The nature and line shapes of charmonium in the \( e^+e^- \rightarrow D\bar{D} \) reactions

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Fano resonances are a general physical phenomenon produced by the interference of a continuum and a closed channel resonant amplitude giving rise to an asymmetric line shape of states. We point out that the \( \psi(3770) \) state observed in the \( e^+e^- \) reactions with an anomalous line shape can be explained naturally as a Fano resonance embedded in the \( D\bar{D} \) continuum. A Fano-type analysis is compared to results from a coupled-channel formalism. Coupled-channels effects are found to be coming from a pole at 3716.0 \( \pm \) 30.0 MeV. As a by-product, the structure identified as \( X(3900) \) in the Belle data, is found to be the tail of the \( \psi(3770) \) state, distorted by the opening of the \( D^*\bar{D} + c.c \) channel and the onset of the \( \psi(4040) \), thus excluding the assignment as a genuine charmonium state.

The proper treatment of near-threshold effects is of particular importance in theoretical studies of charmonium spectroscopy. In fact, most of the experimental candidates for exotic X, Y, Z states lie close to open-charm thresholds. A well known and intensively studied example is \( X(3872) \), positioned within 1 MeV of the nominal \( D^*\bar{D} \) threshold. The newly discovered structure \( Z_c^+ \) \((3900) \) was observed first in the \( J/\psi \tau^+ \) invariant mass spectrum by the BESIII collaboration\(^ {1} \) and soon after confirmed by Belle\(^ {2} \) and CLEO\(^ {3} \), respectively. Its mass is also close to \( D^*\bar{D} \) threshold and the observed decay channel requires a \( c\bar{c}d\bar{d} \) quark flavor structure component. Meanwhile, the BESIII collaboration has observed other charmed charmonium-like states, namely \( Z_c^+ \) \((4020) \)\(^ {4} \) and \( Z_c^+ \) \((4025) \)\(^ {5} \), waiting for confirmation by other experiments. These states contain the same flavor structure as \( Z_c^+ \) \((3900) \) and are located near the \( D^*\bar{D}^* \) threshold.

The internal structure and dynamical properties of a discrete quasi-bound state coupled to a continuum of unbound states are encoded in its line shape. Only in simple potential problems the spectral distribution comes close to the widely used Lorentz- or Breit-Wigner shape, respectively. Under realistic conditions the shape of resonance curves is distorted by the interaction between the discrete and continuum components of the spectra. This kind of interference among states of various configurations is ubiquitous in quantum physics and leads to a plethora of interesting phenomena in nuclear, atomic, condensed matter and quantum optical physics. One of the recent exciting progress in this area is to control the atomic line shapes by the manipulation of the intense laser with different frequencies\(^ {6} \). Obviously, this kind of direct manipulation is not within reach of hadron physics. The basically different situation in hadronic systems is their much shorter lifetime because of their strong interaction with neighbouring states. Hence, the line shapes of hadronic states are often found to be convoluted by overlapping contributions and, additionally, are influenced by coupled-channel effect, as e.g. in baryonic resonances\(^ {7} \).

On the theoretical side, a profound approach accounting properly for interactions between asymptotically open and closed channels was formulated a long time ago in a pioneering work by Fano\(^ {8} \). While the original formulation was in the context of atomic physics, describing there the frequently observed phenomenon of the so-called auto-ionizing states, it was soon recognized that the Fano-mechanism is a phenomenon of compelling generality, appearing in any quantum system at any physical scale when interactions between open and closed reaction channels occur. The unique and new feature of charmonium and hadron physics in general is, however, the interaction of an absolutely confined \( Q\bar{Q} \) configuration, whose constituents are not separable, with the open \( D\bar{D} \) channels, requiring the creation of an additional pair of light quarks out of the vacuum. We also emphasize that two distinct dynamical effects have to be considered in the analysis of quarkonium resonances. First, there are interactions within the \( Q\bar{Q} - D\bar{D} \) system, leading to configuration mixing. Second, we have to take into account channel coupling interactions in the production reaction which superimpose the generic configuration interactions of the previous type. A more recent application to a nuclear physics problem is found in Ref.\(^ {9} \). Very recently, we managed to extract the configuration mixing of open and closed channels from the asymmetric line shapes in hadron physics. With this paper, we intend to shed light
on the special features encountered close to thresholds in the charmonium region, closely following the idea promoted in our previous work [10].

By the pioneering work of Eichten et al. [11], coupled-channel studies were introduced into charm physics from the very beginning. Such investigations have become of new interest in recent years [12, 13] in connection with the newly found charmonium and charmonium-like states. The life time of states below the open charm $D\bar{D}$ threshold is only limited by flavor changing weak interactions, leading to the decay into non-charmed mesons. Charmonium states above the $D\bar{D}$ open charm threshold have a shorter lifetime because of their coupling to hadronic decay channels by strong interactions. The coupling to open channels either via weak or strong interactions contributes dispersive self-energies to the mass operator of charmonium states. These induced self-energies have been studied before and known to be essential for understanding the systematic of the charm spectrum [13].

Corresponding effects are also contributing to the charmonium states above the $D\bar{D}$ threshold but with the important difference that now hadronic decay channels are open explicitly. Hence, decays into the $D\bar{D}$ open charm states mentioned above are possible. As the first charmonium state above the $D\bar{D}$ threshold, the $\psi(3770)$ state is ideal to investigate how this open charm effect influences the properties of states. In fact, its line shape is anomalous in the measured cross section of $e^+e^\to D\bar{D}$ [14–16] and $e^+e^\to hadrons$ [17, 18], indicating coupled-channel effects. In previous studies [19–23] the major source of interactions was attributed to the $\psi(3686)$ state. Our previous paper [10] has pointed out that the $\psi(3770)$ state could be considered as a bare $c\bar{c}$ charmonium state embedded in the $D\bar{D}$ continuum, as schematically depicted in Fig. 1. This idea is easily extended to higher lying states and open charm channels, e.g. the $\psi(4040)$ state and $D^*\bar{D} + c.c$ channel. The $c\bar{c}$ state by itself is a confined QCD-configuration, thus corresponding to a closed channel which, however, is emersed into the open channel $D^*(\bar{D})$ continuum. Suppose that we know the spectrum of $c\bar{c}$ states with wave functions $\phi_c$ and the unperturbed $D\bar{D}$ elastic scattering states with wave functions $\phi_{\omega}$ at energy $\omega = \sqrt{s}$. Following Fano [3] and also as briefly demonstrated in our previous paper [10], the eigenstates of an interacting system with a number of closed channels and one open channel are given by

$$\Psi_\omega = \sum_c z_c(\omega)\phi_c + \int d\omega'\int d\omega'\phi_{\omega'} \phi_{\omega'},$$

$$= x_\omega(\omega)\chi_\omega + x_\omega(\omega)\chi_s$$

(1)

where $\chi_{c,s}$ denote the complete, normalized charmonium and scattering components, respectively, including channel mixing effects. The channel interaction may push one or a few eigenstates below the particle emission threshold, and one may speculate whether $\psi(3686)$ is of such a nature. The important message of Eq. (1) is that the observed charmonium states have to be considered as varying mixtures of $c\bar{c}$ and $D\bar{D}$ configurations. As depicted in Fig. 1, channel coupling is of increasing complexity at higher energies where more and more channels are open.

One purpose of this paper is to point to the relationship of the $c\bar{c}$ states and the resonances observed in the production cross sections by $e^+e^-$-annihilation. Hence, the task is twofold, namely combining properly the reaction model on the one side and the configuration model on the other side. Starting from an initial reaction channel $|\tau\rangle$ at total energy $\omega$, let be $T$ the transition operator for the production of the states $\Psi_\omega$. In the following formulae we omit the indices of partial waves because we are studying the production and decay of the $1^{--}$ charmonium states which is restricted to a particular $P$-wave. But obviously the formalism is easily extended to any other partial waves. The form factor $F_T \equiv M_{\omega T}$ of that transition is given by the production amplitude

$$M_{\omega T}(\omega) = \langle \Psi_\omega | T | \tau \rangle = x_\omega(\omega)\langle \chi_\omega | T | \tau \rangle + x_s(\omega)\langle \chi_s | T | \tau \rangle .$$

(2)

Of course, here we are interested in the case $|\tau\rangle = |e^+e^-\rangle$. However, it is of interest to note that on a quite different scale the same approach applies to the absorption spectroscopy of helium controlled by extreme ultraviolet laser light [6]. In the case here the different final states populated by the $\psi(3770)$ decay are expected to control its line shape because different non-resonant continua and relative phases are present in the various decay channels.

While referring for details to Ref. [8], here we directly display the result. We introduce $q_{\omega T}(s) = \langle \chi_\omega | T | \tau \rangle / \langle \chi_s | T | \tau \rangle$. For the model of Eq. (1) specifically reduced to one confined and one continuum channel, the dressed form factor is found to be determined by [8, 10]

$$|F_T|^2 = |\langle \chi_s | T | \tau \rangle|^2 q^{-2}|q - \varepsilon|^2 1 + \varepsilon^2 .$$

(3)

Above, $q = q_{\omega T}(m_{\psi'})$ is the line shape parameter where $\chi_{c,s}$ is the original confined $c\bar{c}$ state but dressed by a cloud of (virtual) continuum excitations [8, 10] and

$$\varepsilon = cot\delta_m = -\frac{s + m_{\psi'}^2}{m_{\psi'}^2} .$$

(4)
is given by the additional mixing phase shift $\delta_m$ induced by the channel coupling. The coupling to the $\bar{D}D$ continuum assigns to $\phi_c = \psi'$ a spectral width $\Gamma_\psi'(s)$, accompanied by a mass shift $\Delta m_\psi'(s)$ leading to the dynamical mass $m_\psi'(s) = m_0' + \Delta m_\psi'(s)$. Assuming $q \sim \text{const.}$ over the resonance, Eq.(3) shows that the line shape would develop a dip down to zero at energy $s_0$ where $q = \varepsilon(s_0)$. If many open channels are contributing, the interference minimum will be superimposed on a finite background. For $q \gg \varepsilon$, the usual Breit-Wigner shape is recovered, given by the spectral form factor

$$F_\psi'(s) = \frac{A_\psi'}{s - m^2_\psi' + im_\psi' \Gamma_\psi'}.$$  (5)

where the $\Gamma_\psi'(s) = g_{\psi'\bar{D}D}^2(p^2_0 + p^2_1)/6s$ is the energy dependent width for the final $D$-meson carrying charge $q = 0$ and $\pm 1$ with the momenta $p_0 = \sqrt{s/4 - m^2_{\bar{D}D}}$. The known $A_\psi' = m^2_\psi g_{\psi'\bar{D}D}/g_{\psi'\gamma}$, with $g_{\psi'\bar{D}D}$ being the dimensionless coupling constants of $\psi(3770)$ to $\bar{D}D$ channel, allows to determine the overall normalization of Eq.(3) to be $|\langle \chi_s|T|\tau \rangle|^2 = |A_\psi F_{bg}|^2$. Here $F_{bg}$ accounts for the energy dependence of the bare $D\bar{D}$ production amplitude without $c\bar{c}$ coupling. Since the latter, in fact, is unknown, here we use $F_{bg} = 1/(s - m^2_{bg} + im_{bg} \Gamma_{bg})$ simulating the slow variation with energy mainly due to the $t$- and $u$-channel interaction among $D$-mesons. We determine the photon-vector coupling constant $g_{\psi'\gamma}$ by the relation $\Gamma_{\psi'\bar{D}D} = 4\pi\alpha^2 m_\psi/3g^2_{\psi'\gamma}$, with the fine structure constant $\alpha \simeq 1/137$ and the $\Gamma_{\psi'\bar{D}D} = 0.265$ keV from the Particle Data Group [24]. The BES data [14] are surprisingly well described by integrating Eq.(3) in the $D^0\bar{D}^0(D^+D^-)$ two body phase space, as shown in Fig. 2. The fitted parameters are summarized in Table I. The $\chi^2/d.o.f$ is much smaller than the value of 2.72(3.27) for a fit with the simple Breit-Wigner form in Eq.(5). The extracted $m_{bg}$ and $\Gamma_{bg}$ are much bigger than those of the $\psi(3686)$, but their uncertainties are potentially large because of the approximate treatment of the $F_{bg}$. However, as can be seen, the bare masses and widths of the $\psi(3770)$ in the neutral and charged channels are consistent with each other and the different behaviour in production cross sections is almost totally from the phase space variance of the $D^0\bar{D}^0(D^+D^-)$, namely the mass gap of neutral and charged $\bar{D}D$ dynamics. This is verify the statement in our previous paper [16] that Eq.(3) means a factorization of the coupled channels dynamics contained in the Fano parameter $q$ and the $D\bar{D}$ dynamics included in the $|\langle \chi_s|T|\tau \rangle|^2$ term.

For a more extended description of the physical situation in Fig 1 covering a larger energy interval, a treatment of the multi-channel case is necessary. The Fano-approach, in fact, is flexible enough to achieve that goal, finally coming down to the solution of the coupled-channel problem for any number of closed and open channels [8, 9]. However, here we use directly more conventional coupled-channel theory based on the Bethe-Salpeter equation for the full $D^+(\bar{D})T$-matrix, as depicted in the upper panel of Fig. 3. The formalism is easily extended to non-$\bar{D}D$ channels which play a key role in understanding the nature of the $\psi(3770)$ [22, 23]. We follow our practice of Ref. [1], but also used elsewhere, e.g. Ref. [20], and decompose the interaction kernel $V = V + V_R$ into the pure $D\bar{D}$ elastic part $V$ and resonant part $V_R$. This amounts to consider $\psi(3770)$ as a s-channel $D\bar{D}$ resonance which is well justified because of the weak non-$\bar{D}D$ decay of $\psi(3770)$, resembling $\rho(770)$, the latter being produced as an elastic $\pi\pi$ isovector $P$-wave s-channel resonance. Other charmonium states with weaker coupling to the $D\bar{D}$ channel are included in the $V_R$ as inelastic resonances. Applying the two potential formula, the $T$-matrix separates accordingly:

$$T_{ij}(s) = \delta_{ij} \delta_1 \tilde{T}(s) + T_{ij}^{R}(s)$$  (6)

where $\tilde{T}$ is the purely diagonal elastic amplitude and

$$T_{ij}^{R} = \Gamma_{ij}^{\text{out}} \Gamma_{ij}^{\text{out}} T_{ij}^{R} \Gamma_{ij}^{\text{out}}$$  (7)

$$t_{ij}^{R} = [1 - V_R]^{\text{out}} V_{ij}^{R}$$  (8)

with the self-energies $\Sigma_i = G_i \Gamma_i^{\text{out}}$ and $G_i$ being the bare

FIG. 2: The fitted total cross section of $e^+e^- \rightarrow D^0\bar{D}^0$ and $e^+e^- \rightarrow D^+D^-$ reactions with Eq.(3), labeled as the solid and dashed curve, respectively. The data are from the BES collaboration [14].

TABLE I: Fitted parameters in the $e^+e^- \rightarrow D^0\bar{D}^0$ and $e^+e^- \rightarrow D^+D^-$ reactions with the Fano formulae in Eq. 3.
of the formalism, however, are already clearly visible in example on an effective Lagrangians. The main features 1. Here \( \xi \) is a candidate for a Fano resonance with different line shapes formation and decay. We use the relation \( F_V = M + TGM \) to determine the form factor [20]:

\[
F_V(s) = F_{\psi}(s)[1 - V^R\sum_{i,k}^{-1}M_k]
\]

as displayed in the lower panel of Fig. 3. Here \( M_k \) is the electro-production vertex. The unitarity of Eq. (9) could be kept by using the Omnes function satisfying elastic unitarity in the bare form factor [27], as clearly proved in Ref. [24]. However, presently the Breit-Wigner parametrization of the form factor in Eq. (9) is sufficient because of the narrow width of \( \psi(3770) \), much smaller than, e.g. for the \( \rho \)-meson. Another reason we do not use Omnes function at present is that it is related to the \( P \)-wave \( \bar{D}\bar{D} \) phase shift up to high energies, which is unfortunately less theoretically known [21] and experimentally non-achievable nowadays. The measured form factor \( F_V \) is dressed by the contribution from other resonances, as expressed symbolically by the summation in Eq.(9). By comparison to the Fano-formula one finds a close correspondence of the \( q \)-parameter to the terms in the denominator, however, in considerably generalized form. In this context, the \( \psi(3770) \) is expected to be a candidate for a Fano resonance with different line shapes in various final states.

We express the imaginary part of the self-energies as

\[
Im\Sigma = \sigma_i \xi^2 \Gamma_i^{out}(s) \theta(s - s_i)\]

with \( \Gamma_i^{out} = F_{\psi} \) and \( \Gamma_i^{out} = 1 \).

Here \( \xi_i = 48\pi\sqrt{s} \) with the two-body phase space \( \sigma_i = \sqrt{(s - (m_1 + m_2)^2)(s - (m_1 - m_2)^2)} / 48\pi \). We use the subtracted dispersion relation to regularize the self energies loop integral,

\[
\Sigma_i = \eta_i s + \frac{s^2}{\pi} \int_{s_i}^{\infty} ds' \frac{Im\Sigma_i}{s'^2(s' - s - i\epsilon)}
\]

with \( s_i \) being the threshold energy of the \( i \)-th channel and the subtraction constant is denoted by \( \eta_i \).

The model could be constructed dynamically, based for example on an effective Lagrangians. The main features of the formalism, however, are already clearly visible in the more schematic parametrization used here. We write

\[
M_k = c_k - s \sum_i \frac{g_i^2 g_{ij}^2}{s - m_i^2}, \quad V_{ij}^R = - \sum_i \frac{g_i^2 g_{ij}^2}{s - m_i^2}
\]

The linear dependence on \( s \) complies with gauge invariance and using \( c_1 = 1 \) ensures that in Eq. (9) the usual Breit-Wigner form of the \( \psi(3770) \) form factor is retained. The data [14, 15] show two dips in the cross section of \( e^+e^- \rightarrow \bar{D}\bar{D} \) below 4.0 GeV. Herein the attraction of \( P \)-wave \( \bar{D}\bar{D} \) interaction is calculated to be weak [21], and also the two relevant channels are weakly coupled. As a result these dips could not be dynamically originated in this simple case, and we could speculate that the dips are from the electro-production vertices \( M_k \) in Eq. (9).

So the coupling constants, and therefore the interaction matrix elements, are largely determined by the location of the interference minima in the cross section. It should be pointed out that the \( e^+e^- \) collision can lead to the production of sub-threshold charmonium states like \( J/\psi \) and \( \psi(3686) \). Hence, they contribute to the summation in \( M_k \). In the T-matrix, those states are sub-threshold and, if not at all, would correspond to \( \bar{D}\bar{D} \) bound states, i.e. poles of the T-matrix on the negative energy axis. Thus, they are reachable by the production part \( M_k \), but they do not contribute to the \( \bar{D}\bar{D} \) open channel part. In our formulation, the T-matrix obviously accounts only for open channels. As a consequence, if the states corresponding to closed \( \bar{D}\bar{D} \) channels are included into the \( \bar{V}^R \) as the same in \( M_k \), in principle we should explicitly extend its meaning to subthreshold imaginary values of the relative momentum. That is a tempting idea worthwhile to be investigated in the future. At present, however, nothing is known safely about the structure of such sub-threshold charm molecules to which those \( \bar{D}\bar{D} \) bound states would correspond.

We first explore a reduced problem (Fit-I) modelling the data with \( \sqrt{s} \leq 3.8 \) GeV from BES collaboration (14 points) [14] both in the neutral and charged \( \bar{D}\bar{D} \) channels. Including only the \( \bar{D}\bar{D} \) channel and \( \psi(3686) \) state and using the once subtracted self-energies, Eq. (10), the number of free parameters reduces to four. Results are displayed in Fig. 3 and Tab. 1. As shown, we have achieved an excellent description of the line shape of the \( \psi(3770) \), whose \( \chi^2 / d.o.f \) are considerably better than in previous isobar model analyses [19, 20].

Next, we further consider the data set with \( \sqrt{s} \leq 4.0 \) GeV from Belle (13 points) [12] and CLEO (2 points) [16] collaborations by additionally including the \( \psi(4040) \) state, using the twice subtracted self-energies, Eq. (10) in the \( D^*\bar{D} + c.c \) channel (Fit-II). A reasonable agreement with the data is achieved, however, slightly worse than achieved before in Fit-I, as indicated by \( \chi^2 / d.o.f = 1.43 \). It is puzzling that the data point of the Belle collaboration at \( \sqrt{s} = 3.770 \) GeV is obviously much larger than the nearby data of BES and CLEO collaborations, see Fig. 3. If we disregard that data point, the \( \chi^2 / d.o.f \) decreases significantly, as shown in Tab. 1. In Fig. 4 our results are summarized together with the bare line shape of the \( \psi(3770) \). Not only the \( \psi(3770) \) line shape is properly described, but also the broad enhancement around 3.90 GeV is explained equally well. It is fully determined by the tail of the \( \psi(3770) \) bending by the \( D^*\bar{D} + c.c \) threshold opening and the onset of the \( \psi(4040) \), leaving little room for a real resonance, quoted

\[
\begin{align*}
M_h &= c_h - s \sum_i \frac{g_i^2 g_{ij}^2}{s - m_i^2}, \\
V_{ij}^R &= - \sum_i \frac{g_i^2 g_{ij}^2}{s - m_i^2}
\end{align*}
\]
FIG. 4: Total cross section of $e^+e^- \rightarrow D^0\bar{D}^0$ (upper) and $e^+e^- \rightarrow D^+D^-$ (lower) reactions in parameterized coupled-channel model. The dotted curve is the bare $\psi(3770)$ contribution in Fit-II.

in the literature as $X(3900)$.

Our extracted parameters in the neutral and charged $D\bar{D}$ channels are consistent with each other within the uncertainties, as can be seen in Tab. III. This confirms the speculation that the distinct behavior of the $D^0\bar{D}^0$ and $D^+D^-$ channels could be largely explained by the different phase space caused by the mass gap between neutral and charged $D$-mesons [28, 29]. It should be noted that $m_{\psi'}$ can be viewed as the bare mass of the $\psi(3770)$. Its fitted value is a little bigger than that in the isobar models [19, 20], but consistent with that in the parameterized coupled-channels K-matrix model [22]. The pole of the $\psi(3770)$ is found to be stable in the complex plane at the location $(3778.2 \pm 3.8)$ MeV + i $(13.9 \pm 1.4)$ MeV in both fits, indicating its robust properties.

The mass of $\psi(3686)$ is kept fixed in the above fits. If we allow it to vary freely, we find that it is highly uncertain with a value of $3716.0 \pm 30.0$ MeV, whose upper bound is consistent with the value of the above Fano-type formula. Though its lower bound is compatible with the mass of $\psi(3686)$, the existence of a hidden $1^{--}$ charmonium state as indicated by lattice calculation [30] is not
excluded by our analysis. Moreover, the comparatively large uncertainty probably indicates self-energy contributions induced by continuum couplings, e.g. \cite{10}.

In summary, we have analysed the $e^+e^- \rightarrow D^0\bar{D}^0$ and $e^+e^- \rightarrow D^+D^-$ reactions leading to the conclusion that the $\psi(3770)$ state is a Fano resonance embedded in the $D\bar{D}$ continuum. Its properties are complicated by the coupled-channels effects. We have extended this enlightening idea to the energy range above the $\psi(3770)$ state in a parameterized coupled-channel formalism and illustrate that the $X(3900)$ state is of non-resonant nature. In this picture, it is naturally that the line shape of the $\psi(3770)$ state is distinct in various decay channels because of the different non-resonant continua and interfering contributions. This is intriguing in our understanding of the properties of many other hadronic states, especially with respect of the newly found and hardly understood X, Y, Z states.

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| TABLE II: Fitted parameters in different fit schemes. */: excluding one data point of Belle at $\sqrt{s} = 3.770$ GeV. |
|---|---|---|---|---|
| | $D^0\bar{D}^0$ | $D^+D^-$ | $D^0\bar{D}^0$ | $D^+D^-$ |
| $m_\psi$ (MeV) | 3783.2 ± 2.5 | 3782.5 ± 2.9 | 3787.0 ± 2.2 | 3787.8 ± 3.5 |
| $g_{\psi D\bar{D}}$ | 14.1 ± 0.7 | 14.5 ± 0.5 | 11.6 ± 0.2 | 11.6 ± 0.1 |
| $g_1^{\psi} = g_{\psi(3686)D\bar{D}}$ | 0.17 ± 0.02 | 0.17 ± 0.03 | 0.17 ± 0.03 | 0.17 ± 0.05 |
| $g_2^{\psi} = g_{\psi(3686)D^+D^-}$ | — | — | 1.19 ± 0.03 | 1.29 ± 0.06 |
| $g_3^{\psi} = g_{\psi(4040)D^+D^-}$ | — | — | 0.145 ± 0.014 | 0.160 ± 0.021 |
| $g_4^{\psi} = g_{\psi(3686)\gamma}$ | 6.4 ± 0.5 | 6.7 ± 0.9 | 7.6 ± 0.1 | 8.0 ± 0.2 |
| $g_5^{\psi} = g_{\psi(4040)\gamma}$ | — | — | 3.4 ± 0.4 | 2.9 ± 0.7 |
| $c_2$ | — | — | 13.9 ± 1.4 | 17.4 ± 1.2 |
| $\eta_2$ | — | — | 0.39 ± 0.19 | 0.41 ± 0.16 |
| $\chi^2/d.o.f$ | 1.12 | 0.92 | 1.43 (1.00*) | 1.57(1.25*) |

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