We discuss how the possible changes of hadron properties can be tested, associated with the chiral transition in nuclear medium; the hadrons discussed are the vector mesons, the \( \sigma \) meson and the chiral partner of the nucleon. We emphasize that the proper quantity is the spectral function which describes the nuclear medium having a hadron; the analysis of electro-quasielastic reactions, for instance, has a relevance to the physics of CERES. The similarity is indicated between the physics discussed here with that of giant resonances. Experiments using nuclear targets are proposed to observe the \( \sigma \) meson and to explore how the partial chiral restoration in nuclear medium manifests itself.

1. Introduction

Various theoretical approaches suggest that the QCD vacuum undergoes the phase transition with the order parameter \( \langle \bar{q} q \rangle \) at high baryonic density \( \rho_B \) as well as at high temperature \( T \)\cite{1–3}. However, our theoretical understanding on the nature of the chiral transition at \( \rho_B \) is relatively limited, although some suggestive results have been obtained. In other words, there are a lot of things to be clarified and explored on the problem of the chiral transition at \( \rho_B \).

To have an idea on relevant observables to explore the problem, let us start with recalling the very fact that the chiral transition is a phase transition of the QCD vacuum. In condensed matter physics and also in nuclear physics, a change of the ground state (the vacuum) is examined through the study of a possible change of collective excitations on top of the ground states\cite{2, 4}: If the phase transition is of second order or of weak first order, there will appear characteristic changes in some specific collective modes, which are actually the fluctuations of the order parameter or modes coupled to them. Collective excitations in QCD are hadrons, especially the low lying ones. Therefore, the study of hadrons in the nuclear medium has the possibility to give the insight into the nature of the chiral restoration at finite density. Indeed, there are suggestions on how some hadrons change their properties in association with changes of the QCD vacuum \cite{2, 3}.

When a hadron is put in a nucleus, the hadron will couple strongly to various excitations in the system, such as nuclear particle-hole (p-h) and \( \Delta \)-hole excitations, simultaneous excitations of them and mesons and so on: In general,
the hadron may dissociate into complicated excitation to lose its density in the nuclear medium. The relevant quantity is the response function or spectral function of the system when the quantum numbers of the hadron are put in. If the coupling of the hadron with the environment is relatively small, then there may remain a peak with a small width in the spectral function, which correspond to the hadron; such a peak may be viewed as an elementary excitation or a quasi particle known in Landau’s Fermi liquid theory for fermions. Landau gave an argument that there will be a chance to describe a system as an assembly of almost free quasi-particles owing to the Pauli principle when the temperature is low. Symmetries of the system may ensure the existence of an elementary bosonic excitation in the many-body system[4–6].

In this report, we discuss three topics on possible changes of hadron properties in the nuclear medium; the hadrons to be discussed are the $\rho$ meson, the sigma meson and the possible chiral partner of the nucleon.

2. The spectral function in the vector channel and the inclusive electron scattering

Various effective models, the QCD sum rules and the scaling argument all predicted a decrease of the vector meson masses $m_v$. The subsequent measurement of the lepton pairs from the heavy-ion collisions by CERES group [7] seemed to show a decrease of the spectral function in the $\rho$ meson channel, which might be an evidence of the decrease of $m_v$ [8]. Thus the experiment prompted heated discussions on possible interpretation of the data.

The cross section in this reaction is essentially the spectral function

\[
R_{L,T}(\omega, q) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \propto \text{Im}\Pi_{L,T}(\omega, q), \tag{1}
\]

where $\Pi_{L,T}(\omega, q)$ is the longitudinal (transverse) response function in the $\rho$ meson channel; the spectral function is given by the imaginary part of the response function or the retarded Green’s function. At $\rho_B \neq 0$, $\Pi_L(\omega, q) \neq \Pi_T(\omega, q)$ when $q \neq 0$, and manybody effects as well as the lack of the Lorenz invariance gives rise to a significant modification of the spectral function from that in the free space. For example, the coupling of the process $\rho \rightarrow p - h (\Delta - h) + \pi$, which makes a collective mode with the $\rho$ meson quantum number, increases the width of the vector meson. Furthermore the $p$-wave coupling of the $\rho$ meson with baryons induces the momentum dependence of the spectral function. Indeed there exist many baryons, i.e., excited states of the nucleon and the $\Delta$, which couple to the $\rho$ meson; the $(\omega, q)$-regions of some of the baryon-hole excitations overlap with the dispersion relation of $\omega = \sqrt{m_\rho^2 + q^2}$, which makes the spectral function in the $\rho$ meson channel complicated; see Fig.1, where the $(\omega, q)$-regions of various baryon-hole excitations are shown together with the $\rho$ meson dispersion relation (in the free space).
However, there are some ambiguities in the theoretical calculations based on such a hadron scenario; many coupling constants of the vector meson with baryons including their form factors are obviously not available. In fact, S.H. Lee has shown that the QCD sum rule approach gives different momentum dependence of the dispersion relation of the $\rho$ meson[9].

The problem is to extract the global structure of the spectral function in the $\rho$ channel and understand it theoretically. The moral extracted from these studies may be that putting a hadron in a nucleus is nothing but providing the same quantum numbers as those possessed by the hadron. And the proper observables for this physics is the response function or the spectral function in this channel, which might show peaks corresponding to the hadron as an elementary excitation in the system.

The reader should have recognized that the underlying physics of this game is the same as that of giant resonances[10]: The physics of the giant resonances are actually concerned with the global structure of the response function in the relevant channel. The analogy or correspondence goes further: Spin-isospin dependent giant resonances are related with the pion and the $\rho$ meson in nuclei[11, 12]: The possible softening of the longitudinal spin-isospin mode in the large momentum region (or modes with a large angular momentum) is a precursor of the pion condensation, a phase transition of nuclear matter[12]. This is an example for the
general feature that there arises a soft mode as a precritical phenomena of the phase transition, as mentioned before.

We notice that exploring the spectral function (the response function) in the $\rho$ channel has a relevance to a longstanding problem in the inclusive electron scattering[13]: The cross section of the electro-quasi-elastic reaction in the transverse channel has an anomalously large cross section in the energy region between the nucleon- and the $\Delta$-production region. The origin of the anomalous strength may be attributed to a possible $\Delta$-hole collective excitation due to the strong $p$-wave $\rho$-$N\Delta$ tensor coupling[14]; the collective mode supplies a strength in the $(\omega, q)$ region below the free $\Delta$-hole dispersion curve, which region is found to be kinematically hit by the quasi-free electron scattering. Indeed, the attractive $\rho$-baryon tensor coupling $B^\dagger \sigma \tau B' \times \nabla \rho$ in the transverse channel could make the $\rho$-meson condensation in nuclear matter at high density[15].

3. The sigma meson and the spectral function in the sigma channel in the nuclear medium

The sigma meson is the chiral partner of the pion for the $SU_L(2) \otimes SU_R(2)$ chiral symmetry in QCD. The particle representing the quantum fluctuation of the order parameter $\tilde{\sigma} \sim \langle (: \bar{q} q :)^2 \rangle$ is the sigma meson. The sigma meson is analogous to the Higgs particle in the Weinberg-Salam theory. Some effective theories and Weinberg’s mended symmetry predict the sigma meson mass $m_\sigma \sim 500$-$700$ MeV. Such a scalar meson with a low mass can account for various experimental and empirical facts[2, 17].

A tricky point on the sigma meson is that the sigma meson strongly couples to two pions which gives rise to a huge width $\Gamma \sim m_\sigma$. Recent phase shift analyses of the pi-pi scattering in the scalar channel claim a pole of the scattering matrix in the complex energy plane with the real part $\text{Re} m_\sigma = 500$-$700$ MeV and the imaginary part $\text{Im} m_\sigma \simeq 500$ MeV[16].

Since the sigma meson is the fluctuation of the order parameter of the chiral transition, it will become a soft mode and induce characteristic phenomena associated with the chiral restoration in a hot and/or dense nuclear medium. Thus one may expect a better chance to see the sigma meson in a clearer way in a hot and/or dense medium than in the vacuum[2].

The present author proposed three types of experiment to produce the sigma meson in a nucleus[17]; the probes are pions, protons, light nuclei and electrons. To detect the sigma, one may use 4 $\gamma$’s and/or two leptons. The latter process is possible when the $\sigma$ has a three momentum owing to the scalar-vector coupling at $\rho_B \neq 0$.

In the introduction, we emphasized that the relevant observable is the spectral function. Recently, a calculation of the spectral function in the sigma channel has been performed with the $\sigma$-$2\pi$ coupling incorporated in the linear $\sigma$ model at finite $T$; it was shown that the enhancement of the spectral function in the
σ-channel just above the two-pion threshold is the most distinct signal of the softening [18].

More recently, it has been shown [19] that the spectral enhancement associated with the partial chiral restoration takes place also at finite baryon density close to $\rho_0 = 0.17$fm$^{-3}$. Refering to [19] for the explicit model-calculation, let us describe the general features of the spectral enhancement near the two-pion threshold. Consider the propagator of the $\sigma$-meson at rest in the medium:

$$D^{-1}_\sigma(\omega) = \omega^2 - m^2_\sigma - \Sigma_\sigma(\omega; \rho),$$

where $m_\sigma$ is the mass of $\sigma$ in the tree-level, and $\Sigma_\sigma(\omega; \rho)$ is the loop corrections in the vacuum as well as in the medium. The corresponding spectral function is given by

$$\rho_\sigma(\omega) = -\pi^{-1} \text{Im} D_\sigma(\omega).$$

We parametrize the chiral condensate in nuclear matter $\langle \sigma \rangle$ as

$$\langle \sigma \rangle \equiv \sigma_0 \Phi(\rho).$$

In the linear density approximation, $\Phi(\rho) = 1 - C \rho / \rho_0$ with $C = (g_s / \sigma_0 m^2_\sigma) \rho_0$. Instead of using $g_s$, we use $\Phi$ as a basic parameter in the following analysis. The plausible value of $\Phi(\rho = \rho_0)$ is $0.7 \sim 0.9$ [2].

The spectral function together with $\text{Re}D^{-1}_\sigma(\omega)$ calculated with a linear sigma model are shown in Fig.2: The characteristic enhancements of the spectral function just above the $2m_\pi$. The mechanism of the enhancement is understood as follows. The partial restoration of chiral symmetry implies that $m_\sigma^*$ approaches toward $m_\pi$. On the other hand, $\text{Re}D^{-1}_\sigma(\omega)$ has a cusp at $\omega = 2m_\pi$. The cusp point goes up with the density and eventually hits the real axis at $\rho = \rho_c$ because $\text{Re}D^{-1}_\sigma(\omega)$ increases associated with $m_\sigma^* \rightarrow 2m_\pi$. It is also to be noted that even before the $\sigma$-meson mass $m_\sigma^*$ and $m_\pi$ in the medium are degenerate, i.e., the chiral-restoring point, a large enhancement of the spectral function near the $2m_\pi$ is seen.

To confirm the threshold enhancement, measuring $2\pi^0$ and $2\gamma$ in experiments with hadron/photon beams off the heavy nuclear targets are useful. Measuring $\sigma \rightarrow 2\pi^0 \rightarrow 4\gamma$ is experimentally feasible [20], which is free from the $\rho$ meson meson background inherent in the $\pi^+\pi^-$ measurement. Measuring of $2\gamma$'s from the electromagnetic decay of the $\sigma$ is interesting because of the small final state interactions, although the branching ratio is small.$^a$ Nevertheless, if the enhancement is prominent, there is a chance to find the signal. When $\sigma$ has a finite three momentum, one can detect dileptons through the scalar-vector mixing in matter: $\sigma \rightarrow \gamma^* \rightarrow e^+e^-$. We remark that $(d, ^3\text{He})$ reactions is also useful to produce the excitations in the $\sigma$ channel in a nucleus because of the large incident flux, as the $\eta$ production[23].

$^a$One needs also to fight with large background of photons mainly coming from $\pi^0$s.
FIG. 2. Spectral function for $\sigma$ and the real part of the inverse propagator for several values of $\Phi = \langle \sigma \rangle / \sigma_0$ with $m_{\sigma}^{\text{peak}} = 550$ MeV. In the lower panel, $\Phi$ increases from bottom to top.

is estimated to be $1.1 \text{GeV} < E < 10 \text{ GeV}$, to cover the spectral function in the range $2m_\pi < \omega < 750$ MeV.

Recently CHAOS collaboration [21] measured the $\pi^+\pi^\pm$ invariant mass distribution $M_{\pi^+\pi^\pm}$ in the reaction $A(\pi^+, \pi^+\pi^\pm)X$ with the mass number $A$ ranging from 2 to 208: They observed that the yield for $M_{\pi^+\pi^-}^A$ near the $2m_\pi$ threshold is close to zero for $A = 2$, but increases dramatically with increasing $A$. They identified that the $\pi^+\pi^-$ pairs in this range of $M_{\pi^+\pi^-}^A$ is in the $I = J = 0$ state. The $A$ dependence of the the invariant mass distribution presented in [21] near $2m_\pi$ threshold has a close resemblance to our model calculation in Fig.2, which suggests that this experiment may already provide a hint about how the partial restoration of chiral symmetry manifest itself at finite density.\textsuperscript{b}

4. Parity doubling of baryons

So far we have discussed the meson properties in the nuclear medium in relation with the possible chiral restoration. Then, do baryons show up any char-

\textsuperscript{b}See [22] for other approaches to explain the CHAOS data.
acteristic features when the chiral symmetry is (partially) restored? To answer this question, one needs to clarify how baryons can be described in the context of chiral symmetry. There are actually some approaches on this problem; for example, we know the nonlinear representation theory and the Skyrm model of baryons. Here, we introduce a linear sigma model where baryons are described in the linear representation of the chiral symmetry, and discuss its phenomenological consequences.

About a decade ago, motivated by the lattice simulation on the screening masses of the nucleons with positive and negative parity both in chirally broken and restored phases, De Tar and the present author proposed a linear sigma model with parity doubling[24]. The model reads,

\[ L = \bar{\Psi} i \gamma \cdot \partial \Psi - g_1 \bar{\Psi} (\sigma + i \tau \cdot \pi \rho_3 \gamma_5) \Psi, \]

\[ + g_2 \bar{\Psi} (\rho_3 \sigma + i \tau \cdot \pi \gamma_5) \Psi - i M_0 \bar{\Psi} \rho_2 \gamma_5 \Psi \]

\[ + L_M (\sigma, \pi), \] (4)

with \( \Psi = t(\psi_1, \psi_2) \) where \( \psi_1 \) and \( \psi_2 \) are both a Dirac spinor and \( \rho_i \) \((i = 1, 2, 3)\) is the Pauli matrices acting on the spinors composed of \( \psi_1 \) and \( \psi_2 \). \( \psi_i \) \((i = 1, 2)\) are supposed to be the ur-state of the positive-parity and the negative-parity nucleon, respectively; the term with \( \rho_2 \) mixes the two Dirac spinors to make the physical eigenstates \( \Psi^+ \) and \( \Psi^- \) with positive and negative parity, respectively.

This lagrangian can be shown invariant under the extended chiral transformation,

\[ \delta \Psi = 1/2 \cdot i \alpha \cdot \tau \Psi + 1/2 \cdot i \beta \cdot \tau \rho_3 \gamma_5 \Psi. \] (5)

In the tree level, the eigen-value problem is solved yielding,

\[ \Psi^+ (p) = N t(\psi^+_+(p), e^{-\theta} \gamma_5 \psi^+_+(p)), \]

\[ \Psi^- (p) = N t(-e^{-\theta} \gamma_5 \psi^-_-(p), \psi^-_-(p)), \]

with \( \gamma \cdot p \psi^\pm (p) = M^\pm \psi^\pm (p), \) \( M^\pm = \mp g_2 \sigma_0 + \sqrt{(g_\sigma_0)^2 + M_0^2}, \) (6)

where \( N = 1/\sqrt{1 + \exp(-2\theta)} \), which is determined by the normalization conditions \( \Psi^\pm \Psi^\pm = 1 \). We remark that (1) \( M^- > M^+ \) and (2) \( M^\pm \rightarrow |M_0| \neq 0. \)

It can be shown that \( \Psi^- \) has the opposite parity and the charge conjugation to \( \Psi^+ \).

The axial charge matrix is calculated to be

\[ \hat{g}_A = \begin{pmatrix} \tanh \theta & -1/\cosh \theta \\ -1/\cosh \theta & -\tanh \theta \end{pmatrix}, \] (7)

where the 1-1 (2-2) component denotes \((g_A)_{NN} ((g_A)_{N'N'})\), for example. We remark that the negative parity nucleon has a negative axial charge in this model. One can also show that the generalized Goldberger-Treiman relation;

\[ g_{\pi NN} = g_{ANN} M_+/\sigma_0, \]

\[ g_{\pi N'N'} = g_{AN'N'} M_-/\sigma_0, \]

\[ g_{\pi NN'} = g_{ANN'} (M_+ - M_-)/2\sigma_0. \] (8)
So far, we have not identified the negative parity nucleon N’ with any particle. There are two possibilities; one is that N’≡ N(1535) and the other is that N’ is elusive like the sigma meson and has not been observed experimentally, yet. Here, let us assume the first possibility. Then, from the decay width $\Gamma_{\pi N} \simeq 70$ MeV for $N' \rightarrow N + \pi$, and the empirical values $M_+ = 939, \quad M_- = 1535, \quad \sigma_0 = 93$, in MeV, we have $g_{\pi NN'} = 0.70$, which correspond to the values of the model parameters as follows; $\sinh \theta = 5.5, \quad g_1 = 13.0, \quad g_2 = 3.2, \quad M_0 = 270$ (MeV). How can the model be tested experimentally? A characteristic feature of this model is that the axial charge of the nucleon and N(1535) has the opposite sign, provided that N’ is identified with N(1535). Therefore experiments which can see an interference between the two amplitudes including $g_{\pi NN}$ and $g_{\pi NN'}$, respectively, will be interesting.

It is also interesting to explore the properties of the negative-parity nucleon experimentally. Assuming that the sigma condensate $\sigma_0$ decreases with the baryonic density, we can discuss some phenomenological consequences of the model at finite density: Fig.3 shows the $\sigma_0$ dependence of the masses $M_{\pm}(\sigma_0)$. One can see that both the masses $M_N$ and $M_{N'}$ decrease as the chiral symmetry is restored, which suggests that a downward mass shift of both the particles at finite density. The rate of the decrease is larger for N(1535) than the nucleon. The axial charges and the coupling with the pion of both the nucleon decrease as the chiral symmetry is restored, which may give another origin of the quenching of the $\beta$ decay in nuclei and affect the cooling rate of neutron stars; see [25] for recent developments.

![Fig.3 The masses $M_{\pm}(\sigma_0)$ as functions of $\sigma_0$](image_url)
5. Summary

This report may be summarized as follows: (1) The chiral transition (or a change of the QCD vacuum generically) may imply that of properties such as mass shifts and a change of the lifetime of hadrons as (collective) excitations on top of the vacuum. The hadrons discussed in this context are the sigma meson, the $\rho$ meson and the negative-parity nucleon. (2) If a hadron is put in in the nuclear medium, the coupling of the hadron with the environment may be so large that it becomes inadequate to describe the system in terms of the mass and the width of the hadron. The proper quantity to describe the situation properly is the strength function in the hadron channel. (3) Nevertheless, if the hadron exists as a quasi-particle not loosing its identity, it is still adequate to interpret experimental data in terms of the mass and the width of the hadron. (4) Experiments using light ions and electrons as probes are as useful to explore the physics discussed here as super-relativistic heavy ion collisions. They are also complementary to each other. (5) The anomalous enhancement near two-$m_\pi$ threshold in the strength function in $I = J = 0$ channel (the sigma channel) is interesting to see how the partial restoration of the chiral symmetry manifest itself in the nuclear medium. Some experiments have been proposed on this problem.

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