What Physics Does The Charged Lepton Mass Relation Tell Us?

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§0. Prologue

The story begins from a charged lepton mass relation

\[ K \equiv \frac{m_e + m_\mu + m_\tau}{\sqrt{m_e + m_\mu + m_\tau}} = \frac{2}{3}. \]  

It is well known that the formula is excellently satisfied by the observed charged lepton masses

\[ K(m_{ei}^{obs}) = \frac{2}{3} \times (0.999989 \pm 0.000014). \]  

1982, the formula predicted a tau lepton mass

\[ m_{\tau}^{pred} = 1776.97 \text{ MeV}, \]  

by inputting the observed mass values \( m_e \) and \( m_\mu \). On the other hand, the observed mass at 1982 was

\[ (m_{\tau}^{obs})_{old} = 1784.2 \pm 3.2 \text{ MeV}. \]  

Ten years after (1992), an accurate value of \( m_{\tau}^{obs} \) was reported:

\[ (m_{\tau}^{obs})_{new} = 17776.99^{+0.29}_{-0.26} \text{ MeV}. \]  

The new observed value (0.5) was excellently coincident with the prediction (0.3). Therefore, the formula suddenly attracted a great deal of attention as seen in Fig.1.

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1 A talk presented at “7th Workshop on Flavour Symmetries and Consequences in Accelerators and Cosmology” (FLASY 2018).
Thus, it is not that the formula (0.1) was proposed by taking the observed values into consideration.

I would like to emphasize that the formula $K = 2/3$ should be never satisfied with the observed charged lepton masses in the theoretical point of view. This is the main motivation of my present talk.

In general, the “mass” in the relation derived in a field theoretical model means the “running” mass, instead of the “pole” mass. Therefore, the charged lepton mass relation $K = 2/3$ should be never satisfied by pole masses. Nevertheless, the relation is excellently satisfied by the pole masses as seen in Eq.(0.2). This accuracy is excellent enough to believe that the coincidence (0.2) is not accidental, but suggests a nontrivial physics behind it. Thus, if we take the coincidence seriously, we should treat the renormalization group (RG) effects carefully. This was first pointed by Sumino\cite{3}.

Therefore, my talk is arranged as follows: The present topic is not phenomenological one. The purpose of my talk is to review of a field theoretical study by Sumino and recent development.

1. Why is the excellent coincidence so problematic?
2. Derivation of the mass formula
3. Sumino mechanism
4. Modified Sumino model
5. Recent development

§1. Why is the excellent coincidence so problematic?

The charged lepton mass relation was derived based on a field theoretical model as reviewed in the next section. Therefore, we have to use the running masses for the formula $K = 2/3$, not the pole masses. However, if we use pole masses, then, we obtain

$$K(m_{\text{run}}^{\ell_i}) = \frac{2}{3} \times (1.00189 \pm 0.00002),$$

(1.1)

(at $\mu = m_Z$). The agreement is not so excellent.

Are the present observed mass values mistaken? Is the coincidence (0.2) accidental? Will a future experimental value be changed, and will the problem disappeared? However, such a case is not likely.

This is a serious theoretical problem.

§2. Derivation of the mass formula

Prior to review of the Sumino model, let us review the derivation\cite{4} of the formula $K = 2/3$ in briefly.

First, we introduce a scalar $\Phi$ which is a nonet of a family symmetry U(3) and whose vacuum expected value (VEV) is given as

$$\langle \Phi \rangle = v_0 \text{diag}(z_1, z_2, z_3),$$

(2.1)

with $z_1^2 + z_2^2 + z_3^2 = 1$. 


In the model, the charged lepton mass matrix $M_e$ is given by

$$M_e = k_e \langle \Phi \rangle \langle \Phi \rangle.$$  

(2.2)

(The structure (2.2) may be considered from a seesaw like mechanism.)

Then we assume the following scalar potential:

$$V = \mu^2 [\Phi \Phi] + \lambda [\Phi \Phi \Phi \Phi] + \lambda' [\Phi S S][\Phi]^2,$$  

(2.3)

where $\Phi_S$ is an octet part of the nonet scalar $\Phi$,

$$\Phi_S \equiv \Phi - \frac{1}{3}[\Phi]1.$$  

(2.4)

($1$ is a unit matrix: $1 = \text{diag}(1, 1, 1)$.) Here and hereafter, for convenience, we denote $\text{Tr}[A]$ as $[A]$ simply. Then, the condition $\partial V/\partial \Phi = 0$ leads to

$$\frac{\partial V}{\partial \Phi} = 2 (\mu^2 + \lambda [\Phi \Phi] + \lambda' [\Phi]^2) \Phi + \lambda' \left( [\Phi \Phi] - \frac{2}{3}[\Phi]^2 \right) 1.$$  

(2.5)

Hereafter, for convenience, we denote $\langle \Phi \rangle$ as $\Phi$ simply.

We want a solution $\Phi \neq 1$, so that the coefficients of $\Phi$ and $1$ must be zero. Then, we obtain

$$\mu^2 + \lambda [\Phi \Phi] + \lambda' [\Phi]^2 = 0,$$  

(2.6)

and

$$[\Phi \Phi] - \frac{2}{3}[\Phi]^2 = 0.$$  

(2.7)

The relation (2.6) fixes the scale of the VEV value $\Phi$, and the relation (2.7) gives just our mass relation $K = 2/3$. Note that Eq. (2.7) is independent of the potential parameters $\mu$ and $\lambda$.

Also, recently, we have obtained another mass formula

$$\kappa \equiv \frac{\text{det} \Phi}{[\Phi]^3} = \frac{\sqrt{m_em_\mu m_\tau}}{\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}} = \frac{1}{2 \cdot 3^5} = \frac{1}{486},$$  

(2.8)

in addition to the formula $K = 2/3$:

$$K \equiv \frac{[\Phi \Phi]}{[\Phi]^2} = \frac{m_e + m_\mu + m_\tau}{\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}} = \frac{2}{3}.$$  

(2.9)

Note that those relation are invariant under a transformation

$$(m_e, m_\mu, m_\tau) \rightarrow (\lambda m_e, \lambda m_\mu, \lambda m_\tau).$$  

(2.10)

This will become important in order to understand the Sumino mechanism which is reviewed in the next section.
§3. Sumino mechanism

The deviation between $K(m_i(\mu))$ and $K(m_i^{\text{pole}})$ is caused by the logarithmic term the QED correction\cite{7}

$$m_i(\mu) = m_i^{\text{pole}} \left(1 - \frac{\alpha(\mu)}{\pi} \left(1 + \frac{3}{4} \log \frac{\mu^2}{(m_i^{\text{pole}})^2}\right)\right),$$

(3.1)

where $m_i(\mu)$ and $m_i^{\text{pole}}$ are running mass and pole mass, respectively. (Hereafter, for simplicity, we denote $m_i^{\text{pole}}$ as $m_i$.) If the logarithmic term $\log m_i^2$ is absent, then the formula $K = 2/3$ will also be satisfied by the running masses as we seen in Eq.(1.10).

In 2009, Sumino\cite{3} proposed an attractive mechanism: He assume $U(3)$ family gauge bosons (FGBs) $A_i^j$ with their masses $(M_{ij})^2 \propto (m_i + m_j)$. Then, the unwelcome term $\log (m_i/\mu)^2$ in Eq.(3.1) is canceled by the factor $\log (M_{ii}/\mu)^2$ in the radiative mass term due to FGB. (Note that in his model, only FGBs $A_i^j$ with $i = j$ contribute to the radiative diagram.)

However, we should notice that the Sumino model has some serious shortcomings. In order to cancel the $\log m_i$ term in the QED contribution by the $\log M_{ii}$ term in the FGB contribution, we must consider an origin of the minus sign. Sumino has assumed that the left- and right-handed charged leptons $e_L$ and $e_R$ have the same sign coupling constants $e$ and $e$ for photon, respectively, but the coupling constants for FGBs takes $+g$ and $-g$ for $e_L$ and $e_R$, respectively. In other words, Sumino has assigned the charged leptons $e_L$ and $e_R$ to $3$ and $3^*$ of $U(3)$ family, respectively. Therefore, his model is not anomaly free. Besides, in his model, unwelcome decay modes with $\Delta N_{\text{family}} = 2$ inevitably appear.

§4. Modified Sumino model

In order to avoid these defects, Yamashita and YK proposed a modified Sumino model\cite{5} with $(e_L, e_R) = (3, 3)$ of U(3) family. In this model, the minus sign comes from the following idea: The family gauge bosons have an inverted mass hierarchy, i.e.

$$M_{ii}^2 \propto (m_i)^{-1}.$$  

(4.1)

Then, because of $\log M_{ii}^2 \propto -\log m_i$, we can obtain the minus sign for the cancellation without taking $(e_L, e_R) = (3, 3^*)$.

In the modified assignment $(e_L, e_R) = (3, 3)$ with the inverted mass hierarchy (4.1), FGB with the lowest mass is $A_3^3$. Note that the family number $i = 1, 2, 3$ is defined by the charged lepton sector, i.e. $i = (1, 2, 3) = (e, \mu, \tau)$. On the other hand, for the quark sector, we do not have any constraint from experimental observations. Both cases $(u_1, u_2, u_3) = (u, c, t)$ and $(u_1, u_2, u_3) = (t, c, u)$ are allowed. If we choose the latter case, we can expect FGBs with considerably lower masses, e.g. we can suppose $M(A_3^3) \sim$ a few TeV, because the inverted family number assignment for quarks weakens severe constraints from $K^0-\bar{K}^0$ and $D^0-\bar{D}^0$ mixing data, so that we can obtain considerably low FGB masses \cite{8}. Thus, in the modified Sumino model, we can expect fruitful phenomenology. For examples, see Ref.\cite{9} for $\mu \rightarrow e + \gamma$ conversion, and see Ref.\cite{10} for $A_1^1$ production at LHC. However, note that in our model the transition $\mu \rightarrow e + \gamma$ is exactly forbidden.
5. Recent development and summary

There is another effect which disturbs the $K$-relation: $\Phi_8 \leftrightarrow \Phi_0 \equiv |\Phi|/\sqrt{3}$ mixing due to renormalization effect [11]. The $K$ and $\kappa$ relations were derived from potential model under a non-SUSY scenario. Recall that there is no vertex correction in a SUSY model. Therefore, if we derive the relations on the basis of SUSY scenario, then the problem will disappear. Very recently, we succeeded to re-derive the $K$ and $\kappa$ relations on the basis of SUSY scenario [12]. Thus, we can understand why the $K$- and $\kappa$-relations can keep the original forms.

In conclusion, we have discussed why the $K$ relation is so beautifully satisfied by the pole masses, not the running masses. Now we can understand the reason according to the Sumino’s idea and the modified Sumino model.

References

[1] Y. Koide, Lett. Nuovo Cim. 34 (1982) 201; Phys. Lett. B 120 (1983) 161; Phys. Rev. D 28 (1983) 252.

[2] C. Patrignani et al. (Particle Data Group), Chinese Physics C, 40 (2016) 100001.

[3] Y. Sumino, Phys. Lett. B 671 (2009) 477; JHEP 0905 (2009) 075.

[4] Y. Koide, Mod. Phys. Lett. A 5 (1990) 2319.

[5] Y. Koide and T. Yamashita, Phys. Lett. B 711 (2012) 384.

[6] Y. Koide, Phys. Lett. B 777 (2018) 131.

[7] H. Arason, et al., Phys. Rev. D 46 (1992) 3945.

[8] Y. Koide, Phys. Lett. B 736 (2014) 499.

[9] Y. Koide and M. Yamanaka, Phys. Lett. B, (2016).

[10] Y. Koide, M. Yamanaka and H. Yokoya, Phys. Lett. B, (2015).

[11] T. Yamashita, private communication.

[12] Y. Koide and T. Yamashita, arXive1805.09533 (hep-ph).