A New Interaction for the sd Shell?

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Abstract. We discuss the history of the USD interaction for the sd-shell and the progress for determining a new interaction by the inclusion of a more complete set of experimental data including the new data that has accumulated over the last 20 years.

The USD hamiltonian is defined by a set of 63 numbers given for the sd-shell two-body matrix elements (TBME) and three numbers for the $0d_{5/2}$, $0d_{3/2}$ and $1s_{1/2}$ single-particle energies (SPE) given in Table 1 in the paper [1] by Hobson Wildenthal. Since 1983 the USD hamiltonian has been the standard interaction for the sd-shell and has been used in several hundred papers for the interpretation of spectroscopic properties of the nuclei from $A=18$ to $A=38$.

Figure 1. Number of data considered for each nucleus from blue (one level) to red (23 levels)

Figure 2. Rms deviation (MeV) for each nucleus with USD for the 1983 data set.
The USD TBME and SPE were obtained from a least square fit of 447 binding energy and excitation energies for sd-shell nuclei. The rms deviation for the data was about 150 keV. This fit was the culmination of about a decade of work to understand the energy levels and spectroscopic properties for these nuclei in terms of a “unified” hamiltonian to be applied to an untruncated sd-shell model space. The earliest applications of shell model configuration mixing to the sd-shell were limited by computational power and had to be applied to truncations within the sd-shell space. For example, in 1968 Arima et al [2] considered the properties of A=18-20 in the model space of \([0d_{5/2},1s_{1/2}]^n\) in a way that was similar to that applied for the USD interaction. The 447 data are much more sensitive to some linear combinations of the 66 parameters (63 TBME and 3 SPE) than others. The important linear combinations can be obtained by diagonalizing the fit matrix.

In the early 1960’s the effective interaction between nucleons in a model space and the bare nucleon-nucleon (NN) interaction was only qualitatively understood. Thus there was an obvious motivation for determining the effective interaction directly from the experimental energy data. Starting with Kuo-Brown the techniques for calculating the renormalized G matrix became more quantitative. However, questions arose concerning the convergence of the perturbation series [3], and in practice the application of the renormalized G matrix to many-particle sd-shell spectra often gave rather poor results for energy levels [4]. Thus there is still a motivation for determining effective interactions derived from energy spectra. The empirical interaction must take into account higher-order effects including those due to real three-body forces.

The immediate predecessors to USD were the Chung-Wildenthal particle (CWP) and hole (CWH) interactions that were obtained from fits to data in the lower and upper parts of the sd-shell, respectively. As the computational power advanced it became possible around the late 1970’s to consider nuclei in the middle of the sd-shell and eventually the CW interactions could be merged into the “universal” sd interaction USD.

The data that was used for the USD fit is shown in Fig. 1. Most data comes from the regions around \(^{24}\text{Mg}\) and \(^{32}\text{S}\). The data considered for the middle of the shell was limited by

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**Figure 3.** Comparison of experimental and USD theoretical levels for \(^{28}\text{Si}\).

**Figure 4.** Comparison of experimental and USD theoretical levels for \(^{32}\text{Si}\).
Figure 5. Comparison of experimental and USD theoretical levels for $^{30}$P.

Figure 6. Comparison of experimental and USD theoretical levels for $^{33}$P.

Figure 7. Comparison of calculated (USD) and experimental magnetic moments. The left-hand side is obtained with free-nucleon g-factors and the right-hand side is obtained with effective g-factors.

The computational speed. For example, in 1983 it took about a day to calculate the $0^+$ state of $^{28}$Si and the two-body transition density that is required for the fit. The rms deviations for the ground-state binding energies shown in Fig 2 is are consistent with the overall 150 keV rms except for the most neutron-rich nucleus $^{31}$Na. This deviation in the binding energy was one of several features of the properties of these neutron-rich nuclei that lead to the understanding of
nuclei around \(^{31}\text{Na}\) as being part of an “island of inversion” where the dominant structure of the ground state involves the excitation of neutrons into the pf shell \(^{5}\).

In the years immediately following the publication of the USD interaction in \(^{1}\), it was applied to the calculation of the spectra for all sd shell nuclei with the results given on the website \(^{7}\). The calculated levels are compared to all experimental positive parity states up to 10 MeV for some nuclei in the middle of the sd-shell in Figs. 3-6. One observes an overall good one-to-one match of experiment and theory up to 4–6 MeV. Above this energy the experimental level density becomes higher than theory because of “intruder states” due to the excitation of nucleons into or out of the sd-shell. At even higher energy the experimental data ends due to the experimental limitations. For a given nucleus one must understand which levels might be attributed to the sd-shell configuration, in particular for the region above 4–6 MeV where the intruder states start. This was often done on the basis of nucleon transfer experiments where the observed strength and \(\ell\) value is an indication of the configuration. Also one can use beta decay and electromagnetic decay data to determine the character of specific levels.

The wavefunctions obtained with USD were applied to many spectroscopic data; spectroscopic factors, electromagnetic transitions and electron scattering with some results summarized in \(^{4}\). As an example we show in Figs. 7–8 the magnetic and quadrupole moments obtained with free-nucleon and effective operators. The effective operators take into account the configurations outside the sd shell including the mesonic-exchange currents for M1. A global analysis of magnetic moments and Gamow-Teller beta decay allowed us to empirically deduce the importance of various sources of renormalization \(^{4}\). In particular the global quenching of isoscalar moments is attributed to higher-order configuration mixing, the global quenching of Gamow-Teller (GT) matrix elements comes from (about 2/3) higher-order configuration mixing plus about 1/3 from delta-nucleon exchange currents, and the isovector moments (the most complex operator) have the same quenching as GT but is brought back towards the free-nucleon value by the mesonic-exchange current corrections.
Figure 9. Energy levels for the $2^+$ states of the oxygen isotopes. The experimental data are shown by the crosses. The arrow for $\text{N}=16$ indicates the lower limit for energy that starts at the neutron separation energy from the data of [6]. The predictions are those for USD and the Kuo-Brown G matrix interactions.

Figure 10. Energy levels of $^{22}\text{O}$. The experimental data from [6] is compared to the USD and Bonn A G matrix predictions. The line for $S(n)$ indicates the one neutron separation energy.

Figure 11. Rms deviation (MeV) for each nucleus with USD for the 2003 data set.

Figure 12. Rms deviation (MeV) for each nucleus with USD for the 2003 data set.

Energy levels that were calculated subsequent to the USD fit were in agreement with experiment with about the same accuracy (150 keV) as those levels that were included in the
Figure 13. Summed Gamow-Teller strength for the beta decays of $^{38,37,36}$Ca. The data shown by the points is from beta decay data [8], [9] together with $^{38}$Ar(p,n) data [10]. The theoretical results are shown for the USD (solid line) and Chung-Wildenthal hole (dashed lines) interactions.

Figure 14. Comparison of the USD and G matrix two-body matrix elements.

Figure 15. Comparison of the USD and USDA (new) twp-body matrix elements.

447 data set. Hence there was little motivation to obtain a new sd-shell interaction obtained with an updated data set. However, in 1983 there was little data available for the neutron-rich sd-shell nuclei, essentially just the binding energies for the neutron-rich Na isotopes shown in
Fig. 1. In the last 20 years binding energy and energy level data have become available for many more neutron-rich nuclei than shown in Fig. 2. For some cases the predictions made with the USD interaction are in excellent agreement with experiment. For example, the energy levels of the $^3P_2$ state of the oxygen isotopes out to the drip line are shown in Fig. 9, and the energy levels of $^{22}$O are shown in Fig. 10.

The USD interaction applied to the complete set of binding energy data (including the Audi-Wapstra extrapolations) is shown in Fig. 11. One observes a blue region where the experimental energy comes above the theory for N=18-20 and Z=8-9. The region just above this where the experimental energy comes below the USD theory has an interpretation of the intruder states having a lower energy that the sd-shell states. In contrast the blue region must be related to an overbinding of the $d_{3/2}$ neutron orbital in this region. A specific defect related to this overbinding is the binding energy of $^{20}$O that is bound by 1 MeV with USD but is known to be unbound in experiment [11]. Another specific defect of the USD interaction can be observed in the Gamow-Teller strength function in the beta decay of the Ca isotopes shown in Fig. 13 with data from [8], [9], [10]. There is a systematic shift of the strength to lower-excitation energy than predicted and in fact is in better agreement with the older CWH interaction [12].

In this work we reported on the beginning of an effort to obtain a new sd shell interaction. Today the computational effort is trivial — it is possible to obtain a complete set of two-body transition densities for the entire sd-shell (about 1000 levels) in less than one day. The first progress we have made is to include the updated set of ground states binding energies plus some excited states in the neutron-rich oxygen, flourine and neon isotopes. After two iterations that include these data we obtain approximately about 170 keV rms deviation provided that we exclude the nuclei with N=19-20 and Z=10-12. If these nuclei are included in the fit the rms deviation increases significantly with particularly large deviations (up to 2.5 MeV for $^{32}$Mg) for this region of nuclei. This behavior can be used to define the boundaries for the island-of-inversion. The fit for the neutron-rich Z=8−9 nuclei is much improved and in particular all of the isotopes above $^{24}$O are unbound.

The comparison of the USD and renormalized G matrix for the original USD interaction are shown in Fig. 14. One observes differences with an rms of about 200 keV. The comparison of USD and the new interaction USDA is shown in Fig. 15. One observes that USD and USDA are very similar with an rms difference of about 20 keV. Nonetheless the USDA interaction gives a significant improvement for the neutron-rich sd-shell data with changes of up to about 1.5 MeV in the binding energies. Our task over the next year will be to fill in the complete set of sd-shell data that has become available since 1983.

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