Critical dynamics and universality of the random-bond Potts ferromagnet with tri-distributed quenched disorders

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(Dated: November 20, 2018)

Critical behavior in short-time dynamics is investigated by a Monte Carlo study for the random-bond Potts ferromagnet with a trinary distribution of quenched disorders on two-dimensional triangular lattices. The universal dynamic scaling is verified and applied to estimate the critical exponents \(\theta, z\) and \(\beta/\nu\) for several realizations of the quenched disorder distribution. Our critical scaling analysis strongly indicates that the bond randomness influences the critical universality.

PACS numbers: 75.40.Mg, 64.60.Fr, 64.60.Ht
Keywords: short-time dynamics, random-bond Potts model, critical exponent
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I. INTRODUCTION

The study of the critical properties of physical systems with bond randomness on phase transitions is a quite active field of current interest in equilibrium statistical physics [1, 2]. One of the central importance here is to answer the question whether the critical exponents (of a homogeneous pure magnet) change on the addition of quenched impurities and, if so, how do they change? Replying to this question, it was first stated already two decades ago that if the specific heat critical exponent \(\alpha\) of a pure system is positive, then a quenched disorder is a relevant perturbation at the second-order critical point and it causes changes in critical exponents. This statement is known as the Harris criterion [3]. Furthermore, following the earlier work of Imry and Wyart [4], who argued that quenched disorder could smooth first-order phase transitions and thus produce the second-order phase transitions, the introduction of randomness to pure systems undergoing a first-order transition has been comprehensively considered [4, 5]. The theory was initially numerical checked with the Monte Carlo (MC) method by Chen, Ferrenberg and Landau (CFL) [6], who studied the 8-state random-bond Potts model with a bimodal self-dual distribution. On the other hand the experimental evidence has been found that, for the order-disorder phase transitions of absorbed atomic layers on two-dimensions, the critical exponents are changed from the original universality class of the 4-state Potts model on the addition of disorders [4, 5]. However, no modification is found when the pure system belongs to the Ising universality class [4]. Recently such disordered systems are extensively studied by examining how a phase transition is modified by quenched disorder coupling to the local energy density [4], where use of intensive MC simulations is often helpful [2, 5, 7, 8, 9, 10, 11].

The two-dimensional (2D) \(q\)-state random-bond Potts ferromagnet (RBPF) is an interesting framework in the MC research to study the influence of impurity on pure systems. For \(q > 2\) such randomness acts as a relevant perturbation, and for \(q > 4\) it even changes the nature of the transition from first to second order. In Table 1 we list the magnetic scaling index \(\beta/\nu\) of the 2D 8-state RBPF obtained by different groups. Here a disorder amplitude \(r\), defined by the ratio of the strong to weak coupling (distributed according to the bimodal distribution) in the range \(r = 2-20\) appears to be adapted to a numerical analysis and gives a good estimate of the disordered fixed point exponents [2, 17]. A recent work of Olson and Young [16] used a specially continuous self-dual probability distribution of the disordered bonds

\[
P_X(x) = \frac{2\sqrt{q}}{\pi(1-x)^2 + qx^2}
\]

for the Boltzmann factor \(x \equiv e^{-K}\), and performed a MC study of multiscale properties of the correlation functions for several values of \(q\). Their results are very interesting to examine the universality class of the RBPF. Cardy and Jacobsen [18] studied the RBPF based on the connectivity transfer matrix (TM) formalism [19], and their estimates of the critical exponents lead to a continuous variation of \(\beta/\nu\) with \(q\), which is in sharp disagreement with the MC results by CFL (see Table 1).

In this paper, we present a MC study by short-time dynamics (STD) to verify the dynamic scaling features of the RBPF and estimate the critical exponents for a trinary random-bond Potts model on a 2D triangular lattice. It is well known that this model on the 2D triangular lattice belongs to the same universality as the one on the 2D square lattice in equilibrium. However, a difference exists that the transition temperature is changed because the systems on triangular lattices lost the original (square-square) self-duality relation. Instead, a honeycomb-triangular duality (or star-triangular duality) is satisfied [20]. Therefore the multi-disorder amplitudes \(r_i\) can be introduced to study the dynamic universality of the RBPF on such lattices by the STD approach [21, 22]. In particular we will investigate the
However, the quenched random disorders to demonstrate the first-order phase transition in the pure cases, in hope that the dependence of critical exponents on the randomness strength in order to clarify the crossover behavior (pure $\leftrightarrow$ random fixed point $\leftrightarrow$ percolation-like limit) \[17\].

II. THE MODEL

The Hamiltonian of the $q$-state Potts model on a 2D triangular lattice with quenched random interactions can be written as

$$\mathcal{H} = -\beta H = \sum_{<i,j>} K_{ij} \delta_{\sigma_i \sigma_j} = \sum_\{L_0\} K_0 \delta_{\sigma_i \sigma_j} + \sum_\{L_1\} K_1 \delta_{\sigma_i \sigma_j} + \sum_\{L_2\} K_2 \delta_{\sigma_i \sigma_j},$$

(2)

where the spin $\sigma_i$, defined on lattice site $i$, takes the values $1, \cdots, q$, $\beta = 1/\kappa B T$ is the inverse temperature, $\delta$ the Kronecker delta function and the sum is over all the nearest-neighbor (NN) interactions (bonds) $<i,j>$ on the lattice with a size $N = L^2$. There are $3N$ NN bonds (six for each and shared by two spins) and they can be grouped into three classes $\{L_0, L_1, L_2\}$ corresponding to the interactions on them. The dimensionless couplings $K_{ij}$ can be selected from three positive (ferromagnetic) values $K_0$, $K_1 = r_1 K_0$ and $K_2 = r_2 K_0$ according to a probability distribution,

$$P(K) = (1-p_1-p_2) \delta(K-K_0) + p_1 \delta(K-K_1) + p_2 \delta(K-K_2).$$

(3)

When $p_1 = p_2 = 1/3$ it is a trinary random-bond system. By denoting $F_0 = (e^{K_0} - 1)$, $F_1 = (e^{K_1} - 1)$ and $F_2 = (e^{K_2} - 1)$, where $K_0$, $K_1 = r_1 K_0$ and $K_2 = r_2 K_0$, are the critical values of $K_0$, $K_1$ and $K_2$ respectively, the critical point can be determined by a self-dual relation \[24\] \[29\],

$$F_0 F_1 F_2 + F_0 F_1 + F_0 F_2 + F_1 F_2 = q$$

(4)

for the trinary system with the given state parameter $q$.

The strong to weak coupling ratio $r_1 = K_1/K_0$ is called disorder amplitude. The values $r_1 = 0$ or $r_2 = 0$ correspond to the diluted case \[20\], and $r_1 = r_2 = 1$ to the pure case where the phase transitions are first-order for $q > 4$. With the presence of quenched random-bonds, however, the second-order phase transitions are induced for any $q$-state of the Potts model and the critical points can be calculated according to Eq. (1). In our simulations we chose $q = 8$ which is known to have a strong first-order phase transition in the pure cases, in hope that we would find second-order phase transitions induced by the quenched random disorders to demonstrate the influence of quenched impurities on the first-order phase transitions. The strength of disorder is realized by the disorder amplitudes $\{r_1, r_2\}$ as chosen in Table 2.

![Table 1: Magnetic exponent $\beta/\nu$ estimated by different groups for the 2D 8-state RBPF.](image)

| Authors                  | $r$   | $\beta/\nu$ | Technique |
|--------------------------|-------|-------------|-----------|
| Chatelain and Berche     | 1     | 0.1505(3)   | MC        |
| Olson and Young          | 10    | 0.152(3)    | MC        |
| Pico [17]                |       | 0.153(1)    | MC        |
| Cardy and Jacobsen [18]  | 2     | 0.142(4)    | TM        |
| Chatelain and Berche     | 10    | 0.1505(3)   | TM        |
| Ying and Harada          | 10    | 0.151(3)    | STD       |

In Ref. [23], however it was dis-
covered that for an $O(N)$ vector magnetic system in the states with a very high temperature $T \gg T_c$, when it is suddenly quenched to the critical temperature $T_c$ and evolves according to a dynamics of model $A$, a universal dynamic scaling behavior emerges already within the short-time regime,

$$M^{(k)}(t, \tau, L, m_0) = b^{-k\beta/\nu} M^{(k)}(b^{-z} t, b^{1/\nu} \tau, b^{-1} L, b^\gamma m_0),$$

where $M^{(k)}$ is the $k$th moment of magnetization, $t, \tau = (T - T_c)/T_c, L$ and $b$ are time, reduced temperature, linear size of the lattice and scaling factor respectively. $\beta$ and $\nu$ are the well known static critical exponents. The quantity $x_0$, a new independent exponent, is the scaling dimension of the initial magnetization $m_0$. This dynamic scaling form is generalized from finite size scaling in equilibrium.

Up to present, the short-time dynamic MC simulations have been successfully performed to estimate the critical temperatures $T_c$ and the critical exponents $\theta, \beta, \nu$ and the dynamic exponent $z$ for the various models with second order phase transitions [24]. This approach has also been extensively applied for systems with disorders, as the FFXY and spin glass, to estimate both dynamic and static critical exponents [27, 28], and for systems on the 2D triangular lattices to study the dynamic scaling universality [29]. The method is also efficient to study the deterministic dynamics [21].

We carry out simulations in an analogous strategy described in Ref. [21] where the magnetic observables to be measured for the RBPF have been well defined. In this work, however, an additional step is introduced to verify that the second-order phase transitions really are induced when quenched randomness is added. Then we will be particularly concerned to demonstrate the evidence for the dynamic universality of the trinary RBPF. Therefore our MC simulations are mainly performed for the time evolution of the magnetization $M(t)$ from the disordered initial states with small magnetizatons $m_0 \gtrsim 0$, and the completely ordered state with $m_0 = 1$. In the former, the magnetization $M(t)$ undergoes an initial increase at the critical point $K_c$ for $t > t_{mic}$,

$$M(t) \sim m_0^\theta, \quad \theta = (x_0 - \beta/\nu)/z.$$  

For the latter, with $m_0 = 1$, $M(t)$ shows a power-law decay behavior [31]

$$M(t) \sim t^{-c_1}, \quad c_1 = \beta/\nu z,$$

at the critical point. Furthermore the Binder cumulant $U(t, L) = M^{(2)}(t, L)/M^2(t, L)$ shows, for $m_0 = 1$ at criticality, a similar power-law behavior on a large enough lattice,

$$U(t, L) \sim t^{c_u}, \quad c_u = d/z.$$  

Here the magnetization $M(t)$ is defined by

$$M(t) = \frac{1}{N} \left[ \langle qM_0(t) - N \rangle / q - 1 \right].$$  

$M_0$ is the total number of spins in the $k$th state. $N = L^2$ is the total number of spins on the lattice. $\langle \cdots \rangle$ denotes thermal averages over the initial states and/or random number sequences, and $[\cdots]$ the disorder averages over quenched randomness distributions. The time unit $t$ is defined as a MC sweep over all spins on the lattice.

The time-dependent susceptibility (the second moment of the magnetization) is defined as usual

$$M^{(2)}(t) = \frac{1}{N} \left[ \langle M^2(t) \rangle - \langle M(t) \rangle^2 \right].$$  

It has played an important role in equilibrium to determine the critical exponents $\gamma/\nu$ and $\beta/\nu$ [3].

### IV. RESULTS

In our simulations, up to 60,000 MC processes are taken for MC averages (300 samples chosen as disorder averages in a given distribution and 100-200 initial configurations as thermal averages for each disorder configuration realization). Statistical errors are simply estimated by performing three groups of averages with different random number sequences as well as independent initial configurations. To minimize the number of bond configurations needed for the disorder averaging, we confine our study to the bond distributions in which there are the same number of disordered (stronger) NN bonds for each of the three groups $L_0, L_1, L_2$. This procedure is reliable and should reduce the variation between different disorder configurations with no loss of generality. Periodic boundary conditions are applied to the $N = L^2$ lattice. We use $L$ up to 128 and adopt the heat-bath algorithm.

The data for the evolution of $M(t)$ from $m_0 \gtrsim 0$ and $m_0 = 1$ exhibit that the power-law behavior is satisfied at the same point $K_c$ for both the disordered and ordered initial states. This fact gives an evidence that a second-order phase transition is induced at $K_c$ instead of a first-order one: In Ref. [14] it was argued that there are two quasi-critical points if a first-order transition happens.
FIG. 1: $M(t)$ versus MC time plotted on a log-log scale for \( \{r_1=1, r_2=8\} \) from the initial states $m_0 \sim 0.01$, performed at and near the critical point $K_c$ and $K_c^\pm = (1 \pm 0.012)K_c$.

$K_{\text{random}}$ for evolutions from random initial states and $K_{\text{order}}$ from ordered initial states, and $K_{\text{random}} < K_c < K_{\text{order}}$ is satisfied. We plot in Fig.1 and Fig.2 the curves $M(t)$ for disorder amplitudes $\{r_1=1, r_2=8\}$ on a $64^2$ lattice. They exhibit obviously a power-law relaxation at the same critical point for both the $m_0 \sim 0$ and $m_0 = 1$ states. On the other hand the curves deviate from the power-law behavior for a small but nonzero $\tau$ described by $M(t, \tau) = m_0^{\theta} M(t^{1/\nu_z} \tau)$ or $M(t, \tau) = t^{-\alpha} F(t^{1/\nu_z} \tau)$ with respect to the disordered or ordered initial states respectively. This modification has been initially used to determine the critical temperatures \[32, 33\]. The same features, as plotted in Figs. 1 and 2, exist for all other realizations of the randomness listed in Table 2.

Then we perform simulations systematically at the exact critical points $K_c(r_i)$ as a function of $\{r_1, r_2\}$ for magnetization evolutions starting with $m_0 \gtrsim 0$ and $m_0 = 1$. The disordered initial states are prepared from random states with small magnetizations $m_0 = 0.04 - 0.01$. The curves $M(t)$ for the disordered configurations with $r_1 = 5$, $r_2 = 8$ on a $128^2$ lattice are displayed in Fig.3 with a double-logarithmic scale. They exhibit the power-law behavior of Eq.(6) for $t > t_{\text{mic}}$, which depends on $m_0$. Thus the exponent $\theta$ can be estimated with the standard least square fitting algorithm, from the slopes of the curves for $t > t_{\text{mic}}$, and by an extrapolation to the $m_0 \to 0$ limit. The values are included in Table 3.

Next we start from $m_0 = 1$ to observe the power-law de-
TABLE III: The values of indices \( (\theta, c_1, c_u) \) and results of exponents \( (z, \beta/\nu) \) with respect to the disorder amplitudes \( \{r_1, r_2\} \).

| \( \{r_1, r_2\} \) | \( \theta \) | \( c_1 \) | \( c_u \) | \( z \) | \( \beta/\nu \) |
|-------------------|--------|--------|--------|-----|---------|
| I: \( \{1, 5\} \) | 0.238(3) | 0.7780(8) | 0.789(6) | 2.53(4) | 0.202(5) |
| I: \( \{1, 8\} \) | 0.206(2) | 0.6547(7) | 0.611(6) | 3.28(5) | 0.212(5) |
| I: \( \{1, 10\} \) | 0.187(2) | 0.5977(7) | 0.599(6) | 3.34(5) | 0.200(5) |
| I: \( \{1, 1.2\} \) | 0.180(2) | 0.5597(7) | 0.554(6) | 3.61(5) | 0.202(5) |
| II: \( \{2, 5\} \) | 0.268(3) | 0.8664(9) | 0.889(6) | 2.25(4) | 0.194(5) |
| II: \( \{5, 8\} \) | 0.283(3) | 0.8499(9) | 0.893(7) | 2.24(4) | 0.190(4) |
| II: \( \{8, 10\} \) | 0.290(3) | 0.8530(9) | 0.897(7) | 2.23(4) | 0.185(4) |
| III: \( \{5, 5\} \) | 0.305(4) | 0.7848(8) | 0.759(7) | 2.09(4) | 0.164(4) |
| III: \( \{8, 8\} \) | 0.296(3) | 0.8111(9) | 0.907(8) | 2.21(4) | 0.179(4) |
| III: \( \{10, 10\} \) | 0.293(3) | 0.8159(9) | 0.898(8) | 2.23(4) | 0.182(4) |

V. SUMMARY AND CONCLUSION

We have, for the first time, introduced the trinary distribution for the 2D RBPF with multi-disorder amplitudes to study its critical behavior by the STD method. In the MC simulation the dynamic scaling behavior is verified and the power-law behavior is used to estimate both the dynamic and static exponents as a function of the disorder amplitudes \( \{r_1, r_2\} \). It is found that the values of exponents \( \theta \) and \( \beta/\nu \) vary continuously with the disorder amplitudes, and they violate the Ising-like universality. Furthermore the values of dynamic exponent \( z \) are larger than those for systems without disorders and they increase with the strength of disorder amplitudes \( \{r_1, r_2\} \) for the systems I: \( r_1 = 1, r_2 > 1 \) and III: \( r_1=r_2 > 1 \) in Table 3.

On the other hand, for the systems II where 2/3 bonds are strongly coupled with different amplitudes \( 1 < r_1 < r_2 \), they seem to have been located at a “random” regime where their values of the critical exponents \( \beta/\nu \) and \( z \) are nearly independent of disorder amplitudes within the order bars, \( z \sim 2.23 \) and \( \beta/\nu \sim 1.85 \). By comparing these results to \( z=2.23(4) \) and \( \beta/\nu=0.182(5) \) for the \( r_1=r_2=10 \) in III, we could argue that, after \( r_1 = r_2 \geq 10 \) (equivalent to \( r_1=1, r_2 \leq 0.1 \) in I), it will pass the “crossover” region to the random regime from other type, I or III, of the disordered systems, so that \( z \) and \( \beta/\nu \) will be nearly constants. In Fig.6 the \( U(t) \) curves for the dis-
order amplitudes \( \{ r_1 = 5, r_2 = 8 \} \), \( \{ r_1 = 8, r_2 = 10 \} \) and \( \{ r_1 = 10, r_2 = 10 \} \) are almost parallel each other, which shows that the values of exponent \( \beta/\nu \) are nearly same for these disordered systems.

In conclusion, our simulation verifies that second order phase transitions are induced. Then the results show evidence that the dynamic universality class of the trinary RBPF should not belong to that of the Ising model, as inferred by Jacobsen and Cardy [19]. From the work we find it rather encouraging to apply the short-time dynamic MC to simulate the scaling and critical dynamics of disordered spin systems. We will pursue this field to explore the logarithmical slow dynamics [2, 35].

Acknowledgement: We are grateful to the helpful discussions with Q. Wang. H.P.Y would like to thank the Heinrich-Hertz-Stiftung for fellowship and acknowledge the hospitality of Universität Siegen. The work supported in part by the Deutsche Forschungsgemeinschaft, DFG Schu 95/9-3 and by the NNSF of China, 19975041 and 10074055.

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