QCD Analysis of Polarized Scattering Data and New Polarized Parton Distributions

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In this talk results from a new QCD analysis in leading (LO) and next-to-leading (NLO) order are presented. New parametrizations of the polarized quark and gluon densities are derived together with parametrizations of their fully correlated 1σ error bands. Furthermore the value of α_s(M_Z^2) is determined. Finally a number of low moments of the polarized parton densities are compared with results from lattice simulations. All details of the analysis are given in Ref. [1].

1. Formalism

The twist–2 contributions to the structure function g_1(x, Q^2) can be represented in terms of a Mellin convolution of the polarized singlet density ∆Σ, the polarized gluon density ∆G, the polarized non–singlet density ∆q_{jNS}, and the corresponding polarized Wilson coefficient functions ∆C^A_i by

\[ g_1(x, Q^2) = \frac{1}{2} \sum_{j=1}^{N_f} e_j^2 \int_x^1 \frac{dz}{z} \left[ \frac{1}{N_f} \Delta \Sigma \left( \frac{x}{z}, \mu_f^2 \right) \Delta C_q^S \left( z, \frac{Q^2}{\mu_f^2} \right) + \Delta G \left( \frac{x}{z}, \mu_f^2 \right) \right. \]

\[ \times \Delta C_G \left( z, \frac{Q^2}{\mu_f^2} \right) + \Delta q_{jNS} \left( \frac{x}{z}, \mu_f^2 \right) \left. \Delta C_q^{NS} \left( z, \frac{Q^2}{\mu_f^2} \right) \right], \quad (1) \]

where e_j denotes the charge of the jth quark flavor, N_f the number of flavors, and μ_f is the factorization scale. In addition the above quantities depend on the renormalization scale μ_r of the strong coupling constant α_s(μ_r^2) = g_s^2(μ_r^2)/(16π^2).

The change of the parton distributions w.r.t. the factorization scale μ_f^2 = Q^2 is described by the evolution equations which contain the polarized

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splitting functions \( \Delta P_{ij} \). Both the polarized Wilson coefficient [2] and the polarized splitting functions [3] are known in the MS scheme up to NLO.

The evolution equations are solved in MELLIN–N space. A MELLIN–transformation turns the MELLIN–convolution into an ordinary product. The solutions for the evolved parton distributions are structured such that the input and the evolution part factorize [4], which is of key importance for the error calculation. An inverse MELLIN–transformation to \( x \)–space is then performed numerically by a contour integral in the complex plane around all singularities.

2. Parametrization and Error Calculation

The shape chosen for the parameterization of the polarized parton distributions at the input scale of \( Q^2 = 4.0 \) GeV\(^2\) is:

\[
x \Delta q_i(x, Q^2_0) = \eta_i A_i x^{a_i} (1 - x)^{b_i} (1 + \gamma_i x + \rho_i x^{1/2}).
\]

(2)

The normalization constant \( A_i \) is defined such that \( \eta_i \) is the first moment of \( \Delta q_i(x, Q^2_0) \). The densities to be fitted are \( \Delta u_v, \Delta d_v, \Delta \bar{q}, \) and \( \Delta G \).

Assuming \( SU(3) \) flavor symmetry the first moments of \( \Delta u_v \) and \( \Delta d_v \) can be fixed by the \( SU(3) \) parameters \( F \) and \( D \) measured in neutron and hyperon \( \beta \)–decays to \( \eta u_v = 0.926 \) and \( \eta d_v = -0.341 \). The sea-quark distribution \( \Delta \bar{q} \) was assumed to be described according to \( SU(3) \) flavor symmetry. Given the present accuracy of the data we set a number of parameters to zero, namely \( \rho_{u_v} = \rho_{d_v} = 0 \), \( \gamma_{\bar{q}} = \rho_{\bar{q}} = 0 \), and \( \gamma_G = \rho_G = 0 \). Furthermore we adopted the following two parameter relations: \( a_G = a_{\bar{q}} + c \), with \( 0.5 < c < 1.0 \) and \( (b_{\bar{q}}/b_G)(\text{pol}) = (b_{\bar{q}}/b_G)(\text{unpol}) \). These relations were essential to respect positivity for \( \Delta \bar{q} \) and \( \Delta G \) and to achieve the expected similar low–\( x \) behaviour for both parton densities. No positivity constraint was assumed for \( \Delta u_v \) and \( \Delta d_v \). In addition \( \Lambda_{QCD} \) was determined.

The gradients of the polarized parton densities w.r.t. the fitted parameters needed for the Gaussian error propagation are calculated analytically at the input scale \( Q^2_0 \) in MELLIN–N space. Their values at \( Q^2 \) are then given by evolution. For more details see Ref. [1].

To treat all data sets on the same footing only the statistical errors were used. To be able to calculate the fully correlated 1\( \sigma \) error bands via Gaussian error propagation only fits ending with a positive–definite covariance matrix were accepted. We allowed for a relative normalization shift between the different data sets within the normalization uncertainties quoted by the experiments. Thereby the main systematic uncertainties of the data were taken into account. The relative normalization shifts being fitted once and then fixed enter as an additional term (penalty) for each data set in the \( \chi^2 \)–expression.
3. Results

The results reported here are based on 435 data points of asymmetry data, i.e. $g_1/F_1$ or $A_1$, above $Q^2 = 1.0$ GeV$^2$, the world statistics published so far (see Ref. [1] for a list of references). The fits are performed on $g_1$ which is evaluated from the asymmetry data using parametrizations for $F_2$ [5] and $R$ [6]. The data do not constrain the four parameters $\gamma_{uv}$, $\gamma_{dv}$, $\bar{b}_q$, and $b_G$ well enough. Their values have been fixed as obtained in the first minimization. The NLO polarized parton densities at the input scale are presented in Fig. 1 (ISET = 3, see Ref. [1]).

![Diagram](image_url)

Fig. 1. Polarized parton distributions at the input scale $Q^2_0 = 4.0$ GeV$^2$ (solid line) compared to results obtained by GRSV (dashed–dotted line) [7], AAC (dashed line) [8]. The shaded areas represent the fully correlated $1\sigma$ error bands calculated by Gaussian error propagation [1]. The dark dotted lines correspond to the positivity bounds given by the unpolarized densities.

The current data constrain $\Delta u_v$ at best, followed by $\Delta d_v$, $\Delta \bar{q}_v$, and $\Delta G$. Mainly the lack of data at low $x$ leads to the broader error bands for the latter two densities. The comparison with results from other QCD analyses...
shows the variation of the outgoing parton distributions when using different parametrizations at the input scale. The measured structure function $g_1^p$ is well described both as function of $x$ and $Q^2$. The derived parton densities and their error bands have been evolved to $Q^2$ values up to 10,000 GeV$^2$. One finds that $\Delta u_v$ stays positive, $\Delta d_v$ remains negative although less constraint than $\Delta u_v$, and $\Delta G$ stays positive even within the error band for the range $Q^2 \geq 4 GeV^2$. All three densities evolve towards smaller values of $x$. The sea-quark density $\Delta \bar{q}$ is negative for $Q^2 \leq 10^4 GeV^2$ and remains negative within errors for $x \leq 5 \cdot 10^{-2}$ up to $Q^2 = 10^4 GeV^2$, but changes sign for larger values of $x$.

In determining $\alpha_s$ the QCD–parameter $\Lambda_{QCD}$ was fitted. The impact of the variation of both the renormalization and the factorization scale on the value of $\alpha_s$ was investigated. The following result was obtained (for ISET = 3, see Ref. [1], as well for more details):

$$\alpha_s(M_Z^2) = 0.113 \pm 0.004 \text{ (stat)} +0.004 \text{ (fac)} +0.008 \text{ (ren)} \ .$$

This value is compatible within 1σ with the world average of 0.118 ± 0.002 [9], although our central value is somewhat lower. It is also compatible with results from other QCD analyses of polarized and unpolarized data.

In recent lattice simulations [10] low moments for the polarized parton densities $\Delta u_v$, $\Delta d_v$, and $\Delta u - \Delta d$ were determined. In Table 1 these moments are compared with the ones extracted from our NLO polarized densities.

| $\Delta f$ | $n$ | QCD moments at $Q^2 = 4 \text{ GeV}^2$ | lattice results |
|-----------|-----|------------------------------------|----------------|
| $\Delta u_v$ | -1 | 0.926 ± 0.071 | 0.889(29) | 0.860(69) |
| | 0 | 0.163 ± 0.014 | 0.198(8) | 0.242(22) |
| | 1 | 0.055 ± 0.006 | 0.041(9) | 0.116(42) |
| $\Delta d_v$ | -1 | -0.341 ± 0.123 | -0.236(27) | -0.171(43) |
| | 0 | -0.047 ± 0.021 | -0.048(3) | -0.029(13) |
| | 1 | -0.015 ± 0.009 | -0.028(2) | 0.001(25) |
| $\Delta u - \Delta d$ | -1 | 1.267 ± 0.142 | 1.14(3) | 1.031(81) |
| | 0 | 0.210 ± 0.025 | 0.246(9) | 0.271(25) |
| | 1 | 0.070 ± 0.011 | 0.069(9) | 0.115(49) |

Table 1: Moments of the NLO parton densities (for ISET = 3, see [1]) at $Q^2 = 4 \text{ GeV}^2$ and from recent lattice simulations at the scale $\mu^2 = 1/a^2 \sim 4 \text{ GeV}^2$. 
The first moment is denoted with \( n = -1 \). For the \( n = 0, 1 \) values of the QCDSF Collaboration no continuum extrapolation was performed. The values compare within errors. Unlike in the unpolarized case where a strong \( m_\pi \)–dependence is expected in the lattice extrapolation the comparison here suggests a flat behaviour, which also has been found in [11] very recently.

4. Conclusions

A QCD analysis in LO and NLO of the current world–data on polarized structure functions was performed. New parametrizations of the polarized parton densities including their fully correlated \( 1\sigma \) error bands were derived. These parameterizations are available as a fast FORTRAN–routine which makes their application possible in Monte Carlo simulations. The value determined for \( \alpha_s(\mu_Z^2) \) is compatible within 1\( \sigma \) with the world average. Comparing the lowest moments with values from lattice simulations the errors improved during recent years and the values became closer.

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