LOCAL BRS TRANSFORMATIONS AND $OSp(3, 1|2)$
SYMMETRY OF YANG-MILLS THEORIES

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Abstract

We consider a certain local generalization of BRS transformations of Yang-Mills theory in which the anti-commuting parameter is space time dependent. While these are not exact symmetries, they do lead to a new nontrivial WT identity. We make a precise connection between the “local BRS ” and the broken orthosymplectic symmetry recently found in superspace formulation of Yang-Mills theory by showing that the local BRS WT identity is precisely the WT identity obtained in the superspace formulation via a superrotation. This “local BRS ” WT identity could lead to new consequences not contained in the usual BRS WT identity.

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I. INTRODUCTION

The theories with non-abelian local gauge invariance have acquired a central place in particle physics as they enter into the unified description strong, weak and electromagnetic interactions viz. the standard model. Discussion related to the gauge invariance and its preservation to all orders are of a primary importance as the unitarity and renormalizability of all these theories depend critically on these. The consequence of the local gauge invariance are formulated as a series of Ward- Takahashi (WT) identities and these enter largely in the above discussion. While the initial formulations of the consequences of the local invariance (the WT identities ) depend solely on the local gauge transformation of the gauge field \( \int \), these formulation were later replaced by much more elegant techniques using the BRS symmetry discovered later \[2,3\]. This BRS symmetry is a global symmetry (i.e. depends on a space time independent anti-commuting parameter.) The relevent consequences of the local gauge invariance of the basic Yang-Mills action \( \int d^4 x F_{\mu\nu} F^{\mu\nu} \) that are necessary for the discussion of renormalization of gauge theories and their unitarity can also be derived via latter global symmetry, the BRS symmetry \[3\]. A natural question that comes to mind is whether the global BRS transformations yield all the information contained the original local gauge invariance of the theory.

The question becomes relevant in view of a recent observation by the authors \[4\]. It was in connection with the derivation of WT identities in gauge theories in a superspace formulation. A superspace formulation of gauge theories has been constructed that has a (broken) \( OSp(3,1|2) \) invariance corresponding to rotations in a six dimensional superspace that mix \( x^\mu \) and the anti-commuting coordinates \( \lambda \) and \( \theta \). In this formulation the BRS transformation of the original Yang-Mills theory been replaced in effect by a specific kind of superrotation. The broken \( OSp(3,1|2) \) then leads to an equation of the form \[\] \[1\]

\[1\]This differs slightly from the WT identity in Ref. \[4\] partly because \( W \) used is different.
\[ \dot{W}[K,t] = \dot{W}[K',t'] - \ll \int \delta_4 \left[ \frac{\partial}{\partial \theta} (\partial^\mu A_\mu \zeta) \right] d^4x \gg \]  

(1.1)

Then, as a special piece of information, it was shown from this that

\[ \frac{\partial \dot{W}}{\partial \theta} = 0 \]  

(1.2)

and this was shown to embody the WT identities of the gauge theories. The question naturally arises if the Eq. (1.1) embodying broken $OSp(3,1|2)$ symmetry contains any additional information not contained in Eq. (1.2) which are equivalent to the usual BRS WT identities.

In searching for this question, we can do a “backward” derivation of the existence of an “approximate” $OSp(3,1|2)$ invariance that can be associated with the Yang-Mills theory. This derivation makes use of what we call “local BRS” and these are transformations that are a local generalization of the usual BRS transformation viz.

\[ \delta A_\mu (x) = D_\mu c(x) \Lambda (x) \]

\[ \delta c = - \frac{1}{2} g f c c \Lambda (x) \]

\[ \delta \zeta = - \frac{1}{\lambda} \partial \cdot A \Lambda (x) \]  

(1.3)

Here, $\Lambda (x)$ is an x-dependent arbitrary Grassmannian. We hasten to add that the transformations of Eq. (1.3) are not a symmetry of the Yang-Mills effective action $S_{eff}$ [ nor is the first of Eq. (1.3) a special case of the local gauge transformation on $A_\mu$ ]. However, we can look upon transformation of Eq. (1.3) as a “broken” or approximate symmetry of $S_{eff}$ and use it to obtain its consequences via the usual procedure for obtaining WT identities. [Note that any field transformation on the integration variables of the generating functional does give some identity : whether it is useful or not depends on whether the transformation is a “relevant” one for the action under consideration.] The consequence sources obtained are seen in particular to contain the origin of the broken $OSp(3,1|2)$ symmetry found in superspace.

We should remark, however, that the additional results contained in the local BRS WT identity, while they do explain the origin of $OSp(3,1|2)$ symmetry, do not in an easy way,
lend to consequences; as the identity contains new operators which are not already included in the generating functional. However, the new operator is a BRS variation of a simple operator of dimension three and is expected to be multiplicatively renormalizable. Further investigation is needed to come up with the new additional results.

Now we briefly state the plan of the paper. In sec.II we introduce the notations and the “local BRS” transformations. We also introduce briefly the superspace formulation of Ref. 4. In sec. III, we derive the “local BRS” WT identities. In sec. IV, we show the connection between these and the OSp WT identity of Eq. [1]. In sec. V, we summarize the results and make a few observations.

II. PRELIMINARY

A. Notations

In this section we shall introduce the notations and conventions which will be used frequently in this paper. We consider the generating functional for the Yang-Mills theory in linear gauges as follows

$$W = \int [dA dc d\zeta] \exp \left( iS + i \int d^4x \left\{ j_\mu A^\alpha_\mu + \bar{\xi}^{\alpha} c^{\alpha} + \zeta^{\alpha} \zeta^{\alpha} + \kappa^{\alpha \mu} D_{\mu}^{\alpha \beta} c^{\beta} + \frac{g}{2} j^{\alpha} f^{\alpha \beta \gamma} c^{\beta} c^{\gamma} \right\} \right)$$

$$= W \left[ j, \bar{\xi}, \xi, \kappa, -l, t \right]$$

(2.1)

Where the action, $S$ is given by

$$S = S_0 + S_g + S_{gf}$$

(2.2)

with

$$S_0 = \int d^4x \left\{ -\frac{1}{4} F^{\alpha}_{\mu \nu} F^{\alpha \mu \nu} \right\}$$

$$S_g = \int d^4x \left\{ -\partial^{\mu} \zeta^{\alpha} D_{\mu}^{\alpha \beta} c^{\beta} \right\}$$

$$S_{gf} = \left\{ -\frac{1}{2\lambda}(\partial . A^{\alpha} + t^{\alpha})^2 \right\}$$

(2.3)
Here we assume a Yang-Mills theory with a simple gauge group and introduce following notations

\[ \text{Lie Algebra : } [T^\alpha, T^\beta] = i f^{\alpha\beta\gamma} T^\gamma \]

\[ \text{Covariant Derivative : } D_\mu c^\beta = \left( -\partial_\mu \delta^\alpha c^\beta + g f^{\alpha\beta\gamma} A_\mu^\gamma c^\beta \right) \]  \hspace{1cm} (2.4)

\( f^{\alpha\beta\gamma} \) are structure constant of the gauge group and totally anti-symmetric.

**B. Local BRS Transformations**

Now we introduce the local BRS transformations which we shall use in next section to derive WT identities.

The local BRS transformations are,

\[ \delta A_\mu^\alpha = D_\mu^{\alpha\beta} c^\beta \Lambda(x) \]
\[ = D_\mu^{\alpha\beta} \left( c^\beta \Lambda(x) \right) + c^\alpha \partial_\mu \Lambda(x) \]

\[ \delta c^\alpha = -\frac{g}{2} f^{\alpha\beta\gamma} c^\beta c^\gamma \Lambda(x) \]

\[ \delta \zeta^\alpha = -\frac{1}{\lambda} (\partial \cdot A^\alpha + t^\alpha) \Lambda(x) \]  \hspace{1cm} (2.5)

where the anti-commuting BRS parameter, \( \Lambda(x) \) is function of \( x \).

Note in the transformations (2.5) spatial differential operator is involved only in the transformation of gauge field \( A_\mu^\alpha \). We can perform these local BRS transformation in two steps as follows

First consider the transformation, \( \delta_1 \)

\[ \delta_1 A_\mu^\alpha = D_\mu^{\alpha\beta} \left( c^\beta \Lambda(x) \right) \]

\[ \delta_1 c^\alpha = -\frac{g}{2} f^{\alpha\beta\gamma} c^\beta c^\gamma \Lambda(x) \]

\[ \delta_1 \zeta^\alpha = -\frac{1}{\lambda} (\partial \cdot A^\alpha + t^\alpha) \Lambda(x) \]  \hspace{1cm} (2.6)

and then consider the transformation \( \delta_2 \)
\[ \delta_2 A_\mu^\alpha = c^\alpha \partial_\mu \Lambda(x) \]

\[ \delta_2 c^\alpha = 0 \]

\[ \delta_2 \zeta^\alpha = 0 \]  

(2.7)

Where, \( \delta \) of 2.3 is simply \( \delta = \delta_1 + \delta_2 \). Now the transformation 2.6 on \( A_\mu \) is still a gauge transformation and symmetry of \( S_0 \). Hence the effect of local BRS transformation on \( S_0 \) can be obtained directly using transformation 2.7 only which is very simple in nature.

Now we shall see how the action in 2.2 transform under these local BRS transformations. We only write down the changes of the action here.

Under the transformations 2.5

\[ \delta S_0 = \delta_2 S_0 = \int d^4 x \ c \partial_\mu \Lambda(x) \frac{\delta S_0}{\delta A_\mu(x)} \]

\[ = \int d^4 x \partial_\mu (D_{\nu} F^{\mu \nu}) \Lambda(x) \]  

(2.8)

\[ \delta S_g + \delta S_{g.f} = \int d^4 x \left\{ \partial_\mu \left( \frac{(\partial \cdot A + t)}{\lambda} D_{\mu} c \right) \Lambda(x) + \partial_\mu \left( \partial^\mu \zeta^g \frac{g}{2} f_{cc} \right) \Lambda(x) \right\} \]

\[ \equiv \delta_1 (S_g + S_{g.f}) + \delta_2 (S_g + S_{g.f}) \]  

(2.9)

with

\[ \delta_1 (S_g + S_{g.f}) = \int d^4 x \left\{ \partial_\mu \left[ -\frac{g}{2} \partial^\mu \zeta_{f cc} \right] \Lambda \right\}

+ \partial_\mu \left[ \frac{(\partial \cdot A + t)}{\lambda} D_{\mu} c \right] \Lambda + \partial_\mu \left[ \frac{\partial_\mu (\partial \cdot A + t)}{\lambda} c \right] \Lambda \]  

(2.10)

One can check the relations in Eq. 2.9 very easily. We shall use these transformation properties in next section for the derivation of WT identities.

C. The Superspace Formulation

We briefly recapitulate the superspace formulation of Ref. [5]. In this superspace one has four space-time commuting and two anti-commuting dimensions sources that \( \bar{x}^i = (x^\mu, \lambda, \theta) \).

The metric in this space is \( g_{ij} = g_{\mu\nu} = \text{diag}(1,-1,-1,-1) \) for \( 0 \leq i, j \leq 3 \) and \( g_{ij} = -\epsilon_{ij} \) (\( 4 \leq i, j \leq 5 \) with \( \epsilon_{45} = 1 \)). The group of transformation that preserves \( \bar{x}^i \bar{x}_i \) is \( OSp(3,1|2) \). We introduce the superfields \( \bar{A}_i^\alpha(\bar{x}) \) and \( \zeta^\alpha(\bar{x}) \) transforming as a covariant
vector and a scalar under $OSp(3, 1|2)$. The $A_\mu^\alpha(x)$ are identified with the usual gauge fields, $A_5^\alpha(x)$ with the usual ghost fields, $A_4^\alpha(x)$ is an additional ghost field and $\zeta(x)$ is the usual antighost field of the gauge theories. The remaining fields in the expansion of $\bar{A}_i^\alpha(\bar{x})$ and $\zeta^\alpha(\bar{x})$ in terms of $\lambda$ and $\theta$ are certain auxiliary fields. We also introduce a vector and a scalar supersource $\bar{K}_i^\alpha(\bar{x})$ and $t^\alpha(\bar{x})$. As was shown in Ref. [5]. $\bar{K}_i^\alpha(\bar{x})$ contains in it compactly sources for gauge, ghost fields and also sources for composite fields $\kappa_\mu$ and $l$ all in a single supermultiplet of $OSp(3, 1|2)$. $t^\alpha(\bar{x})$ contains in it the source for antighost field and the source for the gauge fixing $\left(\frac{1}{\sqrt{\eta_0}}\partial_\mu A_\mu^\alpha(\bar{x}) + \frac{1}{2\eta_0} \zeta^\alpha(\bar{x}) + t^\alpha(\bar{x})\right)$ in one multiplet. An action for this superfield theory was introduced as

$$\bar{S} = \int d^4x\bar{L}_0[\bar{A}] + \int d^4x \frac{\partial}{\partial \theta} \left\{ \bar{K}_i^\alpha(\bar{x})\bar{A}_i^\alpha(\bar{x}) + \zeta^\alpha(\bar{x}) \left[ \partial_\mu A_\mu^\alpha(\bar{x}) + \frac{1}{2\eta_0} \zeta^\alpha(\bar{x}) + t^\alpha(\bar{x}) \right] \right\}$$

$$\equiv \bar{S}_0 + \bar{S}_1 \quad (2.11)$$

with $\bar{L}_0[\bar{A}] = -\frac{1}{4} g^{ik} g^{jl} \bar{F}_{ij}^\alpha \bar{F}_i^\alpha$ and $\bar{F}_{ij}^\alpha = \partial_i \bar{A}_j^\alpha - \bar{A}_i^\gamma \partial_j + g_0 f^{\alpha\beta\gamma}_i \bar{A}_i^\beta \bar{A}_j^\gamma$ and a generating functional was introduced as (please refer to [5] for details of definition of measure)

$$\bar{W}[\bar{K}, \bar{t}] = \int \{d\bar{A}\} \{\zeta\} \exp \left( iS[\bar{A}, \zeta\bar{K}, \bar{t}] \right) \quad (2.12)$$

In Ref. [3], this generating functional was shown to contain the generating functional of Green’s function of gauge theory, viz.,

$$\bar{W}[\bar{K}, \bar{t}] = \prod_{\alpha, \gamma} \delta(K^{\alpha\gamma}(y)) \prod_{\beta, \delta} K_{\beta\delta}^{\gamma\delta}(z) W[K_\alpha^{\alpha\mu}, K_\alpha^{\alpha5}; -t_\beta^{\alpha\delta}; K^{\alpha\mu}, K^{\alpha5}, t^\alpha] \quad (2.13)$$

where the usual generating functional for gauge theories is defined by Eq. ?? Thus $K^{\alpha\mu}_, K^{\alpha5}_; -t_\beta^{\alpha\delta}$ serving as sources for gauge, ghost and antighost fields respectively and $K^{\alpha\mu}, K^{\alpha5}, t^\alpha$ serving as sources for operators corresponding to BRS variations, all are in compact supermultiplets.

We note some of the virtues of this formulation (not directly used in this work): (i) $\bar{L}_0[\bar{A}]$ is $OSp(3, 1|2)$ invariant. $\bar{S}_0$ is invariant under infinitesimal $OSp(3, 1|2)$. (ii) The terms $\bar{S}_1$ that breaks $OSp(3, 1|2)$ invariance has a relatively simple form and, moreover, has a partial $OSp(3, 1|2)$ invariance [Lorentz transformations, a subgroup of $Sp(2)$ and certain superspace
III. WT IDENTITIES UNDER LOCAL BRS TRANSFORMATIONS

Here in this section we derive the first result of this paper, i.e., the local WT identities satisfied by the generating functional, $W$ using local BRS invariance.

Performing the “local BRS” transformation of Eq. 2.5 in the generating functional of Eq. 2.1 for the Yang-Mills theory and equating the changes to zero, we obtained

$$
\delta S + \int K^\mu \delta (D_\mu c) d^4 x + \int d^4 x \left[ j^\mu \delta A^\mu + \bar{\xi} \delta c + \delta \zeta \xi \right] = 0 \quad (3.1)
$$

where we have used the fact that $\delta \left( \frac{g}{2} f^{\alpha \beta \gamma} c^\alpha c^\beta c^\gamma \right) = 0$ is also valid for local BRS transformations.

This can be written using 2.9 as

$$
\int d^4 x \left[ \partial_\mu (D_\nu c F^{\mu \nu}) \lambda + \frac{1}{\lambda} \partial_\mu (\partial \cdot AD_\mu c) \Lambda - \partial_\mu (K^\mu \frac{g}{2} f c c) \Lambda 
+ j^\mu < D_\mu c \Lambda > + \bar{\xi} < - \frac{g}{2} f c c \Lambda > + < - \frac{1}{\lambda} (\partial \cdot A + t) \Lambda > \xi \right] = 0 \quad (3.2)
$$

This can be further written by using Eqs. 2.8 - 2.10 as

$$
\int d^4 x \left[ \partial_\mu (D_\nu c F^{\mu \nu}) \lambda + \frac{1}{\lambda} \partial_\mu (\partial \cdot AD_\mu c) \Lambda - \partial_\mu (K^\mu \frac{g}{2} f c c) \Lambda 
+ j^\mu < D_\mu c \Lambda > + \bar{\xi} < - \frac{g}{2} f c c \Lambda > + < - \frac{1}{\lambda} (\partial \cdot A + t) \Lambda > \xi \right] = 0 \quad (3.3)
$$

We now use the equation of motion

$$
< \left( \frac{\delta S}{\delta A_\mu} + j_\mu + f K c \right) \delta_2 A_\mu > = 0 \quad (3.4)
$$

Dropping $\Lambda$ from the above expression and making some simplification we can write Eq. 3.3 as

$$
< \partial_\mu (j_\mu c) + \frac{1}{\lambda} \partial_\mu [(\partial \cdot A + t) g f c A_\mu] + \frac{1}{\lambda} \partial_\mu [(\partial \cdot A + t) c] - (\partial \cdot A + t) \partial_\mu c] - 
\partial^\mu \left\{ (K^\mu + \partial^\mu c) \frac{g}{2} f c c \right\} > + j_\mu < D_\mu c > + \bar{\xi} < - \frac{g}{2} f c c > - < - \frac{1}{\lambda} (\partial \cdot A + t) > \xi = 0 \quad (3.5)
$$
Now we note that

\[
\frac{1}{\lambda} \left[ \partial_\mu (\partial \cdot A + t)c - (\partial \cdot A + t)\partial_\mu c \right] - \frac{g}{2} \partial^\mu \zeta fCc + \frac{1}{\lambda} g(\partial \cdot A + t)fcA_\mu \\
= -\frac{1}{\lambda} (\partial_\mu (\partial \cdot A + t)c) + \delta_{\text{BRS}} (2\partial_\mu \zeta c + gf\zeta cA_\mu)
\]  

(3.6)

[ For any function \( f \), change under global BRS \( \equiv \delta_{\text{BRS}} f \Lambda \). Then we can write the local BRS WT identities as

\[
\begin{align*}
\dot{j}_\mu < D_\mu c > + \bar{\xi} < -\frac{g}{2} fCc > + \frac{1}{\lambda} (\partial \cdot A + t) > \xi \\
= -\partial_\mu < j_\mu c > + \frac{g}{2} K^\mu fCc > - \partial_\mu < \delta_{\text{BRS}} O^\mu > + \frac{1}{\lambda} \partial^2 < (\partial \cdot A + t)c >
\end{align*}
\]

(3.7)

with

\[ O^\mu = 2\partial^\mu \zeta c + f\zeta cA^\mu \]

(3.8)

We note that, if we integrate the equation (3.7) the right hand side being a total divergence contributing nothing and we recover the usual global BRS WT identity. The first two terms on the right hand side will however contribute to the “first moment” of the equation obtained by multiplying by \( \epsilon \cdot x \) and integrating over \( d^4x \) [ \( \epsilon_\mu \) is an arbitrary constant four-vector]; while the last term will only contribute to the “second moment”. We shall be particularly interested in the “first moment” equation.

\[
\int d^4x \epsilon \cdot x < j^\mu D_\mu c + \bar{\xi} (-\frac{g}{2} fCc) + \frac{1}{\lambda} (\partial \cdot A + t) \xi > \\
= \int d^4x \epsilon_\mu < j^\mu c + \frac{g}{2} K^\mu fCc + \delta_{\text{BRS}} O^\mu >
\]

(3.9)

This will be exploited in the next section to relate the above equation to the broken OSp invariance.

**IV. CONNECTION WITH BROKEN OSP SYMMETRY**

In this section, we shall make contact with the superspace formulation of Ref. [4]. We shall then see how the equation (3.9) from local BRS is equivalent to the broken OSp symmetry of Eq. [11].
In order to do this, we note the correspondence between the sources in $W$ of Eq. 2.1 and $\bar{W}$, superspace generating functional, as exhibited in Eq. 2.13. This requires $f^\mu \rightarrow K^\mu_\theta, \xi \rightarrow K^5_\theta, \xi \rightarrow -t_\theta, \kappa \rightarrow K, l \rightarrow K^5$. Then Eq. 3.9 reads

$$\int d^4x \epsilon \cdot x < K^\mu_\theta D_\mu c + K^5_\theta \left(-\frac{1}{2}gfcc - \frac{1}{\Lambda}(\partial \cdot A + t)\theta \right>$$

$$= \int d^4x \epsilon_\mu < K^\mu_\theta c + \frac{1}{2}gfcc + \delta_{\text{BRS}}O^\mu >$$

(4.1)

To convert this into an equation for $\bar{W}$ we note the correspondence from Eq. 2.13. In particular note

$$\delta \bar{W} \delta K^\mu = \prod \delta(\bar{K}^4) \prod K^4_\theta \frac{\delta W}{\delta K^\mu} = \prod \delta(\bar{K}^4) \prod K^4_\theta < D_\mu c > i\bar{W}$$

$$= < D_\mu c > i\bar{W} \equiv i \ll D_\mu c \gg$$

(4.2)

and

$$\delta \bar{W} \delta K^5_\theta = \prod \delta(\bar{K}^4) \prod K^4_\theta \frac{\delta W}{\delta K^5_\theta} = \prod \delta(\bar{K}^4) \prod K^4_\theta < c >$$

$$= < c > i\bar{W} \equiv i \ll c \gg$$

(4.3)

and analogous equation for $\delta \bar{W} \delta K^5_\theta$. We multiply Eq. 4.1 by $\prod \delta(\bar{K}^4) \prod K^4_\theta W [ a = \text{an anticommuting constant}]$ and use Eqs. 4.2 and 4.3 in it to obtain

$$\int d^4x \epsilon \cdot x \left[ K^\mu_\theta \frac{\delta \bar{W}}{\delta K^\mu} + K^5_\theta \frac{\delta \bar{W}}{\delta K^5_\theta} + t_\theta \frac{\delta \bar{W}}{\delta t} \right]$$

$$- \int d^4x \left[ \epsilon_\mu aK^\mu_\theta \frac{\delta \bar{W}}{\delta K^5_\theta} - \epsilon_\mu aK^\mu_\theta \frac{\delta \bar{W}}{\delta K^\mu_\theta} \right] = i\int \delta_{\text{BRS}} \ll \epsilon_\mu aO^\mu \gg d^4x$$

(4.4)

On account of the fact that $\bar{W}$ contains $\prod K^4_\theta; K^4_\theta \frac{\delta W}{\delta K^4_\theta} = 0$ and hence this term can be added to the first square bracket on the left hand side of Eq. 4.4. Also for similar reasons, $\epsilon_\mu aK^4_\theta \frac{\delta W}{\delta K^4_\theta} + \epsilon^\mu aK^4_\theta \frac{\delta W}{\delta K^4_\theta}$ can be added to the second square bracket. Then the left hand side of Eq. 4.4 is just the change in $\bar{W}[\bar{K}, t]$ under the infinitesimal transformations:

$$\delta K^\mu(\bar{x}) = \epsilon \cdot x aK^\mu_\theta(\bar{x}) + \epsilon_\mu aK^\mu(\bar{x})$$

$$\delta K^5(\bar{x}) = \epsilon \cdot x aK^5_\theta(\bar{x}) + \epsilon_\mu aK^\mu(\bar{x})$$

$$\delta K^4 = 0, \quad \delta t = \epsilon \cdot x a t_\theta$$

(4.5)
and the ones they, in particular, contain

\[ \delta K^\mu_\theta = -\epsilon_\mu aK^4_\theta \]
\[ \delta K^4_\theta = 0 \]
\[ \delta K^5_\theta = -\epsilon_\mu aK^\mu_\theta \]
\[ \delta t_\theta = 0 \] (4.6)

But these are just the superspace rotation transformations for an anticommuting 4-vector \( K^\mu(\bar{x}) \) and a scalar \( t(\bar{x}) \)

\[ K'^\mu(\bar{x}') = K^\mu(x + \epsilon a\lambda, \lambda, \theta - \epsilon \cdot xa) = K^\mu(\bar{x}) + \epsilon_\mu aK^4(\bar{x}) \]
\[ K'^5(\bar{x}') = K^5(\bar{x}) + \epsilon_\mu aK^\mu(\bar{x}) \] (4.7)

Where we have ignored the translation in \( x \) contained in Eq. (4.7) because it leads to no change in \( \bar{W} \) on account of its translational invariance. Thus Eq. (4.4) just reads

\[ \bar{W}[K', t'] = \bar{W}[K, t] + i \int \delta_{BRS} \ll \epsilon_\mu aO^\mu \gg d^4x \] (4.8)

It can be easily shown that in the superspace formulation of Ref. [4], the last term in Eq. (4.8) is equal to the last term in Eq. (1.1), thus proving the broken \( OSp(3, 1|2) \) identities of Eq. (1.1) directly via “local BRS”.

V. CONCLUSIONS AND OBSERVATIONS

In this section, we summerize the conclusions of this work and make a number of further observations that could lead to new results.

To summerize, we have introduced a straightforward local generalization of BRS transformation, which we call “local BRS”. This is not an exact symmetry of the Yang-Mills effective action. However, it leads to a WT identity which (i) contains the usual BRS WT identity as a special case. (ii) contains an extra operator. The presence of an extra operator at first sight deters one from being able to extract new consequences with case. But as we shall observe later, the renormalization properties of this operator are particularly simple and this may enable one to derive new consequences.
In this work, we have however focussed on another formal aspect, the connection between this broken “local BRS” symmetry and broken OSp symmetry discussed recently in the context of a superspace formulation of gauge theories \cite{4}. We have shown that the “local BRS” WT identity is precisely equivalent to the broken $OSp(3,1|2)$ WT identity obtained earlier using the broken symmetry of superspace formulation under superrotations that mix $x_\mu$ with $\lambda$ and \theta. We thus give a backward derivation of approximate $OSp(3,1|2)$ symmetry of Yang-Mills theories.

Finally we make a number of observations:

(1) It is well known from the renormalization transformation properties of gauge theories \cite{1} that the left hand side of (3.9) suffers an overall renormalization $(\tilde{Z}Z)^{-\frac{1}{2}}$. Hence multiplying the Eq. (3.9) by $(\tilde{Z}Z)^{\frac{1}{2}}$, we obtain, in particular

$$\text{finite} = \tilde{Z} \int \left\{ j^{\mu R} < c >^R + K^{\mu R} < \frac{1}{2} gfcc >^R + < \delta_{BRS}O^\mu > \right\} d^4x$$

$$= \tilde{Z} \int d^4x \left\{ -\frac{\delta\Gamma}{\delta A^R_\mu} c^R - K^{\mu R} \frac{\delta\Gamma}{\delta l^R} + < \delta_{BRS}O^\mu > \right\}$$

(5.1)

This immediately gives the renormalization properties of $< \delta_{BRS}O^\mu >$: At $K = 0$, it receives counter terms proportional to itself and to $\frac{\delta S}{\delta A^\mu}c$ both dependent on $\tilde{Z}$, a “known” renormalization constant. This observation should help in finding possible applications of (3.9)

(2) The operator $O^\mu$ can, in fact, be further simplified. We note

$$\delta_{BRS}O^\mu = \delta_{BRS} \left\{ 2\partial_\mu \zeta c + f\zeta cA^\mu \right\}$$

$$= \delta_{BRS} \left( \partial_\mu \zeta c + \frac{\partial A}{\lambda} A^\mu \right) - \frac{1}{\lambda} \partial DcA^\mu + \partial_\mu \delta_{BRS}(\bar{c}c)$$

(5.2)

When (5.2) is substituted in (3.9), the last term in (5.2) does not contribute being a total derivative; the second to last term can be simplified using equation of motion of $\zeta$. The net result is

$$\int d^4x \epsilon_\mu c \cdot < j^{\mu R}D_\mu c + \bar{\zeta}(-\frac{g}{2} fcc) + \frac{1}{\lambda}(\partial A + t)\xi > =$$

$$\int d^4x \epsilon_\mu c + \frac{1}{2} K^{\mu g} fcc + \frac{1}{\lambda} \xi A^\mu > + \int d^4x \epsilon_\mu < \delta_{BRS}\tilde{O}^\mu >$$

(5.3)
Where $\tilde{O}^\mu$ is the simpler looking operator

$$
\tilde{O}^\mu = \partial^\mu \zeta C + \frac{\partial A}{\lambda} A_\mu
$$

(5.4)

[ We kept $O^\mu$ in 3.9 as it was the relevant one to OSp WT identity].

(3) In view of the fact that $x^\mu \sim i \frac{\partial}{\partial p^\mu}$, the local BRS WT identity of Eq. 3.9 may be useful in studying momentum variation of Green’s functions.
REFERENCES

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