Experimental demonstration of broadband reconfigurable mechanical nonreciprocity

Amin Mehrvarz\textsuperscript{1,\dagger}, Mohammad Javad Khodaei\textsuperscript{1,\dagger}, Amir Darabi\textsuperscript{2}, Ahmad Zareei\textsuperscript{3}, and Nader Jalili\textsuperscript{4,5}

Abstract
Breaking reciprocity has recently gained significant attention due to its broad range of applications in engineering systems. Here, we introduce the first experimental demonstration of a broadband mechanical beam waveguide, which can be reconfigured to represent wave nonreciprocity. This is achieved by using spatiotemporal stiffness modulation with piezoelectric patches in a closed-loop controller. Using a combination of analytical methods, numerical simulations, and experimental measurements, we show that contrary to the conventional shunted piezoelectrics or nonlinearity based methods, our setup is stable, less complicated, reconfigurable, and precise over a broad range of frequencies. Our reconfigurable nonreciprocal system has potential applications in phononic logic, wave diodes, energy trapping, and localization.

Keywords
nonreciprocity, spatiotemporal modulation, mechanical waveguide, smart material, wave propagation

1. Introduction
Reciprocity is a fundamental property of various physical systems, where the transmission of a physical quantity, such as waves, between 2 points in space is symmetrical. Breaking this reciprocity offers an enhanced control over wave signal transmission and has recently become of interest in many branches of physics such as optics Miri et al. (2017); Sounas and Aila (2017), electronics Dobson (1995), thermodynamics Torrent et al. (2018); Nakai and Nagaosa (2019), electromagnetism Mahmoud et al. (2015); Caloz et al. (2018), acoustics Nassar et al. (2020); Fleury et al. (2014), and classical mechanics Sugino et al. (2020); Marconi et al. (2020). In mechanical systems, breaking wave reciprocity has numerous applications in trapping waves for efficient energy harvesting devices and designing transistors in mechanical logic circuits Cummer et al. (2016); Cullen (1958); Hadad et al. (2016); Felsen and Whitman (1970); Auld et al. (1968).

Breaking reciprocity in mechanical systems can be achieved either by (i) passively employing nonlinear elements in an asymmetric structure or (ii) actively varying material properties in space and time periodically Casimir (1945). Passive methods of breaking reciprocity require high wave amplitudes, and as a result are impractical in compact devices Sugino et al. (2020). Additionally, since passively designed structures cannot be reconfigured or reprogrammed due to their static design, creating robust devices for broad frequency ranges arises additional complexities Darabi et al. (2019); Franck et al. (2019); Wu et al. (2018); Boechler et al. (2011). Active metamaterials, on the other hand, are reprogrammable and tunable by

\textsuperscript{1}Department of Mechanical and Industrial Engineering, Northeastern University, Boston, MA, USA
\textsuperscript{2}Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA, USA
\textsuperscript{3}Harvard John A. Paulson School of Engineering and Applied Sciences, Harvard University, Cambridge, MA, USA
\textsuperscript{4}Department of Mechanical Engineering, The University of Alabama, Tuscaloosa, AL, USA
\textsuperscript{5}Department of Mechanical and Industrial Engineering, Northeastern University, Boston, MA, USA

Received: 15 May 2022; accepted: 30 October 2022
\textsuperscript{\dagger}Amin Mehrvarz and Mohammad Javad Khodaei contributed equally to this work

Corresponding authors:
Nader Jalili, Department of Mechanical Engineering, The University of Alabama, Tuscaloosa, AL 3548, USA.
Email: njalili@ua.edu

Ahmad Zareei, Harvard John A. Paulson School of Engineering and Applied Sciences, Harvard University, Cambridge, MA, 02138, USA.
Email: ahmad@seas.harvard.edu

Amir Darabi, Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA, 30332, USA.
Email: amirdarabi@gatech.edu
leveraging the active spatiotemporal modulations. Such active metamaterials offer an effective platform for breaking reciprocity Riva et al. (2019); Trainiti and Ruzzene (2016); Nassar et al. (2017), and as a result, have found many applications in (i) increasing the width of bandgaps Airoldi and Ruzzene (2011), (ii) focusing or redirecting wave propagation Celli et al. (2017); Darabi et al. (2018a); Zareei et al. (2018), (iii) changing the amplitude and phase of transmitted and reflected waves Chen et al. (2018), and (iv) one-way wave blocking and cloaking Ning et al. (2019); Darabi et al. (2018b).

In active metamaterials, active elements such as magnetoelastic Chen et al. (2016); Korivand et al. (2021), photosensitive Yannopoulos and Trunov (2009), or piezoelectric Airoldi and Ruzzene (2011); Mehrvarz et al. (2019) are used to modulate physical properties. In the photosensitive and magnetoelastic materials, the stiffness can be varied by changing the magnetic field Chen et al. (2016); Korivand et al. (2021) and temperature Yannopoulos and Trunov (2009), respectively. In piezoelectric transducers, however, the stiffness is modulated by connecting these patches to shunted circuits Airoldi and Ruzzene (2011); Marconi et al. (2020) with negative capacitance Tateo et al. (2015). Although shunted piezoelectric patches are precise Darabi et al. (2020); Sugino et al. (2020); Trainiti et al. (2019); Chen et al. (2019) and able to function in a wide frequency range, a dramatic change in the equivalent Young’s modulus only happens when the system operates very close to the unstable zones of the circuit Airoldi and Ruzzene (2011). As such, the shunted piezoelectric patches are prone to an error where a small variation of the applied negative capacitance can make the system unstable and result in a large deviation from desired values Tateo et al. (2015). Additionally, implementing a system with shunted piezoelectric patches is extremely challenging since each piezoelectric element is separately connected to its own independent circuit and this requires working with too many elements and connections.

Here, we present a closed-loop feedback control system connected to two sets of parallel piezoelectric patches1 bonded on a host beam to modulate the beam’s stiffness. The controller here continually measures the voltage from one set of PZTs and applies the required analog signals to the other set, and as such, changes the beam’s effective stiffness Li et al. (2017); Sugino et al. (2018). Contrary to shunted PZTs with negative capacitance circuits, a closed-loop stiffness modulation system is stable and precise with reduced complexity and has functionality over a broad range of frequencies. As a result, stiffness modulation with a closed-loop control system offers a robust platform for breaking reciprocity. Here, we first analytically/numerically show that spatiotemporal (or spatial) stiffness modulation in beam results in directional (or total) bandgaps. We then experimentally exhibit the bandgaps using a closed-loop controlled system. We show that different behaviors (i.e., wave transmission, blockage, and nonreciprocal transmission) are all attainable on the same reconfigurable beam setup by only tuning the controller’s parameters.

2. Model

We consider elastic wave propagation in a thin beam with linear mass density $\rho$, rigidity $\frac{E}{\ell}$, second moment of inertia $I_o$, and stiffness $D_0 = E_i d_0$ where the displacement $w$ is governed by the Euler-Bernoulli equation This straightforward equation can clearly show the concept of nonreciprocity.

$$\rho \frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial x^2} \left( D(x,t) \frac{\partial^2 w(x,t)}{\partial x^2} \right) = 0. \quad (1)$$

Assuming a spatiotemporal modulation in stiffness with spatial modulation wavelength $\lambda_m$ and temporal modulation period $T_m$, the stiffness is obtained as $D(x,t) = D_0 (1 + a_m \cos(\omega_m t - \alpha_m x))$, where $\alpha_m = 2\pi/\lambda_m$ is the spatial modulation wavelength, $a_m = 2\pi T_m$ is the temporal modulation angular frequency, and $a_m$ is the modulation amplitude (Figure 1(b)). Inserting the modulation stiffness, $D(x,t)$ into equation (1) and taking the Fourier transform, we find the characteristic equation

$$a_m \gamma^2 \frac{\partial^2}{\partial t^2} w_n + \frac{\partial^2}{\partial x^2} \left( \frac{\partial^2 w_n}{\partial x^2} \right) = 0,$$

where $\gamma = D_0/\rho$ and $w_n$ is the $n^{th}$ mode of the wave amplitude Fourier transform given by $w(x,t) = e^{i(\omega t - kx)} \sum_{n=-\infty}^{+\infty} \tilde{w}_n e^{i(\omega_m t - \alpha_m x)}$.

The band diagram of a spatiotemporally modulated beam is the solution of equation (2) and can be found for different modulation parameters $k_m$, $\omega_m$, and $a_m$. Achieving different behaviors is possible by selecting proper modulation functions and simply changing the controller’s parameters accordingly (see Supplementary Information for the effects of density and stiffness modulation with or without a phase difference between them.) To avoid nonlinear effects, we assume a small fixed value of $a_m = 0.15$ in our modulation. Initially assuming only a spatial modulation, that is, $k_m \neq 0$, $\omega_m = 0$, the band diagram exhibits a full bandgap as shown in Figure 2(a). Next, by introducing and varying temporal modulation, that is, $\omega_m \neq 0$, we find that the bandgap becomes asymmetrical (Figure 2(b)). Furthermore, increasing the temporal modulation results in a fully asymmetrical bandgap, which has a nonreciprocal wave behavior with unidirectional wave propagation (Figure 2(c)). In the case of only temporal modulation, that is, $k_m = 0$, $\omega_m \neq 0$, we find that the bandgaps are shifted to the wavenumbers where certain wavenumbers are not allowed in the system.
Next, we use a closed-loop circuit with two parallel PZT patches (one actuator and one sensor) on both sides of the beam to implement spatiotemporal modulation in stiffness. Based on the Euler-Bernoulli theory, the stiffness in a beam is defined \( M = -D w_{xx} \), where \( M \) is the moment and \( w_{xx} \) is the curvature of the beam. When a PZT actuator and a PZT sensor are attached on each side of a beam extending between \( x_l \) and \( x_r \), the stiffness relation is modified to Jalili (2010)

\[
D = \frac{\int_{x_l}^{x_r} M dx}{\int_{x_l}^{x_r} w_{xx} dx} = D_0 + D_p - \frac{K_p V_a}{\int_{x_l}^{x_r} w_{xx} dx},
\]

where \( D_p \) is the stiffness of the PZT patches, \( V_a \) is the PZT actuator voltage, and \( K_p \) is a factor that depends on PZT parameters (see Supplementary Information for more details). If the actuator’s voltage, \( V_a \), becomes

\[
V_a = \frac{(D_p - aD_0)}{K_p} \int_{x_l}^{x_r} w_{xx} dx,
\]

then the beam’s rigidity changes to \( D = D_0(1 + \alpha) \). Note that \( \alpha \) is the modulation amplitude and can depend on the location of PZT, \( x \), and also time \( t \). To apply the actuator’s voltage in equation (4), the only unknown parameter on the right-hand side is the change in the beam’s slope, that is, \( \int_{x_l}^{x_r} w_{xx} dx \), where it can be obtained based on the PZT sensor and its dielectric permittivity Yi et al. (2019). In a PZT sensor \( V_s = C_p \int_{x_l}^{x_r} w_{xx} dx \), where \( V_s \) is the sensor’s voltage and \( C_p \) is a factor that depends on PZT parameters (see Supplementary Information for the details). As a result, given sensor voltage \( V_s \), if the actuator voltage is applied as

\[
V_a = \frac{(D_p - aD_0)}{K_p C_p} V_s,
\]

the flexural rigidity of the beam becomes \( D = D_0(1 + \alpha) \). Modulating the coefficient \( \alpha \) using \( \alpha = \alpha_m \cos(\omega_m t + \phi) \) for each PZT set (sensor and actuator) and applying the phase \( \phi \) based on the location of the PZT set, a spatiotemporal modulation can be achieved.

**3. Numerical results**

We first numerically test the effect of spatiotemporal modulation of the PZTs on an aluminum beam using Finite Element simulations in COMSOL Multiphysics. Finding the best results which have the closest results to the theoretical results is the goal of this simulation. We consider an aluminum beam with the thickness of 1 mm, width of 2.54 mm, and length of 1.5 m. We further assume 12 pairs of PZT patches perfectly bonded on both sides of the beam.

(see Figure 2(d) Trainiti et al. (2019). (A continuous changing of the modulation parameters (i.e. \( k_m, \omega_m \)) can provide a better insight into spatiotemporal modulation, shown in supplementary videos A and B).
Each PZT covers an area of 21 mm × 21 mm, thickness 0.55 mm, and we have a distance of 5 mm in-between them (see Figure 3(a)). A few of the parameters are general and not dependent on the experimental setup, parameters such as frequency of time modulation and the phase of modulation that can be tuned in the feedback controller. On the other hand, for the other parameters, such as frequency of space modulation we are limited to the experimental setup because it is related to the length of PZTs and the distance between them. So, we choose these numbers based on an experimentally feasible setup. A low reflection boundary is used on both sides of the beam to avoid reflecting the wave. The closed-loop parameters are selected so that the stiffness of the piezoelectric actuators vary by 15%, i.e. \( \alpha_m = 0.15 \). The closed-loop feedback controller based on equation (5) is used to modulate the PZT (see Figure 3(c)). The phase difference between subsequent PZTs is set to \( \phi = 2 \pi/3 \), which results in three PZT pairs per spatial wavelength \( \lambda_s \) (Figure 3(b)). It is to be noted that the equivalent stiffness of each PZT pair, contrary to the shunted circuits, can vary continuously based on the continuous modulation signal (see Figure 3(b)). To test the setup, the wave is initiated on one side of the structure, and measurements are done on both sides of the spatiotemporally modulated section. Finally, a sweep over a range of wave frequencies is done to obtain the transfer function (i) in the absence of any modulation, in the presence of (ii) spatial modulation (i.e., \( \omega_m = 0, k_m \neq 0 \)), or (iii) spatiotemporal modulation (i.e., \( \omega_m \neq 0, k_m \neq 0 \)).

In Figure 3(d)–(e), we plot the transfer function, that is, the ratio of transmitted wave amplitude to incident wave amplitude, in the frequency range 2 – 10 kHz for the spatial/spatiotemporal modulation of the beam. Here, the transmission ratio below \( r_t \leq 0.1 \) is considered blockage because the energy level, which is proportional to the deflection square, drops more than 0.01 Kheif and Adibi (2015). Note that this transmission level corresponds to \( -20dB \) which is commonly considered as bandgap Yi et al. (2017); Zouari et al. (2018); Tang and Cheng (2017). First, in the absence of modulation, no bandgap is observed over the frequency range of interest (black line in Figure 3(d)). In the presence of spatial modulation, however, the transfer function changes and results in identical transfer functions for left-to-right (LR) and right-to-left (RL) wave of propagation (see red and blue lines in Figure 3(d), respectively). Additionally, the results show two identical bandgaps for opposite propagation direction (LR and RL) bounded between 2.5 – 4.5 kHz, marked with yellow boxes in Figure 3(d), and 7–8.5 kHz. Interestingly, the first bandgap contains a localized mode at 3.16 kHz, which dominates the transfer function response, and results in a spike in the transmission ratio in the bandgap. Next, we run the spatiotemporal modulated simulations (i.e., \( \omega_m \neq 0, k_m \neq 0 \)) and report the results in Figure 3(e) for the wave propagation in LR and RL directions. This figure shows the presence of three directional bandgaps (marked with red rectangles), in which the magnitude of the wave for LR is different from RL. As demonstrated, for two of these bandgaps (2.65–2.86 kHz, and 6.46–7.48 kHz), waves only travel for LR, while for the (7.97–8.69 kHz) waves propagate in the opposite direction. Furthermore, bandgaps move to lower frequencies for the RL and higher frequencies for the LR, respectively. Nonetheless, this change is not visible for the first bandgap due to the presence of localized mode in this frequency range. See Supplementary Information for the band structure diagram of FEM.

4. Experimental results

Next, to show the feasibility of having nonreciprocity with the proposed structure, we implement the time-periodic
stiffness modulation of the elastic waveguide beam using an array of PZT actuators and sensors attached on a flexible aluminum beam controlled using closed-loop circuits (Figure 4). The aluminum beam has a rectangular cross-section similar to the numerical simulation, and layers of Butyl rubber at its boundaries damp the wave reflection on the structure. The waves are produced using Macro Fiber Composite (MFC) actuators that are bonded on two ends of the beam, and a laser vibrometer (Polytec CLV-2544) measures the system’s response at a distance of 5 mm before and after the modulated section (Figure 4(a)). The voltages of PZT sensors are measured using a DS1006 R&D controller board. The measured voltages are used in the closed-loop circuits in Simulink-MATLAB with reconfigurable parameters of equation (5) (see Supplementary Information for more details). As can be seen in Figure 4(a), ControlDesk software with DS1006 R&D controller board applies the control voltages to the piezodrive amplifier, and the outputs are applied to the PZT actuators. See Figure 4(b) for the experimental setup. The experimental results for the transformation ratio without any modulation are shown in Figure 4(c) with a solid black line. Introducing spatial modulation with \( k_m = 80.5 \text{ m}^{-1} \), the transmission ratio changes and results in two identical bandgaps for RL and LR wave propagation direction at frequencies 2.86–3.33 kHz and 3.18–3.6 kHz (shown with yellow boxes in Figure 4(c)). Note that in the analytical results, this bandgap is continuous from 2.86 to 3.6 kHz without any break in the middle. However, in the experiments similar to the numerical simulations, this bandgap breaks into two separate bandgaps due to an internal localized mode of the system. Introducing a time modulation in addition to the spatial modulation with \( f_m = 400 \text{ Hz} \) results in asymmetrical bandgaps that depend on the direction of wave propagation. Figure 4(d) shows the presence of four directional bandgaps (red boxes), in which, same as the numerical simulations, the magnitude of the wave for LR is different from RL. Interestingly, two of the bandgaps (specifically 2.84–2.9 kHz, and 3.16–3.27 kHz) allow only LR wave propagation (RL nonreciprocal bandgaps), while on the contrary, the other two bandgaps (i.e., 3.01–3.13 kHz and 3.57–3.6 kHz) allow only RL waves propagation (LR nonreciprocal bandgaps). Moreover, similar to the numerical results, bandgaps move to lower/higher frequencies for the RL/LR bandgaps. It should be noted that two of the directional bandgaps are observed only because the bandgaps are in the vicinity of the interior localized mode. Additionally, we expect the experimental result to defer from the numerics due to the following sources of error, (i) In the numerical simulations, the contacts between the PZTs and beam are assumed to be perfect contacts; however, there is an adhesive layer between the PZTs and aluminum beam that changes the effective thickness of the system but also changes the corresponding density of the system Rabinovitch and Vinson (2002). Additionally, (ii) our experiments include structural damping and nonlinear effects, which are ignored in the simulations and theory. Lastly, (iii) including the equipment’s error (e.g., signal generator, piezodrive amplifier, and controller) would further defer our experimental result from the numerical simulation.
5. Conclusion
In summary, we experimentally implemented a beam structure to achieve total/nonreciprocal bandgaps. For the first time, an active closed-loop feedback control system was employed using an array of PZTs bonded on both sides of an aluminum host layer to implement the stiffness modulation. At first, we investigated the changing effects of spatiotemporal parameters on the bandgaps. Then, we numerically simulated spatial/spatiotemporal stiffness modulation using the closed-loop system and validated our analytical results. Finally, we experimentally showed that actively controlled stiffness modulation using closed-loop circuits is reconfigurable and can be used for both spatial and spatiotemporal modulation and has potential applications in waveguides, diodes, phononic logic circuits, or energy localization.

Declaration of conflicting interests
The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding
The authors received no financial support for the research, authorship, and/or publication of this article.

ORCID iDs
Amin Mehrvarz https://orcid.org/0000-0003-3996-9348
Mohammad Javad Khodaei https://orcid.org/0000-0002-2204-937X
Nader Jalili https://orcid.org/0000-0002-0868-801X

Supplemental Material
Supplemental material for this article is available online.

Note
1. More specifically, the PZT-5J type, for convenience, PZT refers to this material in the rest of this manuscript.

References
Airoldi L and Ruzzene M (2011) Design of tunable acoustic metamaterials through periodic arrays of resonant shunted piezoelectric elements. New Journal of Physics 13(11): 113010.
Auld B, Collins JH and Zapp H (1968) Signal processing in a nonperiodically time-varying magnetoelastic medium. Proceedings of the IEEE 56(3): 258–272.
Boechler N, Theocharis G and Daraio C (2011) Bifurcation-based acoustic switching and rectification. Nature Materials 10(9): 665–668.
Caloz C, Alù A, Tretyakov S, et al. (2018) Electromagnetic nonreciprocity. Physical Review Applied 10(4): 047001.
Casimir HBG (1945) On onsager’s principle of microscopic reversibility. Reviews of Modern Physics 17(2–3): 343–350.
Celli P, Gonella S, Tajeddini V, et al. (2017) Wave control through soft microstructural curling: bandgap shifting, reconfigurable anisotropy and switchable chirality. Smart Materials and Structures 26(3): 035001.
Chen Y, Hu J and Huang G (2016) A design of active elastic metamaterials for control of flexural waves using the transformation method. Journal of Intelligent Material Systems and Structures 27(10): 1337–1347.
Chen Y, Li X, Nassar H, et al. (2018) A programmable metasurface for real time control of broadband elastic rays. Smart Materials and Structures 27(11): 115011.
Chen Y, Li X, Nassar H, et al. (2019) Nonreciprocal wave propagation in a continuum-based metamaterial with space-time modulated resonators. Physical Review Applied 11(6): 064052.
Cullen A (1958) A travelling-wave parametric amplifier. Nature 181(4605): 332–332.
Cummer SA, Christensen J and Alù A (2016) Controlling sound with acoustic metamaterials. Nature Reviews Materials 1(3): 16001.
Darabi A, Fang L, Mohajed H, et al. (2019) Broadband passive nonlinear acoustic diode. Physical Review B 99(21): 214305.
Darabi A, Ni X, Leamy M, et al. (2020) Reconfigurable foquet elastodynamic topological insulator based on synthetic angular momentum bias. Science Advances 6(29): eaaba8566.
Darabi A, Zareei A, Alam MR, et al. (2018a) Broadband bending of flexural waves: acoustic shapes and patterns. Scientific Reports 8(1): 1–7.
Darabi A, Zareei A, Alam MR, et al. (2018b) Experimental demonstration of an ultrabroadband nonlinear cloak for flexural waves. Physical Review Letters 121(17): 174301.
Dobson I (1995) Stability of ideal thyristor and diode switching circuits. IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications 42(9): 517–529.
Felsen L and Whitman G (1970) Wave propagation in time-varying media. IEEE Transactions on Antennas and Propagation 18(2): 242–253.
Fleury R, Soumas DL, Sieck CF, et al. (2014) Sound isolation and giant linear nonreciprocity in a compact acoustic circulator. Science 343(6170): 516–519.
Fronk MD, Tawfick S, Daraio C, et al. (2019) Acoustic nonreciprocity in lattices with nonlinearity, internal hierarchy, and asymmetry: computational study. Journal of Vibration and Acoustics 141(5): 051011 1–11.
Hadad Y, Soric JC and Alu A (2016) Breaking temporal symmetries for emission and absorption. Proceedings of the National Academy of Sciences 113(13): 3471–3475.
Jalili N (2010) Piezoelectric-Based Systems Modeling. Boston, MA: Springer US, pp. 183–232. DOI: 10.1007/978-1-4419-0070-8 8.
Khelif A and Adibi A (2015) Phononic Crystals. Berlin, Germany: Springer.
Korivand S, Mehrvarz A, Candelino N, et al. (2021) Band gap and stiffness tuning of a sandwich plate by magnetostreictive materials. ASME Letters in Dynamic Systems and Control 1(4): 041007.
Li F, Zhang C and Liu C (2017) Active tuning of vibration and wave propagation in elastic beams with periodically placed piezoelectric actuator/sensor pairs. Journal of Sound and Vibration 393: 14–29.
Mahmoud AM, Davoyan AR and Engheta N (2015) All-passive nonreciprocal metastructure. *Nature Communications* 6(1): 1–7.

Marconi J, Riva E, Di Ronco M, et al. (2020) Experimental observation of nonreciprocal band gaps in a space-time-modulated beam using a shunted piezoelectric array. *Physical Review Applied* 13(3): 031001.

Mehrvarz A, Najafi Ardekani A, Khodaei MJ, et al. (2019) Vibration analysis and control of fluid containers using piezoelectrically-excited side wall. *Journal of Vibration and Control* 25(7): 1393–1408.

Miri MA, Ruesink F, Verhagen E, et al. (2017) Optical nonreciprocity based on optomechanical coupling. *Physical Review Applied* 7(6): 064014.

Nakai R and Nagaosa N (2019) Nonreciprocal thermal and thermoelectric transport of electrons in noncentrosymmetric crystals. *Physical Review B* 99(11): 115201.

Nassar H, Xu X, Norris A, et al. (2017) Modulated phononic crystals: non-reciprocal wave propagation and willis materials. *Journal of the Mechanics and Physics of Solids* 101: 10–29.

Nassar H, Yousefzadeh B, Fleury R, et al. (2020) Nonreciprocity in acoustic and elastic materials. *Nature Reviews Materials* 5(9): 667–685.

Ning L, Wang YZ and Wang YS (2019) Active control of elastic metamaterials consisting of symmetric double helmholtz resonator cavities. *International Journal of Mechanical Sciences* 153: 287–298.

Rabinovitch O and Vinson JR (2002) Adhesive layer effects in surface-mounted piezoelectric actuators. *Journal of Intelligent Material Systems and Structures* 13(11): 689–704.

Riva E, Marconi J, Cazzulani G, et al. (2019) Generalized plane wave expansion method for non-reciprocal discretely modulated waveguides. *Journal of Sound and Vibration* 449: 172–181.

Sounas DL and Al`u A (2017) Non-reciprocal photonics based on time modulation. *Nature Photonics* 11(12): 774–783.

Sugino C, Ruzzene M and Erturk A (2018) Design and analysis of piezoelectric metamaterial beams with synthetic impedance shunt circuits. *IEEE/ASME Transactions on Mechatronics* 23(5): 2144–2155.

Sugino C, Ruzzene M and Erturk A (2020) Nonreciprocal piezoelectric metamaterial framework and circuit strategies. *Physical Review B* 102(1): 014304.

Tang L and Cheng L (2017) Broadband locally resonant band gaps in periodic beam structures with embedded acoustic black holes. *Journal of Applied Physics* 121(19): 194901.

Tateo F, Collet M, Ouisse M, et al. (2015) Experimental characterization of a bi-dimensional array of negative capacitance piezo-patches for vibroacoustic control. *Journal of Intelligent Material Systems and Structures* 26(8): 952–964.

Torrent D, Poncelet O and Batsale JC (2018) Nonreciprocal thermal material by spatiotemporal modulation. *Physical Review Letters* 120(12): 125501.

Trainiti G and Ruzzene M (2016) Non-reciprocal elastic wave propagation in spatiotemporal periodic structures. *New Journal of Physics* 18(8): 083047.

Trainiti G, Xia Y, Marconi J, et al. (2019) Time-periodic stiffness modulation in elastic metamaterials for selective wave filtering: theory and experiment. *Physical Review Letters* 122(12): 124301.

Wu Z, Zheng Y and Wang K (2018) Metastable modular metamaterials for on-demand reconfiguration of band structures and nonreciprocal wave propagation. *Physical Review E* 97(2): 022209.

Yannopoulos S and Trunov M (2009) Photoplastic effects in chalcogenide glasses: a review. *Physica Status Solidi (b)* 246(8): 1773–1785.

Yi K, Collet M and Karkar S (2017) Frequency conversion induced by time-space modulated media. *Physical Review B* 96(10): 104110.

Yi K, Ouisse M, Sadoulet-Reboul E, et al. (2019) Active metamaterials with broadband controllable stiffness for tunable band gaps and non-reciprocal wave propagation. *Smart Materials and Structures* 28(6): 065025.

Zareei A, Darabi A, Leamy MJ, et al. (2018) Continuous profile flexural grin lens: focusing and harvesting flexural waves. *Applied Physics Letters* 112(2): 023901.

Zouari S, Brocail J and Génevaux JM (2018) Flexural wave band gaps in metamaterial plates: a numerical and experimental study from infinite to finite models. *Journal of Sound and Vibration* 435: 246–263.