Quantum computation on cluster states has been proposed in a variety of systems, including linear optics, quantum dots, neutral atoms in optical lattices, and flying atom schemes. To date, experiments have been performed using optical lattices, where the cluster state can be created, but the current lack of individual addressing remains the stumbling block and linear optics, where scalability remains a problem due to the need to generate the initial many-photon state from, for example, high orders of the parameterized down conversion process. On the other hand, there have recently been theoretical and experimental breakthroughs into the possibility of direct coupling of high Q cavities and in achieving strong coupling between the cavity mode and an embedded two-level system. A variety of technologies have been employed, namely fiber coupled micro-toroidal cavities interacting with atoms, arrays of defects in photonic band gap materials (PBGs) and superconducting qubits coupled through microwave stripline resonators. This has prompted proposals for the implementation of optical quantum computing, the production of entangled photons and the realization of Mott insulating and superfluid phases. Here we propose the use of such arrays for the realization of cluster state quantum computation.

**System Description:** We start by describing the system and showing how to construct qubits from the hybrid light-matter excitations (polaritons). For simplicity, we describe the system as a linear chain of coupled cavities doped with two level systems, although this is we describe the system as a linear chain of cavities doped with two level systems, although this is

\[ H_{\text{free}} = \omega_d \sum_{k=1}^{N} a_k^\dagger a_k + \omega_0 \sum_k |e\rangle\langle e|_k \]  

\[ H_{\text{int}} = g \sum_{k=1}^{N} \left( a_k^\dagger |g\rangle_k \langle e|_k + a_k |e\rangle_k \langle g|_k \right) \]  

\[ H_{\text{hop}} = A \sum_{k=1}^{N} \left( a_k^\dagger a_{k+1} + a_k a_{k+1}^\dagger \right) \]  

where \( \omega_d \) and \( A \) are the photon frequencies and hopping rates respectively and \( g \) is the light-atom coupling strength. The \( H_{\text{free}} + H_{\text{int}} \) component of the Hamiltonian can be diagonalized in a basis of combined photonic and atomic excitations, called polaritons (Fig. 1). These polaritons are defined by creation operators \( P_k^{(\pm, n)} = n \pm \langle n \rangle_k \), where the polaritons of the \( k \)th atom-cavity system are given by \( n \pm \langle n \rangle_k = (|g\rangle_k \pm |e\rangle_k) / \sqrt{2} \) with energies \( E_n = n\omega_d \pm g\sqrt{n} \), and \( |n\rangle_k \) denotes the \( n \)-photon Fock state. As has been shown elsewhere, a polaritonic Mott phase exists in this system where a maximum of one excitation per site is allowed. This originates from the repulsion due to the photon blockade effect. In this Mott phase, the system’s Hamiltonian can be written in the interaction picture as

\[ H_I = A \sum_{k=1}^{N} P_k^\dagger P_{k+1} + P_k P_{k+1}^\dagger, \]  

where \( P_k = P_k^{(-1)\dagger} \) (Fig. 1). As double or more occupancy of the sites is prohibited, one can identify \( P_k \) with \( \sigma^x_k = \sigma^x_k + i\sigma^y_k \), where \( \sigma^x_k \) and \( \sigma^y_k \) are the standard Pauli operators. The system’s Hamiltonian then becomes the standard XY model of interacting spin qubits with spin up/down corresponding to the presence/absence of a polariton.

\[ H_I = A \sum_{k=1}^{N} \sigma^x_k \sigma^x_{k+1} + \sigma^y_k \sigma^y_{k+1}. \]  

Some applications of XY spin chains in quantum information processing can thus be implemented in this system.
Cluster state generation: The typical implementation of cluster state quantum computing requires initializing all qubits in a 2D lattice in the $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ state and then performing controlled-phase gates (CP) between all nearest-neighbours. In the present system, we have no direct two-qubit gate and the available interaction is not of the Ising type, which straightforwardly gives controlled-phase gates $\sigma_z$, but an ‘always on’ global Hamiltonian coupling of the XY form. It behooves us to consider how we will proceed with the measurement sequence once the cluster state has been generated without the system continuing to evolve. (The need to disable the evolution is an aspect often neglected when discussing the generation of cluster states, either as the ground states of ‘natural’ Hamiltonians \[28\], or from evolution of the Ising Hamiltonian). This requirement can be realized by combining the system’s natural dynamics with a protocol where some of the available physical qubits are used as gate “mediators” and the rest as the logical qubits. The mediator atoms can be Stark shifted on and off resonance from their cavities through the application of an external field, inhibiting the photon hopping and thereby isolating each logical qubit. The same inhibition of couplings will be used to generate the cluster state. We note here that the error introduced in the step is due to a second-order transition between on-resonance qubits (via a dark-passage through the central off-resonant qubit), which is thus suppressed by a factor of order $A/\Delta$, where $\Delta$ is the detuning of the off-resonant cavity.

Before describing the 4-step global gate sequence to create the cluster state, first observe that for the control phase part is enough to localize chains of 3 qubits, let them evolve for a time $t_0 = \pi/(2\sqrt{2}A)$ and then apply a measurement on the middle ‘mediator’ qubit (in the $\sigma_z$ basis). Depending on the measurement result, $|0\rangle$ or $|1\rangle$, a nonlocal gate is generated between the remaining two qubits, either SWAP, $(\sigma_z \otimes \sigma_z).\text{CP}$ or SWAP.\text{CP} respectively\[29, 30\]. In both cases, the gates in addition to the CP are Clifford operations which can be recorded and taken into account during the measurement-based computation.

Our sequence to generate the cluster state initiates by preparing all qubits in the $|+\rangle$ state through the application of global $\pi/2$ pulse. One quarter of the sites will be used as logical qubits and the rest as “mediators” and “off” qubits interchangeably. All qubits addressed by the gates A to D (Fig. 1) are, by default, “off”, thereby isolating all the qubits. Switching on any one of the four gates thus creates chains of 3 qubits, which we use to enact a CP between pairs of qubits (separated by a mediator qubit, which was previously off). Consecutive use of each of the gates A to D serves to enact a CP gate between a particular qubit and all of its nearest-neighbours, and this happens in parallel across the whole device. This entire sequence is illustrated in Supplementary Video 1.

The measurement sequence is then applied as requested by the cluster state algorithm, utilizing the local accessibility of the sites (in any implementation, the cavity-atom systems are well separated compared to the resolution of the external field used for addressing them) \[13, 14, 16, 17, 18, 19\].

In Fig. 2, we calculate the fidelity of generation of a cluster state on a 3x3 array of cavities. More sophisticated schemes have the potential to further reduce the experimental errors. For example, standard Hamiltonian simulation techniques allow us to negate the second order exchange term due to the off-resonance cavities, simply by repeatedly applying $\sigma_z$ gates to every second on-
resonance triplet throughout the evolution. One might even hope that we could use this coherent effect to enhance the scheme through the use of, for example, optimal control techniques. Most of the errors considered here (cavity leakage, spontaneous emission of the atom, and on-off detuning of qubits) are local effects, introducing local noise, which can ultimately be addressed by fault-tolerant techniques \[31\].

Implementing algorithms: Initial experimental algorithmic implementations with coupled cavities can be expected to utilize the most basic building block of our scheme, a $3 \times 3$ grid of cavities, which allows us to generate a four-qubit cluster state. As with the four-photon cluster state recently used by Walther et al. \[12\], this cluster state would be suitable for demonstrating the preparation of an arbitrary one-qubit state, an entangling gate between two qubits, and even the implementation of Grover’s search algorithm on two qubits \[12\]. For example, by applying the local gates $H \otimes H \otimes \sigma_z \otimes \sigma_z$, where $H$ is the Hadamard rotation, we convert of ‘box’ cluster that the $3 \times 3$ grid prepares into the 1D cluster state of 4 qubits, which is given the interpretation of a single qubit, and measurements on the state yield quantum gates on this single qubit. Moreover, generation of this four qubit cluster state is simpler than generation of an arbitrarily sized cluster state because we only need two control steps instead of four, thereby keeping us even further within the decoherence time of the system.

Perhaps the next important step would then be to demonstrate Shor’s factoring algorithm, the factoring of 15 being the standard demonstration. To implement as a cluster state computation, the six computational qubits \[33\] translate into the requirement of a cluster state that is eleven qubits wide. Hence, we need an array which is 21 cavities wide. The breadth of the cluster state, which corresponds to time in the circuit model, is a quantity that we can trade against the time taken for the computation. At one extreme, we can create the whole cluster state in one go, with the simple set of four steps already outlined, and we benefit from the large degree of parallelism available to us. This requires a 2D grid of cavities of size $21 \times 31$ \[34\]. At the other extreme, a grid of $21 \times 3$ cavities suffices. In this case, one starts with the $11 \times 2$ cluster state, and performs one time step of measurement (i.e. measure the 11 qubits in one column). The result remains in the other column. We then repeat the cluster state generation process, reinitialising the measured qubits in the cluster state, and performing the next time step (Fig. 3). This requires 156 consecutive entangling steps, but the reinitialising of the cluster state after measurement eliminates the effect of decoherence over this timescale. Any combination between these two extremes is also possible, and is a necessary property of any scalable implementation of cluster state computation for the sake of preventing decoherence.

Once initial cluster state experiments have been performed, it simply becomes a question of how many cavities one can reasonably couple together. Alternatively, since the two-qubit gate that we can generate is entangling (and hence universal for quantum computation), we can also consider using it directly to implement the circuit model of computation. This has a much smaller overhead of qubits, but instead requires much higher quality cavities. For example, to factor 15 we would only need a $5 \times 3$ grid of cavities to give us six computational qubits. However, we would need approximately 15 consecutive entangling steps (we have attempted to minimise this number by allowing as many of the gates to be applied in parallel as possible, and by optimising the initial labelling of each qubit), hence requiring a time of order $15\pi/(\sqrt{2A})$. Hence, to reduce the effect of dissipative decay, we require an order of magnitude improvement in the decoherence properties of the qubits to compensate...
for the increased running time.

**Experimental implementations:** As previously mentioned, there are three primary candidate technologies: fiber coupled micro-toroidal cavities [14, 15], arrays of defects in PBGs [16, 17, 18] and superconducting qubits coupled through microwave stripline resonators [19]. In order to achieve the required limit of no more than one excitation per site [22], the ratio between the internal atom-photon coupling and the hopping of photons down the chain should be of the order of $g/A \sim 10^{-2} \sim 10^{-1}$ ($A$ can be tuned while fabricating the array by adjusting the distance between the cavities and $g$ depends on the type of the dopant). In addition, the cavity/atomic frequencies to internal coupling ratio should be $\omega_d, \omega_0 \sim 10^8 g, 10^9 g$ and the losses should also be small, $g/\max(\kappa, \gamma) \sim 10^3$, where $\kappa$ and $\gamma$ are cavity and atom/other qubit decay rates. The polaritonic states under consideration are essentially unaffected by decay for a time $10^{-1} / A$ (10ns for the toroidal case and 100ns for microwave stripline resonators). While the decay time of $10^{-1} / A$ may seem uncomfortably close to the preparation time for a cluster state, $\sqrt{2\pi}/A$, the previously described technique (Fig. 8 of continuously reforming the cluster state and connecting it to the output of the previous stage allows a continuous computation that exceeds the decay time for an individual cavity. The required parameter values are currently on the verge of being realised in both toroidal microring systems with atoms and stripline microwave resonators connected to superconducting qubits, but further progress is needed. Arrays of defects in PBGs remain one or two orders of magnitude away, but recent developments, and the integrability of these devices with optoelectronics, make this technology very promising as well. In all implementations the cavity systems are well separated by many times the corresponding wavelength of any local field that needs to be applied in the system for the measurement process.

**Conclusions:** In this paper, we have shown how universal quantum computation could be realized in a coupled array of individually addressable atom-cavity systems, where the qubits are given by mixed light-matter excitations in each cavity site. While single-qubit operations can be locally achieved, the only available interaction between qubits is due to the natural system Hamiltonian. We show how to manipulate this to give a controlled-phase gate between pairs of qubits. This allows computation either using the circuit model, or a measurement-based computation, the latter being most suited to reducing experimental errors. We have discussed possible architectures for implementing these ideas using photonic crystals, toroidal microcavities and superconducting qubits and point out their feasibility and scalability with current or near-future technology.

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[1] Raussendorf, R. & Briegel, H. J., *Phys. Rev. Lett.* 86, 5188 (2001).
[2] Nielsen, M. A., *Phys. Rev. Lett.* 93, 040503 (2004).
[3] Browne, D. E. & Rudolph, T., *Phys. Rev. Lett.* 95, 010501 (2005).
[4] Barrett, S. D. & Kok, P., *Phys. Rev. A* 71, 060310(R) (2005).
[5] Lim, Y., Beige, A. & Kwok, L., *Phys. Rev. Lett.* 95, 030505 (2005).
[6] Benjamin, S. C., Eisert, J. & Stace, T. M., *New J. Phys.* 7, 194 (2005).
[7] Borhani, M. & Loss, D., *Phys. Rev. A* 71, 034308 (2005).
[8] A. Kay, J. K. Pachos & C. S. Adams, *Phys. Rev. A* 73, 022310 (2006).
[9] Blythe P.J. & Varcoe B.T. H., *New J. of Phys.* 8, 231 (2006).
[10] Schön C. et al., *Phys. Rev. Lett.* 11, 110503 (2005).
[11] Mandel, O. et al., *Nature* 425, 937 (2003).
[12] Walther, P. et al., *Nature* 434, 169 (2005).
[13] Pan, J.-W. et al., *Nature Physics* 3, 91 (2007).
[14] Armani, D. K., Kimpenberg, T. J., Spillane, S. M. & Vahala, K. J., *Nature* 421, 925 (2003).
[15] Aoki, T. et al., *Nature* 443, 671 (2006).
[16] Hattice, A., & Vuckovic, J., *Appl. Phys. Lett.* 84, 191 (2004).
[17] Song, B.-S., Noda, S., Asano, T. & Akahane, Y., *Nat. Mater.* 4, 207-210 (2005).
[18] Badolato, A. et al., *Science* 308, 1158-1161 (2005).
[19] Wallraff, A. et al., *Nature* 431, 162 (2004).
[20] Angelakis, D. G., Santos, M. F., Yannopapas, V. & Ekert, A., *Phys. Lett. A* 362, 377 (2007).
[21] Angelakis, D. G. & Bose, S., *J. Opt. Soc. Am. B* 24, 266 (2007).
[22] Angelakis, D. G., Santos, M. F. & Bose, S., Photon blockade induced mott transitions and XY spin models in coupled cavity arrays (2006), quant-ph/0606159.
[23] Hartmann, M. J., Brandao, F. G. S. L. & Plenio, M. B., *Nat. Phys.* 2, 849 (2006).
[24] Greentree, A. D., Tahan, C., Cole, J. H. & Hollenberg, L. C. L., *Nat. Phys.* 2, 856 (2006).
[25] Imamoglu, A., Schmidt, H., Woods, G. & Deutsch, M., *Phys. Rev. Lett.* 79, 1467-1470 (1997).
[26] Birnbaum, K. M. et al., *Nature* 436, 87 (2005).
[27] Bose, S., *Phys. Rev. Lett.* 91, 207901 (2003).
[28] Bartlett, S. D. & Rudolph, T., *Phys. Rev. A* 74, 040302(R) (2006).
[29] Yung, M.-H., Leung, D. W. & Bose, S., *Quant. Infor. and Comp.* 4, 174 (2004).
[30] Albanese, C., Christandl, M., Datta, N. & Ekert, A., *Phys. Rev. Lett.* 93, 230502 (2004).
[31] Raussendorf, R., Harrington, J. & Goyal, K., *New J. Phys.* 9, 199 (2007).
[32] Raussendorf, R., Harrington, J. & Goyal, K., *New J. Phys.* 9, 199 (2007).
[33] Kieling, K., Gross, D. & Eisert, J., *J. Opt. Soc. Am. B* 24(2), 184 (2007).
[34] Lieven, M. K. et al., *Nature* 414, 883 (2001).
a nearest-neighbor, 2-qubit gate algorithm. Hence, the
possibility for some small degree of optimisation in the
number of qubits remains.