Nonlinear dynamics of multi-edge trepanning vibration drilling

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Abstract. In this paper, the dynamics of multi-edge trepanning vibration drilling is considered as multi-edges cutting with regenerative mechanism of oscillations excitation. The equations of new surface formation, the equations of motion for a system with one degree of freedom, and the fractional-rational cutting law are used for the numerical simulation of the dynamics of the cutting process. It presents the results of modeling the variation in chip thickness and shape of the chips.

1. Introduction

One of the main methods for receiving large diameter holes is trepanning drilling. Because, the energy costs in this method are significantly lower than in other methods. One of the main problems encountered during drilling is the removal of chips from the cutting zone and the supply of coolant to this zone due to the drain chips. For solution of this problem, vibration is used in trepanning drilling, which, under certain conditions, allows to obtain segmented (discontinuous) chips. As in other processing methods with the rotation of a tool or workpiece (turning, milling, etc), in trepanning drilling, the regenerative mechanism is the most powerful, leading to the appearance of self-oscillations of tool and the appearance of intermittent cutting [1, 2, 3, 6].

In this paper, we studied the dynamics of vibration trepanning drilling with a multi-edge trepanning drill. Consider a tool in the form of the main body, which appears to be a hollow cylinder (shell), on the end surface of which $n$ cutting elements are mounted (figure 1).

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![Figure 1](image_url)  
**Figure 1.** Schematic of multi-edges trepanning vibration drilling
2. The complete model of nonlinear dynamics

The study of the dynamics of this process is performed through a complete model of nonlinear dynamics, which can be described by a system consisting of three groups of equations: equations of new surfaces generation; cutting law and equations of motion of the technological system [5, 8, 9, 11].

Equations of new surfaces generation. In multi-edge trepanning drilling, j-th cutting edge processes the surface, that was formed by the previous (j-1)-th cutting edge in time \( t - t_{j-1} \). The time \( t_{j-1} \) is the delay between cutting edges (j-1)-th and j-th), that equal to the time the part rotates at angle between the cutting edges \( t_{j-1} = \varphi_{j-1}/\omega \). Equations of new surfaces generation are written as [4, 6, 7, 8, 11, 13, 14]:

\[
\begin{align*}
D_j(t) &= Vt - u(t) - L_{j-1}(t - t_{j-1}) + A - H_j, \\
h_j(t) &= \max\left[0, D_j(t)\right], \\
L_j(t) &= L_{j-1}(t - t_{j-1}) + h_j(t),
\end{align*}
\]

where \( L_j(t) \) - is the axial distances from the free end of the part to the surface under j-th cutting edge and; \( L_{j-1}(t - t_{j-1}) \) - is the axial distances from the free end of the part to the surface under (j-1)-th cutting edge; \( D_j(t) \) - is the axial distance from the j-th cutting edge to the surface being processed \( L_{j-1}(t - t_{j-1}) \); \( u(t) \) - axial deflection (vibration) of the tool; \( h_j(t) \) - uncut chip thickness for the j-th cutting edge; \( A = l - Z_0 \) - constant distance, \( l \) – length of the part being processed, \( Z_0 \) - initial axial position of the first cutting edge (nominal set value); \( H_{0j} \) - initial axial offset of the j-th cutting edge (nominal set value) from the first one, axial for which \( A \) is defied, \( t \) – current time.

Cutting forces. \( F_j \) in the axial direction for each j-th cutting edge are described by the cutting law model in the form of a fractional rational function [4, 6, 10, 12]:

\[
F_j(t) = K_\sigma h_j(t) \left( \frac{c + r h_j(t)}{c + h_j(t)} \right),
\]

where: \( K_\sigma = \gamma \sigma_\gamma B \) - static cutting stiffness, \( \sigma_\gamma \) - characteristic stress value for given material, \( B \) - chip width; \( \gamma, r \) - non-dimensional coefficients, determined experimentally for given process conditions at hand, \( c \) – characteristic linear size of the cutting process.

Equation of motion. In the case of rigid attachment of the cutting edges with the main body of the tool, all cutting edges perform the same vibrations of the tool. We thus have a dynamical system with one degree of freedom (DOF) with the axial vibration of tool. Equation of motion for the axial vibration of the tool has the following form:

\[
m \ddot{u} = -d \dot{u} - ku + F; \quad F = \sum_{j=1} F_j,
\]

with \( m \) - mass of tool, \( d \) and \( k \) - linear damping and stiffness coefficients of fastening of tool with spindle, \( F_j \) - axial component of the cutting force, that acts on the j-th cutting edge.

Equations (1)-(3) constitutes a complete mathematical model for dynamics of the multi-edge vibration trepanning drilling, which is a system of delay differential algebraic equations (DDAE). Solutions of these equations can be carried out by the DDE-BIFTOOL module in the MATLAB system using the \( \varepsilon \)-embedding method [4, 6] for transition from system DDAE to system of delay differential equation (DDE). Thus, equations (1) have the form:
We present equations (2) - (4) to non-dimensional form as the linear scale $X$, feed per revolution of part $h_o$, as the time scale $T_o = 2\pi \sqrt{m/k}$, $T$ - free oscillation period the tool and as the scale of cutting forces $F = K_o h_o$. 

$$\zeta = d/2\sqrt{m/k}, \kappa = K_o/k, \Pi_j = F_j/K_o h_o, \rho = T/T_o = 2\pi/\omega T_o,$$

$$\{\xi_j, \eta_j, \Lambda_j, \Delta_j, \Lambda, H_j\} = 1/h_o \{u_j, h_j, L_j, D_j, A, H_j\}.$$

Here $\rho$ - is the ratio of the natural frequency of the tool to the frequency of rotation of the part (the parameter is a dimensionless cutting speed), the parameter $\kappa$ - is the relative static stiffness of cutting. Then, equations (2) - (4) in dimensionless form take the form:

$$\Delta_j(\tau) = \tau/\rho - \zeta(\tau) - \Lambda_{j-1}(\tau - \tau_{j-1}) + \Lambda - H_j, \quad \sum_{j=1}^{n} \tau_j = \rho,$$

$$\eta_j(\tau) = \max(0, \Delta_j(\tau)), \quad \Pi_j = \eta_j(\eta_j + r\eta_j)/(\eta_j + \eta_j),$$

$$\xi'' - 4\pi \xi' - 4\pi^2 \xi + 4\pi^2 \kappa \Pi, \quad \Pi = \sum_{j=1}^{n} \Pi_j$$

3. Numerical simulation of dynamics

The numerical solution is carried out with the case of three-edge trepanning drilling with unsymmetrical angular disposition of three cutting edges ($\phi_1 = 180^\circ; \phi_2 = \phi_3 = 90^\circ$) and without axial offsets ($H_j = 0$) for machining of hole with diameter of 150 mm in the part made of titanium alloy Ti-6Al-4V with characteristic stress $\sigma_L = 1000$ MPa [15]. In this case, the trepanning drill with mass of 12.6 kg and dimensionless cutting coefficient of cutting edges $\gamma = 8.8456$ [16] is used; the chip width is set to $B = 10$ mm. Therefore, static cutting stiffness can be calculated by $K_o = \gamma \sigma_L B = 8.8456 \times 10^7$ N/m. Numerical calculations, based on the system (5) with small parameter $\varepsilon = 10^{-4}$, have enabled the computation of axial vibration of the tool $\xi$ (figure 2), thickness of chips and shape of chips (figure 3) as a function of dimensionless time $\tau/\rho$ for the following set of system properties: $\rho = 10, 4; \zeta = 0.055; \ r = 0.65; \kappa = 0.15; \eta_r = 0.15$.

In figure 2, it can be seen that with the trepanning drilling, the phenomenon of self-oscillation of the tool occurs under the interaction between the three cutting edges. At first, the amplitude of oscillation of the tool varies greatly and after some rotations of the rotation of part is established (stability).

In figure 3 shows the development process of vibrations (figure 3, a) and the occurrence of segmented chips (figure 3, b) - this is characteristic of the subcritical Poincare – Andronov – Hopf bifurcation [2, 7]. In virtue of the unsymmetrical arrangement of the cutting edges ($\phi_i$) without axial offsets ($H_j$), the shape of the chip sections and their length are different, which can be used to optimize the removal of chips from the cutting zone.
Figure 2. Axial oscillation of the tool with unsymmetrical angular disposition of three cutting edges without axial offsets

Figure 3. Thickness (a) and shape (b) of the chips removed by each cutting edge in the unsymmetrical arrangement of the three cutting edges without axial offsets

Conclusion
Maintaining of vibrations in the form of self-oscillation with three-edge trepanning drilling allows to obtain discontinuous chips for effective removal from the cutting zone.

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