Balls rolling down a playground slide: What factors influence their motion?

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Abstract
Take a selection of balls and marbles along to a nearby playground slide and let students investigate factors that may influence how balls accelerate down an inclined plane. Students can make hypotheses in small groups, plan investigations to test multiple possible explanations and draw conclusions about the importance of different variables. The experiments illustrate the principle of equivalence between inertial and gravitational mass, as well as of the importance of mass distribution. Students can develop an intuitive feeling for these concepts even without dealing with the full mathematical treatment, which involves torque, angular momentum and moment of inertia, typically treated in high-school or introductory university physics courses. The paper discusses this investigation as part of a setup of teacher professional development events on a playground.

Keywords: playground physics, open-ended investigations, inclined plane, moment of inertia, teacher professional development, informal learning environments, ISLE

1. Introduction
What properties of a ball influence how it rolls down an inclined plane? This question can be used to introduce a playground investigation. A group of students is likely to bring up mass, size, material and possibly friction. Depending on their background they may or may not need a prompt to list more specific material properties, including density, elasticity and surface properties. (Preschool children may also expect ‘colour’ to play a role.)

A next step is to come up with experiments to test the different possible effects, with access to a number of different balls, e.g. as shown in figure 1(a). Students may need hints that rolling pairs of balls next to each other, as shown in figure 1(b),
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gives a more direct comparison than measuring the time required for individual balls.

Investigating balls rolling down an inclined plane can be used in many different educational contexts, from traditional lecture demonstrations to enjoyable playground outings [1], with different types of instructions, as discussed below. Although the mathematical treatment involves concepts, such as moment of inertia, angular momentum and torque, that may not be introduced until high school or introductory university courses, an intuitive and conceptual understanding is possible much earlier. The investigations can give practice in dealing with several variables, as well as combining results from different investigations.

2. Background

As a PhD student, I was involved with supervision of inquiry-based experiments, where first-semester engineering physics students were asked to find relations between different factors influencing e.g. oscillation periods of springs or beams or irregular pegboard shapes, or liquid flow in thin tubes. Students typically had two 4 h sessions in the lab to come up with an expression for the relation, converting it to a linear relation between suitable combinations of variables and then insert all their measured data into a single graph, to demonstrate how well the data were described by their mathematical formula.

One of the most fascinating experiments to supervise was the study of the acceleration of cylinders rolling down an inclined plane. The students had access to steel cylinders with different radii and lengths, both solid cylinders and sets of nesting cylinders. In addition, one brass cylinder and one aluminium cylinder were available with the same dimensions as one of the steel cylinders.

That neither mass nor dimension played a role for solid cylinders was an unexpected result, that often took students quite a long time to discern. This important result is related to the principle of equivalence between gravitational and inertial mass, which is relatively unknown among new students, but can be introduced long before a mathematical treatment [1]–[4].

In the year 2000, I was invited as a guest teacher to be involved with a science teacher program at University College of Skövde, where the rolling cylinder experiment setup was not available. Instead we utilised a nearby playground, with variety of object to roll or slide down a long slide. The classical ‘soup can race’ between cans of mushroom soup and chicken broth is easily performed outdoors—although we discovered that repeated rolling of the mushroom soup can down a long slide exposed thixotropic properties of the soup, thus reducing the difference between the behaviour of the cans. An alternative for this demonstration is to use bottles or jars filled with sand and water, respectively, as illustrated in Figure 2. Students may also discover that the bottles do not take notice of their pre-experiment voting on the outcome of the race.

For later occasions, more rolling objects have been added to the bag taken to the playground. Depending on the background of the students, the types of variation in objects may be restricted. A selection of balls with different sizes and materials offers opportunities for systematic investigations, as discussed in this paper. Participating in a workshop on ISLE—Investigative Science Learning environment [5] inspired the current format, where students are first asked to come up with as many factors as possible and then plan investigations together.

3. Equal or different?

Traditional labs often involve performing independent measurements for different objects and recording the data in a table. However, comparing two balls starting together from the top of a slide gives a more direct visual comparison, as in figures 1(b) and 2. This can also give more direct hints if some differences occur in the first part of the slide. Whatever procedure is chosen, a discussion is necessary to establish what constitutes a difference and what results should be considered ‘nearly equal’ [2]. A first indication about uncertainty can be obtained by letting two identical balls roll together and observe possible differences in their motion. It can also be worth repeating the comparison a couple of times, including swapping places to compensate for any unevenness in the slide surface.
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Figure 1. (a) A selection of balls to be used for introduction and investigations (from [1]). (b) An example of two balls of different size rolling together during a teacher activity.

Figure 2. A group of first-year students comparing two bottles rolling down a playground slide. One of the bottles was filled with sand and the other with water.

A complication with common student choices of variables is that it is impossible to change one of the variables mass, size and density without changing (at least) one of the others. However, the collection of balls taken along to the playground slide should preferably include pairs of balls of the same size but with different density and balls of the same material but of different size.

Comparing a bouncing ball and a heavy marble with the same diameter should lead students to realise that mass and density have very little effect on the acceleration of the ball. Depending on the group, this conclusion may be forgotten when the students are asked to predict whether a small or large bouncing ball will reach the ground first, or whether they will come down together. Observing a small marble and a large bouncing ball rolling down side-by-side can still be surprising.

All balls in figure 1(a) roll without sliding on typical playground slides. The different friction coefficients for different surfaces of the balls thus do not lead to differences in motion.

After having established what differences can be expected for identical balls, it is time for students to find pairs of balls to test the different factors, suggested in the initial discussion. An additional hint for the students can be to ask them to create groups of balls that have essentially the same acceleration down the slide, instead of treating each race as a competition between the balls.
Finally, there are some balls that clearly lag the bouncing balls! The students may be left to discuss reasons for a while before being encouraged to compare different air filled balls—only to find that these balls form another group sharing essentially the same acceleration.

At this stage students may need some scaffolding discussion to help them connect the behaviour of the balls to their experiences on a carousel or a pirouette, where the rotational speed increases as the mass shifts closer to the rotation axis. In discussing the rotational energy, I have also on many occasions found it useful to hold a large ball in one hand and indicate with a finger on the other hand how a point close to the edge moves compared to points closer to the centre and compare their velocity, related to the kinetic energy. For teachers from primary to lower secondary school, this level of theory may be sufficient. At higher levels, these investigations can serve as an introduction to a more formal treatment of moment of inertia and angular momentum, discussed in section 4.

The investigations can, of course, also be done indoors, using a shelf plane or a table as inclined plane. This paper focuses only on the factors relating to the balls, but if desired the investigations can be expanded to a quantitative study, by measuring times to travel a given distance for different slopes, and the slope can be determined in terms or the ratio between elevation and length, or by using a protractor or the inclinometer on a smartphone. The motion of the balls can also be compared to bottles, jars or other cylinders.

4. Theory: forces, rotation and moment of inertia

This section presents a theoretical analysis, as a support for teachers’ dialogues with students. Unavoidable energy losses are discussed in section 5.1.

Friction causes a ball to rotate as it moves down an inclined plane. For a ball rolling without slipping, the centre of the ball moves a distance \(2\pi R\) for a full turn of the ball (i.e. an angle \(360^\circ = 2\pi\)). This gives a relation \(v = \omega R\) between the translational speed \(v\) and the angular velocity \(\omega\). The rotation around the centre of mass also involves kinetic energy, which can be expressed in terms of a moment of inertia, \(I_{cm}\) as

\[
E_{rot} = \frac{I_{cm} \omega^2}{2}.
\]  

The total kinetic energy for an object rolling without slipping is a sum of the translational kinetic energy and the rotational energy:

\[
E_k = \frac{mv^2}{2} + E_{rot} = \frac{m(R\omega)^2}{2} + \frac{I_{cm} \omega^2}{2} = \frac{(mR^2 + I_{cm}) \omega^2}{2}.
\]  

The total kinetic energy can also be expressed in terms of a moment of inertia with respect to the contact point: \(E_k = I_{cp} \omega^2 / 2\), where \(I_{cp} = I_{cm} + mR^2\) (parallel-axis theorem).

Figure 4 shows the forces on a ball accelerating down an inclined plane. Since the ball stays on the plane, the size of the normal force \(N\) is given by \(N = |N| = mg \cos \theta\). The size of the acceleration, \(a\), and of the friction force, \(F_{fr}\), are not known before a closer analysis of the motion, but the vector sum of the gravitational force, the normal force and the friction force must be equal to \(ma\) according to Newton’s second law.
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4.1. Torque and angular momentum

The torque on the ball with respect to the point of contact is given by

\[ \tau = mgR \sin \theta \]  (3)

The torque results in a change of angular momentum:

\[ \frac{dL}{dt} = \tau \]  (4)

where the angular momentum is given by \( L = I \omega \), where \( \omega \) is the angular velocity and \( I \) is the moment of inertia.

For motion around the centre of mass \( I_{cm, \text{ball}}/m = 2/5 \ R^2 \) for a compact ball and \( I_{cm, \text{sphere}}/m = 4/5 \ R^2 \) for a thin shell with radius \( R \). However, in this case the motion is around the contact point at a distance \( R \) from the centre, which is accounted for by adding a term \( mR^2 \) to the respective moments of inertia as discussed above: \( I_{cp} = I_{cm} + mR^2 \). This gives \( I_{cp, \text{ball}}/m = 7/5 \ R^2 \) and \( I_{cp, \text{sphere}}/m = 9/5 \ R^2 \) for the moments of inertia relative to the contact point.

Combining equations (3) and (4) gives

\[ \frac{d\omega}{dt} = \frac{1}{I_{cp}} \frac{dL}{dt} = \frac{\tau}{I_{cp}} = \frac{mgR \sin \theta}{I_{cp}} \]  (5)

Using the relation \( v = \omega R \), the expression 5 for the angular acceleration \( d\omega/dt \) can be used to obtain an expression for the linear acceleration, \( a = dv/dt \), giving

\[ a = \frac{d\omega}{dt} R = \frac{mR^2 g \sin \theta}{I_{cp}} \]

The acceleration for a rolling object can thus be written as \( a = C g \sin \theta \), where \( C = mR^2/I_{cp} \). For a compact ball the acceleration becomes

\[ a_{\text{ball}} = \frac{5}{7} g \sin \theta \]  (7)

and for spherical shell

\[ a_{\text{sphere}} = \frac{5}{9} g \sin \theta \]  (8)

It is worth noting that these expressions involve neither mass nor radius.

Newton’s second law then gives the size of the friction force for these two
cases: $F_{fr, ball} = \left(\frac{2}{7}\right)mg \sin \theta$ and $F_{fr, sphere} = \left(\frac{4}{9}\right)mg \sin \theta$. Since the friction force points upward along the slope, the friction force can also be written in terms of the coordinate system indicated in figure 4 as $F_{fr, ball} = -(\frac{2}{7})mg \sin \theta e_x$ and $F_{fr, ball} = -(\frac{2}{7})mg \sin \theta e_y$, where $e_x$ is a unit vector pointing downhill along the plane. For a general rolling object the friction force can be expressed as $F_{fr} = (C - 1)mg \sin \theta e_x$.

4.2. Educational challenges and opportunities

The motion of a ball of radius $R$ rolling down an inclined plane is a standard problem in physics textbooks. Still, student conceptual difficulties in analysing forces on objects rolling on an inclined plane are well documented, even at university level, as discussed e.g. by Rimoldino and Singh [7]. As seen above, working out the size of the friction force on the ball to ensure that the forces in the free-body diagram are drawn to scale, as in figure 4, requires a few steps. Sometimes free-body diagrams are instead drawn with all forces the same length, e.g. by [8] and [9] in their discussions of challenges involving friction on rolling objects.

An important role for the teacher is to help students discern the relevant aspects of the motion and to provide additional representations. Carvalho and Sousa [8] discuss teaching of friction and rotation in secondary school. Modern technology enables additional representations. E.g. Arnone et al [10] use a GeoGebra applet to visualise some of Galileo’s experiments and Phomarach et al [9] demonstrate how video analysis of the rolling motion gives results in good agreement with theory.

The focus in the present work is however on qualitative observation of the motion and the importance of different factors. This can be initiated long before working through a mathematical treatment.

5. Planning a playground outing for teachers

Investigating balls rolling down a slide has been part of playground activities for teacher events in many different forms [1]. Figure 5 shows one group of teachers discussing their plans for investigating balls rolling down a slide.

Even small playgrounds usually include swings, climbing racks and slides in some form, which are sufficient for a small group of teachers, who can be divided in 2–3 groups of 4–5 teachers, taking turns at the different stations.

The instructor then moves between the groups to listen to their suggestions and conclusions, and to provide scaffolding questions if needed. Typically a group spends 15–20 min at each station and should check in with the instructor for a concluding discussion before getting a bag with material for the next station. (It is thus useful to have bags for a couple of extra stations so that not all groups need to switch at the same time.)

A large playground may have several swing sets, slides and climbing racks, as well as other equipment that can be useful for physics investigations, such as carousels [11], and possibly small trampolines [12–14].

On occasions we have had groups of up to $6 \times 6$ teachers—who must, of course, recognise that the playground is primarily intended for children, not for physics experiments. An early morning start is advisable to have maximum access to popular playground devices. However, students often discover that children are curious and happy to assist with experiments. Bringing a couple of extra booklets with experiments to give to interested parents or preschool teachers can be a way to share the joy of physics!
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It is also worth emphasising that for use with a class in school or a preschool group, it makes more sense to use one activity at a time, with preparation and follow-up in the classroom, as described for example by the teachers who planned experiment on friction in slides [2] and investigations of falling objects [3].

5.1. Small effects

In the theoretical discussions above, no account was taken of inevitable energy losses, which may obscure the intended result. Reality can be more complex, and instructors need to be prepared to discuss small effects that may influence or obscure the results. For very light balls air resistance cannot be neglected relative to the force of gravity, causing pingpong balls to roll slower than tennis balls. Softer balls move slower, due to increased rolling friction [15, 16] and bouncing balls may lag slightly behind marbles.

One group of students noted that the sand seemed to move around in bottle that was not completely filled with sand, causing it to move differently from a small sand-filled jar. They quickly made use of the sand on the playground to top up the bottle for a better result.

Sometimes the slide is not perfectly even and, in any case, repeating the experiment is a good practice, possibly also with position interchanges. Many teachers have student use video clips as parts of the documentation [2, 3], which is also helpful in cases where students may have failed to let go of both balls at the same time.

6. Discussion

Balls rolling down an inclined plane is one of the few experiments that Richard Feynman could find in a textbook discussed to in ‘Surely you are joking, Mr Feynman’ [17]—although he discovered that the results presented were fake—having failed to take moment of inertia into account, and obviously not representing real experiments.

What is the role of experiments in physics teaching? In a review about ‘Varieties of Lab-work’ [18] Millar et al note that in many cases ‘the real task for the student is to produce the phenomenon’, that is, to succeed in producing the outcome that is predicted by a well-established scientific explanation’. There are of course many traditional ways to perform investigations or rolling on inclined planes, including ‘confirmation of theory’, as part of the implicit ‘didactic contract’.

On one occasion a lecturer had chosen to work through the theory during the lesson preceding the investigation of rolling objects, which was intended to be open-ended. Still many of the students were surprised to discover that small and large balls rolled side by side: Although the mass and radius are not present in the equations (7) and (8), the intuitive expectation that mass influences motion often remains. After some reflection, one student exclaimed with delight that ‘Oh, they have the same C!’. However, once the theoretical expression had been ‘confirmed’, these students showed no interest in continued investigations.

In the open-ended investigations of the influence of various factors, students should be encouraged to generate many possible explanations for their observations. Etkina and Planinič [6] note that ‘These multiple explanations naturally appear as students work in groups’ and continue ‘The students need to accept all explanations as “correct” for the time being even if they do not like some of those, and then design experiments whose outcomes they can predict using all of the explanations (testing experiments). Thus, they need to learn to differentiate between the explanations and the predictions of the outcomes of the experiments.’

In the investigations of the influence of various factors discussed in this paper, the non-influence of mass first appears as an observation of two balls with different masses rolling together. After the students describe this observation, they could formulate a hypothesis and investigate if it holds for other object they could investigate. Starting with a selection or marbles and bouncing balls of different size should help them realise that neither mass nor density nor radius plays a role. At this stage, the instructor may suggest a comparison with hollow ball, possibly by handing one to them. Often, some of the students in a small group will suggest that the lower mass accounts for the slower motion. Scaffolding discussions with an instructor can then be useful to remind them of
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their earlier conclusion about the non-influence of mass.

A playground offers many opportunities to illustrate the principle of equivalence between gravitational mass (in $m_g$) and inertial mass (in $m_a$), by noting that mass has no (or very limited) influence, for example as you swing together with an empty swing or drop objects from a climbing rack [3, 4], or observe different objects sliding down an inclined plane [2]. This gives students a chance to develop an early intuitive grasp of topics that involve well-documented difficulties also for university students [7]. The deep significance of mass not influencing acceleration by gravity is more likely to be appreciated by students who reach this conclusion from scaffolded investigations than by students who just confirm a theoretical formula where mass does not appear.

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