Noodle Models for Scintillation Arcs

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ABSTRACT

I show that multiple, narrow, parallel strips of phase-changing material, or “noodles,” generically produce parabolic structures in the delay-rate domain. Such structures are observed as “scintillation arcs” for many less-strongly-scattered pulsars. The noodle model assumes that the strips are extremely long, with widths comparable to a pair of Fresnel zone at their separation from the origin. Their lengths are many times a Fresnel scale, or hundreds or thousands times their widths. Physically, the strips may correspond to filaments or sheets of over- or under-dense plasma, with a normal perpendicular to the line of sight. They may lie in reconnection sheets, along magnetic field lines. If so, observations of scintillation arcs would allow visualization of magnetic fields in reconnection regions. Along the strips, the Kirchhoff integral leads to a stationary-phase point where the strip is closest to the line of sight. Across the strip, the integral leads to a 1D Fourier transform to the observer plane. Most observations can measure for each strip only an amplitude, that is related to the width of the strip and its phase contrast with surrounding material, and a phase, that varies with the geometric phase where the strip is closest to the line of sight. Observations suggest a minimum strip width of about 800 km, comparable to the ion cyclotron radius.

Key words: scattering – pulsars – ISM: structure

1 INTRODUCTION

1.1 Background

Scintillation arcs are a remarkable phenomenon in interstellar scintillation of pulsars at decimeter and meter wavelengths. They indicate the presence of extremely compact, sparsely distributed structures that are concentrated in thin screens, and produce angular deflections of radio waves by many milliarcseconds.

Scintillation is often observed in the dynamic spectrum: a time series of spectra, gathered sequentially in time. Thus, the dynamic spectrum represents the intensity of the electric field as a function of observing frequency \( \Delta \nu \) and time \( t \). Scintillation appears as variations of intensity, as scattered rays reinforce or cancel. First described by Stinebring et al. (2001), scintillation arcs appear in the secondary spectrum. The secondary spectrum is the square modulus of the Fourier transform of the dynamic spectrum to the domain of delay \( \tau \) (conjugate to frequency offset \( \Delta \nu \)) and rate \( f \) (conjugate to time \( t \)). Some authors refer to rate as Doppler frequency. A central maximum at the origin of the the secondary spectrum \( (\tau, f) = 0 \) corresponds to the mean intensity of the source, averaged over the observing frequency band and time span. Arcs extend from this maximum, along approximately parabolic paths \( \tau = \pm af^2 \), where \( a \) is a constant, the curvature parameter. They appear for both positive and negative \( \tau \), nearly symmetrically, as required for a real dynamic spectrum (as for single-antenna observations) or a nearly real spectrum (as for a baseline short compared with the lateral scale of scintillation: see Brisken et al. (2010)). Particularly sensitive observations reveal subsidiary arcs, extending from points along each arc, and with the same curvature but in the opposite direction. Figure 1 displays a schematic view of a scintillation arc in the secondary spectrum.

Scintillation arcs have been observed for most nearby pulsars (Stinebring 2007). Even the power summed over all arcs

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Figure 1. Schematic depiction of scintillation arcs in the rate-delay plane ($f, \tau$). The central maximum is the point at $a$ (the first term in Equation 29, from the direct path); the primary arcs are $b$ and its inversion $c$ (from the second and third terms); and examples of secondary arcs are $d$ and $e$, the fourth and fifth terms.

and secondary arcs is much less than the power in the primary maximum (Gwinn et al. 2011). Observations in several bands at individual epochs show that the curvature parameter varies quadratically with observing frequency $\nu$: $a \propto \nu_0^2$ (Hill et al. 2003). Arcs vary in strength from one observation to another, and also vary in the curvature parameter $a$; in some cases, more than one arc is observed at one epoch (Stinebring 2007). The fine structures within the arcs evolve on timescales of months, with individual identifiable features occasionally moving along the arcs, toward increasing $f$ (Hill et al. 2005). Interferometric observations have showed that the scintillation arcs arise from lines of sight separated in angle from the central maximum in the secondary spectrum (Brisken et al. 2010).

Observations imply that the arcs are an interference phenomenon. Their weakness relative to the peak indicate that the amplitude of the scattered field is small, relative to the unscattered field. The wide span of the arcs in delay, far more than the inverse of the observing bandwidth, indicates that phase differences between scattered and unscattered fields amount to many turns of phase. The identification of individual features suggests that the arcs arise from a set of entities $\{q_i\}$, mapped onto individual points of delay and rate $(\tau_i, f_i)$, via functions $\tau_i \propto a q_i^2$ and $f_i \propto q_i$. The fact that interferometry detects phase differences along the arcs suggests that geometric phase, from offsets of scatterers from the line of sight, contributes to delay and rate.

In general, phase-coherent scattering can fall into “strong” and “weak” extremes. In strong scattering the observer receives radiation from along multiple paths, with phases that differ by more than $2\pi$. In weak scattering, phases along paths to the observer differ by less than $2\pi$. Both the geometric path length and the optical path length can contribute to this phase, and both vary with frequency. The rapid change of arc properties with frequency, as evidenced by their large delay $\tau$ relative to the unscattered path, implies that the arcs result from strong scattering, in this sense.

Commonly, interstellar scattering is assumed to be “optically thick” in the sense that every path from the source intercepts the scattering material and is deflected to some degree. The deflection is drawn from a Gaussian distribution, or a distribution that resembles it. This appears not to be the case for the scintillation arcs, at least in many cases: only a small fraction of the radiation from the source suffers enough deflection to contribute to the arcs. The rest resides in the nearly-undeflected radiation that contributes to the maximum at the origin in the secondary spectrum. Thus, the arcs arise from strong but “optically thin” scattering. In this sense, scintillation arcs are an extreme case of scattering by a Levy distribution, where extreme but rare deflections dominate averages (Boldyrev & Gwinn 2003b,a, 2005).

Traditionally, interstellar scattering has been treated as localized in a screen that is thin along the line of sight. This picture includes many features of scattering material that is distributed along the line of sight. The thin-screen picture also describes scintillation arcs well; material that is extended along a large fraction of the line of sight tends to blur the arcs.
1.2 Interpretation

Interpretations of scintillation arcs have focused on discrete compact scattering structures in the interstellar medium. These compact structures deflect pulsar radiation from out of the line of sight to the observer (Walker et al. 2004). The resulting paths interfere with the undiffracted, primary path to produce interference fringes, with fringe phase that varies with frequency and time, and thus to form the scintillation arcs. The arcs remain stable over the time and frequency span of an observation, and overall features can persist for weeks (Hill et al. 2003, 2005). The stability of the arcs can be taken to suggest that the compact structures responsible for the scattering remain nearly fixed, in both structure and position, with changes in frequency and time. Within this interpretation, a number of inversions have found the locations of the compact structures responsible for scattering on the sky, relative to the undiffracted line of sight (Brisken et al. 2010; Pen et al. 2014). The inferred distribution of the compact scattering structures is highly anisotropic (Walker et al. 2004). The gradient of refractive index required to deflect a scattered ray implies a large gradient of electron column, and hence a large column density, even given the small lateral scale.

The physical nature and origin of the compact structures responsible for scintillation arcs has led to much discussion, fueled by the inferences of small size and large column densities. Among the suggestions are evaporating concentrations of primordial hydrogen (Walker 2013) and of quark strangelets (Pérez-García et al. 2013). Pen & Levin (2014) suggest that the scattering responsible for the arcs arises where reconnection sheets in the interstellar plasma happen to lie tangent to the line of sight. A corrugated sheet can align with the line of sight at a number of places; each such location acts as a scatterer (Liu et al. 2016). The extension of the sheet along the line of sight spreads the required electron column over a longer distance. Simard & Pen (2018) propose a specific optical model for such a sheet, matching the splendid observations of scintillation arcs for pulsar B0834+06 by Brisken et al. (2010).

1.3 Outline of Paper

This paper concerns primarily the optics of scintillation arcs. I consider a simple picture in which variations in refractive index take the form of thin strips, or “noodles,” in a screen localized along the line of sight. Figure 2 illustrates the structure. Outside of these relatively long, narrow strips, localized in a thin screen, the medium is uniform. Physically, these strips may be filaments perpendicular to the line of sight, or sheets with normal perpendicular to the line of sight. This model is motivated by the work of Pen & Levin (2014), Liu et al. (2016), and Simard & Pen (2018). In Section 2, I use scalar Kirchhoff diffraction theory to find the field at the observer from an assemblage of parallel strips. I first find the field in the absence of scattering (the “no-screen” case), and then the change in that field from introducing a single strip. The integral along the strip takes the form of a Gaussian function with imaginary variance; the stationary-phase approximation yields the exact result, with magnitude \( \sqrt{2\pi r_F} \), where \( r_F \) is the Fresnel scale, defined by Equation 3 below. The integral across the strip has the form of a Fourier transform, with a magnitude of order its width \( w \), and an overall phase of the geometric phase where the strip lies closest to the line of sight. Figure 2 shows the geometry. The single-strip expression leads to the expression for many such strips, and so to scintillation arcs. In Section 5, I introduce a number of simplifying approximations, and find their consequences for the Kirchhoff integral in Section 6. In Section 7, I show that under these approximations, an assemblage of strips will generically produce scintillation arcs; and then in Section 8 I compare results with observations to justify the approximations made earlier. I find the effects of structures that are slanted, either out of the thin screen or relative to other noodles in the screen, in Section 9. In Section 10, I discuss the physical nature of the “noodles,” perhaps filaments or sheets aligned with magnetic field lines in a reconnection region. Section 11 summarizes the conclusions.

2 KIRCHHOFF INTEGRAL FOR THE NOODLE MODEL

A Kirchhoff integral gives the scattered field in the observer plane (see, for example Johnson & Gwinn 2015). This scattered field \( \psi_{\text{obs}} \) may be one polarization of the electric field of the source, \( \psi_{\text{src}} \), for example.

\[
\psi_{\text{obs}}(b) = \frac{-i}{2\pi r_F^2} \int_{\text{screen}} d^2 x \left| b - x \right|^2 e^{i\left( \frac{x}{r_F} \right) |x-s|^2} \psi_{\text{src}}(s). \tag{1}
\]

Here, the wavenumber is \( k = 2\pi v/c \), where \( v \) is the observing frequency. Figure 3 shows the geometry. The optical axis is the \( z \)-axis. The distance from observer to screen is \( D \), and that from screen to pulsar is \( R \). The transverse coordinates are \( b \) in the observer plane, \( x \) in the screen plane, and \( s \) in the source plane. The scattering screen introduces a phase \( \varphi(x) \). In analogy to the magnification of the scattering screen viewed as a lens, I define:

\[
M = \frac{D}{R} \tag{2}
\]
Figure 2. Structure of the “noodle” model. The medium is uniform, except for narrow parallel strips in a thin plane perpendicular to the line of sight. The plasma phase may vary across the strips, but is uniform along them. The dotted ellipses suggest the their contributions to the Kirchhoff integral, of magnitude $\sqrt{2}\pi$ times the Fresnel scale $r_F$ along the strips, and of their width $w$ across them.

The Fresnel scale is the lateral separation at the scattering screen that corresponds to an additional geometric path length corresponding to one-half radian of phase. That scale is:

$$r_F = \sqrt{\frac{DR}{(D+R)k}} = \frac{1}{(1+M)k}. \quad (3)$$

I suppose that the source is pointlike, so that the integral of $\psi_{\text{src}}$ over the source plane collapses to a single value, at the point $s$. The Kirchhoff integral then takes the form:

$$\psi_{\text{obs}}(b) = \frac{-i}{2\pi r_F^2} e^{\frac{r_F}{\alpha}} \int_{\text{screen}} d^2x \ e^{\frac{r_F}{\alpha}} \left[ -\frac{1}{\pi r_F^2} (b+Ms)x\cdot x + |x|^2 + \phi(x) \right] \psi_{\text{src}}, \quad (4)$$

where I display the quadratic geometric phases at source and observer outside the integral. These quadratic phases express the fact that the observing plane and the source plane are flat, rather than curved like spherical wavefronts centered on the screen at $x = 0$. The Fresnel phase, is:

$$\phi_F = \frac{|x|^2}{2r_F} \quad (5)$$

This is the phase introduced by geometric path length, for a path that passes from a source on the optical axis to an observer on the axis, but through a point on the screen at $x$. A “Fresnel zone” is the annulus between $\phi_F = (N + \frac{1}{2})\pi$ and $\phi_F = (N - \frac{1}{2})\pi$, where $N$ is an integer (except for the first Fresnel zone, which extends from $\phi_F = 0$ to $\phi_F = \frac{1}{2}\pi$). Thus, across a pair of adjacent Fresnel zones, the Fresnel phase changes by $2\pi$.

3 NO SCREEN

For reference and to introduce integrals, I consider the case of zero screen phase: $\phi_s = 0$. This is the case of no screen at all. For this calculation only, without loss of generality, I take $\hat{x}$ parallel to $b + Ms$, so that $b_x + Ms_x = 0$. I use the notation $(b + Ms)\cdot \hat{x} = b_x + Ms_x$. Equation 4 then takes the form:

$$\psi_{\text{NS}}(b\hat{x}) = \frac{-i}{2\pi r_F^2} e^{\frac{r_F}{\alpha}} \int_{-\infty}^{\infty} dx \ e^{\frac{r_F}{\alpha}} \left( -\frac{1}{\pi r_F^2} (b_x + Ms_x)x + x^2 \right) \int_{-\infty}^{\infty} dy e^{\frac{r_F}{\alpha}} |y|^2 \psi_{\text{src}}. \quad (6)$$
The integral over $y$ is that of a Gaussian function with imaginary variance. One can use analytic continuation to extend the integral of a Gaussian function with positive real part for the variance to this case. Alternatively, one can invoke the stationary-phase approximation; indeed, this integral is the archetype for integration using the stationary phase approximation (see Bender & Orszag 1978). The integral over $y$ thus becomes:

$$
\int_{-\infty}^{\infty} dy \, e^{\frac{i}{2\sigma^2} y^2} = \sqrt{\frac{2\pi}{\sigma^2}} e^{\frac{-i\pi}{4}}.
$$

(7)

I complete the square to convert the integral over $x$ to the same form, times a phase factor:

$$
\int_{-\infty}^{\infty} dx \, e^{\frac{i}{2\sigma^2} \left(-\frac{1}{1+M}(b_x+Ms_y)x+x^2\right)} \frac{1}{(1+M)^2} e^{\frac{-i}{2\pi} \frac{(b_x+Ms_y)x^2}{(1+M)^2} + \frac{i}{4}} = \sqrt{\frac{2\pi}{\sigma^2}} e^{\frac{-i\pi}{4}}.
$$

(8)

where $b_{1x} = (b_x + Ms_y)/(1 + M)$. I then find for the field at the observer at $b$:

$$
\psi_{NS} = e^{\frac{i}{2\sigma^2} M[b-y]^2} \psi_{sc} = e^{\frac{i}{2\pi} \frac{(b_x+Ms_y)x^2}{(1+M)^2} + \frac{i}{4}} \psi_{sc}
$$

(9)

where I make use of the condition that $b_x + Ms_y = 0$ to obtain a coordinate-independent form, and I eliminate $M$ and $\sigma^2$ in favor of $k$, $D$, and $R$. Note that in the absence of a screen, the magnitude of the observed field is $|\psi_{sc}|$, and the phase of the observed field depends only on the lateral separation of source and observer $|b-s|$, as expected. The field is independent of the position of the screen, as required; and its magnitude is independent of the distance $D + R$, for the Kirchhoff integral, as normalized in Equation 1. The intensity at the observer at $b$ is simply:

$$
I_{NS} = \psi_{NS}^* \psi_{NS} = |\psi_{sc}|^2
$$

(10)

also as expected.

4 STRIP

I now suppose that the entire screen plane introduces zero phase change $\varphi(x) = 0$, except within a narrow strip of width $w$, where the phase takes the form $\varphi_s(x)$. Without loss of generality I suppose that the strip lies perpendicular to the $x$-axis and a distance $x_j$ away. I assume that this phase depends only on the coordinate perpendicular to the width of the strip, $x$. The phase is uniform along the strip, in $y$. Figure 3 shows the geometry.

I add and subtract regions of the screen to form the integral over its entirety. I divide the integral over the screen plane into 3 parts: the strip with phase $\varphi_s(x)$, contributing $\psi_s$ to the field at the observer; the integral over the entire screen with zero phase, with value $\psi_{NS}$; and the contribution of the strip with zero phase, $\psi_{s0}$, to be subtracted from the other two:

$$
\psi_{obs} = \psi_{NS} + \psi_s - \psi_{s0}
$$

(11)
4.1 Single Strip

The contribution of the strip \( \psi_s \) is given by integration over its portion of the domain of \( x \) in Equation 4:

\[
\psi_s = \frac{-i}{2\pi \tau_F} e^{\frac{1}{2\pi \tau_F} \int_{\tau_F}^{|\Im (\frac{b^2 + M^2 s^2)}{(1+M^2)}|} dx} e^{\int_{-w/2}^{w/2} dy} e^{\frac{1}{2\pi \tau_F} \left(-\frac{b^2 + M^2 s^2}{(1+M^2)}x^2 + y^2\right)} e^{i\psi_s(x,y)} \psi_{sc} \tag{12}
\]

The integral over \( y \) is identical to the integral over \( x \) in the case of no screen, Equation 8. Thus, it contributes a factor of magnitude \( \sqrt{\tau_F} \) and of phase \( \left( \frac{b_y^2}{2\pi} + \pi/4 \right) \), so that:

\[
\psi_s = \frac{1}{\sqrt{2\pi \tau_F}} e^{\frac{1}{2\pi \tau_F} \int_{\tau_F}^{|\Im (\frac{b^2 + M^2 s^2)}{(1+M^2)}|} dx} e^{\int_{-w/2}^{w/2} dy} e^{\frac{1}{2\pi \tau_F} \left(-\frac{b^2 + M^2 s^2}{(1+M^2)}x^2 + y^2\right)} e^{i\psi_s(x,y)} \psi_{sc} \tag{13}
\]

where I conveniently set \( b_{1x} = (b_x + Ms) / (1 + M) \) and \( b_{1y} = (b_y + Ms) / (1 + M) \). I use the substitution \( u = x - x_j \) to find for the contribution of the strip to the field observed at \( b^2 \):

\[
\psi_s = \frac{1}{\sqrt{2\pi \tau_F}} e^{\frac{1}{2\pi \tau_F} \int_{\tau_F}^{|\Im (\frac{b^2 + M^2 s^2)}{(1+M^2)}|} dx} e^{\int_{-w/2}^{w/2} dy} e^{\frac{1}{2\pi \tau_F} \left(-\frac{b^2 + M^2 s^2}{(1+M^2)}x^2 + y^2\right)} e^{i\psi_s(u)} \psi_{sc} \tag{14}
\]

\[
eq \frac{1}{\sqrt{2\pi \tau_F}} e^{i\phi_{e_j} - \frac{1}{2} \pi} e^{\int_{-w/2}^{w/2} du} e^{\frac{1}{2\pi \tau_F} (2(-b_{1x} + x_j)u + u^2)} e^{i\psi_s(u)} \psi_{sc} \tag{15}
\]

where \( \phi_{e_j} \) is the geometric phase at the center of the strip, \( x_j \):

\[
\phi_{e_j} = \frac{1}{2\pi \tau_F} \left( \int \left( \frac{b_x^2 + M^2 s^2}{M (1+M^2)} - b_{1y}^2 - 2b_{1x} x_j + x_j^2 \right) \right) = \frac{1}{2\pi \tau_F} \left( \frac{M(b_{1y}^2 - x_j^2)}{(1+M)} + \frac{(b_x - x_j)^2 + M(s_x - x_j)^2}{(1+M)} \right) \tag{16}
\]

The subscript \( j \) indicates that this phase depends on \( x_j \). The field at the observer is then given by Equation 11: it is the sum of the contribution of the screen without a strip, \( \psi_{NS} \); the contribution of the strip, \( \psi_s \); and the negative of the contribution of the strip with zero screen phase, \( \psi_{0s} \):

\[
\psi_{obs} = \psi_{NS} + \psi_s - \psi_{0s} \tag{17}
\]

\[
= \frac{1}{\sqrt{2\pi \tau_F}} e^{\frac{1}{2\pi \tau_F} \int_{\tau_F}^{|\Im (\frac{b^2 + M^2 s^2)}{(1+M^2)}|} dx} e^{\int_{-w/2}^{w/2} dy} \left[ e^{\frac{1}{2\pi \tau_F} (2(-b_{1x} + x_j)u + u^2)} \right] \psi_{sc} \tag{18}
\]

This is a fundamental result of this paper.

4.2 Fourier transform

One can place Equation 17 into a somewhat more intuitive form by associating the phase quadratic in \( u \) with the screen phase, and expressing the limits of the integral as a boxcar function:

\[
\psi_{obs} = \psi_{NS} + \frac{1}{\sqrt{2\pi \tau_F}} e^{i\phi_{e_j} - \frac{1}{2} \pi} \left( \int_{-w/2}^{w/2} du \cdot e^{\frac{1}{2\pi \tau_F} (2(-b_{1x} + x_j)u + u^2)} \cdot B_w(u) \cdot \left[ e^{i\psi_s(u)} + \frac{i}{2\pi \tau_F} u^2 \right] \right) \psi_{sc} \tag{19}
\]

where the boxcar function is:

\[
B_w(u) = \begin{cases} 1 & \text{if } -w/2 < u < w/2 \\ 0 & \text{otherwise} \end{cases} \tag{20}
\]

The term in braces (...) in Equation 19 has the form of a Fourier transform. The integral Fourier transforms from the domain of the variable \( u = x - x_j \) to the variable \( q_j = \frac{i}{\tau_F} (-b_{1x} + x_j) = \frac{i}{\tau_F} \left( \frac{b_x + Ms}{1+M} + x_j \right) \) \tag{21}

Thus, the Fourier transform converts from the screen plane to the observer plane, as in Fourier optics (Goodman 2005). Like the geometric phase \( \phi_{e_j} \), the variable \( q_j \) depends on \( x_j \). I indicate these dependence with the subscripted quantities \( q_j \) and \( \phi_{e_j} \). The function to be transformed is the product of the boxcar function and the term in square brackets [...] in Equation 19; I define this to be the function \( g(u) \):

\[
g(u) = e^{i\psi_s(u) + \frac{i}{2\pi \tau_F} u^2} - e^{i\phi_{e_j}} \tag{22}
\]

The Fourier transform of the boxcar function is:

\[
\hat{B}_w(q) = w \sin \left( \frac{w q}{2} \right) \tag{23}
\]
where the tilde “{
\hat{\cdot}\n” denotes the Fourier transform, and the sinc function is defined as \(\text{sinc}(t) \equiv \sin(t)/t\). The Fourier transform of \(g(u)\) is \(\hat{g}(q)\). The Fourier transform of the product \(B_w(u) \cdot g(u)\) is the convolution \(B_w(q) \ast \hat{g}(q)\). Here, the symbol \(\ast\) denotes convolution. The convolution must be evaluated at \(q_j\), the particular \(q\) corresponding to \(x_j\), as shown in Equation 17. I thus write the observed field in the form:

\[
\psi_{\text{obs}} = \psi_{\text{NS}} + \frac{w}{\sqrt{2\pi r F}} e^{i\psi_{\text{sc}}} \left( \frac{a}{q} \psi_{\text{Arc}} \right) \tag{25}
\]

where I have defined

\[
\Gamma_j = \frac{w}{\sqrt{2\pi r F}} e^{-\frac{i}{2} q_j} \left( \frac{a}{q} \psi_{\text{Arc}} \right) \tag{26}
\]

\[
\psi_{\text{obs}} = \psi_{\text{NS}} + \Gamma_j e^{i\psi_{\text{Arc}}} \psi_{\text{Arc}} \tag{26}
\]

\[
\psi_{\text{obs}} = \psi_{\text{NS}} + \sum_j \left( \Gamma_j e^{i\phi_{\text{Arc}}} \psi_{\text{Arc}} \right) \tag{28}
\]

Those strips may have different widths \(w_j\), as well as different forms for their internal phases \(\phi_s(x)\), leading to different coefficients \(\Gamma_j\). Moreover, at this stage of the calculation, before approximations, \(\Gamma_j\) and \(\phi_{\text{Arc}}\) depend on frequency and on the positions of source and observer; because source and observer are in motion, they depend on time. The resulting intensity at the observer is the square modulus of the field:

\[
I_{\text{obs}} = |\psi_{\text{arc}}|^2 \left( 1 + \sum_j |\Gamma_j|^2 + 2 \sum_j \text{Re} \left[ \Gamma_j e^{i\phi_{\text{Arc}}} \right] + 2 \sum_{k \neq j} \text{Re} \left[ \Gamma_j \Gamma_k^* e^{i(\phi_{\text{Arc}} - \phi_{\text{Arc}})} \right] \right) \tag{29}
\]

I show below that the first term in rectangular brackets \([\ldots]\) is responsible for the central maximum, the second term is responsible for the primary arcs, and the third term is responsible for the secondary arcs. Note that the first term is of order the intensity of the unscattered source, the second is smaller by a factor of order \(\Gamma\), and the third by \(\Gamma^2\); and that \(\Gamma\) is of order \(w/x_j\), as Equation 27 states. Thus, if \(w \ll r_F\), as we argue below, the second term is much smaller than the first, and the third smaller than the second. The terms also have distinct dependences on the geometric phase \(\phi_{\text{Arc}}\). The visibility takes the form of a similar expression, but with different values for \(\phi_{\text{Arc}}\) and, in principle, for \(\Gamma_j\), at the two ends of the baseline.

\section{Approximations}

Here I investigate the case of greatest interest and applicability to the problem of scintillation arcs: where the strip is narrow relative to variations of geometric phase, far from the undeflected path, and introduces a small phase change relative to the geometric phase. I suppose that the typical observed frequency range is small compared with the observing frequency, and that the displacement of the line of sight during an observation is small compared with the Fresnel scale \(r_F\). From these I obtain simpler forms for the observed field and intensity, that exhibit the properties of scintillation arcs. I show that these assumptions are well justified in the case of observed scintillation arcs in Section 8.

\subsection{Strip Location \(x_j\)}

In the most interesting cases, the strip lies far outside the first Fresnel zone. As I discuss in Section 8, analyses of observations cover cases where \(x_j\) lies between 100\(r_F\) and 1000\(r_F\). Hence I assume that \(x_j \gg r_F\).

The characteristic geometric path length introduced by the strip is \(\sqrt{c r_F} = \sqrt{c^2 r_F} = \sqrt{x_j^2 / (2 r_F^2)}\). The assumption that \(x_j \gg r_F\) thus implies that \(c r_F\) is much greater than a wavelength. This assumption produces strong cancellation or reinforcement of the scattered paths, relative to one another and to the undeflected line of sight. This assumption is slightly different from the traditional assumption of strong scattering: that every path suffers a change in path length much larger than a wavelength, relative to an undeflected path. Here I assume that scattering strips are relatively uncommon in the screen plane, so that most of the power remains in the undeflected path. However, the relatively few deflected paths are longer by many wavelengths.
5.2 Strip Width \( w \)

I consider cases where the strip has width comparable to a pair of Fresnel zones at the distance \( x_j \), or less:

\[
\frac{2\pi r_F^2}{x_j} \lesssim w
\]

This will serve to make the model more transparent, in that I separate the presumably-independent variations of geometric phase \( \phi_{gj} \) and screen phase \( \varphi_s \). Note that because of the assumption that \( x_j \gg r_F \) in Section 5.1, a strip is much narrower than the Fresnel scale:

\[
w \ll r_F.
\]

Wider strips can easily be modeled as a superposition of narrower strips. However, sufficiently wide strips will lose coherence, unless they track the Fresnel phase; and they can track the Fresnel phase over a frequency range that is inversely proportional to the width of the strip. I discuss this constraint in another paper.

5.3 Strip Phase \( \varphi_s \)

We suppose that the phase of the strip is small compared with the geometric phase: \( \varphi_s \ll \phi_g \). As we discuss in Section 7 below; the frequency dependence of the geometric phase gives rise to the scintillation arcs. If strip phase is comparable, its different frequency dependence will deform the arc. The strip phase depends on frequency through the index of refraction of plasma, and so on the column density of electrons:

\[
\varphi_s = \frac{c_0}{v} \int N_e \, dz
\]

where \( c_0 = 2.8 \times 10^{-13} \text{ cm} \) is the classical radius of the electron, \( N_e \) is the number density of electrons, and the integral is through the scatterer. Geometric phase depends on observing frequency through the Fresnel scale; \( \phi_g \propto r_F^{-2} \propto \nu \). The geometric phase is on the order of \( x_j^2/2r_F^2 \); as we discuss in Section 8 below, observations analyze a range of \( x_j/r_F \) of about \( 4.1 \times 10^2 \) to \( 1.4 \times 10^3 \). Consequently, we demand \( \varphi_s \ll 10^4 \), or \( \int N_e \, dz \ll 4 \times 10^{14} \text{ cm}^{-2} \) for a typical observing frequency of \( \nu = 326 \text{ MHz} \). Most models for arcs require far more modest column densities.

5.4 Bandwidth \( B \)

The observer of a source in strong scintillation sees changes in intensity with changes in observing frequency. Observations cover a range of frequencies \( \nu \) within a passband of bandwidth \( B \), from \( \nu_0 - B/2 \) to \( \nu_0 + B/2 \). The frequency at the center of the observing band is \( \nu_0 \), and the offset of a particular frequency from that center is \( \Delta \nu = \nu - \nu_0 \). The observing frequency affects only the Fresnel scale, with value at frequency offset \( \Delta \nu \) of:

\[
n_F^2(\Delta \nu) = n_F^2 \left( 1 + \frac{\Delta \nu}{\nu_0} \right)
\]

where \( n_0 = \sqrt{D/\left((1 + M)\kappa_0\right)} \) is the Fresnel scale at \( \nu_0 \). Here, I suppose that \( \Delta \nu/\nu_0 \ll 1 \). I will compare this with typical observing parameters in Section 8 below.

5.5 Time Range \( T \)

Over the time span of an observation, the observer at \( b \) and source at \( s \) move relative to the screen, with constant velocities. These cause the line of sight to move relative to the scatterers during an observation. Observations are made at times \( t \) ranging from \(-T/2\) to \( T/2 \). I set the coordinates so that the source and observer are at \( s = 0 \) and \( b = 0 \) at \( t = 0 \). The most important change in position is through the \( x \)-component of \( \frac{d}{dt} b_1 \), perpendicular to the strip:

\[
V_s = V \cos \theta = \frac{d}{dt} b_{1x} = \frac{1}{(1 + M)} \left( \frac{db_s}{dt} + M \frac{ds_s}{dt} \right).
\]

Here, \( V = |\mathbf{V}| \) is the speed of the line of sight relative to the scatterers, and \( \theta \) is the angle between the direction of that motion and the normal to the strip, \( \hat{s} \). Figure 4 shows the geometry. As I discuss in Section 8 below, variations of observer and source positions over the course of an observation may be as large as a Fresnel scale, but not much larger, so that \( |V_s T| \lesssim r_F \). In particular, for observations of arcs, Earth-based interferometer baselines are shorter than a Fresnel scale, whereas baselines to telescopes closer than the Moon can reach a few Fresnel scales. Thus, with the origins at the midpoint of the observation in time and frequency, we may assume that \( |b| \lesssim r_F \) and \( |s| \lesssim r_F \). Furthermore, since we assumed in Section 5.1 above that the strip is far from the optical axis, so that \( r_F \ll x_j \), we may further assume that \( |b| \ll x_j \) and \( |s| \ll x_j \). Thus, we may assume that during a single observation,

\[
V_s T \ll x_j.
\]
The geometric phase $\phi$ and frequency scales, that are inaccessible to the limited ranges of frequency and time in an observation. This is a consequence of the assumption that the observer and source move by less than a Fresnel scale during an observation, so that $s/r_0 \lesssim 1$ and $j/r_0 \lesssim 1$. Thus, phases proportional to $b^2/r_0^2$, $s^2/r_0^2$, and $bs/r_0^2$ will vary gently, by no more than a few radians, over the course of an observation. By contrast, $x_j/r_0 \gg 1$, so that phases proportional to $bx_j/r_0^2$ or $sx_j/r_0^2$ will vary much more rapidly, by hundreds or thousands of radians. I therefore ignore the slowly-varying phases in favor of the rapidly-varying ones. Furthermore, because $\Delta \nu \ll \nu_0$, I ignore terms proportional to $(\Delta \nu/\nu_0)(b/r_0)$ and $(\Delta \nu/\nu_0)(s/r_0)$. I then find that the geometric phase as defined in Equation 36 takes the simple form:

$$\phi_{\xi j} \approx \frac{1}{2r_0^2} \left( x_j^2 - 2x_j V_x t + x_j^2 \frac{\Delta \nu}{\nu_0} \right)$$

Thus, the geometric phase increases linearly with time, in the approximation that $x_j \gg V_x t$. The geometric phase increases linearly with frequency in the assumption that $\phi_S \ll \phi_\xi$. Because the strip lies far from the origin, as discussed in Sections 5.1 and 5.5, these variations are hundreds to millions of radians. By contrast, terms of order $b^2/r_0^2$, $b^2/r_0^2$, and $bs/r_0^2$ introduce variations of a radian or so. Locally, one can ignore these slowly-varying phases, while noting that they will reduce coherence of this simple linear model, over the longer span, as noted in Section 11.

### 6.3 Strip Phase and Fourier Transform

Aside from a constant amplitude and phase, any variation of screen phase $\varphi_s$ within the strip will appear only on large spatial and frequency scales, that are inaccessible to the limited ranges of frequency and time in an observation. This is a consequence...
of the narrowness of the strip, and the resulting convolution in the delay-rate domain with a sinc function, in Equation 27.

I make use of the velocity $V_r$ to express the Fourier transform variable $q_j$, as defined in Equation 22, in the form:

$$q_j = \frac{1}{r_0} \left( x_j - V_r t + x_j \left( \Delta \nu / \nu_0 \right) \right)$$

(38)

The argument in the sinc function in the expression for $\psi_{\text{obs}}$, Equation 25, is $wq_j/2$. Using the assumption that the strip has about the width of a pair of Fresnel zones, $w = 2 \pi r_0^2 / x_j$, so that $wq_j/2 = \pi$ at $t = 0, \Delta \nu = 0$. The value of the sinc function is then about $-1/\pi$. More importantly, motion of the line of sight, or changes in frequency, change the argument of the sinc function only by a small fraction: $w\Delta \alpha_j(t) = wV_r t / r_0^2 \ll 1$, and $w\Delta \nu_j(\Delta \nu) = w\Delta \nu / r_0^2 (\Delta \nu / \nu_0) \ll 1$. Thus, the contribution of the sinc function to the observed field remains nearly static over the span of an observation. Of course, the screen phase may vary in an arbitrarily complicated way, so that $\tilde{g}$ might vary strongly; however, convolution with the nearly-constant sinc function will smooth such variations out.

Thus, I make the approximation for the argument of the convolution:

$$q_j \approx q_j \bigg|_{t=0, \Delta \nu=0} = \frac{1}{r_0} x_j$$

(39)

Consequently, the above approximations imply that

$$\Gamma_j \approx \text{const.}$$

(40)

Variations of intensity over the course of an observation arise from the geometric phase $\phi_{\text{gi}}$, and its interference with the undeflected path and the other strips.

### 6.4 Approximated Kirchhoff Integral

I apply the approximations of Sections 5.1, 5.2, and 5.5 and the argument in Section 6.3 to find the corresponding approximate form for the fundamental result Equation 17:

$$\psi_{\text{obs}} \approx \psi_{\text{arc}} + \frac{1}{\sqrt{2 \pi r_F}} e^{i \phi_{\text{gi}}} \frac{1}{2} \left( \int_{-\infty}^{\infty} du e^{i \nu u} e^{i \phi(u)} - 1 \right) \psi_{\text{arc}}$$

(41)

where the geometric phase $\phi_{\text{gi}}$ is given by Equation 37. The quadratic factor of $u^2$ in the exponent of the integrand of Equation 14 is omitted because $u \lesssim w$, and $w \ll b, s, r_F$ as discussed in Section 5.2; and the term $b_{1x} u$ is omitted from that integral because of the argument in Section 6.3. The integral can be regarded as either over the strip $w$, or as over the real line restricted by a boxcar, as in Section 4.2. In either case, the result of the integral is on the order of $w$, so that the contribution of the strip is on the order of $w/r_F$.

### 7 SCINTILLATION ARCS FROM THE NOODLE MODEL

The approximations of Section 5 applied to the noodle model of Section 2, lead to scintillation arcs. I apply the approximate forms for $\phi_{\text{gi}}$ and $\Gamma_j$, as given by Equations 37, 39, and 41, to the expression for the observed field including multiple strips, Equation 28, to find:

$$\psi_{\text{obs}} = \psi_{\text{NS}} + \sum_j \left( \Gamma_j e^{i \phi_{\text{gi}}} \right) \psi_{\text{arc}} = \psi_{\text{NS}} + \sum_j \left( \psi_{\text{arc}} \right) \psi_{\text{arc}}$$

(42)

where I have defined

$$\alpha = \frac{1}{2 r_0^2 v_0}, \quad \tau_j = \alpha x_j^2$$

(43)

$$\beta = \frac{2 V_r \cos \theta}{2 r_0}, \quad f_j = \beta x_j$$

$$\gamma_j = \Gamma_j e^{i \tau_j / 2 r_0^2}$$

Note that $\gamma_j$ represents an amplitude and phase that depend on $j$, and $\tau_j$ and $f_j$ are real variables that depend on $j$, whereas $\alpha$ and $\beta$ are constants independent of $j$, although they depend on the geometry and parameters of the observation. Because $x_j$ is large compared with other transverse dimensions, the geometric phase that contributes to $\gamma_j$ is more or less random.

The intensity is the square modulus of the field:

$$I_{\text{obs}}(s, t) = \psi_{\text{obs}}^* \psi_{\text{obs}}$$

$$\approx |\psi_{\text{arc}}|^2 \left[ 1 + \sum_j |\gamma_j|^2 + 2 \sum_j \text{Re} \left( \gamma_j \exp\left(i(\tau_j \Delta \nu - f_j t)\right)\right) + 2 \sum_{k<j} \text{Re} \left( \gamma_j \gamma_k^* \exp\left(i(\tau_j - \tau_k) \Delta \nu - (f_j - f_k) t\right)\right) \right]$$

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I Fourier transform the intensity to the delay-rate domain:

\[
I_{\text{obs}}(\tau, f) = \int_{-\infty}^{\infty} d\nu e^{2\pi i \nu \tau} \int_{-\infty}^{\infty} dt e^{2\pi i ft} I_{\text{obs}}(\nu, t)
\]

From the integrals:

\[
I_{\text{obs}}(\tau, f) = |\Psi_{\text{src}}|^2 \left\{ \left[ 1 + \sum_j |\gamma_j|^2 \right] \delta(\tau, f) + \sum_j [\gamma_j \delta(\tau - \tau_j) \delta(f + f_j) + \gamma_j^* \delta(\tau + \tau_j) \delta(f - f_j)] + \sum_{k<j} \left[ \gamma_j \gamma_k^* \delta(\tau - (\tau_j - \tau_k)) \delta(f + (f_j - f_k)) + \gamma_j^* \gamma_k \delta(\tau + (\tau_j - \tau_k)) \delta(f - (f_j - f_k)) \right] \right\}
\]

where \(\delta(\cdot)\) is the Dirac delta-function. In practice, the limited duration of the observation in time and limited range in frequency convert the Fourier transform to a discrete Fourier transform; the result is the same combination of delta-functions, convolved with sinc functions in delay and rate, multiplied by the “shah” function \((\text{Bracewell 2000})\).

The secondary spectrum \(\tilde{C}(\tau, f)\) is the square modulus of the intensity in the delay-rate domain:

\[
\tilde{C}(\tau, f) = |\tilde{I}(\tau, f)|^2
\]

This secondary spectrum is an assemblage of delta-functions along parabolic arcs. Figure 1 shows these results in graphical form. The first term is simply a delta-function at the origin, shown as “a” in the figure. For some set of \(x_j\) spanning a range including \(x_0 = 0\), the second term sum sketches two parabolas with apexes at the origin; at \(\tau > 0\) for the first term in square brackets, and at \(\tau < 0\) for the second term. The figure shows these as “b” and “c” respectively. Because I assume \(|\Gamma| \approx w / \nu_0 \ll 1\), the parabolas are fainter than the delta function at the origin. The equation for the parabolas is

\[
\tau_j(f_j) = \pm 2 \alpha x_j^2 = \pm \frac{\alpha}{\beta^2} f_j^2
\]

where Equation 43 defines the constants \(\alpha\) and \(\beta\). Curvature of the parabola increases quadratically with observing frequency:

\[
\frac{\alpha}{\beta^2} \propto \nu_0^{-2}.
\]

The third sum sketches a set of yet fainter parabolas with apexes at each point of those parabolas; opening toward \(-\tau\) from apexes at \(\tau > 0\) for the first term in the square brackets, and toward \(+\tau\) from apexes at \(\tau < 0\) for the second. Figure 1 shows typical examples as “d” and “e”. This is precisely the form of the scintillation arcs. Taking the square modulus of the secondary spectrum leaves the delta-functions intact, while removing any phases.

As this discussion makes clear, the arcs arise from the linear and quadratic dependences on \(x_j\) in Equation 48. The scattering screen includes an assemblage of more or less random values \(x_j\). These random values, times the constants given by Equation 43, appear quadratically in the coefficient of \(\Delta \nu\) in Equation 42, and linearly in the coefficient of \(\nu\) in Equation 42. The 2D Fourier transform to the delay-rate domain then converts these to parabolic arcs.

8 OBSERVED PARAMETERS FOR SCINTILLATION ARCS

To frame my discussion, I present parameters similar to those observed and derived by Brinken et al. (2010) for pulsar B0834+06. They observed well-defined arcs in single-dish and sensitive VLBI observations. They observed in four frequency sub-bands of bandwidth \(B = 8\) MHz, over a frequency range of \(310 \leq \nu \leq 342.5\) MHz. I adopt a reference frequency of \(\nu_0 = 326\) MHz. Thus, the fractional change in observing frequency \(\Delta \nu / \nu_0 < 2.5 \times 10^{-3}\), in accord with the assumption in Section 5.4. Their most sensitive spectrum, from Arcadio to the Green Bank Telescope, had a projected length of \(b \approx 2300\) km. They obtained secondary spectra with resolution of 125 ns in delay \(\tau\) to a maximum delay of 2.05 ms, and a resolution of 0.15 mHz in rate \(f\) over a width of \(\pm 80\) mHz. These parameters correspond to an effective observing bandwidth of \(B = 8\) MHz, and an effective observing time interval of \(T = 6500\) s.

The maximum delay observed for the arc is \(\tau_{\text{max}} \approx 1.0\) msec and the corresponding maximum observed fringe rate is \(f_{\text{max}} \approx 45\) mHz. Brinken et al. (2010) distinguish features in the arc to delays as small as \(\tau_{\text{min}} \approx 0.01\) msec and rates as small as \(f_{\text{max}} \approx 0.5\) mHz; in their holographic inversion of these data, Pen et al. (2014) trace features to about the same minimum. Features with smaller delays are too numerous to separate; the “undiflected line of sight” may simply be a superposition of many arcs, very near the origin. Brinken et al. (2010) find that the curvature of the arc is well modeled as \(\tau = a f_j^2\) with \(a = 5.5577 \times 10^6 / \nu_0^2\), where \(\nu_0\) is the frequency in Hz at the center of the observing band. The arcs maintain their locations in delay-rate space, when observed over an 8-MHz band, to within about 1% over the 6500 s span of the observation. The total flux density of the arc is a few percent of the total flux density of the pulsar (Gwinn et al. 2011). The distance of the pulsar is \(D + R \approx 640\) pc, as determined from the pulsar’s dispersion measure; such measures are unreliable and the distance may be quite different. Brinken et al. (2010) infer a fractional distance of the screen to the pulsar of \(D / (D + R) = 0.353\), or \(M = R / D = 1.83\).

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From comparison of interferometric, temporal, and frequency variations of the scintillation arcs, Brisken et al. (2010) interpret arc as resulting from paths that intersect the screen primarily along a linear structure, with long axis oriented at position angle of 25.2 degrees east of north. This direction is chosen as the x-axis in the noodle model. They find that the scintillation velocity is dominated by the velocity of the pulsar; the effective velocity at the screen is \( V_x = 305 \text{ km s}^{-1} \) parallel to the long axis of the distribution, and \( V_y = -145 \text{ km s}^{-1} \) perpendicular to that axis. This corresponds to a velocity of the source alone of \( V_x = V_{\text{eff}}/M \), with velocity parallel to the long axis of scattering \( V_{\text{eff}} = 170 \text{ km s}^{-1} \) and perpendicular to the long axis \( V_{\text{eff}} = V_{\text{eff}} = 90 \text{ km s}^{-1} \).

From the above parameters, the Fresnel scale at \( v_0 = 326 \text{ MHz} \) is:

\[
r_{\text{F}} = \sqrt{\frac{M}{1 + M}} \frac{D}{K_0} = 8.0 \times 10^{10} \text{ cm}.
\]

In fact, the Fresnel scale is slightly different for each of the four observed 8-MHz bands; I ignore this detail. The net motion of the line of sight at the screen during the observation is \( V_x T \approx 1.2 \times 10^{11} \text{ cm} \), and the motion parallel to the axis of scattering is \( V_x T \approx 1.1 \times 10^{11} \text{ cm} \), both on the order of the Fresnel scale. This is in accord with the assumption in Section 5.5. The interferometer baseline length is much less than a Fresnel scale \( b / r_{\text{F}} \approx 3 \times 10^{-3} \); although a baseline extending to the distance of the Moon, such as those to the RadioAstron spacecraft (Kardashev et al. 2017) would have length about half a Fresnel scale.

The maximum displacement of a scattered path, at the screen, is

\[
x_{\text{max}} = \sqrt{\frac{M}{1 + M}} \sqrt{c \tau_{\text{max}} D} = 1.1 \times 10^{14} \text{ cm} = 1.4 \times 10^{3} r_{\text{F}}.
\]

Analogously, the minimum displacement of a distinguishable feature is

\[
x_{\text{min}} = \sqrt{\frac{M}{1 + M}} \sqrt{c \tau_{\text{max}} D} = 1.1 \times 10^{13} \text{ cm} = 1.4 \times 10^{2} r_{\text{F}}.
\]

Thus, observations indicate that \( x_j \ll r_{\text{F}} \), as assumed in Section 5.1. Features lie closer than this to the origin, as noted above, but are so numerous that they are difficult to disentangle.

9 EXTENSIONS OF THE THIN-SCREEN NOODLE THEORY

9.1 Sheets or Filaments Extending Along the Line of Sight

Sheets or filaments of refracting material (“lasagna” or “spaghetti”) that extend along the line of sight, and are aligned with 2D surfaces of constant Fresnel phase, can play the role as strips in a thin screen. Figure 5 shows the geometry for a filament. Such sheets reduce the required plasma over- or under-density of a strip, by distributing the scattering plasma along the line of sight. The effective column density is

\[
N_e = \frac{\int N_e \csc \eta dz}{\int N_e dz},
\]

where \( \eta \) is the angle between the sheet or filament and the line of sight. Because the cosecant is strongly peaked near \( \eta = 0 \), this effect will strongly select for structures close to the line of sight. This is a fundamental part of the models of Pen & Levin (2014), Liu et al. (2016), and Simard & Pen (2018); in the model of Simard & Pen (2018) a radius of curvature plays the role of \( \text{w csc} \eta \).

Sheets or filaments can extend until curvature of either sheet or Fresnel surface makes the sheet depart from the Fresnel surface by the width of a Fresnel zone. Surfaces of constant Fresnel phase take the form of ellipsoids of rotation about the undeflected line of sight, with the foci of the ellipsoids at the source and observer positions. In the approximations considered in Section 5, the possible distance of such extension is quite long; the Fresnel phase changes by 1 radian in a distance parallel to the optical axis of \( \Delta z = r_{\text{F}} \sqrt{(R + D)/x_j} \). For the parameters of B0834+06 presented in Section 8 this is about \( 4 \times 10^{14} \text{ cm} \) for a sheet at \( x_j = 10^{3} r_{\text{F}} \), with a thickness of \( w \approx r_{\text{F}}^2 / x_j \approx 8 \times 10^{7} \text{ cm} \). Evidently the physics of such structures limits their extent along the line of sight, more than geometric phase.

9.2 Canted Noodles

If one strip lies at an angle to the others, then it will produce a point in the secondary spectrum that lies off the original arc. Figure 6 shows an exaggerated view of the geometry. Mathematically, the offset of the resulting point \((r_{\text{F}}, f_j)\) reflects the inclination \( \theta_p \) for this “canted” strip, resulting in a different value for \( \beta \) in Equation 43 for this particular strip. The secondary arc extending from this point will have the same curvature as the primary arc; and the canted strip will contribute an offset point to the secondary arcs from other strips. (Brisken et al. 2010) observe a similar offset for the 1-ms feature, and (Pen et al. 2014) propose a similar explanation based on stationary, compact structures.
Figure 5. A filament of scattering plasma tilted with respect to the line of sight acts as a strip of the same width in a thin screen; the slant increases the effective column density of the filament. The figure exaggerates the length and thickness of the filament, relative to the length of the line of sight.

Figure 6. Schematic view of scattering by a parallel strip (a) and a canted strip (b). A strip parallel to the rest combines with the undeflected line of sight, and the other strips, to produce a parabolic arc in the secondary spectrum, as discussed in Section 7. A canted strip will combine with the undeflected line of sight to produce a point off the parabolic arc, but with the other strips to produce a secondary arc with apex there, with the same curvature as that of the primary arc. Crosses show points where strips are closest to the undeflected line of sight at times $t = 0, T$. The thickness of the strips, and the motion of the line of sight during an observation, are highly exaggerated.
10 PHYSICAL NATURE OF THE NOODLES

Physically, the strips or “noodles” could be either sheets or filaments, with a normal perpendicular to the line of sight. Thus, sheets would be extended parallel to the line of sight, and filaments might lie at some angle to it. As noted above, Pen & Levin (2014) suggested that reconnection sheets, corrugated along the line of sight, could contain sufficient electron column to produce the arcs. They followed a suggestion by Goldreich & Sridhar (1995) that the interstellar turbulence responsible for scattering might lie in reconnection sheets. Goldreich & Sridhar (1995), Lithwick & Goldreich (2001), and Maron & Goldreich (2001) suggest that the resulting fluctuations in plasma density should lie in long, thin filaments. If noodles are indeed such filaments, observations of scintillation arcs would allow visualization of magnetic fields in reconnection regions.

Such turbulence might be highly intermittent, in the sense that particular degrees of freedom are excited only rarely (Frisch & Kolmogorov 1995). In this case, such filaments would be rare, so that the scattering is “optically thin” in the sense discussed in Sections 1 and 6.1 above. Of course, filaments will not extend as far along the line of sight as sheets with dimensions comparable to the length of a filament. They thus require a higher density contrast to produce the same phase change in a model screen. I will discuss the electron column required to produce such a phase change, in the strip model, in a further paper.

Within the context of the strip model, features in the secondary spectrum at maximum displacement from the origin correspond to the minimum width of strips. The inferred scale of structures required to produce scintillation arcs at the maximum delay $\tau$ is quite small: about $\frac{x^2}{x_{\text{max}}}$, or about 800 km. Minimum scales of turbulence, or inner scales, of this order have been proposed for interstellar turbulence on the basis of other observational evidence. Lithwick & Goldreich (2001) suggest that the minimum width of filaments should be about the proton gyroradius. Spangler & Gwinn (1990) propose on the basis of observations that the minimum length scale should be a few hundred km, about the ion inertial scale, somewhat larger than the gyroradius; Johnson et al. (2018) find a value for the minimum scale of turbulence of 800 km, toward the Galactic center source SgrA*.

11 SUMMARY

I have presented a simple model for scintillation arcs, based on linear structures of over- or under-density in the interstellar plasma: “noodles” or strips. These structures are much longer than they are wide: they extend over many Fresnel zones along their lengths, but have width of about a pair of Fresnel zones perpendicular to their lengths, and to the line of sight. At the inferred separation of the filaments from the undeflected line of sight, this demand lengths hundreds or thousands of times their widths. The noodles may have large extent along the line of sight, but must remain aligned with it to within a fraction of a Fresnel zone. Physically, the noodles may correspond to filaments or sheets of over- or under-dense plasma, with a normal perpendicular to the line of sight. They may lie in reconnection sheets, perhaps along magnetic field lines; observations of scintillation arcs would then allow visualization of fields in reconnection regions. If the distribution of phase change $\varphi_\text{s}$ is constant along the length of such a strip, then the Kirchhoff integral along the strip is the straightforward integration of a complex Gaussian function. The Kirchhoff integral across the strip is the Fourier transform of $\varphi_\text{s}$ minus the contribution of the strip with $\varphi \equiv 0$. The resulting effects of structure within the strip are not accessible present observations, except for the magnitude and phase of that Fourier-transformed function near the origin. Nearby strips can contribute coherently to brighten points in the secondary spectrum, but will reduce coherence in frequency; Macquart has noted such reduced coherence in frequency. Canted strips will produce points off the main arc.

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