Anti-reflection coatings of zero-index metamaterial for solar cells

Cite as: AIP Advances 10, 025010 (2020); https://doi.org/10.1063/1.5129516
Submitted: 30 September 2019 . Accepted: 16 January 2020 . Published Online: 06 February 2020

Muhammad Kamran, and Muhammad Faryad

ARTICLES YOU MAY BE INTERESTED IN

Lattice resonances in dielectric metasurfaces
Journal of Applied Physics 125, 213105 (2019); https://doi.org/10.1063/1.5094122

Dielectric nanoresonators and metamaterials
Journal of Applied Physics 126, 150401 (2019); https://doi.org/10.1063/1.5129100

Hyperbolic metamaterials: From dispersion manipulation to applications
Journal of Applied Physics 127, 071101 (2020); https://doi.org/10.1063/1.5128679
Anti-reflection coatings of zero-index metamaterial for solar cells

Cite as: AIP Advances 10, 025010 (2020); doi: 10.1063/1.5129516
Submitted: 30 September 2019 • Accepted: 16 January 2020 • Published Online: 6 February 2020

Muhammad Kamran and Muhammad Faryad

AFFILIATIONS
1 Department of Electrical Engineering, Lahore University of Management Sciences, Lahore 54792, Pakistan
2 Department of Physics, Lahore University of Management Sciences, Lahore 54792, Pakistan

Author to whom correspondence should be addressed: muhammad.faryad@lums.edu.pk

ABSTRACT
Anti-reflection coatings of zero index metamaterials (ZIMs) are proposed for maximum absorption of light in solar cells. A thin layer of a ZIM is shown to help trap light inside a solar cell. The outer surface of a ZIM layer is planar, and the inner surface has periodic corrugations in order for the incident light to pass through but block the re-transmission of the light back into free space. Using rigorous calculations for light absorption efficiency integrated over the AM1.5 solar spectrum, the basic design of the anti-reflection coating using a ZIM is studied by comparing the results with a common anti-reflection coating and a ZIM layer planar on both sides.

I. INTRODUCTION
The reflection of light from solar cells is one of the important factors in reducing the efficiency of the cell to convert sunlight to electric power. For this reason, anti-reflection coatings are used to minimize the reflections. The simplest anti-reflection coating was first observed by Lord Rayleigh in the 19th century, when he observed that tarnishing on glass increased its transmittance instead of its reflectance. Etching of a surface can also decrease the reflectance from the surface, as observed by Fraunhofer in 1817. The most commonly used anti-reflection coating is a quarter-wave layer matching the impedance of the free-space to that of the substrate. An optimized version of this anti-reflection coating is designed for high efficiency silicon solar cells. Anti-reflection coatings using metamaterials and artificially designed surface relief gratings have also been proposed among others. These techniques have been developed for both crystalline silicon and thin-film solar cells.

In addition to anti-reflection coatings, other light trapping techniques are also being used, for example, localized surface plasmons using metallic nanoparticles and propagating surface plasmon–polariton waves using surface-relief gratings. Furthermore, the texturing of the surface of solar cells is a very well known and efficient technique for light trapping. Recently, photonic crystals have also been proposed for enhanced light trapping in thin-film solar cells. Nano-structuring can also be used to enhance light trapping with the help of silicon nanowire.

Metamaterials are artificially designed materials that get their bulk properties from not only their constituent materials but also their structural arrangements. Common metamaterials include double positive materials, with both permittivity and permeability having positive values, double negative materials, zero index metamaterials (ZIMs) with a near-zero refractive index, and bi-isotropic and bianisotropic materials. A ZIM is a metamaterial in which the permittivity and/or the permeability of the medium are nearly zero, and thus, its refractive index is close to zero. ZIMs find applications in electromagnetic cloaking, directional emission, tunneling effects, transition from total reflection to total transmission, and the reshaping of the phase front. ZIMs can be designed as a mixture of metallic and dielectric materials or as a photonic crystal containing all-dielectric materials working at a frequency close to the Dirac-like point in the photonic band structure. ZIMs using metallic inclusion suffer from ohmic losses and the fact that the impedance is infinite. However, a photonic-crystal-based design does not suffer from both of these issues. However, all designs of a ZIM currently are not broadband.

When the refractive index is near zero, the phase velocity is very large inside the material. Consequently, the wavelength inside
the material is also very large, resulting in the phase being uniform throughout the medium.\textsuperscript{31} Thus, an electromagnetic wave can only penetrate inside a ZIM if the shape of the wave fronts of the incident electromagnetic wave is of the same shape as of the interface of a ZIM. Therefore, if we design a layer of a ZIM medium with a planar surface on the side of the incident solar light and corrugations on the side of active solar material, the light will couple into the solar cell but will not be able to exit. This must help increase the light absorption inside the solar cell. This paper is dedicated to testing this hypothesis theoretically using rigorous calculations. The basic design is shown schematically in Fig. 1. We used the rigorous-coupled wave approach (RCWA) to compute the reflectance, transmittance, and absorbance of the solar cell with a periodically corrugated ZIM layer.\textsuperscript{34–36,40}

The plan of the paper is as follows: since a ZIM has both permittivity and permeability near zero, we present the specialized formulation of the RCWA in Sec. II for isotropic dielectric and magnetic materials. The numerical results are presented and discussed in Sec. III. The conclusions are presented in Sec. IV. An \(\exp(-
abla \omega t)\) time dependency is taken, and it is implicit throughout the paper, where \(\omega\) is the angular frequency. The free-space wave number and free-space wavelength are indicated by \(k_0 = \omega/\sqrt{\varepsilon_0\mu_0}\) and \(\lambda_0 = 2\pi/k_0\), respectively, where \(\varepsilon_0\) and \(\mu_0\) are the permittivity and permeability of free-space. Vectors are represented by boldface, and unit vectors are represented by boldface with a hat on them.

II. PROBLEM DESCRIPTION

Let us consider the schematic of the problem, as shown in Fig. 1. The half-space \(z < 0\) is the incidence medium with the relative permittivity \(\varepsilon_0\) and relative permeability \(\mu_0\). The region between \(z > 0\) and \(z < L_{AR}\) is occupied by a material that serves as an anti-reflection coating, with the relative permittivity \(\varepsilon_{AR}\), relative permeability \(\mu_{AR}\), and thickness \(L_{AR}\). The region between \(z > L_{AR}\) and \(z < L_{AR} + L_z\) is occupied by a one-dimensional surface-relief grating made of ZIM with a relative permittivity \(\varepsilon_z\), relative permeability \(\mu_z\), and thickness \(L_z\). The region between \(z > L_{AR} + L_z\) and \(z < L_{AR} + L_z + L_{GR}\) is a rectangular one-dimensional surface-relief grating along the x axis with a depth \(L_{GR}\), period \(L_p\), and duty cycle \(\zeta \in (0, 1)\), with the relative permittivity \(\varepsilon_G(x, z) = \varepsilon_G(x \pm L, z)\) and the relative permeability \(\mu_G(x, z) = \mu_G(x \pm L, z)\). The region between \(z > L_{GR} + L_z + L_{GR}\) and \(z < L_{GR} + L_z + L_{GR} + L_0\) is occupied by crystalline silicon with a relative permittivity \(\varepsilon_s\), relative permeability \(\mu_s\), and thickness \(L_0\). The region between \(z > L_{GR} + L_z + L_{GR} + L_0\) and \(z < L_s\) is a metallic layer with a relative permittivity \(\varepsilon_l\) and relative permeability \(\mu_l\).

Let us consider a plane wave propagating in the region \(z < 0\) incident on the \(z = 0\) interface making an angle \(\theta\) with the \(z\) axis. The incident, reflected, and transmitted electric and magnetic field phasors in terms of Floquet harmonics can be written as

\[
E_{inc}(r) = \sum_{\ell \in \mathbb{L}} \left[ \tilde{s}_\ell a_\ell(t) + \tilde{p}_\ell d_\ell(t) \right] \exp \left\{ i \left[ k_\ell(x) x + k_{z\ell}(z) \right] \right\}, \quad z \leq 0, \quad (1)
\]

\[
\eta_0 H_{inc}(r) = n_0 \sum_{\ell \in \mathbb{L}} \left[ \tilde{p}_\ell a_\ell(t) - \tilde{s}_\ell d_\ell(t) \right] \exp \left\{ i \left[ k_\ell(x) x + k_{z\ell}(z) \right] \right\}, \quad z \leq 0, \quad (2)
\]

\[
E_{ref}(r) = \sum_{\ell \in \mathbb{L}} \left[ \tilde{s}_\ell t_\ell(t) + \tilde{p}_\ell r_\ell(t) \right] \exp \left\{ i \left[ k_\ell(x) x - k_{z\ell}(z) \right] \right\}, \quad z \leq 0, \quad (3)
\]

\[
\eta_0 H_{ref}(r) = n_0 \sum_{\ell \in \mathbb{L}} \left[ \tilde{p}_\ell t_\ell(t) - \tilde{s}_\ell r_\ell(t) \right] \exp \left\{ i \left[ k_\ell(x) x - k_{z\ell}(z) \right] \right\}, \quad z \leq 0, \quad (4)
\]

\[
E_{tr}(r) = \sum_{\ell \in \mathbb{L}} \left[ \tilde{s}_\ell t_\ell(t) + \tilde{p}_\ell r_\ell(t) \right] \exp \left\{ i \left[ k_\ell(x) x + k_{z\ell}(z - L_z) \right] \right\}, \quad z > L_z, \quad (5)
\]

\[
\eta_0 H_{tr}(r) = n_t \sum_{\ell \in \mathbb{L}} \left[ \tilde{p}_\ell t_\ell(t) - \tilde{s}_\ell r_\ell(t) \right] \times \exp \left\{ i \left[ k_\ell(x) x + k_{z\ell}(z - L_z) \right] \right\}, \quad z > L_z, \quad (6)
\]

where

\[
k_{\ell\ell}(x) = k_0 n_\ell \sin \theta + \ell \kappa_z, \quad \kappa_z = 2\pi/L_z.
\]

\[
k_{z\ell}(z) = \sqrt{\left[ k_0 n_\ell \right]^2 - \left[ k_{\ell\ell}(x) \right]^2},
\]

\[
k_{z\ell}(z) = \sqrt{\left[ k_0 n_\ell \right]^2 - \left[ k_{\ell\ell}(x) \right]^2},
\]

where \(n_\ell\) and \(n_t\) are the refractive indices of the incidence and the transmission medium. In Eqs. (1)–(6), \(a_\ell^0\) and \(d_\ell^0\) are the amplitudes of the \(s\)- and \(p\)-polarized incident plane waves, respectively, and \(a_\ell^t\) and \(d_\ell^t\) are the unknown amplitudes of the Floquet harmonics of order \(\ell\) in the \(s\)-polarized reflected and transmitted fields, respectively. Similarly, \(r_\ell^0\) and \(r_\ell^t\) are the amplitudes of the Floquet harmonics of order \(\ell\) in the \(p\)-polarized reflected and transmitted fields, respectively, where \(\ell\) is the order of Floquet harmonics with \(\ell \in \{0, \pm 1, \pm 2, \pm 3, \ldots\}\). Let us note that \(\ell = 0\) represents the specular component of
the reflected and transmitted field, and $\ell \neq 0$ are the non-spectral components.

The $s$-polarization state is represented by the unit vector

$$\mathbf{s}_s = \hat{u}_s,$$

and the $p$-polarization state of the incident and reflected waves is represented by

$$\mathbf{p}_p = \frac{\pi k_{zp}^{(t)} \hat{u}_x + k_{zp}^{(t)} \hat{u}_y}{k_{0} n_{p}},$$

and the $p$-polarization state of the transmitted wave is represented by

$$\mathbf{p}_t = \frac{k_{zp}^{(t)} \hat{u}_x + k_{zp}^{(t)} \hat{u}_y}{k_{0} n_{t}}.$$

### A. Rigorous coupled-wave approach (RCWA)

The constitutive relations can be written as

$$\mathbf{D}(\mathbf{r}) = \varepsilon(\mathbf{r}, x) \mathbf{E}(\mathbf{r}),$$

$$\mathbf{B}(\mathbf{r}) = \mu(\mathbf{r}, x) \mathbf{H}(\mathbf{r}), \quad x \in [0, L_z].$$

The RCWA demands that all field phasors and permittivity are expressed as Fourier series with respect to $x$ as

$$\varepsilon^{(\ell)}(x, z) = \sum_{\ell = -\infty}^{\infty} \varepsilon^{(\ell)}(z) \exp(i \ell k_x x), \quad z \in [0, L_z],$$

and

$$\mu^{(\ell)}(x, z) = \sum_{\ell = -\infty}^{\infty} \mu^{(\ell)}(z) \exp(-i \ell k_x x), \quad \ell \neq 0, \quad z \in (L_Ax + L_x, L_AR + L_x + L_z),$$

$$\mu^{(\ell)}(x, z) = \sum_{\ell = -\infty}^{\infty} \mu^{(\ell)}(z) \exp(-i \ell k_x x), \quad \ell = 0, \quad z \in (L_Ax + L_x, L_AR + L_x + L_z),$$

for the relative permittivity and the relative permeability can be written as

$$\varepsilon(x, z) = \sum_{\ell = -\infty}^{\infty} \varepsilon^{(\ell)}(z) \exp(i \ell k_x x), \quad z \in [0, L_z],$$

and

$$\mu(x, z) = \sum_{\ell = -\infty}^{\infty} \mu^{(\ell)}(z) \exp(i \ell k_x x), \quad z \in [0, L_z],$$

respectively, where

$$\varepsilon^{(0)}(z) = \begin{cases} \varepsilon_{AR}, & z \in [0, L_{AR}] \\ \varepsilon_z, & z \in [L_{AR}, L_AR + L_z] \\ \varepsilon_{m}, & z \in [L_AR + L_z, L_AR + L_z + L_{AR} + L_z] \end{cases},$$

$$\mu^{(0)}(z) = \begin{cases} \mu_{AR}, & z \in [0, L_{AR}] \\ \mu_z, & z \in [L_{AR}, L_AR + L_z] \\ \mu_{m}, & z \in [L_AR + L_z, L_AR + L_z + L_{AR} + L_z] \end{cases}.$$
where \( X \) matrix

Substituting Eqs. (30) and (31) back in Eqs. (21), (22), (24), and (25) we obtain the matrix ordinary differential equation

\[
\frac{d}{dz} [f(z)] = [P(z)] [f(z)], \quad z \in (0, L),
\]

where \([f(z)]\) is a column vector with 4(2N + 1) components given as

\[
[f(z)] = \left[ [E_r(z)]^T, [E_i(z)]^T, \eta_0[H_i(z)]^T, \eta_0[H_r(z)]^T \right]^T,
\]

and the 4(2N + 1) \times 4(2N + 1) matrix \([P(z)]\) is

\[
[P(z)] = \begin{bmatrix}
0 & 0 & 0 & [P_{m1}](z) \\
0 & 0 & 0 & [P_{m2}](z) \\
0 & 0 & 0 & [P_{m1}](z) \\
0 & 0 & 0 & [P_{m2}](z)
\end{bmatrix},
\]

with

\[
[P_{m1}(z)] = k_0 [\mu(z)] - \frac{1}{k_0^{\prime}} [K_a]^{-1} [g_a],
\]

\[
[P_{m2}(z)] = -k_0 [\mu(z)],
\]

\[
[P_{m3}(z)] = \frac{1}{k_0} [K_a]^{-1} [\mu(z)]^{-1} [K_a] - k_0 [g_a],
\]

where \([0]\) is a (2N + 1) \times (2N + 1) null matrix.

**B. Fields at boundaries**

To implement the boundary conditions at the \( z = 0 \) and \( z = L \) interfaces, column field vectors are required at these boundaries. The column vectors \([f(0)]\) and \([f(L)]\) can be obtained using Eqs. (1)–(6) as

\[
[f(0)] = \begin{bmatrix}
[Y_{inc}^0] \\
[Y_{ref}^0]
\end{bmatrix}, \quad [A] \\
[R]
\]

and

\[
[f(L)] = \begin{bmatrix}
[Y_{inc}^L] \\
[Y_{ref}^L]
\end{bmatrix}, \quad [T],
\]

respectively, where the column vectors for the incident, reflected, and transmitted amplitudes for \( s\) - and \( p\)-polarized components are, respectively, defined as

\[
[A] = \begin{bmatrix}
a_s^{(-N)}, & a_s^{(-N+1)}, & \ldots, & a_s^{(N-1)}, & a_s^{(N)}, & a_s^{(-N)}, \\
ap_p^{(-N+1)}, & a_p^{(-N+1)}, & \ldots, & a_p^{(N-1)}, & a_p^{(N)}, & a_p^{(-N)},
\end{bmatrix}^T,
\]

\[
[R] = \begin{bmatrix}
r_s^{(-N)}, & r_s^{(-N+1)}, & \ldots, & r_s^{(N-1)}, & r_s^{(N)}, & r_s^{(-N)}, \\
r_p^{(-N+1)}, & r_p^{(-N+1)}, & \ldots, & r_p^{(N-1)}, & r_p^{(N)}, & r_p^{(-N)},
\end{bmatrix}^T,
\]

\[
[T] = \begin{bmatrix}
t_s^{(-N+1)}, & t_s^{(-N+1)}, & \ldots, & t_s^{(N-1)}, & t_s^{(N)}, & t_s^{(-N)}, \\
t_p^{(-N+1)}, & t_p^{(-N+1)}, & \ldots, & t_p^{(N-1)}, & t_p^{(N)}, & t_p^{(-N)},
\end{bmatrix}^T.
\]

The other \( (2N + 1) \times (2N + 1) \)-matrices are obtained as

\[
[Y_{inc}] = \frac{1}{k_0^{\prime} \eta_a} \begin{bmatrix}
0 & -[K_{a}] \\
[\xi] & [\xi]
\end{bmatrix},
\]

\[
[Y_{ref}] = \frac{1}{k_0 \eta_a} \begin{bmatrix}
[K_{a}] \\
[0] & -\eta_a k_0 [\xi]
\end{bmatrix},
\]

\[
[Y_{inc}] = \frac{1}{k_0^{\prime}} \begin{bmatrix}
[k_{a}] \\
[0] & [\xi]
\end{bmatrix},
\]

\[
[Y_{ref}] = \frac{1}{k_0^{\prime}} \begin{bmatrix}
[k_{a}] \\
[0] & [\xi]
\end{bmatrix},
\]
\[
\begin{bmatrix}
\mathbf{Y}_s
\end{bmatrix} = \frac{1}{k_0} \begin{bmatrix}
- \mathbf{K}_s & \begin{bmatrix} 0 \\ \mathbf{0} \end{bmatrix} \\
\begin{bmatrix} 0 \\ \mathbf{0} \end{bmatrix} & -n_i k_0 \mathbf{I}_N
\end{bmatrix},
\]

where
\[
\mathbf{K}_s = \text{diag}[k_s^{(f)}],
\]
and \([\mathbf{I}_N]\) is the \((2N_i + 1) \times (2N_i + 1)\) identity matrix.

### C. Absorptance

The unknown reflection and transmission amplitudes can be computed using a stable algorithm described elsewhere. After the amplitudes are computed, the absorptance for the s- and p-polarized incident plane waves can be computed as
\[
A_t = 1 - \sum_{\ell=-N_i}^{N_i} \frac{|t_\ell^{(s)}|^2 + |t_\ell^{(p)}|^2}{|t_\ell^{(s)}|^2 + |t_\ell^{(p)}|^2} \text{Re} \left( \frac{k_\ell^{(s)}}{k_{2at}} \right)
\]
and
\[
A_p = 1 - \sum_{\ell=-N_i}^{N_i} \frac{|t_\ell^{(s)}|^2 + |t_\ell^{(p)}|^2}{|t_\ell^{(s)}|^2 + |t_\ell^{(p)}|^2} \text{Re} \left( \frac{k_\ell^{(s)}}{k_{2at}} \right)
\]
respectively. The solar-spectrum-integrated (SSI) absorption efficiency of light can be defined as
\[
\text{SSI absorption} = \frac{\int (A_t \times AM1.5) d\lambda}{\int (AM1.5) d\lambda}, \quad t \in \{p,s\},
\]
where \(A_t\) is the absorptance, \(t\) in the subscript represents the p- or s-polarized light, and AM1.5 is the solar spectrum.

### III. NUMERICAL RESULTS

To analyze the effect of a periodically corrugated ZIM layer on the enhancement of light absorption in solar cells, we implemented the RCWA presented in Sec. II A in Matlab and computed reflectance, transmittance, and absorbance to elucidate the effect of the ZIM corrugated layer on the top of the solar cell. We chose our one-dimensional grating to be rectangular, with
\[
\varepsilon(x) = \begin{cases} 
\varepsilon_s, & x \in (0, \xi L), \\
\varepsilon_y, & x \in (\xi L, L),
\end{cases}
\]

and
\[
\mu(x) = \begin{cases} 
\mu_s, & x \in (0, \xi L), \\
\mu_y, & x \in (\xi L, L),
\end{cases}
\]
This also helped in speeding up the computations as each region in the solar cell is homogeneous along the z axis and can be considered as comprising one slice for the implementation of the RCWA. For representative results, the solar cell is considered to be made of crystalline silicon. On the top of the ZIM layer, we assumed an anti-reflection layer of magnesium fluoride (MgF\(_2\)) with a thickness of 80 nm. This thin layer of magnesium fluoride helped in minimizing the reflections from the ZIM interface so that maximum light gets into the active layer. The relative permittivity and the relative permeability of the ZIM layer were considered to be 0.01. Let us note that such ZIMs with a constant near-zero index do not exist yet. However, for this proof-of-concept study, this constant value is assumed. The idea is to show the effectiveness of the new design approach for anti-reflection coatings. The consequence of this choice is that the proposed design will only work over the spectral region of the ZIM where the index is nearly constant. The thickness of the ZIM layer and the metallic layer is 80 nm, and for the metallic layer, we used gold (Au). The period \(L\) of rectangular periodic corrugation is 600 nm, the duty cycle is \(\xi = 0.5\), and the depth of periodic corrugation \(k_y\) is considered to be 80 nm. The parameter \(N_i\) is considered to be 15 in the simulation after ascertaining the convergence of the results within 0.5% of the results when \(N_i = 16\) over the whole spectral range considered in this paper. The refractive index of magnesium fluoride is taken from Ref. [44], silicon from Ref. [45], and gold from Ref. [46]. Furthermore, all numerical results are presented for the normal incidence \(\theta = 0\).

To see the effect of the presence of the corrugated ZIM layer, the absorptance \(A_p\) for p-polarized light and \(A_t\) for s-polarized light as a function of wavelength \(\lambda_0\) are presented in Figs. 2 and 3, respectively, for the following four configurations:

- the ZIM layer with periodic corrugations,
- the ZIM layer with planar interfaces,
- the ZIM layer is replaced with MgF\(_2\), and
- the ZIM layer is replaced with MgF\(_2\) or air when \(\lambda_0\) is

\[1.5 \text{ is the solar spectrum.}\]
small. When $\lambda_0$ is larger, absorptance is oscillating as a function of $\lambda_0$. This is because silicon is very lossy at smaller wavelengths, and the losses reduce significantly when $\lambda_0 > 550$ nm. However, the absorptance, in general, is the largest for corrugated ZIM layers. This is more pronounced for the $p$-polarized case than for the $s$-polarized one. This is in line with our expectations that corrugations help trap the light inside the active layer (silicon). Let us note that the oscillations in the absorption, as shown in Figs. 2 and 3, are present regardless of the fact whether the ZIM layer is present or not. This is because the silicon layer at larger wavelengths behaves as a Fabry–Perot cavity. This is also confirmed by the increasing period of oscillation as the wavelength increases because the width of the silicon layer as compared to the operating wavelength decreases.

Since simple absorptance plots fail to show the actual impact of the ZIM layer directly, the SSI absorption efficiency was computed and analyzed. Figure 4 shows the SSI absorption efficiency for the $p$-polarized incident light as a function of period $L$ of the periodic corrugations of the ZIM layer. We can observe that the SSI absorption is considerably higher in the presence of the corrugated ZIM layer, supporting our hypothesis. Also, the variation with respect to the period $L$ of the corrugation is small beyond a threshold value. The optimum value of SSI absorption efficiency is achieved at $L = 600$ nm. Figure 5 shows the SSI absorption efficiency for the $s$-polarized incident light as a function of period $L$ of the periodic corrugation. In this case, again, the SSI absorption efficiency is higher when the ZIM layer is present. Here, again, the SSI absorption increases with the increase in period $L$ and remains more-or-less constant beyond a threshold value. This constant enhancement beyond a threshold value of $L$ indicates the robustness of the ZIM layer as an AR coating. This also makes sense if we consider the properties of the ZIM as they allow the coupling of light only when the incident wavefront of light and the interface has the same shape to allow constancy of the phase inside the ZIM.

To see if the proposed scheme of light trapping using a corrugated ZIM layer is valid for thin-film or thick-film solar cells, we have also plotted SSI absorption efficiency as a function of thickness of silicon layer ($L_s$). Figure 6 shows SSI absorption of $p$-polarized incident light as a function of thickness of the silicon layer ($L_s$) for a fixed value of the period $L$. It can be observed that the ZIM layer...
with periodic corrugation is absorbing maximum light. Also, silicon is the main absorbing layer in the solar cell, and that is why by increasing the thickness of the silicon layer the SSI absorption also increases. Figure 7 shows the SSI absorption for s-polarized incident light as a function of thickness of the silicon layer ($L_s$). Both figures show that the corrugated ZIM layer gives the maximum SSI absorption efficiency for a thickness of the active layer from 400 nm to 2 μm. Therefore, the proposed design of the anti-reflection coating is useful for thin- as well as thick-film solar cells.

To check the dependence of SSI absorption efficiency on the depth of the periodic corrugations $L_g$, Fig. 8 shows the SSI absorption as a function of depth $L_g$. It can be observed that by increasing the depth of periodic corrugations $L_g$, the SSI absorption also increases. Even at a very small value of $L_g$, the SSI absorption is higher in the case of a ZIM layer, and this is the evidence that the presence of periodic corrugations helped in enhancement of the light absorption in solar cells. Figure 9 shows SSI absorption as a function of $L_g$ for s-polarized incident light. In this case, again, we can see that the SSI absorption shows results which are consistent with our hypothesis. Let us also note that the transmittance chosen for our solar cells is negligible for all cases because of a thick metallic back reflector. Therefore, all enhancement in the absorption is due to the decrease in the reflectance.

IV. CONCLUSIONS

We proposed and theoretically analyzed a new concept of anti-reflection coatings for solar cells. These coatings are made of zero index metamaterials (ZIMs), employing the property of these materials that they transmit light through them only if the incident light has the same phase front as the shape of the incidence interface. Therefore, a coating with a planar outer surface and a periodically corrugated inner surface was numerically investigated to see its effect on the integrated absorption efficiency for an AM1.5 incident solar flux. It was found that these coatings enhance the light absorption significantly for both the p- and the s-polarized incident light. Furthermore, it was found to work with the same efficiency, regardless of the thickness of the active layer of the solar cells.

ACKNOWLEDGMENTS

This work is partially supported by the Higher Education Commission (HEC), Pakistan, via Grant No. NRPU 2016-5905.

REFERENCES

1. J. Fraunhofer, Versuche über die Ursachen des Anlaufens und Mattwerdens des Glases und die Mittel, denselben zuvorzukommen (Joseph von Fraunhofer Gesanikte Schriften, Munich, Germany, 1888).
2. G. Chartier, Introduction to Optics (Springer, 2005).
3. J. Zhao and M. A. Green, *IEEE Trans. Electron Devices* **38**, 1925–1934 (1991).
4. J. Zhang, P. A. R. Ade, P. Mauskopf, L. Moncelsi, G. Savini, and N. Whitehouse, *Appl. Opt.* **48**, 6635–6642 (2009).
5. A. Deinega, I. Valuev, B. Potapkin, and Y. Lozovik, *J. Opt. Soc. Am. A* **28**, 770–777 (2011).
6. Y. W. Chen, P. Y. Han, and X.-C. Zhang, *Appl. Phys. Lett.* **94**, 041106 (2009).
7. J. Kroll, J. Darmo, and K. Unterrainer, *Opt. Express* **15**, 6552–6560 (2007).
8. A. Thoman, A. Kern, H. Helm, and M. Walther, *Phys. Rev. B* **77**, 195405 (2008).
9. K. L. Chopra, P. D. Paulson, and V. Dutta, *Prog. Photovoltaics: Res. Appl.* **12**, 69–92 (2004).
10. A. Shah, P. Torres, R. Tscharner, N. Wyrsch, and H. Keppner, *Science* **285**, 692–698 (1999).
11 D. E. Carlson and C. R. Wronski, Appl. Phys. Lett. 28, 671–673 (1976).
12 F. J. Beck, A. Polman, and K. R. Catchpole, J. Appl. Phys. 105, 114310 (2009).
13 V. E. Ferry, M. A. Verschuuren, H. B. T. Li, E. Verhagen, R. J. Walters, R. E. I. Schropp, H. A. Atwater, and A. Polman, Opt. Express 18, A237–A245 (2010).
14 P. Campbell and M. A. Green, J. Appl. Phys. 62, 243 (1987).
15 K. J. Weber and A. W. Blakers, Prog. Photovoltacs: Res. Appl. 13, 691–695 (2005).
16 D. Zhou and R. Biswas, J. Appl. Phys. 103, 093102 (2008).
17 E. Garnett and P. Yang, Nano Lett. 10, 1082–1087 (2010).
18 N. Engheta and R. W. Ziolkowski, Metamaterials: Physics and Engineering Explorations (John Wiley & Sons, Inc., 2006).
19 J. Hao, W. Yan, and M. Qiu, Appl. Phys. Lett. 96, 101109 (2010).
20 A. Alu, M. G. Silveirinha, A. Salandrino, and N. Engheta, Phys. Rev. B 75, 155410 (2007).
21 M. W. Ashraf and M. Faryad, J. Nanophotonics 9, 093057 (2015).
22 F. Wang, M. W. Horn, and A. Lakhtakia, Microelectron. Eng. 71, 34–53 (2004).
23 F. Wang, K. E. Weaver, A. Lakhtakia, and M. W. Horn, Optik 116, 1–9 (2005).
24 M. Faryad and A. Lakhtakia, J. Nanophotonics 5, 053527 (2011).
25 See https://refractiveindex.info/?shelf=main&book=Rodriguez-de_Marcos for the refractive index of magnesium fluoride; accessed 26 July 2019.
26 See https://refractiveindex.info/?shelf=main&book=Schinke for the refractive index of silicon; accessed 26 July 2019.
27 See https://refractiveindex.info/?shelf=main&book=McPeak for the refractive index of gold; accessed 26 July 2019.