Determination of the orbital parameters of binary pulsars

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ABSTRACT

We present a simple method for determination of the orbital parameters of binary pulsars, using data on the pulsar period at multiple observing epochs. This method uses the circular nature of the velocity space orbit of Keplerian motion and produces preliminary values based on two one-dimensional searches. Preliminary orbital parameter values are then refined using a computationally efficient linear least-squares fit. This method works for random and sparse sampling of the binary orbit. We demonstrate the technique on (i) the highly eccentric binary pulsar PSR J0514−4002 (the first known pulsar in the globular cluster NGC 1851) and (ii) 47 Tuc T, a binary pulsar with a nearly circular orbit.

Key words: stars: neutron – pulsars: general – pulsars: individual: PSR J0514−4002 – pulsars: individual: NGC 1851 – pulsars: individual: 47 Tuc T.

1 INTRODUCTION

Knowledge of the orbital parameters of binary pulsars is necessary for coherent timing and for investigation of different properties of the pulsar and the companion star. Determination of the orbital parameters is important for newly discovered pulsars, to plan follow-up observations at different epochs.

With the movement of the binary pulsar in its orbit around the centre of mass, the projected velocity of the pulsar in the line-of-sight direction ($v_1$) changes and as a consequence the observed pulsar period ($P_{\text{obs}}$) changes. The modulation in $v_1$ (i.e. in $P_{\text{obs}}$) is governed by the orbital parameters of the binary system. Thus, it is possible to get information about the orbit by studying the evolution of $P_{\text{obs}}$. Five orbital parameters, namely, the binary orbital period ($P_b$), orbital eccentricity ($e$), projection of the semimajor axis on the line of sight ($a_1 \sin i$, $i$ being the angle between the orbit and the sky plane), longitude of periastron ($\omega$) and the epoch of periastron passage ($T_o$) can be determined from radial velocity/observed pulsar period data (in the Newtonian i.e non-relativistic regime). These orbital parameters of binary pulsar systems can be determined by fitting a Keplerian model to the pulsar period versus epoch of observation data. The usual methods require simultaneous fit to many parameters and need an initial guess. Such methods need dense sampling of period measurements at different epochs during the pulsar orbital period. Overcoming some of these factors, Freire, Kramer & Lyne (2001b) proposed a new method for determination of the orbital parameters of binary pulsars. They utilized information on periods and period derivatives at multiple observing epochs of the kind used in surveys, and extracted orbital parameter values. They successfully determined the orbital parameters of binary pulsars with nearly circular orbits.

2 PRELIMINARY DETERMINATION OF ORBITAL PARAMETERS

2.1 Binary orbital period ($P_b$)

The observed pulsar period ($P_{\text{obs}}$) versus epoch of observation data set is folded with a wide range of trial orbital periods ($P_b$). Corresponding to each trial value of $P_b$, we get $P_{\text{obs}}$ versus orbital phase ($\phi = 2\pi t / P_b, t$ being the time-measured from the periastron). For every set of folded data, we calculate a parameter — roughness ($R$) — which we define as the summation of squared differences of $P_{\text{obs}}$ between the adjacent pairs of $\phi$. Therefore,

$$R = \sum_{i=1}^{n} (P_{\text{obs}}(i) - P_{\text{obs}}(i + 1))^2, \quad (1)$$

where $n$ represents the total number of data points. These points are sorted in order of orbital phase, which will be different for different choices of trial $P_b$. For the optimal choice of the trial folding period $P_b$, the plot of $P_{\text{obs}}$ versus $\phi$ is expected to be the smoothest and hence

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the corresponding roughness parameter \( (R) \) will be minimum. In the search of \( P_b \), the increment \( (\Delta P_b) \) must be chosen to cause small orbital phase shift (i.e. \( \Delta \phi_b T < 1 \)) over the full data length \( T \) [i.e. \( (2\pi/P_b^2)\Delta P_b T \ll 1 \)].

As a cross-check, we apply this method on a simulated Keplerian orbit. First, we simulate sparsely and randomly sampled epoch of observation versus radial velocity data points with a set of arbitrarily chosen \( P_b, e, \omega \) and \( T_b \) values (refer to equation 6 of Section 2.2 for details). Using this kind of randomly generated radial velocity data, spanning over widely separated epochs, as input, we apply the smoothness criterion described in equation (1) and the true binary orbital period is recovered. There are few local minima where \( R \) is lower than the adjacent values but there is no comparable minimum as to the strongest minimum corresponding to true \( P_b \). The method worked for Keplerian orbits generated with various sets of \( P_b, e, \omega \) and \( T_b \) values, and we could reproduce the true periodicity. Hence, to obtain a unique solution for \( P_b \), one needs to search for \( P_b \) within a wide range which includes the actual \( P_b \) with small enough step size determined by the criterion \( (2\pi/P_b^2)\Delta P_b T \ll 1 \).

For preliminary determination of \( P_b \) of PSR J0514–40, we used \( P_{\text{obs}} \) versus epoch of observation data from the GMRT observations. We used 31 such data points, collected over six months, which are similar to the data used for Freire et al. (2004). For the known binary pulsars in globular clusters, the orbital periods lie in the range \( P_b \sim \) few hours to 256 d (refer to table 1.1 of Freire 2000). Initially, we try \( P_b \), starting from few hours and up to 300 d with step size satisfying the criterion \( (2\pi/P_b^2)\Delta P_b T \ll 1 \), and determine \( R \) using equation (1). We then narrowed down our search of the \( P_b \) around the lowest \( R \). Though there are few local minima where \( R \) is lower than the adjacent values, we observe the strongest and rather flat minimum for a range of nearby values of \( P_b \) around 18.79 d, no comparable minimum is observed in the range from few hours to 300 d. Fig. 1 presents the plot of the trial \( P_b \) against the corresponding \( R \), zoomed into a region where \( R \) is minimum. For \( P_b = 18.79 \) d \( R \) is minimum. We fold the data with \( P_b = 18.791 \) d to generate \( P_{\text{obs}} \) versus \( \phi \) data set (see Fig. 2).

For the determination of orbital period of 47 Tuc T, we utilized the nine data points (provided in Freire et al. 2001b) of \( P_{\text{obs}} \) versus epoch of observation. We determine \( P_b = 1.1 \) d which is close to the value estimated by Freire et al. (2001b).

### 2.2 Other orbital parameters from the hodograph

The left-hand panel of Fig. 3 shows the orbit of a binary pulsar around the centre of mass of the system, projected in a plane containing the direction of the Earth and the line of nodes (line of intersection of orbital plane and the sky plane). ‘A’ denotes theperiastron position and \( \theta \) is the angle of the pulsar to the periastron, also known as ‘true anomaly’. ‘B’ and ‘C’ are two other points in the binary orbit. A rather geometric picture of the Kepler’s laws using the idea of velocity space is due to Hamilton (1847). It is not often used and hence described briefly below. According to Newton’s laws for the path of the vector \( [r_{\text{pulsar}}(t) - r_{\text{companion}}(t)] \) (i.e. for the relative orbit of the pulsar with respect to the companion star), the relative velocity

\[
\Delta \mathbf{v} = - \left( \frac{GM}{r^3} \right) \Delta t \mathbf{\hat{r}},
\]

where \( G \) is the Gravitational constant and \( M \) is the total mass of the pulsar and the companion star. From the conservation of angular momentum,

\[
\Delta \theta = \frac{h}{r^2} \Delta t,
\]

where \( h \) is angular momentum per unit mass.

Dividing the absolute value of equation (2) by equation (3), we get

\[
\frac{|\Delta \mathbf{v}|}{\Delta \theta} = \left( \frac{GM}{h} \right) = \text{constant}.
\]

The path followed by the velocity vector of a particle is called the hodograph. \( \Delta \mathbf{v} \) is the arc length and \( \Delta \theta \) is the angle traversed by the pulsar in velocity space. The ratio \( (|\Delta \mathbf{v}|/\Delta \theta) \) is the radius of curvature of the hodograph. Since the radius of curvature is constant, the hodograph is a circle for Keplerian motion. The right-hand panel of Fig. 3 shows the corresponding hodograph of the elliptical binary orbit that is shown in the left-hand panel. The centre of the circle is offset from the origin by \( (eGM/h) \) and the radius of the circle is \( (GM/h) \).

For a particular eccentricity \( (e) \) and longitude of periastron \( (\omega) \), the \( x \)- and \( y \)-components of velocity are given by

\[
v_x = - \frac{GM}{h} \sin \theta, \quad v_y = \frac{GM}{h} (\cos \theta + e).
\]
Hence, the relative radial velocity along the projection of the line of sight into the orbital plane is given by

\[ v_r = [v_r \cos (\pi/2 - \omega) + v_r \sin (\pi/2 - \omega)] \]

\[ = \left( \frac{GM}{h} \right) \sin \theta \sin \omega + (\cos \theta + e) \cos \omega \]

\[ = \left( \frac{GM}{h} \right) v_s. \]  

For \( \omega = 90^\circ \) the observed velocity will be antisymmetric (odd) as a function of \( \theta \) or time-measured from periastron. Similarly, for \( \omega = 0^\circ \) the observed velocity will be symmetric (even). For other intermediate values of \( \omega \), the observed velocity will be a combination of antisymmetric and symmetric parts in the ratio of \( \sin \omega / \cos \omega \).

A plot of the antisymmetric versus the symmetric part will be an ellipse and the parameters of the ellipse will provide preliminary values of the orbital parameters.

As a cross-check, we apply this method on simulated Keplerian orbits. We simulate \( v_r \) for trial value of \( e, \omega \) and \( T_o \). Corresponding to each \( v_r \) value at a particular orbital phase (\( \phi \)), we determine the \( v_r \) at conjugate phase (\( 2\pi - \phi \)), using Lagrange’s interpolation method with three points. The even and odd parts are defined as follows:

\[ v_r^{\text{even}} = \left[ v_r(\phi) + v_r(2\pi - \phi) \right]/2, \]  

\[ v_r^{\text{odd}} = \left[ v_r(\phi) - v_r(2\pi - \phi) \right]/2. \]

A plot of \( v_r^{\text{odd}} \) versus \( v_r^{\text{even}} \) should be an ellipse, for a correct choice of \( T_o \) (Fig. 4). The ratio of the major-axis and minor-axis of the ellipse gives tan \( \omega \), and the shift of the origin of the ellipse gives \( e \).

Using the method illustrated in Appendix A, we fit an ellipse to the \( v_r^{\text{odd}} \) versus \( v_r^{\text{even}} \) data. \( \omega \) and \( e \) are recovered from the parameters of the best-fitting ellipse.

Since \( v_r \) and the observed pulsar period (\( P_{\text{obs}} \)) will have similar modulations, we construct antisymmetric and symmetric parts from the \( P_{\text{obs}} \). Corresponding to each \( P_{\text{obs}} \) value at a particular orbital phase (\( \phi \)), we determine \( P_{\text{obs}} \) at a conjugate phase (\( 2\pi - \phi \)) using Lagrange’s interpolation method with three points. The even and the odd parts are defined as follows:

\[ P_{\text{obs}}^{\text{even}} = \left[ P_{\text{obs}}(\phi) + P_{\text{obs}}(2\pi - \phi) \right]/2, \]

\[ P_{\text{obs}}^{\text{odd}} = \left[ P_{\text{obs}}(\phi) - P_{\text{obs}}(2\pi - \phi) \right]/2. \]

The plot of \( P_{\text{obs}}^{\text{odd}} \) versus \( P_{\text{obs}}^{\text{even}} \) should be an ellipse for correct choice of the periastron passage (\( T_o \)). We vary \( T_o \), corresponding \( P_{\text{obs}}^{\text{odd}} \) versus \( P_{\text{obs}}^{\text{even}} \) plots are generated, and fit an ellipse to the \( P_{\text{obs}}^{\text{odd}} \) versus \( P_{\text{obs}}^{\text{even}} \) plot (Appendix A). The left-hand panel of Fig. 5 is the plot of \( P_{\text{obs}}^{\text{odd}} \) versus \( P_{\text{obs}}^{\text{even}} \) for real data of 47 tuc T with an arbitrary choice of \( T_o \). The right-hand panel of Fig. 5 is the plot of \( P_{\text{obs}}^{\text{odd}} \) versus \( P_{\text{obs}}^{\text{even}} \) for real data of 47 tuc T with optimal choice of \( T_o \) (for which \( \chi^2 \) is minimum after ellipse fitting). Preliminary values of \( e \) and \( \omega \) are obtained from the parameters of the best-fitting ellipse (Appendix A).

### 3 Refinement of the Determined Orbital Parameters

In this section, we take the preliminary determined orbital parameters as the initial guess in a linear least-squares fit. This is now computationally efficient since only a small range of the parameters, near the first guess values, has to be searched. \( P_{\text{obs}} \) is determined by the relation

\[ P_{\text{obs}} = P_o \left( 1 + \frac{12}{e^2} \right), \]  

where \( P_o \) is the period of the unperturbed orbit, and \( e \) is the eccentricity.
where $P_o$ is the rest-frame period of the binary pulsar, $v_1$ is the projected velocity of the pulsar in the line-of-sight direction and $c$ is the velocity of light. This relationship is valid provided $v_1$ is small compared to $c$.

The following are the steps for determination of orbital parameters.

(i) We simulate orbital phase ($\phi$) versus scaled radial velocity ($v_s$) with trial values of $P_o$, $e$, $\omega$ and $T_0$ (using equation 6).

(ii) To compare the simulated data with the observations, we need to find out the simulated $v_s$ at those orbital phase points for which $P_{obs}$ is available. $v_s$ at observed orbital phases is obtained by using Lagrange’s interpolation method with three points.

(iii) Next we fit a straight line to $P_{obs}$ versus $v_s$ and calculate $\chi^2$.

We repeat this procedure for all the trial combinations of orbital parameters. As shown in the Appendix B, for the right choice of the orbital parameters, the plot of $P_{obs}$ versus $v_s$ will be a straight line (see equation B8). Hence, the set of orbital parameters, $P_o$, $e$, $\omega$ and $T_0$, for which the straight line fit is best, that is, the $\chi^2$ value is minimum, will correspond to the optimal choice of orbital parameters. $\chi^2$ is minimized so that the expected value for $N$ independent data points is $N$. A change of 1 then corresponds to a 68 per cent confidence limit (page 694, Press et al. 1992). Given the above criterion for change in $\chi^2$, the optimal grid for any parameter (keeping all the other parameter fixed) would have about three points in an interval over which the minimum $\chi^2$ ($\chi^2_{\text{min}}$) increases by $1\sigma$ ($\sigma \sim \chi^2_{\text{min}} / N$). This is the criterion that decide the step size used for different trial combinations of the orbital parameters. The search for each orbital parameter was continued till $\chi^2$ becomes about 1000$\sigma$ on each side of the minima, keeping all the other parameters fixed. It is possible to use this method to determine the orbital parameters, without assuming the minimum values. However, in that case one has to search a wide range for each of the orbital parameters which would be computationally expensive. The intercept of the best-fitting straight line will give the value of $P_o$. Substituting the values of $P_o$, $e$ and $P_o$ and the slope of the fitted straight line $S_{\text{obs}}$ in equation (B10), we can determine the projected semimajor axis in light seconds, $a; \sin(i)/c$.

3.1 Implementation of the method

(i) $J0514-4002$. Fig. 6 presents the plot of $P_{obs}$ versus $v_s$ (generated with the optimal choice of orbital parameters) and the corresponding straight line fit. The residuals from the best-fitting straight line are small for all the measurements, indicating successful fitting and orbital parameter determination. Table 1 lists the determined orbital parameter values of PSR J0514−4002. The step size used for the different sets of trial of orbital parameters, $P_o$, $e$, $\omega$ and $T_0$, are also listed in Table 1. The uncertainty on the values of each of the orbital parameters is calculated from the change in orbital parameter values required for $1\sigma$ change in the $\chi^2$ value, keeping all the other parameters fixed. The uncertainty quoted in the bracket is on the last significant digit of the concerned parameter.

(ii) 47 Tuc T. Fig. 7 plots $P_{obs}$ versus the optimal $v_s$. It is evident that the observational data are well reproduced. Determined orbital parameter values and the associated errors are listed in Table 2.

4 DISCUSSION

The orbital parameters determined in this paper and those determined by Freire et al. (2004) and Freire, Ransom & Gupta (2007) for PSR J0514−4002 are listed in Table 1. For PSR J0514−4002, we have used similar data to those used by Freire et al. (2004) (Section 2). The orbital parameters determined by us are close to their determination within the error quoted by them. However, our results are more accurate and are close to the values obtained by Freire et al. (2007) who have used a much longer data stretch from regular observations with the GBT for about 2 yr. Table 2 compares the orbital parameters determined by us with those obtained by Freire et al. (2001b) and Freire et al. (2001a) for 47 Tuc T. Our results agree with Freire et al. (2001b), but are more accurate and closer to the values predicted by Freire et al. (2001a), who used coherent timing analysis for orbital parameter determination. Note that the small eccentricity of 47 Tuc T could only be found from the coherent timing solution. Our method of orbital parameter determination has the following features.

(i) The procedure for determination of binary orbital parameters outlined in this paper utilizes the measurements of $P_{obs}$ at a given
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Table 1. Orbital parameters of PSR J0514–4002.

| Parameter                     | Freire et al. (2004) (Period analysis) | Freire et al. (2007) (Coherent timing analysis) | This work |
|-------------------------------|---------------------------------------|------------------------------------------------|-----------|
| Orbital period \( (P_b) \) (d) | 18.7850(8)                           | 18.785 1915(4)                                  | 18.7851(1) |
| Eccentricity \( (e) \)       | 0.889(2)                              | 0.887 9773(3)                                   | 0.8879(2) |
| Longitude of periastron \( (\omega) \) (°) | 82(1)                              | 82.266550(18)                                  | 82.20(6) |
| Semimajor axis of the orbit projected along LOS \( (a_1 \sin (i)/c) \) (light-seconds) | 36.4(2)                              | 36.2965(9)                                     | 36.28(1) |
| Pulsar period \( (P_o) \) (ms) | 4.990 576(5)                          | 4.990 575 114 114(3)                            | 4.990 575(4) |
| Epoch of periastron passage \( (T_o) \) (MJD) | 529 84.46(2)                        | –                                               | 529 84.5(1) |

\[a\] The step size used for comparing the simulation with the observation (Section 3).

Note. The uncertainty quoted in the brackets is on the last significant digit of the concerned parameter.

![Figure 7. The same as Fig. 4 but for 47 Tuc T.](image)

observing epoch and does not require any information about the period derivatives in contrast to the method described by Freire et al. (2001b). It may at first sight be surprising that period derivatives do not help to constrain the final orbital solution. This can be understood by examining the accuracy of the measurement which is limited by the period variation over the length of a single observing session. Clearly, the period derivatives implied by the \( P_{obs} \) versus \( \phi \) curves already have smaller errors than this, since one is looking at period variations over the \( P_b \) time-scale. However, period derivatives clearly play a role in the work by Freire et al. (2001b) in determining orbital phases and \( P_b \), which in our method comes from the roughness search.

(ii) Unlike the method used by Freire et al. (2001b), which works for nearly circular binary orbits, this method works for binary orbit with any eccentricity. For example, our method worked well for the binary orbit with highest known eccentricity (PSR J0514–4002 with \( e \sim 0.888 \)), and also for an orbit with lower eccentricity (PSR 47 Tuc T with \( e \sim 0.0 \)).

(iii) The accuracy of the determined orbital parameter values is subject to the sampling of the binary orbit. Our method works with random sampling of the orbit. A small number of data points are required for determination of orbital parameters in our method. In the case of PSR J0514–4002, our method converged even for five random data points.

Table 2. Orbital parameters of 47 Tuc T.

| Parameter                     | Freire et al. (2001b) (Acceleration analysis) | Freire et al. (2001a) (Coherent timing analysis) | This work |
|-------------------------------|-----------------------------------------------|------------------------------------------------|-----------|
| Orbital period \( (P_b) \) (d) | 1.12(3)                                       | 1.126 176 785(5)                                | 1.126 175(2) |
| Eccentricity \( (e) \)       | –                                             | 0.000 38(2)                                     | 0.0000(8) |
| Longitude of periastron \( (\omega) \) (°) | –                                              | 63(3)                                           | 63.0(1) |
| Semimajor axis of the orbit projected along LOS \( (a_1 \sin (i)/c) \) (light-seconds) | 1.33(4)                                       | 1.338 50(1)                                     | 1.337(2) |
| Pulsar period \( (P_o) \) (ms) | 7.588 476(4)                                  | 7.588 479 792 132(5)                             | 7.588 48(2) |
| Epoch of periastron passage \( (T_o) \) (MJD) | 5100.00.3173(2)                             | 5100.00.317249(2)                               | 5100.00.317(2) |

\[a\] The step size used for comparing the simulation with the observation (Section 3).

Note. The uncertainty quoted in the bracket is on the last significant digit of the concerned parameter.

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(iv) The computation involves only one-dimensional searches and linear least-squares fits.  

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REFERENCES

Camilo F., Lorimer D. R., Freire P. C., Lyne A. G., Manchester R. N., 2000, ApJ, 535, 975
Freire P. C., 2000, PhD thesis, Univ. Manchester
Freire P. C., Camilo F., Lorimer D. R., Lyne A. G., Manchester R. N., D’Amico N., 2001a, MNRAS, 326, 885
Freire P. C., Kramer M., Lyne A. G., 2001b, MNRAS, 322, 885
Freire P. C., Gupta Y., Ransom S. M., Ishwara-Chandra C. H., 2004, ApJ, 606, L53
Freire P. C., Ransom S. M., Gupta Y., 2007, ApJ, 662, 1177
Hamilton W. R., 1847, Proceedings of the Royal Irish Academy, 3, 344, (http://www.maths.soton.ac.uk/EMIS/classics/Hamilton/)
Press W. H., Teukolsky S. A., Vetterling W. T., Flannery B. P., 1992, Numerical Recipes: The art of Scientific Computing, 2nd edn. Cambridge Univ. Press, Cambridge

APPENDIX A: FITTING AN ELLIPSE TO THE EVEN VERSUS ODD DATA

For fitting an ellipse to a set of points \( P_{\text{even}} \) versus \( P_{\text{odd}} \), we use the information that the origin of the ellipse will be at \((0, (eGM/h)\cos\omega)\), and the major-axis and minor-axis of the ellipse will be \((GM/h)\sin\omega\) and \((GM/h)\cos\omega\). Using this information, we get an expression which is linear in parameters and hence is easy to fit. The ellipse will be of the form

\[
X^2 \left( \frac{GM}{h} \sin \omega \right)^2 + Y^2 \left( \frac{GM}{h} \cos \omega \right)^2 = 1.
\]

(A1)

Replacing \((GM/h)\sin\omega = a\), \((GM/h)\cos\omega = b\) and \((eGM/h)\cos\omega = d\), we have

\[
X^2 \frac{(Y-d)^2}{a^2} + \frac{Y^2}{b^2} = 1,
\]

(A2)

which can easily be simplified to the form

\[
AX^2 + BY^2 + CY = 1,
\]

(A3)

where \(A = (1/a^2)/(1 - b^2/d^2), B = (1/b^2)/(1 - b^2/d^2)\) and \(C = -(2d/b^2)/(1 - b^2/d^2)\). We use the singular value decomposition method, as described by Press et al. (1992) (Freire et al. 2001b used this method) to determine \(A, B\) and \(C\). \(\chi^2\) in this case is defined as

\[
\chi^2 = \sum_{i=1}^{N} \left\{ A \left( P_{\text{odd}}^2 \right) + B \left( P_{\text{even}}^2 \right) + C \left( P_{\text{even}} \right) \right\} - 1 \}
\]

(A4)

\footnote{The code we have used consists of several stand-alone programs in the ‘octave’ (MATLAB) like language. These programs have not been linked to make up a pipeline. Readers interested in the code may contact the authors.}

\footnote{While doing the ellipse fitting for the real data we used \( P_{\text{odd}} \) versus mean subtracted \( P_{\text{even}} \) data to avoid numerical problems.}

Here, \(\chi^2\) means deviations of the points normal to the ellipse. A criterion of minimizing the \(\chi^2\) value gave us satisfactory results. From parameters of the fitted ellipse \((A, B\) and \(C)\), we determine \(a, b\) and \(c\) and obtain \(e\) and \(\omega\) values as \(e = d/b\) and \(\omega = \tan^{-1}(a/b)\).

APPENDIX B: ILLUSTRATION OF THE STRAIGHT LINE NATURE OF \( P_{\text{obs}} \) VERSUS \( v_\omega \) PLOT

Here we explain the straight line nature of the \( P_{\text{obs}} \) versus \( v_\omega \) plot and interpret the slope and intercept in terms of the orbital parameters. We consider the binary orbit of the pulsar, where \(m_p\) and \(v_p\) are the mass and velocity of the pulsar and \(m_v\) and \(v_v\) are the same for the companion. \(a\) is the semimajor axis of the pulsar orbit relative to the companion and \(a_1\) is the semimajor axis of the pulsar relative to the centre of mass. Using the standard relation between mass and specific angular momentum in a Kepler orbit, we make the following illustrations for the relative orbit of the pulsar with respect to the companion:

\[
\frac{GM}{h} = \frac{G(m_p + m_v)}{\sqrt{a(1 - e^2)G(m_p + m_v)}},
\]

(B1)

\[
v_\omega = (v_p - v_v) = \frac{m_p + m_v}{m_v} v_p,
\]

(B2)

\[
a = a_1 \frac{m_p + m_v}{m_v}.
\]

(B3)

Substituting \(GM/h\) (from equation B1) in equation (6),

\[
v_\omega = \frac{G(m_p + m_v)}{a(1 - e^2)} v_v.
\]

(B4)

Therefore, velocity of the pulsar \(v_p\) can be obtained from equation (B2) as

\[
v_p = \frac{m_v}{m_p + m_v} \sqrt{\frac{G(m_p + m_v)}{a(1 - e^2)}} v_v.
\]

(B5)

Projected velocity of the pulsar in the line-of-sight direction \(\nu_\omega\) is given by

\[
v_\nu = v_p \times \sin i = \frac{m_v}{m_p + m_v} \sqrt{\frac{G(m_p + m_v)}{a(1 - e^2)}} v_v \times \sin i.
\]

(B6)

Therefore, \(v_\nu\) versus \(v_v\) is a straight line with slope \(S\):

\[
S = \frac{m_v}{m_p + m_v} \sqrt{\frac{Gm_v}{a_1(1 - e^2)}} \sin i.
\]

(B7)

Thus, \(P_{\text{obs}}\) versus \(v_v\) will also be a straight line with slope \(S_{\text{fit}}\):

\[
P_{\text{fit}} = \frac{P_v}{c} \times S.
\]

(B8)

However, \(P_h\) and \(a_1\) are related by

\[
P_h^2 = 4\pi^2 a_1^3 \left( \frac{m_p + m_v}{m_v} \right)^2 G.
\]

(B9)

Therefore, from equations (B8) and (B9)

\[
(\frac{a_1}{\sin i})^2 = \frac{P_h^2 S_{\text{fit}}^2 (1 - e^2) c^2}{4\pi^2 P_v^2}.
\]

(B10)

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