The symmetries of the Fokker - Planck equation in three dimensions

Igor A. Tanski
tanski@protek.ru
ZAO CV Protek

ABSTRACT

We calculate all point symmetries of the Fokker - Planck equation in three-dimensional Euclidean space. General expression of symmetry group action on arbitrary solution of Fokker - Planck equation is presented.

1. The symmetries of the Fokker - Planck equation in three dimensions

The object of our considerations is a special case of Fokker - Planck equation, which describes evolution of 3D continuum of non-interacting particles imbedded in a dense medium without outer forces. The interaction between particles and medium causes combined diffusion in physical space and velocities space. The only force, which acts on particles, is damping force proportional to velocity.

The 3D variant of this equation was investigated in our work [1]. In this work fundamental solution of 3D equation was obtained by means of Fourier transform.

The 1D and 2D variants of this equation were investigated in our works [2-3]. All point symmetries of the Fokker - Planck equation in one-dimensional Euclidean space were calculated.

In present work we continue this investigation for more complex 3D equation.

The Fokker - Planck equation in three dimensions is

\[
\frac{\partial n}{\partial t} + u \frac{\partial n}{\partial x} + v \frac{\partial n}{\partial y} + w \frac{\partial n}{\partial z} - au \frac{\partial n}{\partial u} - av \frac{\partial n}{\partial v} - aw \frac{\partial n}{\partial w} - 3an - k \left( \frac{\partial^2 n}{\partial u^2} + \frac{\partial^2 n}{\partial v^2} + \frac{\partial^2 n}{\partial w^2} \right) = 0. \tag{1}
\]

where
\( n = n(t, x, y, z, u, v, w) \) - density;
\( t \) - time variable;
\( x, y, z \) - space coordinates;
\( u, v, w \) - velocity;
\( a \) - coefficient of damping;
\( k \) - coefficient of diffusion.

The list of symmetries of the Fokker - Planck equation in three dimensions follows. The calculations of symmetries are rather awkward. They are carried out to APPENDIX 1.

Instead of classic "\( \xi - \phi \)" notation we use another ("\( \delta \)" notation. This notation was presented in our work [2].

Addition of arbitrary solution

\[ v_1 = A \frac{\partial}{\partial n}; \tag{2} \]
where $A$ is arbitrary solution of the (1) equation.

Scaling of density

$$v_2 = n \frac{\partial}{\partial n}.$$  \hspace{1cm} (3)

The reason of symmetries (2-3) existence is linearity of PDE (1).

Time shift

$$v_3 = \frac{\partial}{\partial t}.$$ \hspace{1cm} (4)

Space translations

$$v_4 = \frac{\partial}{\partial x}; \hspace{1cm} v_5 = \frac{\partial}{\partial y}; \hspace{1cm} v_6 = \frac{\partial}{\partial z}. \hspace{1cm} (5)$$

Space rotations

$$v_7 = w \frac{\partial}{\partial v} - v \frac{\partial}{\partial w} + z \frac{\partial}{\partial y} - y \frac{\partial}{\partial z}; \hspace{1cm} v_8 = u \frac{\partial}{\partial w} - w \frac{\partial}{\partial u} + x \frac{\partial}{\partial z} - z \frac{\partial}{\partial x}; \hspace{1cm} v_9 = v \frac{\partial}{\partial u} - u \frac{\partial}{\partial v} + y \frac{\partial}{\partial x} - x \frac{\partial}{\partial y}.$$ \hspace{1cm} (6)

Transformations (5) and (6) build three-dimensional Euclidean movements group.

Extended Galilean transformations, which besides time and space coordinates affect the density

$$v_{10} = \frac{\partial}{\partial u} + t \frac{\partial}{\partial x} - \frac{an}{2k} (ax + u) \frac{\partial}{\partial n}; \hspace{1cm} v_{11} = \frac{\partial}{\partial v} + t \frac{\partial}{\partial y} - \frac{an}{2k} (ay + v) \frac{\partial}{\partial n};$$

$$v_{12} = \frac{\partial}{\partial w} + t \frac{\partial}{\partial z} - \frac{an}{2k} (az + w) \frac{\partial}{\partial n}. \hspace{1cm} (7)$$

Negative exponent transformations - they affect time and space coordinates, contain time-dependent common multiplier (negative exponent). They do not affect density.

$$v_{13} = e^{-at} \left(-a \frac{\partial}{\partial u} + \frac{\partial}{\partial x}\right); \hspace{1cm} v_{14} = e^{-at} \left(-a \frac{\partial}{\partial v} + \frac{\partial}{\partial y}\right); \hspace{1cm} v_{15} = e^{-at} \left(-a \frac{\partial}{\partial w} + \frac{\partial}{\partial z}\right). \hspace{1cm} (8)$$

Positive exponent transforms - they affect time, space and density, contain time-dependent common multiplier (positive exponent).

$$v_{16} = e^{at} \left(a \frac{\partial}{\partial u} + \frac{\partial}{\partial x} - \frac{a^2}{k} n v \frac{\partial}{\partial n}\right); \hspace{1cm} v_{17} = e^{at} \left(a \frac{\partial}{\partial v} + \frac{\partial}{\partial y} - \frac{a^2}{k} n v \frac{\partial}{\partial n}\right);$$

$$v_{18} = e^{at} \left(a \frac{\partial}{\partial w} + \frac{\partial}{\partial z} - \frac{a^2}{k} n v \frac{\partial}{\partial n}\right). \hspace{1cm} (9)$$

One-parameter groups, generated by vector fields $v_1 - v_{18}$, are enumerated in the following list. The list contains images of the point $(n, t, x, y, z, u, v, w)$ by transformation $\exp(\varepsilon v_i)$

$G_1$: \hspace{1cm} $(n + \varepsilon A, t, x, y, z, u, v, w);$  

$G_2$: \hspace{1cm} $(e^{\varepsilon n} t, x, y, z, u, v, w);$  

$G_3$: \hspace{1cm} $(n, t + \varepsilon, x, y, z, u, v, w);$  

$G_4$: \hspace{1cm} $(n, t, x + \varepsilon, y, z, u, v, w);$  

$G_5$: \hspace{1cm} $(n, t, x, y + \varepsilon, z, u, v, w);$
\[ G_6: \quad (n, t, x, y, z + \varepsilon, u, v, w); \]
\[ G_7: \quad (n, t, x, (\cos(\varepsilon)y + \sin(\varepsilon)z), (-\sin(\varepsilon)y + \cos(\varepsilon)z), u, (\cos(\varepsilon)v + \sin(\varepsilon)w), (-\sin(\varepsilon)v + \cos(\varepsilon)w)); \]
\[ G_8: \quad (n, t, (\cos(\varepsilon)x - \sin(\varepsilon)z), y, (\sin(\varepsilon)x + \cos(\varepsilon)z), (\cos(\varepsilon)u - \sin(\varepsilon)w), v, (\sin(\varepsilon)u + \cos(\varepsilon)v)); \]
\[ G_9: \quad (n, t, (\cos(\varepsilon)x + \sin(\varepsilon)y), (-\sin(\varepsilon)x + \cos(\varepsilon)y), z, (\cos(\varepsilon)u + \sin(\varepsilon)v), (-\sin(\varepsilon)u + \cos(\varepsilon)v), w); \]
\[ G_{10}: \quad \left( \exp \left[ -\frac{a}{2k} \left( e(ax + u) + \frac{1}{2} e^2(at + 1) \right) \right] \right) n, t, x + \varepsilon t, y, z, u + \varepsilon v, v, w; \]
\[ G_{11}: \quad \left( \exp \left[ -\frac{a}{2k} \left( e(ay + v) + \frac{1}{2} e^2(at + 1) \right) \right] \right) n, t, x, y + \varepsilon t, z, u, v + \varepsilon w, w; \]
\[ G_{12}: \quad \left( \exp \left[ -\frac{a}{2k} \left( e(az + w) + \frac{1}{2} e^2(at + 1) \right) \right] \right) n, t, x, y, z + \varepsilon t, u, v + \varepsilon w, v; \]
\[ G_{13}: \quad (n, t, x + \varepsilon e^{-at}, y, z, u - \varepsilon a e^{-at}, v, w); \]
\[ G_{14}: \quad (n, t, x, y + \varepsilon e^{-at}, z, u, v - \varepsilon a e^{-at}, w); \]
\[ G_{15}: \quad (n, t, x, y, z + \varepsilon e^{-at}, u, v, w - \varepsilon a e^{-at}); \]
\[ G_{16}: \quad (n \exp \left[ -\frac{a^2}{k} e^{-at} \left( \varepsilon u + \frac{1}{2} e^2 a e^{-at} \right) \right] \right) n, t + \varepsilon e^{-at}, y, z, u + \varepsilon a e^{-at}, v, w; \]
\[ G_{17}: \quad (n \exp \left[ -\frac{a^2}{k} e^{-at} \left( \varepsilon v + \frac{1}{2} e^2 a e^{-at} \right) \right] \right) n, t, x, y + \varepsilon a e^{-at}, z, u, v + \varepsilon a e^{-at}, w; \]
\[ G_{18}: \quad (n \exp \left[ -\frac{a^2}{k} e^{-at} \left( \varepsilon w + \frac{1}{2} e^2 a e^{-at} \right) \right] \right) n, t, x, y + \varepsilon e^{-at}, u, v, w + \varepsilon a e^{-at}). \]

For relatively nontrivial integration of \( G_{10} - G_{12}, G_{16} - G_{18} \), we refer to [2] (APPENDIX 3-4).

The fact, that \( G_i \) are symmetries of PDE (1) means, that if \( f(t, x, y, u, v) \) is arbitrary solution of (1), the functions

\[ u^{(1)}: \quad f(t, x, y, z, u, v, w) + \varepsilon A(t, x, y, z, u, v, w); \]
\[ u^{(2)}: \quad e^\varepsilon f(t, x, y, z, u, v, w); \]
\[ u^{(3)}: \quad f(t - \varepsilon, x, y, z, u, v, w); \]
\[ u^{(4)}: \quad f(t, x - \varepsilon, y, z, u, v, w); \]
\[ u^{(5)}: \quad f(t, x, y - \varepsilon, z, u, v, w); \]
\[ u^{(6)}: \quad f(t, x, y, z - \varepsilon, u, v, w); \]
\[ u^{(7)}: \quad f(t, x, (\cos(\varepsilon)y - \sin(\varepsilon)z), (\sin(\varepsilon)y + \cos(\varepsilon)z), u, (\cos(\varepsilon)v - \sin(\varepsilon)w), (\sin(\varepsilon)v + \cos(\varepsilon)w)); \]
that due to these replacements terms with $\varepsilon^2$ in $u^{(10)} - u^{(12)}$ and $u^{(16)} - u^{(18)}$ change their signs.

We systematically replace "old coordinates" by their expressions through "new coordinates". Note, that due to these replacements terms with $\varepsilon^2$ in $u^{(10)} - u^{(12)}$ and $u^{(16)} - u^{(18)}$ change their signs.

We have trivial solution $n = e^{3\alpha t}$ at our disposal. If we act on this solution by transformations (10), we obtain 6 new solutions:

$$n = \exp \left[ 3at - \frac{a}{2k} v \left( e(ax + u) - \frac{1}{2} e^2(at + 1) \right) \right];$$

$$n = \exp \left[ 3at - \frac{a}{2k} v \left( e(ax + u) - \frac{1}{2} e^2(at + 1) \right) \right];$$

$$n = \exp \left[ 3at - \frac{a}{2k} v \left( e(ax + u) - \frac{1}{2} e^2(at + 1) \right) \right];$$

$$n = \exp \left[ 3at - \frac{a}{2k} v \left( e(ax + u) - \frac{1}{2} e^2(at + 1) \right) \right];$$

$$n = \exp \left[ 3at - \frac{a}{2k} v \left( e(ax + u) - \frac{1}{2} e^2(at + 1) \right) \right];$$

where $\varepsilon$ - arbitrary real number, also are solutions of (1). Here $A$ is another arbitrary solution of (1).
\[ n = \exp\left[ 3at - \frac{a^2}{k} e^{at}\left( \epsilon w - \frac{1}{2} \epsilon^2 a e^{at} \right) \right]. \]  

(24)

General expression is

\[ U = e^{t_2} \exp\left[ -\frac{a}{2k} \left( \epsilon_{10}(a t + \tilde{u}) - \frac{1}{2} \epsilon_{10}(at + 1) \right) \right] \exp\left[ -\frac{a}{2k} \left( \epsilon_{11}(a \tilde{v} + \tilde{w}) - \frac{1}{2} \epsilon_{11}(at + 1) \right) \right] \exp\left[ -\frac{a}{2k} \left( \epsilon_{12}(a \tilde{z} + \tilde{v}) - \frac{1}{2} \epsilon_{12}(at + 1) \right) \right] \times \]

\[ \times \exp\left[ -\frac{a}{2k} \epsilon_{16}(a \tilde{r} - \epsilon_{10}) - \frac{1}{2} \epsilon_{16} e^{at} \right] \exp\left[ -\frac{a}{2k} \epsilon_{17} \left( \tilde{r} - \epsilon_{11} \right) - \frac{1}{2} \epsilon_{17} e^{at} \right] \exp\left[ -\frac{a^2}{k} \epsilon_{18}(a \tilde{r} - \epsilon_{12}) - \frac{1}{2} \epsilon_{18} e^{at} \right] \times \]

\[ \times f(t - \epsilon_3, \tilde{r} - \epsilon_4 - \epsilon_13 e^{-\tilde{a}t} - \epsilon_16 e^{at}, \tilde{r} - \epsilon_5 - \epsilon_11 t - \epsilon_14 e^{-\tilde{a}t} - \epsilon_17 e^{at}, \tilde{r} - \epsilon_6 - \epsilon_12 t - \epsilon_15 e^{-\tilde{a}t} - \epsilon_18 e^{at}, \]

\[ \tilde{r} - \epsilon_10 + \epsilon_13 e^{-\tilde{a}t} - \epsilon_16 e^{at}, \tilde{r} - \epsilon_11 + \epsilon_14 e^{-\tilde{a}t} - \epsilon_17 e^{at}, \tilde{r} - \epsilon_12 + \epsilon_15 e^{-\tilde{a}t} - \epsilon_18 e^{at} + \epsilon_1 A(t, x, y, z, u, v, w); \]

where

\[ \tilde{r} = \left( \cos(\epsilon_8) \cos(\epsilon_9) \right) x + \left( \cos(\epsilon_8) \sin(\epsilon_9) \right) y + \left( -\sin(\epsilon_8) \right) z; \]  

(26)

\[ \tilde{v} = \left( \sin(\epsilon_7) \sin(\epsilon_9) \cos(\epsilon_9) - \cos(\epsilon_7) \sin(\epsilon_9) \right) x + \left( \sin(\epsilon_7) \sin(\epsilon_8) \sin(\epsilon_9) + \cos(\epsilon_7) \cos(\epsilon_9) \right) y + \]

\[ + \left( \sin(\epsilon_7) \cos(\epsilon_9) \right) z; \]

(27)

\[ \tilde{z} = \left( \sin(\epsilon_7) \sin(\epsilon_9) + \cos(\epsilon_7) \sin(\epsilon_8) \cos(\epsilon_9) \right) x + \left( -\sin(\epsilon_7) \cos(\epsilon_9) + \cos(\epsilon_7) \sin(\epsilon_8) \sin(\epsilon_9) \right) y + \]

\[ + \left( \cos(\epsilon_7) \cos(\epsilon_9) \right) z; \]

(28)

\[ \tilde{a} = \left( \cos(\epsilon_8) \cos(\epsilon_9) \right) u + \left( \cos(\epsilon_8) \sin(\epsilon_9) \right) v + \left( -\sin(\epsilon_8) \right) w; \]  

(29)

\[ \tilde{v} = \left( \sin(\epsilon_7) \sin(\epsilon_9) \cos(\epsilon_9) - \cos(\epsilon_7) \sin(\epsilon_9) \right) u + \left( \sin(\epsilon_7) \sin(\epsilon_8) \sin(\epsilon_9) + \cos(\epsilon_7) \cos(\epsilon_9) \right) v + \]

\[ + \left( \sin(\epsilon_7) \cos(\epsilon_9) \right) w; \]

(30)

\[ \tilde{w} = \left( \sin(\epsilon_7) \sin(\epsilon_9) + \cos(\epsilon_7) \sin(\epsilon_8) \cos(\epsilon_9) \right) u + \left( -\sin(\epsilon_7) \cos(\epsilon_9) + \cos(\epsilon_7) \sin(\epsilon_8) \sin(\epsilon_9) \right) v + \]

\[ + \left( \cos(\epsilon_7) \cos(\epsilon_9) \right) w. \]  

(31)
DISCUSSION

Looking at the list of all point symmetries of the Fokker - Planck equation in two-dimensional Euclidean space, we see, that there is no simple way to get, for example, fundamental solution of PDE, using these symmetries. We have not at our disposal such an instrument, as scaling of independent variables $t, x, y, z, u, v, w$. The result (19-24) of action of symmetry group on trivial solution is not very interesting from physical point of view.

Indirect way of use of Galilean transformations (7) was demonstrated in [1]. The transformation was used for generalization of solution, which was obtained in the form of exponent of quadratic form of space coordinates and velocities with time dependent coefficients.

There is need of further investigations of Fokker - Planck equation and its set of symmetries, which may lead to another physically interesting results. We can follow the scheme of [8] : to consider invariant solutions for some one-parameter group, thus reduce the independent variables number. To find for obtained in such a way equation all point symmetries - and so long.

In the work [8] this scheme was represented for equations of elasticity and plasticity.

ACKNOWLEDGMENTS

We wish to thank Jos A. M. Vermaseren from NIKHEF (the Dutch Institute for Nuclear and High-Energy Physics), for he made his symbolic computations program FORM release 3.1 available for download for non-commercial purposes (see [9]). This wonderful program makes difficult task of symmetries search more accessible.

REFERENCES

[1] Igor A. Tanski. Fundamental solution of Fokker - Planck equation. arXiv:nlin.SI/0407007 v1 4 Jul 2004
[2] Igor A. Tanski. The symmetries of the Fokker - Planck equation in two dimensions. arXiv:nlin.CD/0412075 v1 31 Dec 2004
[3] Igor A. Tanski. The symmetries of the Fokker - Planck equation in one dimension. arXiv:nlin.PS/0501008 v1 3 Jan 2005
[4] L. P. Eisenhart, Continuous Groups of Transformations, Princeton University Press, Princeton, 1933.
[5] L. V. Ovsyannikov, Group analysis of differential equations, Moscow, Nauka, 1978.
[6] Peter J. Olver, Applications of Lie groups to differential equations. Springer-Verlag, New York, 1986.
[7] N. H. Ibragimov, CRC Handbook of Lie Group Analysis of Differential Equations, V. 3, CRC Press, New York, 1996.
[8] B. D. Annin, V. O. Bytev, S. I. Senashov, Group properties of equations of elasticity and plasticity (Gruppovye svojstva uravnenij uprugosti i plastichnosti), Nauka, Novosibirsk, 1985.
[9] Michael M. Tung. FORM Matters: Fast Symbolic Computation under UNIX. arXiv:cs.SC/0409048 v1 27 Sep 2004
APPENDIX 1

The infinitesimal invariance criteria for PDE (1) is

\[ \delta \frac{\partial}{\partial t} n + \delta u \frac{\partial}{\partial x} n + u \delta \frac{\partial}{\partial x} n + \delta v \frac{\partial}{\partial y} n + v \delta \frac{\partial}{\partial y} n + \delta w \frac{\partial}{\partial z} n + w \delta \frac{\partial}{\partial z} n - (A1-1) \]

\[ -a \delta u \frac{\partial}{\partial u} n - au \delta \frac{\partial}{\partial u} n - a \delta v \frac{\partial}{\partial v} n - av \delta \frac{\partial}{\partial v} n - a \delta w \frac{\partial}{\partial w} n - aw \delta \frac{\partial}{\partial w} n - \]

\[ -3a \delta n - k \left( \frac{\partial^2}{\partial u^2} (\delta t) + \frac{\partial^2}{\partial v^2} (\delta x) + \frac{\partial^2}{\partial w^2} (\delta x) \right) = 0; \]

Expressions for variations of derivatives are presented in APPENDIX 2.

We eliminate \( \frac{\partial}{\partial t} n \) using original equation

\[ \frac{\partial}{\partial t} n = -\left( u \frac{\partial}{\partial x} n + v \frac{\partial}{\partial y} n + w \frac{\partial}{\partial z} n - au \frac{\partial}{\partial u} n - av \frac{\partial}{\partial v} n - aw \frac{\partial}{\partial w} n - 3an - k \left( \frac{\partial^2}{\partial u^2} (\delta t) + \frac{\partial^2}{\partial v^2} (\delta x) + \frac{\partial^2}{\partial w^2} (\delta x) \right) \right); \] (A1-2)

Collecting similar terms, we obtain following equations:

\[ \frac{\partial}{\partial x} \frac{\partial}{\partial u} n \]

\[ -2ku \frac{\partial^2}{\partial \delta n_\delta n_u} (\delta t) + 2k \frac{\partial^2}{\partial \delta n_\delta n_v} (\delta x) = 0; \] (A1-3)

\[ \frac{\partial}{\partial x} \frac{\partial}{\partial u^2} n \]

\[ -ku \frac{\partial^2}{\partial \delta n_\delta n_u} (\delta t) + k \frac{\partial^2}{\partial \delta n^2_\delta n_v} (\delta x) = 0; \] (A1-4)

\[ \frac{\partial}{\partial x} \frac{\partial}{\partial v} n \]

\[ -2ku \frac{\partial^2}{\partial \delta n_\delta n_v} (\delta t) + 2k \frac{\partial^2}{\partial \delta n_\delta n_w} (\delta x) = 0; \] (A1-5)

\[ \frac{\partial}{\partial x} \frac{\partial}{\partial v^2} n \]

\[ -ku \frac{\partial^2}{\partial \delta n_\delta n_v} (\delta t) + k \frac{\partial^2}{\partial \delta n^2_\delta n_w} (\delta x) = 0; \] (A1-6)

\[ \frac{\partial}{\partial x} \frac{\partial}{\delta w} n \]

\[ -2ku \frac{\partial^2}{\partial \delta n_\delta n_w} (\delta t) + 2k \frac{\partial^2}{\partial \delta n_\delta n_u} (\delta x) = 0; \] (A1-7)

\[ \frac{\partial}{\partial x} \frac{\partial}{\delta w^2} n \]

\[ -ku \frac{\partial^2}{\partial \delta n_\delta n_w} (\delta t) + k \frac{\partial^2}{\partial \delta n^2_\delta n_u} (\delta x) = 0; \] (A1-8)

\[ \frac{\partial}{\partial x} \]

\[ -auv \frac{\partial}{\partial v} (\delta t) - auw \frac{\partial}{\partial w} (\delta t) + 3au \frac{\partial}{\partial \delta n} (\delta t) + au \frac{\partial}{\partial \delta n_u} (\delta x) - \] (A1-9)
\[-au^2 \frac{\partial}{\partial u} (\delta t) + av \frac{\partial}{\partial v} (\delta x) + aw \frac{\partial}{\partial w} (\delta x) - 3an \frac{\partial}{\partial n} (\delta x) -
\]
\[-ku^2 \frac{\partial^2}{\partial u^2} (\delta t) - ku \frac{\partial^2}{\partial v^2} (\delta t) - ku \frac{\partial^2}{\partial w^2} (\delta t) + k \frac{\partial^2}{\partial u^2} (\delta x) + 
\]
\[+ k \frac{\partial^2}{\partial v^2} (\delta x) + k \frac{\partial^2}{\partial w^2} (\delta x) + uv \frac{\partial}{\partial y} (\delta t) + uv \frac{\partial}{\partial y} (\delta t) -
\]
\[-u \frac{\partial}{\partial x} (\delta x) + u \frac{\partial}{\partial t} (\delta t) + u^2 \frac{\partial}{\partial x} (\delta t) -
\]
\[-v \frac{\partial}{\partial y} (\delta x) - w \frac{\partial}{\partial z} (\delta x) + \delta u - \frac{\partial}{\partial t} (\delta x) = 0;
\]
\[\frac{\partial n}{\partial y}; \frac{\partial n}{\partial u} \]
\[-2kv \frac{\partial^2}{\partial n \partial u} (\delta t) + 2k \frac{\partial^2}{\partial n \partial u} (\delta y) = 0; \quad (A1-10)
\]
\[\frac{\partial n}{\partial y}; \frac{\partial n}{\partial u^2} \]
\[-kv \frac{\partial^2}{\partial n^2} (\delta t) + k \frac{\partial^2}{\partial n^2} (\delta y) = 0; \quad (A1-11)
\]
\[\frac{\partial n}{\partial y}; \frac{\partial n}{\partial v} \]
\[-2kv \frac{\partial^2}{\partial n \partial v} (\delta t) + 2k \frac{\partial^2}{\partial n \partial v} (\delta y) = 0; \quad (A1-12)
\]
\[\frac{\partial n}{\partial y}; \frac{\partial n}{\partial v^2} \]
\[-kv \frac{\partial^2}{\partial n^2} (\delta t) + k \frac{\partial^2}{\partial n^2} (\delta y) = 0; \quad (A1-13)
\]
\[\frac{\partial n}{\partial y}; \frac{\partial n}{\partial w} \]
\[-2kv \frac{\partial^2}{\partial n \partial w} (\delta t) + 2k \frac{\partial^2}{\partial n \partial w} (\delta y) = 0; \quad (A1-14)
\]
\[\frac{\partial n}{\partial y}; \frac{\partial n}{\partial w^2} \]
\[-kv \frac{\partial^2}{\partial n^2} (\delta t) + k \frac{\partial^2}{\partial n^2} (\delta y) = 0; \quad (A1-15)
\]
\[\frac{\partial n}{\partial y} \]
\[-auv \frac{\partial}{\partial u} (\delta t) + au \frac{\partial}{\partial u} (\delta y) - avw \frac{\partial}{\partial w} (\delta t) + 3avn \frac{\partial}{\partial n} (\delta t) +
\]
\[+ av \frac{\partial}{\partial v} (\delta y) - av \frac{\partial}{\partial v} (\delta t) + aw \frac{\partial}{\partial w} (\delta y) - 3an \frac{\partial}{\partial n} (\delta y) -
\]
\(-kv \frac{\partial^2}{\partial u^2}(\delta t) - kv \frac{\partial^2}{\partial v^2}(\delta t) - kv \frac{\partial^2}{\partial w^2}(\delta t) + k \frac{\partial^2}{\partial u^2}(\delta y) + k \frac{\partial^2}{\partial v^2}(\delta y) + k \frac{\partial^2}{\partial w^2}(\delta y) + uv \frac{\partial}{\partial x}(\delta t) - u \frac{\partial}{\partial x}(\delta y) + \) \\
\(+v \frac{\partial}{\partial y}(\delta t) - w \frac{\partial}{\partial z}(\delta y) + \delta - \frac{\partial}{\partial t}(\delta y) = 0;
\)

\(\frac{\partial n \frac{\partial}{\partial u}}{\partial z \partial u} - 2kw \frac{\partial^2}{\partial n \partial u}(\delta t) + 2k \frac{\partial^2}{\partial n \partial u}(\delta z) = 0;\)  
(A1-17)

\(\frac{\partial n \frac{\partial}{\partial u^2}}{\partial z \partial u^2} - kw \frac{\partial^2}{\partial n^2}(\delta t) + k \frac{\partial^2}{\partial n^2}(\delta z) = 0;\)  
(A1-18)

\(\frac{\partial n \frac{\partial}{\partial v}}{\partial z \partial v} - 2kw \frac{\partial^2}{\partial n \partial v}(\delta t) + 2k \frac{\partial^2}{\partial n \partial v}(\delta z) = 0;\)  
(A1-19)

\(\frac{\partial n \frac{\partial}{\partial v^2}}{\partial z \partial v^2} - kw \frac{\partial^2}{\partial n^2}(\delta t) + k \frac{\partial^2}{\partial n^2}(\delta z) = 0;\)  
(A1-20)

\(\frac{\partial n \frac{\partial}{\partial w}}{\partial z \partial w} - 2kw \frac{\partial^2}{\partial n \partial w}(\delta t) + 2k \frac{\partial^2}{\partial n \partial w}(\delta z) = 0;\)  
(A1-21)

\(\frac{\partial n \frac{\partial}{\partial w^2}}{\partial z \partial w^2} - kw \frac{\partial^2}{\partial n^2}(\delta t) + k \frac{\partial^2}{\partial n^2}(\delta z) = 0;\)  
(A1-22)

\(\frac{\partial n}{\partial z} - aw \frac{\partial}{\partial u}(\delta t) + au \frac{\partial}{\partial u}(\delta z) - aw \frac{\partial}{\partial v}(\delta t) + av \frac{\partial}{\partial v}(\delta z) -\)

\(+3awn \frac{\partial}{\partial n}(\delta t) + aw \frac{\partial}{\partial w}(\delta z) - aw^2 \frac{\partial}{\partial w}(\delta t) - 3an \frac{\partial}{\partial n}(\delta z) -\)

\(-kw \frac{\partial^2}{\partial v^2}(\delta t) - kw \frac{\partial^2}{\partial w^2}(\delta t) + k \frac{\partial^2}{\partial u^2}(\delta z) + k \frac{\partial^2}{\partial v^2}(\delta z) + \)
\[ + k \frac{\partial^2}{\partial w^2} (\delta z) + u w \frac{\partial}{\partial x} (\delta t) - u \frac{\partial}{\partial x} (\delta z) + v w \frac{\partial}{\partial y} (\delta t) - v \frac{\partial}{\partial y} (\delta z) - \]

\[ - w \frac{\partial}{\partial z} (\delta z) + w \frac{\partial}{\partial t} (\delta t) + w^2 \frac{\partial}{\partial z} (\delta t) + \delta w - \frac{\partial}{\partial t} (\delta z) = 0; \]

\[ \frac{\partial n}{\partial u} \frac{\partial n}{\partial v} \]

\[ 2aku \frac{\partial^2}{\partial n^2} (\delta t) + 2akv \frac{\partial^2}{\partial n^2} (\delta t) + 2k \frac{\partial^2}{\partial n^2} (\delta v) + 2k \frac{\partial^2}{\partial n^2} (\delta u) = 0; \quad \text{(A1-24)} \]

\[ \frac{\partial n}{\partial u} \frac{\partial n}{\partial v^2} \]

\[ aku \frac{\partial^2}{\partial n^2} (\delta t) + k \frac{\partial^2}{\partial n^2} (\delta u) = 0; \quad \text{(A1-25)} \]

\[ \frac{\partial n}{\partial u} \frac{\partial n}{\partial w} \]

\[ 2aku \frac{\partial^2}{\partial n^2} (\delta t) + 2akw \frac{\partial^2}{\partial n^2} (\delta t) + 2k \frac{\partial^2}{\partial n^2} (\delta w) + 2k \frac{\partial^2}{\partial n^2} (\delta u) = 0; \quad \text{(A1-26)} \]

\[ \frac{\partial n}{\partial u} \frac{\partial n}{\partial w^2} \]

\[ aku \frac{\partial^2}{\partial n^2} (\delta t) + k \frac{\partial^2}{\partial n^2} (\delta u) = 0; \quad \text{(A1-27)} \]

\[ \frac{\partial n}{\partial u} \frac{\partial^2 n}{\partial u^2} \]

\[ 2k \frac{\partial}{\partial n} (\delta u) + 2k^2 \frac{\partial^2}{\partial n^2} (\delta t) = 0; \quad \text{(A1-28)} \]

\[ \frac{\partial n}{\partial u} \frac{\partial^2 n}{\partial v^2} \]

\[ 2k^2 \frac{\partial^2}{\partial n^2} (\delta t) = 0; \quad \text{(A1-29)} \]

\[ \frac{\partial n}{\partial u} \frac{\partial^2 n}{\partial w^2} \]

\[ 2k^2 \frac{\partial^2}{\partial n^2} (\delta t) = 0; \quad \text{(A1-30)} \]

\[ \frac{\partial n}{\partial u} \frac{\partial^2 n}{\partial u \partial v} \]

\[ 2k \frac{\partial}{\partial n} (\delta v) = 0; \quad \text{(A1-31)} \]

\[ \frac{\partial n}{\partial u} \frac{\partial^2 n}{\partial u \partial w} \]

\[ 2k \frac{\partial}{\partial n} (\delta w) = 0; \quad \text{(A1-32)} \]

\[ \frac{\partial n}{\partial u} \frac{\partial^2 n}{\partial u \partial x} \]
\[
2k \frac{\partial}{\partial n} (\delta x) = 0; \quad (A1-33)
\]
\[
\frac{\partial n}{\partial u} \frac{\partial^2 n}{\partial u \partial \delta y} \quad 2k \frac{\partial}{\partial n} (\delta y) = 0; \quad (A1-34)
\]
\[
\frac{\partial n}{\partial u} \frac{\partial^2 n}{\partial u \partial \delta z} \quad k \frac{\partial}{\partial n} (\delta z) = 0; \quad (A1-35)
\]
\[
\frac{\partial n}{\partial u} \frac{\partial^2 n}{\partial u \partial \delta u} \quad 2k \frac{\partial}{\partial n} (\delta t) = 0; \quad (A1-36)
\]
\[
\begin{align*}
aku \frac{\partial^2}{\partial u^2} (\delta t) + aku \frac{\partial^2}{\partial v^2} (\delta t) + aku \frac{\partial^2}{\partial w^2} (\delta t) + 6akn \frac{\partial^2}{\partial n \partial \delta u} (\delta t) - \\
-aauv \frac{\partial}{\partial y} (\delta t) - aauw \frac{\partial}{\partial z} (\delta t) + au \frac{\partial}{\partial u} (\delta u) - au \frac{\partial}{\partial t} (\delta t) - \\
-aau^2 \frac{\partial}{\partial x} (\delta t) + av \frac{\partial}{\partial v} (\delta u) + aw \frac{\partial}{\partial w} (\delta u) - 3auu \frac{\partial}{\partial n} (\delta u) - \\
-a\delta u + a^2 u \frac{\partial}{\partial v} (\delta t) + a^2 uw \frac{\partial}{\partial w} (\delta t) - 3a^2 u \frac{\partial}{\partial n} (\delta t) + a^2 u \frac{\partial}{\partial u} (\delta t) + \\
+k \frac{\partial^2}{\partial u \partial \delta z} \frac{\partial}{\partial n} (\delta z) - 2k \frac{\partial^2}{\partial n \partial \delta u} (\delta n) + \\
+k \frac{\partial^2}{\partial u^2} (\delta u) + k \frac{\partial^2}{\partial v^2} (\delta u) + k \frac{\partial^2}{\partial w^2} (\delta u) - \\
-u \frac{\partial}{\partial x} (\delta u) - v \frac{\partial}{\partial y} (\delta u) - w \frac{\partial}{\partial z} (\delta u) - \frac{\partial}{\partial t} (\delta u) = 0;
\end{align*}
\]
\[
\begin{align*}
\frac{\partial n}{\partial u^2} \frac{\partial^2 n}{\partial v^2} &= 0; \\
\frac{\partial n}{\partial u^3} \frac{\partial^2 n}{\partial w^2} &= 0; \\
\frac{\partial n}{\partial u^2} &= 0; \\
2aku \frac{\partial^2}{\partial n \partial u} (\delta t) + 3akan \frac{\partial^2}{\partial n^2} (\delta t) - k \frac{\partial^2}{\partial n^2} (\delta n) + 2k \frac{\partial^2}{\partial n \partial u} (\delta u) = 0; \\
\frac{\partial n}{\partial u^3} &= 0; \\
2akv \frac{\partial^2}{\partial n \partial w} (\delta t) + 2akw \frac{\partial^2}{\partial n \partial v} (\delta t) + 2k \frac{\partial^2}{\partial n \partial v} (\delta w) + 2k \frac{\partial^2}{\partial n \partial w} (\delta v) = 0; \\
\frac{\partial n}{\partial v^2} \frac{\partial^2 n}{\partial w^2} &= 0; \\
\frac{\partial n}{\partial v} \frac{\partial^2 n}{\partial u^2} &= 0; \\
2k \frac{\partial^2}{\partial n \partial v} (\delta t) = 0; \\
\frac{\partial n}{\partial v} \frac{\partial^2 n}{\partial v^2} &= 0; \\
2k \frac{\partial}{\partial n} (\delta v) + 2k \frac{\partial^2}{\partial n \partial v} (\delta t) = 0; \\
\frac{\partial n}{\partial v} \frac{\partial^2 n}{\partial w^2} &= 0; \\
2k \frac{\partial^2}{\partial n \partial v} (\delta t) = 0; \\
\frac{\partial n}{\partial v} \frac{\partial^2 n}{\partial u \partial v} &= 0; \\
2k \frac{\partial}{\partial n} (\delta u) = 0; \\
\frac{\partial n}{\partial v} \frac{\partial^2 n}{\partial v \partial w} &= 0;
\end{align*}
\]
\( \frac{\partial n}{\partial v} \frac{\partial^2 n}{\partial v \partial x} = 0; \quad (A1-52) \)

\( \frac{\partial n}{\partial v} \frac{\partial^2 n}{\partial v \partial y} = 0; \quad (A1-53) \)

\( \frac{\partial n}{\partial v} \frac{\partial^2 n}{\partial v \partial z} = 0; \quad (A1-54) \)

\( \frac{\partial n}{\partial v} = 0; \quad (A1-55) \)

\( akv \frac{\partial^2}{\partial u^2} (\delta t) + akv \frac{\partial^2}{\partial v^2} (\delta t) + akv \frac{\partial^2}{\partial w^2} (\delta t) + \)

\( +6kn \frac{\partial^2}{\partial n \partial v} (\delta t) - auv \frac{\partial}{\partial x} (\delta t) + au \frac{\partial}{\partial u} (\delta v) - avw \frac{\partial}{\partial z} (\delta t) + \)

\( +av \frac{\partial}{\partial v} (\delta v) - av \frac{\partial}{\partial t} (\delta t) - av^2 \frac{\partial}{\partial y} (\delta t) + av \frac{\partial}{\partial w} (\delta v) - \)

\( -3an \frac{\partial}{\partial n} (\delta v) - a\delta v + a^2 uv \frac{\partial}{\partial u} (\delta t) + a^2 vw \frac{\partial}{\partial v} (\delta t) - 3a^2 vn \frac{\partial}{\partial n} (\delta t) + \)

\( +a^2 v^2 \frac{\partial}{\partial v} (\delta t) + k \frac{\partial^2}{\partial n \partial z} (\delta t) + k \frac{\partial^2}{\partial u \partial z} (\delta v) - 2k \frac{\partial^2}{\partial n \partial v} (\delta n) + \)

\( +k \frac{\partial^2}{\partial v^2} (\delta v) + k \frac{\partial^2}{\partial w^2} (\delta v) - u \frac{\partial}{\partial x} (\delta v) - v \frac{\partial}{\partial y} (\delta v) - w \frac{\partial}{\partial z} (\delta v) - \frac{\partial}{\partial t} (\delta v) = 0; \quad (A1-56) \)

\( \frac{\partial n}{\partial v^2} = 0; \quad (A1-57) \)

\( k^2 \frac{\partial^2}{\partial n^2} (\delta t) = 0; \quad (A1-58) \)
\[ k^2 \frac{\partial^2}{\partial n^2} (\delta t) = 0; \quad (A1-59) \]
\[ \frac{\partial n}{\partial v^2} \frac{\partial^2 n}{\partial w^2} \]
\[ k^2 \frac{\partial^2}{\partial n^2} (\delta t) = 0; \quad (A1-60) \]
\[ \frac{\partial n}{\partial v^2} \]
\[ 2akv \frac{\partial^2}{\partial n \partial v} (\delta t) + 3akn \frac{\partial^2}{\partial n^2} (\delta t) - k \frac{\partial^2}{\partial n^2} (\delta n) + 2k \frac{\partial^2}{\partial n \partial v} (\delta v) = 0; \quad (A1-61) \]
\[ \frac{\partial n}{\partial v^3} \]
\[ akv \frac{\partial^2}{\partial n^2} (\delta t) + k \frac{\partial^2}{\partial n^2} (\delta v) = 0; \quad (A1-62) \]
\[ \frac{\partial n}{\partial w^2} \frac{\partial^2 n}{\partial u^2} \]
\[ 2k^2 \frac{\partial^2}{\partial n \partial u} (\delta t) = 0; \quad (A1-63) \]
\[ \frac{\partial n}{\partial w^2} \frac{\partial^2 n}{\partial v^2} \]
\[ 2k^2 \frac{\partial^2}{\partial n \partial v} (\delta t) = 0; \quad (A1-64) \]
\[ \frac{\partial n}{\partial w^2} \frac{\partial^2 n}{\partial w^2} \]
\[ 2k \frac{\partial}{\partial n} (\delta w) + 2k^2 \frac{\partial^2}{\partial n \partial w} (\delta t) = 0; \quad (A1-65) \]
\[ \frac{\partial n}{\partial w^2} \frac{\partial^2 n}{\partial u \partial w} \]
\[ 2k \frac{\partial}{\partial n} (\delta u) = 0; \quad (A1-66) \]
\[ \frac{\partial n}{\partial w^2} \frac{\partial^2 n}{\partial v \partial w} \]
\[ 2k \frac{\partial}{\partial n} (\delta v) = 0; \quad (A1-67) \]
\[ \frac{\partial n}{\partial w^2} \frac{\partial^2 n}{\partial w \partial x} \]
\[ 2k \frac{\partial}{\partial n} (\delta x) = 0; \quad (A1-68) \]
\[ \frac{\partial n}{\partial w^2} \frac{\partial^2 n}{\partial w \partial y} \]
\[ 2k \frac{\partial}{\partial n} (\delta y) = 0; \quad (A1-69) \]
\[ \frac{\partial n}{\partial w} \frac{\partial^2 n}{\partial w \partial \delta z} \]

\[ k \frac{\partial}{\partial n} (\delta z) = 0; \quad (A1-70) \]

\[ \frac{\partial n}{\partial w} \frac{\partial^2 n}{\partial \delta w \partial \delta w} \]

\[ 2k \frac{\partial}{\partial n} (\delta t) = 0; \quad (A1-71) \]

\[ \frac{\partial n}{\partial w} \]

\[ akw \frac{\partial^2}{\partial t^2} (\delta t) + akw \frac{\partial^2}{\partial v^2} (\delta t) + akw \frac{\partial^2}{\partial w^2} (\delta t) + 6akn \frac{\partial^2}{\partial n \partial \delta w} (\delta t) - \]

\[ -auw \frac{\partial}{\partial x} (\delta t) + au \frac{\partial}{\partial u} (\delta w) - avw \frac{\partial}{\partial y} (\delta t) + av \frac{\partial}{\partial v} (\delta w) + aw \frac{\partial}{\partial w} (\delta w) - aw \frac{\partial}{\partial t} (\delta t) - \]

\[ -aw^2 \frac{\partial}{\partial z} (\delta t) - 3an \frac{\partial}{\partial n} (\delta w) - a \delta w + a^2 uw \frac{\partial}{\partial u} (\delta t) + a^2 \nu w \frac{\partial}{\partial \nu} (\delta t) - \]

\[ -3a^2 \nu n \frac{\partial}{\partial \nu} (\delta n) + a^2 \nu w \frac{\partial}{\partial \nu} (\delta w) + k \frac{\partial^2}{\partial w \partial \delta z} (\delta n) + k \frac{\partial^2}{\partial w \partial \delta w} (\delta n) + k \frac{\partial^2}{\partial \nu^2} (\delta w) - \]

\[ -2k \frac{\partial^2}{\partial n \partial \delta w} (\delta n) + k \frac{\partial^2}{\partial \nu^2} (\delta w) - n \frac{\partial}{\partial x} (\delta w) - \nu \frac{\partial}{\partial y} (\delta w) - w \frac{\partial}{\partial z} (\delta w) - \frac{\partial}{\partial t} (\delta w) = 0; \quad (A1-72) \]

\[ \frac{\partial n}{\partial w^2} \frac{\partial^2 n}{\partial u^2} \]

\[ k^2 \frac{\partial^2}{\partial t^2} (\delta t) = 0; \quad (A1-73) \]

\[ \frac{\partial n}{\partial w^2} \frac{\partial^2 n}{\partial v^2} \]

\[ k^2 \frac{\partial^2}{\partial t^2} (\delta t) = 0; \quad (A1-74) \]

\[ \frac{\partial n}{\partial w^2} \frac{\partial^2 n}{\partial w^2} \]

\[ k^2 \frac{\partial^2}{\partial t^2} (\delta t) = 0; \quad (A1-75) \]

\[ \frac{\partial n}{\partial w^3} \]

\[ 2akw \frac{\partial^2}{\partial \delta w} (\delta t) + 3akw \frac{\partial^2}{\partial n^2} (\delta t) - k \frac{\partial^2}{\partial n^2} (\delta n) + 2k \frac{\partial^2}{\partial n \partial \delta w} (\delta w) = 0; \quad (A1-76) \]

\[ \frac{\partial n}{\partial w^3} \]

\[ akw \frac{\partial^2}{\partial n^2} (\delta t) + k \frac{\partial^2}{\partial n^2} (\delta w) = 0; \quad (A1-77) \]

\[ \frac{\partial^2 n}{\partial u^2} \]
aku \frac{\partial}{\partial u} (\delta t) + akv \frac{\partial}{\partial v} (\delta t) + akw \frac{\partial}{\partial w} (\delta t) - 3akn \frac{\partial}{\partial n} (\delta t) = (A1-78)

-ku \frac{\partial}{\partial x} (\delta t) - kv \frac{\partial}{\partial y} (\delta t) - kw \frac{\partial}{\partial z} (\delta t) + 2k \frac{\partial}{\partial u} (\delta u) - k \frac{\partial}{\partial t} (\delta t) +

+k^2 \frac{\partial^2}{\partial u^2} (\delta t) + k^2 \frac{\partial^2}{\partial v^2} (\delta t) + k^2 \frac{\partial^2}{\partial w^2} (\delta t) = 0;

\frac{\partial^2 n}{\partial v^2}

aku \frac{\partial}{\partial u} (\delta t) + akv \frac{\partial}{\partial v} (\delta t) + akw \frac{\partial}{\partial w} (\delta t) - 3akn \frac{\partial}{\partial n} (\delta t) = (A1-79)

-ku \frac{\partial}{\partial x} (\delta t) - kv \frac{\partial}{\partial y} (\delta t) - kw \frac{\partial}{\partial z} (\delta t) + 2k \frac{\partial}{\partial v} (\delta v) - k \frac{\partial}{\partial t} (\delta t) +

+k^2 \frac{\partial^2}{\partial u^2} (\delta t) + k^2 \frac{\partial^2}{\partial v^2} (\delta t) + k^2 \frac{\partial^2}{\partial w^2} (\delta t) = 0;

\frac{\partial^2 n}{\partial w^2}

aku \frac{\partial}{\partial u} (\delta t) + akv \frac{\partial}{\partial v} (\delta t) + akw \frac{\partial}{\partial w} (\delta t) - 3akn \frac{\partial}{\partial n} (\delta t) = (A1-80)

-ku \frac{\partial}{\partial x} (\delta t) - kv \frac{\partial}{\partial y} (\delta t) - kw \frac{\partial}{\partial z} (\delta t) + 2k \frac{\partial}{\partial w} (\delta w) - k \frac{\partial}{\partial t} (\delta t) +

+k^2 \frac{\partial^2}{\partial u^2} (\delta t) + k^2 \frac{\partial^2}{\partial v^2} (\delta t) + k^2 \frac{\partial^2}{\partial w^2} (\delta t) = 0;

\frac{\partial^2 n}{\partial u \partial v}

2k \frac{\partial}{\partial v} (\delta u) + 2k \frac{\partial}{\partial u} (\delta v) = 0; (A1-81)

\frac{\partial^2 n}{\partial u \partial w}

2k \frac{\partial}{\partial w} (\delta u) + 2k \frac{\partial}{\partial u} (\delta w) = 0; (A1-82)

\frac{\partial^2 n}{\partial v \partial w}

2k \frac{\partial}{\partial w} (\delta v) + 2k \frac{\partial}{\partial v} (\delta w) = 0; (A1-83)

\frac{\partial^2 n}{\partial u \partial x}

2k \frac{\partial}{\partial u} (\delta x) = 0; (A1-84)

\frac{\partial^2 n}{\partial u \partial y}

2k \frac{\partial}{\partial u} (\delta y) = 0; (A1-85)
\( \frac{\partial^2 n}{\partial u \partial z} \)

\[ k \frac{\partial}{\partial u} (\delta z) = 0; \quad (A1-86) \]

\( \frac{\partial^2 n}{\partial v \partial x} \)

\[ 2k \frac{\partial}{\partial v} (\delta x) = 0; \quad (A1-87) \]

\( \frac{\partial^2 n}{\partial v \partial y} \)

\[ 2k \frac{\partial}{\partial v} (\delta y) = 0; \quad (A1-88) \]

\( \frac{\partial^2 n}{\partial v \partial z} \)

\[ k \frac{\partial}{\partial v} (\delta z) = 0; \quad (A1-89) \]

\( \frac{\partial^2 n}{\partial w \partial x} \)

\[ 2k \frac{\partial}{\partial w} (\delta x) = 0; \quad (A1-90) \]

\( \frac{\partial^2 n}{\partial w \partial y} \)

\[ 2k \frac{\partial}{\partial w} (\delta y) = 0; \quad (A1-91) \]

\( \frac{\partial^2 n}{\partial w \partial z} \)

\[ k \frac{\partial}{\partial w} (\delta z) = 0; \quad (A1-92) \]

\( \frac{\partial^2 n}{\partial t \partial u} \)

\[ 2k \frac{\partial}{\partial t} (\delta t) = 0; \quad (A1-93) \]

\( \frac{\partial^2 n}{\partial t \partial v} \)

\[ 2k \frac{\partial}{\partial t} (\delta t) = 0; \quad (A1-94) \]

\( \frac{\partial^2 n}{\partial t \partial w} \)

\[ 2k \frac{\partial}{\partial t} (\delta t) = 0; \quad (A1-95) \]

\[ 1 \]

\[ 3akn \frac{\partial^2}{\partial u^2} (\delta t) + 3akn \frac{\partial^2}{\partial v^2} (\delta t) + 3akn \frac{\partial^2}{\partial w^2} (\delta t) - \quad (A1-96) \]
\[-3au_n \frac{\partial}{\partial x} (\delta t) - auu \frac{\partial}{\partial u} (\delta n) - 3avn \frac{\partial}{\partial y} (\delta t) - av \frac{\partial}{\partial v} (\delta n) - 3awn \frac{\partial}{\partial z} (\delta t) - aw \frac{\partial}{\partial w} (\delta n) +

+ 3an \frac{\partial}{\partial n} (\delta n) - 3an \frac{\partial}{\partial t} (\delta t) - 3a \delta n + 3a^2 un \frac{\partial}{\partial u} (\delta t) + 3a^2 vn \frac{\partial}{\partial v} (\delta t) + 3a^2 wn \frac{\partial}{\partial w} (\delta t) - 9a^2 n^2 \frac{\partial}{\partial n} (\delta t) +

+ k \frac{\partial^2 n}{\partial u \partial z} \frac{\partial}{\partial u} (\delta z) + k \frac{\partial^2 n}{\partial v \partial z} \frac{\partial}{\partial v} (\delta z) + k \frac{\partial^2 n}{\partial w \partial z} \frac{\partial}{\partial w} (\delta z) -

- k \frac{\partial^2 n}{\partial z^2} (\delta n) - k \frac{\partial^2 n}{\partial v^2} (\delta n) - k \frac{\partial^2 n}{\partial w^2} (\delta n) +

\quad + u \frac{\partial}{\partial x} (\delta n) + v \frac{\partial}{\partial y} (\delta n) + w \frac{\partial}{\partial z} (\delta n) + \frac{\partial}{\partial t} (\delta n) = 0.

From (A1-31 - A1-36), (A1-50 - A1-55), (A1-66 - A1-71), (A1-84 - A1-95) we see that 
\( \delta x = \delta x(x, y, z, t) \); \( \delta y = \delta y(x, y, z, t) \); \( \delta t = \delta t(x, y, z, t) \). We use these expressions to simplify the rest of equations (A1-3 - A1-96).

\[
uv \frac{\partial}{\partial y} (\delta t) + uv \frac{\partial}{\partial z} (\delta t) - u \frac{\partial}{\partial x} (\delta x) + u \frac{\partial}{\partial t} (\delta t) +
\]

\[
+ u^2 \frac{\partial}{\partial x} (\delta t) - v \frac{\partial}{\partial y} (\delta x) - w \frac{\partial}{\partial z} (\delta x) + \delta u - \frac{\partial}{\partial t} (\delta x) = 0;
\]

\[
uv \frac{\partial}{\partial x} (\delta t) - u \frac{\partial}{\partial y} (\delta y) + v w \frac{\partial}{\partial z} (\delta t) - v \frac{\partial}{\partial y} (\delta y) +
\]

\[
+ v \frac{\partial}{\partial t} (\delta t) + v^2 \frac{\partial}{\partial y} (\delta t) - w \frac{\partial}{\partial z} (\delta y) + \delta v - \frac{\partial}{\partial t} (\delta y) = 0;
\]

\[
uv \frac{\partial}{\partial x} (\delta t) - u \frac{\partial}{\partial y} (\delta z) + v w \frac{\partial}{\partial z} (\delta t) - v \frac{\partial}{\partial y} (\delta z) -
\]

\[
-w \frac{\partial}{\partial z} (\delta z) + w \frac{\partial}{\partial t} (\delta t) + w^2 \frac{\partial}{\partial z} (\delta t) + \delta w - \frac{\partial}{\partial t} (\delta z) = 0;
\]

\[
- auv \frac{\partial}{\partial y} (\delta t) - auw \frac{\partial}{\partial z} (\delta t) + au \frac{\partial}{\partial u} (\delta u) - au \frac{\partial}{\partial t} (\delta t) - au^2 \frac{\partial}{\partial x} (\delta t) +
\]

\[
+ auv \frac{\partial}{\partial v} (\delta u) + au \frac{\partial}{\partial w} (\delta u) - a \delta u - 2k \frac{\partial^2}{\partial n^2} (\delta n) +
\]

\[
+ k \frac{\partial^2}{\partial u^2} (\delta u) + k \frac{\partial^2}{\partial v^2} (\delta u) + k \frac{\partial^2}{\partial w^2} (\delta u) - u \frac{\partial}{\partial x} (\delta u) - v \frac{\partial}{\partial y} (\delta u) - w \frac{\partial}{\partial z} (\delta u) - \frac{\partial}{\partial t} (\delta u) = 0;
\]

\[
- k \frac{\partial^2}{\partial n^2} (\delta n) = 0;
\]

\[
- auv \frac{\partial}{\partial x} (\delta t) + au \frac{\partial}{\partial u} (\delta v) - auw \frac{\partial}{\partial z} (\delta t) + av \frac{\partial}{\partial v} (\delta v) - av \frac{\partial}{\partial t} (\delta t) -
\]

\[
- av^2 \frac{\partial}{\partial y} (\delta t) + av \frac{\partial}{\partial w} (\delta v) - a \delta v + k \frac{\partial^2}{\partial u^2} (\delta v) - 2k \frac{\partial^2}{\partial n^2} (\delta v) +
\]
\[ +k \frac{\partial^2}{\partial v^2} (\delta v) + k \frac{\partial^2}{\partial w^2} (\delta v) - u \frac{\partial}{\partial x} (\delta v) - v \frac{\partial}{\partial y} (\delta v) - w \frac{\partial}{\partial z} (\delta v) - \frac{\partial}{\partial t} (\delta v) = 0; \]

\[ -k \frac{\partial^2}{\partial n^2} (\delta n) = 0; \quad (A1-103) \]

\[ -aw \frac{\partial}{\partial x} (\delta t) + au \frac{\partial}{\partial u} (\delta w) - avw \frac{\partial}{\partial y} (\delta t) + av \frac{\partial}{\partial v} (\delta w) + aw \frac{\partial}{\partial w} (\delta w) - \frac{\partial}{\partial t} (\delta w) = 0; \quad (A1-104) \]

\[ -aw \frac{\partial}{\partial t} (\delta t) - aw^2 \frac{\partial}{\partial c} (\delta t) - a\delta w + k \frac{\partial^2}{\partial u^2} (\delta w) + k \frac{\partial^2}{\partial v^2} (\delta w) - \frac{\partial}{\partial t} (\delta w) + k \frac{\partial^2}{\partial w^2} (\delta w) - u \frac{\partial}{\partial x} (\delta w) - v \frac{\partial}{\partial y} (\delta w) - w \frac{\partial}{\partial z} (\delta w) - \frac{\partial}{\partial t} (\delta w) = 0; \quad (A1-105) \]

\[ -ku \frac{\partial}{\partial x} (\delta t) - kv \frac{\partial}{\partial y} (\delta t) - kw \frac{\partial}{\partial z} (\delta t) + 2k \frac{\partial}{\partial u} (\delta u) - k \frac{\partial}{\partial t} (\delta t) = 0; \quad (A1-106) \]

\[ -ku \frac{\partial}{\partial x} (\delta t) - kv \frac{\partial}{\partial y} (\delta t) - kw \frac{\partial}{\partial z} (\delta t) + 2k \frac{\partial}{\partial v} (\delta v) - k \frac{\partial}{\partial t} (\delta t) = 0; \quad (A1-107) \]

\[ -ku \frac{\partial}{\partial x} (\delta t) - kv \frac{\partial}{\partial y} (\delta t) - kw \frac{\partial}{\partial z} (\delta t) + 2k \frac{\partial}{\partial w} (\delta w) - k \frac{\partial}{\partial t} (\delta t) = 0; \quad (A1-108) \]

\[ 2k \frac{\partial}{\partial v} (\delta u) + 2k \frac{\partial}{\partial u} (\delta v) = 0; \quad (A1-109) \]

\[ 2k \frac{\partial}{\partial w} (\delta u) + 2k \frac{\partial}{\partial u} (\delta w) = 0; \quad (A1-110) \]

\[ 2k \frac{\partial}{\partial w} (\delta v) + 2k \frac{\partial}{\partial w} (\delta w) = 0; \quad (A1-111) \]

\[ -3awu \frac{\partial}{\partial x} (\delta t) - au \frac{\partial}{\partial u} (\delta n) - 3avn \frac{\partial}{\partial y} (\delta t) + av \frac{\partial}{\partial v} (\delta n) - 3awn \frac{\partial}{\partial z} (\delta t) = 0; \quad (A1-112) \]

\[ -av \frac{\partial}{\partial w} (\delta n) - 3an \frac{\partial}{\partial n} (\delta n) - 3an \frac{\partial}{\partial t} (\delta t) = -k \frac{\partial^2}{\partial w^2} (\delta n) = 0. \]

From (A1-101), (A1-103) and (A1-105) we conclude that

\[ \delta n = A + nB; \quad (A1-113) \]

where \( A = A(x, y, z, u, v, w, t) \) and \( B = B(x, y, z, u, v, w, t) \). Using these expressions, we simplify the rest of equations (A1-97 - A1-112).

\[ uv \frac{\partial}{\partial y} (\delta t) + uv \frac{\partial}{\partial z} (\delta t) - u \frac{\partial}{\partial x} (\delta x) + u \frac{\partial}{\partial t} (\delta t) + u \frac{\partial}{\partial x} (\delta t) - v \frac{\partial}{\partial y} (\delta x) - w \frac{\partial}{\partial z} (\delta x) + \delta u - \frac{\partial}{\partial t} (\delta x) = 0; \quad (A1-114) \]
\[-20-\]

\[
\begin{align*}
\frac{\partial}{\partial x} (\delta t) - u \frac{\partial}{\partial x} (\delta y) + vw \frac{\partial}{\partial z} (\delta t) - v \frac{\partial}{\partial y} (\delta y) & = 0; \\
\frac{\partial}{\partial y} (\delta t) + v^2 \frac{\partial}{\partial y} (\delta t) - w \frac{\partial}{\partial z} (\delta y) + \delta v - \frac{\partial}{\partial t} (\delta y) & = 0; \\
\frac{\partial}{\partial z} (\delta t) - u \frac{\partial}{\partial x} (\delta z) + vw \frac{\partial}{\partial y} (\delta t) - v \frac{\partial}{\partial y} (\delta z) & = 0; \\
-w \frac{\partial}{\partial z} (\delta z) + w \frac{\partial}{\partial t} (\delta t) + w^2 \frac{\partial}{\partial z} (\delta t) + \delta w - \frac{\partial}{\partial t} (\delta z) & = 0; \\
-aw \frac{\partial}{\partial y} (\delta t) - aw \frac{\partial}{\partial z} (\delta t) - au \frac{\partial}{\partial u} (\delta u) - aw \frac{\partial}{\partial u} (\delta t) - au^2 \frac{\partial}{\partial x} (\delta t) + \\
+av \frac{\partial}{\partial v} (\delta u) + aw \frac{\partial}{\partial w} (\delta u) - a\delta u - 2k \frac{\partial}{\partial u} + \\
+k \frac{\partial^2}{\partial u^2} (\delta u) + k \frac{\partial^2}{\partial v^2} (\delta u) + k \frac{\partial^2}{\partial w^2} (\delta u) - \\
-u \frac{\partial}{\partial x} (\delta u) - v \frac{\partial}{\partial y} (\delta u) - w \frac{\partial}{\partial z} (\delta u) - \frac{\partial}{\partial t} (\delta u) & = 0; \\
-aw \frac{\partial}{\partial x} (\delta t) + au \frac{\partial}{\partial u} (\delta v) - aw \frac{\partial}{\partial z} (\delta t) + av \frac{\partial}{\partial v} (\delta v) - av \frac{\partial}{\partial t} (\delta t) - \\
-av^2 \frac{\partial}{\partial y} (\delta t) + aw \frac{\partial}{\partial w} (\delta v) - a\delta v - 2k \frac{\partial}{\partial v} + \\
+k \frac{\partial^2}{\partial u^2} (\delta v) + k \frac{\partial^2}{\partial v^2} (\delta v) + k \frac{\partial^2}{\partial w^2} (\delta v) - \\
-u \frac{\partial}{\partial x} (\delta v) - v \frac{\partial}{\partial y} (\delta v) - w \frac{\partial}{\partial z} (\delta v) - \frac{\partial}{\partial t} (\delta v) & = 0; \\
-aw \frac{\partial}{\partial x} (\delta t) + aw \frac{\partial}{\partial u} (\delta w) - aw \frac{\partial}{\partial z} (\delta t) + av \frac{\partial}{\partial v} (\delta w) + aw \frac{\partial}{\partial w} (\delta w) - \\
-aw \frac{\partial}{\partial t} (\delta t) - au^2 \frac{\partial}{\partial y} (\delta t) - a\delta w - 2k \frac{\partial}{\partial w} + \\
+k \frac{\partial^2}{\partial u^2} (\delta w) + k \frac{\partial^2}{\partial v^2} (\delta w) + k \frac{\partial^2}{\partial w^2} (\delta w) - \\
-u \frac{\partial}{\partial x} (\delta w) - v \frac{\partial}{\partial y} (\delta w) - w \frac{\partial}{\partial z} (\delta w) - \frac{\partial}{\partial t} (\delta w) & = 0; \\
-ku \frac{\partial}{\partial x} (\delta t) - kv \frac{\partial}{\partial y} (\delta t) - kw \frac{\partial}{\partial z} (\delta t) + 2k \frac{\partial}{\partial u} (\delta u) - k \frac{\partial}{\partial t} (\delta t) & = 0; \\
-ku \frac{\partial}{\partial x} (\delta t) - kv \frac{\partial}{\partial y} (\delta t) - kw \frac{\partial}{\partial z} (\delta t) + 2k \frac{\partial}{\partial v} (\delta v) - k \frac{\partial}{\partial t} (\delta t) & = 0; \\
-ku \frac{\partial}{\partial x} (\delta t) - kv \frac{\partial}{\partial y} (\delta t) - kw \frac{\partial}{\partial z} (\delta t) + 2k \frac{\partial}{\partial w} (\delta w) - k \frac{\partial}{\partial t} (\delta t) & = 0; \\
\end{align*}
\]
Equation (A1-127) is simply Fokker-Planck equation for $A$.

We solve (A1-114 - A1-116) and find $\delta u, \delta v, \delta w$

$$\delta u = \left\{-uv \frac{\partial}{\partial y} (\delta t) + uw \frac{\partial}{\partial z} (\delta t) - u \frac{\partial}{\partial x} (\delta x) + u \frac{\partial}{\partial t} (\delta t) + 
+ u^2 \frac{\partial}{\partial x} (\delta t) - v \frac{\partial}{\partial y} (\delta x) - w \frac{\partial}{\partial z} (\delta x) - \frac{\partial}{\partial t} (\delta x) \right\}$$

$$\delta v = \left\{-uv \frac{\partial}{\partial x} (\delta t) - u \frac{\partial}{\partial x} (\delta y) + vw \frac{\partial}{\partial z} (\delta t) - v \frac{\partial}{\partial y} (\delta y) + 
+ v \frac{\partial}{\partial t} (\delta t) + v^2 \frac{\partial}{\partial y} (\delta t) - w \frac{\partial}{\partial z} (\delta y) - \frac{\partial}{\partial t} (\delta y) \right\}$$

$$\delta w = \left\{-uw \frac{\partial}{\partial x} (\delta t) - u \frac{\partial}{\partial x} (\delta z) + vw \frac{\partial}{\partial y} (\delta t) - v \frac{\partial}{\partial y} (\delta z) - 
-w \frac{\partial}{\partial z} (\delta z) + w \frac{\partial}{\partial t} (\delta t) + w^2 \frac{\partial}{\partial z} (\delta t) - \frac{\partial}{\partial t} (\delta z) \right\}$$

We use these expressions to simplify (A1-117 - A1-126)

$$-2a \frac{\partial}{\partial y} (\delta t) - 2a \frac{\partial}{\partial z} (\delta t) - au \frac{\partial}{\partial t} (\delta t) - 2a^2 \frac{\partial}{\partial x} (\delta t) - a \frac{\partial}{\partial t} (\delta x) - 2k \frac{\partial B}{\partial u}$$

$$-2k \frac{\partial}{\partial x} (\delta t) + 2uv \frac{\partial^2}{\partial y \partial z} (\delta t) + 2uv \frac{\partial^2}{\partial t^2} (\delta t) - 2uv \frac{\partial^2}{\partial x \partial y} (\delta t) + uv^2 \frac{\partial^2}{\partial y^2} (\delta t) + 2uv \frac{\partial^2}{\partial z^2} (\delta t) - 2u \frac{\partial^2}{\partial t \partial x} (\delta t) + 2u^2 v \frac{\partial^2}{\partial x \partial y} (\delta t)$$
\[ +2u^2 \frac{\partial^2}{\partial x \partial z} (\delta t) + 2u^2 \frac{\partial^2}{\partial t \partial x} (\delta t) - u^2 \frac{\partial^2}{\partial x^2} (\delta t) + u^3 \frac{\partial^2}{\partial y \partial z} (\delta t) - 2vw \frac{\partial^2}{\partial y \partial z} (\delta x) - 2v \frac{\partial^2}{\partial y \partial z} (\delta x) - 2w \frac{\partial^2}{\partial z^2} (\delta x) - \frac{\partial^2}{\partial t^2} (\delta x) - \frac{\partial^2}{\partial t^2} (\delta x) = 0; \text{ (A1-137)} \]

\[ -2aw \frac{\partial}{\partial x} (\delta t) - 2aw \frac{\partial}{\partial z} (\delta t) - aw \frac{\partial^2}{\partial t \partial x} (\delta t) - aw \frac{\partial^2}{\partial t \partial z} (\delta t) - akv \frac{\partial}{\partial x} (\delta t) - akv \frac{\partial}{\partial y} (\delta t) - akv \frac{\partial}{\partial z} (\delta t) = \text{ (A1-138)} \]
\[-2kv \frac{\partial}{\partial z} (\delta t) - 2kw \frac{\partial}{\partial y} (\delta t) + 2k \frac{\partial}{\partial z} (\delta y) + 2k \frac{\partial}{\partial y} (\delta z) = 0; \quad (A1-139)\]

\[-au \frac{\partial B}{\partial u} - 3au \frac{\partial}{\partial x} (\delta t) - av \frac{\partial B}{\partial v} - 3av \frac{\partial}{\partial y} (\delta t) - aw \frac{\partial B}{\partial w} - 3aw \frac{\partial}{\partial z} (\delta t) - \quad (A1-140)\]

\[-3a \frac{\partial}{\partial t} (\delta t) - k \frac{\partial^2 B}{\partial u^2} - k \frac{\partial^2 B}{\partial v^2} - k \frac{\partial^2 B}{\partial w^2} + u \frac{\partial B}{\partial x} + v \frac{\partial B}{\partial y} + w \frac{\partial B}{\partial z} + \frac{\partial B}{\partial t} = 0. \quad (A1-140)\]

We collect similar terms in \((A1-134 - A1-139)\) by \(u, v, w\), equate these terms to zero and so split \((A1-134 - A1-139)\) into twenty equations:

\[-5k \frac{\partial}{\partial x} (\delta t) = 0; \quad (A1-141)\]

\[-3k \frac{\partial}{\partial y} (\delta t) = 0; \quad (A1-142)\]

\[-3k \frac{\partial}{\partial z} (\delta t) = 0; \quad (A1-143)\]

\[2k \frac{\partial}{\partial x} (\delta x) - 3k \frac{\partial}{\partial t} (\delta t) = 0; \quad (A1-144)\]

\[-3k \frac{\partial}{\partial x} (\delta t) = 0; \quad (A1-145)\]

\[-5k \frac{\partial}{\partial y} (\delta t) = 0; \quad (A1-146)\]

\[-3k \frac{\partial}{\partial z} (\delta t) = 0; \quad (A1-147)\]

\[2k \frac{\partial}{\partial y} (\delta y) - 3k \frac{\partial}{\partial t} (\delta t) = 0; \quad (A1-148)\]

\[-3k \frac{\partial}{\partial x} (\delta t) = 0; \quad (A1-149)\]

\[-3k \frac{\partial}{\partial y} (\delta t) = 0; \quad (A1-150)\]

\[-5k \frac{\partial}{\partial z} (\delta t) = 0; \quad (A1-151)\]

\[2k \frac{\partial}{\partial z} (\delta z) - 3k \frac{\partial}{\partial t} (\delta t) = 0; \quad (A1-152)\]

\[-2k \frac{\partial}{\partial y} (\delta t) = 0; \quad (A1-153)\]

\[-2k \frac{\partial}{\partial x} (\delta t) = 0; \quad (A1-154)\]

\[2k \frac{\partial}{\partial y} (\delta x) + 2k \frac{\partial}{\partial x} (\delta y) = 0; \quad (A1-155)\]
\[-24-\]

\[-2k \frac{\partial}{\partial z}(\delta t) = 0; \quad (A1-156)\]

\[-2k \frac{\partial}{\partial x}(\delta t) = 0; \quad (A1-157)\]

\[2k \frac{\partial}{\partial z}(\delta x) + 2k \frac{\partial}{\partial x}(\delta z) = 0; \quad (A1-158)\]

\[-2k \frac{\partial}{\partial z}(\delta t) = 0; \quad (A1-159)\]

\[-2k \frac{\partial}{\partial y}(\delta t) = 0; \quad (A1-160)\]

\[2k \frac{\partial}{\partial z}(\delta y) + 2k \frac{\partial}{\partial y}(\delta z) = 0. \quad (A1-161)\]

From (A1-141 - A1-143), (A1-145 - A1-147), (A1-149 - A1-151), (A1-153 - A1-154), (A1-156 - A1-157), (A1-159 - A1-160) we see, that \( \delta t = \delta t(t) \). These expressions result in further simplifications

\[-au \frac{\partial}{\partial t}(\delta t) - a \frac{\partial}{\partial t}(\delta x) - 2k \frac{\partial B}{\partial u} - 2uv \frac{\partial^2}{\partial x\partial y}(\delta x) - 2uw \frac{\partial^2}{\partial x\partial z}(\delta x) + \]

\[+u \frac{\partial^2}{\partial t^2}(\delta t) - 2u \frac{\partial^2}{\partial t\partial x}(\delta x) - u^2 \frac{\partial^2}{\partial x^2}(\delta x) - 2uv \frac{\partial^2}{\partial y\partial z}(\delta x) - 2v \frac{\partial^2}{\partial t\partial y}(\delta x) - \]

\[-v^2 \frac{\partial^2}{\partial y^2}(\delta x) - 2w \frac{\partial^2}{\partial t\partial z}(\delta x) - w^2 \frac{\partial^2}{\partial z^2}(\delta x) - \frac{\partial^2}{\partial t^2}(\delta x) = 0; \quad (A1-162)\]

\[-av \frac{\partial}{\partial t}(\delta t) - a \frac{\partial}{\partial t}(\delta y) - 2k \frac{\partial B}{\partial v} - 2uv \frac{\partial^2}{\partial x\partial y}(\delta y) - 2uw \frac{\partial^2}{\partial x\partial z}(\delta y) - 2u \frac{\partial^2}{\partial t\partial x}(\delta y) - \]

\[u^2 \frac{\partial^2}{\partial x^2}(\delta y) - 2vw \frac{\partial^2}{\partial y\partial z}(\delta y) + v \frac{\partial^2}{\partial t^2}(\delta t) - 2v \frac{\partial^2}{\partial t\partial y}(\delta y) - v^2 \frac{\partial^2}{\partial y^2}(\delta y) - \]

\[-2w \frac{\partial^2}{\partial t\partial z}(\delta y) - w^2 \frac{\partial^2}{\partial z^2}(\delta y) - \frac{\partial^2}{\partial t^2}(\delta y) = 0; \quad (A1-163)\]

\[\frac{\partial}{\partial t}(\delta z) - a \frac{\partial}{\partial t}(\delta z) - 2k \frac{\partial B}{\partial w} - 2uv \frac{\partial^2}{\partial x\partial y}(\delta z) - 2uw \frac{\partial^2}{\partial x\partial z}(\delta z) - \]

\[-2u \frac{\partial^2}{\partial t^2}(\delta t) - u^2 \frac{\partial^2}{\partial x^2}(\delta z) - 2vw \frac{\partial^2}{\partial y\partial z}(\delta z) - 2v \frac{\partial^2}{\partial t\partial y}(\delta z) - v^2 \frac{\partial^2}{\partial y^2}(\delta z) + \]

\[+w \frac{\partial^2}{\partial t^2}(\delta t) - 2w \frac{\partial^2}{\partial t\partial z}(\delta z) - w^2 \frac{\partial^2}{\partial z^2}(\delta z) - \frac{\partial^2}{\partial t^2}(\delta z) = 0; \quad (A1-164)\]

\[2k \frac{\partial}{\partial x}(\delta x) - 3k \frac{\partial}{\partial t}(\delta t) = 0; \quad (A1-165)\]

\[2k \frac{\partial}{\partial y}(\delta y) - 3k \frac{\partial}{\partial t}(\delta t) = 0; \quad (A1-166)\]

\[2k \frac{\partial}{\partial z}(\delta z) - 3k \frac{\partial}{\partial t}(\delta t) = 0; \quad (A1-167)\]
\[ 2k \frac{\partial}{\partial y} (\delta x) + 2k \frac{\partial}{\partial x} (\delta y) = 0; \quad (A1-168) \]
\[ 2k \frac{\partial}{\partial z} (\delta x) + 2k \frac{\partial}{\partial x} (\delta z) = 0; \quad (A1-169) \]
\[ 2k \frac{\partial}{\partial z} (\delta y) + 2k \frac{\partial}{\partial y} (\delta z) = 0; \quad (A1-170) \]
\[-au \frac{\partial B}{\partial u} - aw \frac{\partial B}{\partial w} - aw \frac{\partial B}{\partial w} - 3a \frac{\partial}{\partial t} (\delta t) - k \frac{\partial^2 B}{\partial u^2} - \]
\[-k \frac{\partial^2 B}{\partial y^2} - k \frac{\partial^2 B}{\partial w^2} + u \frac{\partial B}{\partial x} + v \frac{\partial B}{\partial y} + w \frac{\partial B}{\partial z} + \frac{\partial B}{\partial t} = 0. \quad (A1-171) \]

We integrate (A1-164 - A1-167) and obtain
\[ \delta x = C + 3/2x \frac{\partial}{\partial t} (\delta t); \quad (A1-172) \]
\[ \delta y = D + 3/2y \frac{\partial}{\partial t} (\delta t); \quad (A1-173) \]
\[ \delta z = E + 3/2z \frac{\partial}{\partial t} (\delta t); \quad (A1-174) \]

where \( C = C(y, z, t), \quad D = D(x, z, t), \quad E(x, y, t). \) We substitute this expression in (A1-162 - A1-164),
(A1-168 - A1-171) and obtain
\[-3/2ax \frac{\partial^2}{\partial t^2} (\delta t) - au \frac{\partial}{\partial t} (\delta t) - a \frac{\partial C}{\partial t} - 2k \frac{\partial B}{\partial u} - 3/2x \frac{\partial^3}{\partial t^3} (\delta t) - \]
\[-2u \frac{\partial^2}{\partial t^2} (\delta t) - 2wv \frac{\partial^2}{\partial y^2} (\delta x) - 2v \frac{\partial^2 C}{\partial t \partial y} - \]
\[-v^2 \frac{\partial^2 C}{\partial y^2} - 2w \frac{\partial^2 C}{\partial z^2} - w^2 \frac{\partial^2 C}{\partial x^2} = 0; \quad (A1-175) \]
\[-3/2ay \frac{\partial^2}{\partial t^2} (\delta t) - av \frac{\partial}{\partial t} (\delta t) - a \frac{\partial D}{\partial t} - 2k \frac{\partial B}{\partial v} - 3/2y \frac{\partial^3}{\partial t^3} (\delta t) - \]
\[-2aw \frac{\partial^2}{\partial x \partial z} (\delta y) - 2u \frac{\partial^2 D}{\partial x \partial t} - u^2 \frac{\partial^2 D}{\partial x^2} - \]
\[-2v \frac{\partial^2}{\partial t^2} (\delta t) - 2w \frac{\partial^2 D}{\partial t \partial z} - w^2 \frac{\partial^2 D}{\partial z^2} = 0; \quad (A1-176) \]
\[-3/2az \frac{\partial^2}{\partial t^2} (\delta t) - aw \frac{\partial}{\partial t} (\delta t) - a \frac{\partial E}{\partial t} - 2k \frac{\partial B}{\partial w} - 3/2z \frac{\partial^3}{\partial t^3} (\delta t) - \]
\[-2aw \frac{\partial^2}{\partial x \partial y} (\delta z) - 2u \frac{\partial^2 E}{\partial x \partial t} - u^2 \frac{\partial^2 E}{\partial x^2} - \]
\[-2v \frac{\partial^2 E}{\partial t \partial y} - v^2 \frac{\partial^2 E}{\partial y^2} - 2w \frac{\partial^2 E}{\partial z^2} = 0; \quad (A1-177) \]
\[
2k \frac{\partial C}{\partial y} + 2k \frac{\partial D}{\partial x} = 0; \quad (A1-178)
\]
\[
2k \frac{\partial C}{\partial z} + 2k \frac{\partial E}{\partial x} = 0; \quad (A1-179)
\]
\[
2k \frac{\partial D}{\partial z} + 2k \frac{\partial E}{\partial y} = 0; \quad (A1-180)
\]
\[
-av \frac{\partial B}{\partial u} - av \frac{\partial B}{\partial v} - aw \frac{\partial B}{\partial w} - 3a \frac{\partial}{\partial t} (\delta t) - k \frac{\partial^2 B}{\partial u^2} - k \frac{\partial^2 B}{\partial v^2} - k \frac{\partial^2 B}{\partial w^2} = 0. \quad (A1-181)
\]

We have
\[
\frac{\partial C}{\partial y} = \frac{\partial D}{\partial x} = \frac{\partial C}{\partial z} = \frac{\partial D}{\partial z} = \frac{\partial E}{\partial x} = \frac{\partial E}{\partial y} = 0. \quad (A1-182)
\]

and according to (A1-178 - A1-180)
\[
\frac{\partial C}{\partial y} + \frac{\partial D}{\partial x} = \frac{\partial C}{\partial z} + \frac{\partial D}{\partial z} = \frac{\partial C}{\partial x} = \frac{\partial D}{\partial z} = \frac{\partial E}{\partial y} = 0. \quad (A1-183)
\]

Let us think, that \(C, D, E\) are components of some displacement vector. According to (A1-182 - A1-183) all components of corresponding deformations tensor are equal to zero. Therefore displacement vector is time-dependent rotation-translation, which preserves Euclidean metric.
\[
C = S + Qz - Ry; \quad (A1-184)
\]
\[
D = T + Rx - Pz; \quad (A1-185)
\]
\[
E = U + Py - Qx; \quad (A1-186)
\]

where \(P = P(t), Q = Q(t), R = R(t), S = S(t), T = T(t), U = U(t)\).

We substitute \(C, D, E\) to (A1-175 - A1-177) and (A1-181) and obtain
\[
-\frac{3}{2}ax \frac{\partial^2}{\partial t^2} (\delta t) + ay \frac{\partial R}{\partial t} + az \frac{\partial Q}{\partial t} = -av \frac{\partial}{\partial t} (\delta t) - \frac{\partial S}{\partial t} - \frac{\partial D}{\partial x} \quad (A1-187)
\]
\[
-2k \frac{\partial B}{\partial t} - 3/2x \frac{\partial^3}{\partial t^3} (\delta t) + y \frac{\partial^2 R}{\partial t^2} - z \frac{\partial^2 Q}{\partial t^2} - 2u \frac{\partial^2}{\partial t^2} (\delta t) - \frac{\partial^2 R}{\partial t^2} - \frac{\partial^2}{\partial t^2} (\delta t) = 0; \quad (A1-188)
\]
\[
-ax \frac{\partial R}{\partial t} - \frac{3}{2}ay \frac{\partial^2}{\partial t^2} (\delta t) + az \frac{\partial P}{\partial t} + \frac{\partial}{\partial t} (\delta t) - 2v \frac{\partial^2 S}{\partial t^2} = 0; \quad (A1-189)
\]
\[
-2k \frac{\partial B}{\partial t} - x \frac{\partial^2 R}{\partial t^2} - 3/2y \frac{\partial^3}{\partial t^3} (\delta t) + z \frac{\partial^2 P}{\partial t^2} - 2uw \frac{\partial^2}{\partial t^2} (\delta t) = 0; \quad (A1-189)
\]
\[
ax \frac{\partial Q}{\partial t} - ay \frac{\partial P}{\partial t} - 3/2az \frac{\partial^2}{\partial t^2} (\delta t) - \frac{\partial}{\partial t} (\delta t) - \frac{\partial ^2 U}{\partial t^2} = 0; \quad (A1-189)
\]
We find $\frac{\partial B}{\partial u}$, $\frac{\partial B}{\partial v}$, $\frac{\partial B}{\partial w}$ from (187-189):

(A1-190)

We find $\frac{\partial B}{\partial u}$, $\frac{\partial B}{\partial v}$, $\frac{\partial B}{\partial w}$ from (187-189):

(A1-191)

(A1-192)

(A1-193)

Differentiating (A1-191 - A1-193) by $u$ we have

(A1-194)

(A1-195)

(A1-196)
Differentiating (A1-191 - A1-193) by \( v \) we have
\[
\frac{\partial^2 B}{\partial u \partial v} = -2w \frac{\partial^2}{\partial y \partial z} (\delta x) + 2 \frac{\partial R}{\partial t}; \tag{A1-197}
\]
\[
\frac{\partial^2 B}{\partial v^2} = -a \frac{\partial}{\partial t} (\delta t) - 2 \frac{\partial^2}{\partial t^2} (\delta t); \tag{A1-198}
\]
\[
\frac{\partial^2 B}{\partial w \partial v} = -2u \frac{\partial^2}{\partial x \partial y} (\delta z) - 2 \frac{\partial P}{\partial t}. \tag{A1-199}
\]

Differentiating (A1-191 - A1-193) by \( w \) we have
\[
\frac{\partial^2 B}{\partial u \partial w} = -2v \frac{\partial^2}{\partial y \partial z} (\delta x) - 2 \frac{\partial Q}{\partial t}; \tag{A1-200}
\]
\[
\frac{\partial^2 B}{\partial v \partial w} = -2u \frac{\partial^2}{\partial x \partial z} (\delta y) + 2 \frac{\partial P}{\partial t}; \tag{A1-201}
\]
\[
\frac{\partial^2 B}{\partial w^2} = -a \frac{\partial}{\partial t} (\delta t) - 2 \frac{\partial^2}{\partial t^2} (\delta t). \tag{A1-202}
\]

We conclude from (194 - 202), that
\[
\frac{\partial P}{\partial t} = \frac{\partial Q}{\partial t} = \frac{\partial R}{\partial t} = 0; \tag{A1-203}
\]
\[
\frac{\partial^2}{\partial y \partial z} (\delta x) = \frac{\partial^2}{\partial x \partial y} (\delta z) = \frac{\partial^2}{\partial x \partial z} (\delta y) = F. \tag{A1-204}
\]

where \( F = F(x, y, z, t) \).

Under these conditions we can integrate (A1-191 - A1-193) and obtain
\[
B = \frac{1}{2k} \left( -a \frac{\partial}{\partial t} (\delta t) - 2 \frac{\partial^2}{\partial t^2} (\delta t))(u^2 + v^2 + w^2) - 2Fuvw + \right. \tag{A1-205}
\]
\[
+u(-3/2ax \frac{\partial^2}{\partial t^2} (\delta t) - a \frac{\partial}{\partial t} - 3/2x \frac{\partial^3}{\partial t^3} (\delta t) - \frac{\partial^2 S}{\partial t^2}) + \]
\[
+v(-3/2ay \frac{\partial^2}{\partial t^2} (\delta t) - a \frac{\partial}{\partial t} - 3/2y \frac{\partial^3}{\partial t^3} (\delta t) - \frac{\partial^2 T}{\partial t^2}) + \]
\[
+w(-3/2az \frac{\partial^2}{\partial t^2} (\delta t) - a \frac{\partial}{\partial t} - 3/2z \frac{\partial^3}{\partial t^3} (\delta t) - \frac{\partial^2 U}{\partial t^2}) \left. + G; \right.
\]

where \( G = G(x, y, z, t) \).

Substitution of (A1-205) to (A1-171), collecting and equating to zero similar terms by \( u, v \) gives
\[
3ak^{-1} F - k^{-1} \frac{\partial F}{\partial t} = 0; \tag{A1-206}
\]
\[
-k^{-1} \frac{\partial F}{\partial z} = 0; \tag{A1-207}
\]
\[
-k^{-1} \frac{\partial F}{\partial y} = 0; \tag{A1-208}
\]
We find derivatives of $G$ from (A1-218), (A1-220), (A1-222), (A1-224)

$$\frac{\partial G}{\partial x} = -3(3/4 a^2 k^{-1} \frac{\partial^2}{\partial t^2} (\partial t) + 1/2 a^2 k^{-1} \frac{\partial S}{\partial t} - 3/4 k^{-1} \frac{\partial^4}{\partial t^4} (\partial t) - 1/2 k^{-1} \frac{\partial^3 S}{\partial t^3} + \frac{\partial G}{\partial x} = 0; \quad (A1-226)$$
\[
\frac{\partial G}{\partial y} = -(3/4 a^2 k^{-1} y \frac{\partial^2}{\partial t^2} (\delta t)) + 1/2 a^2 k^{-1} \frac{\partial T}{\partial t} - 3/4 k^{-1} z \frac{\partial^4}{\partial t^4} (\delta t) - 1/2 k^{-1} \frac{\partial^3 T}{\partial t^3}; \quad (A1-227)
\]

\[
\frac{\partial G}{\partial z} = -(3/4 a^2 k^{-1} z \frac{\partial^2}{\partial t^2} (\delta t)) + 1/2 a^2 k^{-1} \frac{\partial U}{\partial t} - 3/4 k^{-1} z \frac{\partial^4}{\partial t^4} (\delta t) - 1/2 k^{-1} \frac{\partial^3 U}{\partial t^3}; \quad (A1-228)
\]

\[
\frac{\partial G}{\partial t} = -(3/2 a \frac{\partial}{\partial t} (\delta t)) + 3 \frac{\partial^2}{\partial t^2} (\delta t)); \quad (A1-229)
\]

Cross differentiation of \( G \) by \( t \) and \( x, y, z \) gives

\[
\frac{\partial^3 G}{\partial x \partial t} = -(3/4 a^2 k^{-1} x \frac{\partial^3}{\partial t^3} (\delta t)) + 1/2 a^2 k^{-1} \frac{\partial^2 S}{\partial t^2} - 3/4 k^{-1} x \frac{\partial^5}{\partial t^5} (\delta t) - 1/2 k^{-1} \frac{\partial^4 S}{\partial t^4} = \frac{\partial^3 G}{\partial t \partial x} = 0 (A1-230)
\]

\[
\frac{\partial^3 G}{\partial y \partial t} = -(3/4 a^2 k^{-1} y \frac{\partial^3}{\partial t^3} (\delta t)) + 1/2 a^2 k^{-1} \frac{\partial^2 T}{\partial t^2} - 3/4 k^{-1} y \frac{\partial^5}{\partial t^5} (\delta t) - 1/2 k^{-1} \frac{\partial^4 T}{\partial t^4} = \frac{\partial^3 G}{\partial t \partial y} = 0 (A1-231)
\]

\[
\frac{\partial^3 G}{\partial z \partial t} = -(3/4 a^2 k^{-1} z \frac{\partial^3}{\partial t^3} (\delta t)) + 1/2 a^2 k^{-1} \frac{\partial^2 U}{\partial t^2} - 3/4 k^{-1} z \frac{\partial^5}{\partial t^5} (\delta t) - 1/2 k^{-1} \frac{\partial^4 U}{\partial t^4} = \frac{\partial^3 G}{\partial t \partial z} = 0 (A1-232)
\]

Collecting and equating to zero terms by \( x, y, z \) in (A1-230 - A1-232) gives

\[
+1/2 a^2 k^{-1} \frac{\partial^2 S}{\partial t^2} - 1/2 k^{-1} \frac{\partial^4 S}{\partial t^4} = 0; \quad (A1-233)
\]

\[
+1/2 a^2 k^{-1} \frac{\partial^2 T}{\partial t^2} - 1/2 k^{-1} \frac{\partial^4 T}{\partial t^4} = 0; \quad (A1-234)
\]

\[
+1/2 a^2 k^{-1} \frac{\partial^2 U}{\partial t^2} - 1/2 k^{-1} \frac{\partial^4 U}{\partial t^4} = 0; \quad (A1-235)
\]

\[
3/4 a^2 k^{-1} \frac{\partial^3 S}{\partial t^3} (\delta t) - 3/4 k^{-1} \frac{\partial^5}{\partial t^5} (\delta t) = 0. \quad (A1-236)
\]

From (A1-236), (A1-220), (A1-222), (A1-224) we conclude, that

\[
\delta t = const = C_5. \quad (A1-237)
\]

Integrating (A1-233 - A1-235) we have

\[
S = C_6 + C_7 t + C_8 e^{-at} + C_9 e^{at}; \quad (A1-238)
\]

\[
T = C_{10} + C_{11} t + C_{12} e^{-at} + C_{13} e^{at}; \quad (A1-239)
\]

\[
U = C_{14} + C_{15} t + C_{16} e^{-at} + C_{17} e^{at}. \quad (A1-240)
\]

It follows, that

\[
G = C_{18} - \frac{a^2}{2k} x C_{07} - \frac{a^2}{2k} y C_{11} - \frac{a^2}{2k} z C_{15}. \quad (A1-241)
\]

From (A1-184 - A1-186) we have

\[
C = -y C_3 + z C_2 + t C_7 + C_6 + C_8 e^{-at} + C_9 e^{at}; \quad (A1-242)
\]

\[
D =xC_3 + z C_1 + t C_{11} + C_{10} + C_{12} e^{-at} + C_{13} e^{at}; \quad (A1-243)
\]

\[
E = -xC_2 + y C_1 + t C_{15} + C_{14} + C_{16} e^{-at} + C_{17} e^{at}. \quad (A1-244)
\]
From (A1-172 - A1-174) we find variations of independent variables

\[
\delta x = -yC_3 + zC_2 + tC_7 + C_6 + C_8 e^{-at} + C_9 e^{at}; \quad \text{(A1-245)}
\]

\[
\delta y = xC_3 - zC_1 + tC_{11} + C_{10} + C_{12} e^{-at} + C_{13} e^{at}; \quad \text{(A1-246)}
\]

\[
\delta z = -xC_2 + yC_1 + tC_{15} + C_{14} + C_{16} e^{-at} + C_{17} e^{at}; \quad \text{(A1-247)}
\]

Cross-differentiating (A1-245 - A1-247), we have (see (A1-204))

\[
F = 0.
\]

Variations of velocities are equal (see A1-128 - A1-130)

\[
\delta u = -aC_8 e^{-at} + aC_9 e^{at} - vC_3 + wC_2 + C_7; \quad \text{(A1-248)}
\]

\[
\delta v = -aC_{12} e^{-at} + aC_{13} e^{at} + uC_3 - wC_1 + C_{11}; \quad \text{(A1-249)}
\]

\[
\delta w = -aC_{16} e^{-at} + aC_{17} e^{at} - uC_2 + vC_1 + C_{15}. \quad \text{(A1-250)}
\]

Variation of \( n \) according to (A1-113) is equal to

\[
\delta n = -\frac{an}{2k} (uC_7 + vC_{11} + wC_{15}) - \frac{a^2 n}{2k} (xC_7 + yC_{11} + zC_{15}) -
\]

\[
-e^{at} \frac{a^2 n}{k} (uC_9 + vC_{13} + wC_{17}) + nC_{18} + A.
\]

This is the desired result.
According to [2] (APPENDIX 1, eq. (A1-8) and (A1-18)), we have for variations of derivatives following expression:

\[
\begin{align*}
\delta \frac{\partial n}{\partial x} &= \frac{\partial}{\partial x} (\partial n) + \frac{\partial}{\partial n} (\partial n) \frac{\partial n}{\partial x} + \frac{\partial}{\partial n} (\partial x) + \frac{\partial}{\partial \partial} (\partial x) \frac{\partial n}{\partial x} - \frac{\partial n}{\partial y} (\frac{\partial}{\partial x} (\partial y) + \frac{\partial}{\partial n} (\partial y) \frac{\partial n}{\partial x}) - (A2-1) \\
- \frac{\partial n}{\partial z} &= \frac{\partial}{\partial x} (\partial z) + \frac{\partial}{\partial n} (\partial z) \frac{\partial n}{\partial x} - \frac{\partial n}{\partial y} (\frac{\partial}{\partial x} (\partial y) + \frac{\partial}{\partial n} (\partial y) \frac{\partial n}{\partial x}) - (A2-2) \\
- \frac{\partial n}{\partial u} &= \frac{\partial}{\partial y} (\partial u) + \frac{\partial}{\partial n} (\partial u) \frac{\partial n}{\partial y} - \frac{\partial n}{\partial v} \frac{\partial n}{\partial u} - (A2-3) \\
- \frac{\partial n}{\partial v} &= \frac{\partial}{\partial u} (\partial v) + \frac{\partial}{\partial n} (\partial v) \frac{\partial n}{\partial u} - (A2-4) \\
- \frac{\partial n}{\partial w} &= \frac{\partial}{\partial u} (\partial w) + \frac{\partial}{\partial n} (\partial w) \frac{\partial n}{\partial u} - (A2-5) \\
- \frac{\partial n}{\partial \partial} &= \frac{\partial}{\partial v} (\partial \partial) + \frac{\partial}{\partial n} (\partial \partial) \frac{\partial n}{\partial v} - (A2-6)
\end{align*}
\]
\[
\frac{\partial^2 n}{\partial v^2} \frac{\partial n}{\partial v} \frac{\partial (\delta v)}{\partial n} - \frac{\partial^2 n}{\partial v^2} \frac{\partial n}{\partial v} \frac{\partial (\delta w)}{\partial n} - \frac{\partial^2 n}{\partial n^2} \frac{\partial n}{\partial v} \frac{\partial (\delta t)}{\partial n} + \frac{\partial^2}{\partial n^2} (\delta n) + \\
\frac{\partial^2}{\partial n \partial v} (\delta n) \frac{\partial n}{\partial v} + \frac{\partial^2}{\partial n \partial v} (\delta n) + \frac{\partial^2}{\partial n^2} (\delta n) \frac{\partial n}{\partial v} + \frac{\partial^2}{\partial n \partial x} (\delta n) \frac{\partial n}{\partial v} + \frac{\partial^2}{\partial n \partial y} (\delta n) \frac{\partial n}{\partial v} - \frac{\partial n}{\partial v} \frac{\partial (\delta z)}{\partial v} - \frac{\partial^2}{\partial n \partial v} (\delta z) \frac{\partial n}{\partial v} - \frac{\partial^2}{\partial n \partial t} (\delta t) \frac{\partial n}{\partial v} \\
- \frac{\partial n}{\partial w} \frac{\partial (\delta w)}{\partial w} - \frac{\partial^2}{\partial n \partial w} (\delta w) \frac{\partial n}{\partial w} - \frac{\partial^2}{\partial n \partial t} (\delta t) \frac{\partial n}{\partial w} - \frac{\partial n}{\partial w} \frac{\partial (\delta y)}{\partial w} - \frac{\partial^2}{\partial n \partial w} (\delta y) \frac{\partial n}{\partial w} - \frac{\partial n}{\partial w} \frac{\partial (\delta t)}{\partial w} - \frac{\partial^2}{\partial n \partial w} (\delta t) \frac{\partial n}{\partial w} \\
- \frac{\partial n}{\partial t} \frac{\partial (\delta t)}{\partial t} - \frac{\partial^2}{\partial n \partial t} (\delta t) \frac{\partial n}{\partial t} = 0 \quad (A2-10)
\]
\begin{align*}
- \frac{\partial n}{\partial t} \frac{\partial n}{\partial w} (\frac{\partial^2}{\partial n \partial w} (\delta t)) + \frac{\partial^2}{\partial n^2} (\delta t) \frac{\partial n}{\partial w} - \frac{\partial^2}{\partial w \partial x} (\frac{\partial}{\partial w} (\delta x) + \frac{\partial}{\partial n} (\delta x) \frac{\partial n}{\partial w}) - \frac{\partial^2}{\partial w \partial y} (\frac{\partial}{\partial w} (\delta y) + \frac{\partial}{\partial n} (\delta y) \frac{\partial n}{\partial w}) - \\
- \frac{\partial^2}{\partial w \partial z} (\frac{\partial}{\partial w} (\delta z) + \frac{\partial}{\partial n} (\delta z) \frac{\partial n}{\partial w}) - \frac{\partial^3}{\partial u \partial w} (\frac{\partial}{\partial w} (\delta u) + \frac{\partial}{\partial n} (\delta u) \frac{\partial n}{\partial w}) - \frac{\partial^3}{\partial v \partial w} (\frac{\partial}{\partial w} (\delta v) + \frac{\partial}{\partial n} (\delta v) \frac{\partial n}{\partial w}) - \\
- \frac{\partial^2}{\partial w^2} (\frac{\partial}{\partial n} (\delta w) + \frac{\partial}{\partial n} (\delta w) \frac{\partial n}{\partial w}) - \frac{\partial^2}{\partial t \partial w} (\frac{\partial}{\partial w} (\delta t) + \frac{\partial}{\partial n} (\delta t) \frac{\partial n}{\partial w});
\end{align*}