Symmetry-Protected Spoof Localized Surface Plasmonic Skyrmion

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Electromagnetic (EM) skyrmions are an EM analogue of the skyrmions in condensed matter physics, offering new degrees of freedom to structure light and manipulate light–matter interactions and thus promising various groundbreaking applications in optics and photonics. Recently, there is a growing interest in composing EM skyrmions based on different field vectors of EM waves. Here, an EM skyrmion is realized, i.e., a spoof plasmonic skyrmion (SPS), using the electric field vectors of spoof localized surface plasmons (LSPs) in a planar microwave resonator with rotational and mirroring symmetries. The SPS is constructed by synthesizing a scalar vortex (a topological charge 0) and a polarization vortex (a topological charge 1) in the in-plane and the out-of-plane component of the fields, respectively. Besides an experimental demonstration, group theory is employed and pinpoints the symmetry origin of the skyrmion. This investigation demonstrates the ubiquity of the existence of the skyrmion in any planar EM resonator holding rotational and mirroring symmetries, regardless the dimensions and the operating frequencies. This skyrmion design not only promises novel microwave applications for sensing and transferring information, but also lays down a general guideline for devising skyrmions operating over a broad range in the EM spectra.

1. Introduction

Since first proposed by Skyrme to describe the interactions of pions in nuclear physics,[1,2] skyrmions, topologically stable three-component vector field configurations, have been generalized to various subjects in condensed matter physics, including quantum Hall magnets,[3] Bose–Einstein condensates,[4] nematic liquid crystals,[5] etc. Owing to the topological stability even at small sizes, skyrmions promise applications in spintronics[6] and information storage and transfer.[7–9] Motivated by the advances of skyrmions in condensed matter physics, skyrmions, as a generic wave effect, have been reproduced by acoustic waves[10] and electromagnetic (EM) waves. Especially, largely fueled by the explosive development of singular optics and topological photonics, optical and plasmonic analogues to skyrmions, that is, using EM fields to emulate the vector configurations of skyrmions, have been attracting growing attention and rapidly have become a cutting-edge topic in optics and photonics. Optical skyrmions were first generated by interfering multiple surface plasmon polariton waves.[11] Subsequently, many approaches have been proposed to construct optical skyrmions using different field vectors, such as the electric or magnetic field vectors of free-space or confined waves,[12–16] spin vectors of the evanescent EM waves,[17–23] synthesized Stokes vectors of paraxial vector beams,[24–26] and synthesized pseudospin vectors in photonic crystals,[27] etc. These optical skyrmions exhibit great topologically protected stability and deep-subwavelength features, enable new degrees of freedom to engineer structured light and light–matter interactions, and promise many applications in nanometer- and femtosecond-scale metrology,[24,26] deep-subwavelength microscopy,[18] ultrafast vector imaging,[15] and topological Hall devices.[27]

Very recently, a new method to construct optical skyrmions was reported based on magnetic field vectors of magnetic localized surface plasmons (LSPs).[29] This work opens the avenue for the fundamental and applied explorations of skyrmions at lower frequencies, where the EM skyrmions can be generated and observed in a simpler manner compared with the optical band.[29] EM skyrmions at microwaves have shown potential for use in many advanced microwave applications like ultra-compact and
topologically stable plasmonic devices, including but not limited to label-free dielectric spectroscopy of liquids in microfluids,[30] ultra-small antennas, wearable microwave sensors, extremely multiplexed communications,[31] and so on. However, this work only explored the generation of the EM skyrmions based on the magnetic field vectors of the magnetic LSP. Research on EM skyrmions constructed by the electric field vectors of spoof LSPs, which is the electric counterpart of magnetic LSPs,[32–34] has remained untapped.

Here, we propose a design for generating EM skyrmions using the electric field vectors of spoof LSPs in a planar microwave plasmonic resonator with rotational and mirroring symmetries. We numerically and experimentally demonstrate that a spoof plasmonic skyrmion with the skyrmion number of 1 is supported by the resonator. Such a skyrmion is generated by synthesizing a trivial scalar vortex with a topological charge 0 in the out-of-plane component of the electric fields and a polarization vortex with a topological charge 1 in the in-plane component of the electric fields. Besides, by tracing the symmetry origins of the scalar and the polarization vortices with group representation theory, we explore the symmetry origin of the spoof plasmonic skyrmion. In detail, we demonstrate that the hedgehog-like configuration of the skyrmion is intrinsically associated with the “identity” irreducible representation (irrep) of the group formed by the symmetries of the resonator.[35] Since the identity irrep always exists for a group formed by rotational and mirroring symmetries, we can conclude that the skyrmion ubiquitously exists in any planar EM resonator with the required symmetries. This conclusion is independent from the dimensions and the operational frequencies of the EM resonator. Therefore, with appropriate excitations, the skyrmion can be observed in generic EM systems with rotational and mirroring symmetries, such as microring resonators,[16,37] circular nanopillars,[18,39] plasmonic vortex lenses,[40–42] photonic quasicrystals,[43] circular nanoemitter arrays,[44,45] and circular antenna arrays.[46]

2. Realization of Spoof LSP Skyrmion

We design a five-layer device to generate spoof LSP skyrmions, as shown in Figure 1. The first layer is a microwave plasmonic resonator which is made of copper film and holds eightfold rotational symmetries and mirroring symmetries. The resonator is etched on the second layer made of Rogers 4530B dielectric sheet with 0.508 mm thickness (with relative permittivity 3.48 and loss tangent 0.0037). The same dielectric sheet is used for the third and the fifth layer. Two microstrip lines are etched on the top surface of the third layer and the bottom surface of the fifth layer, respectively, as shown in Figure 1b,c. A via is used to connect the two microstrip lines (via 1 in Figure 1b,c). The fourth layer is made of copper film which serves as the ground of the two microstrip lines. A circular opening is cut in the fourth layer to avoid an electric connection between via 1 and the ground, as shown in Figure 1c. The third to fifth layers contain a feeding network to excite the resonator. The incident waves are fed to the feeding network by a SMA connector from the position marked by “port” in Figure 1a,b. The selection of the feeding topology is closely related with the symmetries of the resonator. This will be postponed to the later part of this work where the symmetry origin of the skyrmion is discussed.

The whole device was designed in CST microwave studio. Corresponding simulation results are shown in Figure 2 and partially in Figure 3, in which Figure 2a depicts the magnitude distribution of the electric field and the configuration of the normalized electric field vectors. From Figure 2a, it can be observed that the electric field vector is “up” at the center where \( r = 0 \), gradually flips as the radius increases, and finally points “down” at \( r = R_s \). This electric field configuration defines an “electric field-based skyrmion.”[11,17]

Further, we evaluate the topological invariant of the “electric field-based skyrmion,” that is, the skyrmion number.[11] The skyrmion number describes, when the vector distribution is mapped onto the unit sphere, how many times the vectors cover a unit sphere. The skyrmion number can be evaluated as below[11]

\[
s = \frac{1}{4\pi} \int \int_A \mathbf{e} \cdot \left( \frac{\partial \mathbf{e}}{\partial x} \times \frac{\partial \mathbf{e}}{\partial y} \right) \, dx \, dy \quad (1)
\]

In the above equation, \( \mathbf{e} \) denotes the normalized electric field vector, that is, \( \mathbf{e} = \text{Re}\{E_x, E_y, E_z\}/|\mathbf{E}| \), where \( E_{xy} = |E_{xy}| e^{-i\theta} \), and \( A \) is the integration area (the circular region with radius \( R_s \).

Figure 1. Device proposed to generate a spoof LSP skyrmion: a) top view, b) bottom view, c) stratified layers. The adopted parameters are: inner radius of the resonator \( r_i = 12 \text{ mm} \), outer radius of the resonator \( r_o = 24 \text{ mm} \), radius of the circular dielectric sheet \( r_d = 52 \text{ mm} \), angular size of one part of the resonator is \( \pi/4 \), the distance between the center of via 1 and the center \( r_f = 4 \text{ mm} \), the distance between the center of via 1 and the port (i.e., the length of the microstrip line on the bottom surface of the fifth layer) \( r_f = 50 \text{ mm} \), and the length of the microstrip line on top of the third layer \( d_i = 4 \text{ mm} \). The width of the two microstrip lines is 1.1 mm yielding the input impedance of 50 \( \Omega \), matched with the SMA connector. In (c), two additional vias marked by “via 2” and “via 3” are added to connect the ground of the SMA connector with the ground of the microstrip lines (i.e., the fourth layer). The radius of the opening is 1.5 mm.
Figure 2. Illustration of the generated spoof LSP skyrmion. The electric field distribution at 3.77 GHz in the plane 10 mm above the resonator (z = 10 mm) is chosen to represent the vector configuration of the generated skyrmion. The frequency is obtained from the simulated reflection coefficient $S_{11}$ of the device, as shown in Figure 3. a) The bottom and top insets show the magnitude distributions of the electric field (i.e., $|E| = \sqrt{E_x^2 + E_y^2 + E_z^2}$) and the normalized electric field vectors in the central region. The central region is defined by a white dashed circle in the magnitude distribution. b) The distribution of the normalized electric field vectors along the radial direction is shown. c) Plots the cosine of the latitude angles $\beta$ of the normalized electric field vectors. In (a) and (b), $r$ is the radial distance, and $R_s = 55$ mm. The colors of the vectors in (a) and (b) are coded from blue to red to denote the latitude angle $\beta$ varying from $\pi$ to 0.

Figure 3. Comparison of experimental and simulation results: a) reflection coefficient $S_{11}$, b) magnitude, and c) phase distributions of the simulated $E_z$, and d) magnitude and e) phase distributions of the measured $E_z$. In (a) the top and bottom views of the fabricated sample are shown. Both the simulated and measured $E_z$ show a trivial scalar vortex mode with topological charge of 0, that is, $l_z = 0$.

Equation (3) shows that the skyrmion number is a product of two independent factors. The first factor is related with the number of flips of the unit vectors along the radial direction. The unit vector is “up” at the center $r = 0$, that is, $\beta = 0$ and $\cos \beta = 1$, and flips “down” at the boundary ($r = R_s$), that is, $\beta = \pi$ and $\cos \beta = -1$, as shown in Figure 2b,c. Consequently, the first factor is equal to 1. The second factor is related with the variation of the unit vectors along the azimuthal direction. It is observed that along this direction, the relative orientation of the electric field vectors is invariant with respect to the radial direction, that is, the axial symmetry mentioned above. In other words, the variation of the longitudinal angle $\alpha(\varphi)$ with respect to the azimuthal angle is constant over the azimuthal direction and, as a result, $da/d\varphi = 1$. Therefore, we obtain a skyrmion number of the generated spoof LSP skyrmion of 1. It is worth noting that the topological charge of the field skyrmion averages to zero over
one oscillation cycle (see Section S2, Supporting Information). This observation agrees with the results of the reported works about the skyrmions composed of plasmonic fields,[11,14] but is different from the spin skyrmion (or meron) texture[23,24,47] which has a stable configuration over an oscillation period.

To verify the above numerical results, the five-layer device was fabricated (see the insets of Figure 3a), and measured. It can be observed that the measured reflection coefficient $S_{11}$ (see Figure 3a) agrees well with the simulated one. The spoof LSP skyrmion is formed at the resonant dip $\lambda = 10$ mm above the device at 3.77 GHz. The perpendicular component of the electric field, $E_z$, is always zero and the polarization azimuth is undefined. [49]

3. Symmetry Origins of the Skyrmion

First, we look for the topological signatures of the electric field distributions underlying the skyrmion excited at 3.77 GHz. On the one hand, it can be observed from Figure 3b,c that the out-of-plane component, that is, the $z$ component, of the electric field always exhibits a trivial scalar vortex mode with the topological charge of 0, that is, a vortex mode carrying an orbital angular momentum (OAM) of 0.[48] On the other hand, it can be observed from Figure 4a,b that the in-plane components, that is, the $x$ and the $y$ components, of the electric field demonstrate a zero magnitude at the center and are linearly polarized over the region of interest. Both aspects indicate that the in-plane components of the electric field form a polarization vortex with a V-type polarization singularity, that is, a V point. At this singularity, by definition, the magnitude is zero and the polarization azimuth is undefined.[49]

By evaluating the Stokes fields (see Section S2, Supporting Information), the topological charge of this polarization singularity is found to be 1. Further, it is well-known that a polarization vortex can be conveniently understood in terms of the left and right circular bases, that is, $\hat{L} = \frac{1}{\sqrt{2}} (\hat{x} + i \hat{y})$ and $\hat{R} = \frac{1}{\sqrt{2}} (\hat{x} - i \hat{y})$, and can be seen as the superposition of the scalar vortex modes for the left and the right circular components [49,50] that is, $E_l$ and $E_R$. If the scalar vortex modes corresponding to the left and right circular components have topological charges $l_l$ and $l_R$, the topological charge of the polarization vortex $l$ can be proven to be $l = l_l - l_R$. By projecting the in-plane components of the electric field in Figure 4a,b onto the circular bases, it is found that the left- and the right-circular components hold topological charges $-1$ and $+1$. Therefore, the excited skyrmion is composed of three scalar vortex modes (in the $E_l, E_R$, and $E_z$ components of the electric field, respectively), and the topological charges of these three scalar vortex modes are $-1, +1$ and 0, respectively.

The topological charges of the scalar vortex modes in $E_z$, $E_R$, and $E_L$ are intrinsically related with the symmetries of the resonator. The rotational and the mirroring symmetries form a $D_8$ point group. To simplify the discussions and to stress the symmetry features (for a more complete discussion, see the Section S4, Supporting Information and the refs.[51–56]), we select eight
In the Ez distance from the center to each golden point is set as $\lambda_d$. The oscillating frequency is chosen as 1 GHz in our simulation. It is worth noting that the oscillating frequency of the dipoles actually can be freely chosen because the existence of the skyrmion is independent of the oscillating frequency. All dimensions used here are scaled with respect to the oscillating wavelength. The electric dipole momentum of the electric dipole located at $r_{m'}$ is marked by $p_{l}(r_{m'})$, where $m' = 0, 1, \ldots, 7$ (see details in Section S4.4, Supporting Information). The distance from the center to each golden point is set as $\lambda_d/8$.

Figure 5. Illustration of the eigen electric dipole set belonging to the identity irrep and the radiated electric field. a) The eight dipoles which are located in the $xoy$ plane and oscillate along the radial direction at frequency $f_d$ (corresponding to the wavelength $\lambda_d$). The oscillating frequency is chosen as 1 GHz in our simulation. It is worth noting that the oscillating frequency of the dipoles actually can be freely chosen because the existence of the skyrmion is independent of the oscillating frequency. All dimensions used here are scaled with respect to the oscillating wavelength. The electric dipole momentum of the electric dipole located at $r_{m'}$ is marked by $p_{l}(r_{m'})$, where $m' = 0, 1, \ldots, 7$ (see details in Section S4.4, Supporting Information). The distance from the center to each golden point is set as $\lambda_d/8$. b) The $E_x$, the $E_y$, and the $E_z$ components of the radiated electric fields in the plane $z = \lambda_d$.

exemplary spatial points, which can be transformed into each other by the symmetry operations forming the $D_8$ point group in the resonator (see Figure 5a), and we impose a polarizable dipole at each point. The discussion on the eight symmetry-related points can be readily extended to the whole resonator, by seeing the resonator as a collection of these points and by reconstructing the currents flowing in the resonator by putting the dipoles at these points. According to the group representation theory, the $D_8$ point group has seven irreps, including four 1D and three 2D irreps.\[35\] As a result, it can be proven that the polarizable dipoles at the eight points can form seven unique orthogonal dipole distributions (see Section S4, Supporting Information, for more information). Especially, the distribution corresponding to the “identity” irrep (see Figure 5a) is of special importance. On the one hand, at each point, the orientation of the dipole is invariant with respect to the radial direction (in the case of Figure 5a, the orientation is along the radial direction). On the other hand, the phases of all the dipoles are the same. The former implies that the electric fields radiated by the dipoles have $E_y$ and the $E_z$ components, and that the $E_x$ component is expected to be significantly smaller, even almost zero (see Figure 5b; Figure S6, Supporting Information). The latter requires that $E_x$ and $E_z$ hold trivial topological charges (see Figure S6, Supporting Information), that is, a constant phase along the azimuthal direction.

Converting to the circular basis, the trivial topological charge in the $E_y$ component leads to the $-1$ and $+1$ topological charges in the $E_x$ and the $E_z$ components, as shown in Figure 5b (also see the derivations in Section S4.5, Supporting Information), respectively. Considering the $E_x$, $E_y$ and $E_z$ components, the electric field vectors may demonstrate an “electric field-based” skyrmion as a whole (see Figure S9, Supporting Information). Since the electric field is radiated by the dipole distribution corresponding to the “identity” irrep, the radiated skyrmion is said to belong to the “identity” irrep. The above discussion reveals the existence of an “electric field-based” skyrmion in the targeted resonator. Since only symmetries are involved and the “identity” irrep always exists in a generic $D_M$ group (where $M$ is an integer),\[35\] the ubiquity of the existence of such skyrmion in any planar EM resonator holding the symmetries forming the $D_M$ group is demonstrated.

To excite the “electric field-based” skyrmion belonging to the “identity” irrep, the symmetries of the resonator require an incident field satisfying a so-called symmetry matching condition (see details in Section S4.3, Supporting Information). In mathematical terms, the incident field must have a non-vanishing projection along the identity irrep, and in physical terms, the symmetry of the incident field is at least partially aligned with the symmetry of the electric field vectors of the skyrmion. Take our design in Figure 1 as an example. In the device, the microstrip line on the top surface of the third layer is used to excite the resonator. This microstrip line is quasi-static at the resonant frequency (a length of 4 mm is much smaller than the resonant wavelength of around 80 mm at 3.77 GHz) and holds a mirroring symmetry with respect to the x-axis (see Figure 1c). As a result, the generated incident field has a mirroring symmetry with respect to the x-axis as well. This symmetry of the incident field is partially aligned with the axial symmetry of the electric field vectors of the skyrmion. Hence, the skyrmion is excited in the device (see details in Section S5, Supporting Information). It is worth mentioning that the single port feeding network in our design is used only for demonstrative purposes, in the sense that the feeding network is the simplest design that can both excite the targeted skyrmion and provide an intuitive picture for the symmetry matching condition. This feeding can be readily generalized to more complex topologies, for example, multiport feeding networks, or, alternatively, more complex incident fields can be used, for example, hypergeometric-Gaussian beams with zero total angular momentum,\[48\] so that the incident field is more efficiently coupled to the skyrmion.

4. Conclusion

We present a design to generate a spoof plasmonic skyrmion by using the electric field vectors of the spoof LSPs. A device consisting of a microwave plasmonic resonator with the $D_8$ group symmetries and a feeding network is designed to launch the
skyrmion. Both the simulation and experimental results well demonstrate the formation of the spoof plasmonic skyrmion. By analyzing the spatial configurations of the out-of-plane and in-plane components of the electric fields, we find that a scalar vortex mode with topological charge of 0 and a polarization vortex with topological charge of 1 are formed in the out-of-plane component and in the in-plane components, respectively. These vortex modes constitute the hedgehog-like vector configuration of the spoof plasmonic skyrmion. By exploiting group representation theory, we find that the constituent vortex modes and the skyrmion are intrinsically related with the “identity” irrep of the group formed by the symmetries of the resonator. Since the “identity” irrep of a $D_m$ group always exists, the excited skyrmion is ubiquitous in generic planar EM systems holding the corresponding symmetries. Our work not only promises many future ultra-compact and flexible devices for microwave applications owing to the virtues of the skyrmions (topological stability) and the spoof LSPs (high-Q resonance and deep-subwavelength field localization),[32–34] but also provides a general guideline for engineering EM skyrmions over a broad range of the EM spectra.

5. Experimental Section

**Numerical Simulations:** Codes were composed in MATLAB to generate the transformation operators, the irreps, and the projection operators for the $D_3$ group, and to calculate the electric fields radiated by the eigen dipole set belonging to the identity irrep.

**Experimental Measurements:** The fabricated samples were characterized by a near-field scanning system in an anechoic chamber. The scanning system consisted of a servo actuator, a coaxial near-field probe, a vector network analyzer (VNA) and connection cables. A 50Ω SMA connector receiving the input signal from the VNA was welded onto the microstrip line. The experiment setup is shown in Figure S11, Supporting Information. In the fabrication, glue was used to bond the resonator and the feeding system together, which might also lead to discrepancies between simulations and experiments.

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Keywords

phase singularity, polarization singularity, spoof localized surface plasmons, spoof plasmonic skyrmions, symmetries

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