Heights of generation of runaway electrons in bright cosmic ray events observed on the ground during thunderstorms

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Abstract. The brightest events with enhancements of the intensity of the soft component of secondary cosmic rays observed during thunderstorms in the Baksan Valley are analyzed. These experimental data were obtained during thunderstorm seasons of 2003-2008. Assuming bremsstrahlung photons from cascades of runaway electrons to be the main source of the enhancements, the height of generation level is estimated for every event. It is shown that for a half of all events the region of particle generation is located in the stratosphere.

1. Introduction
In [1] a description of the Baksan experiment for studying short disturbances of cosmic ray intensity during thunderstorms is given together with examples of observed enhancements of the soft component with energy higher than 10 MeV. For five bright events of such enhancements analyzed in [2] a conclusion was made, based on mathematical modelling of transport of electrons and photons through the atmosphere and their detection in scintillation detectors with different energy thresholds, that power law spectra of generation of photons at different altitudes can describe the data, the higher altitude the steeper spectrum being required (this fact is explained by energy dependence of the absorption path length for photons in air). More sophisticated analysis of experimental data can be made with the use of more realistic spectra of particles in the source taking into account the spectrum evolution during propagation in air and in the matter of a detector.

In this paper we analyze transport of electrons and photons having the standard energy distribution calculated in paper [3] for cascades of runaway particles. A mathematical model of their detection by the scintillation detector is constructed. The analytical solution connecting experimentally detected amplitudes of variations of the counting rate at different energy thresholds of the scintillation detector with the level of generation of photons generated by avalanches of runaway electrons with spectrum calculated in [3] (for this spectrum a one-parameter approximation is suggested). Using thus obtained relations the levels of generation of runaway cascades are determined for 13 bright events observed by the Baksan air shower array in the period 2003-2008.

2. Particle transport equation and its solution
Let \( J'(r, \Omega, \epsilon) \) be intensity of photons and \( J'(r, \Omega, \epsilon) \) be intensity of electrons and positrons. The kinetic equation is the equation of particle balance in phase volume element (i.e., within element \( dV \) in the angle interval \( d\Omega \) and energy range \( d\epsilon \)). Without electric field the transport (kinetic) equation for photons, electrons, and positrons has the following form (in ultra-relativistic limit, quasi-stationary case, and quasi-homogeneous medium):
\[ \Omega \cdot \frac{dJ^-(\bar{r}, \bar{\Omega}, \varepsilon)}{d\bar{r}} = -J^-(\bar{r}, \bar{\Omega}, \varepsilon) \int_{4\pi} d\Phi \left[ \omega_c(\varepsilon, \varepsilon', \Phi) + \omega_p(\varepsilon, \varepsilon', \Phi) \right] d\varepsilon' + \\
+ \int_{4\pi} d\Phi \left[ J^-(\bar{r}, \bar{\Omega} + \Phi, \varepsilon') \omega_c(\varepsilon', \varepsilon, \Phi) + J^-(\bar{r}, \bar{\Omega} + \Phi, \varepsilon') \omega_p(\varepsilon', \varepsilon, \Phi) \right] d\varepsilon'
\]

\[ \Omega \cdot \frac{dJ^+(\bar{r}, \bar{\Omega}, \varepsilon)}{d\bar{r}} = -J^+(\bar{r}, \bar{\Omega}, \varepsilon) \int_{4\pi} d\Phi \left[ \omega_c(\varepsilon, \varepsilon', \Phi) + \omega_p(\varepsilon, \varepsilon', \Phi) \right] d\varepsilon' + \\
+ \int_{4\pi} d\Phi \left[ J^+(\bar{r}, \bar{\Omega} + \Phi, \varepsilon') \omega_c(\varepsilon', \varepsilon, \Phi) + \omega_p(\varepsilon', \varepsilon, \Phi) + \omega_b(\varepsilon', \varepsilon, \Phi) \right] d\varepsilon' + \\
+ \int_{4\pi} d\Phi \left[ J^+(\bar{r}, \bar{\Omega} + \Phi, \varepsilon') \omega_c(\varepsilon', \varepsilon, \Phi) + 2\omega_p(\varepsilon', \varepsilon, \Phi) \right] d\varepsilon' \quad (1) \]

Here, \( \omega_c(\varepsilon', \varepsilon, \Phi) \) is the probability of Compton scattering for a gamma ray through a space angle \( \Phi \) from a state with energy \( \varepsilon' \) into energy state \( \varepsilon \) on a path of 1 g/cm\(^2\), and \( \omega_p(\varepsilon', \varepsilon, \Phi) \) is the corresponding probability of origination of electron or positron in pair production process stimulated by a photon. Similarly, \( \omega_b(\varepsilon', \varepsilon, \Phi) \) are probabilities of scattering through angle \( \Phi \) in the processes of ionization, bremsstrahlung, and production of knock-on electrons. As initial conditions for transport problem we take spectra generated by thunderstorm field at a point \( r_0 \) of exit from it. The spectra measured at the point of their detection \( r_s \) are the problem’s boundary conditions.

In the general case the spectra are formed by particles of energetic secondary rays whose spectrum is transformed by the field and by particles generated by runaway electrons. It has been estimated that the spectrum of generated particles are steeper than the equilibrium spectrum of secondary cosmic rays (which is steep enough by itself). One can demonstrate that for a steep spectrum of photons in the approximation of rectilinear continuous deceleration (neglecting fluctuations of energy losses, Coulomb scattering, and scattering during bremsstrahlung) the solution to system (1) looks like

\[ J^-(h, \bar{\Omega}_R, \varepsilon) = J^+_0(\bar{r}_0, \bar{\Omega}_R, E) \exp \left\{ -h / \lambda_{\alpha} \right\} \]

\[ J^+(h, \bar{\Omega}_R, \varepsilon) = \int_0^{\lambda_\gamma - \bar{r}_0} G(h, \bar{\Omega}_R, \varepsilon) dh \]

\[ G(h, \bar{\Omega}_R, \varepsilon) = J^+(h, \bar{\Omega}_R, \varepsilon - \varepsilon(1 - \nu))/\lambda_{\gamma}(\varepsilon - \varepsilon), \quad \varepsilon = (E + \alpha_i \lambda_i) \exp \left\{ \left( \frac{\bar{r}_0 - \bar{r}}{\lambda_i} - \frac{h}{\lambda_i} \right) - \alpha_i \lambda_i \right\} \]

(with a certain accuracy depending on the spectrum steepness). Here, \( G(h, \bar{\Omega}_R, \varepsilon) \) is the injection function for particles detected with energy \( \varepsilon(h) \) and generated by photons at a distance \( h \) from generation point \( r_0 \) in the direction \( \Omega \). The quantity \( \nu \) characterizes the mean difference in energies of a photon and a charged particle generated by it (as a result of Compton scattering or pair production). In fractions of photon energy \( \nu \approx 0.4 \). \( \lambda_{\gamma}(\varepsilon) \) and \( \lambda_{\alpha}(\varepsilon) \) are the lengths for generation of a charged particle by a photon and for photon absorption, respectively. \( \alpha_i = 1.8 \text{ MeV/g/cm}^2 \) and \( \lambda_i = 25 \text{ g/cm}^2 \) are parameters of mean energy losses of electrons in air.

3. Relation of measured spectrum of energy release to photon spectrum
For the sake of definiteness, let us consider vertical motion of particles. In the general case the spectrum of particles is determined as energy release in a detector (in our case its thickness is 23.4 g/cm\(^2\)).
Here, \( z_0 \) is the level of generation of photons, and \( z_s \) corresponds to the level of observation. \( P(E, \varepsilon, h) \) is the probability for a charged particle produced at height \( h \) with energy \( \varepsilon \) to release energy \( E \) in the detector. Integrating and retaining the major term we have for the spectrum of energy release the following expression

\[
J_A(E) = \int_{z_0}^{z_s} \left[ \int_{E}^{\infty} P(E, \varepsilon, h) \cdot G(h, \varepsilon) \, d\varepsilon \right] dh
\]

where \( \delta(E, \beta) \) is a small function with weak energy dependence, and \( \lambda_{ad} \) is the absorption length of photons in detector matter. Coefficient \( \sigma \) characterizes the mean energy transferred by a photon to charged particles in a single arbitrary interaction. In case of full stop of particles the detected energy is \( E = \sigma \varepsilon^\beta \).

4. A method to determine the height of photon production from measured spectrum

The spectrum of bremsstrahlung photons produced by avalanches of runaway electrons calculated in [3] is characterized by narrow directivity at energies exceeding 10 MeV. However, at wide area of generation and far distance to observation point the spectrum integrated over angle takes on a simple form slightly differing from angle-independent part of distribution. The following exponential function represents this spectrum to accuracy of a few percent:

\[
J^y(\varepsilon, H) = A^y \cdot e^{-\varepsilon/\varepsilon_F}/\varepsilon, \quad \varepsilon_F = 7 \text{ MeV.}
\] (4)

In our experiment three energy thresholds were used: 10, 17, and 30 MeV. So we could select two relatively compact energy intervals \( \Delta E_1 = 7 \) MeV and \( \Delta E_2 = 13 \) MeV with approximately equal background counting rate. Let during a disturbance \( \Delta N_1 \) and \( \Delta N_2 \) be excess counts over the background for the first and second energy regions. Then for each region \( \Delta N/\Delta E \) determines \( J_A(E_\beta) \), where \( E_\beta \) is the median energy in this region at a given spectrum. The true value of \( E_\beta \) is determined by the method of successive approximations (\( \beta_0 = 0 \) is taken as the zero approximation).

**Table 1.** Events with soft component enhancements analyzed.

| Event number | Date       | \( \Delta N_1 \) (m\(^2\)s\(^{-1}\)) | \( \Delta N_2 \) (m\(^2\)s\(^{-1}\)) | \( \beta \) | \( h \) (g/cm\(^2\)) |
|--------------|------------|----------------------------------|----------------------------------|-----------|-------------------|
| 1            | Oct 11, 2003 | 12.17 ± 0.08                     | 3.70 ± 0.08                      | 3.44 ± 0.05 | 709 ± 16         |
| 2            | Sept 11, 2005 | 4.97 ± 0.05                      | 2.20 ± 0.05                      | 2.70 ± 0.05 | 444 ± 18         |
| 3            | Sept 3, 2006  | 5.06 ± 0.09                      | 2.02 ± 0.09                      | 2.90 ± 0.10 | 516 ± 34         |
| 4            | Oct 14, 2007  | 1.77 ± 0.08                      | 1.04 ± 0.08                      | 2.13 ± 0.18 | 244 ± 63         |
| 5            | Oct 15, 2007  | 1.42 ± 0.04                      | 0.73 ± 0.04                      | 2.40 ± 0.12 | 338 ± 43         |
| 6            | June 13, 2008 | 1.31 ± 0.05                      | 0.85 ± 0.05                      | 1.94 ± 0.14 | 174 ± 49         |
| 7            | July 18, 2008 | 1.24 ± 0.04                      | 1.12 ± 0.04                      | 1.27 ± 0.10 | 95 ± 180         |
| 8            | July 31, 2008 a | 1.01 ± 0.07                    | 0.58 ± 0.07                      | 2.18 ± 0.29 | 260 ± 98         |
| 9            | July 31, 2008 b | 1.16 ± 0.08                    | 0.90 ± 0.08                      | 1.58 ± 0.23 | 47 ± 79          |
| 10           | Aug 1, 2008 a  | 0.40 ± 0.06                      | 0.29 ± 0.06                      | 1.71 ± 0.53 | 95 ± 180         |
| 11           | Aug 1, 2008 b  | 0.55 ± 0.04                      | 0.47 ± 0.04                      | 1.38 ± 0.23 | 21 ± 70          |
| 12           | Aug 1, 2008 c  | 2.55 ± 0.10                      | 2.00 ± 0.10                      | 1.55 ± 0.13 | 40 ± 45          |
| 13           | Aug 1, 2008 d  | 1.53 ± 0.10                      | 0.89 ± 0.10                      | 2.15 ± 0.27 | 251 ± 92        |
Assuming median energies to be determined strictly enough, one can write in accordance with established relation (3):

\[
\frac{\Delta N_1}{\Delta E_1} = J_0 \left( \frac{E_{1\beta}}{\sigma}, z_0 \right) \cdot e^{-h/\lambda_a(E_{1\beta}/\sigma)} \cdot P_1, \quad \frac{\Delta N_2}{\Delta E_2} = J_0 \left( \frac{E_{2\beta}}{\sigma}, z_0 \right) \cdot e^{-h/\lambda_a(E_{2\beta}/\sigma)} \cdot P_2,
\]

where

\[
P_1 = \left[ 1 - e^{-H_c/\lambda_a(E_{1\beta}/\sigma)} \right] \cdot \left( 1 + \delta(E_{1\beta}, \beta) \right)
\]

\[
P_2 = \left[ 1 - e^{-H_c/\lambda_a(E_{2\beta}/\sigma)} \right] \cdot \left( 1 + \delta(E_{2\beta}, \beta) \right)
\]

Here, quantity \( h \) represents the distance from a source of generation to a detection point. Substituting expression (4) into (5), dividing and taking logs of obtained expressions, we solve the equation for spectrum exponent \( \beta \) at the observation place and distance to generation point \( h \). Then we have:

\[
h = \frac{E_{2\beta_2} - E_{1\beta_2}}{e_F} - \ln \left( \frac{\Delta N_1}{\Delta E_1} \frac{1 - e^{-H_c/\lambda_a(E_{2\beta_2}/\sigma)}}{\Delta N_2/\Delta E_2} \frac{E_{1\beta_2}/\sigma(E_{2\beta_2})}{E_{2\beta_2}/\sigma(E_{1\beta_2})} \right) \pm \sqrt{\delta_1^2 + \delta_2^2}
\]

Here, \( \delta_1 \) and \( \delta_2 \) are relative errors (statistical and of method) of detection channels for the disturbance period under analysis.

![Figure 1](image)

**Figure 1.** Heights of generation determined for 13 events from Table 1. Dashed horizontal lines represent zero isotherm and effective level of muon generation.

The fact that in two cases the calculated heights turned out to be beyond the atmosphere indicate in our opinion to existence of the errors of method that we failed to take into account. Most probably this is due to routine correction of the soft component for muon variations (a half of electrons and positrons at our observation level is a result of muon decays) and different angular distributions of generated photons and muons.

1. **Results and conclusions**

Figure 1 presents results of the analysis described above for 13 brightest events recorded in the period 2003-2008 whose characteristics are given in Table 1. For each event the calculated height is given in kilometers above sea level (right scale) and as a depth in the atmosphere (left scale). For the event on October 11, 2003 published in [2] the estimate obtained here coincides within statistical accuracy with that given previously. It corresponds to the level of zero isotherm and is in reasonable agreement with
estimated (by sound delay of lightning discharges) distances to a place of photon generation (4.4 and 3.1 km). We can conclude from the obtained results that generation of runaway electrons can proceed over entire atmosphere starting from the zero isotherm level and higher. In approximately half of the investigated sample of events the generation of photons took place in the stratosphere.

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