Constraints on $Z'$ from $W^+W^-$ production at the NLC with polarized beams

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Abstract

We discuss the potential of NLC500 and NLC1000 to probe $Z-Z'$ mixing and mass by the reaction $e^+e^- \rightarrow W^+W^-$ with longitudinally polarized electrons. We perform a model-independent analysis of the deviations from the Standard Model, and apply it to a specific class of extended weak gauge models. Results indicate that the corresponding bounds at the NLC500 complement the present ones obtained from LEP1, and rapidly become quite stringent at the higher energy of the NLC1000. Also, we emphasize the importance of initial beam polarization in improving the sensitivity to mixing.

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1 Introduction

A rather common feature of extended electroweak models is the prediction that one (or more) neutral heavy gauge bosons $Z'$ should exist. The mass of such objects is largely unknown, as it cannot be theoretically estimated by a dynamical calculation, but it might be in the TeV range and thus, hopefully, in the reach of future higher energy machines. Clearly, the knowledge of $Z'$ parameters such as $M_{Z'}$, the $Z'$ couplings to fermions and its mixing angle with the standard $Z$, is essential to test extended theories, and in particular their gauge and Higgs structures. Many attempts have been made to phenomenologically derive indications on the $Z'$ properties from present data, and to develop strategies to search for the manifestations of such particles in the data from next-generation high energy machines.

At present, direct $Z'$ production searches at the $p\bar{p}$ Tevatron collider indicate a lower limit on $M_{Z'}$ of the order of 500 GeV, which means that almost certainly the LEP200 will be below the threshold for directly studying the $Z'$ peak, while the the planned next-linear $e^+e^-$ colliders might be near the discovery level. If the CM energy were still not high enough to produce such heavy particle, $Z'$ searches would focus on possible ‘indirect’ manifestations through deviations of observables from the Standard Model (SM) predictions. If such deviations were not found within the expected accuracy, then constraints on the various parameters could be derived. More optimistically, in the event that some deviation was observed, in principle such information could be used to shed light on the properties of the $Z'$. Of course, a related problem would be to have some criterion to distinguish that effect from alternative sources of nonstandard physics, potentially also contributing deviations from the SM.

Studies of the annihilation $e^-e^+\to f\bar{f}$ at LEP1 have lead to restrictions on the $Z - Z'$ mixing angle and the $Z'$ mass comparable with the direct searches at the Tevatron and, in the perspective both of LEP200 and of the next-linear-collider projects, the sensitivity of such reaction to the $Z'$ has been extensively analysed, also recently [1]-[3].

With the increased $e^+e^-$ energy available at these machines, also the reaction

$$e^+e^-\to W^+W^-$$  \hspace{1cm} (1)

should represent a convenient tool to search for $Z'$ effects [4]. Indeed, in this process,
lack of gauge cancellation among the different amplitudes due to nonstandard physics
should lead to deviations from the SM cross section rapidly increasing with energy
and therefore, in principle, to enhanced sensitivity to the existence of the $Z'$ if efficient
$W^+W^-$ reconstruction could be performed. Moreover, it turns out that the strongest
sensitivity of process (1) to nonstandard effects would be obtained if initial beams
were longitudinally polarized with both kinds of polarization available, so that the
information from the separately measurable cross sections for left-handed and right-
handed electrons could be combined. On the one hand, that would lead to stringent
restrictions on the $Z - Z'$ mixing angle. On the other hand, model-independent
separate bounds on the anomalous $(WW\gamma)$ and $(WWZ)$ couplings could be derived
in case the deviation from the SM was attributed to this kind of source [5]. In this
note, we will discuss manifestations of the $Z'$ in $e^+e^- \rightarrow W^+W^-$ at future $e^+e^-$
colliders taking into account, in addition to propagator effects as in Ref. [3], also the
$Z - Z'$ mixing. Starting from a model-independent parametrization of the deviations
from the SM amplitude, we derive bounds on nonstandard couplings and then apply
such constraints to the class of extended electroweak models based on $E_6$ gauge
symmetry. The high sensitivity of process (1) to the parameters of such extended
models, namely the $Z - Z'$ mixing angle and the heavier neutral gauge boson mass,
will be shown. In particular, the essential role of initial beams polarization in this
kind of analyses will be emphasized.

2 Model independent bounds

The SM amplitude for process (1), in Born approximation, is divided into $t$-channel
amplitude (originating from neutrino exchange) and $s$-channel amplitude (induced
by photon and $Z$ exchange): $\mathcal{M} = \mathcal{M}_t + \mathcal{M}_s$. With $\lambda(= -\lambda') = \pm1/2$ the electron
(positron) helicity, the $t$-channel amplitude has the form

$$\mathcal{M}_t^\lambda = \frac{2\lambda - 1}{4ts_w^2} \times T^\lambda(s, \theta), \quad (2)$$

where $\sqrt{s}$ and $\theta$ are the total energy and the $W^-$ production angle in the CM frame,
$t = M_W^2 - s(1 - \beta W \cos \theta)/2$ with $\beta W = \sqrt{1 - 4M_W^2/s}$, and the explicit form of the
kinematical coefficient $T^\lambda(s, \theta)$ is not essential for our discussion and is omitted for
simplicity, as it can easily be found in the literature [3].
The s-channel amplitude is given by

\[ \mathcal{M}_s^\lambda = \left( -\frac{g_{WW\gamma}}{s} + \frac{g_{WWZ}(v - 2\lambda a)}{s - M_Z^2} \right) \times G^\lambda(s, \theta) \] (3)

and, as in the case of Eq. (2), we refer to [6] for the expression of the kinematical coefficient \( G^\lambda(s, \theta) \).

The SM values of the couplings in Eq. (3) are: \( g_{WW\gamma} = 1 \) and \( g_{WWZ} = \cot \theta_W \) with \( \theta_W \) the electroweak mixing angle (\( \sin^2 \theta_W \approx 0.231 \)); \( v = (T_{3,e} - 2Q_e s_W^2)/2s_W c_W \) and \( a = T_{3,e}/2s_W c_W \) with \( T_{3,e} = -1/2 \) (\( s_W = \sin \theta_W \), \( c_W = \cos \theta_W \); \( M_Z^2 = M_W^2/c_W^2 \)) at the leading order.

In the extended electroweak models considered here, the neutral vector boson sector consists, in addition to the photon, of a light \( Z_1 \) (with mass \( M_{Z_1} \approx M_Z \), to be identified to the \( Z \) in the limit of the SM), and a heavy \( Z_2 \) (with, expectedly, \( M_{Z_2} \) much greater than \( M_Z \)). Both exchanges contribute to the s-channel amplitude, which now reads:

\[ \mathcal{M}_s^\lambda = \left( -\frac{g_{WW\gamma}}{s} + \frac{g_{WWZ_1}(v_1 - 2\lambda a_1)}{s - M_{Z_1}^2} + \frac{g_{WWZ_2}(v_2 - 2\lambda a_2)}{s - M_{Z_2}^2} \right) \times G^\lambda(s, \theta). \] (4)

Here, \( Z_1 \) and \( Z_2 \) denote ‘physical’ mass-eigenstates; \( g_{WWZ_1} \) and \( g_{WWZ_2} \) are the corresponding couplings to \( W^+W^- \), both assumed of the usual Yang-Mills form, and \( (v_1, a_1) \) and \( (v_2, a_2) \) are, respectively, the vector and axial-vector couplings to \( e^+e^- \). Clearly, the values of these couplings depend on the extended model under consideration. In the sequel, the notation \( Z \) and \( Z' \) will indicate the Standard Model \( Z \)-particle and the heavy neutral boson weak gauge-eigenstate.

Concerning the \( Z_1 \) couplings to electrons, present constraints from experimental data [7] indicate that their values should be rather close to the SM values \( v \) and \( a \) listed above. Thus, the deviations \( \Delta v = v_1 - v \) and \( \Delta a = a_1 - a \) should be small numbers, to be treated as a perturbation, and the same is true for the mass-shift \( \Delta M = M_Z - M_{Z_1} \), with \( \Delta M > 0 \) if this is due to \( Z - Z' \) mixing.

In linear approximation, as justified by the expected smallness of these nonstandard effects, all deviations can be conveniently parametrized so as to rewrite the s-channel amplitude (4) in the same form as the SM one:

\[ \mathcal{M}_s^\lambda = \left( -\frac{g_{WW\gamma}}{s} + \frac{g_{WWZ}^*(v - 2\lambda a)}{s - M_Z^2} \right) \times G^\lambda(s, \theta), \] (5)
where the ‘effective’ gauge boson couplings $g_{WW \gamma}^*$ and $g_{WW Z}^*$ are defined as:

$$g_{WW \gamma}^* \equiv 1 + \delta_\gamma \equiv 1 + \delta_\gamma(Z_1) + \delta_\gamma(Z_2),$$

$$g_{WW Z}^* \equiv \cot \theta_W + \delta_Z \equiv \cot \theta_W + \delta_Z(Z_1) + \delta_Z(Z_2),$$

(6)

(7)

with

$$\delta_\gamma(Z_1) = v g_{WW Z_1} \left( \frac{\Delta a}{a} - \frac{\Delta v}{v} \right) (1 + \Delta \chi) \chi; \quad \delta_\gamma(Z_2) = v g_{WW Z_2} \left( \frac{a_2}{a} - \frac{v_2}{v} \right) \chi_2;$$

$$\delta_Z(Z_1) = -\cot \theta_W + g_{WW Z_1} \left( 1 + \frac{\Delta a}{a} \right) (1 + \Delta \chi); \quad \delta_Z(Z_2) = g_{WW Z_2} \frac{a_2 \chi_2}{a} \chi.$$  

(8)

(9)

In Eqs. (8) and (9), neglecting the gauge boson widths:

$$\chi(s) = \frac{s}{s - M_Z^2}; \quad \chi_2(s) = \frac{s}{s - M_{Z_2}^2}; \quad \Delta \chi(s) = -\frac{2 M_Z \Delta M}{s - M_Z^2}.$$  

(10)

As it will be emphasized in the sequel, this general parametrization is rather useful for phenomenological purposes, in order to discuss the deviations from the SM in the context of different classes of extended models contributing to the deviations (8) and (9).

As indicated by the notations, in Eqs. (8) and (9) $\delta_\gamma(Z_1)$ and $\delta_Z(Z_1)$ originate from modifications of the $Z$ couplings plus the, generally possible, $Z$ mass-shift from the SM value accounted by $\Delta M$ in (10). Instead, $\delta_\gamma(Z_2)$ and $\delta_Z(Z_2)$ represent the ‘direct’ contribution of the $Z_2$. One can notice that the resulting expressions for $g_{WW \gamma}^*$ and $g_{WW Z}^*$ coincide with those used in [8] in the limit of retaining only the structure corresponding to $\delta_\gamma(Z_2)$ and $\delta_Z(Z_2)$. From the general form of Eqs. (8)-(9), nonvanishing values of $\delta_\gamma$ and $\delta_Z$ can also occur as the consequence of anomalous trilinear gauge boson couplings. Actually, in effective theories of nonstandard anomalous trilinear gauge boson couplings [8-10], at the leading dimension $\delta_\gamma = 0$.

Assuming that the new physics does not involve the charged current couplings ($W e \nu$), which then retain their SM values, the $t$-channel amplitude remains identical to Eq. (3). Consequently, the differential cross section of process (1) at a given CM energy will have the same form as in the SM, except that in general its values will numerically differ from the SM predictions due to the nonstandard effects introduced above. Accordingly, we can represent such effect by the relative deviation of the

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3The problem of distinguishing this alternative source of nonstandard effects from mixing will be discussed in a separate paper.
cross section (either differential or integrated in some angular range) from the SM prediction:
\[ \Delta \equiv \frac{\Delta \sigma}{\sigma_{SM}} = \frac{\sigma - \sigma_{SM}}{\sigma_{SM}}, \]
which brings information on the free, independent, parameters \( \delta_{\gamma} \) and \( \delta_{Z} \) in Eqs. (6) and (7).

If a nonvanishing value of \( \Delta \) was experimentally measured at some level of accuracy, the values of such parameters could be determined and possibly used to learn about the properties of the related nonstandard physics. Alternatively, in the case of no observation, one could derive numerical bounds on \( \delta_{\gamma} \) and \( \delta_{Z} \), and therefore constrain the various extended models, at some confidence level that depends on the attainable sensitivity of the experiment. In this regard, assuming small deviations, \( \Delta \) is expressed as a linear combination of \( \delta_{\gamma} \) and \( \delta_{Z} \) with coefficients which, generally, increase with \( s \). Conversely, the SM cross section decreases as \( 1/s \) (at least) due to the gauge cancellation among the various amplitudes. Therefore, if we parametrize the sensitivity of process (1) to \( \delta_{\gamma} \) and \( \delta_{Z} \) by, e.g., the ratio \( S = \Delta/(\delta \sigma/\sigma) \) with \( \delta \sigma/\sigma \) the attainable statistical uncertainty on the SM cross section, such sensitivity is expected to increase with energy, even at fixed integrated luminosity (basically, as \( S \propto \sqrt{L_{\text{int}}} \)).

As discussed previously [5], initial electron beam longitudinal polarization, and the related possibility to individually measure the cross sections for \( e_{L}e^{+} \) and \( e_{R}e^{+} \) (\( \sigma^{-} \) and \( \sigma^{+} \)), would substantially improve the sensitivity to the couplings \( \delta_{\gamma} \) and \( \delta_{Z} \). In this regard, the measurement of \( \sigma^{+} \) would be of particular interest in two respects. Firstly, although giving a much lower statistics as being suppressed by \( \gamma-Z \) gauge compensation, it is most sensitive to the nonstandard effects considered here because it is free of the unmodified \( t \)-channel amplitude which numerically dominates \( \sigma^{-} \) and \( \sigma^{\text{unpol}} \) as well \( (\sigma^{\text{unpol}} \approx (1/2)\sigma^{-}) \). Secondly, by specifying \( \lambda \), Eq. (5) directly shows that
\[ \Delta \sigma^{-} \propto \delta_{\gamma} - \delta_{Z} \cdot g_{L} e \chi; \quad \Delta \sigma^{+} \propto \delta_{\gamma} - \delta_{Z} \cdot g_{R} e \chi, \]
where \( g_{L} = v - a = \tan \theta_{W} \approx 0.55 \) and \( g_{R} = v + a = g_{L}(1 - 1/2s_{W}^{2}) \approx -0.64 \). Thus, by themselves, \( \sigma^{-} \) (or \( \sigma^{\text{unpol}} \)) and \( \sigma^{+} \) only provide correlations among \( \delta_{\gamma} \) and \( \delta_{Z} \), rather than true limits. These correlations are represented as bands in the \( \delta_{\gamma} - \delta_{Z} \) plane of Fig. 1, with a width proportional to the corresponding sensitivities, and a relative angle of approximately 60 degrees. The figure clearly illustrates that, in
contrast with the case where only the unpolarized cross section is measured, and therefore in principle an ‘unlimited’ area is allowed to \( \delta \gamma \) and \( \delta Z \), the combination of left-handed and right-handed cross sections is essential to obtain a finite allowed region by the intersection of the corresponding bands.

Numerically, Fig. 1 refers to the NLC energy \( \sqrt{s} = 500 \text{GeV} \), assuming \( L_{\text{int}} = 50 \text{ fb}^{-1} \), and to cross sections integrated over the range \( |\cos \theta| \leq 0.98 \). Concerning polarization, in practice the degree of initial electron longitudinal polarization \( P_L \) will be quite near, but not be exactly equal, to unity. Therefore, the measured cross section will be a linear combination of purely polarized cross sections \[5\]

\[
\frac{d\sigma}{d\cos \theta} = \frac{1}{4} \left[ (1 + P_L) \frac{d\sigma^+}{d\cos \theta} + (1 - P_L) \frac{d\sigma^-}{d\cos \theta} \right].
\] (13)

In Fig. 1, the notation \( \sigma^R \) and \( \sigma^L \) refers to the values \( P_L = 0.9 \) and \( P_L = -0.9 \), respectively. Such values of \( P_L \) seem to be obtainable at the NLC \[11\].

The sensitivity of \( \sigma^L \) and \( \sigma^R \) to \( \delta \gamma \) and \( \delta Z \) has been assessed numerically by dividing the considered angular range into 10 equal ‘bins’, and defining a \( \chi^2 \) function in terms of the expected number of events \( N(i) \) in each bin:

\[
\chi^2 = \sum_i^{\text{bins}} \left[ \frac{N_{\text{SM}}(i) - N(i)}{\delta N_{\text{SM}}(i)} \right]^2,
\] (14)

where the uncertainty on the number of events \( \delta N_{\text{SM}}(i) \) combines both statistical and systematic errors as

\[
\delta N_{\text{SM}}(i) = \sqrt{N_{\text{SM}}(i) + (\delta_{\text{syst}}N_{\text{SM}}(i))^2},
\] (15)

(we assume \( \delta_{\text{syst}} = 2\% \)). In Eq. (14), \( N(i) = L_{\text{int}} \sigma_i \varepsilon_W \), with

\[
\sigma_i \equiv \sigma(z_i, z_{i+1}) = \int_{z_i}^{z_{i+1}} \left( \frac{d\sigma}{dz} \right) dz,
\] (16)

and \( z = \cos \theta \). Also, \( \varepsilon_W \) is the efficiency for \( W^+W^- \) reconstruction, for which we take the channel of lepton pairs \( (e\nu + \mu\nu) \) plus two hadronic jets, giving \( \varepsilon_W \simeq 0.3 \) from the relevant branching ratios. An analogous procedure is followed to evaluate \( N_{\text{SM}}(i) \).

As a criterion to derive allowed regions for the deviations of the coupling constants in the case no deviations from the SM were observed, and in this way to assess the
sensitivity of process (1) to $\delta_\gamma$ and $\delta_Z$, we impose that $\chi^2 \leq \chi^2_{\text{crit}}$, where $\chi^2_{\text{crit}}$ is a number that specifies the chosen confidence level. With two independent parameters in Eq. (12), the 95% CL is obtained by choosing $\chi^2_{\text{crit}} = 6$ (12).

From the numerical procedure outlined above, we obtain the bands allowed to $\delta_\gamma$ and $\delta_Z$ by the polarized cross sections $\sigma^R$ and $\sigma^L$ (as well as $\sigma^{\text{unpol}}$) depicted in Fig. 1. This figure manifestly shows the significant role of the combination of polarized measurements in restricting the limits by the intersection of the two bands.

3 Bounds on extended models and conclusions

In this section, we apply the information on $\delta_\gamma$ and $\delta_Z$ given in Fig. 1 to the tests of extended models where the gauge group contains at least one additional $U(1)'$ factor, therefore one new neutral gauge boson $Z'$ (13). Specifically, we focus on the ‘conventional’ class of models with an $E_6$ origin (either inspired by superstring theories or not), where the Higgs fields transform either as doublets or singlets of $SU(2)_L$.

In general, the neutral gauge boson mass matrix in the basis of weak gauge eigenstates $Z$ and $Z'$ takes the form

$$M^2 = \begin{pmatrix} M^2_Z & \delta M^2 \\ \delta M^2 & M^2_{Z'} \end{pmatrix}. \quad (17)$$

Diagonalization of (17) leads to the mass eigenstates $Z_1$ and $Z_2$ via the rotation

$$Z_1 = Z \cos \phi + Z' \sin \phi$$
$$Z_2 = -Z \sin \phi + Z' \cos \phi, \quad (18)$$

where, by convention, $M_{Z_1} < M_{Z_2}$ and $\phi$ is the mixing angle:

$$\tan^2 \phi = \frac{M^2_{Z_2} - M^2_{Z_1}}{M^2_{Z_2} - M^2_Z} \simeq \frac{2M_Z \Delta M}{M^2_{Z_2}}. \quad (19)$$

The mixing of the SM $Z$ with the heavier $Z'$ leads to an ‘observed’ mass of the lighter neutral gauge boson $Z_1$ shifted from $M_Z$ by the positive mass-shift $\Delta M$ introduced in Eqs. (10) and (19). Basically, model-independent information on $\Delta M$ can be derived, for example along the lines of Ref. [14], from the radiatively corrected value of $M_Z$ evaluated in the SM. The starting point is the relation

$$M^2_W = \frac{A}{s^2_W(1 - \Delta r)}, \quad (20)$$
where $A = \pi \alpha(m_e)/\sqrt{2}G_F$ and $\Delta r$ takes into account radiative corrections (depending on $M_W$, $M_Z$, $m_{top}$ and the Higgs mass $m_H$). Moreover, in the on-shell renormalization scheme [13, 16], the electroweak mixing angle including radiative corrections is expressed as

$$s_W^2 = 1 - M_W^2/M_Z^2,$$

reflecting the weak isospin doublet or singlet character of the Higgs fields. The fact that the $W$ mass and couplings are unaffected (at tree level) by the $Z'$ allows the use of the $M_W$ measured at the Tevatron to extract $s_W^2$ in terms of $M_W$, $m_{top}$ and $m_H$ from Eq. (20). Replacing the so determined $s_W$ into Eq. (21) determines the predicted SM value of $M_Z$ in terms of $m_{top}$ and $m_H$, which must be compared to the value of $M_{Z1}$ determined by LEP data. From the current values of $M_W$, $M_{Z1}$ and $m_{top}$ [12], and for $m_H$ in the range 100-500 GeV, one finds for $\Delta M$ an upper limit of the order of 200 MeV. This is compatible with the updated analysis of $\Delta M$ in Ref. [17]. Furthermore, current limits on the mixing angle $|\phi|$ are in the range $10^{-3} - 10^{-2}$, mostly from LEP data [1, 17, 18].

In addition to the $Z$ mass-shift, the $Z-Z'$ mixing induces a change in the couplings of the $Z$ to fermions. In the considered models, the neutral current coupled to the $Z'$ is parametrized in terms of an angle $\beta$ specifying the orientation of the $U(1)'$ generator in the $E_6$ group space [19]. Explicitly:

$$v' = \cos \beta \frac{c_W}{\sqrt{6}}, \quad a' = \frac{1}{2c_W\sqrt{6}} \left( \cos \beta + \sqrt{\frac{5}{3}} \sin \beta \right).$$

(22)

In general, $\cos \beta$ can range from $-1$ to $+1$. Special values are $\beta = 0; \pi/2; \pi - \arctan \sqrt{5/3} \approx 128^\circ$, which specify the so-called $\chi$, $\psi$ and $\eta$ models, respectively.

While being in general an independent parameter, in specific $E_6$ models where the $SU(2)_L$ doublet and singlet Higgses all arise from the $27$ representation of $E_6$, the mixing angle $\phi$ can be related to the values of $M_{Z1}$ and $M_{Z2}$ [13]. For $M_{Z2}$ much larger than $M_{Z1}$ the relation can be written to a good approximation as:

$$\phi \simeq -\sin^2 \theta_W \frac{\sum_i < \Phi_i >^2 I_{3L}^2 Q_{i}'}{\sum_i < \Phi_i >^2 (I_{3L}^2)^2} = C \frac{M_{Z2}^2}{M_{Z1}^2},$$

(23)

where, depending on the model, $< \Phi_i >$ are the Higgs vacuum expectation values spontaneously breaking the gauge symmetry, and $Q_{i}'$ are their $U(1)'$ charges. For

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4In extended electroweak models, $m_H$ effectively indicates the combined contribution of scalar fields to the radiative corrections to $M_W$. 

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example, in the case of $E_6$ ‘superstring’ inspired models, $C$ can be expressed as

$$C = 4s_W \left( \frac{\cos \beta}{2\sqrt{6}} - \frac{\sigma - 1}{\sigma + 1} \frac{10 \sin \beta}{12} \right),$$

(24)

where $\sigma$ is a ratio of vacuum expectation values squared.

With reference to the couplings introduced in Eq. (1), taking Eq. (19) into account we have to first order in $\phi$, in a self-explaining notation:

$$(v_1, a_1) \simeq (v + v' \phi, a + a' \phi) \Rightarrow (\Delta v, \Delta a) \simeq (v' \phi, a' \phi),$$

(25)

and

$$(v_2, a_2) \simeq (-v \phi + v', -a \phi + a'),$$

(26)

and

$$g_{WWZ_1} \simeq g_{WWZ}; \quad g_{WWZ_2} \simeq -g_{WWZ} \phi.$$  

(27)

Replacing Eqs. (25)-(27) into (8) and (9), one finds the general form of $\delta_\gamma$ and $\delta_Z$ for $E_6$ models:

$$\delta_\gamma = v \cot \theta_W \phi \left( \frac{a'}{a} - \frac{v'}{v} \right) \left( 1 - \frac{\chi}{\chi} + \Delta \chi \right) \chi,$$

(28)

$$\delta_Z = \cot \theta_W \left[ \frac{a'}{a} \left( 1 - \frac{\chi}{\chi} \right) + \Delta \chi \right].$$

(29)

Although present in general, at the order we are working here we could safely neglect $\Delta \chi$ which is of order $\phi^2$ due to (10) and (19). In this case, from Eqs. (28) and (29), there is the linear relation between $\delta_\gamma$ and $\delta_Z$ which only depends on the $Z'$ couplings to fermions and is independent from $\phi$ and $M_{Z_2}$:

$$\delta_Z = \delta_\gamma \frac{1}{v \chi} \frac{(a'/a)}{(a'/a) - (v'/v)}.$$  

(30)

Eq. (30) represents straight lines in the plane of Fig. 1, and we explicitly report the representatives of models $\psi$, $\chi$ and $\eta$. As one can see, sensitivities are different: as indicated by the intersection points of these lines with the allowed bands, while $\sigma^R$ mostly constrains the models $\psi$ and $\eta$, $\sigma^L$ is the most sensitive to the $\chi$ model. Furthermore, the allowed range of variation of $\delta_\gamma$ and $\delta_Z$ for the specific models is defined by the intersections of the corresponding lines with the boundaries of the, model-independent, allowed region obtained from the combination of $\sigma^R$ and $\sigma^L$.

One should also remark that, without the information from $\sigma^R$, models represented

\footnote{Taking $\Delta \chi$ into account would slightly shift the origin in Fig. 1.}
by lines almost parallel to the band determined by $\sigma_{unpol}$ (e.g., the $\psi$ model), would remain essentially unconstrained.

The ranges of $\delta_\gamma$ and $\delta_Z$ allowed to the specific models in Fig. 1 can be translated into limits on the mixing angle $\phi$ and the heavier gauge boson mass $M_{Z_2}$, using Eqs. (28)-(30). Starting our discussion from the $\psi$ model, the resulting allowed region (at the 95% CL) in the $(\phi, M_{Z_2})$ plane is limited in this case by the thick solid line in Fig. 2. We have chosen $\Delta M$ equal to the previously mentioned upper limit of 200 $MeV$, although the limiting curves do not appreciably depend on the specific value of this quantity. Also, the indicative current bound on $M_{Z_2}$ from direct searches is reported in this figure.

Fig. 2 shows that the process $e^+e^- \rightarrow W^+W^-$ at 500 $GeV$ has a potential sensitivity to the mixing angle $\phi$ of the order of $10^{-4} - 10^{-3}$, depending on the mass $M_{Z_2} \gg M_{Z_1}$ ranging up from the current lower bound of 500 $GeV$. Specifically, as it is seen from Eqs. (28) and (29), for the higher masses $M_{Z_2}$ much larger than $\sqrt{s}$ such that the $Z_2$ exchange contribution $|\chi_2/\chi|$ is much less than unity, the limiting contour is mostly determined by the modification (25) of the $Z$ couplings to electrons. Asymptotically, in the limit $M_{Z_2} \rightarrow \infty$ where $(1 - \chi_2/\chi) \rightarrow 1$, the limiting angle is $\phi \sim 10^{-3}$. In the region $\sqrt{s} < M_{Z_2} < \sqrt{2s}$ one has to account for (at least) the imaginary part of the $Z_2$ propagator $\chi_2$ by the replacement $M_{Z_2}^2 \rightarrow M_{Z_2}^2 - i M_{Z_2} \Gamma_{Z_2}$ in Eq. (14). Clearly, the sensitivity to $\phi$ in this mass range is dominated by the interference $\chi_2/\chi$ in Eqs. (28) and (29) and is enhanced with respect to that expected in the higher $Z_2$ mass range by a factor which, for the specific values $M_{Z_2} = \sqrt{s} \pm \frac{\Gamma_{Z_2}}{2}$, can reach the value

$$|\text{Re} \frac{\chi_2}{\chi}| \sim \frac{M_{Z_2}}{2 \Gamma_{Z_2}} \sim 20.$$  \hspace{0.5cm} \hfill (31)

The factor 20 in Eq. (31) conservatively assumes $n_g = 3$ exotic heavy fermions generations that, in addition to the conventional fermions, can contribute to $Z_2$ decay without significant phase space suppression. In this case, one approximately expects $\Gamma_{Z_2} \simeq 0.025 M_{Z_2}$, independent of $\cos \beta$ and $\phi$ [20]. In this situation, from the above factor we can qualitatively estimate a sensitivity to values of $\phi$ of the order of $10^{-4}$ or less. Even more stringent numerical constraints, by a factor of order 2-5, would be obtained if $n_g = 0$, which would imply a smaller value of $\Gamma_{Z_2}$.

To complete the discussion on the bounds from process (1), one should make a comparison of the results presented so far with the maximal allowed region to $\phi$
and $M_{Z_2}$ determined in a model- and process-independent way by the limit on the off-diagonal mass-matrix element. The relevant boundary contours, Eq. (19), are represented in Fig. 2 by the dotted lines, corresponding to the chosen upper limit for $\Delta M$. Clearly, for values of $M_{Z_2}$ higher than the intersection of these lines with the thick solid line, Eq. (19) gives the most stringent bounds on ($\phi, M_{Z_2}$). In this regard, it is useful to compare also with the specific ‘superstring inspired’ model previously introduced, where $\phi$ and $M_{Z_2}$ are uniquely related through Eq. (23). Such relation is illustrated in Fig. 2 by the continuous thin lines, which correspond to $\sigma = 0$ and $\sigma = \infty$ in (24) respectively, as representative cases.

In Fig. 2, we consider also the limits on $\phi$ and $M_{Z_2}$ that are expected from the annihilation into lepton pairs at $\sqrt{s} = 0.5 \, \text{TeV}$. It should be emphasized that this process can give alternative (and competitive) bounds, through the combination of the (almost $\phi$-independent) lower bound on $M_{Z_2}$ with the area allowed by Eq. (19).

Up to this point, the discussion has been based on the assumption that signals of the $Z_2$ were not observed within the accuracy, and the corresponding limits have been assessed. One could consider the reverse scenario, and assume that the $Z_2$ is observed with a mass lower than the discovery limit in Fig. 2. Clearly, from higher peak cross sections and almost $\phi$-independence of the annihilation $e^+e^- \rightarrow f\bar{f}$, we expect that such discovery should be more probable in that reaction. However, in this scenario, the usefulness of $e^+e^- \rightarrow W^+W^-$ would be not only to confirm the existence of the $Z_2$ but, especially important, to ‘probe’ the $Z-Z'$ mixing angle with the high accuracy indicated in Fig. 2. As previously mentioned, the highest sensitivity on $\phi$ would be obtained at CM energy $\sqrt{s} = M_{Z_2} \pm \frac{\Gamma_{Z_2}}{2}$, and quantitatively we represent it in Fig. 3, where the thick and thin solid lines correspond to $\sigma^L$ and $\sigma^R$ respectively, both for variable $\cos \beta$ and for the specific models $\psi$, $\chi$ and $\eta$. In particular, once again Fig. 3 shows also the complementary role of the two possible initial beam longitudinal polarizations. Actually, the tiny ‘around-resonance’ $\phi$-values in Fig. 3 are more illustrative than really quantitative, because in this case the practical analysis should be supplemented by radiative corrections in that energy range, which would require separate consideration. For the ‘off-resonance’ case $\sqrt{s} \ll M_{Z_2}$ this problem should be less important, because the $\phi$-values probed there correspond to relative amplitude deviations from the SM of order $0.1$, probably much larger than the effect.

\[\text{For this process, we use a systematic uncertainty } \delta_{\text{syst}} = 1\%.\]
of electroweak corrections [21].

The analogue of Fig. 2 for the NLC500 is depicted in Fig. 4 for the case $\sqrt{s} = 1\, \text{TeV}$ and $L_{\text{int}} = 200\, \text{fb}^{-1}$. The bounds from process (1) at this energy are well consistent with those in Fig. 2 through the anticipated approximate scaling law $\sqrt{L_{\text{int}} s}$ for the sensitivity $S$. In Figs. 5 to 8 we report the results for the models $\eta$ and $\chi$, analogous to Figs. 2 and 4. The general features pointed out above for the $\psi$ model also hold for these other cases, and the corresponding sensitivities can be directly read from the figures.

In conclusion, we have discussed the possibility of probing $Z-Z'$ mixing at future $e^+e^-$ linear colliders, via the measurement of $e^+e^- \to W^+W^-$ cross sections with longitudinally polarized beams. While the corresponding bounds at the NLC500 are found to complement current ones, they rather rapidly improve at the higher energies. Also, such bounds have been compared to the ones generally obtainable from the consideration of the nondiagonal entries of the $Z-Z'$ mass-matrix, as well as with the ‘superstring inspired’ extended gauge models. Moreover, if the $Z'$ was discovered, presumably in $e^+e^- \to f\bar{f}$, measurements of $e^+e^- \to W^+W^-$ for energy around the $Z'$ peak would provide either, perhaps, a ‘direct’ measurement of $Z-Z'$ mixing or, in any case, a strong limit on such effects. These features of $e^+e^- \to W^+W^-$ seem particularly useful in the case of the specific extended gauge models considered here, and the benefits of initial beams polarization in this kind of analysis are clearly manifest.

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**Figure captions**

**Fig. 1** Upper bounds (95% CL) on non-standard couplings \((\delta_\gamma, \delta_Z)\) from \(e^+e^- \to W^+W^-\) with longitudinally polarized electrons at \(\sqrt{s} = 0.5\, \text{TeV}\) and integrated luminosity \(L_{\text{int}} = 50\, \text{fb}^{-1}\). \(\sigma^L\), \(\sigma^R\) and \(\sigma^{\text{unpol}}\) refer to allowed regions obtained from polarized cross sections with degrees of polarization \(P_L = -0.9; +0.9; 0\), respectively. The straight dotted lines represent relation (30) for specific models: \(\psi, \eta,\) and \(\chi\).

**Fig. 2** Allowed domains (95% C.L.) on \((\phi, M_{Z_2})\) for the \(\psi\) model. The thick solid contour corresponds to the region obtained at the NLC500 from \(e^+e^- \to W^+W^-\). Also, the current limit on \(M_{Z_2}\) and the one expected from \(e^+e^- \to l^+l^-\) at \(\sqrt{s} = 0.5\, \text{TeV}\) are indicated. The dotted lines correspond to the constraints derived from the \(Z - Z'\) mass-matrix [13] with \(\Delta M = 0.2\, \text{GeV}\). The thin solid contour shows the constraint for the ‘superstring’ model case, Eq. (23) for \(\sigma = 0\) and \(\sigma = \infty\).

**Fig. 3** Upper limits (95% C.L.) for \(\phi\) vs. the \(E_6\) model parameter \(\cos \beta\) from \(W\) pair production at \(M_{Z_2} = \sqrt{s} \pm \frac{\Gamma_{Z_2}}{2}\) and with \(P_L = -0.9\) (thick solid line) and \(P_L = +0.9\) (thin solid line).

**Fig. 4** Same as Fig. 2, for the NLC1000 with \(\sqrt{s} = 1\, \text{TeV}\) and \(L_{\text{int}} = 200\, \text{fb}^{-1}\).

**Fig. 5** Same as Fig. 2 for the \(\eta\) model (NLC500).

**Fig. 6** Same as Fig. 2 for \(\eta\) model (NLC1000).

**Fig. 7** Same as Fig. 2 for \(\chi\) model (NLC500).

**Fig. 8** Same as Fig. 2 for \(\chi\) model (NLC1000).
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