Understanding the Relationship Between Observations and Stellar Parameters in an Eclipsing Binary System

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Abstract. We would like to investigate the information contained in our observations and to what extent each of them contributes individually to constraining the physical parameters of the system we are investigating. To do this, we present a study involving the technique of Singular Value Decomposition using as a simple example a detached eclipsing binary system. We intend to apply an extension of this technique to asteroseismic measurements of Delta Scuti stars that are members of eclipsing binary systems.

1. Introduction

Asteroseismology is the study of the oscillations of the stars. It is the only method which allows us to see beyond the photosphere of a star, and learn about their interior conditions. We are interested in $\delta$ Scuti stars, whose pulsation amplitudes are large enough to be detected from the Earth. However, they present a physical problem: not all of the oscillation modes that theory predicts are excited nor observed, which leads to various interpretations of the frequency spectra and thus a poorly-constrained solution (mass, age etc.). If we study $\delta$ Scuti stars in eclipsing binary systems we could accurately derive their stellar masses and radii. Then we could constrain some of the parameters of the system. This in turn would lead to an easier identification of the modes of the stars, allowing us to better determine the real parameters of the system. Combining information from these different sources has been suggested by many authors, and some studies have been conducted with this in mind (Aerts & Harmanec 2004; Rodríguez et al. 2004). In fact, they agree that this is the only way to advance if we would like to test stellar structure and evolution theories.

Following a technique similar to that conducted by Brown et al. (1994), we would like to investigate how the combination of information from the study of binary systems with oscillation frequencies constrains the solution parameters. In particular we would like to understand by how much each observable (radial velocities, oscillation modes etc.) influences each of the parameters of the system (mass, age etc). To approach this problem we use the technique of Singular Value Decomposition (SVD) (Press et al. 1986) and apply it to a simple case of a detached eclipsing binary system.
Table 1. Initial parameters and observational errors

| Parameters     | $P_j$ | Observables | $\sigma_i$ |
|----------------|-------|-------------|------------|
| $R_A(R_\odot)$ | 0.850 | $K_A$ (kms$^{-1}$) | 0.90       |
| $R_B(R_\odot)$ | 0.095 | $K_B$ (kms$^{-1}$) | 0.90       |
| $i(^\circ)$    | 89.300| $D_P$ (mag) | 0.02       |
| $T_2/T_1$      | 0.300 | $L_F$ (hrs) | 0.20       |
| a(AU)          | 0.060 | $L_F$ (hrs) | 0.20       |
| $M_A(M_\odot)$ | 0.970 | $P$(yrs)    | $5 \times 10^7$ |
| $M_B(M_\odot)$ | 0.100 | $D_S/D_P$   | 0.0012     |

2. Mathematical Background

For a more detailed discussion we refer the reader to Brown et al. (1994). Let $B_i, i = 1, M$ be the observables of some system, and $P_j, j = 1, N$ with $M > N$ be the variables (parameters) that define this system. There exists some model or list of equations that when given $P_j$, will result in $B_i$. In real situations we know $B_i$ and we would like to extract $P_j$. If the system or model were linear, this would be achieved relatively easy by solving a system of M linear equations in N variables. However, normally this is not the case, and so by linearizing the system around a reference set of parameters, $P_{j0}$, we can write

$$B_i = B_{i0} + \sum_{j=1}^{N} \frac{\partial B_i}{\partial P_j} \delta P_j$$

where $B_{i0}$ are the observables resulting from the initial parameter estimates $P_{j0}$, $\delta P_j = P_j - P_{j0}$ and each of the derivatives, $\frac{\partial B_i}{\partial P_j}$, is evaluated at $P_j = P_{j0}$. The parameter fitting problem involves finding $\delta P_j$. Given the real observables $\beta_i$ and after some substituting our problem is mathematically equivalent to minimizing

$$\chi^2 = ||D\delta P - \delta B||^2$$

where $\delta B_i = (\beta_i - B_{i0})/\sigma_i$, and

$$D = \frac{1}{\sigma_i} \frac{\partial B_i}{\partial P_j}$$

The solution is $\delta P = VW^{-1}U^T\delta B$, where $D = UWV^T$ is the singular value decomposition of $D$. It is precisely this decomposition that provides us with information regarding the origin of the errors in the parameter solutions. In the absence of the real observables $\beta_i$ we can still construct the derivative matrix $D$ to study the transformation between the observable and the parameter space.

3. Application to a Detached Eclipsing Binary System

Given photometric and radial velocity measurements of a detached eclipsing spectroscopic binary system, we would like to obtain the following fundamental parameters: masses of both components $M_A$ and $M_B$, radii of both stars $R_A$ and $R_B$, effective temperature ratio $T_B/T_A$, inclination of system $i$ and separation of both components $a$. In order to obtain these parameters we need the following observables: $K_A$ and $K_B$ are the semi-amplitudes of the radial velocity curve.
Figure 1. Information from the decomposition matrices of the derivative matrix $D$. Left: Three parameter solutions showing the extent to which the parameters are constrained. Center: Three observable vectors quantifying the amount of information contributed by each observable. Right: The effect of increasing the precision of some observations by a factor of two on each of the solution parameter errors.

From the photometric light curve we obtain the primary eclipse depth $D_P$ and with the secondary $D_S$, a depth ratio $D_S/D_P$. We also observe a total duration of eclipse $L_T$, and a duration of eclipse minimum, $L_F$ (see assumptions below). The photometric period $P$ is obtained from long term observations. Let us also assume for simplicity that there is no eccentricity $e = 0$, that neither of the components have stellar spots, that the secondary eclipse is total and that both stars are well isolated such that there is no mass transfer between them.

The model that relates these given observables to the parameters is a set of equations that can be easily derived. After obtaining observations with errors $\sigma_i$, we make some initial parameter estimates $P_{j0}$ (Table 1). We can then calculate the matrix $D$ (equation 2). Using SVD, we decompose $D$ to obtain its singular values and its corresponding orthogonal matrices. Figure 1 shows the first three solution vectors (left panel) and observable vectors (center panel) of $D$. The vector corresponding to the largest singular value (smallest error) is the top solution vector. According to the figure, the best-constrained parameter is the separation between both components (left panel, top row vector). There is also a small effect from this singular value on both masses. If we inspect the observable vector (center panel, top row vector), we can see clearly that $P$ is the most reliable observation (corresponding to the largest singular value). We know that $P$ is directly related to $a$ and $M_A + M_B$ through Kepler’s Law giving a well-determined $a$. In this case $M_A$ and $M_B$ are mainly constrained by the radial velocity measurements (left and center panel, bottom row vector).

The second solution vector shows that both $L_T$ and $L_F$ are also some of the more important observables in constraining the solution. Figure 1 left panel shows that both $R_A$ and $R_B$ correspond to this singular value, implying that $L_F$ and $L_T$ are mainly responsible for the reduction in the errors of both radii. In fact, $L_F$ and $L_T$ contribute also to $i$ but to a lesser extent (sixth singular value).

Suppose now that the errors in our measurements ($\sigma_i$) were to decrease by a factor of 2. By decreasing each of the $\sigma_i$ individually, we may investigate how each of the errors on the parameters ($\epsilon_j$) improves (noting that combining all $\sigma_i$ should result in a corresponding reduction of 0.5 in all the $\epsilon_j$). Figure 1 right panel indicates the decrease in each $\epsilon_j$ as a fraction of the original error.
(\epsilon_{j0} - \epsilon_{ji})/\epsilon_{j0}, where \epsilon_j comes from the covariance matrix (Press et al. 1986), and \epsilon_{j0} represents \epsilon_j given the original \sigma_{i0}, and \epsilon_{ji} are the resulting \epsilon_j given a decrease in a particular \sigma_i. We show the effects of reducing just \sigma_{DP} (solid) and \sigma_{KA} (dash).

The new \sigma_{DP} is solely responsible for the reduction of both \epsilon_{RA} and \epsilon_{RB} (a total of 0.5). If we were interested in improving the errors on both radii, then we know from this that we need only improve the precision of \sigma_{DP} (note with \sigma_{i0} \sigma_{LF} and \sigma_{LT} were important for \epsilon_{RA} and \epsilon_{RB}). Similarly, \sigma_{KA} is responsible for all of the reduction of \epsilon_{MB}. However, \sigma_{KA} is also partially responsible for reducing both \epsilon_{MA} and \epsilon_a (by 0.2 and 0.1 respectively). This indicates that if we were interested in improving the precision of \epsilon_{MA}, we would need not only an improvement of \sigma_{KA}, but a combination of improved \sigma_i (in this case \sigma_{KB}, \sigma_{LF} and \sigma_{LT}). We should also take into account the actual values of the errors, for example, if \epsilon_{MA} improves from 0.004 to 0.002 \sigma_{L}\odot (which is this case), we could also conclude that these observations may not be worthwhile. So we must look at these results cautiously, and we must also emphasize the sensitivity of these results to \sigma_{i0}.

If we could decrease each \sigma_i by 3, 4, 5... times, we could determine at what error level this observable becomes important in constraining a particular parameter, given all the other \sigma_i. This could indicate if it were worthwhile obtaining better observations. It may also help determine the precision of our observations necessary to obtain a solution to the system with errors to within a certain %. Studying the transformation between parameter and observable space may allow us to reach these conclusions.

4. Conclusions

What we have discussed is just a simple example to illustrate what we would like to achieve in the case of a more complicated system, such as a detached eclipsing binary system where one of the components is a \delta Scuti star. Unfortunately, for that case we can not write simple equations to relate the observables to the parameters, and for this reason a study such as this one is beneficial.

We saw briefly the capacity of SVD to try to explain the relationship between the errors in each of the observables to the errors in each of the parameters. We saw how the effect of an increase in the precision of the observables can impact in a rather complex form on the resulting errors of the parameters.

Ideally we would like to observe a real eclipsing system with a \delta Scuti component. From studying this transformation we hope to be able to optimise our observations to obtain the best constrained parameters. Doing this would allow us to solve this system to the required precision in order to test theories of stellar structure.

References

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