Measuring non-Markovianity of processes with controllable system-environment interaction

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Abstract – Non-Markovian processes have recently become a central topic in the study of open quantum systems. We realize experimentally non-Markovian decoherence processes of single photons by combining time delay and evolution in a polarization-maintaining optical fiber. The experiment allows the identification of the process with strongest memory effects as well as the determination of a recently proposed measure for the degree of quantum non-Markovianity based on the exchange of information between the open system and its environment. Our results show that an experimental quantification of memory in quantum processes is indeed feasible which could be useful in the development of quantum memory and communication devices.

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Introduction. – The theory of open quantum systems describes how a system of interest is influenced by the interaction with its environment [1]. This interaction often leads to a loss of the quantum features of physical states and has a great impact on the dynamical behavior of the open system due to the non-unitary character of the time evolution. Since any realistic physical system is coupled to its surroundings, open quantum systems and their description plays an important role in many applications of modern quantum physics.

While Markovian, or memoryless, quantum processes are well understood in the framework of the theory of quantum Markovian master equations developed during the 70’s [2,3], non-Markovian processes with memory have recently become of central importance in the study of open systems [4–12]. In general, non-Markovian features can arise, e.g., because of a strong system-environment interaction, structured reservoirs, due to initial system-environment correlations, or couplings to a low-temperature environment or spin bath. Moreover, recent developments in experimental technology allow reservoir engineering [13], study of quantum correlations [14], and the development of quantum simulators for open systems [15].

On the one hand, non-Markovian systems are not yet well-understood even on the fundamental level, and many efforts have been devoted to the development of theoretical tools for the treatment of quantum dynamics with memory [6–12]. As a matter of fact, the very definition of quantum non-Markovianity has been recently under a vivid discussion [10–12]. On the other hand, there are indications that non-Markovianity may play a role, e.g., in energy transport in certain photosynthetic complexes [16,17] and can be exploited for quantum metrology [18] and cryptography [19].

In this letter we implement non-Markovian processes for single photons and demonstrate that it is possible to determine experimentally the amount of memory in the system. Our work is based on a recent theoretical proposal to study the information flow between the system and its environment, and in particular to quantify non-Markovianity with the help of the backflow of information from the environment to the system [11].
allows us to identify a specific process having the largest memory among all experimentally implemented processes, and then to quantify the amount of non-Markovianity for this process. There exist earlier experimental works on dephasing in photonic systems [20,21], and results which may be regarded as indications for non-Markovian behavior [22,23]. However, the recent theoretical progress is opening the path for rigorous experimental tests of quantum non-Markovianity, for example, by controlling the initial state of the environment [24], or by modifying the interaction between the system and the environment, as is done here. We also note that a theoretical proposal to witness initial system-environment correlations by studying the information flow between a system and its environment [25] has been recently realized experimentally [26,27].

**Theoretical framework.** – We consider a pure dephasing quantum process for which the density matrix \( \rho \) of the open system evolves according to the master equation

\[
\frac{d\rho(t)}{dt} = -i\frac{\epsilon(t)}{2} [\sigma_z, \rho(t)] + \frac{\gamma(t)}{2} (\sigma_z \rho(t) \sigma_z - \rho(t)).
\]

Here, \( \epsilon(t) \) represents the time-dependent energy shift and \( \gamma(t) \) the time-dependent rate of the decay channel described by the Pauli operator \( \sigma_z \). Generally, non-Markovian dynamics can be described, e.g., by memory-kernel equations or time local master equations where the decay rates depend on time [1], and we have chosen the latter due to its conceptual simplicity for the current purpose. Recently, time local equations have received a great deal of attention [6,8,11,12,28], and for a single channel system, as is the case here, one can directly associate the non-Markovianity with the appearance of negative periods of the decay rate [8,11].

The open two-state system we consider consists of the horizontal and the vertical polarization states of a photon, \(|H\rangle\) and \(|V\rangle\), respectively. The dephasing process influences the coherences between the polarization components and the evolution can be described by the decoherence function \( \kappa(t) = \exp[-\int_0^t dt'(\gamma(t') + i\epsilon(t'))] \) which is connected to the energy shift and the decay rate of the master equation (1) by the relations

\[
\epsilon(t) = -\Im [\kappa(t)/\kappa(t)], \quad \gamma(t) = -\Re [\kappa(t)/\kappa(t)].
\]

The corresponding dynamical map \( \Phi_t \) which maps the initial polarization state \( \rho(0) \) to the state \( \rho(t) = \Phi_t \rho(0) \) at time \( t \) is then given by

\[
\rho_{H,H}(t) = \rho_{H,H}(0), \quad \rho_{V,V}(t) = \rho_{V,V}(0),
\]

\[
\rho_{H,V}(t) = \kappa^*(t) \rho_{H,V}(0), \quad \rho_{V,H}(t) = \kappa(t) \rho_{V,H}(0).
\]

The environment consists of the frequency degrees of freedom \( \omega \) of the photon and we consider an initial product state between the system and the environment \( \rho_{\text{tot}}(0) = \rho(0) \otimes \int d\omega A(\omega) A^*(\omega') |\omega\rangle \langle \omega'|. \) The function \( G(\omega) = |A(\omega)|^2 \) represents the normalized frequency distribution of the photon which in our experiments is a Lorentzian distribution with central frequency \( \omega_0 \) and full width at half maximum 2\( \delta \omega \) (FWHM), \( G(\omega) = \frac{1}{\pi (\omega_0 - \omega)^2 + \delta \omega^2} \). We investigate the case in which the evolution of the frequency distribution depends on the polarization state such that the total system dynamics is described by the unitary operator

\[
U_{\text{tot}}(t) = |H\rangle \langle H| \otimes \int d\omega e^{-i\omega u_1(t)t} |\omega\rangle \langle \omega| + |V\rangle \langle V| \otimes \int d\omega e^{-i\omega u_2(t)t} |\omega\rangle \langle \omega|,
\]

where the factors \( u_1(t) \) and \( u_2(t) \) may depend on time.

In the experiment we determine the measure for the degree of non-Markovianity constructed in ref. [11]. For a quantum process of an open system described by a dynamical map \( \Phi_t \) this measure is defined by

\[
N(\Phi) = \max_{\rho_{1,2}(0)} \int_0^\infty dt \sigma(t, \rho_{1,2}(0)).
\]

Here, \( \rho_{1,2}(0) \) are two initial states of the open system and \( \sigma(t, \rho_{1,2}(0)) = \frac{d}{dt} D(\rho_1(t), \rho_2(t)) \) is the rate of change of the trace distance \( D(\rho_1, \rho_2) = \frac{1}{2} \text{tr} |\rho_1 - \rho_2| \). The trace distance represents a measure for the distinguishability of two quantum states. In eq. (4) the time integral is extended over all intervals in which the trace distance increases and the maximum is taken over all pairs of initial states. The measure \( N(\Phi) \) thus quantifies the maximal total increase of the trace distance during the time evolution, which can be interpreted as the maximal total amount of information that flows from the environment back to the open system [11]. In our experiment the increase of the trace distance signifying non-Markovian behavior is restricted to a single time interval \([t_0, t_1]\) which yields \( N(\Phi) = \max_{\rho_{1,2}(0)} [D(\rho_1(t_1), \rho_2(t_1)) - D(\rho_1(t_0), \rho_2(t_0))] \).

**Experimental framework.** – We measure the non-Markovianity of a quantum evolution process given by the master equation (1) which is implemented for single photons emitted from a quantum dot (QD). The experimental setup is shown in fig. 1 and includes three parts: Preparation, time evolution, and state tomography. The first part consists of the generation of a single photon and its preparation in the pure initial state

\[
|\Psi\rangle = \cos(\phi) |H\rangle + \sin(\phi) |V\rangle,
\]

where \( \phi \) is the angle of the polarization direction of the photon from the horizontal direction. A self-assembled InAs/GaAs QD sample (at a temperature of 7 K) [29,30] provides the single-photon source. A He-Ne laser pumps the sample through a beam splitter (BS, high transmission efficiency of 92%) and a 50× objective. The photon is collected by the same objective and separated by a 785 nm long pass filter (LP) and an interference filter (IF, the
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central wavelength of the pass band is tunable around 950 nm and the FWHM is 0.7 nm. Then, a polarizer and a half-wave plate (HWP1) prepares the photon in the initial state (5), where φ can be changed by rotating HWP1.

The time evolution consists of two main contributions: A delay setup and a polarization-maintaining (PM) optical fiber with a HWP3 between the two contributions. The delay setup includes a polarization beam splitter (PBS1), two 45° (from the horizontal direction) placed quarter-wave plates (QWP1 and QWP2), and two mirrors whose relative delay can be controlled. The decoherence function can generally be written as ν(t) = ∫ dω G(ω) exp(iωt).

Fig. 1: (Color online) The experimental setup consisting of three parts: Preparation of a single-photon state, evolution in the delay setup followed by the PM optical fiber, and state tomography (T1 and T2). Used abbreviations: BS, beamsplitter; LP, long pass filter; IF, interference filter; HWP, half-wave plate; PBS, polarization beam splitter; QWP, quarter-wave plate; PM, polarization maintaining; APD, avalanche photodiode.

which is used for times t0 ≤ t ≤ t1, where t1 is the termination time of the process when the photon exits the fiber, and Δn is the real birefringence.

In the third and last part of the setup a tomography [31] of the final states, i.e., of the states at the end of the fiber is carried out (T1). We also perform a tomography of the states after the delay setup (T2). Both tomography setups are identical utilizing two single-photon avalanche photodiodes (APDs). The count rate is about 7000/s, and the integration time is 4 s.

While the length of the fiber is fixed, we can control the maximal delay t0 before the photons enter the fiber. The evolution of the reduced system follows the master equation (1) for all times, where γ(t) is given by

\[ γ(t) = \begin{cases} δω, & 0 ≤ t < t_0, \\ -δω \frac{δn}{n}, & t_0 ≤ t < t_1, \\ δω \frac{δn}{n}, & t_1 ≤ t < t_2. \end{cases} \]  

These can be obtained by using eqs. (2) and (6), (7), and t1 = (1 + tn/Δn)t0 > t0. We can control the length of the negative decay period by changing t0. Note that if t0 is small enough, then t2 < t1 and the last positive period does not appear since the decay rate is negative during all of the fiber evolution. It is also easy to see from eq. (7) that the period of a negative decay rate corresponds to an increase in |κ(t)|. By fixing t0, corresponding to a fixed delay x0, one also fixes the dynamical map which we denote by Φx0, and there exists a different process for each value of x0.

We are now interested in which one of the processes Φx0 has the highest value of non-Markovianity and what is the corresponding value of x0.

**Results.** To identify the process having the largest value of non-Markovianity, we first fix the angles of the initial-state pair as (θ, ξ) = (135°, 45°) (see eqs. (4) and (5)). After the delay time t0, the photons travel in the fiber and a state tomography is performed both before and after the evolution in the fiber. The difference of the trace distances at the corresponding times t1 and t0 is denoted by ΔD(θ, ξ, x0) and the result is shown in fig. 2(a). We find

![Fig. 2](image-url)
that the maximum is located at \( x_0 = 19.15 \text{ mm} \) and takes the value \( \Delta D(135^\circ, 45^\circ, 19.15 \text{ mm}) = 0.962 \pm 0.011 \). If we subtract the dark and background counts of the APDs (about 150/s), this value is corrected to 0.988 \( \pm \) 0.011. The uncertainty is due to the counting statistics. Figure 2(b) shows the final trace distance \( D_n \) as the function of the used delay \( x_0 \). By fitting the experimental data to the theoretical curve \( D_n(x_0) = \exp(-\delta \omega |\Delta n l - 2x_0|/c) \) (the red solid line in fig. 2(b)) we determine the parameter \( 1/\delta \omega = 35.8 \pm 1.9 \text{ ps} \).

For \( x_0 \geq 19.15 \text{ mm} \) the decay rate is positive when \( t < t_0 \) and negative for all times the photon travels in the fiber, \( t_0 \leq t \leq t_1 \). Hence, the results in fig. 2(a) show that for \( x_0 \geq 19.15 \text{ mm} \), the process \( \Phi^{19.15 \text{ mm}} \) gives the maximal non-Markovianity within this group of processes \( \Phi^{x_0 \geq 19.15 \text{ mm}} \). On the other hand, for the processes with \( x_0 \leq 19.15 \text{ mm} \), the smaller \( x_0 \) the shorter is the negative decoherence period since the negative decay occurs at \( t_0 \leq t < t_1 \). Consequently, the point \( x_0 = 19.15 \text{ mm} \) also gives the maximal non-Markovianity for values \( x_0 \leq 19.15 \text{ mm} \). Thus, we can conclude that \( \Phi^{19.15 \text{ mm}} \) yields the maximal non-Markovianity among all processes experimentally implemented.

In the second step, we fix the delay corresponding to the process \( \Phi^{19.15 \text{ mm}} \) and carry out the maximization over pairs of initial states in the definition (4) of the measure by changing the pair of angles \( (\theta, \xi) \). Experimental results for \( \Delta D(\theta, \xi, 19.15 \text{ mm}) \) are shown in fig. 3. We clearly see that \( \Delta D(135^\circ, 45^\circ, 19.15 \text{ mm}) \) still represents the maximal value among all pairs of initial states of the form given by eq. (5). We do not perform the maximization over the whole state space, because restricting to pairs given by eq. (5) is sufficient for finding the maximizing pair as we will see later on.

Let us discuss this point in more detail. Firstly, we find that \( 2 \times 19.15 \text{ mm} \) is approximately equal to \( \Delta n_0 l = 35 \text{ mm} \). The discrepancy is explained by noting that the birefringence given by the manufacturer of the fiber at its designed wavelength 780 nm is \( \Delta n_0 = 3.5 \times 10^{-4} \). However, the wavelength of our chosen peak is at 946.3 nm and consequently the real birefringence in our experiment is \( \Delta n = 3.83 \times 10^{-4} \) (the value used in eq. (7)). This suggests that the birefringence effect of the PM optical fiber compensates exactly the dephasing caused by the delay setup when \( x_0 = 19.15 \text{ mm} \). This means that the open system has recovered all the information that it lost during the earlier part of the evolution. The experimentally determined value for the measure of non-Markovinity is given by \( N(\Phi) = 0.962 \pm 0.012 \), and by \( N(\Phi) = 0.988 \pm 0.011 \) when accounting for dark and background counts. This result matches very closely the theoretical result \( N(\Phi) = 0.972 \) obtained by using eqs. (6), (7) and the above initial states. The difference between the experimental and theoretical result may be caused by mode flips and temperature changes in the PM fiber. Notice, that increasing the fiber length and correspondingly the delay time ultimately give an increase of 1 for the pair of angles \( (135^\circ, 45^\circ) \). Since the trace distance is by definition bounded between 0 and 1 there exists no pair exceeding this value of increase and thus the pair with the angles \( (135^\circ, 45^\circ) \) must be maximizing.

We can also directly measure the spectrum of the quantum dot, see fig. 4(a). The obtained width \( 1/\delta \omega = 34.0 \pm 1.7 \text{ ps} \) matches very well the theoretical value \( 1/\delta \omega = 35.8 \pm 1.9 \text{ ps} \) (see fig. 2(b)). Moreover, we have performed numerical simulations of \( \Delta D(\theta, \xi, 19.15 \text{ mm}) \) shown in fig. 4(b). The found maximum value at \( (135^\circ, 45^\circ) \) coincides with our experimental result in fig. 3.

Conclusions. – We have experimentally realized a family of pure decoherence processes in a non-Markovian open quantum system. The setup allows to identify the process with strongest memory effects in this family and to determine the corresponding measure for non-Markovianity (4). This clearly demonstrates the measurability of this quantity, including in particular the maximization over the initial states. Our results show that wide scale experimental studies on fundamental and practical aspects of quantum systems with memory are indeed becoming feasible in the wake of the recent vivid.

Fig. 3: (Color online) Difference of the final and initial trace distances. The initial states are prepared with a fixed delay \( x_0 = 19.15 \text{ mm} \) and various pairs of angles \( (\theta, \xi) \). A maximum value is observed for \( (\theta, \xi) = (135^\circ, 45^\circ) \). The inset shows an elaborate measurement around the pair \( (135^\circ, 45^\circ) \).

Fig. 4: (Color online) (a) The measured spectrum of the quantum dot. Fit with Lorentzian curve gives the width \( 1/\delta \omega = 34.0 \pm 1.7 \text{ ps} \). (b) Simulation results for the quantity \( \Delta D(\theta, \xi, 19.15 \text{ mm}) \) corresponding to the experimental data of fig. 3. The colors represent the values of \( \Delta D \).
Measuring non-Markovianity of processes with controllable system-environment interaction in the theoretical debate. Moreover, our results may be helpful in the study of quantum communication processes [32].

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