Probing the Orbital Angular momentum through the Polarized Gluon Asymmetry

Gordon P. Ramsey
Physics Dept., Loyola University Chicago and
HEP Division, Argonne National Lab; †

March 26, 2022

Abstract

The orbital angular momentum is one of the least understood of the spin characteristics of a proton. There are no direct ways to model $L_z$. However, the $J_z = \frac{1}{2}$ sum rule includes an angular momentum component and can provide indirect access to properties of $L_z$. One of the other unknowns in the sum rule is the gluon polarization, $\Delta G$. We can define the gluon spin asymmetry in a proton as $A(x, Q^2) = \frac{\Delta G(x, Q^2)}{G(x, Q^2)}$. This can be written as a sum of a $Q^2$-invariant piece, $A_0(x)$ and a small $Q^2$-dependent term, $\epsilon(x, Q^2)$. The $x$-dependence of $A_0$ can be calculated and a suitable parametrization for $\epsilon(x, Q^2)$ can be made to estimate this asymmetry. When combined with the measured unpolarized gluon density, $G(x, Q^2)$, this provides a model independent prediction for $\Delta G(x, Q^2)$. This eliminates one unknown in the $J_z = \frac{1}{2}$ sum rule and allows a reasonable estimate for the size and evolution of the orbital angular momentum of the constituents, $L_z$.

1 Introduction

High energy spin physics studies have evolved into three primary areas. The first involves helicity components of the constituents of the proton spin. From the initial EMC experiments that defined the ”proton spin problem”, these studies have focussed on determining the proportion of spin carried by the valence and sea quarks in addition to the gluons. The remainder of the nucleon spin is attributed to the

---

*Talk given at the Spin Physics Symposium (SPIN 2005), 16-21 September 2005, Dubna, Russia.
†Work supported by the U.S. Department of Energy, Division of High Energy Physics, Contract W-31-109-ENG-38. E-mail: gpr@hep.anl.gov
orbital angular momentum. The second important area of spin studies involves transversity measurements. These not only involve the transverse motion of the nucleon constituents, but the dynamics of the interactions among them. The Sivers, Collins and Boer-Mulders functions all contain information about transversely polarized quarks in various polarized states of nucleons. In all cases, this information is related to the orbital motion carried by these constituents. Finally, our studies of generalized parton distributions constitute an attempt to put these and other spin phenomena into a general formalism. In all of these areas, the nature of the orbital angular momentum of nucleon constituents plays an important role.

2  Determination of $\Delta G(x)$ with the Gluon Asymmetry

The $J_z = \frac{1}{2}$ sum rule can be used to access the orbital angular momentum phenomenologically. This involves the integrated parton densities.

$$J_z = \frac{1}{2} \Delta \Sigma + \Delta G + (L_z)_{q+G},$$  \hspace{1cm} (1)

where $\Delta \Sigma$ is the total spin carried by all quarks, $\Delta G$ the spin carried by gluons and $(L_z)_{q+G}$ is the orbital angular momenta of the quarks and gluons. Numerous DIS experiments have narrowed the quark spin contribution $(\Delta \Sigma)$ to within a reasonable degree. However, the gluon and orbital angular momentum components of the nucleon spin are virtually unknown.

Experiments are presently underway in various kinematic regions [1, 2] to determine this distribution. Others have been proposed to expand these measurements to other kinematic regions. [3] Meanwhile, there have been many models assumed for $\Delta G$. Our calculation of the gluon asymmetry does not presume any specific model for the polarized gluons, but relies on simple theoretical assumptions and the measurements of the unpolarized gluon distribution.

We define the gluon polarization asymmetry as

$$A(x, t) = \frac{\Delta G(x, t)}{G(x, t)},$$  \hspace{1cm} (2)

where the evolution variable $t$ is defined as $t \equiv \ln[\alpha_s(Q^2_0)/\alpha_s(Q^2)]$. Since there are no overwhelming theoretical arguments favoring any single model for $\Delta G$, we consider a more direct argument for its shape in terms of this asymmetry. The conclusions we draw follow from the observation that, in the absence of a “valence” gluon, both $G(x, t)$ and $\Delta G(x, t)$ exhibit scaling violations which can be associated with measurements resolving radiative diagrams. The diagrams leading to positive and negative helicity gluons are the same. This implies that the relative probability of measuring a gluon of either helicity does not depend upon $t$. Thus, to a first approximation, the gluon polarization asymmetry is predicted to be at most, mildly scale dependent.
To construct a theoretical model of the asymmetry, we assume that it has a scale independent part, $A_0(x)$ plus a small piece that vanishes at some large scale. Thus, we can write the asymmetry as

$$A(x, t) = A_0(x) + \epsilon(x, t) \equiv \Delta G / G. \quad (3)$$

Then, $\Delta G$ can be written in terms of the calculated asymmetry $A_0(x)$ and a difference term as $\Delta G = A_0 \cdot G + \epsilon \cdot G$. The scale invariant $A_0$ is calculable and is independent of theoretical models of $\Delta G$. The second term is interpreted as the difference between the measured polarized gluon distribution and that predicted by the calculated $A_0$ combined with measurement of the unpolarized gluon density. It is reasonable to choose $t = 0$ to coincide with a typical hadronic scale, $Q^2 = m_h^2$.

The calculable part can be found by taking the $t$-derivative of $A_0(x)$:

$$\frac{dA_0}{dt} = G^{-1} \left[ \frac{\Delta G}{dt} - A_0 \cdot \frac{dG}{dt} - \epsilon \cdot \frac{dG}{dt} \right] - \frac{d\epsilon}{dt} = 0. \quad (4)$$

We assume at some scale that the quantity $\epsilon \cdot G$ is $t$-independent. Thus,

$$A_0 = \frac{\Delta \Delta G}{\frac{dG}{dt}}. \quad (5)$$

Both $\frac{\Delta \Delta G}{dt}$ and $\frac{dG}{dt}$ are calculated using the DGLAP evolution equations. So

$$A_0 = \frac{\frac{\Delta \Delta G}{dt}}{\frac{dG}{dt}} = \left[ \Delta P_{Gq} \otimes \Delta q + \Delta P_{GG} \otimes \Delta G \right]. \quad (6)$$

Since $\Delta G$ has not been measured, equation 6 can be converted into an iterative equation for $A(x)$ by inserting $\Delta G(x, t) = A(x) \cdot G(x, t)$ from equation 3 into the convolution,

$$A_{n+1} = \left[ \frac{\Delta P_{Gq} \otimes \Delta q + \Delta P_{GG} \otimes (A_n \cdot G)}{P_{Gq} \otimes q + P_{GG} \otimes G} \right]. \quad (7)$$

This equation then can be solved iteratively. With a suitable assumption for $\epsilon(x, t = 0)$, the asymmetry $A(x, t)$ can be determined at all $x$ and $t$ values. From the counting rules, we bound $\epsilon(x, t)$ by $\epsilon(x, t) \leq c(t) \cdot x(1 - x)$ and require $\epsilon(x, t)$ to be decreasing at some scale, since its evolution is opposite that of the gluon. Then, the form for $\epsilon(x, t = 0) = x(1 - x)^n$, where $n$ is the power of $(1 - x)$ in $G(x)$ and we assume that $c(0) \equiv 1$.

For the starting distributions in equation 6 and the iterations of equation 7, we use the polarized quark distributions outlined by GGR [6] using the CTEQ4M and MRST unpolarized distributions. [7, 10] The evolution was performed in LO, since the NLO contributions to the splitting kernels, calculated in ref.[8], are most dominant at small-$x$, where the asymmetry is the smallest. Work is in progress to ensure that the effects of NLO are not significant for the ratio $\frac{\Delta G}{G}$. The iteration is
Figure 1: Ranges of the Gluon Asymmetry versus $x$.

Figure 2: $x\Delta G$ versus $x$ for the range of asymmetries shown in Figure 1.
relatively stable and converges within a few cycles. Some models of $G(x)$ converge more uniformly than others. The resulting range of shapes for $A(x)$ generated by these distributions is shown in Figure 1. The corresponding extremes of $x\Delta G$ are shown in Figure 2.

The HERMES experimental group at DESY has measured the longitudinal cross section asymmetry $A_{||}$ in high-$p_T$ hadronic photoproduction. [1] From this and known values of $\Delta q$ from DIS, a value for $A_G(x_G)$ are extracted. Here, $x_G = s/2M\nu$ is the nucleon momentum fraction carried by the gluon. The COMPASS group has also measured this asymmetry at a slightly smaller average value of $x_G$. [11] The results of these measurements are

- HERMES: $A_G = 0.41 \pm 0.18$ (stat.) $\pm 0.03$ (syst.) at $<x_G> = 0.17$
- COMPASS: $A_G = 0.06 \pm 0.31$ (stat.) $\pm 0.06$ (syst.) at $<x_G> = 0.095$.

Our range of asymmetries falls within these values.

The corresponding calculation of $L_z$ and its evolution involves using the $J_z = \frac{1}{2}$ sum rule and the DGLAP evolution equations.

$$L_z = \frac{1}{2} - \Delta \Sigma/2 - (A_0 + \epsilon) \cdot G.$$ (8)

From its derivative with respect to $t$ and the evolution equations, the evolution of $L_z$ is

$$\frac{dL_z}{dt} = -[\Delta P_{qq} \otimes \Delta q + \Delta P_{qG} \otimes ((A_0 + \epsilon) \cdot G)]/2 - (A_0 + \epsilon)[P_{Gq} \otimes q + P_{GG} \otimes G].$$ (9)

### 3 Results and tests for $L_z$ and its evolution

A plot of the $L_z(x)$ is shown in Figure 3. Differences in the MRS and CTEQ based distributions can be seen from the range shown. The evolved $L_z$ to 100 GeV$^2$ for the CTEQ based model is also shown in the figure as a dotted line. The range of results for the integral of $L_z$ is consistent with those outlined in reference [12] and the plots of $L_z(x)$ are comparable to those in reference [13]. The evolved $L_z$ to $Q^2 = 100$ GeV$^2$ shows a positive component of $L_z$ at moderate to large $x$. This can be tested in future experiments.

All of the results presented here can be tested by three separate experiments at DESY (HERMES), BNL (RHIC-STAR and PHENIX) and CERN (COMPASS). First, $\Delta G$ can be measured via prompt photon production or jet production. These processes yield the largest asymmetries for the size of $\Delta G$. [2, 9] The kinematic regions of STAR and PHENIX can determine $A$ over a suitable range of $x_{Bj}$ to test this model of the gluon asymmetry. Coupled with additional direct measurements of $A(x_G)$ from HERMES, an appropriate cross check of $\Delta G(x)$ and $G(x)$ can be made. Since the range of values for $\Delta G$ encompass both a comparable and larger
polarized glue than the GGRA model used in ref.[9], all of the asymmetries for direct-$\gamma$ and jet production should be able to narrow the values due to possible enhanced asymmetries.

The COMPASS group at CERN [3] plans to extract $A$ from the photon nucleon asymmetry, $A_{c\bar{c}N}(x_G)$ in open charm muo-production, which is dominated by the photon- gluon fusion process. This experiment should be able to cover a wide kinematic range of $x_G$ as a further check of this model. The combination of these experiments will be a good test of the assumptions of our gluon asymmetry model and a consistency check on our knowledge of the gluon distribution in the nucleon and its polarization. The corresponding predictions for the orbital angular momentum $L_z$ and its evolution can be measured via deeply-virtual Compton Scattering (DVCS) in the HERMES or COMPASS experiments.

We have outlined a possible method, based upon simple theoretical assumptions, to estimate $\Delta G(x)$ and thus provide some insight as to the $x$ and $Q^2$ dependence of the orbital angular momentum. All aspect of this model can be readily tested in the experiments of the HERMES, RHIC (STAR and PHENIX) and COMPASS groups. The results will bring us closer to understanding the nature of spin in fundamental particles.

**References**

[1] A. Airapetian, *et al.*, Phys. Lett. **84**, 2584 (2000). See also U. Stösslein, Proceedings of the SPIN 2004 Symposium, 2005, World Scientific, K. Aulenbacher, *et al.*, Editors.
[2] G. Bunce, N. Saito, J. Soffer and W. Vogelsang, Ann. Rev. Nucl. Part. Phys. 50, 525 (2000), (hep-ph/0007218).

[3] F. Bradamante, Prog. Part. Nucl. Phys. 44, 339 (2000).

[4] F. E. Close and D. Sivers, Phys. Rev. Lett. 39, 1116 (1977).

[5] G. Altarelli and G. Parisi, Nucl. Phys. B126, 298 (1977); Dokshitzer; V.N. Gribov and L.N. Lipatov, Yad. Fiz., 15, 781 (1972) and Sov. J. Nucl. Phys., 15, 438 (1972).

[6] L. E. Gordon, M. Goshtasbpour and G. P. Ramsey Phys. Rev. D58, 094017 (1998), (hep-ph/9803351).

[7] CTEQ Collaboration, H. L. Lai et al., Phys. Rev. and G. Pang and H. Zhao, Phys. Rev. D65, 014012 (2003) D51, 4763 (1995).

[8] W. Vogelsang, Acta. Phys. Polon. B29, 1189 (1998), (hep-ph/9805295).

[9] L. E. Gordon and G. P. Ramsey, Phys. Rev. D59, 074018 (1999).

[10] A. D. Martin, et. al., Eur. Phys. J. C23, 73 (2001).

[11] See Y. Bedfer, this proceedings.

[12] X. Song, hep-ph/9801332.

[13] O. Martin, et.al., Phys. Lett. B448, 99 (1999).