Analysis of sun-moon gravitational acceleration and engineering realization of space target

Xiaobin Huang*, Yan Zhang, Rui Xiao, Li Lu and Yuan Jiang
Air Force Early Warning Academy, 430019 Wuhan, China

*Corresponding author. Email: 532454132@qq.com

Abstract. Space target orbit determination is a core function of space target surveillance radar, which involves very complicated mathematical principles, and it is very difficult for professional radar system designers with no orbital mechanics to develop corresponding software modules. The difficulty in developing orbit determination software is the calculation of perturbation acceleration, which is the basis for realizing high-precision numerical orbit calculations. To this end, this paper takes the solar-moon gravitational perturbation that cannot be ignored in the precision orbit determination of high-orbit satellites as an example, introduces its mathematical principles, gives the program design ideas for acceleration calculation, implements the interface function and integrates it into the author’s early development radar orbit determination library (RadarOrbDet). The simulation results verify the effectiveness of the interface function and show that the orbit determination function library can assist scientific researchers in radar system design. The research results of this paper can be used as a reference for the spacecraft orbit analysis of interplanetary flight.

1. Introduction
Space target orbit determination is a core function of space target surveillance radar. It mainly includes processing steps such as coordinate conversion, initial orbit determination, and orbit improvement, which involves very complex mathematical principles. Therefore, the development of orbit determination function modules is very difficult for radar system designers who are not specialized in orbital mechanics. Although there is some open source orbit determination software for references [1-5], these software is mainly used to process optical observation data and is not suitable for processing radar observation data. To this end, the author developed the RadarOrbDet library [6] to assist non-orbital mechanics personnel in the system design of space target surveillance radars. The coordinate conversion module [7] and the initial orbit determination module [8] have been developed. This article studies the development of the orbit improvement module.

Orbit improvement is the key and difficult point in determining the orbit of a space target. It involves a lot of content, including perturbation analysis, variational analysis of partial differential equations, least squares optimization and other theories [9]. This article focuses on perturbation analysis, which is the basis for realizing high-precision orbit calculation. Its core is to analyse the force model of the target and calculate various perturbation accelerations. In addition to the central gravity of the earth, other small forces that the space targets orbiting the earth subjected to are collectively referred to as perturbations, including non-spherical perturbation of the earth, atmospheric resistance perturbation, sun-moon gravitational perturbation, etc. [9]. This paper studies the sun-moon gravitational perturbation, and introduces the analysis method of acceleration and its engineering
realization, which is especially important for the precise orbit determination of medium and high orbit satellites.

The main content of this paper includes mathematical principles of the sun-moon perturbation acceleration, program design ideas and use of STK software to verify the effectiveness of the interface function.

2. Mathematical principle of the sun-moon gravity perturbation acceleration

According to Newton’s law of universal gravitation, in the geocentric celestial reference system (GCRS) [10], the satellite acceleration caused by the point mass $M$ is as follows [11]

$$\ddot{r} = GM \left( \frac{s - r}{|s - r|^3} - \frac{s}{|s|^3} \right)$$

(1)

Where $r$ and $s$ are the geocentric position vectors of the satellite and $M$ respectively, and $G$ is the gravitational constant.

Since the distance between the sun or the moon and the earth is much greater than that of most satellites relative to the earth, in qualitative analysis, formula (1) can be further simplified to investigate the characteristics of acceleration in the GCRS coordinate system. Formula (1) is expanded by series, and after omitting high-order small quantities, it can be transformed into

$$\ddot{r} \approx \frac{GMr}{s^3} \left[ -e_s + 3e_e (e_e, e_s) \right]$$

(2)

In above formula, the unit vector $e_e = s/s$, $e_s = r/r$.

If $e_e = \pm e_s$, formula (2) becomes

$$\ddot{r} \approx \frac{2GMr}{s^3}$$

(3)

If $e_e e_s = \pm 0$, formula (2) becomes

$$\ddot{r} \approx -\frac{GMr}{s^3}$$

(4)

From formulas (2), (3), (4), it can be seen that the acceleration is proportional to the distance between the satellite and the earth, and inversely proportional to the third power of the distance between the third body and the earth. Moreover, as long as the satellite is collinear with the earth and the third body, the satellite will be subject to acceleration away from the earth, and when the angle between the satellite and the third body is at a right angle to the earth, the satellite will be subject to acceleration directed to the earth.

It can be seen from formula (1) that in order to calculate the acceleration, it is necessary to know the position vector of the third body in the GCRS coordinate system. This can be calculated with the help of the JPL ephemeris introduced by the next section.

3. JPL Ephemeris

In the 1960s, because the accuracy of the ephemeris at that time was difficult to meet the needs of space navigation, an ephemeris development plan was implemented at the Jet Laboratory (JPL) in the United States to support the observation of the solar system and the analysis of observational data. By the early 1970s, the JPL ephemeris had become a world standard. Currently, the planet/moon ephemeris recommended by the International Earth Rotation Service (IERS) is JPL’s DE405/LE405 ephemeris. JPL ephemeris is currently widely used in data analysis for space navigation, planetary exploration, and astronomical precision observations.

3.1. Ephemeris structure

The JPL ephemeris has multiple versions according to the creation time. DE405, which was created in 1997, is widely used here and includes the positions of the nine planets and the moon in the solar system from 1599 to 2201. The core files of DE405 include header.405 and coefficient file
ascp****.405. ‘****’ represents the start time of the coefficient file. Each coefficient file contains the Chebyshev interpolation coefficient of the celestial body position in 20 years. For example, the coefficients from 2000 to 2020 are contained in the file ascp2000.405.

3.1.1. Head file. The head file header.405 of DE405 contains DE405 data information, astronomical constants and data indexes, which are stored in grouped form. The specific meaning is as follows.

1) Group 1040: ephemeris header information
   Contains the ephemeris name, the start and final epochs of the ephemeris in Julian days and Gregorian calendar.

2) Group 1030: ephemeris span information
   Contains the start and final epochs of the ephemeris expressed in Julian days, and the time span of ephemeris records.

3) Group 1040: ephemeris constant name
   Contains the number of ephemeris constants and the names of these constants, such as astronomical unit, earth-moon mass ratio, etc.

4) Group 1041: ephemeris constant values table
   Every three ephemeris constant values are arranged in one row, and the orders of these values are the same as ephemeris constant names in Group 1040.

5) Group 1050: data index table
   The data index in header.405 is a table with 3 rows and 13 columns, for paper layout, table is transposed. As shown in Table 1, each row of data represents the position of a celestial body in the data block in the coefficient file, which is Mercury, Venus, Earth-Moon system, Mars, Jupiter, Saturn, Uranus, Neptune, Pluto, Moon, and Sun. The data in the 12th row represents nutation angles, including two angles: ecliptic nutation $\Psi$ and angular nutation $\epsilon$. The data in row 13 represents lunar librations, including three Euler angles $\zeta$, $\z$, $\theta$. The first value of each row indicates the starting position of the celestial body data in the data block, the second value indicates the order of the Chebyshev polynomial used in interpolation approximation, and the third value indicates that the celestial body data is divided into several sub-intervals. The last line is the dimension of the data (3 means three-axis, this line is not specifically given in the index table, it is the default value).

### Table 1. Ephemeris data index table information.

| Item                | $P_1$ | $P_2$ | $P_3$ | $P_4$ |
|---------------------|-------|-------|-------|-------|
| Mercury             | 3     | 14    | 4     | 3     |
| Venus               | 171   | 10    | 2     | 3     |
| Earth-moon barycenter | 231    | 13    | 2     | 3     |
| Mars                | 309   | 11    | 1     | 3     |
| Jupiter             | 342   | 8     | 1     | 3     |
| Saturn              | 366   | 7     | 1     | 3     |
| Uranus              | 387   | 6     | 1     | 3     |
| Neptune             | 405   | 6     | 1     | 3     |
| Pluto               | 423   | 6     | 1     | 3     |
| Moon                | 441   | 13    | 8     | 3     |
| Sun                 | 753   | 11    | 2     | 3     |
| Nutation            | 819   | 10    | 4     | 2     |
| Lunar libration     | 899   | 10    | 4     | 3     |

Take Mercury's data index: 3, 14, 4 as an example, the details are as follows:
1) The digital 3 indicates that Mercury's Chebyshev coefficient starts from the 3th data in the data block;
2) The digital 14 is the Chebyshev polynomial order, that is, the position data of each axis is represented by 14 Chebyshev coefficients, and there are three axis coefficients in \( x, y \) and \( z \);
3) The digital 4 is the number of divided sub-intervals. Due to the different motion periods of celestial bodies, the number of divided sub-intervals is different. There are more sub-intervals of celestial bodies with shorter periods and irregular motion, the moon has up to 8 sub-intervals.

3.1.2. Coefficient file. Take the DE405 coefficient file ascp2000.405 as an example. It consists of 229 data blocks, each of which represents 32 days and contains 1018 data. The first line of each data block is the serial number and the amount of data, starting from the second line, one row for every three data, the first data is the start time of the data block, the second data is the end time of the data block, and then in turn position data and nutation data of Mercury, Venus, etc., and lunar libration data.

In the ephemeris file, the position of the planet is given in kilometers (the unit of speed is kilometers per second \( \text{km/s} \)). Nutation and lunar libration are given in radians. If the user does not modify it, the planet position given by the FORTRAN version program provided by JPL is automatically converted to the astronomical unit \( \text{Au} \), and the unit of velocity is the astronomical unit per day \( \text{Au/day} \).

The detailed structure of the ascp2000.405 file is shown in Figure 1.

![Figure 1. Data structure of ascp2000.405 coefficient file.](image)

3.2. Ephemeris calculation
JPL ephemeris uses the Julian day form of TDB time as the interpolation time. Versions smaller than 200 series (such as DE118) use the B1950 coordinate system, and DE200 series use J2000 coordinate system. In the latest DE400 series, all data refer to the International Celestial Reference System (ICRS) [10]. The position values obtained by DE405 are referred to ICRS coordinate system (except for the moon, which referred to GCRS coordinate system). The GCRS and the ICRS coordinate system differ only in the origin of the coordinate system. The former is the center of the earth and the latter is the solar system barycenter.

When calculating the perturbation acceleration of the sun and the moon, it is necessary to know the position of the sun and the moon in the GCRS coordinate system. The latter is directly given in the
JPL ephemeris, while the former is not. This requires the calculation of the position of the sun through spatial geometric relations.

Figure 2 shows the geometric position relationship of related celestial bodies. In the figure, $\mathbf{P}_{bs}$ is the vector of the solar system barycenter pointing to the sun, $\mathbf{P}_{be}$ is the vector of the solar system barycenter pointing to the earth-moon barycenter, and $\mathbf{P}_{em}$ is the vector of the earth pointing to the moon. These three vectors are directly given by the JPL ephemeris. We need to solve the vector of the earth pointing to the sun through the geometric relationship shown in Figure 2, namely $\mathbf{P}_{es}$.

From the geometric relationship in the figure, $\mathbf{P}_{es}$ can be obtained as

$$\mathbf{P}_{es} = \mathbf{P}_{eg} + \mathbf{P}_{gs}$$  \hspace{1cm} (5)

Assuming that $m_e$ and $m_m$ are the masses of the earth and the moon respectively, there are

$$\frac{1}{m_e} + \frac{1}{m_m} = \frac{1}{m_{em}} + \frac{1}{m_{es}}$$  \hspace{1cm} (6)

In the formula, $m_e/m_m$ is the earth-moon mass ratio constant, which can be obtained from the JPL ephemeris constant table, which is about 81.3.

In addition, it is easy to get from the geometric relationship

$$\mathbf{P}_{es} = \mathbf{P}_{eg} - \mathbf{P}_{gs}$$  \hspace{1cm} (7)

Substituting formulas (6) and (7) into formula (5), we get

$$\mathbf{P}_{es} = \frac{1}{1 + \frac{m_e}{m_m}} \mathbf{P}_{es} + \frac{1}{m_m} \mathbf{P}_{es} - \mathbf{P}_{gs}$$  \hspace{1cm} (8)

4. Program design

4.1. Interface function implementation

Figure 3 is a program flow chart for calculating the perturbation acceleration of gravity of the sun and moon.
Input: UTC epoch, Satellite geocentric vector
UTC time to TDB time
JPL ephemeris file jpleph.405
Call the jpl_eph library to calculate the geocentric vectors of the sun and moon
Output: Calculate gravitational acceleration by formula (1)

Figure 3. Program calculation flow chart for calculating the acceleration of sun and moon gravity.

Since the JPL ephemeris uses the TDB time format, first convert UTC time to TDB time [10], then call the jpl_eph library [13] to calculate the geocentric vectors of the sun and the moon, and finally use formula (1) to calculate the acceleration.

The jpl_eph library is a C language implementation library for JPL ephemeris calculation. It translates and repackages the FORTRAN version programs provided by JPL, which is very convenient to use. The core function in the library is jpl_pleph, which can directly calculate the position and velocity vector of a celestial body relative to another celestial body.

Figure 4 shows the interface function for calculating the perturbation acceleration of the solar gravitational. The jpl_pleph library is called in this function, which has been integrated into the RadarOrbDet library [6].

4.2. Interface function validity verification

Two satellites are simulated in STK11.2, and their initial orbital elements are set to be the same, as shown in Table 2. The first satellite only sets the 21st-order earth gravity model, and the second satellite sets the 21st-order earth gravity model, plus the sun gravitational perturbation, as shown in Figure 3. Using the report function of STK, the accelerations of the two satellites at 2021-02-01 00:00:00 are obtained respectively, and the two accelerations can be subtracted to obtain the perturbed acceleration of the second satellite caused by the sun's gravity. In addition, call function shown in Figure 4 with the above-mentioned epoch time and the corresponding satellite vector, the solar perturbation acceleration can be calculated. These results are shown in Table 3, using the relative error described in formula (9) for quantitative comparison.

\[
\text{Relative error} = \frac{|a_{\text{software}} - a_{\text{STK}}|}{a_{\text{STK}}} \quad (9)
\]

It can be seen from Table 3 that the error is 0.01367%, indicating that the software algorithm in this paper is effective.
void radAccelSunPerturbation(double utc, double :[3], double a_ei[3])
{
    double tail, tai2, tti, tt, ttt, tdb;
    double rrd[6], d[3], s[3], nd, ns, gns;
    char nmas[10];

    iauUctai(utc, 0, &tail, &tai2);
    iauTaitt(tai1, tai2, &tti, &ttt);
    tt = tti + ttt;
    ttt = (tt - 2651646.0) / 36525.0;
    tdb = tt + (0.0001675*sin(628.3076*ttt + 6.2401) + 0.0000225*sin(575.3385*ttt + 4.2870)
               + 0.0000145*sin(1256.6162*ttt + 6.1950) + 0.0000025*sin(606.9777*ttt + 4.0212)
               + 0.0000055*sin(52.9691*ttt + 6.0444) + 0.0000025*sin(21.3293*ttt + 5.5451)
               + 0.0000010*ttt*sin(0.3307*t + 4.2340)) / 86400;

    jpl_planeph(epeh, tdb, ii, 3, rrd, 0);
    iauSpM(0, rrd, s);
    iauSpM(i, s, d);
    nd = iauSpM(d);
    ns = iauSpM(s);
    iauSpPsp(d, pov(nd / ns, 3), s, a_ei);
    gns = jpl_get_constant(17, epeh, nmas);
    gns = gns + pov(000, 3) / (88460.0 * 88460.0);
    iauExp(-gns / pov(nd, 3), a_ei, a_ei);
}

Figure 4. Interface function for calculation of solar gravitational perturbation acceleration.

Table 2. Satellite orbit elements.

| Item                  | Value                        |
|-----------------------|------------------------------|
| Epoch                 | 2021-02-01 00:00:00          |
| Semi-major axis       | 6678.14km                    |
| Eccentricity          | 0                            |
| Inclination           | 28.5°                        |
| Argument of perigee   | 0°                           |
| Ascension of ascending node | 0°                          |
| True anomaly          | 0°                           |

Figure 5. Satellite force model settings.
Table 3. Comparison table of solar perturbation acceleration.

| category | item | Acceleration(m/s) |
|----------|------|-------------------|
|          |      | x                | y                | z                |
| STK      | S1   | -8.9509728       | -8.706206×10⁻⁵  | -1.245746×10⁻²  |
|          | S2   | -8.9509727       | -8.744095×10⁻⁵  | -1.247388×10⁻²  |
|          | Sb   | 9.579999×10⁻⁵    | -3.788866×10⁻⁴  | -1.642459×10⁻⁴  |
| Software | Sb   | 9.574232×10⁻⁵    | -3.788857×10⁻⁴  | -1.642454×10⁻⁴  |

Relative error 0.01367%

* S1=Satellite 1, S2=Satellite 2, Sb=Sun perturbation

5. Conclusion
This article introduces the mathematical principle of the acceleration of the sun-moon gravitational perturbation, describes the program design ideas and the interface function, and finally verifies the effectiveness of the algorithm with STK software. The analysis of atmospheric drag perturbation acceleration and its engineering realization will be further studied in our next work.

Acknowledgements
The authors thank SHI Bin-bin and OU Yang-yan for useful discussions on the programming of our software.
The research was supported by the College HOUJI Foundation Project (Grant No. HJGC-2021-015).

References
[1] University of Pisa. The OrbFit Software Package[EB/OL].(2017-04-20)[2021-01-30] http://adams.dm.unipi.it/~orbmain/rtorbit/
[2] Pasquale Tricarico. Orbit Reconstruction, Simulation and Analysis ORSA[EB/OL]. (2018-09-14)[2021-01-30] http://orsa.sourceforge.net/index.html
[3] European Space Agency. Asteroid and Comet Trajectory Propagator[EB/OL]. (2015-11-23)[2021-01-30]. http://neo.ssa.esa.int/neo-propagator
[4] The United States Naval Observatory. NOVAS[EB/OL]. (2016-11-02)[2021-01-30]. https://pypl.org/project/novas/
[5] National Aeronautics and Space Administration. Orbit Determination Toolbox[EB/OL]. (2019-05-15)[2021-01-30] https://sourceforge.net/projects/odtbx/
[6] Huang Xiao-bin. Radar orbit determination library (RadarOrbDet)[EB/OL]. (2019-10-20)[2021-01-30]. https://pan.baidu.com/s/19kYFKZy77awg6pNnL-Y73Q
[7] Huang Xiao-zhong, Xiao Xin, Xiao Rui. Discussion on radar orbit determination coordinate transformation of space targets[J]. Journal of Navigation and Positioning, 2020,8(4): 39-43
[8] Zhang Yan, Huang Xiao-bin, Xiao Rui, etc. Discussion on radar orbit determination of space target[A]. Proceedings of the 2013 National Avionics Information Technology High-end Forum (Part 1)[C]. Beijing: China Electronics Technology Group Co., Ltd., 2020
[9] Vallado D. A. Fundamentals of Astrodynamics and Applications[M]. 4th ed. McGraw-Hill, New York, 2013: 129-498
[10] Petit G, Luzum B. IERS Conventions (2010)[EB/OL]. (2010-12-25)[2021-01-30]. http://tai.bipm.org/iers/conv2010/
[11] Oliver Montenbruck, Eberhard Gill. Satellite Orbits Models, Methods and Applications[M]. Springer Verlag, Heidelberg, 2005: 69-76
[12] Ma Gao-feng, Lu Qiang, Zheng Yong. JPL Planetary/Lunar Ephemeris[C]. The First Academic Conference of the Deep Space Exploration Technology Committee of the Chinese Astronautical Society. Harbin, 2005:395-401
[13] Bill-Gray. C/C++ source code for JPL DE ephemerides[EB/OL]. (2016-08-01)[2021-01-31]. http://www.projectpluto.com/jpl_eph.htm