The CMBR ISW and HI 21-cm Cross-correlation Angular Power Spectrum

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Abstract

The late-time growth of large scale structures is imprinted in the CMBR anisotropy through the Integrated Sachs Wolfe (ISW) effect. This is perceived to be a very important observational probe of dark energy. Future observations of redshifted 21-cm radiation from the cosmological neutral hydrogen (HI) distribution hold the potential of probing the large scale structure over a large redshift range. We have investigated the possibility of detecting the ISW through cross-correlations between the CMBR anisotropies and redshifted 21-cm observations. Assuming that the HI traces the dark matter, we find that the ISW-HI cross-correlation angular power spectrum at an angular multipole $\ell$ is proportional to the dark matter power spectrum evaluated at the comoving wave number $\ell/r$, where $r$ is the comoving distance to the redshift from which the HI signal originated. The amplitude of the cross-correlation signal depends on parameters related to the HI distribution and the growth of cosmological perturbations. However, the cross-correlation is extremely weak as compared to the CMBR anisotropies and the predicted HI signal. Even in an ideal situation, the cross-correlation signal is smaller than the cosmic variance and a statistically significant detection is not very likely.

Keywords: Integrated Sachs-Wolfe effect, Inter-galactic medium, Power spectrum
I. INTRODUCTION

In recent times, a host of independent observations, like Supernova-Ia [1, 2], galaxy surveys [3] and Cosmic Microwave Background (CMB) anisotropies [4, 5], have indicated that the expansion of the Universe is accelerating [6]. This can be explained by a dark energy component, with an equation of state $p/\rho = w (< -\frac{1}{3})$. The cosmological constant, $\Lambda$, has emerged as a strong candidate for dark energy, as various observations [3] constrain $w$ to be close to $-1$.

An indirect effect of $\Lambda$ is that it causes a decay of the gravitational potential, when the universe evolves from the matter dominated to the dark energy dominated era. This generates a weak anisotropy in the CMB temperature fluctuation, through the Integrated Sachs Wolfe (henceforth ISW) effect [7]. A non-flat spatial geometry would contribute to the ISW in the same way. However, CMB data largely constrain our universe to be spatially flat (see ref. [5]) so, such effect of spatial curvature can be ignored in the first approximation. The late-time evolution of the gravitational potential is sensitive to the specific dark energy model. Therefore, the associated ISW anisotropy, can in principle be used to probe the nature of dark energy.

It is difficult to separate the ISW signal from the primary CMB anisotropy, because it is intrinsically weak and it appears at large scales, where the error due to cosmic variance is large.

Techniques to measure the ISW, use the cross-correlation of the CMB fluctuations, with fluctuations of some tracer of the large scale structure at a later redshift. Fluctuations in the primary CMB field and in the tracer are uncorrelated, so that this method allows one to single out the contribution solely due to the ISW. It is also important to note here, that the foregrounds and noise are not correlated between independent random fields. Recently ISW-large scale structure and ISW-weak lensing cross-correlations have been studied extensively (see ref. [8, 9, 10, 11, 12, 13, 14, 15]). These studies look at a median $z \sim 1.5$ and are in agreement with the $\Lambda$CDM model at $2\sigma \sim 3\sigma$ levels.

Observations of redshifted 21 cm radiation of the spin-flip hyperfine transition from neutral hydrogen (HI) have the potential of probing the universe over a wide range of redshifts ($200 \geq z \geq 0$): from the dark ages to to the present epoch (eg. [16, 17, 18]). Recently, radio-optical cross-correlation study has detected a positive correlation between the opti-
cal galaxies (6dFGS) and HI fluctuations \[19\]. This vindicates the theoretical predictions \[20, 21\] about the possibility of using HI distribution statistically, as a probe of the large scale structure, without the need to resolve individual galaxies. Cross-correlation technique using the HI 21-cm radiation as one of the fields has been considered for the study of cosmic reionization \[22, 23, 24, 25, 26\].

In this paper we study the use of diffused HI as a tracer of the large scale structure to probe dark energy induced ISW effect. We look at the cross correlation between the post-reionization (\(z \lesssim 6\)) fluctuations in the HI brightness temperature and the CMB.

Redshifted 21 cm observations of neutral HI allow us to probe the universe as a function of redshift. The advantage of using HI tomography is that, we can probe the late-time cosmic history continuously over a range of redshifts. Radio telescopes (eg. currently functioning GMRT \[48\] and upcoming MWA \[49\] & LOFAR \[50\]) are aimed to map the large-scale distribution of HI at high redshifts. At redshifts \(0 \leq z \leq 3.5\) we have \(\Omega_{\text{gas}} \sim 10^{-3}\) (for details see \[27, 28, 29\]). This implies that the mean neutral fraction of the hydrogen gas is \(\bar{x}_{\text{HI}} = 50 \Omega_{\text{gas}} h^2 (0.02/\Omega_{b} h^2) = 2.45 \times 10^{-2}\), which we assume is a constant over the the entire redshift range \(z \leq 6\).

The redshifted 21 cm radiation seen in emission in this redshift range, from individual clouds is rather weak (< 10 \(\mu\)Jy). This makes its detectability dubious, with existing observational facilities. (There might be considerable magnification caused by gravitational lensing \[30\] which may enhance detection chances). Statistical distribution of HI however produces a weak background in radio observations. This radiation has the information about the HI fluctuations in probed redshift range \[20, 21\]. CMB map of a large portion of the sky and a corresponding HI map would allow us to compute the cross-correlation power spectrum and hence independently quantify the cosmic history at redshifts \(z \leq 6\).

II. FORMULATION

The CMB brightness temperature fluctuation along the direction of the unit vector \(\hat{n}\) is described by \[31, 32\]

\[
\Delta T(\hat{n}) = T \left\{ \left( \frac{1}{4} \delta_R + \mathbf{v} \cdot \mathbf{n} + \Phi \right)_{\text{LSS}} + \int_{\eta_{\text{LSS}}}^{\eta_0} d\eta \left[ \dot{\Phi} + \dot{\Psi} \right] \right\}.
\]  

(1)
where $T$ is the CMB temperature at present. Here, under the assumption of instantaneous recombination, the Sachs Wolfe effect (first term) is evaluated at the last scattering surface (LSS) and the ISW effect (second term) is integrated from the LSS to the present epoch. The scalar potentials $\Phi$ and $\Psi$ are the metric perturbations in the conformal Newtonian gauge [33, 34], the dots refer to differentiation with respect to the conformal time $\eta$ and we shall use $r = \eta_0 - \eta$ to denote the comoving distance to the conformal time $\eta$.

In the absence of anisotropic stress we have $\Phi = \Psi$ [33] and the ISW term is

$$\Delta T(\hat{n})^{\text{ISW}} = 2T \int_{\eta_{\text{LSS}}}^{\eta_0} d\eta \dot{\Phi}(r \hat{n}, \eta).$$

Expanding this in the basis of spherical harmonics

$$\Delta T(\hat{n})^{\text{ISW}} = \sum_{\ell,m} a_{\ell m}^{\text{ISW}} Y_{\ell m}(\hat{n})$$

and using the identity

$$e^{ik \cdot r} = 4\pi \sum_{\ell,m} (-i)^\ell j_\ell(kr) Y^*_{\ell m}(\hat{k}) Y_{\ell m}(\hat{n})$$

we have

$$a_{\ell m}^{\text{ISW}} = 8\pi T (-i)^\ell \int \frac{d^3k}{(2\pi)^3} \int_{\eta_{\text{LSS}}}^{\eta_0} d\eta \dot{\Phi}(k, \eta) j_\ell(kr) Y^*_{\ell m}(\hat{k})$$

where $\dot{\Phi}(k, \eta)$ is the Fourier transform of $\Phi(r, \eta)$, and $j_\ell(x)$ is the spherical Bessel function.

For sufficiently sub-horizon scales the gravitational potential can be related to the matter density fluctuations $\delta$ via the Poisson equation. In Fourier space this takes the form

$$\tilde{\Phi}(k, \eta) = -\frac{3}{2} \frac{H_0^2 \Omega_m \delta(k, a)}{c^2 k^2} \frac{\delta(k, a)}{a}$$

Further, retaining only the growing mode of density perturbations $\delta(k, a) = \delta(k) D_+(a)$ we have

$$\dot{\Phi}(k, \eta) = \frac{(f - 1)\dot{a}}{a} \tilde{\Phi}(k, \eta)$$

where

$$f = \frac{d \ln D_+}{d \ln a}$$

which we use in eq. (5) to calculate $a_{\ell m}^{\text{ISW}}$. 


The HI 21-cm brightness temperature fluctuations from redshift \( z_{HI} \) can, in Fourier space, be written as

\[
\Delta_{HI}(k) = \bar{T} \bar{x}_{HI} (b + f \mu^2) \delta(k, a)
\]  

(9)

where \( \bar{x}_{HI} \) is the mean HI fraction, \( \mu = \hat{k} \cdot \hat{n} \) and

\[
\bar{T}(z) = 4.0 \text{ mK} \ (1 + z)^2 \left( \frac{\Omega_b h^2}{0.02} \right) \left( \frac{0.7}{h} \right) \frac{H_0}{H(z)}
\]  

(10)

Here it has been assumed that the HI traces the underlying dark matter distribution with a possible bias \( b \). On the large scales under consideration, where the matter fluctuations are in the linear regime, it is reasonable to assume that the baryonic matter follows the underlying dark matter distribution. The term \( f \mu^2 \) has its origin in the HI peculiar velocities \[20, 35\] which have also been assumed to be caused by the dark matter fluctuations. It should be noted that all the terms on the rhs. of equation (9) are to be evaluated at the redshift \( z_{HI} \) at which the HI signal originated. Note that one should include a normalized window function \( W(z) \) in eq. (9) describing the spectral response of an instrument \[25\]. On scales of our interest (\( \ell \lesssim 100 \)), the spectral resolution of the instrument can however be assumed to be much smaller than the features in the HI signal \[36\] and \( W(z) \) can be approximated by a Dirac delta function, so that eq. (9) is, a reasonably good approximation.

Expanding the HI signal in terms of spherical harmonics and proceeding as before we get

\[
a^{\text{HI}}_{\ell m} = 4\pi \bar{T}(z) \bar{x}_{HI} (-i)^\ell \int \frac{d^3k}{(2\pi)^3} \delta(k, a) I_\ell(kr) Y_{\ell m}^*(\hat{k})
\]  

(11)

where

\[
I_\ell(x) = b j_\ell(x) - f \frac{d^2 j_\ell}{dx^2} .
\]  

(12)

We use equations (5) and (11) to calculate \( C^{HI-ISW}_\ell \) the cross correlation angular power spectrum between the HI 21 cm brightness temperature signal and the CMBR ISW signal defined through

\[
\langle a^{\text{ISW}}_{\ell m} a_{\ell m'}^{\text{HI}} \rangle = C^{HI-ISW}_\ell \delta_{\ell \ell'} \delta_{mm'}
\]  

(13)

Note that \( C^{HI-ISW}_\ell \) also depends on \( z_{HI} \) the redshift from which the HI signal originates, or equivalently on \( \nu = 1420 \text{ MHz}/(1 + z_{HI}) \) the frequency of the HI observations, but we do not show this explicitly here. We obtain

\[
C^{HI-ISW}_\ell = A(z_{HI}) \int dk \left[ P(k) I_\ell(kr_{HI}) \int_{\eta_{LS}}^{\eta_0} d\eta F(\eta) j_\ell(kr) \right]
\]  

(14)
where $P(k)$ is the present day dark matter power spectrum,

$$A(z) = -ar{T}(z) x_{HI} D_+(z) \frac{6H_0^3\Omega_{m0}}{\pi c^3}$$

(15)

and

$$F(\eta) = \frac{D_+(f - 1)H(z)}{H_0}$$

(16)

For large $\ell$ we can use the Limber approximation \[9, 37\] which allows us to replace the spherical Bessel functions by a Dirac deltas $\delta_D(x)$

$$j_\ell(kr) \approx \sqrt{\frac{\pi}{2\ell + 1}} \delta_D(\ell + \frac{1}{2} - kr)$$

(17)

whereby the angular cross-correlation power spectrum takes the simple form

$$C_{HI-ISW}^\ell \approx \frac{\pi A(b + f) F(\ell)}{2\ell^2} P_\ell^r$$

(18)

where $P(k)$ is the present day dark matter power spectrum and all the other terms on the rhs. are evaluated at $z_{HI}$.

III. RESULTS

Figure 1 and Figure 2 respectively show the predicted HI-ISW cross-correlation angular power spectrum $C_{HI-ISW}^\ell$ and the HI-HI angular power spectrum $C_{HI}^\ell$ for a few redshifts $z_{HI}$ in the range $0.5 \leq z_{HI} \leq 5$. We have used equations 14 and 19 to
FIG. 1: The HI-ISW angular power spectrum for redshifts $z = 0.5, 1.0, 2.0, 3.0, 4.0, 5.0$ (top to bottom).

FIG. 2: The HI angular power spectrum $C_{\ell}^{HI}$ at redshifts $z = 0.5, 1.0, 2.0, 3.0, 4.0, 5.0$ (top to bottom).

calculate the cross-correlation angular power spectrum and HI power spectrum respectively. The approximated equation (18) is useful for qualitative description of the results. We have assumed the currently favored ΛCDM cosmological model with parameters $(\Omega_m, \Omega_{\Lambda}, h, \sigma_8, n_s) = (0.28, 0.72, 0.7, 0.82, 0.97)$ [4, 5]. The bias, $b$ for the post reionization HI on large scales is assumed to be linear. We have taken $b = 1$ as the fiducial model.
However, it is important to note that HI in the post reionization epoch is assumed to be distributed in high column density clouds which could be more biased with respect to the underlying cold dark matter distribution.

The shape ($\ell$ dependence) of the cross-correlation signal reflects the shape of the matter power spectrum $P(k)$ (eq. 18). We find a peak in $C_{\ell}^{HI-ISW}$ at $\ell = r_{eq}$, where $k_{eq}$ is the wave vector corresponding to the matter radiation equality. For different redshifts $z_{HI}$ the $\ell$ value corresponding to this peak scales as $\ell \propto r$, the comoving distance to the redshift $z_{HI}$.

The amplitude of the cross-correlation signal $C_{\ell}^{HI-ISW}$ depends on a product of various terms some of which ($\bar{T}, \bar{x}_{HI}, b$) depend on the HI distribution and others ($D_+, f, H$) which depend on the cosmological model. The dimensionless term $f$ quantifies the growth of the dark matter perturbations, and the ISW effect is proportional to $f - 1$. We have $f = 1$ in cosmological models with no dark energy, and we do not expect to have any ISW effect in such models. The term $f - 1$ is a sensitive probe of dark energy. The amplitude of $C_{\ell}^{HI-ISW}$ contains this information combined with unknown parameters related to the HI distribution.

It has been recently proposed that observations of the HI fluctuations at low $z$ can be used to estimate cosmological parameters [40, 41]. It is in principle possible to combine observations of $C_{\ell}^{HI-ISW}$ and $C_{\ell}^{HI}$ to jointly estimate parameters of the HI distribution and the background cosmological model.

**IV. DETECTABILITY AND CONCLUSIONS**

Here we estimate the viability of detecting the HI-ISW cross-correlation signal. The cosmological HI signal is weak and buried under the foregrounds which are orders of magnitude higher than the signal [16, 38, 39, 42, 43]. This is a serious observational problem for auto-correlation studies involving the 21cm radiation. One may separate the foreground components by noting that HI signal (a line emission) decorrelates beyond a certain frequency separation whereas the foregrounds remain correlated over large frequency separations. We shall subsequently assume that foregrounds have been removed. Moreover, the cross-correlation signal is less affected by foregrounds and other systematics. This is because, many of the foregrounds and noise are expected to be uncorrelated between the two maps.

The uncertainty in estimating the cross correlation signal is the sum, in quadrature, of
the instrumental noise and the cosmic variance. While the system noise can, in principle, be reduced by increasing the duration of the observation the cosmic variance sets a fundamental limit in deciding whether the signal can at all be detected or not.

The cosmic variance of the cross-correlation angular power spectrum $C_{\ell}^{HI-ISW}$ is

$$\sigma^2 = \frac{C_{\ell}^{CMB}C_{\ell}^{HI}}{(2\ell + 1)\sqrt{N_c f_s \Delta \ell}}$$

where $C_{\ell}^{CMB}$ is the CMB angular spectrum for which we have used the WMAP5 results [51], $\Delta \ell$ is the width of bands in $\ell$ and $f_s$ is the fraction of the sky common to both the CMBR and HI observations. We have used $\Delta \ell = 10$ for $\ell \leq 100$ and $\Delta \ell = 100$ for $\ell > 100$, and have assumed the most optimistic possibility $f_s = 1$ for our estimates at redshift $z_{HI} = 0.5$. Different frequencies channels in the frequency band of HI observations provide $N_c$ independent estimates of the HI signal which cause a reduction in the cosmic variance by a factor $1/\sqrt{N_c}$. Here we have assumed that the HI observations are carried out across a bandwidth of 32 MHz centered around $z_{HI} = 0.5$ and the HI signal is assumed to be independent at frequency separations of $\sim 1$MHz [21], which gives $N_c = 32$. Using these to estimate the signal to noise ratio $S/N = C_{\ell}^{HI-ISW}/\sigma$ we find that $S/N < 0.45$ for all $z_{HI}$ and $\ell$ and a statistically significant detection is not possible in such cases. It is possible to increase $S/N$ collapsing the signal at different multipoles $\ell$. To test if a statistically significant detection is thus feasible we have collapsed all multipoles less than $\ell$ to evaluate

FIG. 3: The cumulative S/N by collapsing all multiploes less than $\ell$ for different redshifts.
the cumulative S/N defined as \[ S/N_C = \sum_{\ell} \frac{(2\ell + 1)\sqrt{N_{\ell} f_s (C_{\ell}^{HI-I_{SW}})^2}}{C_{\ell}^{CMB} C_{\ell}^{HI}}. \] (21)

Results are shown in Figure 3 for various redshifts (0.4 < z < 3). We find that the contribution in the cumulated S/N comes from \( \ell < 50 \) at all redshifts that we have considered. The cross-correlation signal is largest at (\( z \sim 0.4 \)) and is negligible for (\( z > 3 \)). We further find that although there is an increase in S/N on collapsing the multipoles it is still less than unity. This implies that a statistically significant detection is still not possible. Thus, probing a thin shell of HI doesn’t allow us to detect a cross correlation, the signal being limited by the cosmic variance.

21 cm observations have the advantage that one may probe various redshifts by tuning the frequency of radio observations. This enables us to optimally combine the signal from a large number of thin shells over a continuous range of redshifts. We have considered a range of redshifts (0.4 < z < 3 or 1000 > \( \nu > 350 \)) and combined the signal for independent observations at 32 MHz separations in this range. The S/N cumulated upto a certain redshift is shown in Figure 4. This indicates an increase in the S/N. A cumulated S/N of \( \sim 1.6 \) is attained for redshift upto \( z = 2 \) and there is hardly any increase in S/N on cumulating beyond this redshift. This is reasonable because the contribution from the ISW effect becomes smaller beyond the redshift \( z > 2 \). This S/N is the theoretically calculated
value for an ideal situation and is unattainable for most practical purposes. Incomplete sky coverage, and foreground removal issues would actually reduce the S/N and attaining a statistically significant level is not feasible. We conclude that, within the paradigm of $\Lambda CDM$ cosmology, though there is a weak positive correlation between the CMBR ISW and HI, the signal is much weaker than the individual auto-correlations and a detection is quite unlikely. Certain modified gravity models (eg. [45, 46]) may allow the quantities ($D_+, f, H$) to be different from what they are in the $\Lambda CDM$ model (considered here) [47] and may lead to an increase of the $S/N$. However, since the cross-correlation signal is significant only at large scales we don’t expect the $S/N$ to be much different from the $\Lambda CDM$ predictions.

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