Full-Waveform Inversion for Subsurface Penetrating Radar based on Adaptive Bilateral Total Variation Regularization

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Abstract. Full-waveform inversion (FWI) of subsurface penetrating radar (SPR) is one of the most promising techniques for quantitative reconstruction of high-resolution dielectric properties distribution. However, due to the incomplete observation data of surface detection, there is significant ill-posedness in inversion and the inversion accuracy is lower than expected. In this paper, an FWI method for SPR data with adaptive bilateral total variation (ABTV) regularization is proposed. The method utilizes ABTV regularization to supplement the data constraints and retains the anomaly body boundary information. In addition, the regularization function of adaptive weight factor matrix is designed for the limitation of the standard bilateral total variation (BTV) regularization, which considers both the smoothing effect and the edge preservation effect. Synthetic experiment shows that the proposed method can reconstruct the permittivity distribution more accurately from 2-D SPR data.

1. Introduction

Subsurface penetrating radar (SPR) utilizes the penetrating ability of electromagnetic wave to realize the penetrating detection of medium, which has broad application demand and development prospect in many fields such as military, medical detection and star detection[1]. Full-wave inversion (FWI) uses the full waveform information of observation data, including travel time, amplitude and phase, which has the potential to provide fine subsurface structure and accurate quantitative distribution information of dielectric parameters. Then it has become an important research direction of SPR data analysis and processing. However, in SPR FWI, the ill-posedness is significant due to the weak completeness of observation data[2], which increases the difficulty to construct reliable permittivity model, and hence solving the problem is critical to improve the overall accuracy of inversions.

Regularization is an effective method to fix the ill-posedness of inversions[3]. By adding model constraints to the objective function, some uncertainties are eliminated which limited the solution in an appropriate range, and then the inversion converges to more accurate results. Watson[4] firstly applied total variation regularization (TV) for SPR FWI, which effectively characterized the edge of the target and obtained a more accurate permittivity distribution. Subsequently, Feng et al.[5] used TV regularization for permittivity and conductivity inversion to effectively suppress the oscillation in results. Furthermore, they developed a modified TV regularization to restrain inversion which added Tikhonov regularization term of a priori model to further improve the accuracy and stability of inversion[6]. In addition to constraints of statistical characteristics, an FWI method based on model structure constraints is presented to improve the results of conductivity inversion by Ren[7]. Its theoretical basis is the structural correspondence between permittivity and conductivity of the model.
In this paper, we propose an FWI algorithm based on adaptive bilateral total variation (ABTV) regularization, and the permittivity is inverted with model constraints. An adaptive bilateral total variation regularization function is designed to improve the effect of both artifacts suppression and edges preservation. And multiscale inversion strategy is adapted to effectively avoid plunging into local optimum. Moreover, the synthetic experiments are carried out to verify the effectiveness of the proposed method.

2. Method
In this section, we briefly describe the time-domain FWI forms with regularization and present an adaptive BTV regularization function for SPR FWI in details.

2.1. Inversion Problem
The FWI is usually modelled as an optimization problem, and its objective function includes data misfit term and regularization term, and is defined as:

$$
\Phi = \Phi_d + \lambda R_m
$$

(1)

where $R_m$ is the regularization term and $\lambda = \frac{\Phi_d}{R_m + \xi^2}$ is the regularization factor, which is used to adjust the weight of data misfit and regularization term in the objective function, and its value will affect the performance of the solution. $\xi$ is a small constant. $\Phi_d$ is the inversion objective function without regularization constraint:

$$
\Phi_d(m) = \frac{1}{2}\sum_{s,s',r,r'}\sum_{\tau}\left[ E_{cal}^{s'} - E_{obs}^{s'} \right]^T \delta(x-r,t-\tau) \left[ E_{cal}^{s} - E_{obs}^{s} \right]_{r,t}
$$

(2)

where $m$ is the model parameter vector, $s, r$ and $\tau$ represent the three dimensions of signal sources, receivers and observation time respectively, $E_{cal}^{s'}$ and $E_{obs}^{s'}$ are the forward simulation data and observation data of the whole space-time domain respectively, and $T$ represents the transpose symbol. $\delta$ represents the sampling function at the spatial position $r$ and the time of $\tau$.

The parameter model is updated in an iterative process, which is expressed as[8]:

$$
m_{k+1} = m_k + \alpha_k p_k
$$

(3)

where $p$ represents update direction while $\alpha$ is the update step size.

The nonlinear conjugate gradient method is adopted in SPR FWI, which has the advantages of easy implementation and low storage consumption. With the reference of [9], the step size $\alpha$ can be determined by empirical formula. Its iteration direction needs $p$ to calculate the gradient of $\Phi$,

$$
\nabla \Phi = \nabla \Phi_d + \lambda \nabla R_m
$$

(4)

where $\nabla \Phi_d$ can be effectively calculated with the first-order adjoint state method[10], and the expression is:

$$
\nabla_x \Phi_d(x') = \sum_{s} \int_{s} dt' \left( \partial_s E^{s}(x', t') \right)^T \hat{G}^T R^s
$$

(5)

where $\varepsilon$ is the permittivity, the model parameter we invert. $x'$ is the position vector. $\hat{G}$ denotes the Green’s operator. And $R^s = \sum_{r,r'}\left[ E_{cal}^{s} - E_{obs}^{s} \right]_{r,t}$ represents the residual of wavefield.

2.2. Adaptive Bilateral Total Variation Regularization
Based on the spirit of bilateral filter, bilateral total variation regularization (BTV) can better reconstruct the edge features of parameter model and suppress artifacts. In many image denoising experiments, the effect is better than generalized Tikhonov regularization and TV regularization[11]. Moreover, an adaptive weight factor matrix is designed to replace the fixed weight factor of the standard BTV function to further strengthen the effect of edge enhancement and artifacts suppression.
The ABTV function is improved with adaptive weight factor matrix based on the standard BTV function, which considers the spatial distance weighting mechanism in the neighbourhood and the similarity of structural parameters. For the two-dimensional model parameter matrix $M$, ABTV function is defined as:

$$R_{ABTV}(M) = \sum_{l=1}^{P} \sum_{n=0}^{P} \| \beta{l} || (M - S^t_l S^n_z M) \|_1$$

where $\| \|$ denotes L1-norm, $S^t_l$ and $S^n_z$ operators represent moving $l$ and $n$ elements in horizontal and vertical directions respectively, corresponding to multiple gradient scales. $P$ is the size of the search window. $\beta$ is a scalar weight factor matrix, which is used to provide a spatial attenuation effect for the sum of regularization, and it is designed as a smooth dichotomous function, that is, the weight factor takes a smaller value at the edge to enhance the edge preservation effect, and takes a larger value at the flat place to enhance the smoothing effect.

$$\beta = \left[ 1 - \frac{1}{1 - e^{-\gamma/(1/2)}} \right]$$

where $\gamma$ is the control factor of curve, $\chi = \frac{|M - S^t_l S^n_z M|}{\| M - S^t_l S^n_z M \|_\infty}$ denotes normalized absolute value matrix of gradient.

Then ABTV function’s gradient for model parameter $\varepsilon$ is:

$$\nabla R_{ABTV} = \sum_{l=1}^{P} \sum_{n=0}^{P} \beta{l} || \left( I - S^t_l S^n_z \right) \text{sign} \left( M - S^t_l S^n_z M \right)$$

where $I$ is the identity matrix, $S^t_l$ and $S^n_z$ are the inverse operations of $S^t_l$ and $S^n_z$, $\text{sign}$ denotes the sign function.

3. Synthetic Experiment

A model shown in figure 1, that embedded with two regular targets in homogeneous medium, is considered to analysis the performance of the proposed method in permittivity inversion. The whole simulation area is a 16cm×6cm rectangular area containing the targets. According to the simulation requirements of FDTD, the spatial step is divided into 2mm and the whole area is a discrete grid of 80×30. The background medium is designed with relative permittivity $\varepsilon_r = 4$. For target 1, it has relative permittivity $\varepsilon_r = 2$ and locates at $x = 5.4cm, z = 4cm$, and for target 2, it has relative permittivity $\varepsilon_r = 6$ and locates at $x = 10cm, z = 2.8cm$. Their sizes are both 2.6cm×1cm but located at different depths. The conductivity of the model area is set as $\sigma = 1mS / m$.

![Figure 1. Permittivity distribution of model, which contains two targets.](image)
The synthetic data are generated using 8 sources with “cross” shapes and 40 receivers with “dot” shapes at the upper boundary $z = 6cm$ of the simulation area. The sources are assigned uniformly from $x = 1cm$ to $x = 15cm$ with the interval of 2cm, while the receivers are arranged uniformly from $x = 0.2cm$ to $x = 15.8cm$ with interval of 0.4cm. The source is set as a Ricker wavelet with a central frequency of 6GHz, and the recorded time length is 3ns. The forward modelling is realized with GPRMAX3.0, open electromagnetic calculation software.

To evaluate the performance of proposed method, we develop two inversion experiments applied to the data acquired from above model, which are permittivity inversion without regularization, and permittivity inversion based on ABTV regularization with $\gamma = 10, P = 2$. And the initial model is the homogeneous medium as $\varepsilon_r = 4$ in inversion experiments.

Figure 2 shows the results of above experiments. The inversion result without regularization accurately reconstructs the permittivity of targets, but edges of targets are blurry and there are many sidelobe artifacts in the homogeneous background. And the sidelobe artifacts in background are suppressed and there is high contrast between targets and background in the inversion result with ABTV, which is more similar to true model.

For revealing the differences between inversion results in details, two cross-sections are selected (i.e., black dotted lines AA’ and BB’ in Figure 1). Figure 3(a) is the sections diagram of AA’ located at $z = 4cm$ while Figure 3(b) show the sections diagram of BB’ located at $z = 2.8cm$. The solid line is the true model value; the dashed line represents the standard FWI of permittivity and the dotted line represents the FWI with ABTV regularization of permittivity. The sections results indicate that inversion with ABTV reconstructs more accurately permittivity distribution and preserves sharper edges of high contrast targets.

To evaluate the inversion results quantitatively, reconstruction error is adopted, which is an important index to measure the misfit between inversion models and true model. It can be computed with expression $\frac{\|m_k - m_{true}\|_2}{\|m_k - m_{ini}\|_2}$. Figure 4 shows the reconstruction error curves of inversion methods, which indicates the inversion with ABTV regularization improve significantly the overall accuracy of inversion result.
4. Conclusion
An FWI algorithm based on adaptive bilateral total variation regularization is proposed, and applied to permittivity inversion of SPR data. An adaptive weight factor matrix is designed to improve the effect of both artifacts suppression and edges preservation based on standard BTV function. Synthetic experiments are utilized to reflect the performance of proposed algorithm. Inversion results demonstrate that the proposed algorithm has good ability to suppress the sidelobe artifacts and preserve sharper edges, and reconstruct more accurately the permittivity distribution of subsurface space.

Acknowledgments
This work was supported in part by the National Natural Science Foundation of China under Grant 61901501.

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