Holographic QCD for H-dibaryon (uuddss)

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Abstract. The H-dibaryon (uuddss) is studied in holographic QCD for the first time. In holographic QCD, four-dimensional QCD, i.e., SU(\(N_c\)) gauge theory with chiral quarks, can be formulated with \(S^1\)-compactified D4/D8/D8-brane system. In holographic QCD with large \(N_c\), all the baryons appear as topological chiral solitons of Nambu-Goldstone bosons and (axial) vector mesons, and the H-dibaryon can be described as an SO(3)-type topological soliton with \(B = 2\). We derive the low-energy effective theory to describe the H-dibaryon in holographic QCD. The H-dibaryon mass is found to be twice of the \(B = 1\) hedgehog-baryon mass, \(M_{H} \approx 2 M_{B=1}^{HH}\), and is estimated about 1.7 GeV, which is smaller than mass of two nucleons (flavor-octet baryons), in the chiral limit.

1 Introduction

Nowadays, QCD is established as the fundamental theory of the strong interaction, and all the experimentally observable hadrons have been considered as color-singlet composite particles of quarks and gluons. From QCD, as well as ordinary mesons (\(\bar{q}q\)) and baryons (\(qqq\)) in the valence picture, there can exist “exotic hadrons”\(^{[1]}\) such as glueballs, multi-quarks\(^{[2, 3]}\) and hybrid hadrons, and the exotic-hadron physics has been an interesting field theoretically and experimentally.

The H-dibaryon, \(B = 2\) SU(3) flavor-singlet bound state of uuddss, has been one of the oldest multi-quark candidates, first predicted by R. L. Jaffe in 1977 from a group-theoretical argument of the color-magnetic interaction in the MIT bag model\(^{[2]}\). In 1985, the H-dibaryon was also investigated\(^{[4, 5]}\) in the Skyrme-Witten model\(^{[1, 8]}\). These two model calculations suggested a low-lying H-dibaryon below the \(\Lambda\Lambda\) threshold, which means the stability of H against the strong decay. In 1991, however, Imai group experimentally excluded the low-lying H-dibaryon\(^{[9]}\), and found the first event of the double hyper nuclei, i.e., \(\Lambda\Lambda\) He, instead. Then, the current interest is mainly possible existence of the H-dibaryon as a resonance state.

Theoretically, it is still interesting to consider the stability of H-dibaryons in the SU(3) flavor-symmetric case of \(m_u = m_d = m_s\)\(^{[10, 12]}\), because the large mass of H may be due to an SU(3) flavor-symmetry breaking by the large s-quark mass, \(m_s \gg m_{u,d}\), in the real world. Actually, recent lattice QCD simulations suggest the stable H-dibaryon in an SU(3) flavor-symmetric and large quark-mass region\(^{[10, 11]}\).

So, how about the H-dibaryon in the chiral limit of \(m_u = m_d = m_s = 0\)? Although the lattice QCD calculation is usually a powerful method to evaluate hadron masses, it is fairly difficult to take the chiral limit, because a large-volume lattice is needed for such a calculation to control massless pions.

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In this paper, we study the H-dibaryon and its properties in the chiral limit using holographic QCD [13], which has a direct connection to QCD, unlike most effective models. In particular, we investigate the H-dibaryon mass from the viewpoint of its stability in the chiral limit.

2 Holographic QCD

In this section, we briefly summarize the construction of holographic QCD from a D-brane system [14, 15], and derive the low-energy effective theory of QCD [16] at the leading order of $1/N_c$ and $1/\lambda$ expansions, where the 't Hooft coupling $\lambda \equiv N_c g^2_{YM}$ is given with the gauge coupling $g_{YM}$.

2.1 QCD-equivalent D-brane system

Just after J. M. Maldacena’s discovery of the AdS/CFT correspondence in 1997 [17], E. Witten [14] succeeded in 1998 the formulation of non-SUSY four-dimensional pure SU($N_c$) gauge theories using an $S^1$-compactified D4-brane in the superstring theory. In 2005, Sakai and Sugimoto showed a remarkable formulation of four-dimensional QCD, i.e., SU($N_c$) gauge theory with chiral quarks, using an $S^1$-compactified D4/D8/D8-brane system [15], as shown in Fig. 1. Such a construction of QCD is often called holographic QCD.

This QCD-equivalent D-brane system consists of $N_c$ D4-branes and $N_f$ D8/D8-branes, which give color and flavor degrees of freedom, respectively. In this system, gluons appear as 4-4 string modes on $N_c$ D4-branes, and the left/right quarks appear as 4-8/4-¯8 string modes at the cross point between D4 and D8/D8 branes, as shown in Fig. 1. This D-brane system possesses the SU($N_c$) gauge symmetry and the exact chiral symmetry [15], and gives QCD in the chiral limit.

Figure 1. Construction of holographic QCD with an $S^1$-compactified D4/D8/D8-brane system, which corresponds to non-SUSY four-dimensional QCD with chiral quarks [15, 16]. This figure is taken from Ref. [16].

Figure 2. Holographic QCD after the replacement of large-$N_c$ D4 branes by a gravitational background via the gauge/gravity correspondence [14–16]. This figure is taken from Ref. [16].

In holographic QCD, $1/N_c$ and $1/\lambda$ expansions are usually taken. In large $N_c$, D4-branes are the dominant gravitational source, and can be replaced by their SUGRA solution [15] as shown in Fig. 2 via the gauge/gravity correspondence. In large $\lambda$, the strong-coupling gauge theory is converted into a weak-coupling gravitational theory [14]. In this paper, we consider the leading order of $1/N_c$ and $1/\lambda$ expansions.
2.2 Low-energy effective theory

In the presence of the D4-brane gravitational background $g_{MN}$, the D8/$\overline{D8}$ brane system can be expressed with the non-Abelian Dirac-Born-Infeld (DBI) action,

$$S_{\text{DBI}}^{\text{D8}} = T_8 \int d^8x \, e^{-\phi} \sqrt{-\det(g_{MN} + 2\pi\alpha' F_{MN})},$$  \hspace{1cm} (1)

at the leading order of $1/N_c$ and $1/\lambda$ expansions. Here, $F_{MN} \equiv \partial_M A_N - \partial_N A_M + i[A_M, A_N]$ is the field strength of the $U(N_f)$ gauge field $A_M$ in the flavor space on the D8 brane. The surface tension $T_8$, the dilaton field $\phi$ and the Regge slope parameter $\alpha'$ are defined in the framework of the superstring theory, and, for the simple notation, we have taken the $M_{KK} = 1$ unit, where the Kaluza-Klein mass $M_{KK}$ is the energy scale of this theory $[15]$. 

After some calculations, one can derive the meson theory equivalent to infrared QCD at the leading order of $1/N_c$ and $1/\lambda$ $[15, 16]$. For the construction of the low-energy effective theory, we only consider massless Nambu-Goldstone (NG) bosons and the lightest SU($N_f$) vector meson $\rho_\mu(x) \equiv \rho_\mu(x)^a T^a \in \text{su}(N_f)$, which we simply call "$p$-meson". We eventually derive the four-dimensional effective action in Euclidean space-time $x^\mu = (t, x)$ $[16]$,

$$S_{\text{HQCD}} = \int d^4x \left\{ \frac{f_\pi^2}{4} \text{tr}(L_\mu L_\mu) - \frac{1}{32\pi^2} \text{tr}[L_\mu, L_\nu]^2 + \frac{1}{2} \text{tr}(\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)^2 + m_\rho^2 \text{tr}(\rho_\mu \rho_\mu) 
- ig_3 \text{tr}[(\partial_\mu \rho_\nu - \partial_\nu \rho_\mu) (\rho_\mu, \rho_\nu)] - \frac{1}{2} g_4 \text{tr}[\rho_\mu, \rho_\mu]^2 + 2g_4 \text{tr}[\rho_\mu, \rho_\mu] \right\},$$

where $L_\mu$ is defined with the chiral field $U(x)$ or the NG boson field $\pi(x) \equiv \pi^a(x) T^a \in \text{su}(N_f)$ as

$$L_\mu \equiv \frac{1}{i} U^\dagger \partial_\mu U \in \text{su}(N_f), \quad U(x) \equiv e^{i 2\kappa(x)/f_\pi} \in \text{SU}(N_f).$$  \hspace{1cm} (3)

The axial vector current $\alpha_\mu$ and the vector current $\beta_\mu$ are defined as

$$\alpha_\mu \equiv l_\mu - r_\mu \in \text{su}(N_f)_A, \quad \beta_\mu \equiv \frac{1}{2} (l_\mu + r_\mu) \in \text{su}(N_f)_V,$$  \hspace{1cm} (4)

with the left and the right currents,

$$l_\mu \equiv \frac{i}{2} \varepsilon^\mu \partial_\mu \xi, \quad r_\mu \equiv \frac{i}{2} \xi^\mu \partial_\mu \varepsilon^\mu, \quad \xi(x) \equiv e^{i \alpha(x)/f_\pi} \in \text{SU}(N_f).$$  \hspace{1cm} (5)

Thus, we obtain the effective meson theory derived from QCD in the chiral limit at the leading order of $1/N_c$ and $1/\lambda$ expansions. Note that this theory has just two independent parameters, e.g., the Kaluza-Klein mass $M_{KK} \sim 1\text{GeV}$ and $\kappa \equiv \lambda N_c / 216\pi^3$ $[15, 18]$, and all the coupling constants and masses in the effective action $[15]$ are expressed with them $[16]$. As a remarkable fact, in the absence of the $\rho$-meson, this effective theory reduces to the Skyrme-Witten model $[6]$ in Euclidean space-time,

$$\mathcal{L}_{\text{Skyrme}} = \frac{f_\pi^2}{4} \text{tr}(L_\mu L_\mu) - \frac{1}{32\pi^2} \text{tr}[L_\mu, L_\nu]^2.$$  \hspace{1cm} (6)
3 H-dibaryon as a B=2 Topological Chiral Soliton in Holographic QCD

As a general argument, large-\( N_c \), QCD becomes a weakly interacting meson theory, and baryons are described as topological chiral solitons of mesons \[7\]. In holographic QCD with large \( N_c \), the H-dibaryon is also described as a \( B = 2 \) chiral soliton, and its static profile is expressed with the “SO(3)-type hedgehog Ansatz”, similarly in the Skyrme-Witten model \[4, 5\]. Here, the SO(3) is the flavor-symmetric subalgebra of SU(3), and its generators \( \Lambda_{i=1,2,3} \) are

\[
\Lambda_1 = \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ i & 0 & 0 \end{pmatrix}, \quad \Lambda_2 = -\lambda_5 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \quad \Lambda_3 = \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},
\]

which satisfy the SO(3) algebra and the following relations,

\[
[\Lambda_i, \Lambda_j] = i\epsilon_{ijk}\Lambda_k, \quad (\Lambda \cdot \hat{x})^3 = \Lambda \cdot \hat{x}, \quad \text{Tr}[(\Lambda \cdot \hat{x})^2 - 2/3] = 0,
\]

with \( \hat{x} = x/r \) and \( r \equiv |x| \). The SO(3)-type hedgehog Ansatz \[4, 5, 13\] is generally expressed as

\[
U(x) = e^{i((\Lambda \cdot \hat{x})F(r) + (\Lambda \cdot \hat{x})^2 - 2/3)\varphi(r)} \in SU(3)_f, \quad F(r) \in \mathbb{R}, \quad \varphi(r) \in \mathbb{R},
\]

where \( F(r) \) and \( \varphi(r) \) are the chiral profile functions characterizing the NG boson field. Note that \( U(x) \) in Eq. (9) is the general form of the special unitary matrix which consists of \( \Lambda \cdot \hat{x} \), because of Eq. (8). For the topological soliton, the \( B = 2 \) boundary condition \[4, 5\] is given as

\[
F(\infty) = \varphi(\infty) = 0, \quad F(0) = \varphi(0) = \pi.
\]

On the SU(3)_f ρ-meson field, we use the SO(3) Wu-Yang-t Hooft-Polyakov Ansatz,

\[
\rho_0(x) = 0, \quad \rho_i(x) = \epsilon_{ijk}\hat{x}_jG(r)\Lambda_k \in so(3) \subset su(3), \quad G(r) \in \mathbb{R},
\]

similarly in the \( B = 1 \) case in holographic QCD \[14\]. (This \( G(r) \) corresponds to \( \tilde{G}(r) \) in Ref. \[16\].) Thus, all the above treatments are symmetric in the (u, d, s) flavor space.

Substituting Ansätze (9) and (11) in Eq. (10), we derive the effective action to describe the static H-dibaryon in terms of the profile functions \( F(r), \varphi(r) \) and \( G(r) \) \[13\]:

\[
S_{\text{HQCD}} = \int d^4x \left\{ \frac{2\pi}{4} \left[ \frac{3}{r^2} \varphi^2 + 2F^2 + \frac{8}{r^2}(1 - \cos F \cos \varphi) \right] + \frac{1}{32\epsilon^2} \frac{16}{r^2} \left[ (\varphi^2 + F^2)(1 - \cos F \cos \varphi) + 2\varphi'F' \sin F \sin \varphi + \frac{1}{r^2} \left( (1 - \cos F \cos \varphi)^2 + 3\sin^2 F \sin^2 \varphi \right) \right] + \frac{1}{2} \left[ 8 \left( \frac{3}{r^2}G^2 + \frac{2}{r}GG' + GG'' \right) \right] + m^2_{\pi} [4G^2] + g_3 \left[ \frac{8G^3}{r} \right] + \frac{1}{2} g_4 [4G^4] - g_1 \left[ \frac{16}{r} \left( \frac{1}{r} G + G' \right) (F' \sin F \frac{\varphi}{2} + \varphi' \cos F \sin \varphi \frac{\varphi}{2} + \frac{1}{r^2} G(1 - \cos F \cos \varphi)) \right] - g_2 \left[ \frac{8}{r^2}G^2(1 - \cos F \cos \varphi) \right] + g_3 \left[ \frac{16}{r^2}G \left[ 3 \sin F \sin F \frac{\varphi}{2} + \left( 1 - \cos F \frac{\varphi}{2} \right) (1 - \cos F \cos \varphi) \right] \right] - g_4 \left[ \frac{16}{r^2}G^2 \left( 1 - \cos F \frac{\varphi}{2} \right) \right] - g_5 \left[ \frac{8}{r^2}G^3 \left( 1 - \cos F \frac{\varphi}{2} \right) \right] + g_6 [4G^2 (F'' + \varphi'')] + g_7 \left[ \frac{8}{r^2}G^2 \left[ 3 \sin^2 F \sin^2 \varphi \frac{\varphi}{2} + \left( 1 - \cos F \frac{\varphi}{2} \right)^2 \right] \right] \right\}.
\]

\[
= \int dt \int_0^\infty dr 4\pi r^2 e[F(r), \varphi(r), G(r)].
\]
4 H-dibaryon Solution in Holographic QCD

To obtain the topological soliton solution of the H-dibaryon in holographic QCD, we numerically calculate the profiles $F(r), \varphi(r)$ and $G(r)$ [13] by minimizing the Euclidean effective action [12] under the boundary condition [10] [19]. The two independent parameters, e.g., $M_{KK}$ and $\kappa \equiv \lambda N_c/216\pi^3$, are set to reproduce the pion decay constant $f_\pi=92.4\text{MeV}$ and the $\rho$-meson mass $m_\rho=776\text{MeV}$ [15, 16].

For the H-dibaryon solution in holographic QCD, we obtain the chiral profiles, $F(r)$ and $\varphi(r)$, and the scaled $\rho$-meson profile $G(r)/\kappa^{1/2}$ as shown in Fig. 3 and estimate the H-dibaryon mass of $M_H \approx 1673\text{MeV}$ in the chiral limit. Figure 4 shows the energy density $4\pi r^2 \varepsilon(r)$ in the H-dibaryon. The root mean square radius of the H-dibaryon is estimated as $\sqrt{\langle r^2 \rangle_H} \approx 0.413\text{fm}$ in terms of the energy density. For comparison, we calculate the $B=1$ hedgehog (HH) baryon in holographic QCD with the same numerical condition, and estimate $M_{HH}^{B=1} \approx 836.7\text{MeV}$ and $\sqrt{\langle r^2 \rangle_{HH}^{B=1}} \approx 0.362\text{fm}$. Thus, the H-dibaryon mass is twice of the $B=1$ hedgehog-baryon mass, $M_H \approx 2.00M_{HH}^{B=1}$.

![Figure 3. The chiral profiles, $F(r)$ and $\varphi(r)$, and the scaled $\rho$-meson profile $G(r)/\kappa^{1/2}$ in the H-dibaryon as the SO(3)-type hedgehog soliton solution in holographic QCD. Here, the topological boundary condition of $B=2$ is $F(0) = \varphi(0) = \pi$ and $F(\infty) = \varphi(\infty) = 0$.](image3)

![Figure 4. The energy density distribution $4\pi r^2 \varepsilon(r)$ in the H-dibaryon (solid curve), and that without vector mesons (dashed curve) for comparison.](image4)

We summarize in Table 1 the mass and the radius of the H-dibaryon and the $B=1$ hedgehog baryon in holographic QCD. Since the nucleon mass $M_N$ is larger than the $B=1$ hedgehog mass $M_{HH}^{B=1}$ by the rotational energy [6, 8], the H-dibaryon mass is smaller than mass of two nucleons (flavor-octet baryons), $M_H < 2M_N$, in the chiral limit.

Finally, we examine the vector-meson effect for the H-dibaryon by comparing with the $\rho(x) = 0$ case. As the result, we find that the chiral profiles $F(r)$ and $\varphi(r)$ are almost unchanged and slightly shrink by the vector-meson effect, and the energy density also shrinks slightly, as shown in Fig. 4.
Table 1. The mass $M_H$ and the radius $\sqrt{\langle r^2 \rangle_H}$ of the H-dibaryon in the chiral limit in holographic QCD, together with those of the $B = 1$ hedgehog (HH) baryon.

|       | $M_H$   | $\sqrt{\langle r^2 \rangle_H}$ | $M_{HH}^{B=1}$ | $\sqrt{\langle r^2 \rangle_{HH}^{B=1}}$ |
|-------|---------|---------------------------------|----------------|----------------------------------|
|       | 1673 MeV | 0.413 fm                        | 836.7 MeV      | 0.362 fm                         |

As a significant vector-meson effect, we find that about 100 MeV mass reduction is caused by the interaction between NG bosons and vector mesons in the interior region of the H-dibaryon.

5 Summary and Concluding Remarks

We have studied the H-dibaryon (uuddss) as the $B = 2$ SO(3)-type topological chiral soliton solution in holographic QCD for the first time. The H-dibaryon mass is twice of the $B = 1$ hedgehog-baryon mass, $M_H \approx 2.00M_{HH}^{B=1}$, and is estimated about 1.7 GeV, which is smaller than mass of two nucleons (flavor-octet baryons), in the chiral limit. In holographic QCD, we have found that the vector-meson effect gives a slight shrinkage of the chiral profiles and the energy density, and also gives about 100 MeV mass reduction of the H-dibaryon.

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