Model for T2K indication with maximal $\theta_{23}$ and tri-maximal $\theta_{12}$

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Recently T2K gives hint in favor of large reactor angle $\theta_{13}$. Most of the models, with tri-bimaximal mixing at the leading order, can not reproduce such a large mixing angle since they predict typically corrections for the reactor angle of the order $\theta_{13} \sim \lambda_C^2$, where $\lambda_C \sim 0.2$. In this paper, we discuss the possibility to achieve large $\theta_{13}$ within the T2K region with maximal atmospheric mixing angle, $\sin^2 \theta_{23} = 1/2$, and trimaximal solar mixing angle, $\sin^2 \theta_{12} = 1/3$, through the deviation from the exact tri-bimaximal mixing. We derive the structure of neutrino mass matrix that leads to the large $\theta_{13}$ leaving maximal $\theta_{23}$ and trimaximal $\theta_{12}$. It is shown that such a structure of neutrino mass matrix can arise in a model with $S_4$ flavor symmetry.

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I. INTRODUCTION

The T2K collaboration recently gives that the reactor angle is

\[ 0.087(0.100) \leq \sin \theta_{13} \leq 0.275(0.306), \]

with best fit value of $\sin \theta_{12} = 0.17(0.19)$ for normal (inverted) hierarchy in the neutrino masses. Such a result must be taken as a hint since it is only at 2$\sigma$. We however consider the implications of such an important indication in this paper.

In the last decade, the tri-bimaximal (TB) mixing pattern introduced by Harrison et al. in 2002 $^2$

\[ U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \]

has been used as a guide in neutrino physics for the flavor problem. However if the result of T2K will be confirmed, it will have strong impact on this point of view. In fact, most of the models predicting tri-bimaximal mixing at the leading order are compatible only with small values of the reactor angle. In a generic model, the three mixing angles receive corrections of the same order. Departure of the solar angle from the trimaximal values are at most of $O(\lambda_C^2)$ where $\lambda_C \approx 0.2$. Therefore it is possible to have a deviation of order $\lambda_C^2 \approx 0.04$ only $^3$ for the reactor angle which is about half of the lower bound in $^1$. This is only an estimation and it must be considered individually case by case. We however expect that most of them are on the border of validity if not excluded completely. Therefore it is important to search for the models with large $\theta_{13}$ and with maximal atmospheric mixing angle and trimaximal$^1$ solar angle. Recently, some works fitting T2K result has been presented $^5$. There are some models based on discrete flavor symmetries before T2K data which predicts large reactor mixing angle, for an incomplete list see reference $^6$, and for a classification of models with flavor symmetries classified by its predictions for reactor angle see $^7$.

In this paper, we study the possibility to obtain the large reactor angle with tri-bimaximal values of the solar and atmospheric mixing angles. The lepton mixing matrix with such mixing pattern was first proposed by King in $^8$ and called Tri-bimaximal-reactor (TBR) mixing. Using such mixing matrix, we found the structure of the deviations in the neutrino mass matrix from its TB texture which leads to TBR mixing. We then show that such a particular deviation in neutrino mass matrix can arise in a model with $S_4$ flavor symmetry.

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$^1$The word “trimaximal” is used for different meaning in previous studies $^4$. We mean here $\sin^2 \theta_{12} = 1/3$ by trimaximal solar angle.
The paper is organized as follows. We discuss the conditions to obtain a mass matrix with maximal atmospheric mixing angle, trimaximal solar mixing angle and a non-zero reactor mixing angle within the T2K region in section II. In section III, we present a model based on the group of permutation of four objects, $S_4$ where the neutrino mass matrix with particular form discussed in section II is obtained. Finally, we discuss the phenomenology of the model and conclude in section IV.

II. LARGE REACTOR TRI-BIMAXIMAL MIXING AND NEUTRINO MASS MATRIX

In this section, we study the structure of the neutrino mass matrix (in the diagonal basis of the charged leptons) that gives maximal atmospheric angle $\theta_{23} = \pi/4$, trimaximal solar angle $\sin \theta_{12} = 1/\sqrt{3}$ and an arbitrary reactor angle $\theta_{13} = \lambda$. In the standard PDG parametrization, the lepton mixing matrix with the above values of mixing angles is given by

\[
U_{TBR} = R_{23} \left( \frac{\pi}{4} \right) R_{13}(\lambda) R_{12}(\theta_{12}) = \left( \begin{array}{ccc} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{\sqrt{2}}{2} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{2} \end{array} \right) + O(\lambda^2). \tag{3}
\]

We do not consider the CP violation in the lepton sector assume that the above parameters are real for simplicity. The neutrino mass matrix diagonalized by (3) is given by

\[
m_{\nu}^{TBR} = U_{TBR} \cdot m_{\nu}^{\text{diag}} \cdot U_{TBR}^T = m_{\nu}^{TB} + \delta m_{\nu} \tag{4}
\]

where $m_{\nu}^{\text{diag}}$ is a diagonal matrix with the neutrino mass eigenvalues, $m_{\nu_1}$, $m_{\nu_2}$ and $m_{\nu_3}$. This leads to the following structure of the neutrino mass matrix

\[
m_{\nu}^{TB} = \begin{pmatrix} 2y - x & x & x \\ x & y + z & y - z \\ x & y - z & y + z \end{pmatrix}, \tag{5}
\]

where $x = (m_2 - m_1)/3$, $y = (m_1 + 2m_2)/6$ and $z = m_3/2$ and

\[
\delta m_{\nu} = \lambda \left( \begin{array}{ccc} 0 & \alpha_1 & -\alpha_1 \\ \alpha_1 & \beta_1 & 0 \\ -\alpha_1 & 0 & -\beta_1 \end{array} \right) + \lambda^2 \left( \begin{array}{ccc} \gamma & \alpha_2 & \alpha_2 \\ \alpha_2 & \beta_2 & -\beta_2 \\ \alpha_2 & -\beta_2 & \beta_2 \end{array} \right) + \sum_{n \geq 3} \lambda^n \left( \begin{array}{ccc} 0 & \alpha_n & (-1)^n \alpha_n \\ \alpha_n & 0 & 0 \\ (-1)^n \alpha_n & 0 & 0 \end{array} \right), \tag{6}
\]

with $\alpha_1 = -(x - 2y + z)/\sqrt{2}$, $\beta_1 = \sqrt{2}x$, $\alpha_2 = -x/2$, $\beta_2 = -(x - 2y + z)/2$ and $\gamma = x - 2y + 2z$. Note that $\beta_2$ can be reabsorbed into the TB term $m_{\nu}^{TB}$. The above form of neutrino mass matrix predicts maximal atmospheric mixing angle and trimaximal solar mixing angle if all the terms with all powers of $\lambda$ are taken into account. If one truncates the series in eq. (6) at $n < 3$, the neutrino mass matrix then implies

- (A) negligible deviations from maximality in the atmospheric mixing angle;
- (B) small deviation from trimaximality in the solar mixing angle;
- (C) prediction of $0\nu\beta\beta \propto \lambda^2$.

The prediction (C) is evident from eq. (6) and we verify numerically (A) and (B) in the section IV. We observe that the main structure of the deviation $\delta m_{\nu}$ of order $\lambda$ in eq. (6) is $\mu-\tau$ antisymmetric, see $TBR^2$. Therefore a possible flavor symmetry with neutrino mass matrix texture (4) must contain the group $S_2$ of the $\mu-\tau$ permutation and must be compatible with tri-bimaximal in the unperturbed limit. One possible flavor symmetry with such features is $S_4$ which contains $S_2$ as a subgroup and leads to tri-bimaximal mixing $TBR$.  

2 Note that the main structure of the deviation $\delta m_{\nu}$ of order $\lambda$ in eq. (6) is similar to the one found in the paper by T. Araki in Ref. 1 where (contrary with respect to us) the solar angle is not fixed to be the trimaximal one.
III. THE MODEL

We assume $S_4$ (see appendix) flavor symmetry and extra abelian $Z_N$ symmetry in order to separate the charged leptons from the neutrino sector as usual in models for TB mixing, see for instance [3]. In order to simplify the model as much as possible and to render more clear the main features of the model, we do not enter into the details of the particular $Z_N$ symmetry required in this model. Our purpose is to show that the neutrino mass matrix \([13]\) with the structure given by \([5]\) and \([6]\) can be obtained from symmetry principle. We assume that light neutrino masses arise from both type-I and type-II seesaw and introduce only one right-handed neutrino. The matter content of our model is given in table I.

| $SU_L(2)$ | $\nu_L^c$ | $h$ | $\Delta$ | $\phi^c$ | $\nu^c_l$ |
|-----------|-----------|-----|-----------|----------|-----------|
| $S_4$     | 3         | 3   | 1         | 2        | 3         |

TABLE I: Matter content of the model giving TB mixing at the leading order

In the scalar sector, we have one $SU_L(2)$ triplet $\Delta$ and one singlet $\phi$ in the neutrino sector transforming both as $3_1$ of $S_4$. We have two electroweak singlets $\phi_1$ and $\xi_l$ in the charged lepton sector, transforming as doublet and singlet of $S_4$ respectively. As it has been already mentioned, the two sectors can be separated by introducing an abelian $Z_N$ symmetry under which $l^c$, $\phi_1$ and $\xi_l$ are charged while the other fields could be singlets of $Z_N$. The Yukawa interaction of the model is

$$-\mathcal{L}_t = \frac{1}{\Lambda} y_1 (\overline{L} l_R)_1 h \xi_l + \frac{1}{\Lambda} y_2 (\overline{L} l_R)_2 h \phi_1 + h.c. \quad (7)$$

$$-\mathcal{L}_\nu = y_a L L \Delta + y_b (\overline{L} \phi)_1 \nu R + \frac{1}{2} M \nu^c \nu^c + h.c. \quad (8)$$

where $\Lambda$ is an effective scale. We assume the following $S_4$ alignment in the vacuum expectation values (vevs) of the scalar fields.

$$\langle \Delta^0 \rangle = v_\Delta (1, 1, 1)^T, \quad \langle \phi \rangle = v_\phi (0, 1, -1)^T, \quad \langle \phi \rangle = (v_1, v_2)^T, \quad (9)$$

where $v_1 \neq v_2$. Using the product rules shown in appendix A, one can easily see that the charged lepton mass matrix is diagonal and the lepton masses can be fitted in terms of three free parameters $y_1$, $v_1$ and $v_2$, see [12] for details.

The type-II seesaw gives a contribution to the neutrino mass matrix with zero diagonal entries and equal off diagonal entries since it arises from the product of three $S_4$ triplets. Since we introduced only one right-handed neutrino, Dirac neutrino mass matrix is a column $m_D \sim (0, 1, -1)^T$ and the light-neutrino mass matrix from seesaw relation is given by

$$m_\nu^{\text{type-II}} = \frac{1}{M} m_D m_D^T \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \quad (10)$$

Considering both the type-I and type-II contributions, we have the light neutrino mass matrix which can be diagonalized by TB mixing matrix \([13]\)^3

$$m_\nu^{\text{TB}} = \begin{pmatrix} 0 & a & a \\ a & b & a - b \\ a & a - b & b \end{pmatrix}, \quad (11)$$

The mass eigenvalues of the above matrix are $m_1 = -a$, $m_2 = 2a$ and $m_3 = -a + 2b$. Here $a = y_b v_\Delta$ and $b = y_b^2 v_1^2 v_2^2 / (\Lambda^2 M)$ where $v_b = \langle h^0 \rangle$. This neutrino mass matrix is compatible with the normal hierarchy only and predicts zero neutrinoless double beta decay $m_{ee} = 0$.

3 In [12], similar structure [10] has been obtained through only type-II seesaw and $S_4$ symmetry.
In order to reproduce deviations like eq. (6) in the neutrino mass matrix, we introduce in the scalar sector one Higgs triplet $\Delta_d$ that transforms as a doublet under $S_4$ and an electroweak singlet $\phi_d$ that transforms as a triplet 3 under $S_4$. With inclusion of these fields, the Yukawa interaction Lagrangian $\mathcal{L}_\nu$ contains also the terms

$$- \mathcal{L}_\nu \supset y_\beta LL\Delta_d + \frac{y_\alpha}{\Lambda}(T\phi_d)_{11} \bar{h} \nu_R + h.c.$$  \hspace{1cm} (12)

We assume that $\Delta_d$ and $\phi_d$ take vevs along the following directions

$$\langle \Delta_d^0 \rangle = v_d(1,0)^T, \quad \langle \phi_d \rangle = u_d(1,0)^T.$$  \hspace{1cm} (13)

Here we also assume that $y_{\alpha,\beta} \ll y_{\alpha,\beta}$. This can be realized assuming that $\Delta_d$ and $\phi_d$ are charged under some extra abelian symmetry like $Z_N$ or $U_{FN}(1)$.

After electroweak symmetry breaking and integrating out the right-handed neutrino, eq. (12) gives the following contribution to the neutrino mass matrix

$$\frac{y_\alpha y_\beta v_h^2}{\Lambda^2 M}(\nu_\phi)_{11}, (\nu_\phi)_{11} + \frac{y_\alpha^2 v_h^2}{\Lambda^2 M}(\nu_\phi)_{11}, (\nu_\phi)_{11}.$$  \hspace{1cm} (14)

The second term in eq. (14) is smaller with respect to the first since we have assumed $y_\alpha \ll y_\beta$. In particular assuming $y_\beta \sim 1$ and $y_\alpha \sim \lambda$ the first term is proportional to $\lambda$ and the second term is proportional to $\lambda^2$. The extra contributions to the neutrino mass matrix from the type-I see-saw are as follows

$$\delta m^\text{type-I}_\nu \sim c_1 \lambda \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} + c_2 \lambda^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$  \hspace{1cm} (15)

where, $c_1$ and $c_2$ are coefficients of order $O(1)$. From the extra type-II seesaw term in eq. (12) and using the vev alignments as in (13), the additional contribution to the perturbed neutrino mass matrix will be proportional to $\nu_1 \nu_1 - \nu_2 \nu_2$, therefore the contribution to the neutrino mass matrix coming from Type-II see-saw is

$$\delta m^\text{type-II}_\nu \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$  \hspace{1cm} (16)

Putting all these results together, the structure of the deviation in neutrino mass matrix can be written is

$$\delta m_\nu = \begin{pmatrix} \gamma' & \alpha' & -\alpha' \\ \alpha' & \beta' & 0 \\ -\alpha' & 0 & -\beta' \end{pmatrix},$$  \hspace{1cm} (17)

where $\alpha' = y_\alpha y_\beta v_h^2 v_\phi u_d/\Lambda^2 M$, $\beta' = y_\beta v_d$, $\gamma' = y_\alpha^2 v_h^2 v_\phi u_d/\Lambda^2 M$.

The deviation obtained in our model equal to the neutrino mass deviation in eq. (18) truncated at $\lambda^2$ with $\alpha_2 = 0$. Such a difference does not modify significantly the prediction of maximal atmospheric angle and trimaximal solar angle. In the next section, we study the phenomenological implication of our neutrino mass texture.

### IV. PHENOMENOLOGY

Combining eq. (11) and eq. (17), the resulting neutrino mass matrix in our model is

$$m_\nu = \begin{pmatrix} \gamma' & a + \alpha' & -a - \alpha' \\ a + \alpha' & b + \beta' & a - b \\ -a - \alpha' & a - b & b - \beta' \end{pmatrix},$$  \hspace{1cm} (18)

As mentioned earlier, we assume $y_\alpha, y_\beta \sim O(1)$ and $y_\alpha, y_\beta \sim O(\lambda)$ which implies the hierarchies in the elements $a, b \gg \alpha', \beta', \gg \gamma'$ in the above structure. Since the $(m_\nu)_{11}$ entry is $\gamma \sim O(\lambda^2)$, we have neutrino less double beta decay rate $m_{ee} \propto \lambda^2$ and for small values of $\lambda$ as in the unperturbed case only the normal neutrino mass hierarchy can be fitted. The neutrino mass matrix (18) obtained from the model is equivalent to the matrix (13) up to the correction
of $\mathcal{O}(\lambda^2)$, the neutrino mass matrix \((18)\) is diagonalized by the mixing matrix \((3)\) up to $\mathcal{O}(\lambda^2)$ corrections. Note that the neutrino mass matrix \((18)\) obtained in the model has 5 free parameters while the derived structure in eq. \((3)\) has 4 real parameters \((x, y, z\) and \(\lambda)\). We fix free parameters of eq. \((18)\) in terms of the parameters of eq. \((3)\) by comparing both the structures at each order of \(\lambda\). Comparing the leading order expressions of the neutrino mass matrix in eq. \((11)\) and eq. \((5)\), we restrict our parameters to be

\[
x = 2y; \quad a = 2y; \quad b = y + z.
\]

Further comparing the higher order terms in \(\lambda\), we obtain the relations

\[
\alpha' = -\sqrt{2}z\lambda; \quad \beta' = 2\sqrt{2}y\lambda; \quad \gamma' = 2z\lambda^2.
\]

Note that this is not the most general case for our model, nevertheless we want to point out that even in the case of bimaximal atmospheric mixing angle and trimaximal solar mixing angle it is possible to obtain large reactor mixing angle. Also, we expect the negligible deviations from the tri-bimaximal values for the solar and atmospheric mixing angles due to the fact that our model predicts \(\alpha_2 = 0\) if compared with eq. \((4)\) and it generates the terms only up to the $\mathcal{O}(\lambda^2)$. We analyze such deviations by randomly varying \(y, z\) and \(\lambda\) with the constraints that the square mass differences and the mixing angles are in the observed \((3\sigma)\) range of validity \([14]\). The results of our analysis are shown in figure \([1]\). It is evident from figure \([1]\) that for restricted parameter space as specified earlier, model allows large \(\theta_{13}\) with negligible deviations in atmospheric and solar mixing angles from there bimaximal and trimaximal values respectively. We also check the predictions for the neutrinoless double beta decay rates in our model for restricted parameter space specified above and find that the region for \(4.5\) meV < \(m_{\nu_1}\) < \(5.8\) meV and \(0.5\) meV < \(|m_{\beta\beta}|\) < \(3.5\) meV is allowed for the values of \(\theta_{13}\) in the \(2\sigma\) limits indicated by T2K.

In summary, we found the structure for the deviation in the neutrino mass matrix from the well known TB pattern in such a way that the lepton mixing matrix has large atmospheric mixing angle and trimaximal solar mixing angle with an arbitrary large reactor angle. The deviation must be approximately $\mu$-$\tau$ antisymmetric. This fact suggests us that the flavor symmetry could be some permutation symmetry containing $S_2$ ($\mu$-$\tau$ exchange) subgroup. $S_3$ is too small since it does not give the TB mixing. The smallest permutation group with this property is $S_4$. We provide a candidate model based on $S_4$ where in the unperturbed limit the neutrino mass matrix is TB. Then assuming extra scalar fields we show the possibility to generate deviations from the TB that give a large \(\theta_{13}\) in agreement with T2K result, maximal atmospheric mixing angle and trimaximal solar mixing angle in good agreement with neutrino data.

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Appendix A: $S_4$ product rules

In the basis where the generator of $S_4$ are real, the products of $\mu \times \mu$ (see [11]):

for 2
\[
\begin{align*}
& a_1a'_1 + a_2a'_2 \sim 1_1, \\
& -a_1a'_2 + a_2a'_1 \sim 1_2, \\
& \left( a_1a'_2 + a_2a'_1 \right) \sim 2,
\end{align*}
\]

for $3_1$
\[
\sum_{j=1}^{3} b_jb'_j \sim 1_1,
\]
\[
\left( \frac{1}{\sqrt{6}}(b_2b'_2 - b_3b'_3) \right) \sim 2,
\]
\[
\left( \frac{1}{\sqrt{6}}(-2b_1b'_1 + b_2b'_2 + b_3b'_3) \right) \sim 2,
\]
\[
\begin{align*}
& \left( b_2b'_2 + b_3b'_3 \right) \sim 3_1, \\
& \left( b_1b'_3 - b_3b'_1 \right) \sim 3_2.
\end{align*}
\]

For $3_1 \times 3_2$
\[
\sum_{j=1}^{3} b_jc'_j \sim 1_2
\]
\[
\left( \frac{1}{\sqrt{6}}(2b_1c_1 - b_2c_2 - b_3c_3) \right) \sim 2
\]
\[
\left( \frac{1}{\sqrt{6}}(2b_3c_2 - b_2c_1 - b_1c_2) \right) \sim 2
\]

for $3_2$
\[
\sum_{j=1}^{3} c_jc'_j \sim 1_1,
\]
\[
\left( \frac{1}{\sqrt{6}}(c_2c'_2 - c_3c'_3) \right) \sim 2,
\]
\[
\left( \frac{1}{\sqrt{6}}(-2c_1c'_1 + c_2c'_2 + c_3c'_3) \right) \sim 2.
\]

For $2 \times 3_1$
\[
\left( \frac{a_2b_1}{2}(\sqrt{3}a_1b_2 + a_2b_3) \right) \sim 3_1
\]
\[
\left( \frac{a_1b_1}{2}(\sqrt{3}a_1b_2 - a_1b_3) \right) \sim 3_2
\]

and for $2 \times 3_2$
\[
\left( \frac{a_1c_1}{2}(\sqrt{3}a_2c_2 - a_1c_3) \right) \sim 3_1
\]
\[
\left( \frac{a_2c_1}{2}(\sqrt{3}a_2c_2 + a_2c_3) \right) \sim 3_2.
\]

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