Transverse-Mass Effective Temperature in Heavy-Ion Collisions from AGS to SPS

Yu.B. Ivanov\textsuperscript{1,2} and V.N. Russkikh\textsuperscript{1,2}

\textsuperscript{1}Gesellschaft für Schwerionenforschung mbH, Planckstr. 1, D-64291 Darmstadt, Germany
\textsuperscript{2}Kurchatov Institute, Kurchatov sq. 1, Moscow 123182, Russia

Transverse-mass spectra in Au+Au and Pb+Pb collisions in incident energy range from 2\textsuperscript{A} to 160A GeV are analyzed within the model of 3-fluid dynamics (3FD). It is shown that dynamical description of freeze-out, accepted in this model, naturally explains the incident energy behavior of inverse-slope parameters of these spectra observed in experiment. Simultaneous reproduction of the inverse-slopes of all considered particles (p, π and K) suggests that these particles belong to the same hydrodynamic flow at the instant of their freeze-out.

Experimental data on transverse-mass spectra of kaons produced in central Au+Au\textsuperscript{1} or Pb+Pb\textsuperscript{2} collisions reveal peculiar dependence on the incident energy. The inverse-slope parameter (so called effective temperature $T$) of these spectra at mid rapidity increases with incident energy in the energy domain of BNL Alternating Gradient Synchrotron (AGS) and then saturates at energies of CERN Super Proton Synchrotron (SPS). In Refs.\textsuperscript{3,4} it was assumed that this saturation is associated with the deconfinement phase transition. This assumption was indirectly confirmed by the fact that microscopic transport models, based on hadronic degrees of freedom, failed to reproduce the observed behavior of the kaon inverse slope\textsuperscript{5,6}. Hydrodynamic simulations of Ref.\textsuperscript{7} succeeded to describe this behavior. However, in order to reproduce it these hydrodynamic simulations required incident-energy dependence of the freeze-out temperature which almost repeated the shape of the corresponding kaon effective temperature. This happened even in spite of using equation of state (EoS) involving the phase transition into quark-gluon plasma (QGP). Similar softening is needed for reproduction of recent data on rapidity distributions of net-baryon number in central Pb+Pb collision at energies 20A–80A GeV\textsuperscript{11}.

The transverse-mass spectra are most sensitive to the freeze-out parameters of the model. In fact, inverse slopes of these spectra represent a combined effect of the temperature and collective transverse flow of expansion. Fig.\textsuperscript{11} demonstrates these important interplay.

Had it been only the effect of thermal excitation, inverse slopes for different hadronic species would approximately equal. The collective transverse flow makes them different. These two effects partially compensate each other: the later freeze-out occurs, the lower temperature and the stronger collective flow are. Nevertheless, transverse-mass spectra turn out to be sensitive to the instant of the freeze-out.

3FD results for inverse-slope parameters of transverse-mass spectra of kaons, pions and protons produced in central Au+Au and Pb+Pb collisions are presented in Fig.\textsuperscript{11}. The inverse slopes $T$ were deduced by fitting the calculated spectra by the formula

$$\frac{d^2N}{m_T \, dm_T \, dy} \propto (m_T)^\lambda \exp \left( -\frac{m_T}{T} \right),$$

where $m_T$ and $y$ are the transverse mass and rapidity, respectively. Though the purely exponential fit with $\lambda = 0$ does not always provide the best fit of the spectra, it allows a systematic way of comparing spectra at different incident energies. In order to comply with experimental fits at AGS energies (and hence with displayed experimental points), we also present results with $\lambda = -1$ for pions and with $\lambda = 1$ for protons. These results are obtained with precisely the same EoS and set of parameters (friction, freeze-out and formation time) as those used in Ref.\textsuperscript{8}, which was found to be the best for other observables. No special tuning was done to reproduce these effective temperatures.

Numerical problems, discussed in Ref.\textsuperscript{8}, prevented us from simulations at RHIC energies. Already for the central Pb+Pb collision at the top SPS energy the code requires 7.5 GB of (RAM) memory. At the top RHIC en-
ergy, required memory is three order of magnitude higher, which is unavailable in modern computers.

As seen from Fig. 1, reproduction of effective temperatures is quite reasonable. Moreover, the pion and proton effective temperatures also reveal saturation at SPS energies, if they are deduced from the purely exponential fit with \( \lambda = 0 \). It is important that it is achieved with a single freeze-out parameter \( \varepsilon_{\text{frz}} = 0.4 \text{ GeV/fm}^3 \), the critical freeze-out energy density, which is the same for all considered incident energies above 2.4 GeV, both for chemical and thermal freeze-out. Only for smaller energies we used smaller values: \( \varepsilon_{\text{frz}}(2.4 \text{ GeV}) = 0.3 \text{ GeV/fm}^3 \) and \( \varepsilon_{\text{frz}}(1.4 \text{ GeV}) = 0.2 \text{ GeV/fm}^3 \). In order to clarify why this happens, let us turn to the 3FD freeze-out procedure, which is analyzed in Ref. [15] in more detail.

The freeze-out criterion we use is

\[
\varepsilon < \varepsilon_{\text{frz}}, \tag{2}
\]

where \( \varepsilon = u_\mu T^{\mu\nu} u_\nu \) is the total energy density of all three fluids in the proper reference frame, where the composed matter is at rest. This total energy density is defined in terms of the total energy–momentum tensor \( T^{\mu\nu} = T^{\mu\nu}_u + T^{\mu\nu}_t + T^{\mu\nu}_v \) being the sum of energy–momentum tensors \( T^{\mu\nu}_u \) of separate fluids (projectile-like, target-like and fireball ones) and the total collective 4-velocity of the matter \( u^\mu = u_\mu T^{\mu\nu} / u_\nu T^{\lambda\kappa} u_\kappa \). Note the latter definition is, in fact, an equation determining \( u^\mu \). A very important feature of our freeze-out procedure is an anti-bubble prescription. The matter is allowed to be frozen out only if

(a) either the matter is located near the boarder with vacuum (this piece of matter gets locally frozen out)

(b) or the maximal value of the total energy density in the system is less than \( \varepsilon_{\text{frz}} \)

\[
\max \varepsilon < \varepsilon_{\text{frz}} \tag{3}
\]

(the whole system gets instantly frozen out).

In the 3FD model this freeze-out simultaneously terminates both chemical and kinetic processes.

Before the instant of the global freeze-out, cf. [3], the freeze-out remove matter from the surface of the hydrodynamically expanding system. This removed matter gives rise to observable spectra of hadrons. This kind of freeze-out is similar to the model of “continuous emission” proposed in Ref. [16]. There the particle emission occurs from a surface layer of the mean-free-path width. In our case the physical pattern is the same, only the mean free path is shrunk to zero.

Condition [2] ensures only that the actual freeze-out energy density (let us call it \( \varepsilon_{\text{out}} \), at which the freeze-out actually occurs, is less than \( \varepsilon_{\text{frz}} \). Therefore, \( \varepsilon_{\text{frz}} \) can be called a "trigger" value of the freeze-out energy density. As explained in Ref. [13], a natural value of this actual freeze-out energy density is \( \varepsilon_{\text{out}} \approx \varepsilon_s / 2 \), i.e. at that the middle of the fall from the near-surface value of the energy density, \( \varepsilon_s \), to zero. To find out the actual value of \( \varepsilon_{\text{out}} \), we have to analyze results of a particular simulation.

In our previous paper [8] we have performed only a rough analysis of this kind. This is why in the main text of Ref. [8] we mentioned the value of approximately 0.2 GeV/fm\(^3\) for \( \varepsilon_{\text{out}} \) and in appendix explained how the freeze-out actually proceeded. [In terms of Ref. [8] (\( \varepsilon_{\text{frz}[1]} \) and \( \varepsilon_{\text{frz}[2]} \)) our present quantities are \( \varepsilon_{\text{frz}} = \varepsilon_{\text{frz}[1]} \) and \( \varepsilon_{\text{out}} = \varepsilon_{\text{frz}[1]} \).] Results of more comprehensive analysis for central \((b = 0)\) Pb+Pb collisions are presented in Fig.
which shows the $\langle \varepsilon_{\text{out}} \rangle$ value averaged over space–time evolution of the collision: $\langle \varepsilon_{\text{out}} \rangle$. As seen, $\langle \varepsilon_{\text{out}} \rangle$ reveals saturation at the SPS energies, very similar to that in effective temperatures in Fig. 3. This happens in spite of the fact that our freeze-out condition involves only a single constant parameter $\varepsilon_{\text{frz}}$.

The "step-like" behavior of $\langle \varepsilon_{\text{out}} \rangle$ is a consequence of the freeze-out dynamics, as it was demonstrated in Ref. [14]. At low (AGS) incident energies, the energy density achieved at the border with vacuum, $\varepsilon_s$, is lower than $\varepsilon_{\text{frz}}$. Therefore, the surface freeze-out starts at lower energy densities. It further proceeds at lower densities up to the global freeze-out because the freeze-out front moves not faster than with the speed of sound, like any perturbation in the hydrodynamics. Hence it cannot overcome the supersonic barrier and reach dense regions inside expanding system. With the incident energy rise the energy density achieved at the border with vacuum gradually reaches the value of $\varepsilon_{\text{frz}}$ and then even overshoot it. If the overshoot happens, the system first expands without freeze-out. The freeze-out starts only when $\varepsilon_s$ drops to the value of $\varepsilon_{\text{frz}}$. Then the surface freeze-out occurs really at the value $\varepsilon_s \approx \varepsilon_{\text{frz}}$ and thus the actual freeze-out energy density saturates at the value $\langle \varepsilon_{\text{out}} \rangle \approx \varepsilon_{\text{frz}}/2$. This freeze-out dynamics is quite stable with respect to numerics [15].

This hydrodynamic explanation of the considered "step-like" behavior of effective temperatures is a signal of phase transition into QGP, we should admit that this is not quite clear as yet. It depends on the nature of the freeze-out parameter $\varepsilon_{\text{frz}} = 0.4$ GeV/fm$^3$ which should be further clarified. EoS is not of prime importance for this behavior. The only constrain on the EoS is that it should be in some way reasonable. Moreover, our preliminary results indicate that a completely different EoS [20] with 1st order phase transition to QGP still reasonably reproduces this "step-like" behavior even in spite of that it fails to describe a large body of other data. This happens because the same freeze-out pattern is accepted there.

In fact, EoS is just the pressure as a function of baryon and energy densities: $P(n_B, \varepsilon)$. In this calculation we used hadronic EoS [21] in order to construct the 1st order phase transition to QGP or quasiparticle fits to lattice QCD data [22] in order to construct the 1st order phase transition.
slopes of all considered particles ($\rho$, $\pi$ and $K$) implies that these particles belong to the same hydrodynamic flow at the instant of their freeze-out. An indirect support of this conjecture is the recent success of the GiBUU model\cite{22} in reproduction of kaon inverse-slopes. That was achieved by taking into account three-body interactions, which essentially increased the equilibration rate.

We are grateful to I.N. Mishustin, L.M. Satarov, V.V. Skokov, V.D. Toneev, and D.N. Voskresensky for fruitful discussions. This work was supported the Deutsche Forschungsgemeinschaft (DFG project 436 RUS 113/558/0-3), the Russian Foundation for Basic Research (RFBR grant 06-02-04001 NNIO), Russian Federal Agency for Science and Innovations (grant NSh-8756.2006.2).

[1] L. Ahle, et al., Phys. Lett. B476 (2000) 1.
[2] S. V. Afanasiev, et al., Phys. Rev. C 66 (2002) 054902; C. Alt, et al., J. Phys. G30 (2004 S11); M. Gazdzicki, et al., Phys. G30 (2004) S701.
[3] M.I. Gorenstein, M. Gazdzicki, and K. Bugaev, Phys. Lett. B567 (2003) 175.
[4] B. Mohanty, et al., Phys. Rev. C 68 (2003) 021901.
[5] E.L. Bratkovskaya, M. Bleicher, M. Reiter, S. Soff, H. Stoecker, M. van Leeuwen, S. Bass, and W. Cassing, Phys. Rev. C 69 (2004) 054907; E.L. Bratkovskaya, S. Soff, H. Stoecker, M. van Leeuwen, and W. Cassing, Phys. Rev. Lett. 92 (2004) 032302.
[6] M. Wagner, A.B. Larionov, and U. Mosel, Phys. Rev. C 71 (2005) 034910.
[7] M. Gazdzicki, M.I. Gorenstein, F. Grassi, Y. Hama, T. Kodama, and O. Socolowski Jr, Braz. J. Phys. 34 (2004) 322.
[8] Yu.B. Ivanov, V.N. Russkikh, and V.D. Toneev, Phys. Rev. C 73 (2006) 044904.
[9] V.D. Toneev, Yu.B. Ivanov, E.G. Nikonov, W. Norenberg, and V.N. Russkikh, Phys. Part. Nucl. Lett. 2 (2005) 288; V.N. Russkikh, Yu.B. Ivanov, E.G. Nikonov, W. Norenberg, and V.D. Toneev, Phys. Atom. Nucl. 67 (2004) 199.
[10] V.N. Russkikh and Yu.B. Ivanov, Phys. Rev. C 74 (2006) 034904.
[11] Yu.B. Ivanov and V.N. Russkikh, PoS(CPOD07)008, arXiv:0710.3708 [nucl-th].
[12] V.M. Galitsky and I.N. Mishustin, Sov. J. Nucl. Phys. 29 (1979) 181.
[13] B.B. Back et al., Phys. Rev. C 66 (2002) 054901.
[14] T. Anticic et al., Phys. Rev. C 69 (2004) 024902; C. Alt et al., Phys. Rev. C 73 (2006) 044910.
[15] V.N. Russkikh and Yu.B. Ivanov, Phys. Rev. C 76 (2007) 054907.
[16] F. Grassi, Y. Hama, and T. Kodama, Phys. Lett. B355 (1995) 9; Z. Phys. C73 (1996) 153.
[17] A. Andronic, P. Braun-Munzinger, and J. Stachel, Nucl. Phys. A772 (2006) 167.
[18] J. Cleymans, H. Oeschler, and K. Redlich, J. Phys. G32 (2006) S165; J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, Phys. Rev. C 73 (2006) 034905.
[19] J. Randrup and J. Cleymans, Phys. Rev. C 74 (2006) 047901.
[20] A.S.Khvorostukhin, V.V.Skokov, K.Redlich, and V.D.Toneev, Eur. Phys. J. C48 (2006) 531.
[21] Yu.B. Ivanov, V.V. Skokov, and V.D. Toneev, Phys. Rev. D 71 (2005) 014005; Yu.B. Ivanov, A.S. Khvorostukhin, E.E. Kolomeitsev, V.V. Skokov, V.D. Toneev, and D.N. Voskresensky, Phys. Rev. C 72 (2005) 025804.
[22] A.B. Larionov, O. Buss, K. Gallmeister, and U. Mosel, Phys. Rev. C 76 (2007) 044909.