Electroweak & QCD corrections to
Drell Yan processes

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in honour of the 60th birthday of Prof. S. Jadach
The relevance of single-$W$ and single-$Z$ production processes at hadron colliders is well known: in the present paper the status of theoretical calculations of Drell-Yan processes is summarized and some results on the combination of electroweak and QCD corrections to a sample of observables of the process $pp \rightarrow W^{\pm} \rightarrow \mu^{\pm} + X$ at the LHC are discussed. The phenomenological analysis shows that a high-precision knowledge of QCD and a careful combination of electroweak and strong contributions is mandatory in view of the anticipated LHC experimental accuracy.

One of the authors (O.N.) dedicates these notes to Prof. S. Jadach, in honour of his 60th birthday and grateful for all that Prof. Jadach taught him during their fruitful collaboration.

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1. Introduction

Single-$W$ and single-$Z$ production processes are today considered of utmost relevance for the physics studies at contemporary hadron colliders such as Tevatron and the LHC. Actually, charged and neutral current Drell-Yan (D-Y) processes, i.e. $pp \rightarrow W \rightarrow l\nu + X$, and $pp \rightarrow Z/\gamma \rightarrow l^+l^- + X$ play a very important role, since they have huge cross sections (e.g. $\sigma(pp \rightarrow W \rightarrow l\nu + X) \sim 20$ nb at LHC and about a factor of ten less for $\sigma(pp \rightarrow Z/\gamma \rightarrow l^+l^- + X)$) and are easily detected, given the presence of at least a high $p_T$ lepton, which to trigger on. For this reasons and also because the physics around $W$ and $Z$ mass scale is presently known with high precision after the LEP and Tevatron experience, D-Y processes will provide standard candles for detector calibration during the first stage of LHC running. Moreover, single-$W$ as signal by itself will allow to perform a precise measurement of the $W$ mass with a foreseen final uncertainty of the order of 15 MeV at LHC (20 MeV at Tevatron), a very important ingredient for precision tests of the Standard Model, when associated with a top mass uncertainty of the order of 1-2 GeV. Also, from the forward-backward asymmetry of the charged lepton pair in $pp \rightarrow Z/\gamma \rightarrow e^+e^-$ the mixing angle $\sin^2 \theta_W$ could be extracted with a precision of $1 \times 10^{-4}$. Useful observables for the measurement of the $W$ mass are the transverse mass distribution and the charged lepton transverse momentum distribution. While the latter is in principle experimentally cleaner, the former is less sensitive to the effects of higher order radiative corrections affecting the theoretical predictions.

The few per cent level precision in principle achievable in the cross sections motivated a proposal to use these observables as luminosity monitor for the LHC. Last, single-$W$ and single-$Z$ processes will provide important observables for new physics searches: in fact the high tail of the $l^+l^-$ invariant mass and of the $W$ transverse mass is sensitive to the presence of extra
gauge bosons predicted in many extension of the Standard Model, which could lie in the TeV energy scale detectable at LHC.

For the above reasons, it is of utmost importance to predict the $W$ and $Z$ observables with as high as possible theoretical precision. The sources of uncertainty in the theoretical predictions are essentially of perturbative and non-perturbative origin. The latter ones comprise the uncertainties related to the parton distribution functions and power corrections to resummed differential cross sections, which will not be discussed here. In the following we review the current state-of-the-art on the calculation of higher order QCD and electroweak (EW) radiative corrections and their implementation in simulation tools, and we present some original results about the combination of QCD and EW corrections to $W$ production at the LHC.

2. Status of theoretical calculations

2.1. Higher-order QCD/EW calculations and tools

In the present section, a sketchy summary of the main computational tools for EW gauge boson production at hadron colliders is presented. Concerning QCD calculations and tools, the present situation reveals quite a rich structure, that includes next-to-leading-order (NLO) and next-to-next-to-leading-order (NNLO) corrections to $W/Z$ total production rate [1,2], NLO calculations for $W, Z + 1, 2$ jets signatures (available in the codes DYRAD and MCFM) [3,4], resummation of leading and next-to-leading logarithms due to soft gluon radiation (implemented in the Monte Carlo ResBos) [5,6], NLO corrections merged with QCD Parton Shower (PS) evolution (in the event generators MC@NLO and POWHEG) [7,8], NNLO corrections to $W/Z$ production in fully differential form (available in the Monte Carlo program FEWZ) [9,10], as well as leading-order multi-parton matrix elements generators matched with vetoed PS, such as, for instance, ALPGEN [11], MADEVENT [12], HELAC [13] and SHERPA [14].

As far as complete $O(\alpha)$ EW corrections to D-Y processes are concerned, they have been computed independently by various authors in [15, 16, 17, 18, 19] for $W$ production and in [20, 21, 22, 23] for $Z$ production. EW tools implementing exact NLO corrections to $W$ production are DK [15], WGRAD2 [16], SANC [18] and HORACE [19], while ZGRAD2 [20], HORACE [22] and SANC [23] include the full set of $O(\alpha)$ EW corrections to $Z$ production. The predictions of a subset of such calculations have been compared, at the level of same input parameters and cuts, in the proceedings of the Les Houches 2005 [24] and TEV4LHC [25] workshops for $W$ production, finding a very satisfactory agreement between the various, independent calculations. A first set of tuned comparisons for the $Z$ production process has been performed and is available in [26].
From the calculations above, it turns out that NLO EW corrections are dominated, in the resonant region, by final-state QED radiation containing large collinear logarithms of the form $\log(\hat{s}/m_l^2)$, where $\hat{s}$ is the squared partonic centre-of-mass (c.m.) energy and $m_l$ is the lepton mass. Since these corrections amount to several per cents around the jacobian peak of the $W$ transverse mass and lepton transverse momentum distributions and cause a significant shift (of the order of 100-200 MeV) in the extraction of the $W$ mass $M_W$ at the Tevatron, the contribution of higher-order corrections due to multiple photon radiation from the final-state leptons must be taken into account in the theoretical predictions, in view of the expected precision (at the level of 15-20 MeV) in the $M_W$ measurement at the LHC. The contribution due to multiple photon radiation has been computed, by means of a QED PS approach, in [27] for $W$ production and in [28] for $Z$ production, and implemented in the event generator HORACE. Higher-order QED contributions to $W$ production have been calculated independently in [29] using the YFS exponentiation, and are available in the generator WINHAC. They have been also computed in the collinear approximation, within the structure functions approach, in [30].

It is worth noting that, for what concerns the precision measurement of $M_W$, the shift induced by higher-order QED corrections is about 10% of that caused by one-photon emission and of opposite sign, as shown in [27]. Therefore, such an effect is non-negligible in view of the aimed accuracy in the $M_W$ measurement at the LHC.

A further important phenomenological feature of EW corrections is that, in the region important for new physics searches (i.e. where the $W$ transverse mass is much larger than the $W$ mass or the invariant mass of the final state leptons is much larger than the $Z$ mass), the NLO EW effects become large (of the order of 20-30%) and negative, due to the appearance of EW Sudakov logarithms $\propto - (\alpha/\pi) \log^2(\hat{s}/M_V^2)$, $V = W, Z$ [15, 16, 19, 20, 21, 22]. Furthermore, in this region, weak boson emission processes (e.g. $pp \rightarrow e^+\nu_e V + X$), that contribute at the same order in perturbation theory, can partially cancel the large Sudakov corrections, when the weak boson $V$ decays into unobserved $\nu\bar{\nu}$ or jet pairs, as recently shown in [31].

### 2.2. Combination of EW and QCD corrections

In spite of this detailed knowledge of higher-order EW and QCD corrections, the combination of their effects is presently under investigation. Some attempts have been explored in the literature [32, 33, 34]. Here our approach will be discussed in some detail.

A first strategy for the combination of EW and QCD corrections consists
in the following formula

\[
\left[ \frac{d\sigma}{d\mathcal{O}} \right]_{QCD\&EW} = \left\{ \frac{d\sigma}{d\mathcal{O}} \right\}_{MC@NLO} + \left\{ \left[ \frac{d\sigma}{d\mathcal{O}} \right]_{EW} - \left[ \frac{d\sigma}{d\mathcal{O}} \right]_{Born} \right\}_{HERWIG\ PS} \tag{1}
\]

where \( d\sigma/d\mathcal{O}_{MC@NLO} \) stands for the prediction of the observable \( d\sigma/d\mathcal{O} \) as obtained by means of MC@NLO, \( d\sigma/d\mathcal{O}_{EW} \) is the HORACE prediction for the EW corrections to the \( d\sigma/d\mathcal{O} \) observable, and \( d\sigma/d\mathcal{O}_{Born} \) is the lowest-order result for the observable of interest. The label HERWIG PS in the second term in r.h.s. of eq. (1) means that EW corrections are convoluted with QCD PS evolution through the HERWIG event generator, in order to (approximately) include mixed \( \mathcal{O}(\alpha\alpha_s) \) corrections and to obtain a more realistic description of the observables under study. In eq. (1) the infrared part of QCD corrections is factorized, whereas the infrared-safe matrix element residue is included in an additive form. It is otherwise possible to implement a fully factorized combination (valid for infra-red safe observables) as follows:

\[
\left[ \frac{d\sigma}{d\mathcal{O}} \right]_{QCD\otimes EW} = \left( 1 + \frac{[d\sigma/d\mathcal{O}]_{MC@NLO} - [d\sigma/d\mathcal{O}]_{HERWIG\ PS}}{[d\sigma/d\mathcal{O}]_{Born}} \right) \times \left\{ \frac{d\sigma}{d\mathcal{O}_{EW}} \right\}_{HERWIG\ PS}, \tag{2}
\]

where the ingredients are the same as in eq. (1) but also the QCD matrix element residue is now factorized. Eqs. (1) and (2) have the very same \( \mathcal{O}(\alpha) \) and \( \mathcal{O}(\alpha_s) \) content, differing by terms at the order \( \alpha\alpha_s \). Their relative difference has been checked to be of the order of a few per cent in the peak region, and can be taken as an estimate of the uncertainty of QCD & EW combination.

### 3. Numerical results

In order to assess the phenomenological relevance of radiative corrections to D-Y processes, we show the effect of purely EW corrections to Z-boson production at the LHC (\( \sqrt{s} = 14 \) TeV) in Fig. (1). Input parameters, cuts and lepton identification criteria can be found in ref. (22). The set of PDFs used in our study is MRST2004QED (35). As can be seen, EW corrections give huge contributions around the Z peak, dominated by photonic final state radiation. There are important corrections in the hard invariant mass
Fig. 1. Upper panel: HORACE predictions for the $Z$ invariant mass distribution around the peak (left) and in the high tail (right). Lower panel: relative effect of EW corrections.

tail, mainly due to combined photonic and Sudakov effects. Multiple photon corrections are at the some per cent level.

As far as the combination of QCD & EW corrections is concerned, we study, for definiteness, the production process $pp \rightarrow W^\pm \rightarrow \mu^\pm + X$ at the LHC, imposing the cuts shown in Tab. 1, where $p_T^\mu$ and $\eta_\mu$ are the transverse momentum and the pseudorapidity of the muon, $E_T$ is the missing transverse energy, which we identify with the transverse momentum of the neutrino, as typically done in several phenomenological studies. For set up b., a severe cut on the $W$ transverse mass $M_{WT}$ is superimposed to the cuts of set up a., in order to isolate the region of the high tail of $M_{W}$, which is interesting for new physics searches. The QCD factorization/renormalization scale and the analogous QED scale (present in MRST2004QED) are chosen to be equal, as usually done in the literature, and fixed at $\mu_R = \mu_F = \sqrt{p_{T,W}^2 + M_{\ell \nu}^2}$, where $M_{\ell \nu}$ is the $W$-boson invariant mass.

In order to avoid systematic theoretical effects, all the generators under consideration have been properly tuned to reproduce the same LO/NLO results. A sample of our numerical results is shown in Fig. 2 for the $W$
Table 1. Selection criteria imposed for the numerical simulation of single-\(W\) production process at the LHC.

\begin{table}
\centering
\begin{tabular}{|l|}
\hline
a. \(p_{\mu}^\perp \geq 25\,\text{GeV}\) \(E_T \geq 25\,\text{GeV}\) and \(|\eta_\mu| < 2.5\) \\
\hline
b. the cuts as above \(\oplus M_W \geq 1\,\text{TeV}\) \\
\hline
\end{tabular}
\end{table}

Fig. 2. Upper panel: predictions of MC@NLO, MC@NLO+HORACE and leading-order HORACE+HERWIG PS for the \(M_W^\perp\) (left) and \(p_{\mu}^\perp\) (right) distributions at the LHC, according to the cuts of set up a. Lower panel: relative effect of QCD and EW corrections, and their sum, for the corresponding observables in the upper panel.

transverse mass \(M_W^\perp\) and muon transverse momentum \(p_{\mu}^\perp\) distributions according to set up a. of Tab. 1 and in Fig. 3 for the same distributions according to set up b. In each figure, the upper panels show the predictions of the generators MC@NLO and MC@NLO + HORACE interfaced to HERWIG PS (according to eq. (1)), in comparison with the leading-order result by HORACE convoluted with HERWIG shower evolution. The lower panels illustrate the relative effects of the matrix element residue of NLO QCD and full EW corrections, as well as their sum, that can be obtained by appropriate combinations of the results shown in the upper panels. From Fig. 2 it can be seen that QCD corrections are positive around the jacobian peak and tend to compensate the effect due to EW corrections. Therefore, their interplay is crucial for a precise \(M_W\) extraction at the LHC and their combined contribution can not be accounted for in terms of a pure QCD PS approach, as it can be inferred from the comparison of the predictions of MC@NLO versus the leading-order result by HORACE convoluted with
The interplay between QCD and EW corrections in the region interesting for new physics searches, i.e. in the high tail of $M_W^\perp$ and $p_{\mu}^\perp$ distributions, is shown in Fig. 3. For both $M_W^\perp$ and $p_{\mu}^\perp$, NLO QCD corrections are positive and largely cancel the negative EW Sudakov logarithms. Therefore, a precise normalization of the SM background to new physics searches necessarily requires the simultaneous control of QCD and EW corrections.

4. Conclusions

During the last few years, there has been a big effort towards high-precision predictions for D-Y-like processes, addressing the calculation of higher-order QCD and EW corrections. Correspondingly, precision computational tools have been developed to keep under control theoretical systematics in view of the future measurements at the LHC.

We presented some results about EW and QCD corrections to a sample of observables of the $Z$ and $W$ production processes at the LHC. Our investigation shows that a high-precision knowledge of QCD and a careful combination of electroweak and strong contributions is mandatory in view of the anticipated experimental accuracy. We plan, however, to perform a more complete and detailed phenomenological study, including the predictions of other QCD generators and considering further observables of interest for the many facets of the $W/Z$ physics program at the LHC.
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