Testing a lepton quarticity flavor theory of neutrino oscillations with the DUNE experiment

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Oscillation studies play a central role in elucidating at least some aspects of the flavor problem. Here we examine the status of the predictions of a lepton quarticity flavor theory of neutrino oscillations against the existing global sample of oscillation data. By performing quantitative simulations we also determine the potential of the upcoming DUNE experiment in narrowing down the currently ill-measured oscillation parameters $\theta_{23}$ and $\delta_{\text{CP}}$. We present the expected improved sensitivity on these parameters for different assumptions.

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I. INTRODUCTION

Despite the overwhelming success of the standard model of particle physics, it does not shed any light on the understanding of the masses and mixings of quarks and leptons - the so-called flavor problem. The experimental discovery of neutrino oscillations [1, 2] not only constitutes the first window into particle physics beyond the Standard Model, but also exacerbates the challenge posed by the flavor problem. Indeed, the observed pattern of neutrino oscillation parameters [3] indicates that leptons are very different from quarks insofar as the pattern of their charged current mixing is concerned.

There have been several recent theoretical models proposed in order to address the flavor problem by incorporating various flavor symmetries [4–12] to account for the valuable information that comes from oscillation studies. An alternative approach focusing upon the possible residual CP symmetries characterizing the neutrino mass matrices, irrespective of the details of the underlying theory, has also been considered in [13, 14].

Some of these theoretical constructions [7, 8] have prompted dedicated studies confronting their predictions with global neutrino oscillation data [15–17]. Here we consider a previously proposed neutrino oscillation theory. The flavor model construction implements an $A_4$ flavor symmetry as well as lepton quarticity symmetry [18]. The latter correlates dark matter stability with the predicted Dirac nature of neutrinos [19].

While this is an interesting connection in itself, leading to a viable dark matter scenario, it leads to novel neutrino predictions, for example the presence of neutrinoless quadruple beta decay ($0\nu 4\beta$) signal in the absence of neutrinoless double beta decay ($0\nu 2\beta$) [20]. Considering that Majorana neutrinos have so far remained elusive [21–23], the possibility that the quadruple beta decay might exist on its own [24] is especially intriguing and has already been subject to a dedicated experimental search by the NEMO collaboration [25].

Apart from these interesting features of the model, which arise from the quarticity symmetry, the model has other novel features owing to the presence of $A_4$ flavor symmetry. Thanks to the latter, the tree level dimension-4 Dirac mass terms for the neutrinos are forbidden. However, the $A_4$ symmetry allows us to generate seesaw-induced small neutrino masses. In addition, the model predicts a successful generalized “golden” Bottom-Tau unification formula [26, 29], as well as definite predictions for neutrino oscillations. For example, the scheme leads to normal neutrino mass ordering. It also leads to a strong correlation between the two currently ill-measured oscillation parameters, namely the leptonic CP phase $\delta_{CP}$ and the mixing angle $\theta_{23}$. This correlation in turn implies that CP must be significantly violated in neutrino oscillations, with the atmospheric angle $\theta_{23}$ lying in the second octant.

Owing to the precise predictions made by the model, it constitutes an ideal candidate to be probed at the forthcoming long baseline oscillation experiments aimed at measuring $\delta_{CP}$ and $\theta_{23}$, such as DUNE. Here we scrutinize the neutrino oscillation predictions obtained in the model against the latest available global neutrino oscillation study [3] as well as the future discriminating power of the DUNE experiment. Our strategy here is then, given the
current measurements of the four “well determined” oscillation parameters, i.e. the mass splittings characterizing solar and atmospheric oscillations plus the two mixing angles $\theta_{12}$ and $\theta_{13}$, to determine the potential of the upcoming DUNE experiment in narrowing down the still poorly measured parameters $\theta_{23}$-$\delta_{\text{CP}}$. We do this for the general “unconstrained” three-neutrino oscillation paradigm, as well for our “constrained” scenario in which the model predictions are taken into account. From our results we conclude that substantial improvements are to be expected.

II. CURRENT STATUS OF THE MODEL

In order to define our goal we first determine the current status of the neutrino oscillation parameters within the model, by taking into account the latest global analysis. We provide an improved update of the model neutrino oscillation predictions [18], originally tested against our previous oscillation global fit presented in [30] assuming only the one-dimensional $3\sigma$ intervals. Here we confront the model with the new results [3] and use the more complete $\chi^{2}$-distributions. In order to do so, we generated many points consistent with the model predictions. The latter are obtained by first randomly varying all the free parameters such as Yukawa couplings and new scalar field vevs over their allowed theoretical ranges. The points thus obtained are then tested for their compatibility with the currently well measured observables, such as quark and charged lepton masses [36]. For more details on the model predictions see [18]. Only points within the $3\sigma$ range of these parameters are retained as genuine points. Next, using the results from the global fit to neutrino oscillations in Ref. [3], we assign to each of those points a $\chi^{2}$-value that quantifies their agreement with most recent data. Given the negligible effect of solar parameters in DUNE we have simply selected those oscillation parameter sets with solar parameters within their allowed $3\sigma$ region, as derived in [3]. The resulting 4-dimensional $\chi^{2}$-maps are minimized over $\Delta m_{31}^{2}$ and $\sin^{2}\theta_{13}$, leading to the final distribution as a function of the parameters of interest, $\theta_{23}$ and $\delta_{\text{CP}}$.

The results for the current status of the model are presented in Fig. 1. One finds that, thanks to the new oscillation data, the predicted regions have shrunk significantly with respect to those in Ref. [18], so that CP phase values $\delta_{\text{CP}} \leq \pi$ are allowed only at $3\sigma$. Indeed, one sees that, at $3\sigma$ confidence level, one of the two allowed branches has nearly disappeared. We find that the new $4\sigma$ regions are roughly similar to the old $3\sigma$ regions, and there are no points surviving at the $1\sigma$ confidence level. Note that neither the current best fit point nor the local minimum from the global neutrino oscillation fit in [3] lies within the parameter region predicted by the model. As a result, if new data confirm the current best fit point, the local minimum or nearby values, the model would be strongly disfavored.

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1 The constraints coming from neutrino oscillation parameters are not imposed at this point. However, we do impose the cosmological limit on sum of neutrino masses [31].
III. SIMULATION OF THE DUNE EXPERIMENT

We simulate the DUNE experiment using the GLoBES package [32, 33] and the auxiliary file [34] used to produce the plots in Ref. [35]. In our simulation DUNE is running 3.5 years in the neutrino mode and another 3.5 years in the anti-neutrino mode. Using its 80 GeV beam with 1.07 MW beam power and the 40 kt far detector, this gives an exposure of 300 kton-MW-years, which corresponds to $1.47 \times 10^{21}$ protons on target (POTs) each year.

We consider the disappearance channels for neutrinos and anti-neutrinos, as well as the appearance channels. We also simulate the backgrounds, taking into account several sources of errors in our simulation, where we assign a 2\% error on the signals in the appearance channels and 5\% in the disappearance channels, as indicated in the studies performed by the DUNE Collaboration [35]. Likewise, we have implemented backgrounds ranging between 5\% and 20\%. These include misinterpretation of neutrinos as antineutrinos and vice-versa, contamination of electron neutrinos and antineutrinos in the beam, misinterpretation of muon as electron neutrinos, as well as the appearance and misinterpretation of tau neutrinos and neutral current interactions.

Here we are mainly interested in the currently poorly determined oscillation parameters $\sin^2 \theta_{23}$ and $\delta_{\text{CP}}$. Therefore, in order to simulate the future event rate in DUNE we fix the rest of the parameters to their best fit values reported in [3]. Then, in the statistical analysis performed to determine the DUNE sensitivity, we marginalize over $\theta_{13}$, $\theta_{12}$, $\Delta m^2_{31}$ and $\Delta m^2_{21}$ within their 1\%-ranges, see Table I. Concerning the parameters of interest, we generate future DUNE data by assuming several pairs of $(\theta_{23}^{\text{true}}, \delta_{\text{CP}}^{\text{true}})$. For each set of reconstructed parameters $(\theta_{23}, \delta_{\text{CP}})$ we calculate the $\chi^2$-function, given as

$$\chi^2(\theta_{23}, \delta_{\text{CP}}) = \min_{\theta_{ij}, \Delta m^2_{ji}, \alpha} \sum_{\text{channels}} \sum_{n} 2 \left[ N_{n}^{\text{test}} - N_{n}^{\text{dat}} + N_{n}^{\text{dat}} \log \left( \frac{N_{n}^{\text{dat}}}{N_{n}^{\text{test}}} \right) \right] + \sum_i \left( \frac{\alpha_i}{\sigma_i} \right), \quad (1)$$

Figure 1: Allowed regions at 2, 3 and 4\(\sigma\) in the plane $\theta_{23}$-$\delta_{\text{CP}}$ within the model, given the current global neutrino oscillation analysis.
Table I: Best fit values and 1σ relative uncertainties for the better determined neutrino oscillation parameters from [3].

| Parameter      | Best Fit Value          | Relative Error |
|----------------|-------------------------|----------------|
| $\Delta m^2_{21}$ | $7.56 \times 10^{-5}$eV$^2$ | 2.5%           |
| $\Delta m^2_{31}$ | $2.55 \times 10^{-3}$eV$^2$ | 1.6%           |
| $\sin^2 \theta_{13}$ | 0.02155                 | 3.9%           |
| $\sin^2 \theta_{12}$ | 0.321                   | 5.5%           |

where $\theta_{1j}$, $\Delta m^2_{j1}$ (j=2,3) denote the four well-measured oscillation parameters. Here $N_{\text{dat}}^n$ corresponds to the simulated event number in the $n$-th bin obtained with $\theta_{23}^{\text{true}}$ and $\delta_{\text{CP}}^{\text{true}}$. $N_{\text{test}}^n$ is the event number in the $n$-th bin associated to the parameters ($\theta_{23}, \delta_{\text{CP}}$) and $\alpha_i$ and $\sigma_i$ are the nuisance parameters and their corresponding standard deviations, respectively. Although not explicitly shown, note that $N_{\text{test}}^n$ also depends on $\bar{\alpha}$.

IV. RESULTS

In this section we present the main results of the analyses which we have performed in order to test the neutrino oscillation model in question. We start in Sec. IV A by performing an unconstrained DUNE sensitivity analysis for seven years of run time, assuming 3.5 years runs in both neutrino and anti-neutrino mode. In this analysis we have assumed that $\theta_{23}^{\text{true}}$ and $\delta_{\text{CP}}^{\text{true}}$ lie within the 1σ region obtained in the recent neutrino oscillation global fit [3]. This analysis is performed in order to quantify the projected sensitivity of the DUNE experiment given the current status of these parameters, and is completely model-independent.

In Sec. IV B we present the expected sensitivity on the currently ill-measured parameters after seven years running of DUNE, assuming that $\theta_{23}^{\text{true}}$ and $\delta_{\text{CP}}^{\text{true}}$ lie in the range predicted by the model. In this analysis we have taken into account only the model prediction for these parameters and have not taken into account the current global oscillation fit. Finally, in Sec. IV C we perform a combined analysis of the expected DUNE sensitivity taking into account, as input, both the range predicted by the model as well as the current oscillation global fit. The different analyses are performed in order to highlight the discriminating power of DUNE in various scenarios of interest, both from the model point of view as well from that of the current global fit.

A. Model-independent DUNE sensitivity

As explained above, in this section we study the sensitivity of DUNE to $\theta_{23}$ and $\delta_{\text{CP}}$, taking into account the current status of neutrino oscillations as reported in [3]. Assuming the true oscillation parameters to be the current best fit values would be too strong an
assumption. We have therefore decided to vary $\theta_{23}^{\text{true}}$ and $\delta_{\text{CP}}^{\text{true}}$ within their 1$\sigma$ ranges for two degrees of freedom (d.o.f.), indicated by the dashed black lines in Fig. 2. We have performed this analysis separately for the values in the lower and the upper octant of the atmospheric angle. For this we have defined

$$
\chi^2_{1\sigma}(\theta_{23}, \delta_{\text{CP}}) = \min_{\theta_{23}^{\text{true}}, \delta_{\text{CP}}^{\text{true}}} \chi^2(\theta_{23}, \delta_{\text{CP}}),
$$

where $(\theta_{23}^{\text{true}}, \delta_{\text{CP}}^{\text{true}})$ run first over all the values allowed in the lower octant, and later over all those allowed in the upper octant. Here $\chi^2(\theta_{23}, \delta_{\text{CP}})$ is the function given in Eq. 1.

The results of this minimization can be seen in Fig. 2, where we plot the 1$\sigma$, 2$\sigma$, 3$\sigma$ and 4$\sigma$ allowed regions for 2 d.o.f in the $\sin^2 \theta_{23} - \delta_{\text{CP}}$ plane. The left (right) panel corresponds to the analysis assuming $\theta_{23}^{\text{true}}$ to lie in the lower (upper) octant. At the moment, the lower octant is preferred by the global oscillation data, and therefore there are much more points in this region, resulting in a bigger region in our plot.

Notice that in the left panel, the degenerate solution in the second octant appears only at the 3$\sigma$ confidence level. Conversely, if the true value of the atmospheric mixing angle lies in the small region in the upper octant (see right panel of Fig. 2), the degenerate first-octant solution would be ruled out at more than 4$\sigma$. Maximal mixing is disfavored at more than 3$\sigma$ (5$\sigma$) for $\theta_{23}^{\text{true}}$ in the lower (upper) octant. In both cases, values of $\delta_{\text{CP}} \approx 0.5\pi$ would be excluded with very high significance.
B. Testing the model with DUNE

In order to quantify the sensitivity of DUNE to test the model predictions, we now perform a simulation of DUNE suited to the model of interest. Our procedure will not depend on any input from global neutrino oscillation fits. This means we assume the model prediction for $\theta_{23}$ and $\delta_{\text{CP}}$ to be the true values used to generate DUNE data. As in the last section, we define the $\chi^2$ function as

$$\chi^2_{\text{DUNE+model}}(\theta_{23}, \delta_{\text{CP}}) = \min_{\theta_{23}^{\text{true}}, \delta_{\text{CP}}^{\text{true}}} \chi^2(\theta_{23}, \delta_{\text{CP}}).$$

(3)

In this case, $(\theta_{23}^{\text{true}}, \delta_{\text{CP}}^{\text{true}})$ are not the values from the 1σ regions of $[3]$, but include, instead, all the points predicted by the model and consistent at 3σ with the current global fit, see Fig. 1. The resulting regions corresponding to 1σ to 4σ confidence level for 2 d.o.f. are presented in Fig. 3. These results correspond to the case where only the model predictions are taken into account. In this case one finds that, by themselves, model predictions plus DUNE data would not suffice to determine the octant of the atmospheric angle or a unique preferred range for the CP phase, at least not for all parameter choices.

By including valuable information on the current status of global fits to neutrino oscillations one can sharpen the expected DUNE sensitivity to the oscillation parameters, beyond the results of Fig. 3. This is done in the next section.

Figure 3: DUNE sensitivity to the $(\sin^2 \theta_{23}, \delta_{\text{CP}})$ parameter region predicted by the model consistent with the current global fit at 3σ.

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2 Notice that, ideally, only one set of $(\theta_{23}, \delta_{\text{CP}})$ would be realized in nature as the true value while, to be conservative, here we marginalize over all neutrino oscillation parameters possible in the model.
C. Testing the model with DUNE: the global picture

In order to better quantify the sensitivity of DUNE to test the model predictions we now perform a “constrained global neutrino oscillation fit” suited to the model of interest. We do this by combining the DUNE simulation with the global fit to neutrino oscillations in Ref. [3] in the context of the lepton quarticity flavor model under study. In order to combine the results of the DUNE simulations performed here with the global analysis of neutrino oscillation data we simply sum the \( \chi^2 \) function defined in Sec. IVB with the \( \chi^2 \) grid obtained in the global fit to neutrino oscillations in Ref. [3],

\[
\chi^2_{\text{tot}}(\theta_{23}, \delta_{\text{CP}}) = \chi^2_{\text{DUNE} + \text{model}}(\theta_{23}, \delta_{\text{CP}}) + \chi^2_{\text{fit}}(\theta_{23}, \delta_{\text{CP}}). \tag{4}
\]

The results are presented in Fig. 4 where we plot the regions allowed at 1\( \sigma \) to 4\( \sigma \) confidence level for 2 d.o.f. One sees that by combing all the relevant information the regions shrink with respect to those of Fig. 3 since the global fit to current neutrino data disfavors \( \delta_{\text{CP}} \) in the range \([0, \pi]\). This result is shown in the corresponding panel of Fig. 8 in [3]. One sees that, by properly taking into account the current knowledge of neutrino oscillation parameters and the model under consideration, one concludes that DUNE will determine rather well the CP phase at the 1\( \sigma \) level, excluding CP-conserving scenarios at more than 3\( \sigma \). One sees also that, at the 1\( \sigma \) level, the second octant would be singled out. Besides that, the status of maximal mixing would worsen in comparison to Fig. 3 as a consequence of the recent global fit results, although it would still remain allowed at the 2\( \sigma \) level for certain consistent model parameter choices. As commented before, here we are marginalizing over a large set of true oscillation parameters and therefore, the real sensitivity given in Fig. 4 would still be too conservative.
V. SUMMARY AND DISCUSSION

Neutrino oscillation studies may play a key role in elucidating major aspects of the flavor problem. Here we have provided a quantitative study of the status of the predictions of a lepton quarticity flavor theory of neutrino oscillations. Thanks to the assumed flavor symmetry, the model explains the small neutrino masses as a result of a variant of the seesaw mechanism leading to Dirac neutrinos. Due to quarticity, the model has a viable dark matter candidate stabilized by the Diracness of neutrinos. In addition, it leads to a successful “golden” Bottom-Tau unification formula, as well as definite predictions for neutrino oscillations, first studied in [18]. Here we have reexamined the consistency of neutrino oscillation model predictions in view of the latest global sample of neutrino oscillation data [3]. One finds that the model predicts normal neutrino mass ordering, and significant violation of CP in neutrino oscillations, with the atmospheric angle $\theta_{23}$ lying in the second octant. Our results are given in Fig. 1. By performing dedicated simulations we have also determined the potential of future DUNE data in further restricting the currently ill-measured oscillation parameters, $\theta_{23}$ and $\delta_{CP}$. Fig. 2 illustrates the resulting sensitivity for the “unconstrained” model-independent case, assuming the true $\theta_{23}$ and $\delta_{CP}$ parameters to lie within the 1$\sigma$ region obtained from the recent global fit to neutrino oscillations as given in [3]. By taking into account not only the information from the current neutrino oscillation global fit results but also the specific model predictions, we have shown that DUNE data should unambiguously single out the second octant of $\theta_{23}$ and exclude values of $\delta_{CP}$ below $\pi$ at the 1$\sigma$ level, as seen in Fig. 4. Finally we stress that, as already mentioned, by marginalizing over a large set of potential true oscillation parameter values, we are being conservative in our estimate of the improved sensitivity of DUNE to test the model under consideration.

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