On the reliability of the semiempirical RR Lyrae Period - Blue Amplitude - V-band Luminosity relation.

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ABSTRACT

We investigate the accuracy and reliability of the semiempirical period - blue amplitude - V-band luminosity relationship for ab-type RR Lyrae stars originally obtained by Castellani & De Santis (1994) and De Santis (1996). We infer that the zero point of this relationship does depend on the metallicity, by studying a sample of both field and cluster variables. We also show that the use of this relationship can still be useful for those stellar systems showing an intrinsic metallicity spread, since in this case the metallicity effect has a negligible effect on the final distance modulus estimate.

We compare the adopted semiempirical relationship with the fully empirical one recently provided by Kovács & Walker (2001). When the zero point of the latter relation is fixed consistently with the former one, the two equations are equivalent.

By applying the semiempirical period - blue amplitude - V-band luminosity relation, as well as the technique proposed by Cassisi, De Santis & Piersimoni (2001), to the globular cluster ω Cen, we show that the empirical slope of the relationship between the mass of the fundamental RR Lyrae pulsators and their metallicity, is in fair agreement with the one predicted by updated evolutionary models for Horizontal-Branch stars.

Key words: stars: distances – stars: evolution – stars: horizontal branch – stars: variables: other – globular clusters: general

1 INTRODUCTION

The traditional distance indicator for Population II stellar systems is the magnitude of RR Lyrae variables; for this reason, several observational and theoretical investigations have been devoted to this class of variables (Bono, Castellani & Marconi 2000, Clement et al. 2001).

In spite of the large body of work devoted to their study, some relevant questions are still unanswered. One of the most important problems is the lack of a general agreement about both the slope and the zero-point of the absolute magnitude - metallicity (\(M_V(RR) - [Fe/H]\)) relationship, characteristic of the RR Lyrae stars (e.g., Caputo 1997, Gratton 1998, Cassisi, De Santis, & Piersimoni 2001, Benedict et al. 2002 and references therein). Some methods, like Baade-Wesselink and statistical parallax analyses applied to field RR Lyrae stars, plus observations of field Horizontal Branch (HB) stars with parallax measurements (but with large parallax errors – Gratton 1998) support the short distance scale.

On the other hand, the pulsational properties of cluster RR Lyrae stars (Sandage 1993), the main-sequence fitting to local subdwarfs (Gratton et al. 1997), the calibration of HB luminosity obtained by using the Cepheid distance modulus of the Large Magellanic Cloud (Walker 1992), and analysis based on double-mode RR Lyrae (Kovács 2000, Popielak, Dziembowski & Cassisi 2000) support the long scale.

One possible explanation of such a disagreement could be the existence of a true luminosity difference between field and cluster HB stars, as suggested by Gratton (1998) on the basis of an Hipparcos calibration of the absolute magnitude of field HB stars. However, this evidence is not supported by the analyses performed by Catelan (1998), De Santis & Cassisi (1999, hereinafter DC) and Carretta, Gratton & Clementini (2000). The origin of the distance dichotomy is therefore still unexplained and it has a big impact on a wide range of astrophysical problems such as globular clusters (GCs) age determinations and the extragalactic distances measurements.

A significant contribution to the solution of this problem can be provided by the analysis of the pulsational properties of RR Lyrae stars both in the Galaxy and in the Large
Magellanic Clouds. A first step towards this direction was made by Sandage, Katem & Sandage (1981) who suggested the existence of a tight correlation between temperature and amplitude of RR Lyrae stars. More recently, Caputo & De Santis (1992) showed the existence of a clear correlation between period, blue amplitude and light-mass ratio of the variables. The reliability of these and similar relationships is important, since period and amplitude can be measured with high accuracy, regardless of uncertainties on both distance modulus and reddening. In this field, a pivotal importance is played by any relation connecting pulsational properties to the intrinsic luminosity of RR Lyrae stars.

Theoretical support for the existence of a correlation between visual magnitude, period and blue amplitude (hereinafter PLA relation) for fundamental RR Lyrae (RR_ab) pulsators has been presented by Castellani & De Santis (1994) and by De Santis (1996). They provided also a semiempirical calibration of this relationship by adopting theoretical pulsational models and the observational database available at that time. However, until now, we lack a definitive assessment of the reliability of such relationship.

This paper is the third of a series investigating how the pulsational properties of RR Lyrae stars can be used to constrain their intrinsic luminosity. In particular, DC have used the pulsational behaviour of RR_ab stars to obtain an accurate estimate of the absolute bolometric luminosity of ZAHB stars in GCs. It is worth noticing that their results do not depend on the underlying evolutionary models of HB stars, and this occurrence allowed them to perform a significant comparison with recent theoretical evaluations of the ZAHB luminosity. This, in turn, is important in order to properly evaluate the reliability of current theoretical models of low-mass, He-burning stars (see also the discussion in Vandenberg et al. 2000).

Cassisi, De Santis & Piersimoni (2001, hereinafter CDP) have adopted the same method outlined by DC, in order to derive the absolute visual magnitude of the ZAHB within the RR Lyrae instability strip, for a sample of galactic GCs with accurate photometric data for both variable and non-variable HB stars. After applying a correction for the difference between the mean RR Lyrae magnitude and the ZAHB one, they derived a \( M_V(RR) = -[Fe/H] \) relation and compared it with the most recent empirical ones (see also Caputo et al. 1999). They also discussed a method for determining the GCs distance based only on the pulsational properties of their RR Lyrae population. The advantage of this method is that it does not need an estimate of the ZAHB level, which is particularly difficult task in the case of GCs with blue HB, whose RR Lyrae are suspected to be evolved stars.

The approaches developed by DC and CDP require a preliminary estimate of the cluster metallicity in order to determine the appropriate mass range for RR_ab stars. Due to the non-negligible uncertainties affecting both the GC metallicity scale (see, i.e., Rutledge et al. 1997 and Vandenberg et al. 2000) and their \( \alpha \)-elements distribution, this reduces the accuracy of the method. Therefore, in the present work we investigate the possibility to estimate the absolute visual magnitude of RR_ab stars by adopting a magnitude-period-amplitude relation. This would not require a preliminary evaluation of the stellar metallicity.

In the next section we briefly review the semiempirical PLA relationship adopted in present analysis. In section 3, we apply this relationship to a sample of field RR Lyrae stars, in order to investigate on its accuracy when applied to single stars. In section 4, we follow the same approach in case of a selected sample of galactic GCs, and the derived distance moduli are compared with those provided by CDP. In section 5, the semiempirical PLA relation is compared with the empirical one by Kovacs & Walker (2001). An application of the method to stellar systems showing a spread in the metallicity, like the GC \( \omega \) Cen, is shown in section 6. A brief discussion and conclusions follow in the last section.

2 THE SEMIEMPIRICAL PLA RELATION.

Castellani & De Santis (1994) showed the existence of a tight relation between effective temperature, period and blue amplitude. By combining this relation \( T_e = T_e(P, A_B) \) with the fundamental pulsational equation correlating the period of the variable with its main evolutionary properties such as mass, luminosity and effective temperature (van Albada & Baker 1971), they derived a relationship between the RR Lyrae intrinsic luminosity, period, blue amplitude, mass and visual bolometric correction:

\[
M_V = -1.842 \cdot \log P - 0.137 \cdot A_B + 2.02 \cdot \log M/M_\odot + BC_V + 0.19 \quad (1)
\]

Making use of theoretical results available at that time for both the mass of fundamental pulsators and bolometric corrections, they estimated the term in parentheses to be about 0.2 mag. They derived also an empirical calibration of the zero point of equation (1), by applying it to a sample of RR Lyrae stars with known \( P, A_B \) and absolute visual magnitude, keeping as a free parameter the additional term. For this calibration, the results of Baade-Wesselink analyses summarized by Sandage & Cacciari (1990) were used. The final zero point was equal to 0.45 mag, quite different from the one obtained on the basis of stellar evolution results. Nevertheless, an important result was the evidence that the zero point of relation (1) does not seem to depend on the star metallicity. This topic was further addressed by De Santis (1996, hereinafter DS), who analyzed data for 27 RR_ab in the cluster \( \omega \) Cen (Sandage 1990). DS showed that the dependence on the metallicity of the term in parentheses in equation (1) was completely negligible.

The reliability of this latter result relies on the accuracy of the adopted metallicity scale (Sandage 1990). In the same work, by using current theoretical results for the mass of RR Lyrae variables, the zero point was fixed to 0.31 mag, obtaining:

\[
M_V = -1.842 \cdot \log P - 0.137 \cdot A_B + 0.31 \quad (2)
\]

Since in the following we want to compare the GCs distance moduli obtained by using the PLA relation with the ones obtained by CDP which made use of a more updated evolutionary and pulsational scenario, we have rederived equation (2) by using the updated fundamental pulsational equation...
3 A CHECK OF THE ACCURACY OF THE PLA RELATION.

When the terms in parentheses in equation (1) are fixed, the use of the PLA relation does not require an estimate of the pulsator mass. On the contrary, the approach adopted by DC and CDP for estimating the intrinsic bolometric luminosity of the ZAHB is based on a priori estimates of the allowed mass range for fundamental pulsators. In the above papers, it has been shown that a change of about $+0.01M_\odot$ for both the upper and lower limit of the mass of the fundamental pulsators implies a change in the value of the cluster distance modulus of the order of $+0.01$ mag.

One can therefore check the accuracy of the PLA relation by comparing the pulsator mass obtained by adopting the absolute visual magnitude estimate from the PLA relation, with the value provided by evolutionary HB models. One can estimate the intrinsic bolometric luminosity of a star by using the relation:

$$\log L/L_\odot = -0.4 \cdot (M_V - M_{V,\odot} + BC_V - BC_{V,\odot})$$ (4)

For each variable in the sample the quantity $(BC_V - BC_{V,\odot})$ can be derived from equations (3) listed by DC and CDP, while its effective temperature can be obtained by using the temperature scale by DS. Once obtained the absolute visual magnitude by using relation (3) and estimated its $T_{\text{eff}}$ value, it is easy to derive the mass of the variable from the fundamental pulsational equation (Bono et al. 1997). In the following we adopt this approach for both field and cluster RR Lyrae.

3.1 Field RR Lyrae variables.

By applying this method to a sample of field $RR_{ab}$ stars (Lub 1977), we have derived an estimate of the mass for each variable. These values are plotted in figure 1 as a function of $[\text{Fe/H}]$. From these data the slope of $\log M_{DS}$ (hereinafter $M_{DS}$ denotes the mass of the variable when the absolute visual magnitude has been derived according to relation 3) versus the metallicity is equal to $-0.030 \pm 0.001$. This slope is half of the value predicted by current HB models (see DC).

For a metallicity $[\text{Fe/H}] \approx -2.0$ theoretical models suggest a mass range for fundamental pulsators $0.70 \leq M_{RR}/M_\odot \leq 0.80$, whereas from relation 3) we obtain $M_{DS} \approx 0.72M_\odot$; when $[\text{Fe/H}] \approx -1.5$ evolutionary theory suggests $0.66 \leq M_{RR}/M_\odot \leq 0.72$, whereas relation 3) predicts $M_{DS} \approx 0.69M_\odot$; in the metal-rich regime $[\text{Fe/H}] \approx -0.8$ we can derive from HB models $0.60 \leq M_{RR}/M_\odot \leq 0.63$ which has to be compared with the value $M_{DS} \approx 0.65M_\odot$. A metallicity range of $\pm 0.1$ dex was adopted in calculating the above values of $M_{DS}$.

By adopting the mass range for $RR_{ab}$ stars provided by updated evolutionary predictions, comparing the quoted values for $M_{DS}$ with these mass ranges and remembering that $\Delta M_{RR} = 0.01M_\odot$ implies an uncertainty of $\Delta M_V = 0.01$ mag, one can estimate that the maximum systematic error in the estimated absolute $V$ magnitude is $+0.02/−0.08$ mag in the metal-poor regime, $+0.03/−0.03$ mag for intermediate metallicity variables, and $+0.05$ mag for metal-rich stars.

Thus, even if the metallicity is unknown, the maximum systematic error affecting the absolute $V$ magnitude provided by the PLA relation is $+0.05/−0.08$ mag.

3.2 RR Lyrae variables in galactic GCs.

CDP have shown that by adopting the pulsational properties of GC RR Lyrae stars, it is possible to estimate the GC distances with high accuracy ($\Delta(m - M)_V \leq 0.05$ mag). In this section, we compare results obtained in that work with the ones we obtain by using the PLA relation.
For each cluster in the CDP sample, the mean mass of the variables \(M_{CDP}\) has been obtained by using, for each variable, the approach previously outlined and the \(M_V\) value estimated from the CDP distance modulus.

These values are listed in column 4 of table 1. As one can expect (see the CDP approach), the mean mass results to be equal to the central value of the allowed mass range for fundamental pulsators provided by evolutionary HB models.

By following the same approach outlined in previous section, we have estimated the mean mass \(M_{DS}\) of the variables in each cluster. These values are listed in column 5 of Table 1, and their differences with respect to the \(M_{CDP}\) values are plotted in figure 2 as a function of the metallicity. We obtain that for metal-poor clusters \(M_{CDP}\) values are larger than \(M_{DS}\) by about \(0.03 - 0.04\) \(M_\odot\), whereas for metal-rich clusters \(M_{CDP}\) values are smaller than \(M_{DS}\) by the same amount; for intermediate-metallicity clusters the agreement between the two mass estimates is fine. We conclude that, in the metallicity range \(-2.2 \leq [Fe/H] \leq -0.8\), the distance moduli obtained by using equation 3) are in agreement with the CDP estimates within \(\pm 0.04\) mag.

If one relies on the accuracy of the distance modulus estimates obtained by using the CDP approach, then the results obtained by using the PLA relation depend on the metallicity, since the difference between the CDP distance modulus and \((m - M)_{V}^{DS}\) is correlated with metallicity. Due to the fact that neither the fundamental pulsational equation nor the adopted pulsational temperature scale depend on the star metal content, it is clear that such occurrence has to be related to the assumption that the zero point of the PLA relation does not depend on the metallicity. This shows that the claims by both Castellani & De Santis (1994) and DS about the fact that this zero point shows a negligible dependence on the heavy elements abundance, are no more supported by updated data.

4 A COMPARISON WITH A \(M_V - FOURIER COEFFICIENTS\) RELATION.

Kovács & Walker (2001, hereinafter KW) have recently derived a set of useful relationships connecting the absolute visual magnitude of \(RR\) Lyrae stars with the Fourier parameters of the light curve.

Their main result is that the fundamental contribution
provided by equation (5) and (3). We use a compilation of derivation.

no important physical dependence has been missed in its

\[ \sigma \]

fits the empirical data (Secchi 1926). However, due to the evidence that this formula

KW do not make any attempt to calibrate the zero point of relation (5) \((M_V(KW))\), after fixing its zero point to the value 0.31 mag. We find \(dM_V(KW)/dM_V(DS) = 0.97 \pm 0.02\), with a correlation coefficient r=0.98. This result clearly shows that, when the zero point of relation (5) is fixed consistently with the one adopted for the equation (3), the two relations are substantially equivalent. Since the KW result does not depend on any effective temperature scale for RR Lyrae stars, the consistency between this relation and our PLA provides an independent support for the reliability of the temperature-period-amplitude relation (De Santis 2001) adopted in the present work. It also provides an absolute calibration of the KW relation.

However, the KW relation is affected by the same metallicity effect as the semiempirical PLA relation. This is shown in Figure 4, which displays the data for the RRab stars in the cluster \(\omega\) Cen listed by KW in their table 2. We notice again the existence of a fine agreement between the two sets: 

\[ \frac{dM_V(KW)}{dM_V(DS)} = 1.03 \pm 0.03. \]

In figure 5, the behaviour with the metallicity of the masses obtained by using either equation (3) \((M_V(DS))\) or relation (5) \((M_V(KW))\) is shown. As for the metallicity scale, the relation provided by Jursik & Kovács (1996) between the metallicity and selected Fourier parameters was adopted. One can notice that both relations provide the same dependence of the mass on the metallicity, namely: 

\[ d \log M_{KW}/[Fe/H] = -0.04 \pm 0.01 \] and \[ d \log M_{DS}/[Fe/H] = -0.03 \pm 0.006 \] ¹

5 THE CASE OF THE GC \(\omega\) CENTAURI.

It has been shown in Section 3 that the PLA relation is affected by a systematic metallicity effect. As a consequence, the distance determinations are affected, in comparison with the CDP estimates, by a systematic uncertainty of the order of -0.04 mag for metal-poor systems and of about +0.04

Table 1. The main properties of the RR Lyrae population in the selected sample of globular clusters.

| NGC | Name   | [Fe/H] | \(M_{CDP}/M_\odot\) | \(M_{DS}/M_\odot\) | \((m - M)_V^{CDP}\) | \((m - M)_V^{DS}\) |
|-----|--------|--------|----------------------|----------------------|----------------------|----------------------|
| 6171| M107   | -0.87  | 0.607                | 0.655                | 14.98±0.02           | 15.04±0.07           |
| 6342|        | -0.96  | 0.625                | 0.658                | 14.64±0.01           | 14.68±0.04           |
| 1851|        | -1.08  | 0.642                | 0.670                | 15.51±0.01           | 15.54±0.05           |
| 5904| M5     | -1.11  | 0.636                | 0.665                | 14.47±0.03           | 14.46±0.03           |
| 6981| M72    | -1.30  | 0.689                | 0.675                | 16.42±0.01           | 16.41±0.04           |
| 5272| M3     | -1.34  | 0.664                | 0.674                | 15.10±0.01           | 15.11±0.02           |
| 6934|        | -1.40  | 0.684                | 0.678                | 16.31±0.05           | 16.31±0.04           |
| IC4499|       | -1.46  | 0.682                | 0.684                | 17.10±0.05           | 17.10±0.02           |
| 6715| M54    | -1.55  | 0.691                | 0.689                | 17.62±0.05           | 17.62±0.03           |
| 6333| M9     | -1.57  | 0.698                | 0.684                | 15.75±0.05           | 15.73±0.05           |
| 6809| M55    | -1.65  | 0.702                | 0.695                | 13.90±0.05           | 13.89±0.07           |
| 4590| M68    | -1.99  | 0.756                | 0.708                | 15.19±0.03           | 15.14±0.04           |
| 5466|        | -2.03  | 0.713                | 0.711                | 16.05±0.03           | 16.05±0.03           |
| 6426|        | -2.07  | 0.760                | 0.719                | 17.80±0.07           | 17.76±0.06           |
| 5053|        | -2.10  | 0.744                | 0.721                | 16.13±0.05           | 16.11±0.07           |
| 7078| M15    | -2.12  | 0.754                | 0.711                | 15.36±0.03           | 15.32±0.04           |
| 6341| M92    | -2.15  | 0.743                | 0.715                | 14.71±0.05           | 14.68±0.05           |

Figure 4. As figure 3, but for a sample of RRab variables in the cluster \(\omega\) Cen.

to \(M_V\) comes not only from the period \(P\) and from the first Fourier amplitude \(A_1\), but there are statistically significant contributions also from additional higher order components, mainly from \(A_3\):

\[ M_V = -1.88 \log P - 0.971A_1 + 0.909A_3 + \text{constant} \] ¹

KW do not make any attempt to calibrate the zero point of this relation. However, due to the evidence that this formula fits the empirical data (\(\sigma \approx 0.04\) mag), they conclude that no important physical dependence has been missed in its derivation.

We check now the consistency between the results provided by equation (5) and (3). We use a compilation of Fourier parameters for 59 field RRab variables (Simon & Teays 1982). In figure 3, we show the comparison between the absolute visual magnitudes obtained by using equation (3) \((M_V(DS))\), and those obtained from equation (5) \((M_V(KW))\), after fixing its zero point to the value 0.31 mag. We find \(dM_V(KW)/dM_V(DS) = 0.97 \pm 0.02\), with a correlation coefficient r=0.98. This result clearly shows that, when the zero point of relation (5) is fixed consistently with the one adopted for the equation (3), the two relations are substantially equivalent. Since the KW result does not depend on any effective temperature scale for RR Lyrae stars, the consistency between this relation and our PLA provides an independent support for the reliability of the temperature-period-amplitude relation (De Santis 2001) adopted in the present work. It also provides an absolute calibration of the KW relation.

However, the KW relation is affected by the same metallicity effect as the semiempirical PLA relation. This is shown in Figure 4, which displays the data for the RRab stars in the cluster \(\omega\) Cen listed by KW in their table 2. We notice again the existence of a fine agreement between the two sets: 

\[ dM_V(KW)/dM_V(DS) = 1.03 \pm 0.03. \]

In figure 5, the behaviour with the metallicity of the masses obtained by using either equation (3) \((M_V(DS))\) or relation (5) \((M_V(KW))\) is shown. As for the metallicity scale, the relation provided by Jursik & Kovács (1996) between the metallicity and selected Fourier parameters was adopted. One can notice that both relations provide the same dependence of the mass on the metallicity, namely: 

\[ d \log M_{KW}/[Fe/H] = -0.04 \pm 0.01 \] and \[ d \log M_{DS}/[Fe/H] = -0.03 \pm 0.006 \] ¹

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It has been shown in Section 3 that the PLA relation is affected by a systematic metallicity effect. As a consequence, the distance determinations are affected, in comparison with the CDP estimates, by a systematic uncertainty of the order of -0.04 mag for metal-poor systems and of about +0.04

¹ If the most metal-poor variable and the most metal-rich one are excluded from the linear fit, the slopes do not change significantly, both being equal to -0.04.
mag for metal-rich ones. In the following, we show that this uncertainty is completely negligible when taking into account stellar systems showing an intrinsic metallicity spread.

We have considered the GC ω Cen, which shows an internal metallicity spread in the range $-2 \leq [Fe/H] \leq -0.8$ (Rey et al. 2000 and references therein). According to the same evolutionary models used by DC and CDP, this means that the allowed mass range for RR$_{ab}$ in this cluster is $0.60 \leq M/M_\odot \leq 0.80$. By following the CDP method, we estimate a distance modulus $(m-M)_V = 14.15 \pm 0.05$ mag. Figure 6 shows the behaviour of the reduced period as a function of the effective temperature, i.e., the empirical plane adopted for fixing the cluster distance modulus (see DS and CDP for details). By using the PLA relation for each individual RR Lyrae star, and then computing the mean of the obtained distance moduli, we obtain exactly the same distance estimate, $(m-M)_V = 14.15 \pm 0.03$ mag. In figure 7, for each variable in ω Cen, the absolute visual magnitude obtained by using the CDP distance modulus is compared with its $M_V$ value derived from the PLA relation.

The satisfactory agreement between the two methods applied to stellar systems with an internal metallicity spread, is clearly due to the fact that the metallicity affects the PLA relation in opposite directions for metal-poor and metal-rich variables. If a metallicity spread is present, the two effects cancel out when computing the distance modulus from the mean of the individual values obtained for the whole RR Lyrae sample. This occurrence makes the PLA relation very useful for deriving the distance to stellar systems showing an intrinsic metallicity dispersion. The method is simple and it does not need any preliminary estimate of the metallicity.

We notice that our distance modulus estimate for the GC ω Cen is, within the quoted uncertainty, in good agreement with the direct distance estimate for this clusters obtained by Thompson et al. (2001) by using a detached eclipsing binary system $(m-M)_V = 14.05 \pm 0.11$ mag.

Recently, Caputo, Degl’Innocenti & Marconi (2001) have obtained a fully theoretical PLA relation based on updated pulsational models. They have applied this relation to RR$_{ab}$ stars in ω Cen, obtaining a distance modulus equal to $(m-M)_V = 14.01 \pm 0.11$ mag. This estimate, within the quoted uncertainties, appears in marginal agreement with our result. However, Caputo et al. (2001) have employed a different observational database for the variables in ω Cen in comparison with our investigation. When applying the PLA relation by Caputo et al. (2001) to the data provided by KW, we obtain a distance modulus estimate only $\approx 0.07$ mag lower than the value obtained by using the semi-empirical PLA relation.

In the previous section it has been shown that the RR Lyrae mass derived from the PLA absolute magnitudes shows a dependence on the metallicity $d \log M_{DS}/d [Fe/H] = -0.03$, in disagreement with recent pulsational models. We address this point by considering the variables in the cluster ω Cen.

By using $(m-M)_V = 14.15 \pm 0.04$ mag, we obtain, following the usual approach, the mass of each variable in

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**Figure 5.** Panel a: The behaviour of the logarithm of the RR Lyrae masses in the cluster ω Cen obtained by estimating the intrinsic luminosity of the variable according to equation 3, as a function of the metallicity. Panel b: As in panel a), but estimating the absolute visual magnitude of the variables according to equation 3, as a function of the metallicity. The temperature scale is from De Santis (1996).

**Figure 6.** Comparison in the $(\log P + 0.33 \cdot <V>) - \log T_e$ diagram between the RR$_{ab}$ variables in ω Cen and the results from pulsational theory, having fixed the GC distance modulus to $(m-M)_V = 14.15$, and adopting for the allowed minimum and maximum RR$_{ab}$ mass the values provided by the stellar evolutionary theory. The temperature scale is from De Santis (1996).
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Figure 7. Comparison between the absolute visual magnitude obtained by using the distance modulus evaluated according the CDP approach (see figure 6), and the $M_V$ values obtained according the PLA relation (equation 3), for the RR Lyrae stars in the cluster ω Cen.

In this cluster. In figure 8, we show the trend of the masses as a function of [Fe/H]. In this case we obtain $d \log M/d[Fe/H] = -0.10 \pm 0.03$ which is, within the errors, in satisfactory agreement with the slope predicted by HB stellar models. It is worth emphasizing that an error in the adopted distance modulus does not affect the slope of the $\log M - [Fe/H]$ relation. Since there is only one star in the metal-poor regime ([Fe/H] ≈ −2.1) and just one object in the metal-rich tail ([Fe/H] ≈ −0.8), we performed the same linear regression excluding these two objects: the resulting slope is −0.11, in agreement with that obtained from the whole sample.

This result is important since recently, on the basis of the analysis of double-mode RR Lyrae in the Large Magellanic Cloud, Bragaglia et al. (2001) have found some evidences against the mass-metallicity relation for HB stars predicted by canonical stellar models. In particular, they found no significant differences in mass for RR Lyrae variables in GCs of different metallicity. A similar result was found also by Kovács (2001) by analyzing the properties of the double-mode RR Lyrae variables in the Sculptor dwarf galaxy. A more firm understanding of this issue has to wait for direct spectroscopic measurements of the metallicity for a larger RR Lyrae sample, but the present result does not support their finding.

6 FINAL REMARKS AND CONCLUSIONS.

By applying both the PLA relation and the CDP method to the GC ω Cen, we have shown that the two approaches are in agreement when applied to stellar systems with an internal metallicity spread. This is an advantage of the PLA relationship over the CDP method, since the former does not require any preliminary metallicity estimate. In this respect, some dwarf galaxies in the Local Group (LG) are a fine target to exploit this technique. In fact, even if current metallicity measurements, based mainly on photometric indices, reveal an intrinsic metallicity dispersion of the order of 0.3-0.5 dex (Mateo 1998) in most of the LG dwarfs, more recent high-resolution spectroscopic measurements have disclosed the existence of larger internal spread in the heavy elements abundance, for instance, $\Delta[Fe/H] = 0.73$ dex and 1.53 have been obtained by Shetrone, Cote & Sargent (2001) for Ursa Minor and Draco, respectively.

We have applied the PLA relation to the RR Lyrae sample in three LG dwarfs: Leo II, Sculptor and Sextans. The data are from Siegel & Majewski (2000) for Leo II, from Kaluzny et al (1995) for Sculptor, and from Mateo, Fisher & Krzeminski (1995) for Sextans. For each dwarf we have measured the distance modulus by applying the PLA relation to each individual variable and then computed the mean of the various values.

The resulting distances are $21.67 \pm 0.05$ mag, $19.60 \pm 0.03$ mag, and $19.83 \pm 0.06$ mag for Leo II, Sculptor and Sextans, respectively (the listed error corresponds to the max-
Figure 9. Comparison between the apparent visual magnitude and the $M_V$ value obtained from the PLA relation, for the RR Lyrae variables in the dwarf galaxies Leo II, Sculptor and Sextans.

minimum random error, so it does not account for the observational uncertainty affecting the apparent visual magnitudes). These estimates are in good agreement with currently adopted values (Mateo 1998). In figure 9, we show the satisfactory level of agreement between the apparent visual magnitude for each RR Lyrae variable and the $M_V$ value provided by the PLA relation.

Since the systematic metallicity effect, affecting the PLA relation, works in opposite directions, there is a good agreement between the distance moduli derived by using the CDP or PLA methods. In the case of a single metallicity stellar population, a reliable distance measurement should be still achieved. In fact, the maximum systematic error affecting the GC distance moduli provided by the PLA relation is of the order of ±0.04 mag.

However, the accuracy of the absolute magnitudes provided by the PLA relation strongly relies on the reliability of the adopted blue amplitude for the $RR_ab$ variables. This means that one has to exclude from the adopted data sample those variables affected by amplitude modulation (Blazhko effect).

We wish to summarize the assumptions upon which the PLA relation zero point relies:

- the validity of the adopted temperature scale. Concerning this topic, De Santis (2001) has shown that this relation does not significantly depend on the selected sample of variables, and has discussed its range of validity. Moreover, DC have also shown that, when this pulsational temperature scale is used, consistency is achieved between several sets of $T_{eff}$-color transformations (Buser & Kurucz 1978, Bessel, Castelli & Plez 1998, and Green 1988) and the Lub’s reddening scale. However, a significant error in the zero-point of this reddening scale and/or of the above quoted transformations, would translate into an error for the zero-point of equation (3);

- in section 1, we have noted that the use of the van Albada & Baker (1971) relation for the fundamental pulsational equation makes the PLA relation fainter by 0.05 mag in comparison with the Bono et al’s (1997) relation;

- when calibrating the zero point of the PLA relation, DS adopted 0.69$M_\odot$ as the mean mass of fundamental pulsator for a metallicity $[Fe/H] = -1.5$, while this assumption seems to be supported by recent HB stellar models, it relies on the reliability of the stellar model input physics.

Since all the quoted assumptions seem to be reliable, we trust the accuracy of the semi-empirical PLA relation for deriving the distance to stellar systems such as galactic GCs and LG dwarf galaxies.

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