Calculation of shock absorber parameters of torsional vibrations of screw pumps' rod string

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Abstract. Torsional vibrations of the rod string occur during the operation of surface-driven screw pumping units, which reduces the reliability of the units. It is proposed to include a torsional vibration damper in the rod string to maintain stability. The calculation of the parameters of the shock absorber is presented.

1. Introduction
The use of screw pump installations with a surface drive and having a low rotor speed is a promising method of operating wells with low flow rates and high-viscosity oil. During operation of screw pump installations with a surface drive, oscillatory processes of downhole equipment occur, which reduce the durability of the installation elements, increase the frequency of rod string breakage and wear of the friction surfaces of the screw and the rubber stator lining. Oscillatory processes during operation of screw pump installations with surface generally drive negatively affect the operating parameters of screw pump installations. Elimination of vibration processes in the rod string layout suggests including a torsional vibration damper [1].

Determination of the optimal parameters of the torsional vibration damper is associated with the formulation of the problem of the torsional vibrations of the downhole equipment of the screw pump installations with a surface drive, arising from the falling characteristics of the friction coefficient of the pump screw on the stator rubber cage. The equations of the viscoelastic model describe the model of the operation of a torsional vibration damper, structurally made with an elastic element made of rubber. Figure 1 shows the design layout [2].
2. Materials and methods
The following system of equations describes the above problem:

For a rod string:

$$\frac{\partial^2 \varphi}{\partial t^2} = a^2 \frac{\partial^2 \varphi}{\partial x^2} - 2\nu \frac{\partial \varphi}{\partial t},$$  \hspace{1cm} (1)

For the shock absorber:

$$I_a \frac{d^2 \varphi_a}{dt^2} + C_a \left( \varphi_a - \varphi \right)_{x=l} + h_a \left( \frac{d \varphi_a}{dt} - \frac{\partial \varphi}{\partial t} \right)_{x=l} = -M_{kp}$$  \hspace{1cm} (2)

Initial conditions:

$$\psi(t) \big|_{t=0} = \varphi_a(t) \big|_{t=0} = \varphi(t, x) \big|_{t=0} = 0,$$  \hspace{1cm} (3)

$$\frac{d \varphi_a}{dt} \bigg|_{t=0} = \frac{\partial \varphi}{\partial t} \bigg|_{t=0} = \frac{d \psi}{dt} \bigg|_{t=0} = 0.$$  \hspace{1cm} (4)

Border conditions:

$$\frac{\partial \varphi}{\partial t} \bigg|_{t=0} = \omega_0 = \text{const}$$  \hspace{1cm} (5)

$$G_l \frac{\partial \varphi}{\partial x} \big|_{x=l} = C_a \left( \varphi_a - \varphi \right)_{x=l}.$$  \hspace{1cm} (6)

$$M_{kp} = 4r_v^3 \left( \frac{P}{P_{\psi r}} - \frac{1}{2} \right) \arccos \left( \frac{r_{e} - \delta}{r_v} \right) +$$

$$+ \frac{32 \rho_{ch} geDT}{\pi \eta_s \eta_o} \left[ 250\pi^2 - 7 \left( \frac{\partial \psi}{\partial t} \right)^2 (eDT)^2 \right].$$  \hspace{1cm} (7)
where \( \varphi_a \) is the shock absorber twist angle, \( I_a \) is the moment of inertia of the shock absorber, \( C_a \) is the stiffness of the elastic element of the shock absorber, \( h_a \) is the viscous friction coefficient of the shock absorber, \( l \) is the length of the rod column, \( \psi \) is the screw twisting angle.

Equation (6) is a necessary condition for matching equations (1) and (2).

The equations obtained above can be solved by the grid method, i.e. by replacing differential equations with the corresponding difference equations [3].

The region of integration (0...t; 0...l) can be replaced by a set of equally spaced nodes \((t_i; x_j)\)

\[ t_i = i \Delta t (i = 0, 1, ..., m); \]
\[ x_j = j \Delta x (j = 0, 1, ..., n), \]

where \( \Delta t \) is the partitioning step in time, \( \Delta x \) is the partitioning step along the column length.

The grid function \( u^j \) can replace the function \( \phi(t, x) \):

\[ u^j = \phi(t_j, x_j) = \phi(i \Delta t; j \Delta x). \]  

The function \( \varphi_a(t) \) can be replaced by the function \( v^j \):

\[ v^j = \varphi_a(t_j) = \varphi_a(i \Delta t). \]  

The function \( \psi(t) \) can be replaced by the function \( w^j \):

\[ w^j = \psi(t_j) = \psi(i \Delta t). \]  

Difference analogues replace the equations:

\[ \frac{u^{j+1} - 2u^j + u^{j-1}}{\Delta t^2} = a^2 \frac{u^{j+1} - 2u^j + u^{j-1}}{\Delta x^2} - 2v \frac{u^{j+1} - u^j}{\Delta t}; \]

\[ I_a \frac{v^{j+1} - 2v^j + v^{j-1}}{\Delta t^2} + C_a \left( v^j - u^j \right) + h_a \left( \frac{v^j - v^{j-1}}{\Delta t} - \frac{u^j - u^{j-1}}{\Delta t} \right) = -M_{kp} \]

The initial conditions will take the form:

\[ u^0_j = v^0_j = w^0_j = 0, \]

\[ \frac{u^1_j - u^0_j}{\Delta t} = \frac{v^1_j - v^0_j}{\Delta t} = \frac{w^1_j - w^0_j}{\Delta t} = 0; \]

Border conditions:

\[ \frac{u^i_0 - u^0_i}{\Delta t} = \omega_0, \]

\[ Gl_p \frac{u^i_n - u^{i-1}_n}{\Delta x} = C_a \left( v^i - u^i_n \right), \]

\[ M_{kp} = -4r_s^2 \frac{P}{w - w^{-1}} \frac{0.07}{r_v + 0.28} \arccos \left( \frac{r_e - \delta}{r_v} \right) \]

\[ -32 \rho_g \gamma e DT \left[ 250 \pi^2 - 7 \left( \frac{w - w^{-1}}{\Delta t} \right)^2 (e DT)^2 \right] \]

An algorithm was developed for solving these equations, and a computer program was written in the Delphi programming language, which allows obtaining the results of calculations in graphical form.
and the form of a table [4, 5].

3. Results

Below are the results of solving the equations with the following parameters: shear modulus of the rod material $G=8.06 \times 10^8$ N/m$^2$; material density of rods $\rho_{sh}=7815$ kg/m$^3$; screw tension $\delta = 0.11$ mm; diameter of rods $d_{sh} = 0.022$ m; screw diameter $d_v = 0.06$ m; the length of the rod string $L_{sh} = 1000$ m; screw length $L_v = 2$ m; resistance coefficient $\nu = 2.1$; operating pressure of the pump $P_n = 6 \times 10^6$ Pa; drive rotation speed $n = 100$ rpm; stiffness of the elastic element of the shock absorber $C_a = 2800$ N⋅m / rad; the coefficient of viscous friction of the shock absorber $h_a = 0.88$ N⋅m⋅s / rad.

Figure 2. Dependence of torque on time when a torsional vibration damper is included in the rod string assembly when installed above the pump screw

The magnitude of the non-uniformity of the rod string torque when the torsional vibration damper above the pump screw was installed in the rod string assembly was $m = 0.71$. This indicator is 2.5 times lower than the unevenness of the torque without a shock absorber [6, 7].

Figure 3 shows the design layout with the shock absorber installed directly under the polished stem.

The problem is solved using a system of equations:

For the rod string:

$$\frac{\partial^2 \varphi}{\partial t^2} = \alpha^2 \frac{\partial^2 \varphi}{\partial x^2} - 2 \nu \frac{\partial \varphi}{\partial t}, \quad (20)$$

For the shock absorber:

$$I_a \frac{d^2 \varphi_a}{dt^2} + C_a (\varphi_a - \psi) + h_a \left( \frac{d \varphi_a}{dt} - \frac{d \psi}{dt} \right) = M_a \quad (21)$$

Initial conditions:

$$\psi(t) \big|_{t=0} = \varphi_a(t) \big|_{t=0} = \varphi(t, x) \big|_{x=0} = 0, \quad (22)$$

$$\frac{d \varphi_a}{dt} \big|_{t=0} = \frac{\partial \varphi}{\partial t} \big|_{t=0} = \frac{d \psi}{dt} \big|_{t=0} = 0 \quad (23)$$

Border conditions:

$$GI_p \frac{\partial \varphi_a}{\partial x} \big|_{x=0} = C_a (\varphi_a - \psi) \quad (24)$$
\[
GI_P \left. \frac{\partial \phi}{\partial x} \right|_{x=l} = -4r_v^2 \frac{p}{l} \frac{0,07}{\cos \left( \frac{r_v - \delta}{r_v} \right)} - \\
\frac{32 \rho \phi \eta eDT}{\pi^2 \eta_s \eta_o} \left[ 250 \pi^2 - 7 \left( \frac{\partial \phi}{\partial t} \left|_{x=l} \right. \right)^2 \left( eDT \right)^2 \right].
\]  
\quad (25)

where \( \psi \) is the angle of rotation of the polished rod, \( d\psi / dt = \omega_0 = \text{const} \), the remaining designations are the same as in the previous problem.

**Figure 3.** Scheme for determining the optimal parameters of the shock absorber of torsional vibrations of a rod string when installed under a polished rod.

Equation (24) is an indispensable condition for matching equations (20) and (21).

The equations obtained in this way can also be solved by the grid method, i.e., by replacing the differential equations with the corresponding difference equations [8, 9].

The system of equations is solved using an algorithm similar to the algorithm of the previous problem.

Solution results for the same values: stiffness of the elastic element of the shock absorber \( C_a = 2800 \text{ N-m/rad} \); the coefficient of viscous friction of the shock absorber \( h_a = 0.88 \text{ N-m-s/rad} \).

The magnitude of the torque irregularity (Fig. 4) when installing a torsional vibration damper in the rod string arrangement directly above the pump screw was \( m = 0.73 \), which is 2.4 times lower than the torque irregularity without a shock absorber [10, 11].
Figure 4. Dependence of torque on time when a torsional vibration damper is included in the arrangement when installed under a polished rod

4. Conclusion
The presented study allows concluding that the inclusion of a screw pump with a surface drive of a torsional vibration damper in the rod string arrangement leads to a decrease in the torque unevenness by 2.4-2.5 times.

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