Penguin-dominated $B \to PV$ decays in NLO perturbative QCD

Hsiang-nan Li and Satoshi Mishima

1Institute of Physics, Academia Sinica, Taipei, Taiwan 115, Republic of China,
Department of Physics, National Cheng-Kung University, Tainan, Taiwan 701, Republic of China and
2School of Natural Sciences, Institute for Advanced Study, Princeton, NJ 08540, U.S.A.

We study the penguin-dominated $B \to PV$ decays, with $P$ ($V$) representing a pseudo-scalar (vector) meson, in the next-to-leading-order (NLO) perturbative QCD (PQCD) formalism, concentrating on the $B \to K\phi$, $\pi K^*$, $\rho K$, and $\omega K$ modes. It is found that the NLO corrections dramatically enhance the $B \to \rho K$, $\omega K$ branching ratios, which were estimated to be small under the naive factorization assumption. The patterns of the direct CP asymmetries $A_{CP}(B^0 \to \rho^0 K^\pm) \approx A_{CP}(B^\pm \to \rho^0 K^\pm)$ and $|A_{CP}(B^0 \to \pi^\pm K^{*\pm})| > |A_{CP}(B^\pm \to \pi^0 K^{*\pm})|$ are predicted, differing from $|A_{CP}(B^0 \to \pi^\pm K^{*\pm})| \gg |A_{CP}(B^\pm \to \pi^0 K^{*\pm})|$. The above patterns, if confirmed by data, will support the source of strong phases from the scalar penguin annihilation in PQCD. The results for the mixing-induced CP asymmetries $S_f$ are consistent with those obtained in the literature, except that our $S_{\rho K}$ is as low as 0.5.

PACS numbers: 13.25.Hw, 12.38.Bx, 11.10.Hi

I. INTRODUCTION

It has been a challenge to explain the branching ratios of the penguin-dominated $B \to PV$ decays, such as $B \to K\phi$, $\pi K^*$, $\rho K$, and $\omega K$. The predictions from the naive factorization assumption (FA) [1] and from the QCD-improved factorization (QCDF) approach [2] are usually smaller than the observed values [3, 4, 5]. In order to account for the data in QCDF, the hadronic parameters associated with the penguin annihilation amplitudes must be tuned (i.e., adopting the scenario “S4”), so that these amplitudes are large and constructive to the leading FA ones [6]. On the contrary, the $B \to K\phi$ branching ratios have been predicted correctly [7, 8] in the perturbative QCD (PQCD) approach [8, 10]. The reason is that the leading amplitudes are not factorizable in QCDF based on collinear factorization theorem due to the existence of end-point singularities. Hence, they are characterized by a scale of order $m_B$, $m_B$ being the $B$ meson mass. The characteristic scale of the leading amplitudes in PQCD, being factorizable since they are formulated in $k_T$ factorization theorem, is of order $\sqrt{m_B^2}$, where $\Lambda$ is a hadronic scale. At this lower characteristic scale, the Wilson coefficients of the QCD penguin operators are enhanced, so that the PQCD predictions for the $B \to K\phi$ branching ratios are large enough. This mechanism has been called as the dynamical enhancement of penguin contributions [9].

The PQCD analysis of the $B \to K\phi$ decays in [7, 8] was performed at leading order (LO). Recently, the PQCD formalism has been extended to next-to-leading-order (NLO) accuracy [11], where the contributions from the NLO running of the Wilson coefficients, the vertex corrections, the quark loops, and the magnetic penguin were taken into account. It was found that these NLO contributions affect penguin-dominated decays more than tree-dominated ones. For example, the quark-loop and magnetic-penguin corrections decrease the penguin amplitudes by about 10%, and the $B \to \pi K$ branching ratios by about 20% [11]. However, these NLO contributions have almost no impact on the tree-dominated $B \to \pi \pi$ branching ratios. Therefore, it is essential to examine the NLO effects on LO predictions for other penguin-dominated modes. In this paper we shall study the penguin-dominated $B \to PV$ decays in NLO PQCD, concentrating on $B \to K\phi$, $\pi K^*$, $\rho K$, and $\omega K$. It is expected that the LO PQCD predictions for the tree-dominated modes, such as $B \to \rho\pi$, $\omega\pi$ [12], are stable under the above NLO corrections.

The second motivation to study the $B \to PV$ decays arises from the observed direct CP asymmetries of the $B^\pm \to \rho^0 K^\pm$ decays: the data of $A_{CP}(B^\pm \to \pi^0 K^\pm)$ almost vanish, but those of $A_{CP}(B^\pm \to \rho^0 K^\pm)$ are sizable [13]. The direct CP asymmetries of both the charged $B$ meson decays were predicted to be large in LO PQCD.
However, as demonstrated in [11], the aforementioned NLO corrections can lower the LO predictions for \( |A_{CP}(B^\pm \to \pi^0 K^\pm)| \) to the measured values. The responsible mechanism comes from the vertex corrections, which induce a large and almost imaginary color-suppressed tree amplitude. This amplitude then decreases the relative strong phase between the total tree amplitude and the total penguin amplitude involved in the \( B^\pm \to \pi^0 K^\pm \) modes. Hence, it is natural to investigate whether the LO predictions for \( A_{CP}(B^\pm \to \rho^0 K^\pm) \) and for the direct CP asymmetry of another charged mode, \( A_{CP}(B^\pm \to \pi^0 K^{\pm*}) \), will be modified by the vertex corrections significantly.

The third motivation concerns the extraction of the standard-model parameter \( \sin(2\phi_1) \), \( \phi_1 \) being the weak phase, from the mixing-induced CP asymmetries \( S_f \) of the penguin-dominated \( B \to f \) decays. In principle, the results should be identical to those from the tree-dominated \( b \to c\bar{c}s \) transitions, such as \( B \to J/\psi K^*(\rho) \). Nevertheless, potentially substantial deviations \( \Delta S \equiv S_{\text{penguin}} - S_{\text{tree}} < 0 \) between the above two extractions have been observed [13]. Theoretical estimates [11, 16, 17, 18, 19, 20, 21, 22, 23] of the tree pollution in the penguin-dominated decays are then crucial for justifying these discrepancies as promising new physics signals, which, however, gave tiny and positive \( \Delta S \) for most modes so far. The mixing-induced CP asymmetry of the \( B^0 \to \pi^0 K^0 \) decay has been calculated in NLO PQCD [11], and the result is consistent with those obtained in the literature. It is necessary to complete the survey by computing the mixing-induced CP asymmetries of the penguin-dominated \( B \to PV \) decays. Since the deviations in the penguin-dominated modes are mainly caused by the color-suppressed tree amplitude, which is sensitive to the vertex corrections, we shall adopt the NLO PQCD formalism here.

It will be demonstrated that the dynamical enhancement of the penguin amplitudes in the \( B \to PV \) decays sustains the NLO corrections. Especially, the NLO effects dramatically enhance the \( B \to \rho K, \omega K \) branching ratios, which were predicted to be small in LO PQCD [14] and in FA [2]. The NLO PQCD results for all the considered \( B \to PV \) branching ratios then match the data within theoretical uncertainties. Note that the measured \( B \to \omega K \) branching ratios are close to the \( B \to \rho K \) ones, though the former receive the additional contribution from the Wilson coefficient \( a_5 \). Because the magnitude of \( a_5 \) increases rapidly at a low energy scale, the similarity between these two modes implies that two-body nonleptonic \( B \) meson decays are insensitive to low-energy dynamics. Their data, if persisting, will encourage the application of a perturbation theory to these processes.

The LO PQCD prediction for the direct CP asymmetry \( A_{CP}(B^\pm \to \rho^0 K^\pm) \) [14] turns out to be stable under the NLO corrections. That is, we predict the different patterns \( A_{CP}(B^0 \to \rho^0 K^0) \approx A_{CP}(B^\pm \to \rho^0 K^\pm) \) and \( |A_{CP}(B^0 \to \pi^0 K^0)| \gg |A_{CP}(B^\pm \to \pi^0 K^\pm)| \), which are attributed to the different orientations of the complex penguin amplitudes: the penguin amplitude is inclined to the imaginary (real) axis in the former (latter). When the penguin amplitude is almost imaginary, its strong phase relative to the tree amplitude is maximal. The color-suppressed tree amplitude, despite of being enhanced by the vertex corrections, does not change it much. The configuration of the relevant amplitudes in the \( B \to \pi K^* \) decays is somewhat located between the \( B \to \rho K \) and \( B \to \pi K \) ones, giving \( |A_{CP}(B^0 \to \pi^0 K^{*+})| \gg |A_{CP}(B^\pm \to \pi^0 K^{\pm*})| \). We point out that the above patterns cannot be attained by the other theoretical approaches, including QCDF [2, 6], soft-collinear effective theory (SCET) [20, 21, 24], and QCDF plus final-state interactions (FSI) [22].

As to the mixing-induced CP asymmetries, we obtain \( \Delta S > 0 \) at LO for all the \( B \to PV \) modes. The NLO corrections increase most of \( \Delta S \) except that for \( B^0 \to \rho^0 K_S \), whose mixing-induced CP asymmetry becomes as low as 0.5. The uniqueness of the \( B^0 \to \rho^0 K_S \) decay results from the fact that both the penguin amplitude and the color-suppressed tree amplitude are almost imaginary and parallel to each other at NLO, a configuration which leads to the very negative \( \Delta S \). Among the penguin-dominated decays we have investigated (\( B^0 \to \pi^0 K_S, \phi K_S, \rho^0 K_S, \omega K_S \)), \( B^0 \to \phi K_S \) is the cleanest one with the minimal tree pollution for extracting \( \sin(2\phi_1) \). Roughly speaking, the NLO PQCD predictions for the \( B \to PV \) mixing-induced CP asymmetries are consistent with those in QCDF [17, 18], but differ from those in QCDF plus FSI [19], which gave \( \Delta S \approx +0.04 \) for the \( B^0 \to \rho^0 K_S \) decay [42].

We review the LO PQCD factorization formulas for the \( B \to PV \) decays, and derive the NLO ones in Sec. II. The branching ratios, the direct CP asymmetries, and the mixing-induced CP asymmetries of the penguin-dominated \( B \to PV \) modes are calculated using the NLO PQCD formalism in Sec. III. We have updated the meson distribution amplitudes and the relevant standard-model inputs in the calculation. Section IV is the conclusion.
II. FACTORIZATION FORMULAS

The amplitude for a $B$ meson decay into the two-body final state $M_2M_3$ through the $\bar{b} \to \bar{s}$ transition has the general expression,

$$A(B \to M_2M_3) = V_{ub}^* V_{us} A_{M_2M_3}^{(u)} + V_{cb}^* V_{cs} A_{M_2M_3}^{(c)} + V_{tb}^* V_{ts} A_{M_2M_3}^{(t)}.$$  

(1)

The amplitudes $A_{M_2M_3}^{(u)}$, $A_{M_2M_3}^{(c)}$, and $A_{M_2M_3}^{(t)}$ are decomposed at LO into

$$A_{M_2M_3}^{(u)} = f_M F_e + M_e + f_M F_{eM_3} + M_{eM_3} + f_B F_a + M_a ,$$

$$A_{M_2M_3}^{(c)} = 0 ,$$

$$A_{M_2M_3}^{(t)} = (f_M F_e^P + M_e^P + f_M F_{eM_3}^P + M_{eM_3}^P + f_B F_a^P + M_a^P) ,$$

(2)

where $f_B$ ($f_M$, $f_M$) is the $B$ meson ($M_2$ meson, $M_3$ meson) decay constant, $F_e$ ($M_e$) the color-allowed factorizable (nonfactorizable) tree emission contribution, $F_{eM_3}$ ($M_{eM_3}$) the color-suppressed factorizable (nonfactorizable) tree emission contribution, $F_a$ ($M_a$) the factorizable (nonfactorizable) tree annihilation contribution, and those with the additional superscripts $P$ the contributions from the penguin operators. With the amplitude $A(B \to M_2M_3)$ in Eq. (1), we derive the branching ratios, the direct CP asymmetries, and the mixing-induced CP asymmetries of two-body nonleptonic $B$ meson decays, which have been defined in [11].

|       | $A_{K^+\phi}$ | $A_{K^0\phi}$ |
|-------|---------------|---------------|
| $F_e^P$ | $F_e^4 \left( a_3^{(s)} + a_4^{(s)} + a_5^{(s)} \right)$ | $F_e^4 \left( a_3^{(s)} + a_4^{(s)} + a_5^{(s)} \right)$ |
| $F_a^P$ | $M_{e4} \left( a_3^{(s)} + a_4^{(s)} + a_5^{(s)} \right)$ | $M_{e4} \left( a_3^{(s)} + a_4^{(s)} + a_5^{(s)} \right)$ |
| $M_a^P$ | $M_{a4} \left( a_3^{(s)} + a_4^{(s)} + a_5^{(s)} \right)$ | $M_{a4} \left( a_3^{(s)} + a_4^{(s)} + a_5^{(s)} \right)$ |

TABLE I: $B \to K\phi$ decay amplitudes in LO PQCD.

The LO PQCD factorization formulas for the $B \to K\phi$, $\pi K^*$, and $\rho(\omega)K$ decay amplitudes are summarized in Tables [11] and [11], respectively. The Wilson coefficients $a^{(q)}$ for the factorizable contributions and $a^{(f(q))}$ for the nonfactorizable contributions are the combinations of $a_i$ in the standard definitions [11], where $q = u, d$ or $s$ denotes the quark pair produced in the electroweak penguin. The explicit expression for each amplitude in Tables [11] is similar to that of the $B \to P_2P_3$ modes [11], but with the replacements,

$$\phi_3^A \to \phi_3 , \quad \phi_3^P \to \phi_3^s , \quad \phi_3^T \to \phi_3^l , \quad m_2 \to m_2 , \quad m_3 \to -m_3 ,$$

(3)

for $B \to P_2V_3$ (involving the $B \to P_2$ transition), and

$$\phi_2^A \to \phi_2 , \quad \phi_2^P \to \phi_2^s , \quad \phi_2^T \to \phi_2^l , \quad m_2 \to -m_2 , \quad m_3 \to -m_3 ,$$

(4)

for $B \to V_2P_3$ (involving the $B \to V_2$ transition), where $m_0$ is the chiral enhancement scale associated with a pseudo-scalar meson, and $m$ the vector meson mass. The above replacements also apply to the factorization formulas for the NLO corrections from the quark loops and from the magnetic penguin.

We add the various NLO corrections to the LO factorization formulas in Tables [11]. The NLO corrections to the smaller nonfactorizable contributions will be neglected. It is understood that the Wilson coefficients...
appearing below refer to the NLO ones. The vertex corrections to the $B \to PV$ decays modify the Wilson coefficients for the emission amplitudes in the standard definition into

$$
a_1(\mu) \to a_1(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_F \frac{C_1(\mu)}{N_c} V_1(M),
$$

$$
a_2(\mu) \to a_2(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_F \frac{C_2(\mu)}{N_c} V_2(M),
$$

$$
a_i(\mu) \to a_i(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_F \frac{C_{i+1}(\mu)}{N_c} V_i(M), \quad i = 3 - 10,
$$

where $M$ denotes the meson emitted from the weak vertex, and the upper (lower) sign applies for odd (even) $i$.

| $A_{a+K^+0}^{(u)}$ | $\sqrt{2}A_{a+K^+0}^{(u)}$ |
|---------------------|-----------------------------|
| $F_c$               | 0                           | $F_{c4} (a_1)$ |
| $M_c$               | 0                           | $M_{c4} (a_1')$ |
| $F_{cK^+}$          | 0                           | $F_{cK^+4} (a_2)$ |
| $M_{cK^+}$          | 0                           | $M_{cK^+4} (a_2')$ |
| $F_a$               | $F_{a4} (a_3)$              | $F_{a4} (a_1)$ |
| $M_a$               | $M_{a4} (a_1')$             | $M_{a4} (a_1')$ |
| $A_{a+K^+0}^{(t)}$  | $\sqrt{2}A_{a+K^+0}^{(t)}$ |
| $F_c$               | $F_{c4} (a_1)$              | 0 |
| $M_c$               | $M_{c4} (a_1')$             | 0 |
| $F_{cK^+}$          | 0                           | $F_{cK^+4} (a_2)$ |
| $M_{cK^+}$          | 0                           | $M_{cK^+4} (a_2')$ |
| $F_a$               | 0                           | 0 |
| $M_a$               | 0                           | 0 |
| $A_{a+K^+0}^{(t)}$  | $\sqrt{2}A_{a+K^+0}^{(t)}$ |
| $F_c$               | $F_{c4} (a_1)$              | $F_{c4} (a_1')$ |
| $M_c$               | $M_{c4} (a_1')$             | $M_{c4} (a_1')$ |
| $F_{cK^+}$          | 0                           | $F_{cK^+4} (a_2)$ |
| $M_{cK^+}$          | 0                           | $M_{cK^+4} (a_2')$ |
| $F_a$               | $F_{a4} (a_1)$              | $F_{a4} (a_1)$ |
| $M_a$               | $M_{a4} (a_1')$             | $M_{a4} (a_1')$ |

TABLE II: $B \to \pi K^+$ decay amplitudes in LO PQCD.
When $M$ is a pseudo-scalar meson, the functions $V_i(M)$ are given, in the NDR scheme, by \[2 \, 6\]  

\[
V_i(M) = \begin{cases} 
12 \ln \frac{m_b}{\mu} - 18 + \frac{2\sqrt{2}N_c}{f_M} \int_0^1 dx \, \phi_M^A(x) g(x), & \text{for } i = 1, 4, 9, 10 , \\
-12 \ln \frac{m_b}{\mu} + 6 - \frac{2\sqrt{2}N_c}{f_M} \int_0^1 dx \, \phi_M^A(x) g(1-x), & \text{for } i = 5, 7 , \\
-6 + \frac{2\sqrt{2}N_c}{f_M} \int_0^1 dx \, \phi_M^P(x) h(x), & \text{for } i = 6, 8 ,
\end{cases}
\]  

$x$ being a momentum fraction. For a vector meson $M$, $\phi_M^A (\phi_M^P)$ is replaced by $\phi_M ( - \phi_M^A)$, and $f_M$ by $f_M^A$ in the third line of the above formulas. The explicit expressions of the hard kernels $g$ and $h$ can be found in \[6\].

When a vector meson is emitted from the weak vertex in, for example, the cases of $B \to K \phi, \pi K^*$, the leading

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
 | \(A^{(u)}_{\rho^+ K^0}\) | & \(\sqrt{2} A^{(u)}_{\rho^0 K^0, \omega K^+}\) \\
\hline
\hline
| \(F_c\) | 0 & \(F_{c4} (a_1)\) |
| \(\mathcal{M}_c\) | 0 & \(\mathcal{M}_{c4} (a_1')\) |
| \(F_{cK}\) | 0 & \(F_{cK4} (a_2)\) |
| \(\mathcal{M}_{cK}\) | 0 & \(\mathcal{M}_{cK4} (a_2')\) |
\hline
| \(F_a\) | \(F_{a4} (a_1)\) & \(F_{a4} (a_1)\) |
| \(\mathcal{M}_a\) | \(\mathcal{M}_{a4} (a_1')\) & \(\mathcal{M}_{a4} (a_1')\) |
\hline
\hline
\hline
| \(A^{(t)}_{\rho^+ K^0}\) | & \(\sqrt{2} A^{(t)}_{\rho^0 K^0, \omega K^+}\) \\
\hline
\hline
| \(F_c^P\) | \(F_{c4} (a_1^{(d)}) + F_{c6} (a_6^{(d)})\) & \(F_{c4} (a_1^{(u)}) + F_{c6} (a_6^{(u)})\) |
| \(\mathcal{M}_c^P\) | \(\mathcal{M}_{c4} (a_1^{(d)}) + \mathcal{M}_{c6} (a_6^{(d)})\) & \(\mathcal{M}_{c4} (a_1^{(u)}) + \mathcal{M}_{c6} (a_6^{(u)})\) |
| \(F_{cK}^P\) | 0 & \(F_{cK4} (a_3^{(d)} \mp a_3^{(u)} + a_5^{(d)} \mp a_5^{(u)})\) |
| \(\mathcal{M}_{cK}^P\) | 0 & \(\mathcal{M}_{cK4} (a_3^{(d)} \mp a_3^{(u)} - a_5^{(d)} \pm a_5^{(u)})\) |
\hline
| \(F_a^P\) | \(F_{a4} (a_1^{(d)}) + F_{a6} (a_6^{(d)})\) & \(F_{a4} (a_1^{(u)}) + F_{a6} (a_6^{(u)})\) |
| \(\mathcal{M}_a^P\) | \(\mathcal{M}_{a4} (a_1^{(d)}) + \mathcal{M}_{a6} (a_6^{(d)})\) & \(\mathcal{M}_{a4} (a_1^{(u)}) + \mathcal{M}_{a6} (a_6^{(u)})\) |
\hline
\hline
\hline
\hline
| \(A^{(u)}_{\rho^- K^+}\) | & \(\sqrt{2} A^{(u)}_{\rho^0 K^0, \omega K^0}\) \\
\hline
\hline
| \(F_c\) | \(F_{c4} (a_1)\) & 0 |
| \(\mathcal{M}_c\) | \(\mathcal{M}_{c4} (a_1')\) & 0 |
| \(F_{cK}\) | 0 & \(F_{cK4} (a_2)\) |
| \(\mathcal{M}_{cK}\) | 0 & \(\mathcal{M}_{cK4} (a_2')\) |
| \(F_a\) | 0 & 0 |
| \(\mathcal{M}_a\) | 0 & 0 |
\hline
\hline
\hline
| \(A^{(t)}_{\rho^- K^+}\) | & \(\sqrt{2} A^{(t)}_{\rho^0 K^0, \omega K^0}\) \\
\hline
\hline
| \(F_c^P\) | \(F_{c4} (a_1^{(u)}) + F_{c6} (a_6^{(u)})\) & \(F_{c4} (\mp a_1^{(d)}) + F_{c6} (\mp a_6^{(d)})\) |
| \(\mathcal{M}_c^P\) | \(\mathcal{M}_{c4} (a_1^{(u)}) + \mathcal{M}_{c6} (a_6^{(u)})\) & \(\mathcal{M}_{c4} (\mp a_1^{(d)}) + \mathcal{M}_{c6} (\mp a_6^{(d)})\) |
| \(F_{cK}^P\) | 0 & \(F_{cK4} (a_3^{(u)} \mp a_3^{(d)} + a_5^{(u)} \mp a_5^{(d)})\) |
| \(\mathcal{M}_{cK}^P\) | 0 & \(\mathcal{M}_{cK4} (a_3^{(u)} \mp a_3^{(d)} - a_5^{(u)} \pm a_5^{(d)})\) |
| \(F_a^P\) | \(F_{a4} (a_1^{(d)}) + F_{a6} (a_6^{(d)})\) & \(F_{a4} (\mp a_1^{(u)}) + F_{a6} (\mp a_6^{(u)})\) |
| \(\mathcal{M}_a^P\) | \(\mathcal{M}_{a4} (a_1^{(d)}) + \mathcal{M}_{a6} (a_6^{(d)})\) & \(\mathcal{M}_{a4} (\mp a_1^{(u)}) + \mathcal{M}_{a6} (\mp a_6^{(u)})\) |
\hline
\end{tabular}
\caption{$B \to \rho K, \omega K$ decay amplitudes in LO PQCD with the upper (lower) signs applying to the $\rho K (\omega K)$ modes.}
\end{table}
emission contributions from the Wilson coefficient \( a_{6}^{(g)} \) are absent as indicated in Tables I and II. Including the vertex corrections, the NLO piece \( a_{6\psi}^{(d)} \), containing the second terms of \( a_{6} \) in Eq. (5), contributes through the following additional amplitudes,

\[
K^{+}0\phi : \quad f_{\phi} F_{\rho}^{P} \rightarrow f_{\phi} F_{\rho}^{P} + f_{\phi}^{T} F_{e6} \left( a_{6\psi}^{(s)} \right),
\]

\[
\pi^{+}K^{0}0 : \quad f_{K^{+}K^{0}} F_{\rho}^{P} \rightarrow f_{K^{+}K^{0}} F_{\rho}^{P} + f_{K^{+}K^{0}}^{T} F_{e6} \left( a_{6\psi}^{(d)} \right),
\]

\[
\pi^{0}K^{+}0 : \quad f_{K^{+}K^{0}} F_{\rho}^{P} \rightarrow f_{K^{+}K^{0}} F_{\rho}^{P} + f_{K^{+}K^{0}}^{T} F_{e6} \left( a_{6\psi}^{(u)} \right),
\]

\[
\pi^{0}K^{0}0 : \quad f_{K^{+}K^{0}} F_{\rho}^{P} \rightarrow f_{K^{+}K^{0}} F_{\rho}^{P} + f_{K^{+}K^{0}}^{T} F_{e6} \left( -a_{6\psi}^{(d)} \right).
\]

These additional amplitudes have emerged in the NLO analysis of the \( B \rightarrow VV \) decays [27].

Taking into account the NLO contributions from the quark loops and from the magnetic penguin, the decay amplitudes in Eq. (1) are modified into

\[
A_{K^{+}K^{0}0}^{(u,c)} \rightarrow A_{K^{+}K^{0}0}^{(u,c)} + M_{K^{+}K^{0}0},\quad A_{K^{+}K^{0}0}^{(t)} \rightarrow A_{K^{+}K^{0}0}^{(t)} - M_{K^{+}K^{0}0},
\]

\[
A_{K^{+}K^{0}0}^{(u,c)} \rightarrow A_{K^{+}K^{0}0}^{(u,c)} + M_{K^{+}K^{0}0},\quad A_{K^{+}K^{0}0}^{(t)} \rightarrow A_{K^{+}K^{0}0}^{(t)} - M_{K^{+}K^{0}0},
\]

\[
A_{K^{+}K^{0}0}^{(u,c)} \rightarrow A_{K^{+}K^{0}0}^{(u,c)} + M_{K^{+}K^{0}0},\quad A_{K^{+}K^{0}0}^{(t)} \rightarrow A_{K^{+}K^{0}0}^{(t)} - M_{K^{+}K^{0}0},
\]

\[
A_{K^{+}K^{0}0}^{(u,c)} \rightarrow A_{K^{+}K^{0}0}^{(u,c)} + M_{K^{+}K^{0}0},\quad A_{K^{+}K^{0}0}^{(t)} \rightarrow A_{K^{+}K^{0}0}^{(t)} - M_{K^{+}K^{0}0},
\]

\[
A_{K^{+}K^{0}0}^{(u,c)} \rightarrow A_{K^{+}K^{0}0}^{(u,c)} + M_{K^{+}K^{0}0},\quad A_{K^{+}K^{0}0}^{(t)} \rightarrow A_{K^{+}K^{0}0}^{(t)} - M_{K^{+}K^{0}0},
\]

where the upper (lower) signs apply to the \( \rho K \) (\( \omega K \)) decays. The NLO amplitudes \( M_{M_{2}M_{3}} \), \( M_{M_{2}M_{3}}^{(c)} \), \( M_{M_{2}M_{3}}^{(u)} \), and \( M_{M_{2}M_{3}}^{(d)} \) denote the up-loop, charm-loop, QCD-penguin-loop, and magnetic-penguin corrections, respectively. Their expressions are similar to those for the \( B \rightarrow PP \) decays [11], but with the replacements specified in Eqs. (3) and (4). The magnetic-penguin contribution to the \( B \rightarrow PV \) modes has been computed in [28]. It will be shown that the quark-loop corrections are always constructive, increasing all the \( B \rightarrow PV \) branching ratios considered here. The magnetic-penguin corrections decrease the \( B \rightarrow K\phi, \pi K^{*}, \omega K \) branching ratios, but enhance the \( B \rightarrow \rho K \) ones slightly.

### III. Numerical Results

We perform the numerical analysis in this section based on the factorization formulas derived above. The choices of the \( B \) meson wave function, the \( B \) meson lifetimes, the decay constants and the chiral enhancement scales associated with the pseudo-scale mesons, and the weak phase \( \phi_{1} = 21.7^\circ \) are the same as in [11]. We take \( m_{\rho} = 0.77 \) GeV, \( m_{\pi} = 0.78 \) GeV \( m_{K^{*}} = 0.89 \) GeV, and \( m_{\phi} = 1.02 \) GeV for the vector meson masses [29], and \( m_{b} = 4.8 \) GeV for the \( b \) quark mass appearing in the magnetic-penguin operator \( O_{b\rho} \). The longitudinal decay constants of the vector mesons can be extracted from other decay rates, for example, \( f_{\rho,K^{*}} \) from the measured \( \tau^{-} \rightarrow (\rho^{-}, K^{*-})\nu_{\tau} \) decay rates [24, 30]. The transverse decay constants have been evaluated in QCD sum rules, which are summarized below [31, 32]:

\[
f_{\rho} = (0.209 \pm 0.002) \text{ GeV}, \quad f_{\rho}^{T} = (0.165 \pm 0.009) \text{ GeV},
\]

\[
f_{\pi} = (0.195 \pm 0.003) \text{ GeV}, \quad f_{\pi}^{T} = (0.145 \pm 0.010) \text{ GeV},
\]

\[
f_{K^{*}} = (0.217 \pm 0.005) \text{ GeV}, \quad f_{K^{*}}^{T} = (0.185 \pm 0.010) \text{ GeV},
\]

\[
f_{\phi} = (0.231 \pm 0.004) \text{ GeV}, \quad f_{\phi}^{T} = (0.200 \pm 0.010) \text{ GeV}.
\]

(9)
We employ the updated twist-2 and twist-3 pseudo-scalar meson distribution amplitudes, and the updated twist-2 vector meson distribution amplitudes from QCD sum rules \[31, 33\]. For the twist-3 vector meson distribution amplitudes, we adopt the asymptotic models \[34\], since some of them are still uncertain in sum rules. The explicit expressions of the above distribution amplitudes are collected in the Appendix. The sum-rule analysis of the kaon and $K^*$ meson distribution amplitudes have been also performed in \[35\] and \[36\], respectively. When analyzing the theoretical uncertainty, we consider the following ranges for the Gegenbauer coefficients in the light-meson distribution amplitudes, whose definitions are referred to the Appendix,

\[
\begin{align*}
    a^{\bar{K}}_2 &= 0.25 \pm 0.25, \\
    a^{K}_2 &= 0.25 \pm 0.25, \\
    a_{2p}^{\bar{K}} &= 0.15 \pm 0.15, \\
    a_{2p}^{K} &= 0.11 \pm 0.11, \\
    a_{2\bar{p}}^\parallel &= 0.0 \pm 0.2. \\
\end{align*}
\]

\[\tag{10}\]

Since our results do not vary with the Gegenbauer coefficients $a^{K}_1$ and $a_{1\bar{K}}^\parallel$, much, their values have been fixed at $a^{K}_1 = 0.06$ and $a_{1\bar{K}}^\parallel = 0.03$. All the Gegenbauer coefficients associated with the twist-3 distribution amplitudes are also fixed for simplicity. We update the CKM matrix elements \[29\],

\[
\begin{align*}
    V_{ud} = 0.97377, \\
    V_{us} = 0.2257, \\
    |V_{ub}| = (4.31 \pm 0.30) \times 10^{-3}, \\
    V_{cd} = -0.230, \\
    V_{cs} = 0.957, \\
    V_{cb} = 0.0416, \\
    \phi_3 = (70 \pm 30)^\circ.
\end{align*}
\]

The increase of $|V_{ub}|$ from its old value $(3.67 \pm 0.47) \times 10^{-3}$ will enhance the direct CP asymmetries of $B$ meson decays as shown later.

We then evaluate the relevant transition form factors at maximal recoil,

\[
\begin{align*}
    F_0^{B\pi} &= 0.27^{+0.07}_{-0.05}, \\
    F_0^{BK} &= 0.43^{+0.10}_{-0.08}, \\
    A_0^{B\rho} &= 0.32^{+0.07}_{-0.06}, \\
    A_0^{B\omega} &= 0.29^{+0.06}_{-0.05}, \\
    A_0^{BK^*} &= 0.38^{+0.07}_{-0.06},
\end{align*}
\]

\[\tag{12}\]

where the theoretical uncertainties come from the variation of the parameters associated with the $B$ meson wave function, and from Eqs. \[10\] and \[11\]. The above values of $F_0^{B\pi}$ and $F_0^{BK}$ are higher than $F_0^{B\pi} = 0.24$ and $F_0^{BK} = 0.36$ obtained in \[11\] from the old pseudo-scalar meson distribution amplitudes. The others are similar to $A_0^{B\rho} = 0.30$, $A_0^{B\omega} = 0.28$, and $A_0^{BK^*} = 0.37$ from QCD sum rules \[30\]. Our $A_0^{BK^*}$ is a bit above that from the covariant light-front QCD calculation \[37\]. Note that larger values of the $B \to V$ form factors have been adopted in QCDF \[6\] in order to account for the observed $B \to PV$ branching ratios.

| $\mu$ (GeV) | 0.5 | 1.0 | 1.5 | 2.0 |
|-------------|-----|-----|-----|-----|
| $a_1(\mu)$  | 1.20 | 1.10 | 1.07 | 1.05 |
| $a_{1VC}(\mu)$ | -0.25 + i 0.29 0.00 + i 0.09 0.02 + i 0.06 0.03 + i 0.04 |
| $a_2(\mu)$  | -0.26 | -0.06 | 0.02 | 0.07 |
| $a_{2VC}(\mu)$ | 0.47 - i 0.53 0.00 - i 0.23 -0.08 - i 0.17 -0.12 - i 0.14 |
| $a_3(\mu)$  | 0.03 | 0.01 | 0.01 | 0.01 |
| $a_{3VC}(\mu)$ | -0.06 + i 0.06 0.00 + i 0.02 0.00 + i 0.01 0.01 + i 0.01 |
| $a_4(\mu)$  | -0.14 | -0.08 | -0.06 | -0.05 |
| $a_{4VC}(\mu)$ | 0.03 - i 0.04 0.00 - i 0.01 0.00 + i 0.00 0.00 + i 0.00 |
| $a_5(\mu)$  | -0.15 | -0.04 | -0.02 | -0.02 |
| $a_{5VC}(\mu)$ | 0.29 - i 0.13 0.03 - i 0.03 0.01 - i 0.01 0.00 - i 0.01 |
| $a_6(\mu)$  | -0.37 | -0.14 | -0.10 | -0.08 |
| $a_{6VC}(\mu)$ | 0.00 + i 0.00 0.00 + i 0.00 0.00 + i 0.00 0.00 + i 0.00 |

TABLE IV: RG evolution of the Wilson coefficients involved in the $B \to \omega K$ decays, where $a_1(\mu)$ and $a_{1VC}(\mu)$ represent the first and second terms on the right-hand side of Eq. \[5\], respectively.

We present in Table IV the renormalization-group (RG) evolution of the Wilson coefficients $a_i$ and of the vertex corrections $a_{iVC}$ in the scale $\mu$ for the $B \to \omega K$ decays, which correspond to the first and second terms
on the right-hand side of Eq. (3), respectively. The RG evolution exhibited by the other $B \rightarrow PV$ modes is similar. It is seen that the coefficients $a_{4-6}$ associated with the penguin operators are enhanced more than the leading tree coefficient $a_1$ at a low $\mu$. This mechanism, referred to the dynamical enhancement in [1], gives the large branching ratios of penguin-dominated $B$ meson decays [2, 8, 9]. Most of the vertex corrections remain subleading even at a low $\mu$, except those to $a_2$ and $a_5$. Though the correction $a_{3V}$ to $a_3$ is significant, its effect, much smaller than that of $a_{3VC}$, is negligible. The large $a_{2V}$ has been found to be responsible for the reduction of the LO PQCD predictions for the direct CP asymmetry $|A_{CP}(B^\pm \rightarrow \pi^0 K^\pm)|$ [11] as explained in the Introduction. The new effect observed in this work arises from the large $a_{3VC}$, which will be elucidated below.

|               | $K\phi$   | $\pi K^*$ | $\rho K$   | $\omega K$ |
|---------------|-----------|-----------|------------|------------|
|               | LO +NLO   | LO +NLO   | LO +NLO    | LO +NLO    |
| $T'$          | 0.0 $e^{0.0}$ | 0.4 $e^{-1.9}$ | 12.3 $e^{0.0}$ | 12.0 $e^{0.1}$ | 10.6 $e^{0.0}$ | 10.3 $e^{0.1}$ | 9.6 $e^{0.0}$ | 9.4 $e^{0.1}$ |
| $C'$          | 0.0 $e^{0.0}$ | 0.4 $e^{0.6}$ | 0.5 $e^{-1.9}$ | 2.4 $e^{-1.2}$ | 0.8 $e^{-1.2}$ | 4.4 $e^{-1.3}$ | 0.7 $e^{-1.3}$ | 4.1 $e^{-1.3}$ |
| $P'$          | 36.1 $e^{1.28}$ | 41.2 $e^{1.28}$ | 21.1 $e^{1.26}$ | 23.0 $e^{1.26}$ | 17.1 $e^{1.15}$ | 25.2 $e^{1.14}$ | 15.7 $e^{1.15}$ | 23.2 $e^{1.14}$ |
| $P'_{ew}$     | 0.9 $e^{-1.6}$ | 11.8 $e^{-0.5}$ | —           | —           | —           | —           | 1.8 $e^{-1.0}$ | 19.8 $e^{-1.0}$ |
| $P'_{ew}$     | 11.1 $e^{3.1}$ | 11.4 $e^{-3.1}$ | 5.8 $e^{3.1}$ | 5.6 $e^{-3.0}$ | 9.9 $e^{3.1}$ | 10.1 $e^{3.1}$ | 3.1 $e^{3.1}$ | 3.2 $e^{3.1}$ |

TABLE V: Topological amplitudes in units of $10^{-5}$ GeV.

Since all the meson distribution amplitudes derived from QCD sum rules are defined at the scale 1 GeV, we propose to freeze the RG evolution of the Wilson coefficients at this scale, when the energy runs below it. That is, we set $C_i(\mu) = C_i(1\text{ GeV})$ as $\mu \leq 1\text{ GeV}$. Note that the freezing scale was chosen as 0.5 GeV in [11, 27]. We have checked that the new choice of the freezing scale decreases the NLO PQCD predictions for the $B \rightarrow \pi K$ branching ratios by about 15%, and has a tiny effect on those for the $B \rightarrow \pi \pi$ branching ratios. The final outcomes are still within the theoretical uncertainty in [11, 27]. In order to facilitate the discussion, we present the central values of our predictions in terms of the topological amplitudes in Table [V] where $T'$, $C'$, $P'$, $P'_{ew}$, and $P'_{ew}$ denote the color-allowed tree, color-suppressed tree, QCD penguin, color-suppressed QCD penguin, and electro-weak penguin contributions, respectively. Their definitions are referred to [8]. Note that $P'_{ew}$ exists only in the $B \rightarrow K\phi$ and $B \rightarrow \omega K$ decays due to the flavor-singlet quark content of the $\phi$ and $\omega$ mesons. We have estimated the averaged hard scale $\langle t \rangle$ for each of the above topological amplitude, and found $\langle t \rangle \approx 2.0$ GeV for $T'$ and $P'_{ew}$, $\langle t \rangle \approx 1.5 - 2.0$ GeV for $P'$, and $\langle t \rangle \approx 1.0 - 1.5$ GeV for $C'$ and $P'$. That is, the hard scales, despite of being all of $O(\sqrt{m_B})$, vary a bit among the different topological amplitudes.

The NLO PQCD results for the $B \rightarrow PV$ branching ratios are displayed in Table [VI] where the QCDF predictions from the scenario S4 for the hadronic parameters [6] are shown for comparison. It has been known that QCDF gives the branching ratios similar to those in PQCD, if adopting S4. The QCDF predictions from the default scenario [6] are usually smaller than the data by a factor 2. It is found that the NLO Wilson evolution, labelled by $\text{LO}_{\text{NLOWC}}$, has a more significant effect on the $B \rightarrow K\phi$ decays than on the $B \rightarrow \pi K$ ones [11]. The reason is that the former depends on the Wilson coefficient $a_5$ (see Table [I]), which is enhanced more at NLO compared to $a_6$ in the latter. However, $a_5$ also receives a significant destructive vertex correction as shown in Table [V] such that the $B \rightarrow K\phi$ branching ratios drop rapidly from the column $\text{LO}_{\text{NLOWC}}$ to $+\text{VC}$ in Table [VI]. The magnetic penguin correction, labelled by $+\text{MP}$, further decreases the branching ratios by about 30%. Eventually, the $B \rightarrow K\phi$ branching ratios in NLO PQCD are still consistent with the measured values.

For the $B \rightarrow \pi K^*$ branching ratios, our values in the column $\text{LO}_{\text{NLOWC}}$ are close to the LO PQCD predictions in [39]. The effects from the vertex corrections, the quark loops, and the magnetic penguin follow the pattern appearing in the $B \rightarrow \pi K$ modes [11]. The main difference between these two decays is that the former involve only $a_4$ (no $a_6$). Therefore, the $B \rightarrow \pi K^*$ branching ratios are expected to be much smaller than the $B \rightarrow \pi K$ ones. As indicated in Table [VI] our NLO PQCD predictions (central values) for the former are about 1/3 of those for the latter. Confronting with the data, the NLO predictions for the $B^{\pm} \rightarrow \pi^\pm K^{\mp}$, $B^{\pm} \rightarrow \pi^0 K^{*\pm}$, and $B^{0} \rightarrow \pi^\pm K^{\mp}$ branching ratios reach only 2/3 of the measured values. Nevertheless, considering the uncertainties of both the theoretical predictions and the experimental data, the discrepancy is not serious. On the contrary, the PQCD results for the $B^{0} \rightarrow \pi^0 K^{*0}$ branching ratios are in good agreement with the data.
The relevant factors being higher than the observed ones. It is then likely that two-body nonleptonic meson decays to low-energy dynamics. Note that the destructive combination flips sign of the penguin contribution, and its associated vertex correction exhibit dramatic running effects at a low scale [11].

| Mode | Data [13] | QCDF(S4) [6] | LO | LONLOWC | +VC | +QL | +MP | +NLO |
|------|-----------|--------------|----|----------|-----|-----|-----|------|
| $B^+ \rightarrow K^+\phi$ | 8.30 ± 0.65 | 11.6 | 13.8 | 41.8 | 14.3 | 44.3 | 28.4 | 7.8 \pm 9 (4.6) |
| $B^0 \rightarrow K^0\phi$ | 8.3 ± 1.0 | 10.5 | 12.9 | 39.2 | 13.4 | 41.5 | 26.6 | 7.3 \pm 5 (2.6) |
| $B^\pm \rightarrow \pi^\pm K^0$ | 10.7 ± 0.8 | 8.4 | 5.5 | 10.6 | 9.6 | 11.3 | 6.5 | 6.0 \pm 2 (3.2) |
| $B^\pm \rightarrow \pi^0 K^\pm$ | 6.9 ± 2.3 | 6.5 | 4.0 | 6.8 | 6.2 | 7.1 | 4.5 | 4.3 \pm 5 (1.7) |
| $B^0 \rightarrow \pi^0 K^\pm$ | 9.8 ± 1.1 | 8.1 | 5.1 | 9.3 | 8.8 | 9.9 | 6.0 | 6.0 \pm 3 (2.4) |
| $B^0 \rightarrow \pi^0 K^0$ | 1.7 ± 0.8 | 2.5 | 1.5 | 3.5 | 3.3 | 3.8 | 2.0 | 2.0 \pm 1 (0.9) |
| $B^\pm \rightarrow \rho^\pm K^0$ | < 48 | 9.7 | 3.6 | 6.1 | 7.8 | 6.4 | 6.7 | 8.7 \pm 9 (4.6) |
| $B^\pm \rightarrow \rho^0 K^\pm$ | 4.27 ± 0.54 | 4.3 | 2.5 | 4.0 | 5.0 | 4.3 | 3.8 | 5.1 \pm 1 (3.6) |
| $B^0 \rightarrow \rho^0 K^\pm$ | 9.9 ± 1.5 | 10.1 | 4.7 | 7.0 | 7.8 | 7.2 | 7.9 | 8.8 \pm 6 (6.2) |
| $B^0 \rightarrow \rho^0 K^0$ | 5.6 ± 1.1 | 6.2 | 2.5 | 3.5 | 3.9 | 3.5 | 4.4 | 4.8 \pm 4 (3.2) |
| $B^\pm \rightarrow \omega K^\pm$ | 10.5 ± 0.5 | 5.9 | 2.1 | 5.9 | 9.3 | 6.4 | 4.8 | 4.8 \pm 1 (4.0) |
| $B^0 \rightarrow \omega K^0$ | 4.8 ± 0.6 | 4.9 | 1.9 | 6.0 | 8.6 | 6.6 | 4.8 | 9.8 \pm 8 (6.7) |

Our LO PQCD results for the $B \rightarrow \rho K$ decays in Table VI are close to those obtained in [14]: $B(B^\pm \rightarrow \rho^\pm K^0) = 2.96 \times 10^{-6}$, $B(B^\pm \rightarrow \rho^0 K^\pm) = 2.18 \times 6 \times 10^{-5}$, $B(B^0 \rightarrow \rho^\pm K^\pm) = 5.42 \times 10^{-6}$, and $B(B^0 \rightarrow \rho^0 K^0) = 2.49 \times 10^{-6}$. That is, the LO predictions fall short by a factor 2 compared to the data [13]. The $B \rightarrow \rho K$ branching ratios, also smaller than the $B \rightarrow \pi K$ ones, are attributed to the destructive combination of the Wilson coefficients $a_4 - 2(m_{Q_K}/m_B)a_6$ in the former, and to the constructive combination $a_4 + 2(m_{Q_K}/m_B)a_6$ in the latter [see the replacement $m_{Q_K} \rightarrow -m_{Q_K}$ in Eq. (4)]. The NLO Wilson evolution enhances the $B \rightarrow \rho K$ branching ratios, but not sufficiently. Since the destructive combination flips sign of the penguin contribution to the $B \rightarrow \rho K$ decays (comparing Table V here and Table V in [11]), the NLO effects change accordingly. For example, the magnetic-penguin contribution becomes constructive. The $B \rightarrow \rho K$ branching ratios then increase all the way up to the measured values as shown in Table VI. It turns out that the relation $B(B \rightarrow \rho K) < B(B \rightarrow \pi K^*)$ in LO PQCD and in FA [3] is reversed into $B(B \rightarrow \rho K) > B(B \rightarrow \pi K^*)$ at NLO.

Our LO PQCD results for the $B \rightarrow \omega K$ branching ratios are also consistent with those in [14]: $B(B^\pm \rightarrow \omega K^\pm) = 3.22 \times 10^{-6}$ and $B(B^0 \rightarrow \omega K^0) = 2.07 \times 10^{-6}$, which are smaller than the data [13]. Similar to $B \rightarrow K\phi$, the $B \rightarrow \omega K$ decays involve the Wilson coefficient $a_5$ through the color-suppressed QCD penguin amplitude $P_{K\phi}^P$, in addition to $a_4$ and $a_6$ (see Table IV). Hence, the enhancement of the $B \rightarrow \omega K$ branching ratios from the NLO Wilson evolution is substantial. Because of the sign flip of the QCD penguin contribution as in $B \rightarrow \rho K$, the vertex correction to $a_5$ becomes constructive, opposite to that in the $B \rightarrow K\phi$ case (see the opposite contributions of $P_{K\phi}^P$ in these two modes in Table V). This explains the jump of the $B \rightarrow \omega K$ branching ratios from the column LONLOWC to +VC in Table VI. Though the magnetic penguin correction is destructive, the net NLO predictions for the $B \rightarrow \omega K$ branching ratios remain large, with the central values being higher than the observed ones.

As stated before, $a_5$ and its associated vertex correction exhibit dramatic running effects at a low scale [11]. Therefore, if lifting the hard characteristic scales in the factorization formulas slightly (notice $(t) \approx 1.0 - 1.5$ GeV for $P_{K\phi}^P$), the $a_5$ contribution will be moderated. Then the predicted $B \rightarrow K\phi$ and $B \rightarrow \omega K$ branching ratios will increase and decrease, respectively, approaching the central values of the data. However, we shall not attempt such a fine tuning here, but point out that the comparison of the two branching ratios provides an interesting test on the significance of the $a_5$ contribution, i.e., on the sensitivity of two-body nonleptonic $B$ meson decays to low-energy dynamics. Note that the relevant form factors $A^{B_{\rho\phi}}_0$ and $A^{B_{\omega\phi}}_0$ are roughly equal as indicated in Eq. (12). If allowing the hard scale to be as low as 0.5 GeV, $B(B \rightarrow \omega K)$ would become three times of $B(B \rightarrow \rho^0 K)$. Because the measured branching ratios are close to each other, $a_5$ plays a minor role, and a higher hard scale is implied. It is then likely that two-body nonleptonic $B$ meson decays are insensitive.
to low-energy dynamics, and a perturbation theory is applicable to these processes.

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
\text{Mode} & \text{Data [13]} & \text{QCDF(S4) [6]} & \text{LO} & \text{LO}_{\text{NLO}} & \text{VC} & \text{QL} & \text{MP} & \text{NLO} \\
\hline
B^\pm \to K^\pm \phi & 3.7 \pm 5.0 & 0.7 & -2 & -1 & -1 & 0 & -1 & 1^{+10}_{-9}(-1) \\
B^0 \to K^0 \phi & 9 \pm 14 & 0.8 & 0 & 0 & 0 & 1 & 0 & 3^{+10}_{-11} \\
B^\pm \to \pi^\pm K^{*0} & -8.6 \pm 5.6 & 0.8 & -3 & -2 & -2 & 0 & -2 & 1^{+11}_{-11}(-1) \\
B^0 \to \pi^\pm K^{*0} & 4 \pm 29 & -6.5 & -38 & -31 & -21 & -29 & -45 & 32^{+21}_{-28}(-19) \\
B^0 \to \pi^\pm K^{*0} & -5 \pm 14 & -12.1 & -56 & -40 & -42 & -38 & -60 & 60^{+32}_{-39}(-15) \\
B^0 \to \pi^0 K^{*0} & -1^{+27}_{-26} & 1.0 & -5 & 1 & -13 & 2 & 3 & -11^{+7}_{-11}(-2) \\
B^\pm \to \rho^\pm K^0 & - & 0.8 & 2 & 1 & 1 & 1 & 1 & 1 \pm (1) \\
B^0 \to \rho^0 K^0 & 31^{+11}_{-10} & 31.7 & 79 & 74 & 73 & 71 & 79 & 71^{+25}_{-35}(-14) \\
B^0 \to \rho^0 K^0 & 17^{+15}_{-16} & 20.8 & 83 & 72 & 69 & 71 & 64 & 64^{+24}_{-30}(-11) \\
B^0 \to \rho^0 K^0 & - & -2.8 & 7 & -8 & 3 & -8 & -6 & 5^{+8}_{-5}(-4) \\
B^\pm \to \omega K^\pm & 5 \pm 6 & 19.3 & 82 & 45 & 37 & 43 & 57 & 32^{+15}_{-17}(-5) \\
B^0 \to \omega K^0 & - & 3.7 & -4 & 7 & -2 & 7 & 8 & 3^{+2}_{-4}(-3) \\
\hline
\end{array}
\]

**TABLE VII:** Direct CP asymmetries in the NDR scheme in percentage.

The NLO PQCD predictions for the direct CP asymmetries of the penguin-dominated $B \to PV$ decays are listed in Table VII, where the QCDF results from S4 [6] are also shown. The QCDF results from the default scenario are always opposite in sign compared with the data. Our predictions for the direct CP asymmetries have larger magnitude here, since the updated CKM matrix element $|V_{ub}|$ has increased substantially. If adopting $|V_{ub}|$ extracted from exclusive decays alone, and a lower $B$ meson decay constant, the magnitude of the above direct CP asymmetries could drop by 30% easily. To be cautious, we emphasize only the various patterns of the direct CP asymmetries exhibited by the considered $B \to PV$ decays, instead of their actual values. Because the NLO Wilson evolution increases the penguin amplitudes, i.e., the $B \to PV$ branching ratios, it dilutes the direct CP asymmetries. The effects from the quark-loop and magnetic-penguin corrections can be understood in the same way. The direct CP asymmetries of the $B \to K\phi$ modes, which are almost pure-penguin processes, remain vanishing at NLO as shown in Table VII.

We explain the different effects from the vertex corrections on the direct CP asymmetries $A_{CP}(B^\pm \to \pi^0 K^{*\pm})$, $A_{CP}(B^\pm \to \rho^0 K^{*\pm})$, and $A_{CP}(B^\pm \to \pi^0 K^\pm)$. It has been known [11] that the penguin amplitude $P'$ in the $B \to \pi K$ decays is in the second quadrant, with a strong phase about $15^\circ$ relative to the negative real axis, and that the source of this phase arises mainly from the almost imaginary scalar penguin annihilation. The color-allowed tree amplitude $T'$ is roughly aligned with the positive real axis. Consequently, the $B^0 \to \pi^+ K^\pm$ decays, involving $P'$ and $T'$, show a sizable direct CP asymmetry [11]. On the other hand, the color-suppressed tree amplitude $C'$, enhanced by the vertex corrections, becomes almost imaginary. It then orient the sum $T' + C'$ into the fourth quadrant, such that $T' + C'$ and $P' + P'_w$ more or less line up (point to the opposite directions) in the $B^\pm \to \pi^0 K^\pm$ decays. This is the reason the magnitude of $A_{CP}(B^\pm \to \pi^0 K^\pm)$ tends to vanish in NLO PQCD, leading to the pattern [11],

\[
|A_{CP}(B^\pm \to \pi^0 K^\pm)| \gg |A_{CP}(B^\pm \to \pi^0 K^\pm)|,
\]

which is in agreement with the data.

The pattern of the direct CP asymmetries in the $B \to \pi K^\pm$ decays can be described similarly, but with the distinction that their penguin emission amplitudes contain only $a_4$ (no $a_6$). The almost imaginary scalar penguin annihilation then renders $P'$ more inclined to the positive imaginary axis compared to that in the $B \to \pi K$ case. Hence, $T'$ and $P'$ are more orthogonal to each other in the $B^0 \to \pi^+ K^*\pm$ decays. For this configuration, the modification from $C'$ will be weaker. That is, the total tree amplitude $T' + C'$ and the total penguin amplitude $P' + P'_w$ will not line up completely in the $B^\pm \to \pi^0 K^{*\pm}$ modes. We then predict the larger $B \to K^{*\pm}$ direct CP asymmetries, and the pattern shown in Table VII,

\[
|A_{CP}(B^\pm \to \pi^0 K^{*\pm})| > |A_{CP}(B^\pm \to \pi^0 K^{*\pm})|.
\]
The situation in the $B \to \rho K$ decays is even more extreme, where the real part of $P'$ almost diminishes due to the destructive combination of the Wilson coefficients $a_4 - 2(m_{0K}/m_B)a_6$. Because of the sign flip of the penguin annihilation amplitude $F'_a$ in Table III under the replacements $m_{0K} \to -m_{0K}$ and $m_{0K} \to -m_{0K}$ in Eq. 11, $P'$ is roughly aligned with the negative imaginary axis as indicated in Table V. This explains the predicted sign of $A_{CP}(B^0 \to \rho^0K^\pm)$ opposite to that of $A_{CP}(B^0 \to \pi^\mp K^\pm)$. In this case $T'$ and $P'$ have the maximal relative strong phase, such that the modification from $C'$ is not obvious. We then predict the larger $B \to \rho K$ direct CP asymmetries, and the pattern,

$$A_{CP}(B^\pm \to \rho^0 K^\pm) \approx A_{CP}(B^\pm \to \rho^0 K^\pm).$$

That is, the NLO corrections to the $B \to \rho K$ direct CP asymmetries are minor, and the LO predictions derived in [14] are reliable.

The direct CP asymmetries in the $B \to \omega K$ decays can be understood by means of those in $B \to \rho K$ plus the effect from the color-suppressed QCD penguin amplitude $P''$. Since $P''$ is roughly aligned with the positive real axis as shown in Table V the relative strong phase between the tree and penguin amplitudes decreases from the maximal configuration in the $B \to \rho K$ decays. Therefore, the vertex corrections have some effect on $A_{CP}(B^\pm \to \omega K^\pm)$, which is then expected to be smaller than $A_{CP}(B^\pm \to \rho^0 K^\pm)$, but still sizable as indicated in Table VII. We stress that the above predictions for the direct CP asymmetries, if confirmed by the future data, will support the source of strong phases from the scalar penguin annihilation in PQCD.

| Data [13] | LO | LO + NLO WC + VC + QL + MP + NLO |
|-----------|----|----------------------------------|
| $S_{\phi K_S}$ | 0.47 ± 0.19 0.72 | 0.72 0.72 0.72 0.72 0.71 +0.03 (+0.01) +0.01 (−0.00) |
| $S_{\rho K_S}$ | 0.17 ± 0.58 0.71 | 0.66 0.47 0.67 0.66 0.50 +0.10 (+0.04) −0.06 (−0.04) |
| $S_{\omega K_S}$ | 0.64 ± 0.30 0.76 | 0.69 0.84 0.69 0.70 0.84 +0.03 (+0.00) −0.07 (−0.02) |

TABLE VIII: Mixing-induced CP asymmetries.

The NLO PQCD predictions for the mixing-induced CP asymmetries $S_{MK_S}$, $M = \phi, \rho$, and $\omega$, are collected in Table VIII. It is found that all $S_{MK_S}$ exhibit positive deviations from those of the tree-dominated $b \to c\bar{c}s$ transitions at LO. The deviation in the $B^0 \to \phi K_S$ decay remains negligible at NLO due to the absence of the tree pollution from the color-suppressed tree amplitude. It is interesting to see that the NLO corrections turn $S_{\rho K_S}$ into a much lower value about 0.5. The reason is, again, the destructive combination of the Wilson coefficients $a_4 - 2(m_{0K}/m_B)a_6$, which flips the sign of the penguin amplitude $P'$ as shown in Table V. The color-suppressed tree amplitude $C'$, modified by the vertex corrections, then becomes parallel to $P'$, leading to the very negative deviation. This observation is consistent with the tendency indicated by the data. The deviation of $S_{\omega K_S}$ is positive, since the sign of $P'$ is opposite to that in the $B^0 \to \rho^0 K_S$ decay as shown in Tables III and V. The large deviations exhibited by the $B^0 \to \rho^0 K_S$, $\omega K_S$ decays at NLO are attributed to the involved large $C'$ (see Table V). Our predictions are basically in agreement with those in QCDF [17, 18], but differ from those in QCDF plus FSI [19], which predicted $S_{\rho K_S} \approx 0.76$ with a positive deviation.

IV. CONCLUSION

In this paper we have surveyed the penguin-dominated $B \to PV$ decays in the NLO PQCD formalism, concentrating on the $B \to K\phi, \pi K^*, \rho K$, and $\omega K$ modes. The NLO Wilson evolution enhances all the branching ratios, while the other NLO corrections modulate the branching ratio of each mode in a different way. For example, the vertex corrections to the Wilson coefficient $a_3$ decrease (increase) the $B \to K\phi$ ($B \to \omega K$) branching ratios significantly. The net NLO effects modify the LO predictions for the $B \to K\phi, \pi K^*$ branching ratios a bit, but increase those for the $B \to \rho K, \omega K$ ones by a factor more than 2. Owing to this dramatic enhancement, the NLO PQCD results for these $B \to PV$ branching ratios are in better agreement with the data. Since the penguin emission amplitudes in the $B \to \pi K, \pi K^*$, and $\rho K$ decays are proportional to $a_4 + 2(m_{0K}/m_B)a_6$, $a_4$, and $a_4 - 2(m_{0K}/m_B)a_6$, respectively, their real parts decrease in the above order. The almost imaginary scalar penguin annihilation then makes the total penguin amplitudes more orthogonal to the color-allowed tree amplitudes. As a consequence, the modifications from the NLO color-suppressed
regularization for these singularities have been proposed, such that in collinear factorization theorem due to the existence of the end-point singularities. Recently, a zero-bin in the literature, which can produce the above patterns of the direct CP asymmetries.

We emphasize that PQCD is the only theoretical approach available for extracting the standard-model parameter \( \sin(2\beta) \). Among the decays we have investigated, the two-body nonleptonic \( B \) meson decays, which then also become factorizable [41]. The idea is to subtract soft modes from collinear effective fields to avoid double counting of soft dynamics. When implemented in a factorization formula, this regularization scheme is equivalent to cutoffs at small momentum fractions \( x \). The zero-bin regularization has been extended to annihilation amplitudes in two-body nonleptonic \( B \) meson decays, which then also become factorizable [41]. As expected, the annihilation amplitudes are real, because internal particles, without carrying parton transverse momenta, do not go on mass shell, when one introduces cutoffs at small \( x \). Therefore, whether a physical strong phase is generated by the LO \( B \) meson annihilation amplitude is a distinction between this modified collinear factorization theorem and the \( k_T \) factorization theorem.

We have also predicted the mixing-induced CP asymmetries of the penguin-dominated \( B \to PV \) decays. Among the decays we have investigated \( (B^0 \to \pi^0 K_S, \phi K_S, \rho^0 K_S, \omega K_S) \), \( B^0 \to \phi K_S \) is the cleanest mode for extracting the standard-model parameter \( \sin(2\beta_1) \), since it does not involve the color-suppressed tree amplitude, which can be greatly enhanced by the NLO effects. It has been found that \( S_{\rho K_S} \) exhibits the maximal negative deviation from those of the tree-dominated \( b \to c\bar{c}s \) transitions, and all other \( S_f \) show positive deviations in NLO PQCD. Hence, it is still puzzling, when confronting the theoretical predictions, especially those for \( S_{\phi K_S} \) and \( S_{\pi^0 K_S} \), with the experimental observations.

We thank C.H. Chen, H.Y. Cheng, C.K. Chua, W.S. Hou, C.M. Jen, M. Nagashima, and A. Soni for useful discussions. This work was supported by the National Science Council of R.O.C. under Grant No. NSC-94-2112-M-001-001, by the Taipei branch of the National Center for Theoretical Sciences, and by the U.S. Department of Energy under Grant No. DE-FG02-90ER40542. HNL thanks the Yukawa Institute for Theoretical Physics, Kyoto University, for her hospitality during his visit.

**APPENDIX A: DISTRIBUTION AMPLITUDES**

The pion and kaon distribution amplitudes [33], and the \( \rho \) and \( K^* \) meson distribution amplitudes [31] have been updated up to two-parton twist-3 level in QCD sum rules. The twist-2 pion (kaon) distribution amplitude \( \phi_{\pi(K)}^A \) and the twist-3 ones \( \phi_{\pi(K)}^P \) and \( \phi_{\pi(K)}^{T} \) have been parameterized as [33]

\[
\phi_{\pi(K)}^A(x) = \frac{f_{\pi(K)}}{2\sqrt{2N_c}} \left[ 1 + 6a_2^{\pi(K)} \right],
\]

where

\[
\phi_{\pi(K)}^P(x) = \frac{f_{\pi(K)}}{2\sqrt{2N_c}} \left[ 1 + 3\rho_+^{\pi(K)} \left( 1 + 6a_2^{\pi(K)} \right) - 9\rho_-^{\pi(K)} a_1^{\pi(K)} \right. \\
+ C_1^{1/2}(2x - 1) \left( 27a_2^{\pi(K)} C_1^{1/2}/2 \rho_+^{\pi(K)} a_1^{\pi(K)} - \rho_-^{\pi(K)} a_1^{\pi(K)} \left( 3/2 + 27a_2^{\pi(K)} \right) \right) \\
+ C_2^{1/2}(2x - 1) \left( 30\eta_{3\pi^{\pi(K)}} + 15\rho_+^{\pi(K)} a_2^{\pi(K)} - 3\rho_-^{\pi(K)} a_1^{\pi(K)} \right) \\
+ C_3^{1/2}(2x - 1) \left( 10\eta_{3\pi^{\pi(K)}} + 9\rho_+^{\pi(K)} a_2^{\pi(K)} - 3\rho_-^{\pi(K)} a_1^{\pi(K)} \right) \\
+ \frac{3}{2} \left( \rho_+^{\pi(K)} + \rho_-^{\pi(K)} \right) \left[ 1 - 3a_1^{\pi(K)} + 6a_2^{\pi(K)} \right] \ln x \\
\left. + \frac{3}{2} \left( \rho_+^{\pi(K)} - \rho_-^{\pi(K)} \right) \left[ 1 + 3a_1^{\pi(K)} + 6a_2^{\pi(K)} \right] \ln \frac{1}{x} \right],
\]

\( \text{(A2)} \)
where \( \rho \) respectively, with \( \lambda + K_3 = 0 \) \( \mp \pi = 0 \). \( \lambda_3 \pi = 0 \), \( \lambda_3 K = 1.6 \), \( \omega_3 \pi = -1.5 \), and \( \omega_3 K = -1.2 \) \[33\]. In the above kaon distribution amplitudes the momentum fraction \( x \) is carried by the \( s \) quark \[43\]. The Gegenbauer polynomials are defined by

\[
\begin{align*}
C_0^\lambda (t) &= 1 , \\
C_1^\lambda (t) &= 2 \lambda t , \\
C_2^\lambda (t) &= 2 \lambda (\lambda + 1) t^2 - \lambda , \\
C_3^\lambda (t) &= \frac{4 \lambda}{3} (\lambda^2 + 3 \lambda + 2) t^3 - 2 \lambda (\lambda + 1) t , \\
C_4^\lambda (t) &= \frac{2 \lambda}{3} (\lambda^3 + 6 \lambda^2 + 11 \lambda + 6) t^4 - 2 \lambda (\lambda^2 + 3 \lambda + 2) t^2 + \frac{\lambda}{2} (\lambda + 1) .
\end{align*}
\]

The twist-2 distribution amplitudes for longitudinally polarized vector mesons are parameterized as \[31\]

\[
\begin{align*}
\phi_\rho (x) &= \frac{3 f_\rho}{\sqrt{2} N_c} \pi (1 - x) \left[ 1 + a_{2 \rho} C_2^{3/2} (2x - 1) \right] , \\
\phi_\omega (x) &= \frac{3 f_\omega}{\sqrt{2} N_c} \pi (1 - x) \left[ 1 + a_{2 \omega} C_2^{3/2} (2x - 1) \right] , \\
\phi_{K^*} (x) &= \frac{3 f_{K^*}}{\sqrt{2} N_c} \pi (1 - x) \left[ 1 + a_{1 K} C_1^{3/2} (2x - 1) + a_{2 K} C_2^{3/2} (2x - 1) \right] , \\
\phi_\phi (x) &= \frac{3 f_\phi}{\sqrt{2} N_c} \pi (1 - x) \left[ 1 + a_{2 \phi} C_2^{3/2} (2x - 1) \right] .
\end{align*}
\]
For the twist-3 vector meson distribution amplitudes $\phi_s$ and $\phi_t$, we adopt their asymptotic models \[34\]. Since the Gegenbauer coefficients of these twist-3 distribution amplitudes have not yet been well constrained, the asymptotic models are allowed. Moreover, we shall vary the Gegenbauer coefficients of the twist-2 distribution amplitudes by 100%, which is larger than the error specified in \[31\]. Therefore, the theoretical uncertainty of our predictions from this source is representative enough.

\[1\] M. Bauer, B. Stech, M. Wirbel, Z. Phys. C \textbf{29}, 637 (1985); \textbf{34}, 103 (1987).
\[2\] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Phys. Rev. Lett. \textbf{83}, 1914 (1999); Nucl. Phys. \textbf{B591}, 313 (2000); Nucl. Phys. \textbf{B606}, 245 (2001).
\[3\] A. Ali, G. Kramer and C.D. Lü, Phys. Rev. D \textbf{58}, 094009 (1998).
\[4\] X.G. He, J.P. Ma, and C.Y. Wu, Phys. Rev. D \textbf{63}, 094004 (2001).
\[5\] H.Y. Cheng and K.C. Yang, Phys. Rev. D \textbf{64}, 074004 (2001).
\[6\] M. Beneke and M. Neubert, Nucl. Phys. \textbf{B675}, 333 (2003).
\[7\] C.H. Chen, Y.Y. Keum, and H-n. Li, Phys. Rev. D \textbf{64}, 112002 (2001).
\[8\] S. Mishima, Phys. Lett. B \textbf{521}, 252 (2001).
\[9\] Y.Y. Keum, H-n. Li, and A.I. Sanda, Phys Lett. B \textbf{504}, 6 (2001); Phys. Rev. D \textbf{63}, 054008 (2001).
\[10\] C.D. Lü, K. Ukai, and M.Z. Yang, Phys. Rev. D \textbf{63}, 074009 (2001).
\[11\] H-n. Li, S. Mishima, and A.I. Sanda, Phys. Rev. D \textbf{72}, 114005 (2005).
\[12\] C.D. Lü and M.Z. Yang, Eur. Phys. J. C \textbf{23}, 275 (2002).

Heavy Flavor Averaging Group, hep-ex/0603003; updated in http://www.slac.stanford.edu/xorg/hfag.

\[13\] C.H. Chen, Phys. Rev. D \textbf{58}, 094009 (2001).
\[14\] Z.J. Xiao, X. Liu, and H.S. Wang, Phys. Rev. D \textbf{73}, 114014 (2006).
\[15\] A. Khodjamirian, Th. Mannel, and M. Melcher, Phys. Rev. D \textbf{70}, 094002 (2004).
\[16\] V.M. Braun and A. Lenz, Phys. Rev. D \textbf{70}, 074020 (2004).
\[17\] H.Y. Cheng, C.K. Chua, and C.W. Hwang, Int. J. Mod. Phys. A \textbf{20}, 3648 (2005).

The $K$ meson corresponds to the $K^-$ and $K^0$ ($K^+$ and $K^0$) mesons in \[33\]. The $K$ meson distribution amplitudes are given by $\phi_K(x) = \phi_K(1-x)$ or $\phi_K(x) = \phi_K(x)$ but with a sign flip of $a_1^K$, $\rho_2^K$, and $\lambda_3^K$. 

\[a_1^K\] 
\[\rho_2^K\] 
\[\lambda_3^K\]