Probing large-angle correlations with the microwave background temperature and lensing cross-correlation

A. Yoho, 1, 2⋆ C. J. Copi, 1 G. D. Starkman 1, 2 and A. Kosowsky 3

1 CERCA/ISO, Department of Physics, Case Western Reserve University, 10900 Euclid Avenue, Cleveland, OH 44106-7079, USA
2 CERN, CH-1211 Geneva 23, Switzerland
3 Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA 15208 USA

Accepted 2014 May 8. Received 2014 March 11; in original form 2013 October 31

ABSTRACT
A lack of correlations in the microwave background temperature between sky directions separated by angles larger than 60° has recently been confirmed by data from the Planck satellite. This feature arises as a random occurrence within the standard Λ cold dark matter (ΛCDM) cosmological model less than 0.3 per cent of the time, but so far no other compelling theory to explain this observation has been proposed. Here, we investigate the theoretical cross-correlation function between microwave background temperature and the gravitational lensing potential of the microwave background, which in contrast to the temperature correlation function depends strongly on gravitational potential fluctuations interior to our Hubble volume. For standard ΛCDM cosmology, we generate random sky realizations of the microwave background temperature and gravitational lensing, subject to the constraint that the temperature correlation function matches observational data, and compare with random skies lacking this constraint to illustrate the value added in considering simulations that match observations (rather than standard ΛCDM with cosmic variance). The distribution of large-angle temperature–lensing correlation functions in these two cases is different, and the two cases can be clearly distinguished in around 40 per cent of model realizations. We present an a priori procedure for optimizing statistics for large-angle correlations between other types of data, using unconstrained ΛCDM as generic model for comparison, to determine whether the lack of large-angle correlations is a statistical fluke or points to a shortcoming of the standard cosmological model.

Key words: cosmic background radiation – cosmology: observations – cosmology: theory.

1 INTRODUCTION
Observations of the cosmic microwave background (CMB) have provided cosmologists with a wealth of information about our early Universe. On its own, and especially in concert with data from complementary probes, CMB observations have led to increasingly tight constraints on the inferred values of cosmological parameters, and have allowed us to distinguish among various models of our Universe. This has resulted in a standard cosmological model: inflationary flat Lambda cold dark matter (ΛCDM).

Despite its great successes, ΛCDM has had difficulty explaining certain features in the CMB that were initially characterized by the Cosmic Background Explorer’s Differential Microwave Radiometer (COBE-DMR; Bennett et al. 1996; Hinshaw et al. 1996) or using Wilkinson Microwave Anisotropy Probe (WMAP) observations (Spiegel et al. 2003; Copi et al. 2007, 2009, 2010; Chang & Want 2013) and recently confirmed (Ade et al. 2013; Copi et al. 2013b) with the first release of temperature data from the Planck satellite. These features are predominantly in the large-angle, or low (ℓ ≲ 30) multipole, regime. The anomalies include an ecliptic north–south hemispherical asymmetry (Eriksen et al. 2007), the alignment of the quadrupole and octopole patterns with one another (Copi et al. 2004; de Oliveira-Costa et al. 2004; Land & Magueijo 2005, Copi et al., in preparation), with the ecliptic (Schwarz et al. 2004), and with the cosmological dipole (Copi et al. 2006), and low variance across the sky (Hou, Banday & Gorski 2009). A full list as well as comparison to WMAP observations can be found in Ade et al. (2013).

Another large-angle anomaly, namely the lack of correlation on the CMB sky at angles larger than about 60°, currently has no compelling explanation. The importance of this feature has been detailed in papers over the last several years (Schwarz et al. 2004). Thus far, this anomaly has been observed only in the temperature autocorrelation function. There has been some suggestion that it is merely a statistical fluke (which we call the fluke hypothesis, or the null hypothesis because it is just standard cosmology informed by the experimental data) especially since it was only noted and

⋆E-mail: aey2@case.edu
characterized a posteriori. Moreover, it will be difficult to improve on the current statistical significance purely through further observation of the temperature correlations because of cosmic variance and because the measurements are statistics limited.

The use of temperature data exclusively is due to a lack of high signal-to-noise full-sky maps of other cosmological quantities, such as polarization and lensing potential. With the highly anticipated upcoming release of the Planck polarization map as well as several upcoming lensing experiments, cosmologists will have information on large-angle correlation functions beyond just microwave temperature data. Recently, the ability for a temperature–$Q$ polarization cross-correlation to test the fluke hypothesis was investigated (Copi et al. 2013a). The results of that work showed that there was a possibility for $TT$ correlations to rule out the null hypothesis, but that it would not necessarily be a definitive test. In this work, we will investigate using the cross-correlation between temperature and CMB lensing potential to provide a test of the statistical fluke hypothesis as well as a consistency check for observations. We provide predictions for the distribution of a standard statistic used in two-point correlation function analysis, $S_{1/2}$, as well as define an optimal, a priori statistic for use with temperature–lensing cross-correlations.

While in this paper we present results comparing two particular models – unconstrained and constrained $\Omega$CDM – we use a generic prescription for optimization and for determining the viability of an $S_{1/2}$-like statistic. This analysis can be repeated for any set of models as long as one knows how to produce realizations within that framework. This reason in particular was a driving force in the choice of unconstrained $\Omega$CDM as a generic stand-in comparison model, as generating realizations in unconstrained $\Omega$CDM is straightforward. With the particular choice of $T_\phi$ correlations, the discriminating power of the $S_{1/2}$ statistic will depend on how a particular model suppresses the $\Theta_{ISW}^\phi$ term in the correlation, as this is the piece that dominates the signal, unlike $TT$ correlations. However, this analysis can always be carried out such that the $S_{1/2}$-like statistic can be optimized a priori for correlations between cosmological data sets.

The paper is organized as follows: in Section 2, we will give the theoretical background for CMB two-point correlation function analysis; in Section 3, we will describe how we generate constrained $\Omega$CDM realizations; in Section 4, we will outline our general prescription for calculating statistics; in Section 5, we will present our results for the distributions of the statistic for two models (constrained and unconstrained $\Omega$CDM); and finally, in Section 6, we will summarize and state our conclusions.

## 2 BACKGROUND

The two-point angular correlation function for the CMB temperature is calculated by taking an ensemble average of the temperature fluctuations in different directions:

$$C^{TT}(\theta) = \langle (\Theta(\hat{n}_1)) (\Theta(\hat{n}_2)) \rangle \quad \text{with} \quad \hat{n}_1 \cdot \hat{n}_2 = \cos \theta.$$ (1)

Since we are not able to compute the ensemble average in practice, we instead calculate $C^{TT}(\theta)$, a sky average over the angular separation. In general, any $C(\theta)$ can be expanded in a Legendre series, which we write as

$$C(\theta) = \sum_{\ell} \frac{\ell(\ell+1)}{4\pi} C_\ell P_\ell(\cos \theta),$$ (2)

where the $C_\ell$ on the right-hand side of equation (2) are the measured power spectrum values. On a full sky, the coefficients $C_\ell$ obtained from the estimator,

$$C_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2,$$ (3)

where the $a_{\ell m}$ are the usual spherical harmonic coefficients of a map, are identical to the $C_\ell$ in equation (2). Computationally it is more efficient to use the relations (2) and (3) than to directly correlate pairs of pixels on the observed CMB sky.

The two-point function from the COBE-DMR’s fourth-year data release (Bennett et al. 1996) highlighted a lack of temperature autocorrelations for angular separations larger than 60°. The WMAP first-year data release (Spergel et al. 2003) first quantified this feature using a (a posteriori) statistic that neatly captured the simple observation that $C(\theta)$ nearly vanished at large angles:

$$S_{1/2}^{TT} = \int_{1/2}^{\infty} d(\cos \theta) |C^{TT}(\theta)|^2.$$ (4)

We generalize this to

$$S_{1/2}^{XY} = \int_{1/2}^{\infty} d(\cos \theta) |C^{XY}(\theta)|^2.$$ (5)

where $X, Y$ can be any combination of cosmological quantities for which $C^{XY}(\theta)$ can be calculated. The WMAP team calculated $S_{1/2}^{TT}$ on the most reliable part of the sky – that outside the galactic plane – and found it to be 1152 $\mu$K², much smaller than the expected $\Omega$CDM value of $\sim 50,000$ $\mu$K². They remarked: ‘For our $\Omega$CDM Markov chains... only 0.15 percent of the simulations have lower values of $S$.

The simplest (and perhaps leading) explanation for this anomaly is that it is just a statistical fluke within completely canonical $\Omega$CDM. In this paper, our goal is to define a priori a statistic that can be used with future data to test the fluke hypothesis. We therefore need to find an independent cross-correlation that would respond to the same statistical fluctuations as the temperature two-point function. To this end, we focus on the lensing of the CMB, and construct the cross-correlation, $C^{T\phi}(\theta)$, between the CMB temperature $T$ and the CMB lensing potential $\phi$. Since both derive from the gravitational potential $\Phi$, we expect that if statistical fluctuations generated by primordial physics in $\Phi$ caused the lack of large-angle temperature autocorrelation, then those statistical fluctuations would also imprint themselves in a new, distinct way on $C^{T\phi}(\theta)$.

Since we know that at large angles the CMB signal is dominated by the Sachs–Wolfe (SW) and Integrated Sachs–Wolfe (ISW) contributions, we can write

$$\Theta(\hat{n}) = \Theta_{SW}(\hat{n}) + \Theta_{ISW}(\hat{n})$$ (6)

and can describe $C^{T\phi}(\theta)$ as a correlation of these two terms only. The SW and ISW pieces of the primordial temperature fluctuations in terms of the metric potential, $\Phi$, are

$$\Theta_{SW}(\hat{n}) = \frac{1}{3} \Phi(\chi \hat{n}, \chi)$$ (7)

and

$$\Theta_{ISW}(\hat{n}) = -2 \int_0^{\chi_S} d\chi \Phi(\chi \hat{n}, \chi).$$ (8)
which allows us to write $C^{TT}(\theta)$ in terms of correlations of $\Phi$ as

$$C^{TT}(\theta) = \frac{1}{9} \langle \Phi(\chi, \hat{n}_1, \chi_1) \Phi(\chi, \hat{n}_2, \chi_2) \rangle$$

$$+ \frac{2}{3} \int d\chi_1 \langle \Phi(\chi, \hat{n}_1, \chi_1) \Phi(\chi, \hat{n}_2, \chi_2) \rangle$$

$$+ 4 \int d\chi_1 d\chi_2 \langle \Phi(\chi, \hat{n}_1, \chi_1) \Phi(\chi, \hat{n}_2, \chi_2) \rangle.$$  (9)

Similarly, we can write the CMB lensing potential $\varphi$ in terms of the metric potential,

$$\varphi(\hat{n}) = 2 \int \frac{\cos \chi}{\pi} \Phi(\chi, \hat{n}, \chi),$$  (10)

which allows us to write an expression for the two-point function of the CMB temperature and lensing field:

$$C^{T\varphi}(\theta) = \langle \varphi(\hat{n}_1) \Theta_{\text{SW}}(\hat{n}_2) \rangle + \langle \varphi(\hat{n}_1) \Theta_{\text{ISW}}(\hat{n}_2) \rangle$$

$$= -\frac{2}{3} \int d\chi_1 \langle \Phi(\chi, \hat{n}_1, \chi_1) \Phi(\chi, \hat{n}_2, \chi_2) \rangle$$

$$- 4 \int d\chi_1 d\chi_2 \langle \Phi(\chi, \hat{n}_1, \chi_1) \Phi(\chi, \hat{n}_2, \chi_2) \rangle,$$  (11)

where computations to find $C^{T\varphi}(\theta)$ from data will use sky averages rather than ensemble averages.

From equation (11), we see that the lensing field also accesses the information encoded in $\Phi$. Thus, if the anomalous absence of large-angle correlations in $C^{T\varphi}(\theta)$ is due to statistical fluctuations, then it should be present in a predictable way in the $C^{T\varphi}(\theta)$ cross-correlation. It should be noted, however, that $C^{T\varphi}(\theta)$ traces the same physics in a different way – the terms equations (11) do not exactly match the SW and ISW terms in equation (9). Furthermore, $C^{T\varphi}(\theta)$ is dominated by the ISW–$\varphi$ term, meaning that the behaviour of the two-point cross-correlation is dominated by physics on the interior of our Hubble volume, whereas $C^{T\varphi}(\theta)$ has its largest contribution from the SW–SW term and is therefore dominated by physics at the last scattering surface. This makes $C^{T\varphi}(\theta)$ a complementary probe into the nature of the lack of correlation in $C^{T\varphi}(\theta)$ at large angles.

The way that the cross-correlation of the temperature and lensing fields will be affected depends on the underlying details contained in the metric potential. This means that predictions of how $C^{T\varphi}$ will be affected by new physics can only occur after choosing a particular model. Consequently, in absence of a specific alternative model, we cannot construct a statistic that fits well into a Bayesian statistical approach and differentiates between the fluke hypothesis and all other models generically. The approach we therefore take is to present the distribution of constrained statistics and the values that would rule out the fluke hypothesis at the 99 and 99.9 per cent confidence levels (C.L.). We also present the generic $\Lambda$CDM distribution of statistics for comparison, which will be useful in two ways – it illustrates how imposing constraints on the temperature realizations give a much different prediction for defined statistics, and it acts as a stand-in comparison model that we can use to demonstrate our optimization procedure.

3 CONSTRUANED SKY REALIZATIONS

We know that we live in a Universe with a particular angular power spectrum of the temperature, $C^{TT}_{\Lambda CDM}$, and a particular value of $S_{1/2}^{TT}_{\Lambda CDM}$, and we must see what including this as a prior constraint on the allowed realizations of $\Lambda$CDM does to the probability distribution of values of our target statistic.

It is well known how to create realizations of ordinary $\Lambda$CDM with a fixed set of parameters. It is more unusual to create constrained realizations of $\Lambda$CDM – ones that reproduce, within the measurement errors, the angular power spectrum of the observed sky, and with both a full-sky and a cut-sky $S_{1/2}^{TT}$ no larger than those of the observed sky. A detailed description of how to create constrained realizations is contained in section 2 of Copi et al. (2013a). Briefly, we treat the observational errors in the WMAP-reported $C^{TT}_{\Lambda CDM}$ as Gaussian distributed and generate many random $C^{TT}_{\Lambda CDM}$ from Gaussian distributions centred on the WMAP-reported values, and correct this realization for the slight correlation induced on partial skies. The resulting sky realization is guaranteed to have a full-sky $S_{1/2}^{TT}$ consistent with the small value in the full-sky WMAP ILC map. We only keep realizations with an $S_{1/2}^{TT}$ less than the observed cut-sky-calculated value (1292.6 $\mu$K$^4$ for WMAP-7 and 1304 $\mu$K$^4$ for WMAP-9) for analysis.

With a set of $10^5$ such constrained temperature realizations, $a_{i\alpha}$, we can compute the corresponding set of constrained lensing potential realizations, $\alpha_{i\alpha}$. This is done using standard techniques for generating correlated random variables using the HEALPix package1 (Gorski et al. 2005) and is reviewed in the appendix of Copi et al. (2013a). The required input spectra $C^{TT}_{\Lambda CDM}$ were generated with CAMB2 (Lewis, Challinor & Lasenby 2000).

A full-sky analysis of the CMB relies on a reconstruction of the fluctuations behind the Galactic cut. We instead use the cut-sky-measured value as a threshold to avoid any bias that the reconstruction might induce. We emphasize that no attempt is made (nor is any necessary) to argue that the cut-sky statistic is a better estimator of the value of some full-sky version of the statistic on the full sky (if we could measure it reliably) or on the ensemble. The cut-sky statistic need only be taken at face value as a precise prescription for something that can be calculated from the observable sky. Calculating $S_{1/2}$ with partial-sky data has a well-defined procedure – statistics are just calculated using pseudo-$C_i$s without reference to any full-sky estimators. A more detailed discussion of this can be found in Copi et al. (2013a).

4 CALCULATING $S_{1/2}$ AND $S_\alpha$ STATISTICS

Once we have a set of $a_{i\alpha}$ and $\alpha_{i\alpha}$, we can calculate $S_{1/2}^{XY}_{\Lambda CDM}$ as above (equation 5). However, instead of using $C(\theta)$ directly, we calculate the statistic using $C_{\chi}(\theta)$:

$$S_{1/2}^{XY} = \int_{-1}^{1/2} \cos^2 \theta |C^{XY}(\theta)|^2 d\theta = \sum_{l=2}^{\ell_{\text{max}}} C_{l\chi}^{XY} I_{l\chi} C_{l\chi}^{XY}.$$  (12)

In computing equation (12), we used an $\ell_{\text{max}} = 100$, as the $C_l$ fall sharply and higher order modes have a negligible contribution to the statistic. An explicit definition of the $I_{l\chi}$ matrix can be found in appendix B of Copi et al. (2009).

For temperature–lensing cross-correlations, we can optimize the statistic a priori so that it best discriminates between constrained realizations of $\Lambda$CDM and any other model with which one might choose to compare it. For the purposes of this work, we chose unconstrained $\Lambda$CDM, that is, $\Lambda$CDM without the observed constraints.

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1 http://healpix.sourceforge.net
2 http://camb.info
temperature constraints, as a stand-in comparison model to illustrate our methods. To do this, we generalize $S_{1/2}$ to

$$S_{a,b}^{XY} = \int_a^b d(\cos \theta) [C^{XY}(\theta)]^2 \text{ for } -1 < a < b < 1. \quad (13)$$

For each possible pair of $a = \cos (\theta_a)$ and $b = \cos (\theta_b)$, we calculate the distribution of $S_{a,b}^{XY}$ values for ensembles of both constrained and unconstrained realizations. We compute the 99-percentile value for the constrained distribution (i.e. the value of $S_{a,b}^{XY}$ that is greater than 99 per cent of the members of the constrained ensemble). We then determine the fraction of the values in the $S_{a,b}$ distribution for the unconstrained ensemble that are larger than the 99-per-cent constrained value. The higher the percentile, the better $S_{a,b}^{XY}$ is at discriminating between the constrained and unconstrained models. We repeat the analysis for 99.9 percentile. We choose two different C.L. for analysis here to show that optimization will lead to different $(a, b)$ ranges. The C.L. and corresponding percentage for optimization should always be chosen before any analysis on a particular data set is carried out. This process was performed using reported $C^T_T$ and corresponding error bars from both WMAP 7- and WMAP 9-year releases.\(^3\)

5 RESULTS

The distributions on a cut sky for unconstrained $\Lambda$CDM and for $\Lambda$CDM constrained by the WMAP 9-year power spectrum and $S_{1/2}$ value are shown in Fig. 1. The two distributions have a significant overlap. However, the peak height, peak location, and width are notably different, showing how the inclusion of information about our measured temperature spectrum will lead to an important change in the ensemble of expected statistics.

The analysis shows that 39.6 per cent of the unconstrained $\Lambda$CDM values fall above the 99-percentile constrained value of $1.48 \times 10^{-7}$ $\mu$K$^2$, and 26.7 per cent of the unconstrained $\Lambda$CDM values fall above the 99.9-percentile constrained value of $2.47 \times 10^{-7}$ $\mu$K$^2$. Therefore, if predictions are made using unconstrained $\Lambda$CDM simulations, there is some chance that conclusions could incorrectly confirm $\Lambda$CDM while constrained realizations might rule it out.

We repeated this analysis for the WMAP 7-year release, and found a negligible difference between results generated with the 7- and 9-year data sets. We conclude that the changes in WMAP reported values and error bars and the best-fitting model for $C^T_T$ by release year do not affect the results in any significant way. We have also calculated all of the generalized $S_{a,b}^{XY}$ statistics for the 7- and 9-year data, and found that it as well produced similar results. Therefore, we will proceed with presenting only the results from the WMAP 9-year analysis.

Figs 2 and 3 show only a small improvement by choosing to integrate over a range of angles other than 60–180° for differentiating between constrained $\Lambda$CDM and our stand-in comparison model, unconstrained $\Lambda$CDM. The maximum discriminating power is 40.4 per cent for the statistic integrating over the range $a = \cos (168^\circ), b = \cos (48^\circ)$. This gives $1.43 \times 10^{-7}$ $\mu$K$^2$ as the value for the $S_{a,b}$ statistic from equation (13). For the 99.9 percentile, the maximum discriminating power drops to 27.8 per cent, with the optimal range of angles changing slightly to $a = \cos (127^\circ), b = \cos (53^\circ)$, giving a value of $1.50 \times 10^{-7}$ $\mu$K$^2$ for the $S_{a,b}$ statistic.

\(^3\) http://lambda.gsfc.nasa.gov/
The results for the 99.9-percent optimization are similar enough to the 99-percentile optimization that we do not show the distributions, but instead just quote the appropriate values. Our goal of presenting both of these C.L. is to show that one can choose a priori the C.L. for which they can optimize their pipeline.

We have not repeated the analysis for the first Planck temperature maps because an approximate covariance matrix is not yet available, but we expect the results to be similar based on the close match between large-scale features in the WMAP and Planck sky maps.

6 CONCLUSIONS

In light of the new data from Planck, large-angle anomalies have been gaining more traction in the community as potential evidence of interesting primordial physics that deviates from the widely accepted ΛCDM paradigm. In particular, the observed lack of temperature–temperature correlations at angles larger than 60° has been characterized by the $S_{TT}^1$ statistic, and the measured value has shown to occur in less than 0.1 per cent of standard ΛCDM realizations on a 9-year QK85 masked sky (Copi et al. 2004, 2013b). Since $S_{TT}^1$ was an a posteriori choice of a statistic after seeing the shape of the two-point function from the WMAP data, the possibility that our Universe happens to just be a statistical fluke has been advocated (Efstathiou, Ma & Hanson 2010; Pontzen & Peiris 2010). We wish to test this hypothesis by calculating statistics for a different correlation function, and we make predictions for statistics that are informed by the power spectrum and lack of correlation of the observed microwave background temperature. For this purpose, we have used the cross-correlation between temperature and lensing potential $\varphi$ as an a priori test of the null hypothesis.

We have calculated the distribution of the $S_{T\varphi}^1$ statistic for $10^5$ realizations of unconstrained ΛCDM with the WMAP 7 and WMAP 9 best-fitting cosmological parameters, as well as a similar number of realizations constrained to have a value for $S_{TT}^1$ no larger than the observed value, and found no significant difference between results calculated from each data set despite changes to the reported central values and error bars. We showed how adding observed temperature information produces significant differences for the expected distribution of statistics between constrained realizations and those of standard ΛCDM. Only 39.2 per cent of members of the ensemble of unconstrained realizations had $S_{T\varphi}^1$ greater than the 99-percentile value of $S_{T\varphi}^1$ in the ensemble of constrained realizations. This highlights the benefit of including observational information in simulations when making predictions for statistics of future data sets.

We have also defined a generalized statistic for testing the null hypothesis by investigating which pair of angles used in calculating $S_{a,b}^T$ defined in equation (13) provided the largest percentage of unconstrained ΛCDM $S_{a,b}^T$ lying above the 99-percentile value for the constrained distribution. We find that restricting the integration range from 48° to 168° slightly improves the ability of the statistic to rule out a given measured value being consistent with constrained ΛCDM. To rule out the fluke hypothesis at the 99.9-percentile level, we found that the optimal range of angles is 53°–130°. The contour showing the discriminating power of the $S_{a,b}^T$ statistic for the constrained ΛCDM versus unconstrained ΛCDM is shown in Fig. 2, and the histogram for the $S_{a,b}^T$ statistic for the optimal angular ranges is shown in Fig. 3. However, because the improvement over the $S_{T/2}^T$ statistic is modest, and because $S_{a,b}^T$ is optimized to select between constrained ΛCDM and unconstrained ΛCDM, simplicity argues for using $S_{T/2}^T$ to parallel previous analysis of $C_{TT}^\ell$.

In this analysis, unconstrained ΛCDM is used as a straw man to illustrate our methodology. The outlined procedure for assessing the discriminating power of the $T\varphi$ cross-correlation statistics can be used as a generic prescription for optimizing statistics of any cosmological data. Choice of a particular C.L. to optimize for should always be made before calculations of any $S_{T/2}^T$-like statistics are carried out to avoid any bias in reporting results.

Clearly, some model comparisons will provide statistics from $T\varphi$ that are more useful than others. In particular, since the $\varphi$ field is dominated by effects inside the last-scattering surface, it has a large correlation with $\Theta_{ISW}$. This means that unless a proposed model can find some way to suppress this particular term, there will not be a sharp difference for $S_{T\varphi}^1$ statistics from ΛCDM, which will limit its usefulness if one prefers to compare their model to ΛCDM. Regardless, $T\varphi$ correlations will provide an important consistency check for the data, as the large $\Theta_{ISW}\varphi$ contribution is a probe of physics on the interior of our Hubble volume. It is therefore complimentary to the TT signal which is largely comprised of effects at the last-scattering surface.

The $S_{T\varphi}^1$ statistic is not particularly helpful for testing a hypothesis that the underlying gravitational potential fluctuations lack correlations on scales larger than some comoving scale subtending 60° at the redshift of last scattering. Any suppression in $(\Phi(\theta_1)\Phi(\theta_2))$ that gives rise to the observed $TT$ spectrum would not have any significant effect on the shape of $C_{T\varphi}^\ell(\theta)$ compared to ΛCDM. This fact is precisely due to the large $\Theta_{ISW}\varphi$ term. Similarly, a cutoff in the primordial power spectrum has also been used to investigate another particular CMB anomaly, the low values of the low-multipole temperature power spectrum (Kesden, Kamionkowski & Cooray 2003).

Other cross-correlations may prove to be more fruitful. For example, 21-cm emission correlated with temperature fluctuations will partially mitigate the large $\Theta_{ISW}\varphi$ problem. The 21-cm emission spectrum comes to us from localized region of redshift space, and does not have a component which is an integral along the line of sight. This in particular should reduce the magnitude of the correlation compared to $C_{T\varphi}^\ell(\theta)$. In a future work, we will show how viable this cross-correlation will be for testing the null hypothesis, as well as provide predictions for the shape of $C(\theta)^F$ 21cm with an imposed cutoff.

A related calculation (Copi et al. 2013a) examined the implications of the fluke hypothesis for the temperature–polarization angular correlation function, in particular, the correlation of temperature with the Stokes $Q$ parameter, $C_{TQ}^\ell(\theta)$. This statistic excludes the fluke hypothesis at 99.9 per cent C.L. for 26 per cent of realizations of unconstrained ΛCDM, or at 99 per cent C.L. for 39 per cent of such realizations.

In summary, the temperature autocorrelation of the CMB sky behaves oddly at large angular separations. No satisfactory current theory explains this anomaly, and so the leading explanation is that it is a statistical fluke. In the absence of specific models to compare directly with ΛCDM, the best strategy is to identify other measurable quantities with probability distributions that are affected by the knowledge that $S_{T/2}^T$ is small, and make predictions for the new probability distribution functions. In this way, we can test the fluke hypothesis with current and near-future CMB data sets.

ACKNOWLEDGEMENTS

The authors would like to thank Simone Aiola for useful discussions. The numerical simulations were performed on the facilities

MNras 442, 2392–2397 (2014)
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provided by the Case ITS High Performance Computing Cluster. AY is supported by NASA NESSF Fellowship. CJC, GDS, and AY are supported by a grant from the US DOE to the Particle Astrophysics Theory group at CWRU. AK has been partly supported by NSF grant AST-1108790.

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