Is the Dirac particle composite?

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Abstract

Classical model $S_{Dcl}$ of the Dirac particle $S_D$ is constructed. $S_D$ is the
dynamic system described by the Dirac equation. For investigation of $S_D$
and construction of $S_{Dcl}$ one uses a new dynamic method: dynamic disquan-
tization. This relativistic purely dynamic procedure does not use principles
of quantum mechanics. The obtained classical analog $S_{Dcl}$ is described by a
system of ordinary differential equations, containing the quantum constant
$\bar{\hbar}$ as a parameter. Dynamic equations for $S_{Dcl}$ are determined by the Dirac
equation uniquely. The dynamic system $S_{Dcl}$ has ten degrees of freedom and
cannot be a pointlike particle, because it has an internal structure. There are
two ways of interpretation of the dynamic system $S_{Dcl}$: (1) dynamical inter-
pretation and (2) geometrical interpretation. In the dynamical interpretation
the classical Dirac particle $S_{Dcl}$ is a two-particle structure (special case of a
relativistic rotator). It explains freely such properties of $S_D$ as spin and mag-
netic moment, which are strange for pointlike structure. In the geometrical interpretation the world tube of $S_{Dcl}$ is a "two-dimensional broken band", con-
sisting of similar segments. These segments are parallelograms (or triangles),
but not the straight line segments as in the case of a structureless particle.
Geometrical interpretation of the classical Dirac particle $S_{Dcl}$ generates a new
approach to the elementary particle theory.

Key words: disquantization, Dirac equation, relativistic rotator, geometrical model
1 Introduction

The Dirac particle is the dynamic system $S_D$, described by the Dirac equation. This is one of wide-spread dynamic systems used in theory of quantum phenomena. Mathematical analysis of properties of the dynamic system $S_D$ is of undoubted interest. The Dirac dynamic system $S_D$ was investigated by many researchers. There is no possibility to list all them, and we mention only some of them. First, this is transformation of the Dirac equation on the base of quantum mechanics [1, 2].

The complicated structure of Dirac particle was discovered by Schrödinger [3], who interpreted it as some complicated quantum motion (zitterbewegung). Investigation of this quantum motion and different models of Dirac particle can be found in [4, 5, 6, 7, 8] and references therein. Our investigation differs in absence of any suppositions on the Dirac particle model and in absence of referring to the quantum principles. We use only dynamic methods and investigate the Dirac particle simply as a dynamic system.

Conventionally the analysis of the dynamic system $S_D$ and its dynamic equations is carried out by a use of the quantum mechanics principles. In particular, it means that the quantum constant $\hbar$ is not simply a parameter of the dynamic system $S_D$. The quantum constant $\hbar$ is provided by some additional physical meaning. It is supposed that, if $\hbar \to 0$, all quantum effects are cut off, and dynamic system $S_D$ turns into classical dynamic system $S_{Dcl}$, having six degrees of freedom.

Usually it is supposed that $S_{Dcl}$ is a pointlike relativistic particle of mass $m$, having a spin (angular momentum) $S_D = \hbar/2$ and magnetic momentum $\mu_D = e\hbar/mc$. Procedure of transition from $S_D$ to $S_{Dcl}$ is called the transition to classical description. However, the transition to the limit $\hbar \to 0$ is not carried out in such quantities as spin $S_D = \hbar/2$ and magnetic momentum $\mu_D = e\hbar/mc$, which remain to be quantum in the sense that they contain the unvanishing quantum constant $\hbar$. One states that spin and magnetic moment are quantities, which have no classical analog.

In addition a direct transition to the limit $\hbar \to 0$ in the action for the dynamic system $S_D$ is impossible. Indeed, the action $A_D$ for the dynamic system $S_D$ has the form

$$A_D[\psi, \psi^*] = \int (-m\bar{\psi}\gamma^0 \partial_0 \psi - i/2 \hbar \bar{\psi} \gamma^j \partial_j \psi) d^4x$$  \hspace{1cm} (1.1)

Here $\psi$ is four-component complex wave function, $\psi^*$ is the Hermitian conjugate wave function, and $\bar{\psi} = \psi^* \gamma^0$ is conjugate one. $\gamma^i$, $i = 0, 1, 2, 3$ are $4 \times 4$ complex constant matrices, satisfying the relation

$$\gamma^i \gamma^k + \gamma^k \gamma^i = 2g^{kl}I, \hspace{1cm} k, l = 0, 1, 2, 3.$$  \hspace{1cm} (1.2)

where $I$ is the unit $4 \times 4$ matrix, and $g^{kl} = \text{diag}(c^{-2}, -1, -1, -1)$ is the metric tensor. Considering dynamic system $S_D$, we choose for simplicity such units, where the speed of the light $c = 1$. The action (1.1) generates dynamic equation for the dynamic system $S_D$, known as the Dirac equation

$$i\hbar \gamma^j \partial_j \psi - m\psi = 0$$  \hspace{1cm} (1.3)
and expressions for physical quantities: the 4-flux \( j^k \) of particles and the energy-momentum tensor \( T^k_l \)

\[
j^k = \bar{\psi} \gamma^k \psi, \quad T^k_l = \frac{i}{2} \left( \bar{\psi} \gamma^k \partial_l \psi - \partial_l \bar{\psi} \cdot \gamma^k \psi \right)
\] (1.4)

If we set \( \hbar = 0 \) in the action (1.1), we obtain no description. For transition to the classical description, where \( \hbar = 0 \), we need more subtle methods.

For simplicity we consider a use of these methods in the simple example of the dynamic system \( S_S \), described by the Schrödinger equation. In this case the action has the form

\[
S_S: \quad A_S[\psi, \psi^*] = \int \left\{ \frac{i\hbar}{2} (\psi^* \partial_0 \psi - \partial_0 \psi^* \cdot \psi) - \frac{\hbar^2}{2m} \nabla \psi^* \nabla \psi \right\} dt dx
\] (1.5)

Expressions of the 4-current \( j^k \) and components \( T^0_k \) of energy-momentum tensor have the form

\[
j^k = \{ \rho, j \}, \quad \rho = \psi^* \psi, \quad j = -\frac{i\hbar}{2m} (\psi^* \nabla \psi - \nabla \psi^* \cdot \psi)
\] (1.6)

\[
T^0_0 = \frac{\hbar^2}{2m} \nabla \psi^* \nabla \psi, \quad T^0_\alpha = -\frac{i\hbar}{2} (\psi^* \partial_\alpha \psi - \partial_\alpha \psi^* \cdot \psi), \quad \alpha = 1, 2, 3
\] (1.7)

If we set \( \hbar = 0 \) in the action (1.5), we do not obtain classical description of anything. To obtain the true result, we are to make at first the transformation of the wave function phase

\[
\Psi = \exp \left( \frac{\hbar}{b_0} \ln \frac{\psi}{|\psi|} \right) |\psi|, \quad \psi = \exp \left( \frac{b_0 \hbar}{|\Psi|} \right) |\Psi|
\] (1.8)

in the action (1.5). Here \( b_0 \neq 0 \) is an arbitrary real constant. After transformation the action (1.5) takes the form

\[
S_S: \quad A_S[\Psi, \Psi^*] = \int \left\{ \frac{ib_0}{2} (\psi^* \partial_0 \psi - \partial_0 \psi^* \cdot \psi) - \frac{b_0^2}{2m} \nabla \psi^* \nabla \psi \\
- \frac{\hbar^2 - b_0^2}{8 \psi^* \psi} (\nabla (\psi^* \psi))^2 \right\} dt dx
\] (1.9)

The 4-current (1.6) and components of the energy momentum tensor (1.7) take the form

\[
j^k = \{ \rho, j \}, \quad \rho = \psi^* \psi, \quad j = -\frac{ib_0}{2m} (\psi^* \nabla \psi - \nabla \psi^* \cdot \psi)
\] (1.10)

\[
T^0_0 = \frac{b_0^2}{2m} \nabla \psi^* \nabla \psi + \frac{\hbar^2 - b_0^2}{8 \psi^* \psi} (\nabla (\psi^* \psi))^2,
\] (1.11)

\[
T^0_\alpha = -\frac{ib_0}{2} (\psi^* \partial_\alpha \psi - \partial_\alpha \psi^* \cdot \psi), \quad \alpha = 1, 2, 3
\] (1.12)
If now we set $\hbar = 0$ in the action (1.9), we obtain the action for the pure statistical ensemble $\mathcal{E} [S_{Scl}]$ of dynamic systems $S_{Scl}$. The action for the dynamic system $S_{Scl}$ has the form

$$S_{Scl} : \quad A_{Scl} [x] = \int \frac{m}{2} \left( \frac{dx}{dt} \right)^2 dt \quad (1.13)$$

where $x = x(t) = \{x^1(t), x^2(t), x^3(t)\}$. The action (1.13) describes the free classical nonrelativistic particle.

Thus, the continuous dynamic system $S$, having infinite number of the freedom degrees, associates with the discrete dynamic system $S_{Scl}$, having six degrees of freedom. This circumstance is formulated as follows. Dynamic system $S$ is a result of quantization of the free nonrelativistic particle $S_{Scl}$. One may say also, that the classical dynamic system $S_{Scl}$ is a result of disquantization of the quantum system $S$.

We note two important properties of the disquantization, i.e. transition from the continuous dynamic system $S$ to the discrete dynamic system $S_{Scl}$.

1. The intermediate dynamic system (1.9) is not quantum, because the dynamic equation for the wave function $\Psi$ is nonlinear (but this disagrees with the quantum mechanics principles). Nonlinearity of dynamic equation for $\Psi$ is connected with the fact that transformation (1.8), connecting the wave function $\psi$ and $\Psi$ is nonlinear. Of course, the accordance with the quantum principles must be violated at some moment, as far as the final dynamic system $S_{Scl}$ is not quantum. But it is curious that the accordance is violated not in the time, when we set $\hbar = 0$, but at the earlier stage, when the action (1.5) is transformed to the action (1.9), describing the same dynamic system $S$, as the action (1.5).

2. The transformation (1.8), connecting wave functions $\Psi$ and $\psi$ contains the quantum constant $\hbar$. It becomes to be singular at $\hbar \to 0$. It means essentially, that the quantum constant $\hbar$ is introduced in dynamic variables $\Psi$ and $\Psi^*$. Hereinafter, when $\hbar \to 0$, the quantum constant, contained in $\Psi$ is not changed. In a similar manner we act at disquantization of the dynamic system $S_D$, when we introduce the quantum constant in definition of spin $S_D = \hbar/2$ and magnetic moment $\mu_D = e\hbar/mc$. These quantities are not changed, when we go to the limit $\hbar \to 0$.

Finally, why do we choose the transformation (1.8) for disquantization of the dynamic system $S$, but not some other? What motives are used at the choice of the transformation (1.8)? What transformation should be chosen for disquantization of the dynamic system $S_D$?

We know that the dynamic system (1.9) is a result of quantization of the dynamic system (1.13). The disquantization is the operation reciprocal to the quantization. So we choose the manner of introduction of the quantum constant $\hbar$ into the dynamic variable (wave function) in such a way, to obtain (1.13) as a result of disquantization of dynamic system $S$, described by the action (1.5).

In the case of the Dirac equation the situation is another one. The dynamic system $S_D$ was postulated by Dirac. It was not obtained as a result of quantization of some classical dynamic system. The fact that the result of disquantization of $S_D$ is a relativistic particle with a spin (i.e. the relativistic generalization of the
nonrelativistic particle with a spin, described by the Pauli equation) was considered to be evident. One needs only to invent the proper disquantization procedure.

At the disquantization of the dynamic system $S_D$ different authors use different methods [14, 15, 16, 17]. All this forces one to think that there is no general principle of the dynamic system disquantization. The classical dynamic system (a result of disquantization of quantum system) is obtained from some a priori consideration, and corresponding methods of disquantization are fitted to the a priori result. The common feature of all methods of disquantization is the fact, that the dynamic equations for the disquantized dynamic system do not contain quantum constant $\hbar$, although physical quantities (spin, magnetic moment) may contain unvanishing quantum constant. It is common practice to think that the dynamic system $S_{Dcl}$, obtained as a result of disquantization of $S_D$ is a pointlike relativistic particle with the spin $S_D = \hbar/2$ and magnetic moment $\mu_D = e\hbar/mc$. This belief has historical origin, and one cannot substantiate it mathematically, because the result depends on the applied methods.

Do there exist the principle of disquantization, which satisfies the following conditions?

1. The quantum principles and a reference to the quantum constant are not used. In particular, the transition to the limit $\hbar \to 0$ and the introduction of the quantum constant into dynamic variables, connected with this transition, is not used.

2. Disquantization of continuous dynamic system $S$ determines uniquely a disquantized classical dynamic system $S_{cl}$.

If one succeeded to define the disquantization procedure in accordance with these conditions, this procedure will be the means of investigation of continuous dynamic system $S$ and will realize an interpretation of $S$ in terms of the discrete classical system $S_{cl}$.

To solve this problem we need at first to perceive that the dynamic system $S_{cl}$ is classical and suitable for interpretation of $S$, only in the case, when $S_{cl}$ has finite number of the freedom degrees and, hence, its dynamic equations are ordinary differential equations. It is of no importance whether or not these dynamic equations contain the quantum constant $\hbar$, because effectiveness and simplicity of the dynamic equations analysis is connected with the fact that these equations are ordinary differential equations, but not partial differential equations.

For the system of partial differential equations to be equivalent to a system of ordinary differential equations, it is necessary that the equations contain derivative only in one direction in the space of independent variables. This direction is the direction of the current 4-vector $j^k$, which is determined by the dynamic system $S$. We choose dependent variables of dynamic system $S$ in such a way, that variables $j^k$, $k = 0, 1, 2, 3$ are among them, and make in dynamic equations the change

$$\partial^l \rightarrow \partial^l_\parallel = \frac{j^l j^k}{j^s j^s} \partial_k, \quad l = 0, 1, 2, 3, \quad \partial^k \equiv g^{kl} \partial_l \equiv g^{kl} \frac{\partial}{\partial x_l}, \quad j_l \equiv g_{lk} j^k \quad (1.14)$$

where $g_{ik} = \text{diag}\{c^2, -1, -1, -1\}$, $g^{ik} = \{c^{-2}, -1, -1, -1\}$. The dynamic equations of the system $S$ turn into a system of ordinary differential equations. It is the system
of dynamic equation for the pure statistical ensemble $E_{S_{cl}}$. The dynamic system $S_{cl}$ is classical in the sense, that it has a finite number of the freedom degrees, i.e. its dynamic equations are ordinary differential equations.

Let us make a change of variables in the action (1.9)

$$\Psi = \sqrt{\rho e^{i\varphi}}, \quad \Psi^* = \sqrt{\rho e^{-i\varphi}}$$

We obtain instead of (1.9)

$$S_{S} : \quad A_{S} [\rho, \varphi] = \int \left\{ -b_0 \left( j_0 \partial_0 \varphi + j \nabla \varphi \right) - \frac{\hbar^2}{8m} \frac{(\nabla \rho)^2}{\rho} \right\} \, dt \, dx$$

where in accordance with (1.15) and (1.10)

$$j_0 = \rho, \quad j = \frac{b_0}{2m} \rho \nabla \varphi$$

We make the change (1.14) in (1.16). The first term have the form $j_k \partial_k \varphi$, and the procedure (1.14) does not change it. For the second term we have

$$\frac{(\nabla \rho)^2}{\rho} \to \frac{j^2 (j^k \partial_k \rho)}{\rho (j^s j_s)^2} = \frac{\left( \frac{m}{2m} \right)^2 \rho^2 (\nabla \varphi)^2 \left( \rho \partial_0 \rho + \frac{b_0}{2m} \rho \left( \nabla \varphi \right) \left( \nabla \rho \right) \right)^2}{\rho \left( c^2 \rho^2 - \left( \frac{b_0}{2m} \right)^2 \rho^2 (\nabla \varphi)^2 \right)^2} = O \left( c^{-4} \right)$$

In the nonrelativistic approximation, when $c \to \infty$, this term vanishes. It vanishes independently of whether or not $\hbar \to 0$. But in the relativistic case the result may be different, and dependence of the action on the quantum constant $\hbar$ may remain. We shall see that in the case of the dynamic system $S_{D}$ the quantum constant $\hbar$ remains in the action after the procedure (1.14). Nevertheless the obtained dynamic system is a pure statistical ensemble of classical dynamic systems $S_{D_{cl}}$, whose dynamic equations are ordinary differential equations.

In the nonrelativistic approximation the action (1.16) takes the form

$$E_{S_{cl}} : \quad A_{S_{cl}} [\rho, \varphi] = \int \left\{ -b_0 \rho \partial_0 \varphi - \frac{b_0^2}{2m} \rho \left( \nabla \varphi \right)^2 \right\} \, dt \, dx$$

The action (1.19) generates dynamic equations

$$\frac{\delta A}{\delta \varphi} = b_0 \left( \partial_0 \rho + \nabla \left( \frac{\rho}{m} \nabla \left( b_0 \varphi \right) \right) \right) = b_0 \left( \partial_0 j^0 + \nabla j \right) = 0$$

$$\frac{\delta A}{\delta \rho} = \partial_0 (b_0 \rho) + \frac{1}{2m} \left( \nabla \left( b_0 \varphi \right) \right)^2 = 0$$

Equation (1.20) is the continuity equation, and (1.21) is the Hamilton-Jacobi equation for the free nonrelativistic particle, where $b_0 \varphi$ is the action variable. Introducing Lagrangian variables, one can show [13] that the action (1.19) is a special (irrotational) case of the action

$$E_{S_{cl}} : \quad A_{E_{S_{cl}}} [x] = \int \frac{m}{2} \left( \frac{dx}{dt} \right)^2 \, dt \, d\xi$$
where \( x = x(t, \xi) \), and \( \xi = \{\xi_1, \xi_2, \xi_3\} \) are Lagrangian coordinates, labelling the systems \( S_{\text{Scl}} \), constituting the statistical ensemble. Formally dynamic equations for the statistical ensemble \( E[S_{\text{Scl}}] \) are partial differential equations with independent variables \( t, \xi_1, \xi_2, \xi_3 \). But in fact they contain only derivatives with respect to the variable \( t \), and dynamic equations are ordinary differential equations, because independent variables \( \xi \) are contained in dynamic equations as parameters (in reality dynamic equations do not depend on \( \xi \) explicitly). Formal transformation of the action (1.19) to the form (1.22) is not simple, because the irrotational flow (the property of the dynamic system (1.19)) is described rather easily in the Eulerian coordinates \( \{t, x\} \), but its expression in the Lagrangian coordinates \( \{t, \xi\} \) is not simple.

If we apply transformation
\[
\psi = \sqrt{\rho} e^{i\varphi}, \quad \psi^* = \sqrt{\rho} e^{-i\varphi}
\]
(1.23) to the action (1.5), we obtain the same result (1.20), (1.21) with the constant \( \bar{h} \) instead of the constant \( b_0 \).

Let us compare the dynamic disquantization (1.14) and the conventional method of disquantization (when \( \bar{h} \to 0 \)). The conventional method is not formalized, and only given enough ingenuity, researchers can apply it to new quantum systems, provided the result of disquantization is known a priori. Besides, it refers to the quantum principles. On the contrary, the dynamic disquantization is well defined and formalized. Any literate student can apply this method for disquantization of any new quantum system. He obtains an unique result without any a priori information and without any reference to the quantum principles and quantum constant.

If we want to have a well defined disquantization procedure, we should define it in the form of dynamic disquantization (1.14), but not by means of the limit \( \bar{h} \to 0 \). The dynamic disquantization is not a supposition, which should be founded, or tested. It is simply a method of investigation of a continuous dynamic system \( \mathcal{S} \), associating it with a discrete dynamic system \( S_{\text{D}} \). Application of this method to dynamic system \( S_{\text{S}} \) (1.5) gives the result \( S_{\text{Scl}} \) (1.13), and it allows one to interpret the procedure (1.14) as a disquantization. This disquantization is dynamic, because it uses only properties of the dynamic system \( \mathcal{S} \) and nothing besides them.

We are forced to explain such evident things in details, because the first version [11] of this paper was rejected by several journals on physics and mathematical physics and was not published. Referees of these journals stated that the dynamic disquantization procedure is not substantiated properly and results of this disquantization application to the Dirac particle \( S_{\text{D}} \) are not tested experimentally. Opinion of the referees reflects the viewpoint of the scientific community, and we are forced to explain situation despite absurdity of these objections.

Experimental test is necessary, if one makes some suppositions, and then the experimental test shows, whether or not these suppositions are valid. We make no suppositions. We obtain results by means of logical reasonings and mathematical
calculations. Experimental test of our results is the same as an experimental test of
the Newtonian binomial \((a + b)^2 = a^2 + 2ab + b^2\).

The dynamic disquantization (1.14) is not a supposition. It is a definition of
the procedure. One may test consistency of this procedure. One may apply this
procedure, or not apply it, but one may not demand substantiation of the definition.
One may consider motives of such a definition, and a bit later we consider these
motives, but these motives are not a substantiation of the definition (1.14), and the
definition does not need any substantiation.

Before considerations of these motives, we try to answer the very important
question. Why the disquantization procedure has not been formalized? The dis-
quantization is a very important procedure. The dynamic disquantization (1.14)
is very simple and evident procedure, but it has not been discovered during eighty
years of the quantum mechanics existence. Why? What obstacles did prevent the
disquantization from formalization?

The general answer is as follows. Researchers believed in quantum principles and
in quantum nature of the microcosm. They cannot imagine, that quantum systems
can be investigated without a use of quantum principles. Now details. According to
dominating Copenhagen interpretation the wave function \(\psi\) of a quantum particle
is a specific quantum object, which has not a classical analog. The wave function
is supposed to describe the state of individual quantum particle. On the other
hand, the wave function of an individual quantum particle describes the state of
a continuous dynamic system, having infinite number of the freedom degrees. The
classical particle is described by a discrete dynamic system, having a finite number
of the freedom degrees. To formalize the disquantization procedure, one needs to
formalize the transition from the continuous dynamic system to the discrete one.
How can one formalize the jump from infinite number of the freedom degrees to the
finite one?

The problem is solved as follows. At first, one shows that the wave function is
not a specific quantum object. The wave function is a method of description of any
fluidlike continuous dynamic system [12]. Quantum systems are dynamic systems
of such a kind. But the pure statistical ensembles \(\mathcal{E}[S_{cl}]\) of classical systems \(S_{cl}\) are
also dynamic systems of such a kind. The state of such an ensemble \(\mathcal{E}[S_{cl}]\) may
be also described by the wave function. The classical system \(S_{cl}\) and the statistical
ensemble \(\mathcal{E}[S_{cl}]\) are coupled between themselves in the sense, that the action \(\mathcal{A}_{S_{cl}}\)
for \(S_{cl}\) determines the action \(\mathcal{A}_{\mathcal{E}[S_{cl}]}\) for \(\mathcal{E}[S_{cl}]\) and vice versa the action \(\mathcal{A}_{\mathcal{E}[S_{cl}]}\) determines
the action \(\mathcal{A}_{S_{cl}}\). Dynamic system \(\mathcal{E}[S_{cl}]\) is continuous, and it contains an infinite
number of the freedom degrees, whereas \(S_{cl}\) is a discrete dynamic system which
contains a finite number of the freedom degrees. Connection between \(S_{cl}\) and \(\mathcal{E}[S_{cl}]\)
allows one to overcome the jump between the continuous dynamic system and the
discrete one.

The difference between the quantum system \(S\) and the statistical ensemble \(\mathcal{E}[S_{cl}]\)
lies in the form of dynamic equations. Dynamic equations for \(S\) are partial differential
equations, which cannot be transformed to the form of ordinary differential
equations, because they contain derivatives in different directions, whereas dynamic
equations for $\mathcal{E}[S_{cl}]$ are ordinary differential equations, or partial differential equations, which can be reduced to the ordinary differential equations by means of a change of variables.

At the disquantization procedure all components of derivatives transversal to the vector $j^k$ are suppressed, and the quantum system $S$ turns into the statistical ensemble $\mathcal{E}[S_{cl}]$. Having determined the statistical ensemble $\mathcal{E}[S_{cl}]$, one can determine the classical dynamic system $S_{cl}$. Of course, we must adopt that the quantum dynamic system $S$ is a statistical ensemble $\mathcal{E}[S_{st}]$ of some individual stochastic systems, but not an individual quantum system, because otherwise we cannot explain, how individual quantum particle can turn into the statistical ensemble $\mathcal{E}[S_{cl}]$ of classical particles. It means that the wave function describes the state of the statistical ensemble of particles (classical, or quantum), but not an individual particle. It means that the Copenhagen interpretation is false at the point, when it states that the wave function describes the state of an individual particle. This statement of the Copenhagen interpretation is incompatible with the quantum mechanics formalism [13].

Note that our explanation of the situation with the formalization of the disquantization procedure is purely dynamical. The difference between the quantum system $S$ and the statistical ensemble $\mathcal{E}[S_{cl}]$ lies in the form of dynamic equations, but not in enigmatic quantum principles. Hence, the problem of the disquantization formalization is a dynamical problem, which should be solved by dynamic methods.

Thus, there are three reasons, why the problem of the disquantization formalization has not been solved: (1) belief that the wave function is a specific quantum object, (2) belief that the wave function describes the state of individual quantum particle, (3) belief in principles of quantum mechanics, and attempts to solve the problem on their basis.

Now about physical reasons of the dynamic disquantization (1.14). Conventional approach to disquantization is an attempt of cutting off the quantum stochasticity, setting $\hbar = 0$ in the proper representation (1.9) of the action for $S_{st}$. Dynamic disquantization does not try to cut off the quantum stochasticity. It uses existence of such states of the statistical ensemble of stochastic systems $S_{st}$, where the stochastic component of the particle motion does not influence upon the regular component. Stochasticity influences upon the regular component only in nonuniform states. This statement is valid for all stochastic systems (but not only for quantum ones). For instance, the mean velocity of Brownian particles is determined by the relation

$$v_B = -D \nabla \ln \rho$$

where $\rho$ is the Brownian particle density, and $D$ is the diffusion coefficient. If the ensemble of Brownian particles is uniform, $\rho = \text{const}$, the mean velocity vanishes, although the random motion of Brownian particles remains. Analogously, if in the statistical ensemble $\mathcal{E}[S_{st}]$ $\nabla u = 0$ for all physical quantities $u$, the influence of the stochastic component on the regular component vanishes. The space gradient $\nabla u$ is considered in the coordinate system, where the medium is at rest, and the current vector $j^k$ has the form $j^k = \{j^0, 0, 0, 0\}$. In the arbitrary coordinate system
the condition $\nabla u = 0$ turns into

$$\partial_t^* u = \partial^k u - \frac{j^k j^l}{j^s j_s} \partial_l = 0$$  \hspace{1cm} (1.24)$$

which can be realized by means of the change (1.14). This consideration is only an explanation of the dynamic disquantization, but not its substantiation.

Further we transform the action (1.1) for the Dirac particle $S_D$ to hydrodynamical variables, where the current components $j^k = \bar{\psi} \gamma^k \psi$, $k = 0, 1, 2, 3$ are dependent variables. We produce dynamic disquantization (1.14) and obtain classical dynamic system $S_{Dcl}$, having ten degrees of freedom. We solve the dynamic equations for $S_{Dcl}$ and find that $S_{Dcl}$ can be identified with a rotator which also has ten degrees of freedom.

The goal of investigation is a construction of dynamic system $S_{Dcl}$, associated with $S_D$. Dynamic equations for $S_{Dcl}$ form a system of ordinary differential equations. Further the dynamic system $S_{Dcl}$, will be referred to as the classical Dirac particle. It has finite number of the freedom degrees, and it is simpler for investigation, than $S_D$.

If our statement on the composite structure of the Dirac particle appears to be incompatible with experimental data, this result is an argument against application of the Dirac equation, but not against our investigation of the Dirac equation.

Investigating the Dirac particle $S_D$, we transform the action (1.1) to hydrodynamical variables by means of a change of variables. In terms of the new variables the variables $j^k$, defined by (1.4) are four dependent variables. Form of other four dependent variables is chosen in such a way, to eliminate $\gamma$-matrices from the action. After such a change of variables we can produce the dynamic disquantization in the action, making the change (1.14). Thereafter the action for $S_D$ turns into the action for the statistical ensemble of classical particles $S_{Dcl}$, having ten degrees of freedom. Investigating properties of classical dynamic system $S_{Dcl}$, we investigate properties of the Dirac particle $S_D$. This investigation allows one to interpret properties of the Dirac particle $S_D$ in terms of the classical dynamic system $S_{Dcl}$.

## 2 Transformation of variables

The state of dynamic system $S_D$ is described by eight real dependent variables (eight real components of four-component complex wave function $\psi$). Transforming the action (1.1), we use the mathematical technique [9, 10], where the wave function $\psi$ is considered to be a function of hypercomplex numbers $\gamma$ and coordinates $x$. In this case the dynamical quantities are obtained by means of a convolution of expressions $\psi^* O \psi$ with zero divisors. This technique allows one to work without fixing the $\gamma$-matrices representation.

Using designations

$$\gamma_5 = \gamma^{0123} \equiv \gamma^0 \gamma^1 \gamma^2 \gamma^3,$$

$$\sigma = \{\sigma_1, \sigma_2, \sigma_3, \} = \{-i \gamma_2 \gamma^3, -i \gamma_3 \gamma^1, -i \gamma_1 \gamma^2\}$$

(2.1)  \hspace{1cm} (2.2)
we make the change of variables

\[ \psi = A e^{i \varphi + \frac{1}{2} \gamma_5 \kappa} \exp \left( -\frac{i}{2} \gamma_5 \sigma \eta \right) \exp \left( \frac{i \pi}{2} \sigma n \right) \Pi \]  

(2.3)

\[ \psi^* = A \Pi \exp \left( -\frac{i \pi}{2} \sigma n \right) \exp \left( -\frac{i}{2} \gamma_5 \sigma \eta \right) e^{-i \varphi - \frac{1}{2} \gamma_5 \kappa} \]  

(2.4)

where \(^*\) means the Hermitian conjugation, and

\[ \Pi = \frac{1}{4} (1 + \gamma^0) (1 + z \sigma), \quad z = \{ z^\alpha \} = \text{const}, \quad \alpha = 1, 2, 3; \quad z^2 = 1 \]  

(2.5)

is a zero divisor. The quantities \( A, \kappa, \varphi, \eta = \{ \eta^\alpha \}, \textbf{n} = \{ n^\alpha \}, \alpha = 1, 2, 3, \text{ n}^2 = 1 \) are eight real parameters, determining the wave function \( \psi \). These parameters may be considered as new dependent variables, describing the state of dynamic system \( S_D \). The quantity \( \varphi \) is a scalar, and \( \kappa \) is a pseudoscalar. Six remaining variables \( A, \eta = \{ \eta^\alpha \}, \textbf{n} = \{ n^\alpha \}, \alpha = 1, 2, 3, \text{ n}^2 = 1 \) can be expressed through the flux 4-vector \( j^l = \bar{\psi} \gamma^l \psi \) and spin 4-pseudovector

\[ S^l = i \bar{\psi} \gamma_5 \gamma^l \psi, \quad l = 0, 1, 2, 3 \]  

(2.6)

Because of two identities

\[ S^l S_l \equiv -j^l j_l, \quad j^l S_l \equiv 0. \]

(2.7)

there are only six independent components among eight components of quantities \( j^l \), and \( S^l \).

Matrices \( \gamma_5, \sigma = \{ \sigma_\alpha \}, \alpha = 1, 2, 3 \) are determined by relations (2.1), (2.2) have the following properties

\[ \gamma_5 \gamma_5 = -1, \quad \gamma_5 \sigma_\alpha = \sigma_\alpha \gamma_5, \quad \gamma^{0\alpha} \equiv \gamma_0 \gamma^\alpha = -i \gamma_5 \sigma_\alpha, \quad \alpha = 1, 2, 3; \]  

(2.8)

\[ (\gamma^0)^* = \gamma^0, \quad (\gamma^\alpha)^* = -\gamma^\alpha, \quad \gamma^0 \sigma = \sigma \gamma^0, \quad \gamma^0 \gamma_5 = -\gamma_5 \gamma^0 \]  

(2.9)

According to relations (1.2), (2.1), (2.2) the matrices \( \sigma = \{ \sigma_\alpha \}, \alpha = 1, 2, 3 \) satisfy the relation

\[ \sigma_\alpha \sigma_\beta = \delta_{\alpha\beta} + i \varepsilon_{\alpha\beta\gamma} \sigma_\gamma, \quad \alpha, \beta = 1, 2, 3 \]  

(2.10)

where \( \varepsilon_{\alpha\beta\gamma} \) is the antisymmetric pseudo-tensor of Levi-Chivita \( (\varepsilon_{123} = 1) \).

Using relations (2.8),(2.9), (2.10) and (2.5), it is easy to verify that

\[ \Pi^2 = \Pi, \quad \gamma_0 \Pi = \Pi, \quad \textbf{z} \sigma \Pi = \Pi, \quad \Pi \gamma_5 \Pi = 0, \quad \Pi \sigma_\alpha \Pi = z^\alpha \Pi, \quad \alpha = 1, 2, 3. \]  

(2.11)

(2.12)

Generally, the wave functions \( \psi, \psi^* \) defined by (2.4) are \( 4 \times 4 \) complex matrices. In the proper representation, where \( \Pi \) has the form

\[ \Pi = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \]  

(2.13)
the \( \psi, \psi^\star \) have the form

\[
\psi = \begin{pmatrix}
\psi_1 & 0 & 0 & 0 \\
\psi_2 & 0 & 0 & 0 \\
\psi_3 & 0 & 0 & 0 \\
\psi_4 & 0 & 0 & 0
\end{pmatrix}, \quad \psi^\star = \begin{pmatrix}
\psi_1^\star & \psi_2^\star & \psi_3^\star & \psi_4^\star \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\] (2.14)

Let \( O \) be an arbitrary \( 4 \times 4 \) matrix. The product \( \psi^\star O \psi \) has the form

\[
\psi^\star O \psi = \begin{pmatrix}
a & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix} = a \Pi = \Pi a
\] (2.15)

where \( a \) is a complex quantity. If \( f \) is an analytical function having the property \( f(0) = 0 \), then the function \( f(\psi^\star O \psi) = f(a \Pi) \) of a \( 4 \times 4 \) matrix of the type (2.15) is a matrix \( f(a) \Pi \) of the same type. For this reason we shall not distinguish between the complex quantity \( a \) and the complex \( 4 \times 4 \) matrix \( a \Pi \). In the final expressions of the type \( a \Pi \) (\( a \) is a complex quantity) the multiplier \( \Pi \) will be omitted.

By means of relations (2.8) – (2.12), one can reduce any Clifford number \( \Pi O \Pi \) to the form (2.15), without using any concrete form of the \( \gamma \)-matrix representation. This property will be used in our calculations. Calculating exponents of the type (2.3), (2.4), we shall use the following relations

\[
\exp \left( -\frac{i\pi}{2} \sigma n \right) F(\sigma) \exp \left( \frac{i\pi}{2} \sigma n \right) = F(\Sigma)
\]

where \( F \) is arbitrary function and the quantity

\[
\Sigma = \{ \Sigma_1, \Sigma_2, \Sigma_3 \}, \quad \Sigma_\alpha = \exp \left( -\frac{i\pi}{2} \sigma n \right) \sigma_\alpha \exp \left( \frac{i\pi}{2} \sigma n \right) \quad \alpha = 1, 2, 3;
\] (2.16)

satisfies the same commutation relations (2.10) as the Pauli matrices \( \sigma \).

For variables \( \bar{\psi} \psi, j^l, S^l, l = 0, 1, 2, 3 \) we have the following expressions

\[
\bar{\psi} \psi = \psi^\star \gamma^0 \psi = A^2 \Pi e^{\gamma_5 \kappa} \Pi = A^2 \Pi (\cos \kappa + \gamma_5 \sin \kappa) \Pi = A^2 \cos \kappa \Pi
\]

Taking into account the first relation (2.12), the term linear with respect to \( \gamma_5 \) vanishes, and we obtain

\[
\bar{\psi} \psi = A^2 \cos \kappa \Pi
\]

\[
j^0 \Pi = \bar{\psi} \gamma^0 \psi = A^2 \Pi \exp \left( -\frac{i\pi}{2} \sigma n \right) \exp (-i\gamma_5 \sigma \eta) \exp \left( \frac{i\pi}{2} \sigma n \right) \Pi
\]

\[
= A^2 \Pi \exp (-i\gamma_5 \Sigma \eta) \Pi = A^2 \Pi \left( \cosh \eta - \frac{i\gamma_5}{\eta} \Sigma \eta \sinh \eta \right) \Pi
\]

\[
= A^2 \cosh(\eta) \Pi
\] (2.17)
where
\[ \eta = \sqrt{\eta^2} = \sqrt{\eta^\alpha \eta^\alpha} \]

Again in force of the first relation (2.12) we omit terms linear with respect to \( \gamma_5 \).

In the same way we obtain

\[
j_\alpha \Pi = A^2 \sinh(\eta) v^\alpha \Pi, \quad \alpha = 1, 2, 3
\]

(2.18)

where
\[
v = \{ v^\alpha \}, \quad v^\alpha = \eta^\alpha / \eta, \quad \alpha = 1, 2, 3; \quad v^2 = 1.
\]

(2.19)

Let us introduce designation \( \xi = \{ \xi^\alpha \}, \alpha = 1, 2, 3 \) for the expression

\[
\xi^\alpha \Pi = \Pi \Sigma_\alpha \Pi, \quad \alpha = 1, 2, 3, \quad \xi^2 = \xi^\alpha \xi^\alpha = 1
\]

(2.20)

Then for the spin pseudovector \( S^I \), defined by the relation (2.6), we obtain

\[
S_0 \Pi = \psi^* (-i \gamma_5) \psi = A^2 \Pi (-i \gamma_5) \exp (-i \gamma_5 \Sigma \eta) \Pi =
\]

\[
= A^2 \Pi \sinh(\eta) \Sigma v \Pi = A^2 \sinh(\eta) \xi v \Pi,
\]

(2.21)

Now twice using relations (2.10) for Pauli matrices \( \Sigma_\alpha \), we derive

\[
S^\alpha \Pi = A^2 \Pi \left( \cos^2 \frac{\eta}{2} \Sigma_\alpha + \sinh^2 \frac{\eta}{2} v^\beta v^\gamma (\delta_\alpha_\beta + i \epsilon_\beta_\alpha_\mu \Sigma_\mu) \Sigma_\gamma \right) \Pi =
\]

\[
= A^2 \Pi \left( \cos^2 \frac{\eta}{2} \Sigma_\alpha + \sinh^2 \frac{\eta}{2} \left( v^\alpha v^\gamma \Sigma_\gamma + i \epsilon_\beta_\alpha_\mu \Sigma_\gamma v^\beta \delta_\mu_\gamma + i \epsilon_\mu_\gamma_\nu \Sigma_\nu \right) \right) \Pi =
\]

\[
= A^2 \Pi \left( \cos^2 \frac{\eta}{2} \Sigma_\alpha + \sinh^2 \frac{\eta}{2} \left( v^\alpha v^\gamma \Sigma_\gamma - v^\beta v^\gamma \Sigma_\alpha + v^\beta v^\alpha \Sigma_\beta \right) \right) \Pi
\]

\[
S^\alpha \Pi = A^2 [\xi^\alpha + (\cosh \eta - 1) v^\alpha (v \xi)] \Pi, \quad \alpha = 1, 2, 3.
\]

(2.22)

It follows from relations (2.17), (2.18), (2.19)

\[
j^I j_H \Pi = A^4 \Pi, \quad A = (j^I j_H)^{1/4} = \rho^{1/2}
\]

(2.23)
According to the third equation (2.11), (2.16) and (2.20) one obtains

\[ \xi^a \Pi = \Pi \sigma_n \exp \left( -\frac{i\pi}{2} \sigma_n \right) \sigma_n \exp \left( \frac{i\pi}{2} \sigma_n \right) \Pi = \]

\[ \Pi \left( \cos \frac{\pi}{2} - i\sigma_n \sin \frac{\pi}{2} \right) \sigma_n \left( \cos \frac{\pi}{2} + i\sigma_n \sin \frac{\pi}{2} \right) \Pi = \]

\[ \Pi (\sigma_n) \sigma_n (\sigma_n) \Pi = \Pi n^\mu n^\nu \sigma_\mu \sigma_\nu \Pi = \]

\[ \Pi (n^\alpha n^\nu \sigma_\nu + i\epsilon_{\mu\gamma\alpha} \sigma_\gamma \sigma_\nu n^\mu n^\nu) \Pi = \Pi (n^\alpha n^\nu \sigma_\nu - \epsilon_{\mu\alpha\gamma} \epsilon_{\gamma\nu\beta} \sigma_\beta n^\mu n^\nu) \Pi = \]

\[ \Pi (n^\alpha n^\nu z_\nu - \epsilon_{\mu\alpha\gamma} \epsilon_{\gamma\nu\beta} z_\beta n^\mu n^\nu) \Pi \]

(2.24)

Or

\[ \xi = 2n(nz) - z \]

where \( z \) is defined by (2.5).

3 Transformation of the action

Let us make a change of variables in the action (1.1), using substitution (2.3) – (2.5). The last two terms of the action (1.1) may be written in the form

\[ i \frac{\hbar}{2} \bar{\psi} \gamma^l \partial_l \psi + \text{h.c.} = i \frac{\hbar}{2} \bar{\psi}^* \left( \partial_0 - i\gamma_5 \sigma \nabla \right) \psi + \text{h.c.} \]

\[ = \frac{i}{2} \hbar \psi^* \left( \partial_0 - i\gamma_5 \sigma \nabla \right) \left( i\varphi + \frac{1}{2} \gamma_5 \kappa \right) \psi + \text{h.c.} \]

\[ + \frac{i}{2} \hbar A^2 \Pi \exp \left( -\frac{i\pi}{2} \sigma n \right) \exp \left( -\frac{i}{2} \gamma_5 \Sigma \eta \right) \left( \partial_0 - i\gamma_5 \sigma \nabla \right) \]

\[ \times \left( \exp \left( -\frac{i}{2} \gamma_5 \Sigma \eta \right) \exp \left( \frac{i\pi}{2} \sigma n \right) \right) \Pi + \text{h.c.} \]

where "h.c." means the term obtained from the previous one by the Hermitian conjugation. Calculation of this expression gives the following result (see details of calculation in Appendix A).

\[ i \frac{\hbar}{2} \bar{\psi} \gamma^l \partial_l \psi + \text{h.c.} = F_1 + F_2 + F_3 + F_4 \]

(3.1)

where

\[ F_1 + F_2 = -j^l \partial_l \varphi \Pi - \frac{1}{2} \hbar \xi^l \partial_l \kappa \Pi \]

(3.2)

\[ F_3 = -\frac{\hbar j^l}{2(1 + \xi z)} \epsilon_{\alpha\beta\gamma} \xi^\alpha \partial_l \xi^\beta z^\gamma \Pi \]

(3.3)

\[ F_4 = \left( \frac{\hbar (\rho + j_0)}{2} - \frac{\hbar}{2(\rho + j_0)} \epsilon_{\alpha\beta\gamma} \sigma^\alpha \sigma^\beta \left( \partial_0 j^\gamma \right) + \frac{\hbar}{2(\rho + j_0)} \epsilon_{\alpha\beta\gamma} \left( \partial_0 j^\beta \right) j^\gamma \xi^\gamma \right) \Pi \]

(3.4)
Here $\varepsilon_{\alpha\beta\gamma}$ is 3-dimensional Levi-Chivita pseudotensor.

We see that the expressions (A.6) for $F_1$ and $F_2$ as well as the first term of the action (1.1)

$$-m\bar{\psi}\psi = -m\Pi^{\gamma_5}\Pi = -mA^2 \cos \kappa \Pi = -m\sqrt{j^j_i} \cos \kappa \Pi \equiv -m\rho \cos \kappa \Pi$$

have relativistically covariant form. The terms $F_3$ and $F_4$ have non-covariant form. Introducing the constant unit 4-vector $f^k = \{1, 0, 0, 0\}$, they can be written in the relativistically covariant form (see [11]). The constant 4-vector $f^k$ appears from the matrix 4-vector $\gamma^k$, $k = 0, 1, 2, 3$, which figures in the original action (1.1)

Now we can write the action (1.1) in the hydrodynamical form

$$S_D : \quad A_D[j, \varphi, \kappa, \xi] = \int \mathcal{L} d^4 x, \quad \mathcal{L} = \mathcal{L}_{cl} + \mathcal{L}_{q1} + \mathcal{L}_{q2}$$

$$\mathcal{L}_{cl} = -m\rho - hj^j_i \partial_i \varphi - \frac{h j^j_i}{2(1 + \xi z)} \varepsilon_{\alpha\beta\gamma} \xi^\alpha \partial_i \xi^\beta z^\gamma, \quad \rho \equiv \sqrt{j^j_i}$$

$$\mathcal{L}_{q1} = 2m\rho \sin^2(\frac{\kappa}{2}) - \frac{h}{2} S_l^j \partial_l \kappa,$$

$$\mathcal{L}_{q2} = \frac{h(\rho + j_0)}{2} \varepsilon_{\alpha\beta\gamma} \rho^\alpha \frac{j^\beta}{(j^j_0 + \rho)} \xi^\gamma - \frac{h}{2(\rho + j_0)} \varepsilon_{\alpha\beta\gamma} (\partial^\beta j^\gamma) j^\alpha \xi^\gamma$$

Lagrangian is a function of 4-vector $j^l$, scalar $\varphi$, pseudoscalar $\kappa$, and unit 3-pseudovector $\xi$, which is connected with the spin 4-pseudovector $S^l$ by means of the relations

$$\xi^\alpha = \rho^{-1} \left[ S^\alpha - \frac{j^\alpha S^0}{(j^j_0 + \rho)} \right], \quad \alpha = 1, 2, 3; \quad \rho \equiv \sqrt{j^j_i}$$

$$S^0 = j^l \xi^l, \quad S^\alpha = \rho \xi^\alpha + \frac{(j^l \xi^l)}{\rho + j^j_0}, \quad \alpha = 1, 2, 3$$

### 4 Dynamic disquantization

Let us produce dynamical disquantization of the action (3.6)–(3.9), making the change (1.14). The action (3.6)–(3.9) takes the form

$$\mathcal{A}_{Dqu}[j, \varphi, \kappa, \xi] = \int \left\{ -m\rho \cos \kappa - hj^j_i \left( \partial_i \varphi + \frac{\varepsilon_{\alpha\beta\gamma} \xi^\alpha \partial_i \xi^\beta z^\gamma}{2(1 + \xi z)} \right) \right.$$\n
$$\left. + \frac{h j^j_k}{2(\rho + j_0)} \varepsilon_{\alpha\beta\gamma} \left( \partial_k j^\beta \right) j^\alpha \xi^\gamma \right\} d^4 x$$

Note that the second term $-\frac{h}{2} S^l \partial_l \kappa$ in the relation (3.8) is neglected, because 4-pseudovector $S^k$ is orthogonal to 4-vector $j^j_i$, and the derivative $S^l \partial_l \kappa = S^l \rho^{-2} j^j_i \partial_k \kappa$ vanishes.
Although the action (4.1) contains a non-classical variable $\kappa$, but in fact $\kappa$ is a constant quantity. Indeed, a variation with respect to $\kappa$ leads to the dynamic equation

$$\frac{\delta \mathcal{A}_{\text{Dqu}}}{\delta \kappa} = m \rho \sin \kappa = 0$$

(4.2)

which has solutions

$$\kappa = n \pi$$

(4.3)

where $n$ is integer. Thus, the effective mass $m_{\text{eff}} = m \cos \kappa$ has two values

$$m_{\text{eff}} = m \cos \kappa = \kappa_0 m$$

(4.4)

where $\kappa_0$ is a dichotomic quantity $\kappa_0 = \pm 1$ introduced instead of $\cos \kappa$. The quantity $\kappa_0$ is a parameter of the dynamic system $\mathcal{S}_{\text{Dqu}}$. It is not to be varying. The action (4.1), turns into the action

$$\mathcal{A}_{\text{Dqu}}[j, \varphi, \xi] = \int \left\{ -\kappa_0 m \rho - \hbar j^i \left( \partial_i \varphi + \frac{\varepsilon_{\alpha\beta\gamma} \xi^\alpha \partial_\beta \xi^\gamma}{2 (1 + \xi z)} \right) ight. $$

$$\left. + \frac{\hbar j^k}{2(\rho + j_0) \rho} \varepsilon_{\alpha\beta\gamma} (\partial_k j^\beta) j^\alpha \xi^\gamma \right\} d^4 x$$

(4.5)

Let us introduce Lagrangian coordinates $\tau = \{ \tau_0, \tau \} = \{ \tau_i(x) \}, i = 0, 1, 2, 3$ as functions of coordinates $x$ in such a way that only coordinate $\tau_0$ changes along the direction $j^l$, i.e.

$$j^k \partial_k \tau_\mu = 0, \quad \mu = 1, 2, 3$$

(4.6)

Considering coordinates $x$ to be a functions of $\tau = \{ \tau_0, \tau \}$, one has the following identities

$$\frac{\partial D}{\partial \tau_{0,i}} \tau_{i,k} \equiv \delta^0_i D, \quad i = 0, 1, 2, 3 \quad \tau_{i,k} \equiv \partial_k \tau_i, \quad i, k = 0, 1, 2, 3$$

(4.7)

where

$$D \equiv \frac{\partial (\tau_0, \tau_1, \tau_2, \tau_3)}{\partial (x^0, x^1, x^2, x^3)}, \quad \frac{\partial D}{\partial \tau_{0,i}} \equiv \frac{\partial (x^i, \tau_1, \tau_2, \tau_3)}{\partial (x^0, x^1, x^2, x^3)}.$$ 

(4.8)

Comparing (4.6) with (4.7), one concludes that it is possible to set

$$j^i = \frac{\partial D}{\partial \tau_{0,i}} \equiv \frac{\partial (x^i, \tau_1, \tau_2, \tau_3)}{\partial (x^0, x^1, x^2, x^3)}, \quad i = 0, 1, 2, 3$$

(4.9)

because the dynamic equation

$$\frac{\delta \mathcal{A}_{\text{Dqu}}}{\delta \varphi} = \hbar \partial_l j^l = 0$$

(4.10)

is satisfied by the relation (4.9) identically in force of identity

$$\partial_i \frac{\partial D}{\partial \tau_{k,i}} \equiv 0, \quad k = 0, 1, 2, 3.$$
Let us take into account that for any variable $u$

$$D^{-1}j^i \partial_i u = D^{-1} \frac{\partial D}{\partial \tau_{0,i}} \partial_i u = \frac{\partial (u, \tau_1, \tau_2, \tau_3)}{\partial (\tau_0, \tau_1, \tau_2, \tau_3)} = \frac{du}{d\tau_0}$$

(4.11)

and in particular,

$$D^{-1}j^i = D^{-1} \frac{\partial D}{\partial \tau_{0,i}} = \frac{\partial (x^i, \tau_1, \tau_2, \tau_3)}{\partial (\tau_0, \tau_1, \tau_2, \tau_3)} = \frac{dx^i}{d\tau_0} \equiv \dot{x}^i, \quad i = 0, 1, 2, 3$$

(4.12)

Besides

$$d^4x = D^{-1}d^4\tau = D^{-1}d\tau_0d\tau$$

(4.13)

$$j^i \partial_i \varphi = \frac{\partial (\varphi, \tau_1, \tau_2, \tau_3)}{\partial (x^0, x^1, x^2, x^3)}$$

(4.14)

The action (4.5) can be rewritten in the Lagrangian coordinates $\tau$ in the form

$$A_{\text{Dqu}}[x, \xi] = \int \left\{ -\kappa_0m\sqrt{\dot{x}^i\dot{x}_i} + \hbar \frac{\dot{\xi} \times \xi)z}{2(1 + \xi z)} + \hbar \frac{(\dot{x} \times \dot{x})\xi}{2\sqrt{x^s\dot{x}_s}\sqrt{\dot{x}_s\dot{x}_s + \dot{\xi}^2}} \right\} d^4\tau$$

(4.15)

where the dot means the total derivative $\dot{x}^i \equiv dx^i/d\tau_0$. $x = \{x^0, x\} = \{x^i\}, \quad i = 0, 1, 2, 3, \xi = \{\xi^\alpha\}, \alpha = 1, 2, 3$ are considered to be functions of the Lagrangian coordinates $\tau_0$, $\tau = \{\tau_1, \tau_2, \tau_3\}$. Here and in what follows the symbol $\times$ means the vector product of two 3-vectors. The quantity $z$ is the constant unit 3-vector (2.5). The term $j^i \partial_i \varphi$ is omitted, because it reduces to a Jacobian (4.14), which does not contribute to dynamic equations. In fact, variables $x$ depend on $\tau$ as on parameters, because the action (4.15) does not contain derivatives with respect to $\tau_\alpha, \alpha = 1, 2, 3$. Lagrangian density of the action (4.15) does not contain independent variables $\tau$ explicitly. Hence, it may be written in the form

$$A_{\text{Dqu}}[x, \xi] = \int A_{\text{Dcl}}[x, \xi] d\tau, \quad d\tau = d\tau_1d\tau_2d\tau_3$$

(4.16)

where

$$S_{\text{Dcl}}: \quad A_{\text{Dcl}}[x, \xi] = \int \left\{ -\kappa_0m\sqrt{\dot{x}^i\dot{x}_i} + \hbar \frac{\dot{\xi} \times \xi)z}{2(1 + \xi z)} + \hbar \frac{(\dot{x} \times \dot{x})\xi}{2\sqrt{x^s\dot{x}_s}\sqrt{\dot{x}_s\dot{x}_s + \dot{\xi}^2}} \right\} d\tau_0$$

(4.17)

The action (4.16) is the action for the dynamic system $S_{\text{Dqu}}$, which is a set of similar independent dynamic systems $S_{\text{Dcl}}$. Such a dynamic system is called a statistical ensemble. Dynamic systems $S_{\text{Dcl}}$ are elements (constituents) of the statistical ensemble $E_{\text{Dqu}}$. Dynamic equations for each $S_{\text{Dcl}}$ form a system of ordinary differential equations. It may be interpreted in the sense that the dynamic system $S_{\text{Dcl}}$ may be considered to be a classical one, although its Lagrangian contains the quantum constant $\hbar$. The dynamic system $S_{\text{Dcl}}$ will be referred to as the classical Dirac particle.
Note that the quantum constant $\hbar$ can be eliminated from the action (4.17) by means of the change of variables

$$\xi \to \Xi = \hbar \xi, \quad z \to Z = \frac{1}{\hbar}z, \quad \Xi^2 = h^2, \quad Z^2 = \frac{1}{h^2} \quad (4.18)$$

The first term in the action (4.17) is relativistic. It describes a motion of classical Dirac particle as a whole. The last two terms in the action (4.17) are nonrelativistic. They describe some internal degrees of freedom of the classical Dirac particle. This internal motion (classical zitterbewegung) means that the classical Dirac particle has some internal structure which is described by a method incompatible with relativity principles. Maybe, the classical Dirac particle is composite. It should be considered to be consisting of several pointlike particles. At any rate the classical Dirac particle is not a pointlike particle. It has a more complicated structure which is described by the variable $\xi$ and by the second order derivative $\ddot{\mathbf{x}}$.

It is easy to see that the action (4.17) is invariant with respect to transformation $\tau_0 \to \tilde{\tau}_0 = F(\tau_0)$, where $F$ is an arbitrary monotone function. This transformation admits one to choose the variable $t = \dot{x}_0$ as a parameter $\tau_0$, or to choose the parameter $\tau_0$ in such a way that $\dot{x}_1 \dot{x}_1 = \dot{x}_0^2 - \mathbf{\dot{x}}^2 = 1$. In the last case the parameter $\tau_0$ is the proper time along the world line of classical Dirac particle. Besides, invariance with respect to transformation $\tau_0 \to \tilde{\tau}_0 = F(\tau_0)$ leads to a connection between the components of the canonical momentum

$$p_k = \frac{\partial L}{\partial \dot{x}_k} - \frac{d}{d\tau_0} \frac{\partial L}{\partial \ddot{x}_k}, \quad k = 0, 1, 2, 3$$

where $L$ is the Lagrange function for the action (4.17).

### 5 Solution of dynamic equations for $S_{\text{Dcl}}$

We shall not consider here problems connected with relativistic non-invariance of terms, describing internal degrees of freedom, referring to [11], where these problems are discussed. We obtain dynamic equations generated by the action (4.17), solve them and try to interpret the obtained solution.

Variation of the action (4.17) with respect to $\mathbf{x}$ gives the dynamic equation

$$\frac{d}{d\tau_0} \left( -\kappa_0 m \frac{\dot{\mathbf{x}}}{\sqrt{\dot{x}_s \dot{x}_s}} + \frac{\hbar Q}{2} (\xi \times \ddot{\mathbf{x}}) - \frac{\hbar}{2} \frac{\partial Q}{\partial \dot{x}_0} (\dot{\mathbf{x}} \times \ddot{\mathbf{x}}) \xi + \frac{\hbar}{2} \frac{d}{d\tau_0} (Q(\xi \times \ddot{\mathbf{x}})) \right) = 0 \quad (5.1)$$

where

$$Q = Q(\dot{x}) = \left( \sqrt{\dot{x}_s \dot{x}_s} (\sqrt{\dot{x}_s \dot{x}_s} + \dot{x}_0^2) \right)^{-1}, \quad \dot{x}_s \dot{x}_s = \dot{x}_0^2 - \dot{\mathbf{x}}^2 \quad (5.2)$$

Varying the action (4.17) with respect to $x^0$, we obtain

$$\frac{d}{d\tau_0} \left( \kappa_0 m \frac{\dot{x}_0}{\sqrt{\dot{x}_s \dot{x}_s}} - \frac{\hbar}{2} \frac{\partial Q}{\partial \dot{x}_0} (\dot{\mathbf{x}} \times \ddot{\mathbf{x}}) \xi \right) = 0 \quad (5.3)$$
Varying the action (4.17) with respect to $\xi$, one should take into account the side constraint $\xi^2 = 1$. Setting

$$\xi^\alpha = \frac{\zeta^\alpha}{\sqrt{\zeta^2}}, \quad \alpha = 1, 2, 3 \quad (5.4)$$

where $\zeta$ is an arbitrary 3-pseudovector, one obtains

$$\frac{\delta A_{\text{del}}}{\delta \zeta^\mu} = \frac{\delta A_{\text{del}}}{\delta \zeta^\alpha} \frac{\delta \zeta^\alpha}{\delta \zeta^\mu} = \frac{\delta A_{\text{del}}}{\delta \zeta^\alpha} \frac{\delta \alpha^\mu - \xi^\alpha \xi^\mu}{\sqrt{\zeta^2}} = 0 \quad (5.5)$$

It means that there are only two independent equations among three dynamic equations (5.5). They are orthogonal to 3-pseudovector $\xi$ and can be obtained from equation $\delta A_{\text{del}}/\delta \xi^\alpha = 0$ by means of vector product with $\xi$.

$$-\hbar \frac{(\dot{\xi} \times z) \times \xi}{2(1 + z\xi)} + \hbar \left( -\frac{d}{d\tau_0} \frac{(\xi \times z)}{2(1 + z\xi)} - \frac{(\dot{\xi} \times \xi)z}{2(1 + z\xi)^2} \right) \times \xi + \hbar \frac{(\dot{\xi} \times \dot{x}) \times \xi}{2} Q = 0 \quad (5.6)$$

After transformations this equation reduces to the equation (see Appendix B)

$$\dot{\xi} = - (\dot{x} \times \dot{x}) \times \xi Q, \quad (5.7)$$

which does not contain the vector $z$. It means that $z$ determines a fictitious direction in the space-time. Note that $z$ in the action (3.6) for the system $S_D$ is fictitious also, because the term containing $z$ is the same in both actions (3.6) for $S_D$ and (4.1) for $S_{D_{qu}}$.

Using invariance of the action (4.17) with respect to transformation of the parameter $\tau_0$, we choose $\tau_0$ in such a way, that

$$\sqrt{x^s x_s} = \sqrt{\dot{x}_0^2 - \dot{x}^2} = 1, \quad \dot{x}_0 = \sqrt{1 + \dot{x}^2} \quad (5.8)$$

Then, using condition (5.8), we obtain from (5.2) for quantities $Q$, $\partial Q/\partial \dot{x}_0$, $\partial Q/\partial \dot{x}$

$$Q = \frac{1}{1 + \dot{x}_0}, \quad \frac{\partial Q}{\partial \dot{x}_0} = -1, \quad \frac{\partial Q}{\partial \dot{x}} = \frac{\dot{x}(2 + \dot{x}_0)}{(1 + \dot{x}_0)^2} \quad (5.9)$$

Integration of equation (5.3) leads to

$$\kappa_0 m \dot{x}_0 + \frac{\hbar}{2} (\dot{x} \times \dot{x}) \xi = - p_0 \quad (5.10)$$

where $p_0$ is an integration constant. This constant $p_0$ describes the time component of the dynamic system $S_{\text{Del}}$ canonical 4-momentum.

Integration of equation (5.1) gives

$$-\kappa_0 m \frac{\dot{x}}{\sqrt{x^s x_s}} + \frac{\hbar Q}{2} (\xi \times \dot{x}) - \frac{\hbar}{2} \frac{\partial Q}{\partial \dot{x}} (\dot{x} \times \dot{x}) \xi + \frac{\hbar}{2} \frac{d}{d\tau_0} (Q(\xi \times \dot{x})) = - p = \text{const} \quad (5.11)$$

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where $p$ is the 3-momentum of the dynamic system $\mathcal{S}_{\text{Dcl}}$ as a whole.

Using the gauge (5.2) and relations (5.9), we rewrite the equation (5.11) in the form

$$-m\ddot{x} + \frac{\hbar}{2} \frac{\langle \vec{\xi} \times \dot{x} \rangle}{1 + \dot{x}_0} - \frac{\hbar}{2} \frac{\dot{x}(2 + \dot{x}_0)}{(1 + \dot{x}_0)^2} (\ddot{x} \times \dot{x}) \xi + \frac{\hbar}{2} \frac{d}{d\tau_0} \left( \frac{\langle \vec{\xi} \times \dot{x} \rangle}{1 + \dot{x}_0} \right) = -p \quad (5.12)$$

If we set $\hbar = 0$ in (5.12), we obtain conventional connection $p = m\dot{x}$ between the velocity $\dot{x} = dx/d\tau_0$ and the momentum of a free particle. But the quantum constant $\hbar$ is a coefficient before the highest time derivative, and setting $\hbar = 0$, we suppress some degrees of freedom.

If these additional degrees of freedom are not excited (or suppressed), the classical Dirac particle has six degrees of freedom. We shall see that characteristic energy associated with additional degrees of freedom is of the order of the particle rest energy $m$. At low energetic processes (calculation of atomic spectra, quantum electrodynamics) one may neglect these degrees of freedom, remaining only numerical characteristics (spin, magnetic momentum) of these degrees of freedom. However, in the case of high energies (ultrarelativistic collisions, structure of elementary particles), one cannot neglect these degrees of freedom. Of course, using the Dirac equation, we take into account these additional degrees of freedom automatically. But it is important also to take into account these additional degrees of freedom in our interpretation of the high energetic processes.

Transformation and solution of equation (5.11) is rather bulky. Many efforts is used to prove that the 3-vectors $\vec{\xi}$, $\dot{x}$, and $\ddot{x}$ are mutually orthogonal and their modules are constant [11] in the coordinate system, where $p = 0$. We shall not spend time for this proof. Instead, we choose the coordinate system in such a way that $p = 0$

$$\vec{\xi} = \{0, 0, \varepsilon_0\}, \quad \varepsilon_0 = \pm 1 \quad (5.13)$$

and impose constraints

$$\dot{x}^2 = \text{const}, \quad (\ddot{x} \xi) = 0, \quad (\dddot{x} \xi) = 0, \quad (\dot{x} \times \ddot{x}) \xi = \text{const} \quad (5.14)$$

We use constraints (5.14) in solution of the system of dynamic equations (5.7), (5.10), (5.12) and show that the constraints (5.14) are compatible with dynamic equations (5.7), (5.10), (5.12).

Taking into account (5.14) and (5.8), we introduce new variables

$$y = \frac{\dot{x}}{\sqrt{1 + \dot{x}_0}} = \frac{\dot{x}}{\sqrt{1 + \sqrt{1 + \dot{x}^2}}}, \quad \dot{x} = y \sqrt{y^2 + 2} \quad (5.15)$$

$$\dot{x}_0 = \sqrt{1 + y^2(y^2 + 2)} = y^2 + 1 \quad (5.16)$$

Introducing designation

$$y^2 = \gamma - 1 = \text{const}, \quad (5.17)$$

we obtain

$$\dot{x}_0 = \sqrt{1 + y^2(y^2 + 2)} = y^2 + 1 = \gamma = \text{const} \quad (5.18)$$
Then at $p = 0$ the equation (5.12) takes the form

$$-\kappa_0 my (\gamma + 1) + \frac{h}{2} (\xi \times \dot{y}) - \frac{h}{2} (\gamma + 2) ((y \times \dot{y}) \xi) y + \frac{h}{2} \frac{d}{d\tau_0} ((\xi \times y)) = 0 \quad (5.19)$$

The equation (5.7) takes the form

$$\dot{\xi} = - (y \times \dot{y}) \times \xi = 0 \quad (5.20)$$

because of constraints (5.14). In terms of variables $y$ conditions (5.14) have the form

$$y^2 = \gamma - 1, \quad (\xi y) = 0, \quad (\xi \dot{y}) = 0, \quad (y \dot{y}) = 0 \quad (5.21)$$

where $\gamma$ is a constant of integration. In accordance with (5.18) and (5.21) we obtain

$$(y \times \dot{y}) \xi = \varepsilon_0 \omega (\gamma - 1) \quad (5.22)$$

where $\omega$ is an indefinite constant (some angular velocity).

Substituting (5.22) in (5.19), we obtain after simplification

$$(\xi \times \dot{y}) - \left( \frac{1}{2} (\gamma + 2) (\gamma - 1) \varepsilon_0 \omega + \frac{\kappa_0 m}{h} (\gamma + 1) \right) y = 0 \quad (5.23)$$

As far as $y^2 = \gamma - 1$, the equation (5.22) describes rotation of the vector $y$ with the angular frequency $\omega$. Equation (5.23) describes rotation of the vector $y$ around the vector $\xi$ with the angular frequency $\frac{1}{2} (\gamma + 2) (\gamma - 1) \varepsilon_0 \omega + \frac{\kappa_0 m}{h} (\gamma + 1)$. Equations (5.22) and (5.23) are compatible, if these frequencies coincide. According to (5.21) vectors $y$ and $\dot{y}$ are orthogonal to $\xi$. Then in accordance with (5.13) the vectors $y$ and $\dot{y}$ can be represented in the form

$$y = \left\{ \sqrt{\gamma - 1} \cos \Phi, \sqrt{\gamma - 1} \sin \Phi, 0 \right\} \quad (5.24)$$

$$\dot{y} = \left\{ -\sqrt{\gamma - 1} \omega \sin \Phi, \sqrt{\gamma - 1} \omega \cos \Phi, 0 \right\}, \quad \omega = \frac{d\Phi}{d\tau_0} \quad (5.25)$$

By means of (5.24), and (5.25) the equations (5.23) take the form

$$-\varepsilon_0 \omega y_1 - \left( \frac{1}{2} (\gamma + 2) (\gamma - 1) \varepsilon_0 \omega + \frac{\kappa_0 m}{h} (\gamma + 1) \right) y_1 = 0 \quad (5.26)$$

$$-\varepsilon_0 \omega y_2 - \left( \frac{1}{2} (\gamma + 2) (\gamma - 1) \varepsilon_0 \omega + \frac{\kappa_0 m}{h} (\gamma + 1) \right) y_2 = 0 \quad (5.27)$$

Equations (5.26), (5.27) are satisfied, provided

$$\varepsilon_0 \omega + \left( \frac{1}{2} (\gamma + 2) (\gamma - 1) \varepsilon_0 \omega + \frac{\kappa_0 m}{h} (\gamma + 1) \right) = 0 \quad (5.28)$$

Solution of (5.28) has the form

$$\omega = -\frac{2 \varepsilon_0 \kappa_0 m}{h \gamma} \quad (5.29)$$
According to (5.15) and (5.16) the dynamic equation (5.10) takes the form

\[-p_0 = \kappa_0 m \gamma + \frac{\hbar}{2} (\mathbf{y} \times \dot{\mathbf{y}}) \xi (\gamma + 1)\]  
(5.30)

Using relations (5.22) and (5.29) we obtain from (5.30)

\[-p_0 = \kappa_0 m \left( \frac{\gamma - \gamma^2 - 1}{\gamma} \right) = \frac{\kappa_0 m}{\gamma}, \quad \kappa_0 = \pm 1\]  
(5.31)

Then we obtain for the rest mass \(M\) of the dynamic system \(S_{Dcl}\).

\[M_{Dcl} = \sqrt{p_0^2 - p^2} = |p_0| = \frac{m}{\gamma}\]  
(5.32)

Note, that writing the relation (5.32), we do not act quite consequently. Writing the relation (5.32), we suppose that the dynamic equations (5.10) and (5.11) are relativistically invariant, and solution of equations (5.10), (5.11) for arbitrary \(p\) can be obtained from the solution for \(p = 0\) by means of a corresponding Lorentz transformation. Unfortunately, dynamic equations (5.10), (5.11) are not relativistically invariant, and for arbitrary \(p\) the solution is not a helix, in general, although it is a helix for \(p = 0\). World line is a helix approximately in the nonrelativistic case, when \(|p| \ll m\).

Let us transit from independent variable \(\tau_0\) to the independent variable \(x^0 = t\). We have

\[\Omega t = -\varepsilon_0 \kappa_0 \omega \tau_0, \quad -\varepsilon_0 \kappa_0 \omega = \Omega \dot{x}_0 = \Omega \gamma = \frac{2m}{\hbar \gamma}, \quad \Omega = \frac{2m}{\hbar \gamma^2}\]  
(5.33)

Returning from variables \(\mathbf{y}\) to variables \(\dot{\mathbf{x}}\), we obtain instead of (5.24) and (5.25)

\[
\frac{d\mathbf{x}}{dt} = \begin{cases} 
\sqrt{\gamma^2 - 1} \cos (\Omega t), & -\sqrt{\gamma^2 - 1} \sin (\Omega t), 0 \\
\frac{\hbar \gamma \sqrt{\gamma^2 - 1}}{2m} \sin \left( \frac{2m}{\hbar \gamma^2} t \right), & \frac{\hbar \gamma \sqrt{\gamma^2 - 1}}{2m} \cos \left( \frac{2m}{\hbar \gamma^2} t \right), 0
\end{cases}, \quad \Omega = \frac{2m}{\hbar \gamma^2}\]  
(5.34)

\[
\mathbf{x} = \begin{cases} 
\frac{\hbar \gamma \sqrt{\gamma^2 - 1}}{2m} \sin \left( \frac{2m}{\hbar \gamma^2} t \right), & \frac{\hbar \gamma \sqrt{\gamma^2 - 1}}{2m} \cos \left( \frac{2m}{\hbar \gamma^2} t \right), 0
\end{cases}
\]  
(5.35)

where \(\gamma \geq 1\) is an arbitrary constant.

Thus, in the coordinate system, where the canonical momentum four-vector has the form

\[P_k = \{p_0, \mathbf{p}\} = \left\{ -\frac{\kappa_0 m}{\gamma}, 0, 0, 0 \right\}\]  
(5.36)

the world line of the classical Dirac particle is a helix, which is described by the relation

\[
\{t, \mathbf{x}\} = \{t, a_{Dcl} \sin (\Omega t), a_{Dcl} \cos (\omega_{Dcl} t), 0\}\]  
(5.37)

\[a_{Dcl} = \frac{\hbar \gamma \sqrt{\gamma^2 - 1}}{2m}, \quad \omega_{Dcl} = \frac{2m}{\hbar \gamma^2}\]  
(5.38)
It follows from (5.34) that the classical Dirac particle velocity \( v = dx/dt \) is expressed as follows

\[
v^2 = 1 - \frac{1}{\gamma^2}, \quad \gamma = \frac{1}{\sqrt{1 - v^2}} \tag{5.39}
\]

In other words, the quantity \( \gamma \) is the Lorentz factor of the classical Dirac particle.

We see that the characteristic frequency, connected with the internal degrees of freedom is \( 2m/\gamma^2 \), and the characteristic energy is of the order \( | -m\gamma + m\gamma^{-1} | \).

Parameters \( \gamma \) and \( \omega_{\text{Dcl}} \) as functions of the radius \( a_{\text{Dcl}} \) and the Dirac mass \( m \) have the form

\[
\gamma = \sqrt{\frac{1}{2} \left( 1 + \sqrt{1 + \zeta^2} \right)}, \quad \omega_{\text{Dcl}} = \frac{4m}{\hbar \left( 1 + \sqrt{1 + \zeta^2} \right)}, \quad \zeta = \frac{4ma_{\text{Dcl}}}{\hbar} \tag{5.40}
\]

6 Dynamical interpretation of the classical Dirac particle

If coordinates \( x^k = \{ t, x \} \) are interpreted as coordinates of classical Dirac particle, it seems rather strange, that the world line of a free particle is a helix, but not the straight line. Why does the free classical particle rotate in the coordinate system, where total momentum \( p = 0 \)? Note, that the coordinates of the free quantum Dirac particle \( S_D \) contain oscillating component, whereas momentum of \( S_D \) does not contain oscillating component [1] (sec. 69). This oscillating motion with the frequency \( \omega \geq 2m/\hbar \) is known as zitterbewegung. Usually the zitterbewegung is considered to be a specific quantum phenomenon, but here we obtain classical analog of the zitterbewegung and this classical description contains the quantum constant \( \hbar \).

Classical dynamic system \( S_{\text{Dcl}} \) contains four rotational degrees of freedom in addition to six conventional translation degrees of freedom. It means that the classical dynamic system \( S_{\text{Dcl}} \) is not a pointlike particle, because it has internal degrees of freedom. How does one interpret these additional degrees of freedom? It seems that the dynamic system \( S_{\text{Dcl}} \) consists of several constituents, rotating around its center of inertia. This idea is found in accordance with the contemporary ideas, that such Dirac particles as the proton and the neutron consist of quarks.

We investigate, to what extent the classical dynamic system \( S_{\text{Dcl}} \) may be interpreted as a rotator. Rotator is a dynamic system \( S_r \), consisting of two coupled particles of mass \( m_0 \), which can rotate around their center of mass. The distance between particles is to be constant, i.e. the particles are not to vibrate. Rigid nonrelativistic rotator \( S_{\text{nr}} \) is described by the action

\[
S_{\text{nr}} : \quad A [x_1, x_2, \mu] = \int \left( \sum_{k=1}^{k=2} \frac{m_0 \dot{x}_k^2}{2} + \mu \left( (x_1 - x_2)^2 - 4a^2 \right) \right) dt \tag{6.1}
\]

where \( 2a \) is the distance (length of string) between the particles. The parameter \( a \) is determined by the length of rigid coupling between two particles. It does not
depend on initial conditions. The angular frequency $\omega$ of rotation is an arbitrary constant of integration, which does not connect with the length $2a$ of the string.

If the coupling is elastic, the action for $S_{nr}$ should be written in the form

$$S_{nr} : \mathcal{A}[x_1, x_2, \mu] = \int \left( \sum_{k=1}^{k=2} \frac{m_0 \dot{x}_k^2}{2} - U(|x_1 - x_2|) \right) dt \quad (6.2)$$

where $U$ is the potential energy, describing interaction energy between two particles. The dynamic equations for relative motion of the particles have the form

$$-m_0 (\ddot{x}_1 - \ddot{x}_2) - 2 \frac{(x_1 - x_2)}{|x_1 - x_2|} U'(|x_1 - x_2|) = 0, \quad U'(|x_1 - x_2|) = \frac{\partial U(|x_1 - x_2|)}{\partial |x_1 - x_2|} \quad (6.3)$$

Equations (6.3) describe both rotation and radial vibrations of the particle. Let us imagine that for some reason the vibrations are dumped, and only rotational motion retains. It means that

$$|x_1 - x_2| = 2a = \text{const}, \quad U'(|x_1 - x_2|) = \text{const} \quad (6.4)$$

and dynamic equations (6.3) turns into linear equations for variables $x_1 - x_2$. Solution of these equations describes a rotation in any 2-plane. For instance, in the plane $XY$ we obtain

$$x_1 - x_2 = \{ 2a \cos(\omega t), 2a \sin(\omega t), 0 \} \quad (6.5)$$

where the angular frequency

$$\omega = \omega (a) = \sqrt{\frac{U''(2a)}{m_0a}} \quad (6.6)$$

is not an arbitrary constant. It depends on the form of the potential energy of the elastic string and of radius $a$.

Unfortunately, in the relativistic case one cannot introduce potential energy of interaction between two particles. But we may introduce the rigidity function $f_r(a)$, defining it by the relation

$$f_r (a) = \frac{M - 2m_0}{2m_0} \quad (6.7)$$

where $M$ is the total mass of the rotator at the state of rotation, and $2m_0$ is its mass at rest. In the nonrelativistic case

$$M = 2m_0 + 2m_0 \frac{a^2}{2c^2} = 2m_0 \left( 1 + \frac{a^2\omega^2}{2c^2} \right) = 2m_0 \left( 1 + \frac{a^2}{m_0c^2} U''(2a) \right)$$

$$f_r (a) = \frac{a}{m_0c^2} U''(2a) \quad (6.8)$$

The relation (6.8) connects the rigidity function with the potential energy of the string in the nonrelativistic case. This relation may be extrapolated to the relativistic case, when the potential energy of the string cannot be introduced. The rigidity
function \( f_r(a) \) of the established rotation of the relativistic rotator may be introduced by means of the relation (6.7). Then by means of the relation (6.8) one can introduce formally the potential energy of interaction between the constituents of the relativistic rotator. We may also describe interaction between the constituents directly in terms of the rigidity function.

Solution for the dynamic system (6.1) can be reduced to "established" solution for the dynamic system (6.2), if we introduce a proper coupling between the constants of integration \( a \) and \( \omega \), or introduce a proper rigidity function.

Solution (5.35) for the classical Dirac particle \( S_{Dcl} \) is a helix. It reminds the solution for a rotator, but the identification is rather complicated, because instead of mass \( m_0 \) of the rotator constituents, we have the Dirac mass \( m \), whose meaning is unclear. To produce such an identification, we need to solve dynamic equations for the relativistic rotator of the type (6.1). Comparing the obtained solution with (5.35), we determine the relation between the masses \( m \) and \( m_0 \). Thereafter we can determine the rigidity function and evaluate the character of interaction between the classical Dirac particle constituents.

In the relativity theory a rigid coupling is impossible. Also there are no reasons for introduction of a potential energy of interaction between the particles, because in the relativity theory a long-range action is absent. We are forced to choose another way.

Let us consider established relativistic motion of two particles of mass \( m_0 \), coupled between themselves by a massless elastic string. The condition of established motion means that the particles move in such a way that the length of the string does not change, and one may neglect degrees of freedom, connected with the string. Mathematically it means, that there is such a coordinate system \( K \) (maybe, rotating), where particles are at rest.

Let \( \mathcal{L}_1 \) and \( \mathcal{L}_2 \), be world lines of particles

\[
\mathcal{L}_k : \quad x^i_{(k)}(\tau_k) = x^i_{(k)}(\tau_k), \quad i = 0, 1, 2, 3; \quad k = 1, 2
\]  

(6.9)

where \( \tau_k, \quad k = 1, 2 \) are parameters along these world lines. From geometrical viewpoint the steady (established) motion of particles means that any spacelike 3-plane \( S \), crossing \( \mathcal{L}_1 \) orthogonally, crosses \( \mathcal{L}_2 \) also orthogonally. This circumstance permits one to synchronize events on \( \mathcal{L}_1 \) and on \( \mathcal{L}_2 \).

Let parameters \( \tau_1 \) and \( \tau_2 \) be chosen in such a way that they have the same value \( \tau \) at points \( \mathcal{L}_1 \cap S \) and \( \mathcal{L}_2 \cap S \). The steady state conditions are written in the form

\[
\frac{dx^i_{(1)}(\tau)}{d\tau} (x_{(1)i}(\tau) - x_{(2)i}(\tau)) = \frac{dx^i_{(2)}(\tau)}{d\tau} (x_{(1)i}(\tau) - x_{(2)i}(\tau)) = 0
\]  

(6.10)

Let us describe motion of the two particles (constituents) by the action

\[
\mathcal{S}_{\text{rr}} : \quad \mathcal{A}_{\text{rr}} [x_{(1)}, x_{(2)}] = - \int_{\tau}^{\tau+2} \sum_{k=1}^{2} m_0 \sqrt{\dot{x}^2_{(k)}(\tau)} d\tau
\]  

(6.11)

where variables \( x_{(k)} = \{x^0_{(k)}, x^1_{(k)}, x^2_{(k)}, x^3_{(k)}\}, \quad k = 1, 2 \) are considered to be functions of the same parameter \( \tau \), \( m_0 \) is the mass of the rotator constituent, and the period
denotes differentiation with respect to $\tau$. World lines of constituents are determined as extremals of the functional (6.11) with side constraints (6.10).

We omit details of solution of the dynamic equations for the rotator (6.11), (6.10) (One can find them in sec. 8 of [11]). In the coordinate system, where the rotator is at rest, the solution has the form

$$x^i_{(1)} = \{t, a \cos (\omega_0 t + \phi), a \sin (\omega_0 t + \phi), 0\} \quad (6.12)$$

$$x^i_{(2)} = \{t, -a \cos (\omega_0 t + \phi), -a \sin (\omega_0 t + \phi), 0\} \quad (6.13)$$

Here the quantities $a$, $\phi$, $\omega_0$ are constants of integration. The angular frequency is connected with the total mass $M$ of the rotator, and its rest mass $2m_0$ by means of relations

$$M = \frac{2m_0}{\sqrt{1 - a^2 \omega_0^2}}, \quad \omega_0 = \frac{\sqrt{M^2 - 4m_0^2}}{aM} \quad (6.14)$$

where $v = a\omega_0$ is the velocity of the rotator constituents.

One can see that both world lines $L_1$ and $L_2$ are helices in the space-time. They describe rotation of two particles around their common center of mass along a circle of radius $a$. Such a dynamic system $S_{rr}$ may be qualified as a relativistic rotator. Here $\omega_0$ is an arbitrary quantity, and rotator (6.11), (6.10) is the dynamic system of type of (6.1), but not of (6.2). The state of the rotator may be described by the relative mass increase

$$\alpha = \frac{(M - 2m_0)}{2m_0} = \frac{1}{\sqrt{1 - v^2}} - 1 = \frac{v^2}{\sqrt{1 - v^2} (\sqrt{1 - v^2} + 1)} \quad (6.15)$$

which turns to the rigidity function (6.7), provided we determine $\omega_0$ as a function of the radius $a$. Here $v = a\omega_0$ is the velocity of a constituent in the coordinate system, where center of mass is at rest. The quantity $\alpha$ is a part of total mass, conditioned by rotation.

The radius $a$ and angular frequency $\omega_0$ of rotation are independent constants of integration in the solution (6.12), (6.13). For any real rotator the frequency $\omega_0$ and the radius $a$ cannot be independent. We suppose that the dynamic system $S_{Dcl}$, where angular velocity is coupled with the radius of helix, is a special case of relativistic rotator $S_{rr}$. To determine the rigidity function and the relation between $m$ and $m_0$, we compare relations (6.14), (6.12) with relations (5.38), (5.37) and identify the quantities $\omega_0$, $M$, $a$ of dynamic system $S_{rr}$ respectively with $\omega_{Dcl}$, $M_{Dcl}$, $a_{Dcl}$ of dynamic system $S_{Dcl}$. Using relation (5.40) for $\gamma$ we obtain

$$\omega_0 = \frac{\sqrt{M^2 - 4m_0^2}}{aM} = \frac{4m}{\hbar} \frac{1}{\sqrt{1 + \zeta^2 + 1}} = \omega_{Dcl}, \quad M = \frac{m\sqrt{2}}{\sqrt{(\sqrt{1 + \zeta^2 + 1})}} \quad (6.16)$$

$$a = a_{Dcl}, \quad \zeta = 4a_{Dcl} \frac{m}{\hbar}, \quad c = 1 \quad (6.17)$$
Resolving relations (6.16) with respect to variables $M$ and $m_0$, one obtains all quantities of the relativistic rotator $S_{rr}$ in terms of parameters $\zeta = 4ma_{\text{Dcl}}/\hbar$ and $m$ of the classical Dirac particle $S_{\text{Dcl}}$

$$M = M_{\text{Dcl}} = \frac{m\sqrt{2}}{\sqrt{\left(1 + \zeta^2 + 1\right)}}, \quad m_0 = \frac{m}{\sqrt{\left(1 + \zeta^2 + 1\right)}} \quad (6.18)$$

It follows from the second relation (6.18), that the relation between the Dirac mass $m$ and the constituent mass $m_0$ depends on the state of rotation. In other words, different states of the classical Dirac particle correspond, in general, to different mass $m_0$ of the rotator constituent. In this connection the question arises. What mass is principal $m$ or $m_0$?

One can express parameters $m$, $M_{\text{Dcl}}$, $\omega_{\text{Dcl}}$, $v_{\text{Dcl}}$ of $S_{\text{Dcl}}$ in terms of parameters $v = \zeta_0 = 4am_0/\hbar$, $m_0$ of the relativistic rotator $S_{rr}$. From the second relation (6.18) we obtain relation between $\zeta$ and $\zeta_0$,

$$\zeta_0 = \frac{\zeta}{\sqrt{1 + \zeta^2 + 1}}, \quad \zeta = \frac{2\zeta_0}{1 - \zeta_0^2} \quad (6.19)$$

After some calculations one obtains

$$m = 2m_0 \frac{1}{1 - v^2}, \quad M_{\text{Dcl}} = \frac{2m_0}{\sqrt{1 - v^2}}, \quad \omega_{\text{Dcl}} = \frac{4m_0}{\hbar}, \quad (6.20)$$

where

$$v = \zeta_0 = 4a\frac{m_0}{\hbar} \quad (6.21)$$

It follows from the relations (6.20), (6.21) and (6.7), that the rigidity function $f_{rDcl}(a)$ for the classical Dirac particle has the form

$$f_{rDcl}(a) = \frac{M_{\text{Dcl}} - 2m_0}{2m_0} = \frac{\hbar^2}{\sqrt{\hbar^2 - (4am_0)^2}} - 1 \quad (6.22)$$

As far as two masses $m$ and $M_{\text{Dcl}}$ are distinguished, the question appears, which of the two masses is observable. To solve this question, one needs to consider the Dirac dynamic system in the given electromagnetic field.

We shall not discuss this question here. Instead, we discuss another important problem. What force field does connect constituents of the Dirac particle? Is it possible to separate these constituents as single particles? The direct interaction between the remote constituents seems to be incompatible with the relativity principles. It means that we are to introduce gluons or some other bearers of interaction between the constituents. But even in this case it is rather difficult to understand, why the distance between the constituents is fixed at any state of the dynamic system $S_{\text{Dcl}}$.

We shall see that the only solution of this problem is the geometrical interpretation instead of the dynamical one.
7 Geometrical model of the classical Dirac particle

We explain what is the geometrical interpretation in the example of a classical pointlike particle. Its motion is described by its world line $L$

$$L: \quad x^i = x^i(\tau), \quad i = 0, 1, 2, 3$$  \hfill (7.1)

where $\tau$ is a real parameter along the world line. According to the principles of relativity the world line (WL) is a physical object, whereas the particle (pointlike object in 3-space) is an attribute of WL. We shall use abbreviation WL (instead of world line), when we want to stress that the world line is a physical object, but not an attribute of a particle (its history). WL is not a completely geometrical object, because it contains a non-geometric parameter: mass $m$. To make WL to be a geometrical object we are to geometrize the mass. We make this as follows.

Instead of WL (7.1) we consider the broken line $T_{br}$

$$T_{br} = \bigcup_i T_{[P_iP_{i+1}]} \quad \hfill (7.2)$$

consisting of straight line segments $T_{[P_iP_{i+1}]}$ of the same length $\mu$

$$T_{[P_iP_{i+1}]} = \left\{ R \big| \sqrt{2\sigma(P_i, P_{i+1})} = \sqrt{2\sigma(P_i, R)} + \sqrt{2\sigma(R, P_{i+1})} \right\} \quad \hfill (7.3)$$

where $\sigma$ is the world function of the space-time. Here the space-time points $P_i$, $i = 0, \pm 1, \pm 2...$ are the break points of $T_{br}$. In the case, when the space-time is the Minkowski space, the world function, written in the inertial coordinate system, has the form

$$\sigma = \sigma_M(x, x') = \frac{1}{2}g_{ik}(x^i - x'^i)(x^k - x'^k) \quad \hfill (7.4)$$

In our geometrical description we use the fact that any physical geometry can be described completely in terms of the world function $\sigma$ [20]. This circumstance allows one to use coordinateless description, when all geometric objects and all relations between them are expressed in terms of the world function $\sigma$. This method of description is referred to as the $\sigma$-immanent description. It is convenient in the sense, that a transition from one geometry to another one is carried out by means of a change of the world function $\sigma$.

The vector $P_iP_{i+1} \equiv \overrightarrow{P_iP_{i+1}} = \{P_i, P_{i+1}\}$ is the ordered set of two points. It describes the particle momentum on the segment $T_{[P_iP_{i+1}]}$. The module

$$|P_iP_{i+1}| = \sqrt{2\sigma(P_i, P_{i+1})} = \mu, \quad i = 0, \pm 1, \pm 2... \quad \hfill (7.5)$$

of the vector $P_iP_{i+1}$ is the geometrical mass $\mu$, which is connected with the conventional (physical) mass $m$ by means of the relation

$$m = b\mu, \quad b \approx 10^{-17}\text{g/cm} \quad \hfill (7.6)$$
where $b$ is an universal constant. Analogously, the physical momentum $p_i$ is connected with the geometrical momentum $P_i P_{i+1}$ by means of the relation

$$p_k = b (P_i P_{i+1})_k = b (P_i P_{i+1}, Q_{-1} Q_k), \quad k = 0, 1, 2, 3$$

(7.7)

Here $(P_i P_{i+1})_k$ are covariant coordinates of the vector $P_i P_{i+1}$ in some coordinate system with basic vectors $e_k = Q_{-1} Q_k$, $k = 0, 1, 2, 3$. The coordinate system is determined by five points $\{Q_{-1}, Q_0, Q_1, Q_2, Q_3, \}$ with origin at the point $Q_{-1}$. The scalar product $(P_i P_{i+1}, Q_{-1} Q_k)$ is defined by the relation

$$(P_i P_{i+1}, Q_{-1} Q_k) = \sigma (P_i, Q_k) + \sigma (P_{i+1}, Q_{-1}) - \sigma (P_i, Q_{-1}) - \sigma (P_{i+1}, Q_k)$$

(7.8)

This definition coincide with conventional definition of the scalar product in the proper Euclidean space, or in the pseudo-Euclidean one. It is described in terms of the world function.

If the broken line (7.2) describes the free particle motion, the momenta of adjacent links are parallel: $P_{i-1} P_i \uparrow \uparrow P_i P_{i+1}$, $i = 0, \pm 1, \pm 2...$. Definition of the parallelism of two vectors $P_{i-1} P_i$ and $P_i P_{i+1}$ have the form

$$P_{i-1} P_i \uparrow \uparrow P_i P_{i+1} : \quad (P_{i-1} P_i, P_i P_{i+1}) - |P_{i-1} P_i| |P_i P_{i+1}| = 0, \quad i = 0, \pm 1, \pm 2...$$

(7.9)

Subtracting relations (7.9) for $i$ and $i + 1$ and using the relations (7.5), (7.8), we obtain from (7.9)

$$\frac{(P_i P_{i+1}, P_{i+1} P_{i+2}) - (P_{i-1} P_i, P_i P_{i+1})}{\mu^2} = 0$$

(7.10)

The condition (7.10) is slighter, than the condition (7.9), but it is interesting in the sense, that lhs of (7.10) describes "discrete derivative" of the cosine of the angle between the adjacent links. According to (7.10) this cosine is constant, besides, according to (7.9) this cosine is equal to 1.

Another form of the conditions (7.9) can be obtained, if we use the definition of the scalar product (7.8) and condition (7.5). We obtain instead of (7.9)

$$|P_{i-1} P_{i+1}| = 2\mu, \quad i = 0, \pm 1, \pm 2...$$

(7.11)

We can conclude from (7.11) as well as from (7.9), that for the free particle the broken line (7.2) is a timelike straight line.

The pure geometrical description is useful in the sense that quantum effects can be taken into account by means of a simple change of the space-time geometry. We declare that the real space-time geometry is the Minkowski geometry only approximately. The real space-time geometry is determined by the world function $\sigma_d$

$$\sigma_d = \sigma_M + D (\sigma_M), \quad D (\sigma_M) = \left\{ \begin{array}{ll} \sigma_M + d & \text{if } \sigma_0 < \sigma_M \\ \sigma_M & \text{if } \sigma_M < 0 \end{array} \right.$$  

(7.12)

where $d \geq 0$ and $\sigma_0 > 0$ are some constants. The quantity $\sigma_M$ is the world function in the Minkowski space-time geometry $G_M$, defined by the relation (7.4). Values of
the function $D(\sigma M)$ in interval $(0, \sigma_0)$ are of no importance, provided geometrical mass $\mu$ of the particle satisfies the condition $\mu \geq \sqrt{2\sigma_0}$. The world function $\sigma d$ describes geometry $G_d$ of distorted space-time $V_d$, which is non-Riemannian, if the distortion $d > 0$. The geometry $G_d$ is uniform and isotropic, as well as the Minkowski geometry.

The geometry of the distorted space-time $V_d$ is nondegenerate in the sense, that any link $T_{[P_i P_{i+1}]}$, determined by the relation (7.3) is a 3-dimensional surface (hollow tube) with the characteristic width $\sqrt{d}$. (See figure 1). If $d \to 0$ the tube degenerates to a segment of the one-dimensional straight line. Such a situation is connected with the fact that the definition (7.9) of parallelism is one equation. This equation determines the set of vectors $P_i P_{i+1}$, which are parallel to the fixed vector $P_{i-1} P_i$, as a set of points $P_{i+1}$, which satisfy the equation (7.9). In general, this set is a three-dimensional surface, which degenerates into a one-dimensional line in the case of the Minkowski geometry and timelike vector $P_{i-1} P_i$. As a result position of the point $P_{i+1}$ in $G_d$ appears to be indefinite, even if the position of points $P_{i-1}$ and $P_i$ is fixed. The shape of world tube $T_{br}$ appears to be stochastic. The stochasticity intensity depends on the length $\mu$ of the link. The shorter the length, the larger is stochasticity, because the characteristic wobble angle is of the order $\sqrt{d/\mu}$. If we set

$$d = \frac{\hbar}{2bc}$$

where $\hbar$ is the quantum constant, $c$ is the speed of the light, and $b$ is the universal constant, defined by the relation (7.6), the statistical description of stochastic WLs leads to the quantum description (in terms of the Schrödinger, or Klein-Gordon equation) [18, 21].

Note that the statistical description of stochastic WLs leads to a more general description, than the quantum description. The quantum description is only the simpler part of the statistical description, which can be reduced to linear differential equations for the wave function. The remaining part of the statistical description, which is not reduced to linear differential equations, has not been investigated, in general.

Thus, the world line of a particle without structure is the geometrical object, described as a chain (7.2), consisting of similar 1D links $T_{[P_i P_{i+1}]}$. But, maybe, there exist the chains (broken tubes), consisting of 2D geometrical objects $T_{[P_i P_{i+1} Q_i]}$,

$$T_{br} = \bigcup_{i} T_{[P_i P_{i+1} Q_i]}$$

where $T_{[P_i P_{i+1} Q_i]}$ are triangles with vertices at points $P_i$, $P_{i+1}$, $Q_i$. This broken tube is shown in figure 2. There are two versions of the tube (7.14). The version "dragon" describes the broken tube (7.14) as consisting of triangles. The version "ladder", containing the same characteristic points, describes the broken tube (7.14) as consisting of two world lines $T_1$ and $T_2$

$$T_1 = \bigcup_{i} T_{[P_i P_{i+1}]}, \quad T_2 = \bigcup_{i} T_{[Q_i Q_{i+1}]}$$
Any link $\mathcal{T}_{[P,P_{i+1}]}$ of $\mathcal{T}_1$ is connected with the link $\mathcal{T}_{[Q_i,Q_{i+1}]}$ of $\mathcal{T}_2$ by means of geometrical couplings. The geometrical coupling acts on $\mathcal{T}_1$ in such a way that $\mathcal{T}_1$ becomes to be a helix (more exactly all break points of $\mathcal{T}_1$ lie on the helix (5.35)). In this case we can say, that the broken tube (7.14) describes the Dirac particle. To obtain this result we may suppose that points $P_i, Q_i, P_{i+1}, Q_{i+1}$ lie in one 2-dimensional plane and form a parallelogram. Besides we suppose that all parallelograms $\{P_i, Q_i, P_{i+1}, Q_{i+1}\}$ are similar for $i = 0, \pm 1, \pm 2$...

If we suppose that all parallelograms are similar, and dihedral angles between any pair of adjacent parallelograms are the same, we obtain that all break points of $\mathcal{T}_1$ lie on a helix. This statement is valid also for break points of $\mathcal{T}_2$. Choosing parameters of the broken tube (7.14) in proper way, we can achieve that the break points of $\mathcal{T}_1$ and $\mathcal{T}_2$ lie on the helix (5.35). In this case we may say, that we obtain a geometrical description of the classical Dirac particle. This description is analogous to the simpler case, when the broken tube (7.2) is a geometrical description of the usual particle without an internal structure.

At the geometrical description it is useless to ask, why world lines $\mathcal{T}_1$ and $\mathcal{T}_2$ of constituents interact at a distance and why there are no bearers of the "geometrical interaction". In general, it is useless to explain geometrical facts by means of dynamics, because the geometry is more primary and fundamental, than any dynamics. At the geometrical description any discussion of the dynamical confinement problems becomes to be useless. We cannot say definitely, whether the broken tube (7.14) describes a geometrical coupling of two constituents, describing by (7.15), or it describes a chain of more complicated geometrical objects (triangles). See the left diagram "dragon" in figure 2. Two versions "dragon" and "ladder" distinguish only in their internal geometric couplings, although the characteristic points $P_i, Q_i$ are the same in both diagrams.

The "ladder" is a two-dimensional band in the space-time, which may be regarded as a world tube of a one-dimensional open-ended rotating string. We see that the geometrical model of the Dirac particle opens the door for such notions of the elementary particle theory as string, confinement, quark.

Note that the line segment $\mathcal{T}_{[P,P_{i+1}]}$ is the simplest geometrical object, determined by two points, the triangle $\mathcal{T}_{[P,P_{i+1}]Q_i}$ is the simplest geometrical object, determined by three points, and the tetrahedron $\mathcal{T}_{[P,P_{i+1}Q_iR_i]}$ is the simplest geometrical object, described by four points. If the chain of the line segments $\mathcal{T}_{[P,P_{i+1}]}$ is associated with the spinless particle, the chain of triangles $\mathcal{T}_{[P,P_{i+1}Q_i]}$ is associated with the Dirac particle, we should expect that there exist the chain of tetrahedrons $\mathcal{T}_{[P,P_{i+1}Q_iR_i]}$. Such a chain would be associated with the particles, constructed of three quarks, because such a chain is associated with the composite particle, consisting of three constituents. Compare diagrams "dragon" and "ladder" in figure 3.

Now we formulate mathematically constraints on links of the broken tube (7.14). For simplicity, we consider the case of three-dimensional space-time. In this case mathematical constraints are more simple and demonstrative. Note that the helix
The dihedral angle between any two adjacent triangles $T_i$ axis is directed along the vector $P_i Q_i$.

$$P_i Q_i \uparrow\uparrow P_{i+1} Q_{i+1}, \quad i = 0, \pm 1, \pm 2... \quad (7.16)$$

It means that the vector $P_i Q_i$ is to be timelike

$$|P_i Q_i|^2 = |P_{i+1} Q_{i+1}|^2 = \mu^2 > 0, \quad i = 0, \pm 1, \pm 2... \quad (7.17)$$

because the helix axis in the case (5.35) is described by the timelike momentum vector. It follows from relations (7.16), (7.17) that the points $P_i, Q_i, P_{i+1}, Q_{i+1}$ lie in one 2-dimensional plane and form a parallelogram. Orientation of the parallelogram vector. It follows from relations (7.16), (7.17) that the points $P_i, Q_i, P_{i+1}$ which is defined as the ordered set $\{P_i, Q_i, P_{i+1}\}$ of three points $[20]$.

The adjacent triangles $T_{[P_i Q_i P_{i+1}]}$ is described by the second order multivector $P_i Q_i P_{i+1} = P_i Q_i P_{i+1}$ which is determined by the relation $[20]$

$$(\vec{P}^2, \vec{Q}^2) = \det \left( \begin{array}{cc} P_0 P_1 P_2 \\ Q_0 Q_1 Q_2 \end{array} \right), \quad i, k = 1, 2 \quad (7.18)$$

The module $|\vec{P}^2|$ of the second order multivector $\vec{P}^2$ is defined by the relation

$$|\vec{P}^2|^2 = (\vec{P}^2 \cdot \vec{P}^2) = \det \left( \left| \begin{array}{cc} P_0 P_1 P_2 \\ P_0 P_1 P_2 \end{array} \right| \right), \quad i, k = 1, 2 \quad (7.19)$$

Cosine of the dihedral angle $\theta$ between two second order multivector $\vec{P}^2$ and $\vec{Q}^2$ is determined by the relation

$$\cos \theta = \frac{(\vec{P}^2 \cdot \vec{Q}^2)}{|\vec{P}^2| |\vec{Q}^2|} \quad (7.20)$$

The dihedral angle between any two adjacent triangles $T_{[P_i Q_i P_{i+1}]}$ and $T_{[P_{i+1} Q_{i+1} P_{i+2}]}$ is the same for all pairs of triangles. As far as all triangles are equal, we have in addition to (7.17) the following relations

$$|P_i P_{i+1}| = |P_{i+1} P_{i+2}|, \quad (P_i P_{i+1} P_{i+2}) = (P_{i+1} P_{i+2} P_{i+3}), \quad i = 0, \pm 1, \pm 2... \quad (7.21)$$

The adjacent triangles $T_{[P_i Q_i P_{i+1}]}$ and $T_{[P_{i+1} Q_{i+1} P_{i+2}]}$ are equal and, hence,

$$|P_i Q_i P_{i+1}| = |P_{i+1} Q_{i+1} P_{i+2}|, \quad i = 0, \pm 1, \pm 2... \quad (7.22)$$

Then (7.22) becomes to be a formal mathematical corollary of constraints (7.17) and (7.21). Taking into account (7.21) and (7.20), the condition of the dihedral angle constancy is written in the form

$$(P_i Q_i P_{i+1} P_{i+2}) = (P_{i+1} Q_{i+1} P_{i+2} P_{i+3}), \quad i = 0, \pm 1, \pm 2... \quad (7.23)$$
Finally, we must add expression for the link $\mathcal{T}_{(P_iq_iP_{i+1})}$, which describes the set of points inside the triangle with vortices at the points $P_i, Q_i, P_{i+1}$. In the case of arbitrary geometry this expression is rather bulky [22]. Let $\mathcal{P}^2 = \{P_0, P_1, P_2\}$. Then the set of points $R$ inside the triangle with vertices at the points $P_0, P_1, P_2$ is described by the relation

$$\mathcal{T}[\mathcal{P}^2] = \mathcal{T}_{[\mathcal{P}^2]} = \left\{ R | F_3(R, \mathcal{P}^2) = 0 \bigwedge_{i=0}^{t=2} S_i \geq 0 \right\} \quad (7.24)$$

where

$$F_3(R, \mathcal{P}^2) = \det \| (RP, RP_k) \|, \quad i, k = 0, 1, 2 \quad (7.25)$$

$$S_0 = \left| \mathcal{P}^2 \right|^{-2} \left( \mathcal{P}^2 . RP_1 \mathcal{P}^2 \right), \quad S_1 = \left| \mathcal{P}^2 \right|^{-2} \left( \mathcal{P}^2 . P_0 RP_2 \right), \quad S_2 = \left| \mathcal{P}^2 \right|^{-2} \left( \mathcal{P}^2 . P_0 P_1 R \right) \quad (7.26)$$

The equation (7.25) describes the two-dimensional plane, determined by three points $\mathcal{P}^2 = \{P_0, P_1, P_2\}$. According the first equation (7.26) the condition $S_0 \geq 0$ means that cosine of the dihedral angle between the triangles $\{P_0, P_1, P_2\}$ and $\{R, P_1, P_2\}$ is nonnegative. Hence, the points $R$ and $P_0$ are laid in the plane $\mathcal{T}_{P_0P_1P_2}$ to one side of the $\mathcal{T}_{P_1P_2}$. In a like manner the condition $S_1 \geq 0$ means that the points $R$ and $P_1$ are laid in the plane $\mathcal{T}_{P_0P_1P_2}$ to one side inside of the straight $\mathcal{T}_{P_0P_2}$.

Let us determine connection between the parameters $\gamma, m$ of world line (5.35) of the Dirac particle and the parameters of triangles $\mathcal{T}_{[P_iP_{i+1}Q_i]}$, constituting the world tube (7.14). Projection of the tube (7.14) onto the 2-plane orthogonal to the parallel vectors $P_i, Q_i, i = 0, \pm 1, \pm 2...$ is shown in figure 4 (remember that we are considering now the 3D-case). Points $P_i, Q_i$ are projected into one point. The angle $\theta$ is the dihedral angle between triangles $P_iP_{i+1}Q_i$ and $P_{i+1}P_{i+2}Q_{i+1}$. The point $O$ is the center of the circle, where the points $P_i, Q_i, i = 0, \pm 1, \pm 2...$ are placed. The radius $R = |OP_i|$ of this circle is identified with the radius

$$a = \frac{\hbar \gamma \sqrt{\gamma^2 - 1}}{2m} \quad (7.27)$$

of the world line (5.35). It follows from the figure 4, that

$$R = \frac{2 |P_0P_1|}{\sin \frac{\Delta \theta}{2}} = \frac{2 |P_0P_1|}{\cos \frac{\theta}{2}} \quad (7.28)$$

Displacement $\Delta t$ of the point $P_i$ along the helix axis corresponds to the angle $\Delta \varphi = \pi - \theta$. It is determined by the relation

$$\Delta t = \frac{(P_1P_2P_1Q_1)}{|P_1Q_1|} \quad (7.29)$$
The angular frequency is determined by the relation
\[ \omega = \frac{\Delta \phi}{\Delta t} = \frac{(\pi - \theta) |P_1Q_1|}{(P_1P_2, P_1Q_1)} \] (7.30)

The angular frequency (7.30) should be identified with the angular frequency \( \Omega \), defined by the relation (5.33). It gives
\[ \frac{2m}{\hbar \gamma^2} = \frac{(\pi - \theta) |P_1Q_1|}{(P_1P_2, P_1Q_1)} \] (7.31)

or
\[ \gamma = \sqrt{\frac{2m (P_1P_2, P_1Q_1)}{\hbar (\pi - \theta) |P_1Q_1|}} \] (7.32)

where \( \gamma \), as well as \( \theta \) are parameters, defining internal motion of the Dirac particle. The parameter \( \theta \) is determined by the mutual disposition of links (triangles) in the chain (7.14).

In the 4D space-time constraints on the broken tube (7.14) are to be chosen in such a way that we obtain the diagram of figure 4 for projections of points \( P_i, Q_i \) on some two-dimensional plane which is orthogonal to vectors \( P_iQ_i \) and some spacelike vector \( P_iS_i \). All vectors \( P_iS_i \) are similar, and we consider them as one vector \( \xi \). The vector \( P_iS_i \) satisfies the relations
\[ P_iS_i = P_{i+1}S_{i+1}, \quad (P_iS_i, P_iQ_i) = 0, \quad |P_iS_i|^2 = -1, \quad i = 0, \pm 1, \pm 2... \] (7.33)

In the coordinate system, where
\[ P_iQ_i = \{ \mu, 0, 0, 0 \}, \quad P_iS_i = \xi = \{ 0, 0, 0, 1 \} \] (7.34)

vectors \( P_iP_{i+1}, Q_iQ_{i+1} \) have components
\[ P_iP_{i+1} = Q_iQ_{i+1} = \{ 0, u_1, u_2, 0 \} \] (7.35)

Thus, relations (7.16), (7.17), (7.21) and (7.23) are conditions of the geometrical description of the classical Dirac particle. We can return from them to dynamic description. If we replace the Minkowski world function in relations (7.16), (7.17), (7.21) and (7.23) by the world function (7.12), (7.13), we obtain stochastic world tubes (7.14). Introducing statistical description of the stochastic world tubes, we obtain some version of the quantum description. Does this description coincide with the description in terms of the Dirac equation? Maybe, but it is not necessarily. There are some reasons for such a hesitation. In the case of the spinless particle, described by the world tube (7.2), the statistical description leads to a more general description, than the conventional quantum description in terms of the Schrödinger equation. The quantum description is only a special part of the general statistical description. Second, we cannot be sure, that helices (5.35) can be obtained only at the conditions (7.16), (7.21) and (7.23). Maybe, there are another conditions, which lead to the helices (5.35).
Nevertheless, the approach, founded on the a choice of the proper space-time geometry and proper geometrical objects as candidates for descriptions of elementary particles seems to be rather promising. In this case we do not use enigmatic quantum principles, we do not invent exotic properties of particles and of space-time. We do not invent new hypotheses, we simply look for the proper space-time geometry, and the proper geometrical objects in the set of known geometries and in the set of geometrical objects with known properties. Unfortunately, we cannot restrict ourselves by consideration of the space-time geometry, because now we are able to construct a statistical description only in the framework of dynamics (but not in the framework of geometry). Nevertheless, such a dynamical problem as confinement does not arise, if we start from geometry. Besides, it seems rather probable, that masses of elementary particles are determined mainly by their geometric structure, whereas contribution of quantum effects, conditioned by a distortion (7.12) of the space-time geometry is negligible.

The long-range action is another problem of the relativistic rotator. This is also a dynamical problem, which is absent in the geometrical model.

The relations (7.17), and conditions (7.21), describing that all links (triangles) must be similar seem to be rather reasonable.

In this paper we have investigated well-known dynamic system $\mathcal{S}_D$. We used methods of the model conception of quantum phenomena (MCQP) [23]. We did not used any additional suppositions. Furthermore, we have removed all quantum principles and have not use them in our investigations. Results of investigation of the well-known dynamic system $\mathcal{S}_D$ appeared to be unexpected and encouraging. We have came to the approach, containing a series of notions of the contemporary elementary particles such as string, quark, confinement. Appearance of these concepts is not connected with any additional hypotheses. It is rather reasonable, because their appearance is connected with such a fundamental structure as the space-time geometry.

Why have we obtain these results, which could not be obtained on the basis of the quantum principles? The answer is rather unexpected. Conventional theory of physical phenomena in microcosm contains mistakes, which are compensated by means of the quantum principles. MCQP these mistakes are corrected, and there is no necessity to compensate them. The quantum principles became to be unnecessary. As a result the theory of the microcosm phenomena and its methods become simple and reasonable.
Mathematical Appendices

A Calculation of Lagrangian

Let us calculate the expression

\[ \frac{i}{2} \hbar \bar{\psi} \gamma^l \partial_l \psi + \text{h.c} = F_1 + F_2 + F_3 + F_4 \] (A.1)

where the following designations are used

\[ F_1 = \frac{i}{2} \hbar \psi^* \left( (\partial_0 - i \gamma_5 \sigma \nabla) i \varphi \right) \psi + \text{h.c.} \] (A.2)

\[ F_2 = \frac{i}{2} \hbar \psi^* \left( (\partial_0 - i \gamma_5 \sigma \nabla) \left( \frac{1}{2} \gamma_5 \kappa \right) \right) \psi + \text{h.c.} \] (A.3)

\[ F_3 = \frac{i}{2} \hbar A^2 \Pi \left( \left( \sigma \mathbf{n} \right) \exp \left( -i \gamma_5 \sigma \eta \right) \left( \sigma \mathbf{n} \right) \right) \exp \left( -\frac{i\pi}{2} \sigma \mathbf{n} \right) \times \left( \partial_0 - i \gamma_5 \sigma \nabla \right) \exp \left( \frac{i\pi}{2} \sigma \mathbf{n} \right) \Pi + \text{h.c.} \] (A.4)

\[ F_4 = \frac{i}{2} \hbar A^2 \Pi \exp \left( -\frac{i\pi}{2} \gamma_5 \Sigma \eta \right) \left( \partial_0 - i \gamma_5 \Sigma \nabla \right) \exp \left( -\frac{i}{2} \gamma_5 \Sigma \eta \right) \Pi + \text{h.c.} \] (A.5)

In the last relation the matrix \( \Sigma \) is not differentiated.

Using definitions of \( j^l \) and \( S^l \), the expression \( F_1 \) and \( F_2 \) reduce to the form

\[ F_1 = \frac{i}{2} \hbar \psi^* \left( (\partial_0 - i \gamma_5 \sigma \nabla) i \varphi \right) \psi + \text{h.c.} = -j^l \partial_l \varphi \Pi \] (A.6)

\[ F_2 = \frac{i}{2} \hbar \psi^* \left( (\partial_0 - i \gamma_5 \sigma \nabla) \left( \frac{1}{2} \gamma_5 \kappa \right) \right) \psi + \text{h.c.} \]

\[ = \frac{i}{2} \hbar \psi^* \gamma_5 \gamma^l \partial_l \left( \frac{1}{2} \gamma_5 \kappa \right) \psi + \text{h.c.} \]

\[ F_2 = -\frac{1}{2} \hbar S^l \partial_l \kappa \Pi \] (A.7)

\[ F_3 = \frac{i}{2} \hbar A^2 \Pi \exp \left( -\frac{i\pi}{2} \sigma \mathbf{n} \right) \exp \left( -i \gamma_5 \sigma \eta \right) \left( \partial_0 - i \gamma_5 \sigma \nabla \right) \exp \left( \frac{i\pi}{2} \sigma \mathbf{n} \right) \Pi + \text{h.c.} \]

\[ = \frac{i}{2} h j^l \Pi \sigma_\alpha \sigma_\beta n^\alpha \partial_n^\beta \Pi + \text{h.c.} = \frac{i}{2} h j^l n^\alpha \partial_n^\beta \Pi \left( \delta_\alpha_\beta_+ i \varepsilon_\alpha_\beta_\gamma \sigma_\gamma \right) \Pi + \text{h.c.} \]

\[ = \frac{i}{2} h j^l \left( n^\alpha \partial_n n^\alpha + i \varepsilon_\alpha_\beta_\gamma n^\alpha \partial_n n^\beta \gamma \right) \Pi + \text{h.c.} \]

As far as \( n^2 = 1 \), one obtains

\[ n^\alpha \partial_n n^\alpha = 0 \] (A.8)
Besides it follows from (2.24) that

$$n = \frac{\sigma + z}{\sqrt{2(1 + \sigma z)}}$$  
(A.9)

Then

$$F_3 = -\hbar j^l (\varepsilon_{\alpha\beta\gamma} n^\alpha \partial_l n^\beta z^\gamma) \Pi = -\frac{\hbar j^l}{2(1 + \xi z)} \varepsilon_{\alpha\beta\gamma} \xi^\alpha \partial_l \xi^\beta z^\gamma \Pi$$  
(A.10)

Calculation of $F_4$ leads to the following result

$$F_4 = \frac{i}{2} \hbar A^2 \Pi \exp \left( -\frac{i}{2} \gamma_5 \Sigma \eta \right) \left( \partial_0 - i\gamma_5 \Sigma \nabla \right) \Pi + \text{h.c.}$$

$$= \frac{i}{2} \hbar A^2 \Pi \left( \cosh \frac{\eta}{2} - i\gamma_5 v^\alpha \Sigma_\alpha \sinh \frac{\eta}{2} \right) \times \left( \partial_0 - i\gamma_5 \Sigma \nabla \right) \Pi + \text{h.c.}$$

$$= \frac{i}{2} \hbar A^2 \Pi \left( \cosh \frac{\eta}{2} \sinh \frac{\eta}{2} \partial_0 \eta + \sinh \frac{\eta}{2} v^\alpha \partial_0 v^\beta \Sigma_\alpha \Sigma_\beta \right) \Pi + \text{h.c.}$$

$$+ \frac{i}{2} \hbar A^2 \Pi \left( \cosh \eta \varepsilon_{\beta\alpha\gamma} v^\beta \Sigma_\gamma \partial_\alpha \frac{\eta}{2} \Pi + \text{h.c.} \right)$$

$$+ \frac{i}{2} \hbar A^2 \Pi \sinh \frac{\eta}{2} \Sigma_\alpha \Sigma_\beta \partial_\alpha v^\beta \Pi + \text{h.c.}$$

$$F_4 = \frac{i}{2} \hbar A^2 \Pi \left( \frac{1}{2} \sinh \eta \partial_0 \eta + \sinh \frac{\eta}{2} v^\alpha \partial_0 v^\beta i\varepsilon_{\alpha\beta\gamma} \Sigma_\gamma \right) \Pi + \text{h.c.}$$

$$+ i \hbar A^2 \Pi \left( \cosh \eta v^\alpha + i\varepsilon_{\beta\alpha\gamma} v^\beta \Sigma_\gamma \right) \partial_\alpha \frac{\eta}{2} \Pi + \text{h.c.}$$

$$+ \frac{i}{4} \hbar A^2 \Pi \sinh \eta \left( \partial_\alpha v^\alpha + i\varepsilon_{\alpha\beta\gamma} \partial_\alpha v^\beta \Sigma_\gamma \right) \Pi + \text{h.c.}$$

$$F_4 = -\hbar A^2 \left( \sinh \frac{\eta}{2} v^\alpha \partial_0 v^\beta \varepsilon_{\alpha\beta\gamma} \xi^\gamma \right) \Pi$$

$$- \frac{1}{2} \hbar A^2 \Pi \left( \varepsilon_{\beta\alpha\gamma} v^\beta \xi^\gamma \partial_\alpha \eta + \sinh \eta \varepsilon_{\alpha\beta\gamma} \partial_\alpha v^\beta \xi^\gamma \right) \Pi$$

$$F_4 = -\frac{1}{2} \hbar A^2 \varepsilon_{\alpha\beta\gamma} \left( \partial_\alpha \eta v^\beta + \sinh \eta \partial_\alpha v^\beta + 2 \sinh^2 \left( \frac{\eta}{2} \right) v^\alpha \partial_0 v^\beta \right) \xi^\gamma \Pi$$  
(A.11)

According to the relation (2.23) $A = (j^l j_l)^{1/4} \equiv \rho^{1/2}$, and relation (A.11) may be written in the form

$$F_4 = \left( \frac{\hbar (\rho + j_0)}{2} \varepsilon_{\alpha\beta\gamma} \partial^\alpha \frac{j^\beta}{(j^0 + \rho)} - \frac{\hbar}{2(\rho + j_0)} \varepsilon_{\alpha\beta\gamma} \left( \partial^\alpha j^\beta \right) j^\alpha \right) \xi^\gamma \Pi$$  
(A.12)

To prove this statement, we substitute the expression of $j^i$ via variables $v$, $\eta$

$$j^0 = \rho \cosh \eta, \quad j^\alpha = \rho \sinh \eta v^\alpha$$  
(A.13)
in (A.12). We obtain

\[ F_4 = \frac{\hbar \rho (1 + \cosh \eta)}{2} \varepsilon_{\alpha \beta \gamma} \partial^\alpha \sinh \eta v^\beta \frac{\sinh \eta v^\beta}{(1 + \cosh \eta)} \xi^\gamma \Pi \]

\[ - \frac{\hbar \rho}{2(1 + \cosh \eta)} \varepsilon_{\alpha \beta \gamma} \partial^0 \left( \sinh \eta v^\beta \right) \sinh \eta v^\alpha \xi^\gamma \Pi \]

\[ = \hbar \rho (\cosh \frac{\eta}{2}) \varepsilon_{\alpha \beta \gamma} \partial^\alpha \left( \tanh \frac{\eta}{2} v^\beta \right) \xi^\gamma \Pi - \frac{\hbar \rho \sinh^2 \eta}{2(1 + \cosh \eta)} \varepsilon_{\alpha \beta \gamma} \left( \partial^0 v^\beta \right) v^\alpha \xi^\gamma \Pi \]

\[ = \hbar \rho (\cosh \frac{\eta}{2}) \varepsilon_{\alpha \beta \gamma} \partial^\alpha \left( \tanh \frac{\eta}{2} v^\beta \right) \xi^\gamma \Pi - \hbar \rho \sinh^2 \frac{\eta}{2} \varepsilon_{\alpha \beta \gamma} \left( \partial^0 v^\beta \right) v^\alpha \xi^\gamma \Pi \]

\[ F_4 = \frac{\hbar \rho}{2} \varepsilon_{\alpha \beta \gamma} \left( \partial^\alpha \eta v^\beta + \sinh \eta \partial^\alpha v^\beta - 2 \sinh \frac{\eta}{2} \partial^0 \left( v^\beta \right) v^\alpha \right) \xi^\gamma \Pi \]

The obtained relation coincides with the expression (A.11) for \( F_4 \), if we take into account that \( \partial^\alpha = -\partial_\alpha \).

**B Transformation of equation for variable \( \xi \)**

Multiplying equation (5.6) by \( (1 + z \xi) \) and keeping in mind \( \xi^2 = 1 \) and \( z^2 = 1 \), we obtain

\[ \xi \times \left( -\dot{\xi} \times z + \frac{(z \xi)}{2(1 + z \xi)} \xi \times z - \frac{\dot{\xi} (\xi \times z)}{2(1 + z \xi)} z - \frac{(1 + z \xi)}{2} b \right) = 0, \quad b = -(\dot{x} \times \ddot{x}) Q \tag{B.1} \]

Two middle terms could be represented as the double vector product

\[ \xi \times \left( -\dot{\xi} \times z + \frac{1}{2(1 + z \xi)} \left( \dot{\xi} \times ((\xi \times z) \times z) \right) - \frac{(1 + z \xi)}{2} b \right) = 0 \tag{B.2} \]

This equation can be rewritten in the form

\[ \xi \times \left( \dot{\xi} \times \left( -z + \frac{(z \xi) z - \xi}{2(1 + z \xi)} \right) - \frac{(1 + z \xi)}{2} b \right) = 0 \tag{B.3} \]

Now calculating double vector products and taking into account that \( \xi \dot{\xi} = 0 \), one obtains

\[ -\dot{\xi} - (\xi \times b) = 0 \tag{B.4} \]

or

\[ \dot{\xi} = \frac{1}{2} (\xi \times (\dot{x} \times \ddot{x})) Q \tag{B.5} \]
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Figure 1

a) World line of free particle in the Minkowski space-time
b) World line of free particle in the distorted space-time
Different interpretations of the world tube of composite particle, consisting of two constituents
Different interpretations of the composite particle world tube, when the particle contains three constituents.
Figure 4

Projection of the helix on the two-dimensional plane orthogonal to its axis.