On numerical modeling of natural convection based on the OpenFOAM solver

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Abstract. This work considers numerical modelling of natural incompressible convection problems in the gravity field in the Boussinesq approximation using OpenFOAM. A mathematical model of natural convection in the gravity field, using Boussinesq approximation, was considered. The built-in solver was identified and modified. The exact solution, theoretical data and experiment were used to verify the solver. A numerical study of natural convection in a horizontal liquid layer was performed. Natural convection in a fluid layer between coaxial cylinders was considered.

1. Mathematical model
A mathematical model of natural convection is well known. In the Boussinesq approximation it has the following form [1],

\[
\begin{align*}
\text{div} \vec{U} &= 0, \\
\rho \left( \frac{\partial \vec{U}}{\partial t} + (\vec{U} \cdot \nabla) \vec{U} \right) - \nu \Delta \vec{U} &= -\nabla p + g \left[ \beta (T - T_0) \right], \\
\frac{\partial T}{\partial t} + \vec{U} \cdot \nabla T - \alpha \Delta T &= 0,
\end{align*}
\]

where \( \beta \) — coefficient of thermal expansion of fluid, \( T_0 \) — temperature, \( \alpha \) — the thermal diffusivity, \( \nu \) — effective kinematic viscosity, \( \rho \) — the density of the fluid.

To numerical simulate this system, a modification of the icoFoam solver was used. The heat equation was added and instead of algorithm PISO (Pressure-Implicit with Splitting of Operators), algorithm SIMPLE (Semi-Implicit Method for Pressure-Linked Equation) was used [2].

2. Convection in a horizontal liquid layer
To verify the modified solver, the experiment presented in the article [3] was considered. A rectangular region was investigated, where the upper plane maintains at a given temperature and the lower one was heated. This enclosure was filled with silicone oil (Pr=1780). The initial temperature was taken as the temperature of the upper surface, the remaining boundaries were thermally insulated. The velocity at the initial moment was taken equal to zero; the non-stick condition was used at the boundaries. The general system of boundary conditions was taken as

\[
\begin{align*}
\frac{\partial U_x}{\partial x} &= 0, & \text{at} \quad y = 0, L_y, \\
\frac{\partial U_y}{\partial y} &= 0, & \text{at} \quad x = 0, L_x, \\
U_x &= 0, & \text{at} \quad y = 0, L_y, \\
\frac{\partial T}{\partial y} &= 0, & \text{at} \quad x = 0, L_x, \\
\frac{\partial T}{\partial x} &= 0, & \text{at} \quad y = 0, L_y, \\
\frac{\partial T}{\partial y} &= 0, & \text{at} \quad x = 0, L_x, \\
T &= T_0, & \text{at} \quad y = 0, L_y, \\
\frac{\partial T}{\partial y} &= 0, & \text{at} \quad x = 0, L_x, \\
T &= 0, & \text{at} \quad x = 0, L_x.
\end{align*}
\]
follows:

\[
T(x, 0, t) = T_c, \quad T(x, h, t) = T_h, \quad 0 \leq x \leq l, \quad t > 0,
\]

\[
\frac{\partial T}{\partial x}|_{x=0} = \frac{\partial T}{\partial x}|_{x=l} = 0, \quad 0 \leq y \leq h, \quad t > 0,
\]

\[
\dot{U}|_{\partial \Omega} = 0,
\]

\[
p = f(T|_{\partial \Omega}, \dot{U}|_{\partial \Omega}).
\]

Initial conditions:

\[
T(x, y, t)|_{t=0} = T_0, \quad 0 \leq x \leq l, \quad 0 \leq y \leq h,
\]

\[
\dot{U}(x, y, t)|_{t=0} = 0,
\]

\[
p(x, y, t)|_{t=0} = \rho g(h - y).
\]

The first simulation results [4, 5] were not accurate enough. In studying the causes of deviations, a study was conducted on the grid convergence and the results were clarified. The obtained critical Rayleigh number belonged to the interval \([3042, 3457]\). This deviation was due to unknown material parameters used in the experiment. Therefore, the dependence of the critical Rayleigh number on the kinematic viscosity and thermal diffusivity was studied. The Prandtl number remained unchanged and corresponded to that used in the experiment.

Table 1 shows the kinematic viscosity, thermal diffusivity, the interval for the critical Rayleigh number.

| $\beta$ | $K^{-1}$ | $\alpha$, $m^2/s$ | $\nu$, $m^2/s$ | $Ra_c$ | $Pr$ |
|---|---|---|---|---|---|
| $10^{-3}$ | $1.17 \cdot 10^{-1}$ | $2.08 \cdot 10^{-4}$ | $[3042, 3457]$ | 1778 |
| $10^{-3}$ | $0.983 \cdot 10^{-7}$ | $1.75 \cdot 10^{-4}$ | $[2742, 3126]$ | 1778 |
| $10^{-3}$ | $0.9 \cdot 10^{-7}$ | $1.6 \cdot 10^{-4}$ | $[2103, 2336]$ | 1778 |
| $10^{-3}$ | $0.783 \cdot 10^{-7}$ | $1.392 \cdot 10^{-4}$ | $[2006, 2161]$ | 1778 |

The simulation results showed that with a decrease in the kinematic viscosity and thermal diffusivity, the critical Rayleigh number also decreases.

3. Lorenz attractor

The paper investigates the problem of natural convection motion of a fluid layer enclosed between coaxial cylinders, similar to the Lorentz problem [6]. The enclosure is heated from below and cooled from above. X was taken as fluid flow rate, Y was taken as the temperature at the right point of the ring ($\phi = \frac{\pi}{2}$) and Z was taken as the temperature at lowest point ($\phi = 0$). A set of these variables, reduced to dimensionless quantities, determines the instantaneous state of the system [7]. A two-dimensional model was considered. System of boundary conditions was taken as follows:

\[
T(R1, \phi, t) = T(R2, \phi, t) = T_c, \frac{\pi}{4} \leq \phi \leq \frac{3\pi}{4}, \quad t > 0,
\]

\[
T(R2, \phi, t) = T(R1, \phi, t) = T_h, \frac{3\pi}{4} \leq \phi \leq \frac{5\pi}{4}, \quad t > 0,
\]

\[
\left. \frac{\partial T(r, \phi, t)}{\partial r} \right|_{r=R1}, \left. \frac{\partial T(r, \phi, t)}{\partial r} \right|_{r=R2} = 0, \quad \frac{3\pi}{4} \leq \phi \leq \frac{5\pi}{4}, \quad t > 0
\]

\[
\left. \frac{\partial T(r, \phi, t)}{\partial \phi} \right|_{r=R1}, \left. \frac{\partial T(r, \phi, t)}{\partial \phi} \right|_{r=R2} = 0, \quad -\frac{\pi}{4} \leq \phi \leq \frac{\pi}{4}, \quad t > 0
\]

\[
\dot{U}|_{\partial \Omega} = 0,
\]

\[
p = f(T|_{\partial \Omega}, \dot{U}|_{\partial \Omega}).
\]
Initial conditions:
\[
\begin{aligned}
T(x, y, 0) &= \frac{T_h + T_c}{2}, \quad 0 \leq x \leq l, \ 0 \leq y \leq h, \\
\vec{U}(x, y, 0) &= 0, \\
p(x, y, 0) &= \rho g(h - y).
\end{aligned}
\]

There was a problem with getting the values of X, Y and Z at times of chaos. Samples taken at points A and B had a lot of noise. To fix this problem, a python script was implemented. It calculated the angular velocity for enclosure and the average temperature in a certain neighbourhood. The following cases are shown:

- The direction changed once. Relative to the respective averages, temperature fluctuation decayed out (Case 1. Fig. 1, 2, 3, 10).
- The direction was constantly changing. Continuous unpredictable temperature fluctuation was observed (Case 2 Fig. 4, 5, 6, 11).
- The direction was constantly changing. Periodic temperature and velocity fluctuation were observed. (Case 3. Fig. 7, 8, 9, 12)

The given cases have their intervals of physical values. In the future, it is planned to build a map of the dependence of the type of solutions on the constants. Despite the large intervals of the physical constants used, the Lorentz attractor was not obtained.
4. Conclusion
The processes of natural convection in the gravitational field in the Boussinesq approximation was considered. Tasks in the horizontal fluid layers were considered to verify the program. This results, obtained by modelling, agreed quite well with theoretical data. A numerical study of natural convection in a horizontal liquid layer is carried out. A numerical study of natural convection in a liquid layer between coaxial cylinders is carried out. A problem similar to the
Lorentz problem was investigated. Simulation results are presented. The used solver and test cases were implemented using OpenFOAM.

References
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