number of voxels perceived by observers during the demonstration of 3D images formed using a volumetric display with a rotating screen is given. The calculations can be useful for optimizing the operating parameters of volumetric volumetric displays, and in particular for determining the relationship between the quality of the generated image and the flow of generated and transmitted information.

The next step is to use developed method for a comprehensive assessment of the quality and process of perception of volumetric images displayed using autostereoscopic devices of other types.

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