Simulation Research on Decoupling Control of Active-disturbance-rejection of Omnidirectional Robot

T Shen, X Y Xie, J Y Li*, X B Dong and X H Gao
College of Mechanical Engineering, Beihua University, Jilin City 132021, China
E-mail address: 384582107@qq.com, T Shen; 604204901@qq.com, J Y Li*

Abstract. Active-disturbance-rejection controlling method is proposed in this essay aiming at the problems of omnidirectional robots, such as complex model, strong coupling and controlling difficulty caused by internal and external stochastic disturbance. Taking eight-wheel omnidirectional robots as the example, the coupling method based on active-disturbance-rejection is designed by analysing the kinematic-dynamic model of the robot. Non-linear function is used as the state feedback of the components of the active-disturbance rejection controller, and extended state observe is adopted to estimate the internal and external stochastic disturbances and to provide active disturbance compensation. Through simulation experiment comparing with traditional PID controller, the decoupled active-disturbance-rejection controller can control omnidirectional robots better inhibit the unknown disturbance and improve the performance of the system.

1. Introduction

With the omnidirectional robot soccer game RoboCup becoming more and more popular in China, omnidirectional robots have been applied in various fields of human society[1-3]. To play full advantage of flexible and highly-efficient sport, the technology of its motion control is of great importance. However, all the motor control loops of omnidirectional robots are inter-coupled; at the same time, it also has the characteristics of non-linearity, multivariable as well internal and external stochastic disturbance, which has had system modelling and controller design more complicated. To improve the controlling defects of traditional PID[4], fuzzy PID, neural network and other control algorithms[5-7] have been designed for omnidirectional robots, which have achieved good control effects. However, these methods require plenty of iterative computations, making the control algorithm design more complicated. At the same time, these methods cannot eliminate stochastic disturbance inside and outside the system as well as the influences of strong coupling of the control loops.

Active disturbance rejection controller (ADRC) inherits model independency of traditional PID control law as well as the feature that parameters are easy to be adjusted; in addition, it has the natural advantage of decoupling and can estimate and inhibit actively the influences of unknown disturbance on the system; therefore, ADRC can improve the disturbance adaptability of the overall system and simplify controller design[8-9].

In this essay, eight-mecanum wheel omnidirectional robot is taken as the control plant. By analysing its mathematical model, ADRC decoupling control strategy for robot motor is designed to solve the problems of internal and external stochastic disturbance and coupling of the system, which has improved the stability and anti-interference property of omnidirectional robot motion control.
2. System model analysis

The following assumptions shall be pointed out before the mathematical model is established:

1. The centre of gravity of the omnidirectional robot coincides with its geometric centre;
2. The robot moves only on the surface of two-dimensional rigid body;
3. The friction between the omni-robot and the ground is large and so sliding will not occur.

The eight-wheel omnidirectional robot has altogether 8 Mecanum wheels, which are installed respectively on both sides of the robot chassis. Arrangement of the location and posture of each wheel are shown in Figure 1. Coordinate system \( \Sigma_0 \) is established with the center of the mobile robot as point \( O \), with the right ahead of the robot as positive direction of X-axis and the left ahead as positive direction of Y-axis. It is set the longitudinal distance from number 1, 2, 7 and 8 Mecanum wheels to original point \( O \) is \( L_1 \), the longitudinal distance from number 3, 4, 5, and 6 Mecanum wheels to original point \( O \) is \( L_2 \), and the transverse distance from number 1-8 Mecanum wheels to original point \( O \) is \( L \). The angle between the hub axis of each Mecanum wheel and the roller axis is \( \alpha \), and the hub radius of each Mecanum wheel is \( w_R \).

![Figure 1. Schematic diagram of eight-wheel omnidirectional robot based on Mecanum wheel](image)

The kinematical equation of the robot can be obtained through kinematics analysis:

\[
\begin{pmatrix}
1 & -\tan \alpha & -(L_1 \tan \alpha + L) \\
1 & \tan \alpha & (L_1 \tan \alpha + L) \\
1 & -\tan \alpha & -(L_2 \tan \alpha + L) \\
1 & \tan \alpha & (L_2 \tan \alpha + L) \\
1 & -\tan \alpha & -(L \tan \alpha + L) \\
1 & \tan \alpha & (L \tan \alpha + L)
\end{pmatrix}
\begin{pmatrix}
v_1 \\
v_2 \\
v_3 \\
v_4 \\
v_5 \\
v_6 \\
v_7 \\
v_8
\end{pmatrix}
= \begin{pmatrix}
v_{Ox} \\
v_{Oy} \\
v_{Oz}
\end{pmatrix}
\] (1)

Where, \( v_O = [v_{Ox} \ v_{Oy} \ \omega_z]^T \in \mathbb{R}^3 \) is the velocity of robot centre \( O \) in coordinate system \( \Sigma_0 \); \( v_w = [v_1 \ \cdots \ v_8]^T \in \mathbb{R}^8 \) is the linear velocity of each wheel.
In this essay, Lagrange method is used to analyse robot kinetic model. Lagrange equation under potential force and dissipation force is:

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}_i} \right) - \frac{\partial L}{\partial \theta_i} + \frac{\partial R}{\partial \dot{\theta}_i} = Q_i
\]  

Where, Lag is Lagrange function, which is defined as the difference between the kinetic energy and the potential energy \(U\) of the whole system; \(\theta_i(i=1 \sim n)\) is the generalized coordinate of the whole system; \(\dot{\theta}_i\) is the generalized velocity of the system; \(Q_i\) refers other generalized force except potential force and viscous friction in the system; \(R\) is Rayleigh dissipative function.

Kinetic energy of robot mainly includes the kinetic energy due to the velocities of the robot in X and Y direction, the kinetic energy of omnidirectional robot rotating surrounding center \(O\), and the kinetic energy of 8 Mecanum wheels rotating surrounding its axis, but the kinetic energy of several Mecanum rollers that are contacting with the ground or have just separated from the ground and other negligible kinetic energy are neglected. Friction of the system refers mainly to the friction between each wheel with the ground, but the friction within the system is neglected. Thus, Lagrange function and dissipative function \(R\) of the system can be obtained.

\[
Lag = \frac{J R^2}{128} \left( -\dot{\theta}_1 - \dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4 + \dot{\theta}_5 + \dot{\theta}_6 + \dot{\theta}_7 + \dot{\theta}_8 \right)^2 \\
+ \frac{m R^2}{128} \left( \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4 + \dot{\theta}_5 + \dot{\theta}_6 + \dot{\theta}_7 + \dot{\theta}_8 \right)^2 \\
+ \frac{1}{2} J_u \left( \ddot{\omega}_1^2 + \ddot{\omega}_2^2 + \ddot{\omega}_3^2 + \ddot{\omega}_4^2 + \ddot{\omega}_5^2 + \ddot{\omega}_6^2 + \ddot{\omega}_7^2 + \ddot{\omega}_8^2 \right) - U \\
R = \frac{1}{2} \mu \left( \ddot{\omega}_1^2 + \ddot{\omega}_2^2 + \ddot{\omega}_3^2 + \ddot{\omega}_4^2 + \ddot{\omega}_5^2 + \ddot{\omega}_6^2 + \ddot{\omega}_7^2 + \ddot{\omega}_8^2 \right)
\]  

Where, \(\dot{\omega} = \omega(1 \sim 8)\) is the angular spin rate of Mecanum wheel; \(U\) is the total potential energy of the system. It is set the omnidirectional robot system studied in this essay moves invariably on some horizontal plane, then, the total potential energy \(U\) of the system is constant; \(J_u\) is the inertia of each Mecanum wheel rotating surrounding its centre axis; \(m\) is the total mass of the robot; \(J\) is the inertia of the robot rotating surrounding the central point \(O\); \(\mu\) is the coefficient of rolling friction.

External generalized force is the torque input into each wheel of the motor. Substitute equation (3) and (4) into equation (2), it can be obtained,

\[
M_i \omega + \mu \omega = T_w
\]

Where, \(M_i \in \mathbb{R}^{8 \times 8}\) is the coefficient matrix obtained through calculation. The calculation process will be omitted due to space limitation. \(\omega = [\omega_1 \cdots \omega_8]^T \in \mathbb{R}^8 \); \(T_w = [T_{w1} \cdots T_{w8}]^T \in \mathbb{R}^8\), \(T_{wi}(i=1 \sim 8)\) is the torque functioned on Mecanum wheels when the i-th motor passing the reduction gearbox.

Differential equation of the relation between the control voltage \(U_m\) at both ends of DC gear motor armature and the rotating speed of Mecanum wheel \(\omega\) and the torque \(T_w\) functioned on the wheels is established:
\[
\dot{T}_m = -\frac{R_m}{L_m}T_n - \frac{K_e K_f N^2}{L_m} \omega + \frac{K_f N}{L_m} U_m
\]  

(6)

Where, \( U_m = [U_{m_1} \ldots U_{m_8}]^T \in \mathbb{R}^8 \), \( U_m (i = 1 \sim 8) \) is the control voltage loaded on both ends of the i-th motor armature; \( L_m \) is DC motor armature inductance; \( R_m \) is motor armature resistance; \( K_e \) is the reverse electromotive force constant of the motor; \( K_f \) is the torque constant of DC motor; \( N \) is the reduction ratio of reduction gearbox.

Simultaneous equation (5) and equation (6), the kinetic equation of the system can be obtained:

\[
M_A \omega + M_B \dot{\omega} + M_C \omega = U_m
\]  

(7)

Where, \( M_A, M_B, M_C \in \mathbb{R}^{6 \times 8} \), are the coefficient matrix obtained through calculation. The calculation process will be omitted due to space limitation.

It can be seen from equation (7), the rotational state of each wheel in the entire system not only relates with the control voltage of their motor, but also can affect the control voltage of other motors, so it is a strong coupling second-order MIMO system.

3. Design of active-disturbance-rejection controller

3.1. ADRC fundamentals

ADRC was formally put forward by researcher Han Jingqing in 1999. According to ADRC, uncertainty, internal or external disturbance in dynamic process of the system are named total disturbance of the system. By estimating in real time the total disturbance of the system and compensating and neutralizing actively, nonlinear system with unknown disturbance can be reverted into simple integral series to realize active anti-disturbance. It is composed mainly by Tracking Differentiator (TD), Extended State Observer (ESO), Nonlinear States Error Feedback Control Laws (NLSEF) and disturbance compensation, represented by common second-order system. The principle was shown in Figure 2.

![Functional block diagram of second-order ADRC](image)

Figure 2. Functional block diagram of second-order ADRC

3.2. Design of ADRC-based decoupling controller

Though omnidirectional robot system is three axis motion system, its essential input and output relation is shown in Figure 3. Three axis control of the robot can be completed by designing the nonlinear characteristics of motor controller to the system and the coupling characteristics. According to the separability principle of ADRC, TD, ESO and NLSEF of the motor controller are designed respectively to assemble finally the ADRC of the system.
Inverse kinematics

Expected motion of XYZ-axis

Motor control voltage

Motor

Actual speed of motor

Forward kinematics

Actual motion of XYZ-axis

Expected speed of motor

ADRC

Figure 3. Control input-output relation of the robot

Equation (7) is converted into the following equation:

\[
\ddot{\omega} = -M_d^{-1} M_a^{-1} \dot{\omega} - M_d^{-1} M_c \omega + M_d^{-1} U_m
\]  

(8)

The coupling part \( M_d^{-1} U_m \) in the controlled quantity of the system is called "static coupling", while the coupling part \( -M_d^{-1} M_a^{-1} \dot{\omega} - M_d^{-1} M_c \omega \) in the model excluding the controlled quantity is called "dynamic coupling". Introduce virtual controlled quantity \( U_B = M_d^{-1} U_m \) into equation (8), then, the input-output relation of the i-th channel in the system can be expressed as:

\[
\begin{cases}
\dot{\omega}_i = -M_d^{-1} M_B \omega_i - M_d^{-1} M_c \omega_1 + U_B \\
y_i = \omega_i
\end{cases}
\]  

(9)

It can be seen easily that the virtual controlled quantity \( U_B \) and the output \( y_i = \omega_i \) are in one-to-one correspondence, that is, the controlled output \( y_i = \omega_i \) of the i-th channel and the virtual controlled quantity input \( U_B \) are decoupled, and the "static coupling" of the system is eliminated. The actual controlled quantity \( U_m \) can be calculated only by equation \( U_m = M_d U_B \).

Considering that the system has unknown disturbance \( \chi(t) \), its actuating quantity is extended into an extra state variable \( x_3 \), then, system (8) can be recorded as:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 + M_d^{-1} U_m \\
\dot{x}_3 &= f(t, x_1(t), x_2(t), \chi(t)) \\
y &= x_1
\end{align*}
\]  

(10)

Where, \( f \) is the actuating quantity of the dynamic coupling of and the unknown disturbance \( \chi(t) \) on the system. The extended state variable \( x_3 \) can be observed and estimated by ESO as the sum of the internal and external disturbances, which can be eliminated easily by ADRC.

At the time, the static coupling of the system can be decoupled by introducing the virtual controlled quantity \( U_B \), while the dynamic coupling can be eliminated through system compensation, so that the entire system can be decoupled completely.

So, each part of ADRC can be designed respectively. The state error feedback of second-order TD and second-order NLSEF adopts time-optimal control synthetic function \( fhan(x_1, x_2, r_0, h_0) \):

\[
\begin{align*}
d &= r_0 h_0^2, \quad a_0 = h_0 x_2, \quad y = x_1 + a_0 \\
a_1 &= \sqrt{d(d+8)y}, \quad a_2 = a_0 + \frac{(a_0 - d)}{2} \text{sign}(y) \\
a &= (a_0 + y - a_2) \frac{\text{sign}(y+d) - \text{sign}(y-d)}{2} + a_2 \\
fhan &= -r_0 \frac{a}{d} \frac{\text{sign}(a+d) - \text{sign}(a-d)}{2} - r_0 \text{sign}(a)
\end{align*}
\]  

(11)
Based on the thought of "small error with big gain, big error with small gain", the state error feedback of third-order ESO adopts nonlinear function:

\[
\text{fal}(x, \alpha, \delta) = \begin{cases} 
  \frac{x}{\delta^{1-\alpha}}, & |x| \leq \delta \\
  |x|^\alpha \text{sign}(x), & |x| > \delta 
\end{cases} \quad (12)
\]

The algorithm of dispersed active-disturbance-rejection decoupling control of the i-th motor of omnidirectional robot can be obtained after introducing the virtual controlled quantity.

\[
\begin{align*}
  f_{ih} &= f_{han}(v_{ii}(k) - v_{ai}(k), v_{iz}(k), r_i, h_i) \\
  v_{ii}(k + 1) &= v_{ii}(k) + \alpha v_{iz}(k) \\
  v_{iz}(k + 1) &= v_{iz}(k) + h f_{ih} \\
  e_{ji}(k) &= z_{ji}(k) - \omega_j(k) \\
  z_{ji}(k + 1) &= z_{ji}(k) + h[z_{ji}(k) - \beta_{0i} e_{ji}(k)] \\
  z_{ii}(k + 1) &= z_{ii}(k) + h[z_{ii}(k) - \beta_{00} \alpha \text{fal}(e_{ij}(k), 0.5, \delta) + U_{ii}] \\
  e_{ij}(k) &= v_{ii}(k) - z_{ji}(k) \\
  e_{ij}(k) &= v_{ii}(k) - z_{ji}(k) \\
  U_{iw}(k) &= -f_{han}(e_{ii}(k), c e_{ji}(k), r_i, h_i) - z_{ii}(k)
\end{align*} \quad (13)
\]

Where, \( h \) is sampling step size; \( v_{ai} \) is the expected rotating speed of the i-th wheel; \( v_{ii} \) and \( v_{iz} (i = 1 \sim 8) \) are respectively the transient process arranged by TD of the i-th channel as well as the differential of the signal; \( r_i \) is the factor of tracking velocity; \( h_i \) is the factor of tracking filter; \( z_{ii}, z_{iz}, z_{ii} (i = 1 \sim 8) \) are respectively the state variables observed by ESO of the i-th channel; \( \beta_{0i}, \beta_{02}, \beta_{03} \) are the gain of the state error; \( \delta \) is the filter factor of ESO; \( c \) is the damping coefficient; \( r_i \) is the gain of the controlled quantity, and \( h_i \) is the precision factor.

Considering that ADRC has powerful adaptability to parameter perturbation, the same parameters are adopted in ADRC design of 8 channels in this system.

4. Simulation experiment

To observe the effects of the proposed ADRC decoupling algorithm, comparative simulation experiment is conducted in Simulink by comparing with classical PID. The initial state of three axis velocity is set as 0. The three axes are input the step signal with the starting time of 0.1s and the amplitude of 1m/s and 1rad/s. Physical parameters of omnidirectional robots are shown in Table 1, and ADRC parameters are shown in Table 2.

| Parameter | m(kg) | J(kg\cdot m^2) | \( J_w \) (kg\cdot m^2) | \( R_w \) (m) | \( \alpha \) (rad) | \( L \) (m) | \( L_i \) (m) |
|-----------|-------|----------------|----------------|--------------|---------------|----------|----------|
| Value     | 50    | 15.6           | 0.04           | 0.127        | 0.7854        | 0.289    | 0.303    |

| Parameter | \( L_i \) (m) | \( \mu \) | \( R_m \) (\Omega) | \( L_m \) (mH) | \( K_e \) | \( \gamma \) | N |
|-----------|---------------|------|----------------|----------------|-------|----------|
| Value     | 0.137         | 0.1  | 1.8            | 0.7            | 0.008 | 0.06     | 18 |

| Parameter | h | TD | ESO | NLSEF |
|-----------|---|----|-----|-------|
| Value     | 0.01 | 1.5 | 100  | 120   | 23000 | 0.18 | 5 | 1.2 | 0.2 |

The three axis motion features of the robot are different, so three-axis response curves of PID parameters in the same group differ greatly. Take optimal velocity response of X-axis as the example.
PID parameters are adjusted automatically by Matlab into: $K_p=3.5$, $K_i=15.2$, $K_d=0.03$. The step responses of three axis input velocity are shown in Figure 4. To verify the disturbance adaptability of the system, step responses under disturbance are compared. In the system model, Gaussian white noise with the sampling period of 0.01s and PSD amplitude of 0.05 is superimposed on the motor output torque. The disturbance observation results of the ESO are shown in Figure 5. Three axis response under disturbance is shown in Figure 6.

![Figure 4. Three axis velocity step response](image1)

![Figure 5. Comparison of ESO observation under different disturbances in eight channels](image2)

![Figure 6. Three-axis velocity step response under disturbance](image3)

It can be seen obviously from Figure 4 that, compared with traditional classical PID controller, three axis response curves of ADRC are almost the same and can track the given signals non-
overshooting fast and accurately, indicating that the controller designed in this essay can effectively inhibit the influences caused by coupling in the system. It can be seen from Figure 5 and Figure 6 that when there is unknown stochastic disturbance, the extended state observer can observe the actuating quantity of the disturbance accurately. The maximum error of ADRC is 0.07m/s after the X-axis response enters the steady state under the function of disturbance, while the that of traditional PID reaches 0.24m/s, indicating that ADRC can effectively inhibit the influences of unknown disturbance on the system.

The simulation results show that the controller designed in this essay can inhibit well stochastic disturbance of omnidirectional robots as well as the influences of coupling on the system.

5. Conclusion
ADRC decoupling controller is designed in this essay through kinetic analysis of Mecanum wheel-based 8-wheel omnidirectional robot. The simulation experiment shows that compared with traditional PID controller, ADRC decoupled controller has stronger anti-interference capacity and robustness. Limited by the condition and the time, the study is conducted mainly through simulation, so the proposed algorithm still remains to be verified by experiment in physical system, with the aim to achieve further research results.

Acknowledgement
Project funding: Jilin Province Science and Technology Support Plan Project(20190304131YY); Graduate Innovation Program of Beihua University [2018]042); Graduate Innovation Program of Beihua University [2019]089.

References
[1] Masar, L Design and control of a quasi-omnidirectional mobile robot F.A.A.K. [C]. Proceedings of the IASTED International Conference on Robotics and Applications, 2009: 391-397.
[2] J B Song, K S Byun. Design and control of a four-wheeled omnidirectional mobile robot with steerable omnidirectional wheels[J]. Journal of Robotic systems, 2004, 21(4): 193-208.
[3] Tehrani A F, Doosthosseini, A M, Moballegh, H R, et al. A new odometry system to reduce asymmetric errors for omnidirectional mobile robots RoboCup[J]. 2003: Robot Soccer World Cup VII (Lecture Notes in Artificial Intelligence Vol 3020), 2003: 600-610.
[4] Chang Bingbing. Design and experimental research of three-wheel omnidirectional mobile robot [D]. Harbin Institute of technology, 2019
[5] Huang Ping. Research on Fuzzy PID control of omnidirectional mobile rehabilitation robot chassis [D]. Zhejiang University of technology, 2019.
[6] Sun haoshui, Wang Xiaoping, Wang Xiaoguang, Lin Qining. Omnidirectional chassis control method based on Fuzzy Immune Neural Network PID algorithm [J]. Journal of Air Force Engineering University (NATURAL SCIENCE EDITION), 2018,19 (04): 59-65.
[7] Fan Jinhui, Jia Songmin, Li Xiuzhi. Adaptive control of omnidirectional intelligent wheelchair based on RBF neural network [J]. Journal of Huazhong University of science and Technology (NATURAL SCIENCE EDITION), 2014,42 (02): 111-115.
[8] Han Jingqing. From PID technology to "active disturbance rejection control" technology [J]. Control engineering, 2002 (03): 13-18.
[9] Han Jingqing. Extended state observer for a class of uncertain objects [J]. Control and decision, 1995 (01): 85-88.