On Holography in Brans-Dicke Cosmology*

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Abstract

The holographic bound in Brans-Dicke $k = 1$ matter dominated Cosmology is discussed. In this talk, both the apparent horizon and the particle horizon are taken for the holographic bound. The covariant entropy conjecture proposed by Bousso is also discussed.

1 Introduction

Dilaton field appears naturally both from the Kaluza-Klein compactification and the String spectrum. The simplest way to incorporate the scalar field as the spin-0 partner of spin-2 gravitational field is Brans-Dicke theory in which the gravitational coupling constant is replaced by a scalar field. In [1], the author considered the holographic bound in the region within the particle horizon for a general Brans-Dicke universe. The discussion for the case $k = 1$ matter dominated is not complete because there is no analytical solution for that case. In this paper, we will investigate the holographic bound for the $k = 1$ matter dominated Brans-Dicke universe in regions within particle horizon and apparent horizon. The particle horizon idea was first proposed by Fischler and Susskind [2]. Because the holographic bound in the region within the particle horizon is not satisfied for closed matter dominated universe, Bak and Rey used the apparent horizon to solve the problem [3]. However, Kaloper and Linde showed that the holographic bound in the regions within apparent horizon could be violated in the anti-De-Sitter space [4]. Bousso’s covariant entropy conjecture gave a means to select a null hypersurface starting from any 2 dimensional spacelike surface so that the holographic bound would be satisfied in the null hypersurface [5]. This holographic bound can be applied to general spacetime starting from any surface. A generalized version of this conjecture was proved by Flanagan, Marolf and Wald under some

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assumptions on the entropy flux \[6\]. Other cosmological entropy bounds were discussed in \[7\].

The Friedman-Roberson-Walker metric for \(k = 1\) is

\[
ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - r^2} + r^2 d\Omega^2 \right)
\]

\[
= -dt^2 + a^2(t)(d\chi^2 + \sin^2 \chi d\Omega^2),
\]

where \(r = \sin \chi\). Here I use \(c = \hbar = 1\). The standard cosmological equation and solutions are

\[
H^2 + \frac{1}{a^2} = \frac{8\pi G}{3} \rho,
\]

\[
\rho a^3 = \rho_0 a_0^3,
\]

\[
a(\eta) = \frac{a_{\text{max}}}{2} (1 - \cos \eta) = a_{\text{max}} \sin^2 (\eta/2),
\]

where the cosmic time \(\eta\) is \(d\eta = dt/a(t)\) and \(a_{\text{max}} = 8\pi G \rho_0 a_0^3/3\). Note that \(\eta \to \pi\), \(a \to a_{\text{max}}\). The particle horizon is

\[
\chi_{PH} = \int_0^t \frac{d\tilde{t}}{a(t)} = \eta.
\]

The apparent horizon is

\[
r_{AH} = \frac{1}{a(t) \sqrt{H^2 + 1/a^2(t)}} = |\sin(\eta/2)|,
\]

\[
\chi_{AH} = \eta/2.
\]

The idea of holographic bound is that the matter entropy inside a spatial region \(V\) does not exceed 1/4 of the area \(A\) of the boundary of that region measured in Planck units. From the metric (1), the holographic bound in a region with radius \(r = \sin \chi\) for closed universe is

\[
\frac{S}{GA/4} = \frac{\epsilon V}{GA/4} = \frac{\epsilon(2\chi - \sin 2\chi)}{Ga^2(t) \sin^2 \chi} \leq 1,
\]

where \(\epsilon\) is the constant comoving entropy density.

If we consider the spherical region inside the particle horizon \(\chi_{PH} = \eta\), then the holographic bound (8) becomes

\[
\frac{S}{GA/4} = \frac{\epsilon(2\eta - \sin 2\eta)}{Ga_{\text{max}}^2 \sin^4(\eta/2) \sin^2 \eta} \leq 1.
\]

It is obvious that the bound is violated when \(\eta \to \pi\). Therefore, the holographic bound proposed by Fischler and Susskind does not apply to the closed universe.
If we consider the spherical region inside the apparent horizon $\chi_{AH} = \eta/2$, the holographic bound (8) becomes

$$\frac{S}{GA/4} = \frac{\epsilon(\eta - \sin \eta)}{Ga_{\text{max}}^2 \sin^6(\eta/2)} \leq 1.$$ 

Therefore, the holographic bound is satisfied if it is satisfied initially. So the holographic bound proposed by Bak and Rey applies to the closed universe.

# 2 Brans-Dicke Cosmology

The Brans-Dicke Lagrangian in the Jordan frame is given by

$$L_{BD} = \sqrt{-g} \left[ \phi \tilde{R} - \omega \gamma_{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right] - L_m(\psi, \gamma_{\mu\nu}),$$

with $\langle \phi \rangle = 1/G$.

The cosmological equations are

$$H^2 + \frac{k}{a^2} + \frac{H \dot{\phi}}{\phi} - \frac{\omega}{6} \left( \frac{\dot{\phi}}{\phi} \right)^2 = \frac{8\pi}{3} \rho,$$

$$\ddot{\phi} + 3H \dot{\phi} = \frac{8\pi}{2\omega + 3} (\rho - 3p),$$

$$\rho a^3 = \rho_0 a_0^3.$$  

To solve the above equations for the case $k = 1$ and $p = 0$, we take the solutions for $k = 0$ as the initial conditions.

$$a(t_i) = t_i^p, \quad \phi(t_i) = \frac{4\pi}{2\omega + 3} \frac{4 + 3\omega}{t_i^q},$$

$$p = \frac{2 + 2\omega}{4 + 3\omega}, \quad q = \frac{2}{4 + 3\omega}.$$  

The holographic bound for the particle horizon is shown in Fig. 1. From Fig. 1, we see the bound is violated at later time. The holographic bound for the apparent horizon is shown in Fig. 2. From Fig. 2, we see that the bound is satisfied if it is satisfied initially.

In Einstein frame, the Brans-Dicke Lagrangian is

$$L = \sqrt{-g} \left[ -\frac{1}{2\kappa^2} \tilde{R} - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma \right] - L_m(\psi, e^{-\alpha\sigma} g_{\mu\nu}).$$

The above Lagrangian (15) is from Eq. (9) by the transformations

$$g_{\mu\nu} = e^{\alpha \sigma} \gamma_{\mu\nu},$$

$$\phi = \frac{1}{G} e^{\alpha \sigma},$$

and
Figure 1: The holographic bound for particle horizon in Jordan frame with $\omega = 10$ and $\omega = 1000$ respectively.

Figure 2: The holographic bound for apparent horizon in Jordan frame with $\omega = 10$ and $\omega = 1000$ respectively.

where $\kappa^2 = 8\pi G$, $\alpha = \beta \kappa$, and $\beta^2 = 2/(2\omega + 3)$. Remember that the Jordan-Brans-Dicke Lagrangian is not invariant under the above transformations (16) and (17).

The corresponding cosmological equations are

$$H^2 + \frac{k}{a^2} = \frac{\kappa^2}{3} \left( \frac{1}{2} \dot{\sigma}^2 + e^{-2\alpha \rho} \right),$$

(18)

$$\ddot{\sigma} + 3H\dot{\sigma} = \frac{1}{2} \alpha e^{-2\alpha \rho},$$

(19)

$$\dot{\rho} + 3H \rho = \frac{3}{2} \alpha \dot{\sigma} \rho.$$  

(20)

Here we consider the solutions for the case $k = 1$ and $p = 0$ only. With $8\pi G/3 = 1$, the
Initial conditions are

\[
a(t_i) = \left[ \frac{\sqrt{2}(18 + \alpha^2) t_i}{4\sqrt{18 - \alpha^2}} \right]^{12/(18 + \alpha^2)}, \quad \sigma(t_i) = \frac{\alpha}{3} \ln a(t_i).
\]

The holographic bound for the particle horizon is shown in Fig. 3. From Fig. 3, we see that the bound is violated at later time. The holographic bound for the apparent horizon is shown in Fig. 4. From Fig. 4, we see that the bound is satisfied if it is satisfied initially.

Figure 3: The holographic bound for particle horizon in Einstein frame with \( \omega = 10 \) and \( \omega = 1000 \) respectively.

Figure 4: The holographic bound for apparent horizon in Einstein frame with \( \omega = 10 \) and \( \omega = 1000 \) respectively.
3 Conclusions

The holographic bound for Brans-Dicke cosmology is not satisfied if we use the particle horizon, but it is satisfied if we use the apparent horizon. We can understand the above result by using Bousso’s conjecture: In any spacetime satisfying Einstein’s equation and the dominant energy condition, the total entropy $S$ contained in any null hypersurface $L$ bounded by some connected $(D-2)$ dimensional spacelike surface $B$ with area $A$ and generated by null geodesics with non-positive expansion must satisfy $S \leq A/4G$. In Einstein frame, the spacetime satisfies Einstein’s equation with $\rho_{\text{tot}} = e^{-2\alpha} \rho + \dot{\sigma}/2$ and $p_{\text{tot}} = e^{-2\alpha} p + \dot{\sigma}/2$. Therefore, the matter source satisfies the dominant energy condition. Bousso’s covariant entropy conjecture tells us that the holographic bound is satisfied in the region within the apparent horizon. In this paper, we found that the holographic bound is satisfied for the $k = 1$ matter dominated Brans-Dicke universe. This can be taken as an evidence to support Bousso’s conjecture.

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References

[1] Y. Gong, gr-qc/9909013, to be appear in Phys. Rev. D, (2000).
[2] W. Fischler and L. Susskind, hep-th/9806039.
[3] D. Bak and S. Rey, hep-th/9902173.
[4] N. Kaloper and A. Linde, hep-th/9904120, Phys. Rev. D 60, 103509 (1999).
[5] R. Bousso, hep-th/9905177, JHEP 9907, 004 (1999); R. Bousso, hep-th/9906022, JHEP 9906, 028 (1999).
[6] E.E. Flanagan, D. Marolf and R.M. Wald, hep-th/9908070, see also the talk given by R. Wald during this conference.
[7] S.K. Rama, Phys. Lett. B 457, 268 (1999); S.K. Rama and T. Sarkar, ibid, 450, 55 (1999); R. Easther and D. Lowe, Phys. Rev. Lett. 82, 4967 (1999).