Pre-critical soft photons emission from quark matter

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We compute the soft photon emission rate from the QCD matter in the vicinity of the critical line at moderate density and the temperature approaching the critical one from above. The obtained production rate exhibits a steep rise close to $T_c$.

I. INTRODUCTION

Heavy ion collision experiments carried out at RHIC and LHC over the last two decades brought about the discovery of a new form of matter with unexpected properties. Several probes are used to reveal its nature and characteristics. A special role is played by direct photons. They are produced at all stages of two fireball evolution and can easily escape the collision region without reinteracting. Photons and dileptons production has been studied both experimentally and theoretically for quite a long time. The basic theory concepts have their roots in the studies performed several decades ago [1–5]. The current status of the field is presented in the review article [6]. Up to now the soft photon emission...
has been predominantly studied for hot and low density QGP. In this region of 
the QCD phase diagram perturbative methods including the hard thermal loop 
are the adequate research tools \[7–11\]. Results of several lattice calculations at 
zero chemical potential are also available \[12–14\]. On the other hand during 
the last years it became clear that except for high temperature and low density 
domain the quark matter is a strongly coupled medium \[15\]. There are very few 
calculations of the photon production beyond, or partly beyond, the perturbation 
theory \[16–19\].

The reason is that the finite temperature retarded self-energy of virtual photon 
is known only in perturbation theory \[20, 21\]. Probably the most intriguing region 
of the phase diagram lies in the vicinity of the critical temperature at nonzero 
density. The corresponding research program is planned at NICA and FAIR. In 
this domain the correlation functions are characterized by the presence of a soft 
mode of the fluctuation field. It will be shown below that the propagator of the 
fluctuation mode (FP) has the form

\[ L(q, \omega) = \frac{N}{T - T_c} - i\beta\omega + \xi^2 q^2. \] (1)

The quantities \( N, \beta \) and \( \xi^2 \) will be determined in what follows. One may recognize 
in (1) the linear response function of the phase transition theory \[22, 23\]. At small 
\( \omega \) and \( q^2 \) and close to \( T_c \) the FP (1) can be arbitrary large and is rapidly varying. 
We shall evaluate the soft photon emission rate close to \( T_c \) using the expression 
for the retarded self-energy containing two FP-s. This will lead to the enchanced 
soft photon production rate.

The organization of this paper is as follows. In Section II, we show that there 
is a rather wide fluctuation region above the critical line at moderate density. In 
Section III, using the time-dependent Ginzburg-Landau functional with Langevin 
forces we derive the propagator of the soft collective mode. In Section IV, we
address the retarded photon self-energy in the fluctuation region. In Section V, we compute the soft photon emissivity and confront it with the electrical conductivity computation. We summarize and conclude in Section VI.

II. CRITICAL FLUCTUATIONS

Our focus in this work is on the finite-density pre-critical fluctuation region with \( T \to T_c \) from above. Comprehensive study has shown that at high density and low temperature the ground state of QCD is color superconductor \([27, 28]\). We consider the 2 SC color superconducting phase when \( u \)- and \( d \)- quarks participate in color antitriplet pairing but the density is not high enough to involve the heavier \( s \)- quark. The value of the quark chemical potential under consideration is \( \mu \simeq 200 - 300 \) MeV and the critical temperature \( T_c \simeq 40 \) MeV. The corresponding density is two or three times the normal nuclear density. Both numbers should be considered as educated guess since they rely on model calculations. Prior to forming a condensate the system goes through the phase of pre-formed fluctuation quark pairs. In its basic features this state is very different from the BCS superconductor fluctuation region. In BCS the border between the normal and the superconducting phases is very sharp. In color superconductor it is significantly smeared. Two interrelated explanations of this difference may be given. First, in the BCS the characteristic pair correlation length \( \xi \) is large, \( \xi \simeq 10^{-4} \) cm, so that \( n_e^{1/3} \xi \gg 1 \) where \( n_e \sim 10^{22} \) cm\(^{-3} \) is the electron density. In color superconductor the root-mean-square radius \( \rho \) of the quark pair is \( \rho \simeq (1 - 2) \) fm, the quark density \( n_q \) is (2-3) times larger than the normal nuclear density, so that \( n_q^{1/3} \rho \sim 1 \). Note that \( n^{1/3} \xi \) is the BCS-BEC crossover parameter \([30, 33] \). In the BCS limit the interaction of fermions is weak, the pairs are large and overlap. In the BEC limit the interaction is strong, the pairs are small compared to their separa-
tion and have small overlap (Schafroth pairs [36]). Therefore one may say that at 
\( \mu \sim 300 \text{ MeV}, \ T \sim 50 \text{ MeV} \) the quark system is in the crossover regime [30]. The
second way to reveal the difference between the BCS and color superconductors
is to compare the relative values of of the energy parameters of the two theories.
In the BCS the following scales hierarchy holds \( \Delta : \omega_D : \varepsilon_F \simeq 1 : 10^2 : 10^4 \), where \( \Delta \sim T_c \sim 10^{-4} \text{ eV} \) is the gap/critical temperature, \( \omega_D \sim 10^2 \text{ eV} \) is the Debye energy, \( \varepsilon_F \) is the Fermi energy [29]. In color superconductor the relation is very different, \( \Delta : \Lambda : \mu \simeq 1 : 8 : 5 \), where \( \Delta \sim 0.1 \text{ GeV} \) is the gap, \( \Lambda \sim 0.8 \text{ GeV} \) is the UV cutoff, \( \mu \sim 0.5 \text{ GeV} \) is the quark chemical potential [35]. The width of
the fluctuation region and the fluctuation contribution to the physical quantities
is characterized by the Ginzburg-Levanyuk parameter [25, 29, 30, 35] which can be estimated in two ways using the values of parameters estimated above

\[
G_i \simeq \frac{\delta T}{T_c} \simeq \frac{21\zeta(3)}{64}(\mu g)^{-4} \simeq \frac{27\pi^4}{28\zeta(3)}\left(\frac{T_c}{\mu}\right)^4 \sim 10^{-2},
\]

(2)

where \( \zeta(3) \simeq 1.2 \). For the ordinary superconductors \( G_i \sim 10^{-12} - 10^{-14} \) [29].
The fluctuations region in color superconductors is extremely wide as compared to the ordinary ones.

III. COLLECTIVE MODE PROPAGATOR

The FP of the form \( (1) \) may be derived in several ways. In [37] it was obtained by solving the Dyson equation with relativistic Matsubara quark propagators. Here we shall use the time-dependent Ginzburg-Landau (GL) functional [38, 39] and the stochastic Langevin forces. In absence of the external electromagnetic field the time-dependent GL equation for the fluctuating pair field reads

\[
- \gamma \frac{\partial}{\partial t} \Psi(r, t) = \frac{\delta F[\Psi]}{\delta \Psi^*} + \eta(r, t).
\]

(3)
here $\gamma$ is the order parameter relaxation constant, $\eta(r, t)$ are the Langevin forces. The GL functional with the quartic term dropped has the form \cite{22, 25, 29}

$$F[\Psi] = \nu \int [\varepsilon |\Psi(r, t)|^2 + \xi^2 |\nabla \Psi(r, t)|^2] dV,$$

(4)

where $\nu = \mu p_F/\pi^2$ is the relativistic density of states at the Fermi surface, $\varepsilon = (T - T_c)/T_c$, $\xi$ is the correlation length operator which may be expressed in terms of the diffusion coefficient as $\xi^2 = \frac{\pi}{8 T} D$. According to (4) fluctuations are described by the quadratic Hamiltonian and therefore do not interact. Very close to $T_c$ this approximation breaks down but it is a difficult task to determine when and how it happens. In momentum space (3) takes the form

$$-\left[ \gamma \frac{\partial}{\partial t} + \nu \left( \varepsilon + \frac{\pi}{8 T} q^2 D \right) \right] \Psi(r, t) = \eta(r, t).$$

(5)

From this equation the relaxation time of fluctuations with momentum $q^2$ is

$$\tau = \frac{\gamma}{\nu (\varepsilon + \frac{\pi}{8 T} D q^2)}.$$ 

(6)

The formal solution of (5) may be written as

$$\Psi(r, t) = L \eta(r, t),$$

(7)

where

$$L = -(-i\gamma \omega + \Omega q)^{-1}.$$ 

(8)

with $\Omega q = \nu \left( \varepsilon + \frac{\pi}{8 T} D q^2 \right)$. The fluctuation - dissipation theorem \cite{22, 40} states that the equal time correlator $\langle \Psi^*(r, t) \Psi(r', t) \rangle$ is expressed via the retarded propagator. The formal solution (7) satisfies this requirement provided the correlator of the Langevin forces have a gaussian white noise form

$$\langle \eta^*(r, t) \eta(r't') \rangle = 2 T \gamma \delta(r - r') \delta(t - t').$$

(9)

In this case using (7) we have

$$\langle \Psi^*(r, t) \Psi(r', t) \rangle = \langle \eta^*(r, t) L^* L \eta(r', t) \rangle.$$
Thus, \( L(q,\omega) \) given by (8) meets the needed requirement. The last step is to express the coefficient \( \gamma \) making use of the decay time (6). Keeping in the denominator of (6) only the term \( \nu \varepsilon \), which is equivalent to retaining only the term \( \varepsilon |\psi|^2 \) in (4), and comparing the result with the GL decay time \( \tau_{GL} = \pi [8(T-T_c)]^{-1} \) [25, 29] we obtain \( \gamma = \pi \nu / 8T_c \). This completes the derivation of the FP

\[
L(q,\omega) = - \frac{1}{\nu \left[ \varepsilon + \frac{\pi}{8T_c} (-i\omega + Dq^2) \right]}. (11)
\]

All coefficients in the expression (11) for \( L(q,\omega) \) are now fixed.

IV. THE PHOTON PRE-CRITICAL SELF-ENERGY

To calculate the photon emission rate we have to construct the photon self-energy operator in the pre-critical region. At \( T \rightarrow T_c \) FP (11) shows a singular behavior. Therefore one can expect that the diagrams containing one, or better two, FP-s will play the dominant role.

The choice of the needed diagram has been done long ago in condensed matter theory [25, 40]. This is the celebrated Aslamazov-Larkin (AL) diagram shown in Fig. 1. This diagram gives the strongest temperature dependence and provides a good description of the existing experimental data on the fluctuation regime in superconductors [25, 42].

The AL diagram includes two FP-s depicted by wavy lines inside the diagram in Fig.1. The solid lines correspond to the relativistic Matsubara quark propagators. The detailed calculation of the AL retarded response function presented in Fig.1 has been performed in [37]. For the sake of brevity we expose here the key points of the derivation and refer the reader to [37] for the details. The AL response...
The function corresponding to Fig. 1 reads
\[
\Pi_{lm}(k, \omega_k) = -3Q^2 T \sum_{\Omega_j} \int \frac{dq}{(2\pi)^3} B_l(q, \omega_k) L(k + q, \Omega_j + \omega_k) B_m(q, \Omega_j, \omega_k) L(q, \Omega_j). \tag{12}
\]
Here the factor 3 comes from color, \(Q^2 = \frac{5}{9}e^2\) for two flavors, \(e^2 = 4\pi\alpha\). The trace over the Dirac indices is included into \(B_{l,m}\). The FP-s \(L\) are defined by (11), \(B_{l,m}\) correspond to three Green’s functions blocks. We are interested in the emission of the real photons so that \(k_0 = \omega = i\omega_k = k, k = 0\). Then
\[
B(q, \Omega_j, \omega_k) = T \sum_{\tilde{\epsilon}_n} \lambda(q, \epsilon_n + \omega_k, \Omega_j - \epsilon_n) \lambda(q, \epsilon_n, \Omega_j - \epsilon_n) \cdot \int \frac{dp}{(2\pi)^3} tr_D [\gamma G(p, \tilde{\epsilon}_n) G(p, \tilde{\epsilon}_n + \omega_k) G(q - p, \Omega_j - \tilde{\epsilon}_n)]. \tag{13}
\]
The Matsubara propagators in (13) have the form
\[
G(p, \tilde{\epsilon}_n) = \frac{1}{\gamma_0(i\tilde{\epsilon}_n + \mu) - \gamma p - m}, \tag{14}
\]
where \(\tilde{\epsilon}_n = \epsilon_n + \frac{1}{2\tau} \text{sgn} \epsilon_n, \quad \epsilon_n = \pi T (2n+1), \quad \text{where } \tau \text{ is the momentum relaxation time. From the formal point of view, } \tau \text{ regulates the pinch (collinear) singularities. The factors } \lambda-s \text{ are the vertex renormalization corrections [25, 40]. At } q \to 0, \quad \omega_k \to 0 \text{ the product of the two } \lambda-s \text{ takes the limiting value } |2\tilde{\epsilon}_n|^2/|\epsilon_n|^2 [37].
\]
Integration in (13) is performed using the Fermi surface integration measure
\[
\int \frac{dp}{(2\pi)^3} = \frac{\nu}{2} \int \frac{d\Omega_p}{4\pi} \int_{-\infty}^{\infty} dt, \tag{15}
\]
where \(t = \sqrt{p^2 + m^2 - \mu}, \quad \nu = \frac{\mu P_F}{\pi^2}. \quad \text{The dependence of } L(q, \Omega_j) \text{ and } L(q, \Omega_j - \omega_k) \text{ on } \Omega_j \text{ and } \omega_k \text{ is much stronger than the dependence of the Green’s functions on the}
same quantities. We shall keep in the propagators entering into \( B(q, \Omega_j, \omega_k) \) only
the dependence on the fermionic frequencies \( \tilde{\varepsilon}_n \) and evaluate \( B(q, \Omega_j = \omega_k = 0) \).

Straight forward calculation leads to
\[
B(q) = -\nu T \sum_{\varepsilon_n} \frac{|2\tilde{\varepsilon}_n|^2}{|\varepsilon_n|^2} \int \frac{d\Omega}{4\pi} \frac{(q\Omega)p}{\mu^2} \int_{-\infty}^{\infty} dt \frac{dt}{(t^2 + \tilde{\varepsilon}_n^2)^2},
\]
(16)

Performing the integration one gets
\[
B_l(q_L) = -\frac{7\zeta(3)}{12} \frac{\nu p^2}{\pi^2 T^2} q_l \chi \left( \frac{1}{2\pi T \tau} \right),
\]
(17)

where
\[
\chi(y) = \frac{8}{7\zeta(3)} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2(2n+1+y)}.
\]
(18)

At \( y \to 0, \chi(y) \to 1 \), and at \( y \gg 1, \chi(y) = \frac{\pi^2}{\zeta(3)} y^{-1} \).

The important parameter determining the value of \( B_l \) is momentum relaxation
time \( \tau \). To our best knowledge, a reliable determination of \( \tau \), in particular at
finite density, is lacking. For example, in [43] it varies at \( \mu = 0 \) within the
interval \((0.1 - 0.9) \) fm. We shall assume that \( \tau \approx (0.2 - 0.3) \) fm at the QCD
phase diagram region \((T, \mu) \simeq (50, 300) \) MeV under consideration is the present
work. In this case one can take for \( \chi \) in (17) its asymptotic value at \( y \gg 1 \) and
arrive at
\[
B_l(q) = -q_l \frac{\pi \nu}{2T} \left( \frac{1}{3} \frac{p^2}{\mu^2 \tau} \right) = -4q_l \gamma D,
\]
(19)

where \( D = \frac{1}{3} v^2 \tau \) is the diffusion coefficient, \( v^2 = p^2/\mu^2 \) is the quark velocity near
the Fermi surface, \( \gamma = \pi \nu / 8T \). This definition of the diffusion coefficient is in
line with the relation between the GL functional parameter \( \xi^2 \) and \( D \) given after
Eq. (4). Indeed, for \( T \tau \ll 1 \) one has [25]
\[
\nu \xi^2 = \frac{\pi \nu}{8T} D = \frac{\pi^2 v^2}{9} \left[ \frac{1}{4\pi T \tau} \psi' \left( \frac{1}{2} \right) \right] = \gamma \frac{v^2 \tau}{3},
\]
(20)

where \( \psi(z) = \frac{d}{dz} \ln \Gamma(z), \psi' \left( \frac{1}{2} \right) = 3\zeta(2) = \pi^2 / 2 \).
From (12) and (19) we get

$$\Pi_{lm}(\omega_k) = -12Q^2 T T^2 D^2 \sum_{\Omega_j} \int \frac{dq}{(2\pi)^3} q_l q_m L(q, \Omega_j) L(q, \Omega_j + \omega_k). \quad (21)$$

To evaluate the sum in (21), we can use the technique of replacing the summation by contour integration \[21, 44\]

$$T \sum_{\Omega_j} f(\Omega_j) = \frac{1}{4\pi i} \oint dz \coth \frac{z}{2T} f(-iz), \quad (22)$$

where $z = i\Omega_j$. With the help of (22) we obtain

$$\Pi(\omega) = -\frac{2Q^2 \gamma^2 D^2}{\pi} \int \frac{dq}{(2\pi)^3} q^2 \int_{-\infty}^{\infty} dz \coth \frac{z}{2T} [L^R(q, -iz - i\omega) + L^A(q, -iz + i\omega)] \text{Im} L^R(q, -iz), \quad (23)$$

where $\omega = i\omega_k$, $L^R$ and $L^A$ are the retarded and advanced EP-s. Next we expand the integrand in powers of $\omega$ and subtract the zeroth order term which would lead to Meissner effect above $T_c$. This may be regarded as imposing the Ward identity. Keeping in (23) the term proportional to $\omega$ and integrating by parts one gets

$$\Pi(\omega) = -i\omega \frac{2Q^2 \gamma^2 D^2}{\pi T} \int \frac{dq}{(2\pi)^3} q^2 \int_{-\infty}^{\infty} dz \frac{[\text{Im} L^R(q, -iz)]^2}{sh^2 \frac{z}{4T^2}}. \quad (24)$$

Expanding $sh^2/4T$ at $z \ll 4T$ and integrating over $dz$ we obtain

$$\Pi(\omega) = -i\omega \frac{Q^2 \gamma^2 D^2}{2\nu^2} \int \frac{dq}{(2\pi)^3} \frac{q^2}{(\epsilon + \frac{\pi}{8\nu} dq^2)^3} = -i\omega \frac{3Q^2}{64} \left( \frac{\pi}{8T^2D} \right)^{-1/2} \epsilon^{-1/2}. \quad (25)$$

As expected, the polarization operator is singular function at $T \to T_c$ with the $(T-T_c)^{-1/2}$ singularity.

V. PHOTON EMISSION RATE

The thermal emission rate of soft photons with energy $\omega$ is related to the retarded photon self-energy as \[45\]

$$\omega \frac{dR}{d^3k} = -\frac{2}{(2\pi)^3} \text{Im} \Pi(\omega) \frac{1}{e^{\omega/T} - 1}. \quad (26)$$
Here $\Pi(\omega)$ is the transverse projection of $\Pi_\mu^\nu$, the longitudinal projection vanishes at $\mathbf{k} = 0$.

$$\omega \frac{dR}{d^3k} = \frac{3Q^2T}{28\pi^3} \left( \frac{\pi D}{8T} \right)^{-1/2} \varepsilon^{-1/2}. \tag{27}$$

Equation (27) is valid to order $\epsilon^2$ in electromagnetic interaction and to all orders in strong interaction \cite{5}. Expression (27) corresponds to the diagram shown in Fig.1. It describes the emission of soft photons with $\omega \ll T$ and is applicable within the pre-critical region $\delta T/T_c = Gi \sim 10^{-2}$. The main feature of the emission rate (27) is its steep rise as $T$ is approaching $T_c$ from above. In Fig.2 the photon production rate is plotted as a function of $(T - T_c)$ for $T_c = 50$ MeV and $T_c = 80$ MeV. The last value is chosen in order to compare the result given by (27) with that calculated in \cite{16}. The direct confrontation of the two investigations is not possible. In both cases the density is few times the nuclear saturation density. The difference is that in \cite{16} the quark matter is supposed to be in a color superconducting CFL phase with quarks of three flavors $u$, $d$, and $s$ participating in pairing. In this work we consider the precursor virtual pairing of $u$ and $d$ quarks at the temperature just above the critical one for the formation of the condensate. The bird’s-eye view is that in both studies the characteristics soft photon emission rate is around $10^{-4}$ fm$^{-4}$GeV$^{-2}$, see Fig.2 here and Fig.12 of \cite{16}.

The soft photon emissivity is closely related to the electrical conductivity of quark matter \cite{12, 14, 46, 47}. One can write the following equation for the electric current

$$\mathbf{j}(x) = - \int \Pi(x - y) \mathbf{A}(y) d^4y. \tag{28}$$

Replacing in Fourier transform of (28) $\mathbf{A}(\mathbf{k}, \omega) = \mathbf{E}(\mathbf{k}, \omega)/i\omega$ and comparing with
\[ j = \sigma E \] we obtain \[ \sigma(\omega) = -\frac{1}{\omega} \text{Im} \Pi(\omega). \] (29)

Comparison of (26) and (29) gives

\[ \omega \frac{dR}{d^3k} = \frac{2}{(2\pi)^3} T \sigma(\omega). \] (30)

Note that \( \sigma(\omega) \) is of the same order \( \alpha \) in electromagnetic interaction as \( \omega dR/d^3k \).

The appearance of an additional factor \( \alpha \) in the right-hand side of (II.16) of [13], (7) of [46] and (25) of [14] is unclear to the present author. Possibly this is some problem of notations. One finds a large number of the quark matter electrical conductivity calculations in the literature, see, e.g., [47] and references therein. Most of the results on \( \sigma \) were obtained for \( \mu = 0 \). Equations (25) and (29) yields for \( \sigma \) at \( T = 0.05 \text{ GeV}, \tau = 0.2 \text{ fm}, \varepsilon^{-1/2} = 20 \) the result \( \sigma = 0.09 \text{ fm}^{-1} \). This value was previously obtained in our paper dedicated to the electrical conductivity of quark matter.
VI. CONCLUSIONS

In this paper we have investigated the soft photon emission rate from dense quark matter in the pre-critical region. This part of the QCD phase diagram is up to now to a great extent kept in the dark both from the experimental and theoretical sides. We pursued the approach based on the Aslamazov-Larkin diagram which proved to be very successful in condensed matter theory. For quark matter this attitude allowed to describe the transport anomalies near the phase transition temperature \[ T_c \] \[ \zeta \sim \varepsilon^{-3/2} \] \[ \nu \approx 0.6, \alpha = 0.11 \] predicted in \[ d = 4 - \varepsilon \] renormalisation, modes coupling, or isomorphism between the quark fluid and 3d Ising system \[ 50-54 \].

The most important feature of the soft photon emissivity rate is its rise when the temperature approaches \[ T_c \] from above. The origin of this phenomenon is the formation of the slow fluctuation made in the quark matter. This excitation is described by the fluctuation propagator which is singular at \[ T_c \] in the limit \( \omega \to 0, k \to 0 \). The AL diagram contains two such propagators. The enhancement of the soft photon production near \( T_c \) may be a tentative proposal for the NICA/FAIR investigation.

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