Structural Optimization of Machine Tool Column Based on Dynamic Characteristic Analysis and Response Surface

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Abstract. Based on digital simulation technology, the dynamic characteristics of a NC machine tool column are studied, including modal analysis and harmonic response analysis. Then the response surface of design parameters and indicators is obtained based on the experimental design. Finally, the optimal design is obtained by using genetic algorithm based on response surface.

1. Introduction
An important development trend of modern mechanical equipment is precision and high speed, especially all kinds of machine tools and processing centers. Because of the elasticity of machine tool components, the real-time change of cutting force and the influence of environment in the process of machine tool processing, the machine tool will inevitably produce vibration. Vibration not only reduces the machining accuracy and surface quality of the workpiece, but also brings many hazards to the machine tool itself, such as accelerating tool wear, accelerating fatigue of transmission parts and so on. Dynamic research can effectively alleviate vibration problems and improve the accuracy and life of machinery manufacturing equipment.

In this study, the dynamic characteristics of a NC machine tool column are analyzed based on dynamic numerical simulation technology, and the response surface is obtained based on experimental design. Finally, the structure is optimized.

2. Technological process
The research is based on ANSYS Workbench platform. Because of the use of parametric analysis technology, design variables need to be specified in three-dimensional modeling. The three-dimensional model of the main body of the NC vertical machining tool is shown in Figure 1.

The machine tool bed is mainly composed of base, pillar, spindle box and so on. The pillar is the support of the spindle components, which directly affects the feed rate and cutting depth. Therefore, the stiffness of the pillar has a great impact on the accuracy of the machine. The pillars should be matched with the bed and spindle box, and the relevant dimensions should not be modified as far as possible. Therefore, the following three design variables are determined: the span under the column is marked as a; the wall thickness of the column is marked as b; and the wall thickness of the reinforcement is marked as c, as shown in Figure 2.

Static structure analysis, modal analysis and harmonic response analysis tasks are established in Workbench, which share the same three-dimensional model. Considering that harmonic response analysis is based on modal analysis, it is necessary to import the results of modal analysis into harmonic
response analysis. In each analysis task, the result data focused on are designated as variables, including the maximum static stress value, the first natural frequency in modal analysis, and the response value of the specified position in the harmonic response. These variables are the most important performance indicators for evaluating the dynamic and static characteristics of machine tool columns. On the basis of three analysis tasks, experimental design is carried out. The experimental samples are determined according to the central composite method, and response surface is created according to the specified performance parameters as response values. Optimum design based on response surface can avoid multiple simulation analysis and improve design efficiency.

The model was preprocessed. In order to reduce the amount of calculation, only the column is taken out as the object of analysis. In order to fit the actual load-bearing situation as much as possible, the remote load is adopted, that is, the position of the spindle is taken as the stress point, and the remote load is added to the area where the column and the spindle box cooperate, as shown in Figure 3. The bolt holes are fixed at the bottom of the column. Automated partition of entity grid is adopted.

3. Dynamic characteristic analysis

3.1. Modal analysis

Modal and Harmonious Response Analysis is the Main Method of Dynamics Research. Modal analysis is a numerical technique for calculating the vibration characteristics of structures. Its main purpose is to obtain the natural frequencies and modes of structures. Modal analysis can effectively avoid resonance or vibration at a specific frequency.

The dynamic equation for undamped modal analysis is

$$\{M\}\{\ddot{x}\} + \{K\}\{x\} = \{0\}$$  \hspace{1cm} (1)

The free vibration of the structure is simple harmonic vibration, so the displacement is

$$x = x_0 \sin(\omega t)$$  \hspace{1cm} (2)

The equation (3) is obtained by introducing (2) into (1).

$$\{[K] - \omega^2[M]\}\{x\} = \{0\}$$  \hspace{1cm} (3)

From equation (3), eigenvalues and corresponding eigenvectors can be calculated, in which eigenvalues $\omega_i$ are natural circular frequencies and eigenvectors $\{x_i\}$ are vibration modes.

In ANSYS Workbench, the modal analysis of the column can be performed by calling the Modal module. Although objects have infinite order natural frequencies, from the point of view of mechanical design, we need to focus on the natural frequencies of low frequency band, because it is very resonance-inducing. Therefore, the results of the first six orders are queried, as shown in Table 1 and Figure 4-9.
### Table 1. The first six natural frequencies

| Order | Natural Frequencies /Hz | Mode Shapes                                      |
|-------|-------------------------|---------------------------------------------------|
| 1     | 73.93                   | The column swings around the x-axis               |
| 2     | 124.38                  | The column swings around the z-axis               |
| 3     | 277.78                  | The column swings around the y-axis               |
| 4     | 328.16                  | The upper part of the column moves along the x-axis |
| 5     | 342.31                  | Column twisting around Y axis                     |
| 6     | 392.67                  | Column twisting around z-axis                     |

#### Figure 4. First-order

#### Figure 5. Second-order

#### Figure 6. Third-order

#### Figure 7. Fourth-order

#### Figure 8. Fifth-order

#### Figure 9. Sixth-order

### 3.2. Harmonic response analysis

Harmonic response analysis is to analyze the response of the structure under harmonic loads of different frequencies and amplitudes. By detecting the resonance, the weak stiffness of the structure can be found, so as to achieve targeted structural optimization and improvement. The natural frequencies and modes of the structure are obtained by modal analysis, and the influence of external forces at different frequencies on the structure can be obtained by harmonic response analysis.

When the load of the structure changes with time according to the harmonic law, its equation is as follows:

\[ [M]\{x''\}+[C]\{x'\}+[K]\{x\} = \{F\} \quad (4) \]

Since \( \{F\} \) and \( \{x\} \) represent harmonic load and displacement respectively, the following relations can be obtained:

\[ \{F\} = \{F_0 e^{j\omega t}\} e^{j\omega t} = (\{F_1\} + i\{F_2\}) e^{j\omega t} \quad (5) \]
\[ \{x\} = \{x_0 e^{j\omega t}\} e^{j\omega t} = (\{x_1\} + i\{x_2\}) e^{j\omega t} \quad (6) \]
Where, \( \omega \) is Frequency of Excitation Load; \( \psi \) Phase angle of exciting load; \( \{ F_1 \} \) is The Real Part of the Load; \( \{ F_2 \} \) is Imaginary part of load; \( \{ x_1 \} \) is Real parts of displacement; \( \{ x_2 \} \) is Imaginary parts of displacement.

Equations (5) and (6) are introduced into equation (4) to obtain the characteristic equation of harmonic response analysis.

\[
(K - \omega^2[M] + i\omega[C])(\{ x_1 \} + i\{ x_2 \}) = (\{ F_1 \} + i\{ F_2 \})
\] (7)

Because of the structure and constraints of the machine tool column, the response of each point on it is different. Considering that the area matched with the spindle box directly affects the processing accuracy, the displacement response value of this area is focused on, as shown in Figure 10. Figure 11 shows that the x-direction displacement has greater response when the excitation frequency is about 124 Hz and 328 Hz. Figure 12 shows that the y-direction displacement has greater response when the excitation frequency is about 74 Hz, 124 Hz, 277 and 392. Figure 13 shows that the z-direction displacement has greater response when the excitation frequency is about 74 Hz, 124 Hz and 328 Hz. Comprehensive comparison shows that the structural stiffness of columns in Y and Z directions is relatively weak.

Figure 10. Areas of focus
Figure 11. X-direction displacement response
Figure 12. Y-direction displacement response
Figure 13. Z-direction displacement response

4. Experimental design and response surface construction
Response Surface Design (RSD) is an effective method to establish the relationship between test indices and continuous variables through a small amount of experimental data. Its basic idea is to approximate a polynomial model with explicit expression in a small area to express a complex unknown function.
relationship. This method is based on experimental design and mathematical statistics. It not only considers the random error of experiment, but also calculates simply. It is an effective means to solve the problem.

In this experiment, the design variables are span $a$, column wall thickness $B$ and reinforcement wall thickness $c$, which are recorded as $P1$, $P2$ and $P3$ respectively. The results of analysis which need to be focused on are taken as the response values, including the column mass, the first natural frequency, the maximum static stress and the displacement response values of the specified position in the harmonic response, which are recorded as $P4$-$P9$ in turn.

This method combines the traditional distribution of interpolation nodes with the design of full or partial factors. The experimental data are shown in Table 2.

**Table 2.** Three Factor Central Compound Experiments Table

|     | P1/mm | P2/mm | P3/mm | P4/kg | P5/Hz | P6/MPa | P7/mm | P8/mm | P9/mm |
|-----|-------|-------|-------|-------|-------|--------|-------|-------|-------|
| 1   | 560   | 18    | 15    | 474.7429 | 73.93009 | 66.74668 | 0.052774 | 0.007698 | 0.043699 |
| 2   | 504   | 18    | 15    | 531.0856 | 77.01912 | 66.6999 | 0.0497 | 0.006585 | 0.025106 |
| 3   | 616   | 18    | 15    | 425.9552 | 75.6173 | 62.09837 | 0.048061 | 0.006493 | 0.036552 |
| 4   | 560   | 16.2  | 15    | 546.6729 | 74.03675 | 65.00346 | 0.048319 | 0.007133 | 0.040311 |
| 5   | 560   | 19.8  | 15    | 499.7859 | 72.06995 | 61.02433 | 0.049724 | 0.007537 | 0.042858 |
| 6   | 560   | 18    | 13.5  | 465.5954 | 75.1682 | 72.49665 | 0.054976 | 0.007245 | 0.029157 |
| 7   | 560   | 18    | 16.5  | 483.8331 | 72.7864 | 61.0030 | 0.046217 | 0.006089 | 0.030193 |
| 8   | 514.4701 | 16.53654 | 13.78045 | 493.0992 | 79.2702 | 79.18216 | 0.052986 | 0.007084 | 0.030268 |
| 9   | 605.5299 | 16.53654 | 13.78045 | 405.6569 | 80.68867 | 78.919 | 0.054862 | 0.007475 | 0.030913 |
| 10  | 514.4701 | 19.46346 | 13.78045 | 531.9726 | 75.88097 | 62.89568 | 0.043244 | 0.007081 | 0.039864 |
| 11  | 605.5299 | 19.46346 | 13.78045 | 448.4342 | 77.28036 | 62.84295 | 0.045714 | 0.006138 | 0.02801 |
| 12  | 514.4701 | 16.53654 | 16.21955 | 508.0554 | 77.13119 | 66.21922 | 0.042199 | 0.005662 | 0.027081 |
| 13  | 605.5299 | 16.53654 | 16.21955 | 420.6132 | 78.45866 | 65.58773 | 0.043865 | 0.005999 | 0.034738 |
| 14  | 514.4701 | 19.46346 | 16.21955 | 546.6722 | 74.03675 | 65.00346 | 0.048319 | 0.007133 | 0.040311 |
| 15  | 605.5299 | 19.46346 | 16.21955 | 463.1337 | 75.38955 | 64.74619 | 0.043115 | 0.005767 | 0.029631 |

The expression of response surface is as follows.

$$y(x) = \alpha_0 + \sum_{i=1}^{k} \alpha_i x_i + \sum_{i=1}^{k} \alpha_i^2 x_i^2 + \sum_{i=j} \alpha_{ij} x_i x_j + \varepsilon$$

(8)

Where, $k$ is number of design variables; $x_i$ is design variables; $\alpha_i, \alpha_i^2, \alpha_{ij}$ are undetermined coefficients of polynomials; $\varepsilon$ is error.

After n experiments, the response surface model can be expressed as equation (9).

$$Y = X A + \varepsilon = Y + \varepsilon$$

(9)

Where, $Y$ is AMESIM simulation value; $y$ is response surface approximation function value; $\varepsilon$ is fitting error. The parameters of quadratic polynomial can be estimated by least square method.

$$L = \sum_j \varepsilon_j^2 = (Y - X A)^{\top} (Y - X A)$$

(10)

$$a = (X^{\top} X)^{-1} X^{\top} Y$$

(11)

Based on the above equation, the response surface model can be obtained. Two variables with significant influence on each response value can be displayed as three-dimensional surface, as shown in Figure 14-19.

Through the response surface model, we can more intuitively understand the impact of design variables on indicators. Figure 14 shows that span $a$ has a significant effect on mass. The larger the parameter is, the smaller the mass is. The function shows a parabolic approximation. Other variables have little effect on mass. Figure 15 shows that the stiffener thickness has a significant effect on the
static stress value in a certain range. When the variable is greater than 15, the stress value does not change significantly; when the variable is less than 15, the stress rises sharply.

Figure 14. Response of P4 to P1 and P3

Figure 15. Response of P6 to P1 and P3

Figure 16. Response of P5 to P1 and P3

Figure 17. Response of P7 to P1 and P3

Figure 18. Response of P8 to P1 and P3

Figure 19. Response of P9 to P1 and P3

Figure 16 shows that the influence of span a on the first natural frequency presents a parabolic law. When A is about 550, the natural frequency reaches the lowest. The smaller the reinforcement thickness C is, the higher the first natural frequency value is. From Figure 17, it can be seen that the regularity of the responses of column wall thickness B and reinforcement thickness C to the x-direction harmonic
values is similar to the saddle surface. From Figure 18 and 19, it can be seen that the response values of span a and reinforcement wall thickness C to Y-direction harmonic response and Z-direction harmonic response are parabolic, i.e. the maximum value is reached in the middle region of variables.

5. Structural optimization based on genetic algorithm

Genetic algorithm is a stochastic method to simulate the evolution process of organisms, which is based on natural selection. The algorithm starts with a population representing the potential solution set of the problem. In each generation, individuals are selected according to the fitness of individuals in the problem domain, and genetic operators are used to combine, cross and mutate to produce a population representing the new solution set. According to the principle of survival of the fittest, better and better approximate solutions are gradually generated.

The outstanding advantage of genetic algorithm is that it can find the optimal value in the whole search space, avoiding the problem of getting the optimal value when the parameters fall into the local optimum in the optimization process. Based on the established response surface model, it can be used to optimize the target characteristics of the genetic algorithm population, and no simulation solution is needed, which can greatly improve the computational efficiency.

In order to achieve lightweight design and ensure good comprehensive performance, the optimal design is carried out with the minimum quality as the objective, the maximum static stress not exceeding 80 MPa, the first section natural frequency not less than 80 Hz, and the displacement response in three directions not exceeding 0.05 mm as constraints. Finally, the optimum design parameters are obtained. The span a under the column is 506 mm, the wall thickness B is 16.616 mm, and the wall thickness C is 14.42 mm.

6. Conclusion

Based on the digital simulation technology and experimental design, the dynamic characteristics of the column of a NC machine tool are studied. First, the natural frequencies and modes of the structure are obtained by modal analysis, and then the responses of the structure to external forces at different frequencies are obtained by harmonic response analysis. The response surface was obtained by central composite experiment design, and the genetic algorithm was used to optimize the design based on the response surface. Finally, the lightweight design of the column is realized, and good dynamic characteristics are guaranteed.

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