Predicating magnetorheological effect of magnetorheological elastomers under normal pressure

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Abstract. Magnetorheological elastomers (MREs) present reversible change in shear modulus in an applied magnetic field. For applications and tests of MREs, a normal pressure must be applied on the materials. However, little research paid attention on the effect of the normal pressure on properties of MREs. In this study, a theoretical model is established based on the effective permeability rule and the consideration of the normal pressure. The results indicate that the normal pressure have great influence on magnetic field-induced shear modulus. The shear modulus of MREs increases with increasing normal pressure, such dependence is more significant at high magnetic field levels.

1. Introduction

Magnetorheological (MR) effect is the magnetic field-induced change of rheological properties that exhibited by MR materials. The effect was first discovered by Rabinow in 1948 [1]. Magnetorheological fluids (MRFs) prepared by dispersing large amounts of micro-sized magnetic particles in a non-magnetic liquid are the most common MR materials [2]. They show a reversible and fast transition from a liquid to a nearly solid state when a magnetic field is applied. Therefore, their yield stress and apparent viscosity can be changed by changing magnetic field. This unique property has made them apply in mechanical systems and civil engineering structures that require the transmission of torque or the control of vibration [3-6]. However, the particles tend to settle out in the liquid due to the density mismatch between the particles and the liquid, and the leakage of the fluids must be avoided by seal during their service [7]. Those shortcomings prevent them from certain applications.

Magnetorheological elastomers (MREs) are solid analogs of MR fluids. The most significant difference between MREs and MRFs is that the matrix of MREs is made of natural or synthetic rubber rather than a liquid. As a new branch of MR materials, the shear modulus of MREs can be controlled rapidly and reversibly by the application of an external magnetic field [8-13]. Compared with MRFs, MREs present several merits. For example, the magnetic particles in MREs do not undergo sedimentation, and the devices based on MREs do not need to be sealed. Those advantages make MREs to be widely used in the field that need controllable stiffness, such as automotive bushings,

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tuneable stiffness engine mounts, variable impedance surfaces, and adaptive tuned vibration absorbers [14-16]. MREs can be divided into isotropic and anisotropic based on different curing conditions. The anisotropic MREs are prepared by applying a magnetic field during the cure process of the composites, which gives chain-like columnar structures for the particles in the matrix. Generally, the MR effect of the anisotropic MREs is larger than the isotropic ones, the particles in which are homogeneously distributed [17].

Since 1995, many researchers predicted the properties (especially the field-induced modulus) of MREs by theoretical models. Most of the models are established based on the magnetic dipole interactions between two adjacent particles of the chain. Jolly established a model based on the dipolar interaction of particles within an assumed structure, and the magnetic non-linearities as applied field increase was taken into consideration [18]. Davis used finite element analysis method to predict the shear modulus under a varied magnetic field, the results of which indicated the increase in shear modulus due to inter-particle magnetic forces at saturation is about 50% of the zero-field modulus, and the optimum particle volume fraction for the largest relative MR effect at saturation is 27% [19]. Shen et al. took all the dipole interaction in a chain and the nonlinear properties of the host composite into account, and established a mathematical model to predict the stress-strain relationship of MREs, which agree well with experimental data [20]. Deng and Gong modified the traditional magnetic dipole model by taking the influences of particles in the same chain and the particles in all chains into consideration to compute the magnetic-induced shear modulus of MREs [21]. Recently, Zhang et al. proposed an effective permeability model which taken into account the particle’s saturation to predict the properties of MREs with complex structure and components [22]. However, all of those models ignored the effect of the normal pressure on properties of MREs. In fact, a normal pressure must be applied on MREs when they are used or tested, which can not be overlooked; it influences the distance between adjacent particles, and eventually affects the magnetic field-induced shear modulus.

In this study, a model is established based on the effective permeability rule and the consideration of the normal pressure, which is used to predict the magnetorheological properties of MREs under different normal pressure.

2. Theoretical analysis
The chain-like structural construction of an anisotropic MRE is shown in figure1, which contains two domains. One domain is the chains composed by magnetic particles and matrix among them, another one is the pure matrix composed by elastomers.

![Figure 1. Chain-like structural construction of anisotropic MRE.](image)

For the chains, the effective permeability $\mu_{\text{eff}}$ can be calculated by using Maxwell Garnett mixing rule as,
\[ \mu_{\text{eff}} = \mu_m + 2\phi\mu_m \frac{\mu_p - \mu_m}{\mu_p + \mu_m - \phi(\mu_p - \mu_m)} \]  

(1)

where \( \mu_p \) and \( \mu_m \) are effective permeability of the particles and the matrix, respectively. \( \phi \) is the particles volume fraction in chains. Based on the hypothesis that all the particles are sphere with the same radius \( R \), and the distance between two adjacent particles in a chain is \( d \), \( \phi \) can be expressed as,

\[ \phi = \frac{4R}{3d} \]  

(2)

Based on parallel-connection rule, the effective permeability \( \mu_{\text{eff}} \) along the direction of chains is calculated as,

\[ \mu_{\text{eff}} = \mu_{\text{eff}} \phi_c + \mu_m (1 - \phi_c) \]  

(3)

where \( \phi_c \) is the volume fraction of the chains in the MRE, which can be calculated as,

\[ \phi_c = \frac{\phi_p}{\phi} = \frac{3d\phi_p}{4R} \]  

(4)

where \( \phi_p \) is the particles volume fraction of the MRE. Then the effective permeability along the direction of chains can be obtained by substituting equation (1) and equation (4) into equation (3) as,

\[ \mu_{\text{eff}} = \mu_{\text{eff}} \phi_c + \mu_m (1 - \phi_c) = \mu_m + 2\phi_p\mu_m \frac{\mu_p - \mu_m}{\mu_p + \mu_m - \frac{4R}{3d}(\mu_p - \mu_m)} \]  

(5)

For shear mode, the magnetic field-induced shear stress can be expressed as,

\[ \tau = -\frac{1}{2} \mu_0 \frac{\partial \mu_{\text{eff}}}{\partial \gamma} H_0^2 \]  

(6)

where \( \mu_0 \) is the vacuum permeability, \( H_0 \) is the external magnetic field, and \( \gamma \) is the shear strain. Figure 2 is the shear model of a representative particle chain with two particles; it can be obtained that,

\[ \gamma = \frac{x}{d} \]  

(7)

Then the magnetic field-induced shear stress can be obtained by substituting equation (5) and equation (7) into equation (6) as,

\[ \tau = 12 \mu_0 \mu_m \phi_p H_0^2 R \gamma \frac{1}{d} \frac{(\mu_p - \mu_m)^2}{3(\mu_p + \mu_m) - \frac{4R}{d}(\mu_p - \mu_m)^2} \]  

(8)

The magnetic field-induced shear modulus \( \Delta G \) can be expressed as,

\[ \Delta G = 12 \mu_0 \mu_m \phi_p H_0^2 R \gamma^2 \frac{1}{d} \frac{(\mu_p - \mu_m)^2}{3(\mu_p + \mu_m) - \frac{4R}{d}(\mu_p - \mu_m)^2} \]  

(9)

The equation 9 can not be simplified based on the assumption as \( \mu_p \gg \mu_m \). The reason is the permeability of the particles, which are carbonyl iron particles in most case, is less than 10. Furthermore, the relative permeability of carbonyl iron particles \( \mu_p \) is a function of the magnetic field.
The empirical equation used as the fitting function is given as,

$$\mu_p(H_0) = \frac{H_0(\mu_{p,\text{max}} - 1) + \mu_{p,\text{max}}M_s}{H_0(\mu_{p,\text{max}} - 1) + M_s}$$  \hspace{1cm} (10)$$

where $\mu_{p,\text{max}}$ is the largest relative permeability of particles, and $M_s$ is the saturation magnetization.

The dependence of the relative permeability of carbonyl iron particles on magnetic field is shown in figure 3. As it is shown, the empirical equation fits well with the experimental data, and the relative permeability of carbonyl iron particles decreases with increasing magnetic field intensity; especially, at high magnetic field levels, it approaches 2. Therefore, the assumption $\mu_p >> \mu_m$ is invalid.

Besides, there is polymer matrix between adjacent particles in chains, which indicates the assumption as $R/d \approx 1/2$ is invalid.

To simplify the equation (9), the relative distance between adjacent particles in chains $k$ is defined as the ratio of $d$ to $R$, i.e. $k = d/R$, then the magnetic field-induced shear modulus can be expressed as

$$\Delta G = \frac{12k\mu_p\mu_m\phi_pH_0^2(\mu_p - \mu_m)^2}{\gamma^2[3(\mu_p + \mu_m)k - 4(\mu_p - \mu_m)]^3}$$  \hspace{1cm} (11)$$

Since the Young’s modulus of the particles is much larger than that of the polymer matrix, the deformation of the particles under a normal pressure is negligible. Therefore, only the deformation of the polymer matrix is considered in calculating the adjacent particles distance under the normal pressure. The adjacent particle distance can be calculated by,

$$d = d_0(1 - \varepsilon) = d_0(1 - \frac{\sigma}{E_m})$$  \hspace{1cm} (12)$$

where $d_0$ is the adjacent particles distance under zero pressure, i.e. the initial adjacent particles distance, $\sigma$ is the applied normal pressure, and $E_m$ is the Young’s modulus of the polymer matrix. Based on the assumption that the particle size is not changed under the normal pressure, the relative distance between the adjacent particles in chains is expressed as,
where \( k_0 \) is the initial relative distance between adjacent particles in chains, which is larger than 2.

For an anisotropic MRE with 30\% carbonyl iron particles by volume fraction, its magnetic field-induced shear modulus under various normal pressures is calculated and shown in figure 4. As the magnetic field increases, the shear modulus increases. At the same magnetic field, the shear modulus calculated under high normal pressure is larger than those calculated under low normal pressure. The dependence of the magnetic field-induced shear modulus on normal pressure is further shown in figure 5. It is shown that the shear modulus increases with increasing normal pressure, and the influence of normal pressure on shear modulus at high magnetic field levels is more significant than that at low field levels.

![Figure 4](image1.png)  
**Figure 4.** Dependence of shear modulus on magnetic field under various normal pressures for MRE with 30\% carbonyl iron particles by volume fraction.

![Figure 5](image2.png)  
**Figure 5.** Dependence of the magnetic field-induced shear modulus on normal pressure for MRE with 30\% carbonyl iron particles by volume fraction.

### 3. Conclusion

To predict the influence of normal pressure on magnetorheological properties of anisotropic MREs, a theoretical model is proposed based on the effective permeability rule and the consideration of the normal pressure. The results indicate that the normal pressure have great influence on magnetic field-induced shear modulus; the shear modulus of MREs increases with increasing normal pressure, such dependence is more significant at high magnetic field levels.

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### References

[1] Rabinow J 1948 The Magnetic fluid clutch *AIEE Trans.* 67 1308–15 
[2] Vicente J, Klingenberg D J and Hidalgo-Alvarez R 2011 Magnetorheological fluids: a review *Soft Matter* 7 3701–8 
[3] Ginder J M and Davis L C 1994 Shear stress in magnetorheological fluids: role of magnetic saturation *Appl. Phys. Lett.* 65 3410–8 
[4] Carlson J D and Jolly M R 2000 MR fluid, foam and elastomer devices *Mechatronics* 10 555–69
Dyke S J, Spencer B F Jr and Carlson J D 1996 Modeling and control of magnetorheological dampers for seismic response reduction *Smart Mater. Struct.* **5** 565–75

Chiriac H and Stoian G 2010 Influence of particle size distributions on magnetorheological fluid performances *Journal of Physics: Conference Series* **200** 072095

Davis L C 1999 Model of magnetorheological elastomers *J. Appl. Phys.* **85** 3348–51

Shiga T, Okada A and Kurauchi T 1995 Magnetoviscoelastic behavior of composite gels *J. Appl. Polym. Sci.* **58** 787–92

Jolly M R, Carlson J D and Munoz B C 1996 A model of the behavior of magnetorheological materials *Smart Mater. Struct.* **5** 607–14

M Lokander and Stenberg B 2003 Performance of isotropic magnetorheological rubber material *Polymer Testing* **22** 245–51

Fan Y, Gong X, Xuan S, Zhang W, Zheng J and Jiang W 2011 Interfacial friction damping properties in magnetorheological elastomers *Smart Mater. Struct.* **20** 035007

Chen L, Gong X, Jiang W, Yao J, Deng H and Li W 2007 Investigation on magnetorheological elastomers based on natural rubber *J. Mater. Sci.* **42** 5483–9

Guan X, Dong X and Ou J 2008 Magnetostrictive effect of magnetorheological elastomer *J. Magn. Magn. Mater.* **320** 158–63

Xu Z, Gong X, Liao G and Chen X 2010 An active-damping-compensated magnetorheological elastomer adaptive tuned vibration absorber *J. Intel. Mat. Syst. Str.* **21** 1039–47

Dwivedy S K, Mahendra N and Sahu K C 2009 Parametric instability regions of a soft and magnetorheological elastomer cored sandwich beam *J. Sound Vib.* **325** 686–704

Ginder J M, Schlotter W F and Nichols M E 2001 Magnetorheological elastomers in tunable vibration absorbers *Proc. of SPIE* **4331** 103–10

Wu J, Gong X, Fan Y and Xia H 2010 Anisotropic polyurethane magnetorheological elastomer prepared through in situ polycondensation under a magnetic field *Smart Mater. Struct.* **19** 105007

Jolly M R, Carlson J D and Munoz B C 1996 A model of the behaviour of magnetorheological materials *Smart Mater. Struct.* **5** 607–14

Davis L C 1999 Model of magnetorheological elastomers *J. Appl. Phys.* **85** 3348–51

Shen Y, Golnaraghi M F and Heppler G R 2004 Experimental research and modelling magnetorheological elastomers *J. Intel. Mat. Syst. Str.* **15** 27–35

Deng H and Gong X 2007 Adaptive tuned vibration absorber based on magnetorheological elastomer *J. Intel. Mat. Syst. Str.* **18** 1205–10

Zhang X, Li W and Gong X 2008 An effective permeability model to predict field-dependent modulus of magnetorheological elastomers *Commun. Nonlinear Sci.* **13** 1910–6