PROBING THE MASS FUNCTION OF HALO DARK MATTER VIA MICROLENSING

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ABSTRACT
The simplest interpretation of the microlensing events observed toward the Large Magellanic Clouds is that approximately half of the mass of the Milky Way halo is in the form of massive compact halo objects (MACHOs) with \( M \approx 0.5 \, M_\odot \). It is not possible, owing to limits from star counts and chemical abundance arguments, for faint stars or white dwarves to comprise such a large fraction of the halo mass. This leads us to the consideration of more exotic lens candidates, such as primordial black holes, or alternative lens locations. If the lenses are located in the halo of the Milky Way, then constraining their mass function will shed light on their nature. Using the current microlensing data we find, for four halo models, the best-fit parameters for delta function, primordial black hole, and various power-law mass functions. The best-fit primordial black hole mass functions, despite having significant finite width, have likelihoods that are similar to, and for one particular halo model greater than, those of the best-fit delta functions. We then use Monte Carlo simulations to investigate the number of microlensing events necessary to determine whether the MACHO mass function has significant finite width. If the correct halo model is known, then \( \approx 500 \) microlensing events will be sufficient and will also allow determination of the mass function parameters to \( \approx 5\% \).

Subject headings: dark matter — Galaxy: halo — gravitational lensing

1. INTRODUCTION

The rotation curves of spiral galaxies are typically flat out to about \( \approx 30 \) kpc. This implies that the mass enclosed increases linearly with radius, with a halo of dark matter extending beyond the luminous matter (Ashman 1992; Freeman 1995; Kochanek 1995). The nature of the dark matter is unknown (see e.g., Primack, Sadoulet, & Seckel 1988), with possible candidates including massive astrophysical compact objects (MACHOs), such as brown dwarves, brown dwarfs, brown holes, and elementary particles, known as weakly interacting massive particles (WIMPs), such as axions and neutrinos.

MACHOs with mass in the range \( 10^{-8} \, M_\odot \) to \( 10^3 \, M_\odot \) can be detected via the temporary amplification of background stars that occurs as a result of gravitational microlensing, when the MACHO passes close to the line of sight to a background star (Paczynski 1985). Since the early 1990s several collaborations have been monitoring millions of stars in the Large and Small Magellanic Clouds (LMC and SMC), and a number of candidate microlensing events have been observed.

The interpretation of these microlensing events is a matter of much debate. Whilst the lenses responsible for these events could be located in the halo of our galaxy, it is possible that the contribution to the lensing rate due to other populations of objects has been underestimated (for a discussion see e.g., Bennett 1998; Zhao 1999).

In the case of the two events observed toward the SMC there is significant evidence that, in both cases, the lenses are in fact located within the SMC. The first of these events (97-SMC-1) had a duration of around 217 days, which implies that if the lens is in the Milky Way halo, then it probably has a mass in excess of \( 2 \, M_\odot \) (Palanque-Delabrouille et al. 1997; Alcock et al. 1997b; Sahu & Sahu 1999). Spectroscopy of the source does not show the contamination that would be expected if the lens was a star with such a high mass (Sahu & Sahu 1999). This leaves two possibilities: either the lens is a low-mass star in the SMC, or the lens is in the Milky Way halo but is nonstellar. The second event (98-SMC-1) was a binary, which allowed the time taken for the lens to cross the source star, and hence the projected velocity of the lens, to be measured (Alonso et al. 1998; Alcock et al. 1999). The probability of a standard halo lens having a projected velocity as low as that measured \( (v_{\text{proj}} = 84 \, \text{km s}^{-1}) \) is of order \( 0.2\% \) (Alcock et al. 1999). Owing to tidal disruption, however, the SMC is elongated along the line of sight, and hence its self-lensing rate can be high enough to account for both the observed events (see Gyuk, Dalal, & Griest 1999) and references therein.

Of the eight events observed toward the LMC by the MACHO collaboration during their first two years of observations, one event was due to a binary lens with a very low lens-projected velocity, \( v_{\text{proj}} = 19 \, \text{km s}^{-1} \). Alcock et al. (1996a) argue that if the LMC self-lensing optical depth is large enough to be consistent with all eight lenses being located in the LMC disk, then the probability of finding a projected velocity value this low is less than 0.5\%. Therefore, whilst this lens may itself be in the LMC disk, it is unlikely that all eight events are due to lenses in the LMC disk. If the source star is itself also a binary, however, then the low projected velocity is consistent with the lens being located in either the Milky Way halo or the LMC disk (Alcock et al. 1996a). It has also been argued that there is selection bias against the observation of halo binaries (Honma 1999). These arguments do not, however, rule out self-lensing by other LMC populations. Models of the LMC that have a self-lensing optical depth large enough to account for the observed events have been constructed (Aubourg et al. 1999; Salati et al. 1999; Evans & Kerins 1999). These models require the LMC lenses to be distributed in an extended, shrouded or halo-like distribution. It was previously thought that, if the lenses are stellar, there were...
difficulties reconciling such an extended distribution with the low-velocity dispersions observed (see i.e., Gyuk et al. 1999). Recent analysis of the radial velocities of carbon stars by Graff et al. (1999), however, provides evidence for multiple stellar components. The LMC could also have a dark matter halo with the same MACHO mass fraction as the Milky Way, which would make a significant contribution to the microlensing optical depth (Kerins & Evans 1998). Other possible locations for the lenses include a dark galaxy, or tidal debris, along the line of sight to the LMC (Zaritsky & Lin 1997; Zhao 1998; Zaritsky et al. 1999), a warped and flared MW disk (Evans et al. 1998), and an extended MW protodisk (Gyuk & Gates 1999).

There are several long-term prospects for unambiguously determining the location of the lenses responsible for the LMC events. These include a more sensitive microlensing survey, covering the whole of the LMC, such as the proposed “SuperMACHO survey” (Stubbs 1998), parallax observations by a satellite such as the Space Interformatry Mission (Boden, Shao, & Van Buren 1998; Gould & Salim 1999), and microlensing searches toward M31 (Ansari et al. 1997; Gyuk & Crotts 1999). In the meantime, the location of the lenses is an open question. In this paper we will subsequently assume that the events observed toward the LMC are caused by MACHOs located in the halo of our Galaxy.

Since the duration of a microlensing event depends on the position, transverse velocity and mass of the lens it is not possible to associate a unique MACHO mass with each event. However there are two techniques that can be used, assuming a specific halo model, to probe the mass function of the MACHOs: maximum likelihood fitting of a parameterized mass function (Alcock et al. 1996b, 1997a; Mao & Paczyński 1996) and the method of mass moments (De Rujula et al. 1991; Jetzer 1994; Mao & Paczyński 1996). Mao & Paczyński (1996) found that the maximum likelihood method, while slower than the mass moment method, is more robust. For the standard halo model, a cored isothermal sphere, the most likely MACHO mass function is sharply peaked around 0.5 $M_\odot$, with about half of the total mass of the halo in MACHOs (Alcock et al. 1997a). This poses a problem for stellar MACHO candidates. In the case of white dwarves, an unreasonably large fraction of the baryons in the universe would have had to have been cycled through the MACHOs and their progenitors (see Freese, Fields, & Graff 1999 and references therein). Direct searches place tight limits on the halo fraction in faint stars (Charlot, & Silk 1995), however recent calculations have found that old white dwarves with hydrogen dominated atmospheres may in fact be blue (Hansen 1999a, 1999b), rather than red as previously thought. The Hubble Deep Field South contains a number (~5) of unresolved blue objects with high proper motions that may be old white dwarves (Ibata et al 1999). It is has also been argued, however, that these objects are more likely to be planetary nebulae (Johnson et al. 1999) and, furthermore, no similar objects have been found in ground-based proper motion studies (Flynn et al. 1999).

These problems lead to the consideration of more exotic MACHO candidates such as primordial black holes (PBHs) (Carr 1994). PBHs can be formed in the early universe via a number of mechanisms, the simplest of which is the collapse of large density perturbations produced by inflation. In particular PBHs with mass $M \sim 0.5 M_\odot$ could be formed due to a spike in the primordial density perturbation spectrum at this scale (Ivanov, Naselsky, & Novikov 1994; Yokoyama 1995; Randall, Soljačić, & Guth 1996; Garcia-Bellido, Linde, & Wands 1996) or at the QCD phase transition (Crawford & Schramm 1982; Jedamzik 1997; Schwarz, Schmid, & Widerin 1997; Schmid, Schwarz, & Widerin 1999; Jedamzik & Niemeyer 1999), where the reduced pressure forces allow PBHs to form more easily. In both cases it is not possible to produce an arbitrarily narrow PBH mass function (Niemeyer & Jedamzik 1998, 1999), and the predicted mass function is considerably wider (Green & Liddle 1999) than the sharply peaked mass functions that have been fitted to the observed events to date (Alcock 1997a).

Given the difficulties with stellar MACHO candidates it is therefore important to investigate whether more exotic MACHO candidates, such as PBHs, are compatible with the microlensing events observed toward the LMC. In this paper we first compare the likelihood of the delta function and power-law mass functions, previously fitted to the durations of the observed microlensing events, with that of the PBH mass function. As well as the standard halo model we use Evans’ power-law halo models (Evans 1993, 1994; Alcock et al. 1995, 1996b) and also investigate the effect of incorporating the transverse velocities of the source and observer (Griest 1991). We then use Monte Carlo simulations to address the question of the number of events necessary to determine whether the MACHO mass function has significant finite width.

2. Primordial Black Hole Formation and Mass Function

2.1. Collapse of Density Perturbations during Radiation Domination

In the early universe, where radiation dominates the equation of state, for a PBH to form a collapsing region must be overdense enough to overcome the pressure force resisting its collapse, as it falls within its Schwarzschild radius. This occurs if the size of the perturbation, $\delta = \delta \rho/\rho$, is bigger than a critical size, $\delta_c$, at the time at which it enters the horizon. There is also an upper limit of $\delta < 1$, since a perturbation that exceeded this value would form a separate closed universe (Harrison 1970). Early analytic calculations (Carr & Hawking 1974) found $\delta_c \sim 1/3$ with all PBHs having mass roughly equal to the mass within the horizon at that time, known as the horizon mass $M_{\text{H}}$, independent of the size of the perturbation. Recent studies (Niemeyer & Jedamzik 1998, 1999) of the evolution of density perturbations have found that the mass of the PBH formed in fact depends on the size of the perturbation:

$$M_{\text{BH}} = k M_{\text{H}} (\delta - \delta_c)^3,$$ (1)

where $\gamma \approx 0.37$ and $k$ and $\delta_c$ are constant for a given perturbation shape (for Mexican hat shaped fluctuations $k = 2.85$ and $\delta_c = 0.67$), and

$$M_{\text{H}} \approx \frac{4\pi}{3} \rho (H^{-1})^3,$$ (2)

where $\rho$ is the energy density and $H$ is the Hubble parameter. In order to determine the number of PBHs formed on a given scale, and hence the PBH mass function, we must smooth the density distribution using a window function, $W(kR)$ (see e.g., Green & Liddle 1997). For Gaussian distributed fluctuations the probability distribution of the
smoothed density field \( p[\delta(M_{\text{BH}})] \) is given by

\[
p[\delta(M_{\text{BH}})] \ d\delta(M_{\text{BH}}) = \frac{1}{\sqrt{2\pi\sigma(M_{\text{BH}})}} \exp \left[ -\frac{\delta^2(M_{\text{BH}})}{2\sigma^2(M_{\text{BH}})} \right] d\delta(M_{\text{BH}}) ,
\]

where \( \sigma(M_{\text{BH}}) \) is the mass variance evaluated at horizon crossing defined as in Liddle & Lyth (1993)

\[
\sigma^2(M) = \frac{1}{2\pi^2} \int_0^\infty P(k)W^2(kR)k^2 \, dk ,
\]

where \( P(k) = \langle |\delta_k|^2 \rangle \) is the power spectrum.

The formation of PBHs on a range of scales has recently been studied (Green & Liddle 1999) for both power-law power spectra and flat spectra with a spike on a given scale. In both cases it was found that, in the limit where the number of PBHs formed is small enough to satisfy the observational constraints on their abundance at evaporation and at the present day, it can be assumed that all the PBHs form at a single horizon mass. It is therefore possible to calculate the PBH mass distribution analytically:

\[
\psi(M_{\text{BH}}) = \frac{M_{\text{BH}}}{M_{\text{H}}} p[\delta(M_{\text{BH}})] \frac{d\delta}{dM_{\text{BH}}} ,
\]

which using equations (1) and (3) becomes

\[
\psi(M_{\text{BH}}) = \frac{1}{\sqrt{2\pi\sigma(M_{\text{BH}})}} \left( \frac{M_{\text{BH}}}{kM_{\text{H}}} \right)^\gamma \times \exp \left\{ -\frac{[\delta_c + (M_{\text{BH}}/kM_{\text{H}})]^2}{2\sigma^2} \right\} .
\]

The fraction of the total energy density in the universe, \( \rho_{\text{tot}} \), in the form of PBHs, at the time they form, denoted by “i,” is then given by

\[
\beta_i = \frac{\rho_{\text{BH}}}{\rho_{\text{tot}}} = \int_0^{M_{\text{max}}} \psi(M_{\text{BH}}) dM_{\text{BH}} ,
\]

where \( M_{\text{max}} \) is the mass of the largest PBH that can form at any given \( M_{\text{H}} \):

\[
M_{\text{max}} = kM_{\text{H}}(1 - \delta_i)^\gamma .
\]

The energy density in radiation dilutes as \( \rho_{\text{rad}} \propto a^{-4} \), where \( a \) is the scale factor, whereas in PBHs decreases more slowly, \( \rho_{\text{pbh}} \propto a^{-3} \) so that during radiation domination the fraction of the energy density of the universe in PBHs increases with time:

\[
\beta \propto a \propto t^{1/2} \propto M_{\text{H}}^{-1/2} .
\]

The present day abundance of PBHs must not exceed the maximum value set by the present age and expansion rate of the universe (Carr 1975):

\[
\Omega_{\text{BH},0} = \Omega_{\text{BH},\text{eq}} < 1 ,
\]

where “eq” denotes the epoch of matter-radiation equality, after which the density of PBHs, relative to the critical density, remains constant. This constraint can be evolved backward in time to constrain \( \beta_i \) and hence \( \sigma \):

\[
\Omega_{\text{BH},\text{eq}} = \beta_{\text{eq}} = \beta \left( \frac{M_{\text{eq}}}{M_{\text{H}}} \right)^{1/2} ,
\]

where \( M_{\text{eq}} \sim 10^{56} \) g is the horizon mass at matter-radiation equality. Equation (10) then leads to the constraint \( \sigma(M_{\text{eq}}) < 0.118 \). Because of the exponential dependence of \( \psi(M_{\text{BH}}) \) on \( \sigma \), if \( \sigma(M_{\text{eq}}) \) is reduced below 0.118 by more than a few percent, then the present day density of PBHs becomes negligible. The COBE normalization gives a normalization, on the present horizon scale, of \( \sigma(10^{56} \text{ g}) = 9.5 \times 10^{-5} \), whilst constraints on the abundance of lighter mass PBHs, because of the consequences of their evaporation (see e.g., Carr 1995), prevent \( \sigma \) from increasing rapidly as \( M_{\text{H}} \) is decreased. Therefore to produce a non-negligible density of PBHs with mass \( \sim M_{\text{eq}} \), whilst obeying the COBE normalization and not overproducing lighter PBHs, the primordial density perturbation spectrum must have a spike, with finely tuned amplitude, located at this scale. There are several inflation models that may be capable of produce such a power spectrum (Ivanov et al. 1994; Yokoyama 1995; Randall et al. 1996; Garcia-Bellido et al. 1996).

### 2.2. QCD Phase-Transition

The formation of PBHs at the QCD phase-transition was first suggested by Crawford & Schramm (1982) At a first-order phase transition the pressure response of the radiation to compression is reduced due to the coexistence, in pressure-equilibrium, of a high- and a low-energy phase. This leads to a reduction in \( \delta_c \), and PBHs are formed more easily. The QCD phase-transition occurs at a temperature \( T \sim 100 \text{ MeV} \), when the horizon mass is \( M_{\text{H}} \sim M_{\text{eq}} \). PBHs formed during the QCD phase-transition may therefore naturally have appropriate masses to be viable MACHO candidates.

It is not clear from numerical investigations to date (Jedamzik & Niemeyer 1999) if the PBH scaling law (eq. [1]) holds in this case; the PBHs formed are typically lighter than the horizon mass, and the spread in the masses appears to be even larger than that found for those formed during radiation domination.

### 3. MICROLENSING FORMULAE

In this section we will outline the expressions for the differential microlensing event rate, for lensing toward the LMC (Griest 1991; De Rujula et al. 1991; Alcock et al. 1996b), including a non–delta function mass function. A microlensing event occurs when the MACHO enters the microlensing “tube,” which has radius \( u_T R_E \) where \( u_T \approx 1 \) is the threshold impact parameter for which the amplification of the background star is above the chosen threshold and \( R_E \) is the Einstein radius:

\[
R_E = 2 \left[ \frac{G M x (1 - x) L}{c^2} \right]^{1/2} ,
\]

where \( L \) is the distance to the source. Since the distance to the LMC is much greater than its line-of-sight depth the sources can all be assumed to be at the same distance (~50 kpc) and the angular distribution of sources ignored. The MACHO mass is denoted by \( M \) and \( x \) is the distance of the MACHO from the observer, in units of \( L \). For any non–delta function mass function \( \psi(M)^2 \), such that the fraction, \( f \), of the total mass of the halo in the form of MACHOs is

\[
f = \int_0^\infty \psi(M) dM ,
\]

the differential event rate (assuming a

\[^2\text{ We will assume throughout that the MACHO mass function is independent of position.}\]
spherical halo and an isotropic velocity distribution) is
\[
\frac{d\Gamma}{dt} = \frac{32Ld_{P}}{\bar{v}^{2}C^{4}} \int_{0}^{\infty} \left[ \psi(M) \frac{\rho(x)^{2}}{M} \int_{0}^{x_{a}} \rho(x)\bar{v}_{i}^{2}e^{-Q(x)} \right] \times \left[ e^{-t/\bar{v}_{i}^{2}} - e^{-t/\bar{v}_{i}^{2}} \right] d\Gamma_{0}[P(x)]dx dM ,
\]
where \( x_{a} \approx 1 \) is the extent of the halo, \( \bar{v}_{i} \) is the magnitude of the transverse velocity of the microlensing tube, \( Q(x) = 4R_{d}^{2}(x)/[l^{2}L_{d}^{2}], P(x) = 4R_{d}^{2}(x)d_{P}/[l^{2}L_{d}^{2}] \) and \( I_{0} \) is a Bessel function. We follow the MACHO collaboration and define \( \bar{t} \) as the time taken to cross the Einstein diameter. Other collaborations define the event duration as the Einstein radius crossing time and their timescales are hence smaller by a factor of 2.

For a standard halo, which consists of a cored isothermal sphere,
\[
\rho(R) = \rho_{0} \frac{R_{c}^{2} + R_{s}^{2}}{R^{2}} ,
\]
where \( \rho_{0} = 0.0079 \, M_{\odot} \, pc^{-3} \) is the local dark matter density, \( R_{c} \approx 5 \, kpc \) is the core radius and \( R_{s} \approx 8.5 \, kpc \) is the solar radius, equation (13) becomes
\[
\frac{d\Gamma}{dt} = \frac{512\rho_{0}(R_{c}^{2} + R_{s}^{2})L_{d}^{2}d_{P}}{\bar{v}^{2}C^{4}} \int_{0}^{\infty} \left[ \psi(M) \frac{\rho(x)^{2}}{M} \int_{0}^{x_{a}} \rho(x)\bar{v}_{i}^{2}e^{-Q(x)} \right] \times \left[ e^{-t/\bar{v}_{i}^{2}} - e^{-t/\bar{v}_{i}^{2}} \right] d\Gamma_{0}[P(x)]dx dM ,
\]
where \( A = (R_{c}^{2} + R_{s}^{2})/L_{d}^{2}, B = -2(R_{s}/L_{d}) \cos b \cos l \) and \( b = -33° \) and \( l = 280° \) are the Galactic latitude and longitude, respectively, of the LMC.

3.1. Transverse Velocities of Source and Observer

The transverse velocity of the microlensing tube, \( \bar{v}_{i} \), is often set to zero for simplicity however the motion of the tube through the halo increases the rate of MACHOs entering it from the forward direction and decreases the number leaving it from behind. This results in an increase in the total event rate, and a decrease in the average event duration (Griest 1991). The effect of neglecting the transverse velocity of the microlensing tube, on the determination of the MACHO mass function should therefore be investigated. If the observer has transverse velocity \( \bar{u}_{o} \) and the source has transverse velocity \( \bar{v}_{i} \) then the transverse velocity of the microlensing tube (as a function of position along the tube) is \( \bar{v}_{i} = (1 - x)\bar{v}_{i0} + x\bar{v}_{i,0} \) and its magnitude, \( \bar{v}_{i} = |\bar{v}_{i}| \) is
\[
\bar{v}_{i} = [(1 - x)^{2}\bar{v}_{i,0}^{2} + x^{2}\bar{v}_{i,0}^{2}] + [2x(1 - x)\bar{v}_{i0}\bar{v}_{i,0} \cos \theta]^{1/2},
\]
where \( \theta \) is the angle between \( \bar{v}_{i0} \) and \( \bar{v}_{i,0} \) (Griest 1991).

The transverse velocities of the LMC and Sun, in the rest frame of the galaxy, in coordinates \( (\bar{v}_{i}, \bar{v}_{j}, \bar{v}_{k}) \) where \( \bar{v}_{i} \) is in the direction of the Galactic center, \( \bar{v}_{j} \) is in the direction of the solar rotation and \( \bar{v}_{k} \) is toward the north Galactic pole are \( \bar{v}_{\text{LMC}} = (60, -155,144) \, km \, s^{-1} \) (Jones, Klemola, & Lin 1994) and \( v_{\odot} = (9,231,6) \, km \, s^{-1} \), respectively. The heliocentric position of the center of the LMC, in Galactic coordinates, is 50.1(0.144, -0.824, -0.548) kpc so that the transverse velocities of the LMC and sun, relative to the line of sight between them, are
\[
\begin{align*}
\bar{v}_{\text{LMC}} &= (52, -108,175) \, km \, s^{-1}, \\
v_{\odot} &= (38,68, -92) \, km \, s^{-1}.
\end{align*}
\]

Inserting these values in equation (16) gives
\[
v_{i} = [(1 - x)^{2} + 3.07x^{2} - 2.93x(1 - x)]^{1/2}121 \, km \, s^{-1}.
\]

3.2. Halo Models

The standard halo model used above has a number of deficiencies (see Alcock et al. 1995 and references therein): the halo may not be spherical (N-body simulations of gravitational collapse produce axisymmetric or triaxial halos, and several other spiral galaxies appear to have flattened halos (Sackett et al. 1994), the effect of the Galactic disk is neglected leading to an overestimate of the mass of the halo and the rotation curve of the galaxy may not actually be exactly flat. The power-law halo models of Evans (1993, 1994) provide an analytically tractable framework for investigating the effect of varying the halo model properties on the differential microlensing rate and hence mass function determination (Alcock et al. 1995). The parameters of these models (in addition to the core radius and solar radius) are \( q \) the axis ratio of the concentric equipotential spheroids of the halo, \( \beta \) that governs the asymptotic behavior of the rotation curve \( v_{i} \sim R^{-\beta} \) and \( v_{\odot} \) the normalization velocity that determines the typical MACHO velocities. The expressions for the differential microlensing rate for these models can be found in Appendix B of Alcock et al. (1995).

Other possible halo structures have been considered by various authors. De Paolis, Ingorsso, & Jetzer (1996) have investigated the effect of an anisotropic halo velocity distribution on MACHO mass determination, using the mass moment method. Markovic & Sommer-Larsen (1997) investigated the errors in the determination of the parameters of a power-law mass function that result from assuming a standard halo model if the MACHOs are actually concentrated toward the Galactic center, with a velocity dispersion that changes with radius like that of blue horizontal branch field stars.

It should be noted that analytic descriptions of the halo neglect the substructure that is observed (Helmi et al. 1999), and found in N-body simulations (Moore et al. 1999). Widrow & Dubinski (1998) have investigated hypothetical microlensing observations carried out in a galaxy constructed via a N-body simulation. Whilst they found that the fraction of lines of sight through the halo that intersect clumps of matter is small (~1%), the resulting systematic errors along these lines of sight are large.

4. STATISTICAL METHOD

The maximum likelihood method has been used to determine the best-fit parameters for delta function and power-law mass functions (Alcock et al. 1996b; Mao & Paczynski 1997; Alcock et al. 1997a; Markovic & Sommer-Larsen 1997). We follow the MACHO collaboration and define the likelihood of a given model as the product of the Poisson probability of observing \( N_{\text{obs}} \) events when expecting \( N_{\text{exp}} \) events and the probabilities of finding the observed durations \( t_{j} \) (where \( j = 1, \ldots, N_{\text{obs}} \)) from the theoretical duration distribution, \( \mu_{j} \) (Alcock et al. 1996b; 1997a):
\[
\mathcal{L} = \exp (-N_{\text{exp}})[\prod_{j=1}^{N_{\text{obs}}} \mu_{j}].
\]
The expected number of events is given by

$$N_{\text{exp}} = E \int_0^\infty \frac{d\tilde{t}}{dt} \epsilon(\tilde{t})d\tilde{t}$$

(21)

and \(\mu_j\) by

$$\mu_j = E \epsilon(\tilde{t}_j) \frac{d\tilde{t}(\tilde{t}_j)}{dt},$$

(22)

where \(E = 1.82 \times 10^7\) star years is the exposure, \(\epsilon(\tilde{t})\) is the detection efficiency and \(\tilde{t}\) is the estimated event duration taking account of blending. We use the same analytic form for the detection efficiency as Mao & Paczyński (1996), but with parameters chosen to give a better fit to the photometric efficiency, which allows for the effects of blending, of the MACHO 2 yr data:

$$\epsilon(\tilde{t}) = \begin{cases} 0.3 \exp \left[-0.394(\ln \tilde{t})^{1.7}\right] & \text{if } \tilde{t} > 1, \\ 0.3 \exp \left[-0.281(\ln \tilde{t})^{1.75}\right] & \text{if } \tilde{t} < 1, \\ \end{cases}$$

(23)

where \(\tilde{t}' = \tilde{t} / 75\) days.

5. MASS FUNCTION MODELS

We consider four forms for the MACHO mass function:

1. Delta function (DF)—a delta function mass function at \(M_{DF}\), comprising a fraction \(f\) of the total mass of the halo.
2. Power-law–fixed upper cutoff (PLF)—a power-law mass function, with exponent \(z\), between \(M_{\text{min}}\) and \(M_{\text{max}}\). We follow Alcock et al. (1997a) here and fix \(M_{\text{max}}\) at 12\(M_\odot\):

$$\psi(M) = \begin{cases} A_n M^z & \text{if } M_{\text{min}} < M < M_{\text{max}}, \\ 0 & \text{otherwise}, \\ \end{cases}$$

(24)

where \(A_n\) is a normalization constant.

3. Power-law–variable upper cutoff (PLV)—a power-law mass function, as given by equation (24), but with the upper cutoff mass, \(M_{\text{max}}\), allowed to vary.

4. Symmetric power law (SPL)—a symmetric power-law mass function with center \(M_c\), width 2\(M_d\), slope \(z\) and normalization \(A_n\):

$$\psi(M) = \begin{cases} A_n \left[1 - (M/M_c - M/M_d)\right] & \text{if } M_c - M_d < M < M_c, \\ A_n \left[1 - (M/M_c - M/M_d)\right] & \text{if } M < M_c < M_c + M_d, \\ 0 & \text{otherwise}. \\ \end{cases}$$

(25)

5. Primordial black hole (PBH)—the PBH mass fraction derived in § 2.1, which has two parameters: the horizon mass at the time the PBHs form, \(M_\text{fit}\), and the mass variance, at horizon crossing, on this scale, \(\sigma\). The fraction, \(f\), of the halo mass in MACHOs is identical to \(\beta\) the fraction of the energy density of the universe in PBHs.

6. CURRENT DATA

In their analysis of the 2 yr MACHO collaboration data Alcock et al. (1997a) form a six-event subsample by excluding the binary lens (event 7 in Table 1), which may be in the LMC, as discussed in the introduction, and another event (no. 8 in Table 1), which they consider to be the weakest of the eight events. They argue that this subsample is a conservative estimate of the events resulting from lenses located in the Milky Way halo. The effect of microlensing by other populations (LMC disk and halo; Milky Way disk, spheroid, and bulge) would be more accurately accounted for by including terms representing their contributions to the expression for the total differential rate used in equation (20), as in Alcock et al. (2000). Whilst using this more sophisticated method would increase the uncertainty in, and change the values of, the parameters of the best-fit mass functions it would not change our conclusions about the ability of the current data to differentiate between mass function/halo model combinations.

Since we completed our analysis the MACHO project 5.7 yr results have been released. The total number of events is now 13 or 17, depending on the selection criteria, which corresponds to an optical depth, and hence halo mass fraction, roughly 50% smaller than that of the 2 yr data. The spread in the timescales of all 13/17 events is larger than that of the events observed during the first 2 yr. This is at least partly due to the increase in the sensitivity to longer durations events, which occurs in any survey as the survey duration increases (Evans & Kerins 1999). Repeating our analysis using the new data would change the best-fit mass functions; in particular the values of \(f\) found would be reduced by ~50%, but would not change our general conclusions.

We find the maximum likelihood fit, to the six-event “halo subsample,” for each of the mass functions described above for four sample halo models: the standard halo (SH), the standard halo including the transverse velocity of the line of sight (SHVT) and two power-law halo models. The power-law halo models used are models B (massive halo with rising rotation velocity) and C (flattened halo with falling rotation velocity) from Alcock et al. (1995) with \(\beta = -0.2\) (0.2), \(q = 1\) (0.78), \(R_e = 5\) kpc (10 kpc), \(v_0 = 200\) km s\(^{-1}\) (210 km s\(^{-1}\)) and \(R_0 = 8.5\) kpc (8.5 kpc), respectively. The differential event rate for each of the halo models, for a delta function mass function at \(M = M_\odot\) with \(f = 1\), are shown in Figure 1. To illustrate the uncertainty due to the small number of events, for the standard halo model, we also find the best-fit mass functions for the full eight-event sample.

We find the maximum likelihood fit, for each mass function/halo model combination, relative to that for the delta function mass function and a standard halo, are given in Tables 2 and 3, for the six- and eight-event samples, respectively. The best-fit mass functions, described in § 5 above, are plotted, with the same, arbitrary, normalization but different axes scales, in Figures 2 and 3 for the six-event subsample and in Figure 4 for all eight events and a stan-

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**Table 1**

| Event | \(i\) (days) |
|-------|-------------|
| 1     | 38.8        |
| 2     | 52          |
| 3     | 88          |
| 4     | 100         |
| 5     | 131         |
| 6     | 70          |
| 7     | 143         |
| 8     | 47          |

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**Table 2**

| Event | \(i\) (days) |
|-------|-------------|

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**Table 3**

| Event | \(i\) (days) |
|-------|-------------|

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The maximum value of the likelihood for each mass function/halo model combination, relative to that for the delta function mass function and a standard halo, are given in Tables 2 and 3, for the six- and eight-event samples, respectively. The best-fit mass functions, described in § 5 above, are plotted, with the same, arbitrary, normalization but different axes scales, in Figures 2 and 3 for the six-event subsample and in Figure 4 for all eight events and a stan-

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The maximum value of the likelihood for each mass function/halo model combination, relative to that for the delta function mass function and a standard halo, are given in Tables 2 and 3, for the six- and eight-event samples, respectively. The best-fit mass functions, described in § 5 above, are plotted, with the same, arbitrary, normalization but different axes scales, in Figures 2 and 3 for the six-event subsample and in Figure 4 for all eight events and a stan-

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The maximum value of the likelihood for each mass function/halo model combination, relative to that for the delta function mass function and a standard halo, are given in Tables 2 and 3, for the six- and eight-event samples, respectively. The best-fit mass functions, described in § 5 above, are plotted, with the same, arbitrary, normalization but different axes scales, in Figures 2 and 3 for the six-event subsample and in Figure 4 for all eight events and a stan-
standard halo. The parameters of the best-fit mass functions are given in Tables 4–8. The results for the full eight events and a standard halo are denoted by “SH8.”

Given the small number of events, maximizing the likelihood function depends mainly on reproducing the observed number of events; for each mass function halo model combination the best fit to the six-event halo subsample has $N_{\text{exp}}$ within 1% of 6.00. For the standard halo (both with and without the transverse velocity of the line of sight) and power-law halo B the differential rate for a DF mass function is comparable with the range of the observed timescales, and hence the DF mass function produces the largest maximum likelihood. Despite the large differences in the widths of the best-fit mass functions, the differences between the resulting differential event rates in the region near their peaks, where the event rate is nonnegligible, are very small, especially when the detection efficiency is included. This can be seen in Figure 5 where we plot the differential event rate, with and without the detection efficiency, for the best-fit DF and PBH mass functions for the standard halo. In the case of power-law halo C, the differential rate for the DF mass function is narrower than the range of observed timescales so that broader mass functions are a better fit to the observed events. When the transverse velocity of the line of sight is included, for the standard halo, the DF mass function still has the largest maximum likelihood, however the mean MACHO mass is increased by $\sim 14\%$. We can also see that fixing the upper mass cutoff, as in Alcock et al. (1997a), forces the power-law mass function to fall off more

| Mass function | SH | SHVT | B | C |
|---------------|----|------|---|---|
| DF            | 1  | 0.97 | 0.99 | 0.98 |
| PLF           | $\rightarrow$1 | $\rightarrow$0.97 | $\rightarrow$0.99 | 1.02 |
| PLV           | $\rightarrow$1 | $\rightarrow$0.97 | $\rightarrow$0.99 | 1.12 |
| SPL           | 1  | 0.97 | 1.00 | 1.15 |
| PBH           | 0.92 | 0.96 | 0.92 | 1.10 |

| Halo Model | $M_d/M_\odot$ | $f$ |
|-----------|---------------|-----|
| SH        | 0.44 | 0.50 |
| SHVT      | 0.50 | 0.50 |
| B         | 0.53 | 0.30 |
| C         | 0.22 | 0.041 |
| SH8       | 0.44 | 0.68 |
rapidly than if the upper cutoff is allowed to vary. This is because there are effectively tight limits, due to the absence of long duration events, on the number of large mass ($M \gg M_\odot$) MACHOs. For the full set of eight events and a standard halo model, the maximum likelihood is attained for mass functions with finite width. The DF and PBH mass functions have roughly the same maximum likelihood, with that of each of the power-law mass functions (PLF, PLV, SPL) being roughly 3%–4% greater.

The differences in maximum likelihood between mass function/halo model combinations are small and, unsurprisingly given the small number of events, it is not possible to differentiate between mass functions using the current data, even if the halo model is fixed.

7. MONTE CARLO SIMULATIONS

In order to assess how many microlensing events will be necessary to determine whether the MACHO mass function has significant finite width, we carried out a number of

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TABLE 5

| Parameters, and Halo Fraction and Mean MACHO Mass, of Best-Fit Power-Law Mass Function, with Fixed Upper Cutoff $M_{\text{max}} = 12 M_\odot$ |
| --- | --- | --- | --- | --- | --- |
| Halo Model | $M_{\text{min}}/M_\odot$ | $\alpha$ | $A_\alpha$ | $f$ | $\dot{M}/M_\odot$ |
| SH .......... | 0.44 | $\rightarrow -\infty$ | n/a | 0.50 | 0.44 |
| SHVT ...... | 0.50 | $\rightarrow -\infty$ | n/a | 0.50 | 0.50 |
| B .......... | 0.53 | $\rightarrow -\infty$ | n/a | 0.31 | 0.53 |
| C .......... | 0.15 | -3.2 | $2.7 \times 10^{-5}$ | 0.042 | 0.27 |
| SH8 ........ | 0.34 | -3.9 | $3.0 \times 10^{-8}$ | 0.68 | 0.50 |
Monte Carlo simulations assuming a “broad” MACHO mass function and, for different numbers of events, compared the fit to the data of “broad” and “narrow” mass functions. To minimize the considerable computing time required for these simulations we assume a standard halo and neglect the transverse velocity of the line of sight. The transverse velocity of the line of sight should of course be included in the analysis of a set of real microlensing events, as it leads to a shift in the parameters of the best-fit mass function; however, its inclusion, and the precise form of the halo models chosen, should not change the general conclusions of this section. Furthermore whilst the nature of the halo is not currently well-known leading, as we saw in §6, to large uncertainties in the determination of the MACHO mass function presumably by the time a large (100+ event) survey is completed our knowledge of the halo structure will have improved. We take our broad MACHO mass function to be the best-fit PBH mass function, with parameters $M_H = 0.70 \, M_\odot$ and $\sigma = 0.1153$ as found by the maximum likelihood analysis of the current data and use the DF mass function as a two-parameter “narrow” mass function. Whilst a power-law mass function is perhaps a more realistic “narrow” MACHO mass function we prefer to compare fairly generic “broad”3 and “narrow” mass functions, with the same number of free parameters.

We produced 400 simulations each for $N = 100,316,1000$, and 3162 events, for both a perfect detection efficiency ($\epsilon = 1$ for all $f$) and that of the first 2 yr of the MACHO project, given by equation (23). The actual efficiency of future long duration microlensing searches is likely to be somewhere between these two forms with the peak efficiency, and the duration at which it occurs, increasing with the search duration (Evans & Kerins 1999). For each simulation we find the best-fit PBH and DF mass functions by maximizing the likelihood as defined in equation (20). An alternative definition of the likelihood function (Markovic & Sommer-Larsen 1997) is $\Pi_{i=1}^N (dt_{\text{norm}}/dt)$, where the differential event rate is normalized such that $\int (dt_{\text{norm}}/dt) = 1$. This approach, however, neglects the information about the normalization of the differential event rate that would be obtained in any real microlensing survey, since a particular number of events will be observed during a known exposure time.

For each simulation we compare the theoretical event rate distributions produced by the best-fit mass functions with those “observed” using a modified form of the Kolmogorov-Smirnov (KS) test. We cannot use the standard KS test to compare the theoretical (PBH and DF) event durations with our Monte Carlo “data,” since the parameters of the input mass functions have been estimated from the “data.” Instead we use our simulations to compute the probability distribution of the KS statistic $D_{\text{KS}}$, the maximum distance between the theoretical cumulative distribution function and that of the “data,” for the best-fit real PBH mass function. We then compare this distribution with the values of $D_{\text{KS}}$ of the best-fit “false” DF mass functions. This gives us an indication of the number of events required to differentiate between mass functions. The fraction of the simulations passing the KS test at a given confidence level,4 for both the PBH and DF MACHO mass functions, is shown in Figure 6 for each value of $N$ for both forms of the efficiency. Between 316 and 1000 events should be sufficient to discern whether the MACHO mass function has significant finite width.

\[ \text{TABLE 6} \]

| Halo Model | $M_{\text{min}}/M_\odot$ | $M_{\text{max}}/M_\odot$ | $\alpha$ | $A_\alpha$ | $f$ | $\dot{M}/M_\odot$ |
|------------|----------------|----------------|---------|----------|----|----------------|
| SH .......... | $-0.44$ \quad $-0.44$ | $-\infty$ | n/a | 0.50 | 0.44 |
| SHVT ...... | $-0.50$ \quad $-0.50$ | $-\infty$ | n/a | 0.51 | 0.50 |
| B .......... | $-0.53$ \quad $-0.53$ | $+\infty$ | n/a | 0.30 | 0.53 |
| C .......... | 0.08 \quad 0.44 | 0.2 | $1.7 \times 10^{-41}$ | 0.041 | 0.27 |
| SH8......... | 0.25 \quad 0.66 | 0.5 | $2.75 \times 10^{-30}$ | 0.67 | 0.47 |

\[ \text{TABLE 7} \]

| Halo Model | $M_0/M_\odot$ | $M_*/M_\odot$ | $\alpha$ | $A_\alpha$ | $f$ | $\dot{M}/M_\odot$ |
|------------|--------------|--------------|---------|----------|----|----------------|
| SH .......... | 0.44 \quad 0.04 | 0.4 | $4.5 \times 10^{-33}$ | 0.50 | 0.44 |
| SHVT ...... | 0.50 \quad 0.04 | 0.4 | $4.2 \times 10^{-38}$ | 0.50 | 0.50 |
| B .......... | 0.53 \quad $\rightarrow 0$ | $\rightarrow 0$ | n/a | 0.30 | 0.52 |
| C .......... | 0.27 \quad 0.20 | 3.1 | $5.9 \times 10^{-35}$ | 0.042 | 0.27 |
| SH8......... | 0.30 \quad 0.52 | 8.4 | $5.9 \times 10^{-24}$ | 0.69 | 0.52 |

\[ \text{TABLE 8} \]

| Halo Model | $M_0/M_\odot$ | $\sigma(M_0)$ | $f$ | $\dot{M}/M_\odot$ |
|------------|--------------|--------------|----|----------------|
| SH .......... | 0.70 \quad 0.1153 | 0.51 | 0.50 |
| SHVT ...... | 0.80 \quad 0.1155 | 0.51 | 0.57 |
| B .......... | 0.87 \quad 0.1141 | 0.32 | 0.61 |
| C .......... | 0.38 \quad 0.1071 | 0.042 | 0.26 |
| SH8......... | 0.73 \quad 0.1164 | 0.70 | 0.52 |

3 Since the PBH mass function is close to Gaussian it is a reasonable generic form for a broad mass function.

4 By definition 50% of the “real” best-fit mass functions are accepted at the 50% confidence level.
Fig. 6.—Fraction of best-fit PBH (solid lines) and delta function (dotted line) mass functions passing the KS test at a given confidence level, for $N = 100, 316, 1000,$ and $3162$ events from left to right, for the MACHO 2 yr (upper row) and perfect flat (lower row) efficiencies.

Fig. 7.—Contours containing 68% and 95% of the $N = 100$ (dot-dashed line), 316 (dotted line), 1000 (dashed line) and 3162 (solid line) event best-fit PBH MFs for the MACHO 2 yr (left-hand panel) and perfect flat (right-hand panel) efficiencies.
In Figure 7 we plot 1 and 2σ contours (which contain 68% and 95% of the simulations, respectively) of the parameters of the PBH mass function. The parameters of the input mass function are marked with a cross. In Figure 8 we plot contours of the parameters (f and $M_{\text{DF}}$) of the best-fit delta function mass function and in Figure 9 contours of the mean mass and halo fraction of the best-fit PBH mass functions. Fitting a DF mass function when the true mass func-
tion is the PBH mass function leads to a systematic underestimation of the mean MACHO mass by ~15%. The mean and standard deviation of the best-fit values of $M_H$ and $\sigma$ obtained are displayed in Tables 9 and 10, for the MACHO 2 yr and flat efficiencies, respectively. We find, in agreement with Mao & Paczynski (1996) and Markovic & Sommer-Larsen (1997), that with ~1000 events it will be possible to determine the parameters of the MACHO mass function to a few percent, if the halo structure is known.

### Table 9

| $N_e$ | $\bar{M}_H/M_\odot$ | $SD(M_H/\bar{M}_H)$ | $\bar{\sigma}$ | $SD(\sigma/\bar{\sigma})$ |
|-------|-----------------|---------------------|--------------|-----------------|
| 1000   | 0.681           | 0.125               | 0.11519      | $3.6 \times 10^{-3}$ |
| 316    | 0.676           | 0.068               | 0.11518      | $1.9 \times 10^{-3}$ |
| 10000  | 0.674           | 0.038               | 0.11517      | $1.0 \times 10^{-3}$ |
| 3162   | 0.677           | 0.024               | 0.11518      | $6.9 \times 10^{-4}$ |

### Table 10

| $N_e$ | $\bar{M}_H/M_\odot$ | $SD(M_H/\bar{M}_H)$ | $\bar{\sigma}$ | $SD(\sigma/\bar{\sigma})$ |
|-------|-----------------|---------------------|--------------|-----------------|
| 1000   | 0.714           | 0.125               | 0.11514      | $3.6 \times 10^{-3}$ |
| 316    | 0.704           | 0.065               | 0.11510      | $1.8 \times 10^{-3}$ |
| 10000  | 0.708           | 0.041               | 0.11513      | $1.1 \times 10^{-3}$ |
| 3162   | 0.708           | 0.024               | 0.11533      | $6.9 \times 10^{-5}$ |

8. CONCLUSIONS AND FUTURE PROSPECTS

The MACHO mass ($M \sim 0.5 M_\odot$) and halo fraction ($f \sim 0.5$) favored by current microlensing data pose severe problems for stellar MACHO candidates such as faint stars and white dwarves. It was previously thought (Yokoyama 1998; Green & Liddle 1999) that the relatively broad mass function of primordial black holes was likely to be inconsistent with the durations of the observed microlensing events. In §6 we found that, using the current data, the likelihood of the best-fit primordial black hole mass is comparable to that of the best-fit delta function, to ~<10%. This then led us to investigate the number of events necessary to determine, assuming that the lenses are located in the Milky Way halo and that the halo can accurately be described by a known analytic form, whether the MACHO mass function has significant finite width. Approximately 500 events should be sufficient to answer this question and also determine the parameters of the mass function to ~5%. If the halo model is not known then the number of events necessary is likely to be increased by at least an order of magnitude (Markovic & Sommer-Larsen 1998). If the MACHOs are PBHs, then the gravitational waves emitted by PBH-PBH binaries will allow the MACHO mass distribution to be mapped by the Laser Interferometer Space Antenna (Nakamura et al. 1997; Ioka et al. 1998; Ioka, Tanaka & Nakamura 1999).

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No. 2, 2000  MASS FUNCTION OF HALO DARK MATTER  719

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