Thermal Field Theory in a layer: Applications of Thermal Field Theory methods to the propagation of photons in a two-dimensional electron sheet

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Abstract

We apply the Thermal Field Theory methods to study the propagation of photons in a plasma layer, that is a plasma in which the electrons are confined to a two-dimensional plane sheet. We calculate the photon self-energy and determine the appropriate expression for the photon propagator in such a medium, from which the properties of the propagating modes are obtained. The formulas for the photon dispersion relations and polarization vectors are derived explicitly in some detail for some simple cases of the thermal distributions of the charged particle gas, and appropriate formulas that are applicable in more general situations are also given.
I. INTRODUCTION

It is well known that when elementary particles, such as photons or neutrinos, propagate through a medium, the effects of the background particles influence their properties in important ways. Among the various approaches that exist to study the effects of a medium on the properties and interactions of particles, the methods of Thermal Field Theory (TFT) have proven to be very useful. Largely motivated by the original work of Weldon [1–3], these methods have been applied to the problems mentioned above and other similar ones, and they have been helpful for understanding the physics involved and also from a computational point of view.

The present work is concerned with similar calculations, but with the distinction that the medium consists a gas of particles that are confined to live in a two-dimensional plane sheet, or layer. A typical system of this type is an ordinary plasma in which the electrons are confined to a plane sheet, that we can take to be the $z = 0$ plane. The quantities of interest are the usual ones such as, for example, the dispersion relations of the propagating photon modes, the damping and the transition rates for various processes.

Before continuing we want to stress that this is not the same thing as what is usually called $QED_3$ (or $QED$ in 2+1 dimensions), which has been studied in the literature [4, 5]. $QED_3$ describes a system that, with regard to the space coordinates, has cylindrical symmetry and the physics, being independent of the $z$ coordinate, can be studied by considering a two-dimensional cross section. Thus, for example, the electron in $QED_3$ is really a line of charge in the three-dimensional world, and the Coulomb potential between two such electrons is logarithmic. In contrast, in the system we are considering, the electron is an ordinary point charge, which is confined to the $z = 0$ plane, but the Coulomb potential between two electrons is the usual $1/r$ potential.

The method that we apply here to study these systems could be useful in the context of astrophysical [6–8] as well as plasma physics [9] and condensed matter [10, 11] applications, and they are also interesting in their own right because they can be useful in the study of physical systems of current interest in which a plasma is confined to a layer [12] or a wire [13].

In the context of TFT, the distinctive feature of the system that we are considering is that the medium is not isotropic over the (three-dimensional) space. Consequently, the thermal propagators that are used ordinarily in TFT calculations for the case of homogenous and
isotropic media, are not the appropriate ones for the present case. Therefore, in order to use the TFT methods to study the model of the two-dimensional plasma layer, a crucial requirement is finding the appropriate set of thermal propagators that must be used.

In the present work we considered the simplest situation of an ordinary gas of electrons, which are confined to a plane sheet, but are otherwise free. Our main goal has been to formulate the TFT approach to the model of the two-dimensional plasma layer that we have described. The important steps to this end are taken in the first part of the present paper, where we determine the appropriate set of thermal propagators. The charged particle propagators are very similar in form to the standard three-dimensional form. But, as we will see, the propagation of the photon in the layer is described by an effective field which has a corresponding propagator that is very different from the usual one. The photon propagator is an important quantity because its inverse determines the bilinear part of the effective action or, equivalently, the equation of motion for the photon effective field, from which the dispersion relations and wave functions of the propagating photon modes can be obtained. As an application, in the second part we carry out a one-loop calculation of the photon self-energy in that medium and, as a specific example, we consider in detail the calculation of the longitudinal dispersion relation. There we compare our approach and results for this calculation with the results that are known for this system in literature [14, 15], which have been obtained using the so-called static local field correction approximation [16]. There we show explicitly that our results for this calculation reduce to those known results when the appropriate limits are taken and/or approximations are made, which in particular involves approximating the photon propagator by its static (zero frequency) limit. However, the formulas that we obtain, for the self-energy and dispersion relations in general and for the longitudinal dispersion relation in particular, can be used for a wider range of conditions in which those approximations (such as the static local field correction approximation) and limits are not justified.

II. NOTATION AND KINEMATICS

As usual, we denote by $u^\mu$ the velocity four-vector of the medium. Adopting the frame in which the medium is at rest, we set

$$u^\mu = (1, \vec{0}) ,$$

(2.1)
and from now on all the vectors refer to that frame. Thus, denoting by \( \vec{n} \) the unit vector perpendicular to the plane layer, we introduce the four-vector

\[
n^\mu = (0, \vec{n}) ,
\]

and denote the momentum four-vector of a photon that propagates in the plane is by

\[
k_\perp^\mu = (\omega, \vec{k}_\perp) ,
\]

where

\[
\vec{k}_\perp \cdot \vec{n} = 0 .
\]

It is useful to define

\[
\tilde{u}_\mu \equiv u_\mu - \frac{(u \cdot k_\perp) k_\perp^\mu}{k_\perp^2},
\]

as well as the tensors

\[
\tilde{g}_{\perp \mu \nu} = g_{\mu \nu} - \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} + n_\mu n_\nu ,
\]

\[
Q_{\mu \nu} = \frac{\tilde{u}_\mu \tilde{u}_\nu}{\tilde{u}^2},
\]

\[
T_{\mu \nu} = \tilde{g}_{\perp \mu \nu} - Q_{\mu \nu} .
\]

It is useful to note that the tensors \( Q \) and \( T \) are symmetric and satisfy

\[
k_\perp^\mu T_{\mu \nu} = n^\mu T_{\mu \nu} = \tilde{u}^\mu T_{\mu \nu} = 0 ,
\]

\[
k_\perp^\mu Q_{\mu \nu} = n^\mu Q_{\mu \nu} = 0 ,
\]

as well as

\[
Q_{\mu \lambda} Q^\lambda_\nu = Q_{\mu \nu} ,
\]

\[
T_{\mu \lambda} T^\lambda_\nu = T_{\mu \nu} ,
\]

\[
T_{\mu \lambda} Q^\lambda_\nu = 0 .
\]

III. THE MODEL

A. The electron free field

We envisage a slab of area \( L^2 \) in the \( xy \) plane and thickness \( L_z \). The electrons are confined to live within that slab but are otherwise free to move within it. Eventually, the limits \( L \to \infty \) and \( L_z \to 0 \) will be taken in a suitable way.
Making use of the Furry picture, the electron field is expanded in terms of the one-particle wavefunctions which we take to be of the form

$$H(z)e^{ip_{\perp}\cdot x_{\perp}},$$

(3.1)

where $\vec{p}_{\perp}$ is the component of $\vec{p}$ in the xy plane and similarly for $\vec{x}_{\perp}$. The wavefunctions can be taken to satisfy periodic boundary conditions in the xy plane as usual. In principle the function $H(z)$ is labeled by some quantum number. However, since we are contemplating taking the $L_z \to 0$ limit, for our purposes we assume that only the lowest lying state survives. Moreover, for the same reason, the particular form that it may take is not relevant, as long as it is consistent with our assumptions, and in particular we can adopt

$$H(z) = \begin{cases} 
1 & \text{for } -\frac{L_z}{2} \leq z \leq \frac{L_z}{2} \\
0 & \text{otherwise}
\end{cases}$$

(3.2)

The main assumption of the model is that in the passage to the continuous momentum plane waves in the xy plane ($L \to \infty$), the current density becomes

$$\bar{\psi}\gamma_{\mu}\psi = \left(\frac{H(z)}{\sqrt{L_z}}\right)^2 \bar{\psi}\gamma_{\perp\mu}\psi,$$

(3.3)

where the $\gamma_{\perp}^{\mu}$ are the gamma matrices that are perpendicular to $n^{\mu}$, i.e.,

$$\gamma_{\perp} \cdot n = 0,$$

(3.4)

with the electron field of the form

$$\hat{\psi}(x_{\perp}) = \int \frac{d^2p_{\perp}}{(2\pi)^2} \left[ a(\vec{p}_{\perp}, s)u(\vec{p}_{\perp}, s)e^{-ip_{\perp}\cdot x_{\perp}} + b^*(\vec{p}_{\perp}, s)v(\vec{p}_{\perp}, s)e^{ip_{\perp}\cdot x_{\perp}} \right].$$

(3.5)

In Eq. (3.5), $E = \sqrt{\vec{p}_{\perp}^2 + m^2}$, $x_{\perp}^{\mu} = (x^0, \vec{x}_{\perp})$ and $p_{\perp}^{\mu} = (E, \vec{p}_{\perp})$. The spinors $u$ are the standard Dirac spinors normalized such that

$$u\bar{u} = 2m,$$

(3.6)

and the creation and annihilation operators satisfy

$$\{a(\vec{p}_{\perp}, s), a^*(\vec{p}_{\perp}', s')\} = (2\pi)^2 2E \delta^{(2)}(\vec{p}_{\perp} - \vec{p}_{\perp}')\delta_{s,s'},$$

(3.7)

with analogous relations for the spinors $v$ and the $b$ operators.
The passage to the planar layer is made by taking the $L_z \to 0$ limit at the appropriate stage, which in essence reduces to use

$$\frac{H(z)}{L_z} \to \delta(z)$$

$$H(0) = 1,$$

(3.8)

where $\delta(z)$ is the Dirac delta function, and the free-field current density operator then takes the form

$$\bar{\psi} \gamma_{\mu} \psi = \delta(z) \bar{\psi} \gamma_{\mu} \hat{\psi},$$

(3.9)

For practical purposes Eqs. (3.5) and (3.9) can be taken as the equations that define our model for the plane layer.

B. Electron thermal propagator

From the point of view of TFT, the important ingredient for carrying out the calculation of the photon self-energy is the electron thermal propagator, which in turn can be determined by well established rules in terms of the free-field propagator. The propagator associated with the electron free-field $\psi_\perp$ can be determined in various ways, in analogy with the usual case, and the results look similar. The various components of the thermal propagator matrix are simply

$$S_{111}(p_\perp) = (\not{p}_\perp + m_e) \left[ \frac{1}{p_{\perp}^2 - m_e^2 + i\epsilon} + 2\pi i \delta(p_{\perp}^2 - m_e^2) \eta_e(p_\perp \cdot u) \right],$$

$$S_{122}(p_\perp) = (\not{p}_\perp + m_e) \left[ \frac{-1}{p_{\perp}^2 - m_e^2 - i\epsilon} + 2\pi i \delta(p_{\perp}^2 - m_e^2) \eta_e(p_\perp \cdot u) \right],$$

$$S_{112}(p_\perp) = (\not{p}_\perp + m_e) 2\pi i \left[ \eta_e(p_\perp \cdot u) - \theta(-p_\perp \cdot u) \right],$$

$$S_{121}(p_\perp) = (\not{p}_\perp + m_e) 2\pi i \left[ \eta_e(p_\perp \cdot u) - \theta(p_\perp \cdot u) \right],$$

(3.10)

where

$$\eta_e(p) = \theta(p \cdot u) f_e(p \cdot u) + \theta(-p \cdot u) f_e(-p \cdot u),$$

(3.11)

with

$$f_e(x) = \frac{1}{e^{\beta(x-\mu_e)} + 1}$$

$$f_\bar{e}(x) = \frac{1}{e^{\beta(x+\mu_e)} + 1}$$

(3.12)
and \( \theta(x) \) is the step function. Here \( \beta_e \) and \( \mu_e \) are the inverse temperature and the chemical potential of the electron gas, respectively. The total number of particles in the gas can be calculated from

\[
N = \int d^3x \bar{\psi}\gamma^0\psi \\
= L^2 \int \frac{d^3p}{(2\pi)^3} \text{Tr} \left[ S_{\perp 11}(p_\perp)\gamma^0 \right],
\]

where we have set \( \int dx \, dy \to L^2 \). Using the formulas given above for the propagator, this yields

\[
\frac{N}{L^2} = n_e + \bar{n}_e,
\]

where

\[
n_{e,\bar{e}} = 2 \int \frac{d^2p_\perp}{(2\pi)^2} \frac{1}{e^{\beta(E\pm\mu_e)} + 1},
\]

which represent the surface density of electrons and positrons respectively.

### IV. PHOTON PROPAGATION IN THE LAYER

#### A. Photon effective field

Using Eq. (3.9) as the starting point, the interaction Lagrangian term in the action is taken to be

\[
S_{\text{int}} = -e \int d^3x \perp \bar{\psi}_\perp \gamma^\mu \hat{\psi}_{\perp} \hat{A}_\mu,
\]

where

\[
\hat{A}_\mu \equiv A_{\perp \mu}|_{z=0}.
\]

This indicates that \( \hat{A}_\mu \) is the relevant electromagnetic field variable, and it is the one that we should focus on. Thus, we regard \( \hat{A}_\mu \) as the effective field for the photon, and our goal is to determine its effective action, or equivalently its equation of motion, including the thermal corrections.

Formally, what we want to do is to integrate out all the dynamical field variables except \( \hat{A}_\mu \) itself. Following the usual functional method of quantization of the electromagnetic field[17], and adapting it to the present model, a convenient way to proceed is to introduce in the action an external current

\[
J^\mu(x) = \delta(z)\hat{j}^\mu(x_\perp),
\]
with $\hat{J}^\mu(x_\perp)$ satisfying

$$
\hat{J}(x_\perp) \cdot n = 0,
\partial_\perp \cdot \hat{J}(x_\perp) = 0,
$$

(4.4)

where $\partial_\perp = (\partial_{\perp x_0}, -\partial_{\perp x_\perp})$. Eq. (4.4) ensures that we are selecting only the transverse (gauge invariant) part of $\hat{A}_\mu$, which is the physically meaningful one, since the longitudinal part decouples. The classical field, which we denote by $A_\mu^{(J)}$, in the presence of both, the external current $\hat{J}_\mu$ and the interaction given by $S_{\text{int}}$ in Eq. (4.1), is then defined by

$$
A_\mu^{(J)} = \frac{1}{Z} i\delta Z \hat{J}_\mu.
$$

(4.5)

B. Photon propagator

Following the usual argument, and remembering Eq. (3.8), the generating functional for the free electromagnetic field in the layer is

$$
Z[\hat{J}] \propto \exp \left\{ -\frac{i}{2} \int d^3 x_\perp d^3 x'_\perp \hat{J}^\mu(x_\perp) \hat{\Delta}_F^{\mu\nu}(x_\perp - x'_\perp) \hat{J}^\nu(x'_\perp) \right\},
$$

(4.6)

where $\hat{\Delta}_F^{\mu\nu}(x_\perp - x'_\perp)$ is obtained from the standard photon propagator $\Delta_F^{\mu\nu}(x - x')$ by setting the coordinates normal to the plane ($z$ and $z'$) equal to zero. Therefore, taking into account Eq. (4.4), the propagator for the effective field in the plane is given, in momentum space, by

$$
\hat{\Delta}_F^{\mu\nu}(k_\perp) = \tilde{g}_\perp^{\mu\alpha} \tilde{g}_\perp^{\nu\beta} \left( \int \frac{d\kappa_\parallel}{2\pi} \Delta_\parallel^{\alpha\beta}(k) \right),
$$

(4.7)

where $\tilde{g}_\perp^{\mu\nu}$ has been defined in Eq. (2.6) and, in the integrand, the momentum vector $k$ is decomposed in the form

$$
k_\mu = k_\perp \mu + \kappa_\parallel n_\mu,
$$

(4.8)

with $k_\perp \mu$ as given in Eq. (2.3). Writing

$$
\Delta_F^{\mu\nu}(k) = \frac{-g^{\mu\nu}}{k^2 + i\epsilon} + \text{gauge-dependent terms},
$$

(4.9)

and carrying out the integration over $\kappa_\parallel$, we then obtain the propagator for the effective photon field as

$$
\hat{\Delta}_F^{\mu\nu}(k_\perp) = -\hat{\Delta}(k_\perp) (T^{\mu\nu} + Q^{\mu\nu}),
$$

(4.10)
where the tensors $T$ and $Q$ have been defined in Eq. (2.6), and

$$
\hat{\Delta}(k_{\perp}) = \begin{cases} 
-\frac{i}{2\sqrt{\omega^2 - \kappa^2}} & \text{for } \omega > \kappa \\
\frac{i}{2\sqrt{\kappa^2 - \omega^2}} & \text{for } \omega < \kappa ,
\end{cases} \tag{4.11}
$$

with

$$
\kappa = |\vec{\kappa}_{\perp}|. \tag{4.12}
$$

C. Equation of motion

Armed with the expression for the free photon propagator in the plane, the bilinear part of the effective action for $A^{(J)}_{\mu}$ is then given, in momentum space, by

$$
S^{(2)} = \int \frac{d^3k_{\perp}}{(2\pi)^3} \left\{ \frac{1}{2} A^{(J)*}_{\mu}(k_{\perp}) [D^{\mu\nu}(k_{\perp}) + \hat{\pi}^{\mu\nu}(k_{\perp})] A^{(J)}_{\nu}(k_{\perp}) - A^{(J)*}(k_{\perp}) \cdot \hat{J}(k_{\perp}) \right\}, \tag{4.13}
$$

where $D^{\mu\nu}(k_{\perp})$ is defined

$$
\hat{\Delta}^\mu_{\lambda}(k_{\perp}) D^\lambda_{\nu}(k_{\perp}) = \tilde{g}^\mu_{\nu}, \tag{4.14}
$$

and $\hat{\pi}_{\mu\nu}$ is the photon self-energy in the medium. The dispersion relations, and the corresponding polarization vectors, of the propagating photon modes can be determined by solving the equation of motion for the classical field in the absence of the external current, that is

$$
[D^{\mu\nu}(k_{\perp}) + \hat{\pi}^{\mu\nu}(k_{\perp})] A^{(0)}_{\nu}(k_{\perp}) = 0. \tag{4.15}
$$

D. Photon self-energy and dispersion relations

We denote the components of the thermal self-energy matrix by $\hat{\pi}^{(ab)}_{\mu\nu}$, which to the lowest order are determined by calculating the one-loop diagram shown in Fig. 1. The physical self-energy function that appears in Eq. (4.15) is determined from the relations

$$
\text{Re } \hat{\pi}^{\mu\nu}(k_{\perp}) = \text{Re } \pi^{(11)}_{\mu\nu}(k_{\perp}) \\
\text{Im } \hat{\pi}^{\mu\nu}(k_{\perp}) = \frac{i\pi^{(12)}_{\mu\nu}(k_{\perp})}{2n_\gamma}, \tag{4.16}
$$

where

$$
n_\gamma = \frac{1}{e^{\beta k_{\perp} \cdot \vec{u}} - 1}. \tag{4.17}
$$
In general $\hat{\pi}_{\mu\nu}$ satisfies the transversality condition

$$k_\perp^\mu \hat{\pi}_{\mu\nu} = k_\perp^\nu \hat{\pi}_{\mu\nu} = 0 ,$$

as a consequence of the conservation of the electromagnetic current and, as we will see in Section VII in the one-loop approximation $\hat{\pi}_{\mu\nu}$ satisfies in addition

$$n^\mu \hat{\pi}_{\mu\nu} = n^\nu \hat{\pi}_{\mu\nu} = 0 .$$

It then follows that, in the one loop approximation, $\hat{\pi}_{\mu\nu}$ can be expressed in the form

$$\hat{\pi}_{\mu\nu} = \pi T_{\mu\nu} + \pi L Q_{\mu\nu} .$$

In the more general case, $\hat{\pi}_{\mu\nu}$ can contain more terms such as

$$P_{\mu\nu} \equiv \frac{i}{\kappa} \epsilon_{\mu\alpha\beta} u^\alpha k_\perp^\beta ,$$

and others, but since we are considering only the one-loop approximation to $\hat{\pi}_{\mu\nu}$, for our purposes Eq. (4.20) is the most general form.

Using Eq. (4.10), $D^{\mu\nu}(k_\perp)$ is given by

$$D^{\mu\nu}(k_\perp) = -\hat{\Delta}^{-1}(k_\perp) (T_{\mu\nu} + Q_{\mu\nu}) ,$$

and, remembering Eq. (4.20), the dispersion relations

$$\hat{\Delta}^{-1}(k_\perp) - \pi_L(k_\perp) = 0 ,$$

$$\hat{\Delta}^{-1}(k_\perp) - \pi_T(k_\perp) = 0$$

follow, where $\Delta(k_\perp)$ is given in Eq. (4.11).
V. ONE-LOOP CALCULATION

A. One-loop formula for the self-energy

We want to apply the results of the one-loop calculation of the photon self-energy, to the
dispersion relations given in Eqs. (4.23) and (4.24). We will restrict ourselves to the real
part of the dispersion relations, and therefore we consider here the calculation of
\[ \pi^{(11)}_{\mu\nu} \]
from which the real part of the physical self-energy is determined by means of Eq. (4.16).

Referring to Fig. 1,
\[ i\pi^{(11)}_{\mu\nu}(k) = (-1)(-i)^2 \text{Tr} \int \frac{d^3 p}{(2\pi)^3} \gamma_{\perp\mu} iS_{\perp 11}(p + k) \gamma_{\perp\nu} iS_{\perp 11}(p) . \] (5.1)

When the formula for \( S_{\perp 11} \) given in Eq. (3.10) is substituted in Eq. (5.1), there are three
types of terms. The term that contains two factors of \( \eta_e \) contributes only to the imaginary
part of the self-energy and, since we restrict ourselves here to the real part, we do not
consider it further. The remaining terms then yield
\[ \text{Re} \, \pi^{(11)}_{\mu\nu} = \pi^{(0)}_{\mu\nu} + \pi^{(m)}_{\mu\nu} , \] (5.2)
where \( \pi^{(0)}_{\mu\nu} \) is the standard vacuum polarization term, while the background dependent
contribution is given by
\[ \pi^{(m)}_{\mu\nu} = -4e^2 \int \frac{d^2 \vec{p}}{(2\pi)^2} \frac{L_{\mu\nu}}{k^2 + 2p \cdot k} + (k \rightarrow -k) . \] (5.3)

In this formula,
\[ L_{\mu\nu} = 2p_{\perp \mu} p_{\perp \nu} + p_{\perp \mu} k_{\perp \nu} + k_{\perp \mu} p_{\perp \nu} - g_{\perp \mu\nu} p_{\perp} \cdot k_{\perp} , \] (5.4)
\[ p_{\perp}^2 = (E, \vec{p}) , \quad E = \sqrt{p^2 + m_e^2} , \] (5.5)
and \( f_e, \bar{f}_e \) denote the particle and antiparticle number density distributions given by
\[ f_{e, \bar{e}} = \frac{1}{e^{\beta(E \mp \mu_e)} + 1} \] (5.6)
with the minus(plus) sign holding for the electrons(positrons), respectively. The integral in
Eq. (5.3) is to be interpreted in the sense of its principal value part.

It is easily verified that \( \pi^{(m)}_{\mu\nu} \) satisfies the transversality conditions Eqs. (4.18) and (4.19)
and therefore it can be decomposed in the form
\[ \pi^{(m)}_{\mu\nu} = \pi^{(m)}_T T_{\mu\nu} + \pi^{(m)}_L Q_{\mu\nu} , \] (5.7)
as we already indicated. The functions $\pi_{T,L}^{(m)}$ can be found by projecting Eq. (5.3) with the tensors $T_{\mu\nu}$ and $Q_{\mu\nu}$, and this procedure yields

$$\pi_{T}^{(m)} = -4e^2 \left( A_{\perp e} + \frac{k^2}{\kappa^2} B_{\perp e} \right),$$

$$\pi_{L}^{(m)} = 4e^2 \frac{k^2}{\kappa^2} B_{\perp e},$$

(5.8)

where

$$A_{\perp e} = \int \frac{d^2 \vec{p}_\perp}{(2\pi)^2} \left( f_e + f_\pi \right) \left[ \frac{2m_e^2 - p_\perp \cdot k_\perp}{k^2_\perp + 2p_\perp \cdot k_\perp} + (k_\perp \to -k_\perp) \right],$$

$$B_{\perp e} = \int \frac{d^2 \vec{p}_\perp}{(2\pi)^2} \left( f_e + f_\pi \right) \left[ \frac{2(p_\perp \cdot u)^2 + 2(p_\perp \cdot u)(k_\perp \cdot u) - p_\perp \cdot k_\perp}{k^2_\perp + 2p_\perp \cdot k_\perp} + (k_\perp \to -k_\perp) \right].$$

(5.9)

These two integrals are very similar to the corresponding ones that appear in the three-dimensional case, which have been analyzed in the literature in considerable detail, for various limiting values of the photon momentum and several conditions of the charged particle gas. The methods employed there are applicable here as well. While their evaluation is not possible for the general form of the distribution functions, some useful results can be obtained by considering special cases.

**B. Low momentum limit**

In many situations of interest, the photon momentum is such that

$$\omega, \kappa \ll E_e,$$

(5.10)

where $E_e$ is the typical energy of the electrons in the gas. In this case, borrowing the method used in Ref. [20], we obtain in this case

$$B_{\perp e} = -\frac{1}{2} \int \frac{d^2 \vec{p}_\perp}{(2\pi)^2} \left( \frac{\vec{v}_p \cdot \vec{\kappa}}{\omega - \vec{v}_p \cdot \vec{\kappa}} \right) \frac{d}{dE} (f_e + f_\pi),$$

$$A_{\perp e} = B_{\perp e} + \frac{\omega}{2} \int \frac{d^2 \vec{p}_\perp}{(2\pi)^2} \left( \frac{v^2_p}{\omega - \vec{v}_p \cdot \vec{\kappa}} \right) \frac{d}{dE} (f_e + f_\pi),$$

(5.11)

with $\vec{v}_p = \vec{p}_\perp / E$ denoting the velocity of the particles in the background. The above formula for $B_{\perp e}$ can be rewritten by multiplying the integrand by the factor

$$\frac{1}{\omega} (\omega - \vec{v}_p \cdot \vec{\kappa} + \vec{v}_p \cdot \vec{\kappa}).$$

(5.12)
The first two terms integrate to zero, while the third one leads to

\[ B_{\perp e} = -\frac{1}{2}\omega \int \frac{d^2\vec{p}_\perp}{(2\pi)^2} \left( \frac{(\vec{v}_p \cdot \vec{k})^2}{\omega - \vec{v}_p \cdot \vec{k}} \right) d\omega dE (f_e + f_\pi). \] (5.13)

We stress that the expressions given in Eq. (5.11) are derived from Eq. (5.9) by expanding the integrands in terms of \( k/E \) and retaining only the terms that are dominant when \( k/E \rightarrow 0 \). For a non-relativistic gas, Eq. (5.11) holds for \( \omega, \kappa \ll m_e \). If the gas is extremely relativistic, Eq. (5.11) also holds for \( \omega, \kappa > m_e \), subject to Eq. (5.10).

Furthermore, up to this point no assumption has been made regarding the nature of the electron background. Apart from the restriction on \( k \), Eqs. (5.11) and (5.13) hold for a relativistic or non-relativistic gas, whether it is degenerate or not. Accordingly, they serve as a convenient starting point to find the dispersion relations provided that Eq. (5.10) is verified. As an example we consider one specific situation below.

C. Longitudinal dispersion relation

We consider the case of a non-relativistic electron gas, in the regime in which Eq. (5.10) is valid, and seek the real solutions to the longitudinal dispersion relations in the long wavelength limit,

\[ \omega \gg \kappa v_e, \] (5.14)

where \( v_e \) is the typical velocity of the electrons. Then, neglecting \( f_\pi \) in Eq. (5.13), the calculation of \( B_e \) in this limit gives

\[ B_{\perp e} = \frac{n_e \kappa^2}{4m_e \omega^2}, \] (5.15)

and using Eqs. (4.16), (5.2) and (5.8),

\[ \Re \pi_L = \frac{e^2 n_e}{m_e \omega^2} \left( \omega^2 - \kappa^2 \right). \] (5.16)

A real solution of Eq. (4.23) is obtained for \( \omega < \kappa \) by solving

\[ \left( \frac{e^2 n_e}{2m_e} \right) \frac{\sqrt{\kappa^2 - \omega^2}}{\omega^2} = 1, \] (5.17)

which gives

\[ \omega^2 = \frac{1}{2} \alpha^2 \left[ \left( 1 + \frac{4\kappa^2}{\alpha^2} \right)^{1/2} - 1 \right]. \] (5.18)
where
\[ \alpha = \frac{e^2 n_e}{2m_e}. \] (5.19)

The function \( \omega_\kappa \) is depicted in Fig. 2. It follows from Eq. (5.18) that \( \omega_\kappa \approx \kappa \) for \( \kappa \ll \alpha \), while for \( \kappa \gg \alpha \) it is given approximately by
\[ \omega_\kappa^2 \approx \alpha \kappa. \] (5.20)

This solution is valid as long as
\[ \kappa < \frac{\alpha}{v_e^2}, \] (5.21)
so that the condition \( \omega \gg \kappa v_e \) is satisfied and therefore Eq. (5.15) remains valid.

This result, which implies that in this example system the plasma frequency is momentum-dependent, is known in the plasma physics and condensed matter literature\[14, 15\]. In those contexts, it is derived using the static local field correction approximation, in which the dynamic corrections are replaced by their static (\( \omega \to 0 \)) values\[16\]. In our notation, that approximation corresponds to using the expressions
\[
\hat{\Delta}(k_\perp) \approx \lim_{\omega \to 0} \hat{\Delta}(k_\perp) = \frac{1}{2\kappa}, \\
\Re \pi_L \approx \Re \pi_L|_{\omega \ll \kappa} = -\frac{e^2 n_e \kappa^2}{m_e \omega^2},
\] (5.22)
in Eq. (4.23). The solution of the equation thus obtained is indeed given by Eq. (5.20). But, as we have seen, Eq. (5.20) is an approximation to the solution given in Eq. (5.18), which we have obtained by using the complete expressions for the propagator and self-energy.

Therefore, in our case, by using the complete inverse propagator in Eq. (4.23), we are able to obtain the dispersion relation given in Eq. (5.18), which is valid for all values of $\kappa$, including $\kappa \approx \alpha$ which are outside the range of validity ($\kappa \gg \alpha$) of the static local field correction approximation.

Our motivation for going through this exercise here is to evidence that some known results are not only reproduced, but also generalized, using the method we plan to use, which in turn indicate that they can be applied systematically to study the problems that we have mentioned in this type of system.

VI. CONCLUSIONS

In the present work we have formulated the TFT approach to the model of the two-dimensional plasma layer, that is, a system in which the electrons are confined to a plane sheet. As emphasized in the Introduction, this is not equivalent to what is usually called QED$_3$ (or QED in 2+1 dimensions) which describes a two-dimensional cross section of a system that has cylindrical symmetry and the physics is independent of the $z$ coordinate. Thus, for example, while the electrons in QED$_3$ are really lines of charge in the three-dimensional world and the Coulomb potential between them is logarithmic, in the system we are considering the electrons are ordinary point charges, which are confined to the $z = 0$ plane, but the Coulomb potential between them is the ordinary $1/r$ potential.

An important step in this direction was to determine the appropriate set of thermal propagators. As an application, we performed the one-loop calculation of the photon self-energy in that medium, and we considered in detail the calculation of the longitudinal photon dispersion relation. We made contact with previous calculations of that quantity, that had been obtained using other approaches, and in particular we showed that our results reduce to those known results when the appropriate limits are taken and/or approximations are made. However, the formulas we obtained are more general and can be used for a wider range of conditions in which those approximations and limits are not justified. We considered the simplest situation of an ordinary gas of electrons, which are confined to a plane sheet, but
are otherwise free. However the method allows us to consider variations of the model in a systematic way, such as the effects of anisotropies and/or external fields.

There is a considerable amount of literature on the applications of Thermal Field Theory (TFT) methods to the study of the properties of the electron gas (plasmon dispersion relations and so on) in the three-dimensional space. Although those systems have been studied by a variety of methods, the application of TFT methods to study them have been useful in many ways.

We believe that the applications to the two-dimensional sheet that we have described, are novel TFT calculations which are expected to be equally attractive and useful. For example, they can have astrophysical applications [6–8]; they establish a good point of contact between the applications of TFT methods and dominant themes in plasma physics (such as the effects of anisotropies and instability studies); they may be relevant for laboratory experiments in fundamental disciplines of physics such as plasma physics [9] and condensed matter [10, 11].

Such calculations are also interesting in their own right because they can be useful in the study of physical systems of current interest in which a plasma is confined to a layer [12] or a wire [13], and they may also be helpful for studying and understanding analogous effects and issues in more complicated systems such as the non-abelian plasmas.

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