Stretched horizons, quasiparticles and quasinormal modes

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We propose that stretched horizons can be described in terms of a gas of non-interacting quasiparticles. The quasiparticles are unstable, with a lifetime set by the imaginary part of the lowest quasinormal mode frequency. If the horizon arises from an AdS/CFT style duality the quasiparticles are also the effective low-energy degrees of freedom of the finite-temperature CFT. We analyze a large class of models including Schwarzschild black holes, non-extremal Dp-branes, the rotating BTZ black hole and de Sitter space, and we comment on degenerate horizons. The quasiparticle description makes manifest the relationship between entropy and area.
1 Introduction

The physics of black holes has attracted wide attention as an arena for testing theories of quantum gravity. In particular a great deal of effort has gone towards resolving the puzzles of black hole thermodynamics and information loss. In this context the idea of a stretched horizon arose as a useful tool for thinking about black holes. It originated as a classical description of black holes as seen by outside observers [1], but the concept was later borrowed to help give a consistent quantum mechanical interpretation of black hole physics [2].

Since we now have a complete non-perturbative description of certain spacetimes in the form of the AdS/CFT correspondence and its generalizations [3] it is natural to see if the idea of a stretched horizon is correct. This is not just a hypothetical question, since if one understands how to use the stretched horizon idea, one will gain insight into both the structure of black holes and the structure of the dual quantum field theory.

Recently it was shown that a mean field approximation to the strongly coupled gauge theory which describes non-extremal 0-brane black holes gives results consistent with supergravity expectations [4, 5, 6]. This led to the idea that these finite-temperature gauge theories are well described by a gas of non-interacting quasiparticles with energies of order the temperature and lifetimes of order the inverse temperature. It was then shown that a stretched horizon description (a purely space time perspective) has exactly the same properties [7]. It was also shown that the stretched horizon description correctly accounts for the rate at which energy is thermalized in the gauge theory. This agreement relies on putting the stretched horizon one Planck proper distance away from the event horizon. One goal of the present paper is to understand this better, since the classical notion of a stretched horizon does not fix where it resides [1], while quantum mechanically it has been placed where the proper temperature matches either the Planck [2] or string [8] scale.

The theme of this paper is to show that the degrees of freedom of a stretched horizon can be viewed as a gas of quasiparticles. The quasiparticle picture has the advantage of being extremely simple. It also makes manifest several universal properties of horizons, including the universal relationship between entropy and horizon area.
In developing this picture of a stretched horizon we have drawn heavily from several sources. The basic idea of a stretched horizon originated with the membrane paradigm [1]. Stretched horizons were used to describe the quantum properties of a black hole by Susskind, Thorlacius and Uglum [2], who pointed out several universal thermodynamic properties of the stretched horizon which are fundamental to the quasiparticle description. Stretched horizons were also used by Sen [8] to count the states of a family of degenerate black holes. Although we will not be able to make a precise connection, we find it inspirational to identify the quasiparticles with the open strings stuck on the horizon introduced by Susskind [13]. Finally, we note that the quasiparticle description we will present has some overlap with the work of York [14].

An outline of this paper is as follows. We first show how the quasiparticle picture works for Schwarzschild black holes. We then study non-extremal Dp-branes, as a straightforward generalization of our previous work on D0-branes [7]. In this context the quasiparticles give information about the dual $p + 1$ dimensional gauge theory. We then analyze a model with more parameters, namely the rotating BTZ black hole [9], and show how the stretched horizon picture relates to the dual CFT. The BTZ analysis should serve as a good guide for working out quasiparticle properties in more complicated situations. Finally we show that the static patch of de Sitter space can be described by a stretched horizon with sensible quasiparticle properties. We also comment on the location of the stretched horizon and explain that for degenerate horizons it does not need to be located at the Planck scale. In an appendix we discuss decoupling limits, quasinormal modes and the quasiparticle description for a general class of $p$-brane solitons [10][11][12].

2 Schwarzschild black holes

As our first example, we show that a simple quasiparticle picture can capture the basic properties of a Schwarzschild black hole.

In $d$ spacetime dimensions the Schwarzschild metric reads

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2d\Omega_{d-2}^2$$
where
\[ f(r) = 1 - \frac{\omega_d M}{r^{d-3}} \quad \omega_d = \frac{16\pi G}{(d-2)\text{vol}(S^{d-2})}. \]

The parameter \( M \) is the mass of the black hole. The event horizon is located at the Schwarzschild radius \( R_S = (\omega_d M)^{1/(d-3)} \sim (GM)^{1/(d-3)} \). The Hawking temperature is
\[ T = \frac{d-3}{4\pi R_S} \]
and the Bekenstein-Hawking entropy is
\[ S = \frac{A}{4G} = \frac{\text{vol}(S^{d-2}) R_S^{d-2}}{4G} \sim \left( \ell_{\text{Planck}} M \right)^{(d-2)/(d-3)}. \]

We will introduce a stretched horizon \( H \) at a radius where the proper temperature \( T_{\text{proper}} = T/\sqrt{-g_{tt}} \) is equal to the Planck temperature. As we discuss below, this is necessary to get a consistent description. The Stefan-Boltzmann law gives the rate at which the stretched horizon emits energy in outgoing Hawking radiation.\(^1\)

\[ \frac{dE_{\text{proper}}}{dt_{\text{proper}}} \sim AT_{\text{proper}}^d \] (1)

Multiplying this equation by \(-g_{tt}\) gives the emission rate measured with respect to the Schwarzschild time coordinate \( t \). The proper temperature \( \sim 1/\ell_{\text{Planck}} \), so
\[ \frac{dE}{dt} \sim \frac{A}{\ell_{\text{Planck}}^{d-2}} T^2. \] (2)

We now postulate that the stretched horizon of the black hole can be approximately described as a gas of \( N \) non-interacting quantum mechanical degrees of freedom ("quasiparticles"). The number of quasiparticles depends on the temperature, as we postulate that the total number of quasiparticles matches the entropy of the black hole, \( N \sim S \). Each quasiparticle has an energy \( \epsilon \sim T \) and decays with a characteristic lifetime \( \tau \sim 1/T \). We take the quasiparticle gas to be in thermal equilibrium at temperature \( T \).

\(^1\)In equilibrium this emission rate is balanced by an equal and opposite flux of infalling radiation. Also note that only a tiny fraction of this emitted radiation ever reaches asymptotic infinity; the vast majority of Hawking particles turn around and fall back onto the stretched horizon. See p. 287 of \( \text{[1]} \).
It is easy to see that the quasiparticle gas reproduces the equilibrium thermodynamic properties of the black hole. The energy of the gas

\[ E \approx N \epsilon \sim M \]

agrees with the mass of the black hole, and by construction the entropy of the gas is equal to the Bekenstein-Hawking entropy. These properties of the quasiparticle gas correspond to the universal properties of the stretched horizon noted by Susskind, Thorlacius and Uglum [2]. But the quasiparticle gas also reproduces certain dynamical properties of the black hole, since the rate at which energy gets redistributed in the gas due to the ongoing production and decay of quasiparticles is given by

\[ \frac{dE}{dt} \approx N \epsilon / \tau \sim NT^2. \]  

(3)

This agrees with the rate [2] at which energy gets redistributed on the stretched horizon due to the emission and absorption of Hawking quanta.

Let us make a few comments on these results. Quite generally we expect that any horizon can be described in terms of a membrane located at the stretched horizon. This fact is well-established at the classical level [1], and has been argued to hold in the quantum theory as well [2]. We are claiming that in the quantum theory the membrane has a low-energy effective description in terms of a gas of quasiparticles. Although we cannot make any sort of precise connection, we would like to identify the quasiparticles with the open strings stuck on the horizon introduced by Susskind [13].

We expect the quasiparticle gas to provide a holographic description of the black hole together with certain of its low-energy excitations. If the black hole spacetime admits a dual description along the lines of the AdS/CFT correspondence [3] then one should also be able to identify the quasiparticles in the dual field theory. In [7] we began with a dual field theory of 0-branes and used mean-field methods to extract an effective quasiparticle description. The resulting quasiparticle gas indeed captures the thermodynamic properties of the black hole [15, 16].

In the case of a Schwarzschild black hole no dual description is known,\(^2\) so we are simply postulating that the quasiparticle gas has the right properties

\(^2\)See however [15]. Also note that Schwarzschild black holes are present as excited states in the AdS/CFT correspondence; perhaps the quasiparticles provide an effective description of a sector of the CFT Hilbert space. See [16] for some steps in this direction.
to match the black hole. Some peculiar features of the Schwarzschild black hole, for example its negative specific heat, are simply reflected in the fact that the number of quasiparticles goes down as the temperature increases.

The quasiparticle description does give insight into one key aspect of black hole physics. Note that the black hole emits energy at a rate which is proportional to the area of the event horizon in Planck units. The quasiparticle gas, on the other hand, redistributes energy at a rate which is proportional to the number of quasiparticles. The rates agree provided the entropy of the quasiparticle gas is equal to the area of the horizon in Planck units. This provides a simple origin for the entropy–area relation, which is otherwise very obscure from the point of view of any dual field theory description [7].

We conclude our treatment of Schwarzschild black holes with a discussion of the way in which a perturbed black hole returns to (approximate) thermal equilibrium.\(^3\) The quasiparticle gas should describe low-lying excitations of the stretched horizon. From the quasiparticle point of view such perturbations should decay on a timescale set by the quasiparticle lifetime \(\tau \approx 1/T\). On the other hand, from the gravity point of view such perturbations can be described as quasinormal excitations of the black hole [17]. Quasinormal excitations of Schwarzschild black holes have been studied recently in [18, 19, 20, 21]. The decay of scalar perturbations, for example, is governed by the wave equation

\[
\left( -f(\rho)\partial_\rho f(\rho)\partial_\rho + V(\rho) - (R_S \omega)^2 \right) \psi = 0
\]

\[
f(\rho) = 1 - 1/\rho^2
\]

\[
V(\rho) = \frac{f}{\rho^2} \left[ \ell(\ell + d - 3) + \frac{(d - 2)(d - 4)}{4} + \frac{(d - 2)^2}{4\rho^{d-3}} \right]
\]

where \(\rho = r/R_S\) is dimensionless. The quasinormal frequencies \(\omega\) are determined by solving this equation with boundary conditions that are pure ingoing at the horizon and pure outgoing at infinity. It is obvious on dimensional grounds that

\[
\omega \sim 1/R_S \sim T.
\]

---

\(^3\)The black hole is of course ultimately unstable due to evaporation. The quasiparticle description is only valid, and the notion of equilibrium only applies, on timescales short compared to the black hole lifetime.
The same dimensional analysis works for vector and gravitational perturbations of the black hole [21].

Note that these quasinormal modes are supergravity solutions which govern the late-time behavior of certain bulk closed-string fields [17]. They should not be confused with the quasiparticles, which we regard as arising from open string degrees of freedom on the stretched horizon. But we expect the quasiparticles to provide a holographic description of low-energy excitations near the black hole. In particular the quasiparticle lifetime should govern the decay of the low-lying quasinormal modes. Thus we propose that the quasiparticle inverse lifetime is of order the imaginary part of the lowest quasinormal frequency, $\tau \sim 1/T$.

As in [7], we can use this result for the quasinormal frequencies to show that the stretched horizon must be placed at the Planck temperature. The Stefan-Boltzmann law (1) can be turned into an expression for the rate at which the stretched horizon emits entropy.

$$\frac{dS}{dt} \sim T \cdot AT_{\text{proper}}^{d-2}$$

Given the quasinormal lifetime we expect $dS/dt \sim TS$. Thus we must identify $AT_{\text{proper}}^{d-2}$ with the black hole entropy, which is only consistent provided we place the stretched horizon at the Planck temperature.

3 Non-extremal Dp-branes

As our second example we discuss non-extremal black Dp-branes in type II string theory. We take a near-horizon limit, so that the dual field theory is known [24]. This section generalizes our previous work on D0-brane black holes [7].

4See [14] for an earlier investigation along these lines. It would be interesting to relate this identification to the proposal discussed in [22, 23, 13].
3.1 Gravitational description

We begin with the near-horizon limit of the supergravity solution describing $N$ coincident non-extremal D$p$-branes [24]. The string-frame metric and dilaton are

$$\frac{1}{\alpha'} ds^2 = \frac{U^{(7-p)/2}}{\sqrt{d}e^2} \left( -\left( 1 - \frac{U_0^{7-p}}{U^{7-p}} \right) dt^2 + dx^2_\parallel \right) + \frac{\sqrt{d}e^2}{U^{(7-p)/2}} \left( 1 - \frac{U_0^{7-p}}{U^{7-p}} \right)^{-1} dU^2$$

$$+ \sqrt{d}e^2 U^{(p-3)/2} d\Omega^2_{8-p}$$

$$e^\phi = (2\pi)^{2-p} g^2_{YM} \left( \frac{U^{7-p}}{d} e^2 \right)^{(p-3)/4}$$

where $e^2 = g^2_{YM} N$ and $d_p = 2^{7-2p} \pi^{9-p} \Gamma(\frac{7-p}{2})$. The Einstein metric is

$$ds^2 = \text{const.} U^{(7-p)/2} \left[ -\left( 1 - \frac{U_0^{7-p}}{U^{7-p}} \right) dt^2 + dx^2_\parallel + \frac{d}e^2 U^{7-p} (1 - \frac{U_0^{7-p}}{U^{7-p}}) dU^2$$

$$+ d_p e^2 U^{p-5} d\Omega^2_{8-p} \right]. \quad (4)$$

Up to inessential numerical factors the Hawking temperature, energy density and entropy density depend on the horizon radius $U_0$ according to

$$T \sim \frac{1}{e} U_0^{(5-p)/2} \quad (5)$$

$$E \sim \frac{N^2}{e^4} U_0^{7-p}$$

$$s \sim \frac{N^2}{e^3} U_0^{(9-p)/2}$$

As in the previous section, we introduce a stretched horizon at a radius where the proper temperature has the value $1/\ell_{\text{Planck}}$. The proper rate at which the stretched horizon radiates energy density is given by Stefan-Boltzmann,

$$\frac{dE_{\text{proper}}}{dt_{\text{proper}}} = A T^{10-p}_{\text{proper}}$$

where $A$ is the proper area of an $(8-p)$-sphere with radius $U_0$. Converting to Schwarzschild quantities this means

$$\frac{dE}{dt} = \frac{A}{\ell_{\text{Planck}}^{8-p}} T^2. \quad (6)$$
3.2 Quasinormal modes

We now study quasinormal excitations of the black brane metric (4). Our approach will parallel the study of AdS Schwarzschild black holes in \cite{25,26}. For simplicity we study a massless minimally-coupled scalar field. Separating variables

$$\phi = e^{-i\omega t + ik_i x^i} f(U)Y(\Omega)$$

where $Y(\Omega)$ is an eigenfunction of the Laplacian on $S^{8-p}$ with eigenvalue $-l(l+7-p)$, the scalar wave equation becomes

$$\partial_U U^{8-p} \left( 1 - \frac{U_0^{7-p}}{U^{7-p}} \right) \partial_U f(U) + d_p e^2 U \left( \frac{\omega^2}{1 - \frac{U_0^{7-p}}{U^{7-p}}} - k^2 \right) f(U)$$

$$-l(l+7-p)U^{6-p} f(U) = 0 .$$

We now change variables to

$$z = 1 - \frac{U_0^{7-p}}{U^{7-p}}$$

which gives the equation

$$\left( z \partial_z z \partial_z + \frac{\rho^2 - z \kappa^2}{(1-z)^{2-p}} - \frac{l(l+7-p)z}{(1-z)^2(7-p)^2} \right) f(z) = 0$$

where

$$\rho^2 = \frac{\omega^2 d_p e^2}{(7-p)^2 U_0^{5-p}}, \quad \kappa^2 = \frac{k^2 d_p e^2}{(7-p)^2 U_0^{5-p}}.$$ 

We solve (9) with the Dirichlet boundary condition \cite{25}

$$f(z = 1) = 0$$

at infinity and the purely ingoing condition at the future horizon

$$f(z \to 0) \sim z^{-i\rho} .$$

For given values of $l$ and $\kappa$ there are solutions only for discrete values of $\rho$, labelled by an integer $n$. The $n^{th}$ quasinormal frequency is then

$$\omega_n = \frac{(7-p)U_0^{\frac{6-2p}{2}}}{\sqrt{d_p e^2}} \rho_n .$$
Note that the quasinormal frequencies are proportional to the Hawking temperature, since
\[ \omega_n \sim \frac{1}{e} U_0^{(5-p)/2} \sim T. \]  
(14)

To obtain the coefficient of proportionality we must solve (9) numerically. We focus on the mode with lowest imaginary part, corresponding to \( \kappa = 0 \) and \( l = 0 \). For \( p < 5 \), we use a power series expansion around \( z = 0 \) satisfying the boundary condition (12) matched at a regular intermediate point (e.g. \( z = 1/2 \)) with a Runge-Kutta solution to (9) evolved from \( z = 1 \), and satisfying (11). We solve for \( \rho \) by demanding that \( f'(z)/f(z) \) coincide at the matching point. This method is implemented using Mathematica as described in more detail in appendix A. This method yields dramatically faster convergence than the pure power series solution of [25], producing the results quoted here in just a few seconds. The difficulty with the pure power series solution is that the function is evaluated at its radius of convergence \( (z = 1) \), so convergence of the series can be very slow. The results for the lowest modes are shown in the following table.

| \( p \) | \( \rho \) |
|---|---|
| 0 | 0.554059 \(- 0.930000i\) |
| 1 | 0.616474 \(- 0.887951i\) |
| 2 | 0.693098 \(- 0.816466i\) |
| 3 | 0.779863 \(- 0.686699i\) |
| 4 | 0.834579 \(- 0.435416i\) |

(15)

For \( p = 1, 3, 4 \) these frequencies agree with the results of [25] for large AdS-Schwarzschild black holes in \( AdS_{4,5,7} \) respectively.\(^5\)

For \( p = 5 \), (9) can be solved analytically in terms of hypergeometric functions. The purely ingoing (for \( Im(\rho) < 0 \)) solution on the horizon takes the form
\[ z^{-i\rho}(1 - z)^{\gamma} \frac{\Gamma(\gamma - i\rho, \gamma - i\rho, 1 - 2i\rho; z)}{\Gamma(1)} \]  
(16)

where we have defined
\[ \gamma = \frac{1 + \sqrt{1 - 4(\rho^2 - \bar{\kappa}^2)}}{2}, \quad \bar{\kappa}^2 = \kappa^2 + \frac{l(l + 2)}{4}. \]  
(17)

\(^5\)To see this note that the wave equation (65) in appendix B is invariant under shifting \( D \rightarrow D + 1 \) and \( p \rightarrow p + 1 \), and that in these cases the metric (64) agrees with (2.8) in [25] upon setting \( \rho = 1/r \).
Near infinity $z \to 1$, the solution (16) is a linear combination of a component that is asymptotically $(1 - z)^\gamma$, and one that is $(1 - z)^{1 - \gamma}$. To get a quasi-normal mode with finite flux at infinity, we need to set the coefficient of one of these terms to zero. Therefore we must solve either

$$\frac{\Gamma(1 - 2i\rho)\Gamma(2\gamma - 1)}{\Gamma(\gamma - i\rho)^2} = 0 \tag{18}$$

or

$$\frac{\Gamma(1 - 2i\rho)\Gamma(1 - 2\gamma)}{\Gamma(1 - i\rho - \gamma)^2} = 0 \tag{19}.$$  

Equation (18) is satisfied when $\gamma - i\rho = -n$, with $n$ a positive integer, however this equation has no solutions. Equation (19) has solutions when $1 - i\rho - \gamma = -n$ with $n$ a positive integer. This has purely imaginary solutions for $\rho$, but these always lead to divergent flux at infinity. However if $\rho$ is pure imaginary, then the flux on the horizon vanishes, so one could imagine including an admixture of

$$z^{i\rho}(1 - z)^\gamma _2F_1(\gamma + i\rho, \gamma + i\rho, 1 + 2i\rho; z). \tag{20}$$

We still demand the solution be a function only of $e^{-i\omega t/z - i\rho}$ as $z \to 0$, which rules out this component. We conclude that there are no well-behaved quasinormal modes for black 5-branes.

For $p = 6$ the equation does not appear to be soluble analytically. The solutions have oscillatory behavior as $z \to 1$, so rather different numerical methods are needed to obtain quasinormal mode frequencies reliably. The methods used for Schwarzschild black holes should be useful, since when uplifted to eleven dimensions the $p = 6$ solution is asymptotically flat [24]. We leave further analysis of this case for future work.

### 3.3 Quasiparticle picture for $p \leq 4$

For $p \leq 4$ it is easy to reproduce the thermodynamic properties of the black brane by introducing a quasiparticle gas. We postulate that the longitudinal number density of quasiparticles is equal to the entropy density of the black brane, $n \sim s$. Each quasiparticle has an energy $\epsilon$ of order the temperature and a lifetime $\tau$ set by the imaginary part of the lowest quasinormal frequency. Given our results in section 3.2, this means $\tau \sim 1/T$. 

...
It is easy to see that the energy density and entropy density of the quasi-particle gas agree with the corresponding properties of the black brane. Moreover the rate at which the quasiparticle gas redistributes energy density

\[ \frac{dE}{dt} = n\epsilon/\tau \]

agrees with the black brane radiation rate (6). Note that for Dp-branes the energy and entropy are power-law in the temperature. This property helps make a simple quasiparticle description possible.

Finally, following [7], we show that the quasiparticle picture correctly predicts the horizon radius of a black p-brane. From the point of view of the dual (p + 1)-dimensional gauge theory one would define the radius of the black brane by

\[ R^2_h = \frac{1}{N} \langle \text{Tr}X^2 \rangle. \]

Here \(X\) is one of the transverse matrix-valued scalar fields, related to a canonically normalized field by \(X = eY/\sqrt{N}\). The effective number of entries in the matrix \(Y\) is given in terms of the entropy density by \(N_{\text{eff}} = s/T^p\). Thus

\[ R^2_h = \frac{e^2}{N^2} N_{\text{eff}} \langle y^2 \rangle \]

where \(\langle y^2 \rangle\) measures the fluctuation of a single canonically normalized scalar field. Upon suitably smearing the field to suppress high-frequency contributions [6][7][27], this correlator is given by \(\langle y^2 \rangle \sim T^{p-1}\). Thus the quasiparticle picture predicts

\[ R^2_h \sim \frac{e^2 s}{N^2 T}. \]

This is indeed satisfied for black p-branes, upon identifying \(R_h\) with \(U_0\).

3.4 Comments on \(p = 5\)

For D5-branes we were not able to find quasinormal modes, and a simple quasiparticle description does not appear to be possible. In this case the temperature [5] is independent of the energy density at leading semiclassical order. Sub-leading one-loop effects drive the specific heat negative and
indicate that the thermodynamic ensemble is unstable [28]. As described in [29], a classical instability is present for the D5-brane all the way down to extremality, and this is argued to be equivalent to local thermodynamic instability. This has been conjectured to persist in the decoupling limit [30]. These facts are closely related to the violation of the decoupling of the throat region and the 5-brane described in [31]. The classical instability of the 5-brane implies that thermalization will not occur, so the quasiparticle picture will not work.

4 BTZ black holes

In this section we develop a quasiparticle description of the spinning BTZ black hole [9]. Unlike the previous cases, the BTZ black hole depends on two parameters (mass and angular momentum) and has a free energy which is not a simple power-law in the temperature. This will force us to introduce two distinct species of quasiparticles.

4.1 Gravitational description

Classical and quantum properties of the BTZ black hole are reviewed in [32]. The metric is

$$ds^2 = -V(r)dt^2 + \frac{1}{V(r)}dr^2 + r^2 \left( d\phi - \frac{J}{2r^2}dt \right)^2$$

It has been argued that there is a correspondence between thermodynamic instability and classical instability whenever a non-compact translational symmetry is present [29]. Thus the Schwarzschild black hole is thermodynamically unstable, but there is no translational symmetry, which is consistent with the absence of a classical instability. As discussed in section 2 the quasiparticle picture can make sense here, because the timescale associated with thermalization $O(T^{-1})$ is much shorter than the timescale of evaporation $O(T^{1-d})$. The $p < 5$ D-branes are thermodynamically stable in the decoupling limit, and as described in [29] have no classical instability close to extremality (at least for the $p = 1, 2, 4$ cases studied there).

7We adopt units in which $8G = 1$. 

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where $\phi$ has period $2\pi$ and

$$V(r) = -M + \frac{r^2}{\ell^2} + \frac{J^2}{4r^2}.$$  

The mass $M$ and angular momentum $J$ are related to the horizon radii $r_{\pm}$ by

$$M = \frac{(r_+^2 + r_-^2)}{\ell^2},$$
$$J = \frac{2r_+ r_-}{\ell}$$

where $\ell$ is the asymptotic AdS radius. The horizon rotates with angular velocity

$$\Omega_H = J/2r_+^2 = r_-/\ell r_+.$$

The Hawking temperature of the black hole is

$$T = \frac{r_+^2 - r_-^2}{2\pi r_+ \ell^2}$$

and the Bekenstein-Hawking entropy is

$$S = 4\pi r_+.$$

In the extremal limit a BTZ black hole has $M = J/\ell$ (or equivalently $r_+ = r_-)$ and vanishing temperature. For simplicity we will only consider non-extremal black holes, although we make a few comments on the extremal limit in section 4.4.

We will place the stretched horizon at a radius where the proper temperature $T_{\text{proper}} = T/\sqrt{V(r)}$ is equal to the Planck temperature. We wish to compute the rate at which the stretched horizon emits energy and angular momentum in outgoing Hawking radiation.\textsuperscript{8} To do this we introduce a family of “Rindler observers” who sit on the stretched horizon and rotate with exactly the angular velocity of the event horizon $\Omega_H$. A typical Rindler observer has a worldline

$$t(\tau) = \tau \quad r(\tau) = \text{const.} \quad \phi(\tau) = \text{const.} + \Omega_H \tau.$$  

\textsuperscript{8}In equilibrium these outgoing fluxes are of course balanced by equal and opposite infalling fluxes.
Such observers perceive themselves to be at rest with respect to a thermal bath at temperature $T_{\text{proper}}$. This is easiest to see in a Euclidean formulation: setting $\phi' = \phi - \Omega_H t$ and Wick rotating $t = -it_E$, $\Omega_H = i\Omega_{HE}$ one is forced to identify

$$(t_E, r, \phi') \sim (t_E + 1/T, r, \phi')$$

in order to get a non-singular Euclidean metric \[32\]. The identification is purely in the Euclidean time direction, so Rindler observers who sit at fixed $\phi'$ are instantaneously at rest with respect to the thermal bath.

We should point out that these Rindler observers differ from the fiducial observers (or zero angular momentum observers) introduced in \[1\], who rotate with the local frame-dragging angular velocity, and who see the event horizon as rotating beneath them. Also note that, due to the asymptotically AdS nature of space, a Rindler observer at any radius outside the event horizon moves along a timelike trajectory. For example Rindler observers at spatial infinity co-rotate with the CFT.

To relate local measurements made by Rindler observers to the corresponding conserved Schwarzschild quantities, introduce orthonormal basis vectors $\{\bar{e}_0, \bar{e}_r, \bar{e}_\phi\}$ where

$$\bar{e}_0 \approx \frac{1}{\sqrt{V(r)}} (\partial_t + \Omega_H \partial_\phi)$$

is tangent to the Rindler worldline,

$$\bar{e}_r = \sqrt{V(r)} \partial_r$$

points in the radial direction, and

$$\bar{e}_\phi \approx \frac{1}{r} \partial_\phi$$

points in the $\phi$ direction. (We have only written the leading behavior as $r \to r_+$..) On average, a Rindler observer sees the outgoing Hawking quanta as having an energy-momentum vector of the form

$$\bar{p} = E_{\text{proper}} (\bar{e}_0 + \bar{e}_r).$$

The corresponding outgoing conserved Schwarzschild energy and angular momentum are obtained by taking inner products with the Killing vectors $\partial_t$ and $\partial_\phi$.

$$E_{\text{out}} = -\langle \bar{p}, \partial_t \rangle \quad J_{\text{out}} = \langle \bar{p}, \partial_\phi \rangle$$
Up to corrections which are subleading as \( r \to r_+ \), these quantities are given by

\[
E_{\text{out}} - \Omega_H J_{\text{out}} \approx \sqrt{V(r)} E_{\text{proper}} \tag{22}
\]

\[
J_{\text{out}} \approx \frac{\ell r_+ r_-}{r_+^2 - r_-^2} \sqrt{V(r)} E_{\text{proper}}.
\]

Here we are assuming that the position of the stretched horizon \( r_{\text{stretch}} = r_+ + \Delta r \) is such that \( \Delta r \ll r_+ - r_- \). This assumption is valid since we’re staying away from the extremal limit \( r_+ = r_- \).

A Rindler observer sees the stretched horizon of the black hole radiate energy isotropically at a rate given by the Stefan-Boltzmann law,

\[
\frac{dE_{\text{proper}}}{dt_{\text{proper}}} \sim A T_{\text{proper}}^3
\]

where \( A = 2\pi r_+ \) is the area of the event horizon. Since we place the stretched horizon at a radius corresponding to the Planck temperature, this means

\[
\frac{dE_{\text{proper}}}{dt_{\text{proper}}} \sim \frac{A}{\ell_{\text{Planck}}^2} T_{\text{proper}}^2.
\]

Using the dictionary (22), the relations \( dt_{\text{proper}} = \sqrt{V(r)} dt \) and \( dE_{\text{proper}} = dE/\sqrt{V(r)} \), and recalling \( 8G = 1 \), the Stefan-Boltzmann law implies that

\[
\frac{d}{dt} (E - \Omega_H J) \sim A T^2 \tag{23}
\]

\[
\frac{d}{dt} J \sim \frac{\ell r_+ r_-}{r_+^2 - r_-^2} A T^2.
\]

Note that these equations are equivalent to

\[
\frac{d}{dt} S \sim ST \tag{24}
\]

\[
\frac{d}{dt} J \sim JT
\]

where we have held \( \Omega_H \) fixed and used \( dE - \Omega_H dJ = T dS \) in the first line. Likewise the rate at which energy is radiated can be written as

\[
\frac{d}{dt} E \sim (E - F) T \tag{25}
\]

where \( F = E - TS - \Omega_H J \). These relations suggest that all thermodynamic quantities relax to equilibrium on a time-scale set by \( 1/T \).
4.2 Quasiparticle description

The BTZ metric \(^{21}\) is dual to a conformal field theory on the boundary \(^{33,34}\). We propose that this dual CFT has a low-energy effective description in terms of a gas of weakly-interacting quasiparticles, which are sufficient to describe the black hole and certain of its low-energy excitations. Thus the quasiparticle gas shares some properties with the effective string discussed in \(^{35}\).

To motivate our quasiparticle picture, we define dimensionless right- and left- temperatures in the usual way

\[
r_\pm = \pi \ell (T_R \pm T_L)
\]

Then the black hole energy, entropy, and angular momentum can be split into right- and left- contributions according to

\[
M = 2\pi^2(T_R^2 + T_L^2) \\
S = 4\pi^2\ell(T_R + T_L) \\
J = 2\pi^2\ell(T_R^2 - T_L^2)
\]

(26)

In order to account for both the black hole mass and angular momentum we must introduce two distinct species of quasiparticles. Our precise proposal is that there are \(N_R \approx 4\pi^2\ell T_R\) right-moving quasiparticles, each with energy \(\epsilon_R \approx T_R\) and lifetime \(\tau_R \approx 1/T_L\). We likewise propose that there are \(N_L \approx 4\pi^2\ell T_L\) left-moving quasiparticles each with energy \(\epsilon_L \approx T_L\) and lifetime \(\tau_L \approx 1/T_R\).

A simple argument for the proposed quasiparticle lifetimes is that a right-moving excitation can only thermalize by scattering off a left-mover. This suggests that the thermalization rate for right-movers is proportional to the number of left-moving quasiparticles, or equivalently \(\tau_R \sim 1/N_L \sim 1/T_L\). A more detailed justification is given in section \(^{4.3}\).

In the quasiparticle picture it is straightforward to estimate the energy, entropy, and angular momentum of the CFT.

\[
E_{\text{CFT}} \approx N_R\epsilon_R + N_L\epsilon_L \\
S_{\text{CFT}} \approx N_R + N_L \\
J_{\text{CFT}} \approx N_R\epsilon_R - N_L\epsilon_L
\]

(27)
It is also straightforward to estimate the rate at which energy and angular momentum get redistributed among the CFT degrees of freedom, due to the ongoing production and decay of quasiparticles.

\[
\frac{dE_{\text{CFT}}}{dt_{\text{CFT}}} \approx N_R \epsilon_R/\tau_R + N_L \epsilon_L/\tau_L \quad (28)
\]

\[
\frac{dJ_{\text{CFT}}}{dt_{\text{CFT}}} \approx N_R \epsilon_R/\tau_R - N_L \epsilon_L/\tau_L
\]

We now show that these properties of the quasiparticle gas match the corresponding properties of the black hole. The induced metric on the boundary is conformal to \( ds^2_{\text{CFT}} = -dt^2/\ell^2 + d\phi^2 \), so the dictionary is \( t_{\text{CFT}} = t/\ell \), \( E_{\text{CFT}} = \ell E \), \( J_{\text{CFT}} = J \). Thus the quasiparticle results (27) predict that

\[
E \approx T_R^2 + T_L^2 \\
S \approx \ell(T_R + T_L) \\
J \approx \ell(T_R^2 - T_L^2)
\]

in agreement with the black hole results (26). Likewise the quasiparticles predict that

\[
\frac{dE}{dt} \approx (T_R + T_L)T_RT_L/\ell \\
\frac{dJ}{dt} \approx (T_R - T_L)T_RT_L
\]

which are easily seen to be equivalent to the black hole expressions (23)\(^9\).

Note that these semiclassical expressions for the Hawking flux of energy and angular momentum are only valid if \( N_L \gg 1 \) and \( N_R \gg 1 \).

\(^9\)Note that these semiclassical expressions for the Hawking flux of energy and angular momentum are only valid if \( N_L \gg 1 \) and \( N_R \gg 1 \).
4.3 Correlation times and quasinormal frequencies

In section 4.2 we simply postulated that the lifetimes of the left- and right-moving quasiparticles are given by

\[ \tau_L = 1/T_R \quad \tau_R = 1/T_L. \]

We now provide some justification for this, from both the supergravity and CFT points of view.\(^{10}\) Our discussion in this section closely follows the work of Birmingham, Sachs and Solodukhin.\(^{37,38}\)

Quasinormal modes for bulk fields of various spins were studied in\(^{37,39}\). Working in terms of the CFT time coordinate one set of solutions is

\[ \Phi(t, r, \phi) = e^{-ik(t-\phi)}e^{-t/\tau_R}f(r) \quad k \in \mathbb{Z} \quad (31) \]

where

\[ \tau_R = 1/4\pi T_L(h_L + n) \quad n = 0, 1, 2, \ldots \quad (32) \]

Here \( h_L \) is the conformal dimension of the corresponding operator in the CFT. These solutions describe right-moving excitations which decay on a timescale \( \tau_R \). Likewise there are solutions

\[ \Phi(t, r, \phi) = e^{-ik(t+\phi)}e^{-t/\tau_L}f(r) \quad k \in \mathbb{Z} \quad (33) \]

where

\[ \tau_L = 1/4\pi T_R(h_R + n) \quad n = 0, 1, 2, \ldots \quad (34) \]

These solutions describe left-moving excitations which decay on a timescale \( \tau_L \).

We would like to emphasize that these quasinormal modes are supergravity solutions which govern the late-time behavior of certain bulk closed-string fields\(^{17}\). They should not be confused with the quasiparticles, which we regard as arising from fundamental degrees of freedom on the stretched horizon. But we expect the quasiparticles to provide a holographic description of low-energy excitations near the black hole, so their lifetimes should govern the decay of these low-lying supergravity excitations. It is therefore very

\(^{10}\)A study of horizon degrees of freedom for the BTZ black hole along the lines of\(^{22,23}\) was made in\(^{36}\).
encouraging that the quasinormal lifetimes (32), (34) match our proposed quasiparticle lifetimes.

We can gain more insight into the meaning of these quasinormal frequencies by studying the dual CFT. In a two-dimensional CFT the finite-temperature Euclidean 2-point function of primary fields is determined by conformal invariance:

\[
\langle \mathcal{O}(w_1, \bar{w}_1)\mathcal{O}(w_2, \bar{w}_2) \rangle \sim \left( \frac{\pi T_L}{\sinh \pi T_L (w_1 - w_2)} \right)^{2h_L} \left( \frac{\pi T_R}{\sinh \pi T_R (\bar{w}_1 - \bar{w}_2)} \right)^{2h_R}
\]

where

\[-\infty < \text{Re} \, w < \infty \quad \text{Im} \, w \approx \text{Im} \, w + 1/T_L \quad \text{Im} \, \bar{w} \approx \text{Im} \, \bar{w} + 1/T_R.\]

If one looks at a slice of fixed Euclidean time, say Im \( w = 0 \), then the 2-point function falls off exponentially with spatial separation. Thus at finite temperature there is a finite correlation length [40]. Moreover the correlation function factorizes into holomorphic (left-moving) and anti-holomorphic (right-moving) pieces. The holomorphic part of the correlation function shows that the correlation length in the \( x^+ = t + x \) light-front direction is given by

\[
\xi^+ = 1/2\pi T_L h_L
\]

while the anti-holomorphic part of the correlator shows that the correlation length in the \( x^- = t - x \) direction is

\[
\xi^- = 1/2\pi T_R h_R.
\]

To measure a correlation length in the \( x^+ \) direction one needs to consider excitations which propagate in \( x^+ \). Thus the light-front correlation length for right-moving excitations is governed by \( T_L \), while for left-moving excitations it is governed by \( T_R \). This is the CFT origin of the somewhat puzzling interchange of left and right in the quasiparticle lifetimes.

These correlation lengths can be seen in a more precise way from the results of [38,41], who computed the Fourier transform of the retarded Green’s function in the CFT and found a set of poles at

\[
k_+ = \frac{1}{2}(\omega - k) = -i2\pi T_L (h_L + n) \quad n = 0, 1, 2, \ldots
\]

19
The $n = 0$ pole determines the correlation length in the $x^+$ direction, and therefore sets the lifetime of the right-moving quasiparticles. The poles with $n > 0$ govern the subleading exponential fall-offs in the correlator (35). Likewise there are poles at

$$k_- = \frac{1}{2}(\omega + k) = -i2\pi T_R(h_R + n) \quad n = 0, 1, 2, \ldots$$

(37)

which set the lifetime of the left-moving quasiparticles.

Let us make a few comments on these CFT results. At finite temperature even free field theories have finite correlation lengths. So our discussion is perfectly applicable to any operator with non-trivial scaling dimension, even in a free CFT. Note that it is important to distinguish between the correlation lengths and thermalization times of the system. With generic interactions one would expect these scales to be about the same. However, as we mentioned, correlation lengths are finite even in free field theories which do not thermalize.

These CFT results have a limited range of validity, since the correlator (35) is only exact when the spatial coordinate of the CFT is non-compact. However we are interested in a CFT which lives on a circle, since the $\phi$ coordinate of the BTZ metric is periodic with period $2\pi$. One can only trust the general results (36), (37) as long as $T_R, T_L \gg 1$. Fortunately this is the regime where the BTZ black hole (as opposed to thermal AdS) dominates the partition function [42]. For further discussion of this issue see [38].

4.4 Additional comments

It is interesting to compare the quasiparticle approach with that of [43] where a low-energy effective string description of the dynamics of four and five-dimensional black holes was deduced from supergravity arguments. This picture was far more detailed than the one we have presented. For example in four and five dimensions [43] argued for $(0, 4)$ supersymmetry of the CFT. The resulting CFT required interactions of the left and right moving degrees of freedom to describe Hawking radiation and scattering from the black hole. Our quasiparticle picture is rather more crude, and unlikely to reproduce the detailed low energy scattering predictions of that model. On the other hand the quasiparticle picture is much simpler in that everything is expressed
in terms of quasi-free particles, that is, particles with finite energies and lifetimes but with no other interactions. The quasiparticle picture suffices to describe a limited class of dynamic observables, but as we emphasize in this paper, it is applicable to a much wider range black holes and it also makes the entropy – area relationship manifest.

We would also like to make some comments on the extremal limit \( r_+ \to r_- \). For the BTZ geometry (21) the horizon area remains non-zero in the extremal limit. That is, for BTZ black holes we do not face the difficulties with degenerate horizons that will be discussed in section 5. We therefore expect that the basic quasiparticle description remains valid at extremality. However working out a quasiparticle picture of the extremal limit could be somewhat involved. By staying away from extremality we kept the occupation numbers \( N_R, N_L \gg 1 \), which enabled us to give a simple thermodynamic treatment of the quasiparticle gas. It also allowed us to avoid the breakdown of the semiclassical approximation at extremality discussed in [44].

Finally, the BTZ case makes it clear that the real part of the lowest quasinormal mode frequency \( k \) in (31), (33) cannot be simply identified with the average quasiparticle energy.\(^{11}\) Following [44], the energy gap above the extremal BTZ state is the energy associated with the lowest nontrivial quasiparticle mode when \( N_L \sim 1 \), which implies \( T_L \sim 1/\ell \), so that \( \Delta E_{CFT} \sim 1/\ell \) or \( \Delta E \sim 1/\ell^2 \).\(^{12}\) On the other hand, the real part of the frequency of the lowest nontrivial quasinormal mode is \( k = 1 \), which suggests an energy gap \( \Delta E_{CFT} = 1 \). This is related to the observation of [45] that multiply wound effective strings are needed to correctly describe the low-energy degrees of freedom. The quasiparticles would then be periodic under \( \phi \to \phi + 2\pi \ell \). The operators in the CFT that couple to quasinormal modes are periodic under \( \phi \to \phi + 2\pi \), so must be built out of composites of the elementary quasiparticles. Though the quasiparticle energy gap \( 1/\ell \ll T_L, T_R \) when \( N_L, N_R \gg 1 \), the average quasiparticle energies \( T_L, T_R \) can be deduced by assuming a thermal distribution of left and right moving particles at temperatures \( T_L \) and \( T_R \) respectively and using the equipartition theorem.

\(^{11}\)This was not an issue for the Schwarzschild black holes and the black Dp-branes considered previously, where the real part of the quasinormal mode frequency was of order the quasiparticle energy.

\(^{12}\)Keeping in mind that we are working in units where \( 8G = 1 \), this matches the energy gap of [45].
5 Comments on degenerate horizons

The quasiparticle picture we have developed clearly breaks down in the limit of vanishing horizon area. In this section we comment on some of the subtleties which arise in developing a stretched horizon or quasiparticle description of black holes with degenerate horizons. The main observation is that for degenerate horizons the stretched horizon no longer needs to be placed at the Planck temperature. That is, the arguments to that effect given in section 2 break down in the degenerate limit.

A concrete example of a degenerate horizon is provided by Sen’s black holes which are the most general charged asymptotically flat black hole solutions in heterotic string theory compactified on $T^6$. We concentrate on electrically charged but non-rotating solutions in the extremal or BPS limit.\footnote{Away from extremality we expect a well-defined quasiparticle picture, with multiple species of quasiparticles to account for the electric charge.} This example gives some lessons on the limits of validity of the quasiparticle picture.

Sen pointed out that even though the classical horizon area of these black holes vanishes, the microscopic entropy obtained by counting the corresponding heterotic string states is non-zero.\footnote{The \textit{stringy} stretched horizon is located at the radius where the local temperature coincides with the Hagedorn temperature. See the appendix of \cite{Sen2011}.} To resolve this apparent contradiction he introduced an entropy associated with the horizon area of the \textit{stringy} stretched horizon, and showed that this entropy coincides with the entropy of an elementary heterotic string having the same charge and mass as the black hole. Stretched horizons for extremal black holes have been further considered in [47].

Sen’s analysis seems to conflict with the present paper because we need to put the stretched horizon at the Planck temperature in order to derive the correct entropy–area relation. The conflict can be resolved by noting that the approximations used here and in [7] break down when the horizon area vanishes. In particular the Stefan-Boltzmann law is a thermodynamic formula which doesn’t apply when the size of the black hole is of order string scale. For Sen’s extremal black holes the classical horizon area vanishes and $\alpha'$ corrections to the spacetime action are expected to be important in...
understanding radiation from the stretched horizon.

This breakdown of the Stefan-Boltzmann law due to stringy corrections is quite general: whenever the horizon area becomes of order string scale the simple quasiparticle picture of the horizon breaks down. Fortunately we have other tools for studying such small black holes. In particular the Stefan-Boltzmann law breaks down at the Horowitz - Polchinski correspondence point [48], where one can match to the spectrum of elementary string excitations. Also in the extremal limit one can use supersymmetric non-renormalization theorems to study a microscopic description of the black hole.

6 de Sitter space

In this section we turn from black hole horizons to cosmological horizons, and present a quasiparticle description of the static patch of empty de Sitter space.

There has been considerable debate over the correct quantum description of de Sitter space [49]. Some groups have argued that the inflationary patch of de Sitter is dual to a conformal field theory, in which case the thermodynamic properties of the static patch might arise along the lines of [50]. Other groups have argued that de Sitter space is unstable [51], in which case there could be no precise dual description of the static patch. But even if de Sitter space is ultimately unstable the quasiparticle picture could be valid, in the sense that it provides an effective description of the static patch accurate on timescales short compared to the de Sitter lifetime. This would be analogous to the quasiparticle description of a Schwarzschild black hole developed in section 2.

6.1 de Sitter thermodynamics

The Euclidean metric for $d$-dimensional de Sitter space is

$$ds^2 = \left(1 - \frac{r^2}{\ell^2}\right) dt_E^2 + \left(1 - \frac{r^2}{\ell^2}\right)^{-1} dr^2 + r^2 d\Omega_{d-2}^2$$

(38)
where $\ell$ is the de Sitter radius, related to the cosmological constant by $\Lambda = (d-2)(d-1)/2\ell^2$. This space is smooth at the horizon $r = \ell$ provided the Euclidean time coordinate is periodically identified, $t_E \approx t_E + 2\pi\ell$, with a periodicity corresponding to the de Sitter temperature $T = 1/2\pi\ell$.

To compute thermodynamic quantities we follow [52] and evaluate the Euclidean partition function off-shell. That is we periodically identify $t_E \approx t_E + \beta$ but keep $\beta$ arbitrary; in general this leaves a conical singularity at the horizon. In the semiclassical approximation the partition function is $Z = e^{-I}$ where the Einstein-Hilbert action

$$I = -\frac{1}{16\pi G} \int d^d x \sqrt{g}(R - 2\Lambda).$$

We will regard the action as consisting of two contributions. There is a bulk contribution

$$I_{\text{bulk}} = -\frac{1}{16\pi G} \cdot \frac{\beta \ell A}{d-1} \cdot \frac{2(d-1)}{\ell^2} = -\frac{A}{4G} \cdot \frac{\beta}{2\pi\ell}.$$  \hspace{1cm} (39)

In deriving this we wrote the volume of Euclidean de Sitter space

$$\int d^d x \sqrt{g} = \frac{\beta \ell A}{d-1}$$

where $A = 2\pi^{(d-1)/2} \ell^{d-2}/\Gamma((d-1)/2)$ is the area of the event horizon. We also used the expression for the bulk scalar curvature $R = 2d\Lambda/(d-2)$, with $\Lambda = (d-2)(d-1)/2\ell^2$. The action also gets a contribution from the curvature singularity at the horizon. The horizon contribution has the universal form [52]

$$I_{\text{horizon}} = -\frac{A}{4G} \left(1 - \frac{\beta}{2\pi\ell}\right)$$

Corresponding to this decomposition of the action one finds two contributions to the semiclassical energy and entropy of de Sitter space. The bulk contributions are

$$E_{\text{bulk}} = \left. \frac{\partial}{\partial \beta} \right|_{\beta=2\pi\ell} I_{\text{bulk}} = -\frac{A}{8\pi G \ell}$$

$$S_{\text{bulk}} = \left. \left(\beta \frac{\partial}{\partial \beta} - 1\right) \right|_{\beta=2\pi\ell} I_{\text{bulk}} = 0.$$  \hspace{1cm} (40)
The contributions from the horizon are

\[
E_{\text{horizon}} = \frac{\partial}{\partial \beta} \bigg|_{\beta=2\pi \ell} I_{\text{horizon}} = \frac{A}{8\pi G \ell} \quad (41)
\]

\[
S_{\text{horizon}} = \left( \beta \frac{\partial}{\partial \beta} - 1 \right) \bigg|_{\beta=2\pi \ell} I_{\text{horizon}} = \frac{A}{4G}.
\]

Note that the total energy \( E_{\text{bulk}} + E_{\text{horizon}} \) vanishes \[53\], while the entropy arises solely from the horizon.

To understand this better we turn to the work of Susskind \[13\], who pointed out that from the string theory point of view the bulk contribution arises at closed-string tree level, from spherical worldsheets which do not touch the horizon. We do not expect the quasiparticles to capture the corresponding bulk energy. The contribution from the singularity, on the other hand, arises from spherical worldsheets which are pierced by the horizon. In a Hamiltonian framework such diagrams would describe open strings stuck on the horizon \[13\]. We take the quasiparticles to correspond to these open string degrees of freedom, and therefore expect the quasiparticle gas to reproduce the energy and entropy associated with the horizon.

As usual, we introduce a stretched horizon at a radius where the proper temperature is equal to the Planck temperature. The proper rate at which the stretched horizon emits energy in outgoing Hawking radiation is then

\[
\frac{dE_{\text{proper}}}{dt_{\text{proper}}} \sim AT_{\text{proper}}^d.
\]

To work in terms of energy \( E \) conjugate to the de Sitter time coordinate \( t \) we multiply by the appropriate redshift factors, and find that the rate at which energy is radiated is given by

\[
\frac{dE}{dt} \sim \frac{A}{\ell_{\text{Planck}}^{d-2}} T^2. \quad (42)
\]

### 6.2 Quasiparticle description

It is straightforward to give a quasiparticle description of the static patch of de Sitter space. We postulate that the number of quasiparticles is given by
the de Sitter entropy, \( N \approx S_{\text{horizon}} \). The quasiparticle gas has a temperature equal to the de Sitter temperature \( T = 1/2\pi\ell \). Each quasiparticle has an energy \( \epsilon \approx T \) and a lifetime \( \tau \approx 1/T \).

It is trivial to see that the thermodynamics of such a quasiparticle gas reproduces the energy and entropy \( \epsilon \) associated with the de Sitter horizon. The quasiparticle description also predicts the rate at which the horizon radiates energy.

\[
\frac{dE}{dt} = N\epsilon / \tau
\]

This agrees with the Stefan-Boltzmann law \( \epsilon \), provided one identifies the entropy of the quasiparticle gas \( S \sim N \) with the area of the horizon in Planck units.

### 6.3 de Sitter quasinormal frequencies

To provide further evidence for the proposed quasiparticle lifetimes we proceed to study quasinormal modes in de Sitter space. For simplicity we consider a massless, minimally-coupled scalar field.

The wave equation reads

\[
\left( -\left( 1 - \frac{r^2}{\ell^2} \right)^{-1} \partial_t^2 + \frac{1}{r^{d-2}} \partial_r r^{d-2} \left( 1 - \frac{r^2}{\ell^2} \right) \partial_r + \frac{1}{r^2} \nabla^2_{S^{d-2}} \right) \phi = 0 .
\]

Changing to a dimensionless radial coordinate \( \rho = r/\ell, 0 \leq \rho \leq 1 \) and separating variables \( \phi(t, \rho, \Omega) = e^{-i\omega t} \phi(\rho)Y_j(\Omega) \), where \( Y_j \) is a spherical harmonic on \( S^{d-2} \) with angular momentum \( j \), one obtains the radial wave equation

\[
\left( \frac{1}{\rho^{d-2}} \partial_\rho \rho^{d-2}(1 - \rho^2) \partial_\rho + \frac{\ell^2\omega^2}{1 - \rho^2} - \frac{j(j + d - 3)}{\rho^2} \right) \phi(\rho) = 0 . \tag{43}
\]

Quasinormal frequencies are determined by requiring that \( \phi \) be smooth at \( \rho = 0 \) (the center of the static patch) and purely outgoing at \( \rho = 1 \) (that is, exiting the static patch at the horizon). From (43) it’s clear that the quasinormal frequencies are proportional to the de Sitter temperature.

\[
\omega \sim 1/\ell \sim T
\]
The quasinormal frequencies were determined exactly in [54]. For \( d \geq 3 \), the unique solution which obeys the correct boundary conditions is\(^{15}\)

\[
\phi(\rho) = \rho^j (1 - \rho^2)^{i\ell \omega/2} F(\alpha, \beta, \gamma; \rho^2)
\]

\[
\alpha = \frac{j + d - 1 + i\ell \omega}{2}, \quad \beta = \frac{j + i\ell \omega}{2}, \quad \gamma = j + \frac{d - 1}{2}
\]

where the quasinormal frequencies are given by

\[
\omega = -\frac{i}{\ell} (j + 2n) \quad \text{or} \quad \omega = -\frac{i}{\ell} (j + d + 2n - 1) \quad n = 0, 1, 2, \ldots
\]

(the second possibility is redundant if \( d \) is odd). It is curious that the de Sitter quasinormal frequencies are purely imaginary. We do not have an intuitive understanding of this fact.

To summarize, quasinormal excitations in de Sitter space decay on a timescale set by the inverse temperature, \( \omega^{-1} \sim \ell \sim 1/T \), in agreement with our proposed quasiparticle lifetime.

### 7 Conclusions

Long ago it was argued that the simplest way to think about black holes is in terms of a stretched horizon located some distance outside the true event horizon [1]. The stretched horizon has been shown to be an equally useful concept in the quantum theory [2]. In this paper we have analyzed several examples of black hole and cosmological horizons, and shown that many properties of the stretched horizon can be understood in terms of a simple gas of non-interacting quasiparticles. The quasiparticle gas reproduces the equilibrium thermodynamics of the horizon. In particular it naturally accounts for the universal Bekenstein-Hawking relation between entropy and horizon area. Moreover, by relating the lifetime of the quasiparticles to the imaginary part of the lowest quasinormal frequency, we have shown that the quasiparticle gas correctly describes thermalization on the horizon. In

\(^{15}\)Two dimensional de Sitter space is conformal to a finite cylinder. One can have left- and right-moving waves on the cylinder, but there doesn’t appear to be any sensible definition of quasinormal modes.
particular it correctly accounts for the universal thermalization relations (24), (25).

There are several interesting directions in which the quasiparticle picture could be extended. It would be worthwhile to work out the detailed quasiparticle description of an extremal BTZ black hole. It would also be interesting to study charged black holes, such as the Reissner-Nordstrom solution or the Sen black holes mentioned in section 5. Presumably charged black holes are analogous to the rotating BTZ solution, in that more than one species of quasiparticle is required. Finally, it would be interesting to develop a quasiparticle picture for Rindler space. This would underscore the universality of the quasiparticle description, since all macroscopic horizons are locally approximately Rindler. In attempting this one faces a puzzle, that the redshift factor at the Planck stretched horizon of a Schwarzschild black hole

\[ \frac{1}{\sqrt{-g_{tt}}} = \frac{T_{\text{Planck}}}{T} \sim \left( \frac{\ell_{\text{Planck}} M}{\ell_{\text{Planck}} M} \right)^{1/(d-3)} \]

diverges in the Rindler limit \( M \rightarrow \infty \). A natural way to avoid this difficulty would be to work in terms of proper quantities measured at the stretched horizon. Indeed one might ultimately hope to derive the quasiparticle description starting from an action principle for a membrane located at the stretched horizon [55].

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A **Numerical algorithms for quasinormal frequencies**

We use Mathematica to solve for quasinormal mode frequencies. The wave equation (9) is solved using a power series expansion about the regular singu-
lar point \( z = 0 \). The coefficients are determined using a recursion relation, as shown in the code below, and the boundary condition (12) is selected. This solution is matched at a regular point (e.g. \( z = 1/2 \)), with a numerical solution of (9) obtained via the Runge-Kutta method, starting from \( z = 1 - \epsilon \), \( \epsilon \ll 1 \), with the boundary condition (11). The matching involves setting \( f'(z)/f(z) \) equal, which is achieved by iteratively adjusting \( \rho \) using Mathematica’s FindRoot routine. The FindRoot routine uses an initial guess for the frequency coming from solving (11) using the power series method [25] truncated to 10 terms.

\[
\begin{align*}
n &= 400; \quad p = 0; \quad xx = 1/2; \quad (* \text{Adjustable parameters: } n = \text{number of terms in power series, } p = p\text{-brane, } xx = \text{matching point} *) \\
(b_1[0] &= 0.5581972109 - 0.9281852713 \ I; \\
b_1[1] &= 0.6221606713 - 0.8830128157 \ I; \\
b_1[2] &= 0.6985155813 - 0.8030747084 \ I; \\
b_1[3] &= 0.767368820 - 0.657852677 \ I; \\
b_1[4] &= 0.7647605329 - 0.4398341076 \ I; \\
\end{align*}
\]

\( n=400; \quad p=0; \quad xx = 1/2; \) (* Adjustable parameters: \( n \) = number of terms in power series, \( p \) = p-brane, \( xx \) = matching point *)

(* Construct power series solution using recursion relation *)
ser = Series[1/(1-x)^(9-p)/(7-p)),{x,0,n}];
Do[b[l]=SeriesCoefficient[ser,l],{l,0,n}];
f[rho_]:=Module[{},a[0]=1;
Do[a[k]=N[-1/((k-I rho)^2+rho^2) Sum[a[k-l]b[l]rho^2,
{k,1,k}],100] ,{k,1,n}];
N[Sum[(k-I rho) a[k] (xx)^(k-I rho -1),{k,0,n}],100]/
N[Sum[a[k] (xx)^(k-I rho),{k,0,n}],100]];

(* Solve ODE using Runge-Kutta *)
eps=10^-25;
g[rho_]:= Module[{},
ans = NDSolve[ {w^((9-p)/(7-p)) ((1-w)^2 D[chi[w],w] -
(1-w)chi[w]) + rho^2 phi[w] == 0, D[phi[w],w]==chi[w],
phi[eps]==0,chi[eps]==1}, {phi,chi}, {w,3/5,eps},,
WorkingPrecision->30,Method->RungeKutta,MaxSteps->10000];
N[-chi[1-xx]/phi[1-xx] /. ans[[1]]];
FindRoot[ g[rho]-f[rho]==0, {rho,b[1][p],b[1][p]-.01}, AccuracyGoal->12,
B General $p$-branes

B.1 Decoupling limits

We start with black $p$-brane solutions of the gravity action in $D$ dimensions \[10, 11\]
\[
S = -\frac{1}{2\kappa^2} \int d^D x \sqrt{g} [R - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2(d + 1)!} e^{a \phi} F_{d+1}^2] \tag{44}
\]
where $D = p + d + 3$. The solution with one harmonic function takes the form (in string frame)
\[
ds^2 = H^{\alpha - \frac{2N}{d}}(r)(H^{-N}(r)[-f(r) dt^2 + dy_1^2 + \cdots + dy_p^2] + f^{-1}(r) dr^2 + r^2 d\Omega_{d+1}^2) \tag{45}
\]
where
\[
H(r) = 1 + \frac{\mu^d \sinh^2 \gamma}{r^d} \quad f(r) = 1 - \frac{\mu^d}{r^d}. \tag{46}
\]
The dilaton and gauge potential are
\[
e^{2(\phi(r) - \phi(\infty))} = H^{-aN} \tag{47}
\]
\[
F_{d+1} = \frac{1}{2} \sqrt{N} d \mu^d \sinh 2\gamma \epsilon_{d+1}
\]
and the parameters have a relationship
\[
\alpha = \frac{N(p + 1)}{D - 2}, \quad N = 4[a^2 + \frac{2d(p + 1)}{D - 2}]^{-1}. \tag{48}
\]
In the near extremal limit the charge and thermodynamical functions (per unit volume) are given by \[12\]
\[
q_p = \frac{\omega_{d+1}}{2\sqrt{2\kappa}} d \sqrt{N} \mu^d \sinh 2\gamma
\]
\[
E = \frac{d \lambda \omega_{d+1}}{\sqrt{N} \kappa^2} \mu^d \tag{49}
\]
\[
S = 4\pi d^{-(d+1)/d} \lambda^{-\frac{1}{2d+1}} ((\sqrt{2\kappa})^{2/d-N/2} \left(\frac{q_p}{\sqrt{N}}\right)^{N/2} E^{\lambda}
\]

30
where
\[ \lambda = \frac{d + 1}{d} - \frac{N}{2}. \]  

(50)

We want to go to a near horizon limit for which we can define a decoupled theory. Let us introduce a length scale that we will call \( l_s \) (which would be taken to zero at the end), this could be the string length but could also be the Planck length.

We will change variables
\[ r = U^{2b-1}l_s^{2b} \]  

(51)

where \( b \) at the moment is a free parameter. Another free parameter (at the moment) is \( \beta \) where
\[ \mu^d e^{2\gamma} = g^2 l_s^{2\beta}. \]  

(52)

where \( g^2 \) is some dimensional parameter which is kept fixed in the limit \( l_s \to 0 \). To get a finite relationship between energy and horizon size one has to fix in this limit a parameter which we call \( g_{ym} \), which satisfies
\[ \kappa l_s^{-bd} = g_{ym}. \]  

(53)

To get a metric which scales like \( l_s^2 \) (in the string frame) one has two conditions

\[ N(\beta - db) = -2b \]
\[ a \frac{4}{D} = \frac{p + 1}{D - 2} - 1 + \frac{1}{2b} \]  

(54)

We also want to get a finite dilaton \( e^{\phi} \) after decoupling. For this we need to know how \( g_s = e^{\phi(\infty)} \) scales in the decoupling limit. In \( D \) dimensions

\[ \kappa \sim \frac{g_s l_s^4}{\sqrt{V_{10-D}}} \]  

(55)

where \( V_{10-D} \) is the volume of the \( 10 - D \) dimensions which one has compactified on (not to be confused with \( y_1 \cdots y_p \)). If we do not wish to introduce any more dimensionful parameters then we should take

\[ V_{10-D} \sim l_s^{10-D}. \]  

(56)
In this case the condition for a finite dilaton is

$$\frac{a}{4} = \frac{1}{2b} - \frac{d}{8} + \frac{D - 10}{16b}. \quad (57)$$

Equations (54), (57) have solutions only if $D = 10$ for specific values of $a$, or in $D \neq 10$ with $a = 0$. This means that in the decoupling limit apart from the ten dimensional branes the only other decoupled solutions are those with a constant dilaton. Indeed if there is no dilaton ($a = 0$, or M-branes) then the equations always have a solution

$$b = \frac{D - 2}{2(D - p - 3)}. \quad (58)$$

It turns out that in all cases the conditions (54), (57) are enough to insure a finite relationship between energy density and entropy density.

The metric that one gets after this procedure (in Einstein frame) is given by

$$ds^2 = l_s^2 \left( \frac{g^2}{U^{d(2b-1)}} \right)^{\alpha - N} \left[ -(1 - \left( \frac{U_h}{U} \right)^{d(2b-1)} ) dt^2 + dy_i^2 ight. \\
+ \left. \left( 1 - \left( \frac{U_h}{U} \right)^{d(2b-1)} \right)^{-1} g^{2N} U^{(2b-1)(2-Nd)-2} dU^2 + g^{2N} U^{(2b-1)(2-Nd)} dS_{d+1}^2 \right] \quad (59)$$

We also have the temperature – energy relationship

$$T^{-1} \sim e^N g_{ym}^{2(2/d - N)} E^{\lambda-1} \quad (60)$$

and the energy – radius relationship

$$E \sim g_{ym}^{-4} U_h^{d(2b-1)} \quad (61)$$

which gives a temperature – radius relationship

$$T^{-1} \sim g^N U_h^{d(2b-1)} \quad (62)$$

that is independent of what we called $g_{ym}$.

We now define a new coordinate $\rho$

$$\rho = g^N U_h^{d(2b-1)(\lambda-1)} \quad (63)$$

and the Einstein metric takes the form

$$ds^2 = l_s^2 \left( \frac{g^{N/(\lambda-1)+1}}{\rho^{1/(\lambda-1)}} \right)^{\alpha - N} \left[ -(1 - \left( \frac{\rho h}{\rho} \right)^{1/(\lambda-1)} ) dt^2 + dy_i^2 ight. \\
+ \left. \frac{2d^2(2b-1)}{2 - dN} \left( 1 - \left( \frac{\rho h}{\rho} \right)^{1/(\lambda-1)} \right)^{-1} d\rho^2 + \rho^2 d\Omega_{d+1}^2 \right] \quad (64)$$
B.2 Quasinormal modes

After changing variables to $z = 1 - \left(\frac{\rho h}{\rho}\right)^{1/(\lambda-1)}$, the wave equation for a minimally coupled scalar in the metric (64) becomes

$$z \frac{\partial}{\partial z} \left( z \frac{\partial f}{\partial z} \right) + \rho^2 (1 - z)^{-2+N+\frac{2}{D-1}} f(z) = 0$$

(65)

where

$$\rho^2 = \frac{\omega^2 e^{2N} \mu^{2-Nd} \delta}{d^2}$$

(66)

and $\delta$ may be read off from (64). Again we find that $\omega$ is proportional to the Hawking temperature of the black brane. Let us now consider supersymmetric examples which are non-dilatonic ($a = 0$) for $D < 10$, generalizing the results of section 3.

For $D = 6$, $p = 1$, $N = 2$ the wave equation is soluble analytically. The solution ingoing on the future horizon is

$$f(z) = z^{-i\rho} {}_2F_1(-i\rho, -i\rho, 1 - 2i\rho, z).$$

(67)

Solving the Dirichlet boundary condition at infinity leads to the equation

$$1 - i\rho = -j$$

(68)

where $j$ is a positive integer, so the quasinormal mode frequencies are $\rho = -i(j + 1)$.

For $D = 5$, $n = 3$, $p = 0$ we get the same wave equation for the lowest mode as when $D = 4$, $n = 4$, $p = 0$. The decoupled geometry is simply $AdS_2$ times a sphere of constant size. The wave equation is again soluble analytically

$$f(z) = Ae^{i\rho \log z} + Be^{-i\rho \log z}$$

(69)

with $A$ and $B$ constants of integration. There are therefore no quasinormal modes, only purely ingoing or outgoing solutions.

B.3 Quasiparticle description

In the above cases we find that the quasinormal mode frequency is proportional to the Hawking temperature of the black brane. We can then try to
reproduce the thermodynamic properties of the black brane using a quasiparticle picture. We postulate that the microscopic degrees of freedom giving a holographic description of the black hole (or more generally the degrees of freedom that make up the stretched horizon of the black brane) can be treated as a gas of quasi-free particles in one lower dimension. We associate a quasiparticle lifetime with the imaginary part of the lowest quasinormal mode frequency. We assume the number of quasiparticles accounts for the entropy of the black brane, and that each quasiparticle carries energy of order $T$. The entropy is power-law in the temperature,

$$S \propto E^\lambda. \quad (70)$$

It follows that the thermalization rate is

$$\frac{dE}{dt} \propto AT^2. \quad (71)$$

This agrees with a calculation of the same quantity using the Stefan-Boltzmann law, assuming thermal radiation is emitted off a stretched horizon one Planck distance from the event horizon, as in [7].

The $D = 5$, $n = 3$, $p = 0$ black hole and its cousin the $D = 4$, $n = 4$, $p = 0$ case are rather problematic in the decoupling limit. In each case the geometry is $AdS_2$ times a sphere. In these cases $\lambda = 0$, so (70) implies the temperature diverges. The decoupling limit is rather degenerate in this case, as discussed in [56] where more general decoupling limits are also considered. Furthermore, quantum corrections violate decoupling of the $AdS_2$ throat and the asymptotically flat region [57]. In these cases the properties of the quasiparticles cannot be deduced from the classical geometry alone. However if we include the asymptotically flat region, we expect to get well defined quasinormal modes, and we should be able to construct a sensible quasiparticle description. It would be interesting to examine the quasinormal modes of the general asymptotically flat $D = 4,5$ black holes constructed in [58, 59] and develop a quasiparticle picture for these cases. In these cases, we expect that multiple species of quasiparticles are needed to correctly describe the low energy dynamics of the black holes, as was the case for the BTZ black hole that we analyzed in detail.
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