Ratchet transport of a two-dimensional electron gas at cyclotron resonance

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The driving of charge carriers confined in a quantum well lacking the center of space inversion by an alternating electric field leads to the formation of a direct electric current. We develop a microscopic theory of such a quantum ratchet effect for quantum wells subjected to a static magnetic field. We show that the ratchet current emerges for a linearly polarized alternating electric field as well as a rotating electric field and drastically increases at the cyclotron resonance conditions. For the magnetic field tilted with respect to the quantum well normal, the ratchet current contains an additional resonance at the first subharmonic of the cyclotron resonance.

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I. INTRODUCTION

The response of a conducting system to an alternating electric field is one of the central topics of research in solid-state electrodynamics. Besides the linear response, where the induced electric current oscillates at the frequency of the electric field and its amplitude linearly scales with the field amplitude, ac electric field can give rise to a direct motion of charge carriers. Such an electric rectification (or electronic ratchet effect) naturally occurs in macroscopically inhomogeneous structures such as diodes or field-effect transistors as asymmetric lattices or systems with asymmetric patterning. The ratchet transport of carriers arises also in macroscopically homogenous structures (homogenous in all three dimensions for bulk materials or homogenous in the plane for two-dimensional systems) provided the structures lack the center of space inversion. The ratchet effects are used for the study of spatial symmetry of semiconductor structures and also underlie the operation of fast detectors of microwave and terahertz radiation.

The efficiency of the generation of a dc current by an ac electric field can be considerably enhanced in an external magnetic field if the frequency of the ac field matches the carrier cyclotron frequency. Previously, such an enhancement of the electronic ratchet response at the cyclotron resonance conditions was demonstrated for the spin currents in HgTe quantum wells (QWs) and HgTe-based three-dimensional topological insulators. The magnetic field tilted with respect to the QW normal and the ac electric field polarized in the QW plane. For the latter geometry we find that the dc electric current has an additional resonant contribution. This resonance occurs when the frequency of the ac field matches the double cyclotron resonance, in the spectral range where the free-carrier absorption has no peculiarity. We also calculate the spatial distribution of the electrostatic potential induced by the ratchet current in infinite and finite-size samples and show that the distribution depends on the magnetic field strength, the sample geometry, and the boundary conditions used.

II. PERPENDICULAR MAGNETIC FIELD

We begin the study with the geometry of the static magnetic field \( B \) pointing along the QW normal \( z \), see Fig. 1. In this configuration, a dc electric current \( j^R \) emerges if the ac electric field

\[
E(t) = E_\parallel e^{i\omega t} + E^* e^{i\omega t},
\]

where \( E \) and \( \omega \) are the field amplitude and frequency, has both the in-plane \( E_\parallel \) and out-of-plane \( E_z \) components.

Figure 1 illustrates the microscopic mechanism of the dc current formation. The in-plane component \( E_\parallel(t) \) of the electric field together with the static magnetic field \( B \) causes the motion of electrons in elliptical orbits in the QW plane at the field frequency \( \omega \). Synchronously with this in-plane motions, the electric field out-of-plane component \( E_z(t) \) pushes the electrons to the top or bottom interfaces of the QW depending on the field polarity. The corresponding distributions of the electron density in the QW cross section for positive and negative \( eE_z \), where \( e \) is the electron charge, are sketched in the insets. The shift of the electron density along the \( z \) axis in the asymmetric QW results, in turn, in the modulation of the electron mobility at the field frequency \( \omega \). In the insets in Fig. 1 the QW asymmetry is modeled by placing the \( \delta \)-layer of impurities (black dots), which cause electron scattering and control the electron mobility, closer
which satisfies the Boltzmann equation induced by the electric force and the Lorentz force, respectively, and the static magnetic field is within the classical approximation. The rate of electron scattering in an asymmetric QW to first order in $E_z$ can be presented in the form

$$W_{pp'} = W_{pp'}^{(0)} + e E_z(t) W_{pp'}^{(1)},$$

where $W_{pp'}^{(0)}$ is the rate of scattering.

The rate of electron scattering in an asymmetric QW plane driven by $E_{||}(t)$ together with the mobility modulation at the same frequency induced by $E_z(t)$ leads to the generation of a direct electric current $j_R$. At the cyclotron resonance conditions, the amplitude of the oscillating electron motion in the QW plane increases and so does the efficiency of the dc current generation.

A quasi-classical theory of the orbital ratchet effect described above can be developed in the framework of the Boltzmann transport equation. In this approach, the electric field and the Lorentz force, respectively, acting upon the electrons. The approach is relevant if the collision integral has the form

$$\sum_{\nu} \left( E_{||}(t) + \frac{1}{c} [v \times B] \right) \frac{\partial f_p}{\partial p} = \text{St} f_p,$$

(2)

where $p$ is the momentum, $v = d\varepsilon_p/dp$ and $\varepsilon_p$ are the electron velocity and energy, respectively, and $\text{St} f_p$ is the collision integral. For elastic scattering, the collision integral has the form

$$\text{St} f_p = \sum_{p'} (W_{pp'} f_{p'} - W_{p'p} f_p),$$

(3)

where $W_{pp'}$ is the rate of scattering.

The rate of electron scattering in an asymmetric QW plane subjected to a static magnetic field $B$ at the cyclotron resonance conditions. The dc component of the electric current $j_R$ is much smaller than the mean kinetic energy of the electrons and the magnetic field is within the classical approximation. The rate of electron scattering in an asymmetric QW plane subjected to a static magnetic field $B$ within the classical regime, when $\hbar \nu$ is much smaller than the energy separation between the excited and ground electron subbands.

To solve the Boltzmann equation we decompose the distribution function $f_p(t)$ in the Fourier series of frequency and angular harmonics as follows

$$f_p(t) = \sum_{n,m} f^{n,m}(p) \exp(i m \varphi_p - i \omega t),$$

(7)

where $\varphi_p = \arctan(p_y/p_x)$ is the polar angle of $p$. Accordingly, the collision integral for the scattering rate $W_{pp'}$ dependent on $\varphi_p$ is rewritten in the form

$$\text{St} f_p = - \sum_{n,m} \sum_{m \neq 0} \left[ \frac{f^{n,m}}{\tau_m} + e \zeta_m \left( E_z f^{n-1,m} + E_z^* f^{n+1,m} \right) \right] \times \exp(i m \varphi_p - i \omega t).$$

(8)

where $\tau_m$ is the relaxation time of the $m$-th angular harmonic of the electron distribution function,

$$\tau_m^{-1} = \sum_{p'} W_{pp'}^{(0)} \left[ 1 - \cos m (\varphi_p - \varphi_{p'}) \right],$$

(9)

and

$$\zeta_m = \sum_{p'} W_{pp'}^{(1)} \left[ 1 - \cos m (\varphi_p - \varphi_{p'}) \right].$$

(10)
In the harmonics representation, the Boltzmann equation (2) has the form of the set of linear equations

\[\Gamma^{n,m} f^{n,m} + e\zeta_m (E_z f^{n-1,m} + E_z^* f^{n+1,m}) (1 - \delta_{m,0}) + e E_0 \cdot (\hat{o}_- K^m f^{n-1,m-1} + \hat{o}_+ K^m f^{n-1,m+1}) + e E_0^* \cdot (\hat{o}_- \hat{K}^m f^{n+1,m-1} + \hat{o}_+ \hat{K}^m f^{n+1,m+1}) = 0,\]

where \(\Gamma^{n,m} = 1/\tau_m - in\omega - i\omega c\), \(\omega c = eB_z/(mc)\) is the cyclotron frequency, \(m_e = p/v\) is the cyclotron mass, \(\hat{o}_+ = \hat{o}_x + i\hat{o}_y, \hat{o}_z \) and \(\hat{K}^m\) are the unit vectors along the \(x\) and \(y\) axes, respectively, and \(\hat{K}^m = d/d\pi \pm (m \pm 1)/\pi\).

In thermal equilibrium, when the ac electric field is absent, the distribution function contains only the harmonic \(f^{0,0}\) which is given by the Fermi-Dirac distribution \(f_0(\varepsilon_p)\). To first order in the electric field amplitude, solution of the equation set (11) has the form

\[f^{1,1} = -\frac{e \tau_1 E_0 \cdot \hat{o}_-}{2[1 - i(\omega + \omega c)\tau_1]} \frac{df_0(\varepsilon_p)}{dp},\]

\[f^{-1,-1} = -\frac{e \tau_1 E_0 \cdot \hat{o}_+}{2[1 - i(\omega - \omega c)\tau_1]} \frac{df_0(\varepsilon_p)}{dp},\]

where \(f^{-1,1} = (f^{1,1})^*, f^{1,-1} = (f^{1,1})^*, f^{-1,-1} = (f^{0,1})^*\). The dc electric current is determined by the time-independent asymmetric part of the distribution function described by the harmonics \(f^{0,\pm 1}\) for which one obtains

\[f^{0,1} = -\frac{e \tau_1 \zeta_1}{1 - i\omega c \tau_1} (E_z f^{1,1} + E_z^* f^{1,1}),\]

and \(f^{0,-1} = (f^{0,1})^*\).

The direct current density \(j\) can be then readily found from the general expression

\[j^R = 2e \sum_v [f^{0,1} \exp(i\varphi_p) + f^{0,-1} \exp(-i\varphi_p)],\]

where the factor 2 takes into account the spin degeneracy. Straightforward calculations show that the current density can be presented in the form

\[j^R = L_1 (E_1 E_2^* + E_2 E_1^*) + L_2 \hat{o}_z \times (E_1 E_2^* + E_2 E_1^*) + C_1 i(E_1 E_2^* - E_2 E_1^*) + C_2 \hat{o}_z \times i(E_1 E_2^* + E_2 E_1^*),\]

where \(\hat{o}_z\) is the unit vector along the \(z\) axis. The coefficients \(L_1, L_2, C_1, C_2\) for a degenerate electron gas are given by

\[L_1 = -\frac{e^3 N_e}{2m_e} \frac{\zeta_1^2}{1 + \omega^2 \tau_1^2} \sum_{s = \pm 1} \frac{1 - s\omega_c (\omega + s\omega_c) \tau_1^2}{1 + (\omega + s\omega_c)^2 \tau_1^2},\]

\[L_2 = -\frac{e^3 N_e}{2m_e} \frac{\zeta_1^2}{1 + \omega^2 \tau_1^2} \sum_{s = \pm 1} \frac{\omega_c (\omega - s\omega_c) \tau_1^2}{1 + (\omega - s\omega_c)^2 \tau_1^2},\]

\[C_1 = -\frac{e^3 N_e}{2m_e} \frac{\zeta_1^2}{1 + \omega^2 \tau_1^2} \sum_{s = \pm 1} \frac{\omega_c (\omega + s\omega_c) \tau_1^2}{1 + (\omega + s\omega_c)^2 \tau_1^2},\]

\[C_2 = -\frac{e^3 N_e}{2m_e} \frac{\zeta_1^2}{1 + \omega^2 \tau_1^2} \sum_{s = \pm 1} \frac{s - s\omega_c (\omega + s\omega_c) \tau_1^2}{1 + (\omega + s\omega_c)^2 \tau_1^2},\]

where \(N_e = p_F^2/(2\pi \hbar^2)\) is the electron density, \(p_F\) is the Fermi momentum, and \(\omega_c, \tau_1, \zeta_1\) are taken at the Fermi level.

The coefficients \(L_1\) and \(L_2\) describe the electric current excited by a linearly polarized ac electric field (linear ratchet current) whereas \(C_1\) and \(C_2\) characterize the current contribution which is induced by an elliptically or circularly polarized field and has opposite directions for the right-handed and left-handed polarized radiation (circular ratchet current). In accordance with general symmetry arguments, \(C_1\) and \(L_1\) describing the current component along \(E_\|\) are even functions of the static magnetic field \(B_z\) while \(C_2\) and \(L_2\) describing the perpendicular component of the current are odd functions of \(B_z\).

The dependences of the linear ratchet current and the circular ratchet current on the field frequency \(\omega\) are shown in Figs. 2a and 2b, respectively. At zero frequency, only the linear ratchet current is generated. Its direction in the QW plane with respect to \(E_\|\) is determined by the ratio \((L_2/L_1)_{\omega = 0} = -2\omega c \tau_1/[1 - (\omega_c \tau_1)^2]\). The deflection of the current direction from \(E_\|\) is caused by the Lorentz force acting upon the electrons. The circular ratchet effect emerges at finite frequencies of the ac electric field. The magnitude of the circular ratchet current is proportional to \(\omega\) at small \(\omega\), the current direction is determined by the ratio \(C_2/C_1 = -\omega c \tau_1/[1 - (\omega c \tau_1)^2]\). At the cyclotron resonance conditions, both the linear and the circular ratchet currents drastically increase. Their directions in the QW plane are very sensitive to the frequency detuning \(\omega - \omega_c\), see Figs. 2a and 2b. Finally, far from the cyclotron resonance, the high-frequency asymptotic behavior of the ratchet currents is described by \(L_1 \propto 1/\omega^2, L_2 \propto 1/\omega^4\), and \(C_1, C_2 \propto 1/\omega\).

The magnitude of the ratchet current can be estimated from Eqs. (15) and (16). For the electric field amplitude \(E = 10 \text{kV/cm}\), the static magnetic field \(B = 4 \text{T}\), the momentum relaxation time \(\tau_1 = 10^{-12} \text{s}\), the effective mass \(m_e = 0.07 m_0\) (corresponding to GaAs-based QWs), the carrier density \(N_e = 2 \times 10^{11} \text{cm}^{-2}\), the QW width \(d = 10 \text{nm}\), and the degree of QW structure inversion asymmetry \(\langle \text{Re} V_{11}^\text{R}(p,p') V_{12}(p,p') \rangle/\langle |V_{11}(p,p')|^2 \rangle = 0.1\), an estimation gives \(j^R \sim 2 \mu \text{A/cm}\) for the ratchet current at the cyclotron resonance.

III. TILTED MAGNETIC FIELD

Now we consider the geometry of a tilted static magnetic field \(B\) and ac electric field \(E(t)\) polarized in the QW plane. In this case, an asymmetry of the electron distribution in the momentum space and, hence, a dc electric current may emerge due to the asymmetry of electron scattering induced by the in-plane component \(B_\parallel\) of the magnetic field. Such a magnetic quantum ratchet effect in a purely in-plane magnetic field was theoretically studied in Refs. [15,16] and has been recently observed in graphene [17]. Here, we develop a microscopic theory of the effect for a tilted magnetic field where the in-plane com-
ponent $B_\parallel$ induces the scattering asymmetry while the out-of-plane component $B_z$ causes the cyclotron motion of the electrons and leads to a resonant enhancement of the ratchet current.

To first order in $B_\parallel$, the rate of elastic electron scattering can be generally presented in the form

$$W_{pp'}^{(1)} = W_{pp'}^{(0)} + eB_\parallel W_{pp'}^{(1)},$$

where $W_{pp'}^{(0)}$ is the scattering rate at zero field and $W_{pp'}^{(1)}$ is the field induced correction; $W_{pp'}^{(0)} = W_{p-p,-p}$ and $W_{pp'}^{(1)} = -W_{p-p,-p}$ due to time inversion symmetry. Microscopically, the scattering asymmetry stems from the quantum analogue of the Lorentz force which pushes moving electrons to the top or bottom interfaces of the QW. For a QW structure with the simple parabolic energy spectrum, the term $W_{pp'}^{(1)}$ is given by

$$W_{pp'}^{(1)} = -\frac{4\pi}{\hbar m_e c} \left[ \frac{B_\parallel}{B_\parallel} (p_y + p_y') - \frac{B_y}{B_\parallel} (p_x + p_x') \right] \sum_{\nu \neq \nu'} \xi \left( \text{Re} V_{1\nu}^*(p,p') V_{1\nu}(p,p') \right) \delta(\varepsilon_p - \varepsilon_{p'}),$$

where $m_e$ is the effective mass.

To calculate the dc current due to the magneto-induced ratchet effect we solve the Boltzmann equation \(^2\) with the scattering rate \(^\text{17}\). In the harmonics representation, the Boltzmann equation assumes the form of the linear equation set

$$\Gamma_{n,m} n_{n,m} f_{n,m} + eB_\parallel \sum_l \left( u_{0,l} f_{n,m-l} - u_{m,l} f_{n-m} \right)$$

$$+ \frac{eE_\parallel}{2} \left( o_- K_{n-m-1}^m f_{n,-m-1} + o_+ K_{n-m}^m f_{n,m+1} \right)$$

$$+ \frac{eE_\parallel}{2} \left( o_- K_{n+1,m}^m f_{n+1,m+1} + o_+ K_{n+1,m}^m f_{n-m} \right) = 0,$$

where $u_{n,m}$ are defined by

$$u_{n,m} = \int \frac{d\varphi_p}{2\pi} \sum_{p'} W_{pp'}^{(1)} e^{-i\varphi_p - i\varphi_{p'}}.$$

The harmonics $u_{n,m}$ satisfy the conditions $u_{n,m} = u_{n,-m}$ and $u_{n,m} = (-1)^{n+m+1} u_{n,-m}$ due to the reality of the scattering rate and time inversion symmetry, respectively. It also follows from Eq. \(^\text{15}\) that, for central scattering, the only nonzero harmonics are $u_{n,n+1}$. We seek a solution of the equation set \(^\text{16}\) in the form of a perturbation series in $E$ and $B_\parallel$. To first order in the electric field amplitude, one obtains the harmonics $f_{n+1,\pm 1}^{\pm 1}$ which are given by Eqs. \(^\text{12}\). The second iteration in $E_\parallel$ and $B_\parallel$ yields

$$f_{1,2}^{1,2} = \frac{\tau_2 eB_\parallel (u_{2,-1} - u_{0,1}) f_{1,1}^{1,1}}{1 - i(\omega_c + 2\omega_c)\tau_2},$$

$$f_{1,-2}^{1,-2} = \frac{\tau_2 eB_\parallel (u_{2,1} - u_{0,-1}) f_{1,-1}^{1,-1}}{1 - i(\omega_c - 2\omega_c)\tau_2},$$

$$f_{0,2}^{0,2} = \frac{e\tau_2 (E_\parallel \cdot o_-) K_2 f_{1,1} + (E_\parallel^* \cdot o_-) K_2 f_{1,1}}{2(1 - 2i\omega_c\tau_2)},$$

and $f_{n,-m}^{n,m} = (f_{n,m})^*$.

The dc electric current is determined by the harmonics $f_{n,\pm 1}^{\pm 1} \propto E^2 B_\parallel$; they have the form

$$f_{0,1}^{0,1} = \frac{\tau_1 (E_\parallel \cdot o_+) K_1 f_{1,2}^{1,2} + (E_\parallel^* \cdot o_+) K_1 f_{1,2}^{1,2}}{2(1 - i\omega_c\tau_1)}$$

$$+ \frac{\tau_2 eB_\parallel (u_{1,-2} - u_{0,-1}) f_{0,2}^{0,2}}{1 - i\omega_c\tau_1},$$

and $f_{0,-1}^{0,1} = (f_{0,1}^{0,1})^*$.

Finally, calculating the dc electric current $j^R$ following the general expression \(^\text{14}\), we obtain

$$j_x^R = (\text{Re} M_0 + \xi_1 \text{Re} M_L + \xi_2 \text{Im} M_L + \xi_3 \text{Re} M_C) E^2 B_\parallel,$$

$$j_y^R = (-\text{Im} M_0 + \xi_1 \text{Im} M_L - \xi_2 \text{Re} M_L + \xi_3 \text{Im} M_C) E^2 B_\parallel,$$

where $\xi_1 = (|E_x|^2 - |E_y|^2)/|E|^2$, $\xi_2 = (E_x E_y^* + E_y E_x^*)/|E|^2$, and $\xi_3 = i(E_x E_y^* - E_y E_x^*)/|E|^2$ are the Stokes parameters determining the polarization state of the ac electric field. Accordingly, the parameters $M_0$, $M_L$, and $M_C$ are determined from the harmonics $u_{n,n+1}$.
$M_L$, and $M_C$ describe the magnitudes of the magneto-induced ratchet effect independent of the ac field polarization, the linear magneto-induced ratchet (LMR) effect, and the circular magneto-induced ratchet (CMR) effect, respectively. For a degenerate electron gas, the parameters are given by

$$M_0 = \frac{e^4 N_e \gamma^2 p_F}{4m_e^2} \left( \frac{1}{\Gamma_{0,0}} \right) \left( \frac{1}{\Gamma_{1,1}} + \frac{1}{\Gamma_{1,2}} \right),$$

$$M_L = -\frac{e^4 N_e}{4m_e^2} \left( \frac{\gamma p^2}{\Gamma_{0,1} \Gamma_{1,1}} \right) \left( \frac{1}{\Gamma_{1,1}} + \frac{1}{\Gamma_{1,2}} \right),$$

$$M_C = \frac{e^4 N_e \gamma^2 p_F}{4m_e^2} \left( \frac{1}{\Gamma_{0,0}} \right) \left( \frac{1}{\Gamma_{1,2}} \right) - \frac{1}{\Gamma_{1,2}} \right).$$

Where $\gamma = u_{-2,1} - u_{0,-1}$ is the coefficient describing the electron scattering anisotropy induced by the in-plane component of the magnetic field. All the values in Eq. (24) are taken at the Fermi level. We note that an additional contribution to the polarization dependent current may arise due to asymmetry of the energy relaxation of hot carriers in the magnetic field. This current depends on the details of electron-phonon interaction and is out of the scope of the present paper.

Figure 3 shows the frequency dependence of the LMR and CMR currents which are determined by $M_L(\omega)$ and $M_C(\omega)$, respectively. The curves are calculated for the case when the electron mobility is limited by Coulomb impurities, $\tau_0(\varepsilon_p) = 2\tau_2(\varepsilon_p) \propto \varepsilon_p$, and the intersubband scattering is determined by short-range defects, i.e., $\gamma$ is independent of the electron energy. The coefficient $\gamma$ is assumed to be real which corresponds to the geometry $B_n \parallel y$. One can see that both the LMR and CMR currents drastically increase at cyclotron resonance. However, the resonant contribution to the LMR current exceeds the resonant contribution to CMR current by the factor of $\omega_0\tau_1$. Moreover, the CMR effect emerges due to the energy dependence of the momentum relaxation time. The CMR current has an additional resonance at $\omega = 2\omega_c$, which occurs in the spectral range where the Drude absorbance has no peculiarity. In the quantum-mechanical picture, this additional resonance corresponds to the transitions between the Landau levels $n$ and $n+2$.

**IV. CURRENT DISTRIBUTION IN THE QUANTUM WELL PLANE**

In experiments, ratchet currents can be excited by the ac electric field of terahertz or microwave radiation focused on the sample. The dc electric signal can be measured via the voltage drop across contacts in the open-circuit configuration where the net electric current in the circuit is vanishingly small. The voltage drop is defined by the distribution of the electrostatic potential $\Phi(x,y)$ in the QW plane. The latter is determined by the spatial distribution of the radiation-induced ratchet current $j^R(x,y)$, the drift current $j^{DR}(x,y)$, which tends to compensate the ratchet current, and the boundary conditions at the sample edges.

The continuity equation requires

$$\nabla \cdot (j^R + j^{DR}) = 0. \tag{25}$$

The components of the drift current are given by

$$j^{DR}_\alpha = -\sum_\beta \sigma_{\alpha\beta} \nabla_\beta \Phi, \tag{26}$$

where $\sigma_{\alpha\beta}$ is the conductivity tensor in the magnetic field,

$$\sigma_{xx} = \sigma_{yy} = \frac{\sigma_0}{1 + (\omega_c \tau_1)^2},$$

$$\sigma_{xy} = -\sigma_{yx} = \frac{\omega_c \tau_1 \sigma_0}{1 + (\omega_c \tau_1)^2}, \tag{27}$$

$\sigma_0 = N_e e^2 \tau_1 / m_e$ is the conductivity at zero magnetic field.

From Eqs. (25)-(27) we obtain the Poisson equation

$$\Delta \Phi = (\nabla \cdot j^R) / \sigma, \tag{28}$$
where \( \sigma = \sigma_0/(1 + \omega^2 \tau^2) \). Equation (28) should be solved with the boundary conditions. For finite-size samples, we consider the boundary condition of zero total electric current flowing across the sample edges, \( j^{R \alpha} + j^{DR} = 0 \).

Solution of the Eq. (28) can be generally presented in the form

\[
\Phi(r) = \int j^R(r') \cdot \rho(r, r') dr',
\]

where \( \rho(r, r') \) can be interpreted as the function of nonlocal resistance. The explicit form of the function \( \rho(r, r') \) depends on the sample shape and, in general case, can be calculated numerically. Below we discuss the spatial distribution of the electrostatic potential in infinite, semi-infinite, and rectangular-shape structures and present some analytical results.

For an infinite two-dimensional system, the solution of the two-dimensional Poisson equation can be readily found by Green’s function method, which yields

\[
\rho(r, r') = -\frac{1}{2\pi\sigma} \frac{r - r'}{(r - r')^2}.
\]

The magnetic field \( B_z \) only scales the function \( \rho(r, r') \). However, the field also affect the magnitude and direction of \( j^R(r') \) and, hence, the spatial distribution of the electrostatic potential.

Using the method of mirror images and the function \( \rho(r, r') \) for the infinite system, one can derive an analytical equation for \( \rho(r, r') \) in a semi-infinite system. For the system \( x \geq 0 \) with a single boundary at \( x = 0 \), we obtain

\[
\begin{align*}
\rho_x(r, r') &= -\frac{1}{2\pi\sigma} \left[ \frac{x - x'}{(x - x')^2 + (y - y')^2} \right. \\
&\quad - \left. \frac{(1 - \omega^2 \tau_1^2)(x + x') - 2\omega_1 \tau_1(y - y')}{(1 + \omega^2 \tau_1^2)((x + x')^2 + (y - y')^2)} \right], \\
\rho_y(r, r') &= -\frac{1}{2\pi\sigma} \left[ \frac{y - y'}{(x - x')^2 + (y - y')^2} \right. \\
&\quad + \left. \frac{(1 - \omega^2 \tau_1^2)(y - y') - 2\omega_1 \tau_1(x + x')}{(1 + \omega^2 \tau_1^2)((x + x')^2 + (y - y')^2)} \right].
\end{align*}
\]

By adding three additional boundaries and considering a net consisting of rectangular cells, one can also obtain the function of nonlocal resistance for a finite-size rectangular structure with the edges at \( x = 0 \), \( a \) and \( y = 0 \), \( b \). The function is given by

\[
\begin{align*}
\rho_x &= \frac{1}{4\pi} \sum_{n=-\infty}^{\infty} \left[ \frac{(1/b) \sin(\pi r_-/b)}{\cosh(2\pi na/b - \pi r_-/b) - \cosh(\pi r_-/b)} \\
&\quad + \frac{1 - \omega^2 \tau_1^2}{1 + \omega^2 \tau_1^2} \cosh(2\pi na/b - \pi r_-/b) - \cosh(\pi r_-/b)} \\
&\quad + \frac{2\omega \tau_1}{1 + \omega^2 \tau_1^2} \cosh(2\pi nb/a + i\pi r_-/a) - \cosh(i\pi r_-/a) \right] + \text{c.c.},
\end{align*}
\]

where \( r_\pm = x \pm iy \), \( r'_\pm = x' \pm iy' \); the function \( \rho_y \) is obtained from \( \rho_x \) by replacing \( x \), \( y \), \( x' \), \( y' \), \( a \), and \( b \) with \( y \), \( -x \), \( y' \), \( -x' \), \( b \), and \( a \), respectively.

Figure 4 demonstrates the spatial distribution of the electrostatic potential \( \Phi(x, y) \) in a square-shape structure when the ratchet current is generated in the central area of the structure. The spatial distribution of the ratchet current density is taken in the form of the Gaussian function

\[
\mathbf{j}^R = j^R_0 \exp \left( -\frac{(x - a/2)^2 + (y - a/2)^2}{D^2} \right),
\]

where \( j^R_0 \parallel x \) and \( a \) is the structure size. The potential \( \Phi \) reaches extremal values at the positions close to the border of the current generation spot while at the sample edges it decreases. The magnetic field changes the potential amplitude and also twists the potential spatial distribution that can be seen from equipotential lines. The calculation shows that the voltage drop between the points of the highest and lowest potential is about 10 mV (32) for the spot radius \( D = 1 \text{ mm} \), the ratchet current magni-
tude $j_{\perp}^R = 2 \mu A/cm$ and the parameters of the structure presented in Sec. [1] The calculations also reveal that experimentally measured voltage drops across contacts may drastically depend on the contact positions in the structure and the structure geometry.

V. SUMMARY

We have developed a microscopic theory of the ratchet transport of electrons confined in a quantum well. It has been shown that the magnitude of the direct electric current induced by an alternating electric field is increased in an external static magnetic field at the cyclotron resonance when the electric field frequency matches the cyclotron frequency. The magnetic field gives rise also to an additional mechanism of the ratchet current generation which stems from an asymmetry of electron scattering in the magnetic field. The magneto-induced ratchet effect has a resonant behavior both at the cyclotron resonance and its first subharmonic, with the current being sensitive to the electric field polarization and the mechanism of electron scattering. We have also analyzed the spatial distribution of the electrostatic potential induced by the ratchet current and shown that the voltage drop between contacts is highly sensitive to the sample geometry and the contact positions. The resonant behavior of the ratchet current can be used for the study of electron scattering mechanisms and the development of tunable fast detectors of microwave and terahertz radiation.

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