Intelligent Reflecting Surface for MIMO VLC: Joint Design of Surface Configuration and Transceiver Signal Processing

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Abstract—With the capability of reconfiguring the wireless electromagnetic environment, intelligent reflecting surface (IRS) becomes a new paradigm for designing future wireless communication systems. In this paper, we consider optical IRS for improving the performance of visible light communication (VLC) under a multiple-input and multiple-output (MIMO) setting, where the mean square error (MSE) of the IRS-aided MIMO VLC is minimized by jointly designing the IRS and transceiver signal processing. To this end, the MIMO channel gains of the IRS-aided VLC are first derived under the point source assumption, based on which the MSE minimization problem is formulated subject to the emission power constraints and the IRS configuration constraints. Next, we propose an alternating optimization algorithm, which decomposes the original problem into three subproblems, to iteratively optimize the IRS configuration, the precoding and detection matrices for minimizing the MSE. Moreover, theoretical analysis on the performance of the proposed algorithm in high and low signal-to-noise ratio (SNR) regimes is investigated, revealing that the joint optimization process can be simplified in such special cases, and the algorithm’s convergence property and computational complexity are also discussed. Finally, numerical results show that IRS-aided schemes significantly reduce the MSE as compared to their counterparts without IRS, and the proposed algorithm outperforms other baseline schemes.

Index Terms—Visible light communication (VLC), intelligent reflecting surface (IRS), multiple-input and multiple-output (MIMO), mean square error (MSE), alternating optimization (AO).

I. INTRODUCTION

TO COPE with approximately 50 exabyte-per-month data traffic in the fifth-generation (5G) and beyond [1], wireless communication technologies have been significantly advanced to offer higher spectral efficiency (SE) and energy efficiency (EE), more reliable signal transmission, lower transmission latency, etc. Different from other traditional wireless communication technologies such as millimeter-wave (mmWave) and Terahertz (THz) communications [2], visible light communication (VLC) modulates signals in the untapped frequency band with wavelengths in the range of 380 nm–750 nm, and the non-coherent modulation scheme called intensity modulation (IM) / direct detection (DD) is typically employed in VLC [3]. Owing to the license-free merit of the operating band, abundant spectrum resources including about 400 THz can be safely exploited in VLC, which shows great potential for improving the wireless communication capacity [4], [5]. In the meanwhile, VLC is a green technology due to its low-power transceivers consisting of light-emitting diodes (LEDs) and photodetectors (PDs), and it far outperforms other radio frequency (RF) communications in terms of EE [6]. Moreover, VLC has advantages in dense frequency reuse, ubiquitous LED devices, and inherent physical layer security, which therefore attracts great attention from academia and industry in developing new VLC techniques, e.g., VLC Consortium (VLCC), IEEE 802.15.7 group, and Home Gigabit Access (OMEGA) [6], [7]. Despite the aforementioned benefits, VLC also faces critical challenges, especially in the case of multiple-input multi-output (MIMO) VLC [6]. Specifically, MIMO has been regarded as a revolutionary technique, which can offer multiplexing gain as well as diversity gain, and thus has been widely adopted in 3G to 5G wireless communication systems [2]. However, the mainstream IM/DD (de)modulation scheme in VLC imposes a real-valued and nonnegative constraint on signal amplitudes, which generally follows the typical Lambertian model that highly depends on the...
geometric locations of transceivers [6]. Owing to that, considering a typical scenario where PDs are close to each other, the spatial signature of a specific LED on all PDs is almost homogenous, which results in poor multiplexing and diversity gains of the MIMO VLC [5]. A more detailed explanation is provided in [8], where the singular value decomposition (SVD) of a $4 \times 4$ MIMO VLC channel is carried out to show that its condition number is as large as 189.3, i.e., only 1 data stream can be supported rather than 4. Generally, the performance of MIMO VLC can be improved by designing the shape of the receiver, e.g., the angle diversity receiver, the receiver with narrow field-of-view (FoV) PDs, the imaging receiver, etc [9], [10]. On the other hand, this can also be accomplished by signal processing such as power imbalance and precoding [8]. These methods attempt to improve the MIMO VLC through adjustments of transceivers; however, far little work has been done from the view of optical channel modification.

Benefiting from the wireless environment configuration capability of the emerging intelligent reflecting surface (IRS) [11], the wireless channel evolves to be a controllable variable rather than an uncontrolled barrier that has to be overcome [12]. Basically, IRS is a two-dimensional planar surface composed of periodical artificial atoms, through which the impinging electromagnetic wave can produce controllable induction current patterns, and therefore wave steering, polarizing, and absorbing can be dynamically achieved [13], [14]. As a promising new paradigm, the research on the RF IRS has been in full swing over recent years, including its channel modeling and estimation [15], [16], IRS-aided MIMO communication [17], [18], joint IRS configuration and precoding [19], IRS-enhanced physical layer security [20], etc.

On the other hand, there has been intensive interest recently in optical IRS within the visible light frequency range, which can be physically implemented by mirror array-based design or meta-surface-based design [21]. Reflected channel characterizations of these two designs are elaborated in [22], [23], [24], and [25], and more comprehensive reviews of the optical IRS are provided in [26], [27], and [28]. Given the flexibility of the IRS, it has been widely utilized in various aspects to facilitate VLC, namely IRS deployed at the receiver to actively expand FoV [29], [30], IRS deployed at the transmitter to accomplish beam steering [28], and IRS deployed in the wireless environment to enhance SE/EE and alleviate blockages [31], [32], [33]. Moreover, it is shown in [27] and [28] that the optical IRS can potentially be used in the MIMO VLC system, where the MIMO channel can be modified by IRS and thereby the multiplexing gain is improved.

In this paper, we consider the downlink of an IRS-aided MIMO VLC system, where the signal transmitted by the multi-LED array is received by the multi-PD array through the line-of-sight (LoS) path and the non-LoS (NLoS) paths specularly reflected by optical IRS. Under the point source assumption, the perfect channel state information (CSI) is assumed known to the transmitter, and this can be achieved by various channel estimation techniques [34], [35]. Then, a joint optimization of the IRS configuration and transceiver signal processing including MIMO precoding/detection is investigated for minimizing the mean square error (MSE) of the IRS-aided MIMO VLC system. The main contributions of this paper are summarized as follows:

- First, this paper established a system model of the optical IRS-aided MIMO VLC under the extremely near-field condition, and the MSE minimization problem is formulated in terms of the IRS configuration matrix and the precoding/detection matrices. Remarkably, considering that the power density of the reflected lightwave concentrates in a direction subjected to the generalized Snell’s law of reflection, the IRS configuration in VLC can be specified by the alignments of IRS reflecting units with different transceiver antenna pairs, and thus the optimization of the IRS configuration becomes an association problem.

- Next, we transform the original problem into a more tractable form by variable transformation, based on which an alternating optimization (AO) algorithm is proposed to minimize the MSE by iteratively solving three convex optimization subproblems, namely the IRS configuration subproblem, precoding design subproblem, and detection design subproblem. Theoretical analysis on the performance of the proposed algorithm in high and low signal-to-noise ratio (SNR) regimes is provided to draw essential insights and simplify system design, and the convergence and complexity of the proposed algorithm are also discussed.

- Finally, the performance of the IRS-aided VLC and the proposed AO algorithm are numerically evaluated in an indoor environment. Several baselines are chosen for comparison, including the distance greedy and the random allocation scheme for IRS configuration, and the zero-forcing (ZF) and the minimum mean square error (MMSE) schemes for precoding. From simulation results, it is verified that the MSE and bit error rate (BER) can be significantly lowered by IRS, and the proposed algorithm outperforms other baselines.

The rest of this paper is organized as follows. In Section II, the channel gains of LoS and NLoS paths are attained according to the Lambertian radiation model, based on which the IRS-aided optical MIMO channel model is established. Next, the signal model including the modulation scheme and signal processing is specified in Section III, where the MSE minimization problem is formulated. In Section IV, we propose an AO algorithm to optimize IRS and signal processing matrices, and especially, the situations of the proposed algorithm in the high- and low-SNR regimes are discussed. Numerical results are provided in Section V to evaluate the performance of the proposed algorithm for IRS-aided MIMO VLC. Finally, Section VI concludes the paper.

**Notations:** In this paper, scalar values, vectors, and matrices are denoted by normal letters such as $a$ (or $A$), boldface letters such as $\mathbf{a}$, boldface uppercase letters such as $\mathbf{A}$, respectively. Particularly, $\mathbf{I}_N$, $\mathbf{0}$, and $\mathbf{1}$ indicate the $N \times N$ identity matrix, an all-zero matrix, and an all-one matrix, respectively. The special operators include the Kronecker product $\otimes$, Hadamard product $\circ$, matrix transpose $(\cdot)^T$, vectorization
operator $\text{vec}()$, trace operator $\text{tr}()$, positive definite operator $\succ$, and the differential operator $\partial (\nabla)$. $E[\cdot]$ is used to represent the statistical expectation. Besides, $\text{diag}([a_1, \cdots, a_n])$ is a block diagonal matrix, where the $i$-th main diagonal vector is given by $a_i$, $i = 1, 2, \cdots, n$. The operators $|A|_F$, $|A|_2$, and $|A|$ denote the Frobenius norm, the row norm, and the Frobenius norm, respectively. Moreover, the calligraphic letter $A$ denotes a set with $|A|$ representing the number of elements in the set; $\mathbb{R}$ and $\mathbb{R}_+$ denote the real number set and the real and positive number set, respectively.

II. CHANNEL MODEL

In this section, we describe the LoS and NLoS MIMO channels of the IRS-aided MIMO VLC system, where the channel gains between any transmitter and receiver antennas are given based on the Lambertian radiant model, and then the MIMO VLC channel is derived in terms of the IRS configuration matrix.

A. LoS/NLoS Channel Gains

Before deriving the channel gains, several key geometric parameters need to be specified since the channel gain in indoor VLC follows a geometric-based model [6]. We consider an optical IRS-aided MIMO VLC system as depicted in Fig. 1, where a multi-PD array receives the signal from the multi-LED array in a three-dimensional (3D) Cartesian coordinate system. Particularly, the indices of the antennas at the transmitter, the antennas at the receiver, and the IRS units are denoted by $n_t \in T \triangleq \{1, \cdots, N_t\}$, $n_r \in R \triangleq \{1, \cdots, N_r\}$, and $n \in N \triangleq \{1, \cdots, N\}$, with cardinalities of $|T| = N_t$, $|R| = N_r$, and $|N| = N$, respectively. Let $(n_{t,x}, n_{t,y}, n_{t,z})$, $(n_{r,x}, n_{r,y}, n_{r,z})$, and $(x_t, y_t, z_t)$ be the 3D coordinates of the $n_t$-th LED, the $n_r$-th PD, and the $n$-th IRS unit, respectively. The distance between the considered LED and PD is defined as $d_{n_t,n_r}$, and those of the considered LED-IRS path and IRS-PD path are denoted by $d_{n_t,n_r}$ and $d_{n_r,n}$, respectively. According to geometry, the distance of LoS path can be expressed as

$$D_{n_r,n_t} = \sqrt{(x_t - n_{t,x})^2 + (y_t - n_{t,y})^2 + (z_t - n_{t,z})^2},$$

while the distances of NLoS paths are given by

$$d_{n_r,n_t} = \sqrt{(x_t - n_{t,x})^2 + (y_t - n_{t,y})^2 + (z_t - n_{t,z})^2}, \quad (2)$$

$$d_{n_r,n_t} = \sqrt{(n_{r,x} - x_t)^2 + (n_{r,y} - y_t)^2 + (n_{r,z} - z_t)^2}. \quad (3)$$

Let $\theta_{n_r,n_t}$ be the angle of irradiance for the LoS path from the $n_r$-th LED to the $n_t$-th PD, and $\phi_{n_r,n}$ be the angle of irradiance for the NLoS path from the $n_t$-th LED to the $n$-th IRS unit. Accordingly, the angle of incidence for the LoS path from the $n_t$-th LED to the $n_r$-th PD is defined as $\Phi_{n_r,n_t}$ and the angle of incidence for the NLoS path from the $n_t$-th LED to the $n$-th IRS unit can be represented by $\phi_{n_r,n}$. We assume all normal vectors of the LEDs at the transmitter and the PDs at the receiver are perpendicular to the floor. According to geometry, the angle of irradiance for the NLoS path, which is determined by the coordinates of the $n_t$-th LED and the $n$-th IRS unit, can be expressed as

$$\theta_{n_r,n_t} = \arccos \left( \frac{n_{t,z} - z_t}{d_{n_r,n_t}} \right), \quad (4)$$

and the angle of incidence for the NLoS path is given by

$$\phi_{n_r,n} = \arccos \left( \frac{z_t - n_{r,z}}{d_{n_r,n}} \right). \quad (5)$$

Meanwhile, the angles of irradiance and incidence for the LoS path are given by

$$\Theta_{n_r,n_t} = \Phi_{n_r,n_t} = \arccos \left( \frac{n_{t,z} - n_{r,z}}{D_{n_r,n_t}} \right). \quad (6)$$

1) LoS Channel: In indoor VLC, the LoS channel gain has been derived as the Lambertian radiant formulation, which depends on the locations and geometric parameters of transceivers [4]. Let $A_p$ be the area of PD and $g_{of}$ be the optical filter gain, and moreover, we denote $m$ as the Lambertian index of the Lambertian model. By the above definitions, the LoS channel gain between the $n_t$-th LED and the $n_r$-th PD is given by [6]

$$h_{n_t,n_r}^{(1)} = A_p(m + 1)g_{of} \frac{\cos^m(\Theta_{n_r,n_t})}{2\pi D_{n_r,n_t}^2}\cos(\Phi_{n_r,n_t})f(\Phi_{n_r,n_t}), \quad (7)$$

where the function $f(\cdot)$ denotes the optical concentrator gain, which can be modeled as [4]

$$f(\Phi) = \begin{cases} \frac{q^2}{\sin^2(\Phi_0)}, & \text{if } 0 \leq \Phi \leq \Phi_0, \\ 0, & \text{otherwise}, \end{cases} \quad (8)$$

where $q$ is the refractive index and $\Phi_0$ represents the semi-angle of the FoV.

2) NLoS Channel: In general, the NLoS VLC channels can be categorized into two types, namely, the diffusely reflected path and the specularly reflected path. The gain of the former path has been validated to follow a “multiplicative” model [36], and experiments showed that even the gain of the strongest diffusely reflected path is at least 7 dB lower than that of the weakest LoS path [8]. Therefore, the diffusely reflected NLoS channel is omitted due to its negligible gain,
while only the specularly reflected NLoS channel is considered in the IRS-aided MIMO VLC.

To model the channel gain of the specularly reflected path, we note that the characteristics of the NLoS channel depend on the Rayleigh distance $d_0$ given by [15]

$$d_0 = \frac{2L^2}{\lambda_c},$$

where $L$ and $\lambda_c$ denote the aperture of the antenna array and the wavelength of the carrier wave, respectively. When the propagation distance between the transceivers is larger than $d_0$, the far-field condition is satisfied. As a result, the incident wave can be regarded as a planar wave, and the path loss of the NLoS channel is the product of two cascaded channels [11]. In contrast, the near-field communication occurs when $d_0$ becomes larger than the propagation distance, and the spherical wave model should be adopted. In this case, the path loss of the NLoS channel has been investigated under the near-spherical wave model [12]. According to geometric optics, the beam reflected by optical IRS can be considered as being emitted from an image source, which is symmetrical with respect to the IRS unit [25]. Considering a beam emitted from the $n_t$-th LED, the $n_t$-th IRS unit and arriving at the $n_r$-th PD, the incident light emitted from another LED will be reflected by this IRS unit towards a different direction, and the probability of any PD appearing exactly in this direction is practically very low and thus can be ignored. In other words, the $n_t$-th IRS unit can contribute to the NLoS channel gain if and only if it is aligned with an LED-PD pair. Therefore, the beamwidth of the reflected light is highly spatially confined, based on which the property of no cross-talk can be guaranteed for the NLoS channels in the optical IRS-aided VLC.

Inspired by this property, the NLoS MIMO channel matrix $H_2$ can be characterized based on the alignments of IRS units with the LED/PD pair. Define two binary matrices $G = \{g_{1}, \ldots, g_{N_t}\} \in \{0, 1\}^{N_t \times N_r}$ and $F = \{f_{1}, \ldots, f_{N_t}\} \in \{0, 1\}^{N_t \times N_r}$ to represent the alignments of IRS units with transmitter and receiver antennas, respectively. The $n_t$-th row of $G$ shows the association between the $n_t$-th IRS unit and the antennas at the transmitter, and that of $F$ shows the association with the receiver antennas. Specifically, $g_{n_t,n_r} = 1$ and $f_{n_t,n_r} = 1$ indicate that the $n_t$-th unit is aligned with the $n_t$-th LED and the $n_r$-th PD, respectively. Moreover, the channel gain of each IRS unit can contribute to the NLoS channel gain if and only if it is aligned with an LED-PD pair. Therefore, the beamwidth of the reflected light is highly spatially confined, based on which the property of no cross-talk can be guaranteed for the NLoS channels in the optical IRS-aided VLC.

### B. IRS-Aided MIMO VLC Channel

The MIMO channel between an $N_t$-LED transmitter and an $N_r$-PD receiver is denoted by a matrix $H \in \mathbb{R}^{N_r \times N_t}$, which is comprised of an LoS MIMO channel matrix $H_1$ and an NLoS MIMO channel matrix $H_2$. The channel matrix $H$ can be expressed as

$$H = H_1 + H_2,$$

where $h_{1}^{(1)}$ and $h_{2}^{(2)}$ denote the LoS and NLoS spatial signatures of the $n_t$-th LED, respectively. The matrix $H_1$ can be generated by aggregating $N_t N_r$ channel gains, where the element at the $n_t$-th row and the $n_r$-th column is given by (7), i.e.,

$$h_{1}^{(1)} \triangleq \left[\begin{array}{c}
h_{11}^{(1)} \\
h_{12}^{(1)} \\
\vdots \\
h_{1 N_r}^{(1)}
\end{array}\right].$$

Next, the generation of NLoS MIMO channel matrix depends on the configuration of IRS. To establish the relationship between $H_2$ and IRS, we first discuss the no cross-talk property of NLoS channels in the optical IRS-aided VLC system. Considering the size of IRS unit is infinitely large compared to the wavelength of the lightwave, the geometric optics approximation becomes accurate under such an extremely near-field condition. Numerical results have revealed that the power density of the reflected beam in VLC is far more concentrated than those in mmWave and THz bands, and the energy focuses in a line while hardly leaking into other directions [26]. According to geometric optics, the beam reflected by optical IRS can be considered as being emitted from an image source, which is symmetrical with respect to the IRS unit [25]. Considering a beam emitted from the $n_t$-th LED, reflected by the $n_t$-th IRS unit and arriving at the $n_r$-th PD, the incident light emitted from another LED will be reflected by this IRS unit towards a different direction, and the probability of any PD appearing exactly in this direction is practically very low and thus can be ignored. In other words, the $n_t$-th IRS unit can contribute to the NLoS channel gain if and only if it is aligned with an LED-PD pair. Therefore, the beamwidth of the reflected light is highly spatially confined, based on which the property of no cross-talk can be guaranteed for the NLoS channels in the optical IRS-aided VLC.
III. SIGNAL MODEL AND PROBLEM FORMULATION

In this section, the signal model of IRS-aided MIMO VLC is described, including the modulation scheme, the precoding at the transmitter, and the detection at the receiver. Generally, MSE at the receiver indicates the error performance of the IRS-aided MIMO VLC. The deviation between the encoded and decoded signals becomes smaller when MSE decreases, and the BER performance is improved. Therefore, the MSE is derived and adopted as the objective of the formulated optimization problem, which aims to minimize the MSE of IRS-aided MIMO VLC.

A. Signal Model of IRS-Aided MIMO VLC

The IRS-aided MIMO VLC system with an \( N_T \)-LED transmitter and an \( N_R \)-PD receiver is depicted in Fig. 2. Since the user moving speed in an indoor environment is typically very low, it is assumed that the perfect CSI parameters \( h_{n,r,n,t}^{(1)} \) and \( h_{n,r,n,n}^{(2)} \) can be estimated in real time and the proposed system optimization can be implemented over relatively large intervals. In addition, the bipolar pulse amplitude modulation (PAM) scheme is adopted to modulate the bit stream, which is split into \( N_s \) parallel sub-streams, namely \( b_1 \) to \( b_{N_s} \). In a time slot, \( \log_2 M \) independent bits in the \( n_s \)-th substream are mapped to the symbol \( x_{n_s} \), which can be expressed as

\[
    x_{n_s} \in \mathcal{X} : \{ \pm \sigma_x I, \pm 3 \sigma_x I, \cdots, \pm (M-1) \sigma_x I \},
\]

for \( \forall n_s \in \{1, \cdots, N_s\} \), where \( I = \sqrt{3/(M^2+1)} \) is a normalization parameter to guarantee \( \mathbb{E}(x_{n_s}^2) = \sigma_x^2 \). The generated signal vector \( x \triangleq [x_1, \cdots, x_{N_s}]^T \in \mathbb{X}^{N_s} \) is precoded by a digital precoding matrix \( W \triangleq [w_1^T, \cdots, w_{N_s}^T] \in \mathbb{R}^{N_T \times N_s} \). Next, the coded signal is added by a direct current bias \( r \triangleq [r_1, \cdots, r_{N_s}]^T \in \mathbb{R}^{N_T \times 1} \) to meet with the real-valued and nonnegative constraint on the signal amplitude. Then, after the electro-optic conversion, the signal travels through the wireless MIMO VLC channel, and the received signal \( y \triangleq [y_1, \cdots, y_{N_R}]^T \in \mathbb{R}^{N_R \times 1} \) can be expressed as

\[
    y = H (W x + r) + \omega,
\]

where \( \omega \triangleq [\omega_1, \cdots, \omega_{N_R}]^T \in \mathbb{R}^{N_R \times 1} \) is a zero-mean additive white Gaussian noise (AWGN) with the covariance of \( \sigma_\omega^2 I_{N_R} \). At the receiver, the direct current \( Hr \) is removed before the signal detection, and thereby the recovered symbol vector can be attained by

\[
    \hat{x} = Q (y - H r),
\]

where \( Q \in \mathbb{R}^{N_s \times N_R} \) represents the detection matrix. Finally, \( \hat{x} \triangleq [\hat{x}_1, \cdots, \hat{x}_{N_s}]^T \in \mathbb{R}^{N_s \times 1} \) is demodulated as \( N_s \) sub-streams and the received bits from \( b_1 \) to \( b_{N_s} \) are rearranged by a parallel-to-serial conversion.

B. Problem Formulation

This paper aims to minimize the MSE of IRS-aided MIMO VLC by jointly optimizing the IRS configuration matrices \( F \) and \( G \), the precoding matrix \( W \) at the transmitter, and the detection matrix \( Q \) at the receiver. To this end, we derive the expression of the MSE and show the constraints related to these variables, based on which the optimization problem can be formulated. According to the definition of MSE, \( \text{MSE} = \mathbb{E} \left( \| \hat{x} - x \|_2^2 \right) \), the MSE metric can be expressed by substituting \( \hat{x} \) into (16) and (17), which is given by

\[
    \mathbb{E} \left( \| \hat{x} - x \|_2^2 \right) = \mathbb{E} \left( \| (Q W x + Q r) + Q \omega \|_2^2 \right) = \mathbb{E} \left( \| Q (Q W - I_{N_s}) x + Q \omega \|_2^2 \right)
\]

\[
    + 2 \mathbb{E} \left( \omega^T Q^T (Q W - I_{N_s}) x \right) + \mathbb{E} \left( \omega^T Q^T Q \omega \right).
\]

Due to the fact that \( \text{tr}(CD) = \text{tr}(DC) \), the last term in (18) can be rewritten as

\[
    \mathbb{E} \left( \omega^T Q^T Q \omega \right) = \mathbb{E} \left( \text{tr}(\omega^T Q^T Q \omega) \right) = \text{tr} \left( Q R_{\omega \omega} Q^T \right),
\]

and the MSE can be rewritten as

\[
    \text{MSE} = \text{tr} \left( Q W R_x (Q W)^T \right) + \text{tr} \left( Q R_{\omega \omega} Q^T \right) + \text{tr} (R_x) - 2 \text{tr} (Q W R_x),
\]

where the signal correlation matrix and the noise correlation matrix are denoted by \( R_x = \mathbb{E}[xx^T] = \sigma_x^2 I_{N_s} \) and \( \omega = \mathbb{E}[\omega \omega^T] = \sigma_\omega^2 I_{N_R} \), respectively.

Given the practical considerations for the VLC on both communication and illumination performance, the following

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Fig. 2. The architecture of the proposed IRS-aided MIMO VLC system: baseband digital precoding and detection are adopted at transceivers and IRS is deployed to control the channel gains.
box inequality constraint on each LED guarantees negligible flicker effect, i.e.,
\[ P_{\text{max}} 1 \geq W x + r \geq P_{\text{min}} 1, \quad \forall x \in \mathcal{X}^{N_L} , \]
where \( P_{\text{min}} \) and \( P_{\text{max}} \) denote the minimum and maximum power for the comfort of users’ eyes, respectively. Meanwhile, the total emission power \( P_{\text{total}} \) indicates the maximum power of the MIMO VLC system, which imposes a constraint at the transmitter as
\[ \mathbb{E} \left[ \| W x + r \|^2 \right] \leq P_{\text{total}}. \tag{22} \]
Based on the above discussions, the MSE minimization problem can be formulated as
\[
P : \min_{P,G,W,Q} \text{MSE} (F,G,W,Q) \tag{23}
\]
\[ \text{s.t. } \sum_{n,t=1}^{N_L} g_{n,n_t} \leq 1, \quad \forall n \in N, \tag{24} \]
\[ \sum_{n,t=1}^{N_L} f_{n,n_t} \leq 1, \quad \forall n \in N, \tag{25} \]
\[ f_{n,n_t}, g_{n,n_t} \in \{0,1\}, \quad \forall n_t \in T, \ n_r \in R, \tag{26} \]
\[ P_{\text{max}} 1 \geq W x + r \geq P_{\text{min}} 1, \quad \forall x \in \mathcal{X}^{N_L}, \tag{27} \]
\[ \mathbb{E} \left[ \| W x + r \|^2 \right] \leq P_{\text{total}}, \tag{28} \]
where the constraints in (27) and (28) come from the power limitations and the constraints in (24)-(26) are due to the definitions of \( F \) and \( G \).

IV. PROPOSED ALGORITHMS TO MINIMIZE MSE

Note that \( P \) is a combinational optimization problem and there is no polynomial-time algorithm to attain the globally optimal solution for such a non-deterministic polynomial (NP)-hard problem. Therefore, an AO algorithm is proposed to optimize the relaxed problem of \( P \) in this section, where the IRS configuration matrices \( F \) and \( G \), the precoding matrix \( W \), and the detection matrix \( Q \) are iteratively optimized to minimize MSE.

A. Optimization of IRS Configuration

When the precoding matrix and the detection matrix are given by \( W^{(t)} \) and \( Q^{(t)} \) in the \( t \)-th iteration and kept fixed, the MSE is dependent on IRS configuration matrices \( F \) and \( G \) only. The original problem is thus simplified into the following form
\[
P : \min_{F,G} \text{MSE} (F,G,W^{(t)},Q^{(t)}) , \tag{29}
\]
which is an integer optimization problem and can hardly be optimized in polynomial time. However, the orthogonality among columns of \( f_{n_r} \) and \( g_{n_t} \) makes it feasible to transform \( P1 \) into a more tractable form as
\[
P1 - a : \min_{V} \text{MSE} \left( V, W^{(t)}, Q^{(t)} \right) \tag{30}
\]
\[ \text{s.t. } \sum_{p=1}^{N_L} v_{n,p} \leq 1, \quad \forall n \in N, \tag{31} \]
\[ v_{n,p} \in \{0,1\}, \quad \forall p \in P, \tag{32} \]
where \( \mathcal{P} \triangleq \{1,2,\cdots,N_L,N_r\} \) denotes an index set with the cardinality of \( |\mathcal{P}| = N_L N_r \) and \( V \triangleq [v_1,\cdots,v_{N_L,N_r}] \in \{0,1\}^{N \times N_L N_r} \) is a matrix with each column given by
\[ v_{n_t+(n-1)N_r} = f_{n_r} \odot g_{n_t}, \quad \forall n_t \in T, \ n_r \in R. \tag{33} \]

Proposition 1: The problem \( P1 - a \) is equivalent to \( P1 \).

Proof: Based on the definition of \( F \), the support set of \( f_{n_r} \) is disjoint to those of \( f_i, \forall i \neq n_r \), which reveals the column orthogonality of \( F \). Similarly, the orthogonality among columns of \( g_{n_t} \) can also be guaranteed, and the corresponding properties are formulated as
\[ f_{n_t}^T f_{n_r} = \delta_{n_t-n_r}, \tag{34} \]
\[ g_{n_t}^T g_{n_r} = \delta_{n_t-n_r}, \tag{35} \]
where \( \delta_n \) is the discrete Dirac function [4]. Therefore, the inner product of two arbitrary vectors produced by Hadamard products of \( f_{n_r} \) and \( g_{n_t} \) is given by
\[ (f_{n_t} \odot g_{n_t})^T (f_{n_r} \odot g_{n_r}) = \delta_{n_t-n_r} \delta_{n_t-n_r}, \tag{36} \]
which is nonzero if and only if \( n_t = n_r \) and \( n_r = n_r \). According to (33) and (36), the support set of \( v_p \) is disjoint to that of \( v_i, \forall i \in \mathcal{P}, i \neq p \), assuring that row vectors of \( V \) are one-hot vectors and thus \( V \) has orthogonal columns.

As for the sufficiency, the ones’ locations of \( F \) and \( G \) can be inferred directly from given \( V \). Specifically, \( v_{n,n_r+(n-1)N_r} = 1 \) implies that \( f_{n,n_r} = g_{n,n_r} = 1 \) and \( f_{n,i} = g_{n,i} = 0, \forall i \neq n_r, j \neq n_t \), and thus the proof is thus completed.

Therefore, problem \( P1 \) can be solved by first optimizing \( P1 - a \) and then recovering the IRS configuration matrices \( F \) and \( G \) according to Proposition 1. In general, heuristic algorithms such as the Tabu search algorithm are effective to optimize \( P1 - a \). Nevertheless, in consideration of the computational complexity, a more practical solution is via optimizing the relaxed form of \( P1 - a \), after which the binary result can be obtained according to the minimum distance projection criterion. In the sequel, the relaxed problem of \( P1 - a \) is redefined as
\[
P1 - b : \min_{V: (31)} \text{MSE} \left( V, W^{(t)}, Q^{(t)} \right) \tag{37}
\]
\[ \text{s.t. } 0 \leq v_{n,p} \leq 1, \quad \forall n \in N, \ p \in \mathcal{P}. \tag{38} \]

Lemma 1: For any matrices \( M, C, \) and \( D \) of appropriate dimensions, we have the following result [37]
\[ \text{tr}\{M^T CMD\} = \text{vec}(M)^T (D^T \otimes C) \text{vec}(M). \tag{39} \]

Proposition 2: The objective of problem \( P1 - b \) is convex with respect to \( V \).

Proof: According to Lemma 1, the first term of the MSE function (20) is reformulated as
\[ \text{tr} \left( Q^{(t)} W^{(t)} R_x Q^{(t)} W^{(t)}^T \right) = \text{tr} \left( H^T Q^{(t)T} Q^{(t)} W^{(t)} R_x W^{(t)}^T \right) \]
\[ = \text{vec}(H)^T \left( \left( W^{(t)} R_x W^{(t)} \right)^T \otimes Q^{(t)T} Q^{(t)} \right) \text{vec}(H), \tag{40} \]
where the central square matrix can be rewritten based on the properties of the Kronecker product, i.e.,
\[
\begin{align*}
(W^{(t)} R_x W^{(t)T}) \otimes (Q^{(t)T} Q^{(t)}) & = \\
\left[ (W^{(t)} R_x^2) \otimes Q^{(t)T} \right] \left[ (R_x^2 W^{(t)T}) \otimes Q^{(t)} \right] & = \\
\left[ (R_x^2 W^{(t)T}) \otimes Q^{(t)} \right]^T \left[ (R_x^2 W^{(t)T}) \otimes Q^{(t)} \right]
\end{align*}
\]
(41)
which is a positive definite matrix. Therefore, MSE is convex with respect to \(H\) since another \(H\)-dependent term in (20) is affine. Define an auxiliary matrix \(H_c \in \mathbb{R}^{N_r \times N_t \times N_s}\) as
\[
H_c \triangleq \begin{bmatrix} h_{1,1}^{(2)}, \cdots, h_{N_{r,1}}^{(2)}, h_{1,2}^{(2)}, \cdots, h_{N_{r,1}}^{(2)} \end{bmatrix},
\]
(42)
and then according to the definition of (12), the vectorization of \(H_2\) can be expressed as
\[
\text{vec}(H_2) = \text{diag}(H_c)^T \text{vec}(V),
\]
(43)
which shows that MSE is convex with respect to \(V\) since \(H\) and \(V\) are affine.

Typically, the penalty method and gradient descent algorithm can be used to optimize a convex optimization problem with ensured global optimality [38]. However, the dual projected gradient descent (DPGD) algorithm, which will be elaborated in Section IV-C, is more efficient compared to the aforementioned algorithms [39] in terms of convergence rate. To summarize, the IRS configuration problem \(P_1\) is solved by optimizing its equivalent problem \(P_1 - a\), which is relaxed as \(P_1 - b\) and then optimized by the DPGD algorithm.

B. Optimizations of Precoding and Detection

Designs of the precoding matrix \(W\) and detection matrix \(Q\) are discussed separately in this subsection, after which the overall proposed algorithm is presented to solve \(P\).

1) Optimize \(W\) Given \(V^{(t+1)}\) and \(Q^{(t)}\): With the IRS configuration matrix \(V^{(t+1)}\) and the detection matrix \(Q^{(t)}\) given as fixed, the original problem \(P\) is transformed into a simplified subproblem, which aims to optimize the precoding matrix \(W\). Specifically, the dynamic range of symbol \(x_{n_s}\) is bounded according to (15) by
\[
\sigma_x I(M-1) \geq x_{n_s} \geq -\sigma_x I(M-1),
\]
(44)
based on which the constraint in (27) can be re-expressed as
\[
-\sigma_x I(M-1) \|w_{n_s}\|_1 + r_0 \geq P_{\text{min}},
\]
(45)
\[
P_{\text{max}} \geq \sigma_x I(M-1) \|w_{n_s}\|_1 + r_0,
\]
(46)
which are both convex constraints since the region bounded by the \(l_1\) norm is a convex hull. Meanwhile, the power constraint in (28) can be reformulated due to \(E[|x|] = 0_{N_s \times 1}\) as
\[
E \left[ \|Wx + r\|_2^2 \right] = \text{tr} \left( x^T W^T Wx + 2r^T Wx + r^T r \right) = \sigma_x^2 \|W\|_F^2 + r^T r \leq P_{\text{total}},
\]
(47)
where the second equality is satisfied since \(\text{tr}(W^T W) = \|W\|_F^2\). Therefore, the optimization subproblem for the precoding matrix \(W\) can be expressed as
\[
P_2 : \min_w \text{MSE}(V^{(t+1)}, W, Q^{(t)}) \tag{48}
\]
s.t. \(\|W\|_F^2 \leq \frac{P_{\text{total}} - r^T r}{\sigma_x^2} \),
\[
\|w_{n_s}\|_1 \leq \frac{\min\{r_0 - P_{\text{min}}, P_{\text{max}} - r_0\}}{\sigma_x I(M-1)}, \forall n_s \in T,
\]
(50)
where the objective is convex with respect to \(W\) due to Lemma 1. Consequently, subproblem \(P_2\) is a convex optimization problem, which can also be solved by the DPGD algorithm described in Section IV-C.

2) Optimize \(Q\) Given \(V^{(t+1)}\) and \(W^{(t+1)}\): With the IRS configuration matrix \(V^{(t+1)}\) and the precoding matrix \(W^{(t+1)}\) given as fixed, the problem of optimizing the detection matrix \(Q\) turns to be an unconstrained convex optimization problem given by
\[
P_3 : \min_Q \text{MSE}(V^{(t+1)}, W^{(t+1)}, Q),
\]
(51)
whose globally optimal point can be attained by setting \(\nabla_Q \text{MSE}(V^{(t+1)}, W^{(t+1)}, Q) = 0\), and the optimal result has been derived as [8]
\[
Q^{(t+1)} = R_x \left( H^{(t+1)} W^{(t+1)} \right)^T \left( H^{(t+1)} W^{(t+1)} R_x \right)^{-1} \left( H^{(t+1)} W^{(t+1)} \right)^T + R_{\omega} \right)^{-1},
\]
(52)
which is in closed form.

The algorithm to solve problem \(P\) is provided in Algorithm 1, which iteratively optimizes the IRS configuration matrices, the precoding matrix at the transmitter, and the detection matrix at the receiver. Once MSE converges, IRS configuration matrices \(F\) and \(G\) can be recovered from the relaxed form of \(V\) according to the minimum distance projection criterion. Let \(\vec{v}_n \in \mathbb{R}^{1 \times N_s N_r}\) be the \(n\)-th row vector of \(V\) and \(e_p = [0, \cdots, 1, \cdots, 0] \in \mathbb{R}^{1 \times N_s N_r}\) be the one-hot vector with the \(p\)-th element as 1, with \(e_0 = 0\) for convenience. Then, the index \(p^\dagger \in P \cup \{0\}\) of the optimal projection for the \(n\)-th unit can be expressed as
\[
p^\dagger = \arg \min_p \|\vec{v}_n - e_p\|_2^2,
\]
(53)
for \(e_p \in \{e_0, e_1, \cdots, e_{N_s N_r}\}\), based on which its associated transmitter and receiver can be obtained by Proposition 1 as
\[
n_t = \left\lfloor \frac{p^\dagger - 1}{N} \right\rfloor + 1, \tag{54}
\]
n_r = \text{mod} \left(p^\dagger - 1, N\right) + 1, \tag{55}
where \(\text{mod}(\cdot)\) denotes the modulus operator and \(\lfloor \cdot \rfloor\) is the floor operator. Note that from (33), \(f_{n_r n_t} = 1\) and \(g_{n_t n_s} = 1\), while other elements in the \(n\)-th row of \(F\) and \(G\) are zero due to constraints in (24) and (25). Nevertheless, it is worth noting that \(n_t = n_r = 0\) when \(p^\dagger = 0\), which means this IRS unit should be turned off to minimize the MSE.
Algorithm 1 Proposed AO Algorithm

Input: CSI matrices $H_1$ and $H_c$, direct current $r$, iteration index $t \leftarrow 0$, and IRS unit index $n \leftarrow 1$.

Output: Precoding matrix $W$, detection matrix $Q$, and IRS matrices $F$ and $G$.

1: IRS pre-configuration by minimum distance policy: $G^{(t)}$ and $F^{(t)}$ are initialized by associating each IRS unit to the nearest LED and PD, respectively;
2: $V^{(t)}$ is obtained by (33);
3: $H^{(t)}$ is obtained by (11) and (43);
4: Init: $W^{(t)} \leftarrow H^{(t)+}$, $Q^{(t)} \leftarrow [I_{N_1}, 0_{N_x \times (N_r - N_s)}]$;
5: repeat
6: Solve $P1 - b$ to obtain the optimal $V^{(t+1)}$ under constant $W^{(t)}$ and $Q^{(t)}$;
7: Solve $P2$ to obtain the optimal $W^{(t+1)}$ under constant $V^{(t+1)}$ and $Q^{(t)}$;
8: Solve $P3$ to obtain the optimal $Q^{(t+1)}$ by (52) under constant $V^{(t+1)}$ and $W^{(t+1)}$;
9: Update the index: $t \leftarrow t + 1$;
10: until $\|MSE^{(t)} - MSE^{(t-1)}\| \leq \epsilon$
11: repeat
12: Obtain the optimal index $p^i$ by (53);
13: if $p^i = 0$ then
14: Set elements in the $n$-th row of $F$ and $G$ to zeros;
15: Jump to step 20;
16: end if
17: Obtain $n_s$ and $n_r$ by (54) and (55), respectively;
18: Set the $n$-th row of $F$: $f_{n,n_s} \leftarrow 1$ and $f_{n,j \neq n_s} \leftarrow 0$;
19: Set the $n$-th row of $G$: $g_{n,n_s} \leftarrow 1$ and $g_{n,j \neq n_s} \leftarrow 0$;
20: Update the index: $n \leftarrow n + 1$;
21: until $n > N$

C. DPGD Algorithm to Optimize $P1-b$ and $P2$

Generally, constrained convex optimization problems can be solved by the interior point algorithm, which transforms the constraint into a penalty term in the objective. However, due to the special structure of our considered problem, its Lagrange dual function can be attained in closed form and the gradient projection is implementable. Therefore, this paper exploits the DPGD algorithm to optimize IRS reflection and precoding matrix. Let $z$ and $z^{(i)}$ denote the variable and the fixed variable in the $i$-th iteration, respectively. The DPGD algorithm updates the optimization variables iteratively according to [39]

$$z^{(i+1)} = \arg \min_{z \in \mathcal{Z}} D_h(z, z^{(i)} - \alpha \nabla f(z^{(i)})),$$  \hspace{1cm} (56)

where $\alpha$ denotes the stepsize, $\mathcal{Z}$ is a convex domain of $z$, and $D_h$ indicates a specific distance metric with respect to $z$ and $z^{(i)} - \alpha \nabla f(z^{(i)})$.

As shown in Algorithm 2, we simplify the projection process by optimizing the dual problem of $P1 - b$, whose intrinsic convexity ensures that the dual gap between these two problems is zero [38]. The Lagrangian function $\mathcal{L}(V, \mu_1, \mu_2, \mu_3)$ of problem $P1 - b$ is given by

$$\mathcal{L} = \text{MSE}(V) + \mu_1^T (V W_{1-N N_s} - 1_{N \times 1}) - \mu_2^T \vec{V} + \mu_3^T (\text{vec}(V) - 1_{N+N \times 1}),$$ \hspace{1cm} (57)

where $\mu_1 \in \mathbb{R}^{N \times 1}$, $\mu_2 \in \mathbb{R}^{N \times 1}$, and $\mu_3 \in \mathbb{R}^{N \times 1}$ are Lagrangian multipliers. Next, the Lagrange dual function can be expressed as [38]

$$g(\mu_1, \mu_2, \mu_3) = \inf_{V} \mathcal{L}(V, \mu_1, \mu_2, \mu_3),$$ \hspace{1cm} (58)

where $\inf$ denotes the infimum of the function with respect to $V$. The metric $\text{MSE}(V)$ is in the quadratic form of $V$ according to Proposition 2, implying that the minimum value of the Lagrangian function can be directly obtained by solving $\nabla V \mathcal{L}(V, \mu_1, \mu_2, \mu_3) = 0$, which is detailed in Appendix A. Therefore, the dual optimization problem is given by

$$P1 - c: \max \; g(\mu_1, \mu_2, \mu_3) \quad \text{s.t.} \; \mu_1, \mu_2, \mu_3 \geq 0,$$ \hspace{1cm} (59)

where $g(\mu_1, \mu_2, \mu_3) = \mathcal{L}(V^*, \mu_1, \mu_2, \mu_3)$, and $P1 - c$ is a convex optimization problem [38]. The gradient ascend policy is utilized to reach the maximum value of $P1 - c$, where derivatives with respect to $\mu_1$, $\mu_2$, and $\mu_3$ are given in Appendix B. As for the projection, fortunately, the convex domain has been greatly simplified, and the projection point in $i$-th iteration is given by

$$\mu_j^{(i+1)} = [\mu_j^{(i)} + \alpha \nabla \mu_j g(\mu_1, \mu_2, \mu_3) |_{\mu_j^{(i)}}]^+, \; j \in \{1, 2, 3\},$$ \hspace{1cm} (61)

where $[\cdot]^+ = \max(\cdot, 0)$ denotes the nonnegative function.

Similarly, the DPGD algorithm can also be applied to solve $P2$. A key difference between this problem and the former one lies in the property of the constraint (27). The l-1 norm generally is non-differentiable, whereas its sub-gradient can be derived as

$$\frac{\partial \|w_{n_l}\|_1}{\partial w_{n_l}} = \text{sgn}(w_{n_l}), \; \forall n_l \in T,$$ \hspace{1cm} (62)

where $\text{sgn}(\cdot)$ denotes the sign of the argument. Moreover, another benefit of optimizing the dual problem is the reduction of the number of variables, i.e., only $N_l + 1$ variables are needed instead of $N_l N_s$ of the original problem.

D. Alternative Solutions in High/Low SNR Regimes

The MSE minimization problem $P$ has been solved by the proposed algorithm, which optimizes the IRS configuration matrix, the precoding matrix, and the detection matrix iteratively. Nevertheless, two special cases in the high and low SNR regimes are considered particularly, and the asymptotic analysis of the proposed algorithm is provided in this subsection.
Algorithm 2: Proposed IRS Optimization Algorithm

**Input:** CSI matrices $H_1$ and $H_c$, direct current $r$, iteration index $i \leftarrow 1$.
**Output:** Relaxed variable $V$

1. Calculate the minimum point of the Lagrangian function by (77);
2. The objective function of the dual problem of $P1 - b$ is obtained by (58);
3. **Init:** $\mu_1 \leftarrow e_{1,N \times 1}$,
   $\mu_2, \mu_3 \leftarrow e_{1,N,N_r \times 1}$;
4. **repeat**
5. Calculate $\nabla \mu_1 g(\mu_1, \mu_2, \mu_3)$ by (79);
6. Gradient Projection:
   $\mu_1^{(i+1)} = [\mu_1^{(i)} + \alpha \nabla \mu_1 g(\mu_1, \mu_2^{(i)}, \mu_3^{(i)})]_{\mu_1^{(i)}}$;
7. Calculate $\nabla \mu_2 g(\mu_1^{(i+1)}, \mu_2, \mu_3^{(i)})$ by (83);
8. Gradient Projection: repeat step 6 for $\mu_2$;
9. Calculate $\nabla \mu_3 g(\mu_1^{(i+1)}, \mu_2^{(i+1)}, \mu_3)$ by (84);
10. Gradient Projection: repeat step 6 for $\mu_3$;
11. Update the index: $i \leftarrow i + 1$;
**until** $||MSE^{(i)} - MSE^{(i-1)}|| \leq \epsilon$

1) **High SNR Regime:** The original problem can be simplified in the high-SNR regime, where a more efficient algorithm can be utilized to jointly optimize the precoding and detection matrices. To this end, subproblems $P2$ and $P3$ are combined into the following problem

$$
\begin{align*}
P4 & : \min_{W,Q} \lim_{\sigma_z \to 0} \text{MSE}(V^{(t+1)}, W, Q) \\
& = \frac{\text{tr}\left(QH^{(t+1)}WQH^{(t+1)}W^T\right) - 2\text{tr}\left(QH^{(t+1)}W\right) + N_s}{\sigma_z^2}.
\end{align*}
$$

The SVD of $H^{(t+1)}$ is carried out as $H^{(t+1)} = U^{(t+1)} \Lambda^{(t+1)} V^{(t+1)^T}$, where $\Lambda^{(t+1)}$ is a diagonal matrix with decreasing singular values, and $U^{(t+1)} = [\vec{u}_1^{(t+1)}, \ldots, \vec{u}_{N_s}^{(t+1)}]^T$ and $V^{(t+1)}$ are the left singular matrix and right singular matrix, respectively. Then, a feasible precoding matrix can be expressed as

$$
W^{ZF} = \zeta \vec{V}^{(t+1)} \Lambda^{(t+1)^T} [\vec{u}_1^{(t+1)}, \ldots, \vec{u}_{N_s}^{(t+1)}],
$$

where $\Lambda^{(t+1)^T}$ represents the transpose of $\Lambda^{(t+1)}$ with each element in the reciprocal form and the coefficient $\zeta$ is a constant given by

$$
\zeta = \min \left( \frac{\int \frac{P_{\text{total}} - \mu^T r}{\sigma_z^2 \Lambda^{(t+1)^T} [\vec{u}_1^{(t+1)}, \ldots, \vec{u}_{N_s}^{(t+1)}]^T} \nu_0 / \sqrt{\sigma_z^2} \right),
$$

which is determined to satisfy the power constraints in (49) and (50). In the meanwhile, the detection matrix $Q$ can be chosen as

$$
Q_{ZF}^{(t+1)} = \zeta^{-1} [I_{N_s}, 0_{N_s \times (N_r - N_s)}].
$$

**Proposition 3:** In the high-SNR regime, the proposed precoding matrix $W^{ZF}$ and the detection matrix $Q^{ZF}$ can converge to the optimal solution of $P4$.

**Proof:** According to (64), the MSE metric can be rewritten as $\text{tr}(B \beta^T) - 2\text{tr}(B) + N_s$, where $B = QH^{(t)}W$, which is a convex function with respect to $B$ due to Lemma 1. When constraints of (49) and (50) are ignored, the minimum MSE can be obtained by directly forcing the derivative of $B$ equal to 0, i.e., $B - I_{N_s} = 0$. Considering that $W^{ZF}$ is composed of the first $N_s$ columns of the right pseudo-inverse matrix of $H^{(t)}$, it is easy to confirm that $Q^{ZF} W^{ZF} = I_{N_s}$. Furthermore, emission power constraints can be ensured due to the proper setting of $\zeta$ in (66), implying that the proposed precoding matrix $W^{ZF}$ and detection matrix $Q^{ZF}$ will converge to the optimal solution of $P4$.

2) **Low SNR Regime:** When the noise power is sufficiently high as compared to the signal power, the optimal detection matrix can be expressed according to (52) as

$$
Q^{(t+1)} = R_x \left( H^{(t+1)} W^{(t+1)} \right)^T R^{-1}.
$$

Then, the asymptotic analysis of the MSE performance versus the SNR at the receiver is provided in the following, where the SNR is defined as

$$
\text{SNR} = \frac{E[\|HWx\|^2]}{E[\|\omega\|^2]} = \frac{\sigma_x^2 \|HW\|^2}{\sigma_N^2 N_s}.
$$

**Lemma 2:** $a^T \cdot b \leq |a| \cdot |b|$ for any two vectors $a$ and $b$ (Cauchy-Schwarz inequality [39]).

The normalized MSE, namely MESE, is used to evaluate the MSE performance, which is given by

$$
\text{MSE} = \frac{\text{MSE}(F, G, W, Q)}{\text{tr}(R_x)}
$$

$$
= 1 - \left( 2\text{tr}(QHW) - \|QHW\|^2 \right) - \frac{\sigma_x^2}{\sigma_N^2} ||Q||^2_F
$$

$$
= 1 - \frac{N_r}{N_s} \left( \frac{\sigma_x^2 \|HW\|^2}{\sigma_N^2 N_r} - \frac{\sigma_x^2}{\sigma_N^2} ||Q||^2_F \right)
$$

$$
\leq 1 - \frac{N_r}{N_s} \left( \frac{\sigma_x^2 \|HW\|^2}{\sigma_N^2 N_r} - \frac{\sigma_x^2}{\sigma_N^2} ||HW||^2_F \right),
$$

where the inequality comes from Lemma 2. Therefore, the normalized MSE in the low-SNR regime can be reformulated as

$$
\text{MSE} \approx 1 - \frac{N_r}{N_s} (\text{SNR} - o(\text{SNR})),
$$
where $o(\cdot)$ denotes the higher-order quantity. It can be observed that $\text{MSE}$ decreases linearly with the increase of SNR, and the slope is $N_t/N_r$.

### E. Theoretical Analysis of Proposed Algorithms

The convergence of the proposed AO algorithm is guaranteed due to the convexity of three subproblems. Specifically, the relaxed IRS configuration matrix, the precoding matrix, and the detection matrix in the $t$-th iteration are denoted by $V^{(t)}$, $W^{(t)}$, and $Q^{(t)}$, respectively. In the next iteration, $V^{(t+1)}$ can be obtained by solving the convex optimization subproblem $P1 - b$ and thereby we have $\text{MSE}(V^{(t+1)}, W^{(t)}, Q^{(t)}) \leq \text{MSE}(V^{(t)}, W^{(t)}, Q^{(t)})$. The similar property can be obtained that $\text{MSE}(V^{(t+1)}, W^{(t+1)}, Q^{(t)}) \leq \text{MSE}(V^{(t+1)}, W^{(t+1)}, Q^{(t)}) \leq \text{MSE}(V^{(t+1)}, W^{(t)}, Q^{(t)})$ since $P2$ and $P3$ are also convex optimization subproblems.

Therefore, $\text{MSE}$ will converge to a nonnegative value since it is lower-bounded by 0. However, the optimized solution given by the proposed AO algorithm is not a globally optimal solution in general; instead, it is sub-optimal since $\text{MSE}$ is neither convex nor concave with respect to the optimization variables.

The complexity of the proposed algorithm depends on the number of iterations in $P1 - P3$ and the number of inner loops in each iteration. Define the number of outer loops of Algorithm 1 and the number of inner loops in each iteration of $P1$ and $P2$ as $I_O$, $I_1$, and $I_2$, respectively. The standard big-O metric $O(\cdot)$ is used to evaluate the computational complexity. According to [39], $I_1$ is in the order of $O(\log(1/\epsilon))$ with $\epsilon$ denoting the tolerance error since the Hessian matrix of the objective shown in (41) is a positive definite matrix. This complexity is far lower than $O(1/\epsilon)$ in the gradient descent algorithm [39], showing the superior efficiency of DPGD algorithm. Moreover, the second-order derivative of the objective in $P2$ is given by

\[
\nabla^2_{\text{vec}(W)} \text{MSE}(V^{(t+1)}, W^{(t)}, Q^{(t)}) = R_x \otimes \left( H^{(t+1)}Q^{(t)} H^{(t+1)} \right) = \left( R_x^2 \otimes Q^{(t)} H^{(t+1)} \right)^T \left( R_x^2 \otimes Q^{(t)} H^{(t+1)} \right) \geq \lambda_{\text{min}} I_{N_tN_r},
\]

where $\lambda_{\text{min}}$ is the minimum eigenvalue of the Hessian matrix and $I_2$ is also in the order of $O(\log(1/\epsilon))$ due to the $\lambda_{\text{min}}$-strong convexity [39].

In each iteration, 9 matrix multiplications and 1 matrix inversion are needed to calculate the Lagrange dual function, and 10 matrix multiplications are used to update Lagrangian multipliers for optimizing $P1 - b$. Then, $P2$ is also optimized by the DPGD algorithm, where the number of constraints is reduced from $N^2N_tN_r$ to $N_t+1$ as compared to $P1 - b$. The optimal solution of $P3$ can be directly obtained from (52), which requires 6 matrix multiplications, 1 matrix addition, and 1 matrix inversion. Finally, the complexity of the overall proposed algorithm is $O(I_O(10I_1 + 6I_2 + 19) + N^2N_tN_r)$, where the last term is due to the operations for recovering $F$ and $G$ from the optimized $V$. Moreover, it is worth noting that the complexity of the proposed algorithm can be further reduced by grouping adjacent IRS units as an IRS tile, which is practical for the deployment of large-scale IRS [40].

### V. Numerical Results

In this section, numerical results are provided to evaluate the performance of the proposed algorithm for minimizing the MSE of IRS-aided MIMO VLC. We consider the downlink communication of an indoor MIMO VLC system, where the transmitter equipped with 16 LEDs and the receiver equipped with 4 PDs are both set as uniform planar arrays (UPAs). Set the room size to $6m \times 6m \times 3.2m$, and the coordinates of LEDs are generated by dividing the ceiling into $16$ equal squares and placing LEDs in the centers of them. All units of IRS are deployed equally within the rectangular determined by corners of $(0.0m, 0.0m, 0.85m)$ and $(0.0m, 5.0m, 3.0m)$. As for the receiver, the center of the UPA is located at $(1.5m, 2.4m, 0.75m)$ in the 3D coordinate system, and the normal vector of every PD is perpendicular to the floor. We assume the direct bias on LEDs are the same value of $r_0$, i.e., $r = r_0 \mathbf{1}_{N_t+1} \in \mathbb{R}^{N_t+1}_+$. More detailed settings are listed in Table I, including the VLC channel parameters, the specific modulation scheme, etc. Since the CSI is assumed known to the transceiver, the channel gain in indoor VLC is quasi-static and these parameters remain unchanged in the coherent time. We normalize the CSI matrix by $||H||_F^2 = 1$, and the elements in $H$ are scaled by the same factor. Without loss of generality, the normalized MSE rather than the absolute values are adopted, and the parameter $\sigma_n^2/\sigma_0^2$ given by (69) is set to different values in our simulations.

The performance of the proposed AO algorithm is compared to following alternative IRS configuration schemes: (1) Distance greedy IRS scheme: IRS configuration is carried out by matching each IRS unit and transceiver antennas by the minimum distance criterion, i.e., $f_{n,m} = 1$ and $g_{n,m} = 1$ denote that the $n$-th IRS unit has the shortest distances with the $n_t$-th PD and the $n_t$-th LED, respectively; (2) Random IRS scheme: The binary variable $V$ in $P1$ is randomly generated, and each simulation point is the average result of 5000 independent experiments; (3) No IRS scheme: In this scheme, the MIMO VLC channel gain is given by $H = H_1$ due to $N = 0$. Meanwhile, two commonly adopted precoding solutions are considered to show the performance of the proposed precoding scheme: (1) ZF precoding scheme: The precoding matrix $W$ is determined by the right pseudo-inverse matrix of the MIMO VLC channel matrix $H$, and a coefficient should be multiplied by $W$ to meet the emission power constraints of (49) and (50); (2) MMSE precoding scheme: The precoding matrix is jointly determined by $H$ and the AWGN power $\sigma_n^2$ for maximizing the signal-to-interference-plus-noise ratio (SINR) at the receiver, and the obtained precoding matrix needs to be adjusted similarly to the previous scheme. Moreover, the lower bound of the proposed scheme can be obtained by solving the relaxed form of $P$.

To start with, the convergence performance of the proposed algorithm is depicted in Fig. 3, where the MIMO VLC system
TABLE I
SIMULATION PARAMETERS

| Parameters                              | Values | Parameters                              | Values |
|-----------------------------------------|--------|-----------------------------------------|--------|
| The number of LEDs, \( N_t \)          | 16     | The Lambertian index, \( m \)           | 1      |
| The number of PDs, \( N_r \)           | 4      | The reflectivity of each IRS unit, \( \gamma \) | 0.9    |
| The number of independent data streams, \( N_s \) | 4      | Optical filter gain, \( g_{o_f} \)     | 1      |
| The number of IRS units, \( N \)       | 16     | The area of a single PD, \( A_p \)      | 4 cm²  |
| The area of an IRS unit                 | 10 × 10 cm² | The PD spacing                          | 0.4 m  |
| The operating height of the receiver    | 0.75 m | Semi-angle of the FoV, \( \theta_0 \)   | 75°    |
| Direct current bias, \( \tau \)         | 6 × 1 \( N_t \)×1 | Refractive index of the PD, \( n \)    | 1.5    |
| Total emission power constraint, \( P_{\text{total}} \) | 16 × 50 | The order of bipolar PAM, \( M \)      | 4      |
| Maximum power, \( P_{\text{max}} \)    | 8 W    | Tolerance error, \( \epsilon \)         | \( 10^{-6} \) |
| Minimum power, \( P_{\min} \)          | 4 W    | The covariance of transmitting symbol, \( \sigma_\epsilon^2 \) | 1      |

Fig. 3. The convergence performance of the proposed AO algorithm.

is aided by 16 IRS units for MSE reduction and the parameter \( \sigma_x^2/\sigma_\omega^2 \) is set to 15 dB. As shown in this result, the MSE of the proposed AO algorithm and the proposed scheme when \( N = 0 \) converge to positive values after around 35 and 20 iterations, respectively. The convergence in both two cases is fast, showing that the proposed AO algorithm is computationally efficient. Nevertheless, the number of iterations for the scheme with \( N = 16 \) to converge is larger than that in the no IRS scheme, which takes only 2 iterations to reach a sound MSE. Meanwhile, the result also shows that the MSE of the proposed algorithm is smaller than that of the no-IRS scheme, demonstrating the effectiveness of the optical IRS to reduce the MSE of the IRS-aided MIMO VLC system.

Under the above setup, we investigate the MSE performance at different locations in Fig. 4, since the VLC channel gain depends on the locations of the transceivers. It can be observed that the MSE at the locations close to the IRS can be less than \( 5 \times 10^{-2} \), while the MSE is larger than \( 10^{-1} \) in the center of the room. Even worse, the MSE at the locations furthest to IRS can increase nearly to \( 5 \times 10^{-1} \), and the effect of optical IRS can be ignored there. The result shows that the MSE of MIMO VLC can be improved significantly by the optical IRS. However, the influence of IRS highly depends on the relative position of the optical IRS and receiver. Specifically, the NLoS channel gain obtained by IRS is non-negligible when the angle of incidence at the PD is small and the propagation distance is not too large. Both two conditions require that the receiver should not be too far away from the IRS. Therefore, IRS can locally improve the quality of service (QoS) and the deployment of IRS is practically important in the IRS-aided VLC system.

Next, we investigate the proposed AO algorithm from the aspects of two metrics, namely the MSE calculated by (20) and the BER obtained by averaging over \( 10^7 \) independent and repeated transmissions, to show the performance of IRS by comparing the proposed algorithm with other schemes over different SNRs. Specifically, the normalized MSE versus \( \sigma_x^2/\sigma_\omega^2 \) is depicted in Fig. 5, where the values of parameters remain unchanged as those in Table I and various benchmarks discussed before are provided for comparison. It can be observed from the result that the MSEs of the IRS-aided schemes are lower than those of the counterpart no-IRS schemes, which shows the improvement of the MSE performance due to the optical IRS. Meanwhile, the result also reveals that the normalized MSE of the considered IRS-aided MIMO VLC system can be further improved by optimizing the precoding and detection matrices, since the proposed AO algorithm outperforms other ZF-based schemes and MMSE-based schemes even aided by IRS. Therefore, the MSE reduction of the proposed AO algorithm can be mainly

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attributed to two factors, namely the IRS gain obtained by optimizing the IRS configuration and the SNR gain obtained by optimizing the precoding and detection matrices. Moreover, the MSE gap between the proposed scheme and its relaxed lower bound diminishes in the low-SNR regime. The reason of this result lies in that the objective MSE is mainly dominated by the term $\text{tr} \left( \mathbf{Q} \mathbf{R}_w \mathbf{Q}^T \right)$ when SNR is low, which does not depend on the optical IRS according to (20).

Moreover, the BER performance of the proposed AO algorithm is studied in Fig. 6, where each point is an averaged result. Compared to each no-IRS scheme, the corresponding IRS-aided scheme has an improved BER, showing that the reliability of the IRS-aided MIMO VLC is higher than the no-IRS system. The result also reveals that the BER gain can come not only from the IRS configuration optimization but also from the optimization of the MIMO precoding and detection, which can be observed by comparing the BER of the proposed scheme and other IRS-aided schemes. Moreover, it should be noticed that some points are not shown in the figure, since there is no detection error found even with $10^7$ channel uses.

In Fig. 7, the MSEs of several baselines are simulated under the different number of IRS units $N$, which varies from 0 to 96. Remarkably, it can be observed evidently that the normalized MSE gain of the proposed AO algorithm consists of the IRS gain and the SNR gain, which are in accord with the aforementioned discussions. More specifically, the IRS gain in improving the normalized MSE is larger with the increase of $N$. This is reasonable since the feasible space of the NLoS MIMO channel matrix $\mathbf{H}_2$ is enlarged when more IRS units can be used, and thereby the normalized MSE is monotonously nondecreasing over the number of IRS units $N$. However, the result also shows that the SNR gain obtained by optimizing the precoding and detection matrices is insensitive to $N$, which implies that the SNR gain will get saturated while the IRS gain becomes dominant when $N$ continues to grow.

To investigate the reason of the MSE and BER improvements, the condition numbers of the MIMO channel matrices...
of three IRS-aided schemes are depicted in Fig. 8, where the number of IRS units \( N \) is set from 0 to 32. We can observe from the result that the condition number with the proposed AO algorithm decreases with the increase of \( N \), and it converges to 5.7 when \( N \) is large. Meanwhile, the condition numbers of other two IRS-aided schemes when \( N = 32 \) are 31.3, and 34.9, respectively, which also outperform the condition number 110 of the no-IRS baseline. Nevertheless, the result shows that the condition numbers of the distance greedy scheme and the random IRS scheme do not decrease monotonously. This is because the optimization objective of problem \( P \) is the MSE rather than the condition number. Remarkably, the channel gains in VLC are real-valued and non-negative numbers due to the IM scheme, and thereby the MIMO channel matrix \( \mathbf{H} \) is a non-negative matrix, which depends on the geometric parameters. Thus, the performance of MIMO VLC suffers greatly from strong channel correlation since the large MIMO channel matrix condition number results in a low multiplexing gain [8]. Fortunately, the optical IRS adds a non-negative matrix \( \mathbf{H}_2 \) to the LoS channel \( \mathbf{H}_1 \) according to (11), which can be tuned by adjusting the IRS parameters. Therefore, the singularity of the MIMO channel matrix \( \mathbf{H} \) can be improved and the BER/MSE performances of MIMO VLC can be enhanced.

Finally, the effect of the direct bias \( r_0 \) on the normalized MSE is investigated in Fig. 9, where \( r_0 \) varies from \( P_{\text{min}} \) to \( \sqrt{P_{\text{total}}/N_t} \). The result shows that the MSEs of all schemes decrease versus \( r_0 \) in the low-bias regime, while it inversely increases when \( r_0 \) approaches the maximum of \( \sqrt{P_{\text{total}}/N_t} \). This is because the feasible space of \( \mathbf{W} \) is enlarged when \( r_0 \) increases according to the constraint of (50), while subsequently the space becomes smaller since the constraint of (49) becomes active. Furthermore, the feasible space of \( \mathbf{W} \) under the condition of \( r_0 = \sqrt{P_{\text{total}}/N_t} \) is an empty set, which results in the worst MSE performance. For achieving both communication and illumination functions, the value of direct current bias is important in VLC, and the numerical result shows that there is an optimal bias \( r_0 \) for minimizing the normalized MSE.

**VI. Conclusion**

This paper presented the channel model and signal processing for an IRS-aided MIMO VLC system, where the IRS configuration, precoding at the transmitter, and detection at the receiver are jointly optimized to minimize the detection MSE. Inspired by the concentrated power density of the reflected lightwave, the no cross-talk property of the NLoS channel in IRS-aided VLC was discussed under the point source assumption, and thus the IRS configuration situation is modeled by the alignments of IRS reflecting units with different LED/PD pairs. Then, the non-convex optimization problem was reformulated by variable transformation, and an AO algorithm was proposed to solve it by decoupling the problem into three subproblems, which were shown to be convex and solved efficiently by the DPGD algorithm. In addition, theoretical analysis was provided to ensure the convergence of the proposed algorithm and derive its computational complexity, and furthermore, we discussed simplified algorithms in the high- and low-SNR regimes. Finally, numerical results showed that the optical IRS can potentially reduce MSE and BER at near-IRS user locations and the proposed AO algorithm outperforms other baselines.

**APPENDIX**

A. Global Optimal Point of the Lagrangian Function

In pursuing the dual function based on (58), the minimum value of the Lagrangian function \( \mathcal{L}(\mathbf{V}^*, \mu_1, \mu_2, \mu_3) \) can be attained by setting \( \nabla_{\mathbf{V}} \mathcal{L}(\mathbf{V}^*, \mu_1, \mu_2, \mu_3) = 0 \), which leads to a globally optimal solution since \( \mathcal{L}(\mathbf{V}^*, \mu_1, \mu_2, \mu_3) \) is in a quadratic form of vec\((\mathbf{V})\). According to the chain rule of matrix differentiation, the derivative is further derived as

\[
\nabla_{\text{vec}(\mathbf{V})} \mathcal{L} = \frac{\partial \text{vec}(\mathbf{H})^T}{\partial \text{vec}(\mathbf{V})} \frac{\partial \text{MSE}}{\partial \text{vec}(\mathbf{H})} + \text{vec}(\mu_1 1_{1 \times N_t N_r}) - \mu_2 + \mu_3, \tag{73}
\]

where the second term holds due to the equation \( \nabla_{\text{vec}(\mathbf{V})}(\alpha^T \mathbf{V} b) = \text{vec}(ab^T) \). As for the first term, the MSE metric can be re-expressed as the sum of a quadratic term (40) and an affine term, and thereby its derivative with respect to vec\((\mathbf{H})\) can be expressed as

\[
\frac{\partial \text{MSE}}{\partial \text{vec}(\mathbf{H})} = \left( \mathbf{U} + \mathbf{U}^T \right) \text{vec}(\mathbf{H}) - 2 \text{vec}(\mathbf{Q}^T \mathbf{R}_x \mathbf{W}^T), \tag{74}
\]

where the equation is satisfied due to \( \nabla_{\mathbf{H}} \text{tr}(\mathbf{C} \mathbf{H}) = \mathbf{C}^T \) and \( \mathbf{U} \) is introduced as

\[
\mathbf{U} = \left( \mathbf{W}^T \mathbf{R}_x \mathbf{W}^T \right) \otimes \left( \mathbf{Q}^T \mathbf{Q} \right). \tag{75}
\]

Meanwhile, \( \mathbf{H} \) is related to \( \mathbf{V} \) through (11) and (43), and therefore its derivative with respect to vec\((\mathbf{V})\) results in a Jacobian matrix given by

\[
\frac{\partial \text{vec}(\mathbf{H})}{\partial \text{vec}(\mathbf{V})} = \text{diag}(\mathbf{H}^T). \tag{76}
\]
By substituting (74) and (76) into (73) and setting the derivative of the Lagrangian function to zero, the globally optimal solution can be obtained as 
\[
\text{vec}(V^*) = Z^{-1} \left\{ \text{diag}(H_c) \left[ 2 \text{vec} \left( Q(t) R_s W(t)^T \right) \right. \right.
\]
\[
\left. \left. - \left( U + U^T \right) \text{vec}(H_1) \right] - \text{vec}(\mu_1 1_{1 \times N_r}) + \mu_2 - \mu_3 \right\}, \quad (77)
\]
where $Z$ is a constant matrix given by 
\[
Z = \text{diag}(H_c) \left( U + U^T \right) \text{diag}(H_c)^T. \quad (78)
\]

B. Gradient Derivation of the Dual Function

Given that $g(\mu_1, \mu_2, \mu_3) = \mathcal{L}(V^*, \mu_1, \mu_2, \mu_3)$ and $V^*$ is with respect to $\mu_1$, $\mu_2$, and $\mu_3$ according to (77), the gradient of the dual function is related to both the MSE metric and the Lagrangian multipliers. Therefore, the dual function needs to compute the partial derivative of the vector $\mu_1$, which leads to the following result,
\[
\nabla_{\mu_1} g = \frac{\partial \text{vec}(V^*)}{\partial \mu_1}^T \left[ \frac{\partial \text{MSE}(V^*)}{\partial \text{vec}(V^*)} + \text{vec}(\mu_1 1_{N_r \times N_r}) \right.
\]
\[
\left. - \mu_2 + \mu_3 \right] + V^* 1_{N_r \times 1} - 1_{N \times 1}, \quad (79)
\]
where the Jacobian matrix is expressed as 
\[
\frac{\partial \text{vec}(V^*)}{\partial \mu_1} = -Z^{-1} (1_{N_r \times 1} \otimes I_N). \quad (80)
\]
Similarly, Jacobian matrices for the other two multipliers can be obtained as 
\[
\frac{\partial \text{vec}(V^*)}{\partial \mu_2} = Z^{-1}, \quad (81)
\]
\[
\frac{\partial \text{vec}(V^*)}{\partial \mu_3} = -Z^{-1}, \quad (82)
\]
which are both symmetric since $(Z^{-1})^T = Z^{-1}$. Consequently, the partial derivatives with respect to $\mu_2$ and $\mu_3$ are given by 
\[
\nabla_{\mu_2} g = Z^{-1} \left[ \frac{\partial \text{MSE}(V^*)}{\partial \text{vec}(V^*)} + \text{vec}(\mu_1 1_{N_r \times N_r}) \right.
\]
\[
\left. - \mu_2 + \mu_3 \right] - \text{vec}(V^*) \quad (83)
\]
and
\[
\nabla_{\mu_3} g = -Z^{-1} \left[ \frac{\partial \text{MSE}(V^*)}{\partial \text{vec}(V^*)} + \text{vec}(\mu_1 1_{N_r \times N_r}) \right.
\]
\[
\left. - \mu_2 + \mu_3 \right] + \text{vec}(V^*) - 1_{N \times N_r \times 1}. \quad (84)
\]
respectively, where the derivative of MSE with respect to vec($V^*$) is obtained by (74) and (76).
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