A generalization to all system sizes of DGLV energy loss in the quark-gluon plasma

Isobel Kolbé$^1$ and W. A. Horowitz$^1$

$^1$Department of Physics, University of Cape Town, Private Bag X3, Rondebosch 7701, South Africa
E-mail: isobel.kolbe@gmail.com

Abstract. In the last few years, experiments with small colliding systems (such as $p/d + A$) have yielded some evidence for the presence of small droplets of quark-gluon plasma (QGP). The collective behavior, strangeness enhancement and quarkonium suppression that have been seen beg the question: What of the energy loss? However, in order to answer such a question, one must apply existing perturbative Quantum Chromodynamical (pQCD) energy loss calculations to systems that are much smaller than the systems in which pQCD energy loss calculations were originally so successful, a process that is not currently allowed for in any pQCD energy loss calculation. We relax the large system assumption in the well-known DGLV formalism (an assumption that is also utilized in the ASW, BDMPS-Z, AM and HT calculations), but find, alarmingly, that the correction terms dominate at large energies, resulting in $\sim 100\%$ negative correction, calling into question the validity of the large formation time assumption used in all pQCD-based energy loss calculations. Our results demand a complete overhaul of energy-loss calculations for all system sizes.

1. Introduction

In recent years, alarming questions have been raised concerning the creation of a quark-gluon plasma (QGP) in small colliding systems such as proton or deuteron collisions with heavy nuclei ($p/d + A$) or even proton-proton ($pp$) collisions. A number of signals (such as collective behavior [1], strangeness enhancement [2, 3], and quarkonium suppression [4]) that are traditionally attributed to the presence of a QGP in nucleus-nucleus, $AA$, collisions, have appeared in systems previously thought far too small to create a QGP. During the very successful studies of the QGP in heavy ion collisions, jet quenching proved an important femtoscopic probe of the dynamics of the degrees of freedom of the QGP and tantalizing signs of centrality dependent jets in $p/dA$ collisions at RHIC and LHC [5,6] have surfaced.

Jet tomography in heavy-ion collisions has been extremely successfully described by a number of perturbative Quantum Chromodynamical (pQCD) energy loss models [7–10] that are able to qualitatively describe the momentum dependence and angular distribution of the suppression of high-momentum, $\sim 5 − 150$ GeV single particle pions [11–12] and charged hadrons [13–15] from primordial hard light flavors and gluons and electrons [16–18] as well as $D$ [19] and non-prompt $J/\psi$ mesons [20] from open heavy flavor decays at mid rapidity in $A+A$ systems from $\sqrt{s} = 0.2$ TeV to 2.76 TeV.

The presence of some QGP signals in small colliding systems demands a better understanding of energy loss in small systems, since all of the standard pQCD energy loss models make the...
explicit assumption that the QGP system is large \cite{21}. In the standard opacity expansion developed by Djordjevic, Gyulassy, Levai and Vitev (DGLV) \cite{22,23}, the large system assumption amounts to an assumption that the separation distance $\Delta z \equiv z_1 - z_0 \gg \lambda_{mfp} \gg 1/\mu$ between the initial production position $z_0$ of the hard parent parton and the position $z_1$ where it scatters off a QGP medium quasi-particle is large. The mean free path of the high-$p_T$ particle is

$$\lambda_{mfp} = 1/\rho \sigma \sim 1 - 2 \text{ fm}$$

while the Debye mass in an infinite, static thermal QGP of temperature $T \sim 350 \text{ MeV}$ is $\mu = gT \sim 0.5 \text{ GeV}$, as derived from thermal field theory \cite{24}. In $pp$ and $p/dA$ collisions, one expects a system of radius $\lesssim 2 \text{ fm}$, suggesting that a high-$p_T$ parton that does scatter cannot have a large separation distance between production and scattering.

In order to apply DGLV energy loss to small systems, we derive the $N = 1$ opacity generalization for all separation distances. We find two curious results:

(i) In addition to being suppressed under the large separation distance (along with the usual assumptions of eikonality and collinearity), the majority of the small separation distance terms are also suppressed under the assumption that the gluon formation time is large, which results in only two diagrams with non-zero correction terms.

(ii) The (negative) correction terms dominate even for relatively large system sizes at high parent parton energies.

It has been known for some time that all energy loss formalisms are sensitive to the collinear approximation \cite{25,26}, but we will demonstrate that the present sensitivity to the large formation time approximation is both new and different.

2. Setup

Our computation is available in full detail at \cite{27} and follows DGLV \cite{23} which considers a brick of QGP being traversed by a high-$p_T$ eikonal parton that had been produced at an initial point $(t_0, z_0, x_0)$. In our notation we have described 2D vectors using $p$, 3D vectors using $\vec{p} = (p_z, p)$ and four vectors in Minkowski and light cone coordinates respectively using $p = (p^0, \vec{p}) = [p^0 + p^2, p^0 - p^2, \vec{p}]$. In keeping with DGLV, we describe the scattering potential using a Gyulassy-Wang Debye screened potential \cite{28} with Fourier and color structure given by

$$V_n = V(q_{in})e^{-i\vec{q}_n \cdot \vec{x}_n} = 2\pi \delta(q^0)\nu(q_{in}, q_n^x) e^{-i\vec{q}_n \cdot \vec{x}_n} T_n(R) \otimes T_n(n).$$

(1)

The $SU(N_c)$ generator $T_n(n)$ in the $d_n$ dimensional representation of the target or $T_n(R)$ in the $d_R$ dimensional representation of the high-$p_T$ parent parton manage the color exchanges. The momenta, defined in Fig. 1 of the radiated gluon, the final high-$p_T$ parton, and the exchanged medium Debye quasi-particle are, in lightcone coordinates,

$$k = \left[xP^+, \frac{m^2 + k^2}{xP^+}, \vec{k}\right], p = \left[(1 - x)P^+, \frac{M^2 + k^2}{(1 - x)P^+}, -\vec{k}\right], q = [q^+, q^-, \vec{q}].$$

(2)

We also employ a number of energy rations following \cite{23}: $\omega \approx xE^+/2 = xP^+/2$, $\omega_0 \equiv k^2/2\omega$, $\omega_i \equiv (k - q_i)^2/2\omega$, $\omega_{(ij)} \equiv (k - q_i - q_j)^2/2\omega$, and $\tilde{\omega}_m \equiv (m_g^2 + M^2 x^2)/2\omega$.

We make the following assumptions: 1) the eikonal, or high energy, approximation, for which $E^+$ is the largest energy scale of the problem; 2) the soft (radiation) approximation $x \ll 1$; 3) collinearity, $k^+ \gg k^-; 4$) that the impact parameter varies over a large transverse area; and, most crucially for the present work, 5) the large formation time assumption $\omega_i \ll \mu_i$, where $\mu_i^2 \equiv \mu^2 + q_i^2$.

We have re-evaluated the 10 $N = 1$ opacity diagrams without the large separation distance $\Delta z \gg 1/\mu$ assumption and present the results here.
3. Calculation and Results

![Diagram](https://via.placeholder.com/150)

Figure 1: $\mathcal{M}_{1,0}$ (left panel) and $\mathcal{M}_{2,0}$ (right panel) are the only two diagrams that have non-zero short separation distance corrections in the large formation time limit. $\mathcal{M}_{2,0}$ is the double Born contact diagram, corresponding to the second term in the Dyson series in which two gluons are exchanged with the single scattering center.

The large separation distance assumption allowed the original DGLV calculation to neglect terms that are proportional to $\exp(-\mu_1 \Delta z)$, which we have retained in our implementation of the relaxation of the large separation distance assumption. Nevertheless, an immense simplification occurs due to the large radiated gluon formation time approximation $\omega_1 \ll \mu_1$ in that all but 2 of the 10 diagrams’ 18 new small distance correction pole contributions are suppressed under the large formation time assumption. Fig. 1 shows the two diagrams that contribute non-zero small separation distance, large formation time terms to the result.

The full result for these two amplitudes under our approximation scheme is then

\[
\mathcal{M}_{1,0} \approx -J(p)e^{i(p+0)2qTz_0}c_1 \int \frac{d^2q_1}{(2\pi)^2} v(0, q_1) e^{-iq_1 \cdot b_1} \times \frac{k \cdot \epsilon}{k^2 + m_g^2 + x^2 M^2} \left[ e^{i(\omega_0 + \omega_m)(z_1 - z_0)} - \frac{1}{2} e^{-\mu_1(z_1 - z_0)} \right]
\]

\[
\mathcal{M}_{2,0} \approx J(p)e^{i(p+0)z_0} \int \frac{d^2q_1}{(2\pi)^2} \int \frac{d^2q_2}{(2\pi)^2} e^{-i(q_1 + q_2) \cdot b_1} \times igTc_1 v(0, q_1) v(0, q_1) \left[ e^{i(\omega_0 + \omega_m)(z_1 - z_0)} + e^{-\mu_1(z_1 - z_0)} \left( 1 - \frac{\mu_1 e^{-\mu_2(z_1 - z_0)}}{2(\mu_1 + \mu_2)} \right) \right].
\]

It should be noted that the large formation time assumption allows one to neglect terms in both the original DGLV and the current short separation distance correction. The double differential single inclusive gluon emission distribution is given by

\[
d^3N_g^{(1)} d^3N_f = \frac{d^3\vec{p}}{(2\pi)^3 p^0} \frac{d^3\vec{k}}{(2\pi)^3 2\omega} \left( \frac{1}{d_T} \text{Tr}(|\mathcal{M}_1|^2) + \frac{2}{d_T} \Re \text{Tr}(<\mathcal{M}_0 \cdot \mathcal{M}_2>) \right).
\]

Our main analytic result is then the $N = 1$ first order in opacity small distance generalization of the DGLV induced energy loss of a high-$p_T$ parton in a QGP:
Figure 2: Fractional energy loss of charm and bottom quarks in a QGP with $\mu = 0.5$ GeV and $\lambda_{mfp} = 1$ fm for (a) fixed path length $L = 8$ fm, and (b) fixed energy $E = 10$ GeV. In the figures, “DGLV” dashed curves are computed from the original $N = 1$ in opacity large separation distance DGLV formula while “DGLV + corr” solid lines are from our all separation distance generalization of the $N = 1$ DGLV result, equation (6).

\[
\Delta E^{(1)}_{\text{ind}} = \frac{C_{RO} LE}{\pi \lambda_g} \int dx \int \frac{d^2q_1}{\pi} \frac{\mu^2}{(\mu^2 + q_1^2)^2} \int \frac{d^2k}{\pi} \int d\Delta \bar{\rho}(\Delta z) \left[ - \frac{2(1 - \cos \{ (w_1 + \bar{w}_m)\Delta z \})}{(k - q_1)^2 + m_g^2 + x^2 M^2} \right.
\]
\[
\times \left. \left( \frac{(k - q_1) \cdot k}{k^2 + m_g^2 + x^2 M^2} - \frac{(k - q_1)^2}{(k - q_1)^2 + m_g^2 + x^2 M^2} \right) + \frac{1}{2} e^{-n_1 \Delta z} \left( \frac{k}{k^2 + m_g^2 + x^2 M^2} \right)^2 \left( 1 - \frac{2C_R}{C_A} \right) \left( 1 - \cos \{ (w_0 - \bar{w}_m)\Delta z \} \right) \right.
\]
\[
\left. + \frac{k \cdot (k - q_1)}{(k^2 + m_g^2 + x^2 M^2)(k - q_1)^2 + m_g^2 + x^2 M^2)} \left( \cos \{ (w_0 - \bar{w}_m)\Delta z \} - \cos \{ (w_0 - \omega_1)\Delta z \} \right) \right]\]  (6)

The DGLV result appears in the first two lines of equation (6), while the last two are the short separation distance correction. The correction term behaves as expected by vanishing for both large and vanishing separation distances. However, the correction term also has a number of unexpected features: The correction term contains a color breaking term (proportional to $2C_R/C_A$) and dominates the original DGLV at high energies. This last feature is most apparent in the numerical analysis.

The numerical investigation of equation (6) is performed following [23] and results in figures 2a, 2b, 3a and 3b. The numerical analysis used the following values: $\mu = 0.5$ GeV, $\lambda_{mfp} = 1$ fm, $C_R = 4/3$, $C_A = 3$, $\alpha_s = 0.3$, $m_{charm} \equiv m_c = 1.3$ GeV and $m_{bottom} \equiv m_b = 4.75$ GeV, and the QCD analogue of the Ter-Mikayelian plasmon effect was taken into account by setting $m_{gluon} \equiv m_g = \mu/\sqrt{2}$. As in [24], kinematic upper limits were used for the momentum integrals such that $0 \leq k \leq 2\pi(1 - x)E$ and $0 \leq q \leq \sqrt{3E\mu}$, due to finite kinematics. This choice of $k_{\text{max}}$ guarantees that the final momentum of the parent parton is collinear to the initial momentum of the parent parton and that the momentum of the emitted gluon is collinear to the momentum of the parent parton. The fraction of momentum carried away by the radiated gluon, $x$, was integrated over from 0 to 1. The distribution of scattering centers was assumed to be exponential in order to account for the rapidly expanding medium, $\bar{\rho}(z) = 2 \exp(-2\Delta z/L)/L$.  

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**Figure 2:** Fractional energy loss of charm and bottom quarks in a QGP with $\mu = 0.5$ GeV and $\lambda_{mfp} = 1$ fm for (a) fixed path length $L = 8$ fm, and (b) fixed energy $E = 10$ GeV. In the figures, “DGLV” dashed curves are computed from the original $N = 1$ in opacity large separation distance DGLV formula while “DGLV + corr” solid lines are from our all separation distance generalization of the $N = 1$ DGLV result, equation (6).
Fig. 2a shows the fractional energy loss of charm and bottom quarks of varying energy propagating through an 8 fm long static QGP brick. Clearly visible is that the small distance correction term is generally an energy gain due to the sign of the color triviality breaking term. Furthermore, relative to the large distance DGLV result, the size of the correction increases with energy.

Fig. 2b shows the fractional energy loss of charm and bottom quarks of energy $E = 10 \text{ GeV}$ for path lengths up to 5 fm. It is apparent from Fig. 2b that the small separation distance correction has an appreciable effect even for large path lengths. This feature is well understood as being due to the integration over all separation distances between the production point and the scattering position, but this effect diminishes with path length, as it must.

Fig. 3a illustrates the effect on small systems by showing the fractional energy loss of charm and bottom quarks through a, much smaller, 3 fm thick static QGP brick. Here, the striking effect of the energy of the parent parton on its energy loss in a small system is clearly visible. To illustrate this point further, Fig. 3b shows the fractional energy loss of 100 GeV charm and bottom quarks propagating up to 5 fm through a QGP. Clearly, the small distance “correction” term to the DGLV result dominates, even out to path lengths of $\sim 3 \text{ fm}$, for high energies.

4. Conclusions
It is difficult to reconcile the significant energy gain at high energies with either experimental suppression of charged particles in central AA collisions at LHC [13–15] or an intuitive understanding of the assumption scheme that is employed. As such, a more detailed interrogation of the validity of the large formation time assumption in pQCD-based energy loss calculations is necessary.

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