Adelic Universe and Cosmological Constant

Nugzar Makhaldiani

Laboratory of Information Technologies
Joint Institute for Nuclear Research
Dubna, Moscow Region, Russia
e-mail address: mnv@jinr.ru

Abstract

In the quantum adelic field (string) theory models, vacuum energy – cosmological constant vanish. The other (alternative ?) mechanism is given by supersymmetric theories. Some observations on prime numbers, zeta – function and fine structure constant are also considered.
1. Introduction

There is an opinion that present-day theoretical physics needs (almost) all mathematics, and the progress of modern mathematics is stimulated by fundamental problems of theoretical physics.

In this paper, I would like to show a mechanism of solving of the cosmological constant problem based on the adelic structure of the quantum field (string) theory models. Some speculations on the fine structure constant and the prime numbers are given.

2. Cosmological constant problem

The cosmological constant problem is one of the most serious paradoxes in modern particle physics and cosmology. Some astronomical observations indicate that the cosmological constant is many orders of magnitude smaller than estimated in modern theoretical elementary particles physics.

2.1 In his attempt (1917), to apply the general relativity to the whole universe, A. Einstein invented a new term involving a free parameter \( \lambda \), the cosmological constant (CC),

\[
R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} = \lambda g_{\mu\nu} - 8\pi G T_{\mu\nu}.
\]  

(1)

With this modification he finds a static solution for the universe filled with dust of zero pressure and mass density

\[
\rho = \frac{\lambda}{8\pi G}.
\]

(2)

The geometry of the universe was that of a sphere \( S_3 \) with proper circumference \( 2\pi r \), where

\[
r = \lambda^{-1/2},
\]

(3)

so the mass of the universe was

\[
M = 2\pi^2 r^3 \rho = \frac{\pi}{4}G^{-1}\lambda^{-1/2} \\
\sim r(?!).
\]

(4)

Any contributions to the energy density of the vacuum acts just like CC. By Lorentz invariance, in the vacuum,

\[
\langle T_{\mu\nu} \rangle = -\langle \rho \rangle g_{\mu\nu},
\]

(5)

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so
\[ \lambda_{\text{eff}} = \lambda + 8\pi G < \rho >, \]  
(6)

or the total vacuum energy density
\[ \rho_V = < \rho > + \frac{\lambda}{8\pi G} = \frac{\lambda_{\text{eff}}}{8\pi G}. \]  
(7)

The experimental upper bound on \( \lambda_{\text{eff}} \) or \( \rho_V \) is provided by measurements of cosmological redshifts as a function of distance. From the present expansion rate of the universes [4]
\[ \frac{d\ln R}{dt} \equiv H_0 = 100h \frac{km}{secMpc}, \quad h = 0.7 \pm 0.07 \]  
(8)

we have
\[ H_0^{-1} = (1 \div 2) \times 10^{10} ye, \quad |\lambda_{\text{eff}}| \leq H_0^2, \quad |\rho_V| \leq 10^{-29} g/cm^2 \simeq 10^{-47} GeV^4. \]  
(9)

2.2 The quantum oscillator with hamiltonian
\[ H = \frac{1}{2} P^2 + \frac{1}{2} \omega^2 x^2, \]  
(10)

has the energy spectrum
\[ E_n = \hbar \omega (n + 1/2), \]  
(11)

with the lowest, vacuum, value \( E_0 = \hbar \omega \). Normal modes of a quantum field of mass \( m \) are oscillators with frequencies \( \omega(k) = \sqrt{k^2 + m^2} \). Summing the zero-point energies of all normal modes of the field up to a wave number cut-off \( \Lambda >> m \) yields a vacuum energy density
\[ < \rho > = \int_0^\Lambda \frac{4\pi k^2 dk}{(2\pi)^3} \frac{1}{2} \sqrt{k^2 + m^2} \simeq \frac{\Lambda^4}{16\pi^2}. \]  
(12)

If we take \( \Lambda = (8\pi G)^{-1/2} \), then
\[ < \rho > \simeq 2^{-10} \pi^{-4} G^{-2} = 2 \times 10^7 GeV^4. \]  
(13)

We saw that
\[ |< \rho > + \frac{\lambda}{8\pi G}| \leq 10^{-47} GeV^4 \simeq (10^{-3} eV)^4, \]  
(14)

so the two terms must cancel to better than 100 decimal places! If we take \( \Lambda_{QCD} \), \( < \rho > \simeq 10^{-6} GeV^4 \), the two terms must cancel better to than 40 decimal places. Since the cosmological upper bound on \( < \rho_{\text{eff}} > \) is vastly less
than any value expected from particle theory, theorists assumed that (for some unknown reason) this quantity is zero.

3. Supersymmetric mechanism of solution to the CC problem

A minimal realization of the algebra of supersymmetry

\[
\{Q, Q^+\} = H, \\
\{Q, Q\} = \{Q^+, Q^+\} = 0,
\]  

is given by a point particle in one dimension, [5]

\[
Q = a(-iP + W), \\
Q^+ = a^+(iP + W),
\]  

where \(P = -i\partial/\partial x\), the superpotential \(W(x)\) is any function of \(x\), and spinor operators \(a\) and \(a^+\) obey the anticommuting relations

\[
\{a, a^+\} = 1, \\
\quad a^2 = (a^+)^2 = 0.
\]  

There is a following representation of operators \(a, a^+\) and \(\sigma\) by the Pauli spin matrices

\[
a = \frac{\sigma_1 - i\sigma_2}{2}, \\
a^+ = \frac{\sigma_1 + i\sigma_2}{2}, \\
\sigma = \sigma_3.
\]  

From formulae (15) and (16) then we have

\[
H = P^2 + W^2 + \sigma W_x.
\]  

The simplest nontrivial case of the superpotential \(W = \omega x\) corresponds to the supersymmetric oscillator with Hamiltonian

\[
H = H_B + H_F, \\
H_B = P^2 + \omega^2 x^2, \\
H_F = \omega \sigma,
\]  

wave function

\[
\psi = \psi_B \psi_F,
\]  

and spectrum

\[
H_B \psi_{Bn} = \omega(2n + 1) \psi_{Bn}, \\
H_F \psi_+ = \omega \psi_+, \\
H_F \psi_- = -\omega \psi_-.
\]
The ground state energies of the bosonic and fermionic parts are

\[ E_{B0} = \omega, \quad E_{F0} = -\omega, \quad (23) \]

so the vacuum energy of the supersymmetric oscillator is

\[ <0|H|0> = E_0 = E_{B0} + E_{F0} = 0, \quad |0> = \psi_{B0}\psi_{F0}. \quad (24) \]

**3.1** Let us see on this toy - solution of the CC problem from the quantum statistical viewpoint. The statistical sum of the supersymmetric oscillator is

\[ Z(\beta) = Z_B Z_F, \quad (25) \]

where

\[ Z_B = \sum_n e^{-\beta E_{Bn}} = e^{-\beta \omega} + e^{-\beta \omega(2+1)} + ..., \]
\[ Z_F = \sum_n e^{-\beta E_{Fn}} = e^{\beta \omega} + e^{-\beta \omega}. \quad (26) \]

In the low temperature limit,

\[ Z(\beta) = 1 + O(e^{-\beta^2 \omega}) \to 1, \quad \beta = T^{-1}, \quad (27) \]

so CC

\[ \lambda \sim lnZ \to 0. \quad (28) \]

**3.2** In the case of the adelic solution to the CC problem we will have,

\[ Z(\beta) = \prod_{p \geq 1} Z_p = Z_1 Z_2 Z_3 Z_5 ... , \]
\[ Z_1 \equiv Z_B, \quad Z_F \div Z_2 Z_3 Z_5 ... (?!) \quad (29) \]

**4. p - adic fractal calculus and adelic solution of the cosmological constant problem**

Every (good) school boy/girl knows what is

\[ \frac{d^n}{dx^n}; \quad (30) \]

but what is its following extension

\[ \frac{d^n}{dx^\alpha} = ?, \quad \alpha \in \mathbb{R}. \quad (31) \]
Let us consider the integer derivatives of the monomials
\[
\frac{d^n}{dx^n} x^m = m(m-1)...(m-(n-1))x^{m-n}, \quad n \leq m,
\]
\[
= \frac{\Gamma(m+1)}{\Gamma(m+1-n)} x^{m-n}.
\]

(32)

L. Euler (1707 - 1783) invented the following definition of the fractal derivatives:
\[
\frac{d^\alpha}{dx^\alpha} x^\beta = \frac{\Gamma(\beta + 1)}{\Gamma(\beta + 1 - \alpha)} x^{\beta - \alpha}.
\]

(33)

J. Liouville (1809-1882) takes exponentials as a base functions,
\[
\frac{d^\alpha}{dx^\alpha} e^{ax} = a^{-\alpha}(e^{ax} - e^{ac}).
\]

(34)

J. H. Holmgren (1863) invented the following integral transformation
\[
D_{c,x}^{-\alpha} f = \frac{1}{\Gamma(\alpha)} \int_c^x |x-t|^{\alpha-1} f(t) dt.
\]

(35)

It is easy to show that
\[
D_{c,x}^{-\alpha} x^m = \frac{\Gamma(m+1)}{\Gamma(m+1+\alpha)} (x^{m+\alpha} - c^{m+\alpha}),
\]
\[
D_{c,x}^{-\alpha} e^{ax} = a^{-\alpha}(e^{ax} - e^{ac}),
\]

(36)

so \( c = 0 \), when \( m + \alpha \geq 0 \), in Holmgren’s definition of the fractal calculus, corresponds to the Euler’s definition, and \( c = -\infty \), when \( a > 0 \), corresponds to the Liouville’s definition.

Note also the following slight modification of the \( c = 0 \) case [6]
\[
D_{0,x}^{-\alpha} f = \frac{|x|\alpha}{\Gamma(\alpha)} \int_0^1 |1-t|^{\alpha-1} f(xt) dt
\]
\[
= \frac{|x|\alpha}{\Gamma(\alpha)} B(\alpha, \frac{d}{dx} x f(x)) = |x|\alpha \frac{\Gamma(\frac{d}{dx} x)}{\Gamma(\alpha + \frac{d}{dx} x)} f(x),
\]
\[
f(xt) = x^t f(t) = t^{\frac{dx}{dx}} f(x), (\frac{d}{dx} x)^{-1} = x^{-1} \int_0^x dx.
\]

(37)

Let us consider integer derivatives, \( \alpha = -n \),
\[
D_{0,x}^n f = \frac{1}{x^n \Gamma(-n + \partial x)} f
\]
\[
= x^{-n}(-n + \partial x)(-n + 1 + \partial x)...(-1 + \partial x)f = ... .
\]
\[ f^{(n)} = x^{-n}(-n + 1 + x\partial)x^{n-1}\partial^{n-1}f. \] (38)

Fractal derivatives, when \( \alpha = -n + \varepsilon, 0 < \varepsilon < 1 \), can be calculated as \( D^n D^{-\varepsilon} \).

It is easy to show, that \( D^{-k-1}f = D^{-k}D^{-1}f \), so integrals can be calculated as \( D^{-n}f = (D^{-1})^n f \), where

\[ D^{-1}f = x \frac{\Gamma(\partial x)}{\Gamma(1 + \partial x)} f = x \frac{1}{\partial x} f = (\partial)^{-1} f. \] (39)

4.1 As an example, let us consider Weierstrass C.T.W. (1815 - 1897) fractal function

\[ f(t) = \sum_{n \geq 0} a^n e^{i(b^n t + \varphi_n)}, \quad a < 1, \; ab > 1. \] (40)

For fractals we have no integer derivatives,

\[ f^{(1)}(t) = i \sum (ab)^n e^{i(b^n t + \varphi_n)} = \infty, \] (41)

but the fractal derivative,

\[ f^{(\alpha)}(t) = i^\alpha \sum (ab^\alpha)^n e^{i(b^n t + \varphi_n)}, \] (42)

when \( ab^\alpha = a' < 1 \), is another fractal [6].

4.2 Definition of the p-adic norm, \( | |_p \) for rational numbers \( r \in Q \) is

\[ |r|_p = p^{-k}, \quad r \neq 0; \]
\[ |0|_p = 0, \] (43)

where \( k = ord_p(r) \) is defined from the following representation of the \( r \)

\[ r = \pm p^k \frac{m}{n}, \] (44)

integers \( m \) and \( n \) do not contain as factor \( p \).

The p-adic analog of the fractal calculus (35),

\[ D_x^{-\alpha} f = \frac{1}{\Gamma_p(\alpha)} \int_{Q_p} |x - t|_p^{\alpha-1} f(t) dt, \] (45)

where \( f(x) \) is a complex function of the p-adic variable \( x \), with p-adic \( \Gamma \)-function

\[ \Gamma_p(\alpha) = \int_{Q_p} dt |t|_p^{\alpha-1} \chi(t) = \frac{1 - p^{\alpha-1}}{1 - p^{-\alpha}}, \] (46)
was considered by V.S. Vladimirov [7].

Note also the following slight modification of (45),

\[ D_x^{-\alpha} f = \frac{|x|^\alpha}{\Gamma_p(\alpha)} \int_{Q_p} |1 - t|^{\alpha-1}_p f(xt) dt = |x|_p^\alpha \frac{\Gamma_p(\partial|x|)}{\Gamma_p(\alpha + \partial|x|)} f(x). \]  

(47)

Last expression is applicable for functions of type \( f(x) = f(|x|) \).

4.3 Let us consider the following action

\[ S = \frac{1}{2} \int_{Q_v} dx \Phi(x) D_x^\alpha \Phi, \quad v = 1, 2, 3, 5, \ldots \]  

(48)

In the momentum representation

\[ S = \frac{1}{2} \int_{Q_v} du \tilde{\Phi}(-u)|u|^\alpha_v \tilde{\Phi}(u), \]  

(49)

where

\[ \Phi(x) = \int_{Q_v} du x_v(ux) \Phi(u), \]

\[ D^{-\alpha} x_v(ux) = |u|^{-\alpha}_v x_v(ux). \]  

(50)

The statistical sum of the corresponding quantum theory is

\[ Z_v = \int d\Phi e^{\frac{-i}{2} \int \Phi D^\alpha \Phi} = det^{-1/2} D^\alpha = (\prod_u |u|^{-\alpha}_v)^{-1/2}. \]  

(51)

Note that, by fractal calculus and vector generalization of the model (48), string amplitudes were obtained in [3].

4.4 Adels \( a \in A \) are constructed by real \( a_1 \in Q_1 \) and \( p \)-adic \( a_p \in Q_p \) numbers (see e.g. [9])

\[ a = (a_1, a_2, a_3, a_5, \ldots, a_p, \ldots), \]  

(52)

with restriction that \( a_p \in Z_p = \{ x \in Q_p, |x|_p \leq 1 \} \) for all but a finite set \( F \) of primes \( p \).

\( A \) is a ring with respect to the componentwise addition and multiplication. A principal adel is a sequence \( r = (r, r, \ldots, r, \ldots), r \in Q\)-rational number.

Norm on adels is defined as

\[ |a| = \prod_{p \geq 1} |a_p|_p. \]  

(53)

Note that the norm on principal adels is trivial.
In the adelic generalization of the model (48),

$$\Phi(x) = \prod_{p \geq 1} \Phi_p(x_p), \quad dx = \prod_{p \geq 1} dx_p, \quad D_x^\alpha = \sum_{p \geq 1} D_p^\alpha,$$

(54)

where by $D_x^\alpha$ we denote fractal derivative (37), $x_1$ is real and $||_1$ is real norm.

If

$$\int dx_p |\Phi(x_p)|^2 = 1,$$

(55)

then

$$\int dx |\Phi(x)|^2 = 1, \quad S = \sum_{p \geq 1} S_p,$$

(56)

so

$$Z = \prod_{p \geq 1} Z_p = \prod_{p \geq 1} \left( \prod_{u \geq 1} |u|_p \right)^{-\alpha/2} = \left( \prod_{u} \prod_{p \geq 1} |u|_p \right)^{-\alpha/2} = 1, \quad \lambda \sim \ln Z = 0,$$

(57)

if $u \in Q$.

5. Some observations on zeta function, prime numbers and fine structure constant

Extended particles: nuclei, hadrons, strings,... are characterized by exponential state density

$$\rho(E) \sim e^{\beta H E}.$$  

(58)

Gas of the extended particles described by statistical sum

$$Z = \sum_{n} e^{-\beta E_n} = \sum_{E_n} \rho(E_n) e^{-\beta E_n},$$

(59)

is well defined for $\beta \geq \beta_H$ or $T \leq T_H = 1/\beta_H$ - Hagedorn temperature (see e.g. [8]).

5.1 The following representations of zeta-function [10]

$$\zeta(\beta) = \sum_{n \geq 1} \frac{1}{n^{\beta}} = \sum_{n \geq 1} e^{-\beta E_n} = \prod_{p \geq 2} \frac{1}{1-p^{-\beta}} = \prod_{p \geq 2} \zeta_p,$$

(60)

where $E_n = \ln n$, are defined for $\text{Re}\beta > 1$.

In physical terms, zeta-function is almost a statistical sum of ideal gas of quantum bosonic oscillators with frequencies $\omega = \ln p$. The following modification of the partial zeta-functions,

$$Z_{pB} = p^{-\beta/2} \zeta_p(\beta) = \frac{p^{-\beta/2}}{1-p^{-\beta}} = \frac{1}{p^{\beta/2} - p^{-\beta/2}},$$

(61)
corresponds exactly to the quantum bosonic oscillators. Zeta-function has a pole at $\beta = 1$, ”trivial” zeros at $\beta = -2n, n \geq 1$ and, according to Riemann’s hypothesis, nontrivial (complex) zeros on the imaginary line $\beta = 1/2 + i\lambda_n$.

5.2 In a sense the following reciprocal zeta-function looks more interesting (less reducible):

$$\zeta_r(\beta) = \frac{1}{\zeta(\beta)} = \prod_p (1 - p^{-\beta}) = \sum_{n \geq 1} \frac{\mu(n)}{n^\beta} = (1 - \beta) R(\beta).$$

(62)

Here $\mu(n)$-Mobius arithmetic function is defined on natural numbers as

$$\mu(1) = 1, \quad \mu(n) = (-1)^k,$$

(63)

if the factorized form of $n$, $n = p_1 p_2 \ldots p_k$ contains only different prime factors and is zero if two factors coincide. Partial reciprocal zeta-functions,

$$\zeta_{pr}(\beta) = 1 - p^{-\beta} = e^{-\beta \omega/2} Z_{pF}(\beta),$$

(64)

almost coincide with the fermionic oscillator statistical sum,

$$Z_{pF}(\beta) = \sum_{E_n} \rho(E_n) e^{-\beta E_n} = \sum_{E_n} e^{-\beta F_n},$$

(65)

where the density of the occupied fermionic state is negative

$$\rho(E_1) = -1,$$

(66)

free energy $F_n$ and entropy $S_n$ are

$$F_n = E_n + S_n T, \quad E_n = \omega(n - 1/2), \quad S_n = i\pi n, \quad \omega = ln p, \quad n = 0; 1.$$  

(67)

We can consider mixed quantum gases with different primes if we restrict ourselves with some maximal prime $p_N$,

$$Z_{NB} = \prod_{p=p_1}^{p_N} Z_{pB}, \quad Z_{NF} = \prod_{p=p_1}^{p_N} Z_{pF},$$

(68)

but we cannot consider the quantum systems with the infinite number of prime components without renormalization (simply neglecting) infinite vacuum energy.

For $\zeta_r$-functions we have an adelic identity

$$\prod_{p \geq 1} \zeta_{pr} = 1, \quad \zeta_{1r} \equiv \zeta,$$

(69)

so in the corresponding, ”number - theoretic universe”there is not a CC-problem.
Note that the quantum statistical sums (61,65) are antisymmetric with respect to the dual transformation \( p \rightarrow p^{-1} \). Physical quantities, which are logarithmic derivatives of the statistical sums, remain invariant. The classical limit, \( p \rightarrow 1 \), corresponds to the self-dual point \( p=1 \).

5.3 Following extension of the integer numbers

\[
[n]_p = \frac{p^n - 1}{p - 1} = 1 + p + p^2 + \ldots + p^{n-1},
\]
represents repunits (see e.g. [11]),

\[
[n]_p = 11\ldots1.
\]

In the classical limit, \( p \rightarrow 1 \), \([n]_1 = n\). Note also the identity

\[
[p_1 p_2 \ldots p_k]_q = [p_1]_q [p_2]_q \ldots [p_k]_q p_1 p_2 \ldots p_{k-1}.
\]

This identity in the classical limit, \( q \rightarrow 1 \), reduce to the main arithmetic relation

\( n = p_1 p_2 \ldots p_k \). If we take \( q = \exp\left(\frac{2\pi i}{p}\right) \), then \([n]_q = 0\), when \( p \) is equal to one of the factors of \( n \).

5.4 Now, for a hadronic string model (see e.g. [12]) we know, that the high temperature phase, \( T > T_H \), is the quark-gluon phase or, as it was named by S.B. Gerasimov, Gluqua.

Interesting questions are:
• what is the high temperature phase of the fundamental string (Twistor; Topological; p-adic...) ?
• What is the ”high temperature phase”, \( \beta \leq 1 \), of the zeta-function, what are the constituents of the (prime) numbers ?

The following identity

\[
\frac{1}{1-x} = (1 + x)(1 + x^2)(1 + x^4)\ldots
\]
for \( x = p^{-\beta} \) tells us that (almost) bosonic gas of prime oscillators can be represented as a gas of (almost) fermionic oscillators with frequencies \( \omega = 2^n \ln p \).

This is a hint on a grassmann constituents of primes.

5.5 Function \( R(\beta) \) defined in (62) has the poles in the same points where zeta-function has zeros. So it is natural to investigate R-function by methods of scattering theory [13]. Corresponding resolvent

\[
\hat{R}(\beta) = \frac{1}{\beta - H},
\]
defines a hamiltonian with eigenvalues as zeros of zeta-function.
5.6 For each prime $p$ we have the following representation of $-1$

$$-1 = (p - 1)(1 + p + p^2 + p^4 + ...),$$  \hspace{1cm} (75)

so we can eliminate negative numbers in the field of $p$-adic numbers, for each $p$. Now we can represent $\sqrt{-1}$

$$i = \sqrt{-1} = \sqrt{p - 1\sqrt{1 + p + ...}}.$$  \hspace{1cm} (76)

Thus, for some primes,

$$p = 4k^2 + 1 = 5, \ 17, \ 37, \ 101, \ 197, \ 257, ...$$  \hspace{1cm} (77)

we can also eliminate complex numbers. Next, $\sqrt{-1}$ can be eliminated for primes

$$p = 2^{2^2}k^{2^2} = 17, \ 257, ...$$  \hspace{1cm} (78)

and $\sqrt{-1}$ can be eliminated for primes

$$p = 2^{2^3}k^{2^3} + 1 = 257, ...$$  \hspace{1cm} (79)

Note that the nearest integer to prime 257 is 256 = $2^8$ = 1byte.

Let me also mention that in quantum computing (Quanputing, [14]) we already have quantum logic (dynamics, algorithms,...) but have not yet quantum ethic (save conditions for quanputation, decoherence problems).

In a more general case, $\sqrt[2^n]{-1}$, we come to the primes

$$p = 2^{2^n}k^{2^n} + 1.$$  \hspace{1cm} (80)

The case $k = 1$ in (80) corresponds to the primes of Fermat(1601 - 1665).

5.7 In quantum electrodynamics [15], there is a fundamental constant $\alpha$-fine structure constant. The value of $\alpha^{-1} = 137.036...$ [4] is in a good approximation given by prime $p$=137$^2$. There is no theoretical explanation to this value.

Note that

$$137 = 11^2 + 4^2 = |11 + 4i|^2 = |4 + 11i|...$$  \hspace{1cm} (81)

Now a curious question is: what is the distance between $z_1 = 11 + 4i$ and $z_2 = 4 + 11i$,

$$|z| = |z_1 - z_2| = \sqrt{49 + 49} = \sqrt{100 - 2} = 10(1 - \frac{1}{100} + ...).$$  \hspace{1cm} (82)

$$|z| = 10 - O(1\%)$$  \hspace{1cm} (83)

\footnote{Another prime number that I like is 887 - lifetime of neutron in seconds [4]. I like that $137 + 887 = 1024 = 2^{10} = 1K$.}
If we want to take exactly 10, we must rise 11 a little. This will be in right direction, but gives for \( \alpha^{-1} = 138.5 \). So for more precise value of \( \alpha^{-1} = 137.036 \), we will have a little bigger value of \(|z|\), but less than 10. Let us see also a toy-version of the previous consideration,

\[
37 = 6^2 + 1 = |6 + i| = |1 + 6i| = ...
\]

\[
|z| = \sqrt{5^2 + 5^2} = \sqrt{49 + 1} = 7 + O(1%),
\]

\[
37 + 87 = 124.
\] (84)

If we put on the complex plane all the eight points \( z_1, z_2, ..., z_8 \), and connect the nearest points, we obtain an eightangle with sides with lengths 8 and (almost)10 (2 and 7, in the toy model). It seems interesting that with this figure we can cover the plane on the scale of 10 figures, then the deviation of order 1 (fundamental) unit of length appears. Next characteristic scale is of an order of 100 figures, where deviation of the scale of 1 figure appears.

Some characteristic scales of the quantum theory of particles are: atomic scale \( \sim 10^{-8} \text{cm} \), quantum electrodynamic scale \( \sim 10^{-11} \text{cm} \), strong interaction scale \( \sim 10^{-13} \text{cm} \), week interaction scale \( \sim 10^{-16} \text{cm} \), Plank scale \( \sim 10^{-33} \text{cm} \). There are also other scales including macroscopic and cosmological scales.

5.8 Dirac-Schwinger’s quantization [16, 17]

\[
eg e = n,
\] (85)

says that if there is in Nature even one magnetic monopole, with charge \( g \), electric charge \( e \) is quantized. From (85), when \( n=1 \), we see

\[
\alpha_g = g^2 = e^{-2} = \alpha^{-1} = 137,
\] (86)

and fundamental force between elementary monopoles is

\[
F_g = \frac{g^2}{r^2} = \frac{137}{r^2}.
\] (87)

6. Conclusions and perspectives

There were different attempts to solve the CC-problem (see e.g.,[1]), one of them is on the way of introduction of the several time coordinates [18].

The adelic mechanism considered in this paper can be included also in the adelic generalization of the standard model of cosmology [19].

Zeta-function considerations in this text contain a hint that there is a modification of the quantum field theory not containing divergences.

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3There are some other similar structures, e.g. \( 12^2 + 12^2 = 17^2 - 1 \). This way we come to the rational approximations of the \( \sqrt{2} = 1.41421 \ldots \).
Now let me draw a general picture and make some hand-waving arguments. Classical fields (see e.g. [20]) have a phenomenological, macroscopic, meaning. If we want to understand their fundamental, microscopic, structure, we might find their quants, quantize the fields (see e.g. [15]). If we have problems with quantization of a field, (could not construct corresponding perturbation theory; strong coupling; unrenormalizable interactions, ...) it maybe due to nonfundamental, composite nature of the field, and we should try to find corresponding constituent fields. Sometimes we are forced to take nonconvenient start: with free field (string) theory tachions (Higgs particles! Neutrinos?) (fundamental bosonic string ground state); Big Bang...

In the functional integral formulation [21] of the quantum (as well as classical) theory and renormgroup method [15, 22], for consistency, we should have ultraviolet and infrared attractors (usualy fixed points, but they maybe as well quasi-periodic cycles and even strange attractor-fractals) in the space of coupling constants and fields.

CC–problem is cosmological problem, so we expect, that due to the renor-mgroup evolution, on the cosmological scale, we have an effective theory with supersymmetry and/or adelic structure.

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