Flavor Asymmetry of the Sea Quarks in the Baryon Octet

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Abstract

We show that the chiral $SU(n) \otimes SU(n)$ flavor symmetry on the null-plane severely restricts the sea quarks in the baryon octet. It predicts large asymmetry for the light sea quarks ($u, d, s$), and universality and abundance for the heavy sea quarks. Further it is shown that existence of the heavy sea quarks constrained by the same symmetry reduces the theoretical value of the Ellis-Jaffe sum rule substantially.

1 Introduction

Many years ago, based on the current anticommutation relation on the null-plane [1], the Gottfried sum rule [2] was re-derived. Since the re-derived sum rule had a slightly different physical meaning from the original one, I called it as the modified Gottfried sum rule [3]. Several years ago, this sum rule was found to take the following form [4, 5]:

\[
\int_0^1 \frac{dx}{x} \left\{ F_2^{ep}(x, Q^2) - F_2^{en}(x, Q^2) \right\} = \frac{1}{3} \left( 1 - \frac{4 f_K^2}{\pi} \int_{m_K m_N}^{\infty} \frac{dw}{w} \sqrt{w^2 - (m_K m_N)^2} \left\{ \sigma^{K^+N}(\nu) - \sigma^{K^+p}(\nu) \right\} \right), \tag{1}
\]

where $\sigma^{K^+N}(\nu)$ with $N = p$ or $n$ is the total cross section of the $K^+N$ scatterings and $f_K$ is the kaon decay constant. This gave us a new way to investigate
the vacuum properties of the hadron based on the chiral $SU(n) \otimes SU(n)$ flavor symmetry on the null-plane. In this paper I explain the fact and show that it severely restricts the sea quarks in the $8$ baryon. Before going into details let us first explain some backgrounds about the sum rule (1).

The classical moment sum rules based on the operator product expansion (OPE) were derived only at even integers or at odd integers [6], where the integer $n$ was defined, for example, for the structure function $F_2$ as

$$M(n) = \int_0^1 dx x^{n-2} F_2(x,Q^2). \quad (2)$$

Then we call the integers where the classical moment sum rules do not exist as the missing integers. For $F_2$ in the electroproduction, these missing integers are odd ones. The reason why we lose the moments at missing integers is clear. In the classical derivation, we first do the short-distance expansion. Then we analytically continue it to the light-cone one with use of the dispersion relation and expresses it by the structure function in the $s$ channel. In these processes we need some relation for the structure function originating from the crossing symmetry. We usually take this as the one defined by the current commutation relation because we can use the causality condition directly in this case. Now this relation depends crucially on the way how we define the quantity in the $u$ channel. Hence by the proper consideration of this quantity, it may be possible to get the sum rules at the missing integers. In the perturbative QCD, the situation was resolved by using the analytical continuation with respect to $n$ in the anomalous dimension up to the two loops [4], and as a by-product the Gottfried sum rule was revised as

$$\int_0^1 \frac{dx}{x} \{ F_2^{ep}(x,Q^2) - F_2^{en}(x,Q^2) \} \approx \frac{1}{3},$$

at some low $Q_0^2$, we get

$$\int_0^1 \frac{dx}{x} \{ F_2^{ep}(x,Q^2) - F_2^{en}(x,Q^2) \} \sim \frac{1}{3}. \quad (5)$$
for arbitrary $Q^2$, since the $Q^2$ dependence in Eq.(3) is negligibly small compared with $1/3$. This method of the analytical continuation in $n$ was confirmed directly also in Ref.[8] in the perturbative approach up to the next to leading order. Now the integer $n$ was known to be the $O(4)$ spin, and the moment $M(n)$ should be replaced by the Nachtmann moments [9] which are essentially the $O(4)$ partial waves apart from the trivial kinematical factors. Hence the method of the analytical continuation in $n$ could be checked from the $O(4)$ partial wave expansion in the general context, [11, 12], i.e., independent of the OPE. In this method we first defined the signedatured moments as in the case of the Froissart-Gribov projection in the $O(3)$ partial wave expansion. Then we found that the $O(4)$ partial waves at the wrong signature points exactly corresponded to the Nachtmann moments at the missing integers. This stemmed from the fact that the scattering amplitude was defined by the retarded product whose imaginary part was the commutation relation. Now what we really needed in the inclusive reaction was information of the product of the current. Thus we constructed the amplitude whose imaginary part was the anticommutation relation. By applying the $O(4)$ partial wave expansion to this quantity, we found that the missing integers in the classical derivation corresponded to the right signature points of this expansion. The kinematical form of the moment at $n = 1$ for the structure function $F_2$ obtained in this way in the electroproduction was exactly the one obtained by the analytical continuation of the method in Ref.[7]. However the sign from the crossing symmetry for the structure function was not the usual one defined by the current commutation relation but the one defined by the current anticommutation relation [11]. It is this relation which is necessary to derive the sum rule (1). A physical meaning of the difference will be explained more in the next section. Since a review of the derivation of the current anticommutation relation on the null-plane with consideration for causality and the spectral condition for hadrons is described in detail in Ref.[5], we give here the result

\[
\langle p|\{J_a^{5+}(x), J_b^{5+}(0)\}|p\rangle_{c|x^+=0} = \langle p|\{J_a^{5+}(x), J_b^{5+}(0)\}|p\rangle_{c|x^+=0} \\
= \frac{1}{\pi} P\left(\frac{1}{x^-}\right) \delta^2(\vec{x}^\perp) [d_{abc} A_c(p \cdot x, x^2 = 0) + f_{abc} S_c(p \cdot x, x^2 = 0)] p^+, \quad (6)
\]

where $c$ means to take the connected matrix element and the state $|p\rangle$ is the stable one particle hadron state. Compared with the current commutation relation on the null-plane given as

\[
[J_a(x), J_b(0)]|_{x^+=0} = [J_a^{5}(x), J_b^{5}(0)]|_{x^+=0} = i f_{abc} \delta(x^-) \delta^2(\vec{x}^\perp) J_c^{5+}(0), \quad (7)
\]
the relations (6) are restricted greatly because they are not operator relations. In spite of this limitation we can get many information from them. Let us turn to discuss the difference between Eq.(1) and Eq.(3). The both-hand sides of the sum rule (1) are related to the same quantity

\[ \frac{1}{3\pi} P \int_{-\infty}^{\infty} \frac{d\alpha}{\alpha} A_3(\alpha, 0), \]  

where \( A_0(\alpha, 0) \) on the null-plane \( x^+ = 0 \) can be considered as the quantity defined by

\[
\langle p | \frac{1}{2i} \left[ \bar{q}(x) \gamma^\mu \frac{1}{2} \lambda_a q(0) - \bar{q}(0) \gamma^\mu \frac{1}{2} \lambda_a q(x) \right] | p \rangle_c = p^\mu A_a(px, x^2) + x^\mu \bar{A}_a(px, x^2),
\]  

as far as the moment of the structure function \( F_2 \) at \( n = 1 \) is concerned. We need caution in this identification. The bilocal current on the left-hand side of Eq.(9) is not the regular one in the following sense. Each coefficient of the expansion of the bilocal current on the null-plane into the local operator gets the singular piece due to the anomalous dimension. The \( Q^2 \) dependence given in Eq.(3) corresponding to the moment at \( n = 1 \) is one example which originates from this fact. However this \( Q^2 \) dependent piece was negligibly small. The experimental value of the Gottfried sum \([12]\) substantially violated the originally predicted value \( 1/3 \) \([2]\) and by this negligibly small value it became impossible to explain the defect. On the other hand, the modified Gottfried sum rule explained the defect \([4]\). Thus the term in the expansion of the bilocal current corresponding to the moment at \( n = 1 \) should be the one predicted by our method. In other words, we can consider that the perturbatively predicted \( Q^2 \) dependence is shielded by the large non-perturbative effects, or more practically, we can regard it negligible compared with the non-perturbative contribution as far as the moment at \( n = 1 \) is concerned. Thus our method is complementary to the perturbative QCD which is believed to be valid for the moments above \( n = 2 \) except the large \( n \) region. Finally, it should be noted that the physical origin of the deviation from \( 1/3 \) was the same as that of the axial-vector coupling constant from 1 in the sense that both the deviations are proportional to the square of the pseudo-scalar decay constants.
2 The relation to the quark distribution functions

Here we explain the role of the integral of the type \( P \int_{-\infty}^{\infty} \frac{d\alpha}{\alpha} \ldots \) in Eq.(8). At \( x^+ = 0 \), Eq.(8) corresponds to

\[
\frac{1}{3\pi p^+} P \int_{-\infty}^{\infty} \frac{d\alpha}{\alpha} \langle p|\frac{1}{2i}[\bar{q}(x)\gamma^+\frac{1}{2}\lambda_3 q(0) - \bar{q}(0)\gamma^+\frac{1}{2}\lambda_3 q(x)]:|p\rangle_c. \tag{10}
\]

We expand the quark field at \( x^+ = 0 \) as

\[
q(x) = \sum_n a_n \phi_n^+(x) + \sum_n b_n^\dagger \phi_n^-(x), \tag{11}
\]

where the sum over the subscript \( n \) means the spin sum and the momentum integral collectively. Here the \((+)\) means the positive energy solution and \((-)\) the negative one. The normal ordered product is

\[
: \bar{q}(x)\gamma^+\frac{1}{2}\lambda_3 q(0) := \sum_{n,m} a_n^\dagger a_m \bar{\phi}_n^+(x)\gamma^+\frac{1}{2}\lambda_3 \phi_m^+(0) \\
- \sum_{n,m} b_m^\dagger b_n \bar{\phi}_n^-(x)\gamma^+\frac{1}{2}\lambda_3 \phi_m^-(0). \tag{12}
\]

The first term of the right-hand side of Eq.(12) contributes to the quark distribution function and the second one to the antiquark distribution functions. Let us now define a part of the quark distribution as

\[
f(x) = \frac{1}{2\pi p^+} \int_{-\infty}^{\infty} d\alpha \exp[-i \alpha] \langle p| \sum_{n,m} a_n^\dagger a_m \bar{\phi}_n^+(y^-)\gamma^+\frac{1}{2}\lambda_3 \phi_m^+(0)|p\rangle_c. \tag{13}
\]

with \( \alpha = p^+y^- \). Similarly the antiquark one is

\[
g(x) = -\frac{1}{2\pi p^+} \int_{-\infty}^{\infty} d\alpha \exp[-i \alpha] \langle p| \sum_{n,m} b_m^\dagger b_n \bar{\phi}_m^-(y^-)\gamma^+\frac{1}{2}\lambda_3 \phi_n^-(0)|p\rangle_c. \tag{14}
\]

Then using the integral representation of the sign function \( \epsilon(x) = \frac{1}{i\pi} P \int_{-\infty}^{\infty} \frac{da}{a} \exp[iax] \), we get

\[
\frac{1}{i\pi p^+} P \int_{-\infty}^{\infty} \frac{d\alpha}{\alpha} \langle p| \sum_{n,m} a_n^\dagger a_m \bar{\phi}_n^+(y^-)\gamma^+\frac{1}{2}\lambda_3 \phi_m^+(0)|p\rangle_c \\
= \int_0^1 dx \epsilon(x) f(x) = \int_0^1 dx f(x), \tag{15}
\]
while

$$\frac{1}{i\pi p^+} P \int_{-\infty}^{\infty} \frac{d\alpha}{\alpha} (p|\sum_{n,m} b^\dagger_n b_m \phi_m^-(y^-) (y^-) \gamma^\pm \frac{\lambda_3}{2} \phi_m^-(0) |p) \rangle_c$$

$$= -\int_0^1 dx \epsilon(-x) g(x) = \int_0^1 dx g(x),$$

(16)

Thus we see that the antiquark term gets extra minus sign due to the integral of the type $P \int_{-\infty}^{\infty} \frac{d\alpha}{\alpha} \cdots$. This explains a physical difference between the modified Gottfried sum rule and the Adler sum rule. In the parton model we know

$$\frac{3}{2} \int_0^1 \frac{dx}{x} \{ F_2^{\text{ep}}(x, Q^2) - F_2^{\text{en}}(x, Q^2) \}$$

$$= [f_0^1 dx \{ \frac{1}{2} \lambda_u - \frac{1}{2} \lambda_d \} + f_0^1 dx \{ \frac{1}{2} \lambda_d - \frac{1}{2} \lambda_u \}] - [f_0^1 dx \{ -\frac{1}{2} \lambda_u + \frac{1}{2} \lambda_d \}],$$

(17)

and

$$\frac{1}{4} \int_0^1 \frac{dx}{x} \{ F_2^{\text{ep}}(x, Q^2) - F_2^{\text{en}}(x, Q^2) \}$$

$$= [f_0^1 dx \{ \frac{1}{2} \nu_v - \frac{1}{2} d_v \} + f_0^1 dx \{ \frac{1}{2} d_v - \frac{1}{2} \nu_v \}] + [f_0^1 dx \{ -\frac{1}{2} \lambda_u + \frac{1}{2} \lambda_d \}],$$

(18)

where the subscript $v$ means the valence quark and $\lambda_i$ means the $i$ type sea quark. The difference between the two sum rules is the sign in front of the antiquark distribution. The current anticommutation relation on the null-plane which leads to the modified Gottfried sum rule is proportional to $P \frac{1}{x}$. This factor is the origin of the integral of the type $P \int_{-\infty}^{\infty} \frac{d\alpha}{\alpha} \cdots$. While the current commutation relation on the null-plane which leads to the Adler sum rule is proportional to $\delta(x^-)$. Thus, under the assumption $\int_0^1 dx \lambda(x, Q^2) = f_0^1 dx \lambda_i(x, Q^2)$, the Adler sum rule measures the mean $I_3$ of the {quark} $+$ [antiquark] and hence the one of the valence quarks being equal to the $I_3$ of the proton, while the modified Gottfried sum rule measures the mean $I_3$ of the {quark} $-$ [antiquark] in the proton. In conclusion we can say that the modified Gottfried sum rule naturally explains the physical difference between the Adler sum rule and the Gottfried sum and that it directly probes the vacuum of the proton. This fact is the fundamental importance of the sum rules obtained by the current anticommutation relations (6).
3 The symmetry constraint on the light sea quark distributions in the baryon octet

Many years ago, Weinberg showed that at high energy there is a symmetry closely connected with the dynamic symmetry at low energy \[13\]. The pion coupling matrix discussed there is very similar to the matrix element 
\[
< \beta|J_{5+}^a(0)|\alpha >
\]
on the null-plane at \(q^+ = 0\) where \(q = p_\beta - p_\alpha\) and \(\alpha, \beta\) are one-particle hadron states. This is because this matrix element picks up the non-pole term since 
\[
< \beta|J_{5+}^\mu a(0)|\alpha >= \{g^{\mu\nu} - q^{\mu}q^{\nu}/(q^2 - m_\pi^2)\} < \beta|J_{5+}^\mu(0)|\alpha > N
\]
where \(N\) means the non-pole matrix element. Because of this property the light-like charge algebra plays very similar role as the algebra of the pion coupling matrix. It relates the low energy to the high energy and the relation between the two energy regions is controlled by the symmetry. It is this symmetry which we discuss in this paper.

Now \(A_a(\alpha, 0)\) is governed by this symmetry, i.e., the chiral \(SU(3) \otimes SU(3)\) flavor symmetry on the null-plane. If we take the state on the left-hand side of Eq.(9) as the 8 baryon, it becomes
\[
\langle \alpha, p|\frac{1}{2}\tau| \bar{q}(x)\gamma^\mu\frac{1}{2}\lambda_\alpha q(0) - \bar{q}(0)\gamma^\mu\frac{1}{2}\lambda_\alpha q(x) :|\beta, p\rangle_c
\]
\[
= p^\mu(A_a(px, x^2))_{\alpha\beta} + x^\mu(\bar{A}_a(px, x^2))_{\alpha\beta}, \quad (19)
\]
where \(\alpha, \beta\) are the symmetry index specifying each member of the 8 baryon. Since the matrix element can be classified by the flavor singlet in the product 
\[
8 \otimes 8 \otimes 8, \quad (A_a(\alpha, 0))_{\alpha\beta}\]
decomposed as 
\[
(A_a(\alpha, 0))_{\alpha\beta} = i f_{\alpha\beta\alpha} F(\alpha, 0) + d_{a\alpha\beta} D(\alpha, 0) \quad (20)
\]
for \(a \neq 0\). Using the value of the modified Gottfried sum rule estimated in Ref.\[4\], we obtain
\[
\frac{1}{3\pi} P \int_{-\infty}^\infty \frac{d\alpha}{\alpha} A_8^\alpha(\alpha, 0) = \frac{1}{3\pi} P \int_{-\infty}^\infty \frac{d\alpha}{\alpha}\{F(\alpha, 0) + D(\alpha, 0)\}
\]
\[
= 0.26. \quad (21)
\]
The mean hypercharge sum rule in Ref.\[14\] gives us
\[
\frac{\sqrt{3}}{3\pi} P \int_{-\infty}^\infty \frac{d\alpha}{\alpha} A_8^\alpha(\alpha, 0) = \frac{\sqrt{3}}{3\pi} P \int_{-\infty}^\infty \frac{d\alpha}{\alpha}\sqrt{3}\{F(\alpha, 0) - \frac{1}{3}D(\alpha, 0)\}
\]
\[
= 2.12. \quad (22)
\]
Here we use the notation $A^B(\alpha, 0)$ with $B = (p, n, \Sigma^\pm, \Lambda^0, \Xi^-, \Xi^0)$ to specify each member of the 8 baryon. From Eqs.(21) and (22), we obtain

$$\tilde{F} \equiv \frac{1}{2\pi} P \int_{-\infty}^{\infty} \frac{d\alpha}{\alpha} F(\alpha, 0) = 0.89,$$

(23)

$$\tilde{D} \equiv \frac{1}{2\pi} P \int_{-\infty}^{\infty} \frac{d\alpha}{\alpha} D(\alpha, 0) = -0.50.$$  

(24)

These are the constraints by the chiral flavor symmetry and the discussion can be extended to $SU(4) \otimes SU(4)$ or still higher symmetry with some restrictions explained later.

Let us continue to discuss the $SU(3)$ case. With use of $\tilde{F}$ and $\tilde{D}$ given in Eqs.(23) and (24), it is possible to get the constraints on the sea quarks in each baryon by repeating the same kind of the discussions as those for the proton [5]. We first regularize the sum rule by using analytical continuation from the non-forward direction. This regularization is based on the method in Ref.[15] and its application to our case was explained in detail in Ref.[3].

The important point in this method lies in the fact that the sum rule is convergent under the physically reasonable assumption, i.e., the trajectory of the pomeron satisfies $\alpha_P(t) < 1$ for some small $t$ where $t$ is the momentum transfer. Thus once we can identify the parts which become divergent as $t$ goes to zero, we can safely subtract them from both-hand sides of the sum rule. The soft pomeron by Donnachi and Landshoff [16] is one example which makes it possible to carry out the program easily. Now the assumption $\alpha_P(t) < 1$ for some small $t$ can not be satisfied by the hard pomeron based on the fixed-coupling constant [17]. However, there are great efforts to improve the defect of this pomeron [18]. The next-to-leading corrections seems to suggest a substantial reduction of the value of the intercept [19]. The multiple scatterings of the pomeron gives us important unitary corrections at low $x$ [20]. Thus even in such a perturbative approach there is a hope to satisfy the assumption. We use the soft pomeron to explain the regularization, but in view of the situation, we clarify the quantities which do not depend on the assumed high energy behavior in the following. Now the discussion of the regularization in the non-forward direction is cumbersome kinematically, and the technical aspect of the method can be explained by the effective method in the forward direction, hence we recapitulate it here. However, It should be noted that, corresponding to the finite sum rules in this effective method, there always exist truly convergent sum rules in the above sense. The sum
rule for $F_2^{ep}$ in $SU(3)$ is
\[
\int_0^1 \frac{dx}{x} F_2^{ep} = \frac{1}{18\pi} P \int_{-\infty}^\infty \frac{d\alpha}{\alpha} \{2\sqrt{6}A_0(\alpha, 0) + 3A_3^p(\alpha, 0) + \sqrt{3}A_8^p(\alpha, 0)\}. \tag{25}
\]

We take the leading high energy behavior of $F_2^{ep}$ is given by the pomeron as $(\frac{1}{Q^2})^{\alpha_P(0)-1}\beta_{ep}(Q^2, 1 - \alpha_P(0))(2\nu)^{\alpha_P(0)-1}$, and assume it to be the flavor singlet, where $\alpha_P(0)$ is the intercept of the pomeron. Note that what we assume here is only the high energy behavior $(2\nu)^{\alpha_P(0)-1}$ and no assumption is made about the $Q^2$ dependence, since all the unknown $Q^2$ dependence is absorbed in $\beta_{ep}$. This also applies to the scale factor in $2\nu$. Then the regularization of the sum rule goes as follows. We rewrite the left-hand side of Eq.(25) as
\[
\int_0^1 \frac{dx}{x} F_2^{ep} = \int_0^1 \frac{dx}{x} \{F_2^{ep} - \beta_{ep}(Q^2, 1 - \alpha_P(0))x^{1-\alpha_P(0)}\}
+ \int_0^1 dx \beta_{ep}(Q^2, 1 - \alpha_P(0))x^{-\alpha_P(0)}, \tag{26}
\]
and, since the pomeron term is assumed to be flavor singlet, we rewrite the right-hand side of it as
\[
\frac{\sqrt{6}}{9\pi} P \int_{-\infty}^\infty \frac{d\alpha}{\alpha} \{A_0(\alpha, 0) - f(\alpha)\} = \frac{\sqrt{6}}{9\pi} P \int_{-\infty}^\infty \frac{d\alpha}{\alpha} f(\alpha). \tag{27}
\]
By setting $\alpha_P(0) = 1 + b - \epsilon$, we expand $\beta_{ep}$ as $\beta_{ep}^0(Q^2) - (\epsilon - b)\beta_{ep}^1(Q^2) + O((\epsilon - b)^2)$. The pole term as $\epsilon \to b$ should be canceled out from both-hand sides of Eq.(25) since the sum rules are convergent for the arbitrary finite positive $(\epsilon - b)$ which corresponds to the small negative $t$ in the non-forward case, hence there must exists $f(\alpha)$ such that the quantity $\frac{2\sqrt{6}}{9\pi} f(\alpha) = (\frac{2\sqrt{6}}{9\pi} P \int_{-\infty}^\infty \frac{d\alpha}{\alpha} f(\alpha) - \frac{\beta_{ep}^1}{\epsilon - b})$ becomes finite in the limit $\epsilon \to b$, where $\beta_{ep}^0$ is $Q^2$ independent since Eq.(25) holds at any $Q^2$. After taking out the singular piece we take the limit $\epsilon \to 0$ and obtain
\[
\int_0^1 \frac{dx}{x} \{F_2^{ep} - \beta_{ep}^0x^{-b}\}
= \frac{1}{18\pi} P \int_{-\infty}^\infty \frac{d\alpha}{\alpha} \{2\sqrt{6}S_0(\alpha, 0, Q^2) + 3A_3^p(\alpha, 0) + \sqrt{3}A_8^p(\alpha, 0)\}. \tag{28}
\]

1 Note that the definition of $\beta_{ep}^1$ in Ref.[4] is different from the $\beta_{ep}^1$ in this paper in its sign.
where $S^3_0(\alpha, 0, Q^2)$ and $\tilde{S}^3_0$ are defined as
\[
\frac{2\sqrt{6}}{9} S^3_0 = \frac{2\sqrt{6}}{9} \left[ f(\alpha) + \frac{1}{2\pi} P \int_{-\infty}^\infty \frac{d\alpha}{\alpha} \left\{ A_0(\alpha, 0) - f(\alpha) \right\} \right] + \beta^1_{ep}(Q^2)
\]
\[
= \frac{2\sqrt{6}}{9} \frac{1}{2\pi} P \int_{-\infty}^\infty \frac{d\alpha}{\alpha} S^3_0(\alpha, 0, Q^2).
\]
(29)

Here the superscript 3 in $S^3_0(\alpha, 0, Q^2)$ and $\tilde{S}^3_0$ means the singlet in $SU(3)$. By comparing Eqs.(25) and (28) we see that the regularization of Eq. (25) simply results in the $Q^2$ dependence in the singlet component and that all the relation from the symmetry is inherited. By keeping this fact in mind, we define the sea quark distribution of the $i$ type one in the 8 baryon as $\lambda^i_B$, and regularizes its mean number as
\[
\langle \tilde{\lambda}^i_B \rangle = \int_0^1 dx \{ \lambda^i_B - ax^{-\alpha P(0)} \},
\]
(30)

where $\alpha_P(0)$ here is $\alpha_P(0) = 1 + b$ and is taken as 1.0808, and $a$ is obtained as
\[
\lim_{x \to 0} x^{-\alpha_P(0)} \lambda^i_B = a.
\]
(31)

Note that the superscript $B$ drops out in $a$. This means that the result holds for all members in the 8 baryon. In the following we use the policy to drop the superscript B in the case where the relation holds for all members in the 8 baryon. Further the suffix $i$ is dropped, since the value $a$ is proportional to $\beta^0_{ep}$ and was determined through the sum rule as $a = 1.2$ for $i = u, d, s$ in Ref.[3]. This universality is the result in our formalism obtained from the assumption that the pomeron is flavor singlet. This can be seen by the fact that all the coefficients of the singlet component in the sea quark distributions are the same. Now, in general we can take the high energy behavior more complicated than $\nu^\alpha_P(\nu)^{-1}$. For example in case of $(\ln \nu)^n \nu^\alpha_P(\nu)^{-1}$, we will obtain the expansion of the form $[a_1/(\epsilon - b)^{n+1} + a_2/(\epsilon - b)^n + \cdots + a_n/(\epsilon - b)\}$ in stead of the simple pole $\beta^0_{ep}/(\epsilon - b)$. All these terms belong to the singlet and hence controlled by the symmetry. Thus ambiguities due to high energy behavior can be absorbed into the finite part as $S^3_0(\alpha, 0, Q^2)$ also in this case. Eqs.(30) and (31) should be modified appropriately according to the assumed high energy behavior. Though we lose the explicit value $a = 1.2$ in this case, we still have the relation which relates the coefficient of the leading singularity in $F_2$ and that in the pion nucleon reactions through the sum rule. Now
returning back to the soft pomeron we explain how the constraints on the sea quark distributions are obtained. As an example let us take the proton matrix element in Eq.(19) with \( K = \frac{K_0}{2} = \text{diag}(1 0 0) = \frac{\sqrt{6}}{6} \lambda_0 + \frac{1}{2} \lambda_3 + \frac{\sqrt{2}}{6} \lambda_8 \), where \( \text{diag}(a \ b \ c) \) means the 3×3 matrix whose diagonal element is \( a, b, c \) and all off-diagonal elements are zero. For the proton \( \alpha = \frac{1}{2}(4+i5), \beta = \frac{1}{2}(4-i5) \). Then by taking \( \langle \tilde{\lambda}_B^p \rangle = \langle \tilde{\lambda}_B^p \rangle \), we obtain \( < u^p > + 2 < \tilde{\lambda}_u^p > = \frac{\sqrt{6}}{3} \tilde{S}_3^0 + 2 \tilde{F} + \frac{2}{3} \tilde{D} \).

Since it is straightforward to repeat the same kind of the calculation for other sea quarks, we give the results in Table 1, where we use \( \tilde{S}_3^0 \) defined as \( \tilde{S}_3 = \frac{\sqrt{6}}{3} \tilde{S}_3^0 \). By using the fact that each valence part is merely the number of the valence quark, we get many sum rules from the relations in Table 1. Among them the sum rules for the mean quantum numbers of the light sea quarks are fundamental since they do not depend on \( \tilde{S}_3^0 \). Here the light sea quarks mean the \( u, d, s \) type sea quarks. We summarize them in Table 2. Note that \( \tilde{\lambda}_B^p \) is replaced by \( \lambda_i^p \) because the divergent part is canceled out in each expression through the condition (31). The same kind of the fact holds also in the general case as far as the divergent part is assumed to be flavor singlet. Practically, all the results in Table 2 can be obtained by considering the quantity corresponding to \( \langle I_3 \rangle, \langle Y \rangle \), and \( \langle Q \rangle \) for the light sea quarks directly as in the example in the section 2. In this case the singlet component do not appear explicitly, hence if we assume that the pomeron is flavor singlet and that it contributes universally to every sea quark, we do not encounter the divergence. Though such a practical approach obscures the physical view that at high energy there is a symmetry closely connected with the dynamic symmetry at low energy, it convinces us that the results in Table 2 are insensitive to the regularization. The perturbative QCD corrections to these relations begin from the 2 loops and they enter the same way as the one in the modified Gottfried sum rule. Therefore they are negligibly small compared with the non-perturbative values listed in Table 2.

4 The symmetry constraint on the heavy sea quark distributions in the octet baryon

Here we extend the symmetry from \( SU(3) \otimes SU(3) \) to \( SU(n) \otimes SU(n) \) with \( n \geq 4 \). In general, such symmetry is considered to be badly broken in the Hamiltonian. However, in our case we do not use it explicitly. Our starting

\[ 2 \text{If we set } \tilde{S}_3 = \frac{2}{10} \beta + 1.53, \text{ the results for the proton in Ref.} \]
point is the local current algebra on the null-plane for good-good component. In this case we do not need the equation of motion, hence in this sense all the results in this paper do not depend on the Hamiltonian which badly breaks the symmetry. However we need the assumption concerning the classification of the matrix elements by the symmetry. It is not clear how far such classification holds. We know that the method works well for the badly broken $SU(3)$. Then it may be useful to have physical predictions under this extension. As it is explained later in this section and in the next section the physical results obtained seem to have some experimental relevance. Let us now discuss the heavy sea quarks in the $8$ baryon. For concreteness we take the chiral $SU(4) \otimes SU(4)$ flavor symmetry. In this case the $8$ baryon belongs to $20_M$, and the currents to $15$. The matrix element in this case can be classified by the singlet component in the product $20_M \otimes 20_M \otimes 15$. Since the adjoint representation $15$ appears twice in the product as $20_M \otimes 20_M = 175 \oplus 84 \oplus 45 \oplus 45 \oplus 20 \oplus 15 \oplus 15 \oplus 1$, and since only these two $15$ can make the singlet with the remaining $15$, we have two different terms in the matrix element. Further these two $15$ can be represented by the $4 \times 4$ matrix whose $3 \times 3$ sub-matrix agrees with the $3 \times 3$ matrix in $SU(3)$. Now two different terms have already appeared in Eq.(20) as $F(\alpha, 0)$ and $D(\alpha, 0)$ in $SU(3)$. Hence, even in $SU(4)$, Eq.(20) holds at least for $\alpha, \beta = 1 \sim 8$ and $a = 1 \sim 15$. However, in this generalization, the singlet in $SU(3)$ is not the singlet in $SU(4)$. To see this fact more concretely, we take the matrix $K = diag(1 0 0 0)$, and decomposes it as $K = \frac{\sqrt{2}}{4} \lambda_0^4 + \frac{1}{2} \lambda_3^4 + \frac{\sqrt{6}}{6} \lambda_8^4 + \frac{\sqrt{12}}{12} \lambda_{15}^4$. Here the $\lambda_k^4$ is the Gell-Mann matrix generalized to $SU(4)$. $SU(3)$ singlet part in this decomposition is $\frac{\sqrt{2}}{4} \lambda_0^4 + \frac{\sqrt{6}}{12} \lambda_{15}^4 = \frac{1}{3} diag(1 1 1 0)$. Since the $3 \times 3$ sub-matrix $diag(1 1 1)$ is expressed as $\frac{2}{3} \lambda_0^4$ in $SU(3)$, the coefficient of the singlet part in $SU(3)$ is different from the one in $SU(4)$. On the other hand, $3 \times 3$ sub-matrix in the part $\frac{1}{2} \lambda_3^4 + \frac{\sqrt{6}}{6} \lambda_8^4$ has the same expression in these two cases. Thus we find one relation between the singlet contribution in $SU(3)$ and the one in $SU(4)$. If we denote the $SU(4)$ singlet contribution as $\tilde{S}_0^4$ corresponding to $\tilde{S}_0^3$ in $SU(3)$, we obtain $\frac{\sqrt{2}}{3} \tilde{S}_0^4 = \frac{\sqrt{7}}{2} \tilde{S}_0^3 + \frac{1}{3} \tilde{D}$. Expressed in the parton model, this generalization from $SU(3)$ to $SU(4)$ corresponds to the addition of the charm sea quark with the condition (31) without changing anything in the light sea quarks. After all we find that all the relations in Table 1 and Table 2 hold in $SU(4)$ without changing anything. The discussion here also shows that all the charm sea quark distributions in the $8$ baryon corre-
sponding to the matrix element of $\text{diag}(0\ 0\ 0\ 1)$ are the same. Explicitly in case of the soft pomeron we obtain $2 < \tilde{\lambda}_c > = \tilde{S}^3 - \frac{4}{3} \tilde{D}$ for all members in the $8$ baryon. Since we already have the result that the charm sea quark in the proton is abundant \cite{5, 21}, we reach the conclusion that the charm sea quark is universal and abundant in the $8$ baryon. It should be noted that the gluon fusion like term is in general included in our definition of the charm quark distribution function, but the gluon in our case is not necessarily the perturbative one. Experimentally HERA found \cite{22} abundance of the charm sea quark in the proton which is qualitatively similar to the one investigated in the toy model in Ref.\cite{5} in the following two respects. The one is that abundance of the charm sea quark is correlated with the rapid rise of the structure function $F_2$ in the small $x$ region. The other is that this rise persists even at small $Q^2$, which can naturally be understood by this abundance. Further some unpleasant features of this toy model which come from the constraint obtained by the soft pomeron analysis may be improved if we take into account the perturbative analysis. However, even in such a case the property of abundance of the charm sea quark in the above sense remains.

Now the mean quantum number $< Y >$ and $< Q >$ extended to $SU(4)$ are not so useful since they do not correspond to the traceless matrix. Hence the dependence on $\tilde{S}_0^4$ remains. Rather even in $SU(4)$, the mean quantum numbers of the light sea quarks in Table 2 are useful. The heavy quark effect should be examined by the mean quantum number such as $< \lambda_c - \lambda_s >$, which corresponds to the sum of the mean charm and the mean strangeness.

The same kind of the discussions can be repeated in the $SU(5)$ or $SU(6)$, and we get $2 < \tilde{\lambda}_b > = 2 < \tilde{\lambda}_t > = \tilde{S}^3 - \frac{4}{3} \tilde{D}$ with the constraint (31) for the bottom and the top sea quarks in case of the soft pomeron.

5 Flavor asymmetry of the spin-dependent sea quark distribution

It is interesting to note that similar discussion to extend the symmetry from $SU(3)$ to $SU(4)$ can be applied to the matrix element $< p, s, \alpha | J_5^{\mu}(0) | p, s, \beta >$ which is directly related to the Ellis-Jaffe sum rule\cite{23}. Let us first discuss the $SU(3)$ case. We define

$$\langle p, s, \alpha | J_5^{\mu}(0) | p, s, \beta > = s^\mu A_\alpha^\beta,$$  \hspace{1cm} (32)
where $s^\mu$ is the spin vector, and

$$A^{\alpha\beta}_a = if_{\alpha\beta}F + d_{\alpha\beta}D,$$  \hspace{1cm} (33)

for $a \neq 0$. The Ellis-Jaffe sum for the $8$ baryon is

$$I^B_f = \int_0^1 dx g^B_1(x, Q^2),$$  \hspace{1cm} (34)

where the subscript $f$ specifies the flavor group. $I^B_f$ is proportional to $d_{\alpha\beta}$ and in case of the proton it is well known to take the form

$$I^p_3 = \frac{1}{36}[4\triangle Q^p_0 + 3\triangle Q^p_3 + \triangle Q^p_8],$$  \hspace{1cm} (35)

where $\triangle Q^p_0 = \triangle u^p + \triangle d^p + \triangle s^p$, $\triangle Q^p_3 = \triangle u^p - \triangle d^p$, and $\triangle Q^p_8 = \triangle u^p + \triangle d^p - 2\triangle s^p$, and $\triangle q^p$ is the fraction of the spin of the proton carried by the spin of quarks of flavor $q$. Here $\triangle q^p$ includes the contribution from the antiquark as usual. Since $\triangle Q^p_a$ is proportional to $A^{\alpha\beta}_a$, We can apply Eq.(33) to this quantity. Thus we obtain

$$\triangle u^p = \frac{1}{3}S + \frac{1}{3}D + F, \triangle d^p = \frac{1}{3}S - \frac{2}{3}D, \triangle s^p = \frac{1}{3}S + \frac{1}{3}D - F,$$  \hspace{1cm} (36)

where $\triangle Q^p_0 = S$. It is straightforward to get the spin fraction of the quarks in other baryons. We summarize the result in Table 3. Now in $SU(4)$, $\triangle Q^B_{15}$ can be defined as

$$\triangle Q^B_{15} = \sqrt{6}A^{\alpha\beta}_{15} = \triangle u^B + \triangle d^B + \triangle s^B - 3\triangle c^B.$$  \hspace{1cm} (37)

Applying Eq.(33) to this quantity we obtain $\triangle Q^B_{15} = 2D$ for all members in the $8$ baryon. Since $\triangle Q^B_a$ for $1 \leq a \leq 8$ is the same as in $SU(3)$, we obtain

$$\triangle c = \frac{1}{3}S - \frac{2}{3}D,$$  \hspace{1cm} (38)

for all members in the $8$ baryon. Note that we use the same $S$ as in $SU(3)$. Thus we get

$$I^p_4 = \frac{1}{2}[\frac{4}{5}\triangle u^p + \frac{1}{5}\triangle d^p + \frac{1}{5}\triangle s^p + \frac{4}{5}\triangle c^p]$$

$$= \frac{5}{27}S + \frac{1}{6}F - \frac{5}{54}D.$$  \hspace{1cm} (39)
The generalization to $SU(5)$ or $SU(6)$ is straightforward, and we obtain

$$\Delta b = \Delta t = \frac{1}{3}S - \frac{2}{3}D. \quad (40)$$

Using experimental value of $F = 0.46 \pm 0.01$ and $D = 0.79 \pm 0.01$ [24], we see that for a reasonable value of $S$, the theoretical value of the Ellis-Jaffe sum rule is reduced substantially by the charm quark. It is usually considered that the light sea quark gets contribution from the gluon anomaly because of the small-ness of the quark mass compared with the infra-red cutoff [25]. The magnitude of this gluon contribution is determined by input information. Then, to make the Ellis-Jaffe sum rule consistent with the experiment by this gluon polarization, it must be taken very large. In our case, such large gluon polarization is not necessary. The heavy quark such as the charm one is suffice to make it consistent with the experiment.

6 Conclusions

In conclusion we show that the chiral $SU(n) \otimes SU(n)$ flavor symmetry on the null-plane combined with the fixed-mass sum rule developed in Refs. [1, 3, 4, 5, 14, 21] severely restricts the sea quark in the 8 baryon. It predicts a large asymmetry for the light sea quarks, and universality and abundance for the heavy sea quarks. Further we show that the same symmetry restricts the fraction of the spin of the 8 baryon carried by the quark. Especially we show that this effect is outstanding for the intrinsic charm sea quark in the nucleon and that it plays the role to reduce the theoretical value of the Ellis-Jaffe sum rule substantially.

References

[1] S.Koretune, Phys. Rev. D 21(1980) 820.
[2] K.Gottfried, Phys. Rev. Lett. 19(1967)1174.
[3] S.Koretune, Prog. Theor. Phys. 72(1984) 821.
[4] S.Koretune, Phys. Rev. D 47(1993)2690.
[5] S.Koretune, Phys. Rev. D 52(1995) 44.
[6] D.J.Gross and F.Wilczek, Phys. Rev. D 8, (1973)3633;
H.Georgi and D.Politzer, Phys. Rev. D 9, (1974)416.

[7] D.A.Ross and C.T.Sachrajda, Nucl.Phys.B149, (1978)497.

[8] G.Curci, W.Furmanski, and R.Petronzio, Nucl.Phys.B175(1980)127.

[9] O.Nachtmann, Nucl.Phys.B63(1973)217.

[10] D.Z.Freedman and J.M.Wang, Phys. Rev. D 160(1967)1560.

[11] S.Koretune, Phys.Lett.124B(1983)113;
Prog.Theor.Phys. 69(1983)1764;70(1983)1170E.

[12] P.Amaudruz et al., Phys. Rev. Lett. 66 (1991)2712.

[13] S.Weinberg, in Lectures on Elementary Particles and Quantum Field
Theory, edited by S.Deser, M.Grisaru, and H.Pendelton (MIT Press,
Cambridge,MA,1970),p.283.

[14] S.Koretune, Prog.Theor.Phys.98(1997)749.

[15] S.P.Dealwis, Nucl.Phys. B43(1972)579.

[16] A.Donnachie and P.V.Landshoff, Phys.Lett.B296(1992)227.

[17] E.A.Kuraev, L.N.Lipatov, and V.S.Fadin, Sov.Phys.JETP 45,199(1977);
Ya.Ya.Balitsky and L.N.Lipatov, Sov.J.Nucl.Phys.28(1978)822.

[18] J.R.Forshaw and D.A.Ross, Quantum Chromodynamics and the
Pomeron
(Cambridge Univ. Press,Cambridge,UK,1997),pp.113-138 and pp.204-
237.

[19] M.Ciafaloni and G.Camici, hep-ph9803389.
V.S.Fadin and L.N.Lipatov, hep-ph9802290.

[20] A.H.Mueller, hep-ph9710531.

[21] S.Koretune, Prog.Theor.Phys. 88(1992)63.

[22] H1 collaboration, C.Adloff et al, DESY-96-138, July 1996.
[23] J.Ellis and R.L.Jaffe, Phys. Rev. D 9 (1974)1444; ED 10 (1974)1669.
[24] S.Y.Hsueh et al., Phys. Rev. D 38 (1988)2056.
[25] Mankiewicz and Schäfer., Phys. Lett. B242 (1990)455; and references cited therein.
Table 1: The regularized mean sea quark number. Here the mean valence quark number is nothing but the valence quark number, hence it takes the value 0 or 1 or 2 according to the valence contents of the baryon. Further we set $\tilde{S}^3 = \frac{\sqrt{6}}{3} S_0 ^3$.

| $B$  | $<u_v^B>$ +2 $<\tilde{\lambda}_u^B>$ | $<d_v^B>$ +2 $<\tilde{\lambda}_d^B>$ | $<s_v^B>$ +2 $<\tilde{\lambda}_s^B>$ |
|------|---------------------------------|---------------------------------|---------------------------------|
| $p$  | $\tilde{S}^3 + 2 \tilde{F} + \frac{2}{3} \tilde{D}$ | $\tilde{S}^3 - \frac{4}{3} \tilde{D}$ | $\tilde{S}^3 - 2 \tilde{F} + \frac{2}{3} \tilde{D}$ |
| $n$  | $\tilde{S}^3 - \frac{4}{3} \tilde{D}$ | $\tilde{S}^3 + 2 \tilde{F} + \frac{2}{3} \tilde{D}$ | $\tilde{S}^3 - 2 \tilde{F} + \frac{2}{3} \tilde{D}$ |
| $\Sigma^+$ | $\tilde{S}^3 + \tilde{F} + \frac{2}{3} \tilde{D}$ | $\tilde{S}^3 - \tilde{F} + \frac{2}{3} \tilde{D}$ | $\tilde{S}^3 - \frac{4}{3} \tilde{D}$ |
| $\Sigma^0$ | $\tilde{S}^3 + \frac{2}{3} \tilde{D}$ | $\tilde{S}^3 + \frac{2}{3} \tilde{D}$ | $\tilde{S}^3 - \frac{4}{3} \tilde{D}$ |
| $\Sigma^-$ | $\tilde{S}^3 - \tilde{F} + \frac{2}{3} \tilde{D}$ | $\tilde{S}^3 + \tilde{F} + \frac{2}{3} \tilde{D}$ | $\tilde{S}^3 - \frac{4}{3} \tilde{D}$ |
| $\Xi^-$ | $\tilde{S}^3 - 2 \tilde{F} + \frac{2}{3} \tilde{D}$ | $\tilde{S}^3 - \frac{4}{3} \tilde{D}$ | $\tilde{S}^3 + 2 \tilde{F} + \frac{2}{3} \tilde{D}$ |
| $\Xi^0$ | $\tilde{S}^3 - \frac{2}{3} \tilde{D}$ | $\tilde{S}^3 - 2 \tilde{F} + \frac{2}{3} \tilde{D}$ | $\tilde{S}^3 + 2 \tilde{F} + \frac{2}{3} \tilde{D}$ |
| $\Lambda^0$ | $\tilde{S}^3 - \frac{2}{3} \tilde{D}$ | $\tilde{S}^3 - \frac{2}{3} \tilde{D}$ | $\tilde{S}^3 + \frac{4}{3} \tilde{D}$ |
Table 2: The mean quantum number of the light sea quarks.

|     | \( \langle I_3 \rangle \)          | \( \langle Y \rangle \)          | \( \langle Q \rangle \)          |
|-----|-----------------------------------|-----------------------------------|-----------------------------------|
| \( B \) | \( \frac{1}{2}(\lambda_u^B - \lambda_d^B) \) | \( \frac{1}{3}(\lambda_u^B + \lambda_d^B - 2\lambda_s^B) \) | \( \frac{1}{3}(2\lambda_u^B - \lambda_d^B - \lambda_s^B) \) |
| \( p \) | \( \frac{1}{2}(\bar{F} + \bar{D}) - \frac{1}{4} \) | \( \frac{1}{3}(3\bar{F} - \bar{D}) - \frac{1}{2} \) | \( \frac{1}{3}(3\bar{F} + \bar{D}) - \frac{1}{2} \) |
|     | \( = -0.055 \)                     | \( = 0.56 \)                     | \( = 0.23 \)                     |
| \( n \) | \( -\frac{1}{2}(\bar{F} + \bar{D}) + \frac{1}{4} \) | \( \frac{1}{3}(3\bar{F} - \bar{D}) - \frac{1}{2} \) | \( -\frac{2}{3}\bar{D} \) |
|     | \( = 0.055 \)                     | \( = 0.56 \)                     | \( = 0.34 \)                     |
| \( \Sigma^+ \) | \( \frac{1}{2}\bar{F} - \frac{1}{2} \) | \( \frac{2}{3}\bar{D} \) | \( \frac{1}{6}(3\bar{F} + 2\bar{D}) + \frac{1}{2} \) |
|     | \( = -0.054 \)                     | \( = -0.34 \)                     | \( = -0.22 \)                     |
| \( \Sigma^0 \) | \( 0 \) | \( \frac{2}{3}\bar{D} \) | \( \frac{1}{3}\bar{D} \) |
|     | \( = 0 \) | \( = -0.34 \) | \( = -0.17 \) |
| \( \Sigma^- \) | \( -\frac{1}{2}\bar{F} + \frac{1}{2} \) | \( \frac{2}{3}\bar{D} \) | \( \frac{1}{6}(3\bar{F} + 2\bar{D}) + \frac{1}{2} \) |
|     | \( = 0.054 \) | \( = -0.34 \) | \( = -0.11 \) |
| \( \Xi^- \) | \( \frac{1}{2}(\bar{F} + \bar{D}) + \frac{1}{4} \) | \( -\frac{1}{3}(3\bar{F} + \bar{D}) + \frac{1}{2} \) | \( -\frac{1}{3}(3\bar{F} - \bar{D}) + \frac{1}{2} \) |
|     | \( = -0.45 \) | \( = -0.23 \) | \( = -0.56 \) |
| \( \Xi^0 \) | \( \frac{1}{2}(\bar{F} - \bar{D}) - \frac{1}{4} \) | \( -\frac{1}{3}(3\bar{F} + \bar{D}) + \frac{1}{2} \) | \( -\frac{2}{3}\bar{D} \) |
|     | \( = 0.45 \) | \( = -0.23 \) | \( = 0.34 \) |
| \( \Lambda^0 \) | \( 0 \) | \( -\frac{2}{3}\bar{D} \) | \( -\frac{1}{3}\bar{D} \) |
|     | \( = 0 \) | \( = 0.34 \) | \( = 0.17 \) |

Table 3: The spin fraction of the quarks.

| \( B \) | \( \triangle u^B \) | \( \triangle d^B \) | \( \triangle s^B \) |
|--------|------------------|------------------|------------------|
| \( p \) | \( \frac{1}{3}S + \frac{1}{2}F + \frac{1}{3}D \) | \( \frac{1}{3}S - \frac{2}{3}D \) | \( \frac{1}{3}S + F + \frac{1}{3}D \) |
| \( n \) | \( \frac{1}{3}S - \frac{2}{3}D \) | \( \frac{1}{3}S + F + \frac{1}{3}D \) | \( \frac{1}{3}S - F + \frac{1}{3}D \) |
| \( \Sigma^+ \) | \( \frac{1}{3}S + \frac{1}{2}F + \frac{1}{3}D \) | \( \frac{1}{3}S - \frac{2}{3}F + \frac{1}{3}D \) | \( \frac{1}{3}S - \frac{2}{3}D \) |
| \( \Sigma^0 \) | \( \frac{1}{3}S + \frac{1}{3}D \) | \( \frac{1}{3}S + \frac{1}{3}D \) | \( \frac{1}{3}S - \frac{2}{3}D \) |
| \( \Sigma^- \) | \( \frac{1}{3}S - \frac{1}{2}F + \frac{1}{3}D \) | \( \frac{1}{3}S + \frac{1}{2}F + \frac{1}{3}D \) | \( \frac{1}{3}S - \frac{2}{3}D \) |
| \( \Xi^- \) | \( \frac{1}{3}S - F + \frac{1}{3}D \) | \( \frac{1}{3}S - \frac{2}{3}D \) | \( \frac{1}{3}S + F + \frac{1}{3}D \) |
| \( \Xi^0 \) | \( \frac{1}{3}S - \frac{2}{3}D \) | \( \frac{1}{3}S - F + \frac{1}{3}D \) | \( \frac{1}{3}S + F + \frac{1}{3}D \) |
| \( \Lambda^0 \) | \( \frac{1}{3}S - \frac{1}{3}D \) | \( \frac{1}{3}S - \frac{1}{3}D \) | \( \frac{1}{3}S + \frac{2}{3}D \) |