A Plan to Reclaim Deuterons Escaping from the Loss Cone of a Magnetic Mirror

Mitsuaki Nagata

Soft Creator Company, Kyoto, Japan
Email: nagata@heian-kogyo.jp

Abstract

Research on magnetic mirror reactors has had two serious problems since the beginning stage. One is the magnetohydrodynamic instability due to the negative curvature of the magnetic field lines around the center region of a mirror bottle. Another is the loss of charged particles escaping from the loss cone of a magnetic mirror. We have continued to inquire into a means to solve the latter problem. We here propose a new way which will be able to make a magnitude of a loss angle of a magnetic mirror for deuterons virtually zero.

Keywords
Magnetic Mirror, Control of Escaping Deuterons, Plasma Heating by an Extraordinary Wave

1. Introduction

It would seem that researches for a magnetic mirror fusion reactor are far behind in comparison with ones for Tokamak. The cause may be that a magnetic mirror field has configuration being open-ended [1] [2]. However, recently, a new magnetic mirror reactor scheme [3] was proposed. The scheme tries to solve the problem of the negative curvature by making a mirror length very long so that the region with the negative curvature may nearly vanish. Also, it uses helical winding coils at both ends of the mirror bottle. The helical windings resemble those in Stellarator and will be stronger than a magnetic field of a solenoid, with respect to the magnetohydrodynamic instability due to gravity or charge-separation. However, the helical windings do not intend to suppress the number of escaping charged particles as much as possible. We previously reported a plan [4] in which a supplemental magnetic mirror (called SMM) is
connected to the exit of a magnetic mirror bottle. SMM had the spaces for heating charged particles by cyclotron resonance waves within. The main results are as follows: 1) The magnitude of the loss angle decreased from $14.5^\circ$ (in the exit of the bottle) to about $5^\circ$ (in the exit of SMM) by accelerating velocity components perpendicular to the magnetic field. 2) However, nonrelativistic particles (deuterons) had inherent disadvantage that the acceleration invites deuterons from being outside the loss cone into inside the loss cone. Relativistic particles (electrons) were not related to such an effect. 3) Heavy deuterons required a very long flight length and a very powerful electric field in order for those to get a necessary velocity by the acceleration.

Also, we found a mistake (deuterons are not heated, in a high density plasma, by an extraordinary wave with an ion cyclotron frequency). It may be impossible to make a loss angle for deuterons zero by relying only on acceleration with electric waves. In this work, together with re-consideration of the plasma heating by an extraordinary wave [5] [6] [7], we inquire into a new way of reflecting a deuteron having a very fast velocity by a constant electric field which is not extremely large.

2. A New Plan to Reclaim Escaping Deuterons

We assume that the most part of the bottle is filled with such an ideal plasma consisting of electrons and deuterons as shown below:

- Electron density $n_e = $ deuteron density $n_i = 10^{21}$ m$^{-3}$.
- Plasma temperature $T = 4 \times 10^4$ K.

Then, the most probable thermal velocity $v_{im}$ of deuterons is

$$ v_{im} = \left( \frac{2k_B T}{m_i} \right)^{1/2} = 1.8 \times 10^6 \text{ m/sec} $$

$k_B = 1.38 \times 10^{-23}$ J/K, $m_i$ is a deuteron mass 3680$m_e$ where $m_e$ is the rest mass of an electron $9.1 \times 10^{-31}$ kg.

The mean thermal velocity $\bar{v}_i$ of deuterons is

$$ \bar{v}_i = \left( \frac{8k_B T}{\pi m_i} \right)^{1/2} = 2 \times 10^6 \text{ m/sec}. $$

The mean thermal velocity $\bar{v}_e$ of electrons

$$ \bar{v}_e = \left( \frac{8k_B T}{\pi m_e} \right)^{1/2} = 1.2 \times 10^8 \text{ m/sec}. $$

We show in Figure 1 a plan to reclaim deuterons escaping from a magnetic mirror bottle. In the exit in the left-hand side of the bottle, Apparatuses (A) and (B) (called App (A), App (B)) are installed. Each of those has a square cross section. App (B) is a rolled one of App (A) by 90 degree. App (A) is divided into an upper space and an under space, and in each space the following electric field is imposed:
Figure 1. A schematic diagram of an apparatus to reclaim escaping deuterons. Extraordinary waves with $\omega_2$ (in Equation (10)) and $\omega_1$ (in Equation (9)) are sent to Apparatus (A), Apparatus (B), respectively.

\[
\begin{align*}
\begin{cases}
-\dot{y}E_y + \dot{z}E_z & \text{V/m in the upper space,} \\
-\dot{y}E_y - \dot{z}E_z & \text{V/m in the under space.}
\end{cases}
\end{align*}
\]

The left-hand side of App (B) is connected with a system (corresponding to App (B)) in the right-hand side of the bottle by a solenoid. Coils, Solenoid, App (A) and App (B) must be cooled against collisions with high energy particles.

Since heavy deuterons take electrons together through the Coulomb forces, we roughly regard that, in a steady state, gases within App (A) and App (B) are plasmas. A plasma pressure $P_p$ corresponding to $n_i = n_e = 10^{21}$ m$^{-3}$ in the bottle is

\[
P_p = (n_i + n_e) k_b T = 1.1 \times 10^7 \text{ N/m}^2 = 110 \text{ atmospheric pressure.} \tag{1}
\]

A necessary magnetic pressure $B^2/2\mu$ N/m$^2$ (where, $B$ is a magnetic field and $\mu$ is the permeability of vacuum $4\pi \times 10^{-7}$ mT/A) to stand against $P_p$ is given by

\[
P_p = \frac{B^2}{2\mu} = 1.1 \times 10^7 \text{ N/m}^2 \text{ or } B = 5.26 \text{ T} \tag{2}
\]

So, we assume that a magnetic field in the central part of the bottle is 6 T, also that a magnetic field $B$ “in the exit (plane (a)) of the bottle and accordingly within App (A) and App (B)” is $6 \times 4 \times 4 \text{ T}$. Though a velocity distribution (with respect to the y-components) in plane (a) is unclear, here we consider sending back deuterons with the y-components (denoted by $\nu_y$) of velocities between 0
and \( 2
\nu_{im} \) to plane (a).

Let us aim a deuteron (called D$^+$ ion) starting from plane (a) with \( \nu_0 \) given by \( 2
\nu_{im} = 2 (2k_B T/m_i)^{1/2} \equiv \nu_{im} \) at time \( t = 0 \). D$^+$ ion is a nonrelativistic particle. The velocity \( \nu_y \) of D$^+$ ion decreases according to

\[
\nu_y = \nu_{yo} - \frac{qE_y}{m_i} t
\]

In the flight from plane (a) to plane (c), let us an effective length by which the electric field \( E_y \) acts on D$^+$ ion to be \( \ell \). m.

When \( \nu_y \) of D$^+$ ion becomes zero in plane (c), a mean velocity of \( \nu_y \) in the flight is \( \nu_{yo}/2 \). Then, we have

\[
\nu_{yo} - \frac{qE_y}{m_i} \frac{\ell}{\nu_{yo}/2} = 0
\]

or

\[
E_y \ell = \frac{1}{2} \frac{m_i \nu_{yo}^2}{q} = 1.4 \times 10^5 \text{ V}.
\]

If \( \ell = 10 \text{ m} \), a necessary electric field \( E_y \) is \( 1.4 \times 10^4 \text{ V/m} \). This value will be a producible one. The electric field \( \pm \pm E_z \) in App (A) moves D$^+$ ion in the direction of \( \pm E_z \times \nu B_y \). The movement-length \( \ell_\perp \) is, when \( E_z = 10^4 \text{ V/m} \) and an effective length for \( \pm E_z \) to act in the flight from plane (a) to plane (b) is \( \ell/2 = 5 \text{ m} \),

\[
\ell_\perp = \frac{E_z}{B} \frac{\ell/2}{\nu_{yo}/2} = 4.6 \text{ mm}
\]

The mean Larmor radius \( r_L \) of deuterons is

\[
r_L = \frac{m_i \nu_{yo}}{qB} = 0.78 \text{ mm}
\]

Since the ratio \( \ell_\perp/r_L = 4.6/0.78 \approx 6 \), the most part of deuterons escaping from plane (c) to the solenoid ought not to touch the wall of the solenoid. Even if a part of high-energy deuterons collide with the wall, a sincere problem will not arise if only we make the number of colliding deuterons sufficiently small.

### 3. Plasma Heating by Extraordinary Waves (Called X-Waves)

Since we consider that heating of charged particles should be slowly done outside a main bottle so as not to disturb the stability of a plasma within the bottle, we consider heating deuterons in App (A) with X-wave (the frequency \( \omega_{x2} \) shown after) and electrons in App (B) with another X-wave (the frequency \( \omega_{x1} \) shown after).

The refractive index \( n_x \) for X-wave with a frequency \( \omega \) [8] is given by

\[
n_x^2 = \frac{(1 - \beta^2) (1 - \beta_e^2) - 2 \alpha^2 (1 - \beta_e \beta_e) + \alpha^2}{(1 - \beta^2) (1 - \beta_e^2) - \alpha^2 (1 - \beta_e \beta_e)}
\]

Here,
\[
\beta_e = \frac{\omega_e}{\omega} \quad \left( \omega_e = \frac{qB}{m_e}, m_n = \frac{m_e}{\left(1 - \frac{\beta_e^2}{c^2}\right)^{1/2}} \right)
\]

where \(c\) is the light speed,

\[
\beta_i = \frac{\omega_i}{\omega} \quad \left( \omega_i = \frac{qB}{m_i}, m_i = 3680m_e \right)
\]

\[
\alpha = \frac{\omega_p}{\omega} \quad \left( \omega_p = \left(\frac{n_i q_i^2}{m_e e_0}\right)^{1/2} \right)
\]

Two resonance frequencies are found from

\[
\left(1 - \beta_e^2\right)\left(1 - \beta_i^2\right) - \alpha^2 \left(1 - \beta_e \beta_i\right) = 0
\]

(8)

1) When \(\beta_e^2 = 1\), Equation (8) is simplified to

\[
1 - \beta_e^2 - \alpha^2 = 0 \text{ or } \alpha^2 = \alpha_e^2 + \alpha_i^2 = \alpha_{i1}^2
\]

(9)

2) When \(\beta_e^2 \gg 1\), Equation (8) is simplified to

\[-\alpha_e^2 \left(\alpha^2 - \alpha_i^2\right) - \alpha_i^2 \left(\alpha^2 - \alpha_e \alpha_i\right) = 0
\]

Accordingly,

\[
\omega^2 = \alpha_e^2 + \left(\frac{\omega_i \omega_e - \alpha_i^2}{\omega_i^2 + \omega_p^2}\right) \frac{\alpha_p^2}{\omega_p} \left(1 + \frac{\omega_p^2}{\omega_i \omega_e \left(1 + \frac{\omega_p^2}{\omega_e^2}\right)}\right) = \omega_{i2}^2
\]

(10)

From (10), under the condition \(\omega_{i2}^2 \ll \omega_e^2\), we have

\[
\begin{align*}
\text{When } \omega_p^2 &\ll \omega_e \omega_i, \omega_{i2} = \omega_i, \\
\text{When } \omega_p^2 &\approx \omega_e \omega_i, \omega_{i2} = \sqrt{2} \omega_i, \\
\text{When } \omega_p^2 &\gg \omega_e^2, \omega_{i2} = \left(\omega_e \omega_i\right)^{1/2}.
\end{align*}
\]

(11)

We try concrete numerical calculations on \(\omega_{i1}\) and \(\omega_{i2}\).

We assumed just now that a magnetic field strength \(B\) in the exit of the main bottle is \(B = 6 \times 4 \times 4 = 96\) T. A half vertical angle \(\alpha_h\) of the magnetic mirror is 14.5 degree, from \(\sin \alpha_h = (6/96)^{1/2}\). We roughly regard that a gas within App (A) is a plasma and also that a deuteron density \(n_A\) in a steady-state is a quantity of order of

\[
n_A = 10^{32} \times \frac{14.5}{90} = 0.16 \times 10^{31} \text{ m}^{-3}
\]

Then, within App (A),

\[
\omega_p = \left(\frac{n_A q_i^2}{m_e e_0}\right)^{1/2} = 0.68 \times 10^{12} \text{ sec}^{-1}, \quad \left(e_0 = 8.855 \times 10^{-12} \text{ Farad/m}\right)
\]
\[ \omega_e = \frac{qB}{m_e} = 1.55 \times 10^{13} \text{ sec}^{-1}, \]

\[ \omega_i = \frac{qB}{m_i} = 4.59 \times 10^9 \text{ sec}^{-1}, \]

\[ \omega_{i1} = \left( \omega_i^2 + \omega_e^2 \right)^{1/2} = 1.551 \times 10^{13} \text{ sec}^{-1} = \omega_e \text{ (from (9))}, \]

\[ \omega_{i2} = 2.9 \omega_i \text{ sec}^{-1} \text{ (from (10))}. \]

An ordinary wave (O-wave) can pass through a plasma if \( 1 - \left( \frac{\omega_p^2}{\omega^2} \right) > 0 \). O-wave with \( (\omega = \omega_{i1}) \) is not cut off because \( \omega_p^2/\omega_{i1}^2 \ll 1 \). O-wave with \( (\omega = \omega_{i2}) \) is cut off because \( \omega_p^2/\omega_{i2}^2 \gg 1 \). However, O-wave passing through a plasma will hardly heat the plasma. Energy of a wave with a resonance frequency ought to be absorbed by a plasma. We presume that the wave with \( (\omega = \omega_{i2}) \) will heat mainly deuterons because \( \omega_{i2} \ll \omega_e \).

4. Discussion and Conclusion

In order to reclaim the most part of deuterons escaping from the loss cone of a magnetic mirror bottle, we have proposed the new means based on the idea of decreasing those velocity components parallel to the magnetic field to zero by the external constant electric fields.

The new plan is under the premise that the external electric fields within Apparatus (A) do not suffer a large variation by escaping charged particles. We consider, in the upper space of the lower figure of Figure 1, movements of charged particles in the \( \pm z \)-directions. Electrons drift in the \(-z\)-direction and some electrons arrive at Anode (upper space) after having passed \( (B = 0) \)-space. Deuterons drift in the \( z \)-direction and some deuterons arrive at Cathode (upper space) after having passed another \( (B = 0) \)-space. Let us assume that the number \( n_e \) of electrons arriving at Anode per unit time is more than the number \( n_d \) of deuterons arriving at Cathode per unit time. Then, the \( n_e \) electrons flow as a current in an external circuit, but the \( n_e - n_d \) electrons remain in the \( (B = 0) \)-space and create a negative potential space. As a result, a potential on the \( (B = 0) \)-boundary plane becomes lower than the potential of Anode. However, since the charged particles are tightly restrained by very strong magnetic force lines and drift velocities of deuterons and electrons in the \( \pm z \)-directions ought to be very slow, we consider that such an effect as mentioned above has hardly influence on the magnitudes of \( E_y \) and \( E_z \).

“If the constant electric field \( E_y \) works well”, the apparatus in Figure 1 can make a magnitude of a loss angle for deuterons virtually zero. We consider that a combination of the apparatus in Figure 1 and a long mirror bottle has engineering simplicity.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.
References

[1] Mirnov, V. V. and Ryutov, D. D. (1979) Soviet Technical Physics Letters, 5, 279.

[2] Ågren, O., Moiseenko, V.E., Noack, K. and Hagnestål, A. (2011) Problems of Atomic Science and Technology, 17, 3.

[3] Mazzucato, E. (2019) Fusion Science and Technology, 75, 197-207. https://doi.org/10.1080/15361055.2018.1448202

[4] Nagata, M. (2019) Journal of Modern Physics, 10, 145-156. https://doi.org/10.4236/jmp.2019.102011

[5] Moiseenko, V.E. and Ågren, O. (2012) AIP Conference Proceedings, 1442, 199.

[6] Gospodchikov, E.D. and Smolyakova, O.B. (2015) Radiophys and Quantum Electronics, 57, 857. https://doi.org/10.1007/s11141-015-9570-9

[7] Artemyev, A.V., Aqapitov, O.V. and Krasnoselskich, V.V. (2013) Physics of Plasmas, 20, Article No. 124502. https://doi.org/10.1063/1.4853615

[8] Stix, T.H. (1962) The Theory of Plasma Waves. McGraw-Hill, New York.