Optimal Design of Flexspline in New Modular Robotic Joints Considering Position Keeping Conditions

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Abstract. In order to improve the service life of modular joints and meet the requirements of compact structure, considering the position keeping conditions of modular joints, a multi-objective optimization design method for the maximum stress and tube length of the new modular joint flexspline was proposed. This method established a finite element analysis model based on a new modular joint structure, conducted a thermo-mechanical coupling analysis on the joint under the holding state, and obtained the stress distribution of the flexible wheel. Plackett-Burman factorial design was used to screen the significant factors that affect the maximum stress. Based on the Box-Behnken test method and the least square method, the second-order response surface approximate model of the maximum stress of the flexspline was established. Taking the geometric structure parameters of the harmonic drive components as the design variables, and the maximum stress and tube length of the flexspline as the design goals, the response surface approximate model is optimized through the multi-objective genetic algorithm. The optimization calculation example of the new modular joint shows that the method can reduce the maximum stress of the flexspline while reducing the tube length, achieving the purpose of reducing the joint volume and increasing the service life.

1. Introduction
As light service robot industry develops, it proposes higher requirements for the volume and performance of modular joints [1]. Based on the existing harmonic reducer, the common modular joints integrate the drive motor, sensor, etc., which means that the cost, size and performance of the joints will be restricted by the harmonic reducer [2]. Therefore, it is an important way to reduce the joint volume and improve the comprehensive performance of the product by innovating the structure of harmonic drive and optimizing the design and analysis of the structural parameters of key parts.

Harmonic gear transmission is a common transmission mode of joint, among which fatigue fracture usually leads to flexspline failure [3]. In order to improve the working life of joint, it is necessary to design reasonable structure to reduce the transmission stress of flexspline. From the existing research, the optimization design of flexspline is mostly based on harmonic reducer. Based on response surface method and central composite design (CCD) sampling method, Zhang Lei et al. [4] obtained the response surface of flexspline stress and stiffness to structural parameters, and provided the basis for structural design. Xing Jingzhong et al. [5] established a parameterized model of cup bottom with variable thickness for ultra-short tube flexspline, and optimized the structure of cup bottom to reduce the stress of flexspline. Yu Jinbao et al. [6] analyzed the maximum stress of flexspline under no-load and loading conditions, and optimized the parameters of flexspline cylinder through multi-objective
optimization algorithm. At present, there are few researches on the flexspline during the working of joint. Besides, in order to reduce the maximum stress of the flexspline, the optimization design of the structure is carried out. If all the structural parameters of the flexspline are considered, the amount of calculation will be too large. Since the stress field of the flexspline is an implicit function of the design parameters, it fails to directly establish a parameter optimization model. In order to solve the above problems, this paper, first proposes a new joint scheme integrating harmonic drive components. Considering the position keeping condition, the thermal mechanical coupling model is established to analyze the stress on the flexspline. The Plackett Burman experiment is used to screen the factors with significant impact on the stress, and the response surface model based on box Behnken design is established to analyze the harmonic drive. On this basis, the structure and parameters of flexspline, the key component of harmonic drive, are optimized.

2. New Modular Joint Structure
According to the principle of harmonic drive, a new modular joint structure is proposed, as shown in Figure 1. By directly integrating the harmonic drive components and using a diagonal contact ball bearing on the rigid wheel instead of the cross roller bearing to bear the axial and radial forces, the joint integration can be improved and the cost can be reduced.

![Fig.1 Schematic diagram of modular joint structure](image)

1. Brake 2. Encoder 3. Motor’s stator 4. Motor’s rotor 5. Flexspling 6. Circular spline 7. Motor spindle 8. Flexible bearing
3. Analysis of Structure and Stress of Ring Gear of Top Hat-Shaped Flexspline

3.1 Structural Parameters of Top Hat-Shaped Flexspline

![Fig.2 Structure Diagram of Flexspline]

The top hat-shaped flexspline is developed from cup-shaped flexspline. With extrorse flange, it has higher torsional stiffness, excellent rotation accuracy and better hollow structure. The top hat-shaped flexspline structure is shown in Figure 2. The structure of flexspline references the flexspline of 17-80 harmonic reducer made in China, and the parameter value and variation range are shown in Table 1.

| Symbol | Variable                                      | Initial value | Lower limit | Upper limit |
|--------|----------------------------------------------|---------------|-------------|-------------|
| b₁     | Tooth width/mm                               | 8             | 6           | 10          |
| b₂     | Width of wave generator/mm                   | 6.6           | 5           | 8           |
| L      | Tube length/mm                               | 15            | 12          | 18          |
| d₁     | Inner diameter of Flexspline/mm             | 43            | —           | —           |
| d₂     | Flange inner diameter/mm                    | 60            | 50          | 69          |
| h₁     | Wall thickness of ring gear /mm              | 0.6           | 0.5         | 0.74        |
| h₂     | Wall thickness of cylinder /mm               | 0.3           | 0.2         | 0.5         |
| R₁     | Round corner of cylinder bottom/mm          | 0.9           | 0.3         | 1           |
| R₂     | Flange fillet /mm                            | 0.5           | 0.3         | 0.6         |

3.2 Solution of Flexspline Load Condition

There fails to get an analytical solution to the distribution of the force on the gear teeth. The distribution of the force is determined according to reference [7], as shown in Figure 3.
1) Analysis of meshing force

\[
q_i = q_{r,\text{max}} \cos\left(\frac{\pi}{\alpha} - \varphi_1\right) / (\varphi_3 - \varphi_2)
\]

\[
q_r = q_{r,\text{max}} \tan \alpha \cos\left(\frac{\pi}{\alpha} - \varphi_1\right) / (\varphi_3 - \varphi_2)
\]

In the formula, \(\varphi_1\) is the angle of the symmetrical axis of the distributed load relative to the long axis of the wave generator; \(q_i\) is the circumferential distributed load on the unit width of the ring gear; \(q_r\) is the radial distributed load on the unit width of the ring gear; \(\alpha\) is the torque relationship of tooth angle \(q_{r,\text{max}}\) and transmission bearing. If \(\varphi_2 = \varphi_3\), the load torque

\[
T = 4 \int_{\varphi_1}^{\varphi_1 + \varphi_2} \left(\frac{d_\theta}{2}\right)^2 q_{r,\text{max}} \cos\left(\frac{\pi}{\alpha} - \varphi_1\right) / (2\varphi_2) d\varphi
\]

After integral

\[
q_{r,\text{max}} = \frac{\pi T^2}{\left(2\varphi_2 d_\theta^2 b_w\right)}
\]

In the formula, \(d_\theta\) is the diameter of flexspline indexing circle; \(b_w\) is the width of gear ring.

2) Force on joint

According to reference [5], the force is applied according to the node. Suppose the number of nodes on the section of the ring gear is \(N\), then the angle between adjacent nodes is

\[
f_0 = \frac{360}{N}
\]

The relationship between meshing force and load torque in ring gear is as follows

\[
\sum_{i=j_0}^{i_0} 2 \left(\frac{d_i}{2} + h_i\right) F_{i,\text{max}} \cos \left[\frac{\pi}{\varphi_1 - \varphi_2} \left(f_0 i - \varphi_1\right)\right] = 1000M
\]

In the equation, \(m\) refers to the load torque

According to equation (5), the meshing force applied on the joint is calculated:

\[
F_i = F_{i,\text{max}} \cos \left[\frac{\pi}{\varphi_3 - \varphi_2} \left(f_0 i - \varphi_1\right)\right] / \cos \alpha
\]
4. Optimization Design

4.1 Thermal Mechanical Coupling Simulation Analysis

4.1.1 Establishment of Whole Machine Finite Element Model

If there are gears in contact model, the workload in dividing and calculating will be great. In this paper, according to reference [8], the flexspline gear is treated equivalently and simplified as an equivalent gear ring. The bending stiffness of the wall thickness of the gear ring is about 1.67 times that of the tooth root, which is proportional to the third power of the wall thickness. Therefore, the wall thickness of the equivalent gear ring is twice that of the smooth wall thickness of the tooth root. Since we aim to study the stress of Flexspline during operation, other parts are simplified in modeling to reduce the workload of calculation. As shown in Figure 4 is the mesh model of joint and flexspline.

![Fig.4 Finite Element Mesh Model](image)

(a) 1 / 4 Joint Section Mesh Chart
(b) Flexspline Mesh Chart

4.1.2 Temperature Field Analysis

In this paper, the finite element method is used to simulate the position keeping state of the joint. The position keeping state is an important working state of the joint, during which, the motor is required to output constant torque with zero speed, and the motor will release a lot of heat, so it is necessary to establish the thermal mechanical coupling model of the joint for analysis.

According to the research in reference [9], the stator is the main heat source. The continuous locked rotor power of the selected motor is 30W. After converting it into heat generation rate, the heat load is applied to the stator of the motor. The parameters of boundary conditions and heat load are shown in Table 2, and the joint steady-state temperature field is shown in Figure 5.

![Figure 5](image)
Table 2 Heat Dissipation Coefficient and Heat Rate

| Parameter                                                                 | Calculation result   |
|---------------------------------------------------------------------------|----------------------|
| Heat generation rate of motor stator, W/m³                                  | 1.5x10⁶              |
| Convective heat transfer coefficient between motor stator and rotor, W/(m²·℃) | 22.22                |
| Convective heat transfer coefficient between shell and surrounding air, W/(m²·℃) | 13                   |

4.1.3 Stress Distribution of Flexspline

Fix the joint shell. The flexspline is loaded according to formula (4) to (6), \( \phi_3 = -\phi_2 = \pi/8, \ M = 120Nm \). The input shaft takes the motor continuous stall torque of 1.5nm as the input torque, and the temperature load is introduced as the initial condition. Then we carry out the thermal mechanical coupling analysis.

The stress distribution of flexspline is shown in Figure 6. Since the ring gear and cylinder are the weak parts of top hat-shaped flexspline, the stress at the flange is not considered when fixing the flange. It can be seen that during operation, the circumferential upward stress of flexspline presents asymmetric distribution. The maximum stress is 503.26mpa at the contact position between the flexspline and the rear end of the wave generator on the long axis. There is also a large stress at the two positions inside the front end of the flexible gear ring, reaching 497.53mpa. Besides, the stress concentration occurs at the fillet of the cylinder bottom, and the maximum stress value reaches 300.22mpa.
4.2 Plackett-Burman Design Factor Screening  
There are 8 initial variables. Each factor takes two values above and below the initial value. We conduct 12 times of experiments with Plackett Burman design method, and the experimental plan table is shown in Table 3. In the table, $\sigma$ is the maximum equivalent stress. If we conduct real experiment, there requires time in preparing material, processing and testing, which is unnecessary in the optimization design stage. Therefore, the experiment plan is completed by finite element analysis, and the maximum equivalent stress of each combination is obtained.

| Operation serial number | b$_1$ | b$_2$ | L | d$_2$ | h$_1$ | h$_2$ | R$_1$ | R$_2$ | $\sigma$/MPa |
|-------------------------|------|------|---|------|------|------|------|------|-------------|
| 1                       | 6    | 5    | 12 | 65   | 0.50 | 0.5  | 1.0  | 0.3  | 450.92      |
| 2                       | 6    | 5    | 12 | 50   | 0.50 | 0.2  | 0.3  | 0.3  | 442.34      |
| 3                       | 10   | 5    | 12 | 65   | 0.50 | 0.5  | 0.3  | 0.6  | 374.12      |
| 4                       | 10   | 5    | 18 | 65   | 0.50 | 0.5  | 0.3  | 0.6  | 475.28      |
| 5                       | 10   | 8    | 18 | 65   | 0.50 | 0.2  | 0.3  | 0.3  | 384.24      |
| 6                       | 6    | 8    | 18 | 50   | 0.50 | 0.5  | 1.0  | 0.6  | 475.28      |
| 7                       | 10   | 5    | 12 | 50   | 0.74 | 0.5  | 0.3  | 0.6  | 422.34      |
| 8                       | 10   | 5    | 18 | 50   | 0.74 | 0.2  | 1.0  | 0.6  | 384.24      |
| 9                       | 6    | 5    | 18 | 65   | 0.74 | 0.5  | 1.0  | 0.6  | 380.67      |
| 10                      | 10   | 8    | 12 | 65   | 0.74 | 0.5  | 1.0  | 0.3  | 844.04      |
| 11                      | 6    | 8    | 12 | 65   | 0.74 | 0.2  | 0.3  | 0.3  | 837.54      |
| 12                      | 6    | 8    | 18 | 50   | 0.74 | 0.5  | 0.3  | 0.3  | 554.23      |

It can be seen from Figure 7 that the width of the wave generator, the length of the cylinder and the thickness of the ring gear wall are the most three influential factors on the flexspline stress at the specified significance level. The influence of the other factors on the maximum stress is not significant, which can be ignored in the subsequent optimization process for the purpose of reducing the workload. In the later optimization process, the values of other factors are as follows: B1 is 6.6 mm, D2 is 60 mm, H2 is 0.3 mm, R1 is 0.9 mm, R2 is 0.5 mm.

![Fig.7 Pareto Chart of Standardized Effects](image)

The response is the maximum stress of flexspline $\alpha = 0.25$.
4.3 Establishment of Response Surface Methodology

4.3.1 Response Surface Methodology
Response Surface Methodology, or RSM is a method of optimizing experimental conditions, which is easy to solve the problems related to nonlinear model. Through the data obtained from a reasonable experiment, a mathematical model including the first-order term, the square term and the first-order interaction term between any two factors is fitted. By fitting the process and drawing the contour and response surface, the significant degree of the influence of various factors on the response can be reflected. Response surface method is an effective means to study the relationship between target value and independent variables when the number of experimental independent variables is less than 3 \(^{[10-12]}\). The commonly used second-order response surface model is as follows:

\[
Y = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_1^2 + a_5x_2^2 + a_6x_3^2 + a_7x_1x_2 + a_8x_1x_3 + a_9x_2x_3
\]  

(7)

4.3.2 Response Surface Fitting
During the experiment, the width of the wave generator, the length of the cylinder and the thickness of the ring gear are independent variables. To establish a reasonable response surface model, the response surface method based on Box Behnken experimental design method is used to estimate the influence of these three parameters on the stress value. According to three factors in three levels, 13 experiments are generated. Table 4 shows the value assignment and calculated results.

| Operation serial number | b2  | L  | h1  | σ/MPa |
|-------------------------|-----|----|-----|-------|
| 1                       | 6.5 | 15 | 0.75| 393.9 |
| 2                       | 5   | 12 | 0.75| 365.9 |
| 3                       | 8   | 12 | 0.75| 910.1 |
| 4                       | 5   | 18 | 0.75| 307.9 |
| 5                       | 8   | 18 | 0.75| 625.7 |
| 6                       | 5   | 15 | 0.5 | 423.3 |
| 7                       | 8   | 15 | 0.5 | 737.0 |
| 8                       | 5   | 15 | 1   | 475.0 |
| 9                       | 8   | 15 | 1   | 698.8 |
| 10                      | 6.5 | 12 | 0.5 | 514.7 |
| 11                      | 6.5 | 18 | 0.5 | 408.9 |
| 12                      | 6.5 | 12 | 1   | 459.6 |
| 13                      | 6.5 | 18 | 1   | 391.8 |

The optimal regression equation of response value is as follows

\[
\sigma = 2003.8 - 511.4b_2 + 19.49L - 801.9h_1 - 12.5b_2L - 59.9b_2h_1 + 12.6Lh_1 + 66.2b_2^2 + L^2 + 647.9h_1^2
\]  

(8)

Fig. 8 is the response surface of stress to various factors. It can be seen from figure (a) that when the wall thickness of the ring gear is fixed, the stress increases with the increase of the width of the wave generator and decreases with the increase of the cylinder length. The stress range is 334 mPa to 924 mPa. It can be seen from figure (b) that when the cylinder length is fixed, the stress first decreases and then increases as the width of the wave generator increases, and then it decreases with the increase of the wall thickness of the ring gear. The stress range is 336 mPa to 784 mPa. From figure (c), when the wave
generator is set, the stress decreases with the increase of ring gear wall thickness and increases with the decrease of cylinder length. The stress range is 340 MPa to 540 MPa. Comprehensively, the width of the wave generator has the most significant effect on the stress. The stress increases as the width increases. The influence of tube length serves as the secondary factor that the increase in tube length could significantly reduce the stress when the width of the wave generator is larger. The thickness of the ring gear on the stress is the smallest only when the stress value is large, the change of the thickness of the ring gear has obvious influence on the result.

![Stress response surface with the wall thickness of ring gear in center value](image1)

![Stress response surface with barrel length in center value](image2)

![Stress response surface with wave generator in center value](image3)

Fig.8 Response Surface

4.4 Genetic Algorithm Optimization

Deb proposed a fast non-dominated sorting genetic algorithm NSGA - II based on Pareto, which is considered to be one of the most effective multi-objective evolution methods at present, and has obvious advantages in convergence and convergence speed\[13\].

To reduce the stress and make the structure as compact as possible, the multi-objective optimization design mathematical model, taking the flex spline cylinder length and flex spline stress as the objective function is as follows:

\[
\begin{align*}
\min & \quad \sigma = f(b_2, L, h_j) \\
\min & \quad L \\
\text{s.t.} & \quad 5 \leq b_2 \leq 8, \\
& \quad 12 \leq L \leq 18, \\
& \quad 0.5 \leq h_j \leq 0.74. 
\end{align*}
\]

(9)

The multi-objective genetic algorithm is used to optimize the design of the model shown in equation.
(9), and the calculated parameters are shown in Table 5. After iteration, the design variables, the optimization value of the objective and the final value are shown in Table 6.

### Table 5 Parameters chosen for genetic algorithms

| Parameter               | Value |
|-------------------------|-------|
| Crossover probability   | 0.8   |
| Variation probability   | 0.1   |
| Population size         | 200   |
| Genetic algebra         | 300   |

### Table 6 Optimized results of significant variables

| Design variables                        | initial value | optimization value | final value |
|-----------------------------------------|---------------|--------------------|-------------|
| Width of wave generator b2/mm           | 6.6           | 5.64               | 5.5         |
| tube length L/mm                        | 15            | 12.83              | 13          |
| Wall thickness of ring gear h1/mm       | 0.6           | 0.72               | 0.7         |
| Stress MPa                              | 404           | 335.6              | 385.4       |

We take the optimized structural parameters into the finite element model for calculation, as shown in Figure 9. It can be seen that the stress distribution results are similar with an obvious optimization effect that the maximum stress is reduced from 503mpa to 385mpa, reducing by 23%; the tube length is reduced from 15mm to 13mm, reducing by 13%.

5. Conclusions

1) A new modular joint structure is proposed to serve as a new idea for joint design with low cost and high integration by directly integrating harmonic drive components and using a diagonal contact ball bearing instead of cross roller bearing.

2) Considering the position keeping condition of modular joint, Plackett Burman factorial design and response surface method are introduced into the optimization design of articulated flexspline, and an optimization approximate model of articulated flexspline is established with the structural parameters of harmonic drive parts as design variables and the maximum stress and tube length as design objectives, which reduces the workload of optimization design.

3) The analysis and optimization results of the new modular joint flexspline show that the optimal design of flexspline considering position keeping conditions can effectively reduce the maximum stress of the flexspline and reduce the flexspline tube length, which provides guidance for the pre-design and later improvement of the harmonic drive parts in the joint.
Acknowledgments
This article is one of the phased achievements of the National Natural Science Foundation of China project "Theoretical Research on the Design of High-Efficiency Precision Reducer Based on Line Contact Single Rotational Degree of Freedom Configuration" (52075053).

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