Democratic Approach To Atmospheric 
And Solar Neutrino Oscillations

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Abstract

Working with a $\mathcal{U}(1)$ flavor symmetry, we show how the hierarchical structure in the charged fermion sector and a democratic approach for neutrinos that yields large solar and atmospheric neutrino mixings can be simultaneously realized in the MSSM framework. However, in $SU(5)$ due to the unified multiplets we encounter difficulties. Namely, democracy for the neutrinos leads to a wrong hierarchical pattern for charged fermion masses and mixings. We discuss how this is overcome in flipped $SU(5)$.

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1 Introduction

Recent SuperKamiokande data appear to confirm the existence of both atmospheric [1] and solar [2] neutrino oscillations. From the atmospheric data the preferred oscillation parameters are

\[
\sin^2 2\theta_{\mu\tau} \simeq 1, \quad \Delta m_{\text{atm}}^2 \simeq 3 \times 10^{-3} \text{ eV}^2,
\]

while for solar neutrinos the preferred oscillation scenario is a large angle MSW solution with

\[
\sin^2 2\theta_{e\mu,\tau} \approx 0.8, \quad \Delta m_{\text{sol}}^2 \sim 10^{-4} \text{ eV}^2.
\]

In attempting to simultaneously accommodate the atmospheric and solar neutrino data, one should provide a reasonable theoretical background for understanding the origin of large (in one case even maximal!) mixings in (1) and (2). At the same time, the origin of hierarchies between charged fermion masses and their CKM mixing angles must be explained. Finally, one also must find an explanation of how the third mixing angle \(\theta_{13}\) in the neutrino sector appears to be small \((\lesssim 0.2)\) [3]. For a unified description of quark-lepton sector, one well motivated idea is that of flavor symmetries, with an abelian \(U(1)\) being the simplest possibility. A variety of models for obtaining the desirable fermion mass pattern with \(U(1)\) have been considered [4], [5]. The \(U(1)\) symmetry also can be promising in the neutrino sector [6]-[9], especially for generating nearly maximal mixings between the flavors [7]-[9]. While the atmospheric neutrino data strongly suggests maximal mixing, for the solar neutrinos there is significant deviation from maximal value \((\sin^2 2\theta_{e\mu,\tau} \approx 0.8)\). Because of this, textures leading to bi-maximal neutrino mixings [7]-[9] need to be modified appropriately. This is not always easy in the presence of a flavor symmetry such as \(U(1)\), and one should look for alternative ways for building up the neutrino sector. One alternative (to the maximal mixing texture) is the so called democratic approach [10], in which lepton doublets of different families have the same \(U(1)\) charge. That is, the \(U(1)\) symmetry does not distinguish them from each other and one could naturally expect large neutrino mixings. By the same token, however, the masses of all neutrinos might be of similar magnitude, which would be problematic for obtaining the distinct mass scales relevant for atmospheric and solar neutrinos. This is easily avoided, however, through a careful choice of the singlets (right handed neutrino sector) [11], [9].

In contrast with the left handed lepton doublets, the remaining lepton and quark superfields should have distinct transformation properties under \(U(1)\) in order to obtain desirable hierarchies between their masses and mixings. Following this strategy, we start our considerations with MSSM and show that the democratic approach works out neatly, because MSSM does not provide stringent constraints on the \(U(1)\) charge assignments. However, for GUTs the situation can be drastically changed. Namely, we demonstrate that for \(SU(5)\) GUT [with \(U(1)\) flavor symmetry], the democratic approach gives an acceptably small Cabibbo angle. The root of this problem lies in the unified multiplets and
therefore can be shared by other GUTs unless some additional elements are introduced. While in $SU(5)$ it may be difficult to realize the democratic approach in a natural way, we consider a flipped $SU(5)$ scheme in which the democratic approach for large neutrino mixings is nicely consistent with the hierarchies in the charged fermion sector. We conclude with a brief remark about the third neutrino mixing angle $\theta_{13}$.

2 $U(1)$ Flavor Symmetry: Fermion Masses And Neutrino Oscillations

Let us start our considerations with the MSSM augmented with $U(1)$ flavor symmetry. In addition, we introduce a singlet superfield $X$ with $U(1)$ charge $Q(X) = -1$ and assume that its scalar component has a VEV

$$\frac{\langle X \rangle}{M_{Pl}} \equiv \epsilon \simeq 0.2 .$$

$\epsilon$ plays the role of an expansion parameter and is crucial for the explanation of hierarchies among the charged fermion masses and their mixings. With the following assignment of $U(1)$ charges for the quark-lepton superfields

$$Q[q^{(1)}] = 3 , \quad Q[q^{(2)}] = 2 , \quad Q[q^{(3)}] = 0 , \quad Q[u^{c(1)}] = 4 , \quad Q[u^{c(2)}] = 1 , \quad Q[u^{c(3)}] = 0 ,$$

$$Q[d^{c(1)}] = n + 2 , \quad Q[d^{c(2)}] = Q[d^{c(3)}] = n$$

$$Q[l^{(1)}] = n - n_3 + n_2 + n_1 , \quad Q[l^{(2)}] = n - n_3 + n_2 , \quad Q[l^{(3)}] = n - n_3 ,$$

$$Q[e^{c(1)}] = n_3 - n_2 - n_1 + 5 , \quad Q[e^{c(2)}] = n_3 - n_2 + 2 , \quad Q[e^{c(3)}] = n_3 ,$$

($n, n_{1,2,3}$ are some integers and superscripts stand for generation indices) and the pair of higgs doublets $Q(h_u) = Q(h_d) = 0$, the relevant couplings generating the up, down quark and charged lepton masses respectively are

$$q_1 \left( \begin{array}{ccc} u^c_1 & u^c_2 & u^c_3 \\ \epsilon^7 & \epsilon^4 & \epsilon^3 \end{array} \right) h_u , \quad q_2 \left( \begin{array}{ccc} d^c_1 & d^c_2 & d^c_3 \\ \epsilon^5 & \epsilon^3 & \epsilon^2 \end{array} \right) \epsilon^n h_d ,$$

$$l_1 \left( \begin{array}{ccc} e^{c_1} & e^{c_2} & e^{c_3} \\ \epsilon^{5-n_1} & \epsilon^{n_1+2} & \epsilon^{n_1+n_2} \end{array} \right) e^n h_d ,$$

$$l_2 \left( \begin{array}{ccc} e^{n_3} & e^{n_2} & 1 \\ \epsilon^{5-n_1-n_2} & \epsilon^{2-n_2} & \end{array} \right) e^n h_d .$$

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Note that the entries in textures such as (6) and (7) are real and accompanied by factors of order unity. We will not be concerned with CP violating phases in this work. Upon diagonalization of (6), (7), for the Yukawa couplings we obtain

\[
\begin{align*}
\lambda_t &\sim 1 , \quad \lambda_u : \lambda_c : \lambda_t \sim \epsilon^7 : \epsilon^3 : 1 , \\
\lambda_b &\sim \lambda_\tau \sim \epsilon^n , \quad \lambda_d : \lambda_s : \lambda_b \sim \epsilon^5 : \epsilon^2 : 1 , \\
\lambda_e : \lambda_{\mu} : \lambda_\tau \sim \epsilon^5 : \epsilon^2 : 1 ,
\end{align*}
\]  

while for the CKM matrix elements:

\[
V_{us} \sim \epsilon , \quad V_{cb} \sim \epsilon^2 , \quad V_{ub} \sim \epsilon^3 .
\]  

Thus, the \( U(1) \) flavor symmetry nicely explains the hierarchies between the charged fermion masses and CKM mixing angles.

As far as the lepton mixing matrix is concerned, from (5) and the form of (7), one expects\(^3\) \( \sin^2 2\theta_{\mu\tau} \sim \frac{4^n}{(1+2^n)^2} \) and \( \sin^2 2\theta_{\mu\mu,\tau} \sim \frac{4^n}{(1+2^n)^2} \). With \( n_1 = n_2 = 0 \) which means \( Q[l^{(1)}] = Q[l^{(2)}] = Q[l^{(3)}] \), one expects \( \sin^2 2\theta_{\mu\tau} \sim 1, \sin^2 2\theta_{\mu\mu,\tau} \sim 1 \).

To realize oscillations we have to generate neutrino masses. Introducing an MSSM singlet neutrino \( \mathcal{N} \) with \( U(1) \) charge \( Q(\mathcal{N}) = p \), with couplings

\[
\epsilon^{n+p}(l_1 + l_2 + l_3)\mathcal{N}h_u + \epsilon^{2p}M_N\mathcal{N}^2 ,
\]  

(we assume all entries of order unity) and integrating out \( \mathcal{N} \) leads to a massive state \( m_{\nu_3} \sim \frac{\epsilon^{2n}h_u^2}{M_N} \). For \( M_N/\epsilon^{2n} \sim 10^{14} \) GeV, \( m_{\nu_3} \sim 0.1 \) eV, which is relevant for atmospheric neutrinos. Including a second singlet state \( \mathcal{N}' \) with charge \( Q(\mathcal{N}') = q \) and couplings \( \epsilon^{n+q}(l_1 + l_2 + l_3)\mathcal{N}'h_u + \epsilon^{2q}M_{N'}\mathcal{N}'^2 \), taking \( M_{N'}/\epsilon^{2n} \sim 3 \cdot 10^{15} \) GeV, and integrating out \( \mathcal{N}' \) will introduce into the neutrino mass matrix the deviations \( \sim \frac{\epsilon^{2n}h_u^2}{M_{N'}} \sim 3 \cdot 10^{-3} \) eV. This will create a similar order mass for the second light neutrino state. This mass scale guarantees the large angle MSW oscillations of solar neutrinos. Thus, with this setting the desirable neutrino mass scales can be obtained [11], [9]. Note that large lepton mixings are obtained due to the same \( U(1) \) charge assignments for the left handed lepton doublets, possible in MSSM because there were no constraints on \( n_{1,2,3} \) and \( n \) in (5).

One would naturally wish to extend this mechanism to SUSY GUTs. However, it turns out that due to unified multiplets it is not a straightforward task. For example, in \( SU(5) \) GUT each family of quark-lepton superfields is embedded in an anomaly free \( 10 + 5 \) superfields, where \( 10 = (q, u^c, e^c) \) and \( 5 = (l, d^c) \). Therefore \( Q[q^{(\alpha)}] = Q[u^{c(\alpha)}] = Q[e^{c(\alpha)}] \) and \( Q[l^{(\alpha)}] = Q[d^{c(\alpha)}] \) (\( \alpha \) is a generation index). With universal \( U(1) \) charges for \( l^{(\alpha)} \) states,
one also has the same charges for $d^{c(\alpha)}$ superfields. For obtaining the desirable hierarchies in (10) for charged leptons, one has to take $Q[e^{c(3)}] = 0$, $Q[e^{c(2)}] = 2$, $Q[e^{c(1)}] = 5$. But this means that $Q[q^{(3)}] = 0$, $Q[q^{(2)}] = 2$, $Q[q^{(1)}] = 5$. Although this gives a good estimate for $V_{cb}(\sim \epsilon^2)$, the expected value of Cabibbo angle is $\sim \epsilon^3$, which is smaller by factor $\sim 25$ than the measured value ($\sin \theta_c \simeq 0.2$). Thus, in the framework of minimal SUSY $SU(5)$, it seems difficult to realize the democratic approach discussed above. The reason is the unified multiplets which provide constraints on the $U(1)$ charge assignments of the MSSM chiral superfields. Of course, one can think of a possible extension such that the light $q^{(\alpha)}$ and $e^{c(\alpha)}$ states originate from different unified multiplets. By introducing some additional states it might be possible to realize this. However, it is hard to imagine such a splitting among leptonic and colored states. Note that this situation closely resembles the doublet-triplet (DT) splitting problem in the scalar sector and whose resolution in SUSY $SU(5)$ requires a rather complicated extensions [12]. However, there are GUTs in which DT splitting is acheived in an elegant way and flipped $SU(5)$ GUT is one example [13]. From experience in obtaining a natural DT splitting in the scalar sector of $SU(5) \times U(1)$ through the missing partner mechanism, with introduction of additional vector-like matter we can manage to split the unified matter multiplets in such a way that the democratic approach to neutrino mixings nicely works out. In the next section we present the flipped $SU(5) \times U(1)$ model and its extension.

3 Flipped $SU(5)$ GUT

The 'matter' sector of minimal flipped $SU(5) \times U(1)$ GUT consists of anomaly free $\bar{5}_3 + 10_{-1} + 1_{-5}$ supermultiplets per generation, where the subscripts denote $U(1)$ charges and

$$\bar{5}_3 = (l, u^c), \quad 10_{-1} = (q, d^c, \nu^c), \quad 1_{-5} = e^c.$$  

(13)

The 'higgs' sector contains the following supermultiplets

$$H \sim 10_{-1}, \quad \overline{H} \sim \overline{10}_1, \quad \phi \sim 5_2, \quad \overline{\phi} \sim \overline{5}_{-2}.$$  

(14)

$H, \overline{H}$ are responsible for $SU(5) \times U(1)$ breaking to $SU(3)_c \times SU(2)_L \times U(1)_Y \equiv G_{321}$. $\phi$ and $\overline{\phi}$ contain the MSSM doublet-antidoublet pair $h_d$ and $h_u$ respectively.

Let us first show that the $SU(5) \times U(1)$ model, supplemented with $U(1)$ flavor symmetry and with minimal fermion content (13) neither yilds the desirable hierarchies between charged fermion masses and mixings, nor the two large neutrino mixings. For the CKM mixing angles we need the hierarchies in (11). Taking into account (13) we conclude that

$$Q[10^{(1)}_{-1}] = 3, \quad Q[10^{(2)}_{-1}] = 2, \quad Q[10^{(3)}_{-1}] = 0.$$  

(15)
The down quark masses emerge from $10^{(\alpha)}_{-1}10^{(\beta)}_{-1}\bar{\phi}$ couplings, and with $Q(\phi) = Q(\bar{\phi}) = 0$ and (15) we have

$$
\begin{pmatrix}
10^{(1)}_{-1} & 10^{(2)}_{-1} & 10^{(3)}_{-1} \\
10^{(2)}_{-1} & e^6 & e^5 & e^3 \\
10^{(3)}_{-1} & e^5 & e^4 & e^2 & 1
\end{pmatrix} \phi ,
$$

which gives the unacceptable ratio $m_s/m_b \sim \epsilon^4$ (a reasonable value for the latter would be $\sim \epsilon^2$).

Moreover, the observed hierarchies for up quark masses in (8) (generated through $10^{(\alpha)}\bar{5}^{(\beta)}\bar{\phi}$ couplings) dictates the following assignment

$$Q[\bar{5}^{(1)}_3] = 4 , \quad Q[\bar{5}^{(2)}_3] = 1 , \quad Q[\bar{5}^{(3)}_3] = 0 .$$

Since the $l$ states also come from $\bar{5}_3$-plets [see (13)], according to (17) we will have $Q[l^{(1)}] = 4, Q[l^{(2)}] = 1, Q[l^{(3)}] = 0$. For the lepton mixing elements this gives $V_{23}^l \sim \epsilon$ and $V_{12}^l \sim \epsilon^3$, both of which are in contradiction with observations. We therefore conclude that the matter sector of flipped $SU(5)$ model must be extended if $U(1)$ flavor symmetry is invoked.

### 3.1 Extended Flipped $SU(5)$

In the fermion sector we introduce three families of vector like states $(F + \bar{F})^{(\alpha)}$ $(\alpha = 1, 2, 3)$, where

$$F \sim 5_2 , \quad \bar{F} \sim \bar{5}_{-2} .$$

In terms of $G_{321}$ they decompose as

$$F(5_2) = (l, \bar{d})_F , \quad \bar{F}(\bar{5}_{-2}) = (\bar{l}, d^c)_\bar{F} .$$

With these states and including specific couplings one can arrange that the physical light $l$ and $d^c$ states will come from multiplets different from $5_3$ and $10_{-1}$ respectively. This is realized through a way resembling the missing partner mechanism operative in the higgs sector of $SU(5) \times U(1)$. Let us show this in a one generation example first. Generalization to three families will be straightforward. With couplings

$$H\bar{5}_3\bar{F} + H10_{-1}F + M_F\bar{F}\bar{F} ,$$

and assuming that $\langle H \rangle \gg M_F$, one can easily verify that $l_{5_3}$ and $\bar{l}_\bar{F}$ form a state with mass $\sim \langle H \rangle \sim M_G$. Therefore, the light left handed doublet state resides in $F$. At the same time, $d^c_{10_{-1}}$ and $\bar{d}_\bar{F}$ end up getting mass $\sim \langle H \rangle$, and therefore the light $d^c$ state
comes from $\mathbf{F}$. This gives us the possibility to build a realistic fermion sector with two large neutrino mixings.

Let us then turn to the realistic case of three generations. The $U(1)$ charge prescriptions for $10_{-1}^{(\alpha)}$ and $\bar{5}_{3}^{(\alpha)}$ remain the same as in (15) and (17) respectively. For the other states let us make the assignments

$$Q[1_{-5}^{(1)}] = 5, \quad Q[1_{-5}^{(2)}] = 2, \quad Q[1_{-5}^{(3)}] = 0, \quad Q[F(1)] = Q[F(2)] = Q[F(3)] = 0,$$

$$Q[F(1)] = 2, \quad Q[F(2)] = Q[F(3)] = 0.$$  \hspace{1cm} (21)

From (15), (17), (21) the couplings responsible for the decoupling of appropriate states are schematically

$$F^{(1)} \quad F^{(2)} \quad F^{(3)}$$

$$\bar{5}_{3}^{(1)} \quad \begin{pmatrix} \epsilon^6 \\ \epsilon^4 \\ \epsilon^4 \end{pmatrix}$$

$$\bar{5}_{3}^{(2)} \quad \begin{pmatrix} \epsilon^3 \\ \epsilon \\ \epsilon \end{pmatrix}$$

$$\bar{5}_{3}^{(3)} \quad \begin{pmatrix} \epsilon^2 \\ 1 \\ 1 \end{pmatrix} H, \quad 10_{-1}^{(1)} \quad \begin{pmatrix} \epsilon^3 \\ \epsilon^3 \\ \epsilon^3 \end{pmatrix}$$

$$10_{-1}^{(2)} \quad \begin{pmatrix} \epsilon^2 \\ \epsilon^2 \\ \epsilon^2 \end{pmatrix}$$

$$10_{-1}^{(3)} \quad \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} H.$$  \hspace{1cm} (22)

$$F^{(1)} \quad F^{(2)} \quad F^{(3)}$$

$$\bar{F}^{(1)} \quad \begin{pmatrix} \epsilon^2 \\ 1 \\ 1 \end{pmatrix} M_F.$$

$$\bar{F}^{(2)} \quad \begin{pmatrix} \epsilon^2 \\ 1 \\ 1 \end{pmatrix}$$

$$\bar{F}^{(3)} \quad \begin{pmatrix} \epsilon^2 \\ 1 \\ 1 \end{pmatrix}$$  \hspace{1cm} (23)

Let us assume now that $M_F \ll \langle H \rangle \epsilon^4$. From the couplings in (22), (23) we realize that the light $l^{(\alpha)}$ and $d^{c(\alpha)}$ states respectively come from $F^{(\alpha)}$ and $\bar{F}^{(\alpha)}$

$$F^{(\alpha)} \supset l^{(\alpha)}, \quad \bar{F}^{(\alpha)} \supset d^{c(\alpha)}.$$  \hspace{1cm} (24)

With prescription (21) all $F^{(\alpha)}$ states have the same $U(1)$ charges, and according to (24) the light left handed lepton doublets also have identical (democratic) transformation properties under $U(1)$. The latter guarantee the two neutrino mixings we are after. At the same time, the charged fermion masses and mixings have desirable hierarchies. Namely, the relevant couplings generating up, down quark and charged lepton masses respectively are

$$10_{-1}^{(1)} \quad \begin{pmatrix} \epsilon^7 \\ \epsilon^4 \\ \epsilon^3 \end{pmatrix}$$

$$10_{-1}^{(2)} \quad \begin{pmatrix} \epsilon^6 \\ \epsilon^3 \\ \epsilon^2 \end{pmatrix}$$

$$10_{-1}^{(3)} \quad \begin{pmatrix} \epsilon^4 \\ \epsilon \\ 1 \end{pmatrix} \overline{\phi}.$$  \hspace{1cm} (25)
where $M(\gtrsim M_G)$ is some cut off scale. Substituting appropriate VEVs in (25) and upon diagonalization we find the hierarchies in (8), while diagonalization of (26), (27) yield the hierarchies in (9), (10). Note that (25), (26) also give rise to the CKM mixing angles in (11). At the same time, from (27), one expects

$$\sin^2 2\theta_{\mu\tau} \sim 1, \quad \sin^2 2\theta_{\mu\tau} \sim 1.$$  \hspace{1cm} (28)

Dirac and Majorana couplings $\nu^c l h_u$ and $M_R \nu^c \nu^c$ respectively are generated through $10 \leftrightarrow 10$ and $(10 \leftrightarrow 10)^2$ type couplings. In our scenario all $l^{(a)} (F^{(a)})$ states have the same $U(1)$ charges, and to avoid the same mass scales for atmospheric and solar neutrinos, we will decouple $\nu^{(1)} (\nu^{(2)})$ states (from $10^{(1,2)}$). Introducing two singlets $N_{1,2}$ with charges $Q(N_{1,2}) = -3, -2$, through the couplings $(10^{(1)} N_1 + 10^{(2)} N_2) H$ after substituting $H$'s VEV, the states $\nu^{(1,2)}$ decouple with $N_{1,2}$, and at this stage $\nu_{1,2}$ are massless. From the couplings

$$\frac{1}{M} 10^{(3)} F^{(a)} H \phi + M_R 10^{(3)}_{-1} 10^{(3)}_{-1} \left( \frac{H}{M} \right)^2,$$  \hspace{1cm} (29)

$\nu_3$ obtains a mass $m_{\nu_3} \sim \frac{h_3^2}{M_R}$ which, for $M_R \sim 10^{14}$ GeV, gives 0.1 eV as needed for resolving the atmospheric anomaly. As far as the solar neutrino scale is concerned, introducing an additional singlet $N$ with zero $U(1)$ charge, the relevant couplings will be $F^{(a)} N \phi + M_N N^2$. With $M_N \sim 3 \cdot 10^{15}$ GeV, this gives the desired mass $\sim \frac{h_3^2}{M_N} \approx 3 \cdot 10^{-3}$ eV.

To summarize, an extension of flipped SU(5) GUT by three vector-like ($F + \overline{F}$) states and some singlet states allows us to exploit the $U(1)$ symmetry to generate acceptable masses and mixings both in the charged fermion and neutrino sectors.

## 4 Conclusions

We have shown that the democratic approach for understanding solar and atmospheric neutrino oscillations can be nicely implemented within the MSSM framework and in a
suitably extended flipped $SU(5)$ model through the use of flavor $U(1)$ symmetry. It may be possible to extend our approach to $SO(10)$ which contains flipped $SU(5)$. In the democratic approach described here the small value of the third mixing angle $\theta_{13}(\simeq 0.2 \simeq \epsilon)$ is due to accidental cancellations occurring between quantities that have magnitudes of order unity. In other words, the democratic approach would have to be modified if $\theta_{13}$ turns out to be much smaller than $\epsilon$.

**Acknowledgments**

Q.S. would like to thank Michael Schmidt and Christof Wetterich for their hospitality during his stay at their Institute in Heidelberg, where this work was initiated. We also acknowledge the support of NATO Grant PST.CLG.977666. This work is partially supported in part by DOE under contract DE-FG02-91ER40626.

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