Impact of nuclear effects in the measurement of neutrino oscillation parameters

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Nufact2011
based on E. F. Martinez & DM, 1010.2329, PLB
Main motivation of this exercise

comparing Fermi gas (FG) and advanced nuclear model predictions for physically interesting neutrino observables

- this is relevant because many MonteCarlo codes, used to study the sensitivity to still *unknown* parameters at future $\nu$ facilities are based on FG
- impossible to discuss all recent nuclear models

⇓

focus the attention on *three* different approaches in the *quasi-elastic* regime
Global 3 $\nu$ fit to the world neutrino data

At 1$\sigma$ (3$\sigma$)

- well known parameters

$$\Delta m_{sol}^2 = 6.99 - 8.18 \times 10^{-5} \text{ eV}^2,$$
$$\Delta m_{atm}^2 = 2.06 - 2.67 \times 10^{-3} \text{ eV}^2,$$
$$\sin^2 \theta_{23} = 0.34 - 0.64 (^{+0.054}_{-0.046}),$$
$$\sin^2 \theta_{12} = \begin{cases} 0.259 - 0.359 & \text{(old fluxes)}, \\ 0.265 - 0.364 & \text{(new fluxes)}, \end{cases}$$

- poor and unknown parameters

$$\sin^2 \theta_{13} = \begin{cases} 0.001 - 0.044 & \text{(old fluxes)}, \\ 0.005 - 0.05 & \text{(new fluxes)}, \end{cases}$$
$$\delta_{\text{CP}} \in [0, 360] \text{ (unknown)}$$
$$\text{sign}(\Delta m_{31}^2) \quad \text{octant of } \theta_{23} \quad \text{Majorana or Dirac Neutrinos?}$$

G.L. Fogli et al., arXiv:1106.6028
**Great interest on $\theta_{13}$ and $\delta$**

The appearance neutrino oscillation probability ($\alpha \neq \beta$)

$$P_{\alpha\beta}^\pm (\theta_{13}, \delta) = A_\pm \sin^2 2\theta_{13} \pm B_\pm \sin \delta \sin 2\theta_{13} + Z$$

where

- $\pm$ refers to neutrino and antineutrino
- $A_\pm, B_\pm$ and $Z$ functions of "known" parameters

Many future experiments will look for a precise measurement of $\theta_{13}$.

Large $\theta_{13}$ means good chance to reveal the CP violation in the leptonic sector

One needs to control:

- flux composition
- detector response
- nuclear cross sections
**Elementary charged current neutrino-nucleon cross section**

We focus on charged current interactions at neutrino energies around 1 GeV (CCQE)

\[
\nu_l + n \rightarrow l^- + p
\]

- **Effective Lagrangian:**

\[
\mathcal{L} = \frac{G_F}{\sqrt{2}} \bar{\ell} \gamma^\mu (1 - \gamma_5) \nu J^\text{had}_\mu
\]

- **Feynman Amplitude:**

\[
\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{ud} \bar{\ell}(k') \gamma^\mu (1 - \gamma_5) \nu(k) <N(p')|J^\text{had}_\mu|N(p)>.
\]
**Differential cross section**

- we need a parametrization of the hadronic matrix element

\[
\langle N(p') | J^\mu | N(p) \rangle = \begin{pmatrix}
\bar{u}(p') \\
\gamma^\mu F_1(Q^2) + \frac{i \sigma_{\mu\nu} q^\nu}{2 M_N} F_2(Q^2) \\
\text{hadronic vector current } J^\mu_V \\
\text{hadronic axial current } - J^\mu_A \\
\gamma^\mu \gamma^5 F_A(Q^2)
\end{pmatrix} u(p)
\]

- \( F_{1,2}(Q^2) \) are vector form factors
- \( F_A(Q^2) \) is the axial form factor, depending on a mass parameter: the axial mass \( m_A \)

in principle we should also consider an additional form factor \( F_P(Q^2) \) but its contribution to the cross section is negligible for \( e \) and \( \mu \) (but not for \( \tau \)’s)
Scattering on nuclei in the quasi-elastic (QE) regime

\[ \nu_l + A \rightarrow l^- + X \]

- the description of the interaction is more complicated because nucleons are bound

I see two classes of problems:

1- other processes make difficult to experimentally measure “pure” CCQE

- theoretical calculations should rely on assumptions
The Relativistic Fermi Gas model (RFG)

Problem #2 (assuming experiments are measuring pure CCQE)

- simplest model to account for nuclear effects [R. A. Smith and E. J. Monitz, Nucl. Phys. B 43, 605 (1972)]
- target state as a superposition of non-interacting neutron (and proton) Fermi gases with momentum distribution

\[
\frac{d^2 \sigma}{d \Omega dE_l} = \frac{G_F^2 |V_{ud}|^2}{16 \pi^2} \frac{|k'|}{|k|} L_{\mu \nu} W_{A}^{\mu \nu}
\]

\[
L_{\mu \nu} = 8 \left[ k'_{\mu} k_{\nu} + k'_{\nu} k_{\mu} - (k \cdot k') - i \varepsilon_{\mu \nu \alpha \beta} k'_{\beta} k_{\alpha} \right]
\]

\[
W_{A}^{\mu \nu} = \sum_{X} \langle 0 | J_{A}^{\mu \dagger} | X \rangle \langle X | J_{A}^{\nu} | 0 \rangle \delta^{(4)}(p_0 + q - p_X)
\]

- \( W_{A}^{\mu \nu} \) is the nuclear hadronic tensor
- we need a set of simplifying assumptions to compute \( W_{A}^{\mu \nu} \): the Impulse Approximation

target nucleus seen as a collection of individual nucleons: \( J_{\mu} \rightarrow \sum_{i} \tilde{J}_{\mu}^{i} \)
The Relativistic Fermi Gas model (RFG)

\[ W_A^{\mu\nu} \text{ same structure of the free one } W^{\mu\nu} \text{ in terms of form factors} \]

\[ W_A^{\mu\nu} = \int d\mathbf{p} f(\mathbf{p}, \mathbf{q}) W^{\mu\nu} \]

\[ f(\mathbf{p}, \mathbf{q}) = \left( v_{rel} \right)^{-1} \cdot \frac{n_i(\mathbf{p})}{\text{momentum distribution}} \cdot \frac{[1 - n_f(\mathbf{p} + \mathbf{q})]}{\text{unoccupation probability}} \]

\[ \begin{cases} v_{rel} = f(E_p) \quad \text{where} \quad E_p = \sqrt{p^2 + m_n^2} - \epsilon_b \\ n_i(\mathbf{p}) = \theta(p_F - \mathbf{p}) \\ 1 - n_f(\mathbf{p} + \mathbf{q}) = \theta(\mathbf{p} + \mathbf{q} - p_F) \end{cases} \]

The model depends on two parameters:

- \( \epsilon_b \) = effective binding energy
- \( p_F \) = Fermi momentum

| Nucleus | \( p_F \) (MeV) | \( \epsilon_b \) (MeV) |
|---------|------------------|------------------|
| \( ^{12}C \) | 221              | 25.6             |
| \( ^{16}O \) | 225              | 26.6             |
The Spectral Function Approach

Benhar et al., Phys.Rev.D72:053005,2005

\[ \sum_i \langle 0 | J^\mu | X \rangle \sim \langle 0 | R, p_R; N, -p_R \rangle \sum_i \langle -p_R, N | j^\mu_i | x, p' \rangle \]

\[ \frac{d^2 \sigma_{IA}}{d\Omega dE_l} = \int d^3p \frac{d^2 \sigma_{\text{elem}}}{d\Omega dE_l} \]

\[ P(E, p) \sim \sum_R |\langle 0 | R, -p; N, p \rangle|^2 \]

- \( P(p, E) \) is the \textbf{target spectral function}: probability distribution of finding a nucleon with momentum \( p \) and removal energy \( E \) in the target nucleus.

\textbf{it encodes all the informations about the nuclear dynamics}
overwhelming evidence from electron scattering that the energy-momentum distribution of nucleons in the nucleus is quite different from that predicted by Fermi gas

the most important feature is the presence of strong nucleon-nucleon (NN) correlations (virtual scattering processes leading to the excitation of the participating nucleons to states of energy larger than the Fermi energy)

**spectral function extends to** $|\mathbf{p}| \gg p_F$ and $E \gg \varepsilon$

momentum distribution

$$n(p) = \int dE \ P(p, E)$$
The Random Phase Approximation (RPA)

The Random Phase Approximation (RPA) model based on
M. Martini, M. Ericson, G. Chanfray, J. Marteau, Phys. Rev. C81, 045502 (2010)
M. Martini, M. Ericson, G. Chanfray, J. Marteau, Phys. Rev. C80, 065501 (2009).

\[
\frac{d^2\sigma_{IA}}{d\Omega dE_l} \propto \sum_i K_i R_i
\]

- $K_i$ = kinematical factors
- $R_i$ = response functions,

\[
R(\omega, q) = -\frac{\mathcal{V}}{\pi} \text{Im}[\Pi(\omega, q, q)].
\]

To lowest order the QE cross section is given by the terms in $R_{NN}^{NN}$ $[R_{\tau\tau}^{NN}$ (isovector interaction), $R_{\sigma\tau}^{NN}$ (isospin spin-transverse interaction)]

![Diagram of quantum electrodynamics](image)

Lowest-order contribution from $R_{NN}^{NN}$, $R_{N\Delta}^{NN}$ and $R_{\Delta\Delta}^{NN}$.
Wiggly lines represent the external probe, solid lines correspond to the propagation of a nucleon (or a hole), double lines to the propagation of a $\Delta$ and dashed lines to an effective interaction between nucleons and/or $\Delta$s.

Dotted lines show which particles are placed on-shell.
Specific models

**The Random Phase Approximation (RPA 2p-2h)**

- large contribution from two-body current?!

  M. Martini, M. Ericson, G. Chanfray, J. Marteau, Phys. Rev. C80, 065501 (2009) [arXiv:0910.2622 [nucl-th]].

- wiggly lines represent the external probe,
- solid lines correspond to the propagation of a nucleon (or a hole),
- double lines to the propagation of a $\Delta$
- dashed lines to an effective interaction between nucleons and/or $\Delta$s.

Dotted lines show which particles are placed on-shell
The $\nu$-nucleus cross sections ($\nu A \rightarrow \mu X$)

- some of the *qualitative* impacts of several nuclear models on the $\nu$ observables can already be understood at the "cross section" level
- however the *quantitative* differences should be carefully evaluated

- FG overestimates the $\bar{\nu}$ xsection over the whole QE energy regime
- $m_A \sim 1$ GeV in all models
The $\beta$Beam concept

- the value of the Lorentz boost factor $\gamma$ and the source-detector distance $L$ determine the neutrino spectra
- interested in $\nu_e \rightarrow \nu_\mu$ oscillation
- leading terms in $P_{\nu_e \nu_\mu}$ depend on $\theta_{13}^2$ and $\theta_{13} \cdot \sin \delta$

Here we focus on $(\gamma, L) = (100, 130 \text{ K}m)$

- $(\nu-\bar{\nu})$ spectra very similar
- QE events
- very low backgrounds (from NC $1\pi^0+\text{atmospheric } \mu$)

- use this $\beta$Beam as a prototype!
The CP discovery potential

Definition

for any $\theta_{13}$ is the ensemble of true values of $\delta_{CP}$ for which the $3\sigma$ CL do not touch $\delta_{CP} = 0, \pi, \pm \pi$

1- the FG performs too well compared with the other two models
2- at $\delta \sim \pm \pi/2$ the largest discrepancy: $\sim 2$ better than other computations!
3- notice that RPA $- 2p2h$ does not perform better than FG.
**The CP fraction**

**Definition**

just the total number of “good” $\delta$s divided by $2\pi$
The sensitivity to $\theta_{13}$

- same analysis for $\theta_{13}$

**Definition**

for any $\delta_{CP}$, true values of $\theta_{13}$ for which the $3\sigma$ CL do not touch $\theta_{13} = 0$

- a bit less evident than before: something of $\mathcal{O}(60\%)$
A combined analysis

What about a simultaneous fit to $\theta_{13}$ and $\delta_{CP}$?

To see the impact of various models:

- we first fix some true value $(\theta_{13}, \delta_{CP}) = (0.9^\circ, 30^\circ)$
- then we study the capability of the facility to measure them

general problem of this kind of analysis:

presence of fake points $(\bar{\theta}_{13}, \bar{\delta})$ where

$P_{e\mu}^{\pm}(\bar{\theta}_{13}, \bar{\delta}) = $ to the true transition probability

- no fake solutions for FG

This would point to a $\beta$-beam as a good facility to measure $\theta_{13}$ and $\delta$
Generalizing the previous results

- **same effects with** $^{56}Fe$ **target**
  - blue: FG, red: SF

- **mild dependence on the axial mass**
  - SF with blue: $m_A=1.2$ GeV, red: $m_A=1.1$ GeV
  - black: $m_A=1.0$ GeV

![Graph showing numerical results](image-url)
Summary and perspectives

Summary

- We studied the impact of nuclear effects on the determination of various neutrino parameters.
- In particular, we compare the FG results (widely adopted in MonteCarlo codes) with the SF and RPA approaches.
- The different behaviour of the cross sections translates into overestimated sensitivity to $\theta_{13}$ and $\delta_{CP}$.
- Although we focused on Oxygen, the same pattern is observed for other nuclear targets.
Summary and perspectives

- **Perspectives**

- It could be necessary to implement more realistic nuclear effects in MC codes (plot from http://regie2.phys.uregina.ca/neutrino/qel.html)

- what happens for precision measurements at other facilities?
- what happens at much higher energies (Neutrino Factories)?
\[
\frac{d^2 \sigma_{\text{elem}}}{d\Omega dE_l} = \frac{G_F^2 V_{ud}^2}{32 \pi^2} \frac{|k'|}{|k|} \frac{1}{4 E_p E_{|p+q|}} L_{\mu\nu} W^{\mu\nu}
\]

- The hadronic tensor is decomposed in structure functions as usual

\[
W^{\mu\nu} = -g^{\mu\nu} W_1 + \tilde{p}^\mu \tilde{p}^\nu \frac{W_2}{m_N^2} + i \varepsilon_{\mu\nu\alpha\beta} \tilde{q}^\alpha \tilde{p}^\beta \frac{W_3}{m_N^2} + \tilde{q}^\mu \tilde{q}^\nu \frac{W_4}{m_N^2} +
\]

\[
(\tilde{p}^\mu \tilde{q}^\nu + \tilde{p}^\nu \tilde{q}^\mu) \frac{W_5}{m_N^2}
\]

- the formalism can be applied to both elastic and anelastic processes specifying the form of the structure functions \(W_i\)
details in the SF computation

- using the no-FSI approximation

\[
\sum_X |X\rangle\langle X| \rightarrow \sum_x \int d^3 p_x |x, p_x\rangle\langle p_x, x| \times \sum_{\mathcal{R}} d^3 p_{\mathcal{R}} |\mathcal{R}, p_{\mathcal{R}}\rangle\langle p_{\mathcal{R}}, \mathcal{R}| 
\]

- complete set of free nucleon states, satisfying

\[
\int d^3 p |N, p\rangle\langle p, N| = I
\]

- hadronic matrix element

\[
\langle 0 | J^\mu | X \rangle = \frac{m}{\sqrt{p_{\mathcal{R}}^2 + m^2}} \langle 0 | \mathcal{R}, p_{\mathcal{R}}; N, -p_{\mathcal{R}} \rangle \times \sum_i \langle -p_{\mathcal{R}}, N | j_i^\mu | x, p_x \rangle
\]
The calculation of $P(p, E)$ for any $A$ is a complicated task

For nuclei from Carbon to Gold has been modeled using the Local Densitity Approximation (LDA)

$$P_{LDA}(p, E) = P_{MF}(p, E) + P_{corr}(p, E)$$

- Measured contribution corresponding to low momentum nucleons, occupying the shell model states
- High momentum nucleons calculable using the result of uniform nuclear matter "recomputed" for a finite nucleus of mass number $A$