We present an effective action approach for the problem of Coulomb blocking of tunneling. The method is applied to the “strong coupling” problem arising near zero bias, where perturbation theory diverges. By a semiclassical argument, we obtain electrodynamics in imaginary time, and express the anomaly through exact conductivity of the system $\sigma(\omega, q)$ and exact interaction. The calculation is tested by comparing with the known perturbation theory result for the diffusive anomaly. Also, we use the method to study the anomaly enhancement due to external magnetic field, and the effect of screening by electrodes.

I. INTRODUCTION

Suppression or enhancement of the tunneling conductivity near zero bias is known to be a signature of interaction in the system. It is called “zero-bias anomaly”, and it has been studied in metals and semiconductors since the early sixties \[1\]. Initially, the origin of the anomaly was attributed to the Kondo effect due to magnetic impurities. However, later it was understood that a much more common mechanism is Coulomb blocking of the tunneling. A perturbation theory of this effect was developed by Altshuler, Aronov and Lee \[2\]. The theory deals with the diffusive limit, and shows that the blocking increases at small bias which leads to a singularity in the tunneling conductivity. The theory has been thoroughly tested experimentally and found to be extremely accurate when the anomaly is a weak feature on the top of a large constant conductivity.

In the last years the interest shifted to the systems with strong Coulomb effects, such as disordered metals and semiconductors near metal-insulator transition \[3\]. It is found that the Coulomb anomaly is sharply enhanced near the transition, providing a test of Coulomb correlations. Another important discovery is the observation by Ashoori et al. of the Coulomb blocking of tunneling in a two-dimensional metal in magnetic field \[4\]. In this experiment it is found that at certain magnetic field the zero-bias anomaly abruptly increases and transforms to a “soft Coulomb gap”. It has been pointed out \[3\] that this transition is induced by disorder. More recently, the gap was studied in the systems with higher mobility \[3,12\], where current is almost entirely blocked below certain threshold bias. These findings caused a lot of theoretical work, concerned with the behavior of current near zero bias \[11\], and with determining the gap width \[12\]. The anomaly has been shown to be particularly interesting in the $\nu = 1/2$ Quantum Hall system \[11\] as a sensitive probe of the Quantum Hall state.
Our goal in this contribution is to describe an effective action theory \[13\] that treats the anomaly as cooperative tunneling. In the calculation, the tunneling rate is related to the action for charge spreading and is expressed through the actual conductivity \(\sigma(\omega, q)\) of the system. The treatment is non-perturbative, and remains accurate in the strong coupling regime. For example, at the metal-insulator transition, conductivity has scale invariant form \(\sigma(\omega) \sim \omega^{\alpha}\), and we are able to predict the form of the tunneling \(I - V\) curve. This relation may be used to determine the critical exponent \(\alpha\). We discuss it below, and also apply the method to diffusive anomaly in a two dimensional metal, with and without magnetic field. Our results are consistent with the perturbation theory results \[4,13]\.

II. QUALITATIVE DISCUSSION

Tunneling of an electron into a metal involves two steps: traversing the barrier, followed by spreading within the metal. Typically, the traversal time is much shorter than the relaxation time in the electron liquid. Accordingly, it is legitimate to separate the tunneling into a single-electron and many-electron parts, and to treat them separately. The first contribution is described by the transmission coefficient of the barrier, at small bias being just a constant. We are interested here in the multi-electron effect that involves motion of a large number of electrons in order to accommodate the new electron. At low bias this collective effect may completely control the tunneling rate.

Let us illustrate the effect of charge relaxation on the tunneling rate by using the example of a two-dimensional conducting plane \[7\]. Charge relaxation in a two-dimensional conductor is a classic electrodynamics problem studied by Maxwell who gave a solution in terms of a moving image charge \[14\]. In this problem, a point charge \(e\) is injected into a conducting sheet with conductivity \(\sigma\), and one is interested in the time-dependence of the density and potential of spreading charge. The solution, as Maxwell formulates it, is that the potential within the sheet is given by that of a point charge \(e\) moving along the normal to the plane with the velocity \(v = 2\pi\sigma\). The size of the charge cloud grows as \(r(t) \sim vt\). Let us consider the Coulomb part of the action for the charge:

\[
S(t) \sim \int_{t_0}^{t} \frac{e^2}{r(t')} dt' = \frac{e^2}{2\pi\sigma} \ln \left( \frac{t}{t_0} \right) \tag{1}
\]

In the semiclassical picture the action (1) must be added to the under-barrier action. The divergence of (1) at \(t \to \infty\) indicates that for a charge spreading that takes a long time the spreading action dominates the tunneling rate. We will see that the time of spreading diverges at small bias, \(t_* = e/\sigma V\). From that, near zero bias the tunneling acquires a suppression factor of \(\exp\left(-\frac{1}{\hbar}S(t_*)\right)\). The estimate (1) showing that the action diverges at small bias means that the semiclassical treatment is meaningful even for a well conducting metal. However, in the diffusive limit the estimate (1) does not agree with the perturbation theory. We shall see that the reason is that the main part of the action is rather Ohmic than Coulomb, and that after writing the action properly the semiclassical method completely recovers the perturbation theory result.

The relevance of the semiclassical picture in this problem can be justified by a more general argument, not involving specific features of the charge relaxation in two dimensions. Let us consider a situation when at small bias one electron crosses the barrier. Since the barrier traversal time is much shorter than the relaxation time in the metal, while the electron traverses the barrier other electrons practically do not move. Thus instantly a large electrostatic potential is formed, both due to the tunneling electron itself, and due to the screening hole left behind.
The jump in electrostatic energy by an amount much bigger than the bias $eV$ means that right after the one electron transfer we find the system in a classically forbidden state “under” the Coulomb barrier. In order to accomplish tunneling, the charge yet has to spread over a large area, so that the potential of the charge fluctuation is reduced below $eV$. If the conductivity is finite, the spreading over large distance takes long time, and thus the action of this cooperative under-barrier motion is much bigger than $\hbar$.

III. THE ACTION FOR LONG-WAVELENGTH MODES

For the electrodynamics problem the action can be written in terms of charge and current densities $\rho(r, t)$ and $j(r, t)$. Full action would also contain electromagnetic potentials, but in the quasistationary limit, $c \to \infty$, which we always assume below, the potentials are “slaved” to charges, and thus can be integrated out. As it was argued above, the contribution to the action of the spreading charge is mainly coming from long times when the charge deviation from equilibrium is small. Therefore, we can expand the action in powers of $\rho(r, t)$ and $j(r, t)$, and keep only quadratic terms. The action should reproduce the classical electrodynamics equations: the Ohm’s law and charge continuity.

Of course this requirement is entirely sufficient to determine the form of the action. However, it is more convenient to argue in the following way. We are going to use the action to study the dynamics in imaginary time. Therefore, the action is precisely the one that appears in the quantum partition function. The latter action expanded up to quadratic terms in charge and current density must yield correct Nyquist spectrum of current fluctuations in equilibrium:

$$
\langle \langle g^\alpha_{\omega,q} g^\beta_{\omega,-q} \rangle \rangle = \sigma_{\alpha\beta}|\omega| + \sigma_{\alpha\alpha'} D_{\beta\beta'} q_{\alpha} q_{\beta}'.
$$

(2)

Here

$$
g = j + \dot{D} \nabla \rho
$$

is external current and $D_{\alpha\beta}$ is the tensor of diffusion constants related to the conductivity tensor by the Einstein’s formula: $\hat{\sigma} = e^2 \nu \hat{D}$, where $\nu = dn/d\mu$ is compressibility. Generally, both $\hat{\sigma}$ and $\hat{D}$ are functions of the frequency and momentum. For simplicity, we assume that the temperature is zero and discuss only a two dimensional metal with spatially isotropic and homogeneous conductivity: $\sigma_{xx} = \sigma_{yy}$, $\sigma_{xy} = -\sigma_{yx}$.

The requirement that the action produces correct current fluctuations is essentially equivalent to the fluctuation-dissipation theorem. Thus, the form of the action is fixed by response functions of the system. In imaginary time we get

$$
S = \frac{1}{2} \int \int d^4x_1 d^4x_2 \left[ g_1^T \tilde{K}_{x_1-x_2} g_2 + \frac{\delta_{12} \rho_1 \rho_2}{|r_1-r_2|} \right]
$$

(4)

where $x_{1,2} = (t_{1,2}, r_{1,2})$, and $\delta_{12} = \delta(t_1 - t_2)$. The kernel $\tilde{K}_{r,t}$ is related to the current-current correlator,

$$
(\tilde{K}_{r,q})_{\alpha\beta} = \langle \langle g^\alpha_{i\omega,q} g^\beta_{i\omega,-q} \rangle \rangle
$$

(5)

given by (4), where $\hat{\sigma}$ and $\hat{D}$ are functions of the Matsubara frequency related with the real frequency functions by the usual analytic continuation. We take Coulomb interaction in the second term of the action (4) as non-retarded because we are going to study systems with relatively low conductivity, and thus slow charge relaxation.
One can also justify the form (4) of the action by looking at various particular examples, like the ideal dissipationless liquid of charge, or the phenomenological dissipative Caldeira-Leggett system with spatially distributed coupling to a thermal bath. In the first case the Ohmic part of the action is replaced by the kinetic energy term

\[ S_{\text{kin}} = \int dr dt \frac{j^2}{2e\rho} . \]

In the second case one has

\[ S_{\text{CL}} = \int dr d\omega \frac{\mathbf{j}_\omega(r) \cdot \mathbf{j}_{-\omega}(r)}{4\pi e^2 \eta |\omega|} , \]

where \( \eta \) is the Caldeira-Leggett viscosity. For \( S_{\text{kin}} \) the electric impedance of the system is imaginary, while for \( S_{\text{CL}} \) it is real. From these two “limiting” cases one can conjecture the more general form (4).

**IV. INSTANTON IN IMAGINARY TIME**

To evaluate the tunnelling rate, we use the instanton method and look for a least action “bounce” path in imaginary time [15,16]. Among the bounce paths symmetric in time, \( \rho(r, t) = \rho(r, -t) \), \( \mathbf{j}(r, t) = -\mathbf{j}(r, -t) \), we shall find the least action path which will give a semiclassical estimate of the tunneling rate exponent.

From the variational principle one can derive equation of motion. We note that the action (4) contains the charge and current densities as independent variables, as Eq.(4) was derived by matching with the equilibrium fluctuations in the grand canonical ensemble where charge is not conserved. Therefore, we have to supply the action (4) with the charge continuity constraint:

\[ \dot{\rho} + \nabla \cdot \mathbf{j} = \mathcal{J}(r, t) \],

where

\[ \mathcal{J}(r, t) = e\delta(r)(\delta(t + \tau) - \delta(t - \tau)) . \] (6)

This form of the charge source \( \mathcal{J}(r, t) \) describes electron injected the system at \( r = 0, t = -\tau \), and taken back at \( t = \tau \), at the same point. In principle, one could consider a more general source term \( \mathcal{J} \) of the form \( \mathcal{J} = e(\delta(t + \tau)\delta(r - r_1) - \delta(t - \tau)\delta(r - r_2)) \), which corresponds to the process of one electron entering and then leaving the liquid at different points. However, in real situation the tunnelling occurs preferentially at “hot spots”, or defects, where the tunnelling barrier is low or narrow. Thus we assume \( r_1 = r_2 \), which also accounts for the quantum point contact tunneling experiment.

The charge continuity requirement is incorporated in the action by adding a Lagrange multiplier term:

\[ S_{\text{total}} = S(\rho, \mathbf{j}) + \phi(r, t) (\dot{\rho} + \nabla \mathbf{j} - \mathcal{J}(r, t)) \],

where \( \phi(r, t) \) is an independent variable of the problem. For the least action path, the variation of \( S_{\text{total}} \) relative to infinitesimal change \( \delta \phi, \delta \rho, \) and \( \delta \mathbf{j} \) vanishes. (Note that due to the charge continuity \( \dot{\rho} + \nabla \cdot \delta \mathbf{j} = 0. \) After eliminating \( \phi \) we get the standard equations of classical electrodynamics:

\[ \begin{align*}
\text{(i)} & \quad \dot{\rho} + \nabla \cdot \mathbf{j} = \mathcal{J}(r, t) ; \\
\text{(ii)} & \quad \mathbf{j} + D \nabla \rho = \hat{\sigma}(\omega, q) \mathbf{E} ; \\
\text{(iii)} & \quad \mathbf{E}(r, t) = -\nabla r \int dr' \rho(r', t) U(|r - r'|) .
\end{align*} \]
The equations describe the system trajectory in imaginary time, i.e., the spreading of charge under the Coulomb barrier due to selfinteraction. Then one must solve Eq.(8) for $\rho$ and $j$, and to compute the action (4). For a spatially homogeneous system, by using Fourier transform, we get

$$\rho(\omega, q) = J(\omega) |\omega| + Dq^2 + \sigma_{xx} q^2 U_q,$$

$$j(\omega, q) = -i K^{-1}(\omega, q) q U_q \rho(\omega, q),$$

where $U_q$ is the Coulomb potential formfactor. Substituted in Eq.(4) this yields the action

$$S_0(\tau) = \frac{1}{2} \sum_{\omega, q} \frac{|J(\omega)|^2}{|\omega| + Dq^2} \frac{U_q}{|\omega| + Dq^2 + \sigma_{xx} q^2 U_q}$$

which depends on the accommodation time $\tau$ through Fourier component of the charge source: $J(\omega) = 2ie \sin \omega \tau$.

Finally, to obtain total action of the system we subtract from the action $S_0(\tau)$ of spreading charge the term $2eV \tau$ that accounts for the work done by voltage source: $S(\tau) = S_0(\tau) - 2eV \tau$. Thus the energy conservation at transferring one electron across the barrier is assured. Then one has to optimize $S(\tau)$ in $\tau$. Optimal $\tau_*$ satisfies the relation

$$\frac{\partial S_0(\tau_*(V))}{\partial \tau} = 2eV$$

Having found $\tau_*$ from Eq.(10) one gets the tunneling rate that coincides with tunneling conductivity up to a constant factor:

$$G(V) = G_0 \exp \left[ -\frac{1}{\hbar} \left( S_0(\tau_*(V)) - 2eV \tau_*(V) \right) \right].$$

The optimal $\tau_*$ can be interpreted as the charge accommodation time.

Let us point out here that the accuracy of the term $2eV \tau$ is determined by the assumption that $\tau_* \gg \tau_f$, the time it takes one electron to traverse the barrier. This assumption is valid whenever there is an anomaly: if $\tau_* \approx \tau_f$, then $S(\tau_*) \approx h$, and thus there is no tunneling suppression.

To summarize, the equations (3), (4), and (11), taken together, define tunneling conductivity. Within this quite general framework, one can study the anomaly in different systems. Let us emphasize that after having calculated $\tau_*(V)$ and $S(\tau_*)$ it is essential to check the self-consistency of the assumption that $\tau_* \gg \tau_f$. For example, this assumption will not be fulfilled in a clean metal, i.e., in Fermi liquid without disorder ($D > 1$). The reason is that the conductivity of an ideal conductor is $\sigma(\omega) = ine^2/m\omega$. In this case Eq.(3) gives $S_0(\tau) \approx h$ at any $\tau \gg \tau_f$, and henceforth $\tau_* \approx \tau_f$. This indicates absence of the anomaly in a clean metal, the result familiar from the Fermi liquid picture. On the contrary, for a one-dimensional metal $S_0(\tau) \sim \ln \tau/\tau_f$, which leads to the power-law anomaly known from the Luttinger liquid theory.

**V. COMPARISON WITH DIFFUSIVE ANOMALY**

For a two-dimensional metal with elastic scattering time $\tau_0$ and non-screened Coulomb interaction we set $U_q = 2\pi/|q|$ and $\sigma_{xx} = \sigma$, constant at $|\omega|, v_F|q| \leq 1/\tau_0$. Then Eq.(3) gives

$$S(\tau) = \frac{e^2}{8\pi^2 \sigma} \ln \left( \frac{\tau}{\tau_0} \right) \ln \left( \tau_0 \sigma^2 (\nu e)^2 \right).$$
\( \nu \) is compressibility. From Eq. (10),
\[
\tau^* = \frac{e}{4 \pi^2 V \sigma} \ln(h \nu e/V) .
\]  
(13)

The theory is selfconsistent in the hydrodynamic limit, \( \tau^* \geq \tau_0 \), i.e., at \( eV \leq e^2/\sigma \tau_0 \). Then the least action is
\[
S(V) = \frac{e^2}{8 \pi^2 \sigma} \ln \left( \frac{e}{4 \pi^2 \sigma V \tau_0} \right) \ln \left( \frac{e \tau_0 \sigma (\nu e^2)^2}{4 \pi^2 V} \right) .
\]  
(14)

It is interesting to compare this result with the identical double-log dependence derived by Altshuler, Aronov, and Lee in a different context \[2\]. They calculated perturbatively the correction to the tunneling density of states \( \delta \nu(\epsilon) \) with the assumption that it is small, \( |\delta \nu| \ll \nu_0 \), which is the case only for a weak disorder. It was found that \( \delta \nu(\epsilon) = -\bar{\hbar}^{-1} \nu_0 S(V = \epsilon/e) \), where \( S(V) \) is given by (14). The main difference is that our double-log has to be exponentiated to get the tunnelling density of states, while in \[2\] the double-log itself appears as a correction to the density of states. In the range of the perturbation theory validity the two results agree. From that point of view, our calculation provides description of the diffusive anomaly at low bias, where the perturbation theory diverges.

VI. SCREENING BY ELECTRODES

In a real experiment the charge tunnels between two electrodes, and there are separate contributions to the action due to the relaxation of the electron and hole charges on both sides of the barrier. If the electrodes are close, the charges partially screen the field of each other, which makes their spreading correlated. In this case the least action is smaller than the sum of independent contributions of the electrodes, and thus the anomaly is weakened. For a two dimensional system, quantitatively, the effect will be that the log-divergence of the integral over \( q \) in Eq. (10) will be cut at \( q \approx a^{-1} \), where \( a \) is the distance between the electrodes. As a result, the \( V \) dependence of the second log in Eq. (14) saturates at \( eV \approx V_0 = \bar{\hbar} \sigma/a \).

This “excitonic” correlation effect can be treated straightforwardly by writing the action (10) for each electrode separately, together with the term describing interaction across the barrier. Let us consider an example of two parallel planes with different conductivities and diffusion constants, \( \sigma_1, \sigma_2, D_1, \) and \( D_2 \). It is straightforward to generalize the action and to find the instanton. The least action is

\[
S_0(\tau) = \frac{1}{2} \sum_{\omega,q} |\mathcal{J}(\omega)|^2 \left[ \begin{array}{c} 1 \\ -1 \end{array} \right] \left( |q| + D q^2 \right)^{-1} U \left( |q| + D q^2 + \Sigma q^2 U \right)^{-1} \left[ \begin{array}{c} 1 \\ -1 \end{array} \right] ,
\]  
(15)

where we use matrix notation:
\[
D = \begin{pmatrix} D_1 & 0 \\ 0 & D_2 \end{pmatrix} , \quad \Sigma = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix} , \quad U = \begin{pmatrix} U_q & V_q \\ V_q & U_q \end{pmatrix} ,
\]

\[
U_q = \int e^{iqr} \frac{d^2r}{|r|} , \quad V_q = \int \frac{e^{iqr}}{\sqrt{r^2 + a^2}} d^2r .
\]  
(16)

The rows and columns of the matrices (15) correspond to the two planes.

Let us assume \( e^2 \nu_{1,2}a \gg 1 \), which is the case in almost all experiments. Then, by carrying out matrix inversion and integration we get

\[
S_0(\tau) = a \ln \left( \frac{\tau}{\tau_0} \right) \text{ at } \tau \gg \hbar/eV_0 .
\]  
(17)
Here
\[ \alpha = \frac{e^2}{4\pi^2} \left[ \frac{1}{\sigma_{1,xx}} \ln \frac{4\pi a}{D_2} + \frac{1}{\sigma_{2,xx}} \ln \frac{4\pi a}{D_1} \right], \]
where \( \sigma = \sigma_{1,xx}\sigma_{2,xx}/(\sigma_{1,xx} + \sigma_{2,xx}) \). If the two planes are identical,
\[ \alpha = \frac{e^2}{2\pi^2 \sigma_{xx}} \ln 2 \pi e^2 \nu a. \]  (19)

Thus at \( V < V_0 \) the \( I - V \) curve becomes the power law \( I \sim V^{\alpha+1} \). The tunneling suppression in this case is weaker than for the non-screened interaction.

Like other interaction effects, the anomaly is enhanced if the system dimension is lowered.

Let us consider tunneling into a disordered wire of thickness \( d \) from a well conducting electrode, two- or three-dimensional, parallel to the wire at a distance \( a \gg d \) away from it. In this case the main contribution to the action is due to charge relaxation in the wire. We still can use Eq. (9), where \( \sum \ldots = \int dq / 2\pi \ldots \) and the two-dimensional \( \sigma_{xx} \) is replaced by \( A \sigma \), where \( A \) is the wire crosssection and \( \sigma \) is the three-dimensional conductivity. The Coulomb potential formfactor at small \( k \ll a^{-1} \), using electrostatic image in the electrode, is found to be \( U_0 = 2 \ln a/d \).

By doing the sum in (9) over \( k \) and \( \omega \) we get
\[ S_0(\tau) = B \sqrt{\tau}, \quad B = \frac{U_0 e^2}{\sqrt{\pi D} + \sqrt{\pi (D + \sigma U_0 A)}}. \]  (20)

Then Eq. (10) gives optimal \( \tau_*(V) = B^2/16e^2V^2 \), and thus the conductivity
\[ G(V) \sim \exp \left( -\frac{B^2}{8eV} \right). \]  (21)

The “field+charge” diffusion constant \( \sigma U_0 A \) that appears in the 1D plasmon dispersion relation \( \omega(k) = -i \sigma U_0 A k^2 \) is typically much bigger than \( D \). Then the spreading of charge is effectively one dimensional at \( V \leq V_1 = e^2 U_0/8a \). For a thick wire Eq. (21) holds in a wide voltage range \( V_1 \geq V \geq V_{\text{diff}} \) where one can ignore localization corrections to \( \tau \). By the Thouless criterion, the voltage \( V_{\text{diff}} \) above which the diffusive treatment is valid is estimated by comparing the tunneling time \( \tau_*(V) \) with the inverse level spacing in the region over which the charge spreads, \( \Delta^{-1} = \nu A \sqrt{\sigma U_0 A \tau_*(V)} \). This gives
\[ V_{\text{diff}} = \left( 4\sqrt{\pi} \hbar \nu^2 e DA^2 \right)^{-1}. \]  (22)

On the time scale \( \tau_*(V_{\text{diff}}) \), the transport is diffusive if the wire length \( L \) is much shorter than \( L_{\text{diff}} = \sqrt{\sigma U_0 A \tau_*(V_{\text{diff}})} = \hbar \nu A^2 \sigma U_0 \).

**VII. TUNNELING IN A FINITE SYSTEM**

It is of interest to consider the tunneling problem for a system of small size in which the time of charge spreading is limited by the dimension. Most important realization is tunneling in small Coulomb blockade devices, like quantum dots or very small capacitors. For such systems, at sufficiently low bias, the dynamics of charge spreading is unimportant and the effect of charge relaxation can be taken into account by modelling the conducting part of the system by a single effective resistor.
In this limit, the problem can be treated by the recently developed theory of the “environment effect” on the tunneling conductivity. In the latter the environment of a tunnel junction is replaced by an effective electric circuit that causes voltage fluctuations across the junction. Assuming that the fluctuation spectral density is equilibrium, one calculates the conductivity suppression factor by doing gaussian average of the tunneling current, in a way similar to the Debye-Waller theory of the fluctuation effect on the structure factor of crystals. Alternatively, one can calculate the tunneling transition rate that includes the shake up effect of the plasmon modes in the environment. The results of Refs. can be derived by our method if one takes the zero wavelength limit of the action by keeping only the largest scale electromagnetic mode for each element of the circuit. (For example, in a resistor one takes uniform current distribution and zero charge density, or in a capacitor one assumes uniform charge distribution over the capacitor plates and no current.) Then the instanton calculation will lead to the results identical to those of Refs. This zero wavelength limit describes tunneling at very low voltage, when during the under-barrier motion the tunneling charge spreads over a scale comparable to the size of the system.

Therefore, there are several regimes corresponding to different values of the bias. Let us list them here for the problem of tunneling between two identical conducting sheets $L \times L$ of conductivity $\sigma$, separated by the barrier of thickness $a$.

- Effectively infinite system: $e^2/a > eV > e^2a/L^2$
  \[ G = dI/dV \sim V^\alpha, \quad \alpha = \frac{e^2 \ln(e^2 \nu a)}{4\pi \hbar \sigma}; \] (23)

- Finite system characterized by impedance of environment: $e^2a/L^2 > eV > \hbar D/L^2$
  \[ G \sim V^\alpha, \quad \alpha = \frac{e^2}{4\pi \hbar \sigma} \ln L/d; \] (24)

- Ohm’s law restored at $eV < \hbar D/L^2$: $G=$const

In the expression for $\alpha$ given for the infinite system it is assumed that $e^2 \nu a \gg 1$. The result for finite systems agrees with Refs. The third case describes the limit of very low bias, when the system is characterized by an overall transmission coefficient, which is equivalent to connecting in series the barrier and the metal effective resistors. The three cases describe the change of the conductivity over the whole voltage range, from relatively high voltage where perturbation theory works, through the region of low voltage where the anomaly disappears.

VIII. 2D ELECTRON GAS IN MAGNETIC FIELD

It is straightforward to incorporate magnetic field in the theory by substituting $\sigma_{xx} = \sigma_{xx}(B)$ in the action. (Note that $\sigma_{xy}$ does not enter.) As magnetic field increases, the conductivity drops, and at certain field it reaches the quantum limit $\sigma_q = e^2/\hbar$. In this field range the prefactor $\alpha$ in Eq. becomes of the order of one, and the anomaly in the conductivity changes from weak to strong. The threshold conductivity, according to Eq., is

\[ \sigma_c = \frac{1}{4\pi^2} \frac{e^2}{\hbar} \ln 2\pi e^2 \nu a \] (25)

A transition like that was observed by Ashoori et al. in the tunneling current from a 3D metal into a 2D electron gas. In this experiment, the ohmic conductance was measured as
function of temperature, which corresponds to our zero temperature non-linear current taken at $V \simeq k_B T/e$. The 2D gas was relatively clean with the zero field mobility corresponding to the elastic scattering time $\tau_0 \simeq 4 \cdot 10^{-12}$ s. The Fermi energy calculated from the electron density was $E_F \simeq 10$ mV. By using the result (24) together with the Drude-Lorentz model, $\sigma_{xx}(B) = n e^2 / m \tau_0 \omega_c^2$ at $\tau_0 \omega_c \gg 1$, one finds that the anomaly hardening transition corresponds to the cyclotron frequency $\omega_c^* = (8\pi E_F / \hbar \tau_0)^{1/2} \simeq 8.0$ mV. In terms of the field intensity this is approximately $4.6$ Tesla which is quite close to the transition field reported in Ref [4].

It is interesting to note that in a weakly disordered metal with $E_F \tau_0 \gg \hbar$ the threshold field is small: $\hbar \omega_c^* \ll E_F$. This means that the transition occurs well below the field where the Quantum Hall state is formed. Therefore, our estimate of $\omega_c^*$ based on the “bare” Drude-Lorentz conductivity is meaningful and legitimate. On the other hand, to find the current at very low $V$ one would have to use the conductivity renormalized by localization and interaction effects.

Finally, let us mention a relation with the work by Halperin, He, and Platzman [11] that deals with the anomaly in the $\nu = 1/2$ Quantum Hall state. In this work, the problem was treated by summing linked cluster terms of perturbation theory, with the density response function borrowed from the Chern-Simons Fermi liquid theory [17]. The anomaly was found to have the form:

$$G(V) \sim \exp \left( -\frac{V_0}{V} \right), \quad V_0 = 4\pi \frac{e^2}{\epsilon \sqrt{\pi n}}, \quad (26)$$

where $V \ll V_0$, and $n$ is density. It is interesting to see how this result can be derived from the effective action. It has been shown [17] that conductivity of the $\nu = 1/2$ state has strong spatial dispersion: $\sigma_k = A |k|$, $A = e^2 / 16\pi \epsilon \sqrt{\pi n}$. If this form is inserted in the action (9), one gets $S(\tau) = \pi \sqrt{2\tau} / A$, which leads to the tunneling rate (26).

**IX. CONCLUSION**

The essential feature of the presented approach is that it relates the tunneling anomaly with exact conductivity of the system without calculating it. This would allow a comparison with experiment in the situations where there is no accepted model for conductivity. For example, the tunneling current as function of voltage can be taken from experiment, and directly used to find $S(\tau)$ by inverse Legendre transformation. Thus obtained function $S(\tau)$ can be analyzed to extract the conductivity frequency and wavevector dependence.

To summarize, we argued that the theory of the Coulomb anomaly in the regime of strong suppression of tunneling is semiclassical. The underlying reason is that the transfer of one electron across the barrier is controlled by cooperative motion of many other electrons. We treat this motion as classical electrodynamics in imaginary time, write down the action and find the least action instanton trajectory. The results are compared to several known perturbation theories and an agreement is found. The theory is used to interpret experiments on the coulomb gap induced by magnetic field.

[1] B. L. Altshuler and A. G. Aronov, in: Electron-Electron Interaction in Disordered Systems, eds. A. L. Efros and M. Pollak (North-Holland, 1985); R. C. Dynes and P. A. Lee, Science, vol. 223, p.355 (1984);
Nous présentons une approche de la théorie effective pour le problème du blockage coulombien dans l’effet tunnel. La méthode est appliquée au problème de la limite de fort couplage proche du voltage zero, ou la théorie de la perturbation diverge. Par un argument semiclassique, nous obtenons une théorie électrodynamique en temps imaginaire, et nous exprimons l’anomalie en terme de la conductivité exacte $\sigma(\omega, q)$ du système et de l’interaction exacte. La validité de notre approche est testé par comparaison avec le calcul connu de l’anomalie diffusive obtenu en théorie de la perturbation. Nous utilisons aussi cette méthode pour l’étude de l’accroissement de l’anomalie du à un champs magnétique externe, et à l’écrantage des électrodes.