Gravitational-Wave by Binary OJ287 in 3.5 PN Approximation

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We compute the gravitational radiation of the supermassive binary black hole OJ287. By fitting the data of its recent seven outbursts, we obtain the orbital motion up to 3.5PN (Post-Newtonian) order, the energy and angular momentum fluxes and the waveform up to the 2PN. It is found that the 1PN term of the energy flux, has an opposite sign to the Newtonian flux, and the 2PN term has an opposite sign to the 1PN one. The same pattern is also found for the angular momentum fluxes. The total flux is reduced by 30.8% on average from the Newtonian flux for OJ287, resulting in a smaller orbital decay rate than that of 2.5PN calculation. Consisting with this, it is checked that, in the sequence of non-dissipative PN forces, each term of $i^{th}$ order (dissipative or non-dissipative) has a sign opposite to the $(i - 1)^{th}$ order. The origin of this characteristic is traced to the appearance of the non-diagonal metric component $g_{0i}$ in PN approximation. This feature will have profound impact on estimation of gravitational waves from binary systems.

Introduction Blazar OJ287 at a redshift $z = 0.306$ has long been observed continuously, and has been identified as a binary supermassive black hole system [1–5] with masses $m_1 = 1.84 \times 10^{10} M_\odot$ and $m_2 = 1.46 \times 10^8 M_\odot$. Having a very high orbital velocity $v/c \simeq (0.06 \sim 0.26)$, it serves as a test platform for general relativity up to 3PN order, mainly via the precession effect on the orbit. Nevertheless, OJ287 is also supermassive, its gravitational radiation is very strong with amplitude $h \sim 10^{-15}$ as we shall see, and is a proper target of the low-frequency gravitational wave detectors. In this paper we compute its orbital motion up to 3.5PN, i.e, including the second order of dissipative term. Then we obtain for the first time its gravitational waveform up to 2PN, and demonstrate the energy and angular momentum fluxes up to 2PN correction to the Newtonian fluxes. The result is comprehensive and can be used for waveform templates for detectors.

Orbital Motion to 3.5PN Since $m_1 \gg m_2$, the configuration of the binary is modelled as such that the secondary is orbiting around the primary, the accretion disk of the primary is perpendicular to the orbital plane [3–5]. We do not include the possible spin effects for simplicity [4]. The 3.5PN equation of motion in the center-of-mass frame is given by [3–14],

$$\frac{dv}{dt} = \frac{Gm}{r^2} \times$$

$$[(1 + A_{1PN} + A_{2PN} + A_{3.5PN})e_r$$

$$+ (B_{1PN} + B_{2PN} + B_{2.5PN} + B_{3PN} + B_{3.5PN})v], \quad (1)$$

where $m = m_1 + m_2$, $v$ is the relative velocity, $e_r$ is the radial unit vector, $A_{iPN}$ and $B_{iPN}$ are the PN corrections of the $i$ order. In the polar coordinate $(r, \Phi)$ on the orbital plane, the solution is obtained straightforwardly [14], given an initial condition: $r_0$, $\dot{r}_0$, $\Phi_0$, $\dot{\Phi}_0$. By fitting with the observational data of the last seven outbursts of OJ287 [2–4], and taking into account of the time delay effect [12], using the method of least squares for the difference between the predicted and observed times of seven outbursts, we arrived at the optimal orbital parameters [15].

| Parameter       | Value                  |
|-----------------|------------------------|
| $m_1$           | $1.84 \times 10^{10} M_\odot$ |
| $m_2$           | $1.46 \times 10^8 M_\odot$ |
| period $P$      | 11.9248 years          |
| ellipticity $e$ | 0.6693                 |
| semi – major axis $a$ | 0.0535 pc             |
| $\omega$        | 238.35° at 1971.48     |
| precession rate $\Psi$ | 32.6359°/period       |

The overall behavior of orbital motion is that the secondary is orbiting around with a very large $\Psi$ due to the PN forces, the orbit is continuously shrinking and becomes more circular in the course of time, due to the
reaction of GW radiation. We have checked that the 3.5PN orbital motion fits the data better than the 2.5PN one by the least squares [2, 3]. The detail has been given in Ref. [12]. As Fig. 1 shows, the 3.5PN orbit has a slightly greater precession and gradually surpasses the 2.5PN orbit. More interestingly, as shown in Fig. 2, the mutual separation and velocity in the 3.5PN calculation are shrinking more slowly than those in the 2.5PN one, indicating that the binary described by 2.5PN must radiate away more gravitational energy than the 3.5PN description. Now we investigate this feature in detail.

**The Signs of PN Forces** The first dissipative terms are at 2.5PN ones [3, 13],

\[
A_{2.5\text{PN}} = \frac{1}{c^3} \nu \frac{Gm}{r} \left( \frac{24\nu^2}{5} + \frac{136 Gm}{15} \right),
\]

\[
B_{2.5\text{PN}} = \frac{1}{c^3} \nu \frac{Gm}{r} \left( \frac{8\nu^2}{5} + \frac{24 Gm}{5} \right),
\]

with \( \nu = m_1 m_2 / m^2 \). They cause damping of the orbit and give rise to the Newtonian quadrupole radiation. The next order dissipative terms are 3.5PN,

\[
A_{3.5\text{PN}} = \frac{1}{c^3} \nu \frac{Gm}{r} \left[ \left( \frac{366\nu^4}{35} + 12\nu^4 \right)
- 114\nu^2 r^2 - 12\nu^2 r^2 + 112 r^4 \right]
+ \frac{Gm}{r} \left( \frac{692\nu^2}{35} - \frac{524\nu^2}{15} + \frac{294\nu^2}{5} + \frac{376\nu^2}{5} \right)
+ \frac{G^2 m^2}{r^4} \left( \frac{3956}{35} + \frac{1084\nu}{5} \right),
\]

(4)

Without these four dissipative terms, as has been checked, the 3PN orbit will precess only and the radius will not shrink, so that the energy and angular momentum are conserved.

Fig. 3 demonstrates that, \( A_{2.5\text{PN}} \) and \( A_{3.5\text{PN}} \) always have opposite signs at any instance of time, and the amplitude of \( A_{3.5\text{PN}} \) is \( \sim 0.3 \) that of \( A_{2.5\text{PN}} \). So is with \( B_{2.5\text{PN}} \) and \( B_{3.5\text{PN}} \). Therefore, the 3.5PN force is just in opposite direction to the 2.5PN one, the 3.5PN description yields less orbital damping and less gravitational radiation than the 2.5PN one. Similarly, we find that in the sequence of the non-dissipative PN terms, \( A_{1\text{PN}}, A_{2\text{PN}}, A_{3\text{PN}}, \) each term has a sign just opposite to its precedent term at any instance of time. The same alternating behavior also occurs for the sequence of \( B_{1\text{PN}}, B_{2\text{PN}}, B_{3\text{PN}} \).

**The Energy and Angular Momentum Fluxes** Since the dissipative 3.5PN forces are opposite to the 2.5PN, the work done \( \left( A_{3.5\text{PN}} \mathbf{e}_r + B_{3.5\text{PN}} \mathbf{v} \right) \cdot \mathbf{dr} \) is consequently opposite to \( \left( A_{2.5\text{PN}} \mathbf{e}_r + B_{2.5\text{PN}} \mathbf{v} \right) \cdot \mathbf{dr} \). Thus one expects that, in the total energy flux [10, 13]

\[
\frac{dE}{dt} = \dot{E}_N + \dot{E}_{1\text{PN}} + \dot{E}_{2\text{PN}},
\]

(6)
The dominant Newtonian flux 
\[ E_N = \frac{8}{15} \frac{G^3 m^4 \nu^2}{c^5 r^4} (12 \nu^2 - 11 r^2) \] 
induced by 2.5PN force has an opposite sign to 
\[ \dot{E}_{1\text{PN}} = \frac{8}{15} \frac{G^3 m^4 \nu^2}{c^5 r^4} \times \]
\[ [-2(1487 - 1392 \nu) v^2 r^2 - 160(17 - \nu) m^2 r^2 v^2]
\[ + (785 - 852 \nu) v^4 + 3(687 - 620 \nu) v^4]
\[ + 8(367 - 15 \nu) \frac{m^2}{r^2} + 16(1 - 4 \nu) \left( \frac{m}{r} \right)^2 \] 
induced by 3.5PN force. In turn, for the same reason, \( \dot{E}_{2\text{PN}} \) (see Refs. [16, 17]) induced by 4.5PN force [19] has an opposite sign to \( \dot{E}_{1\text{PN}} \) [16]. The energy fluxes of OJ287 are explicitly demonstrated in Fig. 4 with \( \dot{E}_N > 0, \dot{E}_{1\text{PN}} < 0, \dot{E}_{2\text{PN}} > 0 \), which is in a complete agreement with the relative signs of 3.5PN and 2.5PN forces. Similarly, the angular momentum fluxes [17, 18, 20] \( dJ/dt = J_N + J_{1\text{PN}} + J_{2\text{PN}} \) of OJ287 is computed with a result that \( J_N > 0, J_{1\text{PN}} < 0, J_{2\text{PN}} > 0 \). During each orbiting period the emission occurs mostly around the periastron where \( r \) is minimum and \( v \) attains the maximum. Integrated over a period, the negative \( \dot{E}_{1\text{PN}} \) reduces \( dE/dt \) by \( \sim 30\% \), and \( J_{1\text{PN}} \) reduces \( dJ/dt \) by \( \sim 17\% \). This huge amount of reduction of radiated energy and angular momentum is due to the high orbital speed of OJ287. Fig. 5 shows that the period \( P \) decreases more slowly in 3.5PN than in 2.5PN, a result that is explained by \( \dot{E}_{1\text{PN}} < 0 \) and \( J_{1\text{PN}} < 0 \). This pattern is consistent with the formula (4.29) of \( \dot{P} \) and the formula (4.31) of \( \dot{P} \) in Ref. [17], but different from (4.26) in Ref. [18]. The lesson learned is that, 2.5PN calculations of the Newtonian fluxes \( \dot{E}_N, \dot{J}_N \), and the heuristic rate \( \dot{P} \) [21] are over-estimations. A similar pattern has also been noticed in gravitational recoil due to linear momentum loss, where the Newtonian, 1PN, and 2PN contributions change sign alternatingly [22, 24].

The Waveform Up to 2PN, the traceless-transverse (TT) gravitational waveform is given by [16]
\[ h_{TT}^{ij} = \frac{2Gmν}{R} \left[ Q_{ij} + P^{1/2}Q_{ij} + PQ_{ij} + P^{3/2}Q_{ij} + P^2Q_{ij} \right]_{TT} \]
where \( R \) is the distance from the binary to the observer, \( Q_{ij} \) is the dominant, quadrupole momentum part, and the remaining terms with superscripts denote the the effective PN order (their explicit expressions are given by (6.11) in Ref. [16]). Using the resulting orbital motion, we compute \( h_{TT}^{ij} \) up to 2PN order. Since OJ287 is at cosmic distance with \( z \sim 0.306 \), the effect of cosmic expansion should be taken into account. The distance \( R \) is taken to be the luminosity distance:
\[ D_L(z) = \frac{c}{H_0} (1 + z) \int_0^z \frac{dz'}{\sqrt{Ω_m(1 + z')^3 + Ω_Λ}} \]
in a spatially flat \( Λ \text{CDM} \) Universe with \( Ω_Λ = 0.685, H_0 = 67.3 \text{(km/s)/Mpc} \) and \( Ω_m = 1 - Ω_Λ \) [23], where \( H_0 \) is the Hubble constant. For OJ287, \( R = D_L(0.306) \approx 1646.89 \text{ Mpc} \). The prefactor amplitude of \( h_{ij} \) is \( \frac{2Gmν}{R} \approx 8.42 \times 10^{-15} \). Let the observer’s direction be given by \( \mathbf{n} = \sin \theta \mathbf{e}_x + \cos \theta \mathbf{e}_z \), where \( \mathbf{e}_x \) is the normal vector and \( \mathbf{e}_z \) is along the semimajor axis on the orbital plane. Take an orthonormal basis (\( \mathbf{e}_o, \mathbf{e}_φ, \mathbf{n} \)). The two independent components of GW measured by the observer are [26]
\[ h_{TT}^{θθ} = -h_{TT}^{φφ} = \frac{1}{2} (\cos^2 θ h_{xx} - h_{yy}), \]
\[ h_{TT}^{θφ} = \cos θ h_{xy}. \]
Fig. 6 shows the wave form \( h_{TT}^{θθ} \) and \( h_{TT}^{θφ} \) using the 3.5PN orbital result. The peak amplitude reaches \( |h_{ij}| \sim 1 \times 10^{-15} \) with a frequency \( f \sim 2.66 \times 10^{-9} \text{ Hz} \), higher than the amplitude of relic gravitational waves in this frequency range [27]. Both can be the targets of pulsar timing arrays, such as PPTA [28], EPTA [29], NANOGrav [30], FAST [31] and SKA [32]. But its frequency may be too low for LIGO and VIRGO [33].

The Origin of the Opposite Sign Now let us see the origin of the opposite sign occurring at each orbit. Inspect that, aside the common factor, \( A_{2\text{PN}} \) in Eq. (24) contains the negative factor \( \left(- \frac{1}{8} \frac{c^4}{m} - 1 \frac{G^2}{16 m^2} \right) \), while the dominant terms of \( A_{3\text{PN}} \) in Eq. (11), such as \( v^4, i^4, i^2 v^2, i^2 v^2, (\frac{Gm^2}{c^5})^2 \), are positive. Thus \( A_{3\text{PN}} \) has an opposite sign to \( A_{2\text{PN}} \). Similarly, inspection of
Eq. (3) tells that $\theta = 0$. 3.5PN. The observer is facing the orbital plane with the angle $\theta = 0$. FIG. 5: The period $P$ in 3.5PN decreases at a slower rate than in 2.5PN.

$B_{2.5\text{PN}}$ in Eq. (4), such as $v^4$, $r^4$, $v^2$, and square brackets, all have large negative coefficients, so $B_{2.5\text{PN}} < 0$.

This feature can be inferred generally from the PN approximation, say, in the method of the PN iterative metric parameterized by retarded potentials [14]. The acceleration is determined by $a^i = F^i - \frac{d}{dt}(P^i - v^i)$ (see Eq. (2.13) in Ref. [14]), where

$$F^i \sim \partial_i V + \frac{1}{c^2}(-V \partial_i V + \frac{3}{2} \partial_i V v^2 - 4 \partial_i V v^i) + O(1/c^4),$$

$$(P^i - v^i) \sim \frac{1}{c^2}(v^2 v^i + 3 v^i v^j - 4 V^i) + O(1/c^4),$$

with $V \sim Gm/r \sim v^2$ and $V_j \sim v^j V$ to leading order. For each order, the dominant contribution from $\partial_i V$ is followed by the next order contribution $\frac{1}{c^2}(-V \partial_i V + \frac{3}{2} \partial_i V v^2 - 4 \partial_i V v^i) \sim 10^{-7}$, and $-\frac{d}{dt}(P^i - v^i) \sim 10^{-5}$ having an opposite sign to $\partial_i V$. Thus, for each PN order of $a^i$, the next order has an opposite sign. The cause is traced to $\frac{1}{c^2}(-4 \partial_i V v^i)$ in $F^i$, which originates from the metric component $g_{0i} = -4V_i/c^3 + O(1/c^4)$, i.e., the shift [14].

**Conclusion** We have studied gravitational radiation of the binary black hole OJ287, apply the Post-Newtonian approximation up to 3.5PN order in the orbital motion. By explicit computations, we have found that, at any instance of time, the energy flux $E_{1\text{PN}}$ induced by the 3.5 force is negative, opposite to the Newtonian flux $E_N$ induced by the 2.5PN force, and consistent to this, the dissipative 3.5PN force is always opposite to the 2.5PN one. For OJ287 this reduction of energy flux is as high as $\sim 30\%$. Therefore, for binaries with high orbiting speed, this effect is significant and must be taken into account in evaluation of radiation of GW. We have also demonstrated that, for non-dissipative PN forces, each PN order has a sign opposite to the precedent PN order. The origin this characteristic has been traced to the non-diagonal metric component $g_{0i}$ in PN approximation. With a peak amplitude $|h_{ij}| \sim 10^{-15}$ and low frequencies $f \sim 2.7 \times 10^{-9}$ Hz, the GW from OJ287 is an ideal target for pulsar timing array detectors.

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