Compressed sensing based low complexity 2D-DOA estimation by separation and pair-matching approach

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Abstract: Compressed sensing (CS) based direction-of-arrival (DOA) estimation has the advantage of high resolution and no need for wave number estimation. Although it is applicable for 2 dimensional estimation, the computation complexity is severe problem due to its increased number of array elements and search domain. This letter proposes a separation approach and pair-matching to resolve the above drawback. Since the CS can extract the original signal source, the pair-matching can be simply attained by cross correlation between source estimates of vertical and horizontal arrays. Computer simulation verifies the fundamental effectiveness of the proposed method.

Keywords: DOA estimation, compressed sensing, pair-matching

Classification: Antennas and Propagation

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1 Introduction

A number of researches on direction of arrival (DOA) estimation of radio waves are progressing in many fields, such as radar systems and positioning systems for wireless communication terminals, and applications to mobile communication. Improving estimation accuracy is essential for the rapid spread of wireless communication. Many techniques on DOA estimation have been investigated such as beamformer method and MUltiple SIgnal Classification (MUSIC) method [1]. These methods are frequently used nowadays. However, these methods require information on the number of incoming waves as precursor information.

In recent years, compressed sensing (CS) [2] is applied as a more accurate estimation method. CS can reconstruct the original source from a few observations in which most of components to be estimated are zero, i.e. called sparse. In the CS based DOA estimation, the space around the receiving antenna array is divided into small angular bins. DOAs and complex signals of incident waves can be estimated at the corresponding bins. Its notable feature is that one snapshot is sufficient. When the arrival waves are multi-band signals, the estimation accuracy can be improved more than the case of the single-band signals [3]. Further, it is known to possible to estimate the number of incident waves exceeding the physical degree of freedom of the antenna array. Several algorithms have been developed to obtain the solution of the CS. We focused on half-quadratic regularization (HQR) algorithm which is known to exhibit superior estimation accuracy [4]. However, there is a problem that the amount of calculation becomes enormous due to the increase of search domain, especially in the case of 2D-DOA estimation. Exploiting the nature of CS, this letter proposes a 2D-DOA estimation by separation and pair-matching approaches. The key idea is to calculate a cross correlation between complex-valued source estimates of vertical and horizontal arrays. Simulation result shows that the proposed approach can effectively work under the high SNR environment with reasonable computation complexity.

2 System description

Fig. 1 shows an L-shaped array. Receiver can be equipped with a planar array wherein two ULAs arranged with $M$ and $N$ elements in a row and column are utilized for 2D-DOA estimation. Each inter-element spacing is $d$. Suppose the antenna aperture faces the positive direction of the $Y$-axis perpendicular to the $X$–$Z$ plane. $K$ narrowband plane signals of sources impinging on the array yield distinct directions at elevation and azimuth angles $\{\theta_k\}_{k=1}^K$ and $\{\phi_k\}_{k=1}^K$, respectively. The baseband array input at the $t$-th snapshot along the $Z$-axis is expressed as

$$ x_z(t) = \sum_{k=1}^{K} a(\theta_k)s_k(t) + n_z(t), $$

where $a(\theta_k)$ is so-called steering vector represented as $[a_1(\theta_k), \ldots, a_M(\theta_k)]^T$ with

$$ a_m(\theta_k) = \exp\left\{-j\frac{2\pi}{\lambda} d(m-1) \cos \theta_k \right\}. $$
\( \lambda \) denotes the wavelength and \( \mathbf{n}_z(t) \sim \mathcal{CN}(0, \sigma_{\text{nz}}^2) \) is an additive white Gaussian noise (AWGN) vector. On the other hand, the array input along the \( X \)-axis is also given by,

\[
x_x(t) = a(\theta_k) s_k(t) + \mathbf{n}_x(t),
\]

where \( a(\phi_k) = [a_1(\phi_k), \ldots, a_M(\phi_k)]^T \) and its each element is

\[
a_n(\phi_k) = \exp \left\{-\frac{2\pi}{\lambda} d(n-1) \cos \phi_k \right\}. \tag{4}
\]

\( \mathbf{n}_x(t) \sim \mathcal{CN}(0, \sigma_{\text{nx}}^2) \) is also an AWGN vector. The elevation and azimuth angles can be estimated in a separation manner, however, these pair-matching is required.

### 3 Proposed method

First, we set initial problem to be solved. Let \( L, \hat{s} \in \mathbb{C}^{L \times 1} \) and \( \mathbf{A} \in \mathbb{C}^{M(N) \times L} \) denote the number of bins, sparse vector containing source information and mode matrix, array input can be rewritten as

\[
x(t) = A \hat{s}(t) + \mathbf{n}(t). \tag{5}
\]

Each element of \( \mathbf{A} \) is given by \( a_m(\theta_l) \) or \( a_n(\phi_l) \). Here, HQR method is applied. It transforms a nonquadratic optimization problem into a series of quadratic problems. See [5] for its detailed derivation. Resulting iterative algorithm is given by

\[
H(\hat{s}^{(n)}) \hat{s}^{(n+1)} = E[A^H x(t)], \tag{6}
\]

where \( n \) is the iteration number and \( E[\cdot] \) denotes the ensemble averaging. \( H(\hat{s}^{(n)}) \) is then expressed as

\[
H(\hat{s}^{(n)}) \triangleq A^H A + \alpha \Lambda(\hat{s}^{(n)}),
\]

\[
\Lambda(\hat{s}^{(n)}) \triangleq \text{diag} \left( \frac{q/2}{(|\hat{s}^{(n)}_l|^2 + \varepsilon)^{1-q/2}} \right), \tag{8}
\]

where \( \text{diag}(\cdot) \) is to compose a diagonal matrix \( \Lambda \in \mathbb{C}^{L \times L} \). Calculation of (5) is iteratively performed until
\[
\frac{\|s^{(n+1)} - s^{(n)}\|_2^2}{\|s^{(n)}\|_2^2} < \delta,
\]

is satisfied. $\delta > 0$ is a small constant.

Original signal sources $\hat{s}_{k,\phi}(t)$ and $\hat{s}_{k,\phi}(t)$ can be obtained by applying HQR respective to array inputs $x_z(t)$ and $x_x(t)$. Obtained signals can be found in corresponding bins; DOAs $\hat{\theta}$ and $\hat{\phi}$ can be uniquely determined. Under these conditions, the question of interest is how to make pairs of the corresponding elevation and azimuth angles, i.e., $\{\theta_1, \phi_1\}, \ldots, \{\theta_K, \phi_K\}$, in a computationally efficient manner.

To obtain the relation of the corresponding elevation and azimuth angles, we take the cross correlation matrix $R_{\phi \theta}$ between $\hat{s}_{k,\phi}(t)$ and $\hat{s}_{k,\phi}(t)$, which are the estimation results of HQR.

\[
R_{\phi \theta} = E[\hat{s}_{\theta}(t)\hat{s}_{\phi}^H(t)].
\]

Observing peak values of $R_{\phi \theta}$, these row and column indices are pairs of elevation and azimuth angles for corresponding signal sources.

Since the computation for matrix inversion of $H$ is the most dominant in HQR, its complexity is compared. Suppose the complexity order of Gaussian elimination based matrix inversion, straightforward 2D-DOA estimation requires $O((L_\theta \times L_\phi)^3)$ whereas the proposed method can reduce it to $O((L_\theta + L_\phi)^3)$. These operations must be repeated until the convergence condition is satisfied. Computation complexity becomes huge and thus it can be remarkably reduced by our proposed approach.

4 Computer simulation

4.1 Simulation parameters

This section examines the performance of the proposed 2D-DOA estimation. Evaluation metric is the probability of correct estimation. When the estimated DOA value is within the allowable range, it is determined as successful. Table I lists the simulation parameters. DOAs, $\theta$ and $\phi$, are uniformly distributed from $0^\circ$ to $180^\circ$, respectively. The search domain for both for elevation and azimuth angles are also set to from $0^\circ$ to $180^\circ$ with $1^\circ$ resolution; the space vectors have 181 bins. The probability of correct estimation is evaluated in terms of signal to noise power ratio (SNR), the number of antenna elements $(M, N)$, the number of snap shots $P$, and modulation order. HQR parameters $(\epsilon, q, \alpha)$ were empirically determined prior to the evaluation. All results were averaged via 10000 independent trials.

4.2 Simulation results

First, Fig. 2(a) visualizes the pair matching result of 2D-DOA. This case considers that three signal sources are arrived from $(\theta, \phi) = (160, 16)$, (119, 17), and (42, 117). The number of antenna elements and snapshots are set to $M = N = 20$ and $P = 100$. SNR is 30 dB. The peak values in the figure indicate estimated DOAs. Pair matching is successfully accomplished by our proposed approach.

Following results show the probability of correct estimation when three incident waves are arrived at random angles. Fig. 2(b) shows the probability versus
Table 1. Simulation parameters

| Parameters                  | Values                                             |
|-----------------------------|----------------------------------------------------|
| Data modulation             | BPSK, QPSK, 16QAM, 64QAM                           |
| SNR                         | 0, . . . , 30 dB                                   |
| Inter-element spacing       | λ/2                                                |
| Antenna elements (M, N)     | (10, 10), (20, 20), (30, 30), (40, 40)            |
| Carrier frequency           | 28 GHz                                             |
| Number of sources K         | 3                                                  |
| Number of snapshots P       | 10, . . . , 100                                    |
| Search domain               | 0° < θ < 180°, 0° < φ < 180°                      |
| Number of bins (Lθ, Lφ)     | (181, 181)                                         |
| HQR parameters (ε, q, α)    | (1.0 × 10⁻⁶, 1.0 × 10⁻⁶, 1.0 × 10⁻⁷)             |
| Convergence condition δ     | 1.0 × 10⁻⁶                                         |

SNR with various modulation orders. As shown in the figure, BPSK achieves the highest probability about 77% at SNR = 30 dB when the allowable range is set to ±1°. It indicates that the pair-matching accuracy depends on the modulation order. Signal estimate by CS is also affected by the additive noise effect. The lower the
modulation order has its immunity thanks to the longer Euclidean distance between symbols.

Fig. 2(c) shows the probability versus the number of snapshots at SNR = 30 dB. Here, allowable range is also varied as ±0°, ±0.5°, and ±1°. In high SNR region, estimation accuracy remained almost unchanged even when the number of snapshots is changed [4]. It is possible to estimate DOA even at the small number of snapshots about 10. The advantage of CS-based DOA estimation is kept even in the proposed 2D-DOA separation and pair-matching approach. It should be noted that K snapshots are required at least to realize pair-matching through a cross correlation calculation.

Finally, Fig. 2(d) shows the probability of correct estimation with the number of antennas elements. When sufficient snapshots are available, arbitrary number of signal sources can be estimated by the proposed method. Estimation accuracy can be improved as the number of antenna elements is increased. However, it tends to be saturated for more than 40 antenna elements per edge. Use of such large number of antenna elements also imposes impractical computation complexity in the conventional 2D-DOA estimation. The proposed method enables a considerable complexity reduction while achieving improved estimation accuracy. As a result, the proposed method is expected to be fundamentally effective approach in 2D-DOA estimation using compressed sensing.

5 Conclusion

In this letter, we proposed a practical approach that can simplify the compressed sensing based 2D-DOA estimation in separate manner and cross correlation. Key feature of the proposal is to exploit the nature of the compressed sensing that can extract original complex-valued signal source. It can realize the pair matching as well as computational complexity reduction. We can conclude that the proposed method is the most promising way for 2D-DOA estimation method towards 5G or beyond.

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