Ductile-Brittle Transition in Geomaterials, Caused by Tensile Stress Concentration

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Abstract. The possibility of using known nonlocal and stress gradient fracture criteria to describe brittle, quasi-brittle, and ductile fracture of materials with openings is analyzed. A common property of these criteria is the introduction of intrinsic material length characterizing its structure, which allows one to describe the size effect in conditions of stress concentration. At the same time, the scope of application of nonlocal criteria is limited to cases of brittle or quasi-brittle fracture with a small fracture process zone. To expand the scope of application, new fracture criteria are proposed, which are the development of the average stress criterion, and the stress gradient criterion, and which contain a complex parameter that characterizes the size of the fracture process zone and accounts not only for the material structure, but also plastic properties of the material, geometry of the sample, and its loading conditions. Expressions are obtained for the critical pressure in the problem of the formation of tensile cracks under compression in the samples of geomaterials with a circular hole. The calculation results are in good agreement with the experimental data on the fracture of drilled gypsum plates.

1. Introduction
Ductile-brittle transition is a key problem in arctic materials science. It is directly related to changes in the strength and plastic properties of the material in the case of a low temperature. However, the transition from ductile to brittle fracture can be associated not only with temperature effects, but also with the effect of such structural factors as stress concentration, geometry of structural members, and their loading conditions. Nonlocal fracture criteria are based on the concept of the formation of a fracture process zone in the material, in which there is local redistribution of stresses, whereas the main material is deformed elastically until fracture [1–11]. Typical representatives of such quasi-brittle materials are geomaterials (concrete, gypsum, and rock), composites, high-strength metal alloys, cast iron, graphite, etc. A general property of these criteria is the introduction of intrinsic material length that characterizes its structure, which allows describing the size effect in conditions of stress concentration, thereby expanding the scope of application in comparison with conventional criteria. Pointing to the general property of the nonlocal criteria, Taylor [12] proposes to consider them as partial expressions of some general “theory of critical distances”. Recently, there have been a large number of papers that, in order to describe the fracture of materials with openings, appeal to the theory of critical distances. In 2008, a special issue of the Engineering Fracture Mechanics was devoted to this theory [13]. So-called stress gradient criteria can be classified as a separate group of nonlocal fracture criteria. The most interesting are the criteria developed on the basis of the hypothesis of the
effect of the stress gradient on the local failure stress. Thus, a stress gradient criterion for the tensile cracks initiation under compression is described in [14].

The scope of application of nonlocal criteria is, first of all, the brittle fracture of materials with openings. To describe ductile fracture, additional information is needed on the inelastic behavior of the material in the fracture process zone or on the material constants characterizing its plastic properties. An intermediate position between brittle and ductile fracture is occupied by quasi-brittle fracture region. Paper considers the possibility of expanding the scope of application of nonlocal criteria for cases of fracture with a developed process zone with the aim of predicting the ductile-brittle transition in the fracture, caused by stress concentration.

2. Theoretical approaches

Below, quasi-brittle fracture is understood as a sudden propagation of an unstable crack (also characteristic of brittle fracture), accompanied by the formation of a large fracture process zone. The size \( d \) of the fracture process zone is not associated with the size of the crack, as is customary in fracture mechanics, but with the characteristic size \( d_0 \) of the material structure. We talk about brittle fracture if \( d = d_0 \) and ductile fracture if \( d >> d_0 \). The redistribution of stresses within the limits of \( d_0 \) is not related with the plastic (in the macroscopic sense) deformation of the material. The plastic properties of the material begin to appear when \( d > d_0 \), and the larger \( d \) with respect to \( d_0 \), the more they manifest themselves. Taking this into account, we represent \( d \) in the following form:

\[
d = d_0 + \beta L_c.
\]

Here \( L_c \) is the size of the stress concentration zone, \( \beta \) is the dimensionless parameter that characterizes the plasticity of the material. For brittle materials, \( \beta = 0 \); for plastic materials, \( \beta >> 1 \).

In the case where \( \beta \sim 1 \), the material is characterized by moderate plastic properties.

For ductile fracture, the critical stress does not depend on the size of the opening, so the size of the damage zone is proportional to the size of the opening and, accordingly, to the size \( L_c \) (under unchanged boundary conditions). In brittle fracture, on the contrary, the size of the damage zone does not depend on the size of the opening and is determined by the structure of the material.

During compression, the behavior of the failure stress characterizing the appearance of tensile cracks at the opening has the form shown in Figure 1. For small values of \( L_c \), the material does not feel the stress concentration and collapses as a smooth sample when the applied pressure reaches the compressive strength \( C_0 \). After reaching the critical size of the opening, the failure pressure \( p_c \) decreases, asymptotically approaching the tensile strength of the material \( T_0 \) in the case of brittle failure and the stress \( T_0 \) in the case of ductile fracture.

This behavior of the failure stress agrees with the current understanding of the real solid as containing primary, inherent defects. Accordingly, small artificial defects, whose sizes are comparable with the sizes of the microstructural components of the material, do not affect its strength until their sizes reach a certain (critical) value.
Figure 1. Fracture diagrams: 1 – brittle fracture; 2 – quasi-brittle fracture; 3 – ductile fracture.

2.1. Average stress criterion

The most well-known nonlocal criterion is the average stress criterion (ASC), or integral criterion, which has the form $\langle \sigma_e \rangle < \sigma_0$, where $\langle \sigma_e \rangle$ is the equivalent stress averaged at the distance $d$ along the dangerous cross section and characterizing the internal stressed state of the deformed body. For brittle materials, the size of averaging of $d$ is assumed as a material constant, which characterizes the structure of the material: $d = d_0 = \text{const}$, the material strength $\sigma_0$ is also assumed to be a constant.

Critical pressure for the sample with a circular hole [15]:

$$p_c = \chi C_0 \left( \frac{1 + 2d/l}{1 + d/l} \right)^3, \quad l > l_c.$$  \hspace{1cm} (2)

Here $\chi = \frac{T_0}{C_0}$, $l$ is the hole diameter, and $l_c$ is the critical diameter of the hole.

To describe the quasi-brittle fracture, we determine the size of averaging according to equation (1), in which the size of the stress concentration zone has the form $L_e = \frac{\sigma_e}{\left| \nabla \sigma_e \right|}$. For the problem under consideration, $L_e = l/10$. The critical pressure is determined by substituting equation (1) into equation (2) with account for the obtained estimate for $L_e$:

$$p_c = \chi C_0 \left( \frac{1 + 2d_0/l + 0.2\beta}{1 + d_0/l + 0.1\beta} \right)^3.$$  \hspace{1cm} (3)

2.2. Stress gradient criterion

An alternative point of view on the size effect is that, in brittle materials, it is not associated with redistribution of stresses, but with a real increase in the local failure stress. This hypothesis, in particular, underlies the statistical theories of strength.

On the basis of this hypothesis, an approach was proposed to describe brittle fracture [16], according to which the local strength of the material is assumed to be dependent on the size of the stress concentration zone $L_e$. If the size $L_e$ is sufficiently large in comparison with the size of the structural features of the material, the value of the local strength differs slightly from the value of the
ultimate stress $\sigma_0$ determined under the conditions of uniform stress. Conversely, if $L_e$ is comparable with the size of structural features, their effect on the local strength becomes noticeable. Moreover, the smaller the size $L_e$ with respect to the characteristic size of the material structure $L_0$, the larger this effect. Thus, the local strength of the material depends not only on the size of the stress concentration zone $L_e$, but also on the ratio $L_0 / L_e$ that characterizes the scale in the problem under consideration. In view of this, the local strength condition is represented in the form $\sigma_c < f(\sigma_0, L_0 / L_e)$. The local strength function $f(\sigma_0, L_0 / L_e)$ is determined with account for additional conditions that reflect the specific features of the problem under consideration. For the problem of the tensile cracks initiation under compression, the critical pressure at which tensile cracks start from the hole are determined by the expression [14]:

$$p_c = \chi C_0 \left(1 + \left(\frac{L_0}{L_e}\right)^n\right).$$

For the structural parameter $L_0$, we write the expression similar to equation (1):

$$L_0 = d_0 + \beta L_e.$$  (5)

In the initial formulation of the criterion, the parameter $L_0$ has a meaning of the characteristic size of the material structure $d_0$ and is not associated with the fracture process zone. Taking into account equation (5), $L_0$ acquires the meaning of the parameter $d$ in the nonlocal criteria and the increase in local strength is related not only with the real size effect characteristic of brittle materials, but also to the apparent increase due to the redistribution of elastic stresses in the fracture process zone.

Substitution of equation (5) into equation (4) with account for the fact that, $n = 1$ for brittle materials, gives the following result:

$$p_c = \chi C_0 \left(1 + \beta + 10 \frac{d_0}{l}\right).$$  (6)

The third term in the right side of equation (6) determines the contribution of a real increase in the local strength of the material to the size effect, and the second term in the right side of expression (6) determines the contribution of an apparent increase in the local strength of the material to the size effect.

3. Experimental verification
The applicability of the developed quasi-brittle fracture criteria can be verified by the experimental data on the tensile cracks initiation under compression in the samples of geomaterials with circular holes.

We test samples made of gypsum cement of various compositions. One series of samples is made from a gypsum cement with high (higher than 90%) content of hemihydrate gypsum (Gypsum 1), and the second one from the gypsum cement having a low (within 60–70%) content of hemihydrate gypsum in the initial composition (Gypsum 2). The peculiarities of sample preparation and the experimental procedure are given in [17, 18].

The samples from Gypsum 1 demonstrate brittle fracture. The formation of tensile cracks at the contour of the circular hole is sudden for all the investigated diameters of holes, and the crack length at the time of initiation is 5 to 6 cm. With an increase in the hole diameter, there is a decrease in the critical pressure at the time of crack initiation, and further development of cracks leads to failure of the sample in the form of cleaving into two pieces.

The formation of tensile cracks at the contour of the circular hole in the samples of Gypsum 2 have a different character for small and large holes. Their formation on the contour of a small diameter (up
to 5 mm inclusive) is sudden, and the samples demonstrate brittle fracture as in the samples of Gypsum 1. The formation and propagation of cracks on the contour of a hole of large diameter (10 mm and larger) occurs gradually, which is typical for ductile fracture. After the formation of the remote cracks located at a distance from the hole, the opening of primary tensile cracks decreases, their growth stops, and they no longer affect the further fracture of the sample.

3.1. Average stress criterion

Figure 2 shows the experimental data (solid circles) on the load at the moment of tensile cracks initiation from the hole as a function of its diameter, obtained from the samples of Gypsum 1, and the results of the calculations of the critical pressure (curve 1), carried out according to equation (3) for $\beta = 0$. The size $d_0$ is 1.1 mm and comparable to the size of the largest pores. The dashed line is calculated according to the conventional approach. The experimental data (open circles) and the calculation results for Gypsum 2 for $\beta = 2.5$ (curve 2) are also given in Figure 2. The size $d_0$ is 2 mm. The stress $T_s$ is equal to $T_0$ in the first case and $T_s = 2.7T_0$ (solid straight line) in the second case.

Figure 2 illustrates a significant size effect, i.e. the effect of the diameter of the hole on the local strength of the material. When it decreases, the critical pressure increases, reaching the compressive strength limit. When it increases, the critical pressure asymptotically approaches the tensile strength $T_0$ for Gypsum 1 and the stress $T_s$ for Gypsum 2. This behavior is well described by the modified ASC, in which the averaging size $d$ is determined by equation (1).

![Figure 2. Relationship of critical pressure with hole diameter. Calculated by the modified ASC.](image)

3.2. Stress gradient criterion

Figure 3 illustrates the experimental data (solid circles) on the value of the critical load, obtained from the samples of Gypsum 1, and the results of the calculations (curve 1), carried out according to equation (6) for $\beta = 0$. The size $d_0$ is 0.7 mm. The dashed line is calculated according to the conventional approach. The experimental data (open circles) and the results of the calculations for Gypsum 2, carried out according to equation (6) for $\beta = 1.45$ (curve 2), are also given in Figure 3. The size $d_0$ is 1.8 mm. The stress $T_s$ is equal to $T_0$ in the first case and $T_s = 2.7T_0$ (solid straight line) in the second case.
The experimental results are well described by the stress gradient criterion (SGC) in which the size $L_0$ is determined by equation (5).

![Figure 3. Relationship of critical pressure with hole diameter. Calculated by the modified SGC.](image)

4. Discussion

As can be seen from Figures 2, 3, the experimental data obtained for brittle (Gypsum 1) and ductile (Gypsum 2) crack propagation are equally well described by the modified nonlocal criteria that take into account the change in the size of the fracture process zone in accordance with equations (1) or (5). The plasticity parameter $\beta$ is zero in the first case, and it is determined by the form of the nonlocal criterion and depends on the ratio $T_s/T_0$ in the second case. The stress $T_s$ is determined by the asymptotic form of the dependence $p_c(L_0)$ and equals $2.7T_0$ for both the criteria considered.

The representation of the structural parameter $d$ in the ASC (or $L_0$ in the SGC) as a sum of two terms, one of which characterizes the material structure proper and the other one characterizes the zone of inelastic deformations, allows one to describe the process of material fracture more profoundly (in particular, the transition from brittle to ductile fracture).

For Gypsum 1, the plasticity parameter $\beta$ turns out to be equal to zero, which agrees with the sudden formation of tensile cracks observed in the experiment, which is characteristic of brittle fracture. The redistribution of stresses within the limits of the size $d_0$ is associated with the discreteness of the structure of the material rather than its plastic properties. Gypsum 2 has plastic properties, and $\beta$ is not zero. But these properties begin manifesting only when $\beta L_0$ is larger than $d_0$.

In the samples with small-diameter holes, the plastic zone is also small and does not affect the fracture pattern. With increasing diameter, the size of the plastic zone increases, and it determines the ductile character of the crack propagation, which is observed experimentally. Thus, the condition of brittle fracture of the samples with stress concentrations can be represented in the form $\beta L_0 < d_0$. For $\beta = 0$, the material is brittle by default; for $\beta > 0$, the failure mode (brittle or ductile) is determined by the size and form of the opening, as well as loading conditions (boundary conditions). In the case of a circular hole, one can make the following estimate for the diameter of the hole $l^*$, starting with which the crack propagation becomes ductile:

$$l^* = 10d_0 / \beta.$$  \hspace{1cm} (7)

Diameter $l^*$ calculated for Gypsum 2 according to equation (7) is 8.0 mm for the modified ASC, and 10.6 mm for the modified SGC. Although equation (7) is an estimation expression, the calculated diameters of the hole $l^*$ are in good agreement with what is observed in the experiment.
5. Conclusions
The scope of application of known nonlocal fracture criteria is limited to cases of brittle or quasi-brittle fracture with a small fracture process zone. To expand the scope of application for cases of fracture with a developed fracture process zone, it is proposed to abandon the hypothesis of the size of the process zone as a material constant associated only with its structure. The structural parameter underlying the nonlocal criteria combined into the “theory of critical distances” should be considered as a material constant only in one particular case, which is brittle fracture. For quasi-brittle materials, the size of the fracture process zone is represented as the sum of two terms, one of which characterizes the material structure itself, and the other one refers to the zone of inelastic deformations, whose size is determined by the structural features of the member (notch shape, notch size, loading conditions) and plastic properties of the material. The ratio of these sizes determines which fracture mechanism (brittle or ductile) is to be implemented. The plastic properties that determine the ductile character of fracture manifest only when the size of the zone of inelastic deformations exceeds the characteristic size of the structure of the material. Otherwise, the plastic zone is small and the fracture is brittle.

The proposed approach is used in the development of new (modified) average stress criterion, and stress gradient criterion. The applicability of the developed fracture criteria is verified on the problem of the tensile cracks initiation under compression in the samples of geomaterials with circular holes. It is shown that the modified criteria describe well the experimental data on the fracture of quasi-brittle materials. In addition, the application of the developed criteria makes it possible to explain the change in the fracture mechanism from brittle to ductile with an increase in the size of the hole, observed in the experiment.

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