Cluster structures and monopole transitions of $^{14}$C

Yoshiko Kanada-En’yo
Department of Physics, Kyoto University, Kyoto 606-8502, Japan

Kazuyuki Ogata
Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki 567-0047, Japan
Department of Physics, Osaka City University, Osaka 558-8585, Japan and
Nambu Yoichiro Institute of Theoretical and Experimental Physics (NITEP), Osaka City University, Osaka 558-8585, Japan

Cluster structures of $^{14}$C were investigated with a method of antisymmetrized molecular dynamics (AMD) combined with a $3\alpha + nn$ cluster model while focusing on the monopole excitations and linear-chain $3\alpha$ band. Variation after parity and angular momentum projections was performed in the AMD framework, and the generator coordinate method was applied to take into account various $3\alpha + nn$ cluster configurations. Energy spectra and monopole and $E2$ transition strengths of $0^+$, $2^+$, and $4^+$ states were calculated to assign band structures. The $0^+_g$ state with remarkable monopole transition was obtained as a vibrational mode of the triangle $3\alpha$ configuration. In addition, the linear-chain $3\alpha$ band from the band-head $0^+_g$ state was obtained. $^{10}$Be+\alpha decay widths of $0^+$, $2^+$, and $4^+$ states were evaluated. \alpha inelastic scattering off $^{14}$C was also investigated by the microscopic coupled-channel calculation with the g-matrix folding model to propose possible observation of the $0^+_g$ state via $\alpha$ scattering experiments.

I. INTRODUCTION

$3\alpha$ clustering is one of the hot subjects of C isotopes. A variety of $3\alpha$ cluster structures in $^{12}$C have been investigated (see for examples, review articles of Refs. 1-6 and references therein). Particularly, the $0^+_g$ state of $^{12}$C has been attracting a great interest and discussed as a gas state of $\alpha$ particles. The strong monopole transition from the ground state supports the developed $3\alpha$ state of $^{12}$C. In higher energies than the $0^+_g$ state, other kinds of $3\alpha$-cluster structure such as a triangle, bending-chain, higher-nodal states have been theoretically suggested 7-29.

For neutron-rich C isotopes, further rich phenomena of the $3\alpha$ clustering are expected to appear because of surrounding excess neutrons. One of fascinating topics is the linear-chain $3\alpha$ structure in neutron-rich C. The linear-chain $3\alpha$ clustering in $^{12}$C was originally proposed by Morinaga 30-31, but it is considered to be unstable against bending motion. However in the case of neutron-rich C isotopes, the linear-chain clustering can be stabilized owing to excess neutrons as predicted in theoretical works 32-40. In this decade, many experiments have been performed to observe cluster states in $^{13}$C 41-48. One of good tools to search for new cluster states is $\alpha$ resonant scattering, which has been recently utilized to study cluster states in such unstable nuclei as $^{14}$C. For $^{14}$C, candidate states of the linear-chain band have been reported by recent experiments of the $^{10}$Be+\alpha resonant scattering 45-47, but its rotational band members have yet to be confirmed.

An alternative tool to investigate cluster states is the $\alpha$ inelastic scattering off target nuclei because developed cluster states tend to be populated by the $\alpha$ scattering via isoscalar transitions 49-56. The $\alpha$ scattering has been utilized, in particular, to investigate isoscalar monopole transitions into excited $0^+$ states as done for $^{16}$O to study $0^+$ cluster states 53.

Our aim is to investigate cluster states in $^{14}$C focusing on the monopole excitations as well as $\alpha$-decay property. In preceding works on the linear-chain structure of $^{14}$C 53-55, methods of antisymmetrized molecular dynamics (AMD) 57-59 were applied, but the framework is not sufficient to describe detailed $^{10}$Be+\alpha clustering features. The cluster structures of $^{14}$C have been also studied by the generator coordinate method (GCM) of a $3\alpha+nn$ cluster model 39. However, the cluster model is not able to properly describe low-energy spectra because it is not suitable to describe shell model configurations nor cluster breaking in low-lying states. One of the authors 60 have studied low-lying states of $^{14}$C by a calculation of variation after angular momentum and parity projections (VAP) with the AMD model. The AMD calculation of VAP reasonably described the $0^+$, $2^+$, and $1^+$ spectra and Gamow-Teller transitions from $^{14}$N(1$^+_g$) except for the anomalously hindered $\beta$ decay of the ground state of $^{14}$C.

In the present paper, we apply the AMD method of VAP combining it with the GCM of the $3\alpha + nn$ cluster model, which we call “VAP+cl-GCM” in this paper, and discuss the energy spectra and band structure of $^{14}$C. A particular attention is paid on the monopole excitations and linear-chain band. Cluster features of $^{10}$Be+\alpha and $3\alpha + nn$ clusterings are discussed. The $\alpha$ inelastic scattering off $^{14}$C are also calculated with the microscopic coupled-channel (MCC) calculation using the matter and transition densities obtained by VAP+cl-GCM. The reaction approach is the g-matrix folding model, where $\alpha$-nucleus coupled-channel potentials are microscopically derived by folding the Melbourne g-matrix effective nuclear interaction with input of densities of the target nucleus from the structure model. Similar MCC calculations have been done for study of $^{13}$C($\alpha,\alpha'$) and
0\textsuperscript{+} and 2\textsuperscript{+} states of 14\textsuperscript{C} are calculated with the VAP version of AMD combined with the cluster GCM, in which AMD and 3\textalpha + nn wave functions are superposed.

An AMD wave function is given by a Slater determinant of single-nucleon Gaussian wave functions as

\[
\Phi_{\text{AMD}}(Z) = \frac{1}{\sqrt{A!}} A(\varphi_1, \varphi_2, \ldots, \varphi_A),
\]

\[
\varphi_i = \phi_{x_i} x_i^{\tau_i},
\]

\[
\phi_{x_i}(r_j) = \left( \frac{2\nu}{\pi} \right)^{3/4} \exp[-\nu(\mathbf{r}_j - \mathbf{x}_i)^2],
\]

\[
\chi_i = \left( \frac{1}{2} + \xi_i \right) \chi_\uparrow + \left( \frac{1}{2} - \xi_i \right) \chi_\downarrow.
\]

Here \( A \) is the antisymmetrizer, and \( \varphi_i \) is the \( i \)th single-particle wave function written by a product of spatial (\( \phi_{x_i} \)), spin (\( \chi_i \)), and isospin (\( \tau_i \) fixed to be proton or neutron) wave functions. The width parameter \( \nu \) is chosen to be the same value \( \nu = 0.19 \text{ fm}^{-2} \) as in Ref. [39]. The parameters \( Z = \{ X_1, \ldots, X_A, \xi_1, \ldots, \xi_A \} \) for Gaussian centroid positions and nucleon-spin orientations of all nucleons are treated as variational parameters, which are optimized of each state of 14\textsuperscript{C}. We use five AMD wave functions (\( \Phi_{\text{AMD}}(n) \) \( n = 1, \ldots, 5 \)) obtained for five states, 14\textsuperscript{C}(0\textsuperscript{+}, 2\textsuperscript{+}, 3\textsuperscript{+}, 2\textsuperscript{+}, 1\textsuperscript{+}) with the energy variation after \( J^\pi \) (angular momentum and parity) projection in Ref. [39]. For more details, the reader is referred to this reference.

In addition to the AMD wave functions, 3\textalpha + nn cluster wave functions are superposed to take into account large amplitude motion between \( \alpha \) clusters. The model wave functions for 3\textalpha + nn configurations are almost same as those adopted in Ref. [39]. We first compose wave functions of the subsystem 10\textsuperscript{Be} with a 2\textalpha + nn cluster model considering the \( \alpha-\alpha \) motion in 10\textsuperscript{Be}, and then place the third \( \alpha \) cluster at various distances and angles from the 10\textsuperscript{Be} core as illustrated by a schematic figure shown in Fig. 1. The first and second \( \alpha \) clusters are placed on the \( Z \) axis at \((0,0,\pm d/2)\) with the distance \( d \). For two valence neutrons, we adopt neutron wave functions of an cluster model for 10\textsuperscript{Be} proposed by Itagaki et al. [39], which is an extended version of the Brink-Bloch cluster model. Namely, wave functions for spin-up and -down neutrons are given by Gaussian wave packets at the same position but the momentum opposite to each other. As a result, the 2\textalpha + nn wave function is given as

\[
\Phi_{2\alpha+nn}(d, \phi) = A \left[ \Phi_{\alpha}(R_1) \Phi_{\alpha}(R_2) \varphi_{n\uparrow}(R_{n\uparrow}) \varphi_{n\downarrow}(R_{n\downarrow}) \right],
\]

\[
\varphi_{n\uparrow}(i) (R_{n\uparrow}(i)) = \phi_{R_{n\uparrow}(i) \chi(i)},
\]

with \( R_1 = -R_2 = (0,0,d/2) \) and

\[
R_{n\uparrow} = \rho(\cos \phi, \sin \phi, 0) + i \rho \Lambda(-\sin \phi, \cos \phi, 0),
\]

\[
R_{n\downarrow} = \rho(\cos \phi, \sin \phi, 0) - i \rho \Lambda(-\sin \phi, \cos \phi, 0).
\]

Here \( \Phi_{\alpha}(R_k) \) is the \( \alpha \) cluster wave function given by the (0\textpi\textsuperscript{4}) harmonic oscillator configuration located at \( R_k \) and the parameters \( \rho \) and \( \Lambda \) are optimized to minimize the 10\textsuperscript{Be} energy for each \( \alpha-\alpha \) distance \( d \). Using the optimized values of \( \rho \) and \( \Lambda \), the 3\textalpha + nn wave function is given as

\[
\Phi_{3\alpha+nn}(d, D_{\alpha}, \phi, \theta_{\alpha}) = A \left[ \Phi_{\alpha}(R_1) \Phi_{\alpha}(R_2) \Phi_{\alpha}(R_3) \varphi_{n\uparrow}(R_{n\uparrow}) \varphi_{n\downarrow}(R_{n\downarrow}) \right],
\]

\[
R_3 = (D_{\alpha} \cos \theta_{\alpha}, 0, D_{\alpha} \sin \theta_{\alpha}).
\]

To exactly remove the center of mass motion from the total wave function, the Gaussian center positions are shifted as \( R_k \rightarrow R_k - R_G \) and \( R_{n\uparrow}(i) \rightarrow R_{n\uparrow}(i) - R_G \) with \( R_G \equiv (4 \sum_k R_k + R_{n\uparrow} + R_{n\downarrow})/14 \). The Gaussian widths of the alpha and neutron wave functions are common and taken to be the same \( \nu \) value as that used in the AMD wave function. Note that the present 3\textalpha + nn cluster wave function can be expressed by a specific configuration of the AMD wave function.

The basis wave functions \( \Phi_{3\alpha+nn}(d, D_{\alpha}, \phi, \theta_{\alpha}) \) with various values of four parameters, \( d, D_{\alpha}, \phi, \) and \( \theta_{\alpha} \), are projected to \( J^\pi \) eigen states and superposed in the GCM calculation. We here simply denote the \( m \)th basis wave function as \( \Phi_{3\alpha+nn}^{(m)} \) with the label \( m \) for the parameter set \( (d, D_{\alpha}, \phi, \theta_{\alpha}) \). The final wave function of VAP+cl-GCM
for the $J_K^r$ state is obtained by combining the AMD and the $3\alpha + nn$ wave functions as

$$
\Psi(J_K^r) = \sum_{n,K} c_{n,K}^{(n)} (J_K^r) P_{MK}^{J_K^r} \Phi_{3\alpha + nn}^{(n)} \\
+ \sum_{m,K} c_{m,K}^{(m)} (J_K^r) P_{MK}^{J_K^r} \Phi_{3\alpha + nn}^{(m)},
$$

(11)

where coefficients $c_{n,K}^{(n)} (J_K^r)$ and $c_{m,K}^{(m)} (J_K^r)$ are determined by diagonalization of the norm and Hamiltonian matrices.

In order to evaluate component of a specific $3\alpha + nn$ configuration contained in the $J_K^r$ state, we define the squared overlap of $\Psi(J_K^r)$ with a basis $3\alpha + nn$ wave function as

$$
O_{3\alpha + nn}(d, D_\alpha, \phi, \theta_\alpha) = \\
|\langle \Psi(J_K^r) | P_{MK}^{J_K^r} \Phi_{3\alpha + nn}(d, D_\alpha, \phi, \theta_\alpha) \rangle|^2.
$$

(12)

**B. Effective interactions and parameter setting**

We use the effective nuclear interactions of the MV1 (case 3) central force [67] supplemented by the spin-orbit term of the G3RS force [68, 69] and the Coulomb force. The Bartlett, Heisenberg, and Majorana parameters, $b = h = 0.125$ and $m = 0.62$, in the MV1 force are adopted, and the strengths $u_1 = -u_{11} = 3800$ MeV of the G3RS spin-orbit force are used. These interaction parameters are consistent with those used in Ref. [60] for the study of $^{14}$C with the VAP calculation [60]. It should be commented that we use the MV1 force consisting of the finite-range 2-body and zero-range 3-body forces instead of the Volkov force [70] used in Refs. [34, 39], because spectra of low-lying states in $^{14}$C and also those of cluster states in $^{16}$O were reasonably reproduced by the MV1 force but not by the Volkov force.

For the generator coordinates $d, D_\alpha, \phi,$ and $\theta_\alpha$ of the GCM calculation, we adopt discrete values, $d = 2, 3, 4$ fm, $D_\alpha = 2, 3, \ldots, 7$ fm, $\phi = \pi/8, 3\pi/8, 5\pi/8, 7\pi/8, \pi/2$ and $\theta_\alpha = 0, \pi/8, \pi/4, 3\pi/8, \pi/2$ of the $3\alpha + nn$ wave functions. Note that the angle ranges $0 \leq \phi \leq \pi$ and $0 \leq \theta_\alpha \leq \pi/2$ are equivalent to the full ranges $0 \leq \phi \leq 2\pi$ and $0 \leq \theta_\alpha \leq \pi$ because of the parity and angular momentum projections. In this parametrization, the number of basis $3\alpha + nn$ wave functions $\Phi_{3\alpha + nn}^{(m)}$ ($m = 1, \ldots, m_{\text{max}}$) is $m_{\text{max}} = (3 \times 4 \times 4 + 3) \times 6 = 306$.

The values of $\rho$ and $\Lambda$ of neutron wave functions are listed in Table II. In the first choice (set-1), we optimize $\rho$ and $\Lambda$ to minimize the $0^+$ energy of the subsystem $P_{00}^{3\alpha + nn} \Phi_{2\alpha + nn}$. In the second choice (set-2), we use alternative values of $\rho$ and $\Lambda$ for $d = 3$ fm determined to minimize the $(J^r, K) = (2^+, 2)$ energy of $P_{MK}^{2\alpha + nn}$ so as to globally optimize energies of $^{10}$Be$(0^+_1)$, $^{10}$Be$(2^+_1)$, and $^{10}$Be$(2^+_2)$. As shown in Table II, the second choice gives a better result of the low-energy energy spectra of $^{10}$Be, in particular, the energy of $^{10}$Be$(2^+_2)$. Therefore, we adopt parameters $\rho$ and $\Lambda$ of set-2 in the present calculation of $^{14}$C.

**III. STRUCTURE OF $^{14}$C**

**A. Structure properties**

The energy levels of $^{14}$C obtained by the VAP+cl-GCM calculation are shown in Fig. 2. Root-mean-square radii and monopole transition strengths of the proton and neutron parts for $0^+$ states are listed in Table III and quadrupole $(\lambda = 2)$ transition strengths of the proton and neutron parts are shown in Table III.

![FIG. 2: The calculated energy spectra of $0^+$, $2^+$, and $4^+$ states of $^{14}$C are shown together with the experimental energy spectra assigned to $0^+$, $2^+$, and $4^+$ in Ref. [71]. The experimental levels with asterisk symbols are the states reported in Ref. [77] as candidates of the linear-chain band.](image-url)
TABLE II: Root-mean-square radii of $^{14}\text{C}(0^+)$ and monopole transition strengths from the ground state calculated with VAP+cl-GCM. Proton and neutron parts of radii ($R_{p,n}$) and monopole transition strengths ($B_{p,n}(E0)$) are listed. The experimental proton radius of the ground state reduced from the experimental charge radius $R_p$ is $R_p = 2.37$ fm.

| $R_p$(fm) | $R_n$(fm) | $B_{p}(E0)$(fm$^4$) | $B_{n}(E0)$(fm$^4$) |
|-----------|-----------|----------------------|----------------------|
| $0^+_1$   | 2.52      | 2.65                 | 0.1                  |
| $0^+_2$   | 2.62      | 2.80                 | 1.1                  |
| $0^+_3$   | 2.98      | 2.98                 | 18                   |
| $0^+_4$   | 3.48      | 3.37                 | 1.0                  |

TABLE III: $E2$ transition strengths of $2^+ \rightarrow 0^+$ in $^{14}\text{C}$ calculated with VAP+GCM. In addition to the proton contribution $B(E2)$, the neutron contribution ($B_n(E2)$) of the $\lambda = 2$ transitions is also shown. The strengths for $0^+_1 \rightarrow 2^+_1,2,3,4,5$ and those with $B(E2) \geq 2$ fm$^4$ or $B_n(E2) \geq 2$ fm$^4$ in the transitions $2^+_1,2,3,4,5 \rightarrow 0^+_1,2,3,4$ are given. The experimental value of $B(E2; 2^+_1 \rightarrow 0^+_1)$ is $3.6\pm0.6$ fm$^4$ [71].

| $B(E2)$ (fm$^4$) | $B_n(E2)$ (fm$^4$) |
|------------------|---------------------|
| $2^+_1 \rightarrow 0^+_1$ | 7.8                  |
| $2^+_2 \rightarrow 0^+_1$ | 0.0                  |
| $2^+_3 \rightarrow 0^+_1$ | 0.1                  |
| $2^+_4 \rightarrow 0^+_1$ | 0.0                  |
| $2^+_5 \rightarrow 0^+_1$ | 0.1                  |
| $2^+_1 \rightarrow 0^+_2$ | 0.6                  |
| $2^+_2 \rightarrow 0^+_2$ | 6.8                  |
| $2^+_3 \rightarrow 0^+_2$ | 3.8                  |
| $2^+_4 \rightarrow 0^+_2$ | 3.3                  |
| $2^+_5 \rightarrow 0^+_2$ | 6.0                  |
| $2^+_1 \rightarrow 0^+_3$ | 162                  |
| $2^+_2 \rightarrow 0^+_3$ | 175                  |

and $0^+_5$ states are approximately described by the VAP wave functions showing shell-model like structure but no prominent cluster structure. In the mean-field picture, the $0^+_5$ state is approximately understood as neutron excitation into a $(sd)^2$ configuration and has a normal proton radius as small as that of the ground state. In contrast to the $0^+_5$ state, the $0^+_3$ and $0^+_4$ states have relatively large nuclear radii and spatially developed cluster structures. The $0^+_3$ state dominantly has the linear-chain $3\alpha$ structure and constructs a rotational band.

In the monopole transition strengths listed in Table II, the remarkable strength from the ground state is obtained for the $0^+_5$ state because of its nature of the triangle $3\alpha$ vibration mode. On the other hand, the $0^+_4$ state has weak monopole transition strength even though it is a developed cluster state, because the linear-chain state has the linearly aligned $3\alpha$ configuration, which is much different from the ground state and difficult to be directly excited by the monopole operator.

Let us discuss the band structure based on the $E2$ transition strengths shown in Table III. The $2^+_1$ state is assigned to the ground band and mainly contributed by the proton rotation. The $0^+_1$, $2^+_2$, and $4^+_3$ states with dominant neutron $(sd)^2$ components construct the $K^+ = 0^+_2$ band. The $0^+_3$ state shows no clear signal of band structure. Instead, the $E2'$ strength from this state is fragmented into $2^+_4$ and $2^+_5$ states. The rotational band of the linear-chain structure is built on the $0^+_1$ state with the $2^+_5$ and $4^+_3$ states. This band has remarkably strong $E2$ transitions and large moment of inertia (small level spacing) because of the highly elongated structure. More detailed properties of $\alpha$ decays and cluster structures are discussed in the following sections.

B. $^{10}\text{Be}(0^+_1) + \alpha$ decay widths

In order to discuss $\alpha$ decay properties, we calculate the $^{10}\text{Be}(0^+_1) + \alpha$ component in the $0^+$, $2^+$, and $4^+$ states. The squared overlap of $\Psi(J^+_n)$ obtained by VAP+cl-GCM with the $^{10}\text{Be}(0^+_1) + \alpha$ wave function with the distance $D_\alpha$ is calculated. The result for the $0^+_1,2,3,4$ states is shown in Fig. 3. The ground state has the overlap only in the internal region indicating the $^{10}\text{Be}(0^+_1) + \alpha$ component as the ground state correlation but no spatially developed clustering. In the $0^+_3$ state, $^{10}\text{Be}(0^+_1) + \alpha$ component is relatively minor compared with the $0^+_1$ and $0^+_2$ states. In the $0^+_4$ state, the squared overlap is distributed in a wide range of $D_\alpha$ with the maximum amplitude at $D_\alpha = 4$ fm meaning that the $\alpha$ cluster is moving in the broad region around the $^{10}\text{Be}(0^+_1)$ core. The $0^+_1$ state has large overlap amplitudes at $D_\alpha = 6$ fm and shows remarkable development of the clustering. The significant amplitude at $D_\alpha = 7$ fm may indicate some coupling with $^{10}\text{Be}(0^+_1) + \alpha$
experimental assignment of the linear-chain band from those data because the data are not necessarily consistent between different experiments. The $\theta^2_\alpha(a)$ values at $a = 5$ fm reported in Ref. \[47\] are 0.34(12), 0.091(27), and 0.024(9) for the $0^+$, $2^+$, and $4^+$ states, respectively. The observed $\theta^2_\alpha(a)$ values of the $2^+$ and $4^+$ states are smaller than the present result.

C. 3α + nn cluster structures

As mentioned previously, the remarkable monopole transition strength is obtained for the $0^+_1$ state, whereas the strengths of the $0^+_2$ and $0^+_3$ states are relatively weak. These features of monopole excitations can be understood by the $3\alpha + nn$ clustering. In order to clarify properties of the clustering, we calculate the squared overlap $O^2_{3\alpha+nn}(d, D_\alpha, \phi, \theta_\alpha)$ of $\psi(J^+_\alpha)$ with each $3\alpha + nn$ configuration specified by the parameters $D_\alpha$, $\phi$, and $\theta_\alpha$ as given in Eq. (12). $D_\alpha$ and $\theta_\alpha$ describes the position of the third $\alpha$ cluster around the $^{10}$Be core, and $\phi$ is the parameter for the $nn$ orientation against the $3\alpha$ plane (see Fig. [1]). We categorize $\psi_{3\alpha+nn}(d, D_\alpha, \phi, \theta_\alpha)$ into three kinds of wave functions as “tetrahedral”, “planar”, and “linear” configurations with $(\theta_\alpha, \phi) = (\pi/2, 5\pi/8), (\theta_\alpha, \phi) = (\pi/2, \pi/8)$, and $\theta = 0$, respectively. Each component contained in $J^+_\alpha$ states is evaluated with $O^2_{3\alpha+nn}(d, D_\alpha, \phi, \theta_\alpha)$ for the corresponding configuration. Figure [5] shows components of the tetrahedral, planar, and linear configurations. Obtained values of $O^2_{3\alpha+nn}(d, D_\alpha, \phi, \theta_\alpha)$ are plotted as functions of the distance $D_\alpha$ of the third $\alpha$ from the sub-system $2\alpha + nn$. Here, we show the result for $d = 4$ fm of the $\alpha$-$\alpha$ distance in the $2\alpha + nn$ part in order to discuss prominent cluster features. As shown in Fig. [5](a) for the tetrahedral, the ground state contains component of the compact tetrahedral configuration, and the $0^+_1$ state can be regarded as a vibration mode of the triangle $3\alpha$ in the tetrahedral configuration. This excitation mode of the $0^+_1$ contributes to the remarkable monopole transition strength because it expresses radial excitation of $\alpha$ clusters keeping the same shape as the $0^+_1$ state. The $0^+_1$ state contains dominantly the linear component with $D_\alpha = 5 - 6$ and constructs the linear-chain band with a large momentum of inertia because of the highly elongated shape. The $0^+_2$ state shows weak cluster feature as it is the shell-model state and roughly described by the neutron excitation to the $sd$ shell. However, it contains significant planar component as can be seen in Fig. [5](a).

From this feature found in the mapping onto the cluster model space, one can interpret the $0^+_2$ state as the excitation from the compact tetrahedral configuration of the ground state into the planer configuration. Such excitations into the planer and linear configurations obtained in the $0^+_2$ and $0^+_3$ states involve drastic changes of the geometric structure from the ground state, and describe suppression of monopole transitions in general.

It should be commented that the squared overlap $O^2_{3\alpha+nn}(d, D_\alpha, \phi, \theta_\alpha)$ is not concentrated on a specific
In the present calculation of VAP+cl-GCM, the linear-chain band is built on the $0^+_1$ state in the $3\alpha + nn$ cluster dynamics. This result is qualitatively similar to that of the previous calculation with the $3\alpha + nn$-cluster model. On the other hand, the AMD calculations in Refs. [34, 37, 40] obtained the linear-chain state as the $0^+_2$ state. It means that a low-lying monopole excitation was missing in the AMD calculations because they do not fully take into account $3\alpha + nn$ cluster configurations.

IV. $\alpha$ INELASTIC SCATTERING OFF $^{14}C$

As a probe of cluster states, we investigate the $\alpha$ inelastic scattering off $^{14}C$ with the MCC calculation. A particular attention is paid on the inelastic scattering of the $0^+_2$ having the strong monopole transition, which is expected to be strongly populated by the $\alpha$ scattering. For this state, the monopole transition matrix elements $M_0(E0)$ and $M_2(E0)$ of the proton and neutron parts are predicted to be $M_p(E0) = 4.3$ fm$^2$ and $M_n(E0) = 3.9$ fm$^2$, which are the same order as $M_p(E0) = 5.48 \pm 0.22$ fm$^2$ of $^{12}C(0^+_4)$ measured by electron pair emission [73].

The $\alpha$ inelastic scattering cross sections of the $0^+$ and $2^+$ states of $^{14}C$ are calculated using the matter and transition densities obtained by the present VAP+cl-GCM calculation. The reaction calculation is in principle the same approach as that of our previous works on the $^{12}C(\alpha, \alpha')$ and $^{16}O(\alpha, \alpha')$, which successfully reproduced inelastic cross sections of cluster states. The $\alpha$-nucleus CC potentials are microscopically derived by folding the Melbourne $g$-matrix effective $NN$ interaction [61] with an $\alpha$ density and the matter and transition densities of $^{14}C$. The channel-coupling of $\lambda = 0$ and $\lambda = 2$ transitions for the $0^+_1$, $0^+_2$, $0^+_3$, and $0^+_4$ states theoretically obtained by VAP+cl-GCM are taken into account.

The calculated $\alpha$ scattering cross sections of $^{14}C(0^+)$ at incident energies of $E_\alpha = 140$ and 400 MeV and those of $^{14}C(2^+)$ states are shown in Figs. [a] and [b], respectively. The strong monopole excitation to the $0^+_2$ state by the $\alpha$ scattering is predicted because of the remarkable isoscalar monopole transition. Inelastic cross sections of the $0^+_1$ state are relatively small. This is consistent with the weak isoscalar monopole transitions from the ground states in prediction of the structure calculation. Among the $2^+$ states, the cross sections of the $2^+_2$ state in the ground band are significantly large, but those of other $2^+$ states are relatively small.

There is no $\alpha$ scattering experiment that observed the $0^+_2$ and $0^+_3$ states. From the present prediction of the enhanced $0^+_2$ cross sections, one expects strong population of the $0^+_2$ state in the $\alpha$ scattering. On the other hand, the present result suggests relatively weak productions of the linear-chain states, the $0^+_2$ and $2^+_5$ states of $^{14}C$, even though they have a developed cluster structure. However, as seen in Figs. [a] and [b] the CC effect is minor for the cross sections to those linear-chain states at $E_\alpha = 400$ MeV. This result may suggest a possibility of observing the predicted $0^+_2$ and $2^+_5$ states in the linear-chain band via a multipole decomposition analysis of the $\alpha$ scattering.

V. SUMMARY

Cluster features of $^{14}C$ were investigated with VAP+cl-GCM, which is a method of AMD combined with the $3\alpha + nn$ cluster model. In the structure calculation of $^{14}C$, the AMD wave functions obtained with the variation after
parity and angular momentum projections for $^{14}$C($0^+_1,2$), $^{14}$C($2^+_1,2$), and $^{14}$C($1^+_1$) are used. In addition to the AMD wave functions, $3\alpha+n\alpha$ wave functions are superposed with GCM to take into account large amplitude cluster motion.

The energy spectra and band structures of $0^+$, $2^+$, and $4^+$ states were discussed. The $0^+_1$ and $0^+_2$ states with prominent cluster structures were obtained. The $0^+_1$ state has the remarkable monopole transitions from the ground state and is regarded as the vibrational mode of the triangle $3\alpha$ configuration. The $0^+_2$ state contains dominantly the linear-chain $3\alpha$ structure with two neutrons. The cluster feature of the $0^+_2$ state is qualitatively consistent with the $0^+_1$ state obtained with the $3\alpha+n\alpha$ cluster model calculation in Ref. 39], and similar to the $0^+_3$ state of the AMD predictions in Refs. 24, 37, 40.

The $^{10}$Be($0^+_1$)+$\alpha$ components and the $\alpha$-decay widths were discussed. The calculated result for the linear-chain band was compared with the observed data reported by $\alpha$ resonant scattering experiments [45, 47.

The $\alpha$ inelastic scattering off $^{14}$C at $E_\alpha=140$ and 400 MeV were also calculated by the MCC calculation with the Melbourne $g$-matrix interaction for the folding model by utilizing the matter and transition densities obtained from the structure calculation. The calculation predicts enhanced monopole cross sections for the $0^+_2$ state because of the remarkable monopole transition strength, and suggests a possible observation of the $0^+_2$ state via an $\alpha$ inelastic scattering experiment in future. For the $0^+_1$ and $2^+_1$ states of the linear-chain band, the result shows smaller cross sections compared with the $2^+_1$ and $0^+_3$ states.

In the present calculation, we could not draw a definite assignment of the obtained linear-chain band to observed levels because model ambiguities remains and also experimental information is still limited. In the experimental side, many states have been observed near and above the $^{10}$Be+$\alpha$ threshold energy. The $\alpha$ inelastic scattering can be a good probe for the monopole excitation of the triangle vibration mode predicted as the $0^+_2$ state. For the linear-chain band, the present calculation predicts relatively weaker production of the $0^+_1$ and $2^+_1$ states in the $\alpha$ scattering. Nevertheless, the CC effect is minor for the cross sections to those linear-chain states at $E_\alpha=400$ MeV. This result may suggest a possibility of observing the predicted linear-chain band via a multipole decomposition analysis of the $\alpha$ scattering.

**Acknowledgments**

The computational calculations of this work were performed by using the supercomputer in the Yukawa Institute for theoretical physics, Kyoto University. This
work was partly supported by Grants-in-Aid of the Japan Society for the Promotion of Science (Grant Nos. JP18K03617, JP16K05352, and 18H05407) and by the grant for the RCNP joint research project.

[1] Y. Fujiwara, Y. Suzuki, H. Horiuchi, K. Ikeda, M. Kamimura, K. Katö, Y. Suzuki, and E. Uegaki, Prog. Theor. Phys. Suppl. 68, 29 (1980).
[2] T. Yamada, Y. Funaki, H. Horiuchi, G. Ropke, P. Schuck and A. Tohsaki, Lect. Notes Phys. 848, 229 (2012).
[3] H. Horiuchi, K. Ikeda, and K. Katö, Prog. Theor. Phys. Suppl. 192, 1 (2012).
[4] M. Freer and H. O. U. Fynbo, Prog. Part. Nucl. Phys. 78, 1 (2014).
[5] Y. Funaki, H. Horiuchi and A. Tohsaki, Prog. Part. Nucl. Phys. 82, 78 (2015).
[6] M. Freer, H. Horiuchi, Y. Kanada-En’yo, D. Lee and U. G. Meissner, Rev. Mod. Phys. 90, 035004 (2018).
[7] Y. Fukushima and M. Kamimura, Proc. Int. Conf. on Nuclear Structure, Tokyo, 1977, edited by T. Marumori J. Phys. Soc. Jpn. 44, 225 (1978).
[8] E. Uegaki, S. Okabe, Y. Abe and H. Tanaka, Prog. Theor. Phys. 57, 1262 (1977).
[9] E. Uegaki, Y. Abe, S. Okabe and H. Tanaka, Prog. Theor. Phys. 62, 1621 (1979).
[10] M. Kamimura, Nucl. Phys. A 351, 456 (1981).
[11] P. Descouvemont and D. Baye, Phys. Rev. C 36, 54 (1987).
[12] Y. Kanada-En’yo, Phys. Rev. Lett. 81, 5291 (1998).
[13] A. Tohsaki, H. Horiuchi, P. Schuck and G. Ropke, Phys. Rev. Lett. 87, 192501 (2001).
[14] Y. Funaki, A. Tohsaki, H. Horiuchi, P. Schuck and G. Ropke, Phys. Rev. C 67, 051306 (2003).
[15] T. Neff and H. Feldmeier, Nucl. Phys. A 738, 357 (2004).
[16] S. I. Fedotov, O. I. Kartavtsev, V. I. Kochkin and A. V. Malych, Phys. Rev. C 70, 014006 (2004).
[17] C. Kurokawa and K. Katö, Nucl. Phys. A 738, 455 (2004).
[18] C. Kurokawa and K. Kato, Phys. Rev. C 71, 021301 (2005).
[19] I. Filipkin, V. M. Suslov and B. Vlahovic, J. Phys. G 31, 1207 (2005).
[20] Y. Funaki, H. Horiuchi and A. Tohsaki, Prog. Theor. Phys. 115, 115 (2006).
[21] Y. Kanada-En’yo, Prog. Theor. Phys. 117, 655 (2007) [Erratum-ibid. 121, 895 (2009)].
[22] K. Arai, Phys. Rev. C 74, 064311 (2006).
[23] M. Chernykh, H. Feldmeier, T. Neff, P. von Neumann-Cosel and A. Richter, Phys. Rev. Lett. 98, 032501 (2007).
[24] E. Epelbaum, H. Krebs, T. A. Lahde, D. Lee and U. G. Meißner, Phys. Rev. Lett. 109, 252501 (2012).
[25] A. C. Dreyfuss, K. D. Launey, T. Dytrych, J. P. Draayer and C. Bahri, Phys. Lett. B 727, 511 (2013).
[26] S. Ohtsubo, Y. Fukushima, M. Kamimura, and E. Hiya, Prog. Theor. Exp. Phys. 2013, 073D02 (2013).
[27] S. Ishikawa, Phys. Rev. C 90, no. 6, 061604 (2014).
[28] T. Suhara and Y. Kanada-En’yo, Phys. Rev. C 91, 024315 (2015).
[29] Y. Funaki, Phys. Rev. C 92, no. 2, 021302 (2015).
[30] H. Morinaga, Phys. Rev. 101, 254 (1956).
[31] H. Morinaga, Phys. Lett. 21, 78 (1966).
[32] N. Itagaki, S. Okabe, K. Ikeda and I. Tanihata, Phys. Rev. C 64, 014301 (2001).
[33] N. Itagaki, W. v. Oertzen and S. Okabe, Phys. Rev. C 74, 067304 (2006).
[34] T. Suhara and Y. Kanada-En’yo, Phys. Rev. C 82, 044301 (2010).
[35] T. Suhara and Y. Kanada-En’yo, Phys. Rev. C 84, 024328 (2011).
[36] J. A. Maruhn, N. Loebl, N. Itagaki and M. Kimura, Nucl. Phys. A 833, 1 (2010).
[37] T. Baba, Y. Chiba and M. Kimura, Phys. Rev. C 90, no. 6, 064319 (2014).
[38] T. Baba and M. Kimura, Phys. Rev. C 94, no. 4, 044303 (2016).
[39] Yuta, Yoshida and Yoshiko, Kanada-En’yo, Prog. Theor. Exp. Phys. bfl 2016, 123D04 (2016).
[40] T. Baba and M. Kimura, Phys. Rev. C 95, no. 6, 064318 (2017).
[41] S. Ishikawa, W. G. Bohlen, M. Milin, Tz. Kolakova, S. Thumm, A. Tumino, R. Kalpakchieva, T. N. Massey, Y. Eisermann, G. Graw, T. Faestermann, R. Hertenberger, and H. F. Wirth, Eur. Phys. J. A 21, 193 (2004).
[42] D. L. Price et al., Phys. Rev. C 75, 014305 (2007).
[43] P. J. Haigh et al., Phys. Rev. C 78, 014319 (2008).
[44] M. Freer et al., Phys. Rev. C 90, no. 5, 054324 (2014).
[45] S. Yoshida et al., Phys. Rev. C 93, no. 1, 014321 (2016).
[46] H. Yamaguchi et al., Phys. Lett. B 766, 11 (2017).
[47] Z. Y. Tian et al., Chin. Phys. C 40, no. 11, 111001 (2016).
[48] T. Kawabata, H. Akimune, H. Fujita, Y. Fujita, M. Fujiwara, K. Ihara, K. Hatanaka and M. Itoh et al., Phys. Lett. B 646, 6 (2007).
[49] Y. Kanada-En’yo, Phys. Rev. C 75, 024302 (2007).
[50] T. Yamada, Y. Funaki, H. Horiuchi, K. Ikeda, and A. Tohsaki, Prog. Theor. Phys. 120, 1139 (2008).
[51] Y. Funaki, A. Tohsaki, H. Horiuchi, P. Schuck and G. Ropke, Eur. Phys. J. A 28, 259 (2006).
[52] T. Wakisaka, E. Ihara, K. Fujita, Y. Funaki, K. Hatanaka, H. Horiuchi, M. Itoh and J. Kama, Prog. Phys. Lett. B 653, 173 (2007).
[53] T. Yamada, Y. Funaki, T. Miy, H. Horiuchi, K. Ikeda, G. Ropke, P. Schuck and A. Tohsaki, Phys. Rev. C 85, 034315 (2012).
[54] M. Itoh et al., Phys. Rev. C 84, 054308 (2011).
[55] Y. Chiba and M. Kimura, Phys. Rev. C 91, no. 6, 061302 (2015).
[56] Y. Kanada-En’yo, H. Horiuchi and A. Ono, Phys. Rev. C 52, 628 (1995).
[57] Y. Kanada-En’yo and H. Horiuchi, Prog. Theor. Phys. Suppl. 142, 205 (2001).
[58] Y. Kanada-En’yo, M. Kimura and A. Ono, PTEP 2012 01A202 (2012).
[59] Y. Kanada-En’yo and T. Suhara, Phys. Rev. C 89, no. 4, 044313 (2014).
[60] K. Amos, P. J. Dortmans, H. V. von Geramb, S. Karataglidis, and J. Raynal, Adv. Nucl. Phys. 25, 275...
[62] K. Minomo and K. Ogata, Phys. Rev. C 93, 051601 (2016).
[63] Y. Kanada-En’yo and K. Ogata, Phys. Rev. C 99, no. 6, 064601 (2019).
[64] Y. Kanada-En’yo and K. Ogata, Phys. Rev. C 99, no. 6, 064608 (2019).
[65] N. Itagaki, H. Masui, M. Ito and S. Aoyama, Phys. Rev. C 71, 064307 (2005).
[66] D. M. Brink, Proc. Int. School of Physics Enrico Fermi, Course 36, Varenna, ed. C. Bloch (Academic Press, New York, 1966).
[67] T. Ando, K. Ikeda, and A. Tohsaki, Prog. Theor. Phys. 64, 1608 (1980).
[68] R. Tamagaki, Prog. Theor. Phys. 39, 91 (1968).
[69] N. Yamaguchi, T. Kasahara, S. Nagata, and Y. Akaishi, Prog. Theor. Phys. 62, 1018 (1979).
[70] A. Volkov, Nucl. Phys. 74, 33 (1965).
[71] F. Ajzenberg-Selove, Nucl. Phys. A 523, 1 (1991).
[72] I. Angeli and K. P. Marinova, At. Data Nucl. Data Tables 99, 69 (2013).
[73] J. H. Kelley, J. E. Purcell and C. G. Sheu, Nucl. Phys. A 968, 71 (2017).
[74] Y. Kanada-En’yo, T. Suhara and Y. Taniguchi, PTEP 2014, 073D02 (2014).