Standing on the shoulders of giants: New mass and distance estimates for α Orionis through a combination of evolutionary, asteroseismic, and hydrodynamical simulations with MESA

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(Dated: Accepted XXX. Received YYY; in original form ZZZ)

ABSTRACT

We conduct a rigorous examination of the nearby red supergiant α Orionis, or Betelgeuse, by drawing on the synthesis of new observational data and three different modeling techniques. Our observational results include the release of new, processed photometric measurements collected with the space-based SMEI instrument prior to Betelgeuse’s recent, unprecedented dimming event.

Our theoretical predictions include self-consistent results from multi-timescale evolutionary, oscillatory, and hydrodynamic simulations conducted with the Modules for Experiments in Stellar Astrophysics (MESA) software suite. Significant outcomes of our modeling efforts include a precise prediction for the star’s radius: $750^{+62}_{-30} \, R_\odot$. In concert with additional constraints, this allows us to derive a new, independent distance estimate of $165^{+16}_{-8} \, pc$ and a parallax of $\pi = 6.06^{+0.31}_{-0.52} \, \text{mas}$, in good agreement with Hipparcos but less so with recent radio measurements.

Seismic results from both perturbed hydrostatic and evolving hydrodynamic simulations constrain the period and driving mechanisms of Betelgeuse’s dominant periodicities in new ways. Our analyses converge to the conclusion that Betelgeuse’s $388$ day period is the result of pulsation in the fundamental mode, driven by the $\kappa$-mechanism. Grid-based hydrodynamic modeling reveals that the behavior of the oscillating envelope is mass-dependent, and likewise suggests that the non-linear pulsation excitation time could serve as a mass constraint.

Our results corroborate recent conclusions that Betelgeuse is the outcome of a past merger. We place it definitively in the core helium-burning phase near the base of the red supergiant branch, and we report a present-day mass of $16.5–19 \, M_\odot$—slightly lower than typical literature values.

Keywords: stellar evolution – red giants – stellar oscillations – numerical techniques

1. INTRODUCTION

Since November of 2019, the red supergiant α Orionis—popularly known as Betelgeuse—has experienced an unprecedented brightness drop of nearly 2 magnitudes in the V band. The severity of this decrease and the deviation from its typical pattern of variability have sparked much public speculation about the physics responsible and its likelihood of undergoing a cataclysmic event.

To investigate these questions first requires an understanding of the short-timescale behavior of variable red giants. Such stars are known to exhibit a complex spectrum of variability, where cyclic variations with differ-
ent driving mechanisms occur over a range of timescales. Though we can explain and fully capture some pulsation physics in 1D stellar models (e.g., pressure and gravity modes; see review by Aerts 2019), other mechanisms are not well understood (Wood et al. 2004; Nicholls et al. 2009a). In this latter class fall many of the variations we observe on human timescales, as such behavior is, with rare exception, too rapid to be explained by classical stellar evolution (Molnár et al. 2019). Modeling such processes may require 3 dimensions, time-dependent convection, or otherwise more sophisticated physical formalisms that are beyond the scope of typical 1D stellar evolution programs. Nevertheless, 1D stellar models are among the most powerful devices for gaining insight on the sub-surface physics responsible for observed changes in real stars (Demarque et al. 2004; Pietrinferni et al. 2004; VandenBerg et al. 2006; Cordier et al. 2007; Weiss & Schlattl 2008; Dotter et al. 2008; Townsend & Teitler 2013; Paxton et al. 2018 and others). When conducted on a range of timescales, their calculations can be exploited to great effect.

In red supergiants, the $\kappa$-mechanism drives radial pulsations in the hydrogen ionization zone, and simulations show the emergence of periods and growth rates of the dominant fundamental pulsation mode—typically on the order of years—both in linear and non-linear models, as shown in e.g. Li & Gong (1994), Heger et al. (1997), Yoon & Cantiello (2010), and Paxton et al. (2013). In addition to these, previous modeling work on $\alpha$ Ori and similar red supergiants (RSGs) includes Dolan et al. (2016), Wheeler et al. (2017), Nance et al. (2018), and Goldberg et al. (2020).

In both Yoon & Cantiello (2010) and Paxton et al. (2013), models of rotating and non-rotating RSGs with approximately solar metallicity and initial masses of 25$M_\odot$ were found to exhibit pulsations on the order 1–8 years. Obtaining frequencies of this magnitude required lowering the evolutionary timestep to a fraction of a year during helium burning. The limiting factor on these calculations was the emergence of supersonic radial velocities in the envelope (see Section 6.6 in Paxton et al. 2013 for more details on their example).

A rigorous estimation of the model-derived fundamental parameters of $\alpha$ Ori was undertaken by Dolan et al. (2016). In particular, their models find a best estimate of $20\pm5M_\odot$ for the progenitor mass. They also attempt to model the pulsation properties of $\alpha$ Ori, but find they were unable to reproduce the fundamental and first overtone frequencies with adiabatic models alone. They suggest that interplay between non-adiabatic pulsations and convection could be responsible for some variability, noting that 3D simulations of similar red supergiants show the development of large-scale granular convection that can itself drive pulsation (Xiong et al. 1998; Jacobs et al. 1998; Freytag et al. 2002; Chiavassa et al. 2011; Freytag & Chiavassa 2013; Dolan et al. 2016).

Recent 1D modeling efforts in “The Betelgeuse Project” series and other works suggest that a past merger may be required to explain the present-day surface rotation of $\alpha$ Ori, which is more rapid than standard stellar evolutionary calculations including rotation can reproduce (Wheeler et al. 2017; Nance et al. 2018; Chatzopoulos et al. 2020). The Nance et al. (2018) study also examines the star seismically, but the authors are primarily focused on rapid waves in the convection zone that might precede a cataclysmic event. This concept was also addressed in depth by Goldberg et al. (2020), who modeled the observable features of supernova events as a function of the point of onset during the stellar pulsation.

In this paper, we use a range of tools to investigate the variability of $\alpha$ Orionis. We use the Modules for Experiments in Stellar Astrophysics (MESA, Paxton et al. 2011, 2013, 2015, 2018, 2019) stellar evolution software suite to generate both classical evolutionary tracks and short timescale, hydrodynamic simulations of stars. We likewise use the GYRE pulsation program to construct complementary predictions of the pressure mode (p-mode) oscillations in models of red giants (Townsend & Teitler 2013).

This paper proceeds as follows: In Section 2, we discuss the current knowledge of $\alpha$ Ori’s classical constraints, including pulsation periods, evolutionary stage, radius, temperature, and distance. We present a lightcurve highlighting $\alpha$ Ori’s recent behavior, constructed from data collected from the American Association of Variable Star Observers (AAVSO) and newly processed space-based photometry from the Solar Mass Ejection Imager instrument. In Sections 3, 4, and 5, we discuss our evolutionary, seismic, and hydrodynamic models, respectively. Section 6 concludes our analysis and presents best estimates of its fundamental parameters based on detailed photometric analysis and comprehensive, multi-timescale simulation.

2. OBSERVATIONAL CONSTRAINTS

$\alpha$ Ori is well studied interferometrically; together with R Dor and IRC 10216, it is among the stars with the largest angular diameters ever measured (Bedding et al. 1997; Menten et al. 2012; Stewart et al. 2016). In their Table 3, Dolan et al. (2016) provide a clear summary of previous measurements.

The earliest interferometric measurement from Michelson & Pease (1921) resulted in an angular di-
Table 1. Processed SMEI photometry of α Ori. Observations were corrected for systematics and averaged into 1-day bins. Errors calculated as simple shot noise. V mag is the same light curve, scaled to existing V-band data. The full data set is available in the online journal.

| BJD–V | SMEI | mag | error | mag | error |
|-------|------|------|-------|------|-------|
| 52677.959995 | 0.3759 | 0.0037 | 0.5168 | 0.0037 |
| 52678.983194 | 0.3849 | 0.0039 | 0.5330 | 0.0039 |
| 52680.041678 | 0.3717 | 0.0043 | 0.5094 | 0.0043 |
| 52680.959028 | 0.3801 | 0.0039 | 0.5244 | 0.0039 |
| 52681.911649 | 0.3869 | 0.0035 | 0.5365 | 0.0035 |
| 52682.934838 | 0.3908 | 0.0036 | 0.5436 | 0.0036 |
| 52683.887465 | 0.3991 | 0.0035 | 0.5586 | 0.0035 |
| 52684.875377 | 0.4032 | 0.0035 | 0.5662 | 0.0035 |
| 52685.863260 | 0.4030 | 0.0035 | 0.5657 | 0.0035 |
| 52686.851619 | 0.4068 | 0.0035 | 0.5727 | 0.0035 |
| 52687.415625 | 0.4177 | 0.0093 | 0.5929 | 0.0094 |
| 52689.038675 | 0.4142 | 0.0042 | 0.5863 | 0.0042 |

Radius estimates are further complicated by uncertain parallax measurements, which are made difficult by variability and known >2 au-scale asymmetries on the surface of the star both at optical and radio wavelengths (Young et al. 2000; Kervella et al. 2018). The revised Hipparcos astrometric solution gave an optical-only distance of 153±22 pc (van Leeuwen 2007). Combination of the Hipparcos data with radio observations captured by the Very Large Array (VLA) extended that distance out to 197 ± 45 pc, which was also used by Dolan et al. (2016).

The revised Hipparcos-only value is inconsistent at the 1.7σ level with the most recent radio measurement of 222±34 pc (Harper et al. 2017), which took into account both VLA and Atacama Large Millimeter Array (ALMA) observations but which was also significantly affected by “cosmic noise”.1 The star is well beyond the established brightness limit of Gaia, and data enabling a future parallax measurement were not collected in the first years of the mission. A parallax estimate of Betelgeuse is therefore not included in Gaia Data Release 2 (Gaia Collaboration et al. 2016; Sahlmann et al. 2018). Given the very long time-baselines needed to overcome the effects of photospheric motions and variability, there is unlikely to be a reliable direct parallax measurement of Betelgeuse with < 10% uncertainty in the near term.

Estimates of Betelgeuse’s mass are derived from models and range from roughly 15–25 M⊙, with previous modeling work suggesting that α Ori is in the midst of its core helium-burning giant branch phase (Neilson et al. 2011; Dolan et al. 2016; Wheeler et al. 2017; Nance et al. 2018). However, while Dolan et al. (2016) state that its mass loss rate—the primary piece of evidence supporting the claim that it is on the red supergiant branch (RSB)—is “consistent with having recently begun core helium burning,” they also note that a previous interaction of Betelgeuse with a binary companion could account for similar mass loss rates without necessitating that Betelgeuse currently exist on the RSB. Since nearly half of ~20 M⊙ stars have a companion close enough to induce mass loss, this is, in fact, ambiguous (de Mink et al. 2014). It is demonstrated by Wheeler et al. (2017) that rotating models of α Ori do not produce reasonable evolutionary predictions (a finding consistent with our

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1 “Cosmic noise” is an umbrella term used to describe the elevated dispersion of the residuals of the astrometric solution compared to the formal errors. It can include various physical effects such as source size, unresolved companions, unresolved properties of stars in the stellar models used for fitting, variability of the stellar parameters, and instrumental effects such as excess noise due to saturation.
As it is impossible to measure either mass or evolutionary status directly, and the evidence regarding its phase is not definitive, we do not assume a particular evolutionary phase a priori in our models. Instead, we consider the relative probabilities that \( \alpha \) Ori is in a particular evolutionary stage based on (1) the masses of tracks that match the other observational constraints and (2) the duration of the possible evolutionary stages.

The first-order, theoretical constraints on its mass and age are provided by the linear pulsation calculations, which rule out any model in an evolutionary stage earlier than the RSB. From an observational perspective, we note that Betelgeuse is far in the foreground of the known \(<10\) Myr age young associations in Orion (Großschedl et al. 2018), and it is not known to have kinematics consistent with ejection. In particular, its radial velocity of \(+21.9\pm0.5\,\text{km s}^{-1}\) is consistent with the \(\sim+23\) km s\(^{-1}\) of typical high mass stars in the Orion OB1 association (Morrell & Levato 1991; Famaey et al. 2005), but would differ by \(\sim20\) km s\(^{-1}\) if it had travelled 200 pc in \(\sim20\) Myr. The \((U,V,W)\) space motion of Betelgeuse is \((-22,-10,12)\,\text{km s}^{-1}\) with respect to the Sun, which is \((-11,2,19)\,\text{km s}^{-1}\) with respect to the local standard of rest (Famaey et al. 2005; Schönrich et al. 2010). The high \(W\) velocity in particular is of note, as it is discrepant at 3 \(\sigma\) from the kinematics of the young disk (Robin et al. 2003). If this high \(W\) velocity were due to ejection from a young association lying on the Galactic disk, now falling back through the disk due to vertical epicyclic motion, this would imply an origin of \(\sim50\) Myr ago. With these proper motion estimates in mind, we are left with a few possible scenarios of varying likelihood: (1) Betelgeuse was formed very recently in a region where there is no star formation; (2) it is \(\gtrsim50\) Myr old, or (3) it underwent some kind of binary interaction that propelled its trajectory. Scenario (1) is not reasonable, and scenario (2) would be consistent with a mass below \(10\,M_\odot\)—a possibility that is readily ruled out by our other constraints. We are thus left with the third scenario, which is likewise supported by observations of Betelgeuse’s present-day surface rotation and the inability of 1D, rotating models to reproduce it (see subsequent discussion).

We construct an age-prior function that performs a Monte Carlo interpolation over a grid of stellar tracks with masses ranging from \(16–26\,M_\odot\) (other parameters fixed; \(\alpha_{\text{MLT}} = 2.1\)) and a power law IMF. For two sets of realizations, we adopt a minimum age constraint of \(8\) Myr and permit radii between 600 and \(900R_\odot\). In the first statistical run, masses are heavily skewed towards the head of the distribution, peaking at \(16M_\odot\), and the bulk of the trials fall from \(16–18M_\odot\). This indicates that the lower-mass regime is strongly statistically preferred, which is consistent with our understanding of the prevalence of high-mass stars in general. In the second statistical run, the distribution peaks a bit higher, at \(18M_\odot\), and tapers off rapidly beyond \(17.5\) and \(19\) in either direction. The number of trials that do not fall somewhere on the core helium-burning RSG is negligible regardless of mass, though this is even more strongly the case for trials with masses between 17 and \(19M_\odot\).

As we will conduct estimates of the stellar mass, and many other parameters, in several ways throughout this analysis, we treat the above statistical experiment strictly as sufficient evidence to assume that Betelgeuse is core helium burning in subsequent modeling.

Recent spectral analysis of Betelgeuse presents an effective temperature of \(3600 \pm 25\) K (e.g., Levesque & Massey 2020; Guinan & Wasatonic 2020). As noted by Levesque & Massey (2020), the difference between

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**Table 2.** Observational best values, estimated ranges, or model-derived constraints (where indicated) for \(\alpha\) Ori. The temperature constraints reflect the spectroscopically derived temperature from \(\alpha\) Ori at its brightness minimum, which is not necessarily reflective of its mean temperature. However, even Levesque & Massey (2020)’s 100 K error bars accounting for decadal variations are more restrictive than the theoretical uncertainty imposed by modeled variations in \(\alpha_{\text{MLT}}\). Though we quote a “best” radius and reference a wide range of values, in practice we do not impose any constraints on the radius when modeling. The range of possible radii derived from the models is smaller than the uncertainties reported by many observers.

| Property                      | Value            | Source                          | Comment                                      |
|-------------------------------|------------------|---------------------------------|----------------------------------------------|
| \(T_{\text{eff}}\)           | \(3600 \pm 25\) K | Levesque & Massey (2020)        | range extended by \(\sigma_{\text{theory}}\) to \(\pm200\) K |
| Angular Diameter              | \(42.28\pm0.43\) | Montargès et al. (2014)        | Limb-darkened                                |
| Radius upper limit            | \(\sim1100R_\odot\) | Dolan et al. (2016)            | data collated from many sources              |
| Radius lower limit            | \(500R_\odot\)   | Dolan et al. (2016)            | data collated from many sources              |
| Distance                      | \(197 \pm 45\) pc| Harper et al. (2008)           | parallax data adopted by Dolan et al. (2016) |
| Period of variability         | \(388 \pm 30\) days | Kiss et al. (2006)             | dominant, higher frequency; likely FM        |
| Period of variability         | \(2050 \pm 460\) days | Kiss et al. (2006)           | lower frequency; likely LSP                  |
| Mass                          | \(10–26M_\odot\) | model-derived; various         | range considered in this work                |
the spectroscopically-derived temperature measured in 2004–5 and that measured during its brightness minimum in 2020 is at most a decrease of 100K, and at minimum, negligible. This alone essentially rules out convective turnover as an explanation for its recent dimming, but surface temperature is less informative on other oscillation driving mechanisms (Levesque & Massey 2020).

Critically, the brightness of Betelgeuse varies in a systematic way on at least two different timescales, and these periodicitites were measured with good precision by Kiss et al. (2006) (and later corroborated by Chatys et al. 2019). The shorter occurs with a period of \( \sim 388 \) days and the longer with a period of \( \sim 5.6 \) years (2050 d). The period–luminosity relation depicted in Figure 6 of Kiss et al. (2006) provides some evidence that the 388 d brightness variation is caused by \( p \)-mode pulsation in the fundamental mode (FM). This is likewise supported by various observational and theoretical considerations, including the position of the star on the log \( P–M_{K} \) diagram, where the absolute \( K \) brightness provides the observational proxy for the stellar luminosity. T Kiss et al. (2006) also found that the shorter periods fit the theoretical calculations of Guo & Li (2002), forming an extension to sequences B and C of the supergiant variables observed in the Magellanic Clouds that correspond to the FM and the first overtone (O1) pulsation modes, respectively (Wood et al. 1999; Kiss & Bedding 2003; Soszynski et al. 2007). This also suggests that these variations correspond to \( p \)-mode pulsation.

The longer, \( \sim 2050 \) d periodicity likely falls in a class of signal known as “Long Secondary Periods,” or LSPs. These have been observed in multiple semiregular and red supergiant variables, but the cause of the LSP is still debated (Wood 2000; Chatys et al. 2019). Proposed mechanisms include rotational modulation caused by spots or a nearby companion followed by a dust cloud, among other possibilities (Wood 2000; Soszyński & Udalski 2014). Such signals were observed in the LMC supergiant population as “sequence D,” and the long periods found by Kiss et al. (2006) extend that sequence to higher luminosities (Derekas et al. 2006). Among other things, rotational modulation was proposed as a possible mechanism for the LSP (Percy & Deibert 2016). However, the rotational period of \( \alpha \) Ori has recently been estimated at \( P_{\text{rot}} = 31 \pm 8 \) yr, which is considerably longer than the LSP of the star (Kervella et al. 2018). Models in this work shed more light on the questions of mode classification and driving mechanism.

2.1. Photometric Observations

Both the 388 day and 5.6 yr (2050 d) periods are visible in Figure 1, which shows the longitudinal brightness variations of \( \alpha \) Ori over the last 90 years. These visual brightness estimates were collected in large part by amateur observers and archived by the American Association of Variable Star Observers (AAVSO).

Examining Figure 1 more closely, we see that the amplitude of the brightness drops corresponding to the 388 day pulsation period are about 0.3–0.5 mag in the \( V \) band. The difference between these and the 1 mag drop in 2019–20 is clear. We do note, however, that Betelgeuse has undergone other periods of drastic dimming a few times over the last 100 years. Dimming events of comparable magnitude are visible in Figure 1, for instance, in the mid to late 1980s and arguably in the early 1950s. An argument could be made for the existence of a 35–40 year dimming cycle, particularly if we take into account that the sensitivity of instruments has improved considerably in the last few decades. We note that this 3–4 decade variation is of the same order as the suggested rotational period. While this could potentially be a manifestation of rotational modulation, confirmation will require ongoing observation.

The low amplitude and scarcity of adequate comparison stars make visual estimates less sensitive to smaller changes from one pulsation cycle to another. Digital photometric observations exist for the last three decades, but both the quality and quantity varied over time. Most of the publicly available data have been archived by the AAVSO and provide good coverage from the mid-1980s to the early 2000s and from 2010 onward. To fill in the gap, we supplement the AAVSO data set with the observations taken with the SMEI (Solar Mass Ejection Imager) instrument aboard the Coriolis satellite (Jackson et al. 2004).

2.1.1. SMEI photometry of Betelgeuse

SMEI was designed to follow Coronal Mass Ejections (CMEs) from the Sun, and in order to do this, stellar signals must be removed from its images. About 6000 stellar sources plus the brightest Solar System objects were catalogued and then subtracted from the images. It was soon realized, however, that the source subtraction procedure used by the mission can be processed into time series photometry of the brightest stars in the sky, essentially turning SMEI into one of the early space photometry missions (Buffington et al. 2007; Hick et al. 2007). SMEI observed \( \alpha \) Ori from early 2003 to late 2011 with a cadence of 104 mins. Each year, data collection was split between the three cameras whose outputs needed to be rectified. Yearly systematics arise from the changing thermal conditions in each of the cameras (Tarrant et al. 2008). Slow degradation of the camera sensitivity is also apparent in the data.
Figure 1. Lightcurve of $\alpha$ Ori assembled from publicly available data compiled by the AAVSO, from 1928 to present, and from the SMEI observations. Horizontal axes are marked in both years (top) and JD + 2400000 (bottom). Grey points are visual estimates, blue are $V$-band photometry, from AAVSO. Red points are the SMEI data.

We could not remove the annually repeating instrumental signals directly, as the timescale is on the same order as the variation of $\alpha$ Ori. Therefore, we relied on the ensemble photometry of neighboring stars to derive common instrumental characteristics. We inspected ten nearby bright stars and selected $\gamma$ Ori, $\varepsilon$ Ori and 32 Ori to generate a template for the instrumental signals. We calculated a smoothed systematics curve by calculating the medians of the combined relative intensity data of these three stars in 4-day windows placed around every time stamp of $\alpha$ Ori, and for each camera separately.

The rectified SMEI light curve of $\alpha$ Ori is the result of scaling the raw data with the systematics curve and then transforming it to magnitudes using $m_{\text{SMEI}} = 10.0$ mag as the magnitude zero point. However, the passband of SMEI is not the $V$ band, therefore requiring that we scale and shift the light curve to match the AAVSO data. To compute the appropriate scaling, we determined the brightness difference for six other stars with M1-2 spectral class in the SMEI catalog to be $m_V - m_{\text{SMEI}} \approx 0.15$ mag. We found that we needed to stretch the amplitude by a factor of 1.8 to match the $V$ data points. We then averaged the raw photometry points into 1-day bins. While the shape of the variation could also be passband-dependent to some extent, the scarcity of overlapping $V$ data prohibited us from performing a more detailed comparison. The final light curve is plotted in Fig. 2, along with the AAVSO $V$-band data.

A sample table of the processed and binned SMEI photometry and the scaled $V$-band values can be found in Table 1. Here, we provide formal errors calculated as the shot noise from the number of electron counts.

The photometric light curve reveals a richer set of features than the visual light curve. The SMEI observations, in particular, show both the slow variation from the LSP along with additional smaller, more rapid variations. The SMEI data also put the severity of the recent dimming event in perspective: the brightness of the star did not drop below 1.1 mag in the $V$ band during the last 40 years, whereas the dip commencing in November of 2019 dimmed the star to 1.6 mag in that band. The light curve also highlights some smaller variations on the order of a few hundredths of a magnitude on timescales of days to weeks. Similar variations are present in the SMEI light curves of other nearby stars as well, so we do not consider these to be an intrinsic feature of $\alpha$ Ori.

2.1.2. Frequency analysis of observations
We analyzed the frequency spectrum of the photometric light curve with Period04 (Lenz & Breger 2005). We are able to identify the LSP and pulsation frequency regions easily, as shown in Fig. 3, but the identification of individual frequency components intrinsic to the star was hindered by the presence of yearly aliases. Most notably, the $-f_{\text{LSP}} + 1/\text{yr}$ component coincides with the pulsation frequency region. As the FM pulsation mode itself is only slightly longer than one year, its harmonics and/or overtones could coincide with yearly aliases.

We first apply a de-whitening procedure to the data with LSP components. Figure 3 shows that the LSP is not strictly cyclic and that $\alpha$ Ori hovered in a bright state throughout the 2010s. We test combinations of multiple harmonics and subharmonics of the main $f_{\text{LSP}} = 0.000423 \text{d}^{-1}$ frequency ($P_{\text{LSP}} = 2365 \pm 10 \text{ d}$). We use the 0.5 and 2.5 $f_{\text{LSP}}$ components for the final fit, which successfully reproduces the deep LSP minima in 1985/1989 and in 2001/2006–7. Non-sinusoidal features in LSP light curves are common for smaller red giants in the Magellanic Clouds: one half of the LSP cycle shows an eclipse-like dip, and the other half resembles a plateau. The model proposed by Soszyński & Udalski (2014) to reproduce this shape assumes a nearby orbiting companion and associated dense cloud. Currently, there is no indication of a companion orbiting $\alpha$ Ori, but observations suggest that the recent dimming was likely caused by excess dust (Levesque & Massey 2020).

We detect two significant frequency components ($f_{\text{puls1}} = 0.002469$, $f_{\text{puls2}} = 0.002213 \text{d}^{-1}$) at the pulsation frequency peak, in agreement with the expected short lifetime of the mode. We likewise detect the first...
harmonic \((2f_{\text{puls}})\) of the stronger pulsation component. After pre-whitening the data using these, one significant peak remained at \(f_2 = 0.005392 \pm 0.000002 \text{d}^{-1}\) \((P_2 = 185.5 \pm 0.1 \text{d})^2\). Neither this component nor the harmonic was described by Kiss et al. (2006), nor is it present in the power spectrum of the complete visual light curve. However, \(f_2\) can be identified in some segments. This could suggest the presence of the first overtone with a period ratio of 0.41–0.46 (using the two components identified in \(\alpha\) Ori). However, it is also possible that \(f_2\) corresponds to a yearly alias or a harmonic of the non-coherent pulsation signal that the photometric data does not resolve properly. It would be informative to collect photometric observations of \(\alpha\) Ori throughout the year for as long as possible in order to minimize the gaps in the data and diminish such aliasing in the frequency domain.

Since the pulsation signal is non-coherent, we fit it with a Lorentzian profile as in Kiss et al. (2006), but in combination with a \(1/f\) component to account for the red noise component of the convective motions (bottom panel of Fig. 3). We calculate a pulsation frequency of \(f_{\text{puls}} = 0.00240 \pm 0.00014 \text{d}^{-1}\) from the peak of the profile, corresponding to a period of \(P_{\text{puls}} = 416 \pm 24\text{d}\). We can also use the full width at half maximum \((\Gamma)\) of the profile to estimate a mode lifetime of \(\tau = 1/\pi \Gamma = 1174\text{d}\), or \(\approx 3\) pulsation cycles. The mode lifetime matches the value calculated by Kiss et al. (2006), but the period is longer, though still within their uncertainty range. We note that Dupree et al. (1987) determined a similar, 420 d period, but this was based on only three years of photometric observations.

Apparent changes to the period likely arise from (1) the non-coherent nature and short lifetime of the mode and (2) interference with photometric variations caused by convective motions and the evolution of hot spots. Although the visual data are less accurate than the photometry, the visual data average the pulsation over a considerably longer time. Differences among apparent periods calculated from shorter and longer data sets have been found for other RSGs as well (Chaty et al. 2019). Presently, we report a new period for the photometric data covering only the last three decades; disentangling the temporal evolution of the pulsation is beyond the scope of this work. We therefore prefer to use the \(388 \pm 30\) d period determined by Kiss et al. (2006) as reference in this work.

2 Uncertainties for \(f_2\) were calculated with the assumption of a single coherent Fourier component: more data will be needed to assess the validity of this assumption.

2.1.3. Timing of minima

A standard means of identifying deviations from an assumed periodic signal is the O–C method.\(^3\) Here we attempted to identify and time the various larger and smaller minima in the light curve. The light curve data appear to alternate between two states: one defined by deep minima exceeding 0.5 mag (e.g., at JD 49800, 52750, 54000, 54400, 58500 and the dip itself at 58800), the other by shallower and more frequent meandering (e.g., around JD 51500, 53200, 55000 to 57000). However, the annual gaps make it difficult to identify enough minima accurately, and it is thus possible that we simply miss one type during certain intervals. Time spans between consecutive shallow minima can be as short as 60–100 days—much shorter than the FM pulsation period. We see no indication of discrete frequency components corresponding to these periods in the power spectrum of the star, which suggests they are not high-degree pulsation modes. The timescales and low amplitudes, however, do match the convective turnover times of giant convection cells: our photometric results agree with predictions of timescales from 3D radiative hydrodynamic models and the time-resolved results of spectropolarimetric observations of the surface of the star (Freytag et al. 2002; López Ariste et al. 2018).

The critical observational features of Betelgeuse are summarized in Table 2.

3. CLASSICAL EVOLUTIONARY MODELS

Having carefully collated the set of observational criteria described in Table 2, we proceed in modeling the system. Our numerical efforts include three types of simulation: (1) classical evolutionary tracks; (2) linear pulsation models; and (3) short-timescale, 1D, implicit hydrodynamical evolution. We discuss results from each in this order.

We compute evolutionary tracks for stellar models with initial masses of \(10–26\ M_{\odot}\). Calculations are carried out from the pre-main sequence to the termination of the helium-burning giant branch, with the terminating condition set by the amount of helium remaining in the core of the star \((M(\text{He}) \sim 10^{-8} \ M_{\odot})\). Models in an evolutionary phase more advanced than core helium burning are less favored probabilistically, as the star will spend considerably less time in such phases. As shown above, they are also unlikely to be consistent with the existing array of observational constraints, especially since these constraints prove to be discriminating even within

\(^3\) O–C refers to the observed minus calculated method, where we measure the time differences between observed events (e.g., cycle minima or maxima) and a periodic ephemeris.
Figure 4.
(TOP) A set of classical evolutionary tracks for 10–25 $M_\odot$ computed with MESA. Initial mass per track as indicated. All models shown are computed until the end of helium burning and shown from ZAMS. All tracks in this panel adopt a mixing length of $\alpha_{\text{MLT}} = 2.1$.
(Bottom) Same as above, but with all tracks adopting $\alpha_{\text{MLT}} = 1.8$, to demonstrate the impact of this assumption on predicted temperature.
In all panels, the temperature constraints from Levesque & Massey (2020) are shown as pink vertical bars.

the set of strictly core helium-burning models. As such, we do not consider post-core helium-burning models in further detail.

Figure 4 shows a set of evolutionary tracks evolved from the zero-age main sequence (ZAMS) to the end of core helium burning. Masses indicated refer to the initial mass. In subsequent discussion, we refer to models by their initial masses, though typically the mass of the star will be between $2$–$3$ $M_\odot$ smaller at the termination of its evolution (and onset of its hydrodynamic evolution) due to mass loss during its prior stages.

Our initial grid of models does not invoke rotation and has fixed, solar metallicity represented by a heavy metal fraction of $Z_{\text{in}} = 0.02$. We consider multiple values of the convective mixing length $\alpha_{\text{MLT}}$, ranging from 1.8 to 2.5. As massive stars are quite sensitive to the prescriptions used for convective boundaries and convective overshoot, we adopt convective overshoot settings of $f_{\text{ovs}} = 0.010H_p$\(^4\) surrounding hydrogen- and helium-burning zones (Herwig 2000; Paxton et al. 2018). We account for mass loss in the evolutionary calculations via MESA’s implementation of the “Dutch” wind schemes, a composite of prescriptions summarized in Reimers (1975); de Jager et al. (1988); Bloecher (1995) and van Loon et al. (2005). We model the low-temperature mass loss via the prescription of de Jager et al. (1988), adopting a wind coefficient of $\eta = 0.8$ as default.

We test a range of $\eta$ values and find that while the choice of $\eta$ does impact the terminal mass of the evolutionary model, our results are predominantly sensitive to the radius. The relationship between an evolutionary model’s terminal radius and its input controls—mass, metallicity, mixing length, convective overshoot, mass loss coefficient, etc.—is complex, and we do not gain much insight on this interplay by varying $\eta$. We do not use mass loss or rotation during the hydrodynamic evolution itself, as the impact of these processes on a timescale of several decades is negligible.

A critical component of our classical modeling objective involves reproducing the recently observed temperature of $\alpha$ Ori. However, given limited a priori information on the star’s mass and evolutionary phase, there is a strong degeneracy between choice of $\alpha_{\text{MLT}}$ and predicted temperature. While this issue emerges even for well-constrained systems (Joyce & Chaboyer 2018a,b), the magnitude of the degeneracy is exacerbated as observational constraints loosen and structural complexity of the stellar model increases. To take this into account, a looser interpretation of the temperature constraints can be simulated by extending the observational uncertainties by some approximate theoretical uncertainty. This is done by measuring the shift in temperature a track of given mass undergoes when its mixing length is adjusted to extremal values. In the case of our grid, this shift is calculated for $\alpha_{\text{MLT}} = 1.8$ vs $\alpha_{\text{MLT}} = 2.5$.

A change in mixing length of 1.8 to 2.5 corresponds to a shift in modeled temperature of roughly 0.1 dex in the relevant part of the HR diagram for a track with mid-dling mass 17 $M_\odot$. What this means is that, although the atmospheric models used by Levesque & Massey (2020) can determine the temperature corresponding to the observed line profile with high accuracy, the underlying evolutionary models themselves may shift by about ±200 K. This introduces, at minimum, the same uncertainty on the evolutionary stage at which the star crosses the observed temperature. Variations in $\alpha_{\text{MLT}}$ can thus

\(^4\)Multiples of the pressure scale height, $d \ln P/d \ln T$.\)
be accounted for by using wider temperature range for a set of models with fixed mixing length, as indicated by the blue strip in Figure 4. The effective temperature constraints of Levesque & Massey (2020) alone are shown in the much more restrictive pink band. The observational constraints on luminosity are not strong and do not themselves rule out any of the models shown in Figure 4.

Attempts to reproduce Betelgeuse’s present-day rotation of $\sim 5$ km s$^{-1}$ ($v \sin i = 5.47 \pm 0.25$ km s$^{-1}$, Uitenbroek et al. 1998; Kervella et al. 2018) with single-star evolutionary models are unsuccessful. To this end, we compute tracks that use an initial surface rotation of up to $\Omega = 0.65 \Omega_{\text{crit}}$, or roughly 200 km/s on the ZAMS, in accordance with Wheeler et al. (2017). In cases where the models do not fail outright, the results are not consistent with even the most generous interpretation of the observational constraints. Among tracks that converge, even those with the highest values for $\Omega$, still fail to predict a present-day rotation rate in the vicinity of the observed value.

In particular, tracks with initial rotations approaching breakup velocity ($\Omega/\Omega_{\text{crit}} \sim 0.7$) fail to intersect the (extended) effective temperature regime with large enough present-day surface rotations. The highest values attained by our grid only just reach 1 km/s, and these correspond to models with initial masses as low as 6–10 $M_\odot$. Such low-mass models are easily ruled out by other constraints, especially period.

Our results from this exercise are thus similar to those of Wheeler et al. (2017), who find that “models at the tip of the RSB typically rotate at only $\sim 0.1$ km/s, independent of any reasonable choice of initial rotation.” Though Wheeler et al. (2017) are able to create rotating models consistent with $3\sigma$ uncertainties on their observational constraints at the time, our constraints prioritize the fundamental mode frequency and include a much tighter range on effective temperature. More sophisticated modeling of the rotational aspects of $\alpha$ Ori’s evolution are beyond the scope of this paper.

The terminal models from the evolutionary run provide both the structural input for calculations with the linear pulsation program (next section) and the initial conditions for the hydrodynamic study (Section 5).

4. SEISMIC MODELS

Used in conjunction with classical parameters, synthetic frequencies are an extremely powerful tool for discriminating among possible models of a star. The case of $\alpha$ Ori is no exception.

4.1. Linear perturbations

![Figure 4](image-url)  
Figure 4. Adiabatic $p$-modes are calculated with GYRE for all relevant evolutionary tracks. Periods, in days, of all models consistent with the observed temperature constraints are shown, coded by color for mass and by marker style for mixing length, as indicated. (TOP) Masses range from 10–24 $M_\odot$ at a resolution of 1.0$M_\odot$ and mixing lengths range from 1.8 to 2.5 at a resolution of 0.1. (BOTTOM) Masses range from 10–24 $M_\odot$ at a resolution of 1.0, $M_\odot$ and $\alpha_{\text{MLT}}$ is fixed at 2.1. Here, the observed temperature constraints adjusted to account for the theoretical uncertainty in $\alpha_{\text{MLT}}$. All models shown adopt $\eta = 0.8$ and $Z = 0.02$. The observed seismic constraints from Kiss et al. (2006) are indicated with blue horizontal lines.

We use GYRE to solve the linearized pulsation equations for high-resolution structural models produced during the RSG (Townsend & Teitler 2013). The GYRE program is based on a “Magnus Multiple Shooting” (MMS) scheme and provides both adiabatic and non-adiabatic calculations. We consider only adiabatic results in this analysis. Figures 5 and 6 show results from these calculations.

As a track that intersects the observational requirements will typically do so at multiple evolutionary timesteps, we can produce several pulsation profiles per single track. Where the models are compatible, we gen-
Uncovering $\alpha$ Orionis

To account for the theoretical uncertainty on $T_{\text{eff}}$, we consider two metrics by which an evolutionary track is “compatible” with the observations. In the upper panel of Figure 5, we show the periods of adiabatic $p$-modes versus termination age for a collection of models with a range of initial masses and mixing lengths; here, the structural models used in the seismic analysis have effective temperatures strictly within $3600 \pm 25$ (Levesque & Massey 2020).

In the lower panel of Figure 5, all models use $\alpha_{\text{MLT}} = 2.1$, but are checked for consistency against the extended, theoretical temperature constraints described in Figure 4. In both panels, all masses refer to the initial mass of the model and the $388 \pm 30$ day periodicity is denoted by a blue horizontal band.

In the upper panel of Figure 5, the period–age sequence is tighter and more well-defined, but the results between the upper and lower panels are largely consistent. We note that sub-sequences comprising stars of particular mass are more apparent in the fixed $\alpha_{\text{MLT}}$ case. This visualization more clearly suggests that, at least for masses in the 15–20 $M_\odot$ range, there will necessarily be some point along the helium-burning branch during which the star will pass through the appropriate frequency band. However, the temporal window during which this occurs is quite small in the context of evolutionary timescales—on the order of 0.5–1.0 Myr. The requirement that this time frame align with a particular observed temperature ends up being quite restrictive.

Collectively, these results suggest a model-derived mass of 16–21 $M_\odot$, at a resolution of 1 $M_\odot$. This is broadly consistent with other modellers’ results, though our results are more accommodating at the lower-mass end.

Figure 6 shows other fundamental parameters as a function of period. The FM and its uncertainties are defined by green, vertical bars in all panels.

Models in Graph 1 span the full range of masses and mixing lengths considered in our grid and additionally vary in the prescribed mass loss coefficient ($\eta = 0.2–1.0$), but they are not pre-restricted by agreement with temperature constraints. Instead, these evolutionary tracks are terminated at arbitrary intervals along the helium-burning branch, with spacing set by the degree of helium exhaustion in the core. This is done to produce a more well-populated sequence that incorporates addi-

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5 The lowest radial order for pressure modes, as defined by GYRE
tional sources of uncertainty in the modeling assumptions. Despite this added theoretical noise, the range of possible radii across all models remains heavily restricted by the observational period constraints.

All models in Graph 2 intersect the theoretically extended temperature uncertainties (which essentially sets their termination ages) and adopt $\alpha_{\text{MLT}} = 2.1$ and $\eta = 0.8$.

In the uppermost panel of each Graph in Figure 6, we show radius as a function of period. Though there is some scatter in the synthetic data, the radial span of the period-compatible models is very narrow, especially compared to the range of radial estimates collated in Dolan et al. (2016). We recall from earlier discussion that literature estimates of Betelgeuse’s radius range from $\sim 500 R_\odot$ to nearly $1300 R_\odot$, whereas the results presented here suggest little possibility outside $700–800 R_\odot$.

If we interpret the period measurements as hard limits, our results suggest a radius for $\alpha$ Ori of $750 R_\odot$, with $1\sigma$ uncertainties of roughly $25 R_\odot$ and non-symmetric limits of $R_{\text{max}} = 812 R_\odot$ and $R_{\text{min}} = 720 R_\odot$. We thus report a $3\sigma$, model-derived radius of $750^{+62}_{-56} R_\odot$.

In the middle and lower panels of each graph, we show the models’ initial masses and terminal masses, respectively, as a function of period. In these plots, we emphasize those models that also have radii in our $3\sigma$ uncertainty bounds with larger, darker markers. Considering all possible observational constraints, we report model-derived estimates for the initial and present-day masses of Betelgeuse as approximately $18–21 M_\odot$ and $16.5–19 M_\odot$, respectively. Taking into account the likelihood of a previous merger event, which would significantly complicate any inferences about the state of Betelgeuse at birth, it is our present-day mass estimates that are most pertinent.

4.2. A possible overtone

In Sect. 2.1.2 we present evidence of a possible new frequency component, $f_2$, that corresponds to a periodicity of 185 d. While strong aliasing caused by the annual gaps in the data makes that detection somewhat uncertain, we conduct a cursory analysis to find out whether such a signal would fit an overtone mode in the GYRE results. We find that a narrow range of models indeed reproduces the observed fundamental period of $\alpha$ Ori and the $P_{O1}/P_{YM} \approx 0.46$ period ratio with the first radial overtone. The favored models correspond to an initial mass range of $16–21 M_\odot$, and all are older than 8.0 Myr. These values are consistent with the mass and age constraints we obtained via other channels. It is therefore entirely possible that the first overtone is present in the pulsation of $\alpha$ Ori, but extended and accurate photometry will be required to confirm the presence of additional modes conclusively.

4.3. Seismic parallax and luminosity

With the radius of $\alpha$ Ori heavily constrained by the seismic models, we can calculate the distance to the star based on the measured angular diameter. An angular diameter of 42.28 ± 0.43 mas and a physical radius of $750 \pm 25 R_\odot$ (1$\sigma$) correspond to a linear distance of $165 \pm 7$ pc and a parallax of $\pi = 6.06 \pm 0.27$ mas. Using the $3\sigma$ uncertainty range of the seismic radius estimate, we calculate $165^{+16}_{-8}$ pc for the distance and $\pi = 6.06^{+0.31}_{-0.52}$ mas for the parallax. Our values are in agreement with the parallaxes derived entirely or in large part from the Hipparcos measurements (see van Leeuwen 2007 and Harper et al. 2008), and place $\alpha$ Ori nearer to us. It is, however, somewhat in tension with the more recent results based on radio observations, with disagreement at the 1–2$\sigma$ level (Harper et al. 2017). Figure 7 shows our results in context.

This discrepancy could stem from various observational or theoretical shortcomings. One possibility is that the period shift is caused by large-amplitude, non-linear pulsation. Stellar structure adjusts dynamically to the changes caused by coherent pulsation, which may cause a shift in the eigenfrequencies of the structure. Therefore even if the physical parameters of a linear seismic model agree with those of the star, the calculated and observed periods may not. In the case of $p$-modes, the radius relates to the pulsation period as $R \sim P^{2/3}$ for a given mass. From this alone, we estimate that the linear period of Betelgeuse should be $500 \pm 40$ d if its radius is $887 R_\odot$, as adopted by Dolan et al. (2016). This means that a 20 to 25% non-linear decrease would be required to reproduce the observed $388 \pm 30$ d pul-

![Figure 7](image-url)
Uncovering \( \alpha \) Orionis

5. HYDRODYNAMIC ANALYSIS

The third component of our modeling relies on MESA’s implicit hydrodynamics solver, which we use to probe the non-linear oscillatory behavior of the models’ envelopes on decadal timescales.

5.1. Method

In the implicit hydrodynamical scheme implemented in MESA (version 8118—Paxton et al. 2015), the Euler equations

\[
\frac{D\rho}{Dt} = -\rho \nabla \cdot \vec{v},
\]

\[
\frac{D\epsilon}{Dt} = -\frac{P}{\rho} \nabla \cdot \vec{v} + \dot{\epsilon},
\]

\[
\frac{D\vec{v}}{Dt} = -\frac{1}{\rho} \nabla (P + q) - \frac{GM(r)}{r^2} \hat{r}
\]

are solved in the Lagrangian formalism directly. Here, \( \rho \), \( P \), \( \epsilon \) and \( v \) are the local density, pressure, specific internal energy and velocity of the fluid parcel, respectively. The enclosed mass at a radius \( r \) is given by \( M(r) \), and the gravitational constant by \( G \). The term \( \dot{\epsilon} \) includes all of the energy source terms, such as nuclear reactions, neutrino energy loss, radiative heat loss, and conductive and convective thermal heat transport. The artificial viscosity, \( q \), is necessary for shock-capturing this scheme (Richtmyer & Morton 1967), and \( D/Dt \) is the material derivative defined by \( \partial/\partial t + \vec{v} \cdot \nabla \). We remind the reader that in standard evolutionary calculations, the motion of the Lagrangian mass element appears in the equations of hydrostatic equilibrium as the acceleration term.
The artificial viscosity is defined as

\[ q = \eta_{\text{visc}} 4\pi r^4 \rho \frac{d(v/r)}{dm}, \]  

(4)

and has a unit of energy, with \(dm\) being the mass of a Lagrangian fluid element. The quantity is scaled by the free parameter \(\eta_{\text{visc}}\). We note that although the concept of physical or numerical viscosity depends on turbulence in the local and sub-grid scale, in this work we do not include any time-dependent turbulence in our model. The viscosity defined in Equation 4 is instantaneous.

To capture the shock propagation, artificial viscosity is included in MESA’s hydrodynamical scheme. Furthermore, the code uses the energy-conserving, time-discretization scheme of Grott et al. (2005), which ensures that the models at two consecutive timesteps are consistent with each other. We describe the precise configuration for our simulations in more detail in Appendix A, and provide the inlists necessary for reproduction\(^6\).

It is important to note that MESA’s implicit hydrodynamics scheme does not include the equations of stellar pulsation directly; rather, MESA has a dedicated module, RSP (Radial Stellar Pulsations; Smolec 2016), for calculating non-linear pulsating models. However, RSP is not suitable for stars with the luminosity-to-mass ratios (\(L/M\)) of giants such as \(\alpha\) Ori (Paxton et al. 2019). Previous work has shown that the pulsation modes of stars with \(L/M\) ratios similar to that of Betelgeuse have high enough growth rates to induce shocks even if the implicit solver is employed (Heger et al. 1997; Paxton et al. 2013; Smolec 2016; Yoon & Cantiello 2010; Goldberg et al. 2020).

### 5.2. Period–Radius estimates from hydrodynamic runs

While the main goal of incorporating hydrodynamic simulations into our analysis is to study a canonical model of Betelgeuse’s envelope rigorously, non-tailored hydrodynamic simulations also provide a second method of calculating short-order pulsation modes, and thus a means of independently verifying the linear calculations. From a small grid of cursory hydrodynamic runs of varying mass, we can estimate theoretical pulsation cycle lengths directly.

We first conduct an exploratory investigation of the hydrodynamic evolution for a small subset of the models in our grid, restricting to those with initial masses between 17–23 \(M_\odot\). We require that the timestep not exceed some fixed, small value—typically 5000–10,000 seconds—and compute the temporal evolution for several decades.

Figure 8 demonstrates the oscillatory behavior of two hydrodynamic models with slightly different initial masses. If not handled correctly, the hydrodynamic models will rapidly expand from their evolutionary initial conditions—in most cases, to nearly double our radial limits—before stable pulsations emerge. This is caused by a discrepancy between the luminosity of the inner boundary of the simulated stellar envelope and the actual stellar luminosity. Thus, over time, the star deposits its energy near the surface, making the star expand. This can be mitigated by applying relaxation procedures to the initial hydrostatic model (Wood et al. 2004; Nicholls et al. 2009b; Ireland et al. 2011; Saio et al. 2015).

However, it is still possible to derive the pulsation periods and average radii of these models based upon selected cycles before shocks and/or numerical failure occur, thus providing a cursory but independent validation of the pulsation periods computed with GYRE.

The modeled data are produced by estimating the instantaneous period and radius values from the hydrodynamical models using a combination of quadratic and a sine functions fit to short segments of the radiative evolution. In this way, we can extract multiple theoretical \(R, P\) estimates from one hydrodynamic model. We compare these to a set of direct and inferred observations of pulsation periods and radii of variable stars. The bulk of these data come from the collection of Szatmáry

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\(^6\) We will publicly release the inlist files on Zenodo at the conclusion of the refereeing process.
Figure 9. The upper panel shows the sequence of stellar radii against pulsation periods, extending from RR Lyrae and Cepheid stars to Miras and RSGs. The lower panel highlights the region containing observations of other variable red giants and measurements extracted from the hydrodynamic simulations, demonstrating that Betelgeuse’s 388 day, rather than 2050 day, periodicity lines up better with the modeled sequence. In both panels, gray dots correspond to observations of lower-mass pulsators. Variable red giants in particular are shown in open red circles, with Betelgeuse’s two modes represented by closed red circles. Points derived from models are colored, with the colorbar indicating their mass.

Figure 9 shows a set of synthetic periods and radii derived from the hydrodynamic grid. Also shown in Figure 9 are (1) the period–radius sequence constructed from observations of pulsators across a wide mass range (gray dots); (2) the observed FM and LSP periodicities for the small number of red giants listed above (red, open circles); and (3) the 388 and 2050 day periodicities of Betelgeuse (red, closed circles). Masses of the synthetic stars are indicated via the color bar.

It is well-known that acoustic modes scale with the average density of the star, which itself largely depends on the radius. We should therefore expect a clear correlation between radius and period, as seen both here and in the linear seismic analysis (Figure 6).

As is clear in the upper panel of Figure 9, there is a well-defined \( P, R \) sequence spanning RR Lyrae up to synthetic supergiants. The periods and radii extracted from the hydrodynamic models extend the established sequence of pulsating stars to higher radii in a systematic and continuous way, indicating that our models experience \( p \)-mode pulsation until the final layers are blown off and the numerics break down. In turn, the bottom panel hints at certain mode classifications for some of the observations. The periods for a number of stars fall cleanly on the model sequence, and some fall above: the latter could suggest either pulsations in an overtone or that their radii have been overestimated. Given the complicated circumstellar environment surrounding many supergiants, and our own findings on the radius of \( \alpha \) Ori, the latter is a plausible explanation. Finally, the LSP signals are clearly separate from the model sequence, confirming once again that the 2050 day periodicity is not driven by acoustic variations.

Even before more careful modeling of the hydrodynamic evolution, it is evident that pulsation periods emerging naturally in the simulations are of the same order as Betelgeuse’s 388 day periodicity. The linear and non-linear seismic analysis both demonstrate that this is \( \alpha \) Ori’s fundamental mode, a fact which, when combined with other classical observations, places particularly strong constraints on the star’s radius.

5.3. Possibility of self-excitation due to non-linear effects

As stars evolve across the HRD, they may undergo mode transitions when a new pulsation mode becomes unstable. At this point, the star can switch to the new mode—a phenomenon observed directly in RR Lyrae stars—or transition to a multimode pulsator.

Mode growth rates have various definitions. In the linear framework, they usually represent the natural timescale of changes in the pulsation energy of the star...
(Catelan & Smith 2015). Growth rates are sometimes calculated directly from the change in amplitude between successive cycles, but one must keep in mind that in non-linear calculations, amplitudes do not grow indefinitely. Hence, non-linear growth rates only agree with the linear values initially, eventually fading back to zero (Yoon & Cantiello 2010). Normalized growth rates are thus scaled with the pulsation frequencies. In the case of, e.g., OGLE–BLG–RRLYR–12245, this mode transition lasted for hundreds of cycles, as is consistent with the small growth rates of the modes (Soszyński et al. 2014). But, in contrast to classical pulsators, semiregular stars have very large growth rates that can lead to strong mode interactions, some of which may even become chaotic (Buchler et al. 1996). As such, it is theoretically possible that Betelgeuse has recently experienced a rapid mode transition, or a rapid increase in amplitude of an overtone mode already present, and that the superposition of the resulting modes created the unusually low brightness minimum seen in November of 2019. It is thus worth investigating whether such a situation can be simulated; however, modeling multimode pulsation in the non-linear regime is notoriously difficult; at present, this is only reliably reproducible for stars with much lower $L/M$ ratios (Kolláth et al. 2002; Smolec 2016). It is thus beyond the scope of the current paper to investigate such a situation, though this scenario is one we hope to address in a subsequent investigation.

5.4. Analysis of Canonical Hydrodynamic Model

We consider the evolution of a canonical hydrodynamic model whose initial and terminal evolutionary conditions are consistent (as best as possible) with the parameters reported in Section 4. We construct a star with initial mass 21 $M_\odot$ and terminal mass of 19.54 $M_\odot$ during core He-burning. In order to force the stabilized radius to be consistent with our reported values, we must inflate the mixing length parameter to $\alpha_{\text{MLT}} = 3.0$. However, we still require that our hydrodynamic model intersect the theoretical temperature uncertainties described above, 3600 ± 200 K, during its oscillations.

Following the general methodology outlined in Goldberg et al. (2020), we remove the innermost 6 $M_\odot$, representing the core, at the conclusion of the classical evolutionary run. This value is chosen so that $\sim 1 M_\odot$ of the outer He-layer remains along with the entire H-envelope. The He layer, which sets the base of the hydrodynamical simulation, forms a core-envelope structure with the H-envelope, and the higher density core ensures that the oscillation of the envelope does not interact directly with the mass gap.

In order to maintain a stable configuration after removing the core, we allow the model to settle into a hydrostatic approximation before turning on the hydrodynamic solver. To capture the short timescale motion to adequate resolution, we limit the timestep of the hydrodynamical evolution to a maximum of $10^4$ s. A larger timestep of only $\sim 10^5$ s can already result in the emergence of modes with a sub-annual period; this is due to the implicit nature of the hydrodynamical solver. We note that an implicit hydrodynamic scheme is necessary in order to follow the global motion of the star because the relative distances among mass shells near the surface are small. In terms of the Courant timescale, it is $\sim 10^6$ larger than that required by explicit time discretization ($\sim 0.5$ s). Thus, in order to track the motion of the surface with sufficient temporal resolution, implicit hydrodynamics must be employed.

Our canonical model is evolved for a total of $\sim 10000$ steps from the initiation of hydrodynamics until the point at which stellar expansion begins to disturb the pulsation frequency. We find that beyond $\sim 30$ years, the motion in the star becomes large enough to interact with the convective layer, causing the timestep to drop as low as $1 \times 10^2$ s before the code is unable to evolve the model forward in time. At this point, we stop the simulation.

5.4.1. Global Features

We first discuss the temporal evolution of the critical observables in the canonical model. In the four panels of Figure 10, we plot the luminosity, effective temperature, radius and surface velocity of the star.

The system enters into a state of pulsation with steady but growing cycles a handful of years after the hydrodynamic solver is switched on. Early in the evolution, quasi-annual oscillatory behavior is present in the luminosity and effective temperature, and the stellar radius exhibits a consistent periodic motion on top of a steady exponential expansion. In this work, we will consider the stellar pulsation only when the motion remains linear; we note that once the behavior becomes non-linear, the timestep becomes too small to follow the pulsation effectively. Moreover, non-linear pulsations greatly disturb the profile of the stellar envelope, particularly in terms of opacity and free electron fraction, which makes direct comparison difficult.

In Figure 10, four vertical, dotted lines indicate moments at which we compare the instantaneous values of the four quantities. The red and green lines correspond to timesteps where the luminosity is at a local maximum and minimum, respectively. The blue and purple lines
correspond to timesteps where the surface velocity is at a local minimum and maximum, respectively.

When the star reaches its brightest point in the pulsation cycle, the effective temperature also reaches its maximum. Concurrently, the star is in its most contracted state (radial minimum) and displays nearly zero surface velocity. This is consistent with the behavior of a classical harmonic oscillator where the displacement is largest during one cycle. Conversely, the luminosity and effective temperature are minimal when the star is most radially extended, and when the star is maximal in surface velocity, the luminosity, effective temperature, and radius are near their average values. We then observe that as the star continues to expand, a wobble in its motion emerges. As indicated in the hydrodynamic evolutionary tracks of Figure 8, the star will eventually approach a hydrodynamical instability at the end of its helium-burning phase.

The expansion of the outer radius gradually affects the period–radius relation, as the sound speed travel time increases with increasing distance. Analysis of the radius is additionally complicated by (1) how the outermost boundary of the star is defined and (2) radiation pressure outside the photosphere. A rigorous treatment of radiative transport is necessary in the photosphere regime, and so we stop the simulation to analyze the motion only when the radius is beneath this threshold.

The effective temperature \( T_{\text{eff}} \) and luminosity \( L \) behave similarly to radius in the simulations. In the former case, this is because MESA calculates the effective temperature directly from the luminosity and radius via \( T_{\text{eff}}^4 = \frac{L}{4\pi \sigma_B R^2} \), where \( \sigma_B \) is the Stefan–Boltzmann constant. Given the slow change of the radius, \( T_{\text{eff}} \) primarily mirrors the fast-evolving \( L \).

Regarding the evolution of luminosity, we note that from year 15 onward, the star exhibits periodic motion in its brightness. The early motion is highly regular: as in the preliminary grid of hydrodynamic models (see Figure 8), we observe a quasi-annual rise and fall—the correct timescale for the FM. Near the end of the year...
simulation, the large oscillation begins to trigger non-
oscillatory motion in all quantities, and this is responsi-
ble for the rapid drop in luminosity.

In Figure 11, we plot the structural profiles of six
quantities at points indicated by black circles in Figure
10. The global features of the density and tempera-
ture profiles show that outermost 10% of the stellar
mass has a relatively low density, sitting between \(10^{-9}\)
\(\text{g cm}^{-3}\), while the temperature lies between \(10^{3.5}\)
\(\text{K}\). A small density bump appears at \(\log_{10} 1 - q =
\log_{10} 1 - M(r)/M = -3\), which is accompanied by a
sharp fall in temperature. This occurs in order to main-
tain hydrostatic equilibrium. We note that a density
inversion can occur only when convective mixing is inef-
ficient. When the convection has a timescale compara-
able to the dynamical timescale, the density contrast
can create the Rayleigh-Taylor instability where the
density difference and pressure difference change signs.
Through convective mixing, the excess density can grad-
ually reduce via diffusion. However, modeling this phe-
nomenon would require a detailed time-dependent con-
vective scheme, which is not included in this work. For
our case, the density inversion plays a less important
role in the luminosity evolution, given that this quan-
tity remains steady from \(\log_{10} 1 - q = -1\) upward (see
subsequent discussion).

The free electron fraction shows that up to \(\log_{10} 1 - q =
-3\), the matter is partially ionized. Beyond that,
the low temperature causes the nuclei to recombine with
the free electrons. The opacity profile is richer; rather
than falling monotonically like the free electron fraction,
we see two major opacity bumps near \(\log_{10} 1 - q = -1.2\)
and \(-2.5\). These correspond to the partial ionization
zones of H-HeI and HeII, respectively (Cox et al. 1973;
Kiriakidis et al. 1992).

The velocity and the luminosity vary dynamically dur-
ing the pulsation. When the stellar luminosity is mid-
phase, the whole envelope is contracting with a constant
velocity of \(\sim 0.7 \text{ km s}^{-1}\). The whole star contracts more
slowly when it is close to its luminosity maximum or
minimum. Meanwhile, the luminosity profiles show that
when the star is at its local minimum, the luminosity
near the inner part of the envelope is lower. The oppo-
site applies during its local maximum.

We note that these trends are indicative of the \(\kappa\)-
mechanism, and thus explain why the pulsation gradu-
ally grows over many periods. In particular, during
contraction, the lowered opacity prevents the heat from
being stored in the deeper layers, which in turn prevents
unstable energy extraction by ionization (see previous
discussion for more detail on the the \(\kappa\)-mechanism).

From this collection of profiles, we can deduce that
the \(\sim 1\) year variation is driven by the collective expan-
sion and contraction of the recombined hydrogen layer.
The small motions in the layer interior to this interface
imply a shorter transition time, as the propagation time
required depends on the sound travelling time between
the interface and the surface. We thus conclude that
it is this \(\kappa\)-related interaction driving the fundamental
pulsation mode in Betelgeuse.

### 5.4.2. Literature Comparison

To examine the growth of the oscillation further, we
plot the total kinetic energy of the system against time
in Figure 12. The kinetic energy, which is dominated
mostly by the motion of the atmosphere, is much smaller
than both the total energy and the total gravitational
energy, which are, on average, \(-6 \times 10^{49}\) and \(-1 \times 10^{51}\)
\text{ergs}, respectively. These are several orders of magnitude
larger than the kinetic energy, which is \(10^{42-43}\) \text{erg}, in
agreement with earlier results (see Cox 1980). We note
that it is the outermost \(q = 0.9\) which contributes to the
atmospheric behavior, including the pulsation. This
corresponds to \(\sim 2 \times 10^{47}\) \text{erg} when this matter is moving
at a speed comparable to the escape velocity.

We provide an exponential fit in blue on Figure 12
to characterize the rate of growth of the kinetic en-
ergy. A function of \(\dot{E}_{\text{kin}} = A \exp(bt)\) with param-
eters \(A = 1.2 \times 10^{40}\) \text{erg} and \(b = 0.2\) provide a good
fit to the hydrodynamic component of the simulation.
Naively, this suggests that when the oscillation begins
to grow, we should expect that the outermost layers of
material will be expelled by the pulsations within a time of
\(\sim 83\) years. In reality, however, we see this rate level
off—see, for example, Cox et al. (1966), who showed
this in some early pulsation models. This is because vis-
cous and turbulent dissipation limit the maximum am-
plitude of the star in the non-linear regime. Historically,
the level of dissipation in 1D pulsation models has been
tuned to match the observed amplitudes of RR Lyrae
and Cepheid stars and to reproduce double-mode pulsa-
tion in the models. This dissipation was first applied via
the “artificial viscosity” term and later through the eddy
viscosity and other \(\alpha\) parameters of time-dependent, tur-
bulent convection (see, e.g., Buchler 1990; Takeuti et al.
1998; Koll´ ath et al. 1998; Smolec & Moskalik 2008).

We note that the models in Fig. 13 stop at about \(\pm 2\%\)
luminosity variation or less, whereas Betelgeuse itself
varies by \(\pm 10-30\%\) in \(V\) and about \(\pm 20-30\%\) in near-IR
(the latter being more closely representative of Betel-
geuse’s bolometric variation). As such, there is plenty
time remaining for the kinetic energy and amplitude to
grow and eventually saturate at that level, but this is be-
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Figure 11. Analysis of model star’s structure at three points selected during the pulsation, indicated by black markers in Figure 10. LEFT: The density (top panel), temperature (middle panel) and velocity (lower panel) profiles for the moments at the luminosity minimum (black solid line), midpoint (red dotted line), and maximum (green dashed line). RIGHT: Same as the left panel, but for the luminosity, opacity and free $e^-$ fraction. In both panels, $m$ represents the enclosed mass, $M(r)$.

Figure 12. Kinetic energy as a function of time for the hydrodynamical model. The blue line is an exponential fit of the form $1.2 \times 10^{40} \exp(0.2t)$, with time in years.

Beyond what can be achieved with hydrodynamics today before encountering a numerical runaway episode.

We note that when the oscillation becomes strong, heat deposition effects close to the sharp density gradient near the surface become important. The extra heat can change the opacity of the matter by increasing the ionization fraction, resulting in stronger amplification of the pulsation and in turn accelerating the predicted timescale from the first pulsation until mass ejection. However, similar ejections of the outer layers in models of luminous Cepheid models are known to be related to numerics rather than physics; see Smolec (2016). Regardless, we do not follow the code until this phase because the timestep becomes prohibitively small ($\sim 100$ s). In particular, numerical difficulties arise in the Newton-Raphson iterations, during which the code fails to resolve the formation of convection zones around the shock front. Due to shock compression, the extra heating also invalidates the equilibrium assumptions...
of the mixing length theory (Vitense 1953). To limit the steepness of shockwaves and distribute them over multiple zones, explicit pulsation codes like RSP include either artificial viscosity or eddy viscosity terms (or both), but this is only effective up to certain $L/M$ ratios (Stellingwerf 1975; Smolec 2016).

Also at this stage, non-linear effects become dominant, causing sub-annual features to appear gradually on top of the linear pulsation. As observations of Betelgeuse do not show periodicities on sub-annual timescales, we do not consider this phase of pulsation further, though a study of non-linear pulsation with the dynamical coupling of opacity and ionization will be interesting future work.

By comparing the general features of our hydrodynamical model with the pulsation patterns of Betelgeuse, it becomes clear that the quasi-annual variation is indeed caused solely by the contraction and expansion of the star. It is interesting to note that in this linear oscillation phase, we do not see any evidence of longer timescale variations, such as the 6-year and 35-year periodicities. In fact, the hydrodynamic simulations never reproduce any of the observed variations besides the currently presented quasi-annual pulsation, even when the initial mass is varied. This implies that these periodicities are driven by some mechanism outside the scope of what 1D hydrodynamic simulations can reproduce, i.e., not the $\kappa$-mechanism. It would be interesting to conduct further dynamical studies on how the star relaxes when the opacity effects becomes important; however, work in this domain will require an algorithm to suppress the $\kappa$-mechanism so that the pulsation can be sustained without triggering excessive mass loss.

There are similar works in the literature that focus on the pulsational features of massive stellar envelopes using the stellar evolution code described in Langer et al. (1998). In particular, Heger et al. (1997) present the dynamical evolution of massive stars from 10–20 $M_\odot$ and analyze their linear stability. In Figure 14, we plot the phase diagram of our canonical model’s $\log_{10} L$ against $\log_{10} T_{\text{eff}}$ during the hydrodynamic evolution as a means of comparing directly with Figure 5 in Heger et al. (1997). In their work, the 11 $M_\odot$ Red Supergiant model is followed for about 75 periods of oscillation, whereas ours capture the first 45 periods. Beyond the $45^{\text{th}}$, our models show numerical instability where the expansion and compression interact with the convective mixing zone, which largely suppresses the timestep and creates non-linear behaviour.

Heger et al. (1997) show an approximately circular trajectory that spirals outwards from $\log_{10} T_{\text{eff}}(K) \approx 3.52$ and $\log_{10} L = 4.90/L_\odot$, whereas our model shows an elliptical trajectory, vacillating between high $L$ and high $T_{\text{eff}}$ on one side and low $L$ and low $T_{\text{eff}}$ on the other. The outward spiraling in our work and theirs demonstrates that both stars are undergoing dynamical instability with a growing amplitude. As expected, Heger et al. (1997)’s model has a lower period because it is a lower-mass model. This implies a more compact envelope, which allows all 75 periods of oscillations to happen within 30 years—this is only half the time of our model.

Both models show a clockwise trajectory. Since the radius, temperature, and luminosity are related by the blackbody radiation formula $L = 4\pi\sigma_B R^2 T_{\text{eff}}^4$, this means that when the stellar models resume their initial luminosities, the models achieve a higher maximum $T_{\text{eff}}$ (i.e., smaller $R$) and a lower minimum $T_{\text{eff}}$ (i.e., larger $R$). These features suggest that $T_{\text{eff}}$, $L$ and $R$ achieve their local extrema simultaneously in Heger et al. (1997)’s model, but in our case, this relationship is slightly lagged. As shown in Figure 10, our model approaches its local extrema with a non-zero velocity; thus, the stellar radius, which affects $T_{\text{eff}}$, reaches its local maximum and minimum later than $L$. Our calculations therefore reproduce the phase lag between the luminosity and the velocity that has been observed in many other, smaller pulsators before (Castor 1968; Szabó et al. 2007).

In Yoon & Cantiello (2010), the hydrodynamical features of a 20 $M_\odot$ star with a luminosity of $\log_{10} L = 5.05$ $L/L_\odot$ and temperature of $T_{\text{eff}} = 3198$ K are analyzed. Their stellar parameters are similar to ours, where our hydrodynamical model assumes a 21 $M_\odot$ star with a slightly lower initial hydrostatic luminosity at $\log_{10} L = 5.01$ $L/L_\odot$ and $T_{\text{eff}} = 4000$ K. They model about 50 years of the stellar pulsation; Figure 2 in their work shows the surface velocity and is comparable to Figure 10 in this work. Approximately 20 pulsation cycles are followed in their work, where a higher period of $\sim 1000$ days is observed. Compared to our $\sim 388$ day period, this indicates that their envelope is more relaxed and expanded. Both Yoon & Cantiello (2010) and our work show a consistent growth of the surface velocity. It takes about 5 cycles for the surface velocity to reach a ten-fold of amplification, while our model takes much longer—almost 20 cycles. This suggests that the $\kappa$-mechanism is less efficient in our model, where the star exhibits behavior closer to adiabatic oscillations than driven oscillations. From the growth of kinetic energy of the system, we can estimate that it takes a further $\sim 10$ years for the pulsation to grow to a surface velocity comparable with Yoon & Cantiello (2010). This would correspond to another 11–15 cycles in our case. Despite this, the robust exponential growth of the pul-
sation energy (see Figure 12) in both works implies that the pulsation could potentially remove the outermost layers of the H-envelope from the star. However, this is not consistent with observational evidence; the RSGs we have observed pulsate with limited amplitude for several decades.

Whether or not mass loss can be driven depends on the degree of saturation in the pulsation of the surface layers (King et al. 1966). When the mode is permitted to develop, this process can be influential in the formation of circumstellar matter in Type-IIIn supernovae for massive stars close to $\sim 20 M_{\odot}$ (e.g., Smith 2017). However, given the regulated oscillation amplitude observed in a number of RSGs empirically, additional mechanisms not modeled in this work must become dominant in regulating the growth of these oscillation patterns.

The most recent analysis of this kind can be found in Goldberg et al. (2020). The pulsation of a red supergiant with 16.3 $M_{\odot}$ is computed using MESA with the GYRE extension (version 11701). In contrast to the approaches discussed above, their hydrodynamic models involve an initial perturbation to the density distribution to trigger direct pulsation of not only the fundamental mode, but also the first overtone. They obtain a star to trigger direct pulsation of not only the fundamental mode. However, given the regulated oscillation amplitude observed in a number of RSGs empirically, additional mechanisms not modeled in this work must become dominant in regulating the growth of these oscillation patterns.

We note a severe drop between models of 20.2 and 20.5 $M_{\odot}$ during which the associated number of pulsations also decreases sharply. Below $M = 19 M_{\odot}$, the oscillation does not amplify significantly within 300 years, at which point we terminate the simulation.

5.5. Impact of Initial Mass on Pulsation

Thus far, we have presented one model with an initial mass of 21 $M_{\odot}$. Now, we consider a series of hydrodynamic models of different initial mass and discuss how the progenitor mass affects the pulsation pattern.

We repeat the simulations by varying the progenitor mass, while fixing the mixing length parameter (see Section A for details on the exact configuration) so that we can compare consistently among models. In Table 3, we tabulate the global parameters and pulsation statistics of these models. The data present the following trends: when the progenitor mass increases, the present day mass $M_{\text{fin}}$, helium core mass $M_{\text{He}}$, the radius at the end of He-burning $R$, and its corresponding luminosity $L$ all increase. There is a weak decreasing trend for the effective temperature $T_{\text{eff}}$. Meanwhile, the time required for the non-linear pulsation to emerge decreases. Further, the late stage evolution of high mass stars is especially sensitive to convective parameters; in practice, $\alpha_{\text{MLT}}$ is often tuned arbitrarily until the model converges or behaves as desired. We include the last row of Table 3 to highlight this degeneracy and caution against over-interpretation.

In Figure 13, we present the time evolution of the pulsation pattern for models with the progenitor mass from 19 to 22 $M_{\odot}$. We choose these masses as their timescales are more relevant to that of Betelgeuse. Before non-linearity disturbs the simulation, all models behave similarly in both luminosity and radius.

Despite the fact that the excitation time apparently depends on the progenitor mass and mixing length parameter, the the means by which the star becomes excited—i.e. the pulsation driving mechanism itself—is less sensitive to these choices. The dynamical pulsation always concludes with a significant drop in the stellar luminosity, and the growth rates of the peak luminosity
and maximum radius are similar among all models near the end of the simulation.

To further outline the similarity, we plot in Figure 14 the phase diagram of representative models from 19–23 $M_{\odot}$. Clear similarity can be seen for models above 19 $M_{\odot}$. In particular, for $M = 20 M_{\odot}$, the model has a highly extended $t_{\text{run}}$ of $\sim 166$ year. All models have an elliptical structure, which is actually a clockwise outward-going spiral. They show once again that all stars evolve toward a high $L$ and a high $T_{\text{eff}}$ state simultaneously, or the converse. This suggests that the driving mechanism in all of these models is qualitatively the same, too. A higher progenitor mass gives rise to a sparser trajectory; however, we notice that for $M = 19 M_{\odot}$, there is no regularity in the trajectory. This suggests that the $\kappa$-mechanism fails to stimulate residual numerical noise into periodic motions.

To further characterize the runaway time of the $M = 20 M_{\odot}$ model, we compare the amount of time needed for the star to develop non-linear pulsation ($t_{\text{run}}$) after using the hydrodynamical prescription. For progenitor masses above 20.5 $M_{\odot}$, $t_{\text{run}}$ decreases slowly with time. As shown in Figure 13, nonlinear activation timescales for masses of this range are between 15–40 years. However, below 20.5 $M_{\odot}$, there is a sudden jump in $t_{\text{run}}$, and the star requires more than roughly $\geq 150$ in order for non-linearity to become significant. The sudden jump could signify some qualitative changes in the stellar profile, namely that the $\kappa$-mechanism becomes much less effective in amplifying the acoustic wave inside the star; a detailed comparison to and analysis of the means of formation for the $\kappa$-mechanism will be an interesting future project, but is beyond the scope of the present work. Crucially, this mass-sensitive timescale bifurcation suggests that the time required for the star to develop non-linear pulsation could be a highly discriminating attribute among models of Betelgeuse.

As an order of magnitude estimation, the typical luminosity of our model star is $10^5 L_{\odot}$. The amount of energy dissipated is then $\sim 10^{47}$ erg per year, but the kinetic energy is only on the order of $10^{41-43}$ erg. This is because radiation acts as a damping force through photon emission, and without a consistent driving force for the pulsation, the oscillation would quickly dissipate. It is clear, then, that the 388 pulsation is driven.

In the previous text, we have shown that there are multiple periodicities in Betelgeuse’s lightcurve. These include a quasi-annual mode, a 6-year mode, a 30-year modulation, and, potentially, an overtone mode with a 185 d period. In the hydrodynamic models, we recover only the 388 day period. These results are largely self-consistent, as the 388 d mode is driven by the $\kappa$ mechanism, the LSP is not, and the 30 year modulation is most likely caused by rotation, which is not an internally driven form of variability. In the case of the overtone, however, we must address the question of how multiple modes may appear in the first place.

One possibility is by non-linear mode excitation, as touched upon in Section 5.3. Through large amplitude oscillations, the outer layers can accumulate sufficient energy and momentum to compress matter beneath the stellar surface. This results in compression heating, which in turn raises the local temperature. This may impact the convective structure in the near-surface regions, thus presenting an additional source of energy that alters the net energy flow inside the star. Capturing this scenario numerically is particularly challenging because it involves modeling the dynamics of mixing behaviour in the convection zone. Meanwhile, the standard mixing length theory adopted in our work assumes the convective mixing is in equilibrium (Vitense 1953). Modeling this phase properly would require a more sophisticated approach to time-dependent mixing and a robust solving mechanism.

The short $t_{\text{run}}$ for $M > 21 M_{\odot}$ suggests that, among all models, these are more likely to excite non-linear mode coupling than models with lower $M_{\text{fin}}$. This is because the lower-mass models cannot excite any observable within a time frame of $\sim 30$ years. On the

| $M$ | $\alpha$ | $M_{\text{fin}}$ | $M_{\text{He}}$ | $R$ | $\log_{10} L$ | $T_{\text{eff}}$ | $t_{\text{run}}$ | Pulse |
|-----|---------|----------------|-------------|-----|---------------|----------------|--------------|-------|
| 18  | 3       | 17.12          | 5.57        | 550 | 4.92          | 4160          | $> 300$      | N/A   |
| 19  | 3       | 17.90          | 6.06        | 624 | 5.00          | 4115          | $> 300$      | N/A   |
| 20  | 3       | 18.80          | 6.47        | 655 | 5.04          | 4117          | 166.2 $\sim$ 230 |
| 20.2| 3       | 18.95          | 6.60        | 659 | 5.05          | 4120          | 141.8 $\sim$ 200 |
| 20.5| 3       | 19.17          | 6.75        | 707 | 5.10          | 4081          | 31.5        | 43    |
| 21  | 3       | 19.54          | 7.00        | 721 | 5.11          | 4083          | 27.0        | 40    |
| 22  | 3       | 20.30          | 7.46        | 787 | 5.18          | 4053          | 21.0        | 24    |
| 23  | 3       | 20.92          | 8.04        | 875 | 5.25          | 4088          | 16.7        | 18    |
| 19  | 2.5     | 17.78          | 6.07        | 724 | 5.01          | 3832          | 102.8       | 122   |
| 20  | 2.5     | 18.57          | 6.59        | 794 | 5.08          | 3801          | 41.1        | 37    |
Figure 13. The time evolution of luminosity and stellar radius scaled by its initial values for models with different progenitor masses studied in this work. Time 0 stands for the transition from hydrostatic stellar evolutionary phase at the end of the He-core burning to the hydrodynamical prescription. See also Appendix A for the numerical treatment.

other hand, the higher-mass models can excite energetic motion within a few decades. However, it is unclear whether such dynamical motion can lead to observable mass loss, or whether it is efficiently damped by other atmospheric mechanisms.

Another possible excitation mechanism is wave-driven pulsation, as described in Shiode & Quataert (2014); Fuller (2017); Fuller & Ro (2018). This mechanism proposes that a convective wave in the star can partially penetrate through the evanescent regions\(^8\) and approach the stellar surface. Although wave-driven pulsation was described in the context of very late phases of stellar evolution in those works (i.e., Neon–Oxygen burning, rather than He), the theory suggests that as long as convection is activated, energy can be transferred from the interior convection zone to regions near the surface, where it can then excite surface motion. However, depending on the convective luminosity, such a mechanism would provide a heavily condensed energy deposition near the surface, in turn triggering enormous losses in mass of \(0.01–1 \, M_\odot \, \text{yr}^{-1}\). Mass loss of this order is not observed in Betelgeuse.

6. CONCLUSIONS

We have presented a detailed observational and theoretical analysis of \(\alpha\) Orionis, including the presentation of new photometry and three different types of numerical predictions from classical evolutionary, linear seismic, and hydrodynamic simulations. Our critical results are summarized as follows.

We present a new set of processed, space-based photometric data from the SMEI instrument. These data reveal variation on monthly timescales, which is likely the signature of convective cell turnover. In combination with longitudinal data collected by the AAVSO, the photometry confirms the presence of several key periodicities and contextualizes the recent dimming behavior of \(\alpha\) Orionis in the long-term. In particular, we con-

\(^8\) Zones dominated by thermal radiation.
Figure 14. The phase diagrams for the model with a progenitor mass 19 (top left), 20 (top right), 20.5 (middle left), 21 (middle right), 22 (bottom left), 23 (bottom right) $M_\odot$ respectively. The trajectory is cut when the non-linearity begins to disturb the elliptical pattern in each figure.
firm the presence of a 388 d periodicity, note the 30–40 yr timescale of dimming events similar to the one commencing in November of 2019, and detect a potential 185 d first overtone (O1) mode.

We conduct a grid-based analysis of evolutionary tracks to estimate the fundamental, model-derived parameters of α Ori. Supported by previous studies, we take special account of the theoretical uncertainty imparted by an ad hoc choice of the mixing length parameter, αMLT, and reconsider the uncertainties on Betelgeuse’s effective temperature accordingly (Joyce & Chaboyer 2018a,b; Levesque & Massey 2020). We perform a probabilistic age prior analysis and find good agreement between our estimates of Betelgeuse’s current evolutionary stage (RSB core helium burning) and present-day mass range (16.5–19 $M_\odot$) with previous modeling initiatives (Neilson et al. 2011; Dolan et al. 2016; Wheeler et al. 2017; Nance et al. 2018). However, we find that the observed, present-day rotational velocity of α Ori cannot be reproduced using single-star evolution; a merger or some other source of spin-up is required, in agreement with Dolan et al. (2016); Chatzopoulos et al. (2020). The likelihood of a previous interaction is also supported by our kinematic argument in Section 2.

Linear seismic analysis with GYRE heavily constrains the radius of Betelgeuse, for which we report a value of $750^{+62}_{-36} R_\odot$. Combining this result with existing angular diameter and temperature data, we are able to obtain an independent and precise parallax value for α Orionis based on seismic constraints, resulting in a distance of $165^{+16}_{-8}$ pc. Our results are consistent with reprocessed Hipparcos measurements but in disagreement with recent radio parallax observations (van Leeuwen 2007; Harper et al. 2017), highlighting the difficulty of estimating cosmic noise when deriving the geometric parallax of this star. To the best of our knowledge, this is the first time that a seismic parallax has been obtained for Betelgeuse.

Deeper analysis of emergent periodicities in both hydrostatic seismic and hydrodynamic models, in conjunction with existing observational data on variable stars across the mass spectrum, unambiguously demonstrate that the 388 d period reported by Kiss et al. (2006) is the fundamental $p$-mode pulsation.

Finally, using hydrodynamic models with six different masses, we investigate the physics of these oscillations. All hydrodynamic models in the prescribed mass range manifest similar quasi-annual behavior as the fundamental mode, in agreement with similar studies.

We find that stars with an initial mass below $\sim 20 M_\odot$ take much longer for the pulsation to excite other oscillation modes; in particular, a 19 $M_\odot$ model can take as long as 150 years to build up to non-linearity. The similarity among models suggests that the exact parameters of the model play a less important role in reproducing the fundamental mode of the star. In all cases, the 388 d mode is clearly driven by the $\kappa$-mechanism. Importantly, if non-linear excitation is assumed to be correlated to the $\kappa$-mechanism’s triggering of overtone modes, our hydrodynamic simulations constrain against progenitor masses above $\sim 20 M_\odot$.

It is unclear whether the excited fundamental mode can be modulated by other radiative mechanisms or lead to observable mass loss. If mass loss can be triggered, the short runaway time from the appearance of the first wave until mass ejection suggests that the star can lose a considerable amount of its H-envelope during its post-main-sequence evolution. Our work thus provides an additional indication of the initial mass of Betelgeuse based on the timescale of non-linear excitation.

The sudden bifurcation in excitation time as a function of mass in our hydrodynamical models provides some constraint on Betelgeuse’s upcoming, presupernova evolution. For models with an initial mass above $\sim 20 M_\odot$ (present-day mass 18.8 $M_\odot$), the $\kappa$-mechanism driven pulsation and the mass loss it incites could partially remove the H-envelope prior to the final explosion. This would give rise to a Type-IIp, Type-III and then Type-II supernova. Meanwhile, for models with initial masses below this break-off point, the very long excitation time of the $\kappa$-mechanism means that the star would retain most of its H-envelope. In this case, an alternative mass loss channel would be required for the formation of a circumstellar medium.

Conclusively determining which of these two possible evolutionary channels α Ori will take would require disentangling the degeneracy between mass and mixing length in the simulations, but our work here suggests that a predictive investigation in this vein is possible.

ACKNOWLEDGEMENTS

M.J. was supported the Research School of Astronomy and Astrophysics at the Australian National University and funding from Australian Research Council grant No. DP150100250. M.J. was likewise supported by Ken’ichi Nomoto and invitation to the Kavli Institute for Theoretical Physics at the Institute for the Mathematics and Physics of the Universe (IPMU) at the University of Tokyo in January of 2020. Collaboration with Chiaki Kobayashi was made possible in part through the Stromlo Distinguished Visitors Program.

M.J. wishes to thank Peter Wood and Matteo Cantiello for helpful discussion regarding construction...
and interpretation of hydrodynamic simulations. M.J. further acknowledges Richard Townsend for management of the GYRE forums and the rest of the MESA developers for their support and expert guidance.

This work was also supported by World Premier International Research Center Initiative (WPI), and JSPS KAKENHI Grant Numbers JP17K05382 and JP20K04024. S.C.L. thanks the MESA development community for making the code open-source. S.C.L. acknowledges support by funding HST-AR-15021.001-A.

L.M. was supported by the Premium Postdoctoral Research Program of the Hungarian Academy of Sciences. The research leading to these results received funding from the LP2014-17 and LP2018-7/2019 Lendület grants of the Hungarian Academy of Sciences. L.M. wishes to thank Bernard Jackson for discussions about the SMEI photometry.

CK acknowledges funding from the UK Science and Technology Facility Council (STFC) through grant ST/M000958/1 & ST/R000905/1.

We acknowledge with thanks the variable star observations from the AAVSO International Database contributed by observers worldwide and used in this research. This research has made use of the SIMBAD database, operated at CDS, Strasbourg, France, and NASA’s Astrophysics Data System Bibliographic Services.

Facilities: AAVSO (http://aavso.org), SMEI (Hick et al. 2007)

Software: MESA (Paxton et al. 2018), GYRE (Townsend & Teitler 2013), Period04 (Lenz & Breger 2005), Python: numpy, matplotlib, Bokeh (Oliphant 2006; Hunter 2007); gnuplot

APPENDIX

A. MESA CONFIGURATIONS

In this section, we detail the configuration profile for the evolutionary and hydrodynamical portions of the simulations.

The evolutionary phase inherits settings from the massive_star_defaults inlist. Additionally, we set the “Dutch” mass loss prescription with a parameter 0.8, namely:

hot_wind_scheme = ‘Dutch’
Dutch_scaling_factor = 0.8
hot_wind_full_on_T = 1d0
hot_wind_full_off_T = 0d0

In order to construct a star that maintains the proper radius for hydrodynamic evolution, we must adjust the mixing length parameter:

mixing_length_alpha = 3
MLT_opion = ‘Henyey’

We notice that a larger mixing length parameter results in a smaller radius at the end of the He-burning. The mass of the star is selected such that the luminosity is within the expected range (∼4.8–5.1) and a radius between 700–800 $R_\odot$ for consistency with the seismic parameters.

A requirement of our configuration is that the star should exhibit an observable amount of pulsation within a reasonable amount of time (∼100 years). A small progenitor mass results in a very long quiescent time. Meanwhile, a higher mass can trigger observable pulsation quickly, but its luminosity and radius can be too high. As a result, for the hydrodynamics, we pick the high mass end $M=21M_\odot$ with a large mixing length parameter $\alpha=3$. This is slightly higher than what is used in the evolutionary calculations ($\alpha \leq 2.5$), but our model gives the correct radius at 720 $R_\odot$ and a luminosity $\sim 10^{5.1} L_\odot$. The final mass (present-day mass) is 19.5 $M_\odot$ and the helium core is 7.5 $M_\odot$.

As we require that the stellar profile transition smoothly from the evolutionary phase to the hydrodynamical phase, we use identical settings in the dynamical phase.

T_mix_limit = 0
min_T_for_acceleration_limited_conv_velocity = 0
okay_to_reduce_gradT_excess = .false.

In the hydrodynamical phase, we patch extra settings onto this configuration such that the hydrostatic equilibrium constructed in the previous phase can be well maintained. However, one qualitative change is included, where the mass loss is suspended.
Dutch_scaling_factor = 0.0d0

This is a reasonable approximation given that we are simulating a short period of time: $\sim 100$ years.

To trigger the hydrodynamics, we use the standard settings as provided by the test_suite test case ccsn in MESA version 8118. This includes

use_ODE_var_eqn_pairing = .true.
use_dvdt_form_of_momentum_eqn = .true.
use_dPrad_dm_form_of_T_gradient_eqn = .true.
use_dedt_form_of_energy_eqn = .true.
use_momentum_outer_BC = .true.
use_ODE_form_of_density_eqn = .true.

These settings have been used in our previous work modeling the dynamical pulsation in pulsation pair-instability supernovae. See Leung et al. (2019, 2020) for the application of these setting to the more massive star counterpart.

Furthermore, to ensure the code captures the early oscillation when the simulation has begun, we impose a maximum evolutionary timestep of $10^5$ s.

max_timestep = 100000

We also remove the temperature limitation in which the hydrodynamics is solved. This means the Euler equations are solved throughout the star, without assuming the envelope is in hydrostatic equilibrium:

velocity_logT_lower_bound = 0

At last, we turn on the artificial viscosity so that all potential shocks can be resolved by the simulation. This happens, in particular, near the surface where the density gradient is the highest.

use_artificial_viscosity = .true.
shock_spread_linear = 0
shock_spread_quadratic = 2d-2

We find that a higher artificial viscosity parameter can result in the code crashing earlier in the simulation, whereas a value too small can result in too strong of a shock when the global pulsation amplitude is still weak.

A simulation of $\sim 30$ years requires approximately 10000 timesteps.

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