Dark Matter from Unstable B-balls\textsuperscript{1}

\textbf{KARI ENQVIST}\textsuperscript{2}
Department of Physics and Helsinki Institute of Physics
P.O. Box 9, FIN-00014 University of Helsinki, Finland

March 27, 2022

Abstract

The spectrum of MSSM admits solitons carrying baryonic charge, or B-balls. In an inflationary universe they can be produced in significant numbers by a break-up of a scalar condensate along the flat directions. It is shown that if SUSY breaking is mediated to the observable sector by gravity, B-balls are unstable but decay to baryons and LSPs typically well below the electroweak phase transition. It is argued that B-balls could be the source of most baryons and cold dark matter in the universe, with their number densities related by $n_{\text{LSP}} \simeq 3 n_B$. For B-balls to survive thermalization, the reheating temperature after inflation should be less than about $10^3$ GeV.

\textsuperscript{1} Invited talk at DARK98 conference, Heidelberg, Germany, July 20-25
\textsuperscript{2}enqvist@pcu.helsinki.fi;
1 Introduction

A Q-ball is a stable, charge Q non-topological soliton in a scalar field theory with a spontaneously broken global $U(1)$ symmetry \cite{1}. The Q-ball solution arises provided the scalar potential $V(\phi)$ is such that $V(\phi)/|\phi|^2$ has a minimum at non-zero $\phi$. Although not found in the Standard Model, the spectrum of the MSSM has Q-balls which carry baryonic charge and are therefore called B-balls \cite{2,3}. In a cosmological scenario which includes inflation they can be copiously produced by the breakdown of scalar condensates along the flat directions of the MSSM \cite{4,5}. The properties of the MSSM Q-balls will depend upon the scalar potential associated with the condensate scalar, which in turn depends upon the SUSY breaking mechanism and on the order d at which the non-renormalizable terms lift the degeneracy of the potential; examples are the $H_uL$-direction with $d=4$ and $u^c d^c d^c$-direction with $d=6$ \cite{6}. If SUSY breaking occurs at low energy scales, via gauge mediated SUSY breaking \cite{7}, Q-balls will be stable \cite{4,8}. This is so because for large enough $\phi$, the scalar potential is essentially flat and the resulting Q-ball will be very tightly bound with energy that grows as $\sim Q^{1/4}$. Stable B-balls could have a wide range of astrophysical, experimental and practical implications, extensively discussed in references \cite{3,4,8-12}; for instance, stable B-balls could account for cold dark matter \cite{4}.

In the case of gravity-mediated breaking, studied in \cite{5,13}, B-balls are unstable. However, if they can survive thermalization, they are typically long-lived enough to decay much after the electroweak phase transition, leading to a variant of the Affleck-Dine (AD) mechanism \cite{14} known as B-ball Baryogenesis (BBB).

The requirement that B-balls can survive thermalization implies that B-balls in R-parity conserving models originate from a $d=6$ AD condensate and imposes an upper bound on the reheating temperature of $10^{3-5}$ GeV \cite{5,13}. Such B-balls can protect a B asymmetry originating in the AD condensate from the effects of additional B-L violating interactions, which would otherwise wash out the B asymmetry when combined with anomalous B+L violation \cite{5}. In addition, if the reheating temperature is sufficiently low so that the B-balls decay below the freeze-out temperature of the lightest SUSY particle (LSP ), then cold dark matter can mostly come from B-ball decays rather than from thermal relics. This opens up the possibility of relating the number density of dark matter particles to that of baryons,
allowing for an explanation of their observed similarity for the case of dark matter particles with weak scale masses \[13, 15\].

## 2 Unstable B balls

If SUSY breaking occurs via the supergravity hidden sector, the potential is not flat, but nevertheless radiative corrections to the \(\phi^2\)-type condensate potential allow B-balls to form \[5, 13\]. Along the \(d=6\) \(u^c u^c d^c\) flat direction it reads

\[
V_6 \simeq m_S^2 |\phi|^2 + \frac{\lambda^2 |\phi|^{10}}{M_P^2} + \left( \frac{A \lambda \phi^6}{M_P^3} + h.c. \right),
\]

where \(\lambda\) and \(A\) are coupling constants and the SUSY breaking mass \(m_S^2 \simeq m_0^2 [1 + K \log(|\phi|^2/\phi_0^2)]\), where \(\phi_0\) is the reference point and \(K\) a negative constant (and arises mainly because of gaugino loops), decreases as \(\phi\) grows, thus satisfying the requirement that \(V(\phi)/|\phi|^2\) has a minimum at non-zero \(\phi\). The potential is stabilized by the non-renormalizable term so that inside the condensate the squark field takes the value \(\langle \phi \rangle \simeq 4 \times 10^{14}\) GeV. The decreasing of the effective mass term is also responsible for the growth of any initial perturbation. In particular, there are perturbations in the condensate field inherited from the inflationary period. As was discussed in ref. 13, these will grow and become non-linear when \(H = H_i \simeq 2 |K| m_S \alpha^{-1}\), where \(\alpha \simeq -\log(\delta \phi_0(\lambda_0)/\phi_0)\) with \(\lambda_0\) the length scale of the perturbation at \(H \simeq m_S\), and \(\phi_0\) is the value of \(\phi\) when the condensate oscillations begin. The charge of the condensate lump is determined by the baryon asymmetry of the Universe at the time \(H = H_i\) and the initial size of the perturbation when it goes non-linear \[5, 13\]. The baryon asymmetry of the Universe at a given value of \(H\) during inflaton oscillation domination is given by

\[
n_B = \left( \frac{\eta_B}{2\pi} \right) \frac{H^2 M_{Pl}^2}{T_R} \simeq 1.6 \times 10^{18} H^2 \left( \frac{10^9}{T_R} \right),
\]

where we have taken the baryon to entropy ratio to be \(\eta_B \simeq 10^{-10}\). It can be shown \[13\] that the charge in the initial condensate lump is given by

\[
B = \frac{4\pi^3}{3\sqrt{2}} \frac{\eta_B |K|^{1/2} M_{Pl}^2}{m_S \alpha^2 T_R} = 2 \times 10^{15} |K|^{1/2} \left( \frac{100 \text{ GeV}}{m_S} \right) \left( \frac{10^9 \text{ GeV}}{T_R} \right) \left( \frac{40}{\alpha} \right)^2
\]
where we have used $\alpha(\lambda_0) = 40$ as a typical value.

Once the d=6 AD condensate collapses, a fraction $f_B$ of the total B asymmetry ends up in the form of B-balls. The formation of B-balls from the AD condensate can be shown to be generally effective if the charge density inside the initial lump is small enough; this can be translated to a condition on the reheating temperature which reads

$$T_R \gtrsim \frac{\eta_B m M_{Pl}^2}{8\pi \phi_0^2} = 0.23 \left( \frac{m_S}{100 \text{ GeV}} \right).$$

(4)

After the formation B-balls could be dissociated by the bombardment of thermal particles, or dissolve by charge escaping from the outer layers. Both problems can be avoided provided $T_R \lesssim 10^3 - 10^5$ GeV for $|K|$ in the range 0.01 to 0.1. It then follows that the surviving B-balls must have very large charges, $B \gtrsim 10^{14}$. The decay rate of the B-ball also depends on its charge and takes place at a temperature

$$T_d \simeq 0.01 \left( \frac{f_s}{f_B} \right)^{1/2} \left( \frac{0.01}{|K|} \right)^{3/4} \left( \frac{m}{100 \text{ GeV}} \right) \left( \frac{T_R}{1 \text{ GeV}} \right)^{1/2} \text{ GeV},$$

(5)
where $m$ is the B-ball squark mass and $f_s$ is the possible enhancement factor if the squarks can decay to a pair of scalars rather than to final states with two fermions; we have estimated $f_s \simeq 10^3$ [13]. ($f_B$ and $T_d$ are the only B-ball parameters which enter into the determination of the LSP density from B-ball decay). For example, with $T_R \simeq 1$ GeV, as suggested by the d=6 AD mechanism, and with $f_B$ in the range 0.1 to 1 (in accordance with an argument [13] that B-ball formation from an AD condensate is likely to be very efficient, although the numerical value of $f_B$ is not yet known), $T_d$ will generally be in the range 1 MeV to 1 GeV.

Assuming $f_s = 1$, the decay temperature is depicted in Fig. 1 as a function of the Q-ball charge for both thin and thick-wall Q-balls, which have different surface areas (d=6 B-ball is of the thick-wall variety [4]). As can be seen, for $B \gtrsim 10^{14}$, B-balls will indeed decay well below the electroweak phase transition temperature, providing a new source of baryon asymmetry not washed away by sphaleron interactions. The only requirement is relatively low reheating temperature after inflation, typically of the order of 1 GeV, which is in fact also implied by the observed baryon asymmetry when the CP violating phase responsible for the baryon asymmetry is of the order of 1 [13]. A low reheating temperature can be achieved in the currently popular D-term inflation models [10, 18] as a consequence of R-symmetries needed to protect the flatness of the inflaton potential.

3 Neutralinos from B balls

When the B-ball decays, for each unit of baryon number about 3 units of R-parity will also be produced (B-ball being essentially a condensate made of squarks). As discussed in the previous section, this will typically happen at or below the LSP freeze-out temperature $T_f \simeq m_{LSP}/20$ [13, 20]. The neutralino density will then consist of a possible thermal relic component, $n_{\text{relic}}(T)$, and a component from B-ball decays, $n_{BB}(T)$. The value of $n_{BB}(T)$ will depend upon whether or not the LSPs from B-ball decay can subsequently annihilate. The upper limit on $n_{LSP}(T)$ from annihilations is given by

$$n_{LSP}(T) \lesssim n_{\text{limit}}(T) \equiv \left( \frac{H}{\langle \sigma v \rangle_{\text{ann}}} \right)_T,$$ (6)
where \(< \sigma v >_{ann}\) is the thermal average of the annihilation cross-section times the relative velocity of the LSPs, which can be generally written in the form \(< \sigma v >_{ann} = a + bT/m_{LSP}\) [13]. If \(n_{LSP}(T) \lesssim n_{\text{limit}}(T)\), and if the B-ball formation efficiency \(f_B\) is not too small compared with 1, there will be a natural similarity between the number density of LSPs and that of the baryons. Otherwise the annihilation of neutralinos will suppress the number density of LSPs relative to that of the baryons, although we will still have an interesting non-thermal neutralino relic density.

If the reheating temperature is much less than \(T_f\), there will be essentially no thermal relic background of LSPs, since the additional entropy released during the inflaton matter domination period will strongly suppress the thermal relic density by a factor \((T_R/T_f)^5\). The present direct experimental bound on the LSP mass, valid for any \(\tan \beta\) (but assuming \(m_{\tilde{\nu}} \geq 200\) GeV), is \(m_{LSP} \geq 25\) GeV [21]. If one assumes the MSSM with universal soft SUSY breaking masses and unification, LEP results can be combined to yield an excluded region in the \((m_{LSP}, m_{\tilde{\nu}})\)-plane [22]. In the case of \(\tilde{\nu}_R\), which provides the most stringent bound, the excluded region is roughly parametrized by \(m_{LSP} \lesssim 0.95 m_{\tilde{\nu}_R}\) for \(45\) GeV \(\lesssim m_{\tilde{\nu}_R} \lesssim 78\) GeV (this result holds for \(\tan \beta = 2\) and \(\mu = -200\) GeV) [22]. Therefore the LSP freeze-out temperature is expected to be greater than about 1-2 GeV. Thus there are two possibilities, depending on \(T_R\) and \(T_f\): either the LSP cold dark matter density, \(\Omega_{LSP}\), will be given solely by the LSP density which originated from the B-ball decay, which we denote by \(\Omega_{BB}\), or there will also be a relic density so that \(\Omega_{LSP} = \Omega_{BB} + \Omega_{\text{relic}}\).

Assuming that \(n_{LSP}(T_d) \lesssim n_{\text{limit}}(T_d)\), the LSP density from B-ball decays will be given simply by

\[
n_{BB} = 3f_B n_B.
\]

Thus the B-ball produced LSP and baryonic densities will be related by

\[
\frac{\Omega_B}{\Omega_{BB}} = \frac{m_N}{3f_B m_{LSP}}.
\]

B generation via the AD mechanism requires inflation [14, 16], and although varieties of inflationary models exist with \(\Omega_{tot} < 1\), let us nevertheless adopt the point of view that inflation implies \(\Omega_{tot} = 1\) to a high precision. One
may then write

\[ \Omega_{\text{tot}} = \Omega_0 + \Omega_{\text{LSP}} + \Omega_B \]

\[ = \Omega_0 + \Omega_{\text{relic}} + \left( \frac{3f_B m_{\text{LSP}}}{m_N} + 1 \right) \Omega_B = 1, \quad (9) \]

where \( \Omega_0 \) includes the hot dark matter (HDM) component and/or a possible cosmological constant. Therefore \( \Omega_B \) is fixed by \( \Omega_0, f_B \) and \( m_{\text{LSP}} \) together with the MSSM parameters entering into the annihilation rate. Applying nucleosynthesis bounds \[17\] on \( \Omega_B \) then gives constraints on these parameters. Note that, so long as LSP annihilations after B-ball decay can be neglected, the resulting LSP density is independent of \( T_d \).

Let us first consider the case where the thermal relic density \( \Omega_{\text{relic}} \) is negligible. This would be true if \( T_R \) was sufficiently small compared with the
freeze-out temperature $T_f$. We then obtain the limit \[7\]

$$76.9(1 - \Omega_0)h^2 - 1 \lesssim \frac{3m_{\text{LSP}}f_B}{m_N} \lesssim 208.3(1 - \Omega_0)h^2 - 1.$$  

(10)

With $\Omega_0 = 0$ this would result in a bound on the LSP mass given by

$$3.8f_B^{-1} \text{GeV} \lesssim m_{\text{LSP}} \lesssim 29f_B^{-1} \text{GeV},$$

(11)

where we have used $0.4 \lesssim h \lesssim 0.65$. If $f_B = 1$ this would be only marginally compatible with present experimental constraints and then only if we do not consider universal soft SUSY breaking masses. Larger values of $\Omega_0$ impose even tighter bounds on $m_{\text{LSP}}$, requiring $f_B < 1$. Therefore, in the absence of annihilations after B-ball decays, LSP dark matter from B-balls is likely to be compatible with nucleosynthesis bounds only if a significant fraction of the baryon asymmetry exists outside the B-balls. Reasonable values of $f_B$ can, however, accomodate an interesting range of LSP masses; for example, values in the range 0.1 to 1 allow LSP masses as large as 290 GeV. $f_B$ can be calculated theoretically, but this requires an analysis of the non-linear evolution of the unstable AD condensate. The comparison of the theoretical value with the dark matter constraints will be an important test of this scenario.

Let us next consider the case with $T_R > T_f$. In this case there will be a significant thermal relic density and we can use nucleosynthesis bounds on $\Omega_B$ to constrain the masses of the particles responsible for the LSP annihilation cross-section. The constraints will depend on the identity of the LSP and the masses of the particles entering the LSP annihilation cross-section. In general, this would require a numerical analysis of the renormalization group equations for the SUSY particle spectrum. However, for the case of universal scalar and gaugino masses at a large scale, the LSP is likely to be mostly bino and the lightest scalars are likely to be the right-handed sleptons \[23\]. This is consistent with the requirement that the LSP does not have a large coupling to the Z boson, which would otherwise efficiently annihilate away the thermal relics. However, there will be a small, model-dependent Higgsino component which will be important for LSP masses close to the Z pole. For LSP masses away from this pole, it will be a reasonable approximation to treat the LSP as a pure bino, although the possible suppression of the thermal relic density around the Z pole due to a Higgsino component and the subsequent weakening of MSSM constraints should be kept in mind.
Figure 3: The allowed region in the $(f_B, m_{\text{LSP}})$-plane for fixed $m_{\tilde{l}_R}$, assuming that the total CDM density $\Omega = 0.9$ and the Hubble parameter $h = 0.65$.

For the case of a pure bino, the largest contribution to the annihilation cross-section comes from $t$-channel $\tilde{l}_R$ exchange in $\chi \chi \rightarrow l^+ l^- [23]$. In that case one finds [23]

$$\Omega_{\text{relic}} h^2 = \frac{\Sigma^2}{M^2 m_{\text{LSP}}^2} \left[ \left( 1 - \frac{m_{\text{LSP}}^2}{\Sigma} \right)^2 + \frac{m_{\text{LSP}}^4}{\Sigma^2} \right]^{-1}, \quad (12)$$

where $M \simeq 1$ TeV and $\Sigma = m_{\text{LSP}}^2 + m_{\tilde{l}_R}^2$. Plugging this into Eq. (11) and using the range of $\Omega_B$ allowed by nucleosynthesis [17], one may obtain [15] allowed ranges in the $(m_{\text{LSP}}, m_{\tilde{l}_R})$-plane. These are demonstrated in Fig. 2 for different values of $\Omega_0$ and $h$.

In the conventional MSSM case Eq. (12) would imply that both $m_{\text{LSP}}$ and $m_{\tilde{l}_R}$ should be less than about 200 GeV. Because of the added B-ball contribution a more stringent constraint follows in the present case. If the reheating temperature is larger than the LSP freeze-out temperature, and if we consider the range $0.1 \lesssim f_B \lesssim 1$ to be the most likely, we may conclude
that only a *very light sparticle spectrum* is consistent with $\Omega = 1$; this is so in particular if there is a cosmological constant with $\Omega_0 \simeq 0.7$, as suggested by recent supernova studies \cite{24}. In any case, it is evident that in the case $T_R > T_f$ one obtains significant constraints on the B-ball formation efficiency from MSSM constraints. This is demonstrated in Fig. 3 for the case of $\Omega_0 = 0.1$, where the allowed regions in the $(f_B, m_{\tilde{l}_R})$-plane for fixed values of $m_{\tilde{l}_R}$ are plotted. As can be seen, in the case of $T_R > T_f$ dark matter constrains $f_B$ to be less than about 0.6. If the SUGRA-based LEP limit $m_{\tilde{l}_R} \lesssim 0.95 m_{\tilde{e}_R}$ (45 GeV $\lesssim m_{\tilde{e}_R} \lesssim 78$ GeV) is implemented \cite{22}, the limit on $f_B$ would be even lower. This serves to emphasize the need for an accurate theoretical determination of $f_B$.

4 Conclusions

It is possible to reach only very broad conclusions about the sparticle spectrum at present, as the B-ball decay parameters $f_B$ and $T_d$ are unknown. However, both $f_B$ and $T_d$ are, in principle, calculable in a given model: $f_B$ by solving the non-linear scalar field equations governing the formation of B-balls from the original Affleck-Dine condensate and $T_d$ by calculating the charge and decay rate of the B-balls accurately. $T_d$, which will depend explicitly on the reheating temperature after inflation, is the more model-dependent of the two. The reheating temperature can be estimated under the assumption that the baryon asymmetry originates from an Affleck-Dine condensate with CP violating phase of the order of 1, and, indeed, can be calculated given all the details of an inflation model, but $T_R$ is likely remain an important source of theoretical uncertainty in the B-ball decay scenario. However, it is quite possible that $T_d$ and $T_R$, by being sufficiently small and large relative to $T_f$ respectively, play no direct role in determining the final LSP density.

The B-ball decay scenario for MSSM dark matter is a natural alternative to the thermal relic LSP scenario, and has the considerable advantage of being able to explain the similarity of the baryon and dark matter densities. Should future experimental constraints on the parameters of the MSSM prove to be incompatible with thermal relic dark matter but consistent with B-ball decay dark matter for some set of B-ball parameters, it would strongly support the B-ball decay scenario. In particular, should the LSP mass be determined
experimentally, the ratio of the number density baryons to dark matter would then be constrained by nucleosynthesis bounds on the baryon asymmetry. This would impose significant constraints on the reheating temperature and B-ball parameters, which, by comparing with the theoretical value of $f_B$, could even provide a “smoking gun” for the validity of the B-ball decay scenario, should annihilations happen to play no role in determining the present LSP density.

Acknowledgments

I should like to thank John McDonald for many useful discussions on Q-balls and for an enjoyable collaboration. This work has been supported by the Academy of Finland under the contract 101-35224.

References

[1] S.Coleman, Nucl. Phys. B262 (1985) 263.

[2] A.Cohen, S.Coleman, H.Georgi and A.Manohar, Nucl. Phys. B272 (1986) 301.

[3] A.Kusenko, Phys. Lett. B404 (1997) 285.

[4] A.Kusenko and M.Shaposhnikov, Phys. Lett. B418 (1998) 104.

[5] K.Enqvist and J.McDonald, Phys. Lett. B425 (1998) 309.

[6] See e.g. M. Dine, L. Randall and S. Thomas, Nucl. Phys. B458 (1996) 291.

[7] S. Dimopoulos, M. Dine, S. Raby, S. Thomas and J. D. Wells, Nucl. Phys. Proc. Suppl. A52 (1997) 38.

[8] G.Dvali, A.Kuzenko and M.Shaposhnikov, Phys. Lett. B417 (1998) 99.

[9] A.Kusenko, V.Kuzmin, M.Shaposhnikov and P.G.Tinyakov, Phys. Rev. Lett. 80 (1998) 3185.
[10] A.Kusenko, M.Shaposhnikov, P.G.Tinyakov and I.G.Tkachev, *Phys. Lett.* B423 (1998) 104.

[11] M.Laine and M.Shaposhnikov, [hep-ph/9804237](https://arxiv.org/abs/hep-ph/9804237).

[12] J. Madsen, [hep-ph/9806433](https://arxiv.org/abs/hep-ph/9806433).

[13] K.Enqvist and J.McDonald, [hep-ph/9803380](https://arxiv.org/abs/hep-ph/9803380).

[14] I.A.Affleck and M.Dine, *Nucl. Phys.* B249 (1985) 361.

[15] K. Enqvist and J. McDonald, [hep-ph/9807263](https://arxiv.org/abs/hep-ph/9807263), to appear in *Phys. Lett. B*.

[16] C. Kolda and J. March-Russell, [hep-ph/9802358](https://arxiv.org/abs/hep-ph/9802358).

[17] S.Sarkar, *Rep. Prog. Phys.* 59 (1996) 1493.

[18] K. Enqvist and J. McDonald, [hep-ph/9806213](https://arxiv.org/abs/hep-ph/9806213), to appear in *Phys. Rev. Lett.*.

[19] J.Ellis, J.S.Hagelin, D.V.Nanopoulos, K.Olive and M.Srednicki, *Nucl. Phys.* B238 (1984) 453; G.Jungman, M.Kamionkowski and K.Griest, *Phys. Rep.* 267 (1996) 195.

[20] J.McDonald, *Phys. Rev.* D43 (1991) 1063.

[21] C. Caso *et al.* (Particle Data Group), *Eur. Phys. J.* C3 (1998) 1.

[22] LEP SUSY working group (www.cern.ch/lepsusy).

[23] M. Drees and M. Nojiri, *Phys. Rev.* D47 (1993) 376; J.D.Wells, [hep-ph/9708285](https://arxiv.org/abs/hep-ph/9708285).

[24] S.Perlmutter *et al.* , [astro-ph/9712212](https://arxiv.org/abs/astro-ph/9712212).