Dynamics of a new delayed stage-structured predator-prey model with impulsive diffusion and releasing

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Abstract
In this work, we propose a new delayed stage-structured predator-prey model with impulsive diffusion and releasing. By the stroboscopic map of the discrete dynamical system, we obtain a prey-extinction boundary periodic solution. Furthermore, we prove that the prey-extinction boundary periodic solution is globally attractive. We also prove that the investigated system is permanent by the theory on the delay and impulsive differential equations. Our results indicate that time delay, impulsive diffusion, and impulsive releasing have influence to the dynamical behaviors of the investigated system. The results of this paper also provide a tactical basis for pest management.

Keywords: delayed stage structure; predator-prey model; impulsive diffusion; impulsive releasing; prey-extinction

1 Introduction
Many authors [1–8] and papers [9, 10] have studied the predator-prey, competitive, and cooperative models. Permanence and extinction are significant concepts of those models which also show many interesting results. However, the stage structure of a species has been considered very little. In the real world, almost all animals have the stage structure of being immature and mature. Recently, [11–17] studied the stage structure of species with or without time delays. Aiello et al. [12] considered a time delayed stage structure of being immature and mature of the population model

\begin{align}
\frac{dx_i(t)}{dt} &= \alpha x_m(t) - r x_i(t) - \alpha e^{-r\tau} x_m(t - \tau), \\
\frac{dx_m(t)}{dt} &= \alpha e^{-r\tau} x_m(t - \tau) - \beta x_m^2(t),
\end{align}

(1.1)

where \( x_i(t) \) denotes the immature population density at time \( t \), \( x_m(t) \) denotes the mature population density at time \( t \), \( \alpha > 0 \) represents the birth rate, \( r > 0 \) represents the immature death rate, \( \beta > 0 \) represents the mature death and the overcrowding rate, \( \tau > 0 \), represents the time to maturity rate, the term \( \alpha e^{-r\tau} x_m(t - \tau) \) represents the immature who were born at time \( t - \tau \) and survive at time \( t \) (with the immature death rate \( r \)) and therefore represents the transformation of the immature to the mature.
Dispersal is a ubiquitous phenomenon in the natural world. It is important for us to understand the ecological and evolutionary dynamics of populations mirrored by the large number of mathematical models devoted to it in the scientific literature [13–24]. If the population dynamics with the effects of spatial heterogeneity is modeled by a diffusion process, most previous papers focused on the population dynamical system modeled by the ordinary differential equations. But in practice, it is often the case that diffusion occurs in regular pulse. For example, when winter comes, birds will migrate between patches in search for a better environment, whereas they do not diffuse in other seasons, and the excursion of foliage seeds occurs at a fixed period of time every year. Thus impulsive diffusion provides a more natural description. Lately theories of impulsive differential equations [25, 26] have been introduced into population dynamics. Impulsive differential equations are found in most domains of applied science [15, 19, 20, 24, 27–29].

The organization of this paper is as follows. In the next section, we introduce the model and background concepts. In Section 3, some important lemmas are presented. In Section 4, we give the conditions of global attractivity and permanence for system (2.1). In Section 5, a brief discussion is given in the last section to conclude this work.

2 The model

Wang and Chen [17] considered a single population with impulsive diffusion. Jiao [24] considered a delayed predator-prey model with impulsive diffusion on predator and stage structure on prey. Inspired by [17, 24], we establish a new delayed stage-structured predator-prey model with impulsive diffusion and releasing

\[
\begin{align*}
\frac{dx_1(t)}{dt} &= r_1 y_1(t) - r_{1e^{-w_{1z_1}}} y_1(t - \tau_1) - w_{1x_1}(t), \\
\frac{dy_1(t)}{dt} &= r_{1e^{-w_{1z_1}}} y_1(t - \tau_1) - w_{1y_1}(t) - \beta_{1y_1}(t) z_1(t), \\
\frac{dz_1(t)}{dt} &= k_1 \beta_1 y_1(t) z_1(t) - w_{1z_1}(t), \\
\frac{dx_2(t)}{dt} &= r_{2x}(t) - r_{2e^{-w_{2z_2}}} y_2(t - \tau_2) - w_{2x_2}(t), \\
\frac{dy_2(t)}{dt} &= r_{2e^{-w_{2z_2}}} y_2(t - \tau_2) - w_{2y_2}(t) - \beta_{2y_2}(t) z_2(t), \\
\frac{dz_2(t)}{dt} &= k_2 \beta_2 y_2(t) z_2(t) - w_{2z_2}(t), \\
\Delta x_1(t) &= 0, \\
\Delta y_1(t) &= 0, \\
\Delta z_1(t) &= D(z_2(t) - z_1(t)), \\
\Delta x_2(t) &= 0, \\
\Delta y_2(t) &= 0, \\
\Delta z_2(t) &= D(z_1(t) - z_2(t)),
\end{align*}
\]

\[
\begin{align*}
\Delta x_1(t) &= 0, \\
\Delta y_1(t) &= 0, \\
\Delta z_1(t) &= \mu_1, \\
\Delta x_2(t) &= 0, \\
\Delta y_2(t) &= 0, \\
\Delta z_2(t) &= \mu_2, \\
\end{align*}
\]

\[t = (n + l)\tau, t \neq (n + 1)\tau, \quad t = (n + l)\tau, n \in Z^+, \quad t = (n + 1)\tau, n \in Z^+\]

(2.1)
with initial condition

\[
\begin{align*}
(\psi_1(\xi), \psi_2(\xi), \psi_3(\xi), \psi_4(\xi), \psi_5(\xi), \psi_6(\xi)) &\in C_+ = C([-\tau_1, 0], R^6), \\
\psi_i(0) &> 0, \quad i = 1, 2, 3, 4, 5, 6,
\end{align*}
\]

where system (2.1) is constructed of two patches. \(x_i(t), y_i(t)\) and \(z_i(t)\) represent the immature prey population, mature prey population, predator population in patch \(i = 1, 2\) at time \(t\). It is assumed that birth into the immature prey population is proportional to the existing mature prey population with proportionality constant \(r_i\) in patch \(i = 1, 2\). \(\tau_i\) represents a constant time to maturity of prey population in patch \(i = 1, 2\), that is, immature prey individuals and mature individuals are divided by age \(\tau_i\) in patch \(i = 1, 2\). The natural death rates \(w_{11}, w_{12}\) and \(w_{23}(i = 1, 2)\) are assumed for the immature prey population, mature prey population, and predator population in patch \(i = 1, 2\). \(\beta_i\) \((i = 1, 2)\) is the conversion rate of nutrients into the reproduction of the predator population in patch \(i = 1, 2\). The pulse diffusion occurs every \(\tau > 0\) period. The system evolves from its initial state without being further affected by diffusion until the next pulse appears. \(\Delta y_i(n + l)\tau = y_i((n + l)\tau^+ - y_i((n + l)\tau^-)\) where \(y_i((n + l)\tau^+)\) represents the density of population in the \(i\)th patch immediately after the \(n\)th diffusion pulse at time \(t = (n + l)\tau\), while \(y_i((n + l)\tau^-)\) represents the density of population in the \(i\)th patch before the \(n\)th diffusion pulse at time \(t = (n + l)\tau\) \((n \in \mathbb{Z}_+, 0 < l < 1)\). \(0 < D < 1\) is the dispersal rate of the predator population between two patches. It is assumed here that the net exchange from the \(j\)th patch to the \(i\)th patch is proportional to the difference \(y_j - y_i\) of the predator population densities. The predator population is released with \(\mu_i\) in patch \(i = 1, 2\) at moment \(t = (n + 1)\tau, n \in \mathbb{Z}_+\).

Because \(x_i(t)\) \((i = 1, 2)\) does not affect the other equations of (2.1), we can simplify system (2.1) and restrict our attention to the following system:

\[
\begin{align*}
\frac{d\Delta y_1(t)}{dt} &= r_1e^{-w_{11}l\tau}y_1(t - \tau_1) - w_{12}y_1(t) - \beta_1y_1(t)z_1(t), \\
\frac{d\Delta y_2(t)}{dt} &= k_1\beta_1y_1(t)z_1(t) - w_{13}z_1(t), \\
\frac{d\Delta z_1(t)}{dt} &= r_2e^{-w_{21}l\tau}y_2(t - \tau_2) - w_{22}y_2(t) - \beta_2y_2(t)z_2(t), \\
\frac{d\Delta z_2(t)}{dt} &= k_2\beta_2y_2(t)z_2(t) - w_{23}z_2(t), \\
\frac{\Delta y_1(t)}{t} &= D(z_2(t) - z_1(t)), \\
\frac{\Delta y_2(t)}{t} &= 0, \\
\Delta z_1(t) &= D(z_2(t) - z_1(t)), \\
\Delta z_2(t) &= D(z_1(t) - z_2(t)), \\
\frac{\Delta y_1(t)}{t} &= 0, \\
\frac{\Delta z_1(t)}{t} &= \mu_1, \\
\frac{\Delta y_2(t)}{t} &= 0, \\
\Delta z_2(t) &= \mu_2,
\end{align*}
\]

\((2.2)\)
with initial condition
\[ (\varphi_2(\xi), \varphi_3(\xi), \varphi_5(\xi), \varphi_6(\xi)) \in C, \quad C([-\tau, 0], R^4), \]
\[ \varphi_i(0) > 0, \quad i = 2, 3, 5, 6. \]

3 The lemmas

The solution of (2.1), denoted by \( X(t) = (x_1(t), y_1(t), z_1(t), x_2(t), y_2(t), z_2(t)) \), is a piecewise continuous function \( X : R_+ \to R^6 \). \( X(t) \) is continuous on \( (m\tau, (n + l)\tau] \), \( ((n + l)\tau, (n + 1)\tau] \), \( n \in Z_+ \), and \( X(m\tau) = \lim_{r \to m\tau^+} X(t), X((n + l)\tau) = \lim_{r \to (n + l)\tau^+} X(t) \) exist. Obviously the global existence and uniqueness of solutions of (2.1) are guaranteed by the smoothness properties of \( f \), which denotes the mapping defined by the right side of system (2.1) (see Lakshmikantham, [25]). Before we have the the main results, we need to give some lemmas which will be used in the following.

According to the biological meaning, it is assumed that \( x_i(t) \geq 0, y_i(t) \geq 0, \) and \( z_i(t) \geq 0 \) \( (i = 1, 2) \).

Let \( V : R_+ \times R^6 \to R_+ \), then \( V \) is said to belong to class \( V_0 \), if:

(i) \( V \) is continuous in \( (m\tau, (n + l)\tau] \times R^6 \) and \( ((n + l)\tau, (n + 1)\tau] \times R^6 \), for each \( z \in R^6 \), \( n \in Z_+ \), \( V(m\tau+)z = \lim_{r \to m\tau^+} V(t, y), V((n + l)\tau+, z) = \lim_{r \to (n + l)\tau^+} V(t, y) \) exist.

(ii) \( V \) is locally Lipschitzian in \( z \).

Definition 3.1 \( V \in V_0 \), then, for \( (t, z) \in (m\tau, (n + l)\tau] \times R^6 \) and \( ((n + l)\tau, (n + 1)\tau] \times R^6 \), the upper right derivative of \( V(t, z) \) with respect to the impulsive differential system (2.1) is defined as
\[ D^+ V(t, z) = \lim_{h \to 0^+} \frac{1}{h} [V(t + h, z + hf(t, z)) - V(t, z)]. \]

Lemma 3.2 ([26]) Let the function \( m \in PC[R^+], R \) satisfy the inequalities
\[
\begin{cases}
m'(t) \leq p(t)m(t) + q(t), \\
t \geq t_k, \quad t \neq t_k, \\
m(t_k) \leq d_km(t_k) + b_k, \\
k = 1, 2, \ldots,
\end{cases}
\]
where \( p(q) \in PC[R^+], R \) and \( d_k \geq 0, b_k \) are constants, then
\[
m(t) \leq m(t_0) \prod_{t_0 < t_k < t} d_k \exp \left( \int_{t_0}^t p(s) \, ds \right) \\
+ \sum_{t_0 < t \leq t_k} \left( \prod_{t_0 < t_k < t} d_k \exp \left( \int_{t_0}^t p(s) \, ds \right) \right) b_k \\
+ \int_{t_0}^t \prod_{t_0 < s < t_k} d_k \exp \left( \int_s^t p(\sigma) \, d\sigma \right) q(s) \, ds, \quad t \geq t_0.
\]

Now, we show that all solutions of (2.1) are uniformly ultimately bounded.

Lemma 3.3 There exists a constant \( M > 0 \) such that \( x_i(t) \leq M, y_i(t) \leq M, z_i(t) \leq M \) \( (i = 1, 2) \) for each solution \( (x_1(t), y_1(t), z_1(t), x_2(t), y_2(t), z_2(t)) \) of (2.1) with all \( t \) large enough.
Proof Define

\[ V(t) = \sum_{i=1}^{2} \left[ k_i x_i(t) + k_i y_i(t) + z_i(t) \right], \]

and \( d = \min\{w_{11}, w_{12}, w_{13}, w_{21}, w_{22}, w_{23}\} \), then \( t \neq (n + l)\tau \), \( t \neq (n + 1)\tau \), we have

\[ D^+ V(t) + dV(t) = \sum_{i=1}^{2} k_i r_i x_i(t) \]
\[ - \sum_{i=1}^{2} \left[ k_i(w_{i1} - d)x_i(t) + k_i(w_{i2} - d)y_i(t) + (w_{i3} - d)z_i(t) \right] \]
\[ \leq \sum_{i=1}^{2} k_i r_i x_i(t) < \zeta. \]

When \( t = (n + l)\tau \),

\[ V((n + l)\tau) = \sum_{i=1}^{2} \left[ x_i((n + l)\tau) + y_i((n + l)\tau) + z_i((n + l)\tau) \right] \]
\[ = \sum_{i=1}^{2} \left[ x_i((n + l)\tau) + y_i((n + l)\tau) + z_i((n + l)\tau) \right] = V(n\tau). \]

When \( t = (n + 1)\tau \),

\[ V((n + 1)\tau) = \sum_{i=1}^{2} \left[ x_i((n + 1)\tau) + y_i((n + 1)\tau) + z_i((n + 1)\tau) \right] \]
\[ = \sum_{i=1}^{2} \left[ x_i((n + 1)\tau) + y_i((n + 1)\tau) + z_i((n + 1)\tau) \right] + \mu_1 + \mu_2 \]
\[ = V((n + 1)\tau) + \mu_1 + \mu_2. \]

By Lemma 3.2, for \( t \in (n\tau, (n + 1)\tau] \), we have

\[ V(t) \leq V(0^+) e^{-dt} + \frac{\zeta}{d} (1 - e^{-dt}) + (\mu_1 + \mu_2) \frac{e^{-d(t-\tau)}}{1 - e^{-dt}} + (\mu_1 + \mu_2) \frac{e^d_{e^d - 1}}{e^d_{e^d - 1}}. \]

\[ \rightarrow \frac{\zeta}{d} + (\mu_1 + \mu_2) \frac{e^d_{e^d - 1}}, \text{ as } t \rightarrow \infty. \]

So \( V(t) \) is uniformly ultimately bounded. Hence, by the definition of \( V(t) \), there exists a constant \( M > 0 \) such that \( x_i(t) \leq M/k_i, y_i(t) \leq M/k_i, z_i(t) \leq M \) (\( i = 1, 2 \)) for \( t \) large enough.

The proof is complete. \( \square \)
If \( y_i(t) = 0 \) \((i = 1, 2)\), we have the following subsystem of (2.2):

\[
\begin{align*}
\frac{d z_1(t)}{dt} &= -w_{13} z_1(t), \\
\frac{d z_2(t)}{dt} &= -d_{23} z_2(t), \\
\Delta z_1(t) &= D(z_2(t) - z_1(t)), \\
\Delta z_2(t) &= D(z_1(t) - z_2(t)), \\
\Delta z_1(t) &= \mu_1, \\
\Delta z_2(t) &= \mu_2, \\
\end{align*}
\]

\((3.2)\)

where

\[
\begin{align*}
\frac{dn_1}{dt} &= -w_{13} n_1(t), \\
\frac{dn_2}{dt} &= -d_{23} n_2(t), \\
\Delta n_1(t) &= D(n_2(t) - n_1(t)), \\
\Delta n_2(t) &= D(n_1(t) - n_2(t)), \\
\Delta n_1(t) &= \mu_1, \\
\Delta n_2(t) &= \mu_2, \\
\end{align*}
\]

\((3.3)\)

Considering the third and fourth equations of (3.2), we have

\[
\begin{align*}
z_1((n + l)\tau) &= (1 - D)e^{-w_{13}l\tau} z_1(n\tau^+) + De^{-w_{23}l\tau} z_2(n\tau^+), \\
z_2((n + l)\tau) &= De^{-w_{13}l\tau} z_1(n\tau^+) + (1 - D)e^{-w_{23}l\tau} z_2(n\tau^+).
\end{align*}
\]

\((3.4)\)

Considering the fifth and sixth equations of (3.2), we also have

\[
\begin{align*}
z_1((n + 1)\tau^+) &= z_1((n + l)\tau^+)e^{-w_{13}(1-l)\tau} + \mu_1, \\
z_2((n + 1)\tau^+) &= z_2((n + l)\tau^+)e^{-w_{23}(1-l)\tau} + \mu_2.
\end{align*}
\]

\((3.5)\)

Substituting (3.4) into (3.5), we have the stroboscopic map of (3.2)

\[
\begin{align*}
z_1((n + 1)\tau^+) &= (1 - D)e^{-w_{13}\tau} z_1(n\tau^+) + De^{-w_{13}(1-l)\tau} z_2(n\tau^+) + \mu_1, \\
z_2((n + 1)\tau^+) &= De^{-w_{13}\tau} z_1(n\tau^+) + (1 - D)e^{-w_{23}\tau} z_2(n\tau^+) + \mu_2.
\end{align*}
\]

\((3.6)\)

Equation (3.6) has one fixed point:

\[
\begin{align*}
z_1^* &= \frac{\mu_2(1-A_1) + w_{13}A_2}{(1-A_2)(1-B_2) - A_2B_2} > 0, \\
z_2^* &= \frac{\mu_2B_1 + w_{23}(1-B_2)}{(1-A_2)(1-B_2) - A_2B_2} > 0,
\end{align*}
\]

\((3.7)\)

where

\[
\begin{align*}
A_1 &= (1 - D)e^{-w_{13}\tau} < 1, \\
B_1 &= De^{-w_{13}(1-l)\tau} z_2(n\tau^+) < 1, \\
A_2 &= De^{-w_{23}(1-l)\tau} z_2(n\tau^+) < 1, \\
B_2 &= (1 - D)e^{-w_{23}\tau} < 1.
\end{align*}
\]
Lemma 3.4 The fixed point \((z_1^*, z_2^*)\) of (3.6) is globally asymptotically stable.

Proof For convenience, we use the notation \((z_{n+1}^0, z_{n+1}^0) = (z_1(n\tau^*), z_2(n\tau^*))\). The linear form of (3.6) can be written as

\[
\begin{pmatrix}
z_{n+1}^0 \\
z_{n+1}^0
\end{pmatrix} = M \begin{pmatrix} z_n^0 \\ z_n^0 \end{pmatrix}.
\]

(3.8)

Obviously, the near dynamics of \((z_1^*, z_2^*)\) is determined by linear system (3.6). The stability of \((z_1^*, z_2^*)\) is determined by the eigenvalue of \(M \) less than 1. If \(M\) satisfies the Jury criterion [30], we can know the eigenvalue of \(M\) is less than 1,

\[
1 - \text{tr}M + \det M > 0.
\]

(3.9)

We can easily know that \((z_1^*, z_2^*)\) is unique fixed point of (3.6), and

\[
M = \begin{pmatrix} A_1 & B_1 \\ A_2 & B_2 \end{pmatrix}.
\]

(3.10)

For

\[
1 - \text{tr}M + \det M
\]

\[
= 1 - (A_1 + B_2) + (A_1B_2 - A_2B_1)
\]

\[
= (1 - A_1)(1 - B_2) - A_2B_1
\]

\[
= \left[1 - (1 - D)e^{-w_{13}\tau}\right] \times \left[1 - (1 - D)e^{-w_{23}\tau}\right] - D^2 e^{-(w_{13}+d_2)\tau}
\]

\[
= \left[1 - e^{-w_{13}\tau}\right] \times \left[1 - e^{-w_{23}\tau}\right] - D^2 e^{-(w_{13}+w_{23})\tau}
\]

\[
= (1 - e^{-w_{13}\tau}) \times (1 - e^{-w_{23}\tau}) + De^{-w_{23}\tau}(1 - e^{-w_{13}\tau}) + De^{-w_{13}\tau}(1 - e^{-w_{23}\tau})
\]

\[
> 0.
\]

From the Jury criterion, \((z_1^*, z_2^*)\) is locally stable, then it is globally asymptotically stable. This completes the proof.

\[\square\]

Lemma 3.5 The periodic solution \((\overline{z_1(t)}, \overline{z_2(t)})\) of system (3.2) is globally asymptotically stable, where

\[
\overline{z_1(t)} = \begin{cases}
z_1^* e^{-w_{13}(t-n\tau)}, & t \in [n\tau, (n+l)\tau), \\
z_1^* e^{-w_{13}((n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau),
\end{cases}
\]

\[
\overline{z_2(t)} = \begin{cases}
z_2^* e^{-w_{23}(t-n\tau)}, & t \in [n\tau, (n+l)\tau), \\
z_2^* e^{-w_{23}((n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau),
\end{cases}
\]

(3.11)

where \(z_1^*\) and \(z_2^*\) are determined as (3.7), \(z_1^{**}\) and \(z_2^{**}\) are defined as

\[
\begin{align*}
z_1^{**} &= (1 - D)e^{-w_{13}\tau} z_1^* + De^{-w_{23}\tau} z_2^*, \\
z_2^{**} &= De^{-w_{23}\tau} z_1^* + (1 - D)e^{-w_{23}\tau} z_2^*.
\end{align*}
\]

(3.12)
Lemma 3.6 ([31]) Consider the following equation:

\[
\frac{dx(t)}{dt} = a_1x(t - \omega) - a_2x(t),
\]

where \(a_1, a_2, \omega > 0; x(t) > 0 \) for \(-\omega \leq t \leq 0\), we have:

(i) if \(a_1 < a_2\), then, \(\lim_{t \to -\infty} x(t) = 0\),

(ii) if \(a_1 > a_2\), then, \(\lim_{t \to -\infty} x(t) = +\infty\).

4 The dynamics

From the above discussion, we know there exists a prey-extinction boundary periodic solution \((0, z_1(t), 0, z_2(t))\) of system (2.2). In this section, we will prove that the prey-extinction boundary periodic solution \((0, z_1(t), 0, z_2(t))\) of system (2.2) is globally attractive.

Theorem 4.1 If

\[
\max_{i=1,2} \{r_i e^{-\omega_1 \tau_i} - [w_{i2} + \beta_i (z_i^* + z_i^{**})]\} < 0 \quad (i = 1, 2)
\]

holds, the prey-extinction boundary periodic solution \((0, \tilde{z}_1(t), 0, \tilde{z}_2(t))\) of (2.2) is globally attractive, where \(z_i^*(i = 1, 2)\) is determined by (3.7), \(z_i^{**}(i = 1, 2)\) is defined by (3.12).

Proof From (4.1), we can obtain

\[
r_i e^{-\omega_1 \tau_i} < [w_{i2} + \beta_i (z_i^* + z_i^{**})].
\]

Then, we can choose \(\varepsilon_0\) sufficiently small such that

\[
r_i e^{-\omega_1 \tau_i} < w_{i2} + \beta_i \left[(z_i^* + z_i^{**}) - \varepsilon_0\right].
\]

From the second and fourth equations of system (2.2), we obtain \(\frac{dz(t)}{dt} \geq -w_{i3}z_i(t) (i = 1, 2)\).

So we consider the following comparison impulsive differential system:

\[
\begin{aligned}
\frac{dz_{11}(t)}{dt} &= -w_{13}z_{11}(t), \quad t \neq (n + l)\tau, t \neq (n + 1)\tau, \\
\frac{dz_{21}(t)}{dt} &= -w_{23}z_{21}(t), \\
\Delta z_{11}(t) &= D(z_{21}(t) - z_{11}(t)), \quad t = (n + l)\tau, \\
\Delta z_{21}(t) &= D(z_{12}(t) - z_{21}(t)), \\
\Delta z_{11}(t) &= \mu_1, \\
\Delta z_{21}(t) &= \mu_2, \quad t = (n + 1)\tau, n \in Z^+.
\end{aligned}
\]

In view of Lemma 3.4 and (3.5), we find that the boundary periodic solution of system (4.1)

\[
\begin{aligned}
\tilde{z}_{11}(t) &= \begin{cases} 
\begin{array}{ll}
z_{11}^* e^{-w_{13}(t-n\tau)}, & t \in [n\tau, (n + l)\tau), \\
z_{11}^{**} e^{-w_{13}(t-(n+1)\tau)}, & t \in [(n + l)\tau, (n + 1)\tau),
\end{array}
\end{cases} \\
\tilde{z}_{21}(t) &= \begin{cases} 
\begin{array}{ll}
z_{21}^* e^{-w_{23}(t-n\tau)}, & t \in [n\tau, (n + l)\tau), \\
z_{21}^{**} e^{-w_{23}(t-(n+1)\tau)}, & t \in [(n + l)\tau, (n + 1)\tau),
\end{array}
\end{cases}
\end{aligned}
\]

is globally attractive.
is globally asymptotically stable, where $z^*_1$ and $z^*_2$ are determined as (3.7), $z^{i*}_1$ and $z^{i*}_2$ are defined as (3.12).

From Lemma 3.5 and comparison theorem of impulsive equation [2], we have $z_i(t) \geq z_{i0}(t)$ $(i = 1, 2)$ and $z_{i0}(t) \to z^{\infty}_i(t)$ as $t \to \infty$. Then there exists an integer $k_3 > k_1$, $t > k_3$ such that

$$z_i(t) \geq z_{i0}(t) \geq z^{\infty}_i(t) - \varepsilon_0 \quad (i = 1, 2), n\tau < t \leq (n + 1)\tau, n > k_2,$$

that is,

$$z_i(t) > z^{\infty}_i(t) - \varepsilon_0 \geq \left(z^{i*}_1 + z^{i*}_2\right) - \varepsilon_0 \overset{\Delta}{=} Q_i \quad (i = 1, 2), n\tau < t \leq (n + 1)\tau, n > k_2.$$

From (2.2), we get

$$\frac{d y_i(t)}{dt} \leq r_i e^{-\omega_{1i}} y_i(t - \tau_i) - (w_{12} + \beta \rho_i) y_i(t) \quad (i = 1, 2), t > n\tau + \tau_1, n > k_2. \quad (4.6)$$

Consider the following comparison differential system referring to (4.6):

$$\frac{d R_i(t)}{dt} = r_i e^{-\omega_{1i}} R_i(t - \tau_i) - (w_{12} + \beta \rho_i) R_i(t) \quad (i = 1, 2), t > n\tau + \tau_1, n > k_2. \quad (4.7)$$

From (4.3) and Lemma 3.6, we have $\lim_{t \to \infty} R_i(t) = 0$.

Let $(y_1(t), z_1(t), y_2(t), z_2(t))$ be the solution of system (2.2) with initial conditions and $y_1(\xi) = \varphi_2(\xi)$ ($\xi \in [-\tau_1, 0]$), $y_2(\xi) = \varphi_3(\xi)$ ($\xi \in [-\tau_1, 0]$). $R_i(t)$ $(i = 1, 2)$ is the solution of system (4.7) with initial conditions $R_i(\xi) = \varphi_2(\xi)$ ($\xi \in [-\tau_1, 0]$), $R_2(\xi) = \varphi_5(\xi)$ ($\xi \in [-\tau_1, 0]$). By the comparison theorem, we have

$$\lim_{t \to \infty} y_i(t) < \lim_{t \to \infty} R_i(t) = 0.$$

Incorporating the positivity of $y_i(t)$, we know that $\lim_{t \to \infty} y_i(t) = 0$. Therefore, for any $\varepsilon_1 > 0$ (sufficiently small) and $\varepsilon_1 < \min\{\frac{v_{12}}{\beta}, \frac{v_{23}}{\beta}\}$, there exists an integer $k_3$ $(k_3 \tau > k_2 \tau + \tau_1)$ such that $y_i(t) < \varepsilon_1$ $(i = 1, 2)$ for all $t > k_3 \tau$.

From the second and fourth equations of system (2.2), we have

$$-w_{13} z_i(t) \leq \frac{d z_i(t)}{dt} \leq -(w_{13} - \varepsilon_1) z_i(t) \quad (i = 1, 2). \quad (4.8)$$

Then we have $z_{i0}(t) \leq z_i(t) \leq z^{\infty}_i(t)$ and $z_{i0}(t) \to z^{\infty}_i(t), z_{i0}(t) \to z^{\infty}_i(t)$ $(i = 1, 2)$ as $t \to \infty$.

While $(z_{i2}(t), z_{i2}(t))$ and $(z_{i3}(t), z_{i3}(t))$ are the solutions of

$$\begin{align*}
\begin{cases}
\frac{dz_{i2}(t)}{dt} = -w_{13} z_{i2}(t), & \quad t \neq (n + l)\tau, t \neq (n + 1)\tau, \\
\frac{dz_{i3}(t)}{dt} = -w_{23} z_{i3}(t), & \quad t \neq (n + l)\tau,
\end{cases}
\end{align*}$$

$$\begin{align*}
\Delta z_{i2}(t) &= D(z_{i2}(t) - z_{i2}(t)), & \quad t = (n + l)\tau, \\
\Delta z_{i3}(t) &= D(z_{i1}(t) - z_{i2}(t)), & \quad t = (n + l)\tau,
\end{align*}$$

$$\begin{align*}
\Delta z_{i2}(t) &= \mu_1, & \quad t = (n + 1)\tau, n \in Z^+, \\
\Delta z_{i3}(t) &= \mu_2, & \quad t = (n + 1)\tau, n \in Z^+.
\end{align*} \quad (4.9)$$
and
\[
\begin{align*}
\frac{dz_{13}(t)}{dt} &= -(w_{13} - k_1 e_1)z_{13}(t), \\
\frac{dz_{23}(t)}{dt} &= -(w_{23} - k_2 e_1)z_{23}(t), \\
\Delta z_{13}(t) &= D(z_{23}(t) - z_{13}(t)), \\
\Delta z_{23}(t) &= D(z_{13}(t) - z_{23}(t)),
\end{align*}
\]
respectively, we have
\[
\begin{align*}
z_{13}(t) &= \begin{cases}
  z_{13}^* e^{-(w_{13} - k_1 e_1)(n+1)\tau}, & t \in [(n+1)\tau, n\tau), \\
  z_{13}^* e^{-(w_{13} - k_1 e_1)n\tau}, & t \in [n\tau, (n+1)\tau),
\end{cases} \\
z_{23}(t) &= \begin{cases}
  z_{23}^* e^{-(w_{23} - k_2 e_1)(n+1)\tau}, & t \in [(n+1)\tau, n\tau), \\
  z_{23}^* e^{-(w_{23} - k_2 e_1)n\tau}, & t \in [n\tau, (n+1)\tau),
\end{cases}
\]
where
\[
\begin{align*}
z_{13}^* &= \frac{\mu_2(1-A_{13})+\mu_4 A_{23}}{(1-A_{13})(1-B_{23})-A_{23} B_{13}} > 0, \\
z_{23}^* &= \frac{\mu_2 B_{13}+\mu_4(1-B_{23})}{(1-A_{13})(1-B_{23})-A_{23} B_{13}} > 0,
\end{align*}
\]
and
\[
\begin{align*}
z_{13}^{**} &= (1-D) e^{-(w_{13} - k_1 e_1)\tau} z_{13}^* + D e^{-(w_{23} - k_2 e_1)\tau} z_{23}^*, \\
z_{23}^{**} &= D e^{-(w_{23} - k_2 e_1)\tau} z_{23}^* + (1-D) e^{-(w_{13} - k_1 e_1)\tau} z_{13}^*,
\end{align*}
\]
and
\[
\begin{align*}
A_{13} &= (1-D) e^{-(w_{13} - k_1 e_1)\tau} < 1, \\
B_{13} &= D e^{-(w_{13} - k_1 e_1)(1-\tau)+w_{23} - k_2 e_1)\tau} < 1, \\
A_{23} &= D e^{-(w_{23} - k_2 e_1)(1-\tau)+w_{13} - k_1 e_1)\tau} < 1, \\
B_{23} &= (1-D) e^{-(w_{23} - k_2 e_1)\tau} < 1.
\end{align*}
\]
Therefore, for any \(\varepsilon_2 > 0\) (\(\varepsilon_2\) is small enough), there exists an integer \(k_4, n > k_4\) such that \(z_{13}(t) - \varepsilon_2 < z_{i3}(t) < z_{13}(t) + \varepsilon_2\) (\(i = 1, 2\)). Let \(\varepsilon_1 \to 0\), so we have \(z_{i3}(t) - \varepsilon_2 < z_{i3}(t) < z_{i3}(t) + \varepsilon_2\) (\(i = 1, 2\)), for \(t\) large enough. This implies \(z_{i3}(t) \to \widetilde{z}_{i3}(t)\) (\(i = 1, 2\)) as \(t \to \infty\). This completes the proof. \(\square\)

The next work is to investigate the permanence of system (2.2). Before starting our theorem, we give the following definition.

**Definition 4.2** System (2.2) is said to be permanent if there are constants \(m, M > 0\) (independent of initial value) and a finite time \(T_0\) such that, for all solutions \((y_i(t), z_i(t), \)
If Theorem 4.2. We claim that for any 
\[ y_i(t), z_1(t), z_2(t) \text{ with all initial values } y_i(0^{+}) > 0, z_i(0^{+}) > 0 \ (i = 1, 2), \]
\[ m \leq y_i(t) \leq M, \ m \leq z_i(t) \leq M \ (i = 1, 2) \text{ hold for all } t \geq T_0. \]
Here \( T_0 \) may depend on the initial values \( (y_i(0^{+}), z_i(0^{+}), y_2(0^{+}), z_2(0^{+})) \).

**Theorem 4.3** If
\[
\min_{i=1,2} \left[ r_i e^{-w_1 t_i} - w_{i2} - \beta_i \left( z_{i4}^* e^{-(w_{i3} - k_i \beta_i \gamma)^i t} + z_{i4}^{**} e^{-(w_{i3} + k_i \beta_i \gamma)^i t} \right) \right] > 0,
\]
there is a positive constant \( q \) such that each positive solution \( (y_i(t), z_1(t), z_2(t)) \) of (2.2) satisfies \( y_i(t) \geq q, \) for \( t \) large enough, where \( y_i^* \ (i = 1, 2) \) is determined by
\[
r_i e^{-w_1 t_i} = w_{i2} + \beta_i \left( z_{i4}^* e^{-(w_{i3} - k_i \beta_i \gamma)^i t} + z_{i4}^{**} e^{-(w_{i3} + k_i \beta_i \gamma)^i t} \right) \quad (i = 1, 2),
\]
where \( z_{i4}^* \ (i = 1, 2) \) and \( z_{i4}^{**} \ (i = 1, 2) \) are defined as (4.19) and (4.20), respectively.

**Proof** The second and fourth equations of (2.2) can be rewritten as
\[
\frac{dy_i(t)}{dt} = \left[ r_i e^{-w_1 t_i} - (w_{i2} + \beta_i z_i(t)) \right] y_i(t) - r_i e^{-w_1 t_i} \frac{d}{dt} \int_{t-t_i}^t y_i(u) \, du \quad (i = 1, 2). \tag{4.14}
\]
According to (4.14), \( Q_i(t) \ (i = 1, 2) \) is defined as
\[
Q_i(t) = y_i(t) + r_i e^{-w_1 t_i} \int_{t-t_i}^t y_i(u) \, du \quad (i = 1, 2).
\]
We calculate the derivative of \( Q_i(t) \ (i = 1, 2) \) along the solution of (2.2):
\[
\frac{dQ_i(t)}{dt} = \left[ r_i e^{-w_1 t_i} - (w_{i2} + \beta_i z_i(t)) \right] y_i(t) \quad (i = 1, 2). \tag{4.15}
\]
Since
\[
r_i e^{-w_1 t_i} > w_{i2} + \beta_i \left( z_{i4}^* e^{-(w_{i3} - k_i \beta_i \gamma)^i t} + z_{i4}^{**} e^{-(w_{i3} + k_i \beta_i \gamma)^i t} \right) \quad (i = 1, 2),
\]
we can easily see that there exists a sufficiently small \( \varepsilon > 0 \) such that
\[
r_i e^{-w_1 t_i} > w_{i2} + \beta_i \left( z_{i4}^* e^{-(w_{i3} - k_i \beta_i \gamma)^i t} + z_{i4}^{**} e^{-(w_{i3} + k_i \beta_i \gamma)^i t} + \varepsilon \right) \quad (i = 1, 2).
\]
We claim that for any \( t_0 > 0, \) it is impossible that \( y_i(t) < y_i^* \ (i = 1, 2) \) for all \( t > t_0. \) Suppose that the claim is not valid. Then, there is a \( t_0 > 0 \) such that \( y_i(t) < y_i^* \ (i = 1, 2) \) for all \( t > t_0. \) It follows from the first and third equations of (2.2) that for all \( t > t_0 \)
\[
\frac{dz_i(t)}{dt} < -(w_{i3} - k_i \beta_i y_i^*) z_i(t) \quad (i = 1, 2). \tag{4.16}
\]
Consider the following comparison impulsive system for all $t > t_0$:

$$
\begin{align*}
\frac{d\tilde{z}_{14}(t)}{dt} &= -(w_{13} - k_1\beta_i^*)\tilde{z}_{14}(t), \quad t \neq (n + l)\tau, t \neq (n + 1)\tau, \\
\frac{d\tilde{z}_{24}(t)}{dt} &= -(w_{23} - k_2\beta_i^*)\tilde{z}_{24}(t), \\
\Delta\tilde{z}_{14}(t) &= D(\tilde{z}_{24}(t) - \tilde{z}_{14}(t)), \\
\Delta\tilde{z}_{24}(t) &= D(\tilde{z}_{14}(t) - \tilde{z}_{24}(t)), \\
\Delta\tilde{z}_{14}(t) &= \mu_1, \\
\Delta\tilde{z}_{24}(t) &= \mu_2, \\
\end{align*}
$$

where

$$
\begin{align*}
\tilde{z}_{14}(t) &= \begin{cases} 
\tilde{z}_{14}^* e^{-(w_{13} - k_1\beta_i^*)\tau t}, & t \in [n\tau, (n + l)\tau), \\
\tilde{z}^{**}_{14} e^{-(w_{13} - k_1\beta_i^*)\tau t - \omega (\tau)}, & t \in [(n + l)\tau, (n + 1)\tau), 
\end{cases} \\
\tilde{z}_{24}(t) &= \begin{cases} 
\tilde{z}_{24}^* e^{-(w_{23} - k_2\beta_i^*)\tau t - \nu (\tau)}, & t \in [n\tau, (n + l)\tau), \\
\tilde{z}^{**}_{24} e^{-(w_{23} - k_2\beta_i^*)\tau t}, & t \in [(n + l)\tau, (n + 1)\tau), 
\end{cases}
\end{align*}
$$

here

$$
\begin{align*}
\tilde{z}_{14}^* &= \frac{(l - A_{14})\mu_1 A_{24}}{(l - A_{14})(l - B_{24}) - A_{24} B_{14}}, \quad \tilde{z}_{24}^* > 0, \\
\tilde{z}_{24}^{**} &= \frac{(l - A_{24})\mu_2 A_{14}}{(l - A_{24})(l - B_{14}) - A_{14} B_{24}}, \quad \tilde{z}_{14}^{**} > 0,
\end{align*}
$$

and

$$
\begin{align*}
\tilde{z}_{14}^* &= (1 - D) e^{-(w_{13} - k_1\beta_i^*)\tau t} \tilde{z}_{14}^* + D e^{-(w_{23} - k_2\beta_i^*)\tau t} \tilde{z}_{24}^*, \\
\tilde{z}_{24}^* &= D e^{-(w_{13} + k_1\beta_i^*)\tau t} \tilde{z}_{14}^* + (1 - D) e^{-(w_{23} - k_2\beta_i^*)\tau t} \tilde{z}_{24}^*,
\end{align*}
$$

and

$$
\begin{align*}
A_{14} &= (1 - D) e^{-(w_{13} - k_1\beta_i^*)\tau t} < 1, \\
B_{14} &= D e^{-(w_{13} - k_1\beta_i^*)\tau t - \omega (\tau)} + (w_{23} - k_2\beta_i^*)\tau t < 1, \\
A_{24} &= D e^{-(w_{13} - k_1\beta_i^*)\tau t + (w_{23} - k_2\beta_i^*)\tau t} < 1, \\
B_{24} &= (1 - D) e^{-(w_{23} - k_2\beta_i^*)\tau t} < 1.
\end{align*}
$$

By the comparison theorem for impulsive differential equations [28], we know that there exists a sufficient small $\varepsilon > 0$ and $t_1 > t_0 + r_1$ such that the inequality $z_i(t) \leq \tilde{z}_{i4}(t) + \varepsilon (i = 1, 2)$ holds for $t \geq t_1$, thus $z_i(t) \leq \tilde{z}_{i4}(t) \leq \tilde{z}_{i4}^* e^{-(w_{13} - k_1\beta_i^*)\tau t + \varepsilon (\tau)} e^{-(w_{23} + k_1\beta_i^*)\tau t - \omega (\tau) / \tau} + \varepsilon (i = 1, 2)$ for all $t \geq t_1$. We use the notation $\sigma_i = \frac{\varepsilon}{\tilde{z}_{i4}^* e^{-(w_{13} - k_1\beta_i^*)\tau t + \varepsilon (\tau)} e^{-(w_{23} + k_1\beta_i^*)\tau t - \omega (\tau) / \tau}}$ for convenience. So we have

$$
r_i e^{-w_{13} t_i} > w_{12} + \beta_i \sigma_i \quad (i = 1, 2),
$$

then we have

$$
Q(t) > y_2(t) \left[ r_i e^{-w_{13} t_i} - (w_{12} + \beta_i \sigma_i) \right] \quad (i = 1, 2),
$$
for all \( t > t_1 \). Set \( y_i^m = \min_{t \in [t_1, t_1 + \tau_1]} y_i(t) \), we will show that \( y_i(t) \geq y_i^m \) for all \( t \geq t_1 \). Suppose the contrary, then there is a \( T_0 > 0 \) such that \( y_i(t) \geq y_i^m \) for \( t_1 \leq t \leq t_1 + T_0 \), \( y_i(t_1 + T_0) = y_i^m \) and \( y'_i(t_1 + T_0) < 0 \). Hence, the second and fourth equations of system (2.2) imply that

\[
y'_i(t_1 + T_0) = r_i e^{-\sigma_1 t} y_i(t_1 + T_0) - \left[ w_{i1} + \beta_i z_i(t_1 + T_0) \right] y_i(t_1 + T_0) \geq \left[ r_i e^{-\sigma_1 t_1} - (w_{i2} + \beta_i \sigma_1) \right] m_i^m > 0 \quad (i = 1, 2).
\]

This is a contradiction. Thus, \( y_i(t) \geq y_i^m \) for all \( t > t_1 \). As a consequence, then \( Q_i(t) > y_i^m [r_i e^{-\sigma_1 t_1} - (w_{i2} + \beta_i \sigma_1)] > 0 \) \( (i = 1, 2) \) for all \( t > t_1 \). This implies that as \( t \to \infty \), \( Q_i(t) \to \infty \). It is a contradiction to \( Q_i(t) \leq M(1 + r_i e^{-\sigma_1 t_1}) \). Hence, the claim is complete.

By the claim, we are left to consider two cases. First, \( y_i(t) \geq y_i^* \ (i = 1, 2) \) for all \( t \) large enough. Second, \( y_i(t) \ (i = 1, 2) \) oscillates about \( y_i^* \ (i = 1, 2) \) for \( t \) large enough.

Define

\[
q = \min \left\{ \frac{y_1^*}{2}, \frac{y_2^*}{2}, q_1, q_2 \right\},
\]

where \( q_i = y_i^* e^{-(w_{i1} + \beta_i M) \tau_1} \ (i = 1, 2) \). We hope to show that \( y_i(t) \geq q_i \ (i = 1, 2) \) for all \( t \) large enough. The conclusion is evident in the first case. For the second case, let \( t^* > 0 \) and \( \xi > 0 \) satisfy \( y_i(t^*) = y_i(t^* + \xi) = y_i^* \ (i = 1, 2) \) and \( y_i(t) < y_i^* \ (i = 1, 2) \) for all \( t^* < t < t^* + \xi \) where \( t^* \) is sufficiently large such that \( y_i(t) > \sigma_i \ (i = 1, 2) \) for \( t^* < t < t^* + \xi \) and \( y_i(t) \ (i = 1, 2) \) is uniformly continuous. The positive solutions of (2.2) are ultimately bounded and \( y_i(t) \ (i = 1, 2) \) is not affected by impulses. Hence, there is a \( T \ (0 < t < \tau_1) \) and \( T \) is dependent on the choice of \( t^* \) such that \( y_i(t^*) > \frac{y_i^*}{2} \ (i = 1, 2) \) for \( t^* < t < t^* + T \). If \( \xi < T \), there is nothing to prove. Let us consider the case \( T < \xi < \tau_1 \). Since \( y_i(t) > - (w_{i2} + \beta_i M) y_i(t) \ (i = 1, 2) \) and \( y_i(t^*) = y_i^* \ (i = 1, 2) \), it is clear that \( y_i(t) \geq q_i \ (i = 1, 2) \) for \( t \in [t^*, t^* + \tau_1) \). Then, proceeding exactly as the proof for the above claim, we see that \( y_i(t) \geq q_i \) for \( t \in [t^*, t^* + \tau_1] \). Because the kind of interval \( t \in [t^*, t^* + \xi] \) is chosen in an arbitrary way (we only need \( t^* \) to be large), we conclude that \( y_i(t) \geq q \) for all large \( t \). In the second case, in view of the above discussion, the choice of \( q \) is independent of the positive solution, and we prove that any positive solution of (2.2) satisfies \( y_i(t) \geq q \) for all sufficiently large \( t \). This completes the proof of the theorem.

**Theorem 4.4 If**

\[
\min_{i=1,2} \left[ r_i e^{-\sigma_1 t_1} - w_{i2} - \beta_i (x_{i1} e^{-(w_{i3} + k_i \beta_i \gamma_i)} t_i + z_{i1}^* e^{-(w_{i3} + k_i \beta_i \gamma_i)(1-i \gamma_i)t_i}) \right] > 0,
\]

**system (2.2) is permanent.**

**Proof** Denote \( (y_1(t), z_1(t), y_2(t), z_2(t)) \) for any solution of system (2.2). From system (2.2) and Lemma 3.3, we can easily obtain

\[
\frac{dz_i(t)}{dt} > -w_{i2} z_i(t) \quad (i = 1, 2).
\]
Consider the comparison impulsive system (4.9) for all $t > t_0$. By Lemma 3.5, we obtain

$$
\begin{align*}
\tilde{z}_{12}(t) &= \begin{cases} 
\tilde{z}_1^* e^{-\nu_3(t-t_1)}, & t \in [n\tau,(n+1)\tau), \\
\tilde{z}_1^{**} e^{-\nu_3(t-(n+1)\tau)}, & t \in [(n+1)\tau,(n+1)\tau),
\end{cases} \\
\tilde{z}_{22}(t) &= \begin{cases} 
\tilde{z}_2^* e^{-\nu_3(t-t_2)}, & t \in [n\tau,(n+1)\tau), \\
\tilde{z}_2^{**} e^{-\nu_3(t-(n+1)\tau)}, & t \in [(n+1)\tau,(n+1)\tau),
\end{cases}
\end{align*}
$$

(4.23)

Here $z_1^*$ and $z_2^*$ are defined as (3.7), $z_1^{**}$ and $z_2^{**}$ are defined as (3.12). By the comparison theorem for impulsive differential equation [28], we know that there exists a sufficient small $\varepsilon > 0$ and $t_1 (> t_0 + t_1)$ such that the inequality $z_i(t) \geq \tilde{z}_i(t) - \varepsilon$ ($i = 1, 2$) holds for $t \geq t_1$, thus $z_i(t) \geq [z_i^* e^{-\nu(t-t_1)} + z_i^{**} e^{-\nu_3(1-\tau)t}] - \varepsilon \geq \hat{p}_i$ for all $t \geq t_1$. By Theorem 4.3, Lemma 3.3, and the above discussion, system (2.2) is permanent. The proof of Theorem 4.4 is complete. □

5 Discussion

In this paper, we investigate a new delayed stage-structured predator-prey model with impulsive diffusion and releasing. We analyze that the prey-extinction boundary periodic solution of system (2.2) is globally attractive, and we also obtain the permanent condition of system (2.2). From Theorem 4.1 and Theorem 4.4, we can easily guess that there must exist a threshold $\mu^* (\mu^* = \max_{i=1,2}\mu_i^*)$ and $\mu_i^*$ ($i = 1, 2$) is determined by the condition of Theorem 4.1), if $\mu > \mu^*$, the prey-extinction boundary periodic solution $(0, \tilde{z}_1(t), 0, \tilde{z}_2(t))$ of (2.2) is globally attractive. If $\mu < \mu^*$ ($\mu^{**} = \min_{i=1,2}\mu_i^{**}$) and $\mu_i^{**}$ ($i = 1, 2$) is determined by the condition of Theorem 4.4), system (2.2) is permanent. From Theorem 4.1 and Theorem 4.4, we can also easily guess that there must exist a threshold $D^*$ ($0 < D^* < 1$). If $D < D^*$, the prey-extinction boundary periodic solution $(0, \tilde{z}_1(t), 0, \tilde{z}_2(t))$ of (2.2) is globally attractive. If $D > D^*$, system (2.2) is permanent. This indicates that impulsive diffusion and impulsive releasing can affect the dynamical behaviors of the investigated system (2.2). That is to say, impulsive diffusion and impulsive releasing of the predator population play important roles for the prey-extinction of system (2.2). The parameters as $\tau_i$ ($i = 1, 2$) and $\tau$ can also be discussed, its change also affects the dynamical system of (2.2). The results of this paper provide a tactical basis for pest management.

Competing interests
The authors declare that they have no competing interests.

Authors’ contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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