Economic Ordering Policy for VAR Deterioration Model with Non-stationary Two-warehouse Inventory and Demand

Abdullah M. A. Alshami*1 and Aniket A. Muley2
1Thamar University, Thamar, Yemen, Educational Faculty, Mathematics Department
2Swami Ramanand Teerth Marathwada University, Nanded, India
Mathematical School of Sciences, Statistics 431606 Maharashtra, India
Email: balshami11@gmail.com, aniket.muley@gmail.com

Abstract
This paper adopts the two-warehouse inventory, determination on the first run-time and VAR (Vector Auto Regression) deterioration model. The optimal EOQ in the interval of the finite horizon is determined under critical considerations. The non-stationary two-warehouse inventory, i.e. the inventory and initial inventory are non-stationary at level, but stationary after lag difference similar to demand (demand and initial demand). The output of the proposed model represented the optimal order quantity and optimal first run-time, the optimal total cost as integration of first order with the significant trend and intercept. The optimal demand is decreased during more risk as a deterioration variable to reduce the quantity in the stock. The initial demand is stationary after a first lag and the demand is stationary.

Keywords: initial inventory; optimal of first run-time; EOQ (Economic Ordering Quantity); total cost function (TC).

1. INTRODUCTION

Many types of researchers developed the inventory model, but this paper investigated the inventory model by the new assumptions as of the VAR (Vector Auto Regression) deterioration model.
and non-stationary inventory and initial inventory model. Maiti [1] developed a fuzzy model for deteriorating multi-outlet by applying triangular fuzzy. Chung et al. [2] investigated the model with permissible delay in payment under providing a discount in cash. Pal et al. [3] deals with the price and dependent demand in stock with permissible in delay in payment. Palanivel and Uthayakumar [4] developed the deteriorating model under new assumptions as price and dependent demand in the advertisement but in the output of the model in numerical analysis, the total cost decreased when the deterioration rate increased that is not supported by the inventory theory as Pal et al. [3]. Pal et al. [5] represented the EPQ under constant production across the year, fuzzy demand, and nonlinear deterioration. Omprakash et al. [6] attempted a model for the policy of the retailer’s inventory to minimize the total cost to the retailer’s inventory but the movement in the output is slow especially for the total cost. Jolai et al. [7] the framework of optimization is represented to optimize the production under variable deterioration. Neetu and Tomer [8] the inflation is considered as a variable in the deteriorating inventory and demand as the price of selling function. Tiwari et al. [9] developed the model through new consideration to the two-echelon supply chain; demand is non-constant as function in selling price. Tripathi and Kaur [10] investigated the deteriorating items by linear time-dependent deterioration and demand. In this paper, we propose a model to handle the inventory under the VAR (Vector Auto Regression) deterioration model, non-stationary demands to determine the optimal first run-time and total cost, EOQ (Economic Order Quantity) also, reactivation of the output of the assumed model. The model in this paper used when the industry has stacked with two sub-stocks for the single items.

2. MATERIAL AND METHOD

2.1. Assumptions and notions

2.1.1. Assumptions

The mathematical model is developed within assumptions

1) The planning horizon is finite.
2) Single item inventory control.
3) The demand is non-stationary.
4) Initial demand is non-stationary at level but stationary at lag=1.
5) Deterioration is estimated by VAR(2,2) model as $\theta = a_1 \theta(-1) + a_2 \theta(-2) + b_1 \theta_0(-1) + b_2 \theta_0(-2) + \epsilon$ where $a_1, a_2, b_1, b_2$ are real constants, $\epsilon$ is an error.
6) Initial deterioration is stationary over time.
7) There is no replacement or repair of deterioration items during the supposed period.
8) The shortage is not allowed.
9) The lead time is zero.
10) The inventory level at the end of the planning horizon is zero
11) The cost factors are deterministic.

2.1.2. Notations

$Q_1 = $ The order quantity in stock within $(0, t_1)$.
$Q_0 = $ The order quantity which is in stock within $(0, t_1)$.
$Q_5 = $ The difference of order quantity which is in stock within $(0, t_1)$.
$TC = $ The total relevant cost $(0, t_1)$. 
2.2. Parameters
The mathematical model is representing the following parameters:

- \( A \) = The fixed ordering cost per replenishment $/order.
- \( C \) = The unit purchasing price at time zero $/order.
- \( D_1 \) = The demand per unit time for \( I_1(t) \).
- \( D_0 \) = The initial demand per unit time for \( I_0(t) \).
- \( B_0 \) = positive real constant, \( B_0 > 0 \).
- \( B_1 \) = positive real constant, \( B_1 > 0 \).
- \( I_0(t) \) = The initial inventory level at \( (0, t_1) \).
- \( I_1(t) \) = The inventory level at time during \( (0, t_1) \).
- \( I_h \) = The interest charged per $ per unit by the supplier.
- \( t_1 \) = The first run time of each replenishment cycle for an emergency order.
- \( \theta \) = Deterioration units/unit time.
- \( \theta_0 \) = Initial deterioration units/unit time which is caused in \( \theta \).

3. MATHEMATICAL MODEL

Let \( I_1(t) \) is the inventory level at any time \( t, 0 \leq t \leq t_1 \). Depletion due to demand within first component interval \( 0 \leq t \leq t_1 \). The \( \frac{dI_1(t)}{dt} \) satisfied the Eq. (1) that describes the instantaneous state of \( I_1(t) \) over the open interval \( (0, t_1) \) is given by:

\[
\frac{dI_1(t)}{dt} = \theta \frac{dI_0(t)}{dt} + B_0D_0 + B_1D_1, \quad 0 \leq t \leq t_1, \quad 0 \leq \theta \leq 1,
\]

\[
I_1(t) - \theta I_0(t) = \int_t^{t_1} (B_0D_0 + B_1D_1) du = (B_0D_0 + B_1D_1)(t_1 - t).
\]  

Let \( I_0(t) \) is initial of the inventory level at any time \( t, 0 \leq t \leq t_1 \). Depletion due to initial demand within component interval \( t_1 \leq t \leq T \) and there is initial deterioration. The first-order differential equation Eq. (2) that describes the instantaneous state of \( I_0(t) \) over the open interval \( (0, t_1) \) is given by:

\[
\frac{dI_0(t)}{dt} + \theta_0 I_0(t) = -D_0, \quad 0 \leq t \leq t_1,
\]

where \( I_0(t_1) = 0 \) for equation of number. \( I_0(t) = I_{01}(t) = I_{01}(t) \int_t^{t_1} D_0 e^{\theta_0 u^2} du = \frac{D_0}{\theta_0} (e^{\theta_0(t_1-t)} - 1), \quad I_0(0) = \frac{D_0}{\theta_0} (e^{\theta_0 t_1} - 1), \quad Q_0 = \frac{D_0}{\theta_0} (e^{\theta_0 t_1} - 1) \).

3.1. Fixed ordering cost
We assumed that the fixed ordering cost over the planning horizon \( (0, t_1) \) consideration is:

\( TC_A = A \).
3.2. Purchasing cost

The purchasing cost is calculated as \(TC_p = C \left( (B_0 D_0 + B_1 D_1) t_1 + \frac{\theta D_0}{\theta_0} (e^{\theta_0 t_1} - 1) \right)\).

3.3. Holding cost excluding interest cost

The average inventory quantity is used to obtain holding cost
\[
I = \int_0^{t_1} I_1(t) dt = \int_0^{t_1} (\alpha B_0 D_0 + B_1 D_1) t_1 + \frac{\theta D_0}{\theta_0} (e^{\theta_0 t_1} - 1) dt
\]
\[
= \frac{(B_0 D_0 + B_1 D_1) t_1^2}{2} + \frac{\theta D_0}{\theta_0^2} (e^{\theta_0 t_1} - \theta_0 t_1 - 1).
\]
\[
TC_h = I_h \left[ \frac{(B_0 D_0 + B_1 D_1) t_1^2}{2} + \frac{\theta D_0}{\theta_0} (e^{\theta_0 t_1} - 1) \right],
\]
\[
TC = TC_A + TC_h + TC_p.
\]

Then \(TC = \left[ A + C \left( (B_0 D_0 + B_1 D_1) t_1 + \frac{\theta D_0}{\theta_0} (e^{\theta_0 t_1} - 1) \right) \right] + I_h \left[ \frac{(B_0 D_0 + B_1 D_1) t_1^2}{2} + \frac{\theta D_0}{\theta_0} (e^{\theta_0 t_1} - 1) \right].

3.4. Economic Order Quantity

To find optimal run-time by minimizing the total cost function we found-out the optimal demand and optimal deterioration as the following
\[
TC = \left[ \frac{1}{t_1} \right] A + C \left( (B_0 D_0 + B_1 D_1) t_1 + \frac{\theta D_0}{\theta_0} (e^{\theta_0 t_1} - 1) \right] + I_h \left[ (B_0 D_0 + B_1 D_1) t_1 + \frac{\theta D_0}{\theta_0} (e^{\theta_0 t_1} - \theta_0 t_1 - 1) \right].
\]
\[
dTC \frac{dt_1}{d} = \left[ -\frac{1}{t_1^2} \right] \left[ A + C \left( (B_0 D_0 + B_1 D_1) t_1 + \frac{\theta D_0}{\theta_0} (e^{\theta_0 t_1} - 1) \right) + I_h \left( \frac{(B_0 D_0 + B_1 D_1) t_1^2}{2} + \frac{\theta D_0}{\theta_0} (e^{\theta_0 t_1} - \theta_0 t_1 - 1) \right) \]
\]
\[
+ \frac{\theta D_0}{\theta_0^2} (e^{\theta_0 t_1} - \theta_0 t_1 - 1) \right] \left[ C \left( (B_0 D_0 + B_1 D_1) + \theta D_0 e^{\theta_0 t_1} \right) + \theta D_0 e^{\theta_0 t_1} \right] + I_h \left( (B_0 D_0 + B_1 D_1) t_1 + \frac{\theta D_0}{\theta_0^2} (e^{\theta_0 t_1} - \theta_0 t_1 - 1) \right)
\]
\[
= \frac{\theta D_0}{\theta_0^2} (e^{\theta_0 t_1} - \theta_0 t_1 - 1) \right] \left[ C \left( (B_0 D_0 + B_1 D_1) + \theta D_0 e^{\theta_0 t_1} \right) + \theta D_0 e^{\theta_0 t_1} \right] + I_h \left( (B_0 D_0 + B_1 D_1) t_1 + \frac{\theta D_0}{\theta_0^2} (e^{\theta_0 t_1} - \theta_0 t_1 - 1) \right)
\]

Let \(\frac{dT}{dt_1} = 0\) to find the optimum total cost. Then \(t_1^* = \sqrt{\frac{A}{I_h \left( (B_0 D_0 + B_1 D_1) t_1 + \frac{\theta D_0}{\theta_0} (e^{\theta_0 t_1} - 1) \right)}}\).

To test the \(t_1^*\) by using the second derivative, we found that \(\frac{d^2 TC}{dt_1^2} \bigg|_{t_1^*} = \frac{2A}{t_1^*^2} > 0\).

The total cost has minimum value at \(t_1 = t_1^*\).

4. Sensitivity Analysis

Because the proposed model assumed the non-stationary demand and initial demand is non-stationary, the optimal order quantity and optimal first run-time, the optimal total cost is integrated at first order rank.

Example 1: We assumed the values of \(B_0\) and \(B_1\) as positive real are arbitrary, whether associated costs are too. We choose \(A = 50\), \(C = 150\), \(I_h = 10\), \(C_s = 5\), \(B_0 = 0.2\), \(B_1 = 0.6\). SPSS 26
and EViews 10 software version are used in our analytics. The estimate $VAR(2,2)$ model of deterioration is $\theta = 0.557910880025\theta(-1) + 1.0865152812\theta(-2) + 0.26987524\theta_0(-1) - 0.888957891765\theta_0(-2) + 0.068896924528$.

Table 1. The sensitivity analysis.

| $\theta$ | $\theta_0$ | $D_0$ | $D_1$ | $t_1^*$ | $EOQ_1^*$ | $EOQ_0^*$ | $Q_s^*$ | $TC^*$ |
|----------|------------|------|------|--------|----------|----------|--------|-------|
| 0.00007  | 0.00002    | 250  | 500  | 0.169022 | 59.1608  | 42.5561  | 16.90519 | 53094.23 |
| 0.0007   | 0.001      | 300  | 520  | 0.163864 | 60.9918  | 49.15923 | 11.83258 | 56441.42 |
| 0.07     | 0.01       | 350  | 580  | 0.145195 | 64.25121 | 50.8503  | 13.39618 | 67025.49 |
| 0.1      | 0.015      | 400  | 610  | 0.135582 | 65.89815 | 54.2878  | 11.61035 | 73577.24 |
| 0.2      | 0.15       | 450  | 660  | 0.119629 | 55.72135 | 43.79948 | 11.92187 | 87251.55 |
| 0.3      | 0.25       | 500  | 690  | 0.071814 | 47.78199 | 36.23143 | 11.55056 | 100687.4 |
| 0.4      | 0.35       | 550  | 715  | 0.05514  | 41.9692  | 30.62162 | 11.34757 | 115233.4 |
| 0.5      | 0.45       | 600  | 725  | 0.043832 | 37.60676 | 26.56018 | 11.04658 | 128752.5 |
| 0.6      | 0.55       | 650  | 750  | 0.035817 | 34.8812  | 23.51202 | 11.36918 | 147601.1 |
| 0.7      | 0.65       | 700  | 790  | 0.029949 | 33.20744 | 21.16967 | 12.03777 | 168112.7 |
| 0.8      | 0.75       | 750  | 840  | 0.025521 | 31.25692 | 19.32473 | 12.82546 | 191052.8 |
| 0.9      | 0.85       | 800  | 880  | 0.022093 | 30.36239 | 17.84121 | 13.41571 | 214617.4 |
| 0.99     | 0.95       | 850  | 900  | 0.019471 | 32.15019 | 16.70473 | 13.65766 | 236611.1 |

Table 2. The output of VAR model

| R-squared | Adj. R-squared | Sum sq. Resids | S.E. equation | F-statistic | Log likelihood | Akaike AIC | Schwarz SC |
|-----------|----------------|----------------|---------------|-------------|----------------|------------|------------|
| 0.998513  | 0.997522       | 0.001595       | 0.016305      | 1007.323    | 33.00468       | -5.091759 | -4.910898 |

Table 3. The stationary test of EOQ1 at lag = 1.

| Variable    | Coefficient | Std. Error | t-Statistic | Prob.* |
|-------------|-------------|------------|-------------|--------|
| Augmented Dickey-Fuller test statistic  | -12.54827   | 0.0001     |             |        |
| Test critical values:                  |             |            |             |        |
| 1% level                                | -5.521860   |            |             |        |
| 5% level                                | -4.107833   |            |             |        |
| 10% level                               | -3.515047   |            |             |        |
| Variable    | Coefficient | Std. Error | t-Statistic | Prob.* |
| D(Q1*(-1)) | -1.295156   | 0.103214   | -12.54827   | 0.0002 |
| D(Q1*(-1),2) | 0.215836   | 0.079825   | 2.703876    | 0.0539 |
| D(Q1*(-2),2) | 0.128770   | 0.080352   | 1.602569    | 0.1843 |
| C           | -13.85027   | 1.107274   | -12.50844   | 0.0002 |
| @TREND("1") | 1.120089   | 0.121347   | 9.230464    | 0.0008 |
Table 4. The stationary test of TC* at lag = 1.

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|----------|-------------|------------|-------------|-------|
| D(TC_(-1)) | -2.571953 | 0.605473 | -4.247838 | 0.0132 |
| D(TC_(-1),2) | 1.428007 | 0.480548 | 2.971622 | 0.0411 |
| D(TC_(-2),2) | 0.745408 | 0.248256 | 3.002583 | 0.0398 |
| C | 6757.713 | 1347.356 | 5.015538 | 0.0074 |
| @TREND("1") | 4181.679 | 1079.222 | 3.874715 | 0.0179 |

Table 5. The stationary test of t_1* at lag = 1

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|----------|-------------|------------|-------------|-------|
| D(t_1*(-1)) | -1.738569 | 0.208379 | -8.343314 | 0.0007 |
| D(t_1*(-1),2) | 0.543628 | 0.165157 | 3.291575 | 0.0011 |
| D(t_1*(-2),2) | 0.291296 | 0.117724 | 2.474387 | 0.0686 |
| C | -0.067110 | 0.008077 | 8.308422 | 0.0011 |
| @TREND("1") | 0.005395 | 0.000697 | 7.744471 | 0.0015 |

Figure 1. The deterioration in the stock versus optimal first-run time.

This paper adopted the vector autoregressive inventory model and applied the permissible range for considering the deterioration with the initial determination as $VAR(2,2)$ model. The optimal first-run time, the optimal total cost across the period. The sensitivity analysis considered variable deterioration. The optimal first run-time decreased when the initial and deterioration, increased to satisfy the real situation as figure 1 – 2.
The optimal initial EOQ and EOQ decreased when the initial and deterioration, increased to satisfy the realistic situation, especially when initial deterioration equal to 0.015 or more that, deterioration equal to 0.1 or more than that as figure 3. The optimal total cost increased when the first run-time increased and that expressed a positive relationship based on reality. The optimal of total...
cost increased when optimal of the first run-time was decreased as figure 4. Because the deterioration risk is increasing. The output of the proposed model analyzed by Eviews 10 to check the stationary of EOQ as a table 3, the optimal total cost was stationary at lag=1 with significance trend and intercept as the table 4. Optimal first run-time was stationary at lag=1 with a significant trend and intercept as a table 5.

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