A Deep Dive into Conflict Generating Decisions

Md Solimul Chowdhury, Martin Müller, Jia-Huai You

Department of Computing Science, University of Alberta.
{mdsolimu, mmueller, jyou}@ualberta.ca

Abstract. Boolean Satisfiability (SAT) is a well-known NP-complete problem. Despite this theoretical hardness, SAT solvers based on Conflict Driven Clause Learning (CDCL) can solve large SAT instances from many important domains. CDCL learns clauses from conflicts, a technique that allows a solver to prune its search space. The selection heuristics in CDCL prioritize variables that are involved in recent conflicts. While only a fraction of decisions generate any conflicts, many generate multiple conflicts. In this paper, we study conflict-generating decisions in CDCL in detail. We investigate the impact of single conflict (sc) decisions, which generate only one conflict, and multi-conflict (mc) decisions which generate two or more. We empirically characterize these two types of decisions based on the quality of the learned clauses produced by each type of decision. We also show an important connection between consecutive clauses learned within the same mc decision, where one learned clause triggers the learning of the next one forming a chain of clauses. This leads to the consideration of similarity between conflicts, for which we formulate the notion of conflicts proximity as a similarity measure. We show that conflicts in mc decisions are more closely related than consecutive conflicts generated from sc decisions. Finally, we develop Common Reason Variable Reduction (CRVR) as a new decision strategy that reduces the selection priority of some variables from the learned clauses of mc decisions. Our empirical evaluation of CRVR implemented in three leading solvers demonstrates performance gains in benchmarks from the main track of SAT Competition-2020.

1 Introduction

Boolean Satisfiability (SAT) is a fundamental problem in computer science, with strong relations to computational complexity, logic, and artificial intelligence. Given a formula $F$ over boolean variables, a SAT solver either determines a variable assignment which satisfies $F$, or reports unsatisfiability if no such assignment exists. In general, SAT solving is intractable [11]. Complete SAT solvers based on the framework of DPLL [12] employ heuristics-guided backtracking tree search. CDCL SAT solvers such as GRASP [27] and Chaff [22] substantially enhanced the DPLL framework by adding conflict analysis and clause learning.

Modern CDCL SAT solvers can solve very large real-world problem instances from important domains such as hardware design verification [14], software testing [8], automated planning [25], and encryption [21,28]. The efficiency of modern
solv...
senting how the corresponding variables are assigned. In each branching decision (or just decision), the solver extends the current partial assignment by selecting a single variable \( v \), called a decision variable, from the current set of unassigned variables, and assigns a boolean value to it. A decision is associated with a decision level \( d_l \geq 1 \); the depth of the search tree when the decision was made. Then, unit propagation (UP) is invoked to simplify \( F \) by deducing a new set of implied variable assignments, which are added to the current partial assignment. After UP, if \( F \) is still unsolved and no conflict occurs, the search moves on to the next decision level and the process repeats.

Conflicts and Clause Learning UP may lead to a conflict due to a conflicting clause \( C \), which cannot be satisfied under the current partial assignment. In this case, conflict analysis generates a learned clause and a backtracking level. The learned clause can help to prune the remaining search space.

Most state-of-the-art CDCL SAT solvers employ the first Unique Implication Point (fUIP) scheme to learn a clause. Starting with conflicting clause \( C \), fUIP continues to resolve literals from the current decision level until it finds a clause \( L = R \lor \{\neg f\} \) such that the literal \( f \) was assigned in the current decision level, while all literals in \( R \) were assigned earlier. \( f \) is called the fUIP literal for the current conflict and is contained in every path from the current decision variable to the current conflict. The literals in \( \{r \mid \text{literal } r \in R\} \cup \{f\} \) are called reason literals for the current conflict, since their assignments caused the current conflict. We call \( R \) the reason clause for the current conflict with conflicting clause \( C \).

After \( L \) is learned, search backjumps to a backjumping level \( b \) which is computed from \( \mathcal{L} \). Before backtracking, the clause \( L = R \lor \neg f \) is unsatisfied under the current partial assignment. After backtracking to \( b \), \( \neg f \) is the only unassigned literal in \( L \). The search proceeds by unit-propagating \( \neg f \) from \( L \). The assignment of \( \neg f \) avoids the conflict at \( C \), but may create further conflicts within the same decision, making this a mc decision.

Relevant Notions The following notions have been proposed in the literature, which are relevant for our paper:

Global Learning Rate: The Global Learning Rate (GLR) \([19]\) is defined as \( \frac{c}{d} \), where \( c \) is the number of conflicts generated in \( d \) decisions. GLR measures the average number of conflict per decisions of a solver.

The Literal Block Distance (LBD) Score: The LBD score \([6]\) of a learned clause \( c \) is the number of distinct decision levels in \( c \). If \( \text{LBD}(c) = n \), then \( c \) contains \( n \) propagation blocks, where each block has been propagated within the same decision level. Intuitively, variables in a block are closely related. Learned clauses with lower LBD score tend to have higher quality.

Glue Clauses Glue clauses \([6]\) have LBD score of 2 and are the most important type of learned clauses. A glue clause connects a literal from the current decision

\(^1\) If \( b \) is too far from the current decision level, then performing chronological backtracking results in better solving efficiency \([23]\). Most of the leading CDCL solvers employ a combination of chronological and non-chronological backtracking.
level with a block of literals assigned in a previous decision level. Glue clauses have the strongest potential to reduce the search space quickly.

**Glue to Learned (G2L).** This measure represents the fraction of learned clauses that are glue clauses \[10\]. It is defined as \( \frac{g}{c} \), if there are \( g \) glue clauses among \( c \) learned clauses.

### 2.2 Notation

We denote a CDCL solver \( \psi \) running a given SAT instance \( F \) by \( \psi_F \). Assume that this run makes \( d \) decisions and generates \( c \) conflicts.

**sc and mc Decisions** A sc decision generates exactly one conflict and learns a sc clause, while a mc decision generates more than one conflict and accordingly learns multiple mc clauses. Let \( \psi_F \) takes \( s \) sc decisions and \( m \) mc decisions, learning \( c_s \) and \( c_m \) clauses respectively. Then \( d = m + s \) and \( c = c_s + c_m \).

**Burst of mc Decisions** We define the burst of a mc decision as the number of conflicts (i.e., learned clauses) generated within that mc decision.

For \( \psi_F \), we define

- \( \text{avgBurst} \), the average burst over \( m \) mc decisions as \( \frac{c_m}{m} \)
- \( \text{maxBurst} \), the maximum burst among the bursts of \( m \) mc decisions.
- We define a mapping \( \text{count}_b : Z \mapsto Z \) which takes a burst \( b \geq 2 \) as input and outputs the number of mc decisions with burst \( b \).

**Learned Clause Quality Over sc and mc Decisions** Let \( \text{lbd}(L) \) be the LBD score of a learned clause \( L \). For \( \psi_F \), we define

- the average LBD score \( \text{aLBD} \) over \( c \) learned clauses as \( \frac{\text{sumLBD}}{c} \), where \( \text{sumLBD} \) is the sum of LBD scores over \( c \) learned clauses.
- the average LBD score \( \text{aLBD}_{sc} \) over \( c_s \) sc clauses as \( \frac{\text{sumLBD}_{sc}}{c_s} \), where \( \text{sumLBD}_{sc} \) is the sum of LBD scores over \( c_s \) clauses.
- the average LBD score \( \text{aLBD}_{mc} \) over \( c_m \) mc clauses as \( \frac{\text{sumLBD}_{mc}}{c_m} \), where \( \text{sumLBD}_{mc} \) is the sum of LBD scores over \( c_m \) clauses.

For mc decision \( M \), we denote the minimum LBD score among its learned clauses by \( \text{minLBD}_M \). For \( \psi_F \), \( \text{avgminLBD}_{mc} \) is the average minimum LBD over \( m \) mc decisions.

### 2.3 Test Set, Experimental Setup and Solvers Used

All our experiments use the following setup: The test set consists of 400 benchmark instances from the main-track of SAT Competition-2020 (short SAT20) \[2\]. The timeout is 5,000 seconds per instance. Experiments were run on a Linux workstation with 64 Gigabytes of RAM, processor clock speed of 2.4GHZ, with L2 and L3 caches of size 256K and 20480K, respectively.

We use the following solvers for evaluation: in Section \[3\] we use the solver MapleLCMDiscChronoBT-DL-v3 \[3\] (short MplDL), the winner of SAT Race-2019, and in Section \[6\] we extend three leading CDCL SAT solvers: MplDL and the top two solvers in the main track of SAT Competition-2020, Kissat-sc2020-sat (Kissat-sat) and Kissat-sc2020-default (Kissat-default) \[2\].
3 An Empirical Analysis of sc and mc Decisions

In this section, we present our empirical study of conflict-generating decisions in CDCL search. We use MplDL as CDCL solver and investigate its sc and mc decisions.

Table 1. Conflict Generating Decisions. Columns A to G shows average measures over the number of instances shown in the Count column.

| Type    | Count | Conflict Frequency | Clause Quality | mc Bursts |
|---------|-------|--------------------|----------------|-----------|
|         |       | A: PDSC            | B: PDMC        | C: aLBD   | D: aLBD   | E: avgLBD | F: avgBurst | G: maxBurst |
| SAT     | 106    | 6%                 | 10%            | 22.32     | 32.30     | 18.90     | 2.69        | 33.76       |
| UNSAT   | 110    | 7%                 | 12%            | 236.26    | 389.68    | 80.80     | 2.70        | 52.37       |
| UNSOLVED| 184    | 9%                 | 16%            | 72.14     | 144.75    | 73.38     | 2.60        | 29.70       |
| Combined| 400    | 8%                 | 13%            | 80.68     | 104.07    | 60.86     | 2.65        | 94.51       |

3.1 Distributions of sc and mc decisions

We denote Percentage of Decisions with Single Conflict and Percentage of Decisions with Multiple Conflicts as PDSC and PDMC, respectively. Columns A and B in Table 1 show the average PDSC and PDMC values for the test instances, under SAT, UNSAT and UNSOLVED. Overall, 8% of all decisions are sc and 13% are mc (see the bottom row). Almost two thirds of all conflict producing decisions are mc. On average, about 21% (8%+13%) of the decisions are conflict producing.

This means that on average 79% of all decisions do not create any conflict. However, since the mc decisions produce 2.65 (Column F) conflicts on average, this results in the generation of almost 1 conflict per 2 decisions, on average, which is reflected in the average GLR value of 0.49 for these instances.

3.2 Learned Clause Quality in sc and mc Decisions

Columns C and D in Table 1 compare LBD scores when averaged over sc and mc decisions. On average, sc decisions generate higher quality learned clauses (with lower LBD scores). However, Column E shows that in most cases, the minimum LBD score over the clauses in a single mc decision is lower on average than for sc. The exception is the UNSOLVED category. Fig. 1 shows per-instance details of these three measures in log scale. In almost all instances, LBD scores for mc (blue) are higher than for sc (orange), and minimum mc LBD (green) is lowest.

To summarize, on average mc decisions are conflict-inefficient compared to sc decisions. However, on average the best quality learned clause from a mc decision has better quality than the quality of a sc clause.

3.3 Bursts of mc Decisions

Column F in Table 1 shows the average value of avgBurst for the test set. On average, the burst of mc decisions are quite small, about 2.65. However, as
shown in column G, the average value of \textbf{maxBurst} is very high. The left plot in Fig. 2 compares these values for each test instance in log scale. In almost all cases \textbf{maxBurst} (orange) is much larger than the average (blue). This indicates that while large bursts of \textbf{mc} decisions occur, they are rare, as indicated by the average of 2.65. To analyze this in detail, we count the number of \textbf{mc} decisions for each \textbf{burst} size from 2 to 10.

\textbf{Distribution of mc Decisions by Burst Size} Column G of Table 1 illustrates that \textbf{maxBurst} can be very large. To simplify our quantitative analysis we focus
4 Clause Learning in mc Decisions

In this section, we establish a structural property of the learned clauses in mc decisions.

**Formalization of mc Decisions** Let $v$ be the decision variable for the mc decision $M$ with burst $x \geq 2$. At the time when the search reaches the first conflict $C_1$ in $M$, let $P_0$ be the set of literal assignments that follows from the assignment of $v$. With $1 \leq i \leq x$, let $C_i$ be the $i^{th}$ conflicting clause, from which the clause $L_i = R_i \lor \{\neg f_i\}$ is learned. Here, $R_i$ is the reason clause and $f_i$ is the fUIP literal for this $i^{th}$ conflict. After learning $L_i$, and after backtracking, $\neg f_i$ is the only unassigned literal in $L_i$, and it is immediately unit-propagated from $L_i$. Let $P_i$ be the propagation block that contains literal assignments starting from the assignment of $\neg f_i$ until the search reaches the conflicting clause $C_{i+1}$. Let $L = (L_1, \ldots, L_x)$ be the ordered sequence of $x$ learned clauses in $M$.

![Diagram of clause learning](image)

**Claim 1:** $M$ learns a sequence of clauses $L = (L_1, \ldots, L_x)$, where a clause $L_i$ ($1 \leq i < x$) implicitly constructs $L_{i+1}$, by implying $f_{i+1}$, the fUIP literal for the $(i+1)^{th}$ conflict, from which $L_{i+1}$ is learned.
We justify Claim 1 as follows:

With $1 \leq i < \chi$, after learning the $i^{th}$ clause $L_i = R_i \lor \neg f_i$ and backtracking to a previous level, the literal $\neg f_i$ (the negated literal of the fUIP of $i^{th}$ conflict) is forced in $L_i$. This forced assignment creates a propagation block $P_i$ and reaches the conflicting clause $C_{i+1}$. From $C_{i+1}$ the search learns the next clause $L_{i+1} = R_{i+1} \lor \neg f_{i+1}$ within the current $mc$ decision. Clearly, the fUIP of $(i+1)^{th}$ conflict, $f_{i+1} \in P_i$, as $\neg f_{i+1} \in L_{i+1}$ is the only literal assigned in the current decision level. Fig. 3 shows the connection between $L_i$ and $L_{i+1}$.

We have $(a)(L_i = R_i \lor \neg f_i) \rightarrow f_{i+1}$, $(b)f_{i+1} \in P_i$, and $(c)\neg f_{i+1} \in L_{i+1}$.

Hence, under the current partial assignment, the learning of $L_i$ is a sufficient condition for the learning of $L_{i+1}$. Any pair of consecutive clauses ($L_i, L_{i+1}$) are connected via the pair of assignments ($\neg f_i, f_{i+1}$), where the first assignment in this pair is the negated literal of the fUIP literal for the $i^{th}$ conflict and the second assignment is the fUIP literal for the $(i+1)^{th}$ conflict.

Since the argument applies to all $1 \leq i < \chi$, we have the desired result.

5 Proximity between Conflicts Sequences in CDCL

By Claim 1 we see that learned clauses in a $mc$ decision are connected. This indicates that conflicts in a $mc$ decision are also related, as clauses are learned from conflicts. Here, we first introduce the measure of ConflictsProximity to study proximity between conflict sequences and then present an empirical study to reveal insights on proximity between conflicts sequences in CDCL.

5.1 Conflicts Proximity

The notion of conflicts proximity uses a novel measure called Literal Block Proximity, which measures the commonality of literal blocks between a sequence of reason clauses over a sequence of conflicts.

**Literal Block Proximity** Assume that from a conflicting clause $C$, $L = R \lor \neg f$ is learned, where $R$ is the reason clause for the conflict at $C$ and $f$ is the fUIP literal of current conflict. We define a mapping

$$D : \text{Clause} \rightarrow \{d_1 \ldots d_n\}$$

which maps a given reason clause $R$ to the set of distinct decision levels in $R$. Each $d_i \in D(R)$ corresponds to the block of literals $\text{block}_{d_i}$ in $R$ which were assigned in $d_i$.

Let $\mathcal{R}_C = (R_1, \ldots, R_m)$ be the sequence of reason clauses for the conflicting clauses in $C = (C_1, \ldots, C_m)$, where $R_i \in \mathcal{R}$ is the reason clause for the conflict at $C_i \in C$. We define the set Literal Block Proximity (LBP) for $\mathcal{R}_C$, $\text{LBP}_{\mathcal{R}_C}$ by

$$\text{LBP}_{\mathcal{R}_C} = D(R_1) \cap \cdots \cap D(R_m)$$

That is, $\text{LBP}_{\mathcal{R}_C}$ is the set of decision levels that are common in all clauses in $\mathcal{R}$. Therefore, the assignments in $\text{block}_{d_i}$ with $d_i \in \text{LBP}_{\mathcal{R}_C}$, contribute to the discovery of every conflicting clause in $C$. 
Example 1: Let $R_C = (R_a, R_b)$ be a set of reason clauses for the conflicts at clauses in $C = (C_a, C_b)$. Let $D(R_a) = \{2, 9, 14, 35, 110\}$ and $D(R_b) = \{9, 10, 11, 35, 98, 110\}$ be the sets of decision levels in $R_a$ and $R_b$, respectively. Then $LBP_{R_C} = D(R_a) \cap D(R_b) = \{9, 35, 110\}$. The assignments in $\text{block}_9$, $\text{block}_{35}$, and $\text{block}_{110}$ contribute to the generation of conflicts in both $C_a$ and $C_b$.

ConflictsProximity For a reason clause sequence $R_C = (R_1, \ldots, R_m)$, we define the ConflictsProximity $cp_{R_C}$, with $0 \leq cp_{R_C} \leq 1$ as

$$cp_{R_C} = \frac{|LBP_{R_C}|}{|U_{R_C}|}$$

where $U_{R_C} = D(R_1) \cup \cdots \cup D(R_m)$ is the set of all literal blocks in $R_C$. In Example 1, $cp_{R_C} = \frac{|LBP_{R_C}|}{|U_{R_C}|} = 3/8$.

Intuitively, for any two given reason clause sequences $R_{C_p}$ and $R_{C_q}$, with $|R_{C_p}| = |R_{C_q}|$, if $cp_{R_{C_p}} > cp_{R_{C_q}}$, then the conflicts associated with the reason clauses in $R_{C_p}$ are more closely related to each other than conflicts associated with the reason clauses in $R_{C_q}$. If $cp_{R_{C_p}} > cp_{R_{C_q}}$, then we call the conflicts generated over the clauses in $C_p$ more closely related than the conflicts generated over the clauses in $C_q$.

Example 2: Let $R_{C_p} = (R_{p1}, R_{p2})$ and $R_{C_q} = (R_{q1}, R_{q2})$ be two sequences of reason clauses for conflicts generated at the conflicting clauses in $C_p = (C_{p1}, C_{p2})$ and $C_q = (C_{q1}, C_{q2})$, respectively. Let $cp_{R_{C_p}} = 0.65$ and $cp_{R_{C_q}} = 0.35$. Then conflicts generated over the conflicting clauses in $C_p$ are more closely related than conflicts generated over the conflicting clauses in $C_q$.

We now study proximity of conflicts in CDCL under ConflictsProximity.

5.2 Proximity of Conflicts over sc and mc Decisions

While the learned clauses in a mc decision are connected, each learned clause in a sc decision is learned in isolation. Based on this observation, we propose the following hypothesis:

Hypothesis 1: On average, conflicts in a mc decision with burst $x$ are more closely related than conflicts which are generated in the last $x$ sc decisions.

We support this hypothesis by comparing the ConflictsProximity of reason clauses for conflicts in sc and mc decisions. We performed an experiment with 400 instances from SAT20 with MplDL with a time-limit of 5,000 seconds. For each run of an instance, whenever the search finds a mc decision with burst $x$, we compute the LBP and ConflictsProximity for the reason clauses (i) for these $x$ conflicts and (ii) for the last $x$ conflicts in sc decisions. For this experiment, we collect data for bursts $x \leq 10$. For a run with an instance, we compute the average of ConflictsProximity of reason clauses separately over mc and sc decisions.

Fig. 4 shows that average ConflictsProximity for the reason clauses over mc decisions (blue lines, average is 0.43) are higher than ConflictsProximity
of reason clauses over sc decisions (orange line, average is 0.34) for almost all instances. This validates our hypothesis that conflicts over mc decisions are more closely related than conflicts over sc decisions.

6 The Common Reason Variable Reduction Strategy

6.1 Common Reason Decision Variables

Assume that a mc decision $M$ finds $x \geq 2$ consecutive conflicts within its decision. Let $\mathcal{R} = (R_1, \ldots, R_x)$ be the sequence of reason clauses for these $x$ conflicts. $\text{LBP}_\mathcal{R}$ is the set of common decision levels over $\mathcal{R}$. For each decision level $d_1 \in \text{LBP}_\mathcal{R}$, we call $d_1$, a common reason decision level and the decision variable $v$ at $d_1$, a common reason decision variable (CRV) for $M$. If $|\text{LBP}_\mathcal{R}| = z$, then there are $z$ CRVs in $M$. The CRVs in $M$ are the decision variables from previous decision levels, which contributed to the generation of all the conflicts in $M$.

6.2 Poor mc Decisions

Recall that in Section 3 (Fig.1), we observed that on average, mc decisions (blue line) produce lower quality clauses than sc decisions (orange line). However, the best quality clause (green line) in a mc decision has better average quality than other learned clauses. However, in a poor mc decision its best quality learned clause is worse than the average quality. A mc decision $M$ is poor,

- if the quality of the best learned clause in $M$ is lower than a dynamically computed threshold $\theta$, the average quality of the last $k$ learned clauses.
6.3 The CRVR Decision Strategy

We summarize the previous two subsections as follows:

- Conflicts in a poor mc decision are not likely to be helpful, as the quality of its best learned clause is lower than the recent search average.
- The CRVs in a poor mc decision combinedly contribute to the generation of these conflicts.

Does suppression of such CRVs for the future decisions help the search to achieve better efficiency? We address this question by designing a decision strategy named common reason variable score reduction (CRVR), which can be integrated with any activity based variable selection decision heuristics, such as VSIDS and LRB. The high-level idea of CRVR is as follows: Once a poor mc decision is detected, CRVR (i) finds the CRVs for that poor mc and (ii) marks those CRVs as poor CRVs, and (iii) then reduces the activity scores of those poor CRVs for future decisions. CRVR consists of the two procedures DetectPoorCRV and CRVRBranching, whose pseudo-codes are shown in Table 2.

**DetectPoorCRV** This procedure is invoked at the end of an mc decision \( M \) and just before the next decision. It computes a dynamic conflict quality threshold \( \theta \), the average LBD score of last \( k \) learned clauses. Then it determines if \( M \) is poor by comparing \( \min_{\text{LBD}_M} \) with \( \theta \). In this case, DetectPoorCRV obtains the sequence of reason clauses \( R \) in \( M \) and computes \( \text{LBP}_R \). For each decision level \( d_1 \in \text{LBP}_R \), any decision variable \( v \) at \( d_1 \) is marked as poor.

**CRVRBranching** is shown in the right side of Table 2. This procedure modifies a typical CDCL decision routine to lazily reduce the activity score of poor CRVs. It employs a while loop until a variable is selected, where in each iteration of the loop, it performs the following operations: (i) obtains a free variable \( y \), where \( y \) is the free variable with largest activity score. (ii) checks if \( y \) is marked as poor. If \( y \) is poor, then it computes a fraction of \( \text{activity}[y] \), the current activity score of \( y \), by multiplying it with \((1-Q)\), where \( Q \) is a user defined parameter with \( 0 < Q < 1 \). This fraction, which is lower than \( \text{activity}[y] \), becomes the
new activity score of $y$. (iii) it unmarks $y$ as poor and performs a reordering of the variables, which reorders the variables by their activity scores. The reduction of the activity score of $y$, followed by reordering, decreases the selection priority of $y$.

7 Experimental Evaluation

7.1 Implementation

We implemented CRVR in three state-of-the-art baseline solvers MplDL, Kissat-sat and Kissat-default. We call the extended solvers MplDL$^{crvr}$, Kissat-sat$^{crvr}$, and Kissat-default$^{crvr}$, respectively. The solver MplDL employs a combination of the decision heuristics DIST [29], VSIDS [22] and LRB [18], which are activated at different phases of the search, whereas Kissat-sat and Kissat-default use VSIDS and Variable Move to Front (VMTF) [20] alternately during the search.

The heuristics DIST, VSIDS and LRB share similar computational structures. All three heuristics maintain an activity score for each variable. Whenever a variable involves in a conflict, its activity score is increased. In contrast, VMTF maintains a queue of variables, where a subset of variables appearing in a learned clause are moved to the front of that queue in an arbitrary order.

CRVR is designed to be employed on top of activity-based decision heuristics. Hence in Kissat-sat$^{crvr}$ and Kissat-default$^{crvr}$, we employ CRVR only when VSIDS is active.

In all of our extended solvers, we use the following parameter values: a length of window of recent conflicts $k = 50$ and an activity score reduction factor $Q = 0.1$. Source code of our CRVR extensions are available at [4].

Table 3. Comparison between 3 baselines and their CRVR extensions

| Systems         | SAT | UNSAT | Combined | PAR-2 |
|-----------------|-----|-------|----------|-------|
| MplDL           | 106 | 110   | 216      | 2065  |
| MplDL$^{crvr}$  | 116 (+10) | 107 (-3) | 223 (+7) | 2001  |
| Kissat-sat      | 148 | 118   | 266      | 1552  |
| Kissat-sat$^{crvr}$ | 150 (+2) | 114 (-4) | 264 (-2) | 1565  |
| Kissat-default  | 134 | 126   | 260      | 1624  |
| Kissat-default$^{crvr}$ | 139 (+5) | 125 (-1) | 264 (+4) | 1588  |
7.2 Experiments and Results

We conduct experiments with the same set of 400 instances with a 5,000 seconds timeout per instance. We compare the CRVR extensions and their counterpart baselines in terms of number of solved instances, solving time and PAR-2 score.

Table 3 compares MplDL\text{crvr}, Kissat-sat\text{crvr}, and Kissat-default\text{crvr} with their baselines. All of these extensions show performance gains on SAT instances, but lose on UNSAT instances. The strongest gain is for MplDL\text{crvr}, which solves 10 additional SAT instances, but solves 3 less UNSAT instances, achieves an overall gain of 7 instances compared to its baseline. Kissat-sat\text{crvr} solves 2 more SAT instances, but loose 4 UNSAT instances, with an overall loss of 2 instances compared to its baseline. The third extension Kissat-default solves 5 more SAT instances, but solves 1 less UNSAT instance than its baseline. Overall, Kissat-default\text{crvr} solves 4 more instances than Kissat-default. The PAR-2

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{Solve time comparisons: For any point above 0 in the vertical axis, our extensions solve more instances than their baselines at the time point in the horizontal axis.}
\end{figure}

results are consistent with the solution count results. While Kissat-sat\text{crvr} has a slight increase (a 0.8\% increase) in its PAR-2 score compared to Kissat-sat, both MplDL\text{crvr} and Kissat-default\text{crvr} have significantly lower PAR-2 scores (3.19\% and 2.28\% of reductions, respectively) compared to their baselines, which reflects overall better performance of these two systems compared to their baselines.

Fig. 5 compares the relative solving speed of MplDL\text{crvr} (blue), Kissat-sat\text{crvr} (orange), and Kissat-default\text{crvr} (green) against their baselines by plotting the

\begin{footnote}
PAR-2 score is defined as the sum of all runtimes for solved instances + 2*timeout for unsolved instances \[1\]. Lower scores are better.
\end{footnote}
difference in the number of instances solved as a function of time. If that difference is above 0, then it indicates that an extended solver solves more instances than the baseline at this time point.

\texttt{MplDL} (blue) performs slightly worse than \texttt{MplDL} early on, but beats the baseline consistently after 1,900 seconds. Compared to \texttt{Kissat-default}, \texttt{Kissat-default} (green) is ahead of \texttt{Kissat-default} at most time-points. \texttt{Kissat-sat} (orange) is behind \texttt{Kissat-sat} at most of the time-points.

Overall, compared to their baselines, our extensions perform better on SAT instances, but lose a small number of UNSAT instances.

8 Detailed Performance Analysis of CRVR

For a run of a given solver, the metric GLR measures the overall conflict generation rate of the search, average LBD (aLBD) measures the average quality of the learned clauses and G2L measures the fraction of learned clauses which are glue. All these measures correlate well with solving efficiency \cite{10,19}. Here, we present an analysis that relates the performance of CRVR with these three metrics. We consider two subsets of instances, where \texttt{MplDL} and \texttt{MplDL} show opposite strengths:

- CRVR−bad: 12 instances which are solved by \texttt{MplDL}, but not by \texttt{MplDL}.
- CRVR−good: 19 instances which are solved by \texttt{MplDL}, but not by \texttt{MplDL}.

| Instance Sets | Count (SAT+UNSAT) | average GLR | average aLBD | average G2L |
|---------------|------------------|-------------|--------------|-------------|
| CRVR−good     | 19 (18+1)        | 0.58        | 53           | 499.91      | 147.69      | 0.003 | 0.018 |
| CRVR−bad      | 12 (8+4)         | 0.56        | 0.56         | 18.83       | 18.53       | 0.024 | 0.023 |

The 19 instances in CRVR−good (first row of Table 4) are solved exclusively by \texttt{MplDL}. For this subset, \texttt{MplDL} learns clauses at a slightly lower rate. However, for CRVR−good, the average aLBD (resp. average G2L) is significantly lower (resp. higher) with \texttt{MplDL}. CRVR helps to (i) learn higher quality clauses, and (ii) learn more glue clauses relative to the number of clauses for the subset of instances in CRVR−good, for which \texttt{MplDL} is efficient.

18 of the 19 instances in CRVR−good are SAT. The learning of significantly better quality of clauses with \texttt{MplDL} for these SAT instances may just explain the good performance of CRVR on SAT instances.

For the 12 instances in CRVR−bad (second row of Table 4), \texttt{MplDL} learns clauses at the same rate, but learns clauses which are of slightly lower quality than the clauses learned by \texttt{MplDL}. However, for this set, the average G2L value is slightly higher with \texttt{MplDL} than \texttt{MplDL}. This could explain the better performance of \texttt{MplDL} for this subset.
9 Related Work

Audemard and Simon [5] briefly studied decisions with successive conflicts, which we refer to as mc decisions in this paper. They studied number of successive conflicts in the CDCL solver Glucose on a fixed set of instances. In the current paper, we present a more formal and in-depth study of mc decisions. The authors of [19] relate conflict generation propensity and learned clause quality with the efficiency of several decision heuristics. In contrast, we study and compare the conflict quality of two types of conflict producing decisions for CDCL. Conflicts generation pattern in CDCL is studied in [9] showing that CDCL typically alternates between bursts and depression phases of conflict generation. While that work presented an in-depth study of the conflict depression phases in CDCL, here we study the conflict bursts phases, which are opposite of conflict depression phases. Chowdhury et al. [10] studied conflict efficiency of decisions with two types of variables: those that appear in the glue clauses and those that do not. In the current paper, we compare the conflict efficiency of conflict producing decisions.

10 Conclusions and Future Work

We present a characterization of sc and mc decisions in terms of average learned clause quality that each type produces. Then we analyze how mc decisions with different bursts are distributed in CDCL search. Our theoretical analysis shows that learned clauses in a mc are connected, indicating that conflicts that occur in a mc decision are related to each other. We introduced a measure named ConflictsProximity that enables the study of proximity of conflicts in a given sequence of conflicts. Our empirical analysis shows that conflicts in mc decisions are more closely related than conflicts in sc decisions. Finally, we formulated a novel CDCL strategy CRVR that reduces the activity score of some variables that appear in the clauses learned over mc decisions. Our empirical evaluation with three modern CDCL SAT solvers shows the effectiveness of CRVR for the SAT instances from SAT20.

In the future, we intend to pursue the following research questions:

– Kissat solvers and their predecessors, such as CaDiCaL, employ VMTF as one of their decision heuristics. How to extend VMTF with CRVR is an interesting question that we plan to pursue in future.
– Currently, the user defined parameters $k$ and $Q$ in CRVR are set to fixed values. How to adapt them dynamically during the search? We hope that a dynamic strategy to adapt these parameters will improve the performance of CRVR, specially over the UNSAT instances.

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