Effective phantom dark energy in scalar-tensor gravity

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We revisit the problem of phantom behaviour of effective dark energy in scalar-tensor gravity. The main focus is on the properties of the functions defining the model. We find that models with the present phantom behavior can be made consistent with all constraints, but one of these functions must have rather contrived shape, and the initial data must be strongly fine-tuned. Also, the phantom stage must have begun fairly recently, at \( z \lesssim 1 \). All this disfavors the effective phantom behaviour in the scalar-tensor gravity.

I. INTRODUCTION

Most models of dark energy in the present Universe predict that its effective equation of state satisfies the null energy condition (NEC) \( w_{\text{eff}} = p_{\text{DE}}/\rho_{\text{DE}} \geq -1 \), where \( \rho_{\text{DE}} \) and \( p_{\text{DE}} \) are the effective dark energy density and pressure, respectively. However, the observations do not rule out that dark energy is phantom, i.e., it violates NEC. As an example, the 7-year WMAP+BAO+SN data \(^1\) give the following bound on the equation of state with time-dependent \( w_{\text{eff}} \) at \( z = 0 \):

\[
w_{\text{eff},0} = -0.93 \pm 0.13 \ (68\%CL),
\]

which is not entirely inconsistent with \( w_{\text{eff},0} < -1 \). Even though phantom dark energy can be accomodated within General Relativity \(^2\)\(^-\)\(^5\), it is legitimate to ask whether effective phantom behavior can be obtained in modified gravity theories, such as \( f(R) \) or scalar-tensor gravity \(^6\)\(^-\)\(^11\). This question can be addressed, in particular, within

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a popular approach \cite{6,12,13} employing the reconstruction of model parameters from the redshift expansion of observables, combined with the experimental constraints on non-GR gravity. The main conclusion is that with current observational data, the reconstructed DE behaves more or less like the cosmological constant, but it is still possible to have phantom stage today.

In this paper we follow a somewhat different route and ask what sort of the scalar-tensor Lagrangian can lead to effective phantom DE at the present epoch without violating constraints from Solar system and local gravity experiments, time-(in)dependence of the gravity constant, etc. We also ask whether fine-tuning of initial data is necessary and how long the duration of the phantom stage can be in the past.

Our results are somewhat disappointing. We find that models with the present phantom DE can be made consistent with all constraints, but one of the functions entering the scalar-tensor Lagrangian must have rather specific shape, and the initial data must be strongly fine-tuned. Also, the phantom stage must have begun fairly recently, at \( z \lesssim 1 \). Before that the scalar field was undistinguishable from quintessence. All this disfavors the effective phantom behaviour in the scalar-tensor gravity. In fact, as we point out towards the end of this paper, some of the unpleasant properties we discuss must be present in scalar-tensor models for effective dark energy irrespectively of whether it is phantom or not.

This paper is organized as follows. In Section II we present the equations governing the homogeneous cosmological evolution in the scalar-tensor gravity. We recall in Section III the experimental constraints on the non-GR gravity, that place bounds on the parameters of the theory. In Section IV we define the expansion coefficients of the functions entering the Lagrangian and reformulate the bounds of Section III in terms of these coefficients. In Section V we put together all consistency requirements for the effective phantom dark energy today and arrive at qualitative understanding of the properties of the functions defining the theory. Also, the maximum redshift at which the phantom phase could begin is estimated. Section VI contains a numerical example. We conclude in Section VII.
II. HOMOGENEOUS AND ISOTROPIC EVOLUTION

By definition, the effective dark energy density and pressure $\rho_{\text{eff}}$ and $p_{\text{eff}} = w_{\text{eff}} \rho_{\text{eff}}$ are the quantities entering the GR-looking evolution equations for the homogeneous and isotropic Universe,

$$3H^2 = \rho + \rho_{\text{eff}}$$  \hspace{1cm} (1)

$$-2\dot{H} = \rho + \rho_{\text{eff}} (1 + w_{\text{eff}})$$  \hspace{1cm} (2)

where $\rho$ and $p = 0$ are matter energy density and pressure, and we set $8\pi G_N = 1$. Using (1) and (2), one writes

$$w_{\text{eff}} = -\frac{1}{1 - \Omega_m} \left( 1 + \frac{2}{3} \frac{\dot{H}}{H^2} \right),$$\hspace{1cm} (3)

where $\Omega_m = \rho/3H^2$. We are going to make use of this relation in the context of the scalar-tensor gravity. The action of this theory is

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left( F(\Phi) R - Z(\Phi) g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - 2U(\Phi) \right) + S_m(\psi, g_{\mu\nu}),$$\hspace{1cm} (4)

(mostly positive signature), where the action for the usual matter $S_m$ does not depend on $\Phi$. One can always redefine the field to have a convenient form of either $F(\Phi)$ or $Z(\Phi)$. We will use the general form of $F(\Phi)$ and set

$$Z(\Phi) = 1.$$

From the action (4) one obtains the gravitational equations,

$$F(\Phi) \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) = T_{\mu\nu} + \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} g_{\mu\nu} \left( \partial \Phi \right)^2 + \nabla_\mu \nabla_\nu F(\Phi) - g_{\mu\nu} \Box F(\Phi) - g_{\mu\nu} U(\Phi)$$\hspace{1cm} (5)

and equation of motion for the field $\Phi$,

$$\Box \Phi = -\frac{1}{2} \frac{dF}{d\Phi} R + \frac{dU}{d\Phi}.$$\hspace{1cm} (6)

Let us specify to the homogeneous, isotropic and spatially flat Universe with metric

$$ds^2 = -dt^2 + a^2(t)dx^i dx^i.$$
Since matter does not interact with $\Phi$, the scale factor $a$ has the same meaning as in GR. Using Eqs. (5) and (6) one gets the following set of equations:

$$
3FH^2 = \rho + \frac{1}{2} \dot{\Phi}^2 - 3H\dot{F} + U 
$$

$$
-2F\dot{H} = \rho + \dot{\Phi}^2 + \ddot{F} - H\ddot{F} 
$$

$$
\ddot{\Phi} + 3H\dot{\Phi} = 3(\dot{H} + 2H^2)\frac{dF}{d\Phi} - \frac{dU}{d\Phi}. 
$$

The equation for the matter density has the usual form,

$$
\dot{\rho} + 3H\rho = 0. 
$$

It is convenient for further analysis to switch from the evolution in time to the evolution in redshift. This can be done by using the relation

$$
\frac{d}{dt} = -H(1+z)\frac{d}{dz}. 
$$

In this way one obtains from (7), (8) and (9) the evolution equations in terms of redshift:

$$
3FH^2 = \rho + H^2(1+z)^2\frac{\Phi'^2}{2} + 3H^2(1+z)F' + U 
$$

$$
2FH' = \rho + H^2(1+z)^2\Phi'^2 + H^2(1+z)^2F'' + 
$$

$$
+ [(1+z)^2HH' + 2H^2(1+z)]F' 
$$

$$
H^2(1+z)^2\Phi'' + [HH'(1+z)^2 - 2H^2(1+z)]\Phi' = 3[2H^2 - HH'(1+z)]\frac{F'}{\Phi'} - \frac{U'}{\Phi'}, 
$$

where prime denotes $d/dz$.

### III. CONSTRAINTS

The properties of functions defining the theory are strongly constrained by local and Solar system experiments. This is a major problem for the effective NEC-violating behavior in the scalar-tensor gravity. One important parameter is the Brans–Dicke “constant” $W_{BD}(\Phi)$. It is straightforward to obtain the expression for $W_{BD}$ in our parametrization by redefining the scalar field. One finds

$$
W_{BD} = \frac{F}{(\frac{dF}{d\Phi})^2}. 
$$
The lower bound on the present value of $W_{BD}$ is obtained from the Kassini experiment [14–16]. It reads (the subscript 0 denotes the quantities at the present epoch)

$$W_{BD,0} > 4 \cdot 10^4.$$  \hfill (14)

A bound of another sort follows from the experiments on the time dependence of the gravity constant. In our case the local gravity constant $G_{\text{loc}}$ is given by

$$8\pi G_{\text{loc}} = \frac{1}{F} \left( \frac{2F + 4(dF/d\Phi)^2}{2F + 3(dF/d\Phi)^2} \right) = \frac{1}{F} \frac{2W_{BD} + 4}{2W_{BD} + 3}.$$  \hfill (15)

The experimental constraint on the time evolution of $G_{\text{loc}}$ can be found in Refs. [17, 18]. For $z = 0$ it reads

$$\left( \frac{\dot{G}_{\text{loc}}}{HG_{\text{loc}}} \right)_0 < 0.5 \cdot 10^{-2}.$$  \hfill (16)

We also know that the gravitational constant relevant for cosmology should not change significantly since Big Bang Nucleosynthesis [19],

$$\frac{\Delta G_{\text{cosm}}}{G_{\text{cosm}}} \lesssim 0.1.$$  \hfill (17)

It is the latter constraint that plays a significant role in our analysis, see Section V.

**IV. EXPANDING IN REDSHIFT AND $\Phi$**

A convenient way to analyze the evolution at small redshifts is to expand all functions in the Taylor series in redshift $z$. On the other hand, we are mainly interested in the dependence on $\Phi$, so we will use the mixed expansion. At $z = 0$, without loss of generality we choose

$$\Phi_0 = 0$$

and by definition of the Newton gravity constant we have

$$F_0 = 1.$$
Here is our definition of the expansion coefficients:

\[ F(z) = 1 + F_1 z + \frac{1}{2} F_2 z^2 + \frac{1}{6} F_3 z^3 \ldots \quad (18) \]

\[ U(z)/3H_0^2 = \Omega_{U,0} + U_1 z \quad (19) \]

\[ H^2(z)/H_0^2 = 1 + h_1 z + \frac{1}{2} h_2 z^2 + \ldots \quad (20) \]

\[ \Phi'(z) = \Phi'_0 z + \frac{1}{2} \Phi''_0 z^2 \quad (21) \]

\[ \rho(z)/3H_0^2 = \Omega_{m,0}(1 + z)^3. \quad (22) \]

Without loss of generality we take \( \Phi'_0 > 0 \). From Eq. (10) it is straightforward to obtain the relation between the derivatives at the present time,

\[ \Phi'^2_0 = 6 (1 - \Omega_{U,0} - \Omega_{m,0} - F_1). \quad (23) \]

We will also need \( h_1 \) and \( h_2 \) to obtain the expression for \( w_{eff} \). From Eqs. (10), (11) we get

\[ h_1 = \frac{1}{1 - \frac{F_1}{2}} (6 - 3\Omega_{m,0} - 6\Omega_{U,0} - 4F_1 + F_2), \quad (24) \]

\[ h_2 = \frac{3}{(1 - \frac{F_1}{2})^2} [F_1 \left( \frac{5}{2} F_1 - \frac{3}{2} F_2 - \frac{F_3}{12} + 4\Omega_{U,0} + \frac{U_1}{2} + \frac{11}{4} \Omega_{m,0} - 7 \right) \]
\[ + F_1^2 - \frac{3F_2^2}{2} \Omega_{U,0} - \frac{3F_2}{4} \Omega_{m,0} + 2F_2 + \frac{F_3}{6} - 5\Omega_{U,0} - u_1 - 4\Omega_{m,0} + 5]. \quad (25) \]

Our main purpose is to understand the behavior of \( F \) and \( U \) as functions of the scalar field, so we expand them in \( \Phi \):

\[ F = 1 + f_1 \Phi + \frac{1}{2} f_2 \Phi^2 + \frac{1}{3} f_3 \Phi^3, \quad (26) \]

\[ U = u_0 + u_1 \Phi + \frac{1}{2} u_2 \Phi^2. \quad (27) \]

The relationship between the expansion coefficients entering (18) and (26) is

\[ F_1 = f_1 \Phi'_0 \quad (28) \]

\[ F_2 = f_1 \Phi''_0 + f_2 \Phi'^2_0 \quad (29) \]

\[ F_3 = f_1 \Phi'''_0 + 2f_2 \Phi''_0 \Phi'_0 + f_3 \Phi^3. \quad (30) \]
We now recall the constraint on $W_{BD,0}$, Eq. (14), and make use of Eq. (13). With our normalization $F_0 = 1$, we get very strong upper bound on $f_1$:

$$|f_1| < 0.5 \cdot 10^{-2}. \quad (31)$$

This means that the field $\Phi$ is presently near the extremum of the function $F(\Phi)$. Such a conclusion appears inevitable in modified gravity, see, e.g., Ref. [7].

It follows from Eq. (23) that $\Phi_0^2 \lesssim 1$, so Eq. (28) implies that $F_1$ is also small,

$$|F_1| \lesssim 10^{-2}. \quad (32)$$

This suggests that we can neglect terms with the first derivative of $F$ in the analysis of the present epoch. We note in passing that a small value of $F'(z = 0)$ could have been anticipated, since the GR tests are very precise, and only a slight deviation from GR can be tolerated today.

Using (23) and neglecting the term with $F_1$, we obtain for the present value of the field derivative with respect to redshift

$$\Phi_0'' = 6 (1 - \Omega_{m,0} - \Omega_{m,0}). \quad (33)$$

This simple relation will be instrumental in what follows.

**V. EFFECTIVE PHANTOM BEHAVIOR**

Now we use the expression (3) for $w_{eff,0}$ to find out which parametrs can be responsible for the effective phantom behavior. Making use of Eqs. (28), (29) and (32) we get

$$1 + w_{eff,0} = \frac{f_2(\Phi_0')^2 + 6(1 - \Omega_{m,0} - \Omega_{U,0}) + f_1\Phi_0''}{3(1 - \Omega_{m,0})}. \quad (34)$$

By extracting the second derivative of the field from Eq. (12), we find

$$\Phi_0'' = \left(2 - \frac{h_1}{2}\right) \Phi_0' - U_1.$$

So, there are essentially two parameters that could yield $1 + w_{eff,0} < 0$, namely, $f_2$ and $U_1$. We begin our discussion with the latter.
The possible contribution of the potential to the phantom effective equation of state comes from the third term in (34) (the second term in the numerator is positive in virtue of Eq. (33)) and is given by

$$\Delta U(1 + w_{\text{eff},0}) = -f_1 U_1 \frac{U_1}{3(1 - \Omega_{m,0})}. \quad (35)$$

It is strongly suppressed by small $f_1$, so this contribution can be sizeable only if the potential $U(\Phi)$ is very steep today. However, steep potential would lead to the rapid acceleration of the scalar field, so the phantom phase would be very short in the past. Furthermore, the fast evolution of the scalar field together with large $u_1 = dU/d\Phi(z = 0)$ would imply rapid change in time of the effective dark energy density. To elaborate on the latter point, let us consider the parameter

$$w_1 = \frac{dw_{\text{eff}}}{dz}(z = 0).$$

Observationally, $|w_1|$ is not large: the WMAP analysis [1] gives $-0.31 < w_1 < 1.12$. On the other hand, making use of Eq. (3) and neglecting the terms with $F_1$ we obtain

$$w_1 = w_{1,A} + w_{1,B}, \quad (36)$$

where $w_{1,A}$ and $w_{1,B}$ are:

$$w_{1,A} = \frac{1}{3(1 - \Omega_{m,0})} (F_3 - 6u_1), \quad (37)$$

$$w_{1,B} = \frac{1}{3(1 - \Omega_{m,0})^2} \left[ (1 + 5\Omega_{m,0}) (F_2 - 6\Omega_{U,0} - 3\Omega_{m,0} + 6) - 9\Omega_{m,0} \right] + \frac{1}{3(1 - \Omega_{m,0})} \left[ 12F_2 + \frac{3F_2^2}{2} + 9F_2\Omega_{U,0} - \frac{9}{2}F_2\Omega_{m,0} - 30\Omega_{U,0} \right. \right.$$  

$$+ 24(1 - \Omega_{m,0}) + 6 \left. \right] - \frac{1}{3(1 - \Omega_{m,0})^2} \left( -F_2 + 6\Omega_{U,0} + 3\Omega_{m,0} - 6 \right)^2. \quad (38)$$

The parameter $F_2$ cannot be very large, see below, so for large $u_1 = U_1/\Phi'$ the value of $w_1$ is controlled by $w_{1,A}$ term in the expression (36). To have sizeable contribution (35) and at the same time satisfy the observational constraint on $w_1$, one would need the cancellation between $F_3$ and $6u_1$, which in turn would require strong fine-tuning. Barring this possibility, we arrive at the conclusion that $|f_1 U_1| \ll 1$, so the contribution (35) is very small. From now on we neglect it.
The remaining terms in Eq. (34) can be simplified by using (33):

\[ 1 + w_{eff,0} = \frac{(1 + f_2)(\Phi'_0)^2}{3(1 - \Omega_{m,0})}. \] (39)

Thus, the phantom behaviour today is controlled entirely by \( f_2 = \frac{d^2 F/d\Phi^2(z = 0)}{G_{cosm}(\Phi)} \). To have \( w_{eff} < -1 \) at the present time, one requires that

\[ f_2 < -1. \]

Together with the bound (31), this implies that today the field \( \Phi \) must be close to a relatively sharp maximum of the function \( F(\Phi) \). Clearly, such a special state requires fine-tuning of both the function \( F(\Phi) \) and initial conditions in the theory.

![FIG. 1. The shape of \( F(\Phi) \). The present value of \( \Phi \) must be near the maximum.](image)

We continue the discussion of the shape of \( F(\Phi) \) and recall the constraints on the time-dependence of the gravity constant. Given the small value of \( F_1 \) and large value of \( W_{BD,0} \), the constraint (16) is not hard to satisfy; note that this is in contrast to Ref. [20]. Much less trivial is the fact that the “cosmological” gravity constant has not changed much since BBN. It is clear from (7) that \( G_{cosm}(\Phi) = F^{-1}(\Phi) \). Barring strong cancellations, the constraint (17) together with Eq. (10) imply that the variation of \( F(\Phi) \) has been small since BBN,

\[ \Delta F \lesssim 0.1 \] (40)
and that $F'$ is small at large $z$. Thus, the function $F(\Phi)$ must have the shape shown in Fig. 1.

Let us now estimate the range of redshifts in which the dynamics of $F$ is non-trivial, and the phantom effective equation of state can be realized. We do this by requiring that the value of $F$ does not change much during this period. Since $F_1$ is small, the second term in the expansion of $F$ in redshift is relevant, and the estimate for maximum redshift is found from

$$\frac{1}{2} F_2 z_{\text{max}}^2 \lesssim \Delta F,$$

where the bound on $\Delta F$ is given in Eq. (40). Making use of Eq. (29) we get

$$\frac{1}{2} (|f_2| \Phi_0')^2 z_{\text{max}}^2 \lesssim \Delta F.$$

Let us denote by $\epsilon$ the deviation of $w_{\text{eff}}$ from $-1$ today:

$$1 + w_{\text{eff},0} = -\epsilon.$$

Using (39) we get the estimate $(1 + f_2)(\Phi_0')^2 \approx 3\epsilon$, and hence from (41) we find

$$z_{\text{max}}^2 \lesssim \frac{2\Delta F |1 + f_2|}{3\epsilon |f_2|}.$$

For reasonably strong phantom behavior (i.e., not very small $\epsilon$), $z_{\text{max}}$ is fairly small; roughly speaking, $z_{\text{max}} \lesssim 1$. Note that similar result has been obtained within the reconstruction approach [7, 13, 21]. At larger redshifts, $F$ is frozen out, and the scalar field $\Phi$ reduces to quintessence.

**VI. NUMERICAL EXAMPLE**

Let us give a concrete example of a model with effective phantom behavior today and in the recent past. We note that the relatively large value of the parameter $\epsilon = -(1 + w_{\text{eff},0})$ is obtained for fairly large $\Phi_0'$, otherwise the maximum of $F$ must be very sharp (i.e., $f_2$ must be large), see Eq. (39). So, we take, somewhat arbitrarily, $\Phi_0' = 0.3$.

We choose $\Omega_{\text{m},0} = 0.25$, in rough agreement with observations, then Eq. (10) with $F_1 \ll 1$ gives $\Omega_{U,0} = 0.73$. Fairly strong phantom behavior, $\epsilon = 0.1$, is obtained with $f_2 = -1.75$. The estimate (42) then gives $z_{\text{max}} \sim 0.2$. To satisfy all these requirements, we choose $F(\Phi)$ as shown in Fig. 2.
The shape of the potential $U(\Phi)$ is not constrained particularly strongly; our choice is shown in Fig. 3.

With this choice, the effective equation of state depends on redshift as shown in Fig. 4. As expected, $w_{\text{eff}}$ rapidly tends to $-1$ as $z$ increases, and the field $\Phi$ becomes indistinguishable from quintessence at $z \gtrsim 0.2$. The field value does not change much: the change from redshift 0.2 to the present epoch is about $\Delta \Phi \approx 0.05$.

We have constructed a number of other examples satisfying the constraints of Section III; all of these examples have similar properties.
As we already pointed out, effective phantom dark energy requires both the special form of the function $F(\Phi)$ and fine-tuning of the initial value of the field $\Phi$. To see the latter property explicitly, let us take the same functions $F(\Phi)$ and $U(\Phi)$ as before and consider the evolution from redshift $z = 0.75$ to $z = 0$ for different initial values of the field. If we vary the initial condition for $\Phi$ within 15% interval around the central value yielding Fig. 4 (without varying the initial velocity $\dot{\Phi}$, for the sake of arument), the evolution changes considerably. In particular, the effective equation of state is as shown in Fig. 5.

FIG. 4. $w_{eff}$ as function of redshift.

FIG. 5. $w_{eff}$ for initial conditions for $\Phi$ deviating by $\pm 15\%$ from the central value.
Such a behaviour is not unexpected. The right choice of the initial value of the field ensures that the phantom phase begins in just right time, at some rather small redshift. For different initial values, the onset of the phantom behavior occurs at “wrong” redshifts, so one either has too large deviation of $w_{\text{eff},0}$ from $-1$, or no deviation at all.

VII. CONCLUSIONS

In this paper we revisited the question of the possibility of the present phantom phase in scalar-tensor gravity. We have seen that it is possible to obtain and control effective phantom behavior even in simple scalar-tensor models, but this requires a lot of fine-tuning. First, the large present value of the Brans–Dicke parameter is obtained only if the scalar field $\Phi$ is presently near the extremum of the function $F$ determining the gravity constant. This is consistent with observable phantom property only if this extremum is a sharp maximum. Second, the small variation of the gravity constant since BBN requires that $F$ flattens out at fairly low redshift. Finally, the whole picture is consistent with observations only for fine-tuned initial data.

We conclude by noting that some of the unpleasant properties discussed in this paper must be present in scalar-tensor models for effective dark energy irrespectively of whether it is phantom or not. This remark applies, in particular, to the contrived shape of the function $F$ and fine-tuning of initial conditions. Indeed, the fact that $dF/d\Phi$ must be small today does not rely on the assumption of the phantom behavior. Furthermore, most of the analysis in Section V goes through provided that $|d^2F/d\Phi^2|$ is large enough at the present epoch, while the case $|d^2F/d\Phi^2| \ll 1$ corresponds to quintessence rather than genuine scalar-tensor gravity. All this makes scalar-tensor theory rather unlikely candidate for explaining the accelerated expansion of the Universe.

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