THE FAILURE OF SELF-INTERACTING DARK MATTER TO SOLVE THE OVERABUNDANCE OF DARK SATELLITES AND THE SOFT CORE QUESTION

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ABSTRACT

Self-interacting dark matter was proposed by Spergel & Steinhardt to alleviate two conflicts between cold dark matter (CDM) models and observations. First, CDM N-body simulations predict dark matter halo density profiles that diverge at the center in disagreement with the constant-density cores observed in late-type dwarf and low surface brightness galaxies. Second, N-body simulations predict an overabundance of subhalos in the Galactic halo. Using a simple semianalytical argument we show that weakly self-interacting dark matter models, which can produce halo cores of the sizes observed in dark matter dominated galaxies, are unable to reconcile the number of satellites in the Galactic halo with the observed number of dwarf galaxies in the Local Group.

Subject headings: cosmology: theory — dark matter — galaxies: dwarf — Galaxy: halo

1. INTRODUCTION

Recent improvements in observational and numerical techniques have allowed a comparison between predictions of the cold dark matter (CDM) scenario and observational data on galactic scales. The results point out discrepancies between predictions and observations. High-resolution N-body simulations have shown that, on scales comparable to the Local Group, the predicted number of subhalos is at least a factor of 10 higher than the observed number of dwarf galaxies (Klypin et al. 1999; Moore et al. 1999a). This disagreement, usually called the “satellite question,” can be attributed to the high core densities of satellite dark halos found in cosmological models (Navarro, Frenk, & White 1997; hereafter NFW). These densities, combined with a small central velocity dispersion (Fukushige & Makino 1997), tend to stabilize the satellites against tidal disruption on galactic scales. Another discrepancy emerges when comparing the density profiles of dark matter halos predicted by numerical simulations with observations of H i rotation curves in dwarf galaxies (Moore 1994; Flores & Primack 1994; Burkert 1995). Whereas observations show linearly rising rotation curves out to radii greater than 1 kpc, indicating that the dark matter has a constant-density core (soft core), cosmological simulations predict dark halo density profiles with $ρ \propto r^{-1.5}$ in the central parts (Moore et al. 1999b; Fukushige & Makino 2001). Other N-body simulations appear to converge to halo density profiles described by $ρ \propto r^{-1}$ (Power et al. 2002). These two conflicts, which might be related, the excess of dark satellites and the soft core question, arise because the CDM N-body simulations predict dark matter halos with high core densities.

Each conflict taken individually may not be sufficient to invalidate CDM on galactic scales. Results derived from observed density profiles of the inner regions in galaxies are controversial because of beam smearing effects in H i rotation curves (van den Bosch & Swaters 2001), even though high-resolution observations of Hα also show shallower core densities than those predicted by CDM numerical simulations (e.g., de Blok, McGaugh, & Rubin 2001; Marchesini et al. 2002). Several authors have attempted to reconcile the number of observed Local Group dwarf galaxies with the number predicted by CDM theory through conservative solutions within the framework of the current theory. Early work by Kauffmann, White, & Guiderdoni (1993), using semianalytic models of galaxy formation, found that most subhalos lacked a luminous component. Energetic mechanisms that are more efficient in low-mass systems, such as feedback from evolving stars and heating by an ionizing UV background, were proposed to explain a decoupling of luminous and dark components for low-mass dwarfs (Efstathiou 1992; Bullock, Kravtsov, & Weinberg 2000; Gelato & Sommer-Larsen 1999; Thacker & Couchman 2000). Another solution, proposed by Klypin et al. (1999), suggested an identification of the missing satellites seen in numerical simulations with observed compact high-velocity clouds (Blitz et al. 1999). This proposal may be premature, since it is still unclear whether the high-velocity clouds are galactic or extragalactic in nature. Comparisons of the dark satellite halos in CDM-dominated simulations with the distribution of observed neutral hydrogen high-velocity clouds and compact high-velocity clouds were made by Putman & Moore (2001). Recently, Stoehr et al. (2002) and Hayashi et al. (2003) suggested that the Galactic satellites could be identified with the most massive subhalos of CDM simulations. This would tend to support scenarios in which baryons are lost preferentially from low-mass halos for yet unknown reasons.

Still, the disagreement between observations and predictions might indicate that a revision of the CDM scenario is required. Self-interacting dark matter was proposed by Spergel & Steinhardt (2000) to overcome the satellite question and the soft core question. In this model, dark matter particles experience weak, nondissipative collisions on scales of kiloparsecs to megaparsecs for typical galactic densities. These collisions thermalize the inner regions of the dark halos, producing a soft core. In addition, the excess of subhalos predicted by the CDM models would be reduced. This model has attracted great attention. Numerical simulations (e.g., Burkert 2000; Yoshida et al. 2000; Moore et al. 2000; Firmani, D’Onghia, & Chincarini 2001a; Davé et al. 2001; D’Onghia, Firmani, & Chincarini 2003) demonstrated

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that, in this scenario, soft cores would form naturally after a collisional timescale by energy transport into the cold inner regions. However, after the initial expansion, the cores of isolated halos would evolve toward the core-collapse stage, with final central densities higher than those predicted by NFW (Burkert 2000; Kochanek & White 2000). Ostriker (2000) and Hennawi & Ostriker (2002) pointed out that self-interacting dark matter in a very weak cross section regime in the centers of galaxies reproduces supermassive black hole masses and their observed correlation with the velocity dispersion of the host bulges. However, they point out a possible inconsistency of the collisional scenario; indeed, the model would lead to the exorbitant growth of supermassive black holes, which consequently imposes a very strict upper limit on the collisional cross section. Other limits on the cross section values were derived from the fundamental plane relation for ellipticals in clusters (Gnedin & Ostriker 2001).

Using an analytical approach, this paper explores whether weakly self-interacting dark matter is likely to reconcile the apparent overabundance of subhalos with the small number of visible satellites in the Local Group. Two different disrupting processes are explored: collisions and tidal stripping. This work assumes $H_0 = 70 \text{ km s}^{-1}\text{ Mpc}^{-1}$.

### 2. DISRUPTION BY COLLISIONS

For a satellite dwarf galaxy orbiting the Milky Way, we define $\tau$ to be the time for the dark satellite halo to be destroyed by collisions with the self-interacting dark matter within the halo of the Milky Way,

$$
\tau = \frac{1}{\rho_{MW} \sigma v}, \tag{1}
$$

where $\rho_{MW}$ is the Galactic dark halo density (local density), $\sigma \equiv \sigma_s/m_s$ is the self-interacting cross section per unit mass, and $v$ is the velocity of the satellite relative to the Milky Way. We identify $v$ as the typical Milky Way halo velocity dispersion and assume that one collision for each particle of the satellite is enough to disrupt the satellite within a Hubble time. This assumption implies an optically thin regime (Gnedin & Ostriker 2001).

Let us assume a cross section inversely proportional to the halo velocity dispersion. This choice for the cross section produces smaller, less spherical cores in clusters of galaxies and large cores in dwarf galaxies (Yoshida et al. 2000; Firmani et al. 2001b; Wyithe, Turner, & Spergel 2001), consistent with observations of cluster cores like Cl 0024+1654. It also implies that the product of the cross section times the halo dispersion velocity is constant and independent of the mass. Thus, in the satellite the cross section $\sigma$ times the satellite dispersion velocity $v_0$ has the same value as the product $\sigma v$ in the Milky Way.

Let us suppose that the self-interacting dark matter of the satellite is interacting with itself and at the same time with the Milky Way halo. In the satellite, the effect will be a halo central density decreasing as a consequence of the collisions between dark matter particles. Within a Hubble time $t_H$, the expected average collision time is

$$
\frac{t_H}{N_{\text{coll}}} = \frac{1}{\rho_0 \sigma v}. \tag{2}
$$

Substituting equation (2) in equation (1), under the hypothesis that $\sigma v$ is constant, $\tau$ is given by

$$
\tau = \frac{\rho_0}{\rho_{MW} N_{\text{coll}}}. \tag{3}
$$

We assume for the satellite halo central density the same value observed in late-type dwarf galaxies: $\rho_0 \approx 0.02 M_\odot \text{ pc}^{-3}$ (de Blok et al. 2001; Marchesini et al. 2002), since the choice of $\sigma \propto 1/v$ predicts halo central densities independent of the mass. The average collision rate (the inverse of eq. [2]) is a function of the halo central density, the cross section, and the halo velocity dispersion. Since in this case $\sigma v$ is constant, the average collision rate is a function only of the satellite central density: $N_{\text{coll}}/t_H \propto \rho_0$. Cosmological N-body simulations in which the cross section is assumed to be inversely proportional to the halo velocity dispersion estimate $N_{\text{coll}} \approx 3-4$ for each particle in the core for $\sigma v \approx 0.6$ cm$^2$ g$^{-1}$ in order to reproduce the central densities observed in late-type dwarf galaxies over a Hubble time (D’Onghia et al. 2002). The same estimates are obtained using a dynamical code based on the integration of the Boltzmann equation for the same value of the scattering cross section (Firmani et al. 2001a).

Let us consider a dwarf halo placed at a distance of $25 h^{-1}$ kpc from the center of the Galactic halo. At this radius the Milky Way halo density is predicted to be $\rho_{MW} \approx 4 \times 10^{-3} M_\odot \text{ pc}^{-3}$ (Moore et al. 2001). Hence $\rho_0$ is nearly 5 times larger than the Galactic halo density, $\rho_{MW}$. For $N_{\text{coll}} = 4$ the satellite disruption time at $25 h^{-1}$ kpc is $\tau \approx t_H$. However, at only $30 h^{-1}$ kpc from the center, $\rho_0$ is 10 times larger than the Milky Way density, producing $\tau \approx 2 t_H$. In Figure 1 the time required to destroy satellites in the Galactic halo is shown as a function of the distance from the center of the Milky Way as predicted by our analytic consideration when $\sigma \propto 1/v$ is assumed (filled circles). Note that, at a distance of 50 $h^{-1}$ kpc, 10 Hubble times are required to destroy the satellites, if self-interaction is working to produce the central density we observe in late-type dwarf galaxies.

Let us now analyze the case in which the cross section is independent of velocity: $\sigma \approx \text{const}$. This case has interesting implications for supermassive black hole formation (Ostriker 2000). Equation (3) becomes

$$
\tau = \left( \frac{\rho_0}{\rho_{MW}} \right) \frac{t_H}{N_{\text{coll}}} \frac{v_0}{v}, \tag{4}
$$

with $v_0$ the satellite velocity dispersion. For $\sigma \approx \text{const}$ the halo central densities are no longer independent of the halo mass and $\rho_0 v_0 = \rho_{MW}^0 v$, with $\rho_{MW}^0$ the central density of the Milky Way. Since the halo central density times the halo dispersion velocity is constant, the average collision rate is a function only of the cross section: $N_{\text{coll}}/t_H \propto \sigma$. Cosmological N-body simulations with velocity-independent cross sections of $\sigma \sim 0.6$ cm$^2$ g$^{-1}$ suggest a central density of $\rho_0 \approx 4 \times 10^{-3} M_\odot \text{ pc}^{-3}$ for satellites of $M = 9 \times 10^8 M_\odot$ (Dave et al. 2001), and a typical number of collisions for each particle in the halo core of $N_{\text{coll}} \sim 3-4$ for $\sigma = 0.1$ cm$^2$ g$^{-1}$ (Yoshida et al. 2000). Thus, in equation (4) we assume

In eq. (3) the density profile of the Milky Way, $\rho_{MW}$, in CDM models was adopted from Moore et al. 2001, Fig. 2. In that work, Moore et al. (2001) show that the Milky Way dark density profile in CDM models is well described by $\rho(r) \propto 1/[(r/r_s)^3 + (r/r_s)^5]$, with $r_s$ the scale length.
Figure 1.—Time required to destroy satellites in the Galactic halo by collisions, normalized to the Hubble time, as a function of the distance from the center of the Milky Way. The filled circles show the suppression time if the cross section for self-interacting dark matter is assumed to depend on the halo dispersion velocity: $\sigma \propto v$. The open circles represent the time required to destroy satellites when the cross section is assumed to be velocity independent: $\sigma = \text{const.}$

$N_{\text{coll}} = 4$ and $\rho_0 = 4 \times 10^{-3} M_\odot \text{ pc}^{-3}$. In Figure 1 the disruption time of satellites at different radii from the center of the Milky Way is shown for $\sigma = \text{const}$ (open circles). For dwarf galaxies placed at a distance larger than $100 \ h^{-1} \text{ kpc}$, collisions between dark particles are inefficient in destroying satellites.

Satellite orbits are in general eccentric. As a result, subhalos can be destroyed efficiently when their pericentric distances are within $25$ or $100 \ h^{-1} \text{ kpc}$, depending on whether $\sigma$ is proportional to $1/v$ or not. Let us concentrate on the case in which the cross section decreases with the halo velocity dispersion, since it is in better agreement with the observed size of soft cores in dwarf galaxies. Using a Monte Carlo method we have computed the pericentric distance distribution of the dark satellites found by CDM N-body simulations, assuming orbits with the same eccentricity distribution function found for the halo orbits of cosmological N-body simulations (Ghigna et al. 1998). These simulations yield very eccentric satellite orbits with pericentric over apocentric distance ratios of $R_{\text{peri}}/R_{\text{apo}} \sim 0.2$, whereas observational evidence indicates that dwarf satellite orbits in the Local Group are more circular: $R_{\text{peri}}/R_{\text{apo}} = 0.5$ (Schweitzer et al. 1995). Assuming the eccentric orbits, the chances are high for dark satellites to move through the inner regions with pericentric distances $\leq 25 \ h^{-1} \text{ kpc}$ and to be destroyed. Since satellites spend most of their time at their apocentric distances, we assume that the semimajor axes of their orbits are distributed in the same way satellite distances from the Milky Way center are predicted by N-body simulations. For each satellite predicted by CDM models, knowing its galactocentric distance (courtesy of B. Moore 2002, private communication), we compute with the Monte Carlo technique: (1) its probability of having an eccentricity $e$ predicted by the distribution function found for the halo orbits of N-body simulations by Ghigna et al. (1998) and (2) its probability of having a semimajor axis $a$ as predicted by CDM models and from that its pericentric distance, $R_{\text{peri}} = a(1-e)$. All the satellites with pericentric distances less then $25 \ h^{-1} \text{ kpc}$ are assumed to be destroyed by collisions.

In order to check the validity of our model we have compared the final pericentric distance distribution resulting from our Monte Carlo realization to that found in CDM models by Font et al. (2001). In Figure 2 the masses and pericentric distances of the subhalos within twice the virial radius are shown, resulting from our Monte Carlo realization. The good agreement with the same plot shown in Font et al. (2001) is very encouraging.

What is the percentage of disrupted satellites when the orbital eccentricity distribution is taken into account? In Figure 3 the dashed line shows the cumulative number of dwarf galaxies observed within $250 \ h^{-1} \text{ kpc}$ from the Milky Way center (Grebel 2000). The solid line indicates the cumulative number of satellites predicted by standard CDM (SCDM) models at the same radii. Note that the N-body simulations predict an excess of subhalos at all radii. The dotted line represents the cumulative number of substructures that should survive collisions because their pericentric distances are larger than $25 \ h^{-1} \text{ kpc}$. We still find too many satellites beyond $25 \ h^{-1} \text{ kpc}$. It is clear from Figure 3 that, for $\sigma \approx 1/v$, collisional processes are not efficient enough in destroying substructures at any radii.

If the cross section is assumed to be independent of the relative velocity of the particles, then satellites with pericentric distances within $100 \ h^{-1} \text{ kpc}$ will be destroyed by collisions. Hence, no satellites should exist at radii smaller than $100 \ h^{-1} \text{ kpc}$, in conflict with observations, while the overabundance of satellites at larger radii remains unsolved (Fig. 3, triangles). Note that we neglect those satellites that

\[ \text{predicted by CDM models and from that its pericentric distance, } R_{\text{peri}} = a(1-e). \]

\[ \text{The number of subhalos predicted by standard CDM models is from } B. \text{ Moore (2002, private communication). High-resolution N-body simulations of } \Lambda \text{CDM models predict the same excess of dark satellites in the Local Group (B. Moore 2002, private communication).} \]
spend a very short time within $100\, h^{-1}\, \text{kpc}$; the likelihood of our detecting these satellites at such pericentric distances is small.

3. DISRUPTION BY TIDAL STRIPPING

In a self-interacting scenario the halo central densities are expected to be lower than in CDM models. As a result, tidal stripping should be more rapid and efficient and substructure halos orbiting in the tidal field of the Milky Way are expected to continuously lose mass and to be destroyed as a result of tidal forces. Thus, tidal stripping could be the dominant mechanism for destroying subhalos in a weakly self-interacting scenario. We take this process into account by assuming that the orbiting satellite is tidally truncated at some radius $r_t$, where the differential tidal force of the Milky Way is equal to the gravitational attraction of the satellite. For noncircular orbits, an expression for the tidal radius can be derived from the discussion in Spitzer (1967, p. 105):

$$m(r_t) = (3 + e) \frac{M(R_{peri})}{R_{peri}^3},$$

where $R_{peri}$ is the satellite pericentric distance, $m(r_t)$ the substructure mass within the tidal radius, $M$ the host galaxy mass, and $e$ the satellites orbital eccentricity. Thus, the tidal radius is such that the mean density of the satellite within $r_t$ is proportional to the mean density of the main halo at pericentric distance. In CDM satellites, when the tidally imposed radius approaches a value smaller than the scale radius $r_s$, substructures become unstable. Using $N$-body simulations of tidal stripping, the evolution of substructure halos described by a Hernquist or NFW profile within a static host potential have been explored by Mayer et al. (2001) and Hayashi et al. (2002).

In the self-interacting scenario, both dark satellites and the Milky Way have lower central densities than their CDM counterparts and have core radii $r_c$. Substructures can be tidally stripped when their tidally imposed radii approach values smaller than their core radii $r_c$. A detailed study of this mechanism requires numerical $N$-body simulations and is beyond the scope of this work. However, we note that, choosing a self-interacting cross section of $\approx 0.6\, \text{cm}^2\, \text{g}^{-1}$, which is capable of reproducing the central density of 0.02 $M_\odot\, \text{pc}^{-3}$ and $r_c \approx 2\, h^{-1}\, \text{kpc}$ for dark halos observed in late-type dwarf galaxies, equation (5) is satisfied for substructures with pericentric distances of $R_{peri} \leq 40\, h^{-1}\, \text{kpc}$. Following the same Monte Carlo procedure described in the previous section to derive the pericentric distance distribution for satellite orbits, we have determined the percentage of dark satellites found in the SCDM simulations with $R_{peri} \leq 40\, h^{-1}\, \text{kpc}$. The satellites with pericentric distances less than $40\, h^{-1}\, \text{kpc}$ are assumed to be disrupted by tidal forces.

Filled points in Figure 3 show the cumulative number of substructures surviving the disruption by tidal stripping. Note that in a weakly self-interacting scenario capable of solving the soft core question, tidal stripping is more efficient than collisions in destroying subhalos with pericentric distances within $40\, h^{-1}\, \text{kpc}$ and thus alleviating the excess of satellites at small scales. However, tidal forces cannot solve the excess satellite problem at larger radii.

4. CONCLUSION

In a hierarchical universe, high-resolution $N$-body simulations of Standard CDM models predict an excess of subhalos with respect to the number of dwarf galaxies observed in the Local Group (Klypin et al. 1999; Moore et al. 1999a).

To solve this conflict between predictions and observations a successful theory should reduce the abundance of subhalos at all radii. However, if the “satellite question” remains a problem of CDM models, then our semianalytical argument proves that self-interacting dark matter is unable to solve it.

If the value of the cross section is chosen such that it reproduces soft cores with sizes observed in late-type dwarf and low surface brightness galaxies and assumed to decrease with the halo velocity dispersion, then collisions between particles are effective in disrupting subhalos only within $25\, h^{-1}\, \text{kpc}$ from the Milky Way center. As a result, only a small percentage of all substructures are destroyed by collisions and the overabundance is only slightly reduced, leaving the problem unsolved at all radii. Tidal stripping is more efficient than collisions in destroying subhalos within $40\, h^{-1}\, \text{kpc}$ from the Galactic center, alleviating the problem at small radii. However, the discrepancy between predictions and observations persists.

In summary, finding a process that is able to decrease the halo central density does not appear to be sufficient for reducing the excess of dark satellite halos, especially for substructures placed at large distances from the Milky Way center. Thus, weak self-interaction, which was originally proposed to solve the soft core question in centers of dark matter-dominated galaxies and the overabundance of subhalos in the Local Groups is unable to solve both questions simultaneously.
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REFERENCES

Blitz, L., Spergel, D. N., Teuben, P. J., Hartmann, D., & Burton, W. B. 1999, ApJ, 514, 818
Bullock, J. S., Kravtsov, A. V., & Weinberg, D. H. 2000, ApJ, 539, 517
Burkert, A. 1995, ApJ, 447, L25
Davé, R., Spergel, D. N., Steinhardt, P. J., & Wandelt, B. D. 2001, ApJ, 547, 574
de Blok, W. J. G., McGaugh, S. S., & Rubin, V. C. 2001, AJ, 122, 2396
D’Onghia, E., Firmani, C., & Chincarini, G. 2003, MNRAS, 338, 156
Efstathiou, G. 1992, MNRAS, 256, 43
Firmani, C., D’Onghia, E., & Chincarini, G. 2001a, MNRAS, submitted (astro-ph/0010497)
Firmani, C., D’Onghia, E., Chincarini, G., Hernández, X., & Avila-Reese, V. 2001b, MNRAS, 321, 713
Flores, R. A., & Primack, J. 1994, ApJ, 427, L1
Font, A. S., Navarro, J. F., Stadel, J., & Quinn, T. 2001, ApJ, 563, L1
Fukushige, T., & Makino, J. 1997, ApJ, 477, L9
Gelato, S., & Sommer-Larsen, J. 1999, MNRAS, 303, 321
Ghigna, S., Moore, B., Governato, F., Lake, G., Quinn, T., & Stadel, J. 1998, MNRAS, 300, 146
Gnedin, O. Y., & Ostriker, J. P. 2001, ApJ, 561, 61
Grebel, E. 2000, in ESLAB Symp. 33, Star Formation from the Small to the Large Scale, ed. F. Favata, A. Kaas, & A. Wilson (Noordwijk: ESA), 87
Hayashi, E., Navarro, J. F., Taylor, J. E., Stadel, J., & Quinn, T. 2003, ApJ, 584, 541
Hennawi, J. F., & Ostriker, J. P. 2002, ApJ, 572, 41
Kauffmann, G., White, S. D. M., & Guiderdoni, B. 1993, MNRAS, 264, 201
Klypin, A. A, Kravtsov, A. V., Valenzuela, O., & Prada, F. 1999, ApJ, 522, 82
Kochanek, C. S., & White, M. 2000, ApJ, 543, 514
Marchesini, D., D’Onghia, E., Chincarini, G., Firmani, C., Conconi, P., Molinari, E., & Zacchei, A. 2002, ApJ, 575, 801
Mayer, L., Governato, F., Colpi, M., Moore, B., Quinn, T., Wadsley, J., Stadel, J., & Lake, G. 2001, ApJ, 559, 754
Moore, B. 1994, Nature, 370, 629
Moore, B., Calkano-Roldan, C., Stadel, J., Quinn, T., Lake, G., Ghigna, S., & Governato, F. 2001, Phys. Rev. D. 64, 063508
Moore, B., Gelato, S., Jenkins, A., Pearce, F. R., & Quilis, V. 2000, ApJ, 535, L21
Moore, B., Ghigna, S., Governato, F., Lake, G., Quinn, T., Stadel, J., & Tozzi, P. 1999a, ApJ, 524, L19
Moore, B., Quinn, T., Governato, F., Stadel, J., & Lake, G. 1999b, MNRAS, 310, 1147
Navarro, J., Frenk, C. S., & White, S. D. M. 1997, ApJ, 490, 493 (NFW)
Ostriker, J. P. 2000, Phys. Rev. Lett., 84, 23
Power, C., Navarro, J. F., Jenkins, A., Frenk, C. S., White, S. D. M., Springel, V., Stadel, J., & Quinn, T. 2002, MNRAS, 338, 14
Putman, M. E., & Moore, B. 2001, ASP Conf. Ser. 254, Extragalactic Gas at Low Redshift, ed. J. Mulchaey & J. Stocke (San Francisco: ASP), 245
Schweitzer, A., Cudworth, K. M., Majewski, S. R., & Suntzeff, N. B. 1995, AJ, 110, 2747
Spergel, D. N., & Steinhardt, P. J. 2000, Phys. Rev. Lett., 84, 3760
Spitzer, L., Jr. 1967, Dynamical Evolution of Globular Clusters (Princeton: Princeton Univ. Press)
Stoehr, F., White, S. D. M., Tormen, G., & Springel, V. 2002, MNRAS, 335, L84
Thacker, R. J., & Couchman, H. M. P. 2000, ApJ, 545, 728
van den Bosch, F. C., & Swaters, R. A. 2001, MNRAS, 325, 1017
Wyithe, J. S. B., Turner, E. L., & Spergel, D. N. 2001, ApJ, 555, 504
Yoshida, N., Springel, V., White, S. D. M., & Tormen, G. 2000, ApJ, 544, L87