Development of an accurate DWIA model of coherent $\pi^0$–photoproduction to study neutron skins in medium heavy nuclei

F. Colomer$^{1,2}$, P. Capel$^{1,2}$, M. Ferretti$^2$, M. Thiel$^2$, C. Sfienti$^2$, V. Tsaran$^2$ and M. Vanderhaeghen$^2$

$^1$ Physique Nucléaire et Physique Quantique CP229, Université libre de Bruxelles (ULB), B-1050 Brussels, Belgium
$^2$ Institut für Kernphysik, Johannes Gutenberg-Universität Mainz, D-55099 Mainz, Germany
E-mail: fcolomer@ulb.ac.be

Abstract.
Despite decades of studies which have seen the nuclear charge distribution being measured with increasing precision, the neutron distribution remains elusive. The difference between the neutron and proton distributions is often expressed as the difference of their root mean square radii: the neutron skin thickness. Recently, the A2 collaboration at MaMi has measured the skin thickness in lead through coherent pion photoproduction [1] with a very high precision. However, they do not include theoretical uncertainties, which can be significant for this process.

A new reaction code in the distorted wave impulse approximation (DWIA) is developed to help the (ongoing) analysis of the recent measurement by the A2 collaboration at MaMi of the coherent pion photoproduction cross section on $^{116,120,124}$Sn isotopes [2] and to properly quantify the theoretical uncertainties.

1. Introduction

The symmetry energy is one of the most important ingredients of the equation of state (EOS) of nuclear matter. It governs phenomena in nuclear structure, nuclear reactions as well as nuclear astrophysics. Properties of very small systems such as nuclei ($R \sim 10^{-15}$ m) but also macroscopic objects such as neutron stars ($R \sim 10^4$ m) are governed by its density dependence. Setting constraints on this fundamental quantity has become one of the key objectives of contemporary nuclear physics [3, 4].

The symmetry energy $S$ represents the energy needed by a symmetric system of equal number of protons and neutrons to convert all its protons into neutrons. It is customary to write the EOS and the symmetry energy as an expansion around the symmetric limit and around saturation density $\rho_0$ as [3, 4]

$$E(\rho, \alpha) = E(\rho, \alpha = 0) + \alpha^2 S(\rho) + \mathcal{O}(\alpha^4) \quad \text{with} \quad S(\rho) = J + Lx + \frac{1}{2}K_{\text{sym}}x^2 + \ldots \quad (1.1)$$

where $\alpha = (\rho_N - \rho_Z)/\rho$ is the neutron-proton asymmetry and $x = (\rho - \rho_0)/3\rho_0$ characterizes the deviation from nuclear saturation density. Much effort has been dedicated to constrain the slope of the symmetry energy $L$ which quantifies the difference between the symmetry energy...
at the core of a nucleus (at saturation density) and at its surface (at lower densities). In finite asymmetric nuclear matter, surface tension tends to push the excess neutrons inside the core to have a compact system. On the contrary, the symmetry energy term favours an equal number of protons and neutrons inside the high-density core, and therefore tends to move these excess neutrons towards the peripheral region of the nucleus, where the density is lower. If $L$ is large (the EOS is stiff), the balance goes heavily in favour of the symmetry energy and excess neutrons then tend to form a thick neutron skin. On the contrary, if $L$ is small (the EOS is soft), the skin is thinner. For this reason, $L$ has been shown to be highly correlated to the neutron skin thickness in heavy nuclei [5].

But measuring the neutron skin is a challenge. While the charge distribution has been measured with increasing precision throughout the last century, the neutron distribution remains difficult to measure. Several different experiments have been carried out in the last decades to try to lay a hand on this elusive quantity with more or less success (for a review of the latest efforts and progress in this field, we refer to [4]). In this work, we focus on coherent $\pi^0$-photoproduction, a process in which a photon impinges on a target and produces a $\pi^0$ while the target remains in its ground state. The pion is then measured through its two-photon decay.

In Sec. 2 we introduce the method in a simple first order approximation before extending our model to second order. This model is then compared to the latest experiment carried out on this process [2] in Sec. 3. This experiment has measured the cross section for coherent $\pi^0$-photoproduction on three isotopes of tin (116,120,124Sn). We illustrate the strengths and weaknesses of this method and of the model and what are the prospects of this work.

2. Coherent neutral-pion photoproduction

2.1. Plane wave impulse approximation (PWIA)

The pion photoproduction cross section on a nucleus can be constructed from the standard CGLN amplitudes $F_i$ [6] that describe the elementary process on a nucleon. A good approximation is then to consider that the nuclear photoproduction amplitude is merely the sum of the contributions of the elementary amplitudes on each nucleon: the impulse approximation [7]. In the particular case of coherent photoproduction of neutral pions on spin and isospin zero nuclei, only the isoscalar non-spin-flip part component $F_2$ remains.

We can go further and assume as a first-order approximation that the neutral pion exits the nucleus without any final state interaction, as a plane wave. These two assumptions are commonly known as the plane wave impulse approximation (PWIA). The nuclear pion photoproduction cross section then reads

$$\frac{d\sigma_{\pi\gamma}}{d\Omega} = \frac{1}{2} \frac{k_\pi}{k_\gamma} \sum_\lambda |V^{(\lambda)}_{\pi\gamma}|^2$$

(2.1)

with the amplitude $V^{(\lambda)}_{\pi\gamma}$ reading [7]

$$V^{(\lambda)}_{\pi\gamma}(k_\pi, k_\gamma) = \mathcal{W}_A f_2 \rho_A(q) \left[ \hat{k}_\pi \times \hat{k}_\gamma \right] \cdot \vec{\varepsilon}_\lambda$$

(2.2)

where $\mathcal{W}_A$ is a kinematical factor, $f_2$ is the photoproduction elementary amplitude $F_2$ boosted to the center-of-mass frame and $\rho_A(q)$ is the Fourier transform of the nuclear density. More information on all assumptions that lead to this expression can be found in [7]. Note however that in this work, we choose to consider the spectator on-shell model, in which the $A-1$ nucleus is considered on-shell during the reaction because it seems the most physical. This choice is different to the one considered in [7].

As we can see, the amplitude is proportional to the whole nucleon density $\rho_A$. This illustrates the interest of this process in the measure of the neutron skin: by combining the measurements of
charge density to photoproduction measurements, we should have a direct access to the neutron density and hence, to the neutron skin.

2.2. Distorted wave impulse approximation (DWIA)

In previous section, we considered the pion motion in the outgoing channel to be described by a mere plane wave. In reality, the final state interactions (FSI) play a significant role in the final photoproduction cross section. This is illustrated for example in Fig. 5 of [7]. The influence of the distortion increases with the energy of the incoming photon as we enter into the delta-resonance region. While the PWIA should be a good approximation in the lower energy bins measured in the experiment [2], FSI will have to be considered to accurately capture the reaction process and hence properly extract the neutron skin thickness.

The distorted wave impulse approximation (DWIA) adds a distortion term to $V^{(λ)}_{πγ}$. This term is constructed from the pion-nucleus scattering amplitude. A detailed expression can be found in [7]. Within the impulse approximation and the multiple scattering formalism of KMT [8], the pion-nucleus scattering amplitude can be constructed from the most general expression of s- and p-wave contributions to the elementary $π-N$ scattering amplitude

$$t_{πN}(k_π',k_π) = b_0 + b_1 \hat{t}_π \cdot \hat{τ}_N + (c_0 + c_1 \hat{t}_π \cdot \hat{τ}_N)k_π \cdot k_π' \tag{2.3}$$

where $\hat{t}_π$ and $\hat{τ}_N$ are the isospins of the pion and the nucleon. The values of the parameters of this expansion can then be taken from SAID [9], a partial wave analysis of the $π-N$ elementary scattering phaseshifts.

In this work, we will use the potential of Carr et al. [10], a phenomenological potential derived from Eq. (2.3) and which adds a necessary absorptive part which is not present in the elementary scattering amplitude. The coefficients of this potential have then been fitted to pion-nucleus scattering data on a wide range of nuclei going from carbon to lead. Unfortunately, these fits have been done at a pion laboratory energy limited to 30 and 50 MeV. In the process of pion photoproduction, these energies would roughly correspond to pions produced by photons of energies between 160 and 180 MeV. This is slightly below the energies considered at MaMi. Additionally, while this potential has Eq. (2.3) as a starting point, several simplifying assumptions cause the fitted coefficients to depart from the SAID values. This hinders the extrapolation of these coefficients to higher energies.

3. Results and discussion

Three choices have been made to reproduce the $^{116}$Sn nucleon density: the phenomenological density (a simple Fermi-Dirac shape) of the São Paulo group [11] which exhibits a negative neutron skin of $Δr_{np} = -0.12$ fm and two densities from relativistic mean field (RMF) calculations in the FSU model of Todd-Rutel and Piekarewicz [12] that exhibit neutron skins of $Δr_{np} = 0.17$ fm (FSU000) and $Δr_{np} = 0.10$ fm (FSU040). These densities are represented in Fig. 3.1.

Note that the Sao Paulo density distribution has a negative skin thickness, suggesting that there is actually a proton skin. While this is rather unrealistic for $^{116}$Sn, this increases the difference in skin thickness between all densities considered here. This allows us to better study the sensitivity of our calculations to this observable.

3.1. PWIA cross sections

Let us first start by analysing the cross sections obtained at the PWIA. As these are proportional to the density, this will ease our interpretation of the results. The cross sections are represented on Fig. 3.2 for two bin energies of the impinging photon.
Figure 3.1: Different densities for $^{116}$Sn used in this work. Neutron (red) and proton (black) densities in the phenomenological model (solid) of the Sao Paulo group [11] of thickness $\Delta r_{np} = -0.12$ fm and the two densities from RMF calculations FSU000 $\Delta r_{np} = 0.17$ fm (dashed) and FSU040 $\Delta r_{np} = 0.10$ fm (dash-dotted) in the FSU model [12].

As can be observed, the cross sections vary with the choice of density by about 10% at their peak value at both energies, the densities with the largest skin having the smallest maximum. These differences are comparable in size with the size of the error bars of the preliminary data. It should also be noted that the peak is wider in the case of small skins such as Sao Paulo. This appears like a shift in the larger angle region of the peak of about $2^\circ$irc when we compare extreme values of the skin. This shift is of only $0.5^\circ$irc when we compare realistic RMF densities, much smaller than the experimental resolution of about 1 degree.

At the lower bin energy (Fig. 3.2 left), the model reproduces fairly well the data. The first peak position and its magnitude are well reproduced. In contrast, at the higher bin energy (Fig. 3.2 right), as was expected, the agreement in poor. This is due to the growing significance of the final state interactions as the energy increases as mentioned in last section. In the region
of the first minimum and of the second peak, the agreement is also poor. This is due to the presence of a zero in the Fourier transform of the nuclear density (see Eq. (2.2)). We expect this to change when we include the FSI.

3.2. DWIA cross sections

Let us now include distortion by simulating the FSI with the potential of Carr [10]. The corresponding cross sections are represented in Fig. 3.3.

![Figure 3.3: Photoproduction cross section on a $^{116}$Sn target for the Sao Paulo (black), FSU040 (green) and FSU000 (red) densities at the DWIA together with preliminary data (courtesy of Maria Ferretti) at photon bin energies of 180-190 MeV (left) and 200-210 MeV (right).](image)

As can be seen for the lower bin energy, most of the disagreement with the data that was observed in the PWIA disappears when the FSI are included. The first minimum is filled by the distortion and the second maximum is shifted towards lower angles and increases in magnitude while the first maximum is only slightly influenced by the distortion. It should be noted that at this energy, the model reproduces nearly perfectly the data without any adjustment of the parameters of the potential. Note also that sadly, all curves are nearly superimposed. This is caused by the inversion of the order of all curves. This unfortunate behaviour reduces the sensitivity of this observables to the details of the nucleonic density.

At the higher energy, while the first maximum of data is correctly suppressed, the first minimum and the second maximum are poorly reproduced. This is probably due to the potential that has been used for these calculations. As already mentioned previously, the potential of Carr is fitted at low energies (roughly 180 MeV photon energy) and the simplifying assumptions on which it is based prevent us from extrapolating it towards higher energies. The results obtained at this higher energy bin perfectly illustrate the significant effect of the distortion. Note also that the inversion of the curves that was already seen at lower energy is stronger here. While this was a tiny effect at a low bin energy, the effect is significant here and could actually become handy to choose between different densities. However, in the current state of the data analysis, it seems that the effect of the skin thickness is of the same magnitude if not smaller than the experimental resolution. In order to extract valuable information from the data, a higher experimental resolution would thus be needed.

We have just seen how the potential of Carr sets a limit on the energies at which we can accurately describe the distortion. This shows us that we need a new potential if we want
an accurate description of the reaction process. We are currently deriving such a potential. This new potential would encapsulate the energy dependence without the need of too many fits. While it is derived from the very same Eq. (2.3), it does not make the same simplifying assumptions as Carr’s potential and would thus also allow us to include the influence of realistic densities at first and second orders. While the \( \{b_i, c_i\} \) coefficients should still differ in a nucleus, we expect them to be close to the SAID values. This has been confirmed in preliminary tests.

Finally, as already mentioned, most of the theoretical uncertainty in this model originates from the \( \pi^{-}\)-nucleus potential. Most of this uncertainty comes from the scarce pion scattering data on which the adjustments of the SAID parameters have been performed. We plan on using Bayesian inference to quantify the uncertainties through proper error propagation.

4. Conclusion

We have seen how the coherent neutral pion photoproduction could provide us with valuable information to extract the nucleon density distribution. Indeed, in a first-order approximation such as the PWIA, its amplitude is directly proportional to the nucleon density Fourier transform. Combined with charge distribution measurements, coherent photoproduction would then give us a direct access to the neutron skin. However, PWIA does not capture all the details of the reaction even at low energies, where the approximation holds best and distortion must thus be included in the calculations. At higher energies, it significantly overestimates the magnitude of the cross section.

In order to simulate the interaction of the pion with the nucleus after its production, a phenomenological potential fitted on pion-nucleus scattering data has been used. This potential works well at low photon energies at which it has been fitted and correctly reproduces the details of the cross sections. At higher photon energies however, while it reproduces rather well the magnitude of the DWIA cross section, it does poorly for its details. This motivates the derivation of a new potential that would naturally encapsulate the energy dependence and would allow us to use realistic densities.

This work has also highlighted the difficulties and limitations of the method. As the energy increases, so does the distortion. The latter causes the order of the cross sections calculated at the DWIA for densities with a different skin thickness to be inverted compared to PWIA calculations. At low photon energies, this causes all cross sections to nearly superimpose, removing most of the influence of the skin thickness. At higher energies however, as the effect of the distortion increases, the differences between all cross sections are enhanced. While these results are preliminary because they use a potential that is not fitted at these energies, they suggest that the neutron skin thickness still has an impact on the cross sections. The derivation of a new potential should provide us with a way to properly quantify the uncertainties of this method.

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