Giant dispersion of critical currents in superconductor with fractal clusters of a normal phase

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The influence of fractal clusters of a normal phase on the dynamics of a magnetic flux trapped in a percolative superconductor is considered. The critical current distribution and the current-voltage characteristics of fractal superconducting structures in the resistive state are obtained for an arbitrary fractal dimension of the cluster boundaries. The range of fractal dimensions, in which the dispersion of critical currents becomes infinite, is found. It is revealed that the fractality of clusters depresses of the electric field caused by the magnetic flux motion thus increasing the critical current value. It is expected that the maximum current-carrying capability of a superconductor can be achieved in the region of giant dispersion of critical currents.

One way to increase the critical currents of superconductors consists in creating the artificial pinning centers in the volume of material. The role of such centers can be performed by the clusters of a normal phase formed in the course of the film growth at the sites of defects at the film-substrate interface [1]–[3]. New possibilities for the pinning enhancement are opened in the case of clusters of a normal phase with fractal boundaries [4]–[7] (see also references in [7]). In the present paper the effect of such fractal clusters on the critical currents and current-voltage characteristics of superconductors in the resistive state are considered.

The problem setting is similar to that considered in [6], [7]. A superconductor containing columnar inclusions of a normal phase is cooled in a magnetic field below the critical temperature (in field-cooling regime). As a result, the magnetic flux is trapped in isolated clusters of a normal phase. Then a transport current is passed through the sample across the direction of the magnetic field. It is assumed that a superconducting percolation cluster has been formed in the plane where electric current flows. While the transport current increases, the trapped flux remains unchanged until the vortices begin to break away from the clusters for which the pinning force is smaller than the Lorentz force created by the current. The vortices travel through the superconducting space via weak links between clusters of a normal phase. The weak links are readily formed at various structural defects in high temperature superconductors (HTS’s) characterized by a small coherence length. In usual low-temperature superconductors the weak links can be formed as a result of the proximity effect at the sites of minimum distance between the next normal phase clusters.

Thus, irrespective of their nature, the weak links form channels for the transport of vortices. According to the weak link configuration, each cluster of the normal phase contributes to the total distribution of the depinning critical currents. A geometric probability analysis of the influence of distribution of the points, where vortices can enter into weak links, on the critical currents of clusters was performed in Ref. [7]. As the transport current increases, the vortices break away first from the clusters of a smaller pinning force and, accordingly, a smaller critical current. Therefore, a change in the trapped magnetic flux $\Delta \Phi$ is proportional to the number of normal phase clusters of the critical currents below a preset value of $I$. Therefore, a relative flux decrease is equal to the probability that the critical current $I'$ of an arbitrarily selected cluster is smaller than $I$:

$$\frac{\Delta \Phi}{\Phi} = \int_{0}^{I} f(I') dI'$$

where $f(I)$ is the probability density for the distribution of the critical depinning currents.
At the same time, the magnetic flux trapped in a cluster is proportional to the cluster area $A$. So, a change in the total trapped flux depends on the distribution of areas of the normal phase clusters. Thus, in order to determine how the transport current affects the trapped magnetic flux, it is necessary to find a relationship between distributions of the critical currents of clusters and their areas. This problem was solved in Ref. [8] for the general case of the gamma-distribution of areas of the clusters with fractal boundaries:

$$w(a) = \frac{(g+1)^{g+1}}{\Gamma(g+1)} a^g \exp\left(- (g+1) a\right)$$  \hspace{1cm} (2)

where $w(a)$ is the probability density for the cluster area distribution, $a \equiv A/\overline{A}$ is the dimensionless area of a cluster, $\overline{A}$ is the average cluster area, $g$ is the gamma-distribution parameter which determines the standard deviation of the cluster area $\sigma_a = 1/\sqrt{g+1}$, and $\Gamma(\nu)$ is the Euler gamma–function. In this case, the distribution of critical currents is as follows:

$$f(i) = \frac{2G^{g+1}}{D\Gamma(g+1)} i^{-(2/D)(g+1)-1} \exp\left(-Gi^{-2/D}\right)$$  \hspace{1cm} (3)

where

$$G \equiv \left(\frac{g^g}{\theta^{g+1} - (D/2) \exp(\theta) \Gamma(g+1, \theta)}\right)^{\frac{1}{D}}$$

$$\theta = g + 1 + \frac{D}{2}$$

$\Gamma(\nu, z)$ is the complementary incomplete gamma function, $i \equiv I/I_c$ is the dimensionless electric current, $I_c = \alpha ((g+1)/G\overline{A})^{D/2}$ is the critical current of the transition into the resistive state, $\alpha$ is a form factor, and $D$ is the fractal dimension of the cluster boundary. The fractal dimension determines the scaling relation between perimeter and area:

$$P^{1/D} \propto A^{1/2}$$  \hspace{1cm} (4)

where $P$ is the perimeter of a cluster of area $A$. Relation of Eq. (4) is consistent with the generalized Euclid theorem according to which the ratios of the corresponding geometric measures are equal when reduced to the same dimension [9].

In the particular case of $g = 0$ the gamma-distribution of Eq. (2) reduces to the exponential one, $w(a) = \exp(-a)$, for which $\overline{a} = \sigma_a = 1$. This distribution of areas of the normal phase clusters with fractal boundaries was realized in YBCO-based film structures [6], [7].

The function of the critical current distribution of Eq. (3) allows us to fully describe the effect of the transport current on the trapped magnetic flux. Thus, the density $n$ of vortices broken away from the pinning centers by the current $i$ can be found:

$$n(i) = \frac{B}{\Phi_0} \int_0^i f(i') \, di' = \frac{B}{\Phi_0} \frac{\Gamma(g+1, Gi^{-2/D})}{\Gamma(g+1)}$$  \hspace{1cm} (5)

where $B$ is the magnetic field, $\Phi_0 \equiv hc/(2e)$ is the magnetic flux quantum, $h$ is the Planck constant, $c$ is the speed of light, and $e$ is the electron charge. The integral in the right-hand part of Eq. (5) is the cumulative probability function, which is a measure of the number of clusters of critical current smaller than the preset value of $i$.

Figure 1 shows how the fractal dimension of cluster boundaries affects the critical current distribution. A comparison between curves (1) and (3) shows that an increase in the fractal dimension leads to a significant extension of the “tail” of the distribution $f = f(i)$. This effect is more pronounced for smaller values of $g$-parameter of gamma-distribution.
In the interval of currents $i > 1$ the magnetic flux motion causes the electric voltage across the sample, which goes into the resistive state. Because of the finite resistance any current flow is accompanied now by the energy dissipation. As for any hard superconductor (type-II, with pinning centers) the presence of dissipation in the resistive state does not necessarily implies the destruction of the phase coherence. The superconducting state collapses only when the dissipation exhibits an avalanche-like growth due to the thermo-magnetic instability development.

In the resistive state the superconductor is adequately described by the current-voltage characteristic. Using the fractal distribution of critical currents of Eq. (3), we can find the electric field arising from the magnetic flux motion after the vortices have been broken away from the pinning centers. Since each cluster of the normal phase contributes to the overall distribution of critical currents, the voltage across the superconductor $V = V (i)$ is an integral response to the sum of contributions from all clusters:

$$V = R_f \int_0^i (i - i') f (i') \, di'$$

(6)

where $R_f$ is the flux flow resistance. The similar representation for the voltage across the sample is frequently used in the description of pinning of the vortex filament bundles in superconductors [10] as well as in the analysis of critical scaling of the current-voltage characteristics [11], that is to say, in all cases where there is a distribution of the depinning currents. In the present consideration we will concentrate on the conclusions following from the properties of fractal distribution of Eq. (3), so we will not consider any questions related to the possible dependence of the flux flow resistance $R_f$ on the transport current.

After substitution of the distribution function of Eq. (3) into the convolution integral of Eq. (6) and carrying out the integration, we eventually obtain an expression for the voltage across the sample

$$V = \frac{1}{\Gamma (g + 1)} \left( i \Gamma (g + 1, G_i^{2/D}) - G^{D/2} \Gamma \left( g + 1 - \frac{D}{2}, G_i^{2/D} \right) \right)$$

(7)

Figure 2 shows the current-voltage characteristics calculated by formula of Eq. (7) for a superconductor with fractal clusters of a normal phase. It should be noted that similar current-voltage characteristics were observed in the experiments on the measurement of the dynamic resistance of BPSCCO composites containing silver inclusions [5]. A significant drop in the voltage across a sample for all fractal dimensions is observed beginning with the transport current $i = 1$, which coincides with the current of the transition into the resistive state found in Ref. [6] from the cumulative probability function of the critical current distribution. For smaller currents the trapped flux remains virtually unchanged because there are no clusters with such a small depinning current.

As may be seen from Fig. 2, the fractality of clusters significantly reduces electric field arising from the magnetic flux motion in a superconductor, and this effect is enhanced with decreasing the $g$-parameter. This feature can be explained by peculiarity of the exponential-hyperbolic distribution of Eq. (3). An increase in the fractal dimension leads to expansion of the critical current distribution toward greater values of the current. At the same time, the total area under the curve $f = f (i)$ remains unchanged because the probability function of Eq. (3) is normalized to unity in the whole interval of possible positive critical currents. This implies that an increasing number of clusters, which can best trap the magnetic flux, are involved into the process. Therefore, the number of vortices broken away from the pinning centers by the Lorentz force tends to decrease, so a smaller part of the flux can move and, hence, an electric field of lower magnitude is generated. The lower the electric field, the smaller energy is dissipated as a result of the transport current passage, so a decrease in the heat-evolution, which could induce the transition into the normal state, leads to an increase in the critical current for the superconductor containing such fractal clusters. The maximum current-carrying capability is achieved upon decreasing the $g$-parameter: in this case the clusters of small size, which have the maximum critical currents, make the most contribution to the overall distribution.

The probability re-distribution resulting from a change in the fractal dimension is characterized by the variance of critical currents and can be represented by the standard deviation

$$\sigma_i = G^{D/2} \sqrt{\frac{\Gamma (g + 1 - D)}{\Gamma (g + 1)}} - \left( \frac{\Gamma (g + 1 - D/2)}{\Gamma (g + 1)} \right)^2$$
Dependence of the critical current standard deviation on the fractal dimension has a pronounced superlinear character (Fig. 3). For \( g < 1 \) there exists an interval of \( D \geq g + 1 \), where the dispersion of critical currents exhibits infinite growth. This giant dispersion region is indicated by hatching in Fig. 3. The distributions with infinite variance are well known in the probability theory, a classical example is the Cauchy distribution [12]. However, such a behavior of the exponential-hyperbolic distribution of Eq. (3) is of special interest, since an increase in the variance leads to the increase in critical currents. A statistical distribution with the giant dispersion possesses an extremely extended “tail” corresponding to the contributions from clusters of high depinning currents. To the present, a minimum value of \( g \)-parameter (equal to zero) is realized in the YBCO-based composites with exponential distribution of the areas of normal phase clusters [3]. The films containing such clusters exhibit increased current-carrying capacity. We may expect that superconductors containing the normal phase clusters with the distribution of areas characterized by the values of \( g \leq D - 1 \) would provide for an additional increase in the critical currents.

Thus, the fractal properties of normal phase clusters significantly affect the dynamics of magnetic flux trapped in a superconductor. This phenomenon is related to a radical change in the distribution of critical currents caused by an increase in the fractal dimension of cluster boundaries. The most important result is that the fractality of the normal phase clusters enhances pinning, thus preventing the destruction of superconductivity by the transport current. This effect opens principally new possibilities for increasing the critical currents in composite superconductors by optimization of their geometric-morphological characteristics.

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FIG. 1. Influence of the fractal dimension $D$ and $g$-parameter of gamma-distribution on the distribution of critical currents. The dotted line is drawn for the Euclidean clusters ($D = 1$); and the solid lines for the clusters with the most fractal boundaries ($D = 2$). Curve (1) corresponds to the case of $D = 1$, $g = 0$; curve (2) of $D = 2$, $g = 0.5$; curve (3) of $D = 2$, $g = 0$; curve (4) of $D = 2$, $g = -0.5$. 
FIG. 2. The current–voltage characteristics at different values of $g$-parameter for Euclidean clusters ($D = 1$, dotted line), and for the clusters of the most fractal boundaries ($D = 2$, solid line). Curve (1) corresponds to the case of $D = 1, g = 0$; curve (2) of $D = 2, g = 0.5$; curve (3) of $D = 2, g = 0$; curve (4) of $D = 2, g = -0.5$. 
FIG. 3. Influence of the fractal dimension of the cluster boundary on the standard deviation of critical currents at different values of $g$-parameter. Curve (1) corresponds to the case of $g = 0.25$; curve (2) of $g = 0.5$; curve (3) of $g = 0.75$; curve (4) of $g = 1$. The region of giant dispersion of the critical current ($\sigma \geq g + 1$) is shown by hatching.