Simulation of microwave spatial field of atmospheric brightness temperature under discontinuous cumulus cloudiness

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Abstract. Planck’s model for random discontinuous cloud fields generation in a three-dimensional computational domain is considered. An algorithm has been developed for computation a 2D picture of downwelling microwave radiation brightness temperature distribution from altitude profiles of the main meteorological parameters, including liquid water profile obtained during the generation of cloud field. The inverse problem of integral moisture content parameters distribution retrieval by means of dual-frequency method is solved. Systematic errors in the cloud liquid water content estimation associated with the use of a homogeneous flat-layered model of the cloud field, which ignores its real, inhomogeneous structure, are investigated.

Keywords: brightness temperature, discontinuous cumulus cloudiness, Planck’s model, moisture content parameters, cloud liquid water, dual-frequency method.

1. Introduction
An important problem for remote sensing of the environment is monitoring the state of Earth’s atmosphere and surface. Remote sensing of atmosphere from artificial Earth satellites by means of ”atmosphere – underlying surface” system radio-thermal radiation receiving method opens up new opportunities for studying weather and climate [1]. Microwave radiometric observations of atmosphere from space make it possible to obtain useful information about the state of cloud fields and to retrieve such parameters as total water vapor mass, liquid water content in clouds and effective cloud temperature [2]. A disadvantage of existing methods for processing and interpreting microwave satellite radiometric data is the use of uniform flat-layered model of the cloud field, which ignores its inhomogeneous structure. This leads to systematic errors in the desired meteorological parameters evaluation.

The aim of this work is to study these systematic errors in the moisture content parameters related to the flat-layered model usage and real cloud fields random discontinuity. As the first step of such a study, a numerical simulation of brightness temperatures of discontinuous cumulus cloudiness downwelling radiation at various frequencies of microwave band is performed using Planck’s model of the cloud field [3]. A comparative analysis of the calculated brightness
temperatures and computation results for the model of a flat layer with an equivalent area-averaged liquid water content is carried out.

2. Modeling random cloud fields and brightness temperatures

The field of random discontinuous cloudiness (with gaps between clouds) can be generated according to Planck’s model [3] with the corresponding distribution of clouds by diameters and an unambiguous relation between the power of each cloud and its diameter [4]. Based on the results of processing an extensive database of stereoscopic survey at Florida, USA, Planck (1969) proposed following distribution

\[ n(D) = K \cdot e^{-\alpha D}, \quad 0 \leq D \leq D_m. \]  \hspace{1cm} (1)

Here \( D \) is cloud diameter, \( n(D) \) is number of clouds of a certain diameter, \( D_m \) is the maximum cloud diameter in the ensemble of clouds, \( K \) is normalization coefficient and \( \alpha \) – parameter, which depends on the time of day and various local climatic conditions.

Let cloud power be the difference in altitudes of cloud top and base. The relation between the cloud power and its diameter can be written as

\[ H = \eta D \left( \frac{D}{D_m} \right)^\beta, \]  \hspace{1cm} (2)

where \( \eta \) and \( \beta \) are the dimensionless parameters (see [3]).

Note that there are other models for cloud fields with spaced clouds. For example, in [5], based on the results of processing aircraft measurements in Ukraine, the following distribution of clouds by diameter was proposed

\[ n(D) = K \cdot D \cdot \left( 1 - \frac{D}{D_m} \right)^{p_0}, \quad 0 \leq D \leq D_m, \]  \hspace{1cm} (3)

where \( p_0 \) is the distribution parameter, on average equal to 4.35.

There are variety of cloud species to be discussed, but first we are concerned here, however, with convective clouds of cumulus type. Average values of effective temperature, integral liquid water content, power and cloud base altitudes for Cu hum/med/cong cloud species are listed in [3, 6]. The liquid water profile of such a cloud can be obtained as follows [6]

\[ w(\xi) = w(\xi_0) \frac{\xi^\mu_0 (1 - \xi)^\psi_0}{\xi_0^\mu_0 (1 - \xi_0)^\psi_0} = \frac{W}{H} \cdot \frac{\Gamma(2 + \mu_0 + \psi_0)}{\Gamma(1 + \mu_0) \Gamma(1 + \psi_0)} \frac{\xi^\mu_0 (1 - \xi)^\psi_0}{H^{2.30215}}, \]  \hspace{1cm} (4)

where \( \xi = h/H \) is the reduced height inside the cloud, \( H \) is the power of cloud (km), \( W \) is the integral liquid water content (kg/m²), \( w(\xi) \) represents the profile of liquid water (kg/m³), \( w(\xi_0) \) is the maximum value, and \( \xi_0 \) is the reduced height at which the maximum value is reached, \( \mu_0 \) and \( \psi_0 \) are the dimensionless parameters. According to [2], the values of these parameters are \( \mu_0 = 3.27, \psi_0 = 0.67, \xi_0 = 0.83 \), and the integral liquid water content \( W \) on cloud power \( H \) dependence can be approximated based on given tabular data as

\[ W = 0.132574 \cdot H^{2.30215}. \]  \hspace{1cm} (5)

Introduce a computational domain \( \Omega \) with an area of 50×50 km, 10 km height and a grid of 300×300×500 nodes, respectively. Assume the shape of clouds to be cylindrical, the boundaries not to be intersected and generate a field of discontinuous cumulus cloudiness setting the values of model parameters \( K, D_m, \alpha, \beta, \eta \) and assigning a fixed interval of heights, in which the
cloud base can change. The distribution of clouds by diameter can be calculated from (1) by
substitution \( k \cdot D_m \cdot r^{-1} \) in place of \( D \). Here \( k \) is an integer number, \( 1 \leq k \leq r \), and \( r \) is selected
in accordance with available detalization of computational grid. For instance, \( r = \sqrt{i_x^2 + j_y^2} \),
where \( i_x, j_y \) are the numbers of grid nodes per distance in km along the corresponding directions
to cover a cloud of \( D_m \) diameter. The numerical simulation shows that iterative filling of \( \Omega \) with
clouds in decreasing order of their diameter makes it possible to achieve maximum percentage
of cloud cover (projected on \( h = 0 \) plane) at the level of 65-75% within an acceptable time of
searching suitable random location for each cloud and without using specialized algorithms for
optimal packing. However, it is not possible to obtain a larger cloud cover percentage without
algorithms for optimal packing.

Having cloud field generated, let us define at each point of zero altitude plane the liquid
water altitude profile \( \tilde{w}(h) \) in such a way that \( \tilde{w}(h) = w((h - H_0)/H) \) (see (4)), if \( h \) is inside a
cloud with base altitude equal to \( H_0 \) and power equal to \( H \), but, on the other hand, \( \tilde{w}(h) = 0 \),
if \( h \) is beyond any cloud. In addition to the liquid water profile, it is necessary to know altitude
profiles of thermodynamic temperature \( T(h) \), atmospheric pressure \( P(h) \) and air humidity
if \( h \) is beyond any cloud. In addition to the liquid water profile, it is necessary to know altitude
profiles of thermodynamic temperature \( T(h) \), atmospheric pressure \( P(h) \) and air humidity \( \rho(h) \)
to calculate brightness temperatures \( [7, 8] \). We use the standard atmosphere model with
exponential laws of temperature, pressure and humidity versus altitude.

Brightness temperature of nadir-downwelling radiation of atmosphere as a multilayered
homogeneous medium can be represented as

\[
T_b^\theta(\nu) = \int_0^\infty T(h) \gamma(\nu, h) \sec \theta \exp \left(-\int_0^h \gamma(\nu, z) \sec \theta dz\right) dh.
\]

Here \( T_b^\theta(\nu) \) is brightness temperature at \( \nu \) frequency, \( \gamma(\nu, h) \) is the total for all atmospheric
components path attenuation coefficient (np/km)

\[
\gamma(\nu, h) = \gamma_O^* (\nu, h) + \gamma_\rho^* (\nu, h) + \gamma_w^* (\nu, h),
\]

where \( \gamma_O^* \) and \( \gamma_\rho^* \) are path attenuation coefficients for oxygen and water vapor,
respectively, and \( \gamma_w^* \) is cloud path attenuation. The first two coefficients can be
approximated by theoretical-empirical dependencies \( \gamma_O(\nu, h) = \gamma_O(\nu, T(h), P(h)) \) (dB/km) and
\( \gamma_\rho(\nu, h) = \gamma_\rho(\nu, T(h), P(h), \rho(h)) \) (dB/km) taken from International Telecommunication
Union recommendations [9]. In this case, \( \gamma_O^* = \lambda \cdot \gamma_O \) and \( \gamma_\rho^* = \lambda \cdot \gamma_\rho \),
where \( \lambda = 0.23255814 \) is the conversion factor from decibels (dB) to nepers (np).

Dielectric permittivity of zero salinity water (particles of cloud formation)

\[
\varepsilon_C = \left( \varepsilon_O + \frac{\varepsilon_S - \varepsilon_O}{1 + \Delta \lambda^2} \right) - i \cdot \Delta \lambda \cdot \frac{\varepsilon_S - \varepsilon_O}{1 + \Delta \lambda^2}, \Delta \lambda = \frac{\lambda_S}{\lambda}.
\]

Here \( \varepsilon_C = \varepsilon_C(\lambda, t) \) is the complex dielectric permittivity, \( \lambda \) is wavelength, \( t \) is thermodynamic
temperature, \( \varepsilon_O \) is the ”optical” component of dielectric constant, \( \varepsilon_S = \varepsilon_S(t) \) is the ”static”
component and \( \lambda_S = \lambda_S(t) \) is the characteristic wavelength related to water molecules relaxation
time. The temperature dependencies for \( \varepsilon_O, \varepsilon_S \) and \( \lambda_S \) parameters can be found in [2, 10, 11].
Introduce the weight function of attenuation in clouds \( k_w(\lambda, t_w) \) as

\[
k_w(\lambda, t_w) = \frac{0.6\pi}{\lambda} \cdot K_C,
\]

where \( t_w \) is average effective cloud temperature, and \( K_C \) is a coefficient, which determines
the temperature related cloud attenuation change. For model calculations, it is convenient to use
the following expression

\[
K_C = \text{Im} \left( \frac{\varepsilon_C - 1}{\varepsilon_C + 2} \right) = \frac{3 \cdot (\varepsilon_S - \varepsilon_O) \Delta \lambda}{(\varepsilon_S + 2)^2 + (\varepsilon_O + 2)^2 \Delta \lambda^2}.
\]
Finally, the absorption coefficient $\gamma_w^*(\nu, h)$ is defined as a product of $k_w^*(\nu, t_w) = k_w(c \cdot \nu^{-1}, t_w)$ weight function and $\tilde{w}(h)$ liquid water profile at a certain altitude $h$.

The 2D-picture of brightness temperature distribution (shown in figure 1) at $\nu = 22.2$ GHz frequency in the nadir is calculated taking into account (1), (2), (4)–(10) and the values of model parameters $K = 220, D_m = 3$ km, $\alpha = 1, \beta = 0.5, \eta = 1$. The cloud base altitude varies in the range from 1 to 3 km. The cloud cover percentage (percentage of $h = 0$ area covered by clouds in their vertical projection) is 60.8%. The average over the entire area of $50 \times 50$ km value of liquid water content $W^*$, obtained by integrating $\tilde{w}(h)$ profile over $h$ at each point of zero altitude plane and subsequent averaging, is about $0.32$ kg/m$^2$. A continuous cloud layer with such a liquid water content, according to (5), would have thickness (power) of 1.41 km. However, the average cloud power is 0.97 km when averaged over the area and only 0.61 km when averaged by number of clouds.

3. Integral moisture content parameters retrieval

The dual-frequency radiometric method for integral moisture content parameters retrieval is presented in [2, 11-13]. It is possible to evaluate the values of total water vapor mass $Q$ (g/cm$^2$) and cloud liquid water content $W$ (kg/m$^2$) by means of this method. If cloud effective temperature $t_w$ and average absolute temperature $T_a$ of atmosphere are known, moreover, the total atmospheric opacity is relatively small $\tau \lesssim 1$ np, then it is sufficient to write down and solve by any available method a system of two linear equations to get $Q$ and $W$

$$\tau(\nu_i) = \tau_O(\nu_i) + k_{\rho}(\nu_i) \cdot Q + k_w^*(\nu_i, t_w) \cdot W, \quad i = 1, 2, \quad (11)$$

where $\tau(\nu) = \ln[T_b(\nu)] - \ln[T_a(\nu) - T_b(\nu)]$ is an estimation of total zenith opacity of atmosphere (np), $T_b(\nu)$ is brightness temperature in the nadir, $\tau_O(\nu) = \gamma^*_O(\nu, 0) \cdot H^1_\nu$ is oxygen opacity (np), $\gamma^*_O(\nu, 0)$ is the path attenuation coefficient in oxygen at zero altitude (np/km), $H^1_\nu$ is the characteristic height of attenuation in oxygen (about 5 km), and $k_w^*(\nu, t_w)$ is the previously considered weight function of cloud absorption. The expression for $k_{\rho}(\nu)$ weight function is given below

$$k_{\rho}(\nu) = \gamma_{\rho}^*(\nu, 0) \cdot H^2_\rho \cdot \left(\int_0^\infty \rho(h) \, dh\right)^{-1}. \quad (12)$$
Here $\gamma^*_\rho(\nu,0)$ is the path attenuation coefficient in water vapor at zero altitude (np/km), $H^*_\nu$ is the characteristic height of attenuation in water vapor (from 1.6 to 2.1 km), $\rho(h)$ represents the standard altitude profile of water vapor.

Let us generate a cloud field according to Planck’s model and compute 2D-distributions of nadir-downwelling radiation brightness temperature at frequencies $\nu = 22.2, 27.2$ and $37.5$ GHz. Three brightness temperature arrays $T^*_\nu = T^*_\nu(i,j)$ are obtained, $0 \leq i < 300$, $0 \leq j < 300$. Here we retrieve distribution of integral liquid water content $W_1(i,j)$ from the two of obtained arrays $\tilde{T}^*_\nu$ for the pair of frequencies 22.2 and 27.2 GHz using dual-frequency approach and then repeat all calculations for the pair of 22.2 and 37.5 GHz frequencies to retrieve $W_2(i,j)$. We also consider the average values $W_1^* = \langle W_1(i,j) \rangle_{i,j}$ and $W_2^* = \langle W_2(i,j) \rangle_{i,j}$.

To assess systematic errors in the moisture content parameters evaluation related to the flat-layered cloud field model usage, which ignores its real, discontinuous structure, we apply to the brightness temperature arrays an operation of block averaging (13) with blocks of $n \times n$ nodes for $n = 1, 2, 3 \ldots 300$ sequentially and monitor the dynamics of evaluated liquid water content $W_1^*$ and $W_2^*$ average values.

$$\tilde{T}^*_\nu(i,j) = \sum_{k,l} T^*_\nu(k,l) \frac{n^2}{n^* \times n^*}, \quad \text{(13)}$$

where $n \cdot i^* \leq k < n \cdot (i^* + 1)$, $n \cdot j^* \leq l < n \cdot (j^* + 1)$, and $i^* = [i/n]$, $j^* = [j/n]$ (square brackets represent integer division). Thus, blocks do not have intersections and are understood as elements of resolution (averaging), by means of which a picture of discontinuous cloudiness ($T^*_\nu$) is reduced to a set of plane-layered approximations ($\tilde{T}^*_\nu$), see figure 1 and 2 (a, b). In figure 2, the results $\tilde{T}^*_\nu$ of averaging the earlier obtained brightness temperature distribution $T^*_\nu$, $\nu = 22.2$ GHz (see figure 1), are shown for $K = 220$, $D_m = 3$ km, $\alpha = 1$, $\beta = 0.5$, $\eta = 1$.

**Figure 2.** The model values of brightness temperature $\tilde{T}^*_\nu(i,j)$, [K]. Frequency $\nu$ is 22.2 GHz. The size of resolution element is $n \times n$: a) 10×10 nodes, b) 50×50 nodes.

Averaged values of liquid water content $W_1^*$, $W_2^*$ calculated for various $n$ using dual-frequency method are to be compared to ”real” average liquid water content $W^*$, obtained as earlier by integration $\tilde{w}(h)$ profile over $h$ at each point of zero altitude plane with subsequent averaging.

The relative error value $W_{\text{err}}^{(i)}$ (%), $i = 1, 2$, is introduced in the standard way

$$W_{\text{err}}^{(i)} = \frac{|W_i^* - W^*|}{W^*} \cdot 100\%, \quad i = 1, 2. \quad \text{(14)}$$
Figure 3. The dependence of $W_{err}^{(1)}$ relative error (dual-frequency method, 22.2 and 27.2 GHz) on resolution element $n \times n$, where $n$ is from 1 to 100 nodes, for various cloud cover percentage.

Figure 4. Relative errors $W_{err}^{(1)}$ and $W_{err}^{(2)}$ versus cloud cover percentage for fixed $n = 30$ and $n = 100$ nodes; $K = 50 \ldots 220$, $\eta = 1$.

In figure 3, the influence of resolution element $n \times n$, where $n = 1 \ldots 100$ nodes, on the relative error $W_{err}^{(1)}$ is shown ($D_m = 3$ km, $\alpha = 1$, $\beta = 0.5$, $\eta = 1$). An interpolation of values is done by splines. The x-axis has a logarithmic scale. Curves (1)–(3) are shown for different cloud cover percentage, which is controlled by model parameter $K$. The average absolute temperature of atmosphere $T_a$ is equal to 278 K, the effective cloud temperature is $t_w = -2\,^\circ$C. The minimum allowed cloud base altitude is 1 km, the maximum is 3 km. Clouds do not have intersections and cannot be located one above the other. The dependence of $W_{err}^{(2)}$ relative error (dual-frequency method, 22.2 and 37.5 GHz combination) on the resolution element $n \times n$, where $n = 1 \ldots 100$.
Figure 5. The relative errors $W_{\text{err}}^{(1)}$ and $W_{\text{err}}^{(2)}$ versus cloud cover percentage for fixed $n = 30$ and $n = 100$ nodes; $K = 50...220$; $\eta = 2$.

nodes, follows the same pattern as in figure 3. The difference is that $W_{\text{err}}^{(2)}$ values are always 1-2% greater for small $n$ and 10-15% greater for large $n$ than $W_{\text{err}}^{(1)}$. While resolution element increases in size, the errors (%) in average liquid water content values retrieved by dual-frequency method also increase. For $n \geq 10$ nodes, the cloud cover percentage already noticeably affects the value of relative error.

Consider in figure 4 ($D_m = 3$ km, $\alpha = 1$, $\beta = 0.5$, $\eta = 1$) the dependence of $W_{\text{err}}^{(1)}$ and $W_{\text{err}}^{(2)}$ relative errors (scale on the left) on the cloud cover percentage for fixed sizes of resolution element $n \times n$: 30x30 – curves (1) and (2), 100x100 – curves (3) and (4). Curve (5) on this figure represents linear regression of ”real” average integral liquid water content $W^*$ versus cloud cover percentage. The numerical simulation shows that with a decrease in cloud cover percentage, i.e. with an increase of cloud field discontinuity (in gaps between clouds), a noticeable growth in the averaged liquid water content relative error for $n \geq 10$ is observed, and this error grows faster at a larger element of resolution.

In figure 5 ($D_m = 3$ km, $\alpha = 1$, $\beta = 0.5$, $\eta = 2$) similar dependencies of $W_{\text{err}}^{(1)}$ and $W_{\text{err}}^{(2)}$ relative errors (scale on the left) on cloud cover percentage are shown for comparison for the same $n$, but $\eta = 2$ (see (2)). The other model parameters are left unchanged. Thus, we have clouds of obviously greater power, which is reflected in absolute values of average integral liquid water content (scale on the right). The larger is $W^*$ (see (14)), the less are values of relative errors. However, for $n \geq 10$, they grow as the cloud cover percentage decreases.

4. Conclusion

Planck’s model for generation random discontinuous fields of clouds in a three-dimensional computational domain has been implemented. A numerical simulation of brightness temperature 2D-distributions of discontinuous cumulus cloudiness nadir-downwelling radiation at frequencies 22.2, 27.2 and 37.5 GHz has been carried out. The analysis of systematic errors in cloud liquid water content evaluation related to the inhomogeneous structure of cumulus cloudiness has been performed. It has been shown that the relative error in area-averaged liquid water content obtained by dual-frequency method can significantly increase with an increase in cloud
field discontinuity. Crucial is the size of the selected spatial resolution (averaging) element by means of which a discontinuous random field of clouds is being reduced to a set of flat-layered approximations.

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