EXTRA GENERATIONS,
DISCREPANCIES OF
ELECTROWEAK PRECISION DATA
AND MASS OF THE HIGGS

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Abstract

The latest electroweak precision data are analyzed assuming the existence of the fourth generation of leptons \((N, E)\) and quarks \((U, D)\), which are not mixed with the known three generations. If all four new particles are heavier than \(Z\) boson, quality of the fit for the one new generation is as good as for the Standard Model. In the case of neutral leptons with masses around 50 GeV ("partially heavy extra generations") the minimum of \(\chi^2\) is between one and two extra generations. The predicted value of the higgs mass \(m_H\) in the presence of the fourth generation can be made rather large. The quality of fits drastically improves when the data on b- and c-quark asymmetries and new NuTeV data on deep inelastic scattering are ignored.
My talk rests on the results of two papers written recently in collaboration with L.Okun, A.Rozanov and M.Vysotsky [1, 2].

A Brief History of the SM fits

For more than ten years the Standard Model (SM) enjoyed solid agreement with precision data provided by experiments at LEP, SLC and Tevatron.

The matter came to a climax at the time of La Thuille 2000 conference when the SM fit of the whole set of available electroweak precision data became absolutely perfect: \( \chi^2/n_{d.o.f.} \approx 15/14 \). From the point of view of Statistics no one sample of New Physics could improve such fit - it could make it worse or of the same quality in the best case.

For example, in paper [3] written at that time we reanalyzed the non-decoupled New Physics in a form of additional heavy quark-lepton generations. We confirmed that in the case of all four new fermions (U and D quarks, neutral lepton \( N \) and charged lepton \( E \)) heavier than \( Z \) boson the radiative corrections to low-energy observables were large and the quality of the fit dropped down. As a result such extension of the SM was excluded by the data. In particular we found that one heavy generation was excluded at 2.5 \( \sigma \) level. We also found that corrections due to existence of relatively light neutral lepton \( N \) (\( m_N \approx 50 \text{ GeV} \)) and corrections due to heavy \( U, D \) and \( E \) could compensate each other and that the SM with additional "partially heavy" generation is allowed by precision measurements of low-energy observables as well as the SM itself. This was an example of the conspiracy of New Physics.

From that time situation with the quality of the SM fit has been changed. At the time of Osaka Conference (summer 2000) some of the central values of observables have been shifted within one sigma, some of the error bars have become slightly smaller. Nothing radical happened with any of observables but the coherent result was quite visible and the SM fit became less good: \( \chi^2/n_{d.o.f.} = 21/13 \). The level at which one extra heavy generation was excluded went down to 2\( \sigma \) [7].

For the latest precision data (summer 2001) [8] the SM fit became even worse \( \chi^2 = 24/13 \). As for the fit of the SM with one additional heavy generation it became approximately of the same quality as for the SM.

To see that we present in Fig.1 the exclusion plot for the number \( N_g \) of extra heavy generations.

To produce this plot we take \( m_D = 130 \text{ GeV} \) – the Tevatron lower bound on new quark mass; we use experimental 95\% C.L. bound on higgs mass \( m_H > 113 \text{ GeV} \) [8] and vary \( \Delta m = \sqrt{m_U^2 - m_D^2} \) and number of extra generations \( N_g \). (In order to have two-dimensional plot we arbitrary assumed that \( m_N = m_U \) and \( m_E = m_D \); other choices do not change the obtained results drastically, see discussion below). We see that \( \chi^2 \) minimum corresponds to unphysical point \( N_g = 0.5 \). For 170 GeV < \( m_U \) < 200 GeV we get the same quality of fit in

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1 In this extension of the SM leptons of fourth generation (E,N) should be very weakly mixed with the ordinary ones, while in quark sector (U,D) mixing is limited only by unitarity of 3 \( \times \) 3 CKM matrix. Implications of extra quark-lepton generations for precision data were studied in a number of papers [4] - [6].
Figure 1: Exclusion plot for heavy extra generations with the input: \( m_D = m_E = 130 \text{ GeV}, m_U = m_N \). \( \chi^2 \) minimum shown by cross corresponds to \( \chi^2 / n_{d.o.f.} = 22.2 / 12, N_g = 0.4, \Delta m = 160 \text{ GeV}, m_H = 116 \text{ GeV} \). \( N_g \) is the number of extra generations. Borders of regions show domains allowed at the level 1\( \sigma \), 2\( \sigma \), etc. They correspond to \( \Delta \chi^2 = 1, 4, 9, 16, \) etc.

case \( N_g = 1 \) as that for the SM (\( N_g = 0 \)). We see that both fits are rather bad but they are equally bad. New Physics does not make fit worse as compared with the SM.

Two heavy generations are excluded at more than 3\( \sigma \) level. Nevertheless, two and even three “partially heavy” generations are allowed when neutral fermions are relatively light, \( m_N \approx 55 \text{ GeV} \) (see Fig. 2).

Thus the SM fit is bad, the SM with additional heavy (or partially heavy) generations fit is also bad. In the literature one can find different modifications of the SM that resolve one or another discrepancy with experimental numbers but it seems that not a single modification could fit the whole set of data well. In such situation it is useful to reveal the roots of the bad quality of the SM fit.

The roots of the SM fit troubles.

We see three main discrepancies in the existing data.

Discrepancy 1.

There is discrepancy between the average value of \( s^2_l \) extracted from the pure

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2In ref. 1 one can find a statement that extra heavy generations are excluded by the recent precision electroweak data. However, analysis performed in 2 refers to upper and lower parts of Fig. 1, \( \Delta m > 200 \text{ GeV} \) and \( \Delta m = 0 \), where the existence of new heavy generations is really strongly suppressed. This is not the case for the central part of Fig. 1 (\( \Delta m \approx 150 \text{ GeV} \)).

3Using all existing LEP II statistics on the reactions \( e^+ e^- \rightarrow \gamma + \nu \bar{\nu}, \gamma + NN \) in dedicated search (see 14) one can exclude 3 “partially heavy” generations which contain such a light \( N \) at a level of 3\( \sigma \) (see 10), while one or even two such generations may exist.
leptonic measurements and its value from events with hadrons in final state \( ^{\text{[8]}} \):

\[
\begin{align*}
s_l^2 & \\
\text{Leptons} & = 0.23113(21) \\
\text{Hadrons} & = 0.23230(29)
\end{align*}
\]

This 3.3 \( \sigma \) difference is one of the causes of poor quality of the SM fit.

The value of hadronic contribution to \( s_l^2 \) in \( ^{\text{[8]}} \) is dominated by very small uncertainty of the forward-backward asymmetry in reaction \( e^+e^- \rightarrow Z \rightarrow b\bar{b} \), measured at LEP

\[
(A_{FB}^b)_{\text{exp}} = 0.0990(17)
\]

\textbf{Discrepancy 2.}

There is discrepancy (indirect) between this LEP result and SLC data. Indeed the value of \( A_{FB}^b \) can be calculated by multiplying beauty asymmetry \( A_b \) and leptonic asymmetry \( A_l \) (both measured at SLAC). Then

\[
A_{FB}^b = \frac{3}{4} A_b A_l = 0.1038(25).
\]

The number \( ^{\text{[8]}} \) differs from \( ^{\text{[8]}} \). Thus there is contradiction between LEP and SLC experimental data. Moreover SLC number nicely coincides with the SM fit: \( (A_{FB}^b)_{SM} = 0.1040(8) \) (see e.g. Table 1 from \( ^{\text{[1]}} \)).
Discrepancy 3.

A new result for $s_{W}^{2}(\nu N)$ and hence for $m_{W}(\nu N)$ was published by NuTeV collaboration [12]:

$$s_{W}^{2}(\nu N) = 0.2277(17), \ m_{W}(\nu N) = 80.140(80).$$

(4)

The new value of $m_{W}(\nu N)$ differs from $m_{W}$ measured at LEP II and previously at Tevatron by 3.7 $\sigma$. With new NuTev result we get for the SM fit:

$$m_{H} = 86^{+51}_{-32} \text{ GeV}, \ \chi^{2}/n_{d.o.f.} = 30.3/13.$$  

(5)

At that moment one can stop and wait for the better data that do not contradict each other. We are not going to do that, we are going to proceed with our analisis.

As a guide we take Lev Landau advice to young theorists. According to folklore it sounds like that:

"Look at the data and multiply experimental errors by factor 3"

In general case this advice seems too radical. But in the contradictory situation it seems reasonable to disregard some of the data to understand their relative contribution into trouble. The previous consideration demonstrates that the accuracy of $A_{FB}^{b,c}$ and new NuTeV data are under suspicion. Thus at that point we assume (following Chanowitz [13]) that $A_{FB}^{b}$ has larger uncertainty than given in Eq. (3). If we multiply experimental uncertainties of $A_{FB}^{b,c}$ and $A_{FB}^{e}$ (which are strongly correlated) by a factor 10 and do the same with new NuTeV data, the quality of SM fit improves drastically: $\chi^{2}/n_{d.o.f.}$ shifts from 23.8/13 to 10.9/13. (Landau factor 3 leads to a more or less the same result).

However, a new problem arises after removing $A_{FB}^{b,c}$. It was known for a long time that the SM fit results in prediction of light higgs - the central value of its mass was below the direct lower limit by LEP II. For example in ref. [1] we got for the SM fit that

$$m_{H} = 79^{+47}_{-29} \text{ GeV},$$  

(6)

It is slightly less than one sigma away from 114.1 GeV bound of LEPII. (The discrepancy is smaller in case of inclusion of the new NuTeV result, see Eq. (3)). Thus we have one sigma deviation of the predicted value of higgs mass from the direct LEPII bound. This discrepancy is not too bad, but the $\chi^{2}$ of the SM fit is rather bad.

With our modification of experimental results on $\nu N$ scattering and on $A_{FB}^{b,c}$ the SM fit gives:

$$m_{H} = 42^{+30}_{-18} \text{ GeV},$$  

(7)

with good $\chi^{2} = 10.9/13$, but well below modern LEP II bound. Increasing $m_{H}$ to the LEPII bound leads to an increased $\chi^{2} = 14.5$, thus the difference $\delta \chi^{2} = 14.5 - 10.9 = 3.6$ is close to 1.9 $\sigma$. Two sigma difference is not yet a discovery of the violation of SM, but it is a substantial trouble for the SM.
Fortunately there are ways to avoid this trouble. One possible way to raise the predicted value of $m_H$ is to assume the existence of fourth generation of leptons and quarks [2, 6].

It was noticed in [6] that the predicted mass of the higgs could be as high as 500 GeV. That conclusion was based on a sample of 10,000 random inputs of masses of fourth generation leptons and quarks. In [2] we used our LEPTOP code [14] to find steep and flat directions in the five-dimensional parameter space: $m_H, m_U, m_D, m_E, m_N$. For each point in this space we performed three-parameter fit ($m_t, \alpha_s, \tilde{\alpha}$) and calculated the $\chi^2$ of the fit.

It turns out that the $\chi^2_{\text{min}}$ depends weakly on $m_U + m_D$ and $m_H$, while its dependence on $m_U - m_D, m_E$ and $m_N$ is strong. Therefore to present the result of the complete analysis of the Summer 2001 precision data it is enough to have a few two-dimensional plots. In Figures 3-6 we show $\chi^2_{\text{min}}$ (crosses) and constant $\chi^2$ lines corresponding to $\Delta \chi^2 = 1, 4, 9, 16, ...$ on the plane $m_N, m_U - m_D$ for fixed values of $m_U + m_D = 500$ GeV, $m_H = 120$ (Figs. 3 and 5) and 500 GeV (Figs. 4 and 6) and $m_E = 100$ (Figs. 3 and 4) and 300 GeV (Figs. 5 and 6).

The above choice of masses is based on a large number of fits covering a broad
Figure 4: Exclusion plot on the plane $m_N, m_U - m_D$ for fixed values of $m_H = 500$ GeV, and $m_E = 100$ GeV. $\chi^2_{\text{min}}$ shown by two crosses corresponds to $\chi^2/n_{\text{d.o.f.}} = 21.4/12$. (The left-hand cross is slightly below $m_N = 50$ GeV.) The plot was based on the old NuTeV data. The new NuTeV data preserve the pattern of the plot, but lead to $\chi^2_{\text{min}}/n_{\text{d.o.f.}} = 28.3/12$. If $A_{FB}^1$ and $A_{FB}^2$ uncertainties are multiplied by a factor 10, we get $\chi^2_{\text{min}}/n_{\text{d.o.f.}} = 21.2/12$ for new NuTeV, and $\chi^2_{\text{min}}/n_{\text{d.o.f.}} = 13/12$ for old NuTeV.
Figure 5: Exclusion plot on the plane $m_N, m_U - m_D$ for fixed values of $m_H = 120 \text{ GeV}$, and $m_E = 300 \text{ GeV}$. $\chi^2_{\text{min}}$ shown by cross corresponds to $\chi^2/n_{\text{d.o.f.}} = 23.0/12$.

Figure 6: Exclusion plot on the plane $m_N, m_U - m_D$ for fixed values of $m_H = 500 \text{ GeV}$, and $m_E = 300 \text{ GeV}$. $\chi^2_{\text{min}}$ shown by cross corresponds to $\chi^2/n_{\text{d.o.f.}} = 24.4/12$. 

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space of parameters: $300 \text{ GeV} < m_U + m_D < 800 \text{ GeV}$; $0 \text{ GeV} < m_U - m_D < 400 \text{ GeV}$; $100 \text{ GeV} < m_E < 500 \text{ GeV}$; $50 \text{ GeV} < m_N < 500 \text{ GeV}$; $120 \text{ GeV} < m_H < 500 \text{ GeV}$. Concerning quarks, $m_U + m_D$ is bounded from below by direct searches limit, while from above by triviality arguments. Since $\chi^2$ dependence on $m_U + m_D$ is very weak, our choice of intermediate value $m_U + m_D = 500 \text{ GeV}$ represents a typical, almost general case. For this choice $|m_U - m_D|$ can not be larger than $\sim 200 \text{ GeV}$ because of the mentioned above direct searches bound.

Concerning charged lepton, its mass is taken above LEP II bound. We present fits at two values of $m_E$ (100 GeV and 300 GeV) and one can see how fit is worsening with $m_E$ going up.

Concerning the value of $m_H$, we vary it from the lower LEP II limit up to triviality bound and since the dependence of observables on $m_H$ is flat, one can get $\chi^2$ behavior from two limiting points: $m_H = 120$ and 500 GeV.

For $m_E = 100 \text{ GeV}$ we have the minimum of $\chi^2$ at $m_N \simeq 50 \text{ GeV}$ and:

- for $m_H = 120 \text{ GeV}$: $|m_U - m_D| \sim 50 \text{ GeV}, \frac{\chi^2_{\text{min}}}{n_{\text{d.o.f.}}} = 20.6/12$
- for $m_H = 300 \text{ GeV}$: $|m_U - m_D| \sim 75 \text{ GeV}, \frac{\chi^2_{\text{min}}}{n_{\text{d.o.f.}}} = 20.8/12$
- for $m_H = 500 \text{ GeV}$: $|m_U - m_D| \sim 85 \text{ GeV}, \frac{\chi^2_{\text{min}}}{n_{\text{d.o.f.}}} = 21.4/12$

Thus we have two lines ($m_U > m_D$ and $m_U < m_D$) in the $(m_U - m_D, m_H)$ space that correspond to the best fit of data. Along these lines the quality of the fit is only slightly better for light higgs ($m_H \sim 120 \text{ GeV}$) than for the heavy one ($m_H \sim 300 - 500 \text{ GeV}$).\footnote{Note that the $n_{\text{d.o.f.}}$ is 12, unlike the case of the Standard Model, where it was 13 (ref. \cite{c}). This change occurs because in the present paper $m_H$ is not a fitted, but a fixed parameter (hence 13 becomes 14), while $m_N$ and $m_U - m_D$ are two additional fitted parameters (hence 14 becomes 12).}

For $m_E = 300 \text{ GeV}$ we have the minimum of $\chi^2$ at $m_U - m_D \simeq 25 \text{ GeV}$ and:

- for $m_H = 120 \text{ GeV}$: $m_N \sim 200 \text{ GeV}, \frac{\chi^2_{\text{min}}}{n_{\text{d.o.f.}}} = 23.0/12$
- for $m_H = 300 \text{ GeV}$: $m_N \sim 170 \text{ GeV}, \frac{\chi^2_{\text{min}}}{n_{\text{d.o.f.}}} = 24.0/12$
- for $m_H = 500 \text{ GeV}$: $m_N \sim 150 \text{ GeV}, \frac{\chi^2_{\text{min}}}{n_{\text{d.o.f.}}} = 24.4/12$

Thus, the best fit of the data corresponds to the light $m_E \simeq 100 \text{ GeV}$ and $m_N \simeq 50 \text{ GeV}$. The significance of light $m_N$ (around 50 GeV) was first stressed in \cite{c}. Increase of $m_E$ leads to the increase of $m_N$ and to fast worsening of $\chi_{\text{min}}^2$.

Thus, inclusion of one extra generation improves the quality of the fit (compare $\chi^2/n_{\text{d.o.f.}} = 23.8/13$ for the SM from \cite{c} and $\chi_{\text{min}}^2/n_{\text{d.o.f.}} = 20.6/12$ from Fig. 3), but it remains pretty poor. If one multiplies experimental errors of $A_E^F$ and $A_E^{FB}$ by a factor 10, one gets good quality of SM fit \cite{c} but with extremely light higgs, having only a small (few percent) likelihood to be consistent with the lower limit from direct searches. The fourth generation allows to have higgs as heavy as 500 GeV with a perfect quality of the fit: $\chi_{\text{min}}^2/n_{\text{d.o.f.}} = 13/12$, if one uses old NuTeV data (see caption of Fig. 4). Captions of Figs. 3 and 4 reflect also the recent change in NuTeV data (from $m_W = 80.26 \pm 0.11 \text{ GeV}$...
15 to $m_W = 80.14 \pm 0.08$ GeV 12) which results in drastic worsening of the fit even in the presence of the fourth generation.

To qualitatively understand the dependence of $m_U - m_D$ on $m_H$ in the case of $m_E = 100$ GeV at $\chi^2_{\text{min}}$ let us recall how radiative corrections to the ratio $m_W/m_Z$ and to $g_A$ and $R = g_V/g_A$ (the axial and the ratio of vector and axial couplings of Z-boson to charged leptons) depend on these quantities 5:

$$\delta V^i \approx -\left( \frac{11}{9} s^2 \right) \ln \left( \frac{m_H}{m_Z} \right)^2 + \frac{4}{3} \frac{(m_U - m_D)^2}{m_Z^2} + \left( \frac{16}{9} \frac{s^2}{m_U - m_D} \right)$$

where $i = m, A, R$, while $s^2 \approx 0.23$. Corrections to other observables can be calculated in terms of $\delta V^i$. In the vicinity of $\chi^2_{\text{min}}$ the third term in brackets is much smaller than the second one. Hence the smallness of the left-right asymmetry of the plots of Figs. 1, 2. Since $\frac{11}{9} s^2 \approx s^2 + \frac{1}{9} \approx s^2$, the increase of $m_H$ is compensated by increase of $|m_U - m_D|$ and we have a valley of $\chi^2_{\text{min}}$.

In conclusion I’d like to make two remarks.

1) Note that the often used parameters $S, T, U$ (introduced in 10) are not adequate for the above analysis, because they assume that all particles of the fourth generation are much heavier than $m_Z$, while in our case the best fit corresponds to $m_N \sim m_Z/2$. In the paper 10 modified definitions of $S$ and $U$ were used in order to deal with new particles with masses comparable to $m_Z$. However, both original and modified definitions of $S$, $T$ and $U$ take into account radiative corrections from the “light” 4th neutrino only approximately, while the threshold effects, that are so important for $m_N \approx 50$ GeV, can be adequately described in the framework of functions $V^i$ as it was done in ref. 1, 2). (For narrow region $m_Z/2 < m_N < 46.5$ GeV (that is actually excluded by direct LEP II data on the reactions $e^+e^- \rightarrow \gamma + \nu\bar{\nu}, \gamma + N\bar{N}$) the threshold effects are so large that modify the Breit-Wigner shape of Z-line. To describe this region one needs different formalism.)

2) Note that in the framework of SUSY with three generations radiative corrections due to loops with superpartners also shift upward the mass of the higgs in the case of not too heavy squarks (300-400 GeV, see Table 1 in 17) or light sneutrinos (55-80 GeV, see 15).

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