Theory of fusion hindrance and synthesis of the superheavy elements
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The two-step model for fusion reactions of massive systems is briefly reminded. By the use of fusion probabilities obtained by the model and of survival probabilities obtained by the new statistical code, we predict $\mathit{xn}$ residue cross sections for $^{48}\text{Ca}+\text{actinide}$ systems leading to superheavy elements with $Z=114, 116$ and $118$.

1. INTRODUCTION

Since the discovery of the periodicity in chemical elements by Mendelejeff\textsuperscript{[1]}, our knowledge has been expanded. The heaviest element in nature is known to be Uranium with the atomic number $Z=92$. Heavier elements than that have been synthesized artificially. Thus, it is an intriguing question how many elements can exist or what is the heaviest element we can synthesize. Larger $Z$ values result in the instability due to the larger Coulomb repulsion which would dominate over nuclear attraction. But it is also well-known that closed shells of nucleonic structure give rise to an extra-binding which contributes to the stabilization of atomic nucleus. Many attempts have been made to predict double closed shell nuclei heavier than $^{208}\text{Pb}$\textsuperscript{[2]}. The magic number for proton next to 82 is predicted to be 114, 120 or 126, depending on the nuclear models employed, while that for neutron next to 126 is commonly predicted to be 184. These results suggest that there is a stable region in the nuclear chart far away from that of the known isotopes, which is sometimes called an island of the superheavy elements (SHE). Naturally, enormous experimental efforts have been devoted to answer to the question last few decades. Until now, the elements with up to $Z=112$ have been synthesized by heavy ion fusion experiments with $^{208}\text{Pb}$ target\textsuperscript{[3]}, and about even heavier elements with $Z=114$ and $116$ are reported indicative experimental observations with $^{48}\text{Ca}$ beams on the actinide targets\textsuperscript{[4]}. But reaction mechanisms of nuclear fusion of massive systems are not well known, and thus the experiments have been performed following the systematics of the experimental data available so far. Therefore, it is a very important and urgent subject to develop a theory which permits us to predict which combination of projectile and target is favorable for synthesis of an element under consideration, and at what incident energy residue cross sections are optimized. Of course, in order to predict residue cross sections quantitatively, we have to treat not only fusion processes, but also cooling processes precisely, because compound nuclei formed by fusion reactions are excited. More explicitly, the residue cross section is
given by the following formula, presuming the statistical theory of the compound nucleus,
\[ \sigma_{\text{res}}(E_{\text{c.m.}}) = \pi \lambda^2 \sum_J (2J + 1) \cdot P_J^{\text{fusion}}(E_{\text{c.m.}}) \cdot P_J^{\text{surv}}(E^*) , \]  
(1)

where \( E^* \) is the excitation energy of the compound nucleus and is equal to \( E_{\text{c.m.}} + Q \) with \( Q \) being the \( Q \)-value of the reaction, and \( \lambda \) is the wave length divided by \( 2 \cdot \pi \). The symbol \( J \) is the quantum number of total angular momentum of the system, as usual. \( P_J^{\text{fusion}} \) and \( P_J^{\text{surv}} \) denote the fusion and the survival probabilities, respectively. The latter is the probability for surviving against fission decay and charged particle emissions, which are well described as statistical decays. A new computer program KEWPIE (Kyoto Evaporation Width calculation Program with tIme Evolution)[5] is constructed so as to minimize the ambiguities, which is briefly described later. The remaining unknown is the nuclear masses of the superheavy isotopes, or in other words, their shell correction energies, to which \( xn \) residue cross sections are extremely sensitive.

On the other hand, as stated above, there is no commonly accepted theory for the fusion probability, though it is well-known experimentally that there is a strong hindrance in fusion reactions of massive systems[6]. That is, it does not reach 1/2 at the incident energy equal to an expected barrier height, and in order for that, a large amount of energy is required additionally, so-called extra-push energy[7]. Two possible explanations have been proposed. One is due to the loss of the incident energy by friction which is supposed to act between the ions of the entrance channel[8]. This is inferred from the studies on so-called Deep-Inelastic Collisions (DIC). Gross and Kalinowski proposed surface friction model (SFM)[9] which successfully explained the characteristic aspects of DIC. The other one is due to dissipation of energy of the collective motion which leads the system to the spherical compound nucleus, starting from the pear-shaped configuration formed by the sticking of the ions of the entrance channel[10]. This is conceivable, because there is the strong dissipation in the fissioning motion, i.e., in an “inverse” process. This is confirmed from the studies of anomalous multiplicities of neutrons, gamma rays, etc. emitted prior to fission of excited nuclei[11], with the strong friction given by the one-body wall-and window formula (OBM)[12]. Thus, it is orthodox and comprehensive to consider that both mechanisms are working in the fusion reactions of massive systems. As schematically shown in Fig. 1, two processes undergo in time-sequence, i.e., collision processes are up to the contact point of the projectile and the target to stick each other, and subsequently from the stuck configuration, shape evolution of the united system starts toward the spherical shape over the conditional saddle point. Therefore, the fusion process is described by the two steps, and then the fusion probability \( P_J^{\text{fusion}} \) is given by a product of the sticking probability \( P_J^{\text{stick}} \) and the formation probability \( P_J^{\text{form}} \):

\[ P_J^{\text{fusion}}(E_{\text{c.m.}}) = P_J^{\text{stick}}(E_{\text{c.m.}}) \cdot P_J^{\text{form}}(E_{\text{c.m.}}) \]  
(2)

This is a new two step model[13]. The method of connection is not like that from a diabatic to an adiabatic treatments, but should be called “statistical”, because the first step generally results in statistical distributions of physical quantities of the united system and they should be used as initial conditions for the second step, as will be detailed below.

In order to realize the treatment of the first step, SFM is employed, but in an extended form so as to include the fluctuation forces associated with the friction forces in accord with
Figure 1. Coulomb barrier, sticking configuration, and conditional saddle are shown schematically for massive systems, which illustrates a necessity of two-step treatment for fusion. $E_{\text{shell}}$, $\varepsilon_0$ and $k_0$ represent shell correction energy of the spherical shape, intrinsic excitation energy and remaining kinetic energy at the contact point.

The dissipation-fluctuation theorem. As stated above, the model is used up to the contact point, not for the whole processes like in DIC. Then, the results of the calculations provide the sticking probability $P_{\text{stick}}^J$ for the projectile and the target to stick each other, as well as information on kinetic energy of the radial motion, i.e., information on the distribution of the radial momentum at the contact point. Since they were already reported in detail elsewhere [14], we do not repeat it here, but summarize the results.

1. An energy dependence of the sticking probability shows that a barrier height is effectively shifted to an energy about 10 MeV higher than the original one, which would already provide an explanation of the extra-push energy.

2. A distribution of the radial momentum at the contact point has almost exactly a Gaussian form whose center, i.e., the mean value is equal to zero. Its variance is consistent with the temperature calculated from the energy conservation in average.

3. The orbital angular momentum is found to reach the dissipation limit.

These results indicate the fact that the projectile and the target stick each other at the contact point to form a united system. Therefore, with this distribution as the initial values, we solve shape evolution of the united mono-nuclear system to obtain the formation probability, i.e., the probability that the system overcomes the saddle point to reach the spherical shape, by the use of the same type of Langevin equation which was used for description of fission of excited nuclei [15].
Recently it has been shown analytically that the dissipation manifests its effects in an extremely enhanced way in over-saddle-point problem with a schematic one-dimensional model. This is a modeling of the fusion reactions of massive systems where the sticked configurations are located outside of the conditional saddle point and thus the system has to overcome it in order to reach the spherical shape. Furthermore, we can obtain a simple expression for the extra-push energy. Since they have been reported elsewhere\[16\], in the present paper we concentrate ourselves on realistic calculations of the formation and then the fusion probabilities by the use of the liquid drop model (LDM). In the next section, we describe dynamical evolution of nuclear shapes over the ridge line with Langevin equations for the collective variables. In section 3, we discuss about $^{48}$Ca- induced fusion reactions on the actinide targets, on which experimental fusion excitations are available for comparisons. By the use of KEWPIE, we calculate the survival probability, which is combined with the fusion probability to give $xn$ residue cross sections for SHE with $Z=114, 116$ and $118$. The results are compared with the experimental data available for the first two elements.

2. FORMATION PROBABILITY OF THE SPHERICAL COMPOUND NUCLEUS

Shapes of the united nucleus are described by the two-center parameterization (TCP) \[17\] with the distance between two mass-centers, the mass-asymmetry, and the neck parameter, while the deformations of fragments are neglected. In view of the extremely strong friction for the neck degree of freedom on the basis of OBM, the neck motion is expected to be very slow, which permits us to freeze it in a good approximation. Thus, the parameter $\epsilon$ is taken to be 0.8. It should be worth noticing that we are here interested only in fusion, but if we are interested in properties of fission-like fragments, we have to describe dynamics of the neck degree of freedom as well as those of deformations of the fragments along decaying paths. The shell correction energy is also neglected in formation process, since the united system is already well excited in the preceding processes of overcoming the Coulomb barrier. Therefore, we employ LDM for the potential as well as for the inertia-mass tensor for the collective motion. As for friction, we employ OBM for the calculation of the friction tensor. Now, we need only initial values to start calculations of trajectories with the Langevin equation. As stated in the introduction, our calculations of shape evolution start at the contact point. That is, the initial value of the distance is equal to the distance between the mass centers of the configuration, and that of the mass-asymmetry is equal to the mass-asymmetry of the entrance channel. As for the conjugate momenta, that for the mass-asymmetry is taken to be zero, while that for the radial motion can have various values, due to the Gaussian distribution given by the collision processes over the Coulomb barrier. In order to obtain the formation probability, we have to calculate probabilities $F^J(p_0, T)$ for the system to overcome the ridge line among numerous trajectories, starting with various initial momenta $p_0$, and then, to make an average of $F^J(p_0, T)$ with the weight of the Gaussian distribution of the initial momenta $g^J(p_0, T_c)$ obtained by SFM. Therefore, the formation probability is given as follows,

$$P_{\text{form}}^J(E_{\text{c.m.}}) = \int dp_0 F^J(p_0, T) \cdot g^J(p_0, T_c),$$  \hspace{1cm} (3)
where \( T_c \) denotes the temperature of the system at the contact point. Actually, values of \( T_c \) and \( T \) are very close each other. In Fig. 2, the formation probability calculated for \( ^{48}\text{Ca} + ^{244}\text{Pu} \) system is shown as a function of excitation energy \( E^* \). It is remarkable that it does not increase quickly as transmission coefficient usually used for fusion probability, but remains to be very small over the wide range of energy considered. This is in accord with the typical feature of the fusion hindrance observed.

3. FUSION CROSS SECTIONS AND RESIDUE CROSS SECTIONS OF SHE

Fusion probabilities are obtained with Eq. (2). Then, fusion excitation functions are calculated as usual,

\[
\sigma_f(E_{\text{c.m.}}) = \pi \lambda^2 \sum_j (2J + 1) \cdot P_{\text{fusion}}^J(E_{\text{c.m.}})
\]

(4)

The results are shown in Fig. 3 for \( ^{48}\text{Ca} + ^{238}\text{U}, ^{244}\text{Pu}, ^{248}\text{Cm} \) and \( ^{252}\text{Cf} \) systems, together with the available data. Apparently, it is seen that the calculations reproduce the experimental feature of the energy-dependence, and furthermore, reproduce the absolute values of the first three systems very well systematically. It is worth emphasizing here that there is no adjustable parameter employed all through the calculations, except the probable choice of \( \epsilon \). Experiments on the last system are strongly desired for confirmation of validity of the present model.

From the comparisons made above, it is reasonable to consider that the fusion probabilities calculated by the present model are accurate, and therefore, they can be used for residue calculations with Eq. (1) in a high reliability. The remaining is to calculate accurately the other factor, i.e., the survival probability \( P_{\text{surv}}^J \), which is made by KEWPIE. There, it is aimed to reduce ambiguities often introduced arbitrarily in analyses.

1. The level density parameter \( a \) is calculated with Töke and Swiatecki’s formula for the spherical and the saddle-point shapes. For particle evaporation, it is taken
to accommodate the shell correction energy with Ignatyuk’s prescription\cite{20} with
the standard damping energy of 18.5 MeV.

2. As for the saddle point and fission barrier height, we refer to Cohen-Plasil-Swiatecki’s
LDM\cite{21}. But we also adopt the barrier height systematics by Dahlinger et al. for
SHE, which is supposed to be more realistic.

3. Kramers factor\cite{22} for dynamical fission width is included with the reduced fric-
tion being $5 \cdot 10^{-20}$/sec which is consistent with OBM, together with Strutinski’s
correction factor\cite{23} from inclusion of the collective degree of freedom around the
spherical shape.

The frequencies of the potential at the spherical shape and the saddle point are tem-
porally taken to be the same, the corresponding energy quantum taken equal to 1 MeV.
Of course, one could use more realistic values, but they may give rise to a factor 2 or less.
Thus, there is no free parameter again in the statistical calculations, except unknown
masses or shell correction energies of isotopes of SHE. As stated in the introduction,
there are many structure calculations, most of which predict shell correction energies of
a few to several MeV. Møller et al.’s\cite{24} are larger ones among them, so we use their
values with reduction factor 1/2 in keeping their tendency in mass-dependence. Calcul-
lated excitation functions of $xn$ reactions are shown in Fig. 4 for $Z=114, 116,$ and 118,
together with available experimental points. For $Z=114$ and 116, the calculations appear
not to be inconsistent with the data\cite{4}. But the tendency of the predicted masses is not
satisfactory. Nevertheless, it seems that in $^{48}\text{Ca}+^{248}\text{Cm}$ system a lower energy is more
favourable. For the last system, the model provides predictions, on which experiments
are being waited for.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{$xn$ residue cross sections calculated in the present model are shown for
$^{48}\text{Ca}+^{244}\text{Pu}$, $^{248}\text{Cm}$ and $^{252}\text{Cf}$ systems. The solid dots are experiments\cite{4}.}
\end{figure}

In brief, the present two-step model has turned out to reproduce the experimental fusion
excitation functions systematically for $^{48}\text{Ca}$ induced reactions, as well as the measured $xn$
residue cross sections for SHE with the shell correction energies of Møller et al. with the
reduction factor, the absolute values of which appear to be close to some of the mean-field
calculations\cite{25}. Furthermore, we have made predictions on the fusion excitation function
and the $x_n$ cross sections for $Z=118$, which would stimulate experiments. A systematic study is now being made with typical theoretical predictions of nuclear masses and/or of shell correction energies for SHE. The present model will be applied to other elements and/or other entrance channels soon. Then, we will be able to answer the question raised in the introduction.

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REFERENCES

1. D. Mendelejeff, Z. Chemie 12 (1969) 405.
2. a. P.Møller and R. Nix, J. Phys. G20 (1994) 1681,  
   b. S. Cwiok et al., Nucl. Phys. A611 (1996) 211,  
   c. M. Bender et al., Eur. Phys. J. A7 (2000) 467.
3. S. Hofmann, Rep. Prog. Phys. 61 (1999) 639.
4. Yu. Oganessian et al., Phys. Rev. Lett. 83 (1999) 3154 and private communication.
5. B. Bouriquet et al., to be published.
6. A.B. Quint et al., Z. Phys. A346 (1993) 199.
7. S. Bjornholm and W.J. Swiatecki, Nucl. Phys. A391 (1982) 471.
8. P. Fröbrich et al., Nucl. Phys. A406 (1983) 557.
9. D.H.E. Gross and H. Kalinowski, Phys. Rept. 45 (1978) 175.
10. W.J. Swiatecki, Physica Scripta 24 (1981) 113.
11. T. Wada et al., Phys. Rev. Lett. 70 (1993) 3538.
12. J. Blocki et al., Ann. Phys. (NY) 113 (1978) 330.
13. C. Shen et al., Phys. Rev. C66 (2002) 061602(R).
14. G. Kosenko et al., J. Nucl. and Radiochem. Sci. 3 (2002) 19.
15. Y. Abe et al., J. de Physique 47 (1986) C4-329,  
    Y. Abe et al., Phys. Rept. 275 (1996) Nos. 2 and 3.
16. a. Y. Abe et al., Proc. of YKIS01, Prog. Theor. Phys. Suppl. 146 (2002) 104,  
    Y. Abe et al., Proc. Rauischholzhausen Meeting, to be published,  
    b. Y. Abe et al., Proc. of Zakopane School 2002, to be published in Acta Physica Polonica B (2003),  
    c. W.J. Swiatecki et al., ibid.
17. K. Sato et al., Z. Phys. A290 (1979) 145.
18. a. W.Q. Shen et al., Phys. Rev. C36 (1987) 115,  
    b. M.G. Itkis et al., Il Nuovo Cimento 111A (1998) 783.
19. J. Tóke and W.J. Swiatecki, Nucl. Phys. A372 (1981) 141.
20. A.V. Ignatyuk et al., Sov. J. Nucl. Phys. 21 (1975) 255.
21. S. Cohen et al., Ann. Phys. (NY) 82 (1974) 557.
22. H.A. Kramers, Physica VII 4 (1940) 284.
23. V.M. Strutinski, Phys. Lett. B47 (1973) 121, see also ref. 15.
24. P. Møller et al., Atomic Data and Nuclear Data Tables 59 (1995) 185.
25. T.J. Buervenich, private communication.