Getting more out of $V/V_m$ than just the mean

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ABSTRACT
Banhatti earlier set down the procedure to derive cosmological number density $n(z)$ from the differential distribution $p(x)$ of the fractional luminosity volume relative to the maximum volume, $x \equiv V/V_m(0 \leq x \leq 1)$, using a small sample of 76 quasars for illustrative purposes. This procedure is here applied to a bigger sample of 286 quasars selected from Parkes half-Jansky flat-spectrum survey at 2.7 GHz. The values of $n(z)$ are obtained for 8 values of redshift $z$ from 0 to 3.5. The function $n(z)$ can be interpreted in terms of redshift distribution obtained by integrating the radio luminosity function $\rho(P, z)$ over luminosities $P$ for the survey limiting flux density $S_0 = 0.5$ Jy.

Key words: cosmology: miscellaneous

1 INTRODUCTION
The luminosity-volume or $V/V_m$ test has traditionally been used only through the mean and standard error of $x \equiv V/V_m$ as a test to examine the space distribution of gamma-ray bursts for homogeneity. The test was extensively used for a few years for this purpose (e.g., Dezalay et al 1994 and references therein), but the question of the location of gamma-ray bursts within our Milky Way Galaxy or in other possibly distant galaxies was not resolved from these studies. Only on discovering afterglows of gamma-ray bursts at lower (X-ray, optical, infrared and radio) photon energies in distant galaxies and thereafter measuring redshifts of the parent galaxies did it become clear that they form a cosmological population.

Use of $V/V_m$ distribution as outlined in this paper will be rewarding for a sufficiently large well-defined unbiased sample of gamma-ray bursts. It should be possible to construct such a sample from the results of space and ground-based gamma-ray telescopes by carefully taking into account detection methods and thresholds.

2 SAMPLE OF QUASARS USED AND THE WORLD MODEL
Drinkwater et al (1997) define the survey and list the properties of 323 quasars from which 286 can be used for calculating $x \equiv V/V_m$. The sample used is thus 89% complete relative to the survey, which covers 3.90 sr in the sky. Using the limiting flux density $S_0 = 0.5$ Jy at 2.7 GHz, the limiting redshift $z_m$ is calculated for each quasar from its redshift $z$ by

$$
\nu = 2.7 \text{ GHz flux density } S_0, \ \text{and spectral index } \alpha \ \text{(defined by } \alpha \equiv -d(log S_0)/d(log \nu), \ \text{or equivalently, } S_0 \propto \nu^{-\alpha}).
$$

The world model with the parameters $(q_0, \sigma_0, k, \lambda_0) = (1, 1, 1, 0)$ as defined by von Hoerner (1974) is used for the functions of $z$ needed, viz, the luminosity distance $l_\nu(z)$ and volume $v(z)$. These functions are:

$$
(H_0/c)^2 l_\nu^2(z) = z^2/(1 + z)^{1-\alpha},
$$

$$
(H_0/c)^3 v(z) = (3/2)(\sin^{-1} f(z) - f(z)\sqrt{1 - f(z)^2}),
$$

where $f(z) = z/(1 + z)$.

Here, $c/H_0 = \text{speed of light} / \text{Hubble constant}$, defines the linear scale.
3 DERIVING $n(z)$ FROM $p(V/V_m) \equiv p(x)$

3.1 Binning the $z_m$-values

The quasars are first sorted out in increasing order of $z_m$. The limiting redshift is numerically calculated for each quasar using Newton-Raphson iteration (Rajarevathi 2007).

The $z_m$-bins are then selected, so as to have roughly equal numbers of sources (about 30) each, which is good enough to derive the differential distribution of $x \equiv V/V_m(0 \leq x \leq 1)$ for each of the bins. Details of this binning are given in Table 1. Also listed are numbers proportional to the cosmological number densities $n(z_j)$ corresponding to the bin mid-points $z_j$.

The procedure for calculating $n(z_j)$ is outlined later. Table 2 presents, for comparison, the same results for the smaller sample of 76 from Wills & Lynds (1978) used by Banhatti (2009), although the world model used for these calculations is (von Hoerner (1974)) $(q_0, \sigma_0, k, l_0) = (1/2, 1/2, 0, 0)$, for which the functions $f_z(z)$ and $v(z)$ are different (see Banhatti 2009).

3.2 Differential distributions $p_i(x)$ of $x \equiv V/V_m$ for the nine $z_m$-bins

For each of the 9 bins, indexed by $j = 1$ to 9, $p_i(x)$ histograms are plotted with $\Delta x = 0.2$ from $x = 0$ to 1, making five $x$-bins over the range $[0, 1]$ of $x$. For $p_i(x)$, a curve is drawn by eye. For all other $p_i(x)$, $i = 2$ to 9, the extrapolated frequency polygon, with slightly higher slope than the last segment (to $x = 1$), is used. Cosmological number density $n(z_j)$ is then calculated from the formula (Banhatti 2009):

$$n(z_j) = \frac{(\Omega/3)(c/H_0)^3}{V_j} \sum_{i=1}^{9} N_i \frac{v(z_j)}{v(z_i)} p_j(x_{ij}),$$

where $x_{ij} = v(z_j)/v(z_i)$, and $\Omega$ is the survey solid angle. In this formula, $N_i$ are the bin populations of the 9 bins. Details of $n(z_j)$ calculation are shown in Table 3. Examples are given below:

$n(z_j) \propto \frac{(N_0/\nu(z_j))p_1(x_{1j})}{N_8/\nu(z_8)} + (N_9/\nu(z_9))p_9(x_{9j})$.

The $p_1(x)$ values are interpolated from the $p(x)$ frequency polygon. Thus, for $n(z_j)$ calculation, there are 9 terms to sum (many of which happen to be 0 due to $p_1(x_{1j})$ being 0). For $n(z_2)$ there are 8 terms, and so on. Finally, for $n(z_9)$ there is only one term: $n(z_9) \propto (N_9/\nu(z_9))p_9(x_{99}) = (31/2.112)2.65 = 38.9 \approx 39$.

Using $ln(z_m)$ in place of $z_m$ in the whole analysis leads to essentially the same results.

The cosmological number density $n(z)$ is interpreted as the redshift distribution $n(z; S_0)$, which is the integral of the radio luminosity function $\rho(P, z)$ over all luminosities present in the sample as determined by the flux density limit. Thus,

$$n(z; S_0) = \int_{S_0}^{\infty} \rho(P, z) dP.$$  \hspace{1cm} (4)

This interpretation of $n(z)$ (derived from the differential distribution $p(x) \equiv p(V/V_m)$) as the redshift distribution $n(z; S_0)$ needs to be explored further and utilized in deriving the cosmological evolution of the source population (here quasars).

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Table 1. Limiting redshifts, their bins, mid-points & populations plus derived cosmological number densities using 286 quasars over 3.90 sr in the sky

| $z_m$-bin | 0 to 0.3 | 0.3 to 0.7 | 0.7 to 1.2 | 1.2 to 1.5 | 1.5 to 1.8 | 1.8 to 2.2 | 2.2 to 2.8 | 2.8 to 4.0 | > 4.0 |
|-----------|----------|------------|------------|------------|------------|------------|------------|------------|---|
| $z_j$ (bin mid-pt) | 0.15 | 0.5 | 0.95 | 1.35 | 1.65 | 2.0 | 2.5 | 3.4 | 300* |
| Bin pop. | 29 | 34 | 34 | 32 | 33 | 32 | 30 | 31 | 31 |
| $j$ (bin no.) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| $n(z_j)$ | 48770. | 4717. | 1560. | 1167. | 865. | 642. | 464. | 194. | 39. |
| $\log[n(z_j)]$ | 4.69 | 3.67 | 3.19 | 3.07 | 2.94 | 2.81 | 2.67 | 2.29 | 1.59 |

Table 2. Results of earlier similar calculation for a sample of 76 quasars

| $z_m$-bin | 0 to 0.8 | 0.8 to 1.6 | 1.6 to 2.4 | 2.4 to 3.2 |
|-----------|----------|------------|------------|------------|
| $z_j$ (bin mid-pt) | 0.4 | 1.2 | 2.0 | 2.8 |
| Bin pop. | 19 | 31 | 16 | 10 |
| $j$ (bin no.) | 1 | 2 | 3 | 4 |
| $n(z_j)$ | 1307. | 255. | 67. | 22. |
| $\log[n(z_j)]$ | 3.12 | 2.41 | 1.83 | 1.34 |

Table 3. Calculation of $n(z(j))$. Values in rows labelled $i = 1, i = 2$ & so on are $p(x(ij))$.

| j | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $N_i$ | 29 | 34 | 34 | 32 | 33 | 32 | 30 | 31 | 31 |
| $z_j$ | 0.15 | 0.50 | 0.95 | 1.35 | 1.65 | 2.00 | 2.50 | 3.40 | 300* |
| $v(z_j)$ | 0.002231 | 0.03835 | 0.1251 | 0.2126 | 0.2773 | 0.3492 | 0.4436 | 0.5890 | 2.112 |
| $i = 1$ | 1 |
| $i = 2$ | 0.058 | 1 |
| $i = 3$ | 0.018 | 0.307 | 1 |
| $i = 4$ | 0.010 | 0.180 | 0.588 | 1 |
| $i = 5$ | 0.008 | 0.138 | 0.451 | 0.767 | 1 |
| $i = 6$ | 0.006 | 0.110 | 0.358 | 0.609 | 0.794 | 1 |
| $i = 7$ | 0.005 | 0.086 | 0.282 | 0.479 | 0.625 | 0.787 | 1 |
| $i = 8$ | 0.004 | 0.065 | 0.212 | 0.361 | 0.471 | 0.593 | 0.753 | 1 |
| $i = 9$ | 0.001 | 0.018 | 0.059 | 0.101 | 0.131 | 0.165 | 0.210 | 0.279 | 1 |
| $n(z_j)$ | 48770. | 4717. | 1560. | 1167. | 865. | 642. | 464. | 194. | 39. |