Power Minimization in Multi-pair Two-Way Relaying

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An alternative form of SINR balancing problem is to minimize the total transmit-power at the relay subject to the SINR constraints. The power minimization problem is formulated as follows:

\[
\begin{align*}
    \min_A & \quad \sum_{i=1}^{2K} p_i \|Ah_i\|^2 + \text{tr}(AA_R A^H) \\
    \text{s. t.} & \quad \text{SINR}_i(A) \geq \gamma_i, \quad i = 1, \ldots, 2K.
\end{align*}
\] (1)

Noting \(a = \text{vec}(A)\) and \(X = aa^H\), and dropping the rank constraint of \(X\), we can rewrite (1) as

\[
\begin{align*}
    P_R(\lambda) = \min_{X \succeq 0} & \quad \text{tr}(E_0X) \\
    \text{s. t.} & \quad f_i(X) \geq \lambda \gamma_i, \quad i = 1, \ldots, 2K.
\end{align*}
\] (2)

Clearly, setting the parameter \(\lambda\) to 1 reduces (2) to (1). Here, we allow \(\lambda\) to be an arbitrary positive number for ease of further discussions. We note that the power minimization in (2) can be efficiently solved with a single SDP [2].

We next establish a close relation between the max-min SINR problem (5) in [1] and the power minimization problem in (2). We first show that (2) can be solved via solving (5) in [1]. Let \(\lambda_{\text{opt}}(\hat{P}_R)\) denote the optimal value of (8) in [1] for a given power budget \(\hat{P}_R\). It can be shown that:

**Lemma 1:** \(\lambda_{\text{opt}}(\hat{P}_R)\) is a strictly increasing function of \(\hat{P}_R\).
Proof: Let $X^{\text{opt}}$ denote the optimal solution for (8) in [1] with power budget $\hat{P}_R > 0$. For a $\hat{P}_R' > \hat{P}_R$, let $\alpha = \hat{P}_R' / \hat{P}_R > 1$, and $X' = \alpha X^{\text{opt}}$. Then $X'$ is feasible for (8) in [1] with power budget $\hat{P}_R'$, since $\text{tr}(E_0X') = \alpha \text{tr}(E_0X^{\text{opt}}) \leq \alpha \hat{P}_R = \hat{P}_R'$.

On the other hand,

$$\text{SINR}_i(X') = \frac{f_i(X')}{g_i(X')} = \frac{\text{tr}(E_i^{(1)}X')}{\text{tr}(E_i^{(2)}X') + \sigma_i^2} = \frac{\alpha \text{tr}(E_i^{(1)}X^{\text{opt}})}{\alpha \text{tr}(E_i^{(2)}X^{\text{opt}}) + \sigma_i^2} \geq \frac{\text{SINR}_i(X^{\text{opt}})}{\alpha (\text{tr}(E_i^{(2)}X^{\text{opt}}) + \sigma_i^2)} = \text{SINR}_i(X^{\text{opt}}).$$

Therefore, $\bar{\lambda}^{\text{opt}}(\hat{P}_R') \geq \min_{i=1,\ldots,2K} \frac{\text{SINR}_i(X')}{\gamma_i} > \min_{i=1,\ldots,2K} \frac{\text{SINR}_i(X^{\text{opt}})}{\gamma_i} = \bar{\lambda}^{\text{opt}}(\hat{P}_R)$. □

Relying on the monotonicity of $\bar{\lambda}^{\text{opt}}(\hat{P}_R)$ stated in Lemma 1, we can further show that:

**Lemma 2:** The optimal solution for (2) is the same as the matrix $X^{\text{opt}}$ for (8) in [1] with the power budget $P_R$ that satisfies $\bar{\lambda}^{\text{opt}}(P_R) = 1$.

*Proof:* Let $X^{\text{opt}}$ denote the optimal solution for (8) in [1] with the power budget $P_R$ that satisfies $\bar{\lambda}^{\text{opt}}(P_R) = 1$. Since $\bar{\lambda}^{\text{opt}}(P_R) = 1$ implies $\text{SINR}_i(X^{\text{opt}}) \geq \gamma_i, i = 1,\ldots,2K$, $X^{\text{opt}}$ is in the feasible set of (2). Upon denoting $P_R^{\text{opt}}$ as the optimal value for (2), this in turn implies that $P_R^{\text{opt}} \leq \text{tr}(E_0X^{\text{opt}}) \leq P_R$. Consider (8) in [1] with the power budget $P_R^{\text{opt}}$. By Lemma 2, we must have

$$\bar{\lambda}^{\text{opt}}(P_R^{\text{opt}}) \leq \bar{\lambda}^{\text{opt}}(P_R) = 1$$

due to $P_R^{\text{opt}} \leq P_R$.

On the other hand, let $\tilde{X}^{\text{opt}}$ denote the optimal solution for (2), which is the feasible set of (8) in [1] with the power budget $P_R^{\text{opt}}$ since $\text{tr}(E_0\tilde{X}^{\text{opt}}) = P_R^{\text{opt}}$. For this $\tilde{X}^{\text{opt}}$, we have $\min_{i=1,\ldots,2K} \frac{\text{SINR}_i(\tilde{X}^{\text{opt}})}{\gamma_i} \geq 1$ since $\text{SINR}_i(\tilde{X}^{\text{opt}}) \geq \gamma_i, i = 1,\ldots,2K$. This together with the feasibility of $\tilde{X}^{\text{opt}}$ implies that $\bar{\lambda}^{\text{opt}}(P_R^{\text{opt}}) \geq 1$. Clearly, we have both the latter and (3) satisfied, only when all the inequalities are satisfied with equalities; i.e., $P_R^{\text{opt}} = P_R$, and it is achieved by the beamforming matrix $X^{\text{opt}}$. □
From Lemma 2, the optimal solution to (2) is the same as that to (8) in [1] with the power budget $P_R$ satisfying $\lambda^{opt}(P_R) = 1$. As a result, the optimal solution to (2) can be obtained by solving the equation $\lambda^{opt}(P_R) = 1$, which requires a one-dimensional bisection search.

What remains is to show that (5) in [1] can be solved via solving (2). It can be shown that $P_R(\lambda)$ in (2) is a strictly increasing function of $\lambda$. Together with the fact that, for an arbitrary $\lambda > 0$, (2) is readily solvable using a single SDP, we conclude that (5) in [1] is solvable by a bisection search over $\lambda$ satisfying $P_R(\lambda) = \hat{P}_R$.

So far, we have shown that the power minimization and max-min SINR problems are two alternative forms of the SINR balancing problem. This allows us to freely choose a more tractable form, i.e., a form that is more efficiently solvable, as the corner stone to pursue the optimal beamforming designs under various important optimization criteria, as detailed in [1].

REFERENCES

[1] Z. Fang, X. Wang, and X. Yuan, “Beamforming Design for Multiuser Two-Way Relaying: A Unified Approach via Max-Min SINR,” submitted to IEEE Trans Signal Process., March 2013.

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