Kinetic analysis of spin current contribution to spectrum of electromagnetic waves in spin-1/2 plasma, Part II: Dispersion dependencies

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The dielectric permeability tensor for spin polarized plasmas derived in terms of the spin-1/2 quantum kinetic model in six-dimensional phase space in Part I of this work is applied for study of spectra of high-frequency transverse and transverse-longitudinal waves propagating perpendicular to the external magnetic field. Cyclotron waves are studied at consideration of waves with electric field directed parallel to the external magnetic field. It is found that the separate spin evolution modifies the spectrum of cyclotron waves. These modifications increase with the increase of the spin polarization and the number of the cyclotron resonance. Spin dynamics with no account of the anomalous magnetic moment gives a considerable modification of spectra either. The account of anomalous magnetic moment leads to a fine structure of each cyclotron resonance. So, each cyclotron resonance splits on three waves. Details of this spectrum and its changes with the change of spin polarization are studied for the first and second cyclotron waves. A cyclotron resonance existing at \( \omega \approx 0.001 \left| \Omega_e \right| \) due to the anomalous magnetic moment is also described, where \( \left| \Omega_e \right| \) is the cyclotron frequency. The ordinary waves does not have any considerable modification. The electrostatic and electromagnetic Berstein modes are studied during the analysis of waves propagating perpendicular to the external magnetic field with the electric field perturbation directed perpendicular to the external field. A modification of the oscillatory structure caused by the equilibrium spin polarization is found in both regimes. Similar modification is found for the extraordinary wave spectrum.

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I. INTRODUCTION

The Fermi spin current existing in the magnetic moment (spin) evolution equation is introduced in Ref. [1]. It is an example of the thermal part of spin current existing for degenerate electrons. Similarity to the pressure, the thermal part of the spin current is an independent hydrodynamic function. It requires an equation of state if we need to make a truncation and obtain a closed set of equation.

In Ref. [1], an equation of state for the Fermi spin current is found via the derivation of a non-linear Pauli equation from the separate spin evolution Euler equations for the spin-up and spin-down electrons. Found non-linear Pauli equation allows to derive corresponding magnetic moment evolution equation containing explicit form of the Fermi spin current.

Another derivation of the equilibrium Fermi spin current is presented in Ref. [2], where the Fermi spin current is derived as a moment of the spin distribution function for degenerate electrons with an arbitrary spin polarization.

Single-fluid spin-1/2 quantum hydrodynamic model of electrons with the Fermi spin current is applied in Ref. [1] for study of spectrum of electromagnetic waves, spin waves and their linear interaction at the wave propagation parallel and perpendicular to the external magnetic field in electron-ion and electron-positron plasmas. Separate spin evolution quantum hydrodynamics of electrons (two-fluid model of electrons) with the Fermi spin current and the spin-orbit interaction is derived in Ref. [3], where the extraordinary waves propagating perpendicular to the external magnetic field are studied. The extraordinary spin-electron acoustic wave is found and studied there.

More advanced method of the study of the thermal part of the spin current or the Fermi spin current effect for degenerate gas is a quantum kinetic model containing this effect. Corresponding model is developed in Part I of this paper [4]. Corresponding dielectric permeability tensor for spin polarized plasmas is derived for the oblique propagating perturbations.

The Fermi spin current is obtained from the separate spin evolution kinetic [2] can be more relevant for comparison with the kinetic model results. Consider divergence of the Fermi spin current obtained from the kinetic model and find the following vector

\[
\mathcal{S}_K = (6\pi^2)^4 \frac{\pi \hbar \mu_e}{8m} \times \\
\times \{ n_\uparrow \partial_x n_\uparrow - n_\downarrow \partial_x n_\downarrow, n_\uparrow \partial_y n_\uparrow - n_\downarrow \partial_y n_\downarrow, 0 \},
\]

where \( n_\uparrow (n_\downarrow) \) is the concentration of electrons in spin-up (spin-down) states, \( m \) is the mass of particles, \( \mu_e \) is the magnetic moment of electron including the anomalous
magnetic moment, \( \mu_e = e q_e \hbar / 2mc \), \( q_e \) is the charge of electron \( q_e = - |e| \), \( c \) is the speed of light, and \( \mathbf{J}_K^{\alpha\beta} = \partial_\beta J_\alpha^{K} \).

The application of the non-linear Pauli equation with the spinor pressure gives another form of the Fermi spin current \([1]\):

\[
\mathbf{S}_P = \frac{(3\pi^2)^2 \hbar}{m} \left( n_\uparrow - n_\downarrow \right) [\mathbf{M}, \mathbf{e}_z],
\]

(2)

where \( \mathbf{M} \) is the magnetization of electron gas, \( \mathbf{e}_z \) is the unit vector in the \( z \)-direction.

The spin-1/2 quantum hydrodynamics containing the Fermi spin current \([1]\) or \([2]\) (see for instance \([1]\)) is a generalization of the spin-1/2 quantum hydrodynamics suggested earlier \([2, 3]\).

Equations \([1]\) and \([2]\) show that the Fermi spin current exists due to the difference of the populations of electrons with different spin projections. However, \( \mathbf{S}_K \) and \( \mathbf{S}_P \) have some difference. \( \mathbf{S}_P \) does not contain any derivatives. Hence, the Fourier image of its linear form does not includes the wave vector. Consequently, \( \mathbf{S}_P \) gives contribution for all directions of wave propagation. On the other hand, \( \mathbf{S}_K \) contains derivatives on space variables. Moreover, it contains derivatives on space directions which are perpendicular to the external magnetic field. Consequently, at the analysis of linear waves, \( \mathbf{S}_K \) does not give any contribution in spectrum of waves propagating parallel to the external magnetic field. Vector \( \mathbf{S}_K \) is derived from the kinetic equations as a moment of the equilibrium spin-distribution function for the spin-polarized degenerate electron gas.

The dielectric permeability tensor is obtained in Part I for the general form of isotropic distribution function. An explicit form of the dielectric permeability tensor is obtained for the spin-1/2 partially polarized three-dimensional electron gas, where the following equilibrium distribution functions are used \( f_0(p) = \int \rho(p_{F\uparrow} - p) + \rho(p_{F\downarrow} - p)]/(2\pi \hbar)^3 \) and \( S_{0\uparrow}(p) = \int \rho(p_{F\uparrow} - p) - \rho(p_{F\downarrow} - p)]/(2\pi \hbar)^3 \), \( S_{0\uparrow} = S_{0\downarrow} = 0 \), \( p_{F\uparrow} = (6\pi^2 n_e)^{1/3} \hbar \) is the Fermi momentum for spin-s electrons, with \( s = \uparrow \) or \( \downarrow \). It corresponds to the isotropic equilibrium.

Spin evolution can be anisotropic \( S_{0\uparrow}(p, \varphi) \), \( S_{0\downarrow}(p, \varphi) \) even for isotropic \( f_0(p) \) and \( S_{0\uparrow}(p) \). A description of this regime is given in Appendix A of the Part I \([4]\).

An anisotropic spin distribution function \( \mathbf{S}(\mathbf{p}) \) is used to derive the Fermi spin current \([1]\) via the following definition \( J_\alpha^{K} = \mu_e \int S_0^\alpha(p) \mathbf{v}^3 dp \).

Spinless part of the dielectric permeability tensor contains \( f_0(p) \). The spin dependent part of the dielectric permeability tensor \( \varepsilon^\alpha\beta(\omega, \mathbf{k}) \) has two parts. One of them contains \( S_{0\uparrow}(p) \) and another part is proportional to \( f_0(p) \). The Fermi spin current effects are related to the difference in occupations of spin-up and spin-down states. Hence, the Fermi spin current effects appear from the terms containing the equilibrium spin distribution function \( S_{0\uparrow}(p) \). At the kinetic analysis the Fermi spin current effects exist even for the isotropic distribution functions for all directions of wave propagations, as it is demonstrated in this paper.

Spectrum of waves can be found from the following dispersion equation

\[
\text{det} \left[ k^2 \delta^{\alpha\beta} - k^\alpha k^\beta - \frac{\omega^2}{c^2} \varepsilon^{\alpha\beta}(\omega, \mathbf{k}) \right] = 0,
\]

(3)

where the dielectric permeability tensor \( \varepsilon^{\alpha\beta}(\omega, \mathbf{k}) \) is obtained in \([4]\), \( \mathbf{k} \) is the wave vector, \( k \) is the module of the wave vector, \( \omega \) is the frequency of wave, and \( c \) is the speed of light. In this paper, the dispersion equation \( \text{det}(\mathbf{\varepsilon}) \) is applied for study of the wave spectrum with the Fermi spin current effects.

This paper is organized as follows. In Sec. II is devoted to the waves propagating parallel to the external magnetic field. In Sec. III presents analysis of waves propagating perpendicular to the external magnetic field. In Sec. IV a brief summary of obtained results is presented.

II. WAVE PROPAGATION PARALLEL TO THE EXTERNAL MAGNETIC FIELD

Considering the wave propagation parallel to the external magnetic field \( \mathbf{k} = \{0, 0, k_\parallel\} \), the following dielectric permeability tensor can be derived from equation (52) of the Part I \([4]\):

\[
\varepsilon^{\alpha\beta} = \delta^{\alpha\beta} - \sum_{s=\uparrow, \downarrow} \int \sin \theta d\theta \left[ P_{CI}^{\alpha\beta}(\theta, s) + m^2 \frac{\mu_e^2 c^2}{\pi \hbar^3} \frac{1}{2\omega^2} \sum_{r=+,-} \frac{v_{F\alpha}^3 k_\parallel \cos \theta k_{\parallel\beta}}{\omega - k_\parallel v_{F\alpha} \cos \theta + r \Omega_\mu} \right] - \frac{m^2 \mu_e^2 c^2}{\pi \hbar^3} \frac{1}{2\omega^2} \sum_{r=+,-} \int v_{F\alpha}^3 \left[ \frac{r \kappa^{\alpha\beta} (-1)^{\mu} v_{F\alpha}^2 dv}{\omega - k_\parallel v \cos \theta + r \Omega_\mu} \right],
\]

(4)

where \( \delta^{\alpha\beta} \) is the Kronecker symbol, \( \mu_e \) is the magnetic moment of particles, \( v \) is the module of velocity, \( \theta \) is an angle of the spherical coordinates in the velocity space defined as \( \cos \theta = v_\parallel / v \), \( \Omega_\mu = e q \mathbf{B}_0 / mc \), \( \Omega_\mu = 2 \mu_e B_0 / \hbar \), \( \kappa^{\alpha\beta}_+ = K^{\alpha\beta}_+ \), \( \kappa^{\alpha\beta}_\perp = (K^{\alpha\beta}_\perp)^* \), \( i_\parallel = 0, i_\perp = 1 \),

\[
K_\parallel = k_\parallel^2 \begin{pmatrix} 1 & i & 0 \\ i & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix},
\]

and
\[ \tilde{\Pi}_{Cl}(\theta, s) = \frac{3\omega_F^2}{2\omega} \left( \begin{array}{ccc}
\frac{1}{4}\left( \frac{\sin^2 \theta}{\omega - k_z v_F s \cos \theta - \Omega_e} + \frac{\sin^2 \theta}{\omega + k_z v_F s \cos \theta + \Omega_e} \right) & -i\frac{1}{4}\left( \frac{\sin^2 \theta}{\omega - k_z v_F s \cos \theta - \Omega_e} - \frac{\sin^2 \theta}{\omega + k_z v_F s \cos \theta + \Omega_e} \right) & 0 \\
-i\frac{1}{4}\left( \frac{\sin^2 \theta}{\omega - k_z v_F s \cos \theta - \Omega_e} + \frac{\sin^2 \theta}{\omega + k_z v_F s \cos \theta + \Omega_e} \right) & \frac{1}{4}\left( \frac{\sin^2 \theta}{\omega - k_z v_F s \cos \theta - \Omega_e} - \frac{\sin^2 \theta}{\omega + k_z v_F s \cos \theta + \Omega_e} \right) & 0 \\
0 & 0 & \frac{1}{2}\left( \frac{\sin^2 \theta}{\omega - k_z v_F s \cos \theta - \Omega_e} + \frac{\sin^2 \theta}{\omega + k_z v_F s \cos \theta + \Omega_e} \right) \end{array} \right), \tag{6} \]

where well-known properties of the Bessel functions are used, and \( v_F = p_F / m = (6\pi^2 n_0)^{1/3} \hbar / m, \omega_F^2 = 4\pi e^2 n_0 / m. \)

\( \tilde{\Pi}_{Cl}(\theta, s) \) is similar to the traditional result for degenerate electrons presented in many textbooks (see for instance [31]), but it also includes the spin separation effect. The third and fourth terms in equation (4) are caused by the spin evolution via the dynamics of the spin-distribution function.

Further integration in the dielectric permeability tensor gives the following result:

\[ \varepsilon^{\alpha\beta} = \delta^{\alpha\beta} - \sum_{s=\uparrow, \downarrow} \left[ \tilde{\Pi}_{Cl}(s) + \frac{m^2 v_F s}{\pi \hbar^3} \frac{\mu^2 c^2}{2\omega^2} \sum_{r=+,-} r\kappa_r^{\alpha\beta} (-1)^{\nu} \left( \frac{v_F s}{k_r^2} (\omega + r\Omega) + \frac{(\omega + r\Omega)^2 - (k_r v_F s)^2}{2k_r^4} \frac{\ln \left( \frac{\omega + k_r v_F s + r\Omega}{\omega - k_r v_F s} + r\Omega \right)}{\ln \left( \frac{\omega + k_r v_F s + r\Omega}{\omega - k_r v_F s} + r\Omega \right)} \right) \right] \]

with

\[ \tilde{\Pi}_{Cl}(s) = \frac{3\omega_F^2}{2\omega} \left( \begin{array}{ccc}
\frac{1}{4}[G(\omega - \Omega_e) + G(\omega + \Omega_e)] & -i\frac{1}{4}[G(\omega - \Omega_e) - G(\omega + \Omega_e)] & 0 \\
-i\frac{1}{4}[G(\omega - \Omega_e) - G(\omega + \Omega_e)] & \frac{1}{4}[G(\omega - \Omega_e) + G(\omega + \Omega_e)] & 0 \\
0 & 0 & \frac{\omega}{k_z v_F s} (-2 + \frac{\omega + k_z v_F s + r\Omega}{\omega - k_z v_F s} + r\Omega) \end{array} \right), \tag{8} \]

where

\[ G(\omega \pm \Omega_e) = \frac{1}{k_z v_F s} \left[ \frac{2(\omega \pm \Omega_e)}{k_z v_F s} \right] \]

\[ + \left( 1 - \frac{(\omega \pm \Omega_e)^2}{(k_z v_F s)^2} \right) \ln \left( \frac{\omega + k_z v_F s \pm \Omega_e}{\omega - k_z v_F s \pm \Omega_e} \right). \tag{9} \]

The dispersion equation appears in the following form at the application of the found structure of the dielectric permeability tensor:

\[ \det \begin{pmatrix}
l_k^2 - \frac{\omega^2}{c^2} \varepsilon_{xx} & -\frac{\omega^2}{c^2} \varepsilon_{xy} & 0 \\
-\frac{\omega^2}{c^2} \varepsilon_{yz} & l_k^2 - \frac{\omega^2}{c^2} \varepsilon_{yy} & 0 \\
0 & 0 & -\frac{\omega^2}{c^2} \varepsilon_{zz} \end{pmatrix} = 0. \tag{10} \]

It splits on three equations, one equation is for the longitudinal waves

\[ \varepsilon_{zz} = 0, \tag{11} \]

and two equations are for the transverse waves

\[ \frac{k_z^2 c^2}{\omega^2} - \epsilon = \pm \Xi, \tag{12} \]

where it is used that \( \varepsilon_{xx} = \varepsilon_{yy} = \epsilon, \) and \( \varepsilon_{yz} = \varepsilon_{xy} = \epsilon. \)

Explicit form of dispersion equation (11) appears as follows:

\[ 1 + \sum_{s=\uparrow, \downarrow} \frac{3\omega_F^4}{k_z^2 v_F s} \left( 1 - \frac{\omega}{2k_z v_F s} \ln \left( \frac{\omega + k_z v_F s}{\omega - k_z v_F s} \right) \right) = 0. \tag{13} \]

Consideration of the spin-polarized equilibrium distribution functions as the two Fermi step functions gives same dispersion equation for the longitudinal waves as in the separate spin evolution kinetics [2].

Equation (13) reveals existence of the spin-electron acoustic waves and their Landau damping [2]. The bulk spin-electron acoustic wave propagating parallel to the external magnetic field is found theoretically in terms of the separate spin evolution quantum hydrodynamics [13]. Existence of two spin-electron acoustic waves in the regime of oblique wave propagation is demonstrated in [16]. Reacher spectrum of the spin-electron acoustic waves is found in electron-positron-ion plasmas [17, 18]. A method of account of the Coulomb exchange interaction in the separate spin evolution quantum hydrodynamics is developed in [19] for more detailed analysis of linear and non-linear properties of spin-electron acoustic waves. The surface spin-electron acoustic waves are studied either [20]. A possibility of the linear interaction between the surface plasmon and the surface spin-electron acoustic wave is found. An applications of methods of the condensed matter physics [21, 22] allows to find similar effects in graphenes [23, 24].

Dispersion equations for the transverse waves appear from (12) as follows:

\[ \frac{k_z^2 c^2}{\omega^2} = 1 - \sum_{s=\uparrow, \downarrow} \frac{3\omega_F^4}{4k_z^2 v_F s} \left[ \frac{2(\omega \pm \Omega_e)}{k_z v_F s} \right] \]
\( + \left(1 - \frac{(\omega + \Omega_{e})^{2}}{(k_{z}v_{FS})^{2}}\right) \ln \left(\frac{\omega + k_{z}v_{FS} + \Omega_{e}}{\omega - k_{z}v_{FS} + \Omega_{e}}\right) + C_{s} + D_{s}, \) \hspace{1cm} (14) 

for \( \delta E_{x} = \pm i \delta E_{y} \) correspondingly, where

\[ C_{s}(\pm) = \frac{k_{z}^{2}m^{2}v_{FS}^{2}\mu_{e}^{2}c^{2}}{\pi \hbar^{3} \alpha \omega^{2}} \left[ -2 + \frac{\omega + \Omega_{e}}{k_{z}v_{FS}} \times \ln \left(\frac{\omega + k_{z}v_{FS} + \Omega_{e}}{\omega - k_{z}v_{FS} + \Omega_{e}}\right) \right], \]

and

\[ D_{s}(\pm) = (-1)^{i} \frac{m^{3}}{\pi \hbar^{3} \hbar \omega^{2}} \left(-2v_{FS}(\omega \pm \Omega_{e}) \right) + \frac{1}{k_{z}} \left((\omega \pm \Omega_{e})^{2} - (k_{z}v_{FS})^{2}\right) \ln \left(\frac{\omega + k_{z}v_{FS} + \Omega_{e}}{\omega - k_{z}v_{FS} + \Omega_{e}}\right), \] \hspace{1cm} (15) 

where \( C_{s}(\pm) \) is caused by the spin evolution and appears via \( f_{0} \), and \( D_{s}(\pm) \) is also caused by the spin evolution, but \( D_{s}(\pm) \) appears via \( S_{0z} \). Below, an approximate analytical analysis shows that \( D_{s}(\pm) \) contains effects caused by \( \ln \) function in the Taylor series is considered up to the fifth order on \( k_{z}v_{FS}/(\omega \pm \Omega_{e}) \). Traditionally this expansion is restricted by the third order (see for instance [31]). The spin effects in the last regime exist via the expansion of \( D_{s}(\pm) \) (see equation (14)). In equation (16) it exists as \( \sum_{s=\pm}(-1)^{i} \frac{\omega^{2}}{2m^{2}v_{FS}^{2}} \ln \left(\frac{\omega \pm \Omega_{e}}{\omega} \right) \). The next order of the expansion is considered to include the pressure-like term which is proportional to \( \ln \). This term describes the spin evolution with the accounted of the Fermi spin current.

Equation (16) shows that the effects of the Fermi spin current are small in the limit \( k_{z}v_{FS}/(\omega \pm \Omega_{e}) \ll 1 \), since they appear in the highest order on the small parameter.

### III. Wave Propagation Perpendicular to the External Magnetic Field

This section is devoted to the wave propagation perpendicular to the external magnetic field, \( k = \{k_{z}, 0, 0\} \). In this regime the dielectric permeability tensor given by equation (52) of the Part I [1] modifies to

\[ \varepsilon^{\alpha \beta} = \delta^{\alpha \beta} - \sum_{s=\pm} \left\{ \sum_{n=-\infty}^{\infty} \iota \frac{\sin \theta d\theta \tilde{\mathcal{A}}^{n \beta}(n, s)}{\omega - n \Omega_{e}} \right\}, \]

\[ + \frac{m^{2}v_{FS}^{2} \mu_{e}^{2}c^{2}}{\pi \hbar^{3} 2 \omega^{2}} \mathcal{K}_{\perp} \sum_{r=+,-} \frac{n \Omega_{e} \sin \theta d\theta J_{2}^{2}}{\omega - n \Omega_{e} + r \Omega_{e}}, \]

\[ - \frac{m^{3}}{\pi \hbar^{3} \hbar \omega^{2}} \mathcal{K}_{\perp} \sum_{r=+,-} \frac{r}{\omega - n \Omega_{e} + r \Omega_{e}} \times \int_{0}^{v_{FS}} v^{2} d\theta \int_{0}^{\iota \theta} \sin \theta d\theta J_{2}^{2} \left(\frac{k_{z}v \sin \theta}{\Omega_{e}}\right) \]
where $\sum_{n=-\infty}^{+\infty} J_n^2 = 1$, $\sum_{n=-\infty}^{+\infty} J_n J_n' = 0$ are used in the last group of terms of equation (52) of the Part I [4], and

$$\Lambda^{\alpha\beta}(n, s) = \frac{3 \omega L}{2 \omega v F_s} \Pi^\alpha\beta_{C1}(n, s) + m^2 v F_s \frac{\mu^2}{\pi h^3} \frac{k_n^2}{\omega} \frac{\kappa}{e} m^2 v F_s (-1)^s \Pi^\alpha\beta_S (n, s),$$

(18)

with

$$\Pi^\alpha\beta_{C1}(n, s) = v F_s \times$$

$$\left( \begin{array}{ccc}
\Omega^2_{n} & n^2 J_n^2 & 0 \\
-k_x v F_s & n J_n J_n' & 0 \\
0 & 0 & 0
\end{array} \right),$$

(19)

which has structure similar to well-known from textbooks [31], but it separately describes electrons with spin-up and spin-down, and the tensor

$$\Pi^\alpha\beta_S(n, s) = \left( \begin{array}{ccc}
\Omega & n J_n & 0 \\
-i \Omega & n J_n J_n' & 0 \\
0 & 0 & 0
\end{array} \right),$$

(20)

describes the spin evolution leading to the same resonances as the classic evolution $\omega = k_x v_F \cos \theta + n \Omega_e$. Here, the Bessel functions $J_n$ are functions of $k_x v F_s \sin \theta/ | \Omega_e |$, if a Bessel function has different argument this argument is shown explicitly as it is in the third line in equation (18), and $J_n'$ means derivative of $J_n$ on its argument $k_x v F_s \sin \theta/ | \Omega_e |$. Two spin dependent terms in equation (20) contain the following matrix

$$\hat{K} = k_x \left( \begin{array}{ccc}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{array} \right),$$

(21)

Similarly to the spinless case, the dispersion equation in the regime of wave propagation perpendicular to the external magnetic field splits on two independent equations:

$$k_x^2 e^2 \frac{\varepsilon_{zz}}{\omega^2} = \varepsilon_{zz},$$

(22)

for the transverse waves with the linear polarization $\delta E = \{0, 0, \delta E_z\}$ (ordinary waves), and

$$k_x^2 e^2 \frac{\varepsilon_{xx} \varepsilon_{yy} + \varepsilon_{xy}^2}{\omega^2} = \varepsilon_{xx},$$

(23)

(24)

for the longitudinally-transverse (extraordinary) waves.

Equations (22) and (24) includes the dispersion dependence of the cyclotron and Bernstein modes. Analysis of the longitudinal Bernstein modes in relativistic and non-relativistic regimes of classic plasmas with comparable cyclotron and Langmuir frequencies can be found in [32].

FIG. 1: (Color online) The figure shows the influence of the separate spin evolution on spectrum of the first, second, third and fifth “ordinary” cyclotron waves for $\eta = 0.2$ (the left-hand side column) and $\eta = 0.5$ (the right-hand side column). The separate spin evolution enters via the traditional terms which are not caused by the spin dynamics.

FIG. 2: (Color online) The figure shows the influence of the spin dynamics on the spectrum of under assumption $\Omega_e = \Omega_c$ made in equation (25) for spectrum of the first, second and fifth “ordinary” cyclotron waves for $\eta = 0.2$ (the left-hand side column) and $\eta = 0.5$ (the right-hand side column). In this regime the number of roots is not changed, but spectrum is affected by the spin dynamics.
FIG. 3: (Color online) The figure shows details of the fine structure of the first cyclotron mode. It is shown for two values of the spin polarization.

FIG. 4: (Color online) The figure shows the dispersion curve for the lowest cyclotron wave (the zeroth cyclotron wave). This wave exists due to the spin evolution and has frequency near $0.001 \mid \Omega_e \mid$. It is shown for two values of the spin polarization.

FIG. 5: (Color online) The figure shows details of the fine structure of the second cyclotron mode. It is shown for two values of the spin polarization.

FIG. 6: (Color online) The figure shows the modification of the dispersion curve of the upper line in the triplet of the second cyclotron mode with the change of the spin polarization.

A. Linearly polarized waves: Ordinary waves, cyclotron waves and spin waves

Consideration of the ordinary wave spectrum requires a detailed analysis of equation (23). To this end, an explicit form of equation (23) is presented as follows

$$
\frac{k_x^2 v^2}{\omega^2} = 1 - \sum_{s=\uparrow,\downarrow} \sum_{n=-\infty}^{\infty} \frac{3\omega^2 L_s}{2\omega} \int \sin \theta \cos^2 \theta J_n^2 d\theta \omega - n\Omega_e
$$

$$
+ \frac{m^2 v_F s e^2}{\pi \hbar^3} \frac{\mu^2 e^2}{2\omega^2} k_x^2 \sum_{r=\pm} n\Omega_e \int \sin \theta d\theta J_n^2 \frac{(-1)^i m^3}{\pi \hbar^3} \frac{\mu^2 e^2}{\hbar \omega^2} \times
$$

$$
\times k_x^2 \sum_{r=\pm} \int v_F s^2 v^2 dv \int \sin \theta d\theta J_n^2 \frac{k_x v \sin \theta}{\omega - n\Omega_e + r\Omega_{\mu}} \frac{\mu^2 e^2}{\hbar \omega^2} \frac{(-1)^i m^3}{\pi \hbar^3}. \quad (25)
$$

Equation (25) describes the ordinary waves including the ordinary cyclotron waves. The last two terms in equation (25) are caused by the dynamics of $\delta S_x$ and $\delta S_y$. The last term in equation (25) contains the contribution of spin-up and spin-down electrons with different signs. Hence, the Fermi spin current effects enter the ordinary cyclotron wave spectrum via the last term.

All nontrivial terms on the right-hand side of equation (25) contain effects of the separate spin evolution. Except the separate spin evolution, the second term has the same form as in traditional spinless case. The second term (which is proportional to $\omega^2 L_s$) has resonances on harmonics of the electron cyclotron frequency. The third and fourth terms are caused by the spin dynamics. The resonances in the third and fourth terms are shifted by the magnetic moment cyclotron frequency $\pm\Omega_{\mu}$. The spin dynamics leads to new resonances and formation of the fine structure of the cyclotron waves. It is similar to the Bernstein modes structure found in Ref. [33]. However, Ref. [33] deals with the electrostatic Bernstein modes with the electric field directed along $0x$. 
axis \( \delta E = \{ \delta E_x, 0, 0 \} \). This regime is considered in the next subsection. Here, the ordinary transverse cyclotron waves are under consideration. Presented here results have similarity with Ref. 32, but this is a different phenomenon. The effect of splitting of the electrostatic Bernstein modes is demonstrated in 32 for the second Bernstein mode which includes \( \omega = 2 | \Omega_c |, \omega = 3 | \Omega_c | - | \Omega_\mu |, \omega = | \Omega_c | + | \Omega_\mu | \). The splitting gives three closely located dispersion curves (a triplet) since \( \Omega_\mu \neq \Omega_c \) and \( | \Omega_\mu - \Omega_c | \ll | \Omega_c | \). The Fermi-Dirac distribution modified by a factor describing the spin polarization is applied in Ref. 32 as an equilibrium distribution function.

Considering different waves it would be unnecessary comparing the dispersion equations. However, there are some similarities which can be mentioned. The first term on the right-hand side of equation 26 of Ref. 33 is a classical term. The second and third terms on the right-hand side of equation 26 of Ref. 33 appear due to the spin dynamics. The second and third terms are actually two terms of sum taken at \( n = 1 \) and \( n = 3 \). So, the right-hand side of equation 26 of Ref. 33 consists of two groups of terms: classical and spin terms. The second term on the right-hand side of equation 26 is a classical term. The third term gives in-phase contribution of spin-up and spin down states and resembles a distant similarity to second group of terms in equation 26 of Ref. 33. The fourth term in equation 26 presents antiphase contribution of spin-up and spin down states and has no analogs in equation 26 of Ref. 33.

Solve equation 26 in several regimes. First, dropping the spin dynamics (the third and fourth terms on the right-hand side), study the spectrum change due to the separate spin evolution. Second, include all terms, but drop the anomalous part of the magnetic moment (it gives \( \Omega_c = \Omega_\mu \)). Third, consider all effects presented in equation 26. All regimes require numerical solution of equation 26.

Results of solution of equation 26 in the first regime are presented in Fig. 1. Find the following considering the first regime. Fig. 1 shows that the separate spin evolution effect becomes larger for higher cyclotron waves. Fig. 1 shows that an increase of the spin polarization \( \eta \) leads to a larger modification of spectrum. The numerical analysis in this subsection is made for \( n_0 = 10^{21} \) cm\(^{-3} \) and \( B_0 = 10^7 \) G. This is regime of dense plasma with \( \omega_L \gg \Omega_c \).

First of all this modification appears at relatively large wave vectors \( \vec{k} \). It appears via faster oscillation of the dispersion curve. Moreover, the divergency of the dispersion curve from \( n \Omega_c \) decreases. As is well-known, the cyclotron waves and Bernstein modes appears in kinetic theory due to the detailed description of the distribution function evolution in the momentum space. Hydrodynamic description of these effects requires account of the higher moments of the distribution function, such as pressure evolution, which allows to get the Bernstein mode near \( 2 | \Omega_c | \). The spin polarization modifies the Fermi step to two different steps for the spin-up and spin-down electrons. Expectedly, it modifies the Bernstein mode spectrum.

The second regime reveals the following modification of spectrum of cyclotron waves. The spin dynamics changes form of spectrum of cyclotron waves (see Fig. 2). Particularly, the divergence of a curve from \( n | \Omega_c | \) (for corresponding \( n \)) increases several times.

The account of anomalous magnetic moment leads to the fine structure of the cyclotron waves. Each curve splits on three closely located curves Fig. 3. Resonances near \( n | \Omega_c | \) appears due to the following denominators: \( n - n | \Omega_c |, n - (n + 1) | \Omega_c | + | \Omega_\mu |, n - (n - 1) | \Omega_c | - | \Omega_\mu | \).

Considering \( n = 0 \), find one resonance at a positive frequency \( \omega - 4 | \Omega_\mu | - | \Omega_c | = \omega - 0.001 | \Omega_c | \). Solution of equation 26 confirms that there is a branch near \( 0.001 | \Omega_c | \) (see Fig. 4). This effect is demonstrated in 34 in terms of kinetic model. It is also found in terms of extended quantum hydrodynamics containing equations for the pressure moment and the spin-velocity moment 35. A spin modified Maxwellian distribution is employed in Ref. 34. So, they are consider a non-degenerate plasma. As a result the dispersion curve \( \omega(k) \) monotonically increases, reach maximum value, and monotonically decay. While Figs. 3 and 5 shows that there are oscillations of \( \omega(k) \) after reaching the maximum value. It is in accordance with the oscillations of the dispersion curves of cyclotron and Bernstein modes in degenerate plasmas. Here, this wave found as a part of spectrum of ordinary waves, which is in accordance with Ref. 34.

The spin dynamics contributions in the dielectric permeability tensor appear to be proportionaL to one of the following dimensionless constants: \( \epsilon^2 n_0^{1/3} / mc^2 \) and \( mc n_0^{1/3} / \hbar B_0 \sim \epsilon^2 n_0^{1/3} / \Omega_c \). They present the ratio of the average Coulomb interaction energy to the rest energy of electron, and the ratio of the average Coulomb interaction energy to the quanta of cyclotron oscillation, correspondingly.

There is a change of the triplet structure of the first cyclotron wave due to the spin polarization change (see Fig. 3). Some conclusion is correct for the zeroth cyclotron wave (see Fig. 4). However, there is a modification of dispersion curve of the upper line of the triplet of the second cyclotron wave. The modification of the upper curve due to the spin polarization change is demonstrated in Fig. 6. Change of the light and dark areas shows the oscillatory structure of the dispersion curve and its change. The monotonic increase of the maximal deviation with the increase of the spin polarization is demonstrated either.

The upper line in the triplet of the first cyclotron wave has a deviation towards lower frequencies. The middle and lower lines of the triplet of the first cyclotron wave are deviated towards smaller frequencies. The increase of the spin polarization makes the oscillatory structure of curves is less noticeable, but maximal deviation increases for all waves in the triplet. Similar picture is found for
FIG. 7: (Color online) The figure shows the electrostatic Bernstein modes in two regimes. The dashed red lines show the classic spectrum existing in degenerate electron gas with no spin polarization. The continuous green lines show the spectrum with account of the separate spin evolution. It is plotted for \( n_0 = 10^{21} \text{ cm}^{-3} \) and \( B_0 = 10^7 \text{ G} \).

FIG. 8: (Color online) The figure shows the dispersion curves for Bernstein modes with account of the transverse electric field and the separate spin evolution (green continuous lines) and dispersion curves for Bernstein modes in the electrostatic regime with the account of separate spin evolution (black dashed curves).

FIG. 9: (Color online) The figure shows a small modification of the spectrum of the extraordinary wave at the account of the separate spin evolution. The green continuous lines show spectra of the Bernstein modes and extraordinary wave without of the separate spin evolution. The black dashed lines present the spectra with the account of the separate spin evolution. The figure is plotted for the following parameters: \( n_0 = 10^{19} \text{ cm}^{-3} \) and \( B_0 = 10^7 \text{ G} \).

B. Elliptically polarized modes: Bernstein modes, extraordinary wave

This subsection is devoted to the longitudinally-transverse waves. Neglecting the transverse part they reduces to the electrostatic modes: Langmuir (hybrid) wave, Bernstein modes.

The dielectric permeability tensor elements contained in equation (24) have the following form:

\[
\varepsilon_{xx} = 1 - \sum_{s=\uparrow,\downarrow} \sum_{n=1}^{\infty} \frac{n^2 \Omega_n^2}{k_x^2 v_{Fs}^2} \frac{3 \omega_{Ls}^2}{\omega^2 - n^2 \Omega_n^2} \int \sin^3 \theta J_n^2 d\theta, \tag{26}
\]

\[
\varepsilon_{yy} = 1 - \sum_{s=\uparrow,\downarrow} \left\{ \sum_{n=1}^{\infty} \frac{1}{\omega^2 - n^2 \Omega_n^2} \left[ 3 \omega_{Ls}^2 \int \sin^3 \theta J_n^2 d\theta \right. \right.
\]

\[
\left. + 2 \frac{m^2 v_{Fs}^2}{\pi \hbar^3} k_x^2 c^2 v_{Fs} \int \sin \theta J_n^2 d\theta \right. \right.
\]

\[
\left. + 4(-1)^i \frac{\gamma_e v_{Fe}}{\pi \hbar^3} m^2 v_{Fs}^2 k_x c \int \sin^2 \theta J_n J_n' d\theta \right] + 2 \frac{m^2 v_{Fs}^2}{\pi \hbar^3} \mu_e \frac{k_x^2 c^2}{\omega^2} \int \sin \theta J_n^2 d\theta
\]

\[
\left. + \frac{3 \omega_{Ls}^2}{2 \omega^2} \int \sin^3 \theta J_0^2 d\theta + \frac{\mu_e^2}{\pi \hbar^3} \frac{k_x^2 c^2}{\omega^2} m^2 v_{Fs} \int \sin \theta J_0^2 d\theta \right. \right.
\]

\[
\left. + 2(-1)^i \frac{\gamma_e v_{Fe}}{\pi \hbar^3} m^2 v_{Fs}^2 \frac{k_x c}{\omega^2} \int \sin^2 \theta J_0 J_0' d\theta \right\}, \tag{27}
\]
and

$$\varepsilon_{xy} = - \sum_{s=\uparrow, \downarrow} \sum_{n=1}^{\infty} \frac{2n^2\Omega^2}{\omega^2 - n^2\Omega^2} \left[ \frac{3\omega^2}{2\omega} \frac{1}{k_x v_{F_{s}}} \times \int d\theta \sin^2 \theta J_n^m J_{n'}^m + (-1)^{n} \frac{m^2 v_{F_{s}} q_e \mu_e c}{\omega} \frac{\Omega_{\mu}}{\pi h^3} \int d\theta \sin \theta J_n^2 \right].$$

(28)

$\varepsilon_{xx}$ is the longitudinal dielectric permeability. Equation (20) shows that the spin dynamics gives no contribution in $\varepsilon_{xx}$. The separate spin evolution related to the equilibrium spin polarization modifies $\varepsilon_{xx}$ in compare with the classic model. $\varepsilon_{yy}$ and $\varepsilon_{xy}$ are modified by the spin dynamics. The spin related terms are proportional to $\mu_e$. Hence, they can be easily identified. All nontrivial terms in $\varepsilon_{xx}$, $\varepsilon_{xy}$ and $\varepsilon_{yy}$ are proportional to $(\omega^2 - n^2\Omega^2)^{-1}$, with n=0, 1, 2, ... since equations (20)-(28) are caused by dynamics of $\delta f$ and $\delta S_z$. $\Omega_{\mu}$ does not enter the denominators in equations (20)-(28). So, there is no fine structure of the Bernstein modes appearing as solutions of equation (21). The triplet fine structure of the cyclotron waves is found in the previous subsection. The triplet fine structure of the Bernstein modes is found in Ref. [33] in terms of another form of spin-1/2 kinetic model, but it does not appear in the described model.

In the electrostatic limit the dispersion equation reads $\varepsilon_{xx} = 0$. Several solutions of this equation are shown in Fig. 7. The separate spin evolution modifies the oscillatory structure of the Bernstein modes. The modification becomes more prominent with the increase of the mode number. Next, include the transverse electric field. To this end, the dispersion equation (24) is solved, where $\varepsilon_{xx}, \varepsilon_{xy}$ and $\varepsilon_{yy}$ are considered in accordance with equations (20)-(27) dropping terms proportional to the magnetic moment $\mu_e$. Corresponding results are presented in Fig. 8. It shows that the spectrum of Bernstein modes is not changed, but additional lines appear. Finally, the terms caused by the spin dynamics are to include. In this regime, the full elements of the dielectric permeability tensor presented by equations (20)-(27) are employed. Comparison of coefficients in classic and spin terms shows a relative increase of the spin terms at the large wave vector.

Modification of the extraordinary wave spectrum due to the separate spin evolution is found and presented in Fig. 9. It is rather small, but noticeable contribution. The spin dynamics in considered regime gives even smaller contribution which is not presented in figures.

This subsection shows that the major contribution in the Bernstein modes and extraordinary wave spectra in the considered parameter regime gives the separate spin evolution of electrons.

IV. CONCLUSION

Two groups of results have been found. One is related to the transverse linearly polarized (ordinary) waves propagating perpendicular to the external magnetic field with the electric field perturbation parallel to the external magnetic field. The second group of results is related to the longitudinally-transverse elliptically polarized (extraordinary) waves propagating perpendicular to the external magnetic field with the electric field perturbation perpendicular to the external magnetic field.

The following results are found in the first regime. The spectrum modification of the cyclotron waves and the triplet fine structure for each cyclotron wave existing due to combination of spin effects have been demonstrated. Properties of the lowest spin-cyclotron wave with the frequency near 0.001 $|\Omega_{e}|$ have been studied for the partially spin polarized degenerate electron gas. The anomalous magnetic moment leads to the fine structure formation and appearance of the lowest cyclotron wave. However, several effects contribute into the form of dispersion curves. They are the classic terms modified by the separate spin evolution and terms caused by the spin dynamics (they also contain the separate spin evolution). The spin terms are specific in two ways. They give a noticeable contribution even if the anomalous magnetic moment is neglected, but the anomalous magnetic moment causes additional effects like the fine structure. Relative contributions of these effects are traced analytically and numerically.

The Bernstein modes are considered in the second regime. It has been found that the separate spin evolution modifies the oscillatory structure of the Bernstein modes. The modification increases with the increase of the mode number. This effects remains at the account of the transverse field. Similar modification is found for the extraordinary wave.

First of all the obtained results are important for the magnetically ordered conductors and semiconductors. They can be applied to the degenerate astrophysical plasmas with the large spin polarization of charge carriers either.

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