E-commerce supply chain inventory decisions and contract design considering sales effort and risk aversion

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Accepted: 25 May 2022
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Abstract
We introduce vendor-managed inventory (VMI) modes and retailer-managed inventory modes and use conditional value-at-risk to quantify the inventory manager’s risk aversion. Then, we compare the supply chain (SC) members’ optimal decisions and expected utilities when the online retail platform or the supplier conducts sales effort under different inventory modes. The cost-sharing contracts are introduced to optimize the SC performance. Some interesting findings are achieved: for VMI SC, SC members’ sales-effort preferences change with the wholesale price; the inventory manager’s risk-aversion degree has a significant impact on the SC members’ preferences for inventory management modes; there always exists appropriate cost-sharing ratio regions in which the SC members’ expected utilities can be improved. The corresponding research results can help to improve the operational effectiveness of different kinds of SCs.

Keywords Online retail platform · Supply chain · Sales effort · Risk aversion · Cost-sharing contract

1 Introduction

Due to the huge impact of COVID-19, an increasing number of consumers choose to purchase goods through online retail platforms. Hence, the “new online economy” is experiencing a great development opportunity, which also intensifies online market competition. At the same time, the homogeneity of different kinds of products is becoming increasingly serious. There is no significant difference in efficacy or price

1 https://www.thepaper.cn/newsDetail_forward_9652682.

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Published online: 22 August 2022

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among products provided by different manufacturers, such as milk and other necessary daily consumables. To achieve the goal of attracting consumers and expanding market demand, many online retail platforms, and their suppliers conduct different forms of sales effort continuously to stimulate consumer demand. For the traditional sales mode, the retailer generally conducts sales efforts through advertising, product exhibitions, and hiring sales staff as shopping guides carry out the sales effort in the supply chain (SC) [46]. The new online trading mode has spawned a series of new sales-effort behaviors, such as product recommendations based on big data and e-commerce live streaming, which have been progressively applied by the SC members [24, 30]. At the same time, more and more suppliers are involved in the sales effort. Thus, the sales effort may be conducted by the online retail platform or the supplier.

We examine Yili milk, selling in the Tmall supermarket as an example. To improve product demand during the epidemic, Yili recruits famous stars to promote their products through live streaming as a dairy supplier of the Tmall supermarket. As an online retail platform, Tmall supermarket also promotes product sales through new sales-effort behavior, such as product recommendations based on big data and offering discount coupons and cashback promotions accordingly [6, 52]. However, because the supplier and online retail platform can easily obtain information about the other’s sales efforts when one SC member dominates sales promotions, the other often chooses to take a free ride to reduce the cost. Therefore, SC’s sales effort is usually conducted by one member. From the case about Yili milk, we notice that each SC member is possible to conduct sales effort in reality. Hence, the SC members may want to make it clear that how different sales-effort modes impact the SC performance.

The inventory decision is closely related to the sales effort decision. It is interesting to discuss the sales-effort decision based on a special inventory management mode. In a two-echelon SC, there are two common inventory management modes, i.e., vendor-managed inventory (VMI) and retailer-managed inventory (RMI). The Tmall supermarket mainly introduces VMI mode to construct cooperation mechanisms with its suppliers. The Tmall supermarket has more than 30 Cainiao warehouses in China. Under VMI SC, suppliers are required to put goods into Cainiao warehouses in advance and manage the inventory by themselves. The ownership of the products in the warehouses still belongs to the suppliers, and the Tmall supermarket only pays the suppliers according to the actual sales quantity. Meanwhile, the supplier that takes on inventory risk is usually risk averse. Unlike the Tmall supermarket, JD mainly introduces RMI mode to construct cooperation mechanisms with its suppliers. Under RMI SC, before the selling season, JD directly purchases goods

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2 Tmall supermarket is a new local online retail supermarket built by Taobao.com. https://chaoshi.tmall.com/?targetPage=index
3 https://www.thepaper.cn/newsDetail_forward_8454616.
4 https://m.hexun.com/news/2020-07-27/201768298.html.
5 JD is one of the influential integrated online retailers in China’s e-commerce field. https://www.jd.com/
6 https://baijiahao.baidu.com/s?id=1601489887113360167&wfr=spider&for=pc.
from suppliers through wholesale price contracts for its 8 warehouses in China and conducts inventory management. In this situation, since JD pays for the product inventory, it faces a greater inventory risk and may want to control the risk in a certain extent.

The above cases show that there are different sales-effort modes and inventory management modes in the platform SC. We can analyze the relationship between the online retail platform and the supplier based on supply chain management (SCM) philosophy, which refers to a set of methods used to effectively coordinate suppliers, producers, depots, and stores, in order to reduce system costs while satisfying service level requirements [44]. SCM philosophy can help us propose appropriate operational modes and cooperation mechanisms of the SC members [18]. Key determinants of SCM include the sharing of information, risks and rewards, goals and strategies, as well as relationship management, cooperation, and the integration of processes and behaviors [42]. Through appropriate designs, SCM can improve the SC performance through providing better product and service strategies, demand planning and forecasting, and effective decision making [42]. In this process, game theory is also needed in analyzing the SC members’ optimal decisions. Game theory is a mathematical theory for studying how players in a competitive relationship make decisions [1, 7, 14]. With the development of SCM, the game theory begins to be widely used in the SC models [12, 26].

According to above cases and theories, for most SCs providing consumer goods, such as apparel, electric products, etc., the supplier and the online retail platform may face the following typical management problems:

1. As a decision maker who is risk averse, how to choose optimal supply quantity or optimal sales-effort level under different inventory management modes?
2. Under VMI SC or RMI SC, what are the SC members’ preferences for the sales-effort conductors, i.e., the online retail platform or the supplier?
3. What are the SC members’ preferences for inventory management modes under different sales-effort modes?

To answer these research questions from the perspective of SCM, we construct newsvendor models based on game theory considering the inventory manager’s risk aversion based on the conditional value-at-risk (CVaR) criterion [1, 15]. In these models, we not only discuss different sales-effort modes, for example, the online retail platform conducts sales effort and the supplier conducts sales effort, but also introduce different inventory management modes (VMI and RMI). Simultaneously, we also introduce cost-sharing contracts to optimize the SC performance. We analyze and compare the SC members’ optimal decisions and Pareto improvement schemes of VMI SC and RMI SC considering sales effort and risk aversion, and the results can help SC choose the most appropriate operational modes and introduce effective collaboration mechanisms.

The remaining section of this paper is arranged as follows. In Sect. 2, we review the related literature. In Sect. 3, we give a description of the problem and define the parameters and structures of the model. Section 4 investigates the optimal decisions
under VMI SC when the online retail platform or the supplier conducts sales effort. Section 5 compares and analyzes the decision differences of VMI SC and RMI SC. Section 6 performs numerical simulations, Sect. 7 summarizes this paper and suggests for future related research. Please see the “Appendix” for proofs of propositions and corollaries.

2 Literature review

Three research directions in the existing literature are worth noting, including sales effort in the SC, the cost-sharing contract, and risk measure based on CVaR, which are summarized in this section.

2.1 Sales effort

Both the upstream supplier and downstream retailer may conduct sales efforts in a two-stage SC. However, a large number of current researches are focused on the situation in which the retailer conducts sales efforts under RMI SC. In the traditional offline SC, Taylor [46] finds that the manufacturer can induce the retailer to choose a more reasonable level of sales effort by introducing a channel rebate contract. Basiri and Heydari [3] investigate the green channel coordination issue and assume that the retailer decides both the selling price and sales-effort level for green products, while the products’ green quality is decided by the manufacturer. Cao and Fan [5] investigate the retailer’s optimal sales-effort investment and ordering policies for temperature-sensitive products in a stylized newsvendor setting. Liu et al. [31] discuss the timing effect of a single retailer’s sales-effort commitments (early commitment and delay commitment) on firms’ investment and pricing decisions. Hu et al. [21] compare four investment modes of cost reduction in SCs to show the SC members’ preferences when the retailer conducts sales effort. Lin et al. [29] find two-way compensation confirming warehouse financing can coordinate the SC when the retailer conducts sales effort.

In the wake of developments in e-commerce technology, many scholars begin to pay attention to SC management involving online channels. Ke and Liu [25] discuss the competition between dual-channel SCs with channel preference and sales effort under uncertainty. Zhou et al. [57] study the phenomenon that buyer’s online purchase channel gets pre-sales service free of charge by sharing the retailer’s sales-effort cost, and analyze the influence of this phenomenon on the SC members’ profits, pricing strategy and service strategy. Ranjan and Jha [40] express demand as a linear function related to online or offline sales price, the level of green quality and sales effort, and then discuss the pricing strategy and coordination mechanism among dual-channel SC members. Yan et al. [54] also formulate a dual-channel SC model, and investigate the impact of free-riding on the quantity, market share, structure of channel, and pricing competition level of the offline retailer and the online platform.

The above literature analyzes SC models considering the sales effort of online/offline retailer from different perspectives, but there are few studies considering the
situation in which the supplier conducts sales effort. In our research, we focus on the two-level SC composed of one online retail platform and one supplier under different operational modes and assume that both the online retail platform and the supplier may conduct sales effort. By comparing the sales-effort level and supply quantity under different operational modes, our study enriches the related research on sales effort.

2.2 Cost-sharing contract

Cost-sharing contracts are extensively used for sharing, investment, research, operational, and other costs between the SC members. Empirical studies show that cost-sharing contract is an important strategy for generating a competitive advantage in managing supplier relations [56]. Studies on integrating cost-sharing contracts into the SC to optimize the SC performance have attracted widespread attention in academia. Ghosh and Shah [17] discuss SC coordination issues caused by the green SC initiatives and analyze the effort of the cost-sharing contract on the critical decisions of SC participants who implement green suggestions. Roma and Perrone [43] introduce outcome-based versus ex-ante-based cost-sharing contracts to study the impact of these two contracts on the profitability and total welfare of competing companies by constructing a game model. Liu et al. [33] build a differential game framework and find that the retailer can effectively encourage the manufacturer to improve the green-degree of products by implementing a cost-sharing contract. Wang et al. [47] build single and combined carbon reduction models that consider cap-and-trade regulations and find that the implementation of two-way cost-sharing contracts can improve SC profit, product quantity and the level of carbon emission abatement. This contract can also be seen in other researches [15, 34, 36, 38, 41, 55].

Similar to our research, some studies focus on the supplier’s cost-sharing mechanism when the retailer conducts sales effort. Bai et al. [2] and Lin et al. [29] consider a coordination issue by introducing the cost-sharing contract in a two-echelon SC in which the retailer conducts sales effort. Zhou et al. [57] investigate SC members’ pricing issues when the manufacturer’s online channel shares the cost of retailer’s sales effort and takes the retailer’s pre-sale service for free. Phan et al. [39] build a two-echelon VMI SC model considering the manufacturer’s corporate social responsibility and the level of the retailer’s sales effort, provide a new coordination pattern for the SC combining revenue-sharing contract and cost-sharing contract. He et al. [19] investigate the optimal investment and coordination decisions in an SC of fresh agricultural products under three types of cost-sharing contracts by establishing differential game models. In fact, the retailer can also share the sales-effort cost when the supplier conducts sales effort. In this regard, the above literature is not involved in the mode when the supplier conducts sales effort. Therefore, this paper considers two sales-effort modes to show that both the online retail platform and the supplier may conduct sales effort and introduce cost-sharing contracts to optimize the SC performance. Then, if the supplier conducts sales effort, the online retail platform may provide an appropriate cost-sharing ratio to share the cost of sales effort; if the
online retail platform conducts sales effort, the supplier may seek to optimize the SC performance by providing a certain cost-sharing ratio.

2.3 CVaR risk aversion

In the presence of a variety of uncertainties, risk management is essential in achieving the effective operations of SC [4, 20]. Based on the theories of information processing and the dynamic capability view, some empirical analyses show that effective SC risk management can improve firm performance [23, 37]. On the one hand, a large number of firms are aware of the need for risk control in operations management. On the other hand, A growing number of researchers begin to consider a firm’s degree of risk aversion when constructing a newsboy model. Chen et al. [8] first use the CVaR risk quantification measure in the newsvendor model. The CVaR criterion has been investigated in various extensions of the newsvendor model [9, 35, 49]. Based on the CVaR criterion, Huang et al. [22] investigate coordination and risk-sharing issues of the SC consisting of a dominant retailer and a risk-averse manufacturer, where demand is stochastic and dependent on marketing effort. Yang et al. [55] investigate the influence of firms’ risk-averse attitudes on the SC performance, and show that both the push SC and the pull SC can be coordinated with buy-back contract and three-part tariff revenue sharing contract considering firms’ CVaR. Based on CVaR criterion, Liu et al. [32] study the option pricing and producing, ordering issues in an SC composed of a risk-averse retailer and a risk-neutral supplier. Song et al. [45] investigate the risk-averse retailer’s acquisition of quality information and ordering decisions by formulating a newsvendor model based on the CVaR criterion. Wang et al. [48] use the CVaR criterion to quantify risk and study the risk-averse retailer’s ordering decisions when providing unattended plans. Fan et al. [16] use CVaR to quantify the risk attitudes of the SC members, and then consider the application of option contracts in a buyer-led SC, in which both the supplier and the buyer are risk averse. Similar works can also be found in other studies [10, 27, 50, 51, 53].

2.4 The distinctiveness of this research

The key contribution of this research is taking the lead in analyzing and comparing the optimal decision of VMI SC and RMI SC considering sales effort and risk aversion, and proposing SC Pareto improvement scheme. Our work is most closely related to Chernonog et al. [11] and Zhu and Lee [58]. Chernonog et al. [11] develop a stochastic model of a two-echelon SC with virtual products and focus on studying the SC members’ decisions on pricing and sales-effort level considering demand uncertainty and risk sensitivity. They don’t consider the inventory decisions in the model since the research target is the virtual product. Zhu and Lee [58] construct a Stackelberg game model with a risk-averse retailer and a risk-neutral supplier based on CVaR criterion. They explore the impact of misplaced inventory on the SC and introduce the cost-sharing contract to achieve SC coordination. Sales effort is not introduced in Zhu and Lee [58]. Different from Chernonog et al. [11] and
| Literature          | Inventory management mode | Sales-effort conductor | Risk attitude | Contract design                                      |
|---------------------|---------------------------|------------------------|--------------|-----------------------------------------------------|
| Bai et al. [2]      | RMI                       | Retailer               | Risk neutral | Cost-sharing contract                                |
| Chernonog et al. [11]| Virtual products no need for inventory | Retailer               | Risk averse | −                                                    |
| He et al. [19]      | RMI                       | Retailer               | Risk neutral | Cost-sharing contract                                |
| Hu et al. [21]      | RMI                       | Retailer               | Risk neutral | Revenue-sharing contract                             |
| Huang et al. [22]   | RMI                       | Platform’s self-run stores and third-party stores | Risk averse | Option contract                                      |
| Ke and Liu [25]     | RMI                       | Retailer               | Risk averse | −                                                    |
| Lin et al. [29]     | RMI                       | Retailer               | Risk neutral | Cost-sharing contract                                |
| Phan et al. [39]    | VMI                       | Retailer               | Risk neutral | Cost-sharing contract                                |
| Yang et al. [55]    | RMI and VMI               | −                      | Risk averse | Three-part tariff revenue sharing and buy-back contracts |
| Zhou et al. [57]    | RMI                       | Retailer               | Risk neutral | Cost-sharing contract                                |
| Zhu and Lee [58]    | RMI                       | −                      | Risk averse | Cost-sharing contract                                |
| This paper          | RMI and VMI              | Retailer and supplier  | Risk averse | Cost-sharing contract                                |
Zhu and Lee [58], this paper focuses on comparing two different inventory management modes by assuming the inventory manager is risk averse. We further assume that both two SC members may be in charge of conducting sales effort. By comparing the decisions on sales-effort level and supply quantity under different inventory management modes and sales-effort modes, this paper enriches the current researches. We introduce Table 1 to position the contribution of our research in the literature and exhibit the research gap.

It is worth noting that in the research about sales effort, the free-rider strategy is very popular in many different types of SCs [29, 54, 57]. Therefore, our research assumes that only one SC member is in charge of sales effort. Furthermore, many researches show that matching inventory with market demand can improve the SC operational efficiency and reduce SC losses caused by inventory slack [13, 20]. Due to market demand uncertainty, the inventory manager often needs to bear the losses caused by the inventory slack. Since only one member is in charge of inventory management under VMI SC or RMI SC, we reasonably assume that the inventory manager is risk averse in this paper. In general, by introducing cost-sharing contracts, this paper mainly analyzes and compares optimal decisions and Pareto improvement schemes of VMI SC and RMI SC considering sales effort and risk aversion. In short, this research contributes to the literature in the following three ways.

(1) This paper makes contribution to the existing literature on VMI SC management from a fresh perspective. We examine the effects of decision differences on supply quantity and sales effort when the sales effort is controlled by different VMI SC members. Such a research design grasps the key characteristics of modern VMI SCs. Corresponding research results can help to improve operational effectiveness of different kinds of VMI SCs.

(2) This paper enriches the comparison study on VMI SC and RMI SC by jointly introducing sales effort and risk attitude. By considering different situations in which the supplier or the online retail platform conducts inventory management or sales effort, this paper explores different management insights for the supplier and the online retail platform.

(3) This paper verifies the value of two cost-sharing contracts in improving the performance of VMI SC and RMI SC. We obtain Pareto improvement schemes based on the cost-sharing contracts for VMI SC and RMI SC and provide reasonable suggestions for determining the cost-sharing ratio under different inventory management and sales-effort modes.

3 Model description

This paper focuses on a two-echelon SC in which one supplier (referred to as “he”) sells products to one online retail platform (referred to as “she”) over a single selling season. Before the selling season, both the supplier and the online retail platform are likely to conduct some level of sales effort in order to improve sales quantity. Let $e$ ($e \geq 0$) denote the sales-effort level. Then, the corresponding sales-effort cost is $g(e) = \eta e^2 / 2$, where $\eta$ represents the cost coefficient of the sales effort, $\eta \geq 0$. 
Because both the online retail platform and the supplier may conduct sales effort, we assume that their cost coefficients of the sales effort are $\eta_p$ and $\eta_s$, respectively. In this paper, following the reasons proposed in Sect. 1, we assume that only one member is in charge of the sales effort; a similar assumption can be found in many other works [3, 5, 21, 31]. Therefore, it is clear that market demand is sales-effort dependent. We further assume that the demand comprises two parts: a determinate part $d(e)$ and a stochastic part $\xi$, i.e., $D = d(e) + \xi$, where $d(e) = A + \beta e$ denotes the determinate part of the market demand, in which $A$ stands for the basic market demand, and $\beta$ ($\beta \geq 0$) is the elasticity coefficient of the sales-effort level, which is used to describe the impact of the unit change of sales effort on the demand. The stochastic variable $\xi$ is defined in the region $[0, +\infty)$. Let $F(\xi)$ and $f(\xi)$ denote the cumulative distribution function and probability density function of $\xi$, respectively. The mean of $\xi$ is $\mu$, i.e., $E(\xi) = \mu$. We further denote $q$ as the supply quantity and $r$ as the unit shortage cost. For the supplier, the unit cost of product is $c$, and $w$ is denoted as the wholesale price. The retail price of product sold to consumers by the online retail platform is $p$. Assume $c$, $w$, and $p$ are all exogenous and $p > w > c$.

Generally, there are two SCs with different inventory management modes, VMI SC and RMI SC. Under VMI SC, the supplier decides the supply quantity and bears all inventory risks. Hence, we suppose that the supplier is risk averse and the online retail platform is risk neutral. In contrast, under RMI SC, the online retail platform determines the supply quantity and bears all inventory risks. Hence, we assume that the online retail platform is risk averse and the supplier is risk neutral. To promote sales quantity, both the supplier and the online retail platform can conduct sales efforts under the above two inventory management modes. In this paper, we first consider the model development of VMI SC and introduce cost-sharing contracts to improve VMI SC performance. Then, we exert similar investigation on RMI SC. Finally, detailed comparative analysis between VMI SC and RMI SC is conducted. Different discussion scenarios are presented in Table 2.

Next, we measure the decision maker’s risk attitude base on CVaR criterion. Let $\pi(q, e)$ denote the profit function of the decision maker and $R$ denote the set of real numbers. Then, according to the definition of CVaR in Chen et al. [9], Li et al. [28] and Xue et al. [53], the risk-averse decision maker’s utility function can be expressed as

| Table 2 Different discussion scenarios of the decentralized SC |
|---------------------------------------------------------------|
|                                                              |
| Without the cost-sharing contract | With the cost-sharing contracts |
|-----------------------------------|---------------------------------|
| **VMI**                           | **RMI**                          |
| Online retail platform decides $e$| Online retail platform decides $e$| Section 4.1                   |
| Supplier decides $e$              | Supplier decides $e$             | Section 4.2                   |

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where \( \eta \in (0, 1] \) denotes the degree of risk aversion (the smaller \( \eta \) is, the more risk-averse the decision maker is). When \( \eta \) approaches 0, the decision maker is exceedingly risk averse; in contrast, when \( \eta = 1 \), it means the decision maker is risk neutral. \( \eta \) is also known as a confidence indicator defined by the decision maker to achieve a certain profit level \( v \). When \( \eta = 1 \), the decision maker’s expected utility is equal to the expected profit. Next, when the decision maker is risk neutral, we still use expected utility instead of expected profit to standardize the expressions in this paper. For the sake of narration, we define the relevant notations as follows (see Table 3).

\[
CVaR_\tau(\pi(e, q)) = \max_{q \geq 0, v \in \mathbb{R}} \left\{ v - \frac{1}{\tau} E[v - \pi(e, q)]^+ \right\}, \tau \in (0, 1]
\]  

(1)

where \( \tau \in (0, 1] \) denotes the degree of risk aversion (the smaller \( \tau \) is, the more risk-averse the decision maker is). When \( \tau \) approaches 0, the decision maker is exceedingly risk averse; in contrast, when \( \tau = 1 \), it means the decision maker is risk neutral. \( \tau \) is also known as a confidence indicator defined by the decision maker to achieve a certain profit level \( v \). When \( \tau = 1 \), the decision maker’s expected utility is equal to the expected profit. Next, when the decision maker is risk neutral, we still use expected utility instead of expected profit to standardize the expressions in this paper. For the sake of narration, we define the relevant notations as follows (see Table 3).

| Parameters | Definition |
|------------|------------|
| \( p \)    | Retail price |
| \( w \)    | Wholesale price |
| \( c \)    | Unit production cost |
| \( \xi \)  | Stochastic part of the demand |
| \( A \)    | Basic market demand |
| \( \beta \) | The elasticity coefficient of the sales-effort level |
| \( r \)    | Unit shortage cost |
| \( \eta_i \) | The cost coefficient of the sales effort |
| \( \tau_i \) | The risk-aversion degree |
| \( \phi_i \) | The sales effort cost-sharing ratio |
| \( \pi_{ij} \) | Profit of the SC member without cost-sharing contracts |
| \( \pi_{ij}^\phi \) | Profit of the SC member with cost-sharing contracts |

**Subscripts**
- \( i \): Index of the SC member type; \( i = r, s \) represent online retail platform and supplier, respectively
- \( j \): Index of sales effort type; \( j = 1, 2, 3, 4 \) represent online retail platform conducts sales effort under VMI SC, supplier conducts sales effort under VMI SC, online retail platform conducts sales effort under RMI SC, supplier conducts sales effort under RMI SC, respectively

**Superscripts**
- \( \phi_i \): \( \phi_r, \phi_s \) represent online retail platform and supplier provide cost-sharing contract scenarios, respectively
- *: The optimal value for decision variable or profit function

**Decision variables**
- \( e_i \): Sales-effort level without cost-sharing contracts
- \( e_i^\phi \): Sales-effort level with cost-sharing contracts
- \( q_i \): Supply quantity without cost-sharing contracts
- \( q_i^\phi \): Supply quantity with cost-sharing contracts

TABLE 3 Definition of parameters

| Parameters | Definition |
|------------|------------|
| \( p \)    | Retail price |
| \( w \)    | Wholesale price |
| \( c \)    | Unit production cost |
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| \( \pi_{ij} \) | Profit of the SC member without cost-sharing contracts |
| \( \pi_{ij}^\phi \) | Profit of the SC member with cost-sharing contracts |

For the sake of narration, we define the relevant notations as follows (see Table 3).
4 VMI SC: the online retail platform conducts sales effort versus the supplier conducts sales effort

4.1 The online retail platform conducts sales effort

In this section, we study the scenario where the online retail platform conducts sales effort with the cost function \( g(e) = \eta_r e^2 / 2 \). Then, we show the competition model as follows: (i) The online retail platform acts as a leader by determining sales-effort level; (ii) the supplier decides the supply quantity using the \( \tau \)-CVaR criterion as a follower.

By utilizing backward induction, we find that the supplier’s profit function can be expressed as:

\[
\pi_{s1}(e, q) = w \min(D, q) - cq
\]  

(2)

We use the CVaR measurement of Eq. (1) to quantify the supplier’s risk-averse attitude. Then, the supplier’s expected utility function can be expressed as:

\[
CVaR_{\tau_s} [\pi_{s1}(e, q)] = \max_{q \geq 0, v \in \mathbb{R}} \left\{ v - \frac{1}{\tau_s} E[v - \pi_{s1}(e, q)]^+ \right\}, \tau_s \in (0, 1]
\]  

(3)

Anticipating the supplier’s optimal supply quantity, the online retail platform’s expected utility function can be expressed as:

\[
U[\pi_{r1}(e, q)] = (p - w)E \min(D, q) - rE(D - q)^+ - g(e)
\]  

(4)

**Proposition 1**  Under VMI SC, if the online retail platform decides the sales-effort level, then the optimal decisions of the online retail platform and the risk-averse supplier can be expressed as \( e_r^* = \frac{(p-w)\eta_r}{\eta_r}, q_1^* = F^{-1}\left[ \frac{(w-c)\tau_s}{w} \right] + d(e_r^*) \).

Next, we introduce the cost-sharing contract to improve the SC performance as an incentive mechanism. Based on the cost-sharing contract, the supplier promises a cost-sharing ratio \( \phi_s \) \((0 \leq \phi_s \leq 1)\) to share a part of the online retail platform’s sales-effort cost. Then, if the total sales-effort cost is \( g(e) \), the supplier is willing to share \( \phi_s g(e) \), and the online retail platform actually bears \( (1 - \phi_s) g(e) \). Here we call this a “VMI-\( \phi_s \) contract”. We further assume that \( \phi_s \) is exogenous, and \( \phi_s \) may be determined by negotiation between the supplier and the online retail platform. Then, we can describe the decision process as follows: (i) Given \( \phi_s \), the online retail platform determines sales-effort level; (ii) the supplier determines his supply quantity with the \( \tau \)-CVaR criterion. Let \( \pi_{s1}^{\phi_s}(e, q) \) and \( \pi_{r1}^{\phi_s}(e, q) \) represent the profit function of the supplier and the online retail platform under the VMI-\( \phi_s \) contract, respectively. Similarly, by using backward induction, we can find that the online retail platform’s optimal sales-effort level is \( e_{r1}^{\phi_s} = \frac{(p-w)\eta_r}{\eta_r (1 - \phi_s)\eta_r} \). Then, the supplier’s optimal supply quantity is \( q_1^{\phi_s} = F^{-1}\left[ \frac{(w-c)\tau_s}{w} \right] + d(e_{r1}^{\phi_s}) \).
Corollary 1 Under VMI SC, if the online retail platform determines the sales-effort level, the risk-averse supplier’s optimal supply quantity and the online retail platform’s optimal sales-effort level have the following properties:

(a) \( \frac{\partial e^*}{\partial \phi_s} > 0; \frac{\partial q^*_1}{\partial \phi_s} > 0 \)

(b) \( \frac{\partial e^*}{\partial \beta} > 0 \) and \( \frac{\partial e^*}{\partial \beta} > 0 \); \( \frac{\partial q^*_1}{\partial \beta} > 0 \) and \( \frac{\partial q^*_1}{\partial \beta} > 0 \)

(c) \( \frac{\partial e^*}{\partial \eta_r} < 0 \); \( \frac{\partial e^*}{\partial \eta_r} < 0 \); \( \frac{\partial q^*_1}{\partial \eta_r} < 0 \) and \( \frac{\partial q^*_1}{\partial \eta_r} < 0 \)

(d) \( \frac{\partial q^*_1}{\partial \tau_s} > 0 \) and \( \frac{\partial q^*_1}{\partial \tau_s} > 0 \)

Corollary 1(a) clearly reveals that the online retail platform’s sales-effort level and the supplier’s supply quantity both increase with the cost-sharing ratio \( \phi_s \). This means that if the cost-sharing ratio increases, the online retail platform is motivated to improve the sales-effort level to promote market demand. At the same time, the supplier also chooses to increase supply quantity to meet market demand. Corollary 1(b) shows that for the online retail platform, if the sales-effort level has a relatively high impact on market demand, she chooses a relatively high sales-effort level to promote market demand. At the same time, the supplier chooses a relatively high supply quantity to obtain possible high expected utility. Corollary 1(c) expresses that the online retail platform’s sales-effort level and the supplier’s supply quantity always decrease with \( \eta_r \), since a relatively high \( \eta_r \) means that the online retail platform induces a relatively high sales-effort cost given the same sales-effort level. Corollary 1(d) reveals that the degree of the supplier’s risk aversion negatively influences supply quantity decision. Hence, if the risk-aversion degree is relatively high, the supplier is inclined to reduce the optimal supply quantity to control risk.

Corollary 2 Under VMI SC, if the online retail platform decides the sales-effort level, the effects of the supplier’s risk-aversion degree on the online retail platform’s optimal expected utility and supplier’s optimal expected utility are as follows:

(a) \( \frac{\partial \text{CVaR}_{R_1}(\pi^*_1,c^*_i,q^*_1)}{\partial \tau_s} > 0 \) and \( \frac{\partial \text{CVaR}_{R_1}(\pi^*_1,c^*_i,q^*_1)}{\partial \tau_s} > 0 \)

(b) \( \frac{\partial U[\pi_{R_1}(c^*_i,q^*_1)]}{\partial \tau_s} > 0 \) and \( \frac{\partial U[\pi_{R_1}(c^*_i,q^*_1)]}{\partial \tau_s} > 0 \)

Corollary 2 reveals that as the degree of the supplier risk aversion decreases, both the online retail platform’s and the supplier’s expected utilities increase regardless of whether the VMI-\( \phi_s \) contract is introduced.
4.2 The supplier conducts sales effort

In this section, we study the scenario where the supplier conducts sales effort with the cost function \( g(e) = \eta e^2 / 2 \). Hence, the supplier determines the optimal supply quantity and the sales-effort level with the \( \tau \)-CVaR criterion synchronously.

The profit function of the supplier can be expressed as:

\[
\pi_{s_2}(e, q) = w \min(D, q) - cq - g(e) \tag{5}
\]

Similar to Sect. 4.1, by using the CVaR measurement of Eq. (1), we can get the expected utility function of the supplier as follows:

\[
\text{CVaR}_{\tau_s} [\pi_{s_2}(e, q)] = \max_{q \geq 0, v \in \mathbb{R}} \left\{ v - \frac{1}{\tau_s} E[v - \pi_{s_2}(e, q)]^+ \right\}, \tau_s \in (0, 1] \tag{6}
\]

**Proposition 2** Under VMI SC, if the risk-averse supplier decides the sales-effort level and supply quantity synchronously, then the optimal decisions can be expressed as:

\[
q_2^* = F^{-1} \left( \frac{(w-c)\tau_r}{w} \right) + d(e_s^*), e_s^* = \frac{(w-c)\beta}{\eta_s}.
\]

Given the supplier’s optimal supply quantity and the sales-effort level, the online retail platform’s expected utility function can be expressed as:

\[
U[\pi_{s_2}(e_s^*, q_2^*)] = (p - w) [q_2^* - \int_0^{q_2^*} F(\xi) d\xi] = (p - w) [q_2^* - \frac{q_2^* - d(e_s^*)}{\tau_s}] \tag{7}
\]

Similar to Sect. 4.1, to improve VMI SC members’ expected utilities, the online retail platform introduces a cost-sharing ratio \( \phi_r \) \((0 \leq \phi_r \leq 1)\) to share a part of the sales-effort cost. Then, if the total cost of sales effort is \( g(e) \), the online retail platform is willing to share \( \phi_r g(e) \), and the supplier actually bears \((1 - \phi_r) g(e)\). Here we call this a “VMI-\( \phi_r \) contract”. We further assume that \( \phi_r \) is exogenous, and \( \phi_r \) may be determined by the negotiation between the supplier and the online retail platform. Then, the flow of events can be described as follows: (i) Before the selling season, VMI SC members negotiate the value of \( \phi_r \); (ii) the supplier determines the sales-effort level and supply quantity using the \( \tau \)-CVaR criterion. Similarly, let \( \pi_{s_2}^\phi(e, q) \) and \( \pi_{s_2}^\phi(e, q) \) denote the profit functions of the supplier and the online retail platform under the VMI-\( \phi_r \) contract, respectively. By using the CVaR measurement from Eq. (1), we can obtain that the supplier’s optimal supply quantity and sales-effort level are:

\[
q_2^{\phi_r} = F^{-1} \left[ \frac{(w-c)\tau_r}{w} \right] + d(e_s^{\phi_r}), e_s^{\phi_r} = \frac{(w-c)\beta}{(1-\phi_r)\eta_s},
\]

**Corollary 3** Under VMI SC, if the risk-averse supplier decides the sales-effort level, his optimal supply quantity and sales-effort level have the following properties:

\[
\begin{align*}
(a) \quad \frac{\partial q_2^{\phi_r}}{\partial \phi_r} > 0 \text{ and } \frac{\partial q_2^{\phi_r}}{\partial \phi_r} > 0 \\
(b) \quad \frac{\partial e_s}{\partial \beta} > 0 \text{ and } \frac{\partial e_s}{\partial \beta} > 0; \frac{\partial q_2^*}{\partial \beta} > 0 \text{ and } \frac{\partial q_2^*}{\partial \beta} > 0
\end{align*}
\]
Corollary 3(a) shows that the supplier’s sales-effort level and supply quantity increase with the cost-sharing ratio $\phi_s$. This means that if the cost-sharing ratio increases, the supplier has an incentive to improve the sales-effort level to promote market demand. At the same time, he also chooses to increase supply quantity to meet market demand. Corollary 3(b) reveals that if the sales-effort level has a relatively high impact on market demand, the supplier has the incentive to choose a relatively high sales-effort level and supply quantity to obtain possible high expected utility. Corollary 3(c) shows that the supplier’s sales-effort level always decreases with $\eta_s$ since a relatively high $\eta_s$ means that the supplier induces a relatively high cost from sales effort given the same sales-effort level. Corollary 3(d) shows that the supplier’s degree of risk aversion negatively influences his supply quantity decision. Hence, if the degree of risk aversion is relatively high, the supplier is inclined to reduce the optimal supply quantity to control the risk.

Corollary 4 Under VMI SC, if the risk-averse supplier decides the sales-effort level, the effects of the supplier’s degree of risk aversion on the supplier’s optimal expected utility and the online retail platform’s optimal expected utility are as follows:

(a) \[ \frac{\partial \text{CVaR}_{s_2}}{\partial \tau_r} \left[ \tilde{\pi}_2(e^*_s, q^*_2) \right] < 0, \quad \frac{\partial \text{CVaR}_{s_2}}{\partial \tau_s} \left[ \tilde{\pi}_2(e^*_s, q^*_2) \right] > 0 \]

(b) \[ \frac{\partial U}{\partial \tau_r} \left[ \tilde{\pi}_2(e^*_s, q^*_2) \right] > 0, \quad \frac{\partial U}{\partial \tau_s} \left[ \tilde{\pi}_2(e^*_s, q^*_2) \right] > 0 \]

Corollary 4 reveals that with the decrease in the supplier’s degree of risk aversion, both the online retail platform’s expected utility and the supplier’s expected utility increase regardless of whether the VMI-$\phi_r$ contract is introduced.

4.3 Comparison analysis

In this section, we aim to find more important managerial implications by comparing the above decisions and corresponding expected utilities under VMI SC discussed in Sects. 4.1 and 4.2.

4.3.1 Comparison analysis for sales-effort modes

Proposition 3 Under VMI SC without cost-sharing contracts, if $c < w < \frac{p \eta_r + c \eta_s}{\eta_r + \eta_s}$, then $e^*_r > e^*_s$, $q^*_1 > q^*_2$; if $\frac{p \eta_r + c \eta_s}{\eta_r + \eta_s} \leq w < p$, then $e^*_r \leq e^*_s$, $q^*_1 \leq q^*_2$. 

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According to Proposition 3, compared with the mode in which the supplier conducts sales effort, we find the following features when the online retail platform conducts sales effort: (i) For a relatively low wholesale price, i.e., \(c < \frac{\eta_2 + c_2}{\eta_1 + \eta_2}\), the online retail platform chooses a relatively high sales-effort level, and the supplier chooses a relatively high supply quantity; (ii) For a relatively high wholesale price, i.e., \(\frac{\eta_2 + c_2}{\eta_1 + \eta_2} < \eta < \eta_2\), the online retail platform chooses a relatively low sales-effort level, and the supplier chooses a relatively low supply quantity.

**Proposition 4** Under VMI SC without cost-sharing contracts, by comparing the expected utilities of VMI SC members when the risk-averse supplier conducts sales effort and the online retail platform conducts sales effort, we have the following: (a) If \(c < \eta_2 < \frac{\eta_2 + 2\eta_1}{2\eta_1 + \eta_2}\), then \(\text{CVaR}[\pi_{s1}(e_2^*, q_2^*)] < \text{CVaR}[\pi_{s1}(e_1^*, q_1^*)]\), \(U[\pi_{r2}(e_2^*, q_2^*)] > U[\pi_{r2}(e_1^*, q_1^*)]\); (b) if \(\frac{\eta_2 + 2\eta_1}{2\eta_1 + \eta_2} \leq \eta < \frac{2\eta_1 + \eta_2}{2\eta_2 + \eta_1}\), then \(\text{CVaR}[\pi_{s2}(e_2^*, q_2^*)] < \text{CVaR}[\pi_{s1}(e_1^*, q_1^*)]\), \(U[\pi_{r2}(e_2^*, q_2^*)] > U[\pi_{r2}(e_1^*, q_1^*)]\); (c) if \(\frac{2\eta_1 + \eta_2}{2\eta_2 + \eta_1} \leq \eta < \eta_2\), then \(\text{CVaR}[\pi_{s2}(e_2^*, q_2^*)] > \text{CVaR}[\pi_{s1}(e_1^*, q_1^*)]\), \(U[\pi_{r2}(e_2^*, q_2^*)] > U[\pi_{r2}(e_1^*, q_1^*)]\).

Figure 1 illustrates the comparison result of Proposition 4. We find: (i) For a relatively low wholesale price, i.e., \(c < \frac{\eta_2 + 2\eta_1}{2\eta_1 + \eta_2}\), both the supplier and the online retail platform can obtain more expected utilities under the mode when the online retail platform conducts sales effort. Therefore, we reasonably recommend that the online retail platform takes responsibility of exerting sales effort; (ii) when \(\frac{\eta_2 + 2\eta_1}{2\eta_1 + \eta_2} \leq \eta < \frac{2\eta_1 + \eta_2}{2\eta_2 + \eta_1}\), by comparing VMI SC members’ expected utilities under different sales-effort modes, it is easy to find that no mode under VMI SC can achieve a win–win result for the supplier and the online retail platform; (iii) for a relatively high wholesale price, i.e., \(\frac{2\eta_1 + \eta_2}{2\eta_2 + \eta_1} \leq \eta < \eta_2\), both the supplier and the online retail platform can obtain more expected utilities under the model when the supplier conducts sales effort. Hence, we can reasonably conclude that it is reasonable to arrange for the supplier to be in charge of the sales effort.

**Proposition 5** Under VMI SC with cost-sharing contracts, if \(0 \leq \phi_s < \max\left[0, 1 - \frac{(1 - \phi_r)\eta_1}{(w - c)\eta_1}\right]\) then \(e^*_{\phi_s} < e^*_{\phi_r}, q^*_1 < q^*_2\); otherwise, if \(\max\left[0, 1 - \frac{(1 - \phi_r)\eta_1}{(w - c)\eta_1}\right] \leq \phi_s \leq 1\), then \(e^*_{\phi_s} \geq e^*_{\phi_r}, q^*_1 \geq q^*_2\).
Proposition 5 shows that after introducing the cost-sharing contracts, compared with the model when the supplier conducts sales effort, we find the following features when sales effort is conducted by the online retail platform: (i) if the SC introduces a relatively low sales-effort ratio, i.e., \(0 \leq \phi_s < \max\left[0, 1 - \frac{(1 - \phi_s)(p - w)\eta_s}{(w - c)\eta_s}\right]\), the online retail platform chooses a relatively low sales-effort level and the supplier chooses a relatively low supply quantity; (ii) if the SC introduces a relatively high sales-effort ratio, i.e., \(1 - \max\left[0, \frac{(1 - \phi_s)(p - w)\eta_s}{(w - c)\eta_s}\right] \leq \phi_s \leq 1\), the online retail platform chooses a relatively high sales-effort level and the supplier chooses a relatively high supply quantity. The above results show that after introducing the cost-sharing contracts under two different sales-effort modes, the cost-sharing ratio determined by the SC has an important impact on the members’ decisions on sales-effort levels and supply quantities under VMI SC.

**Proposition 6** Under VMI SC with cost-sharing contracts, (a) if \(\phi_s \in [0, M_1]\), then \(U\left[\pi_{r_1}^{\phi_s}(e_1^{\phi_s}, q_1^{\phi_s})\right] < U\left[\pi_{r_2}^{\phi_s}(e_2^{\phi_s}, q_2^{\phi_s})\right];\) otherwise \(U\left[\pi_{r_1}^{\phi_s}(e_1^{\phi_s}, q_1^{\phi_s})\right] \geq U\left[\pi_{r_2}^{\phi_s}(e_2^{\phi_s}, q_2^{\phi_s})\right];\) (b) if \(\phi_s \in [0, \max (0, M_2)]\) or \(\phi_s \in \left(\min (M_3, 1), 1\right]\), then \(\text{CVaR}_{\phi_s}\left[\pi_{r_2}^{\phi_s}(e_2^{\phi_s}, q_2^{\phi_s})\right] < \text{CVaR}_{\phi_s}\left[\pi_{r_1}^{\phi_s}(e_1^{\phi_s}, q_1^{\phi_s})\right];\) otherwise \(\text{CVaR}_{\phi_s}\left[\pi_{r_2}^{\phi_s}(e_2^{\phi_s}, q_2^{\phi_s})\right] \geq \text{CVaR}_{\phi_s}\left[\pi_{r_1}^{\phi_s}(e_1^{\phi_s}, q_1^{\phi_s})\right].\)

Here, \(M_1 = 1 - \frac{(p - w)^2(1 - \phi_s)^2\eta_s}{2\eta_s(1 - \phi_s)(p - w)(w - c)(w - c)\eta_s\phi_s}\), \(M_2 = 1 - \frac{Z_1 + Z_2 + \sqrt{(Z_1 + Z_2 - 2Z_3)^2 - 4Z_2(Z_1 - Z_3)}}{2Z_2},\)

\(Z_3 = 1 - \frac{Z_3 + Z_1 - \sqrt{(Z_2 + Z_3 - 2Z_3)^2 - 4Z_2(Z_1 - Z_3)}}{2Z_3},\)

\(Z_1 = (w - c)^2\eta_s\eta_s^{\phi_s},\) \(Z_2 = 2(p - w)(w - c)\)

\((1 - \phi_s)\eta_s\eta_s^{\phi_s},\)

\(Z_3 = (1 - \phi_s)\eta_s\eta_s^{\phi_s}(p - w)^2.\)

Proposition 6 shows that after introducing the cost-sharing contracts, (i) if the SC introduces a relatively low sales-effort ratio, i.e., \(\phi_s \in [0, M_1]\) under VMI-\(\phi_s\) contract, then the online retail platform can obtain more expected utilities under the mode when the supplier conducts sales effort than that when the online retail platform conducts sales effort; otherwise, the online retail platform can obtain more expected utilities under the mode when the online retail platform conducts sales effort than that when the supplier conducts sales effort; (ii) if the SC introduces a relatively low or relatively high sales-effort ratio, i.e., \(\phi_s \in [0, \max (0, M_2)]\) or \(\phi_s \in \left(\min (M_3, 1), 1\right]\) under VMI-\(\phi_s\) contract, then the supplier can obtain more expected utilities when the supplier conducts sales effort than that when the online retail platform conducts sales effort; otherwise, the supplier can obtain more expected utilities when the online retail platform conducts sales effort than that when the supplier conducts sales effort. Proposition 6 provides a reference for selecting an appropriate VMI SC member to conduct sales efforts under the different cost-sharing contracts with different sales-effort ratios.

### 4.3.2 Comparison analysis for contract effectiveness

To date, we propose two cost-sharing contracts, i.e., VMI-\(\phi_s\) contract and VMI-\(\phi_r\) contract. It is important to find the conditions in which VMI SC members can benefit from VMI-\(\phi_s\) contract or the conditions in which VMI SC members can benefit from VMI-\(\phi_r\) contract.

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For the VMI-\(\phi_s\) contract, the supplier’s expected utility variation can be characterized by the difference between \(\text{CVaR}_{\tau_2} \left[ \pi_{S_3}^{\phi_s} (e^{\phi_s}_r, q^{\phi_s}_i) \right] \) and \(\text{CVaR}_{\tau_2} \left[ \pi_{S_1} (e^*_r, q^*_i) \right] \). We have \(\Delta \text{CVaR}_{\tau_2} (\pi_{S_3}^{\phi_s}) = \text{CVaR}_{\tau_2} \left[ \pi_{S_3}^{\phi_s} (e^{\phi_s}_r, q^{\phi_s}_i) \right] - \text{CVaR}_{\tau_2} \left[ \pi_{S_1} (e^*_r, q^*_i) \right] \). If \(\Delta \text{CVaR}_{\tau_2} (\pi_{S_3}^{\phi_s}) > 0\), then the supplier can benefit from the VMI-\(\phi_s\) contract. The online retail platform’s expected utility variation can be characterized by the difference between \(U[\pi_{S_3}^{\phi_s} (e^{\phi_s}_r, q^{\phi_s}_i)]\) and \(U[\pi_{S_1} (e^*_r, q^*_i)]\). Let \(\Delta U(\pi_{S_3}^{\phi_s}) = U[\pi_{S_3}^{\phi_s} (e^{\phi_s}_r, q^{\phi_s}_i)] - U[\pi_{S_1} (e^*_r, q^*_i)] = \frac{(p-w)^2 \phi_s^2}{2\eta(1-\phi_s)}\). If \(\Delta U(\pi_{S_3}^{\phi_s}) > 0\), the online retail platform can benefit from the VMI-\(\phi_s\) contract.

**Proposition 7** When the online retail platform conducts sales effort under VMI SC, if \(w \in \left(\frac{p+2c}{3}, p\right)\), there always exists a Pareto improvement region \(\phi_s \in \left[0, \frac{3w-p-2c}{2(w-c)}\right)\), after introducing the VMI-\(\phi_s\) contract.

Under VMI SC when the online retail platform conducts sales effort, \(\Delta U[\pi_{S_3}^{\phi_s}] > 0\) always holds and increases with \(\phi_s\) for the online retail platform. From Proposition 7, if \(w \in \left(\frac{p+2c}{3}, p\right)\), it is possible for the SC to achieve a Pareto improvement by introducing a VMI-\(\phi_s\) contract, in which a relatively low cost-sharing ratio is necessary. In this VMI-\(\phi_s\) contract, a relatively high wholesale price, and a relatively low cost-sharing ratio jointly drive the SC to a win–win result for two VMI SC members.

Proposition 7 shows that a Pareto improvement can be achieved by introducing a VMI-\(\phi_s\) contract when the online retail platform conducts sales effort. Under VMI SC, it is also very important to help the supplier judge the following two situations: (i) Conducting sales effort by himself; (ii) inducing the online retail platform to conduct sales effort by introducing a cost-sharing contract. Let

\[
\Delta \text{CVaR}_{\tau_2} (\pi_{S_1}^{\phi_s}) = \text{CVaR}_{\tau_2} \left[ \pi_{S_1}^{\phi_s} (e^{\phi_s}_r, q^{\phi_s}_i) \right] - \text{CVaR}_{\tau_2} \left[ \pi_{S_2} (e^*_r, q^*_i) \right] = \frac{\beta^2 \left[ 2(p-w)(w-c)(1-\phi_s)\eta_s - (w-c)^2(1-\phi_s)^2\eta_s - (p-w)^2\phi_s^2\eta_s \right]}{2\eta_s(1-\phi_s)^2}.
\]

(8)

If \(\Delta \text{CVaR}_{\tau_2} (\pi_{S_1}^{\phi_s}) > 0\), the supplier obtains more expected utilities by inducing the online retail platform to conduct sales effort with the support of the VMI-\(\phi_s\) contract. Based on the above comparison, Corollary 5 can be obtained.

**Corollary 5** The risk-averse supplier prefers to induce the online retail platform to conduct sales effort based on a VMI-\(\phi_s\) contract rather than conducting sales effort by himself, if \(\phi_s \in (\max[0, H_1), H_2)\). In particular, if \(\eta_s = \eta = \eta, \phi_s \in (\max[0, H_1), H_2)\) equals \(\phi_s \in (H_1, H_2)\) in region \(w \in (w^*_1, p)\), and \(\phi_s \in (\max[0, H_1), H_2)\) equals \(\phi_s \in (0, H_2)\) in region \(w \in \left(\frac{p+2c}{3}, w^*_1\right)\). Here \(H_1 = \frac{N_1 - (p-w)\sqrt{\eta N_1}}{2\eta_s(w-c)^2}, H_2 = \frac{N_1 + (p-w)\sqrt{\eta N_2}}{2\eta_s(w-c)^2}\). \(N_1 = 2\eta_s(w-c)^2 + \eta_s(w^2 - p^2 + 2cp - 2cw),\)
\( N_2 = \eta_c(p + w)^2 - 4\eta_r(w - c)^2 + 4\eta_r c(c - p - w) \), and \( w_1^* \) is given by 
\[ 2cp - 6cw^* + 2c^2 - p^2 + 3w^* - \sqrt{(p + 3w^* - 4c)(p - w^*)}^3 = 0. \]

Corollary 5 shows that whether the supplier is willing to share the cost of the online retail platform’s sales effort is closely related to the value of \( \phi_s \). If \( \phi_s \not\in (\max\{0, H_1\}, H_2) \), the supplier would rather conduct sales effort by himself than share in the online retail platform’s sales-effort cost. Similar to Proposition 7, we next explore the possible Pareto improvement region after introducing the VMI-\( \phi_r \) contract. Then, we can obtain the following proposition:

**Proposition 8** When the supplier conducts sales effort under VMI SC, if \( w \in \left( c, \frac{2p+w}{3} \right) \), there always exists a Pareto improvement region, \( \phi_r \in \left[ 0, \frac{2p-3w+c}{2(p-w)} \right) \), after introducing the VMI-\( \phi_r \) contract.

Proposition 8 shows the necessity of introducing a VMI-\( \phi_r \) contract into VMI SC when the supplier conducts sales effort. If \( w \in \left[ c, \frac{2p+w}{3} \right) \), VMI SC can achieve a Pareto improvement by introducing a VMI-\( \phi_r \) contract, in which a relatively high wholesale price and a relatively low cost-sharing ratio jointly drive VMI SC to a win–win result for two VMI SC members.

Proposition 8 shows that a Pareto improvement can be achieved by introducing a VMI-\( \phi_r \) contract when the supplier conducts sales effort. Similar to Corollary 5, it is also very significant to help the online retail platform judge the following two situations under VMI SC: (i) Conducting sales effort by herself; (ii) inducing the supplier to conduct sales effort by introducing the VMI-\( \phi_r \) contract. Let

\[
\Delta U(\pi_{r1}^{w}) = U[\pi_{r2}^{\phi_s}(e_s^{\phi_s^*}, q_2^{\phi_s^*})] - U[\pi_{r1}(e_r^*, q_r^*)] = \beta^2[2\eta_r(w - c)(p - w)(1 - \phi_r) - (p - w)^2(1 - \phi_r)^2\eta_s - \phi_r\eta_r(w - c)^2] \\
2\eta_r\eta_s(1 - \phi_r)^2
\]

(9)

If \( \Delta U(\pi_{r1}^{w}) > 0 \), the online retail platform obtains more expected utilities by inducing the supplier to conduct sales effort with the support of the VMI-\( \phi_r \) contract. Then, we can gain the following corollary.

**Corollary 6** The online retail platform prefers to induce the risk-averse supplier to conduct sales effort based on a VMI-\( \phi_r \) contract rather than conduct sales effort by herself if \( \phi_r \in (\max\{0, H_3\}, H_4) \). In particular, \( \eta_s = \eta_r = \eta \), \( \phi_r \in (\max\{0, H_3\}, H_4) \) equals \( \phi_r \in (H_3, H_4) \) in region \( w \in (c, w_s^*) \), and \( \phi_r \in (\max\{0, H_3\}, H_4) \) equals \( \phi_r \in [0, H_4) \) in region \( w \in \left[ w_2^*, \frac{2p+w}{3} \right) \). Here \( H_3 = \frac{N_s-(w-c)\sqrt{\eta_sN_s}}{2\eta_s(p-w)^2} \), \( H_4 = \frac{N_s+(w-c)\sqrt{\eta_sN_s}}{2\eta_s(p-w)^2} \), \( N_s = 2\eta_p(p-w)^2 + \eta_c(w^2 - c^2 + 2cp - 2pw) \), \( N_s = \eta_s(c + w)^2 - 4\eta_c(p-w)^2 - 4\eta_p(p - c - w) \), and \( w_2^* \) satisfies \( 2cp - 6pw - c^2 + 2p^2 + 3w^2 - \sqrt{(4p - 3w - c)(w - c)}^3 = 0. \)
Corollary 6 reveals that whether the online retail platform is willing to share the supplier’s sales-effort cost is closely related to the value of $\phi_r$. If $\phi_r \notin (\max\{0, H_3, H_4\})$, the online retail platform would rather conduct sales effort by herself than share in the supplier’s sales-effort cost.

### 5 Comparison of VMI SC and RMI SC

RMI is a traditional inventory management mode in which the online retail platform is in charge of inventory management. Because of its simplicity, RMI SC is popular in practice. Next, we construct RMI SC models and then compare VMI SC and RMI SC. Similar to Sect. 4, we assume that the online retail platform is risk averse and the supplier is risk neutral under RMI SC.

#### 5.1 RMI SC

##### 5.1.1 The online retail platform conducts sales effort under RMI SC

In this section, the online retail platform conducts sales effort with the cost function $g(e) = \eta(e^2/2)$, and her profit function can be expressed as:

$$
\pi_{r3}(e, q) = p \min(D, q) - qw - r(D - q)^+ - g(e_r)
$$

(10)

According to the definition of CVaR, the online retail platform’s expected utility function can be expressed as:

$$
CVaR_{\tau_r}[\pi_{r3}(e, q)] = \max_{q \geq 0, v \in R} \left\{ v - \frac{1}{\tau_r} E[v - \pi_{r3}(e, q)]^+ \right\}, \tau_r \in (0, 1]
$$

(11)

Similar to the proof of Proposition 2, we can obtain that the online retail platform’s optimal sales-effort level and supply quantity can be expressed as $e^*_r = \frac{(p - w)\beta}{\eta}$, $q^*_3 = F^{-1}\left[\frac{(p - w + r)\tau_r}{p + r}\right] + d(e^*_r)$.

Given the online retail platform’s optimal supply quantity and sales-effort level, the supplier’s expected utility function can be expressed as:

$$
U[\pi_{r3}(e^*_r, q^*_3)] = (w - c)q^*_3 = (w - c)\left[F^{-1}\left[\frac{(p - w + r)\tau_r}{p + r}\right] + d(e^*_r)\right]
$$

(12)

To improve RMI SC members’ expected utilities, we assume the supplier proposes a cost-sharing ratio $\phi_3 (0 \leq \phi_3 \leq 1)$ to share with a part of the sales-effort cost. Given $\phi_3$, the online retail platform determines her optimal sales-effort level and supply quantity. We call this an “RMI-\phi_3 contract”. We also assume that $\phi_3$ is exogenous and may be determined by negotiation between the supplier and the online retail platform. Then, the process of events can be expressed as follows: (i) Before the selling season, the RMI SC members negotiate the value of $\phi_3$; (ii) The online retail platform decides the level of sales-effort and supply quantity using the $\tau$-CVaR
criterion. Similarly, let $\pi_{s3}(e, q)$ and $\pi_{r3}(e, q)$ express the supplier’s and the online retail platform’s profit functions under the RMI-$\phi_s$ contract, respectively. By using CVaR measurement from Eq. (1), we can find that the online retail platform’s optimal supply quantity and sales-effort level are $e_{r3}^* = \frac{(p-w)\beta}{(1-\phi_s)\eta_r}$ and $q_{3r}^* = F^{-1}\left[\frac{(p-w+r)\tau_e}{p+r}\right] + d(e_{r3}^*)$, respectively.

5.1.2 The supplier conducts sales effort under RMI SC

In this section, the supplier acts as a leader by determining the sales-effort level, and then the online retail platform decides the supply quantity as a follower. Utilizing backward induction, the profit function of the online retail platform can be expressed as:

$$\pi_{r4}(e, q) = p \min(D, q) - wq - r(D - q)^+$$  \hspace{1cm} (13)

According to the definition of CVaR, the expected utility function of the online retail platform is defined as:

$$CVaR_{\tau_r}[\pi_{r4}(e, q)] = \max_{q \geq 0, v \in \mathbb{R}} \left\{v - \frac{1}{\tau_r}E[v - \pi_{r4}(e, q)]^+\right\}, \tau_r \in (0, 1]$$  \hspace{1cm} (14)

The online retail platform intends to determine the optimal supply that can maximize the utility function. Anticipating online retail platform’s optimal supply quantity, the supplier’s expected utility function can be expressed as:

$$U[\pi_{s4}(e, q)] = (w - c)q - g(e)$$ \hspace{1cm} (15)

Under RMI SC, analogous to the proof of Proposition 1, if the supplier decides the sales-effort level, the optimal decisions of the supplier and the online retail platform can be expressed as $e_s^* = \frac{(w-c)\beta}{\eta_s}$, $q_{4r}^* = F^{-1}\left[\frac{(p-w+r)\tau_e}{p+r}\right] + d(e_{s}^*)$.

Similar to Sect. 4.1, we next introduce the RMI-$\phi_r$ contract, in which the online retail platform proposes a cost-sharing ratio $\phi_r$ ($0 < \phi_r < 1$) to share a proportion of the supplier’s sales-effort cost. Given $\phi_r$, the supplier decides his optimal sales-effort level first, and then the online retail platform decides her optimal supply quantity. Let $\pi_{s4}(e, q)$ and $\pi_{r4}(e, q)$ denote the supplier’s and the online retail platform’s profit functions under the RMI-$\phi_r$ contract, respectively. Similarly, by using backward induction, we can obtain that the supplier’s optimal sales-effort level is $e_{s}^* = \frac{(w-c)\beta}{(1-\phi_r)\eta_s}$. Then, the online retail platform’s optimal supply quantity is $q_{4r}^* = F^{-1}\left[\frac{(p-w+r)\tau_e}{p+r}\right] + d(e_{s}^*)$.

5.2 Compare VMI SC & RMI SC

In this section, we study the impact of different inventory management models on the optimal supply quantities and expected utilities of the SC members.
In the next proposition, we first compare the supply quantities under VMI SC and RMI SC when the online retail platform conducts sales effort; then, we further compare the supply quantities under VMI SC and RMI SC when the supplier conducts sales effort.

**Proposition 9** From the perspectives of the supply-quantity decisions under VMI SC and RMI SC, the following relationships exist: (a) If \( \tau_r > \frac{(p+r)(w-c)}{w(p+r-w)} \), then \( q^*_3 > q^*_1 \), \( q^*_3 > q^*_1 \); \( q^*_2 > q^*_4 \); \( q^*_2 > q^*_4 \); (b) otherwise, \( q^*_3 \leq q^*_1 \), \( q^*_3 \leq q^*_1 \); \( q^*_2 \leq q^*_4 \); \( q^*_2 \leq q^*_4 \).

Recall that Corollaries 1(d) and 3(d) show that the optimal supply quantity of the supplier decreases with the degree of risk aversion (the lower \( \tau_r \)) under VMI SC. Similarly, with the decrease in the online retail platform’s risk aversion (the higher \( \tau_r \)), the online retail platform’s optimal supply quantity is increasing under RMI SC.

Proposition 9 compares the influence of the degree of risk aversion of the online retail platform and the supplier on the supply quantity decisions of VMI SC and RMI SC when the online retail platform or the supplier conducts sales effort. According to Proposition 9, we find (i) for VMI SC and RMI SC when the online retail platform conducts sales effort; if the online retail platform’s degree of risk aversion is relatively low under RMI SC and the supplier’s degree of risk aversion is relatively high under VMI SC, i.e., \( \tau_r > \frac{(p+r)(w-c)}{w(p+r-w)} \), the optimal supply quantity under RMI SC is higher than that under VMI SC. After introducing a cost-sharing contract, the above conclusion still holds; (ii) for VMI SC and RMI SC in which the supplier conducts sales effort, if the supplier’s degree of risk aversion is relatively low under VMI SC and the online retail platform’s risk-aversion degree is relatively high under RMI SC, i.e., \( \tau_s \geq \frac{w(p+r-w)}{(p+r)(w-c)} \tau_r \), the optimal supply quantity under VMI SC is higher than that under RMI SC. After introducing cost-sharing contract, the above conclusion still holds.

In the above analysis, we discover that the degree of risk aversion has an important impact on supply quantity decisions. The next proposition further shows how the degree of risk aversion affects the SC members’ expected utilities under different inventory management modes. Such results may provide a scheme for appropriate inventory management mode selection under different degrees of risk aversion.

**Proposition 10** From the perspectives of expected utilities under VMI SC and RMI SC, the following relationships exist: (a) If \( 0 < \tau_r < \frac{(p+r) F(\xi)}{w(p+r-w)} \), then \( \text{CVaR}_{\tau_r}[\pi_{r3}(e^*_r, q^*_3)] < U[\pi_{r1}(e^*_r, q^*_1)] \) and \( \text{CVaR}_{\tau_r}[\pi_{s4}(e^*_s, q^*_4)] < U[\pi_{s2}(e^*_s, q^*_2)] \); else if \( \frac{(p+r) F(\xi)}{w(p+r-w)} \leq \tau_r \leq 1 \), then \( \text{CVaR}_{\tau_r}[\pi_{r3}(e^*_r, q^*_3)] \geq U[\pi_{r1}(e^*_r, q^*_1)] \) and \( \text{CVaR}_{\tau_r}[\pi_{s4}(e^*_s, q^*_4)] \geq U[\pi_{s2}(e^*_s, q^*_2)] \); (b) if \( 0 < \tau_r < \frac{w}{p+r-w} F[M_5 - \frac{w}{\tau_r(w-c)} F(\xi)] \), then \( U[\pi_{s3}(e^*_r, q^*_3)] \leq \text{CVaR}_{\tau_r}[\pi_{s1}(e^*_r, q^*_1)] \) and \( U[\pi_{s4}(e^*_s, q^*_4)] \leq \text{CVaR}_{\tau_r}[\pi_{s2}(e^*_s, q^*_2)] \).
else, if \( \frac{p_{EE}}{p+rw} F \left[ M_{5} - \frac{w}{r(w-c)} \int_{0}^{M_{*}} F(\xi) d\xi \right] \leq \tau_r \leq 1 \), then \( U \left[ \pi_{s3} \left( e_{r}^{*}, q_{3}^{*} \right) \right] \geq CVaR_{\tau_r} \left[ \pi_{s1} \left( e_{r}^{*}, q_{1}^{*} \right) \right] \)
and
\( U \left[ \pi_{s4} \left( e_{r}^{*}, q_{4}^{*} \right) \right] \geq CVaR_{\tau_r} \left[ \pi_{s2} \left( e_{r}^{*}, q_{2}^{*} \right) \right] \). Here \( M_{4} = F^{-1} \left[ \frac{r_{\left( p+r-w \right)}}{p+r} \right] \) and \( M_{5} = F^{-1} \left[ \frac{r_{\left( w-c \right)}}{w} \right] \).

According to Proposition 10, by comparing the SC members’ expected utilities under VMI SC and RMI SC, we find that (i) for VMI SC and RMI SC, when the online retail platform or the supplier conducts sales effort, if the online retail platform’s risk-aversion degree is relatively high, i.e., \( 0 < \tau_{r} < \frac{\left( p+r \right) \int_{0}^{M_{*}} F(\xi) d\xi}{\left( p+r-w \right) \left( M_{1} - M_{s} + \int_{0}^{M_{*}} F(\xi) d\xi \right)} \),
then the online retail platform’s optimal expected utility under RMI SC is always lower than that under VMI SC; if the online retail platform’s degree of risk aversion is relatively low, i.e., \( \frac{\left( p+r \right) \int_{0}^{M_{*}} F(\xi) d\xi}{\left( p+r-w \right) \left( M_{1} - M_{s} + \int_{0}^{M_{*}} F(\xi) d\xi \right)} \leq \tau_{r} \leq 1 \), then the online retail platform’s optimal expected utility under RMI SC is always higher than that under VMI SC; (ii) for VMI SC and RMI SC, when the online retail platform or the supplier conducts sales effort, if the online retail platform’s risk-aversion degree is relatively high, i.e., \( 0 < \tau_{r} < \frac{\left( p+r \right) F \left[ M_{5} - \frac{w}{r(w-c)} \int_{0}^{M_{*}} F(\xi) d\xi \right]}{\left( p+r-w \right) \left( M_{1} - M_{s} + \int_{0}^{M_{*}} F(\xi) d\xi \right)} \), then the supplier’s optimal expected utility under RMI SC is always lower than that under VMI SC; if the online retail platform’s risk-aversion degree is relatively low, i.e., \( \frac{\left( p+r \right) F \left[ M_{5} - \frac{w}{r(w-c)} \int_{0}^{M_{*}} F(\xi) d\xi \right]}{\left( p+r-w \right) \left( M_{1} - M_{s} + \int_{0}^{M_{*}} F(\xi) d\xi \right)} \leq \tau_{r} \leq 1 \), then the supplier’s optimal expected utility under RMI SC is always higher than that under VMI SC. It is easy to find that, after introducing cost-sharing contracts, the above conclusions still hold. The above findings provide suggestions for the selection of appropriate sales-effort conductors under different degrees of risk aversion and inventory management modes.

### 6 Numerical examples

To further clarify the analysis result, we assume that the determinate component of demand satisfies \( d(e) = 200 + 50e \). We further assume that the stochastic part of demand obeys an exponential distribution with parameter \( \lambda = \frac{1}{1000} \). Then, the probability density function and cumulative distribution function of the stochastic part \( \xi \) are:

\[
\begin{align*}
    f(\xi) &= \begin{cases} 
    \frac{1}{1000} e^{-\frac{\xi}{1000}} & \xi \geq 0 \\
    0 & \xi < 0
    \end{cases}, \\
    F(\xi) &= \begin{cases} 
    1 - e^{-\frac{\xi}{1000}} & \xi \geq 0 \\
    0 & \xi < 0
    \end{cases}
\end{align*}
\]

We have \( f(0) = 0, F(0) = 0 \). The following specific set of parameters is given for the numerical analysis: \( \rho_1 = \{ c = 5, p = 25, r = 2, \tau_s = 0.7, \tau_r = 0.8, \eta_s = 110, \eta_r = 100 \} \). Under VMI SC without a cost-sharing contract, the comparison of sales-effort levels and supply quantities under two sales-effort modes can be seen in Fig. 2(a). If \( 5 < w < 14.52 \), then \( e_{r}^{*} > e_{s}^{*} \) and \( q_{1}^{*} > q_{2}^{*} \); if \( 14.52 < w < 30 \), then \( e_{r}^{*} < e_{s}^{*} \) and \( q_{1}^{*} < q_{2}^{*} \). Next, we assume that the wholesale price is given, i.e.,
$w = 15$ in $\rho_1$. After introducing the cost-sharing contracts, we focus on discussing the influence of $\phi_s$ on the optimal decisions given $\phi_r = 0.3$. With the increase in $\phi_s$, the comparison under two sales-effort modes can be seen in Fig. 2(b). If $0 < \phi_s < 0.36$, then $e_{\phi_s}^{*} < e_{\phi_r}^{*}$ and $q_{1}^{\phi_s} < q_{2}^{\phi_r}$; if $0.36 \leq \phi_s < 1$, then $e_{\phi_s}^{*} > e_{\phi_r}^{*}$ and $q_{1}^{\phi_s} > q_{2}^{\phi_r}$. The results in Propositions 3 and 5 are illustrated below.

Under VMI SC without cost-sharing contracts, the comparison of VMI SC members’ optimal expected utilities under the two sales-effort models can be seen in Fig. 3. If $5 \leq w < 11.25$, both the supplier and the online retail platform can obtain more expected utilities under the mode when the supplier takes on all the cost of sales-effort; else if $17.90 \leq w \leq 25$, both the supplier and the online retail platform can obtain more expected utilities under the mode when the online retail platform takes on all the cost of sales-effort; then if $11.25 \leq w < 17.90$, the online retail platform’s expected utility is higher when the supplier conducts sales effort than when the online retail platform conducts sales effort by herself, and the supplier’s expected utility is higher when the online retail platform conducts sales effort than the supplier conducts sales effort by himself. All the conclusions in Proposition 4 are illustrated below.

After introducing the cost-sharing contracts under VMI SC, we focus on discussing the influence of $\phi_s$ on the SC members’ optimal decisions given $w = 15$ in $\rho_1$. As the results illustrated in Proposition 6, the numerical comparison can be seen in Fig. 4. Setting $\phi_r = 0.15$, we can obtain $M_1 = 0.57$, $M_2 = -0.91$, $M_3 = 0.60$. Consistent with the conclusion of Proposition 6, Fig. 4(a) shows that, for the online retail platform, if $\phi_s \in [0, 0.57)$, then $U[M_1(e_{\phi_s}, q_{1}^{\phi_s})] < U[M_2(e_{\phi_s}, q_{2}^{\phi_s})]$;
if \( \phi_s \in [0.57, 1) \), then \( U[\pi_{s_1}(e_1^*, q_{1_1}^*)] \geq U[\pi_{s_2}(e_2^*, q_{2_2}^*)] \). If \( \phi_s \in [0, 0.60) \), then \( CVaR_{r_1}[\pi_{s_1}(e_1^*, q_{1_1}^*)] > CVaR_{r_2}[\pi_{s_2}(e_2^*, q_{2_2}^*)] \); if \( \phi_s \in [0.60, 1] \), then \( CVaR_{r_1}[\pi_{s_1}(e_1^*, q_{1_1}^*)] \leq CVaR_{r_2}[\pi_{s_2}(e_2^*, q_{2_2}^*)] \). In sum, given \( \phi_r = 0.15 \), there are three sets of utility ranges for SC members: If \( 0 \leq \phi_s < 0.57 \), the online retail platform’s expected utility when the supplier conducts sales effort is higher than that when she conducts sales effort by herself, but the supplier’s expected utility when he conducts sales effort by himself is lower than that when the online retail platform conducts sales effort; if \( 0.57 \leq \phi_s < 0.60 \), both the supplier and the online retail platform can obtain more expected utilities when the online retail platform conducts sales effort; if \( 0.60 \leq \phi_s \leq 1 \), the online retail platform’s expected utility when the supplier conducts sales effort is lower than that when she conducts sales effort by herself, but the supplier can obtain a higher expected utility when he conducts sales effort by himself than that when the online retail platform conducts sales effort. If we set \( \phi_r = 0.5 \), then \( M_1 = 0.55, M_2 = 0.22, M_3 = 0.42 \). Consistent with the conclusion in Proposition 6, Fig. 4(b) shows that, both for a relatively low \( \phi_s \) and

Fig. 3  Comparison of VMI SC members’ optimal expected utilities given different values of \( w \)
a relatively high $\phi_s$, the supplier can obtain a higher expected utility when he conducts sales effort than that when the online retail platform conducts sales effort.

Next, we introduce a new parameter set $\rho_2 = \{c = 5, w = 16, p = 23, r = 2, \tau_s = 0.6, \tau_r = 0.8, \eta_s = 110, \eta_s = 80\}$ for numerical analysis. Figure 5 shows the Pareto improvement regions under VMI SC when the online retail platform or the supplier conducts sales effort, as seen in the results illustrated in Propositions 7 and 8. Figure 5(a) shows that when the online retail platform conducts sales effort under $\rho_2$, both the online retail platform and the supplier can benefit from the VMI-$\phi_s$ contract with $\phi_s \in [0, 0.68)$. Similarly, Fig. 5(b) shows that when the supplier conducts sales effort under $\rho_2$, both the online retail platform and the supplier can benefit from the VMI-$\phi_r$ contract with $\phi_r \in [0, 0.21)$. Further comparing Fig. 5(a) with (b), we can find that when $\phi_s \in (0.11, 0.67)$, the supplier can obtain a higher expected utility by inducing the online retail platform to conduct sales effort with the support of a VMI-$\phi_s$ contract, as explained in the conclusion of Corollary 5.

Next, we discuss the impact of the risk-aversion degree $\tau_r$ on the optimal supply quantity decisions, thereby illustrating Proposition 9. By introducing $\rho_3 = \{c = 5, p = 30, r = 2, \eta_s = 110, \eta_s = 100, w = 16, \phi_s = 0.3, \phi_r = 0.4\}$ for the next numerical analysis, we center on discussing the impact of $\tau_r$ on the optimal decisions given $\tau_s = 0.3$. Fig. 6 shows how the optimal supply quantities change with $\tau_r$. If $\tau_r > 0.41$, the optimal supply quantity under RMI SC is higher than that under VMI SC, regardless of whether the online retail platform or the supplier conducts sales effort. After introducing the cost-sharing contracts, the above conclusion still holds. If $\tau_r \leq 0.41$, we can get the opposite conclusion.
Then, we further compare the expected utilities of the SC members under different degrees of risk aversion. Figure 7(a) reveals how the SC members’ expected utilities change with $\tau_r$ under VMI SC and RMI SC when the online retail platform conducts sales effort. By comparison, we can find that if $0 < \tau_r \leq 0.72$, then $\text{CVaR}_{\tau_r} [\pi_{r3}(e^*_r, q^*_3)] \leq U[\pi_{r1}(e^*_r, q^*_1)]$; if $0 < \tau_r \leq 0.21$, then $U[\pi_{r3}(e^*_r, q^*_3)] \leq \text{CVaR}_{\tau_r} [\pi_{r3}(e^*_r, q^*_1)]$. From the perspective of the SC, for the different degrees of risk aversion of the online retail platform, there exist three sets of utility value regions for SC members: If $0 < \tau_r \leq 0.21$, both the online retail platform and the supplier can obtain a higher expected utility under VMI SC; if

Fig. 5 Pareto improvement regions under VMI SC with cost-sharing contracts

Fig. 6 Comparison of the optimal supply quantities under VMI SC and RMI SC
0.21 < \tau_r \leq 0.72$, the online retail platform can obtain a higher expected utility under VMI SC compared with RMI SC, but the supplier’s expected utility under VMI SC is lower than under RMI SC; if $0.72 < \tau_r \leq 1$, both the online retail platform and the supplier can obtain a higher expected utility under RMI SC. Similar conclusions can be found when the supplier conducts sales effort, as shown in Fig. 7(b). The results in Proposition 10 are illustrated below.

7 Conclusion

This paper constructs a two-level SC composed of one online retail platform and one supplier by assuming the inventory manager is risk averse, and compares the optimal decisions of supply quantity and sales-effort level when the online retail platform or the supplier conducts sales effort. Then, the paper introduces cost-sharing contracts to improve the SC under different operational modes. Pareto improvement regions are obtained after introducing cost-sharing contracts. Furthermore, the paper compares the decision differences between VMI SC and RMI SC. Finally, based on the numerical analysis, the paper provides a scheme for the selection of optimal inventory modes and sales-effort modes under different wholesale prices, the cost-sharing ratios, or the degrees of risk aversion of the inventory manager. The key findings can be summarized as follows:

First, we pursue the decision differences on supply quantity and sales effort when the sales effort is controlled by different VMI SC members. We find that SC members’ preferences for the sales-effort conductor are different under different
wholesale prices. When the wholesale price is relatively low, VMI SC members obtain more expected utilities when the online retail platform conducts sales effort. When the wholesale price is relatively high, both SC members obtain more expected utilities when the supplier conducts sales effort. When the wholesale price locates in a midrange region, by comparing VMI SC members’ expected utilities under different sales-effort modes, it is easy to find that no mode can achieve a win–win result for the supplier and the online retail platform.

Second, we enrich a comparison study on VMI SCs and RMI SCs by jointly introducing sales effort and risk attitudes. We find that the degree of risk aversion of the inventory manager plays an important role in SC members’ preferences for the optimal inventory management mode. When the supplier’s degree of risk aversion is relatively low, both SC members can obtain more expected utilities under VMI SC. In contrast, when the online retail platform’s degree of risk aversion is relatively low, choosing RMI SC can benefit two SC members.

Third, we verify the values of the cost-sharing contracts in improving the performances of VMI SCs and RMI SCs. We obtain Pareto improvement schemes based on the cost-sharing contracts for VMI SC and RMI SC. After introducing the cost-sharing contracts into the different operational modes shown above, we find that appropriate cost-sharing ratio regions always exist, in which the SC members’ expected utilities are improved.

Our research points to three important managerial insights: First, under VMI SC, the SC members’ preferences for conducting sales effort are closely related to the product’s wholesale price. When the wholesale price is relatively low, SC members can obtain higher expected utilities when the online retail platform conducts sales effort; when the wholesale price is relatively high, SC members can obtain higher expected utilities when the supplier conducts sales effort. These research results can help to improve the performances of different kinds of VMI SCs. Second, the inventory manager’s risk-aversion degree has an important influence on the SC members’ preferences for inventory management modes. When the inventory manager’s risk-aversion degree is relatively low, the VMI mode is superior to the RMI mode. While when the inventory manager’s risk-aversion degree is relatively low, the RMI mode is superior to the VMI mode. Finally, we provide reasonable suggestions for determining the cost-sharing ratios under different inventory management modes and sales-effort modes.

Finally, we discuss several potential extensions arising from this research. First, our study only considers demand uncertainty. However, due to the uncertainties in technology level and production environment, yield uncertainty may also be inevitable in the production process. Future research can construct the model considering demand uncertainty and yield uncertainty simultaneously. In addition, our study only considers the risk aversion of inventory manager. However, in a two-level SC composed of a supplier and an online retail platform, all SC members may be risk averse, which can be further analyzed in future research. Finally, our research only considers the situation that the supplier and the online retail platform are completely information symmetric. In fact, the online retail platform is closer to the market and is likely to obtain more private demand information. Hence, introducing information asymmetry into the model is a very interesting work.
Appendix

Proof of Proposition 1 Proof of SC members’ optimal decisions when the online retail platform conducts sales effort under VMI SC.

Define a concave function in term of \( v \),

\[
CVaR_{\tau_s}[\pi_{s1}(e, q)] = \max_{q \geq 0, \nu \in R} \left\{ v - \frac{1}{\tau_s} E[v - \pi_{s1}(e, q)]^+ \right\}, \quad \tau_s \in (0, 1].
\]

Let \( k_s(v, q) = v - \frac{1}{\tau_s} E[v - \pi_{s1}(e, q)]^+ \). Then

\[
k_s(v, q) = v - \frac{1}{\tau_s} \int_{q-d(e)}^{q-d(e)} [v + c q - w(\xi + d(e))]^+ f(\xi) d\xi - \frac{1}{\tau_s} \int_{q-d(e)}^{+\infty} (v - w q + c q)^+ f(\xi) d\xi
\]

(1) When \( v \leq (w - c)q \), we have

\[
k_s(v, q) = v - \frac{1}{\tau_s} \int_{q-d(e)}^{q-d(e)} [v + c q - w(\xi + d(e))]^+ f(\xi) d\xi
\]

Here, \( \frac{\partial k_s(v, q)}{\partial v} = 1 - \frac{1}{\tau_s} F \left( \frac{v + c q - w d(e)}{w} \right) \), \( \frac{\partial^2 k_s(v, q)}{\partial v^2} = -\frac{1}{w \tau_s} f \left( \frac{v + c q - w d(e)}{w} \right) < 0 \), then we have \( \frac{\partial k_s(v, q)}{\partial v} \bigg|_{v=-(w-c)q} = 1 - \frac{1}{\tau_s} F \left[ q - d(e) \right] \).

(2) When \( v > (w - c)q \), we have

\[
k_s(v, q) = v - \frac{1}{\tau_s} \int_{q-d(e)}^{+\infty} [v + c q - w(\xi + d(e))]^+ f(\xi) d\xi - \frac{1}{\tau_s} \int_{q-d(e)}^{+\infty} (v - w q + c q)^+ f(\xi) d\xi
\]

\[
= \frac{v \tau_s - v - c q + w q}{\tau_s} - \frac{w}{\tau_s} \int_{q-d(e)}^{q-d(e)} F(\xi) d\xi
\]

Obviously, \( \frac{\partial k_s(v, q)}{\partial v} = 1 - \frac{1}{\tau_s} < 0 \).

Therefore, when \( 1 - \frac{1}{\tau_s} F \left[ q - d(e) \right] \geq 0 \), \( v^*_1 = (w - c)q, q \leq d(e) + F^{-1} \left( \tau_s \right) \). When \( 1 - \frac{1}{\tau_s} F \left[ q - d(e) \right] < 0 \), \( v^*_2 = w F^{-1} \left( \tau_s \right) + w d(e) - c q, q_{s1} > d(e) + F^{-1} \left( \tau_s \right) \).

(a) When \( v^*_1 = (w - c)q \), \( k_s(v, q) = (w - c)q - \frac{w}{\tau_s} \int_{0}^{q-d(e)} F(\xi) d\xi \).

Taking the first and second partial derivatives with respect to \( q \), we have

\[
\frac{\partial k_s(v, q)}{\partial q} = (w - c) - \frac{w}{\tau_s} F \left[ q - d(e) \right], \quad \frac{\partial^2 k_s(v, q)}{\partial q^2} = -\frac{w}{\tau_s} f \left[ q - d(e) \right] < 0.
\]

Thus \( k_s(v, q) \) is concave in \( q \). By solving \( \frac{\partial k_s(v, q)}{\partial q} = 0 \), we have \( q_1 = F^{-1} \left[ \frac{(w-c)\tau_s}{w} \right] + d(e) \).
(b) When \( v_1^* = w F^{-1}(r_x) + \omega d(e) - c q \), \( k_s(v, q) = w F^{-1}(r_x) + \omega d(e) - c q - \frac{w}{r_x} \int_0^{r_x} F(\xi) d\xi \), then we have \( \frac{\partial k_s(v, q)}{\partial q} = -c < 0 \). Obviously, \( k_s(v, q) \) decreases with \( q \).

Therefore, the supplier’s optimal threshold of loss is obtained from \( v_1^* = (w - c) q \) as \( k_s(v, q) = (w - c) q - \frac{w}{r_x} \int_0^{r_x} F(\xi) d\xi \) with the optimal supply quantity \( q_1^* = F^{-1} \left[ \frac{(w-c)(r_x)}{w} \right] + d(e) \) to maximize it.

Anticipating that the optimal supply quantity is \( q_1^* = F^{-1} \left[ \frac{(w-c)(r_x)}{w} \right] + d(e) \), the online retail platform’s objective function can be rewritten as

\[
U[\pi_r(e, q)] = (p - w) E \min(D, q) - r E(D - q)^+ - g(e) = (p + r - w) \left( F^{-1} \left[ \frac{(w-c)(r_x)}{w} \right] + d(e) - \int_0^{r_x} F(\xi) d\xi \right) - r[d(e) + u] - \frac{\eta_r}{2} e^2
\]

Taking the second-order derivative with respect to \( e \), we have \( \frac{\partial^2 U[\pi_r(e, q)]}{\partial e^2} = -\eta_r < 0 \). Hence, by solving \( \frac{\partial U[\pi_r(e, q)]}{\partial e} = (p - w) \beta - \eta_r e = 0 \), we can get the optimal sales-effort level \( e_r^o = \frac{(p-w)\beta}{\eta_r} \), then the supplier’s optimal supply quantity is \( q_1^* = F^{-1} \left[ \frac{(w-c)(r_x)}{w} \right] + d(e_r^o) \).

This completes the proof. \( \square \)

Proof of Corollary 1 Sensitivity analysis of the optimal supply quantity and sales-effort level under VMI SC when the online retail platform conducts sales effort is as follows.

1. \( \frac{\partial e_r^o}{\partial \phi_r} = \frac{(p-w)\beta}{(1-\phi_r)^2} > 0 \), \( \frac{\partial q_1^*}{\partial \phi_r} = \frac{(p-w)\beta^2}{\eta_r(1-\phi_r)^2} > 0 \);

2. \( \frac{\partial e_r^o}{\partial \beta} = \frac{p - w}{\eta_r} > 0 \), \( \frac{\partial q_1^*}{\partial \beta} = \frac{2\beta(p-w)}{\eta_r} > 0 \);

3. \( \frac{\partial e_r^o}{\partial \eta_r} = -\frac{(p-w)\beta}{\eta_r^2} < 0 \), \( \frac{\partial q_1^*}{\partial \eta_r} = -\frac{(p-w)\beta^2}{(1-\phi_r)\eta_r^2} < 0 \);
When \( q_i^* \) is the solution of the optimality condition, i.e.,

\[
\frac{\partial q_i^*}{\partial \tau_i} = \frac{w-c}{w_f[q_{11}^*-d(e_r^*)]} > 0,
\]

This completes the proof.

**Proof of Corollary 2**

When the online retail platform conducts sales effort under VMI SC, the sensitivity analysis of the optimal expected utility of the supplier and the online retail platform is as follows.

(1) When \( \partial CVaR_{\tau_s} \left[ \pi_{s1}(e_r^*, q_1^*) \right] / \partial \tau_s \) \[ \frac{w}{r^2_s} \int_0^{q_1^*-d(e_r^*)} F(\xi) d\xi > 0 \]

(2) When \( \partial CVaR_{\tau_s} \left[ \pi_{s1}^\phi(e_r^*, q_1^*) \right] / \partial \tau_s \) \[ \frac{w}{r^2_s} \int_0^{q_1^*-d(e_r^*)} F(\xi) d\xi > 0 \]

(3) When \( \partial U[\pi_{s1}(e_r^*, q_1^*)] / \partial \tau_s \) \[ (p + r - w)[1 - F(q_1^*-d(e_r^*))] \frac{\partial q_1^*}{\partial \tau_s} > 0 \]

(4) When \( \partial U[\pi_{s1}^\phi(e_r^*, q_1^*)] / \partial \tau_s \) \[ (p + r - w)[1 - F(q_1^*-d(e_r^*))] \frac{\partial q_1^*}{\partial \tau_s} > 0 \]

This completes the proof.

**Proof of Proposition 2**

Proof of SC members’ optimal decisions when the supplier conducts sales effort under VMI SC.

Define a concave function in term of \( v \), \( CVaR_{\tau_s} \left[ \pi_{s2}(e, q) \right] = \max_{q \geq 0, v \in \mathbb{R}} \left\{ v - \frac{1}{\tau_s} E[v - \pi_{s2}(e, q)]^+ \right\} \). Let

\[ k_s(v, e, q) = v - \frac{1}{\tau_s} E[v - \pi_{s2}(e, q)]^+ \]

(1) When \( v \leq (w - c)q - \frac{\eta_2}{2} e^2 \), we have

\[ k_s(v, e, q) = \frac{w}{\tau_s} \int_0^{w+cq-wd(e)+\eta_2e^2/2} F(\xi) d\xi \]

(2) When \( v > (w - c)q - \frac{\eta_2}{2} e^2 \), we have

\[
\frac{\partial k_s(v, e, q)}{\partial v} \bigg|_{v=(w-c)q-\frac{\eta_2}{2}e^2} = 1 - \frac{1}{\tau_s} F[q - d(e)] < 0,
\]

\[
\frac{\partial k_s(v, e, q)}{\partial q} \bigg|_{q=0} = 1 > 0, \quad \frac{\partial k_s(v, e, q)}{\partial q} \bigg|_{q=(w-c)q-\frac{\eta_2}{2}e^2} = 1 - \frac{1}{\tau_s} F[q - d(e)].
\]
\[ k_s(v, e, q) = v - \frac{1}{\tau_s} \int_0^{q-d(e)} \left[ v + cq + \frac{\eta_s}{2} e^2 - w(\xi + d(e)) \right] f(\xi) d\xi \]
\[ - \frac{1}{\tau_s} \int_{q-d(e)}^{\infty} (v - wq + cq + \frac{\eta_s}{2} e^2) f(\xi) d\xi \]
\[ = v - \frac{v + cq + \frac{\eta_s}{2} e^2 - wq}{\tau_s} - \frac{w}{\tau_s} \int_0^{q-d(e)} F(\xi) d\xi \]

Obviously, \( \frac{\partial k_s(v, e, q)}{\partial v} = 1 - \frac{1}{\tau_s} < 0. \)

Therefore, When \( 1 - \frac{1}{\tau_s} F[q - d(e)] \geq 0, \quad v_3^* = (w - c)q - \frac{\eta_s}{2} e^2, \quad q < d(e) + F^{-1}(\tau_s). \)

When \( 1 - \frac{1}{\tau_s} F[q - d(e)] < 0, \quad v_4^* = wF^{-1}(\tau_s) + wd(e) - cq - \frac{\eta_s}{2} e^2, \quad q > d(e) + F^{-1}(\tau_s). \)

(a) When \( v_3^* = (w - c)q - \frac{\eta_s}{2} e^2, \)

Taking the first and second partial derivatives with respect to \( e \), we have
\[ \frac{\partial k_s(v, e, q)}{\partial e} = (w - c) - \frac{w}{\tau_s} F[q - d(e)], \quad \frac{\partial k_s(v, e, q)}{\partial e} = -\eta_q e - \frac{w \beta}{\tau_s} F[q - d(e)]. \]
\[ \frac{\partial^2 k_s(v, e, q)}{\partial e^2} = -\frac{w}{\tau_s} f[q - d(e)] < 0, \quad \frac{\partial^2 k_s(v, e, q)}{\partial e^2} = -\eta_q - \frac{w \beta^2}{\tau_s} f[q - d(e)] < 0 \]
\[ \frac{\partial^2 k_s(v, e, q)}{\partial q \partial e} = \frac{\partial^2 k_s(v, e, q)}{\partial e \partial q} = \frac{w \beta}{\tau_s} f[q - d(e)] > 0 \]

Consider the first and second partial derivatives with respect to \( e \) and \( q \). The Hessian matrix of \( k_s(v, e, q) \) is
\[ H(e, q) = \begin{pmatrix}
\frac{\partial^2 k_s(v, e, q)}{\partial e^2} & \frac{\partial^2 k_s(v, e, q)}{\partial e \partial q} \\
\frac{\partial^2 k_s(v, e, q)}{\partial e \partial q} & \frac{\partial^2 k_s(v, e, q)}{\partial q^2}
\end{pmatrix} \]

We can find the Hessian matrix \( H(e, q) \) is a negative definite. Hence, \( k_s(v, e, q) \) is strictly concave in \( e \) and \( q \). Let \( \frac{\partial k_s(v, e, q)}{\partial q} = 0, \quad \frac{\partial k_s(v, e, q)}{\partial e} = 0 \) hold together, we can find the supplier’s optimal sales-effort level \( e_s^* \) and supply quantity \( q_2^* \) under CVaR criterion are given by \( e_s^* = \frac{(w - c) \beta}{\eta_q}, \quad q_2^* = F^{-1} \left( \frac{w - c \tau_s}{w} \right) + d(e_s^*). \)

(b) When \( v_4^* = wF^{-1}(\tau_s) + wd(e) - cq - \frac{\eta_s}{2} e^2, \)

\[ k_s(v, e, q) = wF^{-1}(\tau_s) + wd(e) - cq - \frac{\eta_s}{2} e^2 - \frac{w}{\tau_s} \int_0^{F^{-1}(\tau_s)} F(\xi) d\xi \]

Then, we can get \( \frac{\partial k_s(v, e, q)}{\partial q} = -c < 0, \quad \frac{\partial k_s(v, e, q)}{\partial e} = (w - c) \beta - \eta_q e, \quad \frac{\partial^2 k_s(v, e, q)}{\partial e^2} = -\eta_q < 0. \)
Obviously, $k_s(v, e, q)$ decreases with $q$, and we can also get $e_s^* = \frac{(w-c)p}{\eta_s}$. Therefore, the supplier’s optimal threshold of loss is obtained from \( v_3^* = (w-c)q - \frac{\eta_s}{2}e_s^2 \) as
\[
CVaR_{\tau_3} [\pi_{s2}(e, q)] = (w-c)q - \frac{\eta_s}{2}e_s^2 - \frac{w}{\tau_3} \int_{0}^{q-e_s^2} F(\xi) d\xi \]
with the optimal supply quantity $q_2^* = F^{-1}\left(\frac{(w-c)\tau_3}{w}\right) + d(e_s^*)$ to maximize it.

This completes the proof. \( \square \)

**Proof of Corollaries 3 and 4** It is straightforward and details are omitted.

**Proof of Proposition 3** Under VMI SC without the cost-sharing contracts, compare the sales-effort level and supply quantity between the situations when the online retail platform conducts sales effort and the supplier conducts sales effort, we have
\[
e_s^* - e_s^* = \frac{(p-w)\beta}{\eta_r} - \frac{(w-c)\beta}{\eta_s} = \frac{\beta}{\eta_r \eta_s} [(p-w)\eta_s - (w-c)\eta_r]
\]
\[
q_1^* - q_2^* = \beta (e_s^* - e_s^*) = \frac{\beta^2}{\eta_r \eta_s} [(p-w)\eta_s - (w-c)\eta_r]
\]

Obviously, if \( c < w < \frac{p\eta_s + \eta_c}{\eta_s + \eta_r} \), then $e_s^* > e_s^*$, $q_1^* > q_2^*$; if \( \frac{p\eta_s + \eta_c}{\eta_s + \eta_r} \leq w < p \), then $e_s^* \leq e_s^*$, $q_1^* \leq q_2^*$. This completes the proof. \( \square \)

**Proof of Proposition 4** Under VMI SC without the cost-sharing contracts, compare the expected utilities of VMI SC members between the situations when the supplier conducts sales effort and when the online retail platform conducts sales effort.

1. From the supplier’s perspective, we have
\[
CVaR[\pi_{s2}(e_s^*, q_2^*)] - CVaR[\pi_{s1}(e_r^*, q_1^*)] = (w-c)(q_2^* - q_1^*) - \frac{\eta_s}{2}e_s^2
= \frac{(w-c)\beta^2}{2\eta_r \eta_s} [\eta_r (w-c) - 2\eta_s (p-w)]
\]

If \( \frac{2\eta_s \eta_r + \eta_c}{2\eta_s + \eta_r} \leq w < p \), then $CVaR[\pi_{s2}(e_s^*, q_2^*)] \geq CVaR[\pi_{s1}(e_r^*, q_1^*)]$. Hence the supplier is willing to conduct sales effort when the wholesale price is relatively high.

If \( c \leq w < \frac{2\eta_s \eta_r + \eta_c}{2\eta_s + \eta_r} \), then $CVaR[\pi_{s2}(e_s^*, q_2^*)] < CVaR[\pi_{s1}(e_r^*, q_1^*)]$. Hence the supplier isn’t willing to conduct sales effort when the wholesale price is relatively low.

2. Then, from the online retail platform’s perspective, we have
\[
U[\pi_{r2}(e_s^*, q_2^*)] - U[\pi_{r1}(e_r^*, q_1^*)] = \frac{(p-w)\beta^2}{2\eta_r \eta_s} [2\eta_r (w-c) - (p-w)\eta_s]
\]
If $\frac{p_n+2c_n}{2n_++n_1} < w < p$, then $U[\pi_{r2}(e^*_s, q^*_2)] \geq U[\pi_{r1}(e^*_s, q^*_1)]$. Hence the online retail platform isn’t willing to conduct sales effort when the wholesale price is relatively high.

If $c < w < \frac{p_n+2c_n}{2n_++n_1}$, then $U[\pi_{r2}(e^*_s, q^*_2)] < U[\pi_{r1}(e^*_s, q^*_1)]$. Hence the online retail platform is willing to conduct sales effort when the wholesale price is relatively low.

Based on the above analysis, we have: (a) If $c < w < \frac{p_n+2c_n}{2n_++n_1}$, then $\text{CVaR}[\pi_{s2}(e^*_s, q^*_2)] < \text{CVaR}[\pi_{s1}(e^*_s, q^*_1)]$, $U[\pi_{r2}(e^*_s, q^*_2)] < U[\pi_{r1}(e^*_s, q^*_1)]$; (b) compare $\frac{p_n+2c_n}{2n_++n_1}$ and $\frac{2n_p+c_n}{2n_++n_1}$, we can always obtain $\frac{p_n+2c_n}{2n_++n_1} - \frac{2n_p+c_n}{2n_++n_1} = \frac{3(c-p)n_n}{(2n_++n_1)(2n_++n_1)} > 0$.

It means that if $\frac{p_n+2c_n}{2n_++n_1} < w < \frac{2n_p+c_n}{2n_++n_1}$, then $\text{CVaR}[\pi_{s2}(e^*_s, q^*_2)] < \text{CVaR}[\pi_{s1}(e^*_s, q^*_1)]$, $U[\pi_{r2}(e^*_s, q^*_2)] < U[\pi_{r1}(e^*_s, q^*_1)]$; (c) if $w < \frac{p_n+2c_n}{2n_++n_1}$, then $\text{CVaR}[\pi_{s2}(e^*_s, q^*_2)] \geq \text{CVaR}[\pi_{s1}(e^*_s, q^*_1)]$, $U[\pi_{r2}(e^*_s, q^*_2)] > U[\pi_{r1}(e^*_s, q^*_1)]$.

This completes the proof. □

**Proof of Proposition 5** Under VMI SC with the cost-sharing contracts, compare the sales-effort level and supply quantity between the situations when the online retail platform conducts sales effort and the supplier conducts sales effort, we have

$$e^{\phi_r} - e^{\phi_s} = \frac{(p-w)\beta}{(1-\phi_s)\eta_r} - \frac{(w-c)\beta}{(1-\phi_s)\eta_s} = \frac{\beta[(p-w)(1-\phi_s)\eta_r - (w-c)(1-\phi_s)\eta_r]}{(1-\phi_s)(1-\phi_r)\eta_r\eta_s}$$

Obviously, if $1 - \frac{(1-\phi_s)(p-w)n_1}{(w-c)n_1} \leq \phi_s \leq 1$, then $e^{\phi_r} \geq e^{\phi_s}$, $q_1^{\phi_s} \geq q_2^{\phi_r}$; if $0 \leq \phi_s < 1 - \frac{(1-\phi_s)(p-w)n_1}{(w-c)n_1}$, then $e^{\phi_r} < e^{\phi_s}$, $q_1^{\phi_s} < q_2^{\phi_r}$. Due to $\phi_s \in [0, 1]$, we should compare $1$ and $\frac{(1-\phi_s)(p-w)n_1}{(w-c)n_1}$. In summary, under VMI SC with the cost-sharing contracts, if $\max\left[0, 1 - \frac{(1-\phi_s)(p-w)n_1}{(w-c)n_1}\right] \leq \phi_s \leq 1$, then $e^{\phi_r} \geq e^{\phi_s}$, $q_1^{\phi_s} \geq q_2^{\phi_r}$; if $0 \leq \phi_s < \max\left[0, 1 - \frac{(1-\phi_s)(p-w)n_1}{(w-c)n_1}\right]$, then $e^{\phi_r} < e^{\phi_s}$, $q_1^{\phi_s} < q_2^{\phi_r}$.

This completes the proof. □

**Proof of Proposition 6** Under VMI SC with the cost-sharing contracts,

(1) With regard to the online retail platform’s optimal expected utility:

$$U[\pi_{r2}(e^{\phi_r}, q^*_2)] - U[\pi_{r1}(e^{\phi_r}, q^*_1)] = \frac{\beta^2[2n_r\eta_r(1-\phi_r)(1-\phi_r)(p-w)(w-c) - \eta_r(p-w)(1-\phi_r)^2 - \eta_r(1-\phi_r)(w-c)^2]}{2\eta_r(1-\phi_r)^2}$$

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If $U[\pi_{r_2}^{\phi_i}(e_s^{\phi_i^*}, q_{2_2}^{\phi_i^*})] > U[\pi_{r_1}^{\phi_i}(e_s^{\phi_i^*}, q_{1_1}^{\phi_i^*})]$, then $2\eta_r, (1 - \phi_s)(1 - \phi_s)(p - w)\nonumber
\\nonumber(w - c) - n_s(p - w)^2(1 - \phi_s)^2 - \eta_r, (1 - \phi_s)(w - c)^2 > 0$ should be satisfied, we can get $0 \leq \phi_s < M_1$, $M_1 = 1 - \frac{(p - w)^2(1 - \phi_s)^2\eta_r}{2\eta_r, (1 - \phi_s)(p - w)(w - c) - (w - c)^2\eta_r, (1 - \phi_s)}$. It means that if $\phi_s \in [0, M_1)$, then $U[\pi_{r_2}^{\phi_i}(e_s^{\phi_i^*}, q_{2_2}^{\phi_i^*})] > U[\pi_{r_1}^{\phi_i}(e_s^{\phi_i^*}, q_{1_1}^{\phi_i^*})]$, otherwise, $U[\pi_{r_2}^{\phi_i}(e_s^{\phi_i^*}, q_{2_2}^{\phi_i^*})] \leq U[\pi_{r_1}^{\phi_i}(e_s^{\phi_i^*}, q_{1_1}^{\phi_i^*})]$.

(2) With regard to the supplier’s optimal expected utility:

$$CVaR_{\pi_{r_2}}[\pi_{r_2}^{\phi_i}(e_s^{\phi_i^*}, q_{2_2}^{\phi_i^*})] - CVaR_{\pi_{r_1}}[\pi_{r_1}^{\phi_i}(e_s^{\phi_i^*}, q_{1_1}^{\phi_i^*})]$$

$$= \beta^2[(w - c)^2(1 - \phi_s)^2\eta_r, n_s^2 - 2(p - w)(w - c)(1 - \phi_s)(1 - \phi_s)\eta_r, n_s^2 + (1 - \phi_s)\phi_s, \eta_r, n_s^2(p - w)^2]$$

$$2\eta_r, (1 - \phi_s)^2$$

Let $T = (w - c)^2(1 - \phi_s)^2\eta_r, n_s^2 - 2(p - w)(w - c)(1 - \phi_s)(1 - \phi_s)\eta_r, n_s^2 + (1 - \phi_s)\phi_s, \eta_r, n_s^2(p - w)^2$, if $T > 0$, then $CVaR_{\pi_{r_2}}[\pi_{r_2}^{\phi_i}(e_s^{\phi_i^*}, q_{2_2}^{\phi_i^*})] > CVaR_{\pi_{r_1}}[\pi_{r_1}^{\phi_i}(e_s^{\phi_i^*}, q_{1_1}^{\phi_i^*})]$. Set $Z_1 = (w - c)^2\eta_r, n_s^2$, $Z_2 = 2(p - w)(w - c)(1 - \phi_s)\eta_r, n_s^2$, $Z_3 = (1 - \phi_s)\eta_r, n_s^2(p - w)^2$, we can get that if $T > 0$, then $\phi_s \in [0, \max (0, M_2))$ or $\phi_s \in (\min (M_3, 1], 0)$ should be satisfied.

Here, $M_2 = 1 - \frac{Z_2 + Z_s + \sqrt{(Z_2 + Z_s - Z_3)^2 - 4Z_1(Z_2 - Z_3)}}{2Z_1}$, $M_3 = 1 - \frac{Z_2 + Z_s - \sqrt{(Z_2 + Z_s - Z_3)^2 - 4Z_1(Z_2 - Z_3)}}{2Z_1}$.

It means that if $\phi_s \in [0, \max (0, M_2))$ or $\phi_s \in (\min (M_3, 1], 0)$, $CVaR_{\pi_{r_2}}[\pi_{r_2}^{\phi_i}(e_s^{\phi_i^*}, q_{2_2}^{\phi_i^*})]$ $> CVaR_{\pi_{r_1}}[\pi_{r_2}^{\phi_i}(e_s^{\phi_i^*}, q_{1_1}^{\phi_i^*})]$; otherwise, $CVaR_{\pi_{r_2}}[\pi_{r_2}^{\phi_i}(e_s^{\phi_i^*}, q_{2_2}^{\phi_i^*})] \leq CVaR_{\pi_{r_1}}[\pi_{r_2}^{\phi_i}(e_s^{\phi_i^*}, q_{1_1}^{\phi_i^*})]$.

This completes the proof.

\[\Box\]

**Proof of Proposition 7**

(1) Firstly, we compare the supplier’s optimal expected utilities with and without the cost-sharing contracts under VMI SC.

$$\Delta CVaR_{\pi_{r_1}}[\pi_{s_1}^{\phi_i}] = CVaR_{\pi_{r_1}}[\pi_{s_1}^{\phi_i}(e_s^{\phi_i^*}, q_{1}^{\phi_i^*})] - CVaR_{\pi_{r_1}}[\pi_{s_1}^{\phi_i}(e_s', q_{1}')]$$

$$= \frac{\phi_s(p - w)\beta^2}{(1 - \phi_s)^2} \left[ w - c - \frac{p - w}{2(1 - \phi_s)} \right].$$

Only when $CVaR_{\pi_{r_1}}[\pi_{s_1}^{\phi_i}(e_s^{\phi_i^*}, q_{1}^{\phi_i^*})] > CVaR_{\pi_{r_1}}[\pi_{s_1}^{\phi_i}(e_s', q_{1}')]$, the supplier is willing to introduce a cost-sharing contract, then we can get $\phi_s < \frac{3w - p - 2c}{2(w - c)}$. It means that when $\phi_s \in \left[0, \frac{3w - p - 2c}{2(w - c)}\right)$, the supplier will choose to share the online retail platform’s sales-effort cost. Due to $\phi_s \in [0, 1]$, we can set $w \in \left(\frac{p + 2c}{3}, p\right)$.

(2) Secondly, we compare the online retail platform’s optimal expected utilities with and without the cost-sharing contracts under VMI SC.
\[
\Delta U(\pi_{r1}^{\phi}) = U[\pi_{r1}^{\phi} (e_r^{\phi}, q_1^{\phi})] - U[\pi_{r1} (e_r, q_1^*)] = \frac{(p - w)^2 \beta^2}{2 \eta_r (1 - \phi_s)} \]

Because \( \phi_s \in [0, 1] \), \( U[\pi_{r1}^{\phi} (e_r^{\phi}, q_1^{\phi})] \geq U[\pi_{r1} (e_r, q_1^*)] \) always holds. Then taking the first derivative of \( \Delta U(\pi_{r1}^{\phi}) \) with respect to \( \phi_s \), we can get
\[
\frac{\partial \Delta U(\pi_{r1}^{\phi})}{\partial \phi_s} = \frac{(p - w)^2 \beta^2}{2 \eta_r (1 - \phi_s)^2} > 0.
\]

According (1) and (2), we can get: When \( w \in \left( \frac{p + 2c}{3}, p \right) \) and \( \phi_s \in \left[ 0, \frac{3w - p - 2c}{2(w - c)} \right] \), the supplier can gain extra expected utilities. In this case, \( U[\pi_{r1}^{\phi} (e_r^{\phi}, q_1^{\phi})] \geq U[\pi_{r1} (e_r, q_1^*)] \) always holds. To sum up, if the supplier provides the cost-sharing contract to share sales-effort cost, there always exists a Pareto improvement region
\[
\phi_s \in \left[ 0, \frac{3w - p - 2c}{2(w - c)} \right] \text{ when } w \in \left( \frac{p + 2c}{3}, p \right).
\]

This completes the Proof. \( \Box \)

**Proof of Corollary 5** Compare \( CVaR_{r_1} \left[ \pi_{s1}^{\phi} (e_r^{\phi}, q_1^{\phi}) \right] \) and \( CVaR_{r_1} \left[ \pi_{s2} (e_s^*, q_2^*) \right] \), we find
\[
CVaR_{r_1} \left[ \pi_{s1}^{\phi} (e_r^{\phi}, q_1^{\phi}) \right] = CVaR_{r_1} \left[ \pi_{s2} (e_s^*, q_2^*) \right] = \beta^2 \left[ 2(p - w)(w - c)(1 - \phi_s) \eta_s - (w - c)^2 (1 - \phi_s)^2 \eta_r - (p - w)^2 \phi_s^2 \eta_s \right] \\
= \frac{2 \eta_r (1 - \phi_s)^2}{2 \eta_r (1 - \phi_s)^2}.
\]

Only when \( CVaR_{r_1} \left[ \pi_{s1}^{\phi} (e_r^{\phi}, q_1^{\phi}) \right] > CVaR_{r_1} \left[ \pi_{s2} (e_s^*, q_2^*) \right] \), the supplier prefers to introduce the cost-sharing contract to share sales effort rather than conduct the full sales-efforts cost under VMI SC.

Let \( CVaR_{r_1} \left[ \pi_{s1}^{\phi} (e_r^{\phi}, q_1^{\phi}) \right] > CVaR_{r_1} \left[ \pi_{s2} (e_s^*, q_2^*) \right] \), here \( N_1 = 2 \eta_r (w - c)^2 + \eta_s (w^2 - p^2 - 2cp - 2cw) \), \( N_2 = \eta_s (p + w)^2 - 4 \eta_r (w - c)^2 + 4 \eta_s (c - p - w) \), \( H_1 = \frac{N_1 - (p - w) \sqrt{\eta_s \eta_r}}{2 \eta_r (w - c)^2} \), \( H_2 = \frac{N_2 + (p - w) \sqrt{\eta_s \eta_r}}{2 \eta_r (w - c)^2} \).

Compare the Pareto improvement interval 0 and \( H_1 \). We can find the supplier prefers to induce the online retail platform to conduct sales effort based on the cost-sharing contract rather than conduct sales effort by himself if \( \phi_s \in \left( \max [0, H_1], H_2 \right) \).

Especially, if \( \eta_s = \eta_r = \eta \), we compare \( H_1 \) and 0. Let \( T_1 = 2cp - 6cw + 2c^2 - p^2 + 3w^2 - \sqrt{(p + 3w - 4c)(p - w)^2} \), we have \( \frac{\partial H_1}{\partial w} = 6(w - c) + 6 \frac{(p - w)^2 (w - c) + \sqrt{(p + 3w - 4c)(p - w)^2}}{(p + 3w - 4c)(p - w)^2} > 0 \).

Then, we can get \( \frac{\partial H_1}{\partial w} = \frac{3(w - c)^2 (p - w) + \sqrt{(p + 3w - 4c)(p - w)} (p - c)^2 + (p + 3w - 4c)(p - w)^2}{2(w - c)} \). According to the proof of Proposition 7, there is \( w \in \left( \frac{p + 2c}{3}, p \right) \). Then, there is always \( \frac{\partial H_1}{\partial w} > 0 \) in this interval. When \( w \) approaches \( \frac{p + 2c}{3} \), \( H_1 < 0 \); when \( w \) is officially getting closer to
$p, H_1 > 0$. Therefore, there exists a unique $w = w_1^*$ such that $H_1 = 0$. When $w > w_1^*$, $H_1 > 0$. $\phi_s \in (H_1, H_2)$, when $w \leq w_1^*$, $H_1 \leq 0$. $\phi_s \in [0, H_2)$. Here, $w_1^*$ satisfies 
\[2cp - 6cw^* + 2c^2 - p^2 + 3w^2 - (p + 3w^* - 4c)(p - w^*)^3 = 0.\]

To sum up, under VMI SC, we can find that the supplier prefers to induce the online retail platform to conduct sales effort rather than conduct sales effort by himself if $\phi_s \in (\max\{0, H_1\}, H_2)$. Especially, if $\eta_s = \eta_r = \eta$, $\phi_s \in (\max\{0, H_1\}, H_2)$ equals to $\phi_s \in (H_1, H_2)$ in the region $w \in (w_1^*, p)$, and $\phi_s \in (\max\{0, H_1\}, H_2)$ equals to $\phi_s \in [0, H_2)$ in the region $w \in \left(\frac{p + 2c}{3}, w_1^*\right)$. Here $w_1^*$ is given by 
\[2cp - 6cw^* + 2c^2 - p^2 + 3w^2 - (p + 3w^* - 4c)(p - w^*)^3 = 0.\]

The same time, $H_1 = \frac{N_1 -(p - w)\sqrt{\eta_s N_2}}{2\eta_s(w - c)^2}$, $H_2 = \frac{N_1 +(p - w)\sqrt{\eta_s N_2}}{2\eta_s(w - c)^2}$, $N_1 = 2\eta_s(w^2 - p^2 + 2cp - 2cw)$, $N_2 = \eta_s(p + w)^2 - 4\eta_s(w - c)^2 + 4\eta_sc(c - p - w)$.

This completes the proof.

**Proof of Proposition 8**

(1) Firstly, we compare the online retail platform’s optimal expected utilities with and without the cost-sharing contracts when the supplier conducts sales effort under VMI SC.

\[U[\pi_r(e_s^*, q_2^*, \phi_s^*)] - U[\pi_r(e_s^*, q_2^*)] = \frac{\phi_r(w - c)\beta^2}{(1 - \phi_r)\eta_s} \left[p - w - \frac{w - c}{2(1 - \phi_r)}\right].\]

Only when $U[\pi_r(e_s^*, q_2^*, \phi_s^*)] > U[\pi_r(e_s^*, q_2^*)]$, the online retail platform is willing to introduce the cost-sharing contract, then we can get $\phi_r < \frac{2p - 3w + c}{2(p - w)}$. It means that when $\phi_r \in \left[0, \frac{2p - 3w + c}{2(p - w)}\right)$, the online retail platform will choose to share the supplier’s sales-effort cost. Due to $\phi_r \in [0, 1]$, we can get $w \in \left[c, \frac{2p + c}{3}\right]$.

(2) Secondly, compare the supplier’s optimal expected utilities with and without the cost-sharing contracts when the supplier conducts sales effort under VMI SC.

\[\Delta \text{CVaR}_{r_s}(\pi_{s2}^*) = \text{CVaR}_{r_s}[\pi_{s2}(e_s^*, q_2^*, \phi_s^*)] - \text{CVaR}_{r_s}[\pi_{s2}(e_s^*, q_2^*)] = \frac{(w - c)^2 \beta^2 \phi_r^2}{2\eta_s(1 - \phi_r)}.\]

Due to $\phi_r \in [0, 1]$, we can always get $\Delta \text{CVaR}_{r_s}(\pi_{s2}^*) \geq 0$. Then, 
\[\frac{\partial \Delta \text{CVaR}_{r_s}(\pi_{s2}^*)}{\partial \phi_r} = \frac{(w - c)^2 \beta^2 \phi_r (2 - \phi_r)}{2\eta_s(1 - \phi_r)} \geq 0.\]

To sum up, under VMI SC, if the online retail platform provides the cost-sharing contract to share sales-effort cost, there always exist a Pareto improvement region $\phi_r \in \left[0, \frac{2p - 3w + c}{2(p - w)}\right]$ when $w \in \left[c, \frac{2p + c}{3}\right]$.

This completes the proof.
Proof of Corollary 6

Compare \(U[r_2^\phi(e_s^*, q_2^\phi^*)] \) and \(U[r_1(e_r^*, q_1^s)]\), we find that

\[
\Delta U(\pi_{r1}^*) = U[r_2^\phi(e_s^*, q_2^\phi^*)] - U[r_1(e_r^*, q_1^s)] = \beta^2[2\eta_c(w-c)(p-w)(1 - \phi_r) - (p - w)^2(1 - \phi_r)\eta_c - \phi_r\eta_c(w - c)^2] / 2\eta_c\eta_s(1 - \phi_r)^2.
\]

Only when \(U[r_2^\phi(e_s^*, q_2^\phi^*)] > U[r_1(e_r^*, q_1^s)]\), the online retail platform is willing to provide the cost-sharing contracts rather than bear the full sales-efforts cost by herself. Let \(U[r_2^\phi(e_s^*, q_2^\phi^*)] > U[r_1(e_r^*, q_1^s)]\), we can get \(\phi_r \in (H_3, H_4)\). Here \(N_3 = 2\eta_c(p-w)^2 + \eta_s(w^2 - c^2 + 2cp - 2pw)\), \(N_4 = \eta_c(c + w)^2 - 4\eta_s(p-w)^2 - 4\eta_s(p(c - p + w))\), \(H_3 = \frac{N_3 - 2\eta_s(p-w)^2}{2\eta_s(p-w)^2}, H_4 = \frac{N_4 + (w-c)\sqrt{\eta_sN_4}}{2\eta_s(p-w)^2}\).

Compare Pareto improvement interval \(0\) and \(H_3\), We can find the online retail platform prefer provides the cost-sharing contract to share the supplier’s sales-effort cost rather than conducts the sale effort herself if \(\phi_r \in \left(\min\{0, H_3\}, H_4\right)\).

Especially, if \(\eta_s = \eta_r = \eta\), we compare \(H_3\) and 0. Let \(T_3 = 2pc - 6pw - c^2 + 2p^2 + 3w^2 - \sqrt{(4p - 3w - c)(w - c)^2} \), we have \(\frac{\partial T_3}{\partial w} = -6(p-w) - \frac{6(p-w)(w-c)}{\sqrt{(4p - 3w - c)(w - c)}} < 0\).

Then we can get \(\frac{\partial H_3}{\partial w} = \frac{-6(p-w)^2(4 + \frac{1}{4p-3w-c})}{2(p-w)^3} < 0\). Due to \(w \in (c, \frac{2p+c}{3})\), we can always get \(\frac{\partial H_3}{\partial w} < 0\). When \(w\) approaches \(\frac{2p+c}{3}\), \(H_3 = -c^2 - 2p^2 + \frac{4p^2 + 4pc + c^2 - 2\sqrt{3}(p-c)}{3} < 0\);

when \(w\) is officially getting closer to \(c\), \(H_3 = \frac{2p^2 - 4pc + 2c^2}{2(p-c)^2} = 1 > 0\). Therefore, there exists a unique \(w = w_2^*\) such that \(H_3 = 0\). When \(w \geq w_2^*, H_3 \leq 0, \phi_r \in [0, H_4]\); when \(w < w_2^*, H_3 > 0, \phi_r \in (H_3, H_4)\). Here \(w_2^*\) satisfies \(2pc - 6pw_2^* - c^2 + 2p^2 + 3w_2^* - \sqrt{(4p - 3w_2^* - c)(w_2^* - c)^2} = 0\).

To sum up, we can get the online retail platform prefers to share the supplier’s sales-effort cost rather than conduct sales effort by himself if \(\phi_r \in \left(\min\{0, H_3\}, H_4\right)\). Especially, if \(\eta_s = \eta_r = \eta\), we can get \(\phi_r \in (H_3, H_4)\) when \(w \in (c, w_2^*)\), or \(\phi_r \in [0, H_4]\) when \(w \in \left[w_2^*, \frac{2p+c}{3}\right]\).

This completes the proof. \(\Box\)

Proof of Proposition 9

Compare the optimal supply quantity under VMI SC and RMI SC, i.e., \(q_1^s = F^{-1}\left[\frac{(w-c)r_{\tau_s}}{w}\right] + d(e_r^*)\), \(q_2^s = F^{-1}\left[\frac{(p-w+r)c_{\tau_s}}{p+r}\right] + d(e_r^*)\).

we can get \(q_1^s - q_3^s = F^{-1}\left[\frac{(p-w+r)c_{\tau_s}}{p+r}\right] - F^{-1}\left[\frac{(w-c)c_{\tau_s}}{w}\right]\) Since \(F(x)\) is an increasing function, \(F^{-1}(x)\) is also increasing with \(x\).

If \(q_1^s > q_3^s\), then \(F^{-1}\left[\frac{(p-w+r)c_{\tau_s}}{p+r}\right] > F^{-1}\left[\frac{(w-c)c_{\tau_s}}{w}\right]\), i.e. \(\frac{(p-w+r)c_{\tau_s}}{p+r} > \frac{(w-c)c_{\tau_s}}{w}\). Obviously, we can get that when \(\tau_r > \frac{(p+r)(w-c)}{w(p+r-w)} \tau_s\), \(q_3^s > q_1^s\); otherwise, \(q_3^s \leq q_1^s\).
Similarly, given \( q_2^* = F^{-1}\left(\frac{(w-c)\tau_r}{w}\right) + d(e^*_s) \), \( q_4^* = F^{-1}\left(\frac{(p+w+r)\tau_r}{p+r}\right) + d(e^*_s) \), we can get that when \( \tau_r > \frac{(p+r)(w-c)}{w(p+r-w)} \tau_s \), \( q_2^* > q_4^* \); otherwise, \( q_2^* \leq q_4^* \).

After introducing the cost-sharing contracts, the above conclusion still holds. This completes the proof.

Proof of Proposition 10

(1) First, compare the optimal expected utilities of the online retail platform under VMI SC and RMI SC when the online retail platform conducts sales effort:

\[
CVaR_{\tau_r}\left[\pi_{s3}(e^*_r, q_3^*)\right] = U\left[\pi_{s1}(e^*_s, q_1^*)\right] = (p + r - w)(F^{-1}\left(\frac{p - w + r}{p + r}\right) - F^{-1}\left(\frac{(w - c)\tau_s}{w}\right)) - \frac{p + r}{\tau_r} \int_0^{F^{-1}\left(\frac{(p+w+r)\tau_r}{p+r}\right)} F(x)dx
\]

We can get that when \( \frac{(p+r)\int_0^{M_4} F(\xi)d\xi}{(p+r-w)\int_0^{M_4} F(\xi)d\xi} \leq \tau_r \leq 1 \),

\[
CVaR_{\tau_r}\left[\pi_{s3}(e^*_r, q_3^*)\right] \geq U\left[\pi_{s1}(e^*_s, q_1^*)\right], \text{ Here } M_4 = F^{-1}\left(\frac{p+r-w}{p+r}\right).
\]

When the supplier conducts sales effort, the comparison of the online retail platform’s expected utilities is similarly to the above analysis under the two inventory modes.

(2) Second, compare the optimal expected utilities of the supplier under VMI and RMI SC when the online retail platform conducts sales efforts:

\[
U[\pi_{s3}(e^*_r, q_3^*)] = (w - c)q_3^* , \text{ CVaR}_{\tau_s}\left[\pi_{s1}(e^*_s, q_1^*)\right] = (w - c)q_3^* - \frac{w}{\tau_s} \int_0^{(w-c)q_3^*} F(\xi)d\xi
\]

We can get that when \( \frac{(p+w+r)\int_0^{M_5} F(\xi)d\xi}{p+r-w} \leq \tau_r \leq 1 \),

\[
U[\pi_{s3}(e^*_r, q_3^*)] \geq CVaR_{\tau_s}\left[\pi_{s1}(e^*_s, q_1^*)\right]. \text{ Due to } \tau_r \in (0, 1], \text{ We obtain } \tau_r \in \left[\frac{p+r-w}{p+r-w} \int_0^{M_5} F(\xi)d\xi\right], 1]. \text{ When the supplier conducts sales effort, the comparison of the supplier’s expected utilities is similarly to the above analysis under the two inventory modes.}

This completes the proof.

Funding This material is based upon work supported by National Natural Science Foundation of China under the Grant Numbers 71932005, 71972171.
Declarations

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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