Optimization Evaluation of Geometric Error Based on Correctional Simplex Method

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Abstract. This paper researches the theory of geometric error evaluation and its application. On the basis of the geometric model of error evaluation, the features of the geometric error enclosure evaluation are analyzed, and the paper has founded the linear programming model of minimum zone association, maximum inscribed association and minimum circumscribed association. By taking the minimum conditions criterion and the theory on minimizing the extremal difference function as rules of geometric error evaluation, a correctional simplex method for direct solution of the programming model is proposed, and also the process is given. Furthermore, the method is verified by giving an example of the cylindricity error evaluation and comparing the experiment results with the ones obtained from other common methods. In addition, this designed method is also used to other geometric error evaluation in practice. The theoretical analysis and experimental results indicate that, the proposed correctional simplex method does provide well accuracy on geometric error evaluation. The outstanding advantages conclude not only high efficiency and stability but also good universality and practicality.

1. Introduction

The geometric error of workpiece makes great effect to mechanical product quality. Therefore, accurate measurement and evaluation of the geometric error of workpiece are commonly always concerned around the world. Compared with the measure of length and angle, the measurement and evaluation of the geometric error are more difficult. The study of evaluation of geometric error is important for a set of operator for new geometrical characteristics verification in GPS (Geometrical Product Specification and Verification) and a suit of verification software according to the new international GPS standards, especially with using GPS.

It is a complicated work to evaluate the geometric error. Recently, the existing approaches are mainly based on the least squares method [1], which is simple, highly efficient. But it does not follow the evaluation of geometric error in principle, so it is only an approximation method. The geometric error enclosure evaluation can follow the minimum conditions criterion [1], it includes three methods such as minimum zone association, maximum inscribed association and minimum circumscribed association. In fact, the differences between these three methods lie in finding the ideal elements based

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on the different principles, these elements can be obtained by using different algorithms: MINMAX and MAXMIN problems. Geometric error evaluation is based on the optimization problem, thus the computation-based evaluation algorithm must be highly accurate, efficient and universal. Currently the common algorithms are SAM (step accelerated method [2]), CPM (characteristic points method [3]), and GA (genetic algorithm [4]), and so on. A new evaluation method based on correctional simplex method is presented according to the definition and evaluation criterion of geometric error, as well as the corresponding engineering application software. The practice result indicates that this method has merits such as high precise and high speed. At the same time, the verification result can be shown through 3-D display.

2. The programming model of the geometric error enclosure evaluation

Under the ISO and national standards, enclosure evaluation is a method that enclosing the real elements using ideal elements in a minimum enclosure zone, and the enclosure scales are the diameter or width of the enclosure zone. The single enclosure scale is the distance between the ideal elements and the real elements, while the double enclosure scale is the distance between the two ideal elements. According to the relationship between the scale and the extremum of the vertical error, enclosure scale are defined as: the minimum or the maximum of the vertical error, \( \min \Delta \) or \( \max \Delta \) (single enclosure evaluation); the extreme difference of the vertically error, \( \max \Delta - \min \Delta \) (double enclosure evaluation). According to the definition of the enclosure evaluation, the results can be given as follows:

- external enclosure evaluation: \( \min \max_{X} g_i(X) \);
- inner enclosure evaluation: \( \max \min_{X} g_i(X) \);
- double enclosure evaluation: \( \min( \max_{X} \min_{i} g_i(X) - \min_{X} \max_{i} g_i(X)) \).

Where, \( X \) is the position parameter vector of the ideal element, \( X = (\alpha, \beta, \gamma, x, y, z)^T \), where \( \alpha \), \( \beta \), \( \gamma \) are used to indicate the rotation angles of the coordinates on \( X \), \( Y \) and \( Z \) axis, respectively; \( x \), \( y \) and \( z \) are the values shifting along the \( X \), \( Y \) and \( Z \) axis, \( g_i(X) \) is the error between the ideal element and the real element, namely \( g_i(X) = \Delta_i \).

For example, according to the geometric model of the cylindricity error evaluation as shown in Figure 1, through the programming theory model of the surfaces shape error evaluation [5, 6], then the programming theory models of the cylindricity error evaluation are given as follows:

![Figure 1. Geometric model of cylindricity.](image-url)
The minimum circumscribed cylinder (MCC)

\[
\begin{aligned}
\min \ w &= u \\
\text{s.t.} \ u &\geq R_i - x \cos \theta_i - y \sin \theta_i - \alpha z_i \sin \theta_i + \beta z_i \cos \theta_i \\
u, x, y, \alpha, \beta &\geq 0; \quad i = 1, 2, \ldots, n
\end{aligned}
\]

(1)

The maximum inscribed cylinder (MIC)

\[
\begin{aligned}
\min \ w &= -v \\
\text{s.t.} \ v &\leq R_i - x \cos \theta_i - y \sin \theta_i - \alpha z_i \sin \theta_i + \beta z_i \cos \theta_i \\
v, x, y, \alpha, \beta &\geq 0; \quad i = 1, 2, \ldots, n
\end{aligned}
\]

(2)

The minimum zone cylinder (MZC)

\[
\begin{aligned}
\min \ w &= u - v \\
\text{s.t.} \ u &\geq R_i - x \cos \theta_i - y \sin \theta_i - \alpha z_i \sin \theta_i + \beta z_i \cos \theta_i \\
v &\leq R_i - x \cos \theta_i - y \sin \theta_i - \alpha z_i \sin \theta_i + \beta z_i \cos \theta_i \\
u, v, x, y, \alpha, \beta &\geq 0; \quad i = 1, 2, \ldots, n
\end{aligned}
\]

(3)

Where, \( w \) is the objective function, \( u \) and \( v \) are the characteristic parameters, that is, the maximum and minimum radii, respectively. To any sampling point, \((\theta_i, z_i)\) are the angle and position on Z axial line, respectively, and \(R_i\) is the direction from the axial line to the sampling point.

In fact, many form error and position error evaluation problems may be prescribed with the linear programming theory based on two hypotheses [7] (the minor deviation and the minor error).

### 3. The minimum condition of geometric error evaluation

Geometric error evaluation model can be defined as the linear programming model. The programming condition is the minimum condition of the evaluation, which is the basic principle of the geometric error evaluation, and it plays a very important role in the theory comprehension, equation resolution and result judgment. The minimum condition can not only taken as the principle and the criterion for geometric error evaluation, but also verify the correctness of the theory evaluation model [8, 9].

The direction from the evaluation reference to the sampling point can be expressed as equation (4):

\[
f(p; q) = g(p) + q^T h(p)
\]

(4)

Where, \( p \) is the formation variable [10], which is the independent variable can describe the form of the surface; \( q \) is the description variable, which is the independent variable can describe the position of the evaluation reference; \( g(p) \) is the measured data, \( h(p) \) is the phasor function.

According to equation (4), \( f(p; q) \) is a linear function of \( q \), \( f(p; q) \) is a convex function, and \( f(p; q) \) is a concave function. And \( q \) must be satisfy equation (5) for the minimum zone association method.

\[
\text{conv}\{\overline{f}(q)\} \cap \text{conv}\{\overline{f}(q)\} = \phi
\]

(5)

Similarly, \( q \) must be suited for equation (6) according to the maximum inscribed association and minimum circumscribed association method.

\[
0 \in \text{conv}\{\overline{f}(q)\} \quad (0 \in \text{conv}\{\overline{f}(q)\})
\]

(6)

Where, \( \text{conv}\{f(q)\} \) and \( \text{conv}\{\overline{f}(q)\} \) are the convex closure sets of \( R \), \( \overline{f}(q) \) is the mapping set of the high points, \( f(q) \) is the mapping set of the low points.

The minimum condition of the evaluation relates with the set of the high points and the low points. The evaluation will meet the minimum condition if equation (5) and equation (6) are satisfied, which are the necessary and sufficient conditions to meet the minimum condition.

### 4. Implementation techniques of Correctional Simplex Method

According to the two hypotheses conditions, the process of error evaluation is to obtain the solution of a complex linear programming model. The feasible zone of the standard linear programming is a kind of convex polyhedron, and if it is not null, then a basic feasible solution can be surely obtained. Furthermore, if the programming has an optimum solution, it must have an optimum and basic feasible
solution. So it can be concluded that the only thing needed to do is the iterative operation of the basic feasible solution. The process of the simplex method can be described as: Firstly, search for the initial basic feasible solution. Secondly, check whether the current basic feasible solution is optimum or not. Lastly, in order to decrease the value of objective function, the basic feasible solution is changed by iterative operation.

The standard linear programming model is given as [11]:
\[
\min \ Z = C^T X, \quad s.t. \ AX = b; \quad X \geq 0
\]
Or, \[
\min \ Z = C^T X_B + C_N^T X_N \quad s.t. \ B X_B + N X_N = b; \quad X_B, X_N \geq 0
\]  
(7)
Where, \(X_B = (x_1, x_2, \ldots, x_m)^T\) is basic variable, \(X_N = (x_{m+1}, x_{m+2}, \ldots, x_n)^T\) is non-basic variable, \(A = (B, N), \ B = (a_1, a_2, \ldots, a_m)\) and \(X = \begin{bmatrix} X_B \\ X_N \end{bmatrix} = \begin{bmatrix} B^{-1}b - B^{-1}NX_N \\ X_N \end{bmatrix} \).

So the corresponding objective function is given by equation (8)
\[
Z = C_B^T B^{-1} b + \sigma_N^T X_N = Z^{(0)} + \sigma_N^T X_N
\]  
(8)
Where, \(\sigma_N^T = C_N^T - C_B^T B^{-1}N\), \(\sigma_N\) is check vector, corresponding to \(X_N\), if \(\sigma_N \geq 0\), then this basic feasible solution is the optimum solution.

In order to resolve the linear programming with simplex method, the heavy iterative operation is needed. But in fact, we are just concerned with such information as follows:

(1) The basic feasible solution, \(X_B = B^{-1}b\) and the objective function, \(Z = C_B^T B^{-1} b\); 
(2) The check number of the non-basic variable, \(\sigma_N^T = C_N^T - C_B^T B^{-1}N\) and the swap-in variable, \(x_i\); 
(3) The pivot element, \(B^{-1}a_k\) and the swap-out variable, \(x_e\).

According to the above mentioned information, a group of new basic variables and their feasible base \(B_i\) are then formed. If only are the feasible basic obtained, the above information could be computed from the initial information of the linear programming problem. That is, the important information of a basic feasible solution is obtained from the initial linear programming model, which is the important character of the correctional simplex method. The other character of the method is the computation of the \(B^{-1}\), \(B_i^{-1}\) may be obtained with \(B^{-1}\) directly when \(B\) is changed into \(B_i\), and then the heavy calculation load can be obviously cut down.

The method is made up of the following steps:
(1) Transform a general linear programming format into a standard format;
(2) Make certain the initial basic variable and \(B\) according to the standard format, and compute \(B^{-1}\), so the initial basic feasible solution is obtained, \(X_B = B^{-1}b, X_N = 0\);
(3) Compute the simplex multiplier \(\pi = C_B B^{-1}\), get the current value \(Z = C_B^T B^{-1} b = \pi b\);
(4) Compute the check number of the non-basic variable, \(\sigma_N^T = C_N^T - C_B^T B^{-1}N = C_N^T - \pi N\), if \(\sigma_N \geq 0\), then the current solution is the optimum solution, else, continue the next step;
(5) According to min \(\{\sigma_i | \sigma_j < 0\} = \sigma_k\), make certain the non-basic variable being the swap-in variable, compute the \(B^{-1}a_k\), if \(B^{-1}a_k \leq 0\), the e the next step;
(6) According to min \(\frac{(B^{-1}b_i)}{(B^{-1}a_k)} \frac{(B^{-1}a_k)}{(B^{-1}b_i)} > 0\), make certain the basic variable \(x_e\) being the swap-out variable;
(7) Replace \(a_k\) with \(a_k\), and new \(B_i\) is obtained, compute new \(B_i^{-1}\) with \(B_i^{-1} = E_{ok} B^{-1}\), new \(X_B\) with \(X_B = B_i^{-1} b, X_N = 0\), continue steps of 3)-7) until the optimum solution appears.

5. Example for the geometric error evaluation
The geometric error evaluation generally consists of the following steps:
(1) Transform the error evaluation programming model into the standard format;
(2) Input the error values and the coordinates of the sampling points;
(3) Compute the initial value of the ideal elements;
(4) Compute the optimum solution according to the correctional simplex method, $\triangle^*$;
(5) Compute the position of the ideal elements;
(6) Check the solution whether under the minimum condition or not, if not, change the initial values and execute the step 4), else, next step;
(7) Output the position of the ideal element and the result of the error evaluation.

Now, here is a axis with radius 40mm and height 400mm, three sections parallel with $(x, y)$ plane are measured, and the spacing interval between two adjacent sections is 100mm, in each section, there are 1000 sampling points in the same interval. As shown in Figure 2 and Table 1, the results are the least square solution, the minimum zone solution, the minimum circumscribed solution, and the maximum inscribed solution. The enclosure situations of three sections under minimum zone evaluation are also shown in Figure 2. The results of other algorithms are given in Table 2.

![Image](image.png)

**Figure 2.** The results of the cylindricity error evaluation.

| Table 1. Computation results of Cylindricity. | Table 2. Comparison of results. |
|----------------------------------------------|--------------------------------|
| **Method** | **LSC** | **MCC** | **MIC** | **MZC** | **Algorithm** | **f(µm)** |
| $x$(mm) | 1.528 | 2.138 | 1.578 | 2.107 | SAM | 7.9031 |
| $y$(mm) | 1.095 | 0.212 | 1.150 | 0.034 | CPM | 7.8006 |
| $\alpha$(rad) | 0.583 | 0.065 | 0.634 | 0.242 | GA | 7.0622 |
| $\beta$(rad) | 1.170 | 1.409 | 1.200 | 1.388 | CSM | 7.0394 |
| $f$(µm) | 7.9918 | 7.3942 | 7.8996 | 7.0394 | | |
The cylindricity errors in Table 2 are obtained according to different methods under minimum zone condition. The SAM, CPM, and GA are all optimization algorithms, which can be used to solve the form error evaluation. As shown in Table 2, the result obtained by the correctional simplex method (CSM) is better than by SAM or CPM method, and it is a little better than or equal to the one by GA method. From Table 2, it can be concluded that the process of correctional simplex method can obtain a stable result which is the optimum solution. On the other hand, the model of the cylindricity error is verified to be reasonable in solving the evaluation problem, and it is reliable. In addition, the consumed time with this method is shorter than other methods.

6. Conclusion
On the basis of the model of the geometric error evaluation, the correctional simplex method for direct solution of the programming model is proposed. From the experimental results shown in tables and figures, it can be concluded that the proposed method is computationally accurate, robust and efficient in solving the problem of geometric error. In addition, this novel method has also been used to solve other geometric error evaluation problems in practice, being proved universal and practical. By using the evaluation software, the technology of NC and the technology of optics, we can develop the hardware system and improve the current measure instrument, then realize the data collection and evaluation intelligently.

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