Prediction of Collapse Cycles of Shale Wellbores in Bonger Basin of Chad

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Abstract. This paper presents a new collapse cycle model, which is based on the distribution of stress surrounding boreholes under mechanical action and the distribution of pore pressure under chemical action. By considering rock failure criteria, the model has been established for estimating the collapse cycle of shale wellbores. Equation parameters have been verified by experimental studies of Rock mechanics and drilling fluid. The model accounts for the effect of inclination angle and density of drilling fluid on the collapse cycle. The model has been used in 22 directional wells drilled in Chad. According to simulation calculation, the collapse cycle of mudstone wellbores of drilled wells is about 8 d, which is approximately equal to the drilling cycle. The wellbores are stable during drilling process, but they are easy to collapse during well completion, which is consistent with actual drilling data.

1. Introduction
The Bonger Basin in Chad is located in the intersection of the middle and western African valleys, with complex structures and stresses. The Lower Cretaceous formation developed shale, which is easy to spall and disintegrate to result in over pull or stick during drilling. These issues have seriously restricted the progress and quality of safety drilling operations, and become bottlenecks in the development of oilfields in Chad. In this paper, the model has been established for calculating the wellbore collapse cycle. The model has been used to accurately predict the collapse cycle of shale wellbores and can provide guidance for field drilling operations.

2. Formatting the title, authors and affiliations
To effectively improve the stability of shale wellbores, it is necessary to provide the collapse cycle of shale wellbores under water-base drilling fluid and to guide field operations. In this paper, a stress distribution model and a pore pressure distribution model have been established near wellbore in Chad; and based on which the wellbore collapse cycle can be estimated by combining applicable wellbore failure criteria.

2.1. Stress distribution model
The stresses (σ) are consistent with the principal axes of the coordinate system (h, H, v). The deviation of the borehole axis is described by azimuth (dw) and well inclination angle (iw). The well inclination angle is the angle between the borehole axis and the vertical axis (the axis of overburden formation pressure). The azimuth is the angle between the projection of the borehole axis on plane and the horizontal principal stress. When the borehole axis is consistent with Z direction, generally Cartesian
coordinate system (xyz) and cylindrical coordinate system (r θ z) are used to describe the stress field near the wellbore. Converted from a (h, H, v) coordinate system to a (xyz) coordinate system, the second-order stress tensor \( \sigma_{ij} \) expressed by \( \sigma_h, \sigma_H, \sigma_v \) is:

\[
\begin{cases}
\sigma_x = \cos^2\varphi \cos^2\alpha \sigma_h + \sigma_h \sin^2\varphi \cos^2\alpha + \sigma_v \sin^2\alpha \\
\sigma_y = \sigma_h \sin^2\varphi + \sigma_v \cos^2\varphi \\
\sigma_z = \cos^2\varphi \sin^2\alpha \sigma_H + \sigma_h \sin^2\varphi \sin^2\alpha + \sigma_v \cos^2\alpha \\
\tau_{xy} = \frac{1}{2} (\sigma_h - \sigma_H) \cos \alpha \sin^2 \varphi \\
\tau_{xz} = \frac{1}{2} (\sigma_h - \sigma_H) \sin 2 \varphi \sin \alpha \\
\tau_{yz} = \frac{1}{2} (\sigma_h \cos^2 \varphi + \sigma_v \sin^2 \varphi - \sigma_v) \sin 2 \alpha 
\end{cases}
\]

In a cylindrical coordinate system, the total stress caused by in-situ stress is the linear superposition of the stress caused by various in-situ stresses.

The stress caused by \( \sigma_h \) is:

\[
\begin{align*}
\sigma_{rr} &= \left(1 - \frac{\rho^2}{r^2}\right) \sigma_{xx} + \left(1 - 4 \frac{\rho^2}{r^2} + 3 \frac{\rho^4}{r^4}\right) \frac{\sigma_{xx}}{2} \cos 2\theta \\
\sigma_{\theta \theta} &= \left(1 + \frac{\rho^2}{r^2}\right) \frac{\sigma_{xx}}{2} + \left(1 + 3 \frac{\rho^4}{r^4}\right) \frac{\sigma_{xx}}{2} \cos 2\theta \\
\tau_{r \theta} &= -\left(1 + \frac{\rho^2}{r^2} + 3 \frac{\rho^4}{r^4}\right) \frac{\sigma_{xx}}{2} \sin 2\theta
\end{align*}
\]

The stress caused by \( \sigma_H \) is:

\[
\begin{align*}
\sigma_{rr} &= \left(1 - \frac{\rho^2}{r^2}\right) \sigma_{xy} - \left(1 - 4 \frac{\rho^2}{r^2} + 3 \frac{\rho^4}{r^4}\right) \frac{\sigma_{xx}}{2} \cos 2\theta \\
\sigma_{\theta \theta} &= \left(1 + \frac{\rho^2}{r^2}\right) \frac{\sigma_{xx}}{2} + \left(1 + 3 \frac{\rho^4}{r^4}\right) \frac{\sigma_{xx}}{2} \cos 2\theta \\
\tau_{r \theta} &= \left(1 + \frac{\rho^2}{r^2} - 3 \frac{\rho^4}{r^4}\right) \frac{\sigma_{xx}}{2} \sin 2\theta
\end{align*}
\]

The stress caused by \( \sigma_v \) is:

\[
\begin{align*}
\sigma_{zz} &= \sigma_z - \nu \left(2 \left(\sigma_{xx} - \sigma_{yy}\right) \frac{\rho^2}{r^2} \cos 2\theta + 4 \sigma_{xy} \frac{\rho^2}{r^2} \sin 2\theta\right)
\end{align*}
\]

The stress caused by \( \sigma_{xy} \) is:

\[
\begin{align*}
\sigma_{rr} &= \left(1 - 4 \frac{\rho^2}{r^2} + 3 \frac{\rho^4}{r^4}\right) \sigma_{xy} \sin 2\theta \\
\sigma_{\theta \theta} &= -\left(1 + 3 \frac{\rho^4}{r^4}\right) \sigma_{xy} \sin 2\theta \\
\tau_{r \theta} &= \left(1 + \frac{\rho^2}{r^2} - 3 \frac{\rho^4}{r^4}\right) \sigma_{xy} \cos 2\theta
\end{align*}
\]

The stress caused by \( \sigma_{xz} \) is:
3. The stress caused by $\sigma_{yz}$ is:

\[
\begin{align*}
\tau_{yz} &= \sigma_{yz} \left( 1 - \frac{a}{y^2} \right) \sin \theta \\
\sigma_{yz} &= -\sigma_{yz} \left( 1 + \frac{a}{y^2} \right) \cos \theta 
\end{align*}
\]

(6)

The stress caused by $\sigma_{yx}$ is:

\[
\begin{align*}
\tau_{yx} &= \sigma_{yx} \left( 1 - \frac{a}{x^2} \right) \sin \theta \\
\tau_{xy} &= -\sigma_{xy} \left( 1 + \frac{a}{x^2} \right) \cos \theta 
\end{align*}
\]

(7)

In summary, the stress distribution near the wellbore caused by in-situ stress is:

\[
\begin{align*}
\sigma_{yy} &= \left( 1 + \frac{a}{y^2} \right) \frac{\sigma_{xy}}{2} + \left( 1 + 3 \frac{a}{y^2} \right) \frac{\sigma_{yy}}{2} \cos 2\theta + \left( 1 + \frac{a}{y^2} \right) \frac{\sigma_{yy}}{2} - \left( 1 - 4 \frac{a}{y^2} \right) \\
\sigma_{xx} &= \left( 1 - \frac{a}{x^2} \right) \frac{\sigma_{xx}}{2} + \left( 1 + 3 \frac{a}{x^2} \right) \frac{\sigma_{xy}}{2} \sin 2\theta \\
\sigma_{xy} &= \left( 1 + \frac{a}{x^2} \right) \frac{\sigma_{xy}}{2} + \left( 1 + 3 \frac{a}{x^2} \right) \frac{\sigma_{yy}}{2} \cos 2\theta - \left( 1 + 3 \frac{a}{x^2} \right) \sigma_{xy} \sin 2\theta \\
\tau_{xy} &= \left( 1 + 3 \frac{a}{x^2} \right) \sigma_{xy} \cos 2\theta \\
\tau_{yx} &= \sigma_{xx} \left( 1 + \frac{a}{x^2} \right) \cos \theta + \sigma_{yx} \left( 1 - \frac{a}{y^2} \right) \sin \theta \\
\tau_{yx} &= \sigma_{yx} \left( 1 + \frac{a}{x^2} \right) \sin \theta - \sigma_{yx} \left( 1 + \frac{a}{y^2} \right) \cos \theta 
\end{align*}
\]

(8)

2.2. Pore pressure distribution model

Hydraulic pressure difference and chemical potential difference are two main factors that cause the change of pore pressure, thus the volume flow rate through a cell cube can be expressed as:

\[
J = \frac{k}{\mu} \frac{1}{\Delta x} \left( \Delta \rho - \frac{J}{V} \left( \frac{RT}{\rho \mu} \ln \frac{\rho_j}{\rho_i} \right) \right) \quad (9)
\]

where, $J$=volume flow rate, mL/cm³; $k$=permeability of rock, $\mu$m²s⁻¹; $\mu$=fluid viscosity, mPa·s⁻¹.

It can be obtained from balance equation (in a cylindrical coordinate):

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \rho \right) + \frac{\partial}{\partial \theta} (\rho \phi) = 0 \quad (10)
\]

It can be obtained from equation (9) and equation (10):

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \rho \phi \right) + \frac{\partial}{\partial \theta} (\rho \phi) + \frac{\partial}{\partial r} \left( \frac{RT}{\rho \mu} \ln \frac{\rho_j}{\rho_i} \right) = \frac{\partial}{\partial t} \left( \rho \phi \right) \frac{\partial}{\partial t} \left( \frac{\rho \phi}{k} \right) \quad (11)
\]

where, $C$=fluid compressibility; $\phi$=porosity of rock.

Activity degree $\alpha$ should satisfy the diffusion equation:
\[ \frac{\partial \sigma}{\partial t} - D \nabla^2 \sigma = 0 \]  

(12)

where, \(D\) = diffusion coefficient of activity degree.

The boundary condition and initial condition are:

- when \(t=0\), \(r > r_w \leq r \leq \infty\), \(a = a_s\), \(p = p_o\);
- when \(t>0\), \(r = r_w\), \(a = a_s\), \(p = p_w\);
- when \(t<0\), \(r = \infty\), \(a = a_s\), \(p = p_o\).

where, \(r\) = the distance from wellbore axis, m; \(r_w\) = wellbore radius, m; \(p_o\) = bottom hole pressure, MPa.

By combining boundary conditions and equation (11), the distribution law of pore pressure is obtained.

2.3. Total stress distribution

The total stress distribution near wellbore under the effects of hydraulic, chemical and stress is:

\[
\begin{align*}
\sigma_{\theta\theta} &= \left(1 - \frac{r^2}{r_w^2}\right) \sigma_{\theta\theta} + \left(1 - 4 \frac{r^2}{r_w^2} + 3 \frac{r^4}{r_w^4}\right) \sigma_{\theta\theta} \cos 2\theta + \left(1 - \frac{r^2}{r_w^2}\right) \sigma_{\theta\theta} \sin 2\theta \\
&- \left(1 - 4 \frac{r^2}{r_w^2} + 3 \frac{r^4}{r_w^4}\right) \sigma_{\theta\theta} \cos 2\theta + \left(1 - 4 \frac{r^2}{r_w^2} + 3 \frac{r^4}{r_w^4}\right) \sigma_{\theta\theta} \sin 2\theta \\
&+ \frac{\alpha(1-2\nu)}{(1-\nu)^2} \int_0^r \rho'(r,t) r dr
\end{align*}
\]

\(\sigma_{\theta\theta}\) = \(\sigma_{\theta\theta} - \left(1 + \frac{r^2}{r_w^2}\right) \sigma_{\theta\theta} + \left(1 + 3 \frac{r^2}{r_w^2}\right) \sigma_{\theta\theta} \cos 2\theta + \left(1 + 3 \frac{r^2}{r_w^2}\right) \sigma_{\theta\theta} \sin 2\theta 

+ \frac{\alpha(1-2\nu)}{(1-\nu)^2} \int_0^r \rho'(r,t) r dr - \rho'(r,t)

\(\sigma_{xx} = \sigma_{xx} - \nu \left(2 (\sigma_{xx} - \sigma_{yy}) \frac{r^2}{r_w^2} \cos 2\theta + 4 \sigma_{yy} \frac{r^2}{r_w^2} \sin 2\theta\right)

+ \frac{\nu \alpha(1-2\nu)}{(1-\nu)^2} \rho'(r,t) r dr

\(\sigma_{yy} = \sigma_{yy} - \left(1 + \frac{r^2}{r_w^2} - 3 \frac{r^4}{r_w^4}\right) \sigma_{yy} \sin 2\theta + \left(1 + \frac{r^2}{r_w^2} - 3 \frac{r^4}{r_w^4}\right) \sigma_{yy} \sin 2\theta 

+ \left(1 + \frac{r^2}{r_w^2} - 3 \frac{r^4}{r_w^4}\right) \sigma_{yy} \cos 2\theta

\(\sigma_{\theta\theta} = \sigma_{\theta\theta} - \sigma_{\theta\theta} \left(1 + \frac{r^2}{r_w^2}\right) \cos \theta - \sigma_{\theta\theta} \left(1 + \frac{r^2}{r_w^2}\right) \sin \theta

\sigma_{zz} = \sigma_{zz} - \sigma_{zz} \left(1 - \frac{r^2}{r_w^2}\right) \cos \theta + \sigma_{zz} \left(1 - \frac{r^2}{r_w^2}\right) \sin \theta

(15)

(16)

(17)

(18)

2.4. Rock failure criteria

Rock failure criteria are critical conditions for judging whether a wellbore is damaged. In the study, the Mohr-Coulomb criterion was selected for shear failure. By equation derivation, the stress distribution can be obtained within a certain scope near wellbore; by combining the Mohr-Coulomb rock failure criterion, the collapse pressure can be obtained near wellbore under various circumstances. The expression is:
\[ (\sigma_1 - \alpha P_p)(f^2 + 1) - (\sigma_3 - \alpha P_p)(f^2 + 1) = 2\tau_c \]  

(19)

where, \( \sigma_1 \) = the maximum principal stress on a point on the wellbore; \( \sigma_3 \) = the minimum principal stress on a point on the wellbore; \( \alpha \) = effective stress factor; \( P_p \) = formation pore pressure; \( \tau_c \) = cohesive force; \( f \) = internal friction coefficient.

3. Numerical solution and analysis

Based on field data, well logging data and laboratory experiments, basic parameters were obtained for calculating the collapse cycle of shale wellbores in Chad (Table 1).

| Table 1. Formation and petrological parameters |
|-----------------------------------------------|
| Well depth | 1300 m | Well inclination angle | 0°, 30°, 60°, 90° |
| Biot factor | 0.85   | Azimuth                | 0°                 |
| Poisson ratio | 0.234  | Vertical stress        | 2.22 g/cm³         |
| Elastic modulus | 3.40 GPa | Maximum horizontal principal stress | 1.91 g/cm³ |
| Cohesive force | 5.08 MPa | Minimum horizontal principal stress | 1.59 g/cm³ |
| Internal friction angle | 26.97° | Fluid compressibility | 0.8 GPa⁻¹ |
| Porosity | 0.0335 | Membrane efficiency | 0.3 |
| Equivalent density of drilling fluid | 1.5 g/cm³ | Permeability | 2 mD |
| Viscosity of drilling fluid | 5 mPa.s | Activity degree | 0.9 |

The collapse pressure changes with time as follows (Figure 1).

Figure 1 shows that when azimuth is 0, the collapse pressure increases with the increasing inclination angle; when the density of drilling fluid is 1.5 g/cm³ and the inclination angle is 0°, the longest collapse cycle is 12.35 d.
4. Conclusions
(1) The lower Cretaceous in the Bonger Basin in Chad develops thick shales which has high clay mineral content and is easy to disintegrate with hydration. Moreover, with well-developed fractures, drilling fluid filtrate can easily invade formations to reduce rock strength and wellbore stability.

(2) Based the stress distribution model and pore pressure model near wellbore, and combining the Mohr-Coulomb rock failure criterion, the model for calculating the collapse cycle of shale wellbores has been obtained; and based on the model, how collapse pressure changes with time is obtained at different well inclination angles and different densities of drilling fluid.

(3) Based on the statistical drilling cycles and drilling incidents of drilled wells, it has been confirmed the consistence of simulated with actual results, and that the model for calculating collapse cycle and the parameters selected are more reasonable. The research results can accurately predict the collapse cycle of shale wellbores, and provide guidance for designing drilling fluid formula and drilling operations in Chad.

References
[1] Wang Bingyin, Deng Jingen, Zou Lingzhan, et al. Applied research of collapse cycle of shale in wellbore using a coupled physico-chemical model[J]. Acta Petrolei Sinica, 2006, 27(3):130-132.

[2] Deng Jingen, Guo Dongxu, Zhou Jianliang, et al. Mechanics-chemistry coupling calculation model of borehole stress in shale formation and its numerical solving method[J]. Chinese Journal of rock Mechanics and Engineering, 2003, 22(s1):2250-2253.

[3] Cheng Yuanfang, Zhang Feng, Wang Jingyin, et al. Analysis of borehole collapse cycling time for shale[J]. Journal of China University of Petroleum (Edition of Natural Science), 2007, 31(1): 63-66, 71.

[4] Qu Lianzhong, Cheng Yuanfang, Zhao Yizhong, et al. Fluid solid coupling simulation of deliverability of screen out fracturing in permeability anisotropy unconsolidated sandstone[J]. Journal of China University of Petroleum(Edition of Natural Science), 2009, 33(2): 70-74.

[5] Cheng Yuanfang, Li Lingdong, MAHMOOD S, et al. Fluid-solid coupling model for studying wellbore instability in drilling of gas hydrate bearing sediments[J]. Applied Mathematics and Mechanics, 2013, 34(11): 1421-1432.

[6] R.Freij-Ayoub, C.P. Tan. Simulation of Time-Dependent Wellbore Stability in Shales Using A Coupled Mechanical-Thermal-Physico-Chemical Model[J]. SPE 85344, 2003.

[7] M.Yu, G.Chen, M.E.Chenevert. Chemical and Thermal Effects on Wellbore Stability of Shale. SPE71366, 2001.

[8] Liu Xiangjun, Zeng wei, Liang Lixi, et al. Prediction of collapse cycle of shale formation in Longma stream formation[J]. Special oil and gas reservoirs. 2016, 23(5): 1-8.

[9] Yuan Huayu, Cheng Yuanfang, Wang Wei, et al. Analysis on time-dependent wellbore collapse for long horizontal well in shale formation[J]. Science Technology and Engineering, 2017, 17(3): 183-189.

[10] Yan Chunliang, Deng Jingen, Yu Baohua, et al. Wellbore stability analysis and its application in the Fergana basin, central Asia[J]. Journal of Geophysics and Engineering, 2014, 11(1):1-9.

[11] Chen G, Ewy R.T., Yu M. Analytic solutions with ionic flow for a pressure transmission test on shale. Journal of Petroleum Science and Engineering[J]. 2010, 72(1):158-165.

[12] Li Yufei, Fu Yongqiang, Tang Geng, et al. Laws of the effects of earth stress patterns on wellbore stability in a directional well. Gas Industry, 2012, 32(3):78-80.