Study on stability of reinforced soil slope based on upper limit method

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Abstract. Based on the nonlinear Mohr-Coulomb failure criterion, the upper bound method of limit analysis is used to investigate slope stability by introducing new strength parameters $c_1$ and $\phi_1$. It is assumed that the potential weak zone in the soil layer is a straight line, so the straight line fracture surface is used to study the non-reinforced soil and reinforced soil in this study, and the expression of the stability coefficient $N_s$ of the non-reinforced soil slope is derived. In addition, the calculation equations of the safety factor $F$, the ultimate slope height $H$, and the ultimate slope height $H$ of the reinforced soil slope are also derived. The CVX toolbox developed by Stanford University and a MATLAB program developed by the authors are used for illustrations, and good results are obtained. The relationship between the nonlinear parameter $m$ and the limit slope height $H$, the included angle $\theta$, the new strength parameter $\phi_t$, the slope stability coefficient $N_s$, the relationship between the slope angle $\beta$ and the safety factor $F$, and the relationship between different reinforcement tensile strength $K_t$ and the limit slope height $H$ are summarized.

KEYWORDS: Plastic mechanics; Reinforced soil; Limit analysis; Nonlinear Mohr-Coulomb yield conditions; Upper bound theory

1. Introduction

The upper bound method of plastic limit analysis is an effective tool for solving slope engineering problems [1]. The linear failure criterion has been used in many projects, but a large number of test results show that the failure criterion of geotechnical materials is nonlinear. Since Baker [2] introduced the nonlinear yield condition in limit analysis in 1983, the limit analysis method of slope stability based on the nonlinear yield condition has become a hot research topic in academic and engineering circles. Zhang [3] proposed an inverse method to study the stability of the rock slope under the nonlinear failure criterion in 1987. Drescher [4] proposed a tangent method for calculating the stability coefficient of soil slope under the nonlinear failure criterion in 1988; In recent years, Fredlund, YANG X L [5-6] have successively proposed a generalized tangent method and its extended method. However, there are few scientific studies on slope limit analysis under the nonlinear failure criterion, and some of them are difficult to be applied to practical engineering calculations. Therefore, based on the upper bound method of limit analysis, this study investigates the stability of non-reinforced and reinforced soil slopes under the nonlinear Mohr-Coulomb (M-C) yield condition.

2. Nonlinear Mohr-Coulomb yield condition and CVX patch package of MATLAB

As early as 1977, Lade [7] concluded through many experimental observations that the relationship between Shear stress and normal stress is nonlinear when soil fails. Agar [8] proved that this
relationship is nonlinear in the triaxial test of rock in 1985. This nonlinear relationship can be expressed by the following equation:

\[ f(\sigma_{ij}) = \tau - c_0 \left(1 + \frac{\sigma_n}{\sigma_t}\right)^{\frac{1}{m}} \]  

(2-1)

In the equation above, \( \tau \) represents Shear stress; \( \sigma_n \) represents normal stress; \( c_0, \sigma_t, m \) represent material parameters (\( m \) is determined by test).

According to the nonlinear M-C yield criterion, different \( m \) values are taken, and the yield surface pictures are as follows:

![Fig. 2.1 Front view and side view of \( m < 1 \) and Fig. 2.2 Front view and side view of \( m > 1 \)]

According to the calculation results of the yield surface, when \( m = 1 \), it is the most commonly used linear M-C yield criterion. When \( m < 1 \), it does not conform to the principle of convexity of the loading surface in plastic mechanics, so this situation will not occur. When \( m > 1 \), it conforms to the principle of convexity. Therefore, in the plastic limit analysis of reinforced soil slopes under the nonlinear M-C yield criterion, \( m > 1 \).

The CVX toolbox is a toolbox based on MATLAB developed in recent years by Stanford Professor Stephen P. Bold and others that is specially used to solve convex optimization calculations. It can solve many optimization engineering-related problems, and it is relatively easy to program and calculate based on the secondary development of this toolbox. In plastic mechanics, the yield surface of the objective function is convex, so the optimization in the limit analysis of geotechnical plastic mechanics can be solved by programming using the CVX toolbox and MATLAB.

3. The method of circumscribed lines for nonlinear Mohr-Coulomb yield condition

The ultimate load calculated by a convex surface circumscribing or circumscribing the actual yield surface is an upper limit of the actual ultimate load, whereas the ultimate load calculated by an inscribed or inscribed surface is a lower limit of the actual failure load. Draw an outer tangent line to the yield surface of the nonlinear M-C criterion, as shown in the figure below. Since the area contained by the outer tangent line is greater than the area contained by the original yield surface represented by the shaded area in the figure, the solution obtained by the method of outer tangent lines must be an upper limit of the real load.
Fig. 3.1 The Extangent Method of Nonlinear M-C Failure Criterion

According to the above idea, the nonlinear M-C yield criterion commonly used in soil is adopted, as shown in Equation (3-1):

\[
f(\sigma_{ij}) = \tau - c_0 \left(1 + \frac{\sigma_n}{\sigma_t}\right)^\frac{1}{m} = 0 \quad (3-1)
\]

In the equation above, \(\tau\) represents Shear stress; \(\sigma_n\) represents normal stress; \(c_0\), \(\sigma_t\), \(m\) represent material parameters (\(m\) is generally determined by test); \(\sigma_t\) represents the intersection of the curve and horizontal axis.

Let the coordinates of the point \(m\) of the failure surface be \((\sigma_n, \tau)\), the tangent equation passing through point \(M\) is

\[
\tau = \sigma_n \tan \phi_t + c_t \quad (3-2)
\]

In the equation above, \(c_t\) represents the intercept of the circumscribed line; \(\tan \phi_t\) represents the slope of the tangent line. The derivation of Equation (3-1) is

\[
\tan \phi_t = \frac{c_0}{\sigma_t} \left(1 + \frac{\sigma_n}{\sigma_t}\right)^\frac{1}{m} \quad (3-3)
\]

Generally, \(\sigma_n > 0\), and when the tangent passes point \((0, \ c_t)\), \(\tan \phi_t = \frac{c_0}{m\sigma_t}\), that is, the maximum value; when \(\sigma_n \to +\infty\), \(\tan \phi_t = 0\), that is, the maximum value. Hence, we can obtain the range of \(\phi_t\):

\[
0 < \phi_t < \arctan \frac{c_0}{m\sigma_t} \quad (3-4)
\]

By substituting Equation (3-3) into Equation (3-2), the following equation is obtained:

\[
c_t = \frac{m-1}{m} c_0 \left(\frac{m\sigma_t\tan \phi_t}{c_0}\right)^\frac{1}{m} + \sigma_t \tan \phi_t \quad (3-5)
\]

Therefore, the normal stress at the failure point can be obtained first \(\sigma_n\). That is, the normal stress at point \(M\). According to Equation (3-3), \(\tan \phi_t\) can be obtained. Then, substitute the value of \(\tan \phi_t\) into Equation (3-5) to obtain \(c_t\).

Under the nonlinear failure criterion, the definition of the slope stability coefficient \(N_s\) is shown in Equation (3-6):

\[
N_s = \frac{H_{cr} \gamma}{c_t} \quad (3-6)
\]

In the equation above, \(H_{cr}\) represents the critical height of slope; \(\gamma\) represents the gravity of slope soil.

To directly apply the calculation results to engineering practice, the slope stability safety factor \(F\) is introduced into the upper bound theorem using the strength reduction method. When the slope is in the limit state, the expression of the strength index of geotechnical materials is as follows:

\[
c_e = \frac{c_t}{F} (3-7) \tan \phi_e = \frac{\tan \phi_t}{F} \quad (3-8)
\]

With the change in the position of the \(M\) point, the slope \(\tan \phi_t\) and the intercept \(c_t\) of the tangent line constantly change, so the slope \(\tan \phi_e\) and the intercept \(c_e\) of the tangent line after strength reduction also constantly change. In the upper bound analysis, according to the optimization theory, all the unknown quantities in the established model are regarded as variables, and the optimization
calculation is performed by writing a MATLAB program. The minimum upper bound solution and the values of the relevant independent variables can be obtained.

4. Upper bound method for limit analysis of unreinforced slopes with straight failure surfaces

4.1 Theory

For the slope angle, as shown in the figure of $\beta$ Large simple slope, when the slope soil reaches the limit state, the slope is destroyed in the form of a straight fracture surface, and the yield mechanism is formed, as shown in Figure 4.1. When the slope soil slides downward along the straight fracture surface, a rigid wedge ABC is formed, and the downward sliding velocity of wedge ABC is $V$. The angle between fracture surface BC and sliding velocity V is $\phi_i$.

![Figure 4.1 Schematic diagram of a slope sliding along a straight line fracture surface](image)

From the trigonometric function relation, we can obtain the following results:

$$L_{BC} = H \frac{\sin(\beta - \alpha)}{\sin\beta \sin(\theta - \alpha)}$$  \hspace{1cm} (4-1)

$$H_B = H \frac{\sin\beta \sin(\beta - \alpha)}{\sin\beta \sin(\theta - \alpha)} = H \frac{1 - \cot\tan\alpha}{1 - \cot\tan\alpha}$$  \hspace{1cm} (4-2)

In the equation above, $L_{BC}$ represents the length of slope fracture surface BC; $H_B$ represents the vertical projection of slope fracture surface BC.

According to the upper bound theorem of limit analysis, the power of the external force is equal to the product of the vertical component of the self-weight and self-velocity of the soil:

$$W_{\text{external}} = \frac{1}{2} \gamma \frac{H^2(1 - \cot\beta)}{1 - \cot\tan\alpha} \frac{1 - \cot\tan\alpha}{1 - \cot\tan\alpha} \sin(\theta - \phi)$$  \hspace{1cm} (4-3)

The power of the internal force of slope is destroyed along discontinuous fracture surface is equal to the product of cohesive force $c_t$ and velocity component on the fracture surface:

$$W_{\text{inside}} = c_t H \frac{1 - \cot\tan\alpha}{\sin\theta(1 - \cot\tan\alpha)} \cos\phi_t V$$  \hspace{1cm} (4-4)

According to the upper bound theorem of limit analysis, the external force power is equal to the internal dissipation power of the failure mechanism:

$$W_{\text{external}} = W_{\text{inside}}$$  \hspace{1cm} (4-5)

$$\frac{1}{2} \gamma H^2 \left(\cot\beta - \cot\phi_t\right) \sin(\theta - \phi_t) - \frac{c_t H}{\sin\theta} \cos\phi_t = 0$$  \hspace{1cm} (4-6)

$$H = \frac{2c_t \cos\phi_t}{\gamma \sin\theta \sin(\theta - \phi_t) (\cot\beta - \cot\phi_t)} = \frac{c_t}{\gamma} f(\theta)$$  \hspace{1cm} (4-7)

The cohesion of soil $c_t$ and soil weight $\gamma$ are all material parameters, $\alpha$ and $\beta$ represent the inherent parameters of the slope. If Equation (3-5) is substituted into Equation (4-7), the limit height $H$
of the slope only contains \( \theta \) and \( \phi_t \). We can obtain the values of the two variables when \( H \) is the smallest by programming in MATLAB to optimize the above two parameters. The slope stability coefficient \( N_s \) can be obtained by solving Equation (4-8). Because there are few parameters along the straight fracture surface and \( c_t \) is only related to \( \phi_t \), we only need to obtain the minimum value of \( f (\theta) \) if we want to obtain the critical height \( H \) of the slope. Based on \( \frac{\partial f}{\partial \theta} = 0 \), the calculated \( \theta \) value is the angle between the fracture and horizontal planes of the slope:

\[
\theta = \arctan \left( \frac{\tan \phi_t - \cot \beta}{\sqrt{1 + \cot^2 \theta \tan \phi_t}} \right) + \sqrt{1 + \cot \theta \tan \phi_t} + \cot \beta \tan \phi_t \tag{4-9}
\]

Take \( \theta \) Substituting Equation (4-7), we can obtain the critical height of slope \( H \) and \( \phi_t \). According to \( 0 < \phi_t < \arctan \frac{c_0}{m_o} \), \( H \) must have an extreme value. According to the definition of the slope stability coefficient under the nonlinear failure criterion, the slope stability coefficient \( N_s \) can be obtained. The expression is as follows:

\[
N_s = 2 \cdot \cos \phi_t \left( 1 + \sin^2 \phi_t \right) \left( 1 + \cot \beta \right) + \cot \beta + \cot \phi_t \left( 1 + \cot \beta \right) \left( 1 + \cot^2 \phi_t \right) \tan \phi_t \left( 1 + \tan \phi_t \left( 1 + \cot \beta \right) \right) \sin \left( \arctan \left( \frac{1 + \tan \phi_t \left( 1 + \cot \beta \right) \left( 1 + \cot^2 \phi_t \right)}{1 + \tan \phi_t \left( 1 + \cot \beta \right)} \right) \right) \tag{4-10}
\]

The strength reduction method is used to directly apply the calculation results to engineering problems. According to Equations (3-7) and (3-8), the slope stability safety factor \( F \) is introduced. The parameters of the external force power in Equation (4-3) and the internal force power in Equation (4-4) are significantly reduced, and then, a function of \( F \) can be obtained according to Equation (4-5). When the slope height is known, the safety factor of the upper limit is obtained:

\[
\frac{1}{2} \gamma H \left( \cot \theta - \cot \beta \right) \sin \left( \arctan \left( \frac{\tan \phi_t}{F} \right) \right) - \frac{c_0}{F \sin \theta} \cos \arctan \left( \frac{\tan \phi_t}{F} \right) = 0 \tag{4-11}
\]

If the safety factor of slope stability is to meet the requirements of the upper bound theorem, it is necessary to select the minimum value of all \( F \) values, so the expression of the safety factor of slope stability \( F \) should be as follows:

\[
\frac{\partial F}{\partial \phi_t} = 0, \quad \frac{\partial F}{\partial \theta} = 0 \tag{4-12}
\]

\[
\alpha < \theta < \beta \tag{4-13}
\]

\[
0 < \phi_t < \arctan \frac{c_0}{m_o} \tag{4-14}
\]

In summary, the above problem can be equivalent to a constrained nonlinear optimization problem, as follows:

\[
\min F = F (\theta, \phi_t) \quad (4-15) 0 < \phi_t < \arctan \frac{c_0}{m_o} \quad (4-16) \alpha < \theta < \beta \tag{4-17}
\]

### 4.2 Example

The gravity of soil \( \gamma = 20 \text{ kN/m}^3 \), \( \alpha = 0^\circ \), \( \beta = 90^\circ \), \( c_0 = 90 \text{ kPa} \), and \( \sigma_t = 247.3 \text{ kPa} \). The values of \( m \) are 1.2, 1.4, 1.6, 1.8, 2.0, and 2.5. The slope stability coefficient \( N_s \) is calculated; the angle between the fracture surface and the horizontal direction of the slope \( \theta \) is calculated; the angle between the direction of velocity and the fracture surface is \( \phi_t \). The limit slope height \( H \) is calculated. A
MATLAB program is used to optimize all independent variables simultaneously. The calculation results are shown in Table 4-1.

Table 4-1 Comparison between limit analysis upper limit method and traditional limit equilibrium method

| m value | Limit slope height H (m) calculated by limit equilibrium method | The limit slope height H (m) calculated by this method | Floating range |
|---------|---------------------------------------------------------------|------------------------------------------------------|----------------|
| 1.2     | 23.93                                                         | 24.11                                               | 0.75%          |
| 1.4     | 22.79                                                         | 23.02                                               | 1.01%          |
| 1.6     | 22.02                                                         | 22.26                                               | 1.09%          |
| 1.8     | 21.45                                                         | 21.71                                               | 1.21%          |
| 2.0     | 21.02                                                         | 21.27                                               | 1.19%          |
| 2.5     | 20.31                                                         | 20.60                                               | 1.43%          |

Fig. 4.2 Relationship between m and limit slope height H

Table 4-1 and Figure 4.2 show that the limit height H of the slope decreases gradually with an increase in the value of m using the nonlinear M-C yield criterion. This changing trend can provide a basis for us to analyze the limit height of the slope in the high geo-stress area. In this study, the slope limit height calculated by the upper bound method of limit analysis is slightly greater than that calculated by the traditional limit equilibrium method, and the slightly greater range is between 0.88% and 1.43%, which meets the requirements of the upper bound method of limit analysis, verifying the correctness of the calculation method in this study.

Table 4-2 Other parameters of slope obtained by limit analysis method

| m value | θ(radian) | φₜ(radian) | Nₛ in References [3] | Nₛ calculated by this paper method | Floating range |
|---------|-----------|------------|-----------------------|------------------------------------|----------------|
| 1.2     | 0.9260    | 0.2812     | 5.13                  | 5.32                               | 3.57%          |
| 1.4     | 0.9023    | 0.2338     | 4.89                  | 5.06                               | 3.36%          |
| 1.6     | 0.8854    | 0.1999     | 4.73                  | 4.89                               | 3.27%          |
| 1.8     | 0.8726    | 0.1744     | 4.60                  | 4.77                               | 3.56%          |
| 2.0     | 0.8627    | 0.1546     | 4.52                  | 4.67                               | 3.21%          |
| 2.5     | 0.8495    | 0.1209     | 4.35                  | 4.51                               | 3.55%          |
When the fracture surface of the slope is a straight line, as the m value increases, θ the values of $\phi_t$ decrease gradually, as shown in Figure 4.3. Therefore, under the nonlinear M-C yield condition, the main reason why the value of nonlinear parameter m affects the stability of the slope is that its change will change the angle between the failure and horizontal planes of the slope θ and the Shear strength index Ct of soil. Figure 4.4 shows that the maximum error of the slope stability coefficient $N_s$ calculated in this section is less than 3.6% compared with that in reference [3]. Moreover, because the upper bound theorem is adopted, the slope stability coefficient obtained is slightly larger than that in the reference study above, which agrees with the property of the upper bound method. Therefore, the correctness of this method in this study is proved.

Using the soil parameters provided in the above example, suppose $\alpha = 0^\circ$, $\beta = 45^\circ$, $60^\circ$, $90^\circ$, $H = 20$ m is the specified slope height. By solving Equations (3-7) and (3-8), the slope stability safety factor F is introduced. The results obtained after the substitution are shown in Equation (4-11). F is regarded as a dependent variable, and all independent variables are optimized by MATLAB programs. Different slope angles under the straight fracture surface are obtained as β. The corresponding safety factor F is compared with the results obtained using the traditional limit equilibrium method, and the specific results are shown in Table 4-3 and Figure 4.5.

Table 4-3 Comparison between limit analysis upper limit method and traditional limit equilibrium method

| Slope angle β | Safety factor F of limit equilibrium method | Safety factor F of upper bound method in limit analysis | Floating range |
|---------------|---------------------------------------------|-------------------------------------------------------|----------------|
| 45            | 2.970                                       | 2.978                                                 | 0.27%          |
| 60            | 2.127                                       | 2.132                                                 | 0.24%          |
| 90            | 1.348                                       | 1.354                                                 | 0.45%          |

According to Table 4-3 and Figure 4.5, the safety factor calculated by the traditional limit equilibrium method is very close to the safety factor calculated by the limit analysis method in this study, and the fluctuation range is less than 0.5%. Because the upper bound method of plastic limit analysis is used in this study, the results of this study should be slightly larger than those of the limit equilibrium method, which proves the correctness of the method in this section.

5. Upper bound method for limit analysis of reinforced soil slope

5.1 Theory

The power of the external force is
In comparison to the previous calculation's results, \( \alpha = 0 \). Therefore, the expression of the power produced by the external force can be obtained as follows:

\[
W_{\text{external}} = \frac{1}{2} \gamma H^2 (\cot \theta \cot \beta) \sin (\theta - \phi) V \quad (5-1)
\]

For the reinforced soil slope, the internal energy loss rate includes two parts: one is the energy loss rate on reinforcement, and the other is the energy loss rate on the soil. The energy loss rate of soil and reinforcement is expressed as follows:

\[
D_{\text{Soil}} = \int c_v \frac{\sin \theta}{\sin \theta} \cos \phi_1 dL = \frac{c_v H \cos \phi_1}{\sin \theta} (5-3)
\]

\[
D_{\text{rebar}} = \int k_t V \cos \theta \cos (\theta - \phi_1) dL = k_t V H \cos (\theta - \phi_1) \quad (5-4)
\]

In the equation above, \( k_t \) represents the tensile strength of reinforcement per unit section. For uniformly distributed bars, the expression of \( k_t \) is as follows:

\[
k_t = \frac{T}{s} = \frac{T}{h_n} = \frac{T}{H} \quad (5-5)
\]

In the equation, \( T \) represents the tensile strength of reinforcement, kN/m; \( s \) represents the spacing of reinforcement, m; \( n \) represents the number of reinforced layers.

According to the upper bound theorem of limit analysis, Equations (5-2), (5-3), and (5-4) can be used to obtain the limit height of reinforced soil slope:

\[
H_{\text{Reinforced soil}} = \frac{2(c_t \cos \phi_1 + k_t \cos (\theta - \phi_1))}{V(\cot \theta \cot \beta) \sin (\theta - \phi_1)} \quad (5-6)
\]

In the equation, \( c_t = \frac{m-1}{m} c_0 \left( \frac{\sigma_0 \tan \phi_1}{c_0} \right)^{\frac{1}{m}} + \sigma_t \tan \phi_1 \quad (5-7) \); \( \tan \phi_1 = \frac{c_0}{\sigma_0} \left( 1 + \frac{\sigma_n}{\sigma_t} \right)^{\frac{1}{m}} \quad (5-8) \)

If \( K_t \) is 0, it is the expression of ultimate height \( H \) of unreinforced soil, and the result is consistent with that in Section 4. Since the upper limit method of limit analysis is used in this section, the minimum value of all \( H \) needs to be obtained, and \( H \) needs to meet the following conditions:

\[
\frac{\partial H}{\partial \phi_t} = 0 \quad \frac{\partial H}{\partial \theta} = 0 \quad \alpha < \theta < \beta \quad 0 < \phi_t < \arctan \left( \frac{c_0}{\sigma_0} \right) \quad (5-9)
\]

According to the expression of Equation (5-6) and the constraint conditions of Equation (5-9), the minimum value of \( H \) can be obtained by MATLAB programming.

### 5.2 Examples

For the convenience of comparison with the results calculated in Section 4, the same geotechnical material parameters are used, and the tensile strength \( K_t \) of the unit section of reinforcement is 5, 10,
20, 50, and 100 kPa. The calculated ultimate slope height is shown in Table 5-1 and Figure 5.2.

Table 5-1 The ultimate slope height corresponding to different tensile strength

| m value | Calculation limit slope height of unreinforced soil (m) in Section 4 | Limit slope height H (m) calculated using different values of Kt (kPa) |
|---------|---------------------------------------------------------------|---------------------------------------------------------------|
| 1.2     | 24.11                                                       | Kt =5: 24.96, 25.85, 27.61, 32.84, 41.48                      |
| 1.4     | 23.02                                                       | Kt =10: 23.82, 24.62, 26.21, 30.94, 38.71                    |
| 1.6     | 22.26                                                       | Kt =20: 23.01, 23.76, 25.24, 29.65, 36.88                    |
| 1.8     | 21.71                                                       | Kt =50: 22.41, 23.12, 24.53, 28.72, 35.58                    |
| 2.0     | 21.27                                                       | Kt =100: 21.96, 22.64, 23.99, 28.01, 34.61                   |
| 2.5     | 20.60                                                       |                                                             |

As shown in Table 5-1 and Fig. 5.2, when the reinforced soil slope is destroyed by a linear fracture surface, the slope height of the slope gradually decreases with an increase in the value of m. When the value of m is small, the change is steep, and when m increases continuously, the curve gradually becomes gentle. When the value of m is the same, the limit height of the slope increases with an increase in the tensile strength of reinforcement on the unit section of reinforcement, which is consistent with the actual situation in engineering, verifying the correctness of the method in this study.

6. Conclusion

1. The value of nonlinear parameter m has a great influence on the slope. When other conditions remain unchanged, the limit slope height h decreases with an increase in m.

2. With an increase in m value, the values of T and \( \phi_t \) decrease gradually. Therefore, under the nonlinear M-C yield condition, the main reason why the value of nonlinear parameter m affects slope stability is that its change will change the angle between the slope fracture surface and the horizontal plane \( \theta \). The change of the new strength parameter \( c_t \) of soil is also discussed.

3. When other conditions remain unchanged, the slope stability coefficient Ns decreases with an increase in the nonlinear parameter m.

4. When other conditions remain unchanged, including the slope angle \( \beta \), the larger the slope, the lower the safety factor F.

5. In the reinforced soil slope, the slope height h decreases with an increase in m value. When m is small, the change is steep. When m is increasing, the change in the curve is slow. When m is the same, the ultimate height H of the slope increases with an increase in the tensile strength Kt of reinforcement per unit section.

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