Signal of Quark Deconfinement
in the Timing Structure of Pulsar Spin-Down†

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May, 12, 1997

PACS 97.60.Gb, 97.60.Jd, 24.85+p

†This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of Nuclear Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098.
‡A part of this work was done at the ECT*, Villa Tambosi, Trento, Italy.
The conversion of nuclear matter to quark matter in the core of a rotating neutron star alters its moment of inertia. Hence the epoch over which conversion takes place will be signaled in the spin-down characteristics of pulsars. We find that an observable called the braking index should be easily measurable during the transition epoch and can have a value far removed (by orders of magnitude) from the canonical value of three expected for magnetic dipole radiation, and may have either sign. The duration of the transition epoch is governed by the slow loss of angular momentum to radiation and is further prolonged by the reduction in the moment of inertia caused by the phase change which can even introduce an era of spin-up. We estimate that about one in a hundred pulsars may be passing through this phase. The phenomenon is analogous to “bachbending” observed in the moment of inertia of rotating nuclei observed in the 1970’s, which also signaled a change in internal structure with changing spin.

The deconfined phase of hadronic matter called quark matter is believed to have pervaded the early universe and may reside as a permanent component of neutron stars in their dense high-pressure cores [1–6]. However no means of detecting its presence has been found because the properties of neutron stars and those with a quark matter core are expected to be very similar. Alternately, instead of looking to the properties of the star itself, we study the spin-down behavior of a rotating star with the realization that changes in internal structure as the star spins down will be reflected in the moment of inertia and hence in the deceleration.

Pulsars are born with an enormous store of angular momentum and rotational energy which they radiate slowly over millions of years by the weak processes of electromagnetic radiation and a wind of electron-positron pairs [7–10]. When rotating rapidly, a pulsar is centrifugally flattened. The interior density will rise with decreasing angular velocity and may attain the critical density for a phase transition. First at the center and then in an expanding region, matter will be converted from the relatively incompressible nuclear matter phase to the highly compressible quark matter phase. The weight of the overlaying layers of nuclear matter will compress the quark matter core and the entire star will shrink. The mass concentration will be further enhanced by the increasing gravitational attraction of the core on the overlaying nuclear matter. The moment of inertia thus decreases anomalously with decreasing angular velocity as the new phase slowly engulfs a growing fraction of the star.
transition is superposed on the natural response of the stellar shape to a decreasing centrifugal force occasioned by radiation loss. Therefore, to conserve angular momentum not carried off by radiation, the deceleration $\dot{\Omega}$ of the angular velocity must respond by decreasing in absolute magnitude and may actually change sign. The pulsar may spin up for a time, just as an ice skater spins up upon contraction of the arms before air resistance and friction reestablish spin-down.

Such an anomalous decrease of the moment of inertia as described is analogous to the phenomenon of “backbending” in the rotational bands of nuclei predicted by Mottelson and Valatin and observed years ago [12,14]. The moment of inertia of a nucleus changes anomalously because of a change in phase from a nucleon spin-aligned phase at high angular momentum to a pair-correlated superfluid phase at low. The connection between the moment of inertia and the internal structure of a neutron star is shown in Fig. 1 and between moment of inertia and frequency in Fig. 2.

![Graph of moment of inertia vs. frequency](image)

FIG. 2. Moment of inertia as a function of rotational angular velocity over the epoch during which an increasing central volume of the star is passing into the deconfined quark matter phase. The temporal development is from large to small $I$. Spin-up of the pulsar is clearly evident. An analogous phenomenon known as “backbending” is observed in the rotational properties of nuclei.

The property of asymptotic freedom of quarks makes a phase change from confined to deconfined matter inevitable for sufficiently high energy density of hadronic matter. We have therefore referred to the transition as being from the confined to deconfined phase. However, it is apparent in our discussion and results, that the phenomenon we discuss is not unique to this particular transition. It is simply the most plausible in our view, but any phase transition in dense hadronic matter is interesting.

Now we develop the appropriate measure for detecting the occurrence of a phase transition and its envelopment of an increasing proportion of the mass and volume of the star with time. The production of electromagnetic radiation and a wind of relativistic pairs by the rotating magnetic field of the star exert a torque on the star which is characteristic of magnetic dipole radiation. The energy loss equation representing processes of multipolarity $n$ is of the form

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{1}{2}I\Omega^2\right) = -C\Omega^{n+1}$$

where $n = 3$ for magnetic dipole radiation. The rate of change of frequency is governed by

$$\dot{\Omega} = -\frac{C}{I}\left[1 + \frac{I'\Omega}{2I\Omega^2}\right]^{-1}\Omega^n.$$

For low frequency or if changes in $I$ are ignored this reduces to the usual form quoted in the literature. $\Omega = -K\Omega^n$ where $K = C/I$, (cf. Ref. [13,14]).

The dimensionless measurable quantity $\Omega\dot{\Omega}/\dot{\Omega}^2$ is referred to as the “braking index”. It would be equal to the intrinsic index $n$ of the energy-loss mechanism if the frequency were small or if the moment of inertia were a constant. However these conditions are not usually fulfilled and the measurable quantity is not constant. Rather it has the value

$$n(\Omega) \equiv \frac{\Omega\dot{\Omega}}{\dot{\Omega}^2} = n - \frac{3I'\Omega + I''\Omega^2}{2I + I'\Omega}$$

where $I' \equiv dI/d\Omega$ and $I'' \equiv d^2I/d\Omega^2$. The progression of the new phase through the central region of the star will be signaled by an anomalous value of the braking index, far removed from the canonical value of $n$.

Because the pulsar rotational energy is coupled to weak processes, $\Omega$ is small and none of the quantities in (3) will change appreciably over any observational time span. However the signal is carried in non-zero derivatives $dI/d\Omega$ and $d^2I/d\Omega^2$; these are large during the phase transition epoch because of the progressive conversion of nuclear matter into compressible quark matter. As can be seen from (3), large derivatives of the moment of inertia will produce enormous deviations of the braking index from its canonical value while the region of the star occupied by quark matter is growing. Since the growth is paced by the slow spin-down of the pulsar, we will find that the signal is “on” over an extended epoch.

The behavior of the moment of inertia in the critical frequency interval for our model star (which is described later) is shown in Fig. 2. As the pulsar evolves in time (decreasing $I$) the derivative $dI/d\Omega$ passes through two singularities, switching between $+\infty$ and $-\infty$ at each turning point of $\Omega$. From (3) it is clear that the deceleration $\dot{\Omega}$ will pass through zero and change sign at both turning points becoming an acceleration in the central
part of the ‘S’; the pulsar spins up for a time. Moreover, 

\(-I''\) has similar singularities as can be found from

\[-I'' = \left(\frac{dI}{dT}\right)^3 \frac{d^2\Omega}{dT^2}.\]  

(4)

Consequently \(n(\Omega)\) goes to \(\pm \infty\) at the two turning points respectively as shown in Fig. 3. We have plotted the braking index as a function of \(I\) because \(I\) decreases monotonically with the time, unlike \(\Omega\).

(The spin-up referred to above has nothing to do with the minuscule spin-up known as a pulsar glitch. The relative change in moment of inertia in a glitch episode is very small (\(\Delta I/I \sim -\Delta \Omega/\Omega \sim 10^{-6}\) or smaller) and approximates closely a continuous response of the star to changing frequency on any time scale that is large compared to the glitch and recovery interval. Excursions of such a magnitude as quoted would fall within the thickness of the line in Fig. 3.)

A change in moment of inertia owing to a change in phase such as we have described is evidently a robust phenomenon. However, we cannot be sure that nature will respond as strongly as our model does. Backbending, as in Fig. 3, is an extreme response of the moment of inertia to the progression of a phase transition through the central region of the star. Instead, the transition of the moment of inertia from that of a neutron star (at high \(\Omega\)) to a hybrid star may be a single-valued function of \(\Omega\). However the transition between these two moments can still be marked by a large first derivative \(I'\) and a second derivative \(I''\) that is large and changes sign in the transition interval. In this case, the braking index will not swing between \(\pm \infty\) but nonetheless can attain large positive and negative values.

Typically it is difficult to measure \(\ddot{\Omega}\) (and hence the braking index) because of timing noise. Only four braking indices are presently known. However, for a star that is passing through the phase transition epoch, the deceleration \(\ddot{\Omega}\) is reduced markedly (even changing to acceleration). The second derivative must therefore be large in absolute magnitude through all but the central portion of the epoch where spin-up occurs. Consequently, the second derivative of frequency should be easier to measure for pulsars passing through the epoch than for typical pulsars. Hence, difficulty in measuring the second derivative may be used as a de-selection criterion.

We estimate the plausibility of observing in the pulsar population a signal of the kind that we find in our calculation. Assume that neutron stars are created in a narrow mass window (as present evidence suggests). The duration over which the observable index is anomalous is \(\Delta T \approx -\Delta \Omega/\ddot{\Omega}\) where \(\Delta \Omega\) is the frequency interval of the anomaly. The range in which \(n\) is smaller than zero or larger than six (Fig. 3) is \(\Delta \Omega \approx 100\) rad/s. Take a typical period derivative of \(\dot{P} \sim 10^{-16}\) s/s to find \(\Delta T \sim 10^5\) years. The signal would endure for 1/100 of a typical active pulsar lifetime. Similarly we estimate that spin-up would last for about 1/6th of that time. Given that more than 700 pulsars are presently known (200 of which have been discovered in the last several years) about 7 of these may be signaling the growth of a central region of new phase.

We describe briefly the calculation. The equations describing the configurations of rotating stellar structures are solved for a sequence having the same baryon number but different rotational frequencies. The usual expression for the moment of inertia in General Relativity due to Hartle is not adequate for our purpose (see Ref. 17 equations (49) and (62)). Not only are effects of internal constitutional changes in the star absent but also absent are the effects of the alteration of the metric of space-time by rotation, the dragging of local inertial frames, and even centrifugal flattening. Instead, to calculate the moment of inertia, we must use an expression that incorporates the above effects as derived by Glendenning and Weber 18,19.

The stellar model is described as follows. Neutron star matter has a charge neutral composition of hadrons consisting of members of the baryon octet together with leptons when in the purely confined phase. The properties of such matter are calculated in a covariant mean field theory as described in Refs. 20,21. The values of the parameters that define the coupling constants of the theory are certain fairly well constrained properties of nuclear matter and hypernuclei as described in the references: (binding energy of symmetric nuclear matter \(B/A = -16.3\) MeV, saturation density \(\rho = 0.153\) fm\(^{-3}\), compression modulus \(K = 300\) MeV, symmetry energy
coefficient $a_{\text{sym}} = 32.5$ MeV, nucleon effective mass at saturation $m_{\text{sat}}^n = 0.7m$ and ratio of hyperon to nucleon couplings $x_u = 0.6, x_d = 0.653 = x_p$ that yield, together with the foregoing parameters, the correct $\Lambda$ binding in nuclear matter \cite{21}. Quark matter is treated in a version of the MIT bag model with the three light flavor quarks $(m_u = m_d = 0, m_s = 150$ MeV) as described in Ref. \cite{21}. A value of the bag constant $B^{1/4} = 180$ MeV is employed. (See Refs. \cite{22,23,24,25} for a correct treatment of first order phase transitions in multi-component substances such as neutron star matter for which baryon and electric charge are the independent conserved charges.) Very little is known about the high-density properties of matter and our calculation does not imply a prediction of the rotational frequency or stellar mass at which a phase transition will occur. Rather it shows what the signal could be if conditions are attained for the phase change.

So far as we know, there is little difference in the properties of a neutron star that has no quark core and one that has already fully developed one. But a strong signal may be associated with the gradual conversion of matter from one phase into the other as the conversion is paced by the slow loss of angular momentum in the processes of electromagnetic radiation and electron-positron wind. The important observational features by which such an epoch could be identified are:

1. The braking index has a value far from the canonical value, possibly by orders of magnitude and can be of either sign.
2. The epoch over which the braking index is anomalous is long because pulsars spin down slowly under ordinary circumstances but even more slowly when $|P|$ is large (see \cite{2}).
3. The pulsar may be observed to be spinning up. (To avoid confusion with spin-up due to accretion, only isolated pulsars are relevant.)
4. Except for the central part of the spin-up era, the derivative $\Omega$ is large and therefore easy to measure and so also is the braking index.
5. Difficulty in measuring $\dot{\Omega}$ can be used to deselect phase transition candidates.
6. An estimated 1/100 pulsars may be passing through the transition epoch.

Pulsar observations are still in their infancy. It takes a considerable time-span of data to measure the braking index. And many of the presently known pulsars have only recently been discovered. We conclude from our work that it is plausible that the phase transition signal can be observed. It would be a momentous discovery to find that a phase of matter that existed in the early universe inhabits the cores of some neutron stars.

This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of Nuclear Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

A part of this work was done at the ECT*, Villa Tambosi, Trento, Italy. One of us (NKG) is grateful to the hospitality of the center.

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