Seesaw Invariance of Fritzsch-like Neutrino Mass Matrices, Leptogenesis and Lepton Flavor Violation

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Abstract

We propose a simple ansatz for neutrino phenomenology, in which the relevant lepton mass matrices take the universal Fritzsch-like form and the seesaw relation holds under a particular condition. There exist six textures of this nature, but their consequences on neutrino oscillations are exactly the same. We show that our scenario is viable to account for the cosmological baryon number asymmetry via thermal leptogenesis. Its predictions for the lepton-flavor-violating decays $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$ are also presented. We find that the branching ratios of these rare processes depend strongly upon the phase parameters responsible for leptogenesis and for leptonic CP violation in neutrino oscillations.

PACS number(s): 14.60.Pq; 13.10.+q; 25.30.Pt
I. INTRODUCTION

The Super-Kamiokande [1], SNO [2], KamLAND [3] and K2K [4] neutrino oscillation experiments have provided us with very compelling evidence that neutrinos are massive and lepton flavors are mixed. This important result indicates that the minimal supersymmetric standard model (MSSM) is actually as incomplete as the standard model in describing the neutrino phenomenology. A very simple but natural extension of the MSSM is to include one right-handed neutrino in each of three lepton families, while the Lagrangian of electroweak interactions keeps invariant under \(SU(2)_L \times U(1)_Y\) gauge transformation. In this case, the Lagrangian responsible for lepton masses can be written as

\[
-L_{\text{lepton}} = \bar{l}_L Y_l E H_1 + \bar{l}_L Y_\nu N H_2 + \frac{1}{2} \bar{N} M_R N + \text{h.c.},
\]

where \(l_L\) denotes the left-handed lepton doublet, \(E\) and \(N\) stand respectively for the right-handed charged lepton and Majorana neutrino singlets, and \(H_1\) and \(H_2\) (with hypercharges \(\pm 1/2\)) are the MSSM Higgs doublets. After spontaneous gauge symmetry breaking, one obtains the charged lepton mass matrix \(M_l \equiv Y_l \langle H_1 \rangle\) and the Dirac neutrino mass matrix \(M_D \equiv Y_\nu \langle H_2 \rangle\). The scale of \(M_R\) may be considerably higher than \(\langle H_{1,2} \rangle\), because right-handed neutrinos are \(SU(2)_L\) singlets and their mass term is not subject to electroweak symmetry breaking. Thus the effective neutrino mass matrix \(M_\nu\) can be derived from \(M_D\) and \(M_R\) via the seesaw relation \(M_\nu = M_D M_R^{-1} M_D^T\) [5]. Although this elegant relation can qualitatively attribute the smallness of left-handed neutrino masses to the largeness of right-handed neutrino masses, it is unable to make any concrete predictions unless a specific lepton flavor structure is assumed. Hence an appropriate combination of the seesaw mechanism and possible flavor symmetries or texture zeros [6] is practically needed, in order to quantitatively account for the neutrino mass spectrum and lepton flavor mixing.

One purpose of this paper is to incorporate the seesaw mechanism with the Fritzsch-like textures of lepton mass matrices listed in Table 1. Those six patterns of \(M_l\) and \(M_\nu\) are actually isomeric [7]; i.e., they have the same phenomenological consequences, although their structures are apparently different from one another. If \(M_D\) and \(M_R\) take the same Fritzsch-like form as \(M_l\) and \(M_\nu\) do, then the seesaw relation \(M_\nu = M_D M_R^{-1} M_D^T\) will in general be violated. We shall show that this relation can keep unchanged, provided the condition \(B_D/C_D = B_R/C_R\) is satisfied. In this case, each nonvanishing matrix element of \(M_\nu\) has a simple seesaw relation with its counterparts in \(M_D\) and \(M_R\); i.e.,

\[
\begin{align*}
A_\nu &= \frac{A_D^2}{A_R}, & B_\nu &= \frac{B_D^2}{B_R}, & C_\nu &= \frac{C_D^2}{C_R}.
\end{align*}
\]

The textures of \(M_D\), \(M_R\) and \(M_\nu\) in Table 1 are therefore referred to as the seesaw-invariant textures. Because the parameters of \(M_\nu\) can essentially be determined from current neutrino oscillation data [8], it is then possible to impose useful constraints on the parameters of \(M_D\) and \(M_R\) via Eq. (2).

Another purpose of this paper is to account for the cosmological baryon number asymmetry via thermal leptogenesis [9]. Indeed, lepton number violation induced by the third term of \(L_{\text{lepton}}\) allows decays of the heavy Majorana neutrinos \(N_i\) (for \(i = 1, 2, 3\)) to happen.
Since the decay can occur at both tree and one-loop levels, their interference may result in a \(CP\)-violating asymmetry \(\varepsilon_i\) between the \(CP\)-conjugated \(N_i \to l + H_2^S\) and \(N_i \to l^c + H_2\) processes. If the masses of \(N_i\) are hierarchical (i.e., \(M_1 < M_2 < M_3\)), the interactions of \(N_1\) can be in thermal equilibrium when \(N_2\) and \(N_3\) decay. The asymmetries \(\varepsilon_2\) and \(\varepsilon_3\) are therefore erased before \(N_1\) decays, and only the asymmetry \(\varepsilon_1\) produced by the out-of-equilibrium decay of \(N_1\) survives. The point of leptogenesis [9] is that \(\varepsilon_1\) may give rise to a net lepton number asymmetry \(Y_L \equiv (n_l - n_{\bar{l}})/s \propto \varepsilon_1\), which is eventually converted into a net baryon number asymmetry \(Y_B\) via nonperturbative sphaleron processes [10]: \(Y_B \equiv (n_B - n_{\bar{B}})/s \propto Y_L\).

Thus this mechanism provides a natural interpretation of the cosmological matter-antimatter data [11]. We shall show that six Fritzsch-like textures of lepton mass matrices in Table 1 yield the same \(\varepsilon_1\) and \(Y_B\), from which useful constraints on the mass scale of three right-handed neutrinos and the Majorana phase of \(CP\) violation can be obtained.

The third purpose of this paper is to calculate the rare lepton-flavor-violating processes \(\mu \to e\gamma, \tau \to \mu\gamma\) and \(\tau \to e\gamma\). We shall focus our attention on a rather conservative case of lepton flavor violation, based on the scenarios where supersymmetry is broken in a hidden sector and the breaking is transmitted to the observable sector by a flavor-blind mechanism. We find that the values of \(\text{Br}(l_i \to l_j\gamma)\) depend strongly upon the phase parameters responsible for leptogenesis and for \(CP\) violation in neutrino oscillations.

### II. SEESAW-IN VARIANT TEXTURES

Now let us show that the seesaw relation \(M_\nu = M_D M_R^{-1} M_D^T\) will hold for each of the six patterns listed in Table 1, if and only if the condition \(B_D/C_D = B_R/C_R\) is satisfied. To be explicit, we take pattern (A) – namely, the Fritzsch texture [12], for example. Given the Fritzsch form of \(M_D\) and \(M_R\), the seesaw relation leads straightforwardly to

\[
M_\nu = \begin{pmatrix}
0 & C_D^2 & 0 \\
C_D^2 & 0 & B_D C_D \\
0 & B_D C_D & 0
\end{pmatrix} + \frac{C_D^2}{A_R} \left( \begin{pmatrix}
B_D & 0 & 0 \\
0 & B_R & 0 \\
0 & 0 & A_R
\end{pmatrix} \right).
\] (3)

One can see that the second term of Eq. (3) will vanish, if \(B_D/C_D = B_R/C_R\) is taken. In this case, the (2,3) or (3,2) element of \(M_\nu\) turns out to be \(B_D C_D/C_R = B_D^2/B_R\). Then Eq. (3) is simplified to

\[
M_\nu = \begin{pmatrix}
0 & C_D^2 & 0 \\
C_D^2 & 0 & B_D^2/B_R \\
0 & B_D^2/B_R & A_D^2/A_R
\end{pmatrix}.
\] (4)
Comparing between the form of $M_o$ in Eq. (4) and that defined in Table 1, we immediately arrive at the elegant seesaw relations given in Eq. (2). This result implies that

$$ \frac{B_D}{C_D} = \frac{B_R}{C_R} = \frac{B_\nu}{C_\nu} $$

(5)

holds. Eq. (5) imposes a strong constraint on the structure of $M_D$, $M_R$ and $M_\nu$. Because the magnitudes of $A_\nu$, $B_\nu$ and $C_\nu$ only have a quite weak hierarchy as required by current experimental data [7], we are not allowed to assume $B_\nu/C_\nu = B_1/C_1$ in a similar way. Indeed, $|B_\nu|/|C_\nu| \ll |B_1|/|C_1|$ must hold. Without loss of generality, we take $A_\nu$, $A_\nu$, $A_D$ and $A_R$ to be real and positive. Then only the off-diagonal elements of $M_a$ (for $a = l, D, R$ or $\nu$) are complex. It is possible to decompose $M_a$ into $M_a = P_a M_a P_a^T$, where

$$ M_a = \begin{pmatrix} 0 & C_a & 0 \\ C_a & 0 & B_a \\ 0 & B_a & A_a \end{pmatrix} $$

(6)

and $P_a = \text{Diag}\{e^{i(\phi_a - \phi_0)}, e^{i\phi_a}, 1\}$ with $A_a = A_a$, $B_a = |B_a|$, $C_a = |C_a|$, $\phi_a \equiv \text{arg}(B_a)$ and $\varphi_a \equiv \text{arg}(C_a)$. Eq. (2) indicates that $\phi_\nu = 2\phi_D - \phi_R$ and $\varphi_\nu = 2\varphi_D - \varphi_R$ hold. Hence one can get the phase relation $\phi_D - \varphi_D = \phi_R - \varphi_R = \phi_\nu - \varphi_\nu$ from Eq. (5). Because of $\text{Det}(M_a) = -A_a C_a^2 < 0$, it is more convenient to diagonalize $M_a$ by using the transformation

$$ (O_a Q)^T M_a (O_a Q) = \begin{pmatrix} \lambda_1^a & 0 & 0 \\ 0 & \lambda_2^a & 0 \\ 0 & 0 & \lambda_3^a \end{pmatrix} $$

(7)

where $O_a$ denotes a real orthogonal matrix, $Q = \text{Diag}\{1, i, 1\}$ is a pure phase matrix defined to cancel the minus sign of $\text{Det}(M_a)$, and $\lambda_i^a$ (for $i = 1, 2, 3$) stand for the positive eigenvalues of $M_a$. Then we have

$$ A_a = \lambda_1^a - \lambda_2^a + \lambda_3^a, $$

$$ B_a = \left[ \frac{(\lambda_1^a - \lambda_2^a) (\lambda_2^a - \lambda_3^a) (\lambda_1^a + \lambda_3^a)}{\lambda_1^a - \lambda_2^a + \lambda_3^a} \right]^{1/2}, $$

$$ C_a = \left[ \frac{\lambda_1^a \lambda_2^a \lambda_3^a}{\lambda_1^a - \lambda_2^a + \lambda_3^a} \right]^{1/2}. $$

(8)

Defining the dimensionless ratios $x_a \equiv \lambda_1^a/\lambda_2^a$ and $z_a \equiv \lambda_1^a/\lambda_3^a$, we further obtain

$$ O_{11}^a = \left[ \frac{x_a - z_a}{(1 + x_a)(1 - z_a)(x_a - z_a + x_a z_a)} \right]^{1/2}, $$

$$ O_{12}^a = -\left[ \frac{x_a^3 (1 + z_a)}{(1 + x_a)(x_a + z_a)(x_a - z_a + x_a z_a)} \right]^{1/2}, $$

$$ O_{13}^a = \left[ \frac{z_a^3 (1 - x_a)}{(1 - z_a)(x_a + z_a)(x_a - z_a + x_a z_a)} \right]^{1/2}. $$
$(5)$ and $(10)$ are universally valid. We diagonalize in Table 1 are also seesaw-invariant under the condition $B$ and $\beta$.

This result allows us to discuss six isomeric patterns of like textures of lepton mass matrices. Then we find that Eqs. $(8)$ and $(9)$ universally hold.

It is then possible to link the observables of leptogenesis and lepton flavor violation to the masses of three light neutrinos ($m_i$) and three heavy neutrinos ($M_i$) in a rather simple way.

One may straightforwardly show that the other five patterns of neutrino mass matrices in Table 1 are also seenaw-invariant under the condition $B_D/C_D = B_R/C_R$. Thus Eqs. $(2)$, $(5)$ and $(10)$ are universally valid. We diagonalize $M_a$ via the transformation

$$(P_a^* O_a Q)^T M_a (P_a^* O_a Q) = (O_a Q)^T M_a (O_a Q) ,$$

as defined in Eq. $(7)$. The relevant forms of $P_a$ and $O_a$ are listed in Table 1 for six Fritzsch-like textures of lepton mass matrices. Then we find that Eqs. $(8)$ and $(9)$ universally hold. This result allows us to discuss six isomeric patterns of $M_a$ in a uniform way.

The phenomenon of lepton flavor mixing arises from the mismatch between diagonalizations of $M_l$ and $M_\nu$. It is described by the unitary matrix $V = (P_l^* O_l Q)^T (P_\nu^* O_\nu Q)^*$. Explicitly,

$$|V_{pq}| = |O_{1p}^l O_{1q}^\nu e^{i\alpha} + O_{2p}^l O_{2q}^\nu e^{i\beta} + O_{3p}^l O_{3q}^\nu| ,$$

where the subscripts $p$ and $q$ run respectively over $(e, \mu, \tau)$ and $(1, 2, 3)$, and the phases $\alpha$ and $\beta$ are defined by $\alpha \equiv (\varphi_\nu - \varphi_l) - \beta$ and $\beta \equiv (\phi_\nu - \phi_l)$. It is clear that $|V_{pq}|$ depend only upon four free parameters: $x_\nu, z_\nu, \alpha$ and $\beta$, because the mass ratios of charged leptons $x_l$ and $z_l$ are well known. The parameter space of $(x_\nu, z_\nu)$ and $(\alpha, \beta)$ can be determined from current experimental data on solar, atmospheric and reactor neutrino oscillations. A detailed analysis has been done in Ref. [7]. It is found that the Fritzsch-like textures of $M_l$ and $M_\nu$ can fit the present data at the $3\sigma$ level.

Instead of repeating the analysis done before, we shall subsequently concentrate on the consequences of our phenomenological ansatz on thermal leptogenesis and lepton flavor violation.
III. THERMAL LEPTOGENESIS

As argued above, we assume that three heavy right-handed neutrinos have a clear mass hierarchy and the out-of-equilibrium decay of the lightest one ($N_1$) is the only source of lepton number asymmetry. In the flavor basis where both $M_L$ and $M_R$ are diagonal, real and positive, the CP-violating asymmetry between $N_1 \to l + H_2^\pm$ and $N_1 \to l^c + H_2$ processes reads [13]

$$
\varepsilon_1 \equiv \frac{\Gamma(N_1 \to l + H_2^\pm) - \Gamma(N_1 \to l^c + H_2)}{\Gamma(N_1 \to l + H_2^\pm) + \Gamma(N_1 \to l^c + H_2)}
\approx \frac{3}{8\pi} \frac{x_R \text{Im} \left( (\tilde{M}_D^\dagger \tilde{M}_D)_{12} \right)^2 + z_R \text{Im} \left( (\tilde{M}_D^\dagger \tilde{M}_D)_{13} \right)^2}{\langle H_2 \rangle^2 (\tilde{M}_D^\dagger \tilde{M}_D)_{11}},
$$

(13)

in which $x_R \equiv M_1/M_2$ and $z_R \equiv M_1/M_3$ with a normal mass hierarchy $z_R^2 \ll x_R^2 \ll 1$, $\langle H_2 \rangle = v \sin \beta_{\text{susy}}$ with $v \approx 174$ GeV, and $\tilde{M}_D = (P_L^t O_L) M_D (P_R^t O_R Q)$. Note that $\tilde{M}_D^\dagger \tilde{M}_D$ can be expressed as

$$
\tilde{M}_D^\dagger \tilde{M}_D = (P O_R Q)^T \tilde{M}_D^2 (P O_R Q),
$$

(14)

where $P \equiv P_L^T P_R^* = \text{Diag}\{1, e^{i \sigma}, 1\}$ with $\sigma \equiv \phi_D - \phi_R$. In obtaining this result, we have used the phase relation $\phi_D - \varphi_D = \phi_R - \varphi_R$. It is remarkable that only a single phase parameter $\sigma$ contributes to $\tilde{M}_D^\dagger \tilde{M}_D$. If $\sigma$ vanishes, there will be no CP violation in the lepton-number-violating decays of heavy Majorana neutrinos $N_i$. We emphasize that $\sigma$ has no direct connection with the effect of leptonic CP violation in neutrino oscillations. The latter is actually associated with the phase differences $\alpha$ and $\beta$ appearing in Eq. (12) [8]. Only in the special case that $\phi_i$ and $\varphi_i$ are switched off (or fixed to certain values), it is possible to indirectly link $\sigma$ to $\alpha$ and $\beta$. For example, $\phi_i = \varphi_i = 0$ leads to $\beta = \sigma + \phi_D$ and $\alpha = \varphi_\nu - \beta$.

With the help of Eqs. (6) and (14), we obtain the explicit expressions of $(\tilde{M}_D^\dagger \tilde{M}_D)_{11}$, Im$[(\tilde{M}_D^\dagger \tilde{M}_D)_{12}]^2$ and Im$[(\tilde{M}_D^\dagger \tilde{M}_D)_{13}]^2$ as follows:

$$
(\tilde{M}_D^\dagger \tilde{M}_D)_{11} = A_D^2 (O_{31}^R)^2 + B_D^2 \left[ (O_{21}^R)^2 + (O_{31}^R)^2 \right] + C_D^2 \left[ (O_{11}^R)^2 + (O_{21}^R)^2 \right] + 2 A_D B_D O_{21}^R O_{31}^R \cos \sigma + 2 B_D C_D O_{11}^R O_{31}^R,
$$

(15)

and

$$
\text{Im}[(\tilde{M}_D^\dagger \tilde{M}_D)_{12}]^2 = -2 A_D B_D \left( O_{22}^R O_{31}^R - O_{21}^R O_{32}^R \right) \left[ A_D^2 O_{31}^R O_{32}^R + B_D^2 \left( O_{21}^R O_{22}^R + O_{31}^R O_{32}^R \right) + C_D^2 \left( O_{11}^R O_{12}^R + O_{21}^R O_{22}^R \right) \right] \sin \sigma + B_D C_D \left( O_{11}^R O_{32}^R + O_{12}^R O_{31}^R \right) \sin \sigma,
$$

$$
\text{Im}[(\tilde{M}_D^\dagger \tilde{M}_D)_{13}]^2 = +2 A_D B_D \left( O_{23}^R O_{31}^R - O_{21}^R O_{33}^R \right) \left[ A_D^2 O_{31}^R O_{33}^R + B_D^2 \left( O_{21}^R O_{23}^R + O_{31}^R O_{33}^R \right) + C_D^2 \left( O_{11}^R O_{13}^R + O_{21}^R O_{23}^R \right) \right] \sin \sigma + B_D C_D \left( O_{11}^R O_{33}^R + O_{13}^R O_{31}^R \right) \sin \sigma.
$$

(16)
Combining Eqs. (13) and (16), one can clearly see that the CP-violating asymmetry \( \varepsilon_1 \) is proportional to \( A_D|B_D| \sin \sigma \). Hence \( \sigma \) is the only source of CP violation for the decays of heavy right-handed neutrinos in our ansatz.

Apart from \( \sin \beta_{\text{susy}} \) or \( \tan \beta_{\text{susy}} \), the free parameters of \( \varepsilon_1 \) include \( m_i \), \( M_i \) (for \( i = 1, 2, 3 \)) and \( \sigma \). Current neutrino oscillation data allow us to constrain \( m_i \) or equivalently \( x_\nu, z_\nu \) and \( m_3 \) to an acceptable degree of accuracy [7]. The ratio

\[
r \equiv \frac{B_R}{C_R} = \frac{B_\nu}{C_\nu} = \sqrt{\frac{(1 - x_\nu)(1 + z_\nu)(x_\nu - z_\nu)}{x_\nu z_\nu}}
\]

(17)
can then be determined. Note that Eq. (17) remains valid, if \( (x_\nu, z_\nu) \) are replaced by \( (x_R, z_R) \). This implies that \( x_R \) and \( z_R \) are correlated with each other for a given value of \( r \). Indeed, it is straightforward to obtain

\[
z_R = \sqrt{\frac{(1 - x_R)^2 + r^2 x_R^2}{2(1 - x_R)}}
\]

(18)
Taking account of Eq. (18), we are left with four unknown parameters to evaluate the magnitude of \( \varepsilon_1 \); namely, \( M_1 \), \( x_R \), \( \sigma \) and \( \tan \beta_{\text{susy}} \).

In the spirit of thermal leptogenesis [9], the CP-violating asymmetry \( \varepsilon_1 \) may lead to a net lepton number asymmetry \( Y_L \equiv (n_L - n_\bar{L})/s = \varepsilon_1 d/g_s \), where \( g_s = 228.75 \) is an effective number characterizing the relativistic degrees of freedom which contribute to the entropy \( s \) of the early universe, and \( d \) accounts for the dilution effects induced by the lepton-number-violating wash-out processes. This lepton number asymmetry is eventually converted into a net baryon number asymmetry \( Y_B \equiv (n_B - n_\bar{B})/s \) via nonperturbative sphaleron processes [10]: \( Y_B \approx -0.35 Y_L \) in the MSSM with three fermion families and two Higgs doublets. Although the dilution factor \( d \) can be computed by solving the full set of Boltzmann equations, it is more convenient to adopt a simple analytical approximation of \( d \) proposed in Ref. [16]: \( d \approx 0.02 \times (0.01 \text{ eV}/m_1)^{1.1} \), where \( m_1 \equiv (M_D^\dagger M_D)_{11}/M_1 \) is an effective neutrino mass parameter and its plausible magnitude is expected to lie in the range \( 10^{-2} \text{ eV} \lesssim m_1 \lesssim 1 \text{ eV} \) (the so-called strong washout regime [16]). If our phenomenological ansatz of lepton mass matrices is viable for leptogenesis, it should be able to reproduce the observed magnitude of \( Y_B \) (i.e., \( 7 \times 10^{-11} \lesssim Y_B \lesssim 10^{-10} \) [11]).

To evaluate \( \varepsilon_1 \) and \( Y_B \), we adopt \( x_\nu \approx 1/3 \), \( z_\nu \approx 1/12 \) and \( m_3 \approx 0.05 \text{ eV} \) as favorable inputs given in Ref. [7]. In addition, we typically take \( \tan \beta_{\text{susy}} \approx 10 \) and \( x_R \approx 1/4 \). We allow \( M_1 \) and \( \sigma \) to vary, in order to reproduce \( Y_B \) in its afore-mentioned range. Then we arrive at the parameter space of \( M_1 \) and \( \sigma \), as shown in Fig. 1. The lower bound of \( M_1 \) is about \( 2.5 \times 10^{10} \text{ GeV} \), while the lower limit of \( \sigma \) approximately reads \( 8.0^\circ \). When \( M_1 \) is much higher than \( 10^{12} \text{ GeV} \), the value of \( \sigma \) becomes closer to \( 57^\circ \). Note that there is in

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\(^1\)We are grateful to Yanagida for clarifying our misunderstanding of Ref. [14] and calling our particular attention to Hamaguchi’s PhD thesis [15], in which the relationship between \( Y_B(\text{final}) \) and \( Y_L(\text{initial}) \) is clearly discussed.
general a potential conflict between achieving successful thermal leptogenesis and avoiding overproduction of gravitinos in the conventional seesaw model with supersymmetry [17]. If the mass scale of gravitinos is of $O(1)$ TeV, one must have $M_1 \lesssim 10^8$ GeV. This limit is apparently disfavored in our ansatz. Such a problem could be circumvented in other supersymmetric breaking mediation scenarios (e.g., gauge mediation [18]) or in a class of supersymmetric axion models [19], where the gravitino mass can be much lighter in spite of the very high reheating temperature.

It is worth pointing out that our ansatz can clearly be distinguished from the interesting Fukugita-Tanimoto-Yanagida (FTY) ansatz of lepton mass matrices [20] by comparing between their consequences on leptogenesis. In the FTY ansatz, $M_L$ and $M_D$ are also of the Fritzsch texture, but $M_R = M_0 \mathbf{1}$ with $\mathbf{1}$ being the unit matrix has been assumed. Hence the texture of $M_\nu$ is different from that of $M_L$ or $M_D$. The exact mass degeneracy of three right-handed neutrinos (i.e., $M_i = M_0$ for $i = 1, 2, 3$) immediately implies that the rephasing invariant of CP violation

$$\Delta_{\text{CP}} \equiv \text{ImTr} \left[ Y_\nu^T Y_\nu M_R^T M_R M_L^T Y_\nu^T Y_\nu M_R \right]$$

$$= \frac{1}{\langle H_2 \rangle^4} \sum_{i < j} \left( M_i M_j \left( M^2_j - M^2_i \right) \text{Im} \left[ (\tilde{M}_D L \tilde{M}_D)^{ij} \right] \right)^2,$$

(19)

which is relevant for leptogenesis [21], is actually vanishing. We can therefore conclude that there will be no lepton number asymmetry between $N_1 \rightarrow l + H_2^c$ and $N_1 \rightarrow l^c + H_2$ decays in the FTY ansatz, although it does allow leptonic CP violation to show up in low-energy neutrino oscillations. In order to accommodate leptogenesis, a slight modification of the FTY hypothesis is necessary. The simplest way might be to assume that three heavy right-handed neutrinos have a near mass degeneracy [22], such that the cosmological baryon number asymmetry may arise from the resonantly-enhanced thermal leptogenesis [23].

**IV. LEPTON FLAVOR VIOLATION**

We proceed to discuss the consequence of our phenomenological scenario on lepton flavor violation in the framework of MSSM with three heavy right-handed neutrinos. We restrict ourselves to a very simple and conservative case where supersymmetry is broken in a hidden sector and the breaking is transmitted to the observable sector by a flavor blind mechanism, such as gravity [18]. Then all the soft breaking terms are diagonal at high energy scales, and the only source of lepton flavor violation in the charged lepton sector is the radiative correction to the soft terms through the neutrino Yukawa couplings. In other words, the low-energy lepton-flavor-violating processes $l_j \rightarrow l_i \gamma$ (for $i, j = e, \mu, \tau$ and $m_j > m_i$) are induced by the renormalization-group effects of the slepton mixing. The off-diagonal elements of the left-handed slepton mass matrix can be written as [24]

$$\left( M_{\tilde{L}}^2 \right)_{ij} \approx - \frac{3m_0^2 + A_0^2}{8\pi^2 v^2 \sin^2 \beta_{\text{susy}}} \left( \tilde{M}_D L \tilde{M}_D^t \right)_{ij},$$

(20)

in the leading-logarithmic approximation, where $m_0$ and $A_0$ denote the universal scalar soft mass and the trilinear term at the GUT scale, respectively. Note that the diagonal matrix
$L$ in Eq. (19) measures the difference between the scale of heavy Majorana neutrinos and that of GUT,

$$L = \begin{pmatrix}
            \ln \frac{M_{GUT}}{M_1} & 0 & 0 \\
            0 & \ln \frac{M_{GUT}}{M_2} & 0 \\
            0 & 0 & \ln \frac{M_{GUT}}{M_3}
        \end{pmatrix}. \quad (21)$$

The branching ratios of $l_j \to l_i \gamma$ can approximately be given by

$$\text{Br}(l_j \to l_i \gamma) \approx \frac{\alpha^3}{G_F^2} \cdot \frac{|(M_L^2)_{ij}|^2}{m_5^2} \tan^2 \beta \text{susy} \quad (22)$$

with $m_5^2 \approx 0.5m_0^2M_{1/2}^2(m_0^2 + 0.6M_{1/2}^2)^2$, where $M_{1/2}$ denotes the gaugino mass [25]. Once the texture of $\tilde{\phi}$ is specified, it will be possible to get concrete predictions for $\text{Br}(l_j \to l_i \gamma)$.

Given the Fritzsch-like textures of $M_a$ (for $a = l, \nu, D, R$), the quantity $\tilde{M}_D L \tilde{M}_D^\dagger$ can be rewritten as

$$\tilde{M}_D L \tilde{M}_D^\dagger = \left( QO_l^T P M_D P_R \right) L \left( QO_l^T P M_D P_R \right)^\dagger \quad (23)$$

in the chosen flavor basis (i.e., both $M_l$ and $M_R$ are diagonal, real and positive), where $P' \equiv P_D^T P_D = \text{Diag}\{e^{i\alpha}, e^{i\rho}, 1\}$ with $\alpha \equiv (\varphi_D - \phi_D) - (\varphi_l - \phi_l)$ and $\rho \equiv \phi_D - \phi_l$, and all the other matrices have been given before. Note that the phase $\alpha$ defined here is just the one defined in Eq. (12) for the lepton flavor mixing matrix $V$, because the phase equation $\varphi_D - \phi_D = \varphi_l - \phi_l = \varphi_R - \phi_R$ does hold in our ansatz. Furthermore, it is easy to prove

$$\rho + \sigma = 2\phi_D - \phi_R - \phi_l = \phi_R - \phi_l = \beta, \quad (24)$$

where the seesaw phase relation $\phi_\nu = 2\phi_D - \phi_R$ has been used. Let us summarize the phase parameters appearing in three categories of phenomena at this point:

Neutrino oscillations: $\alpha$ and $\beta$;
Thermal leptogenesis: $\sigma$;
Lepton flavor violation: $\alpha$ and $\rho = \beta - \sigma$.

We see that $V$, $Y_B$ and $\text{Br}(l_j \to l_i \gamma)$ totally involve three free phases. Among them, $\alpha$ and $\beta$ can be determined from the precise measurement of lepton flavor mixing and CP violation in neutrino oscillations. Current neutrino oscillation data do favor $\beta \approx \pi$, but they are unable to provide a narrow constraint on $\alpha$ [7]. If the value of $\alpha$ is fixed, nevertheless, one may examine the dependence of $\text{Br}(l_j \to l_i \gamma)$ on $\sigma$ with the help of $\rho \approx \pi - \sigma$.

To illustrate, we take $\alpha \approx 30^\circ$, $\beta \approx 180^\circ$ and $x_R \approx 1/4$ in addition to $x_\nu \approx 1/3$, $z_\nu \approx 1/12$ and $m_3 \approx 0.05$ eV [7]. The supersymmetric parameters relevant to our calculation are typically chosen as $\tan \beta \text{susy} \approx 10$, $A_0 \approx 0$, $m_0 \approx 100$ GeV, $M_{1/2} \approx 300$ GeV and $M_{GUT} \approx 2 \times 10^{16}$ GeV. Then we are left with two free parameters $M_1$ and $\sigma$, just like the case of thermal leptogenesis. Allowing $M_1$ and $\sigma$ to vary, we calculate $\text{Br}(l_j \to l_i \gamma)$ by using Eqs. (19)–(23) and by including the leptogenesis constraint $7 \times 10^{-11} \lesssim Y_B \lesssim 10^{-10}$. The experimental upper bounds of $\text{Br}(l_j \to l_i \gamma)$ should certainly be taken into account [26]:

9
\[
\begin{align*}
\text{Br}(\mu \to e\gamma) &< 1.2 \times 10^{-11}, \\
\text{Br}(\tau \to e\gamma) &< 3.6 \times 10^{-7}, \\
\text{Br}(\tau \to \mu\gamma) &< 3.1 \times 10^{-7}. 
\end{align*}
\] (25)

Our numerical results are shown in Figs. 2–4. Once can see that the predicted branching ratios of \(\mu \to e\gamma\), \(\tau \to e\gamma\) and \(\tau \to \mu\gamma\) are all below their corresponding experimental upper limits. A generic feature of \(\text{Br}(l_j \to l_i\gamma)\) is that they increase with the phase parameter \(\sigma\). In particular, \(\text{Br}(\mu \to e\gamma) \sim 10^{-12}\) is reachable for \(\sigma \sim 56^\circ\). Of course, the magnitudes of \(\text{Br}(l_j \to l_i\gamma)\) will change, if different values of the supersymmetric parameters are input. Instead of examining the full parameter space, here we have paid our main attention to the parameter correlation between leptogenesis and lepton-flavor-violating processes.

Let us remark that the gravity mediation scenario used in evaluating lepton flavor violation is in potential conflict with the lower bound of \(M_1\) obtained from thermal leptogenesis in our ansatz. This problem, usually referred to as the gravitino problem, exists in many supersymmetric seesaw models (see, e.g., Ref. [17] and references therein). One possible way out is to fine-tune the model parameters (i.e., those of \(Y_\nu\) and \(M_R\)) or assume the masses of heavy right-handed neutrinos to be nearly degenerate [23] within the thermal leptogenesis mechanism. As for our universal Fritzsch-like textures of lepton mass matrices, however, we are not left with much room for the fine-tuning of model parameters. The assumption of a near mass degeneracy for heavy Majorana neutrinos seems not to be natural either, although it is not impossible.

We find that there actually exists an interesting solution to the gravitino problem [19], which is in no apparent conflict with our phenomenological scenario. Its key point is that the axino and gravitino can be the lightest (of \(\mathcal{O}(1)\) keV) and the next lightest (of \(\mathcal{O}(10^2)\) GeV) supersymmetric particles, respectively, in a large class of supersymmetric axion models. Thus the reheating temperature is allowed to reach \(10^{15}\) GeV or so [19], high enough for the thermal leptogenesis mechanism to work. Some more discussions in this connection will be presented elsewhere [22], since they are already beyond the scope of the present work.

V. SUMMARY

We have incorporated the seesaw mechanism with six Fritzsch-like textures of lepton mass matrices. It is found that the seesaw relation holds under a particular condition, and the consequences of those textures on lepton flavor mixing are exactly the same. Applying this simple ansatz to thermal leptogenesis, we have shown that CP violation in the lepton-number-violating decays of heavy right-handed neutrinos depends only upon a single phase parameter and the cosmological baryon number asymmetry can naturally be explained. The lepton-flavor-violating processes \(\mu \to e\gamma\), \(\tau \to \mu\gamma\) and \(\tau \to e\gamma\) have also been calculated. An interesting result is that the branching ratios of those rare processes rely strongly upon the phase parameters appearing in leptogenesis and in neutrino oscillations. We expect that a stringent test of our phenomenological scenario will be available in the near future, when more precise experimental data are accumulated.
We are deeply indebted to T. Yanagida for bringing Refs. [15,19] to our attention and clarifying our misunderstanding of the sphaleron and gravitino problems. We are also grateful to W.L. Guo and J.W. Mei for helpful discussions. This work was supported in part by the National Natural Science Foundation of China.
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TABLE I. The Fritzsch-like lepton mass matrix $M_a$ (for $a = l, D, R, \nu$), where $\text{arg}(A_a) \equiv 0$, $\text{arg}(B_a) \equiv \phi_a$ and $\text{arg}(C_a) \equiv \varphi_a$. The phase matrix $P_a$ and the real orthogonal matrix $O_a$ are defined to diagonalize $M_a$ via the transformation $(P^* O_a Q)^T M_a (P^* O_a Q)$ with $Q \equiv \text{Diag} \{1, i, 1 \}$. The explicit expressions of $O^a_{ij}$ (for $i, j = 1, 2, 3$) are given in Eq. (9).

| Pattern | $M_a$ | $P_a$ | $O_a$ |
|--------|------|------|------|
| (A)    | $\begin{pmatrix} 0 & C_a & 0 \\ C_a & 0 & B_a \\ 0 & B_a & A_a \end{pmatrix}$ | $\begin{pmatrix} e^{i(\varphi_a - \phi_a)} & 0 & 0 \\ 0 & e^{i\phi_a} & 0 \\ 0 & 0 & 1 \end{pmatrix}$ | $\begin{pmatrix} O^a_{11} & O^a_{12} & O^a_{13} \\ O^a_{21} & O^a_{22} & O^a_{23} \\ O^a_{31} & O^a_{32} & O^a_{33} \end{pmatrix}$ |
| (B)    | $\begin{pmatrix} 0 & 0 & C_a \\ 0 & A_a & B_a \\ C_a & B_a & 0 \end{pmatrix}$ | $\begin{pmatrix} e^{i(\varphi_a - \phi_a)} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\phi_a} \end{pmatrix}$ | $\begin{pmatrix} O^a_{11} & O^a_{12} & O^a_{13} \\ O^a_{21} & O^a_{22} & O^a_{23} \\ O^a_{31} & O^a_{32} & O^a_{33} \end{pmatrix}$ |
| (C)    | $\begin{pmatrix} 0 & C_a & B_a \\ C_a & 0 & 0 \\ B_a & 0 & A_a \end{pmatrix}$ | $\begin{pmatrix} e^{i\varphi_a} & 0 & 0 \\ 0 & e^{i(\varphi_a - \phi_a)} & 0 \\ 0 & 0 & 1 \end{pmatrix}$ | $\begin{pmatrix} O^a_{11} & O^a_{12} & O^a_{13} \\ O^a_{21} & O^a_{22} & O^a_{23} \\ O^a_{31} & O^a_{32} & O^a_{33} \end{pmatrix}$ |
| (D)    | $\begin{pmatrix} 0 & B_a & C_a \\ B_a & A_a & 0 \\ C_a & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} e^{i\varphi_a} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i(\varphi_a - \phi_a)} \end{pmatrix}$ | $\begin{pmatrix} O^a_{11} & O^a_{12} & O^a_{13} \\ O^a_{21} & O^a_{22} & O^a_{23} \\ O^a_{31} & O^a_{32} & O^a_{33} \end{pmatrix}$ |
| (E)    | $\begin{pmatrix} A_a & 0 & B_a \\ 0 & C_a & 0 \\ B_a & C_a & 0 \end{pmatrix}$ | $\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i(\varphi_a - \phi_a)} & 0 \\ 0 & 0 & e^{i\varphi_a} \end{pmatrix}$ | $\begin{pmatrix} O^a_{11} & O^a_{12} & O^a_{13} \\ O^a_{21} & O^a_{22} & O^a_{23} \\ O^a_{31} & O^a_{32} & O^a_{33} \end{pmatrix}$ |
| (F)    | $\begin{pmatrix} A_a & B_a & 0 \\ B_a & 0 & C_a \\ 0 & C_a & 0 \end{pmatrix}$ | $\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\varphi_a} & 0 \\ 0 & 0 & e^{i(\varphi_a - \phi_a)} \end{pmatrix}$ | $\begin{pmatrix} O^a_{11} & O^a_{12} & O^a_{13} \\ O^a_{21} & O^a_{22} & O^a_{23} \\ O^a_{31} & O^a_{32} & O^a_{33} \end{pmatrix}$ |
FIG. 1. The allowed ranges of $\sigma$ and $M_1$ to reproduce $7 \times 10^{-11} \leq Y_B \leq 10^{-10}$ via leptogenesis, where $m_3 \approx 0.05$ eV, $x_\nu \approx 1/3$, $z_\nu \approx 1/12$, $x_R \approx 1/4$ and $\tan \beta_{\text{ SUSY}} \approx 10$ have typically been input.
FIG. 2. The allowed ranges of $\sigma$ and $\text{Br}(\mu \to e\gamma)$, where $m_3 \approx 0.05$ eV, $x_\nu \approx 1/3$, $z_\nu \approx 1/12$, $x_R \approx 1/4$ and $\tan \beta_{\text{susy}} \approx 10$ have typically been input.
FIG. 3. The allowed ranges of $\sigma$ and $\text{Br}(\tau \to e\gamma)$, where $m_3 \approx 0.05$ eV, $x_\nu \approx 1/3$, $z_\nu \approx 1/12$, $x_R \approx 1/4$ and $\tan \beta_{\text{susy}} \approx 10$ have typically been input.
FIG. 4. The allowed ranges of $\sigma$ and $\text{Br}(\tau \rightarrow \mu\gamma)$, where $m_3 \approx 0.05$ eV, $x_{\nu} \approx 1/3$, $z_{\nu} \approx 1/12$, $x_R \approx 1/4$ and $\tan \beta_{\text{susy}} \approx 10$ have typically been input.