A Sliding-Mode Set-Point Position Controller for Hydraulic Excavators

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ABSTRACT This paper proposes a sliding-mode set-point position controller for hydraulic excavators. The controller employs a simple switching surface that exponentially drives the actuator position to the desired position with a specified time constant. The discrete-time algorithm of the controller is constructed through a double-implicit implementation scheme, which is for implementing a simple sliding-mode controller to nonsmooth actuators, including hydraulic actuators. We employ a nonsmooth quasistatic model of the hydraulic actuator, which analytically accounts for the pressure saturation caused by the relief valves and check valves and the square-root law of the pressure-flowrate relation. This paper elaborates the analytical form of the model to be combined with the double-implicit implementation. A state predictor based on the actuator model extends the controller to compensate for the deadtime in the hydraulic system. The proposed controller is validated with some simulations and also through experiments employing the swing axis of a 13-ton class excavator.

INDEX TERMS Deadtime compensation, hydraulic actuator, hydraulic excavator, sliding-mode control.

I. INTRODUCTION Hydraulic excavators are expected to expand their applicability through the combination with more sophisticated control technology. In particular, automatic, semi-automatic, and teleoperated excavators would be potentially useful in many applications such as construction, disaster restoration, and demolition. Position control technology of hydraulic actuators is an essential building block for such future applications. There have been many position controllers proposed for hydraulic actuators. Many of them are intended for trajectory-tracking control [1]–[6], with which the actuator tracks time-varying position command. In many of the reported experimental results, the excavators track smooth command trajectories without making large positional errors.

In contrast to the trajectory-tracking problem, the set-point control problem, i.e., position control from large positional errors, is also important for semi-automatic or teleoperated hydraulic excavators. Such large positional errors may happen when the desired position discontinuously jumps due to communication failures or in applications where the excavator is commanded to move its bucket back to a registered position. The practical requirement for set-point control of excavators is not only to ensure accurate convergence but also to realize appropriate transient behavior converging to the desired position without producing oscillation or overshoots.

Some previous papers [7]–[11] report experimental results of transient responses to a step-like position command. Many of these papers employ proportional-integral-derivative (PID) controllers with which the control gains are adjusted with numerical, heuristic, or empirical methods, such as particle swarm optimization method [7], [8], genetic algorithm [9], and ant-colony optimization [10]. Kim et al. [11] approach is based on an analytical model of the hydraulic system, being based on a linearized system model with multiplicative uncertainty. As far as the authors are aware, there have been no set-point control methods that analytically account for the nonlinearity of the hydraulic systems, such as the pressure saturation caused by the relief valves and check valves, and the square-root law (see, e.g., [12]) of the pressure-flowrate relation.

One convenient approach to design the transient behavior of set-point control is to employ the concept of sliding-mode control, which constrains the system state to the prescribed convergence law. Sliding-mode control is also convenient
to deal with saturation because the bounds of the control input can be explicitly specified in the controller structure. The use of sliding-mode control from this perspective has been proposed for robotic manipulators [13], [14] but not for hydraulic systems. Previous sliding-mode controllers for hydraulic excavators [1], [5], [6] are for ensuring accurate convergence to the desired position, not for designing the transient behavior.

This paper presents a sliding-mode set-point position controller for hydraulic excavators. The controller employs a simple switching surface that exponentially drives the actuator position to the desired position with a specified time constant. The discrete-time algorithm of the controller is constructed through a double-implicit implementation scheme [15], which is for implementing a simple sliding-mode controller to a particular class of plants. We employ a nonsmooth quasistatic model [16] of the actuator structure illustrated in Fig. 1. This paper elaborates the analytical form of the nonsmooth quasistatic model to be combined with the double-implicit implementation and also extends the controller by a model-based state predictor to compensate for the deadtime in the plant. The controller is tested with simulations and experiments employing the swing axis of a 13-ton class excavator.

One practical benefit of the proposed controller is that most of the parameters can be uniquely chosen according to the hardware specifications, without the necessity of parameter tuning on a trial-and-error basis. It is in contrast to simple linear controllers such as PD, PI, or PID controllers. It may be possible to establish gain-tuning methods for linear controllers such as PD, PI, or PID controllers. It may be possible to establish gain-tuning methods for linear controllers based on locally linearized models [11], [17], [18], but they do not cope with strong nonlinearity or nonsmoothness in the relation between the control input and the actuator force, which are caused by relief valves and check valves. Being based on the double-implicit implementation framework [15], our approach fully accounts for nonlinear factors that are not suited for local linearization, such as the pressure saturation caused by the relief and check valves and the square-root law between the valve flowrate and the pressure drop.

The rest of this paper is organized as follows. Section II provides preliminaries, including the nonsmooth quasistatic model [16] of a hydraulic actuator and the double-implicit implementation [15] of a sliding-mode controller. Section III presents the proposed controller for excavators, which is based on the nonsmooth quasistatic model and the double-implicit implementation, and comprises the state predictor to compensate for the deadtime in the plant. Section IV and Section V presents simulation and experimental results, respectively. Section VI concludes this paper.

II. PRELIMINARIES
A. MATHEMATICAL PRELIMINARIES
In this paper, $\mathbb{R}$ denotes the set of all real numbers, $\mathbb{R}^+$ denotes the set of extended real numbers, $\mathbb{R}_{\geq 0} = [0, \infty)$, $\mathbb{R}_{\leq 0} = (-\infty, 0]$, and $\mathbb{R}_{\pm} = (-\infty, +\infty)$. The set of non-negative real numbers, $\mathbb{R}_{\geq 0}$, denotes the set of all non-negative real numbers, $\mathbb{R}_{\leq 0}$ denotes the set of all non-positive real numbers, and $\mathbb{R}$ denotes the closed unit ball in $\mathbb{R}$, i.e., $B = [-1, 1] \subset \mathbb{R}$.

The following functions are used in this paper:

\begin{align}
\text{sat}_X(x) & \triangleq \begin{cases} \min X & \text{if } x < \min X \\ x & \text{if } x \in X \\ \max X & \text{if } x > \max X \end{cases} \\
\text{sgn}(x) & \triangleq \begin{cases} x / |x| & \text{if } x \neq 0 \\ [-1, 1] & \text{if } x = 0 \end{cases} \\
\text{gsng}(a, b) & \triangleq \begin{cases} b & \text{if } x > 0 \\ \text{conv}(a, b) & \text{if } x = 0 \\ a & \text{if } x < 0 \end{cases} \\
S(x) & \triangleq \text{sgn}(x) x^2 \\
R(x) & \triangleq \text{sgn}(x) \sqrt{|x|} \\
N_X(x) & \triangleq \begin{cases} [0, \infty) & \text{if } x = \max X \\ 0 & \text{if } x \in X^0 \\ (-\infty, 0] & \text{if } x = \min X \\ \emptyset & \text{if } x \notin X \end{cases}
\end{align}

where $X$ is a closed interval in $\mathbb{R}$, and $X^0$ is the interior of $X$. Here, $\text{conv}(a, b)$ stands for the convex closure of the set $[a, b]$, being the closed set $[a, b]$ if $a \leq b$ and $[b, a]$ if $b \leq a$. In addition, the following relation holds true between the functions $N_X$ and $\text{sgn}$:

\[ y \in N_X(x) \iff x \in \text{sgn}(y). \]

A function of one or more sets should be understood in the following manner:

\[ \Phi(X) = \bigcup_{x \in X} \Phi(x) \]
\[ \Phi(X, Y) = \bigcup_{(x, y) \in X \times Y} \Phi(x, y). \]

With a set-valued function $f : C_x \rightrightarrows C_y$ where $C_x \subset \mathbb{R}$ and $C_y \subset \mathbb{R}$, recall that the inverse function $f^{-1} : C_y \rightrightarrows C_x$ is the
function satisfies the following condition:

\[ y \in f(x) \iff x \in f^{-1}(y), \quad \forall \{x, y\} \in \mathcal{C}_x \times \mathcal{C}_y. \quad (10) \]

A set-valued function \( f : \mathcal{C}_x \Rightarrow \mathcal{C}_y \) is said to be total if \( f(x) \neq \emptyset \) for all \( x \in \mathcal{C}_x \). It is said to be surjective if, for all \( y \in \mathcal{C}_y \), there exists \( x \in \mathcal{C}_x \) such that \( y \in f(x) \). If \( f \) is total and surjective, \( f^{-1} \) is also total and surjective. A set-valued function \( f \) is said to be monotone if it satisfies \((x_1 - x_2)(y_2 - y_1) \geq 0\) for all \( x_1, x_2 \in \mathcal{C}_x, y_1 \in f(x_1), \) and \( y_2 \in f(x_2) \). With a monotone function \( f \), the following is satisfied:

\[ f(x_1) \ni y_1 \leq y_2 \ni f(x_2) \iff f^{-1}(y_1) \ni x_1 \leq x_2 \ni f^{-1}(y_2). \quad (11) \]

This paper defines set-valued extensions of min and max operators as follows:

\[
\begin{align*}
\max(\mathcal{X}, \mathcal{Y}) & \equiv \{ \xi \in \mathbb{R} \mid \mathcal{X} \ni \xi \geq \exists y \in \mathcal{Y} \lor \mathcal{X} \ni \exists x \leq \xi \ni \mathcal{Y} \} \quad (12a) \\
\min(\mathcal{X}, \mathcal{Y}) & \equiv \{ \xi \in \mathbb{R} \mid \mathcal{X} \ni \xi \leq \exists y \in \mathcal{Y} \lor \mathcal{X} \ni \exists x \geq \xi \ni \mathcal{Y} \} \quad (12b)
\end{align*}
\]

where \( \mathcal{X} \subset \mathbb{R} \) and \( \mathcal{Y} \subset \mathbb{R} \). If \( \mathcal{X} \) and \( \mathcal{Y} \) are closed intervals in \( \mathbb{R} \), the followings are satisfied:

\[
\begin{align*}
\max(\mathcal{X}, \mathcal{Y}) & = [\max(\min(\mathcal{X}, \min(\mathcal{Y})), \max(\max(\mathcal{X}, \min(\mathcal{Y}))))] \\
\min(\mathcal{X}, \mathcal{Y}) & = [\min(\min(\mathcal{X}, \min(\mathcal{Y})), \min(\max(\mathcal{X}, \min(\mathcal{Y}))))] . \quad (13a) \\
\min(\mathcal{X}, \mathcal{Y}) & = [\min(\min(\mathcal{X}, \min(\mathcal{Y})), \min(\max(\mathcal{X}, \min(\mathcal{Y}))))] . \quad (13b)
\end{align*}
\]

The following lemma is useful to deal with the inverse functions and the min and max operators.

**Lemma 1:** Let \( f_1 : \mathcal{C} \rightrightarrows \mathbb{R} \) and \( f_2 : \mathcal{C} \rightrightarrows \mathbb{R} \) be total, surjective, monotone, set-valued functions where \( \mathcal{C} \subset \mathbb{R} \). Then, \( f(x) \equiv \min(f_1(x), f_2(x)) \) results in \( f^{-1}(y) = \max(f_1^{-1}(y), f_2^{-1}(y)) \) and \( f(x) \equiv \max(f_1(x), f_2(x)) \) results in \( f^{-1}(y) = \min(f_1^{-1}(y), f_2^{-1}(y)) \).

**Proof:** See Appendix A. \( \square \)

### B. NONSMOOTH QUASISTATIC MODEL OF A HYDRAULIC ACTUATOR

This section overviews the nonsmooth quasistatic model [16] of the hydraulic actuator driven by the circuit illustrated in Fig. 1. As can be seen in the figure, the actuator has two chambers separated by a piston, and the circuit has four main control valves, a bleed valve, three relief valves, and three check valves. The cross-sectional areas and the internal pressures of the chambers are denoted by \( A_s \) and \( P_s \) (\( s \in \{h, r\} \)), respectively, where \( h \) means the head-side and \( r \) means the rod-side. The pressure limits of the head- and rod-side relief valves are \( P_{hl} \) and \( P_{rM} \), respectively. The oil flow in the circuit is supplied by a single pump, of which the flowrate is \( Q \). The circuit comprises a pump relief valve, of which the pressure limit is \( P_{M} \), to secure the oil outlet from the pump, and a pump check valve to prevent the backflow into the pump.

For each of the control valves (four main control valves and one bleed valve), the ratio of the valve opening area to its maximum value is denoted by \( u_{s} \in [0, 1] \) (\( s \in \{ph, pr, th, tr, b\} \)). The flowrates \( Q_s \) (\( s \in \{ph, pr, th, tr, b\} \)) through the valves can be assumed to satisfy the following flowrate-pressure relations [12], [19]:

\[
Q_s = c_s u_s R(\Delta P_s) \quad (14)
\]

where \( c_s \equiv C_s a_s \sqrt{2/\rho} \), \( \Delta P_s \) is the pressure drop across the valve, \( \rho \) is the mass density of the oil, \( a_s \) is the maximum opening area of the valve, and \( C_s \) is the discharge coefficient [20] of the valve. The discharge coefficient \( C_s \) is a dimensionless quantity of which the value is typically around 0.6 or 0.7 [21], [22].

The nonsmooth quasistatic model gives the algebraic relation among the actuator force \( f \), the ratios of the valve opening areas \( u_{s} \in [0, 1] \) (\( s \in \{ph, pr, th, tr, b\} \)), and the velocity \( v \). For simplicity, the ratios \( u_{s} \in [0, 1] \) (\( s \in \{ph, pr, th, tr\} \)) can be assumed to be determined by a control input \( u \in B \) as follows:

\[
u_{ph} = u_{tr} = \max(0, u), \quad u_{pr} = u_{th} = -\min(0, u). \quad (15)
\]

The model with this control input \( u \) can be represented by a set-valued (thus nonsmooth) function \( \Gamma : \mathbb{R} \times B \rightrightarrows \mathbb{R} \), with which \( f \), \( v \) and \( u \) are constrained in the form \( f \in \Gamma(v, u) \). The complete analytical expression of the function \( \Gamma \) is presented in [16]. Fig. 2 illustrates a numerical example of the function \( \Gamma \) with the parameter values detailed in Section IV.
The double-implicit implementation scheme proposed in [15], which is for implementing a sliding-mode controller to a plant driven by a nonsmooth actuator. It is an extension of the implicit implementation scheme [23]–[28], which is to implement a nonsmooth controller, such as a sliding-mode controller, to a smooth plant, which can be described by an ordinary differential equation. The implicit implementation scheme employs the implicit Euler discretizations of the controller and the plant to construct a discrete-time algorithm of the controller that does not produce chattering. The double-implicit implementation scheme [15] is its extension to deal with the case where both controller and plant are nonsmooth.

The double-implicit implementation scheme is to deal with the position control of plants that can be written in the following form:

$$M \dot{v} = f + g$$ (16a)
$$v = p$$ (16b)
$$f \in \Gamma(v, u)$$ (16c)

where $p \in \mathbb{R}$, $v \in \mathbb{R}$, and $M \in \mathbb{R}^+$ are the position, the velocity, and the mass of the controlled object, respectively. The controlled object is subjected to the external force $g \in \mathbb{R}$ and the actuator force $f \in \mathbb{R}$, and the force $f$ is generated by the actuator modeled as a nonsmooth function $\Gamma : \mathbb{R} \times \mathbb{B} \Rightarrow \mathbb{R}$. Here, $u \in \mathbb{B}$ is the control input that should be given to the actuator from a controller.

For the position control of the plant (16), a simple sliding-mode controller of the following form is considered:

$$f \in \Gamma(v, \text{sgn}(p_d - p - H v))$$ (17)

where $p_d \in \mathbb{R}$ is the desired position and $H > 0$ is a parameter for the controller. The control input $u$ in (16c) should be chosen so that (17) is realized. Theorem 2 in [15] suggests that, with this controller, the sliding mode can be established on the switching surface $\sigma = p - p_d + H v = 0$ and thus $p$ exponentially converges to $p_d$ with the time constant $H$.

In the double-implicit implementation scheme, a discrete-time algorithm that realizes the controller (17) combined with the plant (16) is derived. It is based on the implicit Euler discretization of the nominal model of the plant (16) and the controller (17), which are written as follows:

$$M(v_{k+1} - v_k)/T = f_k + g_k$$ (18a)
$$v_{k+1} = (p_{k+1} - p_k)/T$$ (18b)
$$f_k \in \Gamma(v_k, u_k)$$ (18c)
$$f_k \in \Gamma(v_k, \text{sgn}(p_d - p_{k+1} - H v_{k+1})).$$ (18d)

Here, $T$ denotes the sampling interval and $k$ denotes the discrete-time index. The control input $u_k$ needs to be obtained from the set of algebraic constraints (18) according to the inputs $p_k$, $v_k$, and $g_k$.

Through tedious but straightforward algebraic manipulations on (18), one obtains the following algorithm to obtain $u_k$:

$$v_{f,k} := v_k + g_k T/M$$ (19a)
$$v_{s,k} := (p_d - p_k)/(H + T)$$ (19b)
$$f_k := \text{sat}_{\Gamma_s(T/M,\sigma,f)}((v_k - v_{f,k})M/T)$$ (19c)
$$u_k := \Theta_s(v_{f,k} + f_k T/M, f_k)$$ (19d)

where $\Gamma_s$, $\Theta_s$, and $\Theta$ are functions that satisfy the followings:

$$f = \Gamma_s(\eta, v, u) \iff f \in \Gamma(v + \eta f, u)$$ (20)
$$u \in \Theta(v, f) \iff f \in \Gamma(v, u)$$ (21)
$$\Theta_s(v, f) \in \Theta\left(v, \text{sat}_{\Gamma_s(B)}(f)\right)$$ (22)

where $\eta > 0$. Here, $\Gamma_s$ is a single-valued function that is uniquely defined by (20) as detailed in Theorem 3 in [15], $\Theta$ is the inverse function of $\Gamma$ with respect to its second argument, and $\Theta_s$ is a single-valued selection of $\Theta$. Although $\Theta_s$ is not unique, its use in the algorithm can be justified by Theorem 1 in [15]. The previous paper [15] provides the algorithm (19) and the relations (20), (21), and (22), but does not provide the analytical expressions of $\Gamma_s$, $\Theta$ or $\Theta_s$ corresponding to the function $\Gamma$ presented in [16].

As for the algorithm (19), it should be noted that $v_{f,k} + f_k T/M$, which is in (19d), can be interpreted as the velocity predicted to be achieved in the next timestep if the desired force $f_k$ is kept by the actuator for the time period $t \in [kT, (k + 1)T]$.

### III. SET-POINT POSITION CONTROLLER FOR HYDRAULIC EXCAVATORS

This section proposes a set-point position controller for hydraulic excavators. Fig. 3 illustrates the overall structure of the controller. The proposed controller consists of a sliding-mode controller and a state predictor to compensate for the deadtime. The sliding-mode controller is implemented through the double-implicit implementation scheme [15], which requires the inverse model of the actuator. This section derives the inverse model of the nonsmooth quasi-static model [16] and presents the controller algorithm.
The algorithm of the state predictor is also presented in this section.

**A. INVERSION OF THE NONSMOOTH QUASISTATIC MODEL**

As overviewed in Section II-B, the actuator model is represented by a function \( \Gamma : \mathbb{R} \times \mathcal{B} \Rightarrow \mathbb{R} \), of which the complete analytical form is given in [16]. This section gives the analytical form of its inverse function \( \Theta \) with respect to the second argument, which satisfies (21).

For the convenience of the derivation of the function \( \Theta \), we rewrite the function \( \Gamma \) in an expression different from that in [16]. To this end, let us define functions \( \gamma_+ : (1+\varepsilon)\mathbb{R} \times \mathcal{B} \times \mathbb{R} \Rightarrow \mathbb{R} \) and \( \gamma_- : (1+\varepsilon)\mathbb{R} \times \mathcal{B} \times \mathbb{R} \Rightarrow \mathbb{R} \) as follows:

\[
\gamma_+(u; \bar{v}, a) \triangleq a + N_{(1+\varepsilon)B}(u)
\]

\[
\gamma_-(u; \bar{v}, a) \triangleq a + N_{(1+\varepsilon)B}(u)
\]

The constants appearing in (24) are defined as follows:

\[
C_{ph} \triangleq \sqrt{A_{ph}^2/c_{ph}^2}, \quad C_{tr} \triangleq \sqrt{A_{tr}^2/c_{tr}^2}
\]

\[
C_{hr} \triangleq \sqrt{C_{hr}^2 + C_{tr}^2}, \quad C_{bb} \triangleq A_{bb}^2/(c_{bb}u_{bb})^2, \quad C_{rb} \triangleq A_{rb}^2/(c_{rb}u_{rb})^2
\]

\[
V_h \triangleq Q/A_h, \quad V_r \triangleq Q/A_r, \quad F_{hp} \triangleq A_{hp}P_{HM}, \quad F_{tr} \triangleq A_{tr}P_{TM}
\]

\[
F_{hm} \triangleq A_{hm}P_{HM}, \quad F_{hr} \triangleq A_{hr}P_{TM}
\]

The graph of \( f \in \Gamma(v, u) \) is illustrated in Fig. 2. As seen from Fig. 2(c), the function \( \Gamma \) is monotone with respect to the argument \( u \) and is single-valued when \( v \neq 0 \).

Theorem 1: Let \( f \in \mathbb{R} \), \( u \in (1+\varepsilon)B \), \( v_+ \in \mathbb{R}_+ \), \( v_- \in \mathbb{R}_- \), and \( a \in \mathbb{R} \). Recall that the functions \( \gamma_+ \) and \( \gamma_- \) are defined as (23a) and (23b), respectively. Then, the following statements hold true:

\[
f \in \gamma_+(u; v_+, a) \iff u \in \theta_+(f; v_+, a)
\]

\[
f \in \gamma_-(u; v_-, a) \iff u \in \theta_-(f; v_-, a)
\]

where the functions \( \theta_+ : \mathbb{R} \times \mathbb{R}_+ \times \mathbb{R} \Rightarrow (1+\varepsilon)B \) and \( \theta_- : \mathbb{R} \times \mathbb{R}_- \times \mathbb{R} \Rightarrow (1+\varepsilon)B \) are defined as follows:

\[
\theta_+(f; \bar{v}, a) \triangleq \begin{cases} 
\max \left( \frac{\bar{v}}{1+\varepsilon}, R(a-f) \right) & \text{if } \bar{v} > 0 \\
\text{sgn}(0, f-a, 1+\varepsilon) & \text{if } \bar{v} = 0 \\
-\bar{v} & \text{if } \bar{v} < 0
\end{cases}
\]

\[
\theta_-(f; \bar{v}, a) \triangleq \begin{cases} 
\min \left( \frac{\bar{v}}{1+\varepsilon}, R(a-f) \right) & \text{if } \bar{v} > 0 \\
\text{sgn}(-1-\varepsilon, f-a, 0) & \text{if } \bar{v} = 0
\end{cases}
\]

Lemma 2 states that \( \theta_+ \) and \( \theta_- \) are the inverse functions of \( \gamma_+ \) and \( \gamma_- \) with respect to the first argument, respectively. Note that \( \Gamma \) in (24) is constructed from \( \gamma_+, \gamma_- \), and \( N_B \), and that the inverse function of \( N_B \) is sgn from (7). Thus,
one can see that the inverse function $\Theta$ of $\Gamma$ can be derived from $\theta_+, \theta_-$, and $\text{sgn}$. Here, the fact that $\gamma_+, \gamma_-$, and $\mathcal{B}_{\mathcal{B}}$ are total, surjective, and monotone allows for the application of Lemmas 1 to obtain the analytical expression of $\Theta : \mathbb{R} \times \mathbb{R} \to \mathcal{B}$ as follows:

$$\Theta(v, f) \triangleq \mathcal{B} \cap \text{sgn}(\Theta_-(v, f), v, \Theta_+(v, f)) \quad (28a)$$

where $\Theta_\pm : \mathbb{R} \times \mathbb{R} \to (1 + e)\mathcal{B}$ are defined as follows:

$$\Theta_+(v, f) \triangleq \min (\max (\Theta^{+0}(v, f), \Theta^{+b}(v, f)), \min (\Theta^{+1a}(v, f), \Theta^{+1b}(v, f)), \min (\Theta^{+2a}(v, f), \Theta^{+2b}(v, f))) \quad (28b)$$

and

$$\Theta^{+0}(v, f) \triangleq \theta_+(f; C_{rK} V, F_{hM}) \quad (28d)$$

$$\Theta^{+b}(v, f) \triangleq (1 + e)\text{sgn}(f - F_{hM} + F_{rM}) \quad (28e)$$

$$\Theta^{+1a}(v, f) \triangleq \theta_+(f; C_{hr} v, -C_{hB} s(v - V_h)) \quad (28f)$$

$$\Theta^{+1b}(v, f) \triangleq \theta_+(f; C_{ph} v, -F_{hM} - C_{hB} s(v - V_h)) \quad (28g)$$

$$\Theta^{+2a}(v, f) \triangleq \theta_+(f; C_{hr} v, F_{hP}) \quad (28h)$$

$$\Theta^{+2b}(v, f) \triangleq \theta_+(f; C_{ph} v, F_{hP} - F_{rM}) \quad (28i)$$

$$\Theta^{+3a}(v, f) \triangleq \theta_+(f; C_{rK} v, 0) \quad (28j)$$

$$\Theta^{+3b}(v, f) \triangleq (1 + e)\text{sgn}(f + F_{rM}) \quad (28k)$$

$$\Theta^{-0}(v, f) \triangleq \theta_-(f; C_{ib} v, -F_{iM}) \quad (28l)$$

$$\Theta^{-b}(v, f) \triangleq (1 + e)\text{sgn}(f - F_{hM} + F_{iM}) \quad (28m)$$

$$\Theta^{-1a}(v, f) \triangleq \theta_-(f; C_{pr} v, -C_{ib} s(v + V_i)) \quad (28n)$$

$$\Theta^{-1b}(v, f) \triangleq \theta_-(f; C_{pr} v, F_{hM} - C_{ib} s(v + V_i)) \quad (28o)$$

$$\Theta^{-2a}(v, f) \triangleq \theta_-(f; C_{hr} v, -F_{pP}) \quad (28p)$$

$$\Theta^{-2b}(v, f) \triangleq \theta_-(f; C_{ph} v, -F_{pP} + F_{hM}) \quad (28q)$$

$$\Theta^{-3a}(v, f) \triangleq \theta_-(f; C_{bh} v, 0) \quad (28r)$$

$$\Theta^{-3b}(v, f) \triangleq (1 + e)\text{sgn}(f - F_{hM}). \quad (28s)$$

Note that all $\Theta_\pm$ are total and surjective with respect to their second argument $f$. The function $\Theta$, however, is not total or surjective because the definition (28a) restricts its return value within $\mathcal{B}$, which means that it may be $\emptyset$ for some pairs of values of $v \in \mathbb{R}$ and $f \in \mathbb{R}$.

### B. SLIDING-MODE CONTROLLER FOR HYDRAULIC ACTUATORS

As has been mentioned in Section III-A, the function $\Theta$ defined in (28) is not convenient for the use in the controller because its output is not always a single value but can be a set or even the empty set. This section presents a single-valued total function $\Theta_\pm$ that is related to $\Theta$ through (22). A careful observation of the definition (24) of $\Gamma$ and Fig. 2 reveals that the set-valuedness of $\Theta(v, f)$ takes place when $f$ is at the maximum $F_{hM}$ or the minimum $-F_{rM}$. When $f$ is above $F_{hM}$ or below $-F_{rM}$, $\Theta(v, f)$ is the empty set. In addition, the definition (24a) implies that $\Theta(v, f)$ may be set-valued at $v = 0$. It may also be the case if $f = F_{hM} - F_{rM}$ because of the definitions (28e) and (28m) of $\Theta_{\pm b}$.

The use of an arbitrary single value within the set $\Theta(v, f)$ is justified in Theorem 1 in [15]. The theorem, however, depends on the assumption that the controller parameters are accurate with respect to the real actuator parameters. The system’s sensitivity to the parametric errors may depend on the choice of the single value within the set. Through preliminary investigations, we propose the function $\Theta_\pm$ obtained by the following modifications of the definition (28) of $\Theta$:

- Replace $\theta_{\pm}$ in (27) by

$$\theta_+^\pm (f; \tilde{v}, a) \triangleq \frac{\tilde{v}}{\max (\tilde{v}, R(a - f))} \quad (29a)$$

$$\theta_-^\pm (f; \tilde{v}, a) \triangleq -\frac{\tilde{v}}{\min (\tilde{v}, R(a - f))} \quad (29b)$$

- Use $\Theta_{\pm}^\pm (\ast \in \{0a, 1a, 1b, 2a, 2b, 3a\})$ in (28d) to (28r) with $\theta_{\pm}$ being replaced by $\theta_{\pm}^\pm$ in (29).

- Replace $\Theta_{\pm b}$ in (28e) and (28m) by

$$\Theta_{\pm b}^\pm (v, f) \triangleq \begin{cases} 1 & \text{if } f \geq F_{hM} - F_{rM} \\ -1 & \text{if } f < F_{hM} - F_{rM} \end{cases} \quad (30a)$$

$$\Theta_{\pm b}^\pm (v, f) \triangleq \begin{cases} 1 & \text{if } f > F_{hM} - F_{rM} \\ -1 & \text{if } f \leq F_{hM} - F_{rM} \end{cases} \quad (30b)$$

![FIGURE 4. Inverse model of the actuator: (a) the graph of $f \in \Theta(v, f)$ defined in (21) and (b)(c)(d) the graph of $f = \Theta_2(v, f)$ defined in (32). The function $\Theta_3$ is obtained by removing the set-valuedness and non-totalness of the function $\Theta$.](image-url)
Y. Yamamoto

where $M$ the algorithm to calculate the control input Appendix B and C of [15].

3 It can be implemented as

the estimated external force, which is set zero if unavailable.

Figs. 4(b), (c) and (d). The motivation for this choice of the single-valued

while the derived singled-valued function $\Theta_s$ is shown in

The original set-valued function is summarized as follows:

Note that the definitions (29) does not consider the case of $v = 0$ because (32) implies that $\Theta_{l\pm}$ is not used when $v = 0$.

The original set-valued function is illustrated in Fig. 4(a) while the derived singled-valued function $\Theta_s$ is shown in Figs. 4(b), (c) and (d). The motivation for this choice of the single-valued $\Theta_s$ is summarized as follows:

Equation (29) is chosen so that $\theta_{l\pm}(f; \bar{v}, a) = \text{sat}_{\mathbb{R}}(\theta_{l\pm}(f; \bar{v}, a))$.

Equation (30) is chosen to deal with the set-valuedness of $\Theta_{l\pm0}$ that happens at $f = F_b - F_M$. It is designed so that it allows $\Theta_{l\pm0}(v, f)$ to be chosen in the min-max logic in (28b) and (28c). Nevertheless, $\Theta_{l\pm0}$ are rarely in effect in the logic in (28b) and (28c) and actually have no effect in the numerical and experimental examples in this paper.

Equation (31) is chosen to set $\Theta_s(v, f) = 0$ when $v > 0$ and $f \leq -F_M$ or $v < 0$ and $f \geq F_M$ because, in these cases, the actuator needs to produce the maximally decelerating force and setting $u = 0$ (closing all main control valves) is the most robust way to realize it against the parametric errors.

Equation (32) is chosen to set $\Theta_s(v, f) = 0$ when $v = 0$ because setting $u = 0$ (closing all main control valves) is the most robust way to achieve $v = 0$ against the parametric errors.

Employing the obtained function $\Theta_s$ in (32), we construct the algorithm to calculate the control input $u$ as follows:

\[
\begin{align*}
\hat{v}_{f,k} & := v_k + \hat{g}_k T/M_c & (33a) \\
v_{x,k} & := (p_d - p_k)/(H + T) & (33b) \\
f_k & := \text{sat}_{\mathbb{R}}(T/M_c, v_{x,k})/(v_{x,k} - \hat{v}_{f,k})M_c/T & (33c) \\
u_k & := \Theta_s(v_{x,k}, f_k T/M_c, f_k) & (33d)
\end{align*}
\]

where $M_c$ is the inertia of the controlled object and $\hat{g}_k$ is the estimated external force, which is set zero if unavailable. The singled-valued function $\Gamma_s$ is the function satisfying (20). It can be implemented as

\[
\Gamma_s(\eta, v, u) = (\Lambda(1/\eta, -v/\eta, u) - v)/\eta & (34)
\]

where the function $\Lambda$ is a function of which the complete analytical form is presented in [29]. Stability proofs for the controller (33) applied to the plant (16) are shown in Appendices B and C of [15].

The controller (33) depends on the inertia parameter $M_c$, which should be known in advance. When it is applied to the swing axis of an excavator, it should be the moment of inertia of the upperstructure. The upperstructure is composed of the cab and the links (the boom, arm, and bucket), which are combined with one another through components having some mechanical compliance. Therefore, it is not straightforward to judge whether $M_c$ should be the moment of inertia of the whole upperstructure or only the cab part. The parameter $M_c$ here is used to predict the actuator velocity $v_{f,k} + f_k T/M_c$ after the time $T$, which is 10 ms in our setup. Therefore, a suitable choice of $M_c$ would depend on how far the effect of the actuator force is propagated within the time $T$ within the structure. From some preliminary simulations and experiments, we conclude that it is better to set $M_c$ as the moment of inertia only of the cab. Some supporting results will be presented in Sections IV and V.

C. STATE PREDICTOR FOR DEADTIME COMPENSATION

Hydraulic systems usually include the deadtime between the control input and the actuation. In order to compensate for the deadtime, the proposed controller includes a state predictor based on the quasistatic model [16] of the actuator.

To construct the predictor, we again consider the plant dynamics model (16). The implicit discretization of the model (16) is as follows:

\[
\begin{align*}
v_{k+1} & = v_k + (f_k + g_k)T/M_c \quad (35a) \\
p_{k+1} & = p_k + v_{k+1}T \quad (35b) \\
f_k & \in \Gamma_s(v_{k+1}, u_k). \quad (35c)
\end{align*}
\]

Eliminating $v_{k+1}$ in (35c) and employing $\Gamma_s$ in (34), we obtain the following expression:

\[
\begin{align*}
f_k & = \Gamma_s(T/M_c, v_k + g_k T/M_c, u_k) \quad (36a) \\
v_{k+1} & = v_k + (f_k + g_k)T/M_c \quad (36b) \\
p_{k+1} & = p_k + v_{k+1}T \quad (36c)
\end{align*}
\]

which can be seen as the algorithm of a one-step predictor of the state $\{p_{k+1}, v_{k+1}\}$ based on the inputs $\{p_k, v_k, u_k, g_k\}$.

By iteratively using the one-step predictor (36), we can construct the algorithm of a multi-step predictor for a look-ahead time $\hat{T}_d$ as follows:

\[
\begin{align*}
& \text{for } i \leftarrow 1 \text{ to floor}(\hat{T}_d/T) - 1 \text{ do} \\
& \hspace{1cm} U[i+1] \leftarrow U[i] \quad (37a) \\
& \text{end for} \\
& U[1] \leftarrow u \quad (37b) \\
& \text{for } i \leftarrow \text{floor}(\hat{T}_d/T) \text{ to } 1 \text{ do} \\
& \hspace{1cm} f \leftarrow \Gamma_s(T/M_p, v + \hat{g}_T T/M_p, U[i]) \quad (37c) \\
& \hspace{1cm} v \leftarrow v + (f + \hat{g}_T)T/M_p \quad (37d) \\
& \hspace{1cm} p \leftarrow p + vT \quad (37e) \\
& \text{end for} \\
& \text{return } p \text{ and } v
\end{align*}
\]
where floor means the maximum integer that does not exceed the argument, $M_p$ is the inertia of the controlled object, and $\hat{g}$ is the estimated external force. Here, $U[i]$ ($i \in \{1, \cdots, \text{floor}(T_d/T)\}$) is the buffer to store the control inputs $u$ of $i$ timesteps ago. As has been illustrated in Fig. 3, the finally obtained $p$ and $v$ are provided to the sliding-mode controller (33) as the inputs.

The predictor (37) also depends on the prior knowledge of the inertia $M_p$ of the controlled object, as is the case with the controller (33). When it is applied to the swing axis of an excavator, we again need to consider that the moment of inertia of which part of the upperstructure should be used as $M_p$. The predictor (37) is for predicting the time $T_d$ later, in our setup, it is about 150 to 450 ms, which is 15 to 45 times of $T$. Considering that the effect of the actuator force in the upperstructure is propagated further in $T_d$ than in $T$, we can see that the moment of inertia $M_p$ for the predictor (37) should be larger than the moment of inertia $M_c$ for the controller (33). Our conclusion from some preliminary simulations and experiments is that it is better to set $M_p$ as the moment of inertia of the whole upperstructure. Supporting results will be shown in Sections IV and V.

IV. SIMULATIONS

A. SIMULATION SETUP

The proposed controller, which is the sliding-mode controller (33) combined with the state predictor (37), was validated with our realtime simulator [30] of a 20-ton class hydraulic excavator. The controller was constructed with MATLAB/Simulink and the simulator is constructed with Microsoft Visual C++. They are connected as illustrated in Fig. 5 through TCP/IP sockets at the cycle of 10 ms, i.e., the controller’s sampling interval is $T = 10$ ms. The simulator’s timestep size is 0.1 ms. The proposed controller was tested with the angle $\theta_1$ of the swing axis, driven by a rotary hydraulic actuator, of the simulated excavator. In the simulations, the desired angle $\theta_d$ and the parameter $H$ were fixed as 90° and 1.5 s, respectively. The estimated external force $\hat{g}$ was set as 0 because the excavator was placed horizontally.

The simulator deals with links as rigid bodies connected by virtual viscoelastic elements through virtual beams as illustrated in the green circle in Fig. 6. The stiffness and the viscosity of the virtual viscoelastic elements are $5.0 \times 10^5$ N/m and $3.0 \times 10^5$ N·m/s, respectively, and the length of the virtual beams is 2.0 m. The frictions in the joints are implemented by the technique presented in [31].

In the simulator, the torque of the rotary hydraulic actuator is calculated based on the nonsmooth quasistatic model explained in Section II-B. The parameters of the actuator are shown in Table 1. The torque is amplified by a geared transmission with the reduction ratio 130. The output shaft of the actuator is connected to the cab through a virtual torsional viscoelastic element with the stiffness $5.0 \times 10^7$ N·m/rad and the viscosity $3.0 \times 10^5$ N·m·s/rad as illustrated in the blue circle in Fig. 6, employing the technique presented in [29].

The rotary hydraulic actuator accepts the control input $u$ of the deadline $T_d$ ago. In addition, the responses of the main control valves are assumed to be lagged by the dynamics of the spool valves. In order to emulate the delay and the lag, we include the following filter between the controller and the actuator as shown in Fig. 5:

$$u_f = L^{-1} \left[ \frac{\alpha_0^2 e^{-T_d s} L[u]}{s^2 + 2\xi \omega_0 s + \omega_0^2} \right] \tag{38}$$

where $L$ represents the Laplace transform. The deadline $T_d$, the cutoff frequency $\omega_0$, and the damping ratio $\xi$ are set as 300 ms, 94.2 ($\approx 30\pi$) rad/s, and 1, respectively.

![FIGURE 5. Simulation setup.](image)

![FIGURE 6. Connections of links in the simulator: the figure in the green circle illustrates the connections among links through the virtual viscoelastic elements and the virtual beams, and the figure in the blue circle illustrates the connection between the cab and the rotary actuator.](image)

![TABLE 1. Parameters of the rotary hydraulic actuator in the simulations.](image)
Some simulations were performed to check the effects of DEADTIME in this paper. Thus, we leave empirical comparisons outside the scope of parameter tuning and careful design of the target trajectory.

Controllers [7]–[10] and sliding-mode controllers [1], [5], [6] may realize similar behaviors, but it would require careful parameter tuning and careful design of the target trajectory. Thus, we leave empirical comparisons outside the scope of this paper.

**B. EFFECTS OF SETTINGS OF INERTIA PARAMETERS AND DEADTIME**

Some simulations were performed to check the effects of the inertia parameters \( M_c, M_p \) and the predictor and the look-ahead time \( \hat{T}_d \). We performed the simulations with the following three settings of the inertia parameters \( M_c, M_p \):

- Setting A: \( M_c = M_p = J_{\text{cab}} \).
- Setting B: \( M_c = J_{\text{cab}} \) and \( M_p = J_{\text{up},*} \).
- Setting C: \( M_c = M_p = J_{\text{up},*} \).

The look-ahead time \( \hat{T}_d \) was set as seven different values from 150 ms to 450 ms. The actuator-model parameters in the controller were set idealistically, being equal to the plant parameter values in Table 1. It should be noted that the second-order lag (38) in the plant is not taken into consideration in the controller.

Fig. 8 shows simulation results in the extended configuration. It shows that the swing angle \( \phi \) converges to the desired angle \( \phi_d \) in all settings. From the comparison among different settings of the inertia parameters, one can see that Setting B is the most suitable because it results in the control input \( u \) being non-oscillatory and the state \((p, v)\) being closest to the switching surface \( \sigma = 0 \). This result is consistent with the discussions in Sections III-B and III-C, i.e., \( M_c \) should be the moment of inertia of the cab and \( M_p \) should be that of the whole upperstructure. The chattering-like behavior in \( u \) of Setting C can be explained as that an excessively large value of \( M_c \) results in an unnecessarily large value of \( f_k \) in (33c), leading to a saturated value of \( u \). From the comparison among the different \( \hat{T}_d \) values, one can see that setting the accurate value to \( \hat{T}_d \) results in accurate sliding on the switching surface.

Fig. 9 shows simulation results in the flexed configuration. Also in this configuration, the swing angle \( \phi \) converges to the desired angle \( \phi_d \) in all parameter settings. It is also apparent that the three settings of the inertia parameters do not cause much difference, resulting in non-oscillatory input \( u \) and reasonably accurate sliding on the switching surface \( \sigma = 0 \). It can be attributed to the fact that the ratio \( J_{\text{up},*}/J_{\text{cab}} = 1.68 \) in the flexed configuration is much smaller than \( J_{\text{up},*}/J_{\text{cab}} = 3.98 \) in the extended configuration. One can also see that \( \hat{T}_d \) closer to \( T_d \) results in the state trajectory closer to the switching surface, as was the case with the extended configuration.

In Figs. 8 and 9, one can see that at least one of the chamber pressures \( P_h \) and \( P_r \) was always saturated, which means that the effects of the relief valves cannot be neglected in situations like these simulations. These results show that the controller appropriately constrains the state to the switching surface \( \sigma = 0 \) even during these saturated periods, exhibiting the benefit of the controller accounting for the strong nonlinearity caused by the valve behaviors.

It should be noted that the speed of convergence is determined by the parameter \( H \) and it can be made faster by setting \( H \) smaller. The value of \( H \), however, should not be set too small because it can result in overshoots, i.e., penetration of the state through the switching surface. Appendix B of [15] details the conditions for the state constrained to the switching surface. Different types of switching surfaces, such as those nonlinear, may be effective to make the convergence faster without producing overshoots, but it is left outside the scope of this paper.

**C. EFFECTS OF MODELING ERRORS**

Another set of simulations was conducted to test the influence of the parametric errors in the actuator model. For
each configuration, 100 trials were performed with all parameters listed in Table 1 being randomly varied by
−15%, 0%, or 15%. The inertia parameters were set as $M_c = J_{cab}$ and $M_p = J_{up}$, i.e., Setting B. The
look-ahead time $\hat{T}_d$ was fixed to 300 ms, i.e., the true
deadtime $T_d$.

Fig. 10 shows the results. In all parameter settings, the
angle $p$ converges to the desired angle $p_d$, although some
settings result in chattering-like behaviors in $u$ and separation
of the state from the switching surface $\sigma = 0$. These results
exhibit a certain robustness of the proposed controller against
the modeling errors.
V. EXPERIMENTS

A. EXPERIMENT SETUP

We tested the proposed controller with a 13-ton class excavator, Kobelco ED160-5 (with its dozer blade removed), shown in Fig. 11. The controller was applied to the angle $p$ of the swing axis driven by a rotary hydraulic actuator, which has a structure similar to the one shown in Fig. 1. In the circuit, a single four-port spool valve plays the role of four main control valves. The spool valve is driven by a pair of electromagnetic control valves that accept the control input $u$ so that the spool displacement is proportional to the control input $u$. Further detailed specifications of the actuator and the excavator are not reported here due to proprietary restrictions.

The controller was constructed with MATLAB/Simulink. It was connected with the excavator through the Control Area Network (CAN) to receive the sensor reading of the angular velocity $v$, which was measured by a microelectromechanical systems (MEMS) gyroscope, and send the control input $u$ to the spool valve at the sampling interval of $T = 10$ ms. The angle $p$ was obtained by simply integrating the measured angular velocity $v$, being reset at the beginning of every trial. The internal pressures of the chambers, unfortunately, could not be obtained for some hardware reasons. The desired angle $p_d$ and the controller parameter $H$ were fixed at $90^\circ$ and 1.5 s, respectively, as was the case with the simulations in Section IV. The estimated external force $\hat{g}$ was set as 0 because the excavator was placed horizontally. We used three different settings for the inertia parameters $\{M_c, M_p\}$, which were Settings A, B, and C introduced in Section IV-B. The values of $J_{up,ex}$, $J_{up,fl}$, and $J_{cab}$ (cf. Section IV-A for definitions) were obtained from the nominal specifications of the excavators, of which the ratios were $J_{up,ex}/J_{cab} = 9.92$ and $J_{up,fl}/J_{cab} = 4.61$. Because the deadtime $T_d$ was not accurately available, the look-ahead time $\hat{T}_d$ was tested at five different values, which were 0 ms (i.e., without the state predictor), 300 ms, 400 ms, 500 ms, and 600 ms.

We did not test other controllers in the experiments for the same reasons as in the simulations in Section IV. Another reason, more practical, was that the behavior of simple linear controllers would be quite unpredictable, especially in the early stage of parameter tuning, posing difficulty in ensuring safety.

B. RESULTS

Fig. 12 shows results in the extended configuration. It shows that the swing angle $p$ converges to the desired angle $p_d$ except the case without the deadtime compensation. It can be seen that Setting B is more suitable than Settings A and C because the state trajectory is the closest to the switching
surface $\sigma = 0$ and the chattering of the control input $u$ is smaller. One can see the importance of the chattering reduction by observing the fluctuation in the velocity $v$ coinciding with the chattering in $u$. From the comparison among the different $\hat{T}_d$ values, one can see that it is safer to set $\hat{T}_d$ larger, especially larger than 400 ms in this setup, to avoid undesirable artifacts. For example, the undesirable velocity fluctuation seen in $t \in [3 \text{ s}, 5 \text{ s}]$ with $\hat{T}_d = 300 \text{ ms}$ is suppressed with larger values of $\hat{T}_d$. Even larger $\hat{T}_d$ were not tested in the experiments, but it would be natural to assume
that the range of suitable values of the look-ahead time $\hat{T}_d$ are upperbounded by the accuracy of the state predictor.

Fig. 13 shows results in the flexed configuration. It shows that the swing angle $p$ converges to the desired angle $p_d$ also in this configuration, as long as the state predictor is used. It can be seen that the setting of the inertia parameters does not largely affect the results in this configuration, similarly to the simulations in Section IV-B, presumably because the ratio $J_{wp,fl}/J_{cab}$ was relatively closer to one. It can also be seen that rather acceptable behavior was realized as long as the look-ahead time $\hat{T}_d$ was between 300 ms to 600 ms.

In both configurations with all settings, the state trajectory penetrated the switching surface. It can be attributed to the inaccuracy of the state predictor, which currently does not consider the dynamics of the spool valve or the pressure dynamics (i.e., the compressibility) of the oil. We do not practically consider it as a primary concern because it does not result in overshooting in the angle or abrupt changes in the velocity. It may however be worth investigating to improve the state predictor by including unmodeled dynamics by extending the quasistatic model [16] of the actuator.

VI. CONCLUSION

This paper has proposed a sliding-mode set-point position controller for hydraulic excavators that realizes appropriate converging behavior to the desired position. The proposed controller consists of a sliding-mode controller constructed with a double-implicit implementation scheme [15] and a state predictor based on a nonsmooth quasistatic model [16] of the hydraulic actuator. The controller has been validated through simulations and experiments, in which the swing angle converged to the desired angle with an appropriate transient behavior along the sliding surface. The effects of the controller’s parameters, especially the inertia parameters and the look-ahead time, have also been investigated in the simulations and the experiments.

Future work should address extending the proposed controller to a multi-DOF controller that simultaneously deals with all actuators (the boom, arm, and bucket cylinders). In many commercial excavators, a single pump drives more than one actuator, and thus the motion of an actuator often affects the motion of another actuator. Such effects would need to be taken into account in multi-DOF position control, for which the model presented in Section II-B of [16] would be useful. Moreover, for enhancing the efficiency of positioning and trajectory-tracking tasks, different types of switching surfaces may need to be employed. A previous theoretical study [15] suggests that a steeper switching surface realizes faster convergence but fails to maintain the sliding mode in a high-velocity region. It is thus logical to consider that a faster convergence would be realized by a nonlinear switching surface whose slope is steeper in a lower-velocity region. Therefore, switching surfaces with saturated velocity [32] and a square-root-like nonlinearity [33] would be worth investigating.

APPENDIX A

PROOF OF LEMMA 1

To provide the proof of Lemma 1, let us define the following operators:

$$\begin{align}
\preceq, \succeq : \mathbb{R} &\rightarrow \mathbb{R} \\
(\mathcal{X} \preceq, \mathcal{Y}) &\triangleq (\mathcal{X} \cap \mathcal{Y}) \cup (\mathcal{X} \cup \mathcal{Y})
\end{align}$$

where $\mathcal{X}, \mathcal{Y} \subset \mathbb{R}$. With these operators, we have the following lemma:

**Lemma 3:** Let $f : \mathcal{C} \rightarrow \mathbb{R}$ be a total, surjective, monotone, set-valued function where $\mathcal{C} \subset \mathbb{R}$. Then, $y \in \preceq f^{-1}(y) \iff x \in \preceq f^{-1}(y)$ and $y \in \succeq f(x) \iff x \in \succeq f^{-1}(y)$ are satisfied.

**Proof:** The first statement can be proven as follows:

$$y \in \preceq f(x) \iff \exists y_0 \text{ s.t. } f(x) \ni y_0 \leq y$$

$$\iff \exists y_0, x_1 \text{ s.t. } f(x_1) \ni y_0 \leq y \in f(x_1) \quad (\therefore \text{surjectivity of } f)$$

$$\iff \exists y_0, x_1 \text{ s.t. } f^{-1}(y_0) \ni x \leq x_0 \in f^{-1}(y) \quad (\therefore \text{monotonicity of } f)$$

$$\iff \exists x_1 \text{ s.t. } x \leq x_1 \in f^{-1}(y) \quad (\therefore \text{surjectivity of } f^{-1})$$

$$\iff x \in \succeq f^{-1}(y). \tag{40}$$

The second statement can also be proven in the same manner.$\square$

The definitions (12) of the min and max operators can be equivalently rewritten as follows:

$$\begin{align}
\max(\mathcal{X}, \mathcal{Y}) &\triangleq (\mathcal{X} \cap \succeq \mathcal{Y}) \cup (\succeq \mathcal{X} \cap \mathcal{Y}) \tag{41a} \\
\min(\mathcal{X}, \mathcal{Y}) &\triangleq (\mathcal{X} \cap \preceq \mathcal{Y}) \cup (\preceq \mathcal{X} \cap \mathcal{Y}) \tag{41b}
\end{align}$$

where $\mathcal{X}, \mathcal{Y} \subset \mathbb{R}$. Now we are in position to provide the proof of Lemma 1.

**Proof of Lemma 1:** If $f(x) \triangleq \min(f_1(x), f_2(x))$, one has the following:

$$y \in f(x) \iff \begin{align}
& (y \in f_1(x) \land y \in \succeq f_2(x)) \\
\lor & (y \in \succeq f_1(x) \land y \in f_2(x))
\end{align}$$

$$\iff \begin{align}
& (x \in f_1^{-1}(y) \land x \in \succeq f_2^{-1}(x)) \\
\lor & (x \in \succeq f_1^{-1}(y) \land x \in f_2^{-1}(x))
\end{align}$$

$$\iff x \in \max(f_1^{-1}(y), f_2^{-1}(y)). \tag{42}$$

The first and second lines of the above are connected by Lemma 3. The case of $f(x) \triangleq \max(f_1(x), f_2(x))$ can also be proven in the same manner.$\square$

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