Research on precision assembly error statistics and sensitivity based on vertical launcher model

Xiaobo Lei¹*, Chaoxun He¹, Junzhou Liu¹, and Nianli Fu¹

¹ Zhengzhou Institute of Mechanical and Electrical Engineering, Zhengzhou, Henan, 450015, China

*E-mail: 815557964@qq.com

Abstract. The installation accuracy of the shipborne vertical launcher and the different state parameters of the launcher's guide rail will have different degrees of impact on the initial accuracy of the test-mounted missile. Therefore, it is of great practical significance to study the assembly error characteristics of the launcher. In this paper, the influence law of equipment installation accuracy and the influence of each component's correlation error on assembly precision are analysed. The error law of the launching device is analysed based on mathematical statistics. At the same time, the sensitivity of the error is studied by multi-body theory method to improve the accuracy, efficiency and stability of the assembly. Through the established error sensitivity analysis model, the geometric error of the main components that have a great influence on the assembly accuracy during the assembly process of the launching device is effectively identified. The results show that the error data probability distribution of the vertical launcher conforms to the normal distribution, and the key factors of the assembly error are effectively detected, which provides a theoretical basis for the assembly precision control research.

1. Introduction

At present, many ship-borne missile weapon systems in the world navy use vertical launch mode, and missiles are launched through vertical launchers. The assembly accuracy of the launcher and the accuracy of the rail mounting are important tactical technical indicators in the vertical launch system, which affect the initial accuracy of the test missile launching to varying degrees, which directly determines the accuracy and killing efficiency of the missile attack target. In the vertical launch system, the launcher and the missile are connected by rails and the loading and launching of the missile is guaranteed [1-3]. Through the large amount of data accumulated in peacetime, the research on precision assembly technology is carried out, and the accuracy distribution law of precision vertical transmission device and the precise control mechanism of error are revealed. Therefore, it is especially important to analyse the main error factors affecting the assembly accuracy of the launcher, establish the error transfer model of the assembly process and perform sensitivity analysis.

In the study of the error statistics law and the error transfer model, domestic and foreign scholars have done a lot of research. Rui X T proposed an analysis method based on the maximum entropy method for the intensity statistics of weapon systems [4]. Zhao S Z proposed a method for visually representing the probability distribution of experimental data through histograms, and fitting the distribution law to goodness [5]. Fan J W proposed a new method based on multi-body system kinematics theory to study the sensitivity of machine tool error [6]. Yan Y proposed a method for sensitivity analysis of assembly geometry errors through error transfer model [7]. Lv C analysed the
problem of assembly joint surface error model and tolerance optimization design between parts under various tolerance coupling conditions [8]. Veitschegger adopted the homogeneous transformation matrix and differential transformation method to describe the method of determining the exact position of the robot's end arm in the presence of geometrical error and kinematic error, which lays the foundation for error research [9]. Habibi M used multi-body system theory to summarize and abstract complex systems, and comprehensively consider the system [10].

In this paper, by collecting a large amount of experimental data, the distribution law of the factors affecting the assembly error is studied. The statistical analysis of the test data is carried out by the software Minitab, and the Kolmogorov-Smirnov (K-S) test method is used to fit the data distribution type. At the same time, the error sensitivity analysis model is established to solve the influence of the precision of the precision assembly geometry on the overall assembly precision.

2. Error data statistics and analysis

Normal distribution is a very important distribution in the fields of mathematics and engineering. Many random variables obey the normal distribution. For example, the size error of the same batch parts processed by the adjusted machine tool approximates the normal distribution. In the case of repeated measurements with equal precision, the measurement error also approximates a normal distribution.

If the probability density of the continuous random variable X is

\[ f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, -\infty < x < +\infty \] (1)

In the formula (1), \( \mu, \sigma (\sigma > 0) \) is a constant, then X obeys the normal distribution of the parameters \( \mu, \sigma \), it recorded as \( X \sim N(\mu, \sigma^2) \).

However, in the process of actually measuring the error data, due to the objective existence of the accidental errors, the obtained experimental values always have a certain discrete type. It is also possible that individual discrete values are present due to error, which are often referred to as bad or suspect values. If you retain this data, it will affect the accuracy of the test results. This article will give priority to the Grubbs method to eliminate these bad values.

The histogram is a commonly used method for preliminary collating data. The drawing procedure is simple and the purpose is clear. More importantly, according to the shape of the histogram, it is possible to preliminarily determine the distribution of the batch data. In this paper, the error data of the vertical launcher guide rail is collated in the form of histogram to judge its distribution law.

In this paper, the partial error data histogram of launcher is drawn.

![Histograms](image1.png)

**Figure 1. Error data histogram**

It can be preliminarily judged from figure 1 that the transmitting device data conforms to the normal distribution type.

Although the histogram can initially determine the type of distribution that the test data fits. However, it is necessary to test whether the distribution is consistent with the initially selected distribution. The basis for the inference is the goodness of fit test. The goodness of fit is the degree of agreement between the distribution of experimental data and the selected theoretical distribution. This paper uses the K-S test to verify the distribution type of error data.
Given the cumulative distribution function \( F(x) \), K-S test statistic is as follows.

\[
D_{KS} = \sup_x |F_n(x) - F(x)|
\]  

(2)

In the formula (2), \( \sup_x \) represents the upper bound, and \( F_n(x) \) is the sample empirical accumulation distribution function. Set N samples \( X_1, X_2, \ldots, X_N \) have the values \( x_1, x_2, \ldots, x_N \), arrange them from small to large and renumber them as \( X(1) \leq X(2) \leq \cdots \leq X(n) \), the empirical accumulation distribution is expressed as

\[
F_N(x) = \begin{cases} 
0, & x < x(1) \\
k/N, & x(k) \leq x \leq x(k+1) \quad k = 1,2,\ldots,N \\
1, & x < x(N) 
\end{cases}
\]  

(3)

In the formula (3), for a given sample \( X \) and a known theoretical distribution function, a test statistic \( D \) and a threshold \( D_{KS} \) can be recorded. If it is greater than this threshold, the sample \( X \) is considered not to obey the distribution \( F(x) \), whereas the sample \( X \) is considered to be subject to the distribution \( F(x) \).

In order to make the test more accurate, the significance level is taken as \( \alpha = 0.05 \). The test data was subjected to a K-S test using Minitab software, which compares the empirical cumulative distribution function of the sample data with the assumed data distribution. If the p values of these tests are greater than the selected \( \alpha \), then the null hypothesis can be affirmed and the overall hypothetical distribution is determined. Otherwise, it is rejected.

The normality test of the error data is now performed, and the results are as follows.

![Data probability map](image)

It is known from figure 2 that the left graph is \( p=0.597 \) and the right graph is \( p=0.741 \). Using the above method to perform normality test on all launcher data. Now summarize the p values in the probability plots of different sample error data.

| P Measuring point | Sample | A | B | C | D |
|-------------------|--------|---|---|---|---|
| Sample 1          | 0.597  | 0.308 | 0.580 | 0.481 |
| Sample 2          | 0.741  | 0.433 | 0.562 | 0.711 |
| Sample 3          | 0.601  | 0.465 | 0.692 | 0.544 |
| Sample 4          | 0.464  | 0.577 | 0.703 | 0.671 |
| Sample 5          | 0.389  | 0.611 | 0.515 | 0.515 |

It is known from Table 1 that the results show that all p values are greater than \( \alpha \), and the distribution of the preliminary hypotheses can be accepted. The error data conforms to the normal distribution type, which lays a good foundation for error sensitivity analysis.

3. Error model and sensitivity calculation

In this paper, the multi-body system theory is used to describe the topological structure of the low-order body array. The homogeneous coordinate transformation matrix is used to describe the pose.
transformation between adjacent bodies, and then the geometric motion model of the whole multi-body system is established.

Table 2. Low-order array of multi-body systems

|   | 1  | 2  | 3  | 4  |
|---|----|----|----|----|
| L^0(k) | 1  | 2  | 3  | 4  |
| L^1(k) | 0  | 1  | 2  | 3  |
| L^2(k) | 0  | 0  | 1  | 2  |
| L^3(k) | 0  | 0  | 0  | 1  |

Table 2 shows the low-order array of multi-body systems.

In this paper, the positional differential change of vertical launcher is used to describe it. Here, a small displacement spin is used to represent that it is a vector composed of a rigid body with six motion components and a small displacement, which is suitable for indicating the deviation of the ideal shape feature. The small displacement rotation can be expressed as \( \mathbf{p} = (\alpha, \beta, \gamma, u, v, w) \), \( \alpha \) represents a small amount of variation about the x-axis, \( \beta \) represents a small amount of variation about the y-axis, \( \gamma \) represents a small amount of variation about the z-axis, and \( u \) represents a small amount of fluctuation along the x-axis, \( v \) represents a small amount of fluctuation along the y-axis, and \( w \) represents a small amount of fluctuation along the z-axis. In the scope of this study, the above small changes are errors.

In the ideal state, the coordinate system first rotates \( \alpha \) around the x-axis, then rotates \( \beta \) around the y-axis, and finally rotates \( \gamma \) around the z-axis to obtain the initial rotation homogeneous transformation matrix of the coordinate system. That is formula 4.

\[
T(R) = T(x, \alpha)T(y, \beta)T(z, \gamma) = \begin{pmatrix}
\cos \beta \cos \gamma & -\sin \beta & \sin \beta & 0 \\
\sin \alpha \sin \gamma + \cos \alpha \sin \beta \cos \gamma & \cos \alpha \cos \gamma - \sin \alpha \sin \beta \sin \gamma & -\sin \alpha \cos \beta & 0 \\
\sin \alpha \sin \gamma - \cos \alpha \sin \beta \cos \gamma & \sin \alpha \cos \gamma + \cos \alpha \sin \beta \sin \gamma & \cos \alpha \cos \beta & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

(4)

The coordinate system is to move \( u \) along the x-axis on the basis of rotation, to move \( v \) along the y-axis, and to move \( w \) along the z-axis, and the homogeneous transformation matrix after the coordinate system synthesis motion is formula 5.

\[
T(R) = T(M)T(R) = \begin{pmatrix}
\cos \beta \cos \gamma & -\cos \beta \sin \gamma & \sin \beta & u \\
\cos \alpha \sin \gamma + \sin \alpha \sin \beta \cos \gamma & \cos \alpha \cos \gamma - \sin \alpha \sin \beta \sin \gamma & -\sin \alpha \cos \beta & v \\
\sin \alpha \sin \gamma - \cos \alpha \sin \beta \cos \gamma & \sin \alpha \cos \gamma + \cos \alpha \sin \beta \sin \gamma & \cos \alpha \cos \beta & w \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

(5)

However, under the actual assembly of two adjacent typical bodies, the combined error transformation matrix caused by three angular displacement errors and three line displacement errors is formula 6.

\[
\Delta T(\Delta R) = \begin{pmatrix}
\cos \Delta \alpha \cos \beta \Delta \gamma & -\cos \Delta \beta \sin \gamma & \sin \beta & \Delta u \\
\cos \Delta \alpha \sin \gamma + \sin \alpha \sin \beta \cos \gamma & \cos \alpha \cos \gamma - \sin \alpha \sin \beta \sin \gamma & -\sin \alpha \cos \beta & \Delta v \\
\sin \alpha \sin \gamma - \cos \alpha \sin \beta \cos \gamma & \sin \alpha \cos \gamma + \cos \alpha \sin \beta \sin \gamma & \cos \alpha \cos \beta & \Delta w \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

(6)

Usually, because the angular error is small, \( \cos \Delta \alpha \approx 1, \cos \Delta \beta \approx 1, \cos \Delta \gamma \approx 1, \sin \Delta \alpha \approx \Delta \alpha, \sin \Delta \beta \approx \Delta \beta, \sin \Delta \gamma \approx \Delta \gamma \), while ignoring high-order infinitesimal terms, it can be expressed as formula 7.
At present, there are three main methods for solving the sensitivity problem: direct method, high-order sensitivity function method and first-order sensitivity function method. The direct method has a large amount of calculation, the calculation result is complex and the feasibility is poor; the high-order sensitivity function method is difficult to solve and has low practicability. At present, the first-order sensitivity function method is mostly used for sensitivity analysis. When the high-order sensitivity function $k=1$, the obtained first-order partial derivative is the first-order sensitivity function. This paper mainly uses the first-order sensitivity function to derive the sensitivity matrix of the vertical launcher error model.

If the function $F(x)$ is derivable, the first order sensitivity can be expressed as

$$S = \frac{\partial F(x)}{\partial U_k} \quad (8)$$

In the formula 8, $S = \frac{\partial F}{\partial U_k}$ is the sensitivity of the error variable $U_k$, and the sensitivity of the system assembly accuracy to each error variable is obtained. This is called the error sensitivity matrix. That is formula 9.

$$S = \frac{\partial F}{\partial U} = \left[ \frac{\partial f_1}{\partial u_1} \frac{\partial f_1}{\partial u_2} \ldots \frac{\partial f_1}{\partial u_6} \right] \left[ \frac{\partial f_2}{\partial u_1} \frac{\partial f_2}{\partial u_2} \ldots \frac{\partial f_2}{\partial u_6} \right] \left[ \begin{array}{c} \vdots \vdots \\ \vdots \vdots \\ \vdots \vdots \\ \frac{\partial f_6}{\partial u_1} \frac{\partial f_6}{\partial u_2} \ldots \frac{\partial f_6}{\partial u_6} \end{array} \right] \quad (9)$$

By establishing the transmission error between the total error of the vertical launching device and the matching characteristic surface error of each component, the sensitivity and sensitivity coefficient of each error component are calculated according to the error sensitivity analysis model, and the quantitative influence degree of each error on the assembly precision of the system is compared. The critical error is determined to provide a theoretical basis for the error control research of the precise assembly of the launcher.

The error of each assembly feature surface is determined based on the error data measured by the test. The feature surface includes a base surface $M_1$, a bottom surface $M_2$ and a top surface $M_3$ of the component body 1, a bottom surface $M_4$ and a top surface $M_5$ of the component body 2, a bottom surface $M_6$ and a top surface $M_7$ of the component body 3, and a bottom surface $M_8$ of the component body 4. The total error vector of the variation can be obtained from the error data of each matching feature surface. The sensitivity to each error component is obtained by determining the components of the total error after assembly.

The sensitivity of the error to the total spatial error in the x-axis translation error component $U$ is $\Delta \beta_{M_1}, \Delta \gamma_{M_2}, \Delta \gamma_{M_3}, \Delta \gamma_{M_6}, \Delta \gamma_{M_7}, \Delta \gamma_{M_8}$ and the sensitivity coefficients are 0.25, 0.15, 0.15, 0.08, 0.08, 0.07. The sum of the sensitivity coefficients of the six errors is 0.78, and the sum of the sensitivity coefficients corresponding to the other errors is 0.22.

These six errors have a large influence on the total spatial error $U$ in the translational error component $\Delta u_x$ in the x-axis direction. Similarly, $\Delta a_{M_2}$ has a greater impact on $\Delta a_{M_1}, \Delta a_{M_3}, \Delta a_{M_4}, \Delta a_{M_5}, \Delta a_{M_6}, \Delta a_{M_7}, \Delta a_{M_8}$, the
sensitivity coefficients of the eight errors to the rotational error component $\Delta \alpha_x$ of the total spatial error $U$ are equal. Similarly, the sensitivity coefficients of $\Delta \beta_x$ and $\Delta \gamma_x$ are also equal.

The translational component of the total spatial error is affected by both the rotational error and the translational error, and is mainly the effect of the rotational error. The rotational component of the total spatial error $U$ is only affected by the rotational error on the respective directional axes, and the sensitivity of each rotational error in a certain direction to the total spatial error in the corresponding direction is comparable. $\Delta \beta_x$ is sensitive to the translation error component $\Delta \gamma_x$ of the total spatial error. Therefore, the value of this error should be controlled in precision design and manufacturing. The rotation error around each axis has the same effect on $\Delta \alpha_x$, $\Delta \beta_x$, $\Delta \gamma_x$ in the corresponding direction.

Therefore, in precision design and manufacturing, the rotational component of the total spatial error in the corresponding direction can be controlled by controlling the rotational error component in each direction.

4. Conclusion
Based on the accuracy error data of shipborne vertical launchers, this paper analyses the data distribution law and the multi-tolerance coupling of geometric elements and the topological relationship between assemblies. The accuracy error distribution type of the vertical launcher and the error transfer model of the assembly are also studied. Through the sensitivity calculation, the error which has great influence on the assembly precision of the system is found, which provides a reliable basis for the error control research of the precision assembly of the vertical launcher.

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