Measurement of the quantum zero-point motion of a nanomechanical resonator

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We present optical sideband spectroscopy measurements of a mesoscopic mechanical oscillator cooled near its quantum ground state. The mechanical oscillator, corresponding to a 3.99 GHz acoustic mode of a patterned silicon nanobeam, is coupled via radiation pressure to a pair of co-localized 200 THz optical modes. The mechanical mode is cooled close to its quantum ground state from a bath temperature of \( T_b \approx 18 \, \text{K} \) using radiation pressure back-action stemming from the optical pumping of one of the optical cavity resonances. An optical probe beam, resonant with the second optical cavity resonance, is used to transduce the mechanical motion and determine the phonon occupancy of the mechanical mode. Measurement of the asymmetry between up-converted and down-converted photons of the probe beam yields directly the displacement noise power associated with the quantum zero-point motion of the mechanical oscillator, and provides an absolute calibration of the average phonon occupancy of the mechanical mode.

The Heisenberg uncertainty principle, one of the fundamental consequences of quantum theory, restricts the certainty with which the position and momentum of an object may be simultaneously known and defined. Consequently, an object confined to a local potential possesses a non-zero ground state energy associated with random quantum fluctuations of its position. For mechanical systems of our daily experience, this so-called quantum zero-point motion is masked by thermal noise resulting from interaction with the environment. To observe quantum motion, the dual and antagonistic feats of isolating a mechanical system from its environment whilst measuring its position must be achieved [1–3]. Recently, coupled electro- and opto-mechanical systems have been cooled into their mechanical quantum ground state by a combination of cryogenic pre-cooling and radiation pressure back-action [4, 5]. In this work, we extend these results by laser cooling the acoustic mode of a silicon optomechanical crystal resonator to near its quantum ground state (occupancy 2.6 ± 0.2 phonons) while simultaneously monitoring its motion with a probe laser resonant with a secondary optical mode of the cavity. Operating in the resolved sideband regime, the spectral selectivity of the secondary cavity mode is used to separate, measure, and compare the intensity of the blue- (anti-Stokes) and red-shifted (Stokes) sidebands created by the mechanical resonator’s motion. The observed asymmetry in the probe sideband amplitudes provides a direct measure of the displacement noise power associated with quantum zero-point fluctuations and allows for an intrinsic calibration of the phonon occupation number of the nanomechanical resonator.

Experiments with trapped atomic ions and neutral atoms [6–8], dating back several decades, utilized techniques such as resolved sideband laser cooling and motional sideband absorption and fluorescence spectroscopy to cool and measure a single trapped particle in its vibrational quantum ground state. These experiments generated significant interest in the coherent control of motion and the quantum optics of trapped atoms and ions [9], and were important stepping stones towards the development of ion-trap based quantum computing [10, 11]. Larger scale mechanical objects, such as fabricated nanomechanical resonators, have only recently been cooled close to their quantum mechanical ground state of motion [4, 5, 12–17]. In a pioneering experiment by O’Connell, et al. [16], a piezoelectric nanomechanical resonator has been cryogenically cooled \( (T_c \approx 25 \, \text{mK}) \) to its vibrational ground state and strongly coupled to a superconducting circuit qubit allowing for quantum state preparation and read-out of the mechanics. An alternate line of research has been pursued in circuit and cavity optomechanics [1], where the position of a mechanical oscillator is coupled to the frequency of a high-Q electromagnetic resonance allowing for back-action cooling [13, 19] and continuous position read-out of the oscillator. Such optomechanical resonators have long been pursued as quantum-limited sensors of weak classical forces [1–3, 14, 20], with more recent studies exploring optomechanical systems as quantum optical memories and amplifiers [21, 24], quantum nonlinear dynamical elements [25], and quantum interfaces in hybrid quantum systems [26, 29].

Despite the major advances in circuit and cavity optomechanical systems made in the last few years, all experiments to date involving the cooling of mesoscopic mechanical oscillators have relied on careful measurement and calibration of the motionally scattered light to obtain the average phonon occupancy of the oscillator, \( \langle n \rangle \). Approach towards the quantum ground state in such experiments is manifest only as a weaker measured signal, with no evident demarcation between the classical and quantum regimes of the oscillator. A crucial aspect of zero-point fluctuations of the quantum ground state, is that they cannot supply energy, but can only contribute to processes where energy is absorbed by the mechanics. This is completely different from classical noise. Techniques that attempt to measure zero-point motion without being sensitive to this aspect (i.e., standard continuous linear position detection)
can always be interpreted classically and described by some effective temperature.

A more direct method of thermometry and characterization of the quantized nature of a mechanical oscillator, one particularly suited to small \( \langle n \rangle \) and utilized in the above-mentioned trapped atom experiments \([6-8]\), is referred to as motional sideband spectroscopy. This method relies on the fundamental asymmetry in the quantum processes of absorption from (proportional to \( \langle n \rangle \)) and emission into (proportional to \( \langle n \rangle + 1 \)) the mechanical oscillator of phonons. In the case of atomic systems, this asymmetry can be measured in the motionally-generated Stokes and anti-Stokes sidebands in either the fluorescence or absorption spectrum of the atom. The ratio of the Stokes to anti-Stokes motional sideband amplitudes \((\langle (n) + 1 \rangle/\langle n \rangle)\) deviates significantly from unity as the quantum ground state is reached \((\langle n \rangle \rightarrow 0)\), and provides a self-calibrated reference for the phonon occupancy. In the present experiment we cool a nanomechanical resonator to near its quantum ground state, and measure the asymmetry in the motional sidebands utilizing a form of resolved sideband spectroscopy based upon the filtering properties of a high-\( Q \) optical cavity with linewidth narrower than the mechanical frequency.

The cavity optomechanical system studied in this work consists of a patterned silicon nanobeam which forms an optomechanical crystal (OMC) \([9]\) capable of localizing both optical and acoustic waves (see Fig. 1). The cavity is designed to have two optical resonances, one for cooling and one for read-out of mechanical motion. The cooling mode is chosen as the fundamental mode of the patterned nanobeam cavity, with a frequency \( \omega_c/2\pi = 205.3 \) THz and a corresponding free-space wavelength of \( \lambda_c = 1460 \) nm. The read-out mode is the second-order mode of the cavity with \( \omega_r/2\pi = 194.1 \) THz \((\lambda_r = 1545 \) nm). An in-plane mechanical breathing mode at \( \omega_m/2\pi = 3.99 \) GHz, confined at the center of the nanobeam due to acoustic Bragg reflection, couples to both optical resonances with zero-point optomechanical coupling rates \( g_c/2\pi = 960 \) kHz and \( g_r/2\pi = 430 \) kHz for the cooling and read-out modes, respectively \((g_c \) and \( g_r \) are measured for each cavity mode from optically-induced damping of the mechanical mode as in Ref. \([5]\)). The Hamiltonian of the coupled system is given by

\[
\hat{H} = \hbar \left( \omega_r + g_c \hat{\xi} / x_{\text{zpf}} \right) \hat{a} \hat{a}^\dagger + \hbar \left( \omega_c + g_r \hat{\xi} / x_{\text{zpf}} \right) \hat{c} \hat{c}^\dagger \\
+ \hbar \omega_m \hat{b} \hat{b}^\dagger,
\]

where we have defined \( \hat{c} (\hat{c}^\dagger) \) and \( \hat{a} (\hat{a}^\dagger) \) to be the annihilation (creation) operators for photons in the cooling and read-out modes of the optical cavity. The optical modes are coupled to the mechanical displacement operator of the breathing mode, \( \hat{\xi} \equiv x_{\text{zpf}} (\hat{b}^\dagger + \hat{b}) \), where \( \hat{b}^\dagger \) and \( \hat{b} \) are the creation and annihilation operators for phonons in the mechanical resonator, and \( x_{\text{zpf}} \) is the zero-point fluctuation amplitude of the mechanical motion.

An illustration of the experimental apparatus used to cool and measure the nanomechanical oscillator is shown in Fig. 2. In order to pre-cool the oscillator, the silicon sample is mounted inside a Helium flow cryostat. For a cold finger temperature of 6.3 K, the bath temperature of the mechanical mode is measured to be 18 K (corresponding to a thermal phonon occupation of \( n_h = 94 \) phonons) through calibrated optomechanical thermometry as described in the Methods section and Ref. \([5]\). At this temperature the mechanical damping rate to the thermal bath is found to be \( \gamma_c/2\pi = 43 \) kHz, corresponding to a intrinsic quality factor \( Q_m = 9.2 \times 10^4 \). The optical resonances of the OMC cavity are measured to have total damping rates of \( \kappa_c/2\pi = 390 \) MHz and \( \kappa_r/2\pi = 1.0 \) GHz for the cooling and read-out modes, respectively. An optical fiber taper, formed from standard single mode optical fiber, is used to optically probe the OMC cavity via evanescent coupling. The component of extrinsic damping resulting from coupling to the optical fiber taper waveguide is measured to be \( \kappa_{e,r}/2\pi = 46 \) MHz for the cooling mode and \( \kappa_{e,c}/2\pi = 300 \) MHz for the read-out mode.

As alluded to above, resolved sideband cooling in optomechanical cavities follows physics which is formally similar to the Raman processes used to cool ions to their motional ground state \([6]\). A cooling laser, with frequency \( \omega_l = \omega_c - \omega_m \), is tuned a mechanical frequency red of the cooling cavity resonance of the OMC, giving rise to an intra-cavity photon population \( n_c \) at frequency \( \omega_c \). The oscillations of the mechanical system cause scattering of the intra-cavity cooling beam laser light into Stokes and anti-Stokes sidebands at \( \omega_c - 2\omega_m \) and \( \omega_c \), respectively. Since the anti-Stokes sideband is resonant with the cavity at \( \omega_c \), and \( \kappa_c < \omega_m \), the anti-Stokes optical up-conversion process is greatly enhanced relative to the Stokes down-conversion process. This leads to cooling of the mechanical mode and can be modeled effectively as an additional mechanical damping term of \( \gamma_c = 4 g_c^2 n_c / \kappa_c \) due to coupling to a near-zero temperature bath represented by the cooling laser. Assuming a deeply resolved sideband system \((\kappa_c/\omega_m \ll 1)\), the back-action cooled mechanical mode occupancy is approximately given by \( \langle n \rangle_c = \gamma_c n_h / (\gamma_c + \gamma_r) \) \([18, 19]\).

Optical scattering of the intra-cavity light field can also be used to read out the motion of the coupled mechanical oscil-
FIG. 2: Experimental setup. The silicon OMC cavity is mounted into a continuous flow Helium cryostat and optically coupled via an optical fiber taper probe. Two narrowband lasers (New Focus Velocity series; linewidth ~ 300 kHz) are used to independently cool and read out the motion of the breathing mechanical mode of the OMC cavity. The 1500 nm (read-out) and 1400 nm (cooling) laser beams are passed through variable optical attenuators (VOAs) to set the laser power, and combined at an optical fiber wavelength multiplexer (λ-MUX) before being sent into the cryostat through an optical fiber. Transmission of the 1500 nm read-out beam through the OMC cavity, collected at the output end of the optical fiber, is filtered from the 1400 nm cooling beam light via an optical fiber bandpass filter, pre-amplified by an Erbium-doped fiber amplifier (EDFA), and detected on a high-speed photodetector (PD2) connected to a real-time spectrum analyzer (RSA). Other components are labeled as amplitude-modulation (a-m) and phase-modulation (φ-m) electro-optic modulators, fiber polarization controller (FPC), swept frequency radio-frequency signal generator (rf-sg), lock-in amplifier (lock-in), optical switches (SW), and calibrated optical power meter (power meter). Further details of the experimental apparatus and measurements are in the Methods section.
occupancy of the cooled mechanical oscillator,
\[
\eta' \equiv \frac{L_r - I_r}{1 + C_r} - \frac{1}{1 - C_r} = \frac{1}{\langle n \rangle_c}.
\]
where for \(C_r \ll 1\) we recover the standard relation given in Equation (6).

Figure 4 summarizes the measurement results of the calibrated mechanical mode thermometry and motional sideband asymmetry for the silicon nanobeam OMC cavity. These measurements were performed with the cooling laser locked a mechanical frequency to the red of the fundamental mode of the OMC cavity, with the cooling laser power swept from \(n_c \sim 1\) to 800. For each cooling laser power, the read-out laser is used to both estimate the mechanical mode phonon occupancy and to compare the motional sideband amplitudes. With the read-out laser set to a detuning from the read-out cavity mode of \(\Delta = \omega_m\), a Lorentzian spectrum with linewidth \(\gamma_+\) and integrated area \(I_+\) is measured in the read-out laser photocurrent PSD, from which a mode occupancy of \(\langle n \rangle_c\) is inferred from independently calibrated system parameters (see Methods and Ref. [3]). Similarly, by placing the the read-out laser at \(\Delta = -\omega_m\) we obtain spectra with linewidth \(\gamma_-\) and integrated area \(I_- \propto \langle n \rangle_c + 1\), from which we estimate \(\langle n \rangle_c\) EIT spectroscopy [22] of the read-out optical cavity mode is also performed, providing a second, and more accurate (given the better signal-to-noise of such measurements) estimate of the mechanical damping rates \(\gamma_{\pm}\).

From the measured mechanical damping rates, the read-out cooperativity \(C_r\) is found using relation (7), and plotted in Figure 4h. The ratios \(\xi = \gamma_-/\gamma_+\) and \(\langle n \rangle_+ / \langle n \rangle_-\) are plotted in Figure 4b, and found to be approximately equal in accordance with theory. From \(\langle n \rangle_{\pm}\), the estimated phonon occupation number in the absence of a read-out beam, \(\langle n \rangle_c\), is calculated and plotted in Figure 4b versus the mechanical damping rate \(\bar{\gamma} = (\gamma_+ + \gamma_-)/2\). As expected, \(\langle n \rangle_c\) drops approximately linearly with \(\bar{\gamma}\), reaching a minimum value of approximately 2.6 \pm 0.2 phonons, limited in this case by available cooling laser power. As mentioned, the read-out beam was kept low in power so as to minimize the back-action in comparison to that of the cooling beam. In particular, for the largest cooling powers \(C_r \sim 2 - 3\%) and \(n_{\pm}\) are within 10% of \(\langle n \rangle_c\).

In Figure 4h, the measured values of the expression \(\eta'\) are plotted versus the calibrated value of \(\langle n \rangle_c\). Also plotted are the classical and quantum values of this expression, 0 and \(1 / \langle n \rangle_c\), respectively. A clear divergence from the classical result of \(\eta' = 0\) is apparent, agreeing with the deviation due to quantum zero-point fluctuations of the mechanical oscillator. At the largest powers, we measure asymmetries in the motional sideband amplitudes on the order of 40% in agreement with the inferred \(\langle n \rangle_c = 2.6\) phonons from calibrated thermometry. This deviation is directly apparent in the measured spectra, shown for \(\langle n \rangle_c = 85, 63,\) and 3.2 phonons in Figure 4b-g, with the shaded region corresponding to the noise power contribution due to quantum zero-point fluctuations.

While the quantum nature of a mechanical resonator will come as little surprise to most physicists, its observation...
through the zero-point motion is a significant step towards observing and controlling the quantum dynamics of mesoscopic mechanical systems. By demonstrating the fundamentally quantum behaviour of an engineered mechanical nanostructure, we have shown that realizable optomechanical systems have the sensitivity and environmental isolation required for such quantum mechanical investigations.

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### METHODS

**Calibrated Thermometry.** In this work, an estimate of the mean phonon occupation number $\langle n \rangle$ is accomplished using the read-out optical cavity following a calibration procedure outlined in Ref. [5]. To briefly summarize (see experimental set-up in Fig. 2), a 1500 nm band read-out laser is sent through an amplitude electro-optic modulator (EOM), and combined with the cooling beam at a fiber optic multiplexer ($\lambda$–MUX). FPCs are used to ensure correct polarization before the EOMs and the OMC device. Calibration of the detuning of the read-out (cooling) laser from the read-out (cooling) cavity is performed using a form of EIT-spectroscopy [5][22] in which the reflection of optical sidebands generated on the read-out laser via a swept frequency radio-frequency signal generator (rf-sg) are monitored on PD1. The intra-cavity photon population generated by either the cooling ($n_c$) or read-out ($n_r$) laser beam is inferred from measurements of the power at the input and output of the taper, combined with a measurement of the asymmetry in the losses of the taper before and after the OMC device. A wavemeter ($\lambda$–meter) is used to calibrate the optical linewidths of each cavity mode and to lock the lasers to within a few MHz of a given frequency. The optomechanical coupling rate of the read-out cavity, $g_r$, is determined from the measured mechanical linewidth versus $n_r$ as measured in the transmitted read-out laser photocurrent spectrum on the RSA. Calibration of the mechanical mode occupancy from the read-out laser photocurrent spectrum also requires knowledge of the optical power to detector voltage conversion ratio of PD2 ($G_v$) and the amplification furnished by the EDFA ($G_{EDFA}$). $G_v$ is determined by using SW3 to measure the total read-out laser intensity on the calibrated optical power meter. $G_{EDFA}$ is determined by using SW2 to measure the amplitude of a signal generated by the rf-sg, with and without the EDFA on line.

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