Theoretical Status of Pentaquarks

Takumi Doi\textsuperscript{1,2,*}

\textsuperscript{1} Dept. of Physics and Astronomy, Univ. of Kentucky, Lexington, KY 40506, USA
\textsuperscript{2} RIKEN BNL Research Center, BNL, Upton, NY 11973, USA

We review the current status of the theoretical pentaquark search from the direct QCD calculation. The works from the QCD sum rule and the lattice QCD in the literature are carefully examined. The importance of the framework which can distinguish the exotic pentaquark state (if any) from the NK scattering state is emphasized.

\section{Introduction}

While QCD was established as a fundamental theory of the strong interaction a few decades ago, its realization in hadron physics has not been understood completely. For instance, (apparent) absence of “exotic” states, which are different from ordinary $q\bar{q}$ mesons and $qqq$ baryons, has been a long standing problem. Therefore, the announcement\textsuperscript{1} of the discovery of $\Theta^+$ (1540), whose minimal configuration is $uudd\bar{s}$, was quite striking. For the current experimental status, we refer to Ref.\textsuperscript{2}

In this report, we review the theoretical effort to search the $\Theta^+$ pentaquark state. The main issue here is whether QCD favors its existence or not, and the determination of possible quantum numbers for the pentaquark families (if any). In particular, in order to understand the narrow width of $\Theta^+$ observed in the experiment, it is crucial to determine the spin and parity directly from QCD.

For this purpose, we employ two frameworks, the QCD sum rule and the lattice QCD, where both allow the nonperturbative QCD calculation without models, and have achieved a great success for ordinary mesons/baryons. Note, however, that neither of them is infallible, and we consider them as complementary to each other.

For instance, the lattice simulation cannot be performed at completely realistic setup, i.e., there often exists the artifact stemming from discretization error, finite volume, heavy u,d-quark masses and neglect of dynamical quark effect (quenching), etc. On the other hand, the sum rule can be constructed at realistic situation, and is free from such artifacts in lattice. Unfortunately, it suffer from another type of artifact. Because a sum rule yields only the dispersion integral of spectrum, an interpretive model function have to be assumed phenomenologically. Compared to the ordinary hadron analyses, this procedure may weaken the predictability for the experimentally uncertain system, such as pentaquarks. Another artifact in the sum rule is the OPE truncation: one have to evaluate whether the OPE convergence is enough or not. We also comment on the important issue common to both of the methods. Recall that the decay channel $\Theta^+ \rightarrow N + K$ is open experimentally. Considering also that both methods calculate a two-point correlator and seek for a pentaquark signal in it, it is essential to develop a framework which can distinguish the pentaquark from the NK state in the correlator. In the subsequent sections, we examine the literatures and see how the above-described issues have been resolved or remain unresolved.

\textsuperscript{*}) e-mail address: doi@pa.uky.edu

\begin{flushright}
\textcopyright{} 2007. typeset using \texttt{P\MakeLowercase{T}P\MakeLowercase{E}}.cls (Ver.0.9)
\end{flushright}
§2. The QCD Sum Rule Work

More than ten sum rule analyses for $\Theta^+$ spectroscopy exist for $J = 1/2$. The first parity projected sum rule was studied by us for $I = 0$. The positivity of the pole residue in the spectral function is proposed as a signature of the pentaquark signal. This is superior criterion to the consistency check of predicted/experimental masses, because it is difficult to achieve the mass prediction within 100MeV ($\sim [m(\Theta^+) - m(NK)]$) accuracy. We also propose the diquark exotic current $J_{5q} = \epsilon^{abc} \epsilon^{def} \epsilon^{cfg} (u_a^T C d_b)(u_d^T C \gamma_5 e) C \bar{s}^T_g$, in order to suppress the NK state contamination. The OPE is calculated up to dimension 6, checking that the highest dimensional contribution is reasonably small. We obtain a possible signal in negative parity.

In the treatment of the NK state, improvement is proposed in Ref. There, NK contamination is evaluated using the soft-Kaon theorem. Note here that the NK contamination calculated by two-hadron reducible (2HR) diagrams in the OPE level is invalid because what have to be calculated is the 2HR part in the hadronic level, not in the QCD (OPE) level. The reanalysis of sum rule up to dimension 6 shows that the subtraction of the NK state does not change the result of Ref.

Yet, as described in Sec.1, the above sum rules may suffer from the OPE truncation artifact. In fact, the explicit calculation up to higher dimension have shown that this is indeed the case. Here, we refer to the elaborated work in Ref. They calculate the OPE for $I(J^P) = 0(1/2^\pm)$ up to dimension $D = 15$. It is shown that the terms with $D > 6$ are important as well, while further high dimensional terms $D > 15$ are not significant. Another idea in Ref. is the use of the combination of two independent pentaquark sum rules. In fact, the proper combination is found to suppress the continuum contamination drastically, which corresponds to reducing the uncertainty in the phenomenological model function. Examining the positivity of the pole residue, they conclude the pentaquark exists in positive parity.

Does the result definitely predict the $J^P = 1/2^+$ pentaquark? At this moment, we conservatively point out remaining issues. The first problem is still the NK contamination. While such contamination is expected to be partly suppressed through the continuum suppression, it is possible that the obtained signal corresponds to just scattering states. In this point, Ref. argues that the signal has different dependence on the parameter $\langle \bar{q} q \rangle$ from the NK state. We, however, consider this discussion uncertain, because $\langle \bar{q} q \rangle$ is not a free parameter independent of other condensates. For further study, the explicit estimate in the soft-Kaon limit is interesting check, but the calculation up to high dimension has not been worked out yet. Second issue is related to the OPE. In the evaluation of the high dimensional condensates, one have to rely on the vacuum saturation approximation practically, while the uncertainty originating from this procedure is not known. Furthermore, there exists an issue for the validity of the OPE itself when considering the sum rule with high dimensionality. In fact, rough analysis of the gluonic condensates show that the nonperturbative OPE may break down around $D \gtrsim 11 - 16$. One may have to consider this effect as well, through, for instance, the instanton picture.

So far, we have reviewed $J = 1/2$ sum rules. While there are $J = 3/2$ works,
it is likely that they suffer from slow OPE convergence. Further progress is awaited.

§3.  The Lattice QCD Work

There are a dozen of quenched lattice calculations, some of them report the positive signal, while others report null results. This apparent inconsistency, however, can be understood in a unified way, by taking a closer look at the “interpretation” of the numerical results and the pending lattice artifact.

As discussed in Sec.1, the question is how to identify the pentaquark signal in the correlator, because the correlator at large Euclidian time is dominated by the ground state, the NK scattering state. In this point, we develop a new method in Ref. Intuitively, this method makes use of that a scattering state is sensitive to the spacial boundary condition (BC), while a compact one-particle state is expected to be insensitive. Practically, we calculate the correlator under two spacial BCs: (1) periodic BC (PBC) for all u,d,s-quarks, (2) hybrid BC (HBC) where anti-periodic BC for u,d-quarks and periodic BC for s-quark. The consequences are as follows. In PBC, all of \( \Theta^+, N, K \) are subject to periodic BC. In HBC, while \( \Theta^+(uudds) \) remains subject to periodic BC, \( N(uud,udd) \) and \( K(\bar{s}d,\bar{s}u) \) are subject to anti-periodic BC. Therefore, the energy of NK will shift by PBC \( \to \) HBC due to the momentum of N and K, while there is no energy shift for \( \Theta^+ \). (Recall that the momentum is quantized on lattice as \( 2\vec{n}\pi/L \) for periodic BC and \( (2\vec{n}+1)\pi/L \) for anti-periodic BC, with spatial lattice extent \( L \) and \( \vec{n} \in \mathbb{Z}^3 \).) In this way, the different behavior between NK and \( \Theta^+ \) can be used to identify whether the signal is NK or \( \Theta^+ \). We simulate the anisotropic lattice, \( \beta=5.75, V=12^3 \times 96, a_\sigma/a_\tau=4, \) with the clover fermion. The conclusion is: (1) the signal in \( 1/2^- \) is found to be s-wave NK from HBC analysis. No pentaquark is found up to \( \sim 200\text{MeV} \) above the NK threshold. (2) the \( 1/2^+ \) state is too massive (\( >2\text{GeV} \)) to be identified as \( \Theta^+(1540) \).

In comparison with other lattice results, we introduce another powerful method to distinguish \( \Theta^+ \) from NK. This method makes use of that the volume dependence of the spectral weight behaves as \( \mathcal{O}(1) \) for one-particle state, and as \( \mathcal{O}(1/L^3) \) for two-particle state. Intuitively, the latter factor \( \mathcal{O}(1/L^3) \) can be understood as the encounter probability of the two particles. The calculation of the spectral weight from \( 16^3 \times 28 \) and \( 12^3 \times 28 \) lattices reveals that the ground states of both the \( 1/2^- \) channels are not the pentaquark, but the scattering states. Further analysis is performed in Ref. There, the 1st excited state in \( 1/2^- \) is extracted with \( 2 \times 2 \) variational method. The volume dependence of the spectral weight indicates that the 1st excited state is not a scattering state but a pentaquark state. This is consistent with Ref., where \( 19 \times 19 \) variational method is used to extract the excited states.

Note here that this results is consistent with the HBC analysis. In fact, HBC analysis exclude the pentaquark up to \( \sim 200\text{MeV} \) above threshold, while the resonance observed in Ref. locates 200-300MeV above the threshold. The question is whether the observed resonance is really \( \Theta^+ \) which experimentally locates 100MeV above the threshold. To address this question, explicit simulation is necessary at physically small quark mass without quenching. In particular, small quark mass
would be important considering that Refs.\textsuperscript{19,23} are simulated at rather heavy quark masses and expected to suffer from large uncertainty in the chiral extrapolation.

Finally, we discuss the $J^P = 3/2^\pm$ lattice results. We performed the comprehensive study\textsuperscript{20} with three different operators and conclude that all the lattice signals are too massive ($> 2\text{GeV}$) for $\Theta^+$, and are identified as not pentaquarks but scattering states from the HBC analysis. On the other hand, Ref.\textsuperscript{25} claims that a pentaquark candidate is found in $3/2^+$. We, however, observe that the latter result are contaminated by significantly large statistical noise, which makes their result quite uncertain. Note also that their criterion to distinguish $\Theta^+$ from scattering states is based on rather limited argument compared to the HBC analysis.

\textbf{§4. Conclusions}

We have examined both of the QCD sum rule and lattice QCD works. In the sum rule, progresses in OPE calculation and continuum suppression have achieved stable analysis, while the subtraction of NK contamination remains a critical issue. In the lattice, the framework which distinguish the pentaquark from NK have been successfully established. In order to resolve the \textit{superficial} inconsistency in the lattice prediction, the calculation at small quark mass without quenching is highly desirable.

\textbf{Acknowledgements}

This work is completed in collaboration with Drs. H.Iida, N.Ishii, Y.Nemoto, M.Oka, F.Okiharu, H.Suganuma and J.Sugiyama. T.D. is supported by Special Post-doctoral Research Program of RIKEN and by U.S. DOE grant DE-FG05-84ER40154.

\textbf{References}

1) LEPS Collaboration, T. Nakano \textit{et al.}, Phys. Rev. Lett. \textbf{91} (2003), 012002.
2) T. Nakano, in these proceedings.
3) S.-L. Zhu, Phys. Rev. Lett. \textbf{91} (2003), 232002.
4) R.D. Matheus \textit{et al.}, Phys. Lett. B \textbf{578} (2004), 323.
5) J. Sugiyama, T. Doi, and M. Oka, Phys. Lett. B \textbf{581} (2004), 167.
6) Y. Kondo, O. Morimatsu, T. Nishikawa, Phys. Lett. B \textbf{611} (2005), 93.
7) S.H. Lee, H. Kim, Y. Kwon, Phys. Lett. B \textbf{609} (2005), 252.
8) Y. Kwon, A. Hosaka and S.H. Lee, \textit{hep-ph/0505040} (2005).
9) M. Eidemuller, Phys. Lett. B \textbf{597} (2004), 314.
10) B.L. Ioffe and A.G. Oganesian, JETP Lett. \textbf{80} (2004), 386, R.D. Matheus and S. Narison, Nucl. Phys. Proc. Suppl. \textbf{152} (2006), 236, A.G. Oganesian, \textit{hep-ph/0510327} (2005).
11) H.-J. Lee \textit{et al.}, Phys. Rev. D \textbf{73} (2006), 014010, \textit{ibid.}, Phys. Lett. B \textbf{610} (2005), 50.
12) T. Kojo, A. Hayashigaki, D. Jido, Phys. Rev. C \textbf{74} (2006), 045206.
13) M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B \textbf{147} (1979), 385, 448.
14) T. Nishikawa \textit{et al.}, Phys. Rev. D \textbf{71} (2005), 016001, 076004.
15) W. Wei, P.-Z. Huang and H.-X. Chen and S.-L. Zhu, JHEP \textbf{0507} (2005), 015.
16) F. Csikor \textit{et al.}, JHEP \textbf{0311} (2003), 070, S. Sasaki, Phys. Rev. Lett. \textbf{93} (2004), 152001.
17) T.W. Chiu and T.H. Hsieh, Phys. Rev. D \textbf{72} (2005), 034505.
18) N. Mathur \textit{et al.}, Phys. Rev. D \textbf{70} (2004), 074508.
19) N. Ishii \textit{et al.}, Phys. Rev. D \textbf{71} (2005), 034001.
20) B.G. Lasscock \textit{et al.}, Phys. Rev. D \textbf{72} (2005), 014502.
21) F. Csikor \textit{et al.}, Phys. Rev. D \textbf{73} (2006), 034506.
22) C. Alexandrou and A. Tsapalis, Phys. Rev. D \textbf{73} (2006), 014507.
23) T.T. Takahashi, T. Kunihiro, T. Onogi, and T. Umeda, Phys. Rev. D \textbf{71} (2005), 114509.
24) K. Holland, and K.J. Juge, Phys. Rev. D \textbf{73} (2006), 074505.
25) B.G. Lasscock et al., Phys. Rev. D 72 (2005), 074507.
26) N. Ishii, T. Doi, Y. Nemoto, M. Oka and H. Suganuma, Phys. Rev. D 72 (2005), 074503.
27) O. Jahn, J.W. Negele and D. Sigaev PoS LAT2005 (2006), 069.
28) C. Hagen, D. Hierl and A. Schafer, Eur. Phys. J. A 29 (2006), 221.