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Generalized barycenters and variance maximization on metric spaces. (English)

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J. Fixed Point Theory Appl. 25, No. 1, Paper No. 5, 12 p. (2023).

Summary: We show that the variance of a probability measure \( \mu \) on a compact subset \( X \) of a complete metric space \( M \) is bounded by the square of the circumradius \( R \) of the canonical embedding of \( X \) into the space \( P(M) \) of probability measures on \( M \), equipped with the Wasserstein metric. When barycenters of measures on \( X \) are unique (such as on CAT(0) spaces), our approach shows that \( R \) in fact coincides with the circumradius of \( X \) and so this result extends a recent result of Lim-McCann from Euclidean space. Our approach involves bi-linear minimax theory on \( P(X) \times P(M) \) and extends easily to the case when the variance is replaced by very general moments. As an application, we provide a simple proof of Jung’s theorem on CAT\((k)\) spaces, a result originally due to Dekster and Lang-Schroeder.

MSC:

60B05 Probability measures on topological spaces
60D05 Geometric probability and stochastic geometry
58C35 Integration on manifolds; measures on manifolds
49Q22 Optimal transportation
53C21 Methods of global Riemannian geometry, including PDE methods; curvature restrictions

Keywords:

variance maximization; probability measures; bi-linear minimax theory

Full Text: DOI arXiv

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