Signature of the N=126 shell closure in dwell times of alpha-particle tunneling

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Abstract

Characteristic quantities such as the penetration and preformation probabilities, assault frequency and tunneling times in the tunneling description of alpha decay of heavy nuclei are explored to reveal their sensitivity to neutron numbers in the vicinity of the magic neutron number $N = 126$. Using realistic nuclear potentials, the sensitivity of these quantities to the parameters of the theoretical approach is also tested. An investigation of the region from $N = 116$ to $N = 132$ in Po nuclei reveals that the tunneling $\alpha$ particle spends the least amount of time with an $N = 126$ magic daughter nucleus. The shell closure at $N = 126$ seems to affect the behaviour of the dwell times of the tunneling alpha particles and this occurs through the influence of the $Q$-values involved.

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1. MAGIC NUMBERS AND ALPHA DECAY OF NUCLEI

One of the most interesting findings of the early years of nuclear physics was the discovery of the existence of magic numbers which found an explanation based on the shell model of the nucleus. Even if a great number of magic nuclei have been identified over the years, their production and detailed study involves technical challenges and the subject as such continues to be a topic of current interest. The standardly recognized magic numbers with neutron or proton numbers of 2, 8, 20, 28, 50, 82, 126, as such, originate from a spherical shell model but a deformed shell model also generates magic numbers with extra stability corresponding to the deformed structure. Indeed, based on empirical evidence, the authors in [1] while studying “superdeformed” nuclei identify \( Z = 30, 38 - 41, 46, 58, 59, 62 \) (and some others depending on the deformation) as magic numbers for nuclei with superdeformed shapes. In recent years, with the advent of radioactive beams, the experimental studies have been extended to the extremes of stability. These studies indicate that the shell structure established for nuclei near the \( \beta \)-stability line may change a lot for exotic nuclei. For instance, the neutron numbers of 8, 20 and 28 are not magic in \(^{12}\text{Be} \) \((N = 8)\), \(^{32}\text{Mg} \) \((N = 20)\) and \(^{42}\text{Si} \) \((N = 28)\) whereas new magic numbers such as \( N = 16, 24 \) emerge in \(^{24}\text{O} \) and \(^{54}\text{Ca} \) [2]. In [3], a new shell closure at \( N = 90 \) was found for neutron rich Sn isotopes [4].

There exists yet another topic in nuclear physics which enjoys this kind of continued interest and this is the alpha decay of radioactive nuclei. Indeed, alpha decay was the very first application of the most exotic phenomenon of quantum mechanics, namely, tunneling as shown by Gamow [5] and Condon and Gurney [6] in their pioneering works in 1928. We have come a long way since the discovery of magic numbers and alpha-tunneling but the interest in the two phenomena taken together seems to be ever increasing [7]. For example, in [8] it was found that even if empirical laws are usually sufficient to explain the alpha decay half-lives without a consideration of the alpha cluster preformation problem, close to the neutron shell closure of \( N = 126 \), one must take into account the shell model and other effects [8].

The present work is aimed at studying the behaviour of the characteristic quantities, appearing in the semiclassical approaches used to describe the alpha decay of heavy nuclei, as a function of the neutron number of the parent nuclei. The idea behind the investigation is to test if one or more of these quantities turn out to be good indicators of the existing
magic numbers. In case they do, one could consider them to be tools for identifying possible shell closures. In particular, the alpha decay of Polonium isotopes in the region around \( N = 126 \) is studied within a standard semiclassical approach involving the tunneling of a preformed alpha through the Coulomb barrier (for other approaches, see [9]). The tunneling probability, cluster preformation factor, assault frequency of the alpha at the barrier and the dwell time are calculated using a nuclear potential which is based on a double folding model with realistic nucleon-nucleon interactions [10]. The potential is fitted to scattering data and has also been tested in the \( \alpha \) decay of several nuclei [9, 11]. In the next section we discuss the theoretical approach used for evaluating the decay width and the different tunneling time concepts used. An interpretation of the results is given in section 3.

2. DECAY WIDTHS AND TUNNELING TIMES

The general formula for the lifetime of a nucleus decaying by \( \alpha \)-decay was obtained on the basis of a Gamow-state formalism [12] in the seventies. Though such formalisms [13] are surely better than a semiclassical JWKB approach in general, for the purpose of the present investigations, it suffices to use the decay widths evaluated within the JWKB approximation [14, 15].

2.1. Alpha-nucleus potential and the JWKB width

As shown in [11], different semiclassical approaches lead to one and the same expression for the decay width, given by,

\[
\Gamma(Q) = P_\alpha \left[ \frac{\hbar}{2\mu} \left[ \int_{r_1}^{r_2} \frac{dr}{k(r)} \right]^{-1} \right] P,
\]

where \( k(r) = \sqrt{2\mu(Q - V(r))} \), \( Q \) (the \( Q \)-value in tunneling) is the amount of energy released in the decay and \( P \) the tunneling probability in the JWKB approximation, i.e.,

\[
P = \exp \left[ -2 \int_{r_2}^{r_3} \kappa(r)dr \right],
\]

with, \( \kappa(r) = \sqrt{2\mu(V(r) - Q)} \). \( P_\alpha \) is the preformation probability of the \( \alpha \) in the parent nucleus and is often chosen as a free parameter to fit the theoretical half lives to the experimental ones, i.e. \( P_\alpha = t_{1/2}^{\text{theory}}/t_{1/2}^{\text{exp}} \) where \( t_{1/2}^{\text{theory}} = \hbar \ln 2/\Gamma \) (with \( \Gamma \) evaluated as in Eq.(1) but
assuming $P_\alpha = 1$). The total potential $V(r)$ is a sum of the centrifugal (CF), nuclear (N) and Coulomb (C) potentials [11]. The $\alpha$ is restricted within the classical turning points $r_1$, $r_2$ and $r_3$ defined by $V(r)$ and the Q values [11]. The factor in square brackets appearing before the penetration probability P in (1) arises due to the normalization of the bound state wave function in the region from $r_1$ to $r_2$. The $\alpha$ is considered to tunnel through the potential

$$V(r) = V_C(r) + \lambda V_N(r) + \frac{\hbar(l + 1/2)^2}{\mu r^2},$$  \hspace{1cm} (3)

where the strength $\lambda$ of the nuclear part $V_N$ is fixed by the Bohr Sommerfeld quantization condition [11]:

$$\int_{r_1}^{r_2} \sqrt{\frac{2\mu}{\hbar^2}} |V(r) - Q| \, dr = (n + 1/2) \pi.$$  \hspace{1cm} (4)

Here, $n = (G - l)/2$, is the number of nodes of the quasibound wave function of $\alpha$-nucleus relative motion and $r_1$ and $r_2$ which are solutions of $V(r) = Q$ are the classical turning points. $G$ is a parameter fitted to data which we will discuss in more detail below. $V_C$ in (3) is the Coulomb potential between the $\alpha$ and the daughter nucleus. The last term in $V(r)$ represents the Langer modified centrifugal barrier [16]. The modification from the standard $l(l + 1) \to (l + 1/2)^2$ is required to ensure the correct behaviour of the JWKB scattered radial wave function near the origin as well as the validity of the connection formulas used. This modification leads to a potential which “appears” to have a centrifugal part even for $l = 0$ as can be seen in the case of the $^{206}$Pb-$^4$He in Fig. 1. Other works on alpha decay using a similar approach can be found in [17, 18].

The folded nuclear part of the potential is given by a six dimensional integral

$$V_N(r) = \int dr_1 \, d\mathbf{r}_2 \, \rho_\alpha(\mathbf{r}_1) \rho_\delta(\mathbf{r}_2) \, v(\mathbf{r}_{12} = \mathbf{r}_2 - \mathbf{r}_1, E)$$  \hspace{1cm} (5)

where $\rho_\alpha$ and $\rho_\delta$ are the densities of the alpha and the daughter nucleus, respectively. $v(\mathbf{r}_{12}, E)$ is the nucleon-nucleon interaction. $|\mathbf{r}_{12}|$ is the distance between a nucleon in the alpha and a nucleon in the daughter nucleus. We follow reference [10] which found that $v(\mathbf{r}_{12}, E)$ can be written as

$$v(\mathbf{r}_{12}, E) = 7999 \frac{\exp(-4|\mathbf{r}_{12}|)}{4|\mathbf{r}_{12}|} - 2134 \frac{\exp(-2.5|\mathbf{r}_{12}|)}{2.5|\mathbf{r}_{12}|} + J_{00} \delta(\mathbf{r}_{12})$$  \hspace{1cm} (6)

$$J_{00} = -276 \left(1 - 0.005 E_\alpha/A_\alpha \right).$$

The respective densities entering equation (5) take the standard form [10]

$$\rho_\alpha(r) = 0.4229 \exp(-0.7024 r^2)$$  \hspace{1cm} (7)
FIG. 1: Alpha-nucleus potential $V(r)$ as given in [3] for the $^{206}$Pb-$^4$He system resulting from the decay of $^{210}$Po. The inset displays the same potential with the classical turning points determined by the $Q$-value in the decay.

and

$$\rho_d(r) = \frac{\rho_0}{1 + e^{p/(c_a)}}. \quad (8)$$

Here $\rho_0$ is obtained by normalizing $\rho_d(r)$ to the number of nucleons $A_d$ and the constants are given as $c = 1.07A_d^{1/3}\text{fm}$ and $a = 0.54\text{fm}$. Finally, denoting by $\rho_\alpha$ and $\rho_d$ the charge densities of the $\alpha$ and daughter nucleus, the double folded the Coulomb potential is,

$$V_C(r) = \int dr_1 dr_2 \rho_\alpha^2(r_1) \rho_d^2(r_2) \frac{e^2}{|r_1r_2|}. \quad (9)$$

The charge distributions are of similar form as the matter distributions above except for the normalization. The six dimensional integrals to evaluate the potentials can be made simpler by expressing the potential as an integral over the Fourier transforms of the densities and $v$. As a result one can write

$$V_N(r) = \frac{1}{2\pi^2} \int k^2 dk \frac{\sin (kr)}{kr} v(k) \rho_\alpha(k) \rho_d(k) \quad (10)$$

for nuclei with $l = 0$ as considered in this work. $V_C(r)$ can be evaluated using similar methods. The details of the procedure can be found in [10].
2.2. Tunneling times and the assault frequency in alpha decay

The concept of quantum tunneling times in connection with the half lives of radioactive nuclei was discussed in detail in [9]. Here we highlight the points relevant for the present work. Within a semiclassical picture one can define a “period” $T$ as twice the time required for the $\alpha$ particle to traverse the region before the barrier, i.e. the distance between the turning points $r_1$ and $r_2$. The assault frequency is then the inverse of this quantity. Expressing the time interval, $\Delta t$, for the particle traversing a distance, $\Delta r$ as,

$$\Delta t = \frac{\Delta r}{v(r)} = \frac{\mu \Delta r}{\hbar k(r)} ,$$

(11)

the assault frequency $\nu$ can be written as the inverse of the time required to traverse the distance back and forth between the turning points $r_1$ and $r_2$ as [15],

$$\nu = T^{-1} = \frac{\hbar}{2 \mu} \left[ \int_{r_1}^{r_2} \frac{dr}{\sqrt{\frac{2\mu}{\hbar^2} (|V(r) - E|)}} \right]^{-1} .$$

(12)

This expression is however nothing but the “normalization factor” which appeared in the square brackets in Eq. (11). One can then rewrite Eq. (11) as

$$\Gamma(Q) = P,_{\alpha} \nu P,$$

(13)

which is the often found form in literature.

Interestingly, this quantity is directly proportional to the time spent by the $\alpha$-particle residing in the potential well. This time is also known as the dwell time $\tau_D$ [19] and is given by the number of particles in a given region of space divided by the incident flux. Thus, $\tau_D = \int_{x_1}^{x_2} |\Psi(x)|^2 dx/j$ for a particle confined to the interval, $(x_1, x_2)$. The dwell time as such is considered to be a measure of the average time spent by a particle in a given region of space. The concept was first introduced by Smith [20] in the context of quantum collisions and to derive a lifetime matrix for multichannel resonances. In the one-dimensional case, it was first introduced by Büttiker [19]. One can further define a “transmission dwell time”, $\tau_{D,T}$, corresponding to the time spent by those particles in a region (say, $(x_1, x_2)$, before the barrier) that managed to tunnel and get transmitted. As discussed in [9],

$$\tau_{D,T} = \frac{\int_{x_1}^{x_2} |\Psi|^2 dx}{j_T} ,$$

(14)

where, $j_T = \hbar k_0 |T|^2 / \mu$ with $k_0 = \sqrt{2\mu E / \hbar}$ corresponding to the free particle energy $E$ and $|T|^2$ the transmission coefficient (which is the same as the penetration probability $P$ of the
present work). In an investigation of the alpha decay half lives of heavy and super heavy nuclei, it was shown in \[9\], that the JWKB decay width discussed in the previous sub-section is given by the inverse of the transmission dwell time in the region in front of the barrier:

$$\Gamma = P_\alpha [\tau_{D,T}]^{-1},$$  \hspace{1cm} (15)

where $\tau_{D,T}$ is the transmission dwell time of the $\alpha$ in the region between $r_1$ and $r_2$.

The half life of the decaying nucleus is thus given by

$$\frac{\tau_{1/2}}{2} = \frac{\hbar \ln 2}{\Gamma} = \frac{\hbar \ln 2}{P_\alpha [\tau_{D,T}]^{-1}}$$  \hspace{1cm} (16)

with $[\tau_{D,T}]^{-1} = \nu P$. Finally, we must mention that it can also be shown \[9\] that $\tau_D = 2\tau_{\text{trav}}$ where $\tau_{\text{trav}}$ is the traversal time defined by Büttiker and Landauer \[21\]. Therefore by calculating the half-lives we make a direct connection to the quantum mechanical tunneling time concept. The behaviour of the alpha decay half lives displays a dependence on the magic numbers and hence through the present work we are attempting to relate the magic numbers to the quantum tunneling times in alpha decay.

Before moving on to the next section, a brief discussion regarding the status of tunneling times is in order. A recent review \[22\] discusses resonant tunneling, tunneling of composite particles and how the coupling to intrinsic and external degrees of freedom can affect the tunneling probabilities. There exist extensive reviews \[23–25\] on the subject and one often finds contradictory remarks regarding the physical interpretation of some of the times. Ref. \[24\] for example considers the problem of tunneling time ill posed but that of an average dwell time well defined. Some of the controversies have arisen due to the Hartman effect \[26\] (the saturation of the phase time with increasing width of the barrier) which leads to interpretations based on superluminal propagation. The transmission dwell time as defined in the present work however does not lead to any such controversies \[27\] but can rather be related to lifetimes of nuclei as shown in \[9\]. The fact that the dwell time and not the phase time emerges as a useful concept was also shown in the context of eta-mesic nuclei in \[28\]. In fact, an extraction of the dwell times as done in \[29\] from the current-voltage characteristics in solid state tunnel junctions led to values very close to those measured from sophisticated experiments \[30\]. Given the fact that the dwell time seems to be emerging as the time concept with a physical meaning, it is timely to investigate in what other ways it manifests itself in tunneling processes. In this sense it is gratifying to find out that the signature of shell closure can also be found in this concept.
3. DISCUSSION AND INTERPRETATION

Within the model described in the previous sections, we calculate the various characteristic quantities in the tunneling of alpha and study their behaviour in the region of the neutron magic number $N = 126$ for Po isotopes.

3.1. Assault frequencies, $G$ value dependence and magic numbers

In Figure 2(d) we plot the assault frequency $\nu$ for Po isotopes as a function of the parent neutron numbers, using two different choices of the $G$ values appearing in the Bohr Sommerfeld condition on the potential. The lower curve represents the results using realistic potentials and the formulae mentioned above but with the values of $G_\prec$, $G_\succ$, as in [7] where a toy model was introduced in order to study the behaviour of the assault frequencies as a function of the neutron number. In spite of the different nuclear input and the different definition of $\nu$ we reproduce the minimum at $N = 126$ (note that the jump from $N = 126$ to 128 in $\nu_M$ of the authors in [7] is obvious since $\nu_M \propto G$ in their model and the authors choose to change $G$ from 20 to 24 for $N > 126$). The upper curve in Figure 2(d) here uses the $G$ value recommendations obtained from a fit [31] to half lives of several nuclei. In [31] the authors find that $G$ should be large (in the range 18 - 24) and that it should increase by two units while going from below the neutron magic number $N = 126$ to above it. Thus, if $G = G_\prec$ corresponds to $82 < N \leq 126$ and $G = G_\succ$ to $N > 126$, then $\Delta G = G_\succ - G_\prec$ should be always equal to 2. Hence we choose, $G = 22$ for $N \leq 126$, $G = 24$ for $N > 126$. With this choice the assault frequency with realistic potentials changes and the minimum occurs now around $N = 128 - 130$. With such a sensitivity at hand $\nu$ might not be the best indicator for a magic neutron number.

3.2. Cluster preformation probability

In Figure 2(c), we also show the preformation probability, $P_\alpha = \ln 2/(\nu P \tau^{exp})$, based on the ratio between the theoretical,

$$\tau^{theory} = \ln 2 \tau_{D,T} = \frac{\ln 2}{\nu P},$$  \hspace{1cm} (17)
and the experimental half lives. Recall here that $\tau_{D,T}$ is the transmission dwell time of the alpha in the region in front of the barrier. As in the case of $\nu$, we see that $P_\alpha$ is, in general, also sensitive to the choice of $G$. However, as long as we restrict ourselves to even $N$ the qualitative behaviour displaying a local minimum at $N = 126$ remains unchanged. We note that there is a clear model dependence in $P_\alpha$ as far as the magnitude is concerned: we differ from Refs \[7\] by one order of magnitude. The order of magnitude of $P_\alpha$ here is similar to that in \[8\].

3.3. Dwell times in the vicinity of the $N = 126$ shell closure

In view of the sensitivity mentioned above, it might be a good idea to look at the behaviour of other quantities related to the $\alpha$-decay and attempt a new interpretation. The curves for half lives in Figure 2(a) have a local maximum (minimum in the tunneling probability $P$) at $N = 125$ and minimum (maximum in $P$) at $N = 128$. To interpret this behaviour it is not enough to say that $N = 125$ is close enough to $N = 126$ and therefore the maximum is intimately related to the magic number. Indeed, one does not expect the magic nucleus to be the most stable and therefore one would also not expect a maximum in lifetime at the magic number. It is rather the steep slope starting at $N = 126$ to the next nucleus ($N = 127$) (passing over orders of magnitude) which is a clear indicator of a magic number since on account of the shell closure one would expect the next nucleus to be much more unstable.

Thus the minimum in half lives occurring at $N = 128$ has also to do with a magic number, but not of the parent nucleus. For $N = 128$ of the parent nucleus the daughter has the magic number $N = 126$. The theoretical half-lives in Fig. 2(a), which are related to the transmission dwell time of the tunneling alpha, display a similar behaviour.

From the example considered above, it seems that the shell closure at $N = 126$ influences the dwell times of the tunneling alphas. The answer as to why the shell closure should affect the dwell time is probably hidden in the dependence of these times on the $Q$-value in the decay. Just as the binding energy of the last neutron, electric quadrupole moments and excitation energies from the ground to the first excited state can be correlated with shell closures, the $Q$-values in the alpha decays can also be considered as the messengers of the information regarding shell closures (as is evident in Fig. 3 discussed below). Thus it
appears to us that an extension of the calculations in the present work but for other nuclei and in other regions of neutron numbers could possibly reveal similar effects of shell closures on the dwell times of the tunneling alpha particles.

What seems so convincing for the neutron magic number, namely the fact that the parent nucleus “prefers to decay into a magic daughter”, is not a general feature when we probe into the proton magic numbers. In order to explore this point further, in Fig. 3, we plot the experimental half lives of nuclei as a function of the neutron and proton numbers in the vicinity of $N = 84$, $N = 128$ (Fig 3(b)) and $Z = 84$ (Fig. 3(d)) which correspond to the parents of magic daughters. Though one observes dips in half lives at $N = 84$ and $N = 126$ no such structure is seen in the case of $Z = 84$. Even if a possible explanation could be that the Coulomb barrier height changes with changing $Z$, it is not obvious why a parent with $Z = 84$ would not decay rapidly to a magic daughter with $Z = 82$ (as seen in the left

FIG. 2: Tunneling variables as a function of the parent neutron number. In (a) we see the experimental half lives and the theoretical ones with $\tau_{\text{theory}} = \ln 2 \tau_{D,T} = \ln 2/\nu P$, (b) the penetration probability $P$ as in (2), (c) the preformation factor $P_\alpha$ and (d) the assault frequency $\nu$, for Po isotopes.
panel, Fig. 3(b), for magic $N = 82$). Although it is beyond the scope of the present work, we think that the different behaviour of neutron and proton magic numbers studied from the point of view of tunneling times calls for an explanation.

![Graphs showing Q values and experimental half lives as a function of neutron and proton numbers.](image)

FIG. 3: Q values and experimental half lives of nuclei as a function of the neutron and proton numbers of the parent nuclei.

4. SUMMARY

The alpha decay of heavy nuclei is commonly treated within a model where the decaying parent nucleus is made up of a cluster of an alpha or $^4$He nucleus and and the daughter in the decay. The decay is possible due to the tunneling of the alpha through the Coulomb barrier produced due to its interaction with the daughter. The present work studies the characteristic quantities in such a tunneling process and in particular their behaviour in the region around the neutron magic number $N = 126$. Calculations using realistic nuclear potentials are presented for isotopes of the Polonium nucleus in the region of neutron numbers $N = 116$ to $N = 132$. The findings of this work can be summarized as follows:
(i) The amount of time spent by an alpha in front of the barrier before tunneling (the transmission dwell time discussed in section 2, $\tau_{\text{theory}}$ in Fig. 2) reaches a minimum at $N = 128$ of the parent nucleus in the region from $N = 116$ to $N = 132$ studied in this work. $N = 128$ of the parent however corresponds to $N = 126$ of the daughter, implying that the alpha spends the least amount of time with the magic daughter.

(ii) The frequency of assaults $\nu$ of the alpha at the barrier is found to be sensitive to the parameter $G$ used in the semiclassical JWKB approach. Though for a certain set of $G_\text{<}, G_\text{>} \ (\text{defined in section 3})$ the minimum in $\nu$ occurs at the magic number $N = 126$ of the parent, this can change with a small change in the values of $G_\text{<}, G_\text{>}$.  

(iii) The assault frequency is shown to be related to the “traversal time” concept defined in \cite{21}, whereas the half life of the decaying nucleus is related to the “transmission dwell time” defined in \cite{9}. From the present calculations, it seems that the transmission dwell time is a clearer indicator of the magic number $N = 126$ rather than the assault frequency or traversal time which are sensitive to the parameters in the JWKB approach.

Though intuitively, one would expect a similar behaviour of the tunneling characteristics for other heavy nuclei, it would be interesting to investigate other nuclei as well as other regions of magic numbers within the approach of the present work in order to confirm the conclusions of this work.

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