1 Introduction

With the development of science and technology, the request on quality of the products has been constantly increasing [1, 2]. Among such precision measurements, the small batch of products that require a higher critical quality usually need full inspection to avoid any misjudgement risks. The qualification of the products is determined according to the comparison between the measurement result of each single product and the product tolerance. However, the misjudgement risk caused by the measurement uncertainty in the product testing process is objective and cannot be ignored [3–5]. At present, academia has given full attention to the applications of measurement uncertainty of the precision measurement engineering in product testing [6, 7]. However, the existing results are still inadequate: first, some studies often ignore the impact of the measurement uncertainty, greatly reducing the quality and reliability of the product inspection results in the precise measurement process. Second, the calculation of the misjudgement rate is rarely involved in the product precision measurement and testing. The misjudgement rate caused by the measurement uncertainty lacks reasonable quantification. Third, the uncertainty assessment testing is often carried out on a single product. As the information included is limited, it is difficult to apply the inspection results of the whole batch of products.

Considering the limitation of the product inspection research on current precision engineering, the current paper analyses the influence of measurement uncertainty on the full qualification determination of the batch product and establishes the misjudgement rate calculation model for the full inspection of product qualification. The paper also uses the measurement system value characteristics analysis method and introduces the Bayesian estimation method to fully integrate the measurement information of products so as to determine the uncertainty of the batch product test.

2 Calculation of misjudgement rate of precision measurement products

Currently, the general method of product quality inspection is to set the tolerance limit \([C_L, C_U]\) as the standard and determine the quality according to the measured value of each product. Products are accepted as qualified when the measurement results are within the tolerance limit. If the measurement results exceed the tolerance limit, the products shall be returned as unqualified. Considering the impact of the measurement uncertainty, there is a certain misjudgement risk in the test results. If the best estimated value of the measurement result of the quality characteristics measured is \(x\), when the production statistical process is controlled, according to the central limit theorem, \(x\) meets the normal distribution \(N(\mu_x, \sigma_x)\). The probability density function is

\[
 f(x) = \frac{1}{\sqrt{2\pi\sigma_x}} \exp\left(-\frac{(x - \mu_x)^2}{2\sigma_x^2}\right) \tag{1}
\]

where \(\sigma_x^2 = \sigma_U^2 + \sigma_c^2\), \(\mu_x\) is the centre value of the quality control and \(\sigma_x\) reflects the dispersion of the best estimated value of the measurement results in the batch product. \(\sigma_U\) refers to the standard deviation of the process and \(u_x\) refers to the uncertainty of the synthesis standard in the measurement process.

If \(y = x \pm U\) is the complete representation of the measurement results after considering the impact of uncertainty, for each given value of \(x\), the distribution density function of \(y\) is \(g(y|x)\). If \(x\) satisfies the normal distribution \(N(x, u_x)\), then the probability density function of \(y\) is:

\[
 g(y|x) = \frac{1}{\sqrt{2\pi u_x}} \exp\left(-\frac{(y - x)^2}{2u_x^2}\right) \tag{2}
\]

2.1 Customer risk

As shown in Fig. 1, when \(C_L \leq x < C_L + U\) or \(C_U - U < x \leq C_U\), taking into account the impact of the uncertainty, the unqualified products may be judged as qualified and customers may bear the risk. For any product with an unknown quality in the batch, the probability of being misjudged as qualified is:

\[
 L = \frac{1}{2} \left(1 + \frac{1}{\sqrt{\pi}} e^{-\frac{u_x^2}{2}} \right) \tag{3}
\]
where $k = (1/(2\pi u_i \sigma_i))$.

For the products which are judged to be qualified in the batch inspection results, the probability of being misjudged to make customers bear the risk is:

$$\begin{align*}
P_{CR1} &= k \int_{C_L}^{C_L+U} \int_{C_L}^{C_L} \exp \left[-\frac{(y-x)^2}{2\sigma^2} \right] dy \exp \left[-\frac{(x-\mu)^2}{2\sigma^2} \right] dx \\
&+ k \int_{C_L}^{C_L+U} \int_{C_L}^{C_L} \exp \left[-\frac{(y-x)^2}{2\sigma^2} \right] dy \exp \left[-\frac{(x-\mu)^2}{2\sigma^2} \right] dx \\
&+ \int_{C_L}^{C_L+U} \exp \left[-\frac{(y-x)^2}{2\sigma^2} \right] dy \exp \left[-\frac{(x-\mu)^2}{2\sigma^2} \right] dx \tag{4}
\end{align*}$$

\[2.2\text{ Supplier risk}\]

As shown in Fig. 2, when $C_L - U < x < C_L$ or $C_U < x < C_U + U$, taking into account the impact of uncertainty, the qualified products may be judged as unqualified and suppliers may bear the risk.

For any product with an unknown quality in the batch, the probability of being misjudged as unqualified is: (see (6)) the following can be obtained:

$$\begin{align*}
P_{PR1} &= k \int_{C_L}^{C_L+U} \int_{C_L}^{C_L} \exp \left[-\frac{(y-x)^2}{2\sigma^2} \right] dy \exp \left[-\frac{(x-\mu)^2}{2\sigma^2} \right] dx \\
&+ k \int_{C_L}^{C_L+U} \int_{C_L}^{C_L} \exp \left[-\frac{(y-x)^2}{2\sigma^2} \right] dy \exp \left[-\frac{(x-\mu)^2}{2\sigma^2} \right] dx \\
&+ \int_{C_L}^{C_L+U} \exp \left[-\frac{(y-x)^2}{2\sigma^2} \right] dy \exp \left[-\frac{(x-\mu)^2}{2\sigma^2} \right] dx \tag{7}
\end{align*}$$

\[3\text{ Evaluation of test uncertainty of precision measurement products}\]

In the product inspection, the key problem in calculating the misjudgement rate of the product qualification judgment is how to evaluate the synthesis standard uncertainty $u_k$ during the measurement process. The uncertainty of the qualification judgment of the precision measurement products is assessed through the measurement system analysis method, using the value characteristics of the measurement data to reflect the influence of each error on the measurement results.
v. $u_g$ the uncertainty component caused by resolution. If the resolution is $a$, then $u_g = a/2\sqrt{3}$.

vi. $u_T$ refers to the uncertainty component caused by the impact of temperature and other factors on the measurement results.

Among them, $x_i$ is the measured value and $x$ is the measured mean value. $n_i$ refers to the number of repeatability assessments and $n$ refers to the number of measurement results of the reproducibility assessment.

When a single product is determined to be qualified, the measurement uncertainty can be assessed by (5). The uncertainty components caused by the indication error, resolution, instrument stability, temperature and other factors can be calculated by using the type B evaluation method, reviewing the relevant test certificate, acquiring the historical data, or by the theoretical formula; the uncertainty components of the measurement process repeatability and reproducibility should be determined through experiments.

3.2 Repeatability and reproducibility component determination based on the Bayesian method

According to the GUM, generally, the best estimated value of the measurement result is the mean $\mu$ of the $n$ measured data, and the standard uncertainty is the standard deviation of the mean $\sigma$. Taking the repeatability measurement data as an example, the first product measurement data of the full inspection is taken as a priori uncertainty of the repeatability information of the integrated two products.

Table 1

| No. | Measurement data, mm |
|-----|----------------------|
| 1   | 27.9814 27.9802 27.9804 27.9808 27.9801 27.9788 27.9807 27.9807 27.9825 27.9810 |
| 2   | 27.9799 27.9796 27.9799 27.9774 27.9790 27.9772 27.9823 27.9778 27.9778 27.9786 |
| 3   | 27.9792 27.9798 27.9801 27.9792 27.9799 27.9771 27.9774 27.9798 27.9808 27.9798 |
| 4   | 27.9783 27.9796 27.9799 27.9799 27.9790 27.9814 27.9778 27.9778 27.9786 27.9786 |
| 5   | 27.9800 27.9810 27.9821 27.9827 27.9820 27.9824 27.9816 27.9824 27.9824 27.9790 27.9704 |

When the first product is determined to be qualified, the measurement uncertainty of the repeatability information of the two products is:

\[
\sqrt{\mu_{1}^2 + \mu_{2}^2 + \mu_{3}^2 + \mu_{4}^2 + \mu_{5}^2} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2 + \sigma_5^2}
\]

Considering it as a priori information, by integrating it with the follow-up uncertainty assessment data of multiple products, the repetitive component which can fully reflect the accuracy level of the batch product inspection process can be obtained [8].

The reproducibility index evaluation is similar to the repeatability evaluation. There are also differences. The reproducibility reflects the differences of the repeated measurements under the same condition, while the reproducibility reflects the differences of measurement results under different conditions. In the reproducibility assessment, the measurement results should be calculated by using the mean so as to eliminate the repetitive effects.

4 Measurement experiments and results

As a commonly used multi-functional, efficient and sophisticated measuring instrument, coordinate measuring machine (CMM) has been widely used in modern machinery manufacturing, precision engineering, and other fields. It has been an indispensable measurement equipment for high-precision product measurement and inspection. The present paper carries out product testing experiment by using the CMM with five products as examples to demonstrate the updated process of the value characteristic index when each product is measured in the full inspection. It also evaluates the uncertainty of the measurement information of the products and calculates the misjudgement rate based on the theoretical analysis [9, 10].

4.1 Measurement uncertainty evaluation

According to the preliminary measurement, based on the known information from the CMM verification certificate, work piece material information, and the temperature changes, the standard uncertainty components caused by the CMM indication error and temperature are calculated as follow: $u_g = 1.8\mu$m and $u_T = 0.27\mu$m. As the stability and resolution have little effect on the CMM size inspection, the uncertainty components caused by these two can be ignored.

The same operator shall use the CMM to measure the aperture values of the five parts for 10 times. The measurement data are shown in Table 1. With the measurement data of the first part as a priori information, the measurement is $\mu_0 = 27.9793\mu$m and the standard deviation is $\tau = 0.96\mu$m. Based on the measurement data of the second part, the standard deviation is calculated to be: $\sigma_1 = 1.17\mu$m. According to (11), the repetitive component of the previous parts before the integration is $u_t = 0.35\mu$m. So on, the repetitive components of the measurement information of the five parts before the integration are calculated as $u_{RT} = 0.17\mu$m.

Different operators shall carry out the independent measurement of the five parts. Calculate the differences of the results of each part measured by different operators. According to the Section 3 calculation process, the reproducibility components of the measurement information of the integrated five parts are $u_{BD} = 0.56\mu$m.

According to (5), the combined standard uncertainty of the part aperture measurement by using the CMM is $u_c = 1.91\mu$m.

4.2 Misjudgement rate calculation

Combined with the product quality control information of the aperture to be measured, the component aperture control centre value is $\mu_s = 27.979\mu$m. The tolerance is $28.300\pm0.010\mu$m. The
standard deviation of the machining process is $\sigma_p = 4.1\, \mu m$. Based on the standard uncertainty evaluated in Section 4.1, in accordance with (1) and (3), the misjudgement risks borne by customers and suppliers are predicted to be $P_{\text{CR}} = 1.11\%$, $P_{\text{PR}} = 0.36\%$.

The calculation results above show that there is a certain misjudgement rate in the full inspection with the consideration of the impact of the measurement uncertainty. For any product with an unknown quality in the batch, the supplier may bear 0.36\% of the risk. The qualified product may be misjudged as unqualified and be rejected. The customer may bear 1.11\% of the risk. The unqualified product may be misjudged as qualified and be accepted.

The best estimated values of the five parts are measured to be $\bar{x}_1 = 27.9807\, mm$, $\bar{x}_2 = 27.9972\, mm$, $\bar{x}_3 = 27.9788\, mm$, $\bar{x}_4 = 27.9794\, mm$, $\bar{x}_5 = 27.9814\, mm$.

According to the product tolerance limit, Parts 1, 3, 4, 5 are qualified while Part 2 is unqualified. According to (2) and (4), the misjudgement rate of the qualified products and unqualified products are $P_{\text{CR2}} = 1.13\%$, $P_{\text{PR2}} = 25.17\%$.

The calculation results above show that the full inspection will have the qualification judgment to each single product in the batch. For the Products 1, 3, 4, 5 which are judged to be qualified, the misjudgement rate is 1.13\%. The products can be accepted. For the Product 2 which is judged to be unqualified, the misjudgement rate is 25.17\%. The product should, therefore, be rejected.

### 5 Conclusion

According to the increasing requirements of modern industrial manufacturing in terms of the quality and reliability of product inspection, and considering the measurement uncertainty as an important quality index of the precision measurement product test, the present paper has drawn the following conclusions through researches and analysis of the measurement examples:

i. Considering the measurement uncertainty, there is a certain probability of misjudgement in the full inspection. Before the product inspection, it is necessary to carry out reasonable estimation and quantification. The misjudgement risk should be within the acceptable range of both the supply and demand sides.

ii. Generally, the full inspection is applicable to the product parameter inspection of key quality characteristics and high precision requirements of the small batch inspection. For products which have been determined to be qualified or unqualified according to the tolerance in the batch products, a certain misjudgement rate remains when considering the uncertainty. The product supply and demand sides can be used to make the decision on whether to accept or reject the product based on the misjudgement rate.

iii. The measurement information of the whole inspection process can be fully integrated by evaluating the repeatability and reproducibility components of the measurement system through the Bayesian Estimation Method so that the uncertainty assessment result of the product inspection can be more representative.

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