An Application Comparison of Two Poisson Models on Zero Count Data

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Abstract. Counting data (including zero counts) appear in a variety of applications, so counting models have become popular in many fields. In statistical fields, count data can be defined as observation types that use only non-negative integer values. Sometimes researchers may Counts more zeros than the expected. You may describe Excess zero as Zero-Inflation, excess zeros cause over-dispersion. So, the objective of this paper is use zero-inflated regression models (Poisson Regression model, Zero-Inflated Poisson (ZIP), and Zero-Altered Poisson (ZAP)) to analyse rainfall data and select the best model that deal with these type of data. It has been shown through the study and practical application that the advantage and quality of the Zero-Altered Poisson Regression (ZAPR) where the Zero-Altered Poisson regression model was the best count data model for our data, Although it is hard to distinguish Zero-Inflated Poisson (ZIP) regression model, it is better than Poisson regression model.

Keywords: Poisson. Zero-Inflated. Zero-Altered. Counts data. Excess zero.

1. Introduction

Count data reflects the number of occurrence of certain characteristic in a fixed period of time, that is, Count data are non-negative integers {0,1,2,3,...}. Count data becomes popular in a wide areas of interesting sciences; such as finance, marketing, health care, weather, and others. In many disciplines, counting excess zero data is very common, and in many scientific fields, sometimes researchers may count more zeros than expected [1]. You may describe Excess zero as Zero-Inflation. Excess zero sometimes may be the reason of occurs Over-dispersion (variance a lot larger than mean) [2, 3]. The concept of over-dispersion is often used in discrete data analysis. Therefore, linear regression is not applicable procedure to estimate the parameters of predictors due to the asymmetric distribution of the response variable [1]. Under these constraints, Poisson regression was used to model the Count data [3, 4, 5]. Lambert (1992) discussed this matter and suggested “zero-inflated Poisson” model with an application in manufacturing quality also suggested by Grene (1994) and the “zero-altered Poisson” model.
(Another popular approach for modelling excess zeros in count data is the use of hurdle models (also referred to as a zero-altered model) developed by Cragg (1971)) that have been proposed to cope with an overabundance of zeros (also called a zero-altered model). The zero-inflation model has become very interesting [2, 6, 1]. The authors published many papers in various scientific fields such as optimization, reliability and operation research (see [7- 26]), but in this work, we focus on the redundant zero case.

In some commonly used discrete distributions, the distribution mean related to variance is the cause of Over-dispersion [27- 29]. That is, Over-dispersion appears in the data in which there is evidence that variance of the dependent variable is greater than the mean.

In health, marketing, finance, econometrics, ecology, statistical quality control, geographical and environmental fields, data with abundant zeros is particularly common when counting the incidence of certain behavioural and natural events, such as the frequency of alcohol consumption, drug use the amount of cigarettes smoked, the occurrence of earthquakes, rainfall, etc.

Famoye and Consul (1992) proposed “generalized Poisson” distribution which can take consideration of “over-dispersion” of Poisson distribution. The extension of “generalized Poisson distribution” is “zero-inflated generalized Poisson” suggested by Famoye and Singh (2006). Excess zeros in count data perhaps leads to bias in the estimation of parameters due to the difficulty of obtaining the probability of zeros, to address the existing of excess zeros, Zero-Inflated regression models (such as the “zero-inflated Poisson” (ZIP model) and “zero-altered Poisson” (ZAP model) are the best option. The “zero-inflated Poisson” (ZIP) model and “zero-altered Poisson” (ZAP) model, have been used to analyse count data with excess zeros. In this article, to analyse rainfall data, We focus on the models, Poisson, ZIP, and ZAP.

2. Poisson Regression Model (PRM)

The Poisson regression model is a regression model that is non-linear (log-linear) and is convenient for analysing count or rate data. Poisson regression is similar to the multiple regression excepting that the response (y) variable is an observed count that follows the “Poisson distribution”. Therefore, the possible values of (y) are “non-negative integers”. Suppose we have a random sample \( y_1, \ldots, y_n \) drawn from Poisson distribution, then the p.m.f of \( y_i \), As follow

\[
p(y_i, \mu_i) = \frac{e^{-\mu_i} \mu_i^{y_i}}{y_i!}; y_i = 0, 1, 2, \ldots
\]  

(1)

By assumptions of GLM [1, 5, 30, 31], We have

\( Y_i \sim P(\mu_i); E(Y_i) = \mu_i, \text{Var}(Y_i) = \mu_i \), and \( \mu_i = e^{\eta_i}; \mu_i = e^{\eta_i} \)

Where \( X\beta = \alpha + \beta_1 x_{i1} + \ldots + \beta_q x_{iq} \) and \( x_{i1}, \ldots, x_{iq} \) are the independent variables.

Given the p.m.f in (1) and using the method of maximum likelihood and assuming independence of the observations, We can estimate regression parameters as follow

\[
L = \prod_{i=1}^{n} \frac{\mu_i^{y_i} e^{-\mu_i}}{y_i!}
\]

Taking the log of both sides,

\[
\log(L) = \sum_{i=1}^{n} \left( \log(y_i) - \log(\mu_i) - \log(\mu_i) \right)
\]

\[
= \sum_{i=1}^{n} \left( \log(\mu_i) + \log(e^{-\mu_i}) - \log(y_i) \right)
\]

\[
= \sum_{i=1}^{n} \left( y_i \log(\mu_i) - \mu_i - \log(y_i) \right)
\]

2
\[
\sum_{i=1}^{n} (y_i X' \beta - e^{X' \beta} - \log(y_i!))
\]

By taking partial derivatives of the parameters and equalizing the likelihood equation to zero

\[
\frac{\partial \log(L)}{\partial \beta} = \frac{\partial}{\partial \beta} \sum_{i=1}^{n} (y_i X' \beta - e^{X' \beta} - \log(y_i!)) = \sum_{i=1}^{n} (y_i X - X e^{X' \beta}) = 0
\]

(2)

Applying numerical methods such as “Newton Raphson” to solve equation (2).

“Poisson regression model” is suitable for Modeling “count data” but in practice, Usually, the variance of count data overrides its mean, resulting Over-dispersion [1, 3, 32]. Count data underlying Over-dispersion and Poisson regression model leads to bias results [1, 3, 4], and under estimation of the parameters which effects on the standard errors and P-value. This Over-dispersion may be due to a random unobserved variation component in the function of X'.

3. Zero-Inflated Models (ZI)

Excess zeros in certain population is lead to Zero-Inflation which is made up two types of data subgroups (data generation), the first subgroup is a set of only zeros count (true zeros and false zeros), and the second subgroup is a set of count variables (with true zeros) that distributed according to Poisson distribution (Lambert 1992, Van den Broek 1995) [1, 3, 27].

4. Zero-Inflated Poisson Regression Model (ZIPR)

The “zero-inflated Poisson” regression is used for modelling count data that show over-dispersion and zero counts (excess zeros). This model consider there are two types of data sources, the first source is zero type and the second is comes from data follow Poisson distribution [1, 29, 33]. According to Lambert (1992), the response variable Yi is independent with Yi~Poisson(\(\mu_i\)) and Yi~Poisson(\(\mu_i\)) with probability (1-\(\theta_i\))

Where \(\theta_i\) is the probability that observation (i) is in the always zeros subgroup.

Therefore,

\[\Pr(Y_i = 0) = \theta_i + (1 - \theta_i) \times \Pr(\text{Count process at (i)gives a zero})\]  

(3)

by assuming the Yi follows a Poisson distribution with mean \(\mu_i\)

\[p(y_i; \mu_i | y_i \geq 0) = \frac{e^{-\mu_i} y_i^{y_i}}{y_i!}\]

Subsequently, the term \(\Pr(\text{Count process at (i)gives a zero})\) is given by

\[p(Y_{i=0}; \mu_i | y_i \geq 0) = \frac{e^{-\mu_i} \mu_i^0}{0!} = e^{-\mu_i}\]

Hence, Equation (3) can now be rewritten as

\[\Pr(Y_i = 0) = \theta_i + (1 - \theta_i) e^{-\mu_i}\]  

(4)

With probability that Yi is a non-zero count, we have

\[\Pr(Y_i = y_i) = (1 - \theta_i) \times \Pr(\text{Count process})\]  

(5)

Hence, Equation (5) can be rewritten as follow

\[\Pr(Y_i = y_i | y_i > 0) = (1 - \theta_i) \frac{e^{-\mu_i} y_i^{y_i}}{y_i!}\]  

(6)

Furthermore, The probability density function for a ZIP model is given by

\[p(Y_i = y_i) = \begin{cases}  
\theta_i + (1 - \theta_i) e^{-\mu_i} & \text{if } y_i = 0 \\
(1 - \theta_i) \frac{e^{-\mu_i} y_i^{y_i}}{y_i!} & \text{if } y_i > 0 
\end{cases}\]

(7)
By GLM[1, 5, 30, 31], \( \mu_i = e^{X_i \beta} \), where \( X_i \) are known independent variables, Lambert (1992) suggested the functional form for modelling the parameter \( \theta_i \) as logistic function, which is given by

\[
\log \left( \frac{\theta_i}{1 - \theta_i} \right) = z' \gamma_i
\]

and therefore,

\[
\theta_i = \frac{e^{z' \gamma_i}}{1 + e^{z' \gamma_i}} > 0
\]

Where; \( z \) : the covariates and \( \gamma \) : are regression coefficients.

The corresponding Log-Likelihood function is given as follow

\[
\log(L) = \sum_i^n \left[ I(y_i = 0) \log(\theta_i + (1 - \theta_i)e^{-\mu_i}) + I(y_i > 0)\left(\log(1 - \theta_i) - \mu_i + y_i \log(\mu_i) - \log(y_i!)\right)\right]
\]

Subsequently

\[
E(\gamma_i|x_i) = \mu_i(1 - \theta_i)
\]

\[
\text{Var}(\gamma_i|x_i) = (1 - \theta_i)(\mu_i + \theta_i \mu_i^2)
\]

5. Zero-Altered Models (ZA)

Zero-altered models known as a two-part models, Where the first part is a binary outcome model governs with binomial probability, and the second part is a truncated count model [1, 27, 34]. In zero-inflated models assumed that count data consist of two types of data subgroups, the first subgroup is a set of only zeros count (true zeros and false zeros), and the second subgroup is a set of count variables (with true zeros) [1, 32]. While, zero-altered models do not discriminate between the types of zeros; they are simply zeros [1]. The basic idea for the zero-altered models is that the outcomes are treated as absence and presence zeros data. This means that the outcomes are divided into two groups, the first includes all zeros, the second includes non-zero counts [1, 29].

Where, The binomial distribution is used to model the absence and presence, and a Poisson distribution for the counts [2, 27]. To measure a non-zero count should be modified the distribution and exclude the possibility of a zero observation, and this is called a zero-truncated distribution.

Assume that the zeros are follow the probability mass function (p.m.f) \( f_1(.) \) with \( P(y = 0) = f_1(0) \) and \( P(y > 0) = 1 - f_1(0) \), while the positive outcomes are formed by the probability mass function truncated at zero given by

\[
f_2(y|y > 0) = f_2(y)/(1 - f_2(0))
\]

Hence, the Hurdle (Altered) probability mass function as follow

\[
P(y) = \begin{cases} 
    f_1(0) & ; y = 0 \\
    1 - f_1(0)/1 - f_2(0) & f_2(y) ; y > 0 
\end{cases}
\]

6. Zero-Altered Poisson Model (ZAPM)

Suppose that the probability of measuring zero observation in the first part of Hurdle structure is modelled with a binomial distribution. Where \( \theta_i \) is the probability that \( y_i = 0 \).

Suppose that be the response variable for the positive counts' (truncated at zero) with Poisson probability mass function (1).

Furthermore, let the probability of observing \( y_i = 0 \) in the first part of Hurdle model (zero count) as follow

\[
P(y_i = 0) = f_1(0) = \theta_i
\]

Where, the probability of observing \( (y_i > 0) \) in the second part of Hurdle model (positive counts) as follow
\[ P(y_i; \mu_1|y_i > 0) = f_2(y) = \frac{\mu_1 e^{-\mu_1}}{y_1!} \]  

(11)

Therefore, substituting (1), (10), and (11) in Zero-Altered (9), we have

\[ P(Y_i = y_i) = \begin{cases} 
\theta_i & ; y_i = 0 \\
(1 - \theta_i) \frac{\mu_1 e^{-\mu_1}}{(1-e^{-\mu_1})} & ; y_i > 0
\end{cases} \]  

(12)

By GLM[1, 5, 30, 31], \( \mu_i = e^{X_i \beta} \) where \( X_i \) are known independent variables, Lambert (1992) suggested the functional form for modelling the parameter \( \theta_i \) as logistic function, which is given by

\[ \log \left( \frac{\theta_i}{1 - \theta_i} \right) = Z_i \gamma_i \]

and therefore,

\[ \theta_i = \frac{e^{Z_i \gamma_i}}{1 + e^{Z_i \gamma_i}} > 0 \]

Where; \( Z \) : the covariates and \( \gamma \) : are regression coefficients.

The corresponding Log-Likelihood function is given as follow

\[ \log(L) = \sum_{i=1}^{n} \left[ I(y_i = 0) \log(\theta_i) + I(y_i > 0)(\log(1 - \theta_i) - \mu_i + y_i \log(\mu_i) - \log(1 - e^{-\mu_i}) - \log(y_i!)) \right] \]  

(13)

The mean and variance for ZAP are

\[ E(Y_i) = 1 - \theta_i \]
\[ \text{Var}(Y_i) = \frac{1 - \theta_i}{1 - e^{-\mu_i}} (\mu_i + \mu_i^2) - \left( \frac{1 - \theta_i}{1 - e^{-\mu_i}} \mu_i \right)^2 \]

7. Model Selection

It is important that we have one or more a criterion to consider the best results and choose the appropriate model for data representation. There are several methods that provide a measure for selecting the appropriate model. The following four methods will be used: AIC is an evaluating model fit for a given data among different types of non-nested models, and its formula is given as \( AIC = -2\log L + 2k \). BIC is another estimator for evaluating model fit for a given data among different types of non-nested models, and its formula is given as \( BIC = -2\log L + k \log n \). Likelihood ratio test (LR) is a statistical test used to compare two nested models, its formula is given as \( LR = -2\log (L_1/L_2) \) and Vuong test (V) is a statistical test used to compare non-nested models, It is defined as

\[ V = \sqrt{n} \left( \frac{1}{n} \sum_{i=1}^{n} m_i \right) / \left( \frac{1}{n} \sum_{i=1}^{n} (m_i - \bar{m})^2 \right) \]

Where \( m_i = \log \left( P_1(Y_i|X_i) \right) - \log \left( P_2(Y_i|X_i) \right) \).

If \( V > 1.96 \), then the first model is preferred. If \( V < -1.96 \), then the second one is preferred. If \( |V| < 1.96 \), none of the models are preferred.

8. Data Analysis

Data were collected from database of the meteorology and seismology organization in Iraq for Hilla weather station. The weather station are located in central Iraq, specifically in the city of Hilla (about 116 kilometers south of Baghdad).

The count response variable of interest to be modeled "Rainfall hours" measured at Hilla weather station. The predictor variables consists of six climate variables derived from Iraqi Meteorological Organization and Seismology database, which include measurements of rainfall, sea pressure, station pressure, wind...
speed, temperature, and humidity, as shown in Table (1). Data contain observations of (731) for two years.

**Table 1.** Summary statistics of explanatory variables and response used in our count data regression models in Hilla weather station.

| variables               | Minimum value | First quarter | Median | Mean    | Third quarter | Maximum value |
|-------------------------|---------------|---------------|--------|---------|---------------|---------------|
| Rainfall (hours)        | 0             | 0             | 0      | 0.6553  | 0             | 20            |
| Wind speed (m/s)        | 0             | 0.6           | 1.4    | 1.619   | 2.3           | 9.3           |
| Temperature (°C)        | 3             | 15.8          | 25     | 23.97   | 32.85         | 40.5          |
| Humidity (%)            | 17            | 31.8          | 40.6   | 44.54   | 56            | 94            |
| Station pressure (1bar/1000) | 0.9908 | 1.0007       | 1.0068 | 1.0074  | 1.0131        | 1.3804        |
| Sea pressure (1bar/1000) | 0.9947  | 1.0046       | 1.0108 | 1.0109  | 1.0171        | 1.0287        |

**Figure 1.** The distribution of the number of non-rainfall hours in Hilla weather stations for the two years.

9. **Poisson Regression**

The model fit statistics and estimated coefficients of Poisson regression model are given in Table 2 and Table 3.

**Table 2.** Fit statistics of Poisson regression model, Rainfall count data criterions, Hilla weather station

| criterions             | Hilla weather station |
|------------------------|-----------------------|
| -2Log Likelihood       | 1466.649              |
| AIC                    | 1478.649              |
| BIC                    | 11506.216             |
Table 3. Estimated coefficients of Poisson regression model, Rainfall count data in Hilla weather station.

| Parameter         | Estimate  | Standard Error | z Value | Pr > |z|
|-------------------|-----------|----------------|---------|-------|
| Intercept         | 43.305973 | 14.941807      | 2.898   | 0.00375 |
| Wind speed        | 0.2477    | 0.0253         | 9.79    | <2e-16 |
| Temperature       | 0.031051  | 0.017090       | 1.817   | 0.06922 |
| Humidity          | 0.096406  | 0.004648       | 20.742  | <2e-16 |
| Station pressure  | -4.573125 | 20.646534      | -0.221  | 0.82471 |
| Sea pressure      | -45.002287| 25.235835      | -1.783  | 0.07454 |

Since the variance of count data usually exceeds the conditional mean, the equality of variance and mean should always be checked after the development of a Poisson regression. We conducted a test of overdispersion and the results of this test are shown below. The critical value of test statistic at the alpha= 0.00 level: 2.7055. For Hilla weather station, Chi-Square test statistic= 579.6014, p-value = <2.2e-16. The significance of $X^2$-statistics implies the existence of over-dispersion. Therefore, in the next section, we develop Zero-Inflated model to handle the issue of over-dispersion.

10. Zero-Inflated Regression Models

To fixable the excess zeros problem in non-Rainfall days (rainfall hours are zeros), We used Zero-inflated regression models.

11. Zero-Inflated Poisson Regression (ZIPR) Model

We used the same explanatory variables in both parts of the ZIPR model. The model fit statistics and estimated coefficients of ZIPR model are given in Table 4 and Table 5.

Table 4. Fit statistics of Zero-Inflated Poisson Regression (ZIPR) model, Rainfall count data

| criterions       | Hilla weather station |
|------------------|-----------------------|
| -2Log Likelihood | 841                   |
| AIC              | 865.0707              |
| BIC              | 880.5665              |

Table 5. Estimated coefficients of Zero-Inflated Poisson Regression (ZIPR) model, Rainfall count data in Hilla weather station

| Parameter         | Estimate  | Standard Error | z Value | Pr > |z|
|-------------------|-----------|----------------|---------|-------|
| Poisson _ Intercept| -1.784e+00| 3.533e+01      | -0.05   | 0.96  |
| Poisson _ Wind speed| -2.866e-02| 2.556e-02      | -1.122  | 0.262 |
| Poisson _ Temperature| 4.288e-02| 6.296e-02      | 0.681   | 0.496 |
| Poisson _ Humidity| 3.300e-02| 4.345e-03      | 7.595   | 3.09e-14 |
| Poisson _ Station pressure| 1.481e+02| 5.419e+03      | 0.027   | 0.978 |
12. Zero-Altered Regression Models (ZARM)
To fixable the excess zeros problem in non-Rainfall days (rainfall hours are zeros), We used Zero-Altered regression models.

13. Zero-Altered Poisson Regression (ZAPR) Model
We used the same "explanatory variables" in both parts of the ZAPR model. The model fit statistics and estimated coefficients of ZAPR model are given in Table 6 and Table 7.

Table 6. Fit statistics of Zero-Altered Poisson Regression (ZAPR) model, Rainfall count data

| criterions          | Hilla weather station |
|---------------------|-----------------------|
| 2Log Likelihood     | 840.8                 |
| AIC                 | 864.7024              |
| BIC                 | 880.3665              |

Table 7. Estimated coefficients of Zero-Altered Poisson Regression (ZAPR) model, Rainfall count data in Hilla weather station

| Parameter           | Estimate  | Standard Error | z Value | Pr > |z| |
|---------------------|-----------|----------------|---------|-------|---|
| Poisson Intercept   | -2.958e+00| 1.631e+01      | -0.181  | 0.856 |
| Poisson Wind speed  | -2.973e-02| 2.592e-02      | -1.147  | 0.251 |
| Poisson Temperature | 4.540e-02 | 1.966e-02      | 2.309   | 0.021 |
| Poisson Humidity    | 3.348e-02 | 4.417e-03      | 7.581   | 3.43e-14 |
| Poisson Station pressure | 2.536e+01 | 9.986e+02   | 0.025   | 0.98 |
| Poisson Sea pressure| -2.355e+01| 1.001e+03      | -0.024  | 0.981 |
| Logit Intercept     | 146.38914  | 49.35864       | 2.966   | 0.00302 |
| Logit Wind speed    | 0.65227   | 0.09910        | 6.582   | 4.64e-11 |
| Logit Temperature   | -0.0656   | 0.04771        | -1.375  | 0.1691 |
| Logit Humidity      | 0.1103    | 0.01536        | 7.182   | 6.89e-13 |
| Logit Station pressure | -1.09452  | 42.64166       | -0.026  | 0.97952 |
| Logit Sea pressure  | -150.90185| 64.65376       | -2.334  | 0.0196 |

14. Model Comparison
We used Vuong test to compare non-nested models and Likelihood ratio test to compare nested models, The results of all the Vuong tests are summarized in Table 8 and the results of all Likelihood ratio tests
are summarized in Table 9. Furthermore, the results of all information criterions (fit statistics) for all models were summarized in Table 10.

**Table 8.** Model comparison by Vuong test for non-nested models for Hilla weather station

| Model     | Vuong Statistic | Best model |
|-----------|-----------------|------------|
| ZIP vs P  | 6.969103        | ZIP        |
| ZIP vs ZAP| -0.476515       | NONE       |

Note: “If V > 1.96, the first model is preferred. If V < -1.96, then the second one is preferred. If |V|<1.96, none of the models are preferred”.

**Table 9.** Model comparison by likelihood ratio test for nested models for Hilla weather station

| Model     | Likelihood Ratio Test (p-value) | Best model |
|-----------|---------------------------------|------------|
| P vs ZAP  | 1.11                            | ZAP        |

Note:  
$H_0$: the simpler model is preferred.  
$H_1$: the more complex model is preferred.

If p-value < 0.05, we reject $H_0$. $H_1$ is preferred.

**Table 10.** Fit statistics comparison of all models, Rainfall count data Hilla weather station

| models       | criterions                      |
|--------------|--------------------------------|
|              | -2Log Likelihood | AIC     | BIC      |
| Poisson regression | 1466.649       | 1478.649 | 11506.216 |
| ZIPR         | 841              | 865.0707 | 880.5665 |
| ZAPR         | 840.8*           | 864.7024*| 880.3665*|

*The best model.

15. **Application results**

After estimating the regression parameters for all models using real counting data. The test criteria values for all models were obtained for the purpose of comparing these models and selecting the best ones to represent our data. The results in Table 10 indicated that Zero-Altered Poisson Regression (ZAPR) regression model was the best count data model for our data. Although it is hard to distinguish Zero-Inflated Poisson (ZIP) regression model, it is better than Poisson regression model.
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