Abstract

We theoretically address squeezed light generation through the spontaneous breaking of the rotational invariance occurring in a type I degenerate optical parametric oscillator (DOPO) pumped above threshold. We show that a DOPO with spherical mirrors, in which the signal and idler fields correspond to first order Laguerre-Gauss modes, produces a perfectly squeezed vacuum with the shape of a Hermite-Gauss mode, within the linearized theory. This occurs at any pumping level above threshold, hence the phenomenon is non-critical. Imperfections of the rotational symmetry, due e.g. to cavity anisotropy, are shown to have a small impact, hence the result is not singular.

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Introduction.– Squeezed light is a central tool in several sophisticated applications of physics, high-precision measurements [1] and quantum information with continuous variables [2] being probably the most outstanding. The quality of squeezing, i.e. how less noisy light is as compared with vacuum (which sets the so-called shot noise level) is a main concern for those applications as any fluctuation level limits their performance. Improving the quality and reliability of squeezing is thus an important goal.

The paradigmatic squeezing process is singlemode quadrature squeezing of an optical field via degenerate parametric down conversion [3,4], a nonlinear process that converts a pump photon of frequency $2\omega_0$ into two photons of frequency $\omega_0$. Although in this process perfect squeezing is achieved only when the pump power goes to infinity, there is a well known technique for increasing the squeezing level that consists in confining the non-linear interaction inside an optical cavity, in which case one deals with a degenerate optical parametric oscillator (DOPO). In DOPOs squeezing is ideally obtained at the oscillation threshold [3]: A critical phenomenon. DOPOs are nowadays customarily utilized as sources of squeezed light, reaching noise reductions as large as 10dB below the shot noise level (90% of squeezing) [3,4].

Recently an alternative way for producing squeezed light was proposed by some of us [5], based on the exploitation of the spontaneous translational symmetry breaking occurring in a broad area, planar DOPO model. Such system supports cavity solitons (CSs) –among other dissipative structures forming across its transverse plane–, which are bright spots surrounded by darkness that can be placed at any point in space, breaking the translational symmetry. A study of their quantum fluctuations [5] reveals that (i) the CS position diffuses because of quantum noise, and (ii) a special transverse mode, namely the $\pm 1$ phase shifted gradient of the CS (its linear momentum), is perfectly squeezed at low fluctuation frequencies, irrespective of the system’s proximity to threshold. This is reminiscent of a Heisenberg uncertainty relation, with the additional and compatible feature that the full indetermination of the CS position in the long time limit is accompanied by the perfect determination (perfect squeezing) of its momentum at low frequencies, like a canonical pair in a minimum uncertainty state. A main limitation of this result is that CSs have not been observed so far in DOPOs. This is not a fundamental limitation but obviously reduces its practical interest. Nevertheless it paves the way (non-critical squeezing via a spontaneous spatial symmetry breaking) to other extensions, like the one we consider here: The spontaneous rotational symmetry breaking of a DOPO, which can be implemented with current technology. We hope that experiments based on this new phenomenon will successfully generate high-quality non-critically squeezed light.

Rotational invariance and squeezing: General description.– Consider a type I DOPO (i.e., with signal and idler photons degenerated both in frequency and polarization) with spherical mirrors and pumped by a resonant, coherent optical field of frequency $2\omega_0$ and Gaussian transverse profile (i.e., pumped with zero orbital angular momentum photons). Such a configuration is invariant under rotations around the cavity axis ($z$-axis). The cavity is assumed to be tuned to the first transverse mode family at the subharmonic frequency $\omega_0$, so the signal/idler field is a superposition of two Laguerre-Gauss (L-G) modes, $L_{+1}(r)$ and $L_{-1}(r)$, which have opposite orbital angular momenta. Any other transverse mode is assumed to be detuned far enough from $\omega_0$. Inside this cavity a $\chi^{(2)}$ crystal down converts pump photons into signal/idler photon pairs, and vice versa. These photons are degenerate in frequency because of energy conservation and, because of orbital angular momentum conservation, each photon pair must comprise one $L_{+1}$ photon plus one $L_{-1}$ photon. Hence the number of $L_{+1}$ photons and $L_{-1}$ photons should be sensibly equal and highly correlated. Another way of looking at this process follows from noticing that the simultaneous emission of a $L_{+1}$ photon and a $L_{-1}$ photon corresponds to the emission of two photons in a first order Hermite-Gauss (H-G), or TEM_{10}, mode, which breaks the rotational symmetry of the system. The orientation of such mode in the transverse plane, measured by the angle $\theta$ in Fig. 1, is determined by the
relative phase between the two subjacent L-G modes. The rotational invariance of the system implies however that \( \theta \) is arbitrary and one expects that quantum fluctuations will induce a random rotation of the TEM\(_{10} \) mode around the cavity axis. In loose terms this means an “indeterminacy” in the value of \( \theta \) that, generalizing Ref. [3], should be accompanied by a reduction of fluctuations in the canonically conjugated variable, the orbital angular momentum, associated to the operator \(-i\partial/\partial\theta\).

We note that the angular gradient of the TEM\(_{10} \) mode is another H-G mode, spatially crossed with respect to it, call it TEM\(_{01} \) mode. Hence a balanced homodyne detection that uses as a local oscillator a \( \pi/2 \) phase shifted TEM\(_{01} \) mode should yield perfect squeezing at zero noise level above threshold. This is the basic idea of squeezing generation via symmetry breaking of the rotational invariance in DOPO, and below we analytically demonstrate that this is what actually occurs [3].

\[
\begin{align*}
2\omega_0 & \quad \text{pump} & \quad \chi^{(2)} & \quad \omega_0 \\
\begin{array}{c}
\text{signal} \\
\text{y} \\
\text{x}
\end{array}
\end{align*}
\]

FIG. 1: Scheme of the DOPO pumped by a Gaussian beam and tuned to the first transverse mode family at the subharmonic. The orientation \( \theta \) is arbitrary.

The model.– Inside the cavity there are three relevant modes: the pumped Gaussian mode at frequency \( 2\omega_0 \), and two L-G modes at the subharmonic frequency \( \omega_0 \). The electric field at the cavity waist (where the nonlinear crystal is assumed to be located) can be written as

\[
\hat{E}(r,t) = i\mathcal{F}_p \hat{A}_p(r,t) e^{-i2\omega_0 t} + i\mathcal{F}_s \hat{A}_s(r,t) e^{-i\omega_0 t} + \text{H.c.}
\tag{1}
\]

where \( \mathcal{F}_p = \sqrt{\hbar \omega_0/2\pi} \) and \( \mathcal{F}_s = \mathcal{F}_p/\sqrt{2} \). \( L \) is the effective cavity length, \( n \) is the crystal refractive index,

\[
\begin{align*}
\hat{A}_p(r,t) &= \hat{a}_0(t) G(r), \\
\hat{A}_s(r,t) &= \hat{a}_{\pm 1}(r) L_{\pm 1}(r) + \hat{a}_0(t) L_{\pm 1}(r),
\end{align*}
\tag{2a, b}
\]

are slowly varying envelopes, and \( \hat{a}_m(t) \) and \( \hat{a}_m^\dagger(t) \) are the interaction picture boson operators for each mode \( (m = 0, \pm 1) \) obeying \( [\hat{a}_m(t), \hat{a}_m^\dagger(t)] = \delta_{mm} \). The Gauss, \( G(r) \), and L-G, \( L_{\pm 1}(r) \), mode envelopes are given by

\[
G(r) = \sqrt{2\pi^{-1/2}} r^{-1/2} e^{-r^2/2\sigma^2} \quad \text{and} \quad L_{\pm 1}(r) = \pi^{-1/2} r e^{-r^2/2\sigma^2} e^{\pm i\phi} \quad r \quad \text{and} \quad \phi \quad \text{are \ the \ polar \ coordinates \ in \ the \ transverse \ plane, \ and} \quad w \quad (\sqrt{2w}) \quad \text{is \ the \ beam \ radius \ of \ the \ pump \ (signal) \ beam \ at \ its \ waist.}
\]

The interaction Hamiltonian describing pumping and the nonlinear mixing processes occurring at the nonlinear crystal reads \( \hat{H} = i\hbar \left( \mathcal{E}_p \hat{a}_0^\dagger + \mathcal{E}_s \hat{a}_{\pm 1}^\dagger \hat{a}_0 + \text{H.c.} \right) \), where \( \mathcal{E}_p \) is the amplitude of the external coherent pump, real without loss of generality, and \( \chi \) is the nonlinear coupling constant [11]. Assuming that pump and signal modes are damped at rates \( \gamma_p \) and \( \gamma_s \), respectively, losses occurring at only one cavity mirror (which needs not be the same for pump and signal), one can write down Langevin equations using well known techniques of quantum optics of open systems. We notice however that our Hamiltonian is formally equivalent to that for the nondegenerate OPO and use the corresponding Langevin equations in the positive \( P \) representation as given in [12]. This representation sets a correspondence between operators \( \{ \hat{a}_m(t), \hat{a}_m^\dagger(t) \} \) and independent c-number stochastic variables \( \{ a_m(t), a_m^\dagger(t) \} \) respectively, so that any stochastic average equals the corresponding normally ordered quantum expectation value. We further simplify the problem by considering the limit \( \gamma_p \gg \gamma_s \), where the pump variables can be adiabatically eliminated as \( a_0 = (\mathcal{E}_p - \chi a_{\pm 1} a_{\pm 1}^\dagger)/\gamma_p \) and \( a_0^\dagger = (\mathcal{E}_p - \chi a_{\pm 1} a_{\pm 1}^\dagger)/\gamma_p \), arriving at our model equations:

\[
\dot{a}_i = \gamma_s (-\alpha_i + \sigma a_j^\pm - 2^{\sigma} a_j^\pm a_j a_i) + \sqrt{\alpha_0 \gamma_s}, \tag{3}
\]

the overdot meaning \( d/dt \), \( i, j = \pm 1 \) (\( i \neq j \)), and the dimensionless parameters

\[
\sigma = \mathcal{E}_p \chi/\gamma_p \gamma_s, \quad g = \chi/\gamma_p \gamma_s, \tag{4}
\]

\( \sigma^2 \) being proportional to the external pump power. Another equation for \( \dot{a}_+^\dagger \) exists that reads \( \dot{a}_-^\dagger = (\dot{a}_+)^\dagger \), where the operation \( "^\dagger " \) acts as a “hermitian-conjugation”: complex numbers get complex-conjugated and any stochastic variable \( v \) is transformed according to \( v \leftrightarrow v^\dagger \). In Eqs. [3], \( \xi_{\pm 1} = \xi_{\pm 1}^\dagger \equiv \xi \), \( \xi_{\pm 1} = (\xi_{\pm 1}^\dagger)\ast \equiv \xi^\dagger \), and \( (\xi, \xi^\dagger) \) are two independent complex noise sources with zero mean and non zero correlations \( \langle \xi(t_1) \xi^\dagger(t_2) \rangle = \langle \xi^\dagger(t_1) [\xi^\dagger(t_2)]^\dagger \rangle = \delta(t_1 - t_2) \).

Classical steady emission.– The classical DOPO dynamical equations are obtained by setting \( \alpha_{\pm 1} = \alpha_{\pm 1}^\dagger \) and ignoring noise terms in Eqs. [3]. Above threshold \( (\sigma > 1) \) the only stable steady state reads

\[
\dot{a}_{\pm 1} = \rho \exp(\mp i\theta), \quad \rho^2 = g^{-2}(\sigma - 1) \tag{5}
\]

with \( \theta \) an arbitrary phase. The corresponding classical slowly varying envelope is obtained from Eq. [2a] after the replacement \( \{ \hat{a}_m, \hat{a}_m^\dagger \} \rightarrow \{ a_m, a_m^\dagger \} \) and reads

\[
A_{\pm 1}^c(r) = \left( 2\pi^{-1/2} w^{-2} \rho \right) r \cos(\phi - \theta) e^{-r^2/2w^2} \tag{6}
\]
which is a first order H-G mode rotated by \( \theta \) with respect to the transverse \( x \) axis, see Fig. 1. The arbitrariness of \( \theta \) reflects the rotational invariance of the problem.

Quantum fluctuations.— The dynamics of quantum fluctuations around the classical solution are studied by writing \( \alpha_{\pm 1} = \alpha_{\pm 1} + \delta \alpha_{\pm 1} \), and deriving evolution equations for the fluctuations \( \delta \alpha \). As \( |\alpha_{\pm 1}|^2 \gg 1 \) (these quantities give the classical number of signal photons in each mode, which are very large above threshold) we assume that \( |\delta \alpha_{\pm 1}|, |\delta \alpha_{\pm 1}'| \ll |\alpha_{\pm 1}| \) and linearize Eqs. \( \text{[3]} \). We find it convenient to write the fluctuations as

\[
\alpha_{\pm 1} = [\rho + b_{\pm 1}(t)] e^{\mp i\theta(t)},
\]

with \( b_{\pm 1} \) (and \( b_{\pm 1}' \)) c-number stochastic variables accounting for quantum fluctuations. Note that angle \( \theta \) is let to vary with time as, owed to rotational invariance, it is an undamped quantity that is driven by quantum noise, as we show below. Inserting \( \text{[7]} \) into \( \text{[3]} \) and linearizing one easily gets the linearized Langevin equations

\[
\dot{\mathbf{b}} - 2i\rho \mathbf{w}_0 \theta = \mathbf{L} \mathbf{b} + \sqrt{\gamma_s} \mathbf{\xi}(t),
\]

\[
\mathbf{b} = \text{col} (b_{1,1}, b_{2,1}, b_{1,-1}, b_{2,-1}), \quad \mathbf{w}_0 = \frac{i}{2} \text{col} (1, -1, -1, 1),
\]

\[
\mathbf{\xi} = \text{col} (\xi^+, \xi^-, \xi^r, [\xi^+]^r),
\]

and the real and symmetric matrix \( \mathbf{L} \) reads

\[
\mathbf{L} = -\gamma_s \begin{pmatrix}
\sigma & 0 & \sigma - 1 & -1 \\
0 & \sigma & -1 & \sigma - 1 \\
\sigma - 1 & -1 & \sigma & 0 \\
-1 & \sigma - 1 & 0 & \sigma
\end{pmatrix}.
\]

The eigensystem of \( \mathbf{L} \) consists of the Goldstone mode \( \mathbf{w}_0 \), whose null eigenvalue reflects the rotational invariance of the system, of vector \( \mathbf{w}_1 = \frac{1}{2} \text{col} (-1, -1, 1, 1) \), with eigenvalue \(-2\gamma_s\), and of two other eigenvectors that are unimportant for our present purposes.

Angular diffusion of the classical field.— In order to catch the dynamics of the pattern orientation angle \( \theta \), see Fig. 1, we project the linear system \( \text{[3]} \) onto the Goldstone mode \( \mathbf{w}_0 \) and obtain \( \text{[13]} \text{[14]} \)

\[
\dot{\theta} = \sqrt{D_{\theta}} \text{Im} (\xi^+ - \xi^-), \quad D_{\theta} = \frac{\chi^2}{4\gamma_p} (\sigma - 1)^2.
\]

Homodyne detection and squeezing spectrum.— In order to demonstrate that the signal field exiting the DOPO exhibits perfect squeezing (within the linear approach) in the empty H-G mode perpendicular to the macroscopically emitted one, we consider a balanced homodyne detection experiment, see e.g. \( \text{[10]} \). The noise spectrum \( V(\omega) \) of the intensity difference between the two output ports of the beam splitter, in which the signal field exiting the DOPO is mixed with a classical, coherent local oscillator (LO) of frequency \( \omega_0 \), is given by \( \text{[3]} \text{[10]} \)

\[
V(\omega) = 1 + 2\gamma_s \int_{-\infty}^{+\infty} dt \langle \delta \mathbf{E}(t) \delta \mathbf{E}(t + \tau) \rangle e^{-i\omega\tau},
\]

where the positive \( P \) representation is used to evaluate the stochastic average, \( \delta \mathbf{E}(t) = \mathbf{E}(t) - \langle \mathbf{E}(t) \rangle \) with

\[
\langle \mathbf{E}(t) \rangle = \mathcal{N}^{-1/2} \int d^2 \mathbf{r} (A_{s}^r A_{s} + A_{s} A_{s}^r),
\]

\[
\mathcal{N} = \int d^2 \mathbf{r} |A_{s}|^2,
\]

where \( A_{s} \) is an argument \( (r,t) \) should be understood in all fields. \( A_{s}(r,t) \) is the LO transverse envelope, \( A_{s}(r,t) = \sum_{j=1}^{\pm 1} \alpha_j(t) L_j(r) \) and \( A_{s}'(r,t) = \sum_{j=1}^{\pm 1} \alpha_j'(t) L_j'(r) \). When the output is coherent, \( V(\omega) = 1 \) for all \( \omega \), defining the shot noise level. On the other hand \( V(\omega_s) = 0 \) signals perfect squeezing (no noise) at \( \omega = \omega_s \) for the quadrature selected by the LO.

Following the Introduction we choose the LO transverse envelope to be \( A_{L}(r,t) \propto \frac{\partial}{\partial \theta} 4\lambda^4_s \mathbf{r} \), i.e.

\[
A_{L}(r,t) = \eta_L e^{i\psi_L} r \sin (\phi - \theta(t)) e^{-r^2/2\omega^2},
\]

with \( \eta_L \) a real amplitude and \( \psi_L \) the LO phase. This LO is a H-G mode orthogonal, at every time, to the macroscopically excited one, Eq. \( \text{[9]} \). Remind that sufficiently above threshold the diffusion of \( \theta \) is negligible and the matching of the LO to the analyzed mode should not represent any practical problem.

By using \( \text{[13]} \) one finds \( \delta \mathbf{E}(t) = \sqrt{\pi} \text{Im} (\psi_L) c_1(t) \), with \( c_1(t) = \mathbf{w}_1 \cdot \mathbf{b}(t) \). Projecting \( \text{[3]} \) onto \( \mathbf{w}_1 \) gives \( \text{[15]} \)

\[
\dot{c}_1 = -2\gamma_s c_1 - i\sqrt{\gamma_s} \text{Im} (\xi^+ + \xi^-),
\]

which allows the evaluation of the squeezing spectrum \( \text{[11]} \). The result reads

\[
V_{\psi_L}(\omega) = \frac{1 - \frac{\sin^2(\psi_L)}{1 + (\omega/2\gamma_s)^2}}{\left(1 + (\omega/2\gamma_s)^2\right)}.
\]

We note that for \( \psi_L = \pi/2 \) Eq. \( \text{[15]} \) coincides exactly with the squeezing spectrum of a usual DOPO at threshold \( \text{[3]} \), displaying perfect squeezing \( (V = 0) \) at \( \omega = 0 \). The phase quadrature of the TEM_{10} mode orthogonal to the TEM_{10} emitted by the DOPO is perfectly squeezed at zero noise frequency \( \text{i.e., perfect squeezing occurs at the optical frequency } \omega_0, \text{ see } \text{[11]} \). The remarkable difference with usual DOPOs is that the result here reported is independent of the system parameters \( \text{e.g., it is not sensitive} \)
to bifurcations): It is thus a non-critical phenomenon or, in other words, squeezing needs not be tuned.

Some extra comments are in order: (i) $V_{\psi_L}(\omega) \leq 1$ for any LO phase $\psi_L$, i.e., any quadrature exhibits noise reduction, but $\psi_L = 0$ for which $V_{\psi_L=0}(\omega) = 1$; (ii) $V_{\psi_L}(\omega) V_{\psi_L+\pi}(\omega) < 1$, i.e., the two quadratures measured by the LO for $\psi_L$ and $\psi_L + \frac{\pi}{2}$ are not a Heisenberg pair, what looks surprising but is understood by the fact that the detected mode (and thus the LO) is rotating randomly (11) and the discussion below after Eq. (17); (iii) The detected squeezing is very weakly dependent on $\psi_L$, e.g., phase uncertainties of $\psi_L$, around $\psi_L = \frac{\pi}{2}$, as huge as 15° lead to $V(\omega = 0) \simeq 0.067$ (more that 11dB of noise reduction), unlike conventional squeezers [3, 4].

Influence of imperfections.— A natural question is whether the above result is singular in the sense that deviations from perfect rotational invariance could destroy it. We address this issue by introducing different cavity losses, $\gamma_x$ and $\gamma_y$, along two orthogonal transverse directions. The corresponding Langevin equations read

$$\dot{\alpha}_i = \gamma_x(-\alpha_i + \kappa \alpha_j + \sigma \alpha_j^+ - g^2 \alpha_j^+ \alpha_j \alpha_i) + \sqrt{2\gamma_x} \xi_i, \quad (16)$$

where noises have been already written in the linear approximation to be used, $\gamma_x = 2\kappa^2/\gamma_\pi \tau_\gamma$, $\kappa = 2\kappa^2/\gamma_\pi \tau_\gamma$ measure how much the rotational symmetry is externally broken, and $\gamma_x > \gamma_y$ for definiteness ($0 < \kappa < 1$). Above threshold ($\sigma > 1 - \kappa$) the classical steady state is $\alpha_{x+1} = \alpha_{y-1} = g^{-1} \sqrt{\sigma + 1}$, corresponding to a horizontal H-G mode [13], parallel to the direction of smaller losses. Use of Eq. (17) (now with $\theta = 0$) and considering a vertical H-G mode as a LO one gets

$$V_{\psi_L=\frac{\pi}{2}}(\omega) = 1 - \frac{(1 - \kappa^2)}{1 + (1 - \kappa^2)^2 (\sigma/\gamma_x)^2}, \quad (17)$$

independently of the pump level, which reduces to (15) for $\kappa = 0$ ($\gamma_y = \gamma_x$). On the other hand one can show easily that $V_{\psi_L=0}(\omega) V_{\psi_L=\frac{\pi}{2}}(\omega) = 1$, corresponding to a minimum uncertainty state, as is usual in DOPOs. This confirms that the apparent violation of the Heisenberg relation in the previous section was related to the detection scheme (now the LO is kept fixed). Note that $\kappa \neq 0$ does not destroy the squeezing phenomenon we have described before: For example, for $\kappa = \frac{1}{4}$ ($\gamma_y = 2\gamma_x$, a huge anisotropy indeed), $V_{\psi_L=\frac{\pi}{2}}(\omega = 0) \simeq 0.11$ ($\approx 10$dB of noise reduction), a very large squeezing level. This simple approach suggests that the phenomenon here presented is very robust.

Concluding remarks.— The spontaneous breaking of the rotational symmetry around the cavity axis in a type I DOPO above threshold has been shown to be a means for squeezing light in a non-critical way. The squeezed mode is a first-order Hermite-Gauss mode orthogonal to the one in which bright emission occurs. Such mode rotates randomly but very slowly, typically having a diffusion coefficient $D_\psi \sim 10^{-8}$s$^{-1}$. Outstandingly the squeezing level, perfect in the linear approach at zero noise frequency, is independent of the system’s distance from threshold. This result is robust versus deviations from perfect rotational symmetry, e.g. due to a cavity anisotropy. The fact that the squeezed mode is a first-order Hermite-Gauss mode can be of utility for precision measurements [19].

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