Thickness-Magnetic Field Phase Diagram at the Superconductor-Insulator Transition in 2D

N. Marković, C. Christiansen and A. M. Goldman
School of Physics and Astronomy, University of Minnesota, Minneapolis, MN 55455, USA
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The superconductor-insulator transition in ultrathin films of amorphous Bi was tuned by changing the film thickness, with and without an applied magnetic field. The first experimentally obtained phase diagram is mapped as a function of thickness and magnetic field in the T=0 limit. A finite size scaling analysis has been carried out to determine the critical exponent product $\nu z$, which was found to be $1.2 \pm 0.2$ for the zero field transition, and $1.4 \pm 0.2$ for the finite field transition. Both results are different from the exponents found for the magnetic field tuned transition in the same system, $0.7 \pm 0.2$.

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Superconductor-insulator (SI) transition in ultrathin films of metals is believed to be a continuous quantum phase transition [1] which can be traversed by changing a parameter such as disorder, film thickness, carrier concentration or the applied magnetic field [2,3]. The scaling theory and a phase diagram for a two-dimensional system as a function of disorder and magnetic field was postulated by Fisher et al. [4], based on the assumption that this transition can be fully described in terms of a model of interacting bosons, moving in the presence of disorder. The dirty boson problem has been extensively studied by quantum Monte Carlo simulations [5-8], real-space renormalization group calculations [9,10], strong coupling expansion [11] and in other ways [12-14], but there is still some disagreement as to the universality class of the transition. Conflicting experimental evidence suggests that the bosonic model might be relevant [15], but does not give the full picture [16]. An alternative model of interacting electrons has also been proposed [17]. Experimentally, the thickness tuned transition has been studied in the context of the scaling theory in zero magnetic field [18]. In the present work, for the first time, the SI transition was tuned by systematically changing the film thickness in a finite magnetic field. This allows us to map a phase diagram as a function of thickness and magnetic field in the T=0 limit and to determine the critical exponents using a finite size scaling analysis at different fields. The results suggest that this transition is similar to the zero field transition, but the exponent is different from that of the magnetic-field tuned transition studied on the same set of films [19].

The ultrathin Bi films were evaporated on top of a 10 Å thick layer of amorphous Ge, which was pre-deposited onto a 0.75 mm thick single-crystal of SrTiO$_3$ (100). The substrate temperature was kept below 20 K during all depositions and all the films were grown in situ under UHV conditions ($\sim 10^{-10}$ Torr). Under such circumstances, successive depositions can be carried out without contamination to increase the film thickness gradually in increments of $\sim 0.2$A. Film thicknesses were determined using a previously calibrated quartz crystal monitor. Films prepared in this manner are believed to be homogeneous, since it has been found that they become connected at an average thickness on the order of one monolayer [20]. Resistance measurements were carried out between the depositions using a standard dc four-probe technique with currents up to 50 nA. Magnetic fields up to 12 kG perpendicular to the plane of the sample were applied using a superconducting split-coil magnet.

The evolution of the temperature dependence of the resistance as the film thickness changes is shown on Fig. 1. The thinnest films show an exponential temperature dependence of the resistance at low temperatures, consistent with variable range hopping, which crosses over to a logarithmic behavior for thicker films [21]. At some critical thickness, $d_c$, the resistance is independent of temperature, while for even thicker films it decreases rapidly with decreasing temperature, indicating the onset of superconductivity. The critical thickness can be determined by plotting the resistance as a function of thickness for different temperatures (inset of Fig. 1) and identifying the crossing point for which the resistance is temperature independent, or by plotting $dR/dT$ as a function of thickness at the lowest temperatures and finding the thickness for which $(dR/dT) = 0$.

In the zero temperature quantum critical regime the resistance of a two dimensional system is expected to obey the following scaling law [14]:

$$ R(\delta, T) = R_c f(\delta T^{-1/\nu z}) $$

(1)

Here $\delta = d - d_c$ is the deviation from the critical thickness, $R_c$ is the critical resistance at $d = d_c$, $f(x)$ is a universal scaling function such that $f(0) = 1$, $\nu$ is the coherence length exponent, and $z$ is the dynamical critical exponent. We rewrite Eq. 1 as $R(\delta, t) = R_c f(\delta t)$, where $t \equiv T^{-1/\nu z}$, and treat the parameter $t(T)$ as an unknown variable which is determined at each temperature to obtain the best collapse of all the data. The exponent $\nu z$ is
then found from the temperature dependence of \( t \), which must be a power law in temperature for the procedure to make sense. This scaling procedure does not require detailed knowledge of the functional form of the temperature or thickness dependence of the resistance, or prior knowledge of the critical exponents. It is simply based on the data which includes an independent determination of \( d_c \).

The collapse of the resistance data as a function of \( \delta t \) in zero field is shown in Fig. 2. The critical exponent product \( \nu \zeta \), determined from the temperature dependence of the parameter \( t \) (inset of Fig. 3), is found to be \( \nu \zeta = 1.2 \pm 0.2 \). This result is in agreement with the predictions of Ref. 5, from which \( z=1 \) would be expected for a bosonic system with long range Coulomb interactions independent of the dimensionality, and \( \nu \geq 1 \) in two dimensions for any transition which can be tuned by changing the strength of the disorder 22. A similar scaling behavior has been found in ultrathin films of Bi by Liu et al. 18, with the critical exponent product \( \nu \zeta \approx 2.8 \) on the insulating side and \( \nu \zeta \approx 1.4 \) on the superconducting side of the transition. The fact that \( \nu \zeta \) was found to be different on the two sides of the transition raises the question of whether the experiment really probed the quantum critical regime. We believe that the scaling was carried out too deep into the insulating side, forcing the scaling form (Eq. 1) on films which were in a fundamentally different insulating regime 22. Such films should not be expected to scale together with the superconducting films, hence the discrepancy on the insulating side of the transition. In the present work, the measurements were carried out at lower temperatures than previously studied and with more detail in the range of thicknesses close to the transition. We were able to scale both sides of the transition with \( \nu \zeta \approx 1.2 \), which is close to the value obtained by Liu et al. on the superconducting side of the transition. Our result is also in very good agreement with the renormalization group calculations 6,14, and close to that found in Monte Carlo simulations by Cha and Girvin 7, Sørensen et al. 8 and Makivič et al. 8.

In addition to the above, the magnetoresistance as a function of temperature and magnetic field was measured for each film. By sorting the magnetoresistance data, one can probe the thickness-tuned superconductor-insulator transition in a finite magnetic field, which has not been studied before. The same analysis as described above was carried out for several fixed magnetic fields, ranging from 0.5 kG up to 7 kG. The normalized resistance data as a function of the scaling variable for six different values of the magnetic field shown on Fig. 3 all collapse on a single curve, which suggests that the scaling function is indeed universal. The critical exponent product determined from the temperature dependence of the parameter \( t \) (inset of Fig. 3) is found to be \( \nu \zeta = 1.4 \pm 0.2 \), apparently independent of the magnetic field. An applied magnetic field is generally expected to change the universality class of the transition, since it breaks the time reversal symmetry. We find, however, that the critical exponent product \( \nu \zeta \) for the thickness driven SI transition in a finite magnetic field is very close to that obtained for a zero-field transition, given the experimental uncertainties. This result is in agreement with Monte Carlo simulations of the \((2+1)\)-dimensional classical \( XY \) model with disorder by Cha and Girvin 7, which find \( \nu \zeta \approx 1.07 \) for the zero-field transition and \( \nu \zeta \approx 1.14 \) in a finite magnetic field. The critical resistance \( R_c \) at the transition is non-universal, as it decreases with increasing magnetic field.

Furthermore, once a magnetic field is applied and the time-reversal symmetry broken, the thickness-tuned transition in the field is expected to be in the same universality class as the transition which is tuned by changing the magnetic field at a fixed film thickness. The magnetic-field tuned transition was studied on the same set of films used in this study 14, which allows a direct comparison of the critical exponents, without having to worry about differences in the microstructure between different samples. The resistance as a function of temperature for five selected films from Fig. 1 was studied in magnetic fields up to 12kG applied perpendicular to the plane of the sample. The critical exponent product was determined using the method described above, but with the magnetic field as the tuning parameter rather than the film thickness. It was found to be \( \nu_B \zeta_B = 0.7 \pm 0.2 \), independent of the film thickness, which strongly suggests a universality class different from that of the thickness-tuned transition, both in zero-field and in the field. This result does not agree with the predictions based on the model of interacting bosons in the presence of disorder 34. It does, however, agree with what might be expected from a similar model without disorder 35. The details of the analysis and this unexpected value of the critical exponent product, as well as the discussion of its disagreement with previous determinations 34, are reported elsewhere 19.

The fact that the SI transition was traversed by changing both the thickness and the magnetic field independently on the same set of films allows us to determine the phase diagram as a function of thickness and the magnetic field in \( T=0 \) limit, which is shown in Fig. 4. The films characterized by parameters which lie above the phase boundary are "superconducting" (\( \delta R/\delta T < 0 \) at finite temperatures), and the ones below it are "insulating" (\( \delta R/\delta T > 0 \) at finite temperatures). The phase boundary itself follows a power law: \( d_c(B) - d_c(0) \propto B_x^\beta \), where \( x \approx 1.4 \). Our results imply that the critical exponent product \( \nu \zeta \) depends on whether this phase boundary is crossed vertically (changing the thickness at a constant magnetic field), in which case \( \nu \zeta \approx 1.4 \), or horizontally (changing the magnetic field at a fixed thickness), in which case \( \nu_B \zeta_B \approx 0.7 \). In other words, relatively
weak magnetic fields which are experimentally accessible to us do not significantly change the universality class of the thickness-tuned transition, but the magnetic-field tuned transition is in a different universality class from the thickness-tuned transition in $B \neq 0$. In the absence of a detailed theory, we can only speculate on the origins of this surprising result.

The first important issue we wish to discuss is the role of the film thickness as the control parameter. Adding metal sequentially to a quench-condensed film has been shown to decrease the disorder, since the increased screening smooths the random potential seen by the electrons. It presumably also increases the carrier concentration, which in the presence of an attractive electron-electron interaction might result in an increased Cooper pair density.

Increasing the film thickness might therefore be thought of as adding Cooper pairs, which condense at some critical density. In a similar way, an applied magnetic field adds vortices, which behave as point particles and also condense at some critical density, making the system insulating. However, this symmetry between charges and vortices is not perfect, since Cooper pairs interact as $1/r$ and vortices interact logarithmically [24].

If the mechanism responsible for the localization in the magnetic-field tuned transition is different from that of the thickness-tuned transition, than having a non-zero magnetic field may not play a major role in the thickness-tuned transition. Also, the correlation length associated with the thickness-tuned transition would then be different from that associated with the magnetic-field tuned transition. The disorder might be important in one case and not in the other, depending on how these correlation lengths compare to the length scale which characterizes the disorder.

A second difference between the thickness-tuned and the field-tuned transitions may be the nature of disorder itself. In the field-tuned transition the geometry of the film is fixed, and the disorder does not change. In the disorder-tuned transition, each film in the sequence of films will have a slightly different microstructure, so that the disorder may have to be averaged over the different configurations. It has been suggested recently [24] that the nature of the disorder averaging might play an important role in determining the critical exponents.

Another possibility is that the localization transition is enhanced by percolation effects [25] as the film thickness is tuned. This approach takes into account local fluctuations of the amplitude of the superconducting order parameter, which are routinely neglected in the scaling theory and the numerical simulations. Percolation of islands with strong amplitude fluctuations might change the localization exponents obtained from the scaling theory [25]. The role of percolation has also been emphasized in recent discussions of low temperature transport in these systems [27].

Finally, one cannot exclude the possibility that to access the quantum critical region these measurements need to be carried out at much lower temperatures or at high frequencies [28].

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FIG. 1. Resistance per square as a function of temperature for a series of bismuth films with thicknesses ranging from 9Å (top) to 15Å (bottom). Inset: Resistance as a function of thickness for the same set of films close to the transition at low temperature. Different curves represent different temperature, ranging from 0.14K to 0.40K.

FIG. 2. Resistance per square as a function of the scaling variable, $|d-d_c|$, for seventeen different temperature, ranging from 0.14K to 0.5K. Here $t = T^{-1/\nu z}$ is treated as an adjustable parameter to obtain the best collapse of the data. Different symbols represent different temperature. Inset: Fitting the temperature dependence of the parameter $t$ to a power law determines the value of $\nu z$.

FIG. 3. Normalized resistance per square as a function of the scaling variable, $|d-d_c|$, at different temperature, ranging from 0.14K to 0.5K. Different symbols represent different magnetic fields, ranging from 0.5kG-7kG. Here $t = T^{-1/\nu z}$ is treated as an adjustable parameter to obtain the best collapse of the data, and $R_c$ is the resistance at $d = d_c$. Inset: Fitting the temperature dependence of the parameter $t$ to a power law determines the value of $\nu z$.

FIG. 4. The phase diagram in the d-B plane in the T=0 limit. The points on the phase boundary were obtained from disorder driven transitions (triangles) and magnetic field driven transitions (circles). The solid line is a power law fit. The values of the critical exponent product are shown next to the arrows giving the direction in which the boundary was crossed. Here $d_c$ is taken to be the critical thickness in zero field.
\[ R(\Omega) \]
\[ T(K) \]
\[ |d - d_c| \]

\[ \nu \approx 1.2 \]
$R/R_c$

$T(K)$

$\nu z \approx 1.4$
$d - d_c \propto B^{1.4}$

$v z \approx 1.4$

$v z \approx 0.7$

SUPERCONDUCTOR

INSULATOR