Error Rate and Power Allocation Analysis of Regenerative Networks under Generalized Fading Conditions

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Abstract

Cooperative communication has been shown to provide significant increase of transmission reliability and network capacity while expanding coverage in cellular networks. The present work is devoted to the investigation of the end-to-end performance and power allocation of a maximum-ratio-combining based regenerative multi-relay cooperative network over non-homogeneous scattering environment, which is the case in realistic wireless communication scenarios. Novel analytic expressions are derived for the end-to-end symbol-error-rate of both $M$-ary Phase-Shift Keying and $M$-ary Quadrature Amplitude Modulation over independent and non-identically distributed generalized fading channels. The offered results are expressed in closed-form involving the Lauricella function and can be readily evaluated with the aid of a proposed computational algorithm. Simple expressions are also

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derived for the corresponding symbol-error-rate at asymptotically high signal-to-noise ratios. The derived expressions are corroborated with respective results from computer simulations and are subsequently employed in formulating a power optimization problem that enhances the system performance under total power constraints within the multi-relay cooperative system. Furthermore, it is shown that optimum power allocation provides substantial performance gains over equal power allocation, particularly, when the source-relay and relay-destination paths are highly unbalanced.

Index Terms

Asymptotic analysis, decode-and-forward, digital modulations, generalized fading channels, maximum-ratio combining, multi-relay systems, optimum power allocation.

I. INTRODUCTION

Cooperative transmission methods have attracted significant interest over the past decade due to their applicability in size, power, hardware and price constrained devices such as cellular mobile devices, wireless sensors, ad-hoc networks and military communications [1]–[9]. Such systems exploit the broadcast nature and the inherent spatial diversity of wireless paths and are typically distinguished between regenerative (decode-and-forward) or non-regenerative (amplify-and-forward) relaying schemes. In general, the digital processing nature of regenerative relaying is considered more efficient than non-regenerative relaying, since the latter typically requires costly RF transceivers in order to avoid forwarding a noisy version of the signal [10]–[16].

The performance of cooperative systems can be substantially improved by optimum allocation of the limited overall power to the source and relays of the network in order to minimize the overall energy consumption for given end-to-end performance specifications. Among others, this can be efficiently achieved by accurately accounting for the detrimental effects of multipath fading [17]–[38] and the references therein. Based on this, the authors in [39] derived upper and lower bounds for the outage probability (OP) of multi-relay decode-and-forward (DF) networks over independent but non-identically distributed (i.n.i.d) Nakagami\(\text{--}\)m fading channels. In the same context, the authors in [40] analyzed the symbol error rate (SER) and OP of DF systems with relay selection over i.n.i.d Nakagami\(\text{--}\)m fading channels, with integer values of \(m\), whereas a comprehensive analytical framework for a dual-hop multi-antenna DF system under multipath fading was derived in [41]. Likewise, the performance of DF systems over different fading environments was investigated in [42]–[45] whereas analysis for the SER of
dual-hop DF relaying for $M$-ary phase-shift keying ($M$–PSK) and $M$-ary quadrature amplitude modulation ($M$–QAM) over Nakagami–$m$ fading channels was reported in [46]. In addition, optimum power allocation (OPA) in dual-hop regenerative relaying with respect to pre-defined thresholds for the SER and OP was analyzed in [46] and [47], respectively. This problem was also addressed in [48] and [49] for multi-node DF relaying based on asymptotic SER and for a given network topology over Rayleigh fading channels, respectively. Finally, the authors in [50] proposed power allocation schemes for the case of multi-relay DF communications in the high-SNR regime over Nakagami–$m$ fading channels.

Nevertheless, all reported investigations have been carried out in the context of either asymptotic or dual-hop scenarios as well as considering only Rayleigh or Nakagami–$m$ fading channels, which is mainly due to the presence of cumbersome integrals that involve combinations of elementary and special functions [51]–[57] and the references therein. However, it is recalled that these fading models are based on the underlying concept of homogeneous scattering environments, which is not practically realistic since surfaces in most radio propagation environments are spatially correlated [58]. This issue was addressed in [59] by proposing the $\eta–\mu$ distribution, which is a generalized fading model that has been shown to provide particularly accurate fitting to realistic measurement results, while it includes as special cases the well known Rayleigh, Nakagami–$m$ and Hoyt distributions [59]–[62]. Based on this, several contributions have been devoted to the analysis of various communication scenarios over generalized fading channels that follow the $\eta – \mu$ distribution, see, e.g., [62]–[68] and the references therein. Motivated by this, the present work is devoted to the evaluation of the end-to-end SER in regenerative cooperative communication systems with multiple relays for $M$–PSK and $M$–QAM constellations over generalized fading channels as well as the corresponding OPA analysis. Specifically, the contributions of this work are summarized below:

- Exact closed-form expressions are derived for the end-to-end SER of $M$–PSK and $M$–QAM based multi-relay regenerative networks over generalized multipath fading environments for both i.n.i.d and i.i.d scenarios.
- Simple asymptotic expressions are derived for the above scenarios for high SNR values.
- The corresponding amount of fading is derived for quantifying the respective fading severity.
- Optimal power allocation based on the convexity of the derived asymptotic expressions is
formulated, to minimize the corresponding SER under sum-power constraint in all nodes.

- The derived expressions are employed in evaluating the performance of the considered system and extracting useful insights.
- It is shown that a 3dB gain is achieved by OPA for even a small number of relays.
- It is shown that post-Rayleigh fading conditions result to an improved performance by up to 4dB compared to communications over severe fading channels.
- It is shown that a maximum gain of about 21dB occurs, compared to ordinary direct communication, even if only few nodes are employed in non-severe fading conditions. This renders the resource constrained communication system a meaningful alternative for increasing the quality of service of demanding emerging wireless systems.
- A simple algorithm is proposed for the computation of the generalized Lauricella function.

The reminder of the present paper is organized as follows: Section II revisits the considered system and channel models. The exact and asymptotic SER expressions for $M$–QAM and $M$–PSK constellations over generalized multipath fading channels are derived in Section III. The optimum power allocation scheme based on sum-power constraint is provided in Section IV while Section V presents the corresponding numerical results along with related discussions and insights. Finally, closing remarks are given in Section VI.

II. SYSTEM AND CHANNEL MODELS

A. System Model

We consider a multi-node cooperative radio access system consisting of a source node $S$, intermediate relay nodes $R_k$, with $k = \{1, 2, \cdots, K\}$, and a destination node $D$, as depicted in Fig.1. Each node in the system is equipped with a single antenna while a half-duplex decode-and-forward protocol is adopted. Furthermore, in order to avoid inter-relay interference the nodes
in the system transmit signals through orthogonal channels. In phase I, the source broadcasts the signal to the destination and to all relay nodes in the network which yields

$$y_{S,D} = \sqrt{P_0\alpha_{S,D}} x + n_{S,D}$$

(1)

and

$$y_{S,R_k} = \sqrt{P_0\alpha_{S,R_k}} x + n_{S,R_k}$$

(2)

where $P_0$ is the source transmit power, $x$ is the transmitted symbol with normalized unit energy in the first transmission phase, $\alpha_{S,D}$ and $\alpha_{S,R_k}$ are the complex fading coefficients from the source to the destination and from the source to the $k^{th}$ relay, respectively, whereas $n_{S,D}$ and $n_{S,R_k}$ represent the corresponding additive-white-Gaussian noise (AWGN) with zero mean and variance $N_0$. In the next time slot, if the $k^{th}$ relay decodes correctly, then it forwards the decoded and re-encoded signal to the destination with power $P_{R_k} = P_{R_k}$; otherwise, $P_{R_k} = 0$ yielding

$$y_{R_k,D} = \sqrt{P_{R_k}\alpha_{R_k,D}} x + n_{R_k,D}$$

(3)

where $\alpha_{R_k,D}$ denotes the complex fading coefficient from the $k^{th}$ relay to the destination and $n_{R_k,D}$ is the corresponding AWGN. It is also assumed that each path experiences narrowband multipath fading that follows the $\eta - \mu$ distribution and that MRC diversity scheme is employed at the destination. As a result, the combined output received signal is expressed as follows

$$y_D = w_0 y_{S,D} + \sum_{k=1}^{K} w_k y_{R_k,D}$$

(4)

where $w_0 = \sqrt{P_0\alpha_{S,D}^*/N_0}$ and $w_k = \sqrt{P_{R_k}\alpha_{R_k,D}^*/N_0}$ denote the optimal MRC coefficients for $y_{S,D}$ and $y_{R_k,D}$, respectively with $(\cdot)^*$ representing the complex conjugate operator.

**B. Generalized Multipath Fading Channels**

It is recalled that $\eta - \mu$ distribution has been shown to account accurately for small-scale variations of the signal in non-line-of-sight communication scenarios. This fading model is described by the two named parameters, $\eta$ and $\mu$, and it is valid for two different formats that correspond to two physical models [59]. The probability density function (PDF) of the instantaneous SNR $\gamma = |\alpha|^2 P/N_0$ is given by [59], [66]

$$f_\gamma(\gamma) = \frac{2\sqrt{\pi} \mu^{\mu+\frac{1}{2}} h^{\mu\gamma-\frac{1}{2}}}{\Gamma(\mu)H^{\mu-\frac{1}{2}} \pi^{\mu+\frac{1}{2}}} \exp\left(-\frac{2\mu\gamma h}{\pi}\right) I_{\mu-\frac{1}{2}}\left(\frac{2\mu H\gamma}{\pi}\right)$$

(5)
TABLE I

RELATION BETWEEN $\eta-\mu$ DISTRIBUTION AND OTHER COMMON FADING DISTRIBUTIONS.

| Fading Distribution | Format-1 | Format-2 |
|---------------------|----------|----------|
| $\eta - \mu$        | $h = (1 + \eta^{-1} + \eta)/4$, $H = (\eta^{-1} - \eta)/4$ | $h = 1/(1 - \eta^2)$, $H = \eta/(1 - \eta^2)$ |
| Nakagami            | $\mu = m$, $\eta \to 0$ or $\eta \to \infty$ | $\eta \to \pm 1$ |
|                     | $\mu = m/2$, $\eta \to 1$ | $\eta \to 0$ |
| Nakagami–q (Hoyt)   | $\mu = 0.5$, $\eta = q^2$ | $q^2 = (1 - \eta)/(1 + \eta)$ |
| Rayleigh            | $\mu = 0.5$, $\eta = 1$ | $\mu = 0.5$, $\eta = 0$ |

where $\gamma = E(\gamma) = (P/N_0)\Omega$ is the average SNR per symbol, with $E(\cdot)$ denoting statistical expectation, and $\Omega = E(|\alpha|^2)$ denotes the channel variance. Furthermore, $\Gamma(\cdot)$ and $I_{\nu}(\cdot)$ denote the Euler gamma function and the modified Bessel function of the first kind, respectively [69]. The parameters $h$ and $H$ are given in terms of $\eta$ in two formats, as depicted in Table I along with the fading distributions that are included in $\eta-\mu$ as special cases. In terms of physical interpretation $\eta$ denotes the scattered-waves power ratio between the in-phase and quadrature components of the scattered waves in each multipath cluster in Format-1 and the correlation coefficient between the in-phase and quadrature components of each multipath cluster in Format-2. Likewise, $\mu = E^2(\gamma)(1 + (H/h)^2)/2V(\gamma)$ is related to multipath clustering in both formats, with $V(\cdot)$ denoting variance operation, respectively [59].

III. EXACT END-TO-END SYMBOL ERROR RATE ANALYSIS

The end-to-end SER for the considered cooperative system can be expressed as [48], [50]

$$P_{SER}^D = \sum_{z=0}^{2^K-1} P(e|A = C_z)P(A = C_z)$$

(6)

where the binary vector $A = [A(1), A(2), A(3), ..., A(K)]$ of dimension $(1 \times K)$ denotes the state of the relay nodes in the system, with $A(k)$ taking the binary values of 1 and 0 for successful and unsuccessful decoding, respectively. For the case of statistically independent channels the joint probability of the possible state outcomes can be represented as follows:

$$P(A) = P(A(1))P(A(2))P(A(3))\cdots P(A(K)) = \prod_{k=1}^{K} P(A(k)).$$

(7)

Furthermore, $C_z = [C(1), C(2), C(3), \cdots, C(K)]$ denotes different possible decoding combinations of the relays with $z \in \{0, 2^K - 1\}$, where $C(k)$ takes the value of either 0 or 1. The conditional error probability $P(e|A = C_z)$ is the error probability conditioned on particular
decoding results at relays while $P(A = C_z)$ is the corresponding joint probability of the decoding outcomes. Based on the MRC method, the instantaneous SNR at the destination for given decoding combination, $C_z$, can be expressed as

$$\gamma_{MRC}(C_z) = |\alpha_{S,D}|^2 \frac{P_0}{N_0} + \sum_{k=1}^{K} C(k)|\alpha_{R_k,D}|^2 \frac{P_{R_k}}{N_0}. \quad (8)$$

It is also recalled that the MGF for independent fading channels in DF scheme is given by [72]

$$M_{\gamma_{MRC}}(s) = M_{\gamma_{S,D}}(s)^{K} \prod_{k=1}^{K} C(k) M_{\gamma_{R_k,D}}(s) \quad (9)$$

which in the present analysis can be expressed according to [66] eq. (6), namely

$$M_{\gamma_{\eta-\mu}} \left( \frac{g_{PSK}}{\sin^2 \theta} \right) = \left( \frac{4\mu^2 h_{PSK}}{(2(h - H)\mu + \frac{g_{PSK}}{\sin^2 \theta})(2(h + H)\mu + \frac{g_{PSK}}{\sin^2 \theta})} \right)^{\mu} \quad (10)$$

It is noted that the above expression is particularly useful in the subsequent SER analysis.

A. End-to-End SER for $M-PSK$ Constellations

1) The case of i.n.i.d $\eta - \mu$ fading channels: The end-to-end error probability for $M-PSK$ constellations over individual $\eta - \mu$ fading link when $\eta, \mu$ and $\gamma$ in each path are not necessarily equal can be expressed as [70] eq. (5.78)]

$$\bar{P}_{M-PSK} = \int_{0}^{(M-1)\pi/M} \int_{0}^{\pi/2} \frac{p_{\gamma}(\gamma)}{\pi e^{\frac{g_{PSK}}{\sin^2 \theta}}} d\gamma d\theta = \frac{1}{\pi} \int_{0}^{\pi/2} M_{\gamma} \left( \frac{\sin^2 \theta}{\sin^2 \theta} \right) d\theta + \frac{1}{\pi} \int_{0}^{(M-1)\pi/M} M_{\gamma} \left( \frac{g_{PSK}}{\sin^2 \theta} \right) d\theta \approx \bar{I}_1 \quad (11)$$

where $g_{PSK} = \sin^2(\pi/M)$ [66]. In order to evaluate (6), we firstly need to determine the error probability for decoding at the destination terminal, using MRC, under given decoding outcomes at nodes i.e., for a given $C_z$ [71]. To this end and based on the MGF approach it follows that

$$P(e|A = C_z) = \frac{1}{\pi} \int_{0}^{\pi/2} \left( \frac{4\mu^2 h_{PSK}(2(h_{S,D} + H_{S,D})\mu_{S,D} + \frac{g_{PSK}}{\sin^2 \theta} \gamma_{S,D})^{-1}}{(2(h_{S,D} - H_{S,D})\mu_{S,D} + \frac{g_{PSK}}{\sin^2 \theta} \gamma_{S,D})} \right)^{\mu_{S,D}}$$

$$\times \prod_{k=1}^{K} C(k) \left( \frac{4\mu^2 h_{R_k,D}(2(h_{R_k,D} - H_{R_k,D})\mu_{R_k,D} + \frac{g_{PSK}}{\sin^2 \theta} \gamma_{R_k,D})^{-1}}{(2(h_{R_k,D} + H_{R_k,D})\mu_{R_k,D} + \frac{g_{PSK}}{\sin^2 \theta} \gamma_{R_k,D})} \right)^{\mu_{R_k,D}} d\theta$$

$$+ \frac{1}{\pi} \int_{\pi/2}^{(M-1)\pi/M} \left( \frac{4\mu^2 h_{PSK}(2(h_{S,D} + H_{S,D})\mu_{S,D} + \frac{g_{PSK}}{\sin^2 \theta} \gamma_{S,D})^{-1}}{(2(h_{S,D} - H_{S,D})\mu_{S,D} + \frac{g_{PSK}}{\sin^2 \theta} \gamma_{S,D})} \right)^{\mu_{S,D}}$$

$$\times \prod_{k=1}^{K} C(k) \left( \frac{4\mu^2 h_{R_k,D}(2(h_{R_k,D} - H_{R_k,D})\mu_{R_k,D} + \frac{g_{PSK}}{\sin^2 \theta} \gamma_{R_k,D})^{-1}}{(2(h_{R_k,D} + H_{R_k,D})\mu_{R_k,D} + \frac{g_{PSK}}{\sin^2 \theta} \gamma_{R_k,D})} \right)^{\mu_{R_k,D}} d\theta. \quad (12)$$
Evidently, the derivation of an analytic solution for \( \mathcal{I}_1 \) is subject to analytic evaluation of the integrals \( \mathcal{I}_1 \) and \( \mathcal{I}_{11} \) in closed-form. To this end, for the case of non-identical fading parameters, i.e., \( \mu_{S,D} \neq \mu_{R_1,D} \neq \cdots \neq \mu_{R_K,D}, \eta_{S,D} \neq \eta_{R_1,D} \neq \cdots \neq \eta_{R_K,D} \) and \( \gamma_{S,D} \neq \gamma_{R_1,D} \neq \cdots \neq \gamma_{R_K,D} \), the \( \mathcal{I}_1 \) term can be alternatively expressed as follows

\[
\mathcal{I}_1 = \frac{1}{\pi} \int_0^{\pi/2} \frac{1}{\left(1 + \frac{A_1}{\sin^2 \theta}\right)^{\mu_{S,D}} \left(1 + \frac{A_2}{\sin^2 \theta}\right)^{\mu_{S,D}}} \prod_{k=1}^{K} \frac{C(k) \, d\theta}{\left(1 + \frac{B_{1k}}{\sin^2 \theta}\right)^{\mu_{R_k,D}} \left(1 + \frac{B_{2k}}{\sin^2 \theta}\right)^{\mu_{R_k,D}}}
\]

where

\[
\{A_1\} = \frac{\gamma_{S,D} \alpha_{PSK}}{2(h_{S,D} \{\mp\} H_{S,D})^{\mu_{S,D}}}
\]

and

\[
\{B_{1k}\} = \frac{\gamma_{R_k,D} \alpha_{PSK}}{2(h_{R_k,D} \{\mp\} H_{R_k,D})^{\mu_{R_k,D}}}
\]

By also setting \( u = \sin^2(\theta) \) and carrying out tedious but basic algebraic manipulations yields

\[
\mathcal{I}_1 = \frac{\beta_{MRC}(g_{PSK})}{2\pi} \int_0^1 \frac{(1-u)^{-\frac{1}{2} \sum_{k=1}^K C(k) \mu_{R_k,D}}}{{(1 + \frac{u}{A_1})^{\mu_{S,D}} (1 + \frac{u}{A_2})^{\mu_{S,D}}}} \prod_{k=1}^{K} \frac{C(k) \, du}{(1 + \frac{u}{B_{1k}})^{\mu_{R_k,D}} (1 + \frac{u}{B_{2k}})^{\mu_{R_k,D}}}
\]

where

\[
\beta_{MRC}(g_{PSK}) = \left(4\mu_{S,D}^2 (h_{S,D}^2 - H_{S,D}^2)\right)^{\mu_{S,D}} \prod_{k=1}^{K} \left(4\mu_{R_k,D}^2 (h_{R_k,D}^2 - H_{R_k,D}^2)\right)^{\mu_{R_k,D}}
\]

Importantly, eq. (16) can be expressed in closed-form in terms of \( \eta \) eq. (7.2.4.57)), yielding

\[
\mathcal{I}_1 = \frac{\beta_{MRC}(g_{PSK}) \Gamma(2\mu_{S,D} + 2 \sum_{k=1}^K C(k) \mu_{R_k,D} + \frac{1}{2})}{2\sqrt{\pi} \Gamma(2\mu_{S,D} + 2 \sum_{k=1}^K C(k) \mu_{R_k,D} + 1)}
\times F^{(2K+2)}_D \left(\begin{array}{c}
2\mu_{S,D} + 2 \sum_{k=1}^K C(k) \mu_{R_k,D} + \frac{1}{2}; \mu_{S,D}, \mu_{S,D}, \mu_{R_1,D}, \cdots, \mu_{R_K,D}, \mu_{R_1,D}, \cdots, \mu_{R_K,D}
\end{array}\right)
\]

where \( F_D(n, \cdot) \) denotes the generalized Lauricella hypergeometric function of \( n \) variables [73].

In the same context, for the \( \mathcal{I}_{11} \) integral we set \( u = \cos^2(\theta) / \cos^2(\pi/M) \) in (12) yielding

\[
\mathcal{I}_{11} = \frac{M_{MRC}(g_{PSK}) \cos(\pi/M)}{2\pi}
\times \int_0^1 \frac{u^{-\frac{1}{2} \sum_{k=1}^K C(k) \mu_{R_k,D} - \frac{1}{2}}; \mu_{S,D}, \mu_{S,D}, \mu_{R_1,D}, \cdots, \mu_{R_K,D}, \mu_{R_1,D}, \cdots, \mu_{R_K,D}}{(1 + \frac{\cos^2(\pi/M) u}{A_1})^{\mu_{S,D}} (1 - \frac{\cos^2(\pi/M) u}{A_2})^{\mu_{S,D}}} \prod_{k=1}^{K} \frac{C(k) \left(1 - \frac{\cos^2(\pi/M) u}{1 + B_{1k}}\right)^{-\mu_{R_k,D}}}{\left(1 - \frac{\cos^2(\pi/M) u}{1 + B_{2k}}\right)^{-\mu_{R_k,D}}} \, du.
\]
Evidently, the above integral can be also expressed in closed-form in terms of the $F_D^{(n)}(\cdot)$ function; therefore, by performing the necessary change of variables and substituting in \([19]\), one obtains
\[
\mathcal{I}_{11} = \frac{M_{\gamma\text{MRC}}(g_{\text{PSK}})}{\pi} \times F_D^{(2K+3)} \left( \frac{1}{2}; \mu_{S,D}, \mu_{S,D}, \mu_{R_1,D}, \cdots, \mu_{R_K,D}, \mu_{R_1,D}, \cdots, \mu_{R_K,D}, \frac{1}{2} - 2\mu_{S,D} + 2 \sum_{k=1}^{K} C(k)\mu_{R_k,D} ; \frac{3}{2} \right);
\]
\[
\cos^2(\pi/M) \cos^2(\pi/M) \cos^2(\pi/M) \cdots \cos^2(\pi/M) \cos^2(\pi/M) \cos^2(\pi/M) \cdots \cos^2(\pi/M) \cos^2(\pi/M) .
\]

It is noted here that the $F_D^{(n)}(\cdot)$ function has been studied extensively over the past decades. Nevertheless, despite its importance it is not unfortunately included as built-in function in popular software packages such as MATLAB, MATHEMATICA and MAPLE. Based on this, a simple MATLAB algorithm for computing this function straightforwardly is proposed in Appendix I.

2) The case of i.i.d $\eta-\mu$ fading channels: For the special case of identical relay to destination paths i.e., $\mu_{R_1,D} = \cdots = \mu_{R_K,D} = \mu_{R,D}, \eta_{R_1,D} = \cdots = \eta_{R_K,D} = \eta_{R,D}, \overline{\gamma}_{R_1,D} = \cdots = \overline{\gamma}_{R_K,D} = \overline{\gamma}_{R,D}$ and thus, $B_{11} = \cdots = B_{1K} = B_1$, and $B_{21} = \cdots = B_{2K} = B_2$, equations \([18]\) and \([20]\) can be readily simplified to the following representations
\[
\mathcal{I}_{1\text{ii},d} = \frac{\beta_{\gamma\text{MRC}}(g_{\text{PSK}}) \Gamma \left( 2\mu_{S,D} + 2\mu_{R,D} \sum_{k=1}^{K} C(k) + \frac{1}{2} \right) F_D^{(4)} \left( 2\mu_{S,D} + 2\mu_{R,D} \sum_{k=1}^{K} C(k) + 1 ; \mu_{S,D}, \mu_{S,D}, K\mu_{R,D}, K\mu_{R,D} ; 2\mu_{S,D} + 2\mu_{R,D} \sum_{k=1}^{K} C(k) + 1, -\frac{1}{A_1}, -\frac{1}{A_2}, -\frac{1}{B_1}, -\frac{1}{B_2} \right)}{2\sqrt{\pi} \Gamma \left( 2\mu_{S,D} + 2\mu_{R,D} \sum_{k=1}^{K} C(k) + 1 \right)} \]
\[
\mu_{S,D}, \mu_{S,D}, K\mu_{R,D}, K\mu_{R,D} ; 2\mu_{S,D} + 2\mu_{R,D} \sum_{k=1}^{K} C(k) + 1, -\frac{1}{A_1}, -\frac{1}{A_2}, -\frac{1}{B_1}, -\frac{1}{B_2} \right)
\]
\[
\mathcal{I}_{1\text{ii},d} = \frac{M_{\gamma\text{MRC}}(g_{\text{PSK}}) \Gamma \left( 3; \mu_{S,D}, \mu_{S,D}, K\mu_{R,D}, K\mu_{R,D} ; 2\mu_{S,D} - 2\mu_{R,D} \sum_{k=1}^{K} C(k) \right)}{2\sqrt{\pi} \Gamma \left( 3; \mu_{S,D}, \mu_{S,D}, K\mu_{R,D}, K\mu_{R,D} ; 2\mu_{S,D} - 2\mu_{R,D} \sum_{k=1}^{K} C(k) \right)} \]
\[
\frac{3}{2}; \frac{\cos^2(\pi/M)}{1 + A_1}, \frac{\cos^2(\pi/M)}{1 + A_2}, \frac{\cos^2(\pi/M)}{1 + B_1}, \frac{\cos^2(\pi/M)}{1 + B_2} \right) .
\]

respectively. As a result, with the aid of the derived expressions for $\mathcal{I}_1$ and $\mathcal{I}_{11}$, the corresponding error probability for $M-$PSK constellations can be determined by $P(\epsilon | A = C_z) = \mathcal{I}_1 + \mathcal{I}_{11}$.

It is recalled that the derivation of the overall SER also requires the determination of the decoding probability of the relay nodes $P(A = C_z)$. This is in fact a direct product of the terms $P(\tilde{\gamma}_{S,R_k}) = P(A(k) = C(k) = 0)$ i.e. decoding error at the relays and $(1 - P(\tilde{\gamma}_{S,R_k})) = \cdots \cdots $
$P(A(k) = C(k) = 1)$ i.e. successful decoding at the relays, which, as already mentioned, is a pre-requisite for forwarding re-encoded signals to the destination. Importantly, this can be also determined in closed-form with the aid of the commonly used MGF approach, namely

$$P(\gamma_{S,R_k}) = \frac{1}{\pi} \int_0^{\pi/2} M_{\gamma_{S,R_k}} \left( \frac{g_{PSK}}{\sin^2 \theta} \right) d\theta + \frac{1}{\pi} \int_{\pi/2}^{(M-1)\pi} M_{\gamma_{S,R_k}} \left( \frac{g_{PSK}}{\sin^2 \theta} \right) d\theta \quad (23)$$

which with the aid of (10) can be equivalently re-written as follows

$$P(A(k) = C(k) = 0) = \frac{1}{\pi} \int_0^{\pi/2} \left( \frac{4\mu_{S,R_k}^2 h_{S,R_k} (2(h_{S,R_k} + H_{S,R_k})\mu_{S,R_k} + \frac{g_{PSK}}{\sin^2 \theta} \gamma_{S,R_k})^{-1}}{(2(h_{S,R_k} - H_{S,R_k})\mu_{S,R_k} + \frac{g_{PSK}}{\sin^2 \theta} \gamma_{S,R_k})} \right) d\theta + \frac{1}{\pi} \int_{\pi/2}^{(M-1)\pi} \left( \frac{4\mu_{S,R_k}^2 h_{S,R_k} (2(h_{S,R_k} + H_{S,R_k})\mu_{S,R_k} + \frac{g_{PSK}}{\sin^2 \theta} \gamma_{S,R_k})^{-1}}{(2(h_{S,R_k} - H_{S,R_k})\mu_{S,R_k} + \frac{g_{PSK}}{\sin^2 \theta} \gamma_{S,R_k})} \right) d\theta \quad (24)$$

Notably, the integrals in (24) have similar algebraic form to $I_1$ and $I_{11}$ since the difference is the absence of $\mu_{R_k,D}$ terms. As a result, the following closed-form expressions can be deduced

$$I_2 = \frac{\beta_{\gamma_{S,R_k}}(g_{PSK})\Gamma(2\mu_{S,R_k} + \frac{1}{2})}{2\sqrt{\pi} \Gamma(2\mu_{S,R_k} + 1)} F_D^{(2)} \left( \begin{array}{c} 2\mu_{S,R_k} + \frac{1}{2}; \mu_{S,R_k} \mu_{S,R_k}, \mu_{S,R_k}; 2\mu_{S,R_k} + 1; \frac{1}{C_1}, -\frac{1}{C_2} \end{array} \right)$$

and

$$I_{22} = \frac{M_{\gamma_{S,R_k}}(g_{PSK})}{\pi} F_D^{(3)} \left( \begin{array}{c} \frac{1}{2}; \mu_{S,R_k}, \mu_{S,R_k}, \frac{1}{2} - 2\mu_{S,R_k}; 3; \frac{1}{2} \cos^2(\pi/M) + \frac{1}{1 + C_1}, \frac{\cos^2(\pi/M)}{1 + C_2}, \cos^2(\pi/M) \end{array} \right)$$

where

$$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \frac{\overline{\gamma}_{S,R_k} g_{PSK}}{2(h_{S,R_k} + H_{S,R_k})}$$

and

$$\beta_{\gamma_{S,R_k}}(g_{PSK}) = \left( \frac{4\mu_{S,R_k}^2 h_{S,R_k}^2 - H_{S,R_k}^2}{\pi^2 S_{R_k} g_{PSK}} \right)^{\mu_{S,R_k}} \quad (28)$$

Therefore, the $P_{SER}^D$ for $M$–PSK is deduced by inserting $P(e|A = C_z)$ and $P(A = C_z)$ in (6).

To the best of the authors’ knowledge, the derived analytic expressions are novel.

\textbf{B. End-to-End SER for $M$–QAM Constellations}

Having derived the SER over i.n.i.d and i.d $\eta - \mu$ fading channels for the case of $M$–PSK modulations, this subsection is devoted to the derivation of the corresponding SER for the case of $M$–QAM constellations.
1) The case of i.n.i.d $\eta-\mu$ fading channels: In the case of independent but not necessarily identically distributed $\eta-\mu$ fading channels and based on \cite{70}, eq. (9.21), it follows that

$$P_{M-QAM} = \frac{4C}{\pi} \int_0^{\pi/2} M_{\gamma MRC} \left( \frac{g_{QAM}}{\sin^2 \theta} \right) d\theta - \frac{4C^2}{\pi} \int_0^{\pi/4} M_{\gamma MRC} \left( \frac{g_{QAM}}{\sin^2 \theta} \right) d\theta$$  \hspace{1cm} (29)

where $g_{QAM} = 3/2(M-1)$ and $C = (1-1/\sqrt{M})$ \cite{71}. Therefore, the probability of decoding error using MRC under given decoding results at the relays can be expressed as follows:

$$P(e|A = C_2) = \frac{4C}{\pi} \int_0^{\pi/2} \left( \frac{4\mu^2_{S,D} h_{S,D} (2(h_{S,D} + H_{S,D}) \mu_{S,D} + \frac{g_{QAM}}{\sin^2 \theta} \mu_{S,D})}{(2(h_{S,D} - H_{S,D}) \mu_{S,D} + \frac{g_{QAM}}{\sin^2 \theta} \mu_{S,D})} \right)^{\mu_{S,D}} \prod_{k=1}^{K} C(k) \left( \frac{4\mu^2_{R_k,D} h_{R_k,D} (2(h_{R_k,D} + H_{R_k,D}) \mu_{R_k,D} + \frac{g_{QAM}}{\sin^2 \theta} \mu_{R_k,D})}{(2(h_{R_k,D} - H_{R_k,D}) \mu_{R_k,D} + \frac{g_{QAM}}{\sin^2 \theta} \mu_{R_k,D})} \right)^{\mu_{R_k,D}} d\theta - \frac{4C^2}{\pi} \int_0^{\pi/4} \left( \frac{4\mu^2_{S,D} h_{S,D} (2(h_{S,D} + H_{S,D}) \mu_{S,D} + \frac{g_{QAM}}{\sin^2 \theta} \mu_{S,D})}{(2(h_{S,D} - H_{S,D}) \mu_{S,D} + \frac{g_{QAM}}{\sin^2 \theta} \mu_{S,D})} \right)^{\mu_{S,D}} \prod_{k=1}^{K} C(k) \left( \frac{4\mu^2_{R_k,D} h_{R_k,D} (2(h_{R_k,D} + H_{R_k,D}) \mu_{R_k,D} + \frac{g_{QAM}}{\sin^2 \theta} \mu_{R_k,D})}{(2(h_{R_k,D} - H_{R_k,D}) \mu_{R_k,D} + \frac{g_{QAM}}{\sin^2 \theta} \mu_{R_k,D})} \right)^{\mu_{R_k,D}} d\theta$$ \hspace{1cm} (30)

Evidently, the derivation of a closed-form expression for (30) is subject to analytic evaluation of the above two integrals that correspond to $I_3$ and $I_4$. Following the same methodology as in the case of $M-PSK$ constellations and using the change of variable of $u = \sin^2 \theta$, one obtains

$$I_3 = \frac{\beta_{\gamma MRC} (g_{QAM})}{2} \int_0^1 \left( 1 - u \right)^{-\frac{1}{2}} u^{2\mu_{S,D} + \sum_{k=1}^{K} C(k) \mu_{R_k,D} } \prod_{k=1}^{K} C(k) du$$ \hspace{1cm} (31)

where the parameters $A_1, A_2, B_{1k}$ and $B_{2k}$ are determined by substituting $g_{PSK}$ with $g_{QAM}$. To this effect and after some algebraic manipulations, the following analytic expression is deduced

$$I_3 = \frac{\beta_{\gamma MRC} (g_{QAM}) \sqrt{\pi} \Gamma \left( 2\mu_{S,D} + 2 \sum_{k=1}^{K} C(k) \mu_{R_k,D} + \frac{1}{2} \right)}{2 \Gamma \left( 2\mu_{S,D} + 2 \sum_{k=1}^{K} C(k) \mu_{R_k,D} + 1 \right)}$$ \hspace{1cm} (32)

$$\times F_D^{(2K+2)} \left( 2\mu_{S,D} + 2 \sum_{k=1}^{K} C(k) \mu_{R_k,D} + 1; -\frac{1}{A_1}, -\frac{1}{A_2}, \cdots, -\frac{1}{B_{11}}, -\frac{1}{B_{21}}, \cdots, -\frac{1}{B_{2K}} \right).$$
Likewise, the $I_4$ integral has the same integrand but a different upper limit of integration. Thus, by following the same methodology and setting $y = 2u$, yields the following closed-form expression

$$I_4 = \frac{\beta_{\gamma_{\text{MRC}}}(g_{\text{QAM}})\Gamma(2\mu_{s,d} + 2 \sum_{k=1}^{K} C(k) \mu_{r,k,d} + \frac{1}{2})}{\Gamma(2\mu_{s,d} + 2 \sum_{k=1}^{K} C(k) \mu_{r,k,d} + \frac{3}{2})^{2^{2(\mu_{s,d} + \sum_{k=1}^{K} C(k) \mu_{r,k,d})} + \frac{1}{2}}} \times F_D^{(2K+3)} \left(2\mu_{s,d} + 2 \sum_{k=1}^{K} C(k) \mu_{r,k,d} + \frac{1}{2}; 2\mu_{s,d}, \mu_{s,d}, \mu_{s,d}, \mu_{r,1,d}, \cdots, \mu_{r,K,d}, \mu_{r,1,d}, \cdots, \mu_{r,K,d}; \frac{1}{2}\right)$$

where

$$\beta_{\gamma_{\text{MRC}}}(g_{\text{QAM}}) = \left(\frac{4\mu_{s,d}^2 h_{s,d}}{\gamma_{s,d}^2 g_{\text{QAM}}^2}\right) \prod_{k=1}^{K} C(k) \left(\frac{4\mu_{r,k,d}^2 (h_{r,k,d}^2 - H_{r,k,d}^2)}{\gamma_{r,k,d}^2 g_{\text{QAM}}^2}\right)^{\mu_{r,k,d}}.$$ 

Notably, the $I_3$ and $I_4$ terms are also expressed in terms of the generalized Lauricella function.

2) The case of i.i.d $\eta-\mu$ fading channels: In this simplified scenario, the corresponding solutions for $I_3$ and $I_4$ for the case of $M-$QAM constellations can be derived by following the same methodology as in the case of $M-$PSK modulations. To this end, after a necessary change of variables and long but basic algebraic manipulations, it immediately follows that

$$I_{3,i.d} = \frac{\beta_{\gamma_{\text{MRC}}}(g_{\text{QAM}})\sqrt{\pi}}{2\Gamma(2\mu_{s,d} + 2 \mu_{r,d} \sum_{k=1}^{K} C(k) + \frac{1}{2})} F_D^{(4)} \left(2\mu_{s,d} + 2 \mu_{r,d} \sum_{k=1}^{K} C(k) + \frac{1}{2}; 2\mu_{s,d}, \mu_{s,d}, \mu_{s,d}, \mu_{r,d}, \mu_{r,d}, \frac{1}{2}; 2\mu_{s,d} + 2 \mu_{r,d} \sum_{k=1}^{K} C(k) + \frac{3}{2}; -\frac{1}{A_1}, -\frac{1}{A_2}, -\frac{1}{B_1}, -\frac{1}{B_2}\right)$$

and

$$I_{4,i.d} = \frac{\beta_{\gamma_{\text{MRC}}}(g_{\text{QAM}})\Gamma(2\mu_{s,d} + 2 \mu_{r,d} \sum_{k=1}^{K} C(k) + \frac{1}{2})}{2\Gamma(2\mu_{s,d} + 2 \mu_{r,d} \sum_{k=1}^{K} C(k) + \frac{3}{2})^{2^{2\mu_{s,d} + 2 \mu_{r,d} \sum_{k=1}^{K} C(k)} + \frac{1}{2}}} F_D^{(5)} \left(2\mu_{s,d} + 2 \mu_{r,d} \sum_{k=1}^{K} C(k) + \frac{1}{2}; 2\mu_{s,d}, \mu_{s,d}, \mu_{s,d}, \mu_{r,d}, \mu_{r,d}, \frac{1}{2}; 2\mu_{s,d} + 2 \mu_{r,d} \sum_{k=1}^{K} C(k) + \frac{3}{2}; -\frac{1}{A_1}, -\frac{1}{A_2}, -\frac{1}{B_1}, -\frac{1}{B_2}, -\frac{1}{B_3}\right)$$

respectively. Hence, the error probability is determined by $P(e|A = C_z) = 4C I_3/\pi - 4C^2 I_4/\pi$.

Likewise, the corresponding decoding error probability at the relay nodes can be expressed as

$$P(\gamma_{s,r_k}) = \frac{4C}{\pi} \int_{0}^{\pi/2} M_{\gamma_{s,r_k}} \left(\frac{g_{\text{QAM}}}{\sin^2 \theta}\right) d\theta - \frac{4C^2}{\pi} \int_{0}^{\pi/4} M_{\gamma_{s,r_k}} \left(\frac{g_{\text{QAM}}}{\sin^2 \theta}\right) d\theta.$$ 

(37)
Based on the approach in (24) and given the non-arbitrary limits of integration, the following exact closed-form expressions are deduced for the integrals $\mathcal{I}_5$ and $\mathcal{I}_6$:

$$
\mathcal{I}_5 = \frac{\beta_{\gamma S,R_k}(g_{\text{QAM}})\sqrt{\pi}\Gamma(2\mu_{S,R_k}\sum_{k=1}^{K} C(k) + \frac{1}{2})}{2\Gamma(2\mu_{S,R_k}\sum_{k=1}^{K} C(k) + 1)} \times F_D^{(2)}(2\mu_{S,R_k}\sum_{k=1}^{K} C(k) + \frac{1}{2}; \mu_{S,R_k}; 2\mu_{S,R_k} + 1; -\frac{1}{C_1}, -\frac{1}{C_2})
$$

(38)

and

$$
\mathcal{I}_6 = \frac{\beta_{\gamma S,R_k}(g_{\text{QAM}})\Gamma(2\mu_{S,R_k}\sum_{k=1}^{K} C(k) + \frac{1}{2})}{\Gamma(2\mu_{S,R_k}\sum_{k=1}^{K} C(k) + \frac{3}{2})\Gamma(2\mu_{S,R_k}\sum_{k=1}^{K} C(k) + \frac{5}{2})} \times F_D^{(3)}(2\mu_{S,R_k}\sum_{k=1}^{K} C(k) + \frac{1}{2}; \mu_{S,R_k}; 2\mu_{S,R_k} + 1; 2\mu_{S,R_k}\sum_{k=1}^{K} C(k) + \frac{3}{2}; \frac{1}{2C_1}, -\frac{1}{2C_2}, 1, 1)
$$

(39)

respectively, where

$$
\beta_{\gamma S,R_k}(g_{\text{QAM}}) = \left(\frac{4\mu_{S,R_k}^2(h_{S,R_k}^2 - H_{S,R_k}^2)}{\gamma_{S,R_k} g_{\text{QAM}}^2}\right)^{\mu_{S,R_k}}
$$

(40)

whereas $C_1$ and $C_2$ are obtained by substituting $g_{\text{PSK}}$ with $g_{\text{QAM}}$ in (27). Therefore, using (38) and (39), the corresponding decoding error probability can be readily expressed as

$$
P(\gamma_{S,R_k} | A(k) = C(k) = 0) = \frac{4C_1}{\pi} \mathcal{I}_5 - \frac{4C_2}{\pi} \mathcal{I}_6.
$$

(41)

Thus, a closed-form expression for the SER is deduced by substituting $P(A = C_z)$ in (6) along with the corresponding $P(e | A = C_z)$ and can be computed using the algorithm in Appendix I.

C. Simple Asymptotic Expressions

The derivation of asymptotic expressions typically leads to useful insights on the impact of the involved parameters on the system performance. This is also the case in the present analysis as simple closed-form asymptotic expressions are derived for high SNR values. To this end, the MGF of $\eta - \mu$ distribution can be accurately approximated as

$$
M_{\gamma - \mu} \left(\frac{g}{\sin^2 \theta}\right) = \left(\frac{4\mu^2 h}{(2(h - H)\mu + \frac{g}{\sin^2 \theta})\gamma(2(h + H)\mu + \frac{g}{\sin^2 \theta})}\right)^{\mu} \approx \left(\frac{4\mu^2 h}{g \gamma}\right)^{\mu} \sin^{4\mu}(\theta).
$$

(42)

Based on this, the conditional error probability $P(e | A = C_z)$ can be approximated as follows:

$$
P(e | A = C_z) \approx \left(\frac{4\mu_{S,D}^2 h_{S,D}^2}{g^2 \gamma_{S,D}^2}\right)^{\mu_{S,D}} \prod_{k=1}^{K} A_{R_k,D}(C_z) \left(\frac{4\mu_{R,D}^2 h_{R_k,D}^2}{g^2 \gamma_{R_k,D}^2}\right)^{\mu_{R_k,D}}
$$

(43)
where \( A_{R_k,D}(C_z) \) for \( M-\)PSK constellations is given by

\[
A_{R_k,D}(C_z) = \frac{1}{\pi} \int_0^{(M-1)\pi} [\sin(\theta)]^{4(\mu_{S,D} + \sum_{k=1}^K C(k)\mu_{R_k,D})} d\theta
\]

which can be equivalently re-written as follows:

\[
A_{R_k,D}(C_z) = \frac{1}{\pi} \int_0^{\pi/2} [\sin(\theta)]^{4(\mu_{S,D} + \sum_{k=1}^K C(k)\mu_{R_k,D})} d\theta + \frac{1}{\pi} \int_{\pi/2}^{(M-1)\pi} [\sin(\theta)]^{4(\mu_{S,D} + \sum_{k=1}^K C(k)\mu_{R_k,D})} d\theta.
\]

The derivation of a closed-form expression for (43) is subject to analytic evaluation of the above trigonometric integrals. Hence, setting \( u = \sin^2(\theta) \) and \( a = 2(\mu_{S,D} + \sum_{k=1}^K C(k)\mu_{R_k,D}) \) yields

\[
I_7 = \frac{1}{2} \int_0^1 \frac{u^{a-\frac{1}{2}}}{(1-u)^{\frac{1}{2}}} du
\]

which can be expressed in closed-form according to [69, eq. (3.191.1)]. To this end, by performing the necessary variable transformation and after basic algebraic manipulations one obtains

\[
I_7 = \frac{\sqrt{\pi} \Gamma(2\mu_{S,D} + 2\sum_{k=1}^K C(k)\mu_{R_k,D} + 1)}{2\Gamma(2\mu_{S,D} + 2\sum_{k=1}^K C(k)\mu_{R_k,D} + 1)}. \tag{47}
\]

Likewise, by setting \( u = \cos^2(\theta)/\cos^2(\pi/M) \) to the second integral in (44), it follows that

\[
I_8 = \frac{\cos(\pi/M)}{2} \int_0^{\cos^2(\pi/M)} \frac{u^{-\frac{1}{2}}}{(1 - u \cos^2(\pi/M))^{\frac{1}{2} - a}} du
\]

which can be expressed in closed-form in terms of the Gauss hypergeometric function, yielding

\[
I_8 = \cos\left(\frac{\pi}{M}\right) {}_2F_1\left(1, 2 - 2\mu_{S,D} + 2\sum_{k=1}^K C(k)\mu_{R_k,D}, \frac{3}{2}; \cos^2\left(\frac{\pi}{M}\right)\right). \tag{49}
\]

To this effect, \( A_{R_k,D}(C_z) \) can be expressed in closed-form by

\[
A_{R_k,D}(C_z) = \frac{I_7}{\pi} + \frac{I_8}{\pi}.
\]

In the same context, for the case of M-\(Q\)AM constellations one obtains

\[
A_{R_k,D}(C_z) = \frac{4C}{\pi} \int_0^{\pi/2} [\sin(\theta)]^{4(\mu_{S,D} + \sum_{k=1}^K C(k)\mu_{R_k,D})} d\theta - \frac{4C^2}{\pi} \int_{\pi/2}^{\pi/4} [\sin(\theta)]^{4(\mu_{S,D} + \sum_{k=1}^K C(k)\mu_{R_k,D})} d\theta.
\]

It is evident that the integrals \( I_7 \) and \( I_9 \) have the same algebraic forms. Hence, it follows that

\[
I_9 = \frac{\sqrt{\pi} \Gamma(2(\mu_{S,D} + \sum_{k=1}^K C(k)\mu_{R_k,D}) + 1)}{2\Gamma(2(\mu_{S,D} + \sum_{k=1}^K C(k)\mu_{R_k,D}) + 1)}.
\]

\[
I_{10} = \frac{\sqrt{\pi} \Gamma(2(\mu_{S,D} + \sum_{k=1}^K C(k)\mu_{R_k,D}) + 1)}{2\Gamma(2(\mu_{S,D} + \sum_{k=1}^K C(k)\mu_{R_k,D}) + 1)}.
\]

\[
I_{11} = \frac{\sqrt{\pi} \Gamma(2(\mu_{S,D} + \sum_{k=1}^K C(k)\mu_{R_k,D}) + 1)}{2\Gamma(2(\mu_{S,D} + \sum_{k=1}^K C(k)\mu_{R_k,D}) + 1)}.
\]
respectively, depending on the selected modulation scheme. To this effect and by substituting (43) and (53) into (6), the SER of the multi-relay regenerative and corresponds to

\[
I_{10} = \frac{2^{-2(\mu_{S,D} + \frac{1}{2})}}{2^{2\sum_{k=1}^{K} C(k) \mu_{R_k,D}}} \frac{2\mu_{S,D} + 2 \sum_{k=1}^{K} C(k) \mu_{R_k,D} + \frac{1}{2}}{2^{2\sum_{k=1}^{K} C(k) \mu_{R_k,D} + \frac{3}{2}}} \frac{1}{\Gamma(2\mu_{S,D} + 2 \sum_{k=1}^{K} C(k) \mu_{R_k,D} + \frac{1}{2})}. 
\]

Therefore, with the aid of (51) and (52), a closed-form expression for \( A_{R_k,D}(C_z) \) for the case of square \( M \)-QAM constellations is given by

\[
A_{R_k,D}(C_z) = \frac{4C I_0/\pi - 4C^2 I_{10}/\pi}{\pi/4} \sin^{4\mu_{S,R_k}}(\theta) d\theta
\]

The decoding error probability of the relays also in the high SNR regime can be expressed as

\[
P(A(k) = C(k) = 0) \approx A_{S,R_k} \left( \frac{4\mu_{S,R_k} h_{S,R_k}}{g^{2\gamma_{S,R_k}}} \right)^{\mu_{S,R_k}}
\]

and

\[
P(A(k) = C(k) = 1) \approx 1 - A_{S,R_k} \left( \frac{4\mu_{S,R_k} h_{S,R_k}}{g^{2\gamma_{S,R_k}}} \right)^{\mu_{S,R_k}}
\]

where \( A_{S,R_k} \) for \( M \)-PSK and \( M \)-QAM constellations is given by

\[
A_{S,R_k} = \frac{1}{\pi} \int_{0}^{(M-1)\pi} \sin^{4\mu_{S,R_k}}(\theta) d\theta
\]

and

\[
A_{S,R_k} = \frac{4C}{\pi} \int_{0}^{\pi/2} \sin^{4\mu_{S,R_k}}(\theta) d\theta - \frac{4C^2}{\pi} \int_{0}^{\pi/4} \sin^{4\mu_{S,R_k}}(\theta) d\theta
\]

respectively. Exact closed-form expressions for \( A_{S,R_k} \) for both modulation schemes can be obtained by following the same methodology and procedure as in the previous case yielding

\[
A_{S,R_k} = \frac{\Gamma(2\mu_{S,R_k} + \frac{1}{2})}{\pi 2^{\gamma_{S,R_k} + \frac{1}{2}}} \frac{1}{\Gamma(2\mu_{S,R_k} + 1)} \left( \frac{1}{2} - 2\mu_{S,R_k} \right)^{\gamma_{S,R_k} + \frac{1}{2}} \cos^2(\pi/M)
\]

for the case of \( M \)-PSK and

\[
A_{S,R_k} = \frac{2CT(2\mu_{S,R_k} + \frac{1}{2})}{\sqrt{\pi} \Gamma(2\mu_{S,R_k} + 1)} - \frac{C^2\Gamma(2\mu_{S,R_k} + \frac{1}{2})}{\pi 2^{\gamma_{S,R_k} + \frac{1}{2}}} \frac{1}{\Gamma(2\mu_{S,R_k} + 1)} \left( \frac{1}{2} - 2\mu_{S,R_k} \right)^{\gamma_{S,R_k} + \frac{1}{2}} \cos^2(\pi/M)
\]

for the case of \( M \)-QAM. It is noted here that at sufficiently high SNR, the probability \( P(A(k) = C(k) = 1) \) is clearly smaller than unity and thus, \( 1 - A_{S,R_k} ((4\mu_{S,R_k} h_{S,R_k})/(g^{2\gamma_{S,R_k}}))^{\mu_{S,R_k}} \approx 1 \). To this effect and by substituting (43) and (53) into (6), the SER of the multi-relay regenerative system over generalized fading channels at the high SNR regime can be expressed as follows:

\[
P_{\text{SER}}^D \approx \left( \frac{4\mu_{S,D} h_{S,D}}{g^{2\gamma_{S,D}}} \right)^{\mu_{S,D}} \sum_{z=0}^{2^K-1} \prod_{k=1}^{K} A_{R_k,D}(C_z) \left( \frac{4\mu_{R_k,D} h_{R_k,D}}{g^{2\gamma_{R_k,D}}} \right)^{\mu_{R_k,D}} \prod_{k=1}^{K} A_{S,R_k} \left( \frac{4\mu_{S,R_k} h_{S,R_k}}{g^{2\gamma_{S,R_k}}} \right)^{\mu_{S,R_k}}
\]

where \( g \) corresponds to \( g_{\text{PSK}} = \sin^2(\pi/M) \) or \( g_{\text{QAM}} = 3/(2(M-1)) \) according to (11) and (29), respectively, depending on the selected modulation scheme.
D. Amount of Fading

It is recalled that the amount of fading (AoF) is a useful metric for quantifying the fading severity in the communication scenarios and is defined according to \[ \text{[70, eq. (1.27)]}, \] namely

\[
\frac{\text{Var}(\gamma_{\text{MRC}})}{(E(\gamma_{\text{MRC}}))^2} = \frac{E(\gamma_{\text{MRC}}^2) - (E(\gamma_{\text{MRC}}))^2}{(E(\gamma_{\text{MRC}}))^2}.
\] (60)

The \( n \)th moment of the \( \gamma_{\text{MRC}} \) under the decode-and-forward strategy can be determined by \[ \text{[72]} \]

\[
\mu_n = (-1)^n \left[ \frac{d^n}{ds^n} \left( M_{\gamma_{S,D}}(s) \prod_{k=1}^{K} C(k) M_{R_k,D}(s) \right) \right]_{s=0}.
\] (61)

Based on this, the first two moments in (61) are obtained for \( n = 1 \) and \( n = 2 \), namely,

\[
E(\gamma_{\text{MRC}}) = \partial M_{\gamma_{S,D}}(s)/\partial s |_{s=0} \quad \text{and} \quad E(\gamma_{\text{MRC}}^2) = \partial^2 M_{\gamma_{S,D}}(s)/\partial^2 s |_{s=0}.
\]

To this effect and recalling \( \text{[8]}-\text{[10]} \) as well as carrying out long but basic algebraic manipulations, the corresponding minimum AoF, if all relays decode correctly, can be represented as follows:

\[
\text{AoF} = \frac{\mu_{S,D}(\mu_{S,D} + 1)\delta_1^2 - 2\delta_2\mu_{S,D} - 2\delta_3\mu_{R,D}}{(\mu_{S,D}\delta_1 + K\mu_{R,D}\delta_4)^2} + \frac{\mu_{R,D}(K\mu_{R,D} + 1)\delta_1^2 + 2\mu_{S,D}\mu_{R,D}\delta_1\delta_4}{(\mu_{S,D}\delta_1 + K\mu_{R,D}\delta_4)^2} K^{-1} - 1
\] (62)

where

\[
\delta_1 = \gamma_{S,D} h_{S,D}/\mu_{S,D}(h_{S,D}^2 - H_{S,D}^2), \quad \delta_2 = \gamma_{S,D}^2/4\mu_{S,D}^2(h_{S,D}^2 - H_{S,D}^2), \quad \delta_3 = \gamma_{R,D}^2/4\mu_{R,D}^2(h_{R,D}^2 - H_{R,D}^2), \quad \delta_4 = \gamma_{R,D} h_{R,D}/\mu_{R,D}(h_{R,D}^2 - H_{R,D}^2).
\]

By recalling that the \( \eta-\mu \) model includes the Nakagami-\( m \), Hoyt and Rayleigh distributions, the AoF for these cases can be readily deduced.

IV. Optimum Power Allocation

This section is devoted to the derivation of the OPA strategy that minimizes the derived asymptotic SER of the considered regenerative system subject to the sum-power constraint of a power budget \( P \). Since the derived SER expression in \[ \text{[59]} \] is a function of the power allocated at the source and the \( K \) relays of the system, the available power should be allocated optimally in order to enhance the end-to-end quality of the transmission. Based on this, the corresponding non-linear optimization problem can be formulated as follows:

\[
a_{\text{opt}} = \arg \min_a P_{\text{SER}}^D
\] (63)

Subject to:

\[
a_0 + \sum_{k=1}^{K} a_{R_k} = 1, \quad a_0 \geq 0, a_{R_k} \geq 0
\]

where \( a = [a_0, a_{R_1}, a_{R_2}, \ldots, a_{R_k}] \) denotes the relative power allocation vector. Importantly, the cost function in (63) is convex in the parameters \( a_0 \) and \( a_{R_k} \) over the feasible set defined by
linear power ratio constraints. The corresponding proof is provided in Appendix II. To this effect and following the definitions in [75], the Lagrangian of this optimization problem is given by

$$L = P_{\text{SER}}^D + \lambda \left( a_0 + \sum_{k=1}^{K} a_{R_k} - 1 \right) - \mu_0 a_0 - \sum_{k=1}^{K} \mu_k a_{R_k}$$  \hspace{1cm} (64)$$

where $\lambda$ denotes the Lagrange multiplier satisfying the power constraint whereas $\mu_0$ and $\mu_k$ parameters serve as slack variables. The nonlinear optimization problem in (63) can be solved using e.g. a line search method. However, to develop some insights for the power allocation policy we apply the Karush-Kuhn-Tucker (KKT) conditions for minimizing the corresponding SER [75]. To this end, it follows that all $\mu_k$ and $\mu_0$ parameters are zero while the following derivatives form the necessary condition for maximizing the performance of the system

$$\frac{\partial P_{\text{SER}}^D}{\partial a_0} = \frac{\partial P_{\text{SER}}^D}{\partial a_{R_k}}, \hspace{1cm} (1 \leq k \leq K)$$  \hspace{1cm} (65)$$

In order to obtain feasible relations between optimal powers of the cooperating nodes, we employ the asymptotic SER in (59). By re-writing this in terms of the power ratios allocated at the transmitting nodes it follows that

$$P_{\text{SER}}^D \approx \left( \frac{4\mu_{S,D}^2 h_{S,D} N_0^2}{g a_0^2 \Omega_D^2 P_2^2} \right)^{\mu_{S,D}} \sum_{z=0}^{2K-1} \prod_{k=1}^{K} A_{R_k,D}(C_z) \left( \frac{4\mu_{R,D}^2 h_{R_k,D} N_0^2}{a_{R_k,D}^2 g^2 \Omega_D^2 P_2^2} \right)^{\mu_{R_k,D}} \times \prod_{k=1}^{K} A_{S,R_k} \left( \frac{4\mu_{S,R_k}^2 h_{S,R_k} N_0^2}{a_{R_k,S}^2 g^2 \Omega_{R_k,S}^2 P_2^2} \right)^{\mu_{S,R_k}}$$  \hspace{1cm} (66)$$

In order to derive an optimal power allocation policy for the DF protocol, we initially restrict our scenario to $K = 1$ and $K = 2$ relay nodes and then we generalize the result for $K$ relays.

1) $K = 1$ Scenario: The possible decoding sets in this case are $C_0 = 0$ i.e., the relay is unable to decode correctly, and $C_1 = 1$ i.e. the relay decodes successfully. Thus, using (66) and neglecting the constant term outside the summation after factoring out $a_0$, it follows that

$$\min \left[ \frac{K_1}{a_0^{2(\mu_{S,D}+\mu_{S,R_1})}} + \frac{K_2}{a_0^{2\mu_{S,D}} a_{R_1}^{2\mu_{R_1,D}}} \right]$$  \hspace{1cm} (67)$$

Subject to : $a_0 + a_{R_1} = 1$

where

$$K_1 = A_{S,D} A_{S,R_1} \left( \frac{4\mu_{S,R_1}^2 h_{S,R_1} N_0^2}{g^2 \Omega_{S,R_1}^2 P_2^2} \right)^{\mu_{S,R_1}}$$  \hspace{1cm} (68)$$
and

\[ K_2 = A_{R_1,D} \left( \frac{4\mu_{R_1,D}^2 h_{R_1,D} N_0^2}{g^2\Omega_{R_1,D}^2 D^2} \right)^{\mu_{R_1,D}}. \]  

(69)

Next, we apply the necessary condition in (65) to determine the relation between optimal power allocations in the two nodes. Thus, the first derivative of \( P_{\text{SER}}^D \) with respect to \( a_0 \) is given by

\[
\frac{\partial P_{\text{SER}}^D}{\partial a_0} = -\frac{2(\mu_{S,D} + \mu_{S,R_1}) K_1}{a_0^{2(\mu_{S,D} + \mu_{S,R_1}) + 1}} - \frac{2\mu_{S,D} K_2}{a_0^{2\mu_{S,D} + 1}} \frac{2\mu_{R_1,D}}{a_{R_1}^{2\mu_{R_1,D}}},
\]  

(70)

whereas the first derivative with respect to \( a_{R_1} \) is given by

\[
\frac{\partial P_{\text{SER}}^D}{\partial a_{R_1}} = -\frac{2\mu_{R_1,D} K_2}{a_0^{2\mu_{S,D}} a_{R_1}^{2\mu_{R_1,D} + 1}}.
\]  

(71)

Equating the above two relations and after some long but basic rearrangements, it follows that

\[
\frac{(\mu_{S,D} + \mu_{S,R_1}) K_1}{K_2} = \frac{a_0^{2\mu_{S,R_1} + 1}}{a_{R_1}^{2\mu_{R_1,D}}} \left( \frac{\mu_{R_1,D}}{a_{R_1}} \frac{\mu_{S,D}}{a_0} \right).
\]  

(72)

It is noticed that the left-hand side of (72) depends only on the channel parameters. As a result, this term is always positive which implies directly a power policy of

\[ a_0 \mu_{R_1,D} \geq a_{R_1} \mu_{S,D}. \]

This relation is also in agreement with the power allocation over Rayleigh, Nakagami\(-m\), and Hoyt (Nakagami\(-q\)) fading distributions under the special cases given in section III.

2) \( K = 2 \) Scenario: In this case, the following four scenarios are valid: \( C_0 = (0,0) \) when the two relays decode incorrectly; \( C_1 = (0,1) \) when the first relay decodes incorrectly and the second relay decodes successfully; \( C_2 = (1,0) \) when the first relay decodes successfully and the second relay decodes incorrectly; and \( C_3 = (1,1) \) when both relays decode successfully. Based on this, the corresponding optimization problem can be expressed as follows

\[
\min \left[ \frac{K_3}{a_0^{2(\mu_{S,D} + \mu_{S,R_1} + \mu_{S,R_2})}} + \frac{K_4}{a_0^{2(\mu_{S,D} + \mu_{S,R_1})}} \frac{2\mu_{R_2,D}}{a_{R_2}^{2\mu_{R_2,D}}} + \frac{K_5}{a_0^{2(\mu_{S,D} + \mu_{S,R_2})}} \frac{2\mu_{R_1,D}}{a_{R_1}^{2\mu_{R_1,D}}} + \frac{K_6}{a_0^{2\mu_{S,D}}} \frac{2\mu_{R_2,D}^2}{a_{R_2}^{2\mu_{R_2,D}}} \frac{2\mu_{R_1,D}}{a_{R_1}^{2\mu_{R_1,D}}}. \right]
\]  

(73)

Subject to: \( a_0 + a_{R_1} + a_{R_2} = 1 \)

where \( K_3, K_4, K_5 \) and \( K_6 \) relate to the channel parameters, which are not affecting the sign of the derivatives in any case. To this effect, the derivative of \( P_{\text{SER}}^D \) with respect to \( a_{R_1} \) is given by

\[
\frac{\partial P_{\text{SER}}^D}{\partial a_{R_1}} = -\frac{2K_5 \mu_{R_2,D}}{a_0^{2(\mu_{S,D} + \mu_{S,R_2})}} \frac{2\mu_{R_1,D} + 1}{a_{R_1}^{2\mu_{R_1,D} + 1}} - \frac{2K_6 \mu_{R_1,D}}{a_0^{2\mu_{S,D}}} \frac{2\mu_{R_2,D}^2}{a_{R_2}^{2\mu_{R_2,D}}} \frac{2\mu_{R_1,D} + 1}{a_{R_1}^{2\mu_{R_1,D} + 1}}
\]  

(74)
whereas the derivative of $P_{SER}^D$ with respect to $a_{R_2}$ is given by

$$\frac{\partial P_{SER}^D}{\partial a_{R_2}} = -\frac{2K_4\mu_{R_2,D}}{a_0^2(\mu_{S,D} + \mu_{S,R_1})} \frac{2\mu_{R_2,D} + 1}{a_{R_2}^2} \frac{2\mu_{R_2,D} + 1}{a_{R_1}^2}. \quad (75)$$

With the aid of the above two representations, a power allocation strategy can be proposed for identical channel conditions as $a_{R_1} = a_{R_2}$. Likewise, applying $\frac{\partial P_{SER}^D}{\partial a_{R_k}} = \frac{\partial P_{SER}^D}{\partial a_{R_{k+1}}}$, $(2 \leq k \leq K)$ it is straightforwardly shown that $a_{R_k} = a_{R_{k+1}}$. This power assignment indicates that under the total power constraint and the assumed channel conditions for the regenerative network, power control mechanism with the relays is not essential, but the remaining power left from the total power budget after the source can be simply allocated uniformly between the nodes. This is further elaborated and largely verified in the following section.

V. NUMERICAL RESULTS

In this Section, the offered analytic results are employed in evaluating the performance of the considered regenerative system for different communication scenarios. To this end, the variance of the noise is assumed to be normalized as $N_0 = 1$ for all considered scenarios while transmit

![Fig. 2. SER in $\eta - \mu$ fading with $\mu = 0.5, \eta = 1$ and $\Omega_{S,D} = \Omega_{S,R_k} = \Omega_{R_k,D} = 0$dB for 4−PSK/4−QAM constellation with different number of relays and EPA.](image)

![Fig. 3. SER in $\eta - \mu$ fading with $\mu = 0.5, \eta = 1$ and $\Omega_{S,D} = \Omega_{S,R_k} = \Omega_{R_k,D} = 0$dB for 4−QAM/4−PSK and 16−QAM, $K = 2$ and EPA.](image)
powers are equally or optimally allocated to the source and the relays. It is also noted that
the presented results are limited to Format−1 of the \( \eta - \mu \) distribution but they can be readily
extended to scenarios that correspond to Format−2 [59].

Fig. 2 illustrates the SER performance as a function of SNR for one, two and three relays
using equal power allocation, i.e., \( P_0 = P_{R_k} = P/(K + 1) \) over symmetric and balanced \( \eta - \mu \)
fading channels for 4–PSK/4–QAM constellation. Also, the \( \eta - \mu \) fading parameters are \( \mu = 0.5 \) and \( \eta = 1 \) while \( \Omega \) parameters are equal to unity. It is shown that the exact results are
bounded tightly by the corresponding asymptotic curves from moderate to high SNR values
while a full diversity order i.e., 2, 3 and 4, can be achieved. Table II depicts the corresponding
diversity gains computed from the slopes for the exact and asymptotic curves along with the
direct transmission scenario, for reference, where it is assumed that \( P_0 = P \). It is observed
that at a target SER of \( 10^{-4} \) the single relay system exhibits a gain of 15dB over the direct
transmission whereas the two and three relay systems outperform the direct scenario by about
19.5dB and 21.5dB, respectively. In the same context, Fig. 3 illustrates the exact and asymptotic

![Fig. 4. SER vs average SNR with \( \mu = \{0.5, 1, 1.5\}, \eta = \{0.1, 0.9\} \) and \( \Omega_{S,D} = \Omega_{S,R_k} = \Omega_{R_k,D} = 0dB \) for 4–PSK/4–QAM, \( K = 2 \) and EPA.](image)

![Fig. 5. SER vs average SNR with \( \mu = \{0.5, 1\}, \eta_{S,D} = 0.9, \Omega_{S,D} = \Omega_{S,R_k} = 0dB, \Omega_{R_k,D} = \{0, 10\}dB \) with different \( \eta_{S,R_k} \) and \( \eta_{R_k,D} \) for 4–PSK/4–QAM, \( K = 2 \) and EPA.](image)
TABLE II
DIVERSITY GAINS FOR ONE, TWO AND THREE RELAYS USING 4−PSK/4−QAM FOR $\mu = 0.5$ AND $\eta = 1$.

| $K$ | Diversity Gain (Exact) | Diversity Gain (Asymp.) |
|-----|-------------------------|--------------------------|
| 1   | 1.96                    | 2                        |
| 2   | 2.91                    | 3                        |
| 3   | 3.85                    | 4                        |

Fig. 6. SER in $\eta - \mu$ fading with $\mu = \{0.5, 1\}$, $\eta_{S,D} = 0.9$ and $\Omega_{S,D} = \Omega_{S,Rk} = 0$dB, $\Omega_{Rk,D} = \{0, 10\}$dB with different $\eta_{S,Rk}$ and $\eta_{Rk,D}$ for 4−PSK/4−QAM signals, $K = 2$ and EPA.

results for 4−QAM/4−PSK and 16−QAM constellations for the case of two relays with equal power allocation over symmetric $\eta - \mu$ fading channels with $\mu = 0.5$ and $\eta = 1$ as well as balanced links i.e., $\Omega_{S,D} = \Omega_{S,Rk} = \Omega_{Rk,D} = \Omega = 0$dB. It is shown that the asymptotic curves are almost identical to the exact ones for SERs lower than around $10^{-3}$. Therefore, it becomes evident that in practical system designs of DF relaying at the high SNR regime, the offered asymptotic expressions can provide useful insights on the system performance.

Fig. 4 illustrates the cooperation performance of 4−QAM/QPSK system in a two relay scenario for $\mu = \{0.5, 1, 1.5\}$ and identical channel variance of $\Omega_{S,D} = \Omega_{S,Rk} = \Omega_{Rk,D} = \Omega = 0$dB with equal power allocation. It is also recalled that the case of $\mu = 0.5$ corresponds to the Nakagami−$q$ (Hoyt) distribution with $q^2 = \eta$. By varying the value of $\eta$, we observe the effect of the scattered-
wave power ratio on the average SER of the considered regenerative system. This verifies that
the SER is inversely proportional to $\eta$ since for an indicative SER of $10^{-4}$, an average gain of
2dB is observed when $\eta$ increases from 0.1 to 0.9 for all values of $\mu$.

Furthermore, average gains of 4dB and 1.75dB are obtained as $\mu$ increases from 0.5 to 1
and from 1 to 1.5, respectively. Likewise, Fig. 5, demonstrates the SER performance in i.n.i.d
$\eta - \mu$ fading channels for 4-QAM/4-PSK constellations for the case of two relays with equal
power allocation. It is assumed that $\eta_{S,D} = 0.9$, $\Omega_{S,D} = \Omega_{S,R_k} = 0$dB, $\Omega_{R_k,D} = \{0, 10\}$dB,
$\mu_{S,D} = \mu_{S,R_k} = \mu_{R_k,D} = \mu = \{0.5, 1\}$ with different values of $\eta_{S,R_k}$ and $\eta_{R_1,D}$ set as
$\{\eta_{S,R_k}, \eta_{R_1,D}, \eta_{R_2,D}\} = \{0.1, 0.1, 0.9\}$ and $\{0.9, 0.8, 0.9\}$, respectively. It is observed that the
performance of the system improves substantially as $\eta_{S,R_k}$ and $\eta_{R_1,D}$ increase as at a SER of
$10^{-4}$ almost 1.25dB and 1.75dB gains are achieved when $\{\eta_{S,R_k}, \eta_{R_1,D}, \eta_{R_2,D}\}$ changes from
$\{0.1, 0.1, 0.9\}$ to $\{0.9, 0.8, 0.9\}$ for $\Omega_{R_k,D} = \{0, 10\}$dB, for the considered values of $\mu$.

Fig. 6 illustrates the SER performance in $\eta - \mu$ fading conditions with $\mu_{S,D} = 0.5$ and $\eta = 0.1$
for 4-QAM/4-PSK constellations and balanced links of relative channel variance 0dB for the
case of two relays using equal power allocation with different $\mu_{S,R_k}$ and non-identical values of
$\mu_{R_k,D}$. The figure shows that increasing at least one of $\mu_{R_k,D}$’s value at a fixed $\mu_{S,R_k}$ or increasing
both $\mu_{S,R_k}$ and $\mu_{R_k,D}$ simultaneously can improve the cooperation performance. Indicatively,
Fig. 8. SER with OPA in $\eta - \mu$ fading with $\eta = 0.5$, $\Omega_{S,D} = \Omega_{S,R_k} = 0$dB and $\Omega_{R_k,D} = 10$dB for different $\mu$ and $4$–QAM/$4$–PSK and $K = 2$.

SER of $10^{-4}$ nearly $1.25$dB and $1.75$dB gains are observed when $\{\mu_{S,R_k}, \mu_{R_1}, \mu_{R_2}\}$ changes from $\{0.5, 0.5, 0.5\}$ to $\{0.5, 0.5, 1\}$ and from $\{0.5, 0.5, 1\}$ to $\{0.5, 1, 1.5\}$, respectively. Also, nearly $0.75$dB and $1$dB gains are achieved when $\{\mu_{R_1}, \mu_{R_2}\}$ varies from $\{1, 1\}$ to $\{1, 1.5\}$ and then to $\{1.5, 2\}$ when $\mu_{S,R_k} = 1$, whereas a gain of $4$dB is shown when both $\mu_{S,R_k}$ and $\mu_{R_k,D}$ increase at the same time, for instance, from $\{0.5, 0.5, 1\}$ to $\{1, 1, 1.5\}$. Likewise, Fig. 7, depicts the SER performance for $4$–QAM/$4$–PSK constellations for $\mu = 0.5$ and $1$, $\eta = 0.5$ and unbalanced channel links employing two relays with equal power allocation. It is shown that increasing either $\Omega_{S,R_k}$ or $\Omega_{R_k,D}$ can improve the SER and that the performance for the case of $\Omega_{S,R_k} = 0$dB and $\Omega_{R_k,D} = 10$dB is better than the reverse scenario i.e. $\Omega_{S,R_k} = 10$dB and $\Omega_{R_k,D} = 0$dB. For example, almost constant gains of $1$dB and $1.75$dB are achieved when $\Omega_{S,R_k} = 0$dB and $\Omega_{R_k,D} = 0$dB increase to $\Omega_{S,R_k} = 10$dB and $\Omega_{R_k,D} = 0$dB and from $\Omega_{S,R_k} = 0$dB and $\Omega_{R_k,D} = 0$dB to $\Omega_{S,R_k} = 0$dB and $\Omega_{R_k,D} = 10$dB, respectively. This verifies that in the considered regenerative protocol, the overall performance improves more when increasing average power from the relays to the destination than from the source to the relays.

Fig. 8, demonstrates the SER performance of the proposed optimal power allocation scheme for different values of the fading parameter $\mu$, and assuming stronger channel variance from the relay nodes to the destination node with constant scattered-wave power ratio for the case of
two relays and 4–QAM/4–PSK constellations. It is also assumed that \( \mu_{S,D} = \mu_{S,R_k} = \mu_{R_k,D} = \mu, \Omega_{S,D} = \Omega_{S,R_k} = 0 \text{dB}, \Omega_{R_k,D} = 10 \text{dB} \) and \( \eta = 0.5 \). It is shown that when \( \mu \) is small, for example \( \mu = 0.5 \), the OPA provides small gain to the cooperation system, which, however, increases as \( \mu \) increases. Indicatively, for a SER of \( 10^{-4} \) the optimal system outperforms the equal power allocation scenario by at least 1.5\( \mu \)dB when \( \mu = 0.5 \) and by 2.5\( \mu \)dB and 3\( \mu \)dB when \( \mu = 1 \) and \( \mu = 1.5 \), respectively. In addition, Fig. 9 depicts the corresponding SER for the case that \( \mu_{S,D} = \mu_{S,R_k} = \mu_{R_k,D} = \mu, \eta = 0.5, \Omega_{S,D} = \Omega_{S,R_k} = 0 \text{dB} \) and \( \Omega_{R_k,D} = \{0,10\} \text{dB} \). It is observed that when \( \Omega_{S,R_k} = \Omega_{R_k,D} = 0 \text{dB} \), OPA does not provide significant performance improvement for the considered DF network. On the contrary, the SER improves as the difference between \( \Omega_{S,R_k} \) and \( \Omega_{R_k,D} \) increases. For example, it is noticed that for a SER of \( 10^{-4} \) and \( \mu = 0.5 \), gains of 0.5\( \mu \)dB and 2\( \mu \)dB are achieved by OPA over the EPA scheme when \( \Omega_{R_k,D} = \{0,10\} \text{dB} \), respectively. Similarly, gains of 0.5\( \mu \)dB and 3\( \mu \)dBs are obtained when \( \mu \) is increased to \( \mu = 1.5 \) while it is generally noticed that OPA is typically more effective than EPA, even in the low-SNR regime.

Fig. 10 demonstrates the SER performance of the derived OPA and EPA scenarios for single, two and three relays over symmetric \( \eta - \mu \) fading scenario, i.e., with constant \( \mu \) and constant scattered-power ratios \( \eta \) and unbalanced channel variances from source-to-relay and from relay-to-destination. It is assumed that \( \mu_{S,D} = \mu_{S,R_k} = \mu_{R_k,D} = 1, \eta = 0.5 \) whereas \( \Omega_{S,D} = \Omega_{S,R_k} = 0 \text{dB} \) and \( \Omega_{R_k,D} = 10 \text{dB} \). It is shown that the OPA strategy clearly outperforms its EPA counterpart since the gain for a SER of \( 10^{-4} \) is 2\( \mu \), 2.5\( \mu \) and 2.75\( \mu \)dB for one, two and three relays, respectively. The characteristics of the OPA strategy are further analyzed with the aid of Tables.
Fig. 9. SER with OPA over $\eta - \mu$ fading for $\mu_{S,D} = \mu_{S,R_k} = \mu_{R_k,D} = \mu, \eta = 0.5, \Omega_{S,D} = \Omega_{S,R_k} = 0$ dB with different $\Omega_{R_k,D}$ for $4-$QAM/$4-$PSK and $K = 2$.

### TABLE IV

| $\Omega_{R_k,D}$ | $P_0/P$ | $P_{R_k}/P$ | $P_0/P$ | $P_{R_k}/P$ | $P_0/P$ | $P_{R_k}/P$ |
|------------------|---------|-------------|---------|-------------|---------|-------------|
| 0.5              | 1       | 0.6270      | 0.3730  | 0.4832      | 0.2584  | 0.4036      | 0.1988      |
|                  | 10      | 0.7968      | 0.2032  | 0.9741      | 0.1513  | 0.6328      | 0.1224      |
|                  | 100     | 0.9181      | 0.0819  | 0.8712      | 0.0644  | 0.8371      | 0.0543      |
| 1                | 1       | 0.5925      | 0.4075  | 0.4368      | 0.2816  | 0.3520      | 0.2160      |
|                  | 10      | 0.8316      | 0.1684  | 0.7343      | 0.1328  | 0.6658      | 0.1114      |
|                  | 100     | 0.9557      | 0.0443  | 0.9247      | 0.0376  | 0.8995      | 0.0335      |
| 1.5              | 1       | 0.5755      | 0.4265  | 0.4131      | 0.2935  | 0.3274      | 0.2242      |
|                  | 10      | 0.8496      | 0.1504  | 0.7549      | 0.1226  | 0.6850      | 0.1050      |
|                  | 100     | 0.9683      | 0.0317  | 0.9439      | 0.0280  | 0.9232      | 0.0256      |

III, IV and V, which depict the optimal power ratios allocated to the source and the relay-nodes in terms of $P_0/P$ and $P_{R_k}/P$. In case of multiple relays, the relays are assigned with equal powers ($P_{R_k}/P = P_{R_k+1}/P$). The power ratios allocated to the source and relay nodes for asymmetric and balanced channel conditions are tabulated indicating that the numerical values are in tight agreement with the formulated power strategy in section IV. Table IV also corresponds to the case that the source is assigned with high proportion of power when $\Omega_{R_k,D}$ is larger than $\Omega_{S,R_k}$, i.e., unbalanced channel links whereas Table V verifies that the optimal power allocation scheme is dependent upon the considered modulation scheme.
TABLE V
OPTIMAL TRANSMIT POWER ALLOCATIONS WITH DIFFERENT MODULATIONS AND
\[
\mu_{S,D} = \mu_{S,R_k} = \mu_{R_k,D} = \mu, \eta = 0.5, \Omega_{S,D} = \Omega_{S,R_k} = \Omega_{R_k,D} = 0 \text{dB for two relays at SNR} = 20 \text{ dB}.
\]

| \(\mu\) | 4-PSK | 16-PSK | 16-QAM |
|-------|-------|--------|--------|
| 0.5   | 0.4832 | 0.4932 | 0.5113 |
| 1     | 0.4368 | 0.4392 | 0.4572 |
| 1.5   | 0.4130 | 0.4138 | 0.4287 |

Fig. 10. SER with OPA over \(\eta - \mu\) fading for \(\mu = 1, \eta = 0.5, \Omega_{S,D} = \Omega_{S,R_k} = 0 \text{dB and} \Omega_{R_k,D} = 10 \text{dB for} 4-\text{PSK}/4-\text{QAM} \text{ and different number of relays.}

VI. CONCLUSION

In this paper, we analyzed the end-to-end performance and optimum power allocation of regenerative cooperative systems over generalized fading channels. Novel exact and asymptotic closed-form expressions for the SER assuming \(M-\text{PSK}\) and \(M-\text{QAM}\) modulated signals were derived over independent and identically distributed as well as independent and non-identically distributed channels. The derived analytic expressions were then used to draw insight of the different fading parameters in the generalized \(\eta - \mu\) fading conditions and their impact on the end-to-end system performance. The offered results were subsequently employed in developing an optimum power allocation scheme which was shown to significantly outperform conventional equal power allocation strategy. It was also shown that the optimum power allocation scheme is
practically independent of the scattered-waves power ratio parameter from source-to-destination, while it is dependent upon the number of multipath clusters as well as the selected modulation scheme and that it overall provides significant performance enhancement.

APPENDIX I
A MATLAB ALGORITHM FOR COMPUTING THE GENERALIZED LAURICELLA FUNCTION

The Generalized Lauricella function is defined by the following non-infinite single integral,

\[ F^{(n)}_D(a, b_1, \ldots, b_n; c; x_1, \ldots, x_n) \triangleq \frac{\Gamma(c)}{\Gamma(a)\Gamma(c-a)} \int_0^1 t^{a-1} \frac{(1-t)^{c-a-1}}{(1-x_1 t)^{b_1} \cdots (1-x_n t)^{b_n}} \, dt \quad (76) \]

The numerical evaluation of the above representation was also discussed in [76, Appendix V] and can be straightforwardly evaluated with the aid of the following proposed MATLAB algorithm:

Function FD = Lauricella(a, b1, ..., bn, c, x1, ..., xn);
    f = gamma(c).*gamma(a).*gamma(c - a);
    Q = @(t) f.*t.^(a - 1).*((1 - t).^(c - a - 1).*...  
         (1 - x1.*t).^(-b1) ... (1 - xn.*t).^(-bn));
    FD = quad(Q,0,1)

APPENDIX II
PROOF OF CONVEXITY OF THE OPTIMIZATION PROBLEM

We provide the proof for the convexity of the SER expression by using (59). For mathematical tractability, we consider the proof for three relay-nodes. Based on this, the proof for larger number of nodes scenario follows immediately. To this end, the asymptotic SER can be expressed as

\[ P_{\text{SER}}^D \approx \frac{K_1}{a_0^{2(\mu_{S,D}+\mu_{S,R_1}+\mu_{S,R_2}+\mu_{S,R_3})}} + \frac{K_2}{a_0^{2(\mu_{S,D}+\mu_{S,R_1}+\mu_{S,R_2})} a_{R_3}} + \frac{K_3}{a_0^{2(\mu_{S,D}+\mu_{S,R_1}+\mu_{S,R_3})} a_{R_2}} + \frac{K_4}{a_0^{2(\mu_{S,D}+\mu_{S,R_3})} a_{R_2} a_{R_3}} + \frac{K_5}{a_0^{2(\mu_{S,D}+\mu_{S,R_2}+\mu_{S,R_3})} a_{R_1}} + \frac{K_6}{a_0^{2(\mu_{S,D}+\mu_{S,R_2})} a_{R_1} a_{R_3}} + \frac{K_7}{a_0^{2(\mu_{S,D}+\mu_{S,R_3})} a_{R_1} a_{R_2}} + \frac{K_8}{a_0^{2(\mu_{S,D}+\mu_{S,R_1})} a_{R_2} a_{R_3}} \quad (77) \]

where \( K_1, \ldots, K_8 \) are related to the channel parameters. Let \( f_1(a_0, \ldots, a_{R_3}), \ldots, f_8(a_0, \ldots, a_{R_3}) \) be functions which represent each term of \( P_{\text{SER}}^D \). For example, we assign

\[ f_1(a_0) = \frac{K_1}{a_0^{2(\mu_{S,D}+\mu_{S,R_1}+\mu_{S,R_2}+\mu_{S,R_3})}}, \quad f_2(a_0, a_{R_3}) = \frac{K_2}{a_0^{2(\mu_{S,D}+\mu_{S,R_1}+\mu_{S,R_2})} a_{R_3}}, \ldots \quad (78) \]
The principal minors of the matrix $\frac{\partial^2 f_1(a_0)}{\partial^2 a_0} = 4K_1(\mu_{S,D} + \mu_{S,R_1} + \mu_{S,R_2} + \mu_{S,R_3})(\mu_{S,D} + \mu_{S,R_1} + \mu_{S,R_2} + \mu_{S,R_3} + \frac{1}{2})$. 

The Hessian matrix of $f_2(a_0, a_{R_3})$, $\nabla^2 f_2(a_0, a_{R_3})$, can be determined as follows:

$$H(a_0, a_{R_3}) = \begin{bmatrix}
\frac{4K_2(\mu_{S,D} + \mu_{S,R_1} + \mu_{S,R_2})(\mu_{S,D} + \mu_{S,R_1} + \mu_{S,R_2} + \frac{1}{2})}{a_0^{2(\mu_{S,D} + \mu_{S,R_1} + \mu_{S,R_2} + 1)}} & \frac{4K_2(\mu_{S,D} + \mu_{S,R_1} + \mu_{S,R_2} + 1)}{a_0} \\
\frac{4K_2(\mu_{S,D} + \mu_{S,R_1} + \mu_{S,R_2})}{a_0^{2(\mu_{S,D} + \mu_{S,R_1} + \mu_{S,R_2} + 1)}} & \frac{4K_2(\mu_{S,D} + \mu_{S,R_1} + \mu_{S,R_2} + 1)}{a_0}
\end{bmatrix}.$$ 

The principal minors of the matrix $H(a_0, a_{R_3})$, $H_{11}(a_0, a_{R_3}) \geq 0$, $H_{22}(a_0, a_{R_3}) \geq 0$, and $H_{12}H_{22} \geq H_{21}$. To this effect, the symmetric Hessian matrix $H(a_0, a_{R_3})$ is positive semi-definite (PSD).

Since $\frac{\partial^2 f_2(a_0)}{\partial^2 a_0} \geq 0$ and $H(a_0, a_{R_3})$ is PSD, i.e., $\nabla^2 f_2(a_0, a_{R_3}) \geq 0$, by the second order test in [75] both $f_1(a_0)$ and $f_2(a_0, a_{R_3})$ functions are convex. Following the same methodology, it is shown that the functions $f_3, \cdots, f_8$ are also convex. Hence, by the sum rule of convexity [75], Section (3.2.1]) it follows that the total function $P_{D_{\text{SER}}}$ is convex w.r.t $a_0, a_{R_1}, a_{R_2}$ and $a_{R_3}$.

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\[ \mu_{SR_k} = 0.5, \mu_{R_1,D} = 0.5, \mu_{R_2,D} = 1 \]
\[ \mu_{SR_k} = 0.5, \mu_{R_1,D} = 1, \mu_{R_2,D} = 1.5 \]
\[ \mu_{SR_k} = 0.5, \mu_{R_1,D} = 1.5, \mu_{R_2,D} = 2 \]