The light scalar $K_0^*(700)$ in the vacuum and at nonzero temperature

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There is mounting evidence toward the existence of a light scalar kaon $\kappa \equiv K_0^*(700)$ with quantum numbers $I(J^P) = \frac{1}{2}(0^+)$. Here, we recall the results of an effective model with both derivative and non-derivative terms in which only one scalar kaonic field is present in the Lagrangian (the standard quark-antiquark ,,seed'' state $K_0^*(1430)$): a second “companion” pole $K_0^*(700)$ emerges as a dynamically generated state. A related question is the role of $K_0^*(700)$ at nonzero $T$: since it is the lightest scalar strange state, one would naively expect that it is relevant for $\pi$ and $K$ multiplicities. However, a repulsion in the $\pi K$ channel with $I = 3/2$ cancels its effect.

1. Introduction

The lightest scalar kaonic state listed in the PDG $[1]$ is $K_0^*(700)$ (previously called $K_0^*(800)$, see PDG 2016 $[2]$ and older versions). This state, sometimes called $\kappa$, still “needs confirmation”, but many works do find a pole in that energy region, see Ref. $[3]$ and refs. therein. The PDG reports at present the following result:

pole $\kappa$ [PDG]: $(630-730) - i(260-340)$ MeV,

(hence, the pole width lies between 520-680 MeV), while the Breit-Wigner (BW) mass and widths are

BW [PDG]: $m_{\kappa,BW} = 824 \pm 30$ MeV , $\Gamma_{\kappa,BW} = 478 \pm 50$ MeV. (2)

The BW and the pole widths are compatible, but the BW mass is somewhat larger. There is however no friction, since BW and pole masses are different

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quantities which coincide only when a resonance is narrow. This is definitely not the case for the $\kappa$, which is a very broad state with a width-to-mass ratio larger than 0.5.

In a certain sense, the light $\kappa$ can be regarded as the “brother” of the light $\sigma \equiv f_0(500)$ meson [1]. This state is also very broad and for a long time it was not clear if there is a pole on the complex plane. Now its existence is confirmed by many studies and the state is listed in the PDG, see also the review paper [4]. The destiny of the light $\kappa$ looks somewhat similar: its final confirmation is probably just a matter of time.

Yet, a different issue is the nature of the $\kappa \equiv K^*_0(700)$ and the $\sigma \equiv f_0(500)$. According to mounting evidence, both states are not simple quark-antiquark states, but are rather four-quark objects, either in the form of a tetraquark nonet together with $a_0(980)$ and $f_0(980)$ [5] or as dynamically generated molecular-like states [6]. The $\kappa$ can be then interpreted as a diquark-antidiquark state ([u,d][\bar{d},\bar{s}] ,...) and/or as $K\pi$ state (mixing among these configurations is of course possible and rather probable to occur). If $\kappa$ is not $\bar{q}q$, where should be the scalar strange quarkonium? According to the quark model [7] and modern chiral approaches [8], the lightest $\bar{q}q$ kaonic state (u$\bar{s}$,...) is the well-established $K^*_0(1430)$ (similarly, the lightest scalar/isoscalar quarkonium is the state $f_0(1370)$). The question that we review in this work is the link between the standard state $K^*_0(1430)$ and the dynamically generated state $K^*_0(700)$. We find (see Sec. 2) that the $\pi K$ loops dressing $K^*_0(1430)$ generate $K^*_0(700)$ as a companion pole (a peculiar four-quark object) [9] (similarly, the $a_0(980)$ emerges as a companion pole of $a_0(1450)$ [10]).

There is however a related important question: if the light $\kappa$ is existent, should it be included into thermal hadronic models [11]? At a first sight, the answer is ‘yes’. In fact, the light $\kappa$ is the second-lightest state with nonzero strangeness, thus potentially relevant. Yet, a detailed analysis of the problem [12] shows that one should better not include this state into a thermal model (see Sec. 3). Namely, also repulsive channels contribute to the thermodynamics [13, 14, 15]. Just as for the $f_0(500)$ whose contribution is cancelled by $\pi\pi$ scattering with $I = 2$, the contribution of the $\kappa$ is cancelled by the repulsion in $\pi K$ channel with $I = 3/2$. Thus, the easiest thing to do is to neglect both the $f_0(500)$ and the $K^*_0(700)$ when studying hadronic thermal models for the late stage of heavy ion collisions.

2. The light $\kappa$ in the vacuum

As a first step, we write down a Lagrangian that contains only one scalar state $K^*_0$, to be identified with $K^*_0(1430)$, coupled to $K\pi$ pairs:

$$\mathcal{L}_{K^*_0} = aK^*_0 K^- \pi^0 + bK^*_0 \partial_\mu K^- \partial^\mu \pi^0 + \ldots ,$$

(3)
where dots refer to other isospin channels. Note, there is no $\kappa \equiv K_0^*(700)$ into the model (yet). There are both derivative and non-derivative terms: the former naturally dominates in the context of chiral perturbation theory and also emerge from the extended Linear Sigma Model \cite{8}. The decay width reads:

$$\Gamma_{K_0^* \rightarrow K\pi}(m) = 3 \frac{|\vec{k}_1|}{8\pi m^2} \left[ a - b \frac{m^2 - M_K^2 - M_\pi^2}{2} \right]^2 F_\Lambda(m), \quad (4)$$

with the vertex function $F_\Lambda(m) = \exp(-2\vec{k}_1^2/\Lambda^2)$. Here, $\Lambda$ is an energy scale describing the nonlocal nature of mesons \cite{16} and $\vec{k}_1$ the three-momentum of one outgoing particle, $M_K$ the kaon mass, and $M_\pi$ the pion mass. (For details and phenomenology of the spectral function, see Refs. \cite{17}).

The propagator of $K_0^*$ is given by

$$\Delta_{K_0^*}(m^2) = \left[ m^2 - M_0^2 + \Pi(m^2) + i\epsilon \right]^{-1},$$

$M_0$ being the bare mass of $K_0^*(1430)$ and $\Pi(m^2)$ the one-loop contribution. The spectral function $d_{K_0^*}(m) = \frac{2\pi}{m^2} |\text{Im}\Delta_{K_0^*}(p^2 = m^2)|$ is the mass probability density (its integral is normalized to unity). Typically, for the “Breit-Wigner” value $M_{BW}$ determined as $M_{BW}^2 - M_0^2 + \text{Re}\Pi(M_{BW}^2) = 0$ the spectral function has a peak’s width $\Gamma_{BW} = \text{Im}\Pi(M_{BW})/M_{BW}$. A useful approximation, valid if the width is sufficiently small, is the relativistic Breit-Wigner expression:

$$d_{K_0^*}(m) \approx d_{K_0^*}^{BW}(m) = N \left[ (m^2 - M_{BW}^2)^2 + M_{BW}^2 \Gamma_{BW}^2 \right]^{-1}. \quad (5)$$

Under this approximation, there is only one pole in the complex plane at $m^2 \simeq M_{BW}^2 - iM_{BW}\Gamma_{BW}$ (hence, $m \simeq M_{BW} - i\Gamma_{BW}/2$). But, when a resonance is broad, these approximations are not anymore valid.

We now turn to $\pi K$ scattering. Within our framework, the pion-kaon phase shift is given by \cite{9}:

$$\delta_{\pi K,\text{swave}}(m) = \delta_{(I=1/2, J=0)}(m) = \frac{1}{2} \arccos \left[ 1 - \pi \Gamma_{K_0^*}(m)d_{K_0^*}(m) \right], \quad (6)$$

where $\delta_{(I,J)}(m)$ is the general phase shift for a given isospin $I$ and total spin $J$. The amplitude of the process and the phase-shift are linked by $a_{(I,J)} = (e^{i\delta_{(I,J)}(m)} - 1)/(2i)$. The parameters $(a, b, M_0, \Lambda)$ entering in Eq. (3) were determined via a fit to $\pi K$ phase-shift data \cite{18}, see Ref. \cite{9} for details. A very good description of data is achieved. A study of the complex plane shows an interesting fact: besides the pole corresponding to the well-known $K_0^*(1430)$ state $(1.413 \pm 0.002) - i(0.127 \pm 0.003)$ GeV, there is a second pole which correspond to $K_0^*(700)$:

$$(0.746 \pm 0.019) - i(0.262 \pm 0.014) \text{ GeV}. \quad (7)$$
The numerical value is compatible with the PDG value of Eq. (1). A large-$N_c$ study confirms that, while the first pole tends to the real axis (and hence is a $\bar{q}q$ state), the second one moves away from it, as it is expected for a dynamically generated state.

In conclusion, the simple model of Eq. (3) is able to describe $\pi K$ scattering data and naturally gives rise to the pole of $K^*_0(700)$ as a companion pole of the predominantly quark-antiquark resonance $K^*_0(1430)$.

3. The light $\kappa$ at nonzero temperature

The partition function of an hadronic gas can be expressed as the sum of the contributions of stable particles and their mutual interactions:

$$\ln Z = \ln Z_{\text{pions}} + \ln Z_{\text{kaons}} + \ldots + \ln Z^{\text{int}}, \quad \ln Z^{\text{int}} = \sum_{I,J} \ln Z_{IJ}.$$ (8)

The first term $\ln Z_{\text{pions}} = 3F_1(m_\pi)$ refers to pions and $\ln Z_{\text{kaons}} = 4F_1(m)$ to kaons, where $F_1(m) = \int \frac{d^3p}{(2\pi)^3} \ln \left[1 - e^{-\sqrt{\vec{p}^2 + m^2}/T}\right]$ is the contribution of a free particle with mass $m$. The term $\ln Z_{IJ}$ refers to the contribution of the interactions in the $(I,J)$ channel [13]:

$$\ln Z_{IJ} = (2I+1)(2J+1) \int_0^\infty \frac{d\delta_{IJ}(m)}{dm} F_1(m).$$ (9)

When in a certain channel a narrow resonance is present, one finds its standard contribution. For instance, for $I = J = 1$ the $\rho$ meson is produced. In the nonrelativistic BW-limit $\frac{1}{\pi} \frac{d\delta_{1/2,0}(m)}{dm} \simeq \frac{\Gamma_\rho}{2\pi} \left(\frac{m - m_\rho}{\Gamma_\rho^2/4}\right)^{-1}$. Moreover, for $\Gamma_\rho \to 0$, $\delta(m - M_\rho)$ emerges: the contribution of a stable $\rho$ is obtained.

However, Eq. (9) is very general and can describe also broad resonances as well as non-resonant channels, such as repulsive ones. This is important for the $\kappa$. In the resonant $I = 1/2, J = 0$ channel in which the $\kappa$ is formed, one has (upon integrating up to 1 GeV) $\ln Z_{(1/2,0)} = \int_0^{1 \text{ GeV}} \frac{2dm}{\pi} \frac{d\delta_{1/2,0}(m)}{dm} F_1(m)$. This is sizable. However, one should also consider the repulsion in the $I = 3/2, J = 0$ channel. Remarkably, the sum

$$\ln Z_{(1/2,0)} + \ln Z_{(3/2,0)} = \int_0^{1 \text{ GeV}} \frac{2dm}{\pi} \frac{d\delta_{1/2,0}(m)}{dm} + \frac{4}{\pi} \frac{d\delta_{3/2,0}(m)}{dm} F_1(m)$$ (10)

is small. Namely, while $\frac{d\delta_{1/2,0}(m)}{dm} > 0$ (attraction), $\frac{4}{\pi} \frac{d\delta_{3/2,0}(m)}{dm} < 0$ (repulsion). Note: $\frac{1}{\pi} \frac{d\delta_{1/2,0}(m)}{dm} \neq d_{K^*_0}(m)$. (This would be true only in the BW limit). In conclusion, the light $\kappa$ can be safely neglected in the construction of thermal hadronic models.
4. Conclusions

We have described the emergence of the state $\kappa \equiv K_0^*(700)$ as a companion pole of $K_0^*(1430)$ by using an effective hadronic model [9]. The numerical value of the pole (7) is in agreement with the present PDG estimate of Eq. (1). On the other hand, contrary to the naive expectations, the light $\kappa$ is not relevant in a thermal hadronic gas. Namely, its influence on thermodynamical properties is cancelled by a repulsion in the $I = 3/2$ channel. Either one includes both the light $\kappa$ and the repulsion, or -even easier- neglects both of them.

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