Early Resolution to the Neutrino Mass Ordering?

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We hereby illustrate and numerically demonstrate via a simplified proof of concept calculation tuned to the latest average neutrino global data that the combined sensitivity of JUNO with NOvA and T2K experiments has the potential to be the first fully resolved (≥5σ) measurement of neutrino Mass Ordering (MO) around 2028; tightly linked to the JUNO schedule. Our predictions account for the key ambiguities and the most relevant ±1σ data fluctuations. In the absence of any concrete MO theoretical prediction and given its intrinsic binary outcome, we highlight the benefits of having such a resolved measurement in the light of the remarkable MO resolution ability of the next generation of long baseline neutrino beams experiments. We motivate the opportunity of exploiting the MO experimental framework to scrutinise the standard oscillation model, thus, opening for unique discovery potential, should unexpected discrepancies manifest. Phenomenologically, the deepest insight relies on the articulation of MO resolved measurements via at least the two possible methodologies matter effects and purely vacuum oscillations. Thus, we argue that the JUNO vacuum MO measurement may feasibly yield full resolution in combination to the next generation of long baseline neutrino beams experiments.

The discovery of neutrino (ν) oscillations phenomenon have completed a remarkable scientific endeavour lasting several decades that has changed forever our understanding of the phenomenology of the leptonic sector of the standard model of elementary particles (SM). A few modifications were accommodated to account for the new phenomenon [1]. This means the manifestation of massive neutrinos and leptonic mixing along with an embedded mechanism for the intrinsic difference between ν and ν due to the violation of charge conjugation parity symmetry, or CP-violation (CPV); e.g. review [2].

Neutrino oscillations imply that the neutrino mass eigenstates (ν, ν, ν) spectrum is non-zero and non-degenerate, so at least two neutrinos are massive. Each mass eigenstate (ν; with i=1,2,3) can be regarded as a non-trivial mixture of the known neutrino flavour eigenstates (ν, ν, ν) linked to the three (e, μ, τ) respective charged leptons. Since no significant experimental evidence beyond three families exists so far, the mixing is characterised by the 3×3 so called Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [3, 4] matrix, assumed unitary, thus parametrised by three independent mixing angles (θ, θ, θ) and one CP phase (δCP). The neutrino mass spectra are indirectly known via the two measured mass squared differences indicated as δm(≡ m - m) and Δm(≡ m - m), respectively, related to the ν/ν and ν/ν pairs. The neutrino absolute mass is not directly accessible via neutrino oscillations and remains unknown, despite major active research [5].

As of today, the field is well established both exper-
imentally and phenomenologically. All key parameters ($\theta_{12}$, $\theta_{23}$, $\theta_{13}$ and $\Delta m^2_{21}$, $\Delta m^2_{32}$) are known to the few percent precision. The $\delta_{\text{CP}}$ phase and the sign of $\Delta m^2_{32}$, the so called Mass Ordering (MO), remain despite existing hints (i.e. $<3\sigma$ effects). CPV arises if $\delta_{\text{CP}}$ is different from 0 or $\pm\pi$; i.e. CP-conserving solutions. The unique binary MO measurement outcome means the resolution between the normal (NMO) and inverted (IMO) mass ordering, respectively implying positive (or $\Delta m^2_{32} > 0$) or negative (or $\Delta m^2_{32} < 0$) signs. NMO implies $\nu_3$ is heavier than $\nu_2$, while IMO is the opposite. The resolution of the positive sign of $\delta m^2_{21}$ is known from solar neutrino data \[^{[6]}\;8\;9\;10\] combined with KamLAND \[^{[11]}\] thus establishing the solar large mixing angle MSW \[^{[12]}\;13\] solution.

### Mass Ordering Knowledge

This publication focuses on the global strategy to achieve the earliest and most robust MO determination scenario. MO has rich implications not only for the terrestrial oscillation experiments, to be discussed in this paper, but also from a fundamental theoretical, e.g. see review \[^{[14]}\], an astrophysical, e.g. see review \[^{[15]}\], and a cosmological, e.g. see review \[^{[16]}\], points of view. Preliminary knowledge from global data \[^{[5]}\;17\;18\;19\] implies a few $\sigma$ hints on both MO and $\delta_{\text{CP}}$, where the latest results were reported at Neutrino 2020 Conference \[^{[20]}\]. According to latest NuFit5.0 \[^{[21]}\] global data analysis, NMO is favoured up to $2.7\sigma$. However, this preference remains fragile, as it will be explained later on.

Experimentally, MO can be addressed via three very different techniques (see e.g. \[^{[22]}\] for earlier work): a) medium baseline reactor experiment (i.e. JUNO) b) long baseline neutrino beams (labelled here LBνB) and c) atmospheric neutrino based experiments. LBνB and atmospheric rely heavily on matter effects \[^{[12]}\;13\] as neutrinos traverse the Earth over long enough baselines. Anti-neutrinos exhibit the opposite effect since the planet is made of matter (not anti-matter). Instead, JUNO is the only experiment able to resolve MO via vacuum dominant oscillations, thus holding a unique insight and role in the MO world strategy. This implies complementarity and possible synergies among the different techniques. The scenario for possible discrepancies is not negligible since the experimental setups and observables are not fully redundant. Indeed, new physics may manifest as differences in the binary outcome of, at least, two well resolved MO measurements.

The relevant LBνB experiments are the running LBνB-II\(^{[2]}\) both NOvA \[^{[27]}\] and T2K \[^{[28]}\] experiments. These are to be followed up by the next generation LBνB-III with DUNE \[^{[29]}\] and the Hyper-Kamiokande (HK) \[^{[30]}\] experiments, which are expected to start taking data around 2027. The possibility of the second HK detector, in Korea, would enhance its MO determination sensitivity \[^{[31]}\]. We will here focus mainly on the immediate impact of the LBνB-II. Nonetheless, we shall highlight the prospect contributions by LBνB-III, due to their leading order implications to the MO resolution. The relevant atmospheric neutrino experiments are Super-Kamiokande \[^{[32]}\] (SK) and IceCube \[^{[33]}\] (both running) as well as future specialised facilities such as INO \[^{[34]}\], ORCA \[^{[35]}\] and PINGU \[^{[36]}\]. Compared to LBνB, one advantage is that of probing many baselines simultaneously but one disadvantage is the larger uncertainties in both baseline and energy reconstruction. The HK experiment may also offer key MO insight via atmospheric neutrinos. Contrary to all those experiments, JUNO \[^{[37]}\] relies on high precision spectral analysis with reactor neutrinos for the extraction of MO sensitivity.

Despite their different MO sensitivity potential and time schedules (discussed in the end), it is worth highlighting the complementarity of each techniques as a function of the most important neutrino oscillation unknowns today. In terms of $\theta_{23}$, the dependence is limited by the so called octant ambiguity\[^{[38]}\]. The MO sensitivity of atmospheric experiments is heavily dependent on this ambiguity solution while LBνB exhibit a smaller dependence. JUNO is totally independent; a unique asset. In terms of the unknown $\delta_{\text{CP}}$, its role in atmospheric and LBνB’s inverts, while JUNO remains again uniquely independent. This way, the MO sensitivity dependence on $\delta_{\text{CP}}$ is less important for atmospherics (i.e. washed out) but LBνB are largely handicapped by the degenerate phase-space competition to resolve simultaneously both $\delta_{\text{CP}}$ and MO. In brief, while the MO sensitivity of ORCA/PINGU swings from $3\sigma$ to about $5\sigma$ based on the value of $\theta_{23}$, LBνB-II sensitivities are effectively blinded to MO for more than half of the $\delta_{\text{CP}}$ phase-space. DUNE though has the unique ability to resolved MO, also via matter effects only, regardless of $\delta_{\text{CP}}$. Although not playing an explicit role, the constraint on $\theta_{13}$, from reactor experiments (Daya Bay \[^{[39]}\] Double Chooz \[^{[40]}\] and RENO \[^{[41]}\]), is critical for the MO (and $\delta_{\text{CP}}$) quest for both JUNO and LBνB experiments.

This publication aims to illustrate, and numerically demonstrate, via a simplified proof-of-concept estimation, the most important ingredients to reach a fully

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1. JUNO has a minor matter effect impact, mainly on the $\delta m^2_{21}$ oscillation while tiny on MO sensitive $\Delta m^2_{32}$ oscillation \[^{[23]}\].

2. The first generation LBνB-I are here considered to be K2K \[^{[24]}\], MINOS \[^{[25]}\] and OPERA \[^{[26]}\] experiments.

3. This implies the approximate degeneracy of oscillation probabilities for the cases between $\theta_{23}$ and $(\pi/4 - \theta_{23})$. 

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resolved (i.e. $\geq 5\sigma$) MO measurement strategy relying, whenever possible, only on existing (or imminently so) experiments to yield the fastest timelines\(^4\). Our approach relies on the latest $3\nu$ global data information\(^{[21]}\), summarised in Table 1 to tune our analysis to the most probable and up to date measurements on $\theta_{23}, \delta_{CP}$ and $\Delta m_{32}^2$, using only the LB$\nu$B inputs, as motivated later.

| NuFit5.0 | $\delta m_{21}^2$ | $\sin^2 \theta_{12}$ | $\sin^2 \theta_{13}$ |
|----------|--------------------|---------------------|---------------------|
| Both MO  | $7.42 \times 10^{-5}$ eV$^2$ | 0.304 | 0.0224 |
| LB$\nu$B | $\Delta m_{32}^2$ | $\sin^2 \theta_{23}$ | $\delta_{CP}$ |
| NMO      | $2.411 \times 10^{-3}$ eV$^2$ | 0.565 | $-0.91\pi$ |
| IMO      | $-2.455 \times 10^{-3}$ eV$^2$ | 0.568 | $-0.46\pi$ |

Table 1: In this work, the neutrino oscillation parameters are reduced to the latest values obtained in the NuFit5.0\(^{[21]}\), where $\Delta m_{32}^2$, $\sin^2 \theta_{23}$ and $\delta_{CP}$ were obtained by using only LB$\nu$B experiments by fixing $\delta m_{21}^2$, $\sin^2 \theta_{12}$ and $\sin^2 \theta_{13}$ to the values shown in this table.

We also aim to highlight some important redundancies across experiments that could aid not only the robustness of the MO resolution but also to exploit – likely for the first time – the MO measurements for high precision scrutiny of the standard $3\nu$ flavour scheme. This way, MO exploration might open the potential for manifestations of beyond the standard model (BSM) physics; e.g. see reviews\(^{[42, 37]}\). Our simplified approach is expected to be improvable by more complete developments; e.g. see reviews\(^{[42, 37]}\). Our simplified approach is expected to be improvable by more complete developments though are considered beyond our scope. We think those are unlikely to significantly change our findings and conclusions, given the level of precision available today. In order to better accommodate the known limitations of our approach, we have intentionally err to a conservative rationale, so that our conclusions are more likely to be only reinforced by future studies and additional information. We shall elaborate these points further during the final results discussion.

**Mass Ordering Resolution Analysis**

Our analysis relies on a simplified combination of experiments able to yield MO sensitivity intrinsically (i.e. standalone) and via an inter-experiment synergies, where the gain may be direct or indirect. The indirect gain implies that the sensitivity improvement occurs due to the combination itself; i.e. hence not accessible to neither experiment alone but caused by the complementary nature of the observables provided by the different experiments. These effects will be carefully studied as a function of the delicate dependences to ensure the most accurate prediction. The existing synergies embody a framework for powerful sensitivity boosting to yield MO resolution upon combination. To this end, we shall combine the running LB$\nu$B-II experiments together with the shortly forthcoming JUNO. The valuable additional information from atmospheric experiments will be considered qualitatively, for simplicity, only at the end during the discussion of results. Unless otherwise stated explicitly, throughout this work, we shall use only the NuFit5.0 best fit values summarised in Table 1 to guide our estimations and predictions by today’s data.

**Mass Ordering Resolution Power in JUNO**

JUNO experiment\(^{[37]}\) is one of the most powerful neutrino oscillation high precision machines as well as the first experiment able to exhibit the spectral distortion due to two simultaneous oscillations; i.e. a bi-oscillation pattern, driven by “solar” $\delta m_{21}^2$ and “atmospheric” $\Delta m_{32}^2$. The JUNO spectral distortion effects are described in Figure 1 and its data-taking is to start by late 2022\(^{[33]}\). Complementary $3\nu$ interference ef-

\(^4\)The timelines of experiments is a complex subject, as the construction schedules may delay beyond the control of the scientific teams. Our approach aims to provide a minimally timing information to contextualise the experiments but variations may be expected.
effects, such as those enabling the $\delta_{CP}$ manifestation, are exploited by the LBvB experiments. JUNO alone can yield the most precise measurements of $\theta_{23}$, $\delta m^2_{31}$ and $|\Delta m^2_{32}|$, at the level of $\leq 1\%$ precision for the first time. This implies JUNO is to lead the measurements of about half (i.e. three out of six) of the parameters in the field.

Figure 2: LBvB-II Mass Ordering Sensitivity. The Mass Ordering (MO) sensitivity of LBvB-II experiments via the appearance channel (AC), constrained to a range of $\theta_{23}$, is shown as a function of the true value of $\delta_{CP}$. The bands represent the cases where the true value of $\sin^2 2 \theta_{23}$ lies within the interval $[0.45, 0.60]$ with a relative experimental uncertainty of $2\%$. The $\sin^2 2 \theta_{23} = 0.60 \pm 0.05$ gives the maximum (minimum) sensitivity for a given value of $\delta_{CP}$. The NuFit5.0 best fitted $\sin^2 2 \theta_{23}$ value is indicated by the black dashed curves. The NMO and IMO sensitivities are illustrated respectively in the (a) and (b) panels. The sensitivity arises from the fake CPV effect due to matter effects, which are proportional to $\sin \delta_{CP}$ (i.e. one-bin counting) analysis, thus neglecting any shape-driven sensitivity gain. This approximation is particularly accurate for off-axis beams (narrow spectrum) specially in the low statistics limit where the impact of systematics remains small (here neglected). The background subtraction was accounted and tuned to the latest experiments’ data. To corroborate the accuracy of our estimate, we reproduced the LBvB-II latest results [20], as detailed in Appendix A. While NOvA AC holds major intrinsic MO information, it is unlikely to resolved ($\Delta \chi^2 \geq 25$) alone. This outcome is similar to that of JUNO. Of course, the natural question may be whether their combination could yield the full resolution. Unfortunately, as it will be shown, this is unlikely but not far. Therefore, in the following, we shall consider their combined potential, along with T2K, to provide the extra missing push. This may be somewhat counter-intuitive, since T2K has just been shown to hold very small intrinsic MO sensitivity; i.e. $\leq 4$ units of $\Delta \chi^2$. Indeed, the role of T2K, along with NOvA, has an alternative path to enhance the overall sensitivity, which is to be described next.
Synergetic Mass Ordering Resolution Power

A remarkable synergy exists between JUNO and LBνB experiments thanks to their complementarity [37]. In this case, we shall explore the contribution via the LBνB’s disappearance channel (DC); i.e. the transitions $\nu_\mu \rightarrow \nu_\mu$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$. Again, this might appear counter-intuitive, since DC is practically blinded (i.e. a <1% effect) to MO; as proved in Appendix-B. Instead, DC provides a complementary precise measurement of $\Delta m^2_{32}$. This information unlocks a mechanism, to be described below, enabling the intrinsic MO sensitivity of JUNO to be enhanced by the external $\Delta m^2_{32}$. This highly non-trivial synergy may yield a MO leading order role but introduces new dependences explored below.

Both JUNO and LBνB analyse data in the 3ν framework so they can provide $\Delta m^2_{32}$ (or $\Delta m^2_{31}$) directly as output. The 2ν approximation leads to effective observables, such as $\Delta m^2_{32}$ and $\Delta m^2_{31}$ [45] detailed in Appendix-C. The LBνB DC information precision on the $\Delta m^2_{32}$ measurement is limited by a $\delta_{CP}$-driven ambiguity. The role of this ambiguity is small, but not fully negligible and will be detailed below. The dominant LBνB-II’s precision is today ∼2.9% per experiment [48,49]. The combined LBνB-II global precision on $\Delta m^2_{32}$ is already ∼1.4% [21]. Further improvement below 1.0% appears possible within the LBνB-II era when integrating the full luminosities. An average precision of ≤0.5% is reachable only upon the LBνB-III generation. Instead, JUNO precision on $\Delta m^2_{32}$ is expected to be well within the sub-percent (<0.5%) level [57].

The essence of the synergy is here described. Upon 3ν analysis, both JUNO and LBνB experiments obtain two different values for $\Delta m^2_{32}$. Since there is only one true solution, either NMO or IMO, the other solution is thus false. The standoff ability to distinguish between those two solutions is the intrinsic MO resolution power of each experiment. The key observation tough is that the general relation between the true/false solutions is different for reactors and LBνB experiments, as illustrated in Figure 3. For a given true $\Delta m^2_{32}$, its false value, referred as $\Delta m^2_{32}$ false, can be estimated, as shown in Appendix C. Regardless, all experiments must agree on the unique true $\Delta m^2_{32}$ solution. As a consequence, the corresponding JUNO ($\Delta m^2_{32}$ false JUNO) and LBνB ($\Delta m^2_{32}$ false LBνB) false solutions will differ, if the overall $\Delta m^2_{32}$ precision allows their relative resolution. This false solution difference can be exploited as an extra dedicated discriminator characterised by the term

$$\Delta \chi^2_{\text{BOOST}} \sim \left( \frac{\Delta m^2_{32} \text{ JUNO} - \Delta m^2_{32} \text{ LBνB}}{\sigma(\Delta m^2_{32})_{\text{LBνB}}} \right)^2.$$  \hspace{1cm} (1)

![Figure 3: JUNO & LBνB Mass Ordering Synergy](image-url)

The behaviours of $\Delta \chi^2$ terms (parabolas) is shown as a function of $|\Delta m^2_{32}|$ for JUNO (black), T2K (or NOvA) and their enhanced combination (red). The $\Delta m^2_{32}$ true is fixed to the NuFit5.0 best value shown, respectively, in panels (a) and (c) for NMO and IMO. The extra gain in $\Delta \chi^\text{BOOST}$ discrimination numerically originates from the fact that the true $|\Delta m^2_{32}|$ solutions should match between JUNO (solid black vertical line) and LBνB (solid blue vertical circle), hence the false solutions (dashed vertical lines) must differ. Panels (b) and (d) illustrate this origin. The relation between true/false $\Delta m^2_{32}$ solutions is different and complementarity for JUNO and LBνB experiments. The difference is large ($\approx 1.5 \times 6m^2_{31}$) for JUNO. Instead, LBνB exhibits a smaller difference that modulates with $\delta_{CP}$. So, the relative difference between $\Delta m^2_{32}$ true and $\Delta m^2_{32}$ false is maximal (minimal) for the $\delta_{CP}$-conserving $+\pi$ ($0$) value. Hence, $\Delta \chi^2_{\text{BOOST}}$ depends on $\delta_{CP}$ and an ambiguity arises (yellow band) from the a priori different values of $\delta_{CP}$ for the true or false solutions. The T2K data (red points) contrasts the precision on $|\Delta m^2_{32}|$ now 50 as compared to needed scenarios ≤1.0% scenario (blue points and parabolas). The precision of each contribution indicated by width of the parabolas, where JUNO is fixed to the nominal value 37.
Figure 4: JUNO and LBνB Mass Ordering Synergy Dependences. The isolated synergy boosting term obtained from the combining JUNO and LBνB experiments is represented by $\Delta \chi_{\text{BOOST}}^2$, as defined in Eq. (1). $\Delta \chi_{\text{BOOST}}^2$ depends on the true value of $\delta_{\text{CP}}$ and $\Delta m_{32}^2$ precision, where uncertainties are considered: 1.0% (a), 0.75% (b) and 0.5% (c). The $\Delta \chi_{\text{BOOST}}^2$ term is almost identical for both NMO and IMO solutions. Two specific effects lead the uncertainty in the a priori prediction on $\Delta \chi_{\text{BOOST}}^2$. (I) illustrates only the ambiguity of the CP phase (yellow band) impact and (II) includes the additional impact of the ±1σ fluctuations of $\Delta m_{32}^2$, as measured by LBνB (orange band). The JUNO uncertainty on $\Delta m_{32}^2$ is considered. The grey bands shows when both effects were taken into account simultaneously. The mean value of the $\Delta \chi_{\text{BOOST}}^2$ term increases strongly with the precision on $\Delta m_{32}^2$. The uncertainties from CP phase ambiguity and fluctuation could deteriorate much of the a priori gain on the prospected sensitivities. The impact of $\Delta m_{32}^2$ fluctuations dominates, while the $\delta_{\text{CP}}$ ambiguity is only noticeable for the best $\Delta m_{32}^2$ precisions. The use of NuFit5.0 data (black point) eliminates the impact of the $\delta_{\text{CP}}$ prediction ambiguity while the impact of $\Delta m_{32}^2$ remains as fluctuations cannot predict a priori. Today’s favoured $\delta_{\text{CP}}$ maximises the sensitivity gain via the $\Delta \chi_{\text{BOOST}}^2$ term. When quoting sensitivities, we shall consider the lowest bound as the most conservative case.

This $\Delta \chi_{\text{BOOST}}^2$ term characterises the rejection of the false solutions (either NMO or IMO), including an explicit hyperbolic dependence on the overall precision on $\Delta m_{32}^2$. The derived MO sensitivity enhancement may be so strong that it can be regarded, and will be referred, as a potential boost effect in the MO sensitivity.

So, the JUNO-LBνB boosting synergy exhibits four main features illustrated in Figure 4. First, there is a major increase of the combined MO sensitivity ($\Delta \chi_{\text{BOOST}}^2 > 0$). This contribution is to be added to the intrinsic MO discrimination $\Delta \chi^2$ terms per experiment described in the previous sections. Second, a strong expected dependence on the precision of $\Delta m_{32}^2$ is present, as shown in Eq. (1). The precision in $\Delta m_{32}^2$ is typically dominated by the poorer LBνB precision as compared to JUNO. Third, the unavoidable ±1σ data fluctuations of $\Delta m_{32}^2$ can have an important impact in the boosted MO sensitivity. And fourth, an explicit dependence on $\delta_{\text{CP}}$ manifests giving rise to an ambiguity. The two latter effects are mainly relevant for a priori predictions of $\Delta \chi_{\text{BOOST}}^2$ without data.

All these effects are quantified and explained in Figure 4. The possible fluctuations due $\Delta m_{32}^2$ uncertainties and the $\delta_{\text{CP}}$ ambiguity could diminish much of the a priori boosting potential. These dependences are inherited by the complex phenomenology of LBνB, typically also suffering from a less precise outcome. The leading order effect is the uncertainty on $\Delta m_{32}^2$, referred as $\sigma(\Delta m_{32}^2)_{\text{LBνB}}$. Three cases are explored in this work 1.0%, 0.75% and 0.5%, including the simultaneous impact of the non-negligible $\delta_{\text{CP}}$ ambiguity. This ambiguity arises from the possible different $\delta_{\text{CP}}$ phases obtained for the true and false solutions, as detailed in Appendix C.

The main consequence is to limit the predictability of $\Delta \chi_{\text{BOOST}}^2$, even if the true value of CP phase was known. Its effect is not negligible when the LBνB precision on $\Delta m_{32}^2$ improves significantly (≤0.5%), as shown in Figure 4. However, the direct use of LBνB data bypasses some of the impact of this prediction limitation. This is why we adopt the NuFit5.0 latest global data, including the pertinent fluctuations. In this way, we are able to conservatively maximise the accuracy of our predictions to the most probable parameter-space, as favoured by the latest world neutrino data. This is particularly important to compute the most accurate $\Delta \chi_{\text{BOOST}}^2$ and hence the final MO determination significance. For the similar reason, our simplified formulation cannot easily account for the atmospherics data whose vast dynamic range in E/L demands a more complete treatment to be able to remain reasonably accurate.

In brief, when combining JUNO to the LBνB experiments, the overall sensitivity works as if JUNO’s intrinsic sensitivity gets boosted, via the external $\Delta m_{32}^2$ information, as illustrated and quantified in Figure 5 as a function of the precision on $\Delta m_{32}^2$ despite the sizeable impact of fluctuations. The LBνB intrinsic AC contribution will be added, as shown in the next section. However, it is via the boosting that the DC information of the LBνB’s could play a major role in the overall MO sensitivity. However, this improvement cannot manifest without JUNO – and vice versa. For a average precision on $\Delta m_{32}^2$ below 1.0%, even with fluctuations, the boosting effect can be large. A $\Delta m_{32}^2$ precision as good as >0.7% may be accessible by LBνB-II while the LBνB-
III generation is expected to go up to \( \leq 0.5\% \) level.

The measurement of \( \Delta m^2_{32} \) depends slightly \( \delta_{CP} \), which is obtained via the AC information, itself sensitivity to matter effects.

In this work, we use the terminologies, AC (appearance channel) and DC (disappearance channel) for simplicity but this does not mean that the relevant information is coming only from AC or DC, but that \( \Delta \chi^2(LBvB-AC) \) comes dominantly from LBvB AC whereas \( \Delta \chi^2(JUNO\oplusLBvB-DC) \) comes dominantly from JUNO + LBvB DC.

Figure 5: JUNO Mass Ordering Sensitivity Boosting. A major increase of JUNO intrinsic sensitivity (\( \Delta \chi^2_{JUNO} \approx 9 \)) is possible upon the exploitation of the LBvB’s disappearance (DC) characterised by \( \Delta \chi^2_{BOOST} \) depending strongly on the uncertainty of \( \Delta m^2_{32} \). Today’s NuFit5.0 average LBvB-II’s precision on \( \Delta m^2_{32} \) is \( \sim 1.4\% \). A rather humble 1.0% precision is possible, consistent with doubling the statistics, if systematics allowed. Since NOvA and T2K are expected to increase their exposures by about factors of \( \sim 3 \times \) before shutdown, sub-percent precision may also be within reach. While, the ultimate precision is unknown, we shall consider a \( \geq 0.75\% \) precision to illustrate this possibility. So, JUNO alone could yield a \( \geq 4\sigma \) (i.e. \( \Delta \chi^2 \geq 16 \)) MO sensitivity, at \( \geq 84\% \) probability, within the LBvB-II era. Fluctuations allowing, a 5\% potential may not be impossible. Similarly, JUNO may further increase in significance in order to resolve (\( \geq 5\sigma \) or \( \Delta \chi^2 \geq 25 \)) a purely vacuum oscillations MO measurement in combination with the LBvB-III’s \( \Delta m^2_{32} \) information.

Since the exploited DC information is practically blinded to matter effects\(^5\), the boosting synergy effect remains dominated by JUNO’s vacuum oscillations nature. This is why the sensitivity performance is almost identical for both NMO and IMO solutions, in contrasts to the sensitivities obtained from solely matter effects, as shown in Figure 2. This is specially noticeable for the case of atmospherics data. The case of T2K is particularly illustrative as its impact to MO resolution is via the boosting term mainly, given its small intrinsic MO information obtained by AC data. This combined MO sensitivity boost between JUNO and LBvB (or atmospherics) is likely one of the most elegant and powerful examples so far seen in neutrino oscillations and it is expected to play a major role for JUNO (always needed) to yield a leading impact on the MO quest. In fact, this effect has already been considered by JUNO to claim its possible median 4\(\sigma \) potential \(^6\) i.e. without the \( \Delta m^2_{32} \) fluctuations. Our results are fully consistent with those results, as described in Appendix D.

**Simplified Combination Rationale**

The combined MO sensitive of JUNO together with LBvB-II experiments (NOvA and T2K) can be obtained from the independent additive of each individual \( \Delta \chi^2 \). Two contributions are expected: a) the LBvB-II’s AC, referred as \( \Delta \chi^2(LBvB-AC) \) and b) the combined JUNO and LBvB-II’s DC, referred as \( \Delta \chi^2(JUNO\oplusLBvB-DC) \). All terms were described in the previous sections\(^7\). Hence the combination can be represented as \( \Delta \chi^2 = \Delta \chi^2(JUNO\oplusLBvB-DC) + \Delta \chi^2(LBvB-AC) \), illustrated in Figure 6 where the orange and grey bands represent, respectively, the effects of the \( \Delta m^2_{32} \) fluctuations and the CP-phase ambiguity. Figure 6 quantifies the MO sensitivity in significance (i.e. numbers of \( \sigma \)’s) obtained as \( \sqrt{\Delta \chi^2} \) quantified in all previous plots. Again, both NMO and IMO solutions are considered for 3 different cases for the uncertainty of \( \Delta m^2_{32} \):

**The \( \Delta \chi^2(LBvB-II-AC) \) Term:** this is the intrinsic MO combined information, largely dominated by NOvA’s AC, as described in Figure 2. The impact of T2K (\( \leq 2\sigma \)) is very small, but in the verge of resolving MO for the first time, T2K may still help here. As expected, this \( \Delta \chi^2 \) depends on both \( \theta_{23} \) and strongly on \( \delta_{CP} \), as shown in Figure 6 represented by the light green band. The complexities of possible correlations and systematics handling of a hypothetical NOvA and T2K combination are disregarded in our study and are considered integrated within the combination of the LBvB-II term, now obtained from NuFit5.0. The full NOvA data is expected to be fully available by 2024 \(^19\), while T2K will run until 2026 \(^18\), upon the beam upgrades (T2K-II) aiming for HK.

**The \( \Delta \chi^2(JUNO\oplusLBvB-DC) \) Term:** this term can be regarded itself as composed of two contributions. The first part is the JUNO intrinsic information; i.e. \( \Delta \chi^2 = 9 \pm 1 \) units after 6 years of data-taking. This contribution is independent from \( \theta_{23} \) and \( \delta_{CP} \), as shown in Figure 6 represented by the blue band. The second part is the JUNO boosting term, shown explicitly in Figure 4 including its generic dependences such as the true value of \( \delta_{CP} \). This term exhibits strong modulation with \( \delta_{CP} \) and
the uncertainty of $\Delta m_{32}^2$, as illustrated in both Figures 4 and 5. The $\Delta\chi^2$($\text{JUNO+LB\nu B-DC}$) term strongly shapes the combined $\Delta\chi^2$ curves (orange). Indeed, this term causes the leading variation across Figure 6 for the different cases of the uncertainty of $\Delta m_{32}^2$: a) 1.0% (top), which is easily reachable by LB\nu B-II, b) 0.75% (middle), which may still be reachable (i.e. optimistic) by LB\nu B-II and c) 0.5% (bottom), which is only reachable by the LB\nu B-III generation.

The combination of the JUNO along with both AC and DC inputs from LB\nu B-II experiments indeed appears on the verge of achieving the first MO resolved measurement with a sizeable probability. The ultimate significance of the combination is likely to mainly depend on the final uncertainty on $\Delta m_{32}^2$ obtained by LB\nu B experiments. The discussion of the results and implications, including limitations, is addressed in the next section.

**Implications & Discussion**

Possible implications arising from the main results summarised in Figure 6 deserved some extra elaboration and discussion for a more accurate contextualisation, including a possible timeline, as well as known limitations associated to our simplified approach. These are the main considerations:

1. **MO Global Data Trend**: today’s reasonably high significance, a priori not far the level reached by the intrinsic sensitivities of JUNO or NOvA alone, is obtained by the most recent global analysis [21] favours NMO up to 2.7$\sigma$. This significance however lowers to 1.6$\sigma$ without SK atmospherics data, thus proving their key value to the global MO knowledge today. The remaining aggregated sensitivity integrates over all other experiments. However, the global data preference is known to be somewhat fragile still between NMO and IMO solutions [17, 51, 21]. The reason behind is actually the corroboration manifestation of the alluded complementarity between LB\nu B-II and reactor [4] experiments. Indeed, while the current LB\nu B data alone favours IMO, the match in $\Delta m_{32}^2$ measurements by LB\nu B and reactors tend to match better for the case of NMO, thus favouring this solution upon combination. Hence, the MO solution currently flips due to the reactor-LB\nu B data interplay. This might happen given the sizeable $\Delta m_{32}^2$ uncertainty fluctuations, as compared to the aforementioned scenario when JUNO is on. This effect is at the heart of the described boosting mechanism and has started manifesting earlier following the expectation a priori [15]. Hence, this can be regarded as the first data-driven manifestation of the $\Delta\chi^2_{\text{BOOST}}$ effect.

![Figure 6: The Combined Mass Order Sensitivity](image_url)

The combination of the MO sensitive of JUNO and LB\nu B-II is illustrated for six difference configurations: NMO (left), IMO (right) considering the LB\nu B uncertainty on $\Delta m_{32}^2$ to 1.0% (top), 0.75% (middle) and 0.5% (bottom). The NuFit5.0 favoured value is set for $\sin^2 2\beta_3$ with an assumed 2% experimental uncertainty. The intrinsic MO sensitivity are shown for JUNO (blue) and the combined LB\nu B-II (green), the latter largely dominated by NOvA. The JUNO sensitivity boosts when exploiting the LB\nu B’s $\Delta m_{32}^2$ additional information via the via the $\Delta\chi^2_{\text{BOOST}}$ term, described in Figure 4 but not shown here for illustration simplicity. The orange and grey bands illustrate the boosting term prediction effects, respectively, the $\pm1\sigma$ fluctuation of $\Delta m_{32}^2$ and the $\delta_{\text{CP}}$ ambiguity in addition. T2K impacts mainly via the precision of $\Delta m_{32}^2$ and the measurement of $\delta_{\text{CP}}$. The combined sensitivity suggests a mean (dashed blue line) $\geq 4\sigma$ significance for any value of $\delta_{\text{CP}}$ even for the most conservative $\sigma(\Delta m_{32}^2) =1\%$. However, a robust $\geq 5.0\sigma$ significance at 84% probability (i.e. including fluctuations) seems possible, should $\delta_{\text{CP}}$ and NMO remain favoured by data, as indicated by the yellow band and black point (best fit). Further improvement in the precision of $\Delta m_{32}^2$ translates into a better MO resolution potential.

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Before JUNO starts, the reactor experiments stand for Daya Bay, Double Chooz, RENO whose lower precision on $\Delta m_{32}^2$ is $\sim$2%.
2. Atmospherics Extra Information: we did not account for atmospheric neutrino input, such as the running SK and IceCube experiments. They are expected to add valuable Δχ², though susceptible to the aforementioned θ_{23} and δ_{CP} dependences. This contribution is more complex to replicate with accuracy due to the vast E/L phase-space, hence we disregarded it in our simplified analysis. Its importance has long been proved by SK dominance of much of today’s MO information. So, all our conclusions can only be enhanced by adding the atmospheric missing contribution. Future ORCA and PINGU has the potential to yield extra MO information, while their combinations with JUNO data is being actively explored [52, 53] to yield full MO resolution.

3. Inter-Experiment Full Combination: a more complete strategy of data-driven combination between JUNO and LBνB-II experiments will be beneficial in the future. Ideally, this could be an official inter-collaboration effort to be able to carefully scrutinise the possible impact of systematics and correlations, including both experimentalists and phenomenologists. At this stage, we do not foresee a significant change in our the findings by a more complex study, including the MO discovery potential here highlighted for the first time, due to the limitations of today’s data and knowledge.

For this reason, our approach did not aim to merely demonstrate the numerical yield of the combination between JUNO and LBνB, but to illustrate and characterise the different synergies manifesting therein. So, our study was tailored to focus on the breakdown of all the relevant contributions in the specific and isolated cases of the MO sensitivity combination of the leading experiments. For example, the impact of the Δχ^{2}_{boost} was isolated while its effect is otherwise transparently accounted by a complete 3ν χ² formulation, such as done by NuFit5.0 or others similar analyses. Last, our study was tuned to the latest data in order to maximise the accuracy of the predictability, expected to be ~0.5σ around the 5σ resolution threshold.

4. Hypothetical MO Resolution Timeline: one of the main observations upon this study is that the MO could be resolved (≥5σ), maybe even comfortably, by the JUNO, NOvA and T2K combination. Considering today’s favoured δ_{CP}, the NMO solution discovery potential has a probability of ≥50% (≥84%) for a Δm^{2}_{32} precision of up to 1.0% (0.75%). In the case of the more challenging IMO, the sensitivity may reach a mean of ~5σ potential only if the Δm^{2}_{32} uncertainty was as good as ~0.75%. If so, still within the same time scale, the atmospheric data is expected to add up to enable a robust 5σ resolution for both solutions. If correct, this is likely to become the first fully resolved MO measurement tightly linked to the JUNO data timeline, as described in Figure 7, which sets the time to be around ∼2028. This possibility may be far from evident in today’s appreciation of the field.

Such a combined MO measurement can be regarded as a “hybrid” between vacuum and matter driven oscillations. In this context, JUNO and NOvA are, unsurprisingly, the driving experiments. Despite holding little intrinsic MO information, T2K plays a key role by simultaneously a) boosting JUNO via its precise measurement of Δm^{2}_{32} (the same as NOvA) and b) aiding NOvA by reducing the possible δ_{CP} ambiguity phase-space. This combined measurement relies on an impeccable 3ν data model consistency across all experiments. Possible inconsistencies may diminish the combined sensitivity.

Since our estimate has accounted for fluctuations (typically, up to ∼84% probability), those inconsistencies should amount to ≥2σ effects for them to matter. Those inconsistencies may however be the first manifestation for new physics. Hence, this inter-experiment combination has an extra relevant role: to exploit the ideal MO binary phase-space solution to test for inconsistencies that may prove the way to possible discoveries beyond today’s standard picture. Other additional contributions, such as the aforementioned atmospherics data, are expected to reinforce the significance and the model consistency scrutiny potential here highlighted.

5. Readiness for LBνB-III: in the absence of any robust model-independent for MO prediction and given its unique binary MO outcome, the articulation of at least two well resolved (i.e. ≥5σ) measurements appears critical for the sake of redundancy and consistency test across the field. In the light of the unrivalled MO resolution power of DUNE, the articulation of another robust MO measurement may be considered as a priority to make the most of DUNE’s insight.

6. Vacuum versus Matter Measurements: since all experiments but JUNO are driven by matter effects, the articulation of a competitive and fully resolved measurement via only vacuum oscillations remains an unsolved challenge to date. Indeed, boosting JUNO sensitivity alone, as described in Figures 4 and 5, to ≥5σ remains likely impractical in the context of LBνB-II, modulo fluctuations. However, this possibility is a priori numerically feasible in combination with the LBνB-III improved precision, as shown in Figure 7. The potential major improvement in the Δm^{2}_{32} precision, up to order 0.5% [29 [30], may prove crucial. Furthermore,
Figure 7: Mass Order Sensitivity & Possible Resolution Timeline. Since all the NOvA and T2K data are expected to be accumulated by ~2024 and ~2026, respectively, the timeline for the combined sensitivity follows the availability of the JUNO data. JUNO is expected to start by late 2022 reaching its statistically dominated nominal MO sensitivity (9 units of $\Delta \chi^2$) within ~6 years. We consider three possible scenarios: (left) JUNO boosted by the LBvB’s DC only (both NMO and IMO) and the JUNO combined to LBvB using both DC and AC, whose performance depends strongly on the solution: NMO (middle) and IMO (right). The evolution of the sensitivity depends largely on the boosting, as proved by considering three different $\Delta m^2_{32}$ uncertainties ($1.0\%$ black line), $0.75\%$ (red line) and $0.5\%$ (magenta points) cases. The effect of $\Delta m^2_{32}$ fluctuations is indicated (orange bands), including that of the variance due to data favoured region for $\delta_{CP}$ (green band). The JUNO intrinsic (blue) and boosted sensitivities are almost independent from NMO and IMO solutions (left). This is almost identical to the IMO case (right), as the role of LBvB-II’s AC is negligible, thus driven by boosted effect only. In both cases, the mean significance is ~$5\sigma$ (red line), hence some additional atmospheric data ($\Delta \chi^2 \geq 7$ units) should suffice to reach a robust $\geq 5\sigma$, including fluctuations and degeneracies (i.e. $\geq 84\%$ probability). Should the solution be NMO though, as somewhat favoured by global data, a robust $\geq 5\sigma$ resolution ($\geq 84\%$ probability) may be comfortably feasible even for the lowest $\Delta m^2_{32}$ precision, thanks to the extra contribution by NOvA mainly. In the LBvB-III era though, the more precise $\Delta m^2_{32}$ could boost JUNO well above the $5\sigma$ level only using Disappearance Channel. This unique possibility goes well beyond the JUNO timeline; thus scaling is irrelevant, so data points are shown instead. In this era, JUNO data could prescind from any LBvB’s AC information, thus enabling a pure vacuum oscillation fully resolved MO measurement.

The comparison between two fully resolved MO measurements, one using only matter effects and one exploiting pure vacuum oscillations is foreseen to be one of the most insightful MO coherence tests. So, the ultimate MO measurements comparison may be the DUNE’s AC alone versus JUNO boosted by HK’s $\Delta m^2_{32}$ precision, thus maximising the depth of the MO-based scrutiny by their stark differences in terms of mechanisms, implying dependences, correlations, etc. The potential for a breakthrough exists, again, should a discrepancy manifest here. The expected improvement in the knowledge of $\delta_{CP}$ by LBvB-III experiments will also play an important role to facilitate this opportunity.

This observation implies that JUNO MO capability, despite its a priori humble intrinsic sensitivity, has the potential to play a critical role throughout the history of MO explorations. Indeed, the first MO fully resolved measurement is likely to depend on JUNO sensitivity (direct and indirectly), hence JUNO should maximise ($\Delta \chi^2 \geq 9$) of its yield. However, JUNO’s ultimate aforementioned role may remain rather unaffected even by a small loss of performance, providing the overall sensitivity remains sizeable (e.g. $\Delta \chi^2 \geq 7$), as illustrated in Figures 5 and 6. This is because JUNO sensitivity could still be boosted by the LBvB experiments via their precision on $\Delta m^2_{32}$, thus sealing its legacy. There is no reason for JUNO not to perform as planned, specially given the remarkable effort for solutions and novel techniques developed for the control of spectral shape.

7. LBvB Running Strategy: since the role of these experiments to yield the maximal combined MO sensitivity is driven by both AC and DC derived informations, both channels should be considered when maximising the global MO sensitivities, as well as the usual optimisation based on $\delta_{CP}$ sensitivity. Indeed, as shown, the precision on $\Delta m^2_{32}$ could even play a more important role than the intrinsic MO resolution, based on the AC data. This is particularly crucial for T2K and HK due to their shorter baselines. So, forthcoming beam-mode running optimisation by the LBvB collaborations could, and likely should, consider that the neutrino mode is likely significantly benefit the DC outcome thanks to its larger signal rate and better signal-to-background ratio. For such considerations, our Figure 5 might offer the needed leading order guidance today.
Conclusions

Our here presented simplified proof-of-concept calculation, tuned to the latest world neutrino data via NuFit5.0, illustrates that the combined sensitivity of JUNO together with NOvA and T2K has the potential to yield the first resolved ($\geq 5\sigma$) measurement of Mass Ordering (MO). The timeline is expected to be around 2028, tightly linked to the JUNO schedule, since both NOvA and T2K data are expected to be available by 2026. Due to the absence of any a priori MO prediction and given its intrinsic binary outcome, we noted the benefit to the field to envisage at least two independent and well resolved ($\geq 5\sigma$) measurements. This is even more important in the light of the powerful outcome from the next generation of long baseline neutrino beams experiments. Such MO measurements could be exploited to over-constrain and test the standard oscillation model, thus opening for discovery potential, should unexpected discrepancies may manifest. The deepest phenomenological insight is however expected to be obtained by having two different and well resolved MO measurements based on either only matter effects and pure vacuum oscillations experimental methodologies. We here describe the feasible path to promote JUNO’s MO measurement to reach a robust $\geq 5\sigma$ resolution level without compromising its unique vacuum oscillation nature. This potential depends on the next generation of long baseline neutrino beams disappearance channel ability to reach a precision of $\leq 0.5\%$ on $\Delta m^2_{32}$.

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APPENDICES

A. Empirical Reproduction of the $\chi^2$ Function for the LBνB-II Experiments

In this section, we shall detail how we computed the number of events for T2K and NOvA. For a constant matter density, without any approximation, appearance oscillation probability for given baseline $L$ and neutrino energy $E$, can be expressed \[35\] as

\[
P(\bar{\nu}_\mu \rightarrow \nu_\mu) = a_\nu + b_\nu \cos \delta_{CP} + c_\nu \sin \delta_{CP},
\]
\[
P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = a_\nu + b_\nu \cos \delta_{CP} + c_\nu \sin \delta_{CP},
\]

where $a_\nu$, $b_\nu$, $c_\nu$, $a_\mu$, $b_\mu$, and $c_\mu$ are some factors which depend on the mixing parameters ($\theta_{12}$, $\theta_{23}$, $\theta_{13}$, $\delta m_{21}^2$ and $\Delta m_{32}^2$), $E$ and $L$. This implies that, even after taking into account the neutrino flux spectra, cross sections, energy resolution, detection efficiencies, and so on, which depend on neutrino energy, and after performing integrations over the true and reconstructed neutrino energies, the expected number of $\nu_\mu$ ($\bar{\nu}_e$) appearance events, $N_{\nu_\mu}$ ($N_{\bar{\nu}_e}$), for a given experimental exposure (running time) have also the similar $\delta_{CP}$ dependence as,

\[
N_{\nu_\mu} = n_0 + n_c \cos \delta_{CP} + n_s \sin \delta_{CP},
\]
\[
N_{\bar{\nu}_e} = \bar{n}_0 + \bar{n}_c \cos \delta_{CP} + \bar{n}_s \sin \delta_{CP},
\]

where $n_0$, $n_c$, $n_s$, $\bar{n}_0$, $\bar{n}_c$ and $\bar{n}_s$ are some constants which depend not only on mixing parameters but also on experimental setups. Assuming that background (BG) events do not depend (or depend very weakly) on $\delta_{CP}$, the constant terms $n_0$ and $\bar{n}_0$ in Eq. (3) can be divided into the signal contribution and BG one as $n_0 = n_0^{\text{sig}} + n_0^{\text{BG}}$ and $\bar{n}_0 = \bar{n}_0^{\text{sig}} + \bar{n}_0^{\text{BG}}$, as an approximation.

In Table 2, we provide the numerical values of these coefficients which can reproduce quite well the expected number of events shown in the plane spanned by $N_{\nu_\mu}^{\text{obs}}$ and $N_{\bar{\nu}_e}^{\text{obs}}$, often called bi-rate plots, found in the presentations by T2K \[48\] and NOvA \[49\] at Neutrino 2020 Conference, for their corresponding accumulated data (or exposures). We show in the left panels of Figures 8 and 9 respectively, for T2K and NOvA, the bi-rate plots which were reproduced by using the values given in Table 2. Our results are in excellent agreement with the ones shown by the collaborations \[48, 49\].

The $\chi^2$ function for the appearance channel (AC), for a given LBνB experiment, T2K or NOvA, which is based on the total number of events, is simply defined as follows, for each MO,

\[
\chi^2_{\text{LBνB}} = \min_{\theta_{23}, \delta_{CP}} \left[ \frac{(N_{\nu_\mu}^{\text{obs}} - N_{\nu_\mu}^{\text{theo}}(\sin^2 \theta_{23}, \delta_{CP}))^2}{N_{\nu_\mu}^{\text{obs}}} \right. \\
\left. + \frac{(N_{\bar{\nu}_e}^{\text{obs}} - N_{\bar{\nu}_e}^{\text{theo}}(\sin^2 \theta_{23}, \delta_{CP}))^2}{N_{\bar{\nu}_e}^{\text{obs}}} \right] + \chi^2_{\text{pull}}(\sin^2 \theta_{23}),
\]

where $N_{\nu_\mu}^{\text{obs}}$ ($N_{\bar{\nu}_e}^{\text{obs}}$) is the number of observed (or to be observed) $\nu_\mu$ ($\bar{\nu}_e$) events, and $N_{\nu_\mu}^{\text{theo}}$ ($N_{\bar{\nu}_e}^{\text{theo}}$) are the corresponding theoretically expected numbers (or prediction), and

\[
\chi^2_{\text{pull}}(\sin^2 \theta_{23}) = \frac{(\sin^2 \theta_{23}^{\text{true}} - \sin^2 \theta_{23}^{\text{obs}})^2}{\sigma^2(\sin^2 \theta_{23})}.
\]

Note that the number of events in Eq. (4) include also background events.

| MO | $n_0^{\text{sig}}$/$n_0^{\text{BG}}$ | $n_c^{\text{BG}}$/$n_c^{\text{BG}}$ | $n_s$/$\bar{n}_s$ |
|----|-------------------------------|-------------------------------|-----------------|
| T2K $\nu$ NMO | 68.6 | 20.2 | 0.2 | -16.5 |
| T2K $\bar{\nu}$ NMO | 6.0 | 12.5 | 0.2 | 2.05 |
| T2K $\nu$ IMO | 58.1 | 20.2 | 0.7 | -15.5 |
| T2K $\bar{\nu}$ IMO | 14.0 | 6.0 | 0.05 | 2.40 |
| NOvA$\nu$ NMO | 70.0 | 26.8 | 3.2 | -13.2 |
| NOvA$\bar{\nu}$ NMO | 18.7 | 14.0 | 1.3 | 3.7 |
| NOvA$\nu$ IMO | 45.95 | 26.8 | -3.25 | -10.75 |
| NOvA$\bar{\nu}$ IMO | 26.2 | 14.0 | -1.5 | 5.0 |

Table 2: NOvA and T2K Oscillation Probability Empirical Parametrisation as of Neutrino 2020 Conference. The numerical values of the factors appear in Eq. (4) are shown, which were adjusted to approximately agree with what have been presented by T2K \[48\] and NOvA \[49\]. These numbers correspond to the exposures of 2.0(1.6) $\times 10^{21}$ protons on target (POT) for $\nu$ ($\bar{\nu}$) mode of T2K and 1.4(1.3) $\times 10^{21}$ POT for $\nu$ ($\bar{\nu}$) mode of NOvA experiments. The 3 factors $n_0^{\text{sig}}$, $n_c$, and $n_s$ correspond to the case where $\sin^2 \theta_{23} = 0.55$ (0.57) for T2K (NOvA) and they scale as $n_0^{\text{sig}} \propto \sin^2 \theta_{23}$ and $n_c, n_s \propto \sin^2 2\theta_{23}$ as $\theta_{23}$ varies. The values of $\Delta m_{32}^2$ are fixed to $\Delta m_{32}^2 = 2.49(-2.46) \times 10^{-3}$ eV$^2$ for NMO (IMO) for T2K \[48\] and $\Delta m_{32}^2 = 2.40(-2.44) \times 10^{-3}$ eV$^2$ for NMO for NOvA \[49, 50\].

Using the number of events given in Eq. (3) with values of coefficients given in Table 2, we performed a fit to the data recently reported by T2K at Neutrino 2020 Conference \[48\] just varying $\sin^2 \theta_{23}$ and $\delta_{CP}$ and could reproduce rather well the $\Delta \chi^2$ presented by T2K in the same conference mentioned above, as shown in the right panel of Fig. 8. We have repeated the similar exercises also for NOvA and obtained the results, shown in the right panel of Fig. 9 which are reasonably in agreement
with what was presented by NOvA at Neutrino 2020 Conference [49]. In the case of NOvA, the agreement is slightly worse as compared to the case of T2K. We believe that this is because, for the results shown in Fig. [9] unlike the case of T2K, we did not take into account the $\theta_{23}$ constraint by NOvA (or we have set $\chi_{\text{pull}}$ in Eq. (5) equals to zero) as this information was not reported in [49].

![Figure 8: Reproduction of T2K Bi-Rate and MO Sensitivity Results as of Neutrino 2020 Conference.](image)

Figure 8: Reproduction of T2K Bi-Rate and MO Sensitivity Results as of Neutrino 2020 Conference.

We note that for this part of our analysis, we considered only the dependence of $\sin^2 \theta_{23}$ and $\delta_{CP}$ and ignore the uncertainties of all the other mixing parameters as we are computing the number of events in an approximated way, as described above, by taking into account only the variation due to $\sin^2 \theta_{23}$ and $\delta_{CP}$ with all the other parameters fixed (separately by T2K [48] and NOvA [49] collaborations) to some values which are close to the values given in Table 1.

In particular, we neglected the uncertainty of $\Delta m^2_{32}$ in the LBvB AC part analysis when it is combined with JUNO plus LBvB DC part analysis to obtain our final boosted MO sensitivities. Strictly speaking, $\Delta m^2_{32}$ must be varied simultaneously (in a synchronised way) in the $\chi^2$ defined in Eq. (6) when it is combined with the $\chi^2$ defined in Eq. (15). However, in our analysis, we simply add $\Delta \chi^2$ obtained from our simplified LBvB AC simulation which ignored $\Delta m^2_{32}$ uncertainty, to the JUNO’s boosted $\chi^2$ (described in detail in the Appendix C). This can be justified by considering that $\sim 1\%$ level (or smaller) magnitude of variations in the appearance oscillation probabilities, which would be significantly smaller than the statistical uncertainties of LBvB-II AC mode, which are expected to reach at most the level or $\sim 5\%$ or larger even in our future projections for T2K and NOvA.

For the MO resolution sensitivity shown in Figure 2 and used for our analysis throughout this work, we define the $\Delta \chi^2$ (labeled as $\chi^2_{AC}^{LBvB}$), as

$$\Delta \chi^2_{LBvB}^{AC}(\text{MO}) \equiv \pm \min_{\sin^2 \theta_{23}, \delta_{CP}} \left[ \chi^2_{LBvB}^{AC}(\text{IMO}) - \chi^2_{LBvB}^{AC}(\text{NMO}) \right],$$

where $\pm$ sign corresponds to the case where the true MO is normal (inverted), and $\chi^2_{LBvB}^{AC}$ is computed as defined in Eq. (1) but with $N_{\text{obs}}$ replaced by the theoretically expected ones for given values of assumed true values of $\theta_{23}$ and $\delta_{CP}$. In practice, since we do not consider the effect of fluctuation for this part of our analysis, $\chi^2_{LBvB}^{AC} = 0$ by construction for true MO.

![Figure 9: Reproduction of NOvA Bi-Rate and MO Sensitivity Results as of Neutrino 2020 Conference.](image)

Figure 9: Reproduction of NOvA Bi-Rate and MO Sensitivity Results as of Neutrino 2020 Conference.

For simplicity, for our future projection, we simply increase by a factor of $3 \times$ both T2K and NOvA exposures, to the coefficients given in Table 2 for both $\nu$ and $\bar{\nu}$ channels. This corresponds approximately to $8.0 \times (6.4) \times 10^{21}$ POT for T2K $\nu$ ($\bar{\nu}$) mode and to $4.1 \times (3.8) \times 10^{21}$ POT for NOvA $\nu$ ($\bar{\nu}$) mode, to reflect roughly the currently considered final exposures for T2K [57] ($\approx 10 \times 10^{21}$ POT in total for $\nu$ and $\bar{\nu}$) and NOvA [56] ($\approx 3.2 \times 10^{21}$ POT each for $\nu$ and $\bar{\nu}$). This approach...
implies that our calculation does not consider future unknown optimisations on the $\nu(\bar{\nu})$ mode running.

B. LB$\nu$B Disappearance MO Sensitivity

In the upper panel of Figure 10, we show the 4 curves of survival oscillation probabilities, $P(\nu_\mu \rightarrow \nu_\mu)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)$ for NMO and IMO, which were obtained by using the best fitted parameters in NuFit5.0 given in Table 1 for the baseline corresponds to NOvA ($L = 810$ km) and with the matter density of $\rho = 2.8$ g/cm$^3$. The NMO and IMO cases are shown, respectively, by blue and red colours whereas the cases for $\nu$ and $\bar{\nu}$ are shown, respectively, by solid thin and dashed thick curves. We observe that all of these 4 curves coincide very well with each other, so differences are very small. In the lower panel of the same Figure 10, we show the differences of these curves, between $\nu$ and $\bar{\nu}$ channels for both NMO and IMO, as well as between NMO and IMO for both $\nu$ and $\bar{\nu}$, as indicated in the legend. We observe that the differences of these oscillation probabilities are $\leq 1\%$ for the energy range relevant for NOvA.

![Figure 10: LB$\nu$B Survival Probability Mass Ordering Dependence.](image)

In this section, we shall detail the relation between true and false $\Delta m^2_{32}$ solutions in the case of JUNO and LB$\nu$B, as they are different. This different is indeed exploited as the main numerical quantification behind the $\Delta X_{\text{boost}}^2$ term.

C. Analytic Understanding of Synergy between JUNO and LB$\nu$B based experiments

C.1 JUNO Relation between True-False $\Delta m^2_{32}$

The $\bar{\nu}_e \rightarrow \nu_e$ survival probability in vacuum can be expressed as [58]

$$P_{\bar{\nu}_e \rightarrow \nu_e} = 1 - c_{13}^4 \sin^2 2\theta_{12} \sin^2 \Delta_{21} - \frac{1}{2} \sin^2 2\theta_{13}$$

$$\times \left[ 1 - \sqrt{1 - \sin^2 2\theta_{12} \sin^2 \Delta_{21} \cos(2\Delta_{ee} \pm \phi) \right],$$

where the notation $c_{ij} \equiv \cos \theta_{ij}$ and $s_{ij} \equiv \sin \theta_{ij}$ is used, and $\Delta_{ij} \equiv \Delta m^2_{ij} L/4E$, $L$ and $E$ are, respectively, the baseline and the neutrino energy, and the effective mass squared difference $\Delta m^2_{ee}$ is given by [55]

$$\Delta m^2_{ee} \equiv c_{12}^2 \Delta m^2_{31} + s_{12}^2 \Delta m^2_{32} = \Delta m^2_{32} + c_{12}^2 \Delta m^2_{21},$$

and $\phi$ is given by

$$\tan \phi = \frac{c_{12}^2 \sin(2s_{12}^2 \Delta_{21}) - s_{12}^2 \sin(2c_{12}^2 \Delta_{21})}{c_{12}^2 \sin(2s_{12}^2 \Delta_{21}) + s_{12}^2 \sin(2c_{12}^2 \Delta_{21})},$$

where $\phi \simeq 0.36$ radian $\simeq 0.11\pi$ for $s_{12}^2 = 0.304$ and $\delta m^2_{21} = 7.42 \times 10^{-5}$ eV$^2$. The $\pm(-)$ sign in front of $\phi$ in Eq. (7) corresponds to the normal (inverted) mass ordering.

Upon data analysis, JUNO will obtain two somewhat different values of $\Delta m^2_{32}$ corresponding to NMO and IMO, which we call $\Delta m^2_{32}^{\text{JUNO NMO}}$ and $\Delta m^2_{32}^{\text{JUNO IMO}}$ where one of them should correspond (or closer) to the true solution. It is expected that by considering $\Delta_{ee}^{\text{NMO}} + \phi = \Delta_{ee}^{\text{IMO}} - \phi$, they are approximately related by

$$\Delta m^2_{32}^{\text{JUNO IMO}} \simeq -\Delta m^2_{32}^{\text{JUNO NMO}} - 2c_{12}^2 \delta m^2_{21} - \delta m^2_{21}^2,$$  (10)
where the approximated value of $\delta m_\odot^2$ can be estimated by choosing the average representative energy of reactor neutrinos ($\sim 4$ MeV) as

$$\delta m_\odot^2 \equiv \frac{4E}{L} \phi \simeq 2.1 \times 10^{-5} \left( \frac{E}{4 \text{ MeV}} \right) \text{eV}^2. \quad (11)$$

We found that for a given assumed true value of $\Delta m_{32}^2 = 2.411 \times 10^{-3} \text{ eV}^2$ (corresponding to NMO), we can reproduce very well the false value of $\Delta m_{32}^2 = -2.53 \times 10^{-3} \text{ eV}^2$ (corresponding to IMO) obtained by a $\chi^2$ fit if we use $E = 4.4 \text{ MeV}$ in Eqs. (10) and (11). The relation between true and false $\Delta m_{32}^2$ for JUNO is illustrated by the vertical black dashed and black solid lines in Fig. 3 (b) and (d).

C.2 LBνB Relation between True-False $\Delta m_{32}^2$

It is expected that for LBνB based experiments like T2K and NOvA whose $L/E$ is adjusted to around the first oscillation maximum, such that $|\Delta s_{31}| \sim |\Delta s_{32}| \sim \pi/2$, only from the disappearance channel, $\nu_\mu \to \nu_\mu$ and $\nu_\mu \to \nu_\tau$, one could determine precisely the effective mass squared difference called $\Delta m_{\mu\tau}^2$ independent of the MO, which can be expressed, as a very good approximations, in terms of fundamental mixing parameters as [45],

$$\Delta m_{\mu\tau}^2 = \Delta m_{32}^2 + (s_{12}^2 + \cos \delta_{CP}s_{13}s_\mu \sin 2\theta_{12} \tan \theta_{23})\delta m_{21}^2. \quad (12)$$

From this relation, one can extract two possible values of $\Delta m_{32}^2$ corresponding to two different MO as

$$\Delta m_{32}^{\text{MO}}_{LB\nu B} = +(-1)\Delta m_{\mu\tau}^2 - (s_{12}^2 + \cos \delta_{CP}s_{13}s_\mu \sin 2\theta_{12} \tan \theta_{23})\delta m_{21}^2, \quad (13)$$

where superscript MO implies either NMO or IMO, and + and - sign correspond, respectively, to NMO and IMO. Note that when the fit is performed assuming LBνB, the best fitted mixing parameters are, except for solar parameters $\theta_{12}$ and $\delta m_{21}^2$, can be different. This relation can be rewritten as

$$\Delta m_{32}^{\text{MO}}_{LB\nu B} = -\Delta m_{32}^{\text{NMO}}_{LB\nu B} - \Delta m_{\mu\tau}^2 \{2s_{12}^2 + \sin 2\theta_{12} \cos \delta_{CP}s_{13}s_\mu \tan \theta_{23} \tan \theta_{23} \tan \theta_{23} \cos \delta_{CP} + \cos \delta_{CP}s_{13}s_\mu \tan \theta_{23} \tan \theta_{23} \tan \theta_{23} \cos \delta_{CP} \}, \quad (14)$$

where in the last line of the above equation, some simplifications were done based on the fact that best fitted values of $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$ in recent global analysis [21] are similar for both MO solutions. By using the relation given in Eq. (14), for a given assumed true value of $\Delta m_{32}^2$ (common for all experiments) we obtain the yellow colour bands shown in Fig. 3 (b) and (d).

C.3 Boosting Synergy Estimation

The extra synergy for MO determination sensitivity by combining JUNO and LBνB DC can be achieved thanks to the mismatch (or disagreement) of the fitted $\Delta m_{32}^2$ values for the wrong MO solutions between these two types of experiments. For the correct MO, $\Delta m_{32}^2$ values coming from different experiments should agree with each other within the experimental uncertainties but for those correspond to the wrong MO do not agree, which can be quantified and used to enhance the sensitivity as follows.

Following the procedure described in [17], we simply try to add the external information on $\Delta m_{32}^2$ coming from LBνB based experiments as a pull term as

$$\chi^2 = \chi^2_{\text{JUNO}} + \frac{\left( \Delta m_{32}^2 - \Delta m_{32}^{NMO \text{ or IMO}} \right)^2}{\sigma(\Delta m_{32}^2)_{LB\nu B}}, \quad (15)$$

where $\chi^2_{\text{JUNO}}$ implies the $\chi^2$ function for JUNO alone computed in a similar fashion as in [37], $\sigma(\Delta m_{32}^2)_{LB\nu B}$ implies the experimental uncertainty for $\Delta m_{32}^2$ to be achieved by LBνB based experiments. As typical values, we consider 3 cases $\sigma(\Delta m_{32}^2)_{LB\nu B} = 1,$ 0.75 and 0.5%.

In order to take into account the possible fluctuation of the central values of the measured $\Delta m_{32}^2_{LB\nu B}$ we define the extra boosting $\Delta \chi^2$ due to the synergy of JUNO and LBνB based experiments as the difference of $\chi^2$ defined in Eq. (15) for normal and inverted MO as,

$$\Delta \chi^2_{\text{boost}} = \pm (\chi^2_{\text{IMO}} - \chi^2_{\text{NMO}}), \quad (16)$$

where $\pm$ sign corresponds to the case where the true MO is normal (inverted). Note that in our simplified phenomenological approach (based on the future simulated JUNO data), for the case with no fluctuation, by construction, $\chi^2_{\text{NMO}}$ (IMO) = 0 for NMO (IMO).

Suppose that we try to perform a $\chi^2$ fit for the wrong MO. Let us first assume that $\sigma(\Delta m_{32}^2_{\text{JUNO}}) \ll \sigma(\Delta m_{32}^2_{\text{LB} \nu B})$ and no fluctuation for simplicity (i.e. $\chi^2_{\text{true MO}} = 0$). The first term in Eq. (15), $\chi^2_{\text{JUNO}}$, forces to drive the fitted value of $\Delta m_{32}^2$ very close to the true one favoured by JUNO or $\Delta m_{32}^2_{\text{JUNO}}$ (otherwise, $\chi^2_{\text{JUNO}}$ value increases significantly). Then the extra increase of $\chi^2$ is approximately given by the second term in Eq. (15), with $\Delta m_{32}^2$ replaced by $\Delta m_{32}^2_{\text{JUNO}}$.

$$\Delta \chi^2_{\text{boost}} \sim \left[ \frac{\Delta m_{32}^2_{\text{JUNO}} - \Delta m_{32}^2_{\text{NMO}}}{\sigma(\Delta m_{32}^2)_{LB\nu B}} \right]^2,$$

$$\sim \left[ \frac{\delta m_{21}^2 + 2\delta m_{21}^2 \cos 2\theta_{12} \sin 2\theta_{12} s_{13} \tan \theta_{23} \cos \delta_{CP}}{\sigma(\Delta m_{32}^2)_{LB\nu B}} \right]^2,$$

$$\sim 4, 9, 16,$$ respectively, for $\delta_{CP} = 0, \pm \pi/2, \pm \pi. \quad (17)$$
where the numbers in the last line were estimated for \(\sigma(\Delta m_{32}^2)_{\text{LB-B}} = 1\%\). The case where \(\delta_{\text{CP}} = \pm \pi/2\) and \(\Delta \chi^2_{\text{boost}} \sim 9\) can be directly compared with more precise results shown in Fig. 4(a), see the blue solid curve at \(\delta_{\text{true}} = \pm \pi/2\) which gives \(\Delta \chi^2_{\text{boost}} \sim 8\) which is in rough agreement. The expression in Eq. (17) is in agreement with the one given in Eq. (18) of [47] apart from the term \(\delta m^2_\phi\) which is not so large.

D. Formulation 3\(\nu\) versus 2\(\nu\) Models

In the previous discussions found in [47, 37], in order to demonstrate the boosting synergy effect between JUNO and LB\(\nu\)B experiments, the effective mass squared differences \(\Delta m_{ee}^2\) and \(\Delta m_{\mu\mu}^2\), defined respectively, in Eqs. (8) and (12) originally found in [45] were used. While we used these parameters in some intermediate steps of our computations, as described in Appendix C, we did not use these parameters explicitly in our combined \(\chi^2\) describing the extra synergy between JUNO and LB\(\nu\)B (DC) based experiments defined in Eq. (15), as well as in the final sensitivity plots presented in this paper.

![Figure 11: \(\Delta \chi^2(\text{JUNO} \oplus \text{LB}\nu\text{B-DC})\) as a Function of the Precision of \(\Delta m_{\mu\mu}^2\).](image)

Figure 11: \(\Delta \chi^2(\text{JUNO} \oplus \text{LB}\nu\text{B-DC})\) as a Function of the Precision of \(\Delta m_{\mu\mu}^2\). Expected MO sensitivity to be obtained by JUNO with external information of \(\Delta m_{\mu\mu}^2\) coming from LB\(\nu\)B experiments following the procedure described in [47, 37], are shown as a function of the precision of \(\Delta m_{\mu\mu}^2\) for \(\cos \delta_{\text{CP}} = \pm 1\) and 0. This plot is similar to Figure 7 in [47], once upgraded to the latest NuFit5.0 global data inputs. We observe that they are consistent with each other, if the curves for the \(\delta_{\text{CP}}\) values of 0° (blue) and 180° (red) were interchanged, as a result of a minor typo in legend [47].

In order to check the consistency between our work and previous studies, we have explicitly verified that the results do not depend on the parameters to be used in the analysis and in the presentation of the final results provided that that comparisons are done properly. In Figure 11, we show \(\Delta \chi^2(\text{JUNO} \oplus \text{LB}\nu\text{B-DC})\) computed by using explicitly \(\Delta m_{\mu\mu}^2\) (instead of using \(\Delta m_{32}^2\)) in our \(\chi^2\) analysis as done in [47, 37], as a function of the precision of \(\Delta m_{\mu\mu}^2\). There is general good agreement with the result shown in Figure 7 of [47], if \(\delta_{\text{CP}}\) curves for 0° and 180° were interchanged, as described in Figure 11.