Abstract

The proliferation of smart meters enables load-serving entities to aggregate customers according to their consumption patterns. We demonstrate a method for constructing groups of customers who will be the cheapest to service at wholesale market prices. Using smart meter data from a region in California, we show that by aggregating more of these customers together, their consumption can be forecasted more accurately, which allows an LSE to mitigate financial risks in its wholesale market transactions. We observe that the consumption of aggregates of customers with similar consumption patterns can be forecasted more accurately than that of random aggregates of customers.

I. INTRODUCTION

With the deregulation of electricity markets, most residential customers in the United States are now served by retail electric utilities. These utilities play the role of intermediary between the end consumers of electricity and the wholesale market. More precisely, a utility represents an aggregate of customers as a single entity in the wholesale market and passes the cost of buying electricity to the end customers. For historical reasons, a single utility typically serves all of the residential customers in a broad geographical area in most states in the U.S. Utilities traditionally do not differentiate between consumers, instead adopting a single rate plan for all residential customers.

Aggregating and treating all customers equally provides two main advantages for a utility. One advantage is that the billing and management services can be streamlined and simplified. The more fundamental and interesting effect is that by presenting customers as an aggregate, the uncertainty in their behavior is reduced dramatically, and the utility faces much lower risk in the wholesale market. It is well known that while the consumption of a single consumer is almost impossible to forecast with any kind of accuracy, that of a large group of users can be forecasted to within a couple of percentage points [1], [2]. This arrangement, however, leads to cost sharing. Some users will consume more of their electricity at times when it is more expensive to purchase on the wholesale market, thereby driving up the aggregate per unit cost of electricity for the utility. The utility, though, charges the same per unit rate to all residential users, so some users end up subsidizing the others.

The multi-staged structure of the current wholesale electricity market makes some level of aggregation necessary because of the risk in purchasing power to serve customers. For example, consider a market with two stages: day-ahead and real-time. At day-ahead, a load serving entity (LSE) forecasts the demand for its customers and
purchases electricity at the day-ahead price [3], [4]. The LSE’s real-time deviation from its forecasted consumption is settled at the real-time price. If the LSE’s forecasts were perfectly accurate, or if the real-time price were always the same as the day-ahead price, the LSE would face no financial uncertainty in its daily operations. However, the combination of forecasting uncertainty with the possibility of high real-time prices introduces risk into the LSE’s operations. For example, if the LSE significantly underestimates its customers’ consumption, it will purchase too little electricity on the day-ahead market, and it will be forced to purchase the remaining consumption at the real-time price, which can spike to thirty or forty times higher than the day-ahead price [5]. Therefore aggregation can reduce the overall cost by lowering uncertainty in the customers’ consumption. Initially, we focus on the problem of designing average cost-based pricing. Such pricing mechanisms are broadly adopted but designed for a whole population [6]. Moreover, consumer pricing response depends on estimated average cost of electricity rather than marginal cost [7].

This paper studies the trade-off between the size of an aggregate of customers and the uncertainty in serving them. When forming the aggregate, we want to group the customers with the lowest cost; that is, we seek customers whose consumption profile is orthogonal to the day-ahead price profile (e.g. households with high night time energy usage). This process is formalized as a fractional integer program, and we provide an optimal solution to find the lowest cost customers for a given aggregation size. As the size of this aggregate grows, the per unit cost of electricity for the group also increases because customers with higher cost are added to the group. On the other hand, the aggregate demand becomes easier to forecast [8], and we quantify how this uncertainty changes with aggregation size. A surprising result is that the relative uncertainty does not decrease monotonically as the size of the aggregate grows. This suggests that after a certain limit, there is no benefit in aggregating a larger group of customers.

We arrive at the above conclusions by analyzing customer consumption patterns. We use hourly smart meter data provided by PG&E to obtain consumption profiles of individual households as well as to evaluate the demand forecasting accuracy for various aggregate sizes of low-cost users.

Note that the idea of segmentation of users has been proposed in the literature and used in practice by some utilities [9]–[12]. For example, PG&E divides Northern California into several zones and charges different rates for customers in each zone. The Electric Reliability Council of Texas (ERCOT) already allows for LSEs that can purchase electricity in bulk on the wholesale market for their customers without owning any physical infrastructure [13]. However, these segmentations are all based on geographical information. We show that there is significant behaviour variation even within in a single zip code.

This paper is organized as follows. In Section II we formulate the aggregation problem for the LSE, and we define a metric for the per unit electricity cost of serving a customer or group of customers. In Section III we provide an optimal algorithm for constructing low-cost aggregates of customers. In Section IV we evaluate the uncertainty in forecasting the consumption of these aggregates as we vary the aggregation size, and we conclude the paper in Section V.
II. MODEL SETUP AND PROBLEM FORMULATION

We use the term load serving entity (LSE) to denote an agent that buys energy in the wholesale markets and serves end customers. To lower its total cost, an LSE would seek out lower-cost residential customers. In general, the wholesale price of electricity is determined by the total demand of all customers in a control area and is higher during the peak consumption times of the day. An LSE would wish to recruit customers with consumption profiles that are misaligned with the wholesale price profile.

In addition to recruiting customers, an LSE must manage the day-to-day operations of participating in the day-ahead and real-time markets. The timeline of these phases is shown in Fig. 1. The main source of financial uncertainty for the LSE is in the operations phase, in which the LSE must hedge against errors in forecasting its customers’ consumption and the risk of high real-time electricity costs. The LSE’s primary opportunity to influence this risk is in the recruitment phase, and we focus on this phase in this paper.

Fig. 1. The timeline for an LSE. Both the planning and operation stages are subject to uncertainties. After recruiting its customers, the LSE must purchase electricity for them on the day-ahead and real-time wholesale electricity markets.

A. Recruitment Optimization

Suppose the LSE is evaluating which residential customers to serve in an area with a total of \( N \) users. Let \( \mathbf{u} \) be an indicator vector of user enrollment. That is, \( u_i = 1 \) if user \( i \) is in the group and \( u_i = 0 \) otherwise. Let \( \{ \mathbf{d}_i \} \) be the set of user demands where \( \mathbf{d}_i \) is a random vector in \( \mathbb{R}^{24} \). Define \( \mathbf{d}_u \) as the total demand of the group of selected users,

\[
\mathbf{d}_u = \sum_{i=1}^{N} u_i \mathbf{d}_i. \tag{1}
\]

Denote the day-ahead wholesale price of electricity as \( \mathbf{p} \), a known (at the day-ahead stage) vector in \( \mathbb{R}^{24} \). The cost of the total electricity purchased by the LSE on the day-ahead market is \( C_u = \mathbf{p}^T \mathbf{d}_u \). At the recruitment stage, we seek to minimize the expected cost per unit of electricity, which is given by

\[
\mathbb{E} \left[ \frac{C_u}{1^T \mathbf{d}_u} \right]. \tag{2}
\]

We seek to control the uncertainty of the expected per unit cost, and this gives the overall optimization problem

\[
\min_{\mathbf{u}} \mathbb{E} \left[ \frac{C_u}{1^T \mathbf{d}_u} \right] \tag{3a}
\]

s.t. \( CV_{\mathbf{d}_u} \leq \sigma^2 \), \hspace{1cm} \tag{3b}
where \( CV_{d_{u}} \) is the coefficient of variation forecasting error for the day-ahead hourly consumption of the group of users. The coefficient of variation is a dimensionless measure of forecast error, defined as the standard deviation of the residuals divided by the mean of the actuals. High forecasting error can lead to high real-time deviation charges for the LSE and consequently a high variance in the expected per unit cost of electricity. Therefore, the LSE sets a limit \( \sigma^2 \) on \( CV_{d_{u}} \) as a way to limit its financial risk in its wholesale market transactions.

There is one other source of uncertainty in the recruitment phase: the LSE is unsure of whether the recruited people will act as their past data suggests. We set this concern aside by assuming that past data accurately predicts expected future behavior and by noting that we are not at this time trying to induce these customers to change their behavior.

B. Data

We demonstrate the recruitment and forecasting methods using data from four zip codes in Bakersfield, California. Specifically, we use anonymized and secure hourly smart meter readings for 1923 PG&E residential customers for a period of one year spanning from 2011 to 2012. We construct a year-long day-ahead price vector based on California ISO data for the same date range. The consumption forecasting method makes use of mean daily and hourly temperature forecasts, so we take this information from WeatherUnderground.com. Temperature serves as an exogenous input for the time series forecasting of electricity consumption. This sample is fairly large, in both space and time, so we believe it serves as a legitimate basis for this proof of concept demonstration.

C. Identifying low-cost users

During recruitment, the LSE starts by identifying the users who are cheapest to service at wholesale prices. Based on the smart meter data, we construct the hourly electricity consumption of the \( i \)th user as a known real vector \( \mathbf{D}_i \) of length \( T \), where \( T \) is the number of hourly time steps in the period of historical data that we evaluate. For the results presented in this paper, \( T = 24 \cdot 365 = 8760 \) hours, and \( \mathbf{D}_i \) is the concatenation of the past 365 \( \mathbf{d}_i \) vectors for a given user.

For the same period of historical data, we construct the vector \( \mathbf{P} \), the day-ahead price of electricity for each hour. Note that \( \mathbf{P} \) is not indexed by user - it is a common price that all buyers of electricity pay on the wholesale market since all the users are at the same geographic location. For a large set of users, the location marginal prices can be used to differentiate locations of different groups of users. Define the per unit cost of servicing a user over the past year as

\[
\lambda_i = \frac{\mathbf{P}^T \mathbf{D}_i}{\mathbf{I}^T \mathbf{D}_i}. \tag{4}
\]

The metric \( \lambda_i \) gives the LSE some sense of what it would have cost, on a per unit basis, to supply user \( i \) with electricity purchased on the wholesale day-ahead market. The LSE can then rank each user in ascending order of \( \lambda_i \). Figure 2 plots \( \lambda_i \) as it increases, from the cheapest user to the most expensive user based on the Bakersfield data. It is not feasible to just choose the lowest cost user or two and purchase electricity for them on the wholesale
market for two reasons. The first is that extremely small groups of users are not a viable basis for an LSE because
the revenue would be too small. The second is that the variance of their consumption would be very high, so the
forecasting error would be high, violating (3b). Therefore, the LSE must evaluate how to reduce the forecasting
error, and build a more viable group, by aggregating more low-cost users together.

![Graph of \( \lambda_i \) vs. user rank, in order of increasingly expensive users. The difference between the cheapest user and the costliest user is approximately 1.1 cent/kWh. The steep slopes at either end of the curve mean that there are relatively few very cheap users, and even fewer very expensive users.]

**III. CONSTRUCTING GROUPS OF LOW-COST USERS**

Figure 2 illustrates that some users are more expensive to service at wholesale prices than others. During its
recruitment phase, the LSE must seek to enroll users who collectively form an aggregate that has a relatively low
per unit cost of electricity. The best aggregation of low-cost users can be posed as an optimization problem.

Let \( K \) be an integer between 1 and \( N \). Choosing the \( K \) best users to minimize the per unit cost of electricity is
given by the following optimization:

\[
\min_u \frac{\sum_{i=1}^{N} u_i P^T D_i}{\sum_{i=1}^{N} u_i 1^T D_i} \tag{5a}
\]

subject to \( u_i \in \{0, 1\} \) \tag{5b}

\( 1^T u = K \) \tag{5c}

where \( u_i \) are the 0, 1 selection variables. Let \( t_i = P^T D_i \) and \( w_i = 1^T D_i \). We rewrite the optimization as:

\[
\min_u \frac{u^T t}{u^T w} \tag{6a}
\]

subject to \( u_i \in \{0, 1\} \) \tag{6b}

\( 1^T u = K \). \tag{6c}
Finally, we introduce a slack variable $\lambda$ and obtain:

\begin{equation}
\min_{\mathbf{u}, \lambda} \lambda \quad (7a)
\end{equation}

\begin{equation}
\text{s.t. } (\mathbf{t} - \lambda \mathbf{w})^T \mathbf{u} \leq 0 \quad (7b)
\end{equation}

\begin{equation}
\mathbf{u}_i \in \{0, 1\} \quad (7c)
\end{equation}

\begin{equation}
\mathbf{1}^T \mathbf{u} = K. \quad (7d)
\end{equation}

We can solve this optimization efficiently using a bisection algorithm for a feasibility problem which tries to find $\mathbf{u}$ and $\lambda$ to satisfy (7b), (7c), and (7d). This can be performed in a greedy fashion per Algorithm 1. For a given $\lambda$, we rank each element of the vector $(\mathbf{t} - \lambda \mathbf{w})$ and choose the smallest $K$ elements with the selection vector $\mathbf{u}$. If $(\mathbf{t} - \lambda \mathbf{w})^T \mathbf{u} \leq 0$, then we have found a feasible solution for the given value of $\lambda$; otherwise, no solution exists for this value of $\lambda$, and we must choose a larger $\lambda$.

The proof of the optimality of Algorithm 1 is given in the Appendix. Note that linear fractional programs are generally NP-complete [14]. The special feature in (7) is that all $w_i$’s are positive, which induces a monotonic property. The runtime of this algorithm is $O(\log(1/\varepsilon)N \log(N))$.

By solving this optimization, the LSE can select the best groups of users of size $K$. By varying $K$, the LSE can vary the consumption forecasting error, and can thereby control the risk it faces in the two-stage electricity market. Varying $K$ also varies the per unit cost of purchasing electricity for the group. Let $M_K$ denote the optimal group of low-cost users of size $K$, constructed per Algorithm 1. The minimized objective in (5a) is the per unit cost of servicing group $M_K$ at wholesale prices, denoted $\lambda_{M_K}^*$. Fig. 3 illustrates that $\lambda_{M_K}^*$ increases with increasing $K$. This makes sense given that groups of small size will include only the lowest-cost users, and as $K$ increases, higher-cost users are added to the group.

There is a possibility that a given user will be cheaper to service some days and more expensive to service on
Result: Assignment \( u^* \) for optimal group \( M_K \) and corresponding per unit electricity cost \( \lambda^*_{M_K} \)

Data: [1] Population servicing cost: \( t \) [2] Population consumption: \( w \) [3] Group size: \( K \) [4] Stopping criterion: \( \tau \)

// Initialize bisection method
\[ \bar{\lambda} = \max\{t_1/w_1, \ldots, t_N/w_N\} \]
\[ \underline{\lambda} = \min\{t_1/w_1, \ldots, t_N/w_N\} \]

// Bisection Method: while method has not converged.

\[ \textbf{while } \bar{\lambda} - \underline{\lambda} > \tau \textbf{ do} \]

\[ \lambda = (\underline{\lambda} + \bar{\lambda})/2 \quad \text{//Update current } \lambda; \]

\textbf{Solve feasibility Problem:}

Compute \( (t - \lambda w) \)

Sort \( (t - \lambda w) \) in ascending order to obtain \( \{i_1 \ldots i_N\} \)

Construct \( u_{\lambda,i} = 1 \) if \( i \in \{i_1 \ldots i_K\} \)

// Test if resulting point is feasible.

\[ \text{if } (t - \lambda w)^T u \leq 0 \text{ then} \]

\[ \text{else} \]

\[ \text{endif} \]

// If no feasible point exists, tighten lower bound. Else, tighten upper bound.

\[ \text{if } u = \emptyset \text{ then} \]

\[ \text{else} \]

\[ \text{endif} \]

Algorithm 1: Efficient customer selection method.

other days; that is, their ranking per the \( \lambda_i \) measure may change from day to day. We definitely observe this in the Bakersfield data set, but we also observe that at group size \( K \approx 0.1N \), the aggregate ranking of \( M_K \) compared to other groups of similar size in the population is very stable over the course of the year. Thus, the LSE should not be concerned with variations in the optimality of its recruited group of users over a year.

IV. Evaluating Forecast Error

For each group size \( K \), the LSE must evaluate the error of forecasting the aggregated consumption for optimal group \( M_K \) in order to find the right balance between increasing per unit cost of electricity and decreasing forecast error. To participate in the day-ahead market, the LSE must submit an hour-by-hour forecast of the aggregate's
consumption for the next day; that is, it must forecast the total consumption of its group of users at each hour of the day for the entire next day by some cut-off time for the day-ahead market (i.e., not on a rolling 24-hour basis). We propose a forecasting method with two components.

The first component is an ARMAX model, regressed by daily mean temperature, which forecasts the total electricity that will be consumed tomorrow ($\hat{y}$). The second component is a vector ARMAX model, regressed by hourly mean temperature, which forecasts the consumption load shape for tomorrow ($\hat{s}$). Given $d_u$, a vector in $\mathbb{R}^{24}$ with the entry $d_{u,h}$ equal to the total consumption during hour $h$ of the group of users selected by $u$, we compute the load shape vector $s$ as follows:

$$s = \frac{d_u}{1^T d_u} \quad (8)$$

Note that $\sum_{h=1}^{24} s_h = 1$, and $s_h$ represents the fraction of the daily total consumed during hour $h$. We believe that it is fair to assume we have the hourly mean temperatures for the next day because day-ahead temperature forecasting is quite accurate [10]. To generate the hour-by-hour forecast of electricity consumption for the group of users for tomorrow, denoted $\hat{d}_u$, we compute $\hat{d}_u = \hat{s} \cdot \hat{y}$.

We run this forecaster over 90 days on the Bakersfield data. We look at 29 values of $K$ ranging from 2 to 1900, and we compare the forecasting error for optimal low-cost groups $M_K$ to that of groups of size $K$ drawn randomly from the population.

In the forecasting figures presented here, we have chosen to plot the group size in terms of electrical power consumption (kW) instead of number of users. On average, each residential user consumes about 1 kW, so the number of users in a group, $K$, is approximately equal to the kW consumed by the group. That said, kW is a more meaningful measure of group size for an LSE and it allows us to control for the variation in consumption rates of different users.

Fig. 4 shows that forecasting for optimal groups of low-cost users $M_K$ is more accurate than for randomly sampled groups of the same size. This result is somewhat counterintuitive. Generally speaking, we would expect that a more diverse group of users could be predicted more accurately because they would be closer to I.I.D. random variables. Conversely, users with very similar behavior (e.g., higher consumption at off-peak hours) should be harder to predict accurately because their correlation should increase the variance of their sum. In this particular instance, it seems that at an adequate level of aggregation, the forecasting algorithms are able to take advantage of similarities in the consumption patterns of low-cost users to accurately predict their aggregated demand. In order to explore this result further, we take the price vector $P$ and shuffle the hours in the day in order to generate a new price vector with a modified profile. We then create new groups of size $K$ by using Algorithm 1 with the shuffled price vector. In each case, when shuffling the price vector 4, 8, 12, 16, or 20 hours at a time, the optimal groups with respect to the shuffled price vector are easier to forecast than randomly sampled groups, though harder to forecast than the true optimal groups $M_K$. Fig. 4 shows the coefficient of variation for forecasting optimal groups created with respect to the 12-hour shuffled price vector. This result reinforces the notion that aggregates of users with similar shapes (whether out-of-phase with $P$ or shuffled versions of $P$) have behavioral similarities that facilitate
A. Forecasting and User Selection

We observe that when we aggregate beyond 250 kW of low-cost users, the forecasting error rises back up to the average forecasting error for the population. This happens because as the aggregate gets larger, it adds in users who aren’t particularly different from the norm. For this dataset, the LSE would not stand to gain by aggregating beyond 250 kW of low-cost users (assuming that is a viably large customer base), because doing so would lead to a simultaneous increase in per unit electricity costs and prediction error. Taken together, Figs. 3 and 4 illustrate the trade-off. Increasing the size $K$ of aggregates of low-cost users leads to higher per unit electricity costs but lower forecasting error. The optimal balance will be at some group size $K$ that depends on the LSE’s budget for risk and revenue targets.

For example, Fig. 4 indicates that low-cost groups of size 250 kW have the lowest forecasting error. Fig. 3 tells us that groups of this size will have a per unit electricity cost of 3.1 cents per kWh on the wholesale day-ahead market, which is a discount of approximately 9% with respect to the per unit cost of supplying the entire population of 1923 users on the wholesale market. If an LSE wants a lower per unit cost, it could decide to serve just 80 users, who would have a cost of 3.0 cents per kWh. Fig. 4 tells us that this group size will have a higher forecasting error, so the LSE will face a greater risk of incurring high real-time deviation charges, and the variance of its per unit electricity cost will increase.

In practice, a load serving entity would service much more than just 2,000 customers. In future studies, we intend to expand our analysis to a much higher number of users. This may show that a significant number of customers could be served at a significantly discounted rate.
V. Conclusion

We have proposed a model for establishing an LSE. Based on smart meter data, the LSE recruits and aggregates residential users whose electricity consumption is cheaper to supply on the wholesale market. We have demonstrated that the LSE can reduce its forecasting error by increasing the number of customers it aggregates, up to a point. Given a tolerance for risk and desired revenue, a load serving entity could find an optimal group size of low-cost users to service.

Appendix

Proof of Optimality of Algorithm 1

We show that there exists an unique minimum $\lambda^*$ as the optimal solution of (7) and Algorithm 1 always succeed in finding it.

Define the feasible set of (7) as $\Lambda = \{\lambda : \exists \mathbf{u} \in \{0, 1\}^N, \mathbf{1}^T \mathbf{u} = K, (t - \lambda \mathbf{w})^T \mathbf{u} \leq 0\}$. Therefore, the solution to (7) is $\lambda^* = \inf\{\lambda \in \Lambda\}$. The bisection method in Algorithm 1 always find $\lambda^*$ if the following monotonicity lemma holds.

Lemma 1. Suppose $w_i > 0$ for all $i$. The feasible region $\Lambda$ satisfy the following: (1) If $\lambda \in \Lambda$, then $\lambda + \epsilon \in \Lambda$ for all $\epsilon > 0$. (2) If $\lambda \notin \Lambda$, then $\lambda - \epsilon \notin \Lambda$ for all $\epsilon < 0$.

Suppose Lemma 1 is true, there is only one local minimum in $\Lambda$ and hence it is the global optimal solution $\lambda^*$. It is clear that the greedy bisection method in Algorithm 1 is guaranteed to find it.

It remains to prove the two statements in Lemma 1. For the first statement, if $\lambda \in \Lambda$, then there exists $\mathbf{u}$ satisfying the constraints. Let $\mathcal{G}_K = \{i_1, \ldots, i_K\}$ denote the $K$ users chosen corresponding to $\mathbf{u}$. The vector $\mathbf{u}$ is feasible for $\lambda + \epsilon$ since $\sum_{i \in \mathcal{G}_K} (t_i - (\lambda + \epsilon)w_i) = \sum_{i \in \mathcal{G}_K} (t_i - \lambda w_i) - \sum_{i \in \mathcal{G}_K} \epsilon w_i \leq 0$. The first term is negative since $\mathbf{u}$ is feasible, the second is always positive since $w_i > 0$ and $\epsilon > 0$.

The second statement in Lemma 1 follows from the first statement. If $\lambda \notin \Lambda$ but $\lambda - \epsilon \notin \Lambda$ for some $\epsilon > 0$, by the first statement $\lambda$ is in $\Lambda$ so we have a contradiction.

References

[1] W. R. Christiaanse, “Short-term load forecasting using general exponential smoothing,” Power Apparatus and Systems, IEEE Transactions on, vol. PAS-90, no. 2, pp. 900–911, 1971.
[2] M. Hagan and S. M. Behr, “The time series approach to short term load forecasting,” Power Systems, IEEE Transactions on, vol. 2, no. 3, pp. 785–791, 1987.
[3] H. Allcott, “Real time pricing and electricity markets,” Harvard University, 2009.
[4] M. Roozbehani, M. Dahleh, and S. Mitter, “Volatility of power grids under real-time pricing,” Power Systems, IEEE Transactions on, vol. 27, no. 4, pp. 1926–1940, 2012.
[5] D. Kirschen and G. Sirbac, Fundamentals of Power System Economics. John Wiley and Sons, 2004.
[6] S. Borenstein and S. Holland, “On the efficiency of competitive electricity markets with time-invariant retail prices,” RAND Journal of Economics, vol. 36, no. 3, pp. 469–493, 2005.
[7] K. Ito, “Do consumers respond to marginal or average price? evidence from nonlinear electricity pricing,” National Bureau of Economic Research, Working Paper, 2012.

[8] R. Sevlian and R. Rajagopal, “Value of aggregation in smart grids,” in Proceedings of the IEEE SmartGridComm, 2013.

[9] J. Kwac, J. Flora, and R. Rajagopal, “Household energy consumption segmentation using hourly data,” to Appear in IEEE Transactions on Smart Grid, SPECIAL ISSUE ON ANALYTICS FOR ENERGY FORECASTING WITH APPLICATIONS TO SMART GRID, 2013.

[10] A. Albert and R. Rajagopal, “Smart meter driven segmentation: What your consumption says about you,” Power Systems, IEEE Transactions on, vol. 28, no. 4, pp. 4019–4030, 2013.

[11] V. Figueiredo, F. Rodrigues, Z. Vale, and J. Gouveia, “An electric energy consumer characterization framework based on data mining techniques,” Power Systems, IEEE Transactions on, vol. 20, no. 2, pp. 596–602, 2005.

[12] C. Alzate, M. Espinoza, B. De Moor, and J. A. K. Suykens, “Identifying customer pro?les in power load time series using spectral clustering,” in Proceedings of the 19th International Conference on Artificial Neural Networks, 2009.

[13] Electric Reliability Council of Texas. Load serving entities. [Online]. Available: http://www.ercot.com/services/rq/lse/

[14] C. Floudas, Deterministic Global Optimization: Theory, Methods and Applications. Boston: Kluwer Academic, 1999.