ν’s in Particle Physics and Cosmology

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Abstract. We propose a minimal scale-invariant (SI) scotogenic model and study its phenomenological implications. The SI scotogenic predicts a singlet scalar (dilaton) that triggers electroweak symmetry breaking and sources the lepton number violation that allows radiative neutrino mass. A viable parameter space is found for dark matter candidate and the observed neutrino oscillation. The model can be probed in lepton flavor violating processes, future dark matter direct detection experiments, and collider searches.

1. Introduction
Although the standard model (SM) of particle physics is very successful in describing interactions of elementary particle up to TeV scale, there are a number of problems remained unsolved. This includes the mechanism of neutrino mass, the nature of dark matter (DM), and the origin of the electroweak symmetry breaking (EWSB).

An attractive way to induce naturally small neutrino mass is the radiative neutrino mass generation, where neutrino mass are generated at loop level\cite{1, 2, 3, 4, 5, 6, 7, 8, 9}. The scotogenic model is a simple radiative neutrino model that aims to offer an explanation of the origin of small neutrino mass and the dark matter \cite{10}. With regards to the issue of the EWSB, a number of authors considered extending the SM with a scale-invariance (SI) symmetry and where the Higgs mass arises via radiative symmetry breaking \cite{11, 12, 13}.

In this talk, we consider the minimal implementation of the SI symmetry of the scotogenic model\cite{1}. The aim is to maintain the appealing features of the scotogenic model, namely the explanation for both neutrino mass and DM, while incorporating a dynamical model for the origin of the weak scale. We will show that there is a viable parameter space in which the short comings of the standard model mentioned above can be explained. Moreover, the model can be probed in different ways, including: lepton flavor violation (LFV) searches of the process $\mu \rightarrow e + \gamma$, future DM direct detection experiments, and high energy collider searches.

2. A Scale-Invariant Neutrino Mass Model
Here we consider the minimal SI extension of the standard model which neutrino mass is generated at one loop. The SM is extended by the addition of two gauge-singlet fermions, $N_{iR} \sim (1, 1, 0)$, where $i = 1, 2, 3$, labels generations, a second SM-like scalar doublet, $S \sim (1, 2, 1)$, and a singlet scalar $\phi \sim (1, 1, 0)$. A discrete $Z_2$ symmetry with action $\{N_{iR}, S\} \rightarrow \{-N_{iR}, -S\}$

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\textsuperscript{1} The SI scotogenic model belongs to a larger family of SI models with one-loop neutrino mass and DM. For details see \cite{14}.
is imposed on the model. The scalar \( \phi \), as well as the SM fields, transform trivially under the \( Z_2 \) symmetry. The lightest \( Z_2 \)-odd particle is a stable DM candidate, which should be taken as either the lightest singlet fermion \( N_1 \) or a neutral component of the doublet \( S \), as discussed below. The scalar \( \phi \) plays the dual role of sourcing lepton number violation to allow neutrino masses and triggering electroweak symmetry breaking.

With this field content, the Lagrangian for the model is the most-general Lagrangian consistent with both the SI and \( Z_2 \) symmetries. It contains the terms

\[
\mathcal{L} \supset i N_{iR} \partial^\mu N_{iR} + \frac{1}{2} (\partial^\mu \phi)^2 + |D^\mu S|^2 - \frac{y_i}{2} \phi N_{iR} \bar{N}_{iR} - g_{ia} N_{iR} L_\alpha S - V(\phi, S, H),
\]

where \( L_\alpha \sim (1, 2, -1) \) denotes the SM lepton doublets, with generations labeled by Greek letters, \( \alpha, \beta = e, \mu, \tau \). We denote the SM scalar doublet as \( H \sim (1, 2, 1) \) and \( V(\phi, S, H) \) is the most-general scalar potential consistent with the symmetries of the model.

### 2.1. Symmetry Breaking

In the absence of dimensional parameters, the scalar potential contains only quartic interactions:

\[
V(\phi, S, H) = \lambda_H |H|^4 + \frac{\lambda_\phi}{4} \phi^4 + \frac{\lambda_S}{2} |S|^4 + \frac{\lambda_{\phi H}}{2} \phi^2 |H|^2 + \frac{\lambda_{\phi S}}{2} \phi^2 |S|^2 + \lambda_3 |H|^2 |S|^2 + \lambda_4 |H^\dagger S|^2 + \frac{\lambda_5}{2} (S^\dagger H)^2 + \text{H.c.}
\]

(2)

where \( \lambda_5 \) can be taken real without loss of generality. The desired VEV pattern has \( \langle S \rangle = 0 \), to preserve the \( Z_2 \) symmetry, with \( \langle H \rangle \neq 0 \) and \( \langle \phi \rangle \neq 0 \) to break both the SI and electroweak symmetries. In order to ensure viable symmetry breaking (equivalently, to give a positively-valued dilaton mass), the radiatively corrected scalar potential should be dominated by loop corrections involving the scalar field \( S \), and hence we can neglect the loop contributions from the SM scalar and the dilaton. In this case the one loop effective potential reads

\[
V_{\text{1-loop}}(h, \phi) = \frac{\lambda_H}{4} h^4 + \frac{\lambda_{\phi H}}{4} \phi^2 h^2 + \frac{\lambda_5}{4} \phi^4 + \sum_{i = \text{all fields}} n_i G \left( M_i^2 (h, \phi) \right),
\]

(3)

where \( n_i \) is a multiplicity factor, \( \Lambda \) is the renormalization scale, and the sum is over all fields barring the light scalars (\( h \) and \( \phi \)) and the light SM fermions (all but the top-quark). The function \( G \) is given by

\[
G(X) = \frac{X^2}{64 \pi^2} \left[ \log \frac{X}{\Lambda^2} - \frac{3}{2} \right],
\]

(4)

while, in the absence of bare dimensionful parameters, the field-dependent masses can be written as

\[
M_i^2 (h, \phi) = \frac{\alpha_i}{2} h^2 + \frac{\beta_i}{2} \phi^2,
\]

(5)

where \( \alpha_i \) and \( \beta_i \) are constants. Analyzing the potential reveals a minimum with both \( \langle \phi \rangle \equiv x \neq 0 \) and \( \langle h \rangle \equiv v \neq 0 \) for \( \lambda_{\phi H} < 0 \). At tree-level, the desired VEV pattern is triggered at the scale \( \Lambda \) where the running couplings obey \( 2 \sqrt{\lambda_H(\Lambda)} \lambda_\phi(\Lambda) + \lambda_{\phi H}(\Lambda) = 0 \). Including the loop corrections modifies this relation to give

\[
2 \left\{ \lambda_H \lambda_\phi + \frac{\lambda_H}{x^2} \sum_i n_i \left( \beta_i - \alpha_i \right) \left( \frac{v^2}{x^2} \right) \right\}^{1/2} + \lambda_{\phi H} + \frac{2}{x^2} \sum_i n_i \alpha_i G' \left( M_i^2 \right) = 0,
\]

(6)
Figure 1. One-loop diagram for neutrino mass in a scale-invariant scotogenic model.

with $G'(X) = \partial G(X)/\partial X$. The further condition

$$-\frac{\lambda_{\phi H}}{2\lambda_H} = \frac{v^2}{x^2} + \sum_i \frac{n_i \alpha_i}{\lambda_H x^2} G'(M_i^2),$$

(7)

is also satisfied. For the stability of the vacuum we require that the couplings obey:

$$\lambda_{H}^{1-l}, \lambda_{\phi}^{1-l}, \lambda_{\phi H}^{1-l} + 2 \sqrt{\lambda_{H}^{1-l} \lambda_{\phi}^{1-l}} > 0,$$

(8)

where the one-loop couplings are defined as

$$\lambda_{H}^{1-l} = \frac{1}{6} \frac{\partial^4 V_1}{\partial h^4}, \quad \lambda_{\phi}^{1-l} = \frac{1}{6} \frac{\partial^4 V_1}{\partial \phi^4}, \quad \lambda_{\phi H}^{1-l} = \frac{\partial^4 V_1}{\partial h^2 \partial \phi^2}.$$

(9)

Eq. (8) guarantees that the masses for the neutral scalars $h$ and $\phi$ are strictly positive, thereby forcing one of the beyond-SM scalars in the doublet $S$ to be the heaviest particle in the spectrum, to overcome top-quark contributions to the dilaton mass. Demanding $\lambda_{\phi H}^{1-l} < 0$ also ensures that the vacuum with $v \neq 0$ and $x \neq 0$ is the preferred over the vacuum with only one nonzero VEV.

2.2. Neutrino Mass

The combined terms in Eqs. (1) and (2) explicitly break lepton number symmetry. Neutrinos therefore acquire mass radiatively at the one-loop level, as shown in Figure 1. Calculating the mass diagram gives

$$\langle M_{\nu} \rangle_{\alpha \beta} = \sum_i \frac{g_{\alpha i} g_{\beta i} M_i}{16\pi^2} \left\{ \frac{M_{S_0}^2}{M_{S_0}^2 - M_i^2} \ln \frac{M_{S_0}^2}{M_i^2} - \frac{M_A^2}{M_A^2 - M_i^2} \ln \frac{M_A^2}{M_i^2} \right\}.$$  

(10)

In the limit that $M_{S_0}^2 \approx M_A^2 \equiv M_0^2$, this simplifies to

$$\langle M_{\nu} \rangle_{\alpha \beta} \approx \sum_i \frac{g_{\alpha i} g_{\beta i} \lambda_5 v^2}{16\pi^2} \frac{M_i}{M_0^2 - M_i^2} \left\{ 1 - \frac{M_i^2}{M_0^2 - M_i^2} \ln \frac{M_0^2}{M_i^2} \right\}.$$  

(11)

Note that the $Z_2$ symmetry prevents mixing between SM neutrino and the exotics $N_i$. 

One can relate the entries in the neutrino mass matrix to the elements of the Pontecorvo-Maki-Nakawaga-Sakata (PMNS) mixing matrix elements. We parameterize the latter as

\[
U_\nu = \begin{pmatrix}
c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta_d} \\
-c_{23}s_{12} - c_{13}s_{123}e^{i\delta_d} & c_{23}c_{13} - s_{13}s_{123}e^{i\delta_d} & -s_{13}c_{23} \\
s_{12}s_{23} - c_{13}s_{123}e^{i\delta_d} & -c_{23}s_{12} - c_{13}s_{123}e^{i\delta_d} & c_{13}c_{23}
\end{pmatrix} U_m,
\]

with \( \delta_d \) being the Dirac phase and \( U_m = \text{diag}(1, e^{i\delta_\alpha}/2, e^{i\delta_\beta}/2) \) encoding the dependence on the Majorana phases \( \theta_{\alpha,\beta} \). The shorthand \( s_{ij} \equiv \sin \theta_{ij} \) and \( c_{ij} \equiv \cos \theta_{ij} \) refers to the mixing angles. In our subsequent numerical scans for viable parameter space in the model, we shall fit the Majorana phases \( \theta_{\alpha,\beta} \).

In order to probe the parameter space that gives viable neutrino masses, we use the Casas-Ibarra parametrization [15]

\[
(M_\nu)_{\alpha\beta} = \sum_i g_{i\alpha}g_{i\beta}A_i = (g^T A g)_{\alpha\beta},
\]

with

\[
A_i = \frac{M_i}{16\pi^2} \left\{ \frac{M_{S0}^2}{M_{S0}^2 - M_i^2} \ln \frac{M_{S0}^2}{M_i^2} - \frac{M_A^2}{M_A^2 - M_i^2} \ln \frac{M_A^2}{M_i^2} \right\}.
\]

According to the Casas-Ibarra parametrization, the coupling \( g \) can be written as

\[
g = D_{\sqrt{\Lambda}} D_{\sqrt{m_\nu}} U_\nu^\dagger,
\]

and where \( D_{\sqrt{\Lambda}} = \text{diag} \left\{ \sqrt{\Lambda_1}, \sqrt{\Lambda_2}, \sqrt{\Lambda_3} \right\} \), \( D_{\sqrt{m_\nu}} = \text{diag} \left\{ \sqrt{m_1}, \sqrt{m_2}, \sqrt{m_3} \right\} \), with are the neutrino eigenmasses and \( \mathcal{R} \) is an orthogonal rotation matrix.

3. Constraints from Lepton Flavor Violation

In this section we present formula for important lepton flavor violating effects in the model.

3.1. \( \mu \rightarrow e\gamma \)

The new fields give rise to one-loop contributions to \( \mu \rightarrow e + \gamma \). Normalized relative to \( \text{Br}(\mu \rightarrow e\nu_e\bar{\nu}_e) \), the branching fraction for \( \mu \rightarrow e + \gamma \) is

\[
\frac{\text{Br}(\mu \rightarrow e\gamma)}{\text{Br}(\mu \rightarrow e\nu_e\bar{\nu}_e)} = \frac{3(4\pi)^3\alpha_{em}}{4G_F^2} |A_D|^2,
\]

where \( A_D \) is the dipole form factor:

\[
A_D = \sum_i g_{i\mu}g_{i\nu} \frac{1}{32\pi^2 M_{S^+}} F^{(n)}(M_i^2/M_{S^+}^2),
\]

with the loop function given by

\[
F^{(n)}(x) = [1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x]/[6(1 - x)^4].
\]
3.2. $\mu \rightarrow \bar{e} e e$

Similarly, the branching fraction for $\mu \rightarrow \bar{e} e e$ is

$$
\frac{\text{Br}(\mu \rightarrow \bar{e} e e)}{\text{Br}(\mu \rightarrow e\nu\bar{\nu}_e)} = \frac{3(4\pi)^3\alpha^2}{8G_F^2} \left\{ |A_D|^2 \left( \frac{8}{3} \ln \frac{m_\mu^2}{m_e^2} - \frac{22}{3} \right) + |A_{ND}|^2 + \frac{1}{6}|B|^2 + \frac{1}{3}(2|F_Z^L|^2 + |F_Z^R|^2) - \left( 2A_{ND}A_D^* - \frac{1}{3}B^*A_{ND} + \frac{2}{3}B^*A_D + \text{H.c.} \right) \right\},
$$

Here $B$ represents the contribution from box diagrams while $A_D$ and $A_{ND}$ are the dipole and non-dipole contributions from photonic penguin diagrams. We also defined

$$
F_{Z}^{L,R} = \frac{F_Z G_{L,R}}{g^{2}M_{Z}^{2}\sin\theta_{W}^{2}},
$$

where $g_{L,R}$ are the couplings of L/R-handed charged leptons to the $Z$ boson and $F_Z$ is due to the $Z$-penguin:

$$
F_{Z} = -\frac{1}{16\pi^{2}} \sum_{i} g_{ei} g_{\mu} \left[ 2g_{ZS} C_{24}(M_{i}, M_{S^{+}}, M_{S^{+}}) + g_{L} B_{1}(M_{i}, M_{S^{+}}) \right],
$$

where $g_{ZS}^{+}$ is the coupling of the Z to $S^{+}$, and $C_{24}$ and $B_{1}$ are standard Passarino-Veltman loop functions (see e.g. Ref. [16]). The photonic non-dipole contribution is

$$
A_{ND} = \sum_{i} \frac{g_{ei}^{*}g_{\mu}}{96\pi^{2}} \frac{1}{M_{S^{+}}^{2}} G^{(n)}(M_{i}^{2}/M_{S^{+}}^{2}),
$$

with the loop factor $G^{(n)}(x) = (2 - 9x + 18x^{2} - 11x^{3} + 6x^{3} \ln x)/[6(1 - x)^{4}]$. The box contribution is

$$
\epsilon^{2} B_{(n)} = \frac{1}{16\pi^{2}} \sum_{i,j} \left[ g_{ei} g_{i\mu} g_{ej} g_{j\mu} \tilde{D}_{0} - M_{ij} g_{e_{i}}^{*} g_{j\mu} g_{j\mu} g_{j\mu} D_{0} \right],
$$

where $D_{0} = D_{0}(M_{i}, M_{j}, M_{S^{+}}, M_{S^{+}})$ and $\tilde{D}_{0} = \tilde{D}_{0}(M_{i}, M_{j}, M_{S^{+}}, M_{S^{+}})$ are standard loop functions, whose form is given in Ref. [16]. We neglect the Higgs penguin contribution due to the Yukawa suppression.

3.3. $\mu - e$ conversion in nuclei

Normalized relative to the muon capture rate, the $\mu - e$ conversion rate is [17] [18]

$$
\text{CR}(\mu - e, \text{nucleus}) = \frac{p_{e}E_{e}m_{\mu}^{3}\alpha_{em}^{3}G_{F}^{2}Z_{eff}^{4}F_{p}^{2}}{8\pi^{2}Z\Gamma_{\text{capt}}^{2}} \times \left\{ [(Z + N)(g_{LV}^{(0)} + g_{LS}^{(0)}) + (Z - N)(g_{LV}^{(1)} + g_{LS}^{(1)})]^{2} + (L \rightarrow R) \right\},
$$

where $Z_{\text{eff}}$ is the effective atomic charge, $Z$ and $N$ are the proton and neutron numbers for the nucleus, the nuclear matrix element is denoted by $F_{p}$, $\Gamma_{\text{capt}}$ is the total muon capture rate, and the electron has momentum (energy) $p_{e}$ ($E_{e}$) (which can be taken as $\sim m_{\mu}$). The factors $g_{XK}^{(0,1)}$ ($K = V, S$ and $X = L, R$) are given by

$$
g_{XK}^{(0)} = \frac{1}{2} \sum_{q=u,d,s} \left\{ g_{XK(q)} G_{K}^{(q,p)} + g_{XK(q)} G_{K}^{(q,n)} \right\},
$$

$$
g_{XK}^{(1)} = \frac{1}{2} \sum_{q=u,d,s} \left\{ g_{XK(q)} G_{K}^{(q,p)} - g_{XK(q)} G_{K}^{(q,n)} \right\}.
$$

(23)
Here, the couplings $g_{XK(q)}$ are defined through the effective Lagrangian for $\mu - e$ conversion,

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_q \{(g_{LS(q)} e L \mu R + g_{RS(q)} \bar{e} R \mu L) \bar{q} q + (g_{LV(q)} e L \gamma^\mu \mu L + g_{RV(q)} \bar{e} R \gamma^\mu \mu R) \bar{q} q \gamma^\mu q\},$$  

(24)

while $G(q,p)$ and $G(q,n)$ are numerical factors, obtained once the quark matrix elements are replaced by nucleon matrix elements in the standard way:

$$\langle n | q \Gamma_{K} q | n \rangle = G_{(q,n)} \bar{n} \Gamma_{K} n, \quad \langle p | q \Gamma_{K} q | p \rangle = G_{(q,p)} \bar{p} \Gamma_{K} p. \tag{25}$$

The $\mu - e$ conversion rate receives contributions from $\gamma$, $Z$ and the Higgs penguins, though the latter can be neglected due to the Yukawa suppression. The model does not generate box contributions. As discussed in Ref. [16], the relevant effective couplings are

$$g_{RV(q)} = g_{LV(q)}^\gamma + g_{LV(q)}^Z,$$  

(26)

with $g_{RV(q)} = g_{RV(q)}|_{L \rightarrow R}$, and $g_{LS(q)} \approx g_{RS(q)} \approx 0$. The couplings appearing in the above are

$$g_{LV(q)}^\gamma = \frac{e^2}{G_F} \sqrt{2} Q_q (A_{ND} - A_D),$$

$$g_{LV(q)}^Z = \frac{\sqrt{2} F_Z g_L^q + g_R^q}{G_F M_Z^2}, \tag{27}$$

where $Q_q$ is the quark electric charge and $g_{L,R}^q$ are the standard couplings of the $Z$ boson to quarks. Finally, numerical factors for the nucleon matrix elements take the values

$$G_V^{(d,p)} = G_V^{(u,n)} = 1, \quad G_V^{(u,p)} = G_V^{(d,n)} = 2. \tag{28}$$

Note that, in general, the scotogenic model is subject to strong LFV constraints, relating to the fact that the DM annihilates via the same Yukawa couplings that mediate LFV processes. However, due to additional DM annihilation processes mediated by the dilation, there will be some degree of decoupling between the LFV processes and DM annihilations, such that the LFV bounds can be easily satisfied.

4. Dark Matter
4.1. Relic Density

When the universe cools down where the temperature drops much below the DM mass, the DM number density is Boltzmann suppressed so that the annihilation rate of the dark matter becomes comparable with the Hubble parameter. At certain temperature value at this stage, the DM particles freeze out and fall out of equilibrium where DM number density in a comoving volume remains constant. Therefore the cold DM relic abundance depends essentially on the total thermally averaged annihilation cross section

$$\langle \sigma(N_{DM} N_{DM}) v_r \rangle = \sum_X \langle \sigma(N_{DM} N_{DM} \rightarrow X) v_r \rangle = \frac{1}{8 T M_{DM}^4 K_2^2 \left(\frac{M_{DM}}{T}\right)} \times$$

$$\sum_X \int_0^\infty ds \sigma_{N_{DM} N_{DM} \rightarrow X(s)} \left(s - 4 M_{DM}^2\right) \frac{\sqrt{s}}{T} K_1 \left(\frac{\sqrt{s}}{T}\right), \tag{29}$$

where $v_r$ is the relative velocity, $s$ is the Mandelstam variable, $K_{1,2}$ are the modified Bessel functions and $\sigma_{N_{DM} N_{DM} \rightarrow X(s)}$ is the annihilation cross due to the $N_{DM} N_{DM} \rightarrow X$ channel at
the CM energy $\sqrt{s}$. At the freeze-out, the thermal relic density can be given in terms of the thermally averaged annihilation cross section by
\[
\Omega_{\text{DM}} h^2 \simeq \frac{(1.07 \times 10^9) x_F}{\sqrt{g_* M_{\text{pl}} (\text{GeV})} \langle \sigma (N_{\text{DM}} N_{\text{DM}}) v_r \rangle},
\]  

(30)

with $M_{\text{pl}}$ is the Plank mass and $g_*$ counts the effective degrees of freedom of the relativistic quantities in equilibrium. The inverse freeze-out temperature $x_F = M_{\text{DM}}/T_F$ can be determined iteratively from the equation
\[
x_F = \log \left( \frac{45 M_{\text{DM}} M_{\text{pl}} \langle \sigma (N_{\text{DM}} N_{\text{DM}}) v_r \rangle}{8 \pi^3 \sqrt{g_* x_F}} \right).
\]  

(31)

In our model, we list three classes of annihilation channels as shown in Fig. 2: (1) charged leptons and neutrinos LFV final states $\ell^- \ell^+$ and $\nu_\alpha \bar{\nu}_\beta$, (2) SM fermions and gauge bosons $b\bar{b}$, $t\bar{t}$, $W^+W^-$, $ZZ$ and the scalars $S\bar{S}$ final states, and (3) the Higgs dilaton final states $h_i h_k$. The first class channels are $h_1, h_2$-mediated $s$-channel processes, the second class ones are $S$-mediated $t$-channel processes while the third class channels are both Higgs $s$- and $t$-channel processes.

![Figure 2. Different diagrams for DM annihilation.](image)

4.2. Direct Detection
Concerning direct-detection experiments, the effective low-energy Lagrangian responsible for interactions between the DM and quarks is given by
\[
\mathcal{L}_{N_{1}q}^{(\text{eff})} = a_q \bar{q} q F_{\text{DM}}^c F_{\text{DM}},
\]  

(32)

with
\[
a_q = \frac{s_{h_i h_k} M_q M_{\text{DM}}}{2 \langle H^0 \rangle \langle \phi \rangle} \left[ \frac{1}{M_{h_1}^2} - \frac{1}{M_{h_2}^2} \right].
\]  

(33)

Consequently, the nucleon-DM effective interaction can be written as
\[
\mathcal{L}_{\text{DM}-N}^{(\text{eff})} = a_N \bar{N} N F_{\text{DM}}^c F_{\text{DM}},
\]
with
\[ \alpha_N = \frac{s_h c_h (M_N - \frac{7}{3} M_B) M_{\text{DM}}}{\langle H^0 \rangle} \left[ \frac{1}{M_{h_1}^2} - \frac{1}{M_{h_2}^2} \right]. \] (34)

In this relation, \( M_N \) is the nucleon mass and \( M_B \) the baryon mass in the chiral limit \([7]\). This leads to the following nucleon-DM elastic cross section in the chiral limit
\[ \sigma_{\text{det}} = \frac{s_h^4 M_N^2 (M_N - \frac{7}{3} M_B)^2 M_{\text{DM}}^4}{\pi (H^0)^4 (M_{\text{DM}} + M_B)^2} \left[ \frac{1}{M_{h_1}^2} - \frac{1}{M_{h_2}^2} \right]^2, \] (35)

where we used the relation \( t_h = \langle H^0 \rangle / \langle \phi \rangle \). The analysis below will show that that the upper bound reported by LUX experiment \([26]\) provides a stringent constraint on the DM-Nucleus scattering cross section.

5. Electroweak Precision Tests

In this model, the electroweak precision measurements can impose more constraints. The oblique parameters corrections due to new physics are given by
\[ \frac{\alpha}{4 s_w^2 c_w^2} S = \frac{A_{ZZ} (M_Z^2) - A_{ZZ} (0)}{M_Z^2} - \frac{\partial A_{\gamma \gamma} (q^2)}{\partial q^2} \Bigg|_{q^2=0} + \frac{c_W^2 - s_W^2}{c_W s_W} \frac{\partial A_{\gamma Z} (q^2)}{\partial q^2} \Bigg|_{q^2=0}, \] (36)
\[ \alpha T = \frac{A_{WW} (0)}{M_W} - \frac{A_{ZZ} (0)}{M_Z^2}. \] (37)

Here, \( \alpha = e^2 / (4\pi) = g_s^2 s_w^2 / (4\pi) \) is the fine-structure constant, \( s_w = \sin \theta_w \) and \( c_w = \cos \theta_w \) are the sine and cosine, respectively, of the Weinberg angle \( \theta_w \), and the functions \( A_{\mu \nu} (q^2) \) are the coefficients of \( g_{\mu \nu} \) in the vacuum-polarization tensors \( \Pi_{\mu \nu} (q) = g^{\mu \nu} A_{\nu \nu} (q^2) + g^{\mu \nu} B_{\nu \nu} (q^2) \), where \( VV' \) could be either \( \gamma \gamma, \gamma Z, ZZ, \) or \( WW \). In our model, following \([7]\) the oblique parameters are given by
\[ T = \frac{1}{16 \pi s_w^2 M_W^2} \left\{ F (M_{S_0}^2, M_{S_+}^2) + F (M_{A_0}^2, M_{A_+}^2) - F (M_{S_0}^2, M_A^2) \right\} + 3 \cos^2 \theta_h \left[ F (M_Z^2, M_{h_1}^2) - F (M_W^2, M_{h_1}^2) \right] + 3 \sin^2 \theta_h \left[ F (M_Z^2, M_{h_2}^2) - F (M_W^2, M_{h_2}^2) \right] - 3 \left[ F (M_Z^2, M_{h_1}^2) - F (M_W^2, M_{h_2}^2) \right], \] (38)
\[ S = \frac{1}{24 \pi} \left\{ (2 s_w^2 - 1)^2 G (M_{S_+}^2, M_{S_+}^2, M_{Z}^2) + G (M_{S_0}^2, M_{S_+}^2, M_{Z}^2) \right\} + \cos^2 \theta_h \log \left( \frac{M_{h_1}^2}{M_{S_0}^2} \right) + \sin^2 \theta_h \log \left( \frac{M_{h_2}^2}{M_{S_0}^2} \right) + \ln \left( \frac{M_{h_1}^2}{M_{S_+}^2} \right) + \cos^2 \theta_h \hat{G} (M_{h_1}^2, M_{Z}^2) + \sin^2 \theta_h \hat{G} (M_{h_2}^2, M_{Z}^2) - \hat{G} (M_{h_1}^2, M_{Z}^2) - \hat{G} (M_{h_2}^2, M_{Z}^2), \] (39)

where the functions \( F, G \) and \( \hat{G} \) are given in the appendix and \( M_h = 125.09 \text{ GeV} \) denotes the reference value.

6. Analysis and Results

We perform a numerical scan for parameter space to determine whether radiative electroweak symmetry breaking is compatible with one-loop radiative neutrino mass (i.e. with viable neutrino masses and mixing), subject to the LFV and muon anomalous magnetic moment
Figure 3. The direct detection cross section versus the DM mass. The dashed line shows the recent constraints from LUX, while the palette gives the mass for the neutral beyond-SM scalar (dilaton), $M_{h_2}$, in units of GeV.

constraints, while simultaneously generating a viable DM relic density and consistent with the direct detection experiments. In the scans, we enforce the minimization conditions, Eqs. (6) and (7), vacuum stability via Eq. (8), and demand that the SM-like Higgs mass is in the experimentally allowed range, $M_{h_1} = 125.09 \pm 0.21$ GeV. Compatibility with constraints from LEP (OPAL) on a light Higgs [19] are enforced, and we consider the constraint from the Higgs invisible decay, $B(h \rightarrow \text{inv}) < 17$ [20]. Dimensionless couplings are restricted to the perturbative range throughout, and we consider values of $100 \text{ GeV} \lesssim \phi \lesssim 5 \text{ TeV}$.

In Figure 3, we plot the direct-detection cross section versus the DM mass with all the other phenomenological constraints satisfied. The mass of the dilaton, $M_{h_2}$, in units of GeV, is shown in the corresponding palette. One immediately observes that direct-detection limits from LUX [21] impose very serious constraints on the model, with a large number of benchmark sets already excluded. The plot shows that the surviving benchmark points mostly occur for $M_{DM} \lesssim 10$ GeV, with a smaller number of viable points found for $M_{DM} \gtrsim 200$ GeV. Benchmarks with intermediate $M_{DM}$ values are excluded. The viable parameter space typically requires a lighter dilaton mass, $M_{h_2} \lesssim 10$ GeV, as all benchmarks with $M_{h_2} \gtrsim 50$ GeV are excluded. It is clear from the figure that the surviving benchmark sets can be probed in forthcoming direct-detection experiments.

7. Conclusion
In this work I presented a detailed study of the minimal scale invariant scotogenic model. Our analysis demonstrates the existence of viable parameter space in which one obtains radiative electroweak symmetry breaking, one-loop neutrino masses and a good DM candidate. The model predicts a new scalar with $O(\text{GeV})$ mass. This field plays the dual roles of triggering electroweak symmetry breaking and sourcing lepton number symmetry violation. The model can give observable signals in LFV searches, and direct-detection experiments. It also predicts a scalar doublet $S$, whose mass is expected to be $\lesssim \text{TeV}$, within reach of collider experiments. The model is subject to strong constraints from direct-detection experiments; viable parameter

\footnote{However, we only find viable benchmark points for $\phi \gtrsim 150$ GeV.}
space was found for $M_{DM} \leq 10$ GeV and $M_{DM} \geq 200$ GeV, while intermediate values for $M_{DM}$ appear excluded.

7.1. Acknowledgments
I would to thank the local organizing committee of FISICPAC for the wonderful conference. Special thanks goes to R. Soualah for the invitation. I also would like to thank my collaborators A. Ahriche and K. McDonald with whom the work presented at this conference has been based on.

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