Phase shifting profilometry with optical vortices

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Abstract. In this work we review principles and applications of a method of phase shifting profilometry with using of optical vortices imbedded into the probe beam. High spatial resolution caused by vortex phase sensitivity is analysable to retrieve the 2D and 3D shape of optically transparent and reflecting surfaces with exceeding of optical diffraction limit. This method applicable for non-destructive testing of thin films, live cells and biological tissues in real-time regime. Automatic processing of vortex interferograms with vortex phase shift analysis allow to achieve a vertical resolution down to 1.75 nm.

1. Introduction
The field of singular optics and its applications is a relatively new branch of development and investigation, but it already is very active and significant research effort has been invested in its physical implications and possible applications, especially in microscopy and digital imaging.

The diffraction limit, discovered in 1873 by Ernst Abbe, is the smallest possible size of a light spot that can be obtained by focusing electromagnetic radiation of a given wavelength in a medium with a refractive index $n$. For more than a century this limit of spatial resolution was an insurmountable obstacle to the improvement of the technique of optical imaging in the far field. Overcoming the diffraction limit is the topic of many fundamental and applied studies of modern optics. High interest is due to the fact that the field of application of super resolution is not limited to the improvement of image quality, but is widely used to recording on optical media, in lithography and nano-structuring, optical manipulation [1-4].

In the present paper we focus our attention on the interferometric analysis of phase evolution of structured light and its application to profilometry. Interferometry is used for control of micro-relief and defects of scattering surfaces, roughness of solid bodies like microchip subtract and thin films [5]. This technique is able to perform measurements of thickness and surface topology without direct contact with an object in non-destructive way with high accuracy. The phase shifting interferometry is widely used and actively developing method significantly increased prevalence of interferometric measurements in research and production applications due to its high noise sustainability, especially with using of low-coherence light sources where measurements are based on analysis of oscillating light intensity envelope during the interference process which is described by function of time and special coherence [6]. The field of special interest is using of low-coherence differential interferometry for diagnostics of stepped structures by 3D-profilometry, optical tomography of semi-transparent biological objects and other applications because it allows to measure high difference (thickness) of the sample which is exceeding $\lambda/2$ [7].
Using of interferometric methods in coupling with computer processing leads to a new opportunities in interferometry of 3D reconstruction, measurement and visualisation of structural and morphological features of studying complex surfaces at micro- and nanoscale [8].

Last researches in field of nanotechnology and nano engineering gave rise to the corresponding control and testing equipment which is used in fundamental research and optical device production [9]. Intensive developing of this branches need a new technique for fast and low-error measurements, therefore optical methods and technical apparatus requires further improvement and attraction of novel physical mechanisms [10-11].

2. Phase shifting profilometry with optical vortices

Methods of obtaining and decoding of interferograms have been widely used in interferometry systems construction in recent years due to the efficiency of phase-sampling. Phase shifting interferometry for fast and non-destructive analysis is extensively adopted in optical sensing and metrology [5]. This method is based on registration of several interference patterns while the reference beam phase was changed on certain values. In this technique the main task in phase recovering procedure remains a definition of phase difference between interfering wave fronts by the values of registered intensity. For this purposes we need no less than a three interferograms for each point of imaging object. Numerous papers considered methods with using of interferograms sequence to extract the phase and surface relief [2-4]. First known algorithms of phase reconstruction used formulas based on three and four phase steps. Increasing of computing performance gave a possibility to use algorithms with a high number of phase shifts: from 15 up to 101 and the objects must remain stationary during the capture of these fringe sequence images. Therefore, any motion of the object could introduce phase error and thus measurement error. In contrast, profilometry with Fourier transform of interference pattern has the advantage of only requiring one single image for phase retrieval, but steel need high quality of interference fringes [9].

Practical application of optical vortex in the vortex scanning optical imaging allows to study, for example, the surface geometry and optical density of the sample by analysis of phase singularity’s distortion [10]. It was shown that vortex phase analysis carrier information about sample topology, and depends of the features of incident beam and different aperture systems. This study demonstrates the response of the optical vortex imbedded in focused Gaussian beam to a produce a small phase shift after spreading through the probe and may be used for high-resolution profilometry [11].

In this paper we consider theoretically and experimentally monochromatic linearly polarized along $x$-axis singular beam with the wavelength $\lambda$ bearing the single-charged optical vortex (a so-called vortex beam) and transmitting through the isotropic glass plate with complex surface relief, which provokes additional phase transformations. Let us consider first the propagation of the paraxial beam along the $z$-axis. The transverse $E_x$ component have a wavenumber $k_x = nk_o$, where $k_o$ is a wavenumber in a free space and $n = \sqrt{\varepsilon} -$ refractivity index of a test sample. In the paraxial approximation, we can treat the linearly polarised component as $E = E_i(x, y, z) \exp(-ik_o z)$. Then, the paraxial equations for the complex amplitude $\hat{E}_x$ is represented in the form:

$$d^2\hat{E}_x + d^2\hat{E}_x - 2ik_x d_z \hat{E}_x = 0.$$ (1)

A particular solution to the paraxial wave equation (1) for the vortex beam can be written in the reference frame $x, y, z$ of the sample:

$$\hat{E}_x = \left(\frac{x - i\xi y}{w_o \sigma_o}\right) \exp\left[-\left(x^2 + y^2\right)/w_o^2\sigma_o\right] / \sigma_o,$$ (2)

where: $\sigma_o = \frac{1 - iz}{z_o}$, $z_o = \frac{k_o w_o^2}{2}$, $w_o$ - is the radius of the beam waist at the plane $(z = 0)$, $\xi = \pm 1 - is
the vortex topological charge. The vortex position is described by the equation
\[ \text{Re} \hat{E}_r(x, y, z) = \text{Im} \hat{E}_r(x, y, z) = 0. \]

Numerical calculation of intensity distribution for Gaussian envelope (according to the second term in equation (2) and intensity of whole vortex beam with typical axial minima are shown on figure 1 (a, b). The phase pattern on figure 1 (c) depicts vortex phase as a helix.

![Figure 1](image1.png)

**Figure 1.** Intensity distribution of Gaussian beam (a) and vortex beam (b) with topological charge \( \zeta = +1 \) and its phase profile (c). Other beam parameters are: \( \omega_o = 30 \, \text{mcm}, \) \( n_r = 1.54, \) \( z = 20 \, \text{mm}, \) \( \lambda = 632.8 \, \text{nm}. \)

Singular beam phase is extremely sensitive to changes of surface and material parameters, like thickness and refractivity index, therefore this phenomenon can be exploited for metrology with high sensitivity which allow to study different physical processes at the nanoscale [12]. Let us consider a case when singular beam propagates through the isotropic plate with a stepped profile so that the reference (zero-order) plane coincides with the lower edge of the surface as shown in figure 2 (a), whereas due to the optical path difference from lower to height surfaces we can observe a phase shift [13]. The singular beam with helical wavefront has a unique feature – the spiral interference pattern and its rotation (illustrated in figure 2 (b, c) can be easy recovered with computer processing in real-time regime.

![Figure 2](image2.png)

**Figure 2.** Qualitative sketch of singular beam with topological charge \( \zeta = +1 \) spreading through the quartz plate with step profile (a) of height \( h \). Intensity distribution (b) of interference between Gaussian and singular beams after a step with \( h = 50 \, \text{nm} \) in comparison to the zero level \( h = 0 \, \text{nm} \) and its phase profiles respectively (c); \( n_r = 1.54, \) \( z = 20 \, \text{mm}, \) \( \lambda = 632.8 \, \text{nm}. \)
3. Experimental results and its discussion

We have experimentally considered the propagation of a paraxial monochromatic vortex beam through the isotropic plate with complex surface relief. An experimental set-up with specially designed interferometer is illustrated in figure 3. Linearly polarized Gaussian beam with a waist radius \( w_0 \approx 0.02 \text{ mm} \) at the wavelength \( \lambda = 0.6328 \mu\text{m} \) acquires an optical vortex with the topological charge \( \xi = \pm1 \) after passing through the optical wedge. Then the vortex beam is launched into the sample. The 60\( \times \) micro-objective projected the central beam region at the output face of the plate onto the input pupil of the CCD camera.

Due to the high sensitivity of singular beam phase to the small distortions of the wavefront, the interference of the reference beam and the vortex beam (Laguerre-Gaussian beam) transmitted through the isotropic thin plate coated with a wedge-shaped layer of \( 0 \sim 500 \text{ nm} \) thickness makes possible to analyse rotation of the spiral phase, depending on the thickness of applied layer, as shown on figure 4.

To enable rapid implementation and universal phase recovery method of vortex beam with images from the CCD camera, we used the method proposed in the works [14, 15]. For the comparison of object beam with the reference one the Mach-Zehnder interferometer was used (Figure 3). The sharp thick curves with a characteristic "spiral", corresponding to optical vortices were imaged by the camera and assayed. Total phase shift which is observable due to angular rotation of interference spiral can be calculated from simple equations: \( 2\pi - \lambda, \Delta\phi = n(d_1 - d_2) = nh \), where \( d_1 - d_2 \) is a difference between observable and neighboring levels of sample surface, \( n \) – is refractivity index of sample material.
4. Conclusion

Overcoming the diffraction limit is the topic of many fundamental and applied studies of modern optics. High interest is due to the fact that the field of application of nanoscale resolution is not limited to the improvement of image quality, but is widely used to recording on optical media, in lithography and nano-structuring and optical manipulation. The field of special interest is using of interferometry for diagnostics of stepped structures by 3D-profilometry, optical tomography of semi-transparent biological objects and other applications because it allows to measure thickness of the sample which is exceeding diffraction limits.

We have theoretically and experimentally considered evaluation of phase sensitivity and have shown that the distinguishable spiral phase rotation occurs at the isotropic plate thicknesses equals to $\lambda/130$, where $\lambda$ – is a wavelength. Proposed technique may be applied to optically transparent and reflecting surfaces exceed optical diffraction limit. Experimental measurement accuracy was reached up to 5 nm and may be improved by using of optical system with short focal distances and blue-light laser sources.

Moreover, this method applicable for non-destructive testing of live cells and biological tissues in real-time regime. Automatic processing of vortex spiral interferograms allow to achieve a vertical resolution down to 1.75 nm for visible light with $\lambda = 310$ nm and longitudinal resolution down to 7 nm.

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