Infinite summation formulas of Srivastava’s general triple hypergeometric function

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Abstract

In this paper we derive the infinite summation formulas of Srivastava’s general triple hypergeometric function. Certain particular cases leading to infinite summation formulas for fourteen Lauricella and three Srivastava’s triple hypergeometric functions are also presented.

Keywords: Srivastava’s general triple hypergeometric function, Lauricella function, Srivastava’s triple hypergeometric function.

1 Introduction

Recently, the authors have studied the Srivastava’s general triple hypergeometric function \[ F^{(3)}[x, y, z] \] from the viewpoint of recursion formulas and \( q \)-derivative with respect to parameters. Further, in [5], we have enumerated finite summation formulas of Srivastava’s general triple hypergeometric function. In the present paper, we obtain the infinite summation formulas satisfied by this function. The particular cases will lead to results involving fourteen Lauricella function and three Srivastava’s triple hypergeometric function.

Earlier, Wang [11] obtained infinite summation formulas of double hypergeometric functions. In [12], finite summation formulas of double hypergeometric functions are derived.

The Srivastava’s general triple hypergeometric function \[ F^{(3)}[x, y, z] \] is defined by

\[
F^{(3)}[x, y, z] = \left[ (a) : (b) ; (b') : (c) ; (c') ; (c'') : (d) ; (d') ; (d'') : x_1, x_2, x_3 \right] \\
= \sum_{m_1, m_2, m_3=0}^{\infty} \Lambda(m_1, m_2, m_3) \frac{x_1^{m_1} x_2^{m_2} x_3^{m_3}}{m_1! m_2! m_3!} \tag{1.1}
\]

where

\[ \Lambda(m_1, m_2, m_3) \]

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The following infinite summation formulas of Srivastava’s general triple hypergeometric function by using the well known binomial summation theorem.

\[ \sum_{k=0}^{\infty} \frac{(a)_{k}}{k!} t^k F^{(3)} \left[ \begin{array}{l} a_k+k, (a')_{k} : (b); (b')_{k} : (c); (c')_{k} \\ (c) : (g); (g')_{k} : (h); (h')_{k} : x_1, x_2, x_3 \end{array} \right] = (1-t)^{-a_k} F^{(3)} \left[ \begin{array}{l} (a) : (b); (b') : (c); (c') : (c'') \\ (c) : (g); (g') : (h); (h') : x_1, x_2, x_3 \end{array} \right], \tag{2.1} \]

and \((a)\) abbreviates the array of \(A\) parameters \(a_1, a_2, \ldots, a_A\), etc. The region of convergence of the general triple hypergeometric series \((1.1)\) is given in the literature \([10]\).

In this paper following abbreviated notations are used. For example, write

\((a+k) := a_1+k, \ldots, a_A+k,\)

\((a') := a_1, \ldots, a_{j-1}, a_{j+1}, \ldots, a_A,\)

\((a^j+k) := a_1+k, \ldots, a_{j-1}+k, a_{j+1}+k, \ldots, a_A+k, \quad j = 1, \ldots, A.\)

Also, we use the notations

\([a]_k := \prod_{i=1}^{A} (a_i)_k, \quad [a^j]_k := \prod_{i=1, i\neq j}^{A} (a_i)_k, \quad \text{etc.}\)

where \(k\) is a non-negative integer and \((a_i)_k\) is the Pochhammer symbol \([2]\).

The subject matter is divided in two sections. The infinite summation formulas for non-terminating Srivastava’s general triple hypergeometric function are discussed in Section 2 and the results for the terminating Srivastava’s general triple hypergeometric function are discussed in Section 3.

## 2 Infinite summation formulas with non-terminating Srivastava’s general triple hypergeometric function

In this section, we establish infinite summation formulas of non-terminating Srivastava’s general triple hypergeometric function by using the well known binomial summation theorem.

**Theorem 2.1** The following infinite summation formulas of Srivastava’s general triple hypergeometric function hold true:

\[ \sum_{k=0}^{\infty} \frac{(b)_{k}}{k!} t^k F^{(3)} \left[ \begin{array}{l} a_k+k, (a')_{k} : (b'); (b'') : (c); (c')_{k} \\ (c) : (g); (g') : (h); (h') : x_1, x_2, x_3 \end{array} \right] = (1-t)^{-b} F^{(3)} \left[ \begin{array}{l} (a) : (b); (b') : (c); (c') : (c'') \\ (c) : (g); (g') : (h); (h') : x_1, x_2, x_3 \end{array} \right], \tag{2.2} \]

where \(i = 1, \ldots, A;\)

\[ \sum_{k=0}^{\infty} \frac{(c)_{k}}{k!} t^k F^{(3)} \left[ \begin{array}{l} a_k+k, (c')_{k} : (b'); (b'') : (c); (c') : (c'') \\ (c) : (g); (g') : (h); (h') : x_1, x_2, x_3 \end{array} \right] = (1-t)^{-c} F^{(3)} \left[ \begin{array}{l} (a) : (b); (b') : (c); (c') : (c'') \\ (c) : (g); (g') : (h); (h') : x_1, x_2, x_3 \end{array} \right], \tag{2.3} \]

where \(i = 1, \ldots, B;\)

\[ \sum_{k=0}^{\infty} \frac{(c)_{k}}{k!} t^k F^{(3)} \left[ \begin{array}{l} a_k+k, (c')_{k} : (b'); (b'') : (c); (c') : (c'') \\ (c) : (g); (g') : (h); (h') : x_1, x_2, x_3 \end{array} \right] = (1-t)^{-c} F^{(3)} \left[ \begin{array}{l} (a) : (b); (b') : (c); (c') : (c'') \\ (c) : (g); (g') : (h); (h') : x_1, x_2, x_3 \end{array} \right]. \tag{2.4} \]
\[ (1-t)^{-c} \cdot F^{(3)} \left[ \left( \begin{array}{c} a \vdash (b) (b') (b''); c (c') (c'''); \\ c (c') (c''') \end{array} \right); (g) (g') (g''); (h) (h') (h'''); \frac{x_1}{t-1}, x_2, x_3 \right], \]  

(2.3)

where \( i = 1, \ldots, C \).

**Proof:** Applying the definition of Srivastava's general triple hypergeometric function \( F^{(3)}[x, y, z] \) and transformation

\[
(a)_k (a + k)_{m_1 + m_2 + m_3} = (a)_{m_1 + m_2 + m_3} (a + m_1 + m_2 + m_3)_k,
\]

the L.H.S of (2.1) can be expressed as

\[
\sum_{m_1, m_2, m_3=0}^{\infty} \wedge(m_1 + m_2 + m_3) \frac{x_1^{m_1} x_2^{m_2} x_3^{m_3}}{m_1! m_2! m_3!} F_0 \left[ a + m_1 + m_2 + m_3; t \right],
\]

where

\[
1 F_0 \left[ a; t \right] = (1 - t)^{-a}.
\]

(2.4)

After some simplification, we get the right side of (2.1). This completes the proof of (2.1). The remaining identities (2.2) and (2.3) are proved in a similar manner.

**Theorem 2.2** The following infinite summation formulas of Srivastava's general triple hypergeometric function hold true:

\[
F^{(3)}[x_1 + t, x_2, x_3] = \sum_{k=0}^{\infty} \frac{[a]_k [b]_k [b']_k [c]_k [c']_k [c''']_k}{[c]_k [g]_k [g']_k [g'']_k [h]_k [h']_k [h'']_k} \cdot \frac{t^k}{k!}
\]

\[
\times F^{(3)} \left[ \left( \begin{array}{c} a+k \vdash (b+k) (b') (b'') \vdash (c+k) (c') (c'') \\ c+k \vdash (g+k) (g') (g'') \vdash (h+k) (h') (h'') \end{array} \right); x_1, x_2, x_3 \right].
\]

(2.5)

\[
F^{(3)}[x_1, x_2 + t, x_3] = \sum_{k=0}^{\infty} \frac{[a]_k [b]_k [b']_k [c]_k [c']_k [c''']_k}{[c]_k [g]_k [g']_k [g'']_k [h]_k [h']_k [h'']_k} \cdot \frac{t^k}{k!}
\]

\[
\times F^{(3)} \left[ \left( \begin{array}{c} a+k \vdash (b+k) (b') (b'') \vdash (c+k) (c') (c'') \\ c+k \vdash (g+k) (g') (g'') \vdash (h+k) (h') (h'') \end{array} \right); x_1, x_2, x_3 \right].
\]

(2.6)

\[
F^{(3)}[x_1, x_2, x_3 + t] = \sum_{k=0}^{\infty} \frac{[a]_k [b]_k [b']_k [c]_k [c']_k [c''']_k}{[c]_k [g]_k [g']_k [g'']_k [h]_k [h']_k [h'']_k} \cdot \frac{t^k}{k!}
\]

\[
\times F^{(3)} \left[ \left( \begin{array}{c} a+k \vdash (b+k) (b') (b'') \vdash (c+k) (c') (c'') \\ c+k \vdash (g+k) (g') (g'') \vdash (h+k) (h') (h'') \end{array} \right); x_1, x_2, x_3 \right].
\]

(2.7)

**Proof:** From the definition \( F^{(3)}[x_1, x_2, x_3] \) and the transformation \((a)_k (a + k)_{m_1} = (a)_{m_1+k} \), the right side of (2.2) can be written as

\[
\sum_{k=0}^{\infty} \wedge(m_1 + k, m_2, m_3) \frac{x_1^{m_1} x_2^{m_2} x_3^{m_3}}{m_1! m_2! m_3!} t^k.
\]

Replacing \( m_1 + k \to l \) in the above result and after some simplification, we get

\[
\sum_{l,m_2,m_3=0}^{\infty} \wedge(l, m_2, m_3) \frac{x_2^{m_2} x_3^{m_3}}{l! m_2! m_3!} \sum_{k=0}^{l} \left( \begin{array}{c} l \\ k \end{array} \right) x_1^{l-k} t^k
\]

\[
= \sum_{l,m_2,m_3=0}^{\infty} \wedge(l, m_2, m_3) \frac{(x_1 + t)^l x_2^{m_2} x_3^{m_3}}{l! m_2! m_3!}.
\]
where, we have applied the special case of the binomial theorem \((2.4)\) as
\[
\sum_{k=0}^{l} \binom{l}{k} x_1^k x_2^{l-k} = (x_1 + x_2)^l
\]
in the inner summation. This completes the proof of \((2.5)\). Identities \((2.6)\) to \((2.7)\) can be proved in an analogous manner.

**Theorem 2.3** The following infinite summation formulas of Srivastava’s general triple hypergeometric function hold true:
\[
\sum_{k=0}^{\infty} \frac{[a]^k [b]^k [b']^k [c]^k (r)_k}{[c] [g][g'][h][k] k!} x_1^k F^{(3)} \left[ \begin{array}{c} a_i, a_i + k \vdash (b+k); (b'); (c); (c'); (c''); \\
(c); (c'+k); (c''); \\
(c''); (c'''); \\
(c''''); (c'''''); \\
(a_i + r, (a'_i); (b'''); (c'''); (c''''); \\
(c'''''''); \end{array} \right]_{x_1, x_2, x_3}
\]
where \(i = 1, \ldots, A;\)
\[
\sum_{k=0}^{\infty} \frac{[a]^k [b]^k [b']^k [c]^k (r)_k}{[c] [g][g'][h][k] k!} x_1^k F^{(3)} \left[ \begin{array}{c} a_i, a_i + k \vdash (b+k); (b'); (c); (c'); (c''); \\
(c); (c'+k); (c''); \\
(c''); (c'''); \\
(c''''); (c'''''); \\
(a_i + r, (a'_i); (b'''); (c'''); (c''''); \\
(c'''''''); \end{array} \right]_{x_1, x_2, x_3}
\]
where \(i = 1, \ldots, C;\)

**Proof:** We first prove identity \((2.3)\). From the definition of \(F^{(3)}[x_1, x_2, x_3]\)-series and the transformation \((a)_k (a+k); m_1 = (a)_{k+m_1};\) the left side of \((2.3)\) can be expressed as
\[
\sum_{m_1, m_2, m_3=0}^{\infty} \frac{\text{and}(m_1+k, m_2, m_3)[r]_k}{(a_i + m_1 + m_2 + m_3)_{k!} m_1! m_2! m_3!} x_1^{m_1+k} x_2^{m_2} x_3^{m_3}
\]
\[
= \sum_{l, m_2, m_3=0}^{\infty} \text{and}(l, m_2, m_3) x_1^l x_2^{m_2} x_3^{m_3} \frac{\text{and}(m_1+k, m_2, m_3)}{l! m_2! m_3!} 2F1 \left[ \begin{array}{c} \frac{-l, r}{-l, -l-m_2-m_3} \end{array} \right]_{1}
\]
where, we have performed the replacement \(m_1 + k \to l\) in the second equation. Obviously the inner summation \(2F1\) can be evaluated by the Vandermonde’s theorem \(2)\)
\[
2F1 \left[ \begin{array}{c} -n, a \end{array} \right]_{c} = \frac{(c-a)n}{(c)_n}
\]
After some simplification, we get right side of \((2.3)\). The identity \((2.10)\) can be proved in an analogous manner.

**Theorem 2.4** The following infinite summation formulas of Srivastava’s general triple hypergeometric function hold true:
\[
\sum_{k=0}^{\infty} \frac{[a]^k [b]^k [b']^k [c]^k (d)_k}{[c] [g][g'][h][k] k!} \left(-x_1\right)^k F^{(3)} \left[ \begin{array}{c} a_i, a_i + k \vdash (b+k); (b'); (c); (c'); (c''); \\
(c); (c'+k); (c'''); \\
(c''); (c'''''); \end{array} \right]_{x_1, x_2, x_3}
\]
where \(i = 1, \ldots, A;\)
\[
\sum_{k=0}^{\infty} \frac{[a]^k [b]^k [b']^k [c]^k (d)_k}{[c] [g][g'][h][k] k!} \left(-x_1\right)^k F^{(3)} \left[ \begin{array}{c} a_i, a_i + k \vdash (b+k); (b'); (c); (c'); (c''); \\
(c); (c'+k); (c'''); \\
(c''); (c'''''); \end{array} \right]_{x_1, x_2, x_3}
\]
where \(i = 1, \ldots, C;\)
Proof: We give the proof of (2.12). This theorem can be proved by Vandermonde’s theorem (2.11). Using the definition of $F^{(3)}[x_1, x_2, x_3]$ and the transformation $(a)_k(a + k)_m = (a)_{k+m}$, then applying replacement $m_1 + k \rightarrow l$, and making some simplification, the left side of (2.12) can be expressed as

$$\sum_{l,m_2,m_3=0}^{\infty} \frac{\wedge(l, m_2, m_3)}{l!m_2!m_3!} x_1^{m_2} x_2^{m_3} 3F_1 \left[ -l, d; 1 \right].$$

Calculating inner summation $3F_1(1)$ by Vandermonde’s theorem (2.11), we get right side of (2.12). Transformation (2.13) can be proved in an analogous manner. We omit these details.

Theorem 2.5 The following infinite summation formulas of Srivastava’s general triple hypergeometric function hold true:

$$\sum_{k=0}^{\infty} \frac{[a]_k [b]_k [b']_k [c']_k (d)_k (r)_k}{[e]_k [g]_k [g']_k [h]_k (d + r + c)_k k!} x_1^{k} F^{(3)} \left[ (a+k) ; (b+k) ; (b'+k) ; (c'+k) ; (c') ; (c') ; (c') ; (g+k) ; (g'+k) ; (h+k) ; (h') ; (h') ; x_1, x_2, x_3 \right]$$

where $i = 1, \ldots, C$.

Proof: The left side of (2.13) can be expressed as

$$\sum_{l,m_1,m_2,m_3=0}^{\infty} \frac{\wedge(l, m_1, m_2, m_3)}{l!m_1!m_2!m_3!} x_1^{m_1} x_2^{m_2} x_3^{m_3} 3F_2 \left[ -l, d, r ; d+r+c, 1-c-i, 1 \right],$$

where, Replaced $m_1 + k \rightarrow l$ in above equation and using the well known Saalschütz formula [6]

$$3F_2 \left[ -n, a, b \right]_{c, 1+d+c-n} = \frac{(c-a)_n (c-b)_n}{(c)_n (c-a-b)_n}$$

and after some simplification, we get right side of (2.14). This completes the proof of the theorem.

Theorem 2.6 The following infinite summation formulas of Srivastava’s general triple hypergeometric function hold true:

$$\sum_{k=0}^{\infty} \frac{[a]_k [b]_k [b']_k [c]_k (d)_k (1 + \frac{d}{2})_k}{[e]_k [g]_k [g']_k [h]_k (\frac{d}{2})_k k!} (-x_1)^k F^{(3)} \left[ (a+k) ; (b+k) ; (b'+k) ; (c+k) ; (c') ; (c') ; (c') ; (g+k) ; (g'+k) ; (h+k) ; (h') ; (h') ; x_1, x_2, x_3 \right]$$

where $i = 1, \ldots, A$;

$$\sum_{k=0}^{\infty} \frac{[a]_k [b]_k [b']_k [c]_k (d)_k (1 + \frac{d}{2})_k}{[e]_k [g]_k [g']_k [h]_k (\frac{d}{2})_k k!} (-x_1)^k F^{(3)} \left[ (a+k) ; (b+k) ; (b'+k) ; (c+k) ; (c') ; (c') ; (c') ; (g+k) ; (g'+k) ; (h+k) ; (h') ; (h') ; x_1, x_2, x_3 \right]$$

where $i = 1, \ldots, C$.

Proof: The left side of (2.14) can be expressed as

$$\sum_{l,m_1,m_2,m_3=0}^{\infty} \frac{\wedge(l, m_1, m_2, m_3)}{l!m_1!m_2!m_3!(\frac{d}{2})_k (a)_k (-1)^k} x_1^{m_1+k} x_2^{m_2} x_3^{m_3} F^{(3)} \left[ -l, d, r ; d+r+c, 1-c-i, 1 \right].$$

and after some simplification, we get right side of (2.14). This completes the proof of the theorem.
where replace \( m_1 + k \rightarrow l \) in the last equation. Applying the nearly poised summation formula \([6]\):

\[
3F_2 \left[ \begin{array}{c} \frac{-n}{b}, \frac{a+\frac{1}{2}}{b} ; 1 \\ \frac{a}{b} \end{array} \right] = \frac{(b-a-1-n)(b-a)}{(b)_n}, \tag{2.18}
\]

and simplifying, we get \((2.16)\). The transformation \((2.17)\) can be proved in an analogous manner.

**Theorem 2.7** The following infinite summation formula of Srivastava’s general triple hypergeometric function holds true:

\[
\sum_{k=0}^{\infty} \frac{a_k b_k [b'_k] c'_k (r_k) (r'_k) k}{m_k! m'_k! (1+r_k + \frac{2}{3}) k!} x^k = F(3) \left[ \begin{array}{c} (a+k); (b+k); (b'+k); (c+k'); (c'+k); (c''); \\ c+k; (g+k); (g'+k); (h+k); (h'+k) ; x_1, x_2, x_3 \end{array} \right],
\]

where \( i = 1, \ldots, C \).

**Proof:** The left side of \((2.19)\) can be expressed as

\[
\sum_{k=0}^{\infty} \frac{\Lambda(l+m_1, m_2, m_3)(r_k) k}{m_1! m_2! m_3! (1+r_k + \frac{2}{3}) k!} x^k = \sum_{l,m_2,m_3=0}^{\infty} \frac{\Lambda(l, m_2, m_3)}{l! m_2! m_3!} x^k F_3 \left[ \begin{array}{c} \frac{-n}{b}, \frac{a+\frac{1}{2}}{b} ; 1 \\ \frac{a}{b} \end{array} \right] = \frac{(a-2b)n(1+\frac{1}{a}-b)n(-b)_n}{(a-b)n(-2b)_n}, \tag{2.20}
\]

and simplifying, we get the right side of \((2.19)\). This completes the proof of this theorem.

**Theorem 2.8** The following infinite summation formula of Srivastava’s general triple hypergeometric function holds true:

\[
\sum_{k=0}^{\infty} \frac{a_k b_k [b'_k] c'_k (d_k) (1+\frac{2}{3}) k}{m_k! m'_k! (1+d_k + \frac{2}{3}) k!} x^k = F(3) \left[ \begin{array}{c} (a+k); (b+k); (b'+k); (c+k); (c'+k); (c''); \\ (c+k); (g+k); (g'+k); (h+k); (h'+k) ; x_1, x_2, x_3 \end{array} \right],
\]

where \( i = 1, \ldots, C \).

**Proof:** From the definition of \( F(3) [x_1, x_2, x_3] \)-series and using the transformation \((a+k)_m = (a+k)_m \), the left side of \((2.21)\) can be simplified as

\[
\sum_{k=0}^{\infty} \frac{\Lambda(m_1+k, m_2, m_3)(c_k) m_1(m_1)(1+\frac{2}{3}) k}{m_1! m_2! m_3! (1+d_k + \frac{2}{3}) k!} x^k = \sum_{l,m_1,m_2,m_3=0}^{\infty} \frac{\Lambda(l, m_2, m_3)}{l! m_2! m_3!} x^k 4F_3 \left[ \begin{array}{c} \frac{-n}{b}, \frac{a+\frac{1}{2}}{b} ; 1 \\ \frac{a}{b} \end{array} \right] = \frac{(a-2b)_n(-b)_n}{(a-b)_n(-2b)_n}, \tag{2.21}
\]

and simplifying, we get \((2.21)\).
3 The infinite summation formulas with terminating Srivastava’s general triple hypergeometric function

In this section, we present infinite summation formulas with terminating Srivastava’s general triple hypergeometric function. We remark that by specializing the parameters in \( F^{(3)} \) we can deduce summation formulas for the fourteen Lauricella functions \([1]\) as well as three Srivastava’s triple hypergeometric functions \( H_A, H_B, H_C \). \([2, 3, 10]\). We have listed only some particular cases leading to results of Lauricella functions and three Srivastava’s triple hypergeometric functions. The infinite summation formulas for the remaining Lauricella functions and three Srivastava’s triple hypergeometric functions can be worked out analogously.

Theorem 3.1 The following infinite summation formula of Srivastava’s general triple hypergeometric function holds true:

\[
\sum_{k=0}^{\infty} \frac{(c)_k}{k!} (-t)^k F^{(3)} \left[ \left( \begin{array}{c} a; b; (b'); (b''); -k, c'; c''; \end{array} \right); \frac{1+t}{t} x_1, x_2, x_3 \right] = (1+t)^{-c_1} F^{(3)} [x_1, x_2, x_3],
\]

where \( i = 1, \ldots, C \).

Proof: From the definition of \( F^{(3)}[x_1, x_2, x_3] \), the left side of (3.1) can be expressed as

\[
\sum_{k=0}^{\infty} \sum_{m_1, m_2, m_3=0} \frac{\Lambda(m_1, m_2, m_3)}{m_1!m_2!m_3!} \frac{(c_1)_k}{(c_1)_{m_1}} (-t)^k.
\]

Replacing \( k = m_1 + l \), changing the summation order and simplifying, we get

\[
\sum_{m_1, m_2, m_3=0} \frac{\Lambda(m_1, m_2, m_3)}{m_1!m_2!m_3!} \left( \frac{1+t}{t} \right)^{m_1} x_1^{m_1} x_2^{m_2} x_3^{m_3} (1+t)^{-m_1} \left( \frac{t}{1+t} \right)^{m_1} F_0 \left[ \left( \begin{array}{c} c_1 + m_1; \end{array} \right); \left( \begin{array}{c} 1; \end{array} \right); \left( \begin{array}{c} -l; \end{array} \right) \right].
\]

Evaluating the inner \( F_0 \)-series in the above equation by binomial theorem \([2, 3]\), we establish another infinite summation formula of Srivastava’s general triple hypergeometric function.

Theorem 3.2 The following infinite summation formula of Srivastava’s general triple hypergeometric function holds true:

\[
\sum_{k=0}^{\infty} \frac{(c)_k}{k!} \left( \frac{1+x_1}{x_1-1} \right)^k F^{(3)} \left[ \left( \begin{array}{c} a; b; (b'); (b''); -k, c'; c''; \end{array} \right); \frac{1+t}{t} x_1, x_2, x_3 \right] = (1+t)^{-c_1} F^{(3)} [x_1, x_2, x_3],
\]

where \( i = 1, \ldots, C \).

Proof: The proof of this theorem is similar to Theorem 3.1. We omit the details.

4 Conclusion

We have obtained several infinite summation formulas involving the Srivastava’s general triple hypergeometric function. We remark that by specializing the parameters in \( F^{(3)}[x_1, x_2, x_3] \), we can deduce summation formulas for the fourteen Lauricella functions \([1]\) as well as three Srivastava’s triple hypergeometric functions \( H_A, H_B, H_C \). \([2, 3, 10]\). We have listed only some particular cases leading to results of Lauricella functions and three Srivastava’s triple hypergeometric functions. The infinite summation formulas for the remaining Lauricella functions and three Srivastava’s triple hypergeometric functions can be worked out analogously.
For example, specializing the parameters in (2.1), (2.2) and (2.3) we get the infinite summation formulas for $F_A^{(3)}$ and $F_D^{(3)}$:

$$\sum_{k=0}^{\infty} \frac{(a)_k}{k!} t^k F_A^{(3)}(a+k, b_1, b_2, c_1, c_2, x_1, x_2, x_3) = (1-t)^{-a} F_A^{(3)}\left(a, b_1, b_2, b_3; c_1, c_2, c_3; \frac{x_1}{1-t}, \frac{x_2}{1-t}, \frac{x_3}{1-t}\right); \tag{4.1}$$

$$\sum_{k=0}^{\infty} \frac{(a)_k}{k!} t^k F_D^{(3)}(a+k, b_1, b_2, c; x_1, x_2, x_3) = (1-t)^{-a} F_D^{(3)}\left(a, b_1, b_2, b_3; \frac{x_1}{1-t}, \frac{x_2}{1-t}, \frac{x_3}{1-t}\right). \tag{4.2}$$

Again, specializing the parameters in (2.5) we obtain the infinite summation formulas for $H_A$:

$$\sum_{k=0}^{\infty} \frac{(a)_k (b_1)_k}{(c_1)_k k!} t^k H_A(a+k, b_1+k, b_2; c_1+k, c_2; x_1, x_2, x_3) = H_A(a, b_1, b_2; c_1, c_2; x_1 + t, x_2, x_3). \tag{4.3}$$

References

[1] G. Lauricella, Sulle funzioni ipergeometriche a più variabili, Rend. Circ. Mat. Palermo, 7 (1893), pp. 111–158.

[2] E.D. Rainville, Special Functions, Chelsea Publishing Company, New York, 1960.

[3] V. Sahai, A. Verma, Recursion formulas for Srivastava’s general triple hypergeometric functions, Asian Eur. J. Math., 9 (2016) 17, 1650063.

[4] V. Sahai, A. Verma, $n$th-order $q$-derivatives of Srivastava’s general triple $q$-hypergeometric series with respect to Parameters, Kyungpook. Math. J., 56 (2016) pp. 911-925.

[5] V. Sahai, A. Verma, Finite summation formulas of Srivastava’s general triple hypergeometric function, Asian Eur. J. Math., To Appear, https://doi.org/10.1142/S1793557119500207.

[6] L. J. Slater, Generalized hypergeometric functions, Cambridge: Cambridge University Press; 1966.

[7] H. M. Srivastava, Hypergeometric functions of three variables, Ganita, 15 (1964), pp. 97–108.

[8] H. M. Srivastava, Generalized Neumann expansions involving hypergeometric functions, Proc. Cambridge Philos. Soc. 63 (1967), pp. 425–429.

[9] H. M. Srivastava, P. W. Karlsson, Multiple Gaussian Hypergeometric Series, Halsted Press (Ellis Horwood Limited, Chichester), John Wiley and Sons, New York, Chichester, Brisbane and Toronto, 1985.

[10] H.M. Srivastava, H. L. Manocha, A Treatise on Generating Functions, Halsted Press (Ellis Horwood Limited, Chichester), John Wiley and Sons, New York, Chichester, Brisbane and Toronto, 1984.

[11] X. Wang, Infinite summation formulas of double hypergeometric functions, Integral Transforms Spec. Funct. 27(5) (2016), pp. 347–364.

[12] X. Wang, Y. Chen, Finite summation formulas of double hypergeometric functions, Integral Transforms Spec. Funct. 28(3) (2016), pp. 239–253.