Effects of boundaries in mesoscopic superconductors

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Abstract

A thin superconducting disk, with radius \( R = 4\xi \) and height \( H = \xi \), is studied in the presence of an applied magnetic field parallel to its major axis. We study how the boundaries influence the decay of the order parameter near the edges for three-dimensional vortex states.

PACS numbers: 74.20.-z,74.60.-w,74.50.+r

Keywords: Vortex state, Mesoscopic superconductor, Ginzburg-Landau
A few years ago the response of a mesoscopic superconducting disk to a magnetic field parallel to its major axis was measured [1] and more recently the detection of giant vortices was made thanks to new advances in small-tunnel-junction technology [2]. The small volume to area ratio of mesoscopic systems brings new and interesting physical properties such as the onset of spontaneous and persistent currents [3] and for this reason the treatment of boundaries must be carefully considered. In this work the three-dimensional Ginzburg-

\[ |\Psi|^2 \text{ vs. the distance from the center of the disk is shown here for the case of zero and one vorticity. The dots correspond to } |\Psi|^2 \text{ values at mesh grid points. The three-dimensional figures correspond to isocontours taken at 20\% of its maximum value.} \]
FIG. 2: Magnetization (arbitrary units) vs. the applied field. Inset shows the $H_{c2}$ to $H_{c3}$ region. The magnetization is normalized by a negative arbitrary constant.

Landau (GL) theory is solved in a parallelepiped cell that contains a smaller mesoscopic superconducting disk inside. Space is discretized and gauge invariance kept on a 61 by 61 by 16 grid. The distance between two consecutive mesh points along any of the major axes is $\xi/5$, $\xi$ being the coherence length at some fixed temperature. Thus the disk occupies the center of a 12.0$\xi$ by 12.0$\xi$ by 3.0$\xi$ cell. The vortex state solutions are obtained by numerical minimization of the GL free energy in the cell through the method of simulated annealing. Coupling of the disk to the outside non-superconducting space is included and the order parameter converges to zero outside the disk at the end of the minimization procedure. This makes the present method somewhat different to those that just seek the solution inside, but not outside, the disk [5]. The present description is restricted to a hard type
FIG. 3: Free energy vs. the applied field is shown here, the inset shows the $H_{c2}$ to $H_{c3}$ region. The free energy is normalized to $H_{c2}^2/8\pi$.

II superconductor whose free energy is simply given by, $f = \int \frac{dV}{V} (\tau\xi^2|(|\nabla - 2\pi i/\phi_0 A)|\psi|^2 - \tau|\psi|^2 + \frac{1}{2}|\psi|^4)$, expressed in reduced units such that the density is normalized to one. The magnetization is determined from $M = const \int dv \mathbf{x} \times \mathbf{J}$, where $\mathbf{J}$ is the supercurrent. The shape of the mesoscopic superconductor enters directly into the free energy through the step-like function $\tau(\mathbf{x})$, equal to one inside the sample, and zero outside. As shown in Ref. [4] this approach yields the de Gennes boundary condition. Here we make the $\tau(\mathbf{x})$ function smooth with an exponential decay linking the two sides. The condition of a vanishing supercurrent component pointing outside the superconductor is only approximately enforced between the disk and the outside external space: $\tau = \exp(-|(\mathbf{x} - \mathbf{R})/\eta|^N)$. We take here for the two adjustable parameters, $\eta = 0.8\xi$ and $N = 8$. Numerically, this smoothness is advantageous...
since a steep $\tau(x)$ function directly couples a mesh point in the disk to an outside point rendering a severe depletion of the order parameter inside. In the present approach the order parameter never reaches its maximum value of one, predicted for a bulk superconductor, regardless of the $\eta$ and $N$ values because the coupling to the outside conspires to lower the superconducting density near the edge of the disk, as exemplified in figure 1 for the cases of none and one vortex inside the disk. The magnetization and free energy curves obtained with the present approach display a total of 13 lines as shown in figures 2 and 3 respectively. Each line corresponds to a distinct vorticity state. The crossing of these lines in case of the free energy defines the so-called matching fields [6]. The present results are in fair agreement to those found by Baelus [7] for an extremely thin disk using a two-dimensional approach for the superconducting density.

We have theoretically studied the vortex states of a mesoscopic disk with finite thickness. Previous studies have also considered a mesoscopic disk but with a vanishing thickness [7]. We conclude that the present minimization procedure of the Ginzburg-Landau free energy is able to obtain the vortex patterns of truly three-dimensional mesoscopic superconductors.

Acknowledgments A. R. de C. Romaguera and M. M. Doria thank CNPq (Brazil), FAPERJ (Brazil) and the Instituto do Milênio de Nanotecnologia (Brazil) for financial support. F. M. Peeters acknowledges support from the Flemish Science Foundation (FWO-Vl), the Belgian Science Policy (IUAP), the JSPS/ESF-NES program and the ESF-AQDJJ network.

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