Calibration of random dot stereograms and correlograms free of monocular cues

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Dynamic random dot stereograms (DRDSs) and correlograms (DRDCs) are cyclopean stimuli containing binocular depth cues that are ideally, invisible by one eye alone. Thus, they are important tools in assessing stereoscopic function in experimental or ophthalmological diagnostic settings. However, widely used filter-based three-dimensional display technologies often cannot guarantee complete separation of the images intended for the two eyes. Without proper calibration, this may result in unwanted monocular cues in DRDSs and DRDCs, which may bias scientific or diagnostic results. Here, we use a simple mathematical model describing the relationship of digital video values and average luminance and dot contrast in the two eyes. Without proper calibration, this may result in unwanted monocular cues in DRDSs and DRDCs, which may bias scientific or diagnostic results. Here, we use a simple mathematical model describing the relationship of digital video values and average luminance and dot contrast in the two eyes. We present an optimization algorithm that provides the set of digital video values that achieve minimal crosstalk at user-defined average luminance and dot contrast for both eyes based on photometric characteristics of a given display. We demonstrated in a psychophysical experiment with color normal participants that this solution is optimal because monocular cues were not detectable at either the calculated or the experimentally measured optima. We also explored the error by which a range of luminance and contrast combinations can be implemented. Although we used a specific monitor and red-green glasses as an example, our method can be easily applied for other filter based three-dimensional systems. This approach is useful for designing psychophysical experiments using cyclopean stimuli for a specific display.

Introduction

Three dimensional display technology is widely used in vision research as well as in medicine, engineering and entertainment. In order to create the illusion experiment with color normal participants that this solution is optimal because monocular cues were not detectable at either the calculated or the experimentally measured optima. We also explored the error by which a range of luminance and contrast combinations can be implemented. Although we used a specific monitor and red-green glasses as an example, our method can be easily applied for other filter based three-dimensional systems. This approach is useful for designing psychophysical experiments using cyclopean stimuli for a specific display.

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of a three-dimensional visual scene on flat displays, dichoptic stimulation is needed, that is, the two eyes have to be presented with different but clearly separated images. There is a wide variety of technical solutions, each of them involving certain compromises against the natural viewing situation. The most perfect image separation can be provided if the images of two separate displays are fused with the help of an optical system. Mirror stereoscopes (Wheatstone, 1838) and recently, virtual reality goggles use this technique. Fusion is still difficult for some viewers (children for instance) due to the unnatural viewing situation [discussed by Markó et al. (2009)]. Techniques using shutters to present the two images in an alternating fashion are prone to flicker, as well as ghosting, the intrusion of the image intended for one eye into the other eye.

The most economical and thus, wide-spread solution to these problems is to present the two images simultaneously and to separate them based on their spectral composition using colored filters (anaglyphic technology) or taking advantage of light polarization and using polarizing filter goggles. Ideally, the filters would guarantee (1) complete separation of the lights intended for each eye (no crosstalk); (2) that the luminance of transmitted light is equal through both filters (equal light attenuation). Because of imperfection of physical filters though, the separation of eye channels is never complete. Furthermore, the luminance of light reaching the two eyes may be different due to differences between the light sources intended for each eye as well as due to unequal transmittance of the filters.

When viewing a conventional stereogram such as in 3D cinema, uncompensated filter crosstalk results in image ghosts where each eye also receives the shifted image intended for the other eye. A certain amount of crosstalk can be neglected in entertainment applications but in vision research, it can severely bias scientific results and conclusions.

Besides ghosting, stereopsis is also sensitive to interocular differences in luminance as well as contrast. A well-known example of the effect of interocular luminance difference is the Pulfrich-effect (Pulfrich, 1922), which is due to luminance dependent differences in processing time in the visual pathways (Froehlich & Kaufman, 1991; Kurita-Tashima, Tobimatsu, Nakayama-Hiromatsu, & Kato, 1992; Markó, Mikó-Baráth, Kiss, Török, & Jandó, 2012). The same effect can deteriorate the depth percept in stereo kinematograms. Interocular contrast differences on the other hand, rapidly deteriorate binocular fusion, whereas equalizing interocular contrast restores it. This phenomenon is often referred to as contrast paradox in stereopsis (Halpern & Blake, 1988; Legge & Gu, 1989; Stevenson & Cormack, 2000). Also, it is often desirable in research applications that the average luminance and contrast of stereoscopic displays be precisely controlled. Finally, it is important to know the extent to which this complex set of requirements can be fulfilled by the specific displays and filters used.

Dynamic random dot stereograms (DRDSs) (Julesz, 1960; Julesz & Miller, 1962) and correlograms (DRDCs) (Julesz, Kropfl, & Petrig, 1980) are visual stimuli originally invented by Béla Julesz to selectively activate low-level disparity sensitive mechanisms of the visual system. A strict requirement for the utility of DRDSs is that the three-dimensional (3D) pattern be only visible by binocular viewing and functional stereoscopic vision whereas monocular viewing or dysfunctional stereopsis would result in a meaningless random dot pattern for either eye (Figure 1). Monocular cues (also called monocular artefacts in the current context) on the other hand, can result in responses from monocular perceptual mechanisms essentially based on luminance contrast (Figure 2). In such cases, the responses, whether behavioral or neurophysiological, could be confounded with responses of the stereoscopic mechanism. Although the problem is generally recognized by researchers (Baker, Kaestner, & Gouws, 2016), yet the techniques of proper calibration are often not published. A notable exception is a study by
Measurement of monitor and filter properties

Methods

Measurement of monitor and filter properties

From the user’s side, the intensity of pixels of a computer display are controlled using digital video values. Our aim is to find digital video values that fulfill certain requirements defined in terms of luminance and luminance contrast. The relationship of digital video values and luminance (termed here as luminance characteristics) must therefore be measured in advance. We recorded the luminance characteristics using an ILT1700 photometer (International Light Technologies, Peabody, MA) equipped with a SED033 detector, Y2 filter and R input optic. An increasing arithmetic progression of digital video values was presented in steps of no more than five units. The presentation of the levels was separated by 7 seconds of mid-gray (RGB = [127, 127, 127]). Each level was presented for 7 seconds and measurement was done during the last second. Each level was measured once. Digitized output of the photometer was read through a USB serial line at three digits’ precision.

We needed four different luminance characteristics arising from the combination of two monitor phosphors (r, g) with the two color filters (ρ, γ). The functions will be referred to as $p_{rg}$ (Red Attenuation), $p_{gr}$ (Red Crosstalk), $p_{gy}$ (Green Attenuation), and $p_{yg}$ (Green Crosstalk).

Luminance values at a specific digital video value can be queried from lookup tables made from the measured characteristics. In order to facilitate nonlinear optimization however, it was necessary to describe this conversion by a continuous monotonic, differentiable function. It is customary to approximate the luminance characteristics of monitors by a power function (the “gamma function”) of the form $L = ax^\gamma + b$, where $L$ is luminance, $x$ is the digital video value of any of the RGB phosphors and $a$, $b$ and $\gamma$ are free parameters. Here, we chose to fit a third-order polynomial $L = ax^3 + bx^2 + cx + d$ instead (with free parameters $a$, $b$, $c$, $d$) to approximate the measured lookup tables. Model parameters were estimated using the fit function from the Curve Fitting Toolbox in Matlab (The MathWorks, Natick, MA). In preliminary calculations for various computer monitors, both models resulted in similarly good fits (Figure 3) but an analytical solution (see Results) was only practicable when using the third-order polynomial approximation.

Psychophysical testing of the calibration procedure

Observers

Altogether 16 participants completed the psychophysical test (seven males, nine females). All participants had a best corrected visual acuity of at least 1.0. All procedures performed in this study involving human participants were in accordance with the ethical standards of the institutional and/or national research committee and with the 1964 Helsinki declaration and
Figure 3. Luminance characteristics of the red and green monitor phosphors of the LG Cinema 3D D2343P (LG Display, South Korea) monitor used in the present study. Data points are measured values; red or green curves are best fit third order polynomials. The best fit gamma functions are plotted in gray but they are mostly covered by the polynomial fits. $R^2$ values were greater than 0.985 for all fitted curves shown.

**Dynamic random dot correlogram**

We used dynamic random-dot correlograms (Julesz et al., 1980) viewed through red-green color goggles. The DRDC stimulus was constructed of two types of regions (Figure 1). Both regions were random dot patterns made of 50% dark and 50% bright, square-shaped dots, each subtending 9.8′ from the viewing distance of 1 m. Random dot images were updated 60 times per second synchronized to the monitor refresh cycle.

In a correlated region, the contrast polarity of each dot was identical for both eyes (correlation of dot luminances equaled +1, Figure 1). Since we used red-green anaglyphic presentation, this was ensured by using dots of black and yellow logical colors. In an anticorrelated region, dots had opposite contrast polarity in the two eyes (correlation of dot luminances equaled −1), which was ensured by using the red and green logical colors.

In the actual stimuli, Snellen E optotypes were shown as an anticorrelated region on a correlated background. The Snellen E targets had a size of 11.5° × 11.5° visual angle and the entire stimulus area had a size of 16° × 16°.

**Test stimulus set**

Stimuli were presented on a LG Cinema 3D D2343P monitor with a refresh rate of 60 Hz. The screen had a resolution of 1920 × 1080 and subtended 27 × 16 degrees of visual angle. The stimuli were generated using Psychophysics Toolbox version 3 extensions (Brainard, 1997; Kleiner, Brainard, & Pelli, 2007; Pelli, 1997) written in Matlab. Participants wore red-green color goggles equipped with R26 low-pass (red) and YG09 band-pass (green) gelatin filters (Tobiás Optic, Ltd., Budapest, Hungary) for the left and right eyes (Markó et al., 2009). Transmittance spectra of the filters are shown in Supplementary Figure S1.

The respective RGB values of the four colors of the anaglyphic stimulus (Table 1) were first determined using the numerical optimization procedure described in the present paper. The required space-averaged luminance was 6 cd/m² and the required Michelson contrast of dark and bright dots was 50%. This parameter combination was within the region where the calculated luminance and contrast errors for the specific monitor were negligible (Table 1, Figure 6).

Each block of stimuli consisted of DRDCs with 41 different combinations of black, yellow, red and green anaglyph colors. One of them corresponded to the optimized RGB values from the model. The other 40 stimuli were intentionally “spoiled” versions of the optimal color combination. These were obtained by increasing or decreasing either the red or the green phosphor luminances in five steps. The $n$th increment had a luminance $1.04^n$ times the optimized value while the $n$th decrement had a luminance $0.96^n$ times the optimized value. One such series was generated for each of the four anaglyph colors resulting in 40 different stimuli.

**Procedure**

The aim of the psychophysical procedure was to assess the detection probability of monocular artefacts. The stimuli were presented in a 4-alternative forced choice procedure with a method of constant stimuli. Specifically, participants had to decide the orientation of the Snellen E (left, right, up, down) and indicate the response by pressing the corresponding arrows on the keyboard. There was no time limit for the presentation and response times and the subsequent stimulus appeared after the response had been made. Testing was carried out in a darkened room.

The 41 stimuli were randomly shuffled in each block and 10 blocks were presented in one session, which lasted about 40 minutes. One of the eyes was covered with an eye patch in each session. Left and right eyes were tested in separate sessions to limit the effect of fatigue. In the session where stimuli were viewed through the left (red) filter, only the red RGB...
component was varied. In the session, where they viewed through the right (green) filter, only the green component was varied. Thus, a total of 820 responses were obtained from each participant. Nine participants (five males and four females) started with the right eye and the others started with the left eye. We did not systematically investigate the effect of varying the RGB component for which the respective filter was “closed” because a 4% luminance increment or decrement was not possible within the gamut of the monitor.

### Fitting of inverted gaussian to the response curve

In order to estimate the empirical optimum of a color value from the psychophysical data, percent correct responses ($y$) were first inverted then fitted by a Gaussian curve (with an additional $y$-shift) of the following form using the `fit` function in Matlab:

$$y = ae^{-\left(\frac{x-\mu}{\sigma}\right)^2} + b$$

where $x$ is the R or G value that was varied, $\mu$ is the center, $\sigma$ is the spread. Parameters $a$ and $b$ were also varied during the fit. The center ($\mu$) was taken as the empirical optimum digital video value.

### Results

#### Problem statement

The left-hand side of Figure 5 illustrates the problem faced when presenting random-dot stereograms or correlograms using the anaglyphic method. What the observer needs to see is patterns of dark and bright dots in both eyes whereby four combinations of dark and bright dots can occur: dark in both eyes, bright in both eyes, dark left and bright right and vice versa (Figure 1). We term the first two cases correlated dots because the contrast polarity of the dot is equal in both eyes. Dots seen in opposite contrast in the two eyes are termed anticorrelated. Figure 2 shows that unwanted monocular cues arise in RDCs when luminance and or contrast of correlated and anticorrelated regions differ. Therefore, we require that the mean luminance ($L_0$) and the contrast ($C_0$) of these types of regions be equal in both eyes (right-hand side of Figure 5).

This sort of dichoptic stimulus can be composed by using the red and green primary lights (phosphors) of a computer monitor ($r$, $g$, respectively) in combination with red and green colored filters ($\rho$, $\gamma$, respectively). The light emitted by the red phosphor mainly can pass through the red filter and light emitted by the green phosphor mainly can pass through the green filter. When viewed without the filter goggles, the display will thus consist of dots with one of four colors, which we term here as logical colors red, green, black and yellow (denoted R, G, B, Y, respectively, Figure 4). However, if the logical colors would naively, be composed purely of the red and green primary lights driven at full intensity, their mean luminances and contrasts would not be equal as required. This is because of (1) unequal luminance of the phosphors, (2) unequal light attenuation and (3) crosstalk between the filters. These problems can be compensated by adjusting the red and green phosphor components of the logical colors but the effects of these adjustments on the mean luminance and contrast of the display must be regarded together.

In mathematical terms, the problem to be solved is essentially to minimize the difference between the actual mean luminance and contrast and their required values ($L_0$, $C_0$, respectively) for all regions of the display.

#### Obtaining the luminance values

Here, we begin with calculating the actual mean luminances and contrasts. Let us denote the digital video values defining the intensities of the red ($r$) and
Figure 4. Appearance of the four so-called logical colors (R, Red; G, Green; B, Black; Y, Yellow) comprising a random-dot stereogram or correlogram through the color filters (red (\(\rho\)) and green (\(\gamma\))).

Since our requirements are stated in terms of luminance, the conversion between digital video values...
and luminance of the output device needs to be known. We acquire these luminance characteristics by measurement (see Methods) but in order to facilitate optimization, we approximate them by third-order polynomials (Figure 3). There are four different luminance characteristics referred to as $p_{r\rho}$ (Red Attenuation), $p_{r\gamma}$ (Red Crosstalk), $p_{g\gamma}$ (Green Attenuation), and $p_{g\rho}$ (Green Crosstalk). The whole set of luminance values calculated from the measured luminance characteristics is defined as follows

$$\begin{align*}
L_{R_{r\rho}} &= p_{r\rho}(r_R) \\
L_{R_{g\rho}} &= p_{g\rho}(g_R) \\
L_{G_{r\rho}} &= p_{r\rho}(g_G) \\
L_{G_{g\rho}} &= p_{g\rho}(g_G) \\
\end{align*}$$

(1)

**Calculation of average luminance and contrast**

First, we calculate the luminance of each logical color as they are seen through each of the color filters. The luminance of the red logical color through the red filter ($L_{R_{r\rho}}$) for example is the sum of luminances passed from the red and green phosphors, where the second term accounts for the crosstalk:

$$L_{R_{r\rho}} = L_{R_{r\rho}} + L_{R_{g\rho}}$$

(2)

Similarly, for the other combinations of logical colors and color filters, we write

$$\begin{align*}
L_{G_{r\rho}} &= L_{G_{r\rho}} + L_{G_{g\rho}} \\
L_{B_{r\rho}} &= L_{B_{r\rho}} + L_{B_{g\rho}} \\
L_{Y_{r\rho}} &= L_{Y_{r\rho}} + L_{Y_{g\rho}} \\
L_{R_{r\gamma}} &= L_{R_{r\gamma}} + L_{R_{g\gamma}} \\
L_{G_{r\gamma}} &= L_{G_{r\gamma}} + L_{G_{g\gamma}} \\
L_{B_{r\gamma}} &= L_{B_{r\gamma}} + L_{B_{g\gamma}} \\
L_{Y_{r\gamma}} &= L_{Y_{r\gamma}} + L_{Y_{g\gamma}} \\
\end{align*}$$

(3)

The anticorrelated regions of a random dot correlogram are composed of pixels of red and green logical colors and the correlated regions consist of the yellow and black logical colors (Figure 1). In the following, the subscripts $RG$ and $YB$ stand for these regions of the stimulus, respectively. Their average luminances can be calculated from Equations 2 and 3 as

$$\begin{align*}
\bar{L}_{R_{r\rho}} &= (L_{R_{r\rho}} + L_{G_{g\rho}})/2 \\
\bar{L}_{R_{g\rho}} &= (L_{R_{r\rho}} + L_{G_{r\rho}})/2 \\
\bar{L}_{Y_{r\rho}} &= (L_{Y_{r\rho}} + L_{B_{r\rho}})/2 \\
\bar{L}_{Y_{g\rho}} &= (L_{Y_{r\rho}} + L_{B_{g\rho}})/2 \\
\end{align*}$$

(4)

where $\bar{L}_{R_{r\rho}}$ and $\bar{L}_{R_{g\rho}}$ are the average luminances of the anticorrelated region seen through the red and green filters, respectively. Similarly, $\bar{L}_{Y_{r\rho}}$ and $\bar{L}_{Y_{g\rho}}$ are the average luminances of the correlated region.

We use Michelson-contrast to quantify the contrast ($C$) of dark and bright dots in each of the regions of the stimulus:

$$\begin{align*}
C_{R_{g\rho}} &= (L_{R_{r\rho}} - L_{G_{g\rho}})/(L_{R_{r\rho}} + L_{G_{g\rho}}) \\
C_{R_{r\gamma}} &= (L_{G_{r\rho}} - L_{R_{r\gamma}})/(L_{G_{r\rho}} + L_{R_{r\gamma}}) \\
C_{Y_{g\rho}} &= (L_{Y_{r\rho}} - L_{B_{r\rho}})/(L_{Y_{r\rho}} + L_{B_{r\rho}}) \\
C_{Y_{r\gamma}} &= (L_{Y_{r\gamma}} - L_{B_{r\gamma}})/(L_{Y_{r\gamma}} + L_{B_{r\gamma}}) \\
\end{align*}$$

(5)

where the subscripts have the same meaning as in Equation 4.

**The goal of the calibration**

The goal of calibration is to calculate digital video values corresponding to logical colors for which the average luminance and contrast are equal to pre-defined values. The deviation from these values can be interpreted as an error. The purpose of the various solution approaches is to make this error minimal or in ideal case, zero. Since the error can be calculated separately for the correlated and anti-correlated sets of pixels, the solution of the problem can be also separated into two independent parts corresponding to these regions.

**Solutions**

**Symbolic solution**

Ideally, we require that all average luminances (Equation 4) be equal to the required mean luminance ($L_0$) and all contrasts (Equation 5) be equal to the required contrast ($C_0$). For the anticorrelated set of pixels, we can therefore write the following system of equations:

$$\begin{align*}
L_0 - \bar{L}_{R_{g\rho}} &= 0 \\
L_0 - \bar{L}_{R_{r\gamma}} &= 0 \\
C_0 - C_{R_{g\rho}} &= 0 \\
C_0 - C_{R_{r\gamma}} &= 0
\end{align*}$$

(6)

Since we approximated the monitor luminance characteristics by third-order polynomials
(Equation 1), the general solution of Equation 6 is rather complicated; yet it can be calculated by a computer algebra system. In this case, the solver finds the algebraic expressions that provide the digital values \( r_R, g_R, g_G, r_G \) when \( L_0, C_0 \) and the coefficients of the third-order polynomials are known. We solved the equation system consisting of Equations 1–6 using Mathematica (Wolfram Research, Champaign, IL). Substituting the known parameters into this expression results in the digital video values satisfying Equation 6 with theoretically, zero error. The precision of the result will nevertheless, depend on two factors; (1) how exactly the third-order polynomials approximate the luminance characteristics and (2) rounding of the real-valued parameters to the integer digital video values (see below).

The same procedure can be carried out for the correlated set of pixels, which should satisfy

\[
\begin{align*}
L_0 - L_{YB\rho} &= 0 \\
L_0 - L_{YB\gamma} &= 0 \\
C_0 - C_{YB\rho} &= 0 \\
C_0 - C_{YB\gamma} &= 0
\end{align*}
\]

(Equation 7)

Numerical solution

In case of a numerical solution, we seek to minimize an error function \( E_{RG} \) for the anticorrelated pixels and an error function \( E_{YB} \) for the correlated pixels, which can be written as the Euclidean norms of deviations of average luminance and contrast from their required values:

\[
E_{RG} = \| (e_{LRG\rho}, e_{LRGY}, e_{CRG\rho}, e_{CRGY}) \| \quad (8)
\]

\[
E_{YB} = \| (e_{LYB\rho}, e_{LYB\gamma}, e_{CYB\rho}, e_{CYB\gamma}) \| \quad (9)
\]

where the terms \( e_L \) and \( e_C \) are fractional luminance and contrast errors for the anticorrelated and correlated pixels as seen through each filter, respectively as follows

\[
e_{LRG\rho} = \frac{L_0 - L_{RG\rho}}{L_0} \quad e_{LRGY} = \frac{L_0 - L_{RG\gamma}}{L_0} \quad e_{LRG\rho} = \frac{C_0 - C_{RG\rho}}{C_0} \quad e_{CRG\rho} = \frac{C_0 - C_{RG\gamma}}{C_0}
\]

\[
e_{LYB\rho} = \frac{L_0 - L_{YB\rho}}{L_0} \quad e_{LYB\gamma} = \frac{L_0 - L_{YB\gamma}}{L_0} \quad e_{CYB\rho} = \frac{C_0 - C_{YB\rho}}{C_0} \quad e_{CYB\gamma} = \frac{C_0 - C_{YB\gamma}}{C_0}
\]

When minimizing \( E_{RG} \), the free parameters of the optimization are the digital video values \( [r_R, g_R, g_G, r_G] \). Similarly, the free parameters \( [r_B, g_B, r_Y, g_Y] \) are optimized when minimizing \( E_{YB} \).

We used the \texttt{fmincon} function of the Optimization Toolbox of Matlab to minimize the error functions. Optional parameters of \texttt{fmincon} were left at their default values (interior-point algorithm; maximum number of iterations 1000; termination tolerances on the function value and on the free parameters were \( 10^{-6} \) and \( 10^{-10} \), respectively). All free parameters (i.e., digital video values) were constrained to the interval [0; 255]. Their starting values were chosen by assuming no crosstalk between the filters. The required digital video values can in this case, easily be obtained from the luminance characteristics for filter attenuation. An implementation of the numerical solution is provided as Matlab source code along with example data on GitHub (https://github.com/JanosRado/StereoMonitorCalibration).

Symbolic solution based on the assumption of channel constancy

There is an alternative approach to the problem suggested by Mulligan (1986), which simplifies the problem in two ways. First, instead of the digital video values, the model predicts the phosphor luminances. Here, we denote the luminances of the red (\( r \)) and green (\( g \)) components of the four logical colors (\( R, G, Y, B \)) by \( \{L_{Rr}, L_{Rg}, L_{Gr}, L_{Gg}, L_{Yr}, L_{Yg}, L_{Br}, L_{Bg}\} \). These can be calculated from the following equation system

\[
L_{Yr} = S_r (L_{Gr} - L_{Gg} S_r + L_{Rr}) + L_{Br}
\]

\[
L_{Yg} = \frac{L_{Gr} + S_r (L_{Gr} - L_{Gg} S_r - L_{Rr})}{S_g + 1}
\]

\[
L_{Br} = S_r (L_{Gr} - L_{Rr} S_r + L_{Gr}) + L_{Br}
\]

\[
L_{Bg} = S_g (L_{Gg} - L_{Gg} S_g - L_{Gr} + L_{Rg}) + L_{Bg}
\]

\[
\begin{align*}
L_{Rr} - L_{Gg} &= L_{Gr} - L_{Yr} + L_{Yg} \\
L_{Rr} + L_{Gg} &= L_{Gr} + L_{Yr} + L_{Yg} \\
\frac{1}{4} (L_{Rr} + L_{Gg} + L_{Yr} + L_{Yg}) &= L_0 \\
L_{Rr} - L_{Gg} &= C_0 \\
L_{Rr} + L_{Gg} &= C_0
\end{align*}
\]

The first five equations correspond to Equations 1–5 of Mulligan (1986), where the so-called selectivity indexes \( S_r, S_y, S_t \) and \( S_g \) are
defined below. To provide the necessary degrees of freedom however, 3 additional equations have to specify the required mean luminance \( L_0 \), the required luminance contrast \( C_0 \) and the equality of luminances in the left and right eyes, respectively.

The second simplification is the assumption of channel constancy meaning that the spectral composition of the primary lights do not change with intensity (Brainard, Pelli, & Robson, 2002). For this reason, we call this approach the “channel constancy model”. The assumption of channel constancy is incorporated in the selectivity indexes. \( S_r = \frac{T_{rG}}{T_{rG}} \) and \( S_y = \frac{T_{rG}}{T_{rG}} \) are the selectivity indexes for filters, \( S_r = \frac{T_{rG}}{T_{rG}} \) and \( S_y = \frac{T_{rG}}{T_{rG}} \) are the selectivity indexes for the phosphors. The \( T \) parameters can be interpreted as transmittances; for example, \( T_{rG} \) represents \( L_{rG} / L_G \), the fraction of light from the green phosphor transmitted by the red filter. However, if channel constancy of a monitor fails as it is sometimes the case (Supplementary Figure S2), the measured filter selectivity indexes will depend on the intensity of the phosphors instead of being constants. Therefore, to evaluate the channel constancy model for our display device, we used the median values of the \( T \) transmittances measured for the entire range of digital video values.

We solved equation system 10 using Mathematica for the luminance values in symbolic form, from which the numeric values of the luminances \( \{ L_{RB}, L_{RG}, L_{GB}, L_{GR}, L_{YR}, L_{YG}, L_{BR}, L_{BG} \} \) can be calculated for a given set of \( T \) parameters, desired luminance and contrast. Finally, the actual, device specific digital video values of the logical colors can be looked up from the measured luminance characteristics of the \( r \) and \( g \) phosphors; for example, \( g_R \) will be the \( x \)-value on the luminance curve for the green phosphor at the luminance \( L_{RG} \), and so on. Note that for this procedure, the luminance curves have to be measured without filters because the filter transmittances have already accounted for by the \( T \) parameters.

**Minimizing the effect of rounding**

Each of the above methods delivers real-valued video values, which must be rounded for usage in a digital computer. Because the solution is an eight-element set and each of them can be rounded up or down, there are \( 2^8 \) rounding patterns, each of them having different effect on the overall error. In order to minimize the effect of rounding, we adaptively chose the rounding pattern from these 256 cases that resulted in the smallest overall error calculated as \( ||(E_{RG}, E_{YB})|| \) from Equations 8 and 9. The benefit of this method of rounding is most significant when low luminance and contrast are required at the same time (data not shown).

**Domain of achievable luminance and contrast values**

To investigate the domain of achievable luminance and contrast values for a real display device and pair of color filters, we carried out a parameter sweep of the desired luminance and contrast values for a specific computer monitor and a pair of red-green color goggles (see Methods). We chose the set of desired luminances and contrasts tested to cover the achievable range in 100 equidistant steps. As a goodness measure, we calculated the Euclidean norm of fractional errors in luminance \( (E_L) \) and contrast \( (E_C) \) separately as follows (using notation introduced above):

\[
E_L = \| (e_{LRG}, e_{LRG}, e_{LYB}, e_{LYB}) \| \quad (11)
\]

\[
E_C = \| (e_{CRG}, e_{CRG}, e_{CYB}, e_{CYB}) \| \quad (12)
\]

Figure 6 shows color coded plots of luminance and contrast errors over the parameter space calculated by each of the three solutions described above. Mean and standard deviation of the plotted values (log transformed) are shown in Table 2. The achievable region of luminance-contrast combinations is limited by the minimum and maximum luminances that the display device can produce. Therefore the range of achievable mean luminances becomes narrower for increasing contrasts. Another factor limiting the achievable contrast is filter crosstalk, which results in certain amount of light being added to the luminance of a dark pixel if the same pixel needs to be bright in the other eye (anti-correlated case). If the two filters have different spectral selectivities, the one with higher crosstalk will limit the maximum achievable contrast. This also means that if very high contrasts are requested, the numerical solution will converge to the maximum achievable contrast at the expense of increasing error (reddish regions at high contrasts in Figures 6C and D). For the symbolic method and the channel constancy model, these constraints mean that there is no valid solution for certain high luminances or contrasts (white regions in Figs. 6A, B, E, F).

Within the valid range of digital video values, rounding to integer numbers is a common source of error for all three solutions. If required, this sort of quantization error may be reduced by using a display with higher intensity resolution or employing one of the workaround techniques for increasing luminance resolution of a computer monitor (e.g., bit-stealing Tyler, Chan, Liu, McBride, and Kontsevich (1992); see Brainard et al. (2002) for an extended list). Assuming a monitor with the same characteristics as the one tested here but 12 bit intensity resolution per channel (an improvement of 4 bits), the error of the solutions would decrease about \( 2^4 = 16 \)-fold (1.2 log units, Table 2).
Figure 6. Contour plots of fractional luminance (A, C, E) and contrast (B, D, F) errors over the range of values achievable on the hardware configuration tested. The three rows show results obtained using the symbolic (A, B) and numeric solutions (C, D) and the channel constancy model (E, F), respectively, each result rounded to the nearest 8-bit integer using the procedure described in the text. Error values calculated according to Equation 11 for luminance and Equation 12 for contrast are represented by color on a logarithmic scale shown on the right. Hotter colors indicate higher fractional deviation from the required luminance or contrast. The white regions correspond to luminance-contrast pairs where a symbolic solution was not possible (A, B, E, F). White broken contour line marks the smallest common area of the 3 valid regions. Error values in Table 2 were calculated from this area.
Apart from rounding errors, the symbolic solution provides the highest possible precision and accuracy. Compared to this benchmark, the numeric optimization gave identical results to at least a precision of 4 digits (Table 2). The most important merit of the numeric solution is, however, its accessibility given that Matlab or Scipy is more widespread in scientific communities than software tools of symbolic mathematics. The channel constancy model provided on average, 3.50% higher error relative to the required luminance and 1.70% higher error relative to the required contrast (Table 2) compared with the symbolic or numeric methods, although for an added benefit of mathematical simplicity. It is important to note that even if the required luminance or contrast cannot be achieved exactly, their actual values can always be obtained from Equations 4 and 5.

Higher deviation from the required luminance or contrast does not necessarily mean that the cyclopean random-dot stimulus is flawed in the sense that features that should only be detected binocularly would be visible monocularly. As shown in Figure 2, the presence of monocular cues can be tested using a stimulus consisting of binocularly correlated and anticorrelated regions. If for example, the mean luminances of the correlated (“yellow-black”) and anticorrelated (“red-green”) regions are different within the correlated and anticorrelated regions. Similarly, if the contrast of dark and bright dots is different within the correlated and anticorrelated regions, they could appear different in one or both eyes causing monocular artefacts. The strengths of these sources of monocular cues can be summarized in the following metric:

$$M = \left\| \left( \frac{\bar{L}_{RG\rho} - \bar{L}_{YB\rho}}{L_0} \right), \left( \frac{\bar{L}_{RGY} - \bar{L}_{YBy}}{L_0} \right), \left( C_{RG\rho} - C_{YB\rho} \right), \left( C_{RGY} - C_{YBy} \right) \right\|$$

(13)

The value of $M$ may be interpreted as contrast between the correlated and anticorrelated regions of the test image, and thus, it is expected to correlate with the likelihood of detectable monocular cues. The calibrated anaglyph colors obtained using the symbolic solution and the numerical approximation both resulted in $M = 1.04 \pm 0.59\%$ as compared with $M = 5.91 \pm 0.77\%$ for the channel constancy model (Figure 7, Table 3).

**Psychophysical test of the stimulus calibration**

The Snellen-E optotype embedded in a well-calibrated dynamic random dot correlogram should be binocularly visible because anticorrelated and correlated regions of the DRDC are perceived differently by stereoscopic vision (Julesz et al., 1980). The perception of a correlated background is a noisy surface in the plane of the monitor while in the anti-correlated region of the Snellen-E, an indefinite (“woolly” according to Julesz et al. (1980)) depth can be perceived. In an optimally calibrated stimulus, the orientation of the target stimulus could be identified only if the observer has intact binocular vision and the image is viewed binocularly. Upon closing either eye, the Snellen-E disappears (Figure 1) and the person cannot detect its orientation.

The aim of our psychophysical test was to identify the minimum of monocular visibility across a range of RGB colors that included the predicted optimum, as well as 40 “spoiled” versions. We expected that for intentionally “spoiled” stimuli, the target stimulus would become monocularly visible to an increasing degree depending on the deviation to any direction from the predicted optimum. The reason for this is that either the contrast between bright and dark dots or the average luminance within the correlated and anticorrelated areas become unequal causing the background and target regions appear visually different (Figure 2). Thus, if our prediction was correct, monocular detectability would show a minimum for the predicted set of RGB values within a reasonable error.

Figure 8 shows percentage of correct responses in a 4-alternative forced choice task where participants had
Table 3. Mean and standard deviation of the estimated strength of monocular cues across the achievable range of luminances and contrasts shown in Figure 7.

|                          | Mean   | SD     |
|--------------------------|--------|--------|
| Symbolic solution rounded to 8 bit integer | 0.0104 | 0.0059 |
| Numeric solution rounded to 8 bit integer   | 0.0104 | 0.0059 |
| Channel constancy model, rounded to 8 bit integer | 0.0591 | 0.0077 |
| Symbolic solution rounded to 12 bit integer  | 0.00062| 0.0004 |

Figure 7. Estimated strength of monocular cues (M) for a random dot correlogram consisting of correlated and anticorrelated regions such as the one shown in Figure 2. The symbolic (A) and numerical (B) solutions and the channel constancy model (C) are compared. Hotter colors indicate higher chance of the cyclopean target to be detected monocularly; white regions correspond to non-achievable luminance-contrast pairs. Values are calculated according to Equation 13 for the monitor and filter goggles used for testing (see text).
Figure 8. Percent correct responses in a psychophysical task measuring the visibility of monocular artefacts in a DRDC. The abscissa shows the digital video values (at 8 bit resolution) with the vertical blue line indicating the values predicted by our model. Each row represents one of the four anaglyphic colors. The left and right columns show results for the two sessions performed with participants viewing through the red (left) and green (right) filters, respectively. We varied only the red component of each color for the red filter and only the green component of each color for the green filter. Gray traces show individual responses of the participants; the red trace is the group average. Horizontal gray line at 60% correct responses indicates the limit of chance level performance for individual participants. Horizontal red line at 32% correct responses indicates the limit of chance level performance when responses of all participants are accumulated. Asterisks show stimulus conditions where the accumulated number of correct responses exceeded the limit of chance level performance.

Figure 9. Number of participants where detection of monocular artefacts exceeded the chance level of 60%. The abscissa and the arrangement of panels correspond to those of Figure 8. For the predicted red and green digital values (vertical blue line), artefact visibility did not exceed chance level for any of the participants.
visibility by fitting inverted Gaussian functions to the individual response curves (see Methods). The center of the fitted Gaussian (μ) of a participant was taken as the optimum R or G value for the respective anaglyph color. Figure 10 shows that the individual empirical optima (blue tick marks) were indeed often different from the prediction by several RGB units. The medians of the optima were nevertheless within 1.25 RGB units for all anaglyphic color and filter combinations (Figure 11).

Discussion

Dynamic random dot stereograms and correlograms are indispensable tools in assessing stereoscopic function in experimental or diagnostic settings. Their utility strongly relies on proper adjustment of the luminance and contrast of the comprising dots in the face of technical limitations of a given display device. Here, we present a numeric approximation method that provides the set of digital video values that achieve a given average luminance and dot contrast with minimal error on a given display device. We demonstrate that this solution is optimal because it is indistinguishable from the analytic solution. The analytic solution is possible but less practical. We also compare the numeric approximation with an alternative analytic solution (Mulligan, 1986), which incorporates the constraint of channel constancy Brainard et al. (2002).

Using a specific monitor and red-green glasses as an example, we explored the error by which a range of luminance and contrast combinations can be implemented. This approach is useful for designing psychophysical experiments using DRDS or DRDC stimuli for a specific hardware.

Finally, we determined by a psychophysical method, the digital video values that result in no visible monocular artefacts and found these to be within 1.5 RGB units (at an intensity resolution of 8 bit per channel) of the optimum values obtained by the numerical solution.

Comparison to previous approaches

The problems of presenting 3D content on computer displays have been studied extensively mainly due to the demands of consumer electronics. Undoubtedly, the most disturbing type of artefact that deteriorates stereoscopic viewing is the appearance of ghosts of the other image due to crosstalk between the eye channels (Woods, 2012). When viewing real-world scenes, ghosting may cause misjudgment of depth (Tsirlin, Wilcox, & Allison, 2011, 2012), general deterioration of image quality (Wilcox & Stewart, 2003) or eye strain (Seuntiens, Meesters, & Ijsselsteijn, 2005). Image ghosts
Figure 11. Boxplots of the difference of empirical optimum R and G digital video values from the predicted optima (shown by the vertical red line at zero). Boxes are bounded by the first and third quartiles, red lines in the boxes show the medians and whiskers show the lowest and highest data points within 1.5 times the interquartile range to the median. The largest deviation of the median empirical optimum from the prediction was $-1.250$ RGB units (upper right data set). Outliers (7 data points altogether) are not shown but they can be found in Figure 10. Arrangement of the data sets is the same as in Figure 8.

are in our terminology, monocular artefacts because they are visible by viewing the display (through the appropriate filters) by one eye.

Methods of crosstalk cancellation have been presented for minimizing image ghosts in 3D displays using passive filter goggles (Woods, 2012) or shutter glasses (Konrad, Lacotte, & Dubois, 2000). The validity of these models is usually tested by subjective rating of the corrected stimuli (Chang, Kim, Choi, & Yu, 2008). Alternatively, a mathematical model of crosstalk magnitude is provided and perceptual optimum is assumed when this quantity is minimized (Sanftmann & Weiskopf, 2011). Our approach is similar insofar as we also minimize a calculated term, namely the error of required mean luminance and contrast but we also performed a rigorous psychophysical test of the results.

The demands of psychophysical studies where cyclopean mechanisms need to be isolated are more specific. Crosstalk does not cause ghosts in DRDSs or DRDCs but it does result in the appearance of monocular cues, which may be confounded with cyclopean perception. This problem is generally recognized by researchers (Baker et al., 2016), yet the techniques of proper calibration are often not published. A notable exception is a study by Mulligan (1986) who presented a mathematical model of perceived luminances of the dots in DRDS displays. The main difference of this model from ours is the assumption of channel constancy (Brainard et al., 2002). We extended the system of equations suggested by Mulligan (1986) with the requirements of a given average luminance and dot contrast in both eyes and calculated solutions for a specific display device and pair of filters. This method resulted in higher errors than our numeric optimization procedure probably because the monitor we used for testing failed to conform to channel constancy (Supplementary Figure S2).

Monocular cues are especially vivid when a pattern containing both correlated and anti-correlated regions is used as test stimulus but they disappear when optimal crosstalk cancellation is achieved. We took advantage of this property of DRDCs to design a sensitive and quantitative psychophysical test of crosstalk cancellation. In addition, this test is also sensitive to variation of contrast between regions of the stimulus, which again, result in monocular cues. Testing for monocular artefacts in DRDC patterns is thus a useful tool in evaluating calibration of 3D displays for demanding psychophysical applications.

Unwanted differences in mean luminance or contrast between the left and right images can only be detected by comparing the two images. Interocular luminance differences result in unequal latencies of evoked neural responses (Froehlich & Kaufman, 1991; Kurita-Tashima et al., 1992) and a similar effect caused by interocular contrast differences of matching features has been described recently (Reynaud & Hess, 2017). A well-known perceptual consequence is the Pulfrich
effect, the illusory appearance of a pendulum moving in depth when observed through a pair of glasses with unequal optical densities (Pulfrich, 1922). The classical Pulfrich effect (i.e., change in perceived depth) cannot occur in DRDS or DRDC. The reason is that the matching features in a pair of DRDS or DRDC only exist for the duration of a single frame and they change randomly over time. Therefore, the apparent time-shift between the frame sequences of the two eyes that result from unequal luminance causes decorrelation of the left and right eye images and the breakdown of the 3D percept instead of the appearance of illusory depth. Theoretically, the loss of the stereo percept could be used to test for interocular luminance or contrast errors in calibration. In practice however, the interocular luminance differences must be 60% or more for such an effect to occur (Boydstun, Rogers, Tripp, & Patterson, 2009; Diehl, 1991) and therefore a calibration error that is worth a few RGB units is unlikely to be detected.

**Potential extensions of the method**

An important point to note is that the space averaged mean luminance of a stimulus display composed of the calibrated logical colors will optimally correspond to the required mean luminance (L₀) only if the ratio of bright and dark dots is 1:1. This assumption is built in Equation 4. If the ratio of bright dots in the entire display is n, the final space averaged luminance can be calculated post hoc as \( L = nL_{\text{bright}} + (1 - n)L_{\text{dark}} \), where \( L_{\text{bright}} = L_0(1 + C_0) \) and \( L_{\text{dark}} = L_0(1 - C_0) \).

Sometimes, stereoscopic displays are composed of bright features on a dark background. Because the total area of the bright features is typically less than 50% in such displays, the above consideration has to be taken into account. Furthermore, the dark background must be brighter than the nominal black produced by the RGB setting of (0; 0; 0). This is because to make crosstalk invisible, the luminance of the dark background pixels has to be increased to the level of the crosstalk caused by bright pixels in the other eye. This issue is discussed in section “Domain and resolution of achievable luminance and contrast values” above.

Another limitation of the present method is that it can only calculate the logical colors if the images presented to each eye are composed of two (dark or bright) states. In an anaglyphic image, this requires the use of \( 2^2 = 4 \) logical colors. Where more than two states of pixels are needed, the algorithm must be run multiple times for different combinations of mean luminance and contrast. For instance, if pixels can have three states (e.g., dark, background and bright) and they can be varied independently in the two eyes, we would require \( 3^2 \) logical colors. To generate the nine logical colors, the algorithm must be run three times, each time for a combination of two different states. In the 12 resulting logical colors, those where the left and right images are in identical state (e.g., dark in both eyes) will be obtained twice.

In many display systems, separation of the two images is achieved by a combination of polarized light emitting displays and polarizing filters. Although crosstalk and asymmetric light attenuation of the two filters are much smaller than they are for color filters, calibration is still necessary if monocular cues have to be excluded. Our algorithm can be used to achieve this for dichoptic grayscale images. Here, the red and green components of the four logical colors are interpreted as gray-scale values set for the left and right eye-channels, respectively. For example, Yellow logical color of a pixel means that it must be bright for both eyes. For a polarized light system, the gray values of the left and right eye-channels must equal to the red and green components of Yellow, respectively.

**Conclusions**

Unwanted monocular cues are a potential source of spurious results when using random dot correlograms and stereograms to study cyclopean vision. Human observers are surprisingly sensitive to such monocular cues therefore proper calibration of RDS and RDC stimuli is essential. A numerical optimization method described in the present paper can find digital video values for a computer monitor that efficiently avoid monocular cues for color normal observers.

Keywords: stereopsis, binocular vision, cyclopean stimuli, perception, luminance, contrast, DRDS, psychophysical test, source code

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