Luescher’s finite volume test for two-baryon systems with attractive interactions

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with T. Doi (Riken) and T. Iritani (StonyBrook U.) for HAL QCD collaboration

A previous talk by T. Iritani in this session.
Motivation
Direct vs Potential : NN systems

Reviewed in T. Doi PoS LAT2012,009 (+ updates)

“di-neutron”

“deuteron”

Potential

\( \Delta E \) [MeV]

\( m_\pi^2 \) [GeV\(^2\)]

Potential method (HALQCD) : unbound

Direct method (Yamazaki et al./NPL/CalLat): bound
Fake plateau problem (direct method)

Plateaux from wall and smeared sources disagree.
One (or both) of them is fake, but we can not judge if they are fake or not.

need a method to see a reliability of data from one source without others.
Finite volume test
Finite volume formula

S-wave

\[ k \cot \delta(k) = \frac{1}{\pi L} \sum_{\vec{n} \in \mathbb{Z}^3} \frac{1}{\vec{n}^2 - q^2}, \quad q = \frac{kL}{2\pi}, \quad \Delta E = 2\sqrt{k^2 + m^2 - 2m} \]

attractive interaction \( \Delta E < 0 \) \[ k^2 < 0 \] \[ \delta(k) \text{ at } k^2 < 0 \] ?

analytic continuation of \( \delta(k) \) at \( k^2 < 0 \)

analytic continuation of \( \delta(k) \) at \( k^2 < 0 \)

finite Ls

infinite L

ground state

scattering state

1st excited state

with bound state

discrete spectra

with bound state

continuum (+ pole)
One can check lattice data at finite volume from ERE behaviors.

ERE (Effective Range Expansion)\[ k \cot \delta(k) = \frac{1}{a_0} + \frac{r_0}{2} k^2 + \cdots \]

\[
\Delta E_L \ [\text{MeV}]
\]

\[
\text{NN (YKU 12) } \frac{k \cot \delta}{m_\pi}
\]

scattering state spectra

L-indep. fake plateaux

if "true" pole

finite vol. spectra

ERE

\[
(k/m_\pi)^2
\]
Results
$N_f = 2 + 1, \ a \simeq 0.09 \ fm, \ m_\pi \simeq 510 \ MeV$

$\Delta E_{NN}(^1S_0) \simeq -7.4(1.3) \ MeV$

$\Delta E_{NN}(^3S_1) \simeq -11.5(1.1) \ MeV$
\[ \Delta E_{NN}(^{1}S_0) \simeq -3.9(1.3) \text{ MeV} \]

\[ \Delta E_{NN}(^{1}S_0) \simeq -0.7(0.8) \text{ MeV} \]

HALQCD 2016

same ensembles of Yamazaki et al. 2012

\[ E_{NN}(^{1}S_0) = \left(1 \pm \frac{1}{2}\right) \text{MeV} \]

Data from a previous talk by T. Iritani

HAL 2016 smeared src. NN(\(^{1}S_0\))

smeared

HAL 2016 wall src. NN(\(^{1}S_0\))

wall

strange behaviors

strange behaviors except two largest volumes
\[ \Delta E_{NN}(^3S_1) \simeq -8.7(0.9) \text{ MeV} \]

\[ \Delta E_{NN}(^3S_1) \simeq -1.4(0.8) \text{ MeV} \]

HAL 2016 smeared src. NN\(^(^3S_1)\)

HAL 2016 wall src. NN\(^(^3S_1)\)

finite volume tests suggest signals for NN bound states are fake.
$\Delta E_{\Xi\Xi}(1S_0) \simeq -5.4(0.8) \text{ MeV}$

$\Delta E_{\Xi\Xi}(1S_0) \simeq -0.3(0.5) \text{ MeV}$

$\Delta E_{\Xi\Xi}(3S_1) \simeq 12.2(0.9) \text{ MeV}$

$\Delta E_{\Xi\Xi}(3S_1) \simeq -0.9(0.6) \text{ MeV}$
Quenched, $a \simeq 0.128$ fm, $m_\pi \simeq 800$ MeV

$N_f = 2 + 1$, $a \simeq 0.09$ fm, $m_\pi \simeq 300$ MeV
All NN bound states from Yamazaki et al. have strange ERE behaviors

1. finite volume formula does not work (too small volumes)  unlikely
2. strange ERE behaviors are correct.  unlikely
3. extracted energy shifts are incorrect  likely, agrees with Iritani’s results

finite volume formula

\[ k \cot \delta(k) = \frac{1}{\pi L} \sum_{\vec{n} \in \mathbb{Z}^3} \frac{1}{\vec{n}^2 - q^2} = \frac{1}{a_0} + \frac{r_0}{2} k^2 + \cdots \]

a very easy and useful test for a reliability of the extracted energy shift

How about other results?
\( N_f = 2 + 1, a_s \simeq 0.123 \text{ fm}, a_s/a_t \simeq 3.5, m_\pi \simeq 390 \text{ MeV} \)

\( N_f = 2 + 1, a \simeq 0.1167 \text{ fm}, m_\pi \simeq 450 \text{ MeV} \)
$N_f = 3$ (SU(3) limit), $a \simeq 0.145$ fm, $m_{PS} \simeq 800$ MeV

NPL 2012 : PRC88(2013)024003

strange ?

possible ?

deeply bound

deeply bound

strange
Summary Plots

\[ \text{NN}(^1S_0) \]

\[ \text{NN}(^3S_1) \]
Conclusion and Discussion
• Finite volume formula give a useful test for the bound states.
  • Yamazaki et al.: very strange behaviors (fail the test)
    • confirmed by HAL smeared data.
  • NPL: some pass, the other fail the test. (Not conclusive)
    • necessary test but can not guarantee the correctness.
    • need further checks (wall vs. smeared, variational method)
  • finite volume test is mandatory for the bound state search in lattice QCD
  • the formula should be used for the infinite volume extrapolation
    • using LO (NLO) ERE

\[
k \cot \delta(k) = \frac{1}{\pi L} \sum_{\vec{n} \in \mathbb{Z}^3} \frac{1}{|\vec{n}|^2 - q^2} = \frac{1}{a_0} + \frac{r_0}{2} k^2 + \cdots
\]
Direct vs Potential : NN systems

“di-neutron”

“deuteron”

Potential

Yamazaki et al.

NPL

CalLat

Potential method (HALQCD) : unbound

Direct method (Yamazaki et al./NPL/CalLat): bound

questionable
$N_f = 3$ (SU(3) limit), $a \simeq 0.145$ fm, $m_{PS} \simeq 800$ MeV

same as NPL 2012

$NN(^1S_0)$
$NN(3S_1)$

NPL 2012 10-bar rep.

NPL 2012

NPL and CalLat seems incompatible.

Second bound state in CalLat?

NLO is large?

Large effective range?
Expectation at physical point $L \approx 8 \text{ fm}$