PROBING SUSY IN FCNC AND CP VIOLATING PHENOMENA

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Abstract

We analyze constraints on low-energy flavour-changing sfermion mass terms, coming from FCNC and CP violating processes, in the model-independent framework of the mass insertion method. We discuss the relevance of these constraints as tests of supersymmetric extensions of the standard model; in particular, we consider grand-unified models and models with non-universal soft breaking terms.

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1 Introduction

Flavour Changing Neutral Currents (FCNC) and CP violating processes are a privileged window on new physics. In fact, tree-level FCNC transitions are forbidden in the Standard Model (SM), and can only occur in higher orders of the perturbative expansion. This implies that they are sensitive to the properties of virtual particles running in loop diagrams. Therefore, FCNC processes might reveal the presence of new physics at energies well below the threshold for direct production of new particles. The same is true for CP violating processes, which are all related, in the SM, to a single phase in the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix, whereas in extensions of the standard model there may be many independent sources of CP violation contributing to low-energy transitions.

For the reasons given above, the importance of a detailed study of FCNC and CP violating low-energy processes is twofold. On one hand, such a study can indicate which of the rare (or forbidden in the SM) and not yet observed processes are most sensitive to the various possible extensions of the SM, and therefore tell us where to look for new physics in the low-energy region. On the other hand, it can exclude wide portions of the parameter space of various models, on the basis of measured low-energy transitions and limits on rare processes.

In the following, we will concentrate on supersymmetric models \cite{1}, which can be considered to be the most likely extensions of the SM, and in particular we will consider a class of FCNC and CP violating contributions which have no analogue in the SM: the gluino- and photino-mediated ones \cite{2}. These sources of flavour change are closely related to the pattern of soft SUSY breaking and to the interactions of fermions and sfermions from the Planck energy scale down to the electroweak one. This means that by considering these contributions one can obtain indirect informations on the physics at the GUT and Planck scales, and eventually discriminate phenomenologically viable models.

In Section \ref{sec:2}, we briefly review the origin of gluino- and photino-mediated FCNC, and we introduce the model-independent formalism that we will use to obtain the low-energy constraints reported in Section \ref{sec:3}. In Section \ref{sec:4}, we compare the predictions of some general SUSY models with the constraints previously obtained. Finally, in Section \ref{sec:5} we draw our conclusions.
2 FCNC in non-universal SUSY

In the SM, flavour-changing charged currents arise because of a mismatch in the mass matrices of up- and down-type quarks. In fact, these two matrices, $m_u$ and $m_d$, are not aligned, and one needs to separately diagonalize them via two different biunitary transformations:

$$U_R^u m_u U_L^u = \text{diag} (m_u, m_c, m_t), \quad U_R^d m_d U_L^d = \text{diag} (m_d, m_s, m_b). \quad (1)$$

This can be achieved by suitably rotating the left- and right-handed quark fields:

$$d_{L,R} = U_{d_{L,R}} d'_{L,R}; \quad u_{L,R} = U_{u_{L,R}} u'_{L,R}. \quad (2)$$

Such rotations leave neutral currents unaffected, but introduce a flavour change in the charged current vertex:

$$\bar{u}_L \gamma^\mu d_L \to \bar{u}'_L U_{u_L}^\dagger \gamma^\mu U_{d_L}^\dagger d'_L \equiv \bar{u}'_L K \gamma^\mu d'_L, \quad (3)$$

where we have introduced the CKM matrix $K \equiv U_{u_L}^\dagger U_{d_L}^\dagger$.

Let us now turn to SUSY extensions of the SM, where supersymmetry is usually supposed to be softly broken at the Plank scale. Soft breaking terms include mass matrices for sfermions: $m_{\tilde{Q}}, m_{\tilde{u}}, m_{\tilde{d}}, m_{\tilde{L}}$ and $m_{\tilde{e}}$, where we have indicated by $Q$ and $L$ the left-handed SU(2) doublets and by $u, d, e$ the right-handed SU(2) singlets.

The supersymmetric interactions of squarks and sleptons with fermions and gauginos include neutral vertices such as $q - \tilde{q} - \tilde{g}$ and $l - \tilde{l} - \tilde{\gamma}$. These vertices are flavour-diagonal in the interaction basis for fermions and sfermions. However, when one rotates the fermion fields to diagonalize their mass matrices, these neutral vertices are not flavour-diagonal any more. One may then choose to keep them diagonal by rotating sfermion fields simultaneously with fermion ones:

$$\tilde{d}_{L,R} = U_{\tilde{d}_{L,R}} \tilde{d}'_{L,R}; \quad \tilde{u}_{L,R} = U_{\tilde{u}_{L,R}} \tilde{u}'_{L,R}. \quad (4)$$

In this basis, the vertices $q - \tilde{q} - \tilde{g}$ and $l - \tilde{l} - \tilde{\gamma}$ are still flavour-diagonal; however, the mass matrices for sfermions are now given by

$$U_{\tilde{d}_{L,R}}^\dagger m_{\tilde{d}_{L,R}}^2 U_{\tilde{d}_{L,R}} \quad (5)$$

and are not diagonal in flavour space unless $m_{\tilde{d}_{L,R}}^2 \propto 1$ (universality) or $m_{\tilde{d}_{L,R}}^2 \propto m_{q_{L,R}} m_{q_{L,R}}^\dagger$ (alignment). In general, one will obtain off-diagonal mass terms $(\Delta_{ij})_{AB}$.
between sfermions of flavour \( i \) and helicity \( A \) and sfermions of flavour \( j \) and helicity \( B \). This \( \Delta \)'s, when inserted in a sfermion propagator, generate a flavour change proportional to \( \delta \equiv \Delta/m_{\tilde{q}}^2 \), where \( m_{\tilde{q}} \) is the average sfermion mass. Under the hypothesis that these \( \delta \)'s are small, one may treat them as perturbations and calculate flavour-changing effects to any given order of perturbation theory in \( \delta \) [3]. Notice that this approach is model-independent, as one does not need the full knowledge of sfermion mass matrices. From the available experimental limits on FCNC processes, one can obtain constraints on the size of these \( \delta \)'s [4, 5, 6]. If one finds that the \( \delta \)'s are constrained to be small, the mass-insertion method is consistently applicable.

In the next Section, we will examine the constraints on these \( \delta \)'s that one can obtain from available low-energy data on FCNC and CP violating processes, analyzing them in the framework of the model-independent mass-insertion method.

3 Phenomenological analysis

We will now present the phenomenological limits on the \( \delta \) parameters at the electroweak scale. These limits are obtained by computing the relevant amplitudes in the framework of the mass-insertion method, by separating the contributions proportional to the various \( \delta \)'s and by imposing that each of these contributions does not exceed in absolute value the experimental limit. Therefore, we neglect any possible interference effect between the various contributions and any accidental cancellation. Further details on the analysis can be found in refs. [3, 6].

Let us start by analyzing hadronic processes. We consider gluino-mediated transitions, and we choose an average squark mass \( m_{\tilde{q}} = 500 \) GeV and a gluino mass \( m_{\tilde{g}} = 500 \) GeV. From \( K - \bar{K} \) mixing, we obtain, from the experimental value of \( \Delta M_K \), the following limits:

\[
\sqrt{\left| \text{Re} \left( \delta_{12}^d \right)^2 \right|_{LL}} < 4.0 \times 10^{-2}, \quad \sqrt{\left| \text{Re} \left( \delta_{12}^d \right)^2 \right|_{LR}} < 4.4 \times 10^{-3},
\]

\[
\sqrt{\left| \text{Re} \left( \delta_{12}^d \right)_{LL} \left( \delta_{12}^d \right)_{RR} \right|} < 2.8 \times 10^{-3}. \quad (6)
\]

Similar constraints can be obtained from the \( B_d - \bar{B_d} \) mixing parameter \( x_d \),

\[
\sqrt{\left| \text{Re} \left( \delta_{13}^d \right)^2 \right|_{LL}} < 9.8 \times 10^{-2}, \quad \sqrt{\left| \text{Re} \left( \delta_{13}^d \right)^2 \right|_{LR}} < 3.3 \times 10^{-2},
\]

\[
\sqrt{\left| \text{Re} \left( \delta_{13}^d \right)_{LL} \left( \delta_{13}^d \right)_{RR} \right|} < 1.8 \times 10^{-2}, \quad (7)
\]
and from $D - \bar{D}$ mixing:

$$\sqrt{\text{Re} \left( \delta_{12}^u \right)^2_{LL}} < 1.0 \times 10^{-1}, \quad \sqrt{\text{Re} \left( \delta_{12}^u \right)^2_{LR}} < 3.1 \times 10^{-2}, \quad \sqrt{\text{Im} \left( \delta_{12}^u \right)_{LL} \left( \delta_{12}^u \right)_{RR}} < 1.7 \times 10^{-2}. \quad (8)$$

We now turn to radiative $B$ decays. From the decay $b \rightarrow s\gamma$, we obtain the following limits on $\delta_{23}^d$:

$$\left| \left( \delta_{23}^d \right)_{LL} \right| < 8.2, \quad \left| \left( \delta_{23}^d \right)_{LR} \right| < 1.6 \times 10^{-2}. \quad (9)$$

The equation above shows that the decay $b \rightarrow s\gamma$ does not limit the $\delta_{LL}$ insertion for a SUSY breaking of $O(500 \text{ GeV})$. Indeed, even taking $m_{\tilde{g}} = 100\text{ GeV}$, the term $(\delta_{23}^d)_{LL}$ is only marginally limited ($\left( \delta_{23}^d \right)_{LL} < 0.3$). Obviously, $(\delta_{23}^d)_{LR}$ is much more constrained since with a $\delta_{LR}$ FC mass insertion the helicity flip needed for $(b \rightarrow s + \gamma)$ is realized in the gluino internal line and so this contribution has an amplitude enhancement of a factor $m_{\tilde{g}}/m_b$ over the previous case with $\delta_{LL}$.

Constraints on $\delta$'s can also be obtained from the CP violating parameters $\varepsilon$ and $\varepsilon'$. Imposing that the gluino-mediated contribution to $\varepsilon$ does not exceed the experimental value, we get the following limits:

$$\sqrt{\text{Im} \left( \delta_{12}^d \right)^2_{LL}} < 3.2 \times 10^{-3}, \quad \sqrt{\text{Im} \left( \delta_{12}^d \right)^2_{LR}} < 3.5 \times 10^{-4}, \quad \sqrt{\text{Im} \left( \delta_{12}^d \right)_{LL} \left( \delta_{12}^d \right)_{RR}} < 2.2 \times 10^{-4}, \quad (10)$$

while the conservative experimental limit on $\varepsilon'$, $\varepsilon' / \varepsilon < 2.7 \times 10^{-3}$, gives the constraints

$$\left| \text{Im} \left( \delta_{12}^d \right)_{LL} \right| < 4.8 \times 10^{-1}, \quad \left| \text{Im} \left( \delta_{12}^d \right)_{LR} \right| < 2.0 \times 10^{-5}. \quad (11)$$

From eqs. (10) and (11) one can distinguish two regimes of CP violation. If $(\delta_{12}^d)_{LL} \gg (\delta_{12}^d)_{LR}$, which is the case in the MSSM and in most other models, then *indirect* CP violation dominates and the model can be considered to be superweak. On the other hand, if one envisages a sizeable $(\delta_{12}^d)_{LR}$, *direct* CP violation is dominant and the model is of milliweak type. However, there is a caveat to this last hypothesis. In fact, there are strong constraints on the imaginary parts of flavour-conserving Left-Right mass insertions, coming from the electric dipole moment of the neutron:

$$\left| \text{Im} \left( \delta_{11}^d \right)_{LR} \right| < 3.0 \times 10^{-6}, \quad \left| \text{Im} \left( \delta_{11}^u \right)_{LR} \right| < 5.9 \times 10^{-6}. \quad (12)$$
Usually, as one can see from eq. (5), the off-diagonal terms are proportional to flavour-diagonal ones via some (small) mixing angle. If this is the case, then the limits in eq. (12) are stronger than the ones in eq. (11), and this seems to suggest that the milliweak scenario is not quite likely.

Let us now turn to the leptonic sector. We now consider photino-mediated FCNC processes, and choose an average slepton mass \( m_{\tilde{l}} = 100 \text{ GeV} \) and a photino mass \( m_{\tilde{\gamma}} = 100 \text{ GeV} \). Constraints on \( \delta_{ij} \) can be obtained from the experimental limits on radiative lepton decays \( \mu \rightarrow e\gamma \), \( \tau \rightarrow \mu\gamma \) and \( \tau \rightarrow e\gamma \):

\[
\begin{align*}
|\langle \delta_{12} \rangle_{LL}| &< 7.7 \times 10^{-3} \quad |\langle \delta_{12} \rangle_{LR}| < 1.7 \times 10^{-6} \\
|\langle \delta_{13} \rangle_{LL}| &< 29 \quad |\langle \delta_{13} \rangle_{LR}| < 1.1 \times 10^{-1} \\
|\langle \delta_{23} \rangle_{LL}| &< 5.3 \quad |\langle \delta_{23} \rangle_{LR}| < 2.0 \times 10^{-2}
\end{align*}
\]

These constraints are very important in the framework of SUSY-GUT’s, as we shall see in the next Section.

4 Non-universal SUSY models and SUSY-GUT’s

In the previous Section, we have derived bounds on flavour-changing (FC) sfermion mass terms. These limits are valid at the electroweak scale. We now want to discuss what informations on the high-energy structure of SUSY models can be extracted from these low-energy constraints. In particular, we will focus our attention on SUSY-GUT’s and on models with non-universal soft breaking terms at the Planck scale.

It has been known since the pioneering works of Duncan and Donoghue, Nilles and Wyler in 1983 [2] that even in the MSSM the running of sfermion masses from the super-large scale where SUSY is broken down to the Fermi scale is responsible for a misalignment of fermion and sfermion mass matrices with the consequent presence of FC in \( \tilde{g} - f - \tilde{f} \) or \( \tilde{\gamma} - f - \tilde{f} \) vertices. However, these FC contributions in the MSSM are well below the experimental limits.

The key-feature of the unification of quark and lepton superfields into larger multiplets in SUSY-GUT’s in relation to the FCNC issue was thoroughly investigated by Hall, Kostelecky and Rabi ten years ago [3]. But it was only recently, with the realization of the large size of the top Yukawa coupling, that it became clear that in SUSY-GUT’s radiative corrections can lead to slepton non-degeneracies so important as to imply \( L_e \) and \( L_\mu \)
violations just in the ballpark of the present or near future experimental range \[\text{\cite{7}}\]. The interested reader can find all the details of this relevant low-energy manifestation of grand unification in the works of refs. \[\text{\cite{8}}\]. Here we will just compare the predictions for leptonic FC mass insertions, \(\delta^l_{ij}\), obtained in the two simplest SUSY-GUT models, minimal SU(5) and SO(10), with the constraints obtained from the analysis of the experimental upper limits for \(\mu \rightarrow e\gamma\) decays (for further details, see ref. \[\text{\cite{6}}\]).

Let us first consider SU(5). We choose \(m_{\tilde{e}} = 100\) GeV, \(m_{\tilde{\gamma}} = 80\) GeV, \(\tan \beta = 10\) and a top Yukawa coupling at the GUT scale \(\lambda_G = 1.4\). The most interesting case is that of a double mass insertion, a Right-Right, flavour-changing one followed by a Right-Left, flavour-diagonal one. The experimental upper bound yields the following limit:

\[
\left| \left( \delta^l_{12} \right)_{RR} \left( \delta^l_{22} \right)_{RL} \right| < 3.2 \times 10^{-6},
\]

while the theoretical prediction is

\[
\left| \left( \delta^l_{12} \right)_{RR} \left( \delta^l_{22} \right)_{RL} \right| = 2.7 \times 10^{-6}.
\]

For lower values of \(\tan \beta\), the theoretical prediction lowers at most by less than one order of magnitude. From the above equations we see that we are in the ballpark of the experimental limit, and that in the future, when the experimental number will improve, we will be able either to observe a signal of new physics, or to constrain the parameter space of SUSY SU(5).

We now turn to SO(10). In this case, the situation is even more favourable, because of the possibility to obtain an amplitude for \(\mu \rightarrow e\gamma\) decay which is proportional to \(m_{\tau}\) instead of \(m_{\mu}\). Choosing \(m_{\tilde{e}} = 300\) GeV, \(m_{\tilde{\gamma}} = 150\) GeV and a top Yukawa coupling at the GUT scale \(\lambda_G = 1.25\), and considering a double mass insertion with an intermediate \(\tilde{\tau}\) propagator, we obtain the prediction

\[
\left| \left( \delta^l_{13} \right)_{RR} \left( \delta^l_{32} \right)_{RL} \right| = 1.4 \times 10^{-5},
\]

to be compared with the limit

\[
\left| \left( \delta^l_{13} \right)_{RR} \left( \delta^l_{32} \right)_{RL} \right| < 6.0 \times 10^{-6},
\]

obtained from the experimental upper limit. Evidently, in this case we are already beyond the experimental upper bound, and in fact a complete analysis shows that the parameter space of SUSY SO(10) is already strongly constrained \[\text{\cite{8}}\]. An improvement of the experimental number by one order of magnitude would practically rule this model out.
However, care must be taken when interpreting these results. In fact, in deriving them one has to rely on three main hypotheses:

1. there is a sizeable gap between the scale at which SUSY is softly broken (which we have taken to be the Planck scale) and the GUT scale;

2. renormalization group equations can be trusted for the evolution from the Planck scale down to the GUT scale;

3. there is no cancellation between various sources of FC effects.

While the third hypothesis is quite realistic, the first two assumptions can be considered to be questionable.

Before closing this section, we want to consider models with non-universal soft breaking terms. In particular, we want to answer the following question: do the constraints on low-energy FC mass terms $\delta_{ij}$ survive as constraints on non-universal soft breaking terms at the high scale, or are they diluted by the evolution? As an example, we consider a simple model with minimal non-universality in the leptonic sector. Let us assume that the soft breaking mass term for left-handed sleptons at the GUT scale has the following form:

$$m_l^2 = \text{diag}(\tilde{m}_0^2 + \Delta m^2, \tilde{m}_0^2, \tilde{m}_0^2 - \Delta m^2),$$

while the mass term for right-handed sleptons is universal. We now assume for simplicity that the Yukawa couplings of leptons are proportional to the ones of $d$-quarks in the basis where the couplings of $u$-quarks are diagonal. Performing the RGE evolution down to the electroweak scale, diagonalizing the lepton mass matrix and rotating sleptons to keep the $l - \tilde{l} - \tilde{\gamma}$ vertex diagonal, we get a flavour-violating mass insertion between s electrons and smuons $\left(\delta_{12}^{l}\right)_{LL}$ which is proportional to $\Delta m^2$. Starting from the limits in eq. 13 we obtain the constraints on $\delta_m = \Delta m^2 / \tilde{m}_0^2$ plotted in figure 1, as a function of $x = m_{\tilde{\gamma}}^2 / m_{l}^2$.

If one compares the results plotted in fig. 1 with those in eq. 13, one finds that the “dilution” of the degeneracy constraint when going from the low to the large scale increases for a more accentuated gaugino dominance. Namely, the larger the gaugino mass at the large scale is, the weaker the constraint on $\delta_m$ becomes.

1For a general analysis of FCNC constraints on non-universal soft-breaking terms, see ref. 10.
Figure 1: The $|\delta_m|$ as a function of $x = m_{\tilde{\tau}}^2/m_l^2$, for an average slepton mass $m_{\tilde{l}} = 100$GeV.
5 Conclusions

We have analyzed the constraints on low-energy off-diagonal sfermion mass terms, coming from gluino- and photino-mediated FCNC and CP violating processes, in the model-independent framework of the mass-insertion method. The more stringent bounds come, in the hadronic sector, from $\Delta M_K$ and from the CP violating parameter $\varepsilon$, while in the leptonic sector they are obtained from the decay $\mu \rightarrow e\gamma$.

While the minimal supersymmetric standard model with universal soft breaking terms passes all these tests unscathed, a comparison between these constraints and the predictions of minimal SUSY SU(5) and SO(10) models shows that the values predicted for lepton radiative decays are in the ballpark of, or even clash with, the present experimental bounds.

The analysis of a simple model with minimal non-universality in the leptonic sector has shown that slepton masses at the large scale are required to be equal within a few percents. This constraint weakens to the level of 20% in a very accentuated gaugino-dominance framework. Similar constraints also hold in the hadronic sector.

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