Optimal control of microalgae growth using linear quadratic regulator method with firefly algorithm optimization

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Abstract. Microalgae is an important potential resource for various industrial applications. In the industry, there are a variety of processed products from algae biomass. Lipids from microalgae used for biofuel, humans and animals in food supplements while in the pharmaceutical, lipids used as a vitamin or protein. The other application of microalgae captures $CO_2$ and wastewater treatment. This research discusses the optimal control of microalgae growth with variable control are carbon dioxide and nutrients. In this case, the methodology is the Linear Quadratic Regulator method. In the Linear Quadratic Regulator method, there are weights in which the value is determined by trial and error in the matrices Q and R. The method of optimizing the weight value is required to optimize the weight. In this paper, the firefly algorithm is used to optimize the matrix values for Q and R. Application of firefly algorithm is able to optimally increase microalgae growth, so that the biomass produced is also abundant. The using of Firefly Algorithms optimization in the LQR method can increase algal concentration by 63.3% of the initial concentration.

1. Introduction
Indonesia is a large country that has a large area of water. This abundant natural resource has not used maximally. Natural resources continue to be exploited is fossil-based energy that the availability is increasingly thinning. Thus, alternative energy is needed to replace fossils able to meet user needs. Microalgae is a potential resource able to produce oil that can be used as biofuel as renewable energy environmentally friendly. Moreover, microalgae processing can use as beauty products to being a biofuel, food supplements, and etc. Apart from microalgae are easy to be cultivated to produce microalgae oil of 16,103,453 kl/year only 274 ha of land needed (Abishek, 2014). Microalgae harvested at the time low growth will result in suboptimal biomass (Danquah, et al, 2009). Therefore, the development of microalgae as a source of raw oil substitutes petroleum has a reliable prospect. There are several kinds of researches regarding microalgae, from optimizing control in algal growth systems to production lipids in microalgae. In the previous research, the researcher explained the model of lipid production in microalgae to get optimal value in the variable state (Prismahardi, Mardlijah, 2018) with optimal control of lipid production in microalgae, the controlling factor is the substrate nutrition and carbon dioxide. As same as Mardlijah, et al. (2015) research about concerning optimal control on the
flow of nutrients used to get algal growth maximum. Meanwhile, Mairret, etc. (2011) research used nitrogen limitations and carbon dioxide as optimal flame parameters. The use of nitrogen and carbon dioxide will use again in the process of algal growth.

From the explanation above, there are many studies about microalgae. Therefore, in this paper, the researchers conducted the research on the optimal control on microalgae growth with variable control of carbon dioxide flow and nutrients to produce maximum algal production. Besides, this study used LQR (Linear Quadratic Regulator) method in completing the case of study. Linear Quadratic Regulator is a control method with a trial error that is simpler because performed on two variables namely Q and R. In this method, there is an objective function which uses a measure of how much system performance. Therefore, the Linear formulation Quadratic Regulator has the ease of analyzing the system using numerical analysis and application (Naidu, 2002). Besides using the Linear Quadratic Regulator method, the researcher also uses the firefly algorithm to optimize the weight value of the matrix Q and R. This algorithm has several advantages including the firefly algorithm has a faster convergence than other algorithms such as genetic algorithms and PSO. The firefly algorithm only influenced by two parameters while in PSO algorithm and genetics influenced by three and four parameters (Mardlijah, 2013).

2. Mathematics Model of Algae Growth

In this paper, the researchers used the Thornton model as the mathematical model of algal growth (Thornton, 2010). The model introduced by Anthony Thornton to describe algal growth influenced by carbon dioxide concentration, nutrients, and glucose. Based on the Thornton model, algal growth illustrated in Figure 1. Carbon dioxide is pumped into water and turned into glucose through photosynthesis. Thus, the nutrients found in factory wastewater and glucose form algae. Furthermore, the algae and glucose stored assumed to be reduce in the presence of death from the harvest. Energy is not only stored in glucose, but glucose is also more complex and oil. The mathematical model of algal growth illustrated in the following figure 1.

![Figure 1. Algae Growth](Thornton, 2010)

Algae production modeled with the concentration of algae (A), nutrients (M), glucose (S), and carbon dioxide (C) in ponds. Thornton assumes that the pool stirred with good and algae growth is very slow. The concentrations above are independent of all state variables and only depends on the time (t). The entry of nutrients and carbon dioxide denoted by $I_m$ and $I_c$. These algae are starving at "rate death" ($d_r$) and harvested ($h_r$), both of which reduce the number of algae and its glucose stored in algae. Furthermore, the glucose produced ($\alpha_s$) is constant. The mathematics models of microalgae growth are as follows (Thornton, 2010):
\[ \dot{A} = \frac{\rho_{\text{max}} M(t)}{M(t) + M_{\text{turn}}} - \alpha_A S(t) - (d_r + h_r) A(t) \quad (1) \]

\[ \dot{M} = -\frac{\rho_{\text{max}} M(t)}{M(t) + M_{\text{turn}}} k_2 \alpha_A S(t) + I_m(t) \quad (2) \]

\[ \dot{S} = \alpha_s C(t) - \frac{\rho_{\text{max}} M(t)}{M(t) + M_{\text{turn}}} k_3 \alpha_A S(t) - (d_r + h_r) S(t) \quad (3) \]

\[ \dot{C} = -k_1 \alpha_s C(t) + I_c(t) \quad (4) \]

### Table 1. Variables and Parameters (Thronton, 2010)

| Parameters                        | Value  |
|-----------------------------------|--------|
| \(\alpha_A\)                     | Rate constant for biomass growth | 10.2   |
| \(\rho_{\text{max}}\)            | maximal nutrient concentration inside algae | 0.4    |
| \(d_r\)                          | algae death rate | 0.46   |
| \(h_r\)                          | algae harvest rate | 2      |
| \(M_{\text{turn}}\)              | half saturation constant for nutrient concentration inside algae | 4      |
| \(I_m\)                          | inflow of nutrient | 4.2    |
| \(\alpha_S\)                     | rate constant for photosynthesis | 67.6   |
| \(k_1\)                          | conversion rate of \(\text{CO}_2\) into \((\text{CH}_2\text{O})_6\) | 0.4    |
| \(k_2\)                          | conversion rate of nutrients into dry algae | 0.05   |
| \(k_3\)                          | conversion rate of \((\text{CH}_2\text{O})_6\) into dry algae | 0.05   |
| \(I_c\)                          | inflow of carbon dioxide | 4      |

### 3. Optimal Control of LQR

LQR is a control system that consists of a system and system gain feedback with maximizing algal growth as an objective function as follows:

\[
J = \frac{1}{2} x^T P_c x + \frac{1}{2} \int_{t_0}^{t_f} (x^T Q_c x + u^T R_c u) dt
\]

\(Q_c\) and \(P_c\) are semi-definite positive, \((Q_c \geq 0, P_c \geq 0)\), so \(Q_c\) and \(P_c\) have values non-negative eigenvalues with \(x^T Q_c x\) and \(x^T P_c x\) are not negative for all \(x(t)\). \(R_c\) is definite positive matrix where \(R_c > 0\), means that has a positive eigenvalue. Thus, \(u^T R_c u > 0\) for all \(u(t) \neq 0\). Whereas \(\frac{1}{2} x^T P_c x\) is the final state that will be Optimal Cost that depends on the selection of related Quadratic Performance indexes with the control. The purpose of study is to get a flow of nutrients and optimal carbon dioxide so that it can maximize the concentration of algae such that algal growth can be maximum. Mathematically, this problem is maximizing objective functions.

\[
J = \frac{1}{2} \int_{t_0}^{t_f} (x^T Q_x + R I_m^2(t) + R I_c^2(t)) dt
\]

\(I_m\) and \(I_c\) are control variable and optimal controls, \(R\) is an element of the weight control matrix.
4. Linear Quadratic Regulator Principle

The first step is to form the hamiltonian function from an objective function, the hamiltonian function was given by:

\[ H(A, M, S, C, I_m, I_c, \lambda) = x^t Q x + R I_m^2(t) + R I_c^2(t) + \Sigma_{i=1}^4 \lambda_i (A x + B u) \]

\[ = x^t Q x + R I_m^2(t) + R I_c^2(t) + \lambda_1 (\rho_{\text{max}} \frac{M}{M + M_{\text{turn}}}) S - (d_r + h_r) A \]

\[ - \lambda_2(t) [-k_2 \rho_A (\rho_{\text{max}} \frac{M}{M + M_{\text{turn}}}) S + I_m(t)] \]

\[ + \lambda_3(t) [\alpha_S C - k_3 \rho_A (\rho_{\text{max}} \frac{M}{M + M_{\text{turn}}}) S - (d_r + h_r) S \]

\[ + \lambda_4(t) [-k_1 \alpha_S C + I_c(t)] \]

So, the state function is given by:

\[ \frac{\partial H}{\partial \lambda_1} = \alpha_A (\rho_{\text{max}} \frac{M}{M + M_{\text{turn}}}) S - (d_r + h_r) A \]

\[ \frac{\partial H}{\partial \lambda_2} = -k_2 \rho_A (\rho_{\text{max}} \frac{M}{M + M_{\text{turn}}}) S + I_m(t) \]

\[ \frac{\partial H}{\partial \lambda_3} = \alpha_S C - k_3 \rho_A (\rho_{\text{max}} \frac{M}{M + M_{\text{turn}}}) S - (d_r + h_r) S \]

\[ \frac{\partial H}{\partial \lambda_4} = -k_1 \alpha_S C + I_c(t) \]

and the costate Function is given by:

\[ \frac{d \lambda_1}{dt} = -\frac{\partial H}{\partial A} = (d_r + h_r) \lambda_1 \]

\[ \frac{d \lambda_2}{dt} = -\frac{\partial H}{\partial M} = -[(\alpha_A \rho_{\text{max}} S \lambda_1) (\frac{M + M_{\text{turn}} - M}{(M + M_{\text{turn}})^2}) - (k_2 \alpha_A \rho_{\text{max}} S \lambda_2) (\frac{M + M_{\text{turn}} - M}{(M + M_{\text{turn}})^2})] \]

\[ - (k_3 \alpha_A \rho_{\text{max}} S \lambda_3) (\frac{M + M_{\text{turn}} - M}{(M + M_{\text{turn}})^2}) \]

\[ = \frac{\alpha_A \rho_{\text{max}} S}{(M + M_{\text{turn}})} (-\lambda_1 + k_2 \lambda_2 + k_3 \lambda_3) + \frac{\alpha_A \rho_{\text{max}} M S}{(M + M_{\text{turn}})} (\lambda_1 - k_2 \lambda_2 + k_3 \lambda_3) \]

\[ \frac{d \lambda_3}{dt} = -\frac{\partial H}{\partial S} = -[\alpha_A (\rho_{\text{max}} M + M_{\text{turn}})] \lambda_1 - k_2 \rho_A (\rho_{\text{max}} \frac{M}{M + M_{\text{turn}}}) \lambda_2 \]

\[ - k_3 \rho_A (\rho_{\text{max}} \frac{M}{M + M_{\text{turn}}}) \lambda_3 - (d_r + h_r) \lambda_3] \]

\[ \frac{d \lambda_4}{dt} = -\frac{\partial H}{\partial C} = -[\alpha_S \lambda_3 - k_1 \alpha_S \lambda_4] \]

Furthermore, determine the stationary conditions to get the optimal control equation:

\[ \frac{\partial H}{\partial I_m(t)} = 0 \]
\[
\begin{align*}
0 &= RI_m(t) + \lambda_2(t) \\
I_m(t) &= -R^1\lambda_2(t) \\
\frac{\partial H}{\partial I_c(t)} &= 0 \\
0 &= RI_c(t) + \lambda_4(t) \\
I_c(t) &= -R^1\lambda_4(t)
\end{align*}
\]

With limit values \(0 < I_m(t) \leq 4.2\) and \(0 < I_c(t) \leq 4\). To find \(\lambda(t)\) used Riccati equation in \textit{Matlab}. After find the optimal control of LQR method, in the next sub-chapter is simulation with \textit{Matlab}.

5. Firefly Algorithm
Firefly algorithm (Firefly Algorithm) is an inspired metaheuristic algorithm by nature that is optimization based on the social behavior of flashing fireflies which is found in the tropics. In this study, the firefly algorithm is used to optimize the Q and R matrix values in the Linear Quadratic Regulator method.

5.1. Tug
Based on the firefly algorithm, the shape of the firefly attraction function is the function decreases monotonically as follows:

\[
\beta(r) = \beta_0 \ast \exp(-\gamma r^m) 
\]

where \(m \geq 1\) means \(r\) is the distance between two fireflies, \(\beta_0\) is the initial attraction at \(r = 0\), and \(\gamma\) is a coefficient absorption which controls the decrease in the intensity of the light whose value ranges from 0 to 1. (Xin She Yang, 2010).

5.2. Distance
The distance between the two firefly \(i\) and \(j\) at position \(x_i\) and \(x_j\) can be determined respectively as Cartesian or Euclidean distance as follows:

\[
r_{ij} = \|x_i - x_j\| = \sqrt{\sum_{k=1}^{d}(x_{i,k} - x_{j,k})^2}
\]

Where \(x_{i,k}\) is the \(k\)-th of the spatial coordinates \(x_i\) of the \(i\) and \(d\) firefly is the number of dimensions (Xin She Yang, 2010).

5.3. Movement
Movement of firefly to-\(i\) who are interested in firefly \(j\) are more interesting (sunnier) is given by the following equation:

\[
x_i = x_i + \beta_0 \ast \exp(-\gamma r_{ij}^2) \ast (x_j - x_i) + \alpha \ast (X - \frac{1}{2})
\]

Coefficient has \(\alpha\) ratio whose value ranges from 0 to 1, while \(X\) is random number generators are uniform distributed at intervals \((0.1)\) (Xin She Yang, 2010). Pseudo code from Firefly Algorithm which has been following formulated shown on figure 2.
6. Simulation Results
This section discusses the simulation results of microalgae growth models with variables nutrition and carbon dioxide control. Initial conditions used to simulate Algae growth is $A_0 = 3$ gram, $M_0 = 0.4$ gram, $S_0 = 10$ gram and $C_0 = 5$ gram for 100 days.

In figure 3 shows the position of the fireflies that converge towards one point. The firefly algorithm simulation is used to find the combination of matrix weight values Q and R which
produce maximum objective functions. Thus, they can optimize microalgae growth.

Figure 4. Comparison of nutrient concentration with control and without control

In figure 4 shows the concentration of nutrients experienced an increase on the second day of observation by 0.835 grams, then decreased up to 0.813 grams on the 3rd day and stable with a concentration of 0.8 grams on the 3rd day until 100th day of observation.

Figure 5. Comparison of glucose concentration with control and without control

In figure 5 shows glucose has decreased the same as without control. Glucose reduction with control reached 2,356 grams from day 1 until the 2nd day of observation and stable with a concentration of 2,254 grams on the 3rd day until the 100th day of observation. This glucose decreases because glucose used as energy in the algae formation and photosynthesis process.
Figure 6. Comparison of carbon dioxide concentration with control and without control

In figure 6 carbon concentrations dioxide with control has decreased as the condition of carbon dioxide without control. The concentration of carbon dioxide under control has decreased to 0.243 grams. This condition is caused by carbon dioxide being used for the formation of glucose through process photosynthesis.

Figure 7. Comparison of algae concentration without control, with LQR control and LQR+Firefly
In figure 7 shows the comparison of uncontrolled microalgae concentrations, with LQR control and LQR control using Fire Algorithm. The simulation results show the concentration of algae without control continues to decrease. However, with LQR control of algae concentration increased to 3,754 grams and was stable at a concentration of 3,653 grams until the 100th day of observation. Algae concentration uses the Fire control Algorithm the LQR method increased by 4,921 grams and stable with a concentration of 4,872 grams. This result shows that with LQR control the algal concentration increases by 21.6% of the initial concentration, while using the Fire Algorithm Optimization on the LQR method can increase the concentration of algae by 63.3% percent from the initial concentration.

7. Conclusion
Microalgae concentration without control continues to decrease. The using of LQR control can increase microalgae concentration by 21.6% from the initial concentration. Meanwhile, the using of Firefly Algorithms optimization in the LQR method can increase microalgae concentration by 63.3% of the initial concentration. Further research can add the effect of temperature on the microalgae growth model that the growth of microalgae can be optimized.

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