Comments on Penrose Limit of $AdS_4 \times M^{1,1,1}$

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abstract

We construct a Penrose limit of $AdS_4 \times M^{1,1,1}$ where $M^{1,1,1} = \frac{SU(3) \times SU(2) \times U(1)}{SU(2) \times U(1) \times U(1)}$ that provides the pp-wave geometry equal to the one in the Penrose limit of $AdS_4 \times S^7$. There exists a subsector of three dimensional $\mathcal{N} = 2$ dual gauge theory which has enhanced $\mathcal{N} = 8$ maximal supersymmetry. We identify operators in the $\mathcal{N} = 2$ gauge theory with supergravity KK excitations in the pp-wave geometry and describe how the gauge theory operators made out of two kinds of chiral fields of conformal dimension $4/9$, $1/3$ fall into $\mathcal{N} = 8$ supermultiplets.
1 Introduction

Recently [1], it was found that the large $N$ limit of a subsector of four dimensional $\mathcal{N} = 4$ $SU(N)$ supersymmetric gauge theory is dual to type IIB string theory in the pp-wave background [2, 3]. In the $N \to \infty$ with the finiteness of string coupling constant, this subspace of the gauge theory and the operator algebra are described by string theory in the pp-wave geometry. By considering a scale limit of the geometry near a null geodesic in $AdS_5 \times S^5$, it leads to the appropriate subspace of the gauge theory. The operators in the subsector of $\mathcal{N} = 4$ gauge theory can be identified with the excited states in the pp-wave background.

There are many papers [4]-[51] related to the work of [1]. There exist some works [7, 8, 10] on the Penrose limit of $AdS_5 \times T^{1,1}$ that gives the pp-wave geometry of $AdS_5 \times S^5$. There is a subsector of $\mathcal{N} = 1$ gauge theory that contains an enhanced $\mathcal{N} = 4$ supersymmetry. The corresponding operators in the gauge theory side were identified with the stringy excitations in the pp-wave geometry and some of the gauge theory operators are combined into $\mathcal{N} = 4$ supersymmetry multiplets [8]. Moreover, it was found in [2] that a subsector of $d = 3, \mathcal{N} = 2$ gauge theory dual to $AdS_4 \times Q^{1,1,1}$ has enhanced $\mathcal{N} = 8$ maximal supersymmetry and the gauge theory operators are combined into $\mathcal{N} = 8$ chiral multiplets.

In this paper, we consider a similar duality that is present between a certain three dimensional $\mathcal{N} = 2$ gauge theory and 11-dimensional supergravity theory in a pp-wave background with the same spirit as in [2, 7, 8, 10]. We describe this duality by taking a scaling limit of the duality between 11-dimensional supergravity on $AdS_4 \times M^{1,1,1}$ where $M^{1,1,1}$ was considered first in [53] and three dimensional superconformal field theory that consists of an $\mathcal{N} = 2$ $SU(N) \times SU(N)$ gauge theory with two kinds of chiral fields $U_i, i = 1, 2, 3$ transforming in the $(\Box, \Box^*)$ color representation and $V_A, A = 1, 2$ transforming in the $(\Box \Box^*, \Box \Box)$ color representation [54]. The complete analysis on the spectrum of $AdS_4 \times M^{1,1,1}$ was found by [55]. This gives the theory of [54] that lives on $N$ M2-branes at the conical singularity of a Calabi-Yau four-fold. The scaling limit is obtained by considering the geometry near a null geodesic carrying large angular momentum in the $U(1)_R$ isometry of the $M^{1,1,1}$ space which is dual to the $U(1)_R$ R-symmetry in the $\mathcal{N} = 2$ superconformal field theory. In section 2, we construct the scaling limit around a null geodesic in $AdS_4 \times M^{1,1,1}$ and obtain a pp-wave background. In section 3, we identify supergravity KK excitations obtained by [55] in the Penrose limit with gauge theory operators. In section 4, we summarize our results.

2 Penrose Limit of $AdS_4 \times M^{1,1,1}$

Let us start with the supergravity solution dual to the $\mathcal{N} = 2$ superconformal field theory [54]. By putting a large number of $N$ coincident M2 branes at the conifold singularity and taking
the near horizon limit, the metric becomes that \[6, 57, 58\] of \(AdS_4 \times M^{1,1,1}\) (See also \[53, 56\])
\[
ds_{11}^2 = ds_{AdS_4}^2 + ds_{M^{1,1,1}}^2,
\]
where
\[
ds_{AdS_4}^2 = L^2 \left( - \cosh^2 \rho \, dt^2 + d\rho^2 + \sinh^2 \rho \, d\Omega_2^2 \right),
\]
\[
ds_{M^{1,1,1}}^2 = \frac{L^2}{64} \left( d\tau + 3 \sin^2 \mu \sigma_3 + 2 \cos \theta d\phi \right)^2 + \frac{3L^2}{4} \left( d\mu^2 + \frac{1}{4} \sin^2 \mu \left( \sigma_1^2 + \sigma_2^2 + \cos^2 \mu \sigma_3^2 \right) \right)
+ \frac{L^2}{8} \left( d\theta^2 + \sin^2 \theta d\phi^2 \right),
\]
where \(d\Omega_2\) is the volume form of a unit \(S^2\) and the curvature radius \(L\) of \(AdS_4\) is given by \((2L)^6 = 32\pi^2 \ell_p^6 N\). Topologically \(M^{1,1,1}\) is a nontrivial \(U(1)\) bundle over \(\mathbb{CP}^2 \times S^2\). The spherical coordinates \((\theta, \phi)\) parametrize two sphere, as usual, and the angle \(\mu\) and three real one forms \(\sigma_1, \sigma_2\) and \(\sigma_3\) parametrize \(\mathbb{CP}^2\) satisfying the \(SU(2)\) algebra and the angle \(\tau\) parametrizes the \(U(1)\) Hopf fiber. The angles vary over the ranges, \(0 \leq \theta \leq \pi, 0 \leq \phi \leq 2\pi, 0 \leq \tau \leq 4\pi\) and \(0 \leq \mu \leq \pi/2\). The \(SU(3) \times SU(2) \times U(1)\) isometry group of \(M^{1,1,1}\) consists of \(SU(3) \times SU(2)\) global symmetry and \(U(1)_R\) symmetry of the dual superconformal field theory of \[54\].

Let us make a scaling limit around a null geodesic in \(AdS_4 \times M^{1,1,1}\) that rotates along the \(\tau\) coordinate of \(M^{1,1,1}\) whose shift symmetry corresponds to the \(U(1)_R\) symmetry in the dual superconformal field theory. Let us introduce coordinates which label the geodesic
\[
x^+ = \frac{1}{2} \left( t + \frac{1}{8} (\tau + 2\phi) \right), \quad x^- = \frac{L^2}{2} \left( t - \frac{1}{8} (\tau + 2\phi) \right),
\]
and make a scaling limit around \(\rho = 0 = \mu = \theta\) in the above geometry \[1\]. By taking the limit \(L \to \infty\) while rescaling the coordinates
\[
\rho = \frac{r}{L}, \quad \mu = \frac{\zeta}{\sqrt{3}L}, \quad \theta = \frac{\sqrt{2\zeta}}{L},
\]
the Penrose limit of the \(AdS_4 \times M^{1,1,1}\) becomes
\[
ds_{11}^2 = -4dx^+ dx^- + \sum_{i=1}^3 \left( dr^i dr^{i'} - r^i r^{i'} dx^+ dx^+ \right)
+ \frac{1}{4} \left( d\xi^2 + \xi^2 d\phi^2 - 2\xi^2 d\phi dx^+ \right) + \frac{1}{4} \left( d\zeta^2 + \zeta^2 \left( \bar{\sigma}_1^2 + \bar{\sigma}_2^2 + \bar{\sigma}_3^2 \right) + 2\xi^2 \bar{\sigma}_3 dx^+ \right)
\]

|1| Sometimes instead of using the notation \(M^{pqr}\), \((m,n)\) space with two parameters \(m\) and \(n\) (that are two winding numbers of the \(U(1)\) gauge field over the \(\mathbb{CP}^2\) and \(S^2\) of the base manifold) rather than three parameters \(p, q, r\) in \(M^{pqr}\) is used where the parameters are related by \(m/n = 3p/(2q)\). The integers \(p, q, r\) and \(r\) characterize the embedding of \(SU(2) \times U(1) \times U(1)\) in \(SU(3) \times SU(2) \times U(1)\). For \(M^{3,2} = M^{1,1,1}\), there is an \(\mathcal{N} = 2\) supersymmetry while for all other \(M(m,n)\) no supersymmetry survives. In particular, \(M(0,1) = M^{0,1,0} = \mathbb{CP}^2 \times S^3\), \(M(1,0) = S^3 \times S^2\) and \(M^{1,0,1} = (S^3/Z_3) \times S^2\).
have conformal weights $\frac{4}{n}$ in the $(\bullet \bullet \bullet)$ of the theory. In each multiplet, we specify a $SU(3)$ and long multiplets with operators in the gauge theory and focus only on the bosonic excitations $SU(3)$ by a Young diagram with 2 complex coordinates $w$ where in the last line we introduce the complex coordinate $\partial/\partial w$ encoded in a quiver diagram with gauge group $U$. The supersymmetry enhancement in the Penrose limit implies that a hidden $R$-charge tells us that the conformal dimensions of $U(N)$ symmetry is present in the corresponding subsector of the dual $\mathcal{N} = 2$ superconformal field theory. We identify states in the supergravity containing both short and long multiplets with operators in the gauge theory and focus only on the bosonic excitations of the theory. In each multiplet, we specify a $SU(3) \times SU(2)$ representation $[\mathfrak{a}]$, conformal weight and $R$-charge.

$$ds^2_{11} = -4dx^+dx^- + \sum_{i=1}^{9} (dr^i dr^i - r^i r^i dx^+ dx^+)$$

$$+ \frac{1}{4} (dwd\bar{w} + i (\bar{w} dw - wd\bar{w}) dx^+) + \frac{1}{4} \sum_{i=1}^{2} (dz_i d\bar{z}_i - i (z_i d\bar{z}_i - \bar{z}_i dz_i)) dx^+$$

where in the last line we introduce the complex coordinate $w = \xi e^{i\phi}$ for $R^2$ and a pair of complex coordinates $z_1$ and $z_2$ for $R^4$. Since the metric has a covariantly constant null Killing vector $\partial/\partial \tau$, it is also pp-wave metric. The pp-wave has a decomposition of the $R^9$ transverse space into $R^3 \times R^2 \times R^4$ where $R^3$ is parametrized by $r^i$, $R^2$ by $w$ and $R^4$ by $z_1$ and $z_2$. The symmetries of this background are the $SO(3)$ rotations in $R^3$. In the gauge theory side, the $SO(3)$ symmetry corresponds to the subgroup of the $SO(2, 3)$ conformal group. Note that the pp-wave geometry $[\mathfrak{a}]$ in the scaling limit reduces to the maximally $\mathcal{N} = 8$ supersymmetric pp-wave solution of $AdS_4 \times S^7$ $[11, 12, 17, 18]$ through $w = e^{i\tau} \bar{w}$ and $z_i = e^{-i\tau} \bar{z}_i$

The supersymmetry enhancement in the Penrose limit implies that a hidden $\mathcal{N} = 8$ supersymmetry is present in the corresponding subsector of the dual $\mathcal{N} = 2$ superconformal field theory. In the next section, we provide precise description of how to understand the excited states in the supergravity theory that corresponds in the dual superconformal field theory to operators with a given conformal dimension.

3 Gauge Theory Spectrum

The 11-dimensional supergravity theory in $AdS_4 \times M^{1,1,1}$ is dual to the $\mathcal{N} = 2$ gauge theory encoded in a quiver diagram with gauge group $SU(N) \times SU(N)$ with two kinds of chiral fields $U_i, i = 1, 2, 3$ transforming in the $(\Box, \Box^*)$ color representation and $V_A, A = 1, 2$ transforming in the $(\Box^2, \Box^2)$ color representation $[54]$. At the fixed point, these chiral superfields $U, V$ have conformal weights $4/9, 1/3$ respectively $[\mathfrak{a}]$ and transform as $(3, 1)$ and $(1, 2)$ under the $SU(3) \times SU(2)$ global symmetry. We identify states in the supergravity containing both short and long multiplets with operators in the gauge theory and focus only on the bosonic excitations of the theory. In each multiplet, we specify a $SU(3) \times SU(2)$ representation $[\mathfrak{a}]$, conformal weight and $R$-charge.

2 There exist two supersymmetric 5-cycles that are the restrictions of the $U(1)$ fibration over $P^{2a}$ and $P^1 \times P^1$. One can determine the dimensions of the baryons by computing the ratio of volume of 5-cycle to the one of $M^{1,1,1}$. It turns out that $N$ product of $U$ is equal to $4N/9$ and $N$ product of $V$ is equal to $N/3$. Therefore this tells us that the conformal dimensions of $U$ and $V$ are $4/9$ and $1/3$ respectively $[54]$.

3 A representation of $SU(3)$ can be identified by a Young diagram and when we denote the Dynkin label $(M_1, M_2)$ so that totally we have $M_1 + 2M_2$ boxes, the dimensionality of an irreducible representation is $N(M_1, M_2) = (1 + M_1)(1 + M_2)(\frac{1 + M_1 + M_2}{2})$. Also an irreducible representation of $SU(2)$ can be described by a Young diagram with $2J$ boxes. Its dimensionality is $2J + 1$. 
The energy spectrum on $SU_4$ of the massless vector multiplet denoted by (8, 1) of the flavor group and its conformal dimension is 1 with vanishing $R$-charge. Therefore massless multiplets 1), 2) and 3) saturate the unitary bound and have a conformal toric description \[54\], it is conserved and corresponds to additional massless vector multiplets.

There exists a stress-energy tensor superfield $T_{\alpha \beta}(x, \theta)$ that satisfies the equation for conserved current $D^\pm T_{\alpha \beta}^\pm(x, \theta) = 0$. This $T_{\alpha \beta}(x, \theta)$ is a singlet with respect to the flavor group $SU(3) \times SU(2)$ and its conformal dimension is 2. Moreover $R$ charge is 0. So this corresponds to the massless graviton multiplet that propagates in the $AdS_4$ bulk.

There exists a conserved vector current, a scalar superfield $J_{betti} = 2\sqrt{\alpha_q - 3VY}$ of the first factor $SU(3)$ of the flavor group and its conformal dimension is 1 with vanishing $R$-charge. This corresponds to the massless vector multiplet (8, 1) propagating in the $AdS_4$ bulk. There is also massless vector multiplet denoted by (1, 3). There exists a corresponding scalar superfield $J_{SU(2)}(x, \theta)$, to the generator of the flavor symmetry group $SU(2)$ satisfying the conservation equations $D^\pm D^\pm J_{SU(2)}(x, \theta) = 0$. This $J_{SU(2)}(x, \theta)$ transforms in the adjoint representation 3 of the $SU(2)$ of the flavor group and its conformal dimension is 1 with vanishing $R$-charge.

3) Two massless vector multiplets: $(1, 1)$, \( \Delta = 1 \), \( R = 0 \).

It is known that the Betti current \( J_{\text{betti}} = 2\sqrt{\alpha_q - 3VY} \) of $M^{1,1,1}$ is obtained from the toric description \[54\], it is conserved and corresponds to additional massless vector multiplets. Therefore massless multiplets 1), 2) and 3) saturate the unitary bound and have a conformal weight related to the maximal spin.

Short multiplets \[55\]

It is known that the dimension of the scalar operator in terms of energy labels, in the dual SCFT corresponding $AdS_4 \times M^{1,1,1}$ is

\[
\Delta = \frac{3}{2} + \frac{1}{2}\sqrt{1 + \frac{m^2}{4}} = \frac{3}{2} + \frac{1}{2}\sqrt{45 + \frac{E}{4} - 6\sqrt{36 + E}}.
\]

The energy spectrum on $M^{1,1,1}$ exhibits an interesting feature which is relevant to superconformal algebra and it is given by \[55\]

\[
E = \frac{64}{3} \left( \frac{1}{M_1 + M_2 + M_1 M_2} + \frac{1}{2} J(J + 1) + \frac{1}{8} Y^2 \right)
\]

Of course, for general $M^{ppr}$ space, it is known that the energy spectrum on this space is given by

\[
E = \gamma^2 \left( \frac{2}{3} \alpha \frac{q^2}{p^2} (M_1 + M_2 + M_1 M_2) + 2\beta \left( J + J^2 - \frac{1}{4} q^2 Y^2 \right) + \frac{1}{4} q^2 Y^2 \right)
\]

where $M_1 = 0, 1, 2, \cdots$, $M_2 = M_1 + \frac{2}{3} p Y$, $J = \frac{1}{2} q Y$, $|\frac{1}{2} q Y| + 1, \cdots$, and $Y = 0, \pm 2, \pm 4, \cdots$. For $Y < 0$, $M_2 = 0, 1, \cdots$ and $M_1 = M_2 + \frac{2}{3} p |Y|$. Here $\alpha$, $\beta$ and $\gamma$ are related to the scale parameters. In particular, on the supersymmetric space $M^{1,1,1}$, we have $p = q = 1$, $\alpha = \frac{1}{4}$, $\beta = \frac{1}{4}$ and $\gamma = 8$. 

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\[4\]
where the eigenvalue $E$ is classified by $SU(3)$ quantum numbers $(M_1, M_2)$, $SU(2)$ isospin $J$ and weak hypercharge $Y$: $M_1 = 0, 1, 2, \cdots$, $M_2 = M_1 + \frac{3Y}{2}$, $J = |Y/2|, |Y/2| + 1, \cdots$ and $Y = 0, \pm 2, \pm 4, \cdots$. For $Y < 0$, $M_2 = 0, 1, 2, \cdots$ and $M_1 = M_2 + \frac{3|Y|}{2}$.

The corresponding eigenmodes occur in $(M_1, M_1 + \frac{3|Y|}{2})$ for $Y > 0$ or, $(M_2 + \frac{3|Y|}{2}, M_2)$ for $Y < 0$ $SU(3)$ representation, the angular momentum $J$ $SU(2)$ representation and $U(1)$ charge $Y/4$. The eigenvalues (4) as a linear combination of the quadratic Casimirs for the symmetry group $SU(3) \times SU(2) \times U(1)$ are the form for a coset manifold [58] sometime ago.

The $U(1)$ part of the isometry group of $M^{1,1,1}$ acts by shifting $U(1)$ weak hypercharge $Y$. The half-integer $R$-charge, $R$ is related to $U(1)$ charge $Y$ by $R = Y$. Let us take $R \geq 0$. One can do similar case for $R < 0$. One can find the lowest value of $\Delta$ is equal to $R$ corresponding to a mode scalar with $M_1 = 0$, $M_2 = 3R/2$ and $J = R/2$ because $E$ becomes $16R(R + 3)$ and plugging back to (3) then one obtains $\Delta = R$.

Thus we find a set of operators filling out a $((1 + \frac{3R}{2})(2 + \frac{3R}{2}), R + 1)$ multiplet of $SU(3) \times SU(2) \times U(1)$ where the number $\frac{(1 + \frac{3R}{2})(2 + \frac{3R}{2})}{2}$ is the dimension of $SU(3)$ representation while $R + 1$ is the dimension of $SU(2)$ representation. The condition $\Delta = R$ saturates the bound on $\Delta$ from superconformal algebra. The fact that the $R$-charge of a chiral operator is equal to the dimension was observed in [59] in the context of $R$ symmetry gauge field.

1) One hypermultiplet:

$$\left( \frac{(1 + \frac{3R}{2})(2 + \frac{3R}{2})}{2}, R + 1 \right), \quad \Delta = R.$$

According to [58], the information on the Laplacian eigenvalues allows us to get the spectrum of hypermultiplets of the theory corresponding to the chiral operators of the SCFT. This part of spectrum was given in [54] and the form of operators is

$$\text{Tr} \Phi_C \equiv \text{Tr}(U^3 V^2)^{R/2} \quad (5)$$

where the flavor $SU(3)$ and $SU(2)$ indices are totally symmetrized and the chiral superfield $\Phi_C(x, \theta)$ satisfies $D^+_a \Phi_C(x, \theta) = 0$. The hypermultiplet spectrum in the KK harmonic expansions on $M^{1,1,1}$ agrees with the chiral superfield predicted by the conformal gauge theory. From this, the dimension of $U^3 V^2$ should be 2 to match the spectrum. In fact, the conformal weight of a product of chiral fields equals the sum of the weights of the single components. This is due to the the relation of $\Delta = R$ satisfied by chiral superfields and to the additivity of the $R$-charge.

2) One short graviton multiplet: $\left( \frac{(1 + \frac{3R}{2})(2 + \frac{3R}{2})}{2}, R + 1 \right)$, $\Delta = R + 2$

The gauge theory interpretation of this multiplet is obtained by adding a dimension 2 singlet operator with respect to flavor group into the above chiral superfield $\Phi_C(x, \theta)$. We consider $\text{Tr} \Phi_{\alpha\beta} \equiv \text{Tr} \left( T_{\alpha\beta} \Phi_C \right)$, where $T_{\alpha\beta}(x, \theta)$ is a stress energy tensor and $\Phi_C(x, \theta)$ is a chiral
Therefore the short superfield \( (5) \). There exists other short multiplet by a conformal dimension related to the \( R \) integer values of operator satisfies \( D_\alpha^+ \Phi^{\alpha\beta}(x, \theta) = 0 \).

3) One short vector multiplet:

\[
\left( \frac{(1 + \frac{3R}{2})(2 + \frac{3R}{2})}{2}, R + 3 \right) \text{ or } \left( 2 + \frac{3R}{2})(4 + \frac{3R}{2}), R + 1 \right), \quad \Delta = R + 1.
\]

One can construct the following gauge theory object, corresponding to the short vector multiplet \( \left( \frac{(1 + \frac{3R}{2})(2 + \frac{3R}{2})}{2}, R + 3 \right) \), \( \text{Tr} \Phi \equiv \text{Tr} (J_{SU(2)}(x, \theta), \text{where} \ J_{SU(2)}(x, \theta) \text{is a conserved vector current transforming in the adjoint representation of SU(2) flavor group and} \ \Phi_c(x, \theta) \text{is a chiral superfield} \ (3). \text{There exists other short multiplet by} \ \left( (2 + \frac{3R}{2})(4 + \frac{3R}{2}), R + 1 \right). \text{Similarly one can consider} \ \text{Tr} \Phi \equiv \text{Tr} (J_{SU(3)}(x, \theta), \text{where} \ J_{SU(3)}(x, \theta) \text{is a conserved vector current transforming in the adjoint representation of SU(3) flavor group. In this case, we have} \ D^{+\alpha}D^+_{\alpha} \Phi(x, \theta) = 0. \text{Therefore the short OSp(2|4) multiplets 1), 2) and 3) saturate the unitary bound and have a conformal dimension related to the} \ R \text{-charge and maximal spin.}

- Long multiplets \[ 5 \]

Although the dimensions of nonchiral operators are in general irrational, there exist special integer values of \( n_i \) such that for \( M_1 = n_1, M_2 = n_1 + 3R/2 \) and \( J = R/2 + n_2, \) one can see the Diophantine like condition(See also \[ 63 \]),

\[
n_1^2 - n_1 + 3 \left( n_2^2 - n_2 \right) - 6n_1n_2 = 0 \tag{6}
\]

make \( \sqrt{36 + E} \) be equal to \( 4R + 2(2n_1 + 2n_2 + 3) \). Furthermore in order to make the dimension be rational(whose conformal dimensions are protected), \( 45 + E/4 - 6\sqrt{36 + E} \) in \( (3) \) should be square of something. It turns out this is the case without any further restrictions on \( n_i \)'s. Therefore we have \( \Delta = R + n_1 + n_2 \) which is \( \Delta_+ \) for \( \Delta \geq 3/2 \) and \( \Delta_- \) for \( \Delta \leq 3/2 \). This is true if we are describing states with finite \( \Delta \) and \( R \). Since we are studying the scaling limit \( \Delta, R \to \infty, \) we have to modify the above analysis. This constraint \( (3) \) comes from the fact that

\[ 5 \] Of course there exist two short gravitino multiplets specified by

\[
\left( 2 + \frac{3R}{2}(4 + \frac{3R}{2}), R + 2 \right), \quad \left( \frac{(1 + \frac{3R}{2})(2 + \frac{3R}{2})}{2}, R \right)
\]

with conformal weight, \( \Delta = R - 1/2 \) and \( R + 3/2 \) respectively.

\[ 6 \] The complete KK spectrum of the round \( S^7 \) compactification consists of short \( OSp(8|4) \) multiplets characterized by a quantization of masses in terms of the \( SO(8) \) R-symmetry representation \( (53) \). All the KK states are BPS states and their spectrum can be obtained by analyzing the short unitary irreducible representation of \( OSp(8|4) \). In the dual theory, all the composite primary operators have conformal weight equal to their naive dimensions. No anomalous dimensions are generated. However, the KK states with lower supersymmetry, for example, in \( AdS_4 \times M^{1,1,1} \) do not fall into short multiplets of \( OSp(2|4) \) and do not necessarily have quantized masses. Their masses depend not only on the \( R \)-symmetry representation but also on the gauge group \( SU(3) \times SU(2) \) in the supergravity side(or flavor group in the dual field theory). This implies that according to AdS/CFT correspondence, anomalous dimensions are generated.
the energy eigenvalue of the Laplacian on $M^{1,1,1}$ for the supergravity mode $\Psi$ takes the form

$$E = \frac{64}{3} n_1^2 + 32 n_2^2 + \frac{64}{3} \left( \frac{3}{2} R + 2 \right) n_1 + 32 (R + 1) n_2 + 16 R (R + 3). \quad (7)$$

One can show that the conformal weight of the long vector multiplet $A$ below becomes rational if the condition (6) is satisfied.

1) One long vector multiplet $A$:

$$\Delta = -\frac{3}{2} + \frac{1}{4} \sqrt{E + 36}. \quad (8)$$

However, as we take the limit of $R \to \infty$, this constraint (6) is relaxed. The combination of $\Delta - R$ is given by

$$\Delta - R = n_1 + n_2 + O\left( \frac{1}{R} \right)$$

where the right hand side is definitely rational and they are integers. So the constraint (3) is not relevant in the subsector of the Hilbert space we are interested in. Candidates for such states in the gauge theory side are given in terms of semi-conserved superfields [54]. Although they are not chiral primaries, their conformal dimensions are protected. The ones we are interested in take the following form,

$$\text{Tr} \Phi_{\text{s.c.}} \equiv \text{Tr} \left[ \left( J_{SU(3)} \right)^{n_1} \left( J_{SU(2)} \right)^{n_2} (U^3 V^2)^{R/2} \right] \quad (9)$$

where the scalar superfields $J_{SU(3)}(x, \theta)$ transform in the adjoint representation of flavor group $SU(3)$ and satisfy $D^+ a D^- a J_{SU(3)}(x, \theta) = 0$ with conformal dimension 1 and zero $U(1)_R$ charge. Similarly, the scalar superfields $J_{SU(2)}(x, \theta)$ transform in the adjoint representation of the flavor group $SU(2)$. Also we have $D^+ a D^- a \Phi_{\text{s.c.}}(x, \theta) = 0$. Since the singleton superfields $U^{ac}_{i,bd}$ carry indices $a, c$ in the $\Box$ of $SU(N)$ and indices $b, d$ in the $\Box^*$ of the $SU(N)$, the fields $V^{bf}_{A,ace}$ carry indices $a, c, e$ in the $\Box \Box^*$ of $SU(N)$ and indices $b, d, f$ in the $\Box \Box$ of the $SU(N)$, one can construct the following conserved flavor currents transforming $(8, 1)$ and $(1, 3)$ under $SU(3) \times SU(2)$ respectively

$$\left( J_{SU(3)} \right)^{\delta_{ij}}_{j_i} = U^j \nabla_{i_1} - \frac{\delta^{j_i}_{i_1}}{3} U \nabla,$$

$$\left( J_{SU(2)} \right)^{\delta_{ij}}_{j_i} = V^j \nabla_{i_2} - \frac{\delta_{ij}}{2} V \nabla,$$

where the color indices are contracted in the right hand side. Note that the conformal dimension of these currents is not the one of naive sum of $U$ and $\nabla$ and $V$ and $\nabla$. As we discussed in the last section, supergravity theory in $AdS_4 \times M^{1,1,1}$ acquires an enhanced $\mathcal{N} = 8$ superconformal symmetry in the Penrose limit. This implies that the spectrum of the gauge theory operators
in this subsector should fall into $\mathcal{N} = 8$ multiplets. We expect that both the chiral primary fields of the form $\text{Tr}(U^3 V^2)^{R/2}$ and the semi-conserved multiplets of the form (3) combine into make $\mathcal{N} = 8$ multiplets in the limit. Note that for finite $R$, the semi-conserved multiplets should obey the Diophantine constraint (3) in order for them to possess rational conformal weights.

In the remaining multiplets we consider the following particular representations in the global symmetry group:

$$\left(\frac{(1+n_1)(1+n_1+3R)}{2}, 2n_2 + R + 1\right).$$

2) One long graviton multiplet $h$: $\Delta = \frac{1}{2} + \frac{1}{4}\sqrt{E + 36}$

For finite $R$ with rational dimension, after inserting the $E$ into the above, we will arrive at the relation with same constraint (3) which is greater than (3) by 2:

$$\Delta - R = 2 + n_1 + n_2 + \mathcal{O}\left(\frac{1}{R}\right).$$

The gauge theory interpretation of this multiplet is quite simple. If we take a semi-conserved current $\Phi_{\text{s.c.}}(x, \theta)$ defined in (3) and multiply it by a stress-energy tensor superfield $T_{\alpha\beta}(x, \theta)$ that is a singlet with respect to the flavor group, namely $\text{Tr}(T_{\alpha\beta}\Phi_{\text{s.c.}})$, we reproduce the right $OSp(2\mid 4) \times SU(3) \times SU(2)$ representations of the long graviton multiplet. Also one can expect that other candidate for this multiplet with different representation by multiplying a semi-conserved current into a quadratic conserved scalar superfield: $\text{Tr}\left(J_{SU(3)} J_{SU(3)} \Phi_{\text{s.c.}}\right)$, $\text{Tr}\left(J_{SU(3)} J_{SU(2)} \Phi_{\text{s.c.}}\right)$ or $\text{Tr}\left(J_{SU(2)} J_{SU(2)} \Phi_{\text{s.c.}}\right)$. In this case, the constraint for finite $\Delta$ and $R$ is shifted as $n_1 \to n_1 + 2$, $n_1 \to n_1 + 1$, $n_2 \to n_2 + 1$ and $n_2 \to n_2 + 2$ respectively.

3) One long vector multiplet $Z$: $\Delta = \frac{1}{2} + \frac{1}{4}\sqrt{E + 32R + 36}$

Although there exists no rational dimension for this case with any choice of $n_i$'s when $\Delta$ and $R$ are finite, the combination of $\Delta - R$ with Penrose limit $R \to \infty$ in the gauge theory side becomes

$$\Delta - R = 3 + n_1 + n_2 + \mathcal{O}\left(\frac{1}{R}\right).$$

Since we do not have any singlet of conformal dimension 3 with respect to the flavor group, one cannot increase a conformal dimension by simply tensoring any extra superfields into a semi-conserved current in order to match the spectrum. So the only way to do this is to increase the number of conserved scalar superfield. In order to produce the following gauge theory operator $\text{Tr}\left(T_{\alpha\beta} J_{SU(3)} \Phi_{\text{s.c.}}\right)$ or $\text{Tr}\left(T_{\alpha\beta} J_{SU(2)} \Phi_{\text{s.c.}}\right)$ corresponding to this vector multiplet, one can think of a higher dimensional representation in the global symmetry $SU(3)$ or $SU(2)$. Then the constraint coming from the requirement of rationality of conformal dimension is also changed for finite $\Delta$ and $R$. One can describe also the product of cubic $J$'s and $\Phi_{\text{s.c.}}$ similarly.

4) One long vector multiplet $W$: $\Delta = \frac{5}{2} + \frac{1}{4}\sqrt{E + 36}$
In this case, we get $\Delta - R$ by adding 2 to the one in (10)

$$\Delta - R = 4 + n_1 + n_2 + O\left(\frac{1}{R}\right).$$

One can describe corresponding gauge theory operator by taking quadratic stress-energy tensor $T_{\alpha\beta}(x, \theta)$ and multiplying it into a semi-conserved current $\Phi_{S.C.}(x, \theta)$ in order to match with the conformal dimension. That is, one obtains $\text{Tr}\left(T_{\alpha\beta}T^{\alpha\beta}\Phi_{S.C.}\right)$. Similarly one can construct the following gauge theory operators related to this vector multiplet $\text{Tr}\left(T_{\alpha\beta}J_{SU(3)}J_{SU(3)}\Phi_{S.C.}\right)$, $\text{Tr}\left(T_{\alpha\beta}J_{SU(3)}J_{SU(2)}\Phi_{S.C.}\right)$ or $\text{Tr}\left(T_{\alpha\beta}J_{SU(2)}J_{SU(2)}\Phi_{S.C.}\right)$. For four $J$’s with $\Phi_{S.C.}$, one can analyze similarly.

5) One long vector multiplet $Z$: $\Delta = \frac{1}{2} + \frac{1}{4}\sqrt{E} + 4$

Although there exists no rational dimension for this case with any choice of $n_i$’s when $\Delta$ and $R$ are finite, the combination of $\Delta - R$ with Penrose limit $R \to \infty$ in the gauge theory side becomes $\Delta - R = 2 + n_1 + n_2 + O\left(\frac{1}{R}\right)$. In addition to the above 1), 2), 3), 4) and 5) multiplets, there are also six long gravitino multiplets.

4 Conclusion

We described an explicit example of an $\mathcal{N} = 2$ superconformal field theory that has a subsector of the Hilbert space with enhanced $\mathcal{N} = 8$ superconformal symmetry, in the large $N$ limit from the study of $AdS_4 \times M^{1,1,1}$. The pp-wave geometry in the scaling limit produced to the maximally $\mathcal{N} = 8$ supersymmetric pp-wave solution of $AdS_4 \times S^7$. The result of this paper shares common characteristic feature of previous case of $AdS_4 \times Q^{1,1,1}$. This subsector of gauge theory is achieved by Penrose limit which constrains strictly the states of the gauge theory to those whose conformal dimension and $R$ charge diverge in the large $N$ limit but possesses finite value $\Delta - R$. We predicted for the spectrum of $\Delta - R$ of the $\mathcal{N} = 2$ superconformal field theory and proposed how the excited states in the supergravity correspond to gauge theory operators. In particular, both the chiral multiplets (5) and semi-conserved multiplets (9) of $\mathcal{N} = 2$ supersymmetry should combine into $\mathcal{N} = 8$ chiral multiplets.

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7 There exist three of them $\chi^+$ characterized by $\Delta = -\frac{1}{2} + \frac{1}{4}\sqrt{E} + 16R + 32$, or $-\frac{1}{2} + \frac{1}{4}\sqrt{E} + 16R$. Similar analysis can be done in this case. Although for finite $\Delta$ and $R$, both do not provide rational conformal dimensions, in the Penrose limit there is no constraint on the integer values and $R \to \infty$ limit will give us a rational conformal dimension.
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