Charge-odd correlation of lepton and pion pair production in
electron-proton scattering

A.I. Ahmadov

Institute of Physics, Azerbaijan National Academy of Science, Baku, Azerbaijan and
JINR-BLTP, 141980 Dubna, Moscow region, Russian Federation

Yu.M. Bystritskiy and E.A. Kuraev

JINR-BLTP, 141980 Dubna, Moscow region, Russian Federation

A. N. Ilyichev

National Scientific and Educational Centre of Particle and High Energy
Physics of the Belarusian State University, 220040 Minsk, Belarus

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Charge-odd correlation of the charged pair components produced at electron-proton scattering can measure three current correlation averaged by proton state. In general these type correlation can be described by 14 structure functions. We restrict here by consideration of inclusive distributions of a pair components, which is the light-cone projection of the relevant hadronic tensor. Besides we consider the point-like approximation for proton and pion. Numerical estimations show that charge-odd effects can be measured in exclusive $ep \rightarrow 2\pi X$ experiments.

I. INTRODUCTION

The real photon production in lepton nucleon scattering allow to extract Deep virtual Compton scattering (DVCS) amplitude describing the real hard photon emission by proton block. In such kind of experiments some additional information compared with one obtained
in Deep inelastic scattering (DIS) experiments can be extracted. Among them the important General Parton Distribution (GPD), which generalize the parton (gluon) distribution inside proton which is extracted from DIS experiments.

In this paper we propose to measure the odd part of DVCS \( \gamma p \rightarrow \gamma^* p \) and \( \gamma p \rightarrow \rho^* p \), which is associated with interference of two types of processes with additional lepton

\[
e(p_-) + p(p) \rightarrow e(p'_-) + l^+(q_+) + l^-(q_-) + p(p'),
\]

or pion

\[
e(p_-) + p(p) \rightarrow e(p'_-) + \pi^+(q_+) + \pi^-(q_-) + p(p')
\]

pair production. Namely we consider the interference of amplitudes of pair creation by so called two-photon (Fig. 1 a)) or photon \( \rho \)-meson (Fig. 2 a)) and bremsstrahlung (Fig. 1 b)-e), Fig. 2 b)-d)) mechanisms. The last one describe emission of virtual photon (or vector meson) by proton with subsequent conversion to lepton or pion pair. To distinguish this mechanism from the other contributions to pair production processes in electron-proton
Fig. 2: Additional pion pair production in electron-proton scattering: a) photon bremsstrahlung and b)-d) bremsstrahlung mechanisms.

scattering we suggest to measure the charge-odd part of differential cross section [2]

\[
\frac{d\sigma}{d\Omega}^{_{e^{-}p\to e^{-}\pi^{+}\pi^{-}p}} = \sigma_{0} A^{\pi}(q_{+}, q_{-}) \frac{1}{M_{\rho}^{4}} dV_{\pm}
\]

\[
\frac{d\sigma}{d\Omega}^{_{e^{-}p\to e^{-}l^{+}l^{-}p}} = \sigma_{0} A^{l}(q_{+}, q_{-}) \frac{1}{m_{l}^{4}} dV_{\pm},
\]

with \(dV_{\pm}\) is the phase volume of the pair created and \(A^{\pi,l}(q_{+}, q_{-})\) is some asymmetry function

\[
A^{\pi,l}(q_{+}, q_{-}) = -A^{\pi,l}(q_{-}, q_{+}),
\]

which incorporate besides QED as well some information about three current correlation of the proton block.

Really the process belongs to ones of higher orders on the fine structure constant \(\alpha\) but it is highly enhanced in the Weizsaecker-Williams (WW) approximation, corresponding to the kinematics when the scattered electron moves to the direction close to the it’s initial one, which is partly compensate the extra power of \(\alpha\).

II. METHOD OF CALCULATION AND NUMERICAL RESULTS

Below we will consider the case when proton state is not enhanced, remains to be proton. The summed over the final and initial spin states the interference of matrix elements for
pion and lepton pair production have a form:

$$\Delta |M_\pi|^2 = 2M_\pi^a |M_\pi^b + M_\pi^c + M_\pi^d|^\dagger = \frac{32(4\pi\alpha)^2(g_\pi g_N)^2}{(q^2)^2(q_1^2 - M_\rho^2)(q_2^2 - M_\rho^2)} \times T_\mu\nu T_\eta^{P\mu\nu} T_\pi^{\mu\nu} T_\lambda^{\eta\nu} G^{\mu\nu\lambda\mu\nu\eta\lambda\eta}.$$  \hspace{1cm} (5)

$$\Delta |M_l|^2 = 2M_l^a |M_l^b + M_l^c|^\dagger = \frac{128(4\pi\alpha)^4}{(q^2)^2 q_1^2 q_2^2} T_\mu\nu T_\eta^{P\mu\nu} T_\pi^{\mu\nu} T_\lambda^{\eta\nu} G^{\mu\nu\lambda\mu\nu\eta\lambda\eta},$$  \hspace{1cm} (6)

with

$$G^{\mu\nu\lambda\mu\nu\eta\lambda\eta} = g^{\mu\mu_1} g^{\nu\nu_1} g^{\lambda\lambda_1} g^{\eta\eta_1},$$  \hspace{1cm} (7)

g_\pi \approx g_N = 10 are coupling constants of \(\rho\) meson with the charged pions and nucleons, the transferred momentum \(q = p_\pi - p'_\pi, q_1 = p_p - p'_p\), with \(p_\pi, p'_\pi\)-the 4 momenta of the scattered electron and proton, \(q_2 = q_+ + q_-\), \(q_\pm\) is the momenta of pair created. The contribution of the diagrams \(M_l^d\) and \(M_l^c\) are negligible since we consider the kinematics of proton fragmentation which is relevant to DVCS measurement.

The cross sections then

$$\Delta d\sigma = \frac{1}{8s} \Delta |M|^2 dV,$$  \hspace{1cm} (8)

where phase volume \(dV\) will be specified below.

The electron tensor have a form:

$$T_{\mu\nu}^e = \frac{1}{4} \text{Tr} \left[ \gamma_{\mu \hat{p}_-} \gamma_{\nu \hat{p}'_-} \right].$$  \hspace{1cm} (9)

Three index tensor with lepton pair is

$$T_{\mu_1\eta_1\lambda_1}^l = \frac{1}{4} \text{Tr} \left[ (-\hat{q}_+ + m)\gamma_{\lambda_1}(\hat{q}_- + m)\frac{\gamma_{\mu_1}(\hat{q}_- - \hat{q}_+ + m)\gamma_{\eta_1}}{D_-} + 
+ (\hat{q}_- + m)\gamma_{\lambda_1}(-\hat{q}_+ + m)\frac{\gamma_{\eta_1}(\hat{q}_- - \hat{q}_+ + m)\gamma_{\mu_1}}{D_+} \right],$$  \hspace{1cm} (10)

$$D_\pm = (q - q_\pm)^2 - m^2, \quad q_\pm^2 = m^2.$$

Proton three indices tensor contains the virtual Compton tensor \(O_{\lambda\sigma}\) and the vertex function of proton:

$$T_{\eta\lambda\nu_1}^P = \frac{1}{4} \text{Tr} \left[ (\hat{p}' + M)\Gamma_{\eta}(\hat{p} + M)O_{\lambda\nu_1} \right].$$
We will specify it for the case of point-like proton below. The three index tensor with charged pions is

\[ T_{\mu \nu \eta \lambda}^{\pi} = \left[ \frac{(2q_- - q)_\mu (q_1 - 2q_\perp)_\nu}{D_-} + \frac{(2q_- - q_1)_\eta (q - 2q_\perp)_\mu}{D_+} - 2g_{\mu \eta} \right] \times (q_1 - q_\perp)_\lambda. \]  

(11)

In the peripheral kinematics

\[ s = (p_- + p)^2 \gg q_2^2 \gg |q^2| \]  

(12)

(which provide the maximal contribution to the cross section) the substitution

\[ G^{\mu \nu \eta \lambda \mu \nu \eta \lambda} = (2/s)^4 p^\mu p^\nu p^\lambda p^\eta p_\mu p_\nu p_\lambda p_\eta, \]  

(13)

is valid. Throughout the paper we will use Sudakov’s [3, 4] (infinite momentum frame) parametrization of momenta of the problem:

\[ q_1 = \alpha_1 \bar{p} + \beta_1 \bar{p}_- + q_\perp, \]
\[ q = \alpha \bar{p} + \beta \bar{p}_- + q, \]
\[ q_\pm = \alpha_\pm \bar{p} + \beta_\pm \bar{p}_- + q_\perp; \]
\[ p' = \alpha' \bar{p} + \beta' \bar{p}_- + p_\perp, \]  

(14)

where we define

\[ q_\perp p_- = q_\perp p = 0, \quad q_\perp^2 = -q^2, \]  

(15)

and \( \bar{p}_-, \bar{p} \) are the light-like 4-vectors

\[ \bar{p}_- = p_- - p \frac{m^2}{s}, \quad \bar{p} = p - p_- \frac{M^2}{s}, \quad 2\bar{p}_- \bar{p} = s. \]  

(16)

We use below the on mass shell conditions

\[ s \alpha_\pm = \frac{1}{\beta_\pm} [q_\perp^2 + m^2], \quad s \beta' = \frac{1}{\alpha'} [p'_{\perp}^2 + M^2] \]  

(17)

and conservation law

\[ q_1 = q_+ + q_-; \quad \alpha_1 = \alpha_+ + \alpha_-; \quad \beta = \beta_+ + \beta_- \]  

(18)
(we use here the WW condition \(|\mathbf{q}| \ll |\mathbf{q}_\pm| \sim |\mathbf{q}_1|\) and the kinematic features of peripheric interaction. The relevant kinematic invariants are:

\[
q_1^2 = -q_1^2 = -(q_+ + q_-)^2;
\]

\[
q_2^2 = (q_+ + q_-)^2 = \frac{1}{x_+ x_-}[m^2 + (x_- q_+ - x_+ q_-)^2],
\]

\[
x_\pm = \frac{\beta_\pm}{\beta} = \frac{pq_\pm}{pq}, \quad 0 < x_\pm < 1, \quad x_+ + x_- = 1. \tag{19}
\]

Besides we use

\[
d^4q = \frac{s}{2} d\alpha d\beta d^2 \mathbf{q}. \tag{20}
\]

Factor \(G\) provides the light-cone projection of tensor product:

\[
T_{\mu\nu}^c T_{\eta\nu_1}^P T_{\mu_1\eta_1}^{\pi,l} G^{\mu\nu\lambda\eta\mu_1\nu_1\lambda_1\eta_1} = \left(\frac{2}{s}\right)^4 s^2 N_e N_{\pi,l} s^3 N^P \sim s^2, \tag{21}
\]

with finite in the limit \(s \to \infty\) quantities \(N_e, N^P, N_{\pi,l}\). We had convinced that the matrix element squared is proportional \(s^2\). Besides we will see that it is proportional to \(q^2\). This fact is the consequence of gauge invariance. The explicit calculations give:

\[
N_e = \frac{1}{4s^2} \text{Tr} \left[\hat{p} \hat{p}_- \hat{p}' \hat{p}'_-\right] = \frac{1}{2}. \tag{22}
\]

The quantity \(N^p\) we will calculate in point like proton approximation:

\[
N^p = \frac{1}{4s^3} \text{Tr} \left[ (\hat{p} + M) \left( \frac{\hat{p}_- (\hat{p} + \hat{q} + M) \hat{p}_-}{D} + \frac{\hat{p}_- (\hat{p}' - \hat{q} + M) \hat{p}_-}{D'} \right) (\hat{p}' + M) \hat{p}_- \right], \tag{23}
\]

with \(D = (p + q)^2 - M^2 \approx s\beta\) and \(D' = (p' - q)^2 - M^2 \approx -s\alpha'\beta + 2p'q\). The result is:

\[
N^p = \frac{\alpha'}{2} \left[ \frac{1}{D} + \frac{\alpha'}{D'} \right] = \frac{q_1 q_1}{(s\beta)^2}. \tag{24}
\]

The expression for \(N^l\) is

\[
N^l = \frac{1}{4s} \text{Tr} \left[ (-\hat{q}_+ + m) \hat{p} (-\hat{q}_- + m) \frac{\hat{p}_- (-\hat{q}_- - \hat{q} + m) \hat{p}_-}{D_-} + \right.
\]

\[
+ (\hat{q}_- + m) \hat{p} (-\hat{q}_+ + m) \frac{\hat{p} (\hat{q}_- - \hat{q}_+ + m) \hat{p}_-}{D_+} \right]. \tag{25}
\]
Using the current conservation conditions

\[ T^{\mu\nu}_{\mu\lambda} q^\lambda = T^{\mu\nu}_{\mu\lambda} q_1^\nu = 0 \]  \hfill (26)

and

\[ q \approx \beta \bar{p} - q_1, \]
\[ q_1 \approx \alpha_1 \bar{p} + q_1, \]

we can replace \( p_+ \rightarrow -q_1/\beta \) and \( \bar{p} \rightarrow -q_1/\alpha_1 \). Simple calculation lead to (we use \( D_\pm = -d_\pm/x_\pm, d_\pm = q_\pm^2 + m^2 \)):

\[ N^t = s \beta \left[ x_+ \frac{q_+ \cdot q}{d_+} - x_- \frac{q_- \cdot q}{d_-} \right] \left[ x_- (q_1 q_+)+ x_+ (q_1 q_-) \right], \]  \hfill (27)

For the case of pion pair production we have

\[ N^\pi = s \beta \left( x_+ - x_- \right) \left[ (d_+ - d_-) \left( x_- \frac{q_- \cdot q}{d_-} + x_+ \frac{q_+ \cdot q}{d_+} \right) + q_1 q_1 \right]. \]  \hfill (28)

The factor \( q_i q_j \) must be extracted, which after angular averaging turns to \( 1/2 \delta_{ij} q^2 \) and the rest part must be evaluated at \( q = 0 \). Next step consists in transformation of phase volume. Introducing as a variables the momentum transferred

\[ d^4 q_1 \delta^4 (p - q_1 - p') d^4 q \delta^4 (p_+ - p'_+ - q), \]

and using the Sudakov parametrization we transform the phase volume

\[ dV = (2\pi)^{4-12} \delta^4 (p_+ + p - p'_+ - q_+ - q_-) \frac{d^3 p'_+ d^3 p' d^3 q_+ d^3 q_-}{2 E'_- 2 E'_+ 2 E_- 2 E_+}, \]  \hfill (29)

as

\[ dV = \frac{(2\pi)^{-8} \pi^3 d^2 q d\beta}{8 s} \frac{d^3 p'_+}{\pi} \frac{d^3 p'}{\pi} \frac{d^3 q_+}{\pi} \frac{d^3 q_-}{\pi} dV_+, \]
\[ dV_+ = \frac{d^2 q_+ d^2 q_-}{\pi^2} \frac{dx_-}{x_+ x_-}. \]  \hfill (30)

Employing the form of the photon momentum squared by electron \( q^2 \):

\[ q^2 \approx - \left[ q^2 + \beta^2 m^2_e \right], \]  \hfill (31)

\[ \int_0^{Q^2} \frac{q^2 dq^2}{(q^2 + m^2_e \beta^2)^2} = \ln \left( \frac{Q^2 s^2}{m^2_e s^2_1} \right) - 2 \ln \left( s \beta/s_1 \right) - 1, \]  \hfill (32)
where $Q^2 \sim M_p^2$. Further integration on $s\beta > s_1 = q^2_+/x_+ + q^2_-/x_-$ leads to

$$\frac{1}{s_1} R = \int_{s_1}^{\infty} \left[ \ln \left( \frac{Q^2 s^2}{m^2 s_1^2} \right) - 2 \ln s\beta/s_1 - 1 \right] \frac{d(s\beta)}{(s\beta)^2} = \frac{1}{s_1} \left[ \ln \frac{Q^2 s^2}{m^2 s_1^2} - 3 \right]. \quad (33)$$

The final results for odd part of cross sections are:

$$\Delta d\sigma^{ep \rightarrow e\pi^+\pi^-} = \sigma_0^\pi \frac{A_\pi}{x_+x_-} \frac{x_+x_-dV_\pm}{M_p^4},$$

$$\sigma_0^\pi = \frac{\alpha^2 (g_\rho g_N)^2}{8\pi^3 M_p^2} = 1.4 \text{ nb};$$

$$A_\pi = (x_- - x_+) \frac{M_p^6 (q^2_2 - M_p^2) R}{(q^2_1 + M_p^2)((q^2_2 - M_p^2)^2 + M_p^2 \Gamma_2^2)} \times$$

$$\times \frac{1}{s_1 (d_+ - d_-) (x_- d_+ q_1 - x_+ d_- q_1) + d_+ d_- q_1^2}, \quad (34)$$

with

$$s_1 = \frac{d_+}{x_+} + \frac{d_-}{x_-}, \quad q^2_2 = \frac{1}{x_+x_-} \left( (x_- q_+ - x_+ q_-)^2 + M^2_\pi \right), \quad (35)$$

and

$$\Delta d\sigma^{ep \rightarrow e\nu\pi^-} = \sigma_0^l \frac{A_l}{x_+x_-} \frac{x_+x_-dV_\pm}{m^4}, \quad \sigma_0^l = \frac{8\alpha^4}{\pi m^2} = 0.2 \text{ nb} \quad \text{for} \quad m = M_\mu,$$

$$A_l = -\frac{m^6 R}{s_1 (d_+ - d_-) q_1 q_2} \left[ x_+ d_- (q_1 q_+) - x_- d_+ (q_1 q_-) \right] \times$$

$$\times \left[ x_- (q_1 q_+) + x_+ (q_1 q_-) \right]. \quad (36)$$

The functions $A_\pi/(x_+x_-), A_l/(x_+x_-)$ are drawn in Fig. 3, 4 for specific values of $q_\pm$ as a function of $x_+.$

### III. CONCLUSION

We investigate the light-cone projection of DVCS rank 3 tensor for specific processes of pair production. The value of charge-odd contributions to the cross sections (see (36), (34)) are sufficiently large to be measured. It can be measured in experiment with inclusive pion and lepton pairs detection.
Fig. 3: The asymmetry (defined in (34)) for pions pair production as a function of $\pi^+$ energy fraction $x_+$. Curve 1 corresponds to case then $|q_+| = |q_-| = 2M_\rho$, Curve 2 corresponds to case then $|q_+| = 2M_\rho$, $|q_-| = 2.5M_\rho$, Curve 3 corresponds to case then $|q_+| = 2M_\rho$, $|q_-| = 3M_\rho$.

Fig. 4: The asymmetry (defined in (36)) for muons pair production as a function of $\mu^+$ energy fraction $x_+$. Curve 1 corresponds to case then $|q_+| = |q_-| = 2m_\mu$, Curve 2 corresponds to case then $|q_+| = 2m_\mu$, $|q_-| = 2.5m_\mu$, Curve 3 corresponds to case then $|q_+| = 2m_\mu$, $|q_-| = 3m_\mu$. 
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[1] M. Diehl, Phys. Rept. 388, 41 (2003), arXiv:hep-ph/0307382.
[2] A. I. Ahmadov, Y. M. Bystritskiy, E. A. Kuraev, E. Zemljana, and T. V. Shishkina, J. Phys. G34, 353 (2007), arXiv:hep-ph/0606004.
[3] V. V. Sudakov, Sov. Phys. JETP 3, 65 (1956).
[4] V. N. Baier, E. A. Kuraev, V. S. Fadin, and V. A. Khoze, Phys. Rept. 78, 293 (1981).