Parametric thermal analysis of the performance of a thermoelectric generator

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Abstract — A parametric analysis is presented for the performance of a thermoelectric generator based on its operating conditions. The mathematical model, consisting of non linear equations, is made dimensionless to allow the characteristic parameters to be evidenced. The proposed parameterization lends generality to the results obtained. In particular the relationships have been investigated between the temperature difference inside the thermoelectric cell and that on the outside of the generator, and the effect of the outer thermal resistances of the generator on the working conditions. These parameters have a significant effect on the efficiency and therefore on the optimization of the operating conditions of the thermoelectric generator.

1.INTRODUCTION

Thermoelectric generators are devices that allow the conversion of thermal energy directly to electrical energy without moving parts. This characteristic makes the thermoelectric generator compact and suitable for waste heat recovery resulting from industrial processes.

Over the last decade there has been a renewed interest in thermoelectric conversion driven by three main factors. These are: the better performance offered by new materials, ascribable to their chemical composition or by new production processes; the decrease of the costs due to a larger scale application; the new opportunities of utilization due to the high costs of the primary energy sources.

The basic component of the generator is the thermoelectric module, formed of a certain number of thermoelectric couples inserted between two ceramic plates thermally conducting, but electrically insulating. Each thermoelectric couple consists of one p-type and n-type semiconductor. The energy conversion is obtained from each thermoelectric couple by the Seebeck effect.

The thermoelectric generators are made by interposing several modules between two surfaces that transfer heat between two different temperature sources. The modules are placed thermally in parallel and connected electrically in series. The heat flow and the consequent temperature difference that is established between the two junctions of each thermoelectric couple, determine the value of the electric voltage to the ends of the generator. In the case of the latter being under load, a circulating electric current is present.

The performance of a thermoelectric generator depends firstly on the temperature difference that is established between the two junctions of the thermoelectric couples and secondly on the effectiveness of the semiconductors.
The outer temperature difference of the generator is determined by the boundary conditions, whereas the inner temperature difference (between the two junctions) depends on several parameters. As a consequence the inner and outer temperature differences need to be correlated.

Only a few authors have proposed models that allow the calculation of the inner temperature difference as a function of the operating conditions.

In the case of a generator without a resistance load the mathematical model proposed by Rowe and Min [1] allows the relationship between the aforesaid temperature differences to be easily obtained.

For the case of a generator under load, in the study of Chen et al. [2] the model in fact is a simplified one because the circulating current remains part of the problem data. Pramanick and Das [3] in their analysis treat the generator from a thermal point of view, but still with the assumption that the circulating current is known. Chen et al. [4] analyze the generator from a thermal viewpoint and correlate the circulating current with the load resistance. However, the mathematical model is not dimensionless and the solution remains limited to a few specific cases. Casano and Piva [5] validate the model of Chen et al. [4] experimentally. In particular the effects of practical but unavoidable details, such as the clamping of bolts and the fitting of thermal insulation between TEMs, are evidenced. Chen [6] improves his model [4] considering the temperature dependency of the electrical interface resistance associated with thermoelectric devices.

In literature the only analysis of the non linear model proposed by Chen et al. [4] in terms of dimensionless parameters is proposed by Casano and Piva [7]. This parameterization lends generality to the results and offers new elements to the optimization of the operating conditions of the thermoelectric generator.

In the present work, following the poster discussion of [7], a modified dimensionless model of the generator is proposed. The results are discussed on the basis of the consequent new dimensionless numbers.

2. MATHEMATICAL MODEL

2.1 Dimensional Equations

The study of the performance of a thermoelectric generator can be expressed in terms of the analysis of its elementary thermoelectric circuit. In this circuit are inserted a thermoelectric couple and a specific load resistance, $R_l$, given by the ratio between the overall load resistance and the number of thermoelectric couples. In this way the circulating current in the load is the same of a simple generator. Fig. 1 shows the scheme of the circuit, which consists of two different temperature sources and of a thermoelectric couple connected in series with the specific load resistance $R_l$.

The thermoelectric couple, placed between the thermal sources of known temperature, receives a thermal flow $Q_H$ from the hot source with temperature $T_H$ and gives a thermal flow $Q_C$ to the cold source with temperature $T_C$.

![Fig. 1 Elementary circuit scheme](image_url)
Upstream and downstream of the thermoelectric couple the heat flows are thus expressed as:

\begin{align*}
Q_H &= K_1(T_H - T_h) \quad (1) \\
Q_C &= K_2(T_C - T_c) \quad (2)
\end{align*}

where \(K_1\) and \(K_2\) indicate the upstream and downstream thermal conductance of the thermoelectric couple, and \(T_h\) and \(T_c\) are the temperatures of the hot and cold junctions, respectively.

In addition the heat flow into the hot junction of the thermoelectric couple is given by:

\[ Q_h = K(T_h - T_c) + \alpha I T_h - 0.5 R I^2 \quad (3) \]

and the heat flow from the cold junction:

\[ Q_c = K(T_h - T_c) + \alpha I T_c + 0.5 R I^2 \quad (4) \]

The three terms on the right hand side of Eqs. (3-4) are the respective contributions due to the Fourier, Peltier and Joule effects. \(K\), \(\alpha\), \(R\) are the thermal conductance, the Seebeck coefficient and the electrical resistance of the thermoelectric couple, respectively. It is assumed that \(K\), \(\alpha\) and \(R\) are independent of the temperature.

The electrical current, \(I\), appearing in Eqs. (3-4) is given by the following relationship:

\[ I = \frac{\alpha(T_h - T_c)}{R + R_1} \quad (5) \]

The energy balance of the described system presupposes the equality both of Eqs. (1) and (3) and of Eqs. (2) and (4). Consequently the equations that govern the physical system and its performance under the operating conditions become:

\begin{align*}
K_1(T_H - T_h) &= K(T_h - T_c) + \alpha T_h I - 0.5 I^2 R \quad (6) \\
K_2(T_C - T_C) &= K(T_h - T_c) + \alpha T_c I + 0.5 I^2 R \quad (7) \\
I(R + R_1) &= \alpha(T_h - T_c) \quad (8)
\end{align*}

where the unknowns are the values of temperature \(T_h\) and \(T_c\) and the electrical current, \(I\). Equations (6-8) constitute a non linear system resolvable by iteration.

It is opportune to appreciate that in the case of the open circuit, where \(I=0\), the system simplifies to the following two equations:

\begin{align*}
K_1(T_H - T_h) &= K(T_h - T_c) \quad (9) \\
K_2(T_C - T_C) &= K(T_h - T_c) \quad (10)
\end{align*}

The solution of the system of Eqs. (9-10) gives the temperature difference between two junctions of the thermoelectric couple, in agreement with the relationship proposed by Rowe and Min [1]:

\[ T_h - T_c = \frac{T_H - T_C}{1 + \frac{K}{K_1} + \frac{K}{K_2}} \quad (11) \]

The open circuit voltage supplied from the elementary thermoelectric generator is given by:

\[ V_O = \alpha(\Delta T')_O \quad (12) \]

where the temperature difference \((\Delta T')_O = T_h - T_c\) is given by Eq. (11).

The useful electrical power in terms of circulating current and load resistance is given by:

\[ P = I^2 R_1 \quad (13) \]
This useful electric power can be also expressed as the difference between the power supplied by the generator and the Joule effect:

\[ P = \alpha I \left( T_h - T_c \right) - RI^2 \]  

(14)

The conversion efficiency is given by the ratio between the useful electrical power and the thermal power input to the thermoelectric generator:

\[ \varepsilon = \frac{P}{Q_h} \]  

(15)

As an alternative, \( \varepsilon \) can be deduced from Eqs. (1-2) via:

\[ \varepsilon = \frac{Q_h - Q_c}{Q_h} \]  

(16)

### 2.2 Dimensionless equations

The physical model consisting of Eqs. (6-8) is made dimensionless, with reference to the temperatures \( T_h \) and \( T_c \), in terms of the dimensionless temperature defined by:

\[ \theta = \frac{T - T_c}{T_H - T_C} \]  

(17)

together with a dimensionless electric current, \( i \), defined as:

\[ i = \frac{1}{I_{ref}} \]  

(18)

As \( I_{ref} \) is chosen the electric current obtained in the case of upstream and downstream thermal conductance, \( K_1 \) and \( K_2 \), tending to infinity:

\[ I_{ref} = \frac{\alpha(T_H - T_C)}{R + R_1} \]  

(19)

Finally, the following set of dimensionless equations is obtained:

\[ i = \Delta \theta \]  

(20)

\[ \kappa_1 (1 - \theta_h) = \Delta \theta + ZT_H \frac{\Delta \theta (1 - e_c (1 - \theta_h))}{1 + \mu} - \frac{1}{2} ZT_H e_c \frac{\Delta \theta^2}{(1 + \mu)^2} \]  

(21)

\[ \kappa_2 \theta_c = \Delta \theta + ZT_H \frac{\Delta \theta (1 - e_c (1 - \theta_c))}{1 + \mu} + \frac{1}{2} ZT_H e_c \frac{\Delta \theta^2}{(1 + \mu)^2} \]  

(22)

In formulating Eqs. (21-22), then, we have the following dimensionless numbers:

\[ \kappa_1 = \frac{K_1}{K}, \ \kappa_2 = \frac{K_2}{K}, \ \gamma_c = \frac{T_H - T_C}{T_H}, \ \mu = \frac{R_1}{R}, \ ZT_H = \frac{\alpha^2}{KR} \]  

(23)

Eq. (14) for the useful electric power, is made dimensionless as:

\[ \pi = \left( \frac{\Delta \theta^2}{1 + \mu - \frac{\Delta \theta^2}{(1 + \mu)^2}} \right) ZT_H e_c \]  

Similarly, Eq. (15) for the conversion efficiency is made dimensionless, as given by:
3. RESULTS AND DISCUSSION

The performance of the thermoelectric generator are discussed for a wide range of the dimensionless quantities. The attention has been focused on the dimensionless temperature difference, $\Delta \theta$, the useful electrical power, $\pi$, and the conversion efficiency of the generator, $\varepsilon$. Due to the number of dimensionless parameters, the analysis is repeated, ceteris paribus, varying just two parameters.

The three dimensionless parameters, $\Delta \theta$, $\varepsilon$ and $\pi$, are presented in Figs. 2(a-b) as a function of the dimensionless thermal conductance upstream of the generator, $\kappa_1$, for a range of values of the dimensionless thermal conductance downstream of the generator, $\kappa_2$. From an examination of Figs. 2(a-b), certain general trends are in evidence. The main feature projected by these figures is that the effect of a high thermal resistance placed upstream and/or downstream of the thermoelectric couple, often unavoidable in the applications, is strong. Both $\Delta \theta$, $\varepsilon$ and $\pi$ increase consistently with increasing values of the thermal conductance $\kappa_1, \kappa_2$. The comparison of the curves for different values of $\kappa_2$ shows that $\Delta \theta$, $\varepsilon$ and $\pi$ drop-off for low values of $\kappa_2$. It is enough that $\kappa_2$ becomes low and the system becomes insensitive to $\kappa_1$, and viceversa.

The three dimensionless parameters, $\Delta \theta$, $\varepsilon$ and $\pi$, are presented in Figs. 3(a-b) as a function of the dimensionless load resistance, $\mu$, for a range of values of the upstream and downstream dimensionless thermal conductance, $\kappa_1=\kappa_2$. From an examination of Figs. 3(a-b), certain general trends are in evidence. The main feature projected by these figures is that the effect of a high thermal resistance placed upstream and/or downstream of the thermoelectric couple, often unavoidable in the applications, is strong. Both $\Delta \theta$ and $\pi$ as also $\varepsilon$, increase consistently with increasing values of the thermal conductance $\kappa_1, \kappa_2$. The comparison of the curves for different values of $\kappa_2$ shows that $\Delta \theta$, $\varepsilon$ and $\pi$ drop-off for low values of $\kappa_2$. It is enough that $\kappa_2$ becomes low and the system becomes insensitive to $\kappa_1$, and viceversa.

The three dimensionless parameters, $\Delta \theta$, $\varepsilon$ and $\pi$, are presented in Figs. 5(a-b) and Figs. 5(c-d) as a function of the dimensionless load resistance, $\mu$, for $\kappa_1=\kappa_2=2$ and 4, respectively. From an overall examination of these figures, $\Delta \theta$ is seen to regularly increase with both $\mu$ and $\varepsilon_c$. All of the curves share a common shape, which is characterised by a steady, regular rise and a concave downward curvature. The value of $\Delta \theta$ tends to that of open circuit

$$
\varepsilon = \frac{Z T H \varepsilon_c \Delta \theta^2}{\Delta \theta + \frac{Z T H \Delta \theta\left(1 - \varepsilon_c (1 - \theta_H)ight)}{1 + \mu}} \left(1 + \mu\right)^2
$$

$$
\varepsilon = \frac{Z T H \varepsilon_c \Delta \theta^2}{\Delta \theta + \frac{Z T H \Delta \theta\left(1 - \varepsilon_c (1 - \theta_H)ight)}{1 + \mu}} \frac{2}{\left(1 + \mu\right)^2}
$$

(24)
4. CONCLUDING REMARKS

In a lumped parameters model of a thermoelectric generator the circulating current in the load resistance needs to be solved together with the thermal parameters. Many are the physical parameters given by Eq. (11) when \( \mu \) tends to infinity and it is equal to 0.5 and 0.67 for \( \kappa_1=\kappa_2=2 \) and 4, respectively. Both the parameters \( \varepsilon \) and \( \pi \) show a maximum for a different value of the dimensionless load resistance, \( \mu \). The peak value of \( \varepsilon \) and \( \pi \) decrease monotonically as the Carnot effect, \( \varepsilon_c \), decrease. As a final comment with regard to the comparison between Figs. 5(a-b) and Figs. 5(c-d), it can be verified that for \( \Delta \theta \), \( \varepsilon \), \( \pi \) the values becomes larger as \( \kappa_1=\kappa_2 \) increase.

For a range of values of the parameter \( ZT_H \), the three dimensionless parameters, \( \Delta \theta \), \( \varepsilon \) and \( \pi \), are presented in Figs. 6(a-b) and Figs. 6(c-d) as a function of the dimensionless load resistance, \( \mu \), for \( \kappa_1=\kappa_2=2 \), and 4, respectively. From an overall examination of these figures, \( \Delta \theta \) is seen to regularly increase with \( \mu \), but to regularly decrease with an increase of \( ZT_H \). Both \( \pi \) and \( \varepsilon \) show a maximum for a certain value of the dimensionless load resistance, \( \mu \). This peak value of \( \varepsilon \) and \( \pi \) increases as \( ZT_H \) increase. As a final comment with regard to the comparison between Figs. 6(a-b) and Figs. 6(c-d), it can be verified that both for \( \pi \) and \( \varepsilon \), the values become larger as \( \kappa_1=\kappa_2 \) increase.

For a range of values of the parameter \( ZT_H \), the three dimensionless parameters, \( \Delta \theta \), \( \varepsilon \) and \( \pi \), are presented in Figs. 7(a-b) and Figs. 7(c-d) as a function of the Carnot effect, \( \varepsilon_c \), for \( \kappa_1=\kappa_2=2 \) and 4, respectively. From an examination of Figs. 7(a-b) and Figs. 7(c-d), certain general trends are in evidence. The curves of the efficiency conversion, \( \varepsilon \) and the dimensionless useful electric power, \( \pi \), are characterised by a steady, regular rise of a concave upward curvature with \( \varepsilon_c \). The main feature projected by these figures is that the effect of a high figure of merit is strong. The parameters \( \varepsilon \) and \( \pi \) increase consistently with increasing values of \( ZT_H \), the opposite for \( \Delta \theta \). As a final comment with regard to the comparison between Figs. 7(a-b) and Figs. 7(c-d), it can be verified that, both for \( \pi \) and \( \varepsilon \), the values become larger as \( \kappa_1=\kappa_2 \) increase.

For a range of values of the parameter \( ZT_H \), the three dimensionless parameters, \( \Delta \theta \), \( \varepsilon \) and \( \pi \), are presented in Figs. 8(a-b) and Figs. 8(c-d) as a function of the dimensionless load resistance, \( \mu \), for \( \kappa_1=\kappa_2=2 \) and 4, respectively. From an examination of Figs. 8(a-b) and Figs. 8(c-d), certain general trends are in evidence. The curves of the efficiency conversion, \( \varepsilon \) and the dimensionless useful electric power, \( \pi \), are characterised by a steady, regular rise of a concave upward curvature with \( \varepsilon_c \). The main feature projected by these figures is that the effect of a high figure of merit is strong. The parameters \( \varepsilon \) and \( \pi \) increase consistently with increasing values of \( ZT_H \), the opposite for \( \Delta \theta \). As a final comment with regard to the comparison between Figs. 8(a-b) and Figs. 8(c-d), it can be verified that, both for \( \pi \) and \( \varepsilon \), the values become larger as \( \kappa_1=\kappa_2 \) increase.
affecting the performance of this model when considering the generator as an overall device. For this reason often in the literature the discussion remains limited to a few specific cases. On the opposite a dimensionless analysis reduces the number of independent parameters and gives more generality to the
First of all, the upstream and downstream thermal conductance plays the most significant role in the optimisation of the thermoelectric conversion device. It is important to maximise this parameter in order to attain the highest efficiencies.

Another important feature is the necessity of a match between load and internal resistances in order to obtain the maximum efficiency of the generator.

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