Non-Supersymmetric Theories with Light Scalar Fields and Large Hierarchies

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Abstract

Various nonsupersymmetric theories at large but finite $N$ are argued to permit light scalars and large hierarchies without fine-tuning. In a dual string description, the hierarchy results from competition between classical and quantum effects. In some cases the flow may end when a string mode becomes tachyonic and condenses, thereby realizing a quantum-mechanically stable Randall-Sundrum hierarchy scenario. Among possible applications, it is suggested that lattice simulation of $\mathcal{N} = 4$ Yang-Mills at large ’t Hooft coupling may be easier than expected, and that supersymmetry may naturally be an approximate symmetry of our world. (This letter is a writeup of work presented at Aspen in summer 2002.)

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Naively, nonsupersymmetric field theories with light scalar fields can only be obtained through fine tuning. A relevant mass operator which is a singlet under all global symmetries can always be obtained by replacing every $\partial_\mu$ in the scalar kinetic terms with a mass parameter $m$. In complete generality, any global-singlet relevant operator (GSRO) naturally has a coefficient of order a power of some cutoff $\Lambda_{UV}$; thus $m^2 \sim \Lambda_{UV}^2$ unless fine-tuning is applied. Indeed, the Higgs boson mass is the only GSRO in the standard model of particle physics, and is the source of the hierarchy problem.

There are well-known ways to evade this constraint. Supersymmetry (SUSY) relates scalar and fermion mass operators; the latter can be forbidden by a chiral symmetry. For Nambu-Goldstone bosons, the shift symmetry $\delta \phi = \text{constant}$ eliminates the mass operator. Conformal symmetry forbids all dimensionful coefficients. However, these symmetries cannot be realized as accidental symmetries in theories with GSROs; if they are not exact, a mass of order $c\Lambda_{UV}^2$, where $c$ is at best an algebraically-small parameter, will be generated.

Another recent example is similar. The planar graphs of a SUSY gauge theory are shared by its “orbifolds” \cite{1}, obtained via a projection operator that eliminates certain fields of the SUSY gauge theory. At $N = \infty$, then, the orbifold theories have $m = 0$. However, nonplanar graphs give $\delta m^2 \sim \Lambda_{UV}^2/N^p$ where $p \geq 1$. Exponential suppression of $m$ requires exponentially large $N$.

All of these ideas have been suggested for solving the standard model hierarchy problem. The SUSY story is well-known; the Higgs-as-pseudo-Goldstone-boson has recently been made to work successfully in deconstruction-motivated models \cite{2}; and conformal symmetry has been suggested by Bardeen \cite{3} and in the context of orbifold models by Frampton and Vafa \cite{4}. Any new ideas might also be useful for model building.

In this article I suggest a novel mechanism which allows light scalars. \cite{22, 23} The above arguments rely on perturbative intuition, for which $m^2$ has positive mass dimension. However, if the mass operator has a large anomalous dimension, it may actually be irrelevant; in this case $m^2$ may be present, but it runs to zero (faster than the Wilsonian renormalization scale) and its infrared effects are suppressed. Using AdS/CFT duality \cite{5}, theories can be constructed which have no GSROs whatsoever; correspondingly, the dual string theories contain no gauge-singlet tachyons. In some cases these models allow (and may generate) exponentially small scalar masses and large hierarchies. (In the string description, this happens through a balancing of classical ($N^0$) and quantum ($N^{-2}$) effects.) These non-SUSY theories
naturally realize Randall-Sundrum-like hierarchies \[6\] without quantum inconsistencies or instabilities.

Consider $SU(N) \; d = 4 \; \mathcal{N} = 4$ SUSY YM, with gauge coupling $g_{YM}$; define $\alpha = g_{YM}^2/4\pi$, and the 't Hooft coupling $\eta \equiv \alpha N$. Its dual is IIB string theory on $AdS_5 \times S^5$, with string coupling $g_s \equiv \alpha$. By hand, add a UV cutoff at which SUSY is completely broken but $SO(6)$ is preserved. The effective action at $\Lambda_{UV}$ includes all possible $SO(6)$-singlet operators with coefficients of a natural size. Classically, this theory has dimension-2 scalar bilinears $\mathcal{K} = N^{-1} \text{tr} \phi^k \phi^k$ and $\mathcal{O}_{2}^{ij} = N^{-1} \text{tr} \phi^i \phi^j - \frac{1}{6} \delta^{ij} \mathcal{K}$, in the 1 and 20’ of $SO(6)$. The former is a GSRO which we cannot forbid by any symmetry. There are no $SO(6)$-singlet dimension-3 operators. At dimension 4 there are several global-singlet marginal operators (GSMOs): the single-trace operators which appear in the Lagrangian, and the double-trace operators $\mathcal{K}^2$ and $\mathcal{O}_{2}^{ij} \equiv \sum_{ij} \mathcal{O}_{2}^{ij} \mathcal{O}_{2}^{ij}$.

For $\eta \ll 1$, the dynamics is entirely determined by the operator $\mathcal{K}$, which is allowed by all symmetries and will rapidly dominate the infrared. The scalars are massive, but the four adjoint fermions remain massless. The IR behavior of this theory is unknown.

The situation at $\eta \gg 1$ is different. Take $N$ large but finite, of order, say, 10-100, and choose $\eta$ large but finite, of order 5-50, at the cutoff $\Lambda_{UV}$. The operators of the gauge theory correspond to modes of supergravity (SUGRA) fields on $AdS_5$. All modes with dimensions of order 1 are classified in \[7\]; there are no GSROs! In fact $\mathcal{K}$ corresponds to a massive string mode (see e.g. \[9\]), and has dimension of order $(4\pi \eta)^{1/4} \gg 1$, greater than four. Consequently the effect of $\mathcal{K}$ decreases in the IR, and the scalars remain light despite the lack of SUSY.

Absent a GSRO, the theory is governed by two GSMOs: the double trace operator $\mathcal{O}_{2}^{2}$ and the $\mathcal{N} = 4$ Lagrangian itself, $\mathcal{O}_{4} = N^{-1} \text{tr}(F_{\mu\nu} F^{\mu\nu} + \ldots)$. With the $\theta$ angle zero for simplicity, the Lagrangian is

$$\mathcal{L} = \frac{N}{2g^2} \left[ \mathcal{O}_{4} - \frac{\hbar^2 N^2}{4g^2 N} \mathcal{O}_{2}^{2} \right]$$

plus boundary terms and terms that vanish on the equations of motion. The expansion parameters for perturbation theory are $\eta = g^2 N/4\pi$ and $\xi \equiv \hbar^2 N^2/4\pi$. The $\xi \ll 1 \ll \eta$ region is controlled by conformal perturbation theory around $\mathcal{N} = 4$ SYM. The operator $\mathcal{O}_{2}^{2}$ is marginally relevant, with $\text{dim}(\mathcal{O}_{2}^{2}) - 4 = -(16/N^2)$ for $\eta \gg 1$ \[4\] and $-(5/2\pi)(\eta/N^2)$ for small $\eta$ \[8\]. [That the correction is of order $1/N^2$ follows from large-$N$ factorization of $\langle \mathcal{O}_{2}^{2}(x) \mathcal{O}_{2}^{2}(0) \rangle$ and nonrenormalization of $\langle \mathcal{O}_{2}^{ij}(x) \mathcal{O}_{2}^{kl}(0) \rangle$. In SUGRA $\mathcal{O}_{2}^{2}$ corresponds to a
non-BPS particle-antiparticle state, with a negative binding energy — hence the sign.

Note we may now (if we wish) take $\Lambda_{UV} \to \infty$, defining without fine tuning a renormalizable theory which is $\mathcal{N} = 4$ SUSY in the UV with SUSY broken by $h \neq 0$. The breaking is soft due to the absence of GSROs. For now we will keep $\Lambda_{UV}$ finite (since in the real world gravity will always add additional physics) while keeping in mind that this limit may always be taken.

I will now address the following issues. First, I will show the beta functions for $\eta$ and $\xi$ are at most of order $1/N^4$. Next, I will argue the theory has no Coleman-Weinberg instabilities except possibly at exponentially small energy scales. Finally, although I will not determine the sign of $\beta_{\eta}$, I will show that for either sign interesting physics, and a large or infinite hierarchy, results.

**Beta Functions I:** The dimensionless coupling which determines the validity of perturbation theory is $h\mu^{-\text{dim}(h)}$. For this reason we redefine $\xi(\mu) = h^2(\mu)\mu^{-2\text{dim}(h)}N^2/4\pi$. This coupling blows up, classically, at $\Lambda_\xi \sim \xi_0^{N^2/16} \Lambda_{UV} \ll \Lambda_{UV}$, where $\xi_0$ is $\xi(\mu = \Lambda_{UV})$. But in fact $\xi$ never becomes large. At order $\xi^2$ the $\beta$ function receives a large positive correction

$$
\beta_\xi = \xi \left( -\frac{16}{N^2} + \frac{\xi}{4\pi} \right).
$$

[10] The order-\xi correction to $\text{dim} \mathcal{O}_2^2$ is

$$
\propto \int d^4z \langle \mathcal{O}_2^2(x)\mathcal{O}_2^2(0)\mathcal{O}_2^2(z) \rangle.
$$

This factorizes, so it is order $N^0$; its sign is as in $\phi^4$ theory.] Thus the flow drives $\xi$ to $\sim 64\pi/N^2$ and $\beta_\xi \lesssim 100/N^4$.

Meanwhile, $\xi = 0$ implies $\beta_{\eta} = 0$ by $\mathcal{N} = 4$, and the leading $\xi$-dependence involves connected graphs, so

$$
\beta_{\eta} = C(\eta) \frac{\xi}{N^2}.
$$

where $C(\eta)$ is an unknown function of order $N^0$. [In SUGRA, the $N^{-2}$ can also be obtained: the addition of $\mathcal{O}_2^2$ to the action [12] involves a change of boundary conditions for the mode $U^{ij}$ corresponding to $\mathcal{O}_2^{ij}$ [10, 11] but for $\langle \mathcal{O}_2^{ij} \rangle = 0$ this can have no classical effect on the dilaton.] Since $\xi$ itself is order $1/N^2$, $\beta_{\eta} \sim 1/N^4$. Thus all beta functions are small and no appreciable change of couplings occurs above scales of order $e^{-cN^4}\Lambda_{UV}$, where $c$ is $N$-independent but could be $\lesssim 10^{-2}$. 

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Stability: Could there be a Coleman-Weinberg instability? The $\mathcal{N} = 4$ SUSY YM has a moduli space; classically, most is lifted by the $h^2\mathcal{O}_2^2$ perturbation, but some flat directions in the potential energy remain. Could quantum corrections make the origin unstable? The approximate conformal invariance at the origin of moduli space implies any effective potential whose overall scale is $v$ scales as $v^4$ near the origin. Along any classically-flat direction, the $SU(N)$ gauge group is broken to a product group which contains decoupled $U(1)$ factors, whose scalars have an $\mathcal{O}_2^2$ interaction. These factors are at small 't Hooft coupling and can be studied in perturbation theory. As with any $\phi^4$ interaction, a positive effective mass-squared $m_{\text{eff}}^2\phi^2$ will be generated; but the relevant cutoff is $v$, so altogether this represents a positive $v^4$ term in the potential. This suggests the theory at $v = 0$ is stable. Of course the argument receives corrections near $v = \Lambda_{\text{UV}}$ (but in this regime, couplings of irrelevant operators can be chosen so as to avoid any minima in the energy) and from the non-zero running there may be effects at exponentially small $v$ (but any minima in this regime would be interesting rather than problematic.)

Beta Functions II. Let us assume the above argument is correct and address the issue of $\beta_\eta$. Unfortunately the computation of this quantity, or even the sign, is problematic, and will not be attempted here. Instead, let us note that both signs are interesting.

$\beta_\eta < 0$: In the IR $\eta$ becomes large; classical string theory breaks down. Since $\mathcal{N} = 4$ has $g_s \to 1/g_s$ duality, the beta function must change sign at large $\eta$, suggesting the presence of at least one IR-stable conformal fixed point. This fixed point will have massless scalars (but no moduli space, see above). It is probably non-SUSY, although there is also the possibility of accidental $\mathcal{N} = 4$ SUSY. If there is no such fixed point, then something else interesting must happen when $\eta \sim N$, leaving the scalars exponentially light and/or with exponentially small vevs.

$\beta_\eta > 0$: Now $\eta$ decreases in the IR. Let us call the scale where $\eta \sim 1$ the cross-over scale $\Lambda_{\text{co}}$. What can happen at this scale? Since dim $\mathcal{O}_2^2 \to 4$ as $\eta \to 0$, the coupling $\xi$ remains small. There might be a fixed point for $\eta \sim 1$, but this seems unlikely, because in this region the operator $\mathcal{K}$ is a GSRO, making accidental conformal invariance impossible. Instead, $\mathcal{K}$ acts as a “dangerous irrelevant” operator; its coefficient $m^2$ shrinks for $\mu > \Lambda_{\text{co}}$ but grows for $\mu < \Lambda_{\text{co}}$, becoming large and dominating the IR physics. Note $m^2$, although multiplicatively renormalized in the SUSY theory, is additively renormalized when $\xi \neq 0$, and is not driven exactly toward zero. Therefore, at $\Lambda_{\text{co}}$, $|m^2| \gtrsim \xi \Lambda_{\text{co}}^2$; it cannot be smaller. (The sign of
the additive renormalization is presumably positive, as in $\phi^4$ theory, though not strictly calculable at $\eta \sim 1$.) Thus, the mass does not generate a scale \textit{exponentially} small compared to $\Lambda_{co}$: the mechanism of Bardeen — in which scalar masses are protected by the scale invariance of the theory — is not quite realized. Rather, the \textit{irrelevance} of $K$ has led to a scale exponentially smaller than $\Lambda_{UV}$ and of order $\Lambda_{co}/N$.

In both cases, any expectation values, dimensionful coupling constants and/or scalar masses lie exponentially [in $N^4$] below $\Lambda_{UV}$, without any fine tuning. Having established this result, I will make a few comments. Thereafter, I will discuss other models with similar properties, as well as variants which more certainly have positive $\beta_\eta$.

\textit{Dual string description:} In the AdS description (with radial coordinate $r \propto \mu$) no GSROs means no gauge-singlet tachyonic modes at large $r$. Classically the only effect (away from $r_{\text{max}} \propto \Lambda_{UV}$) of SUSY breaking is the altered boundary condition on the mode $U^{ij}$ corresponding to $O_2^{ij}$. However this boundary condition affects the the dilaton $S$ at the quantum level, causing it to vary ($dS/dr \propto \beta_\eta$) along with the curvature radius. The absence of a Coleman-Weinberg instability implies the force (classical plus one-loop) between D3-branes in the presence of the new boundary condition is attractive; thus the branes will remain in a single clump and the nearly-AdS region will be preserved. If $\beta_\eta > 0$ the string coupling and curvature radius decrease until, at some small $r_{\text{min}} \propto \Lambda_{co}$, the lightest string state becomes tachyonic, and condenses at and/or inside this region (possibly via both non-normalizable and normalizable modes.) Thereafter the dynamics of the theory becomes difficult to guess, but the space certainly cuts off near this point. If $\beta_\eta < 0$ the string coupling gradually increases as $r$ decreases; when $\eta \sim N$, a fixed point may be reached, or perhaps other dynamics sets in.

\textit{Model building:} With no IR fixed point, the theory generates a low \textit{non-zero} scale automatically; but even when there is a fixed point, it is easy to dynamically generate a low non-zero scale at which scalar masses would appear. Simply couple this theory through $SO(6)$-preserving irrelevant operators to a SUSY or non-SUSY small-$N$ sector which spontaneously breaks $SO(6)$. This would cause dynamical conformal symmetry breaking at a yet lower scale in the large-$N$ sector. In string language, the dynamics of the other sector acts as a small source for an $SO(6)$ non-singlet tachyon, causing it to condense at some small $r$.

\textit{The cosmological constant:} This theory does not have a naturally small cosmological constant. The unit operator is relevant, so any violation of SUSY at the scale $\Lambda_{UV}$ will
generate a cosmological constant of order $\Lambda_{UV}^4$. Even if the theory is SUSY at $\Lambda_{UV}$ (leaving $\Lambda_\xi$, the classical SUSY-breaking scale, finite) we will still get a large cosmological constant of order $h^2 \Lambda_{UV}^{4-2\dim(h)} \gg \Lambda_\xi$.

Supersymmetry: In this theory, SUSY may be badly broken in the ultraviolet, restored up to $1/N^2$ corrections in an intermediate regime, and broken badly in the infrared. Could this be the story in the real world? Could SUSY be everywhere at best approximate, yet still control the physics above the weak scale because of the absence of GSROs? To make a realistic model seems difficult; nonetheless the possibility is thought-provoking. [24]

Lattice gauge theory: Simulating $\mathcal{N} = 4$ SUSY on the lattice seems hopeless, requiring excessive fine-tuning. But if $\eta \gg 1$, the necessary tuning may be minimal; if enough of $SO(6)$ is preserved, then there are no GSROs and few unwanted GSMOs, whose couplings are naturally driven small. How large must $N$ be for this to be useful? Surely not 100, but is $N = 10$ sufficient? Although not feasible at present, it is remarkable that the difficulties of simulating $\mathcal{N} = 4$ at large $\eta$ — and with it the predictions of string theory — might be be algebraic rather than exponential. [Alternatively, one might remove the scalars and replace them with local four-fermion couplings; the scalars might be regenerated as in the Nambu–Jona-Lasinio model.]

More Examples: One may consider other theories which work similarly; in each of these examples the arguments for energetic stability will have to be revisited.

Other nearly-supersymmetric theories: The $\mathcal{N} = 1$ SUSY-preserving $\mathbb{C}^3/\mathbb{Z}_3$ orbifold, which breaks $SO(6) \to SU(3) \times U(1)$, works the same way; there are no GSROs and a couple of GSMOs. [The absence of a dimension-two GSRO follows from the fact that the $20'$ has no singlets under $SO(6) \to SU(3) \times U(1)$.)] However, all $\mathcal{N} = 2$ orbifolds of $\mathcal{N} = 4$ have a GSRO, an operator with positive (negative) $m^2$ for vector-multiplet (hypermultiplet) scalars. (E.g., the $\mathbb{C}^2/\mathbb{Z}_2$ orbifold preserves an $SO(4) \times SO(2)$ of $SO(6)$, under which $20' \to 9_0 \oplus 4_1 \oplus 4_{-1} \oplus 1_2 \oplus 1_{-2} \oplus 1_0$; the last is the GSRO.) Most $\mathcal{N} = 1$ models fail for similar reasons. In short, the physics under discussion is not generic.

Fully non-supersymmetric examples? More interesting would be a non-SUSY orbifold, which would realize the program of Frampton and Vafa. As noted earlier, scalars in these models naturally have masses $\sim \Lambda_{UV}/N$ in perturbation theory, far too large to explain the observed hierarchy. For $\eta \gg 1$, this does not apply if an orbifold can be found with no GSROs. However we must also ensure that there be no disallowed tachyons — instabilities
which would ensure that there is no unitary conformal field theory at all. Unfortunately, most non-SUSY orbifolds have either fixed planes with disallowed tachyons or GSROs; no problem-free example is known.

**Non-supersymmetric conifold:** A different approach is provided by D3 branes at a conifold singularity. Naively this appears doomed: the operator $O_{ij}^2 \sim \text{tr} A^i B^j$ has dimension $\frac{3}{2}$, so its square is a GSRO. However, as in [10, 13], one may add auxiliary fields $Z_{ij}$ and SUSY-violating terms $\Delta L = Z_{ij} O_{ij}^2 + B Z_{ij} Z_{ij}$. The operator $(Z_{ij})^2$ has dimension 5 and $B$ runs to zero in the IR, leaving $O_{ij}^2$ redundant. In SUGRA, this is precisely the reversed quantization condition on $U_{ij}$ discussed in [10]. The global $SU(2) \times SU(2) \times U(1)$ allows no GSROs in this theory; there are two-fermion/two-scalar GSMOs, and the analysis proceeds as in our original example.

**Driving $\eta$ small:** The case $\beta_\eta > 0$ is especially elegant; the theory breaks its own approximate conformal invariance through a dangerous irrelevant operator, without requiring an additional sector. Here are some examples which might realize this scenario even if the original example does not. Again, in all these cases energetic stability must be reconsidered.

**Addition of fermions:** Added hypermultiplets (making the theory non-asymptotically free, but recall $\Lambda_{UV}$ is finite) would give $\beta_\eta > 0$ but would add GSROs, as in all $\mathcal{N} = 2$ theories. However, addition of $N_f > 1$ fermions in the $\mathbf{N} \oplus \overline{\mathbf{N}}$ representation apparently introduces neither GSROs nor GSMOs, while making $\beta_\eta \sim +N_f/N$. Unfortunately the scalar potential at the origin is probably not stable.

**Addition of gauge fields:** A non-SUSY gauging of $SO(3) \times U(1) \subset SO(6)$ introduces no GSROs (though there are additional GSMOs.) The new gauge couplings $\hat{g}$ rapidly run small, so $\beta_\eta \sim \hat{g}^2$ itself rapidly becomes small. Can $\eta$ be driven into a region where $\mathcal{K}$ is relevant? Unfortunately the effect on $\eta$ is borderline, leaving the answer ambiguous. (More generally, many possible small-$N$ field theories could be coupled to the large-$N$ theory while preserving part of $SO(6)$; in some cases $\eta$ might be pushed from large to small.)

**Duality cascade:** Consider the theory of $M > 2$ fractional D3 branes at the conifold singularity [14, 15], with the relevant double-trace operator again removed by Lagrange multipliers $Z_{ij}$. The symmetry group $SU(2) \times SU(2) \times \mathbb{Z}_{2M}$ still forbids all GSROs. A duality cascade ensues [15], reducing $N$ with $g_s$ fixed, and thereby making $\beta_\eta \sim M/N$. (The additional fields $Z_{ij}$ cannot ruin Seiberg duality; at small $\eta$ SUSY would be badly broken, but without GSROs the SUSY breaking is extremely soft.) If the theory is energetically...
stable, there are two possible endpoints for the flow:

\[ g_s M \gg 1: \] in this case, the cascade ends, confinement occurs, and \( Z_{2M} \) is spontaneously broken at a scale \( \Lambda_{UV} e^{-\eta_0/\eta_f^2} \), where \( \eta_0 = \eta(\mu = \Lambda_{UV}) \), \( \eta_f = g_s M \). If \( p \equiv N \mod M \neq 0 \), then as emphasized in [10] the continuous global symmetries are spontaneously broken. Since SUSY is explicitly broken there will be only pseudoscalar Goldstone bosons without their scalar SUSY partners; however, mass splittings, etc., will be small. All of the physics can be reliably computed using SUGRA.

\[ g_s M \lesssim 1: \] In this case, \( \eta \) reaches 1, at a scale \( \Lambda_c \), before \( N \sim M \). At this point, the scalar mass term becomes relevant and the SUSY breaking becomes important; the cascade breaks down, as does the SUGRA description. The low-energy theory will be vaguely standard-model-like; it will have scalar masses and apparent SUSY breaking at this scale, a product gauge group with few colors (if \( g_s \) is not too small), and some light fermions. Precise details of the physics appear difficult to compute but might be worth further study (assuming the theory is energetically stable.)

In this and other models, spontaneous breaking of chiral symmetries is possible or even automatic; are there applications to technicolor? The real problem in technicolor is not symmetry breaking but flavor, and it is far from clear that the breaking dynamics can be coupled to fermions in a realistic manner without introducing GSROs. Moreover, if the standard model is a spectator to the dynamics, then its beta functions tend to be positive and order \( N^2 \) or \( N \); this is a huge obstacle to realism. It seems more likely that the standard model must be embedded inside a large-\( N \) gauge theory for the dynamics described here to be of particle-physics interest. But for any realistic application, it will be necessary to find and understand a wider class of examples. It is not easy to find theories without GSROs; as noted above there are significant constraints.

In summary, theories at large ’t Hooft coupling can violate standard notions of naturalness in interesting ways. At this stage it is impossible to guess whether these phenomena are relevant in the real world. However, with new model-building tools come new ideas, and we may hope for the best.

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[22] Many readers will have seen versions of this talk, given in 2001 and 2002. The early 2001 versions were not quite correct, due to my imperfect understanding of the role of double-trace operators. Even now, the subtleties involved with these operators and the crucial unknown sign of a beta function still leave some important questions unanswered. I would like to thank Markus Luty, who through conversations surrounding his very interesting and highly recommended paper [21] with Goh and Ng, has motivated me to publish this incomplete assessment of these issues. Clearly there is more to say on this subject.

[23] Since the time that this talk was given, a number of other important related papers have appeared [17, 18, 19].

[24] This line of thought has been considered every now and then over the years, the earliest dating back to [20]. A more serious recent discussion, along the lines considered here, has just appeared [21].