Numerical study of natural convection in a rectangular cavity with variation of cavity aspect ratios and cavity inclination angles

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Abstract. Effects of cavity aspect ratios and cavity inclination angles to natural convection in a rectangular cavity are numerically investigated. Investigation is performed at the Rayleigh number ($Ra$) equal to $10^4$, the cavity aspect ratios from 1 to 50 and the cavity inclination angles from 0 to 180°. Consequently, Heat transfer enhancement or decreasing due to the effects is exposed. In addition, streamline contours in the rectangular cavity are illustrated. Multi-cellular flow figuring on the appropriate conditions is exhibited. A new correlation of the average Nusselt number, the cavity aspect ratio and the cavity inclination angle is formulated at $Ra$ equal to $10^4$.

1. Introduction
In many engineering applications relating to thermal energy, forced convection plays as an important role in operating conditions. But in critical conditions, natural convection will be more important for safety [1]. For example, when water circulating pumps of nuclear reactors do not work or ventilation fans of computers fail. In addition, operating or ventilation with natural convection is cost free. In recent decades, energy conservation has been emphasized. And many scientists have been investigating this phenomenon, because natural convection is a phenomenon that can affect both energy saving and energy consumption.

Natural convection in the gap of double glazed units is a case where the phenomenon can enhance or decrease heat gain from solar energy. To understand the causes of this phenomenon, the gap of double glazed units is assumed to be a rectangular cavity filled with air, and the simulation of air flow in the cavity is performed. At high cavity aspect ratios with appropriate Rayleigh numbers, the phenomenon of multi-cellular [2-7] air flow obviously appears. Moreover, many authors [2-26] performed the numerical simulations and experiments of natural convection in a cavity with the various ranges of cavity aspect ratios and cavity inclination angles.

Most [18, 20-23] of the authors mentioned in the foregoing paragraphs performed the numerical simulations of natural convection with the Boussinesq approximation. Although the density in the body force terms is a weak function of the pressure, the pressure still affects to the air density at the high difference of the pressure in the tall cavity.

The objective of this article is to numerically study on the effects of cavity aspect ratios and cavity inclination angles to the results of heat transfer through a rectangular cavity filled with real air, a
function of the temperature and pressure, which the ranges of the study cover greater than the former literature.

2. Problem details

Schematic details of the problem shown in Figure 1. are a two dimensional rectangular cavity filled with air, and the height and width of the cavity are $h$ and $b$, respectively. The cavity aspect ratio is $AR$ ($AR = h/b$). The temperatures of the left and right walls of the cavity are $T_H$ and $T_C$, respectively, and the bottom and top walls of the cavity are adiabatic. Then, the velocities at the boundaries of the cavity are the no-slip condition. The Greek letter $\theta$ symbolizes the inclination angle of the cavity. Thermal radiation is excluded to the problem.

![Figure 1. Schematic geometry of problem](image)

3. Mathematical models

Natural convection in the cavity occurs by circulation of air. Since air flow in the cavity is quite slow, it is reasonable to assume that the flow is incompressible except the densities in the body force terms which induce the buoyancy force. The governing equations defining air flow in the cavity are the energy equation with the viscous dissipation term, the continuity equation and the momentum equations in the $x$ and $y$ directions. The governing equations can be expressed as:

$$
\rho c_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right) 
$$

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
$$

$$
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \rho_{ij} g \cos \theta
$$

$$
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \rho_{ij} g \sin \theta
$$

where $T$, $u$, $v$ and $p$ are the temperature, velocity components in the horizontal and vertical directions and pressure, respectively. Then, $\rho$, $c_p$, $k$, $\mu$ and $g$ are the density, specific heat at the constant pressure, thermal conductivity, viscosity and gravitational acceleration, respectively. And $\rho_{ij}$ is the compressible density.
The air properties except the densities in the body force terms in the momentum equations are imposed from the average temperature between the hot and cold walls of the cavity.

The difference of the vertical wall temperatures causes the difference of the air densities in the body force terms in the momentum equations, and then the buoyancy force results in air circulation in the cavity. Heat transfer from the hot wall to the cold wall occurs due to air circulation called natural convection. The equation of the air densities in the body force terms is a function of the temperature and pressure of air which is obtained from polynomial curve fitting. The data of the air densities, temperatures, and pressures for curve fitting are obtained from the National Institute of Standards and Technology (NIST) Standard Reference Database 23, Version 9.0. The air density equation can be written as

$$\rho_{i,j} = \sum_{mp=1}^{mpden} \left( \sum_{nt=1}^{ntden} a_{nt,mp} T_{nt-i,j} \right) \rho_{mp}^{nt-1}$$

where $a_{nt,mp}$ are the coefficients of the equation, and $ntden$ and $mpden$ are equal to 6. The determination coefficient ($R^2$) of the equation is 0.999.

The final solution of the problem is the Nusselt number, a dimensionless parameter showing the performance of heat transfer through the cavity. The average Nusselt number of the cavity is calculated from

$$\bar{Nu} = \frac{1}{h_y} \int_{y_0}^{y_{hi}} \left( q_{\text{ref}}^* + q_{\text{ref}}^* \right) dy.$$ 

And heat flux on the differentially heated walls of the cavity is obtained from $q^* = -k \frac{dT}{dx}$. Then, reference heat flux which air in the cavity is assumed to be as a solid matter is received from $q_{\text{ref}}^* = k \frac{(T_H - T_C)}{b}$.

4. Numerical implementation

To achieve the purpose of the study, an in-house code was developed by using the FORTRAN computing language. The finite volume method with the LIP scheme [25] were adopted to discretize the values and the derivative values of the fluid variables on the cell faces and the cell centers in the governing equations. The SIMPLE algorithm was used to couple the continuity equation and the momentum equations to determine the pressure field of air flow. The transient condition approached to steady condition was employed to determine the solutions of the problem. The dimensionless time step of the code was 0.001. The convergent criterion for the code was that the maximum residual value of the dimensionless fluid variables at the present iteration and the previous iteration are less than or equal to $10^{-4}$ in ten time continuously. But there were some unstable cases [23], so that for these cases, the code was run until the dimensionless time was equal to 10,000. The Cartesian coordinate with the non-uniform grids were used. The fine grids were applied to the nearby area of the cavity walls, and the coarse grids were applied to the core area of the cavity. In addition, the grid independence test was simultaneously performed to ensure that the solutions did not have any variation due to the grid sizes.

5. Code validation

To ensure that the code gives correct solutions, code validation was carried out by comparison the solutions of the code to the benchmark and published numerical solutions [6, 8, 11-13, 15, 17, 21, 25, 26] and the experimental solutions [9] in the cases of natural convection in a square cavity and a tall cavity with vertical differentially heated walls. The results of the comparison have validated the code.

Table 1. displays comparison of the solutions computed from the code with the benchmark and published numerical solutions for the average Nusselt number $\left( \bar{Nu} \right)$, the maximum horizontal velocity component $\left( u_{\text{max}}^* \right)$ occurring on the vertical cavity centerline $\left( 0.5, y^* \right)$ and the maximum
vertical velocity component \( v_{\text{max}}^* \) occurring on the horizontal cavity centerline \( x^*, 0.5 \) are 3.59%, 2.15% and 2.28%, respectively.

Table 2. shows comparison of the solution computed from the code with and the experimental solution and the published numerical solutions of natural convection in a rectangular cavity at the Rayleigh number and the aspect ratio equal to \( 1.1 \times 10^4 \) and 16, respectively. The maximum differences of the average Nusselt number obtained from the code with the average Nusselt numbers obtained from the experimental and numerical implementation are 6.45% and 0.37%, respectively.

Figure 2. shows temperature and streamline contours of natural convection in a square cavity at the Rayleigh number equal to \( 10^4 \). Overview of Figure 2. is similar the temperature and streamline contours shown in [10].

Figure 3. displays temperature and streamline contours of natural convection in a rectangular cavity at the Rayleigh number and the aspect ratio equal to \( 1.1 \times 10^4 \) and 16, respectively. The contour patterns of Figure 3. look like the patterns of the temperature and streamline contours shown in [6].

Table 1. Comparison of the solutions computed from the code with the benchmark and published numerical solutions of natural convection in a square cavity at \( Ra = 10^4 \).

| Author | \( \overline{Nu} \) | \( u_{\text{max}}^* (0.5, y^*) \) | \( v_{\text{max}}^* (x^*, 0.5) \) |
|--------|------------------|----------------|------------------|
| Present work | 2.2488 | 15.8429 (0.5, 0.8093) | 19.1875 (0.1254, 0.5) |
| [10] | 2.243 | 16.178 (0.5, 0.823) | 19.617 (0.119, 0.5) |
| [12] | 2.2415 | 16.1838 (0.5, 0.8232) | 19.6165 (0.1191, 0.5) |
| [11] | 2.201 | 16.18 (0.5, 0.832) | 19.44 (0.113, 0.5) |
| [13] | 2.245 | - | - |
| [15] | 2.286 | 16.163 (0.5, 0.828) | 19.569 (0.125, 0.5) |
| [17] | 2.168 | - | - |
| [25] | 2.263 | - | - |
| [26] | 2.2448 | 16.1799 | 19.6254 |

Table 2. Comparison of the average Nusselt number obtained from the code with the average Nusselt numbers obtained from the experimental and numerical implementation of natural convection in a rectangular cavity at \( Ra = 1.1 \times 10^4 \) and \( AR = 16 \).

| Author | \( \overline{Nu} \) | Implementation |
|--------|----------------|----------------|
| Present work | 1.5294 | Numerical |
| [9] | 1.4307 | Experimental |
| [6] | 1.525 | Numerical |
| [21] | 1.5351 | Numerical |
6. Results and discussion

Figure 4. and 5. display the variation of the local Nusselt numbers \( Nu = \frac{q^*}{q_{r,ref}} \) on the hot and cold walls with the dimensionless height \( y^* = \frac{y}{b} \) of the cavity at the different cavity aspect ratios and cavity inclination angles, and the Rayleigh number is equal to \( 10^4 \). The wavy graph lines shown in Figure 4.(b), 4.(c), 5.(b) and 5.(c) are appeared because of the multi-cellular effect. The wavy graph lines are more apparent, when the cavity aspect ratios and cavity inclination angles are higher. While the highest local Nusselt numbers occur at the bottom corner on the hot wall and top corner on the cold wall of the cavity, the lowest local Nusselt numbers occur at the top corner on the hot wall and bottom corner on the cold wall of the cavity.
Figure 6. shows the variation of the average Nusselt numbers of the cavity with the different cavity inclination angles and cavity aspect ratios and the Rayleigh number is equal to $10^4$. The average Nusselt numbers at the low cavity aspect ratios have higher values than the average Nusselt numbers at the high cavity aspect ratios, when the cavity inclination angles are between 0° to 150°. And the average Nusselt numbers at the different cavity aspect ratios are almost the same value, when the cavity inclination angles are greater than 150° to 180°.

Figure 7. and 8. show streamline contours in the cavity at the different cavity inclination angles and cavity aspect ratios, and the Rayleigh number is equal to $10^4$. Multi-cellular obviously figure, when the cavity inclination angles become 60° to 180° and the cavity aspect ratios are greater than 30.

Figure 4. Variation of the local Nusselt numbers along the differential heated walls at $Ra = 10^4$, $\theta = 90°$ and the different cavity aspect ratios (a) $AR = 10$, (b) $AR = 30$ and (c) $AR = 50$.

Figure 5. Variation of the local Nusselt numbers along the differential heated walls at $Ra = 10^4$, $AR = 50$ and the different cavity inclination angles (a) $\theta = 45°$, (b) $\theta = 90°$ and (c) $\theta = 135°$. 
Figure 6. Variation of the average Nusselt numbers with the different cavity inclination angles and cavity aspect ratios at $Ra = 10^4$.

Figure 7. Streamline contours in the cavity at $Ra = 10^4$, $AR = 30$ and the different cavity inclination angles (not to scale) (a) $\theta = 0^\circ$, (b) $\theta = 30^\circ$, (c) $\theta = 60^\circ$, (d) $\theta = 90^\circ$, (e) $\theta = 120^\circ$, (f) $\theta = 150^\circ$, and (g) $\theta = 180^\circ$. 

$\theta = 180^\circ$
Figure 8. Streamline contours in the cavity at $Ra = 10^4$, $\theta = 90^\circ$ and the different cavity aspect ratios (not to scale) (a) $AR = 1$, (b) $AR = 10$, (c) $AR = 30$ and (d) $AR = 50$
7. Conclusion
From the simulation, a correlation of the average Nusselt number, the cavity aspect ratio and the cavity inclination angle is formulated in a power law form. The correlation can be written as
\[
\hat{Nu} = (1.73367 AR^{-0.07768} (1.0725 + \cos \theta)^{-0.22963} \text{ at } 1 \leq AR \leq 50 , \quad 0^\circ \leq \theta \leq 180^\circ \quad \text{and} \quad Ra = 10^4 .
\]
The coefficient of determination ($R^2$) of the correlation is 0.77. The 72 sets of data are employed for forming the correlation.

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9. References
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