Confinement from Gauge Invariance in 2+1 Dimensions

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It is shown, in $D = 2 + 1$ dimensions, that by merely imposing non-abelian gauge invariance on the temporal gauge ground state wavefunctional of an abelian gauge theory, a confining state is obtained.

Confined is a property of the vacuum of quantized non-abelian gauge theories. In the Hamiltonian formulation of quantized Yang-Mills theory, in $D = d + 1$ dimensions and temporal gauge, the vacuum is the ground state solution $\Psi_0[A]$ of the time-independent Schrödinger equation

$$H \Psi_0 = E_0 \Psi_0$$

where

$$H = \int d^d x \left\{ - \frac{1}{4} \frac{\delta^2}{\delta A_k^a(x)^2} + \frac{1}{4} F_{ij}^a(x)^2 \right\}$$

Physical states in temporal gauge, in SU(2) gauge theory, are required to satisfy the Gauss Law constraint

$$\left( \partial^\mu \partial_\mu + g e^{abc} A_k^b \right) \frac{\delta}{\delta A_k^a} \Psi = 0$$

which is the condition for invariance under infinitesimal gauge transformations. In strong-coupling lattice gauge theory, the ground state can be solved for systematically, order-by-order in powers of $1/g^4$ [1]

$$\Psi_0[U] = \exp \left[ - \sum_{n=1}^\infty \left( \frac{1}{g^4} \right)^n R_n[U] \right]$$

where the $R_n[U]$ are gauge-invariant expressions formed from Wilson loops on the lattice. If the sum in the exponent is truncated at order $n = M$, the resulting state solves the time-independent Schrödinger equation to $M$-th order in $1/g^4$, but satisfies the physical state condition exactly.

Similarly, we may guess that at weak couplings there is also a weak-coupling expansion for the ground state. This would have the form (in continuum notation, with lattice regularization implicit)

$$\Psi_0[A] = \exp \left[ - \sum_{n=0}^\infty g^{2n} Q_n[A] \right]$$

where the $Q_n[A]$ are gauge invariant, and the approximate solution, with the sum truncated to some order in $g^2$, satisfies the Schrödinger equation to that order. It is awkward to build the $Q_n[A]$ out of Wilson loops at weak couplings; the strong-coupling series suggests that such a construction would involve summation over infinite sets of loops in the continuum limit. Instead, we conjecture that that the building blocks of the $Q_n[A]$ are the covariant derivatives, and their associated covariant Green’s functions. If that is so, then the choice of $Q_0$ is essentially unique. We know that in the $g = 0$ limit, the ground state wavefunctional is just

$$\Psi_0[A] = \exp \left[ - \frac{1}{4} \int d^d x d^d y \left( \partial_i A_j^a(x) - \partial_j A_i^a(x) \right) \times \left( \frac{\delta_{ab}}{\sqrt{-g}} \right) \left( \partial_i A_j^b(y) - \partial_j A_i^b(y) \right) \right]$$

Then a state which solves the Schrödinger equation to zeroth order in the coupling $g$, but satisfies the Gauss Law constraint exactly, is obtained by simply replacing ordinary derivatives by covariant derivatives; i.e.

$$\Psi_0[A] = \exp \left[ - \frac{1}{4} \int d^d x d^d y F_{ij}^a(x) \left( \frac{1}{\sqrt{-g}} \right)^{ab} F_{ij}^b(y) \right]$$

where $D_k$ is the covariant derivative in the adjoint representation of the gauge group. Field strengths are of course commutators of covariant derivatives, so this expression can be regarded as a functional of covariant derivatives only.

The non-confining ground-state solution of the free field theory, eq. (6), is well-known. Many years ago, it was suggested by one of us [2] that, at large distance scales, the effective Yang-Mills vacuum state is simply

$$\Psi_{eff}^{0}[A] \approx \exp \left[ - \mu \int d^d x F_{ij}^a(x) F_{ij}^a(x) \right]$$

This vacuum state has the property of dimensional reduction; i.e. the computation of a space-like loop in $d + 1$ dimensions looks just like the computation of a loop in a $d$-dimensional Euclidean Yang-Mills theory. If both the vacua $\Psi_0^{(3)}$ of the $3 + 1$ dimensional theory, and $\Psi_0^{(2)}$ of the $2 + 1$ dimensional theory have this form, then the computation of a large planar Wilson loop in $3+1$ dimensions reduces, in two steps, to a computation in two-dimensional Yang-Mills theory, i.e.

$$W(C) = \langle \text{Tr}[U(C)] \rangle^{D=4} \approx \langle \Psi_0^{(3)} | \text{Tr}[U(C)] | \Psi_0^{(3)} \rangle \sim \langle \text{Tr}[U(C)] | \Psi_0^{(2)} \rangle \sim \langle \text{Tr}[U(C)] \rangle^{D=2}$$

\textsuperscript{1} See also Halpern [3] and Mansfield [4].
In $D = 2$ Euclidean dimensions the Wilson loop can be calculated analytically, and we know that there is an area-law falloff.

In Hamiltonian lattice gauge theory it is possible to calculate the vacuum state systematically in powers of $1/g^4$, and the result agrees with the above conjecture for the continuum theory. Further support for this form of the ground state wavefunctional was obtained via lattice Monte Carlo simulations [5]. In an interesting paper from 1998, Karabali, Kim, and Nair [6] came up with a strong-coupling expansion for the vacuum state of the continuum Yang-Mills theory in $D = 2 + 1$ dimensions (see also Leigh et al. [7]). This state, to leading order in $1/g^4$, also agrees with eq. (8).

If we take eq. (7) as the zeroth-order approximation to the weak-coupling ground state, and if the kernel $1/\sqrt{-D^2}$ would have a finite range, then the effective ground state at scales much larger than this range would have the dimensional reduction form [8]. In order to make this statement a little more precise in $D = 2 + 1$ dimensions, consider the decomposition of the field strength at fixed time, in the $x-y$ plane, into gauge-covariant and gauge-invariant factors:

$$F^{a}_{12}(x) = \phi^{a}(x) f(x)$$

$$\phi^{a}(x) = \frac{F^{a}_{12}(x)}{D F^{a}_{12}}, \quad f(x) = \sqrt{F^{a}_{12}(x) F^{a}_{12}(x)}$$

(10)

In SU(2) gauge theory $\phi^{a}(x)$ is a unit 3-vector in color space, which can be made to point in any direction, at any position $x$, by a suitable choice of gauge. In the $x-y$ plane, a gauge-invariant distinction between high and low-frequency field-strength components can be made in terms of the Fourier transform $f(k)$ of $f(x)$. For some reference momentum scale $\sigma$, define

$$f^{\text{low}}(x) = \int_{|k|<\sigma} \frac{d^2k}{(2\pi)^2} f(k)e^{ikx}$$

$$f^{\text{high}}(x) = \int_{|k|\geq\sigma} \frac{d^2k}{(2\pi)^2} f(k)e^{ikx}$$

$$F^{\text{low/}}^{\text{high,a}}_{12}(x) = \phi^{a}(x) f^{\text{low/high,a}}(x)$$

(11)

We will also write $\Psi_0 = e^{-R}$ with

$$R = \frac{\gamma}{2} \int d^2x d^2y f(x)\phi^{a}(x) \left( \frac{1}{\sqrt{-D^2}} \right)_{xy}^{ab} \phi^{b}(y) f(y)$$

(12)

Now, in the spirit of self-consistent field approaches, let us approximate the gauge-invariant expression $\phi(1/\sqrt{-D^2})\phi$ by its expectation value

$$S(x-y) = \left< \phi^{a}(x) \left( \frac{1}{\sqrt{-D^2}} \right)_{xy}^{ab} \phi^{b}(y) \right>$$

(13)

If the covariant vacuum kernel $1/\sqrt{-D^2}$ has a finite range $l_{K}$, then $S(x-y)$ has the same range, and its low-momentum components are approximately constant, i.e. $S(k) \approx \gamma$, for $|k| \ll \pi/l_{K}$. If we also choose $\sigma \ll \pi/l_{K}$, we find

\[
R = \frac{\gamma}{2} \int d^2x F^{\text{low,a}}_{12}(x) F^{\text{low,a}}_{12}(x) + \frac{1}{2} \int d^2x d^2y f^{\text{high}}(x) S(x-y) f^{\text{high}}(y)
\]

(14)

Then $f^{\text{high}}(x)$ decouples from $f^{\text{low}}(x)$, and the dependence of the wavefunctional on the long range, “low-momentum” component $F^{\text{low,a}}_{12}(x)$ of the field strength is given by the dimensional reduction form [8]. The fluctuations of $F^{\text{low,a}}_{12}(x)$ are then essentially local, since in $d = 2$ dimensions there is no Bianchi identity among field strengths which would induce a correlation. On the other hand, with a momentum cutoff at $|k| = \sigma$ there is an intrinsic short-distance cutoff, on the order $l = \pi/\sigma$, on the spatial variation of $f^{\text{low}}(x)$. Therefore, the dimensional reduction form for $F^{\text{low,a}}_{12}$ only implies that the correlation length, extracted, e.g., from the correlator

\[
(\text{Tr}[F^{\text{low}}_{12}(x)]^2) - (\text{Tr}[F^{\text{low}}_{12}(x)]^2))^2
\]

is less than $l$. This upper bound on the correlation length is reduced as $\sigma$ is increased, until, at $\sigma = \pi/l_{K}$, the Fourier modes $S(k)$ at $|k| < \sigma$ cease to be constant, and $f^{\text{low}}(x)$ does not fluctuate independently in regions separated by distances $l < l_{K}$. Thus the range $l_{K}$ of the vacuum kernel is also an estimate of the correlation length, which is the inverse of the mass gap. Denoting $m_{K} = l_{K}^{-1}$, we would then have $m_{K} \approx m_{0+}$, where $m_{0+}$ is the mass of the 0th glueball.

At first sight, however, there would seem to be no reason that $1/\sqrt{-D^2}$ should have a finite range. But in fact, for the covariant Laplacian in two Euclidean dimensions, there is good reason to believe that the corresponding Green’s function is finite range for almost any stochastic background $A_{\mu}$ field, no matter how weak. The relevant phenomenon is known as weak localization [10]. Consider a non-relativistic particle moving in a stochastic potential in one or two space dimensions. It is known that the eigenstates of the corresponding Hamiltonian $\mathcal{H}$ are all localized; i.e. they fall off exponentially outside some finite region, no matter how weak the stochastic potential may be. In that case, the inverse operator $\mathcal{H}^{-1}$ has a finite range. Weak localization is expected to occur also in cases where the stochasticity lies in the kinetic, rather than the potential term, and this seems to be relevant to the case at hand, i.e. the two dimensional covariant Laplacian. If all of the eigenmodes of $-D^2$ are localized, then both $1/(\sqrt{-D^2})$ and $1/\sqrt{-D^2}$ are necessarily finite range. In fact, the range is finite even if there is only an interval of localized eigenmodes at the low end of the spectrum, with the bulk of the states non-localized [10,12].

The range of the vacuum kernel $1/\sqrt{-D^2}$ is fixed by self-consistency. In two dimensions there is no Bianchi identity.

\footnote{For this reason it was suggested by Samuel, in ref. [3] that one could introduce a variational parameter $m^2$ into the kernel, i.e. $1/\sqrt{-D^2} + m^2$, as a way of introducing a finite range. The value of $m^2$ would be determined, in principle, by minimizing the energy expectation value of the trial vacuum state.}
so a correlation between field strengths at different points in space, at equal times, can only arise from the non-local character of the vacuum wavefunctional. As argued above, field-strength correlations are negligible on a scale at which the vacuum wavefunctional has the form $\psi_0$, and the wavefunctional $\psi_0$ takes this form on a scale which is roughly equal to the range of the vacuum kernel. This means that the correlation length of field strengths in a typical vacuum fluctuation (i.e. the inverse of the 0+ glueball mass), should approximate the range of the kernel $1/\sqrt{-D^2}$, but the range of the kernel, in turn, depends on the vacuum fluctuation.

The first question is whether, in a vacuum fluctuation typical of massless free field theory, having infinite range correlations among field strengths, the kernel can also have an infinite range. If the kernel has a finite range in such a background, then massless free field fluctuations are inconsistent; they do not arise from the behavior of the kernel that they imply. We check this behavior numerically in the following way: It is required to generate stochastically a gauge field background that would arise in a free theory, from a probability distribution $|\psi_0^{free}|^2$, where, in momentum space,

$$\psi_0^{free}[A] = \exp \left[ -\frac{1}{2} \int d^3k \ k A_i^\dagger(k) A_i^T(-k) \right]$$

The superscript $T$ means “transverse”. In this initial investigation the transversality restriction is ignored, and the continuum is replaced by a lattice. Let $-\nabla^2$ be the (ordinary) lattice Laplacian, with eigenfunctions $\{\phi_n(x)\}$ and eigenvalues $\{\lambda_n\}$, and let $\{s_{n,i}\}$ be a set of random numbers taken from a normal distribution with unit variance. Then a typical free-field vacuum fluctuation, deriving from the probability distribution $|\psi_0^{free}[A]|^2$ (with the transversality condition dropped) is

$$A_i(x) = \sum_{n=1}^{L^3} \frac{s_{n,i}}{\lambda_n^{1/4}} \phi_n(x)$$

This construction is repeated three times, for each of the three color indices, to form the field $A_i^\dagger(x)$.

Although no coupling constant is used to generate the free $A_i$ fields, there is still a Yang-Mills coupling constant present in the definition of the covariant derivative. In lattice regularization, this coupling constant relates the $A_i$-fields to the SU(2) link variables

$$U_i(x) = \exp \left[ ig \sum_{c=1}^{3} A_i^c(x) \sigma^c \right]$$

From the link variables in the fundamental representation, we then obtain link variables $A_i^a$ in the adjoint representation

$$A_i^{a} = \frac{1}{2} \text{Tr} [\sigma^a U_i(x) \sigma^a U_i^\dagger(x)]$$

The covariant lattice Laplacian, in the adjoint representation, is then

$$(\nabla^2)^{a} = \sum_{k=1}^{2} \left[ A_i^{a} k \delta_{\gamma x \gamma k} + A_j^{a} \delta(x-k) \delta_{y x \gamma -k} - 2 \delta_a \delta_{xy} \right]$$

Combining color and space indices, the covariant lattice Laplacian can be regarded as a sparse $3L^2 \times 3L^2$ matrix. Standard numerical routines are used to take the inverse square root of this matrix, to arrive at

$$G_{ab}(x,y) = \left( \frac{1}{\sqrt{-D^2}} \right)_{xy}$$

To calculate the range, we form the gauge-invariant combination

$$\tilde{G}(x,y) = \sqrt{\sum_{a,b} G_{ab}(x,y) G_{ba}(y,x)}$$

and average $\tilde{G}(x,y)$ over all points $x,y$ on the lattice with fixed $|x-y| = R$. Denote the result $\tilde{G}(R)$. A linear fit to $\log[\tilde{G}(R)]$ vs. $R$ determines the inverse $m_k$ of the range $l_k$ of the kernel.

The results, for couplings from $g = 0.1$ to $g = 2.0$ on a $32^2$ lattice are shown in Fig. 1 which plots $G(R)$ vs. $R$ on a semilog scale. The solid line is the result for $g = 0$ in the continuum, which is simply $G(R) = \sqrt{3}/(2\pi R)$. Even at $g = 0.1$, where the average link variable $\langle \frac{1}{2} \text{Tr} |U| \rangle = 0.997$ is very close to unity, there is a noticeable deviation from $1/R$ behavior, and at all larger $g$ values the $32^2$ lattice is big enough to clearly see an exponential falloff in $G(R)^3$.

This data indicates that $1/\sqrt{-D^2}$ is finite range, even in a massless free-field background. This implies that free fields, and probably any sort of non-confining vacuum fluctuations in 2 + 1 dimensions, are inconsistent. The kernel is finite range.

We note that the data at $g > 1.0$ also approaches a limiting line, approximately at the position of the lower line in Fig. 1. This simply means that, in the way we have generated the link variables, taking $g \rightarrow \infty$ does not result in a random link distribution equal to the Haar measure.

FIG. 1: The modulus of the vacuum kernel $1/\sqrt{-D^2}$ in a “free field” background, but with various non-zero values of the Yang-Mills coupling $g$ in the covariant Laplacian. The upper solid line is the kernel $G(R) = \sqrt{3}/(2\pi R)$ corresponding to $A_i^\dagger(x) = 0$ (or, equivalently, $g = 0$). Data at high $g$ tends to fall near the lower dashed line.
in general, dimensional reduction is obtained, and the simple vacuum wavefunctional \( \Psi \) is confining.

Given that the approximate ground state wavefunctional \( \Psi_0 \) of eq. (7) is confining, the next question is how closely this state approximates the true vacuum state of SU(2) Yang-Mills theory. We have already argued that in the state (7), \( m_K \approx m_{0+} \). If \( \Psi_0 \) is a good approximation to the true Yang-Mills vacuum state \( \Psi_{true}^0 \), then in the true vacuum state \( m_K \) and \( m_{0+} \) should still be approximately equal. To test this approximation, we calculate the range of the kernel \( 1/\sqrt{-D^2} \) in a 2D time-slice of a thermalized lattice, generated by Monte Carlo simulation of lattice Yang-Mills theory in D=3 Euclidean dimensions. This procedure makes use of the fact that if \( O[A] \) is an observable defined at fixed time \( t \), then

\[
\langle O \rangle = \frac{1}{Z} \int DA \langle A | e^{-S} \rangle = \langle \Psi_{true}^0 | O | \Psi_{true}^0 \rangle
\]  

Comparison of \( m_K \) with the known lattice results for the mass of the \( 0^+ \) glueball is one measure of how well the zeroth-order state \( \Psi_0[A] \) approximates the true ground state, since if \( \Psi_{true}^0 = \Psi_0 \), then \( m_K \approx m_{0+} \). The procedure for calculating \( m_K \) in the true vacuum is as follows: Link variables are generated by standard lattice Monte Carlo of \( D = 3 \) dimensional lattice gauge theory with a Wilson action, and the covariant Laplacian is evaluated in a time-slice. From this one calculates \( 1/\sqrt{-D^2} \) and \( G(R) \), as described above. Then \( \lim G(R) \) is fit to a straight line in the range \( R \in [\frac{1}{4}, \frac{1}{2}] \); the kernel mass \( m_K \) is just \(-1\) times the slope. The procedure is repeated for a number (> 10) of independent thermalized lattice configurations, and the results are averaged.

The values of \( m_K \) obtained in this way at a number of different lattice couplings \( \beta \) are shown in Fig. 2 where we have also displayed the \( 0^+ \) glueball masses that have been computed via lattice Monte Carlo by Meyer and Teper in ref. [9]. For the evaluations of \( m_K \) we have used lattice extensions \( L = 16, 24, 32, 40 \) at \( \beta = 6, 9, 12, 18 \) respectively. With the exception of \( \beta = 18 \), where Meyer and Teper used a slightly larger lattice \( (L = 50) \), these are the same lattice volumes used in ref. [9]. In the range of \( \beta \) studied, \( m_K \) and \( m_{0+} \) appear to converge as \( \beta \) increases. This is the behavior one would expect, if the true vacuum and the zeroth-order approximation \( \Psi_0 \) to the ground state approach one another in the weak coupling, \( \beta \to \infty \) limit.

Despite this numerical success, it cannot be true that the zeroth-order state and the true ground state agree in all essential ways. Since the zeroth-order state has the dimensional reduction form at large scales, the dependence of the string tension on the group representation of the quark-antiquark sources is the same as in \( D = 2 \) Euclidean dimensions, i.e. Casimir scaling. While this dependence is correct at intermediate distance scales, and also asymptotically in the \( N \to \infty \) limit, it just cannot be true asymptotically at finite \( N \), where the asymptotic string tension depends only on the \( N \)-ality of the quark sources. The simple zeroth-order state appears to be missing the center vortex (or domain) structure, which has to dominate the vacuum state at sufficiently large scales [13, 14]. So dimensional reduction is not the whole story at finite \( N \), and higher-order, \( 1/N^2 \) suppressed terms in the vacuum wavefunctional must come into play at sufficiently large distance scales. At present we have no systematic method for determining these higher-order terms.

It should be noted that there have been a number of efforts to determine the Yang-Mills vacuum wavefunctional beyond weak-coupling perturbation theory. These are mainly variational in character; see in particular [15, 16]. A non-variational approach is followed by Leigh et al. [7] in their recent work on the Yang-Mills ground state in \( 2 + 1 \) dimensions. There are some strong similarities between that work and the present paper. Leigh et al. arrive, by various manipulations, at a ground state which is the exponential of a bilinear expression, involving a finite range kernel connecting gauge-invariant variables related to the field strength. Due to the finite range of the kernel, the vacuum state derived in ref. [7] should have the dimensional reduction form \( \Psi_0 \) when viewed at large scales. This, of course, is very reminiscent of \( \Psi_0 \) in eq. (7). However, it is hard to compare these vacuum states directly. For one thing, the two states are expressed in terms of different variables; ref. [7] makes use of the Karabali-Nair variables in \( 2+1 \) dimensions. For another, the kernel of ref. [7] is gauge-invariant, rather than gauge-covariant, and has a finite range even for a vanishing background field. That is not at all like the behavior of the kernel \( 1/\sqrt{-D^2} \). Still, there are enough superficial similarities between these two states to make one suspect that they are related.

We would like to conclude with some remarks about \( 3+1 \) dimensions. The evidence presented above suggests that confinement in \( 2+1 \) dimensions is simply a consequence of the physical state constraint (i.e. gauge invariance), plus the free-field limit of the vacuum wavefunctional. The key point is that gauge invariance introduces a gauge covariant kernel \( 1/\sqrt{-D^2} \), and this kernel, unlike \( 1/\sqrt{-\nabla^2} \), has a finite range in almost any stochastic background, possibly due to the weak localization property in two Euclidean dimensions. The inter-

\[
0^+ \text{ mass} \\
\text{kernel mass}
\]

FIG. 2: A comparison, at various lattice couplings \( \beta \) in \( D = 2 + 1 \) dimensions, of the inverse range \( m_K \) of the vacuum kernel, and the mass of \( 0^+ \) glueball reported in ref. [8]. Errorbars are comparable to symbol size.
esting question is whether the gauge-covariant kernel still has a finite range, in a free-field background at fixed time, in the 3+1 dimensional theory. It is known that the spectrum of \(-D^2\) has an interval of localized states in \(D = 4\) Euclidean dimensions, in a confining background \([11, 17]\). But it is not known whether this operator has localized states in a free-field background in \(D = 3\) dimensions, or else whether the covariant kernel at \(D = 3\) is finite range for some other reason. If the kernel is, in fact, finite range, then on large scales \(|\Psi_0|^2\) effectively reduces to the probability distribution of a three Euclidean dimensional Yang-Mills theory, and the conjectured two-step dimensional reduction of eq. \(\ref{eq:dimensional_reduction}\), resulting in an area law falloff for large planar Wilson loops, would hold. The matrix operations would have to be carried out on \(3L^3 \times 3L^3\) matrices, rather than the \(3L^2 \times 3L^2\) size that appears in \(2+1\) dimensions. This additional computation cost, while restrictive, is probably not prohibitive.

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