Embedding a Deterministic BFT Protocol in a Block DAG

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This work formalizes the structure and protocols underlying recent distributed systems leveraging block DAGs, which are essentially encoding Lamport’s happened-before relations between blocks, as their core network primitives. We then present an embedding of any deterministic Byzantine fault tolerant protocol \( P \) to employ a block DAG for interpreting interactions between servers. Our main theorem proves that this embedding maintains all safety and liveness properties of \( P \). Technically, our theorem is based on the insight that a block DAG merely acts as an efficient reliable point-to-point channel between instances of \( P \) while also using \( P \) for efficient message compression.

1 INTRODUCTION

Recent interest in blockchain and cryptocurrencies has resulted in a renewed interest in Byzantine fault tolerant consensus for state machine replication, as well as Byzantine consistent and reliable broadcast that is sufficient to build payment systems [2, 12]. A number of designs [21] for such mechanisms depart from the traditional setting of participants directly sending protocol messages to each other, and rely instead in a common higher level abstraction where participants exchange blocks of transactions, linking cryptographically to past blocks—generalizing the idea of a blockchain to a more generic directed acyclic graph embodying Lamport’s happened-before relations [16] between blocks, which we refer to as a block DAG.

Examples of such designs are Hashgraph [1] used by the Hedera network, as well as ALEPH [11], BLOCKMANIA [7], and FLARE [20]. These works argue a number of advantages for the block DAG approach. First, maintaining a joint block DAG is simple and scalable, and can leverage widely-available distributed key-value stores. Second, they report impressive performance results compared with traditional protocols that materialize point-to-point messages as direct network messages. This results from batching many transactions in each block; using a low number of cryptographic signatures, having minimal overhead when running deterministic parts of the protocol; using a common block DAG logic while performing network IO, and only applying the higher-level protocol logic off-line possibly later; and as a result supporting running many instances of protocols in parallel ‘for free’. We take these claimed performance and implementation simplicity advantages as a given and do not examine them further.

We note, however, that while the protocols may be simple and performant when implemented, their specification, and arguments for correctness, safety and liveness are far from simple. Their proofs and arguments are usually inherently tied to their specific applications and requirements, but both specification and formal arguments of HASHGRAPH, ALEPH, BLOCKMANIA, and FLARE are structured around two phases: (i) building a block DAG, and (ii) running a protocol on top of the block DAG. We generalize their arguments by giving an abstraction of a block DAG as a reliable point-to-point link. We can then rely on this abstraction to simulate a protocol \( P \)—as a black-box—on top of this point-to-point link maintaining the safety and liveness properties of \( P \). We hope that this modular formulation of the underlying mechanisms through a clear separation from the high-level protocol \( P \) and the underlying block DAG allows for easy re-usability and strengthens the foundations and persuasiveness of systems based on block DAGs.

In this work we present a formalization of a block DAG, the protocols to maintain a joint block DAG, and its properties. We show that any deterministic Byzantine fault tolerant (BFT) protocol, can be embedded in this block DAG, while maintaining its safety and liveness properties. We demonstrate that the claimed advantageous properties
of block DAG based protocols, such as the efficient message compression, batching of signatures, the ability to run multiple instances 'for free', and off-line interpretation of the block DAG, emerge from the generic composition we present. Therefore, the proposed composition not only allows for straightforward correctness arguments, but also preserves the claimed advantages of using a block DAG approach, making it a useful abstraction not only to analyze but also implement systems that offer both high assurance and high performance.

Overview. Figure 1 shows the interfaces and components of our proposed block DAG framework parametric by a deterministic BFT protocol $\mathcal{P}$. At the top, we have a user seeking to run one or multiple instances of $\mathcal{P}$ on servers $\text{Srvrs}$. First, to distinguish between multiple protocol instances the user assigns a label $\ell$ from a set of labels $\mathcal{L}$. Now, for $\mathcal{P}$ there is a set of possible requests $\mathcal{Rqsts}_\mathcal{P}$. But instead of requesting $r \in \mathcal{Rqsts}_\mathcal{P}$ from $s_i \in \text{Srvrs}$ running $\mathcal{P}$ for protocol instance $\ell$, the user calls the high-level interface of our block DAG framework: $\text{request}(\ell, r)$ in $\text{shim}(\mathcal{P})$. Internally, $s_i$ passes $(\ell, r)$ on to $\text{gossip}(\mathcal{G})$—which continuously builds $s_i$’s block DAG $\mathcal{G}$ by receiving and disseminating blocks. The passed $(\ell, r)$ is included into the next block $s_i$ disseminates, and $s_i$ also includes references to other received blocks, where cryptographic primitives prevent byzantine servers from adding cycles between blocks [17]. These blocks are continuously exchanged by the servers utilizing the low-level interface to the network to exchange blocks. In Section 3 we formally define the block DAG, its properties and protocols for servers to maintain a joint block DAG. Independently, as indicated by the dotted lines, $s_i$ interprets $\mathcal{P}$ by reading $\mathcal{G}$ and running $\text{interpret}(\mathcal{G}, \mathcal{P})$. To do so, $s_i$ locally simulates every protocol instance $\mathcal{P}$ with label $\ell$ by simulating one process instance of $\mathcal{P}(t)$ for every server $s_i \in \text{Srvrs}$. To drive the simulation, $s_i$ passes the request $r$ read from a block in $\mathcal{G}$ to $\mathcal{P}$, and then $s_i$ simulates the message exchange between any two servers based on the structure of the block DAG and the deterministic protocol $\mathcal{P}$. Therefore $s_i$ moves messages between in- and out-buffers $\text{Ms}_\mathcal{P}[\text{in}, t]$ and $\text{Ms}_\mathcal{P}[\text{out}, t]$. Eventually, the simulation $\mathcal{P}(t)$ of the server $s_i$ will indicate $i$ from the set of possible indications $\mathcal{Inds}_\mathcal{P}$. We show how the block DAG essentially acts as a reliable point-to-point link and describe how any BFT protocol $\mathcal{P}$ can be interpreted on a block DAG in Section 4. Finally, after interpret indicated $i$, $\text{shim}(\mathcal{P})$ can indicate $i$ for $t$ to the user of $\mathcal{P}$. From the user’s perspective, the embedding of $\mathcal{P}$ acted as $\mathcal{P}$, i.e. $\text{shim}(\mathcal{P})$ maintained $\mathcal{P}$’s interfaces and properties. We prove this in Section 5 and illustrate the block DAG framework for $\mathcal{P}$ instantiated with byzantine reliable broadcast protocol. We give related work in Section 6, and
conclude in Section 7, where we discuss integration aspects of higher-level protocols and the block DAG framework—including challenges in embedding protocols with non-determinism, more advanced cryptography, and BFT protocols operating under partial synchrony.

**Contribution.** We show that using the block DAG framework of Figure 1 for a deterministic BFT protocol \( \mathcal{P} \) maintains the (i) interfaces, and (ii) safety and liveness properties of \( \mathcal{P} \) (Theorem 5.1). The argument is generic: interpreting the eventually joint block DAG implements a reliable point-to-point link (Lemma 3.7, Lemma 4.3). Using this reliable point-to-point link any server can locally run a simulation of \( \mathcal{P} \) as a black-box. This simulation is an execution of \( \mathcal{P} \) and thus retains the properties of \( \mathcal{P} \). By using the block DAG framework, the user gains efficient message compression and runs many instances of \( \mathcal{P} \) in parallel ’for free’.

## 2 BACKGROUND

**System Model.** We assume a finite set of servers \( \text{Srvrs} \). A correct server \( s \in \text{Srvrs} \) faithfully follows a protocol \( \mathcal{P} \). When \( s \) is byzantine, then \( s \) behaves arbitrarily. However, we assume byzantine servers are computationally bound (e.g. \( s \) cannot forge signatures, or find collisions in cryptographic hash functions) and cannot interfere with the Trusted Computing Base of correct servers (e.g. kill the electricity supply of correct servers). The set \( \text{Srvrs} \) is fixed and known by every \( s' \in \text{Srvrs} \) and we assume \( 3f+1 \) servers to tolerate at most \( f \) byzantine servers. The set of all messages in protocol \( \mathcal{P} \) is \( M_\mathcal{P} \). Every message \( m \in M_\mathcal{P} \) has a \( m.\text{sender} \) and a \( m.\text{receiver} \). We assume an arbitrary, but fixed, total order on messages: \( \prec_\mathcal{P} \). A protocol \( \mathcal{P} \) is deterministic if a state \( q \) and a sequence of messages \( m \in M_\mathcal{P} \) determine state \( q' \) and out-going messages \( M \subseteq 2^{M_\mathcal{P}} \). In particular, deterministic protocols do not rely on random behavior, such as coin-flips. The exact requirements on the network synchronicity depend on the protocol \( \mathcal{P} \), that we want to embed, e.g. we may require partial synchrony [8] to avoid FLP [9]. The only network assumption we impose for building block DAGs is the following:

**Assumption 1 (Reliable Delivery).** For two correct servers \( s_1 \) and \( s_2 \), if \( s_1 \) sends a block \( B \) to \( s_2 \), then eventually \( s_2 \) receives \( B \).

**Cryptographic Primitives.** We assume a secure cryptographic hash function \( \# : A \rightarrow A' \) and write \( \#(x) \) for the hash of \( x \in A \), and \( \#(A) \) for \( A' \) (cf. Definition A.1). We further assume a secure cryptographic signature scheme [14]: given a set of signatures \( \Sigma \) we have functions \( \text{sign} : \text{Srvrs} \times M \rightarrow \Sigma \) and \( \text{verify}_\sigma : \text{Srvrs} \times M \times \Sigma \rightarrow \Sigma \), where \( \text{verify}_\sigma(s, m, \sigma) = \text{true} \) iff \( \text{sign}(s, m) = \sigma \). Given computational bounds on all participants appropriate parameters for these schemes can be chosen to make their probability of failure negligible, and for the remainder of this work we assume their probability of failure to be zero.

**Directed Acyclic Graphs.** A directed graph \( \mathcal{G} \) is a pair of vertices \( V \) and edges \( E \subseteq V \times V \). We write \( \emptyset \) for the empty graph. If there is an edge from \( v \) to \( v' \), that is \((v,v') \in E\), we write \( v \xrightarrow{} v' \). If \( v' \) is reachable from \( v \), then \((v,v') \in E\) is in the transitive closure of \( \rightarrow \), and we write \( \rightarrow^+ \). We write \( \rightarrow^* \) for the reflexive and transitive closure, and \( v \xrightarrow{n} v' \) for \( n \geq 0 \) if \( v' \) is reachable from \( v \) in \( n \) steps. A graph \( \mathcal{G} \) is acyclic, if \( v \xrightarrow{=} v' \) implies \( v \neq v' \) for all nodes \( v, v' \in \mathcal{G} \). We abbreviate \( v \in \mathcal{G} \) if \( v \in V_{\mathcal{G}} \), and \( V \subseteq \mathcal{G} \) if \( v \in \mathcal{G} \) for all \( v \in V \). Let \( \mathcal{G}_1 \) and \( \mathcal{G}_2 \) be directed graphs. We define \( \mathcal{G}_1 \cup \mathcal{G}_2 \) as \((V_{\mathcal{G}_1} \cup V_{\mathcal{G}_2}, E_{\mathcal{G}_1} \cup E_{\mathcal{G}_2})\), and \( \mathcal{G}_1 \preceq \mathcal{G}_2 \) holds if \( V_{\mathcal{G}_1} \subseteq V_{\mathcal{G}_2} \) and \( E_{\mathcal{G}_1} = E_{\mathcal{G}_2} \cap (V_{\mathcal{G}_1} \times V_{\mathcal{G}_1}) \). Note, for \( \preceq \) we not only require \( E_{\mathcal{G}_1} \subseteq E_{\mathcal{G}_2} \), but additionally \( E_{\mathcal{G}_1} \) must already contain all edges from \( E_{\mathcal{G}_2} \) between vertices in \( G_1 \). The following Definition to insert a new vertex \( v \) is restrictive: it permits to extend \( \mathcal{G} \) only by a vertex \( v \) and edges to this \( v \).
Definition 2.1. Let $G$ be a directed graph, $v$ be a vertex, and $E$ be a set of edges of the shape $\{(v_i, v) \mid v_i \in V \subseteq G\}$. We define $\text{insert}(G, v, E) = (V_G \cup \{v\}, E_G \cup E)$.

Lemma 2.2. For a directed graph $G$, a vertex $v$, and a set of edges $E = \{(v_i, v) \mid v_i \in V \subseteq G\}$, the following properties of $\text{insert}(G, v, E)$ hold:

1. if $v \in G$ and $E \subseteq E_G$, then $\text{insert}(G, v, E) = G$;
2. if $E = \{(v_i, v) \mid v_i \in V \subseteq G\}$ and $v \in G$, then $G \subseteq \text{insert}(G, v, E)$; and
3. if $G$ is acyclic, $v \notin G$, then $\text{insert}(G, v, E)$ is acyclic.

To give some intuitions, for Lemma 2.2 (2), if $v \in G$ and $G' = \text{insert}(G, v, E)$, then $E_{G'} \cap (V_{G'} \times V_{G'}) = E_G$ may not hold. For example, let $G$ have vertices $v_1$ and $v_2$ with $E_G = \emptyset$, and $G' = \text{insert}(G, v_2, \{(v_1, v_2)\})$ with $E_{G'} = \{(v_1, v_2)\}$. Now $E_G \neq E_{G'} \cap (V_{G'} \times V_{G'})$. For Lemma 2.2 (3), if $v \in G$, then $\text{insert}(G, v, E)$ may add a cycle. For example, take $G$ with vertices $\{v_1, v_2\}$ and $E_G = \{(v_1, v_2)\}$ then $\text{insert}(G, v_1, \{(v_2, v_1)\})$ contains a cycle.

3 BUILDING A BLOCK DAG

The networking component of the block DAG protocol between servers is defined by gossip in Algorithm 1, and is executed by all correct servers. This protocol is very simple: it has one core message type, namely a block, which is constantly disseminated. It contains simple meta-data, a signature, authentication for references to previous blocks, and requests associated to instances of protocol $P$. Servers only exchange and validate blocks. Now, although servers build their block DAGs locally, eventually correct servers have a joint block DAG $G$. The servers can then independently interpret $G$ as multiple instances of $P$ as defined in Algorithm 2 in Section 4.

Definition 3.1. A block $B \in \text{Blks}$ has (i) an identifier $n$ of the server $s$ which built $B$, (ii) a sequence number $k \in \mathbb{N}_0$, (iii) a finite list of hashes of predecessor blocks $\text{preds} = \{\text{ref}(B_1), \ldots, \text{ref}(B_k)\}$, (iv) a finite list of labels and requests $rs \in 2^{L \times \text{Rqts}}$, and (v) a signature $\sigma = \text{sign}(n, \text{ref}(B))$. Here, ref is a secure cryptographic hash function computed from $n, k, \text{preds}$, and $rs$, but not $\sigma$. By not including $\sigma, \text{sign}(B, n, \text{ref}(B))$ is well defined.

We use $B$ and $\text{ref}(B)$ interchangeably, which is justified by collision resistance of $\text{ref}$ (cf. Definition A.1(3)). We use register notation, e.g. $B.n$ or $B.\sigma$, to refer to elements of a block $B$, and abbreviate $B' \in \{B' \mid \text{ref}(B') \in B.\text{preds}\}$ with $B' \in B.\text{preds}$. Given blocks $B$ and $B'$ with $B.n = B'.n$ and $B'.k = B.k + 1$. If $B \in B'.\text{preds}$ then we call $B$ a parent of $B'$ and write $B'.\text{parent} = B$. We call $B$ a genesis block if $B.k = 0$. A genesis block $B$ cannot have a parent block, because $B.k = 0$ and $0$ is minimal in $\mathbb{N}_0$.

Lemma 3.2. For blocks $B_1$ and $B_2$, if $B_1 \in B_2.\text{preds}$ then $B_2 \notin B_1.\text{preds}$.

Lemma 3.2 prevents a byzantine server to include a cyclic reference between $B$ and $B$ by (1) waiting for—or building itself—a block $B$ with $\text{ref}(B) \in B.\text{preds}$, and then (2) building a block $\hat{B}$ such that $\text{ref}(\hat{B}) \in B$. As with secure timelines [17], Lemma 3.2 gives a temporal ordering on $B$ and $\hat{B}$. This is a static, cryptographic property, based on the security of hash functions, and not dependent e.g. on the order in which blocks are received on a network. While this prevents byzantine servers from introducing cycles, they can still build "faulty" blocks, and hence we impose the following validity conditions:

Definition 3.3. A server $s$ considers a block $B$ valid, written $\text{valid}(s, B)$, if (i) $s$ confirms $\text{verify}_s(B.n, B.\sigma)$, i.e. that $B.n$ built $B$, (ii) either (a) $B$ is a genesis block, or (b) $B$ has exactly one parent, and (iii) $s$ considers all blocks $B' \in B.\text{preds}$ valid.
Especially, (ii) deserves our attention: a server \( s_1 \) may still build two different blocks having the same parent. For example, all blocks in Figure 3 are valid. However, \( s_1 \) will not be able to create a further block to ’join’ these two blocks with a different parent—their successor will remain split. Essentially, this forces a linear history from every block.

We assume, that if a correct server considers a block \( B \) valid, then \( s \) can forward any block \( B' \in B.\text{preds} \). That is, \( s \) has received the full content of \( B' \)—not only \( \text{ref}(B') \)—and persistently stores \( B' \). As there are no cyclic references in blocks, the least and greatest fix-point of valid coincide. From valid blocks and their predecessors, a correct server builds a block DAG:

\[
\text{Definition 3.4. For a server } s, \text{ a block DAG } G \in \text{Dags is a directed acyclic graph with vertices } V_G \subseteq \text{Blks}, \text{ where (i) } \text{valid}(s, B) \text{ holds for all } B \in V_G, \text{ and (ii) if } B \in B'.\text{preds} \text{ then } B \in V_G \text{ and } (B, B') \in E_G \text{ holds for all } B' \in V_G. \text{ Let } B' \text{ be a block such that valid}(s, B') \text{ holds and } B \in G \text{ for all } B \in B'.\text{preds}. \text{ Then } s \text{ inserts } B' \text{ in } G \text{ by insert}(G, B', ((B, B') \mid B \in B'.\text{preds})) \text{ after Definition 2.1 and we write } G.\text{insert}(B). \text{ The preconditions guarantee that } G.\text{insert}(B') \text{ is a block DAG (Lemma A.3).}
\]

\[
\text{Example 3.5. In Figure 2 we show a block DAG with three blocks } B_1, B_2, \text{ and } B_3, \text{ where } B_1 = \{n = s_1, k = 0, \text{preds} = [ \ ], B_2 = \{n = s_2, k = 0, \text{preds} = [ \ ]\}, \text{ and } B_3 = \{n = s_1, k = 1, \text{preds} = [\text{ref}(B_1), \text{ref}(B_2)]\}. \text{ Here, parent}(B_3) = B_1. \text{ Consider now Figure 3 adding the block: } B_4 = \{n = s_1, k = 1, \text{preds} = [\text{ref}(B_1), \text{ref}(B_2)]\}. \text{ With block } B_4, \text{ } s_1 \text{ is equivocating on the block } B_3\text{—and vice versa.}
\]

Algorithm 1 shows how a server \( s \) builds (i) its block DAG \( G \) in lines 4–13, and (ii) its current block \( B \) by including requests and references to other blocks in lines 14–18. The servers communicate by exchanging blocks. Assumption 1 guarantees, that a correct \( s \) will eventually receive a block from another correct server. Moreover, every correct server \( s \) will regularly request disseminate() in line 14 and will eventually send their own block \( B \) in line 17 of Algorithm 1 guaranteed by the high-level protocol (cf. Section 5).

Every server \( s \) operates on four data structures. For one, the data structures shared with interpret in Algorithm 2 given as arguments in line 1: (i) the block DAG \( G \), which interpret will only read, and (ii) a buffer rhsqs, where interpret inserts pairs of labels and requests. On the other hand, \( s \) also keeps (iii) the block \( B \) which \( s \) currently builds (line 2), and (iv) a buffer blks of received blocks (line 3). To build its block DAG, \( s \) inserts blocks into \( G \) in line 7 and line 16. Lemma A.5 guarantees that by inserting those blocks \( G \) remains a block DAG. To insert a block, \( s \) keeps track of its received blocks as candidate blocks in the buffer blks (line 4–5). Whenever \( s \) considers a \( B' \in \text{blks} \) valid (line 6), \( s \) inserts \( B' \) in \( G \) (line 7). However, to consider a block \( B' \) valid, \( s \) has to consider all its predecessors valid—and \( s \) may not have yet received every \( B \in B'.\text{preds} \). That is, \( B' \in \text{blks} \) but \( B \notin \text{blks} \) and \( B \notin G \) (cmp. line 10). Now, \( s \) can request forwarding of \( B \) from the server that built \( B' \), i.e. from \( s' \) where \( B'.n = s' \), by sending \( \text{FWD} \) \( B \) to \( s' \) (lines 10–11). To prevent \( s \) from flooding \( s' \) an implementation would guard lines 10–11, e.g. by a timer \( \Delta B' \). On the other hand, \( s \)
The mechanism, together with the Assumption 1 and the eventual dissemination of block DAGs, allows us to establish the following lemma:

**Lemma 3.6.** For a correct server $s$ executing gossip, if $s$ receives a block $B$, which $s$ considers valid, then (1) every correct server will eventually receive $B$, and (2) every correct server will eventually consider $B$ valid.

In parallel to building $G$, $s$ builds its current block $B$ by (i) continuously adding a reference to any block $B'$, which $s$ receives and considers valid in line 8 (adding at most one reference to $B'$, cf. Lemma A.6), and (ii) eventually sending $B$ to every server in line 17. Just before $s$ sends $B$, $s$ injects literal inscriptions of $(\ell, r_i) \in \text{rqsts}$ into $B$ in line 15. Now $rs$ holds requests $r_i$ for the protocol instances $P$ with label $\ell_i$. These requests will eventually be read by interpret in Algorithm 2. Finally, $s$ signs $B$ in line 15, sends $B$ to every server, and starts building its next $B$ in line 18 by incrementing the sequence number $k$, initializing preds with the parent block, as well as clearing $rs$ and $\sigma$.

So far we established, how $s$ builds its own block DAG. Next we want to establish the concept of a joint block DAG between two correct servers $s$ and $s'$. Let $G_s$ and $G_{s'}$ be the block DAG of $s$ and $s'$. We define their joint block DAG $G'$ as a block DAG $G' \supseteq G_s \cup G_{s'}$. This joint block DAG is a block DAG for $s$ and for $s'$ (Lemma A.7). Intuitively, we want any two correct servers to be able to 'gossip some more' and arrive at their joint block DAG $G'$.

**Lemma 3.7.** Let $s$ and $s'$ be correct servers with block DAGs $G_s$ and $G_{s'}$. By executing gossip in Algorithm 1, eventually $s$ has a block DAG $G'_s$ such that $G'_s \supseteq G_s \cup G_{s'}$.

**Proof.** By Lemma A.5 any block DAG $G'$ obtained through gossip is a block DAG, and by Lemma A.7 $G'$ is a block DAG for $s$. It remains to show that by executing gossip, eventually $G'$ will be the block DAG for $s$. As $s'$ received
and considers all $B \in \mathcal{G}^\prime$ valid, by Lemma 3.6 (2) $s$ will eventually consider every $B$ valid. By executing gossip, $s$ will eventually insert every $B$ in its block DAG and $\mathcal{G}^\prime$ will contain all $B \in \mathcal{G}^\prime$. □

In the next section, we will show how $s$ and $s'$ can independently interpret a deterministic protocol $P$ on this joint block DAG. But before we do so, we want to highlight that the gossip protocol retains the key benefits reported by works using the block DAG approach, namely simplicity and amenability to high-performance implementation. Now, our gossip protocol in Algorithm 1 uses an explicit forwarding mechanism in lines 10–13. This explicit forwarding mechanism—as opposed to every correct server re-transmitting every received and valid block in a second communication round—is possible through blocks including references to predecessor blocks. Hence, every server knows what blocks it is missing and whom to ask for them. But in an implementation, we would go a step further and replace the forwarding mechanism—and messages—as described next.

Each block is associated with a unique cryptographic reference that authenticates its content. As a result both best-effort broadcast operations as well as synchronization operations can be implemented using distributed and scalable key-value stores at each server (e.g. Apache Cassandra, AWS S3), which through sharding have no limits on their throughput. Best-effort broadcasts can be implemented directly, through simple asynchronous IO. This is due to the the (now) single type of message, namely blocks, and a single handler for blocks in gossip that performs minimal work: it just records blocks, and then asynchronously ensures their predecessors exist (potentially doing a remote key-value read) and they are valid (which only involves reference lookups into a hash-table and a single signature verification). Alternatively, best-effort broadcast itself can be implemented using a publish-subscribe notification system and remote reads into distributed key value stores. In summary, the simplicity and regularity of gossip, and the weak assumptions and light processing allow systems’ engineers great freedom to optimize and distribute a server’s operations. Both Hashgraph and Blockmania (which have seen commercial implementation) report throughputs of many 100,000 transactions per second, and latency in the order of seconds. As we will see in the next section no matter which $P$ the servers $s$ and $s'$ choose to interpret, they can build a joint block DAG using the same gossip logic—by only exchanging blocks—and then independently interpret $P$ on $\mathcal{G}$.

4 INTERPRETING A PROTOCOL

To interpret the protocol $P$ embedded in a block DAG $\mathcal{G}$ a server $s$ runs the interpret protocol defined in Algorithm 2. Running interpret is decoupled from running gossip in Algorithm 1 and building the block DAG. To interpret a protocol instance of $P$ with label $t$, $s$ runs locally one process instance of $P$ with label $t$ for each $s_i \in \text{Srvrs}$. Now, $s$ treats $P$ as a black-box which (i) takes a request or a message, and (ii) returns messages or an indication. These requests and messages are embedded in the block DAG $\mathcal{G}$ as (i) requests $B.rs$ embedded in block $B \in \mathcal{G}$, or as (ii) edges by interpreting $B_1 \rightarrow B_2$ as messages sent from $B_1.n$ to $B_2.n$. A server $s$ can fully simulate the protocol instance $P$ for any other server. User requests $r_j$ to $P$ are embedded into blocks, and $s$ reads these requests from the block and passes them on to the simulation of $\mathcal{P}$. Since $P$ is deterministic, $s$ can—after the initial request $r_j$ for $P$—compute all subsequent messages which would have been sent in $P$. There is no need for explicitly sending these messages. And indeed, we show that the interpretation of a deterministic protocol $P$ embedded in a block DAG implements a reliable point-to-point link.

To treat $P$ as a black-box, we assume the following high-level interface: (i) an interface to request $r \in \text{Rqsts}_P$, and (ii) an interface where $P$ indicates $i \in \text{Inds}_P$. When a request $r$ reaches a process instance, we assume that it immediately returns messages $m_1, \ldots, m_k$ triggered by $r$. This is justified, as $s$ runs all process instances locally. As
we would only start process instances for $G$. This is effectively a simplification: we assume a running process instance when \( \bot \).\(^{12} \)\(^{13} \)\(^{14} \)

\[ \text{ requests do not depend on the state of the process instance, also these messages do not depend on the current state of process instance. Then we assume a low-level interface for } P \text{ to receive a message } m. \text{ Again, we assume that when } m \text{ reaches a process instance, it immediately returns the messages } m_1, \ldots, m_k \text{ triggered by } m. \]

Algorithm 2 shows the protocol executed by $s$ for interpreting a deterministic protocol $P$ on a block DAG $G$. Therefore $s$ traverses through every $B \in G$. Through the state of $I$ in line 2, $s$ keeps track of which blocks in $G$ it has already interpreted. Hereby edges in $G$ impose a partial order: $s$ considers a block $B \in G$ as eligible($B$) for interpretation if (i) $I[B] = \text{false}$, and (ii) for every $B_i \in B.\text{preds}$, $I[B_i] = \text{true}$ holds. While there may be more than one $B$ eligible, every $B \in G$ is interpreted eventually (Lemma A.10). For now, let $s$ pick an eligible $B$ in line 3 and interpret $B$ in line 4–12. To interpret $B$, $s$ needs to keep track of two variables for every protocol instance $\ell_j$: (1) $\text{Pls} \{ \ell_j \}$, which holds the state of the process instance $\ell_j$ for a server $s_j \in \text{Srvrs}$, and (2) $\text{Ms} \{ \text{in}, \ell_j \}$ and $\text{Ms} \{ \text{out}, \ell_j \}$, which hold the state of in-going and out-going messages.

Our goal is to track changes to these two variables—the process instances $\text{Pls}$ and message buffers $\text{Ms}$—throughout the interpretation of $G$. To do so, we assign their state to every block $B$. That is, after interpreting $B$, (1) $B.\text{Pls} \{ \ell_j \}$ should hold the state of the process instance $\ell_j$ of the server $s_j$, which built $B$, i.e. $s_j = B.n$, and (2) $B.\text{Ms} \{ \text{in}, \ell_j \}$ should hold the in-going messages for $s_j$ and $B.\text{Ms} \{ \text{out}, \ell_j \}$ the out-going messages from $s_j$ for process instance $\ell_j$.\(^{11} \) We assume $B.\text{Pls} \{ \ell_j \}$ to be initialized with $\bot$, and $B.\text{Ms}[d \in \{ \text{in, out} \}, \ell_j]$ with $\varnothing$, and they remain so while $B$ is eligible. (cf. Lemma A.15).

As a starting point for computing the state of $B.\text{Pls} \{ \ell_j \}$, $s$ copies the state from the parent block of $B$ in line 4. For the base case, i.e. all (genesis) blocks $B$ without parents, we assume $B.\text{Pls} \{ \ell_j \} := \text{{new process}}$ $P(\ell_j, s_i)$ where $s_i = B.n$. This is effectively a simplification: we assume a running process instance $\ell_j$ for every $s_i \in \text{Srvrs}$. In an implementation, we would only start process instances for $\ell_j$ after receiving the first message or request for $\ell_j$ for $s_i = B.n$. Now in our simplification, we start all process instances for every label at the genesis blocks and pass them on from the parent blocks. This leads us to our step case: $B$ has a parent. As $B.\text{parent} \in B.\text{preds}$, $B.\text{parent}$ has been interpreted and

\[^{11} \text{An equivalent representation would keep process instances } \text{Pls} \{ B, \ell_j, B.n \} \text{ and message buffers } \text{Ms} \{ B, d \in \{ \text{in, out} \}, \ell_j \} \text{ explicitly as global state. We believe that our notation accentuates the information flow throughout the } G. \]
moreover $B,\text{parent} = s_1$ (Lemma A.13). Next, to advance the copied state on $B$, $s$ processes (1) all incoming requests $r_j$ given by $B,\text{rs}$ in lines 5–6, and (2) all incoming messages from $B,\text{n} \rightarrow B,\text{n}$ given by $B,\text{i} \rightarrow B$ in lines 8–11. For the former (1), $s$ reads the labels and requests from the field $B,\text{rs}$. Here $r_j$ is the literal transcription of the client’s original request given to $\mathcal{P}$. To give an example, if $\mathcal{P}$ is reliable broadcast, then $r_j$ could read “broadcast(42)” (cf. Section 5). When interpreting, $s$ requests $r_j$ from $B,\text{n}$’s simulated protocol instance: $B,\text{Pls}[\ell_j], r_j$. For the latter (2), $s$ collects (i) in $B,\text{Ms}[\text{in}, \ell_j]$ all messages for $B,\text{n}$ from $B,\text{Ms}[\text{out}, \ell_j]$ where $B,\text{i} \in B,\text{preds}$ in lines 8–9 and then feeds (ii) $m \in B,\text{Ms}[\text{in}, \ell_j]$ to $B,\text{Pls}[\ell_j]$ in lines 10–11 in order $<_{\text{Ms}}$. This (arbitrary) order is a simple way to guarantee that every server interpreting Algorithm 2 will execute exactly the same steps. By feeding those messages and requests to $B,\text{Pls}[\ell_j]$ in lines 6 and 11 $s$ computes (1) the next state of $B,\text{Pls}[\ell_j]$ and (2) the out-going messages from $B,\text{n}$ in $B,\text{Ms}[\text{out}, \ell_j]$. By construction, $m,\text{sender} = B,\text{n}$ for $m \in B,\text{Ms}[\text{out}, \ell_j]$ (Lemma A.14). Once, $s$ has completed this, $s$ marks $B$ as interpreted in line 12 and can move on to the next eligible block. After $s$ interpreted $B$, the simulated process instance $B,\text{Pls}[\ell_j]$ may indicate $i \in \text{lns}$. If this is the case, $s$ indicates $i$ for $\ell_j$ on behalf of $B,\text{n}$ in lines 13–14. Note, that none of the steps used the fact that it was $s$ who interpreted $B \in G$. So, for every $B$, every $s' \in S\text{rvrs}$ will come to the exact same conclusion.

But we glossed over one detail, $s$ actually had to take a choice—more than one $B$ may have been eligible in line 3. This is a feature: by having this choice we can think of interpreting a $G'$ with $G' \supseteq G$ as an ‘extension’ of interpreting $G$. And, for two eligible $B_1$ and $B_2$ it does not matter if we pick $B_1$ before $B_2$. Informally, this is because when we pick $B_1$ in line 3, only the the state with respect to $B_1$ is modified—and this state does not depend on $B_2$ (Lemma A.11). Another detail we glossed over is line 7: when interpreting $B$, $s$ interprets the process instances of every $\ell_j$ relevant on $B$ at the same time. But again, because $\ell_j \neq \ell_j'$ are independent instances of the protocol with disjoint messages, i.e. $B,\text{Ms}[\text{out}, \ell_j]$ in line 9 is independent of any $B,\text{Ms}[\text{out}, \ell_j']$, they do not influence each other and the order in which we process $\ell_j$ does not matter.

Finally, we give some intuition on how Byzantine servers can influence $G$ and thus the interpretation of $\mathcal{P}$. When running gossip, a Byzantine server $\hat{s}$ can only manipulate the state of $G$ by (1) sending an equivocating block, i.e. building a $B$ and $B'$ with $\hat{s} = B,\text{parent} = B',\text{parent}$. When interpreting $B$ and $B'$, $s$ will split the state for $\hat{s}$ and have two ‘versions’ of $\text{Pls}[\ell_j] \rightarrow B',\text{Pls}[\ell_j]$ and $B,\text{Pls}[\ell_j] \rightarrow$ sending conflicting messages for $\ell_j$ to servers referencing $B$ and $B'$. But as $\mathcal{P}$ is a BFT protocol, the servers $s_j$ simulating $\mathcal{P}$ (run by $s$) can deal with equivocation. Then $\hat{s}$ could (2) reference a block multiple times, or (3) never reference a block. But again as $\mathcal{P}$ is a BFT protocol, the servers $s_j$ simulating $\mathcal{P}$ can deal with duplicate messages and with silent servers.

Going back to Algorithm 2, the main task of $s$ interpreting $G$ is to get messages from one block and give them to the next block. So we can see this interpretation of a block DAG as an implementation of a communication channel. That is, for a correct server $s$ executing $s,\text{interpret}(G, \mathcal{P})$ (i) a server $s_1$ sends messages $m_1, \ldots, m_k$ for a protocol instance $\ell_j$ in either line 6 or line 11 of Algorithm 2, and (ii) a server $s_2$ receives a message $m$ for a protocol instance $\ell_j$ in line 11 of Algorithm 2. The next lemma relate the sent and received messages with the message buffers $\text{Ms}$ and follows from tracing the variables in Algorithm 2:

**Lemma 4.1.** For a correct server $s$ executing $s,\text{interpret}(G, \mathcal{P})$

(1) a server $s_1$ sends $m$ for a protocol instance $\ell_j'$ iff there is a $B \in G$ with $B,\text{n} = s_1$ such that $m \in B,\text{Ms}[\text{out}, \ell_j']$ for a $B' \in G$ with $(\ell_j', r) \in B',\text{rs}$ and $B' \rightarrow^* B,\text{i}$.

(2) a server $s_2$ receives a message $m$ for protocol instance $\ell_j'$ iff there are some $B_1, B_2 \in G$ with $B_1 \rightarrow^* B_2$ and $B_2,\text{n} = s_2$ and $m \in B_2,\text{Ms}[\text{in}, \ell_j']$ for a $B' \in G$ such that $(\ell_j', r) \in B',\text{rs}$ and $B' \rightarrow^* B,\text{i}$.
The following lemma shows our key observation from before: interpreting a block DAG is independent from the
server doing to interpretation. That is, $s$ and $s'$ will arrive at the same state when interpreting $B \in G$.

**Lemma 4.2.** If $G \leq G'$ then for every $B \in G$, a deterministic protocol $P$ and correct servers $s$ and $s'$ executing $s$.interpret($G, P$) and $s'$.interpret($G', P$) it holds that $B.\text{Pls}(t_j) = B'.\text{Pls}'(t_j)$ and $B.\text{Ms}(\text{out}, t_j) = B'.\text{Ms}'(\text{out}, t_j)$ for $(t_j, r) \in B_j, r$ with $B_j \rightarrow^n B$ for $n > 0$.

**Proof.** In the following proof, when executing $s'.\text{interpret}(G', P)$ we write $Ms'$ and $\text{Pls}'$ to distinguish from $Ms$ and $\text{Pls}$ when executing $s.\text{interpret}(G, P)$. We show $B_1.\text{Ms}(\text{out}, t_j) = B_1'.\text{Ms}'(\text{out}, t_j)$ and $B_1.\text{Pls}(t_j) = B_1'.\text{Pls}'(t_j)$ by induction on $n$—the length of the path from $B_j$ to $B_1$ in $G$ and $G'$. For the base case we have $B_1 = B$ and $t_j \in \{t_j | (t_j, r_j) \in B_1, \text{rs}\}$. By Lemma A.10, $B_1$ is picked eventually in line 3 of Algorithm 2 when executing $s.\text{interpret}(G, P)$. Then, by line 6 $B_1.\text{Ms}(\text{out}, t_j)$ is $B_1.\text{Pls}(t_j)$, $(B_1, \text{rs})$. By the same reasoning $B_1'.\text{Ms}'(\text{out}, t_j) = B_1'.\text{Pls}'(t_j)$, $(B_1, \text{rs})$ when executing $s'.\text{interpret}(G', P)$. As $B_1.\text{Pls}(t_j), (B_1, \text{rs})$ are deterministic and depend only on $B_1, t_j$, and $P$, we know that $B_1.\text{Pls}(t_j) = B_1'.\text{Pls}'(t_j)$ and $B_1.\text{Pls}(t_j) = B_1'.\text{Pls}'(t_j)$, and conclude the base case. For the step case by induction hypothesis for $B_1 \in B_j, \text{preds}$ with $B_j \rightarrow^{n-1} B_1$ holds (i) $B_1.\text{Ms}(\text{out}, t_j) = B_1'.\text{Ms}'(\text{out}, t_j)$, and (ii) $B_1.\text{Pls}(t_j) = B_1'.\text{Pls}'(t_j)$. Again by Lemma A.10, $B_1$ is picked eventually in line 3 of Algorithm 2 when executing $s.\text{interpret}(G, P)$ and $s'.\text{interpret}(G', P)$.

In line 4 and as $B_1.\text{parent} \in B_1.\text{preds}$ and (ii), now $B_1.\text{Pls}(t_j) = B_1'.\text{Pls}'(t_j)$. Now, as $P$ is deterministic, we only need to establish that $B_1.\text{Ms}(\text{in}, t_j) = B_1'.\text{Ms}'(\text{in}, t_j)$ to conclude that $B_1.\text{Pls}(t_j) = B_1'.\text{Pls}'(t_j)$ and $B_1.\text{Ms}(\text{out}, t_j) = B_1'.\text{Ms}'(\text{out}, t_j)$, which as $(t_j, r) \notin B_1, \text{rs}$, is only modified in this line 11. By Lemma A.9, we know for both executions that $B_1.\text{Ms}(\text{in}, t_j) = B_1'.\text{Ms}'(\text{in}, t_j) = \emptyset$, before $B_1$ is picked. Now, by (i) and line 9 $B_1.\text{Ms}(\text{in}, t_j) = B_1'.\text{Ms}'(\text{in}, t_j)$, and we conclude the proof.

A straightforward consequence of Lemma 4.2 is, that when in the interpretation of $s$, a server $s_1$ sends a message $m$ for $t_j$, then $s_1$ sends $m$ in the interpretation of $s'$ (cf. Lemma A.16). Curiously, $s_1$ does not have to be correct: we know $s_1$ sent a block $B$ in $G$, that corresponds to a message $m$ in the interpretation of $s$. now this block will be interpreted by $s'$ and the same message will be interpreted—and for that the server $s_1$ does not need to be correct By Lemma 4.3 interpret($G, P$) has the properties of an authenticated perfect point-to-point link after [3, Module 2.5, p. 42].

**Lemma 4.3.** For a block DAG $G$ and a correct server $s$ executing $s.\text{interpret}(G, P)$

(1) if a correct server $s_1$ sends a message $m$ for a protocol instance $\ell$ to a correct server $s_2$, then $s_2$ eventually receives

$m$ for protocol instance $\ell$ for a correct server $s'$ executing $s'.\text{interpret}(G', P)$ and a block $\text{DAG } G' \supseteq \text{DAG} G$ (reliable delivery).

(2) for a protocol instance $\ell$ no message is received by a correct server $s_2$ more than once (no duplication).

(3) if some correct server $s_2$ receives a message $m$ for protocol instance $\ell$ with sender $s_1$ and $s_1$ is correct, then the message $m$ for protocol instance $\ell$ was previously sent to $s_2$ by $s_1$ (authenticity).

**Proof Sketch.** For (1), we observe that every message $m$ sent in $s.\text{interpret}(G, P)$ will be sent in $s'.\text{interpret}(G', P)$ for $G' \supset G$ (by Lemma A.16). Now by Lemma 4.3, $s'$ will eventually have some $G' \supset G$. By Lemma 4.1 (1) we have witnesses $B_1, B_2 \in G'$ with $B_1 \rightarrow B_2$, and by Lemma 4.1 (2) we found a witness $B_2$ to receive the message on when executing $s'.\text{interpret}(G', P)$. For (2), we observe, that duplicate messages are only possible if $s_2$ inserted the block $B_1$, which gives rise to the message $m$ in two different blocks built by $s_2$. But this contradicts the correctness of $s_2$ (by Lemma A.6). For (3), we observe that only $s_1$ can build and sign any block $B_1$ with $s_1 = B, n$, which gives rise to $m$. □
Before we compose gossip and interpret in the following next section under a shim, we highlight the key benefits of using interpret in Algorithm 2. By leveraging the block DAG structure together with $\mathcal{P}$’s determinism, we can compress messages to the point of omitting some of them. When looking at line 11 of Algorithm 2, the messages in the buffers $Ms[\text{out}, f]$ and $Ms[\text{in}, f]$ have never been sent over the network. They are locally computed, functional results of the calls $\text{receive}(m)$. The only ’messages’ actually sent over the network are the requests $r_i$ read from $B.rs$ in line 6. To determine the messages following from these request, the server $s$ simulates an instance of protocol $\mathcal{P}$ for every $s_i \in \text{Srves}$—simply by simulating the steps in the deterministic protocol. However, not every step can be simulated: as $s$ does not know $s_i$’s private key, $s$ cannot sign a message on $s_i$’s behalf. But then, this is not necessary, because $s$ can derive the authenticity of the message triggered by a block $B$ from the signature of $B$, i.e., $B.\sigma$. So instead of signing individual messages, $s_i$ can give a batch signature $B.\sigma$ for authenticating every message materialized through $B$. Finally, $s$ interprets protocol instances with labels $\ell_j$ in parallel in line 7 of Algorithm 2. While traversing the block DAG, $s$ uses the structure of the block DAG to interpret requests and messages for every $\ell_j$. Now, the same block giving rise to a request in process instance $\ell_j$ may materialize a message in process instance $\ell_j'$. The (small) price to pay is the increase of block size by references to predecessor blocks, i.e., $B.\text{preds}$. We will illustrate the benefits again on the concrete example of byzantine reliable broadcast in the next Section 5.

5 USING THE FRAMEWORK

The protocol shim($\mathcal{P}$) in Algorithm 3 is responsible for the choreography of the gossip protocol in Algorithm 1, the interpret protocol in Algorithm 2, and the external user of $\mathcal{P}$. Therefore, the server $s$ executing shim($\mathcal{P}$) in Algorithm 3 keeps track of two synchronized data structures (1) a buffer of labels and requests $rqsts$ in line 2, and (2) and the block DAG $\mathcal{G}$ in line 3. By calling $\text{rqsts.put}(f, \ell, r)$, $s$ inserts $(\ell, r)$ in $rqsts$, and by calling $\text{rqsts.get()}$, $s$ gets and removes a suitable number of requests $(\ell_1, r_1), \ldots, (\ell_n, r_n)$ from $rqsts$. To insert a block $B$ in $\mathcal{G}$, $s$ calls $\mathcal{G}.\text{insert}(B)$ from Definition 3.4. We tacitly assume these operations are atomic. When starting an instance of gossip and interpret in line 4 and 5, $s$ passes in references to theses shared data structures. When the external user of protocol $\mathcal{P}$ requests $r \in Rqsts$ for $\ell \in L$ from $s$ via the request request($\ell, r$) to shim($\mathcal{P}$) then $s$ inserts $(\ell, r)$ in $rqsts$ in lines 6–7. By executing gossip, $s$ writes $(\ell, r)$ in $B$ in Algorithm 1 line 15, and as eventually $B \in \mathcal{G}$, $r$ will be requested from protocol instance $\text{Pis}[f]$ when $s$ executes line 6 in Algorithm 2 (cf. Lemma A.17). On the other hand, when interpret indicates $i \in \text{Inds}$, for the interpretation of $\mathcal{P}$ for itself, i.e., $s = s'$, then $s$ indicates to the user of $\mathcal{P}$ in line 8–9 of Algorithm 3 (cf. Lemma A.18).
For $s$ to only indicate when $s = s'$ might be an over-approximation: $s$ trusts $s$'s interpretation of $P$ as $s$ is correct for $s$. We believe this restriction can be lifted (cf. Section 7). Finally, as promised in Section 3, in lines 10–11 $s$ repeatedly requests disseminate from gossip to disseminates $B$. Within the control of $s$, the time between calls to disseminate can be adapted to meet the network assumptions of $P$ and can be enforced e.g. by an internal timer, the block’s payload, or when $s$ falls $n$ blocks behind. For our proofs we only need to guarantee that a correct $s$ will eventually request disseminate. Now, taking together what we have established for gossip in Section 3, i.e. that correct servers will eventually share a joint block DAG, and that interpret gives a point-to-point link between them in Section 4, for shim($P$) the following holds:

**Theorem 5.1.** For a correct server $s$ and a deterministic protocol $P$, if $P$ is an implementation of (i) an interface $I$ with requests $Rqsts_P$ and indications $Inds_P$ using the reliable point-to-point link abstraction such that (ii) a property $\mathcal{P}$ holds, then shim($P$) in Algorithm 3 implements (i) such that (ii) $\mathcal{P}$ holds.

**Proof.** By Lemma A.17 and Lemma A.18, (i) shim($P$) implements the interface $I$ of $Rqsts_P$ and $Inds_P$. For (ii), by assumption $\mathcal{P}$ holds for $P$ using a reliable point-to-point link abstraction. By Theorem 4.3 $\text{interpret}(G, P)$ implements a reliable point-to-point link. As Algorithm 2 treats $P$ as a black-box every $B$.Pls[ℓ] holds an execution of $P$. Assume this execution violates $\mathcal{P}$. But then an execution of $P$ violates $\mathcal{P}$ which contradicts the assumption that $\mathcal{P}$ holds for $P$. □

Our proof relies on a point-to-point link between two correct servers and thus we can translate the argument of all safety and liveness properties, for which their reasoning relies on the point-to-point link abstraction, to our block DAG framework. However, because we provide an abstraction, we cannot guarantee implementation-level properties, e.g. for performance. They rely on the concrete implementation. Also, as discussed in Section 4, properties related to signatures may not easily translate, because blocks and not messages are (batch-)signed.

$P := \text{byzantine reliable broadcast}$. In the remainder of this section, we will sketch how a user may use the block DAG framework. Our example for $P$ is byzantine reliable broadcast—a protocol underlying recently-proposed efficient payment systems [2, 12]. Algorithm 4 in the appendix shows an implementation of byzantine reliable broadcast: this is the $P$, which the user passes to shim($P$), i.e. in the block DAG framework $P$ is fixed to Algorithm 4. The request in Algorithm 4 is broadcast($v$) for a value $v \in \text{Vals}$, so $Rqsts_P = \{\text{broadcast}(v) | v \in \text{Vals}\}$. For simplicity and generality, we assume that $P$—not shim($P$)—authenticates requests, i.e. requests are self-contained and can be authenticated while simulating $P$ (e.g. Algorithm 4 line 3). However, in an implementation shim($P$) may be employed to authenticate requests. On the other hand, Algorithm 4 indicates with deliver($v$), so $Inds_P = \{\text{deliver}(v) | v \in \text{Vals}\}$. The messages sent in Algorithm 4 are $M_P = \{\text{ECHO } v, \text{READY } v | v \in \text{Vals}\}$ where sender and receiver are the $s \in \text{Srvrs}$ running shim($P$). When executing line 9 of interpret($G, P$) in Algorithm 2, then receive(ECHO 42) is triggered, and received ECHO 42 holds in Algorithm 4 (e.g. in line 6). As we assume $P$ returns messages immediately, e.g. when the simulation reaches send ECHO 42, then ECHO 42 is returned immediately (e.g. in line 8 of Algorithm 4). Figure 4 shows a block DAG for an execution of shim($P$) using byzantine reliable broadcast. It further explicitly shows the in- and out-going messages from $M_s[\text{in}, \ell_1]$ and $M_s[\text{out}, \ell_1]$ for a protocol instance $\ell_1$ and the request broadcast(42) at block $B_1$. None of these messages are ever actually sent over the network—every server interpreting this block DAG can use interpret in Algorithm 2 to replay Algorithm 4 and get the same picture. Figure 4 shows only the (unsent) messages for $\ell_1$ and broadcast(42) in $B_1.rs$, but $B_1.rs$ may hold more requests such as broadcast(21) for $\ell_2$, and all the messages of all these requests could be materialized in the same manner—without any messages, or even additional blocks, sent. And not only $B_1$ holds such requests—also $B_3$ does. For example, $B_3.rs$ may contain broadcast(25) for $\ell_3$. Then, for $\ell_3$ on $B_3$
The last years have seen many proposals based on block DAG paradigms (see [21] for an SoK)—some with commercial implementations. We focus on the proposals closest to our work: HASHGRAPH [1], BLOCKMANIA [7], ALEPH [11], and FLARE [20]. Underlying all of these systems is the same idea: first, build a common block DAG, and then locally interpret the blocks and graph structure as communication for some protocol: HASHGRAPH encodes a consensus protocol in block DAG structure, BLOCKMANIA [7] encodes a simplified version of PBFT [4], ALEPH [11] employs atomic broadcast and consensus, and FLARE [20] builds on federated byzantine agreement from STELLAR [18] combined with block DAGs to implement a federated voting protocol. Naturally, the correctness arguments of these systems focus on their system, e.g. the correctness proof in Coq of byzantine consensus in HASHGRAPH [6]. In our work, we aim for a different level of generality: we establish structure underlying protocols which employ block DAGs, i.e. we show that a block DAG implements a reliable point-to-point channel (Section 4). To that end, and opposed to previous approaches, we treat the protocol \( \mathcal{P} \) completely as a black-box, i.e. our framework is parametric in the protocol \( \mathcal{P} \).

To recap, what makes interpreting \( \mathcal{P} \) on a block DAG so attractive: sending blocks instead of messages in a deterministic manner results in a compression of messages—up to their omission. And not only do these messages not have to be sent, they also do not have to be signed. It suffices, that every server signs their blocks. Finally, a single block sent is interpreted as messages for a very large number of parallel protocol instances.

6 RELATED WORK

The last years have seen many proposals based on block DAG paradigms (see [21] for an SoK)—some with commercial implementations. We focus on the proposals closest to our work: HASHGRAPH [1], BLOCKMANIA [7], ALEPH [11], and FLARE [20]. Underlying all of these systems is the same idea: first, build a common block DAG, and then locally interpret the blocks and graph structure as communication for some protocol: HASHGRAPH encodes a consensus protocol in block DAG structure, BLOCKMANIA [7] encodes a simplified version of PBFT [4], ALEPH [11] employs atomic broadcast and consensus, and FLARE [20] builds on federated byzantine agreement from STELLAR [18] combined with block DAGs to implement a federated voting protocol. Naturally, the correctness arguments of these systems focus on their system, e.g. the correctness proof in Coq of byzantine consensus in HASHGRAPH [6]. In our work, we aim for a different level of generality: we establish structure underlying protocols which employ block DAGs, i.e. we show that a block DAG implements a reliable point-to-point channel (Section 4). To that end, and opposed to previous approaches, we treat the protocol \( \mathcal{P} \) completely as a black-box, i.e. our framework is parametric in the protocol \( \mathcal{P} \).

The idea to leverage deterministic state machines to replay the behavior of other servers goes back to PeerReview [13], where servers exchange logs of received messages for auditing to eventually detect and expose faulty behavior. This idea was taken up by block DAG approaches—but with the twist to leverage determinism to not send those messages that can be determined. This allows compressing messages to the extent of only indicating that a message has been sent as we do in Section 4. However, we believe nothing precludes our proposed framework to be adapted to hold equivocating servers accountable, drawing e.g. on recent work from POLYGRAPH to detect byzantine behavior [5].
While our framework treats the interpreted protocol $P$ as a black-box, the recently proposed threshold logical clock abstraction [10] allows the higher-level protocol to operate on an asynchronous network as if it were a synchronous network by abstracting communication of groups. Similar to our framework, also threshold clocks rely on causal relations between messages by including a threshold number of messages for next the time step. This would roughly correspond to including a threshold number of predecessor blocks. In contrast, our framework, by only providing the abstraction of a reliable point-to-point link to $P$, pushes reasoning about messages to $P$.

7 EXTENSIONS, LIMITATIONS & CONCLUSION

We have presented a generic formalization of a block DAG and its properties, and in particular results relating to the eventual delivery of all blocks from correct servers to other correct servers. We then leverage this property to provide a concrete implementation of a reliable point-to-point channel, which can be used to implement any deterministic protocol $P$ efficiently. In particular we have efficient message compression, as those messages emitted by $P$, which are the results of the deterministic execution of $P$ may be omitted. Moreover we are allowing for batching of the execution of multiple parallel instances of $P$ using the same block DAG, and the de-coupling of maintaining the joint block DAG from its interpretation as instances of $P$.

Extensions. First, throughout our work we assume $P$ is deterministic. The protocol may accept user requests, and emit deterministic messages based on these events and other messages. However, it may not use any randomness in its logic. It seems we can extend the proposed composition to non-deterministic protocols $P$—but some care needs to be applied around the security properties assumed from randomness. In case randomness is merely at the discretion of a server running their instance of the protocol we can apply techniques to de-randomize the protocol by relying on the server including in their created block any coin flips used. In case randomness has to be unbiased, as is the case for asynchronous Byzantine consensus protocols, a joint shared randomness protocol needs to be embedded and used to de-randomize the protocol. Luckily, shared coin protocols that are secure under BFT assumptions and in the synchronous network setting exist [15] and our composition could be used to embed them into the block DAG. However we leave the details of a generic embedding for non-deterministic protocols for future work.

Second, we have discussed the case of embedding asynchronous protocols into a block DAG. We could extend this result to BFT protocols in the partial synchronous network setting [8] by showing that the block DAG interpretation not only creates a reliable point-to-point channel but also that its delivery delay is bounded if the underlying network is partially synchronous. We have a proof sketch to this effect, but a complete proof would require to introduce machinery to reason about timing and, we believe, would not enhance the presentation of the core arguments behind our abstraction.

Third, our correctness conditions on the block DAG seem to be much more strict than necessary. For example, block validity requires a server to have processed all previous blocks. In practice this results in blocks that must include at some position $k$ all predecessors of blocks to be included after position $k$. This leads to inefficiencies: a server must include references to all blocks by other parties into their own blocks, which represents a $O(n^2)$ overhead (admittedly with a small constant, since a cryptographic hash is sufficient). Instead, block inclusion could be more implicit: when a server $s$ includes a block $B'$ in its block $B$ all predecessors of $B'$ could be implicitly included in the block $B$, transitively or up to a certain depth. This would reduce the communication overhead even further. Since it is possible to take a block DAG with this weaker validity condition and unambiguously extract a block DAG with the stronger validity condition we assume, we foresee no issues for all our theorems to hold. Furthermore, when interpreting a protocol currently a
server only indicates, when the server running the interpretation indicates in the interpretation. This is to assure that
the server running the interpretation can trust the server in the interpretation, i.e. itself. Again, we believe that this
can be weakened by leveraging properties of the interpreted protocol. However, we again leave a full exploration of
this space to future work.

Limitations. Some limitations of our composition require much more foundational work to be overcome. And these
limitations also apply to the block DAG based protocols which we attempt to formalize. First, there are practical
challenges when embedding protocols tolerating processes that can crash and recover. At first glance safe protocols
in the crash recovery setting seem like a great match for the block DAG approach: they do allow parties that recover
to re-synchronize the block DAG, and continue execution, assuming that they persist enough information (usually
in a local log) as part of $P$. However there are challenges: first, our block DAG assumes that blocks issued have
consecutive numbers. If the higher-level protocols use these block sequence numbers as labels for state machines (as
in Blockmania), a recovering process may have to ‘fill-in’ a large number of blocks before catching up with others. An
alternative is for block sequence numbers to not have to be consecutive, but merely increasing, which would remove
this issue.

However in all cases, unless there is a mechanism for the higher level protocol $P$ to signal that some information
will never again be needed, the full block DAG has to be stored by all correct parties forever. This seems to be a
limitation of both our abstraction of block DAG but also the traditional abstraction of reliable point-to-point channels
and the protocols using them, that seem to not require protocols to ever signal that a message is not needed any more
(to stop re-transmission attempt to crashed or Byzantine servers). Fixing this issue, and proving that protocols can
be embedded into a block DAG, that can be operated and interpreted using a bounded amount of memory to avoid
exhaustion attacks is a challenging and worthy future avenue for work – and is likely to require a re-thinking of how
we specify BFT protocols in general to ensure this property, beyond their embedding into a block DAG.

Finally, one of the advantages of using a block DAG is the ability to separate the operation and maintenance of the
block DAG from the later or off-line interpretation of instances of protocol $P$. However, this separation does not hold
and extend to operations that change the membership of the server set that maintain the block DAG—often referred to
as reconfiguration. How to best support reconfiguration of servers in block DAG protocols seems to be an open issue,
besides splitting protocol instances in pre-defined epochs.

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A APPENDIX

A.1 Ad Section 2: Background

Definition A.1. Let \( \#: A \rightarrow A' \) be a secure cryptographic hash function. We write \( \#: (x) \) for the hash of \( x \in A \), and we write \( \#: (A) \) for \( A' \). By definition [19, p.332], for any \( \# \) it is computationally infeasible

1. to find any preimage \( m \) such that \( \#: (m) = x \) when given any \( x \) for which a corresponding input is not known (preimage-resistance),
2. given \( m \) to find a 2nd-preimage \( m' \neq m \) such that \( \#: (m) = \#: (m') \) (2nd-preimage resistance), and
3. to find any two distinct inputs \( m, m' \) such that \( \#: (m) = \#: (m') \) (collision resistance).

Proof of Lemma 2.2 (1). By definition of \( G \) and insert. \( \square \)

Proof of Lemma 2.2 (2). Let \( G' = \text{insert}(G, v, E) \). By definition of insert, \( V_{G} \subseteq V_{G'} \). Assume \( v \notin G \). As \( E \) contains only edges that \( (v, v) \) such that \( (v, E) \in E_{G} = E_{G'} \cap (V_{G} \times V_{G}) \) holds. \( \square \)

Proof of Lemma 2.2 (3). By definition of \( E \) and insert \( (G, v, E) \) only adds edges from vertices in \( G \) to \( v \). As \( v \notin G \), there is no edge \( (v, v) \) such that \( \) holds. By acyclicity of \( G \), insert \( (G, v, E) \) is acyclic. \( \square \)

Proof of Lemma 1. By assumption \( s \) considers \( B \) valid, hence by lines 6–8 adds a reference to \( B \) to \( B \). As \( s \) is correct, eventually will disseminate \( (B) \), and then \( s \) disseminates \( B \) in line 17. We refer to this disseminated \( B \) as \( B' \).

By Assumption 1, every correct server will eventually receive \( B' \). Assume a correct server \( s' \), which has received \( B' \), but has not received \( B \). As \( s' \) has not received \( B \), by Definition 3.3 (iii), \( s' \) does not consider \( B' \) valid. After time \( \Delta_{B} \) by lines 10–11 \( s' \) will request \( B \) from \( s \) by sending \( \text{FWD} \) \( B \). Again by Assumption 1, after \( s \) receives \( \text{FWD} \) \( B \) from \( s' \) by lines 12–13, \( s \) will send \( B \) to \( s' \), which will eventually arrive, and \( s' \) receives \( B \). \( \square \)

A.2 Ad Section 3: Building a Block DAG

In this section we give the proofs—and lemmas those proofs rely on—which we omitted in Section 3. All proofs refer to Algorithm 1. For the execution we assume, that the body of each handler is executed atomically and sequentially within the handler.

Proof of Lemma 3.2. Let \( x_{1} = \text{ref}(B_{1}) \) and \( x_{2} = \text{ref}(B_{2}) \). By assumption, \( x_{1} \in B_{2}. preds \). Assume towards a contradiction that \( x_{2} \in B_{1}. preds \). Then, to compute \( x_{1} \) we need to know \( x_{2} = \text{ref}(B_{2}) \). But this contradicts preimage-resistance of ref. \( \square \)

Lemma A.2. For a block DAG \( G \) and a block \( B \in G \) holds \( G = G. \text{insert}(B) \), i.e. insert is idempotent.

Proof. As \( E \) is fixed to \( \{ (B, B') | B \in B'. preds \} \) by definition of insert on block DAGs. Since \( B \in G \) also \( \{ (B, B') | B \in B'. preds \} \subseteq E_{G} \) by definition of block DAG. Thus, \( G. \text{insert}(B) = G \) by Lemma 2.2 (1). \( \square \)

Lemma A.3. Let \( G \) be a block DAG for a server \( s \) and let \( B' \) be a block such that valid \( (s, B') \) holds and for all \( B \in B'. preds \) holds \( B \in G \). Let \( G' = G. \text{insert}(B') \). Then \( G' \) is a block DAG for \( s \).

Proof. To show \( G' \) is a block DAG we need to show that \( G' \) adheres to Definition 3.4. For condition (i) we have to show that \( s \) considers all blocks in \( G' \) valid. All blocks in \( G' \) are—by definition of insert—\( \cup \{ (B, B') | B \in B'. preds \} \subseteq E_{G} \). As \( G \) is a block DAG for \( s \), valid \( (s, B) \) holds for all \( B \in V_{G} \) and valid \( (s, B') \) follows from the assumption of the lemma. For condition (ii) we have to show that for every backwards reference to \( B \) from the block \( B' \), the block dag \( G' \) contains
B and an edge from B to B'. The former—for all B ∈ B'.preds we have B ∈ G—holds by assumption of the lemma. The latter—(B, B') ∈ E_G for B ∈ B'.preds—holds by definition of insert. As G is a block DAG, condition (ii) holds for every block in G. It remains to show, that G' is acyclic. If B' ∈ G then by Lemma A.2, G' = G and G is acyclic. If B' ∉ G then G' is acyclic by Lemma 3.

\[\square\]

**Lemma A.4.** For every correct server s executing gossip of Algorithm 1, whenever the execution reaches line 16 then \(\text{valid}(s, B)\) holds.

**Proof.** We need to show, that once the execution reaches line 16 Definition 3.3 (i)–(iii) holds. As s is correct and signs B in line 15 (i) \(\text{verify}_\sigma(s, B, \sigma)\) holds. We prove (ii) and (iii) by induction on the times n the execution reaches line 16. For the base case, B is (a) a genesis block with B.\(k = 0\) as initialized in line 2. Moreover B has no parent. As s is correct and only inserts B' in B.preds in line 8 whenever s considers B' valid in line 6, s considers all B' ∈ B.preds valid. In the step case, B_{n+1} is updated in line 18. We show that (b) B_{n+1} has exactly one parent B_n. By line 18, B_{n+1}.n = B_n.n and B_{n+1}.k = B_n.k + 1. As B_n is inserted B_{n+1}.preds in line 18, by definition B_{n+1}.parent = B_n. By induction hypothesis, s considers B_n valid, and again, as s is correct and only inserts B' in B.preds in line 8 whenever s considers B' valid in line 6, (iii) s considers all B' ∈ B.preds valid.

\[\square\]

**Lemma A.5.** For every correct server s executing gossip of Algorithm 1 G is a block DAG.

**Proof.** We proof the lemma by induction on the times n the execution reaches line 7 or line 16 of Algorithm 1. As G is initialized to the empty block DAG in Algorithm 3 in line 3, G is a block DAG for the base case n = 0. In the step case, by induction hypothesis, G is a block DAG. By Lemma A.3 G.insert(B') is a block DAG if (i) valid(s, B') holds, and (ii) for all B ∈ B'.preds holds B' ∈ G. The former (i), valid(s, B'), holds either by line 6 or by Lemma A.4. As s inserts any block B which s has received and considers valid by lines 6–8, for the latter (ii) it suffices to show that s considers all B ∈ B'.preds valid. As s considers B' valid, by Definition 3.3 (ii), s considers all B ∈ B'.preds valid.

\[\square\]

**Proof of Lemma 3.6 (1).** By assumption s considers B valid, hence by lines 6–8 adds a reference to B to B. By s is correct, s eventually will disseminate(), and then s disseminates B in line 17. We refer to this disseminated B as B'. By Assumption 1, every correct server will eventually receive B'. Assume a correct server s', which has received B', but has not received B. As s' has not received B, by Definition 3.3 (iii), s' does not consider B' valid. After time \(\Delta_B\) by lines 10–11 s' will request B from s by sending FWD B. Again by Assumption 1, after s receives FWD B from s' by lines 12–13, s will send B to s', which will eventually arrive, and s' receives B.

\[\square\]

**Proof of 3.6 (2).** We have to show, that valid(s', B) eventually holds for all correct servers s'. For Definition 3.3 (i), as s considers B valid and s is correct, B has a valid signature. This can be checked by every s'. We show Definition 3.3 (ii) (a) and (iii) by induction on the sum of the length of the paths from genesis blocks to B. For the base case, B does not have predecessors. As s considers B valid, then B is a genesis block, and s' will consider B a genesis block, so Definition 3.3 (ii) (a) and (iii) hold. For the step case, let B' ∈ B.preds. By Lemma 3.6 (1), every correct server s' will eventually receive B'. By induction hypothesis, s' will eventually consider B' valid. The same reasoning holds for every B' ∈ B.preds. It remains to show that B has exactly one parent or is a genesis block. Again, this follows by s considering B valid. As B.parent ∈ B.preds s' also considers B.parent valid.

\[\square\]

**Lemma A.6.** For every B every correct server s executing gossip of Algorithm 1 inserts ref(B) at most once in any block B' with B'.n = s.
Lemma A.7. Let \( s \) and \( s' \) be correct servers with block DAGs \( G_s \) and \( G_{s'} \). Then their joint block DAG \( G = G_s \cup G_{s'} \) is a block DAG for \( s \).

Proof. Let \( bs = B_1, \ldots, B_{k-1} \) be blocks such that \( B_i \in G_{s'} \) but \( B_i \notin G_s \) for \( 1 \leq i < k \). We show the statement by induction on \(|bs|\). As \( G_s \) is a block DAG for \( s \), the statement holds for the base case. For the step case we pick a \( B_i \in bs \) such that \( B_i \in G_{s'} \) and \( B_i \in G_{s} \) are completely disjoint and \( B_i \) is a genesis block in \( G_s \). It remains to show that \( s \) considers \( B_i \) valid and all \( B_i \) are in \( G_s \). Then by Lemma A.3, \( G_s \) is a block DAG and by induction hypothesis the statement holds. For all \( B' \in G_{s} \), \( B' \in G_{s} \) by definition of \( bs \). Moreover, \( G_s \) is the block DAG of \( s \), \( s \) considers every \( B' \) valid. Then by (iii) of Definition 3.3, together with the fact that \( s' \) is correct therefore (i) and (ii) hold for \( s \), \( s \) considers \( B_i \) valid.

Lemma A.8. If \( B_1 \in G \) for the block DAG \( G \) a correct server \( s \), then eventually for a block DAG \( G' \) of \( s \) where \( G' \gg G \) holds \( B_2 \in G' \) and \( B_2.n = s \) and \( B_1 \rightarrow B_2 \).

Proof. For a correct server \( s \) holds that \( B_1 \in G \) only after \( s \) inserted \( B_1 \) either in line 7 or in line 16. Then by either line 8 or 18, respectively, \( B_1 \in B \), \( B \) preds for \( B \) is \( s \). As \( s \) is correct \( s \) will eventually request disseminate() and \( s \) will reach line 16 for \( B \) and insert \( B \) to \( G \) for some \( G' \gg G \).

A.3 Ad Section 4: Interpreting a Protocol

In this section we give the proofs—and lemmas those proofs rely on—which we omitted in Section 3. All proofs refer to Algorithm 2. For the execution we assume, that the body of each handler is executed atomically and sequentially within the handler.

Lemma A.9. For \( B \in G \) if \( I[B] = \text{false} \) then \( B.Ms[d, \ell] = \varnothing \) and \( B.Pls[\ell] = \bot \) for \( \ell \in L \) and \( d \in \{\text{in, out}\} \).

Proof. For every \( B \), \( \ell \in L \), and \( d \in \{\text{in, out}\} \), initially \( B.Ms[d, \ell] = \varnothing \) and \( B.Pls[\ell] = \bot \). Assume towards a contradiction that \( B.Ms[d, \ell] \neq \varnothing \) or \( B.Pls[\ell] \neq \bot \). As \( B.Ms[d, \ell] \) and \( B.Pls[\ell] \) are only modified in lines 11–12 after \( B \) is picked in line 3, then by line 12 \( I[B] = \text{true} \), contradiction \( I[B] = \text{false} \).

Lemma A.10. For \( B \in G \) and a correct server executing \( (G, P) \) in Algorithm 2 every \( B \) is eventually picked in line 3.

Proof. To pick \( B \) in line 3, eligible\( (B) \) has to hold. By \( G \) finite and acyclic, every \( B \in G \) is eligible\( (B) \) eventually.

Lemma A.11. For a block \( B \in G \) and an \( \ell \in L \), if \( I[B] \) holds,

(1) then \( B.Ms[d, \ell] \) will never modified again for every \( d \in \{\text{in, out}\} \).

(2) then \( B.Pls[\ell] \) will never modified again.

Proof. For part 1, assume that \( B.Ms[d, \ell] \) is modified. This can only happen in lines 6, 9, and 11 and only for \( B \) picked in line 3. But as \( I[B] \), \( B \) cannot be picked in line 3, leading to a contradiction. For part 2 assume that \( B.Pls[d, \ell] \) is modified. This can only happen in lines 4 and 11, and only for \( B \) picked in line 3. But as \( I[B] \), \( B \) cannot be picked in line 3, leading to a contradiction.
Lemma A.12. If $m \in B.Ms[\text{out}, t]$ then there is a block $B'$ such that $(t, r) \in B'.rs$ and $B' \rightarrow^* B$.

Proof. In Algorithm 2, $m \in B.Ms[\text{out}, t]$ only after the execution reaches either (1) line 6, and then $B' = B$, or (2) line 11, end then by line 7 exists a $B_r$ such that $(t_j, r) \in B_j.rs$ and $t \in \{t_j \mid (t_j, r) \in B_j.rs \land B_j \in G \land B_j \rightarrow^* B\}$. □

Lemma A.13. For all $B.PI$s[$t$] $\neq \bot$ holds that $B.PI$s[$t$] was started with $\mathcal{P}(t, B.n)$.

Proof. Either $B(t)$ is a genesis block, and then by assumption started with $B.n$ and $t$, (ii) $B$ has a parent and by line 4, $PI$s[$t$] is copied from $B$.parent and as $B$.parent.$n = B.n$, $B.PI$s[$t$] was initialized with $B.n$ and $t$ (Lemma A.15). □

Lemma A.14. If $m \in B.Ms[\text{out}, t]$ then $m$.sender = $B.n$.

Proof. By lines 6 and 11 of Algorithm 2 $m \in B.Ms[\text{out}, t]$ if either $m \in B.PI$s[$t$. $(B.rs)$ or $m \in B.PI$s[$t$. receive($m'$) for some $m'$ of no importance. Important is, that $B.PI$s[$t$] was initialized by $B.n$ by Lemma A.13, and thus every outgoing message $m$ has $m$.sender = $B.n$. It remains to show that every $B$ with $B.n = s$ was build by $s$, which follows by the signature $B.n$. □

Lemma A.15. When the execution of $\text{interpret}(G, \mathcal{P})$ reaches line 7 of Algorithm 2 then for all $t_j \in \{t_j \mid (t_j, r) \in B_j.rs \land B_j \in G \land B_j \rightarrow^* B\}$ holds $B.PI$s[$t_j$] $\neq \bot$.

Proof. We show the statement by induction on the length of the longest path from the genesis blocks to $B$. The base cases $n = 0$ holds by assumption, as $PI$s[$t$] is started on every genesis block. For the step case, by induction hypothesis the statement holds for $B_i \in B$.preds, and as $B$.parent = $B$.preds by line 4 the statement holds. □

Proof of Lemma 4.1(1). By definition $s_1$ sends $m$ for some protocol instance $t'$ if $s$ reaches in Algorithm 2 either in line 6 with $B.rs$, or line 11 with $B.PI$s[$t'$]. receive($m$) for some $B$ picked in line 3. By Lemma A.15 $B.PI$s[$t'$] $\neq \bot$ and $B.PI$s[$t'$].$n = s_1$ by assumption, by Lemma A.13 $B.n = s_1$. $B$ will be our witness for $B_1$. Now $m \in B.Ms[\text{out}, t']$, by the assignment in either line 6 with $(t', r) \in B.rs$ (by line 5), or in line 11 with $(t', r) \in B.rs$ for some $B_j \rightarrow^* B$ (by line 7). $B_j$ is our witness for $B' \neq B_1$. For the other direction, we have $B_1 \in G$ with $B_1.n = s_1$ such that $m \in B_1.Ms[\text{out}, t']$ for a $B' \in G$ with $(t', r) \in B'.rs$ and $B' \rightarrow^* B_1$. By Lemma A.10, eventually $B_1$ is picked in Algorithm 2 line 3 by assumption, $m \in B_1.Ms[\text{out}, t']$ through either (i) line 6, or (ii) as $B' \rightarrow^* B_1$ and thus $t' \in \{t_j \mid (t_j, r) \in B_j.rs \land B_j \in G \land B_j \rightarrow^* B\}$ from line 11. Then, by definition, $s_1$ sends $m$ for protocol instance $t'$.

Proof of Lemma 4.1(2). By Definition $s_2$ receives $m$ in line 11 of Algorithm 2 for protocol instance $t'$ for some $B$ picked in line 3 and $m \in B.Ms[\text{in}, t']$ by line 10. By Lemma A.15 $B.PI$s[$t'$] $\neq \bot$ and $B.PI$s[$t'$].$n = s_2$ by assumption, by Lemma A.13 $B.n = s_2$. $B$ is our witness for $B_2$. Now by line 9 $m \in B.Ms[\text{in}, t']$ only if $m \in B_2.Ms[\text{out}, t']$ for some $B_i$ with $B_i \rightarrow B_2$. $B_i$ is our witness for $B_1$. Finally, by line 7, $t' \in \{t_j \mid (t_j, r) \in B_j.rs \land B_j \in G \land B_j \rightarrow^* B\}$, and $B_1$ is our witness for $B'$. For the other direction we have $B_1, B_2 \in G$ with $B_1 \rightarrow B_2$ and $B_2.n = s_2$ and $m \in B_2.Ms[\text{in}, t']$ for a $B' \in G$ such that $(t', r) \in B'.rs$ and $B' \rightarrow^* B_1$. By Lemma A.10, eventually $B_1$ is picked in Algorithm 2 line 3 and by assumptions eventually reaches line 11 of Algorithm 2. As $m \in B_2.Ms[\text{in}, t']$ by definition, $s_2$ receives $m$ for protocol instance $t'$.

Lemma A.16. For a correct server $s$ executing $s$. interpret$(G, \mathcal{P})$ if a server $s_1$ sends a message $m$ for a protocol instance $t_j$, then $s_1$ sends $m$ for a correct server $s'$. interpret$(G', \mathcal{P})$ for a block DAG $G' \supseteq G$.

Proof. Again, in the following proof, when executing $s'. interpret(G', \mathcal{P})$ we write $Ms'$ and $PI$s' to distinguish from $Ms$ and $PI$s when executing $s$. interpret$(G, \mathcal{P})$. As $s_1$ sends a message $m$ for a protocol instance $t_j$, by Lemma 4.1(1)
there is a $B_1 \in G$ with $B_1.n = s_1$ such that $m \in B_1.Ms[\text{out}, t_j]$ for a $B_j \in G$ with $(t_j, r) \in B.j.rs$ and $B_j \rightarrow^n B_1$ for $n > 0$. By $G' \supset G$, $B_1 \in G$, $B_j \in G$, and the path $B_j \rightarrow^n B_1$ is in $G'$. By Lemma 4.2 $m \in B_1.Ms'[\text{out}, t_j]$, and then by Lemma 4.1(1), $s_1$ sends $s$ for a correct server $s'$ executing $s'.\text{interpret}(G', \mathcal{P})$.

**Proof of Lemma 4.3.1 (Reliable delivery).** By assumption $s_1$ sends a message $m$ to a correct server $s_2$ for a correct server $s$ executing $s'.\text{interpret}(G, \mathcal{P})$. By Lemma 3.7 $s'$ will eventually have some $G_1 \supset G$. Then by Lemma A.16, $s_1$ sends $m$ in $s'.\text{interpret}(G_1, \mathcal{P})$ for $G_1 \supset G$. Then by Lemma 4.1(1) there is a $B_1 \in G_1$ with $B_1.n = s_1$ such that $m \in B_1.Ms[\text{out}, t_j]$ for $B_j \in G_1$ with $(t_j, r) \in B.j.rs$ and $B_j \rightarrow^* B_1$. With $B_1$ we found our first witness. By Lemma A.8, there $G_2 \supset G_1$ such that $B_2 \in G_2$ and $B_2.n = s_2$ and $B_1 \rightarrow B_2$. Then by Lemma 3.7 eventually $s'$ will have some $G' \supset G_2$. By $m \in B_1.Ms[\text{out}, t_j], B_1 \rightarrow B_2$ and $m.\text{receiver} = s_2$ by assumption, by lines 9–10 of Algorithm 2 we have $B.m \in Ms[\text{in}, t_j]$. Now we have found our second witness $B_2$. Now, by Lemma 4.1(2), $s_2$ receives $m$ in $s'.\text{interpret}(G', \mathcal{P})$.

**Proof of Lemma 4.3.2 (No duplication).** Assume towards a contradiction, that $s_2$ received $m$ more than once. Then by Lemma 4.1(2) there are some $B_1, B_2 \in G$ with $B_1 \rightarrow B_2$, $B_2.n = s_2$ and $m \in B_2.Ms[\text{in}, t], \text{and } B'_1 \rightarrow B'_2, B'_2.n = s_2$ and $m \in B'_2.Ms[\text{in}, t]$ for a $B'_j \in G$ such that $(t, r) \in B.j.rs$ and $B_j \rightarrow^* B_1$, but $B_2 \neq B'_2$. That $s_2$ received the exact same message $m$ twice is only possible, if $B_1 = B'_1$. That is, $s_2$ built $B'_2 \neq B_2$ and inserted $B_1$ in both, which contradicts Lemma A.6 as $s_2$ is correct.

**Proof of Lemma 4.3.3 (Authenticity).** By Lemma 4.1(2) there are some $B_1, B_2 \in G$ with $B_1 \rightarrow B_2$ and $B_2.n = s_2$ and $m \in B_2.Ms[\text{in}, t]$ for a $B \in G$ such that $(t, r) \in B.j.rs$ and $B_j \rightarrow^* B_1$. Then by line 9 of Algorithm 2 exist an $B_i \in B_2.preds$ such that $m \in B_i.Ms[\text{out}, t]$. As $m \in B_i.Ms[\text{out}, t]$ by Lemma A.14 $B_i.n = m.\text{sender}$ and as $m.\text{sender} = s_1$, $B_i.n = s_1, B_i$ will be our witness for $B_1$. As $m \in B_i.Ms[\text{out}, t]$ by Lemma A.12 there is a $B' \supset B$ such that $(t, r) \in B'.rs$ and $B' \rightarrow^* B, B'$ is our witness for $B_1$. Hence there is a $B_1 \in G$ with $B_1.n = s_1$ such that $m \in B_1.Ms[\text{out}, t]$ for a $B_1 \in G$ with $(t, r) \in B.j.rs$ and $B_j \rightarrow^* B_1$ and by Lemma 4.1(1) $s_1$ was sent by $s_1$. 

**A.4 Ad Section 5: Using the Framework**

In this section we give the proofs which we omitted in Section 5. All proofs refer to Algorithm 3. For the execution we assume, that the body of each handler is executed atomically. We further give an implementation of authenticated double-echo broadcast in Algorithm 4.

**Lemma A.17. For a correct server $s$ executing shim($\mathcal{P}$), if request($r, t)$ is requested from $s$, then $r$ is requested from $\mathcal{P}$.

**Proof.** By executing shim($\mathcal{P}$), a correct $s$ inserts $(t, r)$ in rqsts in line 6–7 of Algorithm 3. Then executing gossip($s, G, rqsts$), $s$ will eventually disseminate a block $B$ with $B.n = s$ and $(t, r) \in B.j.rs$ in line 15 of Algorithm 1 and $B \in G$ after triggering disseminate in lines 10–11 of Algorithm 3. Now, executing interpret($G, \mathcal{P}$), $s$ for $B \in G$ will call $B.\text{Pls}[t].rs$ line 6 in Algorithm 2.

**Lemma A.18. For a correct server $s$ executing shim($\mathcal{P}$), if $\mathcal{P}$ indicates $i \in \text{Inds}_{\mathcal{P}}$ for $s$, then shim($\mathcal{P}$) indicate($t, i$).

**Proof.** By assumption a correct $s$ indicates $i$ for $t$ and hence indicates in interpret($G, \mathcal{P}$) lines 13–14 of Algorithm 2. Then, by executing shim($\mathcal{P}$), as $s = s'.\text{indicate}((t, i) \in \text{Inds}_{\mathcal{P}})$ by lines 8–9 of Algorithm 3.
module broadcast($s \in \text{Srvrs}$)

    echoed, readied, delivered := false

    broadcast($o \in \text{Vals}$) and authenticate($o$)

        echoed := true

        send to ECHO $o$ to every $s' \in \text{Srvrs}$

    when received ECHO $o$ and not echoed

        echoed := true

        send ECHO $o$ to every $s' \in \text{Srvrs}$

    when received ECHO $o$ from $2f + 1$ different $s' \in \text{Srvrs}$ and not readied

        readied := true

        send READY $o$ to every $s' \in \text{Srvrs}$

    when received READY $o$ from $f + 1$ different $s' \in \text{Srvrs}$ and not readied

        readied := true

        send READY $r$ to every $s' \in \text{Srvrs}$

    when received from READY $o$ from $2f + 1$ different $s' \in \text{Srvrs}$ and not delivered

        delivered := true

        deliver($o$)

Algorithm 4: Authenticated double-echo broadcast after [3].