Cyclotron beam extraction by acceleration

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ABSTRACT: One of the decisive issues in the design and operation of cyclotrons is the choice of the beam extraction method. Typical methods are extraction by electrostatic extractors and by stripping. The former method requires DC high voltage electrodes which are notorious for high-voltage breakdowns. The latter method requires beams of atomic or molecular ions which are notorious for rest gas and Lorentz stripping. Here we discuss the conditions to be met such that a beam will leave the magnetic field of an isochronous cyclotron purely by fast acceleration.

KEYWORDS: Accelerator modelling and simulations (multi-particle dynamics, single-particle dynamics); Beam dynamics

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1 Introduction

The extraction method is a decisive choice for the design of isochronous cyclotrons [1–8]. Specifically in case of high intensity beams, extraction losses have to be minimized in order to avoid activation of the machine components [9]. The extraction methods used most frequently are stripping extraction [10–14] and extraction by electrostatic deflectors [15, 16]. Both methods have their advantages and issues.

Stripping extraction, i.e. the removal of electrons by the passage of an ion beam through thin stripper foils, requires the acceleration of ions that are not fully stripped yet, for instance of $H^-$-ions or $H_2^+$-molecules. But since not fully ionized atoms (or molecules) have a considerably larger scattering cross-section with the rest gas molecules, some losses in the course of acceleration are unavoidable. Besides rest gas stripping also Lorentz stripping may result in losses [17–21]. Therefore stripping extraction requires an excellent vacuum and the Lorentz stripping effect limits both, the maximal beam energy and the maximal magnetic field. Low-loss extraction by electrostatic deflectors, on the other hand, is notorious for beam interruptions by high voltage breakdowns and requires well-separated turns in order to place the extraction septum between turns.

Despite these issues, isochronous cyclotrons are attractive for the production of high intensity CW beams, due to their superior energy efficiency [22], their small footprint, and their relatively low cost. Therefore the use of cyclotrons for the production of high intensity beams has been suggested in various projects, typically aiming for several mA of beam current at energies between 600 MeV and 1 GeV, for instance for ADS systems, neutron production and also neutrino physics [23–26]. A cyclotron facility that often serves as a proof-of-principle-machine for these objectives is the high intensity proton accelerator (HIPA) facility of the Paul Scherrer Institute (PSI) in Switzerland, which provides up to 2.4 mA of protons at 590 MeV [27].

In 1981 Werner Joho formulated “Joho’s $N^3$ law”, which states that the possible beam intensity, for the same losses, increases with the inverse of the third power of the number of turns [28]. This law has been verified with astonishing accuracy at the PSI Ring cyclotron [27]. Hence we can reasonably assume that any high intensity proton machine will aim for the lowest possible number of
turns, that is, for the highest possible acceleration voltage, mostly in order to provide the highest possible turn separation at extraction. The incredible increase of beam current from the PSI Ring machine has been achieved by the insertion of a flat-topping cavity and by the reduction of the number of turns, from originally more than 300 to now \( \approx 180 \) with an upgrade of the rf cavities and amplifier chains. However, the reduction of the turn number has a side effect which has not received much attention, namely that the total phase shift of the beam by the fringe field near the outer radius is significantly reduced.

In 1995, Yves Jongen proposed the so-called “self-extraction”, a method to design cyclotrons such that the beam would leave the field without stripper or electrostatic extractor [29]. A cyclotron build by IBA provided the proof-of-principle [30] for this extraction method. However, a theoretical account of the conditions that have to be met to allow for beam escape has, to the knowledge of the author, not been provided so far.

Here we report about the fact that the design of high energy high intensity separate sector cyclotrons of the PSI type meets the main requirements for auto-escape of the beam without electrostatic deflectors.

2 The cyclotron bending limit

The main aim of the reduction of the turn number in the Ring cyclotron is the increase of the turn separation so that the septum of the electrostatic extractor can be placed between cleanly separated turns. This is required not only in order to avoid an overheating of the septum, but also to minimize beam loss and activation of the septum and subsequent beamline elements. Isochronous cyclotrons are efficient because they operate at constant magnetic field and constant rf-frequency. This allows for narrow-band rf structures with high Q-factors. Furthermore the beam passes the same rf-structure multiple times which also increases the efficiency of the acceleration. However, this method requires that the circulation frequency of the beam stays in sync with the frequency of the rf system. The average magnetic field must then increase radially with the relativistic \( \gamma \)-factor and since the velocity is approximately given by \( v = \omega R \), the (average) field must approximately follow

\[
B \propto \frac{1}{\sqrt{1 - (\omega R/c)^2}}. \tag{2.1}
\]

Therefore the isochronism of a cyclotron can not be sustained in the fringe field due to the radial decrease of the magnetic field. Hence the phase between beam and accelerating rf will shift in the course of extraction and the bunches will get more and more out of sync with the accelerating rf.

Let the energy gain per turn \( dE/dn \) be given by

\[
dE/dn = q V_{rf} \cos (\phi) = \Delta E_{max} \cos (\phi) \tag{2.2}
\]

where \( \phi \) is the phase of the beam (relative to the rf-phase) and \( V_{rf} \) is the maximum accelerating voltage per turn. If the beam is not extracted fast enough, the phase \( \phi \) will be shifted beyond 90° and the beam will loose instead of gain energy when passing the next rf cavity.

Hence there is another important consequence of the reduction of turn number in isochronous cyclotrons: the maximum energy that the beam is able to reach depends crucially of the phase shift in
the fringe field. On the other hand, it is evident that any finite field \( B(R) \) can only hold a circulating beam up to a finite momentum.

The relation between momentum \( p \), radius \( R \) and magnetic field \( B \) is given by

\[
p = q B R \tag{2.3}
\]

so that

\[
\frac{dp}{dR} = q B \left( 1 + \frac{R}{B} \frac{dB}{dR} \right) = q B (1 + k). \tag{2.4}
\]

The factor \( k = \frac{R}{B} \frac{dB}{dR} \) is the field index.\(^1\) The maximum momentum is given by \( \frac{dp}{dR} = 0 \) which corresponds to a field index of \( k = -1 \). Beyond the point, where the radial field gradient in the fringe field region is steeper than \(-B/R\), the beam can not stay within the machine. Hence there is a maximum radius

\[
R_{\text{max}} = -\frac{B(R_{\text{max}})}{\frac{dB}{dR} |_{R_{\text{max}}}} \tag{2.5}
\]

which corresponds to the maximum momentum

\[
p_{\text{max}} = q R_{\text{max}} B(R_{\text{max}}) = -\frac{q B(R_{\text{max}})^2}{\frac{dB}{dR} |_{R_{\text{max}}}}. \tag{2.6}
\]

Let us call this momentum and the corresponding energy the escape momentum and escape energy.

Hence there are two maximum values for the radius, firstly the radius where the phase shift reaches 90° and secondly the radius of the maximum momentum. The decisive question is therefore, which of these radii is larger. This depends on two factors, firstly the exact shape of the fringe field and secondly, the accelerating voltage \( V_{rf} \). The latter is in the general case a function of radius \( V = V(R) \) as well, but since this dependency is usually small (over the region of interest, i.e. the extraction), we shall neglect it in the following.

The reduction of the turn number has, as mentioned before, the main purpose to increase the turn separation. Since energy, radius and momentum have mutual bijective relationships, the radius gain per turn \( \frac{dR}{dn} \) in a cyclotron is, in sectorless approximation, given by

\[
\frac{dR}{dn} = \frac{dE}{n} \left( \frac{dE}{dp} \frac{dp}{dR} \right)^{-1}. \tag{2.7}
\]

From Hamilton’s equation of motion it is known that \( \frac{dE}{dp} = v \), so that one obtains with eq. (2.2) and eq. (2.4):

\[
\frac{dR}{dn} = \frac{V_{\text{rf}} \cos(\phi)}{v B (1 + k)} = \frac{q V_{\text{rf}} \cos(\phi) R}{v p (1 + k)}, \tag{2.8}
\]

which can be reformulated as

\[
\frac{dR}{dn} = \frac{q V_{\text{rf}} \cos(\phi) R \gamma}{E (\gamma + 1) (1 + k)}. \tag{2.9}
\]

Both, energy and radius vary only a little over the region of interest. The dominating factors are therefore \( \cos(\phi) \) and \( 1 + k \). The question is then, whether the radius for \( \phi = 90° \) or the radius for \( k = -1 \) is smaller. When the phase \( \phi \) approaches 90° before \( k \) approaches \(-1\), then the turn separation

\(^1\)The usual convention is \( k = -\frac{R}{B} \frac{dB}{dR} \), but the cyclotron literature mostly uses the positive sign convention.
will typically decrease to zero and then become negative, so that the $E - \phi$-loop closes. However, if the phase $\phi$ stays well below $90^\circ$ when $k$ approaches $-1$, then the beam will escape the field simply because the momentum exceeds the bending limit.

3 Estimation of the acc. voltage required to reach the escape energy

If $\theta$ is the azimuthal angle and $\phi_{rf}$ the phase of the rf, then the phase shift per time $\frac{d\phi}{dt}$ in an isochronous cyclotron can be written as

$$\frac{d\phi}{dt} = \frac{d\phi_{rf}}{dt} - N_h \frac{d\theta}{dt} = \omega_{rf} - N_h \omega_{\text{rev}}$$

(3.1)

where $\omega_{\text{rev}}$ is the actual revolution frequency of the bunch and $N_h$ is the so-called harmonic number, that is the number of rf-cycles per revolution of the bunch. The number of revolutions per time is

$$\frac{dn}{dt} = \frac{1}{T_{\text{rev}}}$$

(3.2)

so that

$$\frac{d\phi}{dn} = T_{\text{rev}} (\omega_{rf} - N_h \omega_{\text{rev}}) = 2\pi \left( \frac{\omega_{rf}}{\omega_{\text{rev}}} - N_h \right)$$

(3.3)

where $T_{\text{rev}} = 2\pi R/v$ is the time required per revolution. The relation between the revolution frequency of a particle with charge $q$ and mass $m$ in the magnetic field $B(R)$ can be written as

$$\omega_{\text{rev}} = \frac{q}{m} \gamma B(R) .$$

(3.4)

In perfectly isochronous machines the field is given by

$$B_{\text{iso}} = \frac{B_0}{\sqrt{1 - (R \gamma R_0)^2}}$$

(3.5)

where $\omega_0 = q/m B_0 = \omega_{rf}/N_h$ is the “nominal” revolution frequency. Often the cyclotron radius $R_{\infty} = c/\omega_0$ is used to write this as

$$B_{\text{iso}} = \frac{B_0}{\sqrt{1 - (R/R_\infty)^2}} = B_0 \gamma_R$$

(3.6)

where $\gamma_R$ is understood purely as a function of the radius.

Let us assume that the fringe field can be approximated by an Enge type function [31, 32] of the simplest form so that the real (azimuthally averaged) magnetic field $B(R)$ is given by

$$B(R) = B_{\text{iso}} f(R)$$

(3.7)

with

$$f(R) = \left( 1 + \exp \left( \frac{R - R_h}{g} \right) \right)^{-1}$$

(3.8)
Figure 1. Top: average fringe field $B_{\text{Ring}}$ of the PSI Ring Cyclotron, corresponding isochronous field $B_{\text{iso}} = 5.45 \, \text{kG} \cdot \gamma R$ and $B(R)$ according to eq. (3.7) with $g = 36 \, \text{mm}$ and $R_h = 4.63 \, \text{m}$ and $R_\infty = 5.7 \, \text{m}$. Bottom: the corresponding phase shift per momentum increase, phase shift per radius increase and function $1 + k$ (scaled by a factor of 5). The positions of $k = 0$ and $k = -1$ are indicated by dashed lines. The radius $R$, the factor $\gamma$ and even a factor $R \gamma R$ (shown) vary only weakly over the fringe field region.

where $g$ is approximately half of the pole air gap$^2$ and $R_h$ is the radius for which the field is half of the isochronous field, i.e. $f(R_h) = 1/2$. The radial derivative of $f(R)$ is then

$$ \frac{df}{dR} = -\frac{1}{g} f \left(1 - f \right). \quad (3.9) $$

Figure 1 shows how this approximation compares to the (azimuthal) average field of the PSI Ring cyclotron. For our purposes the agreement is — within the region of interest — reasonable, even though the parameters obtained from this “fit” do not agree very well with the real Ring cyclotron. The field index is then given by

$$ k = \frac{R}{B} \frac{dB}{dR} = \frac{R}{B} B_0 \left( \frac{d\gamma R}{dR} f + \gamma R \frac{df}{dR} \right) = \gamma_R^2 - 1 - (1 - f) \frac{R}{g}. \quad (3.10) $$

The revolution frequency can then be expressed by

$$ \omega_{\text{rev}} = \frac{q}{m} \gamma R \frac{B_0}{f(R)} = \frac{\gamma R}{\gamma R_0} \omega_0 f(R), \quad (3.11) $$

where $\gamma$ is the usual relativistic factor, that is, a function of velocity:

$$ \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \beta^2}}. \quad (3.12) $$

In the sectorless approximation the velocity is given by

$$ v = \omega_{\text{rev}} R = R \frac{\gamma R}{\gamma} \omega_0 f, \quad (3.13) $
so that \( \gamma \) is given by

\[
\gamma = \frac{1}{\sqrt{1 - \left( \frac{R}{R_{\infty}} \frac{\gamma}{\gamma R f} \right)^2}}. \tag{3.14}
\]

This can be used to find the radial dependency of \( \gamma \):

\[
\gamma = \sqrt{1 + (\gamma_R^2 - 1) f^2}. \tag{3.15}
\]

If the cyclotron is isochronous up to the fringe region, then \( \omega_{rf} = N_h \omega_0 \) and therefore eq. (3.3) yields

\[
\frac{d\phi}{dn} = 2 \pi N_h \left( \frac{\gamma}{\gamma_R f} - 1 \right). \tag{3.16}
\]

The phase shift per energy gain \( \frac{d\phi}{dE} \) can be expressed, using eq. (2.2), by

\[
\frac{d\phi}{dE} = \frac{d\phi}{dn} \frac{1}{dE/dn} = \frac{2 \pi N_h}{q V_{rf} \cos(\phi)} \left( \frac{\gamma}{\gamma_R f} - 1 \right) \tag{3.17}
\]

and per momentum gain by

\[
\frac{d\phi}{dp} = \frac{d\phi}{dE} \frac{dE}{dp} = \frac{d\phi}{dE} v. \tag{3.18}
\]

The velocity \( v = \frac{dE}{dp} \) can be replaced by the use of eq. (3.13) so that:

\[
\frac{d\phi}{dp} = \frac{2 \pi N_h \omega_0}{q V_{rf} \cos(\phi)} \left( 1 - \frac{\gamma_R f}{\gamma} \right) R. \tag{3.19}
\]

In order to obtain the phase shift as a function of radius, one may use eq. (2.4) to obtain

\[
\frac{d\phi}{dR} = \frac{2 \pi N_h m c^2}{q V_{rf} \cos(\phi)} R \left( 1 - \frac{\gamma_R f}{\gamma} \right) \gamma_R f (1+k). \tag{3.20}
\]

Introducing the abbreviation

\[
A = \frac{2 \pi N_h m c^2}{q V_{rf}} \tag{3.21}
\]

together with eq. (3.10) and eq. (3.15) one obtains:

\[
\frac{d\phi}{dR} = A \frac{R}{\cos(\phi) R_{\infty}^2} \left( 1 - \frac{\gamma_R f}{\sqrt{1 + (\gamma_R^2 - 1) f^2}} \right) \times \gamma_R f \left( \gamma_R^2 - (1-f)R/g \right), \tag{3.22}
\]

and hence

\[
d \sin(\phi) = A \frac{R \gamma_R}{R_{\infty}^2} \left( 1 - \frac{\gamma_R f}{\sqrt{1 + (\gamma_R^2 - 1) f^2}} \right) \times f \left( \gamma_R^2 - (1-f)R/g \right) dR. \tag{3.23}
\]

As shown in figure 1, the phase \( \phi \) and the term \( 1 + k \) vary fast over the fringe field region, while the relative change of \( R \) and \( \gamma_R \) are small. Hence it is a reasonable approximation to keep the latter
constant in the integration. This means that we fix $R \approx R_x$ and $\gamma_R \approx \gamma_x$, where the subscript “x” indicates that these are the values at extraction.

From eq. (3.9) one obtains

$$dR = g \frac{df}{f (f - 1)}. \quad (3.24)$$

One may express the phase factor by its Taylor series (using $f$ as variable, located at $f = 1$):

$$\left(1 - \frac{\gamma_R f}{\sqrt{1 + (\gamma_R^2 - 1) f^2}}\right) = \frac{1 - f}{\gamma_R^2} + \frac{3}{2} \frac{\gamma_R^2 - 1}{\gamma_R^4} (1 - f)^2 + \ldots \quad (3.25)$$

and since $1 - f$ is small, one may use the first term only. Then the integrand simplifies to:

$$d \sin (\phi) = -A g \frac{R_x}{\gamma_x^2} \left(\gamma_x^2 + (f - 1) R_x / g\right) df. \quad (3.26)$$

The integration over the fringe field region starts with $f \approx 1$ and ends where $k = -1$, which corresponds (see eq. (3.10)) to $f = 1 - \gamma_R^2 \frac{g}{R}$, so that the leading term after integration is:

$$\sin (\phi_f) - \sin (\phi_i) \approx A \frac{g^2}{R_x^2} \frac{\gamma_x^3}{2} \quad (3.27)$$

where $\phi_i$ is the initial phase (prior to extraction) and $\phi_f$ is the final phase. If the initial phase is zero ($\sin (\phi_i) \approx 0$), the condition $\phi \leq 90^\circ$ yields $\sin (\phi_f) \leq 1$, then the accelerating voltage which suffices to reach the escape energy, is finally (skipping the subscript $x$):

$$q V_{it} \geq \frac{\pi N_h m c^2 \gamma^3 g^2}{R_x^2}. \quad (3.28)$$
With \( \beta \approx R/R_{\text{ext}} \) and \( \Delta E_{\text{max}} = q V_{\text{rf}} \) this can also be written as

\[
\frac{\Delta E_{\text{max}}}{E} \geq \pi N_h \gamma (\gamma + 1) \frac{g^2}{R^2},
\]

where \( E = mc^2 (\gamma - 1) \) and \( R \) are the (kinetic) extraction energy and radius. Since the number of turns is approximately \( N_t \approx E/\Delta E_{\text{max}} \), one finds

\[
N_t \leq \frac{1}{\pi N_h \gamma (\gamma + 1)} \frac{R^2}{g^2}.
\]

Hence it is mostly the squared ratio of extraction radius and pole gap at extraction which determines the maximal number of turns or the minimal energy gain, respectively. The compact superconducting cyclotron COMET [34, 35], which provides the 250 MeV proton beam for the proton therapy facility Proscan at PSI, has a half-gap of 22 mm, an extraction radius of 820 mm and a harmonic number of \( N_h = 2 \). According to eq. (3.30), escape extraction is then possible for a maximum turn number of about 85. This compares to an actual turn number of about 650, i.e. eight times as much. Hence the beam would be able to escape without extractor under these conditions, if the half-gap would be (reduced to) less than 8 mm.

However, for the PSI Ring cyclotron, eq. (3.28) results, using the parameters obtained from the azimuthal average field (i.e. \( g \approx 36 \text{ mm} \)), in a minimal voltage of

\[
V_{\text{rf}} \geq 2.95 \text{ MV}.
\]

The numerical integration of eq. (3.20) for the same parameters yields a voltage of \( V_{\text{rf}} \geq 2.9 \) MV as required to reach the escape energy, which is in reasonable agreement with our approximation. This voltage is close to the average voltage actually used today in the PSI Ring cyclotron, which is about 2.6 MV.

But the azimuthal average of the field is not a good approximation for separate sector cyclotrons. A realistic value of the half-gap at extraction is about 21 mm. As shown in figure 2, the fall-off of the maximal field values is much steeper than that of the azimuthal average. Specifically the parameter \( g \) is less than half of the value obtained from the azimuthal average (and closer to the real half-gap).\(^5\) Hence the required voltage might well be a factor of 4 smaller, i.e. as low as \( V_{\text{rf}} \geq 650 \text{ kV} \) or even less.

Figure 3 shows results of direct orbit tracking\(^6\), starting at 530 MeV and zero phase, which show that beam escape is possible for less than about 90 turns, i.e. for an energy gain per turn of more than \((590 \text{ MeV} - 530 \text{ MeV})/90 = 670 \text{ keV/turn} \). This is indeed by a factor of about 4 below the currently used acceleration voltage.

Therefore the acceleration of the beam in the PSI Ring machine is so fast that the beam approaches the bending limit before the beam phase is shifted to 90°. The beam would escape the magnetic field without any extraction device.

Nonetheless the beam has to pass the fringe field region, where the negative \( k \)-values lead to a strong radial defocusing and vertical focusing. Both effects have to be compensated if one aims to

\(^5\)The most accurate determination of the field shape as “seen by the beam” would derive from the average field along the scalloping closed orbits. But stable closed orbits do not exist beyond \( k = -1 \).

\(^6\)We use CYBORC (“Cyclotron Beam Orbit Calculator”) for the tracking, a “C”-implementation of a 4th-order Runge-Kutta solver for the equations of motion on a polar grid [36]. The field map of the PSI Ring [6] that we used for the tracking is based on measured data [33, 37, 38] and has been used in several other recent publications, for instance refs. [39–41].
Figure 3. Tracking results for escape extraction from the PSI Ring cyclotron. Top: energy vs. turn number for orbits starting at 530 MeV in the PSI Ring cyclotron. The rf-voltage increases from blue to red. Bottom: phase $\tau$ of the rf at $\theta = 0^\circ$ versus energy.

make practical use of escape extraction. Furthermore, since the beam has to pass the $\nu_r = 1$-resonance, it is required to precisely control the first harmonic content of the field prior to extraction.

4 Extraction by acceleration from the PSI Ring cyclotron

Figure 4 shows the iron shape of the PSI Ring cyclotron, some accelerated orbits and the last orbit which escapes the field without electrostatic extractor. The positions of the four accelerating cavities (plus one flat-topping cavity) are indicated by the five rectangular boxes. The cavities provide enough energy gain per turn to extract the beam after about 185 turns [27]. Currently the Ring cyclotron uses an electrostatic extractor to extract the beam at $E \approx 590$ MeV, i.e. before the maximum field is reached. Figures 4 and 5 show the escape extraction of nine orbits. Besides a central (“reference”) orbit, we tracked orbits with a different starting radius ($\pm 3$ mm), orbits with different initial energy ($\pm 0.1$ MeV), initial radial momentum ($\pm 5$ mm) and rf phase ($\pm 3^\circ$). It is not only that the orbit escapes the field, as shown in figure 6, but furthermore the field gradient is positive or zero up to the last sector before escape. Hence the orbit “sees” a substantial negative field gradient only for a short time. Hence a single gradient corrector might be sufficient to keep the beam radially compact. The magnetic field gradient is of the order of $\sim 2$ kG/cm, i.e. a value for which a compensation might be possible, at least in principle.

However, the upper and lower half-poles of the PSI Ring cyclotron are connected by a non-magnetic support at the (inner and) outer pole radius. The orbit of the extracted beam would come close to the support structure, which is welded to the poles as a part of the vacuum chamber (shown in

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7To obtain the momentum in SI units, one has to multiply by $mc/R_\infty$. 

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Figure 4. Median plane of the PSI Ring cyclotron. Some turns of a (centered) accelerated orbit are plotted on top. The first few turns after injection and the last few turns before extraction are shown in gray. The extracted orbits are shown in red. The last turn escapes the magnetic field at $\theta \geq 320^\circ$. The five rectangular boxes indicate the positions and size of the cavities, i.e. four main cavities and one flat-top cavity.

Figure 5. Zoom of figure 4. The thin black lines indicate the positions of the iron bars of possible gradient correctors. They are (partially) in conflict with the non-magnetic mechanical support welded to the poles (shown in dark gray).
Figure 6. Top: magnetic field along the last five turns (from blue to red). Center: same as top, but zoomed in for details. Bottom: radius versus azimuthal angle of the last five turns (blue to red). The last turn escapes the magnetic field at $\theta \geq 320^\circ$. The turn separation between the last two turns at $\theta = 320^\circ$ is more than 25 mm for an almost centered beam — compared to 6 mm at $E = 590$ MeV [27]. The position of the main iron bars of a passive gradient corrector are shown as well.

dark gray in figure 5). This leaves few space for the installation of corrector magnets. A modification of the support structure would imply severe practical difficulties and a long interruption of user data taking. However in case of new cyclotron projects, the sketched extraction method seems to be a promising possibility.

5 Discussion and outlook

In new high intensity cyclotron project the design and shape of the magnetic fringe field and of mechanical components around it could be optimized for escape extraction. Since electrostatic extractors are notorious for high voltage breakdowns, this option might specifically be interesting in cases where the number of acceptable beam interruptions is rigorously limited (as for instance in case of ADS). The negative field gradient in the fringe field requires the use of magnetic gradient correctors [30]. As shown in figure 6, the field of the last turn is larger than that of the previous turn up to the last two sectors and is significantly lower only in the last sector. The extraction is therefore reasonably fast. A gradient corrector magnet, no matter if active or passive, will almost certainly lead to a lower number of beam interruptions than electrostatic extractors. Due to Joho’s “$N^3$ law” one can reasonably assume that any high-intensity and low-loss proton machine aiming for beam power in the multi-MW-range will require a high energy gain per turn. The possibility of escape extraction might, under these conditions, be regarded as a side-effect of high intensity operation.

The voltage and power that can be provided by a single cavity has practical limitations. The PSI main cavities, for instance, are specified for power losses of $\leq 500$ kW [42]. Hence the four main cavities of the PSI Ring cyclotron restrict the maximum power loss to $\approx 2$ MW. If one takes this as the state-of-the-art, then a multi-MW-cyclotron would require substantially more cavities than the PSI Ring cyclotron. Furthermore the power loss in rf cavities is proportional to the square of the cavity voltage [27, 43]. Therefore the cavity wall losses increase by a factor of four when the
voltage is doubled, but only by a factor of two when the number of cavities is doubled. Hence the use of a higher number of cavities is beneficial for the energy efficiency as well. A high number of cavities naturally suggests a high number of sectors for high power proton cyclotrons. This requires space and hence an increase in radius. Though size seems to be an important criterion, the size of a high intensity cyclotrons is negligible when compared to the size of linacs for similar energies. The MYRRHA linac, designed to provide 4 mA at 600 MeV, for instance, has a length of 400 m [44].

In the PSI machine, the effective turn separation between the last two turns is, for a centered beam, 6 mm, which can be enhanced up to 18 mm by betatron oscillations [27]. This is still substantially smaller than the pole gap. Therefore the radial turn separation is as yet the bottleneck for high intensity operation. Hence new high power cyclotrons aiming for power levels in the MW-regime, will likely be designed with a considerably larger extraction radius than the PSI Ring cyclotron, but not necessarily with a much larger pole gap. Then the ratio of pole gap to extraction radius (and hence the required voltage for escape extraction) will naturally be lower than (or equal to) the ratio of the PSI Ring cyclotron and this will facilitate escape extraction even further. Due to these arguments we believe that it is worthwhile to further investigate the feasibility of extraction purely by acceleration — specifically in high power cyclotrons.

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