Gravitational Waves from Supermassive Black Hole Binaries in Ultraluminous Infrared Galaxies

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Received 2018 May 14; revised 2018 August 6; accepted 2018 August 7; published 2018 August 20

Abstract

Gravitational waves (GWs) in the nanohertz band are great tools for understanding the cosmological evolution of supermassive black holes (SMBHs) in galactic nuclei. We consider SMBH binaries in high-\(z\) ultraluminous infrared galaxies (ULIRGs) as sources of a stochastic GW background (GBW). ULIRGs are likely associated with gas-rich galaxy mergers containing SMBHs that possibly occur at most once in the life of galaxies, unlike multiple dry mergers at low redshift. Adopting a well-established sample of ULIRGs, we study the properties of the GBW due to coalescing binary SMBHs in these galaxies. Since the ULIRG population peaks at \(z > 1.5\), the amplitude of the GBW is not affected even if BH mergers are delayed by as long as \(\sim 10\) Gyr. Despite the rarity of the high-\(z\) ULIRGs, we find a tension with the upper limits from pulsar timing array experiments. This result suggests that if a fraction \(f_{\text{m,gal}}\) of ULIRGs are associated with SMBH binaries, then no more than 20\(f_{\text{m,gal}}(\lambda_{\text{Edd}}/0.3)^{3/5}(t_{\text{life}}/30\text{ Myr})\%\) of the binary SMBHs in ULIRGs can merge within a Hubble time, for plausible values of the Eddington ratio of ULIRGs (\(\lambda_{\text{Edd}}\)) and their lifetime (\(t_{\text{life}}\)).

Key words: gravitational waves – infrared: galaxies – quasars: supermassive black holes

1. Introduction

Most massive galaxies in the local universe host supermassive black holes (SMBHs). One fundamental goal in astrophysics is to understand the origins of these SMBHs and their host galaxies. Galaxy mergers play an important role in their evolution on a cosmological timescale (e.g., Kormendy & Ho 2013), in the assembly of massive galaxies, and in fueling gas to nuclear SMBHs. A natural outcome of galaxy mergers containing SMBHs is the formation of binary SMBHs. If the binary SMBHs coalescence within a Hubble time, a significant fraction of their rest-mass energy is emitted as gravitational waves (GWs). The GW emission is a good tool to probe the cosmological evolution of SMBHs in the framework of hierarchical structure formation.

Pulsar timing array (PTA) experiments enable us to directly address the GW emission in the nHz–\(\mu\)Hz band. There are three ongoing PTA experiments: the European Pulsar Timing Array (EPTA), the Australian Parkes Pulsar Timing Array (PPTA), and the North American Nanohertz Observatory for Gravitational Waves (NANOGrav). Their measurements have recently provided upper limits on the strength of the GW background (GBW) from binary SMBHs (Lentati et al. 2015; Shannon et al. 2015; Arzoumanian et al. 2016).

The theoretical GWB signal in the PTA band has been predicted by using semi-analytical calculations (e.g., Jaffe & Backer 2003) and cosmological simulations (e.g., Sesana et al. 2009). Those models incorporated various combinations of physical processes that affect the evolution of binary SMBHs in merging galaxies (Begelman et al. 1980; eccentric binary evolution (Enoki & Nagashima 2007), viscous drag from a circumbinary gaseous disk (Kocsis & Sesana 2011), dynamical friction (DF; McWilliams et al. 2014; Kulier et al. 2015), and multibody BH interactions (Bonnett et al. 2018; Ryu et al. 2018). Combined with cosmological hydrodynamical simulations, Kelley et al. (2017) investigated the impact of various environmental processes on the binary evolution. In spite of great efforts in theory, there are still many uncertainties for model parameters and observed empirical relations (Sesana 2013).

In this Letter, we address this issue with a different approach following observational results. We consider ultraluminous infrared galaxies (ULIRGs), which are one of the best tracers of merging galaxies containing SMBHs, as sources of a GBW. Recent observations by Herschel and the Wide-field Infrared Survey Explorer (WISE) have provided a large sample of bright IR galaxies at 0 < \(z < 4\) and allow us to explore their infrared spectral energy distributions (SEDs; Delvecchio et al. 2014). Adopting the luminosity function (LF) of ULIRGs, we study the development of a GBW due to coalescing binary SMBHs driven by merging ULIRGs. Intriguingly, we find a tension with the most stringent PTA upper limits that constrain the fraction of binary SMBHs that coalesce by the present time, depending on the Eddington ratio of BHs in ULIRGs, the typical lifetime of ULIRGs, and the fraction of ULIRGs associated with SMBH binaries.

2. ULIRGs Hosting SMBHs

We consider here ULIRGs with total infrared luminosities of \(L_{\text{IR,tot}} \gtrsim 10^{12} L_{\odot}\) as good tracers of merging galaxies that host at least one accreting SMBH in each system. At low redshifts of \(z < 1\), infrared observations have revealed that the morphologies of ULIRGs are almost exclusively caused by mergers (e.g., Surace et al. 1998) and that the number fraction containing AGNs increases with distance, reaching almost 1 in ULIRGs (e.g., Veilleux et al. 2002; Ichikawa et al. 2014). At higher redshifts of \(z > 1\), the merger fraction is still uncertain since active star formation in gas-rich galaxies alone could produce a similar level of infrared luminosities observed as ULIRGs (e.g., Kartaltepe et al. 2012).

Recent observations have discovered extremely bright ULIRGs with \(L_{\text{IR,tot}} \approx 10^{13} - 10^{14} L_{\odot}\), the so-called hyperluminous infrared galaxies (HyLIRGs; Assef et al. 2015;
merging galaxies in which coalescing binary SMBHs contribute to the GWB.

The enormous power of ULIRGs for bright AGNs with the radiative transfer equation for a smooth dusty structure becomes \( \sim \) \( L_{\text{bol}} \sim (1-5) \times 10^{14} L_\odot \) (Glikman et al. 2015). Those measurements give a lower limit of the fraction because such merger signatures would smooth out after several dynamical timescales.

The enormous power of ULIRGs is produced by deeply buried and rapidly accreting SMBHs. Most of the AGN radiation is absorbed by surrounding dust and is re-emitted at mid-infrared wavelengths. By decomposing the hot dust emission from their SEDs, the AGN contribution to the total (IR) luminosity increases with \( L_{\text{IR},\text{tot}} \) (e.g., Murphy et al. 2011). Namely, the luminosity ratio is estimated as \( L_{\text{IR,AGN}}/L_{\text{IR},\text{tot}} \approx 0.2-0.3 \) for ULIRGs (e.g., Ichikawa et al. 2014) and reaches \( \approx 0.7-1.0 \) for HyLIRGs (e.g., Jones et al. 2014; Farrah et al. 2017).

Using the SED decomposition technique for a sample of Herschel-selected galaxies within the Great Observatories Origins Deep Survey (GOODS)-South and the Cosmic Evolution Survey (COSMOS) fields, Delvecchio et al. (2014) have reconstructed the AGN bolometric LF at 0.1 \( < z < 3.8 \). The bolometric correction factor is estimated by solving the radiative transfer equation for a smooth dusty structure irradiated by the AGN accretion disk, instead of adopting the bolometric correction shown in Hopkins et al. (2007). The AGN bolometric LF for ULIRGs is well fit by

\[
\frac{d\phi(L, z)}{d\log L} = \phi_\star \left( \frac{L}{L_\star} \right)^\beta \exp\left\{ -\left( \frac{\log(1 + L/L_\star)}{2\sigma^2} \right)^2 - \frac{L}{L_\star} \right\},
\]

where \( L \) is the AGN bolometric luminosity, and the values of fitting parameters (\( \phi_\star, L_\star, \beta, \) and \( \sigma \)) are listed in Table 1 of Delvecchio et al. (2014). Here, we set an exponential cutoff above a critical luminosity \( L_\text{c} \approx 10^{14} L_\odot \) because there is no detection for bright AGNs with \( L > 10^{14} L_\odot \) due to the lack of such bright ones or due to the limited observation volume. In addition, we set a minimum value of the AGN luminosity driven by galaxy major mergers to \( L_{\text{min}} \approx 10^{12} L_\odot \). Considering \( L_{\text{IR,AGN}} \approx 3 \) (Delvecchio et al. 2014) and \( L_{\text{IR,AGN}}/L_{\text{IR,tot}} \approx 0.2-0.3 \) (Ichikawa et al. 2014), \( L_{\text{min}} \approx 10^{12} L_\odot \) corresponds to \( L_{\text{IR,tot}} \approx (1-2) \times 10^{12} L_\odot \). Since the merger fraction of IR galaxies with \( L_{\text{IR,tot}} \approx 10^{12} L_\odot \) may be significantly below unity at high redshift, we do not consider such galaxies as sources of a GWB. Figure 1 (left panel) shows the luminosity function of ULIRGs for different redshifts.

In order to convert the LF to the BH mass function (MF), it is necessary to obtain the Eddington ratio (\( \lambda_{\text{Edd}} = L/L_{\text{Edd}} \)). Since the broad-line regions of ULIRGs are completely obscured in the optical, it is almost impossible to estimate their BH masses and thus the Eddington ratio using the optical spectra. However, a well-defined sample of quasars obtained from the Sloan Digital Sky Surveys (SDSS) catalog suggests that the typical Eddington ratio for those quasars is \( \approx 0.3 \) (Kollmeier et al. 2006). On the other hand, the largest SMBHs have a maximum mass limit at \( M_{\text{BH,max}} \approx 10^{10} M_\odot \), which is nearly independent of redshift (Netzer 2003; Wu et al. 2015). The radiation luminosity from quasars hosting the most massive BHs is estimated as \( L \approx 10^{15} \lambda_{\text{Edd}} L_\odot (M_{\text{BH,max}}/3 \times 10^{10} M_\odot)^{-1} \). Thus, we adopt \( \lambda_{\text{Edd}} = 0.3 \) as our fiducial value. Figure 1 (right panel) shows the BH MF in ULIRGs for different redshifts. This BH MF of ULIRGs is consistent with that of SDSS quasars (QSOs) obtained by Kelly & Shen (2013), where the BH masses are estimated by using the width of the broad emission lines and the AGN continuum luminosity.

Luminous QSOs and ULIRGs are much rarer than normal galaxies, which is expected since those luminous phases have a lifetime shorter than a Hubble time. The lifetime is one of the most fundamental quantities for estimating the intrinsic number density of those luminous objects. The QSO lifetime can be observationally constrained by several methods (Martini 2004 and references therein). Overall, the QSO lifetime lies in the range of \( 1 \text{ Myr} \lesssim \tau_{\text{life}} \lesssim 100 \text{ Myr} \). Using galaxy merger simulations, Hopkins et al. (2006) demonstrated that the lifetime tends to decrease with increasing luminosity, namely,

![Figure 1](image-url)
$t_{\text{life}} \simeq 10$–$50$ Myr for $L_{\text{bol}} > 10^{13} L_\odot$. This shorter lifetime is consistent with 1 Myr $\lesssim t_{\text{life}} \lesssim 20$ Myr obtained from observations of extended Ly$\alpha$ emission near luminous QSOs at $2.5 \lesssim z \lesssim 2.9$ with ultraviolet luminosities of $L_{1400} \sim 10^{14} L_\odot$ (Trainor & Steidel 2013). In this Letter, we adopt a conservative value of $t_{\text{life}} \simeq 30$ Myr as our fiducial case.

3. Gravitational-wave Background

Following Phinney (2001), we estimate the GW energy density per logarithmic frequency interval$^3$ as

$$\Omega_{\text{GW}}(f) = \frac{1}{\rho_c c^2} \int \frac{d^3N}{dM_c dz} \frac{1}{1+z} f E_{\text{gw}} f df_c dM_c dz,$$

(2)

where $\rho_c$ is the critical mass density of the universe at $z = 0$, $f_c$ is the GW frequency in the source’s comoving rest frame, $f = f_c/(1+z)$ is the observed GW frequency, $z$ is the redshift when the GWs are produced, $d^3N/dM_c dz$ is the comoving number density of GW events with chirp masses of $[M_c, M_\text{c} + dM_\text{c}]$ that occurs at cosmic times corresponding to the redshift range between $z$ and $z + dz$,

$$\frac{d^3N}{dM_c dz} dM_c dz = \frac{f_{m,\text{gal}}}{f_{\text{delay}}} \frac{d\phi(L_c,z)}{d\log L} d\log L dz,$$

(3)

where $f_{m,\text{gal}}$ is the merger fraction of galaxies inferred from the morphologies of U/HyLIRGs (we set $f_{m,\text{gal}} \simeq 1$; see Section 2), and $f_{\text{delay}} \equiv t_{\text{life}} / (dz/dt_*)$ is the duty cycle of ULIRGs. Since a constant $t_{\text{life}} \simeq 30$ Myr is adopted, the duty cycle is independent of the luminosity.

The GW emission spectrum from a merging binary in the rest frame is given by

$$\frac{dE_{\text{gw}}}{df_c} = \frac{(\pi G)^{5/3} M_c^{5/3}}{3f_c^{1/3}},$$

(4)

where $E_{\text{gw}}$ is the energy of the GW. The chirp mass is written as $M_c^t \equiv q M_{\text{BH}}^t / (1 + q)^{1/3}$, where $M_{\text{BH}}$ is the mass of the primary SMBH and $q(<1)$ is the mass ratio of two SMBHs. We suppose that the primary BH follows the MF in Figure 1. Thus, we implicitly assume that the primary SMBH is located at the center of a ULIRG after the galaxy mergers and is responsible for the ULIRG activity in a lifetime of $t_{\text{life}}$. While the secondary BH is still located off center in a lower-density region. Since the secondary BH would decay its orbit via DF on a timescale of $t_{\text{DF}} \sim 100$ Myr (e.g., Yu 2002), the binary formation would occur after the ULIRG phase (i.e., $t_{\text{DF}} \gtrsim t_{\text{life}}$). This assumption would be plausible because the number fraction of AGNs that are dual SMBHs is as small as $\sim 10$% at $z < 1$ (Comerford et al. 2013).

As discussed in Section 2, most ULIRGs are triggered by major mergers of galaxies. We set here a minimum value of the BH mass ratio to $q_{\text{min}} \simeq 0.1$. The mass-ratio distribution of SMBHs, $\Phi(q)$ at $q_{\text{min}} \leq q \leq 1$, is uncertain. However, the chirp mass averaged over $q$ is less uncertain, namely, $<M_c^t/M_{\text{BH}}^t> \simeq 0.47, 0.34$, and 0.3 for $\Phi(q) = \text{const.}$, $\Phi(q) \propto 1/q$, and $\Phi(q) \propto \delta(q - 1/3)$, respectively. We adopt the last one for a more conservative estimate.

In the course of a galaxy merger embedding two SMBHs, a variety of physical processes affect the binary evolution to the coalescence in a certain timescale (e.g., Begelman et al. 1980; Merritt 2013). Since the delay time between formation and coalescence of BH binaries is still uncertain, rather than attempting to model this delay, we assume a uniform delay time of $t_{\text{delay}}$. To consider the delay effect on BH mergers, we evaluate the LF and duty cycle in Equation (3) at $z = z + \Delta z$, where $\Delta z$ is determined by solving $t_{\text{delay}} = \int_{z+\Delta z}^z dz$. The delay time tends to be shorter than a Hubble timescale for SMBH binaries with higher total masses ($M_{\text{BH}} > 10^8 M_\odot$) and mass ratios ($q > 0.2$) (Khan et al. 2016; Kelley et al. 2017). Within their model uncertainties, the delay time is estimated as $\sim 0.35$–6 Gyr (see Table B1 in Kelley et al. 2017), which is significantly longer than the SMBH binary formation ($t_{\text{DF}} \sim 100$ Myr) and the ULIRG’s lifetime ($t_{\text{life}} < 100$ Myr).

By using Equations (2)–(4), the energy spectrum of the total GW emission due to merging SMBHs in ULIRGs is calculated as

$$\Omega_{\text{GW}}(f) = 1.53 \times 10^{-9} \mathcal{F}_{\text{GW}} \times \left(\frac{\lambda_{\text{Edd}}}{0.3}\right)^{-5/3} \left(\frac{t_{\text{life}}}{30 \text{ Myr}}\right)^{-1} \left(\frac{f}{10 \text{ nHz}}\right)^{2/3},$$

(5)

and the characteristic strain is estimated as

$$h_c(f) = 3.32 \times 10^{-15} \mathcal{F}_{\text{GW}}^{1/2} \times \left(\frac{\lambda_{\text{Edd}}}{0.3}\right)^{-5/6} \left(\frac{t_{\text{life}}}{30 \text{ Myr}}\right)^{-1/2} \left(\frac{f}{10 \text{ nHz}}\right)^{-2/3},$$

(6)

where $\mathcal{F}_{\text{GW}}$ is the ratio of the GW amplitude with an assumed value of $t_{\text{delay}}$ to that without delay. We also define the number fraction of SMBHs that coalesce within a Hubble time as $\mathcal{F}_{\text{coal}}$.

As shown in Table 1, the coalescence fraction decreases monotonically with $t_{\text{delay}}$ and drops sharply at $t_{\text{delay}} \gtrsim 7$ Gyr. On the other hand, the GW amplitude slightly increases with $t_{\text{delay}}$ (i.e., $\mathcal{F}_{\text{GW}} > 1$) and decreases at $t_{\text{delay}} \gtrsim 10$ Gyr significantly (i.e., $\mathcal{F}_{\text{GW}} < 1$). This is because a short delay time barely reduces the number of merger events occurring in a Hubble time, but induces BH mergers at lower redshift. In Figure 2, we plot the total GW spectrum of interest (solid). The upper limits from the PTA are presented by triangle symbols at frequencies where the limit becomes the most stringent: EPA (Lentati et al. 2015), NANOGray (Arzoumanian et al. 2016), and PPTA (Shannon et al. 2015). Without the delay effect, merging SMBHs in ULIRGs would overproduce a GWB. The GWB has also been overproduced based on the observed (McWilliams et al. 2014) and simulated (Kulier et al. 2015) abundance of massive galaxies, and based on the observed periodic quasar candidates (Sesana et al. 2018), assuming that these objects all host SMBH binary mergers. With these PTA constraints, we obtain an upper limit for $\mathcal{F}_{\text{GW}}$ as

$$\mathcal{F}_{\text{GW}} \lesssim 0.43 \left(\frac{\lambda_{\text{Edd}}}{0.3}\right)^{5/3} \left(\frac{t_{\text{life}}}{30 \text{ Myr}}\right),$$

(7)

which corresponds to $t_{\text{delay}} \gtrsim 10.9$ Gyr and $\mathcal{F}_{\text{coal}} \lesssim 0.16$. We also plot the sensitivity that is achievable with the Square
Table 1
The Relation between the Delay Time, Coalescence Fraction, and GWB Amplitude

| $t_{\text{delay}}$(Gyr) | $\mathcal{F}_{\text{coal}}$ | $\mathcal{F}_{\text{coalescence}}$ | $h_\nu \times 10^{13}$ |
|----------------------|----------------|----------------|------------------|
| 0                    | 1.0            | 1.0            | 3.32             |
| 3                    | 1.0            | 1.16           | 3.57             |
| 7                    | 0.75           | 1.22           | 3.67             |
| 10                   | 0.31           | 0.83           | 3.03             |
| 11                   | 0.14           | 0.39           | 2.06             |
| 12                   | 0.022          | 0.06           | 0.81             |

Note. (1) Delay time, (2) coalescence fraction of BHs, (3) effective coalescence fraction, (4) total GW amplitude, and (5) stochastic GWB amplitude at $f = 10^{-8}$ Hz. We adopt $\lambda_{\text{Edd}} = 0.3$ and $t_{\text{delay}} = 30$ Myr.

Figure 2. Characteristic amplitude of the GWB signal for different delay timescales for BH mergers ($0 < t_{\text{delay}} < 12$ Gyr). Solid curves are the predicted total GW amplitudes. Triangle symbols show the current upper limits from PTA experiments: the EPTA (E), NANOGrav (N), and PPTA (P). To be consistent with the PTA limits, $t_{\text{delay}} > 11$ Gyr is required. Black solid line refers to the expected sensitivity of the SKA assuming monitoring of 50 pulsars at 100 ns rms precision over $T_{\text{obs}} = 10$ years with a cadence of 20 yr$^{-1}$.

Kilometre Array (SKA) with 50 pulsars for a $T_{\text{obs}} = 10$ years observation. If a GWB will not be detected even by such a planned detector, it would imply a strong constraint on $\mathcal{F}_{\text{coal}} < 1\%$.

In Figure 3, we present the evolution of the total stochastic GWB (black), and the GWB due to SMBHs with $M_{\text{BH}} \geq 10^9 M_\odot$ (red) and $< 10^9 M_\odot$ (blue). Without the delay, half of the stochastic GWB energy is produced by merging SMBHs with $M_{\text{BH}} \gtrsim 10^9 M_\odot$ at $z > 1.5$, while others are due to less massive ones at $z < 1.5$. This result reflects the shape and redshift-evolution of the LF of ULIRGs and the MF of SMBHs. In fact, a GWB from higher-mass BHs dominates at a higher redshift. The GWB amplitude decreases significantly at $t_{\text{delay}} \gtrsim 10$ Gyr because a larger fraction of SMBHs in ULIRGs do not merge within a Hubble time. Our results are qualitatively consistent with previous work (e.g., Sesana et al. 2008), concluding that a GBW in the PTA band is dominated by nearby and massive binary SMBHs ($z < 2$ and $M_{\text{BH}} > 10^9 M_\odot$). However, it is worth emphasizing that coalescing binary SMBHs, even in a rare population of high-$z$ ULIRGs associated with gas-rich major mergers, which are quite different from multiple dry mergers occurring at low redshift, can produce a GWB close to the present-day upper limit. Since other types of galaxies unlike ULIRGs may have additional SMBH mergers and may contribute to the GWBs (see, e.g., McWilliams et al. 2014), our result gives a lower limit on the total GWB in the PTA band.

We briefly discuss possible biased estimates of the BH masses caused by a scatter of the Eddington ratio distribution (Shen et al. 2008). For the SDSS quasar sample, the Eddington ratio approximately follows a log-normal distribution with a dispersion of $\sigma_E \simeq 0.25$ dex for a fixed BH mass (see Figure 12 in Kelly & Shen 2013). We estimate the mass bias as $\gamma_\ell \sigma_E^2 \ln 10$ dex, assuming a power-law shape for the underlying true MF and a symmetric Gaussian scatter in $\log M_{\text{BH}}$ around a mass-independent mean value, where $\gamma_\ell (\lesssim 0)$ is the slope of the AGN bolometric LF, and we find that the bias effect reduces the GWB amplitude by half. However, we note that the mean Eddington ratio for SMBHs of interest is significantly lower than our fiducial value (Kelly & Shen 2013). Therefore, the GWB amplitude would be rather enhanced ($\Omega_{\text{GW}} \propto \lambda_{\text{Edd}}^{-5/3}$), and the bias effect would be canceled out.

4. Discussion and Implications

The growth of SMBHs in galactic nuclei can be constrained by observations of present-day BH remnants. Adopting the LF of ULIRGs and a radiative efficiency $\epsilon_r$, we can estimate the BH mass density accreted during the ULIRG phases, and we can compare it to that observed in the local universe (Soltan 1982; Yu & Tremaine 2002). The estimated BH mass density is given by

$$\rho_{\text{BH}} \simeq 3.5 \times 10^4 \left(\frac{1 - \epsilon_r}{\epsilon_r}\right) M_\odot \text{cMpc}^{-3}$$

(Delvecchio et al. 2014). In order not to exceed the value observed in the local universe, $\rho_{\text{BH,obs}} \simeq 4.2^{+1.2}_{-1.0} \times 10^5 M_\odot \text{cMpc}^{-3}$ (e.g., Shankar et al. 2009), the radiative efficiency is required to be $\epsilon_r \gtrsim 0.076^{+0.023}_{-0.018}$. This efficiency is consistent with similar arguments for bright QSOs in the optical and X-ray bands (e.g., Yu & Tremaine 2002; Hopkins et al. 2007; Ueda et al. 2014).
The total present-day energy density in GW radiation is estimated from
\[ \mathcal{E}_{\text{GW}} \equiv \int_0^\infty \frac{\rho c^2 \mathcal{E}_{\text{GW}}(f)}{f} df, \tag{9} \]
and \( \mathcal{E}_{\text{GW}} \approx 4.3 \times 10^{-16} \text{ erg s}^{-1} \text{ cm}^{-3} \) for \( t_{\text{delay}} = 11 \text{ Gyr}, \lambda_{\text{Edd}} = 0.3, \) and \( \lambda_{\text{life}} = 30 \text{ Myr} \). Note that \( \mathcal{E}_{\text{GW}} \propto \lambda_{\text{Edd}}^{-0.98} \lambda_{\text{life}}^{-1} \). As a result, the ratio of the total GW energy to the present-day SMBH rest-mass energy is estimated as
\[ \frac{\mathcal{E}_{\text{GW}}}{\rho_{\text{BH}} c^2} \lesssim \frac{0.20 \epsilon_r}{1 - \epsilon_r}, \tag{10} \]
because \( t_{\text{delay}} \gtrsim 11 \text{ Gyr} \) is required for the PTA observations. We note that for unequal-mass binaries, the GW radiative efficiency from a BH with a mass of \( qM \), which is gravitationally captured in a circular orbit by a BH with mass of \( M \), is given by
\[ \epsilon_{\text{gw}} \approx (0.057 + 0.444 \eta)(1 + \eta)^2/(1 + q) \approx 0.1192, \]
where \( \eta = q/(1 + q)^2 \) and \( q = 1/3 \) is set (Lousto et al. 2010). Approximating \( \epsilon_{\text{gw}} \approx \epsilon_r \), we therefore obtain the interesting constraint that the contribution of BH mergers to the present-day mass density is less than \( \lesssim 20(1 + z) \% \); see Equation (7) in Phinney (2001).

The brightest U/HyLIRGs that have experienced active star formation at high redshift would be observed as massive elliptical galaxies in the local universe. An important consequence from Equation (7) is that \( \gtrsim 80\% \) of the binary SMBHs formed in ULIRGs neither coalesce within a Hubble time nor contribute to the GWB. For SMBH binaries with mass ratios of \( q > 0.1 \), the DF caused by surrounding stars with velocity dispersion \( \sigma_v \) would carry the binary separation down to
\[ a_h \sim \frac{12q}{1 + q} \left( \frac{M_{\text{BH}}}{10^9 M_\odot} \right) \left( \frac{\sigma_v}{300 \text{ km s}^{-1}} \right)^{-2} \text{ pc}, \tag{11} \]
where the binary becomes hard, ejects stars, and stalls the orbital decay (e.g., Merritt 2013). The formation timescale of an SMBH hard binary is typically \( \sim 100 \text{ Myr} \) (e.g., Yu 2002), which is much shorter than both a Hubble time and the delay time \( t_{\text{delay}} \) of interest. This suggests that a remnant population of \( O(1-10) \) pc binaries would be left at the centers of nearby massive ellipticals. Although no such binaries have been detected by PTA to date, this nondetection has already yielded interesting constraints on their mass ratios (Schutz & Ma 2016) and anisotropy in the GWB (Mingarelli et al. 2017).

We thank Jeremiah Ostriker, Alberto Sesana, and Yoshiki Toba for useful discussions and comments. This work is partially supported by the Simons Foundation through the Simons Society of Fellows (K.I.), by JSPS KAKENHI 18K13584 and JST grant “Building of Consortia for the Development of Human Resources in Science and Technology” (K.I.), and by NASA grant NNX17AL82G and NSF grant 1715661 (Z.H.).

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