Placing hidden properties of quantum field theory
into the forefront:
wedge localization and a critical look at past
S-matrix attempts

To the memory of Hans-Jürgen Borchers (1926-2011)

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Abstract

Recent progress about "modular localization" reveals that, as a result of the S-Matrix in its role of a "relative modular invariant of wedge-localization, one obtains a new non-perturbative constructive setting of local quantum physics which only uses intrinsic (independent of quantization) properties. The main point is a derivation of the particle crossing property from the KMS identity of wedge-localized subalgebras in which the connection of incoming/outgoing particles with interacting fields is achieved by "emulation" of free wedge-localized fields within the wedge-localized interacting algebra. The suspicion that the duality of the meromorphic functions, which appear in the dual model, are not related with particle physics, but are rather the result of Mellin-transforms of global operator-product expansions in conformal QFT is thus confirmed. The connection of the wedge-localization setting with the Zamolodchikov-Faddeev algebraic structure is pointed out and an Ansatz for an extension to non-integrable models is presented.

Modular localization leads also to a widening of the renormalized perturbation setting by allowing couplings of string-localized higher spin fields which stay within the power-counting limit. This holds the promise of a Hilbert space formulation which avoids the use of BRST Krein-spaces.
1 Introduction

The course of quantum field theory (QFT) was to a large extend determined by three important conceptual conquests: its 1926 discovery by Pascual Jordan in the aftermath of what in recent times is often referred to as the Einstein-Jordan conundrum [1] [2], the discovery of renormalized perturbation in the context of quantum electrodynamics (QED) after world war II, and the non-perturbative insights into the particle-field relation initiated in the Lehmann-Symanzik-Zimmermann (LSZ) work on scattering theory, its derivation from first principles [6], as well as its application to the rigorous derivation of the particle analog of the Kramers-Kronig dispersion relations and the subsequent successful experimental test of QFT’s foundational causality principle. Their experimental verification strengthened the trust in the causal localization principle of QFT. The later gauge theory of the standard model resulted from an extension of the ideas which already had led to QED. Besides some successes it led to most of the still open problems of actual research.

Jordan’s observations in his dispute with Einstein [2] led to an extension of quantization to matter waves, but its main point, the thermal character of fluctuation in a vacuum state restricted to the local observables of a subvolume which is needed did not receive the conceptual attention which, being a characteristic property which distinguishes QFT from quantum mechanics (QM), it would have deserved. It could have revealed itself as a Gedankenexperiment of the kind which many decades later was proposed by Unruh [3] [4]. Both Gedankenexperiments demonstrate a thermal consequence of causal localization whose early comprehension could have changed the path of QFT history. When Jordan’s incomplete calculation was published as a separate section in the famous 1926 Dreimännerarbeit with Born and Heisenberg, his coauthors had some reservations since it contained problematic aspects which had no place in the previously discovered QM, but which they were not able to clearly formulate and communicate.

Only several years later Heisenberg challenged Jordan in a letter about a missing logarithmic term $\sim -\ln \varepsilon$ in his calculation of the fluctuation spectrum of the 2-dimensional chiral QFT which was Jordan’s favorite model [1], where $\varepsilon$ is a length which characterizes the ”fuzzyness” (deviation from sharp localization) at the endpoints of Jordan’s localization interval. This was the birth of Heisenberg’s discovery of vacuum fluctuation near the localization boundaries with $\varepsilon$ the ”attenuation length” (“fuzzyness” of localization-boundary) conceded to the vacuum polarization cloud.

In his famous paper which he wrote after challenging Jordan, Heisenberg showed that the localization of dimensionless quantum charges (”partial charge”) in QFT behaves quite different from partial charges in QM, for which localization

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1This chiral current model ("2-dim. photon") was the only QFT which Jordan ever used in computations, in particular for his "neutrino theory of light" which was nothing more than what nowadays is called bosonization/fermionization [7]. The prevalent incorrect idea was that QFT like QM does not depend on spacetime dimensions in an essential way which is only correct in QM but breaks down in QFT as soon as the spin $s \geq 1$ (see later).
in the sense of Born is well-defined and complies with naive intuition. He discovered that partial charges lead to a proportionality in terms of a dimensionless area $\frac{\text{area}}{\varepsilon^2}$ which amounts to a quadratic divergence in the sharp boundary limit $\varepsilon \to 0$ (and to the logarithmic divergence in Jordan’s two-dimensional model). With the phenomenon of vacuum polarization Heisenberg exposed one unexpected characteristic consequence of causal localization which separates QFT from QM.

The other inexorable epiphenomenon of localization is “thermalization”, i.e. the fact that the restriction of the global pure vacuum state to observables which are localized in a causally completed spacetime region turns it into an impure state on all local measurements in that region; in fact this impure state is thermal in the sense that it has the KMS property[3] with respect to an intrinsically defined ”modular” Hamiltonian (section 3) which is not necessarily the same as that measured by a thermometer or by the time which describes a Hamiltonian evolution in some non-inertial system.

This knowledge was not accessible at the time of the Einstein-Jordan dispute, but in retrospect it is clear that it would have been the most important conceptual aspect of a complete solution of the so-called Einstein-Jordan conundrum. It is well-known that Einstein steadfastly rejected Born’s assignment of probability to individual events, but it is difficult to imagine that he would have refused a probability as an intrinsic property of ensembles in a KMS state, in particular if it would have been clear to him that this is an unavoidable consequence of the quantum adaptation of Faraday’s and Maxwell’s ”action at the neighborhood principle” and his own achievement of its Minkowski spacetime causality reformulation.

In fact vacuum polarization at the causal boundary, leading to the aforementioned $\varepsilon \to 0$ divergence of the dimensionless area factor in the localization-caused entropy, as well as to the thermal manifestation are two sides of the same coin. In Jordan’s model of QFT the ”localization-thermality” is not only an analogy to Einstein’s statistical mechanics calculation for black body radiation, but it is actually isomorphic to a global heat bath thermal situation; i.e. in this special situation there is a kind of inverse Unruh picture which permits to construct a global heat bath system (Einstein’s side) to Jordan’s localization-caused fluctuations in an interval on a lightlike line associated within his chiral model (which he referred to as the wave quantization of a two-dimensional Maxwell field).

A glance at a recent review of the early work on QFT [1] reveals that even nowadays many authors firmly believe that the quantum mechanical result that a global vacuum passes to a local vacuum (the vacuum of QM factorizes) continues to be true in QFT and that only by coupling the QFT to an external heat bath (applying the rules of statistical mechanics) can the Einstein-Jordan conundrum be fully understood. This incorrect belief is supported by referring to QFT as ”relativistic QM” (with possibly infinite degrees of freedom), a term

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[1] In QM this property only arises in the thermodynamic limit of Gibbs states. The issue of heat bath thermality versus localization caused thermality is at the heart of the Einstein-Jordan conundrum.
minology which unfortunately has entered many articles and even textbooks on QFT. But infinite degrees of freedom as a result of the second quantization formalism applied to QM leading to Schrödinger quantum fields do not change the physical content, and the correct distinction based on fundamental differences about localization is considerable more subtle.

It is an important question to ask why such a foundational aspect of QFT was only understood several decades after it first became visible in particular situations as the Unruh Gedankenexperiment and the issue of quantum matter behind (or in front of) black hole event horizons. The reason has to do with the formulation of renormalized perturbation theory. Whereas the pre-Tomonaga perturbation formalism (which was modeled on the quantum mechanical oscillator formalism) failed whenever vacuum polarization properties entered the higher order perturbative calculation, the covariant formulation, combined with recipes involving ultraviolet cutoffs or regularizations and their final removal through renormalization prescriptions in the works of Tomonaga, Feynman, Schwinger and Dyson, shaped the form of renormalized perturbation. Its first triumph in quantum electrodynamics and subsequently in the more general gauge theoretic formulation of the standard model have made QFT the most successful albeit unfinished theories of particle physics.

A detailed understanding of the derivation of these perturbative rules in terms of the underlying causal localization principle (which could have led to a better understanding of the consequences of causal localization) was not really necessary; the Epstein-Glaser iterative implementation of this principle (“causal perturbation”) has remained little known; renormalized perturbation does not require a foundational understanding of the causal localization principle of QFT, a working knowledge of renormalized perturbation theory in terms of calculational recipes suffices.

The situation changes radically if one comes to problems for which the covariance of the formalism is of not much help, as the Einstein-Jordan conundrum which addresses fluctuations from subvolume localizations. Even for the simplest of all theories, namely that generated by a free scalar covariant field, this is anything but simple. Since it cannot be exactly solved, it is necessary to secure that the approximation is in agreement with the ”holistic” aspects of causal locality in a more direct way, covariance alone does not help. Such aspects are easily overlooked in unguided quantum mechanical calculations based on global oscillator degrees of freedom as in [1]. This problem has its actual counterpart in the impossibility to understand the cosmological constant by occupying global energy levels of particles and enforcing the finiteness of the result by a Planck length cutoff.

The use of the new covariance property was sufficient to liberate the old quantum mechanical based perturbation theory from problems caused by vacuum polarization but misunderstandings about causal quantum localization continued in areas of S-matrix-based particle theories, notably the dual model.

\footnote{In old textbooks of QFT (Heitler, Wenzel) the limitation of the old quantum mechanical inspired formalism is visible in its restriction to terms without vacuum polarization contribution (tree graphs).}
and string theory. It is the main concern of this paper to explain this in detail. Whereas the understanding of these central issue in QFT after its birth in the aftermath of the E-J conundrum was for a long time incomplete, it needed the appearance of the dual model and ST to arrive at it genuine misunderstandings with grave consequences for particle theory (of which only a science outside the natural sciences as mathematics could profit).

Before getting to this issue, we have to recall some basic facts about why particle physicists at the end of the 50s became increasingly interested in attempts to access particle theory through a more direct use of the S-matrix and on-shell approximation methods.

The rather limited, but within its self-proclaimed aims very successful project, resulting from the particle physics adaptation of the (Kramers-Kronig) dispersion theory, led to a revival of older (by that time already abandoned) ideas to formulate particle theory directly in terms of its most important experimentally accessible object: the scattering matrix $S_{\text{scat}}$. All post dispersion theory S-matrix attempts were based on an important new property which had arisen from the setting of dispersion theory as well as from Feynman’s perturbation theory: the use of analytic properties of scattering amplitudes, in particular the analytic continuation of the crossing identity. The first such scheme, the S-matrix bootstrap, was already given up after one decade. The reason was not that any of its basic assumptions turned out to be incorrect, but rather because they resisted a coherent operational formulation which could be the starting point of controlled approximations. Another reason was the strong return of gauge theories, in particular in connection with the property of asymptotic freedom for strong interactions.

For some years S-matrix ideas continued to be of phenomenological interest in connection with a conjectured Regge behavior in certain regimes of scattering amplitudes of strong interactions. However what brought these ideas to an almost 5 decades lasting dominance of string theory was not phenomenology, but rather a mathematical observation by Veneziano that by a clever use of properties of gamma functions it is possible to construct a meromorphic crossing symmetric function of two variables with infinitely many poles in each variable interpreted as particles in an approximation of a new S-matrix theory which was expected to result from an ill-defined unitarization of that "would-be" one-particle approximation called dual model. Apart from the presence of infinitely many pole-terms and the absence of threshold cuts (whose presence turn out to be important for the meanwhile understood true particle physics crossing), this was what Mandelstam expected in his setting of two-variable spectral representations for elastic scattering amplitudes. In any case, the sanctioning by Mandelstam was very important for the increasing popularity of this point of view.

This, at that time quite impressive mathematical construct of a dual model, was interpreted as a lowest order crossing symmetric solution which still has to

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4One may also add the entertainment industry, whose link to natural sciences has been revolutionized in programs directed by Brian Green.
be subjected to an iterative "unitarization". This happened at a time when the conceptionsal origin of the true particle crossing for $S$ and particle formfactors was not yet sufficiently understood. It was seen as a concretization of spectral representations for the elastic scattering amplitude, a project which Mandelstam had previously formulated in an attempt to find a more specific setting beyond the generalities of the bootstrap setting. Its name "dual model" referred to this incorrectly identified crossing property unfortunately with particle crossing in a new (allegedly unique) $S$-matrix setting. The uniqueness was vetoed by several subsequently found dual models, but reinstituted by the requirement of string theory which added the requirement that the "mass spectrum" should come from a positive energy representation of the Poincaré group on the oscillator degrees of freedom contained in the Polyakov action (the square of the Nambu-Goto action); This then led to the unique ten dimensional superstring representation.

The dual model and its extension into string theory appeared much too sophisticated for its phenomenological use in scattering theory of strong interactions. Discrepancies with experimental scattering data and a return of Yang-Mills gauge theories in connection with strong interactions led nearly to its abandonment before it was elevated to a foundational $S$-matrix theory of particle physics. The important point which led to this promotion was the presence of an infinite particle spectrum which, in addition to most of the observed particles also contained zero mass $s=2$ "graviton" together with the promise of an ultraviolet converging approximate description of an Einstein-Hilbert like quantum interaction. Never before in the history of physics any model has been subjected to such a big interpretive jump over so many orders of magnitudes as that of the change of string tension for strong interacting "Regge strings" to the string tension required to describe Planck length physics in a hypothetical quantum gravity.

The social success of such a gigantic jump does not depend on cohesive physical arguments but on the size of the community willing to accept it and the reputation of some of its more charismatic supporters. It was clear that once accepted by part of the particle physics community, this would be the first theory which was under total protection umbrella against observational annoyances. Busting the conceptually tight corset of QFT, ST and its derivatives ("target" embedding of lower dimensional source theories into higher dimensional QFTs, the use of extra dimensions and the claim that Kaluza-Klein dimensional reductions of classical theories commutes with quantization) became a popular area of research in which community protection permitted to ignore restrictions imposed by the successful principles of particle theory.

To a large degree its popularity draws on its commitment to classical mathematical-geometric ideas which are not burdened with the subtle epiphenomena of quantum causal localization as vacuum polarization on its boundaries and thermal manifestations of the localized vacuum state (e.g. topological Lagrangians as the WZW action). In contrast to QFT, which had its problems with interactions

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5 In contrast to the derived Jost-Lehmann representation for matrix-elements of products of two fields which was the rigorous basis of the derivation of the dispersion relations, Mandelstam’s spectral representation was assumed.
involving higher spins and only presently is in the process of finding ways out by using the possibilities of short-distance reducing properties of semiinfinite string-localized fields offered by modular localization, ST became the candidate for the millennium "theory of everything" (TOE).

Although there are by now increasing doubts even within the ST community about whether this enormous popularity was scientifically justified (so that a critical review like this may only support an anyhow ongoing critical trend), it is not the purpose of this article to join the increasing community of individuals who criticize string theory for its lack of success. Rather the main aim is to expose the conceptual contradictions string theory always had with the local quantum physical principles of particle theory and which in normal times would have prevented its ascent. The only potential advantage of its fading popularity in the present context is the small chance that its proponents may be more open to foundational physical critique (instead of propagandistic mudslinging).

Particle theorists often abandon a theory which, after many years of exploration, failed to make contact with laboratory experiments or astrophysical observations. This is not helpful for those who are left behind and who would like to know whether the defeatist stance of their colleagues was mainly resulting from their impatience with a deep theoretical, but physically less successful project, or whether there were deeper more foundational reasons within its conceptual structure which, although difficult to formalize, activated their physical gut feelings telling them that it is time to leave. There is of course the additional personal problem of abandoning part or all of one’s own personal history; for people who dedicated more than 3 decades of their scientific life exclusively to one project, one seems to be asking too much. A theory which had survived for such a long time can (and may be should) not really disappear without a trace, at least historians and philosophers of science will be curious to understand what happened during all that time and what finally contributed to its disappearance.

Since the reasons for the loss of popularity are, unlike the abandonment of the S-matrix bootstrap, not related to the emergence of a successful new theory, a critical foundational review which unearths intrinsic fault lines may even point into new research directions of particle theory.

ST fails on all counts of what its supporters claim it is. What it really represents, namely a special collection of infinitely many oscillators associated to the canonical quantization of the Polyakov action which carries a 10-dimensional positive energy representation of the Poincaré group that is generated by an infinite component point-localized wave function (resulting in an infinite component second quantized pointlike field), is not what can be subsumed under the heading of ST. What its protagonists wanted to see, namely spacetime strings in form of a chiral QFT on a lightlike line (or its circular compactification) as a "source theory" embedded in its own "target space" (alias inner symmetry...)

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6"Normal" are times in which the ongoing computations are on par with their conceptual understanding. "Shut up and calculate" on the other hand is characteristic of the Zeitgeist of ST. Without ST, public relation and entertainment activities on metaphoric subjects as extra dimensions (Lisa Randall) and parallel universes (Brian Green) would not not have appeared.
space), contradicts the quantum principle of causal localization.

One may find the result that the oscillators in the supersymmetric extended Polyakov action can carry a (unique, highly reducible) positive energy representation of the Poincaré group in precisely 10 dimensions highly remarkable, but unlike ST, the peculiarity should be explained in terms of special properties of certain (non-rational) chiral models. It is indeed hard to believe that ST could have attained its popularity on this dry fact which even contradicts its name. Contrary to particle physicist who reject ST on their physical instincts, the author draws his fascination with this subject from the fact that ST fails because the birth-defect of QFT, which was related to the incomplete understanding of the Einstein-Jordan conundrum and which left renormalized perturbation theory largely unscathed, finally led to deep misunderstandings about the dual model and ST.

As often in the history of physics (ether theory,...) the resolution of deep misunderstandings is the source of rapid progress; in section 3 and at the end of the paper there will be some indications that the correct understood string localization of quantum fields (not embedding of QFTs) maybe the way out of the present stagnation of gauge theory. As it is well-known pointlike quantum fields of higher spin \((s \geq 1)\) cause problems with renormalizability. At the bottom of this problem is a clash between the Hilbert space structure (positivity) and causal localization. Lagrangian quantization resolves this clash by working in Krein spaces (Gupta-Bleuler, BRST) and returning to physical pointlike localized subsystems by enforcing BRST gauge invariance at the end of the calculation. Using string-localized potentials in an extended form of causal perturbation theory, one stays within the Hilbert space setting by giving up unreasonable restrictions on localization which come from Lagrangian/functional quantization but have nothing to do with an intrinsic understanding of QFT.

It may be interesting to the reader to familiarize himself with another point of view which has nothing to do with strings but collects in a nutshell what can be rescued from a mathematical observation made by string theorists. This is the solution of an old problem posed by Majorana: find an irreducible algebraic structure which carries a (reducible) positive energy representation of the Poincaré group whose decomposition leads to an interesting one-particle spectrum. He was obviously thinking of the \(O(4,2)\) spectrum of the hydrogen atom. This topic was taken up again in the beginnings of the 60s by Barut, Kleinert, Fronsdal,... The Ansatz in terms of a noncompact group algebra which extends the Lorentz group failed, and the whole project was abandoned. The irreducible system of oscillators in the Polyakov action which leads to the superstring one-particle positive energy representation of the Poincare group is the only known unique (up to a finite number of M-theoretic variations) solution. It is easy to see that one needs a chiral model with a continuous superselected

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7 Even the most hardened reductionist would probably shun away from inverting his expectation that a foundational theory should be rather unique in the sense of permitting no similar theories in its neighborhood. But it is precisely this inversion (rare, unique \(\rightarrow\)fundational) which leads string theorists to believe that we are living on a dimensionally reduced 10-dimensional target space of a conformal QFT.
charge spectrum in order to encounter noncompact group representation as the Poincare group acting on its inner symmetry ("target") space; multicomponent abelian current algebras are the only known models which fulfill these requirements. The former interest in this old project of finding "dynamic"\textsuperscript{8} infinite component field equations is nowadays difficult to convey since these ideas originated at a time long before the subtleties of the particle-field relation in the presence of interactions were appreciated.

Sometimes one can spot the origin of misunderstandings without much effort. When the Lagrangian of a classical relativistic particle $\sqrt{ds^2}$ (see next section) is presented as a "warm up" for a covariant quantum string theory \cite{11} which was expected to result from the quantization of a Nambu-Goto action, the conceptual alarm bell should have started ringing. It was one of the great achievements of Wigner to understand that relativistic particles cannot be described by quantization of a classical covariant particle action, but one rather has to resort to representation theory of the Poincaré group and even that only works in the absence of interactions. The only covariant description which has (asymptotically attached) interacting particles is interacting QFT. Hence why should one believe that covariant string-localized analogs of particles can be obtained from a quantization of the Nambu-Goto action?

To present the problem the other way around: there simply do not exist covariant 4-component covariant operators $\tilde{x}_\mu^\text{op}$ whose spatial components are quantum mechanical position operators. This is well-known and easy to verify by looking at the spectral projections $E$ from the spectral theory of selfadjoint operators

$$\tilde{x}_\mu^\text{op} = \int \tilde{x} dE_\tilde{x}, \ R \subset \mathbb{R}^3 \rightarrow E(R)$$

$$U(a) E(R)U(a)^{-1} = E(R + a), \ E(R)E(R') = 0 \ for \ R \times R'$$

$$(E(R)\psi, U(a)E(R)\psi) = (\psi, E(R)E(R + a)U(a)\psi) = 0$$

where the second line expresses translational covariance and orthogonality of projections for spacelike separated regions. In the third line we assumed that the translation a shifted $E(R)$ spacelike to itself. But since $U(a)\psi$ is analytic in $\mathbb{R}^4 + iV^+ \ (V^+ \ forward \ light \ cone)$ as a result of the spectrum condition, $\|E(R)\psi\|^2 = 0$ for all $R$ and $\psi$ which implies $E(R) \equiv 0$ i.e. covariant position operators do not exist. Hence this analogy is counterproductive for strings based on the quantized Nambu-Goto action. What the correct quantization of the latter really gives has nothing to do with what string theorist expect nor is it a dynamic infinite component solution of the Majorana problem (next section).

QFT was born in the aftermath of the Einstein-Jordan conundrum \cite{12} with an insufficient awareness about the consequences of causal localization \cite{7, 4}. The reason why this is relevant in connection with string theory is that whereas the

\textsuperscript{8}Referring to the requirement that the infinite $(m,s)$ particle tower should arise naturally from a larger indecomposable algebraic structure.

\textsuperscript{9}This dispute led Jordan to the discovery of QFT (at that time matter-wave quantization) \cite{12, 11, 7}. 12
incomplete understanding of the deeper layers of causal-localized QT did not really cause problems with the recipes of renormalized perturbation theory, this is the first time that it had serious consequences. It required the appearance of the dual model and ST and its widespread uncritical acceptance to cause a real derailment in particle theory. There remains of course always the hope that even at this late hour the comprehension and correction of deep misunderstandings may lead to new hints for future directions in particle theory.

Since the correct formulation and conceptual understanding of S-matrix-based ideas is pivotal for the present work, a substantial part will be dedicated to the presentation of particle crossing which, besides scattering theory, is the foundational link between particles and fields (as the generators of local observables). The crossing identity of particle theory and its analytic continuation is one the deepest and subtlest relations between particles and fields. Its derivation from causal localization requires an intrinsic understanding of the latter. This has been achieved in the local quantum physical (LQP) setting of QFT in the form of modular localization as a part of modular theory of operator algebras (next two sections). It is an important property in any new constructive approach to QFT which replaces the functorial relation between Wigner’s intrinsic representation theory of particles and free fields its functorial relation with free fields by an S-matrix based construction of generators of wedge-localized algebras which results from the fact that the S-matrix is a relative modular invariant of wedge-localization.

Without such a conceptual investment it would be hard to understand at what point Mandelstam’s courageous project to attribute a constructive role to the S-matrix (leaving aside all references to Lagrangian/functional quantization and other parallelisms to classical physics) eventually failed; in fact some of the aspects of the new ”top-to-bottom” approach in section 3 may be interpreted as a resurrection and extension of those old on-shell ideas (S-matrix, formfactors) which in the 60s served as an antidote against the (at that time perceived) threat of the ultraviolet catastrophe in off-shell QFT in terms of correlation functions of fields.

Whereas his idea to use spectral representations for scattering amplitudes as a means to control their analytic properties was still well within the spirit of the time, his embracing of Veneziano’s mathematical guesswork on crossing trapped him into a wrong type of crossing which, as we know nowadays, has nothing to do with particle theory. It rather describes the crossing property of Mellin transforms of conformal QFTs, i.e. of theories of scale invariant anomalous dimension QFT which (apart from conformal free fields) do not describe particles (next section). Insufficient understanding of the quantum aspects of causal localization for particle crossing properties did not leave much of a chance to get out of this conceptual trap at that time.

S-matrix-based settings have a long and eventful history, but that of LQP, an intrinsic formulation of QFT which shares with Mandelstam’s S-matrix set-

\footnote{Intrinsic” in the present setting means: independent of the classical parallelism on which Lagrangian quantization is based.}
ting the abstention from quantization analogies with classical field theories, is even longer. It started with Haag’s 1957 attempt to base QFT on intrinsic principles instead of linking a more fundamental theory via a quantization parallelism to a less fundamental one. Hence the terminology LQP stands for a different formulation of QFT which maintains its physical content. Another setting which also did not refer to quantization was Wightman’s formulation of quantum fields in terms of operator-valued Schwartz distributions and their correlation functions.

Both ideas, Mandelstam’s as well as Haag’s (with important early contributions by Borchers [6]), were top to bottom approaches in the sense that one states initially those properties which could be helpful for particle theory before looking for computational tools to implement them. The main difference was that Mandelstam’s proposal was based on the S-matrix, which from the viewpoint of local quantum physics is the observational crown of a QFT (and not its foundation).

Haag’s local quantum physics (LQP) approach was modeled on the enormous successful action at the neighborhood principle of Faraday and Maxwell which he reformulated as a principle of causal localization of quantum matter. Its formulation required a lot of modern (in parts unknown at that time) mathematics. Different to Mandelstam, LQP had no use of the S-matrix as a computational tool. Often the intuitive content of an idea is partially lost, while its mathematical formulation became increasingly precise; this seems to be the fate of all foundational concepts, and the idea of modular localization, which leads to most of the results in this article, is no exception. It is precisely this idea which leads to a completely new constructive role of the S-matrix.

In this paper it will be shown that there is indeed a quite unexpected synthesis of the two views. It is based on the recognition that, besides its prominence in the large timelike asymptotic behavior of scattering theory, the S-matrix is also a "relative modular invariant of wedge localization" [18]. This leads to new nonperturbative dynamical ideas in LQP into which $S_{\text{scat}}$ enters on par with other foundational properties [19]. A first success of this new setting is the existence proof of a particular family of two-dimensional models (factorizing models) with a realistic (noncanonical) short distance behavior [20].

Both attempts, Haag’s and Mandelstam’s, avoid the classical-quantum parallelism of Lagrangian quantization which constitutes the basis for the standard perturbation theory; in Lagrangian quantization one starts from the Lagrangian formulation of classical field theory and explores what one gets by following the quantization rules and imposing reasonable interpretations on the computational results. The same renormalized perturbation theory (but without intermediate cutoffs or regularizations) can be derived from an iterative implementation of causal locality (the Epstein-Glaser approach [22]) starting from a Wick-ordered polynomial in pointlike free fields which specifies the form of the interaction. An on-mass-shell perturbative calculation of the S-matrix only

\footnote{The original version is in French even though most of the talks were in English. Later it was translated back into English in [14].}
(without introducing other on-shell objects) is not possible; only after having computed correlations one can study their restrictions to mass shells.

My own contribution \[18\][19] consisted in trying to bring these two nonperturbative ideas together. As mentioned before, this led in the hands of Lechner \[20\] to an existence proof for certain \(d=1+1\) models with noncanonical short distance behavior. In this way the extensive work on the bootstrap-formfactor project, in which formfactors of \(d=1+1\) factorizing models were successfully calculated on the basis of recipes (for recent review see \[23\]), received a solid conceptual-mathematical underpinning. Another result of this intrinsic way of dealing with QFT is a foundational understanding of the issue of integrability \[24\] and ideas about how to approach nonintegrable theories (which includes all observational relevant models in particle physics).

There is a well-founded hope that an existence proof and a controlled scheme of approximations for the general case may come out of these attempts.\[12\] Far from being a chill pill of finnicking axiomatists, there is good reason to expect that such a step will finally lead to such a conceptual closure of QFT, a role which unfortunately the divergent series of renormalized perturbation theory cannot play.

The incorrect ideas which led to the dual model and string theory took the form of three different but interlinked proposals. The first one is the dual model. There are many dual models; for each conformal QFT there is one, independent of its space time dimension \[25\]. Their definition in terms of Mellin transforms shows that the pole spectrum of the meromorphic functions is given by the anomalous dimensions of (composite) operators appearing in convergent global operator expansions of products of conformal fields; it has nothing to do with the particle spectrum of an approximand of the two-particle scattering amplitude. The relation of Mellin transforms to states and Hilbert spaces is totally different from that of scattering amplitudes. The representation theory of the Poincaré group plays no role in this construction of meromorphic functions.

The second idea is based on the canonical quantization of the Nambu-Goto Lagrangian resulting from ignoring the square root. This approach is more ambitious since it does not only aim at the crossing in terms of poles of particle masses but also at a unitary positive energy representation of the Poincaré group related to the pole positions. This led to a system of oscillators (apart from a quantum mechanical \(p,q\) zero mode) on which one may represent a highly reducible unitary positive energy representation of the Poincaré group in a 10-dimensional spacetime: the famous superstring representation (unique up to a finite number of M-theoretic modifications). The space generated by the oscillators contains in addition to the degree if freedoms which are used to built up the generators of the Poincaré group also operators which connect the different levels of the infinite mass/spin tower. The field which arises from second quantization of the wave function space is a pointlike-localized dynamical infinite component field.

\[12\] Neither in case of classical nor for quantum mechanical nonintegrable systems one can hope for more.
finite component Wightman field (operator-valued Schwartz distribution).

The third construction gives a maximum of insight. It consists in starting from a non-rational (continuously many superselection rules) chiral theory. The concrete model which was used for this purpose is the model of an n-component abelian chiral current. Defining a sigma-model field through the exponentials of the potentials of the currents, one then may ask whether it is possible to have noncompact inner symmetries acting on the index space of the potentials. Surprisingly the answer is indeed positive; it is perfectly possible to represent noncompact inner symmetries acting on the index space of such non-rational chiral theories (referred to as "target space" by string theorists). In fact one can even represent the Poincaré group; and the only real surprise is that on the multi-component chiral current theory there exists a positive energy representation of the Poincaré only for n=10, and it is rather unique (up to a finite number of "M-theoretic" modifications). This construction has the closest relation to the modular localization of the next section. This quantum mechanical representation of the Poincaré group does not contain representations with "infinite spin" which are the only Wigner representation which upon second quantization would lead to semiinfinite spacelike string-localized representations. Hence this representation does not contain stringlike localized components and hence is point-like generated.

This third path to string theory is most revealing because it shows that the quantum mechanical positive energy representation theory of the Poincaré group handles the zero mode of the Fourier decomposition of the chiral current theory in a different way as required by the pointlike nature of chiral model. In other words the pointlike nature in "source" space is not compatible with this representation of the Poincaré group in target space. This is one of the reasons why there is no embedding of the chiral theory in the target theory in a literal sense. A deeper reasoning shows that the holistic nature of localization in QFT [5], in contrast to the Born localization of quantum mechanics, never allows embeddings of lower dimensional into higher dimensional theories; neither is a Kaluza-Klein reduction of dimensions consistent with the holistic quantum structure of causal localization although it is perfectly consistent with classical field theory and the Born localization of QM (and even with quasiclassical approximations of QFT). The embedding of a linear quantum mechanical chain of oscillators into a space of arbitrary dimension is only possible in QM where "localization" lacks an intrinsic meaning.

Before presenting the arguments against ST, it may be helpful to state the conclusions in the form of a collection of theses which will receive detailed attention in the subsequent sections

- The crossing symmetry of the dual model amplitude has no relation to the crossing identity in particle physics. In fact what is interpreted as a \((m,s)\) "particle-spin tower" is really the tower of a \((d_{sd},s)\) (anomalous) scale dimension spectrum which occurs in the global operator expansion of two fields in a conformal QFT [25]. ST shares a particular version of this spectrum.
• Different from what its name suggests, string theory does not describe string-localized objects in spacetime. In particular an embedding of a chiral conformal QFT into a higher dimensional target space is incompatible with the formulation of causal localization in QT.

• ST is a dynamic infinite component QFT, in fact it is the only known solution of an old problem which goes back to Majorana [9], namely a to find an irreducible algebraic structure which carries a representation of the Poincaré group with an interesting \((m,s)\) spectrum.

• The quantization of the diffeomorphism-invariant Nambu-Goto action (the original action for ST) has no relation to conformal chiral theories and leads to very different results from those obtained from the Polyakov action; in particular there is no discrete \((m,s)\) spectrum.

• The Maldacena conjecture (originally a derivation of ST) as a correspondence between two physical theories is contradicted by known facts about the connection between the cardinality of phase space degrees of freedom and the "causal completion property" (timelike causality). Either the conformal QFT has too many degrees of freedom (overpopulation leading to causal "poltergeists") or the upload of a normal conformal QFT leads to a AdS theory which is too "anemic" to support causal localization as we know it (last section).

The incomplete understanding of epiphenomena of modular localization has also led to misunderstandings in derivatives of ST, as Maldacena’s version of the AdS-CFT correspondence, "brane physics" and extensions of the classical Kaluza-Klein dimensional reduction idea to quantum theories based on causal localization\(^{13}\). Although these concepts are only historically and sociologically but not logically tied to ST, their critical evaluation follows similar conceptual reasoning and will be presented after having settled the above claims.

2 Crossing from conformal correlations and crossing in particle physics

One of the manifestations of relativistic QT which is directly related to the foundational is the particle crossing property; it requires to understand the relation between particles and fields beyond their asymptotic relation in terms of scattering theory.

This property, was first observed on Feynman graphs: graphs with a certain number of external lines restricted to the mass-shell describing a cluster of incoming and outgoing particles, can be considered as a relation between a formfactor of an operator which is associated with the remaining unrestricted

\(^{13}\)The reason why we occasionally use this round-about terminology is that the reader may be unaware that our understanding of QFT extends significantly beyond what can be found in textbooks.
(off-shell) lines. Shifting some lines from the incoming to the outgoing particle configuration formally corresponds to a process where a lesser number of particles is incoming and a larger outgoing, except that the shifted outgoing objects are on the backward mass shell and (this is the nontrivial part of "crossing") can be related to the crossed physical formfactor by analytic continuation. The crossing from outgoing to incoming particles is analogous. The S-matrix is the formfactor of the identity operator; in case of the elastic S-matrix the energy-momentum conservation requires to cross a pair of particles, one from the incoming and the other from the outgoing configuration. The formal graphical crossing is trivial; the nontrivial part is the possibility of an analytic continuation inside the complex mass shell; this is nontrivial even in perturbation theory.

In the context of the present paper the main interest is not the result as such (which seems to be "obvious" in a graphical presentation), but rather the subtle concepts used in its proof, as well as the nontrivial thresholds modifications in case that the particle wave functions overlap. The new concept of modular wedge localization leads to a KMS identity in terms of fields, whose appropriate transcription into particle formfactors reveals the true conceptual origin of particle crossing. The ideas are intimately related to a foundational understanding of the formfactor-bootstrap constructions for $d=1+1$ factorizing models [27][20]. In the wider context of integrable and nonintegrable QFTs [24], their presentation will be deferred to the next section.

Whereas the old arguments for crossing in the context of the bootstrap S-matrix project remained vague, the situation changed when this property was used in constructions of "would-be" scattering amplitudes. In the proposed dual model [13] it became sufficiently concrete, so that with the hindsight of the present day knowledge it is fairly easy to see that the dual model crossing has no connection with the particle crossing as first seen in Feynman diagrams and later used as a basic property in the formulation of the S-matrix bootstrap. To see that it has no relation with particle crossing does not require as much preparation as the proof of the real crossing property, and therefore most of the critique of the Dual Model and ST will be presented already in this section.

Even though the foundational origin of particle crossing was not understood at the time of the dual model, it was clear that it describes a subtle interplay between one particle poles of the S-matrix or more general between formfactors which cannot be approximated by a meromorphic function. Veneziano’s meromorphic dual model was the result of an educated mathematical experiment guided by the properties of the Euler beta function. Although the result of this experimental mathematics does not fit what one expects from particle crossing, but one may still ask whether there is anything else in local quantum physics which this model describes. The answer is surprising as well as interesting; the dual model encodes a property of conformal QFT, an area of local quantum physics.
physics which has no (known) relation to particle theory.

Schematically this may be described as follows \[25\]. Quantum fields in conformal QFT have different properties from those which admit a particle interpretation. As a result the only asymptotic converging Wilson-Zimmermann short distance expansions is replaced by a *globally converging* operator expansion in which the coefficient functions have the properties of conformal 3-point functions

\[
A_3(x_3)A_4(x_4)\Omega = \sum_k \int d^4z \Delta_{A_3,A_4,k}(x_1,x_2,y)C_k(z)\Omega \tag{2}
\]

where the second line expresses the fact that, by either using local spacelike commutativity of fields in Minkowski spacetime or commutativity of their euclidean counterparts, one obtains a situation which resembles the three s, t, u Mandelstam variables in the parametrization of the s,t,u physical scattering channels related by analytic continuation. The analogy becomes even closer upon Mellin transforming in the above spacetime variable, in which case the numerical values of the scale dimensions of the composites in the global expansion pass into positions of first order poles; the result consist in three different converging pole expansions of the same meromorphic function into sums over pole contributions. Unlike Fourier transforms, Mellin transforms have no independent operator status in Hilbert space since they are performed on correlation functions and not on individual operators (i.e. there are no "Mellin-operators" in the sense that there exist Fourier transforms and momentum space operators).

This implies in particular that there is no Hilbert space description which can place Mellin-transforms of correlation functions and S-matrix amplitudes under one conceptual roof (which would be the minimum requirement for a basis of unitarization of the dual model amplitude). Independent of the dimensionality of the conformal model, the so obtained functions are always meromorphic with infinitely many first order poles at positions given by the anomalous scale spectrum of those fields which appear in the global operator product decomposition. For mathematical details about Mellin transformations of conformal correlations see \[25\].

The crossing symmetry resulting from the combined result of the global operator expansion with subsequent Mellin transformation may have the effect of an alluring siren/mermaid song, suggesting particle crossing in the sense of particle physics to the conceptually untrained particle physicist. Its wide-spread acceptance showed that the time of the European "Streitkultur" in its use for a conceptual cleansing in particle theory had definitly come to an end\[15\]. Starting with Veneziano’s famous dual model paper, a new Zeitgeist in which the use of analogies was not the exception but rather the rule took over; analogies which were not interdicted on the mathematical side became acceptable.

\[15\] Besides names as Landau, Lehmann, Jost, Kallen, ..this clarifying instrument of the particle-theory discourse also took some roots in the US (Oppenheimer, Feynman, Schwinger,..). It ended with Jost’s masterful polemic article against the S-matrix bootstrap \[28\].
It turned out that there are many more dual models than that found by Veneziano; essentially any conformal QFT in any spacetime dimension led to a meromorphic crossing symmetric functions. As mentioned in the previous section an important additional restriction came from the requirement that the anomalous dimension spectrum \((d_{s\mu}, s)\) of conformal composites should be identical to the \((m, s)\) spectrum of a unitary representation of the Poincaré group. It turns out that the (physically unmotivated) identification of the two requirements is extremely strong; in fact there is no a priori reason why it should have any solution at all, since the two requirements have no visible relation.

The next step which led from the dual model to ST was the search for a Lagrangian whose canonical quantization supports such a situation. The somewhat surprising answer (found by exploring properties of the Virasoro algebra) was that the Polyakov action (the square of the Nambu-Goto action)

\[
\int d\sigma d\tau \sum_{\xi=\sigma, \tau} \partial_{\xi} X_\mu(\sigma, \tau) g^{\nu\tau} \partial^\xi X_\mu(\sigma, \tau)
\]

for \(\mu, \nu = 1, \ldots, 26\) respectively its supersymmetric extension in 10 dimensions does the job; in the latter case one is even in the fortunate situation of a positive energy representation which means (since no infinite spin component occurs in the decomposition into irreducibles) that the localization of 10 dimensional wave function and the associated second quantized free fields is automatically pointlike. For the following we do not have to know what formal arguments led to a representation of the Poincaré group associated with this and how convincing it is; it suffices to accept the result that the oscillator degrees of freedom of a 26 component chiral current or its supersymmetric 10 component extension does the job.

Also the well-known infrared problems of \(d=1+1\) massless fields (potentials of well-defined currents) are irrelevant; the currents as well as their associated charge-carrying sigma-model fields (see below) which feature in our arguments are well-defined (using the appropriate limiting definition starting from finite exponential strings and shifting one charge to infinity), there is no infrared problem.

ST begun with the recognition that the analogy between the poles at certain anomalous dimensions and "would be" particle masses can be strengthened by the construction of a unitary positive energy representation on the oscillators degrees of freedom contained in a (supersymmetrically extended) 10 component chiral current model. Accepting the result of the arguments of string theorists, still leaves two questions connected with such a construction:

1. What is the property which enables chiral current theories to allow representations of noncompact groups on there internal symmetry ("target") space in view of the fact that the inner symmetries of higher dimensional QFT describing particles can only accommodate compact groups.

\[\text{The classical solutions of the Nambu-Goto action coalesce with those of the Polyakov action; but this does not mean that this associated QFTs (see later).}\]
2. Both, the localization of a chiral theory on a lightray (or circle) and the wave functions (and second quantized fields) associated with the pointlike generated target space representation of the Poincaré group realize holistic localization properties in very different spacetime contexts. Do they really use the same oscillator substrate supplied by the Polyakov action?

The only possible "noncompact indices" of a field localized in n-dimensional spacetime \( n > 3 \) are those which refer to their tensorial/spinorial nature in that dimension. How is it possible that for chiral theories they can support noncompact and even Poincaré group representations of higher dimensional theories? Here the distinction between rational and nonrational theories come into play. The characterizing property of rational chiral theories is that their observable local algebras have only a finite number of superselection sectors which are generated by fields with generally plektonic (braid group) commutation structures. The superselected charge spectrum must be continuous in order to accommodate noncompact symmetry groups and the only known QFT (in any spacetime dimensions) which are able to provide such a situation are certain non-rational chiral models, most prominently multi-component current models

\[
\partial \Phi_k(x) = j_k(x), \quad \Phi_k(x) = \int_{-\infty}^{x} j_k(x), \quad Q_k = \Phi_k(\infty), \quad \Psi(x, \vec{q}) = e^{i\vec{q}\vec{\Phi}(x)}
\]

The second line indicates the analogy on which the construction of a 10-dimensional positive energy representation (the superstring representation) of the Poincaré group is based.

Coming to the second question, the answer is the following. The representation of the Poincaré group based on the use of the irreducible algebraic structure from the Polyakov action is sufficiently different from the holistic requirements needed to describe localization of a chiral theory in order to prevent an isomorphism or even an embedding of the chiral theory into its target spacetime (related to the positive energy representation of the Poincaré group). This is obvious from the different spectra of the zero mode operator; whereas the charge spectrum of the oscillator degrees used to implement the holistic localization structure of the chiral current theory is continuous, the energy-momentum spectrum in the target space construction is that of the direct sum of one-particle representations contained in the superstring representation of the Poincaré group. Nevertheless there is one property which the target theory preserves of its chiral avatar: the \((m, s)\) spectrum, alias the \((d_{sc}, s)\) chiral spectrum.

So the remaining question is: can the "picture puzzle" resulting from a \((d_{sc}, s) \sim (m, s)\) kaleidoscope be the starting point of a new foundational particle physics theory? For string theorists the answer obviously affirmative, they are probably not even aware about its existence. For the rest of the world a non-negative response could consist in the perception that, although these analogies do not solve that what the string theorists had hoped for, they do lead to
the first and only illustration of an old forgotten dream by Majorana about a "dynamic infinite component field project" \([10]\) namely to find an irreducible algebraic structure which carries a reducible infinite component one-particle representation of the Poincaré group. This at that time interesting project (in analogy to the \(O(4,2)\) dynamical group of the hydrogen spectrum) from an epoch which had a more naive understanding of QFT (relativistic QM) has disappeared in the maelstrom of time after enjoying a second wave of popularity in the hands of Barut, Fronsdal, Kleinert,.. \([10]\) in the first half of the 60s. In any case it could not have been used as a scientific support for the sexed up colorful science fiction stories of Brian Green and Lisa Randall which appear in TV programs and in interviews and for which the ST Zeitgeist will always be remembered, independent of the scientific future fate of ST.

The second line in \([4]\) contains the definition of the charge-carrying sigma model fields which generate the charge sectors and also play the role of the formal carriers of the Poincaré representation in the nonisomorphic target space interpretation. In contrast to the massless potential \(\Phi\) the sigma model fields \(\Psi\) are well-defined. As a result of the nature of their relation to the current fields they are not local but rather lead to abelian braid group representations. Instead of quantizing the Polyakov action one could also start from the sigma model field in which case the previous observations would amount to saying that the holistic organization of sigma model degrees of freedom which is necessary to support the localization in the wave function space of the Poincaré group representation. Such points are easily overlooked if one naively interprets the similarity of formal appearances as an identity of the conceptual content which is the root cause which led to the incorrect embedding idea. As a result of the wrong embedding picture the spacetime interpretation of the operators \(X_\mu(\sigma)\) as tracing out a stringlike localization is incorrect\(^{17}\) and unfortunately this also damages to the formal Euler beta-function like argument that ST implies QGr.

There remains the philosophical question of what to make out of the near uniqueness of the resulting 10 dimensional superstring representation (more correctly uniqueness modulo a finite number of so-called M-theoretic modifications). If the number would have been zero or infinity the whole story about ST would have ended right there. The metaphoric idea of string theorists that we are living in a (suitably dimensionally reduced) 10 dimensional "target space" of a chiral current theory has to be seen in the context of a widespread belief that foundational theories are unique or nearly unique in the sense that there are no other theories in the immediate neighborhood. Whereas some among us would subscribe to the idea

\[
\text{fundamental theory (TOE)} \rightarrow \text{rarity of realization} \tag{5}
\]

not even the most hardened reductionist would probably accept the inversion of this arrow. But this is precisely what keeps ST going. The wisdom of many

\(^{17}\)Apart from the nullmode which describes the Fourier component of point-localization the "movement" of the other oscillator variables is in an internal space "over" the localization point (that where one pictures spin components).
vernaculars must be called into questions in times of ST. One is certainly the saying many people cannot err. In other times people would have looked for an explanation of the rareness as a peculiar property of nonrational models (whose observable algebras have a continuous cardinality of superselection sectors).

There remains the problem of what becomes of ST if one takes the problem of quantization the Nambu-Goto action (the square root of the Polyakov action) serious. This has been recently answered in a paper by Bahns, Rejzner and Zahn [30]. In that case there is no relation to a conformal QFT as in the case of the Polyakov interaction. The quantization problem is similar to that of quantizing a nonpolynomial interaction as the Einstein-Hilbert action [31]; the overriding problem is whether the aspect of background independence can be separated from the obvious nonrenormalizability of both actions. In both cases there is the indication that this is indeed the case i.e. that diffeomorphism invariance can be implemented without having any restricting effect on the renormalizability problem. Unlike the ST based on the Polyakov Lagrangian, which at least solves the Majorana problem of a rich one particle spectrum from an irreducible algebraic structure, the Nambu-Goto action solves the problem of what remains of a classical Kaluza-Klein dimensional reduction interpreted as a classical diffeomorphism invariant embedding if one subjects it to the rules of diffeomorphism invariant quantization.

Obviously quantization of a classical embedding is not the same as the embedding of a ready made lower dimensional QFT into a larger one. The problem which is at the bottom of this interdiction also seriously impedes its inversion i.e. the direct Kaluza-Klein reduction of a QFT in terms of operators or correlation functions (and not by massaging Lagrangians). As far as the Poincaré group representation in the special case of embedding into a Minkowski spacetime is concerned, the result of some (not completely understood "zero mass stuff") is not encouraging for a ST believer\footnote{In fact the ideology of little strings swirlingly through spacetime is so strong that string theorist do not accept the result of their own correct calculations of pointlike (graded) commutators of what they insist to call string fields. Instead of presenting that point as the middle of an invisible string \footnote{\cite{33,34}}, they should have written (not being able to let loose on the terminology "string") "internal string" which lives there where one spin components live.}

The quantization of a parametrization-invariant classic embedding has a close conceptual (but not mathematical) proximity to an old setting proposed by Pohlmeyer which was based on the observation that the Nambu-Goto Lagrangian describes a integrable classical system. Pohlmeyer \footnote{\cite{32}} identified the infinite number of conserved currents and determined their Poisson bracket relations which he quantized in the spirit of integrable model quantization. He did not identify a concrete representation of the Poincaré group, so that the problem of localization (which in the ST case refers to a concrete positive energy representation) of states remained open.

One would think that finding a physical structure which precisely leads to the dual model (and not to an imagined elastic approximation of a crossing symmetric unitary S-matrix) as the global operator expansion in conformal QFT \footnote{\cite{25}} that this settles the problem; but the interest in conceptual aspects of local
quantum physics obviously reached such a low point ("shut up and calculate"
[74]) that even this does not reach a community which has throne its fortune to
the side of mathematics. It seems that the ST community which started with
Veneziano’s mathematical tinkering is dead-set to also go down with it. How
else, than in this way, can one explain that the vague resemblance of multicom-
ponent chiral currents and anomalous dimensions with particle momenta and
mass spectra of particles can trap so many people (among them brilliant minds)
if it is not the blind faith that mathematics will fix it.

Within all the paean of praise from mathematicians about how much they
owe to ST it may be helpful to point out that the correct use of the multi-current
model which is in harmony with its true role as a ”theoretical laboratory of
QFT” is no way less sexy that its ST avatar. The extensions of the observable
algebra through the addition of sigma model fields with integer scale dimen-
sions are classified in terms of even lattices whereas their charge structure is
characterized in terms of the dual lattice. For quantum field theorists it is
very interesting to have selfdual examples because they are fully ”Haag-dual”
(the only sector is the vacuum sector) which means that not only for simply
connected (interval) but also for multiply connected (multi-interval) localized
regions the commutant is equal to the algebra localized on the complement. As
far as I know these are the only Haag dual algebras within the huge family of
chiral models. The associated lattices and symmetry groups correspond pre-
cisely to the largest semisimple finite groups. This illustrates again the close
connection of causal localization with group theory as a special case of the DHR
superselection theory which classifies local representation equivalence classes of
observable algebras in terms of compact groups.

3 Good news for higher spin interactions from
modular localization

Up to now we only used the critical potential of the modular localization setting.
Unfortunately ST cause a lot of confusion in an area in which bona-fide string-
localization really matters: interactions involving higher spin fields \(m, s \geq 1\), in
particular for gauge theories and their massive counterparts as they are needed
in the standard model.

The project to use the new setting of modular localization to solve remaining
problems of perturbative QFT started with the solution of a conceptual prob-
lem which, since the days of Wigner’s particle classification remained unsolved:
the causal localization of the third Wigner class (the massless infinite spin solu-
tions) of positive energy representations of the Poincaré group whereas the
massive class as well as the zero-mass finite helicity class are pointlike gener-
gated. Spacelike string-generated fields are covariant fields \(\Psi(x,e)\), \(e\) spacelike
unit vector which are localized \(x + \mathbb{R}_+e\) in the sense that the (graded) commu-

\[\text{[19]}\] These concepts were introduced for the one-component current in [35] and generalize to
multicomponent currents in [36][37]
tator vanishes if the full semiinfinite strings (and not only their starting points $x$) are spacelike separated \[49\]

$$[\Psi(x, e), \Phi(x', e')]_{\text{grad}} = 0, \quad x + Re \rangle \langle x' + Re'$$

(6)

Unlike decomposable stringlike fields (line integrals over pointlike fields) such elementary stringlike fields lead to serious problems with respect to the activation of (compactly localized) particle counters.

In the old days \[38\] infinite spin representations were rejected on the ground that nature does not make use of them. But whether nowadays, i.e. in times of dark matter, one would uphold such dismissals is questionable. String-localized quantum fields fluctuate both in $x$ as well as in $e$. The can always be constructed in such a way that their effective short distance dimension is the lowest possible one allowed by positivity, namely $d_{sd} = 1$ for all spins. It is very difficult to construct the covariant ”infinite spin” fields by the group theoretic intertwiner method used by Weinberg \[38\]; in \[49\] the more powerful setting of modular localization was used.

For pointlike generating fields $\Psi^{(A, B)}(x)$ one finds the following two relations between the physical spin (helicity) and the possible range of spinorial indices

$$|A - B| \leq s \leq A + B$$

$$h = A - B, \quad m = 0$$

(7)

In the massive case all possibilities for the angular decomposition of two spinorial indices are allowed whereas in the massless case the values of the helicity $h$ are severely restricted (second line). For $(m = 0, h = 1)$ the formula reproduces the impossibility of reconciling pointlike vector potentials with the Hilbert space positivity. This holds for all $(m = 0, s \geq 0)$: pointlike localized ”field strengths” (in $h=2$, the linearized Riemann tensor) have no pointlike quantum ”potentials” (in $h=2$, the $g_{\mu\nu}$) and represents one solution of the famous clash between localization and the Hilbert space structure. Since the classical theory does not care about positivity, the (Lagrangian) quantization setting inevitably forces the scarification of the Hilbert space in favor of Krein spaces (implemented by the Gupta-Bleuler or BRST formalism). The more intrinsic Wigner representation-theoretical approach keeps the Hilbert space and lifts the unmotivated restriction to pointlike generators in favor of semiinfinite stringlike generating fields.

For $(m = 0, s = 1)$ the stringlike covariant potentials $A_{\mu}(x, e)$ are uniquely determined in terms of the field strength $F_{\mu\nu}(x)$ and a spacelike direction $e$. The idea is somewhat related to Mandelstam’s attempt to formulate QED without the vectorpotentials \[5\]. But even though the string-local potential is uniquely determined in terms of $F_{\mu\nu}, e$, it is much safer to explicitly introduce the $A_{\mu}(x, e)$

\[20\]These long distance (infrared) fluctuations are short distance fluctuation in the sense of the asymptotically associated $d=1+2$ de Sitter spacetime.
because they are a strong reminders that one is dealing with objects which fluctuate in both \( x \) and \( e \); in fact the improvement of the short distance property in \( x \) is paid for by a worsening infrared behavior i.e. the \( A_\mu(x,e) \) is an operator-valued distribution in both \( x,e \). In contrast to the above infinite spin representation which cannot be pointlike generated, all other representations admit pointlike generators and only exclude pointlike potentials.

As an illustrative example let us look at the Aharonov-Bohm effect in QFT\(^{21}\). In terms of Haag’s intrinsic LQP setting of QFT this is a breakdown of Haag duality for a toroidal spacetime localization [39][40].

\[
\mathcal{A}(T'') \subsetneq \mathcal{A}(T)''
\]

\( T \) spatial torus at \( t = 0 \), \( T'' \) its causal completion

For lower spin zero mass fields or for a torus-localized algebra from a massive field of any spin one finds the equality sign (Haag duality). This can be shown in terms of field strengths, but if one (for the convenience of applying Stokes theorem) uses potentials it is easy to see that the indefinite metric potential leads to the wrong result (zero effect) whereas the string-localized potential in the Hilbert space accounts correctly for the A-B effect.

In massive theories there is no such clash: pointlike potentials of field strength exist, but their short distance dimensions increase just like those of field strengths. Nevertheless one can introduce stringlike potentials as a means to lower the short distance dimension in order make couplings fit for renormalization. The connection between the stringlike vectorpotential and its pointlike counterpart (the \( d_{sd} = 2 \) Proca field) leads to a scalar string-localized field (all relations take place in Hilbert space)

\[
A_\mu(x,e) = A^P_\mu(x) + \partial_\mu \phi(x,e)
\]

The strategy for calculations of correlations in e.g. QED is then the following. Use the \( d_{sd} = 1 \) string fields for the perturbative calculations in massive QED. If needed, pass to the pointlike Proca field in every order. But the pointlike Proca field has no zero mass limit (the Hilbert space-localization clash), only the string-localized massive potential passes to its zero mass counterpart.

For the charge-carrying matter fields the counterpart of the additive change from string-localized to point-localized fields is multiplicative. The massive Dirac-charge-carrying string-localized spinor of massive QED\(^{22}\) is then expected to pass to the Dirac+Maxwell charge-carrying ”infraparticle” field, whereas the pointlike matter field corresponds to the pointlike Proca potential.

The Krein space-based BRST setting of gauge theory has a more limited range. Physical charged fields and their (off-shell) correlations are not part of the BRST formalism. Electrically charged particles appear in a somewhat indirect way in form of a prescription for photon-inclusive cross section which, unlike

\(^{21}\)The standard A-B effect is about quantum mechanical charged particle in an external magnetic field.

\(^{22}\)The perturbative transfer of string-localization from the vectorpotential leads to a physical string-localized Dirac field which carries Dirac+Maxwell charge.
the LSZ reduction formalism, has no direct relation to spacetime correlation functions. There is a new setting which takes up an old problem concern[23]ing the use of the restriction of algebras to the forward light cone which casts additional conceptual light on this problem[24].

The more interesting case is that of selfinteracting massive vector mesons. Here the systematic application of Scharf’s version[42] of operator gauge invariance within the BRST setting implements the group symmetry; as already understood by Stora, the gauge group structure does not have to be postulated, it is fixed by other consistency requirements. Different from massive QED, the consistency of the massive BRST formalism also requires the presence of a chargeless scalar field, but without any spontaneous symmetry breaking.

The remaining doubts about whether the presence of a Higgs field is a feature of the BRST quantization formalism (the intermediate use of a Krein setting is imposed by Lagrangian quantization and does not belong to the intrinsic properties of the desired QFT) or a consequence of foundational principles can only be solved in the string-localized setting of this problem. There are other important problems for which one needs this formulation. A derivation of asymptotic freedom from a low order dimensionally regularized beta function is only conceptually acceptable if the beta function is part of a parametric Callen-Symzansk equation; a beta all by itself is a meaningless global quantity. However the derivation of the latter requires the existence of a massive perturbation theory; the prototype of such a computation is the massive Thirring model[43] where $\beta = 0$ to all orders (thus preempting the conformal invariance for vanishing mass). In massless Y-M models even the off-shell correlations are infrared divergent in all covariant gauges. This situation, which is usually blamed on an imagined nonperturbative long-distance behavior (related to confinement) may actually be the undesired consequence of imposing point-localization in a situation which really requires string-localization.

The crucial problem is the reformulation of the iterative Epstein-Glaser renormalization in terms of string-localized fields. This is particularly tricky in massive Y-M interactions where, in contrast to massive QED, the interaction involves several strings. A first incomplete account of these problems was given in[44], but meanwhile this technical aspect of what replaces the diagonal of the pointlike iteration in case of strings has been solved[45]. Therefore we hope to be able to present an account of low order perturbative calculations in the near future[46].

As mentioned in the previous section, the need to understand another more hidden side of local quantum physics did not arise with the appearance of the dual model and string theory, but it already existed in Jordan’s first (1926) model of a QFT[2]. The thermal aspects arising from restricting the QFT vacuum to a spacetime subregion belong to that “other side of QFT” which the

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23 For massive theories this restriction maintains the full information whereas for QED it leads to a natural (geometric) infrared cutoff.

24 This confirms the correctness of Swieca’s viewpoint of a Schwinger-Higgs charge screening mechanism instead of a Goldstone symmetry breaking in which the massless Goldstone boson is subsequently swallowed in a process which converts the photon into a massive vector meson.
standard formalism does not really reveal; for this reason the thermal aspects of the Einstein-Jordan conundrum remained for a long time unexplained or where mistakenly thought to be caused by curvature in gravity theory. They were only understood in a different more algebraic formulation QFT referred to as local quantum physics (LQP), which places the modular localization property into the center stage. Since this setting will not only be useful for the construction of the previously mentioned string-localized fields but even more important to understand the conceptual origin of particle crossing in the next section, we will use the remainder of this section to present some elementary facts about it.

It has been realized, first in a special context in [19], and then in a general mathematical rigorous setting in [47] (see also [48][49]), that there exists a natural localization structure on the Wigner representation space for any positive energy representation of the proper Poincaré group. A convenient presentation can be given in the context of spinless chargeless particle for which the \( (m > 0, s = 0) \) Wigner one-particle space is the Hilbert space \( H_1 \) of (momentum space) wave functions with the inner product

\[
(\varphi_1, \varphi_2) = \int \tilde{\varphi}_1(p) \varphi_2(p) \frac{d^3p}{2p_0}, \quad \tilde{\varphi}(x) = \frac{1}{(2\pi)^{\frac{d}{2}}} \int e^{-ipx} \varphi(p) \frac{d^3p}{2p_0}
\]

In this case the covariant x-space amplitude is simply the on-shell Fourier transform of this wave function whereas for \( (m \geq 0; s \geq 1/2) \) the covariant spacetime wave function is more involved as a consequence of the presence of intertwiners \( u(p, s) \) between the manifestly unitary and the covariant form of the representation [38].

Selecting a wedge region e.g. \( W_0 = \{ x \in \mathbb{R}^d, x^{d-1} > |x^0| \} \), one notices that the unitary wedge-preserving boost \( U(\Lambda_W(\chi = -2\pi t)) = \Delta^{\frac{t}{2}} \) commutes with the antiunitary reflection \( J_W \) on the edge of the wedge (i.e. along the coordinates \( x^{d-1} - x^0 \)). The distinguished role of the wedge region is that they form a commuting pair of (boost, antiunitary reflection). This has the unusual (and perhaps even unexpected) consequence that the unbounded and antilinear operator

\[
S_W := J_W \Delta^{\frac{t}{2}}, \quad S_W^2 \subset 1
\]

which is intrinsically defined in terms of Wigner representation data, is involutive on its dense domain and therefore has a unique closure with \( \text{ran}S = \text{dom}S \) (unchanged notation for the closure).

The involutivity means that the \( S \)-operator has \( \pm 1 \) eigenspaces; since it is antilinear, the +space multiplied with \( i \) changes the sign and becomes the - space; hence it suffices to introduce a notation for just one eigenspace

\[
K(W) = \{ \text{domain of } \Delta^{\frac{t}{2}}_W, \quad S_W \psi = \psi \} \quad (12) \\
J_W K(W) = K(W'), \quad \text{duality} \quad K(W) + iK(W) = H_1, \quad K(W) \cap iK(W) = 0
\]
It is important to be aware that we are dealing here with real (closed) subspaces $K$ of the complex one-particle Wigner representation space $H_1$. An alternative is to directly work with the complex dense subspaces $K(W) + iK(W)$ as in the third line. Introducing the graph norm in terms of the positive operator $\Delta$, the dense complex subspace becomes a Hilbert space $H_{1,\Delta}$ in its own right. The upper dash on regions denotes the causal disjoint (the opposite wedge), whereas the dash on real subspaces means the symplectic complement with respect to the symplectic form $Im(\cdot, \cdot)$ on $H$. All the definition work for arbitrary positive energy representations of the Poincare group.

The two properties in the third line are the defining relations of what is called the standardness property of a real subspace; any abstract standard subspace $K$ of an arbitrary real Hilbert with a $K$-operator space permits to define an abstract $S$-operator in its complexified Hilbert space

$$S(\psi + i\varphi) = \psi - i\varphi, \quad S = J\Delta^{\frac{1}{2}} \quad (13)$$

whose polar decomposition (written in the second line) yields two modular objects, a unitary modular group $\Delta^{it}$ and an antiunitary reflection which generally have however no geometric interpretation in terms of localization. The domain of the Tomita $S$-operator is the same as the domain of $\Delta^{\frac{1}{2}}$, namely the real sum of the $K$ space and its imaginary multiple. Note that for the physical case at hand, this domain is intrinsically determined solely in terms of the Wigner group representation theory.

The $K$-spaces are the real parts of these complex $\text{dom}S$, and in contrast to the complex domain spaces they are closed as real subspaces of the Hilbert space (corresponding to the one-particle projection of the real subspaces generated by Hermitian Segal field operators). Their symplectic complement can be written in terms of the action of the $J$ operator and leads to the K-space of the causal disjoint wedge $W'$ (Haag duality)

$$K'_W := \{ \chi | \text{Im}(\chi, \varphi) = 0, \text{all } \varphi \in K_W \} = J_W K_W = K_W' \quad (14)$$

The extension of W-localization to arbitrary spacetime regions $\mathcal{O}$ is done by representing the causal closure $\mathcal{O}''$ as an intersection of wedges and defining $K_\mathcal{O}$ as the corresponding intersection of wedge spaces

$$K_\mathcal{O} = K_{\mathcal{O}''} \equiv \bigcap_{W \supseteq \mathcal{O}''} K_W, \quad \mathcal{O}'' = \text{causal completion of } \mathcal{O} \quad (15)$$

These $K$-spaces lead via (13) and (15) to the modular operators associated with $K_\mathcal{O}$.

---

25 According to the Reeh-Schlieder theorem a local algebra $\mathcal{A}(\mathcal{O})$ in QFT is in standard position with respect to the vacuum i.e. it acts on the vacuum in a cyclic and separating manner. The spatial standardness, which follows directly from Wigner representation theory, is just the one-particle projection of the Reeh-Schlieder property.
For those who are familiar with Weinberg’s intertwiner formalism for passing from the unitary Wigner to covariant representations in the dotted/undotted spinor formalism, it may be helpful to recall the resulting "master formula"

\[ \Psi^{(A, \dot{B})}(x) = \frac{1}{(2\pi)^2} \int \left( e^{-ipx} \sum_{s_3 = \pm s} u^{(A, \dot{B})}(p, s_3) a(p, s_3) + \right. \]
\[ \left. + e^{ipx} \sum_{s_3 = \pm s} v^{(A, \dot{B})}(p, s_3) b^*(p, s_3) \right) \frac{d^3p}{2\omega} \]
\[ \sum_{s_3 = \pm s} u^{(A, \dot{B})}(p, s_3) a(p, s_3) \rightarrow u(p, e) \cdot a(p) \]

where the a,b amplitudes correspond to the Wigner momentum space wave functions of particles/antiparticles and the u,v represent the intertwiner and its charge conjugate. For the third class (infinite spin, last line) the sum over spin components has to be replaced by an inner product between a p, e dependent infinite component intertwiner u and an infinite component a(p), because in this case Wigner’s "little space" is infinite dimensional. The Ψ(x) respectively Ψ(x, e) are "generating wave functions" i.e. they are wave-function-valued Schwartz distributions which by smearing with O-supported test functions become O-localized wave functions. Adding the opposite frequency anti-particle part one obtains the above formula which by re-interpreting the a#, b# as creation/annihilation operators (second quantization) become point-respectively string-like free fields. The second quantization functor maps the complex amplitudes a, b into creation/annihilation operators. The resulting operator-valued Schwartz distributions are global objects (generators) in the sense that they generate O-localized operators Ψ(f) by "smearing" them with O-supported test functions supp f ∈ O.

Only the massive case the full spectrum of spinorial indices A, \dot{B} is exhausted whereas the massless case leads to huge gaps which come about because pointlike "field-strength" are allowed whereas pointlike "potentials" are rejected. With the awareness about the conceptual clash between localization and the Hilbert space.

The difference to Weinberg’s setting is that, whereas he uses the computational somewhat easier manageable covariance requirement (for wave functions and free fields covariance is synonymous with causal localization, but in the presence of interaction the localization of operators and that of states split apart), the modular localization method uses causal localization directly and bypasses the issue of the nonunique intertwiners by aiming directly at "modular-localized" dense subspaces.

The generating pointlike fields are indispensable in the implementation of perturbation theory. They are the mediators between classical localization (which is used when one specifies zero order interactions in form of invariant

\[ 26 \text{In the case of } [29] \text{ this awareness came from the prior use of "modular localization" starting in } [15], [19] \text{ but foremost (covering all positive energy Wigner representations) in } [47]. \]
Wick-ordered polynomials) and quantum localization, which takes over when one uses the Epstein-Glaser iteration machinery to implement the causal localization principle order by order \[22\]. Modular localization on the other hand is essential in trying to generalize Wigner’s intrinsic representation theoretical approach to the (non-perturbative) realm of interacting localized observable algebras (next section).

In order to arrive at Haag’s setting of local quantum physics in the absence of interactions, one only has to apply the Weyl functor \( \Gamma \) which maps wave functions into operators and wave function spaces into operator algebras (or its fermionic counterpart), symbolically indicated by the functorial relation

\[
K_O \xrightarrow{\Gamma} \mathcal{A}(O)
\]  

(18)

The functorial map \( \Gamma \) also relates the modular operators \( S, J, \Delta \) from the Wigner wave function setting directly with their ”second quantized” counterparts \( S_{Fock}, J_{Fock}, \Delta_{Fock} \) in Wigner-Fock space; it is then straightforward to check that they are precisely the modular operators of the Tomita-Takesaki modular theory applied to causally localized operator algebras.

\[
\sigma_t(\mathcal{A}(O)) \equiv \Delta^{it} \mathcal{A}(O) \Delta^{-it} = \mathcal{A}(O)
\]

(19)

\[
J \mathcal{A}(O) J = \mathcal{A}(O)' = \mathcal{A}(O')
\]

In the absence of interactions these operator relation are consequences of the modular relations for Wigner representations. The Tomita-Takesaki theory secures their general existence for standard pairs \((\mathcal{A}, \Omega)\) i.e. an operator algebra \( \mathcal{A} \) and a state vector \( \Omega \in H \) on which \( \mathcal{A} \) acts cyclic and separating (no annihilators of \( \Omega \) in \( \mathcal{A} \)). The polar decomposition of the antilinear closed Tomita \( S \)-operator leads to the unitary modular automorphism group \( \Delta^{it} \) associated with the subalgebra \( \mathcal{A}(O) \subset B(H) \) and the vacuum state vector \( \Omega \) i.e. with the pair \((\mathcal{A}(O), \Omega)\).

Although \( B(H) \) is generated from the two commuting algebras \( \mathcal{A}(O) \) and \( \mathcal{A}(O') \), they do not form a tensor product in \( B(H) \); hence the standard quantum-information concepts concerning entanglement and density matrices are not applicable. In contrast to QM where one has to average over degrees of freedom in order to convert entangled states into density matrices, modular situations are distinguished in that the averaging is replaced by the trivial operation of just restricting the global ”standard” state (e.g. the vacuum) to the local subalgebra of interest.

The only case for which the modular localization theory (the adaptation of the Tomita-Takesaki modular theory to the causal localization principle of QFT) has a geometric interpretation, independent of whether interactions are present or not and independent of the type of quantum matter, is the wedge region i.e. the Lorentz transforms of the standard wedge \( W = \{ x_0 < x_3 | x_\mu \in \mathbb{R}^3 \} \). In that case the modular group is the wedge-preserving Lorentz boost and the \( J \) represents a reflection on the edge of the wedge i.e. it is up to a \( \pi \)-rotation
equal to the antiunitary TCP operator. The derivation of the TCP invariance as derived by Jost \[28\], together with scattering theory (the TCP transformation of the S-matrix) leads to the relation

\[ J = S_{\text{scat}} J_{\text{in}} \]  

which in \[18\]\[19\] has been applied to constructive problems of integrable QFTs. The is a relation which goes much beyond scattering theory; in fact it only holds in local quantum physics since it attributes the new role of a relative modular invariant of causal localization to the S-matrix.

This opens an unexpected fantastic new possibility of a top to bottom construction of QFT in which the first step is the construction of generators for the wedge-localized algebra \(A(W)\) and the sharpening of localization is done by intersecting wedge algebras. Compact localized double cone algebras and their generating pointlike fields would only appear at the end. In fact according to the underlying philosophy that all relevant physical data can be obtained from localized algebras, thus avoiding the use of individual operators within such an algebra. This is the tenor of the paper "On revolutionizing quantum field theory with Tomita’s modular theory" \[50\] by Borchers, to whose memory I have dedicated this paper.\[27\] The next section presents the first step in such a construction.

The only prerequisites for the general (abstract) case is the "standardness" of the pair \((A, \Omega)\) where "standard" in the theory of operator algebras means that \(\Omega\) is a cyclic and separating vector with respect to \(A\), a property which in QFT is always fulfilled for localized \(A(O)\)'s, thanks to the validity of the Reeh-Schlieder theorem \[6\]. These local operator algebras of QFT are what I referred to in previous publications as a monad; there properties are remarkably different from the algebra of all bounded operators \(B(H)\) which one encounters for Born-localized algebras \[51\]. For general localization regions the modular unitaries have no geometric interpretation (they describe a kind of fuzzy action inside \(O\)) but they are uniquely determined in terms of intersections of their geometric \(W\)-counterparts, a top to bottom strategy which is quite efficient, even in the simpler context of localized subspaces \(K_O\) related to Wigner’s positive energy representation theory for the Poincaré group \[17\].

The most important conceptual contribution of modular localization theory in the context of the present work is the assertion that the reduction of the global vacuum (and also finite energy particle states) to a local operator algebra \(A(O)\) leads to a thermal state for which the "thermal Hamiltonian" \(H_{\text{mod}}\) is the generator of the modular unitary group

\[ e^{-i\tau H_{\text{mod}}} := \Delta^{i\tau} \]

\[ \langle AB \rangle = \langle Be^{-H_{\text{mod}}} A \rangle \]

where the second line is what one obtains for heat bath thermal systems after rewriting the Gibbs trace formula into the state-setting of the open system.\[27\] Please note that the word “revolution” in this context has a completely different meaning from its use in string theory.
formulation of statistical mechanics \[6\]. Whereas the trace formulation breaks down in the thermodynamic limit, this analytic KMS formula (asserting analyticity in \(-1 < \text{Im} \tau < 0\)) remains. It is in this and only in this limit, that QM produces a global monad algebra (different from \(B(H)\)) which is of the same type as the localized monad of QFT.

This underlines again the intimate connection between quantum causal localization and the ensemble nature of measurements in QFT (further remarks in the last section). Note that it is of cause not forbidden to speak about concrete operators in \(\mathcal{A}(O)\), the main difference of the ensemble viewpoint of QFT and the attribution of probabilities to individual events (as a result of the absence of an intrinsic mechanism which requires ensembles) is that the former tries to extract the description of nature from properties of existing localized ensembles. For the case at hand this is the modular Hamiltonian \(H_{\text{mod}}\) which changes together with the standard pair \((\mathcal{A}(O), \Omega)\) in such a way that the Hamiltonian for the algebra with the larger localization can also be applied to an algebra localized inside a causally closed region, but not the other way around. This leads to an infinite supply of modular Hamiltonians which all live in the same Hilbert space; this incredible rich structure has no counterpart in QM.

As well-known Einstein had serious problems with the assignment of probabilities to single events as usually (not by everybody) done in Born's probabilistic interpretation of QM. Since QFT is more foundational than QM one should perhaps consider the extrinsic probability of global QM as a limiting case of the ensemble interpretation in which the thermal aspects of modular localization and vacuum polarization get lost. It can be assumed that Einstein would have accepted the thermal probability arising from localization in QFT if it would have been available during his time.

Closely related is the "GPS" characterization of a QFT, including its Poincaré spacetime symmetry as well as the internal symmetries of its quantum matter content, in terms of modular positioning of a finite number of monads in a shared Hilbert space. For \(d=1+1\) chiral models the number of monads is 2 or 3, depending on the formulation whereas in \(d=1+3\) the smallest number for a GPS construction is 7. This way of looking at QFT is an extreme relational point of view in terms of objects which have no internal structure; this explains the terminology "monad" (a realization of Leibnitz point of view in the context of abstract quantum matter) \[52, 51\]. As life is an holistic phenomenon since it cannot be explained from its chemical ingredients so is QFT which cannot be understood in terms of properties of a monad. This is a philosophical view of QFT which exposes its radically holistic structure in the most forceful way; in praxis one starts with one monad and assumes that one knows the action of the Poincaré group on it \[18, 19\]; this was the way in which the existence of factorizing models was shown \[20\].

As mentioned in the introduction and more forceful in the last section, the intrinsic thermal aspect of localization is the reason why the probability issue in QFT is conceptually radically different from QM for which Born localization does not lead to a probability; the latter rather has to be added.

Although the functorial relation between the Wigner theory and operator
algebras breaks down in the presence of (any) interactions, there is a weak substitute called "emulation" (it emulates W-smeared free field \( \Psi(f) \) inside the interacting \( A(W) \)). It is extremely powerful in terms of integrable systems and promises to have clout even outside this special family; this will be the main topic of next section.

The modular analysis has some simple consequences about the issue of string localization. There is a whole family of Wigner representations (the infinite spin family) for which the intersections \( K_\mathcal{O} \) vanish for compact \( \mathcal{O} \) but not for \( \mathcal{O} = \mathbb{C} \) a spacelike cone \[53\]. This is the origin of the spacelike string generators for spacelike cone localized subspaces \[49\]. The upshot is the existence of generating fields \( \Psi^{(A,B)}(x;e) \) which are localized on the semiinfinite line \( x + \mathbb{R}_+ e \) and fluctuate both in \( x \) and \( e \). Their perturbative use requires a nontrivial extension of the Epstein-Glaser approach. An important "fringe benefit" of the use of string-localized potentials is that the best (smallest allowed by positivity) short distance dimension namely \( d_{sd} = 1 \) can always be attained by the use of suitable potentials instead of field strengths (whose \( d_{sc} \) increases with \( s \)).

This property is preserved in the massive case, although in this case there is no representation-theoretic reason for using such \( A_\mu(x,e) \) potentials. The standard pointlike massive potentials \( A_\mu(x) \) have \( d_{sc} = 2 \); it is only the stringlike potentials which allow a smooth transition in the limit \( m \to 0 \). Whereas for interactions in terms of pointlike fields there is exists in \( d=1+3 \) only a finite number of interactions which stay within the power-counting limit, this limit allows an infinite set of couplings with the help of string-localized fields. Using this new setting, many of the unanswered problems of the gauge theoretic setting (the Higgs issue) hopefully will be laid to rest \[46\].

The problems of gauge theory are very much related to the mentioned clash between pointlike generators with the Hilbert space structure of QT. Although in this paper we focus mainly on thermal manifestations of localization and crossing properties, issues of gauge theories and asymptotic freedom also depend strongly on causal localization, more than most particle physicists might have hitherto imagined. The history of that issue did not start with the famous Politzer-Gross-Wilczek work, but had its precursor in the observation by Parisi and Symanzik that sign of the beta function changes if one inverts the sign in the \( A^4 \) self-coupling.

In that case the computation based on the Callen Symanzik equation is straightforward and since the theory is massive there is no infrared problem which presents a perturbative derivation of the C-S equation. The latter is a parametric differential equation for spacetime correlation functions of pointlike fields whose physical content expresses how a change of coupling constants can be compensated by the change of the other parameters in such a way that one stays inside the finite parametric "island". It leads to a group which only change the parametrization, but not the island itself, and the Callen-Symanzik equation is the differential form of the renormalization group. The Parisi-Symanzik demonstration of asymptotic freedom is very clear, but unfortunately it is a toy model without any physical content.
For scalar fields as they appear in the formulation of critical phenomena it computational quite efficient to follow Wilson and use the so-called dimensional regularization method. This method is based on the formal idea that scalar fields look the same in different spacetime dimensions (the representation of the Wigner "little group" which determines the spinorial/tensorial character is trivial) so that the existence of a smooth interpolation has a certain plausibility. However for $s \geq 1$ fields depend in an essential way on the spacetime dimension. When one applies this method to Yang-Mills gluons there are two obstacles:

- Renormalized correlation functions in the pointlike gauge theoretic setting of Y-M theories have infrared singularities whose physical origin is blamed on the not understood issue of confinement; in contrast to QED these divergencies occur even off mass-shell in all covariant gauges. In a string-localized physical (Hilbert space) setting these divergencies result from fluctuations in the string-direction $\epsilon$ and therefore can be controlled by smearing over spacelike string directions (points in $d=1+2$ de Sitter space). It is precisely these fluctuations which reduce the short distance dimension of vectorpotentials from 2 to 1. (in fact in that description the infrared divergencies are short distance divergencies in the $d=1+2$ de Sitter space of spacelike string directions). Since gauge invariant observables are identical to pointlike localized subobservables within the stringlike setting, a crucial test whether infrared divergencies have their origin in the gauge theoretic treatment of gluons or are fundamentally nonperturbative would be the calculation of pointlike composites $\sum_a \mathcal{N}(F_{\mu\nu}^a(x)F_{\kappa\lambda}^a(x))$ where $\mathcal{N}$ denotes the "normal product". In QED the on-shell infrared divergencies do have their origin in the transfer of semiinfinite string-localization from the stringlike potentials to the charged spinor matter whereas in Y-M theories there is no distinction between the string-localized transmitters (gluons) of interactions and the objects which suffer the interaction (also gluons). One expects a much stronger stringlike localization from a perturbation theory based on the zero mass limiting behavior of massive stringlike Yang-Mills models.

- Even if the infrared problem is solved and renormalized perturbative correlations of gauge-invariant composites fulfilling C-S parametric differential equations exist, the use of dimensional regularization still remains somewhat questionable from the viewpoint of localization for $s \geq 1$ since one is not in Wilson's situation of critical phenomena which are described in terms of scalar fields to which the intuitive idea of a smoothness in the spacetime dimensionality does apply. Already the Wigner representation theory for $s \geq 1$ depends (through the "little group") significantly on spacetime. To the extend that the renormalization can be done by other regularization methods or without regularization a l'a Epstein-Glaser, this caveat is irrelevant.

The asymptotic freedom calculation (where only the sign in beta is important) by Politzer and Gross-Wilczek has led to successful experimental verifica-
tions and is the basis of theoretical precision calculations. From a conceptual point of view the situation has maintained a circular albeit selfconsistent aspect: the beta function by itself (i.e. without correlation functions and the C-S equations of which it is a part) is incomplete and the precision calculations for correlations (which use the perturbative beta function in order to describe the short distance behavior) use the opposite sign from this incomplete result. Both, the infrared behavior and the beta function problem are tied to a deeper understanding of localization for \( s \geq 1 \).

It was the problem of localization for Wigner’s infinite spin class of positive energy representations of the Poincaré group which directed the attention to the issue of string localization [47][49]. The reason why this important issue was discovered rather late is that it does not permit a Lagrangian characterization; even Weinberg’s method of covariant intertwiners based on Wigner’s representation theory encountered difficulties in ”covariantizing” these representations; the appropriate method for their construction (first of their \( K_C \)-spaces and then their string-localized generating wave functions) is modular localization. On the other hand the predominant method in QFT has been Lagrangian/functional quantization which has no access to string-localization. A string field in the sense of ST has no relation with string-localized fields. ”String” in ST refers to the classical Polyakov Lagrangian which contains classical stringlike objects \( X_\mu(\sigma, \tau) \). From the impossibility to understand relativistic quantum particles by quantizing \( L' \sqrt{\text{ds}^2} \), one should be deeply suspicious about attributing the word ”string” to an object on the basis of a questionable quantization; as a covariant particle description cannot be obtained by quantizing this classical relativistic particle Lagrangian it would be foolish to expect a quantum string to arise from the quantization of the Nambu-Goto action.

As genuinely string-localized objects cannot be obtained by quantization (but rather by using Wigner’s representation theory), objects obtained by quantization (as ST) are not string-localized in the quantum sense. The unmanageable infrared divergencies in the gauge-theoretic setting of Yang-Mills theory are a reminder of the presence of the noncompact string-localization of vectors. The reference of string theorists to strings in the sense of gauge bridges between opposite charges is misleading, there are no quantum strings since ST is pointlike generated; this holds not only for the standard ST, but also for recent attempts to free the Nambu-Goto action from the nonsensical spacetime dimensional restriction [30].

4 Generators of wedge algebras, ”Wignerism” in the presence of interactions

The basic idea which underlies the new setting of QFT is to avoid quantization and follow instead Wigner’s representation theoretical setting. As explained in the previous section, this approach leads to free fields in two steps: the classification of positive energy representations of the Poincaré group, and its use
in a functorial setting (second quantization functor) which maps modular localized real subspaces into localized operator algebras (or pointlike wave functions into quantum fields). The main computational work is the classification; knowing modular localization the second functorial step is self-directed. In this way particle state vectors and state vectors obtained by applying free fields to the vacuum become synonymous.

It is well-known that this direct particle-field relation breaks down in the presence of any interaction. The following theorem shows that the separation between the two is very drastic indeed:

**Theorem 1** (Mund’s algebraic extension \[54\] of the old J-S theorem \[16\]) A Poincaré-covariant QFT in \(d \geq 1 + 3\) fulfilling the mass-gap hypothesis and containing a sufficiently large set of "temperate" wedge-like localized vacuum polarization-free one-particle generators (PFGs) is unitarily equivalent to a free field theory.

The only relic of the functorial relation which remains unaffected by this theorem is a rather weak relation between particles and local fields in wedge-localized regions. The idea is to obtain a kind of "emulation" of free incoming fields ("particles") restricted to a wedge regions inside the interacting wedge algebra as a replacement for the nonexisting second quantization functor. This is achieved with the help of modular localization theory.

The starting point is a bijection between wedge-localized incoming fields operators and interacting operators. This bijection is based on the equality of the dense subspace which these operators from the two different algebras create from the vacuum. Since the domain of the Tomita \(S\) operators for two algebras which share the same modular unitary \(\Delta^t\) is the same, a vector \(\eta \in \text{dom}(\Delta_{\text{in}}) = \text{dom}(\Delta_{\text{out}})\) is also in \(\text{dom}(\Delta_{\text{in}}) = \Delta_{\text{out}}\) (in \(\text{dom}(\Delta_{\text{in}}) = \Delta_{\text{out}}\) it was used for one-particle states). In more explicit notation, which emphasizes the bijective nature, one has

\[
\begin{align*}
A |0\rangle &= A_{\text{in}} |0\rangle, \quad A \in A_{\text{in}}(W), \quad A_{\text{in}} \in A(W) \\
S(A)_{\text{in}} |0\rangle &= (A_{\text{in}}) A_{\text{in}} |0\rangle = S_{\text{scat}} A S_{\text{in}} |0\rangle, \quad S = S_{\text{scat}} S_{\text{in}} \\
S_{\text{scat}} A S_{\text{in}}^{-1} &\in A_{\text{out}}(W)
\end{align*}
\]

Here \(A\) is either an operator from the wedge localized free field operator algebra \(A_{\text{in}}(W)\) or an (unbounded) operator affiliated with this algebra (e.g. products of incoming free fields \(A(f)\) smeared with \(f, \text{supp}f \in W\)); \(S\) denotes the Tomita operator of the interacting algebra \(A(W)\). Under the assumption that the dense set generated by the dual wedge algebra \(A(W)\) is in the domain of definition of the bijective defined "emulats" (of the wedge-localized free field operators inside its interacting counterpart) the \(A_{\text{in}}(W)\) are uniquely defined; in order to be able to use them for the reconstruction of \(A(W)\) the domain should be a core for the emulats. Unlike smeared Wightman fields, the emulats \(A_{\text{in}}(W)\) do not define

\[28\] The particle-field relation through scattering theory is asymptotic; here we are interested in relations within localized regions of spacetime.

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a polynomial algebra, since their unique existence does not allow to impose additional properties; in fact they only form a vector space and the associated algebras have to be constructed by spectral theory or other means to extract an algebra from a vector space of closed operators (as Connes reconstruction of an operator algebra from its positive cone state structure).

Having settled the problem of uniqueness, the remaining task is to determine their action on wedge-localized multi-particle vectors and to obtain explicit formulas for their particle formfactors. All these problems have been solved in case the domains of emulats are invariance under translations; in that case the emulats possess a Fourier transform. This requirement is extremely restrictive and is only compatible with $d=1+1$ elastic two-particle scattering matrices of integrable models; in fact it should be considered as the foundational definition of integrability of QFT in terms of properties of wedge-localized generator.

Since the action of emulats on particle states is quite complicated, we will return to this problem after explaining some more notation, formulating the crossing identity in connection with its KMS counterpart and remind the reader of how these properties have been derived in the integrable case.

For integrable models the wedge duality requirement leads to a unique solution (the Zamolodchikov-Faddeev algebra), whereas for the general non-integrable case we will present arguments, which together with the comparison with integrable case determine the action of emulates on particle states. The main additional assumption is that the only way in which the interaction enters the this construction of bijections is through the $S$-matrix. With this assumption the form of the action of the operators $A_{A(W)}$ on multiparticle states is fixed. The ultimate check of its correctness through the verification of wedge duality is left to future investigations.

Whereas domains of emulats in the integrable case are translation invariant, the only domain property which is always preserved in the general case is the invariance of the domain under the subgroup of those Poincaré transformations which leave $W$ invariant. In contrast to QM, for which integrability occurs in any dimension, integrability in QFT is restricted to $d=1+1$ factorizing models.

A basic fact in the derivation of the crossing identity, including its analytic properties which are necessary in order to return to the physical boundary, is the cyclic KMS property. For three operators affiliated with the interacting algebra $A(W)$, two of them being emulates of incoming operators, it reads:

29. This statement, which I owe to Michael Karowski, is slightly stronger than that in [53] in that higher elastic amplitudes are combinatorial products of two-particle scattering functions, i.e. the only solutions are the factorizing models.

30. A very reasonable assumption indeed because this is the only interaction-dependent object which enters as a relative modular invariant the modular theory for wedge localization.

31. There exists also a "free" KMS identity in which $B$ is replaced by $(B)_{A_{\alpha}(W)}$, so everything refers to the algebra $A_{\alpha}(W)$. The derivation of the corresponding crossing identity is rather simple and its use is limited to problems of writing iterating fields as a series of Wick-ordered product of free fields.
\[
\left\langle 0 \left| BA^{(1)}_A(W) A^{(2)}_A(W) \right| 0 \right\rangle_{KMS(A(W))}^{\Delta_{BA}(1)} = \left\langle 0 \left| A^{(2)}_A(W) \Delta BA^{(1)}_A(W) \right| 0 \right\rangle_{KMS(A(W))}^{\Delta_{BA}(1)}
\]

where in the second line the operators were specialized to Wick-ordered products of smeared free fields \(A(f)\) which are then emulated within \(A(W)\). Their use is necessary in order to convert the KMS relation for \(A(W)\) into an identity of particle formfactors of the operator \(B \in A(W)\). If the bijective image acts on the vacuum, the subscript \(A(W)\) for the emulats can be omitted and the resulting Wick-ordered product of free fields acting on the vacuum describe a multi-particle state in \(f\) momentum space wave functions. The roof on top of \(f\) denotes the wave function which results from the forward mass shell restriction of the Fourier transform of \(W\)-supported test function. The result are wave functions in a Hilbert space of the graph norm \((\hat{f}, (\Delta + 1) \hat{f})\) which forces them to be analytic in the strip \(0 < \text{Im} \theta < \pi\).

The derivation of the crossing relation requires to compute the formfactor of the emulate \(A^{(1)}_A(W)\) between \(W\)-localized particle states and a general \(W\)-localized state. For simplicity of notation we specialize to \(d=1+1\) in which case neither the wedge nor the mass-shell momenta have a transverse component and particles are characterized by their rapidity. Using the analytic properties of the wave functions which connect the complex conjugate of the antiparticle wave function with the \(i\pi\) boundary value of the particle wave function, one obtains

\[
\int \ldots \int \hat{f}_1(\theta_1) \ldots \hat{f}_1(\theta_n) F^{(k)}(\theta_1, \ldots, \theta_n) d\theta_1 \ldots d\theta_n = 0
\]

Here \(\Delta^{\frac{1}{2}}\) of \(\Delta\) was used to re-convert the antiparticle wave functions in the outgoing bra vector back into the original particle wave functions. The vanishing of \(F^{(k)}\) is a crossing relation which is certainly sufficient for the validity of (24), but it does not have the expected standard form which would result if we omit the emulation subscript (in which case one obtains the vacuum to n-particle matrixelement of \(B\)). This is not allowed in the presence of interactions. In the following we will show that for special ordered \(\theta\)-configurations the general crossing passes to the standard form.

First we remind the reader how this was achieved in the integrable case \([27]\) when the matrix-elements \(\left\langle 0 \left| B \right| \theta_1, \ldots, \theta_n \right\rangle\) are meromorphic functions. In that case there exists, besides the degeneracy under statistics exchange of \(\theta\)s, also the possibility of a nontrivial exchange via analytic continuation. In that case an analytic interchange of adjacent \(\theta\) produces an \(S(\theta_i - \theta_{i+1})\) factor, where \(S\) is the scattering function of the model (the two-particle S-matrix from which
all higher elastic S-matrices are given in terms of a product formula) [27]. For
general permutations one obtains a representation of the permutation group
which is generated by transpositions. The steps which led to the result can be
summarized as follows:

1. Use the statistics degeneracy to fix a natural order so that the "faster"
particles (bigger $\theta$) are to the left of the smaller $\theta_1 > ... > \theta_n$, so that
in the backward extension of the velocity lines there was no crossing of
velocity lines. Identify the analytic matrix-element in the natural order
with the incoming configuration

$$\langle 0 | B | \theta_1,..\theta_n \rangle = \langle 0 | B | \theta_1,..\theta_n \rangle_{\text{in}}, \quad (26)$$

Any other order is then determined by the analytic exchange rules in terms
of a grazing shot S-matrix $S_{gs}$

$$\langle 0 | B | \theta_1,..\theta_n \rangle = S_{gs} \langle 0 | B | \theta_2,..\theta_1,\theta_{k+1}..\theta_n \rangle_{\text{in}}, \quad S_{gs} = \prod_{l=2}^{k} S(\theta_l - \theta_1) \quad (27)$$

$$\theta_2 > .. > \theta_1 > \theta_{k+1}.. > \theta_n$$

2. The analytic exchange relation can be encoded into algebraic commutation
relations of the Zamolodchikov-Faddeev (Z-F) type

$$Z(\theta) Z^*(\theta') = \delta(\theta - \theta') + S(\theta - \theta' + i\pi) Z(\theta') Z(\theta) \quad (28)$$

$$Z^*(\theta) Z^*(\theta') = S(\theta - \theta') Z^*(\theta') Z^*(\theta)$$

$$Z^*(\theta_1) .. Z^*(\theta_n) \langle 0 | = \langle \theta_1,..\theta_n \rangle_{\text{in}}, \quad \theta_1 > ... > \theta_n \quad (29)$$

where the last line contains the identification with the incoming particles.

3. The Z-F operators are the Fourier components of generating operators of
the interacting wedge-localized algebra [18][19][20]

$$A_{\text{in}}(f_{A(W)}) = \int_{\partial C} Z^*(\theta) e^{ip(\theta)\xi} \hat{f}(\theta) d\theta, \quad C = (0, i\pi) \text{ strip}, \quad Z(\theta) = Z^*(\theta + i\pi) \quad (30)$$

where $\hat{f}(\theta)$ is the mass-shell restriction of the Fourier transform of $f$, $\text{supp} f \in W$.

The consistency of the algebraic structure with wedge-localization and the
proven nontriviality of the intersection of double cone algebras, defined as the
intersection of two wedge-localized algebras, secure the consistency of the an-
alytic assumption as part of the existence a QFT whose S-matrix is the given
scattering function.

The construction has an analog for non-integrable models. The main com-
plication results from the presence of all inelastic threshold singularities of mul-
tiparticle scattering in the "analytic $\theta$-commutation". This leads to a path-
dependence for $\theta$-commutations i.e. the analytic structure cannot be anymore
subsumed into the algebraic structure of a representation of the permutation group. For the special case the shortest path for getting the \( \theta \)s into the natural order corresponds to the commutation of the \( \theta_1 \) with the k-1 cluster \( \theta_2,..\theta_k \). So the first question is whether there exists an analog of the grazing shot S-matrix in the general case. For this purpose it is helpful to rewrite the above integrable \( S_{gs} \) into an expression which only involves the full S-matrices. It is clear that

\[
S_{gs}(\theta_1; \theta_1,..\theta_k) = S(\theta_2,..\theta_k)^*S(\theta_1,..\theta_k) \tag{31}
\]

with \( S \) being the full S-matrices of k respectively k-1 particles does the job. In case the two-particle scattering matrix is not just a scattering function but rather a matrix of scattering functions, one has to use the Yang-Baxter relation in order to cancel all interactions within the k-1 cluster \( \theta_2,..\theta_k \); the remainder describes a "grazing shot" of \( \theta_1 \) on the \( \theta_2,..\theta_k \) cluster. In this form the grazing shot idea permits an adaptation to the general case

\[
S_{gs}^{(m,n)}(\chi_1; \theta) = \sum_l \int \ldots \int d\vartheta_1 \ldots d\vartheta_m \langle \chi_1 \ldots \chi_m | S^* | \vartheta_1, \ldots \vartheta_l \rangle \cdot \langle \vartheta_1, \vartheta_1, \ldots \vartheta_l | S | \vartheta_1, \vartheta_2, \ldots \vartheta_k \rangle \tag{32}
\]

In this case the \( \chi \) represents the \( \chi = \chi_1,..\chi_m \) component of a scattering process in which the grazing shot "bullet" \( \theta_1 \) impinges on a k-1 particle \( \theta \)-cluster consisting of \( \theta_2,..\theta_k \) particles. Here the sum extends over all intermediate particles with energetically accessible thresholds, i.e. the number of intermediate open \( l \)-channels increase with the initial energy. The matrix elements of the creation part of an emulate sandwiched between two multi-particle states can directly be written in terms of the grazing shot S-matrix as

\[
in \langle \chi_1 \ldots \chi_m | Z^* (\theta)_{A(W)} | \theta_1 ; \theta_2, \theta_n \rangle_{in} = S_{gs}^{(m,n)}(\chi, \theta_1 ; \theta) \tag{33}
\]

A similar formula holds for the annihilation part. Once the annihilation operator has been commuted through to its natural position, it annihilated the next particle on the right and contributes a delta contraction. This procedure may be interpreted as a generalization of Wick ordering to interacting emulates.

The general grazing shot S-matrix is the only expression which (a) reduces to the integrable grazing shot S-matrix and (b) fulfills the requirement that the commutation of \( \theta_1 \) with a \( \theta_2,..\theta_k \) cluster can be expressed in terms of S-matrices only. As mentioned this requirement has its origin in the fact that the only way, in which the interaction enters into the theory of modular wedge localization, is through the S-matrix. Under the assumptions (a) and (b) the commutation formula of an emulate \( (A(f))_{A(W)} \) (or its Wick-ordered extension) with a cluster of particles is unique and the resulting formula may be used to evaluate the left hand side of the KMS relation \( (24) \) in terms of vacuum to multi-particle matrix-elements of \( B \). The resulting formula is consistent with the standard form of the crossing identity.
\[ \langle 0 | B|\theta_1, \theta_k, \theta_{k+1}, \theta_n \rangle_{\text{in}} = \langle \bar{\theta}_{k+1}, \ldots, \bar{\theta}_n | U(AW(0,1)(\pi i))B|\theta_1, \ldots, \theta_k \rangle_{\text{in}} \]  
\( B \in \mathcal{A}(\mathcal{O}), \mathcal{O} \subseteq W(0,1), \bar{\theta} = \text{antiparticle of } \theta, \theta_1 > \ldots > \theta_n \)

only in case of the natural order. Any different order between the two clusters will correspond to a different, much more complicated left hand side which will contain contributions from grazing shot S-matrices to arbitrary high particle number. Whereas for the partitioning of n-particle states into two clusters the natural order can always be maintained; in case we start from a general n-k to k formfactor, the relative ordering between out and in \( \theta \) has to be imposed in order to maintain the simple form of crossing. Only in that case the crossing identity retains its simple form without modification from the grazing shot S-matrix.

For the special case of crossing just one particle it reads

\[ \langle \text{out} | \theta_{k+1}, \ldots, \theta_n | B|\theta_1, \ldots, \theta_k \rangle_{\text{in}} = \langle \text{out} | \bar{\theta}_k + i\pi, \theta_{k+1}, \ldots, \theta_n | B|\theta_1, \ldots, \theta_{k-1} \rangle_{\text{in}} \]  

if the \( \theta_k \) is bigger than the outgoing \( \theta_s \).

In the application of the Haag-Ruelle scattering theory to the derivation of the LSZ reduction formalism \[55\] there are threshold modifications from overlapping wave functions which wreck the strong approach of the asymptotes \[56\] in the limit of large times and thus invalidate the LSZ reduction formalism. We believe that they correspond to the opening of threshold in the grazing shot S-matrix which enters in the algebraized analytic changes of the natural order in the presence of overlapping wave functions.

The important new message is that the issue of the general form of the crossing relation together with the computation of the left hand side of the KMS identity (without the ordering restriction) is inexorably linked with a new constructive aspect of the action of emulats on particle states in which the interaction enters in form of the grazing shot S-matrix. The latter couples particle cluster (of those particles through which the emulat has been commuted) to all sectors to which the superselection rules permits such couplings. In other words the analytic exchange of \( \theta_s \) associated with the emulate with those \( \theta_s \) which correspond to a cluster of particle in the incoming state leads to a perfect realization of an on-shell version of "Murphy's law": everything which is not forbidden to couple (subject to the validity of the superselection rules) actually does couple.

In off-shell QFT this is of course well-known, but on-shell (in the sense of formfactors) this is new and somewhat surprising; it is both a blessing and a curse. In the integrable case it leads to a representation of the permutation group \[20\] and the possibility to construct wedge generators for given scattering function by "deformations" of free fields \[21\]; whereas in general the analytic exchange is path-dependent (reflecting the influence of the inelastic threshold cuts) and the generators require the application of the much more complicated emulations. In fact the general situation resembles vaguely that of a d=1+2 Wightman

39
theory with braid group statistics \cite{57} for which the Bargman-Wightman-Hall analyticity domain \cite{16} is not schlicht but contains cuts, and the path-dependent algebraic commutation represents the action of the infinite braid group.

The analytic $\theta$-exchanges was the crucial idea which led the authors in \cite{27} to formulate their bootstrap-formfactor project for factorizing models. In that case the analytic transposition of two adjacent $\theta$ can be encoded into the algebraic Zamolodchikov-Faddeev commutation rules. In the general case there is no transposition rule which leads to a representation of the permutation group, rather the situation becomes analogous to the braid group structure in which a right-left distinction is not sufficient, one must spell out the path which led to final right-left configuration (i.e. analog of the behind/in front move leading to braiding). We would like to think of the process of emulation as being the analog of the functorial construction of free field algebras from the application of the second quantization functor to Wigner’s representation theoretical construction of particles

$$\text{functorial relation} \xrightarrow{\text{interaction}} \text{emulation}$$

According to Mund’s theorem it is impossible to maintain a functorial relation in the presence of interactions; it has to be replaced by a bijection of particles and their free fields into their interacting emulats.

In this way the crossing property becomes an integral part of a new non-perturbative construction of a QFT whose first step is the construction of wedge generators. As in the failed S-matrix bootstrap project, it is an essential part of a new constructive program which in addition to the S-matrix uses on-shell formfactor\cite{32}. There are two unsolved problems with this setting in the non-integrable case

- Show that the action of the emulates in terms of the grazing shot S-matrices leads to wedge duality

$$\left\langle \psi \left| J \left( A_{in}(\hat{f}) \right)_{\mathcal{A}(W)} J, (A_{in}(\hat{g}))_{\mathcal{A}(W)} \right| \varphi \right\rangle = 0, \ J = S_{\text{scat}} J_{\text{in}} \quad (36)$$

where $|\psi\rangle$ and $|\varphi\rangle$ are multi-particle states and the input is an S-matrix which fulfills the crossing property. This is a structural problem.

- Show that the inductive use of the wedge duality starting with a lowest order input for the S-matrix being the lowest nonvanishing mass-shell restriction of the scattering amplitude and computing from this and the validity of (36) the lowest order formfactor and afterwards the next order S-matrix and so on. such an iteration resembles vaguely the Epstein-Glaser iteration based on the recursive implementation of causal locality. The divergence of perturbative series based on singular field has no direct bearing on such an on-shell perturbation.

\footnote{In fact the matrixelements of the S-matrix represent the formfactor of the identity operator.}
The main difference to the old S-matrix program is that it contains much more structure. Without having formfactors in addition to the S-matrix and a relation (36) which contains both, it is not possible to have a constructive iteration. The hope is of course that the iteration converges, which is known not to be true in the case of Epstein-Glaser perturbation which deals with singular fields.

One note of caution. The use of bilinear forms (33) does not mean that the emulates $A(\hat{f})A(W)$ have general n-particle states in their domain. This is only the case for integrable models [53]. The emulated operators as well as their action as operators on multiparticle states are generally only well-defined on W-localized particle states. In computation of norms one should understand these problems in terms of properties of the unboundedness properties of the grazing shot S-matrix. For the check of wedge duality it is important that the operators as well as the states are W-localized. One expects however that in formal computations involving only matrix elements (bilinear forms instead of operators) one can relax those requirements.

The close relation of the crossing with the cyclic KMS identity underlines again that crossing in the sense of particle theory has nothing to do with Veneziano crossing and ST.

The present formalism replaces the old S-matrix attempts (the S-matrix bootstrap, the dual model and string theory). It constitutes a formulation of QFT in terms of on-shell quantities only. But it is merely the first step in a future classification and construction setting (existence and controlled approximations). As in the integrable case where all these steps have been carried out [20], one still needs to show that the double cone intersections of wedge-localized algebras are nontrivial.

The main message of this section is that the failure of the previous S-matrix projects and in particular the deconstruction of string theory based on conceptual misunderstandings about causal localization does not leave one empty-handed. Rather one encounters a completely new window into local quantum physics on the ruins of the old project. In this, and only in this sense, the last 5 decades do not only constitute a loss with respect to foundational aspects of local quantum physics; the resolution of a deep misunderstandings, more than any so-called revolution, could be the seed of deep progress.

5 Resumé and concluding remarks

The main point of the present work was to explain why Mandelstam’s important project of a mass-shell based top-to-bottom approach took a wrong turn when he mistakenly accepted Veneziano’s dual model crossing as a description of the particle physics crossing. As a result Mandelstam’s farsightedness concerning the importance of S-matrix-based on-shell projects in particle physics took a wrong directiony as a result of his belief that this can be accomplished by the dual model and ST.

Our derivation of crossing identities for particle formfactors was based on
the use of two important concepts which both follow from modular localization: *interacting emulats of Wick products of wedge-localized free fields* which describe the particle content of a QFT, and their *use in the KMS identity* of modular localization. We argued that this new construction should be interpreted as an extension of Wigner’s 1939 first intrinsic (quantization-free) construction of interaction-free local quantum physics in terms of positive energy representation of the Poincaré group combined with Weyl’s CCR (or its CAR counterpart) ”second quantization” functor. In contradistinction to Schrödinger’s QM, the functorial relation between localized subspaces of wave functions and local operator subalgebras breaks down in the relativistic case and has to be replaced by the much weaker connection between incoming and interacting wedge algebras presented in the previous section which replaces the functorial relation. The correct particle crossing was shown to be an important side result of this new construction.

We showed that the dual model results from a crossing identity of conformal correlation, using an argument which can be traced back to work by Mack [24]. This is sufficient to show that what Veneziano accomplished in his construction of the dual model has no conceptual relation with the particle crossing seen in Feynman diagrams and used as a defining property by the protagonists of the S-matrix bootstrap as well as in Mandelstam’s proposal of a ”double spectral representation”. However we also added the independent presentation of a recent in order to make our case iron-clad in order to prevent ST to escape through conceptual back-doors or prevent them from following their favorite strategy which Feynman characterized as ”string theorists don’t make predictions, they rather make excuses”.

Our proof of the particle crossing relation also confirms what (since the time of the S-matrix bootstrap and the formulation of Mandelstam’s on-shell project) was always suspected, namely that particle crossing plays an important role in any on-shell top-to-bottom nonperturbative construction within particle theory. Fields without particles as those encountered in interacting conformal QFT may play an important role as ”theoretical laboratories” for studying certain mathematical aspects of QFT (especially if it comes to questions of mathematical existence); but no experiment will ever measure a nucleon field; fields are the best objects to implement localization ideas, but they are simply too fleeting for being directly measurable. Their non-fleeting manifestation are asymptotically stable multi-localized particle states in theories which allow a complete asymptotic particle interpretation. In this respect the underlying philosophy in this paper is very different from that of Wald [58]; the ostentatious absence of Wigner’s concept of particles in curved spacetime is no reason for giving up looking for non-fleeting entities with stable n-fold excitations of a reference state which replaces the Poincaré invariant vacuum which are asymptotically related with fields.

The framework of QFT used in this paper is radically different from quanti-

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The value of historical cohesion in particle physics cannot be overestimated in times of self-proclaimed revolutions.
zation approaches and can be viewed as a synthesis of two hitherto antagonistic settings: Haag’s LQP approach based on localized observables and Mandelstam’s (pre-Veneziano) S-matrix setting. The new framework uses the S-matrix already at the start, its ultimate aim is to construct local observables. Exact solutions will always be limited to integrable models, which in QFT are necessarily two-dimensional [24]. In non-integrable models with a complete particle interpretation one can only hope for mathematically controlled approximations and with some luck, the verification of the (hopefully unique) existence of a QFT for a given crossing symmetric S-matrix as well as controlled arbitrary precise approximations. In view of the suspicion that the divergence of perturbative series may be connected to the fact that correlation of such singular objects as fields (operator-valued distributions) are to blame e for the lack of control, there is even a chance that a perturbation expansion for more rugged non-singular on-shell objects as generators of wedge-algebras may converge; the absence of ultraviolet problems was the historical reason why Heisenberg proposed an S-matrix setting.

Although the new setting provides an optimistic look into a better future for particle theory, there is good reason to be less optimistic when it comes to the computational implementation of these new ideas in the near future. The number of physicists familiar with foundational aspects of local quantum physics has decreased, this is particular evident in the US where the interest in mathematical-geometrical formalisms has overtaken that in the more subtle foundational physical aspect in which geometric properties are always blended with subtle physical consequences of quantum localization.

The absence of any innovative investment into foundational knowledge (“shut up and compute”) within the new globalized communities, and with metaphoric arguments and trendy monocultures on the rise, there is not much reason for optimism. Also the observation that interest in string theory and its derivatives (extra dimensions, branes, TOEs) seems to be waning cannot be a cause for optimism. The metaphoric Zeitgeist of the ST (which seems to parallel the metaphoric nature of financial capitalism) has entered large parts of particle theory (extra dimensions, branes, embedding of QFTs,...). Most supporting quasiclassical arguments collapse if it comes to problems of real quantum matter, e.g. connections between cardinality of degree of freedoms, time-like causality and modular localization for which one needs the full conceptual power of LQP.

Causal localization is certainly the most subtle and far reaching property of local quantum physics; and with the birth of QFT in the aftermath of the incompletely understood Einstein-Jordan conundrum, there was always the latent danger that the incompletely understood sharp conceptual borderline with respect to the Born-localized QM could eventually cause havoc. But apart from transitory problems with the old perturbation theory (see the old pre-renormalization textbooks by Wenzl and Heitler) as well as the misunderstandings about vacuum polarization in connection with the so-called ultraviolet catastrophe [24], one was able to find consistent recipes for the new covariant pertur-
bation without fully understanding the relation between thermal and vacuum polarization aspects with causal localization.

The invocation of covariance which was the turning point in perturbation theory (ascribed to Tomonaga) is however of not much avail if it comes to subvolume fluctuation problems as in the Einstein-Jordan conundrum of the cosmological constant problem; here a more direct understanding of thermalization through causal localization is asked for. In special cases, where modular localization leads to geometric Hamiltonians as in the case of the Unruh effect, this was understood in terms of concrete calculations (and only by a much smaller number of physicists as a result of structural arguments [4]). The insight that important properties of particle theory as the crossing properties of on-shell objects (scattering amplitudes, formfactors), and a new settings for intrinsic on-shell constructions of particle theories (in the spirit of Mandelstam’s post dispersion theory S-matrix attempts i.e. no use Lagrangian or functional quantization) is of a more recent vintage.

Far from leading only to a critical evaluation of what has been done in the past, the purpose of the new approach based on modular localization also includes to extend renormalized perturbation theory involving higher spin fields by lowering their short-distance dimensions by using string-localized "potentials" in Hilbert space instead of pointlike "field strengths" in Krein spaces (BRST-formalism). In the zero mass case this includes also the project of to understand what hitherto has been swept under the rug by referring to "non-perturbative long distance infrared singularities" of Yang-Mills couplings in terms of overlooked long-distance changes caused by semiinfinite string-localized "potentials". This also effects the conceptual position of the Higgs phenomenon and of asymptotic freedom.

Ever since its birth in the aftermath of the 1925 Einstein-Jordan conundrum the insufficient understanding of the physical consequences of quantum causal localization was threatening to limit the full exploration of QFT beyond perturbation theory. Only in the aftermath of the successful particle adaptation of the Kramer-Kronig dispersion relation [59] when physicists started to become interested in non-perturbative on-shell aspects the lack of understanding hit particle theory with full vengeance. As described in detail in section 2 its first casualty was the dual model, or to be more precise the dual model as a description of particle crossing. The correct particle crossing follows from the KMS property of wedge-localization; the same KMS property which, as a thermal epiphenomenon of modular localization, is at the bottom of the Unruh effect and similar situations in curved space time (in which the imagined causal horizon is replaced by a less fleeting event horizon).
It will take a very long time to dispose of all the conceptual clutter created not only by ST, but also by its derivatives which later became disconnected from ST. This includes the idee of embedding of lower dimensional QFTs in higher ones and its inverse namely a QFT version of the quantum mechanical Klein-Kaluza dimensional reduction; but it also extends to Maldacena’s belief that the mathematical AdF-CFT correspondence is capable of relating two physical theories and to the closely related incorrect idea that the restriction of a QFT to a "brane" describes a physical QFT which fulfills timelike causality. Such misunderstandings of local quantum physics are often supported by ”massaging” Lagrangians or using quasiclassical approximations (which contain no information about degrees of freedom their connection with quantum localization) which are contradicting the holistic consequences of causal localization for local observables and their correlations. The best way out of these confusions which arose from the incorrect idea that quantization preserves classical/semiclassical results about embeddings or K-K dimensional reductions, is to follow the guidance from the modular localization theory.

Leaving string theory aside, we permitted ourselves the fascinating historical dream of imagining a changed path of history in which the Einstein-Jordan conundrum was solved at the time of birth of QFT. As it is well known, Einstein had deep-rooted misgivings about Born’s assignment of probability to individual quantum mechanical events. Of course Born did this not out of the blue, but as a consequence of an interpretative necessity in order to relate the (Born approximation of) scattering amplitudes (nowadays the "cross section") to the statistics of scattering events caused by sending a beam onto a target.

Just image that Einstein would have been aware of that intrinsic thermal KMS aspect of localization which is implicit in Jordan’s QFT contribution to the E-J conundrum. The probabilistic aspect of statistical ensembles (different from the assignment of probabilities to single events observed on an individual systems) was Einstein’s centerpiece of his theoretical fluctuation arguments concerning the corpuscular nature of light which got Jordan into the E-J conundrum dispute. The clear recognition that subvolume-reduced QFT which unlike QM represents an ensemble of operators belonging to the same localized algebra and that the reduced vacuum defines a special kind of thermal statistical analog (or even isomorphic) to a global heat-bath statistical mechanics system could have changed history. Certainly Jordan would have appreciated to receive Einstein’s full support in his struggle against the resistance of his coauthors Born and Heisenberg to concede a separate section to his "wave quantization" in the Dreimännerarbeit (which they grudgingly did even without Einstein’s support).

This viewpoint of dealing only with ensembles was fully realized in Haag’s 1957 theory of local nets of operator algebras, in fact it is the central issue which distinguished this approach from other settings as Wightman’s formulation.

It is Born’s quantum mechanical assignment of probabilities to individual

\footnote{In \cite{25} it is shown that the thinning out of degrees of freedom in holographic projections onto null-surfaces does not happen in projections to branes.}

\footnote{The application to Schrödinger wave functions (apparently attributed to Pauli) was later added as a footnote to Born’s famous paper.}
objects which Einstein objected to, and far from criticizing him for his stubbornness on this point, we should admire him for his steadfastness coming from his philosophical realism. It is the more fundamental QFT as compared to QM which brings back to QT some of Einstein’s realism by placing various counter-intuitive effects into a more realistic light by relating the probabilistic ensemble property of QFT as coming directly from the quantum adaptation of the Faraday-Maxwell “action at the neighborhood principle” instead of adding them by fiat as the Born probability in QM.

Not only the many counter-intuitive consequences of QM would have appeared in a different (more Einstein-friendly) philosophical light, but the entire evolution of QFT might have taken a different direction (and certainly ST would have remained without support). Also papers on superluminal propagation as, they appeared in the past (and still appear almost every year), would never have passed the editorial hurdle of PRL.

It is a bit unfair to blame only string theorists for the present schism in particle physics. Those few individuals who always had the knowledge about QFT’s deeper conceptual layers did not leave their ivory tower (which perhaps would have meant a temporary interruption of their own work and a continuation of the old Streitkultur which kept particle physics healthy for several decades starting from the 50s and ending in the 70s) share part of the blame. The present work comes too late to have an effect; but at least there is the small consolation of having tried.

In no country the impact of ST and its derivatives has been as disastrous as in Germany, the country in which QT started. Take as an example the theoretical physics at the university of Hamburg which after its foundation in 1920 adorned itself with an illustrative continuous line of names: Lenz, Pauli, Jordan, Lehmann, Haag and Fredenhagen, but finally could not resist the outside pressure of laboratories of “big science” which, in the tradition of royal courts and their court jesters compete to have their own local string theorists.

Fact is that foundational research on QFT in a country where it began in 1925 and continuously evolved up to the present may not have a place of location in the foreseeable future. It is a small consolation that the situation in Austria, Italy and the UK is not as bad; for the time-being the swan-song of a disappearing foundational research in QFT only threatens to affects the country where it all begun.

Concluding this resumé of foundational aspects of QFT including a scientific critique of ST, it may be helpful to the reader to comment on some sociological

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38 Being the more fundamental theory, the limitation in forcing QFT to dance to the tune of a quantization parallelism would have been perceived much earlier. The “ensemble” point of view in terms of extracting everything from localized subalgebras instead from individual localized operators matches perfectly the way an experimenter does particle physics without knowing (or even wanting to know) the inner workings of his measuring devices.

39 To the exculpation of PRL it should also be mentioned that immediately afterwards they published a correct presentation of Fermi’s argument that subluminal phenomena are absent in QED.

40 After Res Jost’s forceful critique of some of the claims in the context of the the bootstrap approach the Streitkultur of the Pauli era came to an end.
arguments which defenders of ST often use. To be concrete, I explicitly refer to a recent published defense by Duff within a project “Forty Years Of String Theory: Reflecting on the Foundations” [62] as well as on an article by Smolin [63], who in Duff’s article serves as his punching bag only to end in his own article as a fervent subscriber to ST’s most bizarre solution of the background independence problem in form of the “multiverse”[41].

Without wanting to defend Nancy Cartwright’s somewhat extreme points of view on emerging unification[42] and the strife for a “Weltformel” or a TOE (a unique theory of everything) against Duff’s critique, it should be said that at least its inverse, namely to conclude from the existence of a unique or nearly unique realization of some idea that it must have foundational physical significance, is (hopefully) not acceptable to me but also to him. But it appears that string theorists do precisely this when they conclude from the (nearly, up to M-theoretic modifications) unique possibility to represent a unitary positive energy representation of the Poincaré group on what they call the “target space” of a nonrational chiral sigma model (in more old-fashioned terminology the only solution of the Majorana project).

Instead of trying to understand why nonrational chiral theories with their continuously many superselected charge sectors[43] allow representations of noncompact “inner symmetry” groups, string theorists insist to solve the problem by identifying our living spacetime with the noncompact internal symmetry space of a nonrational sigma model. Its continuously superselected charge structure constitutes the only known cases for realizing noncompact inner symmetries on the oscillator content of compactified chiral conformal QFT; the best (and presently only known) illustration for the realization of a positive energy representation of the Poincaré group is the superstring representation on a 10-component abelian chiral current model.

Whereas it is certainly true that one can do this on the “target” space of a (mildly modified) chiral sigma model associated to abelian currents in d=10 spacetime dimensions, the important question is really what to make of such constructions. Should one interpret this property abstracted from a certain way of dealing with the charge spectrum of multi-component current models as defining a foundational spacetime theory (the dog’s tail wiggling with the dog)? Are we living in a dimensionally reduced inner symmetry space of a (modified) nonrational conformal QFT? Apparently Duff [62] believes that we are.

One has all reasons for being somewhat surprised about the near uniqueness (up to a finite number of M-theoretic changes) of the 10 dimensional superstring representation of the Poincaré group on the target space of a nonrational sigma model, but there is no conceptual justification to interpret the rarity of such

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[41] According to the motto: if the tail of the dog does not wiggle, then perhaps the tail’s dog can be made to wiggle.
[42] I think what she is probably criticising is an enforced unification which does not emerge from the natural flow of improved insights into nature.
[43] Different from higher dimensional observable algebras which, at least in theories with compactly localizable superselected charges, only permit extensions to charged algebras with compact internal group symmetries nonrational chiral QFTs are the (little explored) breeding ground for representations of noncompact inner symmetries.
an occurrence (probably related to the fact that non-rational chiral theories have not been the subject of systematic studies) as the harbinger of a new foundational insight into spacetime. Would particle physics in the last 5 decades have developed in a different way if the answer to this question would have been that there is either no or infinitely many representations?

Attributing to this observation the role of a key to the understanding of the universe is not much different than the ontological role attributed to the number 42 as an answer to the ultimate question about "Life, the Universe, and Everything" in Douglas Adam’s scientific fiction comedy "the hitchhiker’s guide through the galaxy".

A closely related remark mentioned in section 2 is that, whereas the principles of classical field theory admit dynamical covariant quantum mechanical variables $X_\mu(\tau)$ which parametrize world-lines in any dimension, there are no covariant quantum position operators which correspond to these classical objects; rather the move with changing $\tau$ takes place in an inner space "over" the localization point (where the spin components are pictured) where localization has no relevance. This is a special case of the general impossibility of quantum field theoretical embeddings and K-K dimensional reductions. In any QFT with $d > 1 + 2$ indices of covariant fields can only refer to compact internal symmetry groups or represent spacetime spinor/tensor indices corresponding to the spacetime living space dimensions of the fields. Classical fields on the other hand can also carry indices on which noncompact groups act, but quantization does not lead to quantum analogs. There is no law which says that the more foundational QFT has to dance to a quantization parallelism.

As there are many classical Lagrangians which cannot be quantized, there are also many QFT which possess no Lagrangians. The best illustration of the latter state of affairs is given by most $d=1+1$ models which have been constructed with the formfactor-bootstrap model and whose existence was secured by Lechner’s application of the modular nuclearity idea about the cardinality of degrees of freedom [64]. The abundance of non-Lagrangian QFTs in $d=1+1$ can be understood in terms of the larger cardinality of crossing symmetric elastic scattering amplitudes as compared to renormalizable Lagrangian couplings.

The sociological criticism contained in some well-known books and articles by Woit, Smolin and others is of no help in this context. Even though this critique may have been well intentioned, string theorists and their adversaries began to live in symbiosis with their opponents. Sociological critique does not penetrate the thick layer of misunderstandings around the subtlest of all principles: quantum causal locality. It rather runs the danger of becoming part of the scientific entertainment industry in an unfortunate mutual dependence between supporters and "opponents".

To many particle physicists string theory appeared as the most surreal if not bizarre subject they ever met. Hopefully this article can convince readers that its surreal nature is not the result of a computational mistake nor an easy to spot conceptual misunderstanding. The present paper exposes some of the scientific causes of these surreal feelings. The sophisticated nature of the conceptual mistake also means that the string theory research during 5 decades does not
amount to a total loss of time, since the derailment of a foundational subject as the Mandelstam on-shell top-to-bottom project is a strong motivations for a second startup using the new property of $S_{\text{scat}}$ as a relative modular invariant of wedge-localization as formulated in the present work. Without the recent concept of modular localization it also would not have been possible to find a new string-localized setting for higher spin fields which lowers their short distance scaling dimension to the $s$-independent value $d_{s.d.} = 1$ which opens the gates for higher spin renormalized perturbation theory. It was the critique of previous S-matrix based attempts (in particular ST) which played an important role in obtaining these new insights and in this way led to a golden opportunity for future progress.

To feel the depth of the crisis into which large parts of particle theory has fallen, it is interesting to present a quotation from Einstein’s talk in the honor of Planck [65].

*In the temple of science are many mansions, and various indeed are they who dwell therein and the motives that have led them thither. Many take to science out of a joyful sense of superior intellectual power; science is their own special sport to which they look for vivid experience and the satisfaction of ambition; many others are found in the temple who have offered the product of their brains on this altar for purely utilitarian purposes. Were an angel of the Lord to come and drive all these people belonging to these two categories out of the temple, the assemblage would be seriously depleted, but there would still be some men, of present and past times, left inside. Our Planck is one of them, and that is why we love him. ...*

But where has Einstein’s *Angel of the Lord*, the protector of the temple of science, gone in the times of string theory and its derivatives? Reading these lines and comparing them with the content of [62] as well as that of his opponents, one cannot help to sense how similar the present Zeitgeist of particle theory has become with that of the financial investment markets. Nowhere is this better reflected than in the alternative to the Nobel physics prize created by the Russian oligarch Yuri Milner. His in principle noble decision to spend part of his gain from financial transactions has been taken in the belief that by awarding the prize to protagonists of ST and related subjects of strong sociological impact (“many people cannot err”) on a large part of the particle physics community one can correct what in his view has been unduly overlooked by the Nobel committee. None of the individuals with a foundational knowledge of local quantum physics of the 60s would have proposed a dual model or ST; the incomplete knowledge or lack of knowledge about the relation of locality with the particle crossing property was the prerequisite for being able to do ST. Only this incorrect idea made it possible to claim that the duality of a meromorphic function constructed by using mathematical properties of the Euler beta-function may be related with the particle crossing property of an elastic scattering amplitude.

A second chance was lost when Fubini et al. [67] used ideas from chiral conformal QFT for its construction and they failed to see/state that what they identified with the particle poles were really the anomalous scale dimensions
of the composites [25] in conformal global operator product expansions; after this missed chance ST became the victim of this admittedly confusing picture puzzle. Milner’s prize will remind future physicists in a post ST era about those parts of particle theory which got lost in the maelstrom of the ST Zeitgeist, it is the first award which is given for making the best use out of one’ lack of knowledge without even being aware after it happened. Such a prize highlights the deep schism within the actual particle physics community.

In the present paper we showed how Mandelstam’s on-shell top-to-bottom idea of accessing nonperturbative particle theory can be saved in a new approach which avoids the dual model/ST picture puzzle confusion by using modular localization which places the $S_{\text{scat}}$-matrix (in its role as a relative modular invariant) together with formfactors (the matrix-elements of $S_{\text{scat}}$ are formfactors of the identity operator!) under one new constructive roof within LQP. The new setting is deep, and since it is still in its beginnings one can expect the appearance of many unaccustomed aspects, but it will never lead to such bizarre consequences as those which came in the wake of ST [66].

The proximity of foundational research to what is perceived as big science is risky, only those intellectual products which find a lobby will continue and once a large enough community has formed they are beyond critical review. What will future historians of physics make of such misunderstood and half-baked ideas as ST, its bizarre derivatives as ”the landscape” or of Tegmark’s even more bizarre credo that every mathematically correct ideas will have a realization in physics one of the zillions of parallel universes?

It is perhaps not an accident that this happens while we live in a time which moves away from the ideals of enlightenment and religious fanaticism is replacing previous economic ideologies. Is the wide-spread support of metaphoric ideas in a particle physics part of a a Zeitgeist-phenomenon ?

In his recent defence of ST Duff should not have left out the name of Feynman who had, probably more driven by its bizarre appearance than its conceptual structure, pointed out that his discussions with proponents of ST convinced him that it is the first construct in particle theory which is not defended by arguments but rather by taking recourse to excuses.

Duff should not be so sure about counting on Weinberg, who on several occasions speculated that string theory may still reveal itself as a camouflaged QFT [29]. In fact this is precisely what the present paper showed: the acceptable observations coming from string theory is its identification with a dynamical infinite component pointlike-localized wave function where, as mentioned before, ”dynamical” means that its wave function space is obtained by solving the ”Majorana project” which consists in constructing irreducible operator algebras which carry a discretely reducible representation of the Poincaré group describing an infinite collection of irreducible positive energy one-particle representation. It was pointed out in this article that the only known solution of this ”infinite component dynamical wave function project” is the ”superstring representation” abstracted from the irreducible oscillator algebra of a d=10 component supersymmetric chiral current model (related to the Polyakov action). Since no zero mass infinite spin representation appears in this decomposition
the wave functions are pointlike generated and the second quantization leads to an infinite component dynamic pointlike quantum field. Such a pointlike field is of course too singular to pass as an operator-valued Schwartz distribution, but by projecting to finite invariant energy subspaces one recovers the standard situation. At the time when Majorana [9] looked for an infinite component relativistic analog of the $O(4,2)$ hydrogen spectrum and also afterwards the search was limited to group representation algebras which extend the Lorentz group. No solution of the Majorana project in this limited context was found and hence the superstring representation from the d=10 Polyakov oscillators (which also intertwine between the levels of the infinite $(m,s)$ tower).

Concerning most of the other names in Duff’s list on whose support of ST he counts, one should perhaps point out that most scientist have a legitimate natural curiosity which leads them to have an unprejudiced look at any new idea which isn’t outright foolish. But they usually do not sacrifice much time to understand the detailed mathematical/conceptual reasons why at second glance ST appears as a mixture of high powered mathematics with somewhat bizarre physics; they rather simply turn away from it. Nowhere in this paper I claim that string theory suffers from a simple-to-recognize mistake of the kind which almost every year leads to new papers on superluminal phenomena.

What makes the critique of ST a very tricky enterprise is the strong analogy, if not to say picture puzzle situation, generated by conserved chiral charges and their associated quadratic anomalous dimensions on one side and particle momenta and their quadratic mass squares on the particle side. It is this similarity which led to the dual model and the incorrect embedding picture underlying ST. The only credible remainder of this analogy is that, whenever it helps to find a representation of the Poincaré group on the substrate of chiral oscillators, the mass spectrum of this representation is a subspectrum of anomalous dimensions of a chiral conformal QFT. Why does one not find this statement together with a commentary in ST papers? It is a clear indication that one is outside the setting of particle crossing.

With respect to Witten, Duff raises another interesting point: the relation of string theory with mathematics. String theoretic pictures have indeed been helpful to generate proven and provable mathematical conjectures, but does this mean (as Duff seems to suggest) that such a strong autonomous science as particle physics should be happy in a role of a subcontractor of mathematics? Mathematicians need not care whether they get their inspirations from (what they conceive as) beautiful castles within flourishing landscapes of particle physics or from its ruins; as long as something represents a fertile soil for their imagination and makes them free to work a bit outside the standard conjecture-theorem-proof pattern they may profit from this source.

One of the populated mathematics-physics meeting grounds since the early 70s is geometry. I hope that the presentation of modular localization in this essay made clear that this means something very different for mathematicians as

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44 I am referring to the Barut, Kleinert, Fronsdal ... constructions using extensions of the noncompact Lorentz group (see appendix in [10]).
45 Actually Fubini and collaborators came quite close [67].
it does (or rather should do) for particle theorists. Whereas in areas of classical physics, notably general relativity and classical gauge theory, the important role of geometry cannot be denied, the connection of QM with geometry is less tight and starts to drift apart in QFT.

The reason is that spacetime geometry in QFT never appears without being burdened with vacuum polarization and thermal KMS properties as localization entropy [24]. To give an illustration, the Atiyah-Singer index theory has applications to QM and free QFT in external fields, but has no place in local quantum physics when localization comes together with vacuum polarization and thermal manifestations. Another illustration comes from the WZWN model. A topological Lagrangian as e.g. the topological Euclidian WZWN action has little to do with the original Wess-Zumino Lagrangian, which at least formally complies with Lagrangian quantization and vacuum polarization (prerequisites for the validity of a causal perturbation theory) and as a consequence admits the applications of perturbation theory. The WZWN topological Lagrangian serves for "baptizing" a model in the traditional way by reading the representation theory of $d=1+1$ chiral currents and the construction of the associated sigma-model fields back into a classical Lagrangian setting at the prize of a topological euclidean extension into a third dimension; but it is precisely this topological aspect which is in conflict with causal localization and prevents the applicability of the standard perturbation formalism. When it comes to computations, one resorts to the representation theoretical methods for currents and their associated sigma-model field (exponentials of potentials of currents). The beauty of a WZWN Lagrangian is entirely on the geometric-mathematical side; it becomes rather useless for the study of localization and its physical consequences as renormalized perturbation theory.

On the other hand mathematicians have undeniably a natural intrinsic interests in such topological Lagrangians; they are not concerned with the problems of quantum localization. With todays hindsight about the past it is clear that the attempts to bring geometry and quantum physics together as expressed in the Atiyah-Witten project of the 70s and its later continuation in the setting of string theory had their greatest success in raising the level of mathematical sophistication of physicists, rather than advancing the course of particle physics.

The geometrical visualization in terms of Riemann surfaces of analyticity properties of duality of chiral models in thermal states has no relation to the "living space" (in the sense of localization) of those models but rather correspond to the Bargman-Hall-Wightman domain of analyticity of correlation functions. Last not least objects of string theory are not localized on strings; the oscillator degrees of freedom which, thinking in terms of quantum mechanical chains of oscillators, ST incorrectly envisages as spacetime string have no bearing on spacetime localization at all, they are simply "sitting" in an inner space which is attached to a spacetime localization point (there were one imagines spin components and matrices acting on them to be living).

The imitation of Feynman rules by world-sheet pictures is a metaphorical step which is not supported by any conceptual understanding of quantum causal localization; it just tries to extend a helpful way to organize perturbation theory...
in terms of graphical rules outside its range of validity. To place this into an ironical historical context, one may say that Stückelberg was very lucky when he extended his graphical illustration of the asymptotic rescattering structure (which follow from his macrocausal ideas about the asymptotic one-particle structure of relativistic scattering theory) to non-asymptotic regions and in this way arrived at the Feynman graphs before Feynman. But this has no repetition on the level of world sheets; the perturbative rules for the string-localized potentials in section 3 are entirely different.

The historical episode where quantum physics and mathematics were totally on par was not (as most people think) the discovery of QM \(^{46}\) rather it was the parallel development in the middle of the 60s of what physicists called ”statistical mechanics of open systems” and mathematicians ”the Tomita-Takesaki modular theory of operator algebras”. It was a taking and giving on totally equal terms between the T-T modular theory on the mathematical side and the statistical mechanics of open systems. It prepared the ground for the modular localization theory which started a decade later and eventually led to the deep connections between operator algebras generated by chiral conformal fields and Vaughan Jones subfactor theory \(^{68}\).

This perfect meeting of mathematical and particle physics minds also finds its expression in the (unjustly little known) Doplicher-Haag-Roberts superselection analysis \(^{69}\) leading to compact group duals in which the Markov traces of the Vaughan Jones theory of subfactors \(^{68}\) and the use of endomorphisms were used independently from that of subfactor theory; after realizing that they hit the same structure in a different context this significantly accelerated the explicit construction of chiral conformal models including proofs of existence.

From a philosophical viewpoint the superselection theory achieved a deep spacetime understanding of the quantum origin of Heisenberg’s phenomenological concept of SU(2) ”isospin” in nuclear physics by showing that group theory is a surprising consequence of the classification of equivalence classes of localizable representations of observable algebras. This is almost as surprising as the intrinsic probability which comes from modular localization without referring to Born’s quantum mechanical probability.

There is one issue in which Smolin \(^{75}\) (together with Arnsdorf), standing on the shoulders of Rehren\(^{45}\) should be supported against Duff. These authors pointed at a kind of conundrum between the bizarre consequences of the string-induced Maldacena conjecture \(^{66}\) and Rehren’s theorem. In a previous paper by Rehren \(^{71}\) it was pointed out that the rigorous correspondence, though mathematical correct, has a serious physical shortcoming. One side of this mathematically well-defined correspondence is always unphysical; if one starts from a physical model on the AdS side, the CFT side will have way to many

\(^{46}\) The Hilbert space theory already existed and it were not the physicists in Hilbert’s Göttingen but rather a research assistant at the technische Hochschule in Stuttgart namely Fritz London, who used ”rotations in Hilbert space” (nowadays unitary operators) for the first time in QM (the first paper on transformation theory).

\(^{47}\) It is truely admirable how, in the face of concentrated misunderstandings, Rehren succeeds to maintain his countenance \(^{70}\).
degrees of freedom many of them are not in the initial data but rather enter
the causal dependency region "sideways" as poltergeists. In the opposite direc-
tion i.e. starting from a physical AdS model will be too "anemic" to support
nontrivial causal localization in compact spacetime regions.

This means that although the correspondence respects local commutativity
(Einstein causality), it violates the quantum analog of what one calls classically
the causal propagation (and becomes the causal closure (or time-slice) prop-
erty in LQP) i.e. the algebra of the causal completion of a region \( O \rightarrow O' \)
is larger than that associated with \( \mathcal{O} : \mathcal{A}(\mathcal{O}) \subseteq \mathcal{A}(\mathcal{O}') \) as a result of additional
degrees of freedom appearing from nowhere. For the "occupants" of its causal
completion (the causally closed double cone world) this is like a "poltergeist"
effect; degrees of freedom seemingly "coming in from nowhere" (not contained
in the slice data) appear in the causal shadow and destroy the validity of time-
like causality. QFT models accessed by Lagrangian quantization do not have
this physical pathology; the time slice postulate of QFT\(^{48}\) was precisely
introduced in order to save from Lagrangian field theory what should be saved
in a world outside quantization (as it is needed in the AdS-CFT problem).

The violation of this property is intimately related to the phase space degrees
of freedom issue which led Haag and Swieca \(^{73}\) to their result that, different
from QM (with or without second quantization) which leads to a finite number
of states per cell in phase space, QFT as we know it from quantization requires
a compact set (later refined to "nuclear" \(^{6}\)). This "mildly" infinite cardinality
of degrees of freedom secures the existence of heat bath temperature states for
arbitrary temperatures as well as causal propagation. It seems that this kind of
insight together with other deep pre-electronic insights got lost in the maelstrom
of time and that especially ST remained ignorant about its existence.

Ignoring the degree of freedom requirement for a moment before return-
ing to it later on, one can ask the question whether it is possible to slightly
modify the AdS-CFT setting, so that a modified and appropriately reformu-
lated Maldacena’s conjecture is in harmony with Rehren’s rigorous theorem.
This is precisely the question Kay and Ortiz asked \(^{76}\). Taking their cue from
prior work on the correspondence principle of Mukohyama-Israel as well from ’t
Hooft’s brick-wall ide\(^{49}\) \(^{77}\), these authors start with a Hartle-Hawking-Israel
like pure state on an imagined combined matter + gravity dynamical system.
They then propose to equate the AdS side of a hypothetical conformal invariant
supersymmetric Yang-Mills model with the restriction of the H-H-I state to a
matter subsystem in accordance with Rehren’s theorem.

It is conceivable that the "degrees of freedom mismatch" in Rehren’s the-
orem, which in the \( \rightarrow \) direction leads an overpopulation of degrees of freedom

\(^{48}\)In a modern setting the principle of causal localization comprises two requirements on
observables: Einstein causality (spacelike commutativity) and Haag duality
\( \mathcal{A}(\mathcal{O}) = \mathcal{A}(\mathcal{O}') \) (timelike causal propagation).

\(^{49}\)A physical argument which leads to a vacuum-polarization driven entropical area law
which seems to be closely related to the dependence on the thickness of the fuzzy surface \( \varepsilon \)
associated with the localization entropy as defined by the split property with \( \varepsilon \) being the size
of the split.\(^{6}\)
and in → to an anemia can be repaired by some extension of the Kay-Ortiz scheme, but is not very plausible. Nevertheless their results, although as stated by the authors not rigorous, are sufficiently interesting and deserve to be taken serious by the community around the Maldacena conjecture. Physics is one of the few science were errors about important properties should receive no lesser attention than correct observations.

In illustrating the "unreasonable power of less than perfect discoveries", Duff points convincingly at the story of antiparticles emerging from Dirac’s hole theory. Indeed some discoveries, especially in the beginning of QFT, did not follow straight logical lines. It was not (as one would have expected) Pascual Jordan, the discoverer of QFT and the positivistic advocate of quantizing everything (Maxwell fields, matter fields) which permits to be quantized, who first saw the relation between charge and antiparticles, rather it was Dirac who distilled the suggestion of antiparticles from his (later abandoned) hole theory. As an intrinsic property of the underlying causal localization aspect of QFT this was shown later by Jost [28].

However it was Dirac’s particle hole theory which gave rise to the idea of antiparticles, even though conceptually it shouldn’t. His philosophical setting for QT was quite different from Jordan’s positivism since he used wave quantization only for classical Maxwell waves and described massive matter in terms of QM. It was somewhat artistic to see antiparticles in the context of hole theory: in fact this setting was later abandoned after it became clear that it becomes inconsistent as soon as vacuum polarization comes into play. Dirac came around to embrace universal field quantization in the early 50s.

As far as stressing old-fashioned virtues in particle theory, there is no problem to agree with Duff. This also includes his refutation of a time-limit on string theory research as expressed in the papers of Woit and Smolin. If string theory really would be what it claims to be, namely a consistent theory which goes beyond QFT, it certainly has the right to take as much time as it needs to settle this problem; but the point is that isn’t.

Duff forgot to tell what he considers to be the string theoretic analog of Dirac’s discovery. Also his mentioning of the Higgs mechanism and gauge theory in connection with string theory warrants some corrective remarks. The present day view of massive vectormesons by a Higgs symmetry breaking and the Higgs particle playing the role of "God’s particle" (giving masses to the other particles) is what the maelstrom of time left over from a much richer past which in the 70ies was referred to as the Schwinger-Higgs screening mechanism [78]. The Higgs model is nothing else than the charge-screened mode of scalar electrodynamics. Whereas the quantum mechanical Debye screening only generates a short-range effective interaction, the QFT screening is more radical in that it affects also the particle spectrum. Charge screening means that the integral over the charge density vanishes, whereas a (spontaneous) symmetry breaking brings about a divergence of this integral (as a result of its bad infrared behav-

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It was still used in the first textbooks by Wenzel and Heitler but did not survive renormalized perturbation theory where vacuum polarization became important.
ior caused by the coupling of the conserved current to the massless Goldstone boson). The problem of why the implementation of screening in massive QED within a BRST setting needs no Higgs particle, whereas its nonabelian massive YM counterpart the BRST consistency requires the presence of such physical freedom can obviously not be answered within the BRST setting. The answer is expected to come from the (ongoing) perturbative computations with stringlike vectorpotentials (section 3).

In contradistinction to the metaphoric idea about the Higgs particle generating the mass of other particles (and presumably also of itself) which is not an intrinsic property and therefore is not accessible to measurements, the divergent charge of a spontaneously broken symmetry and a vanishing charge of a Schwinger Higgs screening mechanism are physically distinct phenomena and according to Swieca’s charge screening theorem a massive photon is inexorably connected with charge screening [78] and not with spontaneous symmetry breaking.

The idea of string-localized vector potentials also re-opens the question of alternatives to the Schwinger-Higgs screening [39][40]. Spinor-QED has a massive counterpart [79] (without introducing S-H screening via an additional Higgs degree of freedom), the so-called massive QED, which in the pointlike formalism needs an intermediate BRST ghost formalism in order to lower the scaling dimension of the effective vectorpotential from 2 to 1. This problem has meanwhile been solved in the setting of string-localized potentials [45]. It may be interesting to try to substitute the BRST formalism by string-localized free vector fields which also have short distance dimension d=1 since only string-localized potentials allow a smooth transition between the massive and the massless case. For Yang-Mills theory there is a ”perturbative theorem” that the consistent use of the BRST formalism requires the presence of additional physical degrees of freedom (the scalar Higgs boson). Alternatives to this construct have never been pursued during the 40 year history of the standard model. This adds an air of desperation to the search of the Higgs boson which is somewhat detrimental for the credibility of a positive result, but perhaps explains the hype around the first traces of a new particle in the expected energy range of the Higgs particle.

String theory has (contrary to what Duff claims) led to stagnation of vital parts of particle physics [50]. What does Duff (or anybody else) expect from a theory which, unlike all other theories has no pre-history and is already misleading in the terminology of its name? The conceptual harbinger of QM was the semiclassical Bohr-Sommerfeld theory and that of QFT the dispute between Einstein and Jordan [2]. A theory which suddenly pops up from nowhere as the dual model (tinkering with mathematics) has a good chance to also go nowhere.

With the widespread acceptation of string theory, Einstein’s epoch of viewing theoretical physics as a process of unfolding physical principles and concepts seems to have come to an end. The total break with this Einsteinian
tradition would be attained with the acceptance of such ideas as the physical realization of any consistent mathematical structure in one universe of a multiverse [74]; already the presentation of the multiverse as a solution of the background indepence problem is way off the mark. In fact this way of thinking became a self-runner, it does not need any more the support of string theory; it has generated its own fantasy world (extra dimensions [80], branes as physical subsysystems, Maldacena’s claim that the AdS-CFT correspondence relates to physical theories [66] etc.)

It will be a long lasting task for the coming new generations to remove all the metaphoric rubble around ”theories of everything” in order to have a chance to successfully confront the LHC experimental results with new ideas going beyond the loose ends of the standard model. The immense progress one could expect from correcting these (by no means trivial) conceptual errors should be a consolation for the many lost decades.

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