The Non-Minimal Supersymmetric Standard Model with
\[ \tan \beta \simeq \frac{m_t}{m_b} \]

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Abstract
We consider the supersymmetric extension of the standard model with an additional singlet \( S \), the Non-Minimal Supersymmetric Standard Model (NMSSM), in the limit \( \tan \beta \simeq \frac{m_t}{m_b} \). We embed this model in a supergravity framework with universal boundary conditions and analyze the renormalization group improved tree-level potential. We examine the relationship between this model and the minimal supersymmetric standard model (MSSM), and discuss the novel connections between the two when \( \tan \beta \) is large. Strong correlations between the free parameters of the nonminimal model are found and the reasons for these discussed. The singlet vacuum expectation value is forced to be large, of the order of 10 \( TeV \). The radiatively corrected mass of the lightest Higgs boson is found to be \( \lesssim 140 GeV \).

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1. Introduction.

The recent past has witnessed much activity in exploring supersymmetric unification[1]. Furthermore, improvements in the determinations of the standard model couplings have given us reason to believe that supersymmetric unification with a SUSY breaking scale of $\sim 1 \ TeV$ is compatible with these measurements[2]. Other predictions from supersymmetric unification are dogged by the lack of knowledge of the crucial parameter $\tan \beta \equiv v_2/v_1$, the ratio of the vacuum expectation values of the two Higgs doublets $H_2$ and $H_1$ required to give masses to the up-type and the down-type (and charged leptons) quarks, respectively (our normalization is $\sqrt{v_1^2 + v_2^2} = 174 \ GeV$ and the mass of the Z boson is defined such that $m_Z^2 = \frac{1}{2}(g^2 + g'^2)v^2$, where $g$ and $g'$ are the gauge couplings of $SU(2)$ and $U(1)$, respectively).

One particularly predictive framework is based on the assumption that the heaviest generation fermions lie in a unique $16$-dimensional representation of the unifying gauge group $SO(10)$ with the Higgs doublets in a $10$-dimensional representation of the group[3]. This implies that the top-quark, b-quark and $\tau$ lepton Yukawa interactions arise from a $h.16.16.10$ term in the superpotential at the unification scale $M_X$ determined from gauge coupling unification. The coupled system of differential equations for the gauge couplings and Yukawa couplings are then evolved down to present energies from $M_X$, and $\tan \beta$ is determined from the accurately measured value of $m_{\tau} = 1.78 \ GeV$. When $h(= h_t = h_b = h_\tau)$ is chosen in such a manner as to yield a value for $m_b(m_b)$ in its “observed” range of $4.25 \pm 0.01 \ GeV$[4], a rather good prediction for the top-quark mass parameter $m_t(m_t)$ is obtained, which with the present central value of $\alpha_S(M_Z) = 0.12$ lies in the range favoured by the experimental data[5]. Here $\tan \beta$ is found to saturate what is considered to be a theoretical
upper bound on its value of $m_t/m_b$ and the Yukawa coupling $h$ is found to come out to be rather large $O(1-3)$ with a certain insensitivity to the exact value since it is near a fixed point of its evolution.

In $SU(5)$ type unification where $\tan \beta$ is free, the region $\tan \beta \approx 1$ is also a region which is favoured for the unification of the b-quark and $\tau$-lepton masses from the observed data[6]. One crucial difference between the two extremes discussed above is that in the $SO(10)$ case the Yukawa couplings of the b-quark (and that of the $\tau$ lepton) always remain comparable to that of the top-quark, with the observed hierarchy in the masses of these quarks arising from the large value of $\tan \beta$, while in the $SU(5)$ case the Yukawa couplings of the b-quark and the $\tau$-lepton are negligible in comparison with that of the top-quark.

The above discussion about unification does not involve in any great detail the remaining aspects of the embedding of the standard model into a supersymmetric grand unified framework. The minimal supersymmetric extension of the standard model requires, besides the superpartners, the introduction of an additional Higgs doublet, and indeed with this matter content and an additional symmetry known as matter parity [1], to forbid couplings that lead to rapid nucleon decay, it is possible to construct a self-consistent and highly successful framework which has come to be known as the Minimal Supersymmetric Standard Model (MSSM)[1]. Here the mass of the lightest Higgs boson, which is unknown in the Standard Model, is related, through the D-term in the potential, to the mass of the Z boson, $m_Z$. It is also related to $\tan \beta$ and $m_A (> > m_z$ as required by $b \rightarrow s\gamma$ constraints), the mass of the CP-odd neutral scalar boson that remains as a physical degree of freedom after the breakdown of $SU(2) \times U(1)$. It has been shown that the mass of the lightest ("Weinberg-Salam") Higgs $m_{h_0}$ in the MSSM for large values of $\tan \beta$, after
inclusion of radiative corrections due to the presence of large top Yukawa coupling, is \( \lesssim 140 \text{ GeV} \)[7]. Furthermore, supersymmetry breaking is understood to arise from embedding MSSM into a supergravity framework and writing down all the possible soft-supersymmetry breaking terms consistent with the gauge and discrete symmetries that define the model. It is often assumed that many of the parameters describing these terms are in fact equal at the unification scale in order to have a predictive framework. Such universal conditions have had to be relaxed in order to avoid fine tuning and minimize the effects of possible radiative corrections to the b-quark mass due to the fine details of the spectrum of this model, especially in the case when \( \tan \beta \) is large[7,8].

Despite its many successes it may be premature to confine our attention only to the MSSM, especially because of the presence of the dimensionful Higgs bilinear parameter \( \mu \) in the superpotential. An alternative model to MSSM that has widely been considered is the one where the Higgs content is extended (economically) by the addition of a gauge singlet \( S \), and assuming a discrete \( Z_3 \) symmetry in order to avoid linear and bi-linear couplings in the superpotential[9], the so called Non-Minimal Supersymmetric Standard Model (NMSSM). This model is also referred to as the next to minimal supersymmetric model or as (M+1)SSM. In particular, this corresponds to forbidding the \( \mu H_1 H_2 \) coupling of the MSSM in the superpotential and instead introduce the couplings

\[
\lambda S H_1 H_2 + \frac{1}{3} k S^3
\]  

with the effective \( \mu \) term generated by the vacuum expectation value \( \langle S \rangle \) \((\equiv s) \neq 0\). This model is particularly interesting since it does not affect the positive features of the MSSM including gauge coupling unification [10] and allows a test of the stability of the features of the MSSM such as the upper bound on the mass
of the lightest Higgs boson with favourable results. It has a significantly richer phenomenology and a typically larger parameter space[11,12,13].

However, in recent studies of this model with high energy inputs[11,14], the interesting case of large \( \tan \beta \) has not been considered. In this paper we discuss this possibility: that of the non-minimal supersymmetric standard model with large \( \tan \beta \). We note that there is a distinct possibility that the desirable features of a good prediction for the top-quark mass may in fact be destroyed by the presence of additional Yukawa couplings \( \lambda \) and \( k \) which couple to the Yukawa couplings of the heaviest generation even at one-loop level, but in actual practice has been found not to be the case[15].

We shall discuss different features of the non-minimal supersymmetric standard model at large values of \( \tan \beta \), and compare and contrast these, as often as possible, with the corresponding features in MSSM. We shall also discuss under what conditions one can obtain the latter as a well-defined limit of the former. We have carried out a renormalization group analysis of this model with universal boundary conditions and analyzed the renormalization group improved tree-level potential at the scale \( Q_0 \). The cut off scale for the renormalization group evolution is chosen to be the geometric mean of the scalar top quark masses which is roughly equal to that of the geometric mean of the scalar b-quark masses as well, since during the course of their evolution the Yukawa couplings of the t and b-quarks are equal up to their hypercharges and the relatively minor contribution of the \( \tau \)-lepton. Whereas in the MSSM the parameters \( \mu \) and \( B \) (the soft susy parameter characterizing the bilinear term in the scalar potential) do not enter into the evolution of the other parameters of the model at one-loop level, the situation encountered here is drastically different with a systematic search in the parameter space having to be performed.
with all parameters coupled from the outset. Our analysis of the minimization conditions that ensure a vacuum gives rise to severe fine tuning problems, that persist in this model, as in the MSSM. The problems are further compounded by having the constraints of three minimization conditions, rather than two such conditions that occur in MSSM. In previous studies of the model where tan $\beta$ was free, the tuning of parameters was possible in order to meet all the requisite criteria, viz., minimization conditions, requirement that the vacuum preserve electric charge and colour, etc. However, in the present where tan $\beta$ is fixed and large, what we find is a highly correlated system. An important conclusion that we draw here is that just as in the case of MSSM, for large values of tan $\beta$, the upper bound on the mass of the lightest Higgs boson is $\lesssim$ 140 GeV.

2. The Model

The model is characterized by the following couplings in the superpotential, where we exhibit the interactions of the heaviest generation and the Higgs (singlet and doublet) sector of the theory:

$$W = h_t Q \cdot H_2 t_R^c + h_d Q \cdot H_1 b_R^c + h_r L \cdot H_1 \tau_R^c + \lambda S H_1 H_2 + \frac{1}{3} k S^3 \tag{2}$$

One has to add to the potential obtained from (2) the most general terms that break supersymmetry softly. We explicitly write down these soft breaking terms here, since it will establish our sign conventions for the relevant parameters. The potential can be computed from (2) by a standard procedure[11]. The relevant soft susy breaking terms are:

$$(h_t A_t Q \cdot H_2 t_R^c + h_d \tilde{Q} H_1 \tilde{b}_R + h_r A_r \tilde{L} H_1 \tilde{\tau}_R + \lambda A H_1 H_2 S + \frac{1}{3} k A S^3) + \text{h.c.}$$

$$+ m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 + m_S^2 |S|^2 + m_Q^2 |\tilde{Q}|^2 + m_{\tilde{t}}^2 |\tilde{t}_R|^2 + m_{\tilde{b}}^2 |\tilde{b}_R|^2 + m_{\tilde{\tau}}^2 |\tilde{\tau}_R|^2.$$
The conventions for the gaugino masses follow those of the MSSM\[16\]. Since part of the discussion that follows rests on the minimization conditions (evaluated at $Q_0$ after all the parameters are evolved via their one-loop renormalization group equations down to this scale) we give them here:

\begin{align*}
m_{H_1}^2 &= -\lambda \frac{v_2}{v_1} s(A_\lambda + ks) - \lambda^2 (v_2^2 + s^2) + \frac{1}{4} (g_2^2 + g'_2) (v_2^2 - v_1^2) \tag{3} \\
m_{H_2}^2 &= -\lambda \frac{v_1}{v_2} s(A_\lambda + ks) - \lambda^2 (v_1^2 + s^2) + \frac{1}{4} (g_2^2 + g'_2) (v_1^2 - v_2^2) \tag{4} \\
m_S^2 &= -\lambda^2 (v_1^2 + v_2^2) - 2k^2 s^2 - 2\lambda s v_1 v_2 - k A_ks - \frac{\lambda A_\lambda v_1 v_2}{s} \tag{5}
\end{align*}

One may rewrite the first two minimization equations to obtain:

\begin{align*}
tan^2 \beta &= \frac{m_{Z/2}^2 + m_{H_1}^2 + \lambda^2 s^2}{m_{Z/2}^2 + m_{H_2}^2 + \lambda^2 s^2} \tag{6} \\
\sin 2\beta &= \frac{(-2\lambda s)(A_\lambda + ks)}{m_{H_1}^2 + m_{H_2}^2 + \lambda^2 (2s^2 + v^2)} \tag{7}
\end{align*}

Equations (6) and (7) give us some crucial insights into the manner in which our solutions may behave like. The first of these guarantees that, as in the MSSM, $\tan \beta$ must lie between 1 and $m_t/m_b$. The proof of this relies once more on the renormalization group equations that govern the behaviour of the mass parameters and may be proved very simply by reductio ad absurdum. For this purpose we need only consider the following equation expressing the momentum dependence of the difference of two supersymmetry breaking scalar mass parameters:

\begin{align*}
\frac{d}{dt}(m_{H_1}^2 - m_{H_2}^2) = \frac{1}{8\pi^2} (-3h_t^2 X_t + 3h_b^2 X_b + h_\tau^2 X_\tau) \tag{8}
\end{align*}

where $t = \log(\mu)$, the logarithm of the momentum scale, and $X_i$, $i = t, b, \tau$, are combinations of scalar masses and tri-linear couplings:

\begin{align*}
X_t &= m_Q^2 + m_t^2 + m_{H_2}^2 + A_t^2 \\
X_b &= m_Q^2 + m_b^2 + m_{H_1}^2 + A_b^2 \\
X_\tau &= m_{\bar{L}}^2 + m_{\bar{\tau}}^2 + m_{H_1}^2 + A_\tau^2
\end{align*}
It must be noted that in order to prove that $\tan \beta > 1$ we neglect $h_b$ and $h_\tau$ and for proving $\tan \beta < m_t/m_b$ we retain them.

The $\tan^2 \beta$ equation (6) shows that, as in the MSSM, in order to guarantee a large $\tan \beta$, with the essential degeneracy of $m_{H_1}^2$ and $m_{H_2}^2$ enforced by the renormalization group equations, the denominator of the equation has to come out to be small at low scale. Therefore, we have the fine-tuning condition that

$$m_{H_2}^2 + \lambda^2 s^2 \approx -m_Z^2/2$$

(9)

Here one notes that the correspondence with the MSSM will occur in a certain well defined manner with the identification of $\lambda s$ with $\mu$. Similarly one has to identify $A_\lambda + k s$ with $B$. What we will show below is that in the large $\tan \beta$ case this identification occurs in a novel way that is not generic to the model, say, in the limit of $\tan \beta \approx 1$.

An investigation of the $\sin 2\beta$ equation yields, when $\beta \approx \pi/2$, as in the case at hand, that one must have the condition

$$A_\lambda \approx -k s$$

(10)

This is similar to the condition in the MSSM that $B \approx 0$. Here the situation is far worse since $A_\lambda$ is not a parameter that is fixed at $Q_0$ but is present from the outset. This is the first of the fine tuning problems that we encounter.

A rearrangement of the $\sin 2\beta$ equation yields:

$$\lambda s (A_\lambda + k s) = \tan \beta (-m_{H_2}^2 - \lambda^2 s^2) \frac{m_Z^2}{2} \left( \frac{\tan \beta^2 - 1}{\tan \beta^2 + 1} \right) - \frac{\lambda^2 v^2 \sin 2\beta}{2}$$

(11)

In this equation it is legitimate to discard the last term for the case of large $\tan \beta$ and one sees here that with the identification of the appropriate parameters in terms of the MSSM parameters as described earlier, one recovers all the analogous
MSSM relations for all values of the other parameters without having to go through a limiting procedure\cite{11}, as is the case when $\tan \beta$ is arbitrary.

The next fine tuning condition we encounter is related to the third minimization condition which we rewrite as:

$$m_S^2 = -\lambda^2 v^2 - 2A_\lambda^2 + \frac{\lambda v^2 \sin 2\beta A_\lambda}{s} + A_\lambda A_k + \frac{\lambda k v^2 \sin 2\beta}{2} \quad (12)$$

Here one may observe that in order to satisfy this condition one must have large cancellations between the fourth and the first two terms since the terms proportional to $\sin 2\beta$ are negligible. This requires that $A_k$ and $A_\lambda$ come out with the same sign and that the product be sufficiently large. As we shall see, it is this condition that leads to problems with finding solutions with sufficiently small tri-linear couplings in magnitude.

3. Results and Conclusions

The starting point of the program is the estimation of the scale $M_X$ with the choice of the SUSY breaking scale $Q_0 \sim 1 \text{ TeV}$. For $\alpha_S(m_Z) = 0.12$, $Q_0 = 1 \text{ TeV}$ and $\alpha = 1/128$, we find upon integrating the one-loop beta functions, $M_X = 1.9 \times 10^{16} \text{ GeV}$ and the unified gauge coupling $\alpha_G(M_X) = 1/25.6$. We then choose a value for the unified Yukawa coupling $h$ of $O(1)$. The free parameters of the model are $(M_{1/2}, m_0, A, \lambda, k)$, which are the common gaugino mass, the common scalar mass, the common tri-linear scalar coupling and the two additional Yukawa couplings respectively. Note that our convention requires us to choose $\lambda > 0$ and $k < 0$ in order to conserve CP in the Yukawa sector of the model\cite{11}.

We then write down the coupled system of renormalization group (RG) equations for the 24 parameters of the model that are coupled to each other (ignoring the parameters of the lighter two generations since they do not couple to the rest of the
parameters at the one-loop level) and evolve this system down to present energies of $Q_0$. These RG equations may be obtained by generalizing the expressions of Ref. [9] to include the contributions of $h_b$ and $h_r$ and can derived from the general expressions of Ref. [17]. We then compare the numerical values of the mass parameters that enter the left hand side of the minimization equations (3) - (5) with the combination of the parameters that enter the right hand side of these equations as obtained from the RG evolution.

In practice, it turns out that the first of the minimization conditions, eq.(3), is the most sensitive to the choice of initial conditions. This reflects the fine tuning condition that we dwelt on in the previous section. We also impose the constraint that $|A| < 3m_0$[18] in order to guarantee the absence of electric-charge breaking vacua. In the case at hand this choice may have to be strengthened further due to the presence of large Yukawa couplings for the b-quark. The situation is considerably less restrictive when mild non-universality is allowed and, for instance, if strict Yukawa unification is relaxed. Given these uncertainties, we choose to work with this constraint.

In Table 1 we present various sets of values of the input parameters that we take and the corresponding output. Our choices are arranged in such a manner so as to show the change in some of the crucial features as we vary certain input parameters, some of which are changed as we go down the columns. Besides the input values we present in Table 1 (a) certain crucial quantities that are calculated from the renormalization group evolution at $Q_0$. These are $\tan \beta$, $r(\equiv s/v)$, $A_\lambda$, $A_k$ and $k_s$. In Table 1 (b), we present the values of $r_1$, $r_2$ and $r_3$ which are defined as the difference between the left and right hand sides of the three minimization equations (3), (4) and (5) divided by the right hand side of each of these equations. A genuine
vacuum corresponds to all three being equal to 0. However, given the extremely fine tuned nature of these conditions, we present points in the parameter space where their variations are most easy to observe. The choices of interest are precisely those where these suffer a change in sign as one of the parameters is changed indicating that such points lie in the neighbourhood of the desired vacuum. In Table 1(b) we then present the corresponding values of $\Delta E$, the difference in the value of the scalar potential computed with the scalar fields attaining their vacuum expectation values $(v_1, v_2$ and $s)$ and its value computed at the origin. A negative $\Delta E$ signifies that the $SU(2) \times U(1)$ breaking vacuum has lower potential energy than the symmetric state $v_1 = v_2 = s = 0$ (since this state is normalized to have a vanishing potential energy), and is favoured. We also compute the mass of the charged Higgs boson

$$m_C^2 = m_W^2 - \lambda^2 v^2 - \lambda(A_\lambda + ks)\frac{2s}{\sin 2\beta}$$

where $m_W$ is the mass of the W-boson. We note that the radiative corrections to the charged Higgs mass are small for most of the parameter range, as in the case of MSSM[19]. The reason for this is that a global $SU(2) \times SU(2)$ symmetry [20] protects the charged Higgs mass from obtaining large radiative corrections. If this quantity were to come out to be negative, then the resulting vacuum would break electric-charge spontaneously and the corresponding point in the parameter space would be excluded. We also compute the squared masses of the neutral pseudoscalar bosons which are assured to be positive[11] for a true vacuum. However, we find that due to the fine-tuning conditions, as we move away from the region where a vacuum is to be found, the smaller eigenvalue of the mass squared matrix of pseudoscalar Higgs bosons in fact changes sign.

We finally present in the last column of the table the quantity that is of the greatest importance, viz., the upper bound (denoted by $M_{h^0}$) on the mass of the
lightest Higgs boson in the model, $m_{h^0}$. Including radiative corrections, this upper bound can be written as [21]:

$$m_{h^0}^2 \lesssim m_Z^2 (\cos^2 2\beta + \frac{2\lambda^2}{g^2} \sin^2 2\beta)$$

$$+ \frac{3g^2}{16\pi^2 m_W^2} (\Delta_{11} \cos^2 \beta + \Delta_{22} \sin^2 \beta + \Delta_{12} \sin 2\beta)$$ (14)

where the $\Delta_{ij}$, $i,j = 1,2$ are the terms that arise from the one-loop radiative corrections and involve $m_t^4/m_W^2$, $m_b^4/m_W^2$ and logarithms of the squark masses. For each of our inputs, we compute the masses of the physical squark eigenstates in order to estimate the upper bound (14). We note that in the limit $\beta \approx \frac{\pi}{2}$, the second term in the first bracket, which is proportional to $\sin^2 2\beta$, is small, so that the upper bound (14) reduces to the corresponding bound on the lightest Higgs mass in the MSSM when appropriate identification of parameters is made. We further note that the bound (14) depends only logarithmically on $r$, and hence on the singlet vacuum expectation value $s$, in the limit of large $r$, which, therefore, decouples from the bound [22].

We broadly discuss the solutions that we find by first noting that the solutions are easier found with $\lambda$ and $k$ chosen to be smaller than 1. Larger values increase their importance in the renormalization group equations of the soft parameters. There is a rough scale invariance enjoyed by the solutions when each of these is scaled by the same parameter. The same is also true of scaling $M_{1/2}$, $m_0$ and $A$. Furthermore if $M_{1/2}$ is taken to be negative and of the same order as $m_0$ and $A = 3m_0$, there is only a weak dependence on the actual value of the parameter. Note that in MSSM $m_0$ is required to be smaller than $M_{1/2}$ due to the correlations between these parameters and the mass of the only pseudoscalar Higgs boson in the spectrum.

For the first 11 rows in Table 1(a) we have taken a unified Yukawa coupling $h = 1.5$ which is a typical value for this parameter that yields a successful prediction
for the top-quark mass [3,8] (when $m_b$ is evolved using two-loop QCD evolution equations this yields $m_b(m_b) = 4.09$ GeV and $m_t(m_t) = 181$ GeV, ignoring small corrections due to the presence of the additional Yukawa couplings $\lambda$ and $k$[15]). This is changed to a somewhat smaller value of 1.0 for the next 4 rows (corresponding to $m_b(m_b) = 4.28$ GeV and $m_t(m_t) = 177$ GeV) and the last four rows to $h = 2.0$ (corresponding to $m_b(m_b) = 4.00$ GeV and $m_t(m_t) = 183$ GeV). The value of $\lambda$ alone is changed as we move down from the 1st to the 3rd rows. The desired features that the $r_i$ ($i = 1, 2, 3$) suffer change of signs are seen. Note that $\Delta E$ and the $m_C^2$ also change sign. This represents the presence of an instability in the vacuum and with the present accuracy it is not possible to ascertain whether any genuine vacuum can be found in this neighbourhood. (A related observation that we make is that in this region the mass squared of the lighter pseudoscalar neutral boson is also found to suffer a change of sign, from positive to negative, as the parameter $\lambda$ is increased, as would be expected due to the fine tuning.) The rows 4 - 6 are similar to 1 - 3 with a smaller value of $|M_{1/2}|$. The qualitative features described for the first three rows persist here and illustrate the weak dependence on the actual value of this parameter.

In rows 7 and 8 we show the changes that occur as $\lambda$ is increased with a fixed ratio $|A/m_0| = 2$, which is a factor of $2/3$ smaller than for the first 6 rows. It is seen that $r_3$ never approaches the neighbourhood of 0 although $r_1$ does suffer a sign change. This illustrates the need for a large value for this ratio of nearly 3. This feature is also observed in the case of small $\tan \beta$[14]. In rows 9 and 10, we scale down the values of the dimensional parameters compared to, say, rows 2 and 3 and varying $\lambda$ in a range that overlaps that of these rows. Similar qualitative features are found to persist showing the rough scale invariance of the system described earlier. Similarly
in row 11, we scale down $\lambda$ and $|k|$ simultaneously by a factor of 10 compared to row 9 so as to demonstrate the relative independence from the absolute values of these parameters. Note that the computed value of $r$ scales inversely by roughly the same factor.

The next four rows 12-15 show the behaviour of our solutions for a somewhat smaller value of $h$ and, qualitatively, the features are no different. The possible solutions now require a smaller value of the ratio $\lambda/|k|$. And, finally the last four rows 16-19 show the behaviour for the largest $h$ that we have chosen for the purposes of illustration.

A point that deserves emphasis is the fact that the quantity $r$ persistently remains rather large corresponding to the vacuum expectation value of the singlet, $s$, being about 10 $TeV$. This is substantially different from the case of $\tan \beta \approx 1[11]$. This implies that for large $\tan \beta$, the vacuum expectation value of the Higgs singlet is forced to be rather large. We note that the singlet vacuum expectation values are not constrained by the experimental data.

As a final, and the most significant, point we go to the last column of Table 1(b) wherein the absolute upper bound $M_{h_0}$ on the lightest Higgs mass is presented for the choice of input parameters of Table 1(a). In Fig. 1 we plot, for typical and reasonable values of the input parameters in the region where the vacuum is expected to lie, the upper bound on the mass of the lightest Higgs boson as a function of the top quark mass $m_t$ in the range that is most favoured under these boundary conditions[3,7,8,15]. Such a linear behaviour has also been observed in the case of the MSSM[23]. It is seen that the upper bound never exceeds 140 $GeV$. We, therefore, conclude that as in the MSSM, even with values of $h$ close to the perturbative bound of 3.3, this bound is not likely to exceed 140 $GeV$. It must be
noted that in MSSM due to the presence of of fewer degrees of freedom at the outset and only two minimization conditions and only one pseudoscalar Higgs boson in the spectrum[7], the constraining of the mass of the lightest higgs scalar is relatively easier.

From the detailed discussion presented above, we see that the non-minimal supersymmetric standard model rests on a rather delicately hinged system of equations and constraints. It has been argued, and found true[23, 24], that the minimization of the tree-level potential in the MSSM is consistent with minimizing the one-loop corrected potential to within $\sim 20\%$ when the cutoff is chosen in the range of the geometric mean of the scalar top-quark masses[18], and the situation in the nonminimal model is expected to be no different. Whereas the non-minimal model provides a good testing ground for the stability of predictions of the MSSM, in practice it deserves great care in its treatment. For instance, whereas in the MSSM the mass of the only pseudoscalar Higgs boson plays an important role in constraining the allowed regions of the parameter space[7], the situation here is significantly more complicated and no simple constraint emerges from our analysis. It appears to us, through the above careful analysis of stability and fine tuning in the nonminimal model, that the framework may have to be broadened via non-universality of soft breaking terms as has been done recently in the case of MSSM. It is hoped that the present work might serve as a springboard for such investigations.

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Table Caption

Table 1 (a). Typical values for inputs $M_{1/2}$, $m_0$, $A$, $h$, $\lambda$, $k$ at $M_X$ and the corresponding low energy values of $\tan\beta$, $r$, $A_\lambda$, $A_k$ and $ks$ (all masses in units of GeV.).

Table 1 (b). The values of $r_1$, $r_2$, $r_3$, $\Delta E$ and $m^2_{C}$ and the upper limit on the mass of the lightest higgs $M_{h^0}$ for the inputs of Table 1a (all masses in units of GeV.).

Figure Caption

Fig. 1. Plot of upper bound on the mass of the lightest higgs, $M_{h^0}$ vs. $m_t(m_t)$ for a typical choice of parameters, $M_{1/2} = -700 \; GeV$, $m_0 = 800 \; GeV$, $A = 2400 \; GeV$ and $\lambda/|k|$ chosen to yield the neighbourhood of a vacuum.
| #  | $M_{1/2}$ | $m_0$ | $A$  | $h$  | $\lambda$ | $k$     | $\tan \beta$ | $r$  | $A_{\lambda}$ | $A_k$ | $k_s$   |
|----|----------|-------|------|------|---------|--------|-------------|-----|------------|-------|---------|
| 1  | -700     | 800   | 2400 | 1.5  | 0.35    | -0.10  | 62          | 50  | 732        | 2257  | -828    |
| 2  | -700     | 800   | 2400 | 1.5  | 0.40    | -0.10  | 62          | 44  | 718        | 2231  | -721    |
| 3  | -700     | 800   | 2400 | 1.5  | 0.45    | -0.10  | 62          | 40  | 703        | 2203  | -637    |
| 4  | -500     | 800   | 2400 | 1.5  | 0.35    | -0.10  | 62          | 40  | 577        | 2258  | -658    |
| 5  | -500     | 800   | 2400 | 1.5  | 0.40    | -0.10  | 62          | 35  | 563        | 2233  | -573    |
| 6  | -500     | 800   | 2400 | 1.5  | 0.45    | -0.10  | 62          | 32  | 548        | 2205  | -507    |
| 7  | -700     | 800   | 1600 | 1.5  | 0.40    | -0.10  | 62          | 44  | 659        | 1485  | -708    |
| 8  | -700     | 800   | 1600 | 1.5  | 0.50    | -0.10  | 62          | 36  | 637        | 1445  | -560    |
| 9  | -350     | 400   | 1200 | 1.5  | 0.40    | -0.10  | 62          | 22  | 359        | 1115  | -358    |
| 10 | -350     | 400   | 1200 | 1.5  | 0.50    | -0.10  | 62          | 18  | 343        | 1086  | -283    |
| 11 | -350     | 400   | 1200 | 1.5  | 0.04    | -0.01  | 62          | 212 | 399        | 1199  | -368    |
| 12 | -700     | 800   | 2400 | 1.0  | 0.20    | -0.10  | 58          | 59  | 835        | 2291  | -986    |
| 13 | -700     | 800   | 2400 | 1.0  | 0.30    | -0.10  | 58          | 40  | 799        | 2226  | -652    |
| 14 | -700     | 800   | 1600 | 1.0  | 0.20    | -0.10  | 58          | 57  | 722        | 1526  | -952    |
| 15 | -700     | 800   | 1600 | 1.0  | 0.30    | -0.10  | 58          | 39  | 697        | 1483  | -629    |
| 16 | -700     | 800   | 2400 | 2.0  | 0.50    | -0.10  | 64          | 47  | 685        | 2235  | -757    |
| 17 | -700     | 800   | 2400 | 2.0  | 0.60    | -0.10  | 64          | 40  | 661        | 2191  | -624    |
| 18 | -700     | 800   | 1600 | 2.0  | 0.50    | -0.10  | 64          | 46  | 643        | 1487  | -749    |
| 19 | -700     | 800   | 1600 | 2.0  | 0.60    | -0.10  | 64          | 39  | 626        | 1457  | -617    |

Table 1 (a)
| #  | $r_1$      | $r_2$       | $r_3$ | $\Delta E (\cdot 10^{11})$ | $m_C^2 (\cdot 10^7)$ | $M_{\nu}$ |
|----|------------|-------------|-------|-----------------------------|----------------------|-----------|
| 1  | 5.6        | $1.6 \cdot 10^{-3}$ | 0.1   | -5.5                        | 0.59                 | 132       |
| 2  | 0.3        | $7.8 \cdot 10^{-5}$  | -4.2  | $10^{-2}$                   | -4.2                 | 0.02      | 132       |
| 3  | -3.6       | $-1.0 \cdot 10^{-3}$ | -0.1  | 3.4                         | -0.39                | 132       |
| 4  | 5.5        | $1.7 \cdot 10^{-3}$  | -0.1  | -2.4                        | 0.39                 | 128       |
| 5  | 0.8        | $2.6 \cdot 10^{-4}$  | -0.1  | 7.4                         | 0.05                 | 128       |
| 6  | -2.5       | $-7.4 \cdot 10^{-4}$ | -0.1  | 14.3                        | -0.20                | 128       |
| 7  | 3.0        | $8.3 \cdot 10^{-4}$  | 0.9   | 216.0                       | 0.29                 | 132       |
| 8  | -4.4       | $-1.3 \cdot 10^{-3}$ | 0.7   | 128.0                       | -0.46                | 132       |
| 9  | $3.2 \cdot 10^{-4}$ | $9.3 \cdot 10^{-6}$ | -4.5  | $10^{-2}$                   | -0.25                | -0.0016   | 122       |
| 10 | -6.6       | $-1.89 \cdot 10^{-3}$ | -0.11 | 0.7                         | -0.18                | 122       |
| 11 | -3.3       | $-9.6 \cdot 10^{-4}$ | -6.6  | $10^{-2}$                   | 12.8                 | -0.09     | 122       |
| 12 | 8.1        | $2.8 \cdot 10^{-3}$  | 0.44  | 26.6                        | 0.83                 | 128       |
| 13 | -7.7       | $-2.6 \cdot 10^{-3}$ | -0.12 | -4.95                       | -0.81                | 128       |
| 14 | 13.2       | $4.3 \cdot 10^{-3}$  | 1.6   | 510                         | 1.23                 | 128       |
| 15 | -3.8       | $-1.2 \cdot 10^{-4}$ | 0.7   | 153                         | -0.36                | 128       |
| 16 | 4.3        | $1.13 \cdot 10^{-3}$ | 1.4   | $10^{-2}$                   | -1.35                | 0.46      | 134       |
| 17 | -2.09      | $-5.53 \cdot 10^{-4}$ | -8.24 | $10^{-2}$                   | 9.43                 | -0.23     | 134       |
| 18 | 6.43       | $1.67 \cdot 10^{-3}$ | 1.01  | 259                         | 0.67                 | 134       |
| 19 | -0.49      | $-1.27 \cdot 10^{-4}$ | 0.76  | 164                         | $-5.7 \cdot 10^{-2}$ | 134       |

Table 1 (b)
Fig. 1

$M_{H^0}$ [GeV] vs. $m_t(m_t)$ [GeV]

Fig. 1