Design principle and meshing analysis of internal gear drive with three contact points

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Abstract
A kind of internal gear drive with three contact points is presented in the article. Firstly, the pinion with circular-arc form and the internal gear with parabola-circular form are proposed, respectively. General equations of tooth profiles are provided. Furthermore, according to gear geometry, mathematical model of the pinion and the internal gear are generated and their tooth surfaces equations are derived. Meanwhile, based on design parameters, 3D solid models of the new gear pair with three contact points are developed utilizing modeling software. Finally, meshing characteristics of generated new gear pair are analyzed, such as sliding ratios and contact stress conditions. General corresponding comparisons with other internal gear drive are also discussed. Research conclusions show that the new internal gear pair with three contact points has low sliding ratios and high contact strength.

Keywords
Internal gear drive, tooth profile equation, mathematical model, sliding ratio, contact stress

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Introduction
As the important mechanical part, internal gears are usually used in planetary gear drive because of the well transmission characteristics. Generally speaking, involute tooth profiles with line contact during design process are commonly tooth profile form. However, the interference effect and tooth profile manufacture are two main problems for practical application.¹ ²

Lots of researches on performance improvement of internal gears were carried out. Litvin and Fuentes³ described and analyzed the undercutting and interference problems of the internal involute gear pair during generation and assembly process. Yang⁴ determined the tooth profile of internal gear with asymmetric involute teeth, and also present its mathematical model by a double envelope method. Utilizing the Archard wear equation, Tunalioglu and Tüz⁵ discussed the influences of tooth profile modification on tooth surfaces wear for internal gears. Cho et al.⁶ established the actual tooth contact models with spring elements for internal gears and the structural analysis method was provided by stiffness analysis of gear teeth. Considering the variable rigidity of the meshing gear pair along the contact locus, Sánchez et al.⁷ established the bending and pitting calculations load conditions based on a new load distribution model for internal gears. Marques et al.⁸ established a basic meshing stiffness analysis model for spur/helical and internal/external gears. Yanase et al.⁹ developed a kind of grinding method for internal gears

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manufacturing through setting a barrel-shaped threaded grinding wheel. Wang et al.\textsuperscript{10–11} presented a kind of high contact ratio internal gear drive. The mathematical models were established. The characteristics were also evaluated and the results show well transmission performance for the new gears. Chen and Shao\textsuperscript{12} researched mesh stiffness of an internal spur gear pair with ring gear rim deformation. The effect of different factors on the mesh stiffness were investigated. Chen et al.\textsuperscript{13} present a manufacturing method for straight internal beveloid gear pair by shaper cutter. Mathematical model of the beveloid gear pair were derived. Gui et al.\textsuperscript{14,15} proposed a mathematical equation of internal compound cycloid gear pair and derived the calculation formula of transverse contact ratio. Otherwise, he also derived the contact analysis models for cycloid internal gear pair under three contact points condition and better mesh characteristics.\textsuperscript{18–20} To further improve the load capacity and contact characteristics, a kind of asymmetric involute internal helical gears and proposed its formation principle of contact tooth profiles.

The authors provided a new single point-contact internal gear form based on curve element.\textsuperscript{17} Compared with the meshing condition of single-point contact, generally multi-point contact has the large contact area and better mesh characteristics.\textsuperscript{18–20} To further improve the load capacity and contact characteristics, a kind of internal gear pair under three contact points condition is proposed. General mathematical model of gear pair is developed, such as the equations of normal tooth profiles and generated tooth surface. Furthermore, 3D solid contact models are established and general simulation analysis is shown to verify the engagement condition. Finally, the meshing characteristics of the gear pair are provided. Also the comparisons between the new internal gears and the involute gears are carried out. This study will provide the useful conclusions for improving the performance of internal gear transmission.

**Design principle of new internal gear pair**

**Normal tooth profile of the pinion**

Normal tooth profile form $\sum_a$ of the pinion is shown in Figure 1. Here, $AB$ is the actual contact tooth profile which is a circular-arc of radius $\rho_{ca}$ and $O_{ca}$ is the center. $BC$ is the fillet tooth profile which is an arc of radius $\rho_{da}$, $\alpha_{da}$ is the parameter angle that determining the point position on $BC$. $h_c$ is the full tooth height. $h_{ca}$ and $h_{cf}$ represent the addendum and dedendum height, respectively. $S_{ca}$ is the tooth thickness. $S_d(O_n - x_n, y_n)$ is the coordinate system of normal tooth profile. $l_{ca}$ is the distance between points $O_{ca}$ and $O_n$.

The actual contact form $AB$ of the pinion is written according to gear geometry in $S_n$ as

$$r_{AB} = \begin{bmatrix} \rho_{ca} \sin \alpha_{ca} \\ \pm(\rho_{ca} \cos \alpha_{ca} - l_{ca}) \\ 0 \\ 1 \end{bmatrix}$$

where $\alpha_{ca}$ is a variable parameter in contact form $AB$ and it should have $\alpha_{min} \leq \alpha_{ca} \leq \alpha_{max}$. The upper and lower sign represent the left and right side of the tooth profile, respectively.

And the fillet tooth profile $BC$ of the pinion is present in $S_n$ as follows.

$$r_{BC} = \begin{bmatrix} -\rho_{da} \cos \alpha_{da} + (\rho_{ca} + \rho_{da}) \sin \alpha_{min} \\ \pm(\rho_{da} \sin \alpha_{da} - (\rho_{ca} + \rho_{da}) \cos \alpha_{min} + l_{ca}) \\ 0 \\ 1 \end{bmatrix}$$

Specially, in equations (1) and (2), the upper and lower symbols represent separately the left and right sides of tooth profile.

**Normal tooth profile of the internal gear**

Figure 2 shows normal tooth profile form $\sum_f$ of the internal gear. Here, $h_f$ is the full tooth height. $h_{fa}$ and $h_{ff}$ represent the addendum and dedendum height, respectively. According to the design method, tooth profile is divided into six parts: $\overline{AB}$, $BP_1$, $P_1P_2$, $P_2P_3$, $P_3C$, and $CD$. Line $\overline{AB}$ is the straight-line section. $\delta$ is the parameter angle that determining the point position on $\overline{AB}$. $BP_1$ and $P_3C$ are the circular-arc working region with radius $\rho_{f1}$ and $\rho_{f2}$, respectively. $O_1$ and $O_2$ are the center points of $BP_1$ and $P_3C$. $P_1P_2$ and $P_2P_3$ are the parabola working region. $P_1$, $P_2$, and $P_3$ are three contact points of tooth profile. $l_{fa}$ is the distance between points $O$ and $O_n$. $S_{cf}$ is the tooth thickness.
Supposed that \( l \) is the distance between \( B \) point to any point on line \( \overline{AB} \), the \( \overline{AB} \) equation can be expressed as follows.

\[
r_{\overline{AB}} = \begin{bmatrix}
\rho_1 \sin \alpha_3 - \left( \rho_1 - \rho_u \right) \sin \alpha_1 - l \cos \delta \\
\left[ l \sin \delta + \rho_1 \cos \alpha_3 - \left( \rho_1 - \rho_u \right) \cos \alpha_1 - l_{fa} \right] \\
0 \\
1
\end{bmatrix}
\]

(3)

where \( \alpha_1 \) and \( \alpha_3 \) are the angles between line \( O_1P_1 \), line \( O_1B \), and the axis \( y_n \), respectively. And the \( BP_1 \) part is described as

\[
r_{BP_1} = \begin{bmatrix}
\rho_1 \sin \alpha - \left( \rho_1 - \rho_u \right) \sin \alpha_1 \\
\left[ \rho_1 \cos \alpha - \left( \rho_1 - \rho_u \right) \cos \alpha_1 - l_{fa} \right] \\
0 \\
1
\end{bmatrix}
\]

(4)

where \( \alpha \) is the angle variable parameter and it has \( \alpha_{3a} \leq \alpha \leq \alpha_{1a} \).

Part \( P_1P_2 \) is a parabola curve and it should be tangent with \( BP_1 \) and \( P_2P_3 \) at points \( P_1 \) and \( P_2 \), respectively. Based on the derived conclusions,\textsuperscript{21} we provide the \( P_1P_2 \) equation in coordinate system \( S_0 \) as follows.

\[
r_{P_1P_2} = \begin{bmatrix}
\lambda_1 \cos \frac{\alpha_2 + \alpha_3}{2} - \frac{\lambda_1^2}{2p_2} \sin \frac{\alpha_2 + \alpha_3}{2} + E_1 \sin \frac{\alpha_2 + \alpha_3}{2} \\
\left( \lambda_1 \sin \frac{\alpha_2 + \alpha_3}{2} + \frac{\lambda_1^2}{2p_2} \cos \frac{\alpha_2 + \alpha_3}{2} - E_1 \cos \frac{\alpha_2 + \alpha_3}{2} + l_{fa} \right) \\
0 \\
1
\end{bmatrix}
\]

(5)

where \( \lambda_1 \) is the parabolic parameter, and it satisfy

\[-p_u \sin \frac{\alpha_3 - \alpha_{3a}}{2} \leq \lambda_1 \leq p_u \sin \frac{\alpha_2 - \alpha_{3a}}{2} \]. \( p_1 \) is the parabolic curve \( P_1P_2 \) coefficient and \( p_1 = \rho_u \cos \frac{\alpha_2 - \alpha_{3a}}{2} \).

Generally, \( P_2P_3 \) is also a parabola curve and it should be tangent with \( P_1P_2 \) and \( P_3C \), respectively. The equation of \( P_2P_3 \) in \( S_n \) is present as

\[
r_{P_2P_3} = \begin{bmatrix}
\lambda_2 \cos \frac{\alpha_2 + \alpha_3}{2} - \frac{\lambda_2^2}{2p_2} \sin \frac{\alpha_2 + \alpha_3}{2} + E_2 \sin \frac{\alpha_2 + \alpha_3}{2} \\
\left( \lambda_2 \sin \frac{\alpha_2 + \alpha_3}{2} + \frac{\lambda_2^2}{2p_2} \cos \frac{\alpha_2 + \alpha_3}{2} - E_2 \cos \frac{\alpha_2 + \alpha_3}{2} + l_{fa} \right) \\
0 \\
1
\end{bmatrix}
\]

(6)

where \( \lambda_2 \) is the parabolic parameter and it has \(-p_u \sin \frac{\alpha_3 - \alpha_{3a}}{2} \leq \lambda_2 \leq p_u \sin \frac{\alpha_2 - \alpha_{3a}}{2} \). \( p_2 \) is the parabolic curve \( P_2P_3 \) coefficient and \( p_2 = \rho_u \cos \frac{\alpha_2 - \alpha_{3a}}{2} \). \( E_2 = \rho_u \sin^2 \frac{\alpha_3 - \alpha_{3a}}{2} + \rho_p \cos \frac{\alpha_2 - \alpha_{3a}}{2} \).

Based on the tooth profile analysis, the parts \( P_3C \) and \( CD \) are also written separately as

\[
r_{P_3C} = \begin{bmatrix}
\rho_2 \sin \alpha' - \left( \rho_2 - \rho_u \right) \sin \alpha_{a2} \\
\left[ \rho_2 \cos \alpha' - \left( \rho_2 - \rho_u \right) \cos \alpha_{a2} - l_{fa} \right] \\
0 \\
1
\end{bmatrix}
\]

(7)

and

\[
r_{CD} = \begin{bmatrix}
\rho_k \left( \cos \theta_k - 1 \right) + h_{fa} \\
- \rho_k \sin \theta_k \\
0 \\
1
\end{bmatrix}
\]

(8)

where \( \alpha' \) is the angle variable parameter and it has \( \alpha_{a2} \leq \alpha' \leq \alpha_{a3} \). \( CD \) is a tooth root transition arc with radius \( \rho_k \) and \( \theta_k \) is the parameter angle that determining the point position on \( CD \).

**Generation of tooth surfaces**

**General derivation**

Here, we establish the coordinate systems \( S(O-x,y,z) \), \( S_1(O_1-x_1,y_1,z_1) \), and \( S_n(O_n-x_n,y_n,z_n) \) in Figure 3. \( x_nO_ny_n \) is the normal plane and axis \( z_n \) is tangent with spiral tooth direction. \( M \) is any point of tooth profile on normal plane. Plane \( xOy \) is the end plane. The angle between planes \( x_nO_ny_n \) and \( xOy \) is \( \beta \), which is also helical angle of the gear. Coordinate system \( S_1(O_1-x_1,y_1,z_1) \) is rigidly attached to the gear and \( S(O-x,y,z) \) makes a spiral motion related to \( S_1(O-x_1,y_1,z_1) \). During this process, \( \phi \) is the rotational angle and its feed distance is \( r \theta \cot \beta \), where \( r \) is the pitch circle radius. Tooth surfaces are developed through series of coordinate transformation if giving the form of tooth profile on normal plane \( x_nO_ny_n \).
So the transformation relationship between different coordinate systems can be written as follows.

\[
M_{0n} = \begin{bmatrix}
1 & 0 & 0 & r \\
0 & \cos \beta & 0 & 0 \\
0 & 0 & \sin \beta & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]  

(9)

\[
M_{10} = \begin{bmatrix}
\cos \phi & -\sin \phi & 0 & 0 \\
\sin \phi & \cos \phi & 0 & 0 \\
0 & 0 & 1 & r \phi \cot \beta \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]  

(10)

where \(M_{0n}\) and \(M_{10}\) are the transformation matrix from \(S_n(O_n - x_n, y_n, z_n)\) to \(S(O - x, y, z)\) and from \(S(O - x, y, z)\) to \(S_1(O_1 - x_1, y_1, z_1)\), respectively. Then, general relation between systems \(S_n(O_n - x_n, y_n, z_n)\) and \(S_1(O_1 - x_1, y_1, z_1)\) can be derived as

\[
\begin{bmatrix}
\rho_1 \sin \alpha_{a3} - \left(\rho_1 - \rho_a\right) \sin \alpha_{a1} - l \cos \delta \sin \phi_2 \pm \left| l \sin \delta + \rho_f \cos \alpha_{a3} \right| \\
-\left(\rho_f - \rho_a\right) \cos \alpha_{a1} - l \sin \delta \cos \phi_2 + \rho_r \cos \phi_2 \\
[\rho_f \sin \alpha_{a3} - \left(\rho_1 - \rho_a\right) \sin \alpha_{a1} - l \cos \delta \sin \phi_2 \mp \left| l \sin \delta + \rho_f \cos \alpha_{a3} \right| - \left(\rho_f - \rho_a\right) \cos \alpha_{a1} - l \sin \delta \cos \phi_2 + \rho_r \cos \phi_2 \\
\pm \left| l \sin \delta + \rho_f \cos \alpha_{a3} - \left(\rho_f - \rho_a\right) \cos \alpha_{a1} - l \sin \delta \cos \phi_2 + \rho_r \cos \phi_2 \right| \sin \beta + r_2 \phi_2 \cot \beta \\
\end{bmatrix}
\]  

(15)

So tooth surfaces of the gear can be developed by

\[
r_1 = M_{1n} r_n
\]  

(12)

**Tooth surfaces equation**

Substituting equations (1) and (2) to equation (12), separately, tooth surfaces of the pinion are solved as

\[
r_{\Sigma B} = [\rho_a \sin \alpha_{a3} \cos \phi_1 \pm (\rho_a \cos \alpha_{a1} - l \sin \delta) \cos \beta \sin \phi_1 + r_1 \cos \phi_1] \\
\rho_a \sin \alpha_{a3} \sin \phi_1 \mp (\rho_a \cos \alpha_{a1} - l \sin \delta) \cos \beta \sin \phi_1 + r_1 \sin \phi_1 \\
\pm (\rho_a \cos \alpha_{a1} - l \sin \delta) \sin \beta + r_1 \phi_1 \cot \beta ]
\]  

(13)

and

\[
r_{\Sigma A} = [-\rho_d \cos \alpha_{a1} + (\rho_a + \rho_d) \sin \alpha_{a1} \sin \phi_1 \pm |\rho_{da} \sin \alpha_{a1}| \sin \phi_1 + r_1 \sin \phi_1] \\
-\rho_d \cos \alpha_{a1} + (\rho_a + \rho_d) \sin \alpha_{a1} \sin \phi_1 \mp |\rho_{da} \sin \alpha_{a1}| \sin \phi_1 \pm |\rho_{da} \sin \alpha_{a1}| \sin \phi_1 \pm |\rho_{da} \sin \alpha_{a1} - (\rho_a + \rho_d) \sin \alpha_{a1} + l \sin \delta | \sin \beta + r_1 \phi_1 \cot \beta ]
\]  

(14)

where \(r_1\) and \(\phi_1\) are the pitch circle radius and rotational angle of the pinion, respectively.

Substituting equations (3) ~ (8) to equation (12), respectively, tooth surfaces of the internal gear are solved as follows.
where \( r_2 \) and \( \phi_2 \) are the pitch circle radius and rotational angle of the internal gear, respectively.

### Solid models

Solid models of tooth surfaces are developed by professional software. According to the basic tooth profile parameters determined in Section “Design principle of new internal gear pair” and tooth surfaces equations in Section “Generation of tooth surfaces,” the MATLAB program is developed to solve the related results. Point set of tooth surfaces are obtained using the mathematical calculation function. Further, utilizing the spline curve and surface modeling function in UG, the final tooth surfaces are established. Given the design parameters in Table 1, the 3D solid models of the new gear pair are generated, as displayed in Figure 4.

Specially, the pinion with small tooth number \((z_1 = 6)\) in this numerical example is designed. No tooth surface interference occurs during the meshing process through motion simulation. And three contact points on the mated tooth surfaces always move along the axial direction.

### Meshing characteristics analysis

**Sliding ratio**

Generally speaking, a lower sliding ratio between two mated tooth surfaces will have the higher transmission efficiency.

### Table 1. Design parameters of the new internal gear pair.

| Design parameters | Values |
|-------------------|--------|
| Gear module \(ml(\text{mm})\) | 6 |
| Pressure angle \(\alpha(\degree)\) | 20 |
| Helix angle \(\beta(\degree)\) | 25.84 |
| Tooth number of pinion 1 \(z_1\) | 6 |
| Tooth number of internal gear 2 \(z_2\) | 30 |
| Center distance \(a(\text{mm})\) | 80 |
| Gear ratio \(i_{21}\) | 5 |
| Tooth width \(B(\text{mm})\) | 38 |
efficiency. In this paper, we provide the three points contact models of tooth profiles. The set of contact points at any motion process is cylinder helix curve. So we develop the calculation thought of sliding ratios considering the contact locus of tooth surfaces according to its contact essence.

Figure 5 shows the sliding conditions of three points contact surfaces. Supposing that the pinion with contact curve $G_1$ transmits motion to the internal gear with the mated contact curve $G_2$, they mesh with each other on point $W$. $\Delta \lambda_1$ and $\Delta \lambda_2$ are the traveling arcs of contact curves $\Gamma_1$ and $\Gamma_2$ after meshing time $\Delta t$, respectively. Supposing that there is relative sliding between contact curves, the arc lengths $WW_1 \neq W W_2$, the sliding arc is the difference between $\Delta \lambda_1$ and $\Delta \lambda_2$. Then the sliding ratios are solved using the following calculation formulas.

$$
\eta_1 = \lim_{\Delta \lambda_1 \to 0} \frac{\Delta \lambda_1 - \Delta \lambda_2}{\Delta \lambda_1} \frac{d \lambda_1 - d \lambda_2}{d \lambda_1} = \frac{d \lambda_1 - d \lambda_2}{d \lambda_1}
$$

$$
\eta_2 = \lim_{\Delta \lambda_2 \to 0} \frac{\Delta \lambda_2 - \Delta \lambda_1}{\Delta \lambda_2} \frac{d \lambda_2 - d \lambda_1}{d \lambda_2} = \frac{d \lambda_2 - d \lambda_1}{d \lambda_2}
$$

where $d \lambda_1 = \sqrt{dx_1^2 + dy_1^2 + dz_1^2} dt$, $d \lambda_2 = \sqrt{dx_2^2 + dy_2^2 + dz_2^2} dt$.

Based on the theoretical derivations by Liang et al., the equations of contact curves, that is, the spatial cylinder helix curves, on the mated internal gear pair can be obtained. We solve the sliding ratios results shown in Figure 6(a) using the parameter data in Table 1. The sliding ratios results always maintain constant conditions if the contact curve parameter $\theta$ changes. The sliding ratios values are calculated as $\pm 0.08$ closed to 0 and there is the approximate pure rolling condition along the axial direction between the new gear pair.

Utilizing the same analytical parameters, sliding ratios of the general involute internal gears are also solved for comparison. The results are displayed in

Figure 6(b). Obviously, if the parameter $\theta$ changes, the sliding ratios results are variable and they will reach the value 0 at pitch point. Compared with the new internal gear pair, there is the greater relative sliding between tooth surfaces.

Stress analysis

Finite element method will be used in this section to analyze the actual contact stress. Here, we provide three kinds of different gear pair: single contact point condition, the three contact points condition, and the involute contact condition.

After finishing the establishment of solid models under the same parameters, the FEM software Work Bench 18.0 is used for calculation. Element solid 186 is considered as the general mesh form. The number of contact elements is 90,720 and the number of nodes is 393,890. The pinion and the internal gear are separately considered as the contact and target object. The material is 40Cr with the properties of Young’s Modulus $E = 206$ GPa and Poisson’s ratio $\lambda = 0.3$.

We use the asymmetric contact method and set the friction coefficient to 0.12. Based on the method of elastic theory, finite element models of three kinds of internal gear pair are processed. The input load torque is 200 Nm on the inner diameter of the pinion. The general analysis model with single tooth is shown in Figure 7.

Figure 8 shows the analytical results of internal gear pair under single point contact condition. The meshing area of gear pair which is elliptical form occupies about 1/2 working tooth height. The long-axis direction of the ellipse is tooth width direction. The maximum value of contact stress is 1266.5 MPa and it appears on the
design position of tooth profile. The maximum values of the von Mises stress of the pinion and the internal gear are 793.69 and 984.14 MPa, separately. The maximum values of shear stress of the gear pair are 411.69 and 371.45 MPa, respectively.

Figure 9 shows the analytical results under three contact points condition. Contact region is almost the entire working tooth height, which is oval with a small upper part and a large lower part. The maximum value of contact stress is 923.26 MPa. The maximum value of the von Mises stress of the gear pair are 449.1 and 928.02 MPa. The maximum value of shear stress of the gear pair are 157.5 and 266.93 MPa, respectively.

Figure 10 shows the analytical results of the involute internal gear pair. Obviously, its contact locus of gear pair is a skew line, and the maximum value of contact stress happening on the position of tooth top is 1639 MPa. The maximum value of the von Mises stress of the gear pair are 1234 and 806.16 MPa. The maximum value of shear stress of the gear pair are 107.39 and 155.34 MPa, respectively.

The comparison analytical results under three kinds of contact conditions are displayed in Table 2. The maximum value of contact stress under three contact points condition is 27.1% and 43.7% lower than that of the internal gear pair under single contact point condition and under the involute contact condition, respectively. The shear stress and von Mises stress of tooth surfaces of the developed gear pair under three contact points condition are also improved. The analytical results are beneficial for the improvement of transmission characteristics. The further study for the dynamic analysis and key manufacturing technology will be carried out. This gear drive is expected to have excellent transmission performance.
Conclusions

(1) We propose a kind of internal gear drive with three contact points. For the pinion, theoretical tooth profile including the actual meshing part and fillet part are proposed. Similarly, for the internal gear, tooth profile including different line segment, circular-arc part, and parabola part are also provided. The general equations of tooth profiles are written, which lay the important foundation for generation of tooth surfaces.

(2) According to gear geometry theory, generation method of general tooth surfaces is put forward. Utilizing aforementioned normal tooth profiles, the final contact tooth surfaces of the pinion and the internal gear are developed, respectively. General expressions of the new internal gear drive are derived.

(3) Numerical example of the developed internal gear pair under three contact points condition is provided. Actual tooth surfaces are established through 3D modeling software UG and mathematical calculation software MATLAB by the given design parameters. Furthermore, the 3D solid models of the gear pair are developed. The pinion with small tooth number is designed and it shows the ideal transmission conditions based on motion simulation.

(4) Sliding ratios of tooth surfaces of the new gear pair are calculated based on the derived conclusions. And the comparison with involute internal gear pair under the same parameters is also

Figure 8. Analytical results under single contact point condition (unit: MPa); (a) contact status, (b) contact stress, (c) von Mises stress of the pinion, (d) shear stress of the pinion, (e) von Mises stress of the internal gear, and (f) shear stress of the internal gear.
discussed. Discussion results show that the approximate pure rolling condition between the new gear pair in the axial direction will happen. Furthermore, the stress analytical process of the generated tooth surfaces is conducted by FEM. The comparison results show that the maximum value of contact stress of the new gear drive is lower than that of other two gears. The shear stress and von Mises stress of tooth surfaces are also improved. The analytical results are beneficial for the improvement of transmission characteristics. Further study on meshing performance and dynamics property will be carried out.

Figure 9. Analytical results under three contact points condition (unit: MPa): (a) contact status, (b) contact stress, (c) von Mises stress of the pinion, (d) shear stress of the pinion, (e) von Mises stress of the internal gear, and (f) shear stress of the internal gear.

Table 2. Stress analysis results of three kinds of contact conditions.

| Gear pair type               | Single-contact point internal gear pair | Three-contact points internal gear pair | Involute internal gear pair |
|-----------------------------|----------------------------------------|----------------------------------------|-----------------------------|
|                             | Pinion | Internal gear | Pinion | Internal gear | Pinion | Internal gear |
| Contact stress value (MPa)  | 1266.5 | 923.26        | 923.26 | 1639          |        |              |
| Shear stress value (MPa)    | 411.69 | 371.45        | 157.5  | 266.93        | 107.39 | 155.34       |
| Von mises stress value (MPa)| 793.69 | 984.14        | 449.1  | 928.02        | 1234   | 806.16       |
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Figure 10. Analytical results of involute internal gear pair (unit: MPa): (a) contact status, (b) contact stress, (c) von Mises stress of the pinion, (d) shear stress of the pinion, (e) von Mises stress of the internal gear, and (f) shear stress of the internal gear.
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