Glue in the light-front pion

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Abstract

It may be possible to approximate the full pion wave function in light-front QCD using only $q\bar{q}$ and $q\bar{g}$ Fock space components. Removing zero modes and using invariant-mass cutoffs that make a constituent approximation possible leads to non-canonical terms in the QCD hamiltonian, and forces us to work in the broken symmetry phase of QCD in which chiral symmetry breaking operators must appear directly in the hamiltonian because the vacuum is trivial. Assuming any candidate interactions are local in the transverse direction, I argue that they are probably relevant operators so that perturbation theory is not modified at high energies. Since light-front chiral symmetry corresponds to quark helicity conservation, we can readily identify candidate chiral symmetry breaking interactions. The only candidate relevant operator is the quark-gluon emission/absorption operator with a quark spin flip. I argue that this operator can only produce the physical $\pi - \rho$ mass splitting if the $q\bar{g}$ component of the pion is significant.

1 A Constituent Pion?

Can the pion be adequately approximated as a quark-antiquark bound state? The constituent quark model (CQM) works well for massive hadrons, but most theorists believe that the pion lies outside the CQM’s range of “validity.” Early attempts to relate the CQM to QCD focussed on the relationship between current and constituent quarks, advocating a picture in which the constituent quarks are dressed current quarks; but only naive transformations were investigated and gluons played little or no role in these investigations.

I believe that a constituent approximation may arise in light-front QCD, as was most convincingly argued by Brodsky, Lepage and collaborators [1]. However, I believe the pion will stand out because it contains a significant amount of glue. Lepage, Brodsky, Huang, and Mackenzie showed that under reasonable assumptions the probability of finding a $q\bar{g}$ component in the pion is of order 1/4; however, their arguments can be turned around to show that higher Fock
space components are important. Moreover, their estimate depends on a model wave function that is not derived from QCD and it is possible that this probability is even higher than 1/4. It should not be surprising that components with glue are large in the pion. What is amazing is that the $q\bar{q}$ component is sizable.

I cannot review the computational scheme that might be used to produce a massless constituent pion [2]. Several theorists, guided by Ken Wilson, have developed renormalization techniques that can be used to attack QCD bound state problems in much the same way as QED bound states can be treated in Coulomb gauge. They first argued that constituent masses and confinement should be added by hand for all partons, and that the strong-coupling, relativistic limit of such models might be full QCD [3]. However, I have shown that confinement does not have to be added by hand, because it appears in the renormalized light-front QCD hamiltonian at second order [2]. Moreover, I have argued that this confining interaction can produce the constituent mass scale as happens in the MIT bag model. Martina Brusudova has shown that the second order hamiltonian produces reasonable results for heavy mesons [4] and Brent Allen has shown that it also produces reasonable glueball results [5], without adding any non-canonical parameters to QCD.

To obtain a constituent approximation, we remove zero modes and are forced to work in the broken symmetry phase of the theory. Interactions that explicitly violate chiral symmetry should be induced by spontaneous chiral symmetry breaking, and in general the new symmetry breaking couplings must be adjusted by hand to obtain the correct $\pi - \rho$ mass splitting.

There are many non-canonical operators induced by renormalization (e.g., a gluon mass term), and in principle the dimensionless couplings for the relevant, $\mu$, and marginal, $\gamma$, operators act as independent parameters that must be tuned to fit data or recover symmetries. We assume that all couplings induced by renormalization are functions of the canonical QCD coupling and that they vanish when this coupling goes to zero. In other words, we allow new relevant couplings, $\mu(g_\Lambda)$ with $\mu(0) = 0$, and marginal couplings, $\gamma(g_\Lambda)$ with $\gamma(0) = 0$. These conditions constitute coupling coherence (or coupling reduction), and we have shown that they produce effective interactions that restore all broken symmetries at leading orders in perturbation theory. I note that for an asymptotically free theory the boundary conditions imply that $\mu_\Lambda \to 0$ and $\gamma_\Lambda \to 0$ as $\Lambda \to \infty$.

I assume that the new chiral symmetry breaking couplings are fixed functions of the canonical coupling, but there is no reason to expect perturbative renormalization group equations to produce the correct dependence, because they are probably non-analytic functions of the coupling.$^1$

$^1$ It is interesting to note that a perturbative RG analysis does lead to a rele-
A marginal operator introduced at low energies will tend to have a comparable effect when the cutoff is increased, and it will typically affect perturbation theory at some finite order. Only relevant operators can be introduced at low cutoffs and necessarily have no effect at any finite order of perturbation theory at higher cutoffs. The only candidate relevant operator is gluon emission/absorption with a quark spin flip, shown below.

It seems that the only simple way in which a constituent picture that includes confinement and chiral symmetry breaking effects is for this single relevant interaction to produce the entire $\pi - \rho$ mass splitting. I assume that the vertex mass, which is $g m_F$ to leading order in perturbation theory, becomes $M_{\chi SB}$ because of spontaneous symmetry breaking.

2 Chiral symmetry in light-front field theory

I first review chiral symmetry without zero modes in light-front QCD, following Mustaki [6]. The fermion field naturally separates into two-component fields $\psi = \psi_+ + \psi_-$ on the light-front, where $\psi_+$ is dynamical and $\psi_-$ is constrained. The light-front chiral transformation applies freely only to the two-component field $\psi_+ = \frac{1}{2} \gamma^0 \gamma^\pm \psi$, because the constraint equations are inconsistent with the normal chiral transformation rules. The minus-component $\psi_-$ is determined by the light-front constraint

$$\psi_- = \frac{1}{i \partial^+} (\alpha_\perp \cdot (i \partial_\perp + gA_\perp) + \gamma^0 m_V) \psi_+.$$  

The light-front chiral transformation acts only on $\psi_+$

$$\psi_+ \longrightarrow \psi_+ + \delta \psi_+ , \quad \delta \psi_+ = -i \theta \gamma_5 \psi_+ ,$$  

and the transformation on $\psi_-$ is given by the equation of constraint

$$\delta \psi_- = \frac{1}{i \partial^+} (\alpha_\perp \cdot (i \partial_\perp + gA_\perp) + \gamma^0 m_F) \delta \psi_+$$

$$= -i \theta \gamma_5 \frac{1}{i \partial^+} \alpha_\perp \cdot (i \partial_\perp + gA_\perp) \psi_+ - i \theta m_F \gamma^0 \gamma_5 \frac{1}{i \partial^+} \psi_+.$$  

Thus, the light-front chiral transformation on $\psi$ is

vant chiral symmetry breaking coupling that scales like $\exp\{-1/(\beta g^2)\}$, but only in asymptotically free theories.
\[ \delta \psi = \delta \psi_+ + \delta \bar{\psi}_- \]

\[ = -i \theta \gamma_5 \psi_+ - i \theta \gamma_5 \frac{1}{i \partial^+} \alpha \cdot (i \partial_\perp + g A_\perp) \psi_+ - i \theta m_F \gamma^0 \gamma_5 \frac{1}{i \partial^+} \psi_+. \]  

(4)

The axial vector current is \( j_5^\mu = \bar{\psi} \gamma^\mu \gamma_5 \psi \), and the conserved light-front axial vector charge is

\[ Q_{LF}^5 = \int dx^- d^2 x_\perp j_5^+(x). \]  

(5)

Explicit calculation using the field expansions and normal ordering leads to

\[ Q_{LF}^5 = \int \frac{dk^+ d^2 k_\perp}{2(2\pi)^3} \sum_\lambda \lambda [b_{\lambda}^\dagger(k)b_{\lambda}(k) + d_{\lambda}^\dagger(k)d_{\lambda}(k)]. \]  

(6)

Thus \( Q_{LF}^5 \) measures the helicity.

What does this teach us? An interaction in which quark helicity changes is required to remove the \( \pi - \rho \) degeneracy produced by massless, canonical light-front QCD.

There is one explicit chiral symmetry breaking term in the canonical Hamiltonian,

\[ g m_V \int dx^- d^2 x_\perp \psi_+^\dagger \sigma_\perp \cdot \left( A_\perp \frac{1}{\partial^+} \psi_+ - \frac{1}{\partial^+} (A_\perp \psi_+) \right); \]  

(7)

and this is the only relevant operator that violates light-front chiral symmetry, although the full interaction contains an unknown function of longitudinal momenta.

3 Crude Estimates

Assume that the \( \pi - \rho \) mass splitting is primarily due to gluon emission/absorption with a helicity flip. This does not necessarily imply that the pion contains significant glue when the cutoff is lowered to \( O(1 \text{GeV}) \) because renormalization produces a direct quark-antiquark interaction of the form:

\[ V_{\chi SB} = c M_{\chi SB}^2 \sigma_\perp \cdot \sigma_\perp . \]  

(8)
The real interaction is not proportional to a constant $c$, but this will give us a crude idea of its effects. The massless spectrum without chiral symmetry breaking is qualitatively similar to the glueball spectrum. We can assume an unperturbed mass of $O(1\text{GeV})$ for the $\pi$ and the $\rho$. However, acting on these states, $V_{\chi SB}$ produces:

$$V_{\chi SB}\mid \pi > = -\frac{c}{2} M_{\chi SB}^{2}\mid \pi > , V_{\chi SB}\mid \rho_0 > = \frac{c}{2} M_{\chi SB}^{2}\mid \rho_m > , V_{\chi SB}\mid \rho_{\pm 1} > = 0 ; \quad (9)$$

where the subscript on the $\rho$ indicates its spin projection.

Since the zeroth-order eigenstates are also eigenstates of this operator, we can immediately determine the masses,

$$M_\pi^2 = M_0^2 - \frac{c}{2} M_{\chi SB}^{2} , M_{\rho_0}^2 = M_0^2 + \frac{c}{2} M_{\chi SB}^{2} , M_{\rho_{\pm 1}} = M_0^2 . \quad (10)$$

If we adjust $M_{\chi SB}$ to produce a massless pion, we apparently destroy rotational symmetry in the $\rho$ multiplet. Violation of few-body kinematic rotational symmetries is common in light-front field theory, where rotations about transverse axes involves parton production and annihilation. If we break rotational invariance in the $q\bar{q}$ sector without gluons by 100%, we can restore it when glue is added, but only if the $q\bar{q}g$ component of the state is comparable to the $q\bar{q}$ component.

Taking into account that mixing $q\bar{q}$ and $q\bar{q}g$ also produces a negative self-energy, and that the combination must be negative, we find that this admixture lowers the mass of the $\rho_0$ less than it lowers the masses of the $\rho_{\pm 1}$ or the $\pi$. The simplest possibility is that the $\rho$ has a much smaller $q\bar{q}g$ component than the $\pi$, and that this component is slightly larger in the $\rho_{\pm 1}$ than in the $\rho_0$. If this is the case, we can consider this admixture in the $\pi$ alone.

Assuming the $q\bar{q}g$ mass is higher than the $q\bar{q}$ mass before the chiral symmetry breaking interaction is turned on, and that confinement produces a mass gap that is linear in the number of constituents (which may well be false) and that there is a large gap between the first two $q\bar{q}g$ states, we can make a crude approximation by considering only one $q\bar{q}$ mode coupled to one $q\bar{q}g$ mode. This produces a drastically simplified hamiltonian (mass-squared operator) of the form,

$$H = \begin{pmatrix} M_0^2 & M_0 M_1 \\ M_0 M_1 & M_1^2 \end{pmatrix} . \quad (11)$$
I have adjusted the off-diagonal matrix element to produce a massless pion, and we find that

$$ P_{q\bar{q}} = \frac{M_1}{M_0 + M_1}, $$

(12)

$$ P_{q\bar{q}g} = \frac{M_0}{M_0 + M_1}. $$

(13)

This drastically oversimplified analysis can only lead to $P_{q\bar{q}} > P_{q\bar{q}g}$, which is not necessary, but this type of analysis leads to the conclusion that the probability of finding glue in the pion may only be of order $1/2$. Nonetheless, it seems unlikely that any model of the pion that does not include constituent glue can be derived from QCD, even if it is possible to derive a valence approximation for other hadrons.

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References

[1] For a review and references see S. J. Brodsky, H.-C. Pauli, and S. Pinsky, Phys. Rept. 301, 299 (1998).

[2] R.J. Perry, in “Proceedings of Hadrons 94,” V. Herscovitz and C. Vasconcelos, eds. (World Scientific, Singapore, 1995).

[3] K. Wilson et al, Phys. Rev. D 49, 6720 (1994).

[4] M. Brisudová and R. J. Perry, Phys. Rev. D 54, 1831 (1996); M. Brisudová, R. Perry and K. Wilson, Phys. Rev. Lett. 78, 1227 (1997).

[5] B. H. Allen and R. J. Perry, Phys. Rev. D 60, 067704 (1999).

[6] Daniel Mustaki, hep-ph/9404206 (1994).