UNIVALENT BAKER DOMAINS AND BOUNDARY OF DEFORMATIONS

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Abstract. For $f$ an entire transcendental map with univalent Baker domain $U$ of hyperbolic type I, we study pinching deformations in $U$, the support of this deformation being a lamination in the grand orbit of $U$. We show that pinching along a lamination that contains the geodesic $\lambda_{\infty}$ (See Section 3.1) does not converges. However, pinching at a lamination that does not contains such $\lambda_{\infty}$, converges and converges to a unique map $F$ if: the Julia set of $f$, $J(f)$ is connected, the postcritical set of $f$ is a positive (plane) distance away from $J(f)$, and it is thin at $\infty$. We show that $F$ has a simply connected wandering domain that stays away from the postcritical set. We interpret these results in terms of the Teichmüller space of $f$, $\text{Teich}(f)$, included in $M_f$ the marked space of topologically equivalent maps to $f$.

1. Introduction

This paper studies some properties of the dynamics of entire transcendental maps $f$ with univalent Baker domains. Baker domains (Definition 1) are components of the Fatou set $F(f)$ of $f$ and the first example of an entire transcendental function with such domain was given by Fatou [F]. Examples of Baker domains of higher period were given by Baker, Kotus and Lü [BKL]. See the survey of Bergweiler [B1], for complete references. Existence of Baker domains without singularities in its interior (univalent Baker domains), were constructed by Bergweiler in [B2] and latter a classification of these kind of domains was given by Baransky and Fagella [BF].

In this paper we are interested in the limit of deformations of entire transcendental maps $f$ with periodic univalent Baker domain $U$ of Hyperbolic Type I, according to Baransky and Fagella. We explain some of these facts in Section 1. In this case, we apply pinching deformations to curves in such domain. This means that we have a continuous path of quasiconformal deformations $(f_t)$ of the original map $f$ supported in curves on $U$. Pinching deformation was introduced by Makienko [M] in the context of rational maps, in analogy of Kleinian groups, subsequently [T], [HT], [BT] and other mathematicians, developed and applied the theory.

A periodic univalent Baker domain $U$, being a finite union of simply connected domains, admits an hyperbolic metric, hence geodesic curves are well defined, in Section 4 we define and study certain geodesic laminations $\Lambda$ in $U$, that we call Baker laminations. The grand orbit of such laminations, $\mathcal{R}(\Lambda)$, is where the pinch deformation occurs. These laminations are of course invariant under the action of the map $f$. We prove in Theorem 1 that for any lamination that contains the geodesic $\lambda_{\infty}$ the deformations $(f_t)$ are divergent. As we explain in 3.1, $\lambda_{\infty}$ are the geodesics that joins the expansive periodic points in the boundary of $U$, with $\infty$. 

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We apply the technique in [HT] to prove that pinching deformations \((f_t)\) along any Baker lamination that does not contain the \(\lambda_\infty\)-geodesic, converges uniformly and that each leaf of the lamination converges to a point. For this we need the basic assumption that \(f\) has the postcritical set \(P(f)\) (as defined in Section 2.1) a positive (plane) distance away from the Julia set. This hypothesis allows the map \(f\) to be expanding in the Julia set according to a theorem of Stallard ([St], Theorem A) and so it is weakly hyperbolic (Section 2.3), hence the convergence techniques in [HT] can be used in our case, with the difference that our pinching curves join two non periodic points of \(\partial U \subset J(f)\), the Julia set of \(f\).

Assume that \(f\) satisfies: (a) \(f\) has a periodic univalent Baker domain of hyperbolic type I, (b) the postcritical set is a positive distance away from the Julia set, (c) \(J(f)\) is thin at \(\infty\) and (d) \(J(f)\) is connected, the examples of Section 2.4 satisfies this conditions. A theorem of G. Stallard ([St], Theorem B) shows that entire transcendental maps satisfying (b) and (c), has the Lebesgue measure of \(J(f)\), equal to zero. This will imply (See Theorem 2) that for our limit map \(F\), the Lebesgue measure of \(J(F)\) is zero also; together with results in [H], we have that the limit map \(F\) is unique. By construction, the fibers of the pinching are the leaves of the lamination, hence we conclude (Theorem 3) that the Baker domain \(U\) ends up in two possible cases: (i) if \(\Lambda\) does intersects the curve \(\lambda_\infty\) then \(U\) is deformed to a family of simply connected wandering domains, meanwhile (ii) if \(\Lambda\) does not intersects \(\lambda_\infty\), it is deformed into a univalent Baker domain together with a family of simply connected wandering domains, in both cases the wandering domains are a positive distance away from the singular set.

In the last section we study the pinching deformations as seeing in the marked Hurwitz space of the map \(f\), here denoted by \(M_f\) and which is defined as the space of all marked maps topologically equivalent to \(f\) (Definition 4). We include the Teichmüller space of \(f\), \(\text{Teich}(f)\) into \(M_f\). Theorem 1 shows that pinching along \(\lambda_\infty\) does not converges to a point in \(M_f\), hence \(\text{Teich}(f)\) is not compactly contained in \(M_f\). Pinching along the other Baker laminations defines (by Theorem 2) points at the boundary of \(\text{Teich}(f)\) contained in \(M_f\). In Proposition 1 we hint if any other possibilities can occur when we consider any path of deformations (supported on \(U\) only). It is shown that if we take a sequence of deformations \((f_t)\) in the slice \(\text{Teich}(U/f)\) of \(\text{Teich}(f)\), with the property that it converges uniformly to \(F\) at the boundary of \(\text{Teich}(f)\) (as subset of \(M_f\)) then, the only development of \(U\) towards a periodic Fatou domain (in case it exists) is to a new periodic univalent Baker domain.

2. Univalent Baker domains

2.1. Notation and definitions.

Our notation is \(\mathbb{H}\) for the upper half plane. \(\mathbb{R}^* = \mathbb{R} \cup \infty\) is the boundary of the upper half plane, \(\mathbb{C}\) the complex plane and \(\hat{\mathbb{C}}\) the Riemann sphere, \(f\) denotes an entire transcendental map, \(d\) denotes the euclidean distance in \(\mathbb{C}\) and \(d_h\) is the hyperbolic metric in \(\mathbb{H}\). If \(U\) is a set with boundary, \(\partial U\) denotes its boundary. We write \(U/f\) for the quotient of \(U\) by the action of \(f\).

The map \(f\) defines a partition of the plane into the Fatou set \(F(f)\), where the iterates of \(f\) form a normal family and the Julia set \(J(f) = \mathbb{C} - F(f)\) is the
complement. Properties of these sets can be found in [B1]. A domain \( W \subset F(f) \) is a wandering domain if for all \( m > n \geq 0 \), \( f^m(W) \cap f^n(W) = \emptyset \).

The singularities of the inverse function are the critical values \( c \) such that \( f'(c) = 0 \) and de asymptotic values. Asymptotic values are points \( a \in \mathbb{C} \) for which there exist a path \( \gamma(t) \to \infty \) as \( t \to \infty \) such that \( f(\gamma(t)) \to a \) as \( t \to \infty \). Let us denote by \( Sing(f^{-1}) \) the closure of the set of critical values and asymptotic values. We denote by \( P(f) \), the postcritical set, which is the closure of the (positive) orbits of points in \( Sing(f^{-1}) \).

2.2. Univalent Baker domains.

The classification of Fatou domains for transcendental maps are well known and are known to be as in the rational case except for one more type of domain:

**Definition 1. (Baker domain)** If \( U \) is a periodic component of the Fatou set of period \( p \), such that there exist \( z_0 \in \partial U \) with \( f^{np}(z) \to z_0 \) for all \( z \in U \) as \( n \to \infty \) but \( f^p(z_0) \) is not defined, \( U \) is called a Baker domain.

It was proved in [Ba] that all Baker domains for entire transcendental maps are simply connected. If \( f \) is an entire transcendental map, then \( z_0 = \infty \). In [EL] it is established that if \( Sing(f^{-1}) \) is bounded then \( f \) has not Baker domains. In case a Baker domain exists it is not necessarily for a Baker domain to have any of the singular values inside. A **Univalent Baker domain** is a periodic Baker domains on which \( f^p \) is univalent and these are the kind of domains that we are interested in this paper.

For \( f \) an entire transcendental map, a classification of such domains is given in [BF] as follows.

Assume that \( f \) has an invariant Baker domain \( U \), then there exist a point \( \xi \in \hat{\mathbb{C}} \) such that the backward iterates under \( (f|_U)^{-1} \) of all points in \( U \) tend to \( \xi \) through the same access (they call it the backward dynamical access), and one of the following occurs:

(a) \( \xi \neq \infty \) is a fixed point in the boundary of \( U \), in this case \( U \) is of *hyperbolic type I*.

(b) \( \xi = \infty \) but the backward dynamical access is different than the forward dynamical access, in this case \( U \) is of *hyperbolic type II*.

(c) \( \xi = \infty \) but the backward dynamical access is the same to the forward one, in this case \( U \) is of *parabolic type*.

This domains have a uniformization \( \Psi : \mathbb{H} \to U \) and a linear map \( G : \mathbb{H} \to \mathbb{H} \) such that \( (f|_U) \circ \Psi = \Psi \circ G \). If \( U \) si of hyperbolic type I or II, then \( G(z) = az \), \( a > 1 \) and for \( U \) of parabolic type, \( G(z) = z + 1 \). Hence the diagram bellow commutes.

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\begin{align*}
U & \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\…
Now, let us denote by $\alpha$ the geodesic $it \in \mathbb{H}$, $t > 0$ and denote by $A_\alpha$ the annulus $\mathbb{H}/G$ with modulus $m(A_\alpha) = \pi/log(a)$, let $c = \alpha/G$ be the core geodesic of $A_\alpha$, with length $log(a)$. The dynamic action of $f$ in $U$ defines the quotient space $U/f$.

Hyperbolic domains of Type I and II, have as quotient space $U/f$, an annulus $A_\alpha$ which is conformally the same that the quotient annulus $\mathbb{H}/G$.

2.3. Hyperbolicity.

In this paper we will be interested in entire transcendental functions with a univalent periodic domain which has the postcritical set at a positive distance away from the Julia set or equivalently $d(P(f), J(f)) > C > 0$, being $d$ the euclidean distance in $\mathbb{C}$. This kind of functions are non-uniformly hyperbolic according to Theorem A in [St], in the sense that for all $z \in J(f)$, $|f^n(z)| \to \infty$ as $n$ tends to $\infty$. However in the context of [HT] it is enough to consider the following condition.

**Definition 2. (Weak hyperbolicity)** We say that $f$ is weakly hyperbolic if there are constants $r > 0$ and $\delta < \infty$ such that, for all $z \in J_f - \{\text{parabolic points}\}$, there is a subsequence of iterates $\{f^{n_k}\}$ such that

$$\deg(f^{n_k} : W_k(z) \to D(f^{n_k}(z), r)) \leq \delta,$$

where $W_k(z)$ is the connected component of $f^{-n_k}(D(f^{n_k}(z), r))$ containing $z$.

Due to the condition of the Postcritical set and the Julia set above, our maps do not have parabolic points.

2.4. Examples of functions with univalent Baker domains.

Examples of entire transcendental maps with univalent Baker domains have been given by Eremenko and Lyubich, Herman, Bergweiler, Stallard and Rippon, etc (see[B1]). Since we are interested in weakly hyperbolic maps, then the Bergweiler example is the typical one.

Let us consider the map $f(z) = 2 - log2 + 2z - e^z$. It was shown in [B2] that $f$ has an invariant Baker domain $U$ containing the half plane $\{Re(z) < -2\}$ and the boundary of $U$ is a Jordan curve, all singular points are in Fatou domains and the plane distance from $\partial U$ to the postcritical set is positive. The Singular points of $f$ are located in one superattracting component and in a union of simply connected wandering domains. Moreover, the Lebesgue measure of $J(f)$ is zero, as it is shown in [St].

In a similar way to Bergweiler, we can also define, for $m > 1$, the set of maps.

$$f_m(z) = (m - 1)log\left(\frac{2}{2m - 1}\right) + \frac{2m - 1}{2} + mz - e^z$$

with the same properties that $f$ except that we have changed the superattracting fixed point by an attracting fixed point at $\frac{2m - 1}{2}$.

For each of these functions, the map $G$ of diagram (1) is $G(z) = mz$.

All these maps satisfies the condition $d(P(f), J(f)) > 0$ and so they are hyperbolic (and weakly hyperbolic).
3. Baker laminations

First, we need good neighborhoods of our arcs on which to support the pinching deformation. Such neighborhoods are closed sets containing the arc, its boundary is a pair of arcs containing the boundary points of the arc and this neighborhoods must be invariant under the map $f$, formally the neighborhoods are constructed as follows.

Let us consider a band $B_\delta = [\pi/2 - \delta, \pi/2 + \delta] \times \mathbb{R}$ in the complex plane, and consider its image under the exponential map $\{ e^z : z \in B_\delta \} := V_\delta$ and choose $\delta$ so $V_\delta \subset \mathbb{H}$. Thus $V_\delta$ is a good neighborhood of thickness $\delta$ of the horizontal geodesic $\alpha$. If $\gamma$ is any other complete geodesic in $\mathbb{H}$, then there is an isometry $A$ of the upper half plane such that $A(\alpha) = \gamma$; we say that $A(V_\delta) := V_\delta(\gamma)$ is a good neighborhood (of thickness $\delta$) for $\gamma$. Here the line $(\pi/2) \times \mathbb{R}$ is sent to the geodesic $\gamma$ under $A \circ \exp(z)$.

Assume as always that $U$ is a univalent invariant Baker domain. Given a geodesic $\lambda \in U$, we have that $\Psi^{-1}\lambda = \gamma$ is a geodesic in $\mathbb{H}$ with a good neighborhood $V_\delta(\gamma)$.

We define $\Psi V_\delta(\gamma) := V_\delta(\lambda)$ as a good neighborhood of $\lambda$ of thickness $\delta$.

Let $\Lambda$ be a set of (complete and open) geodesics in $U$, we say that $\Lambda$ is a Baker lamination of thickness $\delta$ in $U$, if the elements in $\Lambda$ satisfies:

(i) It is invariant under the action of $f$. That is, if $\lambda \in \Lambda$, then $f^n(\lambda) \in \Lambda$ for any $n$ a positive integer. Also if we consider the branch of $f^{-n}$ from $U$ to $U$, we have that $f^{-n}(\lambda) \in \Lambda$.

(ii) For any $\lambda, \lambda' \in \Lambda$, then $\lambda \cap \lambda' = \emptyset$.

(iii) Its end points are well defined points in $\partial U$ and any two geodesics have different end points in $\partial U$.

(iv) There are not self accumulating leaves. That is, all geodesics in $\Lambda$ admit a good neighborhood of thickness $\delta$ with the property that these good neighborhoods are pairwise disjoint.

Let us denote by $\text{End}(\gamma)$ the two end points of the geodesic $\gamma$ and by $\bar{\gamma} = \gamma \cup \text{End}(\gamma)$. Condition (iv) can be restated as $\text{End}(\gamma) \cap \text{End}(\gamma') = \emptyset$ for any $\gamma, \gamma' \in \Lambda$.

If the Baker domain is periodic with $U = \bigcup_{i=1}^{n-1} U_i$ and $fU_i = U_{i+1}$, then $f^n U_1 = U_1$ and we can consider a Baker lamination $\Lambda_1$ in $U_1$, invariant under $f^n$ and so $f^k \Lambda_1 = \Lambda_k$, $k = 1, \ldots, n-1$ define Baker laminations in $U_{k+1}$ respectively, invariant under $f^n$, we call to $\Lambda = \bigcup \Lambda_k$ a Baker lamination in the periodic domain $U$.

The good neighborhood in $\Lambda$ is constructed similarly, first for the Baker lamination in $U_1$ and then by iteration for any element of $\Lambda$.

We will denote by $\mathcal{R}(\Lambda) = \cup_{i=0}^{\infty} f^{-i}(\Lambda)$ the full orbit of $\Lambda$.

3.1. Paths in $\mathcal{R}(\Lambda)$.

In this section we will show that there are not two geodesics of a lamination that touch at a common point.

From [BE] we deduce that the inverse image of $U$ is disconnected. Let $U_1$ one of the components, then if there is a singularity of $f$ in $U_1$ then $f : U_1 \to U$ has degree $d \geq 2$, hence there is a point $w$ in $\partial U_1$ such that $f(w) = \infty$. Therefore, no preimage contains critical points and $f^{-1}(U)$ is made of infinity components.
Recursively, this implies that no critical point is in the inverse branches of \( f \), hence for any inverse branch \( V \) we have that \( V \) is connected, in fact it is simply connected (Theorem 9 in [B1]) and \( f^n : V \to U \) is one to one.

**Remark 1.** The problem, is to know if there are periodic Baker domains or inverse branches that may have some boundary points in common (other than \( \infty \)). In [Si] there is a construction of functions with invariant univalent Baker domains \( U \) and \( U' \) (or more), which touch each other at their common fixed point \( p = \partial U \cap \partial U' \).

The following lemma implies that this is the only possible case.

Assume that \( f \) has a family of periodic univalent Baker domains \( \{ U_i \} \). If \( f^k(U_i) = U_i \), for some \( k \), denote by \( q_i \) the fixed point in \( \partial U_i \) under \( f^k \). Let us denote by \( Q \) the set of such fixed points.

**Lemma 1.** If \( \Lambda \) does not contains \( \lambda_{\infty} \) and \( \gamma_1, \gamma_2 \) are in \( R(\Lambda) \), then \( \bar{\gamma}_1 \cap \bar{\gamma}_2 = \emptyset \)

**Proof.** Suppose that there are \( \gamma_1 \) and \( \gamma_2 \) such that \( \bar{\gamma}_1 \cap \bar{\gamma}_2 = x \). Let us consider two cases:

Case (a): Let \( U, V \) fixed univalent Baker domains and assume that \( \partial U \cap \partial V = x \). Then \( f^n(x) = x_n \) is also in \( \partial U \cap \partial V \). Consider paths \( \sigma_1 \subset U \) and \( \sigma_2 \subset V \) from \( \infty \) to \( x \) and \( x_1 \), respectively and denote by \( D \) the disc which is interior of \( \sigma = \sigma_1 \cup \sigma_2 \), then \( f(D) \) takes its maximum over \( \sigma \), but \( f(\gamma) \) bounds a disc \( D_1 \), hence \( f(D) = D_1 \). Observe that \( f^n(\sigma) \) tends to \( \infty \), by an argument as above, \( f^n(D) \) tends to \( \infty \). That is a contradiction since \( D \) contains points of the Julia set.

Case (b): Let \( U \) and \( V \) univalent Baker domains such that \( f(U) = U \) and there exist \( n > 0 \) with \( f^n(V) = U \) with \( \partial U \cap \partial V = x \). Let \( g = f^n \), then \( g \) sends a neighborhood of \( x \) into its image in a two to one fashion, then \( x \) is a singular point in \( J(g) = J(f^n) = J(f) \) a contradiction.

The general situation is proved by iteration of \( \gamma_1 \) and \( \gamma_2 \) until we fall in Case (a) or (b). \( \square \)

**Remark 2.** From the lemma above, we have that if \( \tau \) is a connected path in \( \overline{R(\Lambda)} \), then between any to geodesics in \( \tau \) there is always another one. Then this path is made up by a numerable set of geodesics belonging to the great orbit of the lamination \( \Lambda \) and a Cantor set which is the set of limits of sequences of geodesics in \( \tau \). This Cantor set belong to the Julia set. In the process of pinching, this geodesics tend to be shorter and shorter until their two ends are identified to a point. At the end of this process the path \( \tau \) is shrinked into a path but not to a point.

### 3.2. The \( \lambda_{\infty} \)-geodesic

Consider the geodesic \( \alpha \) in \( \mathbb{H} \) (Section 2.2), and \( \Psi \) the uniformization of some univalent Baker domain \( U \), of hyperbolic type I. Then \( \Psi(\alpha) \) is a lamination in \( U \), that we will denote by \( \lambda_{\infty} \). Its projection to \( U/f \) is the core geodesic \( c \). If \( f|U \) is of hyperbolic type I, \( \lambda_{\infty} \) joins the fixed point \( p \in \partial U \) with \( \{ \infty \} \) and it is invariant under the action of \( f \), hence a Baker lamination. In the case of a periodic univalent Baker domain \( U_1, ..., U_k \), there is a periodic expanding point \( q_1, ..., q_k \) with \( q_i \in \partial U_i \) and therefore we will consider \( \lambda_{\infty} \) to be the union of the geodesics that joins \( q_i \) with \( \infty \).
Observe that a geodesic in $U$ that contains $\infty$ as end point, projects in $U/f$ to a geodesic such that necessarily accumulates on the core geodesic $c$. Consequently, by property (iv) we have that if a leaf of a Baker lamination has $\infty$ as an end point, it has to be $\lambda_\infty$.

### 3.3. Other examples of Baker laminations

Now consider $U$ a forward invariant univalent Baker domain and two points $u, v$ in $\partial U - \{\infty\}$ and accessible from $U$. These points always exist and dynamically satisfy that $f^n(u) \to \infty$, $f^n(v) \to \infty$ as $n \to \infty$. Consider a geodesic $\gamma_{u,v}$ in $U$ joining $u$ and $v$. Choose the points $u, v$ in such a way that $\gamma_{u,v} \cap f(\gamma_{u,v}) = \emptyset$, then the forward and backward orbit of $\gamma_{u,v}$ in $U$ is a Baker lamination.

In this way it is possible to consider finite couples of points $u_i, v_i$ in $\partial U - \{\infty\}$ and accessible from $U$. If the geodesics joining them are disjoint as well as their images, then their forward and backward orbits in $U$ under $f$ are a Baker lamination.

We can add to such laminations the geodesic $\lambda_\infty$ if the set satisfies (i) to (v) and we obtain laminations that we will consider in Theorem 1.

### 4. Pinching deformation

In this section we have a look to the combinatorics of curves in $\hat{\mathbb{C}}$ that preludes the pinching deformation. We study this combinatorics in our case although the techniques in [HT] apply immediately to our situation.

Let us assume that $f$ is an entire transcendental map satisfying the following conditions:

(a) it has at least one periodic univalent Baker domain $U_1, ..., U_n$ of hyperbolic type I.

(b) the postcritical set is a positive distance away from the Julia set.

Under this conditions $f$ is weakly hyperbolic where we can choose $\delta = 1$ and $r = (1/2)dis(J(f), \text{Post}(f))$.

We have seen that examples of such functions exist in Section 2.1.

A **pinching combinatorics** is defined as follows:

- for $\delta > 0$, a set of finite Baker laminations of thickness $\delta$, $\Lambda_1 \subset U_1, ..., \Lambda_n \subset U_n$ such that $f^k\Lambda_i = \Lambda_j$, $k \geq 0$, $i, j = 1, ..., n$. Observe that each $\lambda_i \in \Lambda$ is in $F(f)$ and disjoint from the orbits of the critical points.

- the $\lambda_i$’s are mutually disjoint.

- $f : \lambda_i \to \lambda_j$ is a homeomorphism.

Set $\hat{R} = \bigcup_{i=1}^n \Lambda_i$, $R = \bigcup_{k} f^{-k}(\hat{R})$.

- An invariant set of disjoint good neighborhoods for the Baker lamination and in consequence for $R$. That is $V(R) = \bigcup_{k} f^{-k}(V_\delta(\Psi \Lambda))$.

For a **pinching deformation**, we take from [HT] the quasiconformal symmetric model on strips, with our $B_\delta$ as the support of the model strip. In that paper is defined the quasiconformal map $\tilde{P}_t(x + iy) = x + iv_t(y)$, which has the following four properties:

1. It commutes with the translation by any real number.
2. It is the identity in the sub-strip $\{-L_y \leq y \leq L_y\}$, being $L_y$ as in [HP].
3. The coefficient of the Beltrami form...
Lemma 2. If \( \gamma \) is a hyperbolic open subset of \( \mathbb{R} \), let \( \lambda \) be a sequence of elements in \( \mathbb{R} \), whose norm is locally uniformly bounded from 1 if \( (t, x) \neq (1, L_r) \) and tends to 1 as \( (t, y) \to (1, L_r) \).

If \( V \) is a connected component of \( \mathcal{V} (\mathcal{R}) \), define the conformal map \( \psi : V \to B_\delta \) as the well defined inverse branch of the map \( \Psi \circ A \circ \text{Exp} : B_\delta \to \mathcal{U} \) that sends the strip to a good neighborhood of an element of the grand orbit of a Baker lamination. For \( t \in [0, 1] \), set \( \sigma_t = (\tilde{P} \circ \psi)^n(\sigma_0) \) to the pull back of the standard complex structure on \( B_\delta \). We spread \( \sigma_t \) to the whole orbit of \( \mathcal{V} \) under the map \( f \). Let \( \sigma_t \) be the extension to the Riemann sphere by setting \( \sigma_t = \sigma_0 \) in the complement of \( \mathcal{V} \). Then, \( \sigma_t \) is an \( f \)-invariant complex structure.

The following notions will be useful, here we follow [T].

Let \( h_t \) be the quasi conformal maps in \( \mathcal{C} \) given by the Measurable Riemann Mapping Theorem that integrates \( \sigma_t \). Also, we can assume that \( h_t \) fixes \( \infty \) and two points \( p, q \in J(f) \).

The composition \( h_t \circ f \circ h_t^{-1} = f_t \) defines a entire transcendental map. We are interested in showing that for some laminations, we have that \( f_t \Rightarrow F \), where the sign \( \Rightarrow \) means uniform convergence.

Observe that if \( f_t \Rightarrow F \), then \( F \) is a meromorphic map with \( \infty \) as essential singularity.

The following notions will be useful, here we follow [T].

Let \( \gamma \) be a geodesic in \( \mathcal{R} \), then \( \gamma \) is in some Fatou component \( W \). For an hyperbolic open subset \( W \) of the Riemann sphere and a curve \( \gamma \in W \) denote by \( l_W (\gamma) \) the hyperbolic length of \( \gamma \) in \( W \).

**Lemma 2.** If \( \gamma \in \mathcal{R} \), then \( \lim_{t \to 1} l_{h_t(\mathcal{V})}(h_t(\gamma)) = 0 \).

**Proof.** Let \( n \) be such that \( \tau = f^n(\gamma) \in \mathcal{U} \) a fixed univalent Baker domain. For \( \Psi^{-1}\tau = \tau_1 \in \mathbb{H} \), choose an hyperbolic transformation \( T \) with axis \( \tau_1 \) and apply \( P_t \) along \( \tau_1 \). By the property (1) above, the hyperbolic length of the segment of curve from any \( z \in \tau_1 \) to \( T(z) \), decreases when we apply \( P_t \) until it is zero as \( t \to 1 \). Therefore, we have that \( \lim_{t \to 1} l_{\mathcal{V}}(P_t(\tau_1)) = 0 \). This implies that \( l_{h_t(\mathcal{U})}(h_t(\tau)) = 0 \), by the f-invariance of the pinch, we have the result. \( \square \)

**Lemma 3.** Let \( f \) satisfying conditions (a), (b) above, then the diameter of any sequence of elements in \( \mathcal{V} (\mathcal{R}) \) tends to 0 if \( \lambda_{\infty} \notin \mathcal{R} \).

**Proof.** The conditions (a), (b) implies that \( f \) is weakly hyperbolic, then the Mañé shrinking lemma in [ST] can be applied to \( f \) in \( \mathcal{C} \). That implies that we have to avoid laminations involving \( \{\infty\} \), by the discussion in Section 3.2, such lamination is \( \lambda_{\infty} \).

\( \square \)

5. Pinching the \( \lambda_{\infty} \)-Lamination

Let us denote by \( \text{diam}_s \) the spherical diameter of a set. Consider \( h_t \) a sequence of quasi conformal maps defined by pinching a Baker lamination as above.

**Lemma 4.** Assume that \( h_t \Rightarrow H \) and \( f_t \Rightarrow F \), then for any \( \lambda \in \mathcal{R} \), \( \lim_{t \to 1} (\text{diam}_s h_t(\gamma)) = 0 \).
Proof. Use our Lemma 2 and the inequalities in Lemma 2.1 of [T].

Next is the main Lemma of this section:

Lemma 5. Let $f$ be an entire transcendental map with a univalent Baker domain $U$ of hyperbolic type I. Then, the pinching process along the lamination $\lambda_\infty$ does not converge.

Proof. Let us assume that $f_t \rightrightarrows F$, by Lemma 5, \( \lim_{t \to 1} (\text{diam}_{h_t}(\lambda_\infty)) = 0 \) which implies that $h_t(\lambda_\infty) \to \infty$. For $P \in \partial U$ the repulsive fixed point of $f$, let $P_t = h_t(P)$. Then $P_t \in \partial(h_t(U))$ is a boundary point of $\lambda_\infty$ other than $\infty$ (along the process of pinching) and we have that $P_t \to \infty$, as $t \to 1$. Since $P_t \in J(f_t)$, by the $f$-invariance of the pinching, we have that any point in the set $f_t^{-n}(P_t)$ tends to $\infty$ under $h_t$, for any $n \geq 0$ and any $t < 1$. However, the set $\cup_n f_t^{-n}(P_t)$ is dense in $J(f_t)$, then, by uniform convergence we have that $J(F) = \infty$. That is a contradiction. 

As a direct consequence of this lemma, we have the following theorem.

Theorem 1. Let $f$ be an entire transcendental map with a periodic univalent Baker domain $U$ of hyperbolic type I. Then, if $\Lambda$ is a Baker lamination on $U$ that contains $\lambda_\infty$, then, the pinching process along the lamination $\Lambda$ does not converge.

6. PINCHING ALONG OTHER LAMINATIONS

In this section, the main task is to prove the following result:

Theorem 2. Let $f$ be an entire transcendental map satisfying properties the following properties:

(a) $f$ has a univalent Baker domain of hyperbolic type I,
(b) the postcritical set $P(f)$, is a positive distance away from the Julia set.
(c) $J(f)$ is thin at $\infty$.
(d) $J(f)$ is connected

Then for $\Lambda$ any Baker lamination in $U$ which does not contains $\lambda_\infty$, the pinching process along $\Lambda$ converges uniformly to an entire transcendental map $F$ which exhibits in its Fatou set a bounded simply connected wandering domain. Moreover such wandering domain is at a (plane) positive distance away from the postcritical set.

Proof. The detailed arguments are in [HT] and we will mention only the required results. By Ascoli’s theorem, the sequence \{\(h_t\)\} converges uniformly if and only if it is equicontinuous and converges pointwise.

The first step is to prove that the quasiconformal homeomorphisms \(h_t\) are equicontinuous. For that [HT] uses two lemmas:

First Lemma: Equicontinuity criterion at a point, which is a criteria based on annuli neighborhoods at a point, and

Second Lemma: One good annulus around each Julia point in $\mathbb{C}$ (see Lemma 2.7 in [HP]). This Lemma is proved using the construction in [HT] which involves Lemma 1 and Remark 2 of this paper. Then the weak hyperbolicit y condition is used to spread these annuli at every point.

At $\infty$ we can always find a bounded annulus $A$ such that $\partial A \cap \mathcal{V}(\mathcal{R}) = \emptyset$, because $\lambda_\infty \notin \Lambda$ and so the geodesics of the lamination either are contained in the annulus
of are disjoint from it. Then there is \( m > 0 \), such that \( \text{mod}(h_t(A)) \geq m \) for all \( t \). Satisfying the one good annulus criteria. Similarly it can be constructed a nested sequence of annuli \( \{A_n\} \), with \( h_t(A_n) \geq m \).

Moreover, Lemma 4 implies that if \( h_{t_n} \rightarrow H \), then \( H \) maps each leaf of \( \mathcal{R} \) to a point. By the discussion in Section 3.2, we have that this are the only fibers of \( H \).

The second step is to prove that the limit map is unique: observe that the limit map \( F \) is an entire transcendental map, it has the singular set a positive distance away from the Julia set (as proved in Section 6.1), \( F \) is expanding in its Julia set and as we will show bellow, it is thin at \( \infty \).

**Definition 3.** (McMullen [Mc]) \( J(F) \) is thin at \( \infty \) if for all \( z \in \mathbb{C} \) there exist \( R \) and \( \epsilon \) such that \( \text{density}(J(F), D_r(z)) < 1 - \epsilon \), for all \( r > R \).

Where \( D_R(z) \) is the disc centered at \( z \) with radius \( R \) and \( \text{density}(E, G) := \frac{\text{vol}(E \cap G)}{\text{vol}(G)} \).

Let us suppose that \( F \) is not thin at \( \infty \), that implies that \( \text{density}(J(f_t), D_r(z)) < 1 - \epsilon_t \) with \( \epsilon_t \rightarrow 0 \) as \( t \rightarrow 1 \). The lamination \( \mathcal{R}(\Lambda) \) as Lebesgue measure 0, hence \( \text{density}(J(F)) \) can not be 1. That is a contradiction, so \( F \) is thin at infinity.

We conclude that the Lebesgue measure of the Julia set, \( m(J(F)) \), is 0 by Theorem B of Stallard [St].

If we assume that there are two sequences \( (t_n) \) and \( (s_n) \) such tending to 1, such that \( h_{t_n} \Rightarrow H_1, h_{s_n} \Rightarrow H_2, f_{t_n} \Rightarrow F_1 \) and \( f_{s_n} \Rightarrow F_2 \), as explained in [HT] there is a homeomorphism \( \varphi \) of \( \hat{\mathbb{C}} \) conformal in \( \hat{\mathbb{C}} - J(F_1) \), such that \( \varphi \circ F_1 \circ \varphi^{-1} = F_2 \). For weakly hyperbolic dynamics, it is proved in [H] (Theorem 1.4, proposition 6.3 and Theorem 6.1) that \( \varphi \) is globally quasiconformal (his proof is general and it does not requires \( f \) to be rational). By [St], we have that \( m(J(F_1)) = 0 \), then \( \varphi \) is a Moebius map. By normalization, our maps \( h_t \) fixes three points, then \( \varphi \) also, consequently it is the identity, this implies that the limit is unique.

After pinching, by the contraction to a point of the leaves of the lamination, we obtain from the Baker domain, that the Fatou set of \( F \) contains disjoint sets of simply connected domains for which any point in this domains tends to \( \infty \) under iteration. Then, we have a wandering domain.

We want to prove now, that \( d(P(F), J(F)) > C > 0 \). Let \( V \) be the denote the union of Fatou components that contains the singular set. First, we have to prove (A): that \( h_t(P(F)) \) does not tends towards \( h_t(\partial V) \) as \( t \rightarrow 1 \) and second (B): that no component of \( V \) shrinks to zero, as \( t \rightarrow 1 \). In the next section we will prove these conditions.

6.1. **Proof of (A) and (B).**

Proof of (A): Let \( A_i \) be annuli, such that (i) \( A_i \subset V \) and such that contains in the interior of its inner boundary a subset of \( P(F) \) and (ii) \( \cup_i A_i \) contains in the interior of their inner boundary, all of \( P(F) \). Then since the quasiconformal maps \( h_t \) are conformal in \( V \), we have that the moduli of the \( A_i \) does not changes when \( t \rightarrow 1 \), so this implies that \( P(F) \) stays away of \( \partial V \). This proves (A).

Notation: For \( S \subset \mathbb{C} \), let us denote by \( |S| \), the euclidean diameter of \( S \). For \( D = \{D_i\} \) a covering of \( J(F) \), a chain \( \{D_j\} \subset D \) from \( D_{k_1} \) to \( D_{k_2} \) is a finite ordered subset of elements in \( D \) with the property that two consecutive elements intersect, being the first one \( D_{k_1} \) and the last one \( D_{k_2} \). Let us denote by \( W_i \) the
components of a wandering domain constructed after pinch a periodic Baker domain $U$, that is $W_i = h_1(G_i)$ with $G_i$ the corresponding subset of $U$. The boundary of $G_i$ is made of some leaves in $\Lambda$ and a piece in $\partial U$ that we denote here by $\partial U_i$ and we have that $h_1(\partial U_i) = \partial W_i$.

Proof of (B): Assume, there exist $S_k \subset V$, such that each $S_k$ is shrinked, under the pinch process, to a small factor; but such that $|h_1(S_k)| \to 0$ as $k \to \infty$.

Fix a covering $N = \cup N_i$ by good annuli in $J(f)$, then $\cup h_i(N_i)$ is a covering of $J(f)$, moreover we know that $\text{mod}(h_i(N_i)) > n_i$ for all $0 \leq t \leq 1$.

For a fixed $j_0$, the boundary of $\partial U_{j_0}$ is covered by good annuli that we will denote by $K_k$, then $\{h_1(K_k)\}$ is a covering of $\partial W_i$. The boundary of $S_i$ is covered by good annuli that we will denote by $L_k$. Since $J(f)$ is connected then for a fixed $l$ and $k$, there is a chain of good annuli from $L_k$ to $K_k$. Here we want to observe that if $|h_t(S_i)|$ decreases, then $|h_t(L_k)|$ decreases also in the same proportion. This shrinking is spread to all annuli in the chain from $L_k$ to $K_k$ and then, to every $K_j$, hence $\partial U_{j_0}$ is shrinked in the same proportion than $S_i$.

By hypothesis we have that for any $\epsilon > 0$, there is an $S_k(\epsilon)$ such that $|h_1(S_k(\epsilon))| < \epsilon$. Since there is a chain from some $S_k(\epsilon)$ to $G_{j_0}$, then under $h_t$ the chain is shrinked in the same proportion near to $\epsilon$. Consequently we have that for any $\epsilon > 0$, $|h_1(\partial U_{j_0})| = |\partial W_{j_0}| < \epsilon$, hence there is no wandering domain, a contradiction with the construction.

The examples of Section 2.4, satisfies (a), (b), (c) and (d).

6.2. Shapes of the limit domains.

In this section we will see that if the limit map $F$ exist as in theorem 2 above, then two things can occur. One is that the Baker domain ends up in a wandering domain and second that the Baker domain ends up into a wandering domain and a Baker domain. We can classify the laminations according to such phenomena in the following way:

- The closure of the geodesic $\lambda_{\infty}$, separates $\partial U$ into the left and right sides.
- A) All the leaves of the lamination $\Lambda$ are in one side or in the other one.
- B) Some leaf of the lamination $\lambda$ intersects $\lambda_{\infty}$.

In case (A), the connected component of $U - \Lambda$ to which $\lambda_{\infty}$ belongs, develops in the limit of the deformation a univalent Baker domain. The other components will be wandering domains.

This observation proves the following:

**Theorem 3.** If $\Lambda$ is as in (A), then $U$ becomes in the limit of the deformation a Baker domain with a family of wandering domains attached to its boundary. If $\Lambda$ is as in (B), then, only a family of wandering domains appears.

7. **Teichmüller space and Hurwitz class**

Let us define the marking classes of entire maps topologically conjugate to $f$ as:

For $p, q$ two fixed points of $f$. 

Definition 4. (Hurwitz marked class). The Hurwitz marked class of an entire transcendental map \( f \) is \( M_f = \{ [g], [\phi] : g \) is entire such that there exist \( \phi, \psi \) homeomorphisms of \( \hat{\mathbb{C}} \) such that \( g = \phi \circ f \circ \psi^{-1} \}. \) [\( \phi \)] means that only the isotopy class and [\( g \)] the conformal class is remembered. That means that \( \phi_1 \sim \phi_2 \) if there is an isotopy of \( \phi_1 \circ \phi_2^{-1} \) to a conformal map of \( \mathbb{C} \) that fixes pointwise the set \( \text{Sing}(f^{-1}) \).

Also, \( g_1 \sim g_2 \) if there is a conformal map \( c \) such that \( g_1 = c \circ g_2 \circ c^{-1} \). Normalize all homeomorphisms so that fix \( p \) and \( q \), as in Section 4.

Eremenko and Lyubich proves in [EL] that if \( \phi_0 \circ f = g_0 \circ \psi_0 \) and \( \phi_1 \circ f = g_1 \circ \psi_1 \) and there is an isotopy \( \phi_t \) connecting \( \phi_0 \) with \( \phi_1 \) fixing the singular values, then \( g_0 = g_1 \).

The topology in \( M_f \) is given by uniform convergence on compacts.

The Teichmüller space of a holomorphic map is according to [Mc]:

\[ \text{Teich}(f)=\{(\phi) : \phi \text{ is an entire map with } g = \phi^{-1} \circ f \circ \phi \}. \]

The brackets are as in the definition of \( M_f \) above.

Let \( \mathcal{J} \) the closure of the set of all periodic points and the postcritical set of \( f \). A theorem of [FH] in full generality and [HaTa] when \( \text{Sing}(f^{-1}) \) is discrete, asserts that in the case of \( f \) an entire map with \( \phi \) a quasiconformal map of \( \mathbb{C} \) satisfying \( f = \phi \circ f \circ \phi \) and \( \phi \sim id \), then it is isotopic to the identity through an isotopy that fixes point wise \( \mathcal{J} \). Hence \( \text{Teich}(f) \subset M_f \). This inclusion is well defined since \( \phi \) defines a quasiconformal Beltrami differential \( \mu \), such that \( f^*(\mu) = \mu \), then if the quasiconformal map \( \psi \) has also Beltrami differential \( \mu \), by the normalization we have that \( \psi = \phi \).

Since our maps satisfy properties (a) and (b) of Section 4 and so \( m(J(f)) = 0 \), according to [McSu] and [FH], we have \( \text{Teich}(f) = \text{Teich}(U/f) \times_i \text{Teich}(D_i/f) \), being \( D_i \) a completely invariant Fatou domains other than the grand orbit of \( U \) (there are no deformations in the Julia set).

For \( f \) with a univalent periodic Baker domain, we have that \( \text{Teich}(f) \) is infinite dimensional complex Banach space. Infinite dimensional Teichmüller spaces have a Teichmüller metric which is non-differentiable and has arbitrarily short simple geodesics [EaLi], but are embedded into certain spaces of quadratic differentials by a theorem of Bers [MS]. Many other properties can be found in [NV], [GH], [GL].

We are interested in the boundary of \( \text{Teich}(f) \) as subspace of \( M_f \). For instance a consequence of Theorem 1 is that \( \text{Teich}(f) \) is not compactly contained in \( M_f \), by Theorem 2, the maps which are limits of pinching along Baker laminations other than \( \lambda_{\infty} \) are in the boundary of \( \text{Teich}(f) \), \( \partial \text{Teich}(f) \) and are in \( M_f \). These limits are similar to the cusp groups in the Bers boundary, whose density was proved by McMullen [Mc2]. Are our pinched limits, dense in \( \partial \text{Teich}(f) \)?.

In the next proposition we are interested in the development of \( U \) to a periodic domain, as \( (f_t) \) tends to the boundary of \( \text{Teich}(f) \), when \( f_t \) deforms in the slice \( \text{Teich}(U/f) \), this deformation is not necessarily by pinching a lamination.

**Proposition 1.** Assume that there is a sequence \( (f_t) \in \text{Teich}(U/f) \) such that \( (f_t) \Rightarrow F \) then, if there are new Fatou periodic domains of \( F \), they are univalent Baker periodic domains.

**Proof.** Since \( F \in M_f \) then \( F \) is entire transcendental. If \( F \) has new Fatou periodic domains, then by the uniform convergence of \( (f_t) \) to \( F \), it has to be a periodic univalent domain without singularities, hence a univalent periodic Baker domain.
The singular set of $F$ is contained in the persistent components of the Fatou domain.

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