Modelling Pentaquark and Heptaquark States

M. Núñez V., S. Lerma H. and P. O. Hess,
Instituto de Ciencias Nucleares,
Universidad Nacional Autónoma de México
Apdo. Postal 70-543, México 04510 D.F.

S. Jesgarz
Instituto de Física, Universidade de São Paulo, CP 66318, São Paulo, 05315-970, SP, Brasil

O. Civitarese and M. Reboiro
Departamento de Física, Universidad Nacional de La Plata,
c.c. 67 1900, La Plata, Argentina.

A schematic model for hadronic states, based on constituent quarks and antiquarks and gluon pairs, is discussed. The obtained hadronic spectrum leads to the identification of nucleon and Δ resonances and to pentaquark and heptaquark states. The predicted lowest pentaquark state \((J^\pi = \frac{1}{2}^+)\) lies at the energy of 1.5 GeV and it is associated to the observed \(\Theta^+(1540)\) state. For heptaquarks \((J^\pi = \frac{3}{2}^+)\) the model predicts the lowest state at 2.5 GeV.

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In a series of previous publications\textsuperscript{1,2,3} a schematic model for QCD was developed. The model was used to test the meson spectrum of QCD. In spite of its schematic nature the model seems to contain the relevant degrees of freedom, as it was shown in the comparison between calculated and experimental meson spectra. This letter is devoted to the extension of the model to accommodate baryonic features. Particularly, we shall concentrate on the appearance of exotic baryonic states, like pentaquark and heptaquark states\textsuperscript{4,5,6,7,8}.

The essentials of the model were discussed in detail in Ref.\textsuperscript{2}. It consists of two fermionic levels in the quark \((q\bar{q})\) and antiquark \((\bar{q}q)\) sector and a gluonic \((g^2)\) state containing pairs of gluons. These are the elementary degrees of freedom of the model. The interaction among these degrees of freedom is described by excitations of pairs of quarks and antiquarks mediated by the exchange of pairs of gluons. The pairs of quarks are classified in a flavor-spin coupling scheme. The pairs of gluons are kept in the angular momentum \((J)\), parity \((\pi)\) and charge conjugation \((C)\) state \(J^{\pi C} = 0^{++}\). The strength of various channels of the interaction, as well as the constituent masses, are taken from a phenomenological analysis. The model describes meson \(((q\bar{q})^n(g^2)^m)\) states and baryonic \((q^3(q\bar{q})^m(g^2)^n)\) states. Among these states we focus on \(q^3(q\bar{q})\) states (pentaquarks) and \(q^3(q\bar{q})^2\) states (heptaquarks), where the configurations indicated represent the leading terms in an expansion over many quark-antiquark and gluon states. The basis states are classified using group theoretical methods\textsuperscript{2}. The interaction of quark-antiquark pairs with gluon pairs is particle non-conserving.

The above described model belongs to a class of exactly solvable models of coupled fermion and boson systems\textsuperscript{9,10,11,12}. Alternative descriptions of pentaquark states were proposed in Ref.\textsuperscript{13}, enforcing particle number conservation.

In what follows we shall classify the basis states and solve the Hamiltonian in the framework of the boson expansion method\textsuperscript{14,15}. Finally, we shall compare the results of the calculations with recently published experimental data\textsuperscript{4,5,6,7}.

The model Hamiltonian is written:

\[
H = 2\omega_f n_f + \omega_b n_b + \sum_{\lambda S} V_{\lambda S} \left\{ \left[ (b_{\lambda S}^\dagger)^2 + 2b_{\lambda S}^\dagger b_{\lambda S} + (b_{\lambda S})^2 \right] \left( 1 - \frac{n_f}{2\Omega} \right) b + b^\dagger \left( 1 - \frac{n_f}{2\Omega} \right) \right\} + \left\{ n_{(0,1)0} \right\} \left( D_1 n_b + D_2 (b^\dagger + b) \right) + n_{(2,0)1} \left( E_1 n_b + E_2 (b^\dagger + b) \right).
\]

(1)

The distance between the fermion levels is \(2\omega_f=0.66\) GeV, \(\omega_b=1.6\) GeV is the energy of the glue ball, \(n_f\) and \(n_b\) are the number operators for fermion and gluon pairs, respectively, \(V_{\lambda S}\) is the strength of the interaction in the flavor(\(\lambda\)) and spin (\(S\)) channel. The actual values \(\lambda = 0, 1\) refer to flavor (0,0) and (1,1) configurations, while the spin channel is \(S=0\) or 1. The adopted values are: \(V_{00}=0.0337\) GeV, \(V_{01}=0.0422\) GeV, \(V_{10}=0.1573\) GeV, and \(V_{11}=0.0177\) GeV. The operators \(b_{\lambda S}^\dagger\) and \(b_{\lambda S}\) are boson images of quark-antiquark pairs. The products which appear inside brackets in \(b\) are scalar products. The factor \(1 - \frac{n_f}{2\Omega}\) results from the boson mapping\textsuperscript{2}. The mapping is exact for the channel \([\lambda, S] = [0, 0]\) and simulates the effect of the boson mapping for the other channels. The operator \(b^\dagger\) (\(b\)) is the boson image of quark pair (quark).
creates (annihilates) gluon pairs with spin-color zero, and \( n_{(\lambda_0, \mu_0)S_0} \) is the number operator of di-quarks coupled to flavor-spin \((\lambda_0, \mu_0)S_0\). The parameters \( D_1(2) \) and \( E_1(2) \) are adjusted to the nucleon and \( \Delta \) resonances. The corresponding terms describe the interaction between valence quarks and gluoballs. The Hamiltonian \( \mathbf{H} \) does not contain terms which distinguish between states with different hypercharge and isospin. It does not contain flavor mixing terms, either. Therefore, the predicted states have to be corrected in the way described in \( \mathbf{[10]} \) to allow a comparison with data. The adopted values of \( D_1(2) \) and \( E_1(2) \) are: \( D_1 = -1.442 \text{GeV}, \) \( D_2 = -0.4388 \text{GeV}, \) \( E_1 = -1.1873 \text{GeV} \) and \( E_2 = -0.3622 \text{GeV} \). The Hamiltonian contains all relevant degrees of freedom requested by QCD.

The complete classification of quark-antiquark configurations was given in Ref. \( \mathbf{[2]} \).

The unperturbed ground state is composed of 18 quarks occupying the lowest fermionic level. The baryonic states are described by three quarks in the upper fermionic level to which we add \((q\bar{q})^n\) states. The group chain which describes these states is

\[
[1^N] \quad [h] = [h_1 h_2 h_3] \quad [h^T] \\
\begin{array}{c}
U(4\Omega) \\ D \supset U(3) \\ U(12)
\end{array}
\]

where \( \Omega = 9 \) accounts for three color and three flavor degrees of freedom. The irreducible representation (irrep) of \( U(4\Omega) \) is completely antisymmetric, and \([h^T]\) is the transposed Young diagram of \([h]\). For \( N \) particles, and due to the antisymmetric irrep \([1^N]\) of \( U(4\Omega) \), the irreps of \( U(\Omega/3) \) and \( U(12) \) are complementary and the irrep of \( U(\Omega/3) \) is the color group, which is reduced to \( SU_C(3) \) with the color irrep \((\lambda_C, \mu_C)\). The \( U(12) \) group is further reduced to

\[
U(12) \supset U_f(3) \otimes U(4) \supset SU_f(3) \otimes SU(2) \\
\begin{array}{c}
[p_1 p_2 p_3 p_4] \quad (\lambda_f, \mu_f) \\ S, M
\end{array}
\]

where \((\lambda_f, \mu_f)\) is the flavor irrep and \([p_1 p_2 p_3 p_4]\) denotes the possible \( U(4) \) irreps. The group reduction is done using the methods exposed in Ref. \( \mathbf{[15]} \). The basis is spanned by the states

\[
|N, [p_1 p_2 p_3 p_4] (\lambda_C, \mu_C), \rho_f (\lambda_f, \mu_f) Y TT z, p S SM > , \quad (4)
\]

where \( N \) is the number of particles, \( Y \) is the hypercharge and \((T_T T_z)\) denotes the isospin and its third component, \( \rho_f \) and \( p_S \) are the multiplicities of the flavor and spin representations. The color labels \((\lambda_C, \mu_C)\) are related to the \( h_i \) via \( \lambda_C = h_1 - h_2 \) and \( \mu_C = h_2 - h_3 \). To obtain the values of \( h_i \) one has to consider all possible partitions of \( N = h_1 + h_2 + h_3 \) which fixes the color. For colorless states we have \( h_1 = h_2 = h_3 = h \). Each partition of \( N \) appears only once. The irrep \([hhh]\) of \( U(3) \) fixes the irrep of \( U(12) \), as indicated in \( \mathbf{[2]} \). For \( \Omega = 9 \) and color \((0,0)\) the irrep of \( U(12) \) is given by \([3^6 0^6] \) for mesons, and by \([3^7 0^7] \) for baryons. As an example, Table \( \mathbf{[16]} \) shows the relevant irreps for mesonic states. (More details are given in Ref. \( \mathbf{[16]} \).)

\[
\begin{array}{cccccccc}
\text{SU}_f(3) & U(4) & |\bar{q} q| & n_q & S_\lambda & |\bar{q} q| & n_q & S_\lambda \text{ or } S \\
\hline
(0,0), (1,1), (2,2) & 8811 & (11) & 2 & 0 & 88 & 2 & 0 & 0 \\
(1,1), (3,0), (0,3) & 9711 & (11) & 2 & 0 & 97 & 2 & 1 & 1 \\
(1,1), (3,0), (0,3) & 8820 & (20) & 2 & 1 & 88 & 2 & 0 & 1 \\
(0,0), (1,1), (2,2) & 9720 & (20) & 2 & 1 & 97 & 2 & 1 & 0, 1, 2 \\
(1,1) & 9810 & (10) & 1 & & 98 & 1 & & 0, 1 \\
(1,1) & 9810 & (11) & 2 & 0 & 97 & 2 & 1 & 1 \\
(1,1) & 9810 & (11) & 2 & 0 & 88 & 2 & 0 & 0 \\
(1,1) & 9810 & (20) & 2 & 1 & 97 & 2 & 1 & 0, 1, 2 \\
(1,1) & 9810 & (20) & 2 & 1 & 88 & 2 & 0 & 1 \\
(0,0) & 9900 & (00) & 0 & 0 & 99 & 0 & 0 & 0 \\
(0,0) & 9900 & (10) & 1 & & 98 & 1 & & 0, 1 \\
(0,0) & 9900 & (20) & 2 & 1 & 97 & 2 & 1 & 0, 1, 2 \\
\end{array}
\]

**TABLE I:** Flavor irreps coupled to the quark-antiquark content of some different \( U(4) \) irreps. Shown are the irreps which contain, at most, two quarks and two antiquarks. The number of quarks (antiquarks) in a given configuration are denoted by \( n_q \) \((n_{\bar{q}})\).

In the boson representation, the states are given by the direct product of one-, three-, eight and 24-dimensional harmonic oscillators \( \mathbf{[2]} \). For each harmonic oscillator the basis states are given by

\[
N_{\Lambda S \nu \lambda S} (\mathbf{b}_\Lambda^2) \frac{\mathbf{X}_{\Lambda S \nu \lambda S} \mathbf{S}_{\nu \lambda S}}{|\nu \lambda S \nu \lambda S > , \quad (5)
\]

where \( N_{\Lambda S} \) is the number of bosons of type \([\lambda, S]\), \( \nu \lambda S \) is the corresponding seniority and \( N_{\Lambda S \nu \lambda S} \) is a normalization constant. The seniority is defined as the number of uncoupled bosons. The quantity \( \nu \lambda S \) represents the other quantum numbers needed to specify a particular harmonic oscillator.

a) Nucleon resonances

The quality of the model predictions, concerning meson states, was discussed in Ref. \( \mathbf{[2]} \). Figure 1 shows the lowest nucleon and \( \Delta \) resonances predicted by the model. In the same Figure are shown the calculated penta- and heptaquark low lying states. For each state we indicate the spin, parity \((J^\pi)\), and the quark and gluon content \((n_q + n_{\bar{q}}, n_q)\). The quantity \( n_q + n_{\bar{q}} \) is the total number of quarks and antiquarks, which is equal to the number of valence quarks \((0 \text{ for mesons, } 3 \text{ for baryons})\) plus the number of quarks and antiquarks of the \( q\bar{q} \)-pairs, and \( n_q \) gives the number of gluons. As shown in the Figure, nucleonic states contain on the average about half an additional quark-antiquark pair (equivalent to one extra quark), and approximately 2.8 gluons. This implies a content of 59% in the quark sector and of 41% in the gluon sector. The theoretical Roper resonance (first excited nucleon resonance) lies near the experimental energy of 1.44 GeV. This is a nice feature of the model, which is shared only by a few other models.
In the present calculation the minimal representation of pentaquark-like states includes the following configurations: $(0,0)\frac{1}{2}^-, (1,1)\frac{1}{2}^-, (3,0)\frac{3}{2}^-, (0,3)\frac{3}{2}^-$ and $(2,2)\frac{1}{2}^-$. Only the $(0,3)\frac{3}{2}^-$ and $(2,2)\frac{1}{2}^-$ configurations contain hypercharge and isospin combinations which can not be obtained in a pure $q^3$ coupling scheme, like $T = 0$, $Y = 2$ in $(0,3)$ and $T = 1$, $Y = 2$ in $(2,2)$. Within the model, the lowest pentaquark state $\Theta^+(1540)$ is interpreted as a coupling of the three valence quarks in $(1,1)\frac{1}{2}^+$ with the $q\bar{q}$ background in $(1,1)0^-$ to the final irrep $(1,1)\frac{1}{2}^-$. Thus, within our model, the calculated pentaquark state at 1.51 GeV may correspond to the observed $\Theta^+(1540)$ state. Anther predicted pentaquark state is shown in Figure. 1.

Within the model, the lowest pentaquark has negative parity in accordance with QCD sum-rules and lattice gauge calculations [22, 23, 24, 25]. If the orbital spin $L$ is included, pentaquark states with positive parity may exist with $L=1$. However, these states include an orbital excitation and should appear at higher energies.

The model contains heptaquarks, characterized by two $q\bar{q}$ pairs added to the three valence quarks. The lowest state has an energy of approximately 2.5 GeV and it has a content of 3.9 $q\bar{q}$ pairs of the type $(1,1)1^+$ coupled to the three valence quarks in the configuration $(1,1)\frac{1}{2}^-$. This coupling scheme yields three exotic flavor irreps: $(3,3)1^+, 3^+$, $(4,1)1^+, 3^+$ and $(1,4)1^+, 3^+$. The lowest heptaquark state contains, basically, three ideal valence quarks, four $q\bar{q}$ pairs and 4.6 gluons. This implies a quark content of 70% and a gluon content of 30%. The model predicts other heptaquark states at higher energies, which are obtained by coupling the three ideal valence quarks with the $(3,0)1^+$ and $(0,3)1^+ q\bar{q}$ configurations. This leads to exotic flavor irreps like $(4,1)$, $(1,4)$ and $(3,3)$ with spin $\frac{3}{2}^+$ and $\frac{5}{2}^+$. The coupling of the three valence quarks with a $q\bar{q}$ irrep $(2,2)S=0, 1, 2$ leads to exotic flavor irreps of the type $(3,3)$, $(1,4)$ with spin-parity $1^+, \frac{3}{2}^+$ and $\frac{5}{2}^+$.

The simplest way to obtain a $\Delta$ resonance is to couple the three valence quarks in the $(3,0)\frac{3}{2}^+$ configuration with $q\bar{q}$ pairs in a $(0,0)J = 0$ configuration. This scheme leads to the $\Delta(1232)$ resonance. The quark-antiquark and gluon content of the calculated $\Delta(1232)$ turns out to be lower than that of the nucleon, while the structure of the state at 1.57GeV can be compared with the Roper resonance. Concerning negative parity states, Fig. 1 shows a $\frac{3}{2}^-$ state at 1.79 GeV. $\Delta$ resonances can also be obtained by coupling the three valence quarks in the $(1,1)\frac{1}{2}^+$ configuration with $(1,1)S$ $q\bar{q}$ states. The lowest state of this type is at 1.51 GeV, and it should be compared with the experimental value ($\Delta$ resonance) at 1.62 GeV [21].

\section{Pentaquarks and heptaquarks}

In this letter we do not go further into the discussion of all possible states with the structure $q^3(q\bar{q})^{2-g}g^{3}$, since the number of these configurations increases with the energy. A more complete overview of these states will be presented in Ref. [10] with its complete classification of states.

To conclude, we have applied a schematic model based on QCD degrees of freedom, to describe nucleon and $\Delta$ resonances and more exotic penta- and heptaquark states. The basis states were classified by applying group theoretical methods. The Hamiltonian, used in the calculation, is a linear combination of valence quarks and gluons, and it includes the effects of QCD degrees of freedom, to describe nucleon and $\Delta$ states.
lations, was tested to the mesonic spectrum, nucleon and Δ resonances. After fixing the parameters in this manner, we have investigated the appearance of penta- and heptaquark states. The results of the calculations show that the model predicts reasonably well the Θ⁺(1540) resonance. The lowest pentaquark state is obtained at an energy of approximately 1.5 GeV and it has $J^π=\frac{1}{2}^-$. The lowest heptaquark state has an energy of approximately 2.5 GeV and $J^π$ either $\frac{1}{2}^+$ or $\frac{3}{2}^+$. In addition, other penta- and heptaquark states are predicted to appear at higher energies.

The model allows for a complete classification of many quark-antiquark and gluon systems. As we have shown, the exotic configurations which appear in our classification scheme can not be obtained in a simple constituent quark picture. The overall agreement with the experimental data supports the claim about the suitability of the procedure.

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