CP-like Symmetry, Family Replication, Charge Quantization

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Bulletin Board: [hep-ph/9405276]

Abstract

We propose that the physics beyond the standard Weinberg-Salam model is such that matter and the CP conjugate anti-matter fields have the same set of charges with respect to the various force groups (upto ordering). We show that this CP-like symmetry leads to some understanding of family replication. We use anomaly constraints to infer the fermion spectra that lead to automatic electric charge quantization. We also find that the unification of coupling constants is possible at $M_U = 2 \times 10^{15}\text{GeV}$.
It is paradoxical that we have many good reasons to believe that many particles not yet discovered actually exist, but hardly any reasons to justify the existence of some of the particles already discovered! Despite lot of effort and progress in particle physics, there have been very few suggestions and ideas as to why nature has picked three families of quarks and leptons, instead of just one family. Neither grandunified theories nor left-right symmetries provide an answer to this question. No definite answer or prediction has come out of supersymmetry or superstring theories. Horizontal symmetries [1, 2, 3, 4] have been proposed between the three families, but the choice of the horizontal group has been largely arbitrary. They provide no explanation to the family triplication puzzle, since the horizontal group is guessed by looking at the families. So far there has been no independent way of guessing this group. There are very few ideas like the supersymmetric preon model [5] and the $SU(3)_L \times SU(3)_c \times U(1)_X$ model [6] which provide possible solutions. Since there is such a dearth of solutions, to what is one of the most intriguing puzzles of the standard model, it is useful to investigate ideas that might throw some light on this problem. In this letter we find that a CP-like discrete symmetry is useful to understand this problem.

In the standard Weinberg-Salam model [7], the $SU(2)_w \times SU(3)_c \times U(1)_Y$ assignments of the quarks and leptons and their CP-conjugate states are as follows:

$$Q_L(2, 3, 1/3) \leftrightarrow Q_R^c(2, 3, -1/3),$$

$$u_R(1, 3, 4/3) \leftrightarrow u_L^c(1, 3, -4/3), \quad d_R(1, 3, -2/3) \leftrightarrow d_L^c(1, 3, 2/3),$$

$$L_L(2, 1, -1) \leftrightarrow L_R^c(2, 1, 1), \quad e_R(1, 1, -2) \leftrightarrow e_L^c(1, 1, 2)$$

(1)

The interesting thing is that the $SU(2)$ quantum numbers for the left(right)-handed states and their corresponding right(left)-handed anti-states is exactly the same for all the above fermions. This is because 2 and 2 are equivalent representations [8] of $SU(2)$. However the $U(1)_Y$ and the $SU(3)_c$ quantum numbers [9] are not the same for the two CP conjugate states of the fermions. We propose to ”symmetrize the charges” of the left-handed particle and right-handed anti-particle states, by suggesting that there are two new forces, $U(1)_{newY}$ and $SU(3)_{newC}$. If any left-handed fermion is assigned to the complex representation $(R, y)$ of the usual $SU(3)_c \times U(1)_Y$ then its CP conjugate state, namely, the right-handed anti-fermion will be assigned to the same representation $(R, y)$ of $SU(3)_{newC} \times U(1)_{newY}$. Further, the gauge coupling constants $g_y = g_{newy}$ and $g_c = g_{newc}$ so that the action is invariant under the following operation (which we will call Strong Parity or S Parity):

$$S \text{ Parity} : \psi_L(x, t) \rightarrow CP(\psi_L) \equiv \psi_R^c(-x, t), \quad \psi_R(x, t) \rightarrow CP(\psi_R) \equiv \psi_L^c(-x, t),$$

$$SU(3)_c \leftrightarrow SU(3)_{newC}, \quad U(1)_Y \leftrightarrow U(1)_{newY}, \quad SU(2)_w \leftrightarrow SU(2)_w$$

(2)

where by $SU(3)_c \leftrightarrow SU(3)_{newC}$ we mean that $G^a_\mu(x, t) \leftrightarrow H^{a\mu}(-x, t)$, by $U(1)_Y \leftrightarrow U(1)_{newY}$ we mean $A_\mu(x, t) \leftrightarrow B^{a\mu}(-x, t)$, and by $SU(2)_w \leftrightarrow SU(2)_w$ we mean $W^a_\mu(x, t) \leftrightarrow W^{a\mu}(-x, t)$. 

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$G^a_\mu, H^a_\mu, A_\mu, B_\mu, \text{ and } W^a_\mu$ are the gauge bosons of $SU(3)_c, SU(3)_{newC}, U(1)_Y, U(1)_{newY}$, and $SU(2)_w$ respectively [14]. Thus S-Parity is the same as the CP operation on fermions, and it is parity followed by an exchange operation on bosons. As a result a fermion and its CP conjugate state have the same charges up to exchange operation between gauge force groups. Thus we propose that the standard model be extended to the gauge group $G_{SP} : SU(2)_w \times SU(3)_c \times SU(3)_{newC} \times U(1)_Y \times U(1)_{newY} \times SParity$. Now, for example, the assignment of the quark doublet to this group is

$$Q_L(2; 3, \bar{3}; 1/3, -1/3) \overset{CP}{\leftrightarrow} Q_R(2; \bar{3}, 3; -1/3, 1/3) \quad (3)$$

which is more symmetric since the set of quantum numbers for the above CP conjugate states is the same up to ordering.

The most interesting consequence of S Parity is that the number of quarks increase by a factor of 3 (dimensionality of $Q_L$ in equation (3) is thrice the dimensionality in equation (1)). We can identify this factor of 3, with the three families if we identify $SU(3)_{newC}$ with a horizontal symmetry $SU(3)_H$. Thus S-Parity leads to an understanding of the triplication of quark families. (In the rest of the paper we will use $SU(3)_H$ instead of $SU(3)_{newC}$). The idea of horizontal symmetries is not new, and has been considered by several authors [1, 2, 3, 4]. However the choice of the horizontal gauge group has been largely arbitrary and has ranged from $U(1)_H$ to $SU(3)_H$. Also the assignment of quarks and leptons to these groups, and the question of whether the horizontal symmetry is vector-like or not, has been done differently by different authors [1, 2, 3, 4]. The nice thing about S Parity is that it not only implies that the horizontal group is $SU(3)_H$ but also implies its vector nature and that it acts only on the quarks and not on the leptons. Thus it is very constraining.

Before we examine the lepton sector, a comment is in order regarding $U(1)_{newY}$. It turns out that symmetrization of the $U(1)$ force can be done without changing any physics. The reason is that since $U(1)_Y$ and $U(1)_{newY}$ are abelian forces, we can consider linear combinations of them, namely, $\frac{Y + Y_{new}}{\sqrt{2}}$ and $\frac{Y - Y_{new}}{\sqrt{2}}$. Owing to the fact that for all fermions the charges $y + y_{new} = 0$ (due to S Parity), this degree of freedom completely decouples from the fermion sector. However, depending on the assignment of $y$ and $y_{new}$ to the Higgs particles in the theory, the new force may or may not decouple entirely. In the rest of this paper we will choose the higgs assignment of $Y_{new}$ such that the combination $y + y_{new} = 0$ for all particles. With this choice, there is only one non-trivial $U(1)$ in the theory which is the usual hypercharge for the standard model. A posteriori, this is nice since it means that the only force that really violates S-parity in the Weinberg-Salam model is $SU(3)$. This is an additional motivation to symmetrize it as we have done.

Anomaly cancellation [11] has been used as an important constraint [12, 13, 14] in the one generation standard model to study, for example, electric charge quantization. Since we have extended the gauge group of the standard model, it would be interesting to keep the
fermion spectrum as an unknown and see how far anomaly cancellation determines it. To begin let us make the following two assumptions.

Assumption A: $SU(3)_c$ and the combination $Q_{em} = I_3 + \frac{Y}{2}$ are vector like and $SU(2)_w$ is chiral and acts on left handed particles.

Assumption B: The fermion content of the theory is such that anomaly constraints alone (together with assumption A) imply electric charge quantization.

Now S-Parity and assumption A imply that $SU(3)_H$ is also vector like. The minimal non-trivial assignment of all the gauge fields of $SU(2)_w \times SU(3)_c \times SU(3)_H \times U(1)_Y$ consistent with assumption A is

$$Q_L = (2; 3, \bar{3}; Y_Q)$$
$$U_R = (1; 3, \bar{3}; Y_Q + 1), D_R = (1; 3, \bar{3}, Y_Q - 1)$$

where we will leave $Y_Q$ arbitrary for now. We note that at this stage there are nine $SU(2)$ doublets and hence the Witten anomaly constraint [15] is not satisfied. Therefore we must introduce an odd number of $SU(2)$ doublets. The assumption of vector-like $Q_{em}$ and chiral $SU(2)$ implies that there must also be $SU(2)$ singlets. Thus the most general assignment of doublets and singlets, consistent with assumption A is

$$L^i_L = (2; 1, 1; Y^i_L), e^i_R = (1; 1, 1; Y^i_L - 1),$$
$$\nu^i_R = (1; 1, 1; Y^i_L + 1), \quad i \leq n'$$
$$L^i_L = (2; 1, 1; 1), \nu^i_R \text{ absent}$$
$$e^i_R = (1; 1, 1; 2), \quad n' < i \leq n''$$
$$L^i_L = (2; 1, 1; -1), \nu^i_R = (1; 1, 1; -2), \quad \nu^i_R \text{ absent}$$
$$n'' < i \leq n \quad (4)$$

where $Y^i_L$ is the hypercharge of the $i^{th}$ doublet and $n$ is odd.

Now the $Tr \, SU(2)_w^2 U(1)$ anomaly cancellation implies

$$9Y_Q + \sum_{i=1}^{n'} Y^i_L + \sum_{i=n'+1}^{n''} 1 + \sum_{i=n''+1}^{n} (-1) = 0 \quad (5)$$

It is easy to check that all other triangle anomalies, including the mixed gravity anomaly, are automatically satisfied. Now assumption B implies that $n' = 0$, and therfore

$$Y_Q = \frac{n - 2n''}{9}, \quad n'' \leq n \quad (6)$$

We verify that $Y_Q$, (and hence $Q_{em}$), is quantized since $n$ and $n''$ are integers. Physically, $n - 2n''$ is the number of chiral lepton families! The values of $n''$ and $n$ that give the observed $Y_Q = 1/3$ are $n = 2n'' + 3$. Substituting this in equation (4), it is easy to check that we
always have 3 chiral lepton families (with no right-handed neutrino) just as in the standard model, with a possibility of \( n'' \) vector-like lepton families \( (n'' \geq 0) \). Table 1 displays our results for the fermion content of the theory.

It is interesting that while we can have vector-like lepton families, additional quark families are not consistent with Assumptions A and B. Basically what happens is that any additional quark (or colored) family \([16]\) adds to the left-hand side of equation (5) an unknown hypercharge associated with that family. All other anomaly constraints will be satisfied due to assumption A, and hence will not constrain the value of this hypercharge. We will therefore lose automatic charge quantization and violate assumption B. Thus absence of any further colored fermions (other than octets) is a prediction of Assumptions A and B. On the other hand, a discovery of a fourth quark family would, according to the ideas of S-Parity, imply a corresponding increase in the dimensionality of the color group - from \( SU(3) \) to \( SU(N > 3) \). This would be a signal to a partial or grand unification of \( SU(3) \) with other forces.

Another observation worth making is that assumptions A and B imply that there is no vector-like anomaly free \( U(1) \) that commutes with all generators of \( SU(2) \) and \( SU(3) \). In fact if we demand that there be no such vector-like \( U(1) \), then assumption A itself will imply charge quantization. For theories with anomaly free vector-like \( U(1) \) (for example \( U(1)_{B-L} \)), a majorana mass term for the neutrino can restore charge quantization as shown in reference \([13]\).

We will now move on to the Higgs sector of the theory \([1, 3, 4]\). We choose the following representations of \( SU(2) \times SU(3)_c \times SU(3)_H \times U(1)_Y \times U(1)_{newY} \) based on the criterion of minimality.

\[
J_1, J_2 = (1; 1, 3; 0, 0) \quad \text{and} \quad K_1, K_2 = (1; 3, 1; 0, 0); \quad \phi = (2; 1, 1; 1, -1); \\
B_1, B_2 = (2; 1, 8; 1, -1) \quad \text{and} \quad C_1, C_2 = (2; 8, 1; -1, 1) \tag{7}
\]

Since S-parity is parity followed by an exchange operator on bosons, we will demand that under S-parity, \( J \leftrightarrow K \), \( B \leftrightarrow C \) and \( \phi \rightarrow \bar{\phi} \) in addition to equation (2). The absence of flavour changing neutral currents \([2, 3, 17]\) suggest that we break \( SU(3)_H \) at a high scale. It requires two \( SU(3)_H \) triplets to do this \([18]\), and so we assume that \( J_1 \) and \( J_2 \) pick up vacuum expectation values at a scale \( M_H \). Note that S-Parity breaks at this scale \([19]\). \( B_1 \) and \( B_2 \) are required to give the quarks masses and mixing angles \([4]\). Several authors have tried to understand fermion masses and mixing angles using horizontal gauge bosons \([1, 2, 3, 4]\). In the most predictive of these attempts (for example, see reference \([3]\)) only one \( B \)-type Higgs is introduced. Most of the masses and the mixing angles have to be generated through radiative corrections. This requires a low scale for horizontal symmetry breaking. Unfortunately these efforts have either proved inconsistent with constraints on
flavour changing neutral currents or do not reproduce quark masses and mixing angles. However if we assume that the horizontal symmetry breaking scale is sufficiently high, then we will require both $B_1$ and $B_2$. Now, one combination of the vacuum expectation values of $B_1$ and $B_2$ can be used to give the up-sector masses, and another linear combination can be used to give masses to the down sector. Their relative orientations can lead to the CKM matrix [20]. There are enough variables in the potential that this will work. The Higgs $\phi$ is required to give the leptons their masses, and can also play a role in the quark mass matrix. It is also worth noting that we can choose to have explicit CP violating terms since S Parity does not necessarily imply CP. We will take up a detailed analysis of the mass matrices in a future study. We also refer the reader to previous work [1, 3, 4] on the quark mass matrices with $SU(3)_H$.

Owing to the fact that the Higgs content of the theory gets enriched by S-parity (see equation (7)), there is the exciting possibility that the unification of all the standard model coupling constants is possible using $G_{SP}$ as an intermediate symmetry. We recall that the precision LEP measurements have proven [21] that there must be new physics, beyond the standard model, if the coupling constants are to unify. To be predictive, we assume that all coupling constants unify at a scale $M_U$ and that there is only one intermediate scale $M_H$ where S-Parity and $SU(3)_H$ break spontaneously. Only the standard model Higgs contributes to the running of coupling constants between $M_Z$ and $M_H$. Beyond $M_H$ all the Higgs in equation (7) contribute, since the effective theory at this scale is given by the group $G_{SP}$. All the quark and lepton families contribute from $M_Z$ to $M_U$, and the gauge bosons contribute from the appropriate scales onwards. We use the usual one-loop renormalization group equation [21] and our result is shown in Figure 1. Inputting the LEP data, we find that $M_H = 10^7 GeV$, $\alpha^{-1}_H(M_H) = 22$, $M_U = 2 \times 10^{15} GeV$ and $\alpha^{-1}_U = 30$. This is very interesting since the above values are consistent both with current bounds on proton life-time [21] and flavour changing neutral currents [3, 4, 17]. If we have more than one intermediate scale, or if we change the Higgs content a bit (for example, make the B-type Higgs lighter and use color sextets to break $SU(3)_H$ instead of triplets), then it is still possible to achieve the unification, consistent with experimental bounds, though the scales of $M_H$ and $M_U$ may change. The crucial point is that due to S-Parity we have the C-type and K-type Higgs fields. The contribution of these fields to the running of the coupling constants is necessary to achieve unification consistent with experiments.

In this letter we have suggested that a Strong Parity symmetry between matter and anti-matter fields may be at the heart of some of the puzzles of the standard model.

It is a pleasure to thank R.N. Mohapatra, J.C. Pati, K.S. Babu, L. Rana, S. Kodiyalam and B. Nath for comments and discussions. This work was supported in part by NSF Grant 9119745.
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Table 1: The fermion content of the theory, as determined by automatic charge quantization condition.

| Fermions | $SU(2) \times SU(3)_c \times SU(3)_H \times U(1)_Y$ |
|----------|--------------------------------------------------|
| $Q_L$    | $(2,3,\bar{3},1/3) \equiv \begin{pmatrix} u & c & t \\ d & s & b \end{pmatrix}_L$ |
| $U_R$    | $(1,3,\bar{3},4/3) \equiv \begin{pmatrix} u & c & t \end{pmatrix}_R$ |
| $D_R$    | $(1,3,\bar{3},-2/3) \equiv \begin{pmatrix} d & s & b \end{pmatrix}_R$ |

$i = 1, 2, 3$

- Three lepton families
- $L^i_L$  $(2,1,1,-1) \equiv \begin{pmatrix} \nu^i \\ e^i \end{pmatrix}_L$
- $e^i_R$  $(1,1,1,-2) \equiv e^i_R$

$j \geq 0$

- Vector-like leptons may exist
- $L^j_{vec}$  $(2,1,1,-1) \equiv \begin{pmatrix} \nu^j \\ e^j \end{pmatrix}_{L,R}$
- $e^j_{vec}$  $(1,1,1,-2) \equiv e^j_{L,R}$

**Octets**

Real $SU(3)$ representations may exist.