Tidal disruption near black holes and their mimickers

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Abstract. Black holes and wormholes are solutions of Einstein’s field equations, both of which, from afar, can look like a central mass. We show here that although at large distances both behave like Newtonian objects, close to the event horizon or to the throat, black holes and wormholes have different tidal effects on stars, due to their respective geometries. We quantify this difference by a numerical procedure in the Schwarzschild black hole and the exponential wormhole backgrounds, and compare the peak fallback rates of tidal debris in these geometries. The tidal disruption rates in these backgrounds are also computed. It is shown that these quantities are a few times higher for wormholes, compared to the black hole cases.

Keywords: Wormholes, GR black holes

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1 Introduction

Tidal forces can tear apart a massive object due to the gravitational influence of another. This fact assumes importance in the context of supermassive black holes (BHs) with masses $M \sim 10^6 \rightarrow 10^{10} M_\odot$ that are believed to exist at the center of most galaxies (see, e.g the reviews of [1–3]). Stars that come in the vicinity of such large masses are often disrupted by extreme gravity ([4–9], see also the more recent review by [10]), when the tidal forces become comparable to, or larger than, the self gravity of the star. Such tidal disruption events (TDEs) of stars in the background of a massive BH often produce an observable luminous flare, that can reveal important properties of the stellar structure ([11–14]), as well as that of the BH ([15, 16]). Indeed, TDEs are known to be one of the main physical processes responsible for the formation of accretion disks around BHs ([17]), a topic that has received considerable attention of late, after the advent of the event horizon telescope ([18–23]). Although several such TDEs have been observed, and seminal works have appeared in the literature over the last several decades, see, e.g [24–28], it is perhaps fair to say that a complete theoretical understanding of these processes is still lacking. This assumes importance in the light of the Large Synoptic Survey Telescope (LSST)\(^1\) which is expected to provide more data on TDEs in the near future.

In the context of a central mass $M$, the tidal radius $R_t \sim R_\ast (M/M_\ast)^{1/3}$, defined as the closest radial distance a stellar object of mass $M_\ast$ and radius $R_\ast$ can exist without getting tidally disrupted, is ubiquitous. This “$1/3$ law” is a standard textbook result where one uses the fact that at the tidal disruption limit, the tidal force equals the force due to self gravity at the surface of a star. This Newtonian result is true for any central mass $M$, regardless of the geometry that it produces, and it is perfectly legitimate to apply this to solutions of GR other than BHs, which might not have any obvious interpretation as a singularity covered by an event horizon. Indeed, while TDEs are believed to be common near galactic centers, relatively less attention has been paid in the literature to the fact that such objects that do not have an event horizon (i.e. are not black holes) can also tidally disrupt stars. One such example is provided by the wormhole (WH) geometry. Wormholes are exotic solutions of GR that connect two different universes or two distant regions of the same universe via a throat, and continue to attract much attention, many decades after their inception. The original idea of

\(^1\)See https://www.lsst.org.
the wormhole was related to a topology change of space-time by [29], and several no attempts
to understand the formation of such wormholes as a result of a phase transition were made in
the early eighties by [30–32]. A traversable wormhole is one in which material particles can
tunnel through the throat, and [33] demonstrated that the Schwarzschild wormhole (or the
Einstein-Rosen bridge) is not traversable. A detailed construction of a traversable wormhole
appeared in the work of [34] and soon after, the construction of a “time machine” based on
this idea appeared in the works of [35], and [36]. For an excellent exposition to related details,
see the book by [37] (see also the work of [38] for related literature). It is well known that
typically the matter required to support wormhole geometries will violate standard energy
conditions in GR ([34]). Various attempts have however been made to evade such violations
and as is well known by now, dynamical solutions ([39]) or modified gravity ([40, 41]) might
offer scenarios in which such a situation might be possible to envisage.

Quite naturally, wormholes continue to be a popular theme ever since these works,
both at the technical level and in popular imagination, and as such are often treated at the
same level as horizonless astrophysical objects. Indeed, a large amount of recent literature
concerns astrophysical signatures of wormholes. For example, gravitational microlensing from
wormholes were considered by [42, 43], while [44] considered the cosmological constraints on
these. Strong gravitational lensing by wormholes were recently studied in [45] and [46].
Accretion disk properties of wormholes have been studied in [47, 48], and the more recent
review of [49].

Part of the reason that wormholes are exciting is that these can effectively mimick black
holes. This was realized first by [50]. These authors pointed out that several detectable fea-
tures of black holes, like quasi-normal modes and accretion properties as well as theoretical
features like no-hair theorems can be very closely mimicked by wormhole solutions as well.
In fact, after LIGO ([51]) announced the first evidences of gravitational wave detection, [52]
reported that a class of wormholes that have a thin shell of matter (with a specific equa-
tion of state) in the throat region can exhibit entirely similar quasi-normal mode ringing at
early times compared to black holes, with differences being clear only at late times. Subse-
tsequently, [53] showed that specific wormhole solutions can in fact exhibit quasi-normal mode
properties that are similar to, or different from, black holes at all times. Clearly then, it is
important and interesting to further the study of similarities between black holes and their
wormhole mimickers, and in this work we study the differences in their TDEs.

As we have mentioned, our understanding of TDEs is far from complete. The main rea-
son for this is the complicated nature of TDEs which rule out an exact theoretical treatment,
and one has to resort to various approximations and costly numerical schemes. A part of
the difficulty comes in due to the fact that one has to factor in effects of general relativity
(GR) if stars come “close enough” to a BH ([10]). To get an estimate of the numbers and
the approximations involved, let us consider a typical $10^6 M_\odot$ BH at the galactic center. The
Schwarzschild radius for such a BH is about a solar radius, $R_\odot$. By a conservative estimate,
we can assume that GR effects will be small beyond $\sim 10R_\odot$, in which region one can safely
resort to a Newtonian approximation. Then, the innermost stable circular orbit of the BH
being at $6R_\odot$, such effects will be important if the TDE occurs in this BH background, be-
tween $6R_\odot$ and $10R_\odot$. For more general parabolic or highly elliptical orbits, the periastron
position in Schwarzschild black hole (SBH) backgrounds can be as small as $4R_\odot$, providing
even further room for the effects of GR to come into the picture. Such effects of GR on tidal
disruption have been studied in several recent works in the context of BHs (see, e.g [54–58]).
The question that we ask here is related to such effects in WH geometries, which are rela-
tively less studied. Indeed, far from a BH or a WH, there is no way to distinguish them from their TDEs, as these are effectively Newtonian, and hence the tidal radius will follow the power law discussed earlier. However, near the WH throat, something different is expected to happen. This is because near the throat of a WH, gravity is essentially repulsive in order to sustain the throat region, i.e. to prevent it from collapsing. We therefore expect that close to the throat, WHs should behave very differently from BHs. It is in this region that GR becomes important, and hence one has to take care of GR effects carefully.

In this paper, we point out that GR might modify the tidal radius exponent, so that a more generic relation \( R_t/R_* = \alpha (M/M_*)^\beta \), with \( \beta \neq 1/3 \) is a more appropriate working hypothesis in the region where GR effects are important. The quantity \( R_* \) in this relation refers to the radius of the star in the absence of strong gravity. What we find from a numerical analysis is that the index \( \beta \), as well as the proportionality constant \( \alpha \) have different values for black hole and wormhole geometries. While for BHs, \( \beta < 1/3 \), for WHs on the other hand, \( \beta > 1/3 \). The index \( \beta \) plays a crucial role in the dynamics of the tidally disrupted matter. As is known, after a TDE, roughly half of the stellar mass remains bound, and starts coming back to the pericenter after a time \( t_{\text{min}} \propto (\Delta \epsilon)^{-3/2} \), where \( \Delta \epsilon \) is the dispersion in the energy of the tidal debris. Now, \( \Delta \epsilon \sim r_t^{-2} (28) \), so clearly any change in the index \( \beta \) from its Newtonian value will be important in determining the accretion rate. In this paper, we perform a numerical analysis adopting the method of [55] and determine the index \( \beta \). This is then used to determine the peak accretion rate, and we show how differences with the Newtonian limit arise for stars that are tidally disrupted at distances close to the black hole event horizons or wormhole throats. Further, we study the rates of tidal disruptions in BH and WH backgrounds. These are also different due to reasons discussed above, and we quantify them in specific backgrounds.

This paper is organized as follows. In the next section, we will briefly review the relevant formalism and some of the main consequences of the deviation from the Newtonian power law. In section 3, we present our numerical analysis and determine this deviation, and revisit the consequences that it has on TDEs in black hole and wormhole backgrounds. Section 4 ends this paper with a summary of the main results.

2 Methodology

We will now discuss the methodology to be followed in this paper. These consist of three main ingredients, the space-time metric, the Fermi-Normal frame, and the numerical method to compute the tidal disruption radius.

2.1 Space-time metrics

The generic rotating vacuum solution to Einstein’s field equations is the Kerr black hole (KBH), represented by the axially-symmetric metric written in Boyer-Lindquist coordinates,

\[
ds^2 = - \left(1 - \frac{2GMr}{c^2\Sigma} \right) c^2 dt^2 - \frac{4GMra \sin^2 \theta}{c\Sigma} dtd\phi + \frac{\Sigma}{\Delta} dv^2 + \Sigma d\theta^2 + \left( r^2 + a^2 \right) \sin^2 \theta d\phi^2 ,
\]

where \( \Sigma = r^2 + a^2 \cos^2 \theta \) and \( \Delta = r^2 + a^2 - 2GMr/c^2 \). Here, \( M \) is the ADM mass, and with \( J \) being the angular momentum of the KBH, \( a = \frac{J}{\mathcal{M}} \) represents the spin parameter such
that $0 \leq a \leq M$. The $a = 0$ limit of the Kerr metric yields the static, spherically symmetric Schwarzschild black hole (SBH) given by the metric

$$ds^2 = -(1 - \frac{2GM}{c^2 r})c^2 dt^2 + \frac{1}{1 - \frac{2GM}{c^2 r}} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 .$$

(2.2)

On the other hand, the quintessential example of a static traversable wormhole of the Morris-Thorne type is exemplified by the metric

$$ds^2 = -e^{2\Phi(r)}c^2 dt^2 + \frac{1}{1 - \frac{b(r)}{r}} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) .$$

(2.3)

The example that we consider here is the exponential wormhole (EWH) given by the functions

$$e^{2\Phi(r)} = e^{-\frac{2GM}{c^2 r}}, \quad b(r) = \frac{2GM}{c^2},$$

(2.4)

where $M$ is the ADM mass of the EWH.

A useful comparison of TDEs in black hole and wormhole backgrounds can be made by considering the SBH and the EWH, and fixing their masses $M$ to be the same. In addition, the position of the throat in the latter geometry is at $r_{th} = \frac{2GM}{c^2}$, which is the same location as the Schwarzschild radius of the SBH. Such a consideration ensures that the differences are coming solely from their respective geometries. From a Newtonian perspective, an object that is captured by a SBH will tunnel through the throat of an EWH of the same mass.

### 2.2 The Fermi-normal frame

We will consider parabolic (or highly elliptical) orbits in the backgrounds of the metrics mentioned above, in Fermi-Normal (FN) coordinates. The general formalism for space-times with spin is a convenient beginning point, from which static results can be obtained in the limit that the spin parameter goes to zero. For stationary backgrounds, we will restrict only to the equatorial plane, where due to the axial symmetry, all the metric components are functions of $r$ only. A star moving in a parabolic orbit close to the horizon of a BH or the throat of a WH will be influenced by the tidal fields of the BH and WH, respectively. GR effects are most conveniently studied by introducing a locally flat frame, called the Fermi Normal frame ([59, 60]), that can move with the star along the geodesic where the time-like basis vector lies along the 4-velocity. The three other space-like vectors would be directed perpendicular to the 4-velocity. This way it is possible to describe the inhomogeneous nature of gravity namely the tidal fields, in terms of the flat space-like coordinates. Exactly on the geodesic, the tidal force is zero. As one moves away perpendicular to the geodesic, the tidal fields increases.

We will here display the FN coordinates for equatorial parabolic orbits in general spherically symmetric, static backgrounds, for simplicity, so that we can directly use them for both SBH and EWH. The same can be constructed for such orbits in the Kerr geometry as well [60], and we will present numerical results on these later. On the equatorial plane, a general spherically symmetric, static metric can be written as

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + C(r)d\phi^2 .$$

(2.5)

Equatorial parabolic orbits specify the energy $E$ and angular momentum $L$ of a test particle as

$$E = 1, \quad L = \sqrt{\frac{C(1 - A)}{A}}$$

(2.6)
Now the Fermi Normal tetrad frame for an equatorial time-like geodesic can be written as

\[
\begin{align*}
\lambda_0^\mu &= \left\{ \frac{E}{A}, \frac{P}{\sqrt{B}}, 0, \frac{L}{C} \right\} \\
\lambda_1^\mu &= \left\{ \sqrt{BCP \cos \Psi}, -\frac{EL \sin \Psi}{\sqrt{A(C+L^2)}}, \frac{E \sqrt{AC \cos \Psi}}{\sqrt{A(C+L^2)}}, -\frac{LP \sin \Psi}{\sqrt{B(L^2+C)}}, 0, -\frac{L \sqrt{L^2+C} \sin \Psi}{LC} \right\} \\
\lambda_2^\mu &= \left\{ 0, 0, \frac{1}{\sqrt{C}}, 0 \right\} \\
\lambda_3^\mu &= \left\{ \sqrt{B \sqrt{A(C+L^2)}}, \frac{EL \cos \Psi}{\sqrt{A(C+L^2)}}, \frac{E \sqrt{AC \sin \Psi}}{\sqrt{A(C+L^2)}}, \frac{LP \cos \Psi}{\sqrt{B(L^2+C)}}, 0, \frac{L \sqrt{L^2+C} \cos \Psi}{LC} \right\}
\end{align*}
\]  

(2.7)

where we have

\[
P = \sqrt{\frac{E^2}{A} - \frac{L^2}{C} - 1}, \quad \frac{d \Psi}{d \tau} = \frac{ELC'}{2 \sqrt{ABC(C+L^2)}}
\]  

(2.8)

The angle \( \Psi \) is introduced in order to parallel transport \( \lambda_1^\mu \) and \( \lambda_3^\mu \) along the time-like geodesic.

### 2.3 Numerical procedure

The components of the Riemann tensor \( R_{\mu\nu\sigma\delta} \) and its covariant derivatives \( R_{\mu\nu\sigma\alpha\beta} \) and \( R_{\mu\nu\sigma\delta;\alpha\beta} \) are first calculated from standard formulae (see, e.g., [61]) using the metrics given in equation (2.1), (2.2) or (2.3). The components of these tensors are then computed in the FN frame using the FN tetrad \( \lambda_0^\mu \) (equation (2.7)) as,

\[
\begin{align*}
R_{abcd} &= R_{\mu\nu\sigma\delta} \lambda_a^\mu \lambda_b^\nu \lambda_c^\sigma \lambda_d^\delta \\
R_{abcede} &= R_{\mu\nu\sigma\alpha\beta} \lambda_a^\mu \lambda_b^\nu \lambda_c^\sigma \lambda_d^\alpha \lambda_e^\beta \\
R_{abcede} &= R_{\mu\nu\sigma\alpha\beta} \lambda_a^\mu \lambda_b^\nu \lambda_c^\sigma \lambda_d^\alpha \lambda_e^\gamma \lambda_f^\beta
\end{align*}
\]  

(2.9)

where, \( a, b, c \ldots \) run from 0, \( \cdots \), 4, and the symbol ‘;’ indicates the covariant derivative. As shown in [55], the tidal potential in the FN frame can be expressed up to fourth order in FN coordinates \( \{ x^0 = \tau, x^1, x^2, x^3 \} \) as,

\[
\begin{align*}
\phi_{\text{tidal}} &= \frac{1}{2} C_{ij} x^i x^j + \frac{1}{6} C_{ijk} x^i x^j x^k \\
&\quad + \frac{1}{24} \left[ C_{ijkl} + 4 C_{(ij} C_{kl)} - 4 B_{(kl)\{i} B_{i\}j\} \right] x^i x^j x^k + O(x^5),
\end{align*}
\]  

(2.10)

where the tensorial coefficients are defined as

\[
C_{ij} = R_{00ij}, \quad C_{ijk} = R_{0((i0)j};k), \quad C_{ijkl} = R_{0((i0)j;k)l}, \quad B_{ijk} = R_{k(ij)0}.
\]  

(2.11)

Here, the Latin indices \( i, j, k \ldots \) run over the spatial components 1 to 3. Moreover, \( R_{0((im)j);kl} \) is a summation over all possible permutations of the indices \( i, j, k, l \) with \( m \) fixed at its position, and then division by the total number of such permutations. Equations (2.9) are used in equations (2.11) which are used, thereafter, in equation (2.10). For orbits where the stellar radius is comparable to the tidal radius, the third and fourth order corrections become important. We consider a fluid star in the FN frame. Since the frame provides a locally flat surroundings around the star, it can be described by Newtonian gravity ([62]).
The hydrodynamic equation for this fluid star in presence of the tidal potential can then be expressed as
\[
\rho \frac{\partial v_i}{\partial \tau} + \rho v^j \frac{\partial v_i}{\partial x^j} = -\frac{\partial P}{\partial x^i} - \rho \frac{\partial (\phi + \phi_{\text{tidal}})}{\partial x^i} + \rho \left[ v^j \left( \frac{\partial A_j}{\partial x^i} - \frac{\partial A_i}{\partial x^j} \right) - \frac{\partial A_i}{\partial \tau} \right]
\]  
(2.12)
where the density, three-velocity \((\frac{dx^i}{d\tau})\) and pressure of the fluid are denoted by \(\rho\), \(v^i\) and \(P\), respectively. The last term on the right hand side of equation (2.12) arises due to the gravito-magnetic force ([63]), and the corresponding vector potential reads, \(A_i = \frac{2}{3}B_{ijk}x^j\). Moreover, \(\phi\) represents the Newtonian self-gravitational potential of the star, which obeys the usual Poisson equation,
\[
\nabla^2 \phi = 4\pi G \rho .
\]
(2.13)
The co-rotational velocity field of the fluid star in the FN frame can be assumed as
\[
v^i = \Omega \left[-\{x^3 - x_c \sin \Psi\}, 0, \{x^1 - x_c \cos \Psi\}\right],
\]
(2.14)
where \(\Omega = d\Psi/d\tau\). In presence of the third order terms of the tidal potential and/or the gravito-magnetic terms, the correction term \(x_c\) is non-zero, which corresponds to the fact that the rotational axis of the fluid star is different from the \(x^2\)-axis and the position of its center of mass slightly deviates from the origin of FN frame. Its magnitude is very small compared to the radius of the star. Now, to simplify our calculation by eliminating \(\Psi\), we consider another frame in which the star is kept fixed so that the frame itself rotates with respect to the original FN frame. Coordinates of the new tilde frame are related to the FN coordinates by
\[
\tilde{x}^1 = x^1 \cos \Psi + x^3 \sin \Psi, \quad \tilde{x}^2 = x^2, \quad \tilde{x}^3 = -x^1 \sin \Psi + x^3 \cos \Psi .
\]
(2.15)
Integrating the hydro-dynamic equation (equation (2.12)) using the expression of \(v^i\) (equation (2.14)), and then converting it to the tilde coordinates \((\tilde{x}^i)\), we obtain
\[
\frac{\Omega^2}{2} \left[(\tilde{x}^1 - x_g)^2 + (\tilde{x}^3)^2\right] = h + \phi + \phi_{\text{tidal}} + \phi_{\text{mag}} + C ,
\]
(2.16)
where \(C\) is an integration constant, \(x_g = 2x_c\), and \(h = \int \frac{dP}{\rho}\). Here, \(\phi_{\text{mag}}\) is the gravito-magnetic scalar potential arising from the last term on the right hand side of equation (2.12) involving \(A_i\). Equations (2.13) and (2.16) are the two equations that we solve numerically to obtain our results. Let us now consider the polytropic equation of state of the star as, \(P = \kappa \rho^{1+1/n}\). Here, \(\kappa\) is called the polytropic constant and \(n\) is the polytropic index. The entire numerical calculation is performed in units, \(c = G = M = 1\). Moreover, we convert the basic equations (equations (2.13) and (2.16)) into dimensionless ones by using \(\tilde{x}^i = pq^i\) to obtain numerical convergence, where the constant \(p\) has dimension of length and \(q^i\)'s are dimensionless. Therefore, in terms of \(q^i\)'s, the basic equations become
\[
\nabla_q^2 \tilde{\phi} = 4\pi \rho ,
\]
(2.17)
\[
\frac{\Omega^2}{2} \rho^2 \left[(q^1 - q_g)^2 + (q^3)^2\right] = h(\rho) + \rho^2 \left( \tilde{\phi} + \tilde{\phi}_{\text{tidal}} + \tilde{\phi}_{\text{mag}}\right) + C ,
\]
(2.18)
where \(q_g = p^{-1}x_g\), \(\tilde{\phi} = p^{-2}\phi\), \(\tilde{\phi}_{\text{tidal}} = p^{-2}\phi_{\text{tidal}}\), \(\tilde{\phi}_{\text{mag}} = p^{-2}\phi_{\text{mag}}\), and \(\nabla_q\) represents the Laplacian operator in \(q^i\) coordinates. Our purpose is to solve equations (2.17) and (2.18)
iteratively to find the critical value (\(\rho_{\text{crit}}\)) of the central density \(\rho_c\) for which the fluid star just remains in stable configuration in presence of the tidal field. A star having central density (\(\rho_c\)) less than \(\rho_{\text{crit}}\) will be tidally disrupted. To obtain the numerical solution, we consider a three dimensional cubical grid system of equal grid size with \(101 \times 101 \times 101\) grid points in every direction. The results obtained using this grid system differ by less than 1% from that with grid size of \(401 \times 401 \times 401\) grid points. To find \(\rho_{\text{crit}}\), we have varied the central density \(\rho_c\) in steps of \(\delta \rho_c \leq 0.0001 \rho_{\text{crit}}\), and for each \(\rho_c\), we have used 60 iterations to solve equations (2.17) and (2.18). It is found that the result, after 60 iterations, converges to a value that differs by 0.2% from that obtained using 1000 iterations. Therefore, for the purpose of this paper, this accuracy is found to be enough. Details of the numerical procedure as well as determination of the constants, viz. \(C, p, q_g\), etc from different boundary conditions can be found in [64].

Once the critical central density (\(\rho_{\text{crit}}\)) and the corresponding density profile of the star are obtained from the numerical analysis mentioned above, we find out the critical mass of the star by integrating the density profile. Specifically, introducing the dimensionless parameters \(\Theta\) and \(\xi\), defined as, \(\rho = \rho_c \Theta^n\) and \(r = \bar{r} \xi\), where \(\rho_c\) is the central density and \(\bar{r}\) is a constant having dimension of length, the critical mass of the star comes out to be

\[
M_\star = \frac{R_\star^3}{\xi R_\star} \rho_c \left[ 4\pi \int_0^{\xi R_\star} \Theta(\xi)^n \xi^2 d\xi \right] = \frac{R_\star^3}{\xi R_\star} \rho_c \mathcal{I},
\]

where in the last relation one has to use the critical value \(\rho_{\text{crit}}\) of the central density \(\rho_c\). We tabulate the values of \(\xi R_\star\) and \(\mathcal{I}\) in table 1, for different values of \(n\). Here, \(\xi R_\star\) corresponds to the value of the Lane-Emden parameter \(\xi\) at the surface of the star. Once we calculate the critical mass of the star, we fit this to a relation of the form

\[
\left( \frac{R_\star}{R_\bullet} \right) = \alpha \left( \frac{M}{M_\star} \right)^\beta.
\]

We emphasize that all terms in equation (2.20) are in \(c = G = M = 1\) units as described above. In figure 1, we show the variation of the critical central density (\(\rho_{\text{crit}}\)) as a function of the periastron position (\(r_p\)) for both the EWH and SBH, where we have taken the masses of the EWH and SBH to be unity, and have also considered, \(R_\bullet = 1\) and the polytropic index is fixed to \(n = 1\).

We now comment on the effect of the higher order terms in the tidal potential. It is known that each higher order term in the potential is less than the previous term by a factor of \(O(R_\bullet/r)\) (this can be checked by a simple dimensional analysis, for example). This indicates that for a star with a radius comparable to the event horizon or the throat radius, the higher order terms become important. On the other hand, for stars with smaller radii, these higher order contributions are small. In the limiting case, where the star radius is very small compared to the event horizon or the throat radius, the higher order terms become insignificant. This limiting case is shown in figure 1 as the second order approximation. In

| \(n\) | \(\xi_{R_\star}\) | \(\mathcal{I}\) |
|-----|-------------|--------|
| 1   | \(\pi\)     | 39.478 |
| 3/2 | 3.654       | 34.106 |
| 3   | 6.897       | 25.362 |

Table 1. Numerical values of \(\xi_{R_\star}\) and \(\mathcal{I}\) for different \(n\).
\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.pdf}
\caption{$\rho_{\text{crit}}$ as a function of the periastron position $r_p$ in logarithmic scale for the EWH and the SBH. Here, the masses of the EWH and the SBH have been set to $M = 1$ and so is the radius of the star. The polytropic index is $n = 1$. The Newtonian power law behaviour is also shown.}
\end{figure}

Table 2. Numerical values of $\alpha(n)$ and $\beta(n)$ for different $n$, for various backgrounds and fitting ranges.

| Object          | Fitting Range | $\alpha(1)$ | $\beta(1)$ | $\alpha(1.5)$ | $\beta(1.5)$ | $\alpha(3)$ | $\beta(3)$ |
|-----------------|---------------|-------------|-------------|---------------|--------------|-------------|-------------|
| SBH (2nd)       | $4 \leq r/M \leq 12$ | 2.752       | 0.303       | 2.740         | 0.303        | 2.763       | 0.305       |
|                 | $100 \leq r/M \leq 2000$ | 2.335       | 0.333       | 2.317         | 0.333        | 2.369       | 0.333       |
| EWH (2nd)       | $2.4 \leq r/M \leq 12$ | 1.842       | 0.374       | 1.832         | 0.375        | 1.835       | 0.380       |
|                 | $100 \leq r/M \leq 2000$ | 2.318       | 0.333       | 2.317         | 0.333        | 2.369       | 0.333       |
| SBH (4th)       | $4 \leq r/M \leq 12$ | 2.866       | 0.298       | 2.852         | 0.299        | 2.862       | 0.302       |
|                 | $100 \leq r/M \leq 2000$ | 2.338       | 0.333       | 2.320         | 0.333        | 2.372       | 0.333       |
| EWH (4th)       | $3 \leq r/M \leq 12$ | 1.951       | 0.366       | 1.934         | 0.367        | 1.948       | 0.370       |
|                 | $100 \leq r/M \leq 2000$ | 2.323       | 0.333       | 2.320         | 0.333        | 2.372       | 0.333       |
| KBH (4th) (a=0.9) | $2.5 \leq r/M \leq 12$ | 2.584       | 0.319       | 2.572         | 0.319        | 2.605       | 0.321       |
|                 | $100 \leq r/M \leq 2000$ | 2.336       | 0.333       | 2.338         | 0.333        | 2.373       | 0.333       |
| KBH (4th) (a=0.999) | $2.1 \leq r/M \leq 12$ | 2.568       | 0.320       | 2.559         | 0.320        | 2.593       | 0.321       |
|                 | $100 \leq r/M \leq 2000$ | 2.336       | 0.333       | 2.338         | 0.333        | 2.373       | 0.333       |

other words, the critical central density of the star at the Roche limit approaches a limiting value (denoted by ‘2nd order’ in figure 1) as the star size decreases. However, even in such cases, there is a significant difference in the critical central density of the star in the EWH background, compared to that in the SBH one. Table 2 shows the results of the fit mentioned in equation (2.20).
| Order | $\rho_c$ | $R_t$ (N) | $R_t$ (SBH) | $R_t$ (EWH) |
|-------|---------|-----------|------------|------------|
| Second | 0.249   | 3.43      | 4.0        | 2.531      |
|        | 0.157   | 4.0       | 4.532      | 3.35       |
| Fourth | 0.223   | 3.545     | 4.24       | 3.0        |
|        | 0.157   | 4.0       | 4.70       | 3.46       |

Table 3. Numerical values of the tidal radius in units of $M = 1$ at second and fourth order with $R_*=1$.

| $M/M_\odot$ | $M_*/M_\odot$ | $R_*$ (cm) |
|-------------|----------------|------------|
| $10^8$      | 1              | $4.69 \times 10^{10}$ |
| $10^6$      | 0.7            | $1.93 \times 10^9$   |
| $10^5$      | 0.7            | $4.16 \times 10^8$   |
| 10          | 1.4            | $1.09 \times 10^6$   |
| 5           | 1.4            | $6.89 \times 10^5$   |

Table 4. Numerical values of the masses and radii of stars with $\rho_c = 0.249$.

3 Analysis

Here, we will present the implications of the discussion above. First, note that for a star to be tidally disrupted by a black hole (rather than being swallowed as a whole), we require the tidal radius to be greater than the radius of the event horizon. In contrast, for a wormhole, if the tidal radius is less than the radius of the wormhole throat, the object can tunnel through the throat into the universe on the other side. For the SBH, as already mentioned, the closest periastron position of the orbit is characterized by $r/M = 4$. From our numerical analysis, we find that for a star to be tidally disrupted at this value of the radius, one has, with $M = 1$, $\rho_c = 0.249$. The tidal disruption radius for the EWH of the same mass for the same value of $\rho_c$ is $r/M = 2.531$. These values are conveniently tabulated in table 3, where we have also shown the value of the tidal radius for which a star is disrupted at $R_t/M = 4$ as given by a purely Newtonian analysis. The differences in the values of the various columns of $R_t$ are then purely due to GR effects. The first two rows in table 3 correspond to a second order GR calculation while in the last two rows we show the results up to fourth order in the tidal potential, which is important only for systems like a SBH-neutron star binary. The value of $\rho_c$ for which tidal disruption occurs at $r/M = 4$ for the SBH gives us the maximum possible difference between effects of TDEs in SBH and EWH backgrounds. From table 3, we see that for a $10^8M_\odot$ SBH and an EWH of similar mass, the difference in the tidal disruption radius of a solar mass star is $\sim 10^{-5}$ light years. In table 4, we show the typical masses and radii of stars for which $\rho_c = 0.249$. For the last two rows in table 4, the fourth order results listed in table 2 are required.

Now, once the star is tidally disrupted, the spread of the specific energy of the debris is given from the expression of [65] as $\Delta \epsilon = GMR_*/R_t^2$. Then, one obtains the minimum time required for the debris to come back to the pericenter ([12]) as

$$t_m = \frac{2\pi GM}{(2\Delta \epsilon)^{3/2}} \propto R_t^3,$$

(3.1)
The peak fallback rate, which is proportional to $M_\ast/t_m$, can then be compared to the Eddington accretion rate, and for $R_\ast \sim R_\odot$ and $M \sim 10^6 M_\odot$, is typically a few orders of magnitude higher than the latter. Then, we see that the maximum value of the ratio

$$\frac{t_{m,\text{EWH}}}{t_{m,\text{SBH}}} = \frac{R_{t,\text{EWH}}^3}{R_{t,\text{SBH}}^3} = \left(\frac{2.53}{4}\right)^3 = 0.253.$$  \hspace{1cm} (3.2)

Denoting the peak fall back rate as $\dot{M}_p$, the maximum ratio of the rates, which is proportional to $1/t_m$ is then obtained as $\dot{M}_{p,\text{EWH}}/\dot{M}_{p,\text{SBH}} = 3.953$.

Finally, we compute the tidal disruption rate of a star by Schwarzschild black hole or exponential wormhole. We assume a Maxwellian distribution of star velocities with a given number density $n_d$ and velocity dispersion $\sigma$ far from the BH or WH so that they follow highly elliptic or parabolic orbits to reach the minimum periastron position $r_p$ near the BH or WH. In the process, stars are either tidally disrupted or directly captured by the BH or the WH depending on the $r_p$. If $r_p$ is less than the tidal radius $R_t$, it can be tidally disrupted. They are directly captured when the orbits enter the event horizon or the throat without tidal disruption. We assume that a star with a specific orbital energy $E$ and angular momentum $L$ approaches a BH or a WH. Following [15], the capture rate is given by,

$$\Gamma_{\text{cap}} = \int_0^{L_{\text{cap}}} \frac{\partial \Gamma}{\partial L} dL = \frac{\sqrt{2\pi n_d L_{\text{cap}}^2}}{\sigma},$$  \hspace{1cm} (3.3)

where, $L_{\text{cap}}$ is the specific orbital angular momentum of the parabolic orbit of a star below which it enters the event horizon or the throat. Similarly, the tidal disruption rate $\Gamma_{\text{tidal}}$ is obtained by evaluating the integration upto $L_{\text{tidal}}$, the specific orbital angular momentum below which the star enters the tidal radius $R_t$. Now, the tidal disruption event rate is obtained as, $\Gamma_{\text{TDE}} = \Gamma_{\text{tidal}} - \Gamma_{\text{cap}}$. In order to find $L_{\text{cap}}$ or $L_{\text{tidal}}$ we first choose the mass and radius of a star in units of $c = G = M = 1$, where $M$ is the BH or WH mass. Then we put these values in equation (2.19) to find out the critical central density for a given value of $n$. Once we have the critical central density, we can calculate the minimum possible $r_p$ which the star can attain without getting tidally disrupted, i.e., $r_p = R_t$. Now, we put this $r_p$ in equation (2.6) to calculate $L_{\text{tidal}}$, whereas $L_{\text{cap}}$ is the value of $L$ for which $r_p = 4M$ for the SBH and $2M$ for the EWH.

Considering a constant density $n_d = 10^5 \text{pc}^{-3}$ and $\sigma = 10^5 \text{cm/s}$, the dependence of TDE rate with $M$ is shown in figures 2 and 3. Variation of the tidal disruption event rate $\Gamma_{\text{TDE}}$ for solar mass stars, white dwarfs and neutron stars with BH or WH mass is shown. Here, the solar mass star with mass $M_\odot$ and radius $R_\odot$, white dwarf with mass $0.7 M_\odot$ and radius $10^8 \text{cm}$, and neutron star with mass $1.4 M_\odot$ and radius $10^5 \text{cm}$ are considered. The polytropic index is fixed at $n = 1.5$ for the first two cases and $n = 1$ for the last one, along with $n_d = 10^5 \text{pc}^{-3}$ and $\sigma = 10^7 \text{cm/s}$. The solid red lines correspond to the EWH and the solid black lines for the SBH. The dashed lines denote capture rate $\Gamma_{\text{cap}}$, with the same color coding.

Note that as the mass of the central object (SBH or EWH) increases, the inhomogeneity of gravity decreases in a local frame, which indicates that the star experiences a weaker tidal field. Consequently, the tidal radius approaches the event horizon or the throat. For a higher central mass, the star reaches the event horizon or the throat before it gets tidally disrupted. That is why the capture rate surpasses the tidal disruption rate as the central mass increases, and eventually, disruption rate decreases drastically. However, if the star
Figure 2. Variation of $\Gamma_{TDE}$ with M (mass of the BH or WH) for a star having (a) mass $M_{\odot}$ and radius $R_{\odot}$, and (b) a white dwarf with mass $0.7M_{\odot}$ and radius $10^9$ cm. The solid red curves represent EWH and the solid black curves correspond to SBH. The dashed lines denote the capture rate $\Gamma_{\text{cap}}$, with the same color coding. For both cases, $n = 1.5$ is used.

Figure 3. Variation of $\Gamma_{TDE}$ with M (mass of the BH or WH) for a star having mass $1.4M_{\odot}$ and radius $10^7$ cm. Here, $n = 1$. The color coding is the same as in figure 2.

gets tidally disrupted very close to the throat or at the throat, the tidal disruption event can give observable effects. The disrupted material can pass through the throat and may create accretion disks on either side of the throat. Such astrophysical phenomena may have observable signatures. A recent discussion on such scenarios can be found in [66].

For solar mass stars, the TDE rate vanishes for $M \sim 10^8 M_{\odot}$ or more. This suggests that solar mass stars are directly captured before getting tidally disrupted, for such central masses. White dwarfs are tidally disrupted if $M \sim 10^5$ or less, and neutron stars are only disrupted by solar mass BHs or WHs of $\mathcal{O}(M_{\odot})$. The capture rate for the EWH is less than that of the SBH. Quantitatively, from figures 2 (a), it can be gleaned that for a solar mass star (with polytropic index $n = 1.5$), the TDE rate vanishes for a central mass $M = 2 \times 10^9 M_{\odot}$ or more for the SBH and for $M = 3 \times 10^8 M_{\odot}$ or more for the EWH. Similarly for white dwarfs, the TDE rate vanishes for a central mass $M = 6 \times 10^5$ or more for SBH and for $M = 9 \times 10^5$ or more for the EWH. Finally, for neutron stars, the corresponding masses are $M = 5.8 M_{\odot}$ for the SBH and $M = 8.1 M_{\odot}$ for the EWH.
This is due to the fact that the capture radius \((2M)\) for exponential WH is smaller than the Schwarzschild BH \((4M)\). If we take a WH spacetime having the throat size of \(4M\), while keeping the orbital parameters \((E\) and \(L)\), at the asymptotic limit, unchanged, the TDE rate goes to zero before that of the SBH. It is expected since a star can get closer to a WH without being tidally disrupted (as \(\rho_{\text{crit}}\) is smaller than SBH in figure 1) increasing the chance of direct capture. This suggests that the background geometry can make a significant role in TDE rates. We perform these comparisons for two specific BH and WH spacetimes.

However, such a consideration does not limit the generality of our results. To make this statement precise, we will briefly discuss the Damour-Solodukhin wormhole (DSWH).

The metric for the DSWH is usually written as:

\[
ds^2 = -(1 - 2M/r + \lambda^2)dt^2 + \frac{dr^2}{1 - 2M/r} + r^2(d\theta^2 + \sin^2 \theta d\phi^2).\tag{3.4}
\]

With this form of the metric, \(t\) does not correspond to the time of an asymptotic observer. Therefore, we redefine \(dt \to dt/\sqrt{1 + \lambda^2}\) and \(M \to M(1 + \lambda^2)\), which enables us to rewrite the metric as:

\[
ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{dr^2}{1 - 2M(1 + \lambda^2)/r} + r^2(d\theta^2 + \sin^2 \theta d\phi^2),\tag{3.5}
\]

which becomes the flat Minkowski metric as \(r \to \infty\). Now, we can define a parabolic orbit of a test particle with energy \(E = 1\). Depending on the angular momentum \(L\), the orbit has different periastron position \(r_p\). The minimum possible periastron position is at \(r_p = 2M(1 + \lambda^2)\), which is also the position of the throat. Thus, the throat size depends on \(\lambda\).

For \(\lambda = 0\) we get SBH, for which minimum \(r_p = 4M\). Therefore, a star approaching the DSWH in a parabolic orbit can, in principle, come very close to the throat and then escape to infinity. We find here that the behaviour of the critical central density is qualitatively similar to the EWH presented in figure 1, i.e., the deviation from the Newtonian result is opposite in nature to the SBH and similar to the EWH.

To give some quantitative estimates, we choose \(\lambda = 1\) in equation (3.5), and use the polytropic index \(n = 1\). Then, as an example, we find that the tidal radius \(R_t\) for a star (assuming \(R_0/R_t \ll 1\) so that the second order approximation is good enough) with central density \(\rho_c = 0.045\) is at \(R_t = 4.8M\) for DSWH (with \(\lambda = 1\)), \(5.55M\) for the EWH and \(6.47M\) for the SBH. Using equation (3.2), we find that for the DSWH, the maximum value of \(\dot{M}_{p,\text{DSWH}}/\dot{M}_{p,\text{SBH}} = 2.449\), which is lower than the maximum value of \(\dot{M}_{p,\text{EWH}}/\dot{M}_{p,\text{SBH}} = 3.953\) that we found for the EWH (see discussion after equation (3.2)). Finally, we comment on the tidal disruption event rates and capture rates (a) for a star with \(M_\odot\) and radius \(R_\odot\) and (b) for a star with mass \(0.7M_\odot\) and radius \(10^9\) cm. For the DSWH with \(\lambda = 1\), we find that the TDE rate vanishes above \(10^8M_\odot\) as compared to \(1.75 \times 10^8M_\odot\) for the SBH and \(2.5 \times 10^8M_\odot\) for EWH in case (a). In case (b), the TDE rate for DSWH vanishes for \(3 \times 10^5M_\odot\) and above, for SBH it is \(5 \times 10^5M_\odot\) and for EWH \(7 \times 10^5M_\odot\).

4 Summary and discussion

Wormholes have various observational features that are similar to black holes and hence these can act as black hole mimickers. In this paper we have studied to what extent these behave differently as far as tidal disruptions are considered. This is important and interesting, as
such tidal disruptions are one of the main reasons for the formation of accretion disks. From a Newtonian perspective, the ubiquitous “1/3 law” mentioned in the introduction determines the tidal radius for a given central mass, irrespective of its geometry. We have shown here that this rule is modified by strong gravity, in a way that can serve as a distinction between BHs and WHs. The physical reason for such a modification is that the physics near the black hole event horizon and that near the wormhole throat are very different. In the latter case, to sustain the throat so that it does not collapse, gravity must behave differently from that near a black hole horizon. Indeed, the deviation from the Newtonian result for the tidal radius is seen to be of opposite signs in the two backgrounds.

These effects might be important for observational distinctions, for example from data expected from the LSST. We have considered two such effects here. Firstly, we find that the peak fall back rate for wormholes might be about 4 times higher than that of a black hole with similar mass. Secondly, the tidal disruption rates for the wormhole examples we consider are greater than those for the corresponding black holes. Our analysis here is numerical, and specialized to a static scenario. A time-dependent analysis of tidal disruptions will be a natural extension of this work.

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