Does Nonaxisymmetric Dynamo Operate in the Sun?

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Abstract

We explore effects of random nonaxisymmetric perturbations of kinetic helicity (the $\alpha$ effect) and diffusive decay of bipolar magnetic regions on generation and evolution of large-scale nonaxisymmetric magnetic fields on the Sun. Using a reduced 2D nonlinear mean-field dynamo model and assuming that bipolar regions emerge due to magnetic buoyancy in situ of the large-scale dynamo action, we show that fluctuations of the $\alpha$ effect can maintain the nonaxisymmetric magnetic fields through a solar-type $\alpha^2\Omega$ dynamo process. It is found that diffusive decay of bipolar active regions is likely to be the primary source of nonaxisymmetric magnetic fields observed on the Sun. Our results show that nonaxisymmetric dynamo models with stochastic perturbations of the $\alpha$ effect can explain periods of extremely high activity (“super-cycle” events) as well as periods of deep decline of magnetic activity. We compare the models with synoptic observations of solar magnetic fields for the last four activity cycles and discuss implications of our results for interpretation of observations of stellar magnetic activity.

Key words: dynamo – stars: activity – stars: magnetic field – stars: solar-type – Sun: activity – Sun: magnetic fields

1. Introduction

Since the seminal papers of Choudhuri (1992) and Hoyng (1993), random variations of kinetic helicity in dynamo processes (the so-called $\alpha$ effect) are often considered as the main source of long-term variations of solar activity cycles (Ossendrijver & Hoyng 1996; Moss et al. 2008; Usoskin et al. 2009; Pipin et al. 2012; Passos et al. 2014). In the standard mean-field framework, turbulent generation of magnetic fields results from reflection-symmetry breaking of helical convection motions (Krause & Rädler 1980). In the mean-field theory, the effect is described by the mean electromotive force,

$$E = (u \times b) = \alpha \mathbf{B} + \ldots,$$

where $u$ is the turbulent velocity, $b$ is the turbulent magnetic field, $\langle B \rangle$ is the large-scale magnetic field, and coefficient

$$\alpha = -\frac{1}{3} (u \cdot \nabla \times u) \tau_{\text{corr}}$$

is a pseudo-scalar proportional to the kinetic helicity, $u \cdot \nabla \times u$, and turbulent correlation times, $\tau_{\text{corr}}$. The amount of convective energy, which can be spent on turbulent generation of the large-scale magnetic field by the $\alpha$ effect, is only few percents of the total convective energy (Parker 1979). Taking this constraint into account, it was shown that magnitude of the $\alpha$ effect can randomly vary in each hemisphere in the range from 10\% to 20\% (see, Choudhuri 1992; Hoyng 1993; Moss et al. 2008). However, the results of Choudhuri (1992) and Ossendrijver & Hoyng (1996) showed that in order to explain the Grand cycles of solar magnetic activity the random fluctuations should be of the same order as the mean magnitude of the $\alpha$-effect. Results of Moss et al. (2008) showed that the timescale of fluctuations should also be taken into account. They found that if the correlation time is comparable to the cycle duration, then the fluctuations with amplitudes of a few dozen percent are sufficient to explain the Grand minima of solar activity.

The current paradigm assumes that sunspots are formed from the large-scale axisymmetric toroidal magnetic field emerging from the solar convection zone where the field is regenerated by hydromagnetic dynamo. Results of Krause & Rädler (1980) and Raedler (1986) showed that, because of the differential rotation, the solar dynamo can not maintain a regular nonaxisymmetric large-scale magnetic field. Nevertheless, large-scale nonaxisymmetric magnetic fields are commonly observed on the Sun, for example, in the form of coronal holes (Glencross 1974), which represent regions of open magnetic flux (Stix 1977). The surface flux-transport models successfully simulate the process of formation of coronal holes from decaying active regions (Wang & Sheeley 1990; Cameron & Schüssler 2017). These models assume that influence of surface nonaxisymmetric magnetic fields on the dynamo action in the deep convection zone is negligible. Moss (1999) and Bigazzi & Ruzmaikin (2004) showed that weak large-scale nonaxisymmetric field structures may be consistent with nonlinear mean-field models of the solar dynamo, in which nonaxisymmetric dynamo modes are maintained by either nonlinearity of $\alpha$-effect quenching, or nonaxisymmetric distribution of $\alpha$. It was suggested that the excitation of the nonaxisymmetric modes can be sensitive to the radial dependence of the rotation law (Moss 1999; Pipin 2017). Pipin & Kosovichev (2015) studied the response of a nonlinear nonaxisymmetric mean-field solar dynamo model to nonaxisymmetric perturbations and showed that the effect can depend on the root depth of the nonaxisymmetric magnetic fields. The nonaxisymmetric dynamo models may be relevant to the problem of solar active longitudes (Berdyugina et al. 2006). Observational results of Stenflo (2012) showed that the presence of background (basal) magnetic flux observed on the surface of the Sun does not depend on the magnetic cycle. From this consideration, it seems that much of the basal flux may well originate from the global dynamo. This flux may persist during the solar minima because diffusion of solar bipolar regions could take a long time.

In this paper, we explore an additional possibility that stems from nonaxisymmetric dynamo action. Longitudinal fluctuations of the $\alpha$ effect are usually ignored in mean-field stellar dynamo models. Using nonlinear mean-field dynamo models,
we show that such nonaxisymmetric random perturbations can maintain large-scale nonaxisymmetric magnetic fields in a solar-type α2Ω dynamo. Our goal is to investigate the process of stochastic excitation of large-scale nonaxisymmetric magnetic field and estimate how it affects the large-scale basal magnetic flux. To compare this mechanism with diffusive decay of bipolar active regions, we simulate active region emergence using the Parker’s magnetic buoyancy effect. To demonstrate the difference between the two competitive mechanisms, we employ a reduced 2D nonlinear and nonaxisymmetric dynamo model that describes the dynamo wave propagation on the spherical surface. The modeling results are compared with synoptic observations of large-scale solar magnetic fields on the Sun.

The paper is organized as follows. Section 2 describes the dynamo model and its parameters. Section 3 presents results of numerical simulations for various model conditions. Section 4 gives an outline of observational data and comparison with the model. The final section summarizes and discusses the main results of our analysis.

2. Nonaxisymmetric 2D Dynamo Model

2.1. Governing Equations

In the framework of mean-field magnetohydrodynamics (Krause & Rädler 1980) the evolution of the large-scale magnetic field, ⟨B⟩, in perfectly conducting media is governed by the induction equation,

\[ \partial_t \langle B \rangle = \nabla \times (\mathcal{E} + \langle U \rangle \times \langle B \rangle), \]

(1)

where \( \mathcal{E} = \langle u \times b \rangle \) is the mean electromotive force with \( u \) and \( b \) standing for the turbulent velocity and magnetic field respectively. It is convenient (Jennings et al. 1990) to represent the large-scale magnetic field induction vector in terms of the axisymmetric and nonaxisymmetric components as follows:

\[ \langle B \rangle = \overline{B} \hat{\phi} + \tilde{B} \]

(2)

\[ \overline{B} = \hat{\phi} B + \nabla \times (A \hat{\phi}) \]

(3)

\[ \tilde{B} = \nabla \times (rT) + \nabla \times \nabla \times (rS), \]

(4)

where \( \overline{B} \) and \( \tilde{B} \) are the axisymmetric and nonaxisymmetric components; \( A, B, T, \) and \( S \) are scalar functions representing the field components; \( \hat{\phi} \) is the azimuthal unit vector; \( r \) is the radius vector; \( \hat{\rho} \) is the radial unit vector; \( \theta \) is the polar angle. Hereafter, the overbar denotes the axisymmetric magnetic field, and tilde denotes nonaxisymmetric properties.

To elucidate basic properties of the nonaxisymmetric dynamo action, we consider a reduced dynamo model in which the radial dependence of the magnetic field is disregarded. In this case, the induction vector of the large-scale magnetic field is represented in terms of the scalar functions as follows:

\[ \langle B \rangle = -\frac{r}{R^2} \frac{\partial \sin \theta A}{\partial \mu} - \frac{\hat{\rho}}{R} \frac{\partial A}{\partial \mu} + \hat{\phi} B \]

\[ -\frac{r}{R^2} \Delta_{\phi} S + \frac{\hat{\rho}}{\sin \theta} \frac{\partial T}{\partial \phi} + \hat{\phi} \sin \theta \frac{\partial T}{\partial \mu}, \]

where \( \Delta_{\phi} \) represents the radius of the spherical surface inside a star where the hydromagnetic dynamo operates; here, \( \Delta_{\phi} = \frac{\partial}{\partial \mu} \cos^2 \theta \frac{\partial}{\partial \mu} + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \) and \( \mu = \cos \theta \). The model employs the following expression of \( \mathcal{E} \):

\[ \mathcal{E} = \alpha \langle B \rangle - \eta \nabla \times \langle B \rangle + V_b \hat{\phi} \times B. \]

(5)

Here, it is assumed that:

\[ \alpha_0 = \alpha_0 \psi(|\langle B \rangle|) \cos \theta \delta_{ij}, \]

(6)

where coefficient \( \alpha_0 \) represents the magnitude of the \( \alpha \)-effect, and \( \psi \) is the standard magnetic quenching function (Pipin 2008). The magnitude, \( \alpha_0 \), is allowed to vary randomly in time \( t \) and longitude \( \phi \):

\[ \alpha_0 = \pi(1 + \xi_0(\phi, t)), \]

(7)

where \( \pi \) is the stationary axisymmetric part of the \( \alpha \)-effect; the function \( \xi_0(\phi, t) \) describes random fluctuations. Details of the fluctuations will be defined further. The second term represents turbulent diffusion with coefficient \( \eta \). The last term in Equation (5) describes the magnetic buoyancy effect. Here, \( \hat{\rho} \) is the radial unit vector, and \( V_b \) is the escape velocity of magnetic field. The escape velocity, \( V_b \), accounts for the loss of generated magnetic flux from the dynamo region (Noyes et al. 1984; Parker 1984; Moss et al. 1990). Parker (1979) suggested that the magnetic buoyancy can result in the formation of bipolar active regions. Currently, it becomes evident that the magnetic buoyancy is not the only mechanism forming emerging active regions of the Sun (Getling 2001; Kitchasvili et al. 2010; Stein & Nordlund 2012; Brandenburg et al. 2013; Leka et al. 2013; Losada et al. 2017; Martin 2018). In this study, we assume that the magnetic buoyancy acts on relatively small-scale parts of the axisymmetric magnetic field, perhaps, because of some kind of nonlinear instability, and contributes to generation of the nonaxisymmetric magnetic field component. It is formulated following Kitchatinov & Pipin (1993):

\[ V_b = \frac{\beta^2 K(\beta)[1 + \xi_0(\phi)]}{\gamma}, \]

if \( \beta \geq \beta_{cr}, \)

(8)

\[ 0, \]

if \( \beta < \beta_{cr}, \)

where \( \beta = |\langle B \rangle|/B_{eq}, \) \( B_{eq} = \sqrt{4\pi \mu_0 I^2}, \) function \( K(\beta) \) is defined in Kitchatinov & Pipin (1993), function \( \xi_0(\phi) \) describes the longitudinal dependence of the instability, and parameter \( \beta_{cr} \) controls the instability threshold. These parameters will be described below. From results of the above cited paper, it follows that for \( \beta \ll 1, K(\beta) \approx 1, \) and for \( \beta > 1, K(\beta) \approx 1/\beta^2. \) In this formulation, the preferable latitude of the “active region emergence” is determined by maximum of the toroidal magnetic field energy, see Equation (8). Parameter \( \beta_{cr} = 0.5 \) is used to prevent emergence of active regions at high latitudes.

The minimal set of the dynamo equations to model the nonaxisymmetric magnetic field evolution can be obtained by generalization of the 1D model suggested by Parker (1993), which has a solution in the form of dynamo waves migrating toward the equator. The model studied extensively, for example, by Kuzanyan (1998), Moss et al. (2008), and Usoskin et al. (2009). In this framework, the radial dependence of magnetic field is disregarded, and it is assumed that the radial gradient of angular velocity is greater than the latitudinal gradient. Applying these simplifications to Equation (1) and Equations (2)–(4), we obtain the following set of dynamo
equations in terms of the scalar functions, $A$, $B$, $S$, and $T$:

\[
\begin{align*}
\partial_t B &= -\sin \theta \frac{\partial \Omega}{\partial r} \frac{\partial (\sin \theta A)}{\partial \mu} + \eta(r) \frac{\sin^2 \theta}{R^2} \frac{\partial^2 (\sin \theta B)}{\partial \mu^2} - \frac{B}{\tau}, \\
&+ \frac{\sin \theta}{R} \frac{\partial}{\partial \mu} \alpha_0 \mu \langle B \rangle + \frac{\alpha_0 \mu}{R} \langle B \rangle - \frac{1}{R} V_\theta \langle B \rangle, \\
\end{align*}
\]

\[
(9)
\]

\[
\begin{align*}
\partial_t A &= \alpha_0 \mu \langle B \rangle + \eta(r) \frac{\sin^2 \theta}{R^2} \frac{\partial^2 (\sin \theta A)}{\partial \mu^2} - \frac{V_\theta}{\tau} A - \frac{A}{\tau}, \\
&+ \frac{1}{R} \frac{\partial}{\partial \mu} \alpha_0 \mu \langle B \rangle + \frac{\alpha_0 \mu}{R} \langle B \rangle + \alpha_0 \mu \sin \theta \langle B \rangle + \mu^2 \langle B \rangle) \\
&- \frac{1}{\sin \theta} \frac{\partial}{\partial \phi} \langle B \rangle V_\theta - \frac{\partial}{\partial \mu} \sin \theta \langle B \rangle V_\theta, \\
\end{align*}
\]

\[
(10)
\]

The $\tau$-terms in Equations (9) and (10) were suggested by Moss et al. (2008) to account for turbulent diffusion in the radial direction. Similarly to the cited paper, we put $\tau = 3.2 \frac{R}{\eta}$. The $\tau$-parameter will be specified in the next subsection. For brevity, some terms in the above equations are given explicitly via components of the magnetic induction vector, $\langle B \rangle$. These terms result in coupling between different modes of the large-scale magnetic field in the case of longitudinal dependence of parameters $\alpha_0$ and $V_\theta$. If the differential rotation is strong, then the nonaxisymmetric modes are stable against the dynamo instability. The nonlinear coupling (e.g., due to the $\alpha$-effect) is not very effective for maintaining nonaxisymmetric dynamo modes (Brandenburg et al. 1989; Raedler et al. 1990; Moss 1999; Pipin & Kosovichev 2015). To simulate stretching of nonaxisymmetric magnetic field by the surface differential rotation, we consider the latitudinal dependence of angular velocity $\delta \Omega = -0.25 \sin^2 \Omega$ in Equations (11) and (12), which are written in the coordinate system rotating with angular velocity $\Omega$. In this paper, we discuss relatively simple 2D models with the constant rotational shear.

The numerical scheme employs a pseudo-spectral approach for integration along latitude. For the nonaxisymmetric components, we employ the spherical harmonic decomposition, i.e., scalar functions $T$ and $S$ are represented in the form:

\[
T(\mu, \phi, t) = \sum \tilde{T}_{l,m}(t) T_{l}^{m \mu} \exp(-im\phi),
\]

\[
(13)
\]

\[
S(\mu, \phi, t) = \sum \tilde{S}_{l,m}(t) P_{l}^{m \mu} \exp(-im\phi),
\]

\[
(14)
\]

where $P_{l}^{m}$ is the normalized associated Legendre function of degree $l \geq 1$ and azimuthal order $m \geq 0$. Note that $\tilde{T}_{l,m} = \tilde{T}_{l,m}$, and the same is valid for $\tilde{T}$. Our typical simulation runs were performed for 990 spherical harmonics (with $l_{\max} = 36, m_{\max} = 18$). In addition, some runs were performed with higher resolution including 1752 spherical harmonics ($l_{\max} = 48, m_{\max} = 24$). All the nonlinear terms are calculated explicitly in the real space. The numerical integration is carried out in latitude from pole to pole.

To quantify asymmetry of the mean radial magnetic field distribution relative to the equator, we introduce the parity index, $P_E$:

\[
P_E = \frac{E_q - E_d}{E_q + E_d},
\]

\[
E_d = \frac{1}{4} \int (\mathcal{B}_t(\mu) - \mathcal{B}_t(-\mu))^2 d\mu,
\]

\[
E_q = \frac{1}{4} \int (\mathcal{B}_t(\mu) + \mathcal{B}_t(-\mu))^2 d\mu,
\]

\[
(15)
\]

where $E_d$ and $E_q$ are the energy of the antisymmetric and symmetric modes, respectively.

We consider decomposition of the radial magnetic field into dynamo modes:

\[
\langle B \rangle = \sum B_{l}^{(m)}(\mu) e^{im\phi},
\]

\[
(16)
\]

where the case of $m = 0$ corresponds to the axisymmetric magnetic field. The degree of nonaxisymmetry of the magnetic field is described by the ratio:

\[
P_X = \frac{E^{(m)}_r}{E^{(m)}_r},
\]

\[
(17)
\]

and $E^{(m)}_r$ is the total energy of the radial component of the magnetic field. The characteristic parameters are the mean strength of the unsigned radial magnetic field:

\[
[\langle B \rangle] = \sqrt{E^{(m)}_r / 2\pi},
\]

\[
(18)
\]

and the unsigned toroidal magnetic field:

\[
[\langle B \rangle] = \sqrt{E^{(m)}_r / 2\pi},
\]

\[
(19)
\]

where $E^{(m)}_r$ is the total energy of the toroidal component. The similar parameters are introduced for the axisymmetric magnetic field.

2.2. Nonaxisymmetric Perturbations

Variations of the $\alpha$-effect are modeled by Equation (7). The timescales in our model are defined in terms the characteristic diffusion time, $\frac{R}{\eta}$. In these units, the basic diffusion period (corresponding to the 11 year cycle) is $P \approx 0.15 \frac{R}{\eta}$. The fluctuations are imposed randomly in time. We consider both short and long correlation times. For the solar case, the short correlation time, $\tau_c = 0.02P$, is roughly equal to the correlation time of nonaxisymmetric modes of solar magnetic fields (Pipin et al. 2014). In addition, we consider the long correlation time, $\tau_c = 0.5P$. The longitudinal dependence of function $\xi_\mu(\phi, t)$ is modeled as a superposition of sinusoidal oscillations with random amplitudes and phases. The strength of fluctuations is
controlled by parameter $\sigma_\xi$, which is the standard deviation (STD) of the random variable, $\xi$. As in the model of Pipin & Sokoloff (2011), we consider a set of $\sigma_\xi = 0.25, 0.5, 1$ for the short and long correlation time $\tau_\xi$ cases. The longitudinal fluctuations of the $\alpha$ effect are modeled as a superposition of random harmonics up to the octupole modes.

In practice, the fluctuations of the $\alpha$ effect are implemented as follows. We use the pseudo-random number generator library of SciPy (scipy.org) to produce a Gaussian set of random intervals with correlation time $\tau_\xi$. Then, during the dynamo run, the distribution of $\xi_\alpha(\phi, t)$ is determined for each of the random time intervals. In this step, we generate a random Gaussian sequence of length $N_\phi$ (the number of mesh points in azimuth) with the STD of $N_\phi\sigma_\xi$. Then, using the fast Fourier transform, we filter out all harmonics higher than the octupole. The obtained distribution of $\xi_\alpha(\phi, t)$ is used in the model until the next random fluctuation.

A snapshot of a random realization of the $\alpha$ effect with $\sigma_\xi = 0.25$ is illustrated in Figure 1.

Magnetic buoyancy instability perturbations are determined by function:

$$\xi_\beta(\phi) = C_\beta \exp( -m_\beta \sin^2 \left( \frac{\phi - \phi_0}{2} \right))$$  \hspace{1cm} (18)

The instability is randomly initiated in the northern or southern hemispheres, and the longitude, $\phi_0$, is also chosen randomly. We arbitrarily chose the fluctuation interval $\tau_\beta = 0.01 P$. After injection of the perturbation, the evolution is solely determined by the dynamo equations. Parameter $m_\beta$ controls the spatial scale of the instability. Theoretically, using high values of $m$, we can reproduce the spatial scale of the solar active regions. However, this requires increasing the resolution in both longitude and latitude, and becomes computationally expensive. We chose the value $m_\beta = 50$, which allows us to accurately resolve the evolving nonaxisymmetric perturbations of magnetic field, and qualitatively reproduce the essential physical effects. Parameter $C_\beta$ controls the amount of the injected magnetic flux. If large-scale toroidal magnetic flux at a given colatitude $\theta$ is transformed into magnetic flux of the perturbation, then $\langle \xi_\beta(\phi, t) \rangle_\phi \approx 1$. This condition corresponds to $C_\beta \approx 15$. In reality, solar active regions are formed by concentration of the toroidal magnetic flux emerging in the photosphere. Turbulent convective motions and other physical processes may take part in the process of formation of solar active regions. Therefore, parameter $C_\beta$ can be higher than the above mentioned value, and we choose it to be about three times greater $C_\beta = 40$ than the value corresponding to the local large-scale field. In the numerical experiments, we found that higher values of $C_\beta$ result in strong cycle-to-cycle variability of the magnetic energy even in the case of the stationary $\alpha$-effect. For the chosen $C_\beta$, the fluctuations of the magnetic field because of the magnetic buoyancy instability are of the order of the axisymmetric toroidal magnetic field strength, which is widely accepted in the literature (Krause & Rädler 1980; Brandenburg & Subramanian 2005). This part of the model can be improved using the bipolar region production algorithms from the flux-transport models.

Equations (9)–(12) are solved numerically in the nondimensional form. As in the model of Moss et al. (2008), we assume that the rotational shear is constant in latitude. The effect of differential rotation is controlled by nondimensional parameter $R_\alpha = \frac{\frac{\partial \Omega}{\partial \phi}}{\frac{\partial \Omega}{\partial \theta}}$, the $\alpha$-effect is measured by parameter $R_\alpha = \frac{\xi_\alpha}{\eta_T}$, the magnetic buoyancy depends on $R_B = \frac{B_{\text{equ}}}{\eta_T}$, and the magnetic field is measured relative to the equipartition strength $B_{\text{eq}} = \sqrt{4\pi \mu_0}$. Following Pipin & Sokoloff (2011), we put $R_\alpha = \frac{R_{\alpha1}}{\eta_T} = 10^3$, $R_\alpha = 1$. This choice describes the $\alpha^2 \Omega$ dynamo regime with differential rotation as the main driver of axisymmetric toroidal magnetic field. The nonaxisymmetric modes do not take part in the dynamo unless some nonaxisymmetric phenomena come into the play. To estimate the magnetic buoyancy parameter, we employ results of Kitchatinov & Pipin (1993) who argued that the maximum buoyancy velocity of large-scale magnetic field of equipartition strength $B_{\text{eq}}$ is of the order of $6 \text{ m s}^{-1}$. In the solar conditions, the magnetic diffusion $\eta_T = 10^{12} \text{ cm}^2 \text{ s}^{-1}$ (Martinez Pillet et al. 1993; Rüdiger et al. 2011), and $R_\beta \approx 500$. In our models, the large-scale magnetic field strength is below $B_{\text{eq}}$. Hence, we use by an order of magnitude smaller value: $R_\beta = 50$. In addition to nonlinear quenching of the $\alpha$-effect, the magnetic buoyancy also causes a nonlinear saturation of the dynamo process.

Table 1 shows the variable parameters of our models: $C_\beta$ controls active region emergence by means of the Parker’s magnetic buoyancy instability; $\sigma_\xi$ is the STD of the random $\alpha$ effect; $\tau_\xi$ is the correlation time of the random $\alpha$ fluctuations, measured relative to dynamo period $P$. In models M2d and M3b, nonaxisymmetric perturbations of $\alpha$ are neglected: $\xi_\alpha = 0$ (however, the axisymmetric $\alpha$ component fluctuates). For the chosen set of parameters, the antisymmetric parity is dominant in stationary dynamo regimes of models M0 and M1a. The magnetic field distribution of model M1a in the

| Model | $C_\beta$ | $\sigma_\xi$ | $\tau_\xi/P$ |
|-------|----------|-------------|--------------|
| M0    | 0        | 0           | ...          |
| M1a   | 40       | 0           | ...          |
| M1b   | 0        | 0.25        | 0.02         |
| M2a   | 40       | 0.25        | 0.02         |
| M2b   | 40       | 0.5         | 0.02         |
| M2c   | 40       | 1.          | 0.02         |
| M2d   | 40       | 1, $\xi_\alpha = 0$ | 0.02         |
| M3a   | 40       | 0.25        | 0.5          |
| M3b   | 40       | 0.25, $\xi_\alpha = 0$ | 0.5         |
stationary stage is used as the initial condition for subsequent runs.

3. Numerical Results

Figure 2 shows evolution of dynamo properties in the reference models, M0 and M1a,b. The mean toroidal magnetic field varies around $0.4B_{eq}$, while the radial magnetic field is by a factor of 50 smaller. Model M0 shows that the nonaxisymmetric magnetic field is decaying with time. Model M1a maintains large-scale nonaxisymmetric magnetic field, which is seeded by diffusive dispersion of bipolar active regions. The mean strength of the nonaxisymmetric magnetic field is a bit less or around the mean strength of the axisymmetric toroidal magnetic field. It is interesting that the nonaxisymmetric perturbations almost do not affect the axisymmetric magnetic field evolution. However, randomness of the active region formation results in some randomness of the overall magnetic activity.

Time–latitude diagrams of the axisymmetric toroidal and radial magnetic fields for models M0 and M1a,b are very similar. Figure 2(b) shows the diagrams for model M1a. We see that the model correctly reproduces the solar-like dynamo waves of toroidal magnetic field, which drifts to the equator during the magnetic cycle. However, the time–latitude evolution of radial magnetic field does not show the polar branch (see Stenflo & Guedel 1988). Also, the phase relation between the radial and toroidal components does not correspond to observations. The observations, e.g., the above cited paper and Stenflo (2013) or Blackman & Brandenburg (2003), show that $\vec{B}_r \vec{B}_\phi < 0$ at low latitudes of the Sun.

Figure 3 shows snapshots of the radial magnetic field distribution for different phases of the magnetic cycle in model M1a. These snapshots show some solar-like features in organization of the bipolar regions, such as the hemispheric Hale polarity rule. It is driven by the axisymmetric toroidal magnetic field. Diffusive decay of the bipolar regions and the differential rotation produce large-scale unipolar regions which may extend from one hemisphere to another, as seen in Figure 3(a) showing a snapshot for the magnetic cycle maximum. Figure 3(b) shows that some remnants of the bipolar regions can survive during the magnetic cycle minimum.

Model M1b shows (Figure 2(a), dashed–dotted line) that short-term stochastic fluctuations of the $\alpha$ effect with $\sigma_\alpha = 0.25$ generate an order of magnitude smaller nonaxisymmetric magnetic field than the bipolar regions in model M1a. Model M1a shows the magnetic cycle modulation of the mean strength of the nonaxisymmetric magnetic field. This feature is seen in the observations (see Section 4), but it is absent in the stochastic nonaxisymmetric dynamo models. The nonaxisymmetric dynamo is sustained by either nonaxisymmetric magnetic buoyancy instability (e.g., model M1a) or the nonaxisymmetric perturbations of the $\alpha$ effect (model M1b). Model M0 shows that the nonaxisymmetric magnetic field vanishes if these two effects are simultaneously neglected. The large-scale nonaxisymmetric magnetic field in model M1b is sustained by the nonaxisymmetric fluctuations of the $\alpha$ effect.
(cf. the results of model M0). In this case, the magnetic buoyancy of the asymmetric magnetic field does not significantly affect the results. Also, the nonlinear interaction between the nonaxisymmetric modes due to magnetic buoyancy is small because the strength of the nonaxisymmetric components is well below the equipartition value.

Figure 4 shows results for the dynamo models, which combine effects of the bipolar regions and stochastic nonaxisymmetric dynamo. Models M2a and M2b employ a moderate level of short-term $\alpha$ fluctuations with $\sigma_\alpha = 0.25$ and $\sigma_\alpha = 0.5$ respectively. In both cases, perturbations of the asymmetric dynamo mode are not strong. This is compatible with the results of the standard 1D models (e.g., Choudhuri 1992; Hoyng 1993; Moss et al. 2008), which showed that small short-term fluctuations of the $\alpha$-effect in $\alpha\Omega$ dynamo models do not lead to substantial variations of the magnetic cycle. Contrary to this, Figures 4(c) and (d) show that the radial magnetic field experiences strong noisy perturbations. In particular, model M2b shows this in both the equatorial and radial regions. The degree of nonaxisymmetry of magnetic activity, parameter $P_x$, varies from about zero during the minimum to 0.95 during the maxima of magnetic cycles. It is found that with the increase of $\sigma_\alpha$ the amount of nonaxisymmetric magnetic field during the cycle minimum increases. Another interesting effect is that the combined action of the bipolar regions and nonaxisymmetric perturbations of the $\alpha$-effect can result in magnetic parity violations. Model M2b shows stronger deviation of parameter $P_x$ from $-1$ than model M2a. The effect becomes clear in model M2c with strong fluctuations of the $\alpha$-effect, $\sigma_\alpha = 1$.

This effect is demonstrated in Figure 5. Model M2c shows strong cyclic variations of the mean total strength of the toroidal magnetic field. In some cases, the two subsequent cycles are merged into one super cycle. In such a cycle, the total magnetic field strength exceeds the strength of the background toroidal magnetic field by a factor of 2. The basal level of nonaxisymmetric magnetic field is well above zero, especially in the activity minima before and after the super cycle. Perturbations of the magnetic parity in model M2c are much higher than in models M2a and M2b. Model M2d is a run without nonaxisymmetric fluctuations of the $\alpha$ effect, but with the same random realization of axisymmetric fluctuations of the $\alpha$ effect as in model M2c. Figure 5(a) shows that the evolutions of $\langle B_r \rangle$ in models M2c and M2d deviate from each other shortly after the beginning. Variations of parameters $P_x$ and $P_E$ differ substantially in models M2c and M2d. In particular, $P_E \approx -1$ in model M2d, but in model M2c parameter $P_E$ substantially deviates from $-1$ indicating asymmetry of the hemispheric magnetic activity.

Another interesting feature of model M2c is the super-cycle event at the time moment, $R^2/\eta_T \approx 1.5$ (Figure 5(a)). It is illustrated in Figure 6. It is found that the super cycle is formed by merging a strong cycle with a weak cycle that occurred during the decaying phase of the strong cycle. The super cycle is characterized by a high level of nonaxisymmetric magnetic field activity. Also, the hemispheric asymmetry of magnetic field distributions is high during the decaying phase of the cycle.

Figures 6(b), (c) and 7 show that the super cycle starts from quite strong $\langle B_r \rangle \approx 0.5B_{eq}$ and also nonaxisymmetric distributions of magnetic field, which is generated during the preceding minimum of magnetic activity. During the cycle maximum, superposition of the strong axisymmetric magnetic field and magnetic field of new bipolar regions produces wide magnetic “nests” with the field strength: $\langle B_r \rangle > B_{eq}$. At the end of the super cycle, the magnetic field remains nonaxisymmetric, and is also antisymmetric relative to the equator. Simultaneously, the strength of the axisymmetric toroidal magnetic field reaches the deep minimum, see Figure 6(b). Remarkably, model M2d does not show similar events. Figure 6(d) shows fluctuations of the axisymmetric and nonaxisymmetric parts of the $\alpha$-effect. The period preceding the super-cycle event is characterized by a moderate level of the random fluctuations. The mean $\alpha$-effect changed its sign from positive to negative for a short period around time $R^2/\eta_T \approx 1.25$. It is unclear if this event triggered the dramatic increase of activity in the subsequent two magnetic cycles.

The case of small long-term fluctuations of $\alpha$ is considered in models M3a and M3b, see Figure 8. These models have very similar time evolutions. The time interval in the model runs covers the period of about 50 cycles, which includes two deep minima of magnetic activity. In model M3a, there is one event that is similar to the super cycle of model M2c. Both models M3a and M3b show a high level of nonaxisymmetry during the

Figure 4. Results for models M2a (black lines) and M2b (red lines): (a) the mean strength of the total toroidal magnetic field (solid lines) and the axisymmetric toroidal magnetic field (dashed lines); (b) parameters $P_x$ (solid lines) and $P_E$ (dashed lines). The time–latitude diagram of the axisymmetric toroidal magnetic field (background image) and the radial magnetic field (contours are plotted in the range of $\pm 0.002$) for (c) model M2a and (d) model M2b.
cycle maxima. In model M3b the nonaxisymmetric dynamo is suppressed. This run shows strictly antisymmetric hemispheric cycle maxima. In model M3b the nonaxisymmetric dynamo is suppressed. This run shows strictly antisymmetric hemispheric cycle maxima. In model M3b the nonaxisymmetric dynamo is suppressed. This run shows strictly antisymmetric hemispheric cycle maxima. In model M3b the nonaxisymmetric dynamo is suppressed. This run shows strictly antisymmetric hemispheric cycle maxima. In model M3b the nonaxisymmetric dynamo is suppressed. This run shows strictly antisymmetric hemispheric cycle maxima.

4. Comparison with Observational Data

To compare our results with observations, we use synoptic maps of radial magnetic field from the KPO, SOLIS, and SDO/HMI data archives (Harvey et al. 1980; Scherrer et al. 2012; Bertello et al. 2014). Using the synoptic maps, we calculate the surface mean of the unsigned radial magnetic field, \( \langle B_r \rangle \) and the same for the axisymmetric radial magnetic field, \( \langle B_{rA} \rangle \), the parity index \( P_E \), the level of nonaxisymmetry \( P_X \), see, Equations (15) and (16). To calculate parameter \( P_X \), we perform the spherical harmonic decomposition up to \( m = 10 \). Results are shown in Figure 9. In agreement with the results of Stenflo (2013), \( \langle B_r \rangle \) reaches about 20–25 G at the solar maximum. The strength of the large-scale magnetic field is about a factor of two smaller than the total magnetic field strength. Figure 9(b) shows that \( P_X \approx 1 \) during the solar maximum and \( P_X \approx 0.5 \) during the cycle minima. This means that the total basal level of the \( m = 1 \ldots 11 \) modes of large-scale nonaxisymmetric magnetic field exceeds the basal level of the axisymmetric component of magnetic field. Parity of the axisymmetric radial magnetic field varies from \( P_E \approx 0 \) during the active phase of the solar cycle to \( P_E \approx -1 \) during the cycle minimum.

Figure 10 shows mean spectra of the azimuthal magnetic field distributions in our models and in the observations. Model M1a (nonaxisymmetric dynamo is suppressed) shows maximum for the modes with \( m = 5 \) and 6. It shows a slow decrease toward small \( m \) and a fast decrease for high \( m \). The spectrum is formed by diffusive decay of the bipolar magnetic active regions. The low-\( m \) branch gradually disappears with the increase of the nonaxisymmetric \( \alpha \)-effect perturbations. Model M1b shows a monotonic decrease of the mean magnetic field strength with the increase of \( m \). To compare with observations, we apply the FFT transform to the set of synoptic magnetograms of the radial magnetic fields. Additionally, we scale the results of our runs for \( \langle B_r \rangle \) in units of \( B_{eq} \) by a factor of 200. Figure 10(c) shows comparison of the models with observations. The observational data show the maximum of the mean magnetic field strength around \( m = 10 \). The decrease toward low \( m \) is not as strong as in model M1a, but it is similar to models M2b and and M2c. We interpret this as the presence of stochastic nonaxisymmetric dynamo action. In the observational data, diffusive decay of the high \( m \)-modes is less prominent than in our models. This can be interpreted in two different ways. First, the diffusion coefficient in the model may be too high. Second, the solar dynamo can be multiscale and intermittent phenomenon. These properties are not captured in our models.
Figures 10(b) and (d) show the normalized (by factor $2\ell + 1$) $\ell$-spectra of spherical harmonic decomposition of radial magnetic field for the models and observations. The observational data are processed using the Python interface of SHTools library (https://shtools.oca.eu/shtools/index.html). Both models and observations show a maximum for the modes of $\ell = 3, 5$ and dips for low-degree even modes. This is in agreement with analyses of Stenflo & Guedel (1988) and Stenflo (2013). As for the $m$-spectra, the magnetic field distribution in the models occupies a smaller spectral interval than in the real data.

5. Discussion

We used a relatively simple 2D mean-field dynamo model to understand whether nonaxisymmetric dynamo modes can be generated and maintained in the presence of random non-axisymmetric perturbations of the $\alpha$-effect and due to diffusive...
dynamo solutions are axisymmetric. Here, for the nonaxisymmetric dynamo modes do not develop, and the decay of emerging bipolar regions. Without such perturbations the nonaxisymmetric dynamo modes do not develop, and the nonaxisymmetric dynamo can happen even for the solar conditions, despite the strong differential rotation.

The reduced 2D nonlinear nonaxisymmetric dynamo models considered in this paper allow us to investigate influence of the perturbations on properties of the dynamo cycles, including the degree of nonaxisymmetry of the dynamo-generated magnetic field and the hemispheric asymmetry in different phases of the magnetic cycles. Despite the simplicity, the models give insight on how large-scale nonaxisymmetric magnetic structures observed on the Sun may develop, as well as directions for development of more realistic nonaxisymmetric dynamo models. Our model of emerging bipolar regions was based on parameterization of the Parker’s buoyancy instability of the axisymmetric toroidal magnetic field. Short- and long-term stochastic large-scale (up to octupole) perturbations of the $\alpha$-effect of various amplitude were considered separately and in combination with the bipolar regions.

It is found that large-scale nonaxisymmetric dynamo modes can be excited and maintained because of diffusion of emerging bipolar magnetic regions. Without perturbations of the $\alpha$-effect the nonaxisymmetric magnetic field evolution affects evolution of the axisymmetric toroidal magnetic field via magnetic buoyancy. This kind of coupling was discussed previously by Pipin & Kosovichev (2015). In general, formation of large-scale nonaxisymmetric magnetic field due to active region decay is usually accepted for granted (Mackay & Yeates 2012). Yet, the origin of the large-scale nonaxisymmetric component during the solar minima is poorly understood. Our results show that remnants of decaying bipolar regions can persists during the solar minima when the cycle duration is determined by the turbulent diffusion scale.

It is found that short-term stochastic nonaxisymmetric fluctuations of the $\alpha$-effect with the STD of 25% relative to the axisymmetric level ($\sigma_\xi = 0.25$) can generate weak nonaxisymmetric magnetic field. Its strength is two orders of magnitude smaller than the strength of magnetic field in magnetic bipolar regions. The magnitude of turbulent $\alpha$-effect fluctuations is unknown. Hoyng (1993) and Ossendrijver & Hoyng (1996) suggested that a high level short-term fluctuations with $\sigma_\xi = 1$ can explain the Grand minima events. The results of Moss et al. (2008) showed that a low level ($\sigma_\xi = 0.1$) of long-term $\alpha$ fluctuations is another option. Our model with strong nonaxisymmetric fluctuations of the $\alpha$-effect ($\sigma_\xi = 1$) shows super-cycle events caused by a nonlinear interaction of the nonaxisymmetric dynamo and the process of formation of bipolar regions. The super-cycle magnitude is more than two times greater than the mean maximum of the magnetic cycles. The model run lasted 250 cycles (Figures 5(a), (b)) shows a few other prolonged cycles of somewhat smaller magnitudes, as well as periods of low magnetic activity, but the super-cycle events are rare.

Comparison of the mean spectra of the latitudinally averaged strength of large-scale radial magnetic field with synoptic observations of solar magnetic fields during the last four cycles showed that the short-term nonaxisymmetric perturbations of the $\alpha$-effect with $\sigma_\xi = 0.5$–1 can be an option to explain the low azimuthal order part of the spectrum. Our simulations show that without the nonaxisymmetric dynamo the magnetic field strength in this part of the spectrum would be a factor of 2–3 lower than it is seen in the observational data. In comparison with our results with solar observations, we have to keep in mind that in our model the toroidal magnetic field generation and the bipolar region formation occur in the same place. Taking into account the strong effect of the differential
rotation in our models and the dominant role of the axisymmetric toroidal magnetic field, the shallow surface tachocline could be considered as a relevant place for our models. This can be different from the solar case (Brandenburg 2005). Also, our results can be applied to stars with shallow convection zones (late-F and early-G spectral classes).

Young solar analogs often show a combination of the “inactive” and “active” branches of the cyclic activity (Oláh et al. 2009). The active branch shows long cycles (Saar & Brandenburg 1999; Böhm-Vitense 2007). Pipin & Kosovichev (2016) conjectured that the active branch can be due to the nonaxisymmetric dynamo. This conclusion is supported by the results of See et al. (2016). Here, for the first time, we demonstrate the nonaxisymmetric dynamo for a solar-type model.

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