Emergence of superconductivity in the cuprates via a universal percolation process

Damjan Pelč, Marija Vučković, Mihael S. Grbić, Miroslav Požek, Guichuan Yu, Takao Sasagawa, Martin Greven & Neven Barišić

A pivotal step toward understanding unconventional superconductors would be to decipher how superconductivity emerges from the unusual normal state. In the cuprates, traces of superconducting pairing appear above the macroscopic transition temperature $T_c$, yet extensive investigation has led to disparate conclusions. The main difficulty has been to separate superconducting contributions from complex normal-state behaviour. Here we avoid this problem by measuring nonlinear conductivity, an observable that is zero in the normal state. We uncover for several representative cuprates that the nonlinear conductivity vanishes exponentially above $T_c$, both with temperature and magnetic field, and exhibits temperature-scaling characterized by a universal scale $\Xi_0$. Attempts to model the response with standard Ginzburg-Landau theory are systematically unsuccessful. Instead, our findings are captured by a simple percolation model that also explains other properties of the cuprates. We thus resolve a long-standing conundrum by showing that the superconducting precursor in the cuprates is strongly affected by intrinsic inhomogeneity.
Despite tremendous experimental and theoretical efforts over the past three decades, the nature of the superconducting fluctuation regime of the cuprates remains intensely debated. Experimentally, the problem has been approached using bulk probes, such as conductivity and other transport properties in a wide frequency range\textsuperscript{2-13}, magnetic susceptibility\textsuperscript{3,14-16}, surface sensitive probes\textsuperscript{17}, local probes, such as muon spin rotation\textsuperscript{18}, and photoemission spectroscopy\textsuperscript{19,20}. Some studies point to the possible persistence of superconducting pairing well above \( T_c \), which has been taken as an indication of preformed Cooper pairs related to the appearance of the pseudogap\textsuperscript{12,21}. Other studies indicate that traces of superconductivity emerge at somewhat lower temperatures, and are most prominent at moderate doping\textsuperscript{9,10,14}. High pairing onset temperatures have been related to exotic normal-state physics\textsuperscript{22,23} and to unconventional preparing\textsuperscript{24,25}, with profound consequences for the mechanism of cuprate superconductivity. However, terahertz and microwave conductivity\textsuperscript{2,5,6,11,12} as well as magnetometry\textsuperscript{15,16} consistently detect superconducting contributions only near \( T_c \), irrespective of doping. The resolution of this puzzle would be a crucial step toward understanding the high-\( T_c \) cuprates.

However, in previous experiments it has often been difficult to reliably establish the nonsuperconducting normal-state contribution in order to extract a superconducting signal. Typical approaches involve the extrapolation of high-temperature behavior or the suppression of superconductivity by a high magnetic field. The situation is further convoluted due to the complexity of the cuprate phase diagram, which features a doping-dependent pseudogap, as well as universal and compound-specific ordering tendencies that manifest themselves differently in different experimental observables\textsuperscript{1}. The presence of various kinds of disorder in these complex oxides poses yet another complication\textsuperscript{26}. Data are often discussed assuming preformed Cooper pairs in an extended temperature range\textsuperscript{9,10,14}, or analyzed within the Ginzburg–Landau (GL) framework with possible corrections to the original mean-field theory\textsuperscript{3,4,7,8,12}, yet this has not resulted in a unified picture.

The absence of any discernible signal due to nonsuperconducting contributions renders the nonlinear conductivity technique uniquely suitable to study and model superconductivity emergence. We apply this probe to a number of cuprate families and a variety of experimental conditions. The measurements unambiguously show that the superconducting precursor is limited to a narrow temperature range above \( T_c \), which rules out extended fluctuations and prepairing regimes. Importantly, we find that the superconductivity emergence range is not controlled by \( T_c \), a crucial qualitative feature of GL theory, but rather by a scale \( \xi_0 \) that is nearly independent of compound and doping (in the studied doping range \( p = 0.08-0.19 \)). This robust experimental finding is an important step toward understanding cuprate superconductivity, as it places strong constraints on any theory. We then use a simple model to explain the data: the superconducting gap is known to be spatially inhomogeneous, which results in a distribution of local transition temperatures, and naturally leads to percolation. Percolation, and the scale-free fractal structures that emerge from it, is a well-known and ubiquitous phenomenon: first investigated in the context of polymer growth, it has since been formulated as a mathematical concept and applied to systems as diverse as random resistor networks, organic molecular gels, dilute magnets, the spread of diseases, and the large-scale structure of the universe\textsuperscript{27,28}. The basic ingredient in percolation theory is inhomogeneity, and we find that evoking \( T_c \) inhomogeneity is essential to understand superconductivity emergence in the cuprates. Remarkably, the minimal percolation model that we employ is sufficient to capture the observed unusual exponential temperature- and magnetic-field dependences of the nonlinear conductivity. We also report complementary linear conductivity measurements and take a fresh look at prior experimental results (torque magnetometry\textsuperscript{15}, resistivity\textsuperscript{7}, Seebeck coefficient\textsuperscript{8}, specific heat\textsuperscript{29}, and tomographic density of states\textsuperscript{30}), to demonstrate that the emergence of superconductivity can be consistently explained with this minimal model. Finally, the universal scale \( \xi_0 \) is shown to be a direct measure of the superconducting gap distribution width. The underlying inhomogeneity therefore is unrelated to material details, and must be an intrinsic, generic feature of cuprate superconductors.

**Results**

**Nonlinear response.** Nonlinear planar response, for current flow along the CuO\(_2\) planes, is measured with a sensitive contact-free method\textsuperscript{30} (see Methods). The response can be analyzed by decomposing the signal into harmonics,

\[
J = \sigma_1 K + \sigma_2 K^n + \sigma_3 K^3 + \ldots, \tag{1}
\]

where \( J \) is the response of the sample to an external field \( K \) (electric or magnetic), \( \sigma_1 \) the linear response tensor, and \( \sigma_2, \sigma_3 \) etc., the correction nonlinear tensors. Here, we discuss the third harmonic \( \sigma_3 \), the lowest-order conventional correction to the linear response (the second-harmonic \( \sigma_2 \) can only appear if time reversal or inversion symmetry is broken\textsuperscript{31} and is not discussed here). In any alternating-field experiment, magnetic and electric fields are related, and therefore it is arbitrary if one designates the signal at frequency \( 3\omega \) as proportional to nonlinear conductivity or (complex) susceptibility. Complementary linear conductivity measurements are performed with a microwave cavity perturbation technique (see Methods).

**Temperature dependence.** Measurements of the in-plane linear and nonlinear response were performed for three representative cuprate families: a nearly optimally doped sample of Hg\(_2\)Ba\(_2\)CuO\(_4+\delta\) (Hg1201), a model cuprate system due to its simple structure, high \( T_c \), and minimal point disorder effects\textsuperscript{32-35}, an optimally doped YBa\(_2\)CuO\(_7\)–\(\delta\) (YBCO) sample with 3% of Cu substituted by Zn (YBCO–Zn), where Zn dramatically affects the superconducting properties\textsuperscript{36}; and La\(_{2-x}\)Sr\(_x\)CuO\(_4\) (LSCO), spanning a wide range of doping across the superconducting dome (see Table 1). For all samples, \( \sigma_3 \) exhibits qualitatively the same temperature dependence (Fig. 1a and Supplementary Figure 1): no signal at high temperatures, a peak at a temperature that we designate as \( T_\sigma \), consistent with previous work (see Methods), and a step-like feature below \( T_c \). The signal magnitude depends on sample size and shape, and thus is normalized to the peak value.

| Sample       | Doping level \( p \) | \( T_c \) (K) | \( \xi_0 \) (K) |
|--------------|----------------------|--------------|----------------|
| Hg1201       | 0.14                 | 94.0         | 25.1 ± 1.4     |
| LSCO-0.08    | 0.08                 | 17.3         | 29.9 ± 1.4     |
| LSCO-0.125   | 0.125                | 30.0         | 28.2 ± 0.9     |
| LSCO-0.15    | 0.15                 | 37.2         | 28.6 ± 0.2     |
| LSCO-0.19    | 0.19                 | 30.1         | 29.3 ± 1.5     |
| YBCO–Zn      | 0.15                 | 60.2         | 26.1 ± 0.6     |

For LSCO, \( p = x \), whereas for Hg1201 and YBCO–Zn, the estimate is based on the findings in refs. 32,36, respectively. As described in the text, \( T_\sigma \) corresponds to the peak in the nonlinear conductivity; we estimate the error to be less than 1%. \( \xi_0 \) is obtained from nonlinear conductivity using a 3D site-percolation model. The uncertainties are from fits to the percolation calculation.
We note that $T_c$ as obtained from $\sigma_3$ agrees with the temperature of the peak in the real part of the linear microwave conductivity, which in turn corresponds to the value determined from magnetic susceptibility measurements.$^{12}$

The measurements clearly show that the nonlinear response decays quickly above $T_c$, which demonstrates the absence of extended fluctuations. Some previous investigations suggested agreement between experiments and GL theory (with various modifications to the theory$^{4,7,8,12}$) for particular cuprate compounds at particular doping levels; in line with these investigations, we have attempted to analyze our results within the GL framework. Within this framework, we would expect an approximately power-law temperature dependence of $\sigma_3$ (see ref. $^{37}$ and Methods), and a scaling of the data for different compounds with the characteristic scale $T_c$. Figure 1b shows our nonlinear conductivity data in dependence on the GL-reduced temperature $\ln(T/T_c)$ compared to a calculation of $\sigma_3$ using anisotropic GL theory beyond mean field (see Methods), similar to ref. $^{12}$. The theory predicts a temperature dependence of $\sigma_3$ that is clearly incompatible with experiment; the agreement cannot be improved by any tuning of the parameters, such as a different definition of $T_c$ (see Supplementary Figure 2). Even more importantly, the expected scaling is absent: $T_c$ is not the characteristic temperature scale for superconductivity emergence. The scaling argument is valid regardless of the manner in which GL theory is modified. However, the data are remarkably similar on an absolute temperature scale: a simple shift by a sample-dependent temperature $T_a$ that gives rise to approximately exponential behavior with a single temperature scale $\xi_0$ underlies the emergence of superconductivity. A similar exponential dependence can also be deduced from...
linear conductivity (inset in Fig. 1c) and torque magnetometry experiments, indicating its robustness. Clearly a framework other than GL is needed to explain the data.

Since nanoscale electronic inhomogeneity is well documented in the cuprates, e.g., from nuclear magnetic resonance and scanning tunneling microscopy (STM) measurements, we now attempt to gain understanding through a simple percolation model. The basic ingredient of the model is spatial inhomogeneity of local superconducting gaps, with a distribution width that is characterized by the scale \( k_0 \Xi_0 \). This distribution corresponds to superconducting patches that proliferate upon cooling, and macroscopic superconductivity then emerges via a percolative process. We calculate the response assuming nearest-neighbor site percolation, although the result does not critically depend on the details of the scenario (see Methods). For simplicity, we take the material to be made of perfectly connected square or cubic patches that are either nonsuperconducting, with a normal resistance \( R_n \), or superconducting, with a nonlinear resistance \( R_s(\sigma) \) that depends on the current through the patch \( j \) (see Methods). Since we normalize the experimental nonlinear conductivity, we can also normalize the resistances by taking \( R_n = 1 \). The fraction \( P \) of superconducting patches depends on temperature: \( P \to 0 \) at high temperatures and \( P \to 1 \) well below \( T_c \). At the critical concentration \( P_c \), the system percolates—a connected, sample-spanning superconducting cluster is formed. \( P_c \) only depends on the dimensionality of the system and on the details of the percolation scenario (e.g., site vs. bond percolation), and it corresponds to the temperature \( T_c \) that can be viewed as the “true” underlying resistive \( T_c \) in the limit of small currents. In principle, the full temperature dependence of \( P \) can be obtained from the underlying gap distribution, but the distribution must be known (or assumed). However, to lowest order, any reasonable distribution yields a linear dependence of \( P \) on temperature close to \( P_c \) (see Fig. 2). We, therefore, approximate \( P_n - P = (T - T_c)/\Xi_0 \). The temperature-dependent linear and nonlinear responses are then obtained via an effective medium calculation (see Methods), which yields functions that decay nearly exponentially, in very good agreement with the experimental \( \sigma_3 \) and \( \sigma_1 \) (Fig. 1c). Within the nearest-neighbor site-percolation model, two values of \( P_c \) are possible: \( \approx 0.3 \) for three-dimensional (3D) and \( \approx 0.6 \) for two-dimensional (2D) percolation. Better agreement is obtained with \( P_c = 0.3 \) (see Supplementary Figure 3), which suggests essentially 3D superconductivity emergence in the samples studied here. We note that we study the in-plane response, and thus the only role of inter-plane coupling in the percolation model is to determine the effective dimensionality, and hence the percolation threshold. For \( P_n = 0.3 \), the resultant characteristic scale \( \Xi_0 \) lies in a narrow range for all investigated samples (see Table 1), \( \Xi_0 = 27 \pm 2 \) K, and hence is de facto universal (the stated uncertainty is 1 s. d. from the mean of the data in Table 1). If we assume 2D percolation and \( P_n = 0.6 \), the agreement between \( \sigma_3 \) and \( \sigma_1 \) is not as good, and the corresponding \( \Xi_0 \) is smaller by about a factor of two. We emphasize that the calculated \( \sigma_3 \) is effectively insensitive to model details such as the parameters of the patch nonlinear response \( R_s \), rendering \( \Xi_0 \) the sole parameter (within a given percolation model). This insensitivity to specifics is a consequence of percolation physics, where model details are unimportant close to the threshold and the response of the largest clusters dominates.

An important feature can be inferred from the comparison of linear and nonlinear response. Within the effective medium calculation, the linear conductivity determines the net current through the sample, given an applied electric field. Yet the nonlinear resistance of the superconducting patches, \( R_s \), is current dependent. The third-harmonic signal therefore depends on the third power of the current (to lowest order), which implies that \( \sigma_3 \propto \sigma_1^3 \). This is indeed borne out by experiment, as seen in Fig. 1d. In contrast, in GL theory both responses are determined by the electric field, and their ratio has a more complex temperature dependence (see ref. 37 and Methods). The apparent characteristic temperature scales for \( \sigma_3 \) and \( \sigma_1 \), therefore differ because of the nonlinear nature of \( \sigma_3 \), but the underlying scale \( \Xi_0 \), which determines the range of superconducting pairing emergence, is the same for both responses. This also implies that the superconducting contribution to the linear response should be discernable up to significantly higher temperatures than the nonlinear part, if the experimental signal-to-noise ratios are
similar. Measurements throughout the phase diagram of LSCO consistently confirm this trend (Fig. 1d), which strongly supports the percolation model.

Magnetic-field effect. As a further test of the model, we investigate the influence of an external magnetic field on the emergence regime. Although the quantitative effects of a magnetic field are difficult to determine within our simple effective medium approach, we can make qualitative predictions. One would expect the field to greatly influence the superconducting percolation process. Both the critical current of a superconducting patch and the number of patches decrease with increasing field. Above a characteristic field $H_0$, the critical currents of all patches are small, except for the sample-spanning cluster at temperatures below $T_m$. At fields significantly above $H_0$, the nonlinear response should therefore only exhibit a step-like feature close to $T_m$. Furthermore, $H_0$ is related to the macroscopic critical field $H_{c2}$, as both fields are determined by the underlying superfluid stiffness: $H_0$ describes the properties of the finite-sized clusters below and above $T_m$, whereas $H_{c2}$ pertains to the sample-spanning cluster below $T_m$. In agreement with these expectations, we find that an external magnetic field strongly suppresses the nonlinear response, rendering it step-shaped far above $H_0$ (Fig. 3a). Once the high-field step-like response is subtracted (see Supplementary Note 4 and Supplementary Figure 5), the data for all samples exhibit universal scaling (Fig. 3b). We apply the same effective medium calculation as for the temperature dependence, assuming a phenomenological power-law dependence of the effective patch critical current on $H/H_0$ (see Supplementary Note 4 and Supplementary Figure 4), and find good agreement with experiment (Fig. 3b). For $H > H_0$, only large superconducting clusters survive. Since the cluster-size distribution in any percolation model is generally exponential for the largest clusters, this leads to an exponential-like field dependence of the high-field response, as also observed in prior torque measurements. $H_0$ is about two orders of magnitude smaller than $H_{c2}$, consistent with the percolation scenario, since $H_0$ is a property of the average (small) cluster. As seen from Fig. 3b, the doping dependencies of the two characteristic fields are remarkably similar, including a minimum close to the “1/8 anomaly” of LSCO and YBCO. The substitution of 3% Cu with Zn in YBCO causes a dramatic decrease of $H_0$, in agreement with established effects of Zn on superconductivity in cuprates.

Discussion

Previous reports suggest that percolation processes might play a role in understanding the properties of the cuprates (see also Supplementary Note 3). However, our work demonstrates for the first time that a universal percolation process can describe the prepairing regime. The percolation picture is in excellent agreement with the temperature and magnetic field dependencies of $\sigma_{||}$ and $\sigma_{\perp}$, and one would expect it to provide an explanation of other experimental results as well. Qualitatively, several previous studies indicate that the superconducting precursor appears within a roughly constant temperature range above $T_m$ similar to Fig. 1d; this is visible, e.g., in high-frequency conductivity measurements, specific heat results, and resistivity curvature plots.

More quantitatively, Fig. 4 demonstrates the similarity of superconducting precursor in several observables. An exponential tail is observed in the dc conductivity of YBCO at various hole doping levels, with a universal slope (Fig. 4b). Notably, YBCO in particular is structurally complex, with alternating CuO$_2$ planes and CuO chains whose filling depends on oxygen concentration; the exponential behaviour, however, is robust and does not depend on the arrangement of the chains. The Nernst effect in Eu-LSCO shows an exponential dependence as well (Fig. 4c).

Although this measurement can be described by 2D Gaussian theory close to $T_m$, where corrections to the simple percolation picture are expected, once the data are plotted on an absolute temperature scale, the exponential tail is apparent and reveals the same underlying temperature/energy scale $\Xi_0$. Torque magnetometry measurements on several cuprate families, including underdoped LSCO, bismuth cuprates and Hg1201, as well as YBCO, exhibit both an exponential signal decrease above $T_c$ (Fig. 4d) and a universal temperature scale $T_d$. The exponential dependencies at temperatures well above $T_m$ are a consequence of the tail of the superconducting gap distribution, and for $\sigma_{||}$ and $\sigma_{\perp}$, the effective medium calculation smoothly continues this dependence down to $T_c$. Finally, roughly exponential tails are observed above $T_c$ in specific heat studies; it is possible to
calculate this within the percolation model by convoluting the mean-field specific heat step at $T_c$ with the gap distribution function (see Methods). This procedure yields agreement with experiment and reveals a scale $\Xi_0$ similar to that obtained from conductivity (Fig. 4e). Notably, critical fluctuations are observed in the specific heat close to the macroscopic $T_c$, which could be also important for other observables in a small temperature range around $T_c$.

We note that the percolation model discussed here is somewhat different from the standard textbook case, in that both the normal and percolating (superconducting) patches have nonzero conductivity. Therefore, instead of a discontinuity at $T_n$ and power-law behavior above the percolation temperature (that is predicted if one of the phases is insulating), the calculation yields smooth exponential-like behavior. Yet the underlying distribution of superconducting cluster sizes should still be scale-free (i.e., follow a power law) close to the percolation threshold. A signature of this might be observed with other experimental probes, e.g., recent optical pump-probe experiments uncovered hitherto unexplained power-law superconducting correlations above $T_c$.

We emphasize that the heterogeneity that gives rise to superconducting percolation is qualitatively different from the disorder discussed previously in the context of “dirty” and granular superconductors, and from inhomogeneity induced by doping. In alloys and films, the electronic mean-free path is extremely shortened by scatterers, while in granular materials differing Josephson couplings between granules cause superconducting percolation. Yet here we find that nanoscale gap inhomogeneity is crucial: the superconducting gap, and hence the local $T_c$, displays spatial variations and causes the percolation we observe. Related gap disorder (on scales much larger than the superconducting coherence length) has been employed previously in modeling the magnetization of select cuprate and other superconductors, but not applied universally or used to calculate transport properties. Inhomogeneity and a residual zero-temperature component of uncondensed carriers has been shown to be essential to understand low-temperature superfluid density and optical response of several cuprates. Spatial gap inhomogeneity also naturally explains the gap filling recently observed in a tomographic density-of-states photoemission experiment. As demonstrated in Fig. 4f, g, a quantitative description of this result can be obtained simply by postulating that the measured density of states is an average over spatial regions with inhomogeneous gaps, again with a distribution width of $k_B\Xi_0 \sim 3$ meV, which further supports the percolation scenario (see Methods for details).
Perhaps the most unexpected result of our study, which covers the doping range from the very underdoped ($p = 0.08$) to the overdoped ($p = 0.19$) part of the phase diagram, is the existence of a (nearly) doping- and sample-independent percolation scale $\Xi_0$, which implies a common intrinsic origin of the gap inhomogeneity in all cuprates, irrespective of material details. Doping does not significantly alter this scale, but affects the macroscopic $T_c$, or, equivalently, the critical percolation temperature. Several distinct types of disorder are generally present in the cuprates: the lamellar structure is intrinsically frustrated, which causes structural inhomogeneity; the hole doping process introduces defects into the crystal structure; and doping a strongly correlated electronic system may induce electronic frustration and inhomogeneity. These different kinds of disorder are typically compound and doping dependent\cite{26,25}, and various experimental techniques have been used to study them. Residual resistivity, a measure of point disorder, is compound-dependent, and can be very small in cuprates such as Hg1201\cite{33,34}. Furthermore, quantum oscillation experiments point to a high degree of doping (hole concentration) homogeneity in oxygen-doped compounds such as Hg1201\cite{35}, thallium cuprates\cite{36}, and YBCO\cite{37}. However, this does not preclude nanoscale electronic inhomogeneity unrelated to point disorder. This reasoning is supported by the fact that we find consistent results for distinctly different cuprates\cite{26}: Hg1201, where doping-related point disorder resides far away from the CuO$_2$ planes; LSCO, which exhibits considerable (La/Sr) disorder structures in both underdoped and overdoped\cite{41}; and YBCO–Zn, where Zn directly introduces point disorder within the CuO$_2$ plane.

The cuprates also appear to exhibit inherent structural inhomogeneity, an elegant demonstration of which comes from conductivity and hydrostatic relaxation experiments that show stretched exponential behavior characteristic of glassy materials\cite{55}. Moreover, X-ray experiments find complex fractal interstitial-oxygen-dopant structures linked to percolative superconductivity\cite{59}. Local electrostatic disorder has been studied via nuclear quadrupole resonance and revealed that LSCO and Bi-based compounds exhibit higher levels of such disorder\cite{38,39} than oxygen-doped cuprates such as Hg1201 and YBCO, where the dopant atoms reside far from the CuO$_2$ planes\cite{39,40}. Importantly, however, none of these experiments directly detect superconducting gap disorder, making it difficult to establish a relationship between electrostatic/doping inhomogeneities and superconducting gap distributions. STM does probe local gap distributions on the sample surface, but has been applied only to a select number of cuprates, and it is not trivial to separate the superconducting gap from the more inhomogeneous higher-energy (pseudo)gap\cite{17}. We emphasize that the gap distribution (with width $k_B \Xi_0$) relevant for our model likely is not precisely the same as the gap distribution seen by STM, but rather a coarsely-grained distribution of mean local gaps (averaged over the local superconducting coherence lengths). Moreover, we expect the distribution to be effectively narrower below $T_c$ because of proximity effects, i.e., large-gap superconducting regions may induce a gap in neighboring regions. Nevertheless, STM clearly reveals disorder structures in both underdoped and overdoped\cite{41} Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ and extended analysis shows a correlation between the presence of inhomogeneous high-energy gaps and superconductivity\cite{60}. A recent phenomenological model\cite{61} based on inhomogeneous, temperature and doping dependent (de) localization of one hole per primitive cell can explain the main features of the cuprate phase diagram and superconductivity. It is conceivable that a universal scale $k_B \Xi_0$ emerges via a complex renormalization of these high-energy localization gaps\cite{61} (see also Supplementary Note 3). In this case, the gap disorder would not necessarily be related to any local doping inhomogeneity: the material may be homogeneously doped, yet possess an underlying gap distribution.

The existence of exponential behavior with a universal scale $\Xi_0$ also shows that expected GL superconducting fluctuations are considerably weaker than inhomogeneity effects. Conversely, if GL fluctuations were important, the simple percolation model with a single $\Xi_0$ would not describe the measurements. This furthermore points to 3D percolation—perhaps in the special case of strong stripe correlations in La-based cuprates\cite{62} such as La$_{1.875}$Ba$_{0.125}$CuO$_4$ and La$_{1.8-n}$Eu$_{0.5}$Sr$_{2-n}$CuO$_{4}$ (see Supplementary Note 2)—since the strong vortex–antivortex fluctuations expected in the 2D case should significantly broaden the onset of superconductivity.

To conclude, we have employed a new approach to the cuprate prepairing problem by studying nonlinear conductivity. The unexpected scaling of nonlinear and linear conductivity for widely different cuprates constitutes a benchmark result for any theory of superconducting prepairing in these materials. Taking into account the well-established fact that significant gap inhomogeneity is present in the cuprates, we have provided a simple framework in which intrinsic and universal superconducting gap inhomogeneity is highly relevant to understanding the superconducting properties of the cuprates.

**Methods**

**Samples.** The Hg1201 and LSCO samples are single crystals of well-established high-quality used in previous works\cite{63} with volumes of about 1 mm$^3$. Hg1201 is grown using an encapsulation method, while LSCO crystals are grown in a traveling floating zone furnace. YBCO–Zn is an oriented powder sample with 3% Cu substituted with Zn, which enables us to discern effects of intentionally introduced CuO$_2$ plane disorder. This sample was prepared using a standard solid-state reaction, used in prior Zn nuclear quadrupole resonance experiments and characterized in detail\cite{6}. See Table 1 for additional sample information.

**Linear and nonlinear conductivity.** Nonlinear response measurements typically require relatively large applied fields in order to detect the small signals. Hence, the most serious problem that plagues nonlinear measurements of conductive systems is Joule heating, i.e., the variation of the conductivity/susceptibility with temperature induced by resistive heating of the sample. If a constant or slowly varying electric field is used to detect nonlinear response, the large current will heat the sample, distorting the measurement, interrupting nonlinear contributions, and inhomogeneity effects. Conversely, if the resistance depends on temperature. Conventionally, millisecond field pulses are used to alleviate the heating problem, but heating still plays a role for highly conducting samples and needs to be disentangled from other possible contributions\cite{64}. A pivotal step in our experiment is the use of a high-frequency excitation field—if the frequency is high enough, the time-dependent temperature change of bulk samples cannot follow the rapidly changing field, and no heating-induced nonlinear signal is observed. An average, time-independent heating is still present, but this does not influence the measurement of the nonlinear response. In principle, time-independent heating may cause a small shift of the sample temperature, but this was determined to be negligible in our case from a comparison of $T_c$ (peak positions) in linear and nonlinear conductivity throughout the phase diagram of LSCO. The nonlinear conductivity experiments are performed with a contactless radio-frequency two-coil setup, with excitation frequency $\omega/2\pi = 17$ MHz and phase sensitive detection at $3\omega/2\pi$ using a Stanford Research Systems SR844 RF lock-in. The coil system is kept at the constant temperature of the liquid helium bath, while the sample temperature is varied independently. We use a nonresonant circuit (a coil with silver paint serving as a distributed capacitance) for excitation, and a tuned resonant LC circuit for detection. A thin-walled glass tube separates the vacuum of the sample space from the liquid helium bath and introduces no distortions to the signal. The sample is mounted on a sapphire holder with temperature control sensitivity better than 1 mK. The setup was previously tested under various conditions\cite{65,66}. Notably, a similar methodology was used in the past to study the nonlinear Meissner effect at low temperatures\cite{66}. The electric fields with the samples may be estimated using Maxwell’s equations, which gives an average $E = -\nabla \Phi$, where $\Phi$ is the magnetic field amplitude, $\omega = 2\pi$ MHz the oscillation frequency, and $L = 2$ mm the typical linear sample dimension. The amplitude of the magnetic field was deliberately kept small, and estimated to be about 0.1 G from the characteristics of the excitation circuit and coil. The electric field amplitude is then $E = 0.02$ V/cm.

We performed complementary microwave (linear) conductivity experiments with a resonant cavity perturbation technique\cite{67} extensively used to study cuprate superconductors\cite{68}. The sample was mounted in an evacuated elliptical microwave
cavity made of copper and immersed in a liquid helium bath. The complex conductivity of the sample was obtained by measuring the temperature dependence of the Q-factor and resonant frequency of the cavity by recording the cavity resonance curve using microwave frequency modulation and employing a demodulator. The signal from the demodulator was fed into a SR830 lock-in amplifier and the Q-factor determined from measurements of the higher harmonic components of the modulation frequency. The microwave cavity resonance was close to 10 GHz, while the modulation frequency was 990 Hz. Similar to the nonlinear conductivity experiment, the cavity was kept at constant temperature, while the sample temperature was varied in a wide range. We obtained the superconducting response above $T_c$ by subtracting the conductivity measured with an external magnetic field of 16 T (perpendicular to the CuO$_2$ planes) from the zero-field conductivity. No appreciable difference in conductivity was observed between 12 and 16 T in the relevant temperature range.

**GL theory.** Classic GL superconducting fluctuations have been extensively investigated in the cuprates using linear response in a wide frequency range$^{27-31,76-79}$. We expect to provide a better probe of fluctuation contributions, since in linear response one must always attempt to determine and subtract a normal-state contribution, a complication that is absent in the third harmonic.

At a quantitative level, the linear and nonlinear GL-fluctuation response has been calculated beyond mean field$^{71}$ and with included anisotropy$^{73,77}$ (only linear conductivity). For an isotropic type-II superconductor, the nonlinear conductivity in both the Gaussian and critical fluctuation regimes is shown to be proportional to the linear conductivity, as follows. In general, one can define a field-dependent conductivity, $\sigma(E) = \sigma(E = 0) E$. The scaling function $\mathcal{G}(E)$ has different forms in different fluctuation regimes and for small/large electric fields, but in the small-field approximation the leading term is always $1 + A E / E_0^2$, where $A$ is a numerical constant and $E_0$ a reference electric field$^{75}$. The field $E_0$ depends on temperature through the mean-field correlation length ($\xi$), as $E_0 \propto \xi^{-3}$. Therefore, $\sigma_1 = A_s A / E_0^2 / \xi^6$. Since, in GL theory, the linear and nonlinear responses are due to the same fluctuation physics, such a scaling relationship between the two should hold as well. But whether $\xi$-plane or $c$-axis field correlation lengths is a numerical constant and $E_0$ is obtained through an expansion in powers of voltage. Due to the parabolic nature of the system, $\sigma_1$ is insensitive to the values of $R_n$ and $I_n$ in the region of interest close to $T_c$ (as long as the current is much smaller than $J_c$). Thus the only parameters entering the calculation of $\sigma_1$ are $\xi$ and the percolation concentration $P$ of the superconducting path, which depends on the number of spatial dimensions, on site vs. bond percolation, etc. $\xi$ is used in the linear response calculation, and was determined to be $0.005 \xi_0$, which is realistic for nanoscale patches at a finite excitation frequency$^{30,31}$.

In order to obtain $\Sigma_0$ and to determine if 3D (with $P = 0.3$) or 2D (with $P = 0.6$) site percolation is more appropriate, we simultaneously calculate the linear and nonlinear conductivity and compare to the data. While the results do not critically depend on $P$, a 3D site-percolation model with $P = 0.31$ yields the best agreement with the data. For example, it enables the linear and nonlinear response in LSCO to be described with a single $\xi_0 = 28.0 \pm 0.4$ K, whereas in the 2D model the discrepancy between $\xi_0$ obtained from linear and nonlinear conductivities differs at least by 25% (Figure S3). With $P = 0.31$ fixed, individual fits to only the nonlinear response of all investigated compounds (Table 1) gives the overall estimate $\xi_0 = 27 \pm 2$ K, whereas the simultaneous calculation of both $\sigma_0$ and $\Sigma_0$ for LSCO gives the highest resolution above. We emphasize that the parameter $\rho_0$ does not influence the determination of $\Sigma_0$, $\Sigma_0$ influences the shape of the linear conductivity curve, whereas $\xi_0$ sets the range of the superconducting contribution. $T_c$ is calculated separately in a model-free way to obtain the best data scaling, with typical uncertainties smaller than 0.05 K. The LSCO-0.15 data are taken as a reference since they exhibit the best signal-to-noise ratio. $\rho_0$ provides a constraint--similar to the 2D case, it cannot be simultaneously obtained with the same $\xi_0$ (the difference being about 10%, larger than the uncertainties). Also, the corrections due to $P$ vs. $T$ nonlinearity may become important. In any case, the change between $P = 0.31$ and $0.25$ is not very significant in view of the crudeness of the modeling, but the data do support a 3D percolation model. The cuprate superconductors are known to be strongly anisotropic, in the site-percolation model, this translates to anisotropy within the patches (i.e., they are elongated in the $c$-direction), but this does not change the percolation threshold. Since we measure in-plane response, the threshold is the only important parameter. A possible exception would be systems with effectively decoupled layers (such as Eu–LSCO and LBCO close to doping 1/8, as discussed).

**Modeling of specific heat.** Specific heat measurements in several cuprates$^{29,48}$ show high-temperature tails above $T_c$. Here, we show that the tails can be modeled in a quantitative fashion by simply convoluting the standard mean-field step in superconducting contributions to the specific heat coefficient by a simple linear dependence below the local $T_c$, $\Delta T_{\text{NL}} = \alpha(T_c - T / 2)$, and take it to be zero above $T_c$.
\[ T_c \text{, Such a form is not correct at low temperatures, but is appropriate close to } T_c. \]

The coefficient in a system with a distribution of \( T_c \) is then just

\[ \Delta y(T) = \int g(T) \Delta y_m(T, T) dT, \]

where \( g \) is the distribution function. The calculated \( \Delta y \) is shown in Fig. 4e in comparison with data on \( Y_{123} \text{CuO}_2 \). The effective gap obtained in these experiments does not close at the macroscopic \( T_c \), but a "filling" of the density of states is observed to extend to temperatures \( \sim 1.2 T_c \) (Fig. 3f). The gap filling was attributed to an increased superconducting pair-breaking rate, and the response above \( T_c \) to preformed pairs. However, as we now show, both effects arise naturally if one assumes a spatial gap distribution. In ref. 19, the density of states was fitted to the standard expression

\[ \rho_{\text{dyn}} = \frac{\omega - i\Gamma}{\sqrt{[\omega - i\Gamma]^2 - \Delta^2}}. \]
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Author contributions
D.P. and M.V. built the nonlinear conductivity setup, performed measurements, and analyzed the data. M.S.G. and M.P. built the microwave conductivity setup and performed linear conductivity measurements. M.P. supervised all conductivity experiments. M.P. and N.B. initiated the parac conductivity studies. D.P., G.Y., M.G., and N.B. conceived the idea to pursue the percolation-based data analysis. D.P. performed the percolation calculations. T.S. prepared the LSCO samples. D.P. prepared the YBCO-Zn sample. G.Y. and N.B. prepared the Hg1201 sample. D.P., M.G., and N.B. wrote the paper with input from all authors.

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