Laser spectroscopy of hyperfine structure in highly-charged ions: a test of QED at high fields

D.F.A. Winters*, M. Vogel
GSI mbH, Planckstrasse 1, Darmstadt D-64291, Germany

D.M. Segal, R.C. Thompson
Blackett Laboratory, Imperial College, Prince Consort Road, London SW7 2BW, United Kingdom

W. Nörtershäuser
Universität Mainz, Fritz-Strassmann-Weg 2, Mainz D-55099, Germany
GSI mbH, Planckstrasse 1, Darmstadt D-64291, Germany

Abstract

An overview is presented of laser spectroscopy experiments with cold, trapped, highly-charged ions, which will be performed at the HITRAP facility at GSI in Darmstadt (Germany). These high-resolution measurements of ground state hyperfine splittings will be three orders of magnitude more precise than previous measurements. Moreover, from a comparison of measurements of the hyperfine splittings in hydrogen- and lithium-like ions of the same isotope, QED effects at high electromagnetic fields can be determined within a few percent. Several candidate ions suited for these laser spectroscopy studies are presented.

Key words: QED, highly-charged ions, hyperfine structure, laser spectroscopy, trapping and cooling
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1. Introduction

Quantum electrodynamics (QED) was the first quantum field theory to be formulated and has successfully passed every experimental test at low and intermediate fields. A well-known example of QED effects at low fields (\(\sim 10^9\) V/cm) is the Lamb shift in hydrogen [1]. At low fields, the QED effects (self-energy and vacuum polarisation) can still be treated as a perturbation, only taking into account lower order terms [2]. However, up to now QED calculations have never been tested at high fields (\(\sim 10^{15}\) V/cm) because such fields cannot be produced in a laboratory, nor by the strongest lasers available. At high fields, perturbative QED is no longer valid and higher order terms become important as well [2]. Experiments carried out at high fields therefore test different aspects of QED calculations and are complementary to high-precision tests of the lower order terms.

Heavy atoms that have been stripped of almost

* Corresponding author.
Email address: d.winters@gsi.de (D.F.A. Winters).
all their electrons, the so-called highly-charged ions (HCI), are ideal ‘laboratories’ for tests of QED at high fields. These ions have, for example, electric field strengths of the order of $10^{15}$ V/cm close to the nucleus [2] and can be produced at high velocities at the Gesellschaft für Schwerionenforschung (GSI) in Darmstadt, Germany.

At the HITRAP facility, which is currently being built at GSI, ions coming from the experimental storage ring (ESR) with MeV energies will be slowed down by linear and radiofrequency stages to keV kinetic energies, trapped and cooled down to sub-eV energies, and finally made available for experiments. Within the HITRAP project, instrumentation is being developed for high-precision measurements of atomic and nuclear properties, mass and g-factor measurements and ion-atom and ion-surface interaction studies [3,4,5].

2. Hydrogen- and lithium-like ions

Hydrogen- and lithium-like ions are the best candidates for our studies, since they have $s$-electrons which are very close to the nucleus. The (higher order) QED effects are most pronounced at the high fields close to the nucleus, therefore the best measurable quantity is the ground state hyperfine splitting (HFS). Due to the simple electronic structure of H- and Li-like species, accurate (higher order) calculations of ground state HFS can be done, which will then be compared with accurate experimental results.

As a first approximation, good within about 4%, the energy of the $(1s)^2S_{1/2}$ ground state HFS of hydrogen-like ions is given by [2,6]:

$$E_{HFS} = \alpha (Z \alpha)^3 g_I \frac{m_e}{m_p} \frac{2(2I+1)}{3} m_e c^2 A_s (1 - \delta)$$

where $\alpha$ is the fine structure constant, $g_I = \mu/(\mu_N I)$ is the nuclear $g$-factor (with $\mu$ the nuclear magnetic moment and $\mu_N$ the nuclear magneton), $I$ the nuclear spin, $m_e$ and $m_p$ are the electron and proton mass, respectively, and $c$ is the speed of light. Equation (1) represents the normal ground state HFS multiplied by a correction $A_s$ for the relativistic energy of the $s$-electron, and by a factor $(1 - \delta)$, which takes the ‘Breit-Schawlow’ (BS) effect into account. The BS effect is due to the spatial distribution of the nuclear charge. It corrects for the fact that we cannot assume a homogeneous charge distribution over a spherical nucleus. The values for $\delta$ were taken from [6], those for $g_I$ and $I$ from [7]. In principle eq.(1) should also contain a correction for the finite nuclear mass, but since this correction is very small it can be neglected [2]. The energy of the $(1s^22s)^2S_{1/2}$ ground state HFS of lithium-like ions only differs from eq.(1) by a factor $1/n^3 = 1/8$ and by the $A_s$-value [2].

However, eq.(1) requires two further important corrections, the one of most interest to us being that which corrects for the QED effects. The other correction takes the ‘Bohr-Weisskopf’ (BW) effect into account [8]. The BW effect is due to the spatial distribution of the nuclear magnetisation and is only known with an accuracy of 20-30 %, which is mainly due to the single-particle model used for its calculation [9]. Unfortunately, the QED effects are of the same order of magnitude as the uncertainty in the BW effect [10]. Thus, from a HFS measurement of a single species (i.e. H- or Li-like) the QED effects cannot be determined accurately.

Equation (1) can also be written as $E_{HFS}^{\text{QED}} = C^{1s} + E_{QED}^{1s}$, where the constant $C^{1s}$ includes everything except the QED effects. Since the equations for the $(1s)$ and $(2s)$ states are so similar, it is possible to write the difference between the two HFS as $\Delta E_{HFS} = E_{HFS}^{2s} - \xi E_{HFS}^{1s} = E_{\text{non-QED}} + E_{QED}$ [10]. The factor $\xi$ only contains non-QED terms and can be calculated to a high precision [10]. From the difference between the HFS measurements of H- and Li-like ions of the same isotope, the QED effects can thus be determined within a few percent. However, this requires measurements of transition wavelengths with an experimental resolution of the order of $10^{-6}$.

The transition lifetime $t$ is defined as $t = A^{-1}$ (see e.g. [11]). The transition probability $A$, for an $M1$ transition from the excited to the lower hyperfine state, is given by [2]

$$A = \frac{4\alpha (2\pi \nu)^3 \hbar^2 I (2I + 1)^2}{27 m_e^2 c^4 (2I + 1)}$$

where $\hbar$ is Planck’s constant divided by $2\pi$ and $\kappa$...
They are singly charged ions, which are easily produced, have M1 transitions at convenient wavelengths, and can be used to test the laser spectroscopy part of the experiment. A measurement of the HFS in $^{207}$Pb$^+$ is of special interest, because it will be possible to extract the value of $\mu$. Currently two different values exist, which unfortunately leads to a 2% difference in the HFS calculations [15].

In principle, similar experiments could be carried out with metastable hafnium ($^{180}$Hf, level energy 1141 keV, half-life 5.5 h [7]). For H-like hafnium, the transition values are $\lambda = 217$ nm and $t = 0.25$ ms. For the Li-like ion, $\lambda = 1434$ nm and $t = 72$ ms are obtained. The difficulty with this isotope is that its nucleus is in an excited state, which is difficult to produce.

Figure 1 shows the calculated transition wavelengths of all H-like lead, and all H- and Li-like bismuth isotopes with half-lives exceeding 10 minutes. (The QED and BW effects are not included.) The isotopes are labelled by their corresponding atomic mass units (in $u$) and the stable isotopes ($^{207}$Pb and $^{209}$Bi) are indicated by the small arrows. For Pb, only the H-like isotopes are accessible with standard lasers, because the transition wavelengths of Li-like isotopes are much longer than 1600 nm. For Bi, many isotopes of both ion species are accessible, although their transition wavelengths differ considerably.

From Fig.1 it is clear that both elements offer many candidates for laser spectroscopy measurements of ground state HFS and that bismuth, in particular, allows for a systematic study of the (higher order) QED effects at high fields. Furthermore, a systematic study of different isotopes of the same species, for example a study of the H-like Pb isotopes, will make it possible to study trends in nuclear properties across a range of isotopes.

There already exist two previous measurements of the 2s ground state HFS in $^{209}$Bi. A direct measurement [16] was carried out using the ESR at GSI (Darmstadt), but unfortunately no resonance was found at the predicted value of $\approx 1554$ nm [10]. An indirect measurement [17] was performed in an electron beam ion trap (EBIT) and yielded a value of $\approx 1512$ nm, but the error in the measurement was rather large ($\approx 50$ nm). In the ESR, the ions have relativistic velocities ($\approx 200$ MeV/u),

Table 1
Calculated HFS transition wavelengths ($\lambda$) and lifetimes ($t$) of the most interesting ion species for systematic studies. Also shown are the nuclear spin ($I$) and magnetic moment ($\mu$), taken from [7]. The half-lives of these species are longer than 10 minutes. (The values listed are truncated and the QED and BW effects are not included.)

| Element   | Ion Type | $\lambda$ (nm) | $t$ (ms) | $I$ | $\mu$ ($\mu_N$) |
|-----------|----------|----------------|---------|-----|-----------------|
| lead      | $^{207}$Pb$^{81+}$ | H-like       | 973     | 45  | 1/2 0.59       |
| bismuth   | $^{209}$Bi$^{82+}$ | H-like       | 239     | 0.38| 9/2 4.11       |
|           | $^{209}$Bi$^{80+}$ | Li-like      | 1469    | 87  |                 |
| protactinium | $^{231}$Pa$^{90+}$ | H-like       | 262     | 0.64| 3/2 2.01       |
|           | $^{231}$Pa$^{88+}$ | Li-like      | 1511    | 123 |                 |
| lead      | $^{207}$Pb$^+$   | $P_{3/2} - P_{1/2}$ | 710     | 41  | 1/2 0.59       |
| chlorine  | $^{35}$Cl$^+$   | $3P_{2} - 1D_{2}$ | 858     | -   | 3/2 0.82       |
| argon     | $^{37}$Ar$^{2+}$ | $3P_{2} - 1D_{2}$ | 714     | -   | 7/2 1.3        |
|           |             | $3P_{1} - 1D_{2}$ | 775     | -   |                 |

is related to the electron’s angular momentum [2]. From eq.(2) it can be seen that $A$ scales with the transition frequency as $\nu^3$, whereas $\nu$ is proportional to $Z^3$, see eq.(1). Therefore, the transition lifetime scales with $Z$ as $t \propto Z^{-9}$ and is roughly of the order of milliseconds for $Z > 70$.

In table 1 the calculated transition wavelengths ($\lambda$) and lifetimes ($t$), together with their corresponding $I$ and $\mu$ values, of the most interesting species for our laser spectroscopy studies are listed. (The QED and BW effects are not taken into account.) The half-lives of these species exceed 10 minutes, which corresponds to the minimum time required for a measurement. Although the wavelengths span a broad range, roughly from 200 to 1600 nm, these transitions are still accessible with standard laser systems. The three species (Pb [12], Cl [13] and Ar [14]) at the bottom of the table are considered as candidates for pilot experiments. They are singly charged ions, which are easily produced, have M1 transitions at convenient wavelengths, and can be used to test the laser spectroscopy part of the experiment. A measurement of the HFS in $^{207}$Pb$^+$ is of special interest, because it will be possible to extract the value of $\mu$. Currently two different values exist, which unfortunately leads to a 2% difference in the HFS calculations [15].

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which are used to shift the transition wavelength to a lower value ($\approx 532$ nm), and the transitions are Doppler-broadened ($\approx 40$ GHz). In the EBIT the ions have temperatures of several hundreds of eV ($\sim 10^6$ K), which lead to considerable Doppler broadening ($\approx 10$ GHz). The resolution obtained in previous measurements at the ESR is of the order of $10^{-4}$, whereas that of the EBIT measurement is of the order of $10^{-2}$.

3. Experiment overview

A detailed description of the proposed experiments, as well as a treatment of the techniques used, can be found elsewhere [18,19]. Briefly, an externally produced bunch of roughly $10^5$ HCI at an energy of a few keV is loaded into a cylindrical open-endcap Penning trap [20] on axis, i.e. along the magnetic field lines. Electron capture (neutralisation) by collisions is strongly reduced by operating the trap at cryogenic temperatures under UHV conditions. The HCI are captured in flight, confined, cooled by ‘resistive cooling’ [21] and radially compressed by a ‘rotating wall’ [22] technique. After these steps a cold and dense ion cloud is obtained. The spectroscopy laser enters the trap axially through an open-endcap and will fully irradiate the ion cloud. The fluorescence from the excited HCI is detected perpendicular to the cooled axial motion (trap axis) through segmented ring electrodes, which are covered by a highly-transparent copper mesh. (The ring is segmented for the rotating wall technique.)

The above mentioned transition lifetimes imply that, for a detection efficiency of $\sim 10^{-3}$, acceptable fluorescence rates, up to a few thousand counts per second, from $M1$ transitions can be expected from a ($\sim 3$ mm diameter) cloud of $10^5$ ions [18,19]. Confining the HCI in a trap, and cooling and compressing the cloud, will thus enable fluorescence detection and ensure long interrogation times by the laser.

However, due to the high density of HCI in the cloud, space charge effects will play a role and will lead to shifts of the motional frequencies of the trapped ions. We have studied this effect in detail and understand the corresponding frequency shifts well [23]. Since these shifts are fairly small, the (frequency dependent) cooling and compression techniques can still be applied.

The HCI also need to be strongly cooled to reduce Doppler broadening of the transitions. This will be achieved by resistive cooling of the (axial) ion motion in the trap. For example, for the $F = 1 \rightarrow F = 0$ transition in $^{207}$Pb$^{81+}$ at \( \nu \approx 3 \times 10^{14} \) Hz, the Doppler broadened linewidth at a temperature of 4 K is $\Delta \nu_D \approx 3 \times 10^7$ Hz. The anticipated resolution is therefore of the order of $10^7/10^{14} = 10^{-7}$. This is three orders of magnitude better than any previous measurement, see e.g. [15,24], and good enough to measure the QED effects within a few percent.

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