Renormalized mean-field $t$-$J$ model of high-$T_c$ superconductivity: comparison with experiment

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Using an advanced version of the renormalized mean-field theory (RMFT) for the $t$-$J$ model, we examine spin-singlet superconducting (SC) state of $d_{x^2-y^2}$-symmetry. Overall doping dependence of the SC gap magnitude is in good agreement with experimental results for Bi$_2$Sr$_2$CaCu$_2$O$_{8+δ}$ (BSCCO) and La$_2-x$Sr$_x$CuO$_4$ (LSCO) compounds at the optimal doping and in the overdoped regime. We also calculate the dispersion relation for the Bogoliubov quasiparticles and compare our findings both with the angle resolved photoemission data for the cuprates, as well as with the variational Monte Carlo and other mean-field studies. Within the method proposed by Fukushima [cf. Phys. Rev. B 78, 115105 (2008)], we analyze different forms of the $t$-$J$ Hamiltonian, i.e. modifications caused by the form of exchange interaction, and by the presence of three-site terms. It is shown that although the former has a small influence, the latter suppresses strongly the superconductivity. We also analyze the temperature dependence of the gap magnitude and compare the results with those of the recently introduced finite-temperature renormalized mean-field theory (TRMFT) of Wang et al. [cf. Phys. Rev. B 82, 125105 (2010)].

I. INTRODUCTION

One of the most characteristic features of high-temperature superconductivity (SC) is that upon hole doping, with the hole concentration $x \gtrsim 0.05$, a generic antiferromagnetic Mott insulating state of e.g. La$_2$CuO$_{4-\delta}$ transforms into SC state. The latter, in turn, after reaching a maximal transition temperature at $x \approx 0.15$, disappears at the upper critical concentration $x_c \gtrsim 0.25 - 0.35$, depending on the system.$^{2,3}$ The last property is particularly surprising, since in the overdoped regime $x \gtrsim 0.15$ the system evolves gradually from a non-Fermi liquid into a quantum liquid that can be regarded as an unconventional Fermi liquid.$^{2}$ The appearance of $x_c$ may speak in favor of real-space type of pairing, as the increased hole doping reduces the pairing correlations in real space. To describe the above features the $t$-$J$ model is often invoked$^{1,2}$ and the kinetic exchange interaction is claimed to induce both antiferromagnetism and the superconductivity.$^{2,3}$

The basic question is whether within the renormalized mean-field theory (RMFT)$^{10,12}$ we can reproduce at least some of the above properties in a semiquantitative manner.$^{12,13}$ as advocated strongly$^{12,13}$ On the other hand, variational Monte Carlo (VMC) methods, within which one treats exactly the double occupancy exclusion, is known$^{14–19}$ to provide a semiquantitative description of the SC correlated state. Hence it is often regarded as being superior to any mean-field (MF) treatment. However, a proper MF approach would have important advantages over VMC. First, its results are not limited to small clusters. Second, it can offer an analytic insight into the physical contents of the model. Third, it allows for a detailed comparison with experiment or, strictly speaking, its critical assessment, as detailed below.

Numerous attempts to improve RMFT have been made$^{20–22}$ in order to take into account also intersite correlations. Here we show that within the MF renormalization scheme proposed recently by Fukushima$^{22}$ supplemented with the maximum-entropy based$^{22}$ self-consistent variational approach,$^{26,27}$ we can produce, among others, the results that are competitive to those of VMC. Specifically, the upper critical concentration $x_c$, as well as the principal features of the excitation spectrum, are shown to agree quite well with experiment in the overdoped regime. Finally, motivated by a recent paper$^{23}$ we comment on the extension of the present treatment to finite temperatures. We study a behavior of the renormalized gap magnitude as a function of the temperature, which is shown to be in quite good agreement with the classic result of the BCS theory.

II. MODEL AND METHOD

We start from $t$-$J$ model$^{5–9}$ is expressed by the following Hamiltonian

$$
\mathcal{H}_{t,J} = \hat{\mathcal{P}} \left( \sum_{\langle i,j \rangle} J_{ij} (\hat{S}_i \cdot \hat{S}_j - \frac{\epsilon_i - \epsilon_j}{4} \hat{\nu}_i \hat{\nu}_j) + c_2 \hat{H}_3 \right) \hat{\mathcal{P}}. \tag{1}
$$

The first term, $\mathcal{H}_t = \sum_{ij\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma}$ is the kinetic energy part, the second expresses the kinetic exchange, and the third the three-site terms. $\sum_{\langle i,j \rangle}$ means the summation pair of sites $\langle i,j \rangle$ (bonds). The Gutzwiller projector $\hat{\mathcal{P}} = \prod_i (1 - \hat{n}_{i\uparrow} \hat{n}_{i\downarrow})$ eliminates double occupancies in real space. Also, explicitly

$$
\hat{H}_3 = \sum_{ijk\sigma} \frac{t_{ij} t_{jk}}{U} \left( b_{i\sigma}^\dagger \hat{S}_j^y b_{k\sigma} - b_{i\sigma}^\dagger \hat{\nu}_j \hat{n}_{j\sigma} b_{k\sigma} \right), \tag{2}
$$

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with the projected fermion operators defined as: $b_{i\sigma} \equiv (1 - \hat{n}_{i\sigma})c_{i\sigma}$, $\hat{v}_{i\sigma} \equiv (1 - \hat{n}_{i\bar{\sigma}})\hat{n}_{i\sigma}$, and $\hat{S}_{i\sigma} \equiv b_{i\sigma}^\dagger b_{i\bar{\sigma}}$. Here the standard fermion creation (annihilation) operators are $c_{i\sigma}$ and $\hat{n}_{i\sigma} \equiv c_{i\sigma}^\dagger c_{i\sigma}$. Hamiltonian (1) in its complete form, i.e. with $c_1 = c_2 = 1$ is derived by applying canonical transformation to the Hubbard Hamiltonian, $\hat{H}_{\text{HF}}$, in the strong coupling ($|t| << U$) limit.

The form (2), apparently different from the usual formulation of RMFT, is fully equivalent to it (a similar approach is discussed in Ref. 23).

We solve this model on a square lattice and in the spatially homogeneous situation, with no coexisting magnetic order. The model parameters are $t_1 = t, t_2 = t'$ and $t_3 = t''$, where $s = 1, 3, 5$ corresponds to sites located at the distances $d(i, j)$ of 1, $\sqrt{2}$, and 2 lattice constants, respectively. Consequently, the following Lagrange multipliers and the corresponding mean fields are assumed as nonzero: $\chi_{ij} = \xi_{ij}$ and $\hat{n}_{ij} = \eta_{ij}$. The averages $\langle \hat{\Delta}_{ij} \rangle$ for $d(i, j) = 1$; $\eta_{ij} = n_{ij} = n/2$ and $\lambda_{ij}^s = \lambda$. In (11) we retain all the terms of the orders of $t^2/U$ and $tt'/U$, and neglect a smaller term of the order $(tt')^2/U$. Then, $\langle \hat{H}_J \rangle_C$ is obtained using the formalism of Ref. 22.

Explicitly,

$$W = \langle \hat{H}_1 \rangle_C + \langle \hat{H}_2 \rangle_C + \langle \hat{H}_3 \rangle_C = \sum_{i<j} (\tau_i \xi_j + 4(\Delta_x^2 + \xi_j)),$$

where $W_{ij} = \Delta_j c_i n_i^2/2$, and $\Lambda$ is the number of lattice sites. In effect, we have that

$$\frac{W_i}{\Lambda} = \frac{16(1 - n)}{2 - n} \left\{ \sum_{s=1,5} t_s \xi_s \left( 1 - \frac{4 \xi_s^2}{(2 - n)^2} \right) \right\},$$

$$\frac{\hat{W}_J}{\Lambda} = -4J \frac{3(\Delta_x^2 + \xi_j^2) + c_1 (1 - n)^2 (\Delta_x^2 - \xi_j^2)}{(2 - n)^2}. $$

The formula for $W_3$, containing three-site terms, is too lengthy to be reproduced here.

The next step is the diagonalization of $\hat{H}_3$ via Bogoliubov-Valatin transformation, which leads to

$$\hat{K}_\lambda = \hat{H}_\lambda - \mu \hat{N}, \quad W \equiv \langle \hat{H}_1 \rangle_C, \quad \eta_{ij} \equiv \langle \hat{n}_{ij} \rangle, \quad \chi_{ij} \equiv \langle c_{i\sigma}^\dagger c_{j\sigma} \rangle, \quad \Delta_{ij} \equiv (c_{ij}^\dagger c_{ij} - c_{ij} c_{ij}^\dagger)/2, \quad \text{and} \quad \tau_{ij} \equiv (\Delta_{ij}).$$
The present formalism, based on the maximum-entropy principle, is formally valid for arbitrary \( T > 0 \). Consequently, we have replaced pure state \( |\psi_0\rangle \) by mixed state represented by \( \hat{\rho}_\lambda \). However, the solutions obtained for non-zero, but sufficiently low \( T \), are for all practical purposes identical to those for \( T = 0 \). This situation is studied in the next Section. The separate question is that of the validity of RMFT approach at \( T > 0 \) from the point of view of physical consistency. This is analyzed in more detail in Section IV.

III. RESULTS FOR \( T \approx 0 \)

An analytical expression for \( W = \langle \hat{H}_0 \rangle_C \) allows us to make qualitative predictions before the numerical analysis is carried out. First, from (10) we expect a strong tendency to the superconductivity suppression for higher doping, as SC order leads to the band energy decrease \( \sim \Delta^2 \). On the other hand, in that regime the renormalized band energy becomes predominant over the exchange part. In effect, the normal state is favored over SC for \( x > x_c \) with \( x_c \) smaller than obtained within previous MF treatments.10-13 Second, from (7) we infer, that the influence of the \( \tilde{n}_i \nu_j / 4 \) term on \( W_{ij} \) is small except for the largest doping; this is due to the presence of the \( (1 - n)^2 \) prefactor (the other term \( \sim n^2 \) merely shifts the chemical potential). On the other hand, \( W_3 \) is multiplied only by \( (1 - n) = x \) prefactor. Consequently, for higher \( x \) this term becomes rather important, due to the number of distinct three-site terms present for a given initial site and spin direction (eight for \( d(i, j) = \sqrt{2} \), and four for \( d(i, j) = 2 \)). Also, this part of \( W \) is expected to suppress SC order, as the term \( \sim \Delta^2 \) in \( W_3 \) contains a factor, which is positive for reasonable values of other mean fields and model parameters.

We solve numerically the part \( \partial_z F = 0 \) of Eqs. (13) for the mean-fields using periodic boundary conditions on the lattice of \( \Lambda = 512^2 \) sites, to minimize finite size effects and for low temperature \( k_B T = 2 \cdot 10^{-3} J \). The solution amounts to solving simultaneously the system of five nonlinear equations using GNU Scientific Library (GSL). In the most cases we take the parameters \( |t| / J = 3 \) (corresponding to \( U / |t| = 12 \) for the Hubbard model), \( t'/t = 0 \) or \( (-0.25) \), and \( \nu'' = 0 \). Additionally, we take also \( |t| = 0.3 \) eV or \( |t| = 0.4 \) eV, which correspond roughly to the lower and the upper limits of the realistic values of this parameter, depending on the compound. Values of \( |t| \) close to 0.4 eV have been determined from the band-structure calculations,25 whereas \( |t| = 0.3 \) eV is used in Refs. 13,14. To highlight the influence of various forms of (1), the results for different values of \( c_1 \) and \( c_2 \) are analyzed. The numbers 1, 2, 3 (4, 5, 6) in Figs. 11 and Table I correspond to the three situations: \( c_1 = c_2 = 0 \) (i.e. with the \( \tilde{S}_i \cdot \tilde{S}_j \) part of the kinetic exchange only, cf. (1)), \( c_1 = 1 \) and \( c_2 = 0 \) (full form of the kinetic exchange), and \( c_1 = c_2 = 1 \) (complete form of the \( t-J \) Hamiltonian with the three-site terms included), each case taken for \( t'/t = 0 \) (\( t'/t = -0.25 \)), respectively.

In Table I we provide the equilibrium values of the mean-fields and of the Lagrange multipliers for cases 1-6 and for \( x = 0.175 \), a representative hole concentration in the overdoped regime.

| \( \varphi \) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| \( \xi_1 \) | 0.1970 | 0.1969 | 0.1990 | 0.1924 | 0.1922 | 0.1944 |
| \( \xi_2 \) | 0.0468 | 0.0465 | 0.0505 | 0.0241 | 0.0239 | 0.0225 |
| \( \xi_3 \) | -0.0080 | -0.0076 | -0.0144 | 0.0337 | 0.0340 | 0.0383 |
| \( \Delta_2 \) | 0.0687 | 0.0708 | 0.0202 | 0.0903 | 0.0919 | 0.0534 |
| \( \eta_1 \) | 0.0080 | 1.0030 | 1.2355 | 1.0031 | 0.9982 | 1.1845 |
| \( \eta_2 \) | 0.0000 | 0.0000 | 0.0803 | -0.2223 | -0.2223 | -0.2118 |
| \( \eta_3 \) | 0.0000 | 0.0000 | 0.0408 | 0.0000 | 0.0000 | 0.0404 |
| \( \gamma_3 \) | 0.1584 | 0.1665 | 0.0320 | 0.2126 | 0.2205 | 0.0834 |
| \( \bar{\mu} \) | -0.4069 | -0.4080 | -0.2935 | -0.8633 | -0.8614 | -0.9406 |

In Fig. 11 we plot the dispersion relation for the Bogoliubov quasiparticles, calculated for the parameters displayed in Table I. The influence of \( \hat{H}_3 \) on \( E_{\tilde{k}} \) is of comparable magnitude to that of having nonzero \( t' \).

Next, we discuss the doping-dependence of the renormalized SC order parameter \( \Delta_{ij}/C \equiv \Delta_C \) (cf. Eqn. (18) of Ref. 22). The numerical results confirm the above
made qualitative predictions. In Fig. 2 we plot $\Delta_C$ for the cases 1-6 specified above, as well as for $t'/t = -0.27$ (value being reasonable for BSCCO compounds) and $J/|t| = 0.3$ (curve 7). Note, that the upper critical concentration $x_c$ for the cases 4 and 5 is close to the VMC result obtained within the Hubbard model and using $|\psi\rangle$ for the corresponding model parameters. Also, nonzero value of $t'$ enhances superconductivity, in agreement with previous VMC results and other calculations. Let us emphasize again, that the presence of the $H_3$ term acts in the opposite direction. The vertical line roughly marks the boundary between under- and over-doped regimes. Importantly, for $|t|/J = 3$, $t'' = 0$ and different $t'/t$ values, $0 \leq t'/t \leq 0.25$, $x_c$ lies in the interval $0.2 \lesssim x_c \lesssim 0.35$, depending on the form of $H_{4,4}$, as illustrated in Fig. 2. As said above, the small differences between the curves 4 and 5 (as well as between 1 and 2) shows an insignificance of the term $\sim c_1$ (cf. also Table I). The results for $x_c$ are in a good overall agreement with the experimental data for the cuprates. To provide an additional support for our results, we list in Table II the values of $x_c$ as a function of $J$, for either $t'/t = -0.1$ (considered to be relevant to the LSCO compound), or $t'/t = -0.27$.

A remark is in place here. The family of curves in Fig. 2 and of $x_c$ values in Table II has the following meaning. Each $x_c$ value singles out either the choice of a model or a particular set of parameters. This detailed analysis is to illustrate that there is a clear upper critical concentration in the proper range, irrespectively of the model details or particular set of parameter values.

Table II. Upper critical concentration $x_c$ vs. $J$ for $t'/t = -0.27$ or $t'/t = -0.1$. The symbol A (B) labels the case $c_2 = 0$ and $c_1 = 0$ ($c_1 = 1$), respectively, whereas C means that $c_1 = c_2 = 1$ is taken in the computation.

| $J/|t|$         | 0.2 | 0.3 | 0.333 | 0.375 | 0.4 |
|----------------|-----|-----|-------|-------|-----|
| $t'/t = -0.1$  | A   | 0.18| 0.26 | 0.29  | 0.32| 0.33|
|                | B   | 0.2  | 0.31| 0.34  | 0.38| 0.4 |
| $t'/t = -0.1$  | B   | 0.18| 0.27| 0.3   | 0.33| 0.35|
|                | B   | 0.2  | 0.33| 0.36  | 0.4 | 0.42|
| $t'/t = -0.1$  | C   | 0.15| 0.21| 0.22  | 0.24| 0.25|
|                | C   | 0.15| 0.23| 0.26  | 0.28| 0.29|

In Fig. 3 we plot $x$-dependence of the SC gap $D_k$ for $k = (\pi, 0)$ (c.f. Eqn. 11) and compare our results with the experimental data. For the selections of $t'/t$ and $J/|t|$ as in Fig. 1 no fully satisfactory agreement with experiment is achieved in the entire range of $x$. However, the agreement with experiment is quite good for the parameters corresponding to the curves 1, 2, 4 and 5 in the overdoped regime, both for $|t| = 0.3$ eV and $|t| = 0.4$ eV. The best overall fit is achieved for the set of parameters represented by curve 7. Note, that in all the cases 1-7 the quasiparticle energies obtained here are decisively lower than those in the standard RMFT formulation. These differences are caused by both the particular selection of the renormalization scheme, as well as by the variational method we use. As a consequence, we obtain also lower values of the Fermi velocity, $v_F = |\nabla_k \xi(k)|_{k=k_F}$, calculated for the nodal $(0, 0) \rightarrow (\pi, \pi)$ direction. The lattice constant has been taken as $a_0 = 4\AA$.

The $x$-dependence of $v_F$ is detailed in Fig. 4 for the same set of parameters as in Figs. 2 and 3 for both $|t| = 0.3$ eV and $|t| = 0.4$ eV, and compared with the data discussed before. The theoretical values are still too low. Also, the $x$-dependence of both $D_{k=\pi}$ and $v_F$, obtained within the MF approaches, is stronger than observed in experiment. This feature is shared with the other mean-field approaches. However, the experimental values for BSCCO $\sim 1.5$ eV $\AA$ have also been reported and are quite close to our results. Note that the disagreement is largest in the underdoped regime (to
FIG. 3: (Color online) Doping dependences of the SC gap $D_k$ at $k = (\pi, 0)$ for cases 1-6 and for $t'/t = -0.27$, and $J/|t| = 0.3$ (filled diamonds). Large filled circles - experimental data.\(^{22}\) Note, that in contradistinction to Ref.\(^{19}\), no ad hoc introduced scaling factor $\alpha = 1/2$ is necessary to obtain a reasonable agreement in the overdoped regime, i.e. to the right of the vertical line.

the left of the vertical line).

FIG. 4: (Color online) Doping dependence of Fermi velocity in the nodal ($(0, 0) \rightarrow (\pi, \pi)$) direction. Experimental data (cf.\(^{34}\) and References therein) are marked by diamonds (YBCO), squares (LSCO) and solid circles (BSCCO).

IV. RESULTS FOR $T > 0$.

So far, we have analyzed only low temperature $\beta J = k_B T / J = 500$, practically equivalent to the true $T = 0$ situation. Obviously, it would be interesting to extend the analysis for higher temperatures, and in particular, to determine the critical temperature $T_c$ as a function of doping. Standard RMFT approach, as based on the Gutzwiller approximation (GA), is devised to study ground-state properties and as such, is not applicable directly for $T > 0$. It may seem, that having a finite-temperature formalism at our disposal, we may examine arbitrary temperature by simply changing value of $\beta$ in $\mathcal{F}(\beta)$ Eq.\(^{12}\). Unfortunately, this may lead to the situation similar to that encountered in the slave-boson mean-field theories, which application at finite temperatures is invalidated by incorrect evaluation of the entropy part of the free energy.\(^{33}\)

Recently, an attempt to extend RMFT to $T > 0$ have been made.\(^{28}\) Within this approach, termed finite temperature RMFT (TRMFT), the term

$$\Delta S = - \sum_i \left( e_i \ln \frac{e_i}{e_{i0}} + q_i \ln \frac{q_i}{q_{i0}} + d_i \ln \frac{d_i}{d_{i0}} \right) . \quad (15)$$

is added to the single-particle entropy $S_0 = -\text{Tr} \rho \ln \rho_0$ of the mean-field model (here, $\rho_0 = \rho_{s0}$). In Eq.\(^{15}\), we have $e_i = \langle \hat{E}_i \rangle_c$, $e_{i0} = \langle \hat{E}_i \rangle$, $q_i = \langle \hat{Q}_i \rangle_c$, $q_{i0} = \langle \hat{Q}_i \rangle$, $d_i = \langle \hat{D}_i \rangle_c$, and $d_{i0} = \langle \hat{D}_i \rangle$ with $E_i = (1 - \hat{n}_i \uparrow)(1 - \hat{n}_i \downarrow)$, $\hat{Q}_i = \hat{n}_i \uparrow(1 - \hat{n}_i \downarrow) + \hat{n}_i \downarrow(1 - \hat{n}_i \uparrow)$ and $\hat{D}_i = \hat{n}_i \uparrow \hat{n}_i \downarrow$. Note, that $\Delta S < 0$. Derivation of\(^{15}\), and its possible generalizations will be discussed elsewhere.\(^{34}\) Here we simply adapt Eq.\(^{15}\), which may be treated as a reasonable Ansatz, similar in spirit to the finite-temperature extensions of the Gutzwiller approximation proposed earlier.\(^{35-37}\)

Within the present formalism, $\Delta S$ can be included by replacing $W(\xi, \Delta, n) \quad (5)$ by $W - T \Delta S$. However, for the $t$-$J$ model, $d_i = 0$, and consequently, for non-magnetic, homogeneous solutions studied here, $\Delta S = \Delta S_{tJ}$ depends only on the total particle number $n$, i.e.

$$\frac{\Delta S_{tJ}}{\Lambda} = (2 - n) \ln \left( 1 - \frac{n}{2} \right) - (1 - n) \ln(1 - n). \quad (16)$$

Therefore, the presence of $\Delta S_{tJ}$ results only in different values of $\mu$, $\lambda$, and thermodynamic potentials $\Omega$ and $F$ \(^{13}\). Still, $\bar{\mu} = \mu + \lambda$, all the mean fields, and the remaining Lagrange multipliers, as well as the free energy difference between the superconducting and the normal solutions, remain unchanged. Hence, $\Delta S$ in the form\(^{15}\) does not lead to any nontrivial modifications of our original formulation.

Nonetheless, it still seems to be interesting to apply the above formalism to examine nonzero temperature situation. This would reveal limitations of the present form of RMFT and should help in formulating more satisfactory finite-temperature mean-field treatment of the $t$-$J$ model. First, in Fig.\(^{14}\) we plot temperature dependence of (renormalized) gap magnitude $\Delta_C(T)$ as a function of $T$ for selected hole concentrations. For the values of model parameters used here, we have $J = 100$ meV $\approx 1160 K$ ($J = 133$ meV $\approx 1550 K$) for $t = 300$ meV ($t = 400$ meV), respectively. Therefore, the critical temperature is underestimated by a factor 3-5 (e.g. $T_c = 340 - 450 K$ at the optimal doping and $T_c \approx 25 - 35 K$ at $x = 0.3$. This is a common feature of all mean-field type approaches, clearly caused by neglecting the fluctuations. This also
shows the insufficiency of the expression $\Delta S$ in the present $t$-$J$ model case.

Nonetheless, one interesting property of the present approach should be noted. Namely, an uncorrelated wave function $|\Psi_0\rangle$, or a related density operator $\hat{\rho}_\lambda$ has an essentially the same form as that coming from the Bardeen-Cooper-Schrieffer (BCS) theory. Therefore, it seems natural to compare our values of reduced renormalized gap magnitude $\Delta_C(T)/\Delta_C(0)$ with the standard BCS results given by $\Delta(T)/\Delta(0) = \tanh(\Delta(T)/t\Delta(0))$, where $t = T/T_C$. The results are shown in Fig. 6. Interestingly, the agreement with the BCS results is quite good, despite the renormalization of $\Delta$ and its nontrivial d-wave symmetry. Uncorrelated (bare) gap ratio $\Delta(T)/\Delta(0)$ exhibits very similar scaling with $T/T_c$.

V. SUMMARY AND OUTLOOK

In this paper, we aimed to describe the doping dependence of selected, experimentally measured quantities, we have decided to systematize the results coming from different versions of $t$-$J$ model, that are discussed in the literature. From that analysis it follows that while the presence of the term $c_1$ does not influence remarkably the results except for the largest doping $x$, the inclusion of the three-site terms ($c_1 = c_2 = 1$) reduces substantially the range in which superconductivity is present.

We have implemented the mean-field renormalization scheme to $t$-$J$ model, in which both the kinetic-exchange and the three-site terms can be taken into account. Such RMFT approach, which is based on an effective single-particle picture, is expected to be valid first of all in the overdoped regime, where an unconventional form of the Fermi liquid is obtained. The variational approach based on the maximum entropy principle has been used to determine mean-field parameters appearing in the model. The theoretical results yield a correct value for the upper critical concentration for the high-$T_c$ d-wave superconductivity disappearance.

Our method provides also lower single-particle energies compared to the previous MF results. Consequently, a good estimate of the experimentally determined gap $D_{k=(\pi,0)}$ is detected experimentally in the overdoped regime for typical values of the model parameters. However, the values of the Fermi velocity in the nodal direction are still slightly too low. We also have examined the temperature dependence of the superconducting gap magnitude and have determined the critical temperature $T_c(x)$ evolution as a function of hole concentration. As may be expected, mean-field results overestimate $T_c$ by a factor 3-5. The present results can be generalized by taking into account more complex lattice or band structure, the broken $C_{4v}$ symmetry (Pomeranchuk instability), and antiferromagnetism. Furthermore, study of the Fermi-surface-topology evolution with the doping is achievable. Finally, a more advanced scheme of calculating the entropy of the correlated state is desired to provide more satisfactory description of the nonzero-temperature situation. We should be able to see progress along these lines soon.

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