INTERACTION CROSS SECTIONS AND SURVIVAL RATES FOR PROPOSED SOLAR SYSTEM MEMBER PLANET NINE

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ABSTRACT

Motivated by the report of a possible new planetary member of the solar system, this work calculates cross sections for interactions between passing stars and this proposed Planet Nine. Evidence for the new planet is provided by the orbital alignment of Kuiper belt objects, and other solar system properties, which suggest a Neptune-mass object on an eccentric orbit with a semimajor axis \( a_0 \approx 400–1500 \) au. With such a wide orbit, Planet Nine has a large interaction cross section and is susceptible to disruption by passing stars. Using a large ensemble of numerical simulations (several million) and Monte Carlo sampling, we calculate the cross sections for different classes of orbit-altering events: (A) scattering the planet into its proposed orbit from a smaller orbit, (B) ejecting it from the solar system from its current orbit, (C) capturing the planet from another system, and (D) capturing a free-floating planet. Results are presented for a range of orbital elements with planetary mass \( m_p = 10 M_\oplus \). Removing Planet Nine from the solar system is the most likely outcome. Specifically, we obtain ejection cross sections \( \sigma_{\text{int}} \sim 5 \times 10^9 \) au\(^5\) (5 \times 10^2 \) au\(^2\)) for environments corresponding to the birth cluster (field). With these cross sections, Planet Nine is likely to be ejected if the Sun resides within its birth cluster longer than \( \Delta t \gtrsim 100 \) Myr. The probability of ejecting Planet Nine due to passing field stars is \( \lesssim 5\% \) over the age of the Sun. Probabilities for producing the inferred Planet Nine orbit are low (\( \lesssim 5\% \)).

Key words: planet–star interactions

1. INTRODUCTION

Evidence for the existence of a new large planet in our solar system has recently been reported (Batygin & Brown 2016). This body, given the working name Planet Nine, provides a viable explanation for the orbital alignment observed for a collection of Kuiper belt objects (KBOs) on wide orbits, the existence of detached Sednoids, and highly inclined Centaurs with large semimajor axes (Marcos de la Fuente & de la Fuente Marcos 2014; Trujillo & Sheppard 2014). To explain these solar system properties, the hypothetical Planet Nine must have mass \( m_9 \approx 10 M_\oplus \), semimajor axis \( a_0 \approx 400–1500 \) au, and eccentricity \( e_0 \approx 0.4–0.9 \). Given the potential importance of a massive new solar system member, any constraints on its orbital properties, dynamical history, and/or formation mechanism are of great interest. With its wide orbit, Planet Nine is highly susceptible to gravitational perturbations from passing stars and binaries (Adams & Laughlin 2001; Adams et al. 2006; Malmberg et al. 2007, 2011; Versas et al. 2009). The goal of this Letter is to calculate the cross sections for interactions between the solar system and passing stars, both in the environment of the solar birth cluster and in the field. These cross sections can then be used to estimate probabilities for Planet Nine to survive or to be captured into its elongated orbit.

Astronomy has a long history of considering the gravitational perturbations due to previously undetected solar system bodies acting on known objects. This course of action led to the prediction and successful detection of the planet Neptune (Adams 1846; Galle 1846; Le Verrier 1846). Later on, the existence of a large Planet X was predicted to reside outside Neptune. Although the original Planet X was never found, the subsequent search ultimately led to the detection of the dwarf planet Pluto (see Tombaugh 1996 for more historical details). However, not all such predictions have been fruitful: Nemesis, a hypothetical red dwarf companion to the Sun with semimajor axis \( a_N \sim 10^5 \) au (Davis et al. 1984), was invoked to explain periodic extinctions in the fossil record and provides a cautionary example. As outlined below, this Letter places significant constraints on possible formation scenarios and dynamical histories for Planet Nine.

Dynamical scattering interactions between the solar system and passing stars can take place over a wide range of parameter space. This paper focuses on interactions that affect the orbit of Planet Nine, which we consider to be dynamically decoupled from the rest of the solar system. Specifically, the giant planets (Jupiter through Neptune) are too far inside the orbit of Planet Nine to affect its dynamics during close encounters, whereas KBOs have too little mass. The results of this Letter support this decoupling hypothesis (Section 2).

Here, we consider two background environments for scattering interactions: the solar birth cluster and the field. Most stars form within stellar clusters of some type (Lada & Lada 2003; Porras et al. 2003). Based on its observed properties, including radioactive enrichment levels and well-ordered orbits, our solar system is likely to have formed within a cluster containing \( N_\ast = 10^3 - 10^4 \) members (Adams & Laughlin 2001; Portegies Zwart 2009; Adams 2010; Pfalzner 2013); these same considerations suggest that the time spent in the birth cluster is \( \sim 100 \) Myr (see also Pfalzner et al. 2015). Within the cluster, with relative velocity dispersion \( \sigma_v \sim 1 \) km s\(^{-1}\), the Sun samples a range of stellar densities, with mean \( \langle n_\ast \rangle \sim 100 \) pc\(^{-3}\). After leaving its birth environment, the solar system has lived an additional \( \sim 4.5 \) Gyr in the solar neighborhood, which is characterized by stellar density \( \langle n_\ast \rangle \sim 0.1 \) pc\(^{-3}\) and velocity \( \sigma_v \sim 40 \) km s\(^{-1}\).
Tremaine 2008). Although the solar system spends more time in the diffuse environment of the field, interactions are more likely in the solar birth cluster due to the increased stellar density and larger cross sections (smaller $v_b$).

Several types of scattering interactions are of interest. First, because planet formation is difficult on wide orbits (Dodson-Robinson et al. 2009), Planet Nine could be formed in a nearly circular orbit with a much smaller semimajor axis ($a_0 \ll a \sim 400 \sim 1500$ au) and/or scattered by the giant planets into an orbit with perihelion $p \sim 30$–40 au (Adams & Laughlin 2003; Veras et al. 2009). The window for scattering a planet into an orbit with $a \sim 1000$ au is narrow (the required energy change is a large fraction of the initial binding energy, so that most planets are ejected instead of retained at large semimajor axis). In any case, we want to calculate cross sections for scattering Planet Nine from such initial orbits into its proposed long-term state. Second, after its formation, while Planet Nine traces through its distant orbit, it can be removed from the solar system by passing stars. We thus need to calculate the cross sections for planet ejection. Finally, Planet Nine, and its orbit, could be produced through a capture event. We thus need the cross sections for capturing Planet Nine from another solar system and/or as a freely floating body within the solar birth cluster.

This Letter is organized as follows. Section 2 reviews the technique used to calculate interaction cross sections and presents results for the channels of scattering interactions outlined above. These cross sections are used in Section 3 to estimate probabilities for producing Planet Nine by scattering from an internal orbit, capturing it from another solar system, capturing it from a freely floating body, and ejecting Planet Nine from the solar system. The paper concludes in Section 4 with a summary of our results and a discussion of their implications.

2. INTERACTION CROSS SECTIONS

In order to calculate the cross sections for interactions between the solar system and passing stars, we perform a large ensemble of numerical experiments using previously developed techniques (for details, see Adams & Laughlin 2001; Li & Adams 2015). This approach is summarized below.

The first step is to specify the initial configuration of the solar system. In this context, we are interested in the related issues of producing the desired orbit of Planet Nine via scattering interactions, removing Planet Nine from its wide orbit and capturing Planet Nine into the solar system. To study the first issue, the initial configurations are circular orbits with semimajor axes $a_0 = 200$, 400, 600, and 800 au. Note that formation of Planet Nine would be difficult for the larger semimajor axes, but interaction cross sections are small for the lower values. For ejection events, we consider starting states with $a_0 = 400$, 600, 800, and 1000 au and orbital eccentricities $e_0 = 0.4$, 0.6, and 0.8. For capture events, we consider the Sun to interact with a passing star that harbors Planet Nine. The initial orbit is assumed to be circular with semimajor axes $a_0 = 50$–200 au. We consider initial stellar hosts with $M_\ast = 1M_\odot$, $M_\ast = 0.3M_\odot$, and with a range of masses sampling the stellar IMF. We also consider capture events where Planet Nine starts as a freely floating body in the solar birth cluster (Perets & Kouwenhoven 2012). The mass of Planet Nine is taken to be $m_9 = 10M_\oplus \ll M_\ast$ for all simulations.

Next, we specify the background environment, i.e., the solar birth cluster or the field. The background determines the velocity distribution from which the encounter speeds are sampled. This distribution is assumed to be Maxwellian with dispersion $v_p = 1$ km s$^{-1}$ for the cluster environment (Lada & Lada 2003; Porras et al. 2003) and $v_p = 40$ km s$^{-1}$ in the solar neighborhood (Binney & Tremaine 2008). Here, we use the distribution of encounter velocities, which has a wider Maxwellian form with relative dispersion $\sqrt{2}v_b$. In addition, the velocity distribution used in our expectation values includes an additional factor of $v$ so that we calculate cross sections $\sigma_{\text{int}} = \langle \nu \sigma \rangle / v_b$, rather than $\langle \nu \sigma \rangle$ (Binney & Tremaine 2008). Note that the stellar density determines the interaction rates and the corresponding probabilities (Section 3) but does not affect calculations of the cross sections.

For the interacting stars, we consider binary systems because a sizable fraction of stars are found in binaries and because they produce significantly larger scattering effects. The binary periods, eccentricities, and mass ratios are sampled from observed distributions (Duquennoy & Mayor 1991); the primary mass is sampled from a standard log-normal form for the stellar initial mass function. The phases of the orbits are uniform-random, and the angles that determine the geometry of the interaction are chosen to produce an isotropic distribution of approach directions. Finally, we must specify the impact parameter $h$ of the interactions. In principle, one should consider the full range of impact parameters out to $h \to \infty$; in practice, however, distant encounters have little effect and do not contribute to the cross sections. Here, we adopt a maximum allowed impact parameter $h_{\text{max}} = 30a_0$, where $a_0$ is the semimajor axis of the binary. This varying distribution of impact parameters must be taken into account in determining the cross sections (Adams & Laughlin 2001). Because the impact parameter has a maximum value, the resulting cross sections represent lower limits to their true values.

With the starting states and interaction environment specified, we perform a large ensemble of numerical simulations, where the relevant parameters are sampled from the distributions described above using a Monte Carlo scheme. For each starting configuration of the solar system and each choice of background environment, we perform $N \approx 500,000$ numerical simulations, where the initial conditions for each case represent an independent realization of the parameters. The results are then used to calculate the interaction cross sections for the events of interest.

The interaction cross sections are listed in Table 1 for the different classes of scattering events. These cross sections listed are the expectation values $\sigma_{\text{int}} = \langle \nu \sigma \rangle / v_b$, where the angular brackets denote averaging over the distribution of encounter speeds. First, we consider cross sections for the formation of the Planet Nine orbit from an interior launching location. For the cross sections listed, the planet starts in a circular orbit with a range of starting semimajor axes, and the solar system is assumed to reside in its birth cluster ($v_b = 1$ km s$^{-1}$). The requirements for successful production of the Planet Nine orbit are that the final semimajor axis lies in the range $400 < a_f < 1500$, with orbital eccentricity $0.4 < e_f < 0.9$ and inclination angle $i_f < 60^\circ$. The resulting cross sections are listed in the first line of Table 1 for four starting values of semimajor axis $a_0 = 200$, 400, 600, and 800 au. These cross sections for producing the required Planet Nine orbit in a cluster environment are approximately the geometric cross
Table 1
Cross Sections for Formation, Ejection, and Capture of Planet Nine

| Event Class | $e_0$ | $\sigma_{\text{f}}(q_1)$ (au$^2$) | $\sigma_{\text{e}}(q_1)$ (au$^2$) | $\sigma_{\text{f}}(q_2)$ (au$^2$) | $\sigma_{\text{e}}(q_2)$ (au$^2$) |
|-------------|-------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| Formation   | 0     | 94600 ± 3400                  | 485200 ± 8300                 | 1020700 ± 12900               | 1261000 ± 14500               |
| Ejection    | 0.4   | 1907900 ± 16200               | 2656700 ± 19400               | 3090800 ± 21500               | 3879400 ± 24400               |
| Ejection    | 0.6   | 2645700 ± 19600               | 3811000 ± 24400               | 4433400 ± 26600               | 5782600 ± 30500               |
| Ejection    | 0.8   | 4834000 ± 28600               | 7017900 ± 35800               | 9386900 ± 40700               | 12226800 ± 48400              |
| Ejection-field | 0.4 | 12000 ± 1100                  | 15900 ± 1200                  | 27500 ± 1500                  | 33800 ± 1800                  |
| Ejection-field | 0.6 | 15600 ± 1200                  | 20800 ± 1400                  | 41900 ± 2000                  | 20800 ± 2100                  |
| Ejection-field | 0.8 | 25600 ± 1500                  | 32100 ± 1600                  | 62500 ± 2500                  | 64600 ± 2600                  |
| Capture(IMF) |      | 29400 ± 1400                  | 55100 ± 2000                  | 72000 ± 2400                  | 85600 ± 2600                  |
| Capture(m = 1) |    | 26800 ± 1300                  | 47500 ± 2000                  | 74900 ± 2500                  | 88300 ± 2600                  |
| Capture(m = 0.3) |    | 29200 ± 1400                  | 54900 ± 2000                  | 78100 ± 2400                  | 97800 ± 2900                  |
| Capture(IMF) | 0.6  | 32700 ± 1600                  | 57700 ± 2500                  | 72900 ± 2600                  | 85000 ± 3200                  |
| Capture(f → b) |    | 11800 ± 1300                  | 16900 ± 1500                  | 22000 ± 1700                  | 79000 ± 2800                  |

Note. For each type of interaction (labeled in the left column), cross sections are given for four values of semimajor axis (which vary with the type of event). Errors represent uncertainties due to incomplete Monte Carlo sampling. For formation events, Planet Nine starts in our solar system with an orbit at $a_0 = 200, 400, 600$, and $800$ au. For ejection events, Planet Nine starts in a circumsolar orbit with $a_0 = 400, 600, 800$, and $1000$ au. For capture events from bound orbits, Planet Nine starts in another solar system with $a_0 = 50, 100, 150$, and $200$ au. For the capture of freely floating planets ($f \rightarrow b$), cross sections are given for final (circumsolar) orbits with $a_f < a_{\text{c}}(\text{max}) = 1000, 1500, 2000$ au, and $\infty$.

Section of the original orbit (specifically, this ratio lies in the range 0.6–1). For completeness, we have calculated cross sections for producing Planet Nine from smaller initial orbits. However, the resulting cross sections are small, $\sigma_{\text{f}} = 133, 974, 3181,$ and $13053$ au$^2$ for $a_0 = 10, 30, 50$, and $100$ au. Finally, given that Planet Nine could have formed in the giant-planet region and been scattered outward, we have calculated cross sections for producing Planet Nine from eccentric orbits with $a_0 = 50–200$ au and perihelion $p = a(1 - e) = 5–40$ au. However, these cross sections are somewhat smaller than those listed in Table 1 for circular starting orbits.

Next, we consider the cross sections for the ejection of Planet Nine. For this class of events, we consider six ensembles of simulations, with three starting values of eccentricity ($e_0 = 0.4, 0.6, 0.8$) and two velocity scales ($v_b = 1$ and $40 \, \text{km} \, s^{-1}$, for the solar birth cluster and the field). Here, we consider four possible values for the semimajor axis of Planet Nine $a_0 = 400, 600, 800$, and $1000$ au (see Table 1). For interactions in a low-speed environment ($v_b = 1 \, \text{km} \, s^{-1}$), the cross sections for ejection of Planet Nine are approximately an order of magnitude larger than the cross sections for scattering Planet Nine into its orbit (see Table 1). This finding suggests that the production of Planet Nine is unlikely to take place by scattering from a close orbit into its required wide orbit.

In principle, Planet Nine could be captured from another solar system. To address this issue, we first assume that the planet initially resides in orbit around another star. For these interactions, the passing “binary” is thus the host star with Planet Nine as its companion. The starting orbit of Planet Nine is taken to have semimajor axes $a_0 = 50, 100, 150$, and $200$ au and vanishing eccentricity $e_0 = 0$. The interactions take place in the solar birth cluster with $v_b = 1 \, \text{km} \, s^{-1}$. We consider the capture of Planet Nine from solar systems with several types of host stars. First, the mass of the star is sampled from the stellar IMF; we also consider cases where the host star has fixed mass $M_*=1M_\odot$ and $M_*=0.3M_\odot$. For comparison, we also consider initial orbits with non-zero eccentricity $e_0=0.6$.

The resulting cross sections for the Sun to capture Planet Nine from another star are given in Table 1. These capture cross sections are much smaller than the ejection cross sections within the solar birth cluster, but are roughly comparable to ejection cross sections in the field (where $v_b = 40 \, \text{km} \, s^{-1}$). The capture cross sections for M-dwarf host stars (with $M_* = 0.3M_\odot$) are nearly the same as those calculated by sampling the stellar IMF, whereas cross sections for $M_* = 1M_\odot$ hosts are mostly smaller. Finally, the cross sections for capture from initial orbits with $e_0=0.6$ are comparable to those with $e_0=0$.

These capture cross sections (listed in Table 1) include all events where the Sun successfully steals Planet Nine from another star, into any bound orbit. However, not all capture events lead to acceptable orbits for a newly acquired planet. Figure 1 shows the post-encounter orbital elements for Planet Nine, in its new orbit about the Sun, for four values of the starting semimajor axis (where the host-star mass is sampled from the IMF). Capture events produce final-state orbits with the full distribution of eccentricities, $0 \leq e \leq 1$, so that the target range for Planet Nine ($0.4 \leq e \leq 0.9$) is readily obtained (see also Jilková et al. 2015). The distribution of final-state semimajor axes is also wide, where the target range $a_f = 400–1500$ au is often realized. Specifically, the fractions of acceptable final-state orbits (out of the total number of capture events) are $3.2\%$ ($a_0 = 50$ au), $14\%$ ($a_0 = 100$ au), $24\%$ ($a_0 = 150$ au), and $32\%$ ($a_0 = 200$ au).

Finally, we consider the capture of free-floating planets. For these simulations, the solar system starts with its four giant planets in their current orbits, and then interacts with a free-floating Planet Nine. The total cross sections for capturing freely floating planets (final line of Table 1) are similar to those for capturing bound planets. However, the fraction of acceptable final states is only $\sim 8.4\%$, so it is more likely to capture a bound planet with large $> 100$ au orbit.

With the cross sections determined, we can assess our assumption that the Planet Nine orbit is dynamically decoupled...
from the inner solar system. The giant-planet orbits are vulnerable to changes during scattering encounters, and the cross sections for their disruption have been calculated previously (Adams & Laughlin 2001; Li & Adams 2015). Specifically, the cross section for doubling the eccentricity of Neptune, or the spread in inclination angles, is \( \sigma_{\text{dis}} \approx (400 \text{ au})^2 \). This cross section is thus smaller than that required to eject Planet Nine by 1–2 orders of magnitude, depending on \( a_9 \) (Table 1). As a result, severe changes to the Planet Nine orbit (ejection) are much more likely than modest changes to the giant-planet orbits.

Although the giant planets are relatively unperturbed, the wider orbits of KBOs can be altered during scattering encounters. We have performed additional ensembles of simulations to address this issue: First, we placed KBOs in orbits at \( a_0 = 200–500 \text{ au} \) in simulations where the solar system captures Planet Nine as a freely floating planet. For KBO orbits with initial eccentricity \( e_0 = 0.7 \), the post-encounter eccentricities always fall within the range \( e_f = 0.55–0.8 \). As a result, Planet Nine capture events produce only moderate eccentricity changes in the wide-orbit KBO population. In contrast, if Planet Nine is captured from a bound orbit in another solar system, the KBOs can be scattered significantly. Specifically, the cross sections for ejecting KBOs (\( a_0 = 200–500 \text{ au}, e_0 = 0.7 \)) are somewhat larger than the cross sections for capturing Planet Nine. Finally, we note that KBO orbits can also be re-populated through close stellar encounters (Kenyon & Bromley 2004).

The simulations of this Letter are limited to 40 dynamical times of the initial (separated) systems (\( t \lesssim 10^5 \text{ years} \)). This interval includes many dynamical times of the giant planets but does not capture long-term secular interactions (see also Li & Adams 2015). The cross sections presented here thus describe the immediate changes in the orbital elements, whereas additional changes can occur over longer timescales. Since Planet Nine is thought to reside in an inclined orbit, e.g., it could enforce Kozai–Lidov oscillations of the giant planets over secular timescales. These longer-term effects, which should be considered in future work, are potentially significant for any scenario that includes Planet Nine.

3. SCATTERING AND SURVIVAL PROBABILITIES

Given the interaction cross sections calculated in Section 2 for different scattering events, we now estimate the probabilities of those events occurring. The scattering rate is given by

\[
\Gamma = n_s \langle v \sigma \rangle = n_s \sigma_{\text{int}} v_b, \tag{1}
\]

where \( \sigma_{\text{int}} \) is the cross section for the particular event and the corresponding optical depth for scattering has the form

\[
P = \int \ln \sigma_{\text{int}} v_b. \tag{2}
\]

For \( P < 1 \), this quantity defines the probability of scattering; for larger values \( P > 1 \), the probability \( \mathcal{P} \) for survival in the face of scattering has the form \( \mathcal{P} = \exp(-P) \). Because the velocity dependence of scattering cross sections has the
approximate power-law form $\sigma_{\text{int}} \propto v^{-7/5}$ (Li & Adams 2015), the product of $v \sigma_{\text{int}} \propto v^{-2/5}$, so that the integrand in Equation (2) does not depend sensitively on the time dependence of the velocity. We then write Equation (2) in the form

$$P = \sigma_{\text{int}}(v_b) \int \mathcal{W} \gamma_{B} \sigma_{\text{int}}(v_b) dv_b \equiv \sigma_{\text{int}}(v_b)(\Delta t) v_b \langle n_b \rangle,$$

where $v_b$ is the velocity dispersion of the environment and the final equality defines the time-and-velocity-averaged density $\langle n_b \rangle$ of the background.

First, we consider the properties of the solar birth aggregate. Previous considerations have placed constraints on this environment and indicate that the Sun was born within a cluster of intermediate stellar membership size $N_s \sim 3000$ (Portegies Zwart 2009; Adams 2010; Pfalzner 2013) where the velocity scale $v_B = 1 \text{ km s}^{-1}$. These same considerations suggest that the solar system is likely to reside in the cluster for an integration time $(\Delta t) \approx 100 \text{ Myr}$, a typical value for the lifetime of an open cluster. The mean stellar density in clusters in the solar neighborhood (Lada & Lada 2003) is nearly constant as a function of stellar membership size $N_s$ (see Figure 1 of Proszkow & Adams 2009) and has a value $\langle n_b \rangle = 100 \text{ pc}^{-3}$, which is used to calculate probabilities for this Letter.

We also consider the solar neighborhood in the Galaxy, where the velocity dispersion $v_B = 40 \text{ km s}^{-1}$ (Binney & Tremaine 2008) and the integration time $(\Delta t) = 4.6 \text{ Gyr}$. The stellar density at the solar circle is difficult to measure. Here, we use recent estimates where $\langle n_s \rangle \approx 0.1 \text{ pc}^{-3}$ (Bovy et al. 2012; McKee et al. 2015). Note that the velocity dispersion of stars tends to increase with stellar age (Nordström et al. 2004), although this complication is not included. As outlined above, the scattering rate $\Gamma \propto (v \sigma_{\text{int}}) \propto v^{-2/5}$ varies slowly with velocity. We expect $v_B$ to vary by a factor $\lesssim 2$ over the age of the solar system in the field, so that the probability estimate changes by a factor $\lesssim 1.3$.

The first class of interactions under consideration is the possible production of Planet Nine orbital elements from less extreme initial conditions. Here, we take the starting configurations to be circular orbits with a range of semimajor axes $a_0 = 200$–800 au. The final system parameters that represent successful production require orbital eccentricity $0.4 < e_f < 0.9$, semimajor axis $a_f = 400$–1500 au, and inclination angle $i_f < 60^\circ$ (Batygin & Brown 2016). Figure 2 shows the scattering optical depths for producing Planet Nine as a function of the starting semimajor axis. The formation probabilities are low unless the starting semimajor axis is large, $a_0 \gtrsim 400$ au, comparable to the required final values of $a_f$. Even for $a_0 = 600$ au, the preferred final value, the probability of scattering Planet Nine into its required orbit is only $\sim 25\%$. The probabilities fall to $\sim 3\%$ for $a_0 = 200$ au. Planets starting with orbits in the giant-planet region of our solar system have a negligible chance of scattering into the required orbit. This finding is consistent with previous results (Adams & Laughlin 2001; Li & Adams 2015), which show that eccentricity and inclination angles are much more readily altered than semimajor axes during scattering interactions.

Next, we consider the probability of removing Planet Nine from its orbit. Here, we consider a range of initial orbital elements, with semimajor axis $a_0 = 400$–1000 au and eccentricity $e_0 = 0.4$–0.8. These ejection simulations are carried out for both the solar birth cluster ($v_B = 1 \text{ km s}^{-1}$) and the solar neighborhood ($v_B = 40 \text{ km s}^{-1}$). Figure 3 shows the resulting optical depths for ejecting Planet Nine from our solar system, plotted versus initial semimajor axis $a_0$. The scattering optical depths are of order unity for the cluster environment (solid curves) but are lower, 0.003–0.03, for the field (dashed curves). Moreover, the interaction rates increase with orbital eccentricity. These results suggest that lower eccentricities and late formation times are preferred.

Next, we consider the capture of Planet Nine from another star, where the planet has starting semimajor axis $a_0$. Within the cluster environment, the total capture optical depth and that for capture into a viable Planet Nine orbit are $(P_c, P_o) = (0.0071, 0.0002, 50 \text{ au})$, $(0.013, 0.0018, 100 \text{ au})$, $(0.017, 0.0042, 150 \text{ au})$, and $(0.021, 0.0066, 200 \text{ au})$. The probability of successful Planet Nine production through capture is thus $\lesssim 1\%$. In addition to the low probability of capture, another problem arises: the largest cross sections for capture occur for systems where Planet Nine already has a wide orbit. Even for $a_0 = 200$ au, the probability of success is only $\sim 0.6\%$, but the formation of Planet Nine in such a wide orbit is problematic. It is thus unlikely that Planet Nine forms in a wide...
orbit around another star and is subsequently captured, as this scenario requires two low probability events to take place.

Finally, for freely floating planets, the total capture optical depth \( \Phi_r = 0.019 \), whereas the optical depth for capture into a viable Planet Nine orbit \( \Phi_b = 0.084 \). These estimates assume that the density and velocity of freely floating planets are comparable to \( (n_s) \) and \( v_b \). Since the capture cross sections are roughly comparable for bound planets in wide orbits and freely floating planets, the resulting probabilities are also comparable (0.2%).

The cross sections in this Letter are calculated for interactions with binaries. However, the correction factor for including single stars is of order unity and has the form

\[
\mathcal{F} = f_b + (1 - f_b) \frac{\sigma_{ss}}{\sigma_b},
\]

where \( f_b \approx 1/2 \) is the binary fraction (Duchêne & Kraus 2013). The ratio of cross sections for impinging single stars versus binaries \( \sigma_{ss}/\sigma_b \approx 1/3 \) (Li & Adams 2015), so that \( \mathcal{F} \approx 0.67 \).

4. CONCLUSION

This paper determines cross sections for scattering interactions between the proposed new solar system member Planet Nine and passing stars (see Table 1). Here, we consider several different classes of interaction events and estimate probabilities for their occurrence.

First, the extreme orbit inferred for Planet Nine can be produced (from an initially closer orbit) via scattering interactions with passing binaries in the solar birth cluster. The cross sections for producing the required orbit increase with starting semimajor axis and the corresponding probability of producing such an orbit is \( \sim 10\% \) (Figure 2). Such a high success rate requires the initial orbit to be rather wide, \( a_0 = 200–400 \) au.

Next we determine cross sections for ejecting Planet Nine from the solar system, where the planet starts in its current (inferred) orbit. For both the solar birth cluster and the field, cross sections are calculated for a range of possible orbital elements \( (a_0, e_0) \). With these cross sections, Planet Nine has a scattering optical depth of order unity (Figure 3) if the Sun resides within its birth cluster for \( (\Delta t) \approx 100 \) Myr. More specifically,

\[
P = 0.78 \left( \frac{\sigma_{int}}{5 \times 10^6 \text{ au}^2} \right) \left( \frac{(n_s)}{100 \text{pc}^{-3}} \right) \left( \frac{v_b}{1 \text{ km s}^{-1}} \right) \times \left( \frac{\Delta t}{100 \text{ Myr}} \right) \left( \frac{\mathcal{F}}{0.67} \right),
\]

where we have used a representative value for the cross section; the factor \( \mathcal{F} \) includes the correction for single stars. The corresponding scattering optical depth \( P \approx 0.002–0.02 \) for interactions in the field (Figure 3). The longer residence time in the field nearly (but not quite) compensates for the lower stellar density and smaller cross section. These results indicate that Planet Nine is most likely produced (either formed or placed in its orbit) after the Sun leaves its birth cluster or near the end of its residence time there. The capture of a free-floating planet naturally occurs during cluster dispersal, but the probabilities remain low.

Given that the current collection of solar system planets is not necessarily the original line-up, we calculate cross sections for capturing Planet Nine. In one scenario, the planet starts with a circular orbit around another star and is captured into its current orbit about the Sun. The cross sections for these capture events are an order of magnitude smaller than the ejection cross sections. In another scenario, Planet Nine starts as a freely floating planet and is captured by our solar system; these events take place with slightly lower probability than capture from a bound orbit. Moreover, capture events are most likely to take place while the solar system is leaving its birth aggregate, equivalently, when the cluster disperses (Perets & Kouwenhoven 2012). If the Sun captures Planet Nine through either channel, the final-state orbital elements often fall in the range necessary to align the KBOs (Figure 1). However, the probability of successful Planet Nine capture is only \( \lesssim 1\% \).

The results of this Letter constrain the dynamical history and possible formation scenarios for Planet Nine. These scattering simulations indicate that survival of Planet Nine is possible but not guaranteed. The ejection cross sections increase (almost linearly) with semimajor axis, so that smaller orbits favor survival and are therefore preferred. This work also suggests that production of the required orbit is somewhat problematic: capture events can produce the right orbital elements, but the overall cross sections for capture are quite low. Scattering Planet Nine into its current orbit from a smaller initial orbit (within the solar system) and capturing it from a freely floating state are also possible, but such events are less likely than ejecting the planet. The formation scenario for Planet Nine thus represents an important challenge for future investigation.

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