**Abstract**

The values of the cosmological constant, $\Lambda$, and the Hubble constant, $H_0$, are determined using the vacuum energy density, $\Delta_e$, equal to $-1.016124 \times 10^{113}\text{J} \text{m}^{-3}$ and the age of the universe, $t_0$, equal to $13.80 \times 10^{10}\text{years}$. The value obtained for $H_0$ is $73.92\left[\text{km s}^{-1} \text{Mpc}^{-1}\right]$ and for $\Lambda$ is $3.520 \times 10^{-52}\text{m}^{-2}$.

**Introduction**

In this work we use the results found in Ferreira [1] to explore the dark matter with a simplified model of the Milky Way. The exploration shows that, for the ratio dark matter / bright matter existing in a distance less than the distance from the center of the Milky Way to the Sun is equal to 89% and a quantity of negative mass equal to $-3.53 \times 10^{-47}\text{kg}$ can be converted into the dark matter for $R_0 = 8.50\text{kpc}$, $V_0 = 220\text{km s}^{-1}$ and $V_L = 160\text{km s}^{-1}$, where $R_0$ is the distance from the center of the Milky Way to the Sun, $V_0$ the speed of the Sun around the center of the galaxy and $V_L$ the velocity of the Sun around the center of the galaxy when only the luminous matter is considered. The main contribution for this negative mass is

$$-\frac{c^2r}{G}$$

with $r = R_0$.

Associated with (1) we have the density

$$\frac{3c^2}{4\pi G r^2}$$

which depends only on the space. This suggests that for $r$ equal to $c t_0$, where $t_0$ is the age of the universe, we obtain the contribution of the vacuum energy to the field equation. Using the value of the vacuum energy found in Ferreira [2] the approximated value of the cosmological constant $\Lambda$ is determined as well as the value of $H_0$.

**The used transformation**

Let $S'$ be a reference system moving with respect to the reference system $S$ with constant speed $u$ in the direction $x^+$. The $y'$-axis and the $z'$-axis are parallel to axis-$y$ and $z$-axis, respectively. Let us suppose that the zero of $t'$ coincides with the zero of $t$ and the origin of $x', y', z'$ coincides with $x, y, z$ when $t = 0$. The Lorentz transformation

$$x' = \frac{x - ut}{(1 - \frac{u^2}{c^2})^{1/2}}$$



---

**How to cite this article:** Ferreira JC. On the cosmological and Hubble constants. Int J Phys Res Appl. 2019; 2: 036-046. https://dx.doi.org/10.29328/journal.ijpra.1001010
On the cosmological and Hubble constants

Published: June 28, 2019

$$y' = y \tag{4}$$

$$z' = z \tag{5}$$

and

$$t' = \frac{t - \frac{u}{c^2}x}{\left(1 - \frac{u^2}{c^2}\right)^{1/2}} \tag{6}$$

is substituted in this work by the following transformation found in Ferreira [1]:

$$x' = (1 - \frac{u^2}{c^2})^{1/2}(x - ut) \tag{7}$$

$$y' = (1 - \frac{u^2}{c^2})^{1/2}y \tag{8}$$

$$z' = (1 - \frac{u^2}{c^2})^{1/2}z \tag{9}$$

and

$$t' = (1 - \frac{u^2}{c^2})^{1/2}t \tag{10}$$

**On the dark matter that exists in the distance of the center of the Milky Way smaller than \(R_0\)**

Let a space probe be in circular orbit in the plane of the Milky Way, being at the same distance from the center of the galaxy in which the Sun is. The probe is in a region where the gravitational attraction is basically due to the existing matter in the galaxy at a distance from its center smaller than the distance from the Sun to the Milky Way center. In the Figure 1 the probe is represented by a blue circle and its velocity around the center of galaxy, denoted by \(v\), is indicated by the red arrow. The center of the galaxy is at rest with respect to the inertial frame \(S\). In the Figure the probe is at the origin of the inertial frame \(S'\). The axis \(x'\) is colinear with the axis \(x\), the plane \(xy\) and the plane \(x'y'\) are in the Milky Way plane.

Let \(M\) be the mass of the galaxy existing at a distance from its center smaller than the distance from the Sun to the Milky Way center, \(r\) is the distance of the probe to the galaxy center and \(G\) is the gravitational constant for the inertial frame \(S\). For the inertial frame \(S'\) the mass \(M'\) is the existing mass in the galaxy at a distance from its center smaller than the distance from the Sun to the Milky Way center, \(r'\) is the distance of the probe to the galaxy center and \(G'\) is the gravitational parameter. The force exerted on the probe of mass \(m\) is

\[
F' = G' \frac{M'mt'}{r'^2} \tag{11}
\]

*Figure 1: Systems S and S' and the probe AM in the Milky Way.*
the orbital period is

$$T' = \sqrt{\frac{2\pi r^3}{GM}}$$  \hspace{1cm} (12)$$

Using the equation obtained in Ferreira [1] the force at the position of the probe shown in Figure 1 is

$$F'_s = (\frac{\gamma v^2}{c^2} + \gamma^2)F_s$$  \hspace{1cm} (13)$$

and the radius

$$r' = \frac{r}{\gamma}$$  \hspace{1cm} (14)$$

The unity of $G$ is $Nm^2/Kg$ or $[LT^{-2}Kg^{-1}]$ and

$$G' = (\frac{v^2}{c^2} + \frac{1}{\gamma^2})G$$  \hspace{1cm} (15)$$

If

$$\sqrt{\frac{u^2}{c^2}} > \frac{v^2}{c^2}$$  \hspace{1cm} (16)$$

then

$$G' = \frac{G}{\gamma^2}$$  \hspace{1cm} (17)$$

With (17) the orbital period of the probe with transformation found in Ferreira [1] is

$$T' = \sqrt{\frac{2\pi r^3}{G'M}} = \frac{T}{\gamma}$$  \hspace{1cm} (18)$$

Let us assume

$$v << c$$  \hspace{1cm} (19)$$

in this case we have $\gamma$ practically equal to 1 and

$$T' = \frac{T}{\sqrt{\frac{v^2}{c^2} + 1}}$$  \hspace{1cm} (20)$$

Now let us assume that

$$2\pi r_a' = 2\pi r_a$$  \hspace{1cm} (21)$$

for any object of the galaxy with $r_a = r$ and $r_{alpha} < r_{max}$. From (20) and (21) we obtain

$$v' = \frac{\sqrt{v^2 + 1}}{c^2}$$  \hspace{1cm} (22)$$

Let us consider that practically all mass is concentrated in the plane defined by the $x$- and $y$-axis; the $z$-axis is directed to the north galactic pole. The Figure 2 shows the $xy$-plane.

Let $\Delta v$ be the difference between the velocities in $r + \Delta r$ and $r$ in the frame system $S$. In the system $S'$ the difference becomes

$$\Delta v' = v_{ext} \frac{v_{ext}^2}{c^2} + 1 - v_{ext} \frac{v_{ext}^2}{c^2} + 1$$  \hspace{1cm} (23)$$
where
\[
v_{\text{ext}} = \sqrt{\frac{G(m + m_r(r) + \Delta m)}{r + \Delta r}}
\] (24)
\[
v_{\text{int}} = \sqrt{\frac{G(m + m_r(r))}{r}}
\] (25)

and \(m_r(r)\) is the luminous matter that exists in the distance of the center of the galaxy smaller than \(r\). Consistently with (21) let \(\Delta v\) be equal to zero. The region where the probe orbits is the same where any object \(r_a\) is:
\[
0 < r_{\text{min}} \leq r \leq r_{\text{max}}
\] (26)

So we have
\[
2 \left( \frac{m_r + m_i + \Delta m_i}{r + \Delta r} \right) \left( \frac{G(m_r + m_i + \Delta m)}{c^2 (r + \Delta r)} + 1 \right) = \frac{m_r + m_i}{r} \left( \frac{G(m_r + m_i)}{c^2 r} + 1 \right)
\] (27)
or
\[
2 \left( \frac{m_i(r) + \Delta m_i}{r + \Delta r} \right) \left( \frac{G(m_i(r) + \Delta m)}{c^2 (r + \Delta r)} + 1 \right) = \frac{m_i(r)}{r} \left( \frac{G(m_i(r))}{c^2 r} + 1 \right)
\] (28)

where
\[
m_i(r) = m_j(r) + m_i(r)
\] (29)

The luminous matter is denoted by \(m_i\) and the dark matter by \(m_j\). The solution of (28) for \(m\) has two roots
\[
m_1 = \frac{\Delta m r}{\Delta r}
\] (30)
and
\[
m_2 = -\frac{r(\Delta m G + c^2 (\Delta r + r))}{G(\Delta r + 2r)}
\] (31)

The equation (30) provides for an increment \(\Delta r\) from a given \(r\) the increment of matter. For
\[
0 < \Delta r << r
\] (32)
we found
\[
m_2 = -\frac{\Delta m}{2} \frac{c^2 r}{2G}
\] (33)
or
\[
m_0(r) + m_1(r) = -\frac{\Delta m}{2} \frac{c^2 r}{2G}
\] (34)

Let \(R_0\) be the distance of the Sun to the center of the Milky Way and \(V_0\) the measured
velocity of the Sun around the center of the Milky Way. The International Astronomical Union (IAU) recommends $R_0 = 8.50\, \text{kpc}$ and $V_0 = 220\, \text{km s}^{-1}$. For $R_0, V_0$ we found in the recent literature $8.34 \pm 0.16\, \text{kpc}, 2408\, \text{km s}^{-1}$ in [3], $8\, \text{kpc}, 200 / 280\, \text{km s}^{-1}$ in [4], $8.34\, \text{kpc}, 240\, \text{km s}^{-1}$ in [5], $8.20 \pm 0.09\, \text{kpc}, 233 \pm 3\, \text{km s}^{-1}$ in [6] and $8.122 \pm 0.031\, \text{kpc}, 233.6 \pm 2.8\, \text{km s}^{-1}$ in [7].

Let $V_L$ be the estimated velocity of the Sun around the center of the galaxy when only the luminous matter is considered. From the Figure 3 of [8] we have that $V_L$ varies from $152 \pm 2\, \text{km s}^{-1}$ to $179 \pm 2\, \text{km s}^{-1}$ for $8.0\, \text{mpc} < R_0 < 8.5\, \text{mpc}$. For two results used in the Figure 3 of [8] the value is $172 \pm 2\, \text{km s}^{-1}$.

Let us assume that $r_{\text{vir}} > 8.50\, \text{kpc}$ and $r_{\text{vir}} \ll r_{\text{vir}}$. The masses $m_L$ and $m_D$ of the galaxy within a radius of $r$ from the center to the Sun (or the probe) are determined with

$$M_{r_{\text{vir}}} - M_{\odot} = \frac{v^2 r}{G}$$

(35)

The Figure 3 shows $m_D$ in function of the velocity of the probe, $v_T$, which ranging from $1.50 \times 10^5\, \text{m s}^{-1}$ to $2.40 \times 10^5\, \text{m s}^{-1}$. The red curve is for $8.5\, \text{kpc}$ and $152\, \text{km s}^{-1}$, the dashed red curve is for $8.15\, \text{kpc}$ and $152\, \text{km s}^{-1}$, the blue curve is for $8.5\, \text{kpc}$ and $172\, \text{km s}^{-1}$ and the dashed blue curve is for $8.12\, \text{kpc}$ and $172\, \text{km s}^{-1}$.

The Figure 4 shows $\frac{\Delta m}{m_0}$ in function of $v_T$. The red curve is for $152\, \text{km s}^{-1}$ and the blue curve is for $172\, \text{km s}^{-1}$. In both $r = 8.5\, \text{kpc}$ is used.

For $R_0 = 8.50\, \text{kpc}$ and $v_L = 160\, \text{km s}^{-1}$ we obtain from (35)

$$1.006 \times 10^{41}\, \text{kg}$$

(36)
and for $V_o = 220 \text{ km/s}$

$$1.902 \times 10^4 \text{ kg}$$

(37)

From (36) and (37) the estimated dark mass of the galaxy within a radius of $R_o$ is

$$8.960 \times 10^4 \text{ kg}$$

(38)

The value of \( \frac{c^2 R_o}{G} \) is

$$3.53194 \times 10^7 \text{ kg}$$

(39)

**The values of \( H_0 \) and \( \Lambda \)**

In the equation (34) the term \(- \frac{c^2 r}{G}\) depends on the spatial dimension \(r\). Let us divide this term by the volume \(\frac{4}{3} \pi r^3\), the result is the specific mass

$$\rho(r) = - \frac{3c^2}{4\pi G r^2}$$

(40)

In [2] we found that the contribution of the vacuum energy to the field equation is

$$\Delta \rho_c = \Delta \rho_e = - \frac{H_0 l_{\text{planck}}}{c} \frac{\Delta e}{c^2}$$

(41)

where \(H_0\) is the Hubble constant, \(l_{\text{planck}}\) is Planck’s length and \(\Delta e\) the vacuum energy. Using

$$r = c t_0 = c \frac{a}{H_0}$$

(42)

where \(t_0\) is the age of the universe and (40), we have

$$\frac{3c^2}{4\pi G} \frac{a^2 \Delta e l_{\text{planck}}^2}{c^2}$$

(43)

The equation (42) results from the hypothesis that \(R \propto t^\alpha\) (that is, \(R = k t^\alpha\)). Solving (43) for \(a\) we obtain

$$a = \frac{c^2}{2 l_{\text{planck}} \sqrt{\frac{3}{\pi \Delta e G}}}$$

(44)

In Ferreira [2] the value of the vacuum energy is obtained with

$$\Delta e = \frac{\pi^3 h c}{45 (l_{\text{planck}})^2}$$

(45)

the Casimir result [10] for the force between perfectly conducting parallel plates was used and the result is

$$\Delta e = 1.016124 \times 10^{11} \text{ J/m}^3$$

(46)

Using (44) and (46) we obtain

$$a = 1.043350$$

(47)

Let us notice that matching (40) with (41) and using (42) we obtain

$$\Delta e = \frac{3c^2}{4\pi a^2 G^2 h}$$

(48)

The adopted age of the universe is equal to $1.380000 \times 10^{10}$ year (13.800 0.024 $\times 10^9$ year [9]). Using $r = c t_0$ into (40) we found
\[
\rho = -\frac{3}{4\pi G t_0^2}
\]

or

\[
\rho = -1.885957 \times 10^{-29} \text{ [kg m}^{-3}]\]

(50)

Using

\[
H_0 = \frac{a}{t_0}
\]

we obtain

\[
H_0 = 73.924 \text{[km s}^{-1}\text{Mpc}^{-1}]\]

(52)

The value given in (52) agrees with calculations based on observations of supernovae and pulsating stars. Reiss et al. [11], using the Cepheids+SN1a method, reduced the uncertainty in the determination of \(H_0\) to 2.3 %. They found \(H_0 = 73.48 \pm 1.66 \text{ km s}^{-1}\). In the Table 1 of [9] the value of \(H_0\) for Planck TT,TE,EE+lowE+lensing is 67.37 \(0.54 \text{ km s}^{-1}\). The discrepancy between these two values is around 3.5 \(\sigma\).

The field equation with the cosmological constant took the form

\[
G^\mu_\nu = \frac{8\pi G}{c^2} T^\mu_\nu - \Lambda \, g^\mu_\nu
\]

(53)

or

\[
\frac{c^2 \, G^\mu_\nu}{8\pi G} = \frac{T^\mu_\nu}{c^4} - \frac{c^2 \Lambda}{8\pi G} g^\mu_\nu
\]

(54)

The estimation of the factor \(-\frac{c^2 \Lambda}{8\pi G}\) that multiplies \(g^\mu_\nu\) in (54) is given in (50), therefore

\[
\Lambda = 1.885957 \times 10^{-29} \text{ [kg m}^{-3}] \times \frac{8\pi G}{c^2}
\]

(55)

or

\[
\Lambda = 3.519884 \times 10^{-52} \text{ [m}^{-2}]\]

(56)

Reference

1. Ferreira JC. On Inertial Frames. 2018.
2. Ferreira JC. The field equation, black holes and vacuum energy. 2018.
3. Reid MJ, Menten KM, Brunthaler A, Zheng XW, Dame TM, et al, Trigonometric Parallaxes of High Mass Star Forming Regions: the Structure and Kinematics of the Milky Way. The Astrophysical Journal. 2014; 783: 130: 14. Ref.: http://bit.ly/31Xlnr0
4. Sofue Y. Rotation and Mass in the Milky Way and Spiral Galaxies. ArXiv e-prints, arXiv:1608.08350v1 [astro-ph.GA]. 2016; Ref.: http://bit.ly/2ZZyPsF
5. Russell D. The Milky Way rotation curve revisited, Astronomy & Astrophysics. 2017; 601: L5. Ref.: http://bit.ly/2zkPc6w
6. Crosta M, Marco G, Lattanzi Mario G, Eloisa P. Shedding light on the Milky Way rotation curve with Gaia DR2. ArXiv e-prints, arXiv:1810.04445v2 [astro-ph.GA]. 2018; Ref.: http://bit.ly/2ZZyPsF
7. Mróz P, Udalski A, Skowron DM, Skowron J, Soszynski I, Pietrukowicz P, et al, Rotation Curve of the Milky Way from Classical Cepheids. The Astrophysical Journal Letters. 2019; 870. Ref.: http://bit.ly/2NjFWO
8. Cisneros S, O’Brien JG, Oblath NS, Formaggio JA. The Luminous Convolution Model for spiral galaxy rotation curves. ArXiv e-prints, arXiv:1506.04587v1 [astro-ph.GA]. 2015; Ref.: http://bit.ly/2FABph2
9. Aghanim N, Akrami Y, Ashdown M, Aumont J, Baccigalupi C, et al. Planck 2018 results. VI. Cosmological parameters, ArXiv e-prints, arXiv:1807.06209v1 [astro-ph.CO]. 2018; Ref.: http://bit.ly/2UbYU
10. Casimir HBG. On the attraction between two perfect conducting plates, Proceedings of the Royal Netherlands Academy of Arts and Sciences. 1948; 51: 793-795.

11. Riess AG, Casertano S, Yuan W, Macri L, Anderson J, et al. New Parallaxes of Galactic Cepheids from Spatially Scanning the Hubble Space Telescope: Implications for the Hubble Constant, ArXiv e-prints, arXiv:1801.01120v2 [astro-ph.SR]. 2018. Ref.: http://bit.ly/2X6tGNF
