Mechanics Modeling and Kinetic Analysis of Sub Marine Tracked Vehicle-Cable Salvage during Steady-state Steering

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Abstract. Based on the demand for deep-sea submarine cable salvage in Northeast Asia, a mathematical model of Sub Marine Tracked Vehicle-Cable Salvage (SMT-CS) was designed in this study. Considering the slip characteristics of the submarine track, the mechanical model of the SMT-CS during steady-state steering was simplified to obtain the kinetic equation of the vehicle body during steady-state steering. According to the steering geometrical relationship, the arbitrary shear rate and shear displacement of the ground plane during the steering process were solved. Combined with the empirical shear model of bottom sediments, the longitudinal driving force, lateral resistance and resistance moment of the larboard and starboard were derived during steady-state steering. The functional relationship between the lateral shift of instantaneous center of the conventional steering rate and the turning radius and angular velocity of the vehicle body as well as that between the slipping rate and the turning radius and angular velocity of the vehicle body was derived according to the geometrical relationship of steering motion. These are of great reference value to the design, performance prediction and maritime application of SMT-CS in the future.

1. Introduction
Northeast Asia is one of the most developed regions in Asia. The power grid interconnection project in Northeast Asia is going to realize the cross-border power grid interconnection between China, South Korea and Japan, which will increase the percentage of cleaner alternative energy sources, adjust the energy structure and optimize energy resource allocation in surrounding areas. Planning to enter the Yellow Sea from Weihai city, the China-South Korea submarine cable project will be laid on the seabed eastward and landed in Incheon, South Korea, with an average water depth of more than 100 meters. Since the cross section of the large-capacity submarine cable conductor and the dead weight of the submarine cable are quite large, whereas the submarine cable salvage technology in China is rather limited, it is of great significance to study the submarine cable salvage technology.

In this study, a steady-state steering mechanics model was designed based on the operational requirements on repairing the faulted submarine cable salvage in Northeast Asia. This model theoretically analyzed the impact of slipping rate on the tangential driving force, turning radius and angular velocity of Sub Marine Tracked Vehicle-Cable Salvage (SMT-CS), simplified the force analysis of SMT-CS during steady-state steering, and conducted a kinetic analysis of the mechanical model.
2. Mechanics Modeling of SMT-CS during Steady-state Steering

2.1 Structure and Assumptions of SMT-CS

The vehicle body of a SMT-CS (Sub Marine Tracked Vehicle-Cable Salvage) primarily consists of track chassis and functional modules, which is shown in Figure 1. The track chassis is mainly rigidly connected by the track and the bracket, and a turn is realized by overcoming the steering resistance through adjusting the driving forces of the larboard and starboard tracks.

Figure 1 Track chassis of a SMT-CS

For the convenience of calculation, the following assumptions were made during the steering operation of the SMT-CS [1-3]:

- The steering of the track chassis is limited to a two-dimensional horizontal plane. When the force is analyzed, the working resistance and resistance moment are projected in the turning plane.
- The SMT-CS has a relatively large turning radius during its operation and maintenance along the submarine cable, with the steering mode mainly dominated by differential steering.
- The submarine cable salvage equipment is symmetric longitudinally and laterally relative to the intermediate plane, and the projection on the horizontal plane of the center of gravity \( CG \) coincides with the geometrical center \( o \) of the vehicle body.
- The ground pressure of the submarine cable salvage equipment is evenly distributed on the surface of the horizontal seabed.

2.2 Mechanics Modeling of SMT-CS during Steady-state Steering

The mass of the SMT-CS is assumed to be \( m_T \), point \( o_{IC} \) denotes the instant turning center, \( D_{IC} \) is the eccentric distance between the turning center and the geometrical center \( o \), \( \omega_{rZ} \) and \( R' \) are the turning angular velocity and radius, respectively.

The operating velocity of the SMT-CS is set as 300m/hr, the turn is steadily made at a low velocity, and the turning radius is large. Therefore, the force borne by the vehicle is considered balanced \((0_T m_x = 0, 0_T m_y = 0)\), the centripetal force can be neglected \((2 / 0_T T m V R' = 0)\), and the moment of force relative to point \( o_{IC} \) is also balanced (the accelerated velocity of the turning angle \( \omega_{rZ} = 0 \)); a force balance equation was established for the SMT-CS, \( \sum F_x = 0, \sum F_y = 0 \), and a moment balance equation was set up relative to point \( o_{IC} \).

\[
\begin{align*}
F_{lT} + F_{dS} = (F_{lL} - R_{rl}) + (F_{lS} - R_{rs}) &= R_w, \\
\int_{(4a+6c)} f_{lL} \cdot dl + \int_{(4a+6c)} f_{lS} \cdot dl &= R_w, \\
F_{lT} (R' - D_{rT}) + F_{dS} (R' + D_{rT}) &= \left( \int_{(4a+6c)} f_{lL} l \cdot dl + \int_{(4a+6c)} f_{lS} l \cdot dl \right) + R_w (D_{wr} + D_{ic})
\end{align*}
\]

2.3 Calculation of the Driving Force, Resistance and Resistance Moment during Steady-state Steering

\( o_{IC} \) is the instant turning center of the track chassis of SMT-CS; \( R' \) is the actual turning radius; \( \omega_{rZ} \) is the turning angular velocity; \( o_{VC} \) is the projection of \( o_{IC} \) on the longitudinal axis \( ox \) of the
vehicle body; $D_{IC}$ is longitudinal eccentric distance of the projection; $o_L$ and $o_S$ are the intersection points between the line connecting $o_{IC}$ and $o_{VC}$, the trajectories of the larboard and starboard, respectively; $o_Lx_Ly_L$ and $o_Sx_Sy_S$ are the body-fitted coordinates established on $o_L$ and $o_S$, respectively; $o_{IC}o_Lo_S$ is the lateral axis of the turning center; $o_L'$ and $o_S'$ are the instantaneous velocity centers of turning shear of the larboard and starboard tracks, respectively, both of which are on the lateral axis of the turning center.

![Diagram](image)

**Figure 2** Geometrical relationship of parameters of SMT-CS during steady-state steering

As shown in Figure 2, $F_{TS}$ and $F_{TL}$ are the driving forces obtained by the larboard and starboard tracks’ shearing of the soil respectively; $R_{TS}$ and $R_{TL}$ are the resistances caused by the deformation of the soil longitudinally squeezed by the larboard and starboard tracks, respectively; $f_{TS}$ and $f_{TL}$ are the distribution of the lateral earth pressure of the larboard and starboard tracks along the grounding direction respectively; wage resistance caused by clearing, additional drag resistance caused by umbilical cable, resistance generated by the mechanical arm and the force of underflow on the vehicle body are combined into the work disturbance resistance whose projections along the body-fitted coordinates are $R_{wx}$ and $R_{wy}$; $D_{WR}$ is the eccentric distance at the action point of the work disturbance resistance.

$dA_L(x_{AL}, y_{AL})$ is a random infinitesimal element on the ground plane of the larboard track, the corresponding shear displacement of which is assumed to be $s_L$; $dA_S(x_{AS}, y_{AS})$ is a random infinitesimal element on the ground plane of the starboard track, the corresponding shear displacement of which is assumed to be $s_S$. The shear force of the selected infinitesimal elements can be obtained based on the shear stress-displacement empirical equation. In this study, the classic shear model[4] proposed by Wong JY was used to solve the interaction force between the tracks and the sediments. Relevant researches have verified that this model is effective for bottom sediments[5,6].

### 2.3.1 Longitudinal Driving Forces of the Larboard and Starboard Tracks

As shown in Figure 2, $s_x$ is the shear displacement of random infinitesimal element along the $x$ axis of the body-fitted coordinates, with that for the larboard track and starboard track denoted as $s_{xL}$ and $s_{xS}$, respectively. $dF_T$ is the longitudinal driving force of the infinitesimal element, and $F_T$, the driving force generated by the tracks’ shearing of bottom sediments, can be obtained with a longitudinal integration along the ground plane.
\[ F_{T*} = \iint_{A} \tau(s_v) \, dA = \int_{-\frac{1}{2}B_l}^{1} \int_{-(L_u-D_c)}^{1} \tau(s_v) \, dx \, dy \]  

(2)

Where \( \tau(s) \) is the shear model proposed by Wong JY.

2.3.2 Lateral Resistance and Resistance Moment of the Larboard and Starboard Tracks. \( s_{y*} \) is assumed to be the shear displacement of random infinitesimal element along the \( y \) axis of the body-fitted coordinates.

\( M_{R*} \), resistance moment of the tracks, can be obtained with a longitudinal integration along the ground plane.

\[ M_{R*} = \int_{-(L_u+D_c)}^{1} f_{T*} x \, dx = \int_{-(L_u+D_c)}^{1} \frac{1}{2} B_s \tau(s_v) x \, dy \, dx \]  

(3)

3. Kinematics Analysis of SMT-CS during Steady-state Steering

3.1 Lateral Shift of Instantaneous Velocity Center of Shear Rate of Tracks

Due to the slip drive and slip resistance, the instantaneous velocity centers of the larboard and starboard tracks experienced a lateral shift. As shown in Figure 3, \( V_{o*} \) is the traction velocity of the tracks, with the traction velocity of the larboard and starboard tracks denoted as \( V_{ol} \) and \( V_{os} \), respectively, \( V_{D*} \) is the velocity of the drive wire, \( V_{s*} \) represents the shear rate.

![Figure 3 Instantaneous velocity center of SMT during a steady-state steering](image)

According to the definition of the slipping rate \( i_* \), the slipping rate of the tracks, can be expressed as follows:

\[ i_* = \frac{V_{D*} - V_{o*}}{V_{D*}} \]  

(4)

The traction velocity \( V_{ol} \) of the tracks can be expressed as,

\[ \begin{align*}
V_{ol} &= (R - D_{cat}) \omega_{YZ} \\
V_{os} &= (R + D_{cat}) \omega_{YZ}
\end{align*} \]  

(5)

The relationship between lateral shift of the instantaneous velocity center and the slipping rate can be expressed as,

\[ \begin{align*}
D'_{ol} &= (R - D_{cat}) \frac{i_*}{1 - i_*} \\
D'_{os} &= (R + D_{cat}) \frac{i_*}{1 - i_*}
\end{align*} \]  

(6)
By analyzing Equations, the following results were obtained:

- when \( i < 0 \), the track is subject to a slip resistance, at which time the instantaneous center of the shear rate is near the turning center of the tracks;
- when \( i > 0 \), the track is subject to a slip drive, at which time the instantaneous center of the shear rate is away from the turning center.

3.2 Shear Displacement of Steady-state Steering

Due to the poor bearing capacity of bottom sediments and the existence of slip drive and slip resistance, the center of gravity \( CG \) often fails to coincide with \( \alpha_{VC} \), the projection of the turning center \( \alpha_{IC} \) on the \( x \) axis, resulting in the eccentric distance \( D_{IC} \). The included angle \( \beta \) between the movement direction of the vehicle body and the \( \alpha_{OX} \) axis of the body-fitted coordinates is defined as the sideslip angle, and the shear rate at points \( \alpha_{L} \) and \( \alpha_{S} \) on the larboard and starboard tracks.

\[
\begin{align*}
V_{\text{rel}} &= R_{sp} \omega_{pl} - \omega_{Tz} (R' - D_{TG}) \\
V_{\text{relS}} &= R_{sp} \omega_{tgs} - \omega_{Tz} (R' + D_{TG})
\end{align*}
\]  

(7)

As shown in Figure 3, \( dA_{L}(x_{AL}, y_{AL}) \) is a random infinitesimal element on the larboard track. According to the kinematic geometry shown in the figure, the turning radius of this point \( R_{AL} = \sqrt{(R' - D_{TG})^2 + x_{AL}^2} \), the turning linear velocity \( V_{AL} = R_{AL} \omega_{Tz} \), then \( V_{AL/L} \) and \( V_{AL/L} \), the components of linear velocity of \( dA_{L} \) in the coordinate system \( \alpha_{L} x_{L}, y_{L} \) can be expressed as follow,

\[
\begin{align*}
V_{AL/L} &= \omega_{Tz} R_{AL} \cos \psi_{AL} = \omega_{Tz} (R' - D_{TG} + y_{AL}) \\
V_{AL/L} &= \omega_{Tz} R_{AL} \sin \psi_{AL} = \omega_{Tz} x_{AL}
\end{align*}
\]  

(8)

Without regard to the overall deformation and assembly clearance of the tracks, the component of linear velocity of a random point on the rib band of the larboard track in the coordinate system \( \alpha_{L} x_{L}, y_{L} \), \( V_{TL/L} = R_{sp} \omega_{pl}, V_{TL/L} = 0 \); and the coordinate component of shear rate of \( dA_{L} \) is,

\[
\begin{align*}
V_{\text{rel}} &= V_{\text{relL}} = R_{sp} \omega_{pl} - \omega_{Tz} (R' - D_{TG} + y_{AL}) \\
V_{\text{rel}} &= V_{\text{relL}} = -\omega_{Tz} x_{AL}
\end{align*}
\]  

(9)

At this time, \( \omega \psi_{AL} \), the turning angle of the larboard track, can be expressed as,

\[
\omega \psi_{AL} = \omega_{Tz} f_{AL} = \frac{L_{T} - D_{IC} - y_{AL}}{R_{sp} \omega_{pl}} \omega_{Tz}
\]  

(10)

In Equation (9), the shear displacement component of each axis in the coordinate system \( \alpha_{L} x_{L}, y_{L} \) can be obtained by integrating time \( t_{AL} \),

\[
\begin{align*}
s_{L} &= [(L_{T} - D_{IC}) - y_{AL}] \left[ 1 - \frac{\omega_{Tz} (R' - D_{TG} + y_{AL})}{R_{sp} \omega_{pl}} \right] \\
s_{L} &= [(L_{T} - D_{IC}) - y_{AL}] \left[ -\frac{\omega_{Tz} x_{AL}}{R_{sp} \omega_{pl}} \right]
\end{align*}
\]  

(11)

As shown in Figure 3, \( dA_{S}(x_{AS}, y_{AS}) \) is a random infinitesimal element on the starboard track. The shear displacement component can be obtained based on the following equation,

\[
\begin{align*}
s_{S} &= [(L_{T} - D_{IC}) - y_{AS}] \left[ 1 - \frac{\omega_{Tz} (R' + D_{TG} + y_{AS})}{R_{sp} \omega_{tgs}} \right] \\
s_{S} &= [(L_{T} - D_{IC}) - y_{AS}] \left[ -\frac{\omega_{Tz} x_{AS}}{R_{sp} \omega_{tgs}} \right]
\end{align*}
\]  

(12)

3.3 Shear Rate and Shear Displacement at Random Points on the Starboard Track

\( \omega_{pl} \) and \( \omega_{tgs} \) are the angular velocity of the driving wheel on the larboard and starboard tracks,
respectively. The geometrical relationship between the central point velocity \((o_x, o_y)\), turning angular velocity and turning radius of the larboard and starboard tracks of the SMT-CS during steady-state steering was analyzed to obtain the turning radius and turning angular velocity of the vehicle body of the SMT-CS.

1) In an ideal state without considering the slipping rate and lateral shift of instantaneous center of the tracks, the turning radius \(R\) and turning angular velocity \(\omega_{rz}\) satisfy the following equations,

\[
\begin{align*}
(o_{ds} - o_{dl}) \cdot R &= (o_{ds} + o_{dl}) \cdot D_{gh} \\
\omega_{rz} &= (o_{ds} + o_{dl}) \cdot \frac{R_{sp}}{2R} = (o_{ds} - o_{dl}) \cdot \frac{R_{sp}}{2D_{gh}}
\end{align*}
\]

According to the above equation:

- When \(o_{ds} = o_{dl}\), the turning radius \(R \to \infty\), and the tracks go straight;
- When \(o_{ds} \cdot o_{dl} = 0\), the track is locked at one side, and \(R = \pm D_{gh}\);
- When \(o_{ds} = -o_{dl}\), the larboard and starboard tracks are driven forward and backward, the vehicle body rotates in situ, and \(R = 0\).

The above three working conditions can be regarded as special working conditions of steady-state steering, which are relatively simple to solve and analyze; here the steady-state steering of non-special working conditions was defined as conventional steering. For ideal steady-state conventional steering:

- When the driving velocity of the larboard and starboard tracks \((o_{ds} \text{ and } o_{dl})\) are fixed, the turning radius \(R\) is proportional to the half gauge of the chassis \(D_{gh}\), and the turning angular velocity \(\omega_{rz}\) is inversely proportional to \(D_{gh}\);
- When the gauge of the chassis \(2D_{gh}\) remains constant, the turning radius is proportional to \((o_{ds} + o_{dl})/(o_{ds} - o_{dl})\), the ratio of the sum of the driving forces of the starboard and larboard tracks to the difference between the two driving forces, and the turning angular velocity is proportional to \((o_{ds} - o_{dl})\), the difference between the driving forces of the starboard and larboard tracks.

\[
\begin{align*}
(o_{ds} - o_{dl}) \cdot R &= (o_{ds} + o_{dl}) \cdot D_{gh} \\
\omega_{rz} &= (o_{ds} + o_{dl}) \cdot \frac{R_{sp}}{2R} = (o_{ds} - o_{dl}) \cdot \frac{R_{sp}}{2D_{gh}}
\end{align*}
\]

For a left turn in conventional steady-state steering, \(o_{dl} \neq 0\), the ratio of the driving force of starboard track to that of larboard track \(K_{dr} = o_{ds}/o_{dl}\), then Equation (13) can be expressed as follows using \(K_{dr}\),

\[
\begin{align*}
(K_{dr} - 1) \cdot R &= (K_{dr} + 1) \cdot D_{gh} \\
\omega_{rz} &= \frac{R_{sp}}{2D_{gh}} \cdot (K_{dr} - 1) \cdot o_{dl}
\end{align*}
\]

2) In conventional steering, considering the slip drive and slip resistance of the tracks, the relationship between the lateral shift of instantaneous center of larboard and starboard tracks \((D'_{st}, D'_{sl})\) and the turning radius \(R'\) as well as the turning angular velocity \(\omega'_{rz}\) was solved; based on the geometrical relationship between relevant parameters, the driving forces of larboard and starboard tracks \((V_{ds} = R_{sp}o_{ds}, V_{sl} = R_{sp}o_{dl})\), the following equation can be worked out,

\[
\begin{align*}
\left((o_{ds} - o_{dl})\right)R' &= \left(o_{ds} + o_{dl}\right)D_{gh} + \left(o_{dl}D'_{sl} - o_{ds}D'_{st}\right) \\
\omega'_{rz} &= \left(o_{ds} - o_{dl}\right) \cdot \frac{R_{sp}}{2D_{gh} + (D'_{sl} - D'_{st})}
\end{align*}
\]
For a left turn in conventional steady-state steering, \( \omega_{lsl} \neq 0 \), by substituting \( K_{Dr} \) into the above equation, the following equation can be obtained,

\[
(K_{p_0} - 1) \cdot R' = (K_{p_0} + 1) \cdot D_{Gh} + (D_{Gh}' - K_{p_0}D_{Gh}')
\]

\[
\omega_{lZ} = \frac{R_{ep}}{2D_{Gh} + (D_{Gh}' - D_{Gh}')} \cdot (K_{p_0} - 1) \cdot \omega_{ol}
\]

(17)

3) In conventional steering, considering the slipping rate \( i \), according to the geometrical relationship between relevant parameters and Equation (14), the turning radius \( R' \) and the turning angular velocity \( \omega_{lZ} \) can be calculated as follows,

\[
\left[ \omega_{ol}(1-i) - \omega_{ol}(1-i_i) \right] \cdot R' = \left[ \omega_{ol}(1-i) + \omega_{ol}(1-i_i) \right] \cdot D_{Gh}
\]

\[
\omega_{lZ}' = \left[ \frac{\omega_{ol}(1-i) + \omega_{ol}(1-i_i)}{2R} \right] \cdot \frac{R_{ep}}{2D_{Gh}} \cdot \left[ \omega_{ol}(1-i) - \omega_{ol}(1-i_i) \right]
\]

(18)

For a left turn in conventional steady-state steering, \( \omega_{lsl} \neq 0 \), by substituting \( K_{Dr} \) into the above equation, the following equation can be obtained,

\[
\left[ \frac{\omega_{ol}(1-i) - \omega_{ol}(1-i_i)}{2R} \right] \cdot R' = \left[ \frac{\omega_{ol}(1-i) + \omega_{ol}(1-i_i)}{2R} \right] \cdot D_{Gh}
\]

\[
\omega_{lZ} = \frac{R_{ep}}{2D_{Gh}} \cdot \left[ K_{p_0}(1-i_i) - (1-i_i) \right] \cdot \omega_{ol}
\]

(19)

4. Conclusion

(1) In this study, a kinetic model was established based on a force analysis of the SMT-CS during steady-state steering. The computational analysis shows that the slipping rate \( i \) of the tracks has an influence on steering performance of the chassis during steady-state steering.

(2) According to the geometrical relationship between parameters of the chassis during steady-state steering, the relational expression between the slipping rate \( i \), drive ratio \( K_{p_0} \) and the actual turning radius \( R' \) was derived.

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