Pretzelosity distribution function $h_{1T}^\perp$

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The 'pretzelosity' distribution $h_{1T}^\perp$ is discussed. Theoretical properties, model results, and perspectives to access experimental information on this leading twist, transverse momentum dependent parton distribution function are reviewed. Its relation to helicity and transversity distributions is highlighted.

Keywords: semi-inclusive deep inelastic scattering (SIDIS), single spin asymmetry (SSA), transverse momentum dependent distribution function (TMD)

1. Introduction

SIDIS allows to access information on TMDs that are defined in terms of light-front correlators 1–6 (with process-dependent paths 7–10)

$$\phi(x, \vec{p}_T)_{ij} = \int \frac{dz}{(2\pi)^2} e^{ipz} \langle N | \bar{\psi}_j(0) W(0, z, \text{path}) \psi_i(z) | N \rangle \bigg|_{z^+=0, p^+=xP^+}$$

where light-cone components $p^\pm = (p^0 \pm p^3)/\sqrt{2}$ are along the virtual photon momentum, and transverse vectors like $\vec{p}_T$ are perpendicular to it. Different TMDs are given by traces of the correlator (1) with specific $\gamma$-matrices. There are 8 leading-twist TMDs, i.e. they give rise to effects that are not power suppressed in the hard scale, and each of them contains independent information about the nucleon structure. All leading-twist TMDs can be accessed unambiguously in SIDIS with polarized leptons and nucleons by observing the azimuthal distributions of produced hadrons.

In this note we will focus on the leading-twist TMD pretzelosity $h_{1T}^{\perp a}$, on which interesting results have been obtained from model calculations. This TMD is responsible for a SSA $\propto \sin(3\phi - \phi_S)$ in SIDIS that could be accessed in experiments — most promisingly at Jefferson Lab.
2. Properties of $h_{1T}^{\perp a}$

Let us list briefly, what we know about the pretzelosity distribution.

I. It can be projected out from the correlator (1) by tracing it with $i\sigma^{\perp+}\gamma_5$
where it appears as the coefficient of the structure $S_T^b(p_T^1-p_T^2-\frac{i}{2}\not{p}_T^2\delta^{ik})$,
and it has a probabilistic interpretation.$^6$

II. It requires nucleon wave-function components with two units orbital
momentum difference,$^{11}$ and 'measures' the deviation of the 'nucleon
shape' from a sphere.$^{12}$ (That is why it is called 'pretzelosity'!)

III. It is expected to be suppressed at small and large
values of $x$ with respect to parton distributions like $f_1^u(x)$, $g_1^u(x)$, $h_1^u(x)$.

IV. It satisfies the positivity condition$^{15}$ $2|h_{1T}^{\perp(1)a}(x)| \leq f_1^a(x) - g_1^a(x)$.
The above and Soffer bound$^{16}$ imply: $|h_{1T}^{\perp(1)a}(x)| + |h_{1T}^a(x)| \leq f_1^a(x)$.

V. In the limit of a large number of colors $N_c$ in QCD it was shown that$^{17}$
$(h_1^{\perp u} + h_1^{\perp d})/(h_1^{\perp u} - h_1^{\perp d}) = \mathcal{O}(1/N_c)$ for $xN_c \sim \mathcal{O}(1)$ and $p_T \sim \mathcal{O}(1)$.
The same pattern is predicted also for antiquarks.$^{17}$

VI. It was observed in the bag model$^{18}$ that $h_{1T}^{\perp(1)q}(x) = g_1^q(x) - h_1^q(x)$
which is confirmed in many$^{18-22}$ (but not all$^{22,23}$) models.

VII. In simple (spectator-type) models, it has been related to chirally odd
generalized parton distributions.$^{22}$

VIII. In SIDIS with unpolarized electrons ($U$) and transversely polarized
nucleons ($T$) it gives rise (in combination with Collins fragmentation
function$^{24,25}$) to an azimuthal modulation of the produced hadrons
proportional to $\sin(3\phi - \phi_S)$. Here $\phi$ ($\phi_S$) is the azimuthal angle of
the produced hadron (nucleon polarization vector) with respect to the
virtual photon. The corresponding SSA is given by

$$A_{UT}^{\sin(3\phi - \phi_S)} = \frac{C_G \sum_a e_a^2 x h_{1T}^{\perp(1)a}(x) H_1^{\perp (1/2)a}(z)}{\sum_a e_a^2 x f_1^a(x) D^a_1(z)} \tag{2}$$

with $h_{1T}^{\perp(1)}(x) = \int d^2\not{p}_T \frac{\not{p}_T^2}{2M_N} h_{1T}^{\perp}(x, \not{p}_T^2)$ the 'transverse moment' of pretzelosity.
Unless the DIS counts are weighted with adequate powers of transverse hadron momenta$^5$ it is necessary to assume a model for the
distribution of transverse parton momenta. In (2) the Gauss model is
assumed. The factor $C_G$ contains the dependence on Gauss model
parameters and is a slowly varying function of these parameters that
can be well approximated for practical purposes$^{18}$ by its maximum
$C_G \leq C_{\text{max}} = 3/(2\sqrt{2})$. Extractions of the (1/2)-moment of the
Collins function $H_1^{\perp (1/2)}(z) = \int d^2\vec{K}_T |\vec{K}_T| H_1^{\perp}(z, \vec{K}_T^2)$ from data$^{26-30}$
were reported elsewhere.$^{31-33}$
3. Pretzelosity in the bag model

In this Section we review the pretzelosity calculation\(^\text{18}\) in the MIT bag model, in which the nucleon consists of 3 non-interacting quarks confined in a spherical cavity.\(^{34-37}\) The momentum space wave function is given by

\[
\varphi_m(\tilde{k}) = i\sqrt{4\pi N} R_0^3 \left( \frac{t_0(|\tilde{k}|) \chi_m}{\vec{q} \cdot \tilde{k} t_1(|\tilde{k}|) \chi_m} \right),
\]

with \(N = \omega^{3/2}(2R_0^3(\omega - 1) \sin^2 \omega)^{-1/2}, \quad \tilde{k} = \vec{k}/|\vec{k}|\). We fix the bag radius \(R_0\) in terms of the proton mass \(M_N\) as \(R_0 M_N = 4\omega\) with \(\omega \approx 2.04\) the lowest root of the bag eigen-equation. Finally \(t_i(\kappa) = \int_0^1 u^2 du j_i(u R_0 \kappa) j_i(u \omega)\) where \(j_i\) are spherical Bessel functions. The bag model wave function (3) contains both \(S\) (represented by \(t_0\)) and \(P\) (represented by \(t_1\)) waves.

With the above wave functions, one obtains the following results\(^\text{18}\)

\[
f_1(x, k_{\perp}) = A \left[ t_0^2 + 2t_0 t_1 \frac{k_z}{k} + t_1^2 \right], \quad g_1(x, k_{\perp}) = A \left[ t_0^2 + 2t_0 t_1 \frac{k_z}{k} + t_1^2 \frac{2k^2}{k^2} - 1 \right] (5)
\]

\[
h_1(x, k_{\perp}) = A \left[ t_0^2 + 2t_0 t_1 \frac{k_z}{k} + t_1^2 \frac{k_z^2}{k^2} \right], \quad h_{1T}^q(x, k_{\perp}) = A \left[ -2 \frac{M_N^2}{k^2} t_1^2 \right] (7)
\]

with \(A = 16\omega^4/[(\pi^2(\omega - 1)) j_0^2(\omega) M_N^2]\) and the flavor dependence given by (we assume an \(SU(6)\) spin-flavor symmetric proton wave function):

\[
f_1^q(x, k_{\perp}) = N_q f_1(x, k_{\perp}), \quad N_u = 2, \quad N_d = 1
\]

\[
g_1^q(x, k_{\perp}) = P_q g_1(x, k_{\perp}), \quad P_u = \frac{4}{3}, \quad P_d = -\frac{1}{3}. \quad (8)
\]

The flavor dependence of \(h_1^q\) and \(h_{1T}^q\) is analogous to \(g_1^q\). The momenta \(k_z\) and \(k\) are defined as \(k_z = x M_N - \omega/R_0\), and \(k = \sqrt{k_z^2 + k_{\perp}^2}\).

Since \(h_{1T}^q \propto t_1^2\) it is proportional to the square of the \(P\)-wave\(^\text{11}\) and thus sensitive to the quark orbital angular momentum in the proton. The results (4-8) satisfy\(^\text{18}\) the positivity conditions in point III, and are consistent\(^\text{18}\) with the predictions from the large-\(N_c\) limit discussed in point IV of Sec. 2.

From the results in Eqs. (4-7) we find that out of the 4 TMDs \(f_1(x, k_{\perp}), g_1(x, k_{\perp}), h_1(x, k_{\perp}), h_{1T}^{I}(x, k_{\perp}) \equiv k_{\perp}^2/(2M_N^2) h_{1T}^{I}(x, k_{\perp})\) only 2 are linearly independent. In QCD the different TMDs are, of course, all independent of each other. But in the bag model all TMDs are expressed in terms of \(t_0\) and \(t_1\) in Eq. (3), which naturally gives rise to bag model relations.
among different TMDs. The most interesting (and possibly more general, see Sec. 4) relation is\textsuperscript{18}

\begin{equation}
    h_{1T}^{q\perp}(x, k_{\perp}) = g_{1}^{q}(x, k_{\perp}) - h_{1}^{q}(x, k_{\perp}).
\end{equation}

Figs. 1a and 1b show results for $h_{1T}^{q\perp}(x) = \int d^{2}k_{\perp} h_{1T}^{q\perp}(x, k_{\perp})$ at the low bag model scale. The pretzelosity distributions $h_{1T}^{q\perp}(x)$ have opposite signs compared to transversity and are larger than $h_{1}^{q}(x)$ in the bag model, even larger than $f_{1}^{q}(x)$. However, $h_{1T}^{q\perp}(x)$ is not constrained by positivity bounds.

The bag\textsuperscript{18} (and spectator\textsuperscript{19}) model results satisfy the positivity bounds for $h_{1T}^{q\perp}(x)$ in point III of Sec. 2. In Fig. 2 the results for $h_{1T}^{q\perp}(x)$ from both models are compared. We observe good qualitative agreement.

The moduli of the transverse moments $h_{1T}^{q\perp}(x)$ of the pretzelosity distribution functions are about 3 or more times smaller than those of the transversity distribution functions $h_{1}^{q}(x)$ with the exception of very small-$x$ where both models are strictly speaking not applicable.\textsuperscript{18}
4. How general is the relation in Eq. (9)?

Eq. (9) is remarkable from the point of view of the observation that \( g_1^q(x) \) and \( h_1^q(x) \) is a measure for relativistic effects in nucleon. This difference is just the transverse moment of pretzelosity!

It is clear that this relation cannot be strictly valid in QCD, where all TMDs are independent. However, could it nevertheless allow to gain a reasonable estimate for \( h_{1T}^1(x) \) in terms of transversity and helicity? Until clarified by experiment, we can address this question only in models.

Interestingly, the relation (9) does not hold only in the bag model, but is found to be satisfied also in the spectator model. In fact, it was conjectured that (9) could hold in a large class of relativistic quark models, which was subsequently confirmed in the constituent quark model and the relativistic model of the proton. But (9) is not satisfied in a different than version of the spectator model.

The limitations of (9) are nicely illustrated in the 'quark target model' where in addition to quarks there are also gluons, and (9) is not satisfied. Thus, the explicit inclusion of gluon degrees of freedom spoils this relation. Of course, as stressed above, we do not expect (9) to be valid in QCD.

It would be interesting to know the necessary and sufficient conditions for the relation (9) to hold in a model.

5. Preliminary COMPASS data & prospects at JLab

At COMPASS the \( \sin(3\phi - \phi_S) \) and other SSAs were measured on a deuteron target. By saturating the positivity bound for \( h_{1T}^1(x) \) (point III in Sec. 2) we estimated the maximum effect for \( A_{UT}^{\sin(3\phi - \phi_S)} \). For that information

![Graph](image-url)
on Collins effect\cite{32,33} and parameterizations\cite{40,41} were used. The results are shown in Fig. 3 and compared to the preliminary data.\cite{39}

At small $x$ the preliminary data favor that pretzelosity does not reach the bound. Whether due to the expected suppression at small $x$ or opposite signs of $u$ and $d$-flavors, see Sec. 2, cannot be concluded.

The important observation is that preliminary COMPASS data\cite{39} do not exclude a sizeable effect in the region $x>0.1$, see Fig. 3, where JLab can measure with precision. This is demonstrated in Fig. 4 showing the $\pi^+$ SSA from a proton target in the kinematics of CLAS with 12 GeV beam upgrade (with error projections for 2000 hours run time\cite{42}).

It could be even more promising to look at SSAs due to Collins effect, like $A_{UT}^{\sin(3\phi-\phi_S)}$, in kaon production. The statistics for kaon production is lower than for pion production, but the SSA might be larger as it is suggested by a model\cite{43} of the Collins function. With a RICH detector at CLAS the kaon SSAs could be measured in the valence-$x$ region.\cite{44}

6. Conclusions

We reviewed the bag model calculation of pretzelosity.\cite{18} An interesting observation is the relation (9) which connects helicity and transversity distributions to $h_{1T}^{\perp}(x)$, and is valid in many\cite{18–22} (though not all\cite{22,23}) models, but not in QCD where all TMDs are independent.\cite{45} Nevertheless (9) could turn out a useful approximation. In view of the numerous novel TMDs, well-motivated approximations are welcome.\cite{46}

In the bag model $h_{1T}^{\perp}$ is proportional to minus the square of the $p$-wave component of the nucleon wave function, and therefore manifestly negative ($h_{1T}^{\perp}$ has opposite sign dictated by SU(6) symmetry, and large $N_c$.\cite{17}). As a consequence of (9) we have $|h_T(x)| > |g_T(x)|$. This is found in models,\cite{47,48}

Forthcoming analyzes and experiments at COMPASS, HERMES and JLab\cite{42,49,50} will provide valuable information on the pretzelosity distribution function, and deepen our understanding of the nucleon structure.
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