Exploratory Calculations of the Effects of Higher Shell Admixtures on Static Electric Quadrupole and Magnetic Dipole Moments of Excited States

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Abstract

Using the interaction $Q \cdot Q + xV_{S.O.}$, where $V_{S.O.}$ is a two-body spin-orbit interaction, we study the effects of varying $x$ on the static electric quadrupole and magnetic dipole moments of the $2^+_1$ and $2^+_2$ states of $^{10}\text{Be}$. This is done both in the valence space $s^4p^6$ and in a larger space in which $2\,\hbar\omega$ excitations are allowed. In the former case, for $x = 0$, we have the Wigner Supermultiplet limit in which the $2^+_1$ and $2^+_2$ states are degenerate and correspond to $K = 0$ and $K = 2$ rotational states, with equal and opposite static quadrupole moments. Turning on the spin-orbit interaction with sufficient strength in the valence space gives an energy splitting to the two $2^+$ states in accord with experiment. When higher shell admixtures are allowed, we get quite a different behaviour as a function of $x$ than in the valence space. Of particular interest is a value of $x$ for which $Q(2^+_1)$ and $Q(2^+_2)$ both nearly vanish, and so does (somewhat coincidentally) $\mu(2^+_1)$.

I. INTRODUCTION

In this work, we make exploratory calculations of the effects of higher shell admixtures on nuclear properties. We will focus on the static electric quadrupole and magnetic dipole moments of the excited states of even-even nuclei. We shall use $^{10}\text{Be}$ as a kind of playing field to see if any new or interesting phenomena emerge. Of course, $^{10}\text{Be}$ is of interest in its own right, and experimental measurements of its excited states may become feasible in the near future.

Why $^{10}\text{Be}$? First of all, it is light enough that we can do shell model calculations with higher shell admixtures. Secondly, because this is a nucleus for which $N \neq Z$, some non-trivial aspects emerge. For instance, whereas in $^{8}\text{Be}$ the magnetic moment of the $2^+$ state is isoscalar and hence less sensitive to the details of the nucleon wave function, in $^{10}\text{Be}$ one also picks up isovector terms which are very sensitive to such details.

In doing the higher shell calculations, we would like to distinguish between ‘expected’ effects and surprising effects. An example of the former is the fact that quadrupole properties
such as $B(E2)$ get enhanced by higher shell admixtures. Indeed, the justification for using effective $E2$ charges in a valence space comes from perturbative calculations in which $\Delta N = 2$ admixtures are allowed. But, as we will see in the following sections, there are some new and less obvious effects.

We call this work exploratory in part because we hope that, although we limit our calculations to $^{10}\text{Be}$, ideas will emerge that will be relevant to other nuclei. Also, since we limit our excitations to $\Delta N = 2$, we cannot be sure what even higher shell admixtures will do.

II. THE INTERACTION AND THE LIMITS

We shall use the interaction $Q \cdot Q + xV_{S.O.}$, where

$$Q \cdot Q = \sum_i \left( \frac{p_i^2}{2m} + \frac{1}{2} m \omega^2 r_i^2 \right) - \chi \sum_{i<j} Q(i) \cdot Q(j)$$

(we do not include $i = j$ terms), and where $V_{S.O.}$ is a two-body spin-orbit interaction that was introduced by Zheng and Zamick [1]. The interactions $Q \cdot Q$ and $V_{S.O.}$ have been previously used in various calculations [2,3]. For $x = 1$, Zheng and Zamick got the best fit to $G$ matrix elements of realistic interactions, but it has been found that several properties in the nuclear medium could be better fit with $x = 1.5$ [4]. We do not introduce a one-body spin-orbit term. Rather, the spin-orbit splitting is implicitly generated by the two-body spin-orbit interaction. Furthermore, we will fix the strength $\chi$ of the $Q \cdot Q$ interaction so that, when used in the Hamiltonian given above with the spin-orbit coupling strength $x$ taken to be equal to its standard of $x = 1$, the excitation energy of the first $2^+$ state comes out to be closest to the experimental value of $3.368 \text{ MeV}$. This is achieved by taking $\chi = 0.5241 \text{ MeV/fm}^4$ in the valence space calculations and $\chi = 0.4157 \text{ MeV/fm}^4$ in the $(0+2) h \omega$ calculations.

We will then study the effects of varying $x$ on the static electric quadrupole and magnetic dipole moments of the $2^+_1$ and $2^+_2$ states.

A. The small $x$ limit

For $x = 0$ in the valence space, we get the Wigner Supermultiplet limit of the $0p$ shell as nicely discussed in the book by Hammermesh [5] (see also [2]). In that limit, the first $2^+$ state of $^{10}\text{Be}$ is doubly degenerate with quantum numbers $[420]$. We can also use the $SU(3)$ quantum numbers $(\lambda, \mu) = (2, 2)$. In fact, the double degeneracy comes from any Wigner plus Majorana interaction, of which $Q \cdot Q$ is a special case.

From the point of view of the rotational model, the $2^+_1$ state is a member of the $K = 0$ ground state band, and the $2^+_2$ state is the lowest member of a $K = 2$ band. They both have the same intrinsic electric quadrupole moment $Q_0$, but they have equal and opposite laboratory electric quadrupole moments $Q$. This can be seen from the rotational formula [6]

$$Q = \frac{[3K^2 - I(I + 1)]}{(I + 1)(2I + 3)} Q_0$$

...
Thus, the small $x$ limit is characterised by $E(2^+_1) = E(2^+_2)$ and by $Q(2^+_1) = -Q(2^+_2)$.

Experimentally, the $2^+_1$ and $2^+_2$ states are not degenerate. The $2^+_1$ state is at 3.368 MeV, and the $2^+_2$ state is at nearly twice the excitation energy of 5.960 MeV. Undoubtedly, this is due to the importance of the spin-orbit interaction, and hence the need to include the latter in our hamiltonian, if only to reproduce energy levels that are close to experiment.

B. The large $x$ limit

When the spin-orbit splitting becomes very large, one gets the $jj$ coupling limit in the small space. The wave function for $^{10}Be$ is then

$$(p_{3/2})_\pi^2(p_{3/2})_\nu^4$$

Note that the neutron shell is closed in this limit and therefore the contribution of the neutrons to the quadrupole moment $Q$ and to the magnetic moment $\mu$ is zero. The value of $Q$ for the two protons is also zero. This is a rather special case where the numbers of holes in the $p_{3/2}$ shell equals the number of particles. In general,

$$Q(j) = -Q(j^{-1})$$

The only way this can be satisfied for $(p_{3/2})_{1}^{l=2}$ is if $Q$ is zero. This is a peculiarity of our choice of $^{10}Be$ as the playing field. But it gives us a bonus in that we have a very simple limit at large $x$ which will guide us as we go from small to large $x$.

As for the magnetic dipole moment, it is easy to evaluate its value in the large $x$ limit. We simply have two $p_{3/2}$ protons coupled to $J = 2^+$, and using the Schmidt value for a single proton $(p_{3/2})_\pi^2$.

$$\mu_{sp} = g_s\pi s + g_l\pi l$$

we have for the $(p_{3/2})_\pi^2$ $J = 2^+$ configuration

$$\mu(J = 2^+) = J J\mu_{sp} = \frac{2}{3/2}(2.793 + 1)$$

yielding $\mu(2^+_1) \to 5.057\ \mu_N$ as $x \to \infty$. This will also be a useful guide in our discussion below, but we must remember that all the above limits are for valence space calculations.

III. DISCUSSIONS OF TABLES AND FIGURES

In Table I, we present the results of electric quadrupole moment calculations for various values of $x$, the strength of the spin-orbit interaction. We give the results in the small space in Table Ia and in the large space in Table Ib. We also show the excitation energies of the two $2^+$ states, as well as the values of the electric quadrupole transition amplitudes $B(E2 : 0^+_1 \to 2^+_1)$ and $B(E2 : 0^+_1 \to 2^+_2)$. Similarly, in Tables IIa and IIb, we give the corresponding magnetic dipole moments $\mu(\text{spin})$, $\mu(\text{orbital})$ and $\mu(\text{total})$ for the $2^+_1$ and $2^+_2$ states.
We also illustrate the results in various figures as follows. In Fig. 1, we give the electric quadrupole moment of the \(2^+_1\) state versus \(x\) as calculated in the small and large spaces. In Fig. 2 we give the small-space results for \(Q(2^+_1)\) and \(Q(2^+_2)\) versus \(x\), and in Fig. 3 we give the corresponding large-space results.

In figures 4, 5 and 6 we present a comparison of the magnetic dipole moment \(\mu(2^+_1)\) as calculated in the small and large model spaces, as a function of \(x\). In Fig. 4 we have \(\mu(2^+_1)_{\text{spin}}\), in Fig. 5 \(\mu(2^+_1)_{\text{orbital}}\), and in Fig. 6 \(\mu(2^+_1)_{\text{total}}\).

### A. Discussion of \(Q(2^+)\) and \(B(E2 : 0^+_1 \rightarrow 2^+)\) as a function of \(x\)

In Fig. 1, we see a large difference in the behaviour of the electric quadrupole moment \(Q(2^+_1)\) of the \(2^+_1\) state versus \(x\), especially for \(x < 1\). In the small space \([(0s)^4(0p)^6]\), the behaviour is much simpler. \(Q(2^+_1)\) is always negative, as we would expect for a prolate nucleus (with a positive intrinsic quadrupole moment \(Q_0\)). We have noted previously that, in the \(jj\) coupling limit, \(Q(2^+_1)\) should vanish, so it is not surprising to find that for the most part (except near \(x = 0\)) the value of \(Q\) decreases steadily from about \(-4 \, e\, fm^2\) towards zero as \(x\) increases.

In the large space, on the other hand, \(Q(2^+_1)\) starts out large and positive \((\approx 5.4 \, e\, fm^2)\). It then decreases and goes through zero at \(x \approx 0.63\), reaches a negative value of about \(-5 \, e\, fm^2\) at \(x = 1.5\), and then starts decreasing to zero. Why are the results in the small and large space so different?

Let us first look at the large \(x\) behaviour. The fact that the magnitude of (negative) \(Q(2^+_1)\) is larger in the large space is understandable. The effects of including \(\Delta N = 2\) admixtures is precisely to change bare \(E2\) charges to effective charges. A popular choice of effective charges is \(e_p = 1.5\) and \(e_n = 0.5\) for the proton and the neutron, respectively. In the calculations, one does not necessarily get exactly these values, but the trend is in that direction. Certainly the isoscalar \(E2\) gets enhanced. Not only \(Q(2^+_1)\) but also \(B(E2)_{0^+_1 \rightarrow 2^+_1}\) should get enhanced if this effective charge argument is correct. From Table 1, we see that for say \(x = 1.5\) \(B(E2)_{0^+_1 \rightarrow 2^+_1}\) is \(18.19 \, e^2 \, fm^4\) in the small space, but is equal to \(42.63 \, e^2 \, fm^4\) in the large space. This then supports the effective charge argument.

However, at small \(x\) the quadrupole moments are of opposite sign in the small and large spaces. Recall that in the small space, at \(x = 0\), the \(2^+_1\) and \(2^+_2\) states are degenerate. We can arrange the two states so that one is a \(J = 2\) \(K = 0\) state and the other \(J = 2\) \(K = 2\). The \(B(E2)\) to the first state should be much larger than to the second. The two states have the same intrinsic quadrupole moment in the rotational model, and hence equal and opposite quadrupole moments in the laboratory. If we turn on the spin-orbit interaction very weakly, the lower energy state is clearly dominantly \(K = 0\) and the upper one \(K = 2\). For example, for \(x = 0.1\), the values are: \(E(2^+_1) = 3.352 \, MeV\), \(Q(2^+_1) = -4.042 \, e\, fm^2\) and \(B(E2)_{0^+_1 \rightarrow 2^+_1} = 21.34 \, e^2 \, fm^4\), whilst for the higher state \(E(2^+_2) = 3.384 \, MeV\), \(Q(2^+_2) = +3.598 \, e\, fm^2\) and \(B(E2)_{0^+_1 \rightarrow 2^+_2} = 3.578 \, e^2 \, fm^4\). There is a small amount of \(K\) mixing due to the spin-orbit interaction, but not very much.

In the large space, what appears to be happening right off the bat at \(x = 0\) is that a state which is dominantly \(K = 2\) is lying lower in energy than one which is dominantly \(K = 0\).

This is evident from the corresponding values for the \((0+2) \, h\omega\) properties of the \(2^+_1\) and \(2^+_2\) states obtained with \(x = 0\):
\(E(2^+_1) = 2.485 \text{ MeV}, \ Q(2^+_1) = 5.453 \ e fm^2\) and \(B(E2)_{0^+_1 \rightarrow 2^+_1} = 5.502 \ e^2 fm^4\).

\(E(2^+_2) = 3.972 \text{ MeV}, \ Q(2^+_2) = -5.559 \ e fm^2\) and \(B(E2)_{0^+_2 \rightarrow 2^+_2} = 50.48 \ e^2 fm^4\).

As we turn on the spin-orbit interaction, a more normal behaviour starts to be restored, one in which \(B(E2)\) is much larger. The reason that at \(x = 0\) the \(J = 2 \ K = 2\) state lies lower than the \(J = 2 \ K = 0\) is not clear. It must be due to \(\Delta N = 2\) mixing from the quadrupole-quadrupole interaction. Such mixing does not maintain the SU(3) symmetry of the Elliott hamiltonian (although in first order perturbation theory all that happens is that the strength of the \(Q \cdot Q\) interaction gets renormalized, so that in that approximation one would still get Elliott’s formula for the energies, and in particular the \(2^+_1\) and \(2^+_2\) states of \(^{10} \text{Be}\) could still be degenerate. Here, however, we are not doing first order perturbation theory, but rather exact \((0+2) \ h \omega\) matrix diagonalization).

**B. Discussion of the magnetic dipole moment \(\mu(2^+)\) as a function of \(x\)**

There are some surprises for the static magnetic dipole moments as well. As seen in Fig. 6, whereas in the small space the magnetic moment of the \(2^+_1\) state increases steadily with \(x\) (and as mentioned previously approaches 5.057 \(\mu_N\) as \(x \rightarrow \infty\)), in the large space \(\mu(2^+_1)\) starts out small and positive but decreases to almost zero at \(x = 0.63\), where there is a near cancellation between the spin and orbital parts of the magnetic moment.

The negative values of \(\mu(2^+_1)_{\text{spin}}\) from \(x = 0\) to \(x = 1.5\) obviously come from the neutrons. At \(x = 0\), \(\mu_{\text{spin}}\) must be zero because in the Wigner Supermultiplet theory the \(2^+_1\) state has \(S = 0\). As \(x\) goes to infinity, we approach the \(jj\) coupling limit, and the neutrons will not contribute to \(\mu_{\text{spin}}\) because they form an \(S = 0\) closed neutron \(p_{3/2}\) subshell. Only the protons will contribute, in that limit, and \(\mu_{\text{spin}}\) for \(p_{3/2}\) protons is positive. Hence, only for some intermediate values of \(x\) will the neutrons’ contribution to \(\mu_{\text{spin}}\) be able to dominate.

For large \(x\), the results are somewhat easier to understand. The fact that \(\mu_{\text{spin}}(\text{large space}) < \mu_{\text{spin}}(\text{small space})\) can be understood by the fact that the higher shell admixtures renormalize and considerably enhance the effective strength of the \(Q \cdot Q\) interaction in the valence space. This makes the ratio \(x/\chi\) of the spin-orbit strength to the \(Q \cdot Q\) strength effectively smaller. Thus, roughly speaking, the value of \(\mu_{\text{spin}}(\text{large space})\) for a given \(x\) in Fig. 4 should be compared with a value of \(\mu_{\text{spin}}(\text{small space})\) for a smaller \(x\) (say \(x/1.5\)). The values of \(\mu_{\text{spin}}\) will then be much closer. Alternatively, one could use a larger value of \(\chi\) in the small space calculation in order to make the comparison with the large space calculation for the same \(x\).

The value of \(\mu_{\text{orbital}}\) in the small space is remarkably constant as a function of \(x\), with a value of about 1.4 \(\mu_N\). However, in the large space and at \(x = 0\) this quantity is much smaller (\(\approx 0.3 \mu_N\)). As \(x\) increases, \(\mu_{\text{orbital}}\) also increases, and for large \(x\), the values in small and large spaces are almost the same.

The spin-orbit interaction is known to give \(K\) mixing and therefore to bring many complications to analyses which start from the point of view of the rotational model. Ironically, here, the spin-orbit interaction with sufficient strength simplifies things. It brings down in energy the expected dominantly \(K = 0 \ J = 2^+\) prolate state, the signatures of which are a strong \(B(E2)\) transition from the ground state and a large negative electric quadrupole moment.
Summarizing this section, we can say that when the coupling of the spin-orbit interaction is sufficiently strong, things tend to settle down.

C. The critical value \( x \approx 0.63 \) where \( Q(2^+_1) \), \( Q(2^+_2) \) and \( \mu(2^+_1) \) nearly all vanish

As we increase \( x \) from zero to 2 in the small space, nothing unusual happens to \( Q(2^+_1) \). It has a large negative value for \( x = 0 \), consistent with the \( SU(3) \) limit and with a prolate nucleus. The value of \( Q(2^+_1) \) then gradually approaches zero as \( x \) approaches infinity, as expected from the discussion of the \( jj \) coupling limit in the previous section.

In the large space, the behaviour is drastically different. Now \( Q(2^+_1) \) starts out positive, goes through zero at about \( x = 0.63 \) and then becomes negative. For large \( x \), the behaviour is less surprising. The curves for both small and large spaces seem to be going to zero. The fact that \( |Q(2^+_1)|_{\text{large}} > |Q(2^+_1)|_{\text{small}} \) for large \( x \) is consistent with the fact that core polarization enhances \( Q \) and \( B(E2) \) i.e. it gives a microscopic justification for using effective charges.

Getting back to the critical region of \( x \approx 0.63 \), we find that not only \( Q(2^+_1) \) but also \( Q(2^+_2) \) becomes very small in magnitude, and likewise \( \mu(2^+_1) \). We can perhaps explain why both \( Q \)’s become small at the same time by noting that in the \( SU(3) \) limit, the two states \( 2^+_1 \) and \( 2^+_2 \) are degenerate and have equal and opposite quadrupole moments. In the rotational model, both the \( K = 0 \) and \( K = 2 \) \( J = 2^+ \) states have the same intrinsic quadrupole moment \( Q_0 \), but the geometric factors cause the laboratory \( Q \)’s to be equal and opposite.

Let us suppose that the spin-orbit interaction mixes the \( K = 0 \) and \( K = 2 \) states:

\[
|2^+_1\rangle = \alpha |K = 0\rangle + \beta |K = 2\rangle
\]

\[
|2^+_2\rangle = -\beta |K = 0\rangle + \alpha |K = 2\rangle
\]

then

\[
Q(2^+_1) = \alpha^2 Q_{K=0} + \beta^2 Q_{K=2} + 2\alpha\beta \langle K = 0 | Q | K = 2 \rangle
\]

\[
Q(2^+_2) = \beta^2 Q_{K=0} + \alpha^2 Q_{K=2} - 2\alpha\beta \langle K = 0 | Q | K = 2 \rangle
\]

Suppose now that \( \langle K = 0 | Q | K = 2 \rangle \) is negligible and that \( Q_{K=0} = -Q_{K=2} \), we then have

\[
Q(2^+_1) = (\alpha^2 - \beta^2) Q_{K=0}
\]

and

\[
Q(2^+_2) = (\beta^2 - \alpha^2) Q_{K=0}
\]

and if we have equal mixing i.e. \( \alpha = \beta = \frac{1}{\sqrt{2}} \) we get

\[
Q(2^+_1) = Q(2^+_2) = 0
\]

To support this \( K \)-mixing argument, we should examine the values of the electric quadrupole transitions \( B(E2)_{0^+_1 \rightarrow 2^+_1} \) and \( B(E2)_{0^+_1 \rightarrow 2^+_2} \). If there were no \( K \)-mixing, the \( B(E2) \)
to the $K = 0 J = 2^+$ state should be much stronger than to the $K = 2 J = 2^+$ state. If there is strong $K$-mixing, the $B(E2)$’s to both states should be comparable.

Before looking at the critical region ($x \approx 0.63$), let us look at the large space results for a value of $x$ that gives the best energies for the $2_1^+$ and $2_2^+$ states. This corresponds to $x = 1.5$ where $E(2_1^+) = 3.513$ MeV and $E(2_2^+) = 6.307$ MeV, and the values of $Q(2_1^+)$ and $Q(2_2^+)$ are -5.039 e fm$^2$ and 5.384 e fm$^2$, respectively. This is consistent with the lowest state being dominantly $K = 0$ and the upper one $K = 2$, and is probably very close to the experimental situation, not only for $^{10}$Be but for most strongly deformed nuclei. Support to this is given by looking at the calculated $B(E2)$’s. We find that $B(E2)_{0_1^+ \rightarrow 2_1^+} = 42.63 e^2 f m^4$ and $B(E2)_{0_1^+ \rightarrow 2_2^+} = 0.989 e^2 f m^4$.

Below the critical $x$, say at $x = 0.1$, we have the reverse situation, with $Q(2_1^+)$ positive, $Q(2_2^+)$ negative (5.345 and -5.230 e fm$^2$ respectively), and $B(E2)_{0_1^+ \rightarrow 2_1^+} = 42.76 e^2 f m^4$ while $B(E2)_{0_1^+ \rightarrow 2_2^+} = 5.83 e^2 f m^4$.

Clearly then, there is a cross-over around $x \approx 0.63$ or so. As mentioned above, the values of $Q(2_1^+)$ and $Q(2_2^+)$ nearly vanish in the critical region $x \approx 0.63$ (they are -0.034 and 0.278 e fm$^2$ respectively). What about the $B(E2)$’s? The values of these are 24.39 e$^2$ fm$^4$ to the $2_1^+$ state and 23.07 e$^2$ fm$^4$ to the $2_2^+$ state, nearly equal indeed. This lends a clear support to the strong $K$-mixing argument.

The fact that $\mu(2_1^+)$ also nearly vanishes where $Q(2_1^+)$ and $Q(2_2^+)$ do is probably coincidental. For the magnetic dipole moment we find that the spin and orbital contributions nearly cancel in the vicinity of $x = 0.63$. As mentioned before, $\mu_{\text{spin}}$ vanishes at $x = 0$. For small, finite $x$, the neutrons tend to dominate to give a negative $\mu_{\text{spin}}$. At very large $x$ however, and at least in the small space, the neutrons again do not contribute because they form a closed $p_{3/2}$ subshell.

**IV. RESULTS THAT ARE OF GENERAL INTEREST**

Some of the results that we have obtained are expected, but some are at least at first thought surprising.

The Wigner Supermultiplet theory, and as a special case $SU(3)$, predicts results that are in disagreement with experiment both in the valence space and the valence plus 2 $\hbar \omega$ space. In the small space, the $2_1^+$ and $2_2^+$ states ($K = 0$ and $K = 2$) are predicted to be degenerate, whereas experimentally the $2_2^+$ state is at about twice the excitation energy of the $2_1^+$ state (3.368 MeV and 5.960 MeV, respectively). Turning on a sufficiently strong spin-orbit interaction ($x \approx 1$ to 1.5) corrects this situation, and furthermore the lower state is dominantly $K = 0$ with a strong $B(E2)$ from the ground state, and the upper one is dominantly $K = 2$ with a weak $B(E2)$ from the ground state.

In the large space, the situation is even worse in the absence of a spin-orbit interaction. The lowest state is dominantly $K = 2$ and the upper one is $K = 0$. As we turn on the spin-orbit interaction, one first gets a crossover in the properties of the lowest $2^+$ state: at about $x = 0.63$ one gets about equal mixing of $K = 0$ and $K = 2$ with the result that the electric quadrupole moments $Q(2_1^+)$ and $Q(2_2^+)$ both nearly vanish. In other regions of $x$ they are both large in magnitude and nearly equal but opposite in sign.

As we increase $x$ further, we slowly converge to the experimental situation (at $x \approx 1.25$)
where the $2_1^+$ and $2_2^+$ states are split by about 3 $MeV$ and the $B(E2)$ to the $2_1^+$ state is much stronger than to the $2_2^+$ state.

Looking at Fig. 1 we see that for the realistic value near $x = 1.5$ we are safely away from the crossover region near $x = 0.63$. But this example suggests that in other nuclei and at other excitation energies, there may be two states of the same angular momentum but with different $K$ or other quantum numbers. In that case, it appears that one can get unexpected and different effects in a valence space calculation as compared with one in which 2 $\hbar\omega$ excitations are allowed. For example, as noted here in the small space calculation with $x = 0.5$, we have $Q(2_1^+) = -3.878$ e fm$^2$ and $Q(2_2^+) = 3.662$ e fm$^2$, whereas in the large space the values are $Q(2_1^+) = 2.109$ e fm$^2$ and $Q(2_2^+) = -1.816$ e fm$^2$. These are drastically different values and the difference cannot be considered to be the result of perturbative corrections. Likewise, for the magnetic dipole moments in the valence space we have at $x = 0.5 \mu(2_1^+)_{spin} = 0.017 \mu_N$, $\mu(2_1^+)_{orbit} = 1.295 \mu_N$ and $\mu(2_1^+)_{total} = 1.312 \mu_N$, whereas in the large space the corresponding values are $-0.387$, $0.462$ and $0.0753 \mu_N$.

When we go to larger $x$, things are not completely chaotic. We expect that 2 $\hbar\omega$ admixtures will cause electric quadrupole moments and $B(E2)$’s to be enhanced, according to the effective charge argument that was given previously. Let us arbitrarily define an electric quadrupole moment enhancement factor as $Q(2_1^+)_\text{large} / Q(2_1^+)_\text{small}$. The values of this enhancement factor for the same values of $x$ as above are respectively $1.41$, $1.99$, $2.45$, $2.76$ and $2.93$. We should remember that in the small space $Q(2_1^+)$ approaches 0 as $x$ becomes very large ($jj$ limit) and this is the reason the enhancement factor, as defined above, increases with $x$.

Let us now consider the corresponding $B(E2)$ enhancement factor $B(E2 : 0^+ \rightarrow 2_1^+)_\text{large} / B(E2 : 0^+ \rightarrow 2_1^+)_\text{small}$. The values of this factor for the same values of $x$ as above are respectively $2.05$, $2.27$, $2.34$, $2.28$ and $2.15$. One might at first expect the $B(E2)$ enhancement factors to be the squares of (and hence much larger than) the quadrupole enhancement factors, but in fact they are smaller.

In summary, we have found some surprising behaviour which, given the large number of nuclei in the periodic table and the large number of excited states, may actually be realized. For $^{10}Be$, we feel that we are realistically in a region of $x$ where, although the effects of higher shell admixtures are very substantial, leading to large $E2$ enhancements and large magnetic dipole suppressions, the results are under control. But it may be possible that for other nuclei we have, for example, nearly degenerate $K = 0$ and $K = 2$ states such that the higher shell admixtures lead to very surprising results. The examples given in this work are the fact that $Q(2_1^+)$ and $Q(2_2^+)$ nearly vanish at $x \approx 0.63$ in the large space, whereas in the small space they are large and nearly equal and opposite. The near vanishing of $\mu(2_1^+)_{total}$ at $x \approx 0.63$ in the large space, and the fact that it takes a long time before $\mu(2_1^+)_{total}$ achieves its full value, are two other examples.

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Figure Captions

**Fig. 1:** The electric quadrupole moment $Q$ of the $2^+_1$ state versus $x$ as calculated in the small space (dashed curve) and in the large space (solid curve).

**Fig. 2:** Small-space calculations of the electric quadrupole moments of the $2^+_1$ state (solid curve) and of the $2^+_2$ state (dashed curve).

**Fig. 3:** Large-space calculations of the electric quadrupole moments of the $2^+_1$ state (solid curve) and of the $2^+_2$ state (dashed curve).

**Fig. 4:** The spin component of the magnetic dipole moment $\mu$ of the $2^+_1$ state versus $x$, as calculated in the small space (dashed curve) and in the large space (solid curve).

**Fig. 5:** Same as Fig. 4, but for the *orbital* component of the magnetic dipole moment $\mu(2^+_1)$.

**Fig. 6:** Same as Fig. 4, but for the *total* magnetic dipole moment $\mu(2^+_1)$. 
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TABLE I. Excitation energies (in $MeV$), electric quadrupole moments $Q$ and transition amplitudes from the ground state $B(E2)$ (in units of $efm^2$ and $e^2fm^4$, resp. and using $e_p = 1.0$ and $e_n = 0.0$), for the first two $2^+$ states in $^{10}Be$ as a function of the strength $x$ of the spin-orbit term in the interaction $Q \cdot Q + x V_{SO}$, in the small and large shell model spaces.

| $x$ | $E(2^+)$ | $Q(2^+)$ | $B(E2: 0^+_1 \rightarrow 2^+)$ | $E(2^+)$ | $Q(2^+)$ | $B(E2: 0^+_2 \rightarrow 2^+)$ |
|-----|-----------|-----------|----------------|-----------|-----------|----------------|
| 0   | 3.356     | -3.913    | 20.95          | 2.485     | 5.453     | 5.502          |
|     | 3.356     | 3.455     | 4.099          | 3.972     | -5.559    | 50.48          |
| 0.1 | 3.352     | -4.042    | 21.34          | 2.504     | 5.345     | 5.829          |
|     | 3.384     | 3.598     | 3.578          | 3.914     | -5.230    | 42.76          |
| 0.25| 3.338     | -4.046    | 21.24          | 2.608     | 4.744     | 7.738          |
|     | 3.532     | 3.668     | 3.137          | 3.819     | -4.445    | 37.30          |
| 0.5 | 3.339     | -3.878    | 20.86          | 2.902     | 2.109     | 16.71          |
|     | 4.022     | 3.662     | 2.571          | 3.876     | -1.816    | 29.87          |
| 0.60| 3.353     | -3.767    | 20.72          | 3.027     | 0.477     | 22.54          |
|     | 4.284     | 3.631     | 2.391          | 3.984     | -0.217    | 24.70          |
| 0.63| 3.358     | -3.729    | 20.08          | 3.063     | -0.034    | 24.39          |
|     | 4.368     | 3.619     | 2.342          | 4.025     | 0.278     | 23.07          |
| 0.70| 3.370     | -3.630    | 20.57          | 3.139     | -1.188    | 28.64          |
|     | 4.573     | 3.588     | 2.238          | 4.138     | 1.408     | 19.15          |
| 0.75| 3.377     | -3.552    | 20.48          | 3.207     | -1.939    | 31.44          |
|     | 4.725     | 3.562     | 2.171          | 4.232     | 2.141     | 16.51          |
| 1.0 | 3.403     | -3.080    | 19.88          | 3.379     | -4.348    | 40.68          |
|     | 5.552     | 3.389     | 1.952          | 4.825     | 4.456     | 7.391          |
| 1.25| 3.382     | -2.549    | 19.05          | 3.486     | -5.078    | 43.32          |
|     | 6.441     | 3.072     | 2.041          | 5.540     | 5.282     | 3.067          |
| 1.50| 3.312     | -2.057    | 18.19          | 3.513     | -5.039    | 42.63          |
|     | 7.331     | 1.667     | 3.427          | 6.307     | 5.384     | 0.989          |
| 1.75| 3.203     | -1.651    | 17.42          | 3.456     | -4.565    | 39.78          |
|     | 8.043     | -0.843    | 4.853          | 7.073     | 4.908     | 0.010          |
| 2.0 | 3.070     | -1.334    | 16.78          | 3.316     | -3.910    | 36.00          |
|      |      |      |      |      |      |
|------|------|------|------|------|------|
| 8.691 | -1.328 | 5.153 |      | 7.727 | 2.925 | 1.860 |
| 3.0   | 2.459 | -0.626 | 15.31 | 2.266 | -1.752 | 23.06 |
|       | 11.519 | -2.060 | 6.162 | 9.506 | -1.967 | 14.15 |
| 4.0   | 1.843 | -0.322 | 14.67 | 1.097 | -0.856 | 17.06 |
|       | 14.563 | -2.352 | 6.551 | 11.870 | -2.878 | 15.30 |
TABLE II. Same as in Table I but for the magnetic dipole moments of the first two $2^+$ states in ${}^{10}\text{Be}$ as a function of the strength $x$ of the spin-orbit term in the interaction $\mathbf{Q} \cdot \mathbf{Q} + xV_{S,O}$, in the small and large shell model spaces.

| $x$ | $E(2^+)$ | $\mu_{\text{spin}}(2^+)$ | $\mu_{\text{orbital}}(2^+)$ | $\mu_{\text{total}}(2^+)$ | $E(2^+)$ | $\mu_{\text{spin}}(2^+)$ | $\mu_{\text{orbital}}(2^+)$ | $\mu_{\text{total}}(2^+)$ |
|-----|-----------|-----------------|-----------------|-----------------|-----------|-----------------|-----------------|-----------------|
| 0   | 3.356     | 0               | 1.244           | 1.244           | 2.485     | 0               | 0.331           | 0.331           |
| 0.1 | 3.356     | -0.014          | 1.274           | 1.259           | 2.504     | -0.014          | 0.332           | 0.318           |
| 0.25| 3.338     | -0.053          | 1.281           | 1.228           | 2.608     | -0.090          | 0.347           | 0.257           |
| 0.5 | 3.339     | 0.017           | 1.295           | 1.312           | 2.902     | -0.387          | 0.462           | 0.075           |
| 0.60| 3.353     | 0.116           | 1.307           | 1.422           | 3.027     | -0.544          | 0.564           | 0.020           |
| 0.63| 3.358     | 0.153           | 1.311           | 1.464           | 3.063     | -0.588          | 0.601           | 0.013           |
| 0.70| 3.370     | 0.253           | 1.321           | 1.574           | 3.139     | -0.677          | 0.692           | 0.015           |
| 0.75| 3.377     | 0.335           | 1.329           | 1.663           | 3.207     | -0.723          | 0.759           | 0.036           |
| 1.0 | 3.403     | 0.830           | 1.363           | 2.193           | 3.379     | -0.714          | 1.044           | 0.331           |
| 1.25| 3.382     | 1.368           | 1.385           | 2.752           | 3.486     | -0.416          | 1.224           | 0.807           |
| 1.50| 3.312     | 1.844           | 1.393           | 3.236           | 3.513     | 0.045           | 1.334           | 1.379           |
| 1.75| 3.203     | 2.221           | 1.392           | 3.613           | 3.456     | 0.604           | 1.396           | 2.001           |
| 2.0 | 3.070     | 2.507           | 1.388           | 3.895           | 3.316     | 1.176           | 1.423           | 2.599           |
| 3.0 | 2.459     | 3.112           | 1.365           | 4.477           | 2.266     | 2.699           | 1.394           | 4.093           |
|   11.519 | 1.620 | 1.159 | 2.779 | 9.506 | 1.488 | 0.952 | 2.440 |
|----------|-------|-------|-------|-------|-------|-------|-------|
| 4.0      | 1.843 | 3.356 | 1.349 | 4.704 | 1.097 | 3.252 | 1.354 | 4.605 |
| 14.563   | 1.450 | 1.210 | 2.660 | 11.870| 1.254 | 1.113 | 2.367 |
Fig. 1
Fig. 2

Q(2, -1, 2)

x

y

Q(2, -1, 2)
Fig. 6