Indirect Kalman Filter in Mobile Robot Application

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Abstract: Problem statement: The most successful applications of Kalman filtering are to linearize about some nominal trajectory in state space that does not depend on the measurement data. The resulting filter is usually referred to as simply a linearized Kalman filter. Approach: This study introduced mainly indirect Kalman filter to estimate robot’s position. A developed differential encoder system integrated accelerometer is experimental tested in square shape. Results: Experimental results confirmed that indirect Kalman filter improves the accuracy and confidence of position estimation. Conclusion: In summary, we concluded that indirect Kalman filter has good potential to reduce error of measurement data.

Key words: Indirect Kalman filter, differential encoder system, sensor data fusion

INTRODUCTION

Kalman’s study of the early 1960s was recognized almost immediate as new and important contributions to least-squares filtering. As a result, there was a renewal of research interest in this area. Research study in this area still continues and new applications and extensions continue to appear regularly in the technical literature (Aggarwal et al., 2005). Some of the most successful applications of Kalman filtering have been in situations with nonlinear dynamics or nonlinear measurement relationships. One is to linearize about some nominal trajectory in state space that does not depend on the measurement data. The resulting filter is usually referred to as simply a linearized Kalman filter. The other method is to linearize about a trajectory that is continually updated with the state estimates resulting from the measurements. This filter is called an extended Kalman filter. Its use in the analysis of visual motion has been documented frequently. The standard Kalman filter derivation is given here as a tutorial exercise in the practical use of some of the statistical techniques (Elfes, 1987; Borenstein and Feng, 1996). Documenting this derivation furnishes the reader with further insight into the statistical constructs within the filter. The filter is constructed as a mean squared error minimization, but an alternative derivation of the filter is also provided showing how the filter relates to maximum likelihood statistics. The purpose of filtering is to extract the required information from a signal, ignoring everything else. How well a filter performs this task can be measured using a cost or loss function. Indeed we may define the goal of the filter to be the minimization of this loss function (Norsuzila et al., 2008; Gao et al., 2007).

MATERIALS AND METHODS

The Kalman filter has long been regarded as the optimal solution to many tracking and data prediction tasks (Figueroa and Mahajan, 1994; Gelb, 1974; Haykin, 1996).

Indirect Kalman filter concept: The process to be estimated and the measurement relationship is written in the form:

\[ \dot{x} = f(x, u, t) + w \]  
\[ z = h(x, t) + v \]

Where:
- \( f \) and \( h \) = Known function
- \( u \) = A deterministic forcing function
- \( w \) and \( v \) = White noise with zero cross-correlation

From the truth trajectory \( x_{ref} \) is referred to as the nominal or reference trajectory and the actual trajectory \( x \) can write as:

\[ x = x_{ref} + \delta x \]

From Eq. 1-3, then become:

\[ \dot{x}_{ref} + \delta x = f(x_{ref} + \delta x, u, t) + w \]  
\[ z = h(x_{ref} + \delta x, t) + v \]

and with Taylor’s series expansion, then the result, retaining only the first-order terms is:
\[
\dot{x}_{\text{Ref}} + \delta \dot{x} = f(x,u,t) + \frac{\partial f}{\partial x} \Delta x + w(t) \tag{6}
\]

\[
z = h(x_{\text{Ref}}, t) + \frac{\partial h}{\partial x} \Delta x + v(t) \tag{7}
\]

Where:

\[
\frac{\partial f}{\partial x} = \begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\
\frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n}
\end{bmatrix}
\]

\[
\frac{\partial h}{\partial x} = \begin{bmatrix}
\frac{\partial h_1}{\partial x_1} & \frac{\partial h_1}{\partial x_2} & \cdots & \frac{\partial h_1}{\partial x_n} \\
\frac{\partial h_2}{\partial x_1} & \frac{\partial h_2}{\partial x_2} & \cdots & \frac{\partial h_2}{\partial x_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial h_n}{\partial x_1} & \frac{\partial h_n}{\partial x_2} & \cdots & \frac{\partial h_n}{\partial x_n}
\end{bmatrix}
\]

From the truth trajectory \(x_{\text{Ref}}\) to satisfy the deterministic differential equation:

\[
\dot{x}_{\text{Ref}} = f(x_{\text{Ref}}, u, t) \tag{8}
\]

Substituting this into Eq. 6 then leads to the linearized model:

\[
\delta \dot{x} = \frac{\partial f}{\partial x} \Delta x + w(t) \tag{9}
\]

\[
(z - h(x_{\text{Ref}}, t)) + \frac{\partial h}{\partial x} \Delta x + v(t) \tag{10}
\]

From Eq. 9 and 10 may be written in the discrete-time form. For process state equation is written as:

\[
\delta x_{k+1} = F_k \delta x_k + \Gamma w_k \tag{11}
\]

and measurement equation as:

\[
\delta z_k = H_k \delta x_k + A v_k \tag{12}
\]

From the basic linearized measurement equation, Eq. 10 the measurement presented to the Kalman filter is \((z - h(x_{\text{Ref}}, t))\). Then the incremental estimate update equation at time \(t_k\) is considered as:

\[
\delta \dot{x}_{k} = \delta \dot{x}_{k-1} + K_k \cdot (\delta z_k - H_k \delta \dot{x}_{k-1}) \tag{13}
\]

From above equation can consider term \(z_k - h(x_{\text{Ref}}, t)\) as \(\delta z_k\) then the equation become:

\[
\delta \dot{x}_{k} = \delta \dot{x}_{k-1} + K_k \cdot (\delta z_k - H_k \delta \dot{x}_{k-1}) \tag{14}
\]

The term \((\delta z_k - H_k \delta \dot{x}_{k})\) is named as Residual and the matrix \(K_k\) is Kalman gain. It can minimize the posteriori covariance of the error estimate. The matrix \(K_k\) is calculated by:

\[
K_k = P_k H_k^T (H_k P_k H_k^T + \Lambda_k R_k \Lambda_k^T)^{-1} \tag{15}
\]

\(R_k\) is the measurement noise covariance at step \(k\). The covariance of the priori estimate of \(\delta \dot{x}\) is calculated as:

\[
P_k = E \left[(\delta x_k - \delta \dot{x}_k)(\delta x_k - \delta \dot{x}_k)^T\right] \tag{16}
\]

The posteriori estimate is:

\[
P_{k}^{-} = E \left[(\delta x_k - \delta \dot{x}_k)(\delta x_k - \delta \dot{x}_k)^T\right] \tag{17}
\]

The equation of extend Kalman Filter for “Measurement update” are:

\[
K_k = P_k H_k^T (H_k P_k H_k^T + \Lambda_k R_k \Lambda_k^T)^{-1} \tag{18}
\]

\[
\delta \dot{x}_{k} = \delta \dot{x}_{k-1} + K_k \cdot (\delta z_k - H_k \delta \dot{x}_{k-1}) \tag{19}
\]

\[
P_{k}^{-} = P_{k}^{-} - K_k H_k P_k^{-} \tag{20}
\]
and the equation of extend Kalman-Filter for “Time update” with the assumption the projection equation is not affected by the cross-correlation between system noise and measurement noise because of the whiteness property of each:

\[
\delta \hat{x}_{k+1} = F_k \delta \hat{x}_k
\]

\[
P_{k+1} = F_k P_k F_k^T + \Gamma_k Q \Gamma_k^T
\]

Generally for the measurement and time update step in the literature start at time \( t_k \) first the “Time update” and then the “Measurement update” shown in Fig. 1.

RESULTS

The experiment is carried out and presented in to demonstrate the feasibility, accuracy and performance using Kalman filter algorithm. The experiment focused on the observation of the position accuracy.

DISCUSSION

The results will be compared and discussed. We start with calibration the accelerometer and controlled the vehicle along a square shape for 3x3 m\(^2\) with start point (0, 0). The trajectory was estimated based on shaft encoders from start point (0, 0) → point (0, 3) → point (3, -3) → point (0, -3) → end point (0, 0). The estimation error from fusion algorithm between encoders and accelerometer at end point in the X-coordinate is 23.5 cm and in Y-coordinate is 32 cm. We see that the estimation with sensor fusion can improve performance of mobile robot’s localization.

CONCLUSION

We have now presented the basics of Kalman filtering and looked at a few examples of how the technique can be applied in physical situations. The Kalman filter is intended to be used for estimating random processes. The Kalman filter is a linear estimator. The filter is optimal in the minimum mean-square-error sense within a class of all estimators, linear and nonlinear. Under certain special circumstances, the Kalman filter yields the same result obtained from deterministic least squares. Kalman filtering is especially useful as an analysis tool in offline error analysis studies. The optimal filter error covariance equation can be propagated recursively without actual measurement data.

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