Viscous and Ohmic dissipation on non-Darcy MHD nanofluid mixed convection flow in porous medium with suction/injection effects

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Abstract - The paper presents the influence of nanofluid flow on MHD mixed convection, incompressible and electrically conducting fluid along a vertical flat plate with viscous and ohmic dissipation effects. The plate is permeable and embedded in a fluid saturated on non-Darcy (Forchheimer flow model) porous medium with suction and injection. Dissipation parameters are described in the energy equation. The governing partial differential equations are transformed into a system of ordinary differential equations using similarity transformation. The non linear ordinary differential equations are linearized by using Quasi-linearization technique and then solved numerically by using implicit finite difference scheme. To compute the numerical values C programming code is used. The numerical results are analyzed for the effects of various physical parameters such as magnetic parameter Ha, Reynolds number Re, Eckert number, Ec thermophoresis parameter Nt, Brownian motion parameter Nb, suction and injection parameter fw and Lewis number Le. The results obtained are discussed with the help of graphical illustrations.

Keywords: MHD, mixed convection, viscous dissipation, thermophoresis parameter, Brownian motion parameter and implicit finite difference scheme.

1. Introduction

The nanofluid is the suspensions of nano-size particles into the conventional fluids which was firstly studied by Choi [1]. The nanoparticles are made of metals, carbides and oxides or carbon nanotubes, and conventional fluids of water, oil and ethylene glycol. Due to suspension of nanoparticle, solid particles are used to enhance the thermal properties of base fluids. This is due to thermal conductivity of nanoparticle is larger than of base fluids. So the existence of nano solid particles in the conventional fluids heat transfer characteristics enhanced. The process of heat and mass transfer is encountered in aeronautics, fluid fuel nuclear reactor, chemical process industries and many engineering applications.

Ganga et al. [2] scrutinized the influence of viscous and Ohmic dissipation, heat generation/absorption on radiative MHD flow of nanofluid over a vertical plate. Recently, the combined effect of viscous-Ohmic dissipation and thermal radiation on hydro-magnetic flow of nanofluid over a stretching/shrinking sheet in the presence of slip and porous medium was studied.
by Ganesh et al. [3]. Makinde and Aziz [4] analyzed boundary layer flow of a nanofluid past a stretching sheet with convective boundary conditions.

Taking consideration of dissipation effects in the study of heat and mass transfer boundary layer problems added new dimension in the domain of research of fluid dynamics. Gebhart [5] was the first who studied the problem by taking into account the viscous dissipation. Very recently the researchers [6,7] investigated about base and nanofluids at different channels.

Abbasi et al. [8] discussed about impact of magnetic field on mixed convective peristaltic flow of water based nanofluids with Joule heating. Kishan Naikoti et al. [9] detailed influence of MHD effects on heat transfer over stretching sheet embedded in porous medium with variable viscosity, viscous dissipation and heat source/sink. Loganathan et al. [10] alayed Ohmic Heating and Viscous Dissipation Effects over a Vertical Plate in the Presence of Porous Medium.

Srinivasachary et al. [11] studied the magnetic and radiation effects on mixed convection heat transfer along a vertical flat plate. The plate is permeable and embedded in a fluid saturated non-Darcy Forchheimer porous medium. Kishan et al. [12] studied influence of thermophoresis and MHD on non-Darcy mixed convection heat and mass transfer along a vertical flat plate with radiation effects. Now this paper presents viscous and ohmic dissipation on MHD nanofluid with suction and injection.

2. Mathematical formulation

Considering steady, laminar, two-dimensional and mixed convection flow over a semi-infinite vertical plate embedded in a free stream velocity of an electrically conducting Newtonian fluid which is embedded in a saturated non-Darcy porous medium. The free stream velocity is \( u_\infty \). The buoyancy force is aiding and opposing the uniform free stream. Far away from the plate, both the surroundings and the fluid are maintained at a constant temperature \( T_\infty \). The coordinate system is taken as \( x \)-axis along the plate and \( y \)-axis normal to the plate. A magnetic field is applied in the \( y \)-direction. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected. The plate is maintained at a constant temperature \( T_w \) (which is higher than the ambient temperature \( T_\infty \)). The suction/injection velocity distribution is assumed to have power function form \( v_\infty(x) = Ax^\lambda \), \( x \) is the distance from the leading edge. The boundary layer equations are framed using assumptions of Boussinesq approximations:

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
\left[1 + \frac{\rho_0 u^2}{\rho v} \right] \frac{\partial u}{\partial y} + \frac{C \sqrt{K}}{\nu} \frac{\partial u}{\partial y} &= \pm \frac{K \rho_0 \beta}{\nu} \frac{\partial T}{\partial y} \\
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho_0} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma_0^2 u^2}{\rho_0} + \tau \left[ D_\beta \frac{\partial C}{\partial y} + \frac{D_\beta}{T_w} \frac{\partial T}{\partial y} \right]^2 \\
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} &= D_c \frac{\partial^2 C}{\partial y^2} + D_T \frac{\partial^2 T}{T_w} \frac{\partial^2 C}{\partial y^2}
\end{align*}
\]

Where \( u \) and \( v \) are the Darcian velocity components along \( x \) and \( y \) directions, \( T \) is the temperature, \( K \) is the permeability constant, \( C \) is an empirical constant, \( \nu \) is the kinematic viscosity, \( g \) is the acceleration due to gravity, \( \beta \) is the coefficient of thermal expansion, \( \rho \) is the density, \( C_p \) is the specific heat at constant pressure, \( \alpha \) is the thermal diffusivity constant, \( \sigma \) is the electrical conductivity of the fluid, \( \beta_0 \) is the strength of the magnetic field.
The boundary conditions are
\[ v = Ax^2, \quad T = T_w \quad \text{as} \quad y \to 0 \]
\[ u = u_w, \quad T = T_{sw} \quad \text{as} \quad y \to \infty \quad \text{where} \quad A \quad \text{is a constant} \]  
(5)

The stream function is defined by
\[ u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \]  
(6)

Substituting (5) in (2-4) using the following similarity transformation
\[ \eta = \sqrt{Pe_x} \frac{y}{x}, \quad \psi = \alpha \sqrt{Pe_x} f(\eta), \]
\[ \theta(\eta) = (T - T_w) / (T_{sw} - T_w), \]
\[ \phi(\eta) = (C - C_w) / (C_{sw} - C_w) \]  
(7)

We get the following system of equations
\[ (1 + Ha^2)f'' + 2 Re_d f' f" = \frac{Ra_d}{Pe_d} \theta'' \]  
(8)
\[ \theta'' + \frac{1}{2} f\theta' + Pr\left(N_b \theta' \phi' + N_f \theta'' + Ec f'' - 2 + Ha^2 Ec f'' \right) = 0 \]  
(9)
\[ \phi'' + Le f' \phi' + N_b \phi'' = 0 \]  
(10)

where
\[ Re_d = u_w d / \nu \quad \text{pore diameter dependent Reynolds number} \]
\[ Ra_d = K g \beta(T_w - T_{sw}) d / \alpha \nu \quad \text{pore diameter dependent Rayleigh number} \]
\[ Pe_d = u_w d / \alpha \quad \text{pore diameter dependent Peclet number} \]
\[ Ha^2 = \sigma \beta_k d^2 k / \rho \nu \quad \text{magnetic parameter (Hartmann number)} \]
\[ Ra_d / Pe_d \quad \text{mixed convection parameter} \]
\[ Ec = u_w^2 / \nu (T_w - T_{sw}) \quad \text{viscous dissipation} \]
\[ N_T = \frac{dD_T (T_w - T_{sw}) / \nu T_x}{\nu} \quad \text{thermophoresis parameter} \]
\[ N_b = \frac{dD_B (C_w - C_{sw}) / \nu}{\nu} \quad \text{Brownian motion parameter} \]

The boundary conditions (5) in terms of \( f \) and \( \theta \) become
\[ f(\eta) = f_w, \quad \theta(\eta) = 1, \quad \phi(\eta) = 1 \quad \text{as} \quad \eta = 0 \]
\[ f(\eta) = 1, \quad \theta(\eta) = 0, \quad \phi(\eta) = 0 \quad \text{as} \quad \eta = \infty \]  
(11)

The suction/injection parameter \( (f_w) \) is defined as
\[ f_w = -\frac{2x}{A \alpha} v_w(x) Pe'_s \]  
(12)
3. Numerical solutions

In order to get physical insight, we integrate the system of ordinary differential equations, Eqs. (8)-(10), with the boundary conditions in Eq. (11) numerically by means of implicit finite difference method. Applying the Quasi-linearization technique, Bellman and Kalaba [13] to the non-linear equation (8) we get

\[
\left(1 + \frac{Ra_d}{Pe_d}\right)f' + 2Ra_d F^n f' = \pm \left(\frac{Ra_d}{Pe_d}\right)f' + 2Ra_d F^n f'
\]  

(13)

Where assumed \( F \) is the value of \( f \) at \( n^{th} \) iteration and \( f \) is at \( (n+1)^{th} \) iteration. The convergence criterion is fixed as \( |F - f| < 10^{-5} \)

Using an implicit finite difference scheme for the equation (13), (9) and (10), we obtain

\[
da[i] = f[i+1] - f[i] + b[i]f[i] + c[i]f[i] + d[i]f[i+1] = d[i]
\]  

(14)

\[
a_1[i] = 1 - 0.5*h*0.5*f[i] + b_1[i] = -2, c_1[i] = 1 + 0.5*h + d_1[i] = 2
\]

(15)

\[
a_2[i] = 1 + 0.5*h (1/2Le)f[i], b_2[i] = -2, c_2[i] = 1 + 0.5*h (1/2Le)f[i]
\]

(16)

The results are computed numerically by using the implicit finite difference scheme along with the Gauss-Seidel method by using the C-programming, for various values of the physical parameters such as magnetic parameter \( Ha \),

4. Results and discussion

The results are computed numerically by using the implicit finite difference scheme along with the Gauss-Seidel method by using the C-programming, for various values of the physical parameters such as magnetic parameter \( Ha \),
Suction and injection parameter $f_w$, viscous dissipation $E_c$, Lewis number $L_e$, thermophoresis parameter $N_t$ and Brownian motion parameter $N_b$. For illustration of the results, numerical values are plotted in figures 1-6.

Effects of magnetic parameter $H_a$ on velocity, temperature and concentration profiles are shown in fig.1. It is noticed that the velocity of a fluid decreases with the effects of magnetic field parameter $H_a$, temperature and concentration of the fluid increases with the increase of the magnetic field parameter $H_a$. The effects of transverse magnetic field to an electrically conducting fluid gives rise to a resistive type force called the Lorentz force. This force has a tendency to drag the motion of the fluid.

The influence of suction and injection parameter parameter on velocity, temperature and concentration profiles is shown in fig.2. It can be seen from figures due to the fluid suction the velocity field decreases.. It is noticed from the figures, the temperature and concentration profiles decrease with the increase of suction and injection parameter.

The effect of thermophoresis parameter $N_t$ is shown in fig. 3. The effect of thermophoresis parameter $N_t$ is to increase the temperature profile and concentration.

Effects of Brownian motion parameter $N_b$ on temperature and concentration profiles are shown in fig. 4. It can be seen that the temperature profile increase with the increase of Brownian motion. From fig.4b concentration profiles decrease with the increase of Brownian motion parameter.

Effects of viscous dissipation is displayed in figure 5. The temperature profile increases with increase of Eckert number.

Effects of Lewis number on concentration profiles are shown in fig.6 it is noticed that the concentration of the fluid decreases with the increase of Lewis number.
Figure 1. Effects of magnetic parameter $Ha$ on Velocity, temperature and concentration

Figure 2. Effects of suction /injection parameter $fw$ on Velocity, temperature and concentration
Nanoparticles

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Figure 3. Effects of thermophoresis parameter Nt on temperature and concentration

Figure 4. Effects of brownian motion parameter on temperature and concentration

Figure 5. Effects of Eckert number on temperature

Figure 6. Effects of Lewis number on temperature

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