Photon Spin and the Shape of the Two-Photon Correlation Function

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Abstract

We use the covariant current formalism to derive the general form of the 2-photon correlation function for fully chaotic sources. Motivated by the recent discussion in the literature we concentrate on the effects from the photon spin on the correlator. We show that for locally thermalized expanding sources, like those expected to be created in relativistic heavy ion collisions, the only change relative to 2-pion interferometry is a statistical factor $\frac{1}{2}$ for the overall strength of the correlation which results from the experimental averaging over the photon spin.
The hot and dense strongly interacting matter created in relativistic heavy-ion collisions has now for many years been a subject of intense investigations. While the bulk of the particles emitted in such collisions are hadrons which due to their late decoupling provide direct information only on the final state if the collision region (its space-time geometry as well as its thermal and dynamical properties), photons and dileptons are emitted directly and thus may serve as a probe also for the early stages of the reaction zone. The absence of any discernible final state interactions makes direct photons a particularly clean and desirable probe for precision experiments like two-particle interferometry to extract the “source size”; their immediate usefulness is, however, severely curtailed by large background contaminations due to electromagnetic decays of hadrons and hadron resonances after freeze-out. 2-photon interferometry has so far been used successfully only at low energies ($E_{\text{lab}} < 100$ MeV/nucleon) where the $\pi^0$ decay background can be controlled [1] but where photon production results from a complicated interplay of coherent and incoherent production mechanisms [2,3]. At ultrarelativistic energies, where incoherent (“chaotic”) emission processes are expected to dominate [1–4], so far no clear direct photon signal has been observed [7].

Nevertheless several authors have considered two-photon correlations for high energy heavy ion collisions theoretically [4–6,8]. In this context a controversy has arisen as to how the photon spin should be correctly accounted for [4,8]. In the simplest version [5] the correlator is simply multiplied by a statistical factor $1/2$ to account for the effect of spin averaging in the detector because only photons with identical spin polarization contribute to the Bose-Einstein correlations. In Refs. [4,8] additional momentum-dependent factors were suggested which were argued to arise from incomplete overlap of the polarization vectors at non-zero relative momentum between the photons and which affect also the shape (not only the normalization) of the correlator. The detailed form of this momentum-dependent modification differs between Refs. [4,8].

In this note we attempt to settle this issue by a careful rederivation of the two-photon correlation function using the covariant current formalism [9,10]. Our final result supports the simple normalization prescription used in Refs. [5]. The additional momentum dependent
terms of Refs. [4,8] are argued to be artifacts, arising from neglecting the constraints of charge conservation in the case of Ref. [4] and of other very general algebraic constraints on the tensor structure of the photon emission function in Ref. [8].

Our treatment will follow rather closely the formalism developed in Refs. [9,10], generalizing it to vector fields. The covariant single- and two-particle photon distributions are defined by

\[ P_1(k, \lambda) = \omega \langle \hat{a}^\dagger(k, \lambda) \hat{a}(k, \lambda) \rangle, \]

\[ P_2(k_a, \lambda_a; k_b, \lambda_b) = \omega_a \omega_b \langle \hat{a}^\dagger(k_a, \lambda_a) \hat{a}^\dagger(k_b, \lambda_b) \hat{a}(k_b, \lambda_b) \hat{a}(k_a, \lambda_a) \rangle. \]

Here \( \hat{a}^\dagger(k, \lambda) \) creates a photon with momentum \( k = (\omega, \mathbf{k}) \), \( \omega = |\mathbf{k}| \), and with polarization \( \lambda \), and \( \langle \ldots \rangle \) denotes the trace over the density matrix of the photon emitting source. Following [9] we consider this source as an ensemble of “elementary” classical electromagnetic currents \( j_\mu(x) \) and construct the density matrix from eigenstates of the solution of Maxwell’s equations (in Lorentz gauge \( \partial \cdot \hat{A}(x) = 0 \))

\[ \Box \hat{A}_\mu(x) = J_\mu(x). \]

The classical source current \( J_\mu(x) \) is taken as a superposition of elementary currents (wave packets) \( j_\mu \), centered at space-time points \( x_n \) in the source and boosted to 4-momenta \( p_n \) relative to a global reference frame, emitting photons with a random initial phase \( \phi_n \):

\[ J_\mu(x) = \sum_{n=1}^N e^{i\phi_n} j_\mu^{\{x_n,p_n\}}(x) \equiv \sum_{n=1}^N e^{i\phi_n} (\Lambda_n j)_\mu(x - x_n). \]

\( x, x_n, p_n \) are coordinates in the global reference frame. \( x_n \) denotes the space-time position of the center of the elementary source \( n \), \( p_n = (E_n, \mathbf{p}_n) \) its 4-momentum on the global frame. \( \Lambda_n \equiv \Lambda(\mathbf{p}_n) \) describes the boost by momentum \( \mathbf{p}_n \) from the global frame into the rest frame of elementary source \( n \), and \( \tilde{\Lambda}_n \equiv \Lambda^{-1}_n = \Lambda(-\mathbf{p}_n) \) its inverse. \( j_\mu \) thus denotes the elementary current in its own rest frame, and \( (\tilde{\Lambda}_n j)_\mu \) is the corresponding 4-vector in the global frame\( ^1 \). All elementary currents \( j_\mu \) have the same internal structure, i.e. the same

\( ^1 \text{(Eq. (4) corrects a notational inaccuracy in Ref. [10] which has, however, no further consequences} \)
functional dependence on the relative coordinate \( x' = \Lambda_n (x - x_n) \) in their own rest frame. Note that the 4-momenta \( p_n \) of the elementary sources are not on the photon mass-shell; on-shell photon momenta will always be labelled by \( k \) or \( \mathbf{k} \). The Ansatz (4) is completely general; in particular, it allows for arbitrary \( x-p \) correlations as they exist, for example, in expanding sources.

The current (4) has the on-shell Fourier transform

\[
\tilde{J}_\mu (k) = \int d^4 x \, e^{i k \cdot x} \, J_\mu (x) = \sum_{n=1}^{N} e^{i \phi_n} e^{i k \cdot x_n} \left( \bar{\Lambda}_n \tilde{j}_\mu \right)_n (k),
\]

where

\[
\left( \bar{\Lambda}_n \tilde{j} \right)_n (k) = \bar{\Lambda}_n \left( \tilde{j}(\Lambda_n k) \right) = \bar{\Lambda}_n \left( \int d^4 x' \, e^{i (\Lambda_n k) \cdot x'} \tilde{j}(x') \right).
\]

Charge conservation implies that not only the total source current \( \tilde{J}_\mu \) is transverse,

\[
k \cdot \tilde{J}(k) = 0,
\]

but that the same is true for the elementary currents whose charge is also conserved:

\[
k' \cdot \tilde{j}(k') = 0.
\]

Due to the randomness of the phases \( \phi_n \) (8) follows directly from (7) upon inserting (5) (with \( k' = \Lambda_n k \)).

It is well known [9] that a classical current \( J^\mu (x) \) generates via Eq. (3) an asymptotic photon field in a (normalized) “coherent” state

\[
|J\rangle = e^{-\bar{n}/2} \exp \left( i \int \frac{d^3 k}{(2\pi)^3 2\omega} \sum_{\lambda=1}^{2} \tilde{J}_\mu (k) \varepsilon^\mu (k, \lambda) \hat{a}^\dagger (k, \lambda) \right) |0\rangle,
\]

with \( \bar{n} = \int \frac{d^3 k}{(2\pi)^3 2\omega} |\tilde{J}(k)|^2 \), which satisfies

\[
\hat{a}(k, \lambda) |J\rangle = i \tilde{J}_\mu (k) \varepsilon^\mu (k, \lambda) |J\rangle.
\]

for the calculations presented there.
Here $\varepsilon^\mu(\mathbf{k}, \lambda)$, $\lambda = 1, 2$, are (real and spacelike) polarization vectors which satisfy the transversality and orthonormality conditions

$$k \cdot \varepsilon(\mathbf{k}, \lambda) = 0, \quad \varepsilon(\mathbf{k}, \lambda) \cdot \varepsilon(\mathbf{k}, \lambda') = g_{\lambda\lambda'}.$$ \hfill (11)

According to (11) the state $|J\rangle$ depends on the parameters $\{x_n, p_n, \phi_n; n = 1, \ldots, N\}$ whose distribution characterizes the photon emitting source: $|J\rangle \equiv |J[N; \{x, p, \phi\}]\rangle$. We take the number $N$ of elementary currents $j_\mu$ to be distributed according to a probability distribution $P_N$, the phases $\phi_n$ to be randomly distributed between 0 and $2\pi$, and the source positions $x_n$ and momenta $p_n$ to be distributed with a classical phase-space density $\rho(x, p)$, with normalizations

$$\sum_{N=0}^\infty P_N = 1, \quad \sum_{N=0}^\infty N P_N = \langle N \rangle, \quad \int d^4x d^4p \rho(x, p) = 1.$$ \hfill (12)

The corresponding ensemble average is given by

$$\langle \ldots \rangle = \sum_{N=0}^\infty P_N \int \prod_{n=1}^N d^4x_n d^4p_n \rho(x_n, p_n) \int_0^{2\pi} \frac{d\phi_n}{2\pi} \langle J[N; \{x, p, \phi\}]| \ldots |J[N; \{x, p, \phi\}]\rangle.$$ \hfill (13)

Using (10) it is then straightforward to show that

$$P_1(\mathbf{k}, \lambda) = \omega \varepsilon^\mu(\mathbf{k}, \lambda) \varepsilon^\nu(\mathbf{k}, \lambda) \langle \bar{J}_\mu(\mathbf{k}) \bar{J}_\nu(\mathbf{k}) \rangle,$$ \hfill (14)

$$P_2(\mathbf{k}_a, \lambda_a, \mathbf{k}_b, \lambda_b) = \omega_a \omega_b \varepsilon^\mu(\mathbf{k}_a, \lambda_a) \varepsilon^\nu(\mathbf{k}_b, \lambda_b) \varepsilon^\rho(\mathbf{k}_b, \lambda_b) \varepsilon^\sigma(\mathbf{k}_a, \lambda_a)$$

$$\times \langle \bar{J}_{\mu}(\mathbf{k}_a) \bar{J}_{\nu}(\mathbf{k}_b) \bar{J}_{\rho}(\mathbf{k}_b) \bar{J}_{\sigma}(\mathbf{k}_a) \rangle.$$ \hfill (15)

In high energy experiments one averages in practice over the helicities of the observed photons. Using

$$\sum_{\lambda=1}^2 \varepsilon^\mu(\mathbf{k}, \lambda) \varepsilon^\nu(\mathbf{k}, \lambda) = -g^{\mu\nu} - \frac{k_\mu k_\nu}{(n \cdot k)^2} + \frac{k_\mu n_\nu + n_\mu k_\nu}{n \cdot k},$$ \hfill (16)

with an arbitrary timelike unit vector $n^\mu$ with $n^2 = 1$, one can see from the transversality condition (7) that the second and third term on the r.h.s. don’t contribute, and we get for the spin-averaged spectra
\[ P_1(k) = -\omega \langle \tilde{J}_\mu^*(k) \tilde{J}_\mu(k) \rangle, \quad \text{(17)} \]
\[ P_2(k_a, k_b) = \omega_a \omega_b \langle \tilde{J}_\mu^*(k_a) J_\nu(k_b) J^\nu(k_b) \tilde{J}_\mu(k_a) \rangle. \quad \text{(18)} \]

The following steps are completely analogous to those presented in Ref. [10] (see [11] for intermediate steps) and will not be repeated in detail. Performing the average over the random phases leads to the factorization of the two-particle spectrum (18) similar to the Wick theorem in vacuum and thermal equilibrium systems:

\[ P_2(k_a, k_b) = \frac{\langle N(N-1) \rangle}{\langle N \rangle^2} \left( P_1(k_a) P_1(k_b) + \omega_a \omega_b \langle \tilde{J}_\mu^*(k_a) \tilde{J}_\nu(k_b) \rangle \langle \tilde{J}_\nu^*(k_b) \tilde{J}_\mu(k_a) \rangle \right). \quad \text{(19)} \]

Here the identical structure of all elementary sources was required to be able to perform the sum over \( N \) in (13) (see [9,11] for details). The average on the right hand side denotes the remaining integrations over the phase-space positions of the elementary sources:

\[ \langle \tilde{J}_\mu^*(k_a) \tilde{J}_\nu(k_b) \rangle = \langle N \rangle \int d^4 x e^{-i(k_a-k_b) \cdot x} \int d^4 p \rho(x, p) \left( \tilde{\Lambda}_p \tilde{j}_\mu^*(k_a) \right) \left( \tilde{\Lambda}_p \tilde{j}_\nu(k_b) \right). \quad \text{(20)} \]

Defining the Wigner density of the source currents according to

\[ S_{\mu\nu}(x, K) = \int d^4 y e^{-iK \cdot y} \left( J_\nu^*(x + \frac{1}{2} y) J_\mu(x - \frac{1}{2} y) \right), \quad \text{(21)} \]

the terms on the r.h.s. of (19) can be rewritten as

\[ \langle \tilde{J}_\mu^*(k_a) \tilde{J}_\nu(k_b) \rangle = \tilde{S}_{\mu\nu}(q, K) \equiv \int d^4 x e^{-iq \cdot x} S_{\mu\nu}(x, K). \quad \text{(22)} \]

Here we defined the off-shell vector \( K = \frac{1}{2} (k_a + k_b) \) as the average of the two on-shell photon momenta and \( q = k_a - k_b \) as their difference. This finally yields the following form for the two-photon correlation function:

\[ C(k_a, k_b) = \frac{(N)^2}{\langle N(N-1) \rangle} \frac{P_2(k_a, k_b)}{P_1(k_a) P_1(k_b)} = \frac{\tilde{S}_{\mu\nu}(q, K) \tilde{S}^\nu_{\mu}(-q, K)}{\tilde{S}^\mu_{\mu}(0, k_a) \tilde{S}_\mu(0, k_b)}. \quad \text{(23)} \]

\(^2\)Please note that (although not immediately apparent from Eq. (20)) the Wigner density in (22) depends only on the average momentum \( K \), and not on \( k_a \) and \( k_b \) separately. This was first derived by Shuryak in his PhD thesis [12] and overlooked in Ref. [8].
The difference between this formula and the corresponding one for two-pion correlations [10] is the tensor structure of the Wigner density [21] which results from the vector nature of the source currents. This is where the spin of the photon leaves its traces. To study spin effects on the 2-photon correlator one must therefore analyze the tensor structure of the Wigner density $S_{\mu\nu}(x,K)$. To this end it is useful to factor the elementary source current vectors $j_\mu$ into their length and a unit vector for their direction. The transversality condition (8) allows to decompose the directional unit vector in the basis spanned by the two polarization vectors:

$$\left(\vec{\Lambda}_n\vec{j}\right)_\mu(k) = \hat{j}_n(k) \left(\cos \psi_n \varepsilon_\mu(k,1) + \sin \psi_n \varepsilon_\mu(k,2)\right)$$

(24)

Here $\hat{j}_n(k)$ is the length of the vector on the l.h.s., and $\psi_n$ is an arbitrary angle between 0 and $2\pi$. In the sum over $n$ in Eq. (5) $\psi_n$ can take a different value for each term, i.e. it may be correlated with the momentum $p_n$ of the source $n$. Eq. (24) is the most general decomposition consistent with charge conservation. It is, however, more restrictive than the decomposition suggested in Eq. (6) of Ref. [4] which uses two angles to parametrize the directional unit vector and thus does not correctly take into account the constraints from current conservation. As is obvious from Eqs. (9) and (10) in that paper [4], this oversight is the origin of the momentum dependent prefactor $\frac{1}{2}[1 + \hat{k}_a \cdot \hat{k}_b]$ in the correlator of Eq. (8) in [4].

We will now consider the simple case where, except for the constraint of transversality, the directions of the elementary current vectors are completely uncorrelated with their momenta and the angle $\psi$ is a random variable. We believe that this is a reasonable assumption if the photon emitting source is locally thermalized. The random nature of $\psi$ is most easily implemented by inserting the Ansatz (24) into (5) and generalizing the ensemble average (13) to include an additional integration over the angles $\psi_n$:

$$\langle \ldots \rangle = \sum_{N=0}^\infty P_N \int \prod_{n=1}^N d^4x_n d^4p_n \rho(x_n,p_n)$$

$$\times \int_0^{2\pi} d\phi_n d\psi_n \frac{1}{2\pi} \frac{1}{2\pi} \langle J[N; \{x, p, \phi, \psi\}] \ldots |J[N; \{x, p, \phi, \psi\}] \rangle . \quad (25)$$
This procedure gives for the correlation function (23) the simple result

\[ C(k_a, k_b) = 1 + \frac{1}{2} \frac{|\tilde{S}(q, K)|^2}{S(0, k_a) S(0, k_b)} \tag{26} \]

where the scalar Wigner density is defined in terms of the moduli of the elementary currents in (24) (c.f. Eqs. (20), (21) and (24)):

\[ \tilde{S}(q, K) = \int d^4 x e^{-iq \cdot x} S(x, K), \tag{27} \]
\[ S(x, K) = \int d^4 y e^{-iK \cdot y} \left\langle J^*(x + \frac{1}{2}y) J(x - \frac{1}{2}y) \right\rangle = \langle N \rangle \int d^3 p \rho(x, p) \hat{J}^\mu_p(k_a) \hat{J}_\nu_p(k_b). \tag{28} \]

The factor $\frac{1}{2}$ in front of the correlation term reflects the fact that only photons in the same helicity state contribute to the Bose-Einstein correlations (i.e. are described with symmetrized wavefunctions), and that we have summed over final state photon helicities. Except for this factor, the correlation function has the same general form as for scalar particles.

We have already explained the origin of the difference between our result and that of Neuhauser as being due to the neglect of current conservation in Ref. [4]. We would like to close the paper with a critical discussion of the work by Razumov and Feldmeier [8] who, using a different derivation, found similar spurious terms in the tensor structure of the photon emission function (21). These authors used the Ansatz (Eq. (16) of Ref. [8])

\[ \langle \tilde{J}^\mu(k_a) \tilde{J}^\nu(k_b) \rangle \overset{\text{Ansatz}}{=} \int d^4 x e^{-iq \cdot x} Q^{\mu\nu}(k_a, k_b| x) \omega(k_a, k_b| x), \tag{29} \]

which (as is also clear from the text of their paper) should be compared with eq. (22) above. They then performed two (in our opinion) erroneous manipulations: using the transversality of the correlator

\[ k_\mu^a \langle \tilde{J}^\mu_a(k_a) \tilde{J}_\nu(k_b) \rangle = \langle \tilde{J}^\mu_a(k_a) \tilde{J}_\nu(k_b) \rangle k_\nu^b = 0 \tag{30} \]

they argued that the tensor $Q^{\mu\nu}$ in (28) should also be transverse to $k_a$ and $k_b$. This, however, does not follow: writing $k_{a,b} = K \pm q/2$ and realizing (see [22]) that $Q(k_a, k_b| x)$ is actually only a function of $K = (k_a + k_b)/2$, one realizes that the transversality condition for $Q$ is actually much more complicated and involves derivates of $Q$ with respect to $x$.  

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Furthermore, in Eq. (20) of Ref. [8] the tensor structure of $Q$ is parametrized in terms of $k_a$ and $k_b$ separately instead of only in terms of $K$, leading to spurious terms in the tensor structure which eventually result in their spurious momentum-dependent prefactor in the second term of the correlation function (26). For a hydrodynamically expanding source their decomposition of the $Q$-tensor also involves the hydrodynamical 4-velocity. From Eqs. (20), (22) it is clear that in a hydrodynamical parametrization of the emission function the hydrodynamical flow velocity enters through the classical phase-space density $\rho(x,p)$ of the elementary source currents, which (in Boltzmann approximation) will have the typical local equilibrium form $\rho(x,p) \sim \exp(-p \cdot u(x)/T(x))$. It is not directly associated with the elementary currents $\tilde{j}_\mu$ which generate the tensor structure $Q^{\mu\nu}(K,x)$ of the emission function.

In summary, we have rederived the general form for the two-photon correlation function for chaotic sources using the covariant current formalism. We found additional constraints on the tensor structure of the photon emission function resulting from conservation of the electromagnetic current which were previously overlooked. For thermalized sources with random orientation of the elementary photon-emitting currents we found that the only effect of photon spin on the correlation function is a reduction of the correlation strength by a factor $\frac{1}{2}$ resulting from spin averaging in the detector. Similarly simple results are expected to hold for quantum statistical correlations between identical fermions, justifying the generally adopted procedure in the analysis of $pp$ and $nn$ correlations.

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