Two different regimes of the turbulent wave cascade decay on the surface of quantum liquids

L V Abdurakhimov, M Yu Brazhnikov, I A Remizov, and A A Levchenko
Institute of Solid State Physics, Russian Academy of Sciences, Chernogolovka, 142432, Russia
E-mail: abdurl@issp.ac.ru

Abstract. We report recent results of the experimental study of capillary turbulence decay on the surface of quantum liquids: normal and superfluid \(^4\)He and liquid hydrogen. In our experiments the turbulent cascade of capillary waves was damped at high frequencies due to viscous dissipation. Turbulent spectrum in high-frequency dissipative region was described by exponential decay function which was in accordance with the theoretical predictions. Moreover, for the first time two different regimes of the turbulent wave cascade decay were observed depending on the type of pumping excitation. When the surface was excited by broad-band noise pumping, a characteristic frequency \(f_d\) of the cascade exponential decay \(\sim \exp(-f/f_d)\) was close to the high-frequency edge of the inertial range. Otherwise, in the case of harmonic pumping, the characteristic frequency \(f_d\) was close to the low-frequency pumping frequency. Thus, the spectrum decay was more dramatic in the case of harmonic pumping than in the case of broad-band pumping. This difference in the values of \(f_d\) can be qualitatively explained in frames of wave turbulence theory by taking into account a non-locality of 3-wave interactions at high frequencies in the case of harmonic pumping.

1. Introduction

It is known that an ensemble of weakly interacting nonlinear waves can be described in frames of wave turbulence (WT) theory [1]. WT is a general phenomenon and has been studied in the variety of wave systems experimentally as well as theoretically since the '60s (spin wave turbulence in solids [2], Langmuir wave turbulence in plasma [3], acoustic wave turbulence in superfluid [4]). In WT theory it is commonly supposed that wave energy is driven into wave system from an external source at large scales and cascaded through inertial range due to nonlinear interactions to small scales, where energy flux is absorbed by viscous dissipation. Theory predicts and many experiments confirm that in the inertial range the energy wave spectrum is described by power law function (Kolmogorov-Zakharov (KZ) spectrum) [1], however, as far as we know, wave spectrum in the dissipation range has not yet been studied experimentally.

Recently, our experimental studies of wave turbulence on the surface of normal and superfluid helium-4 and liquid hydrogen have shown that surface waves on cryogenic liquids are very fruitful model system for testing predictions of WT theory [5, 6]. In this paper we present results of experimental study on capillary wave turbulence decay in the dissipation range on the surface of superfluid helium-4 and liquid hydrogen.
Capillary waves are ripples on the surface of liquid, whose dynamics is determined by surface tension. Frequency of capillary wave $\omega$ is related to its wavevector $k$ by dispersion law $\omega^2 = \frac{\sigma}{\rho} k^2$, where $\sigma$ – surface tension coefficient of liquid, $\rho$ – density of liquid. In 1967 it was shown that capillary wave turbulence can be described by Boltzmann-type kinetic equation in terms of spectral “occupation” numbers $n_k$ in $k$-space (or $n_\omega$ in $\omega$-space, $n_\omega \sim n_k k^2/\omega$) [7]. However, in experiments it is easier to study wave spectrum $I_\omega$ of one-point surface elevations $\eta(t)$, which is related to “occupation” numbers by expression $I_\omega = \langle n_\omega^2 \rangle \sim \frac{\omega}{\omega^*} \sim n_k$. WT theory predicts that capillary turbulent spectrum $I_\omega$ is described by power law function in inertial range, $I_\omega \approx \omega^{-m}$. Index $m$ is equal to 17/6 for broadband pumping [7] (when characteristic pumping frequency $\omega$ is much less than characteristic spectral width of pumping $\Delta \omega$, $\omega \ll \Delta \omega$), and 23/6 for narrowband pumping [8] (when $\omega \gg \Delta \omega$).

The structure of the spectrum in the dissipation region depends on what kind of interaction is dominant for waves in the dissipation region [9]: local interaction between themselves (which is characterized by an interaction time $\tau$) or non-local interaction with waves from the inertial range (which is characterized by an interaction time $\tau_1$). When local interaction prevails ($\tau < \tau_1$), the spectrum in the region of strong dissipation ($\omega > \omega_d$) can be described [9] by “quasi-Planck” distribution

$$n_k = B \omega_k^b \exp(-\omega_k/\omega_d),$$

where $\omega_d$ – cut-off frequency of the inertial range, $B$ and $b$ – some constants. It should be noted, that $\tau > \gamma^{-1}$ in the dissipation range, where $\gamma = 2\nu k^2$ – viscous damping coefficient for surface waves ($\nu$ is kinematic viscosity of liquid).

2. Experimental details
 Experimental setup has been already described elsewhere [5, 6]. Experiments were performed with two different liquids: liquid helium-4 (normal and superfluid) and liquid hydrogen. Experimental results with superfluid helium-4 are shown below, but qualitative similar results have been observed with normal helium-4 and liquid hydrogen.

In our experiments, liquid was condensed into a cylindrical cup (in helium experiments the cup dimensions were 4 mm (depth) × $D = 30$ mm (diameter), in hydrogen experiments – 6 mm × $D = 60$ mm). Liquid surface was charged by positive ions (“snowballs”) with the help of $\beta$-radioactive plate placed at the cup bottom, and, by applying AC electric field perpendicularly to the charged surface, capillary waves were excited on the surface. Surface oscillations were registered by an optical technique. Laser beam was reflected from the oscillating surface, time variations of reflected laser beam power $P(t)$ were recorded, and spectrum $P^2_\omega$ was calculated by FFT. It can be shown, that in our experiments the spectrum of the power $P^2_\omega$ was proportional to the spectrum of surface deviations $I_\omega$, $P^2_\omega \sim I_\omega$.

3. Results & Discussion
 In Fig. 1 turbulent distribution $P^2_\omega$ of capillary waves on superfluid helium-4 surface is shown that was obtained in experiments at the temperature $T = 1.7$ K. Surface was pumped by harmonic (sinusoidal) force at frequency $f_p \approx 80$ Hz. The spectrum consists of set of equidistant harmonics. The first one is situated at the frequency $f_p$ and corresponds to the wave excited by the pumping. Other harmonics are generated due to 3-wave nonlinear interactions on frequencies multiple to the pumping frequency $f_p$. Inertial range is observed from 800 Hz to 4 kHz, where the wave spectrum is described by power-law function $P^2_\omega \sim \omega^{-3.7}$. The value $-3.7$ is in accordance with theoretical predictions for narrow-band pumping [8]. At frequencies greater than 4 kHz the turbulent distribution dramatically falls off due to the viscous dissipation and tends to the level of instrumental noise. Cut-off frequency of the turbulent cascade $f_h \approx 4$ kHz can be regarded as high-frequency boundary of the inertial range. In the dissipation region (at frequencies higher
**Figure 1.** Turbulent spectrum $P^2_\omega$ of capillary waves on the surface of He-II under harmonic pumping at $f_p \approx 80$ Hz. Straight black line corresponds to power law function $P^2_\omega \sim \omega^{-3.7}$. High-frequency edge of inertial range $f_b$ is about 4 kHz. Red line represents exponential decay function $P^2_\omega \sim \exp(-f/f_d)$, with the characteristic frequency $f_d \approx 170$ Hz. It should be emphasized that $f_d$ is close to the pumping frequency $f_p$.

**Figure 2.** Turbulent spectrum $P^2_\omega$ of capillary waves on the surface of He-II under broadband pumping at the frequency band of 90 – 200 Hz. Straight black line corresponds to power law function $P^2_\omega \sim \omega^{-2.8}$. High-frequency boundary of the inertial range $f_b$ is approximately 2 kHz. Red line represents exponential decay function $P^2_\omega \sim \exp(-f/f_d)$, with the characteristic frequency $f_d \approx 700$ Hz. In accordance with theory, $f_d$ is quite close to the cut-off frequency $f_b$. 
than $f_b$) the turbulent distribution can be described by exponential decay law $P_0^2 \sim \exp(-f/f_d)$, with the characteristic frequency $f_d \approx 170$ Hz. It should be emphasized that in the theoretical work [9] it is supposed that the characteristic frequency $f_d$ should be close to the cut-off frequency $f_b$. Instead of this, we obtained that $f_d$ is close to the pumping frequency $f_p$ in the case of harmonic pumping (moreover, we observed that this qualitative relation between $f_d$ and $f_p$ held in various experiments with different frequencies and amplitudes of the harmonic pumping).

When the surface was excited by broadband noisy force, the shape of the wave distribution was changed. In Fig. 2 the spectrum is shown that was obtained when the surface was pumped by noisy force in the frequency band 90-200 Hz. It is seen that the spectrum is continuous, and was changed. In Fig. 2 the spectrum is shown that was obtained when the surface was pumped…

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References
[1] Zakharov V, L’vov V and Falkovich G 1992 Kolmogorov Spectra of Turbulence I (Springer-Verlag)
[2] Zakharov V, L’vov V and Starobinets S 1975 Sov. Phys. Usp. 17 896
[3] Zakharov V 1967 Sov. Phys. JETP 24 455
[4] Ganshin A, Efimov V, Kolmakov G, Mezhov-Deglin L and McClintock P 2010 New J. of Phys. 12 083047
[5] Abdurakhimov L, Brazhnikov M, Remizov I and Levchenko A 2010 JETP Letters 91 291
[6] Brazhnikov M, Abdurakhimov L, Filatov S and Levchenko A 2011 JETP Letters 93 31
[7] Zakharov V and Filonenko N 1967 J. Appl. Mech. Tech. Phys. 4 37
[8] Falkovich G and Shafarenko A 1988 Sov. Phys. JETP 68 1393
[9] Ryzhenkova I and Falkovich G 1990 Sov. Phys. JETP 71 1085
[10] Kolmakov G 2006 JETP Lett. 83 58