Thermodynamics of the Quark-Gluon Plasma in Terms of Quasiparticles and Polyakov Line Condensates

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We study a model of quark-gluon plasma of 2+1 flavors Quantum Chromodynamics in terms of quasiparticles propagating in a condensate of Polyakov loops. The Polyakov loop is coupled to quasiparticles by means of a gas-like effective potential. This study is useful to identify the effective degrees of freedom propagating in the medium above the critical temperature. Our finding is that a dominant part of the phase transition dynamics is accounted for by the Polyakov loop, hence the thermodynamics can be described without the need for rapidly increasing quasiparticle masses as $T \to T_c$, at variance with standard quasiparticle models.

PACS numbers: 25.75.Nq,12.38.Aw,12.38.Mh
Keywords: Quark-gluon-plasma, Quasiparticles.

Introduction. The interest in understanding the thermodynamic properties of the strong interaction theory (QCD) has noticeably increased in the recent years, mainly thanks to improvements of computer facilities which allow to perform lattice simulations of QCD and other Yang-Mills theories, as well as to the possibility to create in laboratories extremely hot environments by means of heavy ion collisions. Lattice simulations of the pure gauge ($SU(3)$) theory have shown a deconfinement phase transition at $T = T_c \approx 270$ MeV [1–4]. Introducing dynamical quarks, the phase transition turns to a smooth crossover [5–7, 9, 49]. In this case it is not possible to define rigorously a transition temperature, because of the absence of a true phase transition; nevertheless a transition region centered on a pseudocritical temperature, which we still denote by $T_c$, can be identified as the one where the thermodynamic quantities have the maximum variation; this definition leads to $T_c \approx 155$ MeV in the case of QCD with $u$, $d$ and $s$ quarks.

For what concerns the high temperature phase it is customary to identify the system above $T_c$ as a plasma of quarks and gluons. However, above the critical temperature the in-medium interactions are nonperturbative, and this makes the identification of the correct degrees of freedom of the quark-gluon plasma, in proximity as well as well beyond the critical temperature, a very complicated task. Resummation schemes have been proposed, based for example on the Hard Thermal Loop (HTL) approach [11–15]. At very high temperature the HTL approach motivates and justifies a picture of weakly interacting quasi-particles, as determined by the HTL propagators.

This quasiparticle description has been assumed to be valid also in the case of $T \approx T_c$ [16–51]. In such an approach, one assumes that the quark-gluon plasma description of the deconfinement phase, with propagating transverse gluons and quarks, is still valid; the strong interaction in this non-perturbative regime is taken into account through a temperature-dependent mass for the propagating degrees of freedom. Within this framework one usually assumes a dependence of the quasiparticle masses on the temperature, leaving free parameters which are then fixed by fitting the thermodynamical data of lattice simulations. This description of the quark-gluon plasma is interesting because it is possible to include the quasiparticle dynamics into a transport theory capable to directly simulate the expanding fireball produced in heavy ion collisions computing the collective properties, as well as the chemical composition of the fireball as a function of time [32–35].

In this brief report we study an extension of the quasiparticle picture of the finite temperature QCD medium, supporting a picture in which quark and gluon quasiparticles propagate in, and interact with, a background Polyakov loop [52–59], following previous studies which within this scheme took into account only the pure glue medium [21, 44, 45]. The standard quasiparticle approach accounts for the dynamics at the onset of deconfinement only by means of temperature dependent masses, which leads to diverging (or steadily increasing) masses as $T \to T_c$. On the other hand, it has been shown that combining a $T$-dependent quasiparticle mass with the Polyakov loop dynamics results in a quite different behavior of the mass itself as $T \to T_c$ [21, 44, 45], at least in the case of the pure glue system; the purpose of the present study is to show that this regular behavior of the quasiparticle masses holds even in the case of QCD with dynamical flavours.

In our study we introduce an effective potential for the Polyakov loop, and couple the latter in a similar manner to what is done within the Polyakov extended Nambu-Jona Lasinio model [10, 11]. We follow the formalism of [14], extending it to the case of QCD with dynamical quarks. Our main finding is that also with dynamical quarks the Polyakov loop background is sufficient to take into account of the nonperturbative aspects of the QCD crossover, and quasiparticle masses are regular as temperature approaches $T_c$.

Quasiparticle model. As explained in the Introduction, the main purpose of our study is to confirm that the pres-
ence of the Polyakov line condensates in the quark-gluon plasma phase mitigates significantly the divergence of the quasiparticle masses in the crossover region. We also find that the above result is independent on the ability to reproduce correctly the lattice results on the expectation value of the Polyakov loop, whenever the latter is smaller than one in the crossover region (if the Polyakov loop was about one in the crossover region as well, then the model would not be different from the pure quasiparticle ones, which predict large masses in that temperature range).

The Polyakov loop in the representation $R$ is defined as $\ell_R = \text{Tr} L_R / d_R$ where

$$L_R(\mathbf{x}) = T \exp \left[ i g \int_0^\beta T_R^a A^a_4(\mathbf{x}) d\tau \right],$$

with $T_R^a$ ($a = 1, \ldots, N_c^2 - 1$) corresponds to the generator of the color group $SU(N_c)$ in the representation $R$, and $d_R$ corresponds to the dimension of the representation. In this study both the loops in the fundamental and adjoint representations will be relevant. We follow here the approach of [44] which has been developed for the pure gauge theory; extending it to the case in which dynamical quarks are also present in the thermal bath. The thermodynamic potential $\Omega$ is given by the sum of several contributions,

$$\Omega = \Omega_\ell + \Omega_g + \Omega_q,$$

where

$$\Omega_\ell = -aT \log (1 - 6\ell_F^2 + 8\ell_F^3 - 3\ell_F^4) + c$$

(3)

corresponds to the pure Polyakov loop potential,

$$\Omega_g = 2T \int \frac{d^3p}{(2\pi)^3} \text{Tr} \log (1 - L_A e^{-\beta \omega_g})$$

(4)

is the transverse gluon quasiparticles potential in the Polyakov loop background, and

$$\Omega_q = -4T \sum_f \int \frac{d^3p}{(2\pi)^3} \text{Tr} \log (1 + L_F e^{-\beta \omega_f})$$

(5)

corresponds to the quark quasiparticles potential. In the above equations $L_F$ and $L_A$ denote the Polyakov line in the fundamental and adjoint representations respectively. The dispersion laws for gluon quasiparticles are given by

$$\omega_g = \sqrt{p^2 + m_g^2}$$

with

$$m_g = \sqrt{\frac{3}{4}} g(T) T,$$

(6)

where we assume

$$g(T)^2 = \frac{8\pi^2}{9} \frac{1}{\log [(T - w)/q]},$$

(7)

On the same footing for the quark quasiparticles we assume $\omega_f = \sqrt{p^2 + m_f^2}$ with

$$m_f = m_q$$

for $f = u, d$, while $m_s = m_0 + m_q$ with $m_0 = 95$ MeV.

In our study we restrict ourselves to the mean field approximation which amounts to replace $\ell_R \rightarrow \langle \ell_R \rangle$ in the thermodynamic potential; to avoid heavy notation we denote the expectation value with $\langle \ell_R \rangle$ from now on, unless otherwise specified. Moreover we consider only a quark-gluon plasma at zero baryon chemical potential, which implies $\ell_F = \ell_F$. The traces in Eqs. (4) and (5) are easily performed in the Polyakov gauge, where the Polyakov lines are diagonal. In agreement with [44] in order to simplify the calculations setup, we assume that the relation

$$(N_c^2 - 1)\ell_A = N_c^2 \ell_F^2 - 1,$$

(9)

which is valid for the actual operators, turns to a relation for the mean fields in which $\ell_F^2 \rightarrow \langle \ell_F \rangle^2$, which permits to express both $\Omega_g$ and $\Omega_q$ in terms of $\ell_F$. This approximation is known to lead to negative values of the adjoint loop [46, 47] in the low temperature phase, and to avoid this problem a matrix model on the lines of [48] should be considered; however we have verified that in our calculations, which refer to the high temperature phase of QCD, the adjoint loop is always positive. We treat $\ell_F$ as a variational parameter, imposing that at any temperature the stationarity condition $\partial \Omega / \partial \ell_F = 0$ is satisfied.

Results and discussion. In this study we have considered two different approaches. In the first one, which we call Model I, we fix the four free parameters of the model by a best fit procedure of the lattice data with the

![FIG. 1: Color online. Pressure and interaction measure as a function of temperature computed within the quasiparticle model. Lattice data [5] are represented by the red diamonds and indigo squares.](image)
computed pressure \( p = -\Omega \). From the operative point of view we proceed as follows. After fixing one set of the values of the parameters, at any temperature we compute the numerical value of the Polyakov loop according to the stationarity condition and then the pressure according to Eq. (2); we iterate this for several values of temperature at which lattice data are available, then computing the mean squared deviation of the computed pressure from the data themselves. We repeat the procedure making a scan of the parameter space, and finally choose the parameter set for which we obtain the minimum value of the mean squared deviation. Within this procedure, the expectation value of the operator \( F \) is an output of the calculation, since we do not impose any constraint on it besides the stationarity condition. This procedure leads to the values \( a = (147.36 \text{ MeV})^3 \), \( c = (64.21 \text{ MeV})^3 \), \( w = 8 \text{ MeV} \) and \( q = 5 \text{ MeV} \).

In Fig. 1 we plot the pressure as a function of temperature obtained within the best fitting procedure described above, and the lattice data for the pressure. For completeness, in the same figure we plot also the interaction measure defined as

\[
\Delta = \frac{\varepsilon - 3p}{T^4},
\]

as a function of temperature, and compare the model result with the lattice data.

In Fig. 2 we plot the ratio of quasiparticle masses over temperature as a function of temperature, for the model with the Polyakov loop. For comparison we also show by thin lines the results obtained in the pure quasiparticle model, which is obtained by our model setting \( \ell_F = 1 \) in the quasiparticle thermodynamic potential and neglecting the pure Polyakov loop potential. The point we stress in this brief report, which is summarized in Fig. 2, is that the presence of the Polyakov loop background avoids the stiff increase of the quasiparticle masses as the critical region is approached from larger temperatures, which instead is a characteristic of the model in which no Polyakov loop background is added.

In the latter model the increasing masses are understood easily since the lattice pressure in the critical region decreases rapidly as the system is cooled down, and this can be reproduced within the quasiparticle model only assuming large increase of the masses which results in the suppression of the states relevant for the thermodynamics. On the other hand, in our model it is no longer necessary that masses become larger and larger as the critical temperature is reached from above, because the states are suppressed statistically thanks to the coupling with the Polyakov loop. This mechanism is similar to the statistical confinement mechanism present in the PNJL model \cite{42,43}, and is understood as follows: the quark quasiparticles (for gluons the discussion is similar) potential can be written as

\[
\Omega_q = -4T \sum_f \int \frac{d^3p}{(2\pi)^3} \log \left(1 + 3\ell_F x + 3\ell_F x^2 + x^3\right),
\]

where \( x = e^{-\beta \omega} \). In the crossover region the Polyakov loop \( \ell_F \ll 1 \), which results in the suppression of the one-quark and two-quark states contributions to the thermodynamic potential, hence suppressing quasiparticle pressure even if \( m/T \) does not become larger.

In Fig. 3 we plot the model prediction for the Polyakov loop expectation value \( \ell_F \) as a function of temperature computed within the model, and compared with the lattice data \cite{44}.
quasiparticle masses are affected by the dynamical detail of $\ell_F$.

To explore this point in more detail, we need a model in which the Polyakov loop is in agreement with lattice data. To this end, we need to modify the best fit procedure above by requiring that $\ell_F$ reproduces the lattice value at any temperature. We are aware this is a very rough procedure, and a more interesting study would be to investigate the reason of the discrepancy with the lattice results. Leaving to a future study a more detailed investigation, we accept here a simplicistic point of view and just try to build up a model in which the Polyakov loop is in agreement with the lattice data, to understand how this affects the quasiparticle masses. We call this model as Model II.

In Fig. 4 we plot the expectation value of the Polyakov loop as a function of temperature for the Model II. In order to reproduce both the pressure and the Polyakov loop lattice data better than we do by Model I we have replaced the parameter $c$ in Eq. (3) by a three parameters function, namely $c(T) = \alpha T^4 \exp(-(x-x_0)/\gamma)^2$ with $\alpha = 1.62, \gamma = 141.4$ MeV and $x_0 = 260.1$ MeV.

In Fig. 5 we plot the ratio of quasiparticle masses over temperature as a function of temperature, for the model II in which the lattice Polyakov loop is used as an input. That quasiparticles are even more statistically suppressed, and to reproduce the total pressure one needs to lower the masses by about 15% to allow the Boltzmann factors to compete with the lowering of $\ell_F$.

**Conclusions.** In this brief report we have studied a model of quark-gluon plasma which combines the description in terms of dynamical quasiparticles with that of a condensate of Polyakov lines. Our main purpose has been to discuss how the presence of the Polyakov loop background affects the quasiparticle masses in the critical region. We have found that the Polyakov loop coupling to the quasiparticles helps to suppress the states in the critical region, permitting the masses to increase not in the same region. This behaviour is different from what is usually found in pure quasiparticle models, where the statistical suppression of states in the critical region can be achieved only by assuming a rapid increase of the quasiparticle masses as the critical temperature is approached from above.

We have found that within our simple model, the computed expectation value of the Polyakov loop is quite different from that computed on the lattice. In order to understand how this discrepancy affects the result on the quasiparticle masses, we have slightly modified our model by using the lattice $\ell_F$ as an input. The result is summarized in Fig. 5 which shows that the masses are only moderately affected by the different Polyakov loop expectation value.

The straightforward step ahead is to investigate on possible different couplings of the Polyakov loop background and the quasiparticles, in order to build a model in which $\ell_F$ is faithful to lattice data in a more natural way. We leave this interesting question to a near future project.
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