Models of Dynamical Supersymmetry Breaking from a SU(2k+3) Gauge Model

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Abstract

We investigate three classes of supersymmetric models which can be obtained by breaking the chiral SU(2k+3) gauge theories with one antisymmetric tensor and 2k-1 antifundamentals. For N=3, the chiral SU(2k)×SU(3)×U(1) theories break supersymmetry by the quantum deformations of the moduli spaces in the strong SU(2k) gauge coupling limit. For N=2, it is the generalization of the SU(5)×U(2)×U(1) model mentioned in the literature. Supersymmetry is broken by carefully choosing the quark-antiquark-doublet Yukawa couplings in this model. For N=1, this becomes the well-known model discussed in the literature.

Submitted to Physics Letters B

*Work supported by Department of Energy contract DE-AC03-76SF00515.
1 Introduction

Supersymmetry is considered to be a leading candidate for resolving the gauge hierarchy problem. It may provide explanations of why the electroweak scale is much smaller than the Planck scale if supersymmetry is dynamically broken within a few orders of magnitude of the weak scale. Supersymmetric gauge theories also have the merit of providing some helpful aspects [1] to the understanding of the non-supersymmetric gauge theories.

There are various models of dynamical supersymmetry breaking. Many of them possess flat directions in field spaces in which the energies vanish classically. The non-perturbative gauge dynamics then lead to SUSY breaking by generating a non-perturbative superpotential which lifts the classical moduli spaces. On the other hand, the gauge dynamics could lead to degeneracy of the quantum moduli space rather than generation of the non-perturbative superpotential in some SUSY theories. A recent report by Intriligator and Thomas [2] observed that supersymmetry can also be broken by the deformation of the quantum moduli space. This occurs if the quantum deformed constraint is inconsistent with a stationary superpotential.

In this paper we discuss the low-energy effective theories with the heavy degrees of freedom being integrated out at the strong gauge scale. The low-energy theory is described by the Kähler potential, the superpotential and the gauge coupling function with the latter two holomorphic to the chiral fields. The holomorphicity of the superpotential combined with the global symmetries then can be used to determine the exact effective superpotential [1]. If one is only interested to know that supersymmetry is broken then the knowledge of the Kähler potential is not necessary provided that the Kähler potential is non-singular. The Kähler potential is necessary only in calculating the vacuum energy and the spectrum [3].

The SU(2k+3) theory is well known to break supersymmetry once appropriate Yukawa terms are added to the superpotential [4a,4b]. It has a zero Witten index [5], which implies that if the SU(2k+3) theory undergoes symmetry breaking, the resulting SU(2k+3-N)×SU(N)×U(1) theory has a zero Witten index as well. This makes the SU(2k+3-N)×SU(N)×U(1) theory a good candidate for Supersymmetry breaking. In the following sections, we will discuss those theories with N≤3 in the limit where the SU(N) coupling is much weaker at the scale at which SU(2k) becomes strong.
This paper is organized as follows. In section two we investigate the SU(2k)\times SU(3)\times U(1) models. Under the strong SU(2k) gauge coupling assumption, the SU(2k) gauge dynamics deformed the quantum moduli space instead of generating a non-perturbative term in the effective superpotential. There are extra SU(3) triplets in this model as compared to the pure SU(2k) gauge model [4]. It will be shown that these triplets play crucial roles in breaking supersymmetry.

In section three, we discuss the resulting SU(2k+1)\times SU(2)\times U(1) gauge theories which were first proposed by M. Dine and his colleagues [6] and a SU(5)\times SU(2)\times U(1) model was discussed and shown to break SUSY in their paper. These models do have R-symmetry. All its flat directions are shown to be lifted once appropriate Yukawa coupling terms are added to the superpotential. For K=1, it becomes the conventional SU(3)\times SU(2) theory except for an extra singlet field S.

In section four we discuss the resulting SU(2k+2)\times U(1) model. This model is already well known in the literature [4,6] with SUSY being dynamically broken. We give a brief summary of the published result in this section.

We conclude the results in section five.

2. SU(2K)\times SU(3)\times U(1)

Recently, Intriligator and Thomas [2] observed that SUSY can be broken by the deformation of the quantum moduli space in the SU(3)\times SU(2) model in the limit \( \Lambda_2 \gg \Lambda_3 \). This can be generalized to SU(2k)\times SU(3)\times U(1) theory. This model contains one antisymmetric tensor \( A_{\alpha\beta} \) with U(1) charge -2, one quark \( Q_{\alpha}^a (2k, 3, -1 + \frac{2k}{3}) \), 2k-1 antiquark \( \bar{Q}_{\alpha j}^a (2k, 1, 1) \), one SU(3) triplet \( \bar{f}_a (1, 3, \frac{4k}{3}) \) and 2k-1 SU(3) triplets \( \bar{F}_{ia} (1, \bar{3}, \frac{-2k}{3}) \). The SU(2k) invariant operators are defined by

\[
\begin{align*}
M_i^a &\equiv \bar{Q}_{\alpha}^a Q_{\alpha}^a \\
X_{ij} &\equiv A_{\alpha\beta} \bar{Q}_{\alpha}^a \bar{Q}_{\beta}^j \\
Y_a &\equiv \varepsilon_{abc} Q_{\alpha_1}^b Q_{\alpha_2}^c \varepsilon^{\alpha_1 \cdots \alpha_{2k}} A_{\alpha_{2k+1} \cdots \alpha_{2k+1}} \\
P f A &\equiv \varepsilon^{\alpha_1 \cdots \alpha_{2k}} A_{\alpha_{2k+1} \cdots \alpha_{2k}}
\end{align*}
\]

(1)

where \( a = 1 \cdots 3 \), \( \alpha = 1 \cdots 2k \) and \( i, j = 1 \cdots 2k - 1 \). The D-flat directions
are parameterized by the following gauge invariant chiral polynomials:

\[ X_{ij}, \quad C_{ijk} \equiv (M_i \cdot M_j \cdot M_k) P f A \]

\[ W_i \equiv Y M_i, \quad V_i \equiv (\bar{f} M_i) P f A, \quad U_{ij} \equiv \bar{F}_i M_j \]

\[ B_i \equiv Y \cdot \bar{f} \bar{F}_i \]

\[ E_{ij} \equiv \bar{f} \cdot \bar{F}_i \cdot \bar{F}_j \]

(2)

where the dot multiplication are defined as 

\[ M_i \cdot M_j \cdot M_k = \varepsilon_{abc} M^a_i M^b_j M^c_k \]

These gauge invariant chiral polynomials are not all independent but subject to constraints. In the limit where the SU(3) gauge coupling is weak at the scale \( \Lambda \), the non-perturbative SU(2k) gauge dynamics deforms the quantum moduli space[4] instead of generating an effective superpotential term.

\[ W_{\text{tree}} = \lambda^i U_{ij} + \lambda^j X_{ij} + \lambda^k E_{ij} \]

\[ W_{\text{eff}} = \lambda^i U_{ij} + \lambda^j X_{ij} + \lambda^k E_{ij} + \mathcal{L} \{ W_{1} \varepsilon_{i_1 \ldots i_{2k-1}} X_{i_2 i_3} \ldots X_{i_{2k-2} i_{2k-1}} - \Lambda_{2k}^4 = 0 \} \]

(3)

This quantum modified constraint can be implemented by adding \( W_{\text{constraint}} \) to the superpotential via a Lagrange multiplier field \( \mathcal{L} \)

\[ W_{\text{tree}} = \lambda^i U_{ij} + \lambda^j X_{ij} + \lambda^k E_{ij} \]

\[ W_{\text{eff}} = \lambda^i U_{ij} + \lambda^j X_{ij} + \lambda^k E_{ij} + \mathcal{L} \{ W_{1} \varepsilon_{i_1 \ldots i_{2k-1}} X_{i_2 i_3} \ldots X_{i_{2k-2} i_{2k-1}} - \Lambda_{2k}^4 = 0 \} \]

(4)

Here we choose to add renormalizable operators to \( W_{\text{eff}} \) that preserve the non-anomalous \( U(1)_R \) global symmetry. Without loss of generality the coupling matrix \( \lambda_1 \) can be assigned a diagonal matrix. It is also easy to see that not all \( E_{ij} \) terms are allowed in the tree superpotential. This is because if all \( E_{ij} \) terms are allowed then all \( U(1)_R \) charges \( R(\bar{F}_i) \) of \( \bar{F}_i \) must be equal which is not consistent with both the anomaly free conditions and the \( R(W_{\text{tree}}) = 2 \) requirement. However, it is still very possible for the tree superpotential to have flat directions after fulfilling the above conditions. To assign the Yukawa coupling matrices that result in no flat direction we need
more information about the flatness conditions.

\[ \lambda_2^{ij} \bar{Q}_i^\alpha Q_j^\beta = 0 \]  \hspace{1cm} (5.1)

\[ \lambda_3^{ij} \varepsilon^{abc} F_{ia} \tilde{F}_{jc} = 0 \]  \hspace{1cm} (5.2)

\[ \lambda_1^{ij} \bar{F}_{ia} Q_j^a = 0 \]  \hspace{1cm} (5.3)

\[ \lambda_1^{ij} \bar{F}_{ia} Q_a^\alpha + 2 \lambda_2^{ij} A_{\alpha\beta} \bar{Q}_i^\beta = 0 \] \hspace{1cm} (5.4)

\[ \lambda_1^{ij} \bar{Q}_a^\alpha \varepsilon^{abc} F_{bc} = 0 \] \hspace{1cm} (5.5)

For simplicity, we may just assign the \( \lambda_1 \) matrix to be the unit matrix without loss of generality. To reduce the complexity it is observed that if the Yukawa couplings \( \lambda_2 \) and \( \lambda_3 \) are of the following forms,

\[ |\lambda_2^{ij}| = \begin{cases} \text{nonzero} & \text{if } i=f(j) \text{ and } j=f(i) \\ 0 & \text{otherwise} \end{cases} \]

\[ |\lambda_3^{ij}| = \begin{cases} \text{nonzero} & \text{if } i=g(j) \text{ and } j=g(i) \\ 0 & \text{otherwise} \end{cases} \]

where \( f \) and \( g \) are one-to-one mappings of indices \( i \) and \( j \), then equations (5.4) and (5.5) imply that operators \( U_{ij} \) will have one-to-one correspondences of operators \( X \) and \( E \) respectively.

\[ U_{jk} = -2\lambda_2^{f(j)j} X_{f(j)k}, \text{ (no summation over } j) \]  \hspace{1cm} (6.1)

\[ U_{kj} = 2\lambda_3^{g(j)j} E_{g(j)k}, \text{ (no summation over } j) \]  \hspace{1cm} (6.2)

\[ U_{g(i),i} = U_{i,f(j)} = 0, \text{ (no summation over } i) \]  \hspace{1cm} (6.3)

By multiplying operators \( \tilde{\bar{Q}}_b^h \varepsilon^{\alpha_2\cdots\alpha_{2k}} A_{\alpha_3\alpha_4} \cdots A_{\alpha_{2k-1}\alpha_{2k}} \) on both sides of the equation (5.4) we find:

\[ B_j + 2\lambda_2^{f(j)j} V_{f(j)} = 0. \text{ (No summation over } j) \]  \hspace{1cm} (7)

From equations (6) and considering the antisymmetry of \( \lambda_2 \) and \( \lambda_3 \) we get
\[ U_{jk} = -\frac{\lambda_{g(k)}}{\lambda_{g^{-1}(j)}} U_{g(k),g^{-1}(j)} = -\frac{\lambda_{f(j)}}{\lambda_{f^{-1}(k)}} U_{f^{-1}(k),f(j)}. \]  
(No summation over j,k) \tag{8}

Equation (8) is very powerful in forcing operators to have vanishing vev’s. To be more explicit, we give the flows of the indices

\[(j, k) \rightarrow (g(k), g^{-1}(j)) \rightarrow (f^{-1}g^{-1}(j), fg(k)) \rightarrow (gf g(k), g^{-1}f^{-1}g^{-1}(j)) \rightarrow \cdots \]

\[(j, k) \rightarrow (f^{-1}(k), f(j)) \rightarrow (g(f(j), g^{-1}f^{-1}(k)) \rightarrow (f^{-1}g^{-1}f^{-1}(k), fgf(j)) \rightarrow \cdots .\]

If any one pair of indices in the follow represents a operator with vanishing vev then all other operators in the same chain will have vanishing vev’s as well. One possible assignment of \(\lambda\) matrices that lifts all flat direction is given by Randall et. al.\[8\]

\[\{(i, f(i))\} = \{(1, 2), (3, 4), \cdots, (2k-3, 2k-2)\}\]
\[\{(i, g(i))\} = \{(2, 3), (3, 4), \cdots, (2k-2, 1)\}\]

Here the ordered pairs represent the unity entities of \(\lambda_2\) and \(\lambda_3\) matrices. Given the above assignments, the equations (5.4) and (5.5) lead to

\[
\bar{Q}_{2k-1}^a Q^a_\alpha = 0 = M_{2k-1} = V_{2k-1} = U_{i,2k-1} = W_{2k-1} = C_{\cdots 2k-1\cdots} \\
\bar{F}_{2k-1,a} Q^a_\alpha = 0 = U_{2k-1,j} = B_{2k-1} \tag{9}
\]

The equations (5.3) and (5.5) give

\[W_i = 2B_{g(i)}\]
\[V_i = 0 \rightarrow B_i = 0 = W_i \quad \text{for all } i. \tag{10}\]
\[U_{11} = U_{22} = \cdots U_{2k-2,2k-2} = 0 \]

To see explicitly why all operators have vanishing vev’s we may take the case \(2k=10\) as an example. In this case, the operators \(U_{ij}\) are determined by using equations (6.1 \sim 10).
\[ U_{ij} = \begin{bmatrix}
    0 & 0 & x & 0 & y & 0 & z & 0 & 0 \\
    0 & 0 & z & 0 & y & 0 & x & 0 \\
    z & 0 & 0 & 0 & x & 0 & y & 0 \\
    0 & x & 0 & 0 & z & 0 & y & 0 \\
    y & 0 & z & 0 & 0 & x & 0 & 0 \\
    0 & y & 0 & x & 0 & 0 & z & 0 \\
    x & 0 & y & 0 & z & 0 & 0 & 0 \\
    0 & z & 0 & y & 0 & x & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 
\end{bmatrix} . \]

Note that the symbols x, y and z used here are denoting the vevs of the same chains. If at least one of x, y and z is nonzero, say \( x \neq 0 \), then we can always choose

\[ (\vec{F}_1)_a = (t_{11}, 0, 0) \quad t_{11} \neq 0. \]

\[ (\vec{F}_2)_a = (t_{21}, t_{22}, 0) \]

and from \( U_{11}, U_{12} \) and \( U_{13} \)

\[ (M_1) = (0, m_{12}, m_{13}), \quad (M_2) = (0, m_{22}, m_{23}) \]

\[ (M_3) = (m_{31}, m_{32}, m_{33}) \]

From \( U_{21} \) and \( U_{22} \) we know that \( t_{22} \neq 0 \) implies \( m_{32} = m_{22} = 0 \) and thus contradicts the pattern that \( U_{11} = 0, U_{42} = x \neq 0 \). If \( t_{22} \) is zero then the vector \( F_2 \) is parallel to the vector \( F_1 \) if \( t_{21} \) is also nonzero. However \( U_{13} = x \) and \( U_{23} = 0 \) tells us the impossibility for both \( t_{11} \) and \( t_{21} \) to be nonzero. Therefore we conclude that the only possible way to satisfy the pattern is to put zero vev’s in all the entries of the \( U_{ij} \) matrix. Now we are left with the operators \( C_{ijl} \). That all the \( C_{ijl} \) also have vanishing vev’s can be seen as follows: to make all the \( U_{ij} \) have vanishing vev’s we could either set all vev’s of \( F_i \) operators to zero or have at least one of them be nonzero. By using equation (15.5) it is easy to check that all \( F_i \) having vanishing vev’s implies the vanishing vev’s of all \( M_i \) operators. On the other hand, if any one of the \( F_i \) operators has nonzero vev then all of the \( M_i \) should be of the following form:
\[ M_i = (0, m_{i2}, m_{i3}). \]

Now we see that all \( M_i \) lie on the same plane in a three-dimensional space and thus make any product \( M_i \cdot M_j \cdot M_l \) to zero. Therefore all of the \( C_{ijkl} \) are also lifted.

Note that the nonzero entities of coupling matrices \( \lambda_2 \) and \( \lambda_3 \) are not necessary to equal to each other in magnitude. The argument after equation (9) is also valid for nonzero coupling matrix entities having different numerical values. In other words, only the index-chain relation is important in breaking SUSY in these models. To form index chains, only even numbers of indices will be used in forming index pairs. On the other side, in order to lessen the total numbers of chains (usually means less unknown nonzero variables after determining \( U_{ij} \) as that in the 2k=10 case) and lift more flat directions we should use as many indices as we can in forming index pairs. Therefore k-1 index pairs are formed in each of the mappings \( f \) and \( g \).

Given the tree level superpotential as above, the classical equations of motion could force all gauge invariant operators to have zero vev’s thus lift all flat directions in the tree level superpotential. On the other hand, the quantum constraint (3) forces some vev’s to be non-zero which spontaneously breaks the \( U(1)_R \) symmetry. Therefore, the following SUSY-breaking criteria are satisfied[7]:

1) There is no classical flat direction.

2) The global \( U(1)_R \) symmetry is broken spontaneously.

According to the above criteria we can see that SUSY is likely broken dynamically.

In this section, we presume an non-anomalous \( U(1)_R \) global symmetry and carefully select renormalizable operators for the tree level superpotential. We see that choosing the Yukawa coupling matrices \( \lambda_2^{ij} \) and \( \lambda_3^{ij} \) is important in lifting classically flat directions. Not all \( U_{ij} \) and \( E_{ij} \) terms can be chosen to preserve the non-anomalous global \( U(1)_R \) symmetry. If we make one-to-one correspondences between operators \( U_{ij} \) and \( X_{ij} \) and \( E_{ij} \) respectively, we can preset some zero vev’s in the \( U_{ij} \) matrix by considering the antisymmetric characteristics of \( X_{ij} \) and \( E_{ij} \). The chain relations between index pairs of \( \lambda_2 \) and \( \lambda_3 \) then help to force all possible nonzero entities of \( U_{ij} \) to zero.
3. SU(2K+1) × SU(2) × U(1)

In this section we consider the SU(2k+3) gauge theory with one antisymmetric tensor and 2k-1 antifundamentals breaking down to SU(2k+1) × SU(2) × U(1). The resulting model contains one antisymmetric tensor field \( (A_{\alpha \beta})_{-2} \), one fundamental \( Q^a_\alpha (2k+1) \), 2k-1 antifundamentals \( \bar{Q}^\beta_i (2k+1, 1) \), 2k-1 doublets \( L^a_i (1, 2) \), and one singlet field \( S (1, 1) \). The numbers on the lower right of parentheses represent charges of the U(1) gauge symmetry. For \( k=1 \), except for the additional singlet field \( S \), this is the well known SU(3) × SU(2) model that has been investigated in the literatures and found to break SUSY either by the dynamically generated superpotential in the limit \( \Lambda_3 \gg \Lambda_2 \) or by the quantum deformation of the moduli space in the limit \( \Lambda_2 \gg \Lambda_3 \). For simplicity, we consider only theories where the SU(2) and U(1) gauge couplings are weak at the scale at which the SU(2k+1) subgroup becomes strong. The SU(2k+1) × SU(2) × U(1) gauge invariant operators are given by

\[
\mathcal{O} = \left\{ (S)_{2k+1}, (C_{ij})_{2k+1} \right\} \times \left\{ (L_{ij})_{-(2k+1)}, (P_i)_{-(2k+1)} \right\},
\]

where operators \( C_{ij}, L_{ij} \) and \( P_i \) are defined by

\[
(C_{ij})_{2k+1} \equiv M_i \cdot M_j, \quad (L_{ij})_{-(2k+1)} \equiv L_i \cdot L_j, \quad (P_i)_{-(2k+1)} \equiv Y \cdot L_i,
\]

and the dot multiplication are given as \( M_i \cdot M_j = \varepsilon_{ab} M_i^a M_j^b \). Note that the SU(2k+1) invariant operators are defined by

\[
M_i^a \equiv \bar{Q}^\alpha_i Q^a_\alpha, \quad a, b=1, 2.
\]
\[
X_{ij} \equiv A_{\alpha \beta}^i \bar{Q}^\alpha_i Q^\beta_j, \quad i, j=1, \cdots, 2k-1.
\]
\[
Y^a \equiv Q^a_{\alpha_2k+1} \varepsilon^{\alpha_1 \cdots \alpha_{2k+1}} A_{\alpha_1 \alpha_2} \cdots A_{\alpha_{2k-1} \alpha_{2k}}.
\]

For \( K \geq 2 \), the SU(2) moduli spaces are not modified quantum mechanically. The SU(2k+1) × SU(2) × (1) invariant operators are subject to the constraints which follow from Bose statistics. If we perturb the superpotential only by renormalizable operators at \( \lambda_i \ll 1 \), the most general superpotential should be:
\[ W_{\text{eff}} = W_{\text{ren}} + W_{\text{dyn}} \]  
(14)

\[ W_{\text{ren}} = \lambda_{1}^{ij} SL_{ij} + \lambda_{2}^{ij} X_{ij} + \lambda_{3}^{ij} N_{ij} \]  
(15)

\[ W_{\text{dyn}} = \frac{\Lambda^{4k+3}}{W_{i_{1}i_{2}...i_{2k-1}}X_{i_{1}j_{1}}...X_{i_{2k-2}j_{2k-1}}}. \]  
(16)

For simplicity, we could choose some special forms of the coupling matrices by

\[ \lambda_{1}^{ij} = \begin{cases} 1 & \text{(i,j)}=(1,2), (3,4) \ldots (2k-3, 2k-2) \\ 0 & \text{otherwise.} \end{cases} \]

\[ \lambda_{2}^{ij} = \begin{cases} 1 & \text{(i,j)}=(2,3), (4,5) \ldots (2k-4, 2k-3),(2k-2,1) \\ 0 & \text{otherwise.} \end{cases} \]

\[ \lambda_{3} : \text{an unit matrix.} \]  
(17)

Note that \( \lambda_{1} \) and \( \lambda_{2} \) are antisymmetric. To see that there is no flat directions classically in this model, we can follow the similar argument as we have done in the section two. The F-flat conditions are:

\[ \lambda_{1}^{ij} L_{i} \cdot L_{j} = 0 \]  
(18.1)

\[ \lambda_{2}^{ij} \bar{Q}_{i}^{\alpha} \bar{Q}_{j}^{\beta} = 0 \]  
(18.2)

\[ \lambda_{3}^{ij} L_{i}^{\alpha} \bar{Q}_{j}^{\alpha} = 0 \]  
(18.3)

\[ 2\lambda_{1}^{ij} SL_{j}^{b} + \lambda_{3}^{ij} \bar{Q}_{j}^{\alpha} Q_{i}^{b} = 0 \]  
(18.4)

\[ 2\lambda_{2}^{ij} A_{\alpha\beta} \bar{Q}_{j}^{\beta} + \lambda_{3}^{ij} L_{i}^{a} Q_{i}^{b} \varepsilon_{ab} = 0 \]  
(18.5)

From equations (18.4) and (18.5), we get \( M_{2k-1}^{b} = 0 = N_{i_{2k-1}} = W_{2k-1} = C_{i_{2k-1}} = C_{2k-1} ; \) and \( P_{i} = 0 = W_{k} \). The relations between \( N_{ij} \), \( X_{ij} \) and \( SL_{ij} \) can be obtained as

\[ N_{ik} = -2X_{kg(i)} \]
\[ N_{ik} = -2 SL_{i(f(k)} \]  
(19)
These relations then are used to determine the vevs of $N_{ik}$ as similar to determining $U_{ij}$ in section two. Take $2k=10$ as an example we have

$$N_{ik} = \begin{bmatrix} 0 & 0 & x & 0 & y & 0 & z & 0 & 0 \\ 0 & 0 & 0 & z & 0 & y & 0 & x & 0 \\ z & 0 & 0 & 0 & x & 0 & y & 0 & 0 \\ 0 & x & 0 & 0 & 0 & z & 0 & y & 0 \\ y & 0 & z & 0 & 0 & 0 & x & 0 & 0 \\ 0 & y & 0 & x & 0 & 0 & 0 & z & 0 \\ x & 0 & y & 0 & z & 0 & 0 & 0 & 0 \\ 0 & z & 0 & y & 0 & x & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$  

The vevs $x$, $y$ and $z$ can be shown to be zero to be consistent with constraint equations (18). That is, all vevs of tensor operators $N_{ik}$, $X_{ik}$ and $SL_{ik}$ vanish. On the other hand, operators $SP_i$ and $W_k$ also have vanishing vevs. We are left with operators $C_{ij}L_{kl}$. By considering that all vevs of $N_{ik} = 0$, the vevs of operators $L_i$ and $M_k$ must be one of the following cases.

1. All $M_k = 0$.
2. All $L_k = 0$.
3. $L_i \parallel M_k$ for those $L_i \neq 0$ and $M_k \neq 0$.

It is easy to see that all $C_{ij}L_{kl}$ vanish in cases 1, 2 and 3. Thus we conclude that classically all vevs of gauge invariant operators vanish and there is no flat direction. From the discussion above we know that by adding appropriate renormalizable tree level superpotential terms to the effective superpotential, we can lift all classical flat directions. In this section, the pattern of tree level Yukawa couplings are the same as those in the section 2. The SUSY criteria guarantee that SUSY is spontaneously broken when the $U(1)_R$ symmetry is spontaneously broken by a dynamically generated non-perturbative superpotential term in this model.

For the case $K=1$, this becomes the well known $SU(3)\times SU(2)$ model except for an additional singlet field $S$. However, the $U(1)$ charge of $S$ hinders the interaction between $S$ and other fields. For $K=2$, it is the $SU(5)\times SU(2)\times U(1)$ model discussed in [6].

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4. \(\text{SU}(2k+2) \times \text{U}(1)\)

This model has been well discussed in the literature \([4b,6]\). We give just a brief summary in this section. When the original SU(2k+3) theory breaks down to SU(2k+2) \(\times\) U(1), it leaves one antisymmetric tensor \((A_{\alpha \beta})_{-2}\), one fundamental \(Q_\alpha (2k+2,1)_{2k+1}\), 2k-1 antifundamentals \(\bar{Q}_i^\alpha (2k+2,1)_{1}\) and 2k-1 singlets \(S_i (1,1)_{-(2k+2)}\) in the resulting theory. The U(1) coupling is presumed to be weak at the scale at which the SU(2) interaction becomes strong. Under the weak U(1) interaction assumption, the SU(2k+2) gauge dynamics generates a non-perturbative superpotential as the one in a SU(2k+2) theory.

\[
W_{\text{dyn}} = \frac{\Lambda^{2k+3}}{|A_i \epsilon^{i_1 \cdots i_{2k-1}} X_{i_2 i_3} \cdots X_{i_{2k-2} i_{2k-1}}|^{\frac{1}{2}}},
\]

where

\[
A_i \equiv M_i P f A, \quad B_{ij} \equiv S_i M_j. \quad i, j = 1, \ldots, 2k - 1.
\]

The singlets \(S_i\) play a crucial role in breaking SUSY. The classical equation of motion of \(S_i\) forces \(A_i\) and \(B_{ij}\) to have zero vev's and thus leads to no flat directions classically. On the other hand, the accidental non-anomalous \(U(1)_R\) symmetry is lifted by the dynamically generated superpotential if the Yukawa coupling of \(B_{ij}\) is a non-degenerate Yukawa matrix. Thus according to the SUSY breaking criteria in section two, SUSY is dynamically broken in this model.

5. Conclusion

The non-perturbatively generated effective superpotential as well as the quantum deformation of a moduli space can lead to supersymmetry breaking. In this paper, we discuss three classes of models which can be constructed by decomposing a chiral gauge SU (2k+3) theory with one antisymmetric tensor and 2k-1 antiquarks. The original SU(2k+3) theory is well known to break SUSY once appropriate Yukawa terms are added to the effective superpotential. By breaking the SU(2k+3) gauge theory down to the various theories with gauge groups as subgroups of SU(2k+3), we have resulting theories with zero Witten indices which thus make them good candidates for
dynamical supersymmetry breaking. The resulting $SU(2k+1) \times SU(2) \times U(1)$ models break SUSY spontaneously in the weak $SU(2)$ limit for $K \geq 2$ by the non-perturbative superpotential generated by the $SU(2k+1)$ dynamics. For $K=1$, this becomes the well known $SU(3) \times SU(2)$ model which breaks supersymmetry either by the non-perturbative superpotential in the $\Lambda_2 \gg \Lambda_3$ limit or by the deformation of the quantum moduli space in the $\Lambda_3 \gg \Lambda_2$ limit. The $SU(2k) \times SU(3) \times U(1)$ models are discussed in the strong $SU(2k)$ limit in this letter. By carefully choosing $E_{ij}$ and $X_{ij}$ terms into the tree level superpotential, the theories can be constructed to be anomaly free and also respect the global $U(1)_R$ symmetry. The $U(1)_R$ symmetry is then spontaneously broken by the deformation of the moduli space and thus fulfill the SUSY-breaking criteria. The $SU(2k+2) \times U(1)$ model discussed in the section 4, it breaks SUSY by the non-perturbative superpotential generated by the $SU(2k+2)$ dynamics. The singlets fields $S_i$ play crucial roles in breaking SUSY. They lead to no flat direction classically and satisfy one of the two SUSY breaking criteria listed in the section three.

During preparation of this letter, we found that Csaki et. al. [8] had finished work specially on the $SU(2k) \times SU(3) \times U(1)$ model.

6. Acknowledgments

We would like to thank M. Peskin, S. Thomas and Y. Shirman for helpful discussions. We also thank A. Rajaraman for providing related information. This work was supported by DOE grant \footnote{DE-AC03-76SF00515.}.
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