Flexible Application of Static Balance Principle in Beam Internal Force Diagram

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\textbf{Abstract:} When using the conventional segmental superposition method to make the bending moment diagram, it is not only complicated to calculate, but also easy to make mistakes. In this paper, it is discussed how to flexibly segment and simplify the calculation with examples. At the same time, according to the differential relationship, integral relationship and static equilibrium conditions between load and shear force, the fast and accurate pithy formula for shear force diagram making is concluded. The deep application of integral relationship between load and shear force is also discussed.

1. Introduction

Making the bending moment diagram of beams with the segmental superposition method is an important content in structural mechanics. And it is also the basis for solving statically indeterminate structures by using force method, displacement method and other methods \cite{1-2}. Especially when calculating the displacement of the structures by using the graph multiplication method, only a deep understanding of the segmental superposition method the complex bending moment diagram can be flexibly divided into some simple diagrams and the calculation process can be simplified \cite{3-5}. Some studies also give general recommendations on how to segment, but it is not flexible and deep \cite{6-9}. Regarding the beam shear force diagram making, most textbooks adopt the control section method, that is, first calculate the shear force of each control section, and then draw the graph. When the load is complicated or the beam is multi-span, the calculation process is more complicated. This article mainly discusses how to flexibly segment as making the bending moment diagram of beams with the segmental superposition method. The simple and quick pithy formula for shear force diagram making is also summarized based on the differential relationship, integral relationship, incremental relationship and static balance conditions between the load and the shear force. At last, the deep application of the integral relationship between load and shear force is analyzed. The conclusions obtained may be used for references by the majority of structural mechanics teaching workers and students.

2. How to segment flexibly with segmental superposition method

Making the bending moment diagram of beams with the segmental superposition method is mainly
divided into two steps: segmentation and superposition. In fact, the beam is divided into a series of simply supported beam elements, and then making the bending moment diagram using the superposition principle. The textbook generally recommends that the discontinuous points of external forces such as the concentrated load application point, the concentrated couple application point, the starting point and the end point of the distributed load, etc., be used as the control section for segmentation. After the segmentation, there is no load in each segment of the beam, and is continuous. This segmentation method is widely applicable but not flexible enough.

For example, the multi-span beam shown in Fig. 1. When discussing how to segment the beam, the engineers often divide the beam into 4 segments AC, CE, ED, and DB for drawing. The bending moment diagram of segment CD is prone to the situation shown in Fig. 2, and the bending moment diagram at point E is not smooth. Obviously it is wrong.

There is a hinge point between point C and D. Can it be regarded as a continuous segment for calculation? The textbook does not make a deeply analysis on this situation. In fact, the internal force of CD segment (Figure 3(a)) is exactly the same as that of the simply supported beam CD (Figure 3(b)). According to the static balance condition, the supporting reaction forces $F_{yC}, F_{yD}$ of the simply supported beam are equal to the vertical forces of CD segment respectively, that is $F_{yC}^0 = 7ql/8, F_{yD}^0 = ql/8$, and the direction are also same. Point E in segment CD is the hinge point, and the bending moment is 0(Figure 3(a)), while the bending moment at point E of the simply supported beam CD is also 0(Figure 3(b)). So, the internal force of segment CD is exactly the same as that of the simply supported beam CD, and the bending moment diagrams of them are also the same as each other.

So, segment CD can be regarded as simply supported beam CD. First make a straight-line bending moment diagram based on the bending moment of point C and D, and then superimpose the bending moment diagram (parabola) of the simply supported beam under uniform load, thus segment CD bending moment diagram is obtained as shown in Figure 4. In this way, there will be no discontinuity at point E, and it will also help learners to have a deep understanding of the segmental superposition method.

Another advantage of non-segmentation at point E is simplifying the calculation process. After the bending moment of section D $M_D = \frac{ql^2}{16}$ is calculated, and the approximate bending moment diagram (dotted line in Fig. 4) can be drawn without calculating $M_C$. According to the conclusion $M_E = 0$, the value of the dotted line at point E is $M_E = \frac{ql^2}{8}, M_C = \frac{5ql^2}{16}$ can be obtained according to the geometric proportional relationship. Thus the calculation of $M_C$ is greatly simplified.

The prerequisite for flexible segmentation is familiar with the bending moment diagrams of simply supported beams under general loads. For example, the multi-span beam shown in Figure 5 is divided...
into three sections, AE, ED, and DB, and the calculation process can be greatly simplified. It is easy to know that the bending moment of section D $M_D = F_p l/4$, and $M_E$ is no need to be calculated temporarily. Firstly, the straight-line bending moment diagram of segment ED is made. Secondly, the bending moment diagram of simply supported beam ED is superimposed, as shown by the dotted line in Figure 6(b). Finally, the bending moment diagram of segment ED is obtained shown in Figure 6(c). Since the bending moment of section C is 0, $M_C = 3F_p l/4$ can be concluded according to the geometric proportional relationship in Fig. 6(c). The bending moment diagram of section AC is a straight-line, so $M_A = 3F_p l/2$ is easy to known.

In summary, when constructing the bending moment diagram of the beam by the subsection superposition method, the beam can be divided into a series of simply supported beam elements at any position of the beam. The only principle is to simplify the calculation. So, memorizing the bending moment diagrams of simply supported beams under various loads will greatly simplify the calculation process.

3. Pithy formula for shear force diagram making

The bending moment diagrams are often emphasized more than shear force diagrams in many structural mechanics papers. Because of the close differential relationship between the shear force and the bending moment of the beam, the bending moment diagram is often difficult to accurately determine without the shear force diagram sometimes [10]. For example, the peak point of the bending moment diagram often depends on the zero point of the shear force diagram. At the same time, the shear force diagram also plays a very important role in the verification of the bending moment diagram.

According to the differential relationship between external load (this article only discusses the common vertical loads in engineering) and shear force, the shear force diagram of any straight bar has
the following characteristics. The shear force diagram of the straight bar is a horizontal line when there is no load on it. The diagram is an oblique (same direction as q) straight line when there is a uniform load on it. According to the differential relationship, the incremental relationship, the integral relationship and the static balance condition, the pithy formula of shear force diagram drawing can be summarized: "From left to right, moving along the force, the diagram is horizontal line between the concentrated forces, and the inclined line between the uniform forces, finally the diagram is the closed frame at the right end ".

The meaning of the pithy formula is as following. "From left to right" means that the shear force diagram of the horizontal bar (beam) is started drawing from the left end, if it is a vertical bar (rigid frame), started from the lower end and drawn from the bottom to the top. The force of "moving along the force" refers to the external loads (vertical concentrated loads and vertical uniformly distributed loads) or the reaction forces of the supports. Under a couple loads, it will not be affected. The corresponding line segment is drawn along the direction of the force at the loading point. The shear force diagram is a horizontal line between the two adjacent concentrated forces, and an oblique downward sloping straight line between uniformly distributed loads. Other connection forms such as hinged joints and directional connections in the segment will not be affected. Finally, the shear force diagram and the bar axis form a closed frame at the right end of the beam. Otherwise the static equilibrium condition will not be met.

For any single-span beams or multi-span beams, only the reaction forces are calculated, and the shear force diagram of the beam can be quickly made using the above-mentioned pithy formula without calculation section by section. For example shown in Fig. 7, only the reaction forces of the supports are calculated $F_{yA} = 32\text{kN}$, $F_{yB} = 28\text{kN}$, the shear force diagram can be made quickly. Starting from point A at the left end, draw a $32\text{kN}$ vertical line segment along $F_{yA}$ direction. There is no uniformly distributed load in AC segment, and the shear force diagram is a horizontal line. There is a downward concentrated load $20\text{kN}$ at C, so the downward abrupt change (Moving forward along the force) occurs at C, i.e. the shear force is reduced by $20\text{kN}$. Thus the shear force is $F_{yc} = 12\text{kN}$ where R is the right of section C. The shear force diagram is a horizontal line in CD segment. There are downward uniform loads acting segment DE, and the shear force diagram slopes downward (forward along the force) from D to E, and the shear force is reduced by $10 \times 4 = 40\text{kN}$ in total at E. Thus the shear force at E is $F_{0e} = 12 - 40 = -28\text{kN}$. The shear force diagram is a horizontal line in segment EB. The shear force is increased by $28\text{kN}$ along $F_{yB}$ direction at B, i.e. the shear force at B is $F_{0B} = 0\text{kN}$. It just forms a closed frame and satisfies the static balance condition. The diagram made above is the shear force diagram shown in Figure 7(b), which is "+" above the bar, and "-" under the bar. Using this pithy formula, only the reaction force is calculated, the shear force diagram of any beam can be quickly drawn without calculating the shear force of each control section. The pithy formula is especially simple for multi-span beams.

![Shear Force Diagram](image-url)
4. Flexible application of the integral relationship between load and shear

From the integral relationship between the external load and the shear force, for any straight bar AB under the vertical load, the bending moment at B is equal to the bending moment at A plus the area enclosed by the shear force diagram and the straight bar AB. Especially when the bending moment at A and B are both 0, the area should be 0. This conclusion has two uses, one of which is to check the shear force diagram. Find two points on the beam where the bending moment is 0, and check whether the area enclosed by the shear force diagram and the bar axis between these two points is 0. If the area is 0, it means that the shear force diagram is correct, otherwise it is wrong.

Another important use of the integral relationship is to simplify the calculation process. For example, when drawing the shear force diagram of the beam shown in Figure 8(a), the diagram can be done without calculating the supporting forces. According to the characteristics of the shear force diagram, the approximate shape is first made as shown in Figure 8(b), where the shear force of the segment BD can be determined. According to the geometric proportional relationship, the shear force of section A is 5x, and the shear force of section C is 5(4-x), and an equation $\frac{1}{2} \times 5x \times x + 10 \times 2 = \frac{5}{2} (4 - x)^2 + 5(4 - x) \times 2$ is established based on the principle that the area enclosed by the shear force diagram between A and B is 0, i.e. the positive and negative areas are equal. It is very easy to solve the equation to obtain x=2m. Accordingly, the shear force diagram can be easily made as shown in Figure 8(b).

5. Conclusions

This article discusses how to flexibly segment and how to make shear force diagrams quickly and accurately in combination with specific examples. When using the segmental superposition method to make the bending moment diagram of the beam, the principle is to simplify the calculation as much as possible. The beam can be segmented at any position of the beam, the beam can be divided into a series of simply supported beam elements, and then the superposition principle is used to draw the graph. Shear force diagram drawing pithy formula is especially suitable for multi-span beams and the beams under complicated loads.

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