A Nonlinear Constitutive Model for Remoulded Fine-Grained Materials Used under the Qinghai–Tibet Railway Line

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Abstract: Using undrained triaxial shear tests, this study investigates the mechanical properties of fine-grained materials (silty clay and sand) which are extensively used for China’s Qinghai–Tibet Railway (QTR) under different confining pressures ($\sigma_3$) and freezing temperatures ($T$). The results show that a reduction in $T$ causes an increase in the shear strength and elastic modulus of all the materials tested in the present study. In addition, the freezing of the silty clay has no significant effect on the type of soil behaviour (strain-hardening), whereas the freezing of the sand changes its strain-hardening behaviour to strain-softening. Supposing that the deviatoric stress–strain curves of the silty clay and sand can be divided into two segments due to a reverse bending point, it was assumed that the first segment follows a hyperbolic function. Meanwhile, the second segment is also a hyperbola, with the reverse bending point as the origin and the residual strength as the asymptote. Accordingly, a nonlinear relation constitutive model that considers $\sigma_3$ and $T$ is derived. All model parameters are identified. The reasonability of the new model was verified using the test results of the materials. A comparison of the predicted and test results shows that this model can well simulate the deviatoric stress–strain response in the failure process of the tested materials. In particular, it can reflect the residual deviatoric stress after the materials’ failure.

Keywords: constitutive model; frozen soil; mechanical behaviour; static triaxial test; Qinghai–Tibet Railway

1. Introduction

The territory covered by frozen soil in China is the third largest in the world and includes $2.15 \times 10^6$ km$^2$ of permafrost areas [1]. Many key projects in China, such as railways, highways, and distance pipelines were built in these regions [2]. Among them, the Qinghai–Tibet Railway (QTR) is the highest and longest plateau railway in the world at present [3–5]. However, it has been reported that a large amount of frost damage has occurred since the QTR was built [6]. Therefore, it was essential for engineering design and maintenance to systematically investigate the mechanical properties of the frozen remoulded fine-grained materials.

In previous studies, the strength and deformation properties of frozen soil under static conditions considering different influencing factors have been researched. The influence of freezing temperatures ($T$) on the physical and mechanical characteristics of silty clay has been studied by many researchers [7–13]. It has been shown that $T$ could significantly improve the physical and mechanical properties of soils due to the formation of a rigid ice–soil matrix [14]. Li et al. [15], Lat et al. [16], and Xu et al. [17] investigated the effects of the confining pressure ($\sigma_3$), $T$, and moisture content on the static strength, stiffness,
and damage behaviour of frozen soil by a series of triaxial static tests. However, the \( \sigma_3 \) of previous work on frozen soil was relatively high, limited by the test equipment. Due to the low \( \sigma_3 \) of the subgrade filling under the actual rail transit subgrade load in cold regions, the test results of a high \( \sigma_3 \) fail to meet the engineering needs.

To master the working state of frozen soil in engineering structures, it is necessary to select the appropriate material constitutive model for analysis according to the actual situation [18]. Based on the experimental results, many constitutive models have been proposed to analytically and numerically study the strength and stress–strain relationship in frozen soil [19–22]. The Duncan–Chang constitutive model has clear concepts and is easy to understand [23,24]; so, it is widely used in hydraulic and geotechnical engineering, because it can better reflect the nonlinear behaviour of soil [25–27]. However, the Duncan–Chang model is not suitable for the evaluation of the frozen soil stress–strain curve, because the softening process of frozen soil cannot be well simulated.

In the following sections, the deformational characteristics of the silty clay and sand under different \( \sigma_3 \) and \( T \) are investigated using static triaxial tests. A simplified nonlinear constitutive model that can capture strain-softening behaviour is developed to analyse the effects of \( \sigma_3 \) and \( T \). Next, the parameters involved in the model are evaluated, and insights and conclusions are drawn from the results.

2. Materials and Methods

2.1. Study Area and Soil Properties

The silty clay and sand of the remoulded samples was collected within the range of 0.5 m under the shoulder at section K1013 along the Qinghai–Tibet Railway, the highest altitude railway in China’s permafrost regions (see Figure 1). The gradation curves of the soil sample are shown in Figure 2. Through laboratory tests, the optimal moisture content of the silty clay was found to be 17.4%, and the corresponding maximum dry density was 1.74 g/cm\(^3\) (see Figure 3). The optimal moisture content of the sand was 11.8%, and the corresponding maximum dry density was 1.88 g/cm\(^3\) (see Figure 3). The two fillings basically accorded with the typical characteristics of the density and optimal water content of silty clay and sand, respectively.

![Figure 1. Remoulded fine-grained materials from Section K1013 along the Qinghai–Tibet railway: (a) frozen soil map of China; (b) Qinghai–Tibet; (c) section K1013 site; (d) remoulded fine-grained materials.](image-url)
Figure 2. Particle size distribution curves of the remoulded fine-grained materials (silty clay and sand).

Figure 3. Compaction curve of the remoulded fine-grained materials (silty clay and sand) used in this study.

2.2. Sample Preparation

According to the Code for Soil Tests of Railway Engineering in China (TB10102-2010) [28], the method of sample compaction at different layers was adopted in the current study. The silty clay and sand obtained from the Qinghai–Tibet Railway was cleaned, dried, and sieved, and only the particles under a 2 mm diameter sieve were collected to make the remoulded samples. The samples were divided into five layers to compact, where the mass of each layer could be obtained considering 95% of maximum dry density ($\rho_{d,\text{max}}$). Subsequently, purified water was added to reach the optimum moisture content ($w_{\text{opt}}$). Later on, the prepared soil mixtures were kept in enclosed bags for 24 h to prevent evaporation. Lastly, the cylindrical material samples with a height of 200 mm and a diameter of 100 mm were compacted layer-by-layer using a standard proctor hammer. After compacting one layer, the layer interface was made sufficiently coarse to ensure
the two layers were integrated. Then, the specimens were wrapped with rubber sleeves, and the top and bottom were covered with epoxy resin platen to prevent water evaporation.

2.3. Test Procedures

The dynamic triaxial tests were conducted on a fully automated Global Digital System (GDS), a cryogenic triaxial apparatus, illustrated in Figure 4. The system is a digitally controlled servo pneumatic system that controls two parameters: axial stress and confining pressure ($\sigma_3$). The system incorporated a control and data acquisition system, which can maintain an auxiliary air receiver with a servo valve for cell pressure control, local deformation measurement in the vertical direction, and a submersible load cell measuring the applied axial load. The stable confining pressure ranged from 20 kPa to 4 MPa, and the maximum axial load and displacement were 40 kN and 85 mm, respectively.

![Figure 4](image)

**Figure 4.** Cryogenic triaxial test system and the tested samples.

According to Wang et al. [29], a closed system was adopted for the test program. A one-dimensional freezing–thawing model was employed to simulate the direction of freezing and thawing from the top to the bottom. To ensure one-dimensional freezing and thawing, only the sample top surface was exposed to the external environment. A 50 mm thick layer of insulating polystyrene was used to protect the perimeter and bottom of the cylinder.

The variation range in ground temperature along the Qinghai–Tibet Railway in China is roughly between $-20^\circ\text{C}$ and $20^\circ\text{C}$, with a large temperature difference. Therefore, the temperatures under the static loading condition were set to $-10^\circ\text{C}$, $-5^\circ\text{C}$, and $-1^\circ\text{C}$. The time needed to freeze the sample was determined by a pre-experiment test. A short resistance temperature detector probe was inserted at the centre of a pre-experiment test sample during the freezing process to show the variation in temperature. It revealed that 12 h was sufficient to freeze the sample to reach $-10^\circ\text{C}$ with slight fluctuations, plus or minus 0.2 $^\circ\text{C}$.

The test sample was isotropically consolidated at $\sigma_3 = 100$ kPa, 150 kPa, or 200 kPa after the sample temperature was under the preset temperature for 12 h. After consolidation, the test specimen was then subjected to shearing at an axial strain rate (1.25 mm per min) until the strain of 15% was reached. The particular test scheme is shown in Table 1.
Table 1. Summary of test schemes and the corresponding model parameters.

| Soil Type | Test No. | Confining Pressure (kPa) | Temperature (°C) | b1 | b2 | b3 | b4 | b5 | Dβ | The Patterns of the Stress–Strain Curves |
|-----------|---------|--------------------------|------------------|----|----|----|----|----|-----|------------------------------------------|
| Sand      | S1      | 100                      | −1               | 0.78 | 0.44 | 0.213 | - | - | 3.86 | Strain-hardening                           |
|           | S2      | 150                      | −5               | 0.54 | 0.46 | 0.165 | - | - | 3.30 | Strain-hardening                           |
|           | S3      | 200                      |                  | 0.39 | 0.43 | 0.174 | - | - | 2.84 | Strain-hardening                           |
|           | S4      | 100                      |                  | 0.78 | 0.38 | 0.0036 | 1.41 | 5.93 | 2.91 | Strain-softening                           |
|           | S5      | 150                      |                  | 0.54 | 0.25 | 0.0077 | 1.22 | 6.39 | 2.53 | Strain-softening                           |
|           | S6      | 200                      |                  | 0.39 | 0.33 | 0.0057 | 1.16 | 6.71 | 2.24 | Strain-softening                           |
|           | S7      | 100                      | −10              | 0.78 | 0.25 | 0.0142 | 3.65 | 11.95 | 2.41 | Strain-softening                           |
|           | S8      | 150                      |                  | 0.54 | 0.33 | 0.0192 | 2.60 | 9.43 | 2.35 | Strain-softening                           |
|           | S9      | 200                      |                  | 0.39 | 0.39 | 0.0446 | 1.83 | 8.44 | 2.40 | Strain-softening                           |
| Silty clay| SC1     | 100                      |                  | 1.26 | 0.42 | 0.19 | - | - | 4.67 | Strain-hardening                           |
|           | SC2     | 150                      |                  | 3.73 | 0.29 | 0.24 | - | - | 4.94 | Strain-hardening                           |
|           | SC3     | 200                      |                  | 4   | 0.02 | 0.16 | - | - | 4.28 | Strain-hardening                           |
|           | SC4     | 100                      |                  | 1.26 | 0.29 | 0.17 | - | - | 3.74 | Strain-hardening                           |
|           | SC5     | 150                      | −5               | 3.73 | 0.16 | 0.26 | - | - | 7.08 | Strain-hardening                           |
|           | SC6     | 200                      |                  | 4   | 0.14 | 0.12 | - | - | 6.54 | Strain-hardening                           |
|           | SC7     | 100                      | −10              | 1.26 | 0.29 | 0.47 | - | - | 4.65 | Strain-hardening                           |
|           | SC8     | 150                      |                  | 3.73 | 0.16 | 0.26 | - | - | 7.08 | Strain-hardening                           |
|           | SC9     | 200                      |                  | 4   | 0.17 | 0.14 | - | - | 6.82 | Strain-hardening                           |

3. Experiment Results and Analysis

3.1. Stress–Strain Behaviour

To illustrate the mechanical behaviour of the remoulded fine-grained materials that were influenced by the freezing temperature and $\sigma_3$, Figure 5 shows the typical stress–strain curves for the samples. The results show that the shear strength of the frozen silty clay was significantly greater than that of the frozen sand at the same conditions, especially at low temperatures. It is seen that all the stress–strain curves for silty clay were of the strain-hardening type, and the principal stress deviation had nonlinear growth with the increase in the axial strain, in which the shape of curves was hyperbolic. The stress–strain curves of the silty clay show that the shape of the stress–strain curve did not change with the variation in the temperature, but the degree of hardening was weakened. Decreasing the freezing temperature to $-10 \, ^\circ\text{C}$ increased the shear strength of the silty clay by 1374% and 2258% with respect to the silty clay of $-1 \, ^\circ\text{C}$ when the $\sigma_3 = 150 \, \text{kPa}$.

As for the sand, when the freezing temperature was the same, the stress–strain curves' variation tendency was similar with the different $\sigma_3$. However, the stress–strain curves gradually changed to strain-softening as the freezing temperature decreased. Moreover, the reduction in the temperature from $-1 \, ^\circ\text{C}$ to $-10 \, ^\circ\text{C}$ caused an increase in the shear strength of 661% for the frozen sand under a $\sigma_3$ of 150 kPa. It can be concluded that the influence of freezing on the increase in shear strength of the silty clay was much greater than that for sand.

3.2. Cohesion and Internal Friction Angle

The results in Figure 5 were processed using the Mohr–Coulomb strength criterion expressed by the “p-q” method to obtain the internal friction angle $\varphi$ and cohesion $c$ in Figure 6. According to Figure 6, the samples of silty clay with more water content than that of sand strengthened the bite friction during the freezing process; so, the cohesion and angle of the internal friction in silty clay were slightly greater than those in sand. Here, the cohesion increased by approximately 5900% and 3100% for silty clay and sand, respectively, comparing the samples at the temperatures of $-1 \, ^\circ\text{C}$ and $-10 \, ^\circ\text{C}$. Similar to the cohesion, the reduction in temperature from $-1 \, ^\circ\text{C}$ to $-10 \, ^\circ\text{C}$ led to an 11.1% and 8.4% increase in the internal friction angle of silty clay and sand, respectively. It is noted that
the rate of increase in the cohesion and angle of internal friction due to the reduction in temperature was almost identical for the silty clay and sand.

Figure 5. Dependence of deviatoric stress on the axial strain of tested remoulded fine-grained materials corresponding to different confining pressures: (a) $T = -1 ^\circ C$, (b) $T = -5 ^\circ C$, and (c) $T = -10 ^\circ C$.

Figure 6. Variation in the cohesion and angle of internal friction versus freezing temperature.
3.3. Elastic Modulus

The effect of \( \sigma_3 \) and freezing temperature reduction on the modulus of elasticity (\( E_e \)) of the silty clay is presented in Figure 7a. The values of \( E_e \) for the silty clay under the temperature of \(-1 \, ^{\circ}C\) for \( \sigma_3 \) values of 100, 150, and 200 kPa were approximately 7.5, 9, and 10 MPa, respectively. After the freezing temperature decreased to \(-5 \, ^{\circ}C\), the growth in \( E_e \) was approximately 900\%, 1567\%, and 1880\%, respectively. As the freezing temperature continued to decrease to \(-10 \, ^{\circ}C\), the values of \( E_e \) increased by 1793\%, 1844\%, and 2230\% with respect to the silty clay at \(-1 \, ^{\circ}C\) for the three \( \sigma_3 \) values, respectively.

Moreover, a decrease in freezing temperature led to a significant increase in the \( E_e \) of sand. Figure 7b presents the effect of the freezing temperature reduction on the \( E_e \) of the sand tested under \( \sigma_3 \) values of 100, 150, and 200 kPa at the freezing temperature of \(-10 \, ^{\circ}C\), \(-5 \, ^{\circ}C\), and \(-1 \, ^{\circ}C\). As expected, the \( E_e \) of sand generally increased with the decreasing freezing temperature. The growth in the \( E_e \) of sand under the freezing temperature of \(-10 \, ^{\circ}C\) relative to the sand at \(-1 \, ^{\circ}C\) was approximately 775\%, 743\%, and 715\%, respectively, for the three \( \sigma_3 \) values.

4. A Nonlinear Constitutive Model for Remoulded Fine-Grained Materials

4.1. Establishment of the Model

As shown in Figure 8, a softening-type stress–strain model with a high degree of accuracy can be used to approximate the nonlinear stress–strain curves for the remoulded fine-grained materials under different freezing temperatures [30]. After careful inspection, the piecewise continuous functions for the proposed constitutive model are expressed as follows:

\[
R_e' = D^*_R \left[ \frac{1 + b_2 R_e}{1 + b_3 R_e} \right] \frac{R_e}{1 + b_1 R_e} \quad (R_e \leq R_{el}) \quad \text{(1a)}
\]

\[
R_e' = R_{el} - \frac{R_e - R_{el}}{b_4 + b_5(R_e - R_{el})} \quad (R_e > R_{el}) \quad \text{(1b)}
\]
where $D_p$, $b_1$, $b_2$, $b_3$, $b_4$, and $b_5$ are undetermined coefficients that can be obtained from the experimental results. The parameters $R_{e\sigma}$, $R_{e\epsilon}$, $R_{et}$, and $R_{etl}$ are defined by Equation (2a) through (2d) as follows:

$$R_{e\sigma} = \frac{\sigma_1 - \sigma_3}{(\sigma_1 - \sigma_3)_p}$$

(2a)

$$R_{e\epsilon} = \frac{\epsilon}{\epsilon_p}$$

(2b)

$$R_{et} = \frac{\sigma_1 - \sigma_3}{(\sigma_1 - \sigma_3)_p}$$

(2c)

$$R_{etl} = \frac{\epsilon_l}{\epsilon_p}$$

(2d)

where $(\sigma_1 - \sigma_3)$ is the deviator stress; $\sigma_1$ and $\sigma_3$ are the major principal stress and minor principal stress, respectively; $\epsilon$ is the axial strain; $(\sigma_1 - \sigma_3)_p$ and $\epsilon_p$ are the deviator stress and deviator strain, respectively, at the peak point (i.e., point $P$ in Figure 8); and $(\sigma_1 - \sigma_3)_l$ and $\epsilon_l$ are the deviator stress and the deviator strain, respectively, at the inflection point (i.e., point $T$ in Figure 8).

Figure 8. Schematic diagram of a nonlinear constitutive model for the remoulded fine-grained materials.

(1) Coefficient determination for Equation (1a).

To accurately represent the relationship between $(\sigma_1 - \sigma_3)$ and $R_{e\epsilon}$, the terms on both sides of Equation (1a) were multiplied by $(\sigma_1 - \sigma_3)_p$ and Equation (1a) was rewritten as follows:

$$\sigma_1 - \sigma_3 = D_p(\sigma_1 - \sigma_3)_p(1 + b_2 R_{e\epsilon}) \frac{1 + b_3 R_{e\epsilon}}{1 + b_1 R_{e\epsilon}}$$

(3)

The compressive strengths of the fillers are assumed to satisfy the Mohr–Coulomb failure criterion. The relationship between the compressive strength $(\sigma_1 - \sigma_3)_p$ and the confining pressure $\sigma_3$ is described by the following equation:

$$(\sigma_1 - \sigma_3)_p = \frac{2(\epsilon_p \cos \varphi_p + \sigma_3 \sin \varphi_p)}{1 - \sin \varphi_p}$$

(4)

in which $\epsilon_p$ and $\varphi_p$ are the Mohr–Coulomb strength parameters.
Next, Equation (4) is substituted for \((\sigma_1 - \sigma_3)_p\) in Equation (3) to yield
\[
\sigma_1 - \sigma_3 = D_\beta \frac{2(C_p \cos \phi_p + \sigma_3 \sin \phi_p)}{1 - \sin \phi_p} \frac{1 + b_2 R_e}{1 + b_3 R_e + 1 + b_1 R_e} \tag{5}
\]
where \(D_\beta = D \times \beta\), with \(\beta\) as a modified coefficient and \(D = E_{\text{max}} / E_p\). \(E_{\text{max}}\) and \(E_p\) (i.e., \(E_p = (\sigma_1 - \sigma_3)_p / \epsilon_p\)) are the initial tangent modulus at the original point (i.e., point O in Figure 8) and the secant modulus at the peak point, respectively. The determination of \(E_{\text{max}}\) is equivalent to the determination of \(E_p\) in the Duncan-Chang model [23].

Performing the differentiation on Equation (1a) produces the following expression
\[
dR_e \sigma_e = D_\beta \left(\frac{1}{1 + b_1 R_e} + \frac{1 + b_2 R_e}{1 + b_3 R_e + 1 + b_1 R_e}\right) \tag{6}
\]

In addition, \(R_e\) can be considered as a function of the parameter \(R_e\) in Equation (1a) and attains its extreme value at the peak. Thus,
\[
dR_e \sigma_e = 0 \tag{7}
\]

Combined with Equations (6) and (7),
\[
\frac{b_1 - b_2}{1 + b_5} + \frac{1 + b_2}{1 + b_1} = 0 \tag{8a}
\]
\[
b_1 = \frac{b_2 b_3 + 2b_2 + 1}{b_3 - b_2} \tag{8b}
\]

In this case, \(R_e = 1\) and \(R_e = 1\) at the peak point based on their definitions in Equation (2). Thus, these values are substituted into Equation (1a) to obtain
\[
1 = D_\beta \cdot \frac{1 + b_2}{1 + b_3} \cdot \frac{1}{1 + b_1} \tag{9}
\]

According to Equation (1a), \(R_e\) and \(R_e\) may be expressed at the inflection point as follows:
\[
R_e = D_\beta \cdot \frac{1 + b_2 R_e}{1 + b_3 R_e} \cdot \frac{1}{1 + b_1 R_e} \tag{10}
\]

By combining Equations (9) and (10), \(b_2\) and \(b_3\) can be expressed as
\[
b_3 = \frac{R_e - 1 + \beta_1 (1 + b_1 R_e) - \beta_1 R_e (1 + b_1 R_e)}{[\beta_1 (1 + b_1 R_e) - \beta_1 (1 + b_1 R_e)] R_e} \tag{11a}
\]
\[
b_2 = \beta_1 (1 + b_1) (1 + b_3) - 1 \tag{11b}
\]

where \(\beta_1 = \frac{R_e}{\beta_1 R_e}\) and \(\beta_1 = \frac{R_e}{\beta_1 R_e}\).

Three undetermined coefficients (i.e., \(b_1, b_2,\) and \(b_3\)) are included in Equation (11). When the original \(b_1\) is given, \(b_2\) and \(b_3\) are evaluated using Equations (11a) and (11b). Subsequently, \(b_2\) and \(b_3\) are substituted into Equation (8) to obtain a new value for \(b_1\). This procedure can be repeated by using the original \(b_1\) until the given and recalculated \(b_1\) are identical.

(2) Coefficients of determination for Equation (1b).

Similarly, the terms on both sides of Equation (1b) are simultaneously multiplied by \((\sigma_1 - \sigma_3)_p\). Next, Equation (1b) is combined with Equation (2a) to (2d) as follows:
\[
\sigma_1 - \sigma_3 = (\sigma_1 - \sigma_3)_p - \frac{\epsilon - \epsilon_i}{E_p b_4 + \frac{1}{E_p b_5} b_3 (\epsilon - \epsilon_i)} \tag{12}
\]
If we let
\[ b_4 = E_p A \]  
\[ b_5 = E_p \varepsilon_p B \]  
then, Equation (12) is rewritten as
\[ \sigma_1 - \sigma_3 = (\sigma_1 - \sigma_3)_1 - \frac{\varepsilon - \varepsilon_1}{A + B(\varepsilon - \varepsilon_1)} \]  
(14)

The tangent modulus \( E \) at any point in the first segment of the curve (i.e., \( R_e \leq R_{et} \)) can be directly represented as
\[ E = \frac{d(\sigma_1 - \sigma_3)}{d\varepsilon} = \frac{d(\sigma_1 - \sigma_3)}{dR_e} \cdot \frac{dR_e}{d\varepsilon} \]  
(15)

The expressions of \( R_\sigma \) and \( R_e \) can be rewritten as
\[ \frac{d(\sigma_1 - \sigma_3)}{dR_e} = (\sigma_1 - \sigma_3)_p \]  
(16a)
\[ \frac{dR_e}{d\varepsilon} = \frac{1}{\varepsilon_p} \]  
(16b)

Equation (16) is substituted into Equation (15) to obtain
\[ \frac{d(\sigma_1 - \sigma_3)}{d\varepsilon} = \frac{(\sigma_1 - \sigma_3)_p}{\varepsilon_p} \cdot \frac{dR_e}{d\varepsilon} = E_p \frac{dR_e}{d\varepsilon} \]  
(17)

Equation (6) is substituted into Equation (17), and the tangent modulus in the first segment of the curve can be expressed as
\[ \frac{d(\sigma_1 - \sigma_3)}{d\varepsilon} = D_{\beta} \frac{E_p}{(1 + b_1 R_e)(1 + b_3 R_e)} \left[ \frac{(b_1 - b_2) R_e}{1 + b_3 R_e} + \frac{1 + b_2 R_e}{1 + b_1 R_e} \right] \]  
(18)

In addition, \( D_{\beta} = D \times \beta, D = E_{max}/E_p \), Equation (18) can be rewritten as
\[ \frac{d(\sigma_1 - \sigma_3)}{d\varepsilon} = \frac{E_{max}}{(1 + b_1 R_e)(1 + b_3 R_e)} \left[ \frac{(b_1 - b_2) R_e}{1 + b_3 R_e} + \frac{1 + b_2 R_e}{1 + b_1 R_e} \right] \]  
(19)

Therefore, the tangent modulus \( (E_{t,1}) \) at the inflection point in the first segment of the curve can be obtained from Equation (19).
\[ E_{t,1} = \frac{E_{max}}{(1 + b_1 R_{et})(1 + b_3 R_{et})} \left[ \frac{(b_1 - b_2) R_{et}}{1 + b_3 R_{et}} + \frac{1 + b_2 R_{et}}{1 + b_1 R_{et}} \right] \]  
(20)

In addition, the tangent modulus \( (E_{t,2}) \) at the inflection point in the second segment of the curve can be expressed from Equation (12) by using Equation (21) in terms of the definition of the derivative.
\[ E_{t,2} = \lim_{\varepsilon \to \varepsilon_1} \frac{(\sigma_1 - \sigma_3) - (\sigma_1 - \sigma_3)_1}{\varepsilon - \varepsilon_1} = \lim_{\varepsilon \to \varepsilon_1} \frac{-1}{A + B(\varepsilon - \varepsilon_1)} = \frac{-1}{A} \]  
(21)

To ensure a continuous condition at the inflection point, the tangent modulus at the inflection point in the two segments of the curve should be equivalent, which indicates that \( E_{t,1} = E_{t,2} \) at the inflection point. Therefore, Equation (21) is substituted into Equation (20) to obtain
\[ A = -\frac{E_{max}}{(1 + b_1 R_{et})(1 + b_3 R_{et})} \left[ \frac{(b_1 - b_2) R_{et}}{1 + b_3 R_{et}} + \frac{1 + b_2 R_{et}}{1 + b_1 R_{et}} \right] \]  
(22)
Consequently, $b_4$ can be evaluated when the value of $A$ is substituted into Equation (13a).

When the deviatoric stress $(\sigma_1 - \sigma_3)_r$, which is the residual strength and its corresponding strain $\varepsilon_r$, at any point in the second segment of the curve, is substituted into Equation (2), $R_{er}$ and $R_{er}$ can be evaluated, and $b_5$ can be obtained using Equation (23). In this study, the residual deviatoric stress $(\sigma_1 - \sigma_3)_r$ is selected when the strain $\varepsilon_r$ is equivalent to 20%.

$$b_5 = -\frac{b_4 + \frac{R_{er} - R_{r}}{R_{er} - R_{r}}}{R_{er} - R_{r}}$$

(23)

4.2. Determination of Model Parameters

Six parameters need to be determined from Equation (1) in the proposed model, including the parameters related to the basic elastic properties ($D_3$), the peak stress state ($b_1$, $b_2$, and $b_3$), and the residual stress state ($b_4$ and $b_5$). Most of these model parameters can be conveniently obtained through the monotonic triaxial tests incorporating $s_3$ and $T$. The initial small stress–strain data can be adopted to calculate the $D_3$. The residual stress state parameters ($b_4$ and $b_5$) can be evaluated through the monotonic triaxial tests results at the residual stress state using Equations (13) and (23). The peak state parameters ($b_1$, $b_2$, and $b_3$) can be determined by a trial-and-error process using Equations (8) and (11), i.e., by comparing the model predictions and laboratory data. Furthermore, the values of the abovementioned undetermined coefficients with the corresponding values are shown in Table 1.

4.3. Model Verification

The test results of the sand and silty clay subjected to different $s_3$ and $T$ are used to validate the proposed model. Figure 9 shows the predicted stress–strain of the sand and silty clay compared to the laboratory observations. Note that the model predictions match well with the test data, and the strain-hardening and strain-softening of the samples subjected to different $s_3$ and $T$ have been captured very well.

![Figure 9](image-url)

Figure 9. Comparison of the predicted and tested data of the tested remoulded fine-grained materials: (a) sand and (b) silty clay.
5. Discussion

5.1. Sensitivity Analysis of the Parameters

In Equation (1a), let $b_3 = 0.7$, $b_2 = -0.44$, and $b_3 = 0.21$; thus, Figure 10a demonstrates the deviatoric stress–strain curves of the proposed constitutive model for different values of the parameter $D\beta$. It can be seen from Figure 10a that the change in the value $D\beta$ has no obvious influence on the shape of the deviatoric stress–strain curve. However, with the increase in the value $D\beta$, the initial elastic modulus of the model increases gradually, indicating that the parameter $D\beta$ mainly reflects the initial elastic modulus of the materials; indeed, the larger the parameter $D\beta$ is, the higher the initial elastic modulus will be.

![Figure 10a](image1)
![Figure 10b](image2)
![Figure 10c](image3)
![Figure 10d](image4)
![Figure 10e](image5)
![Figure 10f](image6)

**Figure 10.** Deviatoric stress–strain curves of the proposed model for different values of parameters: (a) $D\beta$, (b) $b_1$, (c) $b_2$, (d) $b_3$, (e) $b_4$, and (f) $b_5$. 
Furthermore, in Equation (1a), let $D_\beta = 3.86$, $b_2 = -0.44$, and $b_3 = 0.21$; let $D_\beta = 3.86$, $b_1 = 0.78$, and $b_3 = 0.21$; and let $D_\beta = 3.86$, $b_1 = 0.78$, and $b_2 = -0.44$. Thus, Figure 10b,d presents the deviatoric stress–strain curves of the proposed constitutive model for different values of the parameters $b_1$, $b_2$, and $b_3$, respectively. Figure 10b,d show that the change in the values $b_1$, $b_2$, and $b_3$ has a great influence on the shape of the deviatoric stress–strain curve. With a decrease in the values $b_1$ and $b_3$ and an increase in the value $b_2$, the stress–strain curves gradually change from strain-softening to strain-hardening. This indicates that the parameters $b_1$, $b_2$, and $b_3$ mainly reflect the peak strength and brittleness degree of the materials’ failure. The larger the parameters $b_1$ and $b_3$ are, and the smaller the parameter $b_2$ is, the more obvious the brittleness failure characteristic of the materials will be.

In addition, it can be seen from Figure 10 that the variation in $b_4$ and $b_5$ only changes the shape of the residual stress state. With an increase in $b_4$ and $b_5$, the residual stress increases, indicating that the failure characteristic changes from brittleness to ductility.

5.2. Comparison with the Existing Model

In order to further illustrate the rationality of the proposed model, the triaxial compression test results of silty clay and sand, with $\sigma_3 = 100$ kPa and $T = -5$ °C, were employed to validate it. Figure 11 presents the comparison of the theoretical curves from the proposed model and the Duncan–Chang model. It can be seen from Figure 11 that the theoretical curves from the Duncan–Chang model had good agreement with the test data at the prepeak region but poor agreement at the postpeak region for the mechanical behaviours of the materials. In particular, it could not reflect the residual deviatoric stress after the materials’ failure, while the theoretical curves from the model proposed in the current study conform well with the test data in both the prepeak region and postpeak region, no matter what the shape of the stress–strain curves. Therefore, the proposed model has better adaptability.

Figure 11. Comparison of the predicted and tested data of the tested remoulded fine-grained materials under $T = -5$ °C and $\sigma_3 = 100$ kPa.

6. Conclusions

This paper presented the results of a comprehensive experimental investigation to study the effect of confining pressure ($\sigma_3$) and freezing temperature ($T$) on the mechanical behaviour of remoulded fine-grained materials (silty clay and sand) used under the Qinghai–Tibet Railway line. Accordingly, a nonlinear constitutive model, which could reflect the postpeak softening behaviour was established. Conventional monotonic triaxial tests incorporating $\sigma_3$ and $T$ carried out in the current study were used to evaluate all the model parameters. The salient outcomes of the model are summarized below.
(1) All the silty clay exhibited a strain-hardening type of stress–strain curve, but the sand under the temperatures of $-5$ °C and $-10$ °C showed strain-softening. Under the same test conditions, the shear strength of the silty clay was greater than that of the sand. In all cases, the cohesion ($c$) and angle of internal friction ($\phi$) of the silty clay were greater than that of sand. Furthermore, the modulus of elasticity of the materials tested increased due to freezing and the temperature reduction.

(2) A practical constitutive model was developed to represent the nonlinear, stress-dependent, and inelastic stress–strain behaviours of the fillers subjected to freezing and thawing. This model incorporated three important aspects of the stress–strain behaviour, including nonlinearity, strain-dependency softening, and inelasticity. A simple technique was used to interpret the test results and conveniently determine the six parameters in the model.

(3) The triaxial test results of the remoulded fine-grained materials were employed to evaluate the reasonability of the proposed model established in this paper. A comparison of the predicted and test results showed that this model could well simulate the deviatoric stress–strain response in the failure process of the tested materials. In particular, it could reflect the residual deviatoric stress after materials’ failure.

(4) This study analysed the behaviour of the fillers with optimum water content that were exposed to the freeze–thaw cycles to develop a constitutive model. If appropriate experimental results are available, the parameter values in the proposed model can be derived from the triaxial test results. Therefore, additional experiments should be conducted to investigate other parameters, such as the temperature, duration of freezing and thawing, and the moisture content and compactness of the fillers, which are important characteristics of fillers in regions that are subjected to seasonal freezing and thawing.

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References

1. Zhao, F.T.; Chang, L.J.; Zhang, W.Y. Experimental investigation of dynamic shear modulus and damping ratio of Qinghai-Tibet frozen silt under multi-stage cyclic loading. *Cold Reg. Sci. Technol.* **2020**, 170, 102938. [CrossRef]

2. Ma, D.D.; Ma, Q.Y.; Yao, Z.M.; Huang, K. Static-dynamic coupling mechanical properties and constitutive model of artificial frozen silty clay under triaxial compression. *Cold Reg. Sci. Technol.* **2019**, 167, 102858. [CrossRef]

3. Wu, Q.B.; Cheng, G.D.; Ma, W.; Niu, F.J.; Sun, Z.Z. Technical approaches on permafrost thermal stability for Qinghai–Xizang Railroad. *Geomech. Geoenviron.* **2006**, 1, 119–128. [CrossRef]

4. Ma, W.; Cheng, G.D.; Wu, Q.B.; Wang, D.Y. Application on idea of dynamic design in Qinghai–Tibet Railway construction. *Cold Reg. Sci. Technol.* **2004**, 41, 165–173. [CrossRef]

5. Zu, Z.Y.; Ling, X.Z.; Chen, S.P.; Zhang, F.; Wang, L.N.; Wang, Z.Y.; Zou, Z.Y. Experimental investigation on the train-induced subsidence prediction model of Beiluhe permafrost subgrade along the Qinghai–Tibet Railway in China. *Cold Reg. Sci. Technol.* **2010**, 62, 67–75. [CrossRef]

6. Cheng, G.D. A roadbed cooling approach for the construction of Qinghai–Tibet Railway. *Cold Reg. Sci. Technol.* **2005**, 42, 169–176. [CrossRef]

7. Wang, C.; Li, S.; Zhang, T.; You, Z. Experimental Study on Mechanical Characteristics and Fracture Patterns of Unfrozen/Freezing Saturated Coal and Sandstone. *Materials* **2019**, 12, 992. [CrossRef]
8. Sobczyk, K.; Chmielewski, R.; Kruszka, L.; Rekucki, R. Strength Characterization of Soils’ Properties at High Strain Rates Using the Hopkinson Technique—A Review of Experimental Testing. *Materials* 2022, 15, 274. [CrossRef]

9. Wei, X.; Ming, F.; Li, D.; Chen, L.; Liu, Y. Influence of Water Content on Mechanical Strength and Microstructure of Alkali-Activated Fly Ash/GGBFS Mortars Cured at Cold and Polar Regions. *Materials* 2020, 13, 138. [CrossRef]

10. Zhao, X.; Zhou, G.; Wang, J. Deformation and strength behaviors of frozen clay with thermal gradient under uniaxial compression. *Tunn. Undergr. Space Technol.* 2013, 38, 550–558. [CrossRef]

11. Li, H.; Yang, Z.J.; Wang, J. Unfrozen water content of permafrost during thawing by the capacitance technique. *Cold Reg. Sci. Technol.* 2018, 152, 15–22. [CrossRef]

12. Török, Á.; Ficsor, A.; Davarpanah, M.; Vásárhelyi, B. Comparison of mechanical properties of dry, saturated and frozen porous rocks. In Proceedings of the IAE/G/EAG Annual Meeting, San Francisco, CA, USA, 17–21 September 2018; Shakoor, A., Cato, K., Eds.; Springer International Publishing: New York, NY, USA, 2018; Volume 6, pp. 113–118.

13. Esmaeili-Falak, M.; Katebi, H.; Javadi, A.A. Effect of Freezing on Stress–Strain Characteristics of Granular and Cohesive Soils. *Materials* 2022, 15, 519. [CrossRef]

14. Wang, D.Y.; Zhu, Y.L.; Ma, W.; Niu, Y.H. Application of ultrasonic technology for physical–mechanical properties of frozen soils. *Cold Reg. Sci. Technol.* 2020, 164, 103060. [CrossRef]

15. Li, D.W.; Yang, X.; Chen, J.H. A study of Triaxial creep test and yield criterion of artificial frozen soil under unloading stress paths. *Cold Reg. Sci. Technol.* 2017, 141, 163–170. [CrossRef]

16. Lai, Y.M.; Liao, M.K.; Hu, K. A constitutive model of frozen saline sandy soil based on energy dissipation theory. *Int. J. Plast.* 2016, 78, 84–113. [CrossRef]

17. Xu, X.T.; Li, Q.L.; Lai, Y.; Pang, W.T.; Zhang, R.P. Effect of moisture content on mechanical and damage behavior of frozen loess under triaxial condition along with different confining pressures. *Cold Reg. Sci. Technol.* 2019, 157, 110–118. [CrossRef]

18. Peng, W.; Wang, Q.; Liu, Y.F.; Sun, X.H.; Chen, Y.T.; Han, M.X. The Influence of Freeze-Thaw Cycles on the Mechanical Properties and Parameters of the Duncan-Chang Constitutive Model of Remolded Saline Soil in Nong’an County, Jilin Province, Northeastern China. *Appl. Sci.* 2019, 9, 4941. [CrossRef]

19. Li, Z.; Chen, J.; Mao, C. Experimental and Theoretical Investigations of the Constitutive Relations of Artificial Frozen Silty Clay. *Materials* 2019, 12, 3199. [CrossRef]

20. Zhang, Y.; Chen, Y. A Constitutive Relationship for Gravelly Soil Considering Fine Particle Suffusion. *Materials* 2017, 10, 1217. [CrossRef]

21. Xu, P.; Sun, Z.; Shao, S.; Fang, L. Comparative Analysis of Common Strength Criteria of Soil Materials. *Materials* 2021, 14, 4302. [CrossRef]

22. Fu, T.T.; Zhu, Z.W.; Zhang, D.; Liu, Z.J.; Xie, Q.J. Research on damage viscoelastic dynamic constitutive model of frozen soil. *Cold Reg. Sci. Technol.* 2019, 160, 209–221. [CrossRef]

23. Duncan, J.M.; Chang, C.Y. Nonlinear analysis of stress and strain in soils. *J. Soil Mech. Found Div. ASCE* 1970, 96, 1629–1653. [CrossRef]

24. Duncan, J.M. Strength Stress-Strain and Bulk Modulus Parameters for Finite Element Analysis of Stresses and Movements in Soil Masses; VCB/GT/78-02; University of California, Berkeley: Berkeley, CA, USA, 1978.

25. Liu, X.Q.; Liu, J.K.; Tian, Y.H.; Chang, D.; Hu, T.F. Influence of the Freeze-thaw Effect on the Duncan-Chang Model Parameter for Lean Clay. *Transp. Geotech.* 2019, 21, 100273. [CrossRef]

26. Hu, T.F.; Liu, J.K.; Chang, D.; Fang, J.H.; Xu, A.H. Influence of Freeze-thaw Cycling on Mechanical Properties of Silty Clay and Duncan-Chang Constitutive Model. *Zhongguo Gonglu Xuebao/China J. Highw. Transp.* 2018, 31, 298–307.

27. He, Y.; Chen, X. The Application of Improved Duncan-Chang Model in Unloading Soil. *Open Civ. Eng. J.* 2014, 8, 410–415. [CrossRef]

28. TB10102-2010; Code for Soil Test of Railway Engineering. China Railway First Survey and Design Institute Group: Beijing, China, 2010. (In Chinese)

29. Wang, D.; Ma, W.; Niu, Y.; Chang, X.; Wen, Z. Effects of cyclic freezing and thawing on mechanical properties of Qinghai–Tibet clay. *Cold Reg. Sci. Technol.* 2007, 48, 34–43. [CrossRef]

30. Yu, C.Y.; Tian, S.; Tang, L.; Ling, X.Z.; Zhou, G.Q. Finite element analysis on deformation of high embankment in heavy-haul railway subjected to freeze-thaw cycles. *Sci. Cold Arid Reg.* 2015, 7, 421–429. [CrossRef]