BabelCalib: A Universal Approach to Calibrating Central Cameras

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Abstract

Existing calibration methods occasionally fail for large field-of-view cameras due to the non-linearity of the underlying problem and the lack of good initial values for all parameters of the used camera model. This might occur because a simpler projection model is assumed in an initial step, or a poor initial guess for the internal parameters is pre-defined. A lot of the difficulties of general camera calibration lie in the use of a forward projection model. We side-step these challenges by first proposing a solver to calibrate the parameters in terms of a back-projection model and then regress the parameters for a target forward model. These steps are incorporated in a robust estimation framework to cope with outlying detections. Extensive experiments demonstrate that our approach is very reliable and returns the most accurate calibration parameters as measured on the downstream task of absolute pose estimation on test sets. The code is released at https://github.com/ylochman/babelcalib.

1. Introduction

Cameras with very wide fields of view, such as fisheye lenses and catadioptric rigs [44], usually require highly nonlinear models with many parameters. Calibrating these cameras can be a tedious process because of the camera model’s complexity and its underlying non-linearity. If the calibration is inaccurate or even fails, then the user is often required to manually remove problematic images or fiducials, capture additional images, or provide better initial guesses for the unknown model parameters. A second common problem is that the choice of calibration toolbox limits the user to a particular set of supported camera models and extending the toolbox to accommodate more flexible camera models can be a difficult task.

This paper proposes a method that robustly estimates accurate camera models for central projection cameras [39]

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Table 1: Supported camera models. Models compute either radially-symmetric projection, \( r \) or back-projection, \( r : rZ - R \psi_0(r) = 0 \), where \( R \) and \( Z \) are the radial and depth components of a scene point, and \( r \) is the distance from the center of projection of a retinal point. The right column lists functions for published models.

| Model                          | Parameters, \( \theta \) | Radial (Back-)Projection Function |
|-------------------------------|---------------------------|----------------------------------|
| Brown-Conrady (BC) [8]        | \( \{k_1, k_2\} \)       | \( \phi_R(R, Z) = (R/Z) \cdot \left(1 + \sum_{n=1}^{2} k_n (R/Z)^n\right) \) |
| Kannala-Brandt (KB) [20]      | \( \{k_1, \ldots, k_4\} \) | \( \phi_R(R, Z) = \zeta + \sum_{n=1}^{4} k_n (R/Z)^{2n+1} \), \( \zeta = \arctan(2R, Z) \) |
| Unified Camera (UCM) [26]     | \( \{\xi\} \)            | \( \phi_R(R, Z) = R(\xi + 1)/(\xi(\sqrt{R^2 + Z^2}) + Z) \) |
| Field of View (FOV) [7]       | \( \{w\} \)              | \( \phi_R(R, Z) = \frac{1}{\sqrt{2}} \arctan(2R \tan \frac{w}{2}, Z) \) |
| Extended Unified Camera (EUCM) [21] | \( \{\alpha, \beta\} \) | \( \phi_R(R, Z) = R/(\alpha d + (1 - \alpha) Z), \ d = \sqrt{R^2 + Z^2} \) |
| Double Sphere (DS) [41]       | \( \{\xi, \alpha\} \)    | \( \phi_R(R, Z) = R/(\alpha d_2 + (1 - \alpha) Z_2), \ d_2 = \sqrt{R^2 + Z_2^2}, \ Z_2 = \xi \sqrt{R^2 + Z^2} + Z \) |

1.1. Related Work

Camera calibration is an important tool in order to upgrade cameras from pure imaging devices to geometric sensors, and it has led to the development of many parametric models for cameras (and their lens systems) and the introduction of respective toolboxers (e.g. [8, 18, 43, 11, 38, 5, 26, 23, 32]). In order to facilitate the highest accuracy for the calibration parameters, a controlled, usually planar calibration target is employed in many applications. The use of dedicated images (“training data”) for the task of camera calibration distinguishes standard calibration from self-calibration, that extracts calibration parameters from uncontrolled “test” images (e.g. [9, 17, 30, 12, 42, 31, 24]).

Most relevant to our approach in terms of forward projection models are the unified camera model [13, 1, 26], the fisheye projection model by Kannala and Brandt [20], and the double sphere model [41]. Using these models for calibration tasks is not always straightforward and comes with their own set of assumptions.

E.g., the estimator proposed for the Double-Sphere model [41] requires that the circular field-of-view is visible to recover the center of projection and the aspect ratio, that the relative position of the spherical retinas be initialized, and that a non-radial line be identified to recover the focal length. Further, the method proposed for the popular Kannala-Brandt model requires specifying focal length and field of view [20].

The introduction of a linear solver to calibrate the division model in the back-projection framework [34] demonstrates the benefits of using back-projections. This linear method assumes known center of distortion and unit aspect ratio and is extended to a two-stage method in [35] to include estimation of the center of distortion. Urban et al. [40] identifies the shortcomings of this two-stage method.
and suggests joint refinement of all unknowns instead.

Finally, very general non-parametric models for cameras and lenses have been proposed (including [33, 15, 3, 36]). Our experiments indicate that appropriate parametric models are sufficient to model a wide range of cameras and lenses and are therefore—due to Occam’s razor—preferable in general.

2. Preliminaries

Let us define the camera matrix \( P \) mapping from scene coordinates to ray directions in the camera coordinate system as \( P = \text{diag}(f, f, 1) [R \ t] \), where \( f \) is the focal length, \( R = [r_1 \ r_2 \ r_3] \) is the rotation matrix, and \( t = (t_x, t_y, t_z) \) is the translation vector. We build on the omnidirectional camera model of Micusik and Pajdla [27], that relates the image point \( u = (u, v, 1) \) and the scene point \( X \) as

\[
\gamma g(Au) = PX \quad \text{s.t.} \quad \gamma > 0. \tag{1}
\]

In (1) the matrix \( A \) maps from image coordinates to sensor coordinates. Denote the center of projection as \( e = (e_x, e_y, 1) \), the scale factor as \( s \), and the pixel aspect ratio as \( \alpha \). For the initialization method we assume that the pixels are orthogonal, so we have

\[
A = \text{diag}(a^{-1}, 1, 1) \text{diag}(s^{-1}, s^{-1}, 1)T(-e), \tag{2}
\]

where \( T(x) \) is a homogeneous matrix encoding translation by \( x \). The nonlinear function \( g(\cdot) \in \mathbb{R}^3 \rightarrow \mathbb{R}^3 \) in (1) maps from the retinal plane to ray directions in the camera coordinate system. For the initialization method, the typically small distortions caused by lens misalignment are ignored [16] so that we can model back-projection of \( u = (u, v, 1) \) as radially symmetric, \( g(u) = (u, v, \psi(r(u))) \), where the radius of the retinal point is \( r(u) = \sqrt{u^2 + v^2} \).

Back-Projection with Division Model We parameterize \( \psi(\cdot) \) with the division model. It has a good ability to model significant lens distortions and was used in [34] for fish-eye and catadioptric lenses with fields-of-view greater than 180°. The model is defined as

\[
\psi(r) = 1 + \sum_{n=1}^{N} \lambda_n r^{2n}. \tag{3}
\]

Function \( \psi(\cdot) \) is not invertible in general; however, we assume that there is only a single root \( r^* \in [0, r_{\text{max}}] \), where \( r_{\text{max}} \) is the image diagonal. More precisely, let \( x = (x, y, w) \) and let \( h(\cdot) \) be the function projecting \( x \) to the retinal plane,

\[
h(x) = \left( \frac{r^* x}{r(x)}, \frac{r^* y}{r(x)}, 1 \right)^\top, \tag{4}
\]

where \( r^* \) is the only solution of \( \psi(r) = w \) in \([0, r_{\text{max}}]\). Consequently,

\[
u = h(g(u)). \tag{5}
\]

Multiple roots in \([0, r_{\text{max}}]\) imply that a scene point maps to multiple image points, which is an implausible physical configuration.

Radial Fundamental Matrix Without loss of generality, we assume the target to be on the plane \( z = 0 \). Transforming a point \( X \) on the target to a ray direction in the camera coordinate system can be done by the homography \( H = [r_1 \ r_2 \ t] \) constructed from the camera matrix

\[
P X = \text{diag}(f, f, 1) [R \ t] (X, Y, 0, 1)^\top = \text{diag}(f, f, 1) [r_1 \ r_2 \ t] (X, Y, 1)^\top. \tag{6}
\]

Hartley and Kang [15] used the radial fundamental matrix to recover the principal point of distorted pinhole cameras. We extend the radial fundamental matrix to recover the center of projection \( e \) and camera pose \( R, t \) for the back-projection model of [27]. We substitute \( \text{diag}(f, f, 1)Hx \) for \( PX \) in (1) using (6), apply the projection function \( h(\cdot) \) to both sides, and use (5) to eliminate \( g \) giving

\[
u = A^{-1}h(1/\psi)\text{diag}(f, f, 1)Hx = T(e)1/\gamma \psi(1/\gamma)\text{diag}(a(sfr^*), r(x), (sfr^*)/r(x), 1)Hx. \tag{7}
\]

Note that the scaling by \((sfr^*)/(\gamma r(x))\) due to focal length, projection by \( h(\cdot) \), pixel scale \( s \) and depth multiplier \( \gamma \) is entangled and acts radially. Substituting \( \eta = (sfr^*)/(\gamma r(x)) \) into (7) and crossing both sides with \( e \) gives

\[
[e]_x u = [e]_x T(e)\text{diag}(ar, \eta, 1/\gamma)Hx = \gamma [e]_x \text{diag}(ar, \eta, 0)Hx. \tag{8}
\]

The radial line \([e]_x u \) is eliminated by taking the inner product of \( u \) with (8), and \( \eta \) can be eliminated since it is non-zero. Denote the rows of \( H \) such that \( H = [h_1 \ h_2 \ h_3]^\top \). Then (8) simplifies to

\[
0 = u^\top [e]_x (ah_1^\top x, h_2^\top x, 0)^\top, \tag{9}
\]

which can be rearranged to give the radial fundamental matrix \( F_r \) for omni-directional cameras

\[
0 = u^\top [e]_x \begin{bmatrix} ar_{11} & ar_{12} & at_x \\
 r_{21} & r_{22} & t_y \\
 0 & 0 & 0 \end{bmatrix} x. \tag{9}
\]

The aspect ratio is modeled, but cannot be recovered without additional constraints. The radial fundamental matrix \( F_r \) is rank two by construction, and the center of projection \( e \) is a basis for the left null space of \( F_r \).
3. Obtaining the Initial Estimate

The methods proposed in this section ensure that a good initial guess of the camera model is made. The parameters are recovered by a sequence of simple solvers (see Fig. 1). The back-projection model of (1) corresponds image points to ray directions in the camera coordinate system. Given this correspondence, we show that regressing the commonly used projection models can be done easily. This enables the search for good initial guesses for the target projection model in a sampling framework, which increases the robustness of the method.

3.1. Solving the Radial Fundamental Matrix

The radial fundamental matrix is estimated to recover the center of projection and camera pose. The epipolar constraint on the radial fundamental matrix \( u^T F x = 0 \) in (9) can be written as a linear constraint on \( F \)

\[
(x \otimes u)^T \text{vec}(F) = 0. \tag{10}
\]

Following the classic solver for the fundamental matrix in [16], we use at least seven image-to-target point correspondences, denoted \( \{ u_i \leftrightarrow x_i \} \), to compute the null space of stacked constraints of the form (10). The nonlinear constraint \( \det F = 0 \) is enforced to recover at most three real solutions from the null space. The fundamental matrix consistent with the most correspondences is kept.

3.2. Solving the Center of Projection and Pose

As shown in (9), the center of projection is a basis for the left null space of \( F \)

\[
\zeta e = \text{null } F^T. \tag{11}
\]

There is a scale ambiguity, denote it \( \nu \), between the radial fundamental matrix \( F_r \), as formulated in (9) and the fundamental matrix \( F \) recovered by the seven-point method,

\[
F = \nu F_r \quad \text{where } F = (f_{ij}). \tag{12}
\]

Let \( r_{31}, r_{32} \) be the unknown components of the rotation vectors \( r_1 \) and \( r_2 \). If we let \( S = \nu^{-1} \text{diag}(a^{-1}, 1, 1) \), then \( r_j = S (f_{2j}, -f_{1j}, r_{3j})^T \). We use the orthonormality of \( r_1 \) and \( r_2 \) to put quadratic constraints on the unknowns,

\[
\|S(f_{21}, -f_{11}, r_{31})\|^2 = \|S(f_{22}, -f_{12}, r_{32})\|^2 = 0.
\]

\[
(13)
\]

There are four unknowns but only three constraint equations. Additional constraints are needed to recover the aspect ratio. The unknowns \( \{ \nu, a, r_{31}, r_{32}, t_z, \lambda_1, ..., \lambda_n \} \) can be jointly recovered by solving a system of polynomial equations (see Sec. A in Supplemental); however, we chose to sample over the interval of aspect ratios \( a \in [0.5, 2] \) and recover \( \{ \nu, r_{31}, r_{32} \} \) from (13).

![Figure 2: Correcting corners improves the initial guess. We evaluate the accuracy of the center of projection, camera pose and rewarped points using the original and corrected corners. Evaluation is done over 1000 experiments at each noise level. Solid curves are median errors, and shaded regions are interquartile ranges.](image)

3.3. Corner Correction

Corner correction is defined such that given the radial fundamental matrix \( F_r \) and correspondence \( u_i \leftrightarrow x_i \), the corrected corner is \( u_i^* = u_i + \delta u_i \), where \( \delta u_i \) is the smallest displacement such that \( u_i^* \) satisfies the epipolar constraint \( u_i^* F_r x_i = 0 \). The target fiducials \( \{ x_i \} \) are assumed correct since they are noiseless. It can be shown [16] that the corrected corner \( u_i^* \) is recovered by projecting the measured corner \( u_i \) onto the epiline of \( x_i \),

\[
u_i^* = \text{proj}_{F_r, x_i}(u_i).
\]

We refine the radial fundamental matrix \( F_r \) by minimizing the displacements with non-linear least squares. Eight correspondences are sufficient for correcting the sampled corners [16], but it is reasonable to use more since we expect a high inlier ratio for a calibration capture. The rank-two constraint is encoded with the parameterization \( F_r = [e, x, (h_1^*, h_2^*, 0)]^T \). Then the refined fundamental matrix is recovered by solving

\[
e^* h_1^*, h_2^* = \arg\min_{e, h_1, h_2} \sum_i \delta_{u_i}^T \delta u_i. \tag{15}
\]

and reconstructing \( F_r^* \) from \( e^*, h_1^*, h_2^* \). The noisy detected corners are corrected according to (14) using \( F_r^* \).

Evaluation of Corrected Corners Synthetic scenes were used to measure the accuracy gains to camera model estimation from corner correction. The camera was randomly posed to view a chessboard. Image resolution was \( 1200 \times 800 \) pixels, focal length \( 400 \) pixels, and the center of projection was displaced to \( (700, 500) \). We added different levels white noise to the corners: \( \sigma \in \{ 0, 0.1, 0.2, \ldots, 2 \} \).
Camera models were fit using either the original or corrected corners for 1000 images.

Clockwise from the top left, Fig. 2 reports (i) the distance between the estimated and ground truth center-of-projection, (ii) the smallest angle of rotation required to correct the estimated orientation, (iii) the distance between the estimated and ground-truth camera position, and (iv) the RMS reprojection error between an image grid and the reprojection of scene points by the estimated camera that should project onto the image grid. Fig. 2 shows that correcting corners reduces median errors of rotation, translation and RMS reprojection error by 31%, 28%, and 33% on average.

3.4. Solving the Remaining Intrinsics and Depth

The homography mapping from the camera coordinate system to coordinates of the retinal plane can be used to solve for the remaining parameters, \( \gamma g(Au_i) = \text{diag}(f, f, 1)Hx \). Note that \( s \) and \( f \) in \( A \) cannot be disentangled without additional knowledge about the camera such as the pixel size. Further, we assume that this information is unavailable, and let \( f \leftarrow sf \). An unknown \( \gamma \) is eliminated through the cross product,

\[
g \left( \text{diag}(f^{-1}, f^{-1}, 1)u' \right) \times \begin{pmatrix} x' \\ y' \\ z' + tz \end{pmatrix} = 0, \tag{16}
\]

where \( u' = \text{diag}(a^{-1}, 1, 1)T(-e)u \), \( x' = h_1x \), \( y' = h_2x \), and \( z' = (r_{31}, r_{32}, 0)x \). Reparameterizing \( \lambda_k = \lambda_k/f^{2k-1} \) and collecting terms in (16) gives a system linear in the unknowns

\[
\begin{bmatrix}
    x'_{\lambda_1} & x'_{\lambda_1^2} & \ldots & x'_{\lambda_1^{2N}} & -u'_{\lambda_1} \\
    y'_{\lambda_1} & y'_{\lambda_1^2} & \ldots & y'_{\lambda_1^{2N}} & -v'_{\lambda_1} \\
    \vdots & \vdots & \ddots & \vdots \\
    x'_{\lambda_N} & x'_{\lambda_N^2} & \ldots & x'_{\lambda_N^{2N}} & -u'_{\lambda_N} \\
    y'_{\lambda_N} & y'_{\lambda_N^2} & \ldots & y'_{\lambda_N^{2N}} & -v'_{\lambda_N}
\end{bmatrix}
\begin{pmatrix}
    f \\
    \lambda_1 \\
    \vdots \\
    \lambda_N
\end{pmatrix}
= \begin{pmatrix}
    u' \cdot z' \\
    v' \cdot z'
\end{pmatrix}, \tag{17}
\]

where \( r'_i = r(u'_i) \) is the radius of the point \( u'_i \).

Model Selection for the Division Model The degree of division model \( \psi(\cdot) \) defined in (3) needs to be chosen such that it can approximate the extreme radial profiles of fisheye and catadioptric rigs, and so that it does not over-fit to noisy measurements for narrow field-of-view lenses. Clearly these are competing goals. We evaluated \( \psi(\cdot) \) for polynomials of even degrees from two to ten. Model selection is performed on the dataset introduced in Sec. 5, which contains a wide range of lenses as well as catadioptric rigs.

The camera model of (1) is estimated linearly from sampled corner correspondences (denoted Initial in Table 2) as outlined above and fit with non-linear least squares (denoted Refinement in Table 2) using all corners for each calibration capture. The weighted RMS reprojection error and inlier ratio are used to assess the accuracy of each model’s initial guess and refined solution across the entire dataset. Table 2 shows that models of degrees eight and ten significantly deviate from the optimal result, suggesting that they are over-fitting. The fourth-degree division model parameterized by \( \{ \lambda_1, \lambda_2 \} \) is the simplest model that is sufficiently flexible to provide a good initial guess. We estimate the back-projection function (1) using (3) with \( N = 2 \).

| Degree | Initial RMS [px], inl. [%] | Refined RMS [px], inl. [%] |
|--------|--------------------------|--------------------------|
| 2      | 5.342, 26.042            | 0.647, 97.086           |
| 4      | 4.585, 28.991            | 0.587, 97.403           |
| 6      | 4.516, 26.184            | 0.587, 97.399           |
| 8      | 7.429, 13.307            | 1.020, 93.660           |
| 10     | 12.804, 8.448            | 4.812, 61.267           |

Table 2: Model selection for the division model. A polynomial of degree four gives the best results overall.
handle bad detections [10]. The method fits camera models sampled from corner-to-board correspondences. The models are scored with a robust objective. During sampling, the best-so-far model is kept and refined with a maximum-likelihood estimator, which is inspired by the local optimization step in [4]. The output of the model is a maximum-likelihood fit of camera intrinsics and extrinsics. The estimators proposed in Sec. 3 are very simple, so they are fast and are well-suited for use in the model proposal step of RANSAC. Algorithm 1 in Sec. C of Supplemental may be helpful as a cross-reference for the next paragraphs that specify the method.

The input to the method is a set of 2D-3D correspondences that match corner detections with board fiducials, which we denote $X_{ij} \leftrightarrow u_{ijk}$. Indices $i$ and $j$ indicate a particular fiducial $i$ on plane $j$, and $k$ indicates the image of the corner detection $u_{ijk}$ of fiducial $X_{ij}$. For each RANSAC iteration, we sample an image $k$, a plane $j$ visible in image $k$, and a non-minimal sample of 14 correspondences that are used to compute the radial fundamental matrix, center of projection, and corner correction according to (10), (11), and (14). The utilized sample size of 14 correspondences was cross-validated.

The aspect ratio is a necessary parameter for the pose and intrinsics estimators. If the camera has non-square pixels or an anamorphic lens, we sample an aspect ratio from the interval $[0.5, 2]$. Pose and intrinsics are estimated as derived in (13) and (17). The radial profile of the user-selected camera model is regressed against the radial profile of the division model using (18), if the division model is not the desired model. The model-to-model regression generates the

| Dataset        | # cam., # img. | DFOV range | Max. img. size |
|----------------|----------------|------------|----------------|
| Kalibr         | 8, 130 + 60   | 110°—265°  | 1680 × 1680    |
| OCamCalib      | 9, 79 + 40    | 130°—265°  | 3840 × 2880    |
| UZH-DAVIS      | 4, 140 + 60   | 124°—148°  | 346 × 260      |
| UZH-Snapdragon | 4, 140 + 60   | 144°—166°  | 640 × 480      |
| OV-Corner      | 8, 280 + 120  | 109°—109°  | 1280 × 800     |
| OV-Cube        | 4, 105 + 49   | 155°—183°  | 1280 × 800     |
| OV-Plane       | 4, 92 + 41    | 85°—187°   | 1280 × 800     |

Table 3: Calibration dataset details. Train-test split of the images is indicated by +. The diagonal field of view (DFOV) is approximated using intrinsic calibration.

![Figure 3: Projection of calibration target from estimated calibration. Detected corners are red crosses, target projected using initial calibration are blue squares and using the final calibration are cyan circles.](image)

| Dataset | OpenCV-BC | Kalibr-BC | Ours-BC |
|---------|-----------|-----------|---------|
|         | RMS [px], inl. [%] |           |         |
| Cam0    | 0.886, 90.9 | 0.945, 87.3 | 0.704, 96.1 |
| Cam1    | 0.781, 95.2 | 0.893, 88.8 | 0.674, 98.0 |
| Cam2    | 0.773, 96.1 | 0.756, 95.6 | 0.720, 96.9 |
| Cam3    | 0.735, 97.0 | 0.953, 87.6 | 0.710, 96.4 |
| Cam4    | 0.757, 97.3 | 0.828, 93.6 | 0.679, 97.6 |
| Cam5    | 0.772, 96.4 | 0.831, 91.7 | 0.759, 96.0 |
| Cam6    | 0.715, 95.4 | 0.748, 94.5 | 0.677, 96.0 |
| Cam7    | 0.701, 96.6 | 0.855, 90.6 | 0.641, 97.5 |

Table 4: Pose evaluation for the BC model. OV-Corner test images used.

camera geometry portion of the RANSAC model proposal. Given the estimate of intrinsics, the poses for the remaining images are computed using P3P (Perspective-3-Point [28]) from three sampled corner-to-board correspondences. The camera poses are added to the intrinsics to give a RANSAC model proposal.

The reprojection error is evaluated against the entire calibration capture with the robust objective

$$ J(\Theta) = \sum_{ijk} \rho(d(\pi ( [R_{jk} \ t_{jk}] X_{ij}), u_{ijk})) $$

where $\pi(\cdot)$ is the selected projection function, $d(\cdot, \cdot)$ is the Euclidean distance, $\rho(\cdot)$ is the the Huber loss function [19], and $\Theta = \{\theta, \ K, R_{jk}, t_{jk}\}$ are the calibration parameters. In the case of multiple planar targets, the poses $R_{jk}, t_{jk}$ are constructed using the absolute poses of the cameras, $R_{jk}^c, t_{jk}^c$, and the relative poses of the boards with respect to the reference board, $R_{jk}^b, t_{jk}^b$.

$$ R_{jk} = R_{jk}^c R_{jk}^b \quad t_{jk} = R_{jk}^c t_{jk}^b + t_{jk}^c $$

If RANSAC encounters a best-so-far calibration proposal, then the model refinement step, $J(\Theta) \rightarrow \min_{\Theta}$, is invoked. The axis-angle representation is used to minimally parameterize the rotations for the bundle adjustment. Proposals are ranked by their inlier ratio, and the inlier ratio is computed according to

$$ I(\Theta) = \frac{1}{M} \sum_{ijk} \mathbb{1}_{\{l \leq T\}}(d(\pi ([R_{jk} \ t_{jk}] X_{ij}), u_{ijk})) $$

(20)
Table 5: Pose Evaluation for fisheye and catadioptric rigs. Estimated models are (top) KB, UCM, and (bottom) FOV, EUCM, and DS. Each method’s performance on a dataset is given by the weighted RMS reprojection error [px], the inlier ratio [%], and the number of catastrophic failures.

Table 6: Pose evaluation for the DIV model. The test images from OCamCalib were used.
Figure 4: **Performance summary for all calibration captures.** The empirical CDFs show the relative performance of each method across all calibration datasets. The methods are tested for different augmentation types. The probability that the weighted RMS reprojection error is less than $E$ is plotted. The weighted RMS reprojection error is normalized to correspond to a $1000 \times 1000$ px image. BabelCalib performs better for every model type on the original and augmented imagery.

models for all cameras. These measures are reported for each framework-dataset combination in the comparison tables. The winner is boldfaced. A tie is declared if a method is not best on both measures. In this, we mark the highest number of failures in red.

Table 4 reports results for the narrow-medium FOV lenses in the **OV-Corner** dataset. We breakout the results for each camera. The suitable model for this lens type is a pinhole projection with additive BC distortion. BabelCalib outperforms OpenCV and Kalibr. Table 5 reports results for fisheye lenses and catadioptric rigs. Kalibr and OpenCV have both a 24% failure rate for the Kannala-Brandt model, and Kalibr has a 34% failure rate for FOV and 51% failure rate for EUCM. In contrast, BabelCalib has no calibration failures. BabelCalib consistently gives the best results for each model type, even after discarding catastrophic failures for the state-of-the-art.

Table 6 reports calibrations using the DIV model on the **OCamCalib** dataset. OCamCalib-DIV is evaluated only on OCamCalib since it requires all fiducials to be visible across the capture. OCamCalib is the only dataset where this holds. BabelCalib and OCamCalib give comparable results. This dataset includes catadioptric rigs, which shows the flexibility of BabelCalib.

Table 7 summarizes the mean reduction of RMS weighted reprojection error for each dataset-model combination realized by BabelCalib over the state of art. BabelCalib gives a significant error reduction for all framework-model combinations, even after discarding the calibration failures of the state of the art. See Fig. C.2 in Supplemental for more qualitative results and also Sec. D which evaluates calibration performance from a limited amount of images.

**Displaced Center and Non-Square Pixels** We augmented the datasets by adding cropped or stretched images to simulate a displaced projection center or a CCD with rectangular pixels. Displacement was $(0.15w, 0.15h)$ pixels for a $w \times h$ image, and pixel aspect ratio was 1.33:1. Fig. 4 reports the distributions of robust RMS reprojection errors for pose estimation on test images for the original and augmented data. Several model-framework combinations are evaluated. For comparison, the errors are normalized to correspond to a $1000 \times 1000$ pixel image. BabelCalib finds a higher percentage of accurate calibrations on the original and augmented data for all models.

### 6. Conclusion

BabelCalib recovers significantly more accurate calibrations than three widely used frameworks and suffers no catastrophic failures. BabelCalib maintains its dominance for all commonly used models on a large survey of cameras with narrow, wide-angle and fisheye lenses, as well as catadioptric rigs. BabelCalib maintains its performance for cameras with displaced center of projections or non-square pixels. It doesn’t require model initialization nor hyperparameter tuning, so it’s easy to use. Moreover, the regression framework easily admits additional camera models.

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A. Aspect Ratio Solver

The epipolar constraint \( u^T F x = 0 \) is invariant to a projective change of coordinates. This implies that the aspect ratio cannot be recovered using only the constraints available from the radial fundamental matrix (e.g., see (13)). Additional constraints can be obtained by enforcing the concurrence of back-projected corners with their corresponding scene points using (1). The constraints given by (1) generate polynomial equations of the unknown aspect ratio, remaining unknowns of \( H \), and the unknown division model parameters \( \{a, h_{31}, h_{32}, h_{33}, \lambda_1, \ldots, \lambda_n\} \). The formulation introduces \( N + 1 \) unknowns, but the formulation is invariant to the scale of \( H \). The minimal solution requires \( N/2 + 2 \) image-to-target correspondences.

Let \( u' \) be the projection center-subtracted image point \( u' = T(-e)u \). Then we can rewrite (1) as following

\[
\begin{pmatrix}
\frac{u'}{f} \\
\frac{v'}{f} \\
\psi(r(\text{diag}(a^{-1}, 1, 1)u'))
\end{pmatrix}
\times \text{diag}(a, 1, 1)H x = 0. \tag{21}
\]

Solving (10) and (11) gives the radial fundamental matrix \( F \) and projection center \( e \), respectively. We use the relation \( F \sim [e]_x (a h_1^T x, h_2^T x, 0)^T \) to determine the first two rows of the matrix \( H \), up to scale, \( \hat{h}_1^T = (f_{21}, f_{22}, f_{23}) \) and \( \hat{h}_2^T = -(f_{11}, f_{12}, f_{13}) \).

Rewriting (21) in terms of the unknowns gives

\[
\begin{pmatrix}
\frac{u'}{f} \\
\frac{v'}{f}
\end{pmatrix}
\times \begin{pmatrix}
x'

y'
\end{pmatrix}
\times \begin{pmatrix}
\hat{h}_3^T x
\end{pmatrix} = 0, \tag{22}
\]

where \( r_0(a) = \sqrt{(u' a)^2 + (v' a)^2} \), \( x' = \hat{h}_3^T x \), \( y' = \hat{h}_4^T x \).

After reparameterizing with \( b = a^2 \), it can be seen that for \( N = 1 \), (22) is linear in the unknowns and for \( N = 2 \), a polynomial system of degree three must be solved.

We tested the solvers for the cases \( N = 1 \) and \( N = 2 \) on synthetic data and found that they are sensitive to noise. With a sufficiently good guess on aspect ratio, the solver in (17) performs better than the aspect ratio solver (see Fig. C.1). We opted to sample over aspect ratio rather than introduce these solvers. We leave the incorporation of minimal solvers for unknown aspect ratio for future work.

B. Recovering Radial-Projection Functions for User-Selected Camera Models

The detected corners \( u \) can be back-projected to rays \( \gamma g(u) = \gamma (u, v, \psi(r(u)))^T \), \( \gamma > 0 \) in the direction of the board fiducials in the camera’s coordinate system. The polar angle of the ray determines how the points of the ray are projected, so any point of the ray can be used (equivalently, any \( \gamma > 0 \) in (1) can be used). The distances to the optical axis \( R \) and principal plane \( Z \) are computed from the unit vector concurrent with the ray, \( \frac{g(u)}{\|g(u)\|} \). With the radii and depths recovered, least squares can be used to recover the unknowns of the radial projection functions listed in Tables 1. This demonstrates the ease with which BabelCalib is extended.

Division Model to Kannala-Brandt Regression The experiments of Sec. 5 confirm that the Kannala-Brandt (KB) model is the most flexible and accurate over the largest range of lenses, which is consistent with the results of [41]. We also found that the Kannala-Brandt model is also effective for catadioptric rigs (see Table 5).

The number of failed calibrations of OpenCV and Kalibr reported in Table 5 suggests that Kannala-Brandt is one of the hardest camera models to initialize. Directly computing a model proposal for Kannala-Brandt is difficult. The displacement of the projection from the optical center is proportional to a polynomial function of atan2. The trigonometric function atan2 is not easily eliminated since the unknown depth \( Z \) is unique to each fiducial on the chessboard. Thus the problem cannot be solved as a polynomial system. BabelCalib initializes Kannala-Brandt by linearly regressing its parameters against the estimated division model, which is formulated in Sec. 3. We derive the linear system here to show how easy the model-to-model regression is once the back-projection function is known.

We back-project the corners \( u_i = (u_i, v_i, 1)^T \) to ray directions \( x'_i = (x'_i, y'_i, z'_i)^T \) where \( x'_i = g(K^{-1} u_i) \). The radii and depths are computed as \( R_i = \sqrt{x'_i^2 + y'_i^2} \) and \( Z_i = z'_i \), respectively. The polar angles \( \omega_i = \text{atan2}(R_i, Z_i) \) are determined, and the unknown coefficients of the polynomial \( r_i = \omega_i + \sum_{n=1}^{4} k_n \omega_i^{2n+1} \) can be determined linearly.
Solving (23) fully specifies the Kannala-Brandt projection model since the center of projection e and the focal length f are known from estimation of the back-projection model — from (11) and (17), respectively.

C. Algorithm

To summarize the proposed RANSAC-based calibration framework detailed in Sec. 4, we provide Algorithm 1 with all the crucial steps of the BabelCalib framework. The details on the pose optimization are omitted in the algorithm.

\begin{algorithm}[H]
\caption{BabelCalib Camera Calibration}
\textbf{Input:} 2D-3D point correspondences $u_{ijk} \leftrightarrow X_{ij}$
\textbf{Parameters:} Radial projection model $\phi_0$
\textbf{Output:} $\Theta^* = \{\theta^*, K^*, R_{jk}, t_{jk}^*\}$
\begin{algorithmic}
\State $J_0 \leftarrow \infty$, $J^* \leftarrow \infty$
\Repeat
\State Sample image $k'$ and plane $j'$
\State Sample $\{u_{ijk'} \leftrightarrow X_{ij'}\}_{i=1}^N$ with (15)
\State Compute e and F from $\{u_{ijk'} \leftrightarrow X_{ij'}\}_{i=1}^N$ with (17)
\State Correct corners $u_{ijk'}$ to $u_{ijk'}'$ with (16)
\State Compute $\{R_{jk}, t_{jk}\}$ from F with (17)
\State Sample aspect ratio $a$ from $[0.5, 2]$.
\State Compute $\{f, \lambda_1, \ldots, \lambda_N, t_z\}$ with (18)
\State $k \leftarrow \mathbb{T}(e)$ of $\{a f, f, 1\}$, $t_{jk}^* \leftarrow (t_x, t_y, t_z)^T$
\If{$\phi_0^{-1}$ is not the division model}
\State Regress $\theta$ of $\phi_0$ with (19)
\EndIf
\For{$(k, j) \neq (k', j')$}
\State Sample image $k$ and plane $j$
\State Back-project the corners $x_{ijk} \leftarrow g(K^{-1} u_{ijk})$
\State Compute $\{R_{jk}, t_{jk}\}$ from $\{x_{ijk}\}_{i=1}^N$ with [28]
\EndFor
\State $\Theta_0 \leftarrow \{\theta, K, R_{jk}, t_{jk}\}$
\State Compute loss $J_0 \leftarrow J(\Theta_0)$ with (19)
\If{$J_0 < J_0^*$}
\State $J_* \leftarrow J_0$, $I_* \leftarrow I_0$
\State Optimize $\Theta_{LO} \leftarrow \text{argmin}_{\Theta} J(\Theta)$ with (19)
\State Compute loss $J_{LO} \leftarrow J(\Theta_{LO})$ with (19)
\EndIf
\If{$J_{LO} < J^*$}
\State $J^* \leftarrow J_{LO}$, $\Theta^* \leftarrow \Theta_{LO}$
\EndIf
\Until $T$ iterations;
\State Compute inlier ratio $I^* \leftarrow I(\Theta^*)$ with (20)
\end{algorithmic}
\end{algorithm}

Figure C.1: Noise sensitivity. RMS reprojection error on synthetic images with added 1 px noise for (left) the solvers for the division model complexity $N = 1$, and (right) the solvers for the complexity $N = 2$. Red curves correspond to the linear solvers from (17) that do not solve for the aspect ratio, and green curves correspond to the aspect ratio solvers from (22). Solid curves are the median errors and the shaded plots are the corresponding interquartile ranges.

D. Calibration with Limited Data

Calibration accuracy is dependent on good feature coverage. However, fast calibration from few images is also im-
Figure C.3: **Calibration results for limited train data.** Cross-validation with corresponding train size is performed, and the reported errors are normalized in accordance with image resolution $1000 \times 1000$ px. E denotes the RMS weighted reprojection error. OCamCalib-DIV requires at least two images for calibration.

Important for users e.g. for lens inventory purposes. We compared the performance of calibration methods on a limited amount of training data starting from a single image. We randomly drew the subsamples of sizes 1, 2, and 5 images from the training data (10 times for each train size) and evaluated the calibrations on the hold-out test data. The Fig. C.3 reports the distributions of the robust test errors (19). To remove the effect of the image resolution for different cameras, the errors are normalized in accordance with the resolution $1000 \times 1000$ px. There are several limitations found in current frameworks. OCamCalib-DIV requires at least two images for calibration so they don’t have the results for the train size 1. Also, as was mentioned in previous section, this toolbox requires all points to be visible from all views which is a big limitation. All other methods would occasionally fail for particular subsets of images. For such failure cases we set the test error to be the maximum error. The proposed calibration method never fails and provides the best calibration starting from a single image already.