Signatures of octupole shape phase transitions in radioactive nuclei

Kosuke Nomura
Department of Physics, Faculty of Science, University of Zagreb, 10000 Zagreb, Croatia
E-mail: knomura@phy.hr

Abstract. We analyze the octupole deformations and the related collective excitations in medium-heavy and heavy nuclei based on the microscopic framework of the nuclear energy density functional theory. Constrained self-consistent mean-field calculation with a given energy density functional is performed to provide for each nucleus a potential energy surface with axial quadrupole and octupole shape degrees of freedom. Spectroscopic properties are computed by means of the interacting-boson Hamiltonian, which is determined by mapping the fermionic potential energy surface onto the bosonic counterpart. The overall systematics of the calculated spectroscopic observables exhibit phase transitional behaviors between stable octupole deformation and octupole vibration characteristic of the octupole-soft potential within the set of nuclei in light actinide and rare-earth regions, Th, Ra, Sm, Gd, and Ba isotopes, where octupole shapes are most likely to occur.

1. Introduction

The octupole (or pear-like) deformation in nuclei is one of the most prominent and studied themes in nuclear structure physics [1]. Measurement of permanent octupole deformation has an implication for new physics beyond the Standard Model of elementary particles. Experiments using radioactive-ion beams are planned or already operational around the world to find evidence for strong octupole deformation in several mass regions, e.g., $A \approx 220$ and $A \approx 144$. In this context, timely systematic and reliable nuclear structure calculations on octupole deformations and the related spectroscopic properties over a wide range of the chart of nuclides are required. We have carried out large-scale spectroscopic studies on octupole deformations and the related spectroscopic properties over a wide range of the chart of nuclides are required. We have carried out large-scale spectroscopic studies on octupole deformations and the related spectroscopic properties over a wide range of the chart of nuclides are required. We have carried out large-scale spectroscopic studies on octupole deformations and the related spectroscopic properties over a wide range of the chart of nuclides are required. We have carried out large-scale spectroscopic studies on octupole deformations and the related spectroscopic properties over a wide range of the chart of nuclides are required. We have carried out large-scale spectroscopic studies on octupole deformations and the related spectroscopic properties over a wide range of the chart of nuclides are required. We have carried out large-scale spectroscopic studies on octupole deformations and the related spectroscopic properties over a wide range of the chart of nuclides are required. We have carried out large-scale spectroscopic studies on octupole deformations and the related spectroscopic properties over a wide range of the chart of nuclides are required. We have carried out large-scale spectroscopic studies on octupole deformations and the related spectroscopic properties over a wide range of the chart of nuclides are required. We have carried out large-scale spectroscopic studies on octupole deformations and the related spectroscopic properties over a wide range of the chart of nuclides are required. We have carried out large-scale spectroscopic studies on octupole deformations and the related spectroscopic properties over a wide range of the chart of nuclides are required.

arXiv:1909.10161v1 [nucl-th]  23 Sep 2019
2. Octupole shape phase transitions in light actinide and rare-earth nuclei

The axially-symmetric quadrupole and octupole PESs are shown in Figs. 1 and 2 that are computed by the constrained SCMF calculation within the relativistic Hartree-Bogoliubov method with the DD-PC1 EDF [9]. Already at the SCMF level, features of the shape-phase transitions are observed: Non-zero $\beta_{30}$ deformation appears already at $^{224}\text{Th}$, and this octupole minimum becomes much more pronounced at $^{226,228}\text{Th}$, for which rigid octupole deformation is predicted. One then sees a transition to octupole-soft shapes at $^{230,232}\text{Th}$. The quadrupole $\beta_{20}$ deformation stays constant at $\beta_{20} \approx 0.2$ for $A \geq 226$. A similar observation applies to the PESs for the $^{220−230}\text{Ra}$ isotopes. As for Sm isotopes (in Fig. 2), the most pronounced octupole minimum appears at around the neutron number $N = 88$ ($^{150}\text{Sm}$) and, for heavier Sm isotopes, the octupole minimum is no longer present. Somewhat similar systematic is found for Ba.

![Figure 1](image1.png)

**Figure 1.** Axially-symmetric ($\beta_{20}, \beta_{30}$) SCMF PESs for the $^{222−232}\text{Th}$, and $^{222−232}\text{Ra}$ isotopes. Contours are plotted with steps of 0.5 MeV, and the global minimum is identified by open circle.

![Figure 2](image2.png)

**Figure 2.** Same as Fig. 1 but for the $^{222−232}\text{Sm}$, and $^{142−150}\text{Ba}$ isotopes.

For a more quantitative analysis of the QPT, it is necessary to compute spectroscopic properties, including excitation spectra and transition rates, by taking into account dynamical correlations beyond the mean-field approximation, i.e., those arising from symmetry restoration.
and fluctuations for the collective coordinates. To this end, we resort to the diagonalization of the IBM Hamiltonian, which is determined by mapping, at each configuration \((\beta_{20}, \beta_{30})\) the SCMF PES \(E_{SCMF}(\beta_{20}, \beta_{30})\), onto the bosonic one \(E_{IBM}(\beta_{20}, \beta_{30})\), i.e., \(E_{SCMF}(\beta_{20}, \beta_{30}) \approx E_{IBM}(\beta_{20}, \beta_{30})\). The boson system consists of the positive-parity \(0^+ (s)\) and \(2^+ (d)\) and negative-parity \(3^- (f)\) bosons. The bosonic PES is represented by the expectation value of the \(sdf\)-IBM Hamiltonian in the boson coherent state. See Refs. [10, 3] for details of the whole procedure.

![Figure 3](image-url)

**Figure 3.** Excitation energies for the \(\pi = \pm 1\) yrast states in the Th, Ra, Sm, and Ba isotopes.

Figure 3 depicts the calculated positive- and negative-parity low-lying levels in comparison with the experimental data. Firstly one should notice a very nice agreement between our calculation and the data, even though no phenomenological adjustment of the IBM parameters is made. In all the considered isotopic chains, the positive-parity yrast levels become lowered
with increasing neutron number within each isotopic chain, suggesting spherical vibrational to strongly axially deformed states. What is of particular interest is the behaviors of the low-lying negative-parity states. They demonstrate a parabolic systematic as functions of the neutron number, cantered at a particular nucleus, e.g., $^{226}\text{Th}$, where the corresponding PES indicates the most pronounced octupole global minimum. In the Th isotopic chain, for instance, at $^{226}\text{Th}$ the positive- and negative-parity bands are so close in energy to each other and seem to form an approximate alternating parity band typical of the stable octupole deformation. For those nuclei heavier than $^{226}\text{Th}$, however, both the positive- and negative-parity band begin to form separate bands. A similar result is obtained in the Ra isotopic chain. As for Sm, the negative-parity bands become lower in energy toward $^{150}\text{Sm}$ but, from this nucleus on, stays rather constant, which means there is no notable change in the evolution of octupole collectivity. In Ba, the negative-parity levels become lowest at $^{144}\text{Ba}$ and remain constant for the heavier Ba nuclei.

**Figure 4.** The $B(E3)$ and $B(E1)$ values for the considered Th, Ra, Sm, and Ba isotopes.

Next we show in Fig. 4 the $B(E3; 3^{-} \rightarrow 0^{+})$ and $B(E1; 1^{-} \rightarrow 0^{+})$ transition rates. In particular the $B(E3)$ rates are a good measure for the octupole collectivity and, indeed, the predicted $B(E3)$ value becomes maximal at that nucleus where the PES exhibits the most pronounced $\beta_{3} \neq 0$ octupole minimum in each isotopic chain. On the other hand, the $E1$ property is accounted for by single-particle degrees of freedom, which is, by construction, not included in the model, as it is built only on the collective valence nucleons. That is the reason why the calculation fails to reproduce some experimental $B(E1)$ systematics.

As an another signature of the octupole QPT, we show in Fig. 5 the energy ratio $E(I^{\pi})/E(2^{+})$ for the $\pi = \pm 1$ yrast states plotted against the angular momentum $I$. If the nucleus has stable octupole deformation and exhibits alternating-parity band, the ratio increases linearly with $I$. The staggering pattern shown in the figure, that starts from a particular nucleus, e.g., $^{226}\text{Th}$ in the Th chain, indicates that the $\pi = +1$ and $\pi = -1$ yrast bands are decoupled and the octupole vibrational structure emerges.

We have also done a spectroscopic study on the octupole deformations in Sm and Gd isotopic chains by using the non-relativistic, Gogny EDF [4]. There we confirmed the robustness of the mapping procedure: irrespectively of whether relativistic or non-relativistic EDF is employed, a very nice description of the experimental low-lying positive- and negative-parity spectra, as well
as the evolution of octupole deformation is obtained. Another interesting result in Ref. [4] is that many of the excited $0^+$ states in the considered Sm and Gd nuclei could have in their wave functions double octupole phonon (i.e., $f$ boson) component, and this result gives a possible explanation for why so many low-lying excited $0^+$ states are observed in rare-earth nuclei.

3. Octupole correlations in odd-mass systems

Extension to odd-mass system is made by introducing an unpaired nucleon, which is then coupled to the octupole deformed even-even nucleus as a core. The low-lying structure of even-even nucleus is described in terms of the interacting $s$, $d$, and $f$ bosons, and the particle-boson coupling is modeled within the interacting boson-fermion model (IBFM) [11]. The Hamiltonian for the IBFM consists of the $sdf$-IBM Hamiltonian $\hat{H}_B$, the Hamiltonian for the single neutron $\hat{H}_F$, and the term $\hat{H}_{BF}$ that couples the fermion and boson spaces [5]: $\hat{H}_{IBFM} = \hat{H}_B + \hat{H}_F + \hat{H}_{BF}$

Input from the SCMF calculation are the spherical single-particle energies $\epsilon_j$ (needed for $\hat{H}_F$) and occupation numbers $v_j^2$ (for $\hat{H}_{BF}$) for the odd particle in orbital $j$.

Here we illustrate the method in its application to the isotope $^{145}$Ba. Since the corresponding even-even boson core nucleus $^{144}$Ba exhibits an octupole-soft potential at the SCMF level, we also expect that the octupole correlations play an important role in the low-energy spectra of this odd-mass nucleus. The calculated excitation spectrum for $^{145}$Ba is compared to the corresponding experimental bands in Fig. 6. Those calculated positive- and negative-parity bands shown in bold in the figure are made of the one-$f$-boson configuration coupled with a single neutron in the $p_{1/2,3/2}f_{5/2,7/2}h_{9/2}$ and $i_{13/2}$ orbitals, respectively. The corresponding experimental bands that are suggested to be of octupole in nature are also depicted in the figure. The absolute energies of the bandheads and energy spacings within the bands are well reproduced by the calculation. We have also predicted the $B(E3)$ transition rates from the octupole bands to the ground-state bands to be typically within the range 20~30 W.u., which are of the same order of magnitude as the calculated $B(E3; 3^- \rightarrow 0^+)$ value of 23 W.u. for the neighboring even-even nucleus $^{144}$Ba.

Figure 5. Energy ratios $E(I^+)/E(2^+_1)$ for the $\pi = \pm 1$ yrast states with angular momentum $I$.  

Input from the SCMF calculation are the spherical single-particle energies $\epsilon_j$ (needed for $\hat{H}_F$) and occupation numbers $v_j^2$ (for $\hat{H}_{BF}$) for the odd particle in orbital $j$. Here we illustrate the method in its application to the isotope $^{145}$Ba. Since the corresponding even-even boson core nucleus $^{144}$Ba exhibits an octupole-soft potential at the SCMF level, we also expect that the octupole correlations play an important role in the low-energy spectra of this odd-mass nucleus. The calculated excitation spectrum for $^{145}$Ba is compared to the corresponding experimental bands in Fig. 6. Those calculated positive- and negative-parity bands shown in bold in the figure are made of the one-$f$-boson configuration coupled with a single neutron in the $p_{1/2,3/2}f_{5/2,7/2}h_{9/2}$ and $i_{13/2}$ orbitals, respectively. The corresponding experimental bands that are suggested to be of octupole in nature are also depicted in the figure. The absolute energies of the bandheads and energy spacings within the bands are well reproduced by the calculation. We have also predicted the $B(E3)$ transition rates from the octupole bands to the ground-state bands to be typically within the range 20~30 W.u., which are of the same order of magnitude as the calculated $B(E3; 3^- \rightarrow 0^+)$ value of 23 W.u. for the neighboring even-even nucleus $^{144}$Ba.
To examine quality of the model prediction, more experimental information about the \(B(E3)\) transitions for the odd-mass Ba isotopes is expected.

4. Summary

Based on a global theoretical framework of the nuclear DFT, we have analyzed the octupole deformations and the related spectroscopic properties in a large set of medium-heavy and heavy nuclei. The results of the SCMF calculations with a given relativistic and non-relativistic EDF are used to completely determine the Hamiltonian of the IBM, which then provides excitation spectra and transition rates. Evolution of the axially-symmetric quadrupole and octupole PES, the resultant positive- and negative-parity low-lying states, and \(E3\) transition rates and moments, all consistently points to the onset of octupole deformations and the phase transition between stable octupole deformation and octupole soft shapes in the considered Th, Ra, Sm, Gd, and Ba isotopes chain. The octupole correlation plays an important role in describing low-lying spectra in odd-mass nuclei as well as in neighboring octupole deformed even-even nuclei. The theoretical method presented here is general and allows a computationally feasible prediction of octupole collective states and will be, therefore, extended further to study many other radioactive nuclei that are becoming of much more importance for the RIB experiments.

Acknowledgments

The results presented in this contribution are based on the works with D. Vretenar, T. Nikšić, B.-N. Lu, L. M. Robledo, and R. Rodríguez-Guzmán, This work is financed within the Tenure Track Pilot Programme of the Croatian Science Foundation and the ´Ecole Polytechnique Fédérale de Lausanne and the Project TTP-2018-07-3554 Exotic Nuclear Structure and Dynamics, with funds of the Croatian-Swiss Research Programme.

[1] Butler P A and Nazarewicz W 1996 Rev. Mod. Phys. 68 349
[2] Nomura K, Vretenar D and Lu B N 2013 Phys. Rev. C 88 021303
[3] Nomura K, Vretenar D, Nikšić T and Lu B N 2014 Phys. Rev. C 89 024312
[4] Nomura K, Rodríguez-Guzmán R and Robledo L M 2015 Phys. Rev. C 92 014312
[5] Nomura K, Nikšić T and Vretenar D 2018 Phys. Rev. C 97 024317
[6] Bender M, Heenen P H and Reinhard P G 2003 Rev. Mod. Phys. 75 121
[7] Iachello F and Arima A 1987 The interacting boson model (Cambridge: Cambridge Univ. Press)
[8] Cejnar P, Jolie J and Casten R F 2010 Rev. Mod. Phys. 82 02155
[9] Nikšić T, Vretenar D and Ring P 2011 Prog. Part. Nucl. Phys. 66 519
[10] Nomura K, Shimizu N and Otsuka T 2008 Phys. Rev. Lett. 108 142501
[11] Iachello F and Van Isacker P 1994 The interacting boson-fermion model (Cambridge: Cambridge Univ. Press)