Observational signatures of holographic models of inflation

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We discuss the phenomenology of recently proposed holographic models of inflation, in which the very early universe is non-geometric and is described by a dual three-dimensional quantum field theory (QFT). We analyze models determined by a specific class of dual QFTs and show that they have the following universal properties: (i) they have a nearly scale invariant spectrum of small amplitude primordial fluctuations, (ii) the scalar spectral index runs as $\alpha_s = -(n_s - 1)$, (iii) the three-point function of primordial scalar perturbations is of exactly the factorisable equilateral form with $f_{NL}^{equil} = 5/36$. These properties hold irrespective of the details (e.g. field content, strength of interactions etc.) of the dual QFT within the class of theories we analyze. The ratio of tensors-to-scalars is determined by the field content of the dual QFT and does not satisfy the slow-roll consistency relations. Observations from the Planck satellite should be able to confirm or exclude these models.

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Introduction. A wealth of observational data has in recent years transformed cosmology into a precise quantitative science, while new observations promise an unprecedented era of precision cosmology. In particular, the Planck satellite \cite{Planck}, currently operational, is mapping the cosmic microwave background (CMB) anisotropies to an unprecedented degree of sensitivity and angular resolution. The results, expected shortly, will severely tighten existing constraints on fundamental cosmological parameters, and promise to greatly improve our understanding of the primordial inhomogeneities that set the initial conditions for structure formation in our universe. Above all, it is hoped that the results will shed light on the dynamical mechanism through which these primordial cosmological perturbations were generated.

The leading candidate for this mechanism is the theory of inflation, which postulates that the early universe underwent a brief burst of accelerated expansion. The simplest models comprise a single scalar field, the inflaton, coupled to gravity and equipped with a potential function such that the scalar field rolls slowly down to its minimum. In the inflationary paradigm, one starts with a perturbative quantization of fluctuations around the background spacetime described by an accelerating Friedmann-Robertson-Walker (FRW) metric. As the universe inflates, quantum fluctuations produced at very early times grow to superhorizon scales and become classical. They then re-enter the horizon in the post-inflationary era and leave their imprint in the cosmic microwave background, as directly observed today. Different matter content and interactions (potential) lead to a variety of different models with different observational signatures.

An underlying assumption in these scenarios is that it is a valid approximation to use such a perturbative quantization around the background FRW solution in the very early universe. FRW solutions have a curvature singularity at early times, and, moreover, four-dimensional gravity is not UV complete (it is a non-renormalizable theory). Although in some models the perturbative approximation may be justified, in general the quantum theory of the very early universe may be strongly coupled with no useful perturbative gravitational description. In this Letter we discuss models that describe a universe of this type.

Over the last fifteen years a new principle has emerged about the nature of any quantum theory of gravity, namely that it should be holographic. In the context of four-dimensional gravity, this means that there should be an underlying complete description in terms of a three-dimensional QFT without gravity. Concrete holographic models were found in string theory, and, in these models, the holographic correspondence is a strong-weak coupling duality; namely, when the gravitational description is weakly coupled, the dual QFT is strongly coupled and vice versa.

In our models, we will use perturbative three-dimensional QFT to model a putative strongly coupled non-geometric phase of the very early universe. At the end of this epoch, which is the analogue of the conventional inflationary epoch, a weakly coupled geometric description becomes valid and we end up with a specific accelerating FRW spacetime plus a specific set of inhomogeneities. These inhomogeneities are not however linked with a perturbative quantization around the FRW spacetime, but rather, they originate from the dynamics of the dual QFT. This phase should then be matched to conventional hot big bang cosmology. In this paper we will focus only on the phase before the hot big bang. When we refer to late times this means the end of the “holographic inflationary epoch”.

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domain-walls and standard gauge/gravity duality.

FIG. 1: The QFT dual to inflationary cosmology is operationally defined using the correspondence of cosmologies to domain-walls and standard gauge/gravity duality.

**Holographic dualities.** The best understood examples of holographic dualities are the ones obtained by considering brane configurations in string theory and taking appropriate decoupling limits [2]. In recent years, however, the use of holographic methods has been greatly extended by taking a broader perspective and using holography as a model of strong coupling physics. For example, gravitational computations in anti-Sitter (AdS) spacetime were used to model the QCD quark-gluon plasma, and, more recently, similar methods were applied to condensed matter problems.

Here our perspective is similar: we will use perturbative quantum field theory to model strong coupling gravitational dynamics. Furthermore, as in the above examples, our emphasis will be on the phenomenology of the models rather than the theoretical underpinnings. As we will see, our holographic models are consistent with current observations, yet their predictions are different to those of conventional inflation. More importantly, near future observation may verify or exclude them, so the success or failure of these models will ultimately be measured by comparing with observational data. If these models are confirmed by future observations, this would mark a spectacular experimental verification of the idea of holography.

The holographic dualities which are best understood involve spacetimes with negative cosmological constant. For the application at hand, however, we would like to use such a duality for spacetimes that are either de-Sitter or accelerating power-law spacetimes at late times (recall that late times here mean the end of the inflationary epoch). In recent work [3], we proposed that such a duality may be obtained from standard gauge/gravity duality by means of a specific analytic continuation. The idea is sketched in Figure 1.

In the first step (vertical left-hand side of Fig. 1), one uses the fact that there is a one-to-one correspondence between FRW cosmologies and domain-wall spacetimes [4]. This can be understood as a specific analytic continuation [5] that maps FRW spacetimes that at late times approach either de-Sitter or power-law cosmologies, to domain-wall spacetimes that are either asymptotically AdS or else have this property in a specific conformal frame [6]. These spacetimes describe holographic RG flows and there is a well-developed holographic set-up for them [7, 8] (upper horizontal line of Fig. 1). Finally, we express the domain-wall/cosmology analytic continuation in terms of QFT variables (vertical right-hand side of Fig. 1). This amounts to a specific analytic continuation of parameters and momenta that we will discuss in more detail below. Altogether, this gives a way to relate a three-dimensional QFT with a corresponding cosmology.

The main piece of evidence for this holographic duality is that it correctly reproduces conventional inflationary predictions in their regime of applicability, provided standard gauge/gravity duality holds (i.e., the upper horizontal line in Fig. 1 holds). In conventional inflation, the gravitational description is perturbative and thus one should find that inflationary observables can be expressed in terms of (a specific analytic continuation of) strongly coupled correlators of three-dimensional QFT. Indeed, we showed in [3] that the power spectra may be expressed in terms of 2-point functions of the stress-energy tensor, where the latter are computed gravitationally using standard gauge/gravity duality, and the same also holds for the bispectrum [10].

**Holographic dictionary.** The holographic dictionary relates bulk and boundary observables. In particular, bulk fields are related to composite operators of the dual QFT. A prime example is the correspondence between the bulk metric and the stress-energy tensor of the dual QFT. The duality provides a prescription for obtaining quantities in one theory by doing a computation in the other. In our context, we would like to have holographic formulae relating cosmological observables to correlation functions of the dual QFT. Such holographic formulae were derived for the cosmological power spectrum in [3], and results for the scalar bispectrum will appear in [10]. Here, we summarize these results.

The input for the holographic formulae are correlation functions of the stress-energy tensor of the dual QFT. For the power spectra and the bispectrum we only need the 2- and 3-point functions. In momentum space, the general form of the 2-point function is

\[
\langle T_{ij}(\vec{q})T_{kl}(-\vec{q})\rangle = A(q, N)\Pi_{ijkl} + B(q, N)\pi_{ij}\pi_{kl},
\]

where \(\vec{q}\) is the magnitude of the 3-momentum and we indicate explicitly that the QFT coefficients depend on \(N\), the rank of the gauge group of the dual QFT. We suppress the dependence on other parameters (coupling constants, etc.). \(\Pi_{ijkl}\) is the three-dimensional transverse traceless projection operator, \(\Pi_{ijkl} = (\pi_{ik}\pi_{jl} + \pi_{il}\pi_{jk} - \pi_{ij}\pi_{kl})/2\), and \(\pi_{ij} = \delta_{ij} - \vec{q}_i\vec{q}_j/\vec{q}^2\).

The cosmological scalar and tensor power spectra are...
then linked to the 2-point function via
\[
\Delta^2_i(q) = \frac{-q^3/16\pi^2}{\text{Im}B(-iq, -iN)}, \quad \Delta^2_f(q) = \frac{-2q^3/\pi^2}{\text{Im}A(-iq, -iN)},
\]
i.e., we analytically continue the magnitude of the 3-momentum and the rank of the gauge group as
\[
\hat{N} = -iN, \quad \hat{q} = -iq.
\]
The holographic formula for the bispectrum of curvature perturbations is given by [10]
\[
\langle \zeta(q_1)\zeta(q_2)\zeta(q_3) \rangle = -\frac{1}{4} \prod_i \frac{1}{\text{Im}\langle T(q_i)T(-q_i) \rangle} \cdot \text{Im}[\langle T(q_1)T(q_2)T(q_3) \rangle + \sum_i \langle T(q_i)T(-q_i) \rangle - 2\langle T(q_1)T(q_2,q_3) \rangle + \text{cycl. perms}],
\]
where \( T = \delta^i_j T_{ij} \) and on the r.h.s. one should use (3) before taking the imaginary part. The (Fourier transform of the) operator \( T(q_1,q_2) \) is defined by \( T(x_1,x_2) = \delta^i_j \delta^{kl} \delta g_{ij}(x_1)g^{kl}(x_2)g_m = \delta_{mn}, \) i.e., one first obtains the stress-energy tensor \( T_{ij}(g^{kl}) \) of the theory coupled to gravity, differentiates w.r.t. \( g^{kl} \), and then sets the background metric equal to the flat metric. The double bracket notation indicates that we have extracted the momentum-conserving delta function (times \( 2\pi^3 \)), e.g., \( \langle T(q_1)T(q_2)T(q_3) \rangle = (2\pi)^3 \delta^i_j \delta g_{ij}(q_1)T(q_2)T(q_3) \).

As mentioned above, if we are in the regime where gravity is weakly coupled, we may use standard gauge/graviton duality to holographically compute the QFT correlators: the holographic formulae above then reproduce standard inflationary results. In this Letter we are interested in the opposite regime, where gravity is strongly coupled at early times, and we will therefore use perturbative QFT methods to compute the QFT correlation functions.

The model. Ideally, the precise form of the QFT model should be derived from first principles. In the absence of such a derivation, we proceed by using as few assumptions as possible and aim at obtaining universal results that do not depend on the details of the dual QFT. The QFTs that currently have holographic duals are either theories that in the UV become conformal or QFTs with a generalized conformal structure [8, 9]. Here, we focus on the latter class of theories. An example of such a theory is the maximally supersymmetric super-Yang-Mills theory in three dimensions. In such theories, all terms in the Lagrangian scale in the same way, but their dimension is not equal to the spacetime dimension. To be able to use perturbative QFT methods we require standard kinetic terms, so the action is of the form
\[
S = \frac{1}{g_{\text{YM}}^2} \int d^3x \left[ \frac{1}{2} F_{ij}^I F_{ij}^I + \frac{1}{2} (\partial \phi^I)^2 + \frac{1}{2} (\partial \chi^K)^2 + \bar{\psi}^L \gamma^L \psi^L + \text{interactions} \right],
\]
where all fields are massless and we have \( N_A \) gauge fields \( A^I (I=1,\ldots,N_A) \), \( N_{\phi} \) minimally coupled scalars \( \phi^I (J=1,\ldots,N_{\phi}) \), \( N_{\chi} \) conformally coupled scalars \( \chi^K (K=1,\ldots,N_{\chi}) \) and \( N_{\psi} \) fermions \( \psi^L (L=1,\ldots,N_{\psi}) \).

Note that \( g_{\text{YM}}^2 \) has dimension one in three dimensions. The trace over the gauge indices and for concreteness we consider the gauge group to be \( SU(\hat{N}) \) and all fields to transform in the adjoint of \( SU(\hat{N}) \). Any other choices that would admit a large \( \hat{N} \) limit would be as good. One can write down the general form of the interactions such that all terms have dimension four, but in fact (almost) all of the results to follow are independent of the specific form of interactions. QFTs of this form are super-renormalizable and the dimensionful coupling constant acts as an infra-red cut-off [11], so all computations are under control.

QFTs with a generalized conformal structure are dual to domain-walls with asymptotic power-law scaling. Thus, at the end of the holographic inflationary epoch, Einstein gravity becomes a good description again and the universe is now described by a power-law accelerating spacetime
\[
ds^2 = -dt^2 + t^{2n}d\bar{x}^2, \quad n > 1,
\]
with a specific set of inhomogeneities that follow from the holographic computation to be discussed below. Note that (6) is only valid towards the end of this epoch and is the metric that should be matched to the subsequent evolution. Let us emphasize that conventional inflationary theory based on (6) is disfavoured by current observations [12]. The inhomogeneities that we discuss below are not however linked with perturbation theory around (6), but rather, with the dynamics of (5). We will leave \( n \) as a free parameter. We note, however, that the case \( n = 7 \) is distinguished because the corresponding domain-wall solution is the near-horizon limit of the D2-brane solution.

Predictions. We are now ready to present the predictions of this model. For this, we need to compute the 2- and 3-point functions of the stress-energy tensor. The details of the computation of the 2-point function has been reported in [3] and for the 3-point function will appear in [10]. Here, we provide an overview of these results and discuss their implications.

The results for the 2-point function up to 2-loops is
\[
A(q,\hat{N}) = C_A \hat{N}^2 q^2 [1 + D_A g_{\text{eff}}^2 \ln|q/\bar{q}_0| + O(g_{\text{eff}}^4)],
\]
\[
B(q,\hat{N}) = C_B \hat{N}^2 q^2 [1 + D_B g_{\text{eff}}^2 \ln|q/\bar{q}_0| + O(g_{\text{eff}}^4)],
\]
where \( C_A = (N_A + N_{\phi} + N_{\chi} + 2N_{\psi})/256 \), \( C_B = (N_A + N_{\phi})/256 \) and \( g_{\text{eff}}^2 = g_{\text{YM}}^2 \hat{N}/\bar{q}_0 \) is the dimensionless effective coupling constant. The renormalization scale \( \bar{q}_0 \) may be identified with the cosmological pivot scale \( q_0 \) via (3). \( D_A \) and \( D_B \) are numerical coefficients of order one whose value depends on the field content and the precise form of the interactions. To compute \( D_A \) and \( D_B \)
precisely requires summing all the relevant 2-loop diagrams. Notice that the effective coupling remains real under the continuation (3). Inserting now in (2) and using the standard cosmological parameterizations, $\Delta_S^2(q) = \Delta_S^2(q_0)(q/q_0)^{n_S(q)-1}$, $\Delta_T^2(q) = \Delta_T^2(q_0)(q/q_0))^{r(q)}$, where $\Delta_S^2/T(q_0)$ is the scalar/tensor amplitude at some chosen pivot scale $q_0$, and $n_{S/T}(q)$ is the scalar/tensor spectral tilt, we find for the amplitudes

$$\Delta_S^2(q_0) = \frac{1}{16\pi^2 N^2 C_B^2}, \quad \Delta_T^2(q_0) = \frac{2}{\pi^2 N^2 C_A^2},$$

and for the indices

$$n_S(q)-1 = -D_B g_{\text{eff}}^2 + O(q_{\text{eff}}^4), \quad n_T(q) = -D_A g_{\text{eff}}^2 + O(q_{\text{eff}}^4).$$

From the WMAP data [12] we have $\Delta_S^2(q_0) \sim O(10^{-3})$, hence $N \sim O(10^4)$, justifying our use of the large $N$ limit. Furthermore, $(n_S-1) \sim O(10^{-2})$ at $q_0 = 0.002$ Mpc$^{-1}$, and hence $g_{\text{eff}}^4(q_0) \sim O(10^{-2})$ also, justifying our perturbative treatment of the QFT. (Even at the momentum scale $q_0 \sim 2 \times 10^{-4}$ Mpc$^{-1}$ corresponding to the present comoving horizon radius $r_h \sim 14$ Gpc, $g_{\text{eff}}^4(q_0) \sim O(0.1)$ hence all the comoving scales seen in the CMB today lie within the weak-coupling regime where $g_{\text{eff}}^4$ is small). In other words, the two small numbers that appear in the data, the amplitude of the primordial fluctuations and the deviation from scale invariance, appear rather naturally in the dual QFT.

The tensor-to-scalar ratio $r = \Delta_T^2/\Delta_S^2 = 32C_B/C_A$ depends on the field content of the dual QFT; note however that $r$ is not parametrically suppressed as in slow-roll inflation, nor does it satisfy the conventional slow-roll consistency condition $r = -8n_T$. To determine whether the spectral tilts are red or blue requires evaluating the signs of $D_A$ and $D_B$, which in general will depend on the field content and interactions of the QFT. It is nonetheless still possible to extract predictions which are independent of the details of the QFT: for example, in these models, the scalar spectral index runs as

$$\alpha_S = d n_S/d \ln q = -(n_S-1) + O(q_{\text{eff}}^4).$$

This prediction is qualitatively different from slow-roll inflation, for which $\alpha_S/(n_S-1)$ is of first-order in slow-roll [13]. Running of this form is still consistent with the WMAP observational constraints for a wide range of values of $n_S$ and $\alpha_S$ [12], as illustrated in Fig. 2. In the near future, observations from the Planck satellite will determine $n_S$ and $\alpha_S$ to such accuracy that this measurement alone should be able to rule out these models. The expected $1\sigma$ uncertainty in $n_S$ and $\alpha_S$ of $\sim 0.005$ quoted in [14] is illustrated in Fig. 2.

We now move to the discussion of the bispectrum. To

![Figure 2](image-url)
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