Supersymmetry without R-Parity

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I discuss the motivations for supersymmetry, focusing on models with broken R-parity and lepton number. After describing the main theoretical features of these models, I discuss some of the signals expected at colliders such as Tevatron, LEP II, NLC and LHC.

1 Introduction

The Standard Model (SM) is very successful in describing the fundamental elementary particle interactions, except possibly neutrinos. It leaves many unanswered questions and theoretical problems. A basic ingredient in the SM is the breaking of the electroweak symmetry via the Higgs mechanism. One of its most outstanding puzzles is the fact that the mass of the SM Higgs boson is unstable against quantum corrections, a fact known as the hierarchy problem. One of the main theoretical motivations for supersymmetry (SUSY) is that it allows for a stable hierarchy between the electroweak scale $m_{\text{weak}}$ responsible for the W and Z masses and the mass scale of unification. If SUSY holds as a symmetry down to the scale $M_{\text{SUSY}} \sim m_{\text{weak}}$, then the Higgs mass is stabilized under radiative corrections. This happens because the loops containing standard particles are partially cancelled by those containing supersymmetric particles. Supersymmetry now appears as the most natural and well-founded solution to the hierarchy problem, at least in a technical sense.

Another drawback of the SM is that the weak, electro-magnetic and strong interactions are characterized by couplings of different strength. Supersymmetry also allows in an elegant and natural way the unification of the three gauge couplings, when evolved via the renormalization group equations from $m_{\text{weak}}$ up to the unification scale $\mu$.\[1\]

The minimal realization of SUSY is the so-called Minimal Supersymmetric Standard Model (MSSM). Although it has the advantage of being the simplest, the MSSM is an ad hoc choice, which is by no means mandatory. I discuss the simplest effective way to include R-parity violation which mimics the main features of more complete dynamical models where this violation happens due to the non-zero expectation values of sneutrinos. I mention some of the physics motivations and potential

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of various extensions of the MSSM with broken R–parity. Since neutrinos typically have mass in these models, many of the related phenomena are deeply related to the physics of weak interactions and the properties of neutrinos.

2 Supersymmetry and the MSSM

Supersymmetry has several attractive theoretical features. For example, as already mentioned, it allows the resolution of the hierarchy problem, if it survives down to the weak scale, leading to the possibility of discovering a whole plethora of SUSY particles with masses in the TeV scale at the new generation of colliders LEP II and LHC. Moreover, SUSY allows for a very elegant way to break the electroweak symmetry via radiative corrections.

Another attractive feature of SUSY is that precision measurements of the three gauge couplings performed at the CERN $e^+e^-$ collider LEP as well as neutral current data are all in good agreement with the MSSM–GUT with the SUSY scale $M_{SUSY} \lesssim 1$ TeV. This is illustrated in Fig. (1) taken from ref. 6. It is important to stress that the unification scale in SUSY–GUT is high enough to predict a proton decay rate slower than present experimental limits, as opposed to the non–SUSY GUTs, where the proton decays too fast.

The simplest SUSY model is the MSSM. This model realizes SUSY in the presence of a discrete R–parity symmetry. Under this symmetry all standard model particles are even while their partners are odd. As a result of this selection rule SUSY particles are only produced in pairs, with the lightest of them being stable. In the MSSM the Lightest SUSY Particle (LSP for short) is typically a neutralino, for most choices of SUSY parameters. It has been suggested as a candidate for the cold dark matter of the universe and several methods of detection at underground installations have been suggested.
However, one should not forget that R–parity is postulated ad hoc, without a deep theoretical basis. Moreover there are other ways to explain the cold dark matter via the axion. Last, but not least, hot dark matter is needed in any case, not to to mention other existing puzzles in neutrino physics, such as the solar neutrino deficit, which require non-zero neutrino mass. From this point of view the emphasis of the simplest MSSM picture would seem exaggerated.

3 Supersymmetry with Explicitly Broken R–Parity

R–parity could well be broken via tri-linear superpotential couplings of the type

$$\lambda ELL, \; \lambda' DQL, \; \text{and} \; \lambda'' UDD,$$

where the U, D, and E are SU(2) singlet superfields corresponding to the right-handed u, d quarks as well as charged leptons. These could arise from gravitational effects, in which case they are expected to be tiny. There are strong constraints on many of the corresponding couplings, especially the baryon-number violating ones, because of proton stability. There are also many bounds that follow from high energy physics as well as nuclear physics experiments, such as nuclear double beta decays. For a recent compilation see ref. There are, in addition some cosmological and astrophysical limits. For example, preserving a cosmological baryon asymmetry generated at the unification scale severely restricts certain combinations of $\lambda'$s, barring the existence of special symmetries. Alternatively, the baryon asymmetry may also be created at the weak scale. As for astrophysical limits I mention a limit recently derived in ref. It is based on the observation that the tri-linear couplings lead to flavour changing neutral current neutrino interactions which may induce resonant massless-neutrino conversions in a dense supernova medium. As shown in Fig. the restrictions that follow from the observed $\bar{\nu}_e$ energy spectra from SN1987A are much more stringent than those obtained from the laboratory. For the opposite sign of the neutrino mass square difference $\delta m^2$ supernova r-process nucleosynthesis gives complementary restrictions. Altogether, these disfavour a leptoquark interpretation of the recent HERA anomaly.

A simpler and more interesting way to break is via bi-linear superpotential couplings. In this case the superpotential is given by

$$W = h_t \hat{Q}_3 \hat{U}_3 \hat{H}_u + h_b \hat{Q}_3 \hat{D}_3 \hat{H}_d + h_s \hat{L}_3 \hat{R}_3 \hat{H}_d + \mu \hat{H}_u \hat{H}_d + \epsilon_3 \hat{L}_3 \hat{H}_u$$

where the first four terms correspond to the MSSM and the last one is the bi-linear term which violates R–Parity and lepton number explicitly (for three generations there would be three $\epsilon_i$). This superpotential is motivated by models of spontaneous breaking of R–Parity. Contrary to a popular misconception, the bi-linear violation of R–parity implied by the parameter $\epsilon_3$ is physical and can not be rotated.
Whichever way one chooses to parametrize the model there is R–parity violation which also implies a non-zero sneutrino vacuum expectation value $v_3$.

One attractive feature of this model is that it allows the radiative breaking of the electroweak symmetry with the simplest assumption of universal soft SUSY breaking terms at unification. In contrast to the MSSM, however, this model allows for the unification of the bottom and tau Yukawa couplings at the scale $M_{GUT}$ where the gauge couplings unify for any value of $\tan \beta$ provided $v_3$ is chosen appropriately. This is illustrated in Fig. (3), taken from ref. 23. In Fig. (3) the bottom quark and tau lepton Yukawa couplings are unified at $M_{GUT}$ and the horizontal lines correspond to the 1σ experimental $m_t$ determination (for simplicity $M_{SUSY} = m_t$ was assumed). The diagonal band at high $\tan \beta$ values corresponds to $t - b - \tau$ unification, expected in SO(10) models.

Note that $\epsilon_3$ and the $v_3$ are related by a minimization condition. As a result, if we adopt universal conditions for the soft breaking parameters this model contains effectively a single extra free parameter in addition to those of the minimal supergravity model. In this case R–parity violation is induced radiatively, due to the effect of the non-zero bottom quark Yukawa coupling $h_b$ in the running of the renormalization group equations from the unification scale down to the weak scale.

Another important property of the bi-linear model of R-parity breaking is that it provides a very elegant mechanism for the origin of neutrino mass which combines the virtues of the seesaw and the radiative mechanisms of neutrino mass generation.
The tau neutrino $\nu_\tau$ acquires a mass, due to the mixing between neutrinos and neutralinos. If we stick to the simplest unified supergravity version of the model with bi-linear breaking of R-parity and universal boundary conditions for the soft breaking parameters, then the effective neutralino mixing parameter $\xi \equiv (\epsilon_3 v_1 + \mu v_3)^2$ characterizing the violation of R-parity, either through $v_3$ or $\epsilon_3$ will be small since contributions arising from gaugino mixing will cancel, to a large extent, those from Higgsino mixing. This cancellation will happen automatically if the soft breaking parameters are universal. In this case $m_{\nu_\tau}$ will be naturally small and radiatively calculable in terms of the bottom Yukawa coupling $h_b$. This will explain the smallness of the neutrino mass in this model. The above scenario is a hybrid of the see-saw and radiative schemes of neutrino mass generation. The role of the right-handed mass is played by the neutralinos mass (which lies at the weak scale) while the role of the Dirac mass is played by the effective neutralino mixing $\xi$ which is induced radiatively. The $\nu_\tau$ mass induced this way is directly correlated with the magnitude of the effective parameter $\xi$. In Fig. (4) we display the allowed values of $m_{\nu_\tau}$.

It is important to notice that this happens for relatively large values of the relevant model R-parity violation parameter $\epsilon$. As a result many of the corresponding R-parity violating effects can be sizeable even when $m_{\nu_\tau}$ is small. Moreover there can be striking effects of R-parity violation which do not require it to have a large strength. The obvious example is the fact that the lightest neutralino decay will typically decay inside the detector, unless the violation is really tiny as in Fig. (4). Notice that $\nu_e$ and $\nu_\mu$ remain massless in this approximation. They get masses either from scalar loop contributions in Fig. (5)

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1. However, $m_{\nu_\tau}$ can be as large as the present laboratory bound in models with spontaneous breaking of R-parity.
2. Here we use a different basis in which the bi-linear term is removed but re-introduces a tri-linear term $DQL$ whose coefficient is related to the down-type Yukawa couplings.
A more satisfactory picture to R–parity violation would be one in which it is conserved at the Lagrangian level but breaks spontaneously through a sneutrino VEV. Keeping the minimal $SU(2) \otimes U(1)$ gauge structure this also implies the spontaneous breaking of lepton number, which is a continuous ungauged symmetry, and therefore the existence of an associated Goldstone boson (majoron). The breaking of R-parity should be driven by isosinglet right-handed sneutrino vacuum expectation values (VEVS), so as to avoid conflicts with LEP observations of the invisible Z width (in this case the majoron is mostly singlet, and does not couple appreciably to the Z). The theoretical viability of this scenario has been demonstrated both with tree-level breaking of the electroweak symmetry and R–parity as well as in the most attractive radiative breaking approach. This is illustrated in Fig. (6), taken from ref. The existence of the majoron, denoted by $J$, implies a novel Higgs boson decay mode $H \rightarrow JJ$ which is a characteristic feature of $SU(2) \otimes U(1)$ models with spontaneously broken R–parity, as well as in any model with continuous global symmetries spontaneously broken at the weak scale. This has an important impact on Higgs boson search strategies at accelerators such as the Tevatron, LEP II, NLC and LHC.

Again in these models neutrinos will have mass, which will depend in the details.
Figure 6: Scalar potential with radiative R–parity violation.

of the model. One possibility is to add only the $\nu^c$ right-handed neutrino superfields and give them masses a la see-saw $^{15}$. It is conceptually simpler, however, to give masses to the gauge singlets a la Dirac by adding two $SU(2) \otimes U(1)$ singlet superfields $\nu^c$ and $S$ sequentially in each generation $^{13, 14, 28}$. Due to the original lepton number symmetry at the Lagrangean level $^{30}$, neutrinos are massless before breaking R–parity. The magnitude of R–parity violating effects will be directly correlated with the $\nu^c_\tau$ mass which arises due to mixing with neutralinos, as mentioned above. In this approximation the $\nu^c_e$ and $\nu^c_\mu$ remain massless. The $\nu^c_\mu$ may now get mass even in the tree-level approximation by mixing with the singlets $^{31}$ or via SUSY loops.

Another class of models with spontaneous breaking of R–parity consists of models where the gauge symmetry contains lepton number, such as heterotic string inspired $E_6$ models with Calabi-Yau compactifications or left-right symmetric models. These have been discussed in the literature, see ref. $^{16, 17}$. The most important difference with the $SU(2) \otimes U(1)$ models is that in this case there is no physical Goldstone boson (majoron) associated to the breaking of lepton number since it is absorbed by the Higgs mechanism as the longitudinal mode of some extra $Z'$ gauge boson.

In the following few sections I will illustrate with some examples the potential of the present and future colliders in testing supersymmetry with spontaneous or bilinear breaking of R–parity under the assumption that neutrinos acquire mass only due to R–parity violation. For simplicity we will refer to these models generically as RPSUSY models. The characteristic feature of these models is that the pattern of R–parity breaking interactions is determined in terms of relatively few new parameters in addition to those of the MSSM (one in the simplest reference model $^{18, 21}$). This allows for a systematic discussion of the potential of new colliders in searching for broken R–parity SUSY signals. Many of the implications already appear at the level

$^{\dagger}$Strictly speaking, the addition of just one superfield suffices. We prefer, however, to add them sequentially.
of the truncated version of the model in which the bi-linear term mimics the violation of R–parity in an effective sense. As mentioned previously the bi-linear model is consistent in its own right, at least for a given range of $m_{\nu_\tau}$ values, say between 100 keV to an MeV.

Before we start the phenomenological discussion, we note that, even with relatively small strength of R–parity breaking interactions the lightest neutralino is expected to decay inside the existing particle detectors, for the typical energies of interest. This is illustrated in Fig. (7), taken from ref. 14. Although it refers to a particular model with spontaneous radiative breaking of R–parity, similar features arise also in the bi-linear model, or models with spontaneous breaking of R–parity in which lepton-number is part of the gauge symmetry. In this case the neutralino is likely to be the LSP and will decay with a sizeable branching ratio into visible channels. As a result the corresponding effects can be quite striking experimentally, to the extent that the missing momentum signature of the LSP in the MSSM will be substantially diluted in favour of the appearance of novel exotic signatures typically characterized by high fermion multiplicities (see below).

5 R–Parity Violation at LEP

The requirement that SUSY is broken effectively at the weak scale implies that SUSY particles are expected to exist at this scale, thus it makes sense to search for SUSY signatures at colliders such as the present Tevatron and LEP II, as well as the future LHC and NLC colliders.

In the MSSM the usual neutralino pair-production process,

$$e^+e^- \rightarrow \chi\chi$$

where $\chi$ denotes the lightest neutralino, leads to no experimentally detectable signature (other than the contribution to the $Z$ invisible width if $m_Z > 2m_\chi$), as $\chi$ escapes

$\parallel$Lighter $\nu_\tau$ will not decay efficiently in order to cope with the cosmological critical density limits, while heavier ones may have problems with primordial nucleosynthesis.
the apparatus without leaving any tracks. The simplest process that leads to a zen-event topology, with particles in one hemisphere and nothing on the opposite, requires the production of $\chi$ associated to $\chi'$, the next-to-lightest neutralino, i.e. $e^+e^- \rightarrow \chi\chi'$.

In broken R–parity models the $\chi$ may decay into charged particles, so that eq. (3) can lead to zen-events in which one neutralino decays visibly (leptons and jets) and the other invisibly. The topology is the same as in the MSSM but the corresponding rates can be larger than in the MSSM and may occur below the threshold for $\chi'$ production. The missing momentum in these models is carried by the $\nu_\tau$ or by majorons. Another possibility for zen events in RPSUSY is the process $e^+e^- \rightarrow \chi\nu_\tau$. Since this violates R–parity, the rates are somewhat smaller, but might be observable at LEP I.

For the sake of illustration we exhibit in Fig. 8 typical values of the branching ratios of neutralinos and charginos, as a function of $\epsilon$ for $\mu = 150$ GeV, $M_2 = 100$ GeV, and $\tan \beta = 35$. For neutralinos we exhibit its total visible and invisible branching ratios, where we included in the invisible width the contributions coming from the neutrino plus majoron channel ($\chi \rightarrow \nu J$), as well as $\chi \rightarrow 3\nu$.

In Fig. (9) we illustrate the sensitivity of LEP experiments to leptonic signals associated to neutralino pair-production at the Z peak in RPSUSY models. The signal topology used was missing transverse momentum plus acoplanar muon events ($p_T + \mu^+\mu^-$) arising from $\chi\chi$ production followed by $\chi$ decays. The solid line (a) in Fig. (9) is the region of sensitivity of LEP I data of ref. 32 corresponding to an integrated luminosity of 82 pb$^{-1}$, while (b) corresponds to the improvement expected from including the $e^+e^-\nu$ channel, as well as the combined statistics of the four LEP experiments. The dashed line corresponds to the bi-linear model of explicit R–parity violation, allowing $m_{\nu_\tau}$ values as large as the present limit, the dotted one does implement the restriction on $m_{\nu_\tau}$ suggested by nucleosynthesis and the dash-dotted one is calculated in the model with spontaneous breaking of R–parity (majoron model). The inclusion of semi-leptonic decays and of the updated integrated luminosity already achieved at LEP would substantially improve the statistics and thus the sensitivity.
to RPSUSY parameters. The usual chargino pair-production process,

$$e^+ e^- \rightarrow \chi^+ \chi^-$$  \hspace{1cm} (4)

may also provide novel signatures which would not be possible in the MSSM, as the neutralinos produced from chargino decays may themselves decay into jets or leptons leading to exotic channels.

Moreover, in $SU(2) \otimes U(1)$ models with spontaneous violation of R–parity the presence of the majoron implies the existence of two–body chargino decays

$$\chi^\pm \rightarrow \tau^\pm + J$$  \hspace{1cm} (5)

In ref. 34 chargino pair production at LEP II has been studied in supersymmetric models with spontaneously broken $R$–parity. Through detailed signal and background analyses, it was shown that a large region of the parameter space of these models can be probed. The limits on the chargino mass depend on the magnitude of the effective $R$–parity violation parameter $\epsilon$. As $\epsilon \rightarrow 0$ we recover the usual MSSM chargino mass limits, however, for $\epsilon$ sufficiently large, the bounds on the chargino mass can be about 15 GeV weaker than in the MSSM due to the dominance of the two-body chargino decay mode eq. (5). This happens because there is an irreducible background from $W$-pair production with each $W \rightarrow \tau \nu$.

Although the $\nu_\tau$ can be quite relatively heavy in these models, it is consistent with the cosmology critical density as well as primordial nucleosynthesis, due to the existence of the majoron which opens new $\nu_\tau$ decay and annihilation channels. The small mass difference between $\nu_e$ and $\nu_\mu$ may lead to an explanation of solar neutrino deficit by resonant $\nu_\tau$ to $\nu_\mu$ conversions. In this model one may regard the the R–parity violating processes as a tool to probe the physics underlying the solar neutrino conversions. For example, the rates for some RPSUSY rare decays can be used in order to discriminate between large and small mixing angle MSW solutions to the solar neutrino problem.
\[ \tan(\beta) = 2, \, \varepsilon = 1 \text{ GeV} \]

Figure 10: 95% CL excluded region in RPSUSY models in various analyses (dark areas), and the combined excluded region for \( \sqrt{s} = 172 \text{ GeV}, \) and 300 pb\(^{-1}\) integrated luminosity.

6 R–Parity Violation at LHC

It is also possible to find manifestations of R–parity violation at the super-high energies available at hadron super-colliders such as the Tevatron and the LHC. If SUSY particles, gluinos and squarks, are pair produced at hadron collisions, their subsequent cascade decays will not terminate at the lightest neutralino but it will further decay. To the extent that this decay is into charged leptons it will give rise to a quite rich pattern of high multiplicity lepton events. Such pattern of gluino cascade decays in RPSUSY models was studied in detail in ref. \(^40\). The conclusion is that multi-lepton and same-sign dilepton signal rates which can be substantially higher than those predicted in the MSSM. This is illustrated in Fig. (11), which shows the branching ratios for various multi-lepton signals (summed over electrons and muons) with the 3-, 4-, 5- and 6-leptons, for tan \( \beta = 2 \), with other parameters chosen in a suitable way (see ref. \(^40\) for details). We show a) the 3-lepton, b) the 4-lepton, c) the 5-lepton and d) the 6-lepton signal for the MSSM (full line), the majoron-model (dashed line) and the bi-linear model (dashed-dotted line). The shaded area will be covered by LEP II. Note, for example, that for \( \mu < 0 \) the 5-lepton signal is much larger in the majoron-model than in the MSSM, giving about 30 to 1200 events per year for an LHC luminosity of \( 10^5 \text{pb}^{-1} \). The 6-lepton signal has a rate up to \( 5 \times 10^{-5} \) in the range \( -300 \text{ GeV} < \mu < -80 \text{ GeV} \) giving 125 events per year. The multi-lepton rates would be even higher in the bi-linear model. Although with smaller rates, one
also expects in RPSUSY models the single production of the SUSY states in hadron collisions. For example in ref. 41 the single production of weakly interacting SUSY fermions (charginos and neutralinos) via the Drell-Yan mechanism was studied.

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