Properties of highly resistive Josephson junctions

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Abstract. We have investigated indium break junctions and found a strongly reduced critical current as well as a residual resistance $R_0$ when the normal contact resistance $R_N \to R_K/2 = h/(2e)^2$. These results can be described by the horizon model, taking into account the small lead capacitance of the samples. The horizon of a Josephson point contact equals the Compton wavelength of the quasi-particle oscillating in the washboard potential. Our samples had lead capacitances of order $10^{-1}$ pF/m and very low losses with $Q \to \infty$.

1. Introduction
The concept of the horizon is well known for normal tunneling junctions. It was first derived theoretically by Yu. Nazarov [1], and it is at present well established [2]. On the other hand, the horizon of a direct Josephson junction has been barely examined [3]. This is the more surprising since such contacts are very small with lateral dimensions of order 1 nm, and they should have a correspondingly small capacitance. A popular assumption is that the capacitance of a point contact is of order $C = 1\, \text{fF}$ [4] which is large enough to suppress Coulomb-blockade effects. However, the situation of a Josephson point contact is different because it operates at very high frequencies in the GHz or THz range so that the frequency dependence of the capacitance should be taken in to account. We present a simple model that can just do that and compare our experimental data with the predictions of this model.

2. Theory
A real Josephson junction can be described in the RCSJ model [5, 6] as the junction proper with critical supercurrent $I_0$ short-circuited by a quasi-particle damping resistor $R_{qp}$ and a capacitor $C$. This circuit represents a particle of mass $(2e/h)^2 C$ that oscillates at the plasma frequency $\omega_p = \sqrt{2eI_0/hC}$ in the washboard potential.

$$E(I, \varphi) = -\frac{hI}{2e} \varphi - \frac{hI_0}{2e} \cos \varphi$$

where $\varphi$ is the phase difference across the junction and $E_{JE} = hI_0/2e$ is the Josephson coupling energy. The injected current tilts the washboard potential, which affects the depth of the potential minima as well as the eigen frequency. While $I_0$ has been predicted theoretically, capacitance and quasi-particle resistance are a priori unknown.

A classical particle of mass $m$ can interact with a quantum-relativistic field only within the radius of the Compton wavelength $l_C = hc/mc^2$ [7]. A larger horizon of interaction would allow
Figure 1. (a) Tilt of the washboard potential in terms of normalized injected current \( x = I / I_0 \) versus normalized lead capacitance \( \kappa / \kappa_0 \). The Josephson phase has at least one discrete energy level while the Bloch lattice has none. \( E_0 \) and \( E_1 \) mark the destruction of the ground state and the first excited level. (b) Experimental critical current \( I_c \) normalized to the theoretical critical current \( I_0 \) in the clean limit versus normal contact resistance \( R_N \). Open squares are from Ref. [3], solid symbols denote our present results.

for particle creation and destruction. In order for the particle that represents the Josephson junction not to be excited out of its ground state, we interpret the internal energy \( mc^2 \) of the classical particle as the analogue of the energy difference to the first excited level \( \hbar \omega_p \) of the particle in the washboard potential, to obtain the horizon of the Josephson junction [3] \( l_C(\omega) = c / \omega \). Then the dynamic contact capacitance becomes \( C \approx \kappa / \omega_p \) where \( \kappa_0 = e^2 / \hbar c \approx 0.81 \text{ pF/m} \) is the natural unit of the lead capacitance. The washboard potential is not altered by including the frequency dependence of \( C \), but the eigen frequency becomes \( \omega = (2 \kappa_0 I_0 / \kappa e) \sqrt{1 - x^2} \). The existence of a supercurrent requires at least one discrete energy level in the well of the washboard potential or \( 2E(x) \geq \hbar \omega(x) / 2 \) where \( 2E(x) \) is the minimum height of the potential well. This results in a reduced critical current \( x_c = I_c / I_0 \) described implicitly by

\[
\frac{\kappa_0}{\kappa} = \frac{\omega_p E(x_c)}{E_{JE} \omega(x_c)}
\]

Fig. 1 (a) shows that \( I_c \) approaches the theoretical \( I_0 \) only at \( \kappa / \kappa_0 \gg 10 \). No supercurrent can flow when \( \kappa \leq \kappa_0 \) in the Bloch-lattice state.

A small capacitance causes also a finite contact resistance due to quantum tunneling of the phase. The tunneling rate into the continuum is [8, 9]

\[
\Gamma = \gamma \frac{\omega_p}{2\pi} \exp \left( -\frac{14.4E_{JE}}{\hbar \omega_p} [1 + \beta] \right)
\]

where \( \gamma \approx \sqrt{120\pi(14.4E_{JE}/\hbar \omega_p)} \) and \( \beta = 0.87/Q + \ldots \) corrects for damping. The Q factor equals \( \omega R_{qp} C = \kappa e R_{qp} \). Defining the parameter \( K = 1.8\pi(\kappa / \kappa_0)(1 + \beta) \) results in a simple expression
for the differential resistance around zero bias of

$$\frac{dV}{dI} \approx R_0 \cosh (Kx)$$

(4)

with the residual contact resistance $R_0 \approx R_K \sqrt{432\kappa/\pi\kappa_0} \exp (-2K/\pi)$. Here $R_K$ is the von Klitzing resistance. These equations allow us to analyze the spectra of the contacts in terms of lead capacitance and damping parameter.

![Graph](image)

**Figure 2.** (a) $dV/dI$ spectra of several indium contacts versus normalized current $x = I/I_0$. (b) Typical $V(I)$ characteristics of the spectra in (a) show the transition from a Josephson to a tunneling-like behaviour. We have used the clean limit $R_N I_0 = 1.65 \text{ mV}$ of indium.

3. **Experimental setup**

The mechanically controllable indium break junctions were investigated at $T = 0.1 \text{ K}$. The indium wires ($\sim 1 \text{ mm diameter}$) were prepared by cutting a groove and breaking it in vacuum at low temperature. The size of the contacts ($0.3 - 50 \text{ nm}$) was controlled with a piezo tube. The spectra were measured with standard current biasing.

4. **Results and discussion**

Fig. 2 shows typical $dV/dI$ spectra and their $V(I)$ characteristics. By plotting the data in terms of the reduced current we can directly see how the critical current becomes smaller than expected and the contacts more tunneling like when $R_N$ approaches $10 \text{ k}\Omega$. Fig. 1 (b) shows a collection of experimental critical currents versus normal resistance. The theoretical $I_0$ is reached only around $1 - 10 \text{ k}\Omega$ while the supercurrent is completely suppressed at a few k\Omega. We assume that the reduced $I_c$ is due to the small capacitance of the junctions. Comparing then the measured $I_c$ with the phase diagram Fig. 1 (a) yields directly the lead capacitance $\kappa$. For these contacts $\kappa/\kappa_0 \approx 1 - 10$. The corresponding horizon $l_C \approx e\kappa/2\kappa_0 I_0$ varies within $1 - 100 \mu\text{m}$.

At zero bias we can extract further information from the spectra using a two-parameter fit to Eq. 4 which yields $R_0$ and $K$. Fig. 3 shows that they agree rather well with the predictions for $Q \approx 1$. However, when plotting $K$ versus $R_0$ we find that $R_0$ is too small by a factor of
order $10^3$. This large deviation could be attributed to the prefactor $\gamma$ which has been derived for tunneling into the continuum while for the experiments around zero bias tunneling into the neighbouring well would be more appropriate.

We demonstrate how useful the horizon model is by estimating the damping around zero bias. Since the parameter $K$ depends on $\kappa$ and on $Q$, and the lead capacitance is known independently from the reduced critical current, we can extract the $Q$ factor. Fig. 4 shows that $Q \to \infty$, independent of $R_N$. On the other hand, the same contacts slightly above $I_c$ follow $V(I) = R_{qp} \sqrt{I^2 - I_c^2}$ from which one can extract the quasi-particle resistance [5, 6]. We have found that $R_{qp} \approx R_N$ so that $Q = \kappa c R_{qp}$ is very small at these conditions.

5. Conclusion
The horizon model describes our experimental data quite well. It has only two adjustable parameters, the lead capacitance and the damping. Both can be extracted from the measured data, and they give results which appear to be reasonable.

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