Research on Hydroelastic Response of an FMRC Hexagon Enclosed Platform

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Abstract: The numerical hydroelastic method is used to study the structural response of a hexagon enclosed platform (HEP) of flexible module rigid connector (FMRC) structure that can provide life accommodation, ship berthing and marine supply for ships sailing in the deep ocean. Six trapezoidal floating structures constitute the HEP structure so that it is a symmetrical very large floating structure (VLFS). Therefore, this paper studies the structural responses of a hexagon enclosed platform of FMRC structure in waves by means of a 3D potential-flow hydroelastic method based on modal superposition. Numerical models, including the hydrodynamic model, wet surface model and finite element method (FEM) model, are established, a rigid connection is simulated by many-point-contraction (MPC) and the number of wave cases is determined. The load and structural response of HEP are obtained and analyzed in all wave cases, and frequency-domain hydroelastic calculation and time-domain hydroelastic calculation are carried out. After obtaining a number of response amplitude operators (RAOs) for stress and time-domain stress histories, the mechanism of the HEP structure is compared and analyzed. This study is used to guide engineering design for enclosed-type ocean platforms.

Keywords: hexagon enclosed platform; FMRC; hydroelastic; load; stress; frequency-domain; time-domain

1. Introduction

Very large floating structures (VLFSs) are artificial sea land, which could be described as huge plates floating on the sea. A VLFS can be used as a large integrated base for marine development, such as resource development and scientific research, or as a sea airport, sea material transfer base, etc. When coastal cities lack suitable land, VLFSs can be used instead of facilities that should be built on land, such as nuclear power plants, waste treatment plants, etc., in order to reduce urban noise and environmental pollution. The advantages of adopting a symmetric HEP compared to the other already existing VLFSs are that it is easy to assemble due to the connector, and the HEP center has a large moon pool which can provide a shelter wharf and berth for ships. VLFSs have wide application prospects [1]. During 1998~2000, Japan built a super-large floating structure that was 1000 m long and 121 m wide. A series of experiments were carried out to verify whether the platform met the conditions to be used as a sea airport. The test was successful, and it was the first time that human beings had built a sea base of one kilometer order [2]. Different from Japan’s research direction, the United States is committed to the development of offshore mobile bases (MOBs), and tend to use super-large floating bodies as naval military bases. An MOB research program was officially launched by the United States Department of Defense in 1992. The MOB is made of several semi-submersible modules connected by specific connectors [3]. It is a VLFS with autonomous power and can be placed in a designated sea area. Some research institutions in the United States have put forward
different innovative conceptual design schemes: rigid connections between modules and each individual module has its own speed; the connection between semi-submersible modules is a hinge [4]; a combination reinforced concrete and steel structure is used to move the relevant knowledge of architecture into marine equipment [3]; or a flexible connection bridge is used to connect each module [5]. At present, there is no related program into the actual construction and operation [6]. Singapore has built the world’s largest floating stage in the Marina Bay, and is planning to build a large floating fuel storage facility to meet the increasing demand for oil storage capacity (FFSF). Such an FFSF could double the fuel bank and ship mooring system, thereby easing traffic congestion and reducing the turnaround time for ships in Singapore’s seaports [7]. VLFS technology also makes it possible for future humans to live on the surface of the ocean. The Lily Pad floating eco-city proposed by Belgian architect Vincent Callebaut is a huge floating lily-shaped island to accommodate the city’s population.

In addition to the aforementioned large floating body projects in Japan and the United States, countries around the world, especially coastal countries (including Japan, the United States, the United Kingdom, Norway, South Korea, Singapore, etc.) have successively carried out research on large-scale floating bodies and achieved good results, and produced a variety of applications. Among them, Japan and the United States are the representatives, and the two have the most research on super-large floating bodies. Extensive and in-depth research results are the most abundant. In addition, other countries have also made some achievements. The Piper Alpha oil production platform [8] in the United Kingdom was built in 1970. It is a large fixed oil and gas platform located in the North Sea of the United Kingdom. In 1992, the Ysund floating bridge [9] was built in Norway, 933 m long. In 1994, Aas-Jakobsen designed the Nordhordland floating bridge [10,11], which is located in Norway with a length of 1246 m. Brazil’s P-36 oil production platform was made in Italy in 1994 and transformed into an offshore oil platform in 1999. It was put into operation in 2000. It is 112 m long, 119 m high and 1360 m deep. Dutch designer Koen Olthuls proposed the world’s first floating apartment building, “Citadel” [12]. The floating apartment building will use water-cooling technology, saving 25% more energy than traditional land buildings.

As we all know, it is very significant to study the load and response of VLFSs. There are uncontrollable factors such as severe waves and storms in the far sea. VLFSs have been used for a long time in the complex and changeable marine environment. The wave load is still the most important, most complex and longest lasting external load. Many structural failure modes may occur under the action of storm weather conditions, which are far beyond the scope of ships and general marine engineering structures [8]. The HEP studied in this paper has the characteristics of wide horizontal scale, shallow depth and flat structure and shape of enclosure. Its horizontal rigidity is small, so it is necessary to pay attention to its load and response. Domestic research on super-large floating bodies is still in a period of breakthrough in fundamental theory. In the 1980s, academician Wu Yousheng took the lead in the research of super-large floating bodies, and creatively proposed the hydroelasticity theory for analyzing large ocean structures [13]. In 2001, the team of Professor Cui Weicheng [14] of Shanghai Jiaotong University, with the funding of the key project of the Science Foundation, carried out research on the linear hydroelasticity of the super-large box floating structure [15], and achieved excellent research results [16]. In 2003, Professor Yang Jianmin’s team from Shanghai Jiaotong University conducted a VLFS model test study to explore the effect of wave direction, the depth of water, period, etc., on the hydroelasticity of super-large floating bodies [17].

It is evident that the hydroelastic method is an important method to calculate the load and response of VLFS. The analysis concept of hydroelasticity theory contains the idea of fluid–structure interaction, which can more accurately predict wave loads and structural responses, and is more suitable for the development of increasingly large ships and large-scale marine engineering. The theory of hydroelasticity has developed from 2D to 3D, and from linear to nonlinear. In the 1970s, Bishop [18] and others proposed a 2D linear hydroelasticity theory based on the 2D slice theory through summary and research.
With the development of floating structures such as wide and large ships, multi-hull ships and offshore platforms, the 3D hydroelastic theory that can handle floating structures of any shape has gradually become widely used. Du Shuangxing [19] proposed a 3D linear hydroelasticity frequency domain analysis method, which considers the effects of ship speed and non-uniform steady flow field. Wang Dayun and Wu Yousheng [20] proposed a 3D linear hydroelasticity time-domain analysis theory. Wu Yousheng et al. [21–23] proposed 3D nonlinear hydroelasticity theory and numerical methods to analyze the dynamic response of large-scale moving floating structures in high sea conditions. Liu and Sakai [24] combined the hydrodynamic force calculated by the boundary element method (BEM) with finite element analysis (FEA) to explore the 3D hydroelastic response in the time domain. Wu Yousheng (1995) [25] considered the fluid–structure interaction between the fluid and the floating body, and used the hydroelastic theory to calculate the movement and deformation of the floating body module. Riggs (2000) [26] used the simplified FEA model calculated by linear hydroelasticity theory and the rigid module flexible connector (RMFC) model calculated by 3D potential flow theory to simulate MOB, respectively, and compared and analyzed the response and connector load of multiple different floating bodies simulated by these two different models. The results show that when the resonance frequencies of the first 12-order wet structure of the two models are similar in the fluid medium, the RMFC model calculated by the 3D potential flow theory can accurately predict the structural response and connector load. FuS (2007) [27] used the 3D hydroelasticity theory and the RMFC model to calculate the hydroelastic response of a two-module floating body. Gu Jiayang (2015) [28] used the time-domain method to research the dynamic load of the connector of the VLFS under the combined action of the collision load and the wave load.

It is necessary to design an HEP to provide accommodation, berthing and replenishment for many ocean-going ships, and one that is very large in size, so it is classified as a VLFS. Up to now, this regular hexagonal VLFS has not yet been discussed and studied, and the heteromorphic shape of a VLFS as a HEP structure has different loads and unique response characteristics, which present a complex and advanced fluid–structure hydroelastic problem, so it is necessary to study hydroelasticity for the HEP structure. In this paper, a 3D numerical hydroelasticity method of using 3D potential-flow theory and the modal superposition method is used to research the load and structural response of the HEP structure. Numerical models are established and a series of wave cases are determined, frequency-domain hydroelastic calculations and time-domain hydroelastic calculations are carried out, a number of RAOs of stress and time-domain stress histories are obtained and compared and the structural response mechanism and symmetrical load are analyzed for HEP. This study is used to guide engineering design for the symmetrical kind of enclosed-type ocean platform.

2. Numerical Methodology of 3D Hydroelasticity

As the HEP experimental model may have a 3D hydroelastic response in waves, this study adopts the numerical method of 3D hydroelasticity theory. The 3D hydroelasticity software HOMER, which combines the HYDROSTAR, which uses hydrodynamics, and the Nastran, which uses structural dynamics, is used to model the 3D HEP and calculate the hydroelastic response of the HEP experimental model.

2.1. Theory of 3D Hydroelasticity

Bishop and Price firstly proposed a general method for solving the hydroelastic response of a hull girder [29]. Three-dimensional hydroelastic theory for evaluating the structural response of a 3D structure was proposed by Wu via 3D BEM and 3D FEM [30]. Next, ship engineering gradually popularized 3D hydroelasticity theory. Because a VLFS has the characteristics of larger horizontal size and lower overall stiffness, 3D hydroelasticity becomes a critical theory to study the structural response of VLFSs in waves. Compared with the classic rigid body seakeeping model, the 3D hydroelastic theory basically expands the motion representation into additional motion or deformation modes as a series of
the wet structural natural modes. Wet structural modes refers to the vibration modes of the floating structure in water, which need to consider added mass and the damping coefficient. When the large floating structure in waves has small amplitude motion and elastic deformation, the structural response is linear, so the displacement $H(x,y,z)$ can be expressed by the mode superposition method:

$$H(x,y,z) = \sum_{i=1}^{N} \xi_i(t) h_i(x,y,z) = \sum_{i=1}^{N} \xi_i(t) [h^s_i(x,y,z) + h^l_i(x,y,z)]$$ (1)

In Equation (1), $x, y, z$ represents the X axis, Y axis and Z axis, respectively. $h^i(x,y,z)$ represents a general movement or deformation mode including rigidity or elasticity. The decomposition of Equation (1) causes the problem of the additional radiation boundary condition of the elastic mode, and the boundary conditions of the volume undergo the following changes. $\xi$ is defined as modal amplitude, which is used to express the contribution of the mode to the overall deformation or motion. $N$ is the structural modal number, which includes the rigid modal number, 6, and denotes six degrees of freedom of rigid motion and flexible modal number.

The displacement of VLFS can be gained through a quadratic partial differential equation:

$$[a + A] \{H\} + [b + B] \{\dot{H}\} + [c + C] \{\ddot{H}\} = f$$ (2)

In Equation (2), $[a], [b], [c]$ are the generalized structural mass matrix, damping matrix and stiffness matrix, $[A], [B], [C]$ are the generalized added mass matrix, wave damping matrix and restoring force matrix, and $f$ is generalized wave forces. The variable $H$ in Equation (1) can be substituted into Equation (2) to solve for variable $\xi$.

The structural response of a VLFS can be solved by the mode superposition method under waves. The dynamic structural code Nastran is used to obtain stress as:

$$\sigma_x(x,y,z) = \sum_{r=1}^{m} \xi_r \sigma_{x,r}$$ (3)

Another expression of the coupling dynamics of Equation (2) in the frequency domain is as follows:

$$\{-\omega_{ri}^2 (m) + [A]) - i\omega_r [B] + [k] + [C]\} \{\xi\} = \{F^{DI}\}$$ (4)

In Equation (4), $[m]$ denotes modal real mass, $[k]$ denotes the modal structural stiffness, $[A]$ denotes hydrodynamic added mass, $[B]$ denotes the damping of hydrodynamics, $[C]$ denotes the stiffness of hydrostatics, $\{\xi\}$ denotes the modal amplitude and $\{F^{DI}\}$ denotes modal excitation that is obtained by the sum of the integral force, including incident wave velocity potential and the diffraction potential of the ship’s wet surface as Equation (5).

$$\{F^{DI}\} = \left\{ F^{F-K} + F^D \right\} = i\omega \rho \int_{S_b} (\phi_I + \phi_D) n dS$$ (5)

The factor of hydrodynamic added mass $\{A\}$ and the damping of the hydrodynamic wave $\{B\}$ are obtained from the radiation potential integral of Equations (6) and (7). Radiation potential is a professional word in fluid mechanics, and there is a fluid theory named potential flow theory which divides wave velocity potential into inputting, diffraction and radiation potential; radiation potential means the induced velocity potential due to the ship’s motion in static water.

$$A^{ij} = \rho \text{Re} \left\{ \int_{S_B} \Phi_R h' n dS \right\}$$ (6)
\[ B^{ij} = \rho \omega^2 \text{Im} \left\{ \int_S \varphi^*_l h^i n dS \right\} \]  

(7)

The components in Equations (6) and (7) show that diffraction potential and radiation potential can be obtained by BEM or a method of sources, as shown in Equation (8).

\[ \varphi = \int_S \sigma(x_h) G(x_h; x_s) dS \]  

(8)

In Equation (8), \( \sigma(x_h) \) denotes source function. HYDROSTAR is introduced to find the solution of the above hydrodynamic potentials and hydrodynamic coefficients.

The purpose of moving from the frequency domain to the time domain is to effectively simulate the transient response. This will use the method proposed by Cummins \([31,32]\). Equation (9) is another expression of the time-domain equivalent formula of the equation of motion.

\[ (|m| + |A^\infty|) \{ \ddot{\xi}(t) \} + (|k| + [C]) \{ \xi(t) \} + \int_0^t [K(t - \tau)] \{ \ddot{\xi}(\tau) \} d\tau = \{ F^{DI}(t) \} + \{ Q(t) \} \]  

(9)

In Equation (9), over-dots means time derivative, \(|A^\infty|\) is infinite frequency added to the mass matrix and the hydrodynamic load at the structural model coinciding with the acceleration on the left side of the equation and \([K(t)]\) denotes the impulse response functions matrix. The impulse response function can be obtained by the frequency-dependent damping coefficient in Equation (10) \([33]\):

\[ K_{ij}(t) = \frac{2}{\pi} \int_0^\infty B_{ij}(\omega) \cos(\omega t) d\omega \]  

(10)

The total structural response is appropriately divided into quasi-static and dynamic parts, and a convergent stress distribution can be obtained. In the current research results, the division of the response of each part is accomplished by rewriting the equation of motion, Equation (4). The form is as follows:

\[ \begin{bmatrix} R & R \\ E & R \end{bmatrix} \begin{bmatrix} \xi^R_0 \\ \xi^E_0 \end{bmatrix} = \begin{bmatrix} F^R \\ F^E \end{bmatrix}, \]  

(11)

In Equation (11), \( k \) means the matrix of structural modal stiffness. Both \( R \) and \( E \) are a stiffness sub-matrix. Stiffness is the proportional coefficient of structural load and displacement. \( R \) denotes the rigid body parts stiffness matrix, and \( E \) denotes the elastic body stiffness matrix. Next, the total displacement of the structure is divided into two components: quasi-static and dynamic:

\[ \xi^R = \xi^R_0 + \xi^R_d, \xi^E = \xi^E_0 + \xi^E_d \]  

(12)

The quasi-static part of the responses is defined by the following equations:

\[ |R| \{ \xi^R_0 \} = \{ F^R \} \]  

(13)

\[ |k| \{ \xi^E_0 \} = \{ F^E \} - \begin{bmatrix} E & R \end{bmatrix} \{ \xi^R_0 \} \]  

(14)

After inserting Equations (12)–(14) into Equation (11), the following linear system of equations for dynamic parts is obtained:

\[ \begin{bmatrix} R & R \\ E & R \end{bmatrix} \begin{bmatrix} \xi^R_d \\ \xi^E_d \end{bmatrix} = \begin{bmatrix} |R| \{ \xi^E_0 \} \\ |k| \{ \xi^E_0 \} \end{bmatrix}, \]  

(15)
In summary, using the above-mentioned derivation process can retain the classic direct method of the quasi-static part, and clearly treat the dynamic part as a correction term for the quasi-static part. Finally, the recommended decomposition method completely eliminates the problem of convergence [34].

2.2. Introduction of the Hydroelastic Code

The 3D hydroelastic analysis code HOMER was combined with the structural 3D FEM code NASTRAN and hydrodynamic seakeeping code HYDROSTAR to analyze the 3D hydroelastic response of structures. The structural solver and the hydrodynamic solver can be modeled and calculated independently, and HOMER uses their results to solve the hydroelastic response. HYDROSTAR establishes the hydrodynamic model, NASTRAN establishes the structural finite element model and HOMER realizes the coupling through the data transmission between two hydrodynamic and structural models on the HEP wet surface, so as to carry out the hydroelastic calculation. Firstly, the hydrodynamic grid file and finite element grid file are input into LISA for structure preprocessing, and then they are input into HYDROSTAR together with the wave input files such as ship speed, wave frequency and wave height to solve the problem, and the wave load input file is generated. Wave load file, wave input file and modal file are input to HOMER for calculation, and the file containing the final structural load is output. Then, the wave load file is input into NASTRAN for structural response calculation and modal analysis, and the structural response file and modal file are generated. The updated modal file is substituted into HOMER calculation again, and this process is repeated continuously. Finally, the structural response results of hydrodynamic and finite element coupling calculation are output. The hydroelastic method and HOMER were verified to evaluate the hydroelastic response of the HEP model before [35].

3. Numerical Models of HEP Structure and Wave Cases

3.1. Numerical Models of HEP Structure

HEP is composed of six individual trapezoidal modules spliced together, and the main dimensions of an individual module of HEP are presented in Table 1. The diagrammatic sketch of a trapezoidal module is shown in Figure 1a. The HEP is a double-bottom and double-side double-shell structure. A longitudinal bulkhead is set every 10 m in the internal structure of the module. A transverse bulkhead is set every 8.5 m. The inner bottom plate is set at 2 m from the base plane. Design keel plate, transverse partition plate and other transverse longitudinal members are between the bottom plate and the inner bottom plate. The frame space is 2.125 m, and each costal position is provided with longitudinal members such as ribs, strong ribs, beams, strong beams, etc. The width direction sets a longitudinal bone and a truss every 2.5 m, and the height direction every 2 m sets a side longitudinal bone and side longitudinal truss and other components, as shown in Figure 1b. The HEP model material is Q235 steel, density 7.85 t/m$^3$, elastic modulus is $2.1 \times 10^{11}$ Pa and Poisson’s ratio is 0.3. In the process of modeling, considering the difficulty of modeling and the small proportion of superstructure quality in the total displacement, the difference in superstructure between each module is ignored, and the model displacement is replenished by arranging the quality point at the bottom plate.

Table 1. Main dimensions of an individual module of HEP.

| Item      | Real Structure (m) |
|-----------|--------------------|
| Long edge | 221                |
| Short edge| 175                |
| Width     | 40                 |
| Depth     | 7                  |
| Draft     | 3                  |
A conceptual design of HEP which is composed of six individual trapezoidal floating modules, connected by six rigid connectors is proposed, as Figure 2 shows. HEP structure is very different from traditional VLFSs which are of slender and long shape; the HEP structure is a symmetrical hexagon structure with a huge horizontal size and a small depth, and the HEP center has a large moon pool which can provide a shelter wharf and berth for ships. Generally, the platform has a mooring system which consists of a number of relaxed catenary lines connected to the seabed.

3.2. Numerical Models of HEP Structure

The 3D numerical hydroelastic code requires three numerical models, including the hydrodynamic model, wet surface model and structural model. The hydrodynamic model of HEP is used to calculate wave pressures in hydrodynamic code HYDROSTAR on the basis of 3D potential-flow theory and BEM, so the hydrodynamic model of HEP just meshes panels on an external wet surface as Figure 3 shows—4896 hydrodynamic panels are meshed on a hydrodynamic model. The wet surface model is used to transfer wave pressure interpolated to the FEM model, so the wet surface model consists of the whole external surface of HEP where all meshes are applied; about 7357 elements are meshed on all external surfaces of the HEP, as Figure 4 shows.
The FEM model of HEP structure is established based on the structural design plan of HEP. Shell elements are used to simulate plate structure, and beam elements are used to simulate stiffener. There are 38,976 shell elements and 13,248 beam elements in an individual module. The FEM model of integrated HEP structure is shown in Figure 5.

A rigid connection is designed for the HEP structure by using rigid connectors at six common edges of HEP; the rigid connection does not allow relative motion among modules to match all modules as an integrated whole. The rigid connection should be simulated in the FEM model. This study uses MPC to simulate rigid connection; there is an interval space between modules, and a node is placed at the center of the interval, the positive point is determined at the node, then all nodes at common surfaces are selected for negative points and positive point and negative points are combined to model RBE2-type MPC, as Figure 5 shows. The specific method is to establish a rigid connection, whereby all nodes involved in the MPC make up a rigid body which has no deformation and loads are transferred between modules. MPC has an active node and dependent nodes for the rest of the nodes, and all dependent nodes are nodes in the clearance between the two modules which used to be active points, and the rest of nodes are used as slave points. The active point is associated with all six degrees of freedom of the slave point.
All origins of coordinate systems are placed at the center point on the bottom plane of the HEP structure, as Figure 6 shows. The X axis points to a common edge of two modules, Y axis points to the mid-plane and Z axis points to the deck from the HEP bottom. Module number is determined in the FEM model. The HEP is a symmetrical hexagon, so the FEM model is named according to the continuous orders, so No. 1, 2 and 3 modules are posited above the X axis and No. 4, 5 and 6 modules are posited below the X axis. Six stress points which are located on the center points of the deck structure at each module are selected to analyze the strength variation in the HEP, as Figure 5 shows.

![Figure 6](image-url)  
**Figure 6.** The first 6 elastic modals of HEP structure. (a-f) is the first-sixth order modal analysis of HEP.

In this study, two wave inputting angles of $0^\circ$ and $30^\circ$ are selected to determine wave cases due to the symmetry of the HEP structure. Wave direction points to the common edge of the No. 1 module and No. 6 module at a $0^\circ$ wave angle, and another wave direction is perpendicular to the long edge of the No. 6 module at a $30^\circ$ wave angle. A series of wave cases are determined to research the structural responses for HEP at the two wave angles.

### 3.3. Modal Analysis of HEP

Modal analysis of HEP structure is completed before calculating the hydroelastic response for the HEP. The modal superposition method is a basic means to calculate the structural response of the HEP. “Dry frequencies” means the natural vibration frequency of the floating body in the air. “Wet frequencies” means the vibration modes of the floating structure in water, which need to consider the influence of the damping coefficient and additional mass. The difference between the dry frequency and the wet frequency of the HEP is mass. Wet frequency shows that the vibration of the floating structure should consider the influence of water, and the floating structure in the water has an additional mass as shown in Equation (4). According to the frequency solution formula

$$\omega = \sqrt{\frac{k}{m + A}}$$
the total mass is increased as the vibration increases, so the natural frequency of the structure will decrease. Dry frequencies and wet frequencies of HEP are both calculated, and dry frequencies are solved by using the FEM solver Nastran, while wet frequencies of HEP are obtained on the basis of dry frequencies combining hydroelastic code HOMER. The dry frequency and wet frequency of the HEP flexible mode are presented in Table 2. Table 2 shows that the wet frequencies are lower than the dry frequencies in each mode. It is shown that the reasonability of the hydroelastic calculation has been verified in this study, because added mass and wave damping coefficient are considered when calculating wet frequency. The 1st and the 2nd order of dry/wet frequencies are equal, so that HEP is a symmetrical structure from the view of the horizontal plane. The first six order modal analyses of the HEP are given in Figure 6, and it is seen from it that modal analysis of the HEP is very different from the ship structure, which is a slender shape type, and the 1st and 2nd orders are main modal form, because HEP structure possesses wide and flat shape features and stiffness is rather close in all directions at the horizontal plane of the HEP. Therefore, the first six modes are summarized in this study.

Table 2. Dry frequencies and wet frequencies of HEP.

| Flexible Modal Order Number | Dry Frequency (rad/s) | Wet Frequency (rad/s) |
|-----------------------------|-----------------------|-----------------------|
| 1                           | 0.691                 | 0.672                 |
| 2                           | 0.691                 | 0.673                 |
| 3                           | 1.659                 | 0.744                 |
| 4                           | 2.187                 | 0.985                 |
| 5                           | 3.129                 | 1.846                 |
| 6                           | 3.129                 | 1.846                 |

4. Frequency-Domain Results Analysis

This study carries out the frequency-domain hydroelastic response calculation, where six stress points which are marked in Figure 6 are selected to output stress RAOs to analyze the structural response mechanism of the HEP. Two wave angles, 0° and 30°, are both studied when calculating the frequency-domain hydroelastic response. No. 1, 2, 3 and 6 stress points are selected to analyze stress RAOs, because the HEP structure is symmetrical, and so these points which are symmetric around the wave direction should have a similar structural response.

Figure 7 shows the stress RAOs of the HEP at the No. 1, 2, 3 and 6 points under two wave angles, which is analyzed as follows. Firstly, it is revealed from Figure 7 that multiple peaks appear in all stress RAOs. Secondly, among all the measuring points at the 0° wave direction, the RAO stress value of the No. 2 point is the largest, because a rigid connection is used in the HEP of FMRC structure model and the No. 2 point has a more significant middle position at wave angle 0°, so that the stress value of the No. 2 point is higher. Thirdly, the No. 6 point or No. 6 module has a similar stress to the No. 1 point or No. 1 module at wave angle 0° due to transverse symmetry. Fourthly, it is found that many peak frequencies are near the wet frequency of the fourth-order flexible mode of the HEP structure. Because of the little difference in wet frequency of the HEP structure, it is easy to induce the RAO stress with multiple peaks. The multiple peaks are caused by structural natural vibration, which indicates that the HEP structure induces a hydroelastic response to a certain extent.

Stress distribution on the HEP structure is discussed by calculating frequency-domain point stress. At the 0° wave angle, the common side of modules 1 and 6 is heading to the wave, and the No. 1, No. 2 and No. 3 modules are arranged from the bow to the stern, so the No. 1, No. 2 and No. 3 stress points are discussed for the stress level of all modules. Figure 8 gives the stress RAOs of points No. 1, No. 2 and No. 3 at 0° wave, and it is seen that No. 2 has the maximum stress value while No. 1 and No. 3 have a similar stress value. It is seen that the rigid connection is used in the HEP so that the middle structure (No. 2 module) has maximum bending moment and structural deformation, and No. 1 and
No. 3 have similar stress values because the two points are symmetrical about the vertical wave direction. Figure 9 shows the stress RAOs of points No. 1, No. 2 and No. 6 at 30° wave; the No. 6 module faces the wave direction, and the No. 1, No. 2 and No. 6 stress points outputted frequency-domain curves for discussing stress distribution at a 30° wave angle. It is seen that the No. 6 stress point has a lower stress level because the direction of the wave is perpendicular to module 6, and stress points 1 and 2 have similar stress values because the two points are symmetrical about the vertical wave direction.

![Stress RAOs of 4 points. (a) is stress RAOs of point No. 1; (b) is stress RAOs of point No. 2; (c) is stress RAOs of point No. 3; (d) is stress RAOs of point No. 6.](image1)

Figure 7. Stress RAOs of 4 points. (a) is stress RAOs of point No. 1; (b) is stress RAOs of point No. 2; (c) is stress RAOs of point No. 3; (d) is stress RAOs of point No. 6.

![Stress RAOs of points No. 1, 2 and 3 at 0° wave.](image2)

Figure 8. Stress RAOs of point No. 1, 2 and 3 at 0° wave.

![Stress RAOs of point No. 1, 2 and 6 at 30° wave.](image3)

Figure 9. Stress RAOs of point No. 1, 2 and 6 at 30° wave.

Stress symmetry is discussed due to the symmetry of the HEP structure. At the 0° wave angle, No. 2 and No. 5 stress points are symmetric about the wave direction. At the
30° wave angle, No. 1 and No. 5 stress points are symmetric about the wave direction. Figure 10 shows that the stress RAOs of points No. 2 and 5 at 0° wave have the absolute same stress RAO, and Figure 11 shows that the stress RAOs of points No. 1 and 5 at 30° wave have same stress RAO. It is indicated from Figures 11 and 12 that the symmetry of stress on the symmetry point and the HEP structure has strength symmetry.

![Figure 10. Stress RAOs of points No. 2 and 5 at 0° wave.](image)

![Figure 11. Stress RAOs of points No. 1 and 5 at 30° wave.](image)

![Figure 12. Time-domain stress results of No. 1 and No. 2 stress points for wave case 1. (a) is stress histories of No. 1 stress point for wave case 1. (b) is stress histories of No. 2 stress point for wave case 1.](image)
5. Time-Domain Results Analysis

A time-domain hydroelastic solution is carried out to obtain a time-domain structural response for hydroelastic verification. Two stress points are selected to output time-domain hydroelastic stress histories. The wave case uses wave frequency at the RAOs 1st peak value; two time-domain wave cases are defined in Table 3, and two wave angles 0° and 30° are studied for time-domain analysis. The wave parameters are derived from the wave situation near a certain sea in China.

Table 3. Wave cases for time-domain analysis.

| Case | Point | Frequency (rad/s) | Period (s) | Wave Height (m) | Wave Angle (°) |
|------|-------|-------------------|------------|-----------------|---------------|
| 1    | 1,2   | 0.412             | 15.25      | 4               | 0             |
| 2    | 1,2   | 0.412             | 15.25      | 4               | 30            |

Time-domain stress history is discussed to research the structural response of the HEP under wave action. This paper gives X-direction stress, Y-direction stress and XY-shearing stress. Figure 12a reveals the stress history of the No. 1 stress point for wave case 1, for which wave angle is 0°, and it is seen from Figure 12a that significant small period vibrations are aroused in three stress histories of X-direction stress, Y-direction stress and XY-shearing stress. Figure 12b shows the stress history of the No. 2 stress point for case 1, and it is seen from Figure 12b that the X-direction stress has maximum stress and is of very smooth history. It is indicated from Figure 13 that the No. 1 module has a significant local hydroelastic response and the No. 2 module has a low level of hydroelastic response. Figure 13 gives stress cloud images when the ship is under sagging and hogging for case 1, and it is found that the maximum stress distribution appears on the No. 2 module. Figure 14 shows the stress history of the No. 1 and No. 2 points for wave case 2, for which the wave angle is 30°, and it is seen from Figure 14 that significant small period vibrations appear in the stress histories; it is indicated that the hydroelastic response is induced in case 2. Figure 15 gives stress cloud images when the ship is under sagging and hogging for case 2, and it is found that the No. 2 and No. 3 modules have a similar stress distribution.

Frequency-spectrum analysis is performed for the hydroelastic characteristics of the HEP. It is also seen from Figures 12 and 14 that the time-domain stress histories of case 1 and case 2 have complex frequency components, which have an obvious response of resonance vibration. It is very important to explore the hydroelastic properties of the HEP model. The Fourier-transform method is used to transform the time-domain stress history into a spectrum curve, and the hydroelastic properties of the HEP model are analyzed.

Figure 16 presents the amplitude distribution of frequency-spectrum curves of three stress components for the No. 1 and No. 2 points in case 1 and case 2. From Figure 16a,c,d it can be seen that all stress components in the diagram have two peaks of the same frequency. The first peak frequency is 0.375 rad/s, corresponding to the wave frequency. The second peak frequency is 1.877 rad/s, which is about 5 times that of the first peak frequency, close to the 5th and 6th order flexible mode wet frequency of the HEP of 1.846 rad/s. It is shown that the characteristics of nonlinear springing vibration, indicating a few hydroelastic responses, appear. It can be seen from Figure 16b that the stress in X and Y directions has obvious individual peak values, and the shear stress in the XY plane has an obvious double peak value. The first peak frequency of the three stress components is the same, which is 0.375 rad/s, corresponding to the wave frequency. The second peak frequency of shear stress in the XY plane is 1.877 rad/s, which is about 5 times that of the first peak frequency, and is close to the wet frequency of the 5th and 6th flexible modes of the HEP of 1.846 rad/s. It presents the characteristics of nonlinear springing vibration. It is indicated from Figure 16 that case 1 and case 2 truly reflect the hydroelastic response of the HEP, but there are differences between the modules.
Figure 13. Stress cloud images when ship is under sagging and hogging for case 1. (a) is stress cloud images at time point $t_{1-1}$ when ship is under sagging; (b) is stress cloud images at time point $t_{1-2}$ when ship is under hogging.

Figure 14. Time–domain stress results of No. 1 and No. 2 stress points for wave case 2. (a) is stress histories of No. 1 stress point for wave case 2; (b) is stress histories of No. 2 stress point for wave case 2.
Figure 15. Stress cloud images when ship is under sagging and hogging for case 2. (a) is stress cloud images at time point $t_{2.1}$ when ship is under sagging; (b) is stress cloud images at time point $t_{2.2}$ when ship is under hogging.

Figure 16. Frequency-spectrum curves for No. 1 and No. 2 points in case 1 and case 2. (a) is Frequency-spectrum curves for No. 1 point, case 1; (b) is Frequency-spectrum curves for No. 2 point, case 1; (c) is Frequency-spectrum curves for No. 1 point, case 2; (d) is Frequency-spectrum curves for No. 2 point, case 2.
In addition to the low-frequency part representing the frequency of the wave, there are also high-frequency components which are multiples of the frequency of the wave, and the high-frequency component corresponds to a certain order of wet frequency of the HEP structure, showing the characteristics of nonlinear springing vibration. The high-frequency component of the platform response may have a significant impact on the ultimate strength of the structure, which needs attention in the structural design and strength evaluation.

6. Conclusions

This paper adopts 3D hydroelasticity theory to research the hydroelastic response of FMRC-HEP. The HEP structure consists of six individual trapezoidal floating modules, connected by six rigid connectors to form a hexagon enclosed platform. Structural response research for HEP is feasible, forward-looking and reasonable, by using the numerical hydroelastic method which combines 3D potential flow theory with FEM. The numerical model of HEP, including the hydrodynamic model and finite element model, is established. Many wave conditions are designed and simulated, and a large number of research results are analyzed. This study is helpful to study the structural calculation method and hydroelastic response of this kind of large floating body. The HEP studied in this article is expected to become a multi-purpose ocean platform, providing ship docking and marine supplies for ocean-going ships, and a scientific experiment platform for exploring the ocean. HEP is also the first choice for ship repair platforms. HEP has a large moon pool which can provide convenient and safe sheltered waters for ship repairs, and it is proposed to popularize the HEP in far and deep ocean engineering.

There are three conclusions in this study, which are as follows:

(1) The 3D numerical hydroelasticity method can be used to study and analyze the hydroelastic response of the HEP structure;

(2) The frequency domain RAOs of stress have more than one, obvious and similar peaks, and the stress curve of the time domain performed by the Fourier transform and spectrum analysis shows that the hydroelastic response is induced. Therefore, the hydroelastic response cannot be ignored in the platform design;

(3) The HEP structure has regular symmetry. Due to the large response of the structure, the intermediate module should be paid attention from the angle of the incident wave.

This paper studies the response characteristics of a HEP composed of FMRC structure and six trapezoidal floating body models. Future work can study the HEP model with an FMFC structure.

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