Analytic and linear prognostic model for a vehicle suspension system subject to fatigue

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Recent developments in system design technology, such as in aerospace, defense, petro-chemistry and automobiles, are represented earlier in the literature by simulated models during the conception step and this is to ensure the high availability of the industrial systems. Given that the integration of diagnostic–prognostic models in these industrial systems is facilitated by these developments. In fact, the monitoring of the degradation indicators is used indirectly in failure prognostic models and is just a measurement of an unwanted situation. Therefore, the diagnostic is not only a failure detection procedure but it also indicates the actual state and the history of the system. Hence, a predictive maintenance is done by the subsequent prognostic model. Consequently, from a predefined threshold of degradation, the remaining useful lifetime is estimated.

Based on a physical dynamic vehicle suspension system, this research paper elaborates a procedure to create a failure prognostic model. I will adopt here analytic laws of degradation such as the Paris–Erdogan law for fatigue degradation and the Palmgren–Miner law for cumulative damage instead of applying degradation abaci largely used in prognostic studies.

Keywords: analytic laws; diagnostic; fatigue; Palmgren–Miner law; Paris–Erdogan law; degradation; prognostic; remaining useful lifetime

1. Introduction

Predicting the remaining useful lifetime (RUL) of industrial systems becomes currently an important aim for industrialists given that the failure which can occur suddenly is generally very expensive at the level of reparation, of production interruption, and is bad for reputation. The classical strategies of maintenance (Vachtsevanos, Lewis, Roemer, Hess, & Wu, 2006, chaps. 5–7) are no more efficient and practical because they do not take into consideration the instantaneous evolving product state, so it is important to understand the product in real time in order to prevent a failure during operation. In fact, we introduce a prognostic approach that seeks to provide an intelligent maintenance.

In the specialized literature, several studies on the prognostic procedure are presented, among them we mention the model-based, statistic-based and data-based models. The works based on abaci of degradation as in the work of Peysson et al. (Chelidze & Cusumano, 2004; Peysson et al., 2009) are useful at this level. As the latter is related to the three influent components: process, mission and environment, it is a non-analytic-based model founded on expert knowledge and on a large database.

A proposed analytic prognostic methodology based on some laws of damage in fracture mechanics is developed here. The damages are generally: crack propagation, corrosions, chloride attack, creep, excessive deformation and deflection, and damage accumulation. Whenever their analytic laws are available, the advantage of a prognostic approach based on a known damage law for a mechanical system is that it is adaptable to new situations and useful in determining the RUL of the system.

The procedure proposed in this work belongs to the model-based prognosis approach related to the physical model. It is focused on developing and implementing effective diagnostic and prognostic technologies with the ability to detect faults in the early stages of degradation. Early detection and analysis may lead to a better prediction and end-of-life estimations by tracking and modeling the degradation process. The idea is to use these estimations to make accurate and precise prediction of the time to failure of components. Early detection also helps avoid catastrophic failures.

Any prognostic methodology must lie on a type of damage. In mechanical systems, the damage can take many shapes. In this research paper, the case of fatigue degradation has been chosen due to the fact that it can be mathematically formulated by available analytic laws such as Paris–Erdogan’s and Palmgren–Miner’s laws.

This approach seems to be important in ensuring the high availability of industrial systems, such as in aerospace, defense, petro-chemistry and automobiles.

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Among these systems, the petrochemical industries can be cited as an important example of prognostic models due to its favorable economic and availability consequences on their exploitation cost (El-Tawil, Abou Jaoude, Kadry, Noura, & Ouladsine, 2010).

In the automobile industry, for example the suspension component, this approach also shows its importance for the same earlier reasons and it will be explained and elaborated later in this paper. In vehicle suspension study, the results of model simulations are done for three cases of road profile excitations.

An analytic linear prognostic model is developed in this research paper that permits one to predict the RUL of a dynamic suspension system. This model considers the fatigue as a damage parameter, and hence, it is based on existing and well-known damage laws in fracture mechanics, such as the crack propagation law of Paris–Erdogan besides the linear damage accumulation law of Palmgren–Miner. An index of degradation that varies from zero to one will be derived. In fact, my proposed model is based on the link between this damage index \(D\) and the crack length \(a\), given that failure is produced when \(a\) reaches a critical length \(a_C\). Hence, my model is given by a simple function relating the instantaneous degradation to actual crack length as a measurement of actual damage.

This work is organized as follows: first the mechanical model of fatigue is presented in the linear cumulative damage case, then the prognostic model of fatigue failure is developed, and finally a case study of vehicle suspensions is illustrated (Abou Jaoude, 2012).

2. Proposed prognostic model

The purpose of this paper is to construct a process of prognostic models capable of predicting the degradation trajectories of a complex system for a given mission under a given environment and starting from an initial known damage. The complex system is decomposable into subsystems where each one can comprise a damage function.

The fatigue failure is one of the famous damage phenomena in mechanical systems such as in aircraft where the wings are subject to the fluctuation of air pressure between a maximal value \(\sigma_{\text{max}}\) and a minimal value \(\sigma_{\text{min}}\) (Figure 1; Lemaitre & Chaboche, 1990). This type of loadings leads to a crack propagation that can accelerate rapidly. Usually, micro-cracks exist originally in the materials due to the fabrication process where stresses remain after manufacture. These micro-cracks are detected and measured and denoted by \(a_0\).

The advantage of the choice of fatigue damage for the developed prognostic methodology is that it is a failure mechanism very well studied in the literature and described under many known analytic laws. This mechanism has relatively the simplest formulation in comparison to the other damage phenomena. The fatigue characterizes the main failure cause of industrial equipments.

![Figure 1. Load fluctuation.](image)

![Figure 2. Pre-crack fatigue damage.](image)

2.1. Damage evolution law

The fatigue of materials under cyclic loading creates micro-cracks. Starting from an initial length \(a_0\) corresponding to an initial cycle number \(N_0\), the macro-cracks become detectable and unstable. These macro-cracks will grow under loading cycles \(N\) to a critical length \(a_C\) reached at \(N_C\) cycles and creating, thus, fractures that lead to failure. This evolution is represented in Figure 2 in terms of the normalized number of cycles \(N/N_C\) for the simplicity of reading.

It can be assumed that \(a_C = e/8\), where \(e\) and \(\ell\) are, respectively, the device dimension in the crack direction and the perpendicular dimension to the crack direction (Figure 3). \(\Delta a_N\) is the crack length increment due to a loading cycle \(dN\). \(t_N\) is the instant corresponding to a cycle \(N\) and to a crack length \(a_N\).

2.2. Paris–Erdogan’s law

Paris–Erdogan’s law (Paris & Erdogan, 1963) permits one to determine the propagation rate of a crack length \(a\) after its detection. The law of damage growth is given by

\[
\frac{da(N)}{dN} = C \cdot (\Delta K)^m,
\]

where \(C\) and \(m\) are the material and environment parameters: \((0 < C \ll 1)\); \(2 \leq m \leq 4\), respectively, \(a\) is the crack length, \(N\) is the number of cycles (where the RUL is derived directly) and \(\Delta K\) is the stress intensity factor.

It can be distinguished (Figure 4) that:

- The long cracks obey Paris–Erdogan’s law.
The law can be written also as follows:

\[
\log \left( \frac{da}{dN} \right) = \log C + m \log (\Delta K).
\]  \tag{2}

From the general form of Paris–Erdogan’s law, McEvily and Ritchie (1988) have proven the following form:

\[
\frac{da}{dN} = C \cdot (\Delta K_{\text{eff}})^m \cdot (K_{\text{max}})^m,
\]  \tag{3}

where \( \Delta K_{\text{eff}} = K_{\text{max}} - K_{\text{op}} \), \( K_{\text{max}} \) is the maximum stress intensity factor and \( K_{\text{op}} \) is the stress intensity factor required to open the fatigue crack.

So, the decoupled form where two different functions of crack length \( a \) and of load \( P \) can be deduced is given as follows:

\[
\frac{da}{dN} = C \cdot \phi_1(a) \cdot \phi_2(P),
\]  \tag{4}

where the function \( \phi_1(a) = Y(a) \cdot \sqrt{\pi a} \) and the load function \( \phi_2(P) = P^m \), where \( P = K_{\text{max}} \), with \( Y(a) \) being the geometric factor function of the body dimensions and \( P \) being the load parameter.

Palmgren–Miner’s rule can be used now to count the damages (Miner, 1945).

### 2.3. Palmgren–Miner’s rule

Palmgren–Miner’s rule (Miner, 1945) serves to compute the cumulative damage \( d_i \) of different stress levels \( \sigma_i \) \( (i = 1, \ i = 2, \ldots, \ i = k) \) applied for \( n_i \) cycles. Given that \( N_i \) is the total cycle’s number of stresses \( \sigma_i \) to be applied and that lead to failure. The linear cumulative damage relative to applied stresses \( (i = 1 \ to \ k) \) is given by (Figure 5):

\[
D_k = \sum_{i=1}^{k} d_i = \sum_{i=1}^{k} \frac{n_i}{N_i},
\]  \tag{5}

### 2.4. Wöhler’s curve

In material fatigue, it is important to know the critical level of applied stresses. When repeated stresses \( \sigma(t) \) are applied along time under a cyclic model, they are limited between
two extreme values $\sigma_{\text{max}}$ and $\sigma_{\text{min}}$. Wöhler’s curve governs the relation between the applied stress levels $\sigma$ and the critical number of cycles $N_C$ during the fatigue process of the material (Figures 6 and 7). For example, if the equipment is loaded by a stress level $\sigma_1$, then the critical cycle number is $N_{C1}$. Each stress level has its own critical cycle number.

The stress range: $\Delta \sigma = \sigma_{\text{max}} - \sigma_{\text{min}}$

The stress amplitude: $\Delta \sigma/2$

The stress mean: $\overline{\sigma} = (\sigma_{\text{max}} + \sigma_{\text{min}})/2$.

2.5. Stress intensity factor

The stress intensity factor is an important term in Paris’s law expression; it represents the effect of stress concentration in the presence of a flat crack. When a flat crack occurs in the system body, the internal stresses in this section change from a uniform to a non-uniform distribution around the crack. This factor represents an amplification of the stress field near the crack (Figure 8). This change is expressed by a factor $K_I$ called the stress intensity factor (Langon, 1999; Lemaitre & Desmorat, 2005, chap. 6) for mode-I crack opening (mode I: the crack opening is in the same direction of applied stresses), and is given by

$$(K_I)^m = \left(Y(a) \cdot \sqrt{\pi a}ight)^m \cdot (\sigma_{\text{max}})^m = \phi_1(a) \cdot \phi_2(P_j). \quad (6)$$

Note that $K_I$ must verify the inequality:

$$K_I \leq K_{IC} = \sqrt{\frac{J_{IC} \cdot E}{1 - (\nu)^2}},$$

where $Y(a)$ is the geometric factor function of the equipment geometric parameters ($a, e$) and $K_{IC}$ is the tenacity of material (or critical stress intensity factor) and is given by

$J_{IC}$ is the resistant crack force of the material, $E$ is Young’s modulus, $\nu$ is the Poisson ratio.

At failure, the factor $K_I$ is equal to the value of $K_{IC}$, then we can deduce the critical length $a_C$ from $K_I(a) = K_{IC} \Rightarrow a = a_C$; but since it is difficult to deduce it from the last relation, I preferred to adopt the simple form: $a_C = e/8$.

2.6. Additivity rule in Palmgren–Miner’s rule

The case where damage is caused by fatigue is an important application of the additivity rule (Todinov, 2001, 2005). In this case, the measurement of damage is the length of the fatigue crack. The additivity rule in Palmgren–Miner’s rule (Miner, 1945) has been proposed as an empirical rule in the case of damage due to fatigue controlled by the crack.
propagation. The rule states that in a fatigue test at a constant stress amplitude $\Delta \sigma_1$, damage could be considered to accumulate linearly with the number of cycles. Accordingly, if at stress amplitude $\Delta \sigma_1$ the component has $N_1$ cycles of life (where total life corresponds to an amount of damage $a_C$), then after $N_1$ cycles the amount of damage will be $(\Delta n_1/N_1)a_C$. After $N_2$ stress cycles spent at stress amplitude $\Delta \sigma_2$, characterized by a life of $N_2$ cycles, the amount of damage will be $(\Delta n_2/N_2)a_C$.

Failure occurs when, at a certain amplitude $\Delta \sigma_M$, the sum of the partial amounts of damage attains the amount $a_C$, that is, when

$$\frac{\Delta n_1}{N_1}a_C + \frac{\Delta n_2}{N_2}a_C + \cdots + \frac{\Delta n_M}{N_M}a_C = a_C \quad (7)$$

is fulfilled.

As a result, the analytical expression of Palmgren–Miner’s rule becomes

$$\sum_{i=1}^{M} \frac{\Delta n_i}{N_i} = 1, \quad (8)$$

where $N_i$ is the number of cycles needed to reach the specified amount of damage $a_C$ at constant stress amplitude $\Delta \sigma_i$.

Palmgren–Miner’s rule is central to reliability calculations, yet no comments are made whether it is compatible with the damage development laws characterizing the different stages of fatigue crack growth. The necessary and sufficient condition for validity of the empirical Palmgren–Miner’s rule is the possibility of factorizing the rate of damage as a function of the amount of accumulated damage $a$ and the stress or strain amplitude $\Delta \rho$:

$$\frac{da(N)}{dN} = F(a) \cdot G(\Delta \rho). \quad (9)$$

The theoretical derivation of Palmgren–Miner’s rule can be found in Todinov (2001).

A widely used fatigue crack growth model is Paris–Erdogan’s power law given by

$$\frac{da(N)}{dN} = C \cdot (\Delta K)^m, \quad (1)$$

where $\Delta K = Y(a) \cdot \Delta \sigma \cdot \sqrt{a}$ is the stress intensity factor range, $C$ and $m$ are material constants and $Y(a)$ is a parameter which can be expressed as a function of the amount of damage $a$.

Clearly, Paris–Erdogan’s fatigue crack growth law can be factorized as in the previously stated equation and, therefore, it is compatible with Palmgren–Miner’s rule. In the cases where this factorization is impossible, Palmgren–Miner’s rule does not hold. Such as, for example, the fatigue crack’s growth law, given by

$$\frac{da(N)}{dN} = B \cdot \Delta \gamma \cdot a^\alpha - D, \quad (10)$$

discussed by Miller (Todinov, 2001), and which characterizes physically small cracks.

In Equation (10), $B$ and $\beta$ represent the material constants, $\Delta \gamma$ represents the applied shear strain range, $a$ represents the crack length at cycle $N$ and $D$ represents a threshold value.

Thus, following what has been said, the proposed model can use the additivity characteristic of Paris’s law.

2.7. Maintenance and diagnostic/prognostic

It is proved that the schedule-based inspection/maintenance NDI (non-destructive inspection) is less beneficial than the on-demand (or continuous) inspection with permanently installed sensors/condition-based maintenance SHM (structural health monitoring) for many reasons such as the increased availability, the quick assessment of potential/actual damage events, the increasing safety and the performance of advanced materials.

But the major technical challenges for SHM reside in the sensors. The monitoring should be directed to the detection of the cracks and corrosion, the multiple damage modes, the pre-crack fatigue damage and the amount of residual stress.

We can say that the NDI leads to prognostics based on the following:

- NDI performed at the time of fabrication and as in-service inspections.
- Condition-based maintenance-active component monitoring.
- Move from diagnosis to the prediction of the remaining life and the structural health monitoring/management.
- Prognostics (for machinery) are the prediction of a remaining safe or service life, based on an analysis of the system or the material condition, stressors and degradation phenomena.

For example, bearing crack faults may be prognosed by examining and predicting their vibration signals.

The relation between maintenance and prognostic is summarized in Figure 9.

2.7.1. Flow chart of the various components of the diagnostic/prognostic/maintenance process

![Flow chart of the various components of the diagnostic/prognostic/maintenance process](image-url)
2.7.2. Cycle of prognostic/diagnostic/maintenance

![Diagram of diagnostic-prognostic-maintenance process]

2.8. Accumulation of fatigue damage

In fatigue damage, in order to study the prognosis of a degraded component, my idea is to predict and estimate the end of life of an equipment component subject to fatigue by tracking and modeling the corresponding degradation function. To facilitate the analysis, it is convenient to adopt a normalized damage measurement $D \in [0, 1]$ by exploiting the advantage of the cumulative damage law of Palmgren–Miner (Figure 5). In fact, this law helps to estimate the lifetime of components subject to load cycles, and it considers that the damage fraction $d_i$ at stress level $\sigma_i$ is the ratio of $n_i$ over the total cycle number $N_i$ producing failure.

For a body of equipment of thickness $e$, we take the initial crack length as $a_0$ ($a_0 \leq a \leq a_C$). Given that $1.01 \leq e/a \leq 10$ and $e/a_C = 8$, then from Equation (1) a recurrent form of crack length growth $a$ can be deduced as (Abou Jaoude, 2012)

$$\Delta a = a_N - a_{N-1} = C \cdot \phi_1(a_{N-1} - a_0) \cdot \phi_2(P_j),$$

where $(a_0 \approx 0)$.

For the other sequences,

$$a_1 = a_0 + C \phi_1(a_0) \phi_2(P_j)$$

$$a_2 = a_1 + C \phi_1(a_1) \phi_2(P_j)$$

$$\vdots$$

$$a_N = a_{N-1} + C \phi_1(a_{N-1}) \phi_2(P_j)$$

$$\Rightarrow \Delta a = a_N - a_{N-1} = C \cdot (K_1)^{m} \cdot dN$$

where $(K_1(a_N))^{m} = \phi_1(a_N) \cdot \phi_2(P_j)$ is a function of the crack length $a$.

For each cycle, we have $dN = 1$; therefore, $a_N = a_{N-1} + C \cdot (K_i)^{m}$.

As $D_k = \sum_{i=1}^{k} d_i = \sum_{i=1}^{k} \frac{n_i}{N_i}$

(Miner’s law; with $i$ in Miner’s law = $N$ in my model)

and based on the additivity characteristic of Paris’s law, the addition of damages gives the total crack growth at failure point $(a_C - a_0)$ realized at the total number of cycle $N_C$:

$$a_C - a_0 = \sum_{N=1}^{N_C} \Delta a_N = \text{total damage}.$$  

At each $n_i$, the crack grows to length $d_i = da_N$, therefore, Miner’s damage fraction, for any stress level (Figure 10), is given in terms of the crack length by

$$d_i = \frac{n_i}{N_i} = \frac{\Delta a_N}{(a_C - a_0)}.$$   (11)

where $n_i$ is the damage increment due to stress number $i$, $N_i$ is the total damage for stress number $i$. Then, the cumulated total damage at cycle $N$ is given by

$$D_N = \sum_{i=1}^{N} d_i = \sum_{i=1}^{N} \frac{da_i}{a_C - a_0} = \sum_{i=1}^{N} d_i \frac{a_i}{a_C - a_0}.$$  (12)

Figure 9. Diagnostic–prognostic–maintenance. Note: CBM, condition-based maintenance.
Consequently, a recurrent form of degradation can be deduced as follows:

\[
D_N = \frac{a_N}{a_C - a_0} = \frac{a_{N-1} + C \cdot (K_i)^m}{a_C - a_0} = \frac{a_{N-1}}{a_C - a_0} + \frac{C \cdot (K_i)^m}{a_C - a_0} = \frac{a_{N-1}}{a_C - a_0} + \frac{C \cdot (Y(a_{N-1}) \sqrt{\pi} \cdot a_{N-1} \cdot \sigma_j)^m}{a_C - a_0} = \frac{a_{N-1}}{a_C - a_0} + \frac{C \cdot Y(a_{N-1})^m \cdot (\sqrt{\pi} \cdot a_{N-1})^m \cdot \sigma_j^m}{a_C - a_0} = D_{N-1} + \eta \cdot \phi_1(D_{N-1}) \cdot \phi_2(P_j),
\]

(13)

where

\[
D_{N-1} = \frac{a_{N-1}}{a_C - a_0}, \quad \eta = \frac{C}{a_C - a_0}, \quad \phi_1(D_{N-1}) = Y(a_{N-1})^m \cdot (\sqrt{\pi} \cdot a_{N-1})^m, \quad \phi_2(P_j) = \sigma_j^m.
\]

Hence, the new prognostic analytic model is expressed by the general function given by

\[
D_N = D(N) = P_{\text{prog}}(a_N) = \frac{a_{N-1}}{a_C - a_0} + \frac{C}{a_C - a_0} \cdot Y(a_{N-1})^m \cdot (\sqrt{\pi} \cdot a_{N-1})^m \cdot \sigma_j^m. \quad (14)
\]

And therefore, the degradation trajectories \(D(N)\) along the total number of loading cycles \(N\) can be drawn (Abou Jaoude, El-Tawil, Kadry, Noura, & Ouladsine, 2010).

### 2.9. Flow chart of the prognostic model

The following flow chart summarizes all the procedures of the proposed model (Abou Jaoude, Kadry, El-Tawil, Noura, & Ouladsine, 2011):

### 2.10. Environment effects in the proposed prognostic model

The environment effects are taken into account by the two parameters \(C\) and \(m\). These parameters are related to the material interacting with its environment.

Large values of \(m\) (\(m > 40\)) correspond to the case of brittle materials (brittle failure) and small values of \(m\) (\(m \to 0\)) correspond to the case of ductile materials
For each load cycle 
\( N = 1, N_C \)

Calculate crack length by recurrence:
\[ a_N = a_{N-1} + C \cdot \phi_1 \left( a_{N-1} - a_0 \right) \cdot \phi_2 (\sigma_j) \]

Calculate degradation value at cycle \( N \):
\[ D_N = \frac{a_N}{a_C - a_0} \]

If \( D_N < 1 \) then:
- Yes
- \( N_C = N \)

No
- \( N_C = N \)

Plot \((D_N; N) \) \( N = 1, N_C \)

Plot \((\text{RUL}(N); N) \) \( N = 1, N_C \)

Diagnostic/Inspection

Input initial parameters:
\( (a_0, \epsilon, \text{dimensions, } C, m) \)

Estimation \( a_C = \epsilon/8 \)

Analytic simulation of crack growth

Degradation accumulation

Load simulation:
\( \sigma_j \)

Calculate remaining useful lifetime at cycle \( N \): \( \text{RUL}(N) = N_C - N \)

Prognostic

\((m = 2: \text{fully plastic})\). Otherwise for fatigue failure, the range value of \( m \) is \( 2 \leq m \leq 3 \). The parameter \( m \) depends mainly on the specimen length. For lower toughness, steels \( m \) is greater than or equal to 3 (Sukumar, Chopp, & Moran, 2003).

The coefficient \( C \) is affected by the edges and consequently its value depends on whether it is the case of a plane stress or a plane strain. However, for the case of an infinite equipment body and far from the edge effects, the coefficient \( C \) takes a constant value (Newman & Raju, 1981).

Moreover, \( C \) and \( m \) depend on the testing conditions such as the loading ratio \( \sigma_{\text{min}}/\sigma_{\text{max}} \), on the geometry and the size of the specimen, and on the initial crack length.

These two parameters govern the behavior of the material during the fatigue process through the crack propagation. The environment influencing parameters in this process, such as temperature, humidity, geometry dimensions, material nature, water action, soil action, applied load location and body shape, are also represented by these two parameters \( C \) and \( m \).

These two parameters are evaluated by the mean of experiments in true conditions.

Examples (Newman & Raju, 1981; Sukumar et al., 2003):
\[ C = 5.2 \times 10^{-13} \text{ (free air)} \]
\[ C = 1.3 \times 10^{-14} \text{ (under soil)} \]
\[ C = 2 \times 10^{-11} \text{ (offshore)} \]
and \( m = 3 \text{ (metal)} \).
3. Application of the prognostic method to industrial systems

To illustrate the proposed new analytic approach, in this section an important mechanical system will be applied which is the suspension in the automotive industry, given that its corresponding prognostic study is essential for economic reasons.

3.1. Vehicle suspension fatigue life

Fatigue analysis of a vehicle suspension (Figure 11) by finite element models was done in many works (Colquhoun & Draper, 2000) as a consequence of experimental results. It permits one to define the location of potential fatigue cracks. The major aspect of local strain fatigue is determined by the crack initiation and propagation. The original theories that were developed for uniaxial stress conditions were elaborated and improved later to eliminate the errors due to simplified uniaxial conditions.

It was proposed in the literature (Colquhoun & Draper, 2000; Frost & Dugdale, 1957) that for high-cycle fatigue, successful life estimates for biaxial stress conditions could be made using combinations of axial and shear stresses.

There is a lot of experimental evidence from fatigue testing carried out in the middle of the last century showing that stress gradients have an important effect on the total fatigue life of a component. Stress gradients have also been used in an attempt to explain the effect of notch sensitivity.

Moreover, finite element analysis provides surface strains on the model, but for real engineering components it is very difficult to determine the stress concentration factor at a notch (Figure 12).

The stress concentration factor is the same as the stress intensity factor which was explained in Section 2.5.

The endurance limit stress is the stress level for which the critical number of loading cycles tends to infinity (refer to Section 2.4).

Referring to Figure 13, $\sigma_e$ is the smooth specimen endurance limit stress, $S_{th}$ is the threshold stress for non-propagation cracks, that is, below $S_{th}$, fatigue is not influent and $S_e = S_{th}$, $Kt$ is the stress concentration factor, which is given as

$$Kt = \frac{\text{Endurance limit of a notch} - \text{free specimen}}{\text{Endurance limit of a notched specimen}}$$

The endurance limits (Frost & Dugdale, 1957) are obtained from standard rotating beam experiments carried out under certain specific conditions. They are given by $S_e = \sigma_e/Kt$.

As the stress concentration factor increases, where this is the case for many ductile metals, a minimum value of fatigue limit stress that is expressed by $S_{th}$ occurs. Hence, increasing the stress concentration factor by sharpening the notch produces no further reduction in fatigue strength (Figure 13).

The parts forming the vehicle suspension are indicated in Figure 14 where the damper’s element can be seen.

Using test data on plate and round bar, specimens in aluminum alloy and steel materials have shown that if fatigue life to the first crack initiation is considered, then the fatigue strength reduces with increasing stress concentration with no limiting value (Figure 15).
Many works (Conle & Topper, 1980; Duquesnay, Pompetzki, & Topper, 2002; Frost, 1960) have shown that the constant amplitude endurance limit does not apply to the analysis of real service loading if some cycles in the loading exceed the constant amplitude endurance limit stress amplitude. For a finite life design, the larger cycles in the loading cause the endurance limit stress to be reduced significantly, with the result that small cycles contribute to the fatigue damage process.

Figure 15 (Conle & Topper, 1980) shows the results of strain-controlled constant amplitude tests on an aluminum alloy at high temperature. The finite element calculation made by the software SAFE (FE-SAFE) from an elastic finite element analysis (FEA) shows excellent correlation for high-cycle fatigue. For low-cycle fatigue, at 1000 cycles the calculated fatigue life is conservative by a factor of 3. This is a commonly observed phenomenon at such low fatigue lives in components where yielding occurs across the entire section. For comparison, an elastic–plastic FEA analysis of the model was used as an input into the FE-SAFE analysis, and the correlation with the test result was then excellent.

This component was analyzed in FE-SAFE and compared with the results of fatigue testing. A scale factor was applied to the test loading to produce a failure. The correlation between the calculated life of 1631 repeats of the load history and the test life of 1650 repeats is extremely good.

The steel component was analyzed (Duquesnay et al., 2002) with a load–time history in one direction (Figure 16). A scale factor was applied to produce a failure. The analysis used stresses from an elastic FEA; fatigue lives were calculated for each node on the model, using averaged nodal stresses. Experience has shown that this is much more accurate than using stresses at integration points or at the element centroid.

In designing engine crank shafts (Figure 16), the finite elements analysis is used to generate stress solutions. The FEA analysis shows that the principal stresses change their
3.1.1. Types of mechanical effects, their mechanisms and the possible consequences

The following flow chart describes the relationship between the sources, the mechanical effects and the consequences of various loading stresses (Lemaitre & Chaboche, 1990).

3.1.2. Automatic diagnostic of a bad suspension bushing

Automobile suspension bushings come in a variety of shapes, sizes and thicknesses, according to their application. Bushings may be made from several materials, including rubber, polyurethane, urethane and graphite composites. Bushings prevent wear to expensive suspension components by absorbing vertical and lateral forces produced by the vehicle over different terrains. They cushion and absorb shock on the chassis to keep the shock from entering the passenger compartment. While absorbing these vibrations, they still allow limited movement and flex in the suspension joints, keeping the wheels firmly grounded and on track during turning maneuvers. A vehicle’s owner may check all its suspension bushings for proper shape and condition.

3.1.3. Prognostic study for vehicle suspension systems

Let us consider a half-vehicle suspension system (Figure 18) subject to non-regular road surface excitations (Lee, 2004). It is composed of a front part and a rear part. To study the prognostic of this system, it is important to define the mechanical model in order to deduce the output response from the input excitation road. The system has four degrees of freedom that can be reduced to two degrees of freedom by considering the front suspension alone.

The dynamic equations of the system are given by

\[ m\ddot{x} + (f_{ca} + f_{ka}) + (f_{cb} + f_{kb}) = 0, \]
\[ I\ddot{\theta} + l_a(f_{ca} + f_{ka}) - l_b(f_{cb} + f_{kb}) = 0, \]
\[ m_{2a}\ddot{x}_{2a} - (f_{ca} + f_{ka}) + k_{2a}(x_{2a} - w_a) = 0, \]
\[ m_{2b}\ddot{x}_{2b} - (f_{cb} + f_{kb}) + k_{2b}(x_{2b} - w_b) = 0, \]
\[ x = (l_a x_{1a} + l_b x_{1b})/l, \quad \tan \theta \approx \theta = \frac{(x_{1a} - x_{1b})}{l}, \]
\[ l = l_a + l_b, \]
\[ f_{ci} = c_i(x_{1i} - x_{2i}), \quad i = a, b, \]
\[ f_{ki} = k_i(x_{1i} - x_{2i}), \quad i = a, b, \]
The load history can be either push-pull lab test loads at e.g. the tyre contact patch (below), or measured road loads applied e.g. at the wheel centre (right).

Figure 17. Application of force time histories (Duquesnay et al., 2002).

| Stressors                  | Ageing mechanisms | Consequences                      |
|----------------------------|-------------------|-----------------------------------|
| Stress Constant            | Creep             | Degradation (Damage)              |
| Strain Constant            | Relaxation        | Deformation                       |
| Stress variable            | Fatigue           |                                    |
| Temperature                | Thermal Ageing    | Embrittlement and Cracking         |
| Irradiation                | Irradiation Damage|                                    |
| Corrosive medium           | Corrosion         | Material Loss                     |
| Relative Motion of Fluids  | Wear & Erosion    |                                    |
| and Solids                 |                   |                                    |

where $m$ is the vehicle mass; $I$ is the moment of inertia; $m_{2a}$ is the mass of front wheel; $m_{2b}$ is the mass of rear wheel; $\theta$ is the rotary angle of vehicle; $x$ is the vertical displacement; $c_i$ is the friction coefficient of damping ($i = a, b$); $f_{ca}, f_{cb}$ are the damping forces of the front/rear wheel; $f_{ka}, f_{kb}$ are the restoring forces of the front/rear wheel; $k_{1a}, k_{1b}$ are the spring constants of the front/rear suspension; $k_{2a}, k_{2b}$ are the spring constants of the front/rear wheel; $x_{2a}, x_{2b}$ are the vertical displacements of the front/rear wheel; $x_{1a}, x_{1b}$ are the displacements of the vehicle body at front/rear wheel; $l_a, l_b$ are the distances of the front/rear suspension to center; and $w_a, w_b$ are the irregular excitations from the road surface (see Figure 19).
3.1.4 System identification

The model parameters are given by the following numerical data (Lee, 2004):

\[
\begin{align*}
m &= 1200 \text{ kg}, \\
m_{2a} &= 30 \text{ kg}, \\
m_{2b} &= 25 \text{ kg}, \\
c_b &= 4000 \text{ N/m/s}, \\
c_a &= 5000 \text{ N/m/s}, \\
k_{1a} &= 56000 \text{ N/m}, \\
k_{1b} &= 42000 \text{ N/m}, \\
k_{2a} &= k_{2b} = 152 \text{ kN/m}, \\
l_a &= 0.9 \text{ m}, \\
l_b &= 1.2 \text{ m}.
\end{align*}
\]

The matrix form of the previous equations is given by

\[
M \ddot{z} + N \dot{z} + Kz = Eu,
\]

where \(M\) is the mass matrix, \(N\) is the damper coefficients matrix and \(K\) is the stiffness matrix.

The input excitation vector is \(u = [w_a \ w_b]^T\).

The output damper displacement vector is \(z = [x_{1a} \ x_{2a} \ x_{1b} \ x_{2b}]^T\).

The vertical accelerations \(x_{1a}, x_{1b}, x_{2a}, x_{2b}\) are measured variables. The matrices \(M, N, K\) and \(E\), respectively, are given by

\[
M = \begin{bmatrix}
I_b m/l & 0 & 0 & 0 \\
0 & I_b m/l & 0 & 0 \\
0 & 0 & m_{2a} & 0 \\
0 & 0 & 0 & m_{2b}
\end{bmatrix},
\]

\[
N = \begin{bmatrix}
c_a & -c_a & c_b & -c_b \\
l_a c_a & -l_a c_a & -l_b c_b & l_b c_b \\
0 & 0 & -c_b & c_b \\
0 & 0 & 0 & 0
\end{bmatrix},
\]

\[
K = \begin{bmatrix}
k_{1a} & -k_{1a} & k_{1b} & -k_{1b} \\
l_b k_{1a} & -l_b k_{1a} & -l_b k_{1b} & l_b k_{1b} \\
-k_{1a} & k_{1a} + k_{2a} & 0 & 0 \\
0 & 0 & 0 & -k_{1b} & k_{1b} + k_{2b}
\end{bmatrix},
\]

\[
E = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}.
\]

The state vectors (damper displacements and velocity) are

\[
x = \begin{bmatrix} z(t) \\ \dot{z}(t) \end{bmatrix}, \quad \dot{x} = \begin{bmatrix} \ddot{z}(t) \end{bmatrix}.
\]

The state vectors are obtained from the solution of the dynamic equation (15) by a convenient method applied to an ordinary differential equation. As this type of solution is not in the scope of this paper, a simplification was done later (Section 3.1.6) to deduce the damper displacements.

3.1.5 Stress intensity factor of a suspension

The modeling of the suspension damage begins by determining the stress intensity factor composed of the multiplication of two functions:

\[
(K_m)^m = (Y(a) \cdot \sqrt{\pi a})^m \cdot (\sigma_{\text{max}})^m = \phi_1(a) \cdot \phi_2(P_j),
\]

where \(\phi_1(a)\) is the crack length function determined in terms of a geometric function \(Y(a)\) and \(\phi_2(P_j)\) is the loading function.

Determination of the first function \(\phi_1(a)\):

Assume that the front suspension of the system has a crack length \(a\) perpendicular to the exterior load (Figure 20).

Let \(m = \frac{2}{a}\) be the material constant, then \(\phi_1(a) = \frac{Y(a) \cdot \sqrt{\pi a}}{a^2}\).

Experimental and empirical results of the validation of the crack propagation models permit one to define some
studied equipment (El-Tawil, 2004). For the case of sus-
pensions, \( Y(a) \) is defined with a sufficient precision by a
development limited to order 4 (Equation (16)). Therefore,
the first function can be considered as given by

\[
\phi_1(a_N) = (\pi N) \left[ 1.122 - 1.4 \left( \frac{a_N}{e} \right) + 7.33 \left( \frac{a_N}{e} \right)^2 \right.
\]
\[
- 13.08 \left( \frac{a_N}{e} \right)^3 + 14 \left( \frac{a_N}{e} \right)^4 \left. \right]\right] ^2,
\]

(16)

where

\[
Y(a) = 1.122 - 1.4 \left( \frac{a_N}{e} \right)
\]
\[
+ 7.33 \left( \frac{a_N}{e} \right)^2
\]
\[
- 13.08 \left( \frac{a_N}{e} \right)^3 + 14 \left( \frac{a_N}{e} \right)^4
\]

and \( a_N \) is the crack length at cycle \( N \), \( e \) is the width of the
mechanical component of the suspension.

**Determination of the second function \( \phi_2(P_j) \):**

Given that \( P_j \) is the load parameter, and we have
\( \phi_2(P_j) = P_{m} = P_j^2 \), we will simulate the degra-
dation model by generating the load \( P_j \) of road profile \([ w_a \ w_b ]^T \) (Chelildez & Cusumano, 2004) under
the Gaussian Normal law for the three modes of roads
(Table 1).

From the system of equations (15), the solution of this
system of matrices gives the output vector \( z \).

Then, the range of the suspension displacement for the
front wheel is given by

\[
\Delta x_j = x_{1a}^j - x_{2a}^j.
\]

(17)

If we take as mean value \( \bar{x}_j \) and as standard
variation \( \sigma_j \), then we obtain a set of \( \{x_j\} \) for each road
mode \( r = 1, 2, 3 \), given that the load parameter is always
\( P_j \). Hence

\[
\phi_2(P_j) = P_{m} = P_j^2 = \sigma_j^2.
\]

The amplitude of the stresses developed in the suspension
due to \( \Delta x_j \) is simplified by

\[
\Delta \sigma_j = E \times \frac{\Delta x_j}{\ell},
\]

(18)

where \( \ell \) is the length of the suspension device \( (\ell =
500 \text{ mm}) \), \( \Delta x_j \) is the variation of this length (dilation)
under road profile excitation and \( E \) is Young’s modulus of the
suspension material \( (E = 200 \text{ GPa}) \).

3.1.6. Fatigue damage modeling of the suspensions

Assume that the maximum of \( a_N \) is \( a_C = e/8 \) (Lemaitre &
Chaboche, 1990).

We define

\[
D_N = \frac{a_N}{a_C - a_0} \approx \frac{a_N}{a_C} = \frac{8a_N}{e} \quad \text{as} \quad a_0 \ll a_C.
\]

Then, we replace \( \phi_1(a_N = eD_N/8) \) in Equation (13)
and we get

\[
D_N = D_{N-1} + \eta \cdot \phi_1(D_{N-1}) \cdot \phi_2(P_j).
\]

(19)

Moreover, \( \eta \) is a material constant and we have
\( \eta = 8 \times 10^{-6} \) (El-Tawil, 2004).

Thus, we have the recursive formula (20) in terms of
the crack length:

\[
a_N = a_{N-1} + C \cdot \phi_1(a_{N-1}) \cdot \phi_2(P_j).
\]

(20)

With (El-Tawil & Kadry, 2010) \( m = 2, \quad C = \eta \cdot
(a_C - a_0), \quad \eta = 8 \times 10^{-6} \)

\[
\phi_1(a_{N-1}) = (\pi a_{N-1}) \left[ 1.122 - 1.4 \left( \frac{a_{N-1}}{e} \right)
\right.
\]
\[
+ 7.33 \left( \frac{a_{N-1}}{e} \right)^2
\]
\[
- 13.08 \left( \frac{a_{N-1}}{e} \right)^3 + 14 \left( \frac{a_{N-1}}{e} \right)^4
\]
\[
\left. \right]\right] ^2
\]

(21)

and

\[
\phi_2(P_j) = P_{m} = P_j^2 = \sigma_j^2.
\]
Therefore, the recursive expression of the crack length for the suspension model is given by

\[
a_N = a_{N-1} + C \times (\pi a_{N-1}) \left[ 1.122 - 1.4 \left( \frac{a_{N-1}}{e} \right) \right]
\]

\[
+7.33 \left( \frac{a_{N-1}}{e} \right)^2 - 13.08 \left( \frac{a_{N-1}}{e} \right)^3
\]

\[
+14 \left( \frac{a_{N-1}}{e} \right)^4 \right] \times \sigma_j^2.
\]

(22)

From the equation \( D_N = (a_N / a_C - a) \), the recursive expression of the degradation index for the suspension model becomes

\[
D(N) = \frac{a_{N-1}}{a_C - a_0} + \frac{C}{a_C - a_0} \times (\pi a_{N-1}) \times \left[ 1.122 - 1.4 \left( \frac{a_{N-1}}{e} \right) + 7.33 \left( \frac{a_{N-1}}{e} \right)^2
\]

\[
- 13.08 \left( \frac{a_{N-1}}{e} \right)^3 + 14 \left( \frac{a_{N-1}}{e} \right)^4 \right] \times \sigma_j^2.
\]

3.1.7. Simulation of three road profiles

To take into account various states of roads, we consider three different types of roads which are severe, fair and good. In the following table, we indicate the model characteristics of each type of roads.

The parabolic road profile of a vehicle circulation time with \( T = 2 \) s as a recurrent interval is considered. Given that this interval is repeated as needed until reaching the failure level (\( D_C = 1 \)). Figure 21 illustrates the road profile.

Each interval shows that the road profile contains a symmetric curve of width \( T/8 = 0.25 \) s with a peak value followed by a horizontal run of zero amplitude.

Moreover, the period of road profile \( T = 2 \) s must be compared to the proper period of the suspension system to verify if there is a risk of dynamic amplification (i.e. mechanical resonance).

3.1.8. Simulation results

The prognostic study of a suspension is realized by using the degradation simulation (Equation (23)). The methodology is composed of two parts:

- In the first part, the simulation of the road profile for the three modes (severe, fair and good) (Table 1) is done and from which \( \Delta x \) and \( \Delta \sigma \) are deduced.
- In the second part, the crack length \( a_N \) is cumulated at each cycle \( N \) (Equation (22)).

The resulting curves of \( D(N) \) are represented in Figures 22–24.

In mode 1 case (severe), it is noted that (Figure 22) for \( N = 6,836,000 \) cycles, the degradation \( D_N \) reaches the critical value \( D_C = 1 \). The deduced lifetime of the suspension is 6,836,000 cycles of road excitation in mode 1. Moreover, the first sign of damage appears at about 2,500,000 cycles. Starting from 6,000,000 cycles, the slope of the degradation curve becomes very acute; hence, damage is increasing very fast.

In mode 2 case (fair), it is noted that (Figure 23) for \( N = 10,850,000 \) cycles, the degradation \( D_N \) reaches the critical value \( D_C = 1 \). The deduced lifetime of the suspension is 10,850,000 cycles of road excitation in mode 2. Moreover, the first sign of damage appears at about 4,000,000 cycles. Starting from 10,000,000 cycles, the slope of the degradation curve becomes very steep; hence, damage is increasing very fast.

In mode 3 case (good), it is noted that (Figure 24) for \( N = 17,222,000 \) cycles, the degradation \( D_N \) reaches the critical value \( D_C = 1 \). The deduced lifetime of the suspension is 17,222,000 cycles of road excitation in mode...
3. Moreover, the first sign of damage appears at about 6,200,000 cycles. Starting from 16,000,000 cycles, the slope of the degradation curve becomes very acute; hence, damage is increasing very fast.

In addition, Figure 25 recapitulates the three previous figures.

3.1.9. Analysis of the simulation results

The expectation of the lifetime for mode 1 is nearly 63% of that of mode 2 and the expectation of the lifetime for mode 2 is nearly 63% of mode 3 (Figure 25). It can be noticed from the obtained results that the increase in the suspension lifetime relative to the road of mode 3 is as follows: mode (1)/mode (3) \( \approx 152 \) and mode (2)/mode (3) \( \approx 59 \).

From the results above, the three expected lifetimes are as follows: \( N_{C1} = 6,836,000 \) cycles; \( N_{C2} = 10,850,000 \) cycles; \( N_{C3} = 17,222,000 \) cycles. Then, our prognostic procedure yields the RUL for the three modes (Figure 26) that can now be easily deduced from these three curves at any instant or any active cycle \( N \) as follows:

- For mode 1: \( \text{RUL}_1(N) = N_{C1} - N \),
- For mode 2: \( \text{RUL}_2(N) = N_{C2} - N \),
- For mode 3: \( \text{RUL}_3(N) = N_{C3} - N \).

3.1.10. Conversion of RUL into years and km

Given that each cycle duration is 2 s (refer to Figure 21), we convert the suspension lifetime into years units by using: \( \text{RUL}(s) = 2 \times \text{RUL}(N) \). We assume that the suspension time usage is 10% of a day, which corresponds to 2.4 hours/day.

The conversions from cycles to km and to years, for a vehicle running with 50 km/hour, are given by the following literal expressions:
From cycles to km:

\[
\text{RUL}(\text{km}) = \frac{\text{RUL}(\text{cycles}) \times 2(\text{s/cycle}) \times 50(\text{km/hour})}{60(\text{s/min}) \times 60(\text{min/hour})} \\
= \frac{\text{RUL}(\text{cycles})}{36(\text{cycles/km})}.
\]

From km to years:

\[
\text{RUL(\text{years}) = \frac{\text{RUL(\text{km})}}{2.4(\text{hours/day}) \times 50(\text{km/hour}) \times 365(\text{days/year})}} \\
= \frac{\text{RUL(\text{km})}}{43,800(\text{km/year})}.
\]

Therefore, the RUL results can be expressed by the following units: cycles, or km, or years.

Thus, the expected lifetimes’ durations are:

For mode 1:

\[
\frac{6,836,000(\text{cycles}) \times 2(\text{s})}{60(\text{s}) \times 60(\text{min}) \times 2.4(\text{hours}) \times 365(\text{days})} = 4.34 \text{ years} \Rightarrow 190,092 \text{ km},
\]

For mode 2:

\[
\frac{10,850,000(\text{cycles}) \times 2(\text{s})}{60(\text{s}) \times 60(\text{min}) \times 2.4(\text{hours}) \times 365(\text{days})} = 6.88 \text{ years} \Rightarrow 301,344 \text{ km},
\]

For mode 3:

\[
\frac{17,222,000(\text{cycles}) \times 2(\text{s})}{60(\text{s}) \times 60(\text{min}) \times 2.4(\text{hours}) \times 365(\text{days})} = 10.92 \text{ years} \Rightarrow 478,296 \text{ km}.
\]

Moreover, the validation of these results can be found in the work of Vakili-Tahami, Zehsaz, & Alidadi (2009) on the fatigue life of suspensions. An average life of 200,000 km is deduced under severe conditions and which corresponds to 4.57 years for a vehicle running with 50 km/hour and for 2.4 hours/day. The results obtained are thus realistic.

4. Conclusion

An analytic linear prognostic model is developed in this research paper that permits one to predict the RUL of a dynamic suspension system. This model considers the fatigue as a damage parameter, and hence it is based on well-known laws of damage such as Paris’s and Miner’s laws. An index of degradation that varies from zero to one is derived. My proposed model is based on the link between this index \(D\) and the crack length \(a\). Given that failure is produced when \(a\) reaches a critical length \(a_c\). Hence, my model is given by a simple function relating the instantaneous degradation to actual crack length as a measurement of actual damage.

My aim is to evaluate the evolution of the system lifetime at each instant. For this purpose, the degradation trajectories have been used in terms of cycles’ numbers or the time of operation. From these degradation trajectories, the RUL variations were deduced. The prognostics of a complex system can be deduced from the prognostic of its sub-systems when their damage laws are available.

To demonstrate the effectiveness of my model, an industrial example has been considered in the simulation in this paper. This example is the vehicle suspension system where three modes of road profiles are simulated and examined.

In such an industrial system, this model proves that it is very convenient and it provides a useful tool for a prognostic analysis. Moreover, it is less expensive than other models that need a large number of data and measurements.

Disclosure statement

No potential conflict of interest was reported by the author.

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