COLOUR CONFINEMENT IN THE LATTICE LANDAU GAUGE QCD SIMULATION

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The colour confinement criterion proposed by Kugo and Ojima is tested in the lattice Landau gauge QCD simulation. The renormalization effects are studied by measuring the gluon propagator, ghost propagator, three gluon vertex and the ghost-antighost-gluon vertex. The running coupling $\alpha_s$ from ghost-antighost-gluon vertex in the infrared region yields $\Lambda_{MOOM} \approx 1\text{GeV}$, consistent to that from three gluon vertex in high momentum region.

1 The confinement signal and the definition of the gauge field

Two decades ago, Gribov pointed out a possible mechanism of colour confinement in Coulomb gauge or Landau gauge QCD via infrared divergence of the Faddeev-Popov ghost propagator. At nearly the same time, Kugo and Ojima proposed a criterion for the absence of coloured massless asymptoptic states in Landau gauge QCD using the BRST symmetry.

We study the confinement signal in the gluon propagator, the ghost propagator and the Kugo-Ojima parameter, and their dependence on the gauge field. Usually, the gauge field on lattice $A_\mu(x)$ is defined from the link variable $U_\mu(x,\mu) = \frac{1}{2}(U_{x,\mu} - U_{x,\mu}^\dagger)$, which we call $U$-linear version. A more natural definition is $U_{x,\mu} = \exp A_{x,\mu}$, $A_{x,\mu}^\dagger = -A_{x,\mu}$, which we call log $U$ version.

We observe that the Kugo-Ojima parameter $u^{ab}(0)$ at $\beta = 5.5$ is about -0.7 in the log $U$ and about -0.6 in the $U$-linear version.

If the configuration is in the core region, the tensor $G^{ab}_{\mu\nu} = \text{tr}(\lambda^a D_\mu \overleftrightarrow{D_\nu}(\lambda^b)_{xy}$ divided by $N^2 - 1$, where $N$ is the number of colours, is expected to approach a function $E(U)$ defined by the optimizing function. They are $E(U) = \sum_l \frac{1}{N} \text{Re} \text{tr} U_l$ in $U$-linear version, and $\frac{1}{N^2 - 1} \sum_{l,a} \text{tr} \left( \lambda^a S(A_l) \lambda^a \right)$, where $A_l = \text{adj} A_l$ and $S(x) = \frac{x^{\frac{1}{2}}}{\text{tr}(x^{\frac{1}{2}})}$, in log $U$ version.

In the table below $e_1$ and $e_2$ stand for $e = \frac{E(U)}{2}$, in our $16^4$ lattice simulation of the $U$-linear and the log $U$ version of the gauge fields, respectively.
We define the horizon function \( h = \frac{\langle H(U) \rangle}{3(N^2 - 1)} = c - e/d \). In general we expect \( h < 0 \) and in the continuum limit, we expect \( h = 0 \) when the configuration is in the core region.

Table 1: \( \beta \) dependence of the Kugo-Ojima parameter \( c \), trace \( e \) divided by the dimension \( d \), and \( h = c - e/d \). The suffix 1 corresponds to the \( u \)-linear and 2 corresponds to the log \( U \) version. Data are those of \( 16^4 \), except \( \beta = 5.5 \) \( U \)-linear data, which are those of \( 8^4 \).

| \( \beta \) | \( c_1 \) | \( c_1/d \) | \( h_1 \) | \( c_2 \) | \( c_2/d \) | \( h_2 \) |
|---|---|---|---|---|---|---|
| 5.5 | 0.570(58) | 0.780(3) | -0.21 | 0.712(18) | 0.908(1) | -0.20 |
| 6.0 | 0.576(79) | 0.860(1) | -0.28 | 0.628(94) | 0.943(1) | -0.32 |

The gluon propagator is infrared finite and the absolute value in log \( U \) version is about 20% larger than that in \( U \)-linear version. The corresponding difference in the ghost propagator is about 10%.

2 The QCD running coupling

The QCD running coupling \( \alpha(\mu) = g^2/4\pi \) can be measured from the three gluon vertices \( g(\mu^2) = \frac{G_A^{(3)}(p_i^2, p_f^2, p_c^2) Z_3^{(3)}(\mu^2)}{G_A^{(2)}(p_i^2) G_A^{(2)}(p_f^2) G_A^{(2)}(p_c^2)} \). Since the lattice data of \( G_A^{(2)} \) is infrared finite, in contrast to the conjecture that the gluon propagator is infrared vanishing, \( g(\mu^2) \) decreases as \( \mu \) decreases as \( 1.5\mu \). This behaviour does not agree with the results of Dyson-Schwinger approach, which suggest that \( \alpha_s(\mu) \) monotonically increases to a finite constant as \( \mu \) goes to 0.
The running coupling can be obtained also from the ghost-antighost-gluon coupling 
\[ \tilde{g}(\mu^2) = \frac{G_s^{(3)}(p_i^2, p_f^2, p_c^2)Z_t^{(2)}(\mu^2)Z_t(\mu^2)}{} \]. The preliminary results of 
\[ g^2/4\pi \] and \[ \tilde{g}^2/4\pi \] at symmetric momentum points \( \langle p_\mu \rangle = 2a \sin(n_\mu \pi/L) \), \( \sum_\mu n_\mu^2 = 10 \) have a peak which depend on definition of the \( A_\mu \) but at other momentum points \( g^2/4\pi \) and \( \tilde{g}^2/4\pi \) are not so different. When the physical scale is fixed 
by \( a^{-1} = 1.91 \pm 0.1 GeV \), \( \Lambda_{MOM} \) calculated from the \( \tilde{g}^2/4\pi \) at \( \sum_\mu n_\mu^2 = 8 \) and \( 16 \) are about 1GeV, which are consistent to that obtained from the three gluon vertex in high momentum region.

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