Enhanced transmission versus localization of a light pulse by a subwavelength metal slit: Can the pulse have both characteristics?

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The existence of resonant enhanced transmission and collimation of light waves by subwavelength slits in metal films [for example, see T.W. Ebbesen et al., Nature (London) 391, 667 (1998) and H.J. Lezec et al., Science, 297, 820 (2002)] leads to the basic question: Can a light be enhanced and simultaneously localized in space and time by a subwavelength slit? To address this question, the spatial distribution of the energy flux of an ultrashort (femtosecond) wave-packet diffracted by a subwavelength (nanometer-size) slit was analyzed by using the conventional approach based on the Neerhoff and Mur solution of Maxwell’s equations. The results show that a light can be enhanced by orders of magnitude and simultaneously localized in the near-field diffraction zone at the nm- and fs-scales. Possible applications in nanophotonics are discussed.

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I. INTRODUCTION

In the last decade nanostructured optical elements based on scattering of light waves by subwavelength-size metal objects, such as particles and screen holes, have been investigated, intensively. The most impressive features of the optical elements are resonant enhancement and spatial localization of optical fields by the excitation of electron waves in the metal (for an example see, e.g. \cite{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25}). Recently, some nanostructures namely a single subwavelength slit, a grating with subwavelength slits and a subwavelength slit surrounded by parallel deep and narrow grooves attracted a particular attention of researchers. The study of resonant enhanced transmission and collimation of waves in close proximity to a single subwavelength slit acting as a microscope probe \cite{1, 2, 3} was connected with developing near-field scanning microwave and optical microscopes with subwavelength resolution \cite{11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25}. The resonant transmission of light by a grating with subwavelength slits and a subwavelength slit surrounded by grooves is an important effect for nanophotonics \cite{8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25}. The transmissitivity, on the resonance, can be orders of magnitude greater than out of the resonance. It was understood that the enhancement effect has a two-fold origin: First, the field increases due to a pure geometrical reason, the coupling of incident plane waves with waveguide mode resonances located in the slit, and further enhancement arises due to excitation of coupled surface plasmon polaritons localized on both surfaces of the slit (grating) \cite{10, 11, 12}. A dominant mechanism responsible for the extraordinary transmission is the resonant excitation of the waveguide mode in the slit giving a Fabry-Perot like behavior \cite{11}. In addition to the extraordinary transmission, a series of parallel grooves surrounding a nanometer-size slit can produce a micrometer-size beam that spreads to an angle of only few degrees \cite{3}. The light collimation, in this case, is achieved by the excitation of coupled surface plasmon polaritons in the grooves \cite{12}. At appropriate conditions, a single subwavelength slit flanked by a finite array of grooves can act as a ”lens” focusing a light \cite{13}. It should be noted, in this connection, that the diffractive spreading of a beam can be reduced also by using a structured aperture or an effective nanolens formed by self-similar linear chain of metal nanospheres \cite{14, 15}.

New aspects of the problem of resonantly enhanced transmission and collimation of light are revealed when the nanostructures are illuminated by an ultra-short (fs) light pulse \cite{16, 17, 18, 19, 20}. For instance, in the study \cite{16}, the unique possibility of concentrating the energy of an ultrafast excitation of an "engineered” or a random nanosystems in a small part of the whole system by means of phase modulation of the exciting fs-pulse was predicted. The study \cite{17} theoretically demonstrated the feasibility of nm-scale localization and distortion-free transmission of fs visible pulses by a single metal slit, and further suggested the feasibility of simultaneous super resolution in space and time of the near-field scanning optical microscopy (NSOM). The quasi-diffraction-free optics based on transmission of pulses by a subwavelength nano-slit has been suggested to extended the operation principle of a 2-D NSOM to the "not-too-distant” field regime (up to 0.5 wavelength) \cite{18}. Some interesting effects namely the pulse delay and...
long leaving resonant excitations of electromagnetic fields in the resonant-transition gratings were recently described in the studies \[19, 20\].

The great interest to resonant enhanced transmission, spatial localization (collimation) of continuous waves and light pulses by subwavelength metal slits leads to the basic question: Can a light be enhanced and simultaneously localized in space and time by a subwavelength slit? If the field enhancement can be achieved together with nm-scale spatial and fs-scale temporal localizations, this could greatly increase a potential of the nanoslit systems in high-resolution applications, especially in near-field scanning microscopy and spectroscopy. In the present article we test whether the resonant enhancement could only be obtained at the expense of the spatial and temporal broadening of a light wavepacket. To address this question, the spatial distribution of the energy flux of an ultrashort (fs) pulse diffracted by a subwavelength (nanosized) slit in a thick metal film of perfect conductivity will be analyzed by using the conventional approach based on the Neerhoff and Mur solution of Maxwell’s equations. In short, we first will describe the theoretical development of Neerhoff and Mur (Section 2) and the model will then be used to calculate the spatial distribution of the energy flux of the transmitted pulse (wavepacket) under various regimes of the near-field diffraction (Section 3). We will show that a light can be enhanced by orders of magnitude and simultaneously localized in the near-field diffraction zone at the nm- and fs-scales. The implications of the results for diffraction-unlimited near- and far-field optics will then be discussed. In Section 4 we summarize results and present conclusions.

II. THEORETICAL BACKGROUND

An adequate description of transmission a light by a subwavelength nano-sized slit in a thick metal film requires solution of Maxwell’s equations with complicated boundary conditions. The light-slit interaction problem even for a continuous wave can be solved only by extended two-dimensional (x, z) numerical computations. The tree-dimensional (x, z, t) character of the pulse-slit interaction makes the numerical analysis even more difficult. Let us consider the near-field diffraction of an ultrashort pulse (wave-packet) by a subwavelength slit in a thick metal screen of perfect conductivity by using the conventional approach based on the Neerhoff and Mur solution of Maxwell’s equations. Before presenting a treatment of the problem for a wave-packet, we briefly describe the Neerhoff and Mur formulation \[1, 3\] for a continuous wave (a Fourier \(\omega\)-component of a wave-packet). The transmission of a plane continuous wave through a slit (waveguide) of width 2\(a\) in a perfectly conducting screen of thickness \(b\) is considered. The slit is illuminated by a normally incident plane wave under TM polarization (magnetic-field vector parallel to the slit), as shown in Fig. 1. The magnetic field of the wave is assumed to be time harmonic and constant in the \(y\) direction:

\[
\vec{H}(x, y, z, t) = U(x, z)\exp(-i\omega t)\hat{e}_y. \tag{1}
\]

The electric field of the wave is found from the scalar field \(U(x, z)\) using Maxwell’s equations:

\[
E_x(x, z, t) = -\frac{ic}{\omega\epsilon_1}\partial_z U(x, z)\exp(-i\omega t), \tag{2}
\]

\[
E_y(x, z, t) = 0. \tag{3}
\]

\[
E_z(x, z, t) = -\frac{ic}{\omega\epsilon_1}\partial_x U(x, z)\exp(-i\omega t). \tag{4}
\]

Notice that the restrictions in Eq. 1 reduce the diffraction problem to one involving a single scalar field \(U(x, z)\) in two dimensions. The field is represented by \(U_j(x, z)\) (\(j=1,2,3\) in each of the three regions I, II and III). The field satisfies the Helmholtz equation:

\[
(\nabla^2 + k_j^2)U_j = 0, \tag{5}
\]

where \(j = 1, 2, 3\). In region I, the field \(U_1(x, z)\) is decomposed into three components:

\[
U_1(x, z) = U^i(x, z) + U^r(x, z) + U^d(x, z), \tag{6}
\]

each of which satisfies the Helmholtz equation. \(U^i\) represents the incident field, which is assumed to be a plane wave of unit amplitude:

\[
U^i(x, z) = \exp(-ik_1 z). \tag{7}
\]
$U^r$ denotes the field that would be reflected if there were no slit in the screen and thus satisfies
\[ U^r(x, z) = U^d(x, 2b - z). \] (8)

$U^d$ describes the diffracted field in region I due to the presence of the slit. With the above set of equations and standard boundary conditions for a perfectly conducting screen, a unique solution exists for the diffraction problem. To find the field, the 2-dimensional Green’s theorem is applied with one function given by $U(x, z)$ and the other by a conventional Green’s function:
\[ \nabla^2 G_j = -\delta(x - x', z - z'), \] (9)
where $(x, z)$ refers to a field point of interest; $x', z'$ are integration variables, $j = 1, 2, 3$. Since $U_j$ satisfies the Helmholtz equation, Green’s theorem reduces to
\[ U(x, z) = \int_{\text{Boundary}} \left( G \partial_n U - U \partial_n G \right) dS. \] (10)

The unknown fields $U^d(x, z)$, $U_3(x, z)$ and $U_2(x, z)$ are found using the reduced Green’s theorem and boundary conditions on $G$
\[ U^d(x, z) = -\frac{\epsilon_1}{\epsilon_2} \int_{a}^{a} G_1(x, z; x', b) DU_b(x') dx' \] (11)
for $b < z < \infty$,
\[ U_3(x, z) = \frac{\epsilon_3}{\epsilon_2} \int_{a}^{a} G_3(x, z; x', 0) DU_0(x') dx' \] (12)
for $-\infty < z < 0$,
\[ U_2(x, z) = -\int_{a}^{a} \left[ G_2(x, z; x', 0) DU_0(x') - U_0(x') \partial_z G_2(x, z; x', z')|_{z\rightarrow0^+} \right] dx \\
+ \int_{a}^{a} \left[ G_2(x, z; z, b) DU_b(x') - U_b(x') \partial_z G_2(x, z; x', z')|_{z\rightarrow b^-} \right] dx \] (13)
for $|x| < a$ and $0 < z < b$. The boundary fields in Eqs. 11-13 are defined by
\[ U_0(x) = U_2(x, z)|_{z\rightarrow0^+}, \] (14)
\[ DU_0(x) = \partial_z U_2(x, z)|_{z\rightarrow0^+}, \] (15)
\[ U_b(x) = U_2(x, z)|_{z\rightarrow b^-}, \] (16)
\[ DU_b(x) = \partial_z U_2(x, z)|_{z\rightarrow b^-}. \] (17)

In regions I and III the two Green’s functions in Eqs. 11 and 13 are given by
\[ G_1(x, z; x', z') = \frac{i}{4} \left[ H_0^{(1)}(k_1 R) + H_0^{(1)}(k_1 R') \right], \] \[ G_3(x, z; x', z') = \frac{i}{4} \left[ H_0^{(1)}(k_3 R) + H_0^{(1)}(k_3 R') \right], \] (18)
(19)
with
\[ R = [(x - x')^2 + (z - z')^2]^{1/2}, \] (20)
\[ R' = [(x - x')^2 + (z + z' - 2b)^2]^{1/2}, \] (21)
\[ R'' = [(x - x')^2 + (z + z')^2]^{1/2}, \] (22)
where $H_0^{(1)}$ is the Hankel function. In region II, the Green’s function in Eq. 12 is given by
\[ G_2(x, z; x', z') = \frac{i}{4a\gamma_0} \exp(i\gamma_0 |z - z'|) + \frac{i}{2a} \sum_{m=1}^{\infty} \gamma_m^{-1} \cos[m\pi(x + a)/2a] \cos[m\pi(x' + a)/2a] \exp(i\gamma_m |z - z'|), \] (23)
where \( \gamma_m = [k_m^2 - (m\pi/2\alpha)^2]^{1/2} \). The field can be found at any point once the four unknown functions in Eqs. 14-17 have been determined. The functions are completely determined by a set of the four integral equations:

\[
2U_b(x) - U_b(x) = \frac{\epsilon_1}{\epsilon_2} \int_{-a}^{a} G_1(x, b; x', b) DU_b(x') dx',
\]

\[
U_0(x) = \frac{\epsilon_3}{\epsilon_2} \int_{-a}^{a} G_3(x, 0; x', 0) DU_0(x') dx',
\]

\[
\frac{1}{2} U_b(x) = -\int_{-a}^{a} [G_2(x, b; x', 0) DU_0(x') - U_0(x') \partial_x G_2(x, b; x', z')|_{z=0+}] dx'
\]

\[
+ \int_{-a}^{a} [G_2(x, b; x', b) DU_b(x')] dx',
\]

\[
\frac{1}{2} U_0(x) = -\int_{-a}^{a} [G_2(x, 0; x', b) DU_b(x') - U_b(x') \partial_x G_2(x, 0; x', z')|_{z=b-}] dx'
\]

\[
- \int_{-a}^{a} [G_2(x, 0; x', 0) DU_0(x')] dx',
\]

where \( |x| < a \), and

\[
U_b(x) = \exp(-ik_1 b).
\]

The coupled integral equations 24-27 for the four boundary functions are solved numerically. The magnetic \( \vec{H}(x, z, t) \) and electric \( \vec{E}(x, z, t) \) fields of the diffracted wave in region III are found by using Eq. 12. The fields are given by

\[
\vec{H}(x, z, t) = \frac{i}{N} \sum_{j=1}^{N} \left[ H_0^{(1)}(k_3((x - x_j)^2 + z^2)^{1/2}) \right] \times (\vec{D}\vec{U}_0)_j \exp(-i\omega t) \hat{e}_y,
\]

\[
E_x(x, z, t) = -\frac{a}{N} \left( \frac{\epsilon_3}{\epsilon_2} \right)^{1/2} \sum_{j=1}^{N} \frac{z}{((x - x_j)^2 + z^2)^{1/2}} \left[ H_1^{(1)}(k_3((x - x_j)^2 + z^2)^{1/2}) \right] \times (\vec{D}\vec{U}_0)_j \exp(-i\omega t),
\]

\[
E_y(x, z, t) = 0,
\]

\[
E_z(x, z, t) = \frac{a}{N} \left( \frac{\epsilon_3}{\epsilon_2} \right)^{1/2} \sum_{j=1}^{N} \frac{x - x_j}{((x - x_j)^2 + z^2)^{1/2}} \left[ H_1^{(1)}(k_3((x - x_j)^2 + z^2)^{1/2}) \right] \times (\vec{D}\vec{U}_0)_j \exp(-i\omega t),
\]

where \( x_j = 2a(j - 1/2)/N - a \), \( j = 1, 2, ..., N \); \( N > 2a/z \); \( H_1^{(1)} \) is the Hankel function. The coefficients \( (\vec{D}\vec{U}_0)_j \) are found by solving numerically the four integral equations 24-27. For more details of the model and the numerical solution of the coupled integral equations 24-27 see refs. \[11, 12\].

Let us now consider the diffraction of an ultra-short pulse (wave packet). The magnetic field of the incident pulse is assumed to be Gaussian-shaped in time and both polarized and constant in the \( y \) direction:

\[
\vec{H}(x, y, z, t) = U(x, z) \exp[-2\ln(2)(t/\tau)^2] \exp(-i\omega_0 t) \hat{e}_y,
\]

where \( \tau \) is the pulse duration and \( \omega_0 = 2\pi c/\lambda_0 \) is the central frequency. The pulse can be composed in the wave-packet form of a Fourier time expansion (for example, see ref. \[17, 18\]):

\[
\vec{H}(x, y, z, t) = \int_{-\infty}^{\infty} \vec{H}(x, z, \omega) \exp(-i\omega t) d\omega.
\]

The electric and magnetic fields of the diffracted pulse are found by using the expressions (1-32) for each \( \omega \)-Fourier component of the wave-packet (34). This algorithm is implemented numerically by using the discrete fast fourier transform (FFT) instead of the integral (34). The spectral interval \([\omega_{\text{min}}, \omega_{\text{max}}]\) and the sampling points \( \omega_i \) are optimized by matching the FFT result to the original function (33).
III. NUMERICAL ANALYSIS AND DISCUSSION

In this section, we test whether a light pulse can be resonantly enhanced and simultaneously localized in space and time by a subwavelength nano-sized metal slit. To address this question, the spatial distribution of the energy flux of the transmitted pulse under various regimes of the near-field diffraction is analyzed numerically. The electric $\vec{E}$ and magnetic $\vec{H}$ fields of the transmitted pulse in the near-field diffraction zone are computed by solving the equations (1-32) for each Fourier $\omega$-component of the wave-packet (34). The amplitude of a FFT $\omega$-component of the wave-packet transmitted through the slit depends on the wavelength $\lambda = 2\pi c/\omega$. Owing to the dispersion, the Fourier spectra of the transmitted wave-packet changes leading to modification of the pulse width and duration. The dispersion of a continuous wave is usually described by the normalized transmission coefficient $T_{cw}(\lambda)$, which is calculated by integrating the normalized energy flux $S_z/S_z^i$ over the slit value 

$$T_{cw} = -\frac{\sqrt{\varepsilon_i}}{4a\cos \theta} \lim_{z \to 0} \int_{-a}^{a} \frac{1}{\sqrt{a}} |(E_x H_y^* + E_y^* H_x)| dx,$$

where $S_z^i$ is the energy flux of the incident wave of unit amplitude; $S_z$ is the transmitted flux. In order to establish guidelines for the results of our numerical analysis, we computed the transmission coefficient $T_{cw}(\lambda, a, b)$ for a continuous wave ($\omega$-Fourier component) as a function of screen thickness $b$ and/or wavelength $\lambda$ for different values of slit width $2a$. Throughout the computations, the magnitude of the incident magnetic field was assumed to be equal to 1. As an example, the dependence $T_{cw} = T_{cw}(b)$ computed for the wavelength $\lambda=800$ nm and the slit width $2a = 25$ nm is shown in Fig. 2. The dispersion $T_{cw} = T_{cw}(\lambda)$ for $2a = 25$ nm and different values of the screen thickness $b$ is presented in Fig. 3. In Fig. 2, we note the transmission resonances of $\lambda/2$ periodicity with the peak heights $T_{cw} \approx \lambda/2\pi a$ at the resonances. Notice, that the resonance positions and the peak heights are in agreement with the results [2, 3]. The dispersion $T_{cw} = T_{cw}(\lambda)$ (curves A and B) presented in Fig. 3 indicates the wave-slit interaction behavior, which is similar to those of a Fabry-Perot resonator. The transmission resonance peaks, however, have a systematic shift towards longer wavelengths. Our computations showed that the peak heights at the main (strongest) resonant wavelength $\lambda_0^R$ (in the case of Fig. 3, $\lambda_0^R = 500$ or 800 nm) are given by $T_{cw}(\lambda_0^R, a) \approx \lambda_0^R/2\pi a$. This dependence indicates that the optical transmission of a time-harmonic continuous wave can be enhanced by several orders of magnitude by decreasing the slit width and increasing the wavelength. Notice, that the Fabry-Perot like behaviour of the transmission coefficient is in agreement with analytical and experimental results published earlier [21, 22].

The existence of transmission resonances for Fourier’s $\omega$-components of a wave-packet leads to the question: What effect the resonant enhancement has on the spatial and temporal localization of a light pulse? Presumably, the high transmission at resonance occurs when the system efficiently channels Fourier’s components of the wave-packet from a wide area through the slit. At resonance, one might assume that if the energy flow is symmetric about the screen, the pulse width and duration should increase very rapidly past the screen. Thus the large pulse strength associated with resonance could only be obtained at the expense of the spatial and temporal broadening of the wave-packet. To test this hypothesis, the spatial distributions of the energy flux of a transmitted wave-packet were computed for different slit thickness corresponding to the resonance and anti-resonance position. As an example, Figs. 4 (a) and 5 (a) show the energy flux of the anti-resonantly transmitted pulses. Figures 4 (b) and 5 (b) correspond to the case of the waveguide-mode resonance in the slit. Figures 4 and 5 show the transmitted pulses at the distances $|z| = a/2$ and $a$, respectively. The comparison of the flux distribution presented in Fig. 4 (a) with those of Fig. 4(b) shows that, for the parameter values adopted, a transmitted wavepacket is enhanced by one order of magnitude and simultaneously localized in the 25-nm and 100-fs domains of the near-field diffraction zone. Thus at the distance $|z| = a/2$, the slit resonantly enhances the intensity of the pulse without its spatial and temporal broadening. The result can be easily understood by considering the dispersion properties of the slit. For the screen thickness $b = 200$ nm, the amplitudes of Fourier’s components of the wave-packet, whose central wavelength $\lambda_0$ is detuned from the main (at 500 nm) resonance, are practically unchanged in the wavelength region near 800 nm (see, curve B in Fig. 3). This provides the dispersion- and distortion-free non-resonant transmission of the wave-packet (Figs. 4(a) and 5(a)). In the case of the thicker screen ($b = 350$ nm), the slit transmission experiences strong mode-coupling regime at the wavelengths near to 800 nm (see, curve A of Fig. 3) that leads to a profound and uniform enhancement of amplitudes all of the Fourier $\omega$-components of the wave-packet (see curve C in Fig. 3). Thus, the slit resonantly enhances by one order of magnitude the intensity of the pulse without its spatial and temporal broadenings (Figs. 4(a) and 5(a)). Also, notice that at the distance $|z| = a$ (Figs. 5(a) and 5(b)), both the resonantly and anti-resonantly transmitted pulses experience natural spatial broadening in the transverse direction, while their durations are practically unchanged.

By comparing the data for anti-resonant and resonant transmissions presented in Figs. 4 and 5 one can see that at the appropriate values of the distance $|z|$ and the wave-packet spectral width $\Delta \omega$ the resonance effect does not influence the spatial and temporal localization of the wave-packet. To verify this somewhat unexpected result, the FWHMs of the transmitted pulse in the transverse and longitudinal directions were calculated for different values
of the slit width $2a$, central wavelength $\lambda_0$ and pulse duration $\tau \approx 1/\Delta \omega_p$ as a function of screen thickness $b$ at two particular near-field distances $|z| = a/2$ and $a$ from the screen. It was seen that, at the dispersion-free resonant transmission condition $\Delta \omega_p < 0.1 \Delta \omega_r$ (here, $\Delta \omega_r$ is the resonance spectral width), the transmitted pulse indeed does not experience temporal broadening. Thus the temporal localization associated with the duration $\tau$ of the incident pulse remains practically unchanged under the transmission. The value of $\tau$ is determined by the dispersion-free condition $\tau = 1/\Delta \omega_p > 1/0.1 \Delta \omega_r$, where $\Delta \omega_r = \Delta \omega_r(\lambda_r)$ practically does not depend on the screen thickness $b$. We found that the energy flux of the transmitted wave-packet can be enhanced by a factor $T_{cw}(\lambda^R, a) \approx \lambda^R/2\pi a$ by the appropriate adjusting the screen thickness $b = b(\lambda^R)$, for example see Figs. 3-5. Thus the wave-packet can be enhanced by a factor $\lambda^R/2\pi a$ and simultaneously localized in the time domain at the $\tau = \tau(a)$ scale. It was also seen that the FWHM of the transmitted pulse in the transverse direction depends on the wave-packet central wavelength $\lambda_0$ and the distance $z$ from the slit. Nevertheless, the FWHM of the transmitted pulse can be always reduced to the value $2a$ by the appropriate decreasing the distance $|z| = |z|(a)$ from the screen ($|z| = a/2$, in the case of Fig. 4). Thus high transmission can be achieved without concurrent loss in the degree of temporal and spatial localizations of the pulse. In retrospect, this result is reasonable, since the symmetry of the problem for a time-harmonic continuous wave (Fourier’s $\omega$-component of a wave-packet) is disrupted by the presence of the initial and reflected fields in addition to the diffracted field on one side (Eq. 6). As the thickness changes, the field $U^d$ and $U_3$ change only in magnitude, but the field $U_1$ changes in distribution as well since it involves the sum of $U^d$ with unchanging fields $U^i$ and $U^r$. At resonance, the distribution of $U_1$ leads to channeling of the radiation, but the distribution of $U_3$ remains unaffected. By the appropriate adjusting the slit-pulse parameters a light can be enhanced by orders of magnitude and simultaneously localized in the near-field diffraction zone at the nm- and fs-scales.

The limitations of the above analysis must be considered before the results are used for a particular experimental device. The resonant enhancement with simultaneous nm-scale and fs-scale temporal localizations of a light by a subwavelength metal nano-slit is a consequence of the assumption of the screen perfect conductivity. The slit can be made of perfectly conductive (at low temperatures) materials. In the context of current technology, however, the use of conventional materials like metal films at room temperature is more practical. As a general criterion, the perfect conductivity assumption should remain valid as far as the slit width and the screen thickness exceed the extinction length for the Fourier $\omega$-components of a wave-packet within the metal. The light intensity decays in the metal screen at the rate of $I_s = I_0 \exp(-b/\delta)$, where $\delta = \delta(\lambda)$ is the extinction length in the screen. The aluminum has the largest opacity ($\delta < 11$ nm) in the spectral region $\lambda > 100$ nm [20]. The extinction length increases from 11 to 220 nm with decreasing the wavelength from 100 to 50 nm. Hence, the perfect conductivity is a very good approximation in a situation involving a relatively thick ($b > 25$ nm) aluminum screen and a wave-packet of the duration $\tau \approx 1/\Delta \omega_p$ having the Fourier components in the spectral region $\lambda > 100$ nm. However, in the case of thinner screens, shorter pulses and smaller central wavelengths of wave-packets, the metal films are not completely opaque. This would reduce a value of spatial localization of a pulse due to passage of the light through the screen in the region away from the slit. Moreover, the phase shifts of the Fourier components along the propagation path causing by the skin effect can modify the enhancement coefficient and temporal localization properties of the slit.

The above analysis is directly applicable to the two-dimensional near-field scanning optical microscopy and spectroscopy. In a conventional 2D NSOM, a subwavelength ($2a < \lambda$) slit illuminated by a continuous wave is used as a near-field ($|z| < a$) light source providing the nm-scale resolution in space [1, 2, 4, 6]. The non-resonant transmission of fs pulses could provide the super resolution of NSOM simultaneously in space and time [17, 18]. The above-described resonantly enhanced transmission together with nm- and fs-scale localizations in the space and time of a pulse could greatly increase a potential of the near-field scanning optical microscopy and spectroscopy, especially in high-resolution applications. It should be noted in this connection that the high transmission ($T_{cw}(\lambda^R, a) \approx \lambda^R/2\pi a$) of a pulse can be achieved without concurrent loss in the temporal and spatial localizations of the pulse only at the short ($|z| = |z|(a)$) distances from the slit. The presence of a microscopic sample (a molecule, for example) placed at the short distance in strong interaction with NSOM slit, however, modifies the boundary conditions. In the case of the strong slit-sample-pulse interaction, which takes place at the distance $|z| << 0.1a$, the response function accounting for the modification of the quantum mechanical behavior of the sample should be took into consideration.

The potential applications of the effect of the resonantly enhanced transmission together with nm-scale and fs-scale localizations of a pulse are not limited to near-field microscopy and spectroscopy. Broadly speaking, the effect concerns all physical phenomena and photonic applications involving a transmission of light by a single subwavelength nano-slit. A grating with subwavelength slits and a subwavelength slit surrounded by parallel grooves (see, the studies [1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25] and references therein). For instance, the effect could be used for sensors, communications, optical switching devices and microscopes.
IV. CONCLUSION

In conclusion, in the present article we have considered a question whether a light can be enhanced and simultaneously localized in space and time by a subwavelength metal nano-slit. To address this question, the spatial distributions of the energy flux of an ultrashort (fs) pulse diffracted by a subwavelength (nanosized) slit in a thick metal screen of perfect conductivity have been analyzed by using the conventional approach based on the Neerhoff and Mur solution of Maxwell’s equations. The analysis of the spatial distributions for various regimes of the near-field diffraction demonstrated that the energy flux of a wavepacket can be enhanced by orders of magnitude and simultaneously localized in the near-field diffraction zone at the nm- and fs-scales. The extraordinary transmission, together with the nm-scale and fs-scale localizations of a light, make the nano-slit structures attractive for many photonic purposes, such as sensors, communications, optical switching devices and NSOM. We also believe that the addressing of the above-mentioned basic question gains insight into the physics of near-field resonant diffraction.

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FIG. 1: Propagation of a continuous wave through a subwavelength nano-sized slit in a thick metal film.
FIG. 2: The transmission coefficient $T_{cw}$ for a continuous wave ($\omega$-Fourier component of a wave-packet) as a function of screen thickness $b$ computed for the wavelength $\lambda=800$ nm and the slit width $2a = 25$ nm.

FIG. 3: The dispersion $T_{cw} = T_{cw}(\lambda)$ for a continuous wave ($\omega$-Fourier component of a wave-packet) computed for the slit width $2a = 25$ nm and different values of the screen thickness $b$: A - 350 nm and B - 200 nm. The Fourier spectra (curve C) is presented for the comparison. The curve C shows the Fourier spectra of an incident wave-packet with the duration $\tau = 100$ fs and the central wavelength $\lambda_0 = 800$ nm, which was used in the computations presented in Fig. 4 and 5.
FIG. 4: The energy flux of a transmitted pulse at the distances \( |z| = a/2 \). (a) The anti-resonant transmission by the slit (\( 2a = 25 \text{ nm}, b = 200 \text{ nm} \)). (b) The resonant transmission by the slit (\( 2a = 25 \text{ nm}, b = 350 \text{ nm} \)). The incident wave-packet duration \( \tau = 100 \text{ fs} \) and the central wavelength \( \lambda_0 = 800 \text{ nm} \).
FIG. 5: The energy flux of a transmitted pulse at the distances $|z| = a$. (a) The anti-resonant transmission by the slit ($2a = 25$ nm, $b = 200$ nm). (b) The resonant transmission by the slit ($2a = 25$ nm, $b = 350$ nm). The incident wave-packet duration $\tau = 100$ fs and the central wavelength $\lambda_0 = 800$ nm.