Quantum Spin Current Induced Through Optical Fields

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(Dated: March 23, 2022)

We propose a scheme to generate quantum spin current via optical dipole transition process. By coupling a three-level system based on the spin states of charged particles (electrons or holes in semiconductor) to the angular momentum states of the radiation, we show that a pure quantum spin current can be generated. No spin-orbit interaction is needed in this scheme. We also calculate the effect of nonmagnetic impurities on the created spin currents and show that the vertex correction of the spin hall conductivity in the ladder approximation is exactly zero.

PACS numbers: 72.25.Hg, 72.25.Fe, 72.20.My, 73.63.Hs

Spintronics [1, 2], the science and technology of manipulating the spin of the electron for building integrated information processing and storage devices, showed great promise and developed rapidly in recent years. In practical application, one of the most important goals is to create spin currents [3, 4]. For this many interesting and basic phenomena, e.g. the spin hall effect [5], in the system with spin-orbit coupling, have been discovered and further studied. On the other hand, spin currents can also be generated with the interference of two optical fields, [6-8], or with optical Raman scattering effects [9]. Most of these methods in the generating quantum spin currents rely on the spin-orbit coupling, have the effect of nonmagnetic impurities on the created spin currents and show that the vertex correction of the spin hall conductivity in the ladder approximation is exactly zero.

Fig.1(a)). The transition $|s_-\rangle \to |s_0\rangle$ is coupled by a $\sigma_+\$ light with the Rabi-frequency $\Omega_1 = \Omega_1^{(0)} \exp(\mathbf{i} \mathbf{k}_1 \cdot \mathbf{r})$, where $\Omega_1^{(0)}(r)$ is the slowly spatially varying amplitude and $\mathbf{k}_1$ is the wave-vector. Another $\sigma_-\$ light, characterized by the Rabi frequency $\Omega_2 = \Omega_2^{(0)}(r)e^{(i(\phi + \mathbf{k}_2 \cdot \mathbf{r})}$ with $\Omega_2^{(0)}(r)$ the slowly spatially varying amplitude, couples the transition $|s_+\rangle \to |s_0\rangle$, where $\phi = \tan^{-1}(y/x)$ and $\mathbf{k}_2$ is the wave-vector. $l$ indicates that the $\sigma_-\$ photons are assumed to have the orbital angular momentum $\hbar l$ along the $+z$ direction. As in previous works [3, 4, 13], we focus on the generation of spin current without particle-particle interaction. Defining the flip operators as $\hat{\sigma}_{\mu\nu} = |\mu\rangle\langle\nu|$ with energy levels $\mu, \nu = s_0, s_+, s_-$, the Hamilto-

\[ \hat{\sigma}_{\mu\nu} = |\mu\rangle\langle\nu| \]

FIG. 1: (a) Λ-type system based on spin states. (b) An example for dipole transitions in the Λ-type system provided by semiconductor GaAs quantum well, where the excited state $|1/2\rangle$ is a conduction band state, and the two ground states correspond to light-hole state (for $|−1/2\rangle$) and heavy-hole state (for $|3/2\rangle$), see ref. [13] and references therein. (c) Illustration of the two-dimensional system for present model.
where $\nabla_{xy} = \hat{e}_x \partial_x + \hat{e}_y \partial_y$, $V(x) = -eEx$ is the electric potential, parameters $e$, $m_e$, $\mu_B$ and $g_s$ represent the charge, the effective mass of a particle, Bohr magneton and Lande factor, respectively. $A_0$ is the vector potential of the applied magnetic field, i.e. $B_0 \hat{e}_z = \nabla \times A_0$. Note that spin-orbit interaction is not required in our model (we consider a very small or zero Rashba term in present case, see e.g. [14]). To facilitate the discussion, $H_0$ is written in $\mathbf{r}$-representation in eq. (1). The interaction Hamiltonian $H_I$ can be diagonalized with the local unitary transformation: $H_I = U(\mathbf{r})H_I U^\dagger(\mathbf{r})$ with

$$U(\mathbf{r}) = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \sin \theta e^{-iS(\mathbf{r})} & \frac{1}{\sqrt{2}} \cos \theta \\ 0 & \cos \theta & -\sin \theta e^{iS(\mathbf{r})} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \sin \theta e^{-iS(\mathbf{r})} & -\frac{1}{\sqrt{2}} \cos \theta \end{pmatrix}, \quad (2)$$

which generally is a $SU(3)$ non-Abelian gauge potential. The off-diagonal matrix elements of it represent the transitions between each two of $|\Psi_0\rangle$ and $|\Psi_{\pm}\rangle$. However, considering the present system is non-degenerate, we can introduce the adiabatic approximation when both light fields vary sufficiently slowly in space. For a numerical estimate, we consider the case $\frac{\Omega_s^2}{10} = \lambda \rho \propto \rho$ where $\lambda$ is constant and $\rho = \sqrt{x^2 + y^2}$ the distance from $z$ axis [10]. The typical values can be taken as [11]: $\Omega_0^2 = (\Omega_1^2 + \Omega_2^2)^{1/2} \sim 10 \text{ps}^{-1}$, $l \leq 10^4$, $\lambda \sim 10^4 \text{m}^{-1}$ and the radius of the interaction region in the $x-y$ plane is $R \sim 1.0 \text{cm}$, the transition rate can then be evaluated as $\Gamma_{ab} < 10^{-3} \ll 1$ for any two different states ($a \neq b = \pm, 0$). Thus we neglect the off-diagonal matrix elements in eq. (2) and yield a nontrivial adiabatic gauge connection $\hat{A} \rightarrow \hat{A}^{\text{diag}}$, which is associated with a $3 \times 3$ diagonal matrix for the magnetic field with each component ($B_k = \frac{1}{2} e k_{mn} F_{mn} = \frac{1}{2} e k_{mn} [\hat{D}_m, \hat{D}_n]$)

$$B_{11} = B_{33} = (B_0 + B_c) \hat{e}_z, \quad B_{22} = (B_0 - 2B_c) \hat{e}_z \quad (4)$$

where $B_c = \hbar l^2/2$ indicates the strength of additional effective magnetic fields induced by the optical fields depends on the angular momentum $\hbar l$ that can have a large value from a vortex optical beam [17]. For our present purpose we choose $B_0 = B_c/2$ so that $B_{11} = B_{33} = \frac{1}{2} B_0 = \frac{1}{2} B_c/2$. In this case, we obtain an interesting result that the $z$ component of $B_{22}$ is opposite to that of $B_{11}$ (and of $B_{33}$) (see fig.2(a) and (b)), a key point for realizing quantum spin current.

![FIG. 2: (color online) (a)(b) Schematic illustration of the electric field and the effective magnetic field, where $B_0 = B_{22}, B^+ = B_{11}^\dagger$ and $B^- = B_{33}$.](image)
and $\tilde{V}_{11} = \tilde{V}_{22} - \Omega_0$. The momentum operator is defined by $\hat{p} = -i\hbar\hat{D}$ and it has the nontrivial communication relations: $[r_i, \hat{p}_j] = i\hbar\delta_{ij}, [r_i, r_j] = 0, [\hat{p}_i, \hat{p}_j] = -i\hbar\delta_{ij}$. Generally the dynamics of this situation is determined by the $SU(3)$ gauge symmetry. By considering adiabatic condition, the transitions between any two of the states $|\Psi_\alpha\rangle (\alpha = 0, \pm)$ can be neglected and the symmetry $SU(3)$ readily reduces to an Abelian one: $SU(3) \rightarrow U(1) \otimes U(1) \otimes U(1)$. The quantum state of each particle can be expanded by the complete eigenbasis of the interaction Hamiltonian $|\Psi\rangle = \sum_{n=0}^\infty \Phi_n |\Psi_\alpha\rangle$, where the coefficients $\Phi_n$ can be determined by the initial conditions. Note that each eigenstate $|\Psi_\alpha\rangle$ consists of different spin states $|s_0\rangle$ and $|s_\pm\rangle$. However, since $|\Psi_\alpha\rangle$ are eigenstates of the interaction Hamiltonian $H_I$, and $H_0$ does not lead to the transition between $|s_0\rangle$ and $|s_\pm\rangle$, the expectation value of the $z$-axis spin polarization of any state $|\Psi_\alpha\rangle$ is time-independent. Therefore, for convenience, we can treat the states $|\Psi_\alpha\rangle$ approximately as new “effective spin states” with their $z$-axis spin polarization calculated by $S_z^{(e)} = \langle \Psi_\alpha | S_z | \Psi_\alpha \rangle$. It is easy to see that $S_z^{(e)} = \cos^2 \theta_+-\sin^2 \theta_-$, $S_z^{(e)} = S_z^{(e)} = \frac{1}{2} \sin^2 \theta_+ + \cos^2 \theta_-$. Since the particles in different effective spin states $|\Psi_\alpha\rangle$ experience different effective magnetic fields, they may move in opposite directions depending on the index $\alpha$, generating a spin current.

To facilitate the subsequent calculations, we note that the particles with effective spin state $|\Psi_\alpha\rangle (\alpha = 0, \pm)$ interact with the magnetic field $B^a$ where $B^0 = B_{22}$ and $B^2 = B_{11} = B_{33}$. It is convenient to choose the diagonal-elements of the gauge connection $\tilde{A}(r)$ as $\tilde{A}_\alpha = \frac{1}{2} B_\alpha x \hat{e}_y$, so that $p^y_\alpha = k$ is a good quantum number. With the definition $\bar{a}_{\alpha k} = \frac{1}{\sqrt{2}} \left[ (x - \frac{k}{\lambda L}) + (\frac{k}{\lambda L} - \frac{\gamma}{\lambda}) \right]$ where $B = |B|^2 = 3B^2$ and $B^2 = \sqrt{\hbar c/\epsilon B}$, the Hamiltonian for a given $k$ can further be rewritten as

$$H_{eff} = \hbar \omega (\bar{a}_{\alpha k} \bar{a}_{\alpha k} + \frac{1}{2}) - g B_0 S_z^{(e)} + H_c, \quad (6)$$

where $H_c = -\frac{e^2}{2m} mc^2 - \frac{e}{2c} k c$. $g = g_s \mu_B$ and $\omega = \frac{\omega}{m c}$. Operators $\bar{a}_{\alpha k}$ satisfy the relation $[\bar{a}_{\alpha k}, \bar{a}_{\beta k}^\dagger] = \delta_{\alpha \beta} \delta_{k k'}$. The eigenstate of above Hamiltonian can be written as $|n, k, \alpha\rangle$ with its eigenvalue $\epsilon_{n, k, \alpha} = (n + 1/2) \hbar \omega - g B_0 S_z^{(e)} + H_c$. The analytical results allow us to calculate the charge and spin current. The spin current operator for a single particle is defined by $j_s^z = \frac{1}{2} (S_x v_y + v_y S_z)$, where $v_y = [y, H]/i\hbar = p_y + x \omega$ is the velocity operator in the $y$ direction and the corresponding charge current operator reads $j_c = ev_y$.

With the above definitions, for a $N_e$-particle system the average current density can be calculated by

$$J_{c,y} = \frac{1}{N_e} \sum_{n, k, \alpha} \langle j_{c,y} \rangle_{k} f(\epsilon_{n, k, \alpha}), \quad (7)$$

where $\langle j_{c,y} \rangle_{k} = \sum_{\alpha} \langle n, k, \alpha | j_{c,y} | n, k, \alpha \rangle$ is the current carried by one particle in the state $|n, k\rangle$. $f(\epsilon_{n, k, \alpha})$ is the Fermi distribution function, and $N_e = \sum_{n, k, \alpha} f(\epsilon_{n, k, \alpha})$. When the coefficient $\Phi_n$ of the initial state takes on the simple value $|\Phi_0|^2 = 1/2$, $|\Phi_+|^2 = \cos^2 \gamma/2$ and $|\Phi_-|^2 = \sin^2 \gamma/2$, a pure spin current is obtained by

$$(J_{s,y})_{n} = \frac{e E_y}{\hbar \lambda L} (\cos \theta (s_+ - s_-) - s_0), \quad j_c = 0. \quad (8)$$

The spin current in above equation is dependent on the space position $(x, y)$. This is because the effective spin polarization of the state $|\Psi_\alpha\rangle$ is dependent on space (see Fig. 3). Together with the Eq. (7) we further calculate the average spin current

$$J_{s,y} = \frac{e E_y n_e}{6\hbar \lambda L^2} (4s_+ \tan^{-1}(\lambda L_x/2\sqrt{1 + \lambda^2 y^2} - s_- - s_0)). \quad (9)$$

where $s_\pm = s_+ - s_-$, and $n_e$ is the filling of charged particles (unit $1/m^2$). It is interesting that generally the average spin current is still dependent on the position $y$. For practical application, here we consider the case that the $\lambda L_x \gg 1$, i.e. the $\sigma$-field is much stronger than $\sigma_\alpha$ one. Then from the Eq. (9) we obtain the persistent spin current $J_{s,y} = -\frac{e E_y n_e}{6\hbar \lambda L} (s_+ + s_0)$. To provide some numerical evaluations, we set $\lambda = 10(1/m)$ for the blue line, $\lambda = 1000(1/m)$ for the red line. $\rho = \sqrt{x^2 + y^2}$ versus the unit $m$.
have the same probability in state $|\Psi_0\rangle$ with the sum of $|\Psi_+\rangle$ and $|\Psi_-\rangle$. It should be pointed out that, for GaAs quantum well, this numerical evaluation should be revised by a constant factor, because another independent three-level system ($s'_+ = h/2, s'_- = -h/2, s'_0 = -h/2$, noted by \(\Lambda'\) system) can also interact with the optical fields besides the one shown in fig.1(b) \[24\]. If we neglect the detunings of the optical transitions and according to $s'_+ + s'_0 = 3h/2 < s_\pm + s_0$, the \(\Lambda'\) system will slightly decrease the present generated spin currents. 

By now all the discussions are based on a clean semiconductor system. Noting that the vertex correction may decrease the present generated spin currents.

When we turn to the ladder diagrams of the spin Hall charge current in ladder approximation is exactly zero. Hence, the initial condition required for the result in eq. (7). Firstly, the outset.

Because of this vanishing vertex correction, the spin current correction in the charge current only \[18\]. Therefore, we can trap the system in dark state using two light fields. Secondly, we only turn the first light on again. Meanwhile the population in the state \(s_+\) keeps 1/2 unchanged while the state \(s_-\) is coupled to the state \(s_0\) by the \(\sigma_+\) light and the state reads $|\Psi(t_1)\rangle = (\sin \Omega_1(t_1 - t_0)|s_0\rangle + \cos \Omega_1(t_1 - t_0)|s_\pm\rangle)/\sqrt{2}$. Thirdly, we adiabatically turn on the \(\sigma_-\) light field with \(z\)-directional angular momentum and arrive at the adiabatic evolution of the state $|\Psi(t)\rangle = \frac{1}{\sqrt{2}} (\frac{e^{-i\gamma t}}{\sqrt{2}} |\Psi_+\rangle - \frac{e^{i\gamma t}}{\sqrt{2}} |\Psi_-\rangle + |\Psi_0\rangle)$ where $\gamma = \int_{t_0}^t \frac{\Omega_1(t)}{2} dt + \frac{\Omega_2(t)}{2} dt$. Obviously, this is the state needed for the initial condition.

In practice, the spreading of light fields have a boundary in the \(x\)-y plane which may lead to modification in the Landau energy levels. However, based on the previous results \[10\], this boundary effect can be safely neglected when $L_x^2 L_y L_y \gg 1$, which is achieved using a light beam with a large angular momentum. On the other hand, we should emphasize that the particle-particle interaction may lead to a renormalization of the Rabi-frequencies $\Omega_{1,2}$ of the transitions between states $|s_0\rangle$ and $|s_\pm\rangle$ coupled by the light fields \[20, 21\]. However for simplicity, as in previous works \[3, 4, 6, 13\], we do not need to consider such effects in the present model. All these interesting aspects will be further discussed in future publications. In conclusion, we have proposed and demonstrated a means of generating quantum spin current optical dipole transition process. The short-range scattering by the nonmagnetic impurities are discussed and the vertex correction of the spin hall conductivity is shown to be zero in the present optical model.
We thank Prof Mansoor B. A. Jalil, S. -Q. Gong and Dr S. G. Tan for valuable discussions. This work is supported by NUS academic research Grant No. WBS: R-144-000-172-101, and by NSF of China under grants No.10275036.

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