Lopsided Mass Matrices and Leptogenesis in SO(10) GUT

T. Asaka

Institute of Theoretical Physics, University of Lausanne,
CH-1015 Lausanne, Switzerland

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Abstract

Lopsided structure in mass matrices of down quarks and leptons gives a simple explanation for the observed large angles of neutrino mixings. We realize such mass matrices by the Froggatt-Nielsen mechanism in the framework of supersymmetric SO(10) grand unified theory (GUT). It is shown that the model can reproduce the successful mass matrices which have been obtained in SU(5) models. Cosmological implication of the model is also discussed. We show that the hybrid inflation occurs naturally in the model and it offers non-thermal leptogenesis by decays of the next-to-lightest right-handed neutrinos. The present baryon asymmetry is explained by just the oscillation mass scale in the atmospheric neutrinos.
Recent neutrino experiments have provided quite convincing evidence for neutrino masses and their flavour mixings. The atmospheric neutrino anomaly is explained by the $\nu_\mu - \nu_\tau$ oscillations with $\delta m_{\text{atm}}^2 \simeq 2.5 \times 10^{-3}$ eV$^2$ and $\sin^2 2\theta_{\text{atm}} \simeq 1.0$ [1], while the solar neutrino problem is now solved by the so-called “large-mixing angle solution” with $\delta m_{\text{sol}}^2 \sim 7 \times 10^{-5}$ eV$^2$ and $\tan^2 \theta_{\text{sol}} \sim 0.5$ [2]. These developments give us an important clue to physics beyond the Standard Model.

The most natural extension is probably to introduce right-handed neutrinos with superheavy Majorana masses, since they induce the observed suppressed masses of neutrinos through the seesaw mechanism [3]. Further, non-equilibrium decays of right-handed neutrinos in the early Universe offer one natural way to generate the present baryon asymmetry [4]. Then, grand unified theory (GUT) based on SO(10) gauge group [5] is particularly attractive, since all quarks and leptons of each family are unified into a single spinor representation together with the right-handed neutrino. For unification of gauge couplings and stabilization of the gauge hierarchy we had better incorporate supersymmetry.

The observed mixing angles among neutrino flavours are both large, which is completely different from the quark sector. This is a big challenge in constructing a realistic model of SO(10) GUT, since it describes quarks and leptons in unified way. One simple possibility has been proposed in the so-called “lopsided” models [6, 7, 8]. Mass matrices for down quarks and leptons are arranged to have the lopsided structure, i.e., off-diagonal elements appear in a lopsided way such that mixings of left-handed leptons are large while those of left-handed down quarks are small, which give desired mixings of quarks and leptons in the charged current. There have been proposed so far many models to realize the lopsided mass matrices [9].

Such mass matrices can be based on the Froggatt-Nielsen (FN) mechanism [10]. It has been shown in Ref. [6] by using SU(5) GUT that the observed mass hierarchies of quarks and charged leptons are explained by their lopsided charges under some FN flavour symmetry, and the large $\nu_\mu - \nu_\tau$ mixing angle in the atmospheric neutrinos is a direct consequence of such lopsided charge assignment. In this letter we would like to realize this idea in the framework of SO(10) GUT and show one possibility to yield the successful mass matrices obtained in the lopsided SU(5) model. We also discuss cosmological implication of the model, in particular, possible scenarios of leptogenesis in the inflationary Universe.

Masses of quarks and charged leptons approximately satisfy the following relations at the unification scale $M_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV:

\[
\begin{align*}
    m_u : m_c : m_t & \simeq \epsilon^4 : \epsilon^2 : 1 , \\
    m_e : m_\mu : m_\tau & \simeq m_d : m_s : m_b \simeq \epsilon^3 : \epsilon : 1 ,
\end{align*}
\]

where $\epsilon \sim 1/16$. One attractive attempt to understand mass hierarchies and mixings among fermions consists in calling upon the Froggatt-Nielsen (FN) mechanism [10]. This mechanism can be based on U(1)$_{\text{FN}}$ flavor symmetry, which is broken by a vacuum expectation value (vev) of a gauge singlet field $S_{\text{FN}}$ carrying the $U(1)$_{\text{FN}} charge $-1$.\footnote{We take here U(1)$_{\text{FN}}$ as the FN flavour symmetry by way of illustration. This U(1)$_{\text{FN}}$ can be replaced by some discrete symmetry which is anomaly-free.} Matter $f_i$ and Higgs $H$ superfields
are introduced with charges $Q_i$ ($Q_i \geq 0$) and 0, respectively. Then, the $U(1)_{FN}$ flavor symmetry allows the following Yukawa terms

$$W = c_{ij} \left( \frac{S_{FN}}{M_*} \right)^{Q_i+Q_j} H f_i f_j ,$$

where $M_*$ denotes the fundamental cutoff scale of the theory and we regard it as the gravitational scale. By assuming that constant coefficients $c_{ij}$ are of order unity, fermion mass matrices scale as $m_{ij} \propto \epsilon^{Q_i+Q_j}$ depending on the $U(1)_{FN}$ charges, where the parameter $\epsilon$ is defined by

$$\epsilon \equiv \langle S_{FN} \rangle / M_* .$$

We set $\epsilon \sim 1/16$ from the mass relations shown in Eq. (1).

The above FN mechanism can be used to construct lopsided mass matrices for down quarks and leptons, which explain the fermion mass relations in Eq. (1) as well as the observed large neutrino mixing angles while keeping quark mixings small. First, we shall briefly review this point by using SU(5) model, which have been discussed in Refs. [6, 11].

In SU(5) GUT, one family of quarks and leptons can be grouped into the SU(5) multiplets, $10$-plet, $5^*$-plet and $1$-plet. Their Yukawa interactions are given by

$$W = h_{ij}^u H_u^{10} \rho_{ij}^{10} + h_{ij}^d H_d^{10} 5_{ij}^{*} + h_{ij}^D H_u^{10} 5_{ij}^{*} + h_{ij}^N S_{ij}^{1} ,$$

where $H_u$ and $H_d$ denote Higgs fields of $5$-plet and $5^*$-plet and their vevs are denoted by $v_2$ and $v_1$, respectively. Here we assumed that Majorana masses for right-handed neutrinos come from the vev of the singlet Higgs $S$. The hierarchies in the Yukawa couplings $h_{ij}^{u,d,D,N}$ are explained by the FN mechanism. In Table 1 we show the $U(1)_{FN}$ charge assignment for matter fields [6, 11]. Then, we obtain mass matrices for quarks and charged leptons as

$$m^u = v_2 h^u = v_2 \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} , \quad m^e = (m^d)^T = v_1 h^d = v_1 \begin{pmatrix} \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \\ \epsilon & 1 & 1 \end{pmatrix} .$$

It should be noted that every component in these mass matrices contains a coefficient of order unity, which are implicitly assumed here and hereafter. It is found that the “lopsided” charge assignment between $10$-plets and $5^*$-plets leads to the lopsided mass matrices for down quarks and charged leptons (compared with the mass matrix for up quarks), which is crucial for explaining the mass relations given in Eq. (1). The mixing angles for quarks are found to be small.

| $f_i$ | $10_3$ | $10_2$ | $10_1$ | $5^*_3$ | $5^*_2$ | $5^*_1$ | $1_3$ | $1_2$ | $1_1$ |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $U(1)_{FN}$ | 0      | 1      | 2      | $a$    | $a$    | $a+1$  | $b$    | $c$    | $d$    |

Table 1: $U(1)_{FN}$ charge assignment in SU(5) GUT. Here $a = 0$ or 1 and $0 \leq b \leq c \leq d$. 
On the other hand, the charge assignment in Table 1 can also explain large angles of neutrino flavour mixings. Dirac and Majorana mass matrices for neutrinos, $m^D$ and $m^N$, are given by

$$m^D = v_2 h^D = v_2 e^a \begin{pmatrix} \epsilon^{d+1} & \epsilon^d & \epsilon^d \\ \epsilon^{c+1} & \epsilon^c & \epsilon^c \\ \epsilon^{b+1} & \epsilon^b & \epsilon^b \end{pmatrix}, \quad m^N = \langle S \rangle h^N = \langle S \rangle \begin{pmatrix} \epsilon^{2d} & 0 & 0 \\ 0 & \epsilon^{2c} & 0 \\ 0 & 0 & \epsilon^{2b} \end{pmatrix}, \quad (6)$$

where we have chosen a basis where $m^N$ is diagonal and real. It is seen that $m^D$ takes also the lopsided form. Then, the seesaw mechanism [3] brings us the mass matrix for three light neutrinos as

$$m^\nu = -(m^D)^T \frac{1}{m^N} m^D = \frac{v_2^2}{\langle S \rangle} \epsilon^{2a} \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}. \quad (7)$$

Notice that the dependence on the FN charges of right-handed neutrinos drops off in this expression. It is clearly seen that the angle of a $\nu_\mu$-$\nu_\tau$ mixing is large, which is observed in the atmospheric neutrino experiments. This is a direct consequence of the fact that $5^*$-plets in the second and third families have the same FN charges in order to explain the mass hierarchies of down quarks and charged leptons [6, 7, 8]. Furthermore, it have been shown in Ref. [12] that there is no difficulty in this mass matrix to give the large mixing angle in the solar neutrinos.

The above lopsided SU(5) model gives the successful mass matrices for quarks and leptons, to the first approximation. To explain the mass matrices more precisely, we have to include effects of SU(5) breaking. Otherwise, the unwanted SU(5) mass relations $m_e = m_d$ and $m_\mu = m_s$ are obtained. This can be achieved by introducing additional Higgs fields. However, this issue is beyond the scope of this analysis.

In SO(10) GUT, one family of quarks and leptons can be simply grouped into an irreducible spinor representation ($16$-plet).² If it is the case, quarks and leptons in each family possess the same FN charge and all mass matrices of quarks and leptons have the same hierarchical structure. This means that we fail to explain the mass relations (1). To avoid this difficulty, we introduce three $16$-plets $\psi_i$ ($i = 1, 2, 3$) and also one additional $10$-plet $\eta$ as matter fields:

$$\psi_i = (10_i, \ 5^*_i, \ 1_i) \quad \text{and} \quad \eta = (5_4, \ 5^*_4), \quad (8)$$

where we have shown the decomposition under SU(5) group. In Table 2 we show the FN charges for these matter superfields. We determine the charges of $\psi_i$ from the mass hierarchy of up quarks and assign zero charge to $\eta$. In the followings, we will exchange one combination of $5^*_i$ in $\psi_i$ for $5^*_i$ in $\eta$, which have been proposed in the different SO(10) model [13]. By this exchange we can have the lopsided structure between $10$-plets and $5^*$-plets in three families.

²The representations with and without underline correspond to those under SO(10) and SU(5) gauge groups, respectively.
We introduce here the following Higgs superfields

\[ H_1 (\mathbf{10}), \quad H_2 (\mathbf{10}), \quad \Phi (\mathbf{16}), \quad \Phi^c (\mathbf{16}^*), \quad \Sigma (\mathbf{16}) \quad \text{and} \quad \Sigma^c (\mathbf{16}^*) \, . \]  

(9)

Two Higgs doublets which couple to down quarks and up quarks are assumed to be contained in \( H_1 \) and \( H_2 \), respectively.\(^3\) We assume the vevs of these Higgs fields as follows:

\[ \langle H_1 \rangle = v_1, \quad \langle H_2 \rangle = v_2, \quad \langle \Phi \rangle = \langle \Phi^c \rangle = v_\Phi \quad \text{and} \quad \langle \Sigma \rangle = \langle \Sigma^c \rangle = v_\Sigma \, , \]  

(10)

where \( v_1^2 + v_2^2 = (174 \text{ GeV})^2 \) and \( v_\Phi \) and \( v_\Sigma \) are of order of the unification scale \( M_{\text{GUT}} \) which keep an SU(5) subgroup unbroken. All these Higgs fields carry zero charge under the U(1)\(_{FN}\) symmetry (see, however, the discussion in the footnote 4.).

Now we are at the point to discuss mass matrices for quarks and leptons. The Yukawa interactions we shall consider here are given by the following superpotential:

\[ W = \frac{1}{2} h^d_{ij} H_1 \psi_i \psi_j + \frac{1}{2} h^u_{ij} H_2 \psi_i \psi_j + \frac{1}{2} h^n_{ij} \frac{\Phi^c \Phi^c}{M_s} \psi_i \psi_j \]

\[ + g^n_\eta \Sigma \psi_i \eta + g^d_\psi \frac{\Sigma}{M_s} H_1 \psi_i \eta + g^u_\psi \frac{\Sigma^c}{M_s} H_2 \psi_i \eta \, , \]  

(11)

where \( h^{d,u,n} \) and \( g^{n,d,u} \) denote Yukawa coupling constants which are explained by the FN charges of matter fields. In this equation, the first two terms give usual Dirac masses for quarks and leptons, the third term gives Majorana masses for right-handed neutrinos, and the rest three terms denote mass mixings between matter \( \mathbf{16} \)-plets and additional \( \mathbf{10} \)-plet.

Let us first discuss the exchange of \( 5^* \)-plets in matter fields. The vev of the \( \Sigma \) field induces

\[ W = g^n_\psi v_\Sigma \mathbf{5}_4 \mathbf{5}^* \, , \]  

(12)

which gives a Dirac mass for one linear combination (denoted by \( \mathbf{5}^*_H \)) among three \( \mathbf{5}^*_i \) in \( \psi_i \), while \( \mathbf{5}^*_4 \) in \( \eta \) is still massless \(^{13}\). From the FN charge assignment in Table 2 the Yukawa couplings \( g^n_\psi \) are given by \( g^n_\psi = (\epsilon^2, \epsilon, 1) \) with \( \epsilon \sim 1/16 \). Note again that coefficients of order unity are implicitly assumed. We find, then, the dominant component of \( \mathbf{5}^*_H \) is \( \mathbf{5}^*_3 \):

\[ \mathbf{5}^*_H \simeq \epsilon^2 \mathbf{5}^*_1 + \epsilon \mathbf{5}^*_2 + \mathbf{5}^*_3 \simeq \mathbf{5}^*_3 \, . \]  

(13)

\(^3\)In the SO(10) models discussed in Refs. \(^{13}\) \(^{14}\) one \( \mathbf{10} \)-plet and one \( \mathbf{16} \)-plet are introduced for the Higgs doublets. The considering two \( \mathbf{10} \)-plets for the weak Higgs doublets are a natural consequence of the model in six dimensions where SO(10) breaking is achieved by orbifold compactification \(^{13}\) \(^{14}\). In this case, the mass splitting between the weak doublet and the color triplet Higgs fields is realized naturally.
In the following analysis we will take $5^* H = 5^*_3$ for simplicity. Since the mass of $5^*H$ is estimated as $g^3 v_\Sigma \simeq v_\Sigma \sim M_{GUT}$, three families of quarks and leptons at low energies are given by

$$
10_1(2), \quad 10_2(1), \quad 10_3(0),
$$

$$
5^*_1(2), \quad 5^*_2(1), \quad 5^*_4(0),
$$

$$
1_1(2), \quad 1_2(1), \quad 1_3(0).
$$

(14)

Notice that there appears no other massless matter field. Here we have also shown the FN charge of each multiplet. Even after the exchange of $5^*$-plets the FN charge assignment does not have the lopsided structure. As we will explain below, the Higgs field $\Sigma^c$ plays an important role to generate the lopsided mass matrices.

Up quarks obtain Dirac masses from the usual Yukawa term $W = \frac{1}{2} h_{uij}^u H_2 \bar{\psi}_i \psi_j$ in the superpotential (11) and its mass matrix takes the form

$$
m^u = v_2 h^u = v_2 \begin{pmatrix}
\epsilon^4 & \epsilon^3 & \epsilon^2 \\
\epsilon^3 & \epsilon^2 & \epsilon \\
\epsilon^2 & \epsilon & 1
\end{pmatrix}.
$$

(15)

This is the same as in the SU(5) model (cf. Eq. (5)) and gives the approximate mass relations shown in Eq. (1). On the other hand, down quarks and charged leptons receive masses from the following terms in the superpotential (11)

$$
W = \frac{1}{2} h_{dij}^d H_1 \bar{\psi}_i \psi_j + g_{d_i}^d \frac{\Sigma^c}{M_*} H_1 \bar{\psi}_i \eta = h_{dij}^d \langle H_1 \rangle \begin{pmatrix}
10 \\
5^*_1 \\
5^*_2 \\
5^*_4 \\
\cdot \cdot \cdot
\end{pmatrix},
$$

(16)

where $\epsilon^c \equiv v_\Sigma / M_*$. Since $5^*_H = 5^*_3$ decouples from the low energy physics, we have the effective mass terms for down-quarks and charged leptons as

$$
W = \langle H_1 \rangle \begin{pmatrix}
10_1 \\
10_2 \\
10_3
\end{pmatrix} \begin{pmatrix}
\epsilon^4 & \epsilon^3 & \epsilon^2 \\
\epsilon^3 & \epsilon^2 & \epsilon \\
\epsilon^2 & \epsilon & 1
\end{pmatrix} \begin{pmatrix}
5^*_1 \\
5^*_2 \\
5^*_4
\end{pmatrix}.
$$

(17)

Mass matrices for down quarks and charged leptons are

$$
m^e = (m^d)^T = v_1 \begin{pmatrix}
\epsilon^4 & \epsilon^2 \epsilon^c & \epsilon^3 \\
\epsilon^3 & \epsilon \epsilon^c & \epsilon^2 \\
\epsilon^2 & \epsilon^c & \epsilon
\end{pmatrix},
$$

(18)

which yields the following mass relations

$$
m_d : m_s : m_b = m_e : m_\mu : m_\tau \simeq \epsilon^3 : \epsilon^c : 1.
$$

(19)

It should be noted that, when $\epsilon^c$ is equal to $\epsilon$, the mass matrices in Eq. (18) coincide with those in the SU(5) lopsided model with $a = 1$ (see Eq. (5)). In the considering SO(10) model, the
The \( \Sigma^e \) field plays partially the same role as the FN field \( S_{FN} \) only for \( 5^*4 \), and the successful mass matrices for down quarks and leptons are obtained when we arrange the vev of \( \Sigma^e \) such that \( \epsilon_{\Sigma} \simeq \epsilon \). In this analysis we consider that \( 5^*4 \) belongs to the second family as in Refs. [13, 14]. It can, however, belong to the third family as long as \( \epsilon_{\Sigma} \simeq \epsilon \). The ratio of vevs between Higgs doublets is estimated as \( \tan \beta = v_2/v_1 \simeq \epsilon m_t/m_b \sim 8 \) at the unification scale.

We turn to discuss neutrino masses. Dirac mass terms are generated in the similar way to down quarks and charged leptons.

\[
W = \frac{1}{2} h^{u}_{ij} H_2 \psi_i \psi_j + g^{u}_{ij} \frac{\Sigma^e}{M_s} H_2 \psi_i \eta
\]

\[
= \langle H_2 \rangle (1_1, 1_2, 1_3) \begin{pmatrix}
\epsilon^4 & \epsilon^2 \epsilon_{\Sigma} & \epsilon^3 \\
\epsilon^3 & \epsilon \epsilon_{\Sigma} & \epsilon^2 \\
\epsilon^2 & \epsilon \epsilon_{\Sigma} & \epsilon
\end{pmatrix}
\begin{pmatrix}5^*_1 \\ 5^*_4 \\ 5^*_2\end{pmatrix} + \cdots ,
\]

and hence we have

\[
m^D = v_2 \begin{pmatrix}
\epsilon^4 & \epsilon^2 \epsilon_{\Sigma} & \epsilon^3 \\
\epsilon^3 & \epsilon \epsilon_{\Sigma} & \epsilon^2 \\
\epsilon^2 & \epsilon \epsilon_{\Sigma} & \epsilon
\end{pmatrix} .
\]

It is seen that \( m^D \) has the same lopsided structure as \( m^e \) and \( (m^d)^T \). Majorana masses for right-handed neutrinos are induced by the following terms, \( W = h^{n}_{ij} \frac{\Phi^c \Phi^c}{M_s} \psi_i \psi_j \), which leads to

\[
m^n = \frac{v_2^2}{M_s} h^n = \frac{v_2^2}{M_s} \begin{pmatrix}
\epsilon^4 & \epsilon^3 & \epsilon^2 \\
\epsilon^3 & \epsilon \epsilon_{\Sigma} & \epsilon^2 \\
\epsilon^2 & \epsilon \epsilon_{\Sigma} & \epsilon
\end{pmatrix} .
\]

Through the seesaw mechanism we obtain the following Majorana mass matrix for the left-handed neutrinos

\[
m^\nu = \frac{v_2^2 M_s}{v_2^2} \begin{pmatrix}
\epsilon^4 & \epsilon^2 \epsilon_{\Sigma} & \epsilon^3 \\
\epsilon^2 \epsilon_{\Sigma} & \epsilon \epsilon_{\Sigma} & \epsilon^2 \\
\epsilon^3 & \epsilon \epsilon_{\Sigma} & \epsilon
\end{pmatrix} .
\]

Just as before, when we set \( \epsilon_{\Sigma} \simeq \epsilon \), we have the same structure as Eq. (7) in the lopsided SU(5) model. In this case, we can have naturally a large mixing angle for the \( \nu_e-\nu_\tau \) atmospheric neutrino oscillation, and we have no difficulty explaining a large mixing angle in solar neutrino oscillation.

We denote eigenvalues of the mass matrix (23) by \( m_i \) (\( m_1 < m_2 < m_3 \)). Identifying \( m_3 \) with \( \sqrt{\delta m^2_{atm}} \simeq 5 \times 10^{-2} \) eV and requiring \( \epsilon_{\Sigma} \simeq \epsilon \) the vev \( v_\Phi \) is estimated as

\[
v_\Phi \simeq \epsilon \frac{v_2 M_s^{1/2}}{\sqrt{\delta m^2_{atm}}} \approx 2 \times 10^{15} \text{ GeV} ,
\]
where we have taken the cutoff scale $M_*$ as the gravitational scale $M_P \simeq 2.4 \times 10^{18}$ GeV. It is seen that the scale of $v_\Phi$ is one order below the unification scale.\footnote{If the Higgs field $\Phi^c$ carries the FN charge +1, the vev $v_\Phi$ is estimated as $v_\Phi \simeq 4 \times 10^{16}$ GeV and is comparable to the unification scale.} Further, we find from Eq. \footnote{Even if we consider the non-zero FN charge for the Higgs field $\Phi^c$ (see footnote 4), the prediction of Majorana masses $M_i$ does not change.} Majorana masses $M_i$ ($M_1 < M_2 < M_3$) for right-handed neutrinos $n_i$ as

$$M_1 \simeq 4 \times 10^7 \text{ GeV}, \quad M_2 \simeq 9 \times 10^9 \text{ GeV} \quad \text{and} \quad M_3 \simeq 2 \times 10^{12} \text{ GeV} \ .$$

Notice that, in the considering SO(10) model, the FN charges for right-handed neutrinos are fixed by those for up quarks, and hence we can predict Majorana masses uniquely.\footnote{One explanation for $\epsilon_\Sigma \simeq \epsilon \simeq 1/16$ might be obtained by embedding the considering SO(10) GUT model in the higher dimensional theory. We expect the cutoff scale $M_*$ is given by the $(4 + d)$-dimensional Planck scale, and hence $M_* = (M_P^2 M_C^d)^{1/(d+2)}$ where $M_C$ denotes the compactification scale of the extra $d$-dimensional space. We consider $M_C$ is comparable to the unification scale. When the SO(10) GUT is embedded in 6-dimensions ($d = 2$), we find $M_* \simeq 2 \times 10^{17}$ GeV, and then $v_\Sigma \simeq \langle S_{FN} \rangle \simeq 1 \times 10^{16}$ GeV. Moreover, when the FN charge for the Higgs field $\Phi^c$ is +1 (see the footnote 4), $v_\Phi$ is also estimated to be $1 \times 10^{16}$ GeV. This suggests that the SO(10) GUT in 6-dimensions might realize the successful lopsided mass matrices by the vevs $\langle S_{FN} \rangle$, $v_\Sigma$ and $v_\Phi$, which are all of order of the unification scale (or $M_C$). It is very interesting to note that the attractive SO(10) GUT models have recently been constructed in 6-dimensions \footnote{See also the footnote 3.}.} This is completely different from the SU(5) model where they are determined by the unknown FN charges $b$, $c$ and $d$ (see Table 1). This point is crucial for considering possible scenarios of leptogenesis later.

We have shown that the successful mass matrices of quarks and leptons in the SU(5) model can be obtained in the framework of SO(10) GUT when $\epsilon_\Sigma \simeq \epsilon \sim 1/16$. This means the vev of the Higgs field $\Sigma^c$ should be comparable to the vev of the FN singlet field $S_{FN}$ as

$$v_\Sigma \simeq \langle S_{FN} \rangle \simeq \epsilon M_* \simeq 2 \times 10^{17} \text{ GeV} ,$$

where we have taken $M_* = M_P$. It is found that there should be a new physics scale one order above the unification scale. In fact, the SO(10) GUT is realized only above this scale and we have SU(5) as an unbroken group for the scale $v_\Sigma \geq \mu \geq M_{GUT}$. If the origin of the vev $\langle S_{FN} \rangle$ is associated with the SO(10) breaking, this GUT breaking pattern might answer the required condition $\epsilon_\Sigma \simeq \epsilon$, although we have to tune the scale correctly.\footnote{See also the footnote 3.}

Finally, we would like to discuss cosmological implication of the model, in particular, implication in “leptogenesis” \footnote{See also the footnote 3.}. Non-equilibrium decays of right-handed neutrinos $n_i$ give an attractive mechanism to generate dynamically the observed baryon asymmetry in the present Universe. This is because these decays can generate a lepton number in the early Universe, which is partially converted into a baryon number through the electroweak sphaleron processes \footnote{See also the footnote 3.}. The CP asymmetry by the $n_i$ decay can be expressed by the parameter $\epsilon_i$ and is estimated as \footnote{See also the footnote 3.}
\[
\sum_{j \neq i} \text{Im} \left\{ (m^D m^{D\dagger})_{ij} \right\}^2 \right\} \frac{f \left( \frac{M_j^2}{M_i^2} \right)}{8 \pi v^2 (m^D m^{D\dagger})_{ii}},
\]  
\hspace{1cm} (27)

where \( n_i, \ell \) and \( H_u \) denote here scalar or fermionic components of corresponding superfields (\( \ell \) are lepton doublets at low energies) and

\[
f(x) = \sqrt{x} \ln \left( 1 + \frac{1}{x} \right) + \frac{2\sqrt{x}}{x - 1}.
\]

\hspace{1cm} (28)

The SO(10) model described above can predict Majorana masses for right-handed neutrinos as shown in Eq. (25) and their mass ratios are determined by those of up quarks, which is completely different from the SU(5) model. This suggests possible scenarios of leptogenesis are restricted. The CP asymmetry \( \epsilon_1 \) of the lightest right-handed neutrino \( n_1 \) is estimated from Eq. (27) as \( |\epsilon_1| \simeq 1 \times 10^{-8} \), where we have assumed the CP-violating phase of order unity. It have been shown in Ref. [11] that the conventional thermal leptogenesis [19] does not work since \( \epsilon_1 \) is too small to account for the present baryon asymmetry.

We, then, consider non-thermal leptogenesis via inflaton decay [20, 21, 22, 23, 24], where right-handed neutrinos are produced non-thermally in decays of inflaton \( \varphi \). The baryon asymmetry (the ratio of baryon number density \( n_B \) to the entropy density \( s \)) induced by \( n_1 \) is given by\(^7\)

\[
\frac{n_B}{s} \simeq 0.5 \ Br_1 \ \epsilon_1 \ \frac{T_R}{M_\varphi},
\]

\hspace{1cm} (29)

where \( M_\varphi, T_R \) and \( Br_1 \) denote the inflaton mass, the reheating temperature, and the branching ratio of \( \varphi \to n_1 + n_1 \), respectively. In order to ensure the non-thermal production of \( n_1 \) we assume \( M_\varphi > 2M_1 \) and \( T_R \lesssim M_1 \). In the above model we estimate as

\[
\frac{n_B}{s} \simeq 6 \times 10^{-10} \ Br_1 \ \left( \frac{T_R}{10^7 \text{ GeV}} \right) \left( \frac{2M_1}{M_\varphi} \right),
\]

\hspace{1cm} (30)

which should be compared with the observation \((n_B/s)_{\text{OBS}} \simeq (0.1 - 1) \times 10^{-10}\). It is found that the successful leptogenesis is available only for the inflation models which give

\[
M_\varphi \sim 10^9 \text{ GeV}, \quad T_R \sim 10^7 \text{ GeV} \quad \text{and} \quad Br_1 \sim 1.
\]

\hspace{1cm} (31)

Further, as recently proposed in Ref. [25], decays of heavier right-handed neutrinos \( n_2 \) and \( n_3 \) can be a dominant source of the present baryon asymmetry. It is usually considered that decays of the lightest right-handed neutrino are responsible to \((n_B/s)_{\text{OBS}}, \) although decays of heavier ones also induce lepton asymmetry. This is because after the decays of \( n_2 \) and \( n_3 \) the lightest right-handed neutrino can remain in thermal equilibrium and wash out the lepton asymmetry from \( n_2 \) and \( n_3 \). This is true for the conventional thermal leptogenesis. However, in the non-thermal leptogenesis scenarios, as shown in Ref. [25], such dangerous wash-out effects

\(^7\)In this analysis, we neglect the sign of the produced baryon asymmetry.
can be killed just requiring that the reheating temperature is $T_R \lesssim M_1$. We shall illustrate this idea in the described model, especially, leptogenesis by decays of the next-to-lightest right-handed neutrino $n_2$. The successful scenario requires that $M_\nu > 2M_2$ and also $T_R \lesssim M_1$, which means that both $n_1$ and $n_2$ are produced non-thermally in inflaton decays and induce the lepton asymmetry. We find the CP asymmetry for $n_2$ is $|\epsilon_2| \approx 2 \times 10^{-6}$ and

$$\frac{n_B}{s} \sim \left[ 3 \times 10^{-12} Br_1 + 6 \times 10^{-10} Br_2 \right] \left( \frac{T_R}{10^7 \text{ GeV}} \right) \left( \frac{2M_2}{M_\nu} \right).$$

(32)

This equation shows that the dominant contribution to the baryon asymmetry comes from the decays of $n_2$ rather than the lightest one $n_1$ unless $Br_2 \ll Br_1$. In this case, the successful leptogenesis requires

$$M_\nu \sim 10^{11} \text{ GeV}, \quad T_R \sim 10^7 \text{ GeV} \quad \text{and} \quad Br_2 \sim 1.$$  

(33)

We have seen that possible scenarios of leptogenesis are restricted in the SO(10) model even for the non-thermal leptogenesis via inflaton decays, and the present baryon asymmetry can suggest parameters of inflation models as shown in Eqs. (31) and (33). It is quite interesting to observe that the SO(10) model described above provides naturally the hybrid inflation and, moreover, the values given in Eq. (33) are just predicted by this inflation.\(^8\) Let us write the superpotential which gives non-zero vev for the Higgs fields $\Phi$ and $\Phi^c$ as follows;

$$W = \lambda X (\Phi \Phi^c - v_\Phi^2),$$

(34)

where $\lambda$ is the coupling constant. This is nothing but the superpotential for the supersymmetric hybrid inflation\(^9\). The value of $v_\Phi \approx 2 \times 10^{15}$ GeV (see Eq. (24)), which is suggested from the atmospheric neutrino oscillation, gives the coupling constant of $\lambda \sim 10^{-4}$ in order to explain the COBE normalization of the cosmic density fluctuations.\(^10\) (See the detailed analysis of the hybrid inflation in Ref. [24].) The inflaton mass $M_\phi$ is, then, estimated as $M_\phi = \sqrt{2\lambda v_\Phi} \sim 10^{11}$ GeV, which is just the required value in Eq. (33) for the successful leptogenesis. The reheating of inflation takes place via decays through the interactions $W = \frac{1}{2} h_{ij} \Phi^i \Phi^c \psi_i \psi_j$ in Eq. (11). Therefore, the inflaton decays mainly into pairs of right-handed (s)neutrinos and their partial widths are proportional to $M_i^2$.\(^11\) We find from Eq. (25) that $Br_2 \simeq 1$ is ensured for the inflaton with $M_\phi \sim 10^{11}$ GeV. Further, the reheating temperature is estimated as $T_R \sim 10^7$ GeV. With such low reheating temperatures, we can avoid the cosmological gravitino problem.\(^12\)

\(^8\)See Refs. [22, 24] for the similar discussion in the different models.

\(^9\)Supergravity effects are potentially dangerous since they disturb the slow-roll motion of inflation. These effects are induced from the nonrenormalizable interaction in the Kähler potential $K = (k/4) |X|^4 / M_2^2$. For the successful inflation the coupling $k$ should be negative and also $|k| \lesssim 10^{-3}$ when $\lambda \sim 10^{-4}$. We assume it by hand since smallness of couplings in the Kähler potential cannot be explained by the FN mechanism.

\(^10\)The coupling constant $\lambda$ of $10^{-4}$ can be explained by the FN mechanism with the FN charge +3 for $X$.

\(^11\)Although the inflaton decays also through interactions in the Kähler potential, they give negligible corrections to the total width.

\(^12\)We can neglect gravitinos produced non-thermally at the preheating epoch since the coupling $\lambda$ is sufficiently small. Similarly, production of right-handed neutrinos by the preheating can also be neglected.
Therefore, the hybrid inflation provided by the model itself offers the successful non-thermal leptogenesis by decays of the next-to-lightest right-handed neutrinos. The observed baryon asymmetry in the present Universe is explained by just the neutrino mass scale suggested by the atmospheric neutrino experiments.

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