An efficient method to reduce ill-posedness for dynamic load identification in short duration wind tunnels

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Abstract. For the inverse problem of dynamic load identification in short duration wind tunnels, high system ill-posedness is a main cause leading to instability and low accuracy. In this study, an efficient method is proposed to reduce ill-posedness and identify dynamic load stably. The load to be identified is written as superposition of specific basis functions and each basis function is determined by specific parameters. A genetic algorithm is built to solve the parameters and construct the load. Compared with Tikhonov regularization method, the basis function method proposed can reduce the ill-posedness and numerical studies demonstrate its efficiency and accuracy.

1. Introduction
Theoretical analysis, experimental testing, and numerical simulation are three primary means of fluid dynamics research. Wind tunnel tests are one of the most important testing methods and the aerodynamic performance of aircraft is usually tested by it. In wind tunnels, forces are usually measured by strain balances in which strain gauges are used to record local deformation when the force measurement system is under load. The stable running time of some wind tunnels is extremely short (several hundred milliseconds) due to various constraints, and this kind of wind tunnels is usually called as the short duration wind tunnels. The flow field can be established in several milliseconds after the wind tunnel starts and remains basically stable during effective operation time. When affected by the incoming surge, the system begins vibrating and it will continue to vibrate after the wind tunnel stops working. The influence of the reflected airflow still persists in the next few seconds but it is no longer the focus of the test. After a long enough time, the vibration is completely attenuated and the system is restored to rest. The balance signal oscillates violently during the entire test, and eventually decay to zero [1-2]. The relationship between the balance output signal and the dynamic load is not a simple linear relationship according to the theory of forced vibration. How to calculate the load history from the balance signal history has become an urgent problem to be solved [3].

Sanderson and Simmons [4] first introduced the load identification method into wind tunnel force tests and proposed a stress wave balance technique. Abdel et al. [5, 6] discussed the principle and calibration method of the stress wave balance in detail. Hannemann et al. [7-10] designed a multi-component stress wave balance and verified the method using a series of wind tunnel tests. Feng Wang et al. [11-13] of China Aerodynamics Research and Development Centre applied the load identification method to the pulse combustion wind tunnel, and used the method of regularization to
obtain the load history. Jie Liu et al. [14] proposed an analytical method based on the Gegenbauer polynomial expansion theory and regularization method to identify dynamic loads acting on stochastic structures. A global kernel function matrix [15] for load identification was constructed to reduce ill-posedness based on an interpolation method. A regularized cubic B-spine collocation method [16] was studied to overcome the deficiency of ill-posed problem. Shape function methods [17-19] were developed to identify dynamic load. An improved method [20] using moving weighted least square technique was proposed lately and it can identify dynamic load more accurately than Green’s kernel function method and traditional shape function method using moving least square fitting.

This article introduces the method by which the load identification equation is established in section 2.1, and illustrates the necessity of regularization to solve such an inverse problem in section 2.2. A basis function method is proposed and a related algorithm is introduced in section 3. Numerical examples of two force test systems with different characteristics are studied in section 0, which shows the correctness and effectiveness of the method.

2. Problem formulation

2.1. Establishing the load identification equation

The equation of load identification can be established by discretizing the Duhamel's integral.

For a force test system in a wind tunnel, the input is load \( f(t) \), and the output is the balance signal \( y(t) \). \( f(t) \) and \( y(t) \) meet the Duhamel's integral [21]

\[
y(t) = \int_0^t f(\tau) I(t-\tau) d\tau
\]

In which \( I(t) \) is the unit impulse response function of the force measuring system. equation (1) can be discretized in the time domain and written in a matrix product form

\[
If = y
\]

In which

\[
I = \Delta t \cdot \begin{bmatrix} I_1 \\ I_2 & I_1 \\ I_3 & I_2 & I_1 \\ \vdots & \vdots & \vdots & \ddots \\ I_n & I_{n-1} & I_{n-2} & \cdots & I_1 \end{bmatrix}
\]

\[
y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \cdots \\ y_n \end{bmatrix}^T
\]

\[
f = \begin{bmatrix} f_0 & f_1 & f_2 & \cdots & f_{n-1} \end{bmatrix}^T
\]

Subscripts \( 0 - n \) indicate discrete time points. equation (2) is the load identification equation.

2.2. Regularization methods

Section 2.2.1. analyses the problem of solving this type of equation and the necessity of introducing regularization methods. In section 2.2.2. Tikhonov regularization method is introduced, and its regularization parameter is determined by the Generalized Cross Validation Method (hereinafter abbreviated as GCV method).

2.2.1. The necessity of regularization. In equation (2), \( I \) can be obtained by the dynamic calibration [22]. Solving \( f \) from \( I \) and \( y \) belongs to the second kind of inverse problem [23], the direct solution is

\[
f^{\text{naive}} = \arg \min_f \|If - y\|_2
\]

The exact solution satisfies
\[ If_{\text{exact}} = y_{\text{exact}} \]  

And

\[ y = y_{\text{exact}} + n \]  

In which \( n \) denotes the noise. It can be proved [24, 25] that

\[ \frac{\| f_{\text{exact}} - f_{\text{naive}} \|}{\| f_{\text{exact}} \|} \leq \frac{\sigma_1}{\sigma_n} \frac{\| n \|}{\| y_{\text{exact}} \|} \]  

In which \( \sigma_1 \) and \( \sigma_n \) are the largest and smallest singular value of matrix \( I \) respectively. In this problem \( \sigma_1 \) is usually far greater than \( \sigma_n \), and deviation of the direct solution is intolerable even though there is just tiny noise. That’s why we need to use regularization methods.

2.2.2. A Tikhonov method. Tikhonov regularization method works by controlling the magnitude of \( \| f \|_2 \), and the solution is

\[ f_{\lambda} = \arg \min_f \{ \| If - y \|_2^2 + \lambda^2 \| f \|_2^2 \} = (I^T I + \lambda^2 E_n)^{-1} I^T y \]  

In which \( E_n \) is an \( n \times n \) identity matrix and \( \lambda (\lambda > 0) \) is a regularization parameter which balances the weights of \( \| If - y \|_2^2 \) and \( \| f \|_2^2 \). \( \lambda \) is determined by GCV method [26] in this article

\[ \lambda = \arg \min_{\lambda} \{ G(\lambda) \} = \min_{\lambda} \left\{ \frac{\| If_{\lambda} - y \|_2^2}{n - \sum_{i=1}^n \sigma_i^2} \right\} \]  

And 'Tikhonov method' is short for this Tikhonov regularization with parameters determined by GCV method.

3. A basis function method

Section 3 proposes the basis function method and the corresponding algorithm.

\[ f(t) \] is written as a superposition of \( N \) basis functions \( w_j(t) \)

\[ f(t) = \sum_{j=1}^{N} w_j(t) \]  

\( w_1(t) \ldots w_N(t) \) have the same form of basis function \( (w(t)) \) but different parameters. Each basis function \( w_j(t) \) is determined by \( m \) parameters which are recorded by a column vector \( p_j \)

\[ p_j = \begin{bmatrix} c_{j1} & c_{j2} & \cdots & c_{jm} \end{bmatrix}^T \]  

then \( f \) can be written as

\[ f_p = \sum_{j=1}^{N} w_j(p_j) \]  

The relationship between \( w_j \) and \( w_j(t) \) is like that between \( f \) and \( f(t) \), define

\[ P = \begin{bmatrix} p_1 & p_2 & \cdots & p_j & \cdots & p_N \end{bmatrix}^T \]  

Then \( f_p = \text{function}(P) \) and \( If_p \) is used to fit \( y \). Define the objective function as

\[ r(P) = \| y - If_p \|_2 \]  

Then we just need to solve the parameter vector \( P \) to get a sufficiently small \( r \), thus the inverse problem is turned into a problem of parameter optimization. Ultimately, an optimization algorithm is used and the threshold value is \( r_f \). When \( r < r_f \), the optimization is ended and \( f_p \) is obtained.
The choice of the basis function and the algorithm for optimization are critical in the process described above. The basis function should satisfy two conditions: 1) It can represent load characteristics in the short-term wind tunnel; 2) It can be quickly and stably solved. Characteristics of the load are studied based on the operation of the short-term wind tunnel and the results of previous force or pressure measurements. The load usually reaches a certain value instantaneously and fluctuates slightly around it. Considering the sudden start and shutdown of the engine during the engine test, or the rapid adjustment of the flight attitude or the rudder, the loads have obvious step characteristic. Therefore, we take the following ramp step function as the basis function

\[
w(t) = \begin{cases} 
0 & t < c_1 \\
c_1(t - c_1) & c_1 \leq t \leq c_2 \\
c_3(t - c_2) & t > c_2
\end{cases}
\]  

(17)

\(c_1\) is the starting time of the ramp, \(c_2\) is the ending time of the ramp and \(c_3\) is the slope of the ramp. \(w(t)\) is determined by 3 parameters \((c_1, c_2, c_3)\) so the \(m\) in equation (13) is 3 and

\[p_j = [c_1, c_2, c_3]^T\]

(18)

We use the Genetic Algorithm for the optimization, which is a heuristic search inspired by Charles Darwin’s theory of natural evolution. The algorithm reflects the natural selection process in which the fittest individuals are selected for reproduction. In the process of population evolution, the best individual is finally achieved through selection, crossover and mutation [27].

The overall flow chart of the method is shown in Figure 1.

\[\text{Figure 1. Flow chart of the basis function method.}\]
Table 1. Parameters of unit impulse response function of the force measuring system.

| \( i \) | \( f_i (Hz) \) | \( A_i (mV \cdot N^{-1} \cdot s^{-1}) \) | \( \gamma_i \) |
|-------|-----------|----------------|--------|
| 1     | 15.57     | 4.06           | 0.0020 |
| 2     | 40.19     | 1.82           | 0.0018 |
| 3     | 87.38     | 0.54           | 0.0016 |

Table 2. Relative error (\( r \)) of the identified result at different noise levels. (identification of a single-step load)

| noise | 0% | 1% | 2% | 3% | 4% | 5% |
|-------|----|----|----|----|----|----|
| \( \lambda \) (e-04) | 2.95 | 13 | 17 | 23 | 25 | 29 |
| \( r \) (%) | 0.72 | 6.97 | 9.53 | 10.51 | 11.79 | 12.48 |
| \( \gamma \) (N=1) | 0 | 1.42 | 1.85 | 1.29 | 1.68 | 1.06 |

Table 3. Relative error (\( r \)) of the identified result at different noise levels. (identification of a multi-step load)

| noise | 0% | 1% | 2% | 3% | 4% | 5% |
|-------|----|----|----|----|----|----|
| \( \lambda \) (e-04) | 2.95 | 15 | 20 | 27 | 32 | 37 |
| \( r \) (%) | 0.82 | 8.89 | 12.21 | 13.97 | 14.62 | 14.69 |
| \( \gamma \) (N=50) | 2.63 | 2.78 | 2.92 | 3.40 | 3.84 | 4.25 |

\( y_{ji} \) is the object and it is fitted by \( I f_{ji} \) in the flow chart. The first subscript \( j \) represents the current number of basis functions and the second subscript \( i \) represents the current iteration number of the Genetic Algorithm. \( p_j \) (parameters of \( w_j(t) \)) is obtained by the GA operation in which the threshold is \( r_j \), and the residual is calculated after that. \( N \) equals to the last \( j \) and the identification is finished if the residual is smaller than \( r \) (a threshold determined by the accuracy requirement), or the number of basis functions has to be increased (\( j = j + 1 \)) and retry. \( y_{ji} \) equals to \( y \) initially and \( y_{ji} \) equals to the residual of the previous loop. Since the number of load steps is very limited in experimental practice, \( N \) usually turns out to be a relatively small integer under the accuracy requirement. There are \( N \) GA operations and only 3 parameters need to be solved in each GA operation, so the method is efficient.

4. Numerical studies

In this section two measuring conditions with different characteristics are studied. The basic idea of the numerical study is: given the number of sampling points \( n \), sampling interval \( \Delta t \), exact load \( f \), and unit impulse response function \( I \), then \( y \) is calculated by equation (5), and \( y_n \) is obtained by adding noise. The method in section 2.2.2. and section 3. are used to identify \( f_{\text{inverse}} \) respectively. Compare \( f \) and \( f_{\text{inverse}} \) by calculating the relative error \( r \).

\[
    r = \frac{\| f_{\text{inverse}} - f \|_2}{\| f \|_2} \times 100\% 
\]  

(19)

Substitute \( f_{\text{inverse}} \) and \( I \) into equation (5) to calculate the output \( y_{\text{inverse}} \) inversely, then compare \( y \), \( y_n \) and \( y_{\text{inverse}} \).
4.1. Identification of a single-step load

Simulate a relatively simple case: the load is a typical single-step load shown in Figure 2. The force measuring system can be considered as a linear vibration system because only slight amplitude vibration is allowed in the experiment. The vibration can be obtained by the modal superposition method, and the unit impulse response function can be written as [28]

\[ I(t) = \sum_i A_i e^{-2\pi i f_i} \sin(2\pi \sqrt{1-y_i^2} f_i t) \]  

(20)

And the parameters in equation (20) are shown in Table 1. Let \( n = 500 \), \( \Delta t = 0.001s \), and the noise is a random signal, the amplitude of which is a certain percentage of the standard deviation of \( y \).

When the noise amplitude is 2% (which means the amplitude of the random signal is 2% of the standard deviation of \( y \)), \( f_{\text{inverse}}(t) \) and \( f(t) \) are shown in Figure 2. It can be seen that both methods can identify the general trend of the load. The result of Tikhonov method still has obvious fluctuations in the stable section. The result of basis function method is very accurate. When the noise amplitude is 5%, \( y(t) \), \( y_n(t) \) and \( y_{\text{inverse}}(t) \) are shown in Figure 3. \( y_{\text{inverse}}(t) \) is in good agreement with \( y(t) \), and neither fits the high-frequency components of the noise, which means that both methods have a regularization effect. The denoising ability of Tikhonov method is not good enough, so there is obvious fluctuation in the solution, while the result of basis function method is precise. When the noise amplitude is 1% ~ 5% (the amplitude of the random signal is 1% ~ 5% of the standard deviation of \( y \)), the identification results are shown in Table 2, which shows that the relative error in both methods increases with the increase of noise. \( \lambda \) has to be increased continuously to enhance the degree of regularization in order to obtain a more stable solution in Tikhonov method, and the error increases more quickly with the increasing noise. The result of basis function method is very reliable, and the error can still be kept low as the noise increases.

![Figure 2. Real load and identified load. (identification of a single-step load)](image)

![Figure 3. Theoretical, noisy and inverse output. (identification of a single-step load)](image)

4.2. Identification of a multi-step load

Now consider a complicated case: the load has multiple steps during the test. The exact load given has three step changes which is shown in Figure 4. We still use the \( I(t) \) in section 4.1. When the noise amplitude is 2%, \( f_{\text{inverse}}(t) \) and \( f(t) \) are shown in Figure 4, it can be seen that the error of Tikhonov method is significant, while the result of basis function method is quite accurate. When the noise amplitude is 1% ~ 5%, the results are shown in Table 3. Comparing Table 2 and Table 3, the errors...
of both methods increase when the load trends to become more complicated. When the noise amplitude is 5%, $y(t)$, $y_n(t)$ and $y_{inverse}(t)$ are shown in Figure 5. It can be seen that Tikhonov method is overfitted and basis function method is slightly underfitted. The under-fitting situation can be relieved by increasing $N$. As shown in Figure 6, the error increases as the noise increases with a fixed $N$, and the error decreases as $N$ increases if we keep the noise level constant. When $N$ increases to 3, the error decreases significantly, as the true load happens to have 3 obvious step changes. When $N$ is larger than 20, the error decay slowly. As $N$ increases, the identified loads are shown in Figure 7. When the noise amplitude is 2% and $N$ is 50, the relative error is about 4%, which means the identification is fairly accurate.

![Real load and identified load. (identification of a multi-step load)](image1)

![Theoretical, noisy and inverse output. (identification of a multi-step load)](image2)

![Relative error of the identified result when noise level or basis function number changes.](image3)

![Real load and identified load when basis function number changes.](image4)

Since the direct solution (solved without using any regularization method) can deviate to different order of magnitude easily according to equation (9), numerical studies prove that the Tikhonov method does have the effect of reducing the ill-posedness of the problem, but it is easy to overfit and leads to a significant error when the noise level is slightly high or the load is complicated (relative error is more
than 10% when the noise amplitude is above 3% in numerical studies). While the basis function method is much more stable and accurate.

5. Conclusions
In conclusion, the following conclusions can be proposed in this article.
1) The load identification equation for short duration wind tunnels can be established by discretizing the Duhamel's integral in the time domain. Tikhonov regularization with parameters determined by GCV can reduce the ill-posedness of the load identification problem but it is easy to overfit and lead to significant error.
2) The basis function method proposed can reduce the ill-posedness and has high identification accuracy because it makes full use of the priori knowledge about the load.
3) The basis function method proposed is efficient because the number of load steps is very limited in experimental practice, and the load characteristic is depicted well by the basis function which has only 3 parameters need to be solved.

The next step is to improve the measurement technology in the short duration wind tunnel, and continue to refine and verify the identification methods with experimental practice.

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