Measurement of the spectral function for the $\tau \to K^- K_\nu$ decay

J. P. Lees,1 V. Poireau,1 V. Tisserand,1 E. Grauges,2 A. Palano,3 G. Eigen,4 D. N. Brown,5 Yu. G. Kolomensky,5 M. Fritsch,6 H. Koch,6 T. Schroeder,6 C. Hearty,7a,b T. S. Mattison,7b J. A. McKenna,7b R. Y. So,7b V. E. Blinov,8a,8b,8c A. R. Buzykaev,8a
V. P. Druzhinin,8a,b V. B. Golubev,8a,b E. A. Kozyrev,8a,b E. A. Kravchenko,8a,b A. P. Onuchin,8a,b S. I. Serednyakov,8a,8b Yu. I. Skovpen,8a,8b E. P. Solodov,8a,b K. Yu. Todyshov,8a,b A. J. Lankford,9 J. W. Gary,10 O. Long,10 A. M. Eisner,11 W. S. Lockman,11 W. Panduro Vazquez,11 D. S. Chao,12 C. H. Cheng,12 B. Echenard,12 K. T. Flood,12 D. G. Hiltunen,12 J. K. Yin,12 Y. Li,12 T. S. Miyashita,12 P. Omgongkolkul,12 F. C. Porter,12 M. Röhrenk,12 Z. Huard,13 B. T. Meadows,13 B. G. Pushpawela,13 M. D. Sokoloff,13 L. Sun,13 J. G. Smith,14 S. R. Wagner,15 D. Bernard,15 M. Verderi,15 D. Bettoni,16a C. Bozzi,16a R. Calabrese,16a,16b G. Cibinetto,16a,16b E. Fioravanti,16a,16b I. Garzia,16a,16b...

University of California at Santa Cruz, Institute for Particle Physics, Santa Cruz, California 95064, USA

University of British Columbia, Vancouver, British Columbia, Canada V6T 1Z1

INFN Sezione di Bari and Dipartimento di Fisica, Università di Bari, I-70126 Bari, Italy

Ruhr Universität Bochum, Institut für Experimentalphysik 1, D-44780 Bochum, Germany

Lawrence Berkeley National Laboratory and University of California, Berkeley, California 94720, USA

University of British Columbia, Vancouver, British Columbia, Canada V6T 1Z1

University of California at Irvine, Irvine, California 92697, USA

University of California at Riverside, Riverside, California 92521, USA

University of Bergen, Institute of Physics, N-5007 Bergen, Norway

Novosibirsk State University, Novosibirsk 630090, Russia

Novosibirsk State Technical University, Novosibirsk 630092, Russia

University of California at Santa Barbara, Santa Barbara, California 93106, USA

University of California at Santa Cruz, Institute for Particle Physics, Santa Cruz, California 95064, USA

University of California at Riverside, Riverside, California 92521, USA

University of Colorado, Boulder, Colorado 80309, USA

Laboratoire Leprince-Ringuet, École Polytechnique, CNRS/IN2P3, F-91128 Palaiseau, France

(Collaboration)

1Laboratoire d’Annecy-le-Vieux de Physique des Particules (LAPP), Université de Savoie, CNRS/IN2P3, F-74941 Annecy-Le-Vieux, France

2Universitat de Barcelona, Facultat de Física, Departament ECM, E-08028 Barcelona, Spain

3INFN Sezione di Bari and Dipartimento di Fisica, Università di Bari, I-70126 Bari, Italy

4University of Bergen, Institute of Physics, N-5007 Bergen, Norway

5Lawrence Berkeley National Laboratory and University of California, Berkeley, California 94720, USA

6Ruhrg Universität Bochum, Institut für Experimentalphysik 1, D-44780 Bochum, Germany

7Institute of Particle Physics, Vancouver, British Columbia, Canada V6T 1Z1

8Budker Institute of Nuclear Physics SB RAS, Novosibirsk 630090, Russia

9Novosibirsk State University, Novosibirsk 630090, Russia

10University of California at Irvine, Irvine, California 92697, USA

11University of California at Riverside, Riverside, California 92521, USA

12California Institute of Technology, Pasadena, California 91125, USA

13University of Cincinnati, Cincinnati, Ohio 45221, USA

14University of Colorado, Boulder, Colorado 80309, USA

15Laboratoire Leprince-Ringuet, École Polytechnique, CNRS/IN2P3, F-91128 Palaiseau, France

16Laboratoire de Physique des Particules de Franche-Comté, Université de Franche-Comté, CNRS/IN2P3, F-25000 Besançon, France

17University of British Columbia, Vancouver, British Columbia, Canada V6T 1Z1

18INFN Sezione di Bari and Dipartimento di Fisica, Università di Bari, I-70126 Bari, Italy

19University of Bergen, Institute of Physics, N-5007 Bergen, Norway

20University of California at Irvine, Irvine, California 92697, USA

21University of California at Riverside, Riverside, California 92521, USA

22University of California at Santa Cruz, Institute for Particle Physics, Santa Cruz, California 95064, USA

23University of California at Riverside, Riverside, California 92521, USA

24University of Colorado, Boulder, Colorado 80309, USA

25Laboratoire Leprince-Ringuet, École Polytechnique, CNRS/IN2P3, F-91128 Palaiseau, France

1Laboratoire d’Annecy-le-Vieux de Physique des Particules (LAPP), Université de Savoie, CNRS/IN2P3, F-74941 Annecy-Le-Vieux, France

2Universitat de Barcelona, Facultat de Física, Departament ECM, E-08028 Barcelona, Spain

3INFN Sezione di Bari and Dipartimento di Fisica, Università di Bari, I-70126 Bari, Italy

4University of Bergen, Institute of Physics, N-5007 Bergen, Norway

5Lawrence Berkeley National Laboratory and University of California, Berkeley, California 94720, USA

6Ruhrg Universität Bochum, Institut für Experimentalphysik 1, D-44780 Bochum, Germany

7Institute of Particle Physics, Vancouver, British Columbia, Canada V6T 1Z1

8Budker Institute of Nuclear Physics SB RAS, Novosibirsk 630090, Russia

9Novosibirsk State University, Novosibirsk 630090, Russia

10University of California at Irvine, Irvine, California 92697, USA

11University of California at Riverside, Riverside, California 92521, USA

12California Institute of Technology, Pasadena, California 91125, USA

13University of Cincinnati, Cincinnati, Ohio 45221, USA

14University of Colorado, Boulder, Colorado 80309, USA

15Laboratoire Leprince-Ringuet, École Polytechnique, CNRS/IN2P3, F-91128 Palaiseau, France

PHYSICAL REVIEW D 98, 032010 (2018)
INFN Sezione di Ferrara, 1-44122 Ferrara, Italy
Dipartimento di Fisica e Scienze della Terra, Università di Ferrara, 1-44122 Ferrara, Italy
INFN Laboratori Nazionali di Frascati, 1-00044 Frascati, Italy
INFN Sezione di Genova, 1-16146 Genova, Italy
Humboldt-Universität zu Berlin, Institut für Physik, D-12489 Berlin, Germany
Indian Institute of Technology Guwahati, Guwahati, Assam 781 039, India
University of Iowa, Iowa City, Iowa 52242, USA
Iowa State University, Ames, Iowa 50011, USA
Johns Hopkins University, Baltimore, Maryland 21218, USA
Laboratoire de l’Accélérateur Linéaire, IN2P3/CNRS et Université Paris-Sud 11, Centre Scientifique d’Orsay, F-91898 Orsay Cedex, France
Lawrence Livermore National Laboratory, Livermore, California 94550, USA
University of Liverpool, Liverpool L69 7ZE, United Kingdom
Queen Mary, University of London, London E1 4NS, United Kingdom
University of London, Royal Holloway and Bedford New College, Egham, Surrey TW20 0EX, United Kingdom
University of Louisville, Louisville, Kentucky 40292, USA
Johannes Gutenberg-Universität Mainz, Institut für Kernphysik, D-55099 Mainz, Germany
University of Manchester, Manchester M13 9PL, United Kingdom
University of Maryland, College Park, Maryland 20742, USA
Massachusetts Institute of Technology, Laboratory for Nuclear Science, Cambridge, Massachusetts 02139, USA
Institute of Particle Physics, Montréal, Québec, Canada H3A 2T8
McGill University, Montréal, Québec, Canada H3A 2T8
INFN Sezione di Milano, I-20133 Milano, Italy
Dipartimento di Fisica, Università di Milano, I-20133 Milano, Italy
University of Mississippi, University, Mississippi 38677, USA
Université de Montréal, Physique des Particules, Montréal, Québec, Canada H3C 3J7
INFN Sezione di Napoli and Dipartimento di Scienze Fisiche, Università di Napoli Federico II, I-80126 Napoli, Italy
NIKHEF, National Institute for Nuclear Physics and High Energy Physics, NL-1009 DB Amsterdam, The Netherlands
University of Notre Dame, Notre Dame, Indiana 46556, USA
Ohio State University, Columbus, Ohio 43210, USA
INFN Sezione di Padova, I-35131 Padova, Italy
Dipartimento di Fisica, Università di Padova, I-35131 Padova, Italy
Laboratoire de Physique Nucléaire et de Hautes Energies, IN2P3/CNRS, Université Pierre et Marie Curie-Paris6, Université Denis Diderot-Paris7, F-75252 Paris, France
INFN Sezione di Perugia, I-06123 Perugia, Italy
Dipartimento di Fisica, Università di Perugia, I-06123 Perugia, Italy
INFN Sezione di Pisa, I-56127 Pisa, Italy
Dipartimento di Fisica, Università di Pisa, I-56127 Pisa, Italy
Scuola Normale Superiore di Pisa, I-56127 Pisa, Italy
Princeton University, Princeton, New Jersey 08544, USA
INFN Sezione di Roma, I-00185 Roma, Italy
Dipartimento di Fisica, Università di Roma La Sapienza, I-00185 Roma, Italy
Universität Rostock, D-18051 Rostock, Germany
Rutherford Appleton Laboratory, Chilton, Didcot, Oxon OX11 0QX, United Kingdom
CEA, Irfu, SPP, Centre de Saclay, F-91191 Gif-sur-Yvette, France
SLAC National Accelerator Laboratory, Stanford, California 94309 USA
University of South Carolina, Columbia, South Carolina 29208, USA
Southern Methodist University, Dallas, Texas 75275, USA
St. Francis Xavier University, Antigonish, Nova Scotia, Canada B2G 2W5
Stanford University, Stanford, California 94305, USA
State University of New York, Albany, New York 12222, USA
Tel Aviv University, School of Physics and Astronomy, Tel Aviv 69978, Israel
University of Tennessee, Knoxville, Tennessee 37996, USA
University of Texas at Austin, Austin, Texas 78712, USA
University of Texas at Dallas, Richardson, Texas 75083, USA
INFN Sezione di Torino, I-10125 Torino, Italy
The decay $\tau^- \to K^- K_S \nu_\tau$ has been studied using $430 \times 10^6 e^+ e^- \to \tau^+ \tau^-$ events produced at a center-of-mass energy around 10.6 GeV at the PEP-II collider and studied with the BABAR detector. The mass spectrum of the $K^- K_S$ system has been measured and the spectral function has been obtained. The measured branching fraction $B(\tau^- \to K^- K_S \nu_\tau) = (0.739 \pm 0.011({\text{stat}}) \pm 0.020({\text{syst}})) \times 10^{-3}$ is found to be in agreement with earlier measurements.

DOI: 10.1103/PhysRevD.98.032010

I. INTRODUCTION

The $\tau$ lepton provides a remarkable laboratory for studying many open questions in particle physics. With a large statistics of about $10^9 \tau$’s produced in $e^+ e^-$ annihilation at the BABAR experiment, various aspects can be studied, e.g., improving the precision of spectral functions describing the mass distribution of the hadronic decays of the $\tau$. In this work, we analyze the $\tau^- \to K^- K_S \nu_\tau$ decay and measure the spectral function of this channel defined as [1]

$$V(q) = \frac{m_\tau^3}{12\pi C(q)} \left[ \frac{B(\tau^- \to K^- K_S \nu_\tau)}{V_{ud}} \right] \frac{dN}{dq} / N,$$

where $m_\tau$ is the $\tau$ mass [2], $q \equiv m_{K^- K_S}$ is the invariant mass of the $K^- K_S$ system, $V_{ud}$ is an element of the Cabibbo-Kobayashi-Maskawa matrix [2], $(dN/dq)/N$ is the normalized $K^- K_S$ mass spectrum, and $C(q)$ is the phase space correction factor given by the following formula:

$$C(q) = q(m_\tau^2 - q^2)^2(m_\tau^2 + 2q^2).$$

According to the conserved-vector-current hypothesis [1], the $\tau^- \to K^- K_S \nu_\tau$ spectral function is related to the isovector part ($I = 1$) of the $e^+ e^- \to K\bar{K}$ cross section:

$$\sigma_{e^+ e^- \to K\bar{K}}(q) = \frac{4\pi^2 \alpha^2}{q^2} V(q),$$

where $\alpha$ is the fine structure constant. The cross sections $e^+ e^- \to K^+ K^-$ and $e^+ e^- \to K_S K_L$ have been recently measured by the BABAR [3,4] and SND experiments [5]. Combining data from the $\tau^- \to K^- K_S \nu_\tau$ with $e^+ e^- \to K\bar{K}$ measurements, the moduli of the isovector and isoscalar form factors and the relative phase between them can be obtained in a model-independent way.

The branching fraction for the $\tau^- \to K^- K_S \nu_\tau$ decay has been measured with relatively high (3%) precision by the Belle experiment [6]. The $K^- K_S$ mass spectrum was measured by the CLEO experiment [7]. In the CLEO analysis, a data set of $2.7 \times 10^6$ produced $\tau$ pairs was used, and about 100 events in the decay channel $\tau^- \to K^- K_S \nu_\tau$ were selected. In this work, using about $\sim 10^9 \tau$ leptons, we significantly improve upon the measurement of the spectral function for the $\tau^- \to K^- K_S \nu_\tau$ decay.

II. DATA USED IN THE ANALYSIS

We analyze a data sample corresponding to an integrated luminosity of 468 fb$^{-1}$ recorded with the BABAR detector [8,9] at the SLAC PEP-II asymmetric-energy $e^+ e^-$ collider.

For simulation of $e^+ e^- \to \tau^+ \tau^-$ events the KK2f Monte Carlo generator [10] is used, which includes higher-order radiative corrections to the Born-level process. Decays of $\tau$ leptons are simulated using the Tauola package [11]. Two separate samples of simulated $e^+ e^- \to \tau^+ \tau^-$ events are used: a generic sample with $\tau$ decaying to all
significant final states, and the signal channel where \( \tau^+ \rightarrow l^+\nu l^+\nu, \) \( l = e \) or \( \mu \) and \( \tau^+ \rightarrow K^-K_S\nu, \) \( \nu \) is a neutrino. To estimate backgrounds, we use a sample of simulated generic \( e^+e^- \rightarrow \tau^+\tau^- \) events after excluding the signal decay channel \( \tau^+\tau^- \) background and a sample containing all events arising from \( e^+e^- \rightarrow q\bar{q}, q = u, d, s, c \) and \( e^+e^- \rightarrow B\bar{B} \) processes \( (q\bar{q} \text{ background}) \). The \( q\bar{q} \) background events where \( q = u, d, s, c \) are generated using the JETSET generator [12], while \( B\bar{B} \) events are simulated with EVTGEN [13]. The detector response is simulated with GEANT4 [14]. The equivalent luminosity of the simulated sample is 2-3 times higher than the integrated luminosity in data.

III. EVENT SELECTION

We select \( e^+e^- \rightarrow \tau^+\tau^- \) events with the \( \tau^+ \) decaying leptonically \( (\tau^+ \rightarrow l^+\nu l^+\nu, l = e \) or \( \mu \)) and the \( \tau^- \) decaying to \( K^-K_S\nu, \) \( \nu \) is a neutrino. Such events referred to as signal events below. The \( K_S \) candidate is detected in the \( K_S \rightarrow \pi^+\pi^- \) decay mode. The topology of events to be selected is shown in Fig. 1. Unless otherwise stated, all quantities are measured in the laboratory frame. The selected events must satisfy the following requirements:

(i) The total number of charged tracks, \( N_{\text{trk}} \), must be four and the total charge of the event must be zero.

(ii) Among the four charged tracks there must be an identified lepton (electron or muon) and an identified kaon of opposite charge. The track origin point requirements are \( |d_0| < 1.5 \text{ cm} \) and \( |z_0| < 2.5 \text{ cm} \), where \( |d_0| \) and \( |z_0| \) are the distances between the track and the interaction region center in transverse and longitudinal directions with respect to the beams.

(iii) To reject \( \mu \) pairs and Bhabha events, the lepton candidate must have a momentum above \( 1.2 \text{ GeV}/c \), the momentum in the center-of-mass frame (c.m. momentum) must be smaller than \( 4.5 \text{ GeV}/c \), and the cosine of the lepton polar angle \( \cos \theta_l \) must be below 0.9.

(iv) To suppress background from charged pions, the charged kaon candidate must have a momentum, \( p_K \), above \( 0.4 \text{ GeV}/c \) and below \( 5 \text{ GeV}/c \), and the cosine of its polar angle must lie between \( -0.7374 \) and \( 0.9005 \).

(v) The two remaining tracks, assumed to be pions, form the \( K_S \) candidate. The \( \pi^+\pi^- \) invariant mass must lie within \( 25 \text{ MeV}/c^2 \) of the nominal \( K_S \) mass, 497.6 \text{ MeV}/c^2. The \( K_S \) flight length \( r_{K_S} \), measured as the distance between the \( \pi^+\pi^- \) vertex and the collision point, must be larger than 1 cm. The \( r_{K_S} \) distributions for data events and simulated signal events are shown in Fig. 2.

(vi) The total energy in neutral clusters, \( \Sigma E_{\gamma} \), must be less than 2 \text{ GeV} \) (Fig. 3). Here, a neutral cluster is defined as a local energy deposit in the calorimeter with energy above 20 \text{ MeV} and no associated charged track.

(vii) The magnitude of the thrust [15,16] for the event, calculated using charged tracks only, must be greater than 0.875.

(viii) The angle defined by the momentum of the lepton and that of the \( K^-K_S \) system in the c.m. frame must be larger than 110 degrees.

As a result of applying these selection criteria the \( \tau \) background is suppressed by 3.5 orders of magnitude, and the \( q\bar{q} \) background by 5.5 orders.

![FIG. 1. Schematic view of the \( \tau \) decay chains in \( e^+e^- \rightarrow \tau^+\tau^- \) events selected for this analysis. Lepton \( l^+ \) can be electron or muon.](image)

![FIG. 2. The \( K_S \) candidates decay length distribution for data (points with errors), \( \tau^+\tau^- \) simulation events (solid histogram), and \( \tau \) background simulation (dashed histogram). The vertical line indicates the boundary of the selection condition.](image)
IV. DETECTION EFFICIENCY

The detection efficiency obtained after applying the selection criteria is calculated using signal Monte Carlo simulation as a function of the true $m_{K^-K_S}$ mass and is shown in Fig. 4. The efficiency is weakly dependent on $m_{K^-K_S}$. The average efficiency over the mass spectrum is about 13%. It should be noted that the $K^-K_S$ mass resolution is about 2–3 MeV/$c^2$, significantly smaller than the size of the mass bin (40 MeV/$c^2$) used in Fig. 4. Therefore, in the following we neglect the effects of the finite $K^-K_S$ mass resolution.

To correct for the imperfect simulation of the kaon identification requirement, the particle identification (PID) efficiencies have been compared for data and simulation on high purity control samples of kaons from $D^* \rightarrow \pi^+D^0$, $D^0 \rightarrow K^-\pi^+$ decays [17]. We correct the simulated efficiency using the measured ratios of the efficiencies measured in data and Monte Carlo, in bins of the kaon candidate momentum and polar angle. The resulting correction factor as a function of $m_{K^-K_S}$ is shown in Fig. 5.

V. SUBTRACTION OF NON-$K_S$ BACKGROUND

The $\pi^+\pi^-$ mass spectra for $K_S$ candidates in data and simulated signal events are shown in Fig. 6. The data spectrum consists of a peak at the $K_S$ mass and a flat background. To subtract the non-$K_S$ background, the following procedure is used. The signal region is set to $\pi^+\pi^-$ masses within 0.0125 GeV/$c^2$ of the $K_S$ mass (indicated by arrows in Fig. 6), and the sidebands are set to between 0.0125 and 0.0250 GeV/$c^2$ away from the nominal $K_S$ mass. Let $\beta$ be the fraction of events with a true $K_S$ that fall in the sidebands, and let $\alpha$ be the fraction of non-$K_S$ events that fall in the sidebands. The total number of events in the signal region plus the sidebands, $N$, and the number of events in the sidebands, $N_{sb}$, depend on the number of true $K_S$, $N_{K_S}$, and the number of non-$K_S$ background events, $N_b$, according to the following relation:
The value of $\beta$ is determined using $\tau$ signal simulation. It is found to be nearly independent of the $m_{K^-K_S}$ mass and is equal to $0.0315 \pm 0.0015$. The value of $\alpha$ is expected to be 0.5 for a uniformly distributed background. This is consistent with the value $0.499 \pm 0.005$ obtained on simulated $\tau^+\tau^-$ background events. The non-$K_S$ background is subtracted in each $m_{K^-K_S}$ bin. Its fraction is found to be about 10% of the selected events with $m_{K^-K_S}$ near and below 1.3 GeV/$c^2$ and increases up to 50% above 1.6 GeV/$c^2$.

VI. SUBTRACTION OF $\tau$-BACKGROUND WITH A $\pi^0$

Although the studied process $\tau^- \rightarrow K^-K_S\nu_\tau$ is not supposed to contain a $\pi^0$ in the final state, some events from background processes with a $\pi^0$ pass the selection criteria. In the following, we describe how the $\pi^0$ background contribution is subtracted.

The $K^-K_S$ mass spectra for selected data and $\tau^+\tau^-$ simulated events after subtraction of the non-$K_S$ background are shown in Fig. 7. According to the simulation, the number of signal and $\tau$-background events are of the same order of magnitude. The $\tau^+\tau^-$ background consists of events with the decay $\tau^- \rightarrow K^-K_S\pi^0\nu_\tau$ (79%), events with a misidentified kaon from decays $\tau^- \rightarrow \pi^-K_S\nu_\tau$ (10%) and $\tau^- \rightarrow \pi^-K_S\pi^0\nu_\tau$ (3%), and events with a misidentified lepton mainly from the decays $\tau^+ \rightarrow \pi^+\nu_\tau$ and $\tau^+ \rightarrow \pi^0\nu_\tau$ (7%). Thus, more than 80% of the background events contain a $\pi^0$ in the final state. It should be noted that events with a misidentified lepton have the same $m_{K^-K_S}$ distribution as signal events.

The branching fractions for the background modes without a $\pi^0$, $\tau^- \rightarrow \pi^-K_S\nu_\tau$, and $\tau^- \rightarrow \pi^+\nu_\tau$, have been measured with high precision (1.7% and 0.5%) [2]. The hadronic mass spectrum for $\tau^- \rightarrow \pi^-K_S\nu_\tau$ is also well known [18] and this decay proceeds mainly via the $K^0$ (892) intermediate state. Therefore all $\tau^+\tau^-$ background without a $\pi^0$ is subtracted using Monte Carlo simulation. The amount of $q\bar{q}$ background, not shown in Fig. 7, is about 2% of selected data events. The part of this background without a $\pi^0$ is also subtracted using Monte Carlo simulation.

The branching fractions for the background modes $\tau^- \rightarrow K^-K_S\pi^0\nu_\tau$, $\tau^- \rightarrow \pi^-K_S\pi^0\nu_\tau$, and $\tau^- \rightarrow \pi^+\nu_\tau$, are measured with a precision of 4.7%, 3.4%, and 0.4%, respectively. The hadronic mass spectrum is well known only for the last decay [19]. For the two other decays, only low-statistics measurements [7] are available. Therefore, we use the data to subtract the $\tau$-background with a $\pi^0$ from the $K^-K_S$ mass spectrum. To do this, the selected events are divided into two classes, without and with a $\pi^0$ candidate, which is defined as a pair of photons with an invariant mass in the range 100–160 MeV/$c^2$.

![Fig. 6](image_url1)

**Fig. 6.** The $\pi^+\pi^-$ mass spectrum for $K_S$ candidates in data (points with errors) and signal simulation (histogram). Between the two vertical lines there is a signal region used in the procedure of non-$K_S$ background subtraction.

![Fig. 7](image_url2)

**Fig. 7.** The $m_{K^-K_S}$ spectra for data (points with errors), $\tau^+\tau^-$ simulation events (solid histogram), and $\tau$ background simulation (dashed histogram). The non-$K_S$ background is subtracted.
On the resulting sample, the numbers of signal \( N_s \) and background \( \tau^+\tau^- \) events containing a \( \pi^0 \) candidate \( N_b \) are obtained in each \( m_{K^-K_S} \) bin:

\[
N_{0\pi^0} = (1 - \epsilon_b)N_s + (1 - \epsilon_s)N_b, \tag{6a}
\]

\[
N_{1\pi^0} = \epsilon_sN_s + \epsilon_bN_b, \tag{6b}
\]

where \( N_{0\pi^0} \) and \( N_{1\pi^0} \) are the numbers of selected data events with zero and at least one \( \pi^0 \) candidate, and \( \epsilon_s \) \( (\epsilon_b) \) is the probability for signal (background) \( \tau^+\tau^- \) events to be found in events with at least one \( \pi^0 \) candidate calculated using Monte Carlo simulation. The values \( \epsilon_s \) and \( \epsilon_b \) for each bin in \( m_{K^-K_S} \) are measured in Monte Carlo by counting how many signal and background event candidates contain a \( \pi^0 \) candidate. Figure 8 shows the \( \epsilon_s \) and \( \epsilon_b \) measured in Monte Carlo as a function of \( m_{K^-K_S} \). The efficiency \( \epsilon_b \) is corrected to take into account the different \( \pi^0 \) efficiency between data and Monte Carlo as measured on data and simulated control samples in the ISR \( e^+e^- \rightarrow o(783)\gamma \rightarrow \pi^+\pi^-\pi^0\gamma \) process [20]. The average correction is \( \delta = 0.976 \pm 0.008 \). The non-zero value of \( \epsilon_s \) is due to random combinations of two spurious photons originating from beam background or nuclear interactions of charged kaons or pions. The beam-generated background is simulated by using special background events recorded during normal data-taking conditions but with a randomly generated trigger. These events are superimposed on simulated events. The following procedure is used to measure \( \epsilon_s \) on data events. We compare the solution of Eqs. (6a) and (6b) described above with the solution of the same system, in which the number of events with \( \pi^0 \) is determined from the fit to the two-photon invariant mass spectrum of \( \pi^0 \) candidates. Since the mass dependence of \( \epsilon_s \) and \( \epsilon_b \) is mild (Fig. 8), this comparison is performed using the full sample of selected events without splitting the sample into \( K^-K_S \) mass bins. The two-photon mass spectrum of \( \pi^0 \) candidates in data is shown in Fig. 9.

The spectrum in Fig. 9 is fitted by a sum of a Gaussian and a flat component. The numbers \( N_{1\pi^0} \) and \( N_{0\pi^0} \) on the left side of Eqs. (6a) and (6b) are substituted by \( N_{1\pi^0}^* = N_{1\pi^0} - N_{1\pi^0}^\text{lin} \) and \( N_{0\pi^0}^* = N_{0\pi^0} + N_{1\pi^0}^\text{lin} \), where \( N_{1\pi^0}^\text{lin} \) is the number of events under the flat component, obtained after fitting the \( \pi^0 \) spectrum in Fig. 9. The value \( \epsilon_b \) is substituted by \( \epsilon_b^* \), where \( w = 0.682 \pm 0.010 \) is the fraction of events with a reconstructed \( \pi^0 \) for simulated \( \tau^+\tau^- \) background (Fig. 9). The term “reconstructed \( \pi^0 \)” corresponds to \( \pi^0 \) s in the Gaussian part in Fig. 9. The modified system of equations is

\[
N_{0\pi^0}^* = N_s + (1 - \epsilon_b^*)N_b, \tag{7a}
\]

\[
N_{1\pi^0}^* = \epsilon_b^*N_b, \tag{7b}
\]

In Eqs. (7a) and (7b) the top line contains all events without a reconstructed \( \pi^0 \), while the lower line contains events with at least one reconstructed \( \pi^0 \). After subtracting the spurious \( \pi^0 \) s corresponding to the flat background in Fig. 9, Eqs. (7a) and (7b) no longer contains \( \epsilon_s \) nor a contribution from the \( \pi^0 \) background.

The average value of \( \epsilon_b \) from Fig. 8 is \( 0.720 \pm 0.003 \), giving \( \epsilon_b^* = 0.491 \pm 0.008 \) on the average. This value is then corrected by the reconstructed \( \pi^0 \) efficiency correction.

**FIG. 8.** The probabilities \( \epsilon_s \) and \( \epsilon_b \) used in Eqs. (6a) and (6b) as functions of the \( K^-K_S \) mass, measured on simulated events.

**FIG. 9.** Two-photon invariant mass spectrum of \( \pi^0 \) candidates in data. The curve corresponds to the fit function, described in text.
correction in flat background simulation. Then the number of simulated $\tau^+\tau^-$ background events without a $\pi^0$ is multiplied by a factor of $p = 0.92 \pm 0.02$ to take into account the difference between experimental $\tau$ branching fractions and branching fractions used in the Tauola Monte Carlo generator. With these corrected values for $\epsilon_s$ and $\epsilon_b$ we solve Eqs. (6a) and (6b) for each $K^-K_S$ mass bin and obtain mass spectra for signal ($N_s$) and background ($N_b$).

The efficiency corrected signal mass spectrum, using the signal efficiency from Fig. 4, is shown in Fig. 10 (top), in comparison with the simulation. The $\tau$-pair $m_{K^-K_S}$ background spectrum [Fig. 10 (bottom)] is compared with simulation without efficiency correction. Spectra are normalized to the same number of events. We find a substantial difference between data and simulation for the signal spectrum, and better agreement for the background spectrum.

VII. SYSTEMATIC UNCERTAINTIES

This section lists all the uncertainties in the parameters used in this analysis, and estimates the overall systematic uncertainty on the $\tau^+\tau^-$ branching fraction and the $K^-K_S$ mass spectrum.

The subtraction of non-$K_S$ background is described in Sec. V. To check the procedure of the non-$K_S$ background subtraction, we varied the coefficients of $\alpha$ and $\beta$ within their uncertainties, which leads to a systematic uncertainty of 0.4% in the $\tau^+\tau^-\to K^-K_S\nu\bar{\nu}$ branching fraction. This uncertainty is independent of the $K^-K_S$ mass.

The PID corrections were discussed in Sec. IV. The systematic uncertainty due to data-Monte Carlo simulation difference in particle identification is taken to be 0.5%, independent of the $K^-K_S$ mass. The uncertainty on how well the Monte Carlo simulates the tracking efficiency is estimated to be 1%.

Figure 11 shows the $m_{K^-K_S}$ spectra for selected data events with and without a $\pi^0$ candidate near the endpoint $m_{K^-K_S} = m_\tau$ compared to simulated $q\bar{q}$ events. It appears that the number of data and simulated $q\bar{q}$ events are in reasonable agreement at $m_{K^-K_S} > m_\tau$, where all data events are expected to be from the $q\bar{q}$ background. We take the observed difference between data and Monte Carlo near the end point $M_{K^-K_S} = m_\tau$ in Fig. 11 as an uncertainty on the $q\bar{q}$ background. This leads to an uncertainty on $B(\tau^-\to K^-\nu_\tau)$ of 0.5%.

The uncertainty associated with the subtraction of the $\tau^+\tau^-$ background with $\pi^0$'s is estimated by varying the efficiencies $\epsilon_s$ and $\epsilon_b$ used in Eqs. (6a) and (6b) within their systematic uncertainties: 5% in $\epsilon_s$ (uncertainty in the number of spurious $\pi^0$'s) and 6% in $\epsilon_b$ (uncertainty in numbers of both spurious and reconstructed $\pi^0$'s). The corresponding contribution to the systematic uncertainty on $B(\tau^-\to K^-\nu_\tau)$ is 2.3%. For the $m_{K^-K_S}$ spectrum this
uncertainty varies from 9% at $m_{K^-K^0} < 1.1$ GeV/$c^2$ to 1% at 1.7 GeV/$c^2$.

The 2% uncertainty in the correction factor $p$ (Sec. VI), associated with $\tau$ branching fractions without a $\pi^0$, leads to the 0.3% uncertainty in the branching ratio. The mass-dependent uncertainty is 2% at $K^-K^0$ mass below 1.1 GeV and 0.1% for 1.7 GeV/$c^2$.

The systematic uncertainties from different sources, shown in Table I, are combined in quadrature. The total systematic uncertainty for the branching fraction $B(\tau^- \to K^-K^0\nu_\tau)$ is 2.7%. The systematic uncertainties for the mass spectrum are listed in Table II. They gradually decrease from $\approx 9\%$ at $m_{K^-K^0} = 1$ GeV/$c^2$ to 1.5% at $m_{K^-K^0} = m_\tau$. Near the maximum of the mass spectrum (1.3 GeV/$c^2$) the uncertainty is about 2.5%.

\section*{VIII. The Results}

The branching ratio of the $\tau^- \to K^-K^0\nu_\tau$ decay is obtained using the following expression:

\[ B(\tau^- \to K^-K^0\nu_\tau) = \frac{N_{\exp}}{2LB_{\lep}\sigma_{\tau\tau}} = (0.739 \pm 0.011 \pm 0.020) \times 10^{-3}, \]

where $N_{\exp} = 223741 \pm 3461$ (error is statistical) is the total number of signal events in the spectrum in Fig. 12, $L = 468.0 \pm 2.5$ fb$^{-1}$ is the $BABAR$ integrated luminosity [21], $\sigma_{\tau\tau} = 0.919 \pm 0.003$ nb is the $e^+e^- \to \tau^+\tau^-$ cross section at 10.58 GeV [10] and $B_{\lep} = 0.3521 \pm 0.0006$ is
the world average sum of electronic and muonic branching fractions of the \( \tau \) lepton [2]. The first uncertainty in (8) is the statistical, the second is systematic. Our result agrees well with the Particle Data Group (PDG) value \( (0.740 \pm 0.025) \times 10^{-3} \) [2], which is determined mainly by the recent Belle measurement \( (0.740 \pm 0.007 \pm 0.027) \times 10^{-3} \) [6].

The measured mass spectrum \( m_{K^-K_S} \) for the \( \tau^- \rightarrow K^-K_S\nu \tau \) decay is shown in Fig. 12 and listed in Table II. Our \( m_{K^-K_S} \) spectrum is compared with the CLEO measurement [7]. The BABAR and CLEO spectra are in good agreement. The spectral function \( V(q) \) calculated using Eq. (1) is shown in Fig. 13 and listed in Table II. Due to the large error in the mass interval \( 1.66-1.78 \) GeV/c\(^2\), which exceeds the scale of Fig. 13, the value of \( V(q) \) in this interval is not shown in Fig. 13.

**IX. CONCLUSIONS**

The \( K^-K_S \) mass spectrum and vector spectral function in the \( \tau^- \rightarrow K^-K_S\nu \tau \) decay have been measured by the BABAR experiment. The measured \( K^-K_S \) mass spectrum is far more precise than CLEO measurement [7] and the branching fraction \( (0.739 \pm 0.011 \pm 0.020) \times 10^{-3} \) is comparable to Belle’s measurement [6].

**ACKNOWLEDGMENTS**

We are grateful for the extraordinary contributions of our PEP-II colleagues in achieving the excellent luminosity and machine conditions that have made this work possible. The success of this project also relies critically on the expertise and dedication of the computing organizations that support BABAR. The collaborating institutions wish to thank SLAC for its support and the kind hospitality extended to them. This work is supported by the U.S. Department of Energy and National Science Foundation, the Natural Sciences and Engineering Research Council (Canada), the Commissariat à l’Energie Atomique and Institut National de Physique Nucléaire et de Physique des Particules (France), the Bundesministerium für Bildung und Forschung and Deutsche Forschungsgemeinschaft (Germany), the Istituto Nazionale di Fisica Nucleare (Italy), the Foundation for Fundamental Research on Matter (The Netherlands), the Research Council of Norway, the Ministry of Education and Science of the Russian Federation, Ministerio de Economía y Competitividad (Spain), the Science and Technology Facilities Council (United Kingdom), and the Binational Science Foundation (U.S.-Israel). Individuals have received support from the Russian Foundation for Basic Research (Grant No. 16-02-00327), the Marie-Curie IEF program (European Union) and the A. P. Sloan Foundation (USA).
[1] Y. S. Tsai, Phys. Rev. D 4, 2821 (1971); 13, 771(E) (1976).
[2] C. Patrignani et al. (Particle Data Group), Chin. Phys. C 40, 100001 (2016) and 2017 update.
[3] J. P. Lees et al. (BABAR Collaboration), Phys. Rev. D 88, 032013 (2013).
[4] J. P. Lees et al. (BABAR Collaboration), Phys. Rev. D 89, 092002 (2014).
[5] M. N. Achasov et al. (SND Collaboration), Phys. Rev. D 94, 112006 (2016).
[6] S. Ryu et al. (Belle Collaboration), Phys. Rev. D 89, 072009 (2014).
[7] T. E. Coan et al. (CLEO Collaboration), Phys. Rev. D 53, 6037 (1996).
[8] B. Aubert et al. (BABAR Collaboration), Nucl. Instrum. Methods Phys. Res., Sect. A 479, 1 (2002).
[9] B. Aubert et al. (BABAR Collaboration), Nucl. Instrum. Methods Phys. Res., Sect. A 729, 615 (2013).
[10] S. Jadach, B. F. Ward, and Z. Was, Comput. Phys. Commun. 130, 260 (2000).
[11] S. Jadach, Z. Was, R. Decker, and J. H. Kühn, Comput. Phys. Commun. 76, 361 (1993).
[12] T. Sjöstrand, S. Mrenna, and P. Skands, J. High Energy Phys. 05 (2006) 026.
[13] D. J. Lange, Nucl. Instrum. Methods Phys. Res., Sect. A 462, 152 (2001).
[14] S. Agostinelli et al. Nucl. Instrum. Methods Phys. Res., Sect. A 506, 250 (2003).
[15] E. Farhi, Phys. Rev. Lett. 39, 1587 (1977).
[16] S. Brandt, C. Peyrou, R. Sosnowski, and A. Wroblewski, Phys. Lett. 12, 57 (1964).
[17] D. Boutigny et al. (BABAR Collaboration), SLAC Report No. SLAC-R-0504, 2010.
[18] D. Epifanov et al. (Belle Collaboration), Phys. Lett. B 654, 65 (2007).
[19] M. Fujikawa et al. (Belle Collaboration), Phys. Rev. D 78, 072006 (2008).
[20] B. Aubert et al. (BABAR Collaboration), Phys. Rev. D 70, 072004 (2004).
[21] J. P. Lees et al. (BABAR Collaboration), Nucl. Instrum. Methods Phys. Res., Sect. A 726, 203 (2013).