ENERGY CRISIS IN ASTROPHYSICS
(Black Holes vs. N-Body Metrics)

Carroll O. Alley, Darryl Leiter¹, Yutaka Mizobuchi² and Huseyin Yılmaz³
Department of Physics, University of Maryland,
College Park, Maryland 20742

ABSTRACT

The recent observation of the gamma ray burster GRB 990123, requiring at least two $M_\odot c^2$ of energy in gamma radiation alone, created an energy crisis in astrophysics (Schilling 1999). We discuss a theorem which states that, of all four-dimensional curved spacetime theories of gravity viable with respect to the four classical weak field tests, only one unique case, the Yılmaz theory, has interactive N-body (multiparticle) solutions and this unique case has no event horizons. The theorem provides strong theoretical support for Robertson’s explanation of the large energy output of the gamma ray burster GRB 990123 (Robertson 1999b). This explanation requires a switch from black holes (a 1-body solution with horizon) to the case of horizon-free interactive N-body solutions. In addition to the good news that the long sought N-body solutions are found, this unique case enjoys further strong support from other areas of gravitational physics. This development does not rule out GRB models with beaming, which can be used if warranted, but it provides a consistent basis for them, as only in an interactive multiparticle context can such models be constructed.

Subject headings: accretion, accretion disks — black hole physics — gamma rays: bursts — methods: n-body simulations — stars: neutron — X-rays: stars

1. Introduction

The difficulty experienced by general relativity in accounting for the prodigious energy released in the gamma ray bursters, particularly GRB 990123 (Schilling 1999) focuses attention on the problems in the treatment of gravitational energy which the theory has had from its inception. (The quantity proposed by Einstein for the gravitational field stress-energy turned out not to be a tensor (Weyl 1922).) The Schwarzschild metric solution, with the interpretation of ”event horizon” and ”black hole”, limits the mass of a neutron star to about 2.8$M_\odot$. But of even greater importance, the existence of interactive N-body solutions is incompatible with the presence of an event horizon. N-body interactive solutions, however, are necessary to describe the properties of neutron stars, both in the sense of their being a collection of neutrons (Kapusta 1989), and in mergers involving two or more neutron stars as macroscopic N-body systems. We also need such solutions to study the formation of neutron stars themselves during gravitational collapse.

¹9 Marwood Drive, Palmyra, Virginia 22963
²Central Research Laboratory, Hamamatsu Photonics, K.K., Hamakita, 434-8601 Japan
³Hamamatsu Photonics, K.K., Hamamatsu City, 430-8587 Japan and Electro-Optics Technology Center, Tufts University, Medford, Massachusetts
and the conversion of gravitational energy into radiation in the case of merging neutron stars or in accretion onto neutron stars in X-ray binaries. These are just some important examples. All of astrophysics requires interactive N-body solutions which are not present in general relativity.

A resolution of this problem exists in the relativistic curved spacetime gravitation theory of Yilmaz which completes the approach initiated by Einstein. The fundamental differences between the two theories are presented in this paper in the form of a theorem about the N-body solutions. The intent is to provide a brief theoretical underpinning for Robertson’s proposed explanation of GRB 990123 (Robertson 1999b) and to note other astrophysical consequences of this new view.

### 2. The Problem of Interactive N-Body (Multiparticle) Solutions

We begin with the obvious remark that, in order to do physics with any set of objects, we must have more than one object so that we may study their relationships, their interactions, their scatterings, their coalescence, and so on. In other words, an acceptable physical theory must have “interactive N-body solutions.” By interactive, we mean the bodies exert forces on each other, or accelerate in each other’s fields when free of constraints. We therefore propose to investigate whether a theory of gravity has N-body interactive solutions.

In 1974 the British Canadian mathematician Brian O.J. Tupper has shown that in any four-dimensional spacetime theory of gravity viable with respect to the four classical weak field tests the slow motion (sometimes called static) limit $u^i \Rightarrow 0$, $u_0 u^0 \Rightarrow 1$ the field equations are of the form (Tupper 1974a, 1974b)

$$
\frac{1}{2} G_{\mu \nu} = \tau_{\mu \nu} + \lambda t_{\mu \nu},
$$

$$
\sqrt{-g} = \sigma u_{\mu} u^{\mu},
$$

$$
\sqrt{-g_\sigma} = \sum_{A} m_A \delta^3(x - x_A)
$$

where $\tau_{\mu \nu}$ is the Einstein “matter-stress-energy” tensor, and

$$
t^\nu_{\mu} = -\partial_{\mu} \phi \partial^{\nu} \phi + \frac{1}{2} g_{\mu \nu} \partial^{\rho} \phi \partial_{\rho} \phi
$$

is the Yilmaz gravitational “field stress-energy” tensor and $\lambda$ is an arbitrary numerical parameter passing through $\lambda = 0$ (Einstein’s theory) and $\lambda = 1$ (Yilmaz’ theory). The $\phi$ is the low velocity limit of $\phi_0$ when $\phi = \text{trace } \phi_{\mu \nu} = \phi_0^{\mu \nu}$. Thus in this limit $\phi$ is a scalar.

Remarkably, Tupper was able to solve the above equations exactly for arbitrary $\lambda$ as

$$
\frac{1}{2} G_{\mu \nu} = \tau_{\mu \nu} + \lambda t_{\mu \nu}
$$

$$
\sqrt{-g} = \sum_{A} m_A \delta^3(x - x_A)
$$

$$
t^\nu_{\mu} = -\partial_{\mu} \phi \partial^{\nu} \phi + \frac{1}{2} g_{\mu \nu} \partial^{\rho} \phi \partial_{\rho} \phi
$$

where $A = [(1 - \epsilon \phi/2)/(1 + \epsilon \phi/2)]^{2/\epsilon}$, $\lambda = 1 - \epsilon^2$. (We use $2\lambda$ where Tupper used $\lambda$. Note also that Tupper used spherical coordinates whereas we use its transform into cartesian coordinates in order to analyze more simply the N-body solutions.) Tupper noted that for $\lambda = 0$ (that is, $\epsilon = \pm 1$) and for $\lambda = 1$ (that is, $\epsilon = 0$) this metric indeed reduces to the Schwarzschild and Yilmaz metrics respectively. What Tupper did not emphasize is that, while the Schwarzschild metric

$$
ds^2 = [(1 - \phi/2)/(1 + \phi/2)]^2 dt^2 - (1 + \phi/2)^4(dx^2 + dy^2 + dz^2)
$$

was able to solve the above equations exactly for arbitrary $\lambda$ as

$$
ds^2 = Adt^2 - A^{-1}(1 - \epsilon^2 \phi^2/4)^2(dx^2 + dy^2 + dz^2)
$$

$$
\phi = m/r
$$

where $A = [(1 - \epsilon \phi/2)/(1 + \epsilon \phi/2)]^{2/\epsilon}$, $\lambda = 1 - \epsilon^2$. (We use $2\lambda$ where Tupper used $\lambda$. Note also that Tupper used spherical coordinates whereas we use its transform into cartesian coordinates in order to analyze more simply the N-body solutions.) Tupper noted that for $\lambda = 0$ (that is, $\epsilon = \pm 1$) and for $\lambda = 1$ (that is, $\epsilon = 0$) this metric indeed reduces to the Schwarzschild and Yilmaz metrics respectively. What Tupper did not emphasize is that, while the Schwarzschild metric

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which completes the approach initiated by Einstein. The fundamental differences between the two theories are presented in this paper in the form of a theorem about the N-body solutions. The intent is to provide a brief theoretical underpinning for Robertson’s proposed explanation of GRB 990123 (Robertson 1999b) and to note other astrophysical consequences of this new view.
is only a 1-body solution $\phi = m/r$, the Yilmaz metric is an N-body solution (Yilmaz 1958)

$$ds^2 = e^{-2\phi} dt^2 - e^{2\phi} (dx^2 + dy^2 + dz^2)$$

$$\phi = \sum A m_A/r_A + C$$

where $r_A = |x - x_A|$. It reduces to $\phi = m/r$ only as a special case when only one body is present. Note also that in the $\epsilon \neq 0$ case one has an event horizon at $r_{eh} = \epsilon m/2$. In the Yilmaz case ($\epsilon = 0$) there are no event horizons (no black holes). Thus we will show that there is a strict mathematical anticorrelation between having an event horizon and having N-body solutions.

The importance of this result, namely the existence of interactive N-body solutions in Yilmaz’ theory was not appreciated until recently because it was always assumed (or hoped) that such N-body solutions would someday be found in general relativity. However, despite the many able mathematicians and well motivated groups and alliances working on it during the last eighty years, no N-body interactive solution has been found in general relativity. It is only relatively recently that, using the general $\lambda$-parametric solution of Tupper, Yilmaz proved that (Yilmaz 1987, 1992) they do not exist except when $\lambda = 1$. Below we present this important theorem and discuss its consequences, including the energy requirement of GRB 990123.

### 3. Proof of the N-Body Theorem

Since no other generalization than Tupper’s $\lambda$-parametric form is viable, and since the exact solution for arbitrary $\lambda$ is already given, a most interesting thing to do would be to evaluate (taking Eqs. (1) to (5) into account) both sides of the field equations with an unspecified $\phi$ and see what happens (what conditions there are in order for the $\phi$ to be a solution, where the Laplacians occur, etc.). In fact, something remarkable happens – one gets the following exact result:

| Left Hand Side | Right Hand Side |
|---------------|-----------------|
| $\frac{1}{2}\sqrt{-g}G^0_0$ | $-\Delta \phi + \frac{1}{4}(\lambda - 1)\Omega_{00} + \lambda t_{00} = \Sigma A m_A \delta^3(x - x_A) + \lambda t_{00}$ |
| $\frac{1}{2}\sqrt{-g}G^k_i$ | $\frac{1}{4}(\lambda - 1)\Omega_{ik} + \lambda t_{ik} = \lambda t_{ik}$ |

where $\Delta$ is the ordinary Laplacian. $\Omega_{00}$ and $\Omega_{ik}$ are given by

$$\Omega_{00} = -2\phi \Delta \phi$$

$$\Omega_{ik} = \phi \partial_i \partial_k \phi - 3\partial_i \phi \partial_k \phi + \delta_{ik} \partial^j \phi \partial_j \phi - \delta_{ik} \phi \Delta \phi$$

We can see immediately that if $\lambda = 1$ we would have the (needed) N-body solutions of the form (computations are done by a Mathematica symbolic manipulation program)

$$\phi = \sum A m_A/r_A + C$$
but if \( \lambda \neq 1 \), \( \Omega_{00} \) and \( \Omega_{ik} \) will have to vanish. The question is, can we get these two terms to vanish.

Let us first note that, if we ignore the additive constant \( (C = 0) \), this can be done in the case of \( \Omega_{00} \). For, we can take the body as a small sphere of constant matter density in which case the potential may be assumed to start from the center as \( \sigma r^2/6 \), hence \( \Omega_{00} = -\sigma^2 r^2/3 \). Since this expression can be made as small as one likes, the \( G_{0}^{0} \) part of the field equations allows an N-body solution of the type \( \phi = \Sigma A m_{A}/r_{A} \). But no matter how hard we try (including the above trick on its last term) we cannot get \( \Omega_{ik} \) to vanish with an N-body solution where \( N \) is greater than one (for \( N > 1 \), \( \Omega_{ik} \) has no roots). But for \( N = 1 \), that is for \( \phi = m/r \), it can easily be shown that this special case is allowed (because \( \Omega_{ik} = 0 \) for \( \phi = m/r \)) which is the original \( \phi = m/r \) in the Schwarzschild metric. Thus for the \( \lambda = 0 \) case we have a strange situation where the \( G_{0}^{0} \) component of the field equations allows an N-body solution \( \phi = \Sigma A m_{A}/r_{A} \) but the \( G_{k}^{k} \) components of the same equations do not allow that solution. We cannot even argue that, due to nonlinearity, \( \Omega_{ik} = 0 \) may require a form different than the \( \phi = \Sigma A m_{A}/r_{A} \) because the \( \Omega_{00} = 0 \) part already accepts that form.

This failure is normally overlooked. The N-body solution is usually assumed for general relativity in passing to a Newtonian limit. But in general relativity the Newtonian limit is satisfied only in first order. Here we are concerned with second order quantities, \( t_{ik} \), \( \Omega_{00} \) and \( \Omega_{ik} \).

When we include the requirement of the additive constant the situation gets worse. For, in this case we cannot get even the original Schwarzschild solution. The reason is that with the additive constant \( C \), neither \( \Omega_{00} = -2\sigma C \) nor \( \Omega_{ik} = C \phi_{ik} - \delta_{ik} \sigma C \) can be made zero independently of \( C \) so as to satisfy the field equations. On the other hand, the existence of a \( \phi + C \) is of utmost importance. The essence of the additive constant is that if \( \phi \) is a solution to the field equations, then \( \phi + C \) must also be a solution to the same field equations. This means that the equations (and therefore the metric) must not depend on the absolute value of the potential \( \phi \). They must depend only on the “potential differences”. With the \( C \) invariance also imposed in general relativity we cannot get any solution at all. The only way to get the desired N-body solutions is to set \( \lambda = 1 \). In this unique case the event horizon disappears and we have no black holes.

This interpretation is a fundamental prediction of the \( \lambda = 1 \) theory and can be tested experimentally (Yilmaz 1987). To this end, measurements of one-way light propagation times, using 100 picosecond pulses of laser light and transported hydrogen maser clocks, have been made over a 20 km East-West component path. So far the results are inconclusive as to whether the East-West and West-East times are equal or not on the rotating Earth. With improved equipment, now available, conclusive measurements can be obtained (Alley 1992).

4. Interactive Nature of the N-Body Solutions

As we have emphasized in Section 2, to have N-body solutions is not enough. One must also show that the bodies interact, and in a way consistent with observations. At first sight the linearity of the Poisson equation leading to the N-body solutions may give the wrong impression that, in the \( \lambda = 1 \) case, there may be no interaction between the bodies hence no accelerations. Such a conclusion is not correct, because, in the Newtonian theory too, we have such a linear potential and in the Newtonian theory we have interactions and accelerations. Here, as the bodies move, other components \( \phi_{i}^{k} \), \( \phi_{k}^{i} \) of the field also develop.
To exhibit the interactive nature of the N-bodies most clearly we now introduce the more general form of the Yılmaz theory. Written in a Minkowski background (for purposes of correspondence to special relativity, see Note 1) the Yılmaz theory can be given by two equations plus a coordinate (gauge) condition for definiteness, as (Yılmaz 1992)

\[
\frac{1}{2} G^\nu_\mu = \tau^\nu_\mu + t^\nu_\mu \\
\sigma du_\mu / ds = \frac{1}{2} \partial_\mu g_{\alpha\beta}(\tau^{\alpha\beta} + t^{\alpha\beta}) \\
\partial_\nu (\sqrt{-g} g^{\mu\nu}) = 0
\] (14)

(15)

(16)

First of all these equations show a relationship between general relativity and Yılmaz’ theory. The former is a truncated case where the \( t^\nu_\mu \) is removed as in \( t^\nu_\mu \Rightarrow \lambda t^\nu_\mu, \lambda = 0 \). But the presence of \( t^\nu_\mu \) is of crucial importance because there exists an identity which states that (Note 2)

\[
\frac{1}{2} \partial_\mu g_{\alpha\beta}(\tau^{\alpha\beta} + t^{\alpha\beta}) \equiv (\sqrt{-g})^{-1} \partial_\nu (\sqrt{-g} g^{\mu\nu})
\] (17)

hence the equations of motion can also be written as

\[
\sigma du_\mu / ds = (\sqrt{-g})^{-1} \partial_\nu (\sqrt{-g} g^{\mu\nu}).
\] (18)

This equation shows clearly the essential point that the \( t^\nu_\mu \) is the carrier (mediator) of interactions and in its absence there will be no accelerations. This is what we mean with the requirement of “interactive N-body” solutions and here we see that \( \lambda = 1 \) theory has them. Multiplying by \( \sqrt{-g} \) and integrating over the volume containing one of the particles, for example \( m_1 = m \),

\[
mdu_\mu / ds = \Delta \phi \partial_\mu \phi = -m \partial_\mu \phi
\] (19)

which is the equation of motion in the slow motion limit. Since, upon calculation the term \( 1/2 \partial_\mu g_{\alpha\beta}(\tau^{\alpha\beta} + t^{\alpha\beta}) \) gives the same result (note that in this limit \( \partial_\mu g_{\alpha\beta}t^{\alpha\beta} = 0 \)), we have the geodesic limit (\( du_\mu / ds = -\partial_\mu \phi \)) and the strong principle of equivalence satisfied (Yılmaz 1992)

\[
m_i = m_a = m_p
\] (20)

since \(-\Delta \phi = \sqrt{-g} \sigma\) is the density of “active mass”. This calculation also shows that it will be difficult, if not impossible, to satisfy the “strong principle of equivalence” without the \( t^\nu_\mu \) because the active mass comes in by the density divergence of \( t^\nu_\mu \). The theory describes interactive multiparticle dynamics in the sense of Hamiltonian particle mechanics; the continuum limit is allowed by statistical averaging, in which case one needs two or more functions to describe the details of the equation of state.

Can there be noninteractive N-body solutions? It is found that in some simple symmetries, extended bodies such as parallel slabs and spherical shells, there may be N of them even when \( t^\nu_\mu \) is zero. However, by the above equations of motion Eq. (18), the forces between them, hence also their accelerations are zero (Alley 1994). If the \( t^\nu_\mu \) is present, they do interact (consistent with the Newtonian correspondence). These results can be verified by hand or by computer calculations.

If the solution contains only one object, then, of course, there cannot be any interaction as there would be nothing else to interact with. As to the test-body theories having a single central body plus test particles put by hand, they contain an implicit assumption, namely, the central body must have infinite inertial mass and finite active mass which we know is false and is against the strong principle of equivalence. Of course, particles put by hand cannot have active mass and cannot generate gravitational fields. Such particles are called test-particles. A test-particle theory violates the universal interparticle symmetry of gravitation because the central body is in the solution but the test-particles are not (Yılmaz 1988).
The difference between an N-body theory and a test-body theory shows up most dramatically in the calculation of the motions of the planetary perihelia. Thus for example perihelion of Mercury advances 575" per century of which 532" is due to Mercury’s interactions with other planets and 43" per century to relativistic correction. The 532" interactive part is predicted by the N-body theory but not by the test-body theory since test bodies do not interact. The situation is the same for the other eight planets all of which have even larger interactive perihelion shifts. The \( \lambda = 1 \) theory predicts the total perihelion motions in a seamless way.

It is usually believed that in papers published in 1938 and 1940 (Einstein, Infeld & Hoffman 1938, Einstein & Infeld 1940), Einstein, Infeld and Hoffman (EIH) obtained N-body equations of motion in the slow motion limit. This belief is unfounded. As described by P. G. Bergmann in his well-known book (Bergmann 1942), the situation is as follows: With Eqs. (15.12) on page 230, Einstein’s equations are satisfied in first order (right hand sides are put to zero in vacuum), but with Eqs. (15.25) on page 234 they are not satisfied in second order (they are not put to zero in vacuum). They are left unspecified. Yet, as stated on page 232, to obtain the equations of motion one must carry the field equations to second order. Thus the question arises: What should these unspecified second order terms be in order to get the N-body interactive solutions to be used later to obtain the N-body equations of motion (15.49) on page 240? It turns out that they cannot be zero, as Einstein’s theory requires. They rather demand \( \frac{1}{2} G_{\mu}^{\nu} = -t_{\mu}^{\nu} \) in vacuum where the \( t_{\mu}^{\nu} \) is the Yilmaz stress-energy tensor for the N-body field \( \phi = \Sigma A_{A}/r_{A} + C \). (The \(-\) sign is due to the definition of \( G_{\mu}^{\nu} \) in Bergmann’s book as the negative of Yilmaz’ definition). In other words, Eqs.(15.49) are true in Yilmaz’ theory and not in Einstein’s theory. In fact, the Yilmaz exponential metric, our eq.(9), can be derived from the condition that, in the Newtonian limit, the equations of motion will be of the form (15.49) of Bergmann.

5. Discussion

The recent discovery of the gamma ray burster GRB 990123, requiring energies exceeding the limit allowable by general relativity for neutron star mergers, created an energy crisis in astrophysics (Schilling 1999). The limiting factor seems to be that, according to general relativity, a neutron star (or a merger of stars) exceeding a total of \( 2.8M_{\odot} \) would become a black hole and thereafter little radiation could escape whereas the energy required for GRB 990123 seems to be at least \( 2M_{\odot}c^{2} \) to properly account for the gamma and other emissions. In fact, according to the N-body theorem there cannot be such energy producing mergers in general relativity. If the event horizon did not exist, the interaction energy released from the deeper regions, surfaces, magnetic fields, etc., can provide the required energy. (Note that the massive neutron stars can possess magnetic moments – the "black holes have no hair" theorem does not apply in the new theory. Note also that radially directed light can always escape, although substantially redshifted.) In two recent articles by S. L. Robertson such an explanation is already proposed (Robertson 1999a, Robertson 1999b).

Summarizing: a) The long sought N-body interactive solutions in curved spacetime theory of gravity are found which merits immediate attention in its own right. b) The test-body (1-body) nondynamical metrics are replaced by N-body dynamical metrics free of event horizons. A natural explanation of the GRB 990123 energy requirement becomes possible via a merger of two massive neutron stars (called Yilmaz stars by Robertson) which are not black holes. In the past, in times of great theoretical and observational crises, like the ones we are now having, patching up old theories did not help. Instead, a new paradigm emerged which organized known facts in a more systematic manner as well as overcoming the prevalent
difficulties and predicting new effects. We may be witnessing here a similar situation in the equations of general relativity. In both the field equations and the equations of motion, the “matter alone” paradigm is allowed to go over into a new paradigm “matter plus field”. (More precisely, $\tau^{\nu}_{\mu} \Rightarrow \tau^{\nu}_{\mu} + t^{\nu}_{\mu}$. ) This change in paradigm makes it possible to treat the GRB 990123 as a merger or collision of two massive neutron stars, with some beaming if needed, whereas general relativity seems to be in a bind, since it has only a 1-body solution (a solitary black hole) with which none of these models is feasible.

Quite independently of the energy crisis at hand this shift in paradigm has many important consequences in other respects. a) The theory becomes a standard local gauge-field theory in curved spacetime. b) It is a dynamical theory (not a test-body theory), hence the planetary perturbations are treatable in a seamless way along with the relativistic effects. c) It does not lead to event horizons, hence physical properties such as magnetic moments are allowed. d) It has a higher critical mass, hence more energy is available in mergers and collisions. e) As far as we know, it is quantizable (Yılmaz 1997, Alley 1995). These and other important features will be described in a larger paper in preparation.

“The hallmark of a successful theory is that it predicts correctly facts which were not known when the theory was presented or, better still, which were then known incorrectly.”

Francis Crick (Life Itself, Simon and Schuster, 1981)

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Note 1. In any Riemannian spacetime theory, in more general coordinates than the above, a nontensor $z_{\mu}^{\nu}$; $\tau_{\mu}^{\nu} \Rightarrow \tau_{\mu}^{\nu} + z_{\mu}^{\nu}$, $t_{\mu}^{\nu} \Rightarrow t_{\mu}^{\nu} - z_{\mu}^{\nu}$ may appear, but this does not harm the coordinate independence of any theory. It can be removed without harm to anything by replacing the ordinary derivative with respect to the chosen background by the covariant derivative with respect to the same. The reason the Lorentz background is free of $z_{\mu}^{\nu}$ is that in this case these two derivatives coincide (Rosen 1940, Yılmaz 1997).

Note 2. A corollary of the Bianchi and the Freud identities $D_{\nu}(\tau_{\mu}^{\nu} + t_{\mu}^{\nu}) \equiv 0$, $\partial_{\rho}(\sqrt{-g} \tau_{\mu}^{\nu}) \equiv 0$. If the $t_{\mu}^{\nu}$ is not present ($\lambda = 0$), both identities will apply to $\tau_{\mu}^{\nu}$ and/or $C^{\nu}_{\mu} = 2\tau_{\mu}^{\nu}$, leading to an overdetermination (Yılmaz 1992) which is the basic difficulty in the ($\lambda = 0$) theory. The introduction of the appropriate $t_{\mu}^{\nu}$ removes the difficulty. Note that Eqs. (15) and (18) are not contradictory because when $t_{\mu}^{\nu}$ is absent from the field equations, the solution for $\tau_{\mu}^{\nu}$ is such that $\frac{1}{2}\partial_{\mu}g_{\alpha\beta}\tau^{\alpha\beta} = 0$. This is a general result. A specific exact solution clarifying this point has been exhibited (Alley 1994, 1995, Yılmaz 1994).