Enhancement of dark matter relic density from the late time dark matter conversions

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Abstract

We demonstrate that if the dark matter (DM) in the Universe contains multiple components, the possible interactions between the DM components may convert the heavier DM components into the lighter ones. It is then possible that the lightest DM component with an annihilation cross section significantly larger than that of the typical weakly interacting massive particle (WIMP) can obtain a relic density in agreement with the cosmological observations, due to an enhancement of number density from the DM conversion process at late time after the thermal decoupling, which may provide an alternative source of boost factor relevant to the positron and electron excesses reported by the recent DM indirect search experiments.

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1 Introduction

In the recent years, a number of experiments such as PAMELA [1], ATIC [2], Fermi-LAT [3] and HESS [4] etc. have reported excesses in the high energy spectrum of cosmic-ray positrons and electrons over the backgrounds estimated from the traditional astrophysics. Besides plausible astrophysical explanations [5–7], the dark matter (DM) annihilation or decay provides exciting alternative explanations from particle physics.

If the DM particle is a thermal relic such as the weakly interacting massive particle (WIMP), the thermally averaged product of its annihilation cross section with the relative velocity at the time of thermal freeze out is typically $\langle \sigma v \rangle_{F} \simeq 3 \times 10^{-26} \text{cm}^3\text{s}^{-1}$. The positron or electron flux produced by the DM annihilation can be parametrized as

$$\Phi_e = B\, N_e \, \rho_0^2 \langle \sigma v \rangle_{F} / m_D^2,$$

where $\rho_0 \simeq 0.3 \text{ GeV} \cdot \text{cm}^{-3}$ is the smooth local DM energy density estimated from astrophysics, $N_e$ is the averaged electron number produced per DM annihilation which depends on DM models and propagation parameters, and $m_D$ is the mass of the DM particle. The boost factor $B$ is defined as $B \equiv \left( \rho / \rho_0 \right)^2 \langle \sigma v \rangle / \langle \sigma v \rangle_{F}$ with $\rho$ the true local DM density and $\langle \sigma v \rangle$ the DM annihilation cross section multiplied by the relative velocity and averaged over the DM velocity distribution today. Both the PAMELA and Fermi-LAT results indicate that a large boost factor is needed [8, 9]. For a typical DM mass of $\sim 1(1.6) \text{ TeV}$ the required boost factor $B$ is $\sim 500(1000)$ for DM annihilating directly into $\mu^+\mu^-$ and $\rho$ fixed to $\rho_0$ [9].

A large boost factor may arise from the non-uniformity of the DM distribution in the DM halo. The N-body simulations show however that the local cumps of dark matter density are unlikely to contribute to a large enough $\rho / \rho_0$ [10, 11]. An other possibility of increasing the boost factor is that the DM annihilation cross section may be velocity-dependent which grows at low velocity. The DM annihilation cross section today may be much larger than that at the time of thermal freeze out, and thus is not constrained by the DM relic density. Some enhancement mechanisms have been proposed along this line, such as the Sommerfeld enhancement [12–20] and the resonance enhancement [21–23] etc.

In some non-thermal DM scenarios, the number density of the DM particle can be enhanced by the out of equilibrium decay of some heavier unstable particles if the DM particle is among the decay products of the decaying particle [24, 25]. The decay of the unstable particle must take place at very late time. Otherwise the DM particles with the enhanced number density will annihilate into the Standard Model (SM) particles again, which washes out the effect of the enhancement. This requires that the decay width of the unstable particle must be extremely small, typically $10^{-17} \text{ GeV}$ for the mass of the
decaying particle around TeV \[24\], which is much smaller than that of the typical weak interaction.

In this work, we consider an alternative possibility for generating a boost factor, which does not require the velocity-dependent annihilation cross section or the decay of unstable particles. We show that in the scenarios of interacting Milt-component DM, the interactions among the DM components may convert the heavier DM components into the lighter ones, which is not sensitive to the details of the conversion interaction. If the interactions are strong enough and the DM components are nearly degenerate in mass, the conversion can enhance the number density of the lighter DM components at late time after the thermal decoupling. Eventually, the whole DM today in the Universe can be dominated by the lightest DM component with enhanced number density, which corresponds to a large boost factor. The scenarios of multi-component DM have been discuss previously in Refs. \[26–35\]. Note however that the models with simply mixed non-interacting multi-component DM cannot generate large boost factors.

This paper is organized as follows: in section 2, we first discuss the thermal evolution of the DM number densities in generic multi-component DM models. We then give approximate analytic expressions as well as precise numerical calculations of the boost factor in a generic two-component DM model. In section 3, we consider a concrete model containing two fermionic DM particles with extra \(U(1)\) gauge interactions in the hidden sector. The conclusions are given in section 4.

2 Thermal evolution of the interacting multi-component DM

Let us consider a generic model in which the whole cold DM contains \(N\) components \(\chi_i\) \((i = 1, \ldots, N)\), with masses \(m_i\) and internal degrees of freedom \(g_i\) respectively. The DM components are labeled such that \(m_i < m_j\) for \(i < j\), thus \(\chi_1\) is the lightest DM particle. We are interested in the case that \(\chi_i\) are nearly degenerate in mass, namely the relative mass differences between \(\chi_i\) and \(\chi_1\) satisfy \(\varepsilon_i \equiv (m_i - m_1)/m_1 \ll 1\). In this case, we shall show that the interactions between the DM components lead to the DM conversion. The situation is analogous to the neutral meson mixing and neutrino oscillations in particle physics. They all occur at small mass differences. The thermal evolution of the DM number density normalized to the entropy density \(Y_i \equiv n_i/s\) with respect to the rescaled temperature \(x \equiv m_1/T\) is govern by the following Boltzmann
\[
\frac{dY_i(x)}{dx} = -\frac{\lambda}{x^2} \left[ \langle \sigma_i v \rangle (Y_i^2 - Y_{ieq}^2) - \sum_j \langle \sigma_{ij} v \rangle (Y_i^2 - r_{ij}^2 Y_j^2) \right],
\]

(2)

where \( \lambda \equiv x_s / H(T) \) is a combination of \( x \), the entropy density \( s \) and the Hubble parameter \( H(T) \) as a function of temperature \( T \). \( Y_{ieq} \approx (g_i / s)[m_i T / (2\pi)^3]^{3/2} \exp(-\epsilon_i x) \) is the equilibrium number density normalized to entropy density for non-relativistic particles. \( \langle \sigma_i v \rangle \) are the thermally averaged cross sections multiplied by the DM relative velocity for the process \( \chi_i \chi_i \rightarrow XX' \) with \( XX' \) standing for the light SM particles which are in thermal equilibrium, and \( \langle \sigma_{ij} v \rangle \) are the ones for the DM conversion process \( \chi_i \chi_i \rightarrow \chi_j \chi_j \). The quantity

\[
r_{ij}(x) \equiv \frac{Y_{ieq}(x)}{Y_{jeq}(x)} = \left( \frac{g_i}{g_j} \right) \left( \frac{m_i}{m_j} \right)^{3/2} \exp[-(\epsilon_i - \epsilon_j)x]
\]

(3)

is the ratio between the two equilibrium number density functions, In writing down Eq. (2) we have assumed kinetic equilibrium. The first term in the r.h.s. of Eq. (2) describes the change of number density of \( \chi_i \) due to the annihilation into the SM particles, while the second term describes the change due to the conversion to other DM particles.

In the case that the cross section of the conversion process \( \langle \sigma_{ij} v \rangle \) is large enough, the DM particle \( \chi_i \) can be kept in thermal equilibrium with \( \chi_j \) for a long time after both \( \chi_i \) and \( \chi_j \) have decoupled from the thermal equilibrium with the SM particles. In this case, the number densities of \( \chi_{i,j} \) satisfy a simple relation

\[
\frac{Y_i(x)}{Y_j(x)} \approx \frac{Y_{ieq}(x)}{Y_{jeq}(x)} = r_{ij}(x).
\]

(4)

We emphasize that even when \( \chi_i \) is in equilibrium with \( \chi_j \) the ratio of the number density \( Y_i(x)/Y_j(x) \) can be quite different from unity and can vary with temperature. For instance, if \( g_i \gg g_j \) and \( 0 < (\epsilon_i - \epsilon_j) \ll 1 \), from Eq. (3) and (4) one obtains \( Y_i(x) \gg Y_j(x) \) at the early time when \( (\epsilon_i - \epsilon_j)x \ll 1 \). However, at the late time when \( (\epsilon_i - \epsilon_j)x \gg 1 \), one gets \( Y_i(x) \ll Y_j(x) \), which is due to the Boltzmann suppression factor \( \exp[-(\epsilon_i - \epsilon_j)x] \) in the expression of \( r_{ij} \). Thus the heavier particles can be gradually converted into lighter ones through this temperature-dependent equilibrium between \( \chi_i \) and \( \chi_j \).

Since all the DM components \( \chi_i \) are stable, in general the co-annihilation process \( \chi_i \chi_j \rightarrow XX' \) are not allowed as the crossing process \( \chi_i \rightarrow \chi_j XX' \) corresponds to the decay of \( \chi_i \). Furthermore, unlike the case of co-annihilation, \( \chi_i \) and \( \chi_j \) may not necessarily share the same quantum numbers.

An interesting limit to consider is that the rates of DM conversion are large compared with that of the individual DM annihilation into the SM particles, i.e. \( \langle \sigma_{ij} v \rangle \gg \langle \sigma_i v \rangle \).
In this limit, after both the DM particles have decoupled from the thermal equilibrium with the SM particles, which take place at a typical temperature $x = x_{\text{dec}} \approx 25$, the strong interactions of conversion will maintain an equilibrium between $\chi_i$ and $\chi_j$ for a long time until the rate of the conversion cannot compete with the expansion rate of the Universe. Making use of Eq. (4), the evolution of the total density $Y(x) \equiv \sum_{i=1}^{N} Y_i(x)$ can be written as

$$\frac{dY}{dx} = -\frac{\lambda}{x^2} \langle \sigma_{\text{eff}} v \rangle \left( Y^2 - Y_{\text{eq}}^2 \right),$$  

(5)

where $\langle \sigma_{\text{eff}} v \rangle$ is the effective thermally averaged product of DM annihilation cross section and the relative velocity which can be written as

$$\langle \sigma_{\text{eff}} v \rangle = \sum_{i=1}^{N} w_i g_i^2 (1 + \varepsilon_i)^3 \exp(-2\varepsilon_i x) \langle \sigma_1 v \rangle,$$  

(6)

where $w_i \equiv \langle \sigma_i v \rangle / \langle \sigma_1 v \rangle$ is the annihilation cross section relative to that of the lightest one. The total equilibrium number density can be written as

$$Y_{\text{eq}} \equiv \sum_{i=1}^{N} Y_{\text{eq}}(x) \approx g_{\text{eff}} \left( \frac{m_1 T}{2\pi} \right)^{3/2} \exp(-x),$$  

(7)

with effective degrees of freedom $g_{\text{eff}} = \sum_i g_i (1 + \varepsilon_i)^{3/2} \exp(-\varepsilon_i x)$. Note that the conversion terms do not show up explicitly in Eq. (5). Through the conversion processes $\chi_i \chi_i \rightarrow \chi_j \chi_j$ the slightly heavier components will be converted into the lighter ones, because the factor $r_{ij}(x)$ is proportional to $\exp[-(m_i - m_j)/T]$ which suppresses the density of the heavier components at lower temperature. If the conversion cross section is large enough, most of the DM components will be converted into the lightest $\chi_1$ before the interaction of conversion decouples, which may result in a large enhancement of the relic density of $\chi_1$ and leads to a large boost factor.

As an example, let us consider a generic DM model with only two components. For relatively large conversion cross section $u \equiv \langle \sigma_{21} v \rangle / \langle \sigma_1 v \rangle \gtrsim 1$, The effective total cross section is given by

$$\langle \sigma_{\text{eff}} v \rangle = \frac{1 + w g^2 \exp(-2\varepsilon x)}{[1 + g \exp(-\varepsilon x)]^2} \langle \sigma_1 v \rangle,$$  

(8)

where $w \equiv w_2$, $g \equiv g_2/g_1$ and $\varepsilon \equiv \varepsilon_2$. Because of the $x$-dependence in $\langle \sigma_{\text{eff}} v \rangle$, the thermal evolution of $Y(x)$ differs significantly from that of the standard WIMP. In the case that $\chi_2$ has large degrees of freedom but a small annihilation cross section, namely $g \gg 1$, $w \ll 1$ and $wg^2 \ll 1$, the thermal evolution of the total density $Y$ can be approximated by

$$\frac{dY}{dx} \approx -\frac{\lambda}{x^2} \frac{1}{[1 + g \exp(-\varepsilon x)]^2} \langle \sigma_1 v \rangle \left( Y^2 - Y_{\text{eq}}^2 \right),$$  

(9)
the thermal evolution of the total number density can be roughly divided into four stages:

i) At high temperature region where $3 \lesssim x \ll x_{\text{dec}}$, both the DM components are in thermal equilibrium with the SM particles. $Y_i(x)$ must closely track $Y_{\text{eq}}(x)$ which decrease exponentially as $x$ increases. However, since $g \gg 1$ and $\epsilon \ll 1$, the number density of $\chi_2$ is much higher than that of $\chi_1$, i.e. $Y_2(x) \gg Y_1(x)$. ii) When the temperature goes down and $x$ is close to the decoupling point $x_{\text{dec}}$, both the DM components start to decouple from the thermal equilibrium. In the region $x_{\text{dec}} \lesssim x \ll 1/\epsilon$, $\langle \sigma_{\text{eff}} v \rangle$ is nearly a constant and $\langle \sigma_{\text{eff}} v \rangle \approx \langle \sigma_1 v \rangle/(1+g)^2 \ll \langle \sigma_1 v \rangle$, the total density $Y(x)$ behaves just like that of an ordinary WIMP which converges quickly to $Y(x) \approx x_{\text{dec}}/(\lambda \langle \sigma_1 v \rangle)$. iii) As $x$ continues growing, the suppression factor $\exp(-\epsilon x)$ in $\langle \sigma_{\text{eff}} v \rangle$ becomes relevant. The value of $\langle \sigma_{\text{eff}} v \rangle$ grows rapidly especially after $x$ reaches the point $\epsilon x \approx \mathcal{O}(1)$, which leads to the further reduction of $Y(x)$. In this stage, although both $\chi_{1,2}$ have decoupled from the thermal equilibrium with the SM particles. The strong conversion interaction $\chi_2 \chi_2 \leftrightarrow \chi_1 \chi_1$ maintains an equilibrium between the two DM components. According to Eq. (4), the relative number density $Y_2(x)/Y_1(x)$ decreases with $x$ increasing, which corresponds to the conversion from the heavier DM component into the lighter one. At the point $xe = (1/\epsilon) \ln g$ one has $Y_2(x) \approx Y_1(x)$. For the region $x > x_{\text{dec}}$ and $x$ is not close to $x_c$, because of $Y_{\text{eq}}(x) \ll Y(x)$ and $g \exp(-\epsilon x) \gg 1$, the Eq. (4) can be analytically integrated out, using the expression $I(x) = \int x^{-2} \exp(x) dx = Ei(x) - \exp(x)/x$ where $Ei(x)$ is the exponential integral function. The integral has an asymptotic form of $I(x) \approx \exp(x)/x^2$ for $x \gg 1$. Thus $Y(x)$ in this region can be approximated by

$$Y(x) \approx \frac{g^2 x_{\text{dec}}}{\lambda \langle \sigma_1 v \rangle} \left[ 1 + \left( \frac{x_{\text{dec}}}{x_c} \right) \frac{\exp(2\epsilon x)}{2\epsilon x} \right]^{-1}. \quad (10)$$

iv) When $x$ becomes very large $\epsilon x \gg \mathcal{O}(1)$, $\langle \sigma_{\text{eff}} v \rangle$ quickly approaches $\langle \sigma_1 v \rangle$, and becomes independent of $x$ again. The evolution of $Y(x)$ in this region can be obtained by a simple integration as it was done in the stage ii). The solution of $Y(x)$ shows a second decoupling. Finally when the conversion rate cannot compete with the expansion rate of the Universe at some point $x_{\text{F}}$ corresponding to $sY_2(\sigma_{21}v)/H \approx 1$, both $Y_1(x)$ and $Y_2(x)$ remain unchanged as relics. The whole DM can be dominated by $\chi_1$ if the conversion is efficient enough.

By matching the analytic solutions of $Y(x)$ in different regions near the points $x_{\text{dec}}$ and $x_c$, and requiring that the final total relic density is equivalent to the observed $\Omega_{CDM}h^2 \approx 0.11$, we obtain the following approximate expression of the boost factor

$$B \approx g^2 \left[ 1 + \left( \frac{x_{\text{dec}}}{x_c} \right) \left( \frac{\exp(2\epsilon x_c)}{2\epsilon x_c} + g^2 \right) \right]^{-1}. \quad (11)$$

As expected, the enhancement essentially comes from the conversion of the degrees of freedom. Thus the maximum enhancement is $g^2$. The two terms in the r.h.s of the above
equation correspond to the reduction of $Y(x)$ during the late time conversion stages. For large enough $g$, the boost factor can be approximated by $B \approx g^2/(1 + \varepsilon g^2 x_{\text{dec}}/\ln g)$. In order to have a large boost factor, a small $\varepsilon \ll \ln g/(g^2 x_{\text{dec}})$ is also required. As shown in Eq. (11) the boost factor is not sensitive to the exact values of the cross sections as long as the conditions $w \ll 1$ and $u \gg 1$ are satisfied.

We numerically calculate the thermal evolution of $Y_i(x)$ and the boost factor without using approximations for a generic two-component DM model. The results for $w = 10^{-4}$, $u = 10$ and $\varepsilon = 2 \times 10^{-4}$ is shown in Fig. 1. The value of $\langle \sigma v \rangle$ is adjusted such that the final total DM relic abundance is always equal to the observed value $\Omega_{CDM}h^2$. The mass of the light DM particle is set to $m_1 = 1$ TeV. For an illustration the ratio between the internal degrees of freedom is set to be large $g = 60$. From the figure, the four stages of the thermal evolution of $Y(x)$ as well as the crossing point can be clearly seen. The crossing point at $x = x_c \approx 2 \times 10^{-4}$ indicates the time when the number density of $\chi_1$ start to surpass that of $\chi_2$ and eventually dominant the whole DM relic density. In this parameter set a large boost factor $B \approx \langle \sigma_1 v \rangle/\langle \sigma v \rangle F \approx 585$ is obtained which is in a remarkable agreement with Eq. (11) with error less than $\sim 5\%$. For a comparison, in Fig. 1 we also show the cases without conversions.

In Fig. 2 (left), we show how the boost factor $B$ varies with the mass difference $\varepsilon$ for different relative internal degrees of freedom $g$. In general, $B$ becomes larger for smaller $\varepsilon$ and larger $g$. For $\varepsilon = 10^{-4}$ and $g = 60$, the boost factor can reach $B \sim 10^3$. For a much smaller $g = 20$ and a larger $\varepsilon = 8 \times 10^{-4}$, the boost factor can still reach $O(100)$. The dependence of $B$ on the cross sections $u$ and $w$ is shown in Fig. 2 (right). A small $w$ and large $u$ lead to the increasing of $B$. However, for very small $w \lesssim 10^{-4}$ and vary large $u \gtrsim 100$, the value of $B$ becomes insensitive to the exact values of $w$ and $u$, which is also in agreement with the approximate solution given in Eq. (11).

### 3 A simple model with DM conversion

For models with multiple DM components, it is nature that there exists interactions among the DM components which may lead to the conversions among them. In this work we consider a simple interacting two-component DM model by adding to the standard model (SM) with two SM gauge singlet fermionic DM particles $\chi_{1,2}$. The particles $\chi_{1,2}$ are charged under a local $U(1)$ symmetry which is broken spontaneously by the vacuum expectation value (VEV) of a scalar field $\phi$ through the Higgs mechanism. The corresponding massive gauge boson is denoted by $A$ which may cause the interaction $\bar{\chi}_2 \chi_2 \leftrightarrow \bar{\chi}_1 \chi_1$. The stability of $\chi_{1,2}$ is protected by two different global $U(1)$ number symmetries. An SM gauge singlet pseudo-scalar $\eta$ is introduced as a messenger field
which couples to both the dark sector and the SM sector. In order to have the leptophilic nature of DM annihilation, we also introduce an SM $SU(2)_L$ triplet field $\Delta$ with the SM quantum number $(1,3,1)$ and flavor contents $\Delta = (\delta^{++}, \delta^+, \delta^0)$. The triplet carries the quantum number $B−L=2$ such that it can couple to the SM left-handed leptons $\ell_L$ through Yukawa interactions $\bar{\ell}_L \Delta \ell_L$, but cannot couple to quarks directly. The VEV of the triplet has to be very small around eV scale, which is required by the smallness of the neutrino masses. As a consequence, the couplings between one triplet and two SM gauge bosons such as $\delta^{\pm}\pm W^\mp W^\mp$, $\delta^{\pm}W^\mp Z^0$ and $\delta^0 Z^0 Z^0$ are strongly suppressed as they are all proportional to the VEV of the triplet, which makes it difficult for the triplet to decay even indirectly into quarks through SM gauge bosons [36,37]. If $\eta$ has a stronger coupling to $\Delta$ than that to the SM Higgs boson $H$ and $\phi$ then the annihilation products of the dark matter particles $\chi_{1,2}$ will be mostly leptons.

The full Lagrangian of the model can be written as $\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_1$. The new interactions in $\mathcal{L}_1$ which are relevant to the DM annihilation and conversion are given
by

$$\mathcal{L}_1 \supset \bar{\chi}_i (i \not{\partial} - m_i) \chi_i + (D_\mu \phi)^\dagger (D^\mu \phi) - m_\phi^2 \phi^\dagger \phi$$

$$+ \frac{1}{2} \partial_\mu \eta \partial^\mu \eta - \frac{1}{2} m_\eta^2 \eta^2 - y_i \bar{\chi}_i \gamma_5 \eta \chi_i - y_\ell \bar{\ell}_i \Delta \ell_L + h.c$$

$$- (\mu \eta + \xi \eta^2) \left[ \text{Tr}(\Delta^\dagger \Delta) + \kappa (H^\dagger H) + \zeta (\phi^\dagger \phi) \right], \quad (i = 1, 2)$$

(12)

with $D_\mu = \partial_\mu + ig_A A_\mu$ and $g_A$ standing for the gauge coupling constant. Note that $\phi$ and $\eta$ do not directly couple to the SM fermions. After the spontaneous symmetry breaking in $V(\phi)$, the scalar $\phi$ obtains a nonzero VEV $\langle \phi \rangle = v_\phi/\sqrt{2}$ which generates the mass of the gauge boson $m_A = g_A v_\phi$. At the tree level, the three components of the triplet $\delta^{++}, \delta^+,$ and $\delta^0$ are degenerate in mass, i.e. $m_{\delta^{++}} = m_{\delta^+} = m_{\delta^+} \equiv m_\Delta$.

After the spontaneous symmetry breaking in the scalar sectors, the fields $\Delta, H$ and $\phi$ obtain nonzero VEVs, which also generates a linear term in $\eta$ through the last term of Eq. (12). The linear term in $\eta$ in turn leads to a nonzero VEV of $\eta$, i.e., $\langle \eta \rangle = v_\eta \neq 0$, which will give corrections to the masses of $\chi_i$ and may enlarge the mass difference between $\chi_1$ and $\chi_2$. This problem can be avoided by using the above mentioned assumption that $\eta$ has a much stronger coupling to $\Delta$ than that to $H$ and $\phi$, which requires that $\kappa, \zeta \ll 1$.  

Figure 2: Left) boost factor $B$ as a function of the relative mass difference $\varepsilon$ for different relative degrees of freedom $g=80$ (solid), 60 (dashed), 40 (dotted) and 20 (dot-dashed) respectively, for $w = 10^{-4}$ and $u = 10^2$; Right) boost factor as a function of the relative conversion cross section $u$. Four curves correspond to $w = 10^{-5}$ (solid), $10^{-4}$ (dashed), $5 \times 10^{-4}$ (dotted) and $10^{-3}$ (dot-dashed) respectively, for parameters $g = 60, m_1 = 1$ TeV and $\varepsilon = 1 \times 10^{-4}$ respectively.
The VEV of $\eta$ is proportional to the ratio between the linear and quadratic terms in $\eta$, and can be estimated as $v_\eta \approx -\mu(\kappa v_H^2 + \zeta v_\phi^2)/(2(m_\eta^2 + \kappa v_H^2 + \zeta v_\phi^2))$. Since the VEV of the triplet $\Delta$ is extremely small and $v_H \approx \mathcal{O}(10^2)$GeV, if $\mu$, $m_\eta$, and $v_\phi$ are all around TeV scale, for $\kappa \lesssim \mathcal{O}(10^{-2})$ and $\zeta \lesssim \mathcal{O}(10^{-4})$ the VEV of $\eta$ is $v_\eta \lesssim \mathcal{O}(10^{-4})$TeV which is small enough to avoid breaking the degeneracy in the masses of $\chi_{1,2}$.

We assume that $\chi_2$ has large internal degrees of freedom relative to that of $\chi_1$, i.e., $g_2 \gg g_1$, which can be realized if $\chi_2$ belongs to a multiplet of the product of some global nonabelian groups. For instance $g_2 = 4\tilde{g}_2$ with $\tilde{g}_2 = 16$, 8, and 4 if it belongs to the spinor representation of a single group of $SO(8)$, $SO(6)$ and $SO(4)$ respectively. When $\chi_2$ belongs to a representation of the product of these groups, its internal degrees of freedom can be very large.

At the early time when the temperature of the Universe is high enough, the triplet $\Delta$ can be kept in thermal equilibrium with SM particles through the SM gauge interactions. The DM particles $\chi_i$ can reach thermal equilibrium by annihilating into the triplet through the intermediate particle $\eta$. The annihilation $\bar{\chi}_2\chi_2 \rightarrow \eta^* \rightarrow \delta^{\pm\pm}\delta^{\mp\mp}, \delta^{\pm\mp}\delta^{\mp\pm}$ is an s-wave process which is dominant contribution. The cross section before averaging over the relative velocity $v$ is given by

$$\sigma_1v = \frac{N_f y_f^2 \mu^2}{16\pi g_i(s - m_\eta^2)^2} \sqrt{1 - \frac{4m_\Delta^2}{s}},$$

(13)

where $N_f = 3$ is the number of final states, $m_\eta$ is the mass of $\eta$ and $s$ is the square of the total energy in the center of mass frame. For s-wave annihilation we use the approximation that the thermally averaged cross section is the same as the one before the average, i.e., $\langle \sigma v \rangle \simeq \sigma v$. From the above equation the ratio of the two annihilation cross sections is $w = (y_2/y_1)^2(g_1/g_2)$. It is easy to get a very small $w$ provided that $y_2 \ll y_1$ and $g_1 \ll g_2$. In order to have a large enough $\langle \sigma_1 v \rangle \gg \langle \sigma v \rangle_F$ the product of the coupling constants $y_1\mu$ must be large enough, or the squared mass of $\eta$ is close to $s$.

The cross section of the conversion process $\bar{\chi}_2\chi_2 \rightarrow A^* \rightarrow \bar{\chi}_1\chi_1$ is given by

$$\sigma_{12}v = \frac{3g_1^2 m_1^2}{2\pi(s - m_\Delta^2)^2} \left( \frac{g_1}{g_2} \right) \sqrt{1 - \frac{4m_\Delta^2}{s}},$$

(14)

The cross section is suppress by $g_1/g_2$ and also the phase space factor $\sqrt{1 - 4m_1^2/s}$ when $s$ is close to $4m_2^2$ at the very late time of the thermal evolution. However, the cross section be greatly enhanced if $m_A$ is close to a resonance when the relation $s \simeq m_A^2$ is satisfied. In the numerical calculations, we find that for the following selected parameters: $m_1 = 1$TeV, $\epsilon = 1 \times 10^{-4}$, $g_1 = 1$, $g_2 = 60$, $m_\Delta = 500$ GeV, $m_\eta = 1.5$ TeV, $m_A = 2.02$ TeV, $y_1 = 3$, $y_2 = 0.07$, $\mu/m_1 = 3$, and $g_A = 2.5$, the following ratio of the cross section can be obtained

$$w \simeq 1 \times 10^{-5}, \quad u \simeq 0.5, \quad \text{and} \quad \langle \sigma_1 v \rangle / \langle \sigma v \rangle_F \simeq 500.$$
In this parameter set the relative mass difference between $m_A$ and $2m_2$ is around 1%. From Fig. 2 one can see that the corresponding boost factor is $B \sim 500$, which is large enough to account for the PAMELA data for the dark matter mass around TeV.

4 Discussions and Conclusions

The mechanism proposed here does not require velocity-dependent annihilation cross sections which is essential to the Sommerfeld enhancement. There exists stringent constraints from astrophysical observations if the DM annihilation cross section scales with velocity as $1/v$ or $1/v^2$ and saturates at very low velocity. Those constraints involves the bound on the $\mu$-type distortion of CMB spectrum \cite{38, 40} and the bounds on diffuse gamma-rays from the cold structures which have lower velocity dispersion than that in the solar neighborhood in which $v \sim 10^{-3}$. For instance, in the subhalos the average velocity can be as low as $v \sim 10^{-5}$ \cite{41}, and the DM velocity in the protohalos can be even lower $v \sim 10^{-8}$ \cite{42}. If the enhancement is insensitive to the velocity, those astrophysical bounds can be relaxed significantly. Furthermore, unlike the Sommerfeld enhancement, no attractive long-range force between the DM particles is involved. The existence of such a long-range force can change the halo shape and is constrained by observations \cite{43, 44}. The boost factor from DM conversion is free from this type of constraint as well.

In summary, We have considered an alternative mechanism for obtaining boost factors from DM conversions which does not require the velocity-dependent annihilation cross section or the decay of unstable particles. We have shown that if the whole DM is composed of multiple components, the relic density of each DM component may not necessarily be inversely proportional to its own annihilation cross section. We demonstrate the possibility that the number density of the lightest DM component with an annihilation cross section much larger than $\langle \sigma v \rangle_F$ can get enhanced in late time through DM conversion processes, and finally dominates the whole relic abundance, which corresponds to a boost factor needed to explain the excesses in cosmic-ray positron and electrons reported by the recent experiments.

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