THE AVERAGE MASS PROFILE OF GALAXY CLUSTERS

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ABSTRACT

The average mass density profile measured in the Canadian Network for Observational Cosmology cluster survey is well described with the analytic form \( \rho(r) = a_2/\sqrt{a_1 + r} \), as advocated on the basis of n-body simulations by Navarro, Frenk, & White. The predicted core radii are \( a_2 = 0.20 \) (in units of the radius where the mean interior density is 200 times the critical density) for an \( \Omega = 0.2 \) open cold dark matter model and \( a_2 = 0.26 \) for a flat \( \Omega = 0.2 \) model, with little dependence on other cosmological parameters for simulations normalized to the observed cluster abundance. The dynamically derived local mass-to-light ratio, which has little radial variation, converts the observed light profile to a mass profile. We find that the scale radius of the mass distribution, \( 0.20 \leq a_1 \leq 0.30 \) (depending on modeling details, with a 95% confidence range of 0.12–0.50), is completely consistent with the predicted values. Moreover, the profiles and total masses of the clusters as can be acceptably predicted from the cluster rms line-of-sight velocity dispersion alone. This is strong support for the hierarchical clustering theory for the formation of galaxy clusters in a cool, collisionless, dark-matter–dominated universe.

Subject headings: galaxies: clusters: general — large-scale structure of universe

1. INTRODUCTION

A fundamental prediction of the hierarchical clustering paradigm in a collisionless dark-matter–dominated universe is that the average mass profile of dark halos must primarily depend on mass, if the density perturbation spectrum is nearly a power law. The single-parameter surface brightness profile, \( S \propto \exp(-r^{1/4}) \), was proposed by de Vaucouleurs (1948) to describe elliptical galaxies, but it was later found also to be quite an accurate description of isolated n-body collapse simulations, both with and without dissipation (van Albada 1982; Carlberg, Lake, & Norman 1986). This \( r^{1/4} \) “law” has no simple analytic form to describe the volume distribution (Young 1976), which motivated Hernquist (1990) to propose the function \( p_0(r) = 2 \pi a_r^2 \rho(r) r_0^3 \) as a more tractable alternative, which in projection is close to the \( r^{1/4} \) function.

In cosmological halo formation simulations, where external tides and continuing infall are included, the central density profile is found to remain approximately \( r^{-1} \) (Dubinski & Carlberg 1991; Carlberg 1994; Syer & White 1996), although there is growing evidence that the central slope is somewhat steeper (Fukugishie & Makino 1997; Evans & Collett 1997). However, beginning near the virialization radius, the density profile is found to fall somewhat less steeply than in isolated collapses, which led Navarro, Frenk, & White (1996, 1997; the latter is hereafter referred to as NFW) to advocate \( \rho(r) = a_2/r^{2 + a} \) as an empirical description of the halo density profiles. In a series of exhaustive n-body simulations, the relation between the scale radius, \( a_1 \), and the mass has been studied to yield an empirical understanding of its value (Cole & Lacey 1996; NFW). The principal result of interest here is that the scale radius, \( a_1 \), is measured in the simulations to be about 20% of the “virial radius” for the \( 10^{13} M_\odot \) dark halos appropriate to rich clusters. This result has only a weak dependence on either the power spectrum or the world model parameters \( \Omega, A \) for simulations normalized to the observed cluster abundance. Halos of much lower mass, say, the \( 10^{12} M_\odot \) characteristic of individual galaxies, are generally much more centrally concentrated than clusters, with quite a strong dependence on the cosmological details.

In the next section we fit the galaxy number density profile, \( n(r) \), to the NFW function and then present evidence that this profile (specifically for galaxies selected in Gunn r, which as a red passband is relatively insensitive to the current star formation rate) accurately traces the average cluster mass profile, \( \rho(r) \). In particular, we compare the derived surface mass profile with the surface galaxy number profile. Our analysis is independent of the “light traces mass” assumption and allows for a range of velocity dispersion profiles. We have previously shown that this type of analysis works to recover the same mass profile from two vastly different subsamples of the data (Carlberg et al. 1997c). The fitted scale radius of the NFW model is compared to the predicted scale radius. In § 3, we use the scaling from velocity dispersion to radius to predict the form of the profile for each cluster in turn and compare the prediction with the available data. The results are briefly discussed in the final section. We use \( H_0 = 100 \) km s\(^{-1}\) Mpc\(^{-1}\) and \( \Omega_0 = 2q_0 = 0.2 \) for all our calculations, although the results are not very sensitive to these choices.
2. THE AVERAGE MASS PROFILE OF A GALAXY CLUSTER

The Canadian Network for Observational Cosmology (CNOC) cluster survey (Carlberg et al. 1994; Yee, Ellingson, & Carlberg 1996) obtained \( \sim 2600 \) redshifts of Gunn \( r \)-selected galaxies in the fields of 16 high-luminosity X-ray clusters at \( z \approx 1/3 \). This is a relatively homogeneous sample of clusters that are guaranteed to be at least partially virialized on the basis of their X-ray emission. The virial mass-to-light ratios of these clusters were found to be identical, within their measurement errors of \( \sim 25\% \) (Carlberg et al. 1996). The 14 clusters that are “nonbinary” are combined in normalized coordinates to make an ensemble cluster. This diminishes substructure and asphericity to a level at which the galaxies can be treated as if in a spherical distribution that is consistent with dynamical equilibrium (Carlberg et al. 1997c). Galaxies of all colors above a \( k \)-corrected Gunn \( r \) absolute magnitude of \(-18.5\) are included in the average.

2.1. The Average Cluster Number Density Profile

To combine the clusters, the brightest cluster galaxy (BCG) is used as the nominal center of each cluster on the sky following our earlier procedures (Carlberg, Yee, & Ellingson 1997b, hereafter CYE). The galaxy velocities are normalized to \( \sigma_t \), the rms velocity dispersion of the clusters about the cluster mean. The projected radii are normalized to an empirically determined \( r_{200} \), the radius at which the mean interior overdensity is \( 200 \rho_0 \). To derive \( r_{200} \) from the observational virial radius, \( r_v \) (which is largely fixed by the outer boundary of the sample), we assume that \( M(r) \propto r \). This gives

\[
r_{200} = \sqrt[3]{\frac{\sigma_v}{10 H(z)}},
\]

which is completely independent of the observational virial radius. The extrapolation in radius is usually a modest \( 25\% \), so the precise mass profile assumed makes little difference to the result (CYE).

The average projected galaxy number density profile, \( \Sigma(R) \), is fit with the projection of the volume density function,

\[
\nu(r) = \frac{A}{(r + a)^p},
\]

where \( p \) is fixed at either \( p = 2 \) (NFW) or \( p = 3 \) (Hernquist). The results for \( p = 2 \) are shown in Figure 1 as the solid line. The dashed lines are described below. Both the \( p = 2 \) and \( p = 3 \) forms are statistically acceptable fits by the \( \chi^2 \) test. We will consider only the \( p = 2 \) form for the rest of this paper. The fitted scale radius is \( a_v = 0.27 \), with a 95% confidence range of 0.13–0.43 from the \( \chi^2 \) distribution.

2.2. The Relationship between the Mass and Number Density Profiles

There is no dynamical necessity for \( \nu(r) \) to be directly proportional to \( \rho(r) \). The relationship between the two is derived from the projected velocity dispersion, \( \sigma_v(R) \). The dynamical mass profile, \( M_v(R) \), which is the volume integral of \( \rho(r) \), is inferred from the Jeans equation (see, e.g., Binney & Tremaine 1987; CYE),

\[
M_v(R) = \frac{-\sigma^2}{G} \left( \frac{d \ln \sigma^2}{d \ln r} + \frac{d \ln \nu}{d \ln r} + 2\beta \right),
\]

where the velocity anisotropy parameter is \( \beta = 1 - \sigma^2/\sigma_v^2 \). N-body simulations for a variety of cosmologies show that the dependence of \( \beta \) has a nearly universal radial variation (Cole & Lacey 1996), which we somewhat arbitrarily model as

\[
\beta(r) = \beta_m \frac{4r}{r^2 + 4},
\]

which takes on a maximum value of \( \beta_m \) at \( r = 2 \), in units. We demonstrate below that the volume mass density is not very sensitive to the details of the velocity modeling, provided the model is an acceptable statistical description of the data. We use values of 0.3 and 0.5 for the parameter \( \beta_m \), which roughly bracket the range of \( \beta \) seen in the simulations.

We model the radial velocity dispersion as

\[
\sigma^2 = B \frac{cr/(1 + cr) + c_2}{1 + r/b},
\]

where \( B \) and \( b \) are the two parameters adjusted to fit the observed \( \sigma_v(R) \). We include here a new central data point at \( 0.05r_{200} \). The rest of the data are as in CYE. The \([c_1, c_2]\) parameters are externally fixed to allow us to vary the shape of the curve. In Figure 2 we display results in which \([c_1, c_2]\) are \([0, 1] \) and \([1, 0] \) to give the two extremes of the functional form. The \([0, 1] \) form was used in CYE, and \([1, 0] \) gives a dispersion that drops to zero at the center. The intermediate values \([8, 1/2] \) are adopted for reasons made clear below. Both the \([1, 0] \) and \([8, 1/2] \) forms have reduced \( \chi^2 \) near unity. The \([0, 1] \) form is about 4% probable, although this is critically dependent on the 0.05\( r_{200} \) point, the concern being that the old central galaxies may be a distinct, low-velocity-dispersion cluster population.
FIG. 2.—Projected velocity dispersion profile and the projection of the fitted profiles for both $r_{\text{vir}} = 0.3$ and 0.5, which are nearly indistinguishable on this plot. The dotted line is for $[c_1, c_2] = [0, 1]$, the dashed line is for $[1, 0]$ and the solid line is for $[8, 1/2]$ (see text for details).

We show the results of the mass analysis in Figure 3. The quantity plotted is $\rho(r)/\nu(r)$ normalized to $(M/L)L(r)$, computed from the same data. The ratio $\rho(r)/\nu(r)$ is only weakly constrained inside 0.1$r_{200}$, and the assumption of virialization likely fails outside 1.5$r_{200}$. Within this radial range, $\rho(r)/\nu(r)$ varies less than about 30% of its mean value. It should be noted that for most purposes the more relevant quantity is $M_{\text{d}}(r)$, the volume integral of $\rho(r)$, which is substantially more slowly varying. The offset of $\rho(r)/\nu(r)$ below unity indicates that mass and light are similarly distributed but that the virial mass has a scale error. A wide range of positive intermediate values of $c_1$ and $c_2$ gives nearly constant $\beta_\text{m}$ everywhere. We display the $c_1 = 8$, $c_2 = 1/2$ case, which is one of many intermediate sets of parameters that leads to substantial cancellation of the opposite behavior of the two extreme $\sigma(r)$ models in the derived $M_{\text{d}}(r)$.

With the dynamically derived $\rho(r)/\nu(r)$ in hand, we can use it to model the galaxy number profile, $\nu(r)$, assuming that the mass profile is described by the NFW profile. This procedure measures $d_\nu$, the scale radius of the mass distribution. We take $\rho(r)/\nu(r)$ to be equal to the endpoint of the range for $r < 0.1r_{200}$ and $r > 1.5r_{200}$. The results for our three velocity dispersion functions and the two values of $\beta_\text{m}$ are shown in Figure 1 as the dashed lines. The resulting $d_\nu$ values range from 0.20 ($[c_1, c_2] = [0, 1]$) to 0.30 ($[1, 0]$). This expands the 95% confidence range of $d_\nu$ over $a_\nu$ slightly, to 0.12 $\leq a_\nu \leq 0.50$. We conclude that galaxies selected from their red light brightness, i.e., a passband well to the red of the 4000 Å break to minimize sensitivity to star formation (and not necessarily galaxies red in color; Carlberg et al. 1997c), accurately trace the mass profile of the cluster. That is, $\nu(r)$ is directly proportional to $\rho(r)$, within the statistical limits of our measurements.

3. INDIVIDUAL CLUSTER PROFILES

The NFW profile can be tested for its application to the clusters as individuals. The $r_{200}$ for each cluster is derived from the observed velocity dispersion, equation (1). The NFW scale radius is set at our best-fit value, $a_\nu = 0.27$. The fits are restricted to the region inside 1.5$r_{200}$, which is expected to be virialized for $\Omega \approx 0.2$. The resulting $\chi^2$ per degree of freedom (there being 10–16 degrees of freedom) are given in Table 1. The cluster MS 0906+11, which we observed, is excluded because it was not possible to measure a reliable velocity dispersion, which we believe is likely because there is another nearby clusters in the redshift direction (Carlberg et al. 1996). The reduced $\chi^2$ values of 14 of the 15 clusters are 1.1 or less, indicating that the NFW function with a fixed scale radius is a statistically acceptable fit to the density profile. The A2390 cluster has $\chi^2 = 1.38$ per degree of freedom, which indicates that the fit is about 15% probable. A2390 is the cluster with the

| Name            | $\chi^2/\nu$ | Number of Cluster Redshifts |
|-----------------|--------------|----------------------------|
| A2390           | 1.38         | 178                        |
| MS 0016+16      | 0.49         | 47                         |
| MS 0302+16      | 0.88         | 26                         |
| MS 0440+02      | 0.62         | 37                         |
| MS 0451+02      | 0.76         | 114                        |
| MS 0451-03      | 0.83         | 50                         |
| MS 0839+29      | 0.98         | 45                         |
| MS 1006+12      | 0.50         | 28                         |
| MS 1008-12      | 0.70         | 67                         |
| MS 1224+20      | 1.10         | 24                         |
| MS 1231+15      | 0.69         | 76                         |
| MS 1358+62      | 0.57         | 171                        |
| MS 1455+22      | 0.69         | 55                         |
| MS 1512+36      | 0.48         | 38                         |
| MS 1621+26      | 0.88         | 96                         |

TABLE 1

GOODNESS OF FIT OF A UNIVERSAL PROFILE

$r / r_{200}$

Fig. 3.—Derived ratio of the dynamical mass density profile, $\rho(r)$, to $r$-selected galaxy profile, $\nu(r)$, normalized with the virial mass-to-light ratio evaluated inside 1.5$r_{200}$. In each pair of curves, the upper line at small radius is for $\beta_\text{m} = 0.3$ and the lower is for $\beta_\text{m} = 0.5$. The dotted line is for $c_1 = 0$, the dashed line is for $c_2 = 0$, and the solid line is our preferred $c_1 = 8$, $c_2 = 1/2$. A2390

The NFW profile can be tested for its application to the clusters as individuals. The $r_{200}$ for each cluster is derived from the observed velocity dispersion, equation (1). The NFW scale radius is set at our best-fit value, $a_\nu = 0.27$. The fits are restricted to the region inside 1.5$r_{200}$, which is expected to be virialized for $\Omega \approx 0.2$. The resulting $\chi^2$ per degree of freedom (there being 10–16 degrees of freedom) are given in Table 1. The cluster MS 0906+11, which we observed, is excluded because it was not possible to measure a reliable velocity dispersion, which we believe is likely because there is another nearby clusters in the redshift direction (Carlberg et al. 1996). The reduced $\chi^2$ values of 14 of the 15 clusters are 1.1 or less, indicating that the NFW function with a fixed scale radius is a statistically acceptable fit to the density profile. The A2390 cluster has $\chi^2 = 1.38$ per degree of freedom, which indicates that the fit is about 15% probable. A2390 is the cluster with the
most data. If we had about a third less data for this cluster, the fit would have been entirely acceptable. However, as the amount of cluster data increases beyond about 100 galaxies, the substructure in the cluster (Abraham et al. 1996) becomes more clearly detected. By extension, if we had obtained more than 200 or so galaxies in any cluster, the substructure would have become clearly resolved. This sampling strategy was part of the initial layout of the CNOC observations.

4. DISCUSSION AND CONCLUSIONS

We have established that the dynamical mass profile, \( \rho(r) \), is proportional to the galaxy number density profile, \( n(r) \), over the range of radii at which the argument is secure. We further make the modest assumption that \( n(r) \) continues to trace the mass, within the errors, at larger radii. Consequently, we conclude that the NFW function accurately describes the mass field. The measured scale radius is \( 0.20 \leq a_r \leq 0.30 \), with a 95% confidence range of 0.12–0.50. This relatively large confidence range is a result of the relatively small change in profile slope at the characteristic scale for the NFW profile. The Hernquist function has much smaller errors when used in the same fitting procedure. The predicted values for \( \Omega = 0.2 \) and a cold dark matter spectral normalization of \( \sigma_8 = 0.95 \) are 0.20 for an open model (nearly identical to the \( \sigma_8 = 0.6, \Omega = 1 \) value of 0.19) and 0.26 for a flat low-density model (NFW), which are remarkably precise predictions of the observed value.

Overall, it appears that a very good understanding of the evolution of the mass field in clusters is emerging. The value of \( \Omega \) determined from dynamical observations of clusters (Carlberg et al. 1996; CYE) predicts a slow change in the abundance of clusters with redshift, which is in accord with the directly observed value (Carlberg et al. 1997a). Previously, we found that clusters as individuals have masses inside \( r_{200} \) that are accurately predicted from their velocity dispersions (Carlberg et al. 1996). Here we have shown that the predicted scale radius and the universal NFW profile provide a statistically acceptable description of the virialized region, \( r \leq 1.5r_{200} \), of 14 of the 16 clusters selected for the CNOC observational program, with the failures likely being the result of large substructures. The combination of successful predictions of galaxy cluster mass profiles and their cosmological number density evolution is a strong argument for the model of cluster formation in an effectively cold, collisionless, dark-matter–dominated universe. These results could be substantially tightened, in the first instance, through redshift determinations for more cluster galaxies in the outskirts of this sample.

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REFERENCES

Abraham, R. G., et al. 1996, ApJ, 471, 694
Binney, J., & Tremaine, S. 1987, Galactic Dynamics (Princeton: Princeton Univ. Press)
Carlberg, R. G. 1994, ApJ, 433, 468
Carlberg, R. G., Lake, G. L., & Norman, C. A. 1986, ApJ, 300, L1
Carlberg, R. G., Morris, S. L., Yee, H. K. C., & Ellingson, E. 1997a, ApJ, 479, L19
Carlberg, R. G., Yee, H. K. C., & Ellingson, E. 1997b, ApJ, in press (CYE)
Carlberg, R. G., Yee, H. K. C., Ellingson, E., Abraham, R., Gravel, P., Morris, S. L., & Pritchet, C. J. 1996, ApJ, 462, 32
Carlberg, R. G., et al. 1994, JRASC, 88, 39
Carlberg, R. G., et al. 1997c, ApJ, 476, L7
Cole, S., & Lacey, C. G. 1996, MNRAS, 281, 716
de Vaucouleurs, G. 1948, Ann. d’Astrophys. 11, 247
Dubinski, J., & Carlberg, R. G. 1991, ApJ, 378, 496
Evans, N. W., & Collett, J. L. 1997, MNRAS, submitted (astro-ph/9702085)
Fukushige, T., & Makino, J. 1997, ApJ, 477, L9
Hernquist, L. 1990, ApJ, 356, 359
Navarro, J. F., Frenk, C. S., & White, S. D. M. 1996, ApJ, 462, 563
———. 1997, ApJ, submitted (astro-ph/9611107) (NFW)
Syer, D., & White, S. D. M. 1996, MNRAS, submitted (astro-ph/9611065)
van Albada, T. S. 1982, MNRAS, 201, 939
Yee, H. K. C., Ellingson, E., & Carlberg, R. G. 1996, ApJS, 102, 269
Young, P. J. 1976, AJ, 81, 907