Effect of Tensor Correlations on Single-particle and Collective States

H. Sagawa

1 Center for Mathematics and Physics, University of Aizu
Aizu-Wakamatsu, Fukushima 965-8580, Japan

We study the effect of tensor correlations on single-particle and collective states within Skyrme Hartree-Fock and RPA model. Firstly, we study the role of tensor interactions in Skyrme effective interaction on the spin-orbit splittings of N=82 isotones and Z=50 isotope. The isospin dependence of the shell structure is well described as the results of the tensor interactions without destroying good properties of the binding energy and the rms charge radii of the heavy nuclei. Secondly, we performed self-consistent HF+RPA calculations for charge exchange $1^+$ states in $^{90}$Zr and $^{208}$Pb to elucidate the role of tensor interactions on spin dependent excitations. It is pointed out that Gamow-Teller(GT) states can couple strongly with the spin-quadrupole (SQ) $1^+$ states in the high energy region above $E_x=30$ MeV due to the tensor interactions. As the result of this coupling, more than 10% of the GT strength is shifted to the energy region above 30 MeV, and the main GT peak is moved 2 MeV downward.

1. Introduction

One of the current topics in nuclear physics is the role of the tensor interactions on the shell evolution of single-particle states and on spin-isospin excitations. The importance of tensor interactions on nuclear many-body problems was recognized more than 50 years ago. Especially, it plays the essential role to make the binding systems such as the deuteron. As is shown Fig. 1, the tensor interaction acts on the spin triplet state ($S=1$) of two nucleon system. When a proton and a neutron are aligned in the direction of spins, the deuteron gets an extra binding energy by the tensor interaction,

$$V_T = f(r)S_{12}$$

(1)

since $f(r)$ is negative and $S_{12} = 3(\hat{\sigma}_1 \cdot \hat{r})(\hat{\sigma}_2 \cdot \hat{r}) - \hat{\sigma}_1 \cdot \hat{\sigma}_2 = 2$. On the other hand, if a proton and a neutron are perpendicular to the spin direction, the deuteron will lose the binding energy since $S_{12} = -1$ and cannot make a bound state. Thus, the deuteron becomes a strongly deformed prolate shape and makes a binding system only when the tensor correlations are taken into account.

After these findings, the importance of tensor interactions has been widely recognized in nuclear many-body systems, especially in light nuclei. The role of the tensor interactions in the Hartree-Fock calculations was firstly discussed by Stancu et al., thirty years ago [1]. However serious attempts have never been performed until very recently [2–6]. The importance of the tensor correlations on the mean field
Figure 1: Tensor interaction. See the text for details.

revived by the study of the shell evolution of heavy exotic nuclei [7]. In this paper, I will summarize formulas of Skyrme tensor interactions in Section 2. The isotope dependence of the shell structure of Z=50 isotopes and N=82 isotones are studied by using the Hartree-Fock (HF)+BCS calculations in Section 3. The role of tensor interactions on spin-isospin excitations will be examined in Section 4. Summary is given in Section 5.

2. Skyrme tensor interaction

The tensor force was considered in the Skyrme-Landau parameterization and the sum rules of electromagnetic transitions in Ref. [8]. However, the tensor force was essentially dropped in most Skyrme parameter sets which have been used widely in nuclear structure calculations. Recently, in Ref. [3], a Skyrme interaction was fitted including the tensor contribution. Then, tensor terms were added perturbatively in Refs. [2] and [4] to the existing standard parameterizations SIII [9] and SLy5 [10], respectively. Eventually, several new parameter sets have been fitted in Ref. [5] and used for systematic investigations within the Hartree-Fock-Bogoliubov (HFB) framework. The inclusion of tensor terms in the Skyrme HF calculations achieved considerable success in explaining some features of the evolution of single-particle states [2,6]. The Skyrme tensor interaction is given by the triplet-even and triplet-
odd tensor zero-range tensor parts,

\[ v_T = \frac{T}{2} \left\{ (\sigma_1 \cdot k')(\sigma_2 \cdot k') - \frac{1}{3} (\sigma_1 \cdot \sigma_2) k'^2 \right\} \delta(r_1 - r_2) \]

\[ + \delta(r_1 - r_2) \left\{ (\sigma_1 \cdot k)(\sigma_2 \cdot k) - \frac{1}{3} (\sigma_1 \cdot \sigma_2) k^2 \right\} \]

\[ + U \left\{ (\sigma_1 \cdot k') \delta(r_1 - r_2)(\sigma_1 \cdot k) - \frac{1}{3} (\sigma_1 \cdot \sigma_2) k' \cdot \delta(r_2 - r_2) k \right\} \]

(2)

where the operator \( k = (\nabla_1 - \nabla_2)/2i \) acts on the right and \( k' = -(\nabla_1 - \nabla_2)/2i \) on the left. The coupling constants \( T \) and \( U \) denote the strength of the triplet-even and triplet-odd tensor interactions, respectively. We treat these coupling constants as free parameters in the following study. The tensor interactions (2) give the contributions to the binding energy and the spin-orbit splitting proportional to the spin density

\[ J_q(r) = \frac{1}{4\pi r^3} \sum_i v_i^2 (2j_i + 1) \left[ j_i(j_i + 1) - l_i(l_i + 1) - \frac{3}{4} \right] R_i^2(r) \]

(3)

where \( i = n, l, j \) runs over all states and \( q = 0(1) \) is the isospin quantum number for neutrons (protons). The \( v_i^2 \) is the occupation probability of each orbit determined by the BCS approximation and \( R_i(r) \) is the HF single-particle wave function. It should be noticed that the exchange part of the central Skyrme interaction gives the same kind of contributions to the total energy density. The central exchange and tensor contributions give the extra terms to the energy density as

\[ \delta E = \frac{1}{2} \alpha (J_n^2 + J_p^2) + \beta J_n J_p. \]

(4)

The spin-orbit potential is then expressed to be

\[ U_{s.o.}^{(q)}(r) = \frac{W_0}{2r} \left( 2 \frac{d\rho_q}{dr} + \frac{d\rho_{1-q}}{dr} \right) + \left( \alpha \frac{J_q}{r} + \beta \frac{J_{1-q}}{r} \right). \]

(5)

where the first term on the r.h.s comes from the Skyrme spin-orbit interaction and the second term include both the central exchange the tensor contributions \( \alpha = \alpha_c + \alpha_T \) and \( \beta = \beta_c + \beta_T \). In Eq. (5), \( q = 0(1) \) is assigned for neutrons (protons). The central exchange contributions are given by

\[ \alpha_C = \frac{1}{8} (t_1 - t_2) - \frac{1}{8} (t_1 x_1 + t_2 x_2) \]

\[ \beta_C = -\frac{1}{8} (t_1 x_1 + t_2 x_2). \]

(6)

where the parameters are defined in ref. [11]. The tensor contribution are expressed as

\[ \alpha_T = \frac{5}{12} U \]

\[ \beta_T = \frac{5}{24} (T + U). \]

(7)
Before going to detailed study, let us mention two important features of the tensor and the central exchange contributions in Eq. (5) to the spin-orbit splitting. First point is that the mass number dependence of the the first and second terms in Eq. (5). Since the Skyrme spin-orbit force $W_0$ gives the spin-orbit splitting proportional to the derivatives of the densities, the mass number dependence is very modulate in heavy nuclei. On the other hand, the second term in Eq. (5) depends on the spin density $J_q$ which has essentially no contribution for the $l$-$s$ closed shell. The spin density will increase exactly proportional to the number of particles in the open shells if one of the spin-orbit partner is only active. Moreover, the sign of the $J_q$ will change depending upon which orbits are involved in the active shell orbits, i.e., the orbit $j_\geq = l + 1/2$ gives a positive $J_q$ value while the orbit $j_\leq = l - 1/2$ gives a negative $J_q$. This means that the spin-orbit energy will change in the opposite direction according to which orbit is occupied in the open shell nuclei.

We fit the two parameters $T$ and $U$ (equivalently $\alpha_T$ and $\beta_T$) using the recent experimental data of $N = 82$ isotones and $Z = 50$ isotopes. We keep the central part of Skyrme interaction as that of SLy5. The central exchange interactions are $\alpha_c = 80.2 \text{ MeV} \cdot \text{fm}^5$ and $\beta_c = -48.9 \text{ MeV} \cdot \text{fm}^5$ for SLy5. The optimal parameters $\alpha_T$ and $\beta_T$ are determined to be $(\alpha_T, \beta_T) = (-170, 100) \text{ MeV} \cdot \text{fm}^5$. We examine detailed properties of the tensor interactions by using our parameter sets. In Fig. 2 we consider a nucleus where the last occupied orbit is the neutron $j_\geq = l + 1/2$ (for example, imagine $^{90}\text{Zr}$ or $^{48}\text{Ca}$). In. Eq. (5), the $\alpha_T$ increase the spin-orbit splitting because of positive $J_{q=0}$ of $j_\geq = l + 1/2$ orbit, while $\beta_T$ has the opposite sign and decrease the splitting for positive $J_{q=0}$. These effects are illustrated in Fig. 2 where the neutron spin-orbit splitting is increased, while the proton one is decreased. In Fig. 3 the energy differences for proton single-particle states $\Delta e(h_{11/2} - g_{7/2})$ of $Z=50$ isotones are shown as a function of the neutron excess $(N - Z)$. The
original SLy5 interaction fails to reproduce the experimental trend qualitatively and quantitatively. Firstly, the energy differences of the HF results are much larger than the empirical data. Secondly, the experimental data decrease, the neutron excess decreases and reach about 0.5MeV at the minimum value. On the other hand, the energy differences of the original SLy5 increase as the neutron excess decrease and has the maximum at around \((N − Z)=20\). We studied also several other Skyrme parameter sets and found almost the same trends as those of SLy5.

The tensor central exchange interactions are included in the results marked by open circles in Fig. 3. We can see a substantial improvement by introducing the tensor interactions. The set \((\alpha_T, \beta_T) = (-170,100)\)MeV-fm\(^5\) gives a fine agreement with the empirical data from \((N − Z) = 20 \sim 32\) quantitatively and qualitatively.

![Figure 3: Comparison of energy difference between pairs of single-particle 1g\(_{7/2}\) and 1h\(_{11/2}\) proton states of Z=50 isotones.](image)

The HF+tensor results can be qualitatively understood by general argument as follows. Firstly, the strength \(\alpha\) change the depth of the proton spin orbit potential. For Z=50 core, only the proton g\(_{9/2}\) proton orbit dominates the spin density \(J_p\) in Eq. (3) so that with the negative \(\alpha_T=\) value the spin-orbit splittings are increased. Thus, the g\(_{9/2}\) protons increase the spin-orbit splitting between proton \((g_{9/2} - g_{7/2})\) orbits and that of proton h\(_{13/2} - h_{11/2}\) orbits by \(\alpha_T\) effect. As a net effect, the
\( \Delta e(h_{11/2} - g_{7/2}) \) protons decreases substantially. Next let us study the (N-Z) dependence where \( \beta_T \) = plays the essential role. In the case of \( \Delta e(h_{11/2} - g_{7/2}) \) protons on Z=50 core from N-Z=(6-14), the \( g_{7/2} \) neutron orbit is gradually filled. Then the \( \beta_T =100\text{MeV} \cdot \text{fm}^5 \) gives a negative contribution to the spin-orbit potential and increases the spin-orbit splitting. Therefore the energy difference \( \Delta e(h_{11/2} - g_{7/2}) \) is further decreasing. From N-Z=(14 to 20), the \( s_{1/2} \) and \( d_{3/2} \) neutron orbits are occupied. In this region the spin density is not so much changed since \( s_{1/2} \) has zero contribution. For N-Z=(20-32), the \( h_{11/2} \) orbit is gradually filled. Then this orbit gives a positive contribution to the spin-orbit potential , i.e., the the spin-orbit splitting is decreasing. Then the \( \Delta e \) turns out to be increasing. The magnitude of \( \beta \) term determines the slope of the (N-Z) dependence so that a larger \( \beta \) gives a steep slope.

![Figure 4: Comparison of energy difference between pairs of single-particle 1i_{13/2} and 1h_{9/2} neutron states of N=82 isotopes.](image)

Effect of tensor correlations are shown on both p and n spin-orbit splittings.

For the results of Sb-isotones in Fig. 4 the \( \Delta e(h_{11/2} - g_{7/2}) \) for N=82 core is plotted as a function of neutron excess. Essentially, the same argument can be applied for the neutron energy difference \( \Delta e(i_{13/2} - h_{9/2}) \) for N=82 core as that for the \( \Delta e(h_{11/2} - g_{7/2}) \) for Z=50 core. The last occupied neutrons in \( h_{11/2} \) increase the neutron spin-orbit splitting so that the \( \Delta e(h_{11/2} - g_{7/2}) \) become substantially...
smaller by the $\alpha_T$ effect in Eq. (3). The isotope dependence is again explained by the $\beta_T$ effect. The $1g_7/2$ and $2d_5/2$ are almost degenerate above the last occupied proton orbit $1g_9/2$ of Z=50 core. The two proton orbits, $d_5/2$ and $g_7/2$, have opposite effects on the spin orbit potential (5). The occupation probability is larger for larger $j$ orbit so that $1g_7/2$ plays more important role on the spin orbit potential due to the tensor interaction in nuclei with (N-Z)=(32-18) for N=82 isotones. Namely the neutron spin orbit splitting is larger for these isotones so that the $i_{13/2}$ orbit is down and the $h_{9/2}$ is up. These changes make the energy gap $\Delta e(i_{13/2} - h_{9/2})$ smaller for the nuclei from (N-Z)= 32($^{132}$Sn) to (N-Z)=18($^{146}$Gd). Thus the role of the triple-even and triplet-odd tensor interactions are clearly shown in Figs. 3 and 4 for both proton and neutron spin-orbit splittings.

The role of the tensor interaction due to the $\beta$ term is essentially expected from the discussion by Blatt-Weisskopf [13] for the deuteron. The role of $\alpha_T$ is new and has not been examined in a quantitative way in the mean field calculations since this term comes from the triplet-odd tensor interaction. The triplet-odd tensor interaction was not included in the studies of refs. [7,13]. Recently, Brown et al. studied the Skyrme-type tensor interactions in $^{132}$Sn and $^{114}$Sn based on the parameter set SkX. They took both the positive and negative $\alpha_T$ values in the HF calculations and concluded that the negative $\alpha_T$ value gives a better agreement with the experimental data. This is consistent with the present systematic study of Z=50 isotopes and N=82 isotones with HF+BCS model.

3. Tensor effect on Gamow-Teller states

There has been no RPA or QRPA (Quasiparticle Random Phase Approximation) program available to study the effect of the tensor terms on the excited states of nuclei until very recently. The tensor terms of the Skyrme effective interaction was firstly introduced in the self-consistent HF plus RPA calculations [14], in particular, in the GT transitions, which should be affected because of the fact that the corresponding operator is spin-dependent. In the study of GT transitions, the quenching problem is of some relevance. The experimentally observed strength from 10 to 20 MeV excitation energy (with respect to the ground state of the target nuclei) is about 50% of the model-independent non-energy weighted sum rule (NEWSR) [15]. It would be very interesting to study whether the tensor force has an effect in shifting the strength already at one particle-one hole (1p-1h) level. Coupling the GT with two particle-two hole states is essential to describe the resonance width but it is not expected to affect strongly the position of the main GT peak; the effect of the tensor force in connection with the 2p-2h coupling was studied in Ref [16].

It should be noted that $J_q$ gives essentially no contribution in the spin-saturated cases. Therefore, we choose $^{90}$Zr and $^{208}$Pb as examples to be calculated. $^{90}$Zr is a proton spin-saturated nucleus, with a spin-unsaturated neutron orbit $1g_{9/2}$. $^{208}$Pb is chosen as it is not saturated either in protons or neutrons. The two examples
should allow elucidating separately the role of triplet-even and triplet-odd terms. Since the tensor force is spin-dependent and affects the spin-orbit splitting, the spin mode is very likely to receive strong influence. The operator for GT transitions is defined as

\[ \hat{O}_{GT \pm} = \sum_{m} t_{\pm}^{m} \alpha_{m}^{i} \]  

(8)
in terms of the standard isospin operators, \( t_{\pm} = \frac{1}{2}(t_{x} \pm it_{y}) \). In the charge-exchange RPA, the \( t_{-} \) and \( t_{+} \) channels are coupled and the corresponding eigenstates emerge from a single diagonalization of the RPA matrix.

Figure 5: The GT\( ^{-} \) strength in \( ^{90}\text{Zr} \) and \( ^{208}\text{Pb} \). The RPA results are displayed, by smoothing them with Lorentzian function having 1 MeV width. As explained in the text, result labeled by 00 corresponds to neglecting the tensor terms in both HF and RPA; 10 corresponds to including the tensor terms in HF but neglecting them in RPA; finally, 11 corresponds to including the tensor terms in both HF and RPA. The arrow denotes the experimental energy. See the text for details.

The GT\( ^{-} \) strength distributions in \( ^{90}\text{Zr} \) and \( ^{208}\text{Pb} \) are shown in Fig. 5. The calculated results are smoothed by averaging the sharp RPA peaks with Lorentzian function weighting function having 1 MeV width. The tensor force affects these results in two ways. Firstly, it changes the single-particle energies (s.p.e.) in the HF calculation; secondly, it contributes to the RPA residual force. We do three different kind of calculations to analyze separately these effects. In the first one, the tensor terms are not included at all. In the second one, we include tensor terms in HF but drop them in RPA. This calculation is not self-consistent, but it displays the effects of changes in single-particle energies on the strength distribution. In the last one, the tensor terms are included both in HF and RPA calculations. For simplicity, results of the three categories of calculations are labeled by 00, 10 and 11, respectively.
Table 1: Values of the NEWSR $m_-(0)$ and EWSRs $m_-(1)$ for $^{90}$Zr and $^{208}$Pb in different excitation energy regions. The two-body spin-orbit interaction is included in HF but neglected in RPA calculation. The results labeled by 00 correspond to neglecting the tensor terms both in HF and RPA; 10 corresponds to including the tensor terms in HF but neglecting them in RPA; 11 corresponds to including the tensor terms both in HF and RPA. See the text for a discussion of the effects of the tensor terms.

| type of calculation | $m_-(0)$ | $m_-(0)$ | $m_-(1)$ | $m_-(1)$ | $m_-$(1) | $m_+$(1) |
|---------------------|----------|----------|----------|----------|----------|----------|
|         | 0-30MeV | 30-60MeV | 0-30 MeV | 30-60 MeV | total | total |
| $^{90}$Zr 00        | 29.16    | 0.71     | 395      | 26.2     | 421.8   | 10.1     |
| $^{90}$Zr 10        | 29.16    | 0.79     | 444      | 22       | 466     | 11.1     |
| $^{90}$Zr 11        | 27.00    | 2.89     | 366.9    | 122      | 493.2   | 10.3     |
| $^{208}$Pb 00       | 127.54   | 3.43     | 2080     | 124.5    | 2212.8  | 18.8     |
| $^{208}$Pb 10       | 127.38   | 3.68     | 2176     | 93       | 2269    | 21       |
| $^{208}$Pb 11       | 114.10   | 16.58    | 1658     | 694      | 2370    | 19.3     |

We have evaluated the amounts of NEWSR $m_-(0)$ and EWSR $m_-(1)$ in different excitation energy regions, and listed them in Table 1. The EWSR in the energy region below 30 MeV (where the one particle-one hole transitions are located) is decreased, after the inclusion of the tensor term. From Table 1, we also see that an appreciable amount of EWSR is shifted from the lower energy region (0-30 MeV) to the higher energy region (30-60 MeV) by including tensor terms in HF plus RPA calculations.

We also calculated the values of NEWSR in the 0-30 MeV and 30-60 MeV energy regions for $^{90}$Zr and $^{208}$Pb. When the tensor is not included in the residual interaction (i.e., the calculations labeled by 00 and 10), the values of NEWSR in the energy region between 30-60 MeV for both $^{90}$Zr and $^{208}$Pb are small only few percent of the NEWSR(Fig. 5). But in the case 11, about 10% of NEWSR is shifted from the lower energy region to the higher energy region (Corresponding 25% and 29% of EWSR in $^{90}$Zr and $^{208}$Pb, respectively). Moreover, we can see that essentially no unperturbed strength appears in this region (see the Fig. 5). This means that including tensor terms in simple RPA calculation shifts about 10% of the GT strength to the energy region 30-60 MeV. While 2p-2h couplings will increase further these high energy strength, we would like to stress that the tensor correlations move substantial GT strength from the low energy region 0-30 MeV to the high energy region 30-60 MeV even within the 1p-1h model space.

In $^{90}$Zr, one can notice that the GT strength is concentrated in two peaks in the region below 30 MeV. There are only two important configuration involved which are $(\pi 1g_{9/2} - \nu 1g_{9/2}^{-1})$ and $(\pi 1g_{7/2} - \nu 1g_{7/2}^{-1})$ (see the left panel of Fig. 5). When the tensor term is included only in HF and neglected in RPA, the centroid in the energy region of 0-30 MeV are moved upwards by about 1.5 MeV, and the high energy peak
at $Ex \sim 16\text{MeV}$ is moved upwards by only 0.5 MeV, as compared with the results without tensor term. When the tensor term is included both in HF and RPA, the centroid of the GT strength in the energy region 0-30 MeV is moved downwards by about 1 MeV, and the high energy peak is moved downwards about 2 MeV, as compared with the results obtained without tensor term. Including tensor terms in RPA makes the two main separated peaks closer (this situation also happens for $^{48}\text{Ca}$). This result can be attributed from the HF and RPA correlations of the tensor term. When the $\nu_1g_{9/2}$ orbit is filled by neutrons, the tensor correlations give a quenching on the spin–orbit splitting between $\pi_1g_{9/2}$ and $\pi_1g_{7/2}$ orbits so that the unperturbed energies of the two main $p-h$ configurations ($\pi_1g_{7/2} - \nu_1g_{9/2}^{-1}$) and ($\pi_1g_{9/2} - \nu_1g_{9/2}^{-1}$) are closer in energy as is shown in Fig. 5. The RPA results in Fig. 5 with labeled by (00) and (10) reflect these changes of HF single particle energies due the tensor correlations and the energy difference between two peaks is narrower.

In $^{208}\text{Pb}$, from the right panel of Fig. 5 we see that the GT strength is concentrated in two peaks in the low energy region of 0-30MeV for all 00, 10 and 11. There are eleven important configurations which do contribute to these peaks. When the tensor terms are only included in HF and neglected in RPA, the centroid of these peaks is moved upwards about 0.5 MeV, and the higher energy peak at $Ex \sim 18\text{MeV}$ is also raised by about 0.8 MeV. When the tensor terms are included in both HF and RPA calculation, the centroid of these peak moves downward by about 1.5 MeV, and the higher energy peak moves also downwards by about 3.3 MeV, compared with the result obtained without tensor terms. By including tensor terms in RPA calculation, the GT strengths in the energy region of 30-60 MeV are increased substantially by the shift of the strength in the energy region of 0-30 MeV through the tensor force.

### 4. Summary

We study the effect of tensor correlations on single-particle and collective states within Skyrme Hartree-Fock and RPA model. Firstly, We study the role of tensor interactions in Skyrme effective interaction on the isospin dependence of spin-orbit splittings in N=82 isotones and Z=50 isotope. The different role of the triplet-even and triplet-odd tensor forces is elucidated by analyzing the spin-orbit splittings in these nuclei. The experimental isospin dependence of these splittings cannot be described by HF calculations with standard Skyrme forces, but is very well accounted for when the tensor forces are introduced. the GT excitations in $^{90}\text{Zr}$ and $^{208}\text{Pb}$ in the HF plus RPA framework with a Skyrme interaction SIII. If the tensor term is included in both HF and RPA, the centroid of G-T strength in the energy region below 30 MeV is moved downwards by about 1 MeV for $^{90}\text{Zr}$ and 3.3 MeV for $^{208}\text{Pb}$. At the same time, the dominant peak at $E_x \sim 16\text{MeV}(18\text{MeV})$ in $^{90}\text{Zr}(^{208}\text{Pb})$ is also moved downwards by about 2 MeV(3MeV). It is pointed out for the first time that
about 10% of NEWSR is moved in the high energy region of 30-60 MeV by the
tensor correlations in RPA even within $1p-1h$ model space. It was pointed out
recently that the high energy GT strength is shifted by the coupling between GT ana
spin-quadrupole states due to the tensor correlations which has the intrinsic strong
coupling [14] It is interesting to point out that the main GT peak, contrarily, gets
the energy shift downward because of the peculiar feature of the tensor correlations.
The tensor interaction is spin-dependent, so we expect that it can have important
effects not only on the GT transitions, but on spin-dipole and other spin dependent
excitation modes as well.

I would like to thank all the collaborators to proceed the projects of tensor forces
in the mean field models. Especially I would like to thank Gianluca Colò for many
stimulating discussions in various stages of collaborations. This work is supported
in part by the Japanese Ministry of Education, Culture, Sports, Science and Techno-
logy by Grant-in-Aid for Scientific Research under the program number (C (2))
20540277.

[1] Fl. Stancu, D. M. Brink and H. Flocard, Phys. Lett. 68B (1977) 108.
[2] G. Colò, H. Sagawa, S. Fracasso, P.F. Bortignon, Phys. Lett. B646, 227 (2007);
Wei Zou, Gianluca Colò, Zhongyu Ma, Hiroyuki Sagawa and Pier Francesco
Bortignon, Phys. Rev. C77, 014314 (2008)
[3] B.A. Brown, T. Duguet, T. Otsuka, D. Abe, and T. Suzuki, Phys. Rev. C74,
061303 (2006).
[4] D.M. Brink and Fl. Stancu, Phys. Rev. C75, 064311 (2007).
[5] T. Lesinski, M. Bender, K. Bennaceur, T. Duguet, and J. Meyer, Phys. Rev.
C76, 014312 (2007).
[6] Wei Zou, G. Colò, Zhongyu Ma, H. Sagawa and P.F. Bortignon, Phys. Rev.
C77, 014314(2008).
[7] T. Otsuka et al., Phys. Rev. Lett. 95 (2005) 232502.
[8] K.F. Liu, H. Luo, Z. Ma, Q. Shen, S.A. Moszkowski, Nucl. Phys. A534, 1
(1991); K.F. Liu, H. Luo, Z. Ma, Q. Shen, Nucl. Phys. A534, 25 (1991).
[9] M. Beiner, H. Flocard, N. Van Giai, and P. Quentin, Nucl. Phys. A238, 29
(1975).
[10] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and R. Schaeffer, Nucl. Phys.
A627, 710 (1997).
[11] T. H. R. Skyrme, Nucl. Phys. 9 (1959) 615.

[12] J. P. Schiffer et al., Phys. Rev. Lett. 92 (2004) 162501.

[13] J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics (Wiley, New York, 1952)

[14] C.L. Bai, H. Sagawa, H.Q. Zhang, X.Z. Zhang, G. Colò and F.R. Xu, Phys. Lett. B675, 28 (2009); Phys. Rev. C79, 04130(R)(2009).

[15] J. Rappaport et al., Nucl. Phys. A410, 371 (1983); C. Gaarde, in Proceedings of the Niels Bohr Centennial Conference on Nuclear Structure, Copenhagen (North-Holland, Amsterdam, 1985), p. 449c.

[16] G.F. Bertsch and I. Hamamoto, Phys. Rev. C26, 1323 (1982).