STRUCTURE FUNCTIONS AND DISTRIBUTIONS IN SEMILEPTONIC TAU DECAYS

J. H. KÜHN

Institut für Theoretische Teilchenphysik, Universität Karlsruhe, Kaiserstr. 12, 76128 Karlsruhe, Germany.

and

E. MIRKES

Physics Department, University of Wisconsin, Madison, WI 53706, USA

ABSTRACT

Semileptonic decays of polarized \( \tau \) leptons are investigated. The most general angular distribution of three meson final states (\( \tau \rightarrow \pi\pi\nu, K\pi\nu, \pi\pi\pi\nu, K\pi\pi\nu, KK\pi\nu, \eta\pi\pi\nu, \ldots \)) is discussed. It is shown, that the most general distribution can be characterized by 16 structure functions, most of which can be determined in currently ongoing high statistics experiments. Emphasis is put on \( \tau \) decays in \( e^+e^- \) experiments where the neutrino escapes detection and the \( \tau \) rest frame cannot be reconstructed. The structure of the hadronic matrix elements, based on CVC and chiral lagrangians, is discussed.

With the experimental progress in \( \tau \)-decays an ideal tool for studying strong interaction physics has been developed. In this paper we show, that detailed informations about the hadronic charged current for the decay into three pseudoscalar mesons can be derived from the study of angular distributions. Consider the semileptonic \( \tau \)-decay

\[
\tau(l, s) \rightarrow \nu(l', s') + h_1(q_1, m_1) + h_2(q_2, m_2) + h_3(q_3, m_3),
\]

where \( h_i(q_i, m_i) \) are pseudoscalar mesons. The matrix element reads as

\[
\mathcal{M} = \frac{G}{\sqrt{2}} \left( \cos \theta_C \right) M_\mu J_\mu,
\]

with \( G \) the Fermi-coupling constant. The cosine and the sine of the Cabibbo angle (\( \theta_C \)) in \( \left( \cos \theta_C \right) \) have to be used for Cabibbo allowed \( \Delta S = 0 \) and Cabibbo suppressed \( |\Delta S| = 1 \) decays, respectively. The leptonic (\( M_\mu \)) and hadronic (\( J_\mu \)) currents are given by \( M_\mu = \bar{u}(l', s')g_\mu(g_V - g_A\gamma_5)u(l, s) \) and \( J_\mu(q_1, q_2, q_3) = \langle h_1(q_1)h_2(q_2)h_3(q_3)|V^\mu(0) - A^\mu(0)|0 \rangle \). \( V^\mu \) and \( A^\mu \) are the vector and axial vector quark currents, respectively. The most

\footnote{Talk presented by E. Mirkes at the DPF94 Meeting, Albuquerque, New Mexico, USA; August 2-6, 1994.}
general ansatz for the matrix element of the quark current $J^\mu$ is characterized by four formfactors \[ J^\mu(q_1, q_2, q_3) = V_1^\mu F_1 + V_2^\mu F_2 + i V_3^\mu F_3 + V_4^\mu F_4, \] (3)

with

\begin{align*}
V_1^\mu &= q_1^\mu - q_3^\mu - Q^\mu \frac{Q(q_1 - q_3)}{q_2^2}, \\
V_2^\mu &= q_2^\mu - q_3^\mu - Q^\mu \frac{Q(q_2 - q_3)}{q_2^2}, \\
V_3^\mu &= \epsilon^{\mu\alpha\beta\gamma} q_{1\alpha} q_{2\beta} q_{3\gamma}, \\
V_4^\mu &= q_1^\mu + q_2^\mu + q_3^\mu = Q^\mu.
\end{align*}

The formfactors $F_1$ and $F_2$ ($F_3$) originate from the axial vector hadronic current (vector current) and correspond to spin 1, whereas $F_4$ is due to the spin zero part of the axial current matrix element. The formfactors $F_1$ and $F_2$ can be predicted by chiral lagrangians, supplemented by informations about resonance parameters. Parametrizations for the $3\pi$ final states can be found in refs. \[1-3\]. In this case, only the axial vector current formfactors $F_1$ and $F_2$ contribute due to the $G$ parity of the pions. The $3\pi$ decay mode offers a unique tool for the study of $\rho, \rho'$ resonance parameters competing well with low energy $e^+e^-$ colliders with energies in the region below 1.7 GeV. As we will see later, the two body ($\rho$ and $\rho'$) resonances can be fixed by taking ratios of hadronic structure functions, whereas the measurement of four structure functions can be used to put constraints on the $a_1$ parameters. The decay modes involving different mesons (for example $K\pi\pi$, $KK\pi$ or $\eta\pi\pi$) allow for axial and vector current contributions at the same time. Explicit parametrizations for the form factors in these decay modes are presented in refs. \[4,5\]. The vector formfactor $F_3$ is related to the Wess-Zumino anomaly \[3\], whereas the axial vector form factors are again predicted by chiral lagrangians. The latter decay modes allow also for the study of $JP^{PC} = 0^{-+}$ and $JP^{PC} = 1^{++}$ resonances which are not directly accessible from other experiments.

Let us now introduce the formalism of the hadronic structure functions. The differential decay rate is obtained from

\[ d\Gamma(\tau \to \nu_\tau 3h) = \frac{1}{2m_\tau} \frac{G^2}{2} \left( \frac{\cos^2 \theta_c}{\sin^2 \theta_c} \right) \{L_{\mu\nu} H^{\mu\nu}\} dPS^{(4)}, \]

where $L_{\mu\nu} = M_{\mu}(M_\nu)^\dagger$ and $H^{\mu\nu} \equiv J^{\mu}(J^{\nu})^\dagger$. The considered decays \[\|\] are most easily analyzed in the hadronic rest frame $q_1 + q_2 + q_3 = \bar{Q} = 0$. The orientation of the hadronic system is characterized by three Euler angles ($\alpha$, $\beta$ and $\gamma$) as introduced in refs. \[1,3\]. Performing the analysis of $\tau \to \nu_\tau + 3$ mesons in the hadronic rest frame has the advantage that the product of the hadronic and the leptonic tensors reduce to a sum \[\|\]

\[ L^{\mu\nu} H_{\mu\nu} = \sum_X L_X W_X. \]

In fact in this system the hadronic tensor $H^{\mu\nu}$ is decomposed into 16 hadronic structure functions $W_X$ corresponding to 16 density matrix elements for a hadronic system in a spin one $[V_1^\mu, V_2^\mu, V_3^\mu]$ and spin zero state $[V_4^\mu]$ (nine of them originate from a pure
spin one and the remaining are pure spin zero or interference terms). The 16 structure functions contain the dynamics of the three meson decay and depend only on the hadronic invariants $Q^2$ and the Dalitz plot variables $s_i$. The leptonic factors $L_X$ contain the dependence on the Euler angles, (which determine the orientation of the hadronic system), on the $\tau$ polarization, on the chirality parameter $\gamma_{VA}$ and on the total energy of the hadrons in the laboratory frame as well. Analytical expressions for the 16 coefficients $L_X$ are first presented in ref. [1]. They can also be applied to the case, where the $\tau$ rest frame cannot be reconstructed because of the unknown neutrino momentum. The dependence of these coefficients on the $\tau$ polarization allow for an improved measurement of the $\tau$ polarization at LEP [see for example refs. [7,8] and references therein].

The hadronic structure functions $W_X$ on the other hand contain the full dynamics of the hadronic decay and a measurement of these structure functions provide a unique tool for low energy hadronic physics. They can be calculated from a decomposition of the hadronic matrix element $J^\mu$ and can be expressed in terms of the form factors $F_i$. We list here only the result for the pure spin one state.

\[
\begin{align*}
W_A &= (x_1^2 + x_3^2) |F_1|^2 + (x_2^2 + x_3^2) |F_2|^2 + 2(x_1 x_2 - x_3^2) \text{Re}(F_1 F_2^*) , \\
W_B &= x_2 |F_3|^2 , \quad W_C = (x_1^2 - x_3^2) |F_1|^2 + (x_2^2 - x_3^2) |F_2|^2 + 2(x_1 x_2 + x_3^2) \text{Re}(F_1 F_2^*) ,
\end{align*}
\]

\[
\begin{align*}
W_D &= 2 [x_1 x_3 |F_1|^2 - x_2 x_3 |F_2|^2 + x_3(x_2 - x_1) \text{Re}(F_1 F_2^*)] , \\
W_E &= -2x_3(x_1 + x_2) \text{Im}(F_1 F_2^*) , \quad W_F = 2x_4[x_1 \text{Im}(F_1 F_3^*) + x_2 \text{Im}(F_2 F_3^*)] ,
\end{align*}
\]

\[
\begin{align*}
W_G &= -2x_4[x_1 \text{Re}(F_1 F_3^*) + x_2 \text{Re}(F_2 F_3^*)] , \\
W_H &= 2x_3 x_4 [\text{Im}(F_1 F_3^*) - \text{Im}(F_2 F_3^*)] , \\
W_I &= -2x_3 x_4 [\text{Re}(F_1 F_3^*) - \text{Re}(F_2 F_3^*)].
\end{align*}
\]

The remaining structure functions originating from a possible (small) contribution from a spin zero state are presented in [1]. The variables $x_i$ are defined by $x_1 = V_{11}^x = q_1^x - q_3^y, x_2 = V_{22}^x = q_2^x - q_3^y, x_3 = V_{11}^y = -q_3^y, x_4 = V_{33}^y = \sqrt{Q^2}x_3q_3^y$, where $q_1^x$ ($q_3^y$) denotes the $x$ ($y$) component of the momentum of meson $i$ in the hadronic rest frame. The structure functions can be extracted by taking suitable moments with respect to an appropriate product of two Euler angles [3]. An alternative method to extract the structure functions can be achieved by a direct fit to the expressions [4, 5].

As an example, we will now present numerical results for the non vanishing structure functions $W_A, W_C, W_D$ and $W_E$ in the $3\tau$ decay mode, which originates from the spin one part of the hadronic current. Figure 1 shows predictions for the structure function ratios $w_C/w_A, w_D/w_A$ and $w_E/w_A$ as a function of $Q^2$, where we have integrated over the Dalitz plot variables $s_i$ [The integrated structure functions are denoted by lower
Figure 1: Ratio of the spin one hadronic structure functions \( w_C/w_A, w_D/w_A, w_E/w_A \) (from top to bottom) for \( \tau \to \nu \pi \pi \pi \) as a function of \( Q^2 \).

case letter \( w_X \). The results are based on the same parametrization of the formfactors as used in [1]. Although we have lost informations on the resonance parameters in the two body decays by integrating over \( s_1 \) and \( s_2 \), we observe interesting structures. One observes that all normalized structure functions are sizable. \( w_C/w_A \) approaches its maximal value 1 for small \( Q^2 \). Note, that the dependence on the \( a_1 \) mass and width parameters cancel in the ratio \( w_X/w_A \) in fig. 1. On the other hand, the \( Q^2 \) distributions of the structure functions \( w_{A,C,D,E} \) presented in fig. 2 are very sensitive to the \( a_1 \) parameters. As an example, fig. 2 shows predictions for the structure functions, where the different values for the \( a_1 \) width has been used. Therefore, the ratios in fig. 1 can be used to fix the model dependence in the two body resonances, whereas the structure functions itself put then rigid constraints on the \( a_1 \) parameters. It is therefore possible to test the hadronic physics in much more detail than it is possible by rate measurements alone.

The technique of structure functions also allow for a model independent test of the presence of spin zero components in the hadronic current. Such a contribution would lead to additional structure functions to (7) originating from the interference with the (large) spin one contributions.

A detailed discussion of the matrix elements for the decay modes involving different pseudo scalar mesons \([K\pi\pi\nu, KK\nu, \eta\pi\pi\nu]\) together with predictions for the corresponding structure functions and angular distributions is presented in ref. [4]. In this case, all 9 structure functions in (7) are nonvanishing because of the interference of the anomaly with the axial vector contributions. An analyses of these distributions would allow to test the underlying hadronic physics [like the different contributions from the axial and vector (Wess-Zumino anomaly) current] in detail. It is thus possible to con-

\[ ^2 \text{More constraints on the two body resonances can be obtained by analyzing the full dependence on } Q^2 \text{ and } s_i, \text{ which should be accessible with the present high statistic experiments.} \]
Figure 2: Spin one hadronic structure functions $w_A, w_C, w_D, w_E$ (from top to bottom) for $\tau \rightarrow \nu \pi \pi \pi$ as a function of $Q^2$. Results are shown for two sets of $a_1$ parameters: $m_{a_1} = 1.251$ GeV, $\Gamma_{a_1} = 0.599$ GeV (solid) and $m_{a_1} = 1.251$ GeV, $\Gamma_{a_1} = 0.550$ GeV (dashed)

firm (not only qualitatively) the presence of the Wess-Zumino anomaly in the decays modes $\tau \rightarrow \nu K\pi\pi$ and $\tau \rightarrow \nu K\pi\pi$.

In ref. [9], the technique of the structure functions has been extended to the $\tau \rightarrow \nu \omega\pi$ decay mode. This allows to test the model for the hadronic matrix element in this decay mode, which involves both a vector and a second class axial vector current.

References

[1] J.H. Kühn and E. Mirkes, Z.Phys. C56, 661, (1992).
[2] J.H. Kühn and A. Santamaria, Z. Phys. C48, 445, (1990).
[3] J.H. Kühn and E. Mirkes, Phys. Lett. B286, 381, (1992).
[4] R. Decker and E. Mirkes, Phys. Rev. D47, 4012 (1993).
[5] R. Decker, E. Mirkes, R. Sauer and Z. Was, Z. Phys. C58, 445, (1993).
[6] J. Wess and B. Zumino, Phys. Lett. B37, 95, (1971).
[7] P. Privitera, Phys. Lett. B318, 249, (1993).
[8] J. Shukla, these proceedings.
[9] R. Decker and E. Mirkes, Z.Phys. C57, 495, (1993).
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9408400v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9408400v1