Comparative efficiency of wind turbines with different heights of rotor hubs: performance evaluation for Latvia

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Abstract. Performance evaluation of wind turbines (WTs) for different heights of the rotor hub is made based on the wind speed and direction data obtained in 2009 – 2013 on-shore in the north of Latvia using a LOGGER 9200 Symphonie measurement system mounted on a 60 m mast. Based on the measurement analysis results, wind speed distribution curves have been modelled for heights of up to 200 m using power and logarithmic (log) law approximation methods. The curves for the modelled Weibull’s parameters are plotted in dependence on height. The efficiency comparison is made for different WT types taking into account the distribution of the wind energy potential in height in the Latvian territory. The annual electric energy production was calculated for the WTs with different heights of rotor hubs. In the calculations the technical data on the following WT types were used: E-3120 (50 kW, hub height 20.5/30.5/36.5/42.7 m), E-33 (330 kW, hub height 37/44/49/50 m), E-48 (800 kW, hub height 50/60/75 m) and E-82 (2.3 MW, hub height of 78/85/98/108/138 m).

Keywords: long-term wind measurements; wind shear analysis; wind turbine, operational efficiency, power and log law approximations

1. Introduction
Most of the Latvian territory is plain fields covered by forests with trees up to 10 - 15 m; respectively, the impact of terrain to the wind flows is clearly noticeable up to 30 m, and the wind profile tends to have similar properties in the entire territory. These areas are suitable for the construction of wind farms and individual wind turbines (WTs). Therefore, efficiency evaluation is done for WTs with different rated power and different height of the rotor hub.

For this purpose it was necessary to analyze the wind energy streams in the territory of Latvia using the results of wind speed measurements and the models of wind energy density distribution in height. In Latvia, systematic long-term measurements of wind speed taking into account its height distribution have been carried out since 2007 at two sites on the north-west shore of the Baltic Sea in the Ventspils region and on the north of the country in the Ainazi region [1].

In the Ainazi region, the on-site measurements of wind speed were carried out using the certified sensors for wind speed and the sensors indicating the direction of air stream. To collect the wind data, the sensors were mounted at several levels on a 60 m high metallic mast.

For storing data from the sensors at all height levels, a measuring complex NRG LOGGER Symphonie 9200 was used. The complex is powered independently by batteries and stores the average wind speed values obtained with 10 min intervals from nine sensors on its flash memory card.

Retrieval and filtering of the wind data were done using an NRG Symphonie Data Retriever. Further data analysis was made using Microsoft Excel with additional scripts and WAsP Weibull Distribution
Explorer. The installation of the 60 m mast with the measuring complex Symphonie is shown in figure 1 [2].

![Installation of the 60 m mast with a measuring complex Symphonie (Ainazi site)](image)

**Figure 1.** Installation of the 60 m mast with a measuring complex Symphonie (Ainazi site) [2].

2. **Wind shear analysis**

The chart of seasonal fluctuations of the average wind speed $V_{\text{avg}}$, m/s, for heights 10, 30, and 60 m at site (Ainazi) is shown in figure 2 for the monthly averaged measurement time $T$ (04.2009 - 04.2013).

![Average wind speed $V_{\text{avg}}$ for measurement time $T$ (04.2009 - 04.2013) at heights 10, 30 and 60 m (Ainazi site)](image)

**Figure 2.** Average wind speed $V_{\text{avg}}$ for measurement time $T$ (04.2009 - 04.2013) at heights 10, 30 and 60 m (Ainazi site).

Continuous records of the wind speed values with 10-min intervals made it possible to construct the wind speed frequency distribution curves $F(V)$ with high precision [3]. The curves plotted in figure 3 are constructed based on the factual values of wind speed $V$ in the time period 04.2009 - 04.2013.

Having the real frequency distribution of wind speed shown in this figure, we can pass to the model of Weibull’s probability density function $F(V)$ using the parameters $k$ (shape factor) and $c$ (scale factor).
Figure 4 displays similar curves of Weibull’s probability density function $F(V)$ constructed by the classical method [4] for the same time span.

![Figure 3. Wind speed frequency distribution curves, $F(V)$, for the period 04.2009 - 04.2013.](image)

![Figure 4. Weibull probability density function curves, $F(V)$, for the period 04.2009 - 04.2013.](image)

Table 1 contains the measured values of average wind speed, $V_{avg}$, and their standard deviations, $SD$, for heights 10, 20, 30, 40, 50 and 60 m. Based on these data, the values of parameters $k$ and $c$ corresponding to these heights are calculated.

**Table 1.** Average data for wind speed in the Ainazi region for the measurement heights 10, 20, 30, 40, 50, 60 m and the parameters $k$ and $c$ of Weibull’s distribution.

| Period of measurements | Height above ground, m | $V_{avg}$, m/s | $SD$ | Weibull parameters |
|------------------------|------------------------|----------------|------|--------------------|
|                        |                        |                |      | $k$                | $c$    |
| 04.2009 - 04.2013      | 10                     | 2.72           | 0.70 | 1.63               | 3.05   |
|                        | 20                     | 3.28           | 0.70 | 1.85               | 3.70   |
|                        | 30                     | 3.63           | 0.74 | 2.01               | 4.10   |
|                        | 40                     | 4.02           | 0.72 | 2.19               | 4.54   |
|                        | 50                     | 4.38           | 0.73 | 2.31               | 4.94   |
|                        | 60                     | 4.65           | 0.73 | 2.34               | 5.25   |

3. **Modelling of the operational efficiency of WTs taking into account the wind shear**

The distribution of Weibull’s parameters in dependence on height (figure 5) meets a definite regularity, which is well approximated by the power law curves [3]. The expressions for $c_p$ and $k_p$ based on the results of measuring the wind speed in the Ainazi region using the power law approximation are:

$$c_p = c_r \left( \frac{h}{h_r} \right)^{\alpha_c}$$  \hspace{1cm} (1)

$$k_p = k_r \left( \frac{h}{h_r} \right)^{\alpha_k}$$  \hspace{1cm} (2)

where $c_r, k_r$ are the values of the Weibull’s distribution parameters calculated for the reference height $h_r$,

$h$ is the rotor hub height, $m$,

$\alpha_c, \alpha_k$ are the values of approximation coefficients.
Modelling of the distribution curves for \( c_p \) and \( k_p \) parameters in dependence on height \( h \) using approximating functions (1) and (2) allows extrapolation of their values to the heights where the mentioned regularity is supposedly hold.

Analysis of the results obtained at measuring the wind speed (table 1) allows \( c_p \) and \( k_p \) in expressions (1) and (2) to be calculated for Ainazi as

\[
c_p = c_r \cdot \left( \frac{h}{h_r} \right)^{0.359} = \frac{c_r}{h_r^{0.359}} \cdot h^{0.359} = 1.205 \cdot h^{0.359},
\]

\[
k_p = k_r \cdot \left( \frac{h}{h_r} \right)^{0.226} = \frac{k_r}{h_r^{0.226}} \cdot h^{0.226} = 0.937 \cdot h^{0.226},
\]

where \( h \) is the height at the measurement point, m.

Measurements at 10 and 20 m heights are significantly affected by the surface and were not included in the models; therefore, these models were constructed using the data from 30 to 60 m.

Regularities for calculating the \( c \) and \( k \) parameters in dependence on height \( h \), corresponding to the height of the rotor hub, can be presented in an alternative form using a logarithmic (log) law and the known expressions used to characterize the wind shear regarding the Types of Terrain [4]:

\[
c_L = \frac{c_r \ln \left( \frac{h}{z_{0c}} \right)}{\ln \left( \frac{h_r}{z_{0c}} \right)} \quad (5)
\]

\[
k_L = \frac{k_r \ln \left( \frac{h}{z_{0k}} \right)}{\ln \left( \frac{h_r}{z_{0k}} \right)} \quad (6)
\]

where \( c_L, k_L \) are the Weibull parameters in dependence on height \( h \) calculated using the log law approximation function.

\( c_r, k_r \) are the values of Weibull’s distribution parameters calculated for reference height \( h_r \);

\( z_{0c}, z_{0k} \) are the values of the parameters for log law approximation nominally corresponding to the type of ground surface roughness.

Accordingly, expressions (5) and (6) for the Ainazi region can be presented as

\[
c_L = \frac{c_r \ln \left( \frac{h}{z_{0c}} \right)}{\ln \left( \frac{h_r}{z_{0c}} \right)} = \frac{c_r}{\ln (h_r/z_{0c})} \ln \left( \frac{h}{z_{0c}} \right) = 1.65 \cdot \ln \left( \frac{h}{2.5} \right)
\]

\[
k_L = \frac{k_r \ln \left( \frac{h}{z_{0k}} \right)}{\ln \left( \frac{h_r}{z_{0k}} \right)} = \frac{k_r}{\ln (h_r/z_{0k})} \ln \left( \frac{h}{z_{0k}} \right) = 0.52 \cdot \ln \left( \frac{h}{0.63} \right).
\]

Therefore, from \( c \) and \( k \) values (calculated based on the results of physical measurements at the heights where sensors can be placed) it is possible to pass to the model of Weibull’s distribution of the wind speed which would allow its values to be determined for different hub heights. It should be mentioned that extrapolation of the curves enables calculation of wind energy streams at the heights exceeding those of the sensors fixed on the mast. In figure 5 the \( c \) and \( k \) parameters are presented using the power and log law approximations (respectively \( c_p, k_p \) and \( c_L, k_L \)) up to height of 200 m.

Ongoing measurements using a lidar complex Spidar Pentalum allow the authors to conclude that the approximation of Weibull’s distribution for parameter \( c_p \) (scale factor) at heights up to 200 m modelled by the power law corresponds better to real measurements compared with log law approximations (\( c_L \)). At the same time, the \( k_p \) and \( k_L \) values are practically independent of the chosen type of approximation.

In this way – i.e. by calculations using the models considered above – the necessary information can be received on the wind speed frequency distribution for the heights of rotor hub location corresponding
to the type of WTs chosen for the research. This would enable calculation of the annual electricity production (AEP) by generators, making it possible to estimate the efficiency of WT offered for use in the Latvian territory.

![Figure 5. Distribution of Weibull parameters](image)

**Figure 5.** Distribution of Weibull parameters $k$ (shape factor) and $c$ (scale factor) for the measurement heights of 30, 40, 50 and 60 m (points) and their extrapolations up to 200 m using power law (orange and blue lines marked $c_p; k_p$) and log law (dashed orange and blue lines marked $c_l; k_l$).

To evaluate performance of wind turbines at different heights under the Latvian conditions the following WT types are considered: E-3120, Е-33, E-48 and Е-82 [5, 6]. Turbines of the types are equipped with generators having the rated power of 50, 330, 800 and 2300 kW, respectively, which could be disposed on the masts with sizes given in table 2. This table also shows the AEP values calculated by expressions (3), (4) and (7), (8) for the respective heights of rotor hub.

In the AEP estimation of the considered WT types the wind speed frequency distribution function $F(V)$ is used for the corresponding heights of rotor hub approximated by power law models as well as for its combination with power curves $P(V)$ of the generators. The AEP ($W$, MWh) to which the wind stream energy can be converted in a definite period of time is determined by the equality:

$$W = \sum_{i=1}^{n} (P(V_i) \cdot F(V_i)),$$

where the values of function $P(V_i)$ are corresponding to the power curve for the generator, while those of function $F(V_i)$ – to the Weibull’s wind speed frequency distribution curve at the height of rotor hub and respective wind speeds $V_i$, m/s.

To analyze the WT efficiency, functions $P(V)$ and $F(V)$ were used as discrete bins with steps of a definite length ($\Delta V$, m/s) whose number depends on this length and is restricted by the WT generator’s limiting value of wind speed – the so-called “cut-out wind speed”. In figures 3, 4 the step length $\Delta V$ corresponds to 0.5 m/s. The cut-out wind speed for a WT is 25 – 34 m/s (depending on WT type). The wind speed for the $i$-th step is determined from the expression:

$$V_i = \Delta V \cdot i,$$

where $i = 0.5, 1, 1.5 \ldots n$.

Diagrams of figures 6 – 9 show power curves $P(V)$ for WT generators of the types E-3120, E-33, E-48 and E-82, the curves of Weibull’s probability density function $F(V)$. The values of Weibull parameters calculated using the power law models and the step length $\Delta V=1.0$ m/s. On the diagrams of of figures 6 – 9, results are presented for the maximum height of mast which is available for corresponding WT. These figures also shows additional heights for the WT generators of types E-3120,
E-33, E-48 at which their efficiency in the Latvian territory would reach 23.7%. The $W(V)$ curves presents the electric energy production by WTs of the types under the Latvian conditions. At estimation of the AEP (calculated for each WT type by expression (9)) with respect to its maximum that can be obtained from generators for 8760 hours of their uninterrupted operation with the rated power $P_R$, kW, the efficiency of a particular WT can be expressed as

$$C_e = \frac{W}{p_R^{8760}}. \quad (11)$$

The values of parameters $C_{ep}$ and $C_{el}$ calculated by equation (11), respectively using power and log law models, for the WT types under consideration are presented in table 2.

### Table 2. The AEP and the WT operational efficiency at heights of rotor hub for the types E-3120, E-33, E-48 and E-82.

| Type of WT | Rated power of generator, kW | Height of rotor hub, m | Power law model | Log law model | $C_{ep}$/ $C_{el}$ |
|-----------|-------------------------------|------------------------|----------------|--------------|-------------------|
| E-3120    | 50                            | 20.5                   | 30.7           | 7.0          | 28.7              | 6.5              | 1.08            |
|           |                               | 30.5                   | 45.4           | 10.4         | 46.0              | 10.5             | 0.99            |
|           |                               | 36.5                   | 54.3           | 12.4         | 55.5              | 12.7             | 0.98            |
|           |                               | 42.7                   | 63.4           | 14.5         | 63.4              | 14.5             | 1.00            |
|           |                               | 70                     | 103.8          | 23.7         | 98.8              | 22.6             | 1.05            |
| E-33      | 330                           | 37                     | 231.6          | 8.0          | 236.6             | 8.2              | 0.98            |
|           |                               | 44                     | 272.3          | 9.4          | 277.4             | 9.6              | 0.98            |
|           |                               | 49                     | 301.3          | 10.4         | 304.9             | 10.5             | 0.99            |
|           |                               | 50                     | 307.1          | 10.6         | 310.2             | 10.7             | 0.99            |
|           |                               | 118                    | 685.5          | 23.7         | 594.0             | 20.5             | 1.16            |
| E-48      | 800                           | 50                     | 607.1          | 8.7          | 613.8             | 8.8              | 0.99            |
|           |                               | 60                     | 726.4          | 10.4         | 719.2             | 10.3             | 1.01            |
|           |                               | 75                     | 905.1          | 12.9         | 863.4             | 12.3             | 1.05            |
|           |                               | 140                    | 1'658.6        | 23.7         | 1'358.6           | 19.4             | 1.22            |
| E-82      | 2300                          | 78                     | 2'777.2        | 13.8         | 2'633.5           | 13.1             | 1.05            |
|           |                               | 85                     | 3'016.6        | 15.0         | 2'809.7           | 13.9             | 1.08            |
|           |                               | 98                     | 3'457.7        | 17.2         | 3'119.1           | 15.5             | 1.11            |
|           |                               | 108                    | 3'793.3        | 18.8         | 3'341.9           | 16.6             | 1.13            |
|           |                               | 138                    | 4'776.1        | 23.7         | 3'945.2           | 19.6             | 1.21            |

Comparison of the results gives the maximum efficiency ($C_{ep} = 23.7\%$) for the WT of E-82 type with a 138 m high mast. Such operational efficiency can be achieved for WTs E-3120, E-33 and E-48 if their rotor hub heights are raised to 70, 118 and 140 m, respectively.

In table 2 it is seen that at heights up to 100 m the ratio $C_{ep} / C_{el}$ is close to 1.0, while with height increasing the AEP values calculated using the power law model exceed those calculated in the log law model. For example, at height 140 m the ratio $C_{ep} / C_{el}$ is over 20%.

Thus, when evaluating the performance of WTs at heights up to 100 m the choice of models has no significant effect on the results. At the same time, at heights above 100 m there is a discrepancy in the calculation results.

Analysis of Weibull’s probability density function $F(V)$ curves in figures 6 – 9 shows that with increasing height the maximum of this function shifts to greater values of wind speed $V$, which is evidence for stronger wind energy streams. At such wind speeds the WT generators – in compliance with their specific power curves $P(V)$ – operate with better efficiency.
Figure 6. Power curve $P(V)$ for generators of E-3120 type (rated power 50 kW) and Weibull probability density function $F(V)$ curves for wind speeds $V$, m/s at heights 42.7 and 70 m characterizing the AEP $W(V)$, MWh.

Figure 7. Power curve $P(V)$ for generators of E-33 type (rated power 330 kW) and Weibull probability density function $F(V)$ curves for wind speeds $V$, m/s at heights 50 and 118 m characterizing the AEP $W(V)$, MWh.

Figure 8. Power curve $P(V)$ for generators of E-48 type (rated power 800 kW) and Weibull probability density function $F(V)$ curves for wind speeds $V$, m/s at heights 75 and 140 m characterizing the AEP $W(V)$, MWh.

Figure 9. Power curve $P(V)$ for generators of E-82 type (rated power 2300 kW) and Weibull probability density function $F(V)$ curves for wind speeds $V$, m/s at height 138 m characterizing the AEP $W(V)$, MWh.

Figure 10 shows the distribution of $C_{ep}$ coefficients, $\%$, in dependence on the rotor hub height $h$, m for the WTs of E-3120, Е-33, Е-48 and Е-82 type. In this case the height values correspond to the type sizes of masts (see table 2).

For all types of WTs, the operational efficiency increasing with the mast height is linear. However, the technical parameters of WTs E-3120, Е-33, Е-48 do not allow them to operate as efficient as Е-82 type WTs fixed on 138 m high masts.

Analysis of the relevant curves in figure 10 confirms that under conditions characteristic for the distribution of wind energy potential in the Latvian territory the WT efficiency can be raised at the cost of higher masts. For example, to raise the operational efficiency of WTs E-3120, Е-33, Е-48 up to the level comparable with the maximum achievable for the E-82 convertor on a 138 m high mast it is necessary to increase the height of rotor hub up to 70, 118 and 140 m, respectively.
Figure 10. Distribution of $C_{ep}$ coefficients, %, characterizing the efficiency of the WTs of E-3120, E-33, E-48 and E-82 type for different heights of rotor hub $h$, m.

4. Conclusions

- Construction of the models for calculating the efficiency of the wind turbine based on the results of physical measurements is described.
- Comparison has been made between the models of Weibull parameters $c$ and $k$ based on the power law and the log law approximations.
- Assumption is made that the Weibull distributions at heights up to 200 m modelled by the power law approximation better correspond to real measurements compared with the log law approximations.
- Comparison has been made for the operational efficiency of WTs E-3120, E-33, E-48 and E-82 under the conditions characteristic for the distribution of wind energy potential in the Latvian territory. Calculations show that the operational efficiency of WT E-82 with the rated power of 2300 kW and the maximum rotor hub height of 138 m can reach 23.7%.
- The operational efficiency of WTs E-3120, E-33 and E-48 with the rated power of 50, 330 and 800 kW and the maximum rotor hub height of 42.7, 50 and 70 m when used in the Latvian territory does not exceed 14%.

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