Numerical solution of fluid queueing models for communication networks

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Abstract.

Discrete stochastic models of queues are commonly employed in communication networks for their analysis and control. Fluid models, largely stochastic, are being used recently due to their simple character. However, asymptotic analysis of queueing models leads to deterministic fluid models and efficient methods for computing their solution are needed. This paper develops procedures for numerical solution of fluid queueing models whose arrival processes with rates that are piecewise constant. Solutions for fluid queue with single input, queue with two classes under priority and a model with two fluid queues in tandem are developed and implemented. The piecewise linear nature of the dynamics are employed to develop these efficient solution procedures by determining the solution at time instances of rate change of the input. This avoids using numerical solution of differential equations and also greatly reduces the computational complexity. Computer programs are developed implementing the numerical solutions. Motivated by their application to communication network sources, ON/OFF fluid inputs are considered for analysis. The developed numerical solution procedures are applied to the ON/OFF sources as special cases of the piecewise constant rate inputs.

1. Introduction

Queueing models are employed to model and analyse communication networks. A majority of them are stochastic: the arrivals, service times, scheduling as well as routing are random quantities [1]. However, the analysis of stochastic models is mathematically hard; in addition much of the analysis is limited to statistically quantify their steady behaviour. Simulation based study of these complex models is an alternative employed. Again, the statistical quantification of the behaviour is computationally demanding.

Fluid models which characterise the communication networks as a network of nodes processing fluid flowing through them are being employed as they are easier to study and develop scheduling and routing algorithms for. Each communication is modelled as a flow of a liquid and there are no discrete packet instances. This model explains average macroscopic behaviour of network dynamics in terms of relatively simple and deterministic equations with averaged quantities. In fluid models, traffic is continuous rather than discrete. A fluid queue can be imagined as a huge tank with infinite capacity connected to a series of pipes and pumps. Here pipes allow flow and pump removes fluid. Stochastic fluid models, where the arrival and departure rates are modelled as stochastic processes have been studied

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extensively as models for communication networks [2]. Recently, prompted by their role as asymptotic models under fluid scaling, deterministic fluid models are being studied.

The existing literature on fluid models is largely concentrated on either developing numerical methods to solve the differential equations governing the fluid models or employ fluid flow rate simulations. In fact, the simulation technique employs a numerical method as in fluid mechanics to develop a similar time instance as that of the simulation. Fluid simulations are expected to be more efficient than the packet-level stimulation for performance of computer networks and able to accommodate more complex networks at lesser cost [3]. In case of fluid model solution, large number of data packets or network traffic can be treated as a unit flow and its calculative solution is anticipated to be made comparison with packet level simulation. Fluid modelling is expected to be the basis for analysing and solving stability, congestion control and optimization problems.

The advantages of the use of fluid models for communications networks have led to the development of simulation and numerical solution techniques to solve the fluid models. However, evaluating data rate at every instance using complex differential equations would become difficult. It has been observed that the fluid flow based simulations are not always less complex than the packet-level simulations [4]. Hence there is a need for more efficient numerical solution or simulation based analysis of fluid models.

1.1. Related Work
This section provides a concise description on the related work of queueing fluid models, basics of queueing system, fluid model and how they are analysed. Few of the papers make use of simulators tools inorder to validate the fluid models, and others make use of complex differential equations to solve fluid flow models.

In order to simulate big networks at packet level, a vital difficulty that has to be taken into consideration is that the effort that is required to accomplish all the occurrences at that instance in that network [5]. Fluid flow analytical models have been capable of handling the fluctuating TCP packet flows and they can cope well to solve networks which involve flows large in number. Classical packet-level non-continuous simulations give better assumptions of network behaviour, but there can be a gradual raise in the no of flows that are being generated after the solution is provided [6]. In stochastic fluid models the input and output data rates are modelled by a Markov Process.

Vandergraft [7] presents a fluid flow model of networks of queues for modelling flow in a network of queues as deterministic and constant rate fluid queueing models. In [8], a simulation model using SSF simulation is used to analyse the relative error and efficiency of stochastic fluid models and packet based models of communication networks. Integration of fluid-based analytical model with packet-level simulation for analysis of computer networks incorporates fluid flow models into packet level simulation to address the problem of large networks analysis is taken up in [6]. Fluid models and solutions for large scale IP network provides an efficient deterministic fluid model solution technique based on using time-stepped differential equation solution incorporating network topology and TCP flow order for generality and efficiency[9]. Liu’s study [3] evaluates the performance of packet level simulation and event-driven fluid simulation approaches. Incera [10] introduced a simulation tool called FluidSim to analyse realistic fluid models for telecommunication networks. Baccelli and Hong [11] developed flow level fluid simulation approach to model and analyse interaction of huge number of TCP and UDP flows. Huang [12] developed a stochastic fluid model with two levels to model the burstiness of TCP traffic.

In order to analyse a fluid model, the rate at which the fluid enters or exits needs to be computed [3]. Analytical and numerical techniques are employed to solve the stochastic as well as deterministic fluid models. For fluid models employed in modelling large communication networks, simulation based analysis is employed [11] [13]. The simulation based approach developed in [10] [11] employs iterated numerical solution of the deterministic fluid model. However, this approach is not always efficient when compared to the stochastic packet level simulation [3].
1.2. Organisation
The rest of the paper is organised as follows: Section 2 develops numerical solutions for fluid queueing models with single class and multiple classes as well as a tandem of queues. Section 3 documents the results of application of these solution procedures to single, multiple and tandem queueing models with ON-OFF fluid sources. Section 4 concludes the paper.

2. Numerical Solutions for Fluid Queueing Models
In this section Single-Class, Multi-Class and Tandem of Queues fluid models are considered and an efficient numerical solution approach for them under piece-wise constant input rates is developed. Iterative formulas for computing the fluid levels at instances where input rates change and using them, for computing the fluid levels at arbitrary instances of time are derived. The following fluid models are developed as building blocks for developing fluid models for queueing system:

(i) **Single-Class Fluid Model:** In single class of fluids as shown in Figure 1, system is provided with one input and its corresponding output.

(ii) **Multi-Class Fluid Model:** Figure 2 shows a multiclass fluid model, where there can be multiple fluid inputs and each input fluid has its corresponding output.

(iii) **Tandem of Fluid Queues Model:** Tandem of queues is a kind of queueing model where the output of the first queue will be given as an input to the next queue and so on. Figure 3 depicts the model of a tandem of fluid queues.

2.1. Numerical Solution for Single Class fluid Model
Numerical solution for a Single-Class of Fluid Queue is numerically computed by analysing the evolution of the fluid level at two consecutive input rate change instants. Since the evolution of the fluid level between time instances of input rate changes is linear, this information is used to compute the fluid level at an arbitrary time instances. Thus, first fluid level at \( t_i \) is computed from the fluid level at \( t_{i-1} \). Consider a system where the fluid is at level \( x(t_{i-1}) \) as shown in Figure 4. The input to the system is coming at the rate \( a_{i-1} \). In order to find the fluid level at instance \( t_i, x(t_i) \), there are three possible conditions: The input rate can be less than one \( a_{i-1} < 1 \), can be greater than one \( a_{i-1} > 1 \) or, exactly equal to one \( a_{i-1} = 1 \).
These will make \( x(t_i) \) less than, equal to or, greater than \( x(t_{i-1}) \) and the evolution of \( x(t) \) can be obtained considering the effective output rate as \((1 - a_i)\).

During the evolution of the fluid level between the input rate change instants, the fluid level may hit one of the two boundaries: zero or buffer size. These are to be taken into account to derive the solution of the fluid level. In the case when the fluid level value hits zero at a point in \([t_{i-1}, t_i]\), since the fluid level cannot be less than zero, the numerical solution is computed by taking a maximum with respect to zero. There can also be a situation when the fluid hits the maximum buffer size. In that case fluid level is computed using minimum of buffer size and \( x(t_i) \). Considering all these cases, a generalized numerical solution for a Single Class of Fluid Model is given as follows:

\[
x_i = x(t_i) = \min\{B, \max\{0, x_{i-1} - (1 - a_i) \cdot (t_i - t_{i-1})\}\}
\]  

(1)

2.2. Numerical Solution for Multi Class Fluid Model

In a Multiclass Fluid Model, there is more than one input to the queue and each input is processed producing its corresponding output. For the analysis below a two input model is considered. Both the fluids are served with different priorities. Without loss of generality (by renaming if needed) the priority is higher for Class 1. Then, the evolution of the Class 2 will be influenced by the evolution of Class 1 which is in turn influenced by input rate changes of Class 1 fluid. Therefore, in order to compute the level of Class 2 fluid at any instance of time, the previous changes should also have to be taken into consideration. Due to higher priority, the solutions of the Class 1s fluid level at its rate change times as well as at arbitrary instance do not depend on Class 2 fluid and are given by the Single Class Fluid Model solution from Equation (1).

In order to solve for the Class 2 fluid level at its input rate change times, consider the computation of \( x_{2,j} \) from \( x_{2,j-1} \) at instances \( t_{2,j-1} \) and \( t_{2,j} \). Now, from Class 1 evolution, fluid level of Class 2 at \( x_{2,j-1} \) is known and thus level of fluid at \( x_{2,j} \) has to be computed. Let \( t_{1,j}, t_{1,j+1}, \ldots, t_{1,j+l-1} \) be instances of rate changes of Class 1 fluid during the interval \([t_{2,j-1}, t_{2,j}]\). We need to compute the Class 2 fluid level at each of these instances to determine the Class 2 fluid level.

Hence, in order to compute the level of fluid at \( x_{2,j} \), we need to

(i) Determine the change instances of Class 1 fluid and their number (denoted by \( l \)) during the interval \([t_{2,j-1}, t_{2,j}]\)

(ii) For \( k = j, j + 1, \ldots, j + l - 1 \), compute the Class 2 fluid levels at \( t_{2,k} \):

\[
x_2(t_{2,k}) = \begin{cases} 
\min\{B, \max\{0, x_2(t_{2,k-1}) - (1 - a_{1,k-1} - a_{2,i}) \cdot (t_{2,k} - t_{i,k-1})\}\} & \text{if } x_1(t_{1,i}) = 0 \& a_{1,j-1} < 1 \\
\min\{B, \max\{0, x_2(t_{i,k-1}) + a_{2,j} \cdot (t_{i,k} - t_{i,k-1})\}\} & \text{otherwise}
\end{cases}
\]

(2)

(iii) Compute the Class 2 fluid from the last change point:

\[
x_2(t_{2,i}) = \min\{B, \max\{0, x_2(t_{1,j+l-1}) - (1 - (a_1(t_{1,j+l-1} - a_{2,i}) \cdot (t_{2,i} - t_{1,j+l-1}))\}\}
\]

(3)

Equations (2) and (3) together with the solution for Class 1 fluid level from Equation (1) give the numerical solution for the Multi Class Fluid model at the change instances.

2.3. Numerical Solution for Tandem of Fluid Queues

In Tandem of Queues Model, the output (or outputs) from the first queue will be given as an input (resp. inputs) to the second queue, and so on for the other queues in the tandem.

First we discuss the case of single class of fluid flowing through the tandem of two queues. The fluid level in each queue of the tandem is numerically solved by computing the fluid level at instances of input rate change for that queue as a Single Class Fluid Model, namely using the formulas Equations (1).
Let the fluid input at the first queue be termed Class 1 and the output of Class 1 be termed Class 2 which enters the second queue. Once the fluid levels at of Class 1 are computed, the output Class 2 and its rate change instances are to be computed.

In order to compute the output rate of Class 1, it is necessary to check if any of the Class 1 fluid falls to zero. If this happens, there is an additional input rate change for Class 2, which occurs at the instant Class 1 fluid falls to zero.

Let the time instances of output rate changes of Class 1, which are the time instances of input rate changes for Class 2, be denoted by $t_{2,i}$. In addition, let $t'_{1,i} = \text{the instances of } i\text{th output rate change } = t_{2,i}$.

The computation of $x_{2,i} = x_2(t_{2,i}) = x_2(t'_{1,i})$ is carried out as follows:

(i) Compute the fluid level $x_{1,i} = x_1(t_{1,i})$

(ii) Compute $O_{1,i}, i = 0, 1, 2, \ldots$

if $x_{1,i} > 0$ then

$n := x_{1,i}/(1 - a_{1,i})$

if $n < (t_{1,i+1} - t_{1,i})$ then

$O_{1,i} = 1$

Store a new rate change at $t'_{1,i+1} = t_{1,i} + n$ with $O_{1,i+1} = 0$

else

$O_{1,i+1} = 1$

end if

else

$O_{1,i} = \min\{1, a_{1,i}\}$

end if

(iii) Set $a_{2,i} = O_{1,i}$ and $t_{2,i} = t'_{1,i}$ for $i = 0, 1, 2, \ldots$

(iv) Compute $x_{2,i}, i \geq 0$ as well as $x_2(t)$ using the solution procedure for single class queue

Next, we consider the general case of multiple classes of fluids passing through the tandem. If there are multiple classes at the first queue, the output of each of fluids can be sent as an input to the next queue. The analysis and solution of multiclass fluid model can be employed for computing the output rates. Then, the tandem queue fluid model with multiple classes can be solved employing the solution procedure for the tandem fluid queues model presented above.

3. Results and Discussion

This section discusses the results obtained from test input as well as ON-OFF Fluid Model inputs using the developed numerical solutions for various fluid models. The derived numerical solutions will be computed with various input values and corresponding graphs with the expected outputs are analysed. ON-OFF sources are employed for modelling communication networks using fluid models in literature.

3.1. Single Class fluid Model

The results for a Single-Class Fluid Model will be tested by passing a 2D array of ON/OFF graph. The input to the program will be influenced by 4 parameters namely; $T_{ON}, T_{OFF}, a_{ON}$ and $a_{OFF}$. These parameters are shown in Table 1.

The On/Off input values are converted to piecewise constant rate input consisting of time and rate values as shown in Figure 5 are given as input to Single Class Fluid Model. Figure 6 shows the graph of the computed solution $x_{1,i}$. 
Table 1. Single Class ON-OFF Parameters

| Parameter | Value |
|-----------|-------|
| T_{ON}    | 10    |
| T_{OFF}   | 50    |
| a_{ON}    | 10    |
| a_{OFF}   | 0     |

Table 2. Parameters of Class 1 Fluid Input

| Parameter | Value |
|-----------|-------|
| T_{ON}    | 10    |
| T_{OFF}   | 60    |
| a_{ON}    | 1.5   |
| a_{OFF}   | 0     |

Table 3. Parameters of Class 2 Fluid Input

| Parameter | Value |
|-----------|-------|
| T_{ON}    | 10    |
| T_{OFF}   | 60    |
| a_{ON}    | 1.9   |
| a_{OFF}   | 0     |

Figure 5. Single Class Queue Input

Figure 6. Single Class Fluid Model Solution

Figure 7. On/Off Class 1 Input

Figure 8. On/Off Class 2 Input

Figure 9. On/Off Class 1 solution

Figure 10. On/Off Class 2 solution

3.2. Multi Class fluid Model

For a Multi-Class Fluid model, two ON/OFF inputs for Class 1 and Class 2 Fluids with input parameters shown in Table 2 and Table 3, respectively, are used.

The On/Off inputs are converted to a piecewise constant rate inputs as shown in Figures 7 and 8 for Class 1 and Class 2, respectively. Once the converted input for Class 1 and Class 2 are given the fluid levels $x_{1,i}$ and $x_{2,i}$ are computed using the procedure developed in Section 2.2 Figures 9 and 10 show the solutions $x_{1,i}$ and $x_{2,i}$ at their rate change instances, namely $t_{1,i}$ and $t_{2,i}$.

It can be seen that the fluid level evolution of Class 1 influences that of Class 2.

3.3. Tandem of Queues

For a Tandem of Queue, the input parameters for ON/OFF Class1 fluid shown in Table 1 are used. Considering On/Off input as shown in Figure 5 the fluid levels $x_{1,i}$ and the output rates $o_{1,i}$ are computed based on Section 2.3. The graphs of $x_{1,i}$ and $o_{1,i}$ are shown in Figures 5 and 11, respectively.

The output obtained from Class1 Fluids, is fed as input to Class2 Fluid. The output graph for Class 2 Fluid at Queue 2 are shown in Figure 12.

The solution $x_{1,i}$ obtained from Class 1 Fluid shows that there are values of $x_{1,i}$ dropping to zero. Therefore, according to the numerical solution obtained for a Tandem of Queue, new rate change instances are present in the output of Class 1.
4. Conclusion
In this paper, efficient numerical solutions are derived and implemented for queueing models whose inputs are piecewise constant. The queues are solved iteratively instead of applying differential equations taking advantage of linearity. This provided direct iterative solution instead of solution based numerical solution of differential equations.

Fluid queues with piecewise constant input rate which are of interest in communication networks other than ON-OFF sources can be developed and solved using the techniques discussed. Fluid Model with feedback can be developed and its numerical solution can be derived and implemented. The solution of Multi-class Fluid model can be extended to arbitrary number of classes and the solution can then be extended to arbitrary network of queues with and without feedback.

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