Inert-Sterile Neutrino: Cold or Warm Dark Matter Candidate

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In usual particle models, sterile neutrinos can account for the dark matter of the Universe only if they have masses in the keV range and are warm dark matter. Stringent cosmological and astrophysical bounds, in particular imposed by X-ray observations, apply to them. We point out that in a particular variation of the inert doublet model, sterile neutrinos can account for the dark matter in the Universe and may be either cold or warm dark matter candidates, even for masses much larger than the keV range. These Inert-Sterile neutrinos, produced non-thermally in the early Universe, would be stable and have very small couplings to Standard Model particles, rendering very difficult their detection in either direct or indirect dark matter searches. They could be, in principle, revealed in colliders by discovering other particles in the model.

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Since the first indications of the existence of dark matter more than seven decades ago [1], many different strong pieces of evidence in its favor have been accumulated (for reviews, see eg. Refs. [2, 3, 4]). The presence of dark matter has been revealed so far through its gravitational effects. Much effort is being devoted to the detection of dark matter annihilation or decay products or the scattering of dark matter particles off nuclei. However dark matter may consist of particles which will not be revealed (at least in the near future) in this type of searches. We provide here an example of a dark matter candidate found in a simple extension of the Standard Model (SM), whose nature could be indirectly proven only through the discovery and study in colliders of other non-standard particles predicted within the model. The dark matter particle candidate we study here is a sterile neutrino with mass in the tens of keV to the tens of GeV range and produced non-thermally in the early Universe, which can be either warm dark matter (WDM) or cold dark matter (CDM).

One or more gauge singlet right-handed (sterile) neutrinos are included in simple extensions of the SM which can easily accommodate neutrino oscillation data [5, 6, 7, 8, 9, 10, 11]. These data show that at least two of the active neutrinos have a non-zero mass. In many models sterile neutrinos are the right-handed Dirac mass partners of the active neutrinos. In some see-saw-inspired models, sterile neutrinos have large Majorana masses, which leads to three light (mostly active) neutrinos and several heavier (mostly sterile) neutrinos, the lightest of which is an attractive dark matter candidate [8]. Since this candidate necessarily decays into a light neutrino and a photon, to constitute the dark matter its lifetime must be much longer that the age of the Universe. Thus, this dark matter candidate might be detected through the photons produced in its decay in the dark halos of galaxies. Moreover, to have the required dark matter relic density, the lightest sterile neutrino must usually have a mass in the keV range, although this depends on the mechanism through which sterile neutrinos are produced in the early Universe.

Relic sterile neutrinos with only standard model interactions are produced in the early Universe through active-sterile neutrino oscillations. Sterile neutrinos produced through non-resonant oscillations [6, 7, 8, 9, 10, 11] must have masses $M_s$ in the keV range to account for the whole of the dark matter and are WDM. Through a combination of X-ray and structure formation constraints, an upper bound $M_s \leq 3 - 4$ keV has been obtained [7, 8, 9, 10, 11] (see, however, Ref. [12] for a very recent weak hint of a possible signal). Lyman-α forest data has been used to impose the lower bound of $M_s \geq 5.6$ keV [12] (see also Refs. [13, 14, 15] for previous bounds) on non-resonantly produced sterile neutrinos, or the revised limit of $M_s \geq 8$ keV obtained by a new analysis [16], which combined with the previous upper bounds would exclude non-resonantly produced dark matter sterile neutrinos. Even disregarding the controversial Lyman-α bounds, the mass range allowed for these neutrinos is very restricted because there is an independent lower bound $M_s \geq 1.8$ keV [14, 15] derived from the analysis of phase space density evolution of dwarf spheroidal galaxies. In general, these bounds do not consider the possibility of a very large lepton asymmetry. In the presence of a large lepton asymmetry $\mathcal{L} \equiv (n_{\nu_e} - n_{\bar{\nu}_e})/s > 10^{-6}$, where $n_{\nu_e}$ and $n_{\bar{\nu}_e}$ are the number densities of neutrinos and antineutrinos and $s$ is the entropy density in the Universe, sterile neutrinos may be produced in the early Universe through resonant oscillations [14, 15]. Considering the upper limit of the lepton asymmetry imposed by Big Bang Nucleosynthesis (BBN), $\mathcal{L} < 2.5 \times 10^{-3}$ [16], the range 1 keV $\leq M_s \leq 50$ keV is in principle allowed for sterile neutrino dark matter [16, 20, 21]. In slightly more complicated models, sterile neutrino dark matter may be produced as decay products of, for example, a heavy singlet scalar [21, 22, 23], or may not completely thermalize as in low reheating temperature scenarios [24]. Yet, in all these models the X-ray constraints are important.

Here, we consider a small variation of the SM in which the lightest sterile neutrino is stable (hence it does not produce photons as decay products) and may constitute all of the dark matter. We study a variation of the Inert...
Doublet Model \([25, 26]\) (in itself an extension of the model in Ref. \([27]\)). In this model one scalar doublet, \(\eta = (\eta^+, \eta^0)\) and three sterile neutrinos, which we call Inert-Sterile neutrinos, \(N_i\) with \(i = 1, 2, 3\), odd under a new parity \(Z_2\), are added to the SM. All the particles in the SM are even under the additional \(Z_2\) symmetry. These assignments make the new particles “inert” because their couplings to the SM particles are very limited. The leptonic Yukawa couplings in this model are

\[
\mathcal{L}_Y = f_{ij}(\phi^+ \nu_i + \phi^0 l_i) l_j^c + h_{ij}(\nu_3 \eta^0 - l_j \eta^+) N_j + h.c. .
\]  

Here \(\phi = (\phi^+, \phi^0)\) is the SM scalar doublet field, and \(L = (\nu_i, l_i)\) are the SM lepton fields. Under the extended electroweak symmetry \(SU(2)_L \times U(1)_Y \times Z_2\), the fields \(\eta, N, \phi\) and \(L\) are in the \((2,1/2;+), (1,0;-), (2,1/2;+)\) and \((2,-1/2;+)\) representations respectively. The inert and the standard doublet scalar also couple through the scalar potential \([26]\),

\[
V = \mu_2^2 \Phi^1 \Phi + \mu_2^2 \eta^1 \eta + \lambda_1 (\Phi^1 \Phi)^2 + \lambda_2 (\eta^1 \eta)^2 + \lambda_3 (\Phi^1 \Phi)(\eta^1 \eta) + \lambda_4 (\Phi^1 \eta^1 \Phi) + \frac{1}{2} \lambda_5 [(\Phi^1 \eta^2)^2 + h.c.] .
\]  

In particular the last quartic coupling provides the mass splitting between the two physical inert neutral scalar particles \(\eta_H = \sqrt{2} \text{Im}(\eta^0)\) and \(\eta_L = \sqrt{2} \text{Re}(\eta^0)\) \([25, 26]\), which are the heaviest and the lightest for \(\lambda_5 < 0\) (otherwise the two would be exchanged)

\[
m_{\eta_H}^2 - m_{\eta_L}^2 = |\lambda_5| v^2 .
\]  

The masses of the inert scalar bosons are,

\[
\begin{align*}
m_{\eta^+}^2 &= \mu_2^2 + \lambda_3 v^2 / 2, \\
m_{\eta_H}^2 &= \mu_2^2 + (\lambda_3 + \lambda_4 - \lambda_5) v^2 / 2 = \mu_2^2 + (\lambda_L - 2\lambda_5) v^2 / 2, \\
m_{\eta_L}^2 &= \mu_2^2 + (\lambda_3 + \lambda_4 + \lambda_5) v^2 / 2 = \mu_2^2 + \lambda_5 v^2 / 2 .
\end{align*}
\]  

Here \(v/\sqrt{2} = 174\text{ GeV}\) is the vacuum expectation value (VEV) of the SM Higgs field (the inert scalar does not acquire a VEV), \(m_{\eta^\pm}\) is the mass of the charged scalars, \(\lambda_5\) has been chosen to be real and we define \(\lambda_L = \lambda_3 + \lambda_4 + \lambda_5\). The only other terms in the Lagrangian allowed by the \(Z_2\) symmetry are Majorana mass terms for the Inert-Sterile neutrinos,

\[
\frac{1}{2} M_i N_i N_i + h.c. .
\]  

The \(Z_2\) symmetry forbids sterile-active neutrino mixings. The \(N_i\)’s are not the Dirac mass partner of the \(\nu_i\) as in usual extensions of the SM and active neutrino Majorana masses are generated at one-loop level. The active neutrino mass matrix elements are \([26]\)

\[
(\mathcal{M}_\nu)_{ij} = \sum_k h_{ik} h_{jk} \frac{M_k}{16\pi^2} \left[ \frac{m_{\eta_H}^2}{m_{\eta_H}^2 - M_k^2} \ln \left( \frac{m_{\eta_H}^2}{M_k^2} \right) - \frac{m_{\eta_L}^2}{m_{\eta_L}^2 - M_k^2} \ln \left( \frac{m_{\eta_L}^2}{M_k^2} \right) \right] .
\]  

We will assume in what follows that \(m_{\eta_H}\) is of the order of 100 GeV and \(m_{\eta_L}\) of the order of tens of GeV, thus the first term in Eq. \(6\) is dominant.

The \(Z_2\) parity implies that the lightest inert particle is stable and thus a good dark matter candidate. Both the lightest inert scalar \([25, 26, 28, 29, 30, 31, 32, 33]\) and the lightest sterile neutrino \([25, 34, 35]\) could be dark matter candidates. We will assume the second possibility.

In Refs. \([31, 33]\) it was assumed that the mass difference between \(\eta_L\) and \(\eta_H\) is small, i.e. the coupling \(\lambda_5\) is very small. In this case, in order to generate the observed active neutrino masses, the \(h_{ij}\) couplings cannot be very small. In addition, it was assumed that \(m_0 = (m_{\eta_H}^2 + m_{\eta_L}^2)/2 > M_1, M_2, M_3\) and the lightest \(N_i\) is produced thermally. Under these assumptions, Ref. \([31]\) found that the lightest Inert-Sterile neutrino can be CDM and account for the whole of the dark matter if its mass is in the range 7 GeV to 300 GeV.

Here we will explore a range of values of the coupling constants different from those previously considered, namely \(\lambda_5\) not very small and \(h_{ij}\) Yukawa couplings small enough to ensure that the sterile neutrinos \(N_i\) are never in equilibrium in the early Universe. We will not study the flavor structure of the couplings \(h_{ij}\), but only their order of magnitude. We call generically \(h_1, h_2, h_3\) the couplings of \(N_1, N_2\) and \(N_3\), respectively. We assume a hierarchy in the couplings, with \(h_1 < h_2 \approx h_3\). We also assume that only the lightest sterile neutrino, which we take to be \(N_1\), is lighter than the lightest inert scalar \(\eta_L\) and hence, it is the dark matter candidate. The \(\eta_L\) particles are produced thermally in the
early Universe and decouple when they are non-relativistic. The subsequent late decay of the $\eta_L$ produces the Inert-Sterile $N_1$ relic particles that now constitute the dark matter. In this scenario, depending on the mass, abundance and lifetime of $\eta_L$, the $N_1$ can be either CDM or WDM and account for the whole of the dark matter with mass in the range $\sim$few keV to tens of GeV. We will show that all requirements on the model are fulfilled: active neutrino masses of the right order of magnitude are obtained, the upper bound on the $h_{i,j}$ from $\mu \to e\gamma$ is easy to fulfill, all $N_i$ producing reactions in the early Universe are out of equilibrium and the necessary relic density and decay rate of $\eta_L$ for different values of the $\eta_L$ and $N_1$ masses are obtained, while respecting all the collider and other bounds imposed on the model.

Let us see first how large the Yukawa couplings $h$ must be to get reasonable values for the active neutrino masses, i.e. $(M_\nu)_{i,j} \simeq 10^{-1} \text{ eV}$. Using Eq. (5) and assuming that $\eta_L$ is significantly lighter than $\eta_H$, that $M_{2,3}$ is of the same order of magnitude but larger than $m_{\eta_H}$ and that the contributions of $N_2$ and $N_3$ are dominant, we get

$$h_{2,3} \simeq 0.7 \times 10^{-5} \left( \frac{M_{2,3}}{100 \text{ GeV}} \right)^{1/2} \frac{(100 \text{ GeV})}{m_{\eta_H}} \left( \ln \frac{M_{2,3}^2}{m_{\eta_H}^2} \right)^{-1/2} .$$

(7)

Notice that when $m_{\eta_H}$ is large with respect to $m_{\eta_L}$, Eq. (5) implies that $m_{\eta_H} \simeq \sqrt{|\lambda_5|} v$. Moreover, when $M_{2,3}$ are larger than but similar to $m_{\eta_H}$, the logarithm in Eq. (7) is close to 1, thus

$$h_{2,3} \simeq 3 \times 10^{-6} \left( \frac{M_{2,3}}{100 \text{ GeV}} \right)^{1/2} .$$

(8)

Lepton flavor transitions like the $\mu \to e\gamma$ process in Fig. 1 occur in this model. The branching ratio, $B_{\mu \to e\gamma} = \Gamma_{\mu \to e\gamma}/\Gamma_{\mu \to e\nu\nu}$ in the inert doublet model is $[34, 36]$

$$B_{\mu \to e\gamma} = \frac{192\pi^3\alpha}{G_F^2} \left( \sum_j \frac{h_{\mu j} h_{ej}}{4(4\pi)^2 m_{\eta_j}^2} F_2 \left( \sqrt{m_{\eta_j}^2/m_{\eta_j}^2} \right) \right)^2 ,$$

(9)

where $\alpha$ is the fine structure constant and $G_F$ is the Fermi constant. For $M_{2,3} \simeq m_{\eta_L}$ the function $F_2(x) = [1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln(x)]/[6(1-x)^4]^{-1}$ is $F_2(1) \simeq 1/12$, whereas for $M_1 < m_{\eta_L}$ it is $F_2(0) \simeq 1/6$ $[36]$. The experimental upper bound on the branching ratio, $B(\mu \to e\gamma) \leq 1.2 \times 10^{-11}$ $[37]$, implies

$$h_{2,3} \leq 2 \times 10^{-2} \left( \frac{m_{\eta_L}}{100 \text{ GeV}} \right)$$

(10)

for $h_1 << h_{2,3}$.

Let us now see how small the couplings $h_{ij}$ must be in order for the $N_i$ to never be in equilibrium in the early Universe. The upper bounds are particularly important for $N_2$ and $N_3$, whose generic couplings, $h_2$ and $h_3$, are larger than the coupling $h_1$ of $N_1$. The $N_i$ can be produced through the reactions in Fig. 2 i.e. two to two reactions $LL \to N_iN_i$ mediated by any of the four physical inert particles $\eta_H, \eta_L, \eta^+, \eta^-$, which here we call now generically $\eta$, or $\eta \to N_iN_i$ mediated by $L$. The $N_i$ could in principle be produced through the decay $\eta \to N_iL$ and the inverse decay of $\eta L \to N_i$. The production rate for $N_2$, for example, is

$$\Gamma_{N_2} = \sum_L (2 < \sigma v >_{LL \to N_2N_2} + < \sigma v >_{L\bar{L} \to N_2N_3}) n_L^2/n_{eq}^L + \sum_{\eta} (2 < \sigma v >_{\eta \eta \to N_2N_2} + < \sigma v >_{\eta \bar{L} \to N_2N_3}) n_\eta^2/n_{eq}^\eta$$

(11)
where \(n_{eq}^{N_2}\) is the \(N_2\) equilibrium number density which appears in the equation as a normalization factor, and \(n_L\) and \(n_n\) are the number densities of the standard leptons and the inert scalars respectively, at the temperature considered.

Eq. (11) is derived from the Boltzmann equation for the production of \(N_i\) \((i = 1, 2, 3)\) in the process \(ab \rightarrow N_i c\), where \(a, b\) and \(c\) are particles and we assume that only the initial particles, \(a\) and \(b\), have initially a non zero particle density. If the particles \(a\) and \(b\) are in equilibrium, assuming Maxwell-Botzmann density distributions, the time evolution of the number density \(n_{N_i}\) depends on the number densities of particles \(a\) and \(b\) in the following manner (see e.g. Eqs. (5.8) and (5.23) of Chap. 5 of Ref. [38])

\[
\frac{dn_{N_i}}{dt} + 3Hn_{N_i} = \sum_{a,b,c} n_a n_b <\sigma_{ab\rightarrow N_i c}|v| > .
\]

(12)

As usual, it is convenient to change variables to \(Y = n_{N_i}/s\) and \(x = m_{N_i}/T\) (see Eq. (5.16) of Ref. [38]) and obtain

\[
\frac{dY}{dx} = \frac{1}{Hxs} \sum_{a,b,c} n_a n_b <\sigma_{ab\rightarrow N_i c}|v| > .
\]

(13)

Now, dividing and multiplying the right-hand side of Eq. (14) by \(n_{eq}^{N_i}\) as a normalizing function, one gets

\[
\frac{x}{Y^{eq}} \frac{dY}{dx} = \frac{\Gamma_{N_i}}{H}
\]

(14)

where \(\Gamma_{N_i}\) is defined as in Eq. (11) above. Eq. (14) is equivalent in this case to Eq. (5.26) of Ref. [38] and shows that \(Y_{N_i}\) is never significantly different from zero if \(\Gamma_{N_i}/H < 1\).

Following Refs. [39, 40] and making use of Refs. [41, 42], for relativistic \(\eta, N_i\) and \(L\) the thermal averaged cross section of \(LL \rightarrow N_i N_i\) and of \(\eta\eta \rightarrow N_i N_i\) are approximately given by

\[
<\sigma v>_{LL\rightarrow N_i N_i} \simeq 0.7 \times 10^{-1} \frac{h_i^4}{T^2} ,
\]

(15)

\[
<\sigma v>_{\eta\eta\rightarrow N_i N_i} \simeq 3 \times 10^{-1} \frac{h_i^4}{T^2} \ln \left( \frac{4T^2}{M_i^2 + m_\eta^2} \right) ,
\]

(16)

which show that the process \(\eta\eta \rightarrow N_i N_i\) is dominant and

\[
\Gamma_{N_i} \simeq \Gamma_{\eta\eta\rightarrow N_i N_i} \simeq 0.7 \times 10^{-1} h_i^4 T \ln \left( \frac{4T^2}{M_i^2 + m_\eta^2} \right) .
\]

(17)

The production is out of equilibrium if the rate is smaller than the expansion rate of the Universe, \(H\),

\[
\Gamma_{N_i} < H = 1.66 \sqrt{g_\ast} \frac{T^2}{M_{Pl}} ,
\]

(18)

where \(g_\ast\) is the number of degrees of freedom and \(M_{Pl}\) is the Planck mass. Since the right-hand side of Eq. (18) decreases faster than the left-hand side for decreasing \(T\), if the condition is fulfilled for the smallest \(T\) value in the range considered, i.e. the smallest \(T\) for which all the particles involved in the production are relativistic, then it is fulfilled for all larger \(T\).

At high temperatures \(T > M_{2,3} \simeq m_{\eta\eta}\) we need to write the condition in Eq. (18) at \(T \simeq M_k \simeq m_{\eta\eta}\). Thus, the production of relativistic \(N_{2,3}\) is out of equilibrium at \(T > M_{2,3} \simeq m_{\eta\eta}\) if

\[
h_{2,3} < 2 \times 10^{-4} \left( \frac{g_\ast}{106.75} \right)^{1/8} \left( \frac{M_{2,3}}{100 \text{ GeV}} \right)^{1/4} .
\]

(19)
Since we are assuming \( M_1 < m_{\eta_L} << M_{2,3} \), the condition in Eq. (18) for relativistic \( N_1 \) and \( \eta_L \) must be taken at \( T \simeq m_{\eta_L} \), thus the production of relativistic \( N_1 \) from relativistic \( \eta_L \) is out of equilibrium if

\[
h_1 < 2 \times 10^{-4} \left( \frac{g_s}{106.75} \right)^{1/8} \left( \frac{m_{\eta_L}}{10 \text{ GeV}} \right)^{1/4}.
\]  

(20)

At temperatures in the range \( M_{2,3} > T > m_{\eta_L} \), in which the \( N_{2,3} \) are non-relativistic (but \( \eta_L \) and \( L \) are relativistic), the relevant thermal average cross sections for \( N_{2,3} \) production are approximately

\[
< \sigma v >_{\eta_L N_i \rightarrow N_i} \simeq 0.8 \times 10^{-2} \left( \frac{h_i^1}{T^2} \right) \exp \left( -\frac{2M_i}{T} \right),
\]  

(21)

\[
< \sigma v >_{\eta \rightarrow N_i, Ni} \simeq 2 \times 10^{-1} \left( \frac{h_i^1}{T^2} \right) \exp \left( -\frac{2M_i}{T} \right).
\]  

(22)

The production is again dominated by the \( \eta_L \eta_L \rightarrow N_iN_i \) process, thus

\[
\Gamma_{N_i} \simeq \Gamma_{\eta_L \eta_L \rightarrow N_iN_i} \simeq 0.7 \times 10^{-1} h_i^1 \left( \frac{M_{2,3}}{T} \right)^{5/2} \exp \left( -\frac{M_i}{T} \right).
\]  

(23)

Because this rate decreases faster than \( H \), if \( \Gamma_{N_i} < H \) is fulfilled at \( T = M_{2,3} \) where \( \Gamma \) is maximum within the \( T \) interval, the process will be out of equilibrium for lower values of \( T \), thus we obtain

\[
h_{2,3} < 3 \times 10^{-4} \left( \frac{g_s}{106.75} \right)^{1/8} \left( \frac{M_{2,3}}{100 \text{ GeV}} \right)^{1/4}.
\]  

(24)

For still lower temperatures \( T < m_{\eta_{\pm,0}} \), for which all the inert bosons are non-relativistic but the \( N_1 \) are relativistic, we need to verify that the \( N_1 \) are not produced thermally (recall we are assuming that \( m_{\eta_{\pm,0}} > M_1 \)). In this case

\[
< \sigma v >_{\eta \rightarrow N_iN_i} \simeq \frac{3}{4\pi} h_i^4 \frac{M_i^2}{m_\eta^4},
\]  

(25)

and

\[
\Gamma_{N_i} \simeq \Gamma_{\eta \rightarrow N_iN_i} = 10^{-2} h_i^4 \left( \frac{M_i}{m_\eta} \right)^3 \exp \left( -\frac{2m_\eta}{T} \right).
\]  

(26)

This rate decreases faster than \( H \) as \( T \) decreases, thus if \( \Gamma_{N_i} / H < 1 \) at the highest temperature in the range considered, \( T = m_\eta \), the condition is fulfilled at any lower \( T \). Thus,

\[
h_1 < 3 \times 10^{-3} \left( \frac{g_s}{106.75} \right)^{1/8} \left( \frac{m_\eta}{10 \text{ GeV}} \right)^{3/4} \left( \frac{\text{MeV}}{M_1} \right)^{1/2}.
\]  

(27)

After considering all the required upper bounds on the \( h_{ij} \) Yukawa couplings, we conclude that Eq. (19) provides the most restrictive upper bound on the Yukawa couplings of the heaviest inert sterile neutrinos, \( h_{2,3} \), and they are compatible with the value assigned to \( h_{2,3} \) in Eq. (7) which is necessary to account for the active neutrino masses. The most restrictive bound on \( h_1 \), the Yukawa couplings of the lightest Inert-Sterile neutrino \( N_1 \), will be given in Eq. (29) below and is derived from our requirement of a long enough lifetime of the lightest inert bosons \( \eta_L \) into \( N_1 \).

Let us now consider the decays of the \( \eta^\pm \) into Inert-Sterile neutrinos. If \( m_\eta^\pm > m_\eta^0 + m_W \), then the process \( \eta^\pm \rightarrow \eta^0 + W \) can occur. The branching ratio of the decay mode \( \eta^\pm \rightarrow N_iL \) with respect to the dominant \( \eta^\pm \rightarrow \eta^0 + W \) mode is proportional to the ratio of the couplings \( h_i^2 / g_W^2 \), where \( g_W \) is the weak coupling. Using the value of \( h_{2,3} \) necessary to produce the active neutrino masses, given in Eq. (8) with \( |\lambda_5| \approx 0.2 \) for example, \( h_i^2 / g_W^2 \approx 10^{-10} (M_i/100 \text{ GeV}) \), which is negligible. Thus, the heavier Inert-Sterile neutrinos \( N_{2,3} \) are not produced in the decays of the inert charged bosons. Neither the lightest Inert-Sterile neutrino is produced in these decays, since \( h_1 << h_{2,3} \). If, instead \( m_\eta^\pm < m_\eta^0 + m_W \), the 3-body decay \( \eta^\pm \rightarrow \eta^0 + L + \bar{L} \) dominates the decay of \( \eta^\pm \); the branching ratio of \( \eta^\pm \rightarrow N_iL \) then goes as \( h_i^2 / g_W^2 \approx 10^{-11} (M_i/100 \text{ GeV}) \) for the heavier Inert-Sterile neutrinos. The branching ratio is even smaller for \( N_1 \). Again, the decay of the charged inert bosons into the Inert-Sterile neutrinos \( N_i \) is negligible. For the decays of the heavier neutral inert boson \( \eta_H \) the same arguments apply but changing the \( W \)’s by \( Z \)’s. Thus the Inert-Sterile neutrinos are not produced in the decays of \( \eta^\pm \) and \( \eta_H \).
We need to insure that $\eta_L$, the lightest inert scalar particle, is produced thermally in the early Universe and that it is in equilibrium before decoupling while it is already non-relativistic, at freeze-out, $T_{f.o.} < m_{\eta L}$. The dominant processes that maintain the $\eta_L$ particles in equilibrium depend on the couplings of $\eta_L$ with the SM particles. The $\eta_L$ gauge couplings and its couplings in the scalar potential are the same that occur in the inert doublet model in the absence of sterile neutrinos. Using the same couplings, in Ref. 26, 29, 32 $\eta_L$ with mass in the GeV range are found to be good dark matter candidates. We want instead that the $\eta_L$ decay into the lightest inert sterile neutrino $N_1$, which constitutes the dark matter now. After the $\eta_L$ particles decay through the process $\eta_L \rightarrow N_1\nu_1$, there is one $N_1$ per each $\eta_L$. In order for $N_1$ to account for the whole of the dark matter, the number density of $\eta_L$ at their decoupling must be larger for the case considered here than in the scenarios in which they constitute the dark matter 26, 29, 32.

The number density $n_{N_1}$, that is needed for non-relativistic $N_1$ to be the dark matter at present, must be the same relic number density $n_{\eta L}$ the $\eta_L$ should have at present had they not decayed. Thus the relic density of $N_1$ is now $n_{N_1}M_1 = n_{\eta L}M_1$ and

$$\Omega_{N_1}h^2 = \Omega_{\eta L}h^2 \left( \frac{M_1}{m_{\eta L}} \right) ,$$

where $\Omega_{\eta L}h^2$ is the relic density the $\eta_L$ would have at present if they were stable. When the $N_1$ can be either CDM or WDM we require the $N_1$ density to be that of the observed relic density of dark matter $\Omega_{DM}h^2 = 0.1099 \pm 0.0062$ [33]. If the $N_1$ are instead hot dark matter (HDM) we should impose the upper bound $\Omega_{N_1}h^2 \leq 0.014 \equiv \Omega_{HDM−max}h^2$ (the 95% CL on the relic density of light neutrinos) [33].

If $m_{\eta L} > m_W$, the $\eta_L$ annihilate efficiently into two W bosons and their relic density is too small even to constitute the bulk of the dark matter, thus we are not interested in this mass range. When $m_{\eta L} < m_W$, the processes in Fig. 3 and their inverse processes keep $\eta_L$ in equilibrium. The lightest scalar $\eta_L$ coannihilates with the heaviest inert scalar partner $\eta_H$. The coannihilation $\eta_H\eta_L \rightarrow f\bar{f}$ into SM fermions $f$ is mediated by the Z boson and its cross section depends on the mass splitting $\Delta = m_{\eta H} - m_{\eta_L}$ which in turn, depends on $\lambda_5$ (see Eq. 28). The $\eta_L$ also coannihilates with $\eta^\pm$, via $W^\pm$ exchange, with a cross section which depends on the mass split between them. The process $\eta_L\eta_L \rightarrow f\bar{f}$ via Higgs exchange also keeps $\eta_L$ in equilibrium, and in the particular range of masses we explore below is the dominant process. We use the public code MicrOMEGAs 44 to compute the $\eta_L$ relic density.

The decay $\eta_L \rightarrow N_1L$ must happen after the $\eta_L$ freeze-out at $T_{f.o.} = m_{\eta L}/x_f$ where $x_f$ is in the 20 to 30 range. Thus, the decay rate must be $\Gamma_{\eta_L \rightarrow N_1L} \approx \hbar^2 m_{\eta L}/16\pi < H$ for $T > T_{\text{decay}}$ and $\Gamma_{\eta_L \rightarrow N_1L} \approx H$ for $T = T_{\text{decay}}$ with $T_{\text{decay}} < T_{f.o.}$. These conditions lead to the most stringent bound on $h_1$

$$h_1 < 2 \times 10^{-9} \left( \frac{20}{x_f} \right) \left( \frac{m_{\eta L}}{10 \text{ GeV}} \right)^{1/2} \left( \frac{g_*}{10.75} \right)^{1/4} .$$

Note that this bound on $h_1$ is consistent with the previous requirements.

We can now show that the Inert-Sterile neutrinos produced in this model may be either WDM or CDM, which are characterized by the free-streaming length $\lambda_{fs}$ [45, 46]

$$\lambda_{fs} = 2 r t_{EQ} (1 + z_{EQ})^2 \ln \left[ \frac{1}{1 + \frac{1}{r^2 (1 + z_{EQ})^2}} + \frac{1}{r (1 + z_{EQ})^2} \right] .$$

Here the subscript EQ denotes matter-radiation equality and $r = a(t)p(t)/M_1$, where $a(t)$ and $p(t)$ are the scale factor of the Universe and the dark matter particle characteristic momentum at time $t$ respectively. As the Universe expands, the ratio $r$ remains constant. At the time of matter-radiation equality, $\lambda_{fs}$ must be 0.1 Mpc 47 for WDM, which fixes $r \approx 10^{-7}$. At the moment of decay of the $\eta_L$ (we make the approximation of instantaneous decays) the scale factor of the Universe is $a \approx T_0/T_{\text{decay}}$, where $T_0$ is the photon temperature today, and the momentum of the relativistic $N_1$ decay products is $m_{\eta L}/2$. Thus, $r \approx T_0 m_{\eta L}/(2 T_{\text{decay}} M_1)$. Therefore, $r = 10^{-7}$ fixes the mass of $N_1$ to be

$$(M_1)_{WDM} \approx 2.4 \text{ MeV} \left( \frac{m_{\eta L}}{10 \text{ GeV}} \right) \left( \frac{5 \text{ MeV}}{T_{\text{decay}}} \right) .$$
Given a particular $T_{\text{decay}}$, Eq. (31) provides the $N_1$ mass for which the $N_1$ would constitute WDM. Heavier $N_1$ (smaller $\lambda_{fs}$) would be CDM and lighter ones (larger $\lambda_{fs}$) HDM.

We require the decay temperature to be $T_{\text{decay}} \gtrsim 5$ MeV, in order not to affect the success of BBN predictions, and $T_{\text{decay}} < m_{\eta L}/x_f$, because the decays of $\eta L$ happen after they decouple. Thus, the range of masses for which $N_1$ could be a good WDM candidate is

$$24 \text{keV} \left(\frac{x_f}{20}\right) < (M_1)_{\text{WDM}} < 2.4 \text{MeV} \left(\frac{m_{\eta L}}{10 \text{GeV}}\right).$$

Finally, in order to choose suitable sets of parameters there are a number of constraints that need to be considered. The null result for the process $e^+e^- \rightarrow Z^* \rightarrow \eta H \eta L$ in LEP II searches for neutralinos, imposes the bound $m_{\eta H} > 120$ GeV when $m_{\eta L} < 80$ GeV [48]. Alternatively, in a range of parameters we will not explore, the neutral inert boson mass difference must be $m_{\eta L} - m_{\eta L} < 8$ GeV [48] for $m_{\eta L} + m_{\eta H} > m_Z$ due to the LEP I measurement of the Z-width, which implies $m_{\eta L} > 40$ GeV. In addition, the suitable set of parameters should also be within the allowed range provided by electroweak precision measurements [26, 32].

There are also constraints on the $\lambda$ couplings in the scalar potential, Eq. (2). Vacuum stability of the scalar potential imposes $[26]$

$$\lambda_{1, 2} > 0, \lambda_2 < 1$$

and perturbativity of the scalar potential imposes $[26]$

$$\lambda_3^2 + (\lambda_L - \lambda_5)^2 + \lambda_5^2 < 12 \lambda_1^2.$$

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**FIG. 4:** In both panels, $M_{\text{Higgs}} = 160$ GeV, $m_{\eta H} = 125$ GeV and $m_{\eta z} = 130$ GeV. **Upper panel:** The shaded areas correspond to forbidden values of $\lambda_{fs}$ from vacuum stability (cross-hatched violet region) and perturbativity (shaded gray region) arguments. **Lower panel:** From top to bottom, the blue, red and green colored narrow strips show the regions where $N_1$ would have the right dark matter density for the corresponding values of $\lambda_{fs}$. The unshaded background region corresponds to the range in Eq. (32), where the $N_1$ may constitute WDM (above it, $N_1$ can only be CDM and below it, only HDM). For any particular value of $T_{\text{decay}}$ between 5 MeV (upper boundary of unshaded region) and $m_{\eta L}/x_f$ (lower boundary of unshaded region, which depends on $\lambda_{fs}$ through $x_f$) there is one value of $M_1$, given by Eq. (31), within the unshaded background region for which $N_1$ would be WDM (and it would be CDM for all larger values of $M_1$ and HDM for all smaller ones). In order for $N_1$ to be allowed as HDM, its mass must be at least a factor of $\Omega_{DM}h_2^2/\Omega_{HDM_{\text{max}}}h_2 = 0.1099/0.014 \simeq 8$ smaller than that corresponding to the center of the colored bands for a given $m_{\eta L}$.
In Figs. 4 and 5 we show regions of the $m_{\eta L} - M_1$ plane in which $N_1$ has the right dark matter density for two different sets of parameters. The Higgs mass is $M_{\text{Higgs}} = 160$ GeV, $m_{\eta H} = 125$ GeV and $m_{\eta \pm} = 130$ GeV in Fig. 4 and the Higgs mass is $M_{\text{Higgs}} = 500$ GeV, $m_{\eta H} = 150$ GeV and $m_{\eta \pm} = 300$ GeV in Fig. 5. The upper panels of the figures show the bounds on $\lambda_L$ obtained from vacuum stability (cross-hatched violet regions) and pertubativity (shaded gray region) arguments. From top to bottom, the blue, red and green colored narrow bands in the lower panels of Figs. 4 and 5 show the regions in the $m_{\eta L} - M_1$ plane in which $\Omega_{N_1} h^2$ in Eq. 28 is within the $3\sigma$ measured range for the dark matter (either CDM or WDM). The different colors of the narrow bands indicate different values of $\lambda_L$, as shown in the panels. The unshaded background region labeled “WDM Possible” corresponds to the range in Eq. 31 where the $N_1$ may constitute WDM (above it, it can only be CDM and below it, only HDM). For any particular value of $T_{\text{decay}}$ between 5 MeV (which defines the upper boundary of the unshaded region) and $m_{\eta L}/x_f$ (which defines the lower boundary of the unshaded region) there is one value of $M_1$ given by Eq. 31 within the unshaded background region for which $N_1$ would be WDM ($N_1$ would be CDM for all larger values of $M_1$ and HDM for all smaller ones). Notice that the lower boundary of the unshaded region depends on $\lambda_L$ through $x_f$, thus the blue, red, green colors of the lower regions, for which the $N_1$ can only be WDM. Thus, within the unshaded background region $N_1$ could be WDM or CDM, depending on $T_{\text{decay}}$. For a given set of parameters defining the model (and hence a given $T_{\text{decay}}$), in order for $N_1$ to be allowed as HDM, its mass $M_1$ must be, at least, a factor of $\Omega_{\text{DM}} h^2/\Omega_{\text{HDM}} h^2 = 0.1099/0.014 \simeq 8$ smaller than the value at center of the colored band defined by Eq. 28 for a given $m_{\eta L}$. The figures show that the $N_1$ could be HDM even for masses as large as $\sim 1$ keV.

In conclusion, we have shown that Inert-Sterile neutrinos, produced non-thermally in the early Universe, could be a viable WDM or CDM candidate. They are virtually non-detectable in either direct or indirect dark matter searches because of their extremely weak couplings to SM particles. Thus, their existence could be revealed only by discovering other particles of the model in collider experiments. We should keep in mind that the dark matter may consist of an admixture of different types of particles and that particles undetectable in dark matter searches may be part of it. The existence of these particles could only be inferred from collider data, supplemented by the null results from dark matter searches or with results from these searches which find other detectable dark matter components with a density smaller than required to constitute the whole of the dark matter. Unveiling the nature of the dark matter does necessarily require the combination of collider and direct and indirect searches.
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