Theoretical studies on biological additives preparation in a blender with a sinusoidal blade paddle mixer

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Abstract. The purpose of the research is to establish the relationship of the geometric parameters of the sinusoidal mixer paddles with part of the motion indicators of low-altitude radial sectors of bulk material and the possibility of numerical simulation using existing characteristics. The effect of the flat and sinusoidal mixer blades of the blender, in terms of their construction, on the process of moving layers of the mixed material has been theoretically studied. The equations obtained allow us to state that, due to the difference in the geometry of the blades' surface, when the blades are flat, an inertial shock occurs when the blade contacts the material sector due to its sharp acceleration to the peripheral velocity. With sinusoidal blades, acceleration to a specified velocity is stretched in time, taking into account the amplitude of the sinusoid, reducing the energy cost of the friction of material layers. The obtained equations allow us to identify the relationship and numerically simulate the change in the main indicators of the material position in the blender.

1. Introduction

The widespread use of modern technology in all sectors of human activity requires modern materials with specific properties [1-3]. For the preparation of composites and mixtures one uses a variety of mixer constructions [3-5], adapted for mixing materials of different consistencies [4-6] and different particle size distribution [7-9]. Increasing the productivity of farm animals requires providing their organisms with the whole palette of necessary nutrition (biological additives) [10, 11]. To ensure this condition, there are used blenders in agriculture. Given the possibility of diseases at any time, the introduction of therapeutic and other components into the existing nutrition mixture is required. Accordingly, an enterprise should have low-capacity blenders capable of carrying out the necessary preventive portions of the feed mixture. Moreover, the improvement of such devices should be carried out in terms of reducing energy costs to prepare nutrition feed. Energy reduction is possible due to the use of rational-shaped blades.

2. Materials and methods

The purpose of the research is to establish the relationship of the geometric parameters of the sinusoidal mixer paddles with part of the motion indicators of low-altitude radial sectors of bulk
material and the possibility of numerical simulation of the existing characteristics. The research methodology provided for theoretical and numerical studies to establish equations and connections for the parameters of sinusoidal blades and their influence on the motion indicators of low-altitude sectors of the material when the blade mixer is under operation. The previously revealed connections were linked [12, 13] with reference to the sinusoidal paddles of a mixer.

The blender with a vertical blade working body (Figure 1) for biological additives preparation [12] consists of a shaft of the working body 1, fixed in the bearing supports 6 of the blender corpus 2, which is a cylindrical container. Inside the container, a mixer 5 of the blade type is fixed on the shaft. The horizontal part of the blade 4 has a radial shape, and the vertical component is made in the form of a plate having a sinusoidal cross-sectional shape. The blades are fixed on the bushing of the mixer in such a way that their lower part is almost horizontal, and the upper part of the blade is located at the angle of 45 ... 60 degrees relative to the corpus bottom.

The vertical component contributes to material lifting from the bottom along the edges of the bottom and directs the material along the sinusoid upwards, to the subsequent blade of the mixer in the blender. In this case, the horizontal component of the blade surface is the radial component to the center of the container, providing turbulent material circulation. The drive of the working body of the blender is carried out by means of V-belt transmission from the motor shaft.

The blender operation is as follows. In accordance with the formulation of biological additives, the desired proportion of the filler is poured into the blender body, which is used as bran or tart of various compositions, in accordance with the recipe. Then, drugs, premixes and others nutrition components are added. Mixing of the components is carried out for a specified time, sufficient to obtain uneven mixture C\textsubscript{v} in the range of 5-10%. Opening the side gate, the prepared mixture is discharged from the blender during the mixer rotation.

![Figure 1. Scheme of a blender with a blade working body: 1 – shaft of a working body; 2 – a mixing container corpus; 3 – sinusoidal paddle of a mixer; 4 – a blade flat radial mixer; 5 – a mixer sleeve; 6 – lower bearing support.](image)

When the blender was under theoretical consideration, a comparison of the vertical component of the sinusoidal paddle 3 was used, the elementary length section of which was installed at a slight slope to the base of the corpus bottom, in the form of a private fragment of a sinusoid (by analogy with a flat blade 4). The mixing of components is considered by analogy [14] as the relative displacement of the elementary sectors of the material along the path of the vertical section of the material relative to the upper surface of a moving flat rather than circular cross section of the analogue of the blade 4. As the thrown material elementary sectors land down on the following walking blades (or paddles), forces arise that act on the material and cause the normal reaction force of the blade N and the friction force F\text{N}, directed towards the velocity of the subsequent movement of the material. Their resultant R, is turned from N to F\text{N} at an angle of friction of the material on the blade (paddle). The material located
on the blade has internal stress lines along the arches, i.e. relative to the point of contact of the material with the blade at the angle $\varphi'$ from the horizon.

Having considered the movement of the material along a flat blade and determining the acting forces, we proceed to a sinusoidal blade, changing the direction of the normal reaction of the blade (paddle), taking into account the law of changing the angle of the local section. The equations obtained allow us to state that, due to the difference in the geometry of the blades’ surface, when the blades are flat, an inertial shock occurs when the blade contacts the material sector due to its sharp acceleration to the peripheral velocity. With sinusoidal blades, acceleration to a specified velocity is stretched in time, taking into account the amplitude of the sinusoid, reducing the energy cost of the friction of material layers.

\[ \alpha = \arctan \left( \frac{h_1}{L_2} \right) \]  

Figure 2. Scheme of forces acting on the elementary sector of the material: 1 – blade; 2 – bushing of a mixer; 3 – shaft; 4 – the bottom of a blender corpus; 5 – axis of rotation; 6 – elementary sector of the material.

3. Results

The angle (degree) of the paddle installation (as the ratio of the vertical projection of the paddle to its horizontal projection of the paddle’s length):

\[ \alpha = \arctan \left( \frac{h_1}{L_2} \right) \]

The sinusoidal law regulating changes in parameters implies a change in the vertical projection of the hypotenuse-paddle according to the law of sine, and the horizontal projection of the hypotenuse according to the law of cosine.

Take the lower point of sinusoid as $M (O_i-A)$. Then the equation of the paddle relative to the axial longitudinal line of the sinusoid is written as:

\[ z = A \cdot \sin (k y + y_0) \]

where $A$ – half sinusoid amplitude, m; $k$ – sinusoid angular coefficient; $y$ – sinusoid current coordinate, m; $y_0$ – coordinate of the paddle beginning relative to the sinusoid coordinate axes $Y$, m.

Since the front edge of the paddle is at the bottom of the sinusoid, then

\[ y_0 = -\frac{\pi}{2} \text{ and } z = A \cdot \sin \left( k y - \frac{\pi}{2} \right) \]

It remains to determine for sinusoids $A$ and $k$. 

3
By the initial condition: \( z(L_2) = -A + h_{11}, \) \( z'(L_2) = tg(\alpha_0) \),

where \( \alpha_0 \) – the inclination angle of the back part of the paddle from the horizon, radius; \( h_{11} \) - the height of the vertical projection of the paddle, m.

Since \( z = A \cdot k \cdot cos\ (ky - \frac{\pi}{2}) \), we have:

\[
\begin{cases}
A \cdot \sin\left(kL_2 - \frac{\pi}{2}\right) = -A + h_{11}, \\
A \cdot k \cdot \cos\left(kL_2 - \frac{\pi}{2}\right) = tg(\alpha_0).
\end{cases}
\]

Hence \( \sin^2\left(kL_2 - \frac{\pi}{2}\right) + \cos^2\left(kL_2 - \frac{\pi}{2}\right) = \left(\frac{-A+h_{11}}{A}\right)^2 + \left(\frac{tg(\alpha_0)}{A \cdot k}\right)^2 \),

Or \( \left(\frac{-A+h_{11}}{A}\right)^2 + \left(\frac{tg(\alpha_0)}{A \cdot k}\right)^2 = 1. \)

Then \( k = \frac{\pm tg(\alpha_0)}{-2A \cdot h_{11} + h_{11}^2} \cdot \sqrt{2A \cdot h_{11} - h_{11}^2}; \)

Or \( \left(\frac{-A+h_{11}}{A}\right)^2 + \left(\frac{tg(\alpha_0)}{A \cdot k}\right)^2 = 1. \)

From the first equation of the system we interpret \( A = \frac{h_{11}}{cos(kL_2)+1} \).

It is impossible to interpret \( A \) and \( k \) simultaneously. We define both indicators by successive approximation by numerical methods (Fig. 3), where the solution to the equation system is the intersection of the graph lines of the right and left sides of this equation for given geometric parameters.

**Figure 3.** Graph of a step-by-step graphical solution of the equation system for explicating the values of sinusoid angular coefficient \( k \) and half the amplitude of sinusoid \( A \), (modeling in MathCAD program)

The error in calculating \( z \) coordinate is:

\[
\Delta z = \left(\frac{-A+h_{11}}{A}\right)^2 + \left(\frac{tg(\alpha_0)}{A \cdot k} - \sqrt{2A \cdot h_{11} - h_{11}^2}\right)^2 = 1. \quad (6)
\]

Having previously set the values of \( A \), we find the value of \( A \) for the condition: \( \Delta z \rightarrow 0. \) Refine the numerical value of \( k \). Refine the value of \( A \) found from the previous equation when \( \Delta z \rightarrow 0. \)

Acceleration will be interpreted as: \( \ddot{z} = -A \cdot k^2 \cdot \sin\ (ky - \frac{\pi}{2}). \)

Given that the calculation uses a moving coordinate system, we place the origin of \( Y \) and \( Z \) axes at the point when the paddle begins, keeping their original direction.

Then the equations of the paddle surface and its derivatives are written as:

The coordinate will be found, m: \( z(y) = A \cdot \left[\sin\left(ky - \frac{\pi}{2}\right) + A\right]. \)
Velocity will be found, m/s: \[ \dot{z}(y) = A \cdot k \cdot \cos \left( ky - \frac{\pi}{2} \right). \]

Acceleration will be found, m/s²: \[ \ddot{z}(y) = -A \cdot k^2 \cdot \sin \left( ky - \frac{\pi}{2} \right). \]

where 0 is the coordinate of the leading edge of the paddle (initial coordinate) along Y axis, m

We take into account that y changes according to the law:

\[ y_{ji} = V_i \cdot t_j = \omega \cdot R_i \cdot t_j, \]

where i is the number of the elementary sector along the radius of the mixer (along the width of the paddle, along X axis); j – the number of the elementary sector along the paddle length (perpendicular to the radius of the mixer, along Y axis), \( V_i \) – peripheral velocity of i-th elementary sector of the mixer paddle, m/s; \( t_j \) – time to reach j-th elementary sector from the leading edge of the paddle, s:

\[ t_j = \frac{y_j}{\omega R_i}, \text{ where } 0 \leq y_j \leq L_2. \]

Coordinate, m/s²: \[ \ddot{z}(t) = A \cdot k^2 \cdot \omega^2 \cdot R_i^2 \cdot \left[ \sin \left( k \omega \cdot R_i \cdot \frac{y_j}{\omega R_i} - \frac{\pi}{2} \right) \right]. \]

Velocity, m/s²: \[ \dddot{z}(t) = -A \cdot k^2 \cdot \omega^2 \cdot R_i^2 \cdot \left[ \sin \left( k \omega \cdot R_i \cdot \frac{y_j}{\omega R_i} - \frac{\pi}{2} \right) \right]. \]

or after simplifying the equations (example of calculation in Figure 4):

Coordinate, m/s²: \[ \ddot{z}(t) = A \cdot \sin \left( k \omega \cdot R_i \cdot \frac{y_j}{\omega R_i} - \frac{\pi}{2} \right) + 1. \]

Velocity, m/s²: \[ \dot{z}(t) = A \cdot \sin \left( k \omega \cdot R_i \cdot \frac{y_j}{\omega R_i} \right). \]

Acceleration, m/s²: \[ \dddot{z}(t) = -A \cdot k^2 \cdot \omega^2 \cdot R_i^2 \cdot \left[ \sin \left( k \omega \cdot R_i \cdot \frac{y_j}{\omega R_i} - \frac{\pi}{2} \right) \right]. \]

**Figure 4.** Graphs of the natural values of the indicators denoting material movement along the paddle at a shaft rotation frequency of \( n = 340 \text{ min}^{-1} \): (a) – coordinates of the points of position of the paddle \( Z_1 \) (m); vertical projections: (b) — material velocity \( Z_1' \) (m/s); (c) – accelerations of the material \( Z_1'' \) (m/s²).

The acceleration acquired by the material at i point of the paddle length at j point of the cross section is determined, m/s².

\[ a_{H0ij} = z'' = -A \cdot k^2 \cdot \omega^2 \cdot R_i^2 \cdot \left[ \sin \left( k \omega \cdot R_i \cdot \frac{y_j}{\omega R_i} - \frac{\pi}{2} \right) \right]. \]

The vertical velocity of the material at the moment of falling from the paddle, m/s²:

\[ v_0 = z' = A \cdot k \cdot \omega \cdot R_i \cdot \left[ \cos \left( k \cdot L_2 - \frac{\pi}{2} \right) \right]. \]

The total vertical force acting on the sector when moving along the paddle, N:

\[ F_{Ai} = m_c g \cdot \left[ 1 + Ak^2 \omega^2 R_i^2 \cdot \sin \left( k \omega \cdot R_i \cdot \frac{y_j}{\omega R_i} - \frac{\pi}{2} \right) \right] + \rho H_0 \cdot h_0 \cdot (\Delta R_i + \Delta d_j) \cdot \tan(\phi'). \]

Full acceleration acting on the elementary sector when it moves along the paddle, m/s²:
\[ a_{Ai} = \frac{F_{Ai}}{m_c} = g \left[ 1 + Ak^2 \omega^2 R_i^2 \left( \sin \left( k \frac{y_j}{2} \right) + \frac{\rho H_o \cdot L_{R_i} (\Delta R_i + \Delta d_i) \cdot t g(\phi)}{2} \right) \right]. \] (11)

The height of the sector while in-flight will be determined, m

\[ h_{ui1_i} = \frac{\varrho a_i^2}{2 a_{Ai}} = \frac{(A k \omega R_i \cdot \cos (k L_2 - \frac{m}{2}))^2}{2 a_{Ai}}. \] (12)

The lifting time of the material while in-flight will be found with the formula:

\[ \tau_{PB1_i} = \frac{v_{o \perp}}{a_{Ai}} = \frac{A k \omega R_i \cdot \cos (k L_2 - \frac{m}{2})}{g \sqrt{1 + \rho H_o \cdot L_{R_i} (\Delta R_i + \Delta d_i) \cdot t g(\phi)}}. \] (13)

The lifting time of the material while in-flight to the container lid will be, s:

\[ \tau_{PK1_i} = \frac{v_{o \perp}}{a_{Ai}} = \frac{A k \omega R_i \cdot \cos (k L_2 - \frac{m}{2}) \pm \left( A k \omega R_i \cdot \cos (k L_2 - \frac{m}{2}) \right)^2 - 2 g h_{ui1_i}}{g \sqrt{1 + \rho H_o \cdot L_{R_i} (\Delta R_i + \Delta d_i) \cdot t g(\phi)}}. \] (14)

The material in-flight length in low-speed and high-speed operation is:

\[ L_{0_i} = \frac{\omega R_i \left( A k \omega R_i \cdot \cos (k L_2 - \frac{m}{2}) \right)}{g} \sqrt{1 + \rho H_o \cdot L_{R_i} (\Delta R_i + \Delta d_i) \cdot t g(\phi)} \pm \frac{2 (h_{11} + h_{ui1})}{g \sqrt{1 + \rho H_o \cdot L_{R_i} (\Delta R_i + \Delta d_i) \cdot t g(\phi)}}. \] (15)

With increasing the radius (elementary area numbers \( i \) for blades-2 and paddles-1), the peripheral velocity and, accordingly, the vertical component of the material velocity increase. As a result, the material from the blades is thrown to a large height. As soon as the height is limited by the lid, then the lowering / falling / sector time will decrease at the place of reaching the container lid /to1, to2/ and there, where it does not reach the container lid, the lowering / falling / sector time will decrease at the place of reaching the container lid /to1, to2/ and there, where it does not reach the container lid, will increase due to the rise in the in-flight altitude.

![Graph](image)

**Figure 5.** Distance from the container lid to the material at the upper in-flight point of sector (m) on the elementary sections (0 ≤ \( i \) ≤ 10) of the length of paddles and blades: a – with the presence of free space; b – with the absence of free space ‘zero’; c – the in-flight altitude of material on \( i \) sections for paddles –1and blades – 2.

4. **Conclusion**

The effect of the flat and sinusoidal mixer blades of the blender, in terms of their construction, on the process of moving layers of the mixed material has been theoretically studied. The equations obtained
allow us to state that, due to the difference in the geometry of the blades' surface, when the blades are flat, an inertial shock occurs when the blade contacts the material sector due to its sharp acceleration to the peripheral velocity. With sinusoidal blades, acceleration to a specified velocity is stretched in time, taking into account the amplitude of the sinusoid, reducing the energy cost of the friction of material layers. The obtained equations allow us to identify the relationship and numerically simulate the change in the main indicators of the material position in the blender.

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