Predicting geometric parameters of assemblies with neural network models

M A Bolotov¹, V A Pechenin¹, N V Ruzanov¹ and I A Grachev¹

¹Samara National Research University, Moskovskoe Shosse 34, Samara, Russia, 443086

Abstract. The prediction models of assembly processes for critical parts will enable ensuring adaptive control of assembly based on the measured information. Direct simulation of the mating process with employing computational mating models and finite element models of assemblies requires considerable computational resources and is frequently accompanied by the solution convergence problem. To solve the above problems, neural network models may be employed, which describe basic regularities for the mating process based on the cumulative results. The work describes the technique for predicting the mating precision of parts based on real geometric models of surfaces. Real models of parts represents dot arrays of their surfaces. The technique employs the developed model, allowing to calculate the assembly geometric parameters of parts. The results of mating simulation for the disk and the spacer of the turbine rotor are considered. To predict the parameter of “radial run-out” depending on the value and nature of form deviation and tension of mating surfaces, a radial-base neural network was developed and trained.

1. Introduction

Complex industrial and science-intensive products involve high requirements to the geometric accuracy of parts and assembly units. These products include modern aircraft engines, which require high reliability, minimum weight, efficiency and service life. The above characteristics are ensured, among other things, by improving the manufacturing technology for parts and assemblies. Dimensional accuracy is ensured in manufacturing and assembly. The ways to achieve dimensional accuracy are determined at the stage of the manufacturing design engineering [1,2,3]. The solution of these problems at each stage can be simplified due to the availability of mathematical models [4] and tools allowing to predict and determine the actual values of geometric parameters [5,6] characterizing the achievable accuracy.

The technique for estimating the probability spatial mating parameters of parts should allow for evaluating the impact of the geometrical accuracy of surfaces of parts on the accuracy of mating and assembly of parts between each other. The developed technique shall be implemented as a program library. The work suggests a model and software implementation of the “virtual assembly” of assembly units of complex products, which allows modelling the mating process of flat and cylindrical surfaces of parts with the dimensions characteristic for parts of gas turbine engines and calculate the assembly parameters.
2. Object of research
The assembly of parts of the turbine disk and spacer shall represent the object of research. The sketch of the spacer and disc containing the controlled geometric parameters is given in fig. 1. and fig. 2.

For the “spacer” part, the controlled geometric parameters are: linear dimension 101 with a tolerance of -0.14 mm, a perpendicularity of 0.03 mm of the surface D relative to G, a parallelism of 0.03 mm of the end surface mating with the disc relative to the surface G and a radial run-out of 0.03 mm of the opening mating with the disk with respect to the surface D. The disc has is a cylindrical surface B mating with the spacer and the end, whose beating should not exceed 0.02 mm. In addition, the surfaces are rated with respect to the form deviation \( \delta_f \) (cylindricity and flatness deviation) within 0.03 mm. To calculate the resulting assembly parameters, a mathematical model shall be developed for assembling parts by surfaces that have geometric deviations. After assembly, the following assembly parameters are calculated: radial run-out of the disk relative to the surface D \( \Delta r_d \); the parallelism of the end of the disk relative to G \( \Delta_{par,d} \); disc concentricity relative to D \( \Delta_{as,c} \); deviation of the axial clearance \( \Delta_{ag} \). Concentricity influences the unevenness of the end clearance between the disk blades and the motor housing, which should be 0.3-0.5 mm for all stages. In this regard, it is more convenient to operate with the radius-vector of the disk centre deviation, therefore the centre point coordinates are more convenient to be converted into the polar coordinate system \( (\rho_{as,c}, \beta_{as,c}) \).

3. Virtual assembly technique and algorithms
In general, the contact of respectively mating surface pairs of flat and cylindrical surfaces occurs over a finite set of points [7]. Such points are contact points [8]. Analytic solutions of contact problems for surfaces without form deviation are known. In general, for simulating contact problems of surfaces with form deviation, the finite elements method is used. The above method allows to take into account features of surface geometry, solve assembly prediction and tolerance analysis problems [9, 10], contact and strength problems [11].

To simulate mating of parts, a technique has been developed that includes the following steps:
1. Development of real models of parts, assembly forming. Real models of parts represents dot arrays of their surfaces.
2. Setting the calculation model:
   a. Developing triangulation grids on the surfaces of parts.
   b. Setting the contact pairs of surfaces.
   c. Setting the part displacement limitations.
3. Calculation, result saving.
4. Development of a model for processing of calculation results allowing to determine the geometric assembly parameters based on the coordinates of the finite element (triangulation) grid and the displacement, deformation and stress matrices.
5. Calculation of statistical indicators.

The real parts models were developed out in the following sequence. At the first stage, the nominal models of mating parts were developed. At the second stage, the real (model) mating surfaces were calculated from a finite set of points based on nominal equations, form deviation functions and location deviation parameters. One of the methods for formalizing the surface form deviations is the modal approach [12].

The point coordinate of the measured model surface may be determined from the formula:

$$p_m = \left( p_n + n \cdot \delta_f \right) \cdot R + \vec{t},$$  

where

- \( p_m, p_n \) – point coordinate vector \((x, y, z)\) of the measured (simulated) and nominal (CAD) surface, respectively;
- \( n \) - normal vector in the point \( p_n \);
- \( \delta_f \) - form deviation value in the point \( p_n \);
- \( R, \vec{t} \) – rotation matrix and transposition vector of point coordinates \( p_n \) characterizing position deviation.

In the problem being solved comprising the disk and the spacer, a finite element grid was developed, in which a high partition density was set at the contacting end and cylindrical surfaces. The contact parameters were set for the connections: plane-plane (clearance \( G_2 \)) and cylinder-cylinder (clearance \( G_1 \)). The limiting conditions were set for six degrees of freedom for the “Spacer” part. To solve the contact problem using the developed finite element model, an iterative algorithm was developed that allows calculating the mating of parts without taking into consideration the deformation of parts during the assembly process described in detail in [13]. The algorithm for finding the mating state assumes the iterative transposition of one mating surface relative to the other with the surface assembly force vector \( \vec{D}_f \). The notion of clearance function \( G(\vec{V}) \) was introduced characterizing the achievement of the mating state of the surfaces of parts and depending on the relative position vector of the surfaces \( \vec{V} \). To calculate the function \( G(\vec{V}) \) at each stage, the best alignment of the mating surfaces is made. To perform the best alignment procedure, the iterative nearest-point algorithm (ICP) is used [14, 15]. According to this algorithm, the angles for rotation and displacement along the coordinate axes are calculated at each iteration by non-linear optimization search methods. To avoid intersections of two surfaces, the system of inequalities presented in [16] is used, which imposes restrictions on the gap function \( G(\vec{V}) \). Based on the algorithm, the rotation matrix and displacement vector of the moving part are calculated, which determine the transformation of its initial coordinate system into a coordinate system in the assembled state:

$$p_{as} = p_p \cdot R_{as} + \vec{t}_{as},$$

where \( p_{as}, p_p \) – point coordinate vector \((x, y, z)\) parts after assembly and in the initial condition, respectively; \( R_{as}, \vec{t}_{as} \) – rotation matrix and displacement vector of the point coordinates characterizing the part displacement during assembly.

The developed technique and the iterative algorithm were used to calculate the mating accuracy of the assembly unit, including the disk and the spacer in the turbine of the aircraft engine (Figures 1 and 2). Mating of the mentioned parts by plane-cylinder surfaces is considered. A similar connection is considered in the work [17].
4. Virtual assembly application results for predicting assembly parameters

To evaluate the dimensional accuracy parameters of the disk and turbine spacer assembly (radial run-out and parallelism of the disk end), simulation of 72 mating surfaces of parts was performed, for each of which the assembly was simulated in different positions (6 cases for each surface due to different angular positions of the disk). Cylindrical surfaces were set by 720 uniformly distributed points in the cross section \(XOY\) in ten sections, flat mating surfaces were set by points in ten sections. The sections represent the polar distance of the polar surface representation system, the points in the sections are the polar angle. The magnitude of the form deviation amplitude varied according to the normal distribution law.

The limiting values of the parameters of the geometric errors used in simulation of the measured surfaces are given in Table 1.

Table 1. The limit values of error parameters in experiments.

| Parameter                  | \(D\) | \(G\) | Spacing opening | Spacer end | Disc cylindrical surface | Disc end surface |
|----------------------------|-------|-------|-----------------|------------|--------------------------|-----------------|
| Measurement error          |       |       | 1.7(2.5)+L/333 [\(\mu m\)] |            |                           |                 |
| Non-perpendicularity       | 0.03 [mm] | None | None          | None       | None                     | None           |
| Non-parallelism            | None  | None  | Base           | 0.03 [mm]  | None                     | None           |
| Dimensional deviation      | None  | -0.06 [mm] | Base        | 0.14 [mm]  | None                     | None           |
| Radial run-out             | Base  | 0.03 [mm] | None          | None       | None                     | None           |
| Face run-out               | None  | None  | None           | None       | None                     | None           |
| Number of points in cross-section |       |       |                |            |                           | 720             |
| Number of points in longitudinal section |       |       |                |            |                           | 10              |

In the connection between the disc and spacer, the maximum tension of 0.09 shall be ensured subject to the drawing conditions. In this connection, the spacer opening diameter has the tolerance of -0.06 mm, the diameter of the mating cylinder surface of the disc is up to +0.03 mm.

Location deviations were simulated using the definitions from GOST R 53442-2009 [18] through rotation and transposition (displacement) of the points of planes and cylindrical surfaces with the help of rotation matrices and the transposition vector from (1). In simulating the assembly, the disc took different positions relative to the spacer by rotating about the rotation axis in 60 degree increments. An iterative mating algorithm was implemented in the software package MATLAB. The surface models were developed considering the form and location deviations. As a result of mating simulation, the positions of the disk relative to the spacer were determined without taking into consideration the deformations of the parts during the assembly process. The limiting deviation of the form of the mating surfaces of the disc and the spacer was 0.03 mm, the limiting tension in the mating of spacer opening - disc cylindrical surface amounted to 0.08 mm.

Table 2 shows the mathematical expectation \(\mu\), mean square deviation \(\sigma\), minimum and maximum value of deviation of the assembly parameters with the confidence factor of 99.73%.

Let’s consider the degree of influence of the form deviation parameters (described by polynomial equations with degrees \(c_\lambda\) for cylindrical surfaces and \(e_\lambda\) for end surfaces) and the tension values in the mating between the disk and the spacer on the errors of the assembly parameters. The degree and nature of the impact may be calculated using the correlation factors between the uncertainty values and measurement parameters. Table 3 shows the correlation factors between the dimensional accuracy parameters of parts and assembly parameters.
Figure 3. Error histograms of assembly parameters: $\Delta_{r-d}$ (a); $\Delta_{par-d}$ (b); $\rho_{as-c}$ (c); $\Delta_{ag}$ (d).

Table 2. Deviation parameters for form and location of the disc-spacer assembly.

| Parameter      | $\mu$, $\mu$m | $\sigma$, $\mu$m | Min., $\mu$m | Max., $\mu$m |
|----------------|----------------|------------------|--------------|--------------|
| $\Delta_{r-d}$ | 17.29          | 6.95             | 3.69         | 44.88        |
| $\Delta_{par-d}$ | 6.33          | 3.65             | 0.20         | 20.33        |
| $\Delta_{as-c}$ | 1.79           | 1.53             | 0.05         | 13.21        |
| $\Delta_{ag}$  | 29.71          | 23.87            | -9.82        | 86.48        |

Table 3. Correlation factors for assembly parameter errors.

| Parameter                          | Geometric deviations of disc, mm |
|------------------------------------|----------------------------------|
|                                    | $\Delta_{r-d}$ | $\Delta_{par-d}$ | $\rho_{as-c}$ | $\Delta_{ag}$ |
| Tension $G_z$                      | 0.02              | -0.03            | -0.07         | 0.00          |
| Form deviation in cylinder         | 0.75              | -0.22            | -0.03         | -0.12         |
| Form deviation in ends             | 0.67              | -0.06            | -0.08         | -0.17         |
| Polynomial degree for cylinders $\lambda_c$ | 0.02              | -0.24            | -0.02         | -0.04         |
| Polynomial degree for ends $\lambda_e$ | 0.56              | -0.22            | -0.24         | -0.19         |

The form deviation parameters of the mating surfaces have a strong influence on the radial run-out of the disk. The nature of the form deviation at the ends also has an average value of the correlation relationship with the parameter and a weak connection with the remaining parameters. As follows from the results of the correlation analysis, the value is quite strongly interrelated with the form deviations on the mating surfaces, and hence it may be predicted from these parameters.

Based on the above, to predict the parameter $\Delta_{r-d}$ measurement uncertainty, the following regression expression may be used:

$$\Delta_{r-d} = f(G, 0.\Delta_f, \lambda),$$  \hspace{1cm} (3)

The next step in developing a regression model is to develop a model for uncertainty dependencies on the listed parameters. To solve the set task for predicting uncertainties, the Generalized Regression Neural Networks (GRNN) were employed.
A radial-base neural network was developed, having 2 layers – a hidden radial base layer with \( Q \) neurons, and an output linear layer with \( S \) neurons. Schematically, it is represented in fig. 4. The radial-base neuron transforms the distance from the given input vector to its corresponding “centre” through some non-linear law (Gaussian function).

The number of neurons of the input layer \( P \) is equal to the number of parameters used for prediction: the tension in the connection of cylindrical surfaces; total form deviation of cylindrical surfaces; total form deviation of the end surfaces; the polynomial degree describing the deviation on cylindrical surfaces; the polynomial degree describing the deviation on the end planes. The number of neurons of the radial basis layer \( Q \) is equal to the number of elements of the training set, that is, the number of assembly cases employed for training the network. The number of neurons in the second, linear layer is equal to the number of predicted parameters. In our case, it is the parameter.

Based on the simulation results (Tables 1 and 2, Figure 3), a base neural network was developed and trained in the MATLAB software package. To train the network, 66 assembly cases were employed (in each case there were 6 possible positions, so the resulting value was chosen as the average of the 6 positions). The prediction was made for the remaining 6 cases, as a result of which the run-out values are calculated \( \Delta_{r_{\text{pr}}} \). The values of relative errors for determining the “radial run-out” values were calculated \( \Delta_{\text{rel}} \) using the formula:

\[
\Delta_{\text{rel}} = \left( \Delta_{r_{\text{d}_{\text{pr}}}} - \Delta_{r_{\text{d}}} \right) / \Delta_{r_{\text{d}}} \cdot 100\%,
\]

Table 4 shows the moment characteristics (mathematical expectation, \( \mu \) standard deviation \( \sigma \)), as well as the minimum and maximum relative errors in predicting the parameter \( \Delta_{r_{\text{d}}} \).

| Parameter | \( \mu \), % | \( \sigma \), % | Minimum, % | Maximum, % |
|-----------|--------------|----------------|------------|------------|
| Training  | 0.00         | 0.00           | 0.00       | 0.00       |
| Prediction| -3.24        | 4.27           | -9.08      | 2.77       |

As it follows from the results, prediction errors are within 10%.

5. Conclusion
The work describes the technique for predicting the mating precision of parts based on real geometric models of surfaces. The results of mating simulation for the disk and the spacer of the turbine rotor are considered. The analysis of the degree of influence of the form deviations and dimensions of the mating surfaces on the assembly parameters of the product was performed. The data obtained through simulation are used to develop and teach a radial-base neural network allowing to predict errors in the
assembly parameters of a disc by a set of geometry deviation parameters of the measured surfaces of parts.

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