A Model Independent Approach to Solar Neutrino Experiments

S.M. Bilenky\(^{(a,b)}\)\(^{†}\) and C. Giunti\(^{(a)}\)\(^{‡}\)

\(^{(a)}\) INFN, Sezione di Torino, and Dipartimento di Fisica Teorica, Università di Torino, Via P. Giuria 1, I–10125 Torino, Italy  
\(^{(b)}\) Joint Institute for Nuclear Research, Dubna, Russia

Abstract

In the first part of this report we present the results of a model independent analysis of the existing solar neutrino data. We obtained forbidden regions in the plane of the parameters $\Delta m^2$ and $\sin^2 2\theta$ in two cases: A) Without any restrictions on the values of the solar neutrino fluxes from different reactions; B) With some restrictions that take into account the predictions of all the existing solar models. We show that the existing solar neutrino data allow to exclude rather large regions in the plane of the parameters $\Delta m^2$ and $\sin^2 2\theta$ (especially in case B). In the second part of this report we present a general method for the analysis of solar neutrino data that can be applied to future solar neutrino experiments (SNO, Super-Kamiokande, Icarus) in which high energy $^8$B neutrinos will be detected. We show that these experiments will allow: 1) To reveal in a model independent way the presence of sterile neutrinos in the flux of solar neutrinos on the earth and to obtain lower bounds for the probability of transition of $\nu_e$’s into sterile states; 2) To obtain directly from the experimental data the initial $^8$B $\nu_e$ flux and the probability of $\nu_e$’s to survive (if there are no transitions of $\nu_e$’s into sterile states).
1 Introduction

Solar neutrino experiments are very important for the investigation of neutrino mixing, as well as for the investigation of the sun. These experiments are sensitive to very small values of the neutrino mass difference squared $\Delta m^2 \equiv m_2^2 - m_1^2$ (down to $\Delta m^2 \simeq 10^{-10}$ eV$^2$) and to a wide region of mixing angles $\theta$, including very small $\theta$'s. On the other side, solar neutrino experiments allow us to detect neutrinos from different reactions of the thermonuclear solar cycles, including neutrinos from $^8$B decay, whose flux is about $10^{-4}$ of the total flux. The problem is that we cannot determine from the existing solar neutrino experiments [1–4] separately the values of neutrino masses and mixing angles and the values of the neutrino fluxes. Usually, to obtain information about the values of $\Delta m^2$ and $\sin^2 2\theta$, it is assumed that the values of the neutrino fluxes are given by the Standard Solar Model (SSM) [5–9]. It is well known, however, that the neutrino fluxes calculated in the framework of the SSM are subject to many sources of uncertainties, mainly due to a poor knowledge of some input parameters (especially nuclear cross sections and opacity).

In this report we will present:

1. The results of a solar model independent analysis of the existing solar neutrino data [10].

2. A model independent approach to future solar neutrino experiments in which solar neutrinos will be detected through the observation of CC, NC and neutrino-electron elastic scattering (ES) processes [11, 12].

We will show that future solar neutrino experiments (SNO [13], Super-Kamiokande [14], ICARUS [15]), in which high energy $^8$B neutrinos will be detected, will allow to answer in a model independent way the question whether there are transitions of solar $\nu_e$'s into sterile states. If there are only active neutrinos $\nu_e$, $\nu_\mu$, $\nu_\tau$, in the flux of solar neutrinos on the earth, future solar neutrino experiments will allow to determine directly from the experimental data the initial flux of $^8$B $\nu_e$'s and the probability of $\nu_e$'s to survive as a function of neutrino energy.

There exist at present data of three radiochemical solar neutrino experiments (Homestake [1], GALLEX [2] and SAGE [3]) and the water Cherenkov direct counting Kamiokande experiment [4]. In the Homestake experiment solar neutrinos are detected through the observation of the Pontecorvo-Davis reaction $\nu_e + ^{37}\text{Cl} \rightarrow e^- + ^{37}\text{Ar}$, whose threshold is 0.81 MeV. Thus $pp$ neutrinos, which compose the main part of the solar neutrino flux, are not detected in this experiment and the main contribution to the Homestake event rate comes from $^8$B and $^7$Be (according to BP [5] 78% and 15%, respectively). In the GALLEX and SAGE experiments solar neutrinos are detected through the observation of the reaction $\nu_e + ^{71}\text{Ga} \rightarrow e^- + ^{71}\text{Ge}$, whose threshold is 0.23 MeV. The main contributions to the GALLEX and SAGE event rates come from $pp$, $^7$Be and $^8$B (according to BP 54%, 27% and 8%, respectively). In the Kamiokande experiment solar neutrinos are detected through the observation of the process $\nu e \rightarrow \nu e$. The electron energy threshold in this experiment is
Table 1: Data of solar neutrino experiments and rates predicted by BP [5], TL [6] and CDF [7]. The Kamiokande result is presented as the ratio of the observed rate to the rate predicted by BP.

| Experiment     | Event Rate (SNU) | SSM Predictions (SNU) |
|----------------|------------------|-----------------------|
|                |                  | BP        | TL        | CDF        |
| Homestake      | \(N_{\text{HOM}}^{\text{exp}} = 2.32 \pm 0.23\) | 8.0 ± 1.0 | 6.4 ± 1.4 | 7.8        |
| GALLEX         | \(N_{\text{GAL}}^{\text{exp}} = 79 \pm 10 \pm 6\) | 131.5\(^+7\)\(_{-6}\) | 123 ± 7 | 131        |
| SAGE           | \(N_{\text{GAL}}^{\text{exp}} = 69 \pm 11 \pm 6\) |           |           |            |
| Kamiokande     | \(N_{\text{KAM}}^{\text{exp}}/N_{\text{KAM}}^{\text{BP}} = 0.51 \pm 0.04 \pm 0.06\) | 1 ± 0.14 | 0.8 ± 0.2 | 0.98       |

about 7 MeV. Thus only \(^8\)B neutrinos are detected in the Kamiokande experiment. In Table 1 the results of all four solar neutrino experiments are given. In the last three columns of Table 1 the values of the event rates predicted by Bahcall and Pinsonneault (BP) [5], Turck-Chièze and Lopes (TL) [6] and Castellani, Degl’Innocenti and Fiorentini (CDF) [7] are given. As it is seen from Table 1, the event rates measured in all four solar neutrino experiments are significantly less than the event rates predicted by the existing Standard Solar Models.

Pontecorvo neutrino mixing (see, for example, Refs.[16, 17]) is apparently the most natural explanation of the possible “deficit” of solar neutrinos. It was shown in Refs.[18–24] that all existing solar neutrino data can be described by the resonant MSW mechanism [25] in the simplest case of mixing between two neutrino types\(^1\). The following two MSW solutions of the solar neutrino problem were found in the case of \(\nu_e-\nu_\mu(\tau)\) mixing:

1. A small mixing angle solution: \(\Delta m^2 \simeq 5 \times 10^{-6} \text{eV}^2\) and \(\sin^2 2\theta \simeq 8 \times 10^{-3}\)
2. A large mixing angle solution: \(\Delta m^2 \simeq 10^{-5} \text{eV}^2\) and \(\sin^2 2\theta \simeq 0.8\).

Thus, some indications in favor of neutrino mixing follow from the existing solar neutrino data. We would like to emphasize, however, that these indications are model dependent: the analysis of the data is based on the assumption that the values of the neutrino fluxes from the different sources are given by the Standard Solar Models.

\(^1\)The data can be described also by vacuum oscillations [21] with a fine-tuning between \(\Delta m^2\) and the sun-earth distance. For the parameters \(\Delta m^2\) and \(\sin^2 2\theta\) the following values were obtained: \(\Delta m^2 \simeq 8 \times 10^{-11} \text{eV}^2\) and \(\sin^2 2\theta \simeq 0.8\).
2 A model independent analysis of solar neutrino data

In this section we present the results of a model independent analysis [10] of the data of the existing solar neutrino experiments. We will consider the simplest case of mixing between two types of active neutrinos ($\nu_e - \nu_\mu$ ($\tau$)) and assume that the MSW mechanism takes place. Our analysis is based on the fact that the shapes of the spectra of neutrinos from the reactions of the thermonuclear cycle are known. These spectra are determined by the interactions responsible for the reactions and, as it was shown in Ref.[26], are negligibly affected by the conditions in the interior of the sun. Thus, the event rates of the solar neutrino experiments are determined by the values of the parameters $\Delta m^2$ and $\sin^2 2\theta$ and by the values of the total neutrino fluxes (mainly $pp$, $^7$Be and $^8$B). We will consider the total neutrino fluxes as unknown parameters. From the existing solar neutrino data we cannot determine allowed regions of the values of the parameters $\Delta m^2$ and $\sin^2 2\theta$ and of the neutrino fluxes. Instead, we will obtain the regions of values of the parameters $\Delta m^2$ and $\sin^2 2\theta$ that are forbidden for all possible values of the initial neutrino fluxes (or for the initial neutrino fluxes constrained within wide limits chosen in such a way to include the predictions of all the existing Standard Solar Models). We will show that the existing solar neutrino data allow to exclude rather large regions of the values of the parameters $\Delta m^2$ and $\sin^2 2\theta$.

Let us write the initial spectrum of $\nu_e$ from the source $r$ ($r = pp, pep, ^7$Be, $^8$B, Hep, $^{13}$N, $^{15}$O, $^{17}$F) in the form

$$\phi_{\nu_e}^r (E) = X^r(E) \Phi^r,$$  \hspace{1cm} (2.1)

where $E$ is the neutrino energy, $X^r(E)$ is a known function normalized by the condition $\int X^r(E) \, dE = 1$ (see Ref.[5]) and $\Phi^r$ is the total initial neutrino flux from the source $r$.

The integral event rate in any experiment $a$ ($a = \text{HOM (Homestake)}, \text{KAM (Kamiokande)}, \text{GAL (GALLEX+SAGE)}$) is given by the expression

$$N_a = \sum_r Y_a^r \Phi^r.$$  \hspace{1cm} (2.2)

In the case of radiochemical experiments only solar $\nu_e$ are detected. We have

$$Y_a^r = \int_{E_{\text{th}}^a} \sigma_a(E) X^r(E) P_{\nu_e \rightarrow \nu_e}(E) \, dE,$$  \hspace{1cm} (2.3)

with $a = \text{HOM, GAL}$. Here $\sigma_a(E)$ is the cross section of the reaction $\nu_e + ^{37}$Cl $\rightarrow e^- + ^{37}$Ar in the case of the Homestake experiment and the cross section of the reaction $\nu_e + ^{71}$Ga $\rightarrow e^- + ^{71}$Ge, in the case of the GALLEX and SAGE experiments, $P_{\nu_e \rightarrow \nu_e}(E)$ is the probability of $\nu_e$ to survive and $E_{\text{th}}^a$ is the threshold energy. For the calculation of the $\nu_e$ survival probability we used the formula given in Ref.[27], which is valid for an exponentially decreasing electron density.

\textsuperscript{2}In our calculations we use the combined GALLEX–SAGE data: $74 \pm 9$ SNU.
Table 2: Solar neutrino fluxes (with 1σ errors) predicted by BP; $\langle E \rangle$ is the average neutrino energy, $Y_{\text{LUM}} = Q/2 - \langle E \rangle$, where $Q = 26.73 \text{ MeV}$, and $\xi_{\text{min}}$ and $\xi_{\text{max}}$ determine the limits for the values of the total neutrino fluxes in case B.

In the the Kamiokande experiment $\nu_e$ as well as $\nu_\mu$ (and/or $\nu_\tau$) are detected. We have

$$Y_{\nu_e}^r = \int_{E_{\text{th}}}^E \sigma_{\nu_\ell e}(E) X^r(E) \sigma_{\nu_\ell e}(E) dE$$

$$+ \int_{E_{\text{th}}}^E \sigma_{\nu_\mu e}(E) X^r(E) \sum_{\ell=\mu,\tau} \sigma_{\nu_\ell e}(E) dE,$$

where $\sigma_{\nu_\ell e}(E)$ is the cross section of the process $\nu_\ell e \rightarrow \nu_\ell e$ ($\ell = e, \mu$), $E_{\text{th}}$ is the recoil electron energy threshold and $\sigma_{\nu_\ell e}(E)$ is the probability of the transition $\nu_\ell e \rightarrow \nu_\ell$ ($\ell = e, \mu$). In our calculation we took into account the efficiency and the energy resolution of the Kamiokande detector [4]. The fluxes of neutrinos produced in the thermonuclear $pp$ and CNO cycles must satisfy the following relation:

$$N_{\text{LUM}} = \sum_r Y_{\nu e}^r \Phi^r .$$

Here $N_{\text{LUM}} = L_{\odot}/4\pi d^2 = (8.491 \pm 0.018) \times 10^{11} \text{ MeV cm}^{-2} \text{ sec}^{-1}$, where $d = 1 \text{ AU} = 1.496 \times 10^{13} \text{ cm}$ is the average sun-earth distance, $L_{\odot} = (3.826 \pm 0.008) \times 10^{33} \text{ erg sec}^{-1}$ [28], is the luminosity of the sun and $Y_{\text{LUM}}^r = Q/2 - \langle E \rangle^r$, where $Q = 4m_p + 2m_e - m_{^4\text{He}} = 26.73 \text{ MeV}$ and $\langle E \rangle^r$ is the average energy of neutrinos from the source $r$. The values of $\langle E \rangle^r$ and $Y_{\text{LUM}}^r$ are given in Table 2.

Our procedure for the analysis of the solar neutrino data is the following. At fixed values of the parameters $\Delta m^2$ and $\sin^2 2\theta$ we calculate the $\chi^2$ for all possible values of the neutrino fluxes. For each value of the parameters $\Delta m^2$ and $\sin^2 2\theta$ and of the neutrino fluxes we estimate the “goodness-of-fit” by calculating the confidence level (CL) corresponding to the calculated $\chi^2$. Since we do not determine any parameter, the number of degrees of freedom of the $\chi^2$ distribution is equal to the number of data points (i.e. four: three neutrino rates and the solar luminosity constraint). If all the confidence levels found for a
Figure 1: Excluded regions in the $\sin^2 2\theta - \Delta m^2$ plane for MSW transitions due to $\nu_e - \nu_\mu$ mixing in case A. The region F is excluded at 95% CL within the solid line and at 95% CL within the dotted line. The allowed regions found with the BP neutrino fluxes are also shown (shaded areas).

Figure 2: Excluded regions in the $\sin^2 2\theta - \Delta m^2$ plane for MSW transitions due to $\nu_e - \nu_\mu$ mixing in case B. The regions F are excluded at 95% CL within the corresponding solid line and at 95% CL within the corresponding dotted line. The allowed regions found with the BP neutrino fluxes are also shown (shaded areas).

Given value of $\Delta m^2$, $\sin^2 2\theta$ and all possible values of the neutrino fluxes are smaller than $\alpha$ (we choose $\alpha = 0.1$, 0.05, 0.01), then the corresponding point in the $\Delta m^2 - \sin^2 2\theta$ plane is excluded at $100(1 - \alpha)$% CL. In this way we obtain the exclusion plots presented in Figs.1 and 2. Let us notice that for the purpose of determination of the excluded regions in the parameter space our approach is the most conservative: any decrease of the number of degrees of freedom would increase the excluded regions.

For the exclusion plot presented in Fig.1 the only requirement was that all the total neutrino fluxes are positive. Let us call this case A. However, it is interesting and instructive to investigate how the forbidden regions in the $\Delta m^2 - \sin^2 2\theta$ plane change if some limits on the allowed values of the neutrino fluxes are imposed. Thus we also considered the following case B: the different solar neutrino fluxes are constrained in the interval $\xi^r_{\min} \Phi^r(BP) \leq \Phi^r \leq \xi^r_{\max} \Phi^r(BP)$, where $\Phi^r(BP)$ are the BP values of the neutrino fluxes and the factors $\xi^r_{\min}$...
and $\xi_{\text{max}}$ are chosen in such a way to include the predictions of the existing solar models [5–9]. The values of these factors are given in Table 2. We determined the minimum (maximum) values for the $pp$, $pep$, $^7\text{Be}$ and $\text{Hep}$ fluxes by subtracting (adding) 3 times the range of solar model predictions to the minimum (maximum) predicted flux (notices that this range is larger than the 1$\sigma$ error given by BP). Since it has been recently suggested [29] that the value of the astrophysical factor $S_{17}(0)$ could be significantly lower than that used in SSM calculations, we let the $^8\text{B}$ flux to be arbitrarily small. Since the CNO fluxes have large uncertainties, we allow also them to be arbitrarily small. We determined the maximum values of the $^8\text{B}$ and CNO fluxes by adding 3 times the 1$\sigma$ error of BP to the BP average value. Let us emphasize that the limits on the allowed values of the neutrino fluxes which we imposed in case B are rather large. The excluded regions of the parameters $\Delta m^2$ and $\sin^2 2\theta$ in case B are presented in Fig.2. As it can be seen from a comparison of this figure with Fig.1 the excluded regions in case B are much larger than in the case where no limitation is imposed on the values of the neutrino fluxes.

### 3 Are there sterile neutrinos in the flux of solar neutrinos on the earth?

The problem of existence of sterile neutrinos\(^3\) is very important for the theory beyond the Standard Model. Neutrino masses and mixing can be generated in the framework of the standard model if right-handed neutrino fields (singlets) together with left-handed doublets enter in the Yukawa interaction. In this case the total lepton charge $L = L_e + L_{\mu} + L_{\tau}$ is conserved, neutrinos with definite masses are Dirac particles and transitions only between flavor neutrinos ($\nu_\ell \rightarrow \nu_{\ell'}$ with $\ell, \ell' = e, \mu, \tau$) are allowed. In the models beyond the Standard Model transitions of active flavor neutrinos into sterile neutrino states become possible (see, for example, Refs.[16, 17]). So a discovery of such transitions will be a discovery of new physics.

Here we will show that future solar neutrino experiments could allow to reveal the presence of sterile neutrinos in the solar neutrino flux on the earth independently on any assumption about the values of the initial neutrino fluxes [11, 12].

The solar neutrino experiments of the next generation (scheduled to start in 1996) are the SNO [13] and the Super-Kamiokande [14] experiments. In the SNO experiment solar neutrinos will be detected through the observation of three different processes:

1. The CC process

   $$\nu_e + d \rightarrow e^- + p + p ; \quad (3.1)$$

2. The NC process

   $$\nu + d \rightarrow \nu + p + n ; \quad (3.2)$$

\(^3\)Sterile neutrinos are quanta of right-handed neutrino fields. In the Dirac-Majorana mixing scheme, these fields are mixed with the left-handed fields (see, for example, Refs.[16, 17]). Sterile neutrinos do not interact with matter via standard CC and NC interactions.
3. The elastic scattering (ES) process

\[ \nu + e^- \rightarrow \nu + e^- . \]  

(3.3)

In the Super-Kamiokande (S-K) experiment solar neutrinos will be detected through the observation of the process (3.3) with an event rate about 30 times larger than in the current Kamiokande experiment. In both the SNO and S-K experiments, due to the high energy thresholds (\( \simeq 6 \text{ MeV} \) for the CC process, \( 2.2 \text{ MeV} \) for the NC process and \( \simeq 5 \text{ MeV} \) for the ES process), only neutrinos coming from \(^8\text{B} \) decay will be detected. The energy spectrum of the initial \(^8\text{B} \) \( \nu_e \)'s is given by

\[ \phi^0_{\nu_e}(E) = X(E) \Phi . \]  

(3.4)

The function \( X(E) \) is the normalized neutrino spectrum from the decay \(^8\text{B} \rightarrow ^8\text{Be} + e^+ + \nu_e \), which is determined by the phase space factor (small corrections due to forbidden transitions where calculated in Ref.[30]). The factor \( \Phi \) in Eq.(3.4) is the total flux of initial \(^8\text{B} \) solar \( \nu_e \)'s.

Consider first the NC process (3.2). Using \( \nu_e - \nu_\mu - \nu_\tau \) universality of NC for the total NC event rate in the SNO experiment \( N_{\text{NC}} \), we have

\[ N_{\text{NC}} = \int_{E_{\text{NC}}^{\text{th}}} \sigma_{\nu d}(E) X(E) \sum_{\ell = e, \mu, \tau} P_{\nu_e \rightarrow \nu_\ell}(E) \, dE \Phi , \]  

(3.5)

where \( \sigma_{\nu d}(E) \) is the cross section for the process \( \nu d \rightarrow \nu n p \), \( E_{\text{NC}}^{\text{th}} \) is the threshold neutrino energy and \( P_{\nu_e \rightarrow \nu_\ell}(E) \) is the probability of transition of solar \( \nu_e \)'s into \( \nu_\ell \) (\( \ell = e, \mu, \tau \)). It is useful to introduce the average total probability of transitions of \( \nu_e \) into other active neutrinos:

\[ \left< \sum_{\ell = e, \mu, \tau} P_{\nu_e \rightarrow \nu_\ell} \right>_{\text{NC}} = \frac{1}{X_{\nu d}} \int_{E_{\text{NC}}^{\text{th}}} \sigma_{\nu d}(E) X(E) \sum_{\ell = e, \mu, \tau} P_{\nu_e \rightarrow \nu_\ell}(E) \, dE , \]  

(3.6)

where

\[ X_{\nu d} \equiv \int_{E_{\text{NC}}^{\text{th}}} \sigma_{\nu d}(E) X(E) \, dE . \]  

(3.7)

Using the results of a recent calculation of the cross-section \( \sigma_{\nu d}(E) \) [31] we obtained \( X_{\nu d} = 4.72 \times 10^{-43} \text{ cm}^2 \). From Eqs.(3.5) and (3.6) we get

\[ \left< \sum_{\ell = e, \mu, \tau} P_{\nu_e \rightarrow \nu_\ell} \right>_{\text{NC}} = \frac{N_{\text{NC}}}{X_{\nu d} \Phi} . \]  

(3.8)

In the general case of transitions of \( \nu_e \)'s into active as well as into sterile neutrinos

\[ \sum_{\ell = e, \mu, \tau} P_{\nu_e \rightarrow \nu_\ell}(E) = 1 - P_{\nu_e \rightarrow \nu_S}(E) , \]  

(3.9)
where $P_{\nu_e \rightarrow \nu_S}(E)$ is the total probability of transition of $\nu_e$’s into all possible sterile states$^4$. From Eqs.(3.8) and (3.9) it follows that

$$1 - \langle P_{\nu_e \rightarrow \nu_S}\rangle_{NC} = \frac{N^{NC}}{X_{\nu e} \Phi}.$$  

(3.10)

Thus, from the measurement of the NC event rate $N^{NC}$ it is impossible to reach any conclusions about transitions of solar $\nu_e$’s into sterile states without assumptions about the value of the total flux $\Phi$. However, if solar $^8$B neutrinos are detected not only through the observation of NC but also through the observation of the ES and CC processes the problem of existence of sterile neutrinos could be solved in a completely model independent way.

Let us consider the ES process (3.3). The total number of ES events is equal to

$$N^{ES} = \int_{E_{th}^{ES}}^{E_{th}^{ES}} \sigma_{\nu_e e}(E) P_{\nu_e \rightarrow \nu_l}(E) X(E) dE \Phi + \int_{E_{th}^{ES}}^{E_{th}^{ES}} \sigma_{\nu_e e}(E) \sum_{\ell=\mu,\tau} P_{\nu_e \rightarrow \nu_{\ell}}(E) X(E) dE \Phi.$$  

(3.11)

From Eq.(3.11) we have

$$\Sigma^{ES} = \int_{E_{th}^{ES}}^{E_{th}^{ES}} \sigma_{\nu_e e}(E) \sum_{\ell=\mu,\tau} P_{\nu_e \rightarrow \nu_{\ell}}(E) X(E) dE \Phi.$$  

(3.12)

Here

$$\Sigma^{ES} \equiv N^{ES} - \int_{E_{th}^{ES}}^{E_{th}^{ES}} (\sigma_{\nu_e e}(E) - \sigma_{\nu_{\ell} e}(E)) \phi_{\nu_e}(E) dE ,$$  

(3.13)

where $\phi_{\nu_e}(E) = P_{\nu_e \rightarrow \nu_e}(E) X(E) \Phi$ is the flux of $\nu_e$ on the earth. The quantity $\Sigma^{ES}$ can be obtained from the data of the SNO and S-K experiments. In fact, $N^{ES}$ will be measured in both experiments. In the SNO experiment the spectrum of the electrons in the CC process (3.1) will be measured and the spectrum of solar $\nu_e$ on the earth, $\phi_{\nu_e}(E)$, will be determined. From Eq.(3.12) we obtain the relation

$$\left\langle \sum_{\ell=\mu,\tau} P_{\nu_e \rightarrow \nu_{\ell}}\right\rangle_{ES} = \frac{\Sigma^{ES}}{X_{\nu_{\ell} e} \Phi} ,$$  

(3.14)

which is similar in form to the relation (3.8). The quantities in the relation (3.14) are determined as follows:

$$X_{\nu_{\ell} e} \equiv \int_{E_{th}^{ES}}^{E_{th}^{ES}} \sigma_{\nu_{\ell} e}(E) X(E) dE$$  

(3.15)

and

$$\left\langle \sum_{\ell=\mu,\tau} P_{\nu_e \rightarrow \nu_{\ell}}\right\rangle_{ES} = \frac{1}{X_{\nu_{\ell} e}} \int_{E_{th}^{ES}}^{E_{th}^{ES}} \sigma_{\nu_{\ell} e}(E) X(E) \sum_{\ell=\mu,\tau} P_{\nu_e \rightarrow \nu_{\ell}}(E) dE.$$  

(3.16)

$^4$We assume that neutrinos are stable particles. For a discussion of neutrino instability see Ref.[32] and references therein.
For $E_{th}^{ES} = 5.94$ MeV (which corresponds to a kinetic energy threshold $T_{th} = 4.5$ MeV for the electrons in the CC process) we have $X_{\nu_\mu e} = 2.08 \times 10^{-45}$ cm$^2$. From Eqs.(3.8) and (3.14) we have

$$\frac{1 - \langle P_{\nu_\mu \rightarrow \nu_S} \rangle_{ES}^{ES}}{1 - \langle P_{\nu_\mu \rightarrow \nu_S} \rangle_{NC}^{NC}} = R_{NC}^{ES},$$

(3.17)

where

$$R_{NC}^{ES} = \frac{\sum_{\ell=e,\mu,\tau} X_{\nu_{\ell} d}}{X_{\nu_{\mu} e} N_{NC}^{NC}}.$$

(3.18)

Let us stress that in the ratio $R_{NC}^{ES}$ only measurable and known quantities enter (the flux $\Phi$ cancels in the ratio). From Eq.(3.17) it is evident that if

$$R_{NC}^{ES} \neq 1$$

(3.19)

it would mean that there are transitions of solar $\nu_e$'s into sterile states.

In the case $R_{NC}^{ES} = 1$ no conclusion about sterile neutrinos can be reached. In fact, the ratio $R_{NC}^{ES}$ is equal to one if $\langle P_{\nu_\mu \rightarrow \nu_S} \rangle_{NC}^{NC} = \langle P_{\nu_\mu \rightarrow \nu_S} \rangle_{ES}^{ES}$. This relation is satisfied at $P_{\nu_\mu \rightarrow \nu_S}(E) = 0$ as well as at $P_{\nu_\mu \rightarrow \nu_S}(E) = \text{const} \neq 0$. Thus, if $R_{NC}^{ES} \neq 1$ it would mean not only that sterile neutrinos exist, but also that the probability of the transition of solar $\nu_e$'s into sterile states depends on neutrino energy.

If the inequality (3.19) takes place, it is possible to obtain lower bounds for the average values of the probability of transition of $\nu_e$'s into sterile states. In fact, we have

$$\langle \sum_{\ell=e,\mu,\tau} P_{\nu_\mu \rightarrow \nu_{\ell}} \rangle_{ES}^{ES} \leq \frac{\sum_{\ell=e,\mu,\tau} P_{\nu_\mu \rightarrow \nu_{\ell}}}{\sum_{\ell=e,\mu,\tau} P_{\nu_\mu \rightarrow \nu_{\ell}}} = R_{NC}^{ES}.$$

(3.20)

From this inequality it follows that

$$\langle P_{\nu_\mu \rightarrow \nu_S} \rangle_{ES}^{ES} \geq 1 - R_{NC}^{ES}.$$

(3.21)

Thus, if the inequality

$$R_{NC}^{ES} < 1$$

(3.22)

is satisfied, from Eq.(3.21) we obtain a non-zero lower bound for the average probability $\langle P_{\nu_\mu \rightarrow \nu_S} \rangle_{ES}^{ES}$.

In the case of transitions of solar $\nu_e$'s into sterile states the initial $^8\text{B}$ $\nu_e$ flux cannot be determined from the experimental data. However, in this case we can obtain from the data a lower bound for this flux. In fact, from Eqs.(3.14), (3.18) and (3.20) we have

$$\Phi \geq \frac{\sum_{\ell=e,\mu,\tau} X_{\nu_{\ell} d}}{X_{\nu_\mu e} R_{ES}^{ES}} = N_{NC}^{NC} \frac{N_{NC}}{X_{\nu d}}.$$

(3.23)
\[ \nu_e \to \nu_S \quad \Delta m^2 (\text{eV}^2) \quad \sin^2 2\theta \quad [P_{\nu_e \to \nu_e}]_{\text{max}} \quad R_{\text{NC}}^{\text{CC}} \]

|            | MSW      | VACUUM OSC. |
|------------|----------|-------------|
| \( \nu_e \to \nu_S \) | 4.5 \times 10^{-6} | 6.3 \times 10^{-11} |
| \( \Delta m^2 \)       | 7.0 \times 10^{-3} | 0.85         |
| \( \sin^2 2\theta \)   | 0.57     | 0.56        |
| \( R_{\text{NC}}^{\text{CC}} \) | 0.71     | 0.50        |

Table 3: Results of the calculation of \([P_{\nu_e \to \nu_e}]_{\text{max}}\) and \(R_{\text{NC}}^{\text{CC}}\) in the model with \(\nu_e - \nu_S\) mixing. The values of \(\Delta m^2\) and \(\sin^2 2\theta\) used are also given. These values were obtained from the analysis of the existing experimental data (Ref.[20] for the MSW transitions and Ref.[21] for the vacuum oscillations).

Further, we have

\[
\left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_e \to \nu_\ell} \right\rangle_{\text{NC}} \leq \frac{\left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_e \to \nu_\ell} \right\rangle_{\text{NC}}}{\left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_e \to \nu_\ell} \right\rangle_{\text{ES}}} = (R_{\text{NC}}^{\text{ES}})^{-1} .
\] (3.24)

From Eq.(3.24) it follows that if the inequality

\[
R_{\text{NC}}^{\text{ES}} > 1
\] (3.25)

is satisfied, then for the average probability \(\langle P_{\nu_e \to \nu_S}\rangle_{\text{NC}}\) and for the total flux \(\Phi\) we have the following lower bounds:

\[
\langle P_{\nu_e \to \nu_S}\rangle_{\text{NC}} \geq 1 - (R_{\text{NC}}^{\text{ES}})^{-1} ,
\] (3.26)

\[
\Phi \geq \frac{X_{\nu d}}{N_{\text{NC}}} \left(\frac{R_{\text{NC}}^{\text{ES}}}{\sum_{\nu_S} X_{\nu}\nu}\right)^{-1} .
\] (3.27)

It is instructive to calculate the measurable ratio \(R_{\text{NC}}^{\text{ES}}\) (and other ratios that we will consider later) in some model. We considered the simplest model with \(\nu_e - \nu_S\) mixing. The parameters of the model \(\Delta m^2\) and \(\sin^2 2\theta\), which are given in Table 3, were obtained from the fit of the existing solar neutrino data under the assumption of MSW transitions or vacuum oscillations (assuming the SSM neutrino fluxes). The ratio \(R_{\text{NC}}^{\text{ES}}\) as a function of the recoil electron threshold energy is depicted in Fig.3. From Fig.3 it can be seen that in the case of MSW transitions the ratio \(R_{\text{NC}}^{\text{ES}}\) is bigger than one for all the considered threshold energies. In both the MSW and vacuum oscillation cases the ratio \(R_{\text{NC}}^{\text{ES}}\) increases...
with $T_{th}$. From Fig.3 it follows that a high threshold energy is preferable for revealing the existence of sterile neutrinos.

A detailed investigation of the spectrum of the recoil electrons in the process $\nu e \rightarrow \nu e$ will be carried out in the S-K and SNO experiments. We will discuss now what additional model independent tests of the existence of sterile neutrinos can be performed when the recoil electron spectrum will be available. The spectrum of recoil electrons is given by

$$n^{ES}(T) = \int_{E_m(T)} \frac{d\sigma_{\nu_{\ell}e}}{dT}(E, T) P_{\nu_{\ell} \rightarrow \nu_{\ell}}(E) X(E) \ dE \Phi + \int_{E_m(T)} \frac{d\sigma_{\nu\ell e}}{dT}(E, T) \sum_{\ell=e, \mu, \tau} P_{\nu_{\ell} \rightarrow \nu_{\ell}}(E) X(E) \ dE \Phi . \quad (3.28)$$

Here $T$ is the kinetic energy of the recoil electrons, $E_m(T) = T \left(1 + \sqrt{1 + 2 m_e/T}\right)/2$ and $d\sigma_{\nu_{\ell}e}/dT(E, T)$ is the differential cross section of the process $\nu_{\ell}e \rightarrow \nu_{\ell}e \ (\ell = e, \mu)$. With the help of Eq.(3.28) we get the following relation:

$$\left\langle \sum_{\ell=e, \mu, \tau} P_{\nu_{\ell} \rightarrow \nu_{\ell}}(T) \right\rangle_{ES; T} = \frac{\Sigma^{ES}(T)}{\Phi X_{\nu_{\ell}e}(T)} . \quad (3.29)$$
Here

\[ X_{\nu e}(T) \equiv \int_{E_m(T)} d\sigma_{\nu e}(E, T) X(E) \, dE \]  

(3.30)

is a known function, which is plotted in Fig. 4. Other quantities in Eq. (3.29) are determined as follows:

\[ \Sigma^{ES}(T) \equiv n^{ES}(T) - \int_{E_m(T)} \left[ \frac{d\sigma_{\nu e}}{dT}(E, T) - \frac{d\sigma_{\nu e}}{dT}(E, T) \right] \phi_{\nu e}(E) \, dE \]  

(3.31)

and

\[ \left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_{\ell} \to \nu_{\ell}} \right\rangle_{ES;T} \equiv \frac{1}{X_{\nu e}(T)} \int_{E_m(T)} \frac{d\sigma_{\nu e}}{dT}(E, T) X(E) \sum_{\ell=e,\mu,\tau} P_{\nu_{\ell} \to \nu_{\ell}}(E) \, dE \, . \]  

(3.32)

Let us stress that to determine the quantity \( \Sigma^{ES}(T) \) it is necessary to know the recoil electron energy spectrum \( n^{ES}(T) \) as well as the spectrum of \( \nu_e \) on the earth \( \phi_{\nu e}(E) \).

Let us consider now the relation (3.29). If the quantity \( \Sigma^{ES}(T)/X_{\nu e}(T) \) depends on the energy \( T \), it would mean that transitions of solar \( \nu_e \)'s into sterile neutrinos take place (if
there are no such transitions, then \[ \langle \sum_{\ell=e,\mu,\tau} P_{\nu_\ell \to \nu_\ell} \rangle_{ES;T} = 1 \] and \[ \Sigma^{ES}(T)/X_{\nu_\ell e}(T) = \text{const}. \]

In order to derive a lower bound for the average value of the probability of transition of \( \nu_e \)'s into sterile states we will use the following inequality:

\[
\left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_\ell \to \nu_\ell} \right\rangle_{ES;T} \leq \left[ \left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_\ell \to \nu_\ell} \right\rangle_{ES;T} \right]_{\text{max}} = R^{ES}(T). \tag{3.33}
\]

Here
\[
R^{ES}(T) \equiv \frac{\Sigma^{ES}(T)/X_{\nu_\ell e}(T)}{\left[ \Sigma^{ES}(T)/X_{\nu_\ell e}(T) \right]_{\text{max}}}.
\tag{3.34}
\]

is a measurable quantity and the subscript max indicates the maximum value in the explored energy range. From Eq.(3.33) it follows that

\[
\langle P_{\nu_\ell \to \nu_S} \rangle_{ES;T} \geq 1 - R^{ES}(T). \tag{3.35}
\]

For the total initial flux of \( ^8B \) neutrinos, from Eqs.(3.29) and (3.33) we obtain the following inequality:

\[
\Phi \geq \left[ \frac{\Sigma^{ES}(T)}{X_{\nu_\ell e}(T)} \right]_{\text{max}}. \tag{3.36}
\]

The results of the calculation of the ratio \( R^{ES}(T) \) in the model with \( \nu_e - \nu_S \) mixing is presented in Fig.5. From this figure it is seen that a detailed investigation of the ratio \( R^{ES}(T) \) near the threshold could be a promising way to search for \( \nu_e \to \nu_S \) transitions.

We will obtain now other inequalities the test of which could allow to obtain model independent informations about the existence of sterile neutrinos. We have

\[
\left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_\ell \to \nu_\ell} \right\rangle_{ES;T} \leq \left[ \left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_\ell \to \nu_\ell} \right\rangle_{ES;T} \right]_a = R^{ES}_a(T). \tag{3.37}
\]

Here \( a = \text{NC, ES} \) and the ratios

\[
R^{ES}_{\text{NC}}(T) \equiv \frac{\Sigma^{ES}(T) X_{\nu d}}{X_{\nu_\ell e}(T) N^{\text{NC}}}, \tag{3.38}
\]

\[
R^{ES}_{\text{ES}}(T) = \frac{\Sigma^{ES}(T) X_{\nu_\ell e}}{X_{\nu_\ell e}(T) \Sigma^{ES}}. \tag{3.39}
\]
are measurable quantities. 

If the inequality

$$R_{NC}(T) < 1$$  \hspace{1cm} (3.40)$$

takes place in some region of $T$, it would mean that there are transitions of solar $\nu_e$'s into sterile states. From Eqs.(3.29) and (3.37) we find the following lower bounds for $\langle P_{\nu_e \rightarrow \nu_S} \rangle_{ES:T}$ and for the initial total $\nu_e$ flux:

$$\langle P_{\nu_e \rightarrow \nu_S} \rangle_{ES:T} \geq 1 - R_{NC}^{ES}(T),$$  \hspace{1cm} (3.41)

$$\Phi \geq \frac{N^{NC}}{X_{\nu_d}}.$$  \hspace{1cm} (3.42)

Analogously, if in some region of $(T)$ the inequality

$$R_{ES}(T) < 1$$  \hspace{1cm} (3.43)$$

is satisfied, we have the following lower bounds for $\langle P_{\nu_e \rightarrow \nu_S} \rangle_{ES:T}$ and $\Phi$:

$$\langle P_{\nu_e \rightarrow \nu_S} \rangle_{ES:T} \geq 1 - R_{ES}^{ES}(T),$$  \hspace{1cm} (3.44)

$$\Phi \geq \frac{\Sigma^{ES}}{X_{\nu_{\mu,e}}}.$$  \hspace{1cm} (3.45)

Further, we have

$$\langle \sum_{\ell = e, \mu, \tau} P_{\nu_e \rightarrow \nu_{\ell}} \rangle_a \leq \frac{\langle \sum_{\ell = e, \mu, \tau} P_{\nu_e \rightarrow \nu_{\ell}} \rangle}{\langle \sum_{\ell = e, \mu, \tau} P_{\nu_e \rightarrow \nu_{\ell}} \rangle_{ES:T}} = \frac{1}{R_a^{ES}(T)},$$  \hspace{1cm} (3.46)$$

where $a = NC, ES$. Thus, sterile neutrinos exist if in some region of the variable $T$ the inequality

$$R_a^{ES}(T) > 1$$  \hspace{1cm} (3.47)$$

is satisfied. For the averaged probability of the transition of $\nu_e$ into $\nu_S$ and for the total flux we have

$$\langle P_{\nu_e \rightarrow \nu_S} \rangle_a \geq 1 - (R_a^{ES}(T))^{-1},$$  \hspace{1cm} (3.48)

$$\Phi \geq \left( \frac{\Sigma^{ES}(T)}{X_{\nu_{\mu,e}}(T)} \right)_{\text{max}}.$$  \hspace{1cm} (3.49)$$

In Figs.6 and 7 we plotted the ratios $R_{NC}^{ES}(T)$ and $R_{ES}^{ES}(T)$ calculated in the model with $\nu_e - \nu_S$ mixing (the parameters of the model are given in Table 3). It can be seen from these figures that in the model under consideration the ratios $R_{NC}^{ES}(T)$ and $R_{ES}^{ES}(T)$ are larger
than one in a large part of the kinematical region. The deviation of \( R_{ES}^{NC}(T) \) and \( R_{ES}^{ES}(T) \) from one is larger in the case of vacuum oscillations than in the MSW case.

We have derived several different inequalities whose test could allow to reveal the presence of sterile neutrinos in the solar neutrino flux on the earth in a model independent way. Additional inequalities of this type can be obtained from the knowledge of the spectrum of solar neutrinos on the earth \( \phi_{\nu_e}(E) \). The relation

\[
P_{\nu_e \rightarrow \nu_e}(E) = \frac{\phi_{\nu_e}(E)}{\Phi X(E)}
\]

and the relations (3.8), (3.14) and (3.29) have a similar structure. We have

\[
\left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_\ell \rightarrow \nu_e} \right\rangle_a \leq \frac{\left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_\ell \rightarrow \nu_e} \right\rangle_a}{[P_{\nu_e \rightarrow \nu_e}]_{\text{max}}} = R_{CC}^a,
\]

where \( a = NC, ES \) and the ratios

\[
R_{CC}^{NC} \equiv \frac{N_{NC}/X_{\nu d}}{[\phi_{\nu_e}/X]_{\text{max}}},
\]

\[
R_{CC}^{ES} \equiv \frac{\Sigma_{ES}/X_{\nu_\ell e}}{[\phi_{\nu_e}/X]_{\text{max}}}.
\]
Figure 8: Results of the calculation of the ratio $R_{\text{ES}}^{\text{NC}}$ (see Eq.(3.53)) in the model with $\nu_e-\nu_S$ mixing ($T_{\text{th}}$ is the kinetic energy threshold of the recoil electrons in the ES process). The curves correspond to MSW transitions and vacuum oscillations.

are measurable quantities. The quantity $[\phi_{\nu_e}/X]_{\text{max}}$ in Eq.(3.52) and (3.53) is the maximal value of the function $\phi_{\nu_e}(E)/X(E)$ in the explored energy range. From Eq.(3.51), for the averaged values of the probability $P_{\nu_e\rightarrow\nu_S}$ and for the total flux $\Phi$ we have

$$\langle P_{\nu_e\rightarrow\nu_S}\rangle_a \geq 1 - R_{\text{CC}}^a \quad (a = \text{NC, ES}) , \quad (3.54)$$

$$\Phi \geq \left[ \frac{\phi_{\nu_e}}{X} \right]_{\text{max}} . \quad (3.55)$$

The results of our calculations of the ratio $R_{\text{ES}}^{\text{NC}}$ in the model with $\nu_e-\nu_S$ mixing are given in Table 3, whereas the results of our calculations of the ratio $R_{\text{ES}}^{\text{NC}}$ as a function of the kinetic energy threshold of the recoil electrons in the ES process, $T_{\text{th}}$, are depicted in Fig. 8 (the kinetic energy threshold of the electrons in the CC process is assumed to be 4.5 MeV). It can be seen from Fig. 8 that in the model under consideration the ratio $R_{\text{CC}}^{\text{ES}}$ depends strongly on $T_{\text{th}}$ (especially in the case of vacuum oscillations). It is preferable to search for $\nu_e \rightarrow \nu_S$ transitions at relatively small thresholds.

Finally, from Eqs.(3.29) and (3.50) it follows

$$\left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_e\rightarrow\nu_\ell} \right\rangle_{\text{ES};T} \leq R_{\text{CC}}^{\text{ES}}(T) , \quad (3.56)$$
where the ratio
\[ R_{ES}^{CC}(T) \equiv \frac{\sum_{\nu}^{ES}(T)}{X_{\nu e}(T) \left[ \phi_{\nu e}/X \right]_{\text{max}}} \] (3.57)
is a measurable quantity. From Eqs.(3.29) and (3.56) we have
\[ \langle P_{\nu_e \rightarrow \nu_S} \rangle_{ES;T} \geq 1 - R_{ES}^{CC}(T), \] (3.58)
\[ \Phi \geq \left[ \phi_{\nu e}/X \right]_{\text{max}}. \] (3.59)
The results of the calculations of the ratio \( R_{ES}^{CC}(T) \) in the model with \( \nu_e-\nu_S \) mixing are presented in Fig.9. In this model there are large deviations from one of the ratio \( R_{ES}^{CC}(T) \) for values of \( T \) close to the threshold (especially in the case of vacuum oscillations).

In conclusion, we would like to stress that all the inequalities discussed here are based on the conservation law (3.9). The concrete mechanism for the transition of \( \nu_e \)'s into sterile states is not important for the tests proposed here. Thus, there could be not only \( \nu_e \rightarrow \nu_S \) transitions due to Dirac-Majorana neutrino mixing, but also \( \nu_e \rightarrow \nu_S \) transitions due to neutrino Dirac magnetic moments [33]. In the latter case the measurable ratios \( R_{NC}^{ES}, R^{ES}(T), \ldots \) will depend periodically on time [12, 34].

4 The case of absence of transitions of solar \( \nu_e \)'s into sterile states

If the test of all the inequalities obtained in the previous section will not reveal the presence of sterile neutrinos in the solar neutrino flux on the earth, it will be natural to consider the possibility of a model independent treatment of solar neutrino data under the assumption that there are no transitions of solar \( \nu_e \)'s into sterile states\(^5\). In this case
\[ \left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_e \rightarrow \nu_{\ell}} \right\rangle_a = 1 \quad (a = \text{NC, ES}), \] (4.1)
\[ \left\langle \sum_{\ell=e,\mu,\tau} P_{\nu_e \rightarrow \nu_{\ell}} \right\rangle_{ES;T} = 1, \] (4.2)
and the SNO and S-K data will allow to determine the initial flux of \( ^8 \text{B} \) \( \nu_e \)'s with three methods:

1. By means of the measurement of the NC event rate \( N^{NC} \). In fact, from Eqs.(3.8) and (4.1) we have
\[ \Phi = \frac{N^{NC}}{X_{\nu d}}, \] (4.3)
where \( X_{\nu d} \) is given by Eq.(3.7).

\(^5\)Let us stress that all the inequalities considered in the previous section will not reveal the presence of sterile neutrinos if \( P_{\nu_e \rightarrow \nu_S}(E) = \text{const.} \).
2. By means of the measurement of the total number of ES events $N^{\text{ES}}$ and the electron spectrum in the CC process (from which the spectrum of $\nu_e$ on the earth, $\phi_{\nu_e}(E)$, will be determined). From Eqs.(3.14) and (4.1) we have

$$\Phi = \frac{\Sigma^{\text{ES}}}{X_{\nu_e}} ,$$

where $\Sigma^{\text{ES}}$ is given by Eq.(3.13) and $X_{\nu_e}$ by Eq.(3.15).

3. By means of the measurement of the recoil electron spectrum $n^{\text{ES}}(T)$ in the ES process and the electron spectrum in the CC process. In fact, from Eqs.(3.29) and (4.2) we have

$$\Phi = \frac{\Sigma^{\text{ES}}(T)}{X_{\nu_e}(T)} ,$$

where $\Sigma^{\text{ES}}(T)$ is given by Eq.(3.31) and $X_{\nu_e}(T)$ by Eq.(3.30).

All these different methods of determination of the total flux $\Phi$ will give results which must be in agreement with each other (otherwise the ratios $R_{\text{NC}}^{\text{ES}}$, $R^{\text{ES}}(T)$, ... considered in the previous section would be different from one). Let us stress that the proposed methods for the determination of the flux of $^8\text{B}$ neutrinos do not depend on the mechanism of transition of solar $\nu_e$’s into other active states. It is evident that a comparison of the flux $\Phi$ determined directly from the experimental data with the $^8\text{B}$ flux predicted by solar models will be an important test of these models.

The SNO and S-K experiments will allow also to determine in a model independent way the probability of $\nu_e$’s to survive, $P_{\nu_e \rightarrow \nu_e}(E)$. In fact, we have

$$P_{\nu_e \rightarrow \nu_e}(E) = \frac{\phi_{\nu_e}(E)}{\Phi X(E)} ,$$

where $\phi_{\nu_e}(E)$ is the flux of solar $\nu_e$’s on the earth and the total flux $\Phi$ is given by Eqs.(4.3), (4.4) and (4.5). If it will occur that the probability $P_{\nu_e \rightarrow \nu_e}(E)$ is less than one, this will be a model independent proof that $\nu_e$’s transform into other states. A detailed investigation of the dependence on $E$ of the probability $P_{\nu_e \rightarrow \nu_e}(E)$ will allow to distinguish different mechanisms of neutrino transitions (MSW, just-so vacuum oscillations and others) and to determine the corresponding parameters.

The detection of solar neutrinos through the observation of $\nu e \rightarrow \nu e$ scattering will allow to obtain additional informations about the $\nu_e$ survival probability. Let us define the average value of the probability of $\nu_e$ to survive $\langle P_{\nu_e \rightarrow \nu_e}\rangle_{\text{ES}}$ as

$$\langle P_{\nu_e \rightarrow \nu_e}\rangle_{\text{ES}} \equiv \frac{1}{X_{\nu_e} - X_{\mu_e}} \int_{E_{\text{th}}}^{E_{\text{ES}}} \left[ \sigma_{\nu_e}(E) - \sigma_{\mu_e}(E) \right] X(E) P_{\nu_e \rightarrow \nu_e}(E) \, dE ,$$

where

$$X_{\nu_e} \equiv \int_{E_{\text{th}}}^{E_{\text{ES}}} \sigma_{\nu_e}(E) X(E) \, dE$$
\[ \nu_e \rightarrow \nu_\mu(\tau) \quad \Delta m^2 (\text{eV}^2) \quad \sin^2 2\theta \quad \langle P_{\nu_e \rightarrow \nu_e} \rangle_{\text{ES}} \]

|                  | \( \Delta m^2 \) | \( \sin^2 2\theta \) | \( \langle P_{\nu_e \rightarrow \nu_e} \rangle_{\text{ES}} \) |
|------------------|-----------------|-----------------|-----------------|
| SMALL MIX MSW    | \( 6.1 \times 10^{-6} \) | \( 6.5 \times 10^{-3} \) | 0.32            |
| LARGE MIX MSW    | \( 9.4 \times 10^{-6} \) | \( 0.62 \)       | 0.19            |
| VACUUM OSC.      | \( 8.0 \times 10^{-11} \) | \( 0.80 \)       | 0.31            |

Table 4: Results of the calculations of the quantity \( \langle P_{\nu_e \rightarrow \nu_e} \rangle_{\text{ES}} \) in the simplest model with \( \nu_e - \nu_\mu(\tau) \) mixing. The values of \( \Delta m^2 \) and \( \sin^2 2\theta \) used are also given. These values were obtained from the analysis of the existing experimental data (Ref.[20] for the MSW transitions and Ref.[21] for the vacuum oscillations).

and \( X_{\nu_e} \) is given by Eq.(3.15) (for \( E_{\text{th}}^{\text{ES}} = 4.74 \text{ MeV} \), which corresponds to a kinetic energy threshold \( T_{\text{th}} = 4.5 \text{ MeV} \) for the recoil electrons, we have \( X_{\nu_e} = 2.12 \times 10^{-44} \text{ cm}^2 \) and \( X_{\nu_\mu} = 3.23 \times 10^{-45} \text{ cm}^2 \).

The quantity \( \langle P_{\nu_e \rightarrow \nu_e} \rangle_{\text{ES}} \) is connected with the total number of ES events \( N^\text{ES} \) and the total flux \( \Phi \) by the following relation:

\[
\langle P_{\nu_e \rightarrow \nu_e} \rangle_{\text{ES}} = \frac{1}{X_{\nu_e} - X_{\nu_\mu}} \left[ \frac{N^\text{ES}}{\Phi} - X_{\nu_\mu} \right]. \tag{4.9}
\]

The right-hand side of this relation contains only measurable and known quantities (the total flux \( \Phi \) is given by Eqs.(4.3), (4.4) and (4.5)). If the right-hand side of Eq.(4.9) will be found to have a value less than one, it will be a model independent proof that solar \( \nu_e \)'s transform into other active states.

Finally, a measurement of the recoil electron spectrum \( n^\text{ES}(T) \) will allow to determine the average

\[
\langle P_{\nu_e \rightarrow \nu_\mu} \rangle_{\text{ES};T} = \frac{1}{X_{\nu_e}(T) - X_{\nu_\mu}(T)} \int_{E_{\text{m}}(T)} \left[ \frac{d\sigma_{\nu_e}(E, T)}{dT} - \frac{d\sigma_{\nu_\mu}(E, T)}{dT} \right] X(E)P_{\nu_e \rightarrow \nu_\mu}(E) dE, \tag{4.10}
\]

where

\[
X_{\nu_e}(T) \equiv \int_{E_{\text{m}}(T)} \frac{d\sigma_{\nu_e}(E, T)}{dT} X(E) dE \tag{4.11}
\]

and \( X_{\nu_\mu}(T) \) is given by Eq.(3.30). The functions \( X_{\nu_e}(T) \) and \( X_{\nu_\mu}(T) \) are plotted in Fig.4. From Eq.(3.28) we easily obtain

\[
\langle P_{\nu_e \rightarrow \nu_\mu} \rangle_{\text{ES};T} = \frac{1}{X_{\nu_e}(T) - X_{\nu_\mu}(T)} \left[ \frac{n^\text{ES}(T)}{\Phi} - X_{\nu_\mu}(T) \right]. \tag{4.12}
\]
Figure 10: Results of the calculations of the probability of $\nu_e$'s to survive, $P_{\nu_e \rightarrow \nu_e}(E)$, as a function of the neutrino energy $E$, in the model with $\nu_e - \nu_\mu (\tau)$ mixing. The curves correspond to small and large mixing angle MSW transitions and to vacuum oscillations. The values of the parameters $\Delta m^2$ and $\sin^2 2\theta$ used in the calculation are given in Table 4.

Figure 11: Results of the calculation of $\langle P_{\nu_e \rightarrow \nu_e} \rangle_{ES;T}$ (see Eq.(4.10)) in the model with $\nu_e - \nu_\mu (\tau)$ mixing ($T$ is the kinetic energy of the recoil electrons in the ES process). The curves correspond to small and large mixing angle MSW transitions and to vacuum oscillations. The values of the parameters $\Delta m^2$ and $\sin^2 2\theta$ used in the calculation are given in Table 4.

If the right-hand side of Eq.(4.12) will be found to have a dependence on $T$, it will be a model independent proof that there are transitions of solar $\nu_e$'s into other active neutrinos.

We have calculated the survival probability $P_{\nu_e \rightarrow \nu_e}(E)$ and the quantities $\langle P_{\nu_e \rightarrow \nu_e} \rangle_{ES}$ and $\langle P_{\nu_e \rightarrow \nu_e} \rangle_{ES;T}$ in the model with $\nu_e - \nu_\mu (\tau)$ mixing. The values of the parameters $\Delta m^2$ and $\sin^2 2\theta$ and the values of the quantity $\langle P_{\nu_e \rightarrow \nu_e} \rangle_{ES}$ are given in Table 4. The values of the mixing parameters were obtained from a fit of the existing solar neutrino data under the assumption that the neutrino fluxes are given by the SSM. The results of the calculations of $P_{\nu_e \rightarrow \nu_e}(E)$ and $\langle P_{\nu_e \rightarrow \nu_e} \rangle_{ES;T}$ are presented in Figs.10 and 11. These figures illustrate the possibility to distinguish the different mechanisms of neutrino transitions through the investigation of the energy dependence of the probability of $\nu_e$'s to survive, $P_{\nu_e \rightarrow \nu_e}(E)$, and of the averaged probability $\langle P_{\nu_e \rightarrow \nu_e} \rangle_{ES;T}$.

5 Conclusions

We have shown that the existing solar neutrino data allow to exclude in a model independent way large regions of the values of the parameters $\Delta m^2$ and $\sin^2 2\theta$. We have also shown that from future solar neutrino experiments (SNO, Super-Kamiokande, Icarus) it will be
possible: 1) To reveal in a solar model independent way the presence of sterile neutrinos in the flux of solar neutrinos on the earth; 2) To determine directly from the experimental data the flux of initial $^8\text{B} \nu_e$'s; 3) To obtain directly from the data the probability of $\nu_e$ to survive. Thus, the future solar neutrino experiments will allow to check the predictions of the SSM independently from the properties of neutrinos and will allow to investigate neutrino mixing in a model independent way.

References

[1] R. Davis Jr., Talk presented at the 6th International Workshop on Neutrino Telescopes, Venezia, March 1994.

[2] GALLEX Collaboration, Phys. Lett. B 285, 376 (1992); Phys. Lett. B 314, 445 (1993); Phys. Lett. B 327, 377 (1994).

[3] SAGE Collaboration, in Proc. of the 27th Int. Conf. on High Energy Physics, Glasgow, July 1994.

[4] K. S. Hirata et al., Phys. Rev. Lett. 65, 1297 (1990); Phys. Rev. D 44, 2241 (1991); Y. Suzuki, Talk presented at the 6th International Workshop on Neutrino Telescopes, Venezia, March 1994.

[5] J.N. Bahcall and R. Ulrich, Rev. Mod. Phys. 60, 297 (1988); J.N. Bahcall, Neutrino Physics and Astrophysics, Cambridge University Press, 1989; J.N. Bahcall and M.H. Pinsonneault, Rev. Mod. Phys. 64, 885 (1992).

[6] S. Turck-Chièze, S. Cahen, M. Cassé and C. Doom, Astrophys. J. 335, 415 (1988); S. Turck-Chièze and I. Lopes, Astrophys. J. 408, 347 (1993); S. Turck-Chièze et al., Phys. Rep. 230, 57 (1993).

[7] V. Castellani, S. Degl’Innocenti and G. Fiorentini, Astronomy & Astrophysics 271, 601 (1993); S. Degl’Innocenti, INFN-FE-07-93.

[8] I.J. Sakmann, A.I. Boothroyd and W.A. Fowler, Astrophys. J. 360, 727 (1990).

[9] G. Berthomieu et al., Astronomy & Astrophysics 268, 775 (1993).

[10] S.M. Bilenky and C. Giunti, DFTT 32/94 (hep-ph@xxx.lanl.gov/9407379).

[11] S.M. Bilenky and C. Giunti, Phys. Lett. B 311, 179 (1993); Phys. Lett. B 320, 323 (1994); Nucl. Phys. (Proc. Suppl.) 35, 430 (1994).

[12] S.M. Bilenky and C. Giunti, Astropart. Phys. 2, 353 (1994).

[13] The Sudbury Neutrino Observatory Collaboration, Phys. Lett. B 194 (1987) 321; H.H. Chen, Nucl. Instr. Meth. A 264 (1988) 48; G.T. Ewan, Proceedings of the 4th International Workshop on Neutrino Telescopes, Venezia, March 1992.
[14] A. Suzuki, *Proc. of the Workshop on Elementary Particle Picture of the Universe*, KEK, Japan, 1987; Y. Totsuka, ICRR-report-227-90-20 (1990); C.B. Bratton et al., *Proposal to Participate in the Super-Kamiokande Project*, Dec. 1992.

[15] ICARUS Collaboration, ICARUS I: An Optimized Real Time Detector of Solar Neutrinos, LNF-89/005(R) (1989).

[16] S.M. Bilenky and B. Pontecorvo, Phys. Rep. 41, 225 (1978); S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. 59, 671 (1987).

[17] C.W. Kim and A. Pevsner, *Neutrinos in Physics and Astrophysics*, Contemporary Concepts in Physics, Vol. 8, (Harwood Academic Press, Chur, Switzerland, 1993).

[18] GALLEX Collaboration, Phys. Lett. B 285, 390 (1992).

[19] X. Shi, D.N. Schramm and J.N. Bahcall, Phys. Rev. Lett. 69, 717 (1992).

[20] S.A. Bludman, N. Hata, D.C. Kennedy and P.G. Langacker, Phys. Rev. D 47, 2220 (1993); N. Hata and P.G. Langacker, Phys. Rev. D 48, 2937 (1993); Phys. Rev. D 50, 632 (1994).

[21] P.I. Krastev and S.T. Petcov, Phys. Lett. B 299, 99 (1993); Phys. Rev. Lett. 72, 1960 (1994); SISSA 41/94/EP (hep-ph@xxx.lanl.gov/9408234).

[22] L.M. Krauss, E. Gates and M. White, Phys. Lett. B 299, 94 (1993); Fermilab-Pub-94/176-A (hep-ph@xxx.lanl.gov/9406396).

[23] G.L. Fogli and E. Lisi, Astropart. Phys. 2, 91 (1994).

[24] G. Fiorentini et al., Phys. Rev. D 49, 6298 (1994).

[25] L. Wolfenstein, Phys. Rev. D 17, 2369 (1978); Phys. Rev. D 20, 2634 (1979); S.P. Mikheyev and A.Yu. Smirnov, Yad. Fiz. 42, 1441 (1985) [Sov. J. Nucl. Phys. 42, 913 (1985)]; Il Nuovo Cimento C 9, 17 (1986).

[26] J.N. Bahcall, Phys. Rev. D 44, 1644 (1991).

[27] S.T. Petcov, Phys. Lett. B 200, 373 (1988); P.I. Krastev and S.T. Petcov, Phys. Lett. B 207, 64 (1988); T.K. Kuo and J. Pantaleone, Phys. Rev. D 39, 1930 (1989); Rev. Mod. Phys. 61, 937 (1989).

[28] Review of Particle Properties, Phys. Rev. D 45, Part II (1992).

[29] T. Motobayashi et al., Phys. Rev. Lett. 73, 2680 (1994).

[30] J.N. Bahcall and B.R. Holstein, Phys. Rev. C 33, 2121 (1986).

[31] K. Kubodera and S. Nozawa, Int. J. Mod. Phys. E 3, 101 (1994).
[32] Z. Berezhiani, M. Moretti and A. Rossi, Z. Phys. C 58, 423 (1993); Z. Berezhiani and A. Rossi, Nucl. Phys. B (Proc. Suppl.) 35, 469 (1994).

[33] A. Cisneros, Astrophys. Space. Sci. 10 (1970) 87; M.B. Voloshin, M.I. Vysotsky and L.B. Okun, Zh. Eksp. Teor. Fiz. 91 (1986) 754 [Sov. Phys. JETP 64 (1986) 446]; C.S. Lim and W.J. Marciano, Phys. Rev. D 37 (1988) 1368; E.Kh. Akhmedov, Phys. Lett. B 213 (1988) 64.

[34] S.M. Bilenky and C. Giunti, Proc. of the 6th International Workshop on Neutrino Telescopes, Venezia, March 1994.