Secure Distributed Membership Tests via Secret Sharing

How to Hide Your Hostile Hosts: Harnessing Shamir Secret Sharing

Abstract—Data security and availability for operational use are frequently seen as conflicting goals. Research on searchable encryption and homomorphic encryption are a start, but they typically build from encryption methods that, at best, provide protections based on problems that are computationally hard. By contrast, data encoding methods such as secret sharing provide information-theoretic data protections. Archives that distribute data using secret sharing can provide data protections that are resilient to malicious insiders, compromised systems, and untrusted components.

In this paper, we create the Serial Interpolation Filter, a method for storing and interacting with sets of data that are secured and distributed using secret sharing. We provide the ability to operate over set-oriented data distributed across multiple repositories without exposing the original data. Furthermore, we demonstrate the security of our method under various attacker models and provide protocol extensions to handle colluding attackers. The Serial Interpolation Filter provides information-theoretic protections from a single attacker and computationally hard protections from colluding attackers.

I. INTRODUCTION

Computer and data compromises have become so frequent it is almost cliché to cite them as motivation for cyber-security research. Rather than believing every attack can be prevented, effective organizations must operate under the expectation of eventual compromise. Typical data protection methods such as symmetric encryption depend on the enduring privacy of a single key and thus can be insufficient for long-term data security.

Distributed repositories using data splitting and encoding techniques have been presented as promising ways to provide data security even with compromised components. These systems use encodings such as secret sharing to split the bytes of a file into shares, any $k$ of which can be used to recreate the original data. Once the shares have been generated, they are then distributed across multiple repositories. This approach creates an archive that is resilient to insider threat and is able to ensure data privacy and integrity with as many as $k-1$ repositories compromised. Additionally, secret sharing has the benefit of being information-theoretically secure, unlike many cryptographic methods which are based on problems assumed to be computationally hard.

Most previous work on secret sharing has assumed access to the stored information will occur through full reassembly of the data. As a result, this data would then exist in a single location and be significantly more vulnerable than when it was split into shares. For several use cases, such as set membership, full assembly of the secret may not be necessary. Thus, performing a full assembly is an unjustifiable security risk.

To illustrate how such a system would work, we present an example of five companies who want to share a list of known bad IP addresses. No one company trusts any other individual company, but they trust the group as a whole. Any company should be able to query and insert IP addresses, but none should be able to access the entire list. Using secret sharing, the companies split, exchange, and store the addresses. While the addresses are secure at rest, a mechanism for query without reconstruction is necessary.

To address this need, we present the Serial Interpolation Filter (SIF), which allows collaborators to store a set of values and support membership queries and inserts without exposing the original values. Our contributions include:

- an information-theoretically secure method for operating on set-oriented data stored across multiple repositories.
- a security and performance analysis of our method, demonstrating the ability to maintain data confidentiality in noncollusive environments while maintaining performance.
- an extension to SIF using the discrete logarithm as a cryptographic trapdoor to mitigate colluding adversaries.

The rest of the paper is organized as follows: we review related work and background information in Section II, describe the Serial Interpolation Filter in Section III, analyze our
method in Section V discuss handling collusion in Section VI and conclude our work in Section VII.

II. BACKGROUND AND RELATED WORK

In this section, we present an overview of secret sharing, including recent advances in the area, and compare our work with searchable encryption.

A. Secret Sharing

Shamir [6] originally developed secret sharing as an information-theoretically secure approach to share and store a secret amongst a group with \( N \) members but reconstruct the information with only \( k \) of the members. The algorithm shares a secret amongst multiple participants by selecting a polynomial of at most degree \( k - 1 \), setting the \( y \)-intercept of the polynomial to be the desired secret, and distributing points on the polynomial with non-zero \( x \) values.

To demonstrate how Shamir’s algorithm works, suppose Christine has a safe with the combination \( d = 854 \). In an emergency, Christine wants to allow any three of the five people from her office to combine their information to open her safe. She would also like to prevent any fewer than three people from gaining any information about her combination. To do so, she encodes the combination, \( d = 854 \), into \( N = 5 \) shares and requires \( k = 3 \) of these shares to recover the secret. To do this, she generates a random polynomial of degree at most \( k - 1 \), since any three points can uniquely identify this polynomial:

\[
p(x) = d + 276x + 53x^2.
\]

Christine’s safe combination is encoded as the value of a polynomial curve at \( x = 0 \). Christine now creates shares by evaluating the polynomial at \( x \)-values other than \( 0 \). By combining any three of these points, Christine’s officemates can solve for the original polynomial and recover the combination. However, with only two shares, there are as many possible intercepts as there are possible values for \( d \), all equally likely. Fewer than three shares divulges no information about the combination. Figure 1 illustrates how, given two specific points, any possible value of \( d = p(0) \) is an equally likely solution.

In practice, we typically perform all secret sharing operations over a finite field. This allows us to choose polynomial coefficients randomly over a uniform distribution over the elements of the field and obviates floating point errors [6].

This area continues to be an active field of research with many different variations of secret sharing [6]–[11]. Notably, Narayanan et al. discuss information-theoretically secure membership tests for a secret-shared set [12]. However, the work assumes the set is static and the user issuing queries cannot collude with shareholders.

B. Searchable Encryption

This work is akin to searchable encryption [13] and homomorphic encryption algorithms [14], [15]. Such algorithms allow a user to perform certain operations such as remote keyword searches or computing functions on encrypted data on an untrusted server without decrypting the information or pulling all of the encrypted data back to the user. Recent advances in searchable encryption include multi-keyword ranked searching on cloud data with low computation [16] and work with limited size on a user’s mobile device [17].

However, the fundamental security protections provided by encrypted data have many limitations. An attacker that compromises the single host gains access to all of the data. Solutions to searching encrypted data are computationally secure rather than information-theoretically secure and depend on the assumed hardness of certain problems. Homomorphic encryption is typically very expensive, and while specialized hardware exists, it is limited in utility. Even encryption with multiple hosts does not capture the benefits of secret sharing because the data is replicated instead of split into shares. To the best of our knowledge there has been little work on operating on data in a secret-shared archive.

III. SERIAL INTERPOLATION FILTER

In this section, we provide a detailed description of the Serial Interpolation Filter method. We show this method can enable operational use of the data while maintaining information-theoretic data security, a protection well above commonly used symmetric encryption techniques.

A. Using Secret Sharing to Store a Set

We draw from previous work [2]–[5] that proposed using secret sharing to create a secure, distributed archive. While the archive is usable for any type of data representable as a set, we use IP addresses as our exemplar.

A trusted user creates a new entry after externally determining a specific IP is untrusted. To securely store this element, it is split into \( N \) shares using a \((k, N)\) linear secret sharing scheme [6]. As shown in Figure 2 once the shares are created, they are then distributed across \( N \) separate repositories to create the archive holding the list of known bad IP addresses. For example, if there are 20 bad addresses, each of the \( N \) repositories would hold 20 shares unique to that repository. The aggregate of these repositories is an archive securely storing 20 addresses. Once the shares have been distributed,
the user deletes the original data. After this, the list exists solely as an abstract concept and no single repository holds data recognizable as an IP address (or component thereof) in isolation. It is this unique property that separates such solutions from systems using encryption for data security.

When one of the repositories is inevitably compromised \[1\], the attacker learns nothing beyond the size of the set of elements. Even additional compromises, as long as they number less than \(k\), do not let the attacker gain any additional information about the list contents. Ideally, the consortium is able to refresh the shares with new polynomials using techniques such as those by Herzberg et al. \[11\] before the intruders have enough shares to reconstruct the data.

### B. Terminology

We define the following terms:

- **Element**—One entry in the set of data. We wish to simultaneously protect and use these elements.
- **Share**—A resulting datum from using secret sharing to encode an element into \(N\) pieces.
- **Threshold**—The number of shares, denoted as \(k\), needed to recover the original element.
- **Repository**—A remote server storing a unique set of shares.
- **Archive**—The collection of all \(N\) repositories. Together they create a system for securely storing and operating on data elements.

We perform all secret sharing operations over a finite field \(F\). The data is a set \(D = \{d_\ell|d_\ell \in F, 1 \leq \ell \}\) and is of size \(|D|\). We split each data element \(d_\ell \in D\) using a polynomial, \(p_\ell(x)\), of order \(k - 1\). The polynomial takes the form

\[
p_\ell(x) = d_\ell + a_{\ell,1}x + \cdots + a_{\ell,k-1}x^{k-1}
\]

where \(a_{\ell,i} \in F\) is a coefficient chosen uniformly at random for \(\ell \in \{1, \ldots, |D|\}, i \in \{1, \ldots, k - 1\}\).

We denote the list of shares given to Repository \(r\) as \(\tilde{p}(x_r)\), which is the vector of all of the polynomials evaluated at \(x_r\). We use the vector notation to represent interpolation performed on all shares concurrently.

### C. Serial Lagrangian Interpolation

In traditional secret sharing, given a set of \(k\) points, \(S = \{x_1, \ldots, x_k\}\) and their corresponding shares \((p(x_r)\) for \(x_r \in S\)), we can use Lagrangian interpolation to reconstruct the generating polynomial, \(p(x)\) as

\[
p(x) = \sum_{i=1}^{k} L_{i,S}(x)p(x_i),
\]

where

\[
L_{i,S}(x) = \prod_{j \in S, j \neq i} \frac{x - x_j}{x_i - x_j}.
\]

This allows for recovery of the original element by evaluating \(p(0)\).

The archive stores the elements as the \(y\)-intercepts of polynomials, \(\tilde{d} = \tilde{p}(0)\). Here, we use the vector notation to denote operation over all elements in the set. In our example we select a finite field able to represent \(2^{32}\).

To avoid reconstructing the polynomials in a single location, we perform Lagrangian interpolation serially across a subset of \(k\) out of \(N\) repositories. The list of \(k\) repositories to be used is provided as a part of the SIF query and known to all of the repositories involved. Furthermore, the data of each repository, \(\tilde{p}(x_r)\), is private and known only to the repository owning that data. Therefore, while all of the repositories can compute any of the \(L_{i,S}(x)\), only Repository \(r\) has access to \(\tilde{p}(x_r)\). Thus it is possible to calculate \(\tilde{p}(0)\) across three repositories \((k = 3)\) as follows:

\[
\tilde{p}(0) = \frac{x_2x_3}{(x_1 - x_2)(x_1 - x_3)}\tilde{p}(x_1) + \frac{x_1x_3}{(x_2 - x_1)(x_2 - x_3)}\tilde{p}(x_2) + \frac{x_1x_2}{(x_3 - x_1)(x_3 - x_2)}\tilde{p}(x_3).
\]

Nevertheless this calculation would reassemble the original secret in a single location. To perform a set membership test while not exposing the polynomial or query term, we must perturb both the query and the polynomial reconstruction with a vector of nonces, or randomly generated constants. This process is described in detail in the next section.
D. Generalized Algorithm

Once the shares have been distributed, we can now use the SIF to test for set membership. We present the query algorithm a user at a repository would perform to determine if the value \( Z \) is present in the archive. An example query round and user response for a \((k = 3, N = 5)\) secret sharing can be seen in Figure 3.

1) A user at repository \( R_1 \) initiates the query with a unique transaction identifier \( q \). \( R_1 \) then selects \( k \) repositories including itself and creates an ordered list of these repositories \( S = \{x_1, \ldots, x_k\} \). It generates a nonce vector \( (\nu') \) containing a different random pad for each element and calculates \( \bar{\gamma}_1 \) as follows:

\[
\bar{\gamma}_1 = L_{1,S}(0)\bar{p}(x_1) + \nu'
\]

2) \( R_1 \) sends a membership test message \((m_1 = [q, \bar{\gamma}_1, S])\) to the next repository listed in \( S \). In Figure 3, \( R_1 \) sends the message to \( R_2 \).

3) \( R_1 \) calculates a nonce query term \( \bar{Q} = Z\bar{1} + \nu' \) and sends a separate message, \( m_Q = [\bar{Q}, \bar{r}_1] \), to the \( k^{\text{th}} \) repository. As \( k = 3 \) in Figure 3, this message is sent from \( R_1 \) to \( R_3 \).

4) \( R_i \) for \( i \in \{2, \ldots, k-1\} \) receives the message from \( R_{i-1} \) and serially calculates its portion of the interpolation polynomials

\[
\bar{\gamma}_i = L_{i,S}(0)\bar{p}(x_i) + \bar{\gamma}_{i-1}
\]

and sends \( m_i = [q, \bar{\gamma}_i, S] \) to repository \( R_{i+1} \).

5) \( R_k \): Calculates the final contribution to the interpolations

\[
\bar{\gamma}_k = L_{k,S}(0)\bar{p}(x_k) + \bar{\gamma}_k
\]

and compares each component in \( \bar{\gamma}_k \) to each corresponding component of the query terms \( (\bar{Q}) \) received in message \( m_Q \) in step 3. If a match is found, the query response is set to true and false otherwise.

6) \( R_k \) sends the results of the query to the request originator \( R_1 \) \((m_r = [q, \text{True} | \text{False}])\). This final response is seen as the gray arrow returning from \( R_3 \) to the user at \( R_1 \) in Figure 3.

The user \( U \) can determine if the value \( Z \) is in the set while no other user learns the value of \( Z \). Additionally, none of the secrets \((\bar{d} = \bar{p}(0))\) are ever reconstructed in a single location. In short, the SIF protocol has enabled \( U \) to actively query the data while maintaining information-theoretic levels of data protection. In Section IV we will discuss these protections in detail and explain some limitations around adversarial collusion.

IV. Analysis

In this section, we present the threat models we will use for subsequent security and performance analyses.

A. Threat Model

In analyzing the Serial Interpolation Filter, we focus on the confidentiality of the stored data and minimizing release of information due to legitimate user queries. We assume users are authenticated to the service and all legitimate users have the right to place queries, which results in the user learning if the item was contained (or not) in the set, but learning nothing else about the set. We assume the presence of secure communications channels.

With this in mind, we consider two adversary models:

- Honest-but-curious participants
- Byzantine participants

We start with the most restrictive model from the attacker’s standpoint and gradually relax these assumptions while strengthening our protocol. In the honest-but-curious model, the attacker (i.e., any participating party including the end-user) must correctly follow all parts of the protocol, which includes sending correct protocol responses. The attacker is allowed to perform extra calculations and store previous protocol values, but is not allowed to actively aggregate information it would not normally have received during protocol participation (i.e., no collusion). Relaxing our assumptions under a Byzantine model, we consider a system containing \( N \) participants and a bounded number of malicious nodes \( 0 \leq f < k \), where \( k \) is the threshold value for the scheme. The malicious nodes behave arbitrarily and are only limited by the constraints of any cryptographic methods deployed \([18]\), which are assumed resistant to tampering. The set of malicious nodes may collude.

As we focus on the confidentiality of the data and the privacy of the queries, attacks targeting the integrity (e.g., reporting incorrect shares) or availability (e.g., withholding shares) of the secret sharing algorithm are outside the scope of this paper. Techniques such as proactive, public, or verifiable secret sharing \([3]–[11]\) can be used to augment our solution to alleviate many of these issues, but are left for future work.
B. Security

Fundamentally, the security of the data rests on the privacy of each repository’s shares, \( p(x_i) \). It is the ability to aggregate or otherwise calculate these values that constitutes a loss of data protections. We also note that given fewer than \( k \) shares, as proved by Shamir and others, an attacker gains nothing. Intuitively, this is the same as defining a specific parabola given only two points as shown in Figure 1 (i.e., there are as many possible coefficients as there are distinct elements in the field).

For the honest-but-curious model, the repositories are unable to calculate any specific shares held by other repositories. For \( R_2 \) through \( R_k \), \( \gamma_i \) is additively perturbed by \( \vec{v} \). Assuming strong random number generation, this provides the same protections as a one-time pad for all repositories except \( R_1 \). Since \( R_1 \) only receives a true/false result it has no further information about the shares at any of the other repositories. The result is our system maintains information-theoretic protections on data confidentiality.

Under our Byzantine model, \( \gamma_k \) is protected only by \( \vec{v} \) and malicious repositories may share partial results. Thus, collusion between \( R_1 \) who holds \( \vec{v} \) and \( R_k \) would provide access to the complete set of secrets. While this vulnerability to collusion provides data exposure, we note this is not the case for the data at rest and we still provide data protections better than those of typical encryption-based methods. Nevertheless, in Section V we present a method using discrete logarithms to provide computationally hard protections against collusion.

C. Performance

Recall from the generalized SIF algorithm that \( k \) denotes the number of repositories involved in one query round. Furthermore, we use \(|D|\) to denote the number of data elements in the list. We assume it takes \( O(1) \) to compute a nonce and to send a single data element. It takes \( O(k) \) work to evaluate the Lagrange polynomial at each of the \( k \) repositories in the round, resulting in \( O(k^2 + |D|) \) work to carry out the entire interpolation.

Additionally, each message passed contains \( O(|D|) \) data elements and \( O(k) \) repository labels and therefore takes \( O(|D| + k) \) work per message. Therefore, the work for message transfers in a given round of the protocol is \( O(|D|k + k^2) \).

The total work includes both the transfers and interpolation giving a final result of \( O(|D|k + k^2) \). We generally expect \(|D| > k\), that is, the number of data elements vastly exceeds the number of repositories necessary to carry out a query. The computation is dominated by the messages sent rather than the calculations done at each repository.

Similar arguments can be made for insertion using \( O(N) \) messages and \( O(N) \) computation.

V. MITIGATING BYZANTINE ADVERSARIES

As was demonstrated in Section IV-B, the SIF protocol is resilient to honest-but-curious adversaries but does not hold these guarantees in the face of adversarial collusion. In this section, we present a method for maintaining security in Byzantine environments based on computational guarantees.

By utilizing a cryptographic trapdoor function based on the discrete logarithm problem [19], we are able to create computational SIF (cSIF) that is secure given at most \( k - 1 \) adversaries. Consider a (multiplicative) cyclic group \( \mathbb{C}_q \) of order \( q \) with generator \( g \) where the discrete logarithm problem is assumed hard [20]. Under this scenario, we now present the cSIF query algorithm a user at a repository would perform to determine if the value \( Z \) is present in the archive. Original share distribution and storage remains unchanged.

We present some notation regarding vectors. \( g^\vec{v} \) represents a vector where the \( i \)-th component is \( g^{\vec{v}_i} \), the generator raised to the power of the \( i \)-th component of \( \vec{v} \). The operator \( \odot \) denotes component-wise multiplication. For example, each component \( c_i \) of \( \vec{c} = \vec{a} \odot \vec{b} \) is defined as \( c_i = a_i b_i \).

1) A user at repository \( R_1 \) initiates the query with a unique transaction identifier \( q \). \( R_1 \) then selects \( k \) repositories including itself and creates an ordered list of these repositories \( S = \{x_1, \ldots, x_k\} \). It then generates a nonce vector \( \vec{v} \) and calculates \( \gamma_i \) as follows:

\[
\gamma_i' = g^{f_{i,0}(\vec{v})p(x_i) + \vec{v}}
\]

2) \( R_1 \) sends a membership test message \( (m_1 = [q, \gamma_1, S]) \) to the next repository listed in \( S \) (\( R_2 \)).

3) \( R_1 \) calculates a nonce query term \( Q = g^{x_1 + \vec{v}} \) and sends a separate message to the \( k \)-th repository \( (m_Q = [q, Q]) \).

4) \( R_i \) for \( i \in \{2, \ldots, k - 1\} \) receives the message from \( R_{i-1} \) and serially calculates its portion of the interpolation polynomial

\[
\gamma_i = \gamma_i' \odot g^{f_{i,0}(0)p(x_i)}
\]

and sends \( m_i = [q, \gamma_i, S] \) to repository \( R_{i+1} \).

5) \( R_k \): Calculates the final contribution to the interpolation

\[
\gamma_k = \gamma_k' \odot g^{f_{k,0}(0)p(x_k)}
\]

and compares each component in \( \gamma_k \) to each corresponding component of the query terms \( Q \) received in step 3. If a match is found, the query response is set to true and false otherwise.

6) \( R_k \) sends the results of the query to the request originator \( R_1 \) \((m_r = [q, True|False])\).

A. Security Sketch

The constituents of \( \gamma_i \) for \( i \in \{1, \ldots, k\} \) are now protected as exponents of the generator function (e.g., \( g^{f_{i,0}(0)p(x_i)} \)).

Assuming the discrete logarithm is hard, the colluding repositories cannot compute the logarithm of the messages and find the original share values. This would be equivalent to solving the Computational Diffie-Hellman problem [21]. Thus, even under the Byzantine model which allows for collusion between \( R_1 \) who holds \( \vec{v} \) and \( R_k \) who holds \( \gamma_k \), the two repositories cannot gain any additional stored secrets. This holds true for any subset of \( k - 1 \) colluding repositories which do not have enough shares to recreate the original data. The primary use of the nonce vector in this case is to blind query messages and responses.
VI. CONCLUSIONS AND FUTURE WORK

Modern systems desperately need a way of securing data in spite of compromise. While techniques like Shamir’s Secret Sharing have been around for decades, the ability to operationally use data without exposing the original information provides an important capability in efforts to ensure secure and resilient computer systems. Our Serial Interpolation Filter uses the strong data protections from secret sharing to secure data at rest. In addition, we enable active use of data while maintaining information-theoretic levels of protection during a query. Although collusion between two members can expose the original data set, we show how a cryptographic trapdoor based on discrete logarithms can be used to enable computationally hard resilience to up to $k - 1$ colluding adversaries.

Future work includes minimizing the amount of data the SIF protocol must store and transfer using advanced data structures such as a quotient filters [22] or cuckoo filters [23]. Additionally, while the cSIF algorithm was able to tolerate collusion, we hope to extend SIF in a manner tolerating collusion among attackers while maintaining information-theoretic security guarantees.

ACKNOWLEDGMENTS

We would like to thank Cindy Phillips, Jonathan Berry, Michael Bender, Rob Johnson, and Jared Saia for their insights and thoughtful discussions. This work was supported by the Laboratory Directed Research and Development Program at Sandia National Laboratories.

REFERENCES

[1] L. Zaichkowsky. (2013) Active combat in the era of continuous compromise. [Online]. Available: http://www.infasecinsight.com/2013/ active-combat-in-the-era-of-continuous-compromise
[2] G. Ganger, P. Khosla, M. Bakkaloglu, M. Biggins, G. Goodson, S. Oguz, V. Pandurangan, C. Soules, J. Strunk, and J. Wylie, “Survivable storage systems,” in DARPA Information Survivability Conference amp; Exposition, 2001.
[3] A. Subbiah and D. M. Blough, “An approach for fault tolerant and secure data storage in collaborative work environments,” in ACM workshop on Storage security and survivability, 2005.
[4] M. W. Storer, K. M. Greenan, E. L. Miller, and K. Voruganti, “POTSHARDS—a secure, recoverable, long-term archival storage system,” ACM Transactions on Storage, vol. 5, no. 2, pp. 5:1–5:35, 2009.
[5] T. M. Kroeger, J. C. Frank, and E. L. Miller, “The case for distributed data archival using secret splitting with percival,” in International Symposium on Resilient Control Systems, 2013.
[6] A. Shamir, “How to share a secret,” Communications of the ACM, vol. 22, no. 11, pp. 612–613, 1979.
[7] J. Benaloh, “General linear secret sharing,” Microsoft Research, Tech. Rep., 1996.
[8] A. Beimel, “Secret-sharing schemes: a survey,” in Coding and cryptography. Springer, 2011.
[9] B. Chor, S. Goldwasser, S. Micali, and B. Awerbuch, “Verifiable secret sharing and achieving simultaneity in the presence of faults,” in IEEE Symposium on Foundations of Computer Science, 1985.
[10] R. Gennaro, M. O. Rabin, and T. Rabin, “Simplified vss and fast-track multiparty computations with applications to threshold cryptography,” in ACM symposium on Principles of distributed computing, 1998.
[11] A. Herzberg, S. Jarecki, H. Krawczyk, and M. Yung, “Proactive secret sharing or: How to cope with perpetual leakage;” in Advances in Cryptology. Springer, 1995.
[12] G. S. Narayanan, T. Aishwarya, A. Agrawal, A. Patra, A. Choudhary, and C. P. Rangan, “Multi party distributed private matching, set disjointness and cardinality of set intersection with information theoretic security,” in Cryptology and Network Security. Springer, 2009, pp. 21–40.
[13] D. X. Song, D. Wagner, and A. Perrig, “Practical techniques for searches on encrypted data,” in IEEE Symposium on Security and Privacy, 2000.
[14] C. Gentry et al., “Fully homomorphic encryption using ideal lattices,” in ACM Symposium on Theory of Computing, 2009.
[15] C. Gentry, “Computing arbitrary functions of encrypted data,” Communications of the ACM, vol. 53, no. 3, pp. 97–105, 2010.
[16] N. Cao, C. Wang, M. Li, K. Ren, and W. Lou, “Privacy-preserving multi-keyword ranked search over encrypted cloud data,” Parallel and Distributed Systems, IEEE Transactions on, vol. 25, no. 1, pp. 222–233, 2014.
[17] Y.-C. Chang and M. Mitzenmacher, “Privacy preserving keyword searches on remote encrypted data,” in Applied Cryptography and Network Security, 2005.
[18] D. Dolev and A. C. Yao, “On the security of public key protocols,” Information Theory, IEEE Transactions on, vol. 29, no. 2, pp. 198–208, 1983.
[19] D. R. Stinson, Cryptography: theory and practice. CRC press, 2005.
[20] A. M. Odlyzko, “Discrete logarithms in finite fields and their cryptographic significance,” in Advances in cryptology, 1985.
[21] D. Boneh, “The decision diffie-hellman problem,” in Algorithmic number theory. Springer, 1998.
[22] M. A. Bender, M. Farach-Colton, R. Johnson, R. Kraner, B. C. Kuszmaul, D. Medjedovic, P. Montes, P. Shetty, R. P. Spillane, and E. Zadok, “Don’t thrash: how to cache your hash on flash,” Proceedings of the VLDB Endowment, vol. 5, no. 11, pp. 1627–1637, 2012.
[23] B. Fan, D. G. Andersen, M. Kaminsky, and M. D. Mitzenmacher, “Cuckoo filter: Practically better than bloom,” in ACM International on Conference on emerging Networking Experiments and Technologies, 2014.