Analysis of the nonet scalar mesons as tetraquark states with new QCD sum rules

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Abstract

In this article, we take the scalar diquarks as point particles and describe them as basic quantum fields, then introduce the SU(3) color gauge interaction and new vacuum condensates to study the nonet scalar mesons as tetraquark states with the QCD sum rules. Comparing with the conventional quark currents, the diquark currents have the outstanding advantage to satisfy the two criteria of the QCD sum rules more easily.

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1 Introduction

The light flavor scalar mesons present a remarkable exception for the naive quark model, and the structures of those mesons have not been unambiguously determined yet. The numerous candidates with $J^{PC} = 0^{++}$ below 2 GeV cannot be accommodated in one $qar{q}$ nonet, some are supposed to be glueballs, molecular states and tetraquark states (or their special superpositions) [1, 2, 3]. The $a_0(980)$ and $f_0(980)$ are good candidates for the $K\bar{K}$ molecular states [4], however, their cousins $\sigma(600)$ and $\kappa(800)$ lie considerably higher than the corresponding thresholds, it is difficult to identify them as the $\pi\pi$ and $\pi K$ molecular states, respectively. There may be different dynamics which dominate the $0^{++}$ mesons below and above 1 GeV respectively, and result in two scalar nonets below 1.7 GeV [1, 2, 3]. The strong attractions between the diquark states $(qq)_3$ and $(q\bar{q})_3$ in relative $S$-wave may result in a nonet tetraquark states manifest below 1 GeV, while the conventional $^3P_0$ $q\bar{q}$ nonet have masses about $(1.2 - 1.6)$ GeV, and the well established $^3P_1$ and $^3P_2$ $q\bar{q}$ nonets with $J^{PC} = 1^{++}$ and $2^{++}$ respectively lie in the same region. Furthermore, there are enough candidates for the $^3P_0$ $q\bar{q}$ nonet mesons, $a_0(1450)$, $f_0(1370)$, $K^*(1430)$, $f_0(1500)$ and $f_0(1710)$ [5].

In the tetraquark scenario, the structures of the nonet scalar mesons in the ideal

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mixing limit can be symbolically written as \[ \sigma(600) = ud \bar{u} d, \quad f_0(980) = \frac{us \bar{s} + ds \bar{d}}{\sqrt{2}}, \] \[ a_0^-(980) = ds \bar{u} s, \quad a_0^+(980) = ds \bar{d}, \quad a_0^0(980) = us \bar{u} s, \]
\[ \kappa^+(800) = udd \bar{s}, \quad \kappa^0(800) = ud \bar{u} s, \quad \bar{\kappa}^0(800) = us \bar{d}, \quad \kappa^-(800) = ds \bar{u} d. \] (1)

The four light isospin-\(\frac{1}{2}\) \(K\pi\) resonances near 800 MeV, known as the \(\kappa(800)\) mesons, have not been firmly established yet, there are still controversy about their existence due to the large width and nearby \(K\pi\) threshold [5].

In general, we may expect constructing the tetraquark currents and studying the nonet scalar mesons below 1 GeV as the tetraquark states with the QCD sum rules \[7, 8\]. For the conventional mesons and baryons, the “single-pole + continuum states” model works well in representing the phenomenological spectral densities, the continuum states are usually approximated by the contributions from the asymptotic quarks and gluons, the Borel windows are rather large and reliable QCD sum rules can be obtained. However, for the light flavor multiquark states, we cannot obtain a Borel window to satisfy the two criteria (pole dominance and convergence of the operator product expansion) of the QCD sum rules [9]. In Ref. [10], T. V. Brito et al take the quarks as the basic quantum fields, and study the scalar mesons \(\sigma(600), \kappa(800), f_0(980)\) and \(a_0(980)\) as the diquak-antidiquark states with the QCD sum rules, and cannot obtain Borel windows to satisfy the two criteria, and resort to a compromise between the two criteria. For the heavy tetraquark states and molecular states, the two criteria can be satisfied, but the Borel windows are rather small [11].

We can take the colored diquarks as point particles and describe them as the basic scalar, pseudoscalar, vector, axial-vector and tensor fields respectively to overcome the embarrassment [12]. In this article, we construct the color singlet tetraquark currents with the scalar diquark fields, parameterize the nonperturbative effects with new vacuum condensates besides the gluon condensate, and perform the standard procedure of the QCD sum rules to study the nonet scalar mesons below 1 GeV. The QCD sum rules are “new” because the interpolating currents are constructed from the basic diquark fields instead of the quark and gluon fields.

Whether or not the colored diquarks can be taken as basic constituents is of great importance, because it provides a new spectroscopy for the mesons and baryons \[6, 13\].

The article is arranged as follows: we derive the new QCD sum rules for the nonet scalar mesons in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusions.
2 The nonet scalar mesons with QCD Sum Rules

In the following, we write down the interpolating currents for the nonet scalar mesons below 1 GeV,

\[ J_{f_0}(x) = \frac{D^a(x)\bar{D}^a(x) + U^a(x)\bar{U}^a(x)}{\sqrt{2}}, \]

\[ J_{a_0}(x) = \frac{D^a(x)\bar{D}^a(x) - U^a(x)\bar{U}^a(x)}{\sqrt{2}}, \]

\[ J_\kappa(x) = S^a(x)\bar{U}^a(x), \]

\[ J_\sigma(x) = S^a(x)\bar{S}^a(x), \]

(2)

where

\[ U^a(x) \propto \tilde{U}^a(x) = \epsilon^{abc}d_b^T(x)C\gamma_5s_c(x), \]

\[ D^a(x) \propto \tilde{D}^a(x) = \epsilon^{abc}u_b^T(x)C\gamma_5s_c(x), \]

\[ S^a(x) \propto \tilde{S}^a(x) = \epsilon^{abc}u_b^T(x)C\gamma_5d_c(x), \]

(3)

the \( a, b, c \) are color indices, the \( C \) is the charge conjugation matrix, the \( U^a(x), D^a(x) \) and \( S^a(x) \) are basic scalar diquark fields, while the \( \tilde{U}^a(x), \tilde{D}^a(x) \) and \( \tilde{S}^a(x) \) are the corresponding scalar two-quark currents. In this article, we take the isospin limit for the \( u \) and \( d \) quarks, and denote the fields \( U^a(x) \) and \( D^a(x) \) as \( Q^a(x) \).

For the general color antitriplet bilinear quark-quark fields \( q(x)\bar{q}(y) \) and color singlet bilinear quark-antiquark fields \( q(x)\bar{q}(y) \), where the flavor, color and spin indexes are not shown explicitly for simplicity, we can project them into a local and a nonlocal part, after bosonization, the two parts are translated into a basic quantum field and a bound state amplitude, respectively,

\[ q(x)\bar{q}(y) \rightarrow \mathbb{D}\left(\frac{x+y}{2}\right)\Gamma_D(x-y), \]

\[ q(x)\bar{q}(y) \rightarrow \mathbb{M}\left(\frac{x+y}{2}\right)\Gamma_M(x-y), \]

(4)

where the \( \mathbb{D}(x) \) and \( \mathbb{M}(x) \) denote the diquark and meson fields respectively, the \( \Gamma_D(x) \) and \( \Gamma_M(x) \) denote the corresponding Bethe-Salpeter amplitudes respectively [14, 15]. In Ref. [16], we study the structures of the pseudoscalar mesons \( \pi, K \) and the scalar diquarks \( U^a, D^a, S^a \) in the framework of the coupled rainbow Schwinger-Dyson equation and ladder Bethe-Salpeter equation using a confining effective potential, and observe that the dominant Dirac spinor structure of the Bethe-Salpeter amplitudes of the scalar diquarks is \( C\gamma_5 \). If we take the local limit for the nonlocal Bethe-Salpeter amplitudes, the dimension-1 scalar diquark fields \( U^a, D^a \) and \( S^a \) are recovered.

\[ \text{If we take the local limit in the scalar channels, the two-quark currents } \tilde{U}^a(x), \tilde{D}^a(x) \text{ and } \tilde{S}^a(x) \text{ are recovered.} \]
proportional to the dimension-3 scalar two-quark currents $\tilde{U}^a$, $\tilde{D}^a$ and $\tilde{S}^a$, respectively. A dimension-1 quantity $\Lambda$ can be introduced to represent the hadronization $\tilde{U}^a \approx \Lambda^2 U^a$, $\tilde{D}^a \approx \Lambda^2 D^a$ and $\tilde{S}^a \approx \Lambda^2 S^a$.

The attractive interaction of one-gluon exchange favors formation of the diquarks in color antitriplet $\mathbf{3}_c$, flavor antitriplet $\mathbf{3}_f$ and spin singlet $1_s$ $[3, 17]$. Lattice QCD studies of the light flavors indicate that the strong attraction in the scalar diquark channels favors the formation of good diquarks, the weaker attraction (the quark-quark correlation is rather weak) in the axial-vector diquark channels maybe form bad diquarks, the energy gap between the axial-vector and scalar diquarks is about $\frac{2}{3}$ of the $\Delta$-nucleon mass splitting, i.e. $\approx 0.2$ GeV $[18]$, which is also expected from the hypersplitting color-spin interaction $\frac{1}{m_m} \vec{T}_i \cdot \vec{T}_j \cdot \vec{\sigma}_i \cdot \vec{\sigma}_j$, $[3, 17]$. On the other hand, the studies based on the random instanton liquid model indicate that the instanton induced quark-quark interactions are weakly repulsive in the vector and axial-vector channels, strongly repulsive in the pseudoscalar channel, and strongly attractive in the scalar and tensor channels $[19]$. So it is sensible to use the scalar diquark fields to construct the tetraquark currents.

The two-point correlation functions $\Pi_i(p)$ can be written as

$$\Pi_i(p) = i \int d^4x \ e^{ipx} \langle 0 | T[J_i(x) J_i(0)] | 0 \rangle,$$

where the current $J_i(x)$ denotes $J_{f_0}(x)$, $J_{a_0}(x)$, $J_\kappa(x)$ and $J_\sigma(x)$.

We can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $J_i(x)$ into the correlation functions $\Pi_i(p)$ to obtain the hadronic representation $[7, 8]$. Isolating the ground state contributions from the pole terms of the nonet scalar mesons, we obtain the results,

$$\Pi_i(p) = \frac{\lambda_i^2}{M_i^2 - p^2} + \sum_h \frac{\langle 0 | J_i(0) | h(p) \rangle \langle h(p) | J_i(0) | 0 \rangle}{M_h^2 - p^2},$$

where the $M_i$ are the ground state masses, the $\lambda_i$ are corresponding pole residues defined by $\langle 0 | J_i(0) | S(p) \rangle = \lambda_i$, the thresholds $\Theta_i^2 = (2m_s + 2m_q)^2$, $(m_s + 3m_q)^2$, $(4m_q)^2$ in the channels $f_0/a_0(980)$, $\kappa(800)$, $\sigma(600)$ respectively, the $s_i^0$ are the thresholds for the higher resonances and continuum states $| h \rangle$, and the $\rho_i^h(s)$ are the corresponding hadronic spectral densities.

We introduce the following new Lagrangian $\mathcal{L}$,

$$\mathcal{L} = \frac{1}{2} \mathcal{D}_\mu U^\dagger \mathcal{D}^\mu U + \frac{1}{2} \mathcal{D}_\mu D^\dagger \mathcal{D}^\mu D + \frac{1}{2} \mathcal{D}_\mu S^\dagger \mathcal{D}^\mu S - \frac{1}{2} m_U^2 U^\dagger U - \frac{1}{2} m_D^2 D^\dagger D - \frac{1}{2} m_S^2 S^\dagger S + \cdots,$$
where $D_\mu = \partial_\mu - ig_\mu G_\mu$, and carry out the operator product expansion with the basic diquark fields $U, D$ and $S$ instead of the quark fields $u, d$ and $s$, and we have neglected the terms concerning the heavy diquark fields in the Lagrangian. In the QCD, the basic quantum fields are the quark and gluon fields, the attractive interactions in the color antitriplet $\bar{3}_c$, flavor antitriplet $\bar{3}_f$ and spin singlet $1_s$ quark-quark channels favor formation of the scalar diquarks, we can absorb some effects of the quark-gluon interactions into the effective diquark masses, which are characterized by the correlation length $L \sim \frac{1}{D}$. At the distance $l > L$, the $\bar{3}_c$ diquark state combines with one quark or one $3_c$ antidiquark to form a baryon state or a tetraquark state, while at the distance $l < L$, the $\bar{3}_c$ diquark states dissociate into asymptotic quarks and gluons gradually, the strength of the quark-quark correlations is very weak. Just like the quarks, the diquarks have three colors, and can be gauged with the same $SU(3)$ color group to embody the residual quark-gluon interactions.

In calculations, we take into account all diagrams like the typical ones shown in Fig.1, introduce new vacuum diquark condensates $\langle \bar{Q}Q \rangle$ and $\langle \bar{S}S \rangle$ besides the gluon condensate to parameterize the nonperturbative QCD vacuum, and consider the vacuum condensates up to dimension four. In the QCD sum rules, the high dimensional vacuum condensates are usually suppressed by large denominators or additional powers of the inverse Borel parameters $\frac{1}{T^2}$, the net contributions are very small. For example, in the present case the contributions of the dimension-6 vacuum condensates can be estimated as $\langle \bar{Q}Q \rangle \langle \frac{\alpha_s G G}{\pi} \rangle \frac{1}{T^2} \approx \frac{0.000007 \text{ GeV}^{-6}}{T^2}$, which is a tiny quantity. If additional suppressions originate from the denominators are taken into account, the contributions are even smaller, and can be safely neglected.
Once the analytical results are obtained, then we can take the dualities below the thresholds $s_0$ and perform the Borel transform with respect to the variable $P^2 = -p^2$, finally we obtain the following sum rules,

$$\chi_i^2 e^{-\frac{M_i^2}{\pi^2}} = \int_{\Delta_i^2}^{s_0} ds e^{-\frac{s}{\pi^2}} \rho_i(s), \quad (8)$$

where the $\rho_i(s)$ denote the QCD spectral densities $\rho_{f_0/a_0}(s)$, $\rho_\sigma(s)$ and $\rho_\kappa(s)$,

$$\rho_{f_0/a_0}(s) = \frac{3}{16\pi^2} + 2\langle QQ\rangle \delta(s - m_Q^2) - \frac{m_Q^2}{96T^4} \left(\frac{\alpha_sGG}{\pi}\right) \int_0^1 dx \frac{1}{x^3} \delta(s - \bar{m}_Q^2),$$

$$\rho_\sigma(s) = \frac{3}{16\pi^2} + 2\langle SS\rangle \delta(s - m_S^2) - \frac{m_S^2}{96T^4} \left(\frac{\alpha_sGG}{\pi}\right) \int_0^1 dx \frac{1}{x^3} \delta(s - \bar{m}_S^2),$$

$$\rho_\kappa(s) = \frac{3}{16\pi^2} + \langle QQ\rangle \delta(s - m_Q^2) + \langle SS\rangle \delta(s - m_S^2) - \frac{1}{192T^4} \left(\frac{\alpha_sGG}{\pi}\right) \int_0^1 dx \left[ \frac{m_Q^2}{x^3} + \frac{m_S^2}{1-x^3} \right] \delta(s - \bar{m}_Q^2), \quad (9)$$

$\bar{m}_{Q/S}^2 = \frac{m_{Q/S}^2}{x(1-x)}$, $\bar{m}_S^2 = \frac{m_S^2}{x} + \frac{m_S^2}{1-x}$, $\Delta_i^2 = (2m_Q)^2$, $(m_Q + m_S)^2$, $(2m_S)^2$ in the channels $f_0/a_0(980)$, $\kappa(800)$, $\sigma(600)$ respectively, and the $T^2$ is the Borel parameter. The threshold parameters $\Delta_i^2$ are different from the corresponding $\Theta^2$ in Eq.(6), because we absorb some QCD interactions into the effective diquark masses.

Differentiate Eq.(8) with respect to $\frac{1}{\pi^2}$, then eliminate the pole residues $\lambda_i$, we can obtain the sum rules for the masses of the nonet scalar mesons,

$$M_i^2 = \frac{\int_{\Delta_i^2}^{s_0} ds \frac{d}{ds} e^{-\frac{s}{\pi^2}} \rho_i(s)e^{-\frac{s}{\pi^2}}}{\int_{\Delta_i^2}^{s_0} ds \rho_i(s)e^{-\frac{s}{\pi^2}}}. \quad (10)$$

### 3 Numerical Results

We estimate the vacuum diquark condensates $\langle QQ\rangle$ and $\langle SS\rangle$ with the assumption of the vacuum saturation, which works well in the large $N_c$ limit,

$$\langle QQ\rangle = \frac{\langle \bar{q}q\rangle \langle ss\rangle}{6} = \Lambda^4 \langle QQ\rangle,$$

$$\langle SS\rangle = \frac{\langle \bar{q}q\rangle^2}{6} = \Lambda^4 \langle SS\rangle,$$ 

where the $\Lambda$ is a quantity has dimension of mass and can be taken as the confinement energy scale $\Lambda = 0.3 \text{ GeV}$. At the energy scale $\mu = 1 \text{ GeV}$, $\langle ss\rangle = 0.8 \langle \bar{q}q\rangle$, $\langle \bar{q}q\rangle = -(0.24 \text{ GeV})^3$, $\langle \alpha_sGG \rangle = (0.33 \text{ GeV})^4$ [7, 8], and $\langle QQ\rangle = 0.8 \langle SS\rangle = 0.0031 \text{ GeV}^2$.

The quark condensates play a special role being responsible for the spontaneous breaking of the chiral symmetry, and relate with the masses and decay constants of
the light pseudoscalar mesons through the Gell-Mann-Oakes-Renner relation [20]. The values of other vacuum condensates, such as the mixed condensates, the four quark condensates and the gluon condensates, cannot be obtained from the first principles, we usually calculate them with the lattice QCD, the instanton models, or determine them empirically by fitting the QCD sum rules to the experimental data. In this article, we introduce the diquark condensates $\langle \bar{Q}Q \rangle$ and $\langle \bar{S}S \rangle$ to parameterize the nonperturbative QCD vacuum, and assume that they relate with the four quark condensates (therefore they have implicit relations with the spontaneous breaking of the chiral symmetry), and take the dimension-one parameter $\Lambda$ to be the confinement energy scale, as the scalar mesons $f_0(980)$, $a_0(980)$, $\kappa(800)$ and $\sigma(600)$ are bound states which consist of the confined quarks and gluons, and some parameters are needed to embody the confinement. On the other hand, we can understand the parameter $\Lambda = 0.3 \text{ GeV}$ as a fitted value, which happens to be the confinement energy scale, the crude estimation works well.

We take the updated values of the diquark masses from the QCD sum rules for consistency, where the interpolating currents $\tilde{U}^a(x)$, $\tilde{D}^a(x)$ and $\tilde{S}^a(x)$ are used [21], $m_Q = 0.46 \text{ GeV}$ and $m_S = 0.40 \text{ GeV}$; the scalar diquarks were originally studied with the QCD sum rules about twenty years ago [22]. There have been several theoretical approaches to estimate the diquark masses, for example, the simple constituent diquark mass plus hyperfine spin-spin interaction model [23]. The $f_0(980)$ and $a_0(980)$ are well established, and the existence of the $\sigma(600)$ meson is confirmed, although there are controversy about its mass and width, the values listed in the Review of Particle Physics are $(400 - 1200) \text{ MeV}$ and $(600 - 1000) \text{ MeV}$ respectively [5]. As far as the $\kappa(800)$ are concerned, there are still controversy about their existence, we take them as the $S$-wave isospin-$\frac{1}{2}$ $K\pi$ resonance with the Breit-Wigner mass about 850 MeV. The E791 collaboration observed a low-mass scalar $K\pi$ resonance with the Breit-Wigner mass $M = (797 \pm 19 \pm 43) \text{ MeV}$ and width $\Gamma = (410 \pm 43 \pm 87) \text{ MeV}$ respectively in the decay $D^+ \rightarrow K^-\pi^+\pi^+$ [24], and the BES collaboration observed a clear low mass enhancement in the invariant $K\pi$ mass distribution in the decay $J/\psi \rightarrow K^*(892)K^+\pi^-$ with the Breit-Wigner mass $M = (878 \pm 23^{+64}_{-52}) \text{ MeV}$ and width $\Gamma = (499 \pm 52^{+55}_{-87}) \text{ MeV}$, respectively [25]. Recently, the BES collaboration reported the charged $\kappa(800)$ in the decay $J/\psi \rightarrow K^*(892)^\pm K_s\pi^\mp$ with the Breit-Wigner mass $M = (826 \pm 49^{+49}_{-34}) \text{ MeV}$ and width $\Gamma = (449 \pm 156^{+144}_{-81}) \text{ MeV}$, respectively [26]. It is sensible to estimate $M_{f_0/a_0} - M_\kappa = m_s - m_q = 0.14 \text{ GeV}$. On the other hand, the QCD sum rules for the tetraquark states indicate that $M_\kappa = (0.80 - 0.88) \text{ GeV}$ and $M_\sigma = (0.72 - 0.80) \text{ GeV}$ [27].

Assuming the energy gap between the ground and first radial excited tetraquark states is about 0.5 GeV, we can tentatively determine the threshold parameters $s_{f_0/a_0} = (1.0 + 0.5)^2 \text{ GeV}^2$, $s_\kappa = (0.85 + 0.5)^2 \text{ GeV}^2$, and $s_\sigma = (0.75 + 0.5)^2 \text{ GeV}^2$.

The convergence behavior of the operator product expansion is very good, the contributions from the different terms have the hierarchy: perturbative-term $> \langle \bar{Q}Q \rangle \gg \langle \frac{m_Q}{2} \rangle$. In calculation, we take uniform minimum value for the Borel
parameters $T_{\text{min}}^2 = \mu^2 = 1.0 \text{ GeV}^2$. The perturbative continuum $\frac{3}{16\pi^2} \Theta(s - s_0)$ is suppressed by the factor $e^{-\frac{s}{T^2}}$, the contributions from the pole terms are very large, see Fig.2. In this article, we take uniform maximum value for the Borel parameters, $T_{\text{max}}^2 = 1.9 \text{ GeV}^2$, the contributions from the pole terms are about $(61 - 84)\%$, $(54 - 79)\%$ and $(51 - 75)\%$ in the channels $f_0(980)/a_0(980)$, $\kappa(800)$ and $\sigma(600)$, respectively. The two criteria (pole dominance and convergence of the operator product expansion) of the QCD sum rules are well satisfied.

If the conventional quark currents are chosen, the multiquark states i.e. tetraquark states, pentaquark states, hexaquark states, etc, have the spectral densities $\rho \sim s^n$ with the largest $n \geq 4$, the integral $\int_0^\infty s^n e^{-\frac{s}{T^2}} ds$ converges slowly [9]. If one don’t want to release the criterion of pole dominance, we have to either postpone the threshold parameter $s_0$ to very large value or choose very small value for the Borel parameter $T_{\text{max}}^2$. With large value of the threshold parameter $s_0$, for example, $s_0 \gg M_{\text{gr}}^2$, here gr stands for the ground state, the contributions from the high resonance states and continuum states are included in, we cannot use single-pole (or ground state) approximation for the spectral densities; on the other hand, with very small value of the Borel parameter $T_{\text{max}}^2$, the Borel window $T_{\text{max}}^2 - T_{\text{min}}^2$ shrinks to zero or very small values.

The numerical values of the masses and pole residues are presented in Table 1 and Figs.3-4. From Table 1, we can see that the present predictions are compatible with (or not in conflict with) the experimental data [5, 24, 25, 26] and theoretical estimations [27].

The scalar tetraquark currents $J_{f_0}/a_0(x)$, $J_{\kappa}(x)$ and $J_{\sigma}(x)$ maybe have non-vanishing couplings with the scattering states $\pi\pi$, $K\bar{K}$, $K\pi$, $K\eta$, $\pi\eta$, $\eta\eta$, etc, for example,

$$
\langle 0|J_{f_0}(0)|\pi\pi(p)\rangle = \lambda_{f_0\pi\pi}, \quad \langle 0|J_{f_0}(0)|K\bar{K}(p)\rangle = \lambda_{f_0K\bar{K}}. \quad (12)
$$

If the couplings denoted by the $\lambda_{f_0\pi\pi}$ and $\lambda_{f_0K\bar{K}}$ are strong enough, the contaminations from the continuum states are expected to be large. In the following, we study the contributions of the intermediate pseudoscalar meson loops to the correlation
function $\Pi_{f_0}(p)$ in details as an example,

$$\Pi_{f_0}(p) = \frac{\lambda_{f_0}^2}{M_{f_0}^2 - p^2} - i \frac{\lambda_{f_0}}{p^2 - M_{f_0}^2} g_{f_0\pi\pi} \Sigma_{\pi\pi}(p) g_{f_0\pi\pi} \frac{\lambda_{f_0}}{p^2 - M_{f_0}^2}$$

$$- \frac{i \lambda_{f_0}}{p^2 - M_{f_0}^2} g_{f_0KK} \Sigma_{KK}(p) g_{f_0KK} \frac{\lambda_{f_0}}{p^2 - M_{f_0}^2}$$

$$- \frac{\lambda_{f_0}}{p^2 - M_{f_0}^2} g_{f_0\pi\pi} \Sigma_{\pi\pi}(p) T_{\pi\pi\to\pi\pi} \Sigma_{\pi\pi}(p) g_{f_0\pi\pi} \frac{\lambda_{f_0}}{p^2 - M_{f_0}^2}$$

$$- \frac{\lambda_{f_0}}{p^2 - M_{f_0}^2} g_{f_0KK} \Sigma_{KK}(p) T_{KK\toKK} \Sigma_{KK}(p) g_{f_0KK} \frac{\lambda_{f_0}}{p^2 - M_{f_0}^2}$$

$$- \frac{\lambda_{f_0}}{p^2 - M_{f_0}^2} g_{f_0KK} \Sigma_{KK}(p) T_{KK\toKK} \Sigma_{KK}(p) g_{f_0KK} \frac{\lambda_{f_0}}{p^2 - M_{f_0}^2} + \cdots \quad (13)$$

where

$$\Sigma_{\pi\pi}(p) = \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_{\pi}^2} \frac{1}{(p-q)^2 - m_{\pi}^2}$$

$$\Sigma_{KK}(p) = \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_{K}^2} \frac{1}{(p-q)^2 - m_{K}^2} \quad (14)$$

the $g_{f_0\pi\pi}$, $g_{f_0KK}$ are the strong coupling constants between the $f_0(980)$ and the pseudoscalar meson pairs $\pi\pi$, $KK$ respectively, and the $T_{\pi\pi\to\pi\pi}$, $T_{\pi\pi\toKK}$, $T_{KK\toKK}$ are the scattering amplitudes among the pseudoscalar meson pairs $\pi\pi$ and $KK$. The couplings $\lambda_{f_0\pi\pi}$ and $\lambda_{f_0KK}$ are complicated functions of the $M_{f_0}$, $p^2$, $\lambda_{f_0}$, $T_{\pi\pi\to\pi\pi}$, $T_{\pi\pi\toKK}$, $T_{KK\toKK}$, $g_{f_0\pi\pi}$ and $g_{f_0KK}$, the explicit expressions are difficult to obtain. We should bear in mind that the intermediate meson loops contribute a self-energy to the scalar meson $f_0(980)$, and therefore the scalar meson $f_0(980)$ develops a Breit-Wigner width. In fact, the scalar mesons $f_0/a_0(980)$, $\kappa(800)$ and $\sigma(600)$ below 1 GeV can be generated dynamically from the unitarized scattering amplitudes of the pseudoscalar mesons [28]. We can take into account those meson loops effectively by taking the following replacement for the hadronic spectral density,

$$\delta\left(s - M_{f_0}^2\right) \rightarrow \frac{1}{\pi} \frac{\sqrt{s} \Gamma_{f_0}}{\left(s - M_{f_0}^2\right)^2 + s \Gamma_{f_0}^2}, \quad (15)$$

here we neglect the complicated renormalization procedure, take the physical values, and ignore the energy scale dependence of the mass $M_{f_0}$ and pole residue $\lambda_{f_0}$ for simplicity; the approximation works well. The QCD sum rules for other scalar mesons are treated with the same routine. In Ref.[29], Dai et al perform the renormalization procedure in details to take into account the contributions from the continuum states.
Table 1: The masses and the pole residues of the nonet 0++ tetraquark states. The star denotes the modified masses from the sum rules where the finite widths shown in the bracket are taken into account in the hadronic spectral densities.

| tetraquark states | mass (GeV) | pole residue (GeV) |
|-------------------|------------|--------------------|
| $f_0(980)/a_0(980)$ | 0.94 – 1.04 | 0.16 – 0.17 |
| $\kappa(800)$ | 0.84 – 0.93 | 0.15 – 0.16 |
| $\sigma(600)$ | 0.77 – 0.84 | $\approx 0.15$ |
| $f_0(980)/a_0(980)$ | 0.94 – 1.04* | |
| $\kappa(800)$ | 0.79 – 0.88* | |
| $\sigma(600)$ | 0.77 – 0.84* | |

The widths listed in the Review of Particle Physics are $\Gamma_{f_0} = (40 – 100)$ MeV, $\Gamma_{a_0} = (50 – 100)$ MeV, $\Gamma_{\kappa} = (550 \pm 34)$ MeV and $\Gamma_{\sigma} = (400 – 1200)$ MeV, respectively [5]. Taking into account the finite widths, we can obtain the modified masses from the corresponding QCD sum rules, which are shown in Table 1 and Fig.5. In calculation, we observe that the narrow width $\Gamma_{f_0/a_0}$ modifies the mass $M_{f_0/a_0}$ slightly and the effect can be neglected safely, while the broad widths $\Gamma_{\kappa}$ and $\Gamma_{\sigma}$ reduce the masses $M_{\kappa}$ and $M_{\sigma}$ about 55 MeV and $(0 – 75)$ MeV, respectively. Comparing with the experimental data [5, 24, 25, 26], the modified masses are better.

The scalar tetraquark currents $J_{f_0/a_0}(x)$, $J_{\kappa}(x)$ and $J_{\sigma}(x)$ maybe also have non-vanishing couplings with the higher resonances $a_0(1450)$, $f_0(1370)$, $K^{*}(1430)$, $f_0(1500)$ and $f_0(1710)$, which are supposed to be the $^3P_0$ $q\bar{q}$ states or glueballs, the couplings should be very small. Furthermore, the threshold parameters are $s_0 = 2.25$ GeV$^2$, 1.82 GeV$^2$ and 1.56 GeV$^2$ in the channels $f_0/a_0(980)$, $\kappa(800)$ and $\sigma(600)$, respectively, the contaminations should be very small.

4 Conclusions

In this article, we take the scalar diquarks as the point particles and describe them as the basic quantum fields, then introduce the $SU(3)$ color gauge interactions, and construct the tetraquark currents which consist of the scalar fields to study the nonet scalar mesons as tetraquark states with the new QCD sum rules. The numerical values are compatible with (or not in conflict with) the experimental data and theoretical estimations. Comparing with the conventional quark currents, the diquark currents have the outstanding advantage to satisfy the two criteria of the QCD sum rules more easily, the new sum rules can be extended to study other multiquark states.
Figure 2: The contributions from the pole terms. The $A$, $B$ and $C$ correspond to the channels $f_0(980)/a_0(980)$, $\kappa(800)$ and $\sigma(600)$ respectively.

Figure 3: The masses of the nonet scalar tetraquark states. The $A$, $B$ and $C$ correspond to the channels $f_0(980)/a_0(980)$, $\kappa(800)$ and $\sigma(600)$ respectively.
Figure 4: The pole residues of the nonet scalar tetraquark states. The $A$, $B$ and $C$ correspond to the channels $f_0(980)/a_0(980)$, $\kappa(800)$ and $\sigma(600)$ respectively.

Figure 5: The modified masses of the nonet scalar tetraquark states. The $A$, $B$ and $C$ correspond to the channels $f_0(980)/a_0(980)$, $\kappa(800)$ and $\sigma(600)$ respectively. The values in the bracket denote the finite widths in the hadronic spectral densities.
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