Tunable optical delay lines based on a system of coupled whispering gallery mode resonators

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Abstract. The paper presents an analysis of optical delay lines based on coupled whispering gallery mode resonators. The induced optical delay is caused by slow light regime in high finesse resonators. The analysis is based on slowly varying envelopes approach and includes evaluation of possibilities to adjust the induced delay time. We also investigate the properties of transfer coefficient of system of coupled whispering gallery mode resonators.

1. Introduction
For the construction of many modern optical devices, such as systems for the formation of the radiation pattern of phased antenna arrays, optical signal processing devices, optical delay lines are necessary[1, 2]. At the same time, for the adaptability of the characteristics and, as a consequence, greater functionality of the designed devices, it is necessary to provide the ability to control the delay time. One of the most promising methods for constructing optical delay lines is slow light systems, that is, optical systems in which the group velocity of light propagation is significantly less than the phase velocity [3, 4, 5, 6]. To achieve such conditions, it is necessary to create a medium with a large value of the dispersion of the refractive index, such conditions take place if the light frequency is close to the resonances of the media. Optical resonances are formed on different principles:

- due to own energy transitions of atoms and molecules [7, 8],
- nonlinear interaction of coherent optical fields [9, 10, 11],
- in specially created high-Q optical resonators due to multiple interference of radiation [1].

The first type of optical resonances has significant limitations on the change of resonance parameters, and, consequently, the characteristics of the delay lines formed by them. In the second type of resonances, such processes as stimulated Brillouin amplification and four-wave mixing are usually used. Their disadvantage is the limited delay time. Slow light systems based on optical resonators have the greatest potential to vary the parameters and are able to provide significant delay times. It is proposed that the use of systems of coupled resonators[12, 13, 14] will sufficiently improve the properties produced delay lines. Whispering gallery mode (WGM) resonators[1, 15, 16] are ideal candidates for use in optical delay lines due to their high quality factors (up to $10^9$ as reported in [17]). An unsolved problem is the creation of a system capable of controlling the delay time of light within a sufficiently wide range and not distorting the
shape of the delayed optical pulses. Thus, in the spectral region close to a single resonance, in addition to the second-order dispersion (the linear dependence of the group velocity on the wavelength), there is a sufficiently strong third-order dispersion, which leads to a distortion of the pulse shape passing through the system. On the other hand, the presence of a third-order dispersion allows to change the value of the second-order dispersion by changing the resonance frequency or the wavelength of the radiation and, consequently, changing the group velocity and the time delay of the transmitted light. In systems of coupled resonators, the problem of distortion of the pulse shape can be solved by selecting the magnitudes of dispersion of second and third order by regulating the coupling between the resonators and the resonant frequencies of the individual resonators. At the same time, in the system of coupled resonators, the group velocity and time delay depend, among other things, on the phase ratios of the waves that have passed different paths in the coupled system. Thus, by varying the optical dimensions of the resonators themselves and/or the optical distances of the connecting elements, it is also possible to control the optical delay introduced. In turn, one of the most convenient ways to control the refractive index of fiber-optic components is the use of controlled deformations based on PZT elements, widely used for decades in fiber-optic phase modulators[18].

2. Transfer function of coupled WGM resonators

In this section we will consider the transfer function of a pair of identical WGM resonators, coupled to a tapered fiber [13, 19], as shown in figure 1. Using slowly varying amplitude approach, one can write the equation for modes in resonator as

\[
\frac{d a_{cw}}{d t} = -i (\omega_{c,a} a_{cw} + g a_{ccw}) - \frac{k_a}{2} a_{cw} - \sqrt{k_a} a_{in_{cw}},
\]

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\]

where \(a_{cw}\) and \(a_{ccw}\) clock-wise and counter clock-wise mode amplitudes; \(k_{cw(ccw)} = k_{cw(ccw),0} + k_{cw(ccw),1}\) describes mode dissipation in the first resonator; \(g\) – coupling coefficient between CW and CCW modes.

For simplicity, we assume that CW and CCW modes have the same dissipation \(k_{cw,0} = k_{cw,1} = k_a, 0 + k_{a,1} = k_a\).

![Figure 1. Schematic drawing of a system of two coupled WGM resonators.](image)

Output fields \(a_{cw}^{out}\) and \(a_{ccw}^{out}\) are linked with input fields \(a_{cw}^{in}\) and \(a_{ccw}^{in}\) (at carrier frequency \(\omega\)) via \(a_{cw}^{out(cw)} = a_{cw}^{in(cw)} + \sqrt{k_{a,1} a_{cw}}\). Considering the steady-state mode, one can apply Fourier transform to eq. (1)

\[
\alpha_{a} a_{cw} - i g a_{ccw} - \sqrt{k_{a,1} a_{cw}} = 0
\]

\[
\alpha_{a} a_{ccw} - i g a_{cw} - \sqrt{k_{a,1} a_{ccw}} = 0
\]

where \(\alpha_{a} = i (\omega - \omega_{c,a}) - k_a/2\). Output fields are
\[ a_{\text{out}}^{cw} = \frac{igak_{a,1}a_{\text{in}}^{cw} + \beta_{a}^{2}a_{\text{in}}^{cw}}{g_{a}^{2} + \alpha_{a}^{2}} \]
\[ a_{\text{out}}^{ccw} = \frac{igak_{a,1}a_{\text{in}}^{ccw} + \beta_{a}^{2}a_{\text{in}}^{ccw}}{g_{a}^{2} + \alpha_{a}^{2}} \]

(3)

where \( \beta_{a}^{2} = g_{a}^{2} + \alpha_{a}(\alpha_{a} + k_{a,1}) \). We rewrite (2) in a matrix form

\[
\begin{pmatrix}
    a_{\text{out}}^{cw}(\omega) \\
    a_{\text{in}}^{cw}(\omega)
\end{pmatrix}
= T_{a}
\begin{pmatrix}
    a_{\text{in}}^{cw}(\omega) \\
    a_{\text{out}}^{cw}(\omega)
\end{pmatrix}
\]

(4)

where the transfer matrix is

\[
T_{a} = \begin{pmatrix}
    g_{a}^{2} + (\alpha_{a} + k_{a,1})^{2} & igak_{a,1} \\
    -igak_{a,1} & g_{a}^{2} + \alpha_{a}^{2}
\end{pmatrix} / \beta_{a}^{2}
\]

(5)

It is general description of the first resonator which is coupled to fiber. Analogically it can be done for the second resonator

\[
T_{b} = \begin{pmatrix}
    g_{b}^{2} + (\alpha_{b} + k_{b,1})^{2} & igbk_{b,1} \\
    -igbk_{b,1} & g_{b}^{2} + \alpha_{b}^{2}
\end{pmatrix} / \beta_{b}^{2}
\]

(6)

Here, \( g_{b}, k_{b,1}, \alpha_{b} \) and \( \beta_{b} \) are equivalent to the analogical parameters of the first resonator. Thus, the coupling between input and output fields in the coupled by the fiber microresonators can be expressed in the next form

\[
\begin{pmatrix}
    b_{\text{out}}^{cw}(\omega) \\
    b_{\text{in}}^{cw}(\omega)
\end{pmatrix}
= T
\begin{pmatrix}
    a_{\text{in}}^{cw}(\omega) \\
    a_{\text{out}}^{cw}(\omega)
\end{pmatrix}
\]

(7)

where full transfer matrix

\[
T = T_{b}T_{0}T_{a}
\]

(8)

while \( T_{0} \) is a fiber transfer matrix

\[
T_{0} = \begin{pmatrix}
    e^{ikL} & 0 \\
    0 & e^{-ikL}
\end{pmatrix}
\]

(9)

By combining eqs. (8), (5), (6) and (9) and with taking into account that the system transfer coefficient for a field is

\[
t_{E} = T_{0,0} - T_{0,1}T_{1,0}/T_{1,1}
\]

(10)

Substituting eqs. (5), (6) and (9) to eq. (8) and (10), one obtains the following

\[
t_{E} = \frac{e^{ikL}(g_{a}^{2} + \alpha_{a}^{2} + a_{a}K_{a})(g_{b}^{2} + \alpha_{b}^{2} + a_{b}K_{b})}{e^{i2kL}g_{a}g_{b}K_{a}K_{b} + (\alpha_{a}^{2} + g_{a}^{2})(\alpha_{b}^{2} + g_{b}^{2})}
\]

(11)

The equation (10) can be expressed in form [20]

\[
t_{E} = \sqrt{\frac{1}{T(k)}} \cdot \exp(jkL)
\]

(12)

where \( k \) is propagation constant of light in fiber, \( L \) is the distance between the resonators. On the other hand, group velocity of light can be expressed as

\[
v_{g} = d\omega/dk
\]

(13)
Numeric values of group velocity can therefore be found as

\[ v_g = L \cdot \frac{\text{diff}(\omega_i)}{\text{diff}(	ext{angle}(t_E))}. \]  

(14)

As can be seen from eq. (12), transfer function for the intensity of light, passing the considered system can be written as

\[ t_I = |t_E|^2, \]  

(15)

an explicit equation can be obtained by substituting eq. (11) into eq. (15). The equations above can be used for analysis of proposed delay lines, including their abilities of delay time adjustment and transfer coefficient magnitude and uniformity.

3. Analysis of attainable delay line properties

In this section we will analyze the feasibility of building adjustable optical delay lines based on pairs of coupled WGM resonators. We will consider the influence of distance between the resonators \( L \) on the group velocity \( v_g \) and transfer coefficient \( I \). As an example, we have chosen two similar WGM resonators with radius \( R = 10\mu m \), quality factor \( Q = 10^8 \), coupling coefficients between resonators and fiber and between CW and CCW modes \( \kappa = g = \frac{10^3}{\pi \cdot Q} \cdot \omega_0 \), refractive index of resonators \( n_{WGM} = 1.47 \), refractive index of fiber \( n_f = 1.48 \), distance between the resonators \( L \approx 50\mu m \). A dependency of group velocity of light in system of coupled resonators, normalized on group velocity of light in optical fiber on light wavelength is shown in figure 2. It can be seen that \( v_g \) becomes close to zero near resonant wavelength \( \lambda_0 \approx 1.539\mu m \), indicating that the slow light propagation mode indeed takes place in the considered system.

![Figure 2](image)

Figure 2. Dependency of normalized group velocity of light in system of coupled resonators on light wavelength.

Dependencies of normalized group velocity on distance between the resonators \( L \) in case of different frequency offsets from the resonance are shown in figure 3. For plot in figure 3 (a) the offset was \( \sim 170\text{MHz} \), for plot in figure 3 (b) the offset was \( \sim 700\text{MHz} \). As can be seen from these figures, a three-fold adjustment of delay time can be achieved by small (\( \sim 50\text{nm} \)) variations of the distance between the resonators.

For practical use of such devices, sufficient transmission coefficient must be ensured in order to allow the delayed signal be successfully received. Dependencies of transmission coefficient on the distances between the resonators for frequency offsets from the resonance \( \sim 170\text{MHz} \) and \( \sim 700\text{MHz} \) are shown in figure 3 (c) and (d). It can be seen that in case of 700MHz offset, the
variations of transmission coefficient are negligible, while for 170MHz offset it changes in much greater bounds. However, in areas of the steepest dependency $v_g(L)$, transmission coefficient is about 0.25, which is sufficient in most cases.

Figure 3. Dependency of normalized group velocity of light (a and b) and transfer coefficient (c and d) of considered pair of coupled resonators.

4. Conclusion
We have considered a system of a pair of coupled whispering gallery mode resonators and analyzed an ability to use it as a tunable optical delay line. The tunability is achieved by varying the distance between the resonators, enabling a three time adjustment of the induced delay. To the best of our knowledge, transfer coefficient of the considered slow light-based optical delay system is considered for the first time. Further work can be dedicated to analysis of distortion of transmitted optical signals and to experimental validation of our analytical results.

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