A New Robust Adaptive Decentralized Tube Model Predictive Control of Continuous Time Uncertain Nonlinear Large-Scale Systems

Samane Fazeli¹, Naiem Abdollahi², Hashem Imani Marrani³*, Hamid Malekizade⁴ and Hasan Hosseinzadeh⁵

Abstract: In this paper, a new decentralized model predictive control has been proposed for continuous-time nonlinear large-scale systems made of multiple interconnected subsystems and uncertain systems with disturbances. This approach is characterized by: (I) consideration of input and state constraints, (II) no requirement for information transmission between local control rules, (III) robustness of local controllers to uncertainty in model and disturbances; (IV) bounded disturbances in subsystems, but their upper bound is not specified; (V) a robust invariant set for each controller; (VI) proven closed-loop system overall stability and convergence. In order to show the key features and the performance of the proposed algorithm, a simulation model has been provided.

Subjects: Mechanical Engineering; Systems & Control Engineering; Systems & Controls

Keywords: decentralized model predictive control; tube-MPC; robust control; adaptive control

ABOUT THE AUTHORS

Samane Fazeli was born in Torbat-e Heydarieh, Iran. She is Ph.D. student and received M.Sc. degrees with honors in electrical engineering from Islamic Azad University, Science and Research Branch, Tehran, Iran.

Naiem Abdollahi was born in Ardebil, Iran. He is Master in Energy Engineering from the Islamic Azad University, South Tehran branch, Tehran, Iran.

Hashem Imani Marrani is Ph.D. in Control and System Engineering. His research interests include Nonlinear Control, MPC and Nonlinear Fiber Optics.

Hamid Malekizade is an assistant professor at the Department of Electrical Engineering, Imam Khomeini Maritime University, Nowshahr, Iran. His main fields of interest are stabilizing marine systems and robust model predictive control.

Hasan Hosseinzadeh was born in 1979 in Ardabil, Iran. He received his BSc in 2003 University in Payam Noor University Ardabil, MSc in Sistine and Baluchistan University in 2006 and Ph.D. in Karaj Azad University in 2013. His research interests are Mathematical analysis and its applications, differential equations, image processing and other related mathematical fields.

PUBLIC INTEREST STATEMENT

Uncertain large-scale nonlinear systems control is one of the real challenges in the industry. In addition to the problem of optimizing the control signal, issues of multiple interconnected subsystems, disturbance, and uncertainty should also be taken into account. This paper presents a new robust decentralized model predictive control for covering the above effects.
1. Introduction

It has been well documented that a host of interconnected systems, such as multi-robot systems, power systems, communication networks, and transportation networks, are a composition of a group of interconnected subsystems. While in a centralized control framework, the designed controller for each subsystem should use the whole system's data; in decentralized control schemes, the controller designed for each subsystem can just use its local data. Given the high dependency of decentralized controllers, since the 1960s, some people have tried to make design approaches that can guarantee the system stability and performance. For example, some of them were based on vector Lyapunov functions (Siljak, 1991), on sequential design (Hovd & Skogestad, 1994), on optimization (Davison & Ferguson, 1981; Scattolini & Schiavoni, 1985), and on overlapping decompositions (İftar, 1993; Ikeda, Siljak, & White, 1982; Ikeda, Šiljak, & White, 1981). Old books about decentralized control are (Lunze, 1992; Siljak, 1991), while a significant article in this area is (Sandell, Varaiya, Athans, & Safonov, 1978).

In the last few years, some survey papers (Bakule & Papik, 2012; Šiljak, 1996; Šiljak & Zečević, 2005) have addressed the decentralized control, which can provide the latest list of references. The most important characteristic of a decentralized control scheme is its independency of communication between the various local controllers.

Today, MPC is regarded as an efficient method with confirmed theoretical frameworks and proven ability to control a great number of industrial control issues. In fact, by overlooking the mutual relationships and the application reported in (Elliott & Rasmussen, 2008), the local MPC rule can be calculated via standard MPC algorithms (Acar, 1992). Although, in spite of the high significance of this topic, little attention has been paid to decentralized MPC algorithms with proven characteristics have been developed up to now. This could have happened as a result of some reasons. First, because of the multi-variable nature of MPC, one can easily commission centralized regulators with numerous input and output variables, so that decentralization is not always the main issue. Nonetheless, for extensive systems composed of various weakly connected process units, dismantling the optimization problem related to the design of a unique centralized controller into some smaller problems is more convenient; in other words, it is better to have a decentralized control scheme. A second reason for the instability of decentralized algorithms is that the feedback from MPC rule is not explicit and the control variables obtained from an optimization process rather than the calculation resulted from an explicit control rule. This is why it is very hard to analyze the closed-loop system with MPC, and the main stability results are calculated using the optimal cost as a Lyapunov function (Mayne, Rawlings, Rao, & Scokaert, 2000). Although, generalizing this analysis method to decentralized control framework is not a simple task.

Making coordinated decentralized control systems requires the dynamic interrelationship between various units to be taken into mind in the designing process of the control systems. This issue of detecting dynamic interrelationships between various units was investigated in (Gudi & Rawlings, 2006). In (Alessio, Barcelli, & Bemporad, 2011), a decentralized MPC scheme was designed for extensive linear processes with limited inputs. In this study, the general pattern of the process is estimated by numerous (which can be overlapping) smaller subsystem models, utilized for local predictions, and the extent of dissociation among the subsystem models can be a tunable parameter in the scheme. In (Alessio & Bemporad, 2008), probable date packet omissions in the relationship between the divided controllers were taken into consideration in the context of linear systems, and their effect on the closed-loop system stability was studied and investigated.

Magni & Scattolini (2006) has suggested a decentralized state-feedback MPC algorithm for nonlinear discrete-time systems with fading disturbances, where the closed-loop stability is gained through the consideration of a contraction limit in the optimization problem to be calculated at any moment. This limit moves the state routes of the subsystems controlled by local MPC regulators towards the origin, despite the disturbing effect of the mutual interrelationships and of the disturbances. Raimondo, Magni, & Scattolini (2007) has utilized the new theory of powerful
MPC (Magni & Scattolini, 2007) in order to stabilize decentralized state-feedback regulators for nonlinear discrete-time systems, which contain uncertainty. In this method, the relationships between different units are regarded as disturbances that should be avoided.

Another significant study on decentralized control is about the development of a quasi-decentralized control scheme for multi-unit plants that reaches the best closed-loop goals with the lowest cross-communication between the plant units under state-feedback control (Sun & El-Farra, 2008). In this study, the basic concept is to merge in the local control system of each unit a group of dynamic models that present an estimation of the interrelationships between the various subsystems when local subsystem states are not altered between different subsystems and to update the state of each model through state information exchanged when communication is reestablished.

In Tuan, Savkin, Nguyen, & Nguyen (2015), a new decentralized predictive control scheme has been purposed for a plant made of interconnected systems. The objective of Tuan et al. (2015) is to provide a method to stabilize a nominal large-scale plant using limited and decentralized controllers. The plant-wise stability is confirmed online by the recently presented asymptotically positive realness limit (APRC), which can be decentralized with little effort in the local system platform. In fact, it is clear that the stability of each system cannot always lead the plant-wise stability. But, APRC of each system provides the plant-wise APRC that results in the plant-wise stability. Thus, the plant-wise stability is obtained in real time through just local APRC-based controllers. Such APRCs can be naturally incorporated into stability limits for decentralized MPC and so can be an appropriated base for limited and decentralized stabilizing groups of intricate interconnected systems.

Generally, the constraint on the available data and the lack of interrelationship between various controllers can reduce the total closed-loop performance under a decentralized control scheme (Cui & Jacobsen, 2002). This can result in the development of model predictive control structures, where various MPCs adjust their activities through establishing relations in order to share subsystem state and control action data.

In the reviewed papers, some important points should be made regarding the design of the decentralized predictive controllers:

1. The controlled systems have been modeled in a linear manner, a method which overlooks some nonlinear dynamics and views them as disturbance.
2. When nonlinear dynamics are considered, it has been tried to choose those functions that satisfy Lipchitz’s condition. This constraint leads to a smaller class of nonlinear controllable systems.
3. Parametric uncertainties are not considered in the nonlinear system, and in most of the previous works, a complete split in each sub-system’s model is assumed.
4. The system disturbances have not been investigated separately, and in case they are investigated the upper bound of disturbance is specified. In the previous works, the only considered disturbance is that of other sub-systems on their own neighbors.

In this paper, a special class of nonlinear systems has been investigated with the uncertainty in each sub-system. As such, bounded disturbances without a specified upper bound are investigated in the controlled system. The effects of neighboring sub-systems are also provided in the large-scale system. Then, a new decentralized predictive controller is presented that has combined adaptive control and tube-MPC methods. In this new controller, an adaptive method has been used to estimate the mode uncertainties. Also $H_{\infty}$ method has been used to find the appropriate yield in Tube-MPC and to avoid the disturbances and effects of other sub-systems in order to assure the system stability, and finally, a robust invariant set against other subsystem disturbances and effects is presented.
The rest of this paper is organized as follows: The preliminaries are defined in Section 2. The new decentralized robust predictive controller is presented in Section 3. Some numerical examples are provided in Section 4 to prove the efficiency of the new controller. Finally, Section 5 concludes the paper.

2. Preliminaries
Consider the following continuous nonlinear system
\[
\dot{x}(t) = f(x(t), u(t), w(t))
\]
where \(x(t) \in \mathbb{R}^n\) shows the system states, \(u(t) \in \mathbb{R}^n\) the control input, the signal \(w(t) \in \mathbb{R}^m\) is the disturbance or model-plant mismatch, which is unknown but bounded, and lies in a compact set, \(W = \{w(t) \in \mathbb{R}^m | \|w\| \leq w_{\text{max}}\}\)

The system has the following limitations
\[
x(t) \in X, u(t) \in U, \forall \ t > 0
\]
where \(X \subset \mathbb{R}^n\) is bounded and \(U \subset \mathbb{R}^n\) is compact (Yu, Böhm, Chen, & Allgöwer, 2010). The following lemma provides us a way to construct a robust control invariant set for the system (1).

**Lemma 1** (Yu et al., 2010). Let \(S : \mathbb{R}^n \rightarrow [0, \infty)\) be a continuously differentiable function and \(a_1(x) < S(x) < a_2(x)\), where \(a_1, a_2\) are class \(k_\infty\) functions. Suppose \(u : \mathbb{R} \rightarrow \mathbb{R}^n\) is chosen, and there exist \(\lambda > 0\) and \(\mu > 0\) such that
\[
\dot{S}(x) + \lambda S(x) - \mu \dot{w}(t) \leq 0
\]
with \(x \in X, d \in D\). Then, the system trajectory starting from \(x(t_0) \in \Omega \subseteq X\), will remain in the set \(\Omega\), where
\[
\Omega = \left\{x \in \mathbb{R}^n | S(x) \leq \frac{\mu w_{\text{max}}^2}{\lambda}\right\}
\]

**Lemma 2** (Poursafar, Taghirad, & Haeri, 2010). Let \(M, N\) be real constant matrices and \(P\) be a positive matrix of compatible dimensions. Then
\[
M^T P N + N^T P M \leq \varepsilon M^T P M + \varepsilon^{-1} N^T P N
\]
holds for any \(\varepsilon > 0\).

3. Robust adaptive decentralized tube-MPC
Consider a process plant \(\sum\) consisting of \(h\) interconnected nonlinear systems, each denoted as \(s_i, i = 1, \ldots, h\) in the continuous-time state space
\[
\dot{s}_i(t) = A_i x_i(t) + B_i u_i(t) + f_i(x_i) \theta_i + d_i(t) + g_i(x)
\]
where \(x_i(t) \in \mathbb{R}^n\) shows the system states, \(u_i(t) \in \mathbb{R}^n\) the control input, \(f_i(x) : \mathbb{R}^n \rightarrow \mathbb{R}^n\) continuous nonlinear function, \(\theta_i(t) \in \mathbb{R}^n\) uncertainty in the system, \(d_i(t) \in \mathbb{R}^n\) bounded and unknown system disturbances, \(g_i(x) : \mathbb{R}^n \rightarrow \mathbb{R}^n\) interactive (or coupling) continuous nonlinear function, and \(x(t) \in \mathbb{R}^n\) shows the process plant \(\sum\) states. The disturbances are considered in the following set
\[
D_i = \{d_i(t) \in \mathbb{R}^n | \|d_i\| \leq d_{\text{max}}\}
\]

The system has the following limitations
\[
x_i(t) \in X_i, u_i(t) \in U_i, \forall \ t > 0
\]
where \(X_i \subset \mathbb{R}^n\) is bounded and \(U_i \subset \mathbb{R}^n\) is compact. Now, the nominal model of the system is given as
\[ \dot{x}_i(t) = A_i x_i(t) + B_i \hat{u}_i(t) + f_i(\tilde{x}_i(t)) \hat{\theta}_i \]  

(10)

where \( x_i(t) \in R^{n_i} \) shows the nominal model states, \( \hat{u}_i(t) \in R^{m_i} \) the control input of nominal model and \( \hat{\theta}_i(t) \in R^{n_\theta} \) estimator of uncertainty in the system. By defining the cost function as

\[ J_i(\tilde{x}_i, \hat{u}_i) = \int_{t_k}^{t_k + T_p} \left( x_i(\tau; \tilde{x}_i(t_k), t_k) - \hat{x}_i(\tau; \tilde{x}_i(t_k), t_k) \right)^T Q_i x_i(\tau; \tilde{x}_i(t_k), t_k) + \hat{u}_i(\tau; \hat{x}_i(t_k), t_k) \right)^T R_i \hat{u}_i(\tau; \hat{x}_i(t_k), t_k) \, d\tau \]  

(11)

where \( T_p \) is the prediction horizon. We solve the following problem to find \( \hat{u}_i(t) \)

**Problem 1**

minimize \( J_i(\tilde{x}_i(t_k), \hat{u}_i(0; \hat{x}_i(t_k), t_k)) \)

\( \hat{u}_i(0; \hat{x}_i(t_k), t_k) \)

subject to

\[ \dot{x}_i(\tau; \hat{x}_i(t_k), t_k) = A_i x_i(\tau; \hat{x}_i(t_k), t_k) + B_i \hat{u}_i(\tau; \hat{x}_i(t_k), t_k) + f_i(\hat{x}_i(\tau; \hat{x}_i(t_k), t_k)) \hat{\theta}_i(t_k) \]

\[ x_i(\tau; \hat{x}_i(t_k), t_k) \in \tilde{X}_i \quad \tau \in [t_k, t_k + T_p] \]

\[ \hat{u}_i(\tau; \hat{x}_i(t_k), t_k) \in \tilde{U}_i \quad \tau \in [t_k, t_k + T_p] \]  

(12)

Where \( \tilde{X}_i \subset R^{n_i}, \tilde{U}_i \subset R^{m_i} \) and \( \hat{u}_i(0; \hat{x}_i(t_k), t_k) \) are the given control inputs from \( \hat{x}_i(t_k) \) state in \( t_k \), and \( \hat{x}_i(0; \hat{x}_i(t_k), t_k) \) is (10) nominal system trajectory, started from \( \hat{x}_i(t_k) \) state in \( t_k \) and \( \hat{u}_i(0; \hat{x}_i(t_k), t_k) \) control input.

Problem 1 is solved in discrete time with a sample time of \( \delta_i \), and the nominal control during the sample interval \( \delta_i \) is

\[ \hat{u}_i(\tau) = \hat{u}_i^*_i(\tau; \hat{x}_i^*(t_k), t_k) \quad \tau \in [t_k, t_k + \delta_i], i = 1, 2, \ldots, h \]  

(13)

Where \( \hat{u}_i^*_i(\tau; \hat{x}_i^*(t_k), t_k) \) shows the optimum solution of the optimization problem in \( t_k \), and \( \hat{x}_i^*(\tau; \hat{x}_i^*(t_k), t_k) \) is the nominal system trajectory.

The overall applied control input for the actual system (7) during the sampling interval \( \delta_i \) consequently is

\[ u_i(\tau) = \hat{u}_i(\tau) + K_i e_i(\tau) \quad \tau \in [t_k, t_k + \delta_i], i = 1, 2, \ldots, h \]  

(14)

where

\[ e_i(t) = x_i(t) - \hat{x}_i(t) \]  

(15)

According to (7), (10) and (14), the dynamic error equation is given as

\[ \dot{e}_i(t) = (A_i + B_i K_i) e_i(t) + f_i(x_i(t)) \hat{\theta}_i - f_i(\hat{x}_i(t)) \hat{\theta}_i + d_i(t) - g_i(x(t)) \]  

(16)

By defining \( \delta_i = \theta_i - \hat{\theta}_i \), we have

\[ \dot{\delta}_i(t) = (A_i + B_i K_i) e_i(t) + B_{w_i} w_i(t) \]  

(17)

where

\[ m_i(t) = (f_i(x_i(t)) - f_i(\hat{x}_i(t))) \hat{\theta}_i + d_i(t) + g_i(x(t)). w_i(t) = \begin{bmatrix} \hat{\theta}_i \\ m_i(t) \end{bmatrix} \begin{bmatrix} B_{w_i} \\ 0_{n_i \times n_i} \end{bmatrix} \]  

(18)

Under the definitions
\[ A := \text{diag}[A_i]_{i}^{h}, \quad B := \text{diag}[B_i]_{i}^{h}, \quad K := \text{diag}[K_i]_{i}^{h}, \quad B_w := \text{diag}[B_w]_{i}^{h} \]  
\[ e(t) := \begin{bmatrix} e_1(t) \\ \vdots \\ e_h(t) \end{bmatrix}, \quad m(t) := \begin{bmatrix} m_1(t) \\ \vdots \\ m_h(t) \end{bmatrix}, \quad \dot{\theta}(t) := \begin{bmatrix} \dot{\theta}_1(t) \\ \vdots \\ \dot{\theta}_h(t) \end{bmatrix}, \quad w(t) := \begin{bmatrix} w_1(t) \\ \vdots \\ w_h(t) \end{bmatrix} \]  

(19)

The dynamic error equation of the large-scale plant \( \Sigma \) is

\[ \dot{e}(t) = (A + BK)e(t) + B_ww(t) \]  

(21)

Lemma 3. Suppose that there exit positive definite matrix \( X_i \in R^{\nu_i \times \nu_i} \), non-square matrix \( Y_i \in R^{\nu_i \times \nu_y} \), and scalars \( \alpha_i > 0, \beta_i > 0, \lambda > 0, \gamma_i > 0, \varepsilon_i = \frac{\mu_i}{\lambda} > 1, \ldots, h \) and \( \mu \geq \max \left( \frac{1}{\alpha_i + \beta_i} \right) \) such that

\[ (A_iX_i + B_iY_i)^T + A_iX_i + B_iY_i + (\alpha_i + \lambda)X_i \leq 0, \quad i = 1, \ldots, h \]  

(22)

\[ X_i = \frac{1}{\alpha_i} I, \quad i = 1, \ldots, h \]  

(23)

Then, the set \( \Omega = \left\{ e(t) \in R^{\nu_i} | S(e(t), \dot{\theta}) \leq \frac{\mu \gamma_i}{\lambda} \right\} \) is a robust invariant set for the error system (17), where

\[ u(t) = \tilde{u}(t) + Ke(t) \]  

(24)

\[ u(t) := \begin{bmatrix} u_1(t) \\ \vdots \\ u_h(t) \end{bmatrix}, \quad \tilde{u}(t) := \begin{bmatrix} \tilde{u}_1(t) \\ \vdots \\ \tilde{u}_h(t) \end{bmatrix} \]  

(25)

\[ S(e(t), \dot{\theta}) = \sum_{i=1}^{h} S_i(e_i(t), \dot{\theta}_i) = \sum_{i=1}^{h} \left( e_i(t)^T P_i e_i(t) + \frac{1}{\alpha_i} \dot{\theta}_i^T \dot{\theta}_i \right) \]  

(26)

And

\[ P_i = X_i^{-1}, \quad K_i = Y_iX_i^{-1} \]  

(27)

Proof.

According to Lemma 1, for system (17), we have

\[ \dot{S}(e(t), \dot{\theta}) + \lambda S(e(t), \dot{\theta}) - \mu w(t)^T w(t) \leq 0 \]  

(28)

Then, according to (26), we have

\[ \sum_{i=1}^{h} \left( e_i(t)^T P_i e_i(t) + e_i(t)^T P_i \dot{e}_i(t) - \frac{1}{\alpha_i} \dot{\theta}_i^T \dot{\theta}_i - \frac{1}{\lambda} \dot{\theta}_i^T \dot{\theta}_i \right) + \lambda \left( e_i(t)^T P_i e_i(t) + \frac{1}{\alpha_i} \dot{\theta}_i^T \dot{\theta}_i \right) - \mu w(t)^T w(t) \leq 0 \]  

(29)

\[ \sum_{i=1}^{h} \left( e_i(t)^T \left( (A_i + B_iK_i)^T P_i + P_i(A_i + B_iK_i) + \lambda P \right) e_i(t) + \frac{1}{\alpha_i} \dot{\theta}_i^T \dot{\theta}_i + m_i(t)^T m_i(t) \dot{\theta}_i + m_i(t)^T m_i(t) \dot{\theta}_i + m_i(t)^T m_i(t) \dot{\theta}_i + m_i(t)^T m_i(t) \dot{\theta}_i \right) \]  

(30)

By choosing

\[ \dot{\theta}_i = \eta_i \dot{f}_i(x_i(t)) P_i e_i(t) \]  

(31)

We have
\[
\sum_{i=1}^{h} \left( e_i^T(t) \left( (A_i + B_i K_i)^T P_i + P_i (A_i + B_i K_i) + \lambda P_i \right) e_i(t) + \frac{\lambda}{m_i} \tilde{y}_i^T \tilde{y}_i + m_i^T(t) P_i e_i(t) + e_i^T(t) P_i m_i(t) - \mu \tilde{y}_i^T \tilde{y}_i + \tilde{m}_i^T(t) \tilde{y}_i + m_i^T(t) m_i(t) \right) \leq 0
\]  

(32)

According to Lemma 2, we have
\[
m_i^T(t) P_i e_i(t) + e_i^T(t) P_i m_i(t) \leq \alpha_i e_i^T(t) P_i e_i(t) + \alpha_i^{-1} m_i^T(t) P_i m_i(t) \tilde{y}_i(t) + m_i^T(t) \tilde{y}_i(t) \leq \beta_i m_i^T(t) m_i(t)
\]

(33)

By substituting (33) in (32), it is obtained that
\[
\sum_{i=1}^{h} \left( e_i^T(t) \left( (A_i + B_i K_i)^T P_i + P_i (A_i + B_i K_i) + (\alpha_i + \lambda) P_i \right) e_i(t) + \left( \frac{\lambda}{m_i} - \mu (1 + \beta_i) \right) \tilde{y}_i^T \tilde{y}_i + \alpha_i^{-1} m_i^T(t) P_i m_i(t) - \mu (1 + \beta_i^{-1}) m_i^T(t) m_i(t) \right) \leq 0
\]

(34)

Consider
\[
P_i \leq \lambda_{\text{max}} I \leq \varepsilon I
\]

(35)

where \( \lambda_{\text{max}} \) is the maximum eigenvalue of \( P_i \) and \( \varepsilon I \) is the corresponding upper bound (Poursafar et al., 2010), then
\[
\sum_{i=1}^{h} \left( e_i^T(t) \left( (A_i + B_i K_i)^T P_i + P_i (A_i + B_i K_i) + (\alpha_i + \lambda) P_i \right) e_i(t) + \left( \frac{\lambda}{m_i} - \mu (1 + \beta_i) \right) \tilde{y}_i^T \tilde{y}_i + \alpha_i^{-1} m_i^T(t) P_i m_i(t) - \mu (1 + \beta_i^{-1}) m_i^T(t) m_i(t) \right) \leq 0
\]

(36)

By choosing
\[
\mu \geq \max \left( \frac{\lambda}{m_i (1 + \beta_i)} \right)
\]

(37)
\[
e_i = \frac{\mu}{\lambda m_i}, i = 1, \ldots, h
\]

Equation (36) is reduced to
\[
\sum_{i=1}^{h} \left( e_i^T(t) \left( (A_i + B_i K_i)^T P_i + P_i (A_i + B_i K_i) + (\alpha_i + \lambda) P_i \right) e_i(t) \right) \leq 0
\]

(38)

Equation (38) becomes negative if Equation (39) is satisfied for each \( i = 1, \ldots, h \).
\[
e_i^T(t) \left( (A_i + B_i K_i)^T P_i + P_i (A_i + B_i K_i) + (\alpha_i + \lambda) P_i \right) e_i(t) \leq 0, \quad i = 1, \ldots, h
\]

(39)

By multiplying (22) from left and right by \( \text{diag}(P_i) \) and substituting \( P_i = X_i^{-1} \) and \( K_i = Y_i X_i^{-1} \) we have
\[
\left( (A_i + B_i K_i)^T P_i + P_i (A_i + B_i K_i) + (\alpha_i + \lambda) P_i \right) \leq 0, \quad i = 1, \ldots, h
\]

(40)

By multiplying inequality (40) from left by \( e_i^T(t) \) and from right by \( e_i(t) \), (39) is obtained. Also, Equation (23) is obtained from (35).

According to Lemma 1, there is a set \( \Omega = \{ e(t) \in R^n | S(e(t), \tilde{y}) \leq \frac{\mu \lambda_{\text{max}}}{\lambda} \} \) so that it is a robust invariant set for system (17).

Finally, the following controlling algorithm is employed to stabilize the system (Yu, Maier, Chen, & Allgöwer, 2013).

Algorithm 1
Step 0. At time $t_0$, set $\bar{x}_i(t_0) = x_i(t_0), i = 1, 2, ..., h$ in which $x_i(t_0)$ is the current state.

Step 1. At time $t_k$, and current state $(\bar{x}_i(t_k), x_i(t_k))$, solve problem 1 to obtain the nominal control action $\bar{u}_i(t_k)$ and the actual control action $u_i(t_k) = \bar{u}_i(t_k) + K_i e_i(t_k)$.

Step 2. Apply the control $u_i(t_k)$ to the system (7), during the sampling interval $[t_k, t_{k+1}]$, where $t_{k+1} = t_k + \delta_t$.

Step 3. Measure the state $x_i(t_{k+1})$ at the next time instant $t_{k+1}$ of the system (7) and compute the successor state $\bar{x}_i(t_{k+1})$ of the nominal system (10) under the nominal control $\bar{u}_i(t_k)$.

Step 4. Set $(\bar{x}_i(t_k), x_i(t_k)) = (\bar{x}_i(t_{k+1}), x_i(t_{k+1}))$, $t_k = t_{k+1}$, and go to step 1.

4. Numerical example

Recently, the control of surge instability in compressor system has been studied in several articles (Ghanavati, Salahshoor, Jahed Motlagh, Ramezani, & Moarefianpour, 2017; Ghanavati, Salahshoor, Jahed-Motlagh, Ramezani, & Moarefianpour, 2018; Ghanavati, Salahshoor, Motlagh, Ramazani, & Moarefianpour, 2018; Imani, Jahed-Motlagh, Salahshoor, Ramazani, & Moarefianpour, 2017; Imani, Jahed-Motlagh, Salahshoor, Ramezani, & Moarefianpour, 2017, 2018; Imani, Malekizade, Asadi Bagal, & Hosseinizadeh, 2018; Marrani, Fazeli, Malekizade, & Hosseinizadeh, 2019; Taleb Ziabari, Jahed-Motlagh, Salahshoor, Ramezani, & Moarefianpour, 2017; Taleb Ziabari, Jahed-Motlagh, Salahshoor, Ramezani, & Moarefianpour, 2017). In this section, given the proposed decentralized predictive controller, a surge controller is designed for serial and parallel compressors. First, a compressor model is investigated and then a decentralized controller is designed for these combinations: two parallel compressors and one serial compressor. Pure surge model of Moore and Greitzer for the centrifugal compressor is as the follow:

$$\psi = \frac{1}{4B^2 l_c} (\phi - \phi_T(\psi) - d_\phi(t))$$

$$\dot{\phi} = \frac{1}{l_c} (\psi_c(\phi) - \psi + d_\psi(t))$$

(41)

where $\psi$ is the coefficient of increase in compressor pressure, $\phi$ is the coefficient of compressor’s mass flow, $d_\phi(t)$ and $d_\psi(t)$ are the disturbances of flow and pressure. Also, $\phi_T(\psi)$ is the characteristic of throttle valve and $\psi_c(\phi)$ is the characteristic of the compressor. $B$ is the Greitzer’s parameter and $l_c$ shows the length of canals (ducts). Moor and Greitzer’s (Pradhan, Ramamurthy, & Mayavanshi, 2004) compressor characteristic is defined as

$$\psi_c(\phi) = \psi_{c0} + H \left( 1 + \frac{3}{2} \left( \frac{\phi}{W} - 1 \right) - \frac{1}{2} \left( \frac{\phi}{W} - 1 \right)^2 \right)$$

(42)

where $\psi_{c0}$ is the value of the characteristic curve in zero db, $H$ is half of the height of the characteristic curve, and $W$ is the half of the width of the characteristic curve. The equation for throttle valve characteristic is also derived from (Gravdahl & Egeland, 2012) and is as follows:

$$\phi_T(\psi) = \gamma_T \sqrt{\psi}$$

(43)

where $\gamma_T$ is also the valve’s yield. Values of compressor parameters used in simulation according to (Greitzer, 1976).

$$B = 1.8, l_c = 3, H = 0.18, W = 0.25, \psi_{c0} = 0.3$$

(44)

Figure 1 is the diagram of compression system with close couple valve (CCV).
The system model equations, considering a CCV, are

\[
\psi = \frac{1}{4B^2l_c} (\phi - \phi_T(\psi) - d_\psi(t)) \\
\dot{\psi} = \frac{1}{l_c} (\psi_c(\phi) - \psi - \psi_v(\phi) + d_\psi(t))
\]  

(45)

Considering \(\psi_v(\phi)\) as the input for system control.

\[
\psi = \frac{1}{4B^2l_c} (\phi - \phi_T(\psi) - d_\psi(t)) \\
\dot{\psi} = \frac{1}{l_c} (\psi_c(\phi) - \psi - u + d_\psi(t))
\]  

(46)

According to Figure 2, the equations for the two parallel compressors and one serial compressor are

\[
\psi_1 = \frac{1}{4B^2l_c} (\phi_1 - \phi_T(\psi_1) - d_{\phi_1}(t) - \psi_1\phi_2) \\
\dot{\psi}_1 = \frac{1}{l_c} (\psi_{c_1}(\phi_1) - \psi_1 - u_1 + d_{\psi_1}(t) + \psi_2 - \psi_{SB}(\phi_3))
\]
\[ \psi_2 = \frac{1}{4B^2L_c}(\phi_2 - \phi_{T_2}(\psi_2) - d_{\phi_2}(t) - v_3\phi_1) \]

\[ \dot{\phi}_2 = \frac{1}{L_c}(\psi_{c_2}(\phi_2) - \psi_2 - u_2 + d_{\psi_1}(t) + \dot{v}_6\psi_1 - \psi_{SB}(\phi_3)) \]

\[ \psi_3 = \frac{1}{4B^2L_c}(\phi_3 - \phi_{T_1}(\psi_3) - d_{\phi_3}(t) - v_5(\phi_1 + \phi_2)) \]

\[ \dot{\phi}_3 = \frac{1}{L_c}(\psi_{c_3}(\phi_3) - \psi_3 - u_3 + d_{\psi_1}(t) + \dot{v}_6\psi_1) \]

where \( v_j, j = 1, \ldots, 6 \) are uncertain parameters and

\[ d_{\phi_1}(t) = 0.02 \sin(0.1t) + 0.02 \cos(0.4t) \]

\[ d_{\psi_1}(t) = 0.02 \sin(0.1t) + 0.02 \cos(0.4t) \]

\[ d_{\phi_2}(t) = 0.02 \sin(0.1t) + 0.02 \cos(0.4t) \]

\[ d_{\psi_2}(t) = 0.02 \sin(0.1t) + 0.02 \cos(0.4t) \]

\[ d_{\phi_3}(t) = 0.15e^{-0.015c} \cos(0.2t) \]

\[ d_{\psi_3}(t) = 0.1e^{-0.005c} \sin(0.3t) \]

The characteristics of the spill back valve are

\[ \psi_{SB}(\phi_3) = \gamma_{SB}\phi_3^2 \]

where \( \gamma_{SB} \) is also the valve's yield. In designing a surge controller in the compressor systems (49), it is assumed that the value of throttle valve, as well as the compressor characteristic, is not known. So, the fuzzy system (Wang, 1995) is used to approximate the compressor characteristic. To do this, a 9-membership is used, with the following equations

\[ p_j(\phi) = e^{-(\phi-0.1j)^2}, \quad j = 1, 2, \ldots, 9 \]

\[ \psi_{c}(\phi) = W^T P(\phi) + \Delta\psi_{c}(\phi) \]

\[ W^T = [W_1, \ldots, W_9] \]

\[ P(x) = [p_1(\phi), \ldots, p_9(\phi)]^T \]

In which \( P(\phi) \) is a fuzzy basis function vector, \( W^T \) component vector and \( \Delta\psi_{c}(\phi) \) satisfies \( \Delta\psi_{c}(\phi) < \epsilon \)

where \( \epsilon > 0 \) is a real number (Wang, 1995). Then, the equations of compressor systems are obtained:

\[ \psi_1 = \frac{1}{4B^2L_c}(\phi_1 - \phi_{T_1}(\psi_1) - d_{\phi_1}(t) - v_1\phi_2) \]

\[ \dot{\phi}_1 = \frac{1}{L_c}(W_1^T P(\phi_1) + \Delta\psi_{c_1}(\phi_1) - \psi_1 - u_1 + d_{\psi_1}(t) + \dot{v}_2\psi_2 - \psi_{SB}(\phi_3)) \]

\[ \psi_2 = \frac{1}{4B^2L_c}(\phi_2 - \phi_{T_1}(\psi_2) - d_{\phi_2}(t) - v_3\phi_1) \]
\[
\dot{\phi}_2 = \frac{1}{\tau_c} (W^T \delta \phi_2 + \Delta \psi_{c_1}(\phi_2) - \psi_2 - u_2 + d_{\psi_2}(t) + v_4 \psi_1 - \psi_{SB}(\phi_3)) \\
\dot{\psi}_3 = \frac{1}{4B^T \delta_L} (\phi_3 - \phi_1(\psi_3) - d_{\phi_3}(t) - \Delta \psi_5(\phi_1 + \phi_2)) \\
\dot{\phi}_3 = \frac{1}{\tau_c} (W^T \delta \phi_3 + \Delta \psi_{c_1}(\phi_3) - \psi_3 - u_3 + d_{\psi_3}(t) + \psi_{SB}(\psi_4))
\]

Equations (7) and (54) are rewritten for each subsystem \(i = 1, 2, 3\) as

\[
A_i = \begin{bmatrix} 0 & \frac{1}{\tau_c} & 0 \\ 0 & -\frac{1}{\tau_c} & 0 \\ 0 & 0 & -\frac{1}{\tau_c} \end{bmatrix}, B_i = \begin{bmatrix} 0 \\
-\frac{1}{\sqrt{\phi_i}} \\
\frac{1}{\sqrt{\phi_i}} \end{bmatrix}
\]

\[
f_i(\phi_i, \psi_i) = \begin{bmatrix} 0 & \frac{1}{\sqrt{\phi_i}} & 0 \\ -\frac{1}{\sqrt{\phi_i}} & 0 & \frac{1}{\sqrt{\phi_i}} \end{bmatrix} P(\psi_i), \quad \theta_i = \begin{bmatrix} \gamma_i \\ W_i \end{bmatrix}
\]

\[
d_i(t) = \begin{bmatrix} \sqrt{\phi_i} \\ -\frac{1}{\sqrt{\phi_i}} \end{bmatrix} \begin{bmatrix} d_{\phi_3}(t) \\ \psi_{SB}(\phi_3) \end{bmatrix}
\]

\[
g_1(\phi_1, \psi_1, \phi_2, \psi_2, \phi_3, \psi_3) = \begin{bmatrix} -v_1 \phi_2 \\ v_2 \psi_2 - \psi_{SB}(\phi_3) \end{bmatrix}
\]

\[
g_2(\phi_1, \psi_1, \phi_2, \psi_2, \phi_3, \psi_3) = \begin{bmatrix} -v_3 \phi_1 \\ v_4 \psi_1 - \psi_{SB}(\phi_3) \end{bmatrix}
\]

\[
g_3(\phi_1, \psi_1, \phi_2, \psi_2, \phi_3, \psi_3) = \begin{bmatrix} -v_5(\phi_1 + \phi_2) \\ v_6 \psi_1 \end{bmatrix}
\]

Next, the existing constraints in the compressor systems should be incorporated in the optimization problem (12). The first existing constraint is on the controlling input. Since the controlling signal has a CCV output, so we have

\[
\dot{\psi}_i(t) > 0, \quad i = 1, 2, 3
\]

The next constraint and limitation is that the flow has some maximum and minimum values. This constraint should also be considered.

\[
-\phi_{mi} \leq \dot{\phi}_i(t) \leq \phi_{Choke}, \quad i = 1, 2, 3
\]

In order to simulate the compressors behavior, the starting point has been considered on the left of the surge line. The open-loop optimization problem described by problem 1 is solved in discrete time with a sample time of \(h = 0.1, i = 1, 2, \ldots, h \) time units and prediction horizon of \(T_p = 0.3 \) time units.

The value of weight matrices \(Q_i, R_i, i = 1, 2, 3\) in the objective function is considered as

\[
Q_i = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}, \quad R_i = 1
\]

By choosing

\[
\alpha_i = 10^{-3}, \quad \beta_i = 10^{-7}, \quad \epsilon_i = 10^{-2}, \quad \eta_i = 10^3, \quad \lambda = 10^{-3}, \quad \mu = 10^{-6}
\]

Solving LMI for (22,23) relations, \(P_i, K_i, i = 1, 2, 3\) are obtained as

\[
P_i = \begin{bmatrix} 0.4 & 0.1 \\ 0.1 & 0.4 \end{bmatrix} \times 10^{-3}
\]

\[
K_i = \begin{bmatrix} -0.49 & 1.63 \\ 3.25 & -2.55 \end{bmatrix}
\]

\[
\]
In $t = 100$ s the first compressor throttle valve value reduces from $\gamma_{T1} = 0.65$ to $\gamma_{T1} = 0.6$, which leads to surge in the first compressor. As such, in $t = 110$ s, the second compressor throttle valve value reduces from $\gamma_{T2} = 0.7$ to $\gamma_{T2} = 0.6$ and the second compressor also experiences surge. Also, in $t = 120$ s, the third compressor throttle valve value reduces from $\gamma_{T3} = 0.75$ to $\gamma_{T3} = 0.6$ and the third compressor also leads to surge. Finally, in $t = 150$ s the spill back valve value changes from $\gamma_{SB} = 0$ to $\gamma_{SB} = 0.2$. After applying the presented predictive controller and simulating the compressors behavior, the following results are obtained. Figures 3–8 show the first, second, and third compressor’s pressure and flow behavior. The control signals obtained from the controller are shown in Figures 9–11. Also, simulations were performed with different prediction step sizes to demonstrate its importance on system performance. As it can be seen in these figures, the decentralized controllers have managed to prevent the surge and stabilize the system. As it can be seen in Figures 3–8, the flow and pressure for compressors 1, 2, and 3 have been stabilized by using the designed decentralized controller and the system has been controlled in the presence of disturbance and coupling effects.

5. Conclusion

A control invariant-based new decentralized Tube-MPC scheme for continuous-time nonlinear large-scale systems with bounded disturbances has been proposed in this article. This scheme is characterized by:

1. This algorithm is developed for a wide range of large-scale systems and can also be used for interactive subsystems and systems with coupled constraints.

2. Each sub-system in this algorithm is only required to have the information regarding the model regulating its dynamics, and it is assumed that no data is transmitted between the local control rules.

3. The presented controller is robust to subsystem model uncertainty as well as bounded disturbances with an unspecified upper bound.

4. It is guaranteed that the system converges under appropriate assumptions, and conditions on state and input variables are controlled.

One of those conditions is the need to develop a decentralized additional control rule, which is characterized by:

1. When the interrelationships between the sub-systems and the overall extensive system are overlooked, it can stabilize both the local sub-systems.

2. Its Lyapunov function corresponds to a measured sum of local Lyapunov functions.

![Figure 3. Pressure of compressor 1.](image-url)
Figure 4. Flow of compressor 1.

![Flow of compressor 1 graph](image)

Figure 5. Pressure of compressor 2.

![Pressure of compressor 2 graph](image)

Figure 6. Flow of compressor 2.

![Flow of compressor 2 graph](image)
Figure 7. Pressure of compressor 3.

![Pressure of compressor 3 graph]

Figure 8. Flow of compressor 3.

![Flow of compressor 3 graph]

Figure 9. Control signal of compressor 1.

![Control signal of compressor 1 graph]
Future works can investigate the designing process of distributed model predictive control for large-scale systems including interrelated sub-systems and uncertainty and disturbance in sub-system models.

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**Author details**
Samane Fazeli
E-mail: samane.fazeli2@gmail.com
Naiem Abdollahi
E-mail: abdolahinaiemkh@yahoo.com
Hashem Imani Marrani
E-mail: imani.hashem@gmail.com
ORCID ID: http://orcid.org/0000-0002-6458-6851
Hamid Malekzadeh
E-mail: h.malekzadeh@yahoo.com
Hasan Hosseinzadeh
E-mail: h.hosseinzadeh@iauardabil.ac.ir

1 Department of Electrical Engineering, Science and Research branch, Islamic Azad University, Tehran, Iran.
2 Department of Engineering, South Tehran branch, Islamic Azad University, Tehran, Iran.
3 Young Researchers and Elite Club, Ardabil Branch, Islamic Azad university, Ardabil, Iran.
4 Control Engineering Department of Electrical Engineering, Imam Khomeini University of Maritime Sciences, Noshahr, Mazandaran, Iran.
5 Department of Mathematics, Ardabil Branch, Islamic Azad university, Ardabil, Iran.

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