Effects of periodic kicking on dispersion and wave packet dynamics in graphene

Adhip Agarwala\textsuperscript{1}, Utso Bhattacharyya\textsuperscript{2}, Amit Dutta\textsuperscript{2}, and Diptiman Sen\textsuperscript{3}

\textsuperscript{1}Department of Physics, Indian Institute of Science, Bengaluru 560012, India
\textsuperscript{2}Department of Physics, Indian Institute of Technology, Kanpur 208016, India
\textsuperscript{3}Centre for High Energy Physics, Indian Institute of Science, Bengaluru 560012, India

We study the effects of \(\delta\)-function periodic kicks on the Floquet energy-momentum dispersion in graphene. We find that a rich variety of dispersions can appear depending on the parameters of the kicking: at certain points in the Brillouin zone, the dispersion can become linear but anisotropic, linear in one direction and quadratic in the perpendicular direction, gapped with a quadratic dispersion, or completely flat (called dynamical localization). We show all these results analytically and demonstrate them numerically through the dynamics of wave packets propagating in graphene. We propose experimental methods for producing these effects.

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I. INTRODUCTION

Quantum systems driven periodically in time have been extensively studied in recent years from many points of view, such as the generation of defects\textsuperscript{1,2}, coherent destruction of tunneling\textsuperscript{3,4} and dynamical freezing\textsuperscript{5,6}, dynamical saturation\textsuperscript{7,8} and localization\textsuperscript{9,10}, dynamical fidelity\textsuperscript{11,12}, edge singularity in the probability distribution of work\textsuperscript{13} and thermalization\textsuperscript{14} (see Ref. 15 for a review). These studies have become more important following the experimental studies for future work. In this work, we will study the effects of periodic kicks on the Floquet energy-momentum dispersion in graphene. We find that a rich variety of dispersions can appear depending on the parameters of the kicking: at certain points in the Brillouin zone, the dispersion can become linear but anisotropic, linear in one direction and quadratic in the perpendicular direction, gapped with a quadratic dispersion, or completely flat (called dynamical localization). We show all these results analytically and demonstrate them numerically through the dynamics of wave packets propagating in graphene. We propose experimental methods for producing these effects.

The plan of our paper is as follows. In Sec. II we recall the Hamiltonian and energy dispersion of graphene. In Sec. III we describe the ideas of periodic \(\delta\)-function kicking of the Hamiltonian and the quasi-energy dispersion for the most general form of kicking. Sec. IV describes how the dynamics of a Gaussian wave packet can be studied numerically. In Sec. V we examine separately the effects of periodic kicking of three different parameters in the Hamiltonian. We show that a rich variety of quasi-energy dispersions can be obtained depending on the kicking parameters. The effects of these dispersions on the wave packet dynamics will be shown in some particularly interesting cases. In Sec. VI we present a general way of understanding the phenomenon of dynamical localization. We end in Sec. VII with a summary of our main results, possible experimental realizations of periodic kicking of the different parameters, and some directions for future work.
belonging to sublattices A and B and are separated by a distance $a$.

II. HAMILTONIAN OF GRAPHENE

We begin with a brief description of the band structure of graphene. The Hamiltonian is given by a tight-binding model with nearest-neighbor hopping on a hexagonal lattice.\[H = -\gamma \sum_{i,j} \sum_{s = \uparrow, \downarrow} (c_{i,s}^\dagger c_{j,s} + H.c.),\]

where the sum over $i, j$ goes over the nearest neighbors (see Fig. 1), the hopping amplitude $\gamma \approx 2.8$ eV, the nearest neighbor spacing is $a \approx 0.14$ nm, and $s$ denotes the spin component. The midpoint of a unit cell labeled as $\vec{n}$, we denote the two sites, by the following term in the Hamiltonian

$$H = \gamma \sum_{i,j} \sum_{s = \uparrow, \downarrow} (c_{i,s}^\dagger c_{j,s} + H.c.),$$

and $\sigma$ denotes the spin component in, say, the $z$ direction. We will henceforth set $\hbar = 1$. Each unit cell of the hexagonal lattice consists of two sites; we denote the two sites, belonging to sublattices A and B, by $a_n$ and $b_n$ respectively. We introduce the Pauli matrices $\sigma$ with $\sigma^z = \pm 1$ denoting sites on the A and B sublattices respectively. The midpoint of a unit cell labeled as $\vec{n}$ is located at $\vec{n} = (3a/2)(n_1, 2n_2 + n_1)/\sqrt{3}$, where $n_1, n_2$ take integer values. The spanning vectors of the lattice are $\vec{M}_1 = (3a/2)(1, 1)/\sqrt{3}$ and $\vec{M}_2 = (3a/2)(1, -1)/\sqrt{3}$. The reciprocal lattice vectors can be chosen to be $\vec{G}_1 = (2\pi/3a)(1/2, \sqrt{3}/2)$ and $\vec{G}_2 = (2\pi/3a)(1/2, -\sqrt{3}/2)$.

In momentum space, the Hamiltonian in Eq. (1) takes the form

$$H_k = -\gamma \begin{pmatrix} 0 & e^{ik \cdot \vec{M}_1} + e^{ik \cdot \vec{M}_2} \\ e^{-ik \cdot \vec{M}_1} + e^{-ik \cdot \vec{M}_2} & 0 \end{pmatrix},$$

where

$$g_k = 1 + \cos(\vec{k} \cdot \vec{M}_1) + \cos(\vec{k} \cdot \vec{M}_2),$$

and

$$h_k = \sin(\vec{k} \cdot \vec{M}_1) + \sin(\vec{k} \cdot \vec{M}_2).$$

As is well-known, the energy dispersion is given by $\pm E_k = \gamma \sqrt{g_k^2 + h_k^2}$.

The two bands touch each other at two inequivalent points; these are located at the wave vectors $\pm(0, 4\pi/(3\sqrt{3}a))$ and are called $\vec{K}$ and $\vec{K}'$. Around these points, the effective low-energy continuum theory of graphene electrons takes the form of a two-dimensional Dirac Hamiltonian with

$$H_D = \sum_k \psi_k^\dagger \begin{pmatrix} \sigma x k_x + \tau s y k_y \end{pmatrix} \psi_k,$$

where $\psi_k = 3\gamma a/2$ is the Fermi velocity, $\tau^z = \pm 1$ at $\vec{K}$ ($\vec{K}'$) respectively (these are called valleys), and $\psi_k \equiv \psi_k^{\sigma \tau s}$ denote eight-component electron annihilation operators with the components corresponding to sublattice ($\sigma$), valley ($\tau$), and spin ($s$) degrees of freedom.

Equation (5) is the Dirac Hamiltonian and the dispersion is given by $\pm E_k = \pm \sqrt{v_F^2 k^2 + f^2}$; this has a gap of $2|f|$ at $\vec{k} = 0$.

In the next section we study what happens when various parameters in the Hamiltonian are given $\delta$-function kicks with a time period $T$.

III. FLOQUET HAMILTONIAN FOR PERIODIC KICKING

We now apply $\delta$-function kicks to the system, described by the following term in the Hamiltonian

$$H_{kick} = \sum_{\vec{n}} \left( \alpha_x \sigma x + \alpha_y \sigma y + \alpha_z \sigma z \right) \sum_{m = -\infty}^{\infty} \delta(t - mT).$$

We assume that the periodic kicks are the same for all unit cells $\vec{n}$; hence they add the term

$$H_{\delta, kick} = \left( \alpha_x \sigma x + \alpha_y \sigma y + \alpha_z \sigma z \right) \sum_{m = -\infty}^{\infty} \delta(t - mT)$$

to the momentum space Hamiltonian in Eq. (2).

The Floquet (stroboscopic) operator $U_{XYZ}$ which evolves the system through one time period $T$ is given by

$$U_{XYZ} = U_{\delta, kick} U_{\delta} = e^{-i\vec{\sigma} \cdot \vec{F} \tau} e^{-iH_k T} = e^{-iH_{XYZ} T}.$$
in momentum space. This also defines for us the effective stroboscopic Hamiltonian $H_{XYZ}$. The eigenvalues of $H_{XYZ}$ can be found by using the following identity for Pauli spin matrices,

$$e^{i\alpha(n,\vec{r})}e^{i\beta(m,\vec{s})} = e^{i\epsilon(k,\vec{x})},$$

where $\{a, b, c\}$ are scalars and $\{\hat{n}, \hat{m}, \hat{k}\}$ are unit vectors. Here $c$ and $k$ can be found in terms of $a, b, n$ and $m$ using the following relations:

$$\cos c = \cos a \cos b - \hat{n} \cdot \hat{m} \sin a \sin b,$$

$$\hat{k} = \frac{1}{\sin c}(\hat{n} \sin a \cos b + \hat{m} \sin b \cos a - \hat{n} \times \hat{m} \sin a \sin b).$$

Hence the eigenvalues of $H_{XYZ}$ are equal to $\pm \epsilon$, where the quasi-energy $\epsilon$ is given by

$$\epsilon = \frac{1}{T} \arccos \left[ \cos \alpha \cos(E_{\vec{k}}T) + \frac{\gamma(a_{x}g_{k} - \alpha_{y}h_{k})}{\alpha E_{\vec{k}}} \sin \alpha \sin(E_{\vec{k}}T) \right],$$

with $\alpha = \sqrt{\alpha_{x}^{2} + \alpha_{y}^{2} + \alpha_{z}^{2}}$. In the special case of only $\alpha_{z} \neq 0$, i.e., the kicking is applied to only the on-site energies of the graphene sublattice, the quasi-energy takes the particularly simple form $\epsilon = \frac{1}{T} \arccos[\cos \alpha_{z} \cos(E_{\vec{k}}T)]$. We will discuss this in more detail below.

IV. WAVE PACKET DYNAMICS

To corroborate our results from the analytical results presented in the later sections, we numerically study the time evolution of a wave packet on the graphene lattice. We visualize the packet only at times which are integer multiples of $T$, consistent with our understanding of the effective dispersion given by $\epsilon$.

Consider a wave packet $\Psi$ in two spatial dimensions, with an initial momentum $\vec{k}_{0} = (k_{ox}, k_{oy})$ and a width $\sigma$. Namely,

$$\Psi(\vec{r}, t = 0) = \frac{1}{\sqrt{2\pi\sigma^{2}}} \exp(-\frac{\vec{r}^{2}}{4\sigma^{2}}) \exp(i\vec{k}_{0} \cdot \vec{r}),$$

which is normalized such that $\int d\vec{r} |\Psi|^{2} = 1$. The Fourier transform of $\Psi$ is given by,

$$\Psi(\vec{k}, t = 0) = \sqrt{8\pi\sigma^{2}} \exp[-\sigma^{2} \{(k_{x} - k_{ox})^{2} + (k_{y} - k_{oy})^{2}\}].$$

such that $\frac{1}{2\pi\sigma^{2}} \int dk_{x}dk_{y} |\Psi(\vec{k})|^{2} = 1$.

Since the system has translational symmetry even in the presence of kicking, we can study the time evolution of each $k$ mode separately. Thus a wave packet centered at $\vec{k}_{0}$ can be evolved via $U_{XYZ}$ and then Fourier transformed to the real space lattice. At each $\vec{k}$ mode we have a two-component vector associated with the occupancy of the graphene bands. Note that in the absence of any kicking, the upper and lower bands (positive and negative energies) have opposite velocities. Hence the wave packet movement will depend on the band which is initially occupied. In this work we always consider wave packets built out of the lower band (negative energy).

It may be useful to say a few words about our numerical procedure. We impose periodic boundary conditions in both real and momentum space. In momentum space, the Brillouin zone is a rhombus whose corners are at $(\pi/a)(-1, 1, 1, 1, 1, 1, 1, 1)$, $(\pi/a)(-1, 1, 1, 1, 1, 1, 1, 1)$, $(\pi/a)(1, 1, 1, 1, 1, 1, 1, 1)$, and $(\pi/a)(1, 1, 1, 1, 1, 1, 1, 1)$. If the number of unit cells is $N^{2}$ (the number of sites is $2N^{2}$), the real space lies in a rhombus whose center lies at $(0, 0)$ and corners are at $(3aN/2)(-1, 1, 1, 1, 1, 1, 1, 1)$, $(3aN/2)(1, 1, 1, 1, 1, 1, 1, 1)$, $(3aN/2)(1, 1, 1, 1, 1, 1, 1, 1)$, and $(3aN/2)(1, 1, 1, 1, 1, 1, 1, 1)$. In Figs. 3 and 4 we have taken $a = 1$ and $N = 40$.

V. DISPERSION FOR UNIDIRECTIONAL KICKING

We now look at the effects of kicking in each of the three directions in the subsections below. We will show that kicks along $\sigma^{x}$, $\sigma^{y}$ and $\sigma^{z}$ can have quite different effects.

A. X-kicking: $\alpha_{x} \neq 0$, $\alpha_{y} = \alpha_{z} = 0$

We first consider the case when the kicking is applied only in the $\sigma^{x}$ direction. In this case, we find that the quasi-energy spectrum given in Eq. (11) is gapless (i.e., $\epsilon = 0$), when $h_{k} = 0$, $E_{\vec{k}} = \gamma|g_{k}|$, and $E_{\vec{k}}T = \alpha = |\alpha_{x}|$. This leads to the equations

$$2 \cos(\frac{3k_{x}a}{2}) \cos(\frac{\gamma k_{x}a}{2}) = \frac{\alpha_{x}}{\gamma T} - 1,$$

$$2 \sin(\frac{3k_{x}a}{2}) \cos(\frac{\gamma k_{x}a}{2}) = 0.$$  

This gives the following gapless points

$$\cos(\frac{\sqrt{3}k_{x}^{2}a}{2}) = \frac{1}{2} (\frac{\alpha_{x}}{\gamma T} - 1),$$

$$\sin(\frac{3k_{x}^{2}a}{2}) = 0.$$  

Clearly this can only be satisfied if $-\gamma T \leq \alpha_{x} \leq 3\gamma T$. We can also see that only $k_{y}^{2}$ can be modulated using $\alpha_{x}$. The low-energy dispersion about these gapless points can be found by expanding $\epsilon$ as follows,

$$\epsilon(T) = \cos(\frac{\alpha_{x}}{2E_{\vec{k}}}T) + (1 - \frac{\hbar^{2}}{2E_{\vec{k}}^{2}}) \sin(\alpha_{x} \sin(\frac{\epsilon_{\vec{k}}}{E_{\vec{k}}}T),$$



which implies that
\[ \varepsilon^2 = \frac{1}{T^2} \left[ (\alpha_x - E_k T)^2 + \frac{\gamma^2 T^2 h_k^2}{\alpha_x^2} \sin^2 \alpha_x \right]. \]  

(17)

The effective velocities about the gapless points are given by
\[ v_x = \frac{3a}{2T} \sqrt{\frac{(\alpha_x - \gamma T)^2 \sin^2 \alpha_x}{\alpha_x^2}}, \]
\[ v_y = \frac{3a}{2T} \sqrt{\frac{3\gamma^2 T^2 + 2\gamma T \alpha_x - \alpha_x^2}{3}}. \]

(18)

When we consider the case when the kicking is applied only in the \( \sigma^y \) direction, i.e., \( \alpha_y \neq 0 \) and \( \alpha_x = \alpha_z = 0 \). We obtain the effective dispersion about these gapless points.

\[ \cos(\varepsilon T) = \cos \alpha_y \cos(E_k T) \]
\[ + (1 - \frac{g_y^2}{2h_k^2}) \sin \alpha_y \sin(E_k T). \]

(21)

Next we obtain the effective dispersion about these gapless points. Similar to the previous section, the low-energy expansion leads to

\[ \varepsilon^2 = \frac{1}{T^2} \left[ (\alpha_y - E_k T)^2 + \frac{\gamma^2 T^2 h_k^2}{\alpha_y^2} \sin^2 \alpha_y \right]. \]

(22)

B. Y-kicking: \( \alpha_y \neq 0, \alpha_x = \alpha_z = 0 \)

\[ 2 \cos(\frac{3k_y a}{2}) \cos(\frac{\sqrt{3}k_y a}{2}) = -1, \]
\[ 2 \sin(\frac{3k_y a}{2}) \cos(\frac{\sqrt{3}k_y a}{2}) = -\frac{\alpha_y}{\gamma T}. \]

(19)

These can be solved to obtain gapless points at \((k_x^g, k_y^g)\), where
\[ \cos(\frac{\sqrt{3}k_y^g a}{2}) = -\frac{1}{2} \sqrt{1 + \frac{\alpha_y^2}{\gamma^2 T^2}}, \]
\[ \sin(\frac{3k_y^g a}{2}) = \frac{\alpha_y}{\sqrt{\gamma^2 T^2 + \alpha_y^2}}. \]

(20)

We see that a gapless point exists only if \(|\alpha_y| \leq \sqrt{3}\gamma T\). In contrast to the earlier case with only \( \alpha_x \neq 0 \), both \( k_x^g \) and \( k_y^g \) can be modulated here using \( \alpha_y \).

Next we consider the case when the kicking is applied only in the \( \sigma^y \) direction, i.e., \( \alpha_y \neq 0 \) and \( \alpha_x = \alpha_z = 0 \). Substituting the above values in Eq. (11), the gapless points are determined by \( g_k = 0, E_k = \gamma |h| \), and \( E_k T = |\alpha_y| \). We then get

\[ 2 \cos(\frac{3k_x a}{2}) \cos(\frac{\sqrt{3}k_y a}{2}) = -1, \]
\[ 2 \sin(\frac{3k_x a}{2}) \cos(\frac{\sqrt{3}k_y a}{2}) = -\frac{\alpha_x}{\sqrt{\gamma T}}, \]

(25)

This is shown in Fig. 4, where we have chosen \( \gamma T = 1 \).

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**FIG. 2**: Quasi-energy dispersion \( \varepsilon \) (in units of \( 1/T \)) when \( \alpha_x = \gamma T = 1 \) and \( \alpha_y = \alpha_z = 0 \). There is a dispersionless line along the \( x \) direction when \( k_0 a = \pm \pi/\sqrt{3} \). A wave packet localized on this gapless line moves only in the \( y \) direction (see Fig. 3).

Interestingly, at the special value of \( \alpha_x = \gamma T \), we obtain a dispersion which is gapless along \( k_x \) and disperses only along \( k_y \) at \( k_y = k_y^g \). This dispersionless line is shown in Fig. 2 where we have chosen \( \gamma T = 1 \). Therefore a wave packet which is centered at \( k_{0y} = k_y^g \) will move only in the \( y \) direction in real space for a wave packet which is centered at \( k_{0y} = k_y^g \) will move only in the \( y \) direction in real space in the presence of such a kicking. This is demonstrated in Fig. 3.

Another interesting case occurs as \( \alpha_x \) is increased further. At \( \alpha_x = 3\gamma T \), we find that the two Dirac points merge at \((k_x^g, k_y^g) = (0, 0)\) and we get a semi-Dirac dispersion there with \( v_x = (a/T) \sin(3\gamma T) \) and \( v_y = 0 \). The dispersion is linear in \( k_y \) and quadratic in \( k_x \), given by \( \varepsilon = (3k_x^2 a^2)^2/4 \) for \( k_x = 0 \).

We would like to mention here that the merging of Dirac points in graphene resulting in a topological merging transition from a gapless to a gapped system has been studied extensively in recent years. Further, the same phenomenon has been studied inside the gapless phase of the Kitaev model on the hexagonal lattice in Ref. 72.
FIG. 3: Time evolution of a wave packet initially centered at $\vec{r} = (0,0)$, at $t = T$, $4T$, $16T$, $32T$, $64T$ for no kicking (upper panel) and for $\alpha_x = \gamma T = 1$ and $\alpha_y = \alpha_z = 0$ (lower panel), with $k_{ox}a = 1$, $k_{oy}a = \frac{\pi}{\sqrt{3}}$, and $\sigma = \frac{10}{\sqrt{2}}a$. The upper panel shows that in the absence of kicking the wave packet spreads out in both $x$ and $y$ directions. In the lower panel we see that the wave packet only moves in the $y$ direction. This occurs because the wave packet is located at the dispersionless line in the $k_x$ direction (see Fig. 2).

FIG. 4: Quasi-energy dispersion $\varepsilon$ (in units of $1/T$) when $\alpha_y = \sqrt{3}\gamma T = \sqrt{3}$ and $\alpha_x = \alpha_z = 0$. While the dispersion is linear in the $k_x$ direction, it is quadratic in the $k_y$ direction near the gapless points (see text). This realizes a semi-Dirac dispersion.

C. Z-kicking: $\alpha_z \neq 0$, $\alpha_x = \alpha_y = 0$

Finally we consider a kicking which is applied in only the $\sigma^z$ direction; this corresponds to applying a staggered potential on the $A$ and $B$ sublattices. The dispersion in this case is given by

$$\varepsilon = \frac{1}{T} \arccos[\cos \alpha_z \cos(E_{\vec{k}}T)].$$

(26)

At the Dirac points, $E_{\vec{k}} = 0$, we see that $\varepsilon = \alpha^2/T$; hence this kicking opens up a gap in the dispersion. Interestingly, at $\alpha_z = \pi/2$, we find that $\varepsilon = \pi/(2T)$, independent of the value of $\vec{k}$. Thus the dispersion becomes absolutely flat and therefore leads to dynamical localization. This is clearly shown in Fig. 5 where a wave packet gets localized in real space. Note that this is happening even though the system has no disorder and has translational symmetry.

At all other values of $\alpha_z \neq n\pi$ (where $n$ is an integer), the original Dirac points at $\vec{K}$ and $\vec{K}'$ have a gap proportional to $\alpha_z/T$ and a low-energy dispersion which is quadratic in $k$. The effective dispersion is given by

$$\varepsilon = \frac{\alpha_z}{T} + \frac{1}{2} \cot \alpha_z E_{\vec{k}}^2. $$

(27)

VI. DYNAMICAL LOCALIZATION

In this section we will present a general understanding of dynamical localization due to periodic kicking. Suppose that there is a time-independent Hamiltonian $H$ whose eigenstates and eigenvalues come in pairs, namely, $\psi_j$ and $\psi'_j$ with energies $E_j$ and $-E_j$. Let us assume that there is a unitary operator $P$ which produces this
transformation, namely, \( P H P^{-1} = -H \), \( P \psi_j = \psi'_j \) and \( P \psi'_j = \psi_j \); hence \( P^2 = I \). Then we can show as follows that a periodic kick with \( P \) will produce dynamical localization after two time periods. We begin with an eigenstate \( \psi_j \) and evolve it with the Hamiltonian \( H \) for a time \( T \); hence it picks up a phase \( e^{-iE_j T} \). Then we kick it with \( P \) which converts it to the state \( \psi'_j \). Upon evolving this with \( H \) for a time \( T \), it picks up the phase \( e^{iE_j T} \); the two phases cancel each other exactly. Then another kick with \( P \) brings it back to the state \( \psi_j \). Thus any eigenstate \( \psi_j \) will remain unchanged after a time \( 2T \). Since any state can be written as a superposition of eigenstates of \( H \), we see that any state will remain the same after a time \( 2T \); this implies dynamical localization.

VII. CONCLUDING REMARKS

We have shown that applying \( \delta \)-function kicks in different directions in the sublattice space in graphene can lead to interesting physics including dispersionless lines in momentum space, semi-Dirac dispersion and even a completely flat dispersion. We have shown that these lead to rich possibilities for the time evolution of wave packets, such as motion along only one direction or a complete dynamical localization. Given the widespread interest in graphene, the ability to tune its dispersion and obtain a range of dynamical behaviors should lead to applications in a variety of settings.

We note that a dynamical localization-to-delocalization transition has been observed in a quantum kicked rotor. Such a system is realized by placing cold atoms in a pulsed, far-detuned, standing wave, and the transition is detected by measuring the number of zero velocity atoms under the influence of a quasi-periodic driving\(^{63}\).

We would like to mention possible experimental realizations of periodic driving of a graphene system.

(i) If a metallic sheet carrying a uniform current in the \( x \) direction is placed parallel to the graphene (but displaced from it by some distance in the \( z \) direction), this produces a constant magnetic field in the \( y \) direction. The corresponding vector potential can be chosen to be in the \( x \) direction with a magnitude which is linear in the \( z \) coordinate. Hence it will be a constant vector in the graphene plane. This vector potential can be introduced

FIG. 5: Time evolution of a wave packet initially centered at \( \vec{r} = (0, 0) \), at \( t = T, 4T, 16T, 32T, 64T \) for no kicking (upper panel) and for \( \alpha_z = \pi/2 \) (lower panel), with \( k_oxa = 1 \) and \( k_oya = 0 \) and \( \sigma = \frac{10}{\sqrt{2}}a \). The upper panel shows that the wave packet evolves with a net velocity in the \( x \) direction. The lower panel clearly demonstrates that the wave packet is localized. Notice that this happens in the absence of any disorder, so that translational symmetry is preserved. This is called dynamical localization.

For any bipartite lattice with hopping only between sites belonging to different sublattices (graphene is a special case of this), we can see that an operator \( P \) which changes the phase of an eigenstate on only one sublattice by \(-1\) produces another eigenstate with the opposite energy. A kick with a staggered potential of strength \( \pi/2 \) is precisely equivalent to such an operator \( P \) (up to an overall phase of \( i \)). This explains why we observe dynamical localization when \( \alpha_x = \pi/2 \) and \( \alpha_z = \alpha_y = 0 \). In fact, dynamical localization will occur even if we consider a finite piece of graphene with an arbitrary size and shape.
as a Peierls phase in the nearest neighbor hoppings. If we now vary the current periodically in time, we will have a periodically varying hopping phase which cannot be removed by a global gauge transformation. This provides a possible experimental route to achieve the temporal driving in the $\sigma^z$ and $\sigma^y$ directions that we have studied in this paper.

(ii) A kicking proportional to $\sigma^z$ can be experimentally set up as follows. h-BN (a form of boron nitride with a hexagonal lattice structure) and graphene have lattice spacings which are nearly equal; hence one of them can be placed on the other. The boron and nitrogen atoms exert different van der Waals forces on the two graphene carbon atoms in an unit cell, thus creating an effective sublattice potential. A periodic application of the pressure on these two layers (from the top and the bottom) should modulate the distance between the layers and thus lead to a periodic modulation of the sublattice potential.

Finally, we point out some possible directions for future studies. One can study what kinds of edge states can be generated in graphene by periodic kicking of different kinds. In the absence of kicking, it is known that graphene has states on a zigzag edge but not on an armchair edge. It would be interesting to know if periodic driving can change this situation, as is known to happen in the Kitaev model on the hexagonal lattice. It would also be very interesting to analyze the effects of interactions in periodically kicked graphene. One of the central results of this paper is that at $\alpha_z = \pi/2$ the quasi-energy spectrum becomes completely dispersionless. Under these conditions any interaction energy scale in the problem will be dominant due to quenching of the “effective” kinetic energy. It can therefore be intriguing to understand the stroboscopic evolution of a many-particle state in such a system. The presence of a highly anisotropic Dirac dispersion and dispersionless lines in the spectrum may also produce exotic many-body phases in the presence of interactions.

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