Orthogonal code symbols’ synchronization in communication systems

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Abstract. The objective of the article consists in a development and a research of the offered by the author orthogonal coding as a way of noise immunity’s increase using the example of the information communication systems with phase-shift keying. The orthogonal coding is an analogue of convolutional coding over the rational numbers’ field. Properties of orthogonality of proposed codes allow to strengthen significance of useful signal with simultaneous weakening of influence of noise. The orthogonal coding allows to provide the required quality of communication with a smaller energy cost. The main attention is paid to an actual problem of synchronization of signals in combination with orthogonal codes.

1. Introduction
For formation of orthogonal codes [1-3], it is required to implement the synthesis of square matrices so that their product is a unit matrix multiplied by a monomial characterizing the corrective ability of the code.

The encoding \( G(D) \) and decoding \( H(D) \) matrices of the delay variable \( D \) shall satisfy the relation

\[
G(D) \cdot H(D) = \rho \cdot D^x \cdot I,
\]

where \( I \) is the identity matrix. The multiplier \( \rho \cdot D^x \) indicates the amplitude increase of the input signal at \( \rho \) times and that the symbols in the receiver are obtained with a delay of \( x \) time units.

In the process of joint application of orthogonal coding and differential phase-shift keying (DPSK) [3, 4] we will use matrices \( H(D) \) with polynomials in variable \( D \) of degree 1.

2. Code generation example
According to the proposed by the author algorithm [6], at the first step of the synthesis of a matrix \( H(D) \) of order \( n \) the first \( z = 2k \) elements of the main diagonal get values \( 1+D^z \), \( z \leq n \), the integer \( z \) will be called the depth of the matrix. The following elements of the main diagonal assigned value 1. In the last step, we assign values \( 1-D \) to the elements of the odd rows to the right and the odd columns under the main diagonal, and values \( 1+D \) to the elements of even rows to the right and even columns below the main diagonal.
For example, consider the decoding $H(D)$ of order 4 depth 2 [7].

$$H(D) = \begin{pmatrix} 1+D & 1-D & 1-D & 1-D \\ 1-D & 1+D & 1+D & 1+D \\ 1-D & 1+D & 1 & 1-D \\ 1-D & 1+D & 1-D & 1 \end{pmatrix}$$

and the corresponding to it encoding matrix $G(D)$

$$G(D) = \begin{pmatrix} 3+3D & -3+3D & 0 & 0 \\ -3+3D & -5+3D & 4 & 4 \\ 0 & 4 & 4 & -8 \\ 0 & 4 & -8 & 4 \end{pmatrix}.$$

Multiplication of the matrices $G(D)$ and $H(D)$ gives

$$G(D) \cdot H(D) = \begin{pmatrix} 12D & 0 & 0 & 0 \\ 0 & 12D & 0 & 0 \\ 0 & 0 & 12D & 0 \\ 0 & 0 & 0 & 12D \end{pmatrix}.$$

3. Joint use of orthogonal coding and differential phase-shift keying

$G(D)$ and $H(D)$ can be represented as encoding and decoding matrices $G$ and $H$ of size (4 x 8) and (8 x 4) respectively:

$$G = \begin{pmatrix} 3 & -3 & 0 & 0 & 3 & 3 & 0 & 0 \\ -3 & -5 & 4 & 4 & 3 & 3 & 0 & 0 \\ 0 & 4 & 4 & -8 & 0 & 0 & 0 & 0 \\ 0 & 4 & -8 & 4 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad H = \begin{pmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & 0 & -1 \\ -1 & 1 & -1 & 0 \end{pmatrix}.$$

To implement orthogonal coding, it is necessary to use DPSK of very high multiplicity [8]. So, in this example, the possible number of phase shifts is 45 (doubled maximum amount of one column absolute values of the encoding matrix $G$ plus one).

Let the information vector $u$ be transmitted, which is multiplied by the encoding matrix $G$. As a result, we obtain a code vector $c = uG$. It comes at the modulator, which implements DPSK with 45 positions. In this case, adjacent elements of the vector are added together. The result of addition, if it goes beyond the interval $[-22; 22]$, is normalized by adding 45 or -45, respectively [9]. Receive the vector $v$, which is used to form the transmitted signals: $v_i = v_{i-1} + c_i$, $v_{-1} = 0$. From the coding matrix' form follows the possible values of the phases at the modulator output are $0, \pm 1, \pm 2, ..., \pm 22$, multiplied by $\frac{2\pi}{45}$. 
The element with the index $i$ of the vector $v$ is transmitted in the shape of a cosine wave $\cos\left(4\pi t + \frac{v_i \cdot 2\pi}{45}\right)$. The received signal is sampled at the receiving side. As a result of sampling, a sample vector $s$ is obtained. In the study and simulation, the vector length is chosen equal to 64: $s = (s_0, s_1, \ldots, s_{63})$.

The phase difference between two cosine waves, received one after another, is determined by the received samples. The phase difference, as follows from the method of forming the vector $v$ and the corresponding signal, is another element of the vector $c$. For the first cosine wave, the phase difference is calculated with a harmonic of the form $\cos(4\pi t)$, since the first element of the vector $v$ was obtained by adding with zero.

Thus calculated vector $c'$ is multiplied by the matrix $H$, as a result of which the vector $u'$, which differs (in the case of synchronized transmission without noise) from the original vector $u$ only in that its elements are 12 times the elements of the vector $u$. Comparing the values of the vector’ $u'$ elements with a threshold value of zero, we change each element by 1 or -1.

4. Transmission synchronization using orthogonal coding

In communication systems with orthogonal coding, the task of synchronizing signals in combination with orthogonal codes must be solved [10]. Consider the harmonic of the form $\cos(4\pi t + \varphi)$. At first, it will be considered that synchronization is established, and there is no noise in the channel. Harmonics are sampled in 64 samples in increments of 1/64 of the signal transmission time (equal to 1 in the simulation). Then the difference between two such samples will be

$$
\Delta = \cos\left(4\pi \cdot \frac{i+1}{64} + \varphi\right) - \cos\left(4\pi \cdot \frac{i}{64} + \varphi\right) \text{ or otherwise } \Delta = -2\sin\left(\frac{4\pi}{64}\right) \cdot \sin\left(\frac{4\pi \cdot 2i+1}{64} + 2\varphi\right).
$$

The first sine of this expression is approximately equal to 0.09801, the second sine, due to the fact that $i$ can take different values, can only be estimated by the interval $[-1; 1]$. As a result, the expression itself is evaluated by the interval $\Delta = [-0.197; +0.197]$, that is $|\Delta| \leq 0.197$.

In any case, if there is no noise and no desynchronization in the channel, for any two adjacent elements of the sample vector $s$ it will be true that their difference in absolute value does not exceed 0.197.

Consider the situation of the presence of additive noise in the channel. We determine the maximum allowable value, by which any sample can change under the influence of noise. Denote this value as $e$, then the absolute value estimate of the difference between two adjacent samples can change by the maximum of $2e$ (when $e$ is added to one of the samples and subtracted from the other), that is $|\Delta| \leq 0.197 + 2e$. This inequality will be guaranteed to be satisfied for any pair of adjacent samples, if the magnitude of each noise component affecting on the taken samples does not exceed a given value $e$. If the signal amplitude is different from unity, then the estimate will be, respectively, $|\Delta| \leq A \cdot 0.197 + 2e$, where $A$ is the signal amplitude.

Now we will assume that there is no noise in the channel, but there is a desynchronization of signal reception. Then in the samples vector $s$ there will be samples of two signals at once. By analogy with the made conclusions, the difference between the last “correct” sample (taken from the desired harmonic) and the first “wrong” (taken from the next) can be found. This difference will be equal to

$$
\cos\left(4\pi \cdot \frac{i-63}{64} + \varphi_2\right) - \cos\left(4\pi \cdot \frac{i-63}{64} + \varphi_1\right) = \Delta, \quad i \in \{63; 64\}.
$$
This expression takes into account that \( i \) can be a non-integer number, that is, the desynchronization can begin at any time.

If the phase difference of the two signals is sufficiently large, then the left and right boundaries of the interval \( \Delta = [\Delta_1; \Delta_2] \) always have the same sign and slightly differ from each other (that is, a change in \( i \) from 63 to 64 weakly affects the expression value: this will be seen further in Figure 1). Assume that these boundaries are known.

In the case of noise, the lower boundary may decrease by \( 2e \), and the upper boundary may increase by the same amount, so there will be new boundaries \( \Delta = [\Delta_1 - 2e; \Delta_2 + 2e] \). If the amplitude of the signal is not equal to unity, then \( \Delta = [A \cdot \Delta_1 - 2e; A \cdot \Delta_2 + 2e] \).

As a result, we have two estimates of the difference in samples – for synchronized reception for any pair of adjacent samples and for unsynchronized reception for a pair of samples at the “junction” of two signals. Now, if the noise components do not exceed the specified acceptable value, and the lower limit of the difference in samples in the case of asynchronous reception is more than the upper limit of the difference in the case of synchronized sampling, it can be argued that the vector analysis of 64 samples will always accurately determine the desynchronization. In other words, the desynchronization will be determined if the inequality \( A \cdot 0.197 + 2e < A \cdot \Delta_1 - 2e \) holds, that is, if the inequality \( e < \frac{A \cdot (\Delta_1 - 0.197)}{4} \) holds.

The determination of the desynchronization in this case consists in sequentially calculating the differences between the next received sample and the previous one. If the difference turns out to be more than the value \( A \cdot 0.197 + 2e \), then the next received sample belongs to a new signal, and this means that there is no synchronization. Otherwise, the next sample belongs to the previously received signal, and nothing can be said about the lack of synchronization.

It can be seen that the value \( \Delta_1 \) depends only on the difference and the sum of the signals’ phases (\( i \) must always be chosen so that the value of the expression is minimal). In general, the bigger the difference in the signals’ phases, the greater the difference will be in adjacent samples at the “junction” of signals in time. The maximum difference will be given by the difference in phases in 22 sectors of the circle (for a given DPSK). For this purpose, for example, the signal with a zero phase and the signal with a phase of 22 sectors must follow each other. It is difficult to obtain such a pair of signals, since it is desirable that the pair is repeated with any period during the installation of synchronization. If there is one such pair during transmission, then if for some reason the receiver skips it, there will be no guarantee of setting synchronization.

By a simple selection, it was possible to obtain a pair of signals repeating in time with phases in 20 and 4 sectors (that is, a pair with a difference of 16 sectors). For this, all 4 inputs of the encoder must be fed with a synchro signal of the form \((+1,+1,+1,+1,+1,+1,+1,+1, -1, -1, -1, -1, +1,+1,+1,+1, -1, -1, -1, -1)\). In this case, the transmission will include a periodically repeating pair of signals for which the phase difference is 16 sectors. \( \Delta_1 \) can be counted for such a pair. It is equal to approximately 1.835, so an estimate for the noise components maximum value will be \( |e| < 0.41 \cdot A \).

5. Simulation modeling results
The modeling [11, 12] has shown that while maintaining this inequality, desynchronization is always detected. Inequality gives an estimate for the signal-to-noise ratio, if we assume that for a certain signal-to-noise ratio it acts as the signal’ amplitude, and the absolute magnitude of the noise does not exceed unity. Then \( \frac{E}{N_0} > 3.873 \) dB.
The determination of the desynchronization by the described method indicates the sample number corresponding to the beginning of the “wrong signal”, and, therefore, the number of samples by which offset time has occurred regarding to the correct moment of reception. This immediately allows to determine the number of samples that must be skipped in order to start the correct reception of signals.

The synchro signal was originally selected for simulation. Strictly speaking, it is not necessary to use, or use any other, occupying only one input of the encoder. The main thing is always have a guarantee that among the transmitted signals there will certainly be a pair with a phase difference sufficient for a given noise level.

6. Conclusions
Technical implementation of orthogonal coding is characterized by low complexity: decoding is reduced to calculation of several dot products and comparison with the zero threshold [13, 14]. For this reason, the offered way of coding and design of receiving and transmitting devices can be implemented in various communication systems. The proposed way of orthogonal coding is a variety of reception of M-ary DPSK signals with an optimum choice of a modulation code that matches the binary combinations of the source with the phase shift of the DPSK signal. The reason for this optimization is to average the error probability over all bits of the M-ary code [15].

With use of an example of the orthogonal code based on pair of coding and decoding matrices the solution of a problem of synchronization of signals in combination with orthogonal codes is
considered and results of simulation confirming practical applicability of the proposed technical solution are given.

Acknowledgments
The paper was prepared with the financial support of the Russian Foundation for Basic Research (RFBR), project No. 18-07-01298, “Development and research of the newest noise immunity increase methods of spectral effective modulation schemes on the basis of the orthogonal coding” and the financial support of the Ministry of Science and Higher Education and of the Russian Federation, grant agreement No. 2.9214.2017, “Research of perspective classes of algebraic noise-resistant codes and the development of algorithms for their fast decoding for onboard equipment of long-term spacecraft”.

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