Analysis of IP response characteristics in wells on undulating topography

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Abstract. This paper studies the effect of undulating topography on resistivity method and induced polarization (IP) method between the wells. The 3D forward modeling of borehole resistivity/induced polarization method is realized by abnormal potential algorithm based on unstructured finite-element method (FEM). The algorithm is verified in layered medium and spherical abnormal bodies. We studied the response characteristics of apparent resistivity and apparent polarizability in well-well mode and the situation when the source is at different positions in the well, and analyzed the effects of undulating topography on resistivity and induced polarization. The results show that the apparent polarizability and apparent resistivity response have a great relationship with the depth of the emission source in the well, the relative position of the emission source and the measurement electrode. The undulating topography only affects the measuring electrodes at a short distance, and it has a greater impact on the low-resistance anomalous body.

1. Introduction
Induced polarization between wells is used to find hidden ore bodies next to the wells, determine their spatial location, delineate the scope of the ore body or mineralization zone, and determine the electrical continuity of rock and ore layers between adjacent boreholes. In recent years, the theoretical research on resistivity and polarizability between wells has received great attention in and abroad [1].

In the late 1970s, Dey and Morrison applied the idea of absorbing boundary conditions to the three-dimensional simulation of the resistivity method, which effectively reduced the influence of the number of grids and boundary effects, and it achieved good simulation results [2]; Fox studied the influence of undulating topography on resistivity and induced polarization [3]; Lowry et al. and Zhao and Yedlin decomposed the potential into the superposition of the background value and the outlier, and used the anomalous potential method to reduce the error caused by the singularity of the source [4-5]; Rücker and Günther et al. used unstructured tetrahedral meshes to realize finite element simulation under complex topography [6]; Wu X P used the finite difference method to realize the study the IP forward modeling and inversion of undulating topography [7]; Qiang J K introduced the triangular prism element into 3D resistivity FEM forward modeling of undulating topography [8]. Because the influence of topography on the resistivity/IP method between wells has not been considered by predecessors, this paper uses the FEM to study the influence of undulating topography on the well-well observation.
2. Principle of algorithm
Partial differential equation of the potential in rectangular coordinate system [9]:
\[
\nabla^2 \phi + \sigma \frac{\partial \phi}{\partial x} + \sigma \frac{\partial \phi}{\partial y} + \sigma \frac{\partial \phi}{\partial z} = -\frac{4\pi}{\alpha} \delta(x_a) \delta(y_a) \delta(z_a)
\]
(1)

In the numerical calculation of resistivity method, the total potential \( U \) consists of two parts:
\[
U = u_0 + u
\]
(2)
where \( u_0 \) is the normal potential which is caused by the power supply in the homogeneous half space or the whole space, \( u \) is the abnormal potential which is caused by the inhomogeneous medium. The normal potential can be calculated using the analytical solution. The boundary value problem of abnormal potential is equivalent to the following variational problem:
\[
\begin{aligned}
F(u) &= \int_{\Omega} [1/2\sigma \nabla \phi^2 + \sigma' \nabla \cdot \nu \nabla \phi] d\Omega + \int_{\Gamma} \frac{\sigma \nu^2 \cos(r.n)}{2r} + \frac{\sigma' \nu_0 \nu \cos(r.n)}{r} d\Gamma \\
\sigma F(u) &= 0
\end{aligned}
\]
(3)

Because the variation problem of abnormal potential does not include the power supply, the calculation accuracy will be improved. Although the variational problem of abnormal potential has one more area integral and boundary integral, it does not increase the calculation assumption.

According to Seigel theory [10], time domain IP forward modeling is based on resistivity forward modeling. During the forward modeling, the potential of the primary field was solved and the equivalent resistivity \( \rho^* \) was used instead of the original model resistivity \( \rho \). We perform the second solution to obtain the total field potential (polarized field potential). Subtract the primary field potential from the total field potential to obtain the secondary field potential. Using \( \rho^* \) to represent the equivalent apparent resistivity and \( \eta \) to represent the polarizability, the following relations can be obtained:
\[
\rho^* = \frac{\rho}{1 - \eta}
\]
(4)
\[
\rho^*_S = K \frac{\Delta \rho}{\Delta S}
\]
(5)
\[
\eta = \frac{\Delta \rho^*_S}{\Delta \rho} \times 100% = \frac{\Delta \rho^*_S - \Delta \rho^*_1}{\Delta \rho} \times 100% = \frac{\rho^*_S - \rho^*}{\rho^*_1} \times 100%
\]
(6)

Integrating the four terms in equation (3) can be obtained:
\[
F(u) = \sum \frac{\rho^*}{2} u^T K_{1e} u + \sigma' u^T K_{1e} u_0 + \frac{\sigma}{2} u^T K_{2e} u + \sigma' u^T K_{2e} u_0 = \frac{1}{2} u^T K u + u^T K^* u_0
\]
(7)

We make the above equation to be zero, then we get:
\[
K u = K^* u_0
\]
(8)

Where \( u_0 \) can be obtained by analytical solution (9), then the abnormal potential \( u \) at each grid node can be obtained by solving the equations (8). The total potential is calculated by formula (2).
\[
U = \frac{\phi}{4\pi} \left( \frac{1}{R} + \frac{1}{R'} \right)
\]
(9)

Where \( R \) is the distance from the measuring point to the point current source A, and \( R' \) is the distance from the measuring point to the virtual point current source A'.
Figure 1. Schematic of an unstructured grid.

3. Numerical verification
We demonstrate the method in this paper with the half-space layered medium as shown in Figure 2 (left), where the resistivity of the first layer is $10\Omega \cdot m$ and the thickness is 10m; the resistivity of the second layer is $100\Omega \cdot m$, and the emission current of the point power source A is 1A.

Another test is a spherical abnormal body as shown in Figure 2 (right). There is a low-resistance sphere in the homogeneous half-space, the sphere radius is 10m, the center of the sphere is buried (0, 0, -15), the sphere resistivity is $1\Omega \cdot m$, and the uniform half-space resistivity is $100\Omega \cdot m$; The emission current of the point power source A is 1A.

We use a two-pole device for numerical calculations, and M is the measurement point in the X-Axis direction. Comparing the numerical calculation results with the analytical solutions, it can be seen from Figure 3 that the calculation results of the two models fit well.

Figure 2. Schematic diagram of layered media and spherical abnormal body.

Figure 3. Comparison of analytical and numerical solutions (layered medium on the left and spherical abnormal body on the right).

4. Numerical simulation
As shown in Figure 4, in a homogeneous half space where the resistivity of the surrounding rock is $\rho_1 = 100\Omega \cdot m$ and the polarization rate is $\eta_1 = 0.1$. There is a resistivity of $\rho_2 = 10\Omega \cdot m$ ($\rho_2 = 1000\Omega \cdot m$) and the polarization rates are both $\eta_2 = 0.5$. The size of the conductive anomaly is $50m \times 50m \times 50m$, and the center burial depth is 100m; there are two wells on both sides of 50m from the center of the anomaly. The location of the current source is $A_1(-50, 0, -60)$, $A_2(-50, 0, -80)$, $A_3(-50, 0, -100)$, $A_4(-50, 0, -120)$, $A_5(-50, 0, -140)$; the location of the
survey line is (50,0, Z); We perform numerical simulation in the shape of a pyramid-shaped peak (the side of the base is 80m, the top is 40m, and the height is 20m) or there is no topography.

Figure 4. Schematic diagram of the peak model.

Figure 5. Low-resistance abnormal response without topography. (a) apparent resistivity curve; (b) apparent polarizability curve.

Figure 6. Low-resistance abnormal response with topography. (a) apparent resistivity curve; (b) apparent polarizability curve.

From Figures 5 and 6, we can see that there is basically no difference between the apparent resistivity curve and the apparent polarizability curve corresponding to the two figures. This shows that the peak topography has a small impact on the abnormal body. Although Fig. 5 and Fig. 6 are the response results of the low-resistance abnormal body, the apparent resistivity response is the high-resistance abnormal body. This is due to the attraction of the low resistance abnormal body to the current, which causes the position current density on the same side as the point power source decreasing, and the position current density on the opposite side as the point power source increasing. Causes the measured value on the opposite side of the power supply to appear as a high-resistance anomaly. We found that with the emission source descends along the well, the maximum values of its apparent resistivity and apparent polarizability anomalous response will increase first and then decrease. The position and size of the peak value of the apparent resistivity curve are related to the depth of the point power supply. When the current source is located 100m below the well, the abnormal response reaches the maximum. We can see that the location of the current source is horizontal with the center of the abnormal body, but the maximum position value of the anomaly
response is slightly greater than 100m. This is because this paper uses a half-space calculation method (different surface boundary conditions).

![Apparent Resistivity Curves](image1)

**Figure 7.** High-resistance abnormal response without topography. (a) apparent resistivity curve; (b) apparent polarizability curve

![Apparent Resistivity Curves](image2)

**Figure 8.** High-resistance abnormal response with topography. (a) apparent resistivity curve; (b) apparent polarizability curve

From Figures 7 and 8, we can also see that the influence of the peak topography on the anomalous response is small. Although Figures 5 and 6 are the response results of the high-resistance abnormal body, but the apparent resistivity response is a low-resistance anomaly. We can still obtain the above conclusion by analyzing the reasons. And from the abnormal response of apparent polarizability, we can know that in general, low-resistance bodies have high polarization characteristics, and high-resistance bodies have low polarization characteristics. Moreover, the apparent polarizability of the high-resistance anomaly and low-resistance anomaly is smaller than that of the surrounding rocks, which is also caused by the emission source and the survey line being located on both sides of the abnormal body.

![Schematic Diagram of the Valley Model](image3)

**Figure 9.** Schematic diagram of the valley model.
When the location of the current source is 100m downhole, other parameters remain unchanged. Numerical simulations are performed on factors such as high-resistance abnormal body, low-resistance abnormal body, and topography (peak, valley). Based on the above discussion, from Figure 10 (a), we can see that when the emission source and the measurement line are located on the opposite side of the abnormal body, the response of the low-resistance abnormal body is a high-resistance abnormality, and the response of the high-resistance abnormal body is a low-resistance abnormality. The apparent polarizability anomalies are smaller than the polarizabilities of the surrounding rocks, however, they can reflect the approximate position of the abnormal body. In conjunction with Figure 10(a) and (b), we can see that the topography only affects shallower measurement points. With the distance between the survey line and the topography increasing, the influence of the topography on the apparent resistivity and apparent polarization can be ignored, and topography has a greater impact on low-resistance abnormal body. Comparing the response curves of the peak and valley, we can see that the influence of the valley topography is more obvious, which will cause greater errors when the well depth does not exceed 40m.

5. Conclusion
This paper presents three-dimensional boundary value problems and variational equations with potential satisfying. The abnormal potential method is used to solve the finite element equations, the singular value problem of the point power source location is solved, and an unstructured tetrahedral grid is used. Finally, the 3D IP finite element forward modeling was realized, and the following conclusions were obtained:

1. The position of the maximum value of the apparent resistivity curve is related to the depth of the point source. When the position of the current source is horizontal with the center of the abnormal body, the position of the maximum value of the abnormal response is slightly biased to the position of the abnormal body. It is caused by different calculation methods (full space and half space calculation).

2. When the emission source and the survey line are on the opposite side of the abnormal body, the response of the low-resistance anomaly is a high-resistance anomaly, the response of the high-resistance anomaly is a low-resistance anomaly, and the apparent polarizability anomalies are all smaller than the polarizability of the surrounding rocks.

3. The topography only affects shallower measuring points. With the depth of the measuring point increases, the influence of the terrain on the apparent resistivity and apparent polarizability can be ignored, and the influence of the topography on the low-resistance abnormal body is greater. Topography has a greater impact on IP between wells.

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