Abstract

The interaction of light quarks and instanton liquid in the phase of nonzero values of chiral condensate is studied at finite quark/baryon density. Calculating the generating functional in the tadpole approximation we investigate the behaviour of dynamical quark mass and chiral condensate together with instanton liquid density (which shows insignificant increase) as a function of quark/baryon chemical potential. We argue on the noticeable increase of quark density at the point of developing the nonzero magnitude of diquark condensate due only to the quark interaction with the instanton liquid.

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Since the discovery of instanton solutions in the Yang-Mills theory the functional integral language has become very popular and the most operational instrument of quantum chromodynamics (QCD) phenomenology. This formulation tolerates the nonperturbative identification of the field configurations dominating the action and manifests the fundamental physics features of QCD distinguishing it from other complex systems with many degrees of freedom. In an overwhelming number of these theories the quanta exchanged between interacting fermions according the initial Lagrangian lead to the formulation of the concept of interaction potential and the definition of its concrete form. The dominating role of topological excitations of the gluon field and quantum oscillations about them in the QCD physics allowed us to understand and successfully explain new physical effects related, first of all, to the process of chiral symmetry breaking. Moreover, it compels us to have some doubts on the potential concept (perturbative description) pertinent.

In the intervening years considerable theoretical efforts were devoted to extend the analytical description beyond dilute instanton gas approach [1]. It was shown that an effective instanton-anti-instanton interaction could stabilize the medium of these pseudoparticles (PP) [2] at the level of pretty reasonable values of major parameters (average sizes and distances between PPs) forming rather instanton liquid [3]. The approach developed has succeeded in making interesting (even quantitative) predictions although the lattice QCD studies only managed to put the problem of investigating the instanton role on the serious theoretical ground [4]. While much has been learned about the QCD phase structure at finite temperature, this structure at nonzero quark/baryon densities has been significantly less explored because of very serious technical difficulties, mainly, with putting the fermions (and respective chemical potential) on the lattice. Nowadays, the possibility of direct experimental measurements of instanton-induced effects is even discussed in view of the future program for deep-inelastic scattering (small size instantons) at HERA [5] and for ultrarelativistic heavy ion collisions at RHIC and LHC [6].

This paper is addressed to investigate the light quark interactions with the instanton liquid (IL) in the phase of nonzero chiral condensate values at finite quark/baryon densities and continue the analysis we began in our preceding paper [7] where a new method was developed to estimate the IL
back-reaction on the quark presence. Before it was always considered to be negligible. It is natural
to start with the generating functional of the IL theory having the factorized form [4]

\[ Z = Z_g \cdot Z_\psi , \]

where the factor \( Z_g \) originates information on the gluon condensate while the fermion component \( Z_\psi \) provides the opportunity to study the quark behaviour in the instanton medium and, in particular, the chiral and diquark condensates together with their excitations [4]. The factor \( Z_g \) is treated in quasi-classical approximation supposing the superposition of pseudo-particle (PP) fields \( A_{II}(x; \gamma) \) which are the Euclidean solutions of the Yang-Mills equations called the (anti-)instantons (\( \bar{I}I \)) to be dominant saturating configurations. Such a solution is characterized by the parameters \( \gamma = \rho, z, U \)
where \( \rho \) denotes the PP size centered at the coordinate \( z \) with the colour orientation defined by
the matrix \( U \) in the space of colour group \( SU(N_c) \). Let us suppose, for clarity, \( N \) pseudo-particles
available in four-volume \( V \) and a liquid is topologically neutral \( N_I = N_f = N/2 \). Formulating the
variational maximum principle the following practical estimate has been obtained

\[ Z_g \simeq e^{-\langle S \rangle} , \]

with the average IL action \( \langle S \rangle \) defined by the additive functional

\[ \langle S \rangle = \int dz \int d\rho \, n(\rho) \, s(\rho) , \tag{1} \]

and with averaging the action per one instanton

\[ s(\rho) = \beta(\rho) + 5 \ln(\Lambda\rho) - \ln \bar{\beta}^{2N_c} + \beta\xi^2\rho^2 \int d\rho_1 \, n(\rho_1)\rho_1^2 , \tag{2} \]

weighted with instanton size distribution function

\[ n(\rho) = C \, e^{-s(\rho)} = C \, \rho^{-5}\bar{\beta}^{2N_c}e^{-\beta(\rho) - \nu^2/\rho^2} , \tag{3} \]

where \( \bar{\rho}^2 = \int d\rho \, \rho^2 \, n(\rho)/n = (\frac{\nu}{\beta\xi^4n})^{1/2} \), \( n = \int d\rho \, n(\rho) = \frac{N}{V} \), \( \nu = b - 4 \), \( b = \frac{11N_c - 2N_f}{3} \)
and \( N_f \) is the number of flavours. The constant \( C \) is defined by the variational principle in the selfconsistent
way and \( \beta(\rho) = \frac{8\pi^2}{g^2} = -\ln C_{N_c} - b\ln(\Lambda\rho) \) \( (\Lambda = \Lambda_{MS} = 0.92\Lambda_{P.V.}) \) with another constant \( C_{N_c} \)
depending on the renormalization scheme, in particular, here \( C_{N_c} \approx \frac{4.66 \exp(-1.68N_c)}{\pi^2(N_c-1)!}\frac{1}{(N_c-2)!} \). The
parameters \( \beta = \beta(\rho) \) and \( \bar{\beta} = \beta + \ln C_{N_c} \) are fixed at the characteristic scale of the average PP size \( \bar{\rho} \).

The constant \( \xi^2 = \frac{27}{4\cdot N_c^2 - 1} \pi^2 \) gives information on interaction in the stochastic ensemble of
PPs. Then the IL parameters extracted, for example, average PP size \( \bar{\rho} \) and the IL density \( n \) agree reasonably with their values obtained from the phenomenological studies of the QCD vacuum.

In the quark determinant \( Z_\psi \) the quark fields are considered being influenced by stochastic PP
ensemble but the quark backreaction upon the instanton medium is usually ignored, i.e.

\[ Z_\psi \simeq \int D\bar{\psi} D\psi \langle \langle e^{S(\psi,\bar{\psi},A)} \rangle \rangle_A , \]

Small values of the packing fraction parameter \( n\bar{\rho}^4 \) allow us to neglect the correlations between
PPs and one usually deals with the limit \( N_c \to \infty \). The dominant contribution to the action of fermion
fields mainly comes from the zero modes \( \Phi(x - z; \mu) \) which are the solutions of the Dirac equation

\[ [i\bar{D}(A_{II}) - i\mu\gamma_4] \Phi_{II} = 0 , \]

with the chemical potential \( \mu \) in the (anti-)instanton field \( A_{II} \), and their Weyl components
The complex function \( \gamma \) is the antisymmetric tensor. In particular, the quark determinant at \( |x| = 0 \) may be written as 

\[
\Phi(x; \mu) = \left| \begin{array}{c} \Phi_I(x; \mu) \\ \Phi_I(x; -\mu) \end{array} \right| ,
\]

where the factor \( R \) provides the dimensionless result. The preexponential factors are responsible just for the instanton-induced interaction of quarks.

The colour orientation averaging is performed by integrating over \( dU \). The designation \( \tilde{\Phi} \) is introduced for the conjugated zero mode \( \tilde{\Phi}_I(x; \mu) = \Phi_I(x; -\mu) \). \( \mu^\pm \) and similar designations are used for the four-vectors spanned on the matrices \( \sigma^\nu_\pm \), i.e. \( \mu^\pm = \mu^\nu \sigma^\nu_\pm, \mu_\nu = (0, \mu) \).

Integrating over the auxiliary parameter \( \lambda \) brings the quark determinant \( Z_\psi \) to the exponential form convenient for the saddle point calculation.

\[
Z_\psi \sim \int \frac{d\lambda}{2\pi} \exp \left\{ N \ln \left( \frac{N}{i\lambda VR} \right) - N \right\} \times \int D\bar{\psi} D\psi \exp \left\{ \int dx \left[ \bar{\psi} \gamma(k) \psi \right] \pi x \right\}.
\]

The complex function \( \gamma_0(k; \mu) \) of this part of the generating functional is defined by the Fourier components of the zero modes

\[
\gamma_0(k; \mu) = (k + i\mu)_a (k + i\mu)_a h_\beta(k; \mu) h_\beta(k; \mu) , \quad \Phi(k; \mu)^\alpha = h_\mu(k; \mu) (\sigma^\nu_\mu)^j \varepsilon^{jk} U^\alpha_k ,
\]

\[
h_4(k_4, k; \mu) = \frac{\pi \rho^3}{4k^4} \left\{ (k - \mu - ik_4)(2k_4 + i\mu)f^-_1 + i(k - \mu - ik_4)f^+_2 \right\} + (k + \mu + ik_4)(2k_4 + i\mu)f^+_1 - i(k + \mu + ik_4)f^-_2 ,
\]

\[
h_i(k_4, k; \mu) = \frac{\pi \rho^3 k_i}{4k^2} \left\{ (2k - \mu)(k - \mu - ik_4)f^-_1 + (2k + \mu)(k + \mu + ik_4)f^+_1 + \left[ 2(k - \mu)(k - \mu - ik_4) - \frac{1}{k}(\mu + ik_4)[k^2_4 + (p - \mu)^2] \right] f^-_2 + \left[ 2(k + \mu)(k + \mu + ik_4) + \frac{1}{k}(\mu + ik_4)[k^2_4 + (p + \mu)^2] \right] f^+_2 \right\} ,
\]

where \( k = |k| \) for the spatial components of four-vector,

\[
f^\pm = \frac{I_1(z^\pm)K_0(z^\pm) - I_0(z^\pm)K_1(z^\pm)}{z^\pm} ,
\]
\[ f^\pm_2 = \frac{I_1(z^\pm)K_1(z^\pm)}{z^2_\pm}, \quad z^\pm = \frac{\rho}{2} \sqrt{k_1^2 + (k \pm \mu)^2}, \]

and \( I_i, \ K_i \) (i = 0, 1) are the modified Bessel functions.

In order not to overload the following formulae with the unnecessary coefficients we introduce the dimensionless variables

\[ \frac{k_4 \rho}{2} \to k_4, \quad \frac{k_\rho}{2} \to k, \quad \frac{\lambda \rho^3}{2N_c} \to \lambda, \quad \psi(k) \to \bar{\rho}^{5/2} \psi(k). \]

Then it is easy to get the saddle point equation taking the form

\[ \frac{\bar{\rho}^4}{\lambda} - 2N_c \int \frac{dk}{\pi^4} \frac{[\lambda^2 \gamma_0^2(k; \mu)']^2}{(k + i\mu)^2 + \lambda^2 \gamma_0^2(k; \mu)} = 0, \tag{5} \]

here the prime is attributed to differentiating in \( \lambda \).

It has been shown in Refs. [7], [10] that the backreaction of quarks upon IL may be reproduced perturbatively with the small variations of the IL parameters \( \delta n \) and \( \delta \rho \) around their equilibrium values \( n \) and \( \bar{\rho} \). These variations are actually accommodated by the approach if the deformable (crumpled) (anti-)instantons of size \( \rho \), being the function of \( x \) and \( z \), i.e. \( \rho \to \rho(x, z) \) are utilized as the saturating configurations. If the wave length of excitations is much larger than the characteristic (anti-)instanton size \( \bar{\rho} \) (for example, for \( \pi \)-meson) the mean action per one instanton gains the extra term (looking like kinetic energy) generated by the deformable (anti-)instantons

\[ \langle S \rangle \simeq \int dz \int d\rho \ n(\rho) \left\{ \frac{\kappa}{2} \left( \frac{\partial \rho}{\partial z} \right)^2 + s(\rho) \right\}, \tag{6} \]

where \( \kappa \) is the kinetic coefficient being derived within the quasi-classical approach. If we strive for holding the precision declared in this paper the kinetic coefficient should be fixed on a characteristic scale, for example, as \( \kappa \sim \kappa(\bar{\rho}) \). Our estimates give the value of a few single instanton actions \( \kappa \sim c / \beta \)

(with the factor \( c \sim 1.5 - 6 \) depending on the ansatz for the saturating configurations). Collecting the terms of the second order in deviation from the point of action minimum only \( \left. \frac{ds(\rho)}{d\rho} \right|_{\rho_c} = 0 \), and taking approximately

\[ s(\rho) \simeq s(\bar{\rho}) + \frac{s^{(2)}(\bar{\rho})}{2} \varphi^2, \tag{7} \]

where \( s^{(2)}(\bar{\rho}) \simeq \left. \frac{d^2s(\rho)}{d\rho^2} \right|_{\rho_c} = \frac{4\nu}{\bar{\rho}^2} \), and the scalar field \( \varphi = \delta \rho = \rho - \rho_c \simeq \rho - \bar{\rho} \) is the field of deviations from the equilibrium value of \( \rho_c = \bar{\rho} \left(1 - \frac{1}{2\nu} \right)^{1/2} \simeq \bar{\rho} \), it becomes clear the deformation field is described by the following Lagrangian density [7]:

\[ \mathcal{L} = \frac{n \kappa}{2} \left\{ \left( \frac{\partial \varphi}{\partial z} \right)^2 + M^2 \varphi^2 \right\}, \]

with the mass gap of phononlike excitations \( M^2 = \frac{s^{(2)}(\bar{\rho})}{\kappa} = \frac{4\nu}{\kappa \bar{\rho}^2} \), \( M \approx 1.21 \Lambda \) for IL in the "quenched" approximation with the parameters fixed as \( N_c = 3, \quad c = 4, \quad \bar{\rho} \Lambda \approx 0.37, \quad \beta \approx 17.5, \quad n \Lambda^{-4} \approx 0.44 \) [10].

Including the variations of zero modes in the quark determinant results in changing those for

\[ \Phi_{II}(x - z, \rho) \simeq \Phi_{II}(x - z, \rho_c) + \Phi_{II}^{(1)}(x - z, \rho_c) \delta \rho(x, z), \]

where \( \Phi_{II}^{(1)}(u, \rho_c) = \partial \Phi_{II}(u, \rho)/\partial \rho|_{\rho = \rho_c} \) and

\[ 1^{1}) \text{ It seems to us the physical meaning of the deformation field is analogous to the phonons of solid state physics. It is why we called them phononlike excitations of IL [10].} \]
because of the adiabaticity constraint it is valid \( \delta \rho(x, z) \simeq \delta \rho(z, z) = \varphi(z) \). The extra contributions of scalar field generate the corrections in the kernels of factors \( Y^\pm \) which are treated in the linear approximation in \( \varphi \) and it is supposed \( \rho_c \simeq \bar{\rho} \) everywhere. As result we have the effective Lagrangian with the Yukawa interaction of quarks and colourless scalar field

\[
Z \simeq \int d\kappa \bar{Z}_g \int D\psi D\bar{\psi} D\varphi \exp \left\{ N \left( \ln \frac{n\rho^4}{\lambda} - 1 \right) - \int \frac{dk}{\pi^2} \frac{1}{2} \varphi(-k) \left[ k^2 + M^2 \right] \varphi(k) \right\} \times \\
\times \exp \left\{ \int \frac{dkdl}{\pi^8} \left[ \pi^4 \delta(k-l) (-\bar{k} - i\mu + i\lambda \gamma_0(k; \mu) + i\lambda \gamma_1(k, l; \mu) \varphi(k-l)) \right] \psi(l) \right\}.
\]

Let us mention the dimensionless variables as the scalar field \( \varphi(k) \rightarrow (n\kappa)^{-1/2} \bar{\rho}^3 \varphi(k) \) and its mass \( M\bar{\rho} \rightarrow M \) were introduced here. \( \bar{Z}_g = e^{(S(\bar{\rho}))} \) denotes the part of the gluon component of the generating functional which survives expanding the action per one instanton in small deviation from the instanton equilibrium size. The general form of the kernel \( \gamma_1(k, l; \mu) \) is not of crucial importance for us here. We may simplify calculation essentially because of the presence of quark condensate and collect the leading contributions to the functional Eq. (8) which are just the tadpole graphs. In this particular case the following perturbative scheme (in the deviation from the condensate magnitude) is applicable

\[
\psi\bar{\psi} \varphi = \langle \psi\bar{\psi} \rangle \varphi + \{ \psi\bar{\psi} - \langle \psi\bar{\psi} \rangle \}\varphi,
\]

and for the vertex we then have

\[
\gamma_1(k; \mu) = \frac{2}{(n\rho^4\kappa)^{1/2}} \left\{ [kh + (k_4 + i\mu)h_4][k\delta h + (k_4 + i\mu)\delta h_4] + [(k_4 + i\mu)h - kh_4][(k_4 + i\mu)\delta h - k\delta h_4] \right\}.
\]

Here we introduced the component \( h \) spanned on the unit vector \( \frac{k_4}{k} \) \( h_i = \frac{k_i}{k} h \) and \( \delta h \) and \( \delta h_4 \) are produced by \( h \) and \( h_4 \) while the substitutions \( f_i^\pm \rightarrow g_i^\pm \) where

\[
g_1^\pm = f_1^\pm + 2(I_0^\pm K_0^\mp - I_1^\pm K_1^\pm), \quad g_2^\pm = -f_1^\pm - f_2^\pm,
\]
done.

In the present paper two corrections are taken into account. One of them is related to the changes of dynamical quark mass and chiral condensate due to the interaction with the scalar field but another one comes from changing the IL density invoked by the shift of the PP equilibrium size. The former is given by the following contribution

\[
2 \left( i\lambda \right)^2 \int \frac{dkdl}{\pi^8} \frac{dk'dl'}{\pi^8} \gamma_1(k, l; \mu) \gamma_1(k', l'; \mu) \psi\bar{\psi}(k)\psi\bar{\psi}(l) \psi\bar{\psi}(k')\psi\bar{\psi}(l') \langle \varphi(k-l) \varphi(k'-l') \rangle \simeq \\
\simeq 4 \lambda^2 \int \frac{dk}{\pi} \gamma_1(k; \mu) \psi\bar{\psi}(k) \int \frac{dl}{\pi} \gamma_1(l; \mu) \text{Tr} S(l) D(0),
\]

where the conventional definitions are introduced for the convolution of scalar field

\[
\langle \varphi(k) \varphi(l) \rangle = \pi^4 \delta(k+l) D(k), \quad D(k) = \frac{1}{4 (k^2 + M^2)},
\]

and for the quark Green function \( S(k) \)

\[
\langle \psi\bar{\psi}(k)\psi\bar{\psi}(l) \rangle = -\pi^4 \delta(k-l) \text{Tr} S(k).
\]

All the factors surrounding \( \psi\bar{\psi}(k)\psi\bar{\psi}(k) \) in Eq. (10) give the extra contribution to the quark dynamical mass \( \lambda \gamma_0(k; \mu) \rightarrow \lambda \Gamma(k; \mu) = \lambda \gamma_0(k; \mu) + \lambda \delta \gamma_0(k; \mu) \), where

\[
\delta \gamma_0(k; \mu) = \gamma_1(k; \mu) (-2i\lambda) \int \frac{dl}{\pi} \gamma_1(l; \mu) \text{Tr} S(l) D(0).
\]
Let us introduce now the auxiliary function $c(\lambda)$ in order to emphasize the obvious parameter dependence of $\delta \gamma$, i.e.

$$
\delta \gamma_0(k; \mu) = \frac{N_c}{(n\bar{\rho}^4\kappa)^{1/2}} \frac{\kappa}{\nu} \lambda^2 c(\lambda) \gamma_1(k; \mu).
$$

If the quark Green function is known $c(\lambda)$ could be defined by the complete integral equation

$$
c(\lambda) = (n\bar{\rho}^4\kappa)^{1/2} \int \frac{dk}{\pi^4} \gamma_1(k; \mu) \frac{\Gamma(k; \mu)}{(k + i\mu)^2 + \lambda^2 \Gamma^2(k; \mu)},
$$

what helps also to calculate $S(k)$.

It was shown in Ref. [7] the variation of equilibrium PP size generates the IL density variation and can be given by the tadpole contribution $\varphi(0) \Delta$ with $\Delta = \frac{4N_c}{n\bar{\rho}^4} \lambda^2 c(\lambda)$ which produces an extra term in the mean action per one instanton

$$
\langle S \rangle = \int dz \, n \left\{ \langle s \rangle - \langle \Delta \frac{\rho - \rho_c}{\bar{\rho}} \rangle \right\}.
$$

The maximum variational principle helps to find the PP average size $\bar{\rho} \Lambda = \exp \left\{ -\frac{2N_c}{2\nu - 1} \right\}$ and to obtain the following equation to calculate the IL density [7]

$$
(n\bar{\rho}^4)^2 - \frac{\nu}{\beta \xi^2} n\rho^4 = \frac{\Delta}{\beta \xi^2} \frac{\Gamma(\nu + 1/2)}{2\sqrt{\nu} \Gamma(\nu)}.
$$

Eventually, we come to the equation for the saddle point of generating functional Eq. (8) in the form

$$
\frac{n\bar{\rho}^4}{\lambda} - 2N_c \int \frac{dk}{\pi^4} \frac{2\lambda \Gamma^2(k; \mu) + \lambda^2 \Gamma(k; \mu) \Gamma'(k; \mu)}{(k + i\mu)^2 + \lambda^2 \Gamma^2(k; \mu)} - (n\bar{\rho}^4)' \ln \frac{n\bar{\rho}^4}{\lambda} = 0,
$$

where the IL density variations are incorporated. The prime here signifies the derivative in $\lambda$. But we need to know another derivative $c'$ to have the complete equation. With the definition of $c(\lambda)$ as given by Eq. (12), we obtain

$$
(1 - \lambda^2 A(\lambda)) \, c'(\lambda) = 2\lambda \, A(\lambda) \, c(\lambda) + B(\lambda)
$$

where the functions $A(\lambda)$ and $B(\lambda)$ are the following integrals

$$
A(\lambda) = \alpha(\lambda) \frac{N_c \kappa}{\nu} \int \frac{dk}{\pi^4} \frac{\gamma_1^2(k; \mu)}{[(k + i\mu)^2 + \lambda^2 \Gamma^2(k; \mu)]^2},
$$

$$
B(\lambda) = -2\lambda \, (n\bar{\rho}^4\kappa)^{1/2} \int \frac{dk}{\pi^4} \frac{\gamma_1(k) \Gamma^3(k)}{[(k + i\mu)^2 + \lambda^2 \Gamma^2(k; \mu)]^2},
$$

and the factor $\alpha(\lambda)$ reads

$$
\alpha(\lambda) = 1 - \lambda^2 \frac{N_c \Gamma(\nu + 1/2)}{\beta \xi^2 \nu^{1/2} \Gamma(\nu)} \frac{c(\lambda)}{n\bar{\rho}^4 (n\bar{\rho}^4 - \nu \beta \xi^2)}.
$$

The corresponding results for multilavour approach $N_f > 1$ could be obtained with the simple substitutions [7], [11]. We have to introduce the factor $N_f$ into the extra contribution to the dynamical mass in Eq. (10) and to the factor $\Delta$ in the tadpole contribution. Besides, the logarithm in Eq. (14) should be modified according to the prescription $\ln \frac{n\bar{\rho}^4}{\lambda} \to \ln \left( \left( \frac{n\bar{\rho}^4}{2} \right)^{1/N_f} \frac{1}{\lambda} \right)$. For clarity we
Figure 1: The dynamical quark masses (solid lines) and chiral condensates (dashed lines) as the functions of chemical potential $\mu$ at $\Lambda = 280$ MeV at $N_c = 3$, $N_f = 2$. Both lower curves correspond to the calculation with the modified generating functional.

take the factor $R$ to be equal one (see, for example, Eq. (4)) everywhere. However, this parameter could be treated as the free one. Then the dimensional parameters of this approach are given by the corresponding powers of $\Lambda_{QCD}$, for instance, $\rho\Lambda$ and could be self-consistently obtained if the saddle point was already known.

In Fig. 1 we expose the calculation results with the positive root of Eq. (13)

$$n\rho^4 = \frac{\nu}{2\beta\xi^2} + \left[ \left( \frac{\nu}{2\beta\xi^2} \right)^2 + \frac{\Delta}{\beta\xi^2} \frac{\Gamma(\nu + 1/2)}{2\sqrt{\nu} \Gamma(\nu)} \right]^{1/2},$$

for the dynamical quark mass $\lambda \Gamma(0,\mu)$ (solid lines) and for the chiral condensate $-i\langle \psi^\dagger \psi \rangle$ (dashed lines) as the functions of chemical potential $\mu$ at $\Lambda = 280$ MeV and for $N_c = 3$, $N_f = 2$. Both lower curves correspond to the calculation with modified generating functional when the tadpole contributions are taken into account. Apparently, the results obtained are well within the scope of QCD vacuum phenomenology and, in principle, the perfect fit is achievable with slight variation of $\Lambda_{QCD}$.

Fig. 2 demonstrates rather insignificant change of the IL density when the quarks are in the phase where the chiral condensate develops non-zero values though the quark influence on IL is transformed into the small strengthening of gluon condensate manifesting itself in the supplementary attraction available in the system of quarks and gluons. These features fully agree with the corresponding results of Ref. [12] where the IL behaviour was studied in the so-called cocktail model (with (anti-)$\text{instanton molecule admixed})$. In that model the IL density in the phase of broken chiral symmetry is almost constant up to the critical values of chemical potential.

The results of studying the saddle point parameter $\lambda$ (which is proportional to the free energy within the precision of one loop approximation) at $N_c = 3$, $N_f = 2$ are shown in Fig. 3 where the upper curve was calculated without corrections coming from the quark feedback taken into account. In fact, these results allow us to make one substantial qualitative prediction for the behaviour of critical chemical potential $\mu_c$ while transiting to the colour superconductivity phase. Indeed, keeping in mind the results of Refs. [12], [13] we should foresee the point $\mu_c$ shifting perceptibly to the larger values because the parameter $\lambda$ for the diquark phase at precritical values of $\mu$ is always disposed above than for the corresponding curves of broken chiral symmetry phase. It means the crossing
Figure 2: The IL density in the phase of nonzero chiral condensate values at $N_c = 3$, $N_f = 2$ as function of chemical potential $\mu$.

point with the lower curve is always situated further along the $\mu$-axis than the crossing point with the upper curve. Such a situation signals the density of quark matter in the critical interval could be comparable (or even larger) than normal nuclear density. In the meantime, we should remember that the IL model itself should obey some limitation for the chemical potential values. It is just the region where the transition to perturbative quark-gluon matter phase takes place. With quark matter density increasing the average interquark distance may be so small that the interquark gluon ('Coulomb') field strengths become comparable or even exceed the instanton ones. Then it is invalid to consider the (anti-)instanton ensemble as the saturating configuration and the IL approach is made irrelevant in this region.

Figure 3: Saddle point $\lambda$ as function of $\mu$ in the phase of broken chiral symmetry at $N_c = 3$, $N_f = 2$. The upper curve corresponds to the solution with no perturbation of IL.

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