A principal goal of relativistic heavy ion physics is to determine bulk properties of the quark-phonon plasma (QGP), i.e. matter where the density exceeds the point at which individual hadrons can be defined. Interesting properties include the equation of state, charge susceptibility, quark-antiquark condensate, viscosity, diffusivity and jet-energy loss. Several of these properties can be reliably extracted from lattice gauge theory, but even in these cases it is important to extract the properties from experiment to test whether the idealization of a locally equilibrated QGP has indeed been realized in the collision. Careful comparisons of experiment to theoretical models have so far constrained the equation of state [1], charge susceptibility [2], viscosity [1, 3, 4], jet-energy loss [5, 6] and the diffusivity for heavy quarks [7]. Here, we describe how the diffusivity for light quarks can be added to this list.

For temperatures near or above 200 MeV, a range explored in heavy-ion collisions at the Relativistic Heavy Ion Collider (RHIC) and by the LHC at CERN, the charge susceptibility of light quarks is consistent with a picture which treats the light quarks (up, down, and strange) as well-defined quasi-particles, in accordance with lattice calculations [8]. This is despite the fact that the medium is strongly interacting. The shear viscosity and diffusivity represent measures of how strongly the matter interacts. Because there are three charges, or flavors, the diffusivity, $D_{ab}$, is a three-by-three matrix,

$$\dot{j}_a = -D_{ab} \nabla \delta \rho_b,$$

(1)

where $\rho_a$ and $j_a$ are the charge and current densities. As the medium is heated above the hadron/QGP transition temperature ($\sim 160$ MeV), the three flavors behave similarly for zero chemical potential, and the matrix becomes proportional to the unit matrix, $D_{ab} \approx D \delta_{ab}$. Such conditions are realized in the mid-rapidity regions for top RHIC and LHC beam energies. Theoretical calculations of the diffusivity are typically based on the Kubo relation for the conductivity tensor, which translates into the diffusivity,

$$\dot{j}_a = -\sigma_{ab} \nabla \mu_b,$$

(2)

$$\sigma_{ab} = \frac{1}{3T} \int_{t>0} dt \, d^3r \, \langle j_a(0) \cdot j_b(t, r) \rangle$$

$$D_{ab} = \sigma_{ac} \chi_{cb},$$

(3)

where $\mu_a$ is the chemical potential for flavor $a$, and $\chi$ is the susceptibility, or charge fluctuation matrix. The electric conductivity, i.e. the response to a gradient of the charge density for uniform strangeness and baryon number, is

$$D_E = (2D_{uu} + D_{dd} - 2D_{ud} - D_{du} - D_{su} + D_{sd})/3.$$

At high temperatures, where $D$ becomes proportional to the unit matrix, $D_E \approx D_{uu} \approx D_{dd} \approx D_{ss}$.

Lattice results for the diffusivity have been obtained [9, 10] despite the challenges in extracting transport coefficients from lattice calculations due to the difficulty in evaluating real-time correlations. For a strongly coupled liquid, AdS/CFT provides a value of $1/2\pi T$ for light quarks [11], whereas for heavy quarks the value can approach zero for infinite couplings or colors [12]. The values extracted from lattice are indeed of the order of $1/2\pi T$, suggesting that the diffusivity, like the viscosity, is characteristic of strong coupling. For the lattice calculations plotted as a function of $T$, $D$ dips near $T_c$ and falls below the AdS/CFT value as shown in Fig. 1.

One can also estimate the conductivity from a quasi-particle picture if one has an estimate of the relaxation time,

$$\sigma_{ab} \approx \int \sum_h \frac{d^3p}{(2\pi)^3} f_h(p) q_{ha} q_{hb} \frac{|v_p|^2}{3} \tau_h(p),$$

(4)

where $\tau_h$ is the relaxation time for a hadron of type $h$, spin degeneracy $d_h$ and charge $q_{ha}$, to lose its correlation with its original velocity. For fixed cross s-wave cross sections, one can estimate the lifetime for a given momentum by calculating the rate at which the correlation,
FIG. 1. (color online) The electric diffusivity, scaled by $2\pi T$, from lattice gauge theory as calculated in [9] (green points). A hadron gas with a fixed 25-mb cross section (red line) has significantly higher diffusivity. For AdS/CFT, the value is unity (blue dashes).

\[ \langle v(0)v(t) \rangle, \text{ changes at } t = 0 \text{ for each mode } p, \text{ then use the inverse rate as the lifetime above. Such a calculation is displayed in Fig. 1 for 25 mb cross sections, a value consistent with expectations for a hadron gas. This estimate lies significantly above the lattice prediction.} \]

Here, a method is proposed for determining the diffusivity of the light $(u,d,s)$ quarks. Local charge conservation demands that quarks are produced simultaneously with antiquarks, and if one knows the times at which such production occurs, one can constrain the diffusivity by measuring the relative momentum of balancing charges, which are highly correlated with separation in coordinate space due to the effects of collective flow. In [13] a detailed simulation of the production and diffusion of balancing charges was presented. The evolution of charge correlations was superimposed onto a state-of-the-art description of the dynamics, based on hydrodynamics for higher temperatures and using a hadronic simulation for the hadronic stage and for breakup. The source function for balancing charge pairs was determined from the evolution of charge correlations that drive them are concentrated at early times, according to the diffusivity. When the differential charges, \( dq_a \), enter the hadron phase they are translated into differential hadron yields, \( dN_h \), using thermal arguments [13]. In the hadron phase, correlations evolve according to a simulation, and are manifested as charge balance functions,

\[ B(\Delta \phi) \equiv \langle N_{+-}(\Delta \phi) + N_{-+}(\Delta \phi) \rangle - N_{++}(\Delta \phi) - N_{--}(\Delta \phi)/(N_+ + N_-). \]

Here, \( N_{qq'}(\Delta \phi) \) denotes the sum over all pairs of charges \( qq' \) separated by azimuthal angle \( \Delta \phi \), \( N_q \) is the number of charges of type \( q \), and the average covers all events of a given centrality class. Charge balance functions have been measured as a function of relative rapidity, pseudo-rapidity and azimuthal angle, and the charges restricted to specific hadron species. Data from both RHIC and from the LHC have been analyzed [15–23].

Not surprisingly, \( B_{K^+K^-} \) focuses on the correlation between strange quarks [14]. The source function for strangeness, \( S_{ss} \), is dominated by the first surge of charge production as the system is initially equilibrated in the first \( \lessapprox 1 \) fm/c of the collision. During the evolution of an idealized QGP of massless quarks and gluons, entropy conservation maintains the number of quarks and \( S_{ab} \) vanishes. Once hadrons form the source function again becomes strong because hadrons carry multiple quarks, and due to entropy conservation, the number of hadrons roughly equals the number of quarks in the QGP [24, 25]. In contrast to the source functions for up and down quarks, the source function for strangeness remains small during hadronization [8, 14] due to the larger mass of strange hadrons, which suppressed the production of strange quarks. Even though up and down quarks are copiously created during hadronization, the effective source function for baryon number stays fairly constant during hadronization, and even becomes negative below \( T_c [14] \), due to the high mass of baryons in the hadronic stage. Thus, the \( K^+K^- \) and \( pp \) balance functions should be more sensitive to the diffusivity because the source functions that drive them are concentrated at early times, allowing the diffusivity to play a stronger role.

Figure 2 shows balance functions for three cases: all positive/negative particles, \( K^+K^- \) and \( pp \). The methods are the same as described above and applied in [13]. In the hadronic stage, during which the matter is largely in the QGP phase, the charge-charge correlations were evolved according to four different choices for the diffusivity. First, they were evolved according to \( D(T) \) reported from lattice calculations [9], exactly as in [13]. Then, the calculations were repeated with half that value, double that value, and finally, four times the lattice diffusivity. The analysis was restricted to very central events, 0-5% centrality. In each case the balance functions are broader for the larger diffusivities. The balance function for all charges is least sensitive because it is dominated by later-stage production of charge associated with hadronization. In contrast, the \( K^+K^- \) and \( pp \) balance functions broaden significantly. Unfortunately, experimental results for \( K^+K^- \) and \( pp \) balance functions have only been reported binned by relative rapidity thus far. Preliminary results for all charges have been reported by

(\( v(0)v(t) \)), changes at \( t = 0 \) for each mode \( p \), then use the inverse rate as the lifetime above. Such a calculation is displayed in Fig. 1 for 25 mb cross sections, a value consistent with expectations for a hadron gas. This estimate lies significantly above the lattice prediction.

Here, a method is proposed for determining the diffusivity of the light \((u,d,s)\) quarks. Local charge conservation demands that quarks are produced simultaneously with antiquarks, and if one knows the times at which such production occurs, one can constrain the diffusivity by measuring the relative momentum of balancing charges, which are highly correlated with separation in coordinate space due to the effects of collective flow. In [13] a detailed simulation of the production and diffusion of balancing charges was presented. The evolution of charge correlations was superimposed onto a state-of-the-art description of the dynamics, based on hydrodynamics for higher temperatures and using a hadronic simulation for the hadronic stage and for breakup. The source function for balancing charge pairs was determined from the evolution of the susceptibility, assuming the matter maintains local chemical equilibrium [13, 14].

\[ S_{ab}(r, t) = (\partial_t + v \cdot \nabla + \nabla \cdot v)\chi_{ab}(r, t). \]
The results of Fig. 2 suggest that both $K^+K^-$ and $p\bar{p}$ are promising for constraining the diffusivity of the QGP. This was expected, given that the source functions driving the those balance functions were concentrated at early times. However, the $p\bar{p}$ results are strongly sensitive to the choice of transition temperature. Because of the large baryon mass, the equilibrium number of baryons falls rapidly with falling temperature once one enters the hadronic phase, which corresponds to the introduction of negative source functions. Equivalently, in the hadron stage the effects of baryon annihilation can significantly alter the shape of the charge balance function, leading to a dip of the balance function at small relative angle, as well as (due to normalization constraints) an accompanying increase at large relative angle. A careful analysis of annihilation effects requires consistently accounting for regeneration [26–28], and until such an analysis is performed, one must remain cautious in interpreting the $p\bar{p}$ balance function results.

This analysis is based on relative azimuthal angle rather than relative rapidity, because balancing charges produced in the first 1 fm/$c$ might separate significantly along the beam direction by the time the hydrodynamic description is instantiated. Due to the large velocity gradient along the beam axis at early times, $dv_z/dz \approx 1/\tau$, a separation of 1/2 fm at a time $\tau = 1/2$ fm/$c$ translates to a separation of an entire unit of spatial rapidity. Disentangling the longitudinal separations related to pre-equilibrium dynamics from the effects of diffusion could be problematic. Because there are no large transverse velocity gradients at early times, the transverse separation should be dominated by the effects of diffusion, especially for the large sources in central collisions.

The strong sensitivity of the $K^+K^-$ balance function to the diffusivity, combined with a relative lack of complicating factors, suggests that the diffusivity of strange quarks of the QGP can be robustly constrained by experiment. Given that the diffusivity of up, down, and strange quarks should all be similar in the QGP, this should effectively constrain the diffusivity of all three flavors of light quarks. Although carrying some caveats, the $p\bar{p}$ balance functions also show promise. It would not be surprising if the approaches presented here might ultimately constrain the diffusivity of the QGP, a fundamental property of the QGP that has not yet been constrained experimentally, to the $\gtrsim 50\%$ level. This resolution would be similar to how well the shear viscosity, another fundamental transport coefficient of the QGP, has been determined.

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