Superradiance Exclusions in the Landscape of Type IIB String Theory

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We obtain constraints from black hole superradiance in an ensemble of compactifications of type IIB string theory. The constraints require knowing only the axion masses and self-interactions, and are insensitive to the cosmological model. We study more than $10^9$ Calabi-Yau manifolds with Hodge numbers $1 \leq h^{1,1} \leq 491$ and compute the axion spectrum at two reference points in moduli space for each geometry. Our computation of the classical theory is explicit, while for the instanton-generated axion potential we use a conservative model. The measured properties of astrophysical black holes exclude parts of our dataset. At the point in moduli space corresponding to the tip of the stretched Kähler cone, we exclude $\approx 40\%$ of manifolds in our sample at $95\%$ C.L., while further inside the Kähler cone, at an extremal point for realising the Standard Model, we exclude a maximum of $\approx 11\%$ of manifolds at $h^{1,1} = 5$, falling to nearly zero by $h^{1,1} = 100$.

It has long been a dream of physicists working on quantum gravity and string theory to test their theories against observational and experimental data. In this regard, string theory has proven rather stubborn. Instead of a single low energy effective theory in four-dimensional spacetime, string theory provides a landscape of possible theories, including compactifications of superstring theories on six-manifolds such as Calabi-Yau threefolds (CY\textsubscript{3}'s) [1] and discrete quotients thereof. The effective theory depends on the topology of the internal space, for which there are astronomically many possibilities. Testing string theory is therefore challenging for several reasons: the key phenomena are of gravitational strength, there is a vast set of theories to explore, and most predictions rest on model-dependent constructions of cosmology and of the visible sector.

In the present work, we develop a statistical test of part of the landscape based on the spectrum of axions, which can be related directly to the properties of the CY\textsubscript{3}'s [2–7]. We use the observed properties of astrophysical black holes to put limits on axions whose masses and self-interactions are in a range allowing for superradiant instabilities. Such limits do not depend on the cosmological history, or on the detailed realization of the visible sector: they depend only on the Lagrangian of the dark sector, specifically that of axions, which we will compute in an ensemble of CY\textsubscript{3} compactifications.

\textit{Axions from the Kreuzer-Skarke Database:} CY\textsubscript{3} hypersurfaces can be constructed from suitable triangulations of four-dimensional reflexive polytopes. A complete database of all such polytopes, numbering 473,800,776, was constructed by Kreuzer and Skarke [8], and has been the subject of numerous studies [9–14].

Type IIB string theory contains a four-form field, $C_4$, in ten dimensions. Dimensional reduction of this four-form yields a number of axion-like fields (in the sense that they are pseudo-scalar phases) [9, 10], $\theta^i$:

$$\theta^i := \int_{D_i} C_4,$$

where $D_i$ is a closed four-dimensional submanifold. The size of a basis of such submanifolds is given by the Hodge number $h^{1,1}$, so the index $i$ labelling axions takes on values from 1 to $h^{1,1}$. In the Kreuzer-Skarke list, one finds $1 \leq h^{1,1} \leq 491$. This choice of solutions of string theory thus predicts an axiverse [2–7], a low energy theory containing a possibly large number of axions. The axions are one part of a complexified Kähler modulus field $T^i = \tau^i + i\theta^i$.

The axion fields have the Lagrangian:

$$\mathcal{L} = -\frac{M^2}{2} K_{ij} g^{\mu\nu} \partial_\mu \theta^i \partial_\nu \theta^j - \sum_{a=1}^{\infty} \left(1-\Lambda_a \cos(Q^a_n \theta^i + \psi^a)\right),$$

where $g^{\mu\nu}$ is the inverse of the spacetime metric, $K_{ij}$ is the Kähler metric, and the second term is the instanton potential. The instanton potential contains energy scales $\Lambda_a$, charges $Q^a$, and phases $\psi^a$. In Ref. [7] we demonstrated how all the ingredients in the axion potential can be computed. Given a triangulation of a reflexive polytope from the Kreuzer-Skarke database, one can directly compute the Kähler metric as a function of the moduli fields $\tau^i$.

For the instanton potential, we use a well-motivated model, namely that a generating set of homolorphic cycles — specifically, prime toric divisors — support the leading instantons [7]. Then the scales $\Lambda_a$ are computable in terms of the topological data of the CY\textsubscript{3} and the vevs of the moduli $\tau^i$. A strong deviation from this model, involving for example dominant contributions from instantons on non-holomorphic cycles, would be a striking finding in its own right [13]. In this work we set the phases $\psi^a \to 0$, which is well-justified when the number

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FIG. 1: Summary Statistics: a) The kernel density estimate (KDE) shows the distribution of $f_K$ for all triangulations in our dataset, up to $h_{1,1} = 491$. b) KDE of axion masses above the Hubble scale $H_0$ for $2 \leq h_{1,1} \leq 100$, with the approximate BHSR region shaded. c) Bar plot showing the fraction of effectively massless axions, and the fraction of axions in the BHSR region. d) Box plot for $f_K$ as a function of $h_{1,1}$, with median and interquartile ranges marked, and outliers shown as isolated points. e) Joint distribution of mass and perturbative decay constant, $(m, f_{\text{pert}})$ for all triangulations in the subsample with $1 \leq h_{1,1} \leq 15$. f) As (d) for $f_{\text{pert}}$ distributions of the same triangulations. Note that the colour scale labelling $h_{1,1}$ is different in the left hand column plots compared to the right.

of significant instantons is $\leq h_{1,1}$. We treat the general case in [16].

The moduli fields, $\tau_i$, in general need to be stabilised, i.e. given a potential and fixed to certain values. Schemes for this procedure are known for special cases [17, 18], but the problem of moduli stabilisation is not solved in generality. In Ref. [17] and in the following, we simply examine the axion theory at specific points in the moduli space of the $\tau_i$. The resulting theories may have light scalar fields $\tau_i$ and so are not necessarily realistic, but this does not preclude us from computing superradiance constraints from the axion sector at these points.

We place the moduli at specific locations in the stretched Kähler cone (SKC), which is the region of moduli space within which the curvature expansion of string theory is well-controlled. (As in [19], these restrictions
would need to be modified if the string coupling were extremely small.) We consider two points in the SKC. The first is the tip of the SKC, i.e. the point closest to the origin. The second is an interior point defined by rescaling the Kähler parameters until the volume \( \tau_{\text{min}} \) of the smallest prime toric divisor is \( \tau_{\text{min}} = 25 \approx 1/\alpha_{\text{GUT}} \) \cite{20,22}, such that D7-branes wrapping this divisor could support a visible sector with a realistic grand unified gauge coupling \( \alpha_{\text{GUT}} \). In more general constructions of the Standard Model the correct couplings may occur elsewhere in the SKC, but \( \tau_{\min} \approx 25 \) still provides a reasonable estimate of the point beyond which further dilation of the CY3 would make the visible sector too weakly coupled.

**Axion Spectra:** We construct the axion potential for these two points in the SKC for all CY3 hypersurfaces with \( h^{1,1} \leq 5 \), and for a random sample of 1000 manifolds for every \( 6 \leq h^{1,1} \leq 100 \). Using the optimisation suite PYGMO \cite{23}, we were able to minimise, to float64 precision, the axion potentials for the studied triangulations. Tests show that all axions that are not effectively massless (defined as \( m > H_0 \) with \( H_0 \approx 10^{-33} \) eV the Hubble scale today \cite{24}) are minimised at \( \theta = 0 \). We thus set \( \bar{\theta} = 0 \) here; cf. \cite{16}.

We construct the distributions of three quantities that describe the axion physics. First we compute the eigenvalues of \( K_{ij} \), which do not depend on the axion vevs. The canonically normalised field \( \phi^j \) is related to \( \theta^j \) by \( \theta^j = F^j_{i} U_{ik} \phi^k \), where \( F^j_{i} = \text{diag}[1/M_{\text{pl}} \sqrt{\text{eigs}(K)}] \) and \( U^{ij} \) is the unitary matrix that diagonalises \( K_{ij} \). Next we Taylor expand the potential around the minimum and compute the following quantities:

\[
H_{ij} = \frac{\partial^2 V}{\partial \theta^i \partial \theta^j}, \quad \lambda^j_{ijkl} = \frac{\partial^4 V}{\partial \theta^i \partial \theta^j \partial \theta^k \partial \theta^l}.
\]  
(3)

The first term is the Hessian matrix. Multiplying by the transformation matrices to the canonical basis gives the mass matrix: \( M_{ab} = H_{ij} U^i_a U^j_b F_{a} F_{b} \). The eigenvalues of \( M_{ij} \) give the axion masses-squared, \( \{m^2_a\} \), and \( M_{ij} \) is diagonalised by the unitary matrix \( V^{ij} \), defining the mass eigenbasis, \( \phi^i \), from \( \phi^j = V^j_i \phi^i \). Rotating to the canonically normalised mass eigenbasis leads to the interaction term in the Lagrangian \( V_{\text{int}} = \frac{1}{2!} \lambda^i_{ijkl} \phi^i \phi^j \phi^k \phi^l \), where \( \lambda^i_{ijkl} \) is the axion self-interaction tensor in the canonical basis. We define the perturbative decay constant, \( f_{\text{pert},i} \), using the diagonal elements of \( \lambda \) in the mass eigenbasis: \( f_{\text{pert},i}^2 = m_i^2 / \lambda_{iii} \). Due to the numerical cost of evaluating \( \lambda_{ijkl} \) accurately in the presence of large hierarchies in the instanton scales, we were only able to compute \( f_{\text{pert}} \) for \( h^{1,1} \leq 15 \).

A statistical summary of our results for the Kähler eigenvalues (expressed as \( f_K = M_{\text{pl}} \sqrt{\text{eigs}(K)} \)), axion masses, and perturbative decay constants, \( f_{\text{pert}} \) (where available) is shown in Fig. 1 for moduli at the tip of the SKC. A full discussion of our results is presented in Ref. \cite{16}. Summary statistics are collated for fixed \( h^{1,1} \) only for presentation: for constraints we study each CY3 separately.

![FIG. 2: Example exclusion functions for two typical BH's. The left panel shows a supermassive BH, and the right panel a stellar mass BH. BHSR operates over a resonant region in \( m \), and is shut off by the Bosenova process at large values of the quartic coupling, parameterised by \( f_{\text{pert}} \).](image)

The distributions for an individual CY3 display mild scatter around the average for \( h^{1,1} \). As \( h^{1,1} \) increases, the distribution of masses approaches an almost universal shape that is close to log-flat. The universal nature can be seen, for example, from observing that as \( h^{1,1} \) increases the fraction of massless axions, and of axions in any fixed mass window, approach almost constant values.

The quantities \( f_K \) and \( f_{\text{pert}} \) follow statistically similar distributions and trends with \( h^{1,1} \). Both are approximately log-normal distributed, with the mean decreasing as \( h^{1,1} \) increases. This trend can be understood geometrically from the increasing cycle sizes required by the greater topological complexity \cite{7}. Inside the SKC, the results are qualitatively the same, but with all \( f^j \)’s shifted down by approximately two orders of magnitude, and with more than 95% of fields having \( m < H_0 \).

**Black Hole Superradiance (BHSR):** Given the axion masses and quartic couplings, one can compute the effect of the axions on astrophysical black holes (BH’s). A Kerr BH has an ergoregion in which causal curves must co-rotate, leading to a spacelike time-translation Killing vector field external to the event horizon \cite{25,27}. The geometric nature of this region leads to growth of bosonic vacuum fluctuations \cite{28,33}. Quasi-bound-state modes, with a frequency \( \omega \) satisfying the condition \( \omega < \mu \Omega_H \), with \( \Omega_H \) the angular velocity of the BH horizon and \( \mu \) the angular momentum about the BH spin axis, return an associated negative Killing energy flux at the horizon. Energy conservation dictates that the external field source observed at spatial infinity must grow, at the cost of a reduction of the BH’s angular momentum.

This process leads to the superradiant instability, which occurs when the axion Compton wavelength is approximately equal to the radius of the ergoregion. The evolutionary timescale, \( \Gamma_{\text{SR}} \), can be estimated via analytical approximations \cite{25,34,35}. Comparing these so-
lutions to characteristic timescales for the evolution of a BH, and utilising measurements of known BH masses and spins, leads to exclusions on the axion mass, $m$.

Non-linear phenomena\cite{25,29,40} may inhibit the exponential amplification of the dominant state, quenching the instability. When the self-interactions between the bosons are attractive, the cloud undergoes a rapid collapse known as a Bosenova, shutting down the superradiant instability at a critical occupation number proportional to the strength of the quartic coupling $\lambda_{49}$. This leads to a two-dimensional exclusion function in the domain of the axion mass and self-interaction strength, with the latter parameterised by $f_{\text{pert}}$.

The tensor $\lambda_{ijkl}$ contains off-diagonal components that allow axions in the superradiant cloud to decay and annihilate to lighter axions, which could compete with SR. We find that the off-diagonal components are smaller than the diagonal ones by many orders of magnitude, and so these processes can be neglected.

We adopt the detailed model for superradiance, and the BH data compilations, presented in Refs. 50, 51. We pre-compute the exclusion probability for each BH in the plane $(m, f_{\text{pert}})$: an example is shown in Fig. 2 for two typical BH’s. For any manifold, once the set of $(m, f_{\text{pert}})$ is determined, then the model is excluded if even one axion falls into the exclusion region.

As noted, we were only able to compute $f_{\text{pert}}$ for $h^{1,1} \leq 15$. However, since $f_K$ were found to follow the same distribution as $f_{\text{pert}}$, we estimated $f_{\text{pert}}$ for $h^{1,1} > 15$ by random permutations of the $f_K$, which are available for all $h^{1,1}$. To account for a possible $(m, f)$ correlation at large $h^{1,1}$, we sampled for two values of the correlation coefficient: $r = 0$ (uncorrelated), and $r = 0.25$, which is the $(m, f_{\text{pert}})$ correlation we observed at the largest $h^{1,1}$ for which $f_{\text{pert}}$ is available. Changing the correlation did not impact our constraints.

Constraints on the Landscape: Fig. 3 shows our main result: the fraction of excluded CY’s at each value of $h^{1,1}$ for moduli at the tip of the SKC, and for moduli inside the SKC. We show the constraints both with and without the effect of self-interactions, illustrating the importance of the latter. The self-interactions lead to a weakening of constraints at higher $h^{1,1}$, where the trend for lower $f_{\text{pert}}$ and $f_K$ leads to stronger self-interactions, and thus higher probability of a Bosenova.

At the tip of the SKC, the excluded fraction of CY’s rises rapidly with $h^{1,1}$ as the mass distribution spreads to encompass the BHSR region, reaching $\approx 50\%$ at $h^{1,1} = 100$. For $h^{1,1} > 100$, $f_K$ continues to decrease, so that self-interactions eventually compete with BHSR and flatten the curve.

Inside the SKC, the axion masses are already much lighter due to the increased cycle volumes, with the bulk of the distribution lying below the BHSR region. Similarly, the decay constants are two orders of magnitude smaller than at the tip, such that self-interactions shut off the BHSR bounds at a lower value of $h^{1,1}$. The exclusion fraction of CY’s reaches a maximum of only $\approx 11\%$ at $h^{1,1} = 5$, and declines to almost zero by $h^{1,1} = 100$. Thus, at this point in moduli space, CY’s with large $h^{1,1}$ are essentially unconstrained by BHSR.

Moving Beyond BHSR: Superradiance constraints rely only on gravitational interactions and vacuum fluctuations, and so provide a comparatively model-independent test of the string landscape 5, 6. However, for very large $h^{1,1}$ other constraints may become important. In a sample of 10,000 geometries with $h^{1,1} = 491$, the mean $f_K$ was $(f_K) = 10^{10}$ GeV at the tip of the SKC. The axion-photon coupling is $g = c_{\text{mix}}c_{\text{em}}/2\pi f_K$, where $c_{\text{em}}$ is the electromagnetic fine structure constant, and $c_{\text{mix}} \sim \mathcal{O}(1)$ arises from mixing between dark and visible $U(1)$’s. Using $(f_K)$ in this estimate leads to values consistent with those found in Ref. 54. A visible sector coupling of this magnitude for a massless axion is in tension with various astrophysical constraints 54, which demand $g \lesssim 10^{-12} - 10^{-13}$ GeV$^{-1}$. This suggests that further study of visible sector couplings at large $h^{1,1}$ could lead to significant constraints on the landscape.

Our results show that it is possible to make quantitative progress in constraining the landscape. The more challenging problem is to look for evidence in favour of string theory in the remaining — and still vast — parts of the landscape. The axion spectra we have computed may hold the answers.

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