Comment on ”Scalar-tensor gravity coupled to a global monopole and flat rotation curves” by Lee and Lee.

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The recent paper by Lee and Lee (2004) may strongly leave the impression that astronomers have established that the rotation curves of spiral galaxies are flat. We show that the old paradigm of Flat Rotation Curves lacks, today, any observational support and following it at face value leads to intrinsically flawed alternatives to the Standard Dark Matter Scenario. On the other side, we claim that the rich systematics of spiral galaxy rotation curves, that reveals, in the standard Newtonian Gravity framework, the phenomenon of dark matter, in alternative scenarios, works as a unique benchmark.

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I. THE ISSUE

Lee and Lee (2004) consider a global monopole as a candidate for the galactic dark matter riddle and solve the Einstein Equations in the weak field and large r approximations, for the case of Scalar Tensor Gravity (where $G = G_s(1 + \alpha^2)$, with $G_s$ the bare Gravitational Constant). The potential of the triple of the scalar field is written as: $V_M(\Phi^2) = \lambda/4(\Phi^2 - \eta^2)^2$, the line element of the spherically symmetric static spacetime results as: $ds^2 = -N(r)dt^2 + A(r)dr^2 + B(r)r^2d\Omega^2$ where the functions $N(r), A(r), B(r)$ are given in their eq. 19. From the above, Lee and Lee (2004) write the geodesic equations, whose solution, for circular motions, reads:

$$V^2(r) \simeq 8\pi G\eta^2\alpha_s^2 + GM_*(r)/r$$

where $M_*(r)$ is the ordinary stellar mass distribution. In the above equation, they interpret the first (constant) term, that emerges in addition to the standard Keplerian term, as the alleged constant (flat) value $V(\infty)$ that the circular velocities are thought to asymptotically reach in the external regions of galaxies, where the (baryonic) matter contribution $GM_*/r$ has decreased from the peak value by a factor of several. Furthermore, they compare the quantity $8\pi G\eta^2\alpha_s^2$ with the spiral circular velocities at outer radii and estimate: $\eta \sim 10^{17}GeV$. The crucial features of their theory (at the current stage) are: the ”DM phenomenon” always emerges at outer radii $r$ of a galaxy as a constant threshold value below which the circular velocity $V(r)$ cannot decrease, regardless of the distance between $r$ and the location of the bulk of the stellar component.

The theory implies (or, at its present stage, seems to imply) the existence of an observational scenario in which the rotation curves of spirals are asymptotically flat and the new extra-Newtonian (constant) quantity appearing in the modified equation of motion, can be derived from the rotation curves themselves. As a result, the flatness of a RC becomes a main imprint for the Nature of the ”dark matter constituent”.

The aim of this Comment is to show that the above ”Paradigm of Flat Rotation Curves” of spiral galaxies (FRC) has no observational support, and to present its inconsistency by means of factual evidence. Let us notice that we could have listed a number of objects with a serious gravitating vs. luminous mass discrepancy having steep (and not flat) RC’s and that only a minority of the observed rotation curves can be considered as flat in the outer parts of spirals. However, we think that it is worth to discuss in detail the phenomenology of the spirals’ RC’s, in that we believe that it is the benchmark of any (traditional or innovative) work on ”galactic dark matter”, including that of Lee and Lee (2004).

The ”Phenomenon of Dark Matter” was discovered in the late 70’s (Bosma 1981, Rubin et al. 1980) as the lack of the Keplerian fall-off in the circular velocity of spiral galaxies, expected beyond their stellar edges $R_{opt}$ (taken as 3.2 stellar disk exponential scale-lengths $R_D$). In the early years of the discovery two facts led to the concept of Flat Rotation Curves:

1) Large part of the evidence for DM was provided by extended, low-resolution HI RC’s of very luminous spirals (e.g. Bosma 1981) whose velocity profile did show small radial variations.
2) Highlighting the few truly flat rotation curves was considered a way to rule out the claim that non-Keplerian velocity profiles originate from a faint baryonic component distributed at large radii.

It was soon realized that HI RC’s of high-resolution and/or of galaxies of low luminosity did vary with radius, that baryonic (dark) matter was not a plausible candidate for the cosmological DM, and finally, the prevailing Cosmological Scenario (Cold Dark Matter) did predict galaxy halos with rising as well as with declining rotation curves (Navarro, Frenk and White, 1996). The FRC paradigm was dismissed by researchers in galaxy kinematics in the early 90’s (Persic et al. 1988, Ashman, 1992), and later by cosmologists (e.g. Weinberg, 1997).
Today, the structure of the DM halos and their rotation speeds is thought to have a central role in Cosmology and a strong link to Elementary Particles via the Nature of their constituents, (e.g. Olive 2005) and a careful interpretation of the spirals’ RC’s is considered crucial.

II. THE OBSERVATIONAL SCENARIO

Let us stress that a FRC is not a proof of the existence of dark matter in a galaxy. In fact, the circular velocity due to a Freeman stellar disk has a flattish profile between 2 and 3 disk scale-lengths. Instead, the evidence in spirals of a serious mass discrepancy, that we interpret as the effect of a dark halo enveloping the stellar disk, originates from the fact that, in their optical regions, the RC are often steeply rising.

Let us quantify the above statement by plotting the average value of the RC logarithmic slope, \( \nabla \equiv \langle \frac{d\log V}{d\log R} \rangle \) between two and three disk scale-lengths as a function of the rotation speed \( V_{opt} \) at the disk edge \( R_{opt} \). We remind that, at 3 \( R_D \) in the case of no-DM self-gravitating Freeman disk, \( \nabla = -0.27 \) in any object, and that in the Lee and Lee proposal \( \nabla \sim 0 \) (see eq. 1).

We consider the sample of 130 individual and 1000 coadded RC’s of normal spirals, presented in Persic, Salucci & Stel (1996) (PSS). We find (see Fig. 1b):

\[
\nabla = 0.10 - 1.35 \log \frac{V_{opt}}{200 \text{ km/s}} \quad (2a)
\]

(r.m.s. = 0.1), where 80 km/s \( \leq V_{opt} \leq 300 \text{ km/s} \). A similarly tight relation links \( \nabla \) with the galaxy absolute magnitude (see Fig. 1a). For dwarfs, with 40 km/s \( \leq V_{opt} \leq 100 \text{ km/s} \), we take the results by Swaters (1999):

\[
\nabla = 0.25 - 1.4 \log \frac{V_{opt}}{100 \text{ km/s}} \quad (2b)
\]

(r.m.s. = 0.2) that results in good agreement with the extrapolation of eq. 2. The large range in \( \nabla \) and the high values of these quantities, implied by eq. 2 and evident in Fig. 1, are confirmed by other studies of independent samples (e.g. Courteau 1997, see Fig. 14 and Vogt et al. 2004, see figures inside).

Therefore, in disk systems, in region where the stars reside, the RC slope takes values in the range:

\[-0.2 \leq \nabla \leq 1\]

i.e. it covers most of the range that a circular velocity slope could take (-0.5 (Keplerian), 1 (solid body)). Let us notice that the difference between the RC slopes and the no-DM case is almost as severe as the difference between the former and the alleged value of zero.

It is apparent that only a very minor fraction of RC’s can be considered as flat. Its rough estimate can be derived in simple way. At luminosities \( L < L_\star \) (\( L_\star = 10^{10.4} L_{B\odot} \) is the knee of the Luminosity Function in the B-band) the spiral Luminosity Function can be assumed as a power law: \( \phi(L)dL \sim L^{-1.35}dL \), then, by means of the Tully-Fisher relationship \( L/L_\star \sim (V_{opt}/(200 \text{ km/s}))^3 \) (Giovanelli et al., 1997) combined with eq. 2a, one gets: \( n(\nabla)d\nabla \propto 10^{0.74\nabla}d\nabla \) finding that the objects with a solid-body RC \( (0.7 \leq \nabla \leq 1) \) are one order of magnitude more numerous than those with a ”flat” RC \((-0.1 \leq \nabla \leq 0.1) \). In short, there is plenty of evidence of galaxies whose inner regions show a very steep RC, that in the Newtonian + Dark Matter Halos framework, implies that they are dominated by a dark component, with a density profile much shallower than the ”canonical” \( r^{-2} \) one.

At outer radii (between 6-10 disk scale-lengths) the observational data are obviously more scanty, however, we observe a varied and systematics zoo of rising, flat, and declining RC’s profiles (Gentile et al. 2004; Donato et al. 2004).
The evidence from about 2000 RC’s of normal and dwarf spirals unambiguously shows the existence of a systematics in the rotation curve profiles inconsistent with the Flat Rotation Curve paradigm. The non stellar term in eq. 1 must have a radial dependence in each galaxy and vary among galaxies. To show this let us summarize the RC systematics. In general, a rotation curve of a spiral, out to 6 disk scale-lengths, is well described by the following function:

\[ V(x) = V_{\text{opt}} \left[ \frac{1.97x^{1.22}}{(x^2 + 0.782)^{1.43}} + (1 - \beta)(1 + a^2) \frac{x^2}{x^2 + a^2} \right] \]

where \( x \equiv R/R_{\text{opt}} \) is the normalized radius, \( V_{\text{opt}} = V(R_{\text{opt}}) \), \( \beta = V_d^2/V_{\text{opt}}^2 \), \( a = R_{\text{core}}/R_{\text{opt}} \) are free parameters, \( V_d \) is the contribution of the stellar disk at \( R_{\text{opt}} \) and \( R_{\text{core}} \) is the core radius of the dark matter distribution. Using a sample of \( \sim 1000 \) galaxies, PSS found that, out to the farthest radii with available data, i.e. out to \( 6 R_D \), the luminosity specifies the above free parameters i.e. the main average properties of the axisymmetric rotation field of spirals and, therefore, of the related mass distribution. In detail, eq. 2 becomes the expression for the Universal Rotation Curve (URC, see Fig. 2 and PSS for important details). Thus, for a galaxy of luminosity \( L/L_* \) (B-band) and normalized radius \( x \) we have (see also Rhee, 1996):

\[
V_{\text{URC}}(x) = V_{\text{opt}} \left[ \left( \frac{0.72 + 0.44 \log L/L_*}{L_*} \right) \frac{1.97x^{1.22}}{(x^2 + 0.782)^{1.43}} + \right.
\]
\[
\left. \left( 0.28 - 0.44 \log L/L_* \right) \left[ 1 + 2.25 \left( \frac{L}{L_*} \right)^{0.4} \right] \frac{x^2}{x^2 + 2.25 (L/L_*)^{0.4}} \right]^{1/2}
\]

The above can be written as: \( V^2(x) = G(kM_*/x + M_h(1)F(x, L)) \) where \( M_h(1) \) is the halo mass inside \( R_{\text{opt}} \) and \( k \) is the order of unity. Then, differently from the Lee and Lee (2004) claim and the FRC paradigm, the “dark” contribution \( F(x, L) \) to the RC varies with radius, namely as \( x^2/(x^2 + a^2) \), \( a = \text{const} \) in each object. Finally, also the extrapolated ’asymptotic amplitude \( V(\infty) \) varies, according to the galaxy luminosity, between \( 50 \text{ km/s} \) and \( 250 \text{ km/s} \) (see also PSS) in disagreement with the Lee and Lee (2004) predicted constant value of \( 8\pi G \eta^2 \alpha_0^2 \sim 300 \text{ km/s} \).

Let us conclude with an important point: this paper is not intended to discourage testing out whether a theory, alternative to the DM paradigm, can account for an outer flat rotation curve, but to make us sure that this is the (simplest) first step of a project meant to account for the actual complex phenomenology of rotation curves of spirals and of the implied physical relevance of the mass discrepancy (e.g. Gentile et al. 2004).

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FIG. 2: The Universal Rotation Curve

III. DISCUSSION

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