SUPERSYMMETRY, TRACE ANOMALY
AND NAKED SINGULARITIES

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ABSTRACT

We discuss stationary supersymmetric bosonic configurations of the Einstein-Maxwell theory embedded in $N = 2$ supergravity. Some of these configurations, including the Kerr-Newman solutions with $m = |q|$ and arbitrary angular momentum per unit mass $a$, exhibit naked singularities. However, $N = 2$ supergravity has trace anomaly. The nonvanishing anomalous energy-momentum tensor of these Kerr-Newman solutions violates a consistency condition for a configuration to admit unbroken supersymmetry. Thus, the trace anomaly of this theory prevents the supersymmetric solutions from exhibiting naked singularities.
1 Introduction

In this paper we would like to continue our analysis of the relation between supersymmetry and cosmic censorship, which we started in [1]. We have observed that the parameters of the static dilaton black holes [2] considered as bosonic solutions of \( N = 4 \) supergravity are constrained due to the existence of supersymmetric positivity bounds. The effect of imposing these supersymmetric bounds on the parameters of black hole solutions is the same as imposing cosmic censorship: they prevent the solutions from exhibiting naked singularities. Based on this example which generalizes the Reissner-Nordström black hole case considered in the framework of \( N = 2 \) supergravity [3] we conjectured that, in general, supersymmetry may act as a cosmic censor for static configurations in asymptotically flat spaces.

The generic feature of theories with global supersymmetry (without gravity) is the fact that the energy is non-negative, since the Hamiltonian is a square of supersymmetry charges [4]. It looks plausible that the cosmic censorship role of local supersymmetry is the generalization of the role of global supersymmetry as warrant of the positivity of energy in supersymmetric non-gravitational theories.

It may also happen that supersymmetry will help us in justifying the cosmic censorship hypothesis for certain nonsupersymmetric theories, just as it happened with the proof of the positivity of energy in General Relativity. In that case it was enough to know that this theory can be consistently embedded into supergravity [5].

In our previous work we have investigated only static (non-rotating) black holes. The supersymmetric positivity bound of the Einstein-Maxwell theory embedded in \( N = 2 \) ungauged supergravity implies \( m^2 \geq q^2 \) [3], [1], which guarantees that the static candidates to end-points of black hole evaporation (i.e. the Reissner-Nordström solutions) have an event horizon covering the singularity.

In the stationary case, though, it was not quite clear whether one could derive the analogous bound \( m^2 \geq a^2 + q^2 \) for rotating Kerr-Newman (KN) black holes from supersymmetry alone. (Here \( a \) is angular momentum per unit mass.) Moreover, it was proven by Tod [6] that all the KN solutions with \( m^2 = q^2 \) admit Killing spinors. The KN black hole is a configuration with \( m^2 - a^2 - q^2 \geq 0 \). The extreme one has \( m^2 - a^2 - q^2 = 0 \). Any configuration with \( m^2 = q^2 \) and non-vanishing angular momentum is far below extremality, which means that the singularity is not covered by any event horizon.

In fact Tod proved that a whole class of stationary metrics including Israel-Wilson-Perjes metrics [6] admit \( N = 2 \) supergravity Killing spinors. These solutions have been shown by Hartle and Hawking [8] to have, in general, naked singularities. Therefore in [1] we restricted our conjecture about supersymmetry as the cosmic censor only to static (and not stationary) asymptotically flat solutions.
Note, however, that the appearance of a naked singularity at $m^2 < a^2 + q^2$ is a very subtle effect. At $m^2 = a^2 + q^2$ the singularity is deeply hidden under the horizon. An infinitesimally small decrease of mass (or increment of angular momentum) immediately destroys the horizon and makes the singularity naked. In situations in which small causes may have large effects, quantum corrections may be very important.

In particular, all supersymmetric KN black holes with a given charge have the same mass, $m = |q|$, independently of their angular momentum. In other words, they correspond to degenerate energy eigenstates. This degeneracy, being a consequence of supersymmetry, can sometimes be removed by quantum effects. And indeed, as we will see, with an account taken of the trace anomaly, only the state with $a = 0$ (the nonrotating Reissner-Nordström black hole) remains supersymmetric.

## 2 Supersymmetry of Israel-Wilson-Perjes metrics

We will start by rederiving Tod’s result using the standard language of field theory rather than Newman-Penrose spinor language. The first analysis of supersymmetric configurations of $N = 2$ supergravity was performed by Gibbons and Hull in [3] using the standard field theory spinors. They found that the static Papapetrou-Majumdar (PM) metrics, and in particular, the extremal Reissner-Nordström black hole metrics are supersymmetric. Later Tod [6] found that, in addition to these configurations, some other configurations, in particular, some stationary metrics, also admit supercovariantly constant spinors. The class of such metrics admitting $N = 2$ Killing spinors is known in General Relativity as the class of conformal-stationary Einstein-Maxwell fields with conformally flat 3-dimensional space, or Israel-Wilson-Perjes (IWP) metrics. The PM metrics are just the static IWP metrics.

The IWP metrics and the corresponding electromagnetic fields can be completely described in terms of a time-independent complex function $V$:

$$
\begin{align*}
    ds^2 &= (V \bar{V})(dt + \bar{w}d\bar{x})^2 - (V \bar{V})^{-1}(d\bar{x})^2, \\
    \nabla \times \bar{w} &= -i(V \bar{V})^{-1}\nabla \log (V/\bar{V}), \\
    F_{0i} &= E_i = \frac{1}{2} \partial_i (V + \bar{V}), \\
    \ast F_{0i} &= iB_i = \frac{1}{2} \partial_i (V - \bar{V}).
\end{align*}
$$

(1)

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3 We will not consider pp-wave spaces in this paper.
4 Our notation are given in [8], and [10]. In particular, hatted indices are the curved space ones.
This configuration will be a solution of the Einstein-Maxwell equations of motion in absence of matter if the complex function $V$ is chosen to be the inverse of a harmonic function:

$$
\Delta V^{-1} = 0 , \quad \Delta \bar{V}^{-1} = 0 ,
$$

(2)

where $\Delta$ is the flat-space Laplacian in $\vec{x}$.

For real $V$ these configurations are stationary and correspond, as we have said, to the PM solutions [9]. These are the only regular black hole solutions in the IWP class. All solutions with complex $V$ have naked singularities, according to Hartle and Hawking [8]. In particular, the solution presented in eqs. (1) includes the charged KN solution with arbitrary angular momentum and charge equal to its mass, $m^2 = q^2$. The KN charged rotating black hole solution is given by

$$
ds^2 = \left( 1 - \frac{2m r - q^2}{r^2 + a^2 \cos^2 \theta} \right) dt^2 - \frac{2a \sin^2 \theta (2m r - q^2)}{r^2 + a^2 \cos^2 \theta} dtd\phi - (r^2 + a^2 \cos^2 \theta) \left( \frac{dr^2}{r^2 + a^2 + q^2 - 2mr} + d\theta^2 \right) - \sin^2 \theta \left( r^2 + a^2 + \frac{a \sin^2 \theta (2mr - q^2)}{r^2 + a^2 \cos^2 \theta} (2mr - q^2) \right) d\phi^2 .
$$

(3)

When $m^2 = q^2$ and $a$ is arbitrary, this metric can be brought to the form of eq. (2) [7]. In Cartesian coordinates the complex harmonic function is

$$
V = 1 + \frac{m}{\sqrt{x^2 + y^2 + (z - ia)^2}} .
$$

(4)

In terms of more suitable oblate spheroidal coordinates $x + iy = [(r - m)^2 + a^2]^{1/2} \sin \theta e^{i\phi}$, $z = (r - m) \cos \theta$, the function $V$ takes the form

$$
V = 1 + \frac{m}{r - m - ia \cos \theta } ,
$$

(5)

and

$$
V\bar{V} = \frac{(r - m)^2 - a^2 \cos^2 \theta}{r^2 + a^2 \cos^2 \theta} ,
$$

(6)

so the Euclidean 3-metric becomes

$$
(d\vec{x})^2 = \left( (r - m)^2 + a^2 \cos^2 \theta \right) \left( \frac{dr^2}{(r - m)^2 + a^2} + d\theta^2 \right) + \left( (r - m)^2 + a^2 \right) \sin^2 \theta d\phi^2 .
$$

(7)

The corresponding $\vec{\omega}$ is

$$
\vec{w} \cdot d\vec{x} = \frac{(2mr - m^2) a \sin^2 \theta}{(r - m)^2 + a^2 \cos^2 \theta} d\phi .
$$

(8)

Substituting eqs. (3), (7) and (8) into eq. (2) and comparing the result with eq. (3) one can see that this particular IWP metric with function $V$ given by eq. (4) coincides with that of a KN charged rotating black hole with $m^2 = q^2$. 

4
With this in mind we can analyse the problem of supersymmetry for the general class of metrics (1), since the KN solution with $m^2 = q^2$ is a particular case of such solutions. We will come back to this specific solution when analysing the contribution of the trace anomaly to the equations of motion.

Consider now the supersymmetry transformation of the gravitino field in $N = 2$ supergravity:

$$\frac{1}{2} \delta_\epsilon \Psi_{\mu I} = \nabla_\mu \epsilon_I - \frac{1}{2} \epsilon_{IJ} \sigma^{ab} F_{ab} \gamma_\mu \epsilon^J, \quad I, J = 1, 2.$$  \hspace{1cm} (9)

We want to find time-independent Killing spinors $\epsilon^I$, i.e. spinors for which the above expression vanishes,

$$\delta_\epsilon \Psi_{\mu I} = 0 ,$$  \hspace{1cm} (10)

and whose partial time derivative vanishes too,

$$\partial_0 \epsilon_I = 0 .$$  \hspace{1cm} (11)

We already know from Tod’s work that these equations will have nontrivial solutions for IWP metrics, and so we will substitute eqs. (1) in it, and we will look for the Killing spinors which we know to exist. But we will not require the field configurations to satisfy any specific equations of motion like the Einstein-Maxwell equations of motion in absence of matter or any other equations. Thus $V$ will not be constrained to be the inverse of a harmonic function as in eq. (2) and will remain arbitrary for most of our discussion.

It is convenient to express the supersymmetry transformation of the gravitino in terms of Dirac spinors $\epsilon = \epsilon^1 + \epsilon^2$ and $\psi_\mu = \psi^1_\mu + \psi^2_\mu$,

$$\frac{1}{2} \delta_\epsilon \psi_\mu = \nabla_\mu \epsilon + \frac{1}{2} \sigma^{ab} F_{ab} \gamma_\mu \gamma_5 \epsilon .$$  \hspace{1cm} (12)

If we express the chiral Majorana spinors $\epsilon_I$ in terms of two-component Weyl spinors $\tilde{\epsilon}_I$ according to our conventions we have

$$\epsilon_I = \begin{pmatrix} \bar{\epsilon}_I \\ 0 \end{pmatrix}, \quad \epsilon^I = \begin{pmatrix} 0 \\ \bar{\epsilon}_I \end{pmatrix}, \quad \epsilon = \begin{pmatrix} \bar{\epsilon}_2 \\ \bar{\epsilon}_1 \end{pmatrix}. $$  \hspace{1cm} (13)

First we take the time component of the Killing equation $\delta_\epsilon \Psi_{0I} = 0$. Using the time-independence of the spinors we are looking for, eq. (2), we arrive to

$$\begin{pmatrix} \sigma^I [\omega_0^{+0i} \bar{\epsilon}_2 - i F^{+0i} \bar{\epsilon}_1] \\ \sigma^I [\omega_0^{+0i} \bar{\epsilon}_1 - i F^{+0i} \bar{\epsilon}_2] \end{pmatrix} = 0 ,$$  \hspace{1cm} (14)

which, upon use of eqs. (2), implies the following relation between the Killing spinors:

$$\bar{\epsilon}_1 = -i \left( \nabla / V \right)^{\frac{1}{2}} \bar{\epsilon}_2 ,$$  \hspace{1cm} (15)
or, in terms of the chiral Majorana spinors
\[ \epsilon^1 + (V/V)^{1/2} \gamma^0 \epsilon_2 = 0. \]  \hspace{1cm} (16)

Now we take the spatial components of the Killing equation \[ \delta_i \Psi_i = 0. \] Using eqs. (1) and the relation between the spinors eq. (15) we get the following two equations:
\[ \partial_i (V_{1/2} \tilde{\epsilon}^1) = 0, \]
\[ \partial_i (V_{1/2} \tilde{\epsilon}^2) = 0, \]  \hspace{1cm} (17)

which imply for the chiral Majorana spinors,
\[ \epsilon^1 = V_{1/2} \epsilon_{(0)}^1, \]
\[ \epsilon^2 = V_{1/2} \epsilon_{(0)}^2, \]  \hspace{1cm} (18)

where \( \epsilon_{(0)}^1 \) and \( \epsilon_{(0)}^2 \) are constant chiral Majorana spinors. These equations will be consistent with eq. (16) if the constant spinors themselves satisfy
\[ \epsilon_{(0)}^1 + \gamma^0 \epsilon_{(0)}^2 = 0. \]  \hspace{1cm} (19)

Let us stress that the fundamental difference between supersymmetric configurations with naked singularities and without them among the IWP class is the presence or absence of imaginary part in the function \( V \). This is the only function in our Ansatz, which solves Killing spinor equations and allowed Tod to find supersymmetric configurations without reference to any equation of motion.

### 3 Consistency condition for unbroken supersymmetry

Consider the classical Einstein-Maxwell action
\[ S_{EM} = -\frac{1}{4} \int d^4x \sqrt{-g} \left( R + F^2 \right), \]  \hspace{1cm} (20)

which is the bosonic sector of \( N = 2 \) supergravity. The effective equations of motion are
\[ \frac{\delta S_{EM}}{\delta g_{\mu \nu}} = J_{\mu \nu}, \quad \frac{\delta S_{EM}}{\delta A_\mu} = J^\mu. \]  \hspace{1cm} (21)

The two tensors \( J_{\mu \nu} \) and \( J^\mu \) are the “right-hand side” of the metric and electromagnetic vector potential equations of motion. These two tensors vanish for classical (on-shell) configurations but we are going to consider general configurations obeying the equations of motion with \( J_{\mu \nu} \) and \( J^\mu \) nonvanishing in general. The notation emphasizes the fact that \( J_{\mu \nu} \) is different from the
classical electromagnetic energy-momentum tensor that appears in the Einstein-Maxwell theory. Later on we will be interested in semiclassical configurations for which these tensors are induced by quantum corrections.

In [11] we have derived some consistency conditions (Killing Spinor Identities) that any supersymmetric configuration has to satisfy. We are going to show that the only configurations which satisfy these identities are those with $V$ real.

To find the $N = 2$ supergravity Killing Spinor Identities we need the function

$$\Omega \equiv \sum_b J_b \delta \phi^b = J_{\mu \nu} \delta \phi^{\mu \nu} + J^\mu \delta \phi A_{\mu},$$  \hspace{1cm} (22)

where the supersymmetry transformation of the metric is denoted by $\delta \phi^{\mu \nu}$, and that of the vector field by $\delta \phi A_{\mu}$. Now we have to differentiate this function over the gravitino field, and the result has to vanish when $\epsilon^I$ is a Killing spinor. From now on we will assume this to be so. Then the Killing Spinor Identities take the form

$$J^\mu \epsilon^I \gamma_\nu + \frac{1}{2} J^\mu \epsilon^J \epsilon^{IJ} = 0.$$  \hspace{1cm} (23)

This equation was derived from supersymmetry and therefore the spinor in this equation is anticommuting. However, the identity must hold for commuting spinors as well. Using commuting spinors it is simple to derive the consequences of the Killing Spinor Identities for IWP configurations. Using the algebraic relation (16), which is valid also for commuting Killing spinors, one can derive the following relation between the function $V$ and the bilinear combinations of commuting Killing spinors:

$$\bar{\epsilon}^I \gamma_a \epsilon_I = (2i |V|, \bar{0}), \quad \bar{\epsilon}^I \epsilon_J \epsilon^{IJ} = -2i V.$$  \hspace{1cm} (24)

Now we may consider eq. (23), where the spinor is commuting. We multiply this equation by the commuting spinor $\epsilon_I$, sum over the index $I$, and we get for the IWP metrics

$$J^\mu |V| - \frac{1}{2} J^\mu V = 0,$$  \hspace{1cm} (25)

which implies, for complex $V$, $J^\mu |V| = J^\mu = 0$. We are left with

$$J^\mu \epsilon^I \gamma_\nu = 0.$$  \hspace{1cm} (26)

Now we can multiply this equation by a spinor $\eta_I$ such that $\bar{\epsilon}^I \gamma_\nu \eta_I \equiv p_\nu \neq 0$. This gives

$$J^I p_I = 0,$$  \hspace{1cm} (27)

which means that

$$J^{ij} = (\eta^{ij} - p_i p_j / p^2) f.$$  \hspace{1cm} (28)
Finally, if we multiply eq. (26) by $\gamma^{\nu} \eta^J \epsilon_{IJ}$ and take into account that $J^{\mu\nu}$ is a symmetric tensor, we get

$$J^{\mu\nu} \epsilon^I \gamma^\nu \gamma_{\mu} \eta^J \epsilon_{IJ} = J^{\mu\nu} g_{\mu\nu} \epsilon^I \gamma^\nu \eta^J \epsilon_{IJ} = 0.$$  \hspace{1cm} (29)

Since $\epsilon^I \gamma^\nu \eta^J \epsilon_{IJ} \neq 0$, this implies that $J^{\mu\nu}$. This fact, together with eq. (28) proves that, if $V$ is complex, $J^{\mu\nu} = J^\nu = 0$.

Thus, for configurations with $V \neq \bar{V}$, which in general have naked singularities, the consistency conditions for supersymmetry lead to relations between the energy-momentum tensor and the Maxwell current which include a complex function $(V/\bar{V})^{1/2}$. This is not acceptable, and the consequence is that the right-hand sides of the Einstein and Maxwell equations have to vanish for supersymmetric configurations with complex $V$:

$$J^{\mu\nu} = J^\mu = 0.$$  \hspace{1cm} (30)

In particular we have to require the absence of quantum corrections to the right-hand side of the trace of Einstein equation for supersymmetric configurations with complex $V$:

$$R = g^{\mu\nu} J_{\mu\nu} = 0.$$  \hspace{1cm} (31)

4 The trace anomaly

The trace anomaly (also called Weyl anomaly) in gravitational four-dimensional theories was discovered by Capper and Duff about twenty years ago \cite{12}. The existence of this anomaly means that the conformal invariance under Weyl rescaling of classical gravitational field systems does not survive in the quantum theory.

The trace anomaly of the one-loop on-shell supergravity is given by the following expression \cite{12}:

$$T = g^{\mu\nu} < T_{\mu\nu} > = \frac{A}{32\pi^2} R_{\mu\nu\lambda\delta}^* R^{\mu\nu\lambda\delta}.$$  \hspace{1cm} (32)

The coefficient $A$ is known for all fields interacting with gravity.

The integrated form of the anomaly in Euclidean space expresses the trace of the energy-momentum tensor through the Euler number of the manifold,

$$\int d^4x \sqrt{-g} T = A \chi.$$  \hspace{1cm} (33)

The fields of $N = 2$ supergravity include a graviton, 2 types of gravitino and a vector field. As we see in the table, the anomaly coefficient $A = 11\frac{1}{12}$ of pure $N = 2$ supergravity does not vanish.

\footnote{Observe that purely classical KN configurations have vanishing $J_{\mu\nu}$ and $J^\mu$. Therefore, from the purely classical point of view they are supersymmetric.}
The function $*R_{\mu\nu\lambda\delta} R^{\mu\nu\lambda\delta}$ does not vanish in general for IWP configurations. In particular, one can calculate this function for the charged KN solution and check that for arbitrary angular momentum and charge equal to its mass $m^2 = q^2$ this function does not vanish. One can use for this purpose the values of non-vanishing components of the Weyl tensor $C_{abcd}$ and Maxwell tensor $F_{ab}$ given for this solution in [9] in an isotropic tetrad basis. The expression for the anomaly is given by

$$*R_{\mu\nu\lambda\delta} R^{\mu\nu\lambda\delta} = 24 (\Psi_2 \Psi_2 + h.c.) - 32 (\Phi_1 \bar{\Phi}_1)^2,$$

where

$$\Psi_2 = - \frac{m(r + ia \cos \theta) - q^2}{(r - ia \cos \theta)^3(r + ia \cos \theta)},$$

$$\Phi_1 = \frac{q}{\sqrt{2} (r - ia \cos \theta)^2}.$$

We have checked that the function (34) does not vanish for any KN solution with arbitrary values of $m, q, a$ and in particular for $m = |q|$. As an additional consistency check we have calculated the integrated form of the eq. (34) for Reissner-Nordström black hole with $m \geq |q|, a = 0$. The result is 2, which agrees with the well-known Euler characteristic of the Schwarzschild and Kerr black holes:

$$\chi = \frac{1}{32\pi^2} \int R^{ab} \wedge R^{cd} \epsilon_{abcd} = 2.$$

How does this affect the conclusion of the previous section? Let’s consider now semiclassical configurations of this theory, that is, configurations which satisfy the semiclassical equations of motion obtained by adding first-order quantum corrections to the right-hand side of the classical equations of motion. These semiclassical configurations, then, satisfy the equations (21) where the trace of $J_{\mu\nu}$ is identified with the trace anomaly. This is indeed a very small correction which should not produce big changes in the metric of classical configurations. In particular, it is reasonable to expect that classical configurations with nonvanishing imaginary part of $V$ (as

| Field | $N = 2$ supergravity | $N = 2$ Yang-Mills | $N = 2$ hypermultiplet | $N = 2$ supergravity | $N = 4$ Yang-Mills |
|-------|----------------------|-------------------|----------------------|----------------------|----------------------|
| $e_\mu^a$ | 848 | 1 | 0 | 0 | 1 | 0 |
| $\psi_\mu$ | -233 | 2 | 0 | 0 | 4 | 0 |
| $A_\mu$ | -52 | 1 | 1 | 0 | 6 | 1 |
| $\chi$ | 7 | 0 | 2 | 2 | 4 | 4 |
| $\phi$ | 4 | 0 | 2 | 4 | 1 | 6 |
| $\phi_{\mu\nu}$ | 364 | 0 | 0 | 0 | 1 | 0 |

| A=11/12 | A=-1/12 | A=1/12 | A=0 | A=0 |

Table 1: Anomalies in N=2 and N=4 supermultiplets.
the $m = |q|$ KN configurations) will continue to have a nonvanishing imaginary part of $V$ after the quantum corrections have been taken into account.

The presence of $J_\mu^a \neq 0$ and complex $V$ is incompatible with the supersymmetry consistency conditions. Thus, when we embed the Einstein-Maxwell theory in a supersymmetric theory for which $A \neq 0$ (i.e. the trace anomaly does not vanish) the semiclassical KN configurations (now including those with naked singularities $m = |q|$) are not supersymmetric anymore.

The question arises immediately how to make the anomaly coefficient $A$ to vanish. Looking on the table we may observe that the anomaly vanishes for any theory which is build out of the $N = 4$ multiplet of supergravity and arbitrary number of $N = 4$ Yang-Mills multiplets. If we do not want to increase the number of supersymmetries, we may add to $N = 2$ supergravity $11 + n$ $N = 2$ vector multiplets and $n$ hypermultiplets. The anomaly of such system of fields vanishes. What happens, however, with our naked singularity solutions?

We have found that for all above mentioned theories where the anomaly is cancelled, supersymmetric configurations with naked singularities are not solutions of the classical equations of motion anymore. The simplest explanation of this mechanism can be given for the $N = 4$ theory. We would like to add new fields to the theory in such a way that they propagate in the loop diagrams and cancel the anomaly. Simultaneously we want to make only minimal changes in classical field equations, in order to preserve our previous solutions. This is not possible. Indeed, in the $N = 4$ case there is one new equation for the dilaton field of the form

$$\nabla^2 \phi - \frac{1}{2} e^{-2\phi} F^2 = 0.$$  \hspace{1cm} (37)

The configurations with naked singularities which we have considered before had a constant (space- and time-independent) value of the dilaton field and a non-vanishing value of $F^2$. Thus they do not satisfy equation (37). In other words, by adding new fields which cancel the anomaly, we are adding new equations which are not satisfied by our old solutions with naked singularities. This effect is a consequence of the general structure of the supersymmetric coupling of matter multiplets to vector fields in gravitational multiplets [13]. In particular, the coupling of $N = 2$ matter multiplets (and we need at least 11 vector multiplets to cancel the anomaly) will also result in additional equations of the type (37) which will invalidate the naked singularity solutions.

Thus, we have found that in the theory under consideration there are no stationary supersymmetric solutions with naked singularities. In the case of static solutions studied in [1] this was enough to show that for nonsupersymmetric configurations the singularities are even deeper hidden by the horizon, which means that supersymmetry works as a cosmic censor. It remains to be seen whether an analogous statement is true for general stationary solutions. In any case, the results obtained above confirm that there exists some deep and previously unexplored relation between the absence of naked singularities and supersymmetry.

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