Spheres, Deficit Angles and the Cosmological Constant

Ignacio Navarro*
IPPP, University of Durham, DH1 3LE Durham, UK.

Abstract

We consider compactifications of six dimensional gravity in four dimensional Minkowski or de Sitter space times a two dimensional sphere, \( S^2 \). As has been recently pointed out, it is possible to introduce 3-branes in these backgrounds with arbitrary tension without affecting the effective four dimensional cosmological constant, since its only effect is to induce a deficit angle in the sphere. We show that if a monopole like configuration of a 6D \( U(1) \) gauge field is used to produce the spontaneous compactification of the two extra dimensions in a sphere a fine tuning between brane and bulk parameters is reintroduced once the quantization condition for the gauge field is taken into account, so the 4D cosmological constant depends on the brane tension. This problem is absent if instead of the monopole we consider a four form field strength in the bulk to obtain the required energy-momentum tensor. Also, making use of the four form field, we generalize the solution to an arbitrary number of dimensions \( (\geq 6) \), keeping always four noncompact dimensions and compactifying the rest in a \( n \)-dimensional sphere. We show that a \((n+1)\)-brane with arbitrary tension can be introduced in this background without affecting the effective 4D cosmological constant.

*E-mail: ignacio.navarro@durham.ac.uk
1 Introduction

The old idea that the universe has more than four dimensions has been the subject of intense research activity in recent years, due mainly to the observation that some fields can be confined in a submanifold of the total space. This has opened new avenues in (brane world) model building and some of the major problems in theoretical physics have been addressed in this fashion with very interesting results. For instance, regarding the hierarchy problem, in the models of [1,2] it has been shown how the scale of quantum gravity can become a free parameter. Theories in which Standard Model fields are confined in a brane embedded in a higher dimensional bulk space have also been used to attack the cosmological constant problem. In [3,4], solutions were found in a 5D model where a 3-brane is kept flat for any value of its tension making use of a scalar field with particular couplings in the bulk and to the brane tension. However, this kind of models always involve a naked spacetime singularity at finite proper distance from the brane [5], and the resolution of this singularity in a more fundamental theory of gravity can reintroduce a fine tuning between bulk parameters and the brane tension [6]. Recently a similar property was found in 6D models [7,8] that are free of singularities\(^1\). These models have two extra compact dimensions and it has been shown that it is possible to introduce three branes in this background with arbitrary tensions and the effective 4D cosmological constant is completely independent these brane tensions, only depending on bulk parameters. The only effect of the branes is to introduce a deficit angle in the compactification manifold (taken to be a sphere or a disk). This opens the possibility of building models in which the required fine tunings are the consequence of some unbroken symmetry in the bulk (e.g. supersymmetry) while this symmetry can be badly broken in the brane without affecting the effective cosmological constant.

The cosmology and stability of the scenarios sketched above has been studied in [10], where it is argued that the model is stable since all scalar fluctuations of the metric are massive and only the graviphoton and the 4D graviton are massless. Also, non

\(^1\) In [9], it was noticed that the only effect of a three brane in a 6D bulk that is rotationally symmetric around the brane axis would be to induce a deficit angle in the transverse space, but the compactifications considered always involved a spacetime singularity or did not maintain the nice property that the effective cosmological constant in 4D was independent of the brane tension. In contrast, the models of [7,8] do not involve any spacetime singularities except for a conical one that can be easily smoothed out if we allow for a finite brane thickness.
standard cosmological properties are emphasized. In this paper we will continue the exploration of these models. We will consider different mechanisms for producing the required spontaneous compactification of the extra dimensions, and we will generalize the solution to an arbitrary number of dimensions. In the next section we show that if a monopole like configuration of a $U(1)$ gauge field is used for producing the spontaneous compactification of two extra dimensions in a sphere as in [7,11], a fine tuning between bulk and brane parameters is reintroduced once we consider the quantization condition for the gauge field. This happens because the quantization condition for the monopole configuration depends on the deficit angle of the sphere, so to obtain a small enough effective cosmological constant in 4D we must fine tune the 6D cosmological constant against the $U(1)$ charge and the brane tension (via the deficit angle). We will show that this problem is not present if instead of considering a two form with a vacuum expectation value in order to generate the required energy-momentum tensor we consider a four form with a Freund-Rubin configuration [12] in the bulk. In this case the fine tuning required in order to obtain flat 4D space does not involve brane parameters. In section 3 we will generalize the solution to an arbitrary number of dimensions. Making use of a four form field we will obtain solutions in which the space is the direct product of 4 dimensional Minkowski or de Sitter space ($dS_4$) times a $n$ dimensional sphere, $S^n$. In order to obtain Minkowski space at low energies we have to fine tune the $d = 4 + n$ dimensional cosmological constant with the vacuum expectation value (v.e.v.) of the four form field.

Again, we will be able to introduce a $(d-3)$-brane in this background with arbitrary tension and we will see that the effective 4D cosmological constant is independent of this tension, since the effect of the brane is to induce a deficit angle in the two dimensional transverse space. The tuning required in order to obtain flat 4D space is independent of brane parameters also in this general case, since it only involves the bulk cosmological constant and the four form v.e.v.. The brane will have four noncompact flat dimensions and $n-2$ dimensions compactified in a sphere $S^{n-2}$, wrapped around the original $n$-dimensional sphere. Section 4 is devoted to the conclusions.
2 Compactification in $S^2$ Using Two and Four Form Fields.

2.1 The 6D background solution.

We will start by briefly reviewing the basic features of the 6D model of \( \mathbf{8} \), without assuming a particular origin for the energy-momentum tensor. The metric ansatz assumes a factorizable geometry,

$$ds^2 = \gamma(x)_{\mu\nu} dx^\mu dx^\nu + \kappa(z)_{ij} dz^i dz^j.$$  \hspace{1cm} (1)

Lower case latin indices run over the two extra dimensions, greek indices over the four conventional ones and upper case letters over all the coordinates. The required form of the energy momentum tensor is

$$T_{MN} = - \left( \gamma_{\mu\nu} \Lambda_\gamma \kappa_{ij} \Lambda_\kappa \right).$$  \hspace{1cm} (2)

The solution is simply the direct product of 4D Minkowski or $dS_4$ space times a 2 sphere,

$$ds^2 = dt^2 - e^{Ct} d\vec{x}^2 - R_0^2(d\theta^2 + \beta^2 \sin^2 \theta d\varphi^2)$$  \hspace{1cm} (3)

where $\theta$ ranges from 0 to $\pi$ and $\varphi$ takes values in $[0, 2\pi)$. The constants $C$ and $R_0$ are given by

$$C^2 = -\frac{2}{3} \Lambda_\kappa, \quad R_0^{-2} = \frac{1}{2} \Lambda_\kappa - \Lambda_\gamma.$$  \hspace{1cm} (4)

In here and in the following we will take units in which the higher dimensional Planck mass is one, so powers of this mass should be understood where needed. If $\beta \neq 1$ there is a conical singularity at those points where $\sin(\theta)$ vanishes. This can also be cast as a deficit angle redefining the coordinate $\varphi$ so it ranges from 0 to $2\pi \beta$ and then the metric for this new coordinate system would take a conventional form (like (3) with $\beta = 1$). The Einstein equations will be satisfied in all the spacetime if we consider the presence of three branes at this locations with a tension given by \( \mathbf{7,8} \)

$$1 - \beta = \frac{T_0}{2\pi}.$$  \hspace{1cm} (5)
We see that if the compactification manifold is a sphere we need to consider configurations with two branes of equal tensions at antipodal points of the sphere. This could raise concerns since it can be viewed as a fine tuning involving the brane tensions. However, it is also possible to consider compactifications in the disk. The disk is the orbifold $S^2/Z_2$, obtained from the sphere under the identification of points in the northern and southern hemispheres that are symmetric under reflections through the equator. In this case no second three brane is needed, and it was shown in [8] that the matching conditions in the orbifold fixed line (the equator) are trivially satisfied, so we do not need to consider the presence of any four brane at these points that could reintroduce a fine tuning between brane and off-brane parameters [9]. The interesting feature of this model is that the fine tuning required to obtain flat 4D space ($\Lambda_\kappa = 0$, as can be seen from eq.(4)) does not involve brane parameters, and we can find solutions with flat 4D space for any value of the brane tension once we have made this tuning. The only effect of the brane is to induce a deficit angle in the sphere, without affecting the effective 4D cosmological constant. Supersymmetry is probably the best candidate for ensuring that the bulk theory provides an energy-momentum tensor with $\Lambda_\kappa \simeq 0$ after including radiative corrections if it is (approximately) preserved in the bulk, like in the supergravity models of [13,14]. In these models flat 4D space is a direct consequence of the SUSY 6D Lagrangian, with no fine tuning involved. They use a monopole configuration to compactify two extra dimensions in a sphere, and N=1 supersymmetry is preserved by the compactification. However, once we introduce a brane with nonzero tension, the flat 4D solution is no longer preserved for reasons similar to those explained in the next subsection, since the monopole configuration is affected by the deficit angle induced by the brane. If another 6D supersymmetric model is found that produces naturally flat 4D space before supersymmetry breaking, and the solution is not affected by a deficit angle in the transverse space, supersymmetry could be broken in the brane without affecting the effective cosmological constant$^2$.

$^2$In any complete model one should of course compute the transmission of supersymmetry breaking from the brane to the bulk and make sure that this symmetry breaking effects do not produce a cosmological constant in 4D that is too large.
2.2 Monopole vs. Four Form

In the previous section, as well as in [8], we did not assume a particular mechanism for generating the required v.e.v. of the energy-momentum tensor, eq.(2). Since we are decomposing the total 6D space in 4+2 dimensions, following [12], a natural candidate for generating this inhomogeneity would be the v.e.v. of a 2-form or a 4-form field strength. In [7,11] the two form option was chosen, and the resulting configuration is that of a magnetic monopole field. Adding a bulk cosmological constant and tuning its value against the two form v.e.v. one can find flat 4D solutions. Here we will show that once the quantization condition is imposed on the monopole field, since the 2-form v.e.v. depends on the deficit angle, the required tuning involves the brane tension, so the effective 4D cosmological constant does depend on brane parameters, spoiling the nice features of the model. This problem can be overcome using a 4-form field with a v.e.v. that will not depend on the deficit angle of the transverse space.

2.2.1 Monopole.

The Lagrangian in this case is that of Einstein-Maxwell theory with a cosmological constant

\[ S = \int d^6x \sqrt{-g} \left( -\frac{1}{2} R - \frac{1}{4} F_{MN} F^{MN} - \Lambda \right). \]  

(6)

The metric (3) will be a solution to the coupled Einstein-Maxwell equations in this theory if the assumed v.e.v. of the $U(1)$ gauge field is that of a magnetic monopole [11],

\[ A_M dx^M = B(\cos(\theta) \pm 1) d\varphi, \]

(7)

with $F_{MN} = \partial_M A_N - \partial_N A_M$. The plus and minus signs are needed to cover the upper and lower hemispheres of the sphere so the vector field is well defined everywhere. The necessity of a quantization of the constant $B$ can be seen by requiring that a single valued gauge transformation relates the two representations of the gauge field where they overlap, in the equator:

\[ A^+_\varphi = A^-_\varphi + \partial_\varphi \alpha(\varphi) \]

(8)
so $\alpha(\varphi) = 2B\varphi$. Now imposing that the gauge transformation $e^{i\alpha(\varphi)}$ is single valued we get \[\text{(9)}\]

$$B = \frac{n}{2g}$$

with $n$ an integer. The energy-momentum tensor we get from the action \[\text{(6)}\] with the ansatz \[\text{(7)}\] is

$$T_{MN} = \gamma_{\mu\nu} \left( \frac{n^2}{8g^2\beta^2R_0^2} + \Lambda \right) \kappa_{ij} \left( -\frac{n^2}{8g^2\beta^2R_0^2} + \Lambda \right).$$

\[\text{(10)}\]

In order to obtain flat 4D space we must fine tune the coefficient of $\kappa_{ij}$ above to zero, so the required fine tuning involves the deficit angle$^3$. Once this tuning is made for a value of the brane tension, any other value of this tension would generate an effective cosmological constant. The reason for this is clear: the monopole configuration knows about the deficit angle of the sphere since it obeys a topological quantization condition, and it reacts under changes in this parameter. So if we want to maintain the nice property that the effective 4D vacuum energy is independent of the brane tension we need another mechanism for generating an inhomogeneous energy momentum tensor.

### 2.2.2 Four Form Field Strength.

The difficulties we found in the previous construction can be overcome if we consider a four form field with a Freund-Rubin configuration \[\text{(12)}\] instead of the monopole. Four form fields appear naturally in supergravity theories and have been used in work related to the cosmological constant problem \[\text{(15,16)}\]. The action we take in this case is

$$S = \int d^6x \sqrt{-g} \left( -\frac{1}{2} R - \frac{1}{48} F_{MNPQ} F^{MNPQ} - \Lambda \right).$$

\[\text{(11)}\]

Again, the metric \[\text{(3)}\] is a solution of the field equations if the four form has a v.e.v. given by

$$F^{\mu\nu\lambda\sigma} = \frac{E}{\sqrt{-\gamma}} \epsilon^{\mu\nu\lambda\sigma},$$

\[\text{(12)}\]

$^3$We could also have used a coordinate system in which the parameter $\beta$ does not appear in the metric, but the maximum allowed value for $\varphi$ is $2\pi\beta$. In this case the factor of $\beta$ in eq.\[\text{(10)}\] would not come from the metric, but from the quantization condition eq.\[\text{(9)}\], that now would have a factor of $1/\beta$ in the right hand side.
where $\epsilon^{\mu\nu\lambda\sigma}$ is the totally antisymmetric tensor, $E$ is an arbitrary constant\(^4\) and indices run over the four conventional dimensions only. The rest of the elements of $F^{MNPQ}$ are zero. This ansatz solves the four form field equations and yields the following energy-momentum tensor

$$T_{MN} = \left( \gamma_{\mu\nu} \left( \frac{E^2}{2} + \Lambda \right) \kappa_{ij} \left( -\frac{E^2}{2} + \Lambda \right) \right). \quad (13)$$

By comparing this with the formulas of subsection 2.1 we see that the tuning required to obtain flat 4D space is now $E \simeq \sqrt{2\Lambda}$, that is independent of the deficit angle. Some mechanism is required to ensure this cancellation and its stability under radiative corrections to fully address the cosmological constant problem (as we have already commented a good candidate could be supersymmetric models perhaps along the lines of [13,14]). The important point now, and the reason why these models can be regarded as progress in finding an eventual solution to this problem is that once this tuning is made, the cosmological constant is zero for any value of the brane tension.

### 3 Generalizing to $d=4+n$ dimensions.

The four form has also another advantage with respect to the monopole configuration, and it is that the solution can be easily generalized to an arbitrary number of dimensions, keeping always four noncompact dimensions, and compactifying the rest in a $n$-dimensional sphere. The reason for this is that the Freund-Rubin configuration we are assuming for the four form distinguishes four dimensions from the rest and provides an inhomogeneous energy-momentum tensor that differentiates these four dimensions. In this way we can find solutions of the $d$-dimensional Einstein equations with four flat dimensions, while the rest are compactified in a manifold of constant curvature, like a sphere. In this background we will be able to introduce a $(n+1)$-brane with arbitrary tension, without affecting the 4D cosmological constant, since its only effect will be to produce a deficit angle in $S^n$. For the deficit angle mechanism to work we need a codimension two brane, so some of the dimensions of the brane will be compact, forming a $(n-2)$-dimensional sphere that wraps around the original $n$-dimensional sphere.

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\(^4\)In [16] it was argued that this flux has a quantization condition once we embed this construction in M theory. Whether this flux is quantized or not is not important for our purposes, since in any case the quantization condition would be independent of the deficit angle.
3.1 The Background Solution.

The starting point here will be the action

\[ S = \int d^4x \sqrt{-g} \left( -\frac{1}{2} R - \frac{1}{48} F_{MNPQ} F^{MNPQ} - \Lambda \right), \]  

(14)

where \( d = 4 + n \). We will look for solutions in which the total space is the direct product of two Einstein manifolds: 4D Minkowski or \( dS_4 \) times \( S^n \). The metric ansatz is again

\[ ds^2 = \gamma(x)_{\mu\nu} dx^\mu dx^\nu + \kappa(z)_{ij} dz^i dz^j, \]  

(15)

where now the lower case latin indices run over the \( n \) extra dimensions. The ansatz for the four form field is as in eq.(12), and it can be easily seen that it solves the four form equation of motion [12]. Now Einstein equations are

\[- \left( \frac{\gamma_{\mu\nu}}{4} R(\gamma) + \frac{1}{2} R(\kappa) \right) \kappa_{ij} \left( \frac{1}{2} R(\gamma) + \frac{n-2}{2n} R(\kappa) \right) = \left( \frac{\gamma_{\mu\nu}}{2} + \Lambda \right) \kappa_{ij} \left( -\frac{E^2}{2} + \Lambda \right). \]  

(16)

\( R(\gamma) \) and \( R(\kappa) \) are the curvatures of the respective submanifolds. It is easy to solve for these curvatures, we get

\[ R(\gamma) = -8 \frac{n-1}{n+2} \Lambda + 4 \frac{n-1}{n+2} E^2, \quad R(\kappa) = -2n \left( \Lambda + \frac{3}{2} E^2 \right). \]  

(17)

We can write the metric in the form

\[ ds^2 = dt^2 - e^{2Ct} d\vec{x}^2 - R_0^2 d\Omega_n^2 \]  

(18)

where \( d\Omega_n^2 \) is defined as

\[ d\Omega_n^2 = d\theta_{n-1}^2 + \sin^2 \theta_{n-1} d\Omega_{n-1}^2 \]  

(19)

with \( d\Omega_1^2 = \beta^2 d\varphi^2 \). All \( \theta_i \) range from 0 to \( \pi \), while \( \varphi \) goes from 0 to \( 2\pi \). The constants \( C \) and \( R_0 \) can be determined as

\[ C^2 = \frac{8}{3(n+2)} \Lambda - \frac{4(n-1)}{3(n+2)} E^2, \quad R_0^2 = \frac{2}{(n-1)(n+2)} \left( \Lambda + \frac{3}{2} E^2 \right). \]  

(20)
If we want flat 4D space we have to tune \( E = \sqrt{\frac{2A}{n-1}} \). It is interesting to note that we can tune the effective cosmological constant to zero using the four form v.e.v., that is not a parameter in the Lagrangian, but a dynamical variable \([15]\) (see also \([16]\)).

Notice that in the definition of \( d\Omega_1^2 \) we allowed for an arbitrary parameter \( \beta \) (a deficit angle in \( \varphi \)). Values of this parameter different from one will be associated with the presence of a \((n+1)\)-brane at the points with \( \sin^2 \theta_1 = 0 \) with nonzero tension, as we show in the next subsection.

### 3.2 Introducing \((n+1)\)-branes.

To show that the only effect of a \((n+1)\)-brane with nonzero tension in this background will be to induce a conical singularity in the two dimensional transverse space, without affecting the 4D Hubble expansion, we will follow \([8]\), and we will consider a regularization of a \( \delta \)-like brane placed at the points where \( \sin(\theta_1) = 0 \). We will solve Einstein equations in all the spacetime (that now will not have any singularity) and we will take the infinitely thin limit to obtain the relation between the deficit angle and the brane tension. Notice that for \( n > 2 \), the \((d-2)\)-dimensional submanifold defined as the points in which \( \sin(\theta_1) = 0 \) is connected, so we have to introduce a single brane in this background, while for \( n = 2 \) we had to introduce two 3-branes (at the northern and southern poles of the sphere) \([7,8]\). To be able to introduce a single brane when \( n = 2 \) it is necessary to consider compactifications in a disk \([8]\). The reason for this is that the points with \( \sin(\theta_1) = 0 \) form the manifold \( \mathcal{M}_4 \times S^{n-2} \), with \( \mathcal{M}_4 \) being 4D Minkowski or \( dS_4 \), and all \( S^n \) are connected except for \( S^0 \), that consists of just two disconnected points (if we define \( S^n \) as the set of points in \( R^{n+1} \) at unit distance from the origin).

So we add a term to the energy momentum tensor, \( \Delta T_{MN}(\theta_1, \epsilon) \), such that it is finite everywhere (when \( \epsilon > 0 \)), zero for \( \sin(\theta_1) > \epsilon \) and

\[
\lim_{\epsilon \to 0} \Delta T_{MN}(\theta_1, \epsilon) = -\left( \epsilon \frac{g_{ab}T_n}{2\epsilon \sqrt{\kappa}} \delta(\sin(\theta_1)) \begin{pmatrix} 0 \\ \end{pmatrix} \right),
\]

where the indices \( a, b \) run over all the dimensions except \( \theta_1 \) and \( \varphi \), and \( \kappa \) is the deter-

\[\text{\footnote{Alternatively we could compute the the Einstein tensor for the metric \([15]\), along the lines of \([17]\), and show that it has a term proportional to \( \delta(\sin(\theta_1)) \), that would be cancelled if we consider the contribution to the energy-momentum tensor of a \((n+1)\)-brane located at \( \theta_1 = 0, \pi \) with tension proportional to \( (1 - \beta) \).]}\]
ominant of the $2 \times 2$ metric in the two dimensional transverse space spanned by $(\theta_1, \varphi)$. In this way the energy momentum tensor in the right hand side of eq.(21) would be derived from a term in the action given by

$$\Delta S = \int d^d x \sqrt{|g|} \frac{T_0}{2 \pi \sqrt{\kappa}} \delta (\sin(\theta_1)), \tag{22}$$

that is, a $\delta$-like $(n+1)$-brane source (see the appendix of [18] for a discussion of delta functions in curved manifolds). When $\sin(\theta_1) > \epsilon$ the solution is given by eqs.(18,20), while for $\sin(\theta_1) < \epsilon$ we will consider solutions of the form

$$ds^2 = dt^2 - e^{Ct} d\vec{x}^2 - R_0^2 d\tilde{\Omega}_n^2 \tag{23}$$

where $d\tilde{\Omega}_n^2$ is defined again as

$$d\tilde{\Omega}_n^2 = d\theta_{n-1}^2 + \sin^2 \theta_{n-1} d\tilde{\Omega}_{n-1}^2$$ \tag{24}

but now $d\tilde{\Omega}_2^2 = d\theta_1^2 + \rho(\theta_1)^2 d\varphi^2$. The constants $C$ and $R_0$ are given again by eq.(20). It is always possible to consider regularizations of the brane such that one finds solutions of the form (23) for some $\rho(\theta_1)$. Einstein equations for this ansatz can be written as

$$- \left( g_{\alpha \beta} \frac{1}{\sqrt{\kappa}} (\rho(\theta_1) + \rho''(\theta_1)) \right)_{\alpha \beta} = \Delta T_{MN}(\theta_1, \epsilon). \tag{25}$$

It is clear that when $\Delta T_{MN} = 0$ the metric in eq.(18) (i.e. $\rho(\theta_1) = \beta \sin(\theta_1)$) is a solution. To obtain the matching condition that relates the brane tension with the deficit angle we integrate the previous equation in $\theta_1$ and take the limit $\epsilon \to 0$. For doing this we do not need to assume any particular regularization of the brane, since in the thin limit the result will be independent of the regularization used. We only have to take into account that $\rho(0) = 0$, $\rho'(0) = 1$ (any other value would produce a conical singularity and we are assuming that the regularized solution is regular everywhere) and when $\sin(\theta_1) > \epsilon$ the solution is given by eq.(18). Doing this and taking the thin limit we get

$$1 - \beta = \frac{T_0}{2 \pi}. \tag{26}$$

The brane will have four noncompact dimensions and $n-2$ dimensions compactified in $S^{n-2}$ with a radius given by eq.(20). This implies that when $n > 2$, since Standard
Model fields should be able to propagate in all the dimensions of the brane, $R_0$ has to be small, of the order of $TeV^{-1}$, to avoid light KK states of observable particles. A solution to the hierarchy problem making use of large extra dimensions like in [1] is only possible in this scenario when $n=2$.

4 Conclusions

In this paper we continued the exploration of the scenarios proposed in [7,8]. We have shown that if a monopole like configuration of a $U(1)$ gauge field is used to compactify the two extra dimensions in a sphere like in [7,11], the fine tuning required in order to obtain a small 4D vacuum energy involves brane parameters, spoiling the nice features of the model. If a four form field strength is used instead, the required tuning is completely independent of the brane tension, and so is the effective 4D cosmological constant. It would be very interesting to embed this scenario in a supersymmetric theory, since the required tuning could be the consequence of supersymmetry, like in [13,14].

We also generalized the solution to an arbitrary number of dimensions, keeping four noncompact dimensions and compactifying the rest in $S^n$. We have seen that it is also possible to introduce a $(n+1)$-brane with arbitrary tension in these backgrounds, and the effective 4D cosmological constant is not affected by this tension.

In this letter we have not addressed the important issue of the stability of the solutions. A complete analysis of the spectrum of fluctuations is beyond the scope of the present work, but we will make some remarks about this subject. It is generally nontrivial to check whether a spontaneously compactified solution is stable or not in general relativity [10,19,20,21,22]. Recently an analysis of the fluctuation spectrum of the model presented here for $n = 2$ was carried out in [10], where it was concluded that the model is stable. However, it has been shown in [22] that this kind of solutions can be unstable for certain values of the gauge flux and certain dimensionalities. In any case, we regard these constructions as intermediate steps towards finding a more complete theory that addresses the problems left here. For instance, as we have said

\[\text{As we have commented previously, once we introduce a brane with nonzero tension in this model, solutions with flat 4D space are no longer preserved, for reasons similar to those explained in section 2, since a monopole configuration of a } U(1) \text{ gauge field is used to obtain the spontaneous compactification in } S^2.\]
before, it would be desirable to embed these solutions in supergravity, since this is the most promising way to cancel the bulk contributions to the effective 4D cosmological constant and this could change the stability properties of the solutions (one could argue that for the better). Also, a new possible source of instability when \( n > 2 \) corresponds to deformations that shrink the size of the extra dimensions of the brane to zero (this is always possible since we know from homotopy theory that \( \pi_{n-2}(S^n) \) is trivial). In any particular model it would be necessary to check that the modes associated with these deformations have a positive mass squared.

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Note Added

After this work was finished ref.\[23\] appeared, where the introduction of branes in the SUSY model of \[13,14\] was considered. In this reference it was also noticed that the introduction of branes affects the solution and generates an effective 4D cosmological constant proportional to the deficit angle in the simplest realization of the model due to the quantization condition of the gauge field (section 4.2). However, we have shown here that this problem is absent if we consider a four form field strength instead of the monopole to spontaneously compactify the extra dimensions in a sphere.

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