Design-ballistic analysis of a mission for active removal of space debris from geostationary orbit

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Abstract—The paper considers the service spacecraft (SSC) with electric propulsion system (EPS) to approach the objects of space debris (OSD) and its transportation into a disposal orbit. It suggests the SSC operation algorithm and different ways to ensure its implementation. The paper also provides the equations of SSC motion at optimal control, gives design-ballistic analysis alongside with the calculation of the required resources for the SSC operation.

Keywords—spacecraft, space debris, geostationary orbit, service spacecraft, optimal control, fuel budget.

I. INTRODUCTION

The prevention of near-space pollution with space debris is one of the most relevant tasks in the attempt to ensure safe access of space flights in the long-term perspective. The exhaust blocks of launch vehicles, now-defunct spacecrafts (SC), dropped elements of launch vehicles typical for the majority of space launches become the objects of space debris (OSD). The geostationary orbit (GSO) is one of the near-earth orbits mostly populated with SC and OSD.

To prevent the GSO pollution, the international rules and national standards imply some measures, including the deorbiting of launch vehicles and worked-out SC from the GSO vicinity. Unfortunately, due to technical reasons, the deorbiting of SC and launch vehicles is not always possible. During the last 50 years of GSO utilization, a large number of OSD has accumulated and their deorbiting is only possible through specially developed technical means, for example the service spacecrafts (SSC), which approach the OSD, ensure its capture and transportation.

According to the Interagency Agreement on the Prevention of Space Pollution, the protected GSO area is the area limited by altitude corresponding to ±200 km from GSO altitude and by width corresponding to ±15° [1, 2]. Taking into account the long-term evolution of an orbit under the influence of natural perturbation, the near-circular orbit located 300 km higher than the GSO is usually considered a safe disposal orbit.

The paper considers the SSC equipped with electric propulsion system (EPS) to approach the OSD and ensure its transportation to the disposal orbit. It gives preliminary design-ballistic analysis to assess the SSC with regard to EPS time and fuel consumption and hence to the ability to perform major dynamic operations and opportunities of consecutive transportation of a number of OSD from the GSO vicinity into a disposal orbit.

II. SSC FUNCTIONAL SCHEME AND SOURCE DATA

The following SSC functional scheme is considered:

1. SSC is launched into a near-circular parking orbit, which altitude is 300 km higher than the GSO altitude, and the inclination and longitude of ascending node are close to inclination and longitude of the first OSD ascending node, which is to be taken to the disposal orbit. Measuring tools of the SSC ground complex software (GCS) are used at this stage for attitude sensing of the SSC in the orbit.

2. After some time necessary to ensure the SSC phasing with OSD, the SSC flies towards the OSD using the EPS. Besides, the EPS is used to approach the SSC with the OSD up to several kilometers, thus defining the mutual position of the SSC and the OSD by the integrated data of the SSC GCS and the near-space tracking system (for example, automated system to prevent hazardous situations in the near-space).

3. SSC monopropellant hydrazine engines are used on a close maneuvering site and during the OSD capture with the SSC specialized means, and the coordinated referencing is ensured through the range equipment, which each SSC shall be equipped with (a radio range finder or a lidar).

4. After the OSD is captured, the SSC transfers from the OSD into the near-circular disposal orbit, which altitude is 300 km higher the GSO. The EPS ensures transfer to the orbit, while the SSC OSD controls the SSC GCS trajectory.

5. In the disposal orbit the OSD detaches from the SSC, which, in turn, via the monopropellant hydrazine engines is taken to safe distance several kilometers away from the OSD thus excluding their collision. The distance between the SSC and the OSD is controlled by the SSC GCS.

6. SSC repeats operations starting from item 2 with the next OSD until the depletion of fuel or onboard equipment of the SSC.

For design estimates it is assumed that each OSD represents a worked-out SC not transferred by its propulsion system into the disposal orbit. The mass of each OSD is taken to be equal to 2,000 kg.
The original SSC mass is taken to be equal to 1,000 kg. For far rendezvous of the SSC with the OSD, the EPS SC consisting of one SPD-100 electric propulsion engine is used. The EPS thrust is taken to be equal to 83 mN, the specific impulse – 1,520 s.

The approach and capture of each OSD are ensured via monopropellant hydrazine engines, which specific impulse is taken to be equal to 220 s and the required reserves of delta-V are evaluated at 10 m/s.

III. EVOLUTION OF OSD ORBITS IN THE GSO VICINITY AND ASSESSMENT OF OSD ORBITAL PARAMETERS

The analysis of long-term evolution of the OSD in the GSO vicinity under the influence of major perturbing factors (gravitational attraction of moon and sun, non-centrality of the earth’s gravitational field, light pressure) states that when the OSD is formed on the GSO its orbit remains near-circular and the period is close to a star day [3]. The major semi-axis of the OSD orbit deviates to not more than ±20 km from the GSO altitude with the period slightly exceeding 2 years. Long-period perturbations lead to eccentricity change in the range from 0.00002 … 0.00035 to 0.0015 … 0.0023 over the one-period perturbations.

The longitude of ascending node (~ 53.4 years) the inclination of the orbit plane towards the GSO vicinity under the influence of major perturbing factors (gravitational attraction of moon and sun, non-centrality of the earth’s gravitational field, light pressure) states that when the OSD is formed on the GSO its orbit remains near-circular and the period is close to a star day [3]. The major semi-axis of the OSD orbit deviates to not more than ±20 km from the GSO altitude with the period slightly exceeding 2 years. Long-period perturbations lead to eccentricity change in the range from 0.00002 … 0.00035 to 0.0015 … 0.0023 over the one-period perturbations.

Thus, let us consider the OSD orbits with major semi-axis 42164±20 km, maximum eccentricity 0.0023, mean eccentricity 0.001, inclination in the range from 0 to 15° and longitude of ascending node from minus 90 to plus 90°. At the same time it should be assumed that the angle between the orbit planes of consistently transported OSD does not exceed 2°.

IV. CALCULATION OF FLIGHT TRAJECTORIES OF SERVICE SC FROM THE WAITING ORBIT TO THE OSD

As a rule, the trajectories of interorbital flights of the EPS-based SC are characterized by a big range angle. To increase the calculation velocity and accuracy of numerical integration of such trajectories it is suggested to use differential equations of the SC disturbance motion written in osculating orbital elements. From a variety of options regarding the set of orbital elements it is advisable to use equinoctial orbital elements having no features in the vicinity of circular and equatorial orbits, in particular, in the GSO vicinity.

The undisturbed motion equations of EPS-based SC in equinoctial elements in inertial earth centered equatorial system J2000 are as follows [4]:

\[
\begin{align*}
\frac{d\xi}{dt} &= \Omega, \\
\frac{di}{dt} &= \frac{1}{m} \cos \psi e, \\
\frac{di}{dt} &= \frac{1}{m} \cos \psi e, \\
\frac{d\psi}{dt} &= \frac{1}{m} \sin \psi e, \\
\frac{d\nu}{dt} &= \frac{1}{m} \nu, \\
\frac{d\omega}{dt} &= \frac{1}{m} \omega, \\
\frac{d\theta}{dt} &= \frac{1}{m} \theta, \\
\frac{d\varphi}{dt} &= \frac{1}{m} \varphi, \\
\end{align*}
\]

where \( \varphi \) – pitch angle (angle between a thrust vector projection to the plane of the SC osculating orbit and transversal direction), \( \psi \) – yaw angle (angle between a thrust vector and a plane of the SC osculating orbit), \( \mu = \sqrt{\frac{\mu}{2}} \)

\( e_x = e \cos(\Omega + \omega), \quad e_y = e \sin(\Omega + \omega), \quad i_x = \tan \frac{i}{2} \cos \Omega, \quad i_y = \tan \frac{i}{2} \sin \Omega \) and \( F = \nu + \omega + \Omega \) – equinoctial elements, \( p \) – focal parameter, \( e \) – eccentricity, \( \omega \) – argument of perigee, \( i \) – inclination, \( \Omega \) – longitude of ascending node, \( \nu \) – true anomaly, \( \xi = 1 + e_x \cos F + e_y \sin F \cdot i_x \sin F - i_y \cos F \cdot \phi = 1 + i_x^2 + i_y^2 \).

Fig. 1. Inclination vector hodograph in the interval of 53.4 years at passive movement of the OSD located on the GSO at the initial time.
Projections of the EPS jet acceleration to unit vectors of orbital coordinate system are as follows:

\[ a_r = \frac{\partial P}{m} \cos \varphi \cos \psi, \quad a_\varphi = \frac{\partial P}{m} \sin \varphi \cos \psi, \quad a_\psi = \frac{\partial P}{m} \sin \psi \]  

(2)

where \( a_r, a_\varphi, a_\psi \) are respectively transversal, radial and binormal projections of jet acceleration.

There is a need to transfer SC from the initial orbit \( h=h_0, e=e_0, e^\prime=e_0, i=i_0, \varphi=i_0 \) (3) to the target orbit \( h=h_f, e=e_f, e^\prime=e_f, i=i_f, \varphi=i_f \) (4) in a minimum time \( T \).

The problem of minimizing the functional is thus considered

\[ J = \int_0^T dt \rightarrow \min \]  

(5)

The maximum principle is used to solve the problem (1), (3)-(5).

The Hamiltonian of the considered optimal control problem is as follows [4]

\[ H = -1 + \frac{e^2}{h} \dot{p}_r + \frac{\partial P}{m} \frac{h}{\xi} (A_r \cos \varphi \cos \psi + A_\varphi \sin \varphi \cos \psi + A_\psi \sin \psi) - \frac{\partial P}{m} \frac{h}{\xi} \frac{\partial P}{m} \]  

(6)

where

\[ A_r = h p_\varphi + \left[ (\xi + 1) \cos F + e_\varphi \right] p_\psi + \left[ (\xi + 1) \sin F + e_\psi \right] p_\varphi, \]

\[ A_\varphi = \xi (\sin F \cdot p_\varphi - \cos F \cdot p_\psi), \]

\[ A_\psi = \xi (\cos F \cdot p_\varphi + \sin F \cdot p_\psi) + \frac{1}{2} \xi (\cos F \cdot p_\varphi + \sin F \cdot p_\psi) + e \cdot p_r, \]

and \( p_r, p_\varphi, p_\psi, p_\varphi, p_\psi, p_\varphi, p_\psi, p_\varphi, p_\psi \) - variables conjugate to phase coordinates, \( h, e_\varphi, e_\psi, i, i, i, i, i, i \) and \( F \) and \( m \) respectively.

Optimal control \( \delta \dot{r}(t), \delta \dot{\varphi}(t), \delta \varphi(t) \) is defined according to the condition of the Hamiltonian maximum (6):

\[ \cos \varphi = \frac{A_r}{\sqrt{A_r^2 + A_\varphi^2}}, \quad \sin \varphi = \frac{A_\varphi}{\sqrt{A_r^2 + A_\varphi^2}}, \quad \cos \psi = \frac{A_\psi}{\sqrt{A_r^2 + A_\varphi^2}}, \quad \sin \psi = \frac{A_\varphi}{\sqrt{A_r^2 + A_\varphi^2}}, \]

(7)

(8)

\[ \delta = \begin{cases} 1, \psi > 0 \\ 0, \psi \leq 0, \end{cases} \]

(9)

where \( \psi = -\frac{p_\varphi}{c} \frac{h}{m^2} \left( A_r + A_\varphi \right)^{\frac{1}{2}} \) - switching function.

In the problem on flight within a minimum time, taking into account the transversality condition \( p_m(t_f) = 0 \) and nonnegativity constraint \( dp_m/dt \), the condition (9) results in equality

\[ \delta = 1, \quad \dot{\psi} = 1, \]  

(10)

thus, the differential equations for variables \( m \) and \( p_m \) may be neglected using explicit dependence of the SC mass \( m \) on time:

\[ m = m_0 - (T/c) \cdot f. \]  

(11)

The substitution of expressions for optimal control (7), (8) and (10) in (6) taking into account (11) leads to the expression for the optimal Hamiltonian:

\[ H = -1 + \frac{T}{m(t)} \frac{h}{\xi} \left( A_r^2 + A_\varphi^2 + A_\psi^2 \right) + \frac{\xi^2}{h^2} p_r = kT + H_f, \quad (12) \]

where \( k = \frac{1}{m(t)} \frac{h}{\xi}, \quad A = \left( A_r^2 + A_\varphi^2 + A_\psi^2 \right)^{\frac{1}{2}}. \]

The maximum Hamiltonian (12) after true averaging is as follows:

\[ \delta H = \frac{\partial H}{\partial \pi} = \delta T \left[ k \left( A_r \frac{\partial A_r}{\partial \pi} + A_\varphi \frac{\partial A_\varphi}{\partial \pi} + A_\psi \frac{\partial A_\psi}{\partial \pi} \right) A^{-1} \right], \]

(13)

and the motion equations (13) may be rewritten as follows:

\[ \frac{d \xi}{dt} = \frac{\partial H}{\partial \pi} = T \left[ k \left( A_r \frac{\partial A_r}{\partial \pi} + A_\varphi \frac{\partial A_\varphi}{\partial \pi} + A_\psi \frac{\partial A_\psi}{\partial \pi} \right) A^{-1} \right], \]

(14)

(15)

Due to smallness of the problem of jet acceleration to gravitational it is possible to use the averaging of differential optimal motion equations. The averaging allows considerably increasing the step length of numerical integration and thus reducing the computing complexity of the problem. However, the main aspect of applying averaging in the given problem is its regularized role: the averaged system of differential optimal motion equations is numerically more stable than the non-averaged one.

Time-averaging within the orbital period of the SC is used, which corresponds to averaging by mean anomaly of the SC, known in gravitational astronomy [5]. The asymptotic justification of this scheme is well understood: the solution of differential equations averaged following this scheme is a zero term of expansion of the solution of non-averaged equations into the Fourier series according to multiple of the SC mean
the averaged values obtained through the boundary optimal control problem for averaged motion equations, and the initial value for \( p_f \) is taken to be equal to 0.

V. ASSESSMENT OF POSSIBILITY TO REMOVE SEVERAL OSD FROM THE GSO VICINITY TO A DISPOSAL ORBIT

Let us consider the possibility of consecutive deorbiting of several OSD using one SSC from the GSO vicinity into a near-circular disposal orbit, which is 300 km higher than the GSO orbit. It is assumed that the SSC flight from a disposal orbit (also referred to as the waiting orbit) to the OSD is done in three stages. At the first stage the SSC using the EPS flies from the waiting orbit into a circular orbit with major semi-axis, inclination, and longitude of ascending node coinciding with the corresponding orbital parameters of the OSD. To bring the SSC to the OSD vicinity there is a need to ensure matching of mean longitudes of the SSC and the OSD at the end of the first stage of a flight. The fulfillment of this condition is guaranteed by the SSC phasing in the waiting orbit during the required time interval (mean drift velocity of the SSC in relation to the OSD located on an orbit over a diurnal period is 3.819° per day). The boundary optimal control problem described above is solved to define the flight parameters. At the second stage the SSC using the EPS levels the eccentricity and longitude of a perigee. For this the thrust vector of the SSC EPS is oriented in the orbit plane orthogonally to the apse line of the OSD orbit. The direction of the EPS thrust vector remains invariable in inertial space and the EPS operates in apogee and perigee sites with equal range angle of eccentric anomaly. At the end of the second stage the SSC approaches the OSD at a distance of several kilometers. Further approach of the SSC with the OSD until its capture (third stage) is carried out by the monopropellant hydrazine propulsion system. The required delta-V for the third stage is estimated at 10 m/s, and the stage duration – 10 days. Upon capturing the OSD the SSC transfers it to the disposal orbit being in the plane of the initial OSD orbit (fourth stage). The transfer is ensured by continuously operating SSC EPS, the EPS thrust is in transversal direction.

Table 1 shows the main characteristics of the first stage of the SSC flight from the waiting orbit to the first OSD depending on the OSD orbit plane inclination to the plane of the SSC waiting orbit (initial OSD mass is equal to 1,000 kg). The initial phase angle in this table is understood as the difference between mean longitudes of the OSD and the SSC at the beginning of a flight thus ensuring their equality in the end of a flight. The EPS engine time is equal to the minimum duration of the first stage of a flight (is applicable in case of equality of the initial phase angle to the required value). The maximum time of a flight includes phasing time at 360° and flight time from the operating EPS (engine time).
The preliminary design-ballistic analysis of the SSC functioning regarding active removal of space debris from the geostationary orbit suggests the SSC scheme implying five main stages, each of which containing the SSC and the OSD trajectories control systems, as well as the type of the propulsion system used for maneuvering.

The analysis of long-term evolution of quasi-geostationary orbits made it possible to define the range of the OSD orbits applicable for the SSC. The major semi-axis of these orbits falls within 42164±20 km, the maximum eccentricity is 0.0023, the mean eccentricity – 0.001, and the inclination shall be in the range from 0° to 15°. The calculations of propellant costs required to transfer the OSD into the disposal orbit were made assuming that the angle between the orbit planes of consistently transported OSD does not exceed 2°.

The maximum principle was used to obtain the system of optimal control equations allowing minimizing the propellant costs for the SSC maneuvering both at its independent movement and at joint movement with the OSD following the undisturbed motion equations of EPS-based SC in equinoctial elements in inertial earth centered equatorial system of coordinates.

The preliminary design-ballistic analysis of the SSC motion with optimal control demonstrates the possibility of transferring at least 20 OSD (2,000 kg each) from the GSO vicinity into disposal orbits thus the lifetime of the SSC shall be not less than 7.5 years. The longer lifetime will provide for service interruption of the SSC operation, which can be used to manage emergency situations during orbital operation.

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