Quantum Mechanics of Black Holes

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Abstract

These lectures give a pedagogical review of dilaton gravity, Hawking radiation, the black hole information problem, and black hole pair creation. (Lectures presented at the 1994 Trieste Summer School in High Energy Physics and Cosmology)

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1. Introduction

Hawking’s 1974 discovery\cite{1} that black holes evaporate ushered in a new era in black hole physics. In particular, this was the beginning of concrete applications of quantum mechanics in the context of black holes. But more importantly, the discovery of Hawking evaporation has raised a sharp problem whose resolution probably requires a better understanding of Planck scale physics, and which therefore may serve as a guide (or at least a constraint) in our attempts to understand such physics. This problem is the information problem.

In brief, the information problem arises when one considers a Gedanken experiment where a black hole is formed in collapse of a carefully arranged pure quantum state $|\psi\rangle$, or in terms of quantum-mechanical density matrices, $\rho = |\psi\rangle\langle\psi|$. This black hole then evaporates, and according to Hawking’s calculation the resulting outgoing state is approximately thermal, and in particular is a mixed quantum state. The latter statement means that the density matrix is of the form $\sum_\alpha p_\alpha |\psi_\alpha\rangle\langle\psi_\alpha|$, for some normalized basis states $|\psi_\alpha\rangle$ and some real numbers $p_\alpha$ more than one of which is non-zero. In the present case the $p_\alpha$ are approximately the usual thermal probabilities. If Hawking’s calculation can be trusted, and if the black hole does not leave behind a remnant, this means that in the quantum theory of black holes pure states can evolve to mixed. This conflicts with the ordinary laws of quantum mechanics, which always preserve purity. Comparing pure and mixed states, we find that there is missing phase information in the latter. A useful measure of the missing information is the entropy, $S = -\text{Tr}\rho \ln \rho = -\sum_\alpha p_\alpha \ln p_\alpha$.

Hawking subsequently proposed\cite{2} that quantum mechanics be modified to allow purity loss. However, as we’ll see, inventing an alternative dynamics is problematical. This has lead people to consider other alternatives, namely that information either escapes the black hole or that it is left behind in a black hole remnant. Both of these possibilities also encounter difficulties, and as a result we have the black hole information problem.

These lectures will develop these statements more fully.\cite{3} In the past few years a much advanced understanding of black hole evaporation has been obtained through investigation of two-dimensional models, and because of this and due to their greater simplicity we’ll start by reviewing these models. Next will be a detailed treatment of Hawking radiation from the resulting two-dimensional black holes, followed by a generalization to the derivation of four-dimensional Hawking radiation. As black holes Hawking radiate they

\footnote{Other reviews include\cite{4,5} and more recently\cite{6,7}.}
shrink, and a subsequent section is devoted to a semiclassical description of such black hole evaporation. There follows a review of the black hole information problem, its various proposed resolutions, and the problems with these proposals. Finally we turn to a treatment of another non-trivial aspect of the quantum mechanics of black holes, black hole pair production. Besides its intrinsic interest, this process can potentially shed light on the proposed remnant resolution of the information problem, and a brief discussion of how it does so is given.

2. Two-dimensional dilaton gravity

In 2d, formulating gravity with just a metric gives trivial dynamics; for example, the Einstein action is a topological invariant. Instead we consider theories with the addition of a scalar dilaton $\phi$. A particular simple theory has action

$$S = \frac{1}{2\pi} \int d^2 x \sqrt{-g} \left[ e^{-2\phi} (R + 4(\nabla \phi)^2 + 4\lambda^2) - \frac{1}{2} (\nabla f)^2 \right], \quad (2.1)$$

where $\lambda^2$ is an analogue to the cosmological constant and $f$ is a minimally coupled massless matter field that provides a source for gravity. Note that $e^\phi$ plays the role of the gravitational coupling, as its inverse square appears in front of the gravitational part of the action.

In two dimensions the general metric $ds^2 = g_{ab} dx^a dx^b$ can always locally be put into conformal gauge,

$$ds^2 = -e^{2\rho} dx^+ dx^- , \quad (2.2)$$

with the convention $x^\pm = x^0 \pm x^1$. The equations resulting from the action (2.1) are most easily analyzed in this gauge. The matter equations are

$$\partial_+ \partial_- f = 0 , \quad (2.3)$$

with general solution $f_i = f_+(x^+) + f_-(x^-)$. Next, the relation

$$\sqrt{-g} R = -2 \Box \rho \quad (2.4)$$

allows rewriting of the gravitational part of the action,

$$S_{\text{grav}} = \frac{1}{2\pi} \int d^2 x \left\{ 2\nabla (\rho - \phi) \nabla e^{-2\phi} + 4\lambda^2 e^{2(\rho - \phi)} \right\} . \quad (2.5)$$
The equation of motion for $\rho - \phi$ is therefore that of a free field, with solution
\[ \rho - \phi = \frac{1}{2} \left[ w_+(x^+) + w_-(x^-) \right]. \tag{2.6} \]

The equation for $\phi$ then easily gives
\[ e^{-2\phi} = u_+ + u_- - \lambda^2 \int^{x^+} dx^+ e^{w_+} \int^{x^-} dx^- e^{w_-} \tag{2.7} \]
where $u_\pm(x^\pm)$ are also free fields. Finally, varying the action (2.1) with respect to $g^{++}$, $g^{--}$ gives the constraint equations,
\[ \delta g^{++} : G^{++} \equiv -e^{-2\phi} (4\partial_+ \rho \partial_+ \phi - 2\partial_+^2 \phi) = \frac{1}{2} \partial_+ f \partial_+ f \]
\[ \delta g^{--} : G^{--} \equiv -e^{-2\phi} (4\partial_- \rho \partial_- \phi - 2\partial_-^2 \phi) = \frac{1}{2} \partial_- f \partial_- f. \tag{2.8} \]
These determine $u_\pm$ in terms of $f_\pm$ and $w_\pm$:
\[ u_\pm = \frac{M}{2\lambda} - \frac{1}{2} \int dx^\pm e^{w_\pm} \int dx^\pm e^{-w_\pm} \partial_\pm f \partial_\pm f, \tag{2.9} \]
where $M$ is an integration constant. In the following we will choose units so that $\lambda = 1$.

The theory is therefore completely soluble at the classical level. The unspecified functions $w_\pm$ result from the unfixed remaining gauge freedom; conformal gauge (2.2) is unchanged by a reparametrization $x^\pm = x^\pm(\sigma^\pm)$. This freedom may be used to set $w_+ + w_- = \sigma^+ - \sigma^-$, for example. In this gauge the general vacuum solution is[8,9]
\[ ds^2 = -\frac{d\sigma^+ d\sigma^-}{1 + Me^{\sigma^+ - \sigma^-}} \]
\[ \phi = -\frac{1}{2} \ln \left( M + e^{\sigma^+ - \sigma^-} \right). \tag{2.10} \]

The case $M = 0$ corresponds to the ground state,
\[ ds^2 = -d\sigma^+ d\sigma^- \]
\[ \phi = -\sigma. \tag{2.11} \]
This is the dilaton-gravity analogue of flat Minkowski space, and is called the linear dilaton vacuum.

The solutions for $M > 0$ are asymptotically flat as $\sigma^+ - \sigma^- \to \infty$. At $\sigma^+ - \sigma^- \to -\infty$ they are apparently singular, but regularity is restored by the coordinate transformation
\[ x^+ = e^{\sigma^+}, x^- = -e^{-\sigma^-}. \tag{2.12} \]
A true curvature singularity appears at $x^+x^- = M$, and $x^\pm = 0$ is the horizon. The corresponding Penrose diagram is shown in Fig. 1; the solution is a black hole and $M$ is its mass. Notice the important relation $e^{2\phi}_{\text{horizon}} = \frac{1}{M}$. For $M < 0$ the solution is a naked singularity.
Next consider sending infalling matter, \( f = F(x^+) \), into the linear dilaton vacuum. This will form a black hole, as shown in Fig. 2. Before the matter infall the solution is given by (2.11). Afterwards it is found by using (2.6)-(2.9),

\[
e^{-2\phi} = M + e^{\sigma^+} \left( e^{-\sigma^-} - \Delta \right)
\]

\[
ds^2 = -\frac{d\sigma^+ d\sigma^-}{1 + M e^{\sigma^- - \sigma^+} - \Delta e^{\sigma^-}}
\]

where one can easily show

\[
M = \int d\sigma^+ T_{++}
\]

\[
\Delta = \int d\sigma^+ e^{-\sigma^+} T_{++}
\]

\[ (2.13) \]

\[ (2.14) \]
and where
\[ T_{++} = \frac{1}{2}(\partial_+ F)^2 \] is the stress tensor. The coordinate transformation
\[ \xi^- = -\elln \left( e^{-\sigma^--\Delta} \right), \; \xi^+ = \sigma^+ \]
returns the metric (2.13) to the asymptotically flat form
\[ ds^2 = -\frac{d\xi^+ d\xi^-}{1 + M e^{\xi^- - \xi^+}}, \] showing that indeed we have formed a black hole.

3. Hawking radiation in two dimensions

With a collapsing black hole in hand, we can study its Hawking radiation.\footnote{For other references see [1,10,11].} The quickest derivation of the Hawking flux arises by computing the expectation value of the matter stress tensor. Consider the stress tensor for right-movers; before the hole forms they are in their vacuum, and
\[ \langle T_{--} \rangle = \lim_{\hat{\sigma}^- \to 0} \langle 0 | \frac{1}{2} \partial_- f(\hat{\sigma}^-) \partial_- f(\sigma^-) | 0 \rangle \]
\[ = \lim_{\hat{\sigma}^- \to 0} \frac{-1}{4(\hat{\sigma}^- - \sigma^-)^2}, \] where the second line uses the 2d Green function,
\[ \langle 0 | f(\hat{\sigma}) f(\sigma) | 0 \rangle = -\frac{1}{2} \left[ \ln(\hat{\sigma}^+ - \sigma^+) + \ln(\hat{\sigma}^- - \sigma^-) \right]. \]
As usual, one removes the infinite vacuum energy by normal-ordering:
\[ :T_{--} :_{\sigma} = T_{--} + \frac{1}{4} \frac{1}{(\hat{\sigma}^- - \sigma^-)^2}. \]
The formula (3.1) can also be applied at \( \mathcal{I}^+ \), but the flat coordinates are now \( \xi^\pm \). Therefore to compare the stress tensor to that of the outgoing vacuum on \( \mathcal{I}^+ \), we should subtract the vacuum energy computed in the \( \xi \) coordinates,
\[ :T_{--} :_{\xi} = T_{--} + \frac{1}{4} \frac{1}{(\xi^- - \xi^+)^2}. \]
The corresponding expectation value is

\[
\langle T_{--}:\xi \rangle = \lim_{\xi^- \to \xi^-} \langle 0 | \frac{1}{2} \partial_{\xi}^2 f \left( \sigma^{-}(\hat{\xi}^-) \right) \partial_{\xi}^2 f \left( \sigma^{-}(\xi^-) \right) | 0 \rangle + \frac{1}{4(\hat{\xi}^- - \xi^-)^2}
\]

\[
= -\frac{1}{4} \lim_{\xi^- \to \xi^-} \partial_{\xi}^2 \partial_{\xi}^2 \ln \left( \sigma^{-}(\hat{\xi}^-) - \sigma^{-}(\xi^-) \right) + \frac{1}{4(\hat{\xi}^- - \xi^-)^2}.
\]

(3.5)

Next one expands \( \sigma^{-}(\hat{\xi}^-) \) about \( \xi^- \), and in a few lines finds

\[
\langle T_{--}:\xi \rangle = -\frac{1}{24} \left[ \frac{\sigma^{-}'''}{\sigma^-} - \frac{3}{2} \left( \frac{\sigma^{-}''}{\sigma^-} \right)^2 \right]
\]

(3.6)

where prime denotes derivative with respect to \( \xi^- \).

Using the relation (2.16) between the two coordinate systems gives the outgoing stress tensor from the black hole,

\[
\langle T_{--}:\xi \rangle = \frac{1}{48} \left[ 1 - \frac{1}{1 + \Delta e^{\xi^-}} \right]
\]

(3.7)

This exhibits transitory behavior on the scale \( \xi^- \sim - \ln \Delta \), but as \( \xi^- \to \infty \) it asymptotes to a constant value \( 1/48 \). As will be seen shortly, this corresponds to the thermal Hawking flux at a temperature \( T = 1/2\pi \).

A more detailed understanding of the Hawking radiation arises from quantizing the scalar field. Recall the basic steps of canonical quantization:

1. Find a complete orthonormal basis of solutions to the field equations.
2. Separate these solutions into orthogonal sets of positive and negative frequency.
3. Expand the general field in terms of the basis functions with annihilation operators as coefficients of positive frequency solutions and creation operators for negative frequency.
4. Use the canonical commutation relations to determine the commutators of these ladder operators.
5. Define the vacuum as the state annihilated by the annihilation operators, and build the other states by acting on it with creation operators.

In curved spacetime general coordinate invariance implies that step two is ambiguous: what is positive frequency in one frame is not in another. Consequently the vacuum state is not uniquely defined. These two observations are at the heart of the description of particle creation in curved spacetime. This ambiguity was responsible for the different normal-ordering prescriptions in our preceding derivation.
Now follow this recipe. Begin by noting that the equations of motion imply existence of the conserved Klein-Gordon inner product,

$$ (f,g) = -i \int_{\Sigma} d\Sigma^{\mu} f^{\nu} \nabla_{\mu} g^* $$

for arbitrary Cauchy surface $\Sigma$.

Steps 1 and 2: A convenient basis for right-moving modes in the “in” region near $I^-\!$ are

$$ u_{\omega} = \frac{1}{\sqrt{2\omega}} e^{-i\omega\sigma^--} , \; u^*_{\omega} = \frac{1}{\sqrt{2\omega}} e^{i\omega\sigma^-}. $$

These have been normalized so that

$$ (u_{\omega}, u_{\omega'}) = 2\pi \delta(\omega - \omega') = -(u^*_{\omega}, u^*_{\omega'}) , \; (u_{\omega}, u^*_{\omega'}) = 0. $$

Furthermore, note that they are naturally separated according to positive or negative frequency with respect to the time variable $\sigma^0$.

Step 3: The field $f$ has expansion in terms of annihilation and creation operators

$$ f_- = \int_{0}^{\infty} d\omega \left[ a_{\omega} u_{\omega} + a^\dagger_{\omega} u^*_{\omega} \right] \text{ (in)} . $$

Step 4: The canonical commutation relations

$$ [f_-(x), \partial_0 f_-(x')]_{x^0 = x'^0} = \frac{1}{2} [f(x), \partial_0 f(x')]_{x^0 = x'^0} = \pi i \delta(x^1 - x'^1) $$

together with the inner products (3.10) imply that the operators $a_{\omega}$ satisfy the usual commutators,

$$ [a_{\omega}, a^\dagger_{\omega'}] = \delta(\omega - \omega') , \; [a_{\omega}, a_{\omega'}] = 0 , \; [a^\dagger_{\omega}, a^\dagger_{\omega'}] = 0. $$

Step 5: The in vacuum is defined by

$$ a_{\omega} |0\rangle_{\text{in}} = 0; $$

other states are built on it by acting with the $a^\dagger_{\omega}$’s.

To describe states in the future regions it is convenient to follow these steps with a different set of modes. Modes are needed both in the “out” region near $I^+\!$ and near the singularity. The former are the obvious analogues to (3.9),

$$ v_{\omega} = \frac{1}{\sqrt{2\omega}} e^{-i\omega\xi^-} , \; v^*_{\omega} = \frac{1}{\sqrt{2\omega}} e^{i\omega\xi^-}. $$
The latter are somewhat arbitrary as the region near the singularity is highly curved. A convenient coordinate near the singularity proves to be

$$\hat{\xi}^- = \ln(\Delta^2 e^{\sigma^-} - \Delta) ,$$

(3.16)

and corresponding modes $\hat{v}_\omega$ and $\hat{v}^*_\omega$ are given by a formula just like (3.15).

In terms of these modes, $f$ is written

$$f_\pm = \int_0^\infty d\omega \left[ b_\omega v_\omega + b_\omega^† v^*_\omega + \hat{b}_\omega \hat{v}_\omega + \hat{b}_\omega^† \hat{v}^*_\omega \right] \text{ (out + internal)} .$$

(3.17)

These modes are normalized as in (3.10), and the corresponding field operators obey commutators as in (3.13). The vacua $|0\rangle_{\text{out}}$ and $|0\rangle_{\text{internal}}$ are also defined analogously to (3.14).

The non-trivial relation (2.16) between the natural timelike coordinates in the in and out regions – or equivalently between the symmetry directions with respect to which positive and negative frequencies are defined – imply that a positive frequency solution in one region is a mixture of positive and negative frequency in another region. This mixing implies particle creation. For example, positive frequency out modes can be expressed in terms of the in modes,

$$v_\omega = \int_0^\infty d\omega' \left[ \alpha_{\omega\omega'} u_{\omega'} + \beta_{\omega\omega'} u^*_{\omega'} \right] .$$

(3.18)

The Fourier coefficients $\alpha_{\omega\omega'}$, $\beta_{\omega\omega'}$ are called Bogoliubov coefficients, and they can be calculated by inverting the Fourier transform, or equivalently from

$$\alpha_{\omega\omega'} = \frac{1}{2\pi} (v_\omega, u_{\omega'}) , \quad \beta_{\omega\omega'} = -\frac{1}{2\pi} (v_\omega, u^*_{\omega'}) .$$

(3.19)

In the present model they can easily be found in closed form in terms of incomplete beta functions[11]

$$\alpha_{\omega\omega'} = \frac{1}{2\pi} \sqrt{\frac{\omega'}{\omega}} \Delta^{i\omega} B \left( -i\omega + i\omega', 1 + i\omega \right)$$

$$\beta_{\omega\omega'} = \frac{1}{2\pi} \sqrt{\frac{\omega'}{\omega}} \Delta^{i\omega} B \left( -i\omega - i\omega', 1 + i\omega \right) .$$

(3.20)

Although we will not use these formulas directly they are exhibited for completeness.

To investigate the thermal behavior at late times, $\xi^- \gg -\ln \Delta$, $\sigma^- \simeq -\ln \Delta$, we could examine the asymptotic behavior of (3.21), but a shortcut is to use the asymptotic form of (2.16) found by expanding around $\sigma^- = -\ln \Delta$,

$$-e^{-\xi^-} = \Delta - e^{-\sigma^-} \simeq \Delta(\sigma^- + \ln \Delta) \equiv \tilde{\sigma}^- .$$

(3.21)
Note that this is the same as the relation between Rindler and Minkowski coordinates in the context of accelerated motion. Likewise one finds
\[ e^{\hat{\xi}^-} \simeq \tilde{\sigma}^- . \] (3.22)

Next, a trick\[12\] can be used to find the approximate form for the outgoing state. Notice that functions that are positive frequency in \( \tilde{\sigma}^- \) are analytic in the lower half complex \( \tilde{\sigma}^- \) plane. Therefore the functions
\[
\begin{align*}
    u_{1,\omega} &\propto (-\tilde{\sigma}^-)^{i\omega} \propto \hat{v} + e^{-\pi \omega} \hat{v}^* \\
    u_{2,\omega} &\propto (\tilde{\sigma}^-)^{-i\omega} \propto \hat{\tilde{v}} + e^{-\pi \omega} \hat{\tilde{v}}^*
\end{align*}
\] (3.23)
are positive frequency. This means that the corresponding field operators \( a_{1,\omega} \) and \( a_{2,\omega} \) must annihilate the in vacuum. The inverse of the transformation (3.23) gives the relation between field operators, as can easily be seen by reexpanding (3.17) in the \( u_{i,\omega} \)'s and \( a_{i,\omega} \)'s:
\[
\begin{align*}
    a_{1,\omega} &\propto b_{\omega} - e^{-\pi \omega} \hat{b}^\dagger_{\omega} \\
    a_{2,\omega} &\propto \hat{b} + e^{-\pi \omega} b^\dagger_{\omega}
\end{align*}
\] (3.24)

These then determine the vacuum, since it obeys
\[
0 = (a^\dagger_{1,\omega} a_{1,\omega} - a^\dagger_{2,\omega} a_{2,\omega})|0\rangle \\
\propto (\hat{b}^\dagger_{\omega} b_{\omega} - \hat{\tilde{b}}^\dagger_{\omega} \hat{\tilde{b}}_{\omega})|0\rangle \\
\propto (N_{\omega} - \hat{N}_{\omega})|0\rangle ,
\] (3.25)
where \( N_{\omega}, \hat{N}_{\omega} \) are the number operators for the respective modes. The latter equation implies that
\[
|0\rangle = \sum_{\{n_{\omega}\}} c(\{n_{\omega}\}) |\{\hat{n}_{\omega}\}\rangle |\{\tilde{n}_{\omega}\}\rangle \] (3.26)
for some numbers \( c(\{n_{\omega}\}) \). These numbers are determined up to an overall constant from the recursion relation following from the equation \( a_{1,\omega}|0\rangle = 0 \):
\[
c(\{n_{\omega}\}) = c(\{0\}) \exp \left\{-\pi \int d\omega n_{\omega} \right\} . \] (3.27)

Thus the state takes the form
\[
|0\rangle = c(\{0\}) \sum_{\{n_{\omega}\}} e^{-\pi \int d\omega n_{\omega}} |\{\hat{n}_{\omega}\}\rangle |\{n_{\omega}\}\rangle . \] (3.28)
It is clear from this relation that the state inside the black hole is strongly correlated with the state outside the black hole. Observers outside the hole cannot measure the state inside, and so summarize their experiments by the density matrix obtained by tracing over all possible internal states,

\[ \rho_{\text{out}} = \text{Tr}_{\text{inside}} |0\rangle \langle 0| = |c \{ \{ 0 \} \} |^2 \sum_{\{ n_\omega \}} e^{-2\pi \int d\omega \omega n_\omega} \langle \{ n_\omega \} | \langle \{ n_\omega \} |. \] (3.29)

This is an exactly thermal density matrix with temperature \( T = 1/2\pi \). The corresponding energy density is that of a right moving scalar field,

\[ \mathcal{E} = \int_{0}^{\infty} \frac{d\omega}{2\pi} \frac{\omega}{e^{\omega/T} - 1} = \frac{\pi}{12} T^2 = \frac{1}{48\pi}, \] (3.30)

which agrees with (3.7) if we account for the unconventional normalization of the stress tensor used in \[7\],

\[ \mathcal{E} = \frac{1}{\pi} T_{00}. \] (3.31)

Both the total entropy and energy of this density matrix are infinite, but that is simply because we have not yet included backreaction which causes the black hole to shrink as it evaporates. Before considering these issues, however, we will extend these results to four-dimensional black holes.

### 4. Hawking radiation in four dimensions

Fig. 3 shows the Penrose diagram for a collapsing four-dimensional black hole; this picture is analogous to fig. 2. Making the assumption of radial symmetry implies the metric takes the form

\[ ds^2 = g_{tt} dt^2 + g_{rr} dr^2 + R^2(r,t) d\Omega_2^2 \] (4.1)

where \( g_{tt}, g_{rr}, \) and \( R \) are functions of \( r \) and \( t \) and \( d\Omega_2^2 \) is the line element on the two-sphere. Notice that as before reparametrizations of \( r \) and \( t \) allow us to rewrite (4.1) in “conformal gauge”,

\[ ds^2 = e^{2\rho} \left[ -(dx^0)^2 + (dx^1)^2 \right] + R^2(x) d\Omega_2^2. \] (4.2)

In particular, outside the black hole the metric is time independent by Birkhoff’s theorem, and can be written

\[ ds^2 = -g_{tt} \left( -dt^2 + dr^*^2 \right) + r^2 d\Omega_2^2 \] (4.3)
Fig. 3: The Penrose diagram for a collapsing four-dimensional black hole. Also indicated are lines of constant $\sigma^-$ (solid) and lines of constant $\xi^-$ (dashed), as well as the ray $\mathcal{R}$ described in the text.

where the tortoise coordinate $r^*$ is defined by

$$\frac{dr^*}{dr} = \sqrt{g_{rr}/-g_{tt}}.$$  \hfill (4.4)

(For the Schwarzschild black hole $r^* = r + 2M \ln(r - 2M)$.)

Now consider propagation of a free scalar field,

$$S = -\frac{1}{2} \int d^4x \sqrt{-g} (\nabla f)^2.$$ \hfill (4.5)

The solutions take the separated form

$$f = \frac{u(r, t)}{R} Y_{\ell m}(\theta, \phi)$$ \hfill (4.6)

in terms of spherical harmonics and two-dimensional solutions $u(r, t)$. To compare with our two-dimensional calculation, first rewrite (4.5) in two-dimensional form. For example, in the outside coordinates (1.3),

$$\int d^4x \sqrt{-g} (\nabla f)^2 \propto \int dr^* dt \left[ (\partial_t u)^2 - (\partial_{r^*} u)^2 - V(r^*)u^2 \right]$$ \hfill (4.7)
where the effective potential is

\[
V(r^*) = -g_{tt} \frac{\ell(\ell + 1)}{r^2} + \frac{\partial^{2}_{r^*} r}{r}.
\]  

(4.8)

The latter term vanishes at \( r = r^* = \infty \) as \( \frac{1}{r^3} \), and both terms vanish as \( r^* \to -\infty \). For Schwarzschild the maximum of the potential is of order \( \ell(\ell + 1)/M^2 \), and occurs where \( r^* \sim M \). These features are sketched in fig. 4.

\[ \text{Fig. 4: A sketch of the effective potential as a function of tortoise coordinate } r^*. \text{ (Shown is the case of the Schwarzschild black hole.)} \]

In the limits \( r^* \to \pm \infty \) the solutions are therefore plane waves. However, an outgoing plane wave from \( r^* = -\infty \) is only partially transmitted. As we’ll see, the Hawking radiation can be thought of as thermally populating the outgoing modes, and these energy-dependent “gray body” transmission factors lead to deviations from a pure black body spectrum. These factors are, however, negligible for \( \omega^2 \gg V_{\text{max}} \), where for Schwarzschild \( V_{\text{max}} \sim \ell(\ell + 1)/M^2 \).

The Hawking effect is again found by relating the in modes at \( \mathcal{I}^- \) to the out modes at \( \mathcal{I}^+ \). The good asymptotically flat coordinates outside the collapsing body were given in (4.3):

\[
ds^2 = -g_{tt} \, d\xi^+ d\xi^- + r^2 d\Omega^2_2 , \quad \xi^\pm = t \pm r^* .
\]  

(4.9)

These coordinates are suitable for the in or out regions. We will also use conformal coordinates for the interior of the collapsing body,

\[
ds^2 = -\Omega^2(\sigma) \, d\sigma^+ d\sigma^- + r^2 d\Omega^2_2 .
\]  

(4.10)

In particular, we can label the slices so that near the point where the surface of the collapsing matter crosses the horizon

\[
\sigma^\pm = \tau \pm r
\]  

(4.11)
and so that $\sigma^-|_{\text{horizon}} = 0$, for some time coordinate $\tau$. These coordinates pass continuously through the horizon, as shown in fig. 3.

As in two dimensions, the state is defined so that the in modes are in their vacuum, and we wish to compare the resulting state to the out vacuum. The in coordinate with respect to which the vacuum is defined is $\xi^+$. This is related to $\sigma^+$, then to $\sigma^-$, which is then related to $\xi^-$. Let’s follow this chain backwards. First, as $\xi^- \to \infty$, that is near the horizon, $\sigma^-(\xi^-)$ is given by

$$
\frac{d\sigma^-}{d\xi^-} = \frac{1}{2} \left( \frac{\partial \sigma^-}{\partial t} - \frac{\partial \sigma^-}{\partial r^*} \right) = \frac{1}{2} \left( \frac{\partial \tau}{\partial t} - \frac{\partial \tau}{\partial r^*} + \frac{\partial r}{\partial r^*} \right).
$$

(4.12)

This is simplified using

$$
0 = \frac{d \sigma^+}{d \xi^-} \propto \frac{\partial \sigma^+}{\partial t} - \frac{\partial \sigma^+}{\partial r^*} = \frac{\partial \tau}{\partial t} - \frac{\partial \tau}{\partial r^*} - \frac{\partial r}{\partial r^*}
$$

(4.13)

and we find

$$
\frac{d \sigma^-}{d \xi^-} = \frac{\partial r}{\partial r^*}.
$$

(4.14)

Expanding about $r = r_{\text{horizon}}$ then gives

$$
\frac{d \sigma^-}{d \xi^-} \approx \frac{\partial r}{\partial r^*}(r = r_{\text{horizon}} - \sigma^-/2) \approx -\frac{1}{2} \frac{\partial}{\partial r} \left( \frac{\partial r}{\partial r^*} \right)_{\text{horizon}} \sigma^-.
$$

(4.15)

The quantity

$$
\kappa = \frac{1}{2} \frac{\partial}{\partial r} \left( \frac{\partial r}{\partial r^*} \right)_{\text{horizon}}
$$

(4.16)

is the usual surface gravity, and (4.13) integrates to

$$
\sigma^- = -B e^{-\kappa \xi^-}
$$

(4.17)

for some constant $B$. This formula exhibits the expected exponentially increasing redshift near the horizon. The coordinate $\sigma^-$ is matched to $\sigma^+$ at $r = 0$ and then $\sigma^+$ to $\xi^+$ at the surface of the body. For large $\xi^-$ we only need to relate $\sigma^- \to \sigma^+$ and $\sigma^+ \to \xi^+$ in the vicinity of the ray $R$ shown in fig. 3:

$$
\sigma^- \simeq \frac{d \sigma^-}{d \xi^+} \frac{d \sigma^+}{d \xi^+} \xi^+ + \text{const}.
$$

(4.18)
The factor \( \frac{d\sigma^-}{d\xi^+} \bigg|_\mathcal{R} \) gives a constant blueshift, and simply modifies \( B \):

\[
\xi^+ \simeq B' e^{-\kappa \xi^-}.
\] (4.19)

This formula is nearly identical to (3.21). The remaining derivation of the Hawking radiation follows as in two-dimensions, with the replacement \( \omega \to \omega/\kappa \). Therefore the temperature is

\[
T = \frac{\kappa}{2\pi};
\] (4.20)

for a Schwarzschild black hole this gives the familiar

\[
T = \frac{1}{8\pi M}.
\] (4.21)

The outgoing state is again thermal, except for the grey body factors which are negligible for all but low-frequency modes. Up to a numerical constant, the entropy of the Hawking radiation is the same as that of the black hole, which is easily computed:

\[
dS = \frac{dE}{T} = 8\pi M dM,
\] (4.22)

so

\[
S = 4\pi M^2 = \frac{A}{4},
\] (4.23)

where \( A \) is the black hole area. This is the famous Bekenstein-Hawking [13,11] entropy.

We can also easily estimate the lifetime of the black hole. The energy density per out mode is \( \omega/2\pi \); this is multiplied by the thermal factor, the velocity \((c = 1)\), and the transmission coefficient \( \Gamma_{\omega,\ell} \) through the barrier, then summed over modes:

\[
\frac{dM}{dt} = \sum_\ell (2\ell + 1) \int \frac{d\omega}{2\pi} \frac{\omega}{e^{8\pi M\omega} - 1} \Gamma_{\omega,\ell}.
\] (4.24)

The transmission factor can be roughly approximated by no transmission below the barrier and unit transmission above,

\[
\Gamma_{\omega,\ell} \sim \Theta(a\omega M - \ell)
\] (4.25)

for some constant \( a \). \( M \) scales out of the integral (4.24), and we find

\[
\frac{dM}{dt} \propto \frac{1}{M^2}.
\] (4.26)
This gives a lifetime
\[ \tau \sim \left( \frac{M}{m_{\text{pl}}} \right)^3 t_{\text{pl}}, \]
(4.27)
which is comparable to the age of the universe for \( M \sim 10^{-18} M_{\text{sun}} \).

Finally, note that as in two dimensions there are correlations between modes on either side of the horizon, and these mean missing information from the perspective of the outside observer. Also, note that although the calculation refers to ultrahigh frequencies, in essence all that is being used is that the infalling state near the horizon is approximately the vacuum at high frequencies.

5. Semiclassical treatment of the backreaction

So far black hole shrinkage from Hawking emission has been neglected. This section will treat this backreaction in two dimensions and semiclassically; for attempts at construction of a full quantum theory of 2d black hole evaporation see e.g. [14][17].

The effect of the Hawking radiation on the geometry is determined by its stress tensor. Recall that the asymptotic stress tensor was computed in sec. 3 by relating the normal ordering prescriptions in the two different coordinate systems. The metric in the out region asymptoted to
\[ ds^2 = -d\xi^+ d\xi^- = -e^{2\rho} d\sigma^+ d\sigma^- . \]
(5.1)
where
\[ e^{-2\rho} = \frac{d\sigma^+}{d\xi^+} \frac{d\sigma^-}{d\xi^-} . \]
(5.2)
With a little effort, (3.6) can be rewritten in terms of \( \rho \). Working in \( \sigma \) coordinates (but keeping track of which coordinates are used for normal ordering) gives
\[ : T_{--} :\xi = : T_{--} :\sigma + \frac{1}{12} \left[ \partial^- \rho - (\partial_- \rho)^2 \right] \equiv : T_{--} :\sigma + t_{--} \]
(5.3)
where we define \( t_{ab} \) to be the difference between the two normal-ordered stress tensors. This formula is valid for an arbitrary Weyl rescaling \( \rho \), and allows us to determine the full stress tensor given the stress tensor in \( \sigma \) coordinates. In particular, suppose that the initial state satisfies
\[ : T_{++} :\sigma = 0 \]
(5.4)
in accord with conformal invariance of the scalar field. Then the conservation law
\[ \nabla_+ T_{--} + \nabla_- T_{++} = 0 , \]
(5.5)
which should hold for the stress tensor in any generally coordinate invariant regulation scheme, implies
\[ 0 = \frac{1}{12} \partial_+ \left[ \partial_-^2 \rho - (\partial_- \rho)^2 \right] + \partial_- t_{+-} - \Gamma_{--} t_{+-} , \] \hfill (5.6)
where we have used \( \Gamma_{++} = \Gamma_{+-} = 0 \). The Christoffel symbol \( \Gamma_{--} \) is easily computed, giving
\[ \Gamma_{--} = g^{++} \Gamma_{--} = 2 \partial_- \rho . \] \hfill (5.7)

Eq. (5.6) then integrates to
\[ t_{+-} = -\frac{1}{12} \partial_+ \partial_- \rho , \] \hfill (5.8)
so
\[ : T_{+-} : \xi = -\frac{1}{12} \partial_+ \partial_- \rho . \] \hfill (5.9)
The right side is proportional to the curvature \( R \), and we have
\[ \langle T \rangle = \frac{1}{24} R . \] \hfill (5.10)
This is the famous conformal anomaly: the regulated trace of the stress tensor varies with the metric used to define the regulator.

Eq. (5.9) can be integrated to find the quantum effective action of the scalar field, using
\[ \langle T_{+-} \rangle = -\frac{1}{i} \frac{2\pi}{\sqrt{-g}} \frac{\delta}{\delta g^{+-}} \ln \left[ \int Df \, e^{-\frac{1}{4\pi} \int (\nabla f)^2} \right] \]
\[ = -\frac{\pi}{4} \frac{\delta}{\delta \rho} S_{\text{eff}} . \] \hfill (5.11)

We find
\[ S_{\text{eff}} = -\frac{1}{6\pi} \int d^2 \sigma \, \rho \, \partial_+ \partial_- \rho = -\frac{1}{24\pi} \int d^2 x \, \sqrt{-g} \, \rho \, \Box \rho , \] \hfill (5.12)
or, using the relation \( R = -2 \Box \rho \) and the Green function \( \Box^{-1} \) for \( \Box \),
\[ S_{\text{eff}} = -\frac{1}{96\pi} \int d^2 x \sqrt{-g} \int d^2 x' \sqrt{-g'} R(x) \Box^{-1} (x, x') R(x') . \] \hfill (5.13)
\[ \equiv S_{\text{PL}} \]
This expression is known as the Polyakov-Liouville action.[18]

The quantum mechanics of the evaporating black hole is described by the functional integral
\[ \int \mathcal{D}g \mathcal{D}\phi \, e^{\frac{i}{\hbar} S_{\text{grav}}} \int \mathcal{D}f \, e^{\frac{i}{\hbar} S_f (\ldots)} . \] \hfill (5.14)
Here we reinstate $\hbar$, and the ellipses denote matter sources that create the black hole. In particular, if these are taken to be classical, (5.14) becomes

$$\int Dg D\phi e^{i\hbar S_{\text{grav}} + \frac{i}{\hbar} S_{\text{ct}} + iS_{\text{PL}}}.$$  

(5.15)

The Hawking radiation and its backreaction is encoded in $S_{\text{PL}}$, which corrects the functional integral at one loop. However, there are other one loop corrections arising from $\phi$ and $g$, and the Hawking radiation gets lost among them. To avoid this, consider instead $N$ matter fields $f_i$, so (5.15) becomes

$$\int Dg D\phi e^{i\hbar S_{\text{grav}} + \frac{i}{\hbar} S_{\text{ct}} + iS_{\text{PL}} + i(N\hbar)S_{\text{PL}}}.$$  

(5.16)

This has a semiclassical limit exhibiting Hawking radiation for large $N$: $N\hbar$ must be fixed as $\hbar \to 0$. The semiclassical equations of motion are

$$G_{++} = T_{++}^{\text{ct}} + \frac{N}{12} \left[ \partial^2 \rho - (\partial_+ \rho)^2 \right] - e^{-2\phi} \left( 2\partial_+ \partial_- \phi - 4\partial_+ \partial_- \rho - \lambda^2 e^{2\rho} \right) \equiv G_{++} = -\frac{N}{12} \partial_+ \partial_- \rho$$  

(5.17)

and likewise for $G_{--}$; the dilaton equation is unmodified.

The equations (5.17) are no longer soluble. They have been studied numerically [19-21], but instead we’ll take a different approach. Quantization of (5.16) requires addition of counterterms, and specific choices of these exist [23-25] that magically restore solubility! These choices of counterterms don’t spoil the essential features of the evaporation, and the resulting theories are thus soluble models for evaporating black holes.

We will focus on the prescription of Russo, Susskind, and Thorlacius[25], who add counterterms that ensure the current $\partial_\mu (\rho - \phi)$ is conserved at the quantum level. To see how to accomplish this, add (2.5) and (5.12) to get the action including Hawking radiation effects,

$$S_{\text{sc}} = \frac{1}{2\pi} \int d^2 x \left\{ 2\nabla (\rho - \phi) \cdot \nabla e^{-2\phi} + 4 e^{2(\rho - \phi)} \right\} + \frac{N\hbar}{12} \nabla \rho \cdot \nabla \rho.$$  

(5.18)

Conservation of $\partial_\mu (\rho - \phi)$ can clearly be reinstated by adding the counterterm

$$-\frac{\hbar N}{48\pi} \int d^2 x \sqrt{-g} \phi R = -\frac{1}{2\pi} \int d^2 x \frac{N\hbar}{12} \nabla \phi \cdot \nabla \rho.$$  

(5.19)

3 For a general discussion of physical constraints on such counterterms see [22].
and the action becomes
\[ S_{\text{RST}} = \frac{1}{2\pi} \int d^2 x \left\{ 2 \nabla (\rho - \phi) \cdot \nabla \left( e^{-2\phi} + \frac{\kappa}{2} \rho \right) + 4 e^{2(\rho - \phi)} \right\}. \] (5.20)

where we define \( \kappa = N\hbar/12. \)

Eq. (5.20) is the same as (2.5) with the replacement
\[ e^{-2\phi} \to e^{-2\phi} + \kappa\rho/2 , \] (5.21)

so the solutions are found directly from (2.9) and (2.13),
\[ \rho - \phi = \frac{1}{2}(\sigma^+ - \sigma^-) \]
\[ e^{-2\phi} + \frac{\kappa}{2}\rho = M + e^{\sigma^+}(e^{-\sigma^-} - \Delta). \] (5.22)

Note in particular that
\[ e^{-2\phi} + \frac{\kappa}{2}\phi = M + e^{\sigma^+}(e^{-\sigma^-} - \Delta) - \frac{\kappa}{4}(\sigma^+ - \sigma^-). \] (5.23)

The left hand side has a global minimum at
\[ \phi_{cr} = -\frac{1}{2} \ell n(\kappa/4) , \] (5.24)

so the solution becomes singular where the right hand side falls below this. Here additional quantum effects should become important. A Kruskal diagram for the formation and evaporation is sketched in fig. 5.

A sensible definition of the apparent horizon is the place where lines of constant \( \phi \) become null, \( \partial_+ \phi = 0 \) since if these lines are spacelike it means one is inevitably dragged to stronger coupling as in the classical black hole. Differentiating (5.23), the equation for the horizon is
\[ e^{-\sigma^-} = \Delta + \frac{\kappa}{4} e^{-\sigma^+}. \] (5.25)

The singularity becomes naked where it and the horizon meet,
\[ \sigma^-_{\text{NS}} = -\ell n \left( \frac{\Delta}{1 - e^{-4M/\kappa}} \right). \] (5.26)

With the singularity revealed, future evolution outside the black hole can no longer be determined without a complete quantum treatment of the theory: the semiclassical approximation has failed. The last light ray to escape before this happens is called the effective horizon. These features are also shown in fig. 6.
Fig. 5: Shown is a Kruskal diagram for collapse and evaporation of a 2d dilatonic black hole. The matter turns the singularity at $\phi_{cr}$ spacelike, and forms an apparent horizon. The black hole then evaporates until the singularity and apparent horizon collide. The effective horizon is just outside the line $\sigma^-= -\ln \Delta$.

The Hawking flux is computed as before; as $\sigma^+ \to \infty$, the metric asymptotes to the previous form (2.17). The flux $\langle :T_{--} : \rangle$ is therefore still given by (3.7), and asymptotes to $\frac{1}{48}$ as $\xi^- \to \infty$. As a check, we can compute the mass lost up to the time (5.26) where our approximations fail:

$$\int_{-\infty}^{\xi_{NS}} d\xi^- \langle :T_{--} : \rangle = M - \frac{\text{constant}}{\kappa}.$$

As anticipated, the semiclassical approximation breaks down when the black hole reaches the analogue of the Planck scale, here $M_{bh} \sim 1/\kappa$. Furthermore, as in the preceding section it appears that there are correlations between the Hawking radiation and the internal state of the black hole, and that the Hawking radiation is approximately thermal. An estimate of the missing information comes from the thermodynamic relation

$$dS = \frac{dE}{T},$$

(5.28)
Fig. 6: The Penrose diagram for the evaporating two-dimensional black hole. The lower part of the diagram has been truncated at \( \phi_{cr} \), and the upper part where the semiclassical approximation fails due to appearance of large curvatures.

which gives

\[
S = 2\pi M .
\]  

(5.29)

It was mentioned that attempts have been made to go beyond the semiclassical approximation and define a complete quantum theory by choosing a boundary condition \[14-17\], for example reflecting, at \( \phi_{cr} \). These attempts have met with various obstacles, for example of instability to unending evaporation. It is perhaps not surprising that a simple reflecting boundary condition has had difficulty in summarizing the non-trivial dynamics of strong coupling. Such a boundary condition presumnes that there are no degrees of freedom at strong coupling, yet if one thinks of the connection of these solutions to four-dimensional extremal black holes, it seems quite possible that there are either a large number of degrees of freedom or very complicated boundary interactions.

6. The Information Problem

6.1. Introduction

The preceding sections have outlined a semiclassical argument that the Hawking radiation is missing information that was present in the initial state. We’d like to know what
happens to this information. One possibility was proposed by Hawking: the black hole disappears at the end of evaporation and the information is simply lost. Although on the face of it this is the most conservative solution, it is really quite radical; information loss is equivalent to a breakdown of quantum mechanics.

This prompts us to investigate other alternatives. A second possibility is that the information is returned in the Hawking radiation. This could result from a mistake in our semiclassical argument involving a fundamental breakdown of locality and causality, as advocated in [26,13,17,27]. Another possibility is that the information is radiated after the black hole reaches $M \sim m_{pl}$ and the semiclassical approximation fails. Here ordinary causality no longer applies to the interior of the black hole, and it’s quite plausible that the information escapes.

![Fig. 7: A Penrose diagram appropriate to a long-lived remnant scenario. The singularity is replaced by a planckian region near $r = 0$.](image)

However, the amount of information to be radiated is, in four dimensions, given by $S \sim M^2$, and the amount of available energy is just the remaining black hole mass, $M \sim m_{pl}$. The only way that the outgoing radiation can contain such a large amount of information with such a small energy is if it is made up of a huge number of very soft particles, for example photons. Each photon can carry approximately one bit of information, and so $M^2$ photons are required. Their individual energies are therefore...
$E_\gamma \sim 1/M^2$, and the decay time to emit one such photon is bounded by the uncertainty principle,

$$\tau_\gamma \sim \frac{1}{E_\gamma} \sim M^2. \quad (6.1)$$

It then takes a time

$$\tau_{\text{rem}} \sim \left(\frac{M}{m_{\text{pl}}}\right)^4 t_{\text{pl}} \quad (6.2)$$

to radiate all of the information in $M^2$ photons\cite{28,29}. For example, this approximates the lifetime of the universe for a black hole whose initial mass is that of a typical building. This implies the third alternative: that the black hole leaves a long-lived, or perhaps stable, remnant. An example is exhibited in the Penrose diagram of fig. 7: after the Hawking radiation ceases, a remnant is left at the origin. A qualitative picture of such an object is shown in fig. 8, which shows the geometry of the time slice $t_{\text{evap}}$ in fig 7. The remnant is a long planckian fiber with radius $\sim r_{\text{pl}}$, and the infalling matter concentrated at its tip. Describing such a configuration certainly requires planckian physics, and it is quite plausible that this physics allows the remnant to slowly decay.

![Fig. 8: A late time slice through fig. 7 shows a long Planck sized fiber attached to an asymptotically flat geometry.](image)

Each of these three scenarios has staunch advocates. Each also appears to violate some crucial principle in low-energy physics. The resulting conflict is the black hole information problem. Let’s investigate this in more detail.

### 6.2. Information Loss

Information loss violates quantum mechanics, but worse, it appears to violate energy conservation, and this is a disaster. The heuristic explanation for this is that information
transmission requires energy, so information loss implies energy non-conservation. Suppose for example that we attempt to summarize the formation and evaporation process by giving a map from the initial density matrix to the final outgoing density matrix,

$$\rho_f = \mathcal{S} \rho_i ,$$

(6.3)

where $\mathcal{S}$ is a linear operator. Such operators, if not quantum mechanical,

$$\rho_f \neq S \rho_i S^+ ,$$

(6.4)

typically either violate locality or energy conservation [30].

A useful way of explaining the connection between information and energy is to model the black hole formation and evaporation process as the interaction of two Hilbert spaces $\mathcal{H}_o, \mathcal{H}_h$. The first includes the states of the outside world, and the second the internal states of the black hole which are thought of as “lost”. Suppose these interact via a conserved hamiltonian

$$H = H_o + H_i + H_h$$

(6.5)

where $H_o$ acts only on $\mathcal{H}_o$, $H_h$ only on $\mathcal{H}_h$, and the interaction hamiltonian $H_i$ acts on both. $H_i$ summarizes the physics that transfers information from the outside Hilbert space to the hidden one.

Now, the black hole formation and evaporation process involves loss of information during a time $\tau \sim M^3$. On larger time scales the process can be repeated at the same location; that is, another independent black hole formation and evaporation can take place. In general, we say that the information loss is *repeatable* if the information loss in $n$ such experiments is $n$ times the information loss in a single experiment

$$\Delta S_n = n \Delta S_0 .$$

(6.6)

One can plausibly argue [32] if not prove that repeatable information loss on a time scale $\Delta t$ indicates energy losses

$$\Delta E \propto 1/\Delta t$$

(6.7)

from the observable Hilbert space (our world) to the hidden one (the interior of the black hole). Suppose, for example, that there is no energy loss, $H_h = 0$. A simple example for the hidden Hilbert space is the states of a particle on a line, $\mathcal{H}_h = \{ |x\rangle \}$, and an example hamiltonian is

$$H_i = \mathcal{O} \hat{x}$$

(6.8)
for some operator $\mathcal{O}$ acting on the observable space. The wavefunction $|\Psi\rangle$ of the coupled system obeys the Schrödinger equation with Hamiltonian (6.3), and the outside density matrix is defined as

$$\rho_o = tr_h |\Psi\rangle \langle \Psi| .$$

One can readily show [33] that in $n$ scattering experiments, the information loss per experiment declines:

$$\lim_{n \to \infty} \frac{\Delta S_n}{n} = 0 .$$

This is not repeatable information loss.

The explanation for this is familiar from wormhole physics [33,34]. If we started in an $\hat{x}$ eigenstate,

$$|\Psi\rangle = |\Psi_0\rangle |x\rangle ,$$

then the sole observable effect of the interaction is a modification of the external Hamiltonian:

$$H = H_o + \mathcal{O}x .$$

The eigenvalue $x$ simply plays the role of a coupling constant. If we instead started in a superposition of $\hat{x}$ eigenstates, then we simply have a probability distribution for coupling constants. For example, suppose we begin in the superposition

$$\alpha |x_1\rangle + \beta |x_2\rangle .$$

The Hamiltonian does not change this internal state; there is a superselection rule. If we compute the expectation value of some local operators $\mathcal{O}_1, \cdots, \mathcal{O}_n$ in this state, it gives

$$|\alpha|^2 \langle x_1 | \mathcal{O}_1 \cdots \mathcal{O}_n | x_1 \rangle + |\beta|^2 \langle x_2 | \mathcal{O}_1 \cdots \mathcal{O}_n | x_2 \rangle .$$

Performing repeated experiments simply correlates the outside state with the value of the effective coupling constant $x$, that is, measures the coupling constant. Once these are determined there is no further loss of information.

These arguments generalize to more interesting Hilbert spaces $\mathcal{H}_h$, and more general interaction Hamiltonians,

$$H_i = \sum_{\alpha} \mathcal{O}_\alpha I_\alpha$$

where $\mathcal{O}_\alpha$ and $I_\alpha$ act on $\mathcal{H}_o$ and $\mathcal{H}_h$ respectively. Finally, note that if $H_h \neq 0$ but the operators $I_\alpha$ only act between energy levels with energy separation $\lesssim \Delta E$, then the information loss will not be repeatable on time scales $\Delta t \lesssim \frac{1}{\Delta E}$. This indicates a basic connection between information loss and energy non-conservation.
Worse still, by basic quantum principles, if information and energy loss is allowed, it will take place in virtual processes. An example is shown in fig. 9. For such a virtual process on time scale $\sim \Delta t$ one expects energy loss $\Delta E \sim 1/\Delta t$. Furthermore, there is no obvious source for suppression from such processes occurring at a rate of one per Planck volume per Planck time with $\Delta t \sim m_{\text{pl}}$. Allowing such energy fluctuations is analogous to putting the world in contact with a heat bath at temperature $T \sim T_{\text{pl}}$: this is in gross contradiction with experiment.

Indeed, [30] argued that a general class of local $\mathcal{S}$ matrices mimic such thermal fluctuations. It can be shown [32] that these correspond to a thermal distribution at infinite temperature.

Attempts to turn Hawking’s picture of information loss into a consistent scenario are found in [35] and very recently in [36], where a theory of long-lived remnants emerges (see section 6.4).

6.3. Information return

Our semiclassical description, together with the assumption that the black hole disappears, clearly led to information loss — the Hawking radiation at any given time was thermal with no correlations with either the infalling matter or with earlier Hawking radiation. Furthermore, the semiclassical approximation appears valid up until the time when the black hole reaches the Planck mass; before this time curvatures are everywhere small. In the two-dimensional model of section 2, the rate of mass loss of the black hole
is proportional to $N$, and using the relation \((5.29)\) between information and energy, one finds that if the information is to be radiated before the Planck scale it must be at a rate

$$\frac{dI}{dt} \propto N$$

(6.16)

by the time the black hole has reached a fraction of its original mass, say $M/10$. No such information return at this order in the large-$N$ expansion is seen.

In fact, there is another argument that the Hawking radiation doesn’t contain information. For it to do so, the outgoing state would have to be some modification of the Hawking state. If we evolve such a state backwards in time until it reaches the vicinity of the horizon, this state will differ from the local vacuum, and because of the large redshift the difference will be in very high energy modes. An infalling observer would encounter these violent variations from the vacuum at the horizon. This conflicts with our belief that there is no local way for a freely falling observer to detect a horizon: we could all be falling through the horizon of a very large black hole at this very moment, and may not know it until we approach the singularity.

\[\text{Fig. 10:} \] Locality in field theory implies that observations made at $x$ and $y$ with spacelike separations must commute.

\(\text{\footnote{4} For a related argument that information return must begin by this time see \cite{37}.}\)
That information is not returned is a consequence of locality and causality. To see this, consider the spacelike slice $S$ of fig. 10. Locality/causality in field theory is the statement that spacelike-separated local observables commute, so

$$[\mathcal{O}(x), \mathcal{O}(y)] = 0 . \quad (6.17)$$

This means that up to exponentially small tails, the state on the slice $S$ can be decomposed

$$\psi'' = \sum_\alpha |\psi_\alpha\rangle_{\text{in}} \langle \psi_\alpha|_{\text{out}} \quad (6.18)$$

where $|\psi_\alpha\rangle_{\text{in}}, |\psi_\alpha\rangle_{\text{out}}$ are states with support inside or outside the black hole. As we’ve said, infalling observers don’t encounter any particular difficulty at the horizon, and so their measurements reveal different internal states $|\psi_\alpha\rangle_{\text{in}}$ depending on the details of the collapsing matter, etc. Thus when we trace over internal states to find the density matrix relevant to the outside observer, it has missing information, $\Delta I \sim M^2$. The only obvious way one could avoid correlations leading to this impurity is if the internal state of the black hole is unique, i.e.

$$\psi = |\psi_0\rangle_{\text{in}}|\psi\rangle_{\text{out}} , \quad (6.19)$$

and in particular is independent of the infalling matter. In this case an infalling observer would be “bleached” of all information when crossing the horizon. Since there should be no way to discern a horizon locally this appears impossible.

A possible weakness of this argument is the notorious problem of defining observables in quantum gravity.\footnote{For related discussion in two-dimensional models, see [17].} Perhaps there are no truly local observables. However, within regions where the semiclassical approximation is valid one expects the observables of quantum gravity to reduce to ordinary field theoretic observables, up to small corrections.

An extreme example of what information return in the Hawking radiation entails comes from the observation that the vicinity of the horizon for a large mass black hole is well approximated by flat space. Indeed, in the limit $|r - 2M| << 2M$, the Schwarzschild metric

$$ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} + r^2 d\Omega_2^2 \quad (6.20)$$

is well approximated by

$$ds^2 = - \frac{x^2}{16M^2} dt^2 + dx^2 + 4M^2 d\Omega_2^2 , \quad (6.21)$$
as can be shown using the substitution $x^2 = 8M(r - 2M)$. For large $M$, $4M^2d\Omega_2^2 \simeq dy^2 + dz^2$, and the line element \((6.21)\) is that of flat Minkowski space as seen by a family of accelerated observers. This is known as Rindler space, and is shown in fig. 11. If the information from the infalling matter is in the Hawking radiation, it must be accessible to observers outside the horizon, and this should also hold true as $M \to \infty$. Then information about states in the entire left half of Minkowski space is accessible to observers in the right wedge of Rindler space. If I walk past the horizon and keep going for a billion light-years to point $x$, the observer at $y$ still has full access to my internal state. This represents a gross violation of the locality/causality, \((6.17)\).

Fig. 11: The right wedge of Minkowski space corresponds to Rindler space; it is the region observable by uniformly accelerated observers. Taking the $M \to \infty$ limit of the black hole nonlocality arguments shows that information at an arbitrary point $x$ is fully accessible in the vicinity of the Rindler horizon.

This hypothetical “holographic” property of spacetime, that all information in a three-dimensional region is encoded on the surface of that region, requires drastically new physics and has been advocated by ’t Hooft\[26\] and more recently by Susskind, Thorlacius, and Uglum \[27\]. ’t Hooft efforts involve the hypothesis that the world is at a fundamental level similar to a cellular automaton\[38\]. Susskind et. al.’s attempts instead rely on string theory.
Indeed, unlike point particles, strings are intrinsically non-local objects. Measurements made in the vicinity of points $x$ and $y$ of fig. 11 in string theory are expected not to commute. But these violations of locality are expected to generically fall off exponentially fast on the string scale, which is approximately the Planck length,

$$[\mathcal{O}(x), \mathcal{O}(y)] \sim e^{-(x-y)^2/\ell_s^2},$$  \hspace{1cm} (6.22)

and as such are too small to be relevant to the information problem. However, the Rindler observers are moving at huge velocities relative to those freely-falling, and Lowe, Susskind, and Uglum [39] argue that these enormous relative boosts compensate for the exponential fall off and make

$$[\mathcal{O}(x), \mathcal{O}(y)] \sim 1$$  \hspace{1cm} (6.23)

for suitable ultra high-energy observations at $y$.

So in string theory, it is possible that in some sense the information from the left half of the world is encoded in the right half. In fact, all of the information in the Universe might be expected to be encoded in the surface of a piece of chalk, a post-modern take on Huxley [40]! However, an open question is whether it is encoded in any realistically accessible fashion — (6.23) would hold only for particular ultra high-energy observations, and it is not clear that the information available from such unrealistic observations is imprinted on the Hawking radiation. At present no detailed mechanism to transmit the information to the Hawking radiation has been found. In fact, a logical possibility is that the information never escapes from the horizon, and this is consistent with a remnant picture.

In conclusion, information return requires violations of locality/ causality. Since locality and causality are at best poorly understood in string theory, this opens the possibility of information return. However, an explicit quantitative picture of how this happens is yet to be produced.

6.4. Remnants

If black hole remnants, either stable or long lived, resolve the information problem, then there must be an infinite number of remnant species to store the information from an arbitrarily large initial black hole. For neutral black holes Hawking’s calculation fails at the Planck mass, so this means an infinite species of Planck-mass particles. These are expected to be infinitely pair produced in generic physical processes — a clear disaster.
This statement is most easily illustrated if we imagine that remnants carry electric charge; we return to the neutral case momentarily. Stating that a remnant is charged is equivalent to assuming that there is a non-zero minimal-coupling to low-frequency photons, as shown in fig. 12. By crossing symmetry, this coupling implies that pairs will be produced by Schwinger production in a background electric field. The total production rate is

$$\Gamma_{\text{vac}} \sim N \ e^{-\pi m^2/qE}$$

where $m$ is the remnant mass, $E$ is the field strength, and $N$ is the number of species. Although the exponential may be small it is overwhelmed in the case of infinite species.

![Fig. 12: Using crossing symmetry, coupling of a particle to the electromagnetic field implies Schwinger pair production.](image)

Neutral remnants have the same problem. For example, the gravitational analog of Schwinger production is Hawking radiation, for which we’d find a rate

$$\frac{dM}{dt} \propto N \cdot \frac{1}{M^2}$$

for a black hole to decay. $N = \infty$ gives an infinite rate. Likewise pairs could be produced in other everyday physical processes that have sufficient available energy. Although such production is highly suppressed by small form factors, this suppression is overcompensated by the infinite states.

6.5. Summary

| proposal          | principles violated                  |
|-------------------|--------------------------------------|
| information loss  | unitarity, energy conservation       |
| information radiated | locality/causality                  |
| remnants          | crossing symmetry                    |
Table 1

A summary of the proposed fates of information is shown in Table 1, along with the corresponding objections. Each of these objections can be phrased solely in terms of low-energy effective physics, and is apparently independent of any hypothesized Planck-scale physics. For this reason this conflict has been called the information *paradox*. The matter would be reduced to merely a problem if a convincing way were found whereby Planck-scale physics might evade one of these objections.

7. Black hole pair production

An aspect of the quantum mechanics of black holes that is of interest in its own right is the pair production of black holes. This phenomenon is also of direct relevance to the information problem. Indeed, let us assume the validity of unitarity (no information destruction) and locality (no information return in Hawking radiation). We’ve seen that these imply evaporating black holes leave an infinite variety of neutral remnants. The same reasoning in the charged sector implies that there are an infinite number of internal states of an extremal, $M = Q$, Reissner-Nordstrom black hole. Indeed, assume we begin with an extremal hole; we may then feed it a huge amount of information, say, by dropping in the planet Earth. The black hole then Hawking radiates back to $M = Q$ and leaves an extremal black hole with the extra information encoded in its internal states. This process may be continued indefinitely, implying an infinite number of internal states of Reissner-Nordstrom black hole.

Infinite states suggest unphysical infinite Schwinger pair production. There are two possibilities. The first is that a careful calculation of the production rate gives a finite answer. In this case Reissner-Nordstrom black holes would provide an example of how to avoid the infinite states/infinite production connection that could likely be translated into a theory of neutral remnants with finite production. Alternatively, the assumption of infinite states may indeed imply infinite production. This then would appear to rule out either unitarity or locality, and remove the raison d’être for neutral remnants.

An instanton exists describing such pair production, but let us first discuss this process following Schwinger’s original arguments. Since Reissner-Nordstrom black holes — or neutral remnants — are localized objects and must have a Lorentz invariant, local and

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6 Dilaton black holes have also been investigated in this context [42,43].
causal, and quantum mechanical description, they should be described by an effective quantum field. This field should have a species label, 
\[ \phi_a(x) , \] (7.1)
and let these have masses \( m_a \). We will take these to be electrically charged with charge \( q \), although the case of magnetic monopole production in a magnetic field is equivalent by electromagnetic duality. This means that there should be a minimal coupling interaction with the electromagnetic field, plus higher dimension operators:
\[
S_{\text{eff}} = \int d^4x \sum_a \left( -|D_\mu \phi_a|^2 - m_a^2 |\phi_a|^2 \right) + \cdots
\] (7.2)
with \( D_\mu = \partial_\mu + iq A_\mu \).

The decay rate into species \( a \) of the vacuum consisting of a background electric field with vector potential \( A_0^\mu \) is given by the vacuum-to-vacuum amplitude:
\[
e^{-\frac{\Gamma_a}{2} V T - i E_0 T} = \langle 0|0 \rangle_{A_0} = \frac{\int D\tilde{A}_\mu D\phi_a \ e^{iS_{\text{eff}}[A_0+\tilde{A},\phi_a]} \int D\tilde{A}_\mu D\phi_a \ e^{iS_{\text{eff}}[\tilde{A},\phi_a]} }{ } ,
\] (7.3)
where \( V \) is the volume, and where we have separated off electromagnetic fluctuations \( \tilde{A} \).

To leading order in an expansion in the charge, this gives
\[
\Gamma_a \cdot V T = -2 \text{Re } \elln \text{ Det} \left( \frac{-D_0^2 + m_a^2}{-\partial^2 + m_a^2} \right)
\]
\[
= \text{Re } \text{Tr} \elln \left( \frac{-D_0^2 + m_a^2}{-\partial^2 + m_a^2} \right).
\] (7.4)

**Fig. 13:** Above the slice \( S \) (dotted) is shown the lorentzian trajectories of a pair of oppositely charged particles in an electric field. Below \( S \) is the euclidean continuation of this solution. Matching the euclidean and lorentzian solutions smoothly along \( S \) gives a picture of Schwinger production followed by subsequent evolution of the pair of created particles.

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7 See, e.g., [43].
The trace can be rewritten in terms of a single-particle amplitude:

$$\Gamma_a \cdot V T = \text{Re} \int d^4x \int_0^\infty \frac{dt}{t} \left( \langle x | e^{iH(A^0) t} | x \rangle - \langle x | e^{iH(0) t} | x \rangle \right)$$  \hspace{1cm} (7.5)

with

$$\langle x' | e^{iH(A^0) t} | x \rangle = \int_x^{x'} Dx e^{i \int_0^t d\tau \left( \frac{\dot{x}^2}{2} + iqA_\mu \dot{x}^\mu + m_a^2 \right)}.$$  \hspace{1cm} (7.6)

This functional integral has a dominant euclidean saddlepoint corresponding to circular motion of the particle in the field. This instanton is the euclidean continuation of a solution where a pair of charged particles started at rest run hyperbolically to opposite ends of the background field, as shown in fig. 13. Then if the euclidean trajectory is cut on the dotted line, where the velocities vanish, it smoothly matches to the lorentzian solution: pairs are created and run to infinity.

The action of the instanton is easily found to be

$$S_E = \frac{\pi m_a^2}{qE},$$  \hspace{1cm} (7.7)

giving an approximate decay rate into species $a$,

$$\Gamma_a \sim e^{-\pi m_a^2/qE} + \ldots .$$  \hspace{1cm} (7.8)

Corrections arise both from the higher order terms in (7.2) and from subleading quantum corrections. These are typically subleading in an expansion in $qE$. The total decay rate is

$$\Gamma \sim \sum_a e^{-\pi m_a^2/qE} .$$  \hspace{1cm} (7.9)

An infinite spectrum of remnants with nearly degenerate masses clearly gives an infinite decay rate. Note in particular that if the spectrum is of the form $m_a = m_0 + \Delta m_a$, where $\Delta m_a \ll m_0$ are small mass splittings, then the decay rate is proportional to the partition function for internal states:

$$\Gamma \sim e^{-\pi m_0^2/qE} \text{Tr}_a e^{-\beta \Delta m} ,$$  \hspace{1cm} (7.10)

with $\beta = 2\pi m_0/qE$. 

33
Turning now to black holes, the remarkable fact is that solutions analogous to those in fig. 13 are explicitly known \[46-50\] for charged black holes! Although the solutions are known for arbitrary strength coupling to a dilatonic field (as in string theory), here we’ll consider only the simpler case without a dilaton, and for magnetic black holes in a background magnetic field. The solutions are (don’t worry about how they were found!)

\[
ds^2 = \frac{\Lambda^2}{A^2(x-y)^2} \left[ G(y) dt^2 - G^{-1}(y) dy^2 + G^{-1}(x) dx^2 \right] + \frac{G(x)}{\Lambda^2 A^2(x-y)^2} d\phi^2 ,
\]

\[
A_\phi = -\frac{2}{BA} \left[ 1 + \frac{1}{2} Bqx \right] + k .
\]

Here

\[
G(\xi) = (1 - \xi^2 - r_+ A\xi^3)(1 + r_- A\xi) \equiv -r_+ r_- A^2 \prod_{i=1}^{4} (\xi - \xi_i) ,
\]

with ordered zeros

\[
\xi_1 = -\frac{1}{r_- A} , \xi_2, \xi_3, \xi_4 ,
\]

and

\[
\Lambda(x,y) = (1 + qBx)^2 + \frac{B^2 G(x)}{4A^2(x-y)^2} .
\]

Choice of \(k\) corresponds to convention for location of Dirac string singularities. \(A, B, r_+, r_-\), and \(q\) are parameters, to be thought of roughly as the acceleration, the magnetic field, the inner and outer horizon radii, and the charge. These identifications become exact in the limit \(qB \to 0\). The latter three are related by

\[
q = \sqrt{r_+ r_-}
\]

as for a free Reissner-Nordstrom black hole. Furthermore, the charge, magnetic field, mass and acceleration should be related by an expression that simply reduces to Newton’s law \(mA = qB\) at low accelerations, \(B \ll 1\). Without this relation the solution has a string-like singularity connecting the black holes.
The solution (7.11) appears rather complicated, but its structure can be deduced by examining various limits. As \( x \to y \) the solution asymptotes to

\[
\begin{align*}
  ds^2 &= \left(1 + \tilde{B}^2 \rho^2 \right)^2 \left[ -dt^2 + dz^2 + d\rho^2 \right] + \left(1 + \frac{\tilde{B}^2 \rho^2}{4} \right)^{-2} \rho^2 \, d\phi^2, \\
  A_\phi &= \frac{\tilde{B} \rho^2}{2} \left(1 + \frac{\tilde{B}^2 \rho^2}{4} \right)^{-1},
\end{align*}
\]

(7.16)

where \( \tilde{B} = \tilde{B}(r_+, r_-, B) \simeq B \), and where a change of coordinates has been made. This is the Melvin solution\[51] — due to the energy density of the magnetic field, this is the closest one can come to a uniform magnetic field in general relativity. The solution (7.11) naturally yields a Rindler parametrization, and only covers half of the Melvin solution. The rest is found by continuation. The region near the black hole corresponds to \( y \simeq \xi_2 \), and in this limit the solution asymptotes to the Reissner-Nordstrom solution. The black hole follows a trajectory with asymptotes \( y = \xi_3 \), corresponding to the acceleration horizon. These features are illustrated in fig. 14.

\[8\] For more details, see e.g. \[50\].
Analytic continuation, $t = i\tau$, gives a black hole moving on a circular trajectory, as in the lower half of fig. 13. Regularity of the solution at the acceleration horizon requires a specific periodicity for $\tau$ as in standard treatments of Rindler space. However, regularity of the solution at the black hole horizon $y = \xi_2$ also requires periodic identification of $\tau$, in general with a different period. Demanding that these identifications match gives another condition on the parameters,

$$\xi_1 - \xi_2 - \xi_3 + \xi_4 = 0 .$$

This can be thought of as a condition matching the Hawking temperature of the black hole and the acceleration temperature from its motion, which is necessary to find a stationary solution with the black hole in thermodynamic equilibrium.

**Fig. 15:** A sketch of the temperature vs. mass curve for a Reissner-Nordstrom black hole.

**Fig. 16:** The spatial geometry of the time-symmetric slice through the wormhole instanton.

A sketch of the temperature vs. mass curve is in fig. 15. The solution\[48,49\] to $T_{BH} = T_{accel}$ is given by taking $M$ slightly larger than $Q$. On the symmetric slice $S$ of fig. 13, the three geometry is that of fig. 16—near the black hole it corresponds to a symmetric slice through the black hole horizon, as in fig. 17. Thus it’s geometry is that of a Wheeler wormhole with ends of charge $\pm Q$. Another solution\[47,50\] takes advantage of
the ambiguity in the extremal limit \(M \to Q\): here the “throat” near the horizon becomes infinitely long, and the periodic identification is no longer fixed at the horizon. The geometry of the symmetric slice is sketched in fig. 18.

\[\Gamma \propto e^{-S_{\text{inst}}} . \]  \(7.18\)

One can then show [48,52] that in terms of the physical charge \(Q\) and field \(B\),

\[S_{\text{inst}} = \frac{\pi Q}{B} \ (1 + O(QB)) , \]  \(7.19\)

in agreement with (7.7). However, as above one would expect the rate to also be proportional to the number of black hole internal states, which we have argued is infinite.
In order to see how the infinite states contribute, it’s useful to think about their description. In particular, suppose we start with an extremal black hole, throw some matter into it, and let it evaporate back to extremality. The Penrose diagram for this is shown in fig. 19. In describing the evolution we are allowed to choose any time slicing, and from our earlier discussion of the instanton we expect a slicing with spatial slices that stay outside the horizon to be a useful choice. At long times compared to the evaporation time scale \( \sim Q^3 \), the mass excess from the infalling matter will have been re-radiated in the form of Hawking radiation, and this will asymptote to infinity. The state inside a finite radius \( R \) is therefore that of an extremal black hole until the slice nears the incoming matter; see fig. 20. As \( t \to \infty \), this matter is infinitely far down the black hole throat, and is infinitely blueshifted. Clearly a description of it using our time slicing requires planckian physics.

**Fig. 19:** A portion of the Penrose diagram of an extremal Reissner-Nordstrom black hole, into which some matter has been dropped, and which subsequently evaporates back to extremality.

**Fig. 20:** A schematic picture of the state of the black hole of fig. 19 as described on a late time slice that stays outside the horizon.
The contribution of such states to the pair-production rate should come from the functional integral about the instanton, or, at linear level, from the fluctuation determinant. The reason is that the instanton only describes tunneling to the classical turning point, but in computing the full tunneling rate we should consider tunneling to the nearby configurations with gravitational and matter perturbations, as described above. Tunneling to nearby states is via paths near to the instanton, and it can be shown that contributions of such paths gives the fluctuation determinant at the linear level[54], or the full functional integral if interactions are included.

The necessity for the outside observer to use Planck physics in describing these states hints that perhaps Planck physics could play an important role in computing the production rate[32], and in particular one might hope for it to be finite.

However, a careful examination of the instanton reveals that for weak fields, $QB<<1$, the instanton closely approximates the euclidean solution for a free black hole near the horizon. The contribution to the functional integral from the vicinity of the black hole should therefore be essentially the same as that to $Tr e^{-\beta H}$ from a free black hole, where the trace is over black hole internal states, $H$ is the hamiltonian, and $\beta$ is given by the acceleration temperature. The appearance of such a factor is in agreement with the effective field theory computation, (7.10). If the number of black hole internal states is literally infinite, this factor would be expected to be infinite as would be the production rate.

These issues are still being investigated[55], and a definitive verdict on remnants is not in. Despite the apparent need for a planckian description of these objects, the argument for infinite production has endangered the viability of remnants. It may be that black holes are only infinitely produced if they truly have a finite number of internal states.

8. Conclusions

The quantum mechanics of black holes will no doubt remain a fascinating open window on Planck scale physics for some time to come. In particular, we can hope to sharpen our knowledge of the properties of quantum gravity in our continued confrontation with the information problem. A diversity of opinions on its ultimate resolution abounds, and this suggests the answer might be quite interesting when finally discovered.

Black hole pair production should either tell us how to kill the remnant scenario or how to save it. In either case, it remains a rich and interesting process, nontrivially combining
the phenomena of Hawking and acceleration radiation as well as other aspects of quantum gravity.

The quantum mechanics of black holes has much left to teach us.

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