Research on tomography algorithm of furnace flame temperature field based on acoustic method in thermal power station

Zhe Kan¹, Shuqiang Ding¹, Xiaolei Wang²*

¹ School of Information and Control Engineering, Liaoning Petrochemical University, Fushun, China
² School of Computer and Communication Engineering, Liaoning Petrochemical University, Fushun, China
*Corresponding author email: kanzhe@lnpu.edu.cn

Abstract. An improved Tikhonov regularization algorithm MCTR based on Markov radial basis function is proposed to obtain the feedback of closed-loop control for pulverized coal injection process in thermal power plant, so as to solve the ill posed problem in temperature field reconstruction in furnace. The algorithm constructs a new regularization matrix to replace the standard Tikhonov regularization element matrix, so as to achieve the correction effect of the new algorithm. MCTR algorithm determines the boundary singular value of small singular value defined in this paper by determining the proportion between the standard deviation component corresponding to small singular value and the sum of standard deviation components corresponding to all singular values after singular value decomposition of coefficient matrix. After determining the boundary, a new regularization matrix is constructed according to the eigenvector corresponding to small singular value. Compared with the MTR algorithm using the identity matrix as the regularization matrix, MCTR algorithm only selectively modifies the parameters corresponding to small singular values after determining the regularization parameters, which improves the stability of the parameter solution and the reconstruction accuracy of the temperature field, and is helpful to the automatic control of thermal power plants and the efficiency of coal combustion.

1. Introduction

Temperature is of great significance in industrial automation control and daily life. At present, most of the temperature measurements are indirect. By measuring the changes of sensor parameters with specific relationship with temperature, the change of temperature can be deduced [1]. According to whether the measured object is in contact with the corresponding sensor during measurement, the temperature measurement technology can be divided into two categories: contact and non-contact. Contact temperature measurement includes thermocouple temperature measurement, thermal resistance temperature measurement, optical fiber temperature measurement, etc. The non-contact temperature measurement technology does not require the sensor to directly contact the object to be measured, and has the characteristics of non-invasive and non-interference [2]. At the same time, the non-contact temperature measurement technology has small thermal inertia, small time delay and can dynamically reflect the temperature field change of the object to be measured. Common non-contact temperature measurement technologies mainly include radiation temperature measurement technology,
thermal imaging temperature measurement technology, acoustic temperature measurement technology, etc.

Many scholars are exploring acoustic reconstruction of temperature field through various methods (Mayer, S. F. Green, M. Bramanti, Barth et al). Their research results enable other scholars to better apply acoustic temperature field tomography algorithm. However, in the process of temperature field reconstruction algorithm, there are still some unsolved problems, such as improving the stability of parameter solution, so as to improve the reconstruction accuracy of temperature field and achieve better anti noise ability. Therefore, many scholars still need to carry out some simulations and experiments to study and overcome the problems in acoustic reconstruction of temperature field.

Based on the MTR algorithm using identity matrix as regularization matrix, a new algorithm MCTR is proposed and discussed in this paper. At the same time, through the calculation of correcting the square difference of the regularization matrix, the errors of the two are compared, and the advantages of the new reconstruction algorithm are obtained through MATLAB simulation analysis. The part 2 is the principle of measuring temperature field by acoustic method. The part 3 is the regularization matrix correction process to determine the boundary of small singular values defined in this paper. The part 4 is the simulation analysis. The advantages of the new reconstruction algorithm are obtained through the simulation effect. The part 5 is the conclusion.

2. The Principle of Acoustic Temperature Field Reconstruction

When the sound wave passes through the measurement plane of the furnace flame temperature field, its propagation speed has a specific functional relationship with the absolute temperature of the flue gas in the furnace[3]. Therefore, in the process of acoustic method temperature field reconstruction, the temperature distribution in the furnace can be obtained according to the measured sound wave path in the furnace temperature field propagation velocity[4-6]. The sound velocity c in a gaseous medium at an absolute temperature T is given by

\[ c = \sqrt{\frac{\gamma RT}{m}} = Z\sqrt{T} \]  

(1)

Where \( Z \) is a constant decided by gas composition. The value of \( Z \) for air is 20.05.

The reconstruction of acoustic temperature field is mainly divided into two steps: the first step is to establish the forward problem model, obtain the coefficient matrix describing the acoustic system according to the layout of the temperature acoustic transceiver and the division of grid pixels, and use the measured acoustic flight time to establish a linear relationship to model the problem to be solved. In the second step, the mathematic model describing the problem is solved, and the appropriate temperature field reconstruction algorithm is used to reconstruct the temperature distribution of the furnace flame. When reconstructing the actual temperature field, whether it is the simulation verification of the computer in the laboratory or the actual measurement of the utility boiler, the reconstruction matrix describing the acoustic system must be obtained and solved first.

The typical two-dimensional temperature field reconstruction and three-dimensional temperature field reconstruction acoustic path are shown in the figure below. In this paper, the two-dimensional temperature field is mainly reconstructed.

![Fig. 1. Two-dimensional acoustic path distribution](image1)

![Fig. 2. Three-dimensional acoustic path distribution](image2)
3. Construction of new regularization matrix

The standard Tikhonov method takes the identity matrix as the regularization matrix, which can be used to a great extent to reduce the variance of parameter estimation and improve the stability of parameter solution. According to the transmission law of covariance, we can know the standard Tikhonov's covariance formula is[7]:

\[
\text{cov}(\bar{e})=\sigma_0^2 (A^T A+\alpha I)^{-1} A^T A (A^T A+\alpha I)^{-1}
\]  

(2)

According to the covariance formula, the trace of the coefficient matrix is:

\[
D(\bar{e})=\text{tr}\left[\sigma_0^2 (A^T A+\alpha I)^{-1} A^T A (A^T A+\alpha I)^{-1}\right]
\]  

(3)

The eigenvalues of square array \( A^T A \) are decomposed as follows:

\[
A^T A=G^T G^T=\sum_{i=1}^n G_i \land_i G_i^T
\]  

(4)

Where eigenvalue \( \land_i=\lambda_i^2 \), the eigenvalue of the square matrix \( A^T A \) is the square of the singular value of the matrix \( A \), and the eigenvalue of \( \land_i \) is the right singular vector of the singular value \( \lambda_i \). Substitute Formula (4) into formula (3) to obtain:

\[
D(\bar{e})=\text{tr}\left[\sigma_0^2 \left(\left( (A^T A)^{-1}\left(\sum_{i=1}^n G_i \land_i G_i^T+\alpha \sum_{i=1}^n G_i G_i^T\right)\right)^{-1}\right) (A^T A+\alpha I)^{-1}\right]
\]

\[
=\sigma_0^2 \sum_{i=1}^n \frac{\land_i}{(\land_i+\alpha)^2}
\]  

(5)

The standard Tikhonov is a biased estimation and the deviation formula is:

\[
\text{(bias}(\bar{e}_0)\text{)}^2=\alpha^2 (A^T A+\alpha I)^{-1} \bar{e} \bar{e}^T (A^T A+\alpha I)^{-1}
\]  

(6)

The trace can be obtained by decomposing the eigenvalues in the above formula:

\[
\text{tr}\left[(\text{bias}(\bar{e}_0))^2\right]=\sum_{i=1}^n G_i \frac{\alpha^2}{(\land_i+\alpha)^2} G_i^T \bar{e}
\]  

(7)

It can be seen from formula (5) that the standard Tikhonov obtains a stable parameter solution after reducing the parameter variance by modifying the eigenvalue of the square matrix \( A^T A \). Because the unit matrix is used, after the regularization parameters are determined, each eigenvalue is modified to the same extent[8]. According to formula (7), standard Tikhonov regularization introduces deviation when reducing variance estimation[9]. The size of the introduced deviation is related to the selected regularization matrix. Different regularization matrices will correct the eigenvalues to varying degrees and the regularization method can be further modified.

Formula 5 shows that the influence caused by the serious ill condition of the equation is fully reflected in the amplification of the variance of parameter estimation by the eigenvalue with small value. According to \( \land_i=\lambda_i^2 \), the influence of the eigenvalues on variance can be transformed into the influence of singular values on standard deviation[10-11].

\[
\text{sta}=\left\{\left(\frac{1}{\lambda_1},\frac{1}{\lambda_2},\ldots,\frac{1}{\lambda_n}\right),\ldots,\left(\frac{1}{\lambda_1},\frac{1}{\lambda_2},\ldots,\frac{1}{\lambda_n}\right)\right\}
\]  

(8)

According to the above formula, the smaller the singular value, the greater the impact on the standard deviation, according to the characteristics of several orders of magnitude difference between large singular values and small singular values of ill posed matrix, when the sum of standard deviation components of small singular values accounts for more than 95% of the standard deviation, the boundaries of these singular values are determined and regularized to reduce their impact on the
standard deviation[12]. The judgment conditions for singular value boundaries are expressed as follows:

$$\sum_{i=k}^{m} \sigma_i > 95% \sum_{i=1}^{m} \sigma_i$$  \hspace{1cm} (9)

In the matrix $S$ composed of singular values, $\lambda_1 > \lambda_2 > \cdots > \lambda_k > \cdots > \lambda_m$, $\lambda_k$ is the boundary value of the small singular value, and the eigenvector corresponding to the small singular value is selected to construct the regularization matrix:

$$R=\sum_{i=k}^{m} G_i G_i^T$$  \hspace{1cm} (10)

The regularization of the newly constructed regularization matrix can be expressed as:

$$\arg \min_\varepsilon = \| A\hat{\varepsilon} - g \|^2 + \alpha \varepsilon R \varepsilon$$  \hspace{1cm} (11)

$$\hat{\varepsilon} = (A^T A + \alpha \sum_{i=k}^{m} G_i G_i^T)^{-1} A^T g$$  \hspace{1cm} (12)

By substituting the regularization matrix $R$ into formula (5), the trace of the variance matrix is:

$$D(\varepsilon) = \text{tr} \left[ \sigma_{\varepsilon}^2 \left( \left( (A^T A)^{-1} \right) \left( \sum_{i=1}^{n} G_i \lambda_i G_i^T + \alpha \sum_{i=1}^{n} G_i G_i^T \right) \right)^{-1} (A^T A + \alpha I)^{-1} \right]$$

$$= \sigma_{\varepsilon}^2 \left[ \sum_{i=1}^{k} \frac{1}{\lambda_i} + \sum_{i=k}^{m} \frac{\lambda_i}{(\lambda_i + \alpha)^2} \right]$$  \hspace{1cm} (13)

By substituting the newly constructed regularization matrix into equation (7), the deviation formula of regularization can be obtained as follows:

$$\text{tr} \left[ (\text{bias}(\varepsilon_{\theta}))^2 \right] = \sum_{i=k}^{m} e^T G_i \frac{\sigma_e^2}{(\lambda_i + \alpha)^2} G_i^T e$$  \hspace{1cm} (14)

Comparing formula (5) with formula (14), it can be seen that the deviation value corresponding to formula (14) is always less than that corresponding to formula (5). It is proved that the regularization method using the newly constructed regularization matrix can reduce the variance of parameter estimation and reduce the introduction of deviation compared with the standard Tikhonov regularization method, so as to improve the stability of the undetermined parameter solution.

4. Simulation Research on the New Reconstruction Algorithm

In order to verify the reconstruction effect and accuracy of the two algorithms, simulation needs to be carried out in different temperature field models verification:

Single peak symmetric temperature model:

$$T(x,y) = a + b \times \cos(k_1 \pi x) \times \cos(k_2 \pi y)$$  \hspace{1cm} (15)

Single peak bias temperature model:

$$T(x,y) = a + b \times \cos(k_1 \pi (x-x_1)) \times \cos(k_2 \pi (y-y_1))$$  \hspace{1cm} (16)

Bimodal temperature model:

$$T(x,y) = a_1 \left[ (x-x_1)^2 + \left( \frac{y-y_1}{c_1} \right)^2 \right]^{\frac{i}{2}} + a_2 \left[ (x-x_2)^2 + \left( \frac{y-y_2}{c_2} \right)^2 \right]^{\frac{i}{2}}$$  \hspace{1cm} (17)

Where $a$ and $b$ are constants, and $a_1$ represents the average temperature distribution of the temperature field; $b$ represents the temperature gradient of the temperature field; $k_1$ and $k_2$ are constants, indicating the peak value of the temperature field; $(x_1, y_1)$ represents the peak position of the single peak bias temperature field; $(x_1, c_1 y_1)$, $(x_2, c_2 y_2)$ respectively represent the positions of the two peaks; $a_1$ and $b_1$ represent the temperature gradient of the temperature field.
In order to analyze the reconstruction accuracy of MTR and MCTR algorithms, simulation verification is carried out in three temperature field models respectively. To evaluate the quality of the reconstructions, the root-mean-squared percent error of the reconstructed field is used, which is defined as follows.

\[
E_{\text{rms}} = \sqrt{\frac{1}{N} \sum_{j=1}^{N} (T(j) - \hat{T}(j))^2} \times 100\%
\]

(18)

Where \(T(j)\) represents the model temperature of the first network, \(\hat{T}(j)\) represents the reconstruction temperature of the second grid, and \(T_{\text{ave}}\) represents the average temperature of the model temperature field, set the mesh division to 8*8 and the noise standard to 5E-4.

### Table 1. Error analysis of reconstruction results

| Model                  | Reconstruction algorithm | Meshing | Noise standard | \(E_{\text{rms}}\) (%) |
|------------------------|--------------------------|---------|----------------|------------------------|
| Single peak symmetrical| MTR                      | 8*8     | 5E-4           | 2.87                   |
| Single peak offset     | MCTR                     | 8*8     | 5E-4           | 6.63                   |
| Double peak            |                          |         |                | 3.29                   |

From the above simulation studies, the simulation results of MTR and MCTR are made respectively. The simulation results of MTR and MCTR under the same grid division and the same noise standard under the conditions of Single peak symmetry, Single peak offset and Double peak are given. The results show that the new algorithm is more suitable for the reconstruction of complex temperature field, and the new algorithm can also reconstruct complex field well.
5. Conclusion
According to the simulation results of root mean square error of three temperature field models in the above table, MCTR effectively improves the accuracy of temperature field reconstruction. Compared with MTR algorithm, MCTR algorithm has further improved the reconstruction accuracy under different models and different noise levels, which shows that this algorithm has better anti noise ability, and proves the reconstruction advantages of the new algorithm compared with MTR algorithm.

In order to realize the large-scale commercial application of acoustic temperature measurement technology in furnace flame temperature measurement of thermal power station, there are still many problems to be studied. The accurate measurement of acoustic flight time is directly related to the reconstruction accuracy of temperature field. At present, the acoustic line bending effect based on variational principle will be considered in the measurement of acoustic flight time. However, in the actual measurement process, the improvement is not significant when the temperature gradient is small. How to accurately measure and reasonably correct the acoustic flight time in order to improve the applicability of different temperature field reconstruction is also the key research content.

Acknowledgments
This work was supported by the General Project of the Liaoning Province Education Department (L2020019), China.

References
[1] Hua Yan, Guannan Chen, Yang Qi, Lijun Liu. Research on complicated temperature field reconstruction based on acoustic CT[J]. Chinese Journal of Acoustics, 2013, 32(01): 51–62.
[2] Dapeng Yan, Feng Liu, Zhendong Wang, Anzhi He, Wu Ding. An improved algebraic iterative reconstruction technique and its application in 3D temperature field reconstruction [J]. Actaoptica Sinica, 1996 (09): 82–86.
[3] Rong Bao Chen, Ci Bian Jing, Fanghui Meng. Research on Closed-Loop Control System Based on Image-Signal Processing of Furnace Flame[J]. Advanced Materials Research, 2013, 16–21.
[4] Liwei Zhang, Mingji Wang, Wen Cao. Experimental Measurement of Two-Dimensional Circular Boundary Temperature Field by Acoustic Method[J]. Advanced Materials Research, 2014, 31–49.
[5] Lina Jia, Yan Gao, Decai Lu. Two-Dimension Temperature Field Reconstruction in Furnace by Acoustic Method[J]. Applied Mechanics and Materials, 2013, 21–42.
[6] Hongbo Lu, Weihua Sun, Zhihe Li. Research on Reconstruction Algorithm of Temperature Field by Acoustic Measurement in Boiler[J]. Advanced Materials Research, 2012, 11–16.
[7] Yunzhong Shen, Peiliang Xu, Bofeng Li. Bias-corrected Regularized Solution to Inverse Ill-posed Models [J]. Journal of Geodesy, 2012, 86(8): 597–608.
[8] Qian Kong, Genshan Jiang, Yuechao Liu, Jianhao Sun, Fausto Arpino. 3D Temperature Distribution Reconstruction in Furnace Based on Acoustic Tomography[J]. Mathematical Problems in Engineering, 2019, 19–21.
[9] Yanqiu Li, Shi Liu, Schlaberg H. Inaki. Dynamic Reconstruction Algorithm of Three-Dimensional Temperature Field Measurement by Acoustic Tomography[J]. Sensors, 2017, 17(9).
[10] R.X. Wang. Investigations on Acoustic Detecting Techniques and Simulation of 3-D Temperature Field. Daqing Petroleum institute (2007).
[11] M. Barth, A. Raabe. Acoustic tomographic imaging of temperature and flow fields in air[J]. Measurement Science and Technology, 2011, 22(3): 1–13.
[12] Kunpu Ji, Yunzhong Shen. Unbiased estimation of unit weight variance of TSVD canonical solution method [J]. Journal of Wuhan University (Information Science Edition), 2020, 45 (04): 626–632.