TransitFit: an exoplanet transit fitting package for multi-telescope datasets and its application to WASP-127 b, WASP-91 b, and WASP-126 b

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ABSTRACT
We present TransitFit‡, an open-source Python 3 package designed to fit exoplanetary transit light-curves for transmission spectroscopy studies. TransitFit employs nested sampling to offer efficient and robust multi-epoch, multi-wavelength fitting of transit data obtained from one or more telescopes. TransitFit allows per-telescope detrending to be performed simultaneously with parameter fitting, including the use of user-supplied detrending algorithms. Host limb darkening can be fitted either independently ("uncoupled") for each filter or combined ("coupled") using prior conditioning from the PHOENIX stellar atmosphere models. For this TransitFit uses the Limb Darkening Toolkit (LDTk) together with filter profiles, including user-supplied filter profiles. We demonstrate the application of TransitFit in three different contexts. First, we model SPEARNET broadband optical data of the low-density hot-Neptune WASP-127 b. The data were obtained from a globally-distributed network of 0.5m–2.4m telescopes. We find clear improvement in our broadband results using the coupled mode over uncoupled mode, when compared against the higher spectral resolution GTC/OSIRIS transmission spectrum obtained by Chen et al. (2018). Using TransitFit, we fit 26 transit observations by TESS to recover improved ephemerides of the hot-Jupiter WASP-91 b and a transit depth determined to a precision of 170 ppm. Finally, we use TransitFit to conduct an investigation into the contested presence of TTV signatures in WASP-126 b using 126 transits observed by TESS, concluding that there is no statistically significant evidence for such signatures from observations spanning 31 TESS sectors.

Key words: planets and satellites: atmospheres – software: data analysis – software: public release – methods: analytical – methods: data analysis

1 INTRODUCTION
Over the last few decades, the study of exoplanetary atmospheres though transmission spectroscopy studies has been a growing and maturing field, seeing a significant increase in the number of surveys targeting transiting exoplanets. Dedicated space-based transit surveys, such as Kepler Space Telescope (Borucki et al. 2010), and the more recent Transiting Exoplanet Survey Satellite (TESS, Ricker et al. 2014) together with ground-based surveys such as the Next Generation Transit Survey (NGTS, Wheatley et al. 2013), have provided an ever-growing list of targets for study. These surveys have contributed significantly to the growing number of confirmed exoplanets: at the time of writing, there are over 4300 confirmed exoplanets, with over

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‡ Available at https://github.com/joshjchayes/TransitFit and https://github.com/SPEARNET/TransitFit, with documentation available at https://transitfit.readthedocs.io/en/latest/.
3000 of these exhibiting observable transits\(^1\). Early surveys, such as the Wide Angle Search for Planets (WASP, Pollacco et al. 2006), were ground-based and therefore most of the exoplanets discovered in the early days of the field were bright enough to allow for ground-based follow-up studies.

Many of these new targets can be observed with both ground- and space-based telescopes, and as such, the field of transmission spectroscopy is leaving the “target-starved” era and entering an “asset-starved” era. The limiting factor to exoplanetary studies is now no longer the availability of viable targets, but instead the availability of ground-based facilities to conduct follow up. To adapt to this change, new tools and techniques need to be developed to best utilise available resources.

Currently a variety of atomic and molecular species have been identified through analysis of transmission spectra, including potassium (i.e. Sing et al. 2011; Wilson et al. 2015), sodium (Redfield et al. 2008), water (Tinetti et al. 2007; Konopacky et al. 2013; Birkby et al. 2013), titanium oxide (Sedaghati et al. 2017; Nugroho et al. 2017), carbon monoxide (Snellen et al. 2010; Brogi et al. 2012), HCN (Hawker et al. 2018), methane (Guilluy et al. 2019), helium (Allart et al. 2018), vanadium oxide (Evans et al. 2016), and iron (Hoeijmakers et al. 2018).

The identification of these species relies on two stages of retrieval. First, a spectrum must be acquired from measurements of the radius of a planet at different observation wavelengths, and then an atmospheric model can be used to obtain atmospheric parameters for the planet. Obtaining accurate planet-host radius ratios from light curves is a significant challenge as many factors affect the shape of a transit light curve, including atmosphere and orbital parameters of the exoplanet, the behaviour of the host, and instrumental and terrestrial factors.

There are currently a few publicly available codes designed for fitting light curves of transiting exoplanets. PyLightCurve (Tsiaras et al. 2016) is a complete forward model and retrieval package, which uses an MCMC routine to fit transit light curves. PyLightCurve can also simultaneously remove trends from data using a 2nd-order polynomial and offers a variety of limb-darkening laws. EXOPLANET (Foreman-Mackey et al. 2020) is a toolkit for modelling transit and radial velocity observations of exoplanets, and is built with multi-planet systems in mind. Similarly to PyLightCurve, it uses MCMC methods to fit transit curves, and has a variety of limb-darkening laws, though it does not offer detrending functionality.

Approaches to fitting limb-darkening coefficients (LDCs) vary, with some researchers fixing values before retrieval and some instead fitting them as free parameters as part of retrieval. Espinoza & Jordán (2015) discuss this and conclude that it is best to freely-fit LDCs as fixing them can lead to biases of up to 3% in measurements of \( R_p/R_\star \), which can have significant effects on retrieved spectra. Chen et al. (2018) demonstrated that by using information about the host star, namely temperature, mass (or surface gravity), and metallicity, it is possible to improve the fitting of LDCs and consequently the fitting of transit light curves in general. Whilst fitting LDCs as free parameters is common practise, it is clear that LDCs depend on observation wavelength and on the properties of the host star and therefore it is worth trying to develop tools that can exploit this additional information.

In this paper, we present TransitFIt, an open-source Python 3 package for robust multi-wavelength, multi-epoch fitting of transiting exoplanet light curves obtained from one or more telescopes. TransitFIt has been developed in response to the fast-growing numbers of available transmission spectroscopy targets, as part of the Spectroscopy and Photometry of Exoplanetary Atmospheres Research Network (SPEARNET), a survey that is employing automated transmission spectroscopy target selection for follow-up by a globally dispersed and heterogeneous telescope network (Morgan et al. 2019).

In Section 2, we discuss the implementation of TransitFIt, including the approach to limb-darkening, simultaneous detrending, and its handling of large multi-epoch, multi-wavelength datasets. In Section 3 we demonstrate the application of TransitFIt to three different situations. We illustrate the use of TransitFIt on multi-epoch, multi-wavelength, multi-epoch observations of WASP-127b obtained by SPEARNET. We then look at two examples of the use of multi-epoch TESS data, producing improved ephemerides for WASP-91 b and conducting a sensitive investigation into the presence of contested TTV signatures from WASP-126 b. We offer our conclusions in Section 4.

2 IMPLEMENTATION OF TransitFIt

TransitFIt is an open-source, pure Python 3.x package designed specifically with transmission spectroscopy studies in mind and uses transit observations at different wavelengths and epochs from different telescopes simultaneously to fit transit parameters using nested sampling retrieval. This approach allows lightcurves to be fitted by exploiting coupled information across wavelength and epoch. TransitFIt offers a wavelength-coupled approach to LDC fitting, which we discuss in depth in Section 2.1. We also discuss how TransitFIt can detrend and normalise light curves simultaneously with fitting other parameters in Section 2.2. Different light curves being fitted simultaneously may benefit from using different detrending functions, especially if obtained from different telescopes. TransitFIt can also deal with multi-epoch observations from systems which exhibit transit timing variations (TTVs), although, as highlighted in Section 2.3, there are some current limitations to fitting these systems.

The forward model used to calculate transit light curves in TransitFIt is the batman Python package (Kreidberg 2015). We have chosen batman as it is well-established within the community, and the open-source nature of it means that we can easily incorporate it into an open-source retrieval code. Since the choice of forward model is what limits the parameters which can be fitted, using batman means that the physical parameters retrievable by TransitFIt are: orbital period, \( P \); the time of inferior conjunction, \( t_0 \); the planet radius \( R_p \), given as a fraction of the host radius, \( R_\star \); the semi-major axis of the planet orbit, \( a \); the orbital inclination, \( i \); the orbital eccentricity, \( e \); the longitude of periastron, \( \omega \); and a set of limb-darkening coefficients. Along with these, TransitFIt can calculate detrending parameters and normalisation constants for individual light curves.

Retrieval is conducted using nested sampling routines (Skilling 2004; Skilling 2006) implemented with the dynesty Python package (Speagle 2020), with uncertainties on the final best-fit results calculated from the square-root of the leading diagonal of the covariance matrix.

2.1 Limb-darkening

At the time of writing, no publicly available transit fitting code provides the functionality to couple the fitting of limb-darkening parameters across wavelengths. TransitFIt achieves this by using the Limb Darkening Toolkit (LDTK) (Parviainen & Aigrain 2015; Husser

\[^1\] The Extrasolar Planet Encyclopedia: http://exoplanet.eu/
et al. 2013) to calculate the likelihood of sets of limb-darkening coefficients given planet host characteristics and the wavelengths of observations. As well as this “coupled” approach, TransitFit can compute limb-darkening coefficients independently for each filter, corresponding to the more traditional “uncoupled” approach that allows for easy comparison with other models. As a third option, TransitFit can use a “fixed” mode where LDC are fit in only one waveband and LDTK is used to compute LDC values for other observed wavebands. This latter mode is offered for use in situations where coupling is desired, but a large number of filters leads to an unreasonable number of parameters to be fitted. However, such a fixed LDC approach can lead to biased results (Espinoza & Jordán 2015). We therefore advocate using TransitFit in coupled mode whenever feasible.

2.1.1 Limb-darkening models

Stellar intensity varies between the centre and the edge of the disk, which often results in the base of a transit curve being rounder than if the stellar disk were a uniform brightness. Typically, these variations in intensity are described by analytical functions $I_1(\mu)$, where $\mu$ is the cosine of the angle between the line of sight and the emergent intensity. $\mu$ can also be expressed as $\mu = \sqrt{T - r^2}$ where $r$ is the unit-normalised radial coordinate on the stellar disk, and as such, all limb-darkening models must be valid for $0 \leq \mu < 1$.

Multiple limb-darkening models have been implemented in TransitFit. These are the linear law (Schwarzschild & Villiger 1906)

$$I(\mu) = I(1) \cdot (1 - u_{0,l} (1 - \mu)),$$

(1)

the quadratic law (Kopal 1950),

$$I(\mu) = I(1) \cdot (1 - u_{0,q} (1 - \mu)) - u_{0,q} (1 - \mu)^2,$$

(2)

the square-root law (Diaz-Cordoves & Gimenez 1992),

$$I(\mu) = I(1) \cdot (1 - u_{0,\sqrt{r}} (1 - \mu)) - u_{1,\sqrt{r}} (1 - \sqrt{\mu}),$$

(3)

the power-2 law (Morello et al. 2017)

$$I(\mu) = I(1) \cdot (1 - u_{0,p2} (1 - \mu^{u_{1,p2}})), $$

(4)

and the non-linear law (Claret 2000a)

$$I(\mu) = I(1) \cdot (1 - u_{0,nl} (1 - \mu^{1/2}) - u_{1,nl} (1 - \mu) - u_{2,nl} (1 - \mu^{3/2}) - u_{3,nl} (1 - \mu^2)).$$

(5)

Each of $u_0, u_1, u_2,$ and $u_3$ are the limb-darkening coefficients (LDCs), which must be fitted simultaneously with other transit parameters during retrieval. The LDCs are, however, dependent upon wavelength, and consequently must be fitted for each filter used in an observation or set of observations.

2.1.2 Constraining limb-darkening coefficients

The most basic approach for fitting limb-darkening coefficients is to sample them independently and find the best fit values, but this can allow unphysical values to be trialled with no penalty. This is an issue which was addressed by Kipping (2013), who imposed two conditions on limb-darkening profiles to ensure that they are physically allowed:

(i) The intensity profile must be always positive, or

$$\frac{I(\mu)}{I(1)} > 0 \quad 0 \leq \mu < 1$$

(6)

(ii) The intensity profile must be monotonically decreasing from the centre to the edge of the stellar disk, meaning that

$$\frac{\partial I(\mu)}{\partial \mu} > 0 \quad 0 \leq \mu < 1.$$  

(7)

Kipping (2013) showed that by applying these conditions, it is possible to place constraints on allowed values of the LDCs for the two-parameter quadratic and square root laws. To improve the efficiency of sampling within this restricted space, rather than sampling the LDCs $\{u_0, u_1\}$, Kipping (2013) instead reparameterises the laws in terms of the coefficients $\{p(u_0, u_1), q(u_0, u_1)\} \in [0, 1]$. This reparameterisation ensures a sampling efficiency of 100 per cent (i.e. all samples are within the physically allowed region), without the need for checking that sampled values of $u_0$ and $u_1$ follow the imposed constraints.

We have implemented this parameterisation in TransitFit, and have extended the method to the power-2 law in Equation 4. In the limit $\mu \to 0$, Condition (i) yields the constraint

$$u_{0,p2} < 1.$$  

(8)

Condition (ii) implies that

$$\frac{\partial I(\mu)}{\partial \mu} = u_{0,p2}\mu^{u_{1,p2}-1} > 0 \quad 0 \leq \mu < 1.$$  

(9)

This does not give us anything overtly useful in the limit $\mu \to 0$ due to the cross terms, however, as $\mu \to 1$, we see that

$$u_{0,p2}u_{1,p2} > 0.$$  

(10)

This places the constraint that the power-2 LDCs must have the same sign, and can be viewed as a ‘quadrant limiting’ constraint. Mathematically, there is no lower bound on $u_{0,p2}$, and there are no bounds at all on $u_{1,p2}$ but, from a computational perspective, we must place limits on them in order to be able to sample values. Therefore, we can say that $u_{0,p2} \in [u_{0,p2}^{min}, u_{0,p2}^{max}]$ and $u_{1,p2} \in [u_{1,p2}^{min}, u_{1,p2}^{max}]$. We implement this by fitting $\{p_{p2}, q_{p2}\} \in [0, 1]$ and using the conversions

$$u_{0,p2} = p_{p2} \left(1 - u_{0,\min}^{p_{p2}}\right) + u_{0,\min}^{p_{p2}}$$

(11)

and

$$u_{1,p2} = \begin{cases} \frac{u_{1,p2}^{\min} \left(1 - q_{p2}\right)}{q_{p2}}, & \text{for } u_{0,p2} < 0 \\ u_{1,p2}^{\max} q_{p2}, & \text{for } u_{0,p2} \geq 0 \end{cases}.$$  

(12)

Provided that $u_{1,p2}^{\min} = -u_{1,p2}^{\max}$, it can be shown that this method uniformly samples in all the allowed regions in the $\{u_{0,p2}, u_{1,p2}\}$ plane, with 100 per cent efficiency.

It is trivial to also apply this method to the linear law, which places the constraint

$$0 < u_{0,1} < 1,$$

(13)

and this has been implemented in TransitFit. However, this method has yet to be successfully applied to the non-linear law. The current best attempt is by Kipping (2016), where the methodology is extended to the three-term law of Sing et al. (2009), which drops the $\mu^{1/2}$ term from the non-linear law in Equation 5. Consequently, TransitFit does not use the Kipping parameterisation to limit sampling of non-linear LDCs to a physically-allowed region of parameter space.
2.1.3 Coupling limb-darkening coefficients across wavelengths

Multiple codes use the Kipping parameterisation to constrain the LDC values to those which are physically allowed, but it is possible to improve the quality of LDC fitting further. All of the currently-available transit-fitting codes fit LDCs for each filter independently. This means that for each filter a transit is observed at, the best-fit LDCs for each filter may not be physically consistent with each other. Parviainen & Aigrain (2015) developed the Limb Darkening Toolkit (LDTk) to allow researchers to address this problem, but we have been unable to find a publicly available transit fitting code that makes use of LDTk.

LDTk uses the library of PHOENIX stellar atmospheres and synthetic spectra (Husser et al. 2013) to estimate the likelihood of a set of stellar LDCs for a given set of observation filters. Using this, we have given TransitFit the functionality to couple LDCs across multiple filters, which can then be fitted simultaneously. This allows for the refinement of the limb-darkening physics included in transmission spectroscopy studies by ensuring that the retrieved LDC values are statistically tensioned across filters in a manner consistent with stellar atmosphere models. The filter profiles used by TransitFit can be either uniform, box filter profiles, which may be suitable to represent individual spectroscopic channels, or user-supplied filter profiles for broadband photometric studies. In the case where a specific filter profile cannot be obtained, we recommend using the equivalent width of a filter. TransitFit is distributed with filter profiles for the Johnson-Cousins UBVRI set, and the SLOAN-SDSS u’g’r’i’z’ set, as well as profiles for Kepler and TESS. All of these were obtained from the SVO Filter Profile Service (Rodrigo et al. 2012; Rodrigo & Solano 2020).

Since there may be edge cases where this coupling is not desired, for instance where host information is unavailable, or computational limitations in the case of fitting a very large numbers of observation filters, such as spectroscopic observations, TransitFit offers three modes of LDC fitting:

Independent: This is the traditional approach of fitting LDCs for each filter separately. TransitFit still uses the Kipping parameterisations laid out in Section 2.1.2, but LDTk is not used to couple LDCs across filters.

Coupled: Using the Kipping parameterisations, each LDC is fitted as a free parameter, with LDTk being used to estimate the likelihood of sets of LDCs, using information on the host star and the observation filters.

Single filter: When fitting with multiple wavebands, the number of parameters required to be fitted can increase dramatically when using the coupled mode. Consequently, we have provided a method of only freely fitting the LDCs of one filter, and using LDTk to extrapolate LDC values for the remaining filters. For the i-th coefficient of a filter f, \( e_{i,f} \), this extrapolation is calculated by

\[
e_{i,f} = u_i \times \left( \frac{e_{i,f}}{u_i} \right)
\]  

where \( u_i \) is the sampled value of the i-th LDC in the actively fitted filter, and \((e_{i,f})\) and \((u_i)\) are the maximum likelihood values initially suggested by LDTk.

2 http://swo2.cab.inta-csic.es/theory/fps/

2.2 Detrending and normalisation of light curves

Transit light curves are sensitive to a variety of factors which stop the out-of-transit baseline being flat. These can range from host variations through to internal reflections within the telescope. These trends must be removed in order to obtain accurate parameters from observations. Additionally, many transit models, including BATMAN, normalise the out-of-transit baseline to a flux of 1. In some cases, detrending and normalisation is conducted before any further analysis, but ideally, detrending and normalisation coefficients should be fitted simultaneously with other model parameters to ensure that light curve features of interest are not inadvertently removed.

TransitFit offers functionality to simultaneously detrend and normalise light curves during retrieval. Built into the package are flux-conserving, nth order detrending functions, and the user can supply their own custom functions if more complicated detrending is required.

The nth order functions are calculated by writing the detrended flux values of a time series t, \( D(t) \) as

\[
D(t) = F(t) - d(t)
\]

where \( F(t) \) are the raw flux values and \( d(t) \) is some detrending function. We place the constraint of flux conservation upon the detrended light curves such that

\[
\sum_{i=0}^{N} D(t_i) = \sum_{i=0}^{N} F(t_i),
\]

which gives us the constraint that

\[
\sum_{i=0}^{N} d(t_i) = 0.
\]

By applying this conservation of flux, we place constraints on the 0-th component (intercept) of the detrending function. In the case of a linear detrending function, where

\[
d(t) = at + b,
\]

applying Equation 17 yields

\[
\sum_{i=0}^{N} (at_i + b) = 0,
\]

from which we can cast the 0-th order term \( b \) in terms of the other parameters to give

\[
b = -\frac{a}{N} \sum_{i=0}^{N} t_i = -\bar{t}
\]

which can be substituted into Equation 18, resulting in

\[
d(t) = a \left( t - \bar{t} \right).
\]

This can be generalised to nth order (for \( n > 0 \)) detrending functions given by

\[
d(t_i) = \sum_{j=0}^{n} a_j t_i^j
\]

as

\[
d(t_i) = \sum_{j=1}^{n} \left[ a_j \left(t_i^j - \bar{t}^j\right) \right].
\]

where \( a_j \) are the detrending coefficients and the exponent of the time series is bit-wise.
This method allows us to also fit a normalisation constant without falling foul of degeneracy between the scaling due to the normalisation constant and the shift that a freely-fitted 0-th order detrending term introduces. 0-th order detrending cannot be applied due to this degeneracy, and we assume that these light curves are detrended, but not necessarily normalised. In the case of a user-defined detrending function, we strongly recommend following the above flux conservation procedure in order to avoid the risk of degenerate solutions.

2.3 Dealing with systems exhibiting TTVs

The basic implementation of TransitFit assumes that there are no transit timing variations (TTVs) within multi-epoch observations and fits one value of \( t_0 \), assumed to be consistent across all epochs. In the event that a system does exhibit TTVs, this method will fail to produce an accurate result. Consequently, in these cases, TransitFit takes a slightly different approach.

(i) First, we consider each filter separately. We run retrieval on all the curves in this filter, using all the data to fit \( R_p \) and limb-darkening coefficients. However, we fit a separate \( t_0 \) for each observation epoch within the filter, and cannot directly fit a period, \( P \), in this mode, which must instead be provided.

(ii) Using the results from these single-filter retrievals, we detrend and normalise each light curve and then use the retrieved \( t_0 \) values to produce a phase-folded light curve for each filter. The observation times \( t \) for each light curve are folded to give \( t' \), where \( t_0 - \frac{P}{2} < t' \leq t_0 + \frac{P}{2} \), using

\[
t' = t - P \times \left[ \frac{t - (t_0 + P/2)}{P} \right] - C
\]

(24)

where

\[
C = t_0 - P \times \left[ \frac{t_0 - (t_0 + P/2)}{P} \right] - t_{0,\text{base}}
\]

(25)

accounts for the offset caused by the different \( t_0 \) values for each epoch. By choosing a value for \( t_{0,\text{base}} \) this term ensures that all the light curves are centred on \( t_{0,\text{base}} \).

(iii) Fit the folded light curves using the standard TransitFit approach, coupling LDCs where required.

As stated above, when allowing for the presence of TTVs, TransitFit cannot fit for \( P \), which must be provided. For consistency, we recommend first running TransitFit on data assuming that no TTVs are present, in order to obtain an appropriate value for \( P \). TransitFit cannot automatically detect TTVs in data, and must be instructed explicitly to allow for them. In the case where TTVs are present but TransitFit is not allowing for them, the retrieved results will be incorrect. Additionally, TransitFit does not solve the system dynamics associated with any present TTVs, as the purpose of TransitFit is to produce robust transit fitting for the purposes of transmission spectroscopy studies.

2.4 Batched retrieval: fitting a large number of parameters

As with any retrieval algorithm, increasing the dimensionality of the parameter space leads to instability in the nested sampling routines and can lead to inaccurate results. Since TransitFit is anticipated to be used in transmission spectroscopy studies, where many tens, or even hundreds of light curves may need to be fitted, we have provided a solution to this in the form of “batched” retrieval.

In this mode, the user can specify the maximum number of parameters for TransitFit to fit at one time. The light curves are then grouped by observation filter and split into multi-filter batches, where the number of parameters being fitted in each batch is less than the user-set limit. The batches are calculated to try and ensure that filters are present in multiple batches, which results in coupling between them. The exception to this is in the case that one filter has a high enough number of observations in it that the number of parameters required exceeds the user-set limit. In this case, this filter is fitted independently and does not benefit from any coupling. In these cases, we recommend using the “folded” mode. After retrieval has been run on all the batches, a set of summary results are produced by calculating a weighted mean of all parameters.

2.5 Folded retrieval: Producing folded light curves

With the launch of large surveys such as TESS, many exoplanets have multiple-epoch observations in a single filter. TransitFit can make use of these through a two-step retrieval process. In the first step, TransitFit runs a retrieval on each filter independently, and uses the results to produce a phase-folded light curve for each filter. In the second step, TransitFit runs a standard multi-wavelength retrieval using the batched algorithm above. This mode of retrieval allows for the production of high-quality folded light curves from non-detrended data, as well as providing a method where observations from long-term, single-band surveys such as TESS can be easily combined with single-epoch observations at multiple wavelengths, such as from ground-based spectrographic follow-up.

3 APPLICATION OF TransitFit TO OBSERVATIONAL LIGHT CURVES

TransitFit was initially designed for use in spectroscopy studies, but also be applied to temporal studies, either in updating ephemerides of planets, or in studying systems for TTVs, which can be indicative of other planets in a system.

In this section, we will demonstrate the application of TransitFit in three different scenarios, illustrating the impact of using LDTk to inform LDC fitting. First, we will discuss the fitting of multi-wavelength, ground based photometric observations of the low-density hot Neptune WASP-127 b (Lam et al. 2017), using previously unpublished data acquired from the SPEARNET network of telescopes. We then move to applying TransitFit to TESS observations of the warm Jupiter WASP-91 b (Anderson et al. 2017) and provide updated ephemerides and orbital parameters for the system. Finally, we analyse TESS observations of the hot Jupiter WASP-126 b (Maxted et al. 2016), a system which contentiously exhibits TTVs (Pearson 2019; Maciejewski 2020), and use TransitFit to show that there is no statistically significant evidence of TTVs within 126 transits observed by TESS. For all three systems, we assume circular orbits and use TransitFit to fit the global parameters of \( P, t_0, a/R_\star \), and \( i \), as well as the filter-specific \( R_p/R_\star \) and LDC values.

3.1 Application of TransitFit to multi-filter photometric observations of WASP-127 b

SPEARNET is a prototype transmission spectroscopy survey which is utilising a globally-distributed network of heterogeneous optical telescopes, the locations of which are shown in Figure 1. It was conceived to anticipate and address the challenges that the transition into the so-called “asset-starved” era poses, primarily by designing tools which allow for increased utility of resources, both before (Morgan...
et al. 2019) and after (Hayes et al. 2020) transit observations. TRANsFr was conceived as part of the SPEARNET suite of tools to handle transit data from non-homogeneous observations, to facilitate transmission spectroscopy studies in the asset-staved era, as time on larger telescopes is becoming ever-more competitive and studies will have to frequently rely on data taken from a combination of telescopes.

As part of the network operation, Morgan et al. (2019) developed a metric for ranking candidates for observation, effectively pairing targets with telescopes in a way which maximises the signal-to-noise of the observations. The motivation behind this metric is to remove the multiple unquantifiable biases in manual transmission spectroscopy target selection. Since the selection function is known, it is possible to make population-corrected statistical statements based on observations that are guided by the metric. In Table 3 of Morgan et al. (2019), we show that WASP-127 b is consistently ranked in the top three targets for a variety of telescopes when known planet masses are included in the metric calculations, and as such it has become a target of interest for SPEARNET.

With a density of $0.07 \pm 0.01 \rho_{\text{up}}$ (Lam et al. 2017), WASP-127 b is one of the lowest-density planets so far discovered, and occupies the ‘short-period Neptune desert’ (Mazeh et al. 2016), which is notable since most planets with its characteristics are not expected to survive due to photo-evaporation (Haswell et al. 2012). Its low density also makes WASP-127 b an idea target for transmission spectroscopy due to its large scale height, and several studies have been completed, with potential detections of water (Chen et al. 2018; Skaf et al. 2020), and statistically significant detections of sodium, potassium, and lithium ($5\sigma$, $3\sigma$, and $4\sigma$ respectively, Chen et al. 2018). No significant evidence for helium in the upper atmosphere has been found (dos Santos et al. 2020) and it has been proposed that this is due to unfavourable photo-ionisation conditions.

The approaches to LDC fitting in these previous studies differ. Skaf et al. (2020) fix the LDCs for all spectral channels at the white-light values predicted for WASP-127 using the quadratic law of Claret (2000b). Chen et al. (2018) also use a quadratic limb-darkening law, but instead find the highest likelihood values for each channel and fit using a Gaussian prior of width 0.1, sourcing the initial predictions from the Kurucz ATLAS9 stellar atmosphere models (Kurucz 2017).

Using the SPEARNET telescope network, we have obtained eight photometric light curves in six different wavebands from six transits of WASP-127 b, including the first published transits observed in the $u'$-band. The four telescopes used in these observations were:

**The 2.4m Thai National Telescope (TNT):**
Located at the Thai National Observatory (TNO), the TNT observations of WASP-127b were conducted using ULTRASPEC (Dhillon et al. 2014), which uses a 1024×1024 pixel high-speed frame-transfer EMCCD camera with a field-of-view of $7.68 \times 7.68$ arcmin$^2$. The dead time between exposures on this setup is 14 ms.

**A 0.5 m telescope at the Thai National Observatory (TRT-TNO):**
The TRT-GAO is also part of the Thai Robotic Telescope Network, and is located at Gao Mei Gu observatory in Lijiang, China. Observations were taken using an Andor iKon-L 936 2048 × 2048 CCD with a field of view of $20.9 \times 20.9$ arcmin$^2$.

**A 0.7 m telescope at Gao Mei Gu observatory (TRT-GAO):**

The approaches to LDC fitting in these previous studies differ. Skaf et al. (2020) fix the LDCs for all spectral channels at the white-light values predicted for WASP-127 using the quadratic law of Claret (2000b). Chen et al. (2018) also use a quadratic limb-darkening law, but instead find the highest likelihood values for each channel and fit using a Gaussian prior of width 0.1, sourcing the initial predictions from the Kurucz ATLAS9 stellar atmosphere models (Kurucz 2017).

The obtained light curves, shown in 2(a), were then run through TRANsFr, using the ‘batched’ mode, a 2nd-order detrending function, and both ‘coupled’ and ‘independent’ LDC fitting approaches so as to be able to identify the improvement from using filter and host parameters to inform the likelihood of LDC values. For the coupled LDC approach, we adopted the stellar parameters of Lam et al. (2017), namely $T = 5620 \pm 85$ K, $R_\ast = 1.39 \pm 0.03 \ R_{\odot}$, $M_\ast = 1.08 \pm 0.03 \ M_{\odot}$, $\log g = 4.18 \pm 0.01$ cgs, and [Fe/H] = $-0.18 \pm 0.06$ dex.

The resulting detrended light curves from the coupled LDC fitting are shown with the best-fit model in Figure 2(b), using the flux ratio between WASP-127 b and reference stars with their error are shown in Table 2.

Since our data, and those of Chen et al. (2018) and Skaf et al. (2020) have all been analysed separately, there is an intrinsic offset between all the data. In order to visualise the three analyses, we have rescaled the data of Chen et al. (2018) to use the stellar radius of the relevant filter profile by the spectral energy distribution (SED) of WASP-127, predicted using the PHOENIX models.

The 0.6 m PROMPT-8 telescope
Located at the Cerro Tololo Inter-American Observatory (CTIO) in Chile, PROMPT-8 is a 0.6 m telescope operated by the Skynet Robotic Telescope Network. Imaging is conducted on this telescope using a 2048 × 2048 pixel CCD camera with a scale of 0.624 arcseconds/pixel.

Figure 1 shows the location of the primary telescopes in the SPEARNET network, with the telescopes used in this study of WASP-127 b highlighted. The precise details of the observations taken are given in Table 1.

The images obtained by SPEARNET were calibrated using IRAF routines along with astrometric calibration using Astrometry.net (Lang et al. 2010). In order to obtain the light curve, aperture photometry was carried out using sextractor (Bertin & Arnouts 1996) using an adaptive scaled aperture based on the seeing in an individual image. Reference stars were chosen to have magnitudes similar to WASP-127 ($\Delta M < 3$) and no intrinsic variation. The observed time in BJD with flux ratio between WASP-127 b and reference stars with their error are shown in Table 2.

Figure 3(a) shows the location of the primary telescopes in the SPEARNET network, with the telescopes used in this study of WASP-127 b highlighted. The precise details of the observations taken are given in Table 1.

The images obtained by SPEARNET were calibrated using IRAF routines along with astrometric calibration using Astrometry.net (Lang et al. 2010). In order to obtain the light curve, aperture photometry was carried out using sextractor (Bertin & Arnouts 1996) using an adaptive scaled aperture based on the seeing in an individual image. Reference stars were chosen to have magnitudes similar to WASP-127 ($\Delta M < 3$) and no intrinsic variation. The observed time in BJD with flux ratio between WASP-127 b and reference stars with their error are shown in Table 2.

The obtained light curves, shown in 2(a), were then run through TRANsFr, using the ‘batched’ mode, a 2nd-order detrending function, and both ‘coupled’ and ‘independent’ LDC fitting approaches so as to be able to identify the improvement from using filter and host parameters to inform the likelihood of LDC values. For the coupled LDC approach, we adopted the stellar parameters of Lam et al. (2017), namely $T = 5620 \pm 85$ K, $R_\ast = 1.39 \pm 0.03 \ R_{\odot}$, $M_\ast = 1.08 \pm 0.03 \ M_{\odot}$, $\log g = 4.18 \pm 0.01$ cgs, and [Fe/H] = $-0.18 \pm 0.06$ dex.

The resulting detrended light curves from the coupled LDC fitting are shown with the best-fit model in Figure 2(b), where the success of the detrending is particularly apparent in the TRT-GAO-R data. We also show the resulting transit depths from both the coupled and independent LDC mode retrievals alongside the Hubble Space Telescope (HST) spectrum from Skaf et al. (2020), and the Gran Telescopio Canarias (GTC) and Nordic Optical Telescope (NOT) spectral data and best fit model from Chen et al. (2018) in Figure 3(a).

Since our data, and those of Chen et al. (2018) and Skaf et al. (2020) have all been analysed separately, there is an intrinsic offset between all the data. In order to visualise the three analyses, we have rescaled the data of Chen et al. (2018) to use the stellar radius. R = 1.409 R_{\odot} from Skaf et al. (2020). We also added an offset of $\Delta(R_p/R_\ast) = 0.00385$ from all of our retrieved transit depths. This was calculated to minimise the $\chi^2$ between the rescaled Chen et al. (2018) data and our retrieved spectrum in Figure 3(b). This spectrum was obtained by excluding partially-covered transits from the fitting, which is discussed more in Section 3.1.1. The wavelength positions of the SPEARNET observations in Figure 3 are derived by weighting the relevant filter profile by the spectral energy distribution (SED) of WASP-127, predicted using the PHOENIX models.

Looking at Figure 3(a), it is clear that using the coupled LDC fitting approach consistently results in a smaller value of $R_p/R_\ast$ than the independent LDC fitting method, with a discrepancy of 7 per cent in the $u'$-band. This difference is larger than the 3 per cent bias that Espinoza & Jordán (2015) find can be introduced from not fitting LDCs at all, and clearly illustrates the impact that using host characteristics and filter profiles can have when fitting spectroscopic and photometric measurements.
Figure 1. The location and size of the telescopes currently within the SPEARNET network. Telescopes with a red circle are those used to take the observations of WASP-127 b discussed in Section 3.1. The 0.5 m TRT-TNO telescope at the Thai National Observatory which we used has since been upgraded to a 1 m aperture, and is shown as such. (Image credit: NARIT)

Table 1. Details of the SPEARNET observations of WASP-127 b, obtained using the Thai National Telescope (TNT), telescopes from the Thai Robotic Telescope network at Gao Mei Gu (TRT-GAO) and the Thai National Observatory (TRT-TNO), and the PROMPT-8 telescope at the Cerro Tololo Inter-American Observatory. Since the observations were taken, TRT-TNO has been upgraded to a 1 m aperture telescope.

| Date       | Telescope | Filter | Aperture (m) | Number of photometric points | Exposure time (s) | Transit coverage |
|------------|-----------|--------|--------------|------------------------------|-------------------|------------------|
| 15-02-2017 | TNT       | 𝑖     | 2.4          | 24 720                       | 0.78              | Full             |
| 15-02-2017 | TRT-GAO   | 𝑅     | 0.7          | 1 223                        | 15, 20            | Full             |
| 15-02-2017 | TRT-TNO   | 𝑉     | 0.5          | 317                          | 60                | Full             |
| 21-03-2017 | PROMPT-8  | 𝑃     | 0.6          | 413                          | 5                 | Egress only      |
| 29-03-2017 | TRT-TNO   | 𝐼     | 0.5          | 383                          | 20                | Egress only      |
| 26-02-2018 | TNT       | 𝑟     | 2.4          | 68 480                       | 0.38              | Full             |
| 09-03-2019 | TNT       | 𝑢     | 2.4          | 741                          | 12.8              | Ingress only     |
| 30-03-2019 | TNT       | 𝑢     | 2.4          | 1 531                        | 12.8              | Full             |

Table 2. The photometric data of WASP-127 b obtained from TNT and TRTN.

| BJD  | Flux (p.p.m.) | Flux error (p.p.m.) | Filters | Telescopes |
|------|---------------|---------------------|---------|------------|
| 2 457 800.1182452 | 2.387 | 0.007 | i’ | TNT         |
| 2 457 800.1182542 | 2.424 | 0.008 | i’ | TNT         |
| 2 457 800.1182633 | 2.367 | 0.007 | i’ | TNT         |
| 2 457 800.1182724 | 2.394 | 0.008 | i’ | TNT         |
| 2 457 800.1182815 | 2.379 | 0.007 | i’ | TNT         |
| 2 457 800.1182905 | 2.358 | 0.007 | i’ | TNT         |
| 2 457 800.1182996 | 2.414 | 0.008 | i’ | TNT         |
| 2 457 800.1183087 | 2.430 | 0.008 | i’ | TNT         |
| 2 457 800.1183178 | 2.408 | 0.007 | i’ | TNT         |

Note. The full table is available in electronic format.

3.1.1 A cautionary tale - using partial transit coverage

What is also clear in Figure 3(a) is that the I-band observations are in poor agreement with the established models. This is likely due to the incompleteness of the observation, as across all observations, the I-band does not have a complete transit observed. When using the coupled LDC approach, the fact that this observation does not have a complete transit will have an effect on the parameters retrieved for all the filters. We therefore ran the same retrieval again, but excluded any data from filters which did not have at least one complete transit observed, namely the V- and I-band observations.

The final light curves from using the coupled LDC approach are shown in Figure 2(c), and the resulting spectrum is shown in Figure 3(b). In this case, the values of $R_P/R_\star$ retrieved using the coupled LDC method match the established data even more closely, and the discrepancy between the $u’$-band depths obtained by the coupled and independent LDC fitting has grown to 9 per cent.

Table 3 shows the physical results obtained by each of the four retrievals described above. The difference in the precision of LDC fitting is clearly shown in these results, as the uncertainties on the $u_0$ and $h_1$ values obtained using the coupled LDC approach are up to two orders of magnitude smaller than those obtained by the independent method.

From this investigation, we can conclude that TransitFit is able to use photometric observations made with heterogeneous telescopes to produce results which agree with spectroscopic single-telescope observations, provided that the LDCs are coupled across wavelength. When coupling LDCs however, it is important to exclude filters which lack an observation of a complete transit, as the incomplete data can bias the fitting of other filters.
Figure 2. (a): The raw data from the SPEARnet observations of WASP-127 b outlined in Table 1, normalised using the median values for each observation and phase folded using the model obtained in (b). We have binned the TNT observations to a 2-minute cadence, with the raw observations shown in grey. (b): The detrended light curves and associated best-fit models obtained using TransFiT in “batched” mode with a coupled quadratic LDC fitting and simultaneous normalisation and 2nd-order detrending. (c): as (b), but excluding the $V$- and $I$-band observations because they do not contain a complete transit. Each curve has an arbitrary offset from a normalised baseline of 1 and has been phase-folded to centre $t_0$ at a phase of 0.5. For clarity, error bars on data points have been excluded and an average error bar has been provided as a black point to the right of each curve. The best fit transit model from the retrieval is over-plotted.

3.2 Temporal study of WASP-91 b to produce updated ephemerides

WASP-91 b is a 1.34 $M_{\text{Jup}}$ warm Jupiter with a 2.8 day orbit around a K3 host star (Anderson et al. 2017). With a radius of 1.03 $R_{\text{Jup}}$, WASP-91 b is a smaller example of a hot Jupiter, and has not been the subject of further studies since its discovery.

TESS observed WASP-91 b in Sectors 1, 27, and 28, capturing a total of 49,989 photometric data points covering 26 transits, the data from which we acquired from the Barbara A. Mikulski Archive for Space Telescopes (MAST) portal\(^3\). Since these data contain vast

\(^3\) https://mast.stsci.edu
numbers of observations outside of transit, and there are multiple offsets and other trends within the data which cannot be modelled with a simple polynomial, we estimate the transit duration $t_{14}$, assuming a circular orbit with a 90 degree inclination, using

$$t_{14} = \frac{R_p + R_*}{a \pi} P$$

(26)

and discard any data which are more than 2.5$t_{14}$ away from the mid-transit times predicted by the ephemerides given in Anderson et al. (2017). This results in individual transits, capturing a combined total of 4,854 photometric points, which can then be individually normalised and detrended with lower-order polynomials. This splitting of data is provided in TransitFit through the split_lightcurve_file function.

After splitting the full data into individual transit observations, we use the “folded” mode of TransitFit to fit these data, using both the “coupled” and “independent” LDC fitting modes, assuming a circular orbit and using the quadratic limb-darkening model from Equation 2 and a second-order detrending polynomial. For the “coupled” mode, we inform LDC fitting using the stellar parameters found by Anderson et al. (2017), given in Table 4, and the TESS filter profile given on by the SVO Filter Profile Service4 (Rodrigo et al. 2012; Rodrigo & Solano 2020).

\footnote{http://svo2.cab.inta-csic.es/theory/fps/}

Figure 4 shows the fitting process for the “coupled” run of TransitFit at various stages. Figure 4(a) and (b) show the raw TESS observations which clearly exhibit various offsets and long-term trends. In order to reduce the impact of these, we split the data into individual transits using the approach described above, and Figure 4(b) shows the resulting raw data for the first transit. In the “folded” fitting mode, each transit is normalised and detrended, and Figure 4(c) shows the first transit after this first stage of processing. We have overlaid the final best fit model to help illustrate this step, but it should be noted that this model is calculated from all the transits, not just this single epoch. Once normalisation and detrending has been fitted for each transit, all the light curves are folded together and this curves is analysed to retrieve the final best-fit model. Figure 4(d) shows this folded light curve, along with the final best-fit transit model and residuals, which have an rms of 0.00168. We also show the same folded data binned to a cadence of two minutes, to demonstrate the improvement in observation precision when compared to the single-transit TESS observations like the one shown in Figure 4(c). An average of 25.0 photometric points are contained within each of the bins, which through naive root-N statistics suggests a maximum improvement in precision of factor 5. The rms of the residuals of this binned light curve is 0.00032, which is an improvement of factor 5.25. The similarity of these two factors suggest that the improvement due to folding is near maximal, and thus the binned data are not systematics-limited.

We present the results from both runs of TransitFit alongside the results from Anderson et al. (2017) in Table 4, which are all

\begin{table}
\centering
\caption{Output parameters from using TransitFit to run retrieval on photometric observations of WASP-127b, in both coupled and independent LDC fitting mode. We ran this using all observations, and then excluding the $V$- and $J$-band observations because they do not contain a complete transit. Note that the $R_p/R_*$ values plotted in Figure 3 have an offset of $\Delta(R_p/R_*) = 0.00385$ added to them. For comparison, we provide the orbital parameters obtained by Chen et al. (2018) and Skaf et al. (2020). We have converted the Chen et al. (2018) value of $a/R_*$ into AU using the Lam et al. (2017) value of $R_*=1.39 \pm 0.03 R_\odot$.}
\begin{tabular}{lcccc}
\hline
\hline
& \multicolumn{2}{c}{All SPEARNET observations} & \multicolumn{2}{c}{Excluding $V$- and $J$-bands} \\
\cline{2-5}
 & Coupled LDC fitting & Independent LDC fitting & Coupled LDC fitting & Independent LDC fitting \\
\hline
$P$ [days] & 4.1780776 $\pm$ 1.1 $\times 10^{-6}$ & 4.1780698 $\pm$ 1.0 $\times 10^{-6}$ & 4.1780660 $\pm$ 1.3 $\times 10^{-6}$ & 4.1780647 $\pm$ 1.1 $\times 10^{-6}$ \\
$t_0$ [BJD] & 2457248.73832 $\pm$ 0.00022 & 2457248.74026 $\pm$ 0.00021 & 2457248.74232 $\pm$ 0.00026 & 2457248.74141 $\pm$ 0.00022 \\
$i$ [deg] & 89.92 $\pm$ 0.14 & 86.67 $\pm$ 0.07 & 89.94 $\pm$ 0.12 & 86.89 $\pm$ 0.10 \\
$a/R_*$ & 8.038 $\pm$ 0.007 & 7.528 $\pm$ 0.025 & 7.884 $\pm$ 0.007 & 7.613 $\pm$ 0.037 \\
$a$ [AU] & 0.0520 $\pm$ 0.0012 & 0.0487 $\pm$ 0.0011 & 0.0510 $\pm$ 0.0011 & 0.0492 $\pm$ 0.0011 \\
$R_p/R_*$ [V-band] & 0.1101 $\pm$ 0.0015 & 0.1177 $\pm$ 0.0020 & 0.1074 $\pm$ 0.0019 & 0.1176 $\pm$ 0.019 \\
$R_p/R_*$ [J-band] & 0.0906 $\pm$ 0.0028 & 0.0909 $\pm$ 0.0034 & - & - \\
$R_p/R_*$ [K-band] & 0.0988 $\pm$ 0.0001 & 0.1046 $\pm$ 0.0002 & 0.0976 $\pm$ 0.0002 & 0.1041 $\pm$ 0.0003 \\
$R_p/R_*$ [H-band] & 0.0978 $\pm$ 0.0008 & 0.1044 $\pm$ 0.0009 & 0.0955 $\pm$ 0.0010 & 0.1025 $\pm$ 0.0012 \\
$R_p/R_*$ [I-band] & 0.1046 $\pm$ 0.0002 & 0.1094 $\pm$ 0.0003 & 0.1017 $\pm$ 0.0002 & 0.1010 $\pm$ 0.0003 \\
$R_p/R_*$ [F-band] & 0.1186 $\pm$ 0.0030 & 0.1213 $\pm$ 0.0035 & - & - \\
$u_0$ [V-band] & 0.0793 $\pm$ 0.0015 & 0.0781 $\pm$ 0.0016 & 0.0683 $\pm$ 0.0013 & 0.1080 $\pm$ 0.0114 \\
$u_1$ [V-band] & $-0.094$ $\pm$ 0.015 & $-0.434$ $\pm$ 0.113 & $-0.105$ $\pm$ 0.015 & $-0.519$ $\pm$ 0.161 \\
$u_0$ [I-band] & 0.0055 $\pm$ 0.0019 & 0.0508 $\pm$ 0.0019 & - & - \\
$u_1$ [I-band] & 0.0055 $\pm$ 0.0019 & 0.0508 $\pm$ 0.0019 & - & - \\
$u_0$ [K-band] & 0.556 $\pm$ 0.0007 & 0.562 $\pm$ 0.0048 & 0.593 $\pm$ 0.0010 & 0.477 $\pm$ 0.063 \\
$u_1$ [K-band] & 0.241 $\pm$ 0.0007 & 0.236 $\pm$ 0.0063 & 0.291 $\pm$ 0.012 & 0.110 $\pm$ 0.062 \\
$u_0$ [R-band] & 0.751 $\pm$ 0.0038 & 0.583 $\pm$ 0.0112 & 0.927 $\pm$ 0.015 & 0.433 $\pm$ 0.161 \\
$u_1$ [R-band] & 0.019 $\pm$ 0.007 & 0.186 $\pm$ 0.121 & 0.080 $\pm$ 0.0015 & 0.002 $\pm$ 0.0072 \\
$u_0$ [J-band] & 0.470 $\pm$ 0.007 & 0.075 $\pm$ 0.034 & 0.583 $\pm$ 0.12 & 0.061 $\pm$ 0.036 \\
$u_0$ [F-band] & 0.173 $\pm$ 0.008 & 0.171 $\pm$ 0.079 & 0.284 $\pm$ 0.013 & 0.174 $\pm$ 0.086 \\
$u_1$ [F-band] & 0.515 $\pm$ 0.008 & 0.307 $\pm$ 0.201 & 0.507 $\pm$ 0.007 & 0.0507 $\pm$ 0.007 \\
$u_1$ [J-band] & 0.312 $\pm$ 0.008 & 0.312 $\pm$ 0.365 & - & - \\
\hline
\end{tabular}
\end{table}

\footnote{Skaf et al. (2020) adopt the value of $a/R_*$ directly from Chen et al. (2018)
consistent with each other. Due to the large time-span covered by the TESS observations, the analysis with TransITFitt has resulted in an order-of-magnitude improvement on the precision of the measured period of WASP-91 b. Precise ephemerides are required for accurate study of TTVs, and updating ephemerides by applying TransITFitt to planets within TESS data will prove invaluable to future surveys.

The uncertainties on the two TransITFitt retrievals are generally comparable, with the notable exception of the LDCs. Through using LDTk to calculate LDC likelihoods, we see a 1–2 orders-of-magnitude increase in the precision of LDC values, which demonstrates the impact of introducing host parameters and filter information into transit-fitting routines.
3.3 TTV study of WASP-126 b

Orbiting a type G2 star with a period of $3.2888 \pm 0.0008$ days, WASP-126 b (Maxted et al. 2016) is a $0.28 \pm 0.04 \, M_{\text{Jup}}$ hot Jupiter which has been identified as potentially exhibiting TTVs. Through Bayesian N-body simulation coupled with machine learning analysis of Sectors 1–3 of the TESS observations of WASP-126 b, Pearson (2019) showed that there was evidence of a TTV signal with amplitude of $\sim 1$ minute and a period of $\sim 25$ days, which could be attributed to a non-transiting planet with $M_p = 0.202 \pm 0.0774 \, M_{\text{Jup}}$ on a 7.63 $\pm 0.17$ day orbit, dubbed WASP-126 c. Maciejewski (2020) studied the TESS observations from sectors 1–13 and found that when the extra sectors were included, the TTV signal was not present at a statistically significant level.

Here we use the ability of TransitFit to account for TTVs to analyse the TESS observations of WASP-126 b from sectors 1–13, and 27–31 to further investigate the presence of TTVs within these data, and to produce the most up-to-date values for the planetary and orbital parameters of WASP-126 b. As with the analysis of WASP-91 b, we discard all data that is more than 2.5 times predicted by Maxted et al. (2016), which gives 126 individual transits. We then run analysis using TransitFit in both “coupled” and “independent” LDC fitting modes, using the quadratic limb-darkening law, a 2nd order detrending polynomial and assuming a circular orbit. When using LDTk to calculate LDC likelihoods, we use the host parameters given in Maxted et al. (2016), which are presented in Table 5, and the TESS filter profile from the SVO Filter Profile Service. As discussed in Section 2.3, we first run analysis of the data assuming that there are no TTVs, and we use these results to provide priors to parameterise the ephemerides for the analysis where we allow for the presence of TTVs. Since TransitFit requires a fixed period to be provided when considering TTVs, this step should always be used to ensure complete consistency of results. The priors used in this initial step are based on the ephemerides of Maxted et al. (2016). The resulting ephemerides from this initial analysis are given in Table 5, and we use these results when allowing for the presence of TTVs, fixing the period at the values given and using the retrieved $t_0$ values as the mean of a Gaussian prior with a width of 0.007 days.

The orbital parameters of WASP-126 b retrieved by TransitFit when allowing for TTVs are given in Table 5, for both “coupled” and “independent” LDC fitting modes. We present these alongside the results of Pearson (2019), Maciejewski (2020), and Maxted et al. (2016) for comparison. We find that the results from both runs are generally consistent but, as in the WASP-91 b analysis, the uncertainties on the LDCs for the “coupled” run are significantly smaller.

We present the O-C plots for both modes of the TransitFit analysis in the left plots of Figure 5, with the associated Lomb-Scargle periodograms on the right. The top row are the results for the “coupled” LDC run, and the bottom row are the results for “independent” LDCs. The solid horizontal lines on the Lomb-Scargle periodograms represent the false alarm probabilities of 10, 5, and 1 per cent from bottom to top, calculated using astropy routines (Astropy Collaboration et al. 2013, 2018). We find that there are no periodicities of statistical significance within the O-C data for either the “coupled” or “independent” LDC runs, and consequently conclude that there is no evidence of TTVs that would be indicative of a second planet in the WASP-126 system, in agreement with the findings of Maciejewski (2020).

An interesting observation from the O-C plots in Figure 5 is that there is a clear offset from zero in the case of the “independent” LDC results. This would suggest that the predicted ephemerides obtained without using LDTk to inform the LDC fitting are less accurate. For the “independent” LDCs, the O-C plot has a reduced chi-squared value of $\chi^2 = 5.15$, whilst the “coupled” LDC O-C gives $\chi^2 = 3.12$.

The approach to fitting taken by TransitFit differs to the previous studies of Pearson (2019) and Maciejewski (2020). Pearson (2019) uses a simultaneous 2nd order detrending polynomial but, unlike TransitFit, it is not explicitly constructed to be flux-conserving. They do however use LDTk in their handling of LDCs, but not to directly inform the likelihood of trial parameters. Instead, Pearson (2019) uses LDTk to find the highest likelihood LDC values for the host parameters and TESS filter and fixes the values here. We note however that the host parameters used by Pearson (2019) for this do not exactly match those of (Maxted et al. 2016), as indicated in Table 5, and it is unclear where the alternative value of host metallicity originates from.

Detrending of the TESS light curves by Maciejewski (2020) is done before the fitting of physical parameters, but the LDCs are fitted as part of the retrieval. Maciejewski (2020) freely fits the LDCs without using the Kipping (2013) parameterisation built into TransitFit, and does not use any host information to inform the likelihoods, which is reflected in the significantly larger LDC uncertainties. Maciejewski

| TransitFit: coupled LDCs | TransitFit: independent LDCs | Anderson et al. (2017) |
|--------------------------|-----------------------------|------------------------|
| $P$ [days]               | $2.79857948 \pm 4.1 \times 10^{-7}$ | $2.7985782 \pm 4.1 \times 10^{-7}$ | $2.798581 \pm 3 \times 10^{-6}$ |
| $t_0$ [BJD]              | $2456297.7191 \pm 0.0003$ | $2456297.7198 \pm 0.0004$ | $2456297.7190 \pm 0.0002$ |
| $i$ [deg]                | $87.28 \pm 0.12$ | $87.19 \pm 0.14$ | $86.8 \pm 0.4$ |
| $a/R_*$                  | $9.299 \pm 0.078$ | $9.260 \pm 0.074$ | $9.251 \pm 0.408$ |
| $a$ [AU]                 | $0.0372 \pm 0.0013$ | $0.0372 \pm 0.0007$ | $0.037 \pm 0.001$ |
| $R_p/R_*$                | $0.1201 \pm 0.0003$ | $0.1205 \pm 0.0007$ | $0.1225 \pm 0.0012$ |
| $R_p/\langle R_{\text{Jup}} \rangle$ | $1.005 \pm 0.035$ | $1.008 \pm 0.036$ | $1.03 \pm 0.04$ |
| $u_0$                    | $0.508 \pm 0.009$ | $0.528 \pm 0.127$ | - a |
| $u_1$                    | $0.108 \pm 0.009$ | $0.058 \pm 0.115$ | - a |
| $T_{\text{eff},*}$ [K]   | - | - | $4.92 \pm 0.80$ |
| $M_*$ [M$_\odot$]        | - | - | $0.84 \pm 0.07$ |
| $R_*$ [R$_\odot$]        | - | - | $0.86 \pm 0.03$ |
| [Fe/H]                   | - | - | $0.19 \pm 0.13$ |

a Anderson et al. (2017) use the non-linear limb-darkening law but do not provide coefficients to compare with.
(2020) compares their final LDC values to those predicted by bi-linearly interpolating the LDC tables provided by Claret & Bloemen (2011) for the Cousins $R$ and $I$ bands and the Sloan Digital Sky Survey $z$ band and then averaging the results to approximate the TESS filter, which suggests values of $u_0 = 0.30$ and $u_1 = 0.28$. These predicted values agree with the results of the TransItFit "independent" LDC run, but are in significant tension with the results from the "coupled" LDC run. This suggests that the approximation used by Maciejewski (2020) is not as accurate as directly computing LDCs using the TESS filter profile.

In short, since Pearson (2019) does not fit the LDCs and Maciejewski (2020) does not simultaneously detrend the light curves, both approaches have weaknesses when compared to that of TransItFit, and we therefore argue that the planetary parameters and ephemerides obtained through the "coupled" LDC retrieval are the most accurate available to date, and, in agreement with Maciejewski (2020), we find no statistically significant TTVs and conclude that there is no evidence of a second planet in the WASP-126 system.

4 CONCLUSIONS

We have presented TransItFit, a new open-source code for fitting exoplanetary transit light curves using nested sampling routines. TransItFit has been designed for transmission spectroscopy surveys employing multiple telescopes, and allows coupling of limb-darkening coefficients across observation wavelengths by utilising information on the host star and the LDTK Python package (Parviainen & Aigrain 2015).

TransItFit has been developed in anticipation of a new "asset-starved" era of transmission spectroscopy studies, where limited observational time and resources mean that studies will frequently have to combine data of various quality, wavelength coverage, and sources. One such example of this is SPEARNET, a survey which is using a heterogeneous distributed network of small- to mid-sized ground-based telescopes to conduct atmospheric studies of transiting exoplanets.

Using TransItFit and observations from the SPEARNET telescope network, we have presented analysis of new data of the hot-Jupiter WASP-127 b, which includes the first $u'$-band observations of the planet. We have shown that TransItFit produces results which are consistent with existing studies of WASP-127 b, and that introducing a wavelength-coupled approach to LDC fitting can result in changes as large as 9 per cent in the retrieved value of $R_p/R_*$, or 20 per cent in measured transit depth.

We have demonstrated the application of TransItFit in more temporal-focused studies, analysing TESS observations of 26 transits of WASP-91 b to produce updated planetary ephemerides with an order-of-magnitude improvement in the precision of the period of WASP-91 b.

This level of improvement in the calculation of ephemerides will prove invaluable in analysis of planets in the TESS catalogue, allowing for easy searches for TTV signatures. We have used TransItFit to analyse 126 transits of WASP-126 b observed by TESS and have found no statistically significant evidence for the presence of TTVs proposed by Pearson (2019).

TransItFit is a versatile software package with many applications in exoplanetary studies. Through its wavelength-coupling of LDC fitting, it can be used to improve the consistency of multi-wavelength transit depth measurements, and through its handling of multi-epoch observations, can provide significant improvements in precision of planetary ephemerides and TTV measurements.

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Figure 5. *Left:* The O-C plots of 126 transits of WASP-126 b observed by TESS, obtained from results of using TransitFit in “coupled” (top) and “independent” (bottom) LDC modes whilst allowing for the presence of TTVs. The dotted lines represent the 1σ or uncertainty on the predicted ephemerides derived from using TransitFit without allowing for TTVs. *Right:* The associated Lomb-Scargle periodograms for each of the O-C plots. The horizontal grey lines represent the false alarm probabilities of 10, 5, and 1 per cent from bottom to top.

Table 5. The planetary and orbital parameters of WASP-126 b derived using TransitFit in both “coupled” and “independent” LDC fitting modes on observations from sectors 1–13 and 27–31 of the TESS mission. We present these alongside the values found by Pearson (2019), Maciejewski (2020), and Maxted et al. (2016). All fits assume zero orbital ellipticity, use a quadratic limb-darkening model, and unless otherwise stated assume the same host parameters as given by Maxted et al. (2016).

| Parameter | TransitFit: coupled LDCs | TransitFit: independent LDCs | Pearson (2019) | Maciejewski (2020) | Maxted et al. (2016) |
|-----------|--------------------------|-------------------------------|-----------------|---------------------|----------------------|
| $P$ [days] | 3.28878530 ± 4.9 × 10$^{-7}$ | 3.2887692 ± 4.8 × 10$^{-7}$ | 3.2888 ± 1.94 × 10$^{-5}$ | - | 3.28880 ± 0.00001 |
| $n_0$ [BJD] | 2456890.32163 ± 0.00027 | 2456890.32087 ± 0.00027 | - | 2456890.32004 ± 0.00061 | 2456890.31911 ± 0.0006 |
| $R_p/R_*$ | 0.0774 ± 0.0003 | 0.0783 ± 0.0002 | 0.0783 ± 0.0002 | 0.07712(±0.0006) | 0.0781(±0.0013) |
| $R_p/R_{Hep}$ | 0.966 ± 0.076 | 0.967 ± 0.076 | 0.964 ± 0.076 | 0.953 ± 0.075 | 0.965 ± 0.077 |
| $a/\rho_*$ | 7.898 ± 0.050 | 7.637 ± 0.086 | 7.887 ± 0.040 | 7.80±0.11 | 7.63±0.23 |
| $a$ [AU] | 0.0466 ± 0.0037 | 0.0451 ± 0.0036 | 0.0466 ± 0.0037 | 0.0461 ± 0.0038 | 0.0451 ± 0.0052 |
| $i$ [deg] | 89.32 ± 0.30 | 87.94 ± 0.36 | 89.51 ± 0.44 | 88.7±0.9 | 87.9±1.5 |
| $u_0$ | 0.407 ± 0.002 | 0.333 ± 0.048 | 0.43 b | 0.32±0.03 | - |
| $u_1$ | 0.142 ± 0.002 | 0.204 ± 0.063 | 0.14 b | 0.25±0.13 | - |
| $\tau_{eff,*}$ [K] | - | - | - | - | 5800 ± 100 |
| $M_*$ [$M_\odot$] | - | - | - | - | 1.12 ± 0.06 |
| $R_*$ [$R_\odot$] | - | - | - | - | 1.27 ± 0.1 |
| $[\text{Fe/H}]$ | - | - | - | - | 0.17 ± 0.08 |

a These values were derived assuming that no TTVs were present.
b These values are those predicted by LDTk, using host parameters from Maxted et al. (2016) with [Fe/H] = −0.06.
c The value of $P = 2.8493819 ± 0.0000013$ days provided in Maciejewski (2020) appears to be a typo as it exactly matches the period given for WASP-100 b.
