Flat Potential for Inflaton with a Discrete \( R \)-invariance in Supergravity

Kazuya Kumekawa, Takeo Moroi and Tsutomu Yanagida

Department of Physics, Tohoku University,
Sendai 980-77, Japan

Abstract

We show that a very flat potential of inflaton required for a sufficient inflation is naturally obtained in \( N = 1 \) supergravity by imposing a discrete \( R \)-invariance \( Z_n \). Several cosmological constraints on parameters in the inflaton superpotential are derived. The reheating temperature turns out to be \((1 - 10^8) \text{GeV}\) for the cases of \( n=3-10 \). Baryogenesis in this model is also discussed briefly.
1. Introduction

Much effort has been done in building realistic particle-physics models based on \( N = 1 \) supergravity [1]. Although these theories provide a natural framework for producing soft supersymmetry (SUSY) breaking terms at low energies, there are potential cosmological problems. One of them is a difficulty to construct inflationary scenarios of the universe. Namely, to generate a sufficiently large expansion of the universe one must require an extreme fine tuning of several parameters making a very flat potential for the inflaton [2]. However, there has not been found any natural explanation on the existence of such a flat potential.

In this paper, we show that a discrete \( R \)-symmetry \( Z_n \) automatically leads to a flat potential at the origin of the inflaton field \( \phi \) as long as the minimum Kähler potential \( K(\phi, \phi^*) = \phi \phi^* \) is used. We derive cosmological constraints on parameters in the inflaton superpotential.

In this model, the inflaton superpotential does not vanish at the minimum of the inflaton scalar potential and hence the inflaton \( \phi \) plays a role of the Polonyi field for generating a gravitino mass [3]. Together with the cosmological constraints, all relevant parameters in the superpotential are fixed such that the gravitino mass \( m_{3/2} \) lies in 100GeV – 10TeV. With the obtained parameters in the superpotential we argue that the most natural choice for the discrete \( R \)-symmetry is \( Z_4 \), in which the reheating temperature is \( T_R \sim O(1 – 100) \text{TeV} \) depending on the gravitino mass \( m_{3/2} = 100 \text{GeV} – 10 \text{TeV} \). We give a brief comment on a possible scenario for baryogenesis in such a low temperature universe. Particle-physics problems related to the SUSY breaking in this model are also discussed.

2. Discrete \( R \)-symmetries and the inflaton potential

Let us define a discrete \( Z_n \) \( R \)-transformation on the inflaton field \( \phi(x, \theta) \) as

\[
\phi(x, \theta) \rightarrow e^{-i\alpha} \phi(x, e^{i\alpha/2} \theta), \quad \alpha = \frac{2\pi k}{n},
\]

with \( k = 0, \pm 1, \pm 2, \ldots \). Assuming positive power expansion of a Kähler potential \( K(\phi, \phi^*) \) and a superpotential \( W(\phi) \) for the inflaton field \( \phi \), the general forms which
have the $Z_n$ $R$-invariance are given by

$$K(\phi, \phi^*) = \sum_{m=1}^{\infty} a_m (\phi \phi^*)^m,$$

(2)

and

$$W(\phi) = \phi \sum_{l=0}^{\infty} b_l \phi^l.$$

(3)

We take the minimum Kähler potential ($a_1=1$, other $a_i=0$) for simplicity. (We have found that the scenario described below does work with more general Kähler potential in eq.(3), as far as the coefficients $|a_2| < 3/(8n(n+1)$ and $|a_i| \sim O(1)(i \geq 3)$.)

In the $N = 1$ supergravity \[1\], a scalar potential $V(\phi)$ is written as

$$V(\phi) = \exp \left( \frac{K(\phi, \phi^*)}{M^2} \right) \left\{ (K^{-1})_{\phi^*} (D_\phi W)(D_\phi W)^* - \frac{3|W|^2}{M^2} \right\},$$

(4)

with

$$D_\phi W \equiv \frac{\partial W}{\partial \phi} + \frac{\partial K}{\partial \phi} \frac{W}{M^2},$$

(5)

$$(K^{-1})_{\phi^*} \equiv \left( \frac{\partial^2 K}{\partial \phi \partial \phi^*} \right)^{-1},$$

(6)

where $M$ is the gravitational mass $M = M_{Planck}/\sqrt{8\pi} \simeq 2.4 \times 10^{18}$GeV. With the general form of $W(\phi)$ in eq.(3), we find the inflaton potential $V(\phi)$ has automatically flat directions at the origin. That is, the condition

$$\frac{\partial V}{\partial \phi} = \frac{\partial^2 V}{\partial \phi \partial \phi^*} = 0,$$

(7)

is always satisfied at the origin $\phi = 0$. Notice that the mass term $\phi \phi^*$ from the $\exp(K/M^2)$ is cancelled out by that in the curly bracket in eq.(4). This cancellation is very important for the inflaton $\phi$ to do its required job, and in all previous works \[2\] this cancellation is achieved only by a fine tuning of parameters in the superpotential $W(\phi)$.\[1\]

\[1\]There has been proposed an alternative approach, in which non-minimum Kähler potentials are used \[3\]. In this class of models, a fine tuning is necessary in the Kähler potential to achieve a very flat potential for the inflation.
To find the approximate form of \( V(\phi) \) near the origin \( \phi \sim 0 \) we write the superpotential as

\[
W(\phi) = \left( \frac{\lambda}{v^{n-2}} \right) \left( v^n \phi - \frac{1}{n+1} \phi^{n+1} \right) + \cdots.
\]  

(8)

Here, \( \cdots \) represents higher power parts \( (\phi^{kn+1}) \) with \( k \geq 2 \). Since these parts are all irrelevant to the present analysis, we neglect them, hereafter. We easily see that the inflaton potential (4) near \( \phi \sim 0 \) is well approximated by

\[
V(\phi) \simeq \left( \frac{\lambda}{v^{n-2}} \right)^2 \left\{ v^{2n} + \frac{1}{2} v^{2n} \left( \frac{\phi}{M^2} \right)^2 - v^n (\phi^n + \phi^* n) \right\}.
\]  

(9)

Clearly, \( n=2 \) does not give a flat potential and hence we do not consider the \( n=2 \) case. For \( n=3 \) and 4, we can neglect the \( (|\phi|^2/M^2)^2 \) term, since \( v \ll M \) as we will see later. For \( n \geq 5 \), we also neglect this term assuming that the initial amplitude of \( \phi \) is greater than \( v(v/M)^{4/(n-4)} \). Thus, in any case of \( n \geq 3 \) we neglect the \( (|\phi|^2/M^2)^2 \) term in analyzing the inflaton dynamics. In Fig. 1 we depict the exact \( V(\phi) \) along the real axis \( \text{Re}\phi \), which shows that the inflaton potential has always a very flat region near \( \phi \sim 0 \). We have also checked that \( \phi \simeq v \) is the global minimum of the inflaton potential.

Whether the inflation occurs or not depends on initial conditions for the inflaton field \( \phi \). In the present model the initial amplitude of \( \phi \) must be localized very near the origin in order to have a sufficiently large expansion of the universe. We simply assume \( \phi \) and \( \dot{\phi} \) satisfy the desired initial conditions \( |\phi| \sim 0 \) and \( \dot{\phi} \sim 0 \). We have no answer to a question of what physics the initial conditions for \( \phi \) was set by, but if \( \phi \) and \( \dot{\phi} \) satisfy the initial condition, the maximum inflation takes place and this exponentially expanding part of the universe dominates the others.

Setting the phase of \( \phi \) to vanish\(^3\) we identify the inflaton field with \( \varphi \) where \( \varphi \) is a real component of \( \phi \) (\( \varphi \equiv \sqrt{2} \text{Re}\phi \)). Thus, the relevant potential for \( \varphi \) is now given by

\[
V(\varphi) \simeq \lambda^2 \varphi^4 \left\{ 1 - 2 \left( \frac{\varphi}{v} \right)^n \right\},
\]

(10)

\(^2\)This assumption on the initial amplitude \( \varphi_i (\equiv \sqrt{2} \text{Re}\phi_i) \) is consistent with \( \varphi_i < \varphi_N \) (with \( N \sim 40 \)) given in eq.\(^1\)) as far as \( v \ll M \) and \( n \geq 5 \). This initial condition \( \varphi_i < \varphi_N \) is derived to have a sufficient inflation as we will explain later.

\(^3\)Notice that the phase \( \chi(x) \) (\( \phi = \sqrt{2} \text{Re} e^{i\chi} \)) has a positive mass at \( \varphi \neq 0 \). Thus if one chooses the initial value of \( \chi(x) \sim 0 \), it stays there during the inflation.
with
\[ \tilde{\lambda} \equiv \frac{1}{2} \lambda, \quad \tilde{v} \equiv \sqrt{2} v. \] (11)

3. Cosmological constraints

The equation of motion for \( \varphi \) in the expanding universe is given by
\[ \ddot{\varphi} + 3H\dot{\varphi} + \frac{dV}{d\varphi} = 0, \] (12)
where \( H \) is the Hubble expansion rate. During the slow rolling regime of \( \varphi (\ddot{\varphi} \ll 3H\dot{\varphi}) \) the energy density of the universe is dominated by the inflaton potential \( V(\varphi \sim 0) \), which gives a nearly constant expansion rate
\[ H^2 \simeq \frac{V(\varphi \sim 0)}{3M^2} \simeq \frac{\tilde{\lambda}^2 \tilde{v}^4}{3M^2}. \] (13)
In this inflationary epoch, the \( \ddot{\varphi} \) term in eq.(12) can be neglected so that
\[ \dot{\varphi} \simeq - \frac{V'(\varphi)}{3H} \simeq \frac{2n\tilde{\lambda}M}{\sqrt{3} \tilde{v}^{n-2}} \varphi^{n-1}. \] (14)
The slow rolling regime ends at \( \varphi_f \),
\[ \varphi_f^{n-2} \simeq \frac{3}{2n(n-1)M^2} \tilde{v}^n. \] (15)

The cosmic scale factor grows exponentially \( \sim e^N \) till the end of the inflation. With the above approximation, the \( e \)-folding factor \( N \) between the time \( t_f \) and \( t_N \) is given by
\[ N = H(t_f - t_N) \simeq \frac{n - 1}{3(n - 2)} \left\{ \left( \frac{\varphi_N}{\varphi_f} \right)^{2-n} - 1 \right\}, \] (16)
where \( \varphi_f (\varphi_N) \) represents the amplitude of the field variable \( \varphi \) at the time \( t_f \) (\( t_N \)). For a large \( N \), \( \varphi_N \) is given by
\[ \varphi_N \simeq \left\{ 2Nn(n-2) \right\}^{1/(2-n)} \left( \frac{\tilde{v}}{M} \right)^{2/(n-2)} \tilde{v}. \] (17)

To solve the flatness and horizon problems, a sufficiently large expansion of the universe is required during the inflation [5]. In our model, the Hubble radius of the present
universe crossed outside of the horizon $N \sim 40$ e-folds before the end of the inflation.\footnote{The reason why e-folding factor $N \sim 40$ is smaller than the usual value $N \sim 60$ is because the reheating temperature $T_R$ in the present model is relatively low ($T_R \lesssim 100\text{TeV}$ for $n = 3 - 5$). For $n \geq 6$, $T_R$ is $(10^2 - 10^4)\text{TeV}$ and in these cases, $N$ becomes $N \sim 50$.}

This suggests that the initial amplitude of $\varphi$ should be smaller than $\varphi_N$ with $N \sim 40$.

During the de Sitter phase, the density perturbation $(\delta \rho/\rho)$ arises from quantum fluctuations \footnote{This value of $\varphi_N$ is much larger than the quantum fluctuation $\delta \varphi \sim H/2\pi$ unless $\lambda$ is large $\lambda \gtrsim (M/v)^{3-2}$. Furthermore, the change of $\varphi$ in one expansion time is much larger than the quantum fluctuation. Thus, the evolution of $\varphi$ can be discussed by solving the classical equation of motion in eq.(12).} of the inflaton field $\varphi$. It is roughly given by

$$\left( \frac{\delta \rho}{\rho} \right)_N \approx \frac{3}{5\pi} \frac{H^3}{|V'(\varphi_N)|} \approx \left( \frac{\tilde{\lambda} \tilde{v}^3}{10 \sqrt{3\pi n M^3}} \right) \left\{ \frac{\tilde{v}^2}{2 N n (n - 2) M^2} \right\}^{(1-n)/(n-2)} . \quad (18)$$

The relation between the density perturbation and the quadrupole of the temperature fluctuation of cosmic microwave background (CMB) $\sqrt{\langle a_T^2 \rangle}$ is given by \footnote{From the data on anisotropy of the CMB, $\sqrt{\langle a_T^2 \rangle} \approx 6 \times 10^{-6}$, observed by COBE, we derive a constraint

$$\left( \frac{\delta T}{T} \right) \approx \sqrt{\frac{\langle a_T^2 \rangle}{4\pi}} \approx 6 \times 10^{-6} . \quad (20)$$

$$\frac{\tilde{\lambda} \tilde{v}^3}{10 \sqrt{3\pi n M^3}} \left\{ \frac{\tilde{v}^2}{2 N n (n - 2) M^2} \right\}^{(1-n)/(n-2)} \bigg|_{N \sim 40} \approx 2 \times 10^{-5} . \quad (21)$$}

$$\sqrt{\langle a_T^2 \rangle} = \sqrt{\left( \frac{\delta \rho}{\rho} \right)_{N \approx 40}} . \quad (19)$$

From the data on anisotropy of the CMB,

$$\left( \frac{\delta T}{T} \right) \approx \sqrt{\frac{\langle a_T^2 \rangle}{4\pi}} \approx 6 \times 10^{-6} , \quad (20)$$

we derive a constraint

$$\left( \frac{\delta T}{T} \right) \approx \sqrt{\frac{\langle a_T^2 \rangle}{4\pi}} \approx 6 \times 10^{-6} . \quad (20)$$

We are now at the point to discuss the SUSY breaking in the present model. Interesting is that the superpotential $W(\phi)$ does not vanish at the potential minimum $\phi \simeq v$,

$$W(\phi \simeq v) \approx \frac{n}{\sqrt{2(n+1)}} \tilde{\lambda} \tilde{v}^3 . \quad (22)$$

The gravitino mass $m_{3/2}$ is, then, given by

$$m_{3/2} \simeq \frac{e^{(K/2M^2)} W}{M^2} \approx \frac{n}{\sqrt{2(n+1)}} \tilde{\lambda} \tilde{v} \left( \frac{\tilde{v}}{M} \right)^2 . \quad (23)$$
Thus, the inflaton field $\phi$ is regarded as the Polonyi field for producing the gravitino mass \[3\].

From eqs. (21) and (23), we determine $\lambda$ and $v$ for a given sets of $N$ and $m_{3/2}$. The results for $N = 40$ are shown in Table 1. For the obtained $\lambda$ and $v$, we calculate the mass of $\phi$ as

$$m_\phi \simeq \lambda nv.$$  \hspace{1cm} (24)

As seen in Table 1, the inflaton mass $m_\phi$ is predicted as,

$$m_\phi \simeq \left(10^7 - 10^{12}\right) \text{GeV}.$$  \hspace{1cm} (25)

This seems to contradict with the claim \[9\] that the mass of the Polonyi field is always at the gravitino mass scale $m_{3/2} \sim 100 \text{GeV} - 10 \text{TeV}$. However, our result (25) is not inconsistent with their claim, since we have not demanded the cosmological constant to vanish. In fact, we have a non-zero cosmological constant at the inflaton potential minimum,

$$\Lambda_\phi^{\text{cos}} = V(\phi \simeq v) \simeq -3m_{3/2}^2M^2 \sim -(10^{10} - 10^{11}\text{GeV})^4.$$  \hspace{1cm} (26)

Therefore, we need to invoke some mechanism to cancel this negative cosmological constant. The simplest way is to introduce a U(1) gauge multiplet $V(\theta, x)$ in the hidden sector and add a Fayet-Iliopoulos $D$-term\[10, 11, 12\], $\xi D$, which shifts up the vacuum energy density by an amount of

$$\delta \Lambda_\cos^D = \frac{1}{2} \xi^2.$$  \hspace{1cm} (27)

\[6\]In supergravity, the Fayet-Iliopoulos $D$-term can be written as (see Ref.\[13\] for notations)

$$\frac{3}{4} \int d^2 \Theta M^2 \mathcal{E}(\bar{D}D - 8R) \exp \left(-\frac{1}{3} \frac{\xi V}{M^2}\right).$$

Clearly, this term is not invariant under the U(1) gauge transformation $V \rightarrow V + \Lambda + \Lambda^\dagger$. Thus, we assume, here, that this $D$-term is induced by some mechanism for the U(1) breaking. Another solution to this problem may exist if the superpotential has a continuous $R$-symmetry \[11, 12\]. However, this symmetry conflicts with our case, since the superpotential (3) does not have the continuous $R$-symmetry. Therefore, we must also consider that the continuous $R$-symmetry is broken down to our discrete $R$-symmetry by some underlying physics at the Planck scale. A detailed argument on this problem will be given in future communication \[14].
Thus we can always choose $\xi$ so that the total cosmological constant vanish,

$$\Lambda_{\text{cos}} = \Lambda^\phi_{\text{cos}} + \delta \Lambda^D_{\text{cos}} = 0.$$  \hfill (28)

This looks very artificial, but notice that a serious cosmological problem \cite{13} associated with the light Polonyi field is not present due to the relatively large mass $m_\phi$ given in eq.\((24)\). In any case, the presence of non-vanishing cosmological constant $\Lambda^\phi_{\text{cos}}$ does not affect our inflation scenario, since $\Lambda^\phi_{\text{cos}}$ in eq.\((26)\) is always negligible\footnote{Notice that we have not required, in our analysis, the cosmological constant to be negligibly small. The main reason why we have $\Lambda^\phi_{\text{cos}} \ll V(\phi \sim 0)$ is that the vacuum-expectation value $v$ is very small, $(v/M) \sim 10^{-3} - 10^{-2}$.} compared with the inflaton energy density $V(\phi \sim 0)$ in the inflationary epoch, as seen in Fig. 1.

Notice that the SUSY breaking due to the above $D$-term dominates over the $F$-term breaking by the inflaton ($F = D_\phi W$, see eq.\((23)\)). Thus, the physical gravitino field $\psi'_\mu$ is composed mainly of the $\psi_\mu$ and $\lambda$ through the super Higgs mechanism \cite{1} as

$$\psi'_\mu = \psi_\mu + \frac{1}{6} \frac{e^{-(K)}/2M^2}{M} \frac{\langle D \rangle}{\langle W \rangle} \sigma_\mu \bar{\lambda}. \hfill (29)$$

with

$$\langle D \rangle = \xi, \hfill (30)$$

where $\psi_\mu$ and $\lambda$ are the gravitino and the U(1) gaugino fields, respectively (in the two component Weyl representation). This physical gravitino field has a mass term

$$e^{(K)/2M^2} \frac{\langle W \rangle}{M^2} \psi'_\mu \sigma^{\mu\nu} \psi'_\nu + h.c. \hfill (31)$$

In the Minkowski space-time this gravitino mass becomes a physical pole mass of the gravitino propagator.\footnote{Notice that the presence of the gravitino mass given in eq.\((23)\) does not mean the SUSY breaking in the anti-de Sitter space-time.} Furthermore, if we use the minimum Kähler potential for quark and lepton fields, the soft-SUSY breaking masses for squarks and sleptons come also from $W$ in eq.\((22)\).

Since the inflaton $\phi$ couples to the light observable sector very weakly (with strength $\sim (\lambda v^2/M^2)$), the decay width $\Gamma_\phi$ is very small as

$$\Gamma_\phi \sim \left( \frac{\lambda v^2}{M^2} \right)^2 m_\phi. \hfill (32)$$
For the $n \leq 5$ case, this leads to a reheating temperature $T_R$,

$$T_R = O(10^{-2} - 10)\text{GeV},$$

(33)

which is too low to create enough baryon-number asymmetry of the universe. In the case of $n \geq 6$, the reheating temperature is $O(100)\text{GeV}$. But as we will see later, models with $n \geq 5$ have a physical cut-off smaller than the Planck scale. Therefore, if one requires the cut-off scale to be larger than the Planck scale, one should take $n=3$ or 4 and, hence a new interaction is necessary to get a sufficiently high reheating temperature for baryogenesis.

To have a faster decay of $\phi$, we introduce a new singlet chiral supermultiplet $N(x, \theta)$ with a half $Z_n$ charge of $\phi$, that is

$$N(x, \theta) \rightarrow e^{-i\alpha/2}N(x, e^{i\alpha/2}\theta).$$

(34)

Then, we have a new interaction term in Kähler potential

$$K(\phi, \phi^*, N, N^*) = \frac{g}{M}\phi^*NN + h.c.,$$

(35)

and $N$ can have a mass term

$$W(N) = \frac{m_N}{2}NN.$$ 

(36)

Provided $2m_N < m_\phi$, we have a much faster $\phi$ decay mode, $\phi \rightarrow NN$, with decay rate

$$\Gamma_{\phi \rightarrow NN} = \frac{g^2 m^3_\phi}{8\pi M^2} \left(1 - \frac{4m^2_N}{m^2_\phi}\right)^2.$$ 

(37)

With this decay width, we estimate the reheating temperature $T_R \sim (1 - 10^5)\text{TeV}$ for $n=4$–10 (see Table 1). Only for the case of $n=3$, the reheating temperature seems to be too low ($T_R \sim (1 - 100)\text{GeV}$) for the baryogenesis as discussed below.

It is a straightforward task to incorporate the Fukugita-Yanagida mechanism [18] for baryogenesis in the present model, identifying the singlet $N$ with three families of

---

9In the recent article Fischler have derived a new constraint on the reheating temperature $T_R \lesssim (10^2 - 10^5)\text{GeV}$ [14] to solve the gravitino problem [17]. If one adopts this constraint, there are left consistent only the $n = 3$, 4 and 5 cases. However, it is not clear to us if this new constraint on $T_R$ is relevant.
the right-handed neutrinos. Taking $Z_n$ charges for all quarks $Q$ and leptons $L$ to be the same as that of $N$ and assuming Higgs multiplets $H$ and $H$ have zero-$Z_n$ charges, we find that the $N$'s can have the standard Yukawa couplings
\[ W_{\text{Yukawa}} = g_{ij} L_i N_j H, \]
where $i$ and $j$ represent the family indices. The decay $N \to LH$ produces a lepton-number asymmetry if $g_{ij}$ has a CP violating phase. The produced lepton-number asymmetry can be converted to the baryon-number asymmetry through the anomalous electroweak processes at high temperature $T \gtrsim O(100\text{GeV})$. Therefore, the reheating temperature should be higher than $O(100\text{GeV})$. As seen in Table 1 this condition is satisfied in models with $n \geq 4$, while in the $n = 3$ case the gravitino mass less than $O(10\text{TeV})$ is unlikely.

4. Conclusion

Some comments are in order.

10 Through the seesaw mechanism, the light neutrino mass $m_\nu$ is given by $m_\nu \simeq m_D^2/m_N$ with $m_D$ being the Dirac mass of neutrino. If all $m_{N_i} \sim 10^{9}\text{GeV}$ the MSW solution to the solar $\nu$ problem suggests $m_D \simeq 0.1 - 0.01\text{GeV}$.

11 The invariant mass term $W = \mu H H$ is forbidden by the $Z_n$-symmetry. If one considers the exact $Z_n$-symmetry one needs a Yukawa interaction $W = h\phi H$ to produce the invariant mass. To give a week-scale mass for $H$ and $\bar{H}$ one must choose $h$ very small, $h \sim 10^{-14} - 10^{-12}$.

12 If $2m_{N_1} < m_\phi < 2m_{N_{2,3}}$, only the $N_1$ is responsible for the baryogenesis. Suppose a hierarchy in the Yukawa coupling, $|g_{33}| \gg \text{other } |g_{ij}|$, one obtains the lepton-number asymmetry through the $N \to LH$ and $L^* H^*$ decay processes as
\[ \Delta L/s \sim 10^{-5} \frac{1}{8\pi} |g_{33}|^2 \delta \left( \frac{m_{N_1}}{m_{N_3}} \right), \]
where $10^{-5}$ is a dilution factor due to the entropy production of $N$ decays, and $\delta$ is the CP-violating phase of $g_{ij}$. This lepton-number asymmetry is converted to the baryon-number asymmetry and one gets
\[ \Delta B/s \simeq \frac{4n_f + 4}{12n_f + 13} \Delta L/s, \]
with $n_f$ being the number of families ($n_f = 3$).

With this result, one may easily explain the observed baryon-number asymmetry $\Delta B/s \simeq 10^{-10} - 10^{-11}$, taking $g_{33} = O(1)$ and $\delta(m_{N_1}/m_{N_3}) \sim 10^{-4}$. Notice that the $N_1$ decay processes are always out of equilibrium since $m_{N_1} \gg T_R$ and hence we do not have any additional constraint (so-called out-of-equilibrium condition) on $m_{N_1}$.

13 If one uses the Affleck-Dine mechanism for the baryogenesis, the $n = 3$ case is not ruled out.
(i) Since the superpotential \( W(\phi) \) contains a non-renormalizable term \( \frac{\lambda}{v^{n-1}} \phi^{n+1} \), the Born unitarity is violated in the process \( \phi + \phi^* \to (n-1)\phi + (n-1)\phi^* \) above a very high-energy scale. Thus we need a physical cut-off scale \( \Lambda \). By using a simple power counting, we estimate the cut-off scale to be

\[ \Lambda \sim \lambda^{1/(1-n)} v. \]  

(39)

If one imposes \( \Lambda \geq M \) or \( M_{\text{Planck}} \), one finds that only the \( n=3 \) and \( n=4 \) cases are consistent (see Table I).

(ii) The gaugino masses come from the non-minimal kinetic term of gauge multiplet \( W_\alpha \),

\[ \frac{1}{8g^2} \int d^2 \Theta \xi f(\phi)W^\alpha W_\alpha + \text{h.c.}, \]  

(40)

where \( f \) is the kinetic function for the gauge multiplet. Eq.(40) induces the gaugino mass term

\[ \frac{1}{4} \left( \frac{\partial f}{\partial \phi} D\phi W \right) \tilde{g}^\alpha \tilde{g}_\alpha + \text{h.c.} \]  

(41)

Suppose \( f = \kappa \phi / M \), we get a very small gaugino mass\(^{14}\)

\[ m_{\tilde{g}} \simeq \frac{\kappa \lambda n v^4}{4(n+1)M^3} \sim \begin{cases} 10^{-3}m_{3/2}, & \text{for } n = 4, \\ 10^{-2}m_{3/2}, & \text{for } n = 3, \end{cases} \]  

(42)

with \( \kappa = O(1) \). Therefore, to obtain enough large gaugino masses\(^{15}\) we must take account of radiative corrections from some heavy particles\(^{16}\). This situation is very much similar to in the scenario of dynamical SUSY breaking\(^{27}\). Whether the radiative corrections give rise to sufficiently large masses for the gauginos depends on detailed physics at the high-energy (perhaps the Planck) scale. Therefore, it may be safe to conclude that the gaugino masses are much smaller than those of the squarks and sleptons unless a huge number of heavy particles exists at the high-energy scale.

In conclusion, we have shown that a discrete \( R \)-symmetry \( Z_n \) naturally produces a very flat potential for the inflation. Particularly for the case of \( n = 4 \), the effective

\(^{14}\)We thanks M.Yamaguchi for pointing out this problem.

\(^{15}\)For \( n=3 \) and \( m_{3/2}=10\text{TeV} \), the predicted masses for gauginos in eq.(42) are not excluded by the present experiment\(^{24}\). 

\(^{16}\)The gravitino loop may also generate gaugino masses as pointed out in Ref. \(^{11}\).
cut-off scale has turned out to be at the Planck scale. Therefore, it will be interesting to related our discrete $R$-symmetry to some physics at the Planck scale. A possible candidate is the superstring theory, since it is well known that various discrete (non-$R$ and $R$-)symmetries arise from compactified manifolds [28] of 10-dimensional space-time. Further investigation on this intriguing possibility will be given elsewhere.

Acknowledgments

We thank M. Yamaguchi for useful discussions.
References

[1] E. Cremmer, S. Ferrara, L. Grardello and A. van Proeyen, Nucl. Phys. B212 (1983), 413.

[2] R. Holman, P. Ramond and G.G. Ross, Phys. Lett. B137 (1984), 343.
   B.A. Ovrut and P.J. Steinhardt, Phys. Rev. D30 (1984), 2061.
   A.B. Goncharov and A. Linde, Phys. Lett. B139 (1984), 27.

[3] J. Polonyi, Budapest preprint KFK-1977-93 (unpublished).

[4] G. Gelmini, G. Kounnas and D.V. Nanopoulos, Nucl. Phys. B250 (1985), 177.
   A. Linde, Particle Physics and Inflationary Cosmology, (Harwood, 1990).
   H. Murayama, H. Suzuki, T. Yanagida and J. Yokoyama, Yukawa Inst. Uji preprint
   YITP-U-93-29 (1993).

[5] For review, E.W. Kolb and M.S. Turner, The Early Universe, (Addison-Wesley, 1990).

[6] S.W. Hawking, Phys. Lett. B115 (1982), 295.
   A.A. Starobinsky, Phys. Lett. B117 (1982), 175.
   A.H. Guth and S.-Y. Pi, Phys. Rev. Lett. 49 (1982), 1110.

[7] P.J.E. Peebles, Ap. J. 263 (1982), L1.

[8] G.F. Smoot et al., Ap. J. 396 (1992), L1.

[9] T. Banks, D.B. Kaplan and A.E. Nelson, Phys. Rev. D49 (1994), 779.
   B. de Carlos, J.A. Casas, F. Quevedo and E. Roulet, Phys. Lett. B318 (1993),
   447.

[10] P. Fayet and J. Iliopoulos, Phys. Lett. B51 (1974), 461.

[11] R. Barbieri, S. Ferrara, D.V. Nanopoulos and K.S. Stille, Phys. Lett. B113 (1982),
    219.

[12] J.A. Bagger, Nucl. Phys. B211 (1983), 302.

[13] J. Wess and J. Bagger, Supersymmetry and Supergravity, (Princeton University
    Press, 1992).
[14] T. Moroi and T. Yanagida, in preparation.

[15] G.D. Coughlan, N. Fischler, E.W. Kolb, S. Raby and G.G. Ross, Phys. Lett. B131 (1983), 59.

[16] W. Fischler, Univ. of Texas preprint UTTG-04-94, (1994).

[17] S. Weinberg, Phys. Rev. Lett. 48 (1982), 1303.

[18] M. Fukugita and T. Yanagida, Phys. Lett. B174 (1986), 45.

[19] T. Yanagida, Proc. of the Workshop on the Unified Theory and Baryon Number in the Universe, ed. by O. Sawada and A. Sugamoto (KEK report 79-18, 1979) 95. M. Gell-Mann, P. Ramond and R. Slansky, in Supergarvity, ed. by P. van Nieuwenhuizen and D.Z. Freedman (North Holland, Amsterdam, 1979).

[20] L. Wolfenstein, Phys. Rev. D17 (1978), 2369. S.P. Mikheyev and A.Yu. Smirnov, Yad. Fiz. 42 (1985), 1441 [Sov. J. Nucl. Phys. 42 (1985), 913].

[21] V.A. Kuzmin, V.A. Rubakov and M.E. Shaposhnikov, Phys. Lett. B155 (1985), 36.

[22] J.A. Harvey and M.S. Terner, Phys. Rev. D42 (1990), 3344.

[23] I. Affleck and M. Dine, Nucl. Phys. B249 (1985), 361.

[24] Particle Data Group, Phys. Rev. D45 Part II (1992), 1.

[25] J. Ellis, L.E. Ibanez and G. Ross, Nucl. Phys. B221 (1983), 29.

[26] J. Hisano, H. Murayama and T. Goto, Phys. Rev. D49 (1994), 1446.

[27] I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. B256 (1985), 557.

[28] B.R. Greene, K. Kirklin, P. Miron and G.G.Ross, Nucl. Phys. B278 (1986), 667. D. Gepner, Nucl. Phys. B311 (1988), 191. N. Ganoulis, G. Lazarides and Q. Shafi, Nucl. Phys. B323 (1989), 374.
Figure caption

Fig. 1

Inflaton potentials $V(\phi)$ along the real axis $\text{Re}(\phi)$ are shown for the cases (a) $n = 3, v = 1.9 \times 10^{16}\text{GeV}$, (b) $n = 4, v = 2.5 \times 10^{15}\text{GeV}$, (c) $n = 5, v = 9.0 \times 10^{14}\text{GeV}$. Notice that these sets of parameters are given in Table I with $m_{3/2}=1\text{TeV}$ and the shape of the normalized potentials in this figure are independent of the parameter $\lambda$. 


\begin{table}
\begin{tabular}{lcccccc}
\hline
\textbf{$m_{3/2}$=100GeV} \\
\hline
$n$ & $\lambda$ & $v$[GeV] & $\Lambda$[GeV] & $m_\phi$[GeV] & $T_R$[GeV] \\
3 & $6.5 \times 10^{-10}$ & $1.1 \times 10^{16}$ & $1.6 \times 10^{25}$ & $2.1 \times 10^{7}$ & 3.8 \\
4 & $5.0 \times 10^{-7}$ & $1.1 \times 10^{15}$ & $1.6 \times 10^{18}$ & $2.3 \times 10^{9}$ & $4.4 \times 10^{3}$ \\
5 & $1.3 \times 10^{-5}$ & $3.8 \times 10^{14}$ & $1.6 \times 10^{16}$ & $2.5 \times 10^{10}$ & $1.6 \times 10^{5}$ \\
6 & $8.9 \times 10^{-5}$ & $2.0 \times 10^{14}$ & $2.0 \times 10^{15}$ & $1.1 \times 10^{11}$ & $1.4 \times 10^{6}$ \\
7 & $3.1 \times 10^{-4}$ & $1.3 \times 10^{14}$ & $6.6 \times 10^{14}$ & $2.8 \times 10^{11}$ & $5.9 \times 10^{6}$ \\
8 & $7.2 \times 10^{-4}$ & $9.7 \times 10^{13}$ & $3.2 \times 10^{14}$ & $5.6 \times 10^{11}$ & $1.7 \times 10^{7}$ \\
9 & $1.3 \times 10^{-3}$ & $7.9 \times 10^{13}$ & $2.0 \times 10^{14}$ & $9.5 \times 10^{11}$ & $3.7 \times 10^{7}$ \\
10 & $2.1 \times 10^{-3}$ & $6.7 \times 10^{13}$ & $1.5 \times 10^{14}$ & $1.4 \times 10^{12}$ & $6.9 \times 10^{7}$ \\
\hline
\textbf{$m_{3/2}$=1TeV} \\
\hline
$n$ & $\lambda$ & $v$[GeV] & $\Lambda$[GeV] & $m_\phi$[GeV] & $T_R$[GeV] \\
3 & $1.2 \times 10^{-9}$ & $1.9 \times 10^{16}$ & $1.6 \times 10^{25}$ & $6.6 \times 10^{7}$ & $2.1 \times 10^{4}$ \\
4 & $5.0 \times 10^{-7}$ & $2.5 \times 10^{15}$ & $3.5 \times 10^{18}$ & $4.9 \times 10^{9}$ & $1.4 \times 10^{4}$ \\
5 & $9.9 \times 10^{-6}$ & $9.0 \times 10^{14}$ & $4.2 \times 10^{16}$ & $4.4 \times 10^{10}$ & $3.7 \times 10^{5}$ \\
6 & $5.6 \times 10^{-5}$ & $5.0 \times 10^{14}$ & $5.7 \times 10^{15}$ & $1.7 \times 10^{11}$ & $2.7 \times 10^{6}$ \\
7 & $1.7 \times 10^{-4}$ & $3.4 \times 10^{14}$ & $1.9 \times 10^{15}$ & $4.1 \times 10^{11}$ & $1.1 \times 10^{7}$ \\
8 & $3.7 \times 10^{-4}$ & $2.6 \times 10^{14}$ & $9.7 \times 10^{14}$ & $7.8 \times 10^{11}$ & $2.8 \times 10^{7}$ \\
9 & $6.5 \times 10^{-4}$ & $2.2 \times 10^{14}$ & $6.2 \times 10^{14}$ & $1.3 \times 10^{12}$ & $5.7 \times 10^{7}$ \\
10 & $9.9 \times 10^{-4}$ & $1.9 \times 10^{14}$ & $4.4 \times 10^{14}$ & $1.9 \times 10^{12}$ & $1.0 \times 10^{8}$ \\
\hline
\textbf{$m_{3/2}$=10TeV} \\
\hline
$n$ & $\lambda$ & $v$[GeV] & $\Lambda$[GeV] & $m_\phi$[GeV] & $T_R$[GeV] \\
3 & $2.1 \times 10^{-9}$ & $3.4 \times 10^{16}$ & $1.6 \times 10^{25}$ & $2.1 \times 10^{8}$ & $1.2 \times 10^{2}$ \\
4 & $5.0 \times 10^{-7}$ & $5.3 \times 10^{15}$ & $7.5 \times 10^{18}$ & $1.1 \times 10^{10}$ & $4.4 \times 10^{4}$ \\
5 & $7.4 \times 10^{-6}$ & $2.1 \times 10^{15}$ & $1.1 \times 10^{17}$ & $7.9 \times 10^{10}$ & $8.8 \times 10^{5}$ \\
6 & $3.5 \times 10^{-5}$ & $1.2 \times 10^{15}$ & $1.6 \times 10^{16}$ & $2.7 \times 10^{11}$ & $5.5 \times 10^{6}$ \\
7 & $9.7 \times 10^{-5}$ & $8.9 \times 10^{14}$ & $5.6 \times 10^{15}$ & $6.0 \times 10^{11}$ & $1.9 \times 10^{7}$ \\
8 & $1.9 \times 10^{-4}$ & $7.0 \times 10^{14}$ & $2.9 \times 10^{15}$ & $1.1 \times 10^{12}$ & $4.5 \times 10^{7}$ \\
9 & $3.2 \times 10^{-4}$ & $5.9 \times 10^{14}$ & $1.9 \times 10^{15}$ & $1.7 \times 10^{12}$ & $8.8 \times 10^{7}$ \\
10 & $4.6 \times 10^{-4}$ & $5.2 \times 10^{14}$ & $1.4 \times 10^{15}$ & $2.4 \times 10^{12}$ & $1.5 \times 10^{8}$ \\
\hline
\end{tabular}
\end{table}

Table 1: For gravitino masses $m_{3/2}$=100GeV, 1 and 10TeV, coupling constants $\lambda$ and vacuum-expectation values $v$ are calculated with $T/T = 6.0 \times 10^{-6}$ fixed. We have taken $n=3$–10, and physical cut-off scales $\Lambda$, masses of $\phi$ field $m_\phi$ and the reheating temperatures $T_R$ are also shown for each $n$. 

15
\[
\frac{\text{Re}(\phi)}{v} \quad \frac{V(\phi)}{V(\phi=0)}
\]
(a) \(n=3\)
(b) \(n=4\)
(c) \(n=5\)