Gravitational Waves from Stellar Mass Black Holes Around SgrA*

Razieh Emami† and Abraham Loeb†

Center for Astrophysics, Harvard-Smithsonian, 60 Garden Street, Cambridge, MA 02138, USA

We consider the detectability of the gravitational wave (GW) signal from an orbiting stellar-mass black hole (BH) with a mass of order 40 $M_\odot$ around Sgr A* at the center of the Milky Way galaxy. We simulate the sinking BHs to the center through dynamical friction and GW emission. We predict that LISA will detect of order 10 BHs at any given time with a signal-to-noise ratio $S/N > 10$.

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Gravitational Wave (GW) astronomy was ushered with the direct detection of coalescing binary compact objects by LIGO [1–4]. GWs at the low frequency range of, ~ $10^{-5} - 0.1$ Hz are expected to be detected within a decade by the Laser Interferometer Space Antenna (LISA) [5–18].

In this Letter, we consider the detectability of GWs from inspiralling stellar mass black holes (BHs) around the supermassive black hole (SMBH) at the center of the Milky Way galaxy, Sgr A*. We compute the frequency and amplitude of the GWs for the expected eccentric orbits of these BHs. We characterize the parameters of each orbit by its semi major axis $a$ and the eccentricity $e$.

The evolution of $a$ and $e$ depends on the stellar environment. We use a publicly available code [20] to simulate the galactic center and supplement it with the effect of GW emission. We follow the dynamics of the stellar BHs and create a two dimensional grid in the initial values of $a_{in}$ and $e_{in}$ to find the fraction of the BHs that yield a detectable LISA signal. This fraction is then converted to the average expected number of BHs observable by LISA. We choose $t = 10$ Gyr as the final time of our simulation to take account the uncertainty in the BH formation histories.

First, we compute the frequency and the amplitude of GWs emitted by an eccentric BH orbit. Although there are different methods to compute the GW signal [7, 21], we use the standard approach of expanding the waves in harmonics with characteristic harmonic number $n_{ch}$ [22]. This leads to a GW frequency $f = n_{ch} f_{orb}$, with $f_{orb} = (1/2\pi)(GM_{\text{smbh}}/a)^{1/2}$. In the limit of the circular orbit, the peak emission is at $n = 2$ with the GW frequency equal to twice the orbital frequency. However, for eccentric orbits there is wide range of the frequencies emitted and the peak emission occurs at higher harmonics. As shown in Ref. [22], the fitting formula for $n$ as a function of $e$ is given by,

$$n = 2 \left( \frac{1 + e^{1.1954}}{1 - e^2} \right)^{3/2}, \quad (1)$$

yielding [23],

$$f(e) = \frac{\sqrt{GM_{\text{smbh}}}}{\pi} \left( \frac{1 + e^{1.1954}}{a(1 - e^2)^{3/2}} \right). \quad (2)$$

The amplitude of GWs on the eccentric orbits [26] differs from the circular orbits [24] and is given by,

$$A_k(f) = \sqrt{\frac{5}{24}} \frac{(GM/c)^{5/6}}{f^{-7/6}} \frac{g(n,e)}{F(e)(n/2)^{2/3}}, \quad (3)$$

where $g(n,e)$ and $F(e)$ are defined in Eqs. (2) and (4) of Ref. [25], with the chirp mass defined as $M = (m_{bh}M_{\text{smbh}})^{2/5}/(m_{bh} + M_{\text{smbh}})^{3/5}$.

The above signal should be compared to the sensitivity curve of LISA [25, 27], based on the power spectral density of the noise. We adopt the official model for the instrumental noise,

$$S_n(f) = \frac{10}{L^2} \left[ P_{\text{OMS}} + 2 \left( 1 + \cos^2 f/f_s \right) P_{\text{acc}} / (2\pi f)^5 \right] \left( 1 + \frac{6}{10} \left( f/f_s \right)^2 \right), \quad (4)$$

where $L = 2.5$ Gm and $f_s = 19.09$ mHz. In addition, $P_{\text{OMS}}$ and $P_{\text{acc}}$ are defined in Eqs. (10) and (11) of Ref. [25] and refer to the single link “optical metrology noise” and the single test “mass acceleration noise”, respectively.

This must be supplemented by the noise from the unresolved Galactic white dwarf binaries [25, 27],

$$S_c(f) = A f^{7/3} e^{-f^2/2\beta \sin^2 \gamma} \left[ 1 + \tanh \gamma (f_k - f) \right] \mathrm{Hz}^{-1}, \quad (5)$$

where $A$, $\alpha$, $\beta$, $\kappa$, $\gamma$ and $f_k$ are parameters of this model and are listed in Table 1 of Ref. [25]. We use the data for the 4 year detection sensitivity of LISA.

The total noise is $S_N(f) = S_n(f) + S_c(f)$, with the dimensions of $\mathrm{Hz}^{-1}$. It is customary to calibrate this function by $f$ and define a dimensionless parameter, $S_{N,c}(f) = f S_N(f)$. We define the strain as $\text{Strain} \equiv \sqrt{S_{N,c}(f)} = \sqrt{f S_N(f)}$.

As for the signal, following Ref. [25] we define the dimensionless parameter $h_{N,c}(f) \equiv (16/5)^{2/3} A f_k$, where the factor $16/5$ originates from averaging over the sky location, polarization and inclination.

The semi major axis and eccentricity are affected by the gravitational interactions with the surrounding sea of stars. We model the center of the galaxy as a star cluster composed of main sequence stars as well as the BH remnants. The BHs sink toward the center by dynamical friction. The detailed modeling of the system based on the PhaseFlow code, hereafter PF [20], are described in Ref. [28].
We use the PF code to predict the evolution of BH orbits toward the Galactic center. Since the PF code treats the BHs through their density profile, we split the BH profile into radial bins. The resolution of radial shells is chosen so that each shell contains of order one BH with a mass $m_{bh} = 40M_\odot$. We focus on the most massive BHs with this mass because segregation is more important for heavier BHs. The number of BHs in our simulation are consistent with the Kroupa mass function of stellar progenitors. This association was calculated in detail in Ref. [28] and for LIGO sources in Ref. [29].

Each of the radial shells drifts toward the center as a result of dynamical friction. Throughout our analysis, we create a two dimensional grid in the initial $a_i$ and $e_i$ space. We supplement the PF results with the effect of GW emission. Combining these components we get,

$$\dot{a}(t) = -\frac{64 G^3 m_{bh} M_{smbh} M_{tot}}{5} \left[ \frac{1}{c^2 a^3 (1-e^2)^{1/2}} \left( 1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right) + \dot{a}_{pf} \right],$$

$$\dot{e}(t) = -\frac{304 G^3 m_{bh} M_{smbh} M_{tot}}{15} \frac{1}{c^2 a^5 (1-e^2)^{3/2}} \left( 1 + \frac{121}{304} e^2 \right) e,$$

where $M_{tot} \equiv m_{bh} + M_{smbh}$, and $\dot{a}_{pf}$ is associated with the time derivative of the semi-major axis based on the PF code.

We evolve the system for 10 Gyr and find a fraction of the initial grid for every shell that yields $S/N > 10$, for LISA where $S/N \equiv h_{eff}/Strain$. This is defined as the allowed region in what follows.

Here we consider the center of the Milky Way as containing Sgr A* with $M_{smbh} = 4 \times 10^6 M_\odot$ located at $D = 8$ kpc from Earth. For the stellar BHs, we adopt an initial spheroidal density profile with the slope $\gamma \approx 1$ [30].

First, we compare the inspiral time scale with the entire time in our simulation, 10Gyr. Due to the emission of the GWs, we expect a decrease in semi-major axis as well as eccentricity which brings the stellar mass BHs in close proximity to SgrA* until they are swallowed by it. This mechanism decreases the number of remaining BHs substantially. We approximate this timescale as $\tau_{GW} = a/\dot{a}$. We compute the average of $\tau_{GW}$ in every shell of the allowed range of parameters, $\bar{\tau}_{GW}$. Figure 1 shows the number of $40M_\odot$ BHs, $N_{BH}$, with an inspiral timescale $\tau_{GW}$ in units of 10Gyr. The plot implies that the timescale of the emission of GW is shorter than 10Gyr, and shows that the inspiral phase occurs generally within the age of the universe.

We average over all radial shells and find an average timescale for GW inspiral, hereafter $\bar{\tau}_{GW}$, which is defined as $\bar{\tau}_{GW} \equiv \left( \int_{0}^{N} \tau_{GW}(x) dx \right) / N$. From this we compute a percentage of BHs which have inspiral timescale less than the age of the universe, 10Gyr, $Perc \equiv \bar{\tau}_{GW}/(10Gyr) \approx 0.026\%$. Hence, most of the allowed region should have had an inspiral timescale less that the age of the universe.

Next, we compute the allowed range of $a_{in}$ and $e_{in}$ for having $S/N > 10$ for LISA in different radial shells as presented in Figure 2. On the right panel, we present the allowed region in terms of the frequency for different shells. Different shells, colored differently, have a universal behavior.

Next, we compute the percentage of the allowed region in parameter space. In each shell we find the probability for detection by LISA from the fraction of the allowed region relative to the total parameter range in the grid.

Figure 3 shows that the innermost couple of layers have the best chance of being detected by LISA. In order to count the number of detectable BHs we use the following method. First, we interpolate the probability function for the entire BH sample which we denote as $P_i(i = 1, ..., N)$ and then we take the average of the above function as $Ave = \sum_{i=1}^{N} P_i/N$. The total fraction of the remaining BHs is then,

$$N_{det} = \sum_{i=1}^{N} P_i \approx 9.$$

This implies that LISA could detect 1.5% of the entire population of $40M_\odot$ BHs in the inner region.

In conclusion, we find that of order 10 BHs should be detectable by LISA with $S/N > 10$ at any given time.

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FIG. 2: Left: Signal to noise ratio for LISA for BH orbits characterized by initial semi-major and eccentricity values $a_\text{in} - e_\text{in}$. From bottom up the semimajor axis increases. Right: allowed value of the GW signal amplitude relative to the LISA noise for different BHs on the left panel as a function of GW frequency.

FIG. 3: Probability of detection with LISA in each radial shell as a function of the enclosed BH number, $N_{\text{BH}}$.

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