Magnetohydrodynamic chemically reactive Sisko liquid flow through a stretching surface with nonlinear thermal radiation

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Abstract. Present article reveals results of steady 2D magnetohydrodynamic boundary layer flow of a chemically reactive Sisko liquid due to a stretching surface with nonlinear thermal radiation. The governing equations are computed with R-K 4th order method with BVP4C technique. Numerical evaluations have been accomplished for disparate values of physical constraints with abet of delineates. Additionally, the coefficient of drag force, local Nusselt and Sherwood numbers have been evaluated for disparate parameters and scrutinized for engineering interest.

Keywords: Sisko fluid, MHD and nonlinear thermal radiation

1. Introduction
The non-Newtonian fluid flows are most authoritative predicaments in few contemporary years, owing to its ample variety of applications in diverse provinces like chemical engineering, petroleum production, food engineering and power engineering. Also, the current evolutions in industry and developing engineering prowess motivated many researchers to analyze the features of non-Newtonian liquids in methodical way. As non-Newtonian liquids have plenty of heterogeneity in nature, so countless immanent equations were initiated to analyze substantial properties of these liquids. Most of the these fluids used in chemical engineering like lubricating greases, waterborne coatings, multiphase mixers etc. are perform the Sisko liquid model. This model was introduced by Sisko [1], he introduced the properties of lubricating greases. Masood Khan and Rabia Malik [2] probed results on a forced convective heat transfer flow of an electrically conducting non-Newtonian Sisko sap over an penetrable stretching cylinder along axial direction subject to viscous dissipation. They observe that the impact of sundry parameters is favourable for the shear thinning fluid than that of the shear thickening fluid and the temperature is lackle in the matter of flow past a flat plate as correlated to the stretching cylinder. The steady 2D flow and heat transfer on MHD Sisko fluid over a non-isothermal stretching sheet in the existence of convective boundary condition is perused by Rabia Malik et al. [3]. They found that the velocity enhances with material parameter where as it portrays inimical nature with power law index parameter. For higher values of stretching parameter velocity and temperature diminish. The temperature distribution enhance with magnetic parameter and Biot number where as it diminuate with Prandtl number. Awais et al. [4] studied mixed convection heat transfer laminar flow of a Sisko fluid over a stretching cylinder near the axisymmetric stagnant point subject ot magnetic field. To employ the numerical technique they considered an immense mesh size and high liberality error. Their results elucidates that the heat transfer rate increase with enhancing mixed convection parameter and velocity ratio parameter. Asif Munir and Masood Khan [5] analyzed numerical results on stable boundary layer flow of Sisko sap over a rotating disk. Arif Hussain et al. [6] perused the impact of internal friction and temperature dependent thermal conductivity of Sisko fluid flow through a stretching cylinder in the existence of magnetic field. From this study they noticed that the velocity field enhance with lofty values of material constantand curvature parameter whereas the rate of heat transfer enhance with curvature parameter and Eckert number and reduce with Prandtl number and thermal conductivity variable. Sari et al. [7] analysed Lie group theory and boundary layer equations of Sisko sap. They perceived that the thickness of the boundary enhances with non-Newtonian parameters and diminishes with power law index parameter. Masood Khan and Azeem Shahzad [8] analyzed the steady...
laminar flow of a non-Newtonian liquid due to a nonlinear stretching sheet by considering the Sisko sap model, which is amalgamation of power-law and Newtonian fluids in which the liquid may evince shear thinning/thickening performance. Eswaramoorthy et al. [9] probed the impact of convective boundary constrain on steady non-Newtonian liquid flow over a stretching surface with viscous dissipation.

Radiative heat transfer has gigantic applications in thermal therapeutic operations, fluidized bed heat exchangers, space research like aerodynamic rockets, Nuclear power plants and turbid water bodies. Bhatti et al.[10] deliberated a steady heat and mass transfer flow of dusty Jeffrey fluid with non linear thermal radiation on sinusodal locomotion of magnetic soled particles with Joule heating and Hall currents. They noticed that velocity diminuates with influence of particle volume fraction parameter where as it shows paradoxical behavior with hall parameter and the pressure rise enhance with slip and volume fraction parameter while decrease with Hartmann number. Satish Kumar et al. [11] probed3-D boundary layer flow of a Sisko sap over a bidirectional stretching sheet subject to non-linear thermal radiation with nanoparticles. The heat energy of the fluid enhance with volume fraction parameter and thermal radiation parameter. Ramzan et al. [12] analyzed flow of Eyring Powell liquid due to a constant moving surface in the existence of variable thermal conductivity and chemical reaction. The energy equation is modeled by considering non-linear thermal radiation. They found that thermal radiation parameter depicts enhancing nature with temperature while concentration diminishes with chemical reaction parameter. The consequence of viscous dissipation and Joule heating of a natural convective flow past aeroct plate in the existent of nonlinear thermal radiation and magnetic field is demonstrated by Meraj Mustafa et al. [13]. Ganesh Kumar et al. [14] perused two-dimensional flow of a nonlinear radiative Williamson fluid subject to particle liquid suspension over a stretching sheet. The impact of thermal radiation on unsteady chemical reactive viscoelastic liquid flow past a stretchy paper with Dufour and Soret effects was scrutinized by Bhuvaneswari et al [15].

In this analysis we made an attempt to analyse influence of non-linear thermal radiation on the steady boundary layer flow heat and mass transfer of a Sisko liquid over a stretching sheet with magnetic field and first order chemical reaction.

2. Mathematical Formulation:

Stress tensor of the non-Newtonian Sisko model was inscribed by Sisko (1958)

\[
\tau = -\left\{ c + d \sqrt{n} \left( \frac{n}{2} \Delta : \Delta \right) (n-1) \right\} \Delta, (1)
\]

where \( \tau \) and \( \Delta \) are the stress tensor and deformation tensor rate, respectively; \( c \), \( d \) and \( n \) are expounded differently for various fluids.

Let us contemplate steady, 2D and incompressible flow of Sisko sap due to a stretching surface as depicted in figure 1. The stretching velocity of the surface is defined as \( u(x, y) = A_1 x^s \), where \( A_1 \) and \( s \) are nonnegative real numbers and \( (x, y) \)designates the Cartesian coordinates. An unvarying magnetic field is appealed normal to the direction of the flow and transverse magnetic field is appealed under the supposition of insignificant magnetic Reynolds number.

The govening equations for 2D boundary layer flow are

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}
\]

\[
u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial y} \left( -\frac{\partial P}{\partial y} \right) - \frac{\sigma B^2}{\rho} u \tag{3}
\]
\[
\frac{\partial T}{\partial \tilde{x}} + v \frac{\partial T}{\partial \tilde{y}} = \frac{\partial}{\partial \tilde{x}} \left[ \left( \alpha + \frac{16\sigma^* T^3}{3k_1^*} \right) \frac{\partial T}{\partial \tilde{y}} \right] + \frac{c}{\rho c_p} \left( \frac{\partial u}{\partial \tilde{y}} \right)^2 + \frac{d}{\rho c_p} \left( -\frac{\partial u}{\partial \tilde{y}} \right)^{n+1} \tag{4}
\]

\[
\frac{\partial C}{\partial \tilde{x}} + v \frac{\partial C}{\partial \tilde{y}} = D_p \frac{\partial^2 C}{\partial \tilde{y}^2} - K(C - C_\infty) \tag{5}
\]

The associated boundary constraints are

\[
u = u(\tilde{x}, \tilde{y}) + L \left( \frac{\partial u}{\partial \tilde{y}} \right), V = 0, T = T_w, C = C_w \text{ at } \tilde{y} = 0 \tag{6}
\]

\[
u = u(\tilde{x}, \tilde{y}) = Cx', u \to 0, v \to 0, T \to T_\infty, C \to C_\infty \text{ as } \tilde{y} \to \infty \tag{7}
\]

3 Method of Solution

The similarity variables are

\[
\eta = \frac{\tilde{y}}{\tilde{x}}, \psi = -\tilde{x}^2 U \text{Re}_n^{-1} g(\eta), u = U g'(\eta) = A_x x' g',
\]

\[
v = -U \text{Re}_n^{-1} \frac{1}{n+1} \left[ \left\{ s(2n-1) \right\} g(\eta) + \left\{ s(2n-1) \right\} \eta g'(\eta) \right]
\]

\[T = T_w \left( 1 + (\theta_w - 1) \theta \right) \text{ with } \theta_w = \frac{T_w}{T_\infty}, \theta_w > 1, \ \phi = \frac{C - C_w}{C_\infty - C_w} \tag{6}
\]

The transformed ordinary differential equations are

\[A g'^* + n(-g')^{n-1} g + \left( \frac{s(2n-1)+1}{n+1} \right) gg'^* - s(g')^2 - Mg'^* = 0 \tag{8}
\]

\[\left[ \left[ 1 + Nr \left( 1 + (\theta_w - 1) \theta \right)^3 \right] \theta' \right] + \text{Pr} \left( \frac{s(2n-1)+1}{n+1} \right) g\theta' + \text{Pr} Ec \left[ A \left( g' \right)^2 - (-g')^{n+1} \right] = 0 \tag{9}
\]

\[\phi' + \text{Pr} Sc \left( \frac{s(2n-1)+1}{n+1} \right) g\phi' - \gamma \phi = 0 \tag{10}
\]

Transformed boundary constraints are

\[g = 0, \ g'^* = 1 + h_1 g''(0), \ \theta(0) = 1, \ \phi(0) = 1 \tag{11}
\]

\[g'(\infty) \to 0, \ \theta(\infty) \to 0, \ \phi(\infty) \to 0 \tag{12}
\]

Here,

\[\text{Re}_c = \frac{\rho \tilde{x} U}{c} \text{ is the Reynolds number of Newtonian fluid, Re}_d = \frac{\rho \tilde{x}^2 U^{2-n}}{d} \text{ is the Reynolds number of power-law liquid, A} = \frac{\text{Re}_d^{n+1}}{\text{Re}_c} \text{ is the material parameter, M} = \frac{\text{Re}_c \sigma B_0^2}{\text{Re}_d^{n+1}} \text{ Hartmann parameter,}
\]

\[\text{Pr} = \frac{\tilde{x} U \text{Re}_d^{n+1}}{c} \text{ generalized Prandtl number, Nr} = \frac{16\sigma^* T_w^3}{3k_1^* k^*} \text{ thermal radiation number, Sc} = \frac{\nu}{D_a} \text{ Schmidt}
\]
number, \( Ec = \frac{U^2}{C_p(T_f - T_w)} \) is the Eckert number, \( \gamma = \frac{k_\alpha Le}{c} \) is the Chemical reaction variable, \( h_1 = L \left( \frac{V}{c} \right)^{\frac{1}{2}} \) is the slip parameter.

The engineering interest quantities are skin friction \( C_{fr} \), Nusselt number \( Nu_T \) and Sherwood number \( Sh_T \) are defined as

\[
C_{fr} = \frac{\tau_w}{\frac{1}{2} \rho U^2}, \quad Nu_T = \frac{\bar{x}q_y}{k_1 (T_w - T_x)} \quad \text{and} \quad Sh_T = \frac{\bar{x}j_y}{k_1 (C_w - C_x)} \quad \text{at} \quad \bar{y} = 0
\]

(13)

where shear stress \( \tau_w \), heat flux \( q_y \) and mass flux \( j_y \) are stated as follows

\[
\tau_w = \left( c + d \frac{\partial u}{\partial \bar{y}} \right) \frac{\partial u}{\partial \bar{y}}, \quad q_y = -k_1 \frac{\partial T}{\partial \bar{y}}, \quad j_y = -D_b \frac{\partial C}{\partial \bar{y}} \quad \text{at} \quad \bar{y} = 0
\]

(14)

The dimensionless equations of skin friction, Nusselt number and Sherwood number are

\[
C_{fr} = 2 \left[ Ag^n(0) \left[-g^n(0)\right]^n \right] \operatorname{Re}^{-\frac{1}{n+1}}, \quad Nu_T = \left[-\left(1 + Nr \theta_{w}^{n}\right) \theta'(0)\right] \operatorname{Re}^{-\frac{1}{n+1}}, \quad Sh_T = -\phi' \operatorname{Re}^{-\frac{1}{n+1}}
\]

4 Results and Discussions:

In all supra explorations of Sisko sap, majority of the researchers disregarded the impact of nonlinear thermal radiation and viscous dissipation on Sisko sap. It is noticed that the influence of viscous dissipation and nonlinear thermal radiation amends considerably against dissimilarity in temperature. Also, it can be seen that flow attributes are substantially transference by considering above disclosed impacts into account. Due to this certainty, the predominant attentiveness of this probe is to examine the amalgamated influence of nonlinear thermal radiation and viscous dissipation on Sisko liquid. The computational numerical results of nonlinear partial differential equations are established with fourth order R-K method by disparate sundry parameters.

The impact of the Hartmann number (M) on velocity, temperature and concentration has plotted in Figs. 1-3. From these sketches it is perceived that an enhancement in Hartmann number diminishes the velocity and enhances temperature and concentration due to the fluid flow is resisted by the Lorentz force generated by the magnetic field. Hence the thickness of the hydrodynamic boundary diminishes while thermal and solutal boundary enhance. Figs. 4-6elucidate the velocity, temperature and concentration sketches for various power law index parameter. From Fig. 4 we observed that velocity enhances near the boundary of the fluid flow where as it presents paradoxical behavior at distant from the boundary. The temperature and concentration portrays diminish for greater power law index parameter. This deceleration is owing to the reduction in viscosity opposes the heat transfer for shear thickening fluid. Furthermore, the thermal and mass boundary layer thickness decreases. The velocity, temperature and concentration profiles for disparate values stretching variable is elucidated in Figs. 7 – 9. The velocity diminishes with lofty values of stretching parameter whereas the temperature and concentration enhance. Moreover thermal and solutal boundary layer thickness enhances.

Fig. 10presents the velocity ability verses non dimensional parameter \( \eta \) for various A. This figure reveals that the velocity of the liquid is utmost at \( \eta = 0 \), but it penures and approximates to nought asymptotically at \( \eta = 8 \). Hence, Fig. 12 portrays that larger value of material parameter favorsto
momentum boundary. This physical fact due to A is the proportion between the viscosities of lofty shear rate and power law region. The temperature and concentration distributions demonstrate inimical behavior to velocity distribution for higher values of material variable.

It can be clear that with greater generalized Pr temperature decreases. (Figs. 13). A greater value of Prandtl number produces lckle thermal boundary layer thickness. This deceleration is due to physically lofty Pr fluids have lckle thermal conductivity. Fig. 14 presents the influence of Ec on the temperature sketch. The Eckert number contributes the alliancebetwixt the enthalpy and the kinetic strength in the flow. It can be clear that the influence of Ec inchoates to a rise in temperature distribution. Additionally, it is perceived that the thermal boundary layer thickness is more for the larger values of Eckert number. The impact of thermal radiation parameter $Nr$ on temperature portray is depicted in Fig. 15. The temperature shows augmentation phenomena for lofty thermal radiation parameter. Physically, greater values of Nr supplies additional heat to the fluid, hence the temperature and associated boundary layer thickness elongates. Fig. 16 exemplify the variation of temperature ratio variable on temperature. It is seen that the temperature too thermal boundary thickness rises with higher $\theta_w$. As $\theta_w$ enhances the temperature of the ambient fluid is much higher than the sap temperature which reveals the enhancement in thermal state of the sap. The impact of Schmidt’s number and chemical reaction variable on species concentration is depicted in Fig. 17. It is evident that the species concentration reduces significantly with increase in Schmidt’s number as increasing Schmidt’s number corresponds to decrease in mass diffusivity. From Fig. 18 we observed that concentration portray diminishes with greater values of chemical reaction parameter.

5 Conclusions:
The analysis presents the effects of nonlinear thermal radiation, viscous dissipation and chemical reaction on the non-Newtonian incmpressible Sisko fluid flow over a stretching sheet with slip velocity. From the computational results the following conclusions are, the magnetic parameter and slip variable diminish the velocity whereas it enhance with power law index parameter and material parameter. The stretching parameter lessen the temperature and species concentration and increase velocity. The material parameter lessen the thickness of the thermal and solutal boundary layer. The Ec and Nr, temperature ratio parameters enhance the temperature. The Schmidt’s number and chemical reaction parameter have a diminishing impact on concentration ensuing in slender solutal boundary layers.

![Fig. 1 Velocity portray for disparate M](image1.png)

![Fig. 2 Temperature portray for disparate M](image2.png)
Fig. 3 Concentration portrait for disparate $M$

Fig. 4 Velocity portrait for disparate $n$

Fig. 5 Temperature portrait for disparate $n$

Fig. 6. Concentration portrait for disparate $n$

Fig. 7. Velocity portrait for disparate $s$

Fig. 8. Temperature portrait for disparate $s$

Fig. 9. Concentration portrait for disparate $s$

Fig. 10. Velocity portrait for disparate $A$
Fig. 11. Temperature portrayal for disparate $A$

Fig. 12. Concentration portrayal for disparate $A$

Fig. 13. Temperature portrayal for disparate $Pr$

Fig. 14. Temperature portrayal for disparate $Ec$

Fig. 15. Temperature portrayal for disparate $Nr$

Fig. 16. Temperature portrayal for disparate $\theta$

Fig. 17. Concentration portrayal for disparate $Sc$

Fig. 18. Concentration portrayal for disparate $\gamma$
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