An analytical solution for the anti-sliding properties between cable and saddle equipped with vertical plate in extra-dosed bridge

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Abstract. A new method to enhance the anti-sliding force between cable and saddle equipped with vertical plate in extra-dosed bridge is proposed and the analytical solution for the anti-sliding properties is derived in this paper. The validity of the analytical solution is verified by comparison with FEM results. Results show that the new method can largely enhance the anti-sliding force of the saddle by installing a vertical plate. This analytical method can be also used to back-calculate the frictional coefficient in model tests.

1. Introduction
An extra-dosed bridge employs a structure which combines the main elements of both a prestressed box girder bridge and a cable-stayed bridge [1]. Compared to a cable-stayed or rigid girder bridge of comparable span, an extra-dosed bridge uses much shorter stay-towers or pylons than the cable stayed bridge, and a significantly shallower girder than used on the girder bridge[2]. Because of the appealing outlook and economy, the extra-dosed bridge has gained popularity in the word with a span range of 100 meters to 250 meters [3].

For extra-dosed bridge, the main cables are passed through saddles and fixed on the box girder by anchorages. On both sides of the saddle, anti-slip anchor tubes are installed. After the bridge is constructed, the anti-slip tubes are grouted with epoxy mortar to resist the unbalanced force of cables in vehicle load. However, the anti-slip anchor tubes are not grouted in the bridge construction process. Hence, the unbalanced tensile force between both sides of the cable is resisted by the friction between cable and saddle [4]. Researchers have tried ways to better understand the friction mechanism and to enhance the friction force between cable and saddle. Takena et al.[5] conducted an experimental investigation using a test device that simulates the state in a real bridge. Through the experiment, frictional coefficients in the case of a saddle coated with zinc-rich primer and zinc-metalized are measured respectively. Hasegawa et al. [6] used a horizontal plate to increase the frictional resistance. An experimental investigation was also conducted, followed by proposing an analytical model of friction mechanism on the bases of the experimental findings. To analyze the influence of cable-saddle slid on the whole structural system, Chung et al.[7] developed a three dimensional cable finite element that considers the sliding effect and used a geometry nonlinear cable finite element based on elastic catenary theory. Experiments were also conducted to verify the validity of the three dimensional model. Zhang et al.[8] proposed an analytical model for evaluating the frictional resistance of suspension bridges and evaluated the enhancement on frictional resistance by incorporating vertical friction plates. The results revealed that the vertical plate can enhance the friction resistance effectively. However, most of the current research focuses on the frictional resistance of suspension bridge, the frictional
resistance of extra-dosed bridge gains seldom attention. If unexpected incidents occur when the bridge is under construction, the unbalanced force caused by these incidents is mainly resisted by friction between cables and saddle. Therefore, the frictional resistance of extra-dosed bridge needs to be taken seriously.

This paper proposed a method to enhance frictional resistance using circular branch pipes with vertical friction plate. An analytical solution for the anti-sliding properties was proposed. The analytical results were compared with FEM (finite element methods) results for verifying the validity of the proposed analytical solution.

2. New method to enhance frictional resistance

The commonly used method to provide friction force in extra-dosed bridge is installing strand pipes. A strand is set in a circular pipe and the friction is provided by the contact of the strand and the base of the pipe (Fig. 1). A newly developed strand pipe with a vertical plate is proposed in this paper (Fig. 2). A strand pipe is divided into two semi-circles. Each semi-circle holds one strand. The friction is provided by the contact of the strand and the base plate, as well as the contact of the strand and the vertical plate.

3. Analytical solution for the frictional resistance

A single strand pipe is extracted from the proposed strand pipes. The cross section of the stand pipe ant the top of the saddle and the force analysis of the cross section are shown in Fig. 3. Ignoring the impact of the radial shrinkage of the strand, the contact points between strand and the pipe and the circle center of the strand are regard as fixed points. The force analysis of the strand in the longitudinal direction is drawn in Fig. 4. Then the equilibrium equation can be derived from Fig. 3 and Fig. 4.

As shown in Fig. 3, equilibrium equation can be described as follows:

\[
\begin{align*}
-q + N_1 \cos \theta &+ f_2 + f_1 \sin \theta = 0 \\
N_1 \sin \theta - N_2 - f_1 \cos \theta &= 0
\end{align*}
\]  

(1)

Where, \(f_1, f_2\) represent the vertical frictional force of the strand provided by the circular base and the vertical plate respectively. Hence, \(f_1, f_2\) can be termed as \(f_1 = \mu N_1, f_2 = \mu N_2\). \(N_1, N_2\) represent the normal force of the strand caused by the circular base and the vertical plate respectively. \(q\) is the vertical component of external force and is a function of included angle \(\alpha\) and radius of the saddle \(r\). So \(q\) is described as \(q = q(\alpha, r)\). \(\mu\) is the frictional efficiency of the strand pipe.

By solving Eq. (1), \(N_1, N_2\) can be derived as follows:

\[
\begin{align*}
N_1 &= \frac{q}{(1 - \mu^2) \cos \theta + 2 \mu \sin \theta} \\
N_2 &= \frac{(\sin \theta - \mu \cos \theta) q}{(1 - \mu^2) \cos \theta + 2 \mu \sin \theta}
\end{align*}
\]  

(2)

Hence, the longitudinal frictional force because of the contact of the strand and the pipe is calculated as:
The maximum anti-sliding force of the saddle is the integral of $f$ moving through the whole length of the saddle, that is:

$$R_f = \int_{a_1}^{a_2} f(r) r da = \int_{a_1}^{a_2} \mu (\sin \theta - \mu \cos \theta + 1) q(\alpha, r) \cos \theta + 2 \mu \sin \theta dr$$

(4)

From Fig. 4, we can obtain the following equation:

$$\int_{a_1}^{a_2} q(\alpha) \cos \alpha \cos \alpha = T_1 \sin \alpha_1 + T_2 \sin \alpha_2$$

(5)

For most extra-dosed bridge, the saddle is flat. So, $\alpha$ is a small value in the whole region of the saddle. Therefore, substituting $\int_{a_1}^{a_2} q(\alpha, r) \cos \alpha \cos \alpha$ for $\int_{a_1}^{a_2} q(\alpha) \cos \alpha \cos \alpha$ is conservative when calculating the maximum anti-sliding force. Consequently, Eq. (5) is derived as follows:

$$\int_{a_1}^{a_2} q(\alpha) \cos \alpha \cos \alpha = T_1 \sin \alpha_1 + T_2 \sin \alpha_2$$

(6)

Combining Eq. (4) and (6), we can obtain:

$$R_f = \mu (\sin \theta - \mu \cos \theta + 1) \cos \theta + 2 \mu \sin \theta \int_{a_1}^{a_2} q(\alpha) \cos \alpha \cos \alpha T_1 \sin \alpha_1 + T_2 \sin \alpha_2$$

(7)

Moreover, the maximum anti-sliding force can be described as follows:

$$T_2 - T_1 = R_f$$

(8)

Combining Eq. (7) and (8), we can describe $R_f$ in terms of $T_1$:

$$R_f = \frac{(\sin \alpha_1 + \sin \alpha_2) \beta(\theta)}{1 - \beta(\theta) \sin \alpha_2} T_1$$

(9)

Where $\beta(\theta) = \frac{\mu (\sin \theta - \mu \cos \theta + 1) \cos \theta + 2 \mu \sin \theta}{1 - \mu \sin \alpha_2}$.

For a strand pipe without vertical plate, marking $\beta(\theta) = \mu$, the maximum anti-sliding force is:

$$R_{f0} = \frac{(\sin \alpha_1 + \sin \alpha_2) \mu}{1 - \mu \sin \alpha_2} T_1$$

(10)

4. Validity of the analytical solution

To verify the validity of the analytical solution, FEM (Finite Element Method) was adopted to make comparison with analytical results. According to Ma and Fei [9], tensile force $T_2$ was applied to one end of the strand to 65% of its designed force ($f_{pk}$). The values of $T_1$ and $R_f$ ($R_{f0}$) are calculated by the proposed analytical solution and FEM. The finite element model and the stress distribution of the
strand pipe are shown in Fig. 4 and Fig. 6 respectively. The anti-sliding forces of a strand pipe with and without a vertical plate are compared in Fig. 7.

It can be seen from Fig. 7 that the analytical results and the FEM results show little difference. The FEM results are slightly higher than that of analytical results. This is mainly because the analytical solution adopts an approximate substitution \( \int_{-a_1}^{a_2} q(\alpha, r) \cos \alpha \, d\alpha \) for \( \int_{-a_1}^{a_2} q(\alpha, r) \, d\alpha \). However it is conservative to calculate the anti-sliding force. Hence, the analytical solution is valid to predict the anti-sliding force of a strand pipe with a vertical plate.

In addition, Fig. 7 also tells that with the increase of the tensile force \( T_2 \), the anti-sliding force increases correspondingly with an approximate linear trend. The anti-sliding force of the strand pipe with a vertical plate is obviously larger than that without vertical plate. When \( T_2 \) reaches 0.65\( f_{pk} \), the analytical anti-sliding force increases by 69.6% because of the vertical plate.

5. Conclusions
A new type of strand pipe is proposed for enhancing the anti-sliding force between cable and the saddle in an extra-dosed bridge. The analytical solution for calculating the anti-sliding force is derived and validity is verified by comparison with FEM results. Results show that strand pipe with a vertical plate can provide much larger anti-sliding force than that without a vertical plate. This finding indicates the proposed strand pipe is applicable in engineering practice. The analytical solution can be also used to back-calculate the frictional coefficient of a saddle in model tests.

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