Optimization for Mass Hierarchy

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The $\Delta m_{13}^2$ oscillation frequency for reactor neutrinos differs by 6.4% between normal and inverted mass hierarchies. This frequency difference accumulates to a phase difference over distance and time. The optimal distance is when the maximum phase difference between hierarchies occurs near the peak in the observable reactor neutrino spectrum.

Recent developments in neutrino mixing include measurement of $\theta_{13}$ \cite{1} \cite{2} and refinement of parameters \cite{3}. Outstanding questions include the mass hierarchy, the CP violating phase $\delta$, sterile neutrinos, the Majorana nature of neutrino mass and the overall neutrino mass scale. Experiments are underway to better understand these questions.

The measurement of $\theta_{13}$ has done a great deal to permit the field to expand quickly. Plans to measure the mass hierarchy frequently involve ambiguities with $\delta$. Given the unexpectedly large value for $\theta_{13}$ it is worth reconsidering strategies to determine the mass hierarchy.

The transition probability to find a neutrino of type $\beta$ after a time $t$ when starting with a neutrino of type $\alpha$ in vacuum is given by \cite{4}:

$$P_{\alpha \rightarrow \beta} = |<\nu_\beta|\nu_\alpha(t)>|^2 = |\sum_j U_{\alpha j}^* U_{\beta j} e^{-i m_j^2 L/2E}|^2$$

Where $U_{\beta j}$ are elements of the complex PMNS matrix, the $m_j^2$ are the square of the masses of the $j$’th neutrino mass eigenstate and $L \approx ct$. $\overline{\nu_e}$ disappearance experiments to measure the mass hierarchy have an advantage that the measurement is independent of CP violating phases so those ambiguities can be avoided.

In the case of an electron antineutrino disappearance experiment this can be written as \cite{5}:

$$P_{\overline{\nu}_e \rightarrow \overline{\nu}_e} = 1 - (\cos^4(\theta_{13}) \sin^2(2\theta_{12}) \sin^2(\Delta_{21}))$$
$$+ \cos^2(\theta_{12}) \sin^2(2\theta_{13}) \sin^2(\Delta_{31})$$
$$+ \sin^2(\theta_{12}) \sin^2(2\theta_{13}) \sin^2(\Delta_{32}))$$

where $\Delta_{ij} = 1.267(m_i^2 - m_j^2) L/E$. Since $\cos^2(\theta_{12}) \approx 0.7$ and $\sin^2(\theta_{12}) \approx 0.3$ the high frequency oscillation is dominated by $\Delta_{31}$. The L/E plot shows (figure 1) a low amplitude high frequency oscillation at $\approx \Delta_{31}$ added to a high amplitude low frequency oscillation at $\Delta_{21}$.

A popular method to determine the mass hierarchy \cite{5} \cite{6} is to position a large reactor antineutrino detector near the solar neutrino oscillation ($\Delta m_{12}^2$) minimum and to look at ripples in the spectrum caused by $\theta_{13}$ oscillations. The combination of large distances and the oscillation minimum leads to very low rates, resulting in the need for a very large detector and long exposure times. The oscillation frequency, the ripple spacing, also in the need for a very large detector and long exposure times. The oscillation frequency, the ripple spacing, also

The portion of the transition probability sensitive to the mass hierarchy can be isolated from equation \cite{4}:

$$D = \sin^2(2\theta_{13}) \cos^2(\theta_{12}) \sin^2(\Delta_{31})$$
$$= \frac{\sin^2(2\theta_{13}) \cos^2(\theta_{12})(1 - \cos(2\Delta_{31}))}{2}$$
Now $\Delta_{31} = \Delta_{32} + \Delta_{21}$ so $\cos(2\Delta_{31}) = \cos(2\Delta_{32} + 2\Delta_{21}) = \cos(2\Delta_{32}) \cos(2\Delta_{21}) - \sin(2\Delta_{32}) \sin(2\Delta_{21})$. This gives:

$$D = \frac{\sin^2(2\theta_{13})}{2} \cos^2(\theta_{12})$$

$$1 - (\cos(2\Delta_{32}) \cos(2\Delta_{21}) - \sin(2\Delta_{32}) \sin(2\Delta_{21}))$$

The mass hierarchy is the sign of $\Delta_{32}$. The only term in $D$ odd in $\Delta_{32}$ is $\sin^2(2\theta_{13}) \cos^2(\theta_{12})$ $\sin(2\Delta_{32}) \sin(2\Delta_{21})$. The difference between normal and inverted hierarchy (figure 2) is:

$$|D_N - D_I| = \cos^2(\theta_{12}) \sin^2(2\theta_{13}) |\sin(2|\Delta_{32}|) \sin(2\Delta_{21})|$$

The maximum size of this difference is $\cos^2(\theta_{12}) \sin^2(2\theta_{13})$. Since the $\Delta_{21}$ oscillation is fairly slow this maximum difference can be found near $|\sin(2\Delta_{21})| = 1$. The smallest value maximizing it is $2\Delta_{21} = \pi/2$. For $\Delta m^2_{21} = 7.54 \times 10^{-5}$ eV$^2$ this suggests an optimal $L/E$ near 8200 m/MeV, figure 2.

$$\sin(2|\Delta_{32}|) \sin(2\Delta_{21}) = \frac{1}{2} \cos(2|\Delta_{32} - \Delta_{21}|) - \cos(2|\Delta_{32} + \Delta_{21}|))$$

The largest observable difference between the two mass hierarchies occurs when the two predictions are 180 degrees out of phase.

$$2(|\Delta_{32} - \Delta_{21}|) = n\pi$$

$$2(|\Delta_{32} + \Delta_{21}|) = (n + 1)\pi$$

$$4|\Delta_{32}| = (2n + 1)\pi$$

$$4\Delta_{21} = \pi$$

The two oscillation frequencies for the two possible mass hierarchies differ by about 6.4% so the optimal phase difference would first occur at about 7.8 oscillations.

$$\frac{|\Delta_{32}|}{\Delta_{21}} = 2n + 1$$

Which gives $n=15.6$, $L/E=8200$ m/MeV.

The extrema of $|D_N - D_I|$ are the solutions to the equation

$$\tan(2\Delta m^2_{12} L/E) = -\tan(2|\Delta m^2_{32}| L/E)$$

A numerical search (figure 2) gives the L/E to the first global maximum at $L/E=8418$. The smallest L/E which is over 90% of this maximum separation is at $L/E=5861$.

The flux times cross section for a typical reactor neutrino spectrum peaks at about 3.66 MeV. A neutrino propagation length of about 30 km would provide optimal conditions in the vicinity of this peak. The actual shape of the spectrum is fuel dependent and depends on reactor burnup so precise optimization is not possible. But the
most of the observable spectrum would be sensitive to the mass hierarchy.

Since the neutrino mass parameters are only approximately known the estimate given here is not precise. But given the factor of 5 in the accessible neutrino energy range the position optimization described here should be adequate to get the optimal L/E very near the peak in the spectrum. The value of $\Delta m^2$ in Fogli et al. [3] has been used for our value of $\Delta m^2_{32}$. Fogli et al. has $\Delta m^2_{32} = \Delta m^2 + \delta m^2/2$. Most measurements of $\Delta m^2_{32}$ come from muon neutrino disappearance experiments [8].

Systematic errors on $\Delta m^2_{32}$ may be problematic. Since the experiment can not measure the normal and inverted mass hierarchy and compare them, comparison must be made to distributions based on an assumed value of $\Delta m^2_{32}$ and a mass hierarchy.

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After posting the first draft of this note I became aware of some recent work on this question [9]. I would like to thank J. Evslin for correspondence concerning the mass ambiguity.

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