Abstract

Primordial magnetic fields present between the eras of neutrino decoupling and of positron-electron annihilation can lead to the emission of neutrino-anti neutrino pairs or axions by electrons in excited Landau levels, thereby changing the electron energy spectrum. The rate is calculated and shown to not affect significantly the standard Big Bang picture.
In a neutron star, a large ambient magnetic field may cause electrons to precess in cyclotron orbits. Under the proper conditions, electrons in excited states corresponding to higher Landau levels decay to the ground state by the emission of a neutrino-anti-neutrino pair which escapes from the neutron star. This particular form of energy loss has been studied for almost three decades, subject to refinements due to better models of neutron stars and weak interactions.

The effects of a large primordial magnetic field on Big Bang Nucleosynthesis (BBN) have also been considered for almost thirty years, starting with the initial works of Matese and O’Connell and of Greeenstein. This research centered on possible changes in the Universe’s expansion rate due to the magnetic field energy and on shifts in the weak interaction rates due to changes in available phase space, degeneracy etc.

As is well known, conditions inside neutron stars bear some resemblance to those prevailing in the early Universe at the time of BBN. It therefore seems worthwhile to examine if the primordial magnetic fields presumed extant in the early universe could have led to a degradation of electron energy by cyclotron emissivity. It is clear that one needs to focus on the period after neutrino decoupling since before then electrons and positrons are in thermal equilibrium with neutrinos and anti-neutrinos and a shift in the annihilation rate has no effect. It is equally clear that the mechanism has negligible effect after the annihilation of electrons and photons into photons since the electron(positron) population drops dramatically at that point.

The period we focus on then corresponds to the roughly one hundred seconds when the Universe drops from a temperature of $2 \cdot 10^{10} K$ to $2 \cdot 10^9 K$, or equivalently from an energy of about three electron masses, $3m_e$, to an energy of one third of an electron mass, $m_e/3$. In fact, during this time the primordial magnetic field is decreasing as well: assuming the scale factor $R$ is related to temperature by $RT$ remaining constant and assuming also that the magnetic flux remains constant, we expect the magnetic field $B \sim T^2$. $B$ changes by a factor of one hundred as $T$ changes by a factor of ten.

The first studies of the effect of a primordial magnetic field on BBN analyzed modifications of phase space due to the $B$ field and shifts in the expansion rate of the Universe because of the added magnetic field energy. These early calculations have been refined by several authors in increasingly detailed calculations, not without some controversy. The present limits, as quoted by Grasso and Rubinstein, are that, for $T = 10^9 K$, we must have
\( B \leq 10^{12}G \) (where of course we are measuring \( B \) in Gauss) for a magnetic field coherence length \( L \ll 10^{11}cm \). If the coherence length is taken to be the horizon length, the limits on \( B \) are more severe, i.e. \( B \leq 10^{11}G \). For our purposes, we need the first limit since we only require that the magnetic field be homogeneous over an electron cyclotron orbit. Assuming, as we said in the previous paragraph, that \( B \sim T^2 \), we see that at \( T = 10^{10}K \), the maximum allowed magnetic field is \( B = 10^{14}G \).

The energy spectrum of an electron moving in a magnetic field directed along the z axis is quantized according to the formula

\[
E(p_z, n, s_z) = \sqrt{p_z^2 + m_e^2 + eB(2n + 1 + s_z)}
\]  

(1)

where \( n = 0, 1, 2, 3, ... \) label the energy levels, \( p_z \) is the z component of momentum and \( s_z \) is the z component of the electron spin. Clearly the larger the \( B \) field, the larger the energy released in a transition from a higher to a lower energy level. On the other hand, if the \( B \) field is too large, the higher levels are not excited thermally; the optimal value is \( \frac{eB}{m_e T} \approx 1 \) when \( T \leq m_e \) and \( \frac{eB}{T^2} \approx 1 \) otherwise.

Conventionally \( B_c = 4.4 \cdot 10^{13}G \) is called the critical field. It corresponds to a magnetic field \( eB_c = m_e^2 \), clearly the order of magnitude we are considering. We have already seen that our limits on \( B \) indicate that \( B \leq 2B_c \) for \( T \approx 10^{10}K \approx 2m_e \) so in fact we are in the regime where \( \frac{eB}{m_e T} \approx 1 \). We may therefore expect cyclotron emissivity to play a significant role, but how significant is the question. For convenient comparison with the literature, we will quote temperature in units of \( T_9 \) which means multiples of \( 10^9 \ K \) and magnetic fields in units of \( B_{13} \), obviously multiples of \( 10^{13}G \). We normalize our energies to \( \rho_e \), the energy in the electron field, which we take as

\[
\rho_e \approx 7 \cdot 10^{22}T_9^4 \frac{\text{ergs}}{\text{cm}^3}
\]  

(2)

Using the above formula as a rough guideline, we can estimate the electron energy density in the region of interest to us. For e.g. \( T \sim 10^{10}K \), we see that \( \rho_e \sim 10^{27} \frac{\text{ergs}}{\text{cm}^3} \) and that \( n_e \approx 10^{32}/\text{cm}^3 \), an electron density similar to that near the edge of a neutron star. As an estimate of the energy loss due to emission by electrons of neutrino-anti-neutrino pairs, we therefore use the formulas derived for neutron stars by Kaminker, Levenfish and Yakolev.

They introduce parameters

\[
\xi = \frac{T_p}{T} \quad T_p \approx 2 \cdot 10^9 B_{13} X^2 K. \quad X_F = \frac{p_F}{m_e}
\]  

(3)
and go on to consider the limits $\xi \ll 1$ and $\xi \gg 1$. In our case, the limit $\xi \gg 1$ is applicable (extrapolating the $\xi \ll 1$ gives a comparable value). In the $\xi \gg 1$ case the formula for the rate of energy loss is

$$Q_\nu \approx 10^{15} B_{13}^2 T_9^5 \frac{\text{ergs}}{\text{cm}^3 \cdot \text{sec}}.$$  \hspace{1cm} (4)

The best way to estimate the magnitude of the effect is by comparing $Q$ to $\dot{\rho} = \frac{d\rho}{dt}$, the rate of change in electron energy density due to the expansion of the universe (in the above and from now on we denote $\rho_e$ simply as $\rho$). Using time as defined by this expansion rate

$$t = \left(\frac{3g^*}{4}\right) \frac{m_{\text{Planck}}}{T^2} \sim \left(\frac{T}{\text{Mev}}\right)^{-2}$$  \hspace{1cm} (5)

we find that

$$\frac{d\rho}{dt} \sim \frac{d\rho}{dT} \frac{dT}{dt} \sim \frac{2\rho}{t}.$$  \hspace{1cm} (6)

Since $B \sim T^2$, we see that $Q_\nu \sim T_9^5$, while, using the previous expressions for $\rho$ and for $t$, we see that $\dot{\rho} \sim T_9^6$. Their ratio is therefore a function of $T$ or equivalently, of time. Scaling $B$ by setting $B = 10^{12}G$ for $T = 10^9 K$ and using $1\text{Mev.} = 1.2 \cdot 10^{10} K$ to scale $t$, we see that $\frac{Q_\nu}{\dot{\rho}}$ may be written as

$$\eta_\nu = \frac{Q_\nu}{\dot{\rho}} \sim \frac{10^{13}T_9^9}{10^{20}T_9^6} \sim 10^{-7}T_9^3.$$  \hspace{1cm} (7)

The ratio of rates of electron energy decrease due to emission of neutrino-anti-neutrino pairs and expansion of the universe ranges from about $10^{-3}$ for $T \sim 2 \cdot 10^{10} K$, to about $10^{-6}$ at electron-positron annihilation. At this level, the result does not affect the standard BBN model. Even if neutrinos decoupled at a temperature a factor of three higher and if the present formula for $Q_\nu$ could be extrapolated to those higher values of $B$, the result for $\eta_\nu$ would not significantly affect BBN.

A few caveats/questions are in order: 1) is the formula for $Q_\nu$ valid? The answer is probably yes within the limits of the calculation. As Kaminker et al. emphasize, the result is insensitive to the value of the electron Fermi momentum and the extrapolation to larger values of $B$ also appears valid 2) could $B$ be even larger than $10^{12} G$ at $T \simeq 10^9 K$? Of course this limit is only valid over a coherence length much smaller than the horizon, but it is conceivable that even larger of $B$ are present over distances small enough
to affect $\rho_{\nu}$. Turning back to the formula for the electron energy levels, we see that even for $p_z^2 \sim T^2 \gtrsim m_e^2$, the electrons are all in the ground state and hence do not radiate unless $eB \lesssim T^2$. This corresponds to $B \lesssim 10^{12} G$, so we see that a larger magnetic field would not lead to more significant limits.

We would like to turn now to a related topic, the limits on electron-axion couplings imposed by the cyclotron emission of axions. The situation is much like the case we have just discussed. The corresponding $Q_a$ for $e \rightarrow e + a$ was derived by Borisov and Grishina.

$$Q_a = 1.6 \cdot 10^{40} g_{ae}^2 X_F \frac{T_9^{13}}{B_{13}} \frac{ergs}{cm^3 \cdot sec}$$

where $g_{ae}$ is the axion-electron coupling constant and $X_F$ was defined earlier. This formula has been recently recently reanalyzed by Kachelriess et al. who have shown that it holds reasonably well even for values of $B$ which are substantially larger than $B_c$. In the above equation we insert the previously discussed limit for the magnetic field, $B \leq 10^{12} G$ at $T = 10^9 K$ and assume $B \sim T^2$. Using the standard formulas for fermionic energy density and number density, we obtain $n_e \simeq 1.5 \cdot 10^{28} T_9^{-3} cm^{-3}$. With the relation $p_F = (3\pi^2 n_e)^{\frac{1}{3}}$, we find $p_F \sim 0.4 \cdot T_9 m_e$.

Putting this all together we obtain

$$\eta_a = \frac{Q_a}{\rho} \simeq \frac{10^{40} g_{ae}^2 T_9^5}{10^{20} T_9^6} = 10^{20} g_{ae}^2 T_9^{-1}$$

Demanding $\eta_a \lesssim 1$ leads to a bound on the axion-electron coupling $g_{ae} \lesssim 10^{-10}$, comparable to the neutron star bound obtained by Kachelriess et al. As one sees in the above equations this bound does not depend drastically on the value of the magnetic field: it is only weakened by a factor of order two if $B \sim 10^{11} G$ at $T = 10^9 K$. On the other hand the bound is much weaker than that obtained by studying red giant stars or even, as emphasized by Kachelriess et al., using the same process of cyclotron emissivity for magnetic white dwarf stars. Finally we would like to mention that similar types of calculations have been carried out by Kuznetzov and Mikheev.

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