Contact Problem with Friction of Rolls

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Abstract: Based on the engineering background of the roller of steel rolling, researching the contact problem of two long parallel cylinders and considering the infection of friction. Gained the stress expressions when the vertical load and the friction act on the rolls separately, then according with superposition theorem of elastic mechanics, gained the stress expressions with the contact pressure and the friction force combined action.

1. Introduction
Roller is an important part widely used in the steel rolling process of various steel mills. At present, the roller damage caused by friction wear and friction fatigue is quite serious, the steel mills must be replaced regularly, with high replacement frequency and short cycle, greatly increasing the production cost and affecting the rolling efficiency. The study of friction wear of roll needs to be breakthrough. This includes the research of the important basic theory of stress distribution during rolling. Taking the roll rolling as the engineering background, the mechanical model is established for the Hertz contact stress distribution and friction, which provides the theoretical basis for the study of friction wear and friction fatigue. In this paper, we abstract the roller as two free solid rollers at both ends. See Fig. 1, treated the rolling process as the normal pressure perpendicular to the contact surface and the tangential force cut to the contact surface and studied the stress distribution. Based on the stress distribution under the combination of two types of external forces, thus the main stress and the maximum shear stress distribution.

2. Stress distribution caused by the cylindrical normal contact force

2.1. Model building
The simplified model of the working roller and the support roller under the normal contact force (rolling force) is shown in Fig. 1. According to the Hertz assumption, the contact surface changes from the original line contact to the surface contact, suppose the contact surface is rectangular and the length of the contact surface is \(L\) and the width is \(a\).

The deformation of the contact part shows the maximum deformation on the centerline of the contact area, gradually decreasing to the two sides, to the region edge is zero. Similarly, the distribution of the pressure strength is the same as this law. According to the Hertz contact theory, it can be assumed that the width of the contact surface is \(2a\) circular in diameter, and the pressure strength is proportional to the circular longitudinal coordinates \(\sqrt{a^2 - x^2}\), as shown in Fig. 2.
Fig. 1 Simplified the roll mechanical model

\[ p(x) = \frac{P_{\text{max}}}{a} \sqrt{a^2 - x^2} \]  

(1)

The maximum contact stress\(^2\) is,

\[ P_{\text{max}} = \frac{2P}{\pi a} \]  

(2)

Place (2) into (1), available,

\[ p(x) = \frac{2P}{\pi a} \sqrt{a^2 - x^2} \]  

(3)

Among them,

\[ a = \sqrt{\frac{4PR_1R_2}{\pi(R_1 + R_2)} \left(\frac{1 - \mu_1^2}{E_1} + \frac{1 - \mu_2^2}{E_2}\right)} \]  

(4)

2.2. Stress distribution during vertical force action

According to the elastic theory, when the wedge top receives a concentrated force \( P \), and evenly dividing the wedge top angle, the stress distribution in the wedge\(^3\), If the wedge top angle is \( \pi \), the stress distribution of the normal concentration on the boundary can be obtained, as shown in Fig. 3:

\[ \begin{align*}
\sigma_r &= -\frac{2P \cos \theta}{\pi r} \\
\sigma_\theta &= r_{\rho \rho} = 0
\end{align*} \]  

(5)

In Fig. 4, If the distribution force on the semi-plane is \( p(\eta) \), the acting on \( d\eta \) can be regarded as concentrated force \( p(\eta)d\eta \). According to the principle of elastic mechanical superposition, under the action of distribution force \( p(\eta) \), the stress distribution at any point \( K(x, y) \) is:

Fig. 3 Case of wedge concentration force P

Fig. 4 Case of wedge distribution force P(\(\eta\))
\[
\begin{align*}
\sigma_x &= -\frac{2}{\pi} \int_{-\infty}^{\infty} p(\eta) \frac{\cos^3 \theta}{r} d\eta \\
\sigma_y &= -\frac{2}{\pi} \int_{-\infty}^{\infty} p(\eta) \frac{\sin^2 \theta \cos \theta}{r} d\eta \\
\tau_{xy} &= -\frac{2}{\pi} \int_{-\infty}^{\infty} p(\eta) \frac{\sin \theta \cos^2 \theta}{r} d\eta
\end{align*}
\]

(6)

Notice that: \(\eta = y - x \tan \theta \), \(d\eta = -\frac{x}{\cos^2 \theta} d\theta\); If \(\eta = -\infty\), \(\theta = \frac{\pi}{2}\); if \(\eta = \infty\), \(\theta = -\frac{\pi}{2}\); Generation formula (6) is available:

\[
\begin{align*}
\sigma_x &= -\frac{2}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} p(\eta) \cos^2 \theta d\theta \\
\sigma_y &= -\frac{2}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} p(\eta) \sin^2 \theta d\theta \\
\tau_{xy} &= -\frac{2}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} p(\eta) \sin \theta \cos \theta d\theta
\end{align*}
\]

(7)

As shown in Fig.5, on the semi-plane boundary interval \(-a \leq \eta \leq a\), when subject by the distribution load \(p(\eta)\), the stress distribution in the half-plane is solved as follows:

Corresponding to the interval \(-a < \eta < a\), even if \(\theta_1 < \theta < \theta_2\), over here \(\theta_1 = \arctan \frac{y-a}{x}\), \(\theta_2 = \arctan \frac{y+a}{x}\), And in the interval \(-\frac{\pi}{2} < \theta < \theta_1\) and \(\theta_2 < \theta < \frac{\pi}{2}\), \(p(\eta) = 0\); therefore, In the interval \(-a < \eta < a\), assume \(\theta_1 < \theta < \theta_2\), received by (3):

\[
p(\eta) = p(y - x \tan \theta) = \frac{2P}{\pi a^2} \sqrt{a^2 - (y - x \tan \theta)^2}
\]

Put it into (7):

\[
\begin{align*}
\sigma_x &= -\frac{4P}{\pi^2 a^2} \int_{\theta_1}^{\theta_2} \sqrt{a^2 - (y - x \tan \theta)^2} \cos^2 \theta d\theta \\
\sigma_y &= -\frac{4P}{\pi^2 a^2} \int_{\theta_1}^{\theta_2} \sqrt{a^2 - (y - x \tan \theta)^2} \sin^2 \theta d\theta \\
\tau_{xy} &= -\frac{4P}{\pi^2 a^2} \int_{\theta_1}^{\theta_2} \sqrt{a^2 - (y - x \tan \theta)^2} \sin \theta \cos \theta d\theta
\end{align*}
\]

(8)

3. Stress distribution when subject to friction

With the friction force, because the friction force cuts down along the contact surface, when acting together with the normal pressure, under the normal pressure. The Herty assumes the classical assumption and is generally accepted and validated, so the plane assumption still holds when the tangential friction is superimposed. It can be assumed that the contact surface is still in the plane, and the tangential friction force does not affect the shape and size of the contact surface. The friction coefficient \(f\) between the rollers is constant, and the friction force is assumed to be:

\[
t(\eta) = fp(\eta) = f \frac{2P}{\pi a^2} \sqrt{a^2 - (y - x \tan \theta)^2}
\]
As shown in Fig. 6, according to the shear concentration force $T$ of the relevant wedge wedge top in the elastic theory, and the stress $T$ distribution of the wedge body\cite{5}. If the wedge top angle is $\pi$, the stress distribution of the acting tangential concentration force on the semi-plane boundary can be obtained:

$$
\begin{aligned}
\sigma_r &= -\frac{2T \sin \theta}{\pi r} \\
\sigma_\theta &= \tau_{r\theta} = 0
\end{aligned}
$$

\(9\)

If the force acting on the semi-plane is the distribution force $fp(\eta)$, the force $fp(\eta)d\eta$ acting on $d\eta$ it can be regarded as a concentrated force. According to the principle of elastic mechanical superposition\cite{6}, stress distribution at any point under the action of distribution force $fp(\eta)$:

$$
\begin{aligned}
\sigma_x &= -\frac{2}{\pi} \int_{-\infty}^{\infty} p(\eta) \frac{\sin \theta \cos^2 \theta}{r} d\eta \\
\sigma_y &= -\frac{2}{\pi} \int_{-\infty}^{\infty} p(\eta) \frac{\sin^3 \theta}{r} d\eta \\
\tau_{xy} &= -\frac{2}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} p(\eta) \frac{\sin^2 \theta \cos \theta}{r} d\eta
\end{aligned}
$$

\(10\)

Note this: $\eta = y - x\tan \theta$, $d\eta = -\frac{x}{\cos^2 \theta} d\theta$.

If $\eta = -\infty$, $\theta = \frac{\pi}{2}$;

and if $\eta = \infty$, $\theta = -\frac{\pi}{2}$ put them into (10), get to:

$$
\begin{aligned}
\sigma_x &= -\frac{2}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} p(\eta) \sin \theta \cos \theta d\theta \\
\sigma_y &= -\frac{2}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} p(\eta) \sin^2 \theta \tan \theta d\theta \\
\tau_{xy} &= -\frac{2}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} p(\eta) \sin^2 \theta d\theta
\end{aligned}
$$

\(11\)
The stress distribution formula under the friction force (Fig. 7) is:

\[
\begin{align*}
\sigma_x &= -f \frac{4P}{\pi^2 a^2} \int_{\theta_1}^{\theta_2} \sqrt{a^2 - (y - x \tan \theta)^2} \sin \theta \cos \theta d\theta \\
\sigma_y &= -f \frac{4P}{\pi^2 a^2} \int_{\theta_1}^{\theta_2} \sqrt{a^2 - (y - x \tan \theta)^2} \sin^2 \theta \tan \theta d\theta \\
\tau_{xy} &= -f \frac{4P}{\pi^2 a^2} \int_{\theta_1}^{\theta_2} \sqrt{a^2 - (y - x \tan \theta)^2} \sin^2 \theta d\theta
\end{align*}
\]  

In which \( \theta_1 = \arctan \frac{y-a}{x} \), \( \theta_2 = \arctan \frac{y+a}{x} \).

4. Conclusion
To lay the internal stress distribution expression considering friction:

\[
\begin{align*}
\sigma_x &= -\frac{4P}{\pi^2 a^2} \int_{\theta_1}^{\theta_2} \sqrt{a^2 - (y - x \tan \theta)^2} \cos \theta (\cos \theta + f \sin \theta) d\theta \\
\sigma_y &= -\frac{4P}{\pi^2 a^2} \int_{\theta_1}^{\theta_2} \sqrt{a^2 - (y - x \tan \theta)^2} \sin^2 \theta (1 + f \tan \theta) d\theta \\
\tau_{xy} &= -\frac{4P}{\pi^2 a^2} \int_{\theta_1}^{\theta_2} \sqrt{a^2 - (y - x \tan \theta)^2} \sin \theta (\cos \theta + f \sin \theta) d\theta
\end{align*}
\]

After the stress distribution considering friction, the stress at any point can be obtained, and the main and maximum shear stress distribution can be obtained. The location of maximum positive stress, maximum shear stress and Von–Mises maximum stress\cite{7} can be determined, allowing for further study on the location and conditions of the damage points of the material, producing stripping, holes or cracks. The research of this paper provides the basis for the stress analysis.

Roller is an important tool for rolling and one of the main consumption spare parts in rolling production. Mass production practice has proved that the wear of rollers is one of the main reasons for its failure. The quality of the roller, especially its internal quality, is not only related to the consumption and production cost of large tools, but also because the wear of the roller destroys the initial shape of the working roll, worsens the quality of the roll, which brings great difficulties to the quality control of the product. The author conducted the roller friction wear test during the graduate years, and judging from the observed stripping void or crack distribution, it assumes that the stress distribution caused by friction is close to the engineering practice.
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