Spectra and decays of hybrid charmonia

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QCD string model is employed to calculate the masses and spin splittings of lowest charmonium hybrid states with a magnetic gluon. Relative decay rates into various $S$– and $P$–wave $D$–meson pairs are calculated for these hybrids.

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I. INTRODUCTION

There exist strong arguments in favour of hybrid assignment for the recently observed $Y(4260)$ state [1]. Indeed, this $Y$–meson is definitely a vector one, as it is seen in the initial state radiation process $e^+e^- \rightarrow \gamma\pi^+\pi^-J/\psi$, but its $e^+e^-$ width is too small for a conventional $c\bar{c}$ vector, and there is no visible decay into $D\bar{D}$ pairs, in spite of the large phase space available. It is the latter feature that has prompted the hybrid interpretation of the $Y(4260)$ [2], as the selection rule is established — see, for example, [3, 4, 5, 6, 7] — which forbids the decay of the vector hybrid into a $D^{(*)}\bar{D}^{(*)}$ final state.

Competing models for the $Y(4260)$ exist. One is the $[cs] - [\bar{c}\bar{s}]$ diquark–antidiquark model [8]. On the other hand, the $Y(4260)$ is not far from the $D\bar{D}_1$ threshold, where $D_1$ is a $P$–wave $1^+$ charmed meson, so the $Y(4260)$ state could be associated with the opening of a new $S$–wave $D\bar{D}_1$ threshold [9]. In this regard it is important to assess the consequences of the hybrid assignment for the $Y$.

Hybrids can be considered as bound states of a quark–antiquark pair and a gluon with quantum numbers

$$P = (-1)^{l_{q\bar{q}} + j}, \quad C = (-1)^{l_{q\bar{q}} + s_{q\bar{q}} + 1},$$

for the magnetic gluon ($l_g = j$), and

$$P = (-1)^{l_{q\bar{q}} + j + 1}, \quad C = (-1)^{l_{q\bar{q}} + s_{q\bar{q}} + 1},$$

for the electric gluon ($l_g = j \pm 1$), where $l_g$ is the relative angular momentum between the $q\bar{q}$ pair and the gluon, $j$ is the total angular momentum of the gluon, $l_{q\bar{q}}$ is the orbital
momentum in the quark–antiquark subsystem, and \( s_{q\bar{q}} \) is the spin of the quark–antiquark pair. For a magnetic gluon, the lowest states correspond to \( l_{q\bar{q}} = 0 \), with the \( 1^{--} \) hybrid being a spin–singlet state with respect to the quark spin, while the \( J^{++}, J = 0, 1, 2 \), hybrids being spin triplets. These four states are expected to be degenerate in the heavy–quark limit, with the degeneracy removed by spin–dependent quark–gluon interactions. In other words, if \( Y(4260) \) is indeed a \( c\bar{c}g \) vector hybrid, three \( J^{++} \) hybrid charmonia, including the exotic \( 1^{++} \) one, should reside not very far from it. Decays of all these states obey the above–mentioned selection rule.

It is the latter feature, which makes these states potentially interesting. Indeed, the same quantum numbers can be also achieved with the electric gluon, \( 1^{--} \) hybrid being a spin–triplet state with respect to the quark spin, and \( J^{++} \) hybrids being spin–singlets. However, hybrids with the electric gluon couple too strongly to two \( S \)–wave final–state mesons \( (D^{(*)}\bar{D}^{(*)}) \) and, as estimated in Ref. [4], do not exist as resonances. On the contrary, for hybrids with the magnetic gluon, the \( D^{(*)}\bar{D}^{(*)} \) modes are forbidden, and the lowest possible open–charm modes are the ones with an \( S \)–wave \( D^{(*)} \)–meson and a \( P \)–wave \( D_J \)–meson, with the thresholds being close to the masses of such hybrids. Due to a limited phase space, one expects a considerable suppression of the corresponding decay width, so that hybrids with the magnetic gluon could manifest themselves as resonant states, and, as such, could be of immediate relevance to the charmonium spectroscopy issues.

In the present paper we calculate spin splittings in the \( c\bar{c}g \) system with a magnetic gluon in the framework of the QCD string model based on the Field Correlator Method (FCM) for QCD (for a review of the FCM see Ref. [10]). In this method, confining dynamics is encoded in gluonic field correlators which are responsible for area law asymptotic for the Wilson loop. Starting from the Feynman–Schwinger representation for the quark and gluon propagators in the external field, one can extract hadronic Green’s functions and calculate the spectra. The QCD string model corresponds to the limit of a small (vanishing) gluonic correlation length — the so-called string limit of QCD. Then the effective string–type Lagrangian of a colourless object (meson, baryon, hybrid, and so on) can be derived. The QCD string model was successfully applied to calculate spectra and other properties of \( q\bar{q} \) mesons, see, for example, Refs. [11, 12]. To account for hybrid excitations one populates the QCD string with constituent perturbative gluons [13]. This approach was used before in order to investigate the properties of hybrids [14], and the form of the static interquark potentials in
hybrids was studied in detail and compared with the lattice simulations in Ref. [15].

II. THE MODEL

In the framework of the FCM, hybrid is viewed as a gluon with two fundamental strings attached, with the quark and the antiquark at the ends (see Fig. 1). Thus the starting point of our analysis is the effective Lagrangian of such a system:

\[ L - \frac{m_q^2}{2\mu_q} - \frac{m_{\bar{q}}^2}{2\mu_{\bar{q}}} - \frac{\mu_q + \mu_{\bar{q}} + \mu_g}{2} + \frac{\mu_q r_q^2 + \mu_{\bar{q}} r_{\bar{q}}^2 + \mu_g r_g^2}{2} - \sigma |\vec{r}_q - \vec{r}_g| \int_0^1 d\beta_1 \sqrt{1 - l_1^2} - \sigma |\vec{r}_{\bar{q}} - \vec{r}_g| \int_0^1 d\beta_2 \sqrt{1 - l_2^2}, \]

(3)

where \( \vec{r}_q, \vec{r}_{\bar{q}}, \) and \( \vec{r}_g \) are the quark, the antiquark, and the gluon coordinates, \( m_q \) and \( m_{\bar{q}} \) are the quark and the antiquark current masses (the gluon is massless), dots stand for time derivatives, and the minimal surface is approximated by the straight–line string. The Lagrangian (3) is written in the einbein field form [16]; the einbein fields (or simply einbeins) \( \mu_q, \mu_{\bar{q}}, \) and \( \mu_g \) are introduced to deal with relativistic kinematics. No time derivatives of the einbeins enter the Lagrangian, and the corresponding equations of motion yield second–class constraints (see Ref. [17] for the details of the constrained systems formalism and for the corresponding terminology):

\[ \frac{\partial L}{\partial \mu_q} = \frac{\partial L}{\partial \mu_{\bar{q}}} = \frac{\partial L}{\partial \mu_g} = 0. \]

(4)

In principle, one can eliminate einbeins using Eq. (4). However, only the einbein form of the Lagrangian provides a meaningful dynamics for a massless gluon. Application of the einbein
field formalism to the QCD string was suggested in Ref. [18], and further developments can be found in Ref. [11].

Starting from the Lagrangian (3) one can arrive at the Hamiltonian of the $q\bar{q}g$ system. The general procedure outlined in Refs. [11, 18] is rather complicated due to the presence of square roots in (3). However, if one is interested in the low–lying part of the spectrum, the potential–type regime can be considered. To this end notice that the angular velocities $l_{1,2}$ in the Lagrangian (3) describe the contribution of the proper inertia of the rotating string. For the lowest approximation it is sufficient to retain only the first terms (linear confinement) in the expansion of the string terms in powers of $l_{1,2}^2$. Corrections to this potential regime coming from the further expansion of the string terms are a footprint of the underlying string dynamics. The leading correction of this type, of order $l_{1,2}^2$, is known as the string correction [18] — it will be taken into account later.

Then the zero–order Hamiltonian takes the form:

$$H_0 = \mu_q + \mu_{\bar{q}} + \mu_g + \frac{m^2 + p_{\bar{q}}^2}{2\mu_q} + \frac{m^2 + p_q^2}{2\mu_{\bar{q}}} + \frac{p_g^2}{2\mu_g} + \sigma|\vec{r}_q - \vec{r}_g| + \sigma|\vec{r}_{\bar{q}} - \vec{r}_g| + V_{\text{Coul}},$$

(5)

where the long–range confining force is augmented by the short–range Coulomb potential,

$$V_{\text{Coul}} = -\frac{3\alpha_s}{2|\vec{r}_q - \vec{r}_g|} - \frac{3\alpha_s}{2|\vec{r}_{\bar{q}} - \vec{r}_g|} + \frac{\alpha_s}{6|\vec{r}_q - \vec{r}_{\bar{q}}|}.$$  

(6)

The coefficients in Eq. (6) correspond to the colour content of the $q\bar{q}g$ system [19].

The einbein fields are to be found from the constraint conditions [16]:

$$\frac{\partial H_0}{\partial \mu_q} = \frac{\partial H_0}{\partial \mu_{\bar{q}}} = \frac{\partial H_0}{\partial \mu_g} = 0.$$  

(7)

Thus, to quantise the system, one should find the einbeins from Eq. (7) and substitute them back to the Hamiltonian (5). In such a way, einbeins would become entangled functions of coordinates and momenta. To avoid these complications, an approximate einbein field method is used in the QCD string model calculations: einbeins are treated as $c$–number variational parameters. The eigenvalues of the spinless Hamiltonian (5) are found as functions of $\mu_q$, $\mu_{\bar{q}}$, and $\mu_g$ and, finally, minimised with respect to the einbeins. With such simplifying assumptions the spinless Hamiltonian $H_0$ takes an apparently nonrelativistic form, with einbein fields playing the role of the constituent masses of the quarks and the gluon. These quantities, however, are not introduced as model parameters, but are calculated in a relativistic formalism. Indeed, the procedure of taking extrema in the einbeins is nothing
but the summation of the entire series of relativistic corrections to the leading would-be nonrelativistic eigenenergies.

The einbein field method allows one to estimate corrections to the leading potential regime \(^{(5)}\). First, as was discussed before, one takes into account the contribution of the string inertia, expanding the square roots in the expression \(^{(3)}\) to the first order in \(l_{1,2}^2\) — this gives the string correction \(V_{\text{str}}\). Second, employing the Feynman–Schwinger representation for the Green’s functions of spinning quarks and gluons one can extract the spin–dependent part of interaction, as described in Ref. \(^{[20]}\), hereafter denoted as \(V_{\text{SD}}\). Finally, the FCM method allows one to calculate the nonperturbative selfenergy of the quarks which, as was shown in Ref. \(^{[21]}\), provides an overall shift of the hadron mass, as required by phenomenology. We use the notation \(C\) for this contribution. Thus the full form of the hybrid Hamiltonian reads:

\[
H = H_0 + V_{\text{str}} + V_{\text{SD}} + C. \tag{8}
\]

To specify extra terms in Eq. \(^{(8)}\) let us first introduce various angular momentum operators:

\[
\vec{L}_{1g} = [\vec{r}_{31}\vec{p}_{g}], \quad \vec{L}_{2g} = -[\vec{r}_{23}\vec{p}_{g}], \quad \vec{L}_{1q} = -[\vec{r}_{31}\vec{p}_{q}], \quad \vec{L}_{2\bar{q}} = [\vec{r}_{23}\vec{p}_{\bar{q}}], \quad \vec{L}_{q} = [\vec{r}_{12}\vec{p}_{q}], \quad \vec{L}_{\bar{q}} = -[\vec{r}_{12}\vec{p}_{\bar{q}}], \tag{9}
\]

where

\[
\vec{r}_{12} = \vec{r}_{q} - \vec{r}_{\bar{q}}, \quad \vec{r}_{31} = \vec{r}_{g} - \vec{r}_{q}, \quad \vec{r}_{23} = \vec{r}_{q} - \vec{r}_{g}. \tag{10}
\]

In terms of these coordinates and angular momenta the string correction \(V_{\text{str}}\) is trivially calculated from Eq. \(^{(3)}\) and takes the form (due to the symmetry of the problem we set \(\mu_q = \mu_{\bar{q}} = \mu\)):

\[
V_{\text{str}} = -\frac{\sigma}{6r_{13}} \left( \frac{L_{1g}^2}{\mu_g^2} - \frac{\vec{L}_{1g}\vec{L}_{1q}}{\mu_{\mu_g}^2} + \frac{L_{1q}^2}{\mu^2} \right) + \left( 1 \to 2 \right) + \left( q \to \bar{q} \right). \tag{11}
\]

The derivation of the spin–dependent potential can be found in Ref. \(^{[20]}\), with the result:

\[
V_{\text{SD}} = V_{LS}^{(q\bar{q})} + V_{LS}^{(q\bar{q})} + V_{SS} + V_{ST}^{(q\bar{q})} + V_{ST}^{(g)}, \tag{12}
\]

where the superscript LS stands for the spin–orbit interaction, SS — for the hyperfine interaction, ST — for the spin–tensor, and

\[
V_{LS}^{(q\bar{q})} = -\frac{\alpha_s}{4\mu_{12}^2r_{12}^5} \left( \vec{L}_q\vec{s}_{\bar{q}} - \vec{L}_{\bar{q}}\vec{s}_q \right), \tag{13}
\]
Formally, these spin–dependent terms coincide with the well-known Eichten–Feinberg–Gromes ones [22]. Notice, however, that the Eichten–Feinberg–Gromes spin-dependent potential comes out from an expansion of the interaction in the inverse powers of quark masses, whereas the potential $V_{SD}$ derived above contains effective constituent masses $\mu$ and $\mu_g$ in the denominators instead of current masses. Due to confinement, these constituent-like masses are always large, of order of the confinement scale ($\approx \sqrt{\sigma} \approx 400$ MeV) or larger, even for massless particles. So the result for $V_{SD}$ is applicable to the case of light quark flavours, as well as to the case of massless gluons. The only approximation made in order to derive the spin-dependent potentials (13)-(17) is the Gaussian approximation for field correlators — see Ref. [23] for the discussion.

Finally, the constant $C$ is

$$C = -\frac{3\sigma}{\mu\pi} \eta(m, T_g),$$

where $\eta(m, T_g)$ is a universal function of the quark mass $m$ and the gluonic correlation length $T_g$ — see Ref. [21] for the explicit form of this function and for further details of the formalism (in Ref. [21] the gluonic correlation length is denoted as $\delta$).

### III. THE SPECTRUM OF HYBRIDS

The form (15) allows one to separate the centre-of-mass motion in a standard way, introducing the Jacobi coordinates as

$$\vec{r} = \vec{r}_q - \vec{r}_q', \quad \vec{b} = \vec{r}_g - \frac{\mu_q \vec{r}_q' + \mu_g \vec{r}_q}{\mu_q + \mu_g} = \vec{r}_g - \frac{\vec{r}_q' + \vec{r}_g}{2}. \quad (19)$$

In terms of these Jacobi variables the Hamiltonian $H_0$ in the centre-of-mass frame can be written as

$$H_0 = \frac{m^2}{\mu} + \mu + \frac{\mu_g}{2} + \frac{p^2}{2\mu_{12}} + \frac{Q^2}{2\mu_{12,3}} + \sigma r_{31} + \sigma r_{23} + V_{Coul},$$

(20)
where
\[ \mu_{12} = \frac{\mu}{2}, \quad \mu_{12,3} = \frac{2\mu\mu_g}{M}, \quad M = 2\mu + \mu_g, \]
and \( \vec{p} \) and \( \vec{Q} \) are the momenta conjugated to the Jacobi coordinates \( \vec{r} \) and \( \vec{\rho} \), respectively.

First, we note that the relative angular momenta \( l_{q\bar{q}} \) and \( l_{q} \) are not conserved in the three–body \( q\bar{q}g \) system, though the requirement of a given \( J^P C \) imposes restrictions on their possible values. On the other hand, the zero–order Hamiltonian \((20)\) conserves the total quark spin \( s_{q\bar{q}} \). In accordance with Eqs. \((1)\) and \((2)\), states with magnetic and electric gluons have different total quark spin, so they are not mixed in the leading order. Thus one can employ the well–known hyperspherical formalism to calculate the zero–order spectrum and w.f..

As we are interested in hybrids with a magnetic gluon, we use trial w.f.’s of the form
\[
|1^-\rangle_m = \Phi(r, \rho)S_0(q\bar{q}) \sum_{\nu_1\nu_2} C^l_{1\nu_11\nu_2} \rho Y_{1\nu_1}(\rho) S_{1\nu_2}(g), \tag{22}
\]
for the vector hybrid, and
\[
|J^+\rangle_m = \Phi(r, \rho) \sum_{\mu_1\mu_2} C^J_{1\mu_11\mu_2} S_{1\mu_1}(q\bar{q}) \sum_{\nu_1\nu_2} C^l_{1\nu_11\nu_2} \rho Y_{1\nu_1}(\rho) S_{1\nu_2}(g), \tag{23}
\]
for its siblings. Here \( S_{1\nu}(g) \) is the spin w.f. of the gluon, \( S_0(q\bar{q}) \) and \( S_{1\nu}(q\bar{q}) \) are the singlet and triplet spin w.f.’s of the \( q\bar{q} \) pair. The “radial” w.f. \( \Phi(r, \rho) \) depends on its arguments in the form of the hyperspherical radius \( R \),
\[
R^2 = \frac{\mu_{12}}{M} r^2 + \frac{\mu_{12,3}}{M} \rho^2. \tag{24}
\]
Necessary formulae of hyperspherical formalism in three–body systems can be found in Appendix \( \Delta \) (see also Ref. \[24\] for more details). In actual calculations the radial w.f. was chosen in the Gaussian form,
\[
\Phi(r, \rho) = \exp\left( -\frac{1}{2} \beta^2 MR^2 \right), \tag{25}
\]
with \( \beta \) being the variational parameter. Then the eigenvalue \( M_0 \) is given by
\[
M_0 = h(\mu_0, \mu_g, \beta_0), \tag{26}
\]
where
\[
h(\mu, \mu_g, \beta) = \frac{\langle \Psi | H_0 | \Psi \rangle}{\langle \Psi | \Psi \rangle}, \quad \Psi(\vec{r}, \vec{\rho}) = \rho Y_{1\nu}(\rho) \Phi(r, \rho). \tag{27}
\]
and $\mu_0$, $\mu_{g0}$, and $\beta_0$ come as a selfconsistent solution of the set of three coupled equations,

$$
\frac{\partial h}{\partial \beta} = 0, \quad \frac{\partial h}{\partial \mu} = 0, \quad \frac{\partial h}{\partial \mu_g} = 0.
$$

(28)

The model parameters $\{m, \sigma, \alpha_s, \eta\}$ are fixed by evaluating the spectrum of conventional $c\bar{c}$ $1S$– and $1P$– states within the same formalism (in Appendix B we give necessary details of calculations for the conventional charmonium). The set of parameters is given in Table I and the results for charmonia levels are listed in Table II. The value of $\eta = 0.29$ corresponds to the charmed quark mass given in Table I and to the gluonic correlation length $T_g \approx 0.2$ fm, which complies well with the one measured on the lattice (around $0.2 \div 0.3$ fm [25]).

With these parameters the variational procedure described above yields

$$
M_0 = 4.573 \text{ GeV},
$$

(29)

for the zero–order hybrid mass, while the extremal values of the effective masses are

$$
\mu_0 = 1.598 \text{ GeV}, \quad \mu_{g0} = 1.085 \text{ GeV}.
$$

(30)

Notice that the gluon effective mass $\mu_{g0}$ appears to be rather large, and of the same order of magnitude as the effective charmed quark mass $\mu_0$.

All corrections to the leading regime (29) are calculated as perturbations, with the substitution $\mu \to \mu_0$, $\mu_g \to \mu_{g0}$, $\beta \to \beta_0$. There are two types of such corrections: one that does not depend on quark spin and the other, which depends. The former correction provides an overall shift with respect to zero–order regime (29). The spin-dependent correction removes
the degeneracy between the four states \([22]\) and \([23]\) and, in principle, is responsible for the mixing of the magnetic gluon states with electric gluon ones. In what follows we neglect such a mixing.

The simplest correction of the first type is the selfenergy correction which shifts the zero–order hybrid mass downwards,

\[
\Delta M_{\text{selfenergy}} = -28 \text{ MeV}. \tag{31}
\]

The string correction does not depend on spins either, and it is calculated to be

\[
\Delta M_{\text{string}} = -52 \text{ MeV}. \tag{32}
\]

There is also a mass shift due to the gluon spin–orbit force, common for all four states \([22]\) and \([23]\), which comes from the terms in \([14]\) proportional to the operator \(\vec{s}_g\). This yields:

\[
\Delta M_{(g)\text{LS}} = -103 \text{ MeV}. \tag{33}
\]

Finally, to calculate the spin splittings, it is convenient to rewrite the operators \(\vec{s}_q\) and \(\vec{s}_\bar{q}\) in terms of operators \(\vec{s}_{qq}\) and \(\vec{\Sigma}\),

\[
\vec{s}_{qq} = \frac{1}{2}(\vec{s}_q + \vec{s}_{\bar{q}}), \quad \vec{\Sigma} = \frac{1}{2}(\vec{s}_q - \vec{s}_{\bar{q}}). \tag{34}
\]

We notice then that, once the operator \(\Sigma\) is antisymmetric with respect to the permutation \(q \leftrightarrow \bar{q}\), it flips the spin of the \(qq\) pair and, as such, is neglected in our calculations. Then, after tedious but straightforward calculations, one arrives at the following expressions for spin splittings:

\[
\begin{align*}
\Delta M(1^{-}) & = +\frac{3}{4}\Delta_{\text{SS}}^{(qq)}, \\
\Delta M(0^{-}) & = -\frac{1}{4}\Delta_{\text{SS}}^{(qq)} - 2\Delta, \\
\Delta M(1^{+}) & = -\frac{1}{4}\Delta_{\text{SS}}^{(qq)} - \Delta, \\
\Delta M(2^{-}) & = -\frac{1}{4}\Delta_{\text{SS}}^{(qq)} + \Delta,
\end{align*}
\tag{35}
\]

where

\[
\Delta_{\text{SS}}^{(qq)} = 9 \text{ MeV} \tag{36}
\]

comes from the quark–antiquark spin–spin interaction, and

\[
\Delta = \Delta_{\text{SS}}^{(qq)} + \Delta_{\text{LS}}^{(q)} + \Delta_{\text{ST}}. \tag{37}
\]
Individual contributions in Eq. (37) are

$$\Delta_{SS}^{(gg)} = 9 \text{ MeV}, \quad \Delta_{LS}^{(g)} = 24 \text{ MeV}, \quad \Delta_{ST} = 35 \text{ MeV},$$

coming from the quark–gluon spin–spin interaction, the spin–orbit interaction proportional to the quark and the antiquark spin, and the spin–tensor interaction, respectively. These all together give

$$\Delta = 68 \text{ MeV}. \quad (39)$$

The ultimate numerical results for the hybrid masses are given in Table III; spin splittings are established to yield:

$$M(0^{-+}) < M(1^{-+}) < M(1^{--}) < M(2^{-+}). \quad (40)$$

### IV. COMPARISON TO OTHER APPROACHES AND LATTICE CALCULATIONS

The story of hybrid meson studies started with the bag model calculations [26], where the lowest charmonium hybrid mass of about 4 GeV was obtained, and the splitting pattern (40) was found.

In the flux tube model [27] the hybrid excitations are visualised as phonon–type excitations of the string connecting the quark–antiquark pair, and a certain correspondence is established in Ref. [28] between the excited flux tube and the constituent gluon approaches. Eight lowest $c\bar{c}g$ hybrids are predicted [29] to reside around $4.1 \div 4.2$ GeV, and $1^{--}$ and $J^{++}$ states are among those. However, there is a discrepancy in quantum numbers of the flux tube hybrids and the ones with a constituent gluon: the constituent gluon carries colour and spin. As a result, the $P$–even flux tube hybrids have

$$J^{PC} = 0^{+-}, 1^{+-}, 2^{+-}, 1^{++} \quad (41)$$
quantum numbers, while in constituent gluon models these are

\[ J^{PC} = 0^{++}, 1^{++}, 2^{++}, 1^{+-} \]  

hybrids with the electric gluon. Spin splittings of the flux tube hybrids due to the long-range Thomas precession were calculated in Ref. [30] to be small, while the splittings reported in the present paper are much larger, and come mostly from perturbative short-ranged forces.

The constituent gluon model with pairwise forces was presented in the pioneering work [19], in Ref. [3], and in Ref. [31]. The mixing between magnetic and electric gluon hybrids was calculated in Ref. [31], with rather controversial results: in the first entry of Ref. [31] the mixing was found to be small, while in the second entry it is claimed to be substantial (though the details are not given there). The unmixed states are found in the same mass region as in the present work.

There exist results [32] of the QCD string model calculations in the einbein field formalism. The \(1^{--}\) charmonium hybrid mass was found to be 4.2 \(\pm\) 0.2 GeV. However, this result cannot be directly compared with ours, as the adiabatic approximation for quarks is employed there.

Finally, in a potential model, with the \(c\bar{c}\) pair considered as a colour–octet source, the tensor hybrid was predicted at 4.12 GeV [33].

Lattice simulations deal mostly with exotic quantum numbers, with \(1^{--}\) charmonium hybrid residing at about 4.4 GeV — see Ref. [34] and references therein. Among more recent results we would like to mention Ref. [35], which gives 4.405 \(\pm\) 0.038 GeV for the \(1^{--}\) charmonium hybrid, and Ref. [36] where, for the first time, a \(1^{--}\) state was found, excited by the hybrid meson operator, with the mass of 4.379 \(\pm\) 0.149 GeV (the authors claim that they have found a radially excited vector hybrid; such an interpretation was criticised in Ref. [37]). As seen from our Table III, the agreement of lattice results and our findings is quite good.

V. HYBRID DECAYS

The selection rule forbidding the decay of \(1^{--}\) and \(J^{--}\) hybrids with magnetic gluon into the \(D^{(*)}D^{(*)}\) final states was established for the constituent glue model in Refs. [3, 5, 6]. As the decay takes place via gluon \(\rightarrow\) quark–antiquark pair transition, the amplitude is
TABLE IV: Spin–recoupling coefficients for the hybrid states listed in Table III. Here $D^{(*)}$ is an $S$–wave $D^{(*)}$-meson and $D_J$ is a $P$–wave $D$–meson with the total momentum $J$. A proper charge conjugation is implied.

| $\bar{D}D_0$ | $\bar{D}^*D_0$ | $\bar{D}D_1(1P_1)$ | $\bar{D}^*D_1(1P_1)$ | $\bar{D}D_1(2P_1)$ | $\bar{D}^*D_1(2P_1)$ | $\bar{D}D_2$ | $\bar{D}^*D_2$ |
|---------------|----------------|--------------------|--------------------|--------------------|--------------------|---------------|---------------|
| 1--           | $\frac{1}{\sqrt{6}}$ | $-\frac{1}{2}$     | $\frac{1}{2}$     | $\frac{1}{2\sqrt{2}}$ | $-\frac{1}{2\sqrt{6}}$ |               |               |
| 0++           | $-\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ |               |                     |                     |               |               |
| 1++           | $-\frac{1}{\sqrt{3}}$ | $-\frac{1}{2}$     | $\frac{1}{2\sqrt{2}}$ | $\frac{1}{2\sqrt{2}}$ | $\frac{1}{4}$ | $-\frac{1}{4\sqrt{3}}$ |               |
| 2++           | $-\frac{1}{2\sqrt{2}}$ | $\frac{3}{4}$     | $\frac{1}{2\sqrt{2}}$ | $\frac{1}{2\sqrt{2}}$ | $\frac{\sqrt{3}}{4}$ |               |               |

There exists another selection rule based on the spin content of quarks in the final state, established in Refs. [5, 38] and, quite recently, in Ref. [39]. Assuming that i) the spin of the initial $q\bar{q}$ pair does not flip in the decay, and ii) the $q\bar{q}$ pair created in the decay is in spin–triplet, one can define the relative strength of matrix elements for decays into various final meson pairs. This selection rule is rather powerful: for example, for a vector hybrid, the initial quark pair is in spin–singlet, while for a conventional vector quarkonium it is in spin–triplet, with clear discrimination between two possibilities. Thus, measuring the relative rates of various $S$– and $P$– meson pairs, one could distinguish between a vector quarkonium and a vector hybrid [39, 40].

For hybrid decays under consideration, only $S$–wave amplitudes are of immediate relevance (the $D$–wave ones are suppressed because of a limited phase space), and the corresponding spin–recoupling coefficients are given in Table IV (decay rates are proportional to the squares of these coefficients). The coefficients exhibit a sum rule: if the masses and w.f.’s of the initial and final states involved could be taken identical, the total effect of coupling to open charm mesons is identical within the whole hybrid multiplet (though individual
contributions from various $D$-meson channels differ). A similar sum rule holds true for the conventional $c\bar{c}$ charmonium couplings to $D$-mesons, as found in Refs. [41, 42] and formulated in Ref. [43] as a general theorem. Most straightforward consequence of this sum rule is the following observation: the effects of mesonic loops over the spectra could be quite large, but numerically they are similar for all low-lying charmonia. For hybrids, however, the latter is not the case as, in accordance with the results of Table III, different members of hybrid multiplet reside among different thresholds.

VI. DISCUSSION

First, we notice that both lattice calculations and our findings place the vector hybrid at 4.4 GeV, substantially higher than the $Y(4260)$. One should have in mind, however, that the lattice result [36] for a vector hybrid comes with a large error, and the accuracy of the einbein field method is not better than 5% in the binding energy, as estimated in Ref. [11], so the discrepancy could appear to be not very significant. On the other hand, the influence of open charm channels is not taken into account in the present approach, and the said influence could be large.

Indeed, as seen from Table IV, there is a significant coupling of the vector hybrid to the $\bar{D}D_1(3^P_1)$ channel. There are two $D_1$-mesons, a narrow one with the mass of 2420 MeV and the width of 20 MeV, and a broad one with the mass of 2430 MeV and the width of 380 MeV [44], which are (unknown) mixtures of the $^3P_1$ and $^1P_1$ states. Thus the vector hybrid state should be attracted to the corresponding thresholds, which are tantalizingly close to the measured mass of the $Y(4260)$.

It is interesting to mention in this regard another enigmatic vector state (or, maybe, even two states!), namely $Y(4325)$ from BaBar [45] and $Y(4360)$ from Belle [46], both seen in the initial state radiation process, with the masses consistent with each other and with the width of the Belle state being two times smaller than that of the BaBar one. The masses of these new $Y$’s are close to another relevant threshold, the $\bar{D}^*D_0$ one (we assume, following the Belle paper [47], the scalar $D$-meson to be at 2308 MeV, with the width of about 270 MeV). So the coupling of the hybrid vector to $\bar{D}D_1$ and $\bar{D}^*D_0$ could be responsible for the formation of two near-threshold states.

The case of the pseudoscalar hybrid is simpler: in accordance with Table IV, half of the
decay strength goes to the $DD_0$ channel with the nominal threshold at 4.18 GeV, so this hybrid would feed the structure in the $DD\pi$ final state, with this structure being broad due to large $D_0$ width.

Exotic hybrid is estimated to be 80 MeV lighter than the vector one, with the mass very close to the $\bar{D}^*D_0$ threshold. As seen from Table IV, the coupling to $\bar{D}^*D_0$ is two times larger than the one for the vector case, so, as the $D_0(2308)$ is very broad, this hybrid has more chances to disappear in the $\bar{D}^*D\pi$ continuum.

As to the tensor hybrid, with the bare mass of 4.457 GeV, it could survive as a resonance, because the most prominent channel, $\bar{D}^*D_1$, opens nominally only at 4.43 GeV.

VII. CONCLUSIONS

In this paper we calculated the masses of low–lying charmonium hybrids with magnetic gluon in the framework of the Field Correlator Method for QCD. The QCD string approach is employed to estimate spin-dependent corrections for the $1^{--}$, $0^{-+}$, $1^{-+}$, and $2^{+-}$ hybrid states. The spectrum is calculated without fitting parameters, as all the parameters of the effective Hamiltonian are fixed by reproducing $c\bar{c}$ charmonium levels. Our results are in good agreement with lattice data.

Decay modes of hybrids are investigated and predictions are made for the relative rates of the decays into $S$- and $P$-wave $D$-mesons for all four states of the lowest hybrid multiplet.

The calculated mass of the vector hybrid is 4.397 GeV, substantially higher than the mass of a promising hybrid candidate $Y(4260)$. We argue that strong coupling of the vector hybrid to the $DD_1$ and $D^*D_0$ modes can cause considerable threshold attraction, making vector hybrid bare state responsible for the formation of near–threshold $Y(4260)$ and $Y(4325)$ states.

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APPENDIX A: SOME DETAILS OF THREE–BODY KINEMATICS

In a three–body system of particles with masses \( \mu_1, \mu_2, \mu_3 \) three sets of Jacobi coordinates can be defined. One is

\[
\vec{r}_{12} = \vec{r}_1 - \vec{r}_2, \quad \vec{\rho}_3 = \vec{r}_3 - \frac{\mu_1 \vec{r}_1 + \mu_2 \vec{r}_2}{\mu_1 + \mu_2},
\]

and others (\( \vec{r}_{31}, \vec{Q}_2 \) and \( \vec{r}_{23}, \vec{Q}_1 \)) are obtained from it by cyclic permutations of the particle indices. The hyperspherical radius

\[
R^2 = \frac{\mu_{12}^2}{M} \vec{r}_{12}^2 + \frac{\mu_{12,3}^2}{M} \vec{\rho}_3^2,
\]

where

\[
\mu_{12} = \frac{\mu_1 \mu_2}{\mu_1 + \mu_2}, \quad \mu_{12,3} = \frac{(\mu_1 + \mu_2) \mu_3}{M},
\]

is invariant under such cyclic permutations. Similarly,

\[
d^3r_{12}d^3\rho_3 = d^3r_{31}d^3\rho_2 = d^3r_{23}d^3\rho_1.
\]

Angular momenta \( \vec{L}_q \) can be conveniently represented as

\[
\vec{L}_{1g} = \vec{r}_{31} \times \vec{p}_{31} - \frac{\mu_g}{\mu + \mu_g} \vec{r}_{31} \times \vec{Q}_2, \quad \vec{L}_{1q} = \vec{r}_{31} \times \vec{p}_{31} + \frac{\mu_g}{\mu + \mu_g} \vec{r}_{31} \times \vec{Q}_2,
\]

\[
\vec{L}_{2g} = \vec{r}_{23} \times \vec{p}_{23} + \frac{\mu_g}{\mu + \mu_g} \vec{r}_{23} \times \vec{Q}_1, \quad \vec{L}_{2q} = \vec{r}_{23} \times \vec{p}_{23} - \frac{\mu_g}{\mu + \mu_g} \vec{r}_{23} \times \vec{Q}_1,
\]

\[
\vec{L}_q = \vec{r}_{12} \times \vec{p}_{12} - \frac{1}{2} \vec{r}_{12} \times \vec{Q}_3, \quad \vec{L}_q = \vec{r}_{12} \times \vec{p}_{12} + \frac{1}{2} \vec{r}_{12} \times \vec{Q}_3,
\]

where the momenta \( \vec{p}_{ij} \) and \( \vec{Q}_k \) conjugated to corresponding Jacobi coordinates are introduced, and \( \mu_1 = \mu_2 = \mu, \mu_3 = \mu_g \) is substituted.

The formula

\[
\vec{\rho}_3 = \frac{M}{2(\mu + \mu_g)} \vec{r}_{31} - \frac{1}{2} \vec{\rho}_2 = -\frac{M}{2(\mu + \mu_g)} \vec{r}_{23} - \frac{1}{2} \vec{\rho}_1
\]

is extensively used in calculations of various matrix elements.
APPENDIX B: SPECTRUM OF CONVENTIONAL CHARMONIA

In this Appendix we give some details of evaluation of the spectrum of conventional charmonia which was used in order to fix the set of model parameters. Further details of calculations for conventional mesons can be found in Refs. [11, 12].

The spinless centre-of-mass Hamiltonian of the charmonium reads:

$$H_0^{(cc)} = 2\sqrt{p^2 + m^2} + \sigma r - \frac{4\alpha_s}{3} r,$$  \hspace{1cm} (B.1)

where $r$ is the interquark distance. The einbein $\mu$ is introduced then,

$$2\sqrt{p^2 + m^2} \rightarrow \frac{m^2}{\mu} + \frac{p^2}{\mu}.$$  \hspace{1cm} (B.2)

The variational procedure for the charmonium is similar to the one described in the text body for the hybrid, that is, we take a Gaussian trial w.f.,

$$\Psi^{(cc)}(r) = \exp\left(-\frac{1}{2}\mu\beta^2 r^2\right),$$  \hspace{1cm} (B.3)

and evaluate the average

$$h^{(cc)}(\mu, \beta) = \frac{\langle \Psi^{(cc)} | H_0^{(cc)} | \Psi^{(cc)} \rangle}{\langle \Psi^{(cc)} | \Psi^{(cc)} \rangle}.$$  \hspace{1cm} (B.4)

Then, taking extrema in the variational parameters $\mu$ and $\beta$, we find the “bare” charmonium mass and the actual value $\mu_0$ to be substituted to the corrections to the Hamiltonian (B.1).

As in case of the hybrid, these corrections can be classified as spin-independent and spin-dependent.

The spin-independent corrections to the Hamiltonian (B.1) are the string correction [18],

$$V_r^{(cc)} = -\frac{\sigma L^2}{6\mu r},$$  \hspace{1cm} (B.5)

with $\vec{L}$ being the quark–antiquark angular momentum, and the selfenergy correction $C$ which coincides with the one for the hybrid, given by Eq. (18) (this is due to the fact that the quark content is the same for both the charmonium and charmonium hybrid [21]).

The spin-dependent corrections are:

$$V_\text{SD}^{(cc)} = V_\text{LS}^{(cc)} + V_\text{SS}^{(cc)} + V_\text{ST}^{(cc)},$$  \hspace{1cm} (B.6)

where

$$V_\text{LS}^{(cc)} = -\frac{\sigma}{2\mu^2 r} \vec{L} \hat{S} + \frac{2\alpha_s}{\mu^2 r^3} \vec{L} \hat{S},$$  \hspace{1cm} (B.7)
\[ V_{SS}^{(cc)} = \frac{32\pi\alpha_s}{9\mu^2} (\vec{s}_q\vec{s}_q)\delta(\vec{r}), \quad (B.8) \]
\[ V_{ST}^{(cc)} = \frac{4\alpha_s}{3\mu^2 r^3} [3(\vec{s}_q\vec{r})(\vec{s}_q\vec{r}) - r^2(\vec{s}_q\vec{s}_q)], \quad (B.9) \]

with \( \vec{s}_{q/\bar{q}} \) and \( \vec{S} \) being the quark/antiquark spin and the total spin, respectively.

In Table III we compare the predicted masses of the 1S- and 1P-wave charmonium states with the experimental data. The set of parameters used in the calculations is given in Table III. We find a good agreement of our predictions with the data and use the same set of parameters to evaluate the masses of the hybrids.
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