Construction of Fungal Decomposition Model

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Abstract: Fungi, as the key factor to decompose lignocellulosic fibers, play an important role in the carbon cycle of the whole earth, and are an important part of life on the earth. In this paper, we consider all kinds of factors affecting the decomposition rate of fungi, combined with Monod equation, and fit the linear relationship between the decomposition rate of a single fungal population and time through data; transform the temperature and humidity that independently affect the growth rate into a product relationship with higher coupling degree; establish a fungal classification model of competition among multiple populations, and extend the existing conclusions.

1. Introduction

In recent years, domestic and foreign researchers have been devoted to the artificial culture, physicochemical properties and molecular structure of various fungal filaments. However, there are few studies on the decomposition and growth dynamics of fungi, which can be used to predict the structure of fungal community and the impact on carbon cycle. In this paper, according to the various factors affecting the decomposition rate of fungi, combined with Monod equation, fitting the relevant data, it is determined that the decomposition rate of a single fungal population has a linear relationship with time under the condition of the external environment remains unchanged. On this basis, the temperature and humidity conditions that independently affect the growth rate are transformed into a more reasonable product relationship with higher coupling degree. Finally, according to the logistic law and the competition index, the two population competition model is extended to the multi population competition model, and the fungus decomposition model of multi species competition is established.

2. The establishment of the model

The factors that affect the decomposition rate of fungi are biological factors (water resistance and growth rate of fungi) and external factors (temperature, humidity, quality of rotten wood and fallen leaves). Firstly, the relationship between growth rate and decomposition rate of fungi was studied. Daniel S. Maynard et al. Measured the growth rate and decomposition rate by experiments[1]. Fig. 1 is the result of data fitting.
Fig. 1 The growth rate of various fungi (mm/day) was fitted with wood decomposition rate (mass loss% in 122 days) under different temperatures.

Fig. 1 The results show that the positive correlation between the wood decomposition rate and the growth rate of various fungi tends to be flat under three different temperatures, and it is approximately linear in the double logarithmic coordinate system. Therefore, the relationship between wood decomposition rate and growth rate can be defined as:

$$d \ln v_{\text{dec}} = \alpha \cdot d \ln v_{\text{ext}} (\alpha \text{ is a constant coefficient}),$$

Inside: $v_{\text{dec}}$ Represents the rate of wood decomposition, $v_{\text{ext}}$ Indicates the growth rate of fungi.

The differential equation is rewritten as a dominant function of wood decomposition rate and growth rate:

$$v_{\text{dec}} = C \cdot v_{\text{ext}}^\alpha, \tag{1}$$

In order to explore the change of wood residual quality with time, Monod equation was used for reference\cite{2}:

$$\mu = \frac{\mu_m \cdot S}{S + K_s}, \tag{2}$$

Inside: $\mu$ is the growth rate, $\mu_m$ is the maximum growth rate, $S$ Is the current matrix concentration, $K_s$ is the saturation constant.

By substituting the relationship between decomposition rate and growth rate in equation (1) into equation (2), get:

$$v_{\text{dec}} = C \cdot \left(\frac{\mu_m \cdot S}{K_s + S}\right)^\alpha, \tag{3}$$

The two ends of formula (3) are treated with difference at any time node:

$$v_n - v_{n-1} = C \cdot \left(\frac{\mu_m \cdot S}{K_s + S}\right)^\alpha \cdot \Delta t.$$

Because $\Delta t$ is an extremely small invariant, it can be combined with constant coefficient $C$ in quasi-static analysis:

$$v_n - v_{n-1} = C' \cdot \left(\frac{\mu_m \cdot S}{K_s + S}\right)^\alpha \tag{4}$$

Obviously, if the continuous time is divided into infinitely many time infinitesimals, the wood
decomposition rate at any time can be expressed by the recursive iteration method.

In the given data, the decomposition rate takes 122 days as the time unit. Therefore, a time infinitesimal $\Delta t$ is regarded as 1 day. Since the time unit at the left end of the equation is (days), it can be concluded that equation (4) can be rewritten as:

$$v_n - v_{n-1} = C \cdot \left( \frac{\mu_m \cdot S}{K_s + S} \right)^\alpha.$$  \hspace{1cm} (5)

Next, the simulated annealing algorithm is used to find the optimal $\mu_m$ and $K_s$.

Substituting the approximate solution of the parameter into equation (4), the curve of wood quality with time is obtained as Fig. 2.

As can be seen from Fig 2, the trend of change is approximately a straight line. Because the model only needs to simulate the process of fungi degradation of organic matter, in order to simplify the calculation, the relationship between wood quality and time is considered as linear. That is, the decomposition rate of fungi does not change with time when the temperature and humidity are constant.

Considering the influence of temperature and humidity on the decomposition rate of fungi, the decomposition model of fungi can be written as follows:

$$v_{dec} = k_1 \cdot v_{dec}(T) + k_2 \cdot v_{dec}(p) \quad (k_1, \ k_2 \text{ are constant coefficients}),$$  \hspace{1cm} (6)

According to equation (6), when the fungi are at extreme temperature, if the humidity is appropriate, they can still have a higher decomposition rate, contrary to common sense. The temperature and humidity should act on the decomposition rate of fungi in a certain restriction relationship, so it is assumed that:

$$v_{dec} = k \cdot v_{dec}(T) \cdot v_{dec}(p) \quad (k \text{ is a constant coefficient}).$$  \hspace{1cm} (7)

According to the curve data in reference[23], which takes temperature and humidity as independent variables and growth rate as dependent variables, the graph is drawn and fitted according to the characteristics of the image, as shown in Fig. 3.
The expression of the growth rate with respect to temperature and humidity by fitting is as follows:

\[
\begin{align*}
 v_{ext}(p) &= a_0 \cdot p^6 + a_1 \cdot p^5 + a_2 \cdot p^4 + a_3 \cdot p^3 + a_4 \cdot p^2 + a_5 \cdot p \\
 v_{ext}(T) &= \gamma \cdot e^{-\frac{(T-\lambda)^2}{\kappa}} \\
\end{align*}
\]

Combining formulas (1), (7) and (8) can solve the fungal decomposition model:

\[
v_{dec} = C' \cdot \left[ \gamma \cdot e^{-\frac{(T-\lambda)^2}{\kappa}} \cdot (a_0 \cdot p^6 + a_1 \cdot p^5 + a_2 \cdot p^4 + a_3 \cdot p^3 + a_4 \cdot p^2 + a_5 \cdot p)^\gamma \right]
\]

When solving practical problems, fungi often exist in multiple populations at the same time. Different populations of fungi often have differences in growth rate and water tolerance. Next, the established fungal decomposition model will be further improved. The water tolerance and decomposition rate have a linear positive correlation trend on the logarithmic coordinate \([1]\), and the functional relationship between the decomposition rate and the growth rate has been solved in the previous model, so it can be judged that the water tolerance and the growth rate have a one-to-one mapping relationship, so only the difference in water tolerance needs to be taken into consideration:

\[
\frac{\partial \ln v_{dec}}{\partial \delta} = c (c \text{ is a constant coefficient and } c > 0)
\]

Inside: \(\delta\) is the water tolerance.

Solving the equation (9) can be obtained:
$v_{dec} = c' \cdot e^{-\delta}$ ($c'$ is a constant coefficient and $c' > 0$).

When the population structure in the community is known, the average water tolerance of the community can be calculated by solving the mean value. Therefore, the influence of population structure differences in the community on the decomposition rate can be characterized by $\overline{\delta}$. Improve the model as follows:

$$v_{dec} = k' \cdot v_{dec}(T) \cdot v_{dec}(p) \cdot e^{-\overline{\delta}}.$$  \hspace{1cm} (11)

According to (8), (9) and (11), the expression of the decomposition model after the first improvement is:

$$v_{dec} = k' \cdot \left[ \gamma \cdot e^{-\frac{(T-k)^2}{\kappa}} \cdot \left( a_6 \cdot p^6 + a_5 \cdot p^5 + a_4 \cdot p^4 + a_3 \cdot p^3 + a_2 \cdot p^2 + a_1 \cdot p \right) \right] \cdot e^{-\overline{\delta}}$$

Next, take the impact of inter-species competition into consideration, and then make further improvements to the existing model.

If there are $n$ populations, the changes in their numbers when they live alone all obey the Logistic law:

$$\frac{dx_i}{dt} = r_i \cdot x_i \left( 1 - \frac{x_i}{n_i} \right) \quad i = 1, 2, 3, \ldots, n.$$  \hspace{1cm} (12)

When $n$ populations co-exist, analyze a certain population separately, and the blocking effect of other populations on the population is proportional to the number of other populations. So there are the following expressions:

$$\frac{dx_i}{dt} = r_i \cdot x_i \left( 1 - \frac{x_i}{n_i} - \sum_{m=1}^{n} \frac{x_m}{n_m} \right)$$

Inside: $x_m$ means that the consumption of the $X_m$ population per unit amount is a multiple of the $X_i$ population. $n_i$ is the maximum capacity of the population. $r_i$ is the current growth rate.

In this way, we extend the dual-species competition model to multi-species.

For fungal communities, because the growth rate is affected by water tolerance, the faster the growth rate, the weaker the individual's ability to adapt to the environment, so $n_i$ and $\delta$ are considered to be linearly negatively correlated. It can be seen that the essence of interspecies competition in the dynamics research process is the change of population structure (characterized by $\overline{\delta}$).

In order to verify whether the model is ideal, the fungal community structure is set here as the three populations with water tolerance of -0.5, -0.2, and 0.2, each accounting for 1/3, at a constant temperature of 30°C and a humidity of -4MPa Increase by 0.01MPa every day for 300 days, and use formula (11) to simulate the changes in the number of individuals in the three populations. The result is shown in Fig. 4:
Fig. 4 Improved model to simulate fungal population growth curve

It can be seen from Fig. 4 that when the decomposition rate and the iterative effect of the community structure are not considered, the model better simulates the changes in the fungal community structure caused by the competition relationship over time, thereby affecting the average water tolerance $\delta$ in the community value. Therefore, the decomposition rate is affected by the changing $\delta$ over time.

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