Optimal Beamforming for IRS-Assisted SWIPT System with an Energy-Harvesting Eavesdropper

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Abstract: In this paper, we study a simultaneous wireless information and power transfer (SWIPT) system aided by the intelligent reflecting surface (IRS) technology, where an AP transmits confidential information to the legitimate information receiver (IR) in the presence of an energy harvesting (EH) receiver that could be a potential eavesdropper. We aim to maximize the secrecy rate at the legitimate IR by jointly optimizing the information beamforming vector and the energy transfer beamforming vector at the access point (AP), and the phase shift matrix at the IRS, subject to the minimum harvested power required by the EH receiver. The semi-definite relaxation (SDR) approach and the alternating optimization (AO) method are proposed to convert the original non-convex optimization problem to a series of semi-definite programs (SDPs), which are solved iteratively. Numerical results show that the achievable secrecy rate of the proposed IRS-assisted SWIPT system is higher than that of the SWIPT system without the assistance of the IRS.

Keywords: intelligent reflecting surface; SWIPT; secrecy rate; energy harvest

1. Introduction

The increasing demand of high data rates on future communication systems leads to more complex signal processing hardware and more energy consumption. Green communication will play an increasing role in future communication networks. The radio signal enabled energy harvesting (EH) technology has drawn great interest in recent years. In simultaneous wireless information and power transfer (SWIPT) systems, the radio frequency signal plays a role in both information transmission and energy transfer [1–5].

Recently, intelligent reflecting surfaces (IRS), a planar array composed of a great number of reconfigurable elements, is regarded as a potential solution for achieving higher efficiency of both the energy and spectrum for the future communication systems, and has been proposed and studied in [6,7]. A sufficient survey on the topic of IRS can be found in [6]. The IRS passively reflects the RF signals with programmable phase shifts of the reflecting elements. By adaptively tuning the phase shifts, the IRS can help to strengthen the desired signals or suppress the undesired signals. Therefore, the beamforming vector at the source can be jointly optimized with the passive reflection coefficients of the IRS to improve, e.g., the signal to noise ratio (SNR) at the receiver for the multiple-input single-output (MISO) channel [7].

The IRS has recently been considered as a new technology for 6G [8,9]. As one of the requirements for 6G is to support air-to-ground communications, the IRS can be deployed to extend the coverage of the cellular networks to the aerial users (AUs) and improve the air-ground communication network performance [9]. Another promising technology for 6G is the Terahertz (THz) communication. However, the THz signals are readily blocked by obstacles, which leads to communication interruption. The IRS can be explored in the 6G system to assist the THz system [10,11]. The fundamental limit of the capacity of an IRS-assisted multiple-input multiple-output (MIMO) channel was characterized by jointly optimizing the transmit covariance matrix and the IRS reflection coefficients.
in [12]. Assuming the channels were frequency-selective, the authors in [13] studied an IRS-enhanced orthogonal frequency division multiplexing (OFDM) system, in which the IRS elements were grouped, and a practical channel estimation transmission protocol was proposed to reduce the training overhead. Based on the grouping method, the achievable rate was maximized by jointly optimizing the IRS reflection coefficients and the transmit power allocation.

Furthermore, the efficient beamforming gain of the IRS not only benefits the information transmission but is also appealing for the wireless power transfer. By optimizing the energy transfer beamforming vector and the reflecting coefficients of the IRS, the RF signal attenuation of long-distance can be compensated; thus, the efficiency of the RF-enabled wireless energy transfer can be increased. In [14], an IRS-assisted SWIPT system was considered, where an access point (AP) served several information-receiving users and several EH receivers. The weighted sum-power maximization problem under the individual SINR constraints for the information decoding receivers was proposed, and a sub-optimal solution was derived. In particular, it was sufficient that the AP transmits only the information signals while satisfying both the information transmission and energy transfer requirements. Similar to the problem considered in [14], the authors in [15] considered an IRS-assisted SWIPT system assuming the information-receiving users and the EH receivers are equipped with multiple antennas. The weighted sum rate was maximized by jointly optimizing the transmit precoding matrices at the AP and the passive phase shift matrix at the IRS. A more general IRS-assisted SWIPT system was considered in [16], where the energy transfer and information transmission from an AP to multiple energy and information users were assisted by deploying multiple IRSs. A penalty-based optimization method was proposed to solve the joint optimization problem of transmitting precoders and reflecting phase shifts at all IRSs.

However, due to the open nature of the wireless channel, the security problem presents a serious threat to the confidential information transmission in the SWIPT system [17]. Most of the existing works focusing on the physical security layer of the SWIPT systems can be divided into two classes: (1) maximizing the achievable secrecy rate, while the minimum harvested energy requirements are satisfied at the EH receiver [18]; (2) maximizing the energy harvested at the EH receiver, while the achievable secrecy rate of the legitimate users must be larger than the minimum requirement [19,20]. In previous work, considering physical layer security, transmit beamforming vectors were designed to maximize the SNR at the legitimate receivers and keep the SNR at the eavesdroppers as small as possible. However, if the wiretap channel of the eavesdropper is highly correlated with the main channel of the legitimate users, the SNR at the eavesdropper may also be increased with the beamforming vectors designed to maximize the SNR of the legitimate receiver. When the IRSs are deployed, due to the configurable reflection coefficients of the IRS, as mentioned above, we can jointly design information beamforming vectors and the reflection coefficients such that the SNR at the legitimate receiver is increased and the SNR at the eavesdropper is suppressed. Thus, the secrecy performance can be improved. In [21,22], an IRS-assisted secure wireless communication was considered, assuming the eavesdropping channel was correlated with the legitimate channel and was stronger than the legitimate channel. An IRS-aided MISO broadcast system with multiple eavesdroppers was considered in [23], where a minimum-secrecy rate maximization problem was formulated under practical constraints on the reflecting coefficients of the IRS. The authors in [24,25] considered an IRS-assisted MISO wiretap channel and an IRS-assisted MIMO multi-antenna-eavesdropper (MIMOME) wiretap, respectively. In the former scenario, the secrecy rate under a delay-limited quality-of-service (QoS) was maximized by optimizing the beamforming vector at the transmitter and the phase shifts at the IRS. In the latter scenario, the input covariance matrix and the IRS phase shifts were jointly optimized to maximize the secrecy rate. Iterative optimizations were proposed to solve the optimization problems, and numerical results were shown to demonstrate the superiority of the proposed methods. The work in [26–28] considered the IRS-assisted SWIPT sys-
tems in the MISO channels with external eavesdropper(s) assuming a different number of energy-harvesting receivers, eavesdroppers, or IRSs, respectively. Different methods were proposed to solve the secrecy rate maximization problems. The secrecy rate maximization problem for an IRS aided MIMO channel with SWIPT was considered in [29], assuming all nodes were equipped with multiple antennas. Xue et al. [30] proposed a hybrid scheme to jointly optimize the hybrid precoders of the transceiver and phase shift of the IRS to maximize the secrecy in the mmWave MIMO system with SWIPT.

In this paper, we consider an IRS-assisted SWIPT system with an external EH eavesdropper. The multi-antenna AP transfers energy to the single-antenna EH receiver and simultaneously transmits confidential information to the legitimate information receiver. The EH receiver is also a potential eavesdropper. The confidential information should be kept as secret as possible from the eavesdropper.

We aim at maximizing the achievable secrecy rate of legitimate users, while satisfying the minimum harvested energy requirements at the EH receiver. Specifically, we jointly optimize the energy transfer beamforming vector and the information transmit beamforming vector at the AP and phase shift matrix of the IRS. This is a non-convex optimization problem, which is hard to solve directly. To solve this non-convex optimization problem, we introduce the semi-definite relaxation (SDR) method and alternating optimization (AO) approaches to convert the original optimization problem to a series of semi-definite programs (SDPs), which are solved iteratively. Numerical results are then presented to show the achievable secrecy rate performance of the IRS-assisted SWIPT system. The main contributions of this paper are summarized as follows:

1. The IRS-assisted SWIPT system—where the energy harvesting receiver is simultaneously a potential eavesdropper—was considered to show the possible application of the IRS in wireless energy transfer and secrecy information transmission.
2. A joint optimization problem was designed to optimize the active and passive beamforming. The AO algorithm is proposed to iteratively solve the optimization problem by optimizing the active beamforming vectors and the phase shift matrix.
3. We also show that when information should be kept secret to the energy harvesting receiver, a dedicated energy beam should be designed, which is different to that in the IRS-assisted SWIPT system without eavesdroppers.

The organization of the rest of the paper is as follows. The system model and problem formulation are presented in Section 2. The solution to the optimization problem is given in Section 3. In Section 4, we present the numerical results. Section 5 concludes the paper.

Notations: Scalars, vectors, and matrices are denoted by italic letters, bold-face lowercase letters, and bold-face uppercase letters, respectively. The Euclidean norm of a complex-valued vector $\mathbf{x}$ is denoted by $||\mathbf{x}||$. The notation $\text{diag}(\mathbf{x})$ denotes a diagonal matrix with each diagonal element being the corresponding element in $\mathbf{x}$, and $\text{ang}(\mathbf{x})$ is a vector, each element of which is the phase of the corresponding element in $\mathbf{x}$. The space of $m \times n$ complex-valued matrices is denoted by $\mathbb{C}^{m \times n}$. $\text{Tr}(\mathbf{A})$ and $\mathbf{A}^{-1}$ means the trace and inverse of a square matrix $\mathbf{A}$. $\mathbf{A} \succeq 0$ means the matrix $\mathbf{A}$ is positive and semi-definite. The conjugate, transpose, and rank of any matrix $\mathbf{A}$ are denoted by $\mathbf{A}^*, \mathbf{A}^H$, and $\text{rank}(\mathbf{A})$, respectively.

2. System Model and Problem Formulation

2.1. System Model

The IRS-assisted SWIPT system considered in this paper is as shown in Figure 1, where an AP equipped with $N_t$ antennas transmit secret information to a single-antenna legitimate information receiver (IR) and simultaneously transfers the energy to an EH node, which tries to decode the confidential information from the AP. The EH node is a potential eavesdropper and is denoted by EH Eve. It is equipped with only one antenna and adopts a power split approach to split the received signal into two streams—one for information decoding and the other for energy harvesting. An IRS consisting of intelligent reflecting elements is deployed to assist the secret information transmission and energy transfer for the SWIPT system. A smart controller, which communicates with the AP to
exchange information via a separate link, controls the phase shifts of the passive reflecting elements of the IRS.

Denoted by $h_{sd} \in \mathbb{C}^{N_t \times 1}$ and $h_{se} \in \mathbb{C}^{N_t \times 1}$, the channels from the AP to the legitimate information receiver IR and the EH Eve respectively, are $h_{rd} \in \mathbb{C}^{N_t \times 1}$ and $h_{re} \in \mathbb{C}^{N_t \times 1}$, while the channels from the IRS to the IR and Eve, and $G \in \mathbb{C}^{N_t \times N_t}$ is the channel between the AP and IRS. It is assumed that the AP knows the channel state information (CSI) perfectly, for all channels involved. Note that although Eve is a passive eavesdropper, it is possible to assume that the CSI of the EH Eve is known to the AP, since Eve wants to harvest energy from the energy signal transferred from the AP. The AP transmits secret information to the IR and transfers the energy to the EH Eve, simultaneously. The transmitted signal from AP is $x_t = w_s + v_e$, where $w \in \mathbb{C}^{N_t \times 1}$ is the information beamforming vector, $s \sim \mathcal{CN}(0, 1)$ is a circularly symmetric complex Gaussian (CSCG) random variable denoting the information carrying signal, $v \in \mathbb{C}^{N_t \times 1}$ is the energy beamforming vector, and $e \sim \mathcal{CN}(0, 1)$ is a CSCG random variable that denotes the energy-carrying signal intended for the EH Eve. Without any loss of generality, it is assumed that $s$ and $e$ are independent random variables. Therefore, the transmit power at AP is given by:

$$E(x_t^H x_t) = \|w\|^2 + \|v\|^2. \quad (1)$$

Each element of the IRS reflects the received signal from the AP with an adjustable phase shift and amplitude. The reflection-coefficient matrix is denoted by $\Theta = \text{diag}(\beta_1 e^{j\theta_1}, \beta_2 e^{j\theta_2}, ..., \beta_N e^{j\theta_N})$, where $\beta_n \in [0, 1]$, and $\theta_n \in [0, 2\pi)$, are the reflection amplitude and phase shift of the $n$-th element, respectively. In the sequel, we set $\beta_n = 1, \forall n$ for simplicity. Neglecting the signal reflected by the IRS two or more times, the received signals at IR and Eve are respectively expressed as follows:

$$y_d = (h_{rd}^H \Theta G + h_{sd}) x_t + z_d, \quad (2)$$

$$y_e = (h_{re}^H \Theta G + h_{se}) x_t + z_e, \quad (3)$$

Figure 1. System model.
where $z_d$ and $z_e$ are independent and identically distributed (i.i.d.) Gaussian random noises at the IR and Eve with zero means and variances of $\sigma_d^2$ and $\sigma_e^2$, respectively. The energy harvested at the EH Eve is shown as:

$$E = \zeta \left( |(h_{zd}^H \Theta G + h_{zd}^H)w|^2 + |(h_{ze}^H \Theta G + h_{ze}^H)v|^2 \right)$$

$$= \zeta \left( |h_{zd}^H w|^2 + |h_{ze}^H v|^2 \right),$$

(4)

where $h_{zd}^H = h_{zd}^H \Theta G + h_{zd}^H$ and $0 \leq \zeta \leq 1$ means the energy conversion efficiency. The signal-to-interference-and-noise (SINR) at IR and Eve is respectively given by:

$$\gamma_d = \frac{|(h_{zd}^H \Theta G + h_{zd}^H)w|^2}{|(h_{zd}^H \Theta G + h_{zd}^H)v|^2 + \sigma_d^2} = \frac{|h_{zd}^H w|^2}{|h_{zd}^H v|^2 + \sigma_d^2},$$

(5)

$$\gamma_e = \frac{|(h_{ze}^H \Theta G + h_{ze}^H)w|^2}{|(h_{ze}^H \Theta G + h_{ze}^H)v|^2 + \sigma_e^2} = \frac{|h_{ze}^H w|^2}{|h_{ze}^H v|^2 + \sigma_e^2},$$

(6)

where $h_{zd}^H = h_{zd}^H \Theta G + h_{zd}^H$, $h_{ze}^H = h_{ze}^H \Theta G + h_{ze}^H$. Note that the energy beam not only carries energy, but also acts as a jamming signal to interfere the receiving signal at Eve. Unlike the pure energy harvesting system in which the energy-carrying signal is assumed to be known at the intended nodes and can be cancelled before information decoding, the energy-carrying signal in this system is a random variable that cannot be known in advance at the intended nodes. Thus, the achievable secrecy rate of the confidential information transmitted from the AP to the IR is:

$$R_s = \left[ \log_2(1 + \gamma_d) - \log_2(1 + \gamma_e) \right]^+, \hspace{1cm} (7)$$

where $(x)^+ = \max\{x, 0\}$. Note that the secrecy rate in Equation (7) is a conservative one, since it is unlikely that the EH Eve only eavesdrops information while giving up harvesting energy completely.

### 2.2. Problem Formulation

In this work, we try to maximize the secrecy rate $R_s$ by jointly optimizing the energy beamforming vector $v$, the information beamforming vector $w$, and the phase shift matrix $\Theta$, under the transmit power constraint and the minimum required harvested energy at the EH Eve. Thus, the maximization problem is given by:

$$(P1): \max_{w, v, \Theta} R_s\hspace{1cm} \text{s.t.} \hspace{0.5cm} \|w\|^2 + \|v\|^2 \leq P_s \hspace{1cm} E \geq \bar{E} \hspace{1cm} 0 \leq \theta_n < 2\pi, \forall n = 1, 2, \ldots, N$$

(8)

where $P_s$ is the transmit power at the AP and $\bar{E}$ is the minimum required harvested energy at the EH Eve.

**Remark 1.** If the EH receiver is a legitimate energy harvesting receiver, i.e., the information is not needed to be kept secret, the optimization problem (P1) is reduced to a special case of the work in [14]. In this case, as proven in [14], it is sufficient to send information signals only for achieving the maximum power at the EH receiver. However, if the EH receiver is a malicious node to which the information should be kept secret, the energy signals may play two roles: energy transfer and artificial noise, which will interfere with the information being received at the EH node. Therefore, the information beamforming vector and the energy beamforming vector should be optimized jointly to achieve a maximal secrecy rate while satisfying the minimum required harvested energy at the EH receiver.
3. Solution to the Secrecy Rate Maximization Problem

The problem (P1) is a non-convex optimization problem, since the beamforming vectors \( \mathbf{v} \) and \( \mathbf{w} \) are in quadratic terms and the optimization variables involved are coupled. In addition, the objective function, which is the difference of two logarithmic functions, is a non-convex one. It is hard to solve this optimization problem directly. In this paper, we introduce an SDR-based algorithm to convert the original optimization problem (P1) to SDPs and apply an alternating optimization approach to solve the SDPs by optimizing the phase shift matrix \( \Theta \) and the beamforming vectors (\( \mathbf{w} \) and \( \mathbf{v} \)) iteratively with the other fixed.

First, as shown in [31], there always exists a constraint \( \Gamma > 0 \) such that the following problem,

\[
(P2) : \max_{\mathbf{w}, \mathbf{v}, \Theta} \frac{\| (\mathbf{h}_d^H b f G + h_{sd}^H) \mathbf{w} \|^2 + \| (\mathbf{h}_n^H \Theta G + h_{se}^H) \mathbf{v} \|^2 + \sigma_D^2}{\Gamma \left( \| (\mathbf{h}_d^H \Theta G + h_{sd}^H) \mathbf{v} \|^2 + \sigma_D^2 \right)}
\]

s.t. \( \| \mathbf{w} \|^2 + \| \mathbf{v} \|^2 \leq P_s \)

\[\zeta \left( \| (\mathbf{h}_d^H \Theta G + h_{sd}^H) \mathbf{w} \|^2 + \| (\mathbf{h}_n^H \Theta G + h_{se}^H) \mathbf{v} \|^2 \right) \geq \bar{E} \]

\[1 + \frac{\| (\mathbf{h}_n^H \Theta G + h_{se}^H) \mathbf{v} \|^2}{\| (\mathbf{h}_n^H \Theta G + h_{se}^H) \mathbf{v} \|^2 + \sigma_D^2} \leq \Gamma
\]

\[0 \leq \theta_n < 2\pi, \forall n = 1, 2, ..., N \]

has the same optimal solution as (P1), where \( \Gamma \) is a slack variable introduced to avoid the non-convexity caused by the differences of two logarithmic functions of the objective function and the third constraint is introduced to restrict the maximum allowable information leaked to the EH Eve. For any given \( \Gamma \geq 1 \), let the optimal solution to the optimization problem (P2) be denoted by \( \gamma^*(\Gamma) \). Then the optimal value of the original optimization problem is \( (P1) \bar{R}_s = \max \log_2 \left( 1 + \gamma^*(\Gamma) \right) \). Therefore, by a one-dimensional search over \( 1 \leq \Gamma \leq \Gamma_m \), the maximum secrecy rate \( \bar{R}_s \) can be derived, where \( \Gamma_m \) is determined by:

\[
1 + \frac{\| \mathbf{h}_d^H \mathbf{w} \|^2}{\| \mathbf{h}_d^H \mathbf{v} \|^2 + \sigma_D^2} = 1 + \frac{\| (\mathbf{h}_d^H \Theta G + h_{sd}^H) \mathbf{w} \|^2}{\| (\mathbf{h}_d^H \Theta G + h_{sd}^H) \mathbf{v} \|^2 + \sigma_D^2} \\
\leq 1 + \frac{\| \mathbf{h}_d^H \Theta G + h_{sd}^H \| \| \mathbf{w} \|^2}{\sigma_D^2} \\
\leq 1 + \frac{\| \mathbf{u}^H \mathbf{Z} \| + \| \mathbf{h}_{se} \| \| \mathbf{w} \|^2}{\sigma_D^2} \\
1 + \frac{\| \mathbf{u}^H \mathbf{Z} \| + \| \mathbf{h}_{se} \| \| \mathbf{w} \|^2}{\sigma_D^2} \leq \Gamma_m
\]

where \( \mathbf{u}^H = (e^{i\theta_1}, ..., e^{i\theta_N}) \), \( \mathbf{Z} = \text{diag}(\mathbf{h}_{se}^H) \mathbf{G} \) and \( \| \mathbf{Z} \| = \sqrt{\sum_{n=1}^{N} \| \mathbf{z}(n) \|^2 + \| \mathbf{h}_{se} \|} \) is the n-th column vector of the matrix \( \mathbf{Z} \), the second inequality is due to the facts that \( \| \mathbf{h}_d^H \Theta G + h_{sd}^H \|^2 \leq \left( \| \mathbf{h}_d^H \Theta G \| + \| h_{sd}^H \| \right)^2 \) and \( \| \mathbf{w} \|^2 \leq P_s \).

Next, we turn to solve the problem (P2). The AO approach is applied to solve problem (P2) by iteratively optimizing the phase shift matrix \( \Theta \) and the beamforming vectors \( \mathbf{w} \) and \( \mathbf{v} \). First, for any fixed \( \Theta \), we employ the SDR-based algorithm [32] to solve the problem (P2). Defining \( \mathbf{W} = \mathbf{w} \mathbf{w}^H \) and \( \mathbf{V} = \mathbf{v} \mathbf{v}^H \), the optimization problem (P2) is re-expressed as:
where $H_d = h_d h_d^H$ and $H_v = h_v h_v^H$. The rank-one constraints $\text{Rank}(W) = 1$ and $\text{Rank}(V) = 1$ are omitted. However, as proved in [32], $\text{Rank}(W) = 1$ and $\text{Rank}(V) = 1$ always hold at the optimum of problem (P3) for any fixed $\Gamma$. The proof is omitted here for brevity.

The problem (P3) is now a linear fractional optimization problem. We use the Charnes-Cooper transformation [33] to transform the problem (P3) to an SDP. Introducing an auxiliary variable $\varphi$, defining the transformation $V = \hat{V}/\varphi$ and $W = \hat{W}/\varphi$, and utilizing the Charnes-Cooper transformation, we have:

\[
\begin{align*}
(P4) : \max_{W, \varphi} & \quad \frac{\text{Tr}(H_d W) + \text{Tr}(H_d V)}{\text{tr}(H_d V) + \sigma_d^2} \\
\text{s.t.} & \quad \text{Tr}(W) + \text{Tr}(V) \leq P_s \\
& \quad \zeta \left( \text{Tr}(H_d W) + \text{Tr}(H_r \bar{V}) \right) \geq \tilde{\xi} \\
& \quad \text{Tr}(H_r \bar{W}) - (\Gamma - 1) \left( \text{Tr}(H_r \bar{V}) + \varphi \sigma_d^2 \right) \leq 0 \\
& \quad W \succeq 0, V \succeq 0, \varphi \geq 0
\end{align*}
\]

The equivalence between the problems (P2) and (P4) for any fixed $\Theta$ and $\Gamma$ can be easily established. The problem (P4) is a semi-definite program and can be solved by the numerical solver CVX [34].

In the following steps, we optimize the problem (P2) over the phase shift matrix for any fixed active beamforming vectors. Again, let $u = (u_1, u_2, \ldots, u_N)^H$ where $u_n = e^{j\theta_n}$. The involved expressions in (9) are re-expressed as $h_d^H \Theta G = u^H \text{diag}(h_d^H) G$ and $h_v^H \Theta G = u^H \text{diag}(h_v^H) G$. Thus, we have $\left( h_d^H \Theta G + h_v^H \Theta G \right) w = \left( u^H \text{diag}(h_d^H) G + h_v^H \text{diag} \right) w = \left( u^H a_{rdw} + b_{sdw} \right)^2$, where $a_{rdw} = \text{diag}(h_d^H) G w$ and $b_{sdw} = h_d^H w$. Similarly, we have $\left( h_v^H \Theta G + h_v^H \Theta G \right) v = \left( u^H a_{rew} + b_{sew} \right)^2$, $\left( h_v^H \Theta G + h_v^H \Theta G \right) v = \left( u^H a_{rew} + b_{sew} \right)^2$, where $a_{rdv} = \text{diag}(h_d^H) G v$, $b_{sdv} = h_d^H v$, $a_{rew} = \text{diag}(h_v^H) G w$, $b_{sew} = h_v^H v$. With these definitions, we further define $\bar{u} = \left[ u^H \right]^T$, $B_{dw} = \left[ a_{rdw} \quad a_{rdw} h_d^H \right]$, $B_{dv} = \left[ a_{rdv} \quad a_{rdv} h_d^H \right]$, $B_{ew} = \left[ a_{rew} \quad a_{rew} h_v^H \right]$, $B_{ev} = \left[ a_{rew} \quad a_{rew} h_v^H \right]$. For any given beamforming vectors $w$ and $v$, the problem (P2) is equivalent to:

\[
\begin{align*}
(P5) : \max_{\bar{u}} & \quad \frac{\text{Tr}(H_d \bar{w}) + \text{Tr}(H_d \bar{v})}{\text{tr}(H_d \bar{v}) + \sigma_d^2} \\
\text{s.t.} & \quad \zeta \left( \text{Tr}(H_d \bar{w}) + \text{Tr}(H_r \bar{v}) \right) \geq \tilde{\xi} \\
& \quad 1 + \frac{\text{Tr}(H_d \bar{w}) + \text{tr}(H_d \bar{v}) + \sigma_d^2}{\text{Tr}(H_d \bar{v}) + \text{tr}(H_d \bar{v}) + \sigma_d^2} \leq \Gamma \\
& \quad |\bar{u}_n| = 1, \forall n = 1, 2, \ldots, N + 1
\end{align*}
\]
We employ the SDR-based algorithm [32] to solve the problem (P5). Defining \( \hat{U} = \bar{u} \bar{u}^{\dagger} \), the optimization problem (P5) is re-expressed as:

\[
(P6): \max_{\hat{U}} \quad \frac{\text{Tr}(B_{dw} \hat{U}) + |b_{sdw}|^2}{\text{Tr}(B_{dw} \hat{U}) + |b_{sdw}|^2 + \sigma_d^2} \\
\text{s.t.} \quad \zeta \left( \text{Tr}(B_{ew} \hat{U}) + \text{Tr}(B_{cv} \hat{U}) + |b_{sew}|^2 + |b_{sev}|^2 \right) \geq \hat{E} \\
\text{Tr}(B_{ew} \hat{U}) + |b_{sew}|^2 - (\Gamma - 1) \left( \text{Tr}(B_{cv} \hat{U}) + |b_{sev}|^2 + \sigma_e^2 \right) \leq 0 \\
\hat{U} \succeq 0, \hat{U}_{nn} = 1, \forall n = 1, ..., N + 1.
\]

The constraint \( \text{Rank}(\hat{U}) = 1 \) is omitted. The problem (P6) is a linear fractional program. Taking similar steps as problem (P4), defining the transformation \( \bar{U} = \hat{U}/\lambda \), and utilizing the Charnes-Cooper transformation [33], we have:

\[
(P7): \max_{\bar{U}, \lambda} \quad \text{Tr}(B_{dw} \bar{U}) + \lambda |b_{sdw}|^2 \\
\text{s.t.} \quad \zeta \left( \text{Tr}(B_{ew} \bar{U}) + \text{Tr}(B_{cv} \bar{U}) + \lambda |b_{sew}|^2 + \lambda |b_{sev}|^2 \right) \geq \lambda \hat{E} \\
\text{Tr}(B_{ew} \bar{U}) + \lambda |b_{sew}|^2 - (\Gamma - 1) \left( \text{Tr}(B_{cv} \bar{U}) + \lambda |b_{sev}|^2 + \lambda \sigma_e^2 \right) \leq 0 \\
\bar{U} \succeq 0, \bar{U}_{nn} = \lambda, \forall n = 1, ..., N + 1, \lambda \geq 0.
\]

The problem (P7) is an SDP and can be solved optimally by a numerical solver such as the CVX [34]. However, the relaxed constraint \( \text{Rank}(\hat{U}) = 1 \) may not be satisfied in general. In order to derive a rank-one solution from the optimal solution to problem (P7), the Gaussian randomization method, as shown in [7], was used, and the main steps are as follows. First, the optimal solution \( \hat{U}^* \) to the problem (P7) is the eigenvalue decomposed as \( \hat{U}^* = U \Sigma U^{\dagger} \). A sub-optimal solution to (P5) is constructed as \( \bar{U} = U \Sigma r \) where \( r \in \mathbb{C}^{N+1} \) is a Gaussian variable satisfying \( \mathcal{CN}(0, I_{N+1}) \) with zero meaning and covariance matrix \( I_{N+1} \). Accordingly, the sub-optimal solution to the problem (P2) for any fixed beamforming vectors \( w \) and \( v \) is \( \Theta^* = \text{diag}(u^*) \), where \( u^* = e^{\text{ang}(\bar{u}/\bar{u}_{N+1})[1:N]} \), the function \( \text{ang}(x) \) means the angle of the complex variable \( x \). Independently generating the random vector \( r \) a sufficiently large number of times, the vector \( u^* \), which achieves the maximal value of the problem (P2) is selected as the solution to (P5) for any fixed \( w \) and \( v \).

Finally, with the solutions to problems (P4) and (P7), problem (P2) for any fixed \( \Gamma \) can be solved via alternating optimizations, in which problems (P4) and (P7) are solved iteratively until the convergence. By a one-dimensional search over \( 0 \leq \Gamma \leq \Gamma_{nt} \), with methods such as the bisection golden search, the optimal value \( R^*_* \) can be achieved. The AO algorithm is summarized in Algorithm 1.

Next, we discuss the convergence of the proposed AO algorithm. The AO algorithm, as shown in Algorithm 1, can be shown to converge as follows. Denote the objective function of the problem (P2) by \( \gamma(w, v, \Theta, \Gamma) \). For any given \( \Gamma \), the solution to (P2) is derived by solving problem (P4) and (P7) iteratively. It follows that:

\[
\gamma(w^{i+1}, v^{i+1}, \Theta^{i+1}, \Gamma) \geq \gamma(w^{i+1}, v^{i+1}, \Theta^{i}, \Gamma) \geq \gamma(w^{i}, v^{i}, \Theta^{i}, \Gamma),
\]

where the first inequality comes from (12) since \( w^{i+1} \) and \( v^{i+1} \) correspond to the maximizer of \( \gamma(w, v, \Theta^{i}, \Gamma) \) and the second inequality comes from (15) due to a similar reason.

Since the proposed optimal scheme is based on SDP, its computational complexity of each iteration critically depends on the size and number of the constraints and variables. The total size and number of the constraints and variables of the problems (P4) and (P7) of each iteration are shown in Table 1.
Algorithm 1: Alternating optimization

1: Initialization:
2: Set \( k = 0, \Theta^{(0)} = I_N \).
3: repeat
4: Set \( k = k + 1 \).
5: With given \( \Theta = \Theta^{(k-1)} \), solve problem (P4) to derive \( W^{(k)} \) and \( V^{(k)} \), find the eigenvectors corresponding to the largest eigenvalues of \( W^{(k)} \) and \( V^{(k)} \) respectively, and denote them by \( \lambda_{w}^{(k)}, \lambda_{v}^{(k)}, \omega_{w}^{(k)}, \omega_{v}^{(k)} \) respectively.
6: Set \( w^{(k)} = \sqrt{\lambda_{w}^{(k)}}\omega_{w}^{(k)} \).
7: With given \( W^{(k)} \) and \( V^{(k)} \), solve problem (P7) to derive \( U^{(k)} \), and let the eigenvalue decomposition of \( U^{(k)} \) be \( U^{(k)} = \Sigma V^{H} \).
8: Applying Gaussian randomization method to derive \( u^{(k)} = e^{i\text{ang}(\bar{u}/\bar{v}_{N+1}|N|)} \).
9: Set \( \Theta^{(k)} = \text{diag}(u^{(k)}) \).
10: Set \( R^{(k)} = \left[ \log_{2}(1 + \gamma_{d}^{(k)}) - \log_{2}(1 + \gamma_{e}^{(k)}) \right]^{+} \) according to (7).
11: until \( R^{(k)} - R^{(k-1)} \leq \epsilon \)

Table 1. Complexity of the optimization algorithm.

| Number of Constraints | Number of Variables |
|-----------------------|---------------------|
| complex semi-definite | \((N+1) \times (N+1)\) (P7) \(N_{l} \times N_{l}\) (P4) positive scalar (P4) & (P7) complex semi-definite \((N+1) \times (N+1)\) (P7) \(N_{l} \times N_{l}\) (P4) positive scalar (P4) & (P7) |
| 4                     | 6                   | \(N+3\) | 1 | 2 | 2 |

4. Numerical Results

In this section, we present numerical results to show the achievable secrecy rate performance of the IRS-assisted SWIPT system with an energy-harvesting eavesdropper. In the simulation, we consider a two-dimensional coordinate system where the AP with \( N_{l} = 6 \) antennas, the legitimate user, and the energy harvesting eavesdropper (each equipped with one antenna) were located at \((1, 0), (x_{d}, y_{d})\), and \((x_{e}, y_{e})\) in meter \((m)\) in the X-Y plane, respectively. It is assumed that the IRS composed of \( N = 50 \) reflecting elements, the center of which is deployed at \((0, 30)\) in the X-Y plane and is 3 meters high from the X-Y plane. We consider two cases: (1) the location of the energy-harvesting eavesdropper fixed at \((-1, 30)\), while the legitimate user moves along the horizontal line parallel to the y-axis in the X-Y plane from the location \((1, 5)\) to \((1, 55)\); (2) the location of the legitimate user fixed at \((1, 30)\), while the energy-harvesting eavesdropper moves along the horizontal line parallel to the y-axis from the location \((-1, 5)\) to \((-1, 55)\). The simulation setup is shown in Figure 2. In both cases, we compare the performance of the IRS-assisted SWIPT system considered in this paper with that of the SWIPT system without IRS.

The channels between the AP-User, AP-eavesdropper, IRS-user, and IRS-eavesdropper are assumed to be Rayleigh fading. The LoS channel is assumed between the AP and IRS. The path loss model is given by:

\[
L(d) = C_{0} \left( \frac{d}{D} \right)^{-\alpha},
\]

where \(d\) is the distance of the link, \(\alpha\) is the path loss exponent and \(C_{0}\) is the signal attenuation at a reference distance \(D = 1\) meter. We set \(C_{0} = -30\) dB and \(\alpha = -2\) for all channels. The average transmit power at the AP is \(P_{t} = 30\) dBm and the minimum required power at the energy-harvesting eavesdropper is \(\hat{E} = 5\) \(\mu\)W. The number of Gaussian randomizations of the beamforming vectors is set to be 1000. We set \(\epsilon = 10^{-6}\) as the stopping threshold for the alternating optimization approach. The other parameters are set as follows:
\[ \sigma_D^2 = \sigma_E^2 = -50 \text{ dBm}, \quad \zeta = 0.8. \] All simulation results are averaged over 1000 random fading realizations.

Figure 2. Simulation setup.

Figures 3–6 compare the secrecy performance, transmitted power, and harvested power of the schemes with or without IRS under the above two cases. In all cases, the IRS-assisted scheme considered in this paper achieves a higher secrecy rate than that without the assistance of an IRS.

From Figure 3, it can be found that the achievable secrecy rate decreases as the legitimate user moves away from the AP. This fact is apparent since the achievable rate of the legitimate user gets smaller as it moves far away from the source. It is worthy of noting that when the legitimate user is located close to the IRS and Eve, i.e., at the location (1, 30) in the X-Y plane, the secrecy rate decreases rapidly. This is quite different from the results derived in the literatures [6] where no extra Eve exists. It was shown in [6] that when the information decoder is close to the IRS, it leads to a better user SNR. However, when there’s an Eve, as shown in this paper, if the legitimate user and the Eve are both close to the IRS, the Eve SNR may also be better, which causes more information leakage. Therefore, the secrecy rate is decreased in this case. The harvested power shown in Figure 4 also illustrates this fact. In can be observed that the harvested power at the Eve is the largest, though the energy transfer power is the lowest when the legitimate user moves to the location (1, 30) in the X-Y plane. We also show the secrecy rate assuming no energy signals. The achievable secrecy rate with no energy signals is much lower than that when the energy signals are jointly optimized with the information signals.

Figure 3. The achievable secrecy rate versus the Y-coordinate of the legitimate user.
Figures 5 and 6 show the performance comparison when the location of the legitimate user is fixed, while the Eve moves from \((-1, 5)\) to \((-1, 55)\) in the X-Y plane. The secrecy rates increase as the Eve moves far away from the AP. It is interesting to find from Figure 6 that the harvested power of both schemes, with or without IRS, are nearly equal to each other. However, the energy transfer power with IRS is larger than that without IRS. This means that by optimizing the phase shift matrix of the IRS and the beamforming vector at the AP, efficient power allocation can be achieved and a higher secrecy rate can be obtained by the IRS-assisted system. From Figure 5, it is interesting to find that the secrecy rate with no energy signals decreases as the EH Eve moves closer to the IRS. This is quite different from the case when the energy and information signals are jointly optimized with or without IRS. As the EH Eve moves far apart from the IRS, the secrecy rate increases. The existence of the IRS makes the channels of the legitimate IR and EH Eve be correlated. This increases the possibility for the EH Eve to eavesdrop the confidential information, especially when the EH Eve and legitimate IR are close to the IRS. This result further verifies the analysis in Remark 1. It is necessary to send dedicated energy signals and jointly optimize them with the information signals. Next, we give an analysis on Remark 1.

Figure 4. The transmit power at the AP and the harvested power at the Eve versus the Y-coordinate of the legitimate user.

Figure 5. The achievable secrecy rate versus the Y-coordinate of the Eve.
In order to show the secrecy rate with different SNR levels, we fix the locations of the IR and EH Eve at (1, 30) and (−1, 30), respectively. The transmit power Ps at the AP are set to 30 dBm, such that the minimum harvested energy requirements can be satisfied. We illustrate the achievable secrecy rate in Figure 7, assuming the mean power of the Gaussian noise varies from −10 dBm to −30 dBm, i.e., the transmit SNR from 50 dB to 80 dB. It is clear that the achievable secrecy rate increases with the SNR in both cases with or without the assistance of the IRS. From Figure 7, it can be found that the gap of the secrecy rates between the two cases becomes larger as the SNR increases. This further verifies the effectiveness of the proposed IRS-aided SWIPT system considered in this paper.

Figure 6. The transmit power at the AP and the harvested power at the Eve versus the Y-coordinate of the Eve.

Figure 7. The achievable secrecy rate versus the transmitted SNR.

5. Conclusions

In this paper, we consider an IRS-assisted SWIPT system with an external energy-harvesting eavesdropper, which simultaneously harvests energy from the RF signal and attempts to eavesdrop information of the legitimate users. A joint optimization problem is proposed to maximize the secrecy rate under the minimum required harvested energy at the energy-harvesting eavesdropper. The SDR-based alternating optimization approach is introduced to solve the original non-convex problem by solving a series of SDPs iteratively. Numerical results show that the proposed IRS-aided scheme achieves higher secrecy rate performance than that without IRS.

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