How flat is the Universe?

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Abstract

In order to answer this question, we combine ten independent astrophysical constraints in the space of the density parameters $\Omega_m$ of gravitating matter and $\Omega_\Lambda$ of vacuum energy. We find that $\Omega_m = 0.31 \pm 0.07$, $\Omega_\Lambda = 0.63 \pm 0.21$, and thus $\Omega_m + \Omega_\Lambda = 0.94 \pm 0.22$. The total $\chi^2$ is 4.1 for 8 degrees of freedom, testifying that the various systematic errors included are generous. We also determine $\Omega_m$ in the exactly flat case. Five supplementary flat-case constraints can then be included in our fit, with the result $\Omega_m = 1 - \Omega_\Lambda = 0.337 \pm 0.031$. It follows that the age of the Universe is $t_0 = 13.5 \pm 1.3 (0.68/h)$ Gyr.

Keywords: Methods:data analysis, Cosmology:observations.

1 Introduction

If the dynamical parameters describing the cosmic expansion were known to good precision, we would know whether the Universe is open or closed, or whether its geometry is in fact exactly flat as inflationary theory wants it. To know the answer we need at least (i) the Hubble constant $H_0$, usually given in the form $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$, (ii) the dimensionless density parameter $\Omega_m$ of gravitating matter, comprising baryons, neutrinos and some yet unknown kinds of dark matter, and (iii) the density parameter $\Omega_\Lambda$ of vacuum energy, related to the cosmological constant $\Lambda$ by

$$\Omega_\Lambda = \frac{\Lambda}{3H_0^2}. \quad (1)$$

A flat universe is defined by the condition

$$\Omega_m + \Omega_\Lambda = 1. \quad (2)$$

When $H_0, \Omega_m$ and $\Omega_\Lambda$ are known, the age of the Universe, $t_0$, can be obtained from
the Friedman-Lematre model as

\[ t_0 = \frac{1}{H_0} \int_0^1 dx [(1 - \Omega_m - \Omega_\Lambda) + \Omega_m x^{-1} + \Omega_\Lambda x^2]^{-1/2}. \] (3)

In a previous publication (Roos & Harun-or-Rashid 1998) we tried to determine the preferred region in the \((\Omega_m, \Omega_\Lambda)\)-plane by combining three independent observational constraints and a value for \(H_0\). Over the years more observational constraints have become available, (some of them summarized in our unpublished preprint Roos & Harun-or-Rashid 1999) so that we now can make use of fifteen independent constraints meeting our criteria. Since we combine the data in a least-squares fit, we can only make use of observations quoting a value and an error, but in addition many interesting limits also exist.

In Section 2 we describe the fifteen observational constraints entering our least-squares fit. Ten constraints are valid in the space of \(\Omega_m\) and \(\Omega_\Lambda\); the remaining five are only valid on the flat line Eq. (2) and will be included only when we fix the fit to that line. In Section 3 we describe the results of our fit in the \((\Omega_m, \Omega_\Lambda)\)-plane as well as along the flat line Eq. (2). We then also use Eq. (3) to determine \(t_0\).

## 2 Observational Constraints

For the Hubble constant we use the value \(h = 0.68 \pm 0.05\) from the analysis of Nevalainen & Roos (1998) in which the Cepheid period-luminosity relation is corrected for metallicity dependence. This agrees well with more recent precise determinations. For instance Mould & al (1999) find, when similarly corrected, \(h = 0.67\) (our evaluation).

### 2.1 Cosmic Microwave Background Radiation

The observations of anisotropies in the CMBR are commonly presented as plots of the multipole moments \(C_\ell\) against the multipole \(\ell\), or equivalently, against the FWHM value of the angular anisotropy signal. In general, the theoretical models for the power spectrum may depend on up to 9 parameters. Lineweaver (1998) and Tegmark (1998) have combined the data from MAP and PLANCK into a confidence region in the marginal subspace of the \((\Omega_m, \Omega_\Lambda)\)-plane. An independent analysis, combining the angular power spectrum of the BOOMERANG experiment (Melchiorri et al. 1999) with that of COBE, furnishes us a second constraint.
2.2 Gas fraction in X-ray clusters

Matter in an idealized, spherically symmetric cluster is taken to be made up of a nearly hydrostatic inner body surrounded by an outer envelope, infalling with the cosmic mix of the components. The baryonic component of the mass in galaxy clusters is dominated by gas which can be observed by its X-ray emission. Thus by measuring the gas fraction, one expects to obtain fairly unbiased information on the ratio of $\Omega_m$ to the cosmic baryonic density parameter $\Omega_b$. Using a very large sample of clusters, Evrard (1997) has obtained a 'realistic' value of

$$\frac{\Omega_m h^{-4/3}}{\Omega_b} \approx (11.8 \pm 0.7).$$

Taking $\Omega_b = 0.024 \pm 0.006 h^{-2}$ from the low primordial deuterium abundance (Tytler, Fan & Burles 1996), one obtains

$$\Omega_m = 0.36 \pm 0.09,$$

which we use as our constraint.

This constraint are restricted to a flat Universe and we only use them together with the assumption of flat cosmology in section 3.

2.3 Cluster mass function and the Lyα forest

In theories of structure formation based on gravitational instability and Gaussian initial fluctuations, massive galaxy clusters can form either by the collapse of large volumes in a low density universe, or by the collapse of smaller volumes in a high density universe. This is expressed by the cluster mass function which constrains a combination of $\Omega_m$ and the amplitude $\sigma$ of mass fluctuations (normalized inside some volume). The amplitude $\sigma$ is given by an integral over the mass power spectrum. From an analysis by Weinberg & al. (1998) combining the cluster mass function constraint with the linear mass power spectrum determined from Lyα data, one obtains the relation

$$\Omega_m + 0.18\Omega_\Lambda = 0.46 \pm 0.08$$

which we use as one constraint.
2.4 X-ray cluster evolution

Clusters of galaxies are the largest known gravitationally bound structures in the Universe. Since they are thought to be formed by contraction from density fluctuations in an initially fairly homogeneous Universe, their distribution in redshift and their density spectrum as seen in their X-ray emission gives precious information about their formation and evolution with time. Thus by combining the evolution in abundance of X-ray clusters with their luminosity-temperature correlation, one obtains a powerful test of the mean density of the Universe.

The results of Bahcall, Fan and Cen (1997) can be summarized in the relation

$$\Omega_m = 0.195 \pm 0.11 + 0.071\Omega_\Lambda$$

which we use as one constraint.

The results of Eke & al. (1998) can be summarized in the relation

$$\Omega_m = 0.44 \pm 0.20 - 0.077\Omega_\Lambda$$

which we use as one constraint.

Donahue & Voit (1999) constrain $\Omega_m$ through a maximum likelihood analysis of temperatures and redshifts of the high redshift clusters from the Extended Medium Redshift Survey, as well as from a low redshift sample (Markevitch 1998), finding

$$\Omega_m = 0.27 \pm 0.10 ,$$

This constraint are restricted to a flat Universe and we only use them together with the assumption of flat cosmology in section 3.

2.5 Gravitational lensing

The number of multiply imaged QSOs found in lens surveys is sensitive to $\Omega_\Lambda$. Models of gravitational lensing must, however, explain not only the observed probability of lensing, but also the relative probability of showing a specific image separation. The image separation increases with increasing $\sigma^*$, the characteristic velocity dispersion. Thus the results can be expressed as likelihood contour plots in the two-dimensional parameter space of $\sigma^*$ and $\Omega_m = 1 - \Omega_\Lambda$ (Chiba & Yoshii 1999).

We integrate out $\sigma^*$, and we thus obtain the constraint

$$\Omega_\Lambda = 0.70 \pm 0.16$$
Im, Griffiths & Ratnatunga (1997) use seven field elliptical galaxies to determine

\[ \Omega_{\Lambda} = 0.64 \pm 0.15 \]  \hspace{1cm} (11)

in the flat model.

We use these two constraints together with the assumption of a flat cosmology in section 3.

### 2.6 Classical double radio sources

There are two independent measures of the average size of a radio source, where size implies the separation of two hot spots: the average size of similar sources at the same redshift, and the product of the average rate of growth of the source and the total time for which the highly collimated outflows of that source are powered by the AGN. This outflow leads to the large scale radio emission. The two measures depend on the angular size distance to the source in different ways, so equating them allows a determination of the coordinate distance to the source which, in turn, can be used to determine pairs of \( \Omega_m, \Omega_{\Lambda} \)-values.

By using 14 classical double radio galaxies, Daly, Guerra & Wan Lin (1998) determine an approximately elliptical 68\% confidence region in the \((\Omega_m, \Omega_{\Lambda})\)-plane centered at \((0.05, 0.32)\). In our fit we use this constraint.

### 2.7 Supernovæ of type Ia

Type Ia supernovae can be calibrated as standard candles, and have enormous luminosities. These two features make them a near-ideal tool for studying the luminosity-redshift relationship at cosmological distances.

The factor relating brightness to redshift is a function of \( \Omega_m \) and \( \Omega_{\Lambda} \). The High-z Supernova Search Team Riess et al. (1998) have used 10 SNe Ia in the redshift range 0.16 – 0.62 to place constraints on the dynamical parameters and \( t_0 \). In the \((\Omega_m, \Omega_{\Lambda})\)-plane their 68\% confidence region is an ellipse centered at \((0.20, 0.65)\) which we use as one constraint.

The Supernova Cosmology Project (Perlmutter et al. 1998) has published an analysis based on 42 supernovæ in the high-redshift range 0.18 – 0.83. In the \((\Omega_m, \Omega_{\Lambda})\)-plane the 68\% confidence range is an ellipse centered at \((0.75, 1.36)\) which we use as one constraint.
2.8 Power-spectrum of extragalactic objects

Matter in every direction appears to be distributed in high-density peaks separated by voids. The average separation distance is $\sim 130h^{-1}\text{Mpc}$, which translates into a peak in the power spectrum of mass fluctuations. This provides a co-moving scale for measuring cosmological curvature. Broadhurst & Jaffe (1999) used a set of Lyman galaxies at $z \sim 3$ finding a constraint of the form

$$\Omega_m = 0.20 \pm 0.10 + 0.34\Omega_\Lambda$$

(12)

Roukema & Mamon (1999) have carried out a similar analysis of quasars, finding

$$\Omega_m = 0.24 \pm 0.15 + (0.10 \pm 0.08)\Omega_\Lambda$$

(13)

We use these two results as constraints.

2.9 Galaxy peculiar velocities

The large-scale peculiar velocities of galaxies correspond via gravity to mass density fluctuations about the mean, and depend also on the mean density itself. Two catalogs of galaxies have been analyzed for these velocities in order to provide information on $\Omega_m$: the Mark III catalog (Willick & al. 1997) of about 3000 galaxies within a distance of $\sim 70h^{-1}\text{Mpc}$, and the SFI catalog (Borgani & al. 1999) of about 1300 spiral galaxies in a similar volume. Combining the results in these catalogs, Zehavi & Dekel (1999) quote the constraint

$$\Omega_m h_{65}^{1.3} n^2 \simeq 0.58 \pm 0.12 ,$$

(14)

in the case of flat cosmology, where the error corresponds to a 90% confidence range. Taking the index $n$ of the mass-density fluctuation power spectrum to be $n = 1.0 \pm 0.1$ (Bond & Jaffe 1998), one obtains the constraint

$$\Omega_m = 0.55 \pm 0.14 ,$$

(15)

where the error corresponds to a 68% confidence range. This constraint we only use together with the assumption of a flat cosmology in section 3.
3 Results and Discussion

3.1 Fits

We perform a least-squares fit to the above ten constraints in the space of the two free parameters $\Omega_m, \Omega_\Lambda$. The Hubble constant is not treated as a free parameter, but is fixed to the value Nevalainen & Roos (1998).

Paying rigorous attention to statistical detail, we use the standard minimization program MINUIT (James & Roos 1975). The best fit value is then found to be

$$\Omega_m = 0.31 \pm 0.07, \quad \Omega_\Lambda = 0.63 \pm 0.21,$$

$$\chi^2 = 4.1.$$  \hspace{1cm} (16)

In Fig.1 we plot the shape of the $1\sigma$ and $2\sigma$ contours.

The above values can be added to yield

$$\Omega_m + \Omega_\Lambda = 0.94 \pm 0.22.$$  \hspace{1cm} (17)

From this we conclude that (i) the data require a flat cosmology, (ii) the Einstein-de Sitter model is very convincingly ruled out, and (iii) also any low-density model with $\Omega_\Lambda = 0$ is ruled out.

If we assume exact flatness and refit the previous ten constraints as well as the five one-dimensional constraints (5),(6),(10),(11),(15), the result is

$$\Omega_m = 0.337 \pm 0.031, \quad \Omega_\Lambda = 0.663 \pm 0.031,$$

$$\chi^2 = 7.3.$$  \hspace{1cm} (18)

The value of $\Omega_\Lambda$ in the two-dimensional fit, Eq. (16), is determined mainly by the constraint from the Supernovae Cosmology Project (Perlmutter et al. 1998). However, on the flat line several other constraints contribute much more strongly, so that the result in Eq. (18) is very precise, even indepently of the supernovae constraint.

Let us now substitute the above parameter values into Eq. (3). Adding a 7.4% $H_0$ error quadratically to the density parameter errors in Eq. (16), propagated through the integral (3), we find as a value for the age of the Universe

$$t_0 = 13.5 \pm 1.3 \ (0.68/h) \ \text{Gyr}.$$  \hspace{1cm} (19)

For an exactly flat Universe, only the error changes slightly to 1.1.
3.2 Systematic errors

With results as precise as those for the flat model, a question arising is, what about neglected systematic errors?

The constraints we use are indeed pulling the results in every direction. CMBR is orthogonal to the supernova constraints, the gravitational lensing constraint is exactly orthogonal to the gas fraction in X-ray clusters and the remaining constraints represent bands of several different directions. Thus we think that it is justified to consider the systematic errors as random and the total effect of possibly neglected systematic errors to be mutual cancellation.

Moreover, the two-dimensional fit, Eq. (16), $\chi^2$ is 4.1 for 8 degrees of freedom, and in the one-dimensional fit, Eq. (18), $\chi^2$ is 7.3 for 14 degrees of freedom, much too low for statistically distributed data. Thus we can conclude that the various errors quoted for our fifteen constraints are not statistical: they have been blown up unreasonably by the systematic errors added, and there is no motivation for blowing them up further by adding arbitrary systematic errors.

3.3 Comparison with other data

There are some categories of data which we have not used, but to which it is nevertheless interesting to compare our results.

Totani, Yoshii & Sato (1997) have tested cosmological models for the evolution of galaxies and star creation against the evolution of galaxy luminosity densities. They have found $\Omega_\Lambda > 0.53$ at 95% confidence in a flat universe.

Falco, Kochanek & Munoz (1998) have determined the redshift distribution of 124 radio sources and used it to derive a limit on $\Omega_m$ from the statistics of six gravitational lenses. In a flat universe their best fit yields $\Omega_m > 0.26$ at 95.5% confidence.

Since we also determine a value for the age of the Universe, it is of interest to look at other recent $t_0$ determinations. By using several techniques Chaboyer (1998) determine the age of the oldest globular clusters (GC) to $t_{GC} = 11.5 \pm 1.3$ Gyr. Another estimate of the age of the oldest globular clusters is due to Jimenez (1998). He quotes the 99% confidence range $t_{GC} = 13.25 \pm 2.75$ Gyr, which translates into the 68% confidence range $t_{GC} = 13.3 \pm 1.1$ Gyr. Taking a mid-value and adding 0.9 Gyr systematic error to account for the spread of values, we have
\[ t_{GC} = 12.4 \pm 1.3 \pm 0.9 \text{ Gyr} \quad (20) \]

In order to obtain \( t_0 \) from the GC value, however, one must add the time it took for the metal-poor stars to form. Estimates are poor due to the lack of a good theory, so Chaboyer (1998) recommends adding 0.1 to 2 Gyr. We presume that this is a 90% CL estimate, \(+1 \pm 1\) Gyr. Thus one arrives at the 68% confidence range

\[ t_0 = 13.4 \pm 1.3 \pm 0.9 \pm 0.6 \text{ Gyr} = 13.4 \pm 1.7 \text{ Gyr} \quad (21) \]

Jimenez (1998) also finds an age of \( t_0 = 13 \pm 2 \) Gyr for the galaxy 53W069 at \( z = 1.43 \). This value as well as Eq. (21) are in excellent agreement with our value in Eq. (19)

**Acknowledgements**

The authors wish to thank J. Nevalainen, Helsinki, for useful comments. S.M.H. is indebted to the Magnus Ehrnrooth Foundation for support.

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Figure 1: The $1\sigma$ and $2\sigma$ statistical confidence regions in the $(\Omega_m, \Omega_\Lambda)$-plane are shown. The ’+’ marks the best fit: $(\Omega_m, \Omega_\Lambda) = (0.31, 0.63)$. The diagonal line corresponds to a flat cosmology.