Factorization and the Soft-Collinear Effective Theory: 
Color-Suppressed Decays

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Abstract. We discuss the soft-collinear effective theory (SCET) and kinematic expansions in $B$-decays, focusing on recent results for color suppressed $B \to D^{(*)}X$ decays. In particular we discuss model independent predictions for $B^0 \to D^0 \pi^0$ and $\bar{B}^0 \to D^{(*)0} \pi^0$, and update the comparison using new experimental data. We show why HQET alone is insufficient to give these results. SCET predictions are also reviewed for other $B$ and $\Lambda_b$ decay channels that are not yet tested by data.

INTRODUCTION

The soft-collinear effective theory (SCET) provides a formalism for systematically investigating processes with both energetic and soft hadrons based solely on the underlying structure of QCD. Essentially all known methods for simplifying QCD predictions, without introducing model dependent assumptions, depend on exploiting hierarchies of mass scales. For predictions based on SU(3) symmetry we exploit the fact that $m_{u,d,s}/\Lambda \ll 1$, and expect corrections at the $\sim 30\%$ level. In lattice QCD simulations we choose our lattice spacing $a \ll 1/\Lambda$ and volume $V \gg 1/\Lambda^3$ so that we can focus on non-perturbative effects at scales $\sim \Lambda$. In SCET we expand in $\Lambda/Q \ll 1$, with the large momentum of an energetic hadron or jet being $\sim Q$. For $B$ decays corrections will be at the $\sim 20$–$30\%$ level depending on the energy scale $Q$.

Most effective theories that we are familiar with are designed to separate the physics for hard $p^\mu_Q \sim Q^2$ and soft $p^\mu_s \ll Q^2$ momenta. Examples include the electroweak Hamiltonian, chiral perturbation theory, heavy quark effective theory, and non-relativistic QCD. In SCET we incorporate an additional possibility, namely energetic hadrons where the constituents have momenta $p^\mu_\eta$ nearly collinear to a light-like direction $n^\mu$. Both the energetic hadron and its collinear constituents have $\vec{n} \cdot p_\eta \sim Q$, where we have made use of light-cone coordinates $(p_\eta^+, p_\eta^-, p_\eta^z) = (n \cdot p_\eta, \vec{n} \cdot p_\eta, p_\eta^z)$. The collinear constituents still have small offshellness $p_\eta^z \sim p_\eta^2$. The process of disentangling the interactions of hard-collinear-soft particles is known as factorization, and is simplified by the SCET framework.

Much like any effective theory the basic ingredients of SCET are its field content, power counting, and symmetries. The Lagrangian and operators, are organized in a series where only $L^{(0)}$ and $O^{(0)}$ are relevant at LO, an additional $L^{(1)}$ or $O^{(1)}$ is needed at NLO, etc. The expansion parameter will be $\lambda = \sqrt{\Lambda_{QCD}/Q}$ or $\eta = \Lambda_{QCD}/Q$ depending on whether the collinear fields describe an energetic jet of hadrons or an individual energetic hadron. The effective theory with an expansion in $\lambda$ is called SCET$_I$, while the one with an expansion in $\eta$ is called SCET$_II$. In processes such as color-suppressed decays the separation of scales is $Q^2 \gg \Lambda^2$ and the chain QCD–SCET$_I$–SCET$_II$ proves to be useful. The intermediate theory SCET$_I$ provides the dynamics to rearrange soft and collinear quark lines so that they can end up in soft and energetic hadrons. The final theory SCET$_II$ describes the universal low energy hadronic matrix elements. In the case of color-suppressed decays $B \to D^{(*)}M$ these are light-cone distribution functions $\phi_M(x)$ where $M = \pi, \rho, K,$ or $K^*$ and two generalized parton distribution functions $S^{(0,2)}(k_F^+ , k_F^2)$ for the $B \to D^{(*)}$ transition.

COLOR-SUPPRESSED DECAYS AND SCET

Color-suppressed decays were investigated in Ref. [5] using SCET. For $B \to D \pi$ decays the four quark oper-
the amplitudes we use \( A_{+}=A(B^{0}\rightarrow D^{+}\pi^{-}) \), \( A_{0}=A(B^{-}\rightarrow D^{0}\pi^{-}) \), and \( A_{00}=A(B^{0}\rightarrow D^{0}\pi^{0}) \). Written in terms of isospin amplitudes

\[
A_{+}=T+E=\frac{1}{\sqrt{3}}A_{3/2}+\sqrt{\frac{2}{3}}A_{1/2},
\]

\[
A_{0}=T+C=\sqrt{3}A_{3/2},
\]

\[
A_{00}=\frac{C-E}{\sqrt{2}}=\sqrt{\frac{2}{3}}A_{3/2}-\frac{1}{\sqrt{3}}A_{1/2}.
\]

The amplitudes for decays to \( B\rightarrow D^{(*)}\rho \) are defined in a similar fashion.

In the large \( N_{c} \) limit \( C/T\sim E/T\sim 1/N_{c} \) (where we take \( C_{1}\sim 1 \) and \( C_{2}\sim 1/N_{c} \)). The color-allowed amplitudes \( A_{+} \) and \( A_{0} \) are described by a factorization theorem [3,6,8], proven with SCET [9]

\[
A^{(s)}=N^{(s)}x(w_{\text{max}})\int_{0}^{1}dxT^{(s)}(x,m_{c}/m_{b})\phi_{\pi}(x)+\ldots,
\]

where \( \xi(w_{\text{max}}) \) is the Isgur-Wise function at maximum recoil, \( \phi_{\pi}(x) \) is the light-cone distribution function for the pion, \( T=1+O(\alpha_{s}) \) is the hard scattering kernel, and \( N^{(s)}=\frac{G_{F}}{\sqrt{2}}V_{ub}V_{ud}^{*}f_{\pi}f_{\pi}m_{b}m_{D}(1+m_{b}/m_{D}) \). The ellipses in Eq. (3) denote terms suppressed by \( \Lambda/Q \) where \( Q=m_{b},m_{c},E_{\pi} \). In the heavy quark limit, Eq. (3) predicts \( A=A^{*} \), so \( Br(B^{0}\rightarrow D^{+}\pi^{-})=Br(B^{0}\rightarrow D^{*}\pi^{-}) \) and \( Br(B^{-}\rightarrow D^{0}\pi^{-})=Br(B^{-}\rightarrow D^{0}\pi^{-}) \). This agrees well with the experimental results [12,13], which yield

\[
\frac{Br(B^{0}\rightarrow D^{+}\pi^{-})}{Br(B^{0}\rightarrow D^{+}\pi^{-})}=1.03\pm0.14,
\]

\[
\frac{Br(B^{-}\rightarrow D^{0}\pi^{-})}{Br(B^{-}\rightarrow D^{0}\pi^{-})}=0.93\pm0.11.
\]

Eq. (3) also predicts \( A_{+}=A_{0} \), however experimentally \( |A_{0}/A_{+}|=0.77\pm0.05 \) for \( D\pi \) and \( 0.81\pm0.05 \) for \( D^{*}\pi \). We will see that the reduction of these numbers from 1 are explained by an SCET power correction. Other mechanisms for testing factorization for color-allowed decays include using multibody states to make tests as a function of \( q^{2} \) or \( w_{\text{max}} \), looking for decays which do not occur in naive-factorization [13], or tests using inclusive \( B\rightarrow D^{(*)}\pi \) spectra or the equality of rates for particular multibody final states [14].

The color-suppressed amplitude \( A_{00} \) has contributions from \( C \) and \( E \), but not \( T \). With large \( N_{c} \) very little can be said about the \( C \) and \( E \) contributions, besides the fact that we expect \( A_{00}<A_{+}\sim A_{0} \). In SCET the amplitudes \( C \) and \( E \) are suppressed by \( \Lambda/\Lambda_{Q} \) relative to \( T \). Despite this power suppression, predictive power is retained since only a single type of SCET time ordered product contributes to the proper quark rearrangement, \( T(Q_{8}^{(s)}(0),i.L_{q}^{(1)}(x),i.L_{q}^{(1)}(y))|O_{r}(0) \). This combination contributes to both \( C \) and \( E \) as shown in Fig. 2.
The agreement can also be shown graphically. The isospin relations between isospin amplitudes. The predictions in Eq. (7) have corrections at $O(\alpha_s(Q))$ and $O(\Lambda/Q)$. For $M = \pi^0, \rho^0$ the long distance amplitude is suppressed by $\alpha_s(Q)$. The current experimental data [13 16 17] gives the world averages that the data is not yet available. For instance, the analysis above also applies for the $\rho$ predicting

$$\delta(D^\ast \rho) = \delta(D\rho),$$  

$$\frac{Br(B^0 \rightarrow D^0 \rho)}{Br(B^0 \rightarrow D^0 \rho^0)} = \frac{Br(B^0 \rightarrow D^0 \rho)}{Br(B^0 \rightarrow D^0 \rho^0)}.$$

A similar prediction can be made for decays to $D^{(s)}_{\ast} K^{(s)}$ except in this case the long distance contributions to the amplitudes are not suppressed. This means that both longitudinal and perpendicular polarizations occur at the same order. The analog of Eq. (11) is therefore:

$$\frac{Br(B^0 \rightarrow D^\ast_s K^\ast)}{Br(B^0 \rightarrow D^s_s K^\ast)} = \frac{Br(B^0 \rightarrow D^s_s K^\ast)}{Br(B^0 \rightarrow D^0 K^\ast)}.$$  

where these color-suppressed decays are not part of an isospin triangle. Cabibbo suppressed decays to kaons are more analogous to $D \pi$ and $D \rho$, except that they also have long distance contributions which are not suppressed. For instance, the analog of Eq. (11) is

$$\frac{Br(B^0 \rightarrow D^0 K^0)}{Br(B^0 \rightarrow D^0 K^0)} = \frac{Br(B^0 \rightarrow D^0 K^0)}{Br(B^0 \rightarrow D^0 K^0)}.$$  

The predictions in Eqs. (11), (12), and (13) will be tested once data on $B^0 \rightarrow D^{(s)} \rho^0$, $B^0 \rightarrow D^{(s)}_{\ast} K^{(s)}$, and $B^0 \rightarrow D^{(s)}_{\ast} K^{(s)}$ become available. The significance of the long distance terms will be tested by comparing $Br(B^0 \rightarrow D^0_{\ast} \bar{K}^0_{\ast})$ to $Br(B^0 \rightarrow D^0_{\ast} \bar{K}^0_{\ast})$ or $Br(B^0 \rightarrow D^{(s)} \bar{K}^{(s)}_{\ast})$ to $Br(B^0 \rightarrow D^{(s)} \bar{K}^{(s)}_{\ast}).$
The full factorization theorem for color suppressed decays takes the form [13]

\[ A^{D^{(*)}M}_{00} = N^M_0 \int dx dz dk^+_1 dk^+_2 \int_{L-R} T_{L-R}^{(i)}(z) J^{(i)}(x, k^+_1, k^+_2) \times S^{(i)}(k^+_1, k^+_2) \phi_M(x) + A^{D^{(*)}M}_{long}. \]  

(14)

where \( T_{L-R}^{(i)} \) are hard scattering kernels and \( N^M_0 = G_F V_{cb} V^*_{ub} f_M \sqrt{m_B m_D} / 2 \). The non-perturbative dynamics is contained in \( \phi_M \), the light-cone distribution function for meson \( M \), and \( S^{(i)}, i = 0, 8 \), a generalized parton distribution function for the \( B \to D^{(*)} \) transition with \( k^+_1 \) and \( k^+_2 \) being momentum fractions of the light spectator quarks. Finally, the jet function \( f^{(i)} \) is sensitive to physics at the \( \mu^2 \simeq E_M \Lambda \) scale and is responsible for the quark rearrangement.

The predictions discussed above are all valid independent of the form of \( f^{(i)} \), meaning to all orders of \( \alpha_s(\mu_0) \) at the intermediate scale, \( \mu_0^2 \simeq E_M \Lambda \). If we expand \( J \) in powers of \( \alpha_s(\mu_0) \) then this introduces additional uncertainty, but gives further predictions. At lowest order we have

\[ A^{D^{(*)}M}_{00}(0) = \frac{16 \pi \alpha_s(\mu_0)}{9} s_{\text{eff}}(\mu_0) \langle x^{-1} \rangle_M, \]

(15)

where \( C_L^{(0)} = C_1 + C_2 / 3, \langle x^{-1} \rangle_M = \int dx/x \phi_M(x), \) and \( s_{\text{eff}} = -s^{(0)} + C_2 / (4 C_1 + 4/3 C_2) s^{(8)} \) with \( s^{(8)} = \langle d k^+_1 d k^+_2 / (k^+_1 k^+_2) \rangle S^{(8)}(k^+_1, k^+_2). \) Corrections to \( \phi \) in Fig. 3 are \( O(\alpha_s(\mu_0)) \) and \( O(\Lambda/Q) \). At this order the strong phase \( \phi \) is generated by \( s_{\text{eff}} \) and so is independent of whether \( M = \pi \) or \( M = \rho \). Therefore, we predict that \( \phi \) is universal for \( D^{(*)} \) and \( D^{(*)} \rho \).

For the ratio of charged amplitudes Eq. (15) can be used to predict the leading power correction,

\[ R^{D^{(*)}M}_c = \frac{A(B^{0} \to D^{(*)} + M^0)}{A(B \to D^{(*)} M^0)} = 1 - \frac{16 \pi \alpha_s m_D^{(c)}}{9 (m_B + m_D^{(c)})^2} \xi(\alpha_0) / E_M. \]

(16)

A value \( s_{\text{eff}} \simeq (430 \text{ MeV})^d 4^{d/4} \) gives \( |R^{D\pi}_c| \simeq 0.8 \), fitting the experimental values given below Eq. (4) with parameters of natural size. In naive factorization the correction term in \( R_c \) would depend on the decay constant \( f_M \), however for the true factorization theorem that gives Eq. (16) this turns out not to be the case. The observed similarity between \( R_c \) for \( D\pi \) and \( D\rho \) can be explained by having \( \langle x^{-1} \rangle_{\pi} \simeq \langle x^{-1} \rangle_{\rho} \) and is not spoiled by the fact that \( f_{\rho} / f_{\pi} \simeq 1.6 \). Experimentally

\[ \frac{|R^{D\pi}_c|}{|R^{D\rho}_c|} = 0.96 \pm 0.13. \]

(17)

With this approximate equality and the \( \phi^\pi = \phi^\rho \) prediction, we would expect that the strong phase \( \sigma^{D\pi} \simeq \delta^{D\pi} \), or in other words that the \( D\pi \) and \( D\rho \) triangles (as in Fig. 3) will be similar. If this turns out not to be the case then it would indicate that there are substantial \( \alpha_s^2(\mu_0) \) corrections to \( J^{(0,8)} \). This would mean that the subset of predictions that follow from Eq. (15), which depend on a perturbative expansion for \( J^{(0,8)} \), should not be trusted. Predictions for color-suppressed decays using other methods have been discussed in Refs. [23, 11, 14, 20, 21, 22].

**BARYON DECAYS**

Recently, the authors in Ref. [24] have used SCET to make model-independent factorization predictions for baryon decays. The main results for \( \Lambda_b \to \Lambda_c \pi, \Lambda_c \rho, \Sigma_c^{(*)} \pi, \Sigma_c^{(*)} \rho, \) and \( \Xi_c^{(*)} K \) are briefly summarized here. The notation is \( \Sigma = (2455) \) and \( \Sigma^* = (2520) \).

The electroweak Hamiltonian for these baryon decays is in Eq. (1). The diagrams for the flavor contractions differ from the meson decays and are shown in Fig. 4. The decays to \( \Lambda_c \) get contributions from \( T, C, E, \) and \( B \), decays to \( \pi \) have contributions from \( C, E, \) and \( B \), and decays to \( \Xi_c \) only from the \( E \) and \( B \) amplitudes.

In the large \( N_c \) limit \( C/T \sim E/T \sim N_c^0 \), while \( B/T \sim N_c \). Thus, even \( \Lambda_b \to \Lambda_c \pi \) decays do not factorize in the large \( N_c \) limit. The extra factors of \( N_c \) arise from the choice of which of the \( N_c \) quarks in the \( \Lambda_b \) and/or \( \Lambda_c \) participate in the weak interaction.

Expanding in \( \Lambda/Q \) where \( Q = \{m_b, m_c, E_\pi \} \) using SCET one finds \( C/T \sim E/T \sim N_c \) and \( B/T \sim N_c \). For \( \Lambda_b \to \Lambda_c \pi \) the leading order result is from \( T \) and gives

\[ A^{\Lambda_c \pi} = N_c \zeta(\max) \int_0^1 dx T_L(x, m_c / m_b) \phi_\pi(x). \]  

(18)
dicts that state \([5, 24]\). This included the decays—non-leptonic decays with charmed hadrons in the final state—up to corrections suppressed by \(\Lambda/Q\) or \(\alpha_s(Q)\). Here \(\Sigma^c\pi = \Sigma^{c(0)}\pi^0 + \Sigma^{c(+)}\pi^-\). A similar prediction is also made for decays to a \(\rho\),

\[
\frac{\text{Br}(\Lambda_b \to \Sigma^c\rho)}{\text{Br}(\Lambda_b \to \Sigma^c\rho')} = 2.
\]

Using \(\Sigma^{c(0)}\rho^0 \to \Lambda_c \pi^+\pi^-\pi^-\) Eq. (20) may be easier to test experimentally than Eq. (19). For decays involving cascades there can be sizeable long distance contributions, but we still expect

\[
\frac{\text{Br}(\Lambda_b \to \Sigma^cK)}{\text{Br}(\Lambda_b \to \Sigma^cK')} = 2,
\]

\[
\frac{\text{Br}(\Lambda_b \to \Sigma^cK)}{\text{Br}(\Lambda_b \to \Sigma^cK')} = 2.
\]

The \(\text{Br}(\Lambda_b \to \Sigma^cK)\) is also expected to be of the same order of magnitude since it occurs at this order in the power counting.

**CONCLUSION**

In this talk we reviewed the SCET predictions for non-leptonic decays with charmed hadrons in the final state \([2, 24]\). This includes the decays \(B^0 \to D^{(*)\rho^0}\), \(B^- \to D^{(*)\rho^-}\), and \(\Lambda_b \to \Lambda_c\pi\) which occur at leading order, as well as decays which are power suppressed, \(B^0 \to D^{(*)\pi^0}\) and \(\Lambda_b \to \Sigma^{(*)\pi}\). Analogous decays where the \(\pi\) is replaced by a \(\rho\) or kaon were also discussed. For \(B^0 \to D^{(*)\pi^0}\) we updated the experimental comparison in Fig. 3 and Eq. 45 to take into account the new BaBar results [17].

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