Comparison of characteristics of variable magnetic field magnetoelectric sensors based on bidomain lithium niobate, with active magnetic mass and self-biased Ni / Metglas gradient structure

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Abstract. The article is devoted to a theoretical comparison of the characteristics of two magnetoelectric sensors of an alternating magnetic field, operating without an external magnetizing field. The first sensor uses a bending mode in a bimorph lithium niobate with an active magnetic mass at the free end. The second sensor consists of lithium bimorph niobate and a self-biased Ni-Metglas gradient structure. The performed estimates show that a sensor with an active magnetic mass is more than 4 times superior to a sensor with a self-biased gradient structure in sensitivity to an alternating magnetic field. However, it has the advantage that by changing the mass of the magnets, the resonant frequency of the sensor can be controlled. In turn, a sensor with a self-biased gradient structure wins in terms of mass and size characteristics. The results obtained can be used to build promising variable magnetic field sensors for use in biomedicine.

1. Introduction
In recent years, many works have appeared on the development of magnetoelectric magnetic field sensors [1–4]. Along with high sensitivity, magnetoelectric sensors based on a layered magnetostrictive-piezoelectric structure require a constant magnetizing field. The magnetizing field is needed to maximize the pseudo-piezo magnetic coefficient of the magnetostrictive phase. The presence of a source of a constant bias field increases the weight and size parameters of the sensor. Recently, there has been an intense search for opportunities to create magnetic field sensors that do not need a source of a constant bias field. This allows reducing the weight and size of the sensor significantly. In [5], the authors consider two options for creating such improved sensors. A flexural vibration mode is used, with one end of the working fluid rigidly fixed and the other one is free. The first option is characterized by the use of bimorph PZT and the presence of an active magnetic mass at the free end. In the second version, a three-layer structure of PZT/Metglas/Ni is used; an active magnetic mass can also be located at the free end.

In this work, we will consider similar sensor options, but based on a bimorph lithium niobate (Y + 128°cut) [6]. This material is more environmentally friendly compared to PZT, as it does not contain lead. It also has a significantly lower relative dielectric constant compared to PZT, which makes it possible to obtain a higher sensitivity of sensors.
2. Sensor with active magnetic mass

Total piezoelectric thickness is equal to

\[ t = t^1 + t^2, \]

where \( t^1 = 0.15 \text{ mm} \) is the thicknesses of oppositely polarized lithium niobate layers. Two NdFeB magnets are used as active magnetic masses. They are discs of \( d = 5 \text{ mm} \) in diameter and height of \( h = 10 \text{ mm} \). Mass of each magnet is 2.5 g.

![Figure 1. Structural diagram of an alternating magnetic field sensor based on a bimorph lithium niobate with active magnetic masses.](image)

Longitudinal component of the strain tensor

\[ S_1 = -z \frac{\partial^2 w}{\partial x^2}, \]

where \( w \) is the transverse displacement.

Longitudinal component of the stress tensor and the third component of the electric voltage vector of the piezoelectric are

\[ \begin{align*}
\rho T_1 &= e_{11}D_1 - h_{31}S_1, \\
E_3 &= -h_{31}S_1 + \overline{e}_3S_3,
\end{align*} \]

where

\[ \begin{align*}
\overline{e}_{11} &= \left( \rho S_{11} - \frac{\rho^1 d_{31}}{e_{33}e_0} \right)^{-1}, \\
\rho^1 h_{31} &= \frac{\rho d_{31}}{e_{33}e_0}, \\
\rho^2 h_{31} &= \frac{\rho^2 d_{31}}{e_{33}e_0}, \\
\overline{e}_{33} &= 1 + \frac{\rho^1 h_{31} \rho^1 d_{31}}{e_{33}e_0}.
\end{align*} \]

Torque is expressed as

\[ M = \int b z \rho T_1 dz = \int b z \left( -e_{11} z \frac{\partial^2 w}{\partial x^2} - h_{31}D_3 \right) dz = -b \frac{\partial^2 w}{\partial x^2} \int b z^2 dz - bD_3 \int z h_{31} dz = -b \frac{\partial^2 w}{\partial x^2} \int b z^2 D - b \rho^2 \left( h_{31} \right) D_3, \]

where
\[
\langle h_{31} \rangle = \frac{1}{\rho t^2} \int_{-\rho t}^{\rho t} \mathcal{F}_{31} dz = \left( \int_{-\rho t}^{0} z \rho t \mathcal{F}_{31} dz + \int_{0}^{\rho t} z \rho t \mathcal{F}_{31} dz \right) = \frac{1}{4} \rho t \mathcal{F}_{31},
\]

(6)

\[
D = \frac{1}{12} \pi t^3 \rho t^3
\]

is the cylindrical stiffness of the piezoelectric, \( b \) is the width of bimorph lithium niobate.

Let us find the voltage across the piezoelectric

\[
U = \int_{-\rho t}^{\rho t} E_d dz = \int_{-\rho t}^{\rho t} \left( \mathcal{F}_{31} \frac{\partial^2 w}{\partial x^2} + \mathcal{F}_{33} D_3 \right) dz = \rho t \langle h_{31} \rangle \frac{\partial^2 w}{\partial x^2} + \rho t \langle h_{33} \rangle D_3,
\]

(7)

where

\[
\langle \beta_{33} \rangle = \frac{1}{\rho t} \int_{-\rho t}^{\rho t} \mathcal{F}_{33} dz = \mathcal{F}_{33}.
\]

(8)

Hence, we express the electrical displacement in the piezoelectric

\[
D_3 = \frac{U}{\rho t \langle \beta_{33} \rangle} - \rho t \langle h_{33} \rangle \frac{\partial^2 w}{\partial x^2}
\]

(9)

and substitute in the expression for the torque

\[
M = -b \rho t^3 \langle c_{11} \rangle \frac{\partial^2 w}{\partial x^2} - b \rho t \langle h_{33} \rangle U,
\]

(10)

where

\[
\langle c_{11} \rangle = \frac{1}{\rho t^3} \left( D - \rho t^3 \langle h_{31} \rangle^2 \right) = \frac{1}{12} \frac{\rho t^3}{\mathcal{F}_{11}} - \langle h_{31} \rangle ^2.
\]

(11)

Transverse force has the form

\[
V = \frac{\partial M}{\partial x} = -b \rho t^3 \langle c_{11} \rangle \frac{\partial^3 w}{\partial x^3}.
\]

(12)

Bending vibration equation is

\[
\rho \beta \frac{\partial w}{\partial t^2} = \frac{\partial V}{\partial x}.
\]

(13)

Let us substitute (13) into (14) and we will obtain

\[
\rho t^2 \langle c_{11} \rangle \frac{\partial^4 w}{\partial x^4} + \beta \frac{\partial w^2}{\partial t^2} = 0.
\]

(14)

The time dependence is harmonic \( w(x, t) e^{i\omega t} \), therefore

\[
\frac{\partial^4 w}{\partial x^4} - k^4 w = 0,
\]

(15)

where

\[
k = \left( \frac{\rho}{\rho t^2 \langle c_{11} \rangle} \omega^2 \right)^{1/2}.
\]

(16)

General solution of the equation of motion is

\[
w = C_1 \cosh (kx) + C_2 \sinh (kx) + C_3 \cos (kx) + C_4 \sin (kx).
\]

(17)

Open circuit condition is

\[
\int_0^l D_3 dx = 0.
\]

(18)

Let us integrate (8) over \( x \)

\[
UL = \rho t^2 \langle h_{31} \rangle \frac{\partial w}{\partial x} \bigg|_0 = \rho t^2 \langle h_{31} \rangle k \left[ C_1 r_2 + C_2 (r_1 - 1) - C_3 r_4 + C_4 (r_1 - 1) \right].
\]

(19)
where

\[ \begin{align*}
    r_1 &= \cosh (kl) \\
    r_2 &= \sinh (kl) \\
    r_3 &= \cos (kl) \\
    r_4 &= \sin (kl)
\end{align*} \tag{20} \]

Boundary conditions for rigid clamping of the left end of the working body and attachment to the right end of the active magnetic mass are

\[ \begin{align*}
    w(0) &= 0 \\
    \frac{\partial w}{\partial x}(0) &= 0 \\
    V(l) &= m\omega^2 w(l) \\
    M(l) &= J\nu h_l
\end{align*} \tag{21} \]

where \( m = 5 \ g \) is the total mass of magnets, \( v = \frac{\pi d^3 h}{2} \) is the total volume of active magnetic masses, \( J = 1.5T \) is the magnetization of active magnetic mass.

Combining the boundary conditions with (20), we obtain a linear system of five inhomogeneous algebraic equations with respect to five unknowns \( C_1, C_2, C_3, C_4, U \).

Solving this system, considering that

\[ \begin{align*}
    r_1^2 - r_3^2 &= 1 \\
    r_5^2 + r_4^2 &= 1
\end{align*} \tag{22} \]

we will find the voltage across the piezoelectric, and then the ME voltage coefficient

\[ \alpha_v = -\frac{Jv\langle h_{31}\rangle\langle \beta_{33}^5 \rangle}{G}\langle c_{11}\rangle b^4 k^4 p^3 \rho^3 \left( b_{33}^5 \left[ r_4 + 2r_3 \right] + m\omega^2 \left[ r_3 r_5 - 1 \right] \right) \], \tag{23} \]

where

\[ \begin{align*}
    G &= b^4 p^3 \rho^3 \left( c_{11}^2 \right) b^4 k^4 l^3 p^3 \left( b_{33}^5 \left( 1 + r_3 \right) + \langle c_{11}\rangle \langle h_{31}\rangle^2 b^4 k^4 p^3 \left( r_4 + 2r_3 \right) + \langle c_{11}\rangle k l \left( b_{33}^5 \right) m\omega^2 \left( r_3 r_5 - r_4 \right) + \langle h_{31}\rangle^2 m\omega^2 \left( r_3 r_5 - 1 \right) \right)
\end{align*} \tag{24} \]

The graph below shows the dependence of the ME voltage coefficient on the frequency of the alternating magnetic field. If we take into account losses in the calculation, it is assumed \( \omega = 2\pi \left( 1 + \frac{1}{2Q} \right) f \), where the quality factor of the resonance is \( Q = 100 \).
Figure 2. Dependence of the ME voltage coefficient on the frequency of an alternating magnetic field for a sensor with an active magnetic mass.

In the calculation we used the following parameter values for Y + 128° cut of bimorph lithium niobate $\rho = 4647 \, \text{kg} / \text{m}^3$, $\varepsilon_{11}^E = 5.83 \times 10^{-12} \, \text{m}^2 / \text{N}$, $\varepsilon_{33}^T = 49.8$, $d_{31}^1 = -27.24 \times 10^{-12} \, \text{m} / \text{V}$, $d_{31}^2 = 27.24 \times 10^{-12} \, \text{m} / \text{V}$, $b = 10^{-2} \, \text{m}$. Sample length of bimorph lithium niobate is $l = 5 \times 10^{-2} \, \text{m}$.

3. Sensor with self-biased gradient structure Ni/Metglas

A nickel layer with a thickness $m_1 t = 1.4 \times 10^{-6} \, \text{m}$ is required [7,8] to create an internal magnetic field in a 2826 MB Metglas with the thickness $m_2 t = 2.9 \times 10^{-5} \, \text{m}$ that provides the maximum pseudo-piezomagnetic coefficient. The same sample of bidomain niobate lithium will be used as the piezoelectric phase as for the sensor with an active magnetic mass.

Total thickness of the gradient magnetostrictive phase is equal to

$$m_t = m_1 t + m_2 t. \quad (25)$$

Total composite thickness is

$$t = p t + m t. \quad (26)$$

Volume fractions of piezoelectrics and ferromagnets are

$$m_1 V = \frac{m_1 t}{t}, \quad m_2 V = \frac{m_2 t}{t}. \quad (27)$$

Effective density of the composite is

$$\rho = \left(\rho_1 V + \rho_2 V\right) \rho + m_1 V \rho_1 + m_2 V \rho. \quad (28)$$

We will draw the axis $X$ along the neutral line of the composite beam.
Figure 3. Position of the interface between lithium niobate and nickel relative to the neutral line in a three-layer composite.

Longitudinal component of nickel stress tensor is

$$m^1 T_4 = m^1 Y^B \left( S_4 - (q_{11})_4 h_1 \right), \quad (29)$$

where

$$m^1 Y^B = \frac{m^1 Y}{1 - m^1 K_{11}^2},$$

$$m^1 K_{11}^2 = \frac{m^1 Y^B q_{11}^2}{\mu_0}.$$ \hspace{1cm} (30)

Let us substitute (3) into (30) and we will get

$$m^1 T_4 = -z m^1 Y^B \frac{\partial^2 W}{\partial x^2} - (q_{11})_4 h_1, \quad (31)$$

where

$$(q_{11})_4 = m^1 Y^B \frac{1}{q_{11}}.$$ \hspace{1cm} (32)

Similarly, we obtain for the Metglas

$$m^2 T_4 = -z m^2 Y^B \frac{\partial^2 W}{\partial x^2} - 2 (q_{11})_4 h_1,$$ \hspace{1cm} (33)

where

$$m^2 Y^B = \frac{m^2 Y}{1 - m^2 K_{11}^2},$$

$$m^2 K_{11}^2 = \frac{m^2 Y^B q_{11}^2}{2 \mu_0}.$$ \hspace{1cm} (34)
Torque is expressed as
\[ M = \int_{z_0}^{z_1} b z \alpha^P T_i dz + \int_{z_0}^{z_0+z_1} b z m^1 T_i dz + \int_{z_0}^{z_0+z_1} b z m^2 T_i dz = -b \frac{\partial^2 W}{\partial x^2} D - b m t^2 \langle h_{1i} \rangle D_3 - b m t^2 \langle q_{1i} \rangle h_i \]  
(35)

where
\[ \langle q_{1i} \rangle = \frac{1}{m t^2} \left( \int_{z_0}^{z_1} z^1 \bar{q}_{1i} dz + \int_{z_0}^{z_0+z_1} z^2 \bar{q}_{1i} dz \right) = \frac{1}{m t^2} \bar{q}_{1i} m t \left( 2z_0 + m t \right) + \frac{1}{2} q_{1i} m^2 t \left( m^2 t + 2z_0 + 2m t \right). \]  
(36)

The total cylindrical stiffness of the composite beam is expressed as
\[ D = p D + m^1 D + m^2 D, \]  
(37)

where
\[ p D = \frac{1}{3} c_{11} \alpha^P t \left( p t^2 - 3 p t z_0 + 3 z_0^2 \right) \]
\[ m^1 D = \frac{1}{3} m^1 Y B m^1 l \left( m^1 t^2 + 3 m^1 t z_0 + 3 z_0^2 \right) \]
\[ m^2 D = \frac{1}{3} m^2 Y B m^2 l \left( m^2 t^2 + 3 m^2 t \left( z_0 + m^1 t \right) + 3 \left( z_0 + m^1 t \right)^2 \right). \]  
(38)

The position of the interface between lithium niobate and nickel with respect to the neutral line is determined from the condition of the minimum total cylindrical stiffness of the composite beam
\[ z_0 = \frac{\pi D p + m^1 Y B m^1 l^2 + m^2 Y B m^2 l^2}{2 \left( m^1 Y B m^1 l + m^2 Y B m^2 l + \pi D p \right)} \]  
(39)

Let us substitute (10) into (36) and we will obtain
\[ M = -b t^3 \langle c_{1i} \rangle \frac{\partial^3 W}{\partial x^3} - \frac{b \alpha^P t \langle h_{1i} \rangle}{\beta_{1i}} U - b m t^2 \langle q_{1i} \rangle h_i. \]  
(40)

Transverse force is
\[ V = \frac{\partial M}{\partial x} = -b t^3 \langle c_{1i} \rangle \frac{\partial^3 W}{\partial x^3}. \]  
(41)

We will substitute (42) into the bending vibration equation
\[ \rho b t \frac{\partial^2 W}{\partial \tau^2} = \frac{\partial V}{\partial x} \]  
(42)

and we will obtain
\[ t^2 \left( c_{11} \right) \frac{\partial^4 w}{\partial x^4} + \rho \frac{\partial^2 w}{\partial t^2} = 0. \]  \hspace{1cm} (43)

By analogy with the calculation for the first version of the sensor, we obtain the same equation (15)

\[ \frac{\partial^4 w}{\partial x^4} - k^4 w = 0, \]  \hspace{1cm} (44)

but for the wave number we have

\[ k = \left( \frac{\rho}{t^2 \left( c_{11} \right) \omega^2} \right)^{\frac{1}{4}}. \]  \hspace{1cm} (45)

The general solution to the equation of motion has the same form (18). The open circuit condition is the same (19), so equation (20) is also valid.

The boundary conditions for the left end of the composite are the same as before, and for the right free end

\[ V(l) = 0 \]
\[ M(l) = 0. \]  \hspace{1cm} (46)

In exactly the same way, we find the voltage across the piezoelectric and then the ME voltage coefficient

\[ \alpha_v = -\frac{m^2 r^2 \left( q_1 \right) \left( h_{33} \right) \left( \beta_{33}^5 \right) \left( r_4 + r_5 \right)}{t \left[ \left( c_{11} \right) k t^3 \left( \beta_{33}^5 \right) \left( 1 + r_3 \right) + \rho t^3 \left( h_{31} \right) \left( r_4 + r_5 \right) \right]} \]  \hspace{1cm} (47)

The graph below shows the dependence of the ME voltage coefficient on the frequency of the alternating magnetic field. If we consider the losses in the calculation, it is also assumed that \( \omega = 2 \pi \left( 1 + \frac{1}{2Q} \right) f \), where the quality factor of the resonance is \( Q = 100 \).

**Figure 4.** Dependence of the ME voltage coefficient on the frequency of the alternating magnetic field for a sensor with a self-biased gradient structure Ni/Metglas.
The following parameters of nickel and Metglas were used in the calculation: \( m_1 \rho = 8902 \, \text{kg/m}^3 \), 
\( m_1 Y = 5 \times 10^{10} \, \text{Pa} \), 
\( 1 \mu = 300 \), 
\( 1 q_{11} = -1.9 \times 10^{-9} \, \text{m/A} \); 
\( m_2 \rho = 7900 \, \text{kg/m}^3 \), 
\( m_2 Y = 1.0 \times 10^{11} \, \text{Pa} \), 
\( 2 \mu = 10^4 \), 
\( 2 q_{11} = 3.7 \times 10^{-8} \, \text{m/A} \).

4. Conclusion
The calculation results show that a sensor with an active magnetic mass is more than 4 times superior to a sensor with a self-biased gradient structure in sensitivity to an alternating magnetic field. It also has the advantage that the resonant frequency of the sensor can be adjusted by changing the mass of the magnets. At the same time, a sensor with a self-biased gradient structure wins in terms of mass and size characteristics. The considered sensors can be used in biomedicine.

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