Private Edge Computing for Linear Inference Based on Secret Sharing

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Abstract—We consider an edge computing scenario where users want to perform a linear computation on local, private data and a network-wide, public matrix. The users offload computations to edge servers located at the edge of the network, but do not want the servers, or any other party with access to the wireless links, to gain any information about their data. We provide a scheme that guarantees information-theoretic user data privacy against an eavesdropper with access to a number of edge servers or their corresponding communication links. The proposed scheme utilizes secret sharing and partial replication to provide privacy, mitigate the effect of straggling servers, and to allow for joint beamforming opportunities in the download phase, in order to minimize the overall latency, consisting of upload, computation, and download latencies.

I. INTRODUCTION

Edge computing has established itself as a pillar of the 5G mobile network [1] to guarantee very low-latency and high-bandwidth computing services. The key idea is to move the computation power from the cloud closer to where data is generated, by pooling the available resources at the network edge.

Processing data in a distributed fashion over a number of edge servers poses significant challenges. In particular, edge servers may fail, be inaccessible, or struggle. The straggler problem has recently been addressed in the context of distributed computing in data centers in the presence of straggling servers [10], [11]. These works use secret sharing ideas to provide both privacy and robustness against stragglers.

In this paper, we propose a privacy-preserving edge computing scheme that exploits straggler coding and partial replications across servers to reduce latency. To the best of our knowledge, this problem has not been considered before in the literature. In particular, we consider a similar scenario to the one in [8] where multiple users wish to perform a linear inference on some local data given a network-wide, public matrix. Practical examples where such a scenario arises include recommender systems via collaborative filtering.

For this scenario, we present a scheme that guarantees information-theoretic user data privacy against an eavesdropper with access to a number of edge servers or their corresponding communication links. The proposed scheme utilizes secret sharing to provide both privacy and mitigate the effect of straggling servers. Furthermore, by replicating computations across different servers the scheme allows for joint beamforming opportunities. The proposed scheme entails an inherent tradeoff between computational latency due to stragglers, communication latency, and user data privacy. For a given privacy level, we optimize the parameters of the scheme in order to minimize the overall latency incurred by the upload and download of data as well as the computation. For the lowest privacy level, i.e., privacy against a single untrusted server, the proposed scheme yields an increase in latency in the worst case by a moderate factor of about 2.4 compared to the non-private scheme in [8] for the selected system parameters.

Notation: Vectors and matrices are written in lowercase and uppercase bold letters, respectively, e.g., $\mathbf{a}$ and $\mathbf{A}$. The transpose of vectors and matrices is denoted by $(\cdot)^\top$. $GF(q)$ denotes the finite field of order $q$ and $\mathbb{N}$ denotes the positive integers. We use the notation $[a]$ to represent the set of integers $\{0, 1, \ldots, a - 1\}$. Furthermore, $[a/b]$ is the smallest integer larger than or equal to $a/b$, $\lceil a/b \rceil$ is the largest integer smaller than or equal to $a/b$, and $(a)_{b}$ is the integer $a$ modulo $b$. We represent permutations in cycle notation, e.g., the permutation $\pi = (0 \ 2 \ 1 \ 3)$ maps $0 \mapsto 2$, $2 \mapsto 1$, $1 \mapsto 3$, and $3 \mapsto 0$. In addition, $\pi(i)$ is the image of $i$ under $\pi$, e.g., $\pi(0) = 2$. The expected value of a random variable $X$ is denoted by $\mathbb{E}[X]$.

II. SYSTEM MODEL

We consider the system in Fig. 1 with $u$ users $u_{0}, \ldots, u_{u-1}$, where the data of user $u_{i}$ is represented by the vector
\[ x_i = (x_{i,0}, \ldots, x_{i,r-1})^\top \in \text{GF}(q)^r. \] Each user \( u_i \) wants to perform a computation-intensive linear inference \( W x_i \), where \( W \in \text{GF}(q)^{m \times r} \), in a distributed fashion over \( e \) edge nodes (ENs) \( e_0, \ldots, e_{e-1} \) located at the edge of the network. For ease of notation, we will refer to the set \( \{W x_i \mid i \in [u]\} \) as \( \{W x_i\} \) and to \( \{x_i \mid i \in [u]\} \) as \( \{x_i\} \). The matrix \( W \) stays constant for a sufficiently long period of time, and each EN has a storage capacity corresponding to a fraction \( \mu \), \( 0 < \mu \leq 1 \), of the matrix \( W \), which is assumed to be public. Moreover, we assume that each user is connected by \( e \) unicast wireless links to the \( e \) ENs.

A. Computation Runtime Model

The ENs may straggle, which is represented by a random setup time \( \lambda \) for each EN \( e_j \). The setup time is the time it takes an EN to start computing after it has received the necessary data. As in [8], [12], [13], we assume that the setup times are independent and identically distributed (i.i.d.) according to an exponential distribution with parameter \( \eta \), such that \( \mathbb{E}[\lambda_j] = 1/\eta \). The time it takes an EN to compute one inner product in \( \text{GF}(q)^r \) for each of the users is deterministic and denoted by \( \tau \). Thus, the latency incurred by EN \( e_j \) to compute \( d \) inner products for each user \( (u \cdot d \text{ inner products in total}) \) is

\[ L_{\text{comp}}^j = \lambda_j + d\tau. \]

We define the normalized computation latency of EN \( e_j \) as

\[ \tilde{L}_{\text{comp}}^j = \frac{L_{\text{comp}}^j}{\tau} = \frac{\lambda_j + d}{\tau}. \]

B. Communication

Both the upload of data from the users to the ENs and the download of the results of the computations from the ENs to the users is considered. We denote by \( \gamma \) the normalized communication latency of unicasting \( u \) symbols from \( \text{GF}(q) \) in the upload or download. In the uplink, each user unicasts its data vector for computation to the ENs. In the downlink, ENs having access to the same symbol can collaboratively transmit to multiple users at the same time and thereby reduce the communication latency by exploiting joint beamforming opportunities [6–9], [14], [15]. In particular, a symbol available at \( \rho \) ENs can be transmitted simultaneously to \( \min\{\rho, u\} \) users with a normalized communication latency of \( \gamma/ \min\{\rho, u\} \) in the high signal-to-noise (SNR) region. The normalized communication latency, in the high SNR region, of transmitting \( v \) symbols, where symbol \( \alpha_i, i \in [v] \), is available at \( \rho_i \) ENs, is

\[ L_{\text{comm, down}}^i = \gamma \sum_{i=0}^{v-1} \frac{1}{\min\{\rho_i, u\}}. \]

C. Privacy and Problem Formulation

We consider a scenario where some of the ENs or their corresponding communication links are compromised. In particular, we assume the presence of an eavesdropper with access to any \( z \) ENs or their corresponding communication links.

The goal is to offload computations to the (untrusted) ENs in such a way that they do not gain any information in an information-theoretic sense (zero mutual information) about neither the user data \( \{x_i\} \) nor the results of the computations \( \{W x_i\} \), while minimizing the overall normalized latency, consisting of upload, computation, and download latencies.

III. PRIVATE DISTRIBUTED LINEAR INFERENCE

In this section, we present a distributed linear inference computation scheme that provides user data privacy against an eavesdropper with access to any \( z \) ENs or their corresponding communication links. At the heart of the proposed scheme lies Shamir’s secret sharing scheme (SSS) [16]. An SSS with parameters \((n, k), n \geq k\), ensures that some private data can be shared with \( n \) parties in such a way that any \( k-1 \) colluding parties do not learn anything about the data. On the other hand, any set of \( k \) or more parties can recover the data.

For each user \( u_i \), an \((n, k)\) SSS is used to compute \( n \) shares of its private data \( x_i = (x_{i,0}, \ldots, x_{i,r-1})^\top \). In particular, for user \( u_i \), we encode each data entry \( x_{i,t} \) along with \( k-1 \) i.i.d. uniform random symbols \( r_{i,1}^{(1)}, \ldots, r_{i,k-1}^{(1)} \) from \( \text{GF}(q) \) using an \((n, k)\) Reed-Solomon (RS) code to obtain \( n \) coded symbols \( s_{i,0}^{(1)}, \ldots, s_{i,n-1}^{(1)} \). For each \( h \in [n] \), the \((h+1)\)-th share of user \( u_i \) is

\[ s_i^{(h)} = \left(s_{i,0}^{(h)}, \ldots, s_{i,n-1}^{(h)}\right)^\top. \]

Finally, define the matrix of shares

\[ S^{(h)} = \left(s_0^{(h)}, s_1^{(h)}, \ldots, s_{n-1}^{(h)}\right) \in \text{GF}(q)^{r \times u} \]

as the matrix collecting the \((h+1)\)-th share of all users.

The following theorem proves that the original computations \( \{W x_i\} \) of all users can be recovered from a given set of computations based on the matrices of shares \( S^{(0)}, \ldots, S^{(n-1)} \), while providing privacy against an eavesdropper with access to at most \( k-1 \) distinct matrices of shares.

Theorem 1. Consider \( u \) users with their respective private data \( x_i \in \text{GF}(q)^r, i \in [u] \). Use Shamir’s \((n, k)\) SSS on each \( x_i \) to obtain the matrices of shares \( S^{(0)}, \ldots, S^{(n-1)} \) in (1). Let \( W \in \text{GF}(q)^{m \times r} \) be a public matrix and \( I \subseteq [n] \) a set of...
indices with cardinality $|I| = k$. Then, the set of computations $\{WS^{(h)} \mid h \in I\}$ allows to recover the computations $\{Wx_i\}$ of all users. Moreover, for any set $J \subseteq [n]$ with $|J| < k$, $\{WS^{(h)} \mid h \in J\}$ reveals no information about $\{Wx_i\}$.

Proof: Let $C$ be the $(n, k)$ RS code used in the SSS. For each $h \in [n]$, the entries of the rows of $S^{(h)}$ are codewords in position $h$ of codewords from $C$ pertaining to different users. More precisely, for each user $u_i$, each row of the matrix $\{s^{(0)}_i, s^{(1)}_i, \ldots, s^{(n-1)}_i\}$ of all $n$ shares of $u_i$ is a codeword from $C$. Since $C$ is a linear code, each of the $m$ rows of the matrix $W \left( s^{(0)}_i, s^{(1)}_i, \ldots, s^{(n-1)}_i \right)$ is a codeword of $C$. Furthermore, the messages obtained by decoding these codewords are the rows of $\left( Wx_i, Wr^{(1)}_i, \ldots, Wr^{(k-1)}_i \right)$, where $\{r^{(\kappa)}_i \mid \kappa \in [k]\{0\}\} = \{r^{(\kappa)}_i \mid \kappa \in [k]\{0\}\} \top$ is the set of vectors of uniform random symbols used by user $u_i$ in the computation of the shares $s^{(h)}_i$, $h \in [n]$. Then, decoding the vectors in the set $\{WS^{(h)} \mid h \in I\}$ gives $Wx_i$, and it follows that $\{WS^{(h)} \mid h \in I\}$ gives $\{Wx_i\}$.

From the properties of Shamir’s SSS it follows that the mutual information between $\{S^{(h)} \mid h \in J\}$ and $\{x_i\}$ is zero. Subsequently, from the data processing inequality it follows that $\{WS^{(h)} \mid h \in J\}$ reveals no information about $\{x_i\}$.

The following corollary gives a sufficient condition to recover the private computations $\{Wx_i\}$.

**Corollary 1 (Sufficient recovery condition).** Consider an edge computing scenario, where the public matrix $W$ is partitioned into $b$ disjoint submatrices $W_l \in GF(q)^{w \times r}$, $l \in [b]$, and the private data is $\{x_i\}$. Then, the private computations $\{Wx_i\}$ can be recovered from the computations in the sets $\mathcal{S}_I \triangleq \{WS^{(h)} \mid h \in I\}$, $l \in [b]$, for any fixed set $I \subseteq [n]$ with cardinality $|I| = k$.

Proof: From Theorem 1, for a given $l \in [b]$, the computations in the set $\{W_l x_i\}$ can be recovered from the computations in the set $\mathcal{S}_I$. Then, we obtain $Wx_i = \left( (W_0 x_i) \top, (W_1 x_i) \top, \ldots, (W_{b-1} x_i) \top \right) \top$, $\forall i \in [u]$.

In the following, we present a scheme that fulfills the sufficient recovery condition in Corollary 1. Note that it may be beneficial to repeat shares over several ENs in order to exploit broadcasting opportunities during the download phase. This presents difficulties in the design of a private scheme, because repeating shares at different nodes results in a privacy level $z$ lower than that of the SSS ($k$). For example, if all ENs have access to two matrices of shares, the scheme only provides privacy against any $z = \lfloor (k-1)/2 \rfloor$ colluding ENs.

Given the underlying SSS, the proposed scheme can be broken down into two combinatorial problems. The first corresponds to the assignment of the submatrices $\{W_l \mid l \in [b]\}$ to the $e$ ENs such that no EN stores more than a fraction $\mu$ of $W$. The second corresponds to the assignment of the $n$ matrices of shares $\{S^{(h)} \mid h \in [n]\}$ to the ENs such that the users are guaranteed to obtain the computations in (2).

### A. Assignment of $W$ to the Edge Nodes

We start by explaining the assignment of the submatrices of $W$ to the ENs such that no EN stores more than a fraction $\mu$ of $W$, while the users are guaranteed to recover their computations $\{Wx_i\}$. Additionally, we would like to allow for replications across different ENs to allow for joint beamforming in the download phase.

In order to satisfy the storage requirement, we select $p \in \mathbb{N}$ such that $p/e \leq \mu$ and partition $W$ into $b = e$ submatrices as $W = \left( W_0^\top, W_1^\top, \ldots, W_{e-1}^\top \right)^\top$.

We then assign $p$ submatrices to each of the $e$ ENs. The assignment has the following combinatorial structure. Consider a cyclic permutation group of order $e$ with generator $\pi$. We construct an index matrix $I_w \triangleq \begin{pmatrix} \pi^0(0) & \pi^0(1) & \cdots & \pi^0(e-1) \\ \pi^1(0) & \pi^1(1) & \cdots & \pi^1(e-1) \\ \vdots & \vdots & \ddots & \vdots \\ \pi^{p-1}(0) & \pi^{p-1}(1) & \cdots & \pi^{p-1}(e-1) \end{pmatrix}$ (3)

and define the set of indices $\mathcal{I}_w = \{\pi^0(j), \ldots, \pi^{p-1}(j)\}$ (4) for $j \in [e]$ as the set containing the elements in column $j$ of $I_w$. Then, we assign the submatrices $\{W_l \mid l \in \mathcal{I}_w\}$ to EN $e_j$.

For example, if $\pi = (0 e - 1 e - 2 \cdots 1)$, we have $I_w = \begin{pmatrix} 0 & 1 & \cdots & e - 1 \\ e - 1 & 0 & \cdots & e - 2 \\ \vdots & \vdots & \ddots & \vdots \\ e - p + 1 & e - p + 2 & \cdots & e - p \end{pmatrix}$, and EN $e_1$ stores $W_1, W_0, W_{e-1}, \ldots, W_{e-p+2}$.

The ENs process the assigned submatrices of $W$ in the same order as their indices appear in the rows of $I_w$, and we define $\phi^*_j(l)$ for $l \in [p]$ to be the map to the index of the $(l+1)$-th assigned submatrix of EN $e_j$.

### B. Assignment of Shares to the Edge Nodes

Given the assignment of the submatrices of $W$, we now have to assign the shares in such a way that we can guarantee that the users obtain the computations in (2). The users upload their shares to the $e$ ENs according to the following assignment. Given the generator $\pi$ used to assign the submatrices of $W$ to the ENs, we construct a $(\beta + 1) \times e$ index matrix $I_s = \begin{pmatrix} \pi^0(0) & \pi^0(1) & \cdots & \pi^0(e-1) \\ \pi^{e-p}(0) & \pi^{e-p}(1) & \cdots & \pi^{e-p}(e-1) \\ \vdots & \vdots & \ddots & \vdots \\ \pi^{\beta(e-p)}(0) & \pi^{\beta(e-p)}(1) & \cdots & \pi^{\beta(e-p)}(e-1) \end{pmatrix}$.


where $\beta = \lceil e/p \rceil - 1$. Define the set of indices
\[ I^e_j = \{ \pi^0(j), \ldots, \pi^{\beta(e-p)}(j) \} \in \{ n, n + 1, \ldots, e - 1 \} \] (6)
as the subset of elements in column $j$ of $I_s$ that are in $[n]$. User $u_i$ transmits the shares $\{ s_{s_i}^{(h)} \mid h \in I^e_j \}$ to EN $e_j$. Thereby, every EN receives $a = \lceil e/p \cdot n/e \rceil$ shares from each user as we keep only a fraction $n/e$ of the shares corresponding to the $\beta + 1 = \lceil e/p \rceil$ used permutations in $I_s$, i.e., of the indices in $[e]$ we keep only those in $[n]$. As for the submatrices of $W$, the shares are processed in the same order as their indices appear in the rows of $I_s$, and we define $\phi^e_j(h)$ for $h \in [a]$ to be the map to the index of the $(h + 1)$-th assigned matrix of shares of EN $e_j$. For a given matrix of shares assigned to an EN, all assigned submatrices of $W$ are processed before moving on to the next matrix of shares. What remains to be shown is that these combined assignments of submatrices and shares to the ENs allow all users to obtain enough partial computations from the $e$ ENs to retrieve their desired computations.

**Theorem 2.** Consider an edge computing network consisting of $u$ users and $e$ ENs, each with a storage capacity corresponding to a fraction $\mu$, $0 < \mu \leq 1$, of $W$, and an $(n, k)$ SSS, with $n \leq e$. For $j \in [e]$, EN $e_j$ stores the submatrices of $W$ from the set $\{ W_l \mid l \in I^e_j \}$ with $I^e_j$ defined in (4). Furthermore, it receives the matrices of shares from the set $\{ S_{l}^{(h)} \mid h \in I^e_j \}$ with $I^e_j$ defined in (6), and computes and returns the set $\{ W_l S_{l}^{(h)} \mid l \in I^e_j, h \in I^e_j \}$ to the users. Then, all users can recover their desired computations $\{ W x_i \}$.

Due to lack of space, we omit the proof of Theorem 2. We motivate the theorem, however, with the following example.

**Example 1.** Consider $e = n = 5$, $p = 3$, and $\pi = (0\ 3\ 1\ 4\ 2)$, the generator of a cyclic permutation group of order 5. From (3) and (5), we have
\[ I^e_a = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 3 & 4 & 0 & 1 & 2 \\ 1 & 2 & 3 & 4 & 0 \end{pmatrix} \] and $I^e_s = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 0 \end{pmatrix}$.

We focus on the matrix of shares $S^{(0)}$. It is assigned to EN $e_0$ and gets multiplied with the submatrices of $W$ indexed by the elements of the set
\[ I^e_0 = \{ \pi^0(0), \pi(0), \pi^2(0) \} = \{ 0, 3, 1 \}. \]

Note that the set $I^e_0$ contains three recursively $\pi$-permuted integers of $0$ ($\pi^0(0)$, $\pi^1(0)$, and $\pi^2(0)$). Now, consider EN $e_4$, which is also assigned the matrix of shares $S^{(0)}$. We have
\[ I^e_4 = \{ \pi^0(4), \pi(4), \pi^2(4) \} = \{ 4, 2, 0 \}. \]

Notice that $\pi^0(4) = 4$ is the fourth recursively $\pi$-permuted integer of $0$. Hence, the set $I^e_0 \cup I^e_4$ contains in total six recursively $\pi$-permuted integers of $0$, which is sufficient to give the set $[5]$, since the group generated by $\pi$ is transitive. In a similar way, it can be shown that the same property holds for all other matrices of shares. Therefore, each matrix of shares is multiplied with all submatrices of $W$, and the sets in (2) are obtained.

**IV. Communication and Computation Scheduling**

In this section, we describe the scheduling of uploading the assigned shares to the ENs, performing the computations, and downloading a subset of $\{ W_l S_l^{(h)} \mid l \in I^e_j, h \in I^e_j \}$ at $j \in [e]$. In the following, we refer to a single $W_l S_l^{(h)}$ as an intermediate result (IR).

**A. Upload and Computation**

As $W$ stays constant for a long time, the assignment of the submatrices $\{ W_l \mid l \in [e] \}$ can be done offline and does not affect the overall latency. The online phase starts with the upload of the shares. In contrast to the nonprivate scheme in [8], user $u_i$ can not broadcast one vector to all ENs. Instead, the user has to unicast a number of shares to each EN to assure that any $v$ ENs do not obtain any information about $x_i$. In general, broadcasting a message to $e$ receivers is more expensive than transmitting a single unicast message to one receiver. As in [2], we assume that broadcasting to $e$ receivers is a factor $\log(e)$ more expensive in terms of latency than a single unicast. Recall that the cost (or normalized latency) of uncasting $u$ symbols from GF($q$) is $\gamma$. Hence, in the nonprivate scheme the normalized latency of every user broadcasting one vector from GF($q$) to all $e$ ENs is $\hat{L}^{\text{up}} = \gamma \cdot \log(e)$.

In contrast, the normalized latency of uncasting $u$ shares, one from each user, which are elements in GF($q$) to, to one EN is $\gamma r$. Recall that each EN receives $a$ matrices of shares. We assume that each user can upload only one share to one EN at a time. The upload is illustrated in Fig. 2, in which the blue segments correspond to the upload phase. We start by uploading the first matrix of shares to EN $e_0$, continue with EN $e_1$, and proceed until all ENs have received their first matrix of shares. This process is repeated with the remaining matrices of shares until EN $e_j$ has received the $a$ matrices of shares $\{ S_l^{(h)} \mid h \in I^e_j \}$, $j \in [e]$. EN $e_j$ receives its $(h + 1)$-th matrix of shares $S_l^{(h+1)}$ at normalized time
\[ \hat{L}^{\text{up},h} = \gamma r (eh + j + 1), \]
and the total normalized upload latency of the private scheme becomes $\hat{L}^{\text{up}} = \gamma \cdot r \cdot e \cdot a$.

After an EN has received its first matrix of shares, it enters the computation phase. As mentioned earlier, the ENs experience a random setup time before they can start their computations. This is illustrated by the red segments in Fig. 2. For EN $e_j$ this phase incurs a normalized latency of $\lambda_j / \tau$. Once set up, the ENs start their computations on the first assigned matrix of shares. In total, $p$ IRs of the form $W_l S_l^{(h)}$ have to be computed for each assigned matrix of shares $S_l^{(h)}$ by EN $e_j$, where $l \in I^e_j$ and $h \in I^e_j$. This incurs a normalized latency of $p \cdot m/e$, because each $W_l$ has $m/e$ rows and hence, the ENs compute $u \cdot m/e$ inner products for each of the $p$ IRs.

In the case an EN has not received another matrix of shares before finishing the currently assigned computations, it remains idle until it receives another matrix of shares to compute on. This can be seen in yellow in Fig. 2. For $h \in [a]$,
the normalized time at which EN $e_j$ starts to compute on the ($h+1$)-th assigned matrix of shares, i.e., on $S^{(\phi_j(h))}$, is

$$\gamma_{\text{start},h} = \max \left\{ \gamma_{\text{start},h-1} + \frac{m}{e} \gamma_{\text{up},h} \right\}, \text{ for } h > 0,$$

with

$$\gamma_{\text{start},0} = \frac{\lambda_j}{\tau} + \gamma_{\text{up},0}.$$  

The computational phase continues at least until the computations in (2) are obtained, i.e., until there are at least $k$ distinct IRs of the form $W \cdot S^{(h)}$, $h \in [n]$, for each $l \in [e]$. This ensures that a given user $u_i$ can recover $W \cdot x_i$. It can be beneficial to continue computing products to reduce the communication latency in the download phase, as we discuss next.

### B. Download

In the download phase we can make use of joint beamforming opportunities to reduce the latency by serving multiple users at the same time. An IR $W \cdot S^{(h)}$ that is computed at $\rho_{l,h}$ ENs incurs a normalized communication latency of $\gamma / \min\{\rho_{l,h}, u\}$. Hence, a higher multiplicity of computed IRs across different ENs will reduce the communication latency in the download phase. At the same time, the repeated IRs have to be computed first, thereby increasing the computational latency. This tradeoff can be optimized to reduce the overall latency. Assume the optimum is reached after EN $e_j$, has computed the IR $W \cdot S^{(\phi_j(h^*))}$. This gives a normalized computational latency of

$$\bar{\gamma}_{\text{comp}} = \gamma_{\text{start},h^*} + (l^* + 1) \frac{m}{e}.$$  

After the computation phase has finished, the ENs cooperate to send the computed IRs $W \cdot S^{(h)}$ simultaneously to multiple users in descending order of their multiplicity $\rho_{l,h}$ until the computations in (2) are available to the users. More precisely, for each $W \cdot S^{(h)}$ the ENs send the $k$ IRs with the highest multiplicities to the users. Then, a given user $u_i$ can decide the SSS to obtain the desired computation $W \cdot x_i$. For a fixed $l$, let $\mathcal{H}_{l}^{\text{max}} = \arg \max_{h \in [n], |A| = k} \sum_{h \in A} \rho_{l,h}$ be the set of indices $h$ of the $k$ largest $\rho_{l,h}$. This results in a normalized communication latency of

$$\bar{\gamma}_{\text{comm}} = \gamma \sum_{l=0}^{e-1} \frac{1}{\min\{\rho_{l,h}, u\}},$$

and the overall normalized latency becomes

$$\bar{L} = \gamma_{\text{start},h^*} + (l^* + 1) \frac{m}{e} + \gamma \sum_{l=0}^{e-1} \sum_{h \in \mathcal{H}_{l}^{\text{max}}} \frac{1}{\min\{\rho_{l,h}, u\}}. \quad (7)$$

### V. Optimization and Numerical Results

We start by explaining how to choose the parameters of the proposed scheme so that the overall normalized latency $\bar{L}$ in (7), consisting of upload, computation, and download latencies, is minimized for a given privacy level $\lambda$. To reduce the upload latency, it may be beneficial to contact fewer ENs than the maximum number of ENs available, denoted by $e_{\text{max}}$, to which a user can connect. Additionally, storing fewer than $\mu e$ submatrices of $W$ at the ENs can be advantageous, because the ENs will start computations sooner on the later shares. Thus, we can choose $p \leq \mu e$.

From the combinatorial designs, it follows that the number of shares $n$ per user can be at most equal to $e$, while the value of the SSS threshold $k$ is constrained by the choices of $z$, $e$, $n$, and $p$. First, recall that the total number of shares per user assigned to each EN is $a = \lceil e/p \rceil \cdot n/e$, which means that any $z$ ENs have access to $a \cdot z$ possibly distinct shares of each user. Given that this set of shares must not leak any information about the private data $\{x_i\}$, we have to pick $k \geq az + 1$. According to Corollary 1, for a given $W$, waiting for $k$ distinct products allows to recover the computation $W \cdot x_i$ for each user $u_i$. Note that there is no reason to pick $k$ larger than $az + 1$, since then the users have to wait for more products, leading to reduced straggler mitigation and increased computational latency. Therefore, we set $k = az + 1$. Finally, we need to verify that all constraints on $n$ are fulfilled for the chosen parameter values, i.e., $k \leq n \leq e$ (for the scheme to be feasible), $n \geq k$ (for the SSS to work), and $n \leq e$ (from the combinatorial designs).

We have chosen $\pi = (0.0005 - 1.0005 - 2.0005 - 3.0005 - 1)^T$ and performed an exhaustive search for the minimum expected overall normalized latency $\bar{L}$ given in (7) over all valid parameter tuples $(e, n, p)$ for a given privacy level $\lambda$. For each tuple we varied the number of total (not necessarily distinct) IRs to wait for across all ENs for each $W$, in order to minimize the latency. We generated $10^6$ instances of the random setup times $\{\lambda_j\}$ in the simulation of the scheme in order to obtain an accurate estimate of the expected overall normalized latency.

In Fig. 3, we compare the expected overall normalized latency of the proposed private scheme with the nonprivate MDS-repetition scheme in [8]. We plot the overall normalized latency versus $\gamma$ for different privacy levels $\gamma$. For the presented scenario, the users have access to a maximum of $e_{\text{max}} = 9$ ENs, which can store up to a fraction of $\mu = 2/3$ of the matrix $W$ with dimensions $m = 600$ and $r = 50$. The ENs need $\tau = 0.0005$ time units to compute one inner product over
GF(q)^50 for each of the users, and the straggling parameter is set to η = 0.8. Providing privacy against a single EN (z = 1) yields an increase in latency for γ = 8 by a factor of about 2.4 compared to the nonprivate MDS-repetition scheme in [8]. For z = 2, the latency increases by a factor of about 3.5, while it increases to 5.7 and 10.0 for z = 3 and 4, respectively.

One of the factors that leads to an increased latency is the upload. In the nonprivate scheme, the users can broadcast their private data vectors to all ENs simultaneously, whereas in the private scheme, the users have to unicast their shares to the ENs sequentially. In Fig. 4, we show the impact of the upload on the proposed private scheme and the nonprivate MDS-repetition scheme in [8]. For both schemes, for γ = 8, the upload takes about 13% of the overall latency, which means that for the private scheme the latency increases by around 1500 time units, whereas for the nonprivate scheme it increases by only 700 time units.

VI. CONCLUSION

We presented a privacy-preserving scheme that allows multiple users in an edge computing network to offload computations to edge servers for distributed linear inference, while keeping their data private to a number of edge servers or their corresponding communication links. The proposed scheme uses secret sharing to provide user data privacy and mitigate the effect of straggling servers, and partial repetitions to enable joint beamforming opportunities in the download phase in order to reduce the communication latency. The parameters of the scheme were optimized in order to minimize the overall latency incurred by the upload of data to the servers, the computation, and the transmission of partial computations back to the users.

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