Channel-Coupling Fano Resonance and Acoustic Metadamping

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Abstract

Fano resonance featuring asymmetric spectral profiles originates from the interference of local resonances and background continuum. Its narrow-band nature looks seemingly adverse to broadband noise cancellation purposes. In this study, we report theoretically on an intriguing acoustic metamaterial capable of generating multiple Fano-like resonances to realize a broadband sound barrier with satisfactory transmission loss performance. Our proposed design involves an effective channel characterized by effective parameters and short channels filled with air. The effective channel support both monopolar and dipolar modes which interact with the continuum state admitted by the short channels to generate a pair of Fano-like resonances. Due to the destructive interference of sound waves, the two resonances result in transmission loss overall exceeding 10 dB over a broad range 0.6-1.1 kHz. In order to further optimize the overall performance, we introduce metadamping by integrating additional viscous foams in the proposed unit cell. Furthermore, for future experimental tests, the dampened design is decoded into a real space-coiling cell which exhibits identical functionality and is assembled into a partition wall to ensure transmission loss over 10 dB across the range 0.32-4 Hz. Lastly, acoustic negative refraction is accessible by deploying two coupled space-coiling channels in a similar fashion. We believe this work paves the way for realizing effective broadband sound insulation devices with efficient ventilation.
1. Introduction

It has been challenging to manipulate efficiently the sound wave at low frequency [1-4], due to the limitation of traditional materials possessing innate properties and thus exhibiting performance restricted by nature principles [5-9]. Breakthroughs have been made using artificial macroscopic materials. Extreme, unnatural constitutive parameters can be effectively realized, such as negative density and bulk modulus originating from dipolar and monopolar resonances [10, 11], negative refractive index [12], extraordinary reflection and refraction from phase modulating materials [13], hyper-damping from structure buckling [14], and so forth. Local resonance is key to generating these unnatural properties. In acoustics, local resonances are generally established by satisfying the input impedance matching conditions (zero reactance) [5, 15]. The coupling resonance, on the other hand, is a result of the interaction of two or more sources under certain matching conditions. Antiresonance [16, 17] resulting from coupling between two adjacent resonant modes shows acoustically rigid response in a thin elastic membrane [10, 18]. Hybrid resonance in a perfect metasurface absorber is produced by the surface impedance matching [17]. Coupling resonance between two Helmholtz resonators commits perfect absorption as well [19] and can also create multiple double negative bands [20].

Fano resonance, first observed by Fano [21], is a special class of coupling resonance which, in most cases, refers to the interference of a local resonance and a background continuum. It comes with narrow-linewidth asymmetric absorption profile, induced by the intensity interaction and the phase-shift involved in inelastic scatterings of electrons in a quantum mechanical picture. In solid mechanics, the Fano or Fano-like profile has been discussed by several studies. It can be produced either by the scattering from the sided branch scatterer into the waveguide [22, 23] or by the destructive interference in a system of two coupling mass-spring resonators with external driving force and out-of-phase reaction force [24, 25]. However, in acoustics, to the best of our knowledge, a complete analysis on the formation of Fano resonances by the interference of monopolar/dipolar modes and a background one (continuum), along with the potential applications, are still yet to be established.

In this study, we propose theoretically an acoustic metamaterial supporting multiple Fano-like resonances to realize sound insulation application and acoustic double negative refraction. The unit cell design involves an effective channel and short channels. The effective channel generates monopolar and dipolar modes, while the short ones create a continuum state. They coupled to each other to generate a pair of Fano-like resonances, thanks to the destructive interference of sound waves. This pair of Fano resonances leads to acceptable transmission loss within a broadband range. We also use additional acoustic damping foam and decoding approach to optimize the overall sound insulation performance and provide a real strategy to construct the related acoustic partition wall. Lastly, acoustic negative refraction is numerically accessed by including two coupled space-coiling channels inside a single unit cell. We believe this work paves the way for realizing effective broadband sound insulation devices with efficient ventilation.
2. Coupling Fano resonances

Fig. 1(a) illustrates the schematic of an acoustic partition wall proposed to efficiently block the incoming sound waves. The periodically arranged, subwavelength meta unit [see the inset of Fig. 1(a)] serving as the building block is the basic functional unit of sound insulation application. They are designed to support Fano resonances which will be discussed later. To analyze the functionality of the proposed structure, we start with the unit cell in a single-mode tube environment. As shown in Fig. 1(b), the unit cell consists of two short channels (SCs), filled with air, and an effective long acoustic channel (EC), characterized by the effective constitutive parameters ($\rho_c$ and $c_c$). The SCs have shorter acoustic lengths than the EC does, and both of them can be treated later as Fabry-Perrot (FP) channels [5, 26, 27]. Specifically, the EC is a linear resonator supporting both monopolar and dipolar resonances. Since the SCs only resonate at high frequencies, they allow the acoustic pressure to flow through linearly in terms of phase change at low frequencies. The low frequency resonances supported by the EC thus interfere with the linear flow produced by the SCs. Around the resonances, phase profiles change dramatically. Once the pressure fields at the right ends of EC and SCs are out-of-phase, the degeneration occurs and the Fano resonance takes place. To fully investigate the resulting Fano resonance, we first derive a theoretical model of the proposed structure.
Fig. 1: (a) Schematic of the proposed acoustic partition wall for sound insulation application. The unit cell consists of an effective channel and the rigid parts. (b) Schematic of the coupling unit positioned in a single-mode acoustic tube. The effective and the short channels are indicated. (c) and (d) indicates the transformation from the coupling unit to the effective unit cell governed by $\rho_e$ and $c_e$. 
The averaged pressure fields at the left (inlet) and the right (outlet) ends of SCs and EC are expressed as

\[
P_{c1} = 2P_i + \frac{1}{b} \int_{-b/2}^{b/2} p_c^l(0,y) \, dy \\
P_{s1} = 2P_i + \frac{2}{d} \int_{a/2-d/2}^{a/2} p_s^l(0,y) \\
P_{c2} = \frac{1}{b} \int_{-b/2}^{b/2} p_c^r(0,y) \, dy \\
P_{s2} = \frac{2}{d} \int_{a/2-d/2}^{a/2} p_s^r(0,y)
\]

(1)

Here, \(P_i\) is the amplitude of the incident plane wave. The subscription “c”, “s”, “1”, and “2” denote the EC, SC, inlet and outlet, respectively. \(p_c^l\) and \(p_c^r\) are the radiation pressures at the left (upstream) and the right (downstream) domains, which relate to the velocities at the ends of the channels through Green’s functions [28]:

\[
p_c^l(x,y) = -\frac{i \rho c k \nu_c}{\alpha_c} \int_{-b/2}^{b/2} G_t(x,y|0,y_0) \, dy_0 - 2i \frac{\rho c k \nu_s}{\alpha_s} \int_{a-d/2}^{a} G_t(x,y|0,y_0) \, dy_0 \\
p_c^r(x,y) = \frac{i \rho c k \nu_c}{\alpha_c} \int_{-b/2}^{b/2} G_r(x,y|L,y_0) \, dy_0 + 2i \frac{\rho c k \nu_s}{\alpha_s} \int_{a/2-d/2}^{a/2} G_r(x,y|L,y_0) \, dy_0
\]

(2)

The Green’s functions in the upstream and downstream can be presented through the mode shapes of the waveguide [16, 29]:

\[
G_t(x,y|0,y_0) = \frac{1}{ik\alpha} \left\{ e^{ikx} + \sum_{n=1}^{\infty} \frac{\varphi_n(y) \varphi_n(y_0)}{\alpha_n \langle \varphi_n^2(y) \rangle} e^{\alpha_n x} \right\} \\
G_r(x,y|L,y_0) = \frac{1}{ik\alpha} \left\{ \frac{1}{2} \left[ e^{-ik|y-L|} + e^{-ik|y-x|} \right] + i \sum_{n=1}^{\infty} \frac{\varphi_n(y) \varphi_n(y_0)}{\alpha_n \langle \varphi_n^2(y) \rangle} \left[ e^{-\alpha_n|y-L|} + e^{-\alpha_n|y-x|} \right] \right\}
\]

(3)

Here, \(\varphi_n(y)\) and \(\alpha_n\) are the \(n\)th-order mode shape function and the eigenfrequency of the waveguide cross section [30, 31], respectively, and read

\[
\varphi_n(y) = \cos \left( \frac{2n\pi}{a} y \right) \\
\alpha_n = \sqrt{\left( \frac{2n\pi}{a} \frac{c}{\omega} \right)^2 - 1} \\
\langle \varphi_n^2(y) \rangle = \frac{1}{a} \int_{-a/2}^{a/2} \varphi_n^2(y) \, dy
\]

(4)

From Eq. (2), the pressures involved in Eq. (1) can be rewritten as

\[
P_{c1} = 2P_i - \rho c \{ \sigma_c \nu_c(1 + i\delta_{cc}) + \sigma_s \nu_s(1 + i\delta_{cs}) \}
\]

(5)
\[ P_{s1} = 2P_i - \rho c \{ \sigma_c v_{c1} (1 + i \delta_{cs}) + \sigma_s v_{s1} (1 + i \delta_{ss}) \} \]
\[ P_{c2} = \rho c \{ \sigma_c v_{c2} (1 + i \delta_{cc}) + \sigma_s v_{s2} (1 + i \delta_{cs}) \} \]
\[ P_{s2} = \rho c \{ \sigma_c v_{c2} (1 + i \delta_{cs}) + \sigma_s v_{s2} (1 + i \delta_{ss}) \} \]

where \( \sigma_c = b/a \) and \( \sigma_s = d/a \) are the space opening fractions of the EC and SCs, respectively. \( \delta_{cc}, \delta_{cs} \) and \( \delta_{ss} \) represent the crossing factors representing near-field interaction between channels, and are defined using the deviation of Green’s functions \( \delta G \) [16, 32]:

\[ \delta_{cc} = \frac{1}{b} \int_{-b/2}^{b/2} dy \int_{-b/2}^{b/2} \delta G(y, y_0) dy_0 \]
\[ \delta_{cs} = \frac{1}{b} \int_{-b/2}^{b/2} dy \int_{a/2-d/2}^{a/2} \delta G(y, y_0) dy_0 \]
\[ \delta_{ss} = \frac{2}{d} \int_{a/2-d/2}^{a/2} dy \int_{a/2-d/2}^{a/2} \delta G(y, y_0) dy_0 \]

Employing the expression of the mode shape given in Eq. (4), the crossing factors can be simplified as

\[ \delta_{cc} = \sum_{n=1}^{\infty} 2 \frac{\sin(n\pi \sigma_c)}{n\pi \sigma_c} \]
\[ \delta_{cs} = \sum_{n=1}^{\infty} \frac{2}{n\pi \sigma_s} \frac{\sin(n\pi \sigma_s)}{n\pi \sigma_s} \]
\[ \delta_{ss} = \sum_{n=1}^{\infty} (-1)^n \frac{2}{n\pi \sigma_c} \frac{\sin(n\pi \sigma_c)}{n\pi \sigma_c} \frac{\sin(n\pi \sigma_s)}{n\pi \sigma_s} \]

Assuming the channels are narrow, the pressures inside them can be estimated by the fundamental mode

\[ p_c(x) = A_c e^{-ik_c x} + B_c e^{ik_c (x-L)} \]
\[ p_s(x) = A_s e^{-ik x} + B_s e^{ik(x-L)} \]

Employing Euler’s equation, \( \rho \frac{\partial \vec{v}}{\partial t} = -\nabla p \), the coefficients in Eq. (8) can be presented, in terms of the velocities at the ends of the channels, as

\[ A_c = \rho c \frac{v_{c1} - \lambda_c v_{c2}}{1 - \lambda_c^2} \]
\[ B_c = \rho c \frac{\lambda_c v_{c1} - v_{c2}}{1 - \lambda_c^2} \]
\[ A_s = \rho c \frac{v_{s1} - \lambda v_{s2}}{1 - \lambda^2} \]
\[ B_s = \rho c \frac{\lambda \sigma_{s1} - \sigma_{s2}}{1 - \lambda^2} \]

in which \( \lambda = e^{-ikL} = e^{-i\alpha} \) and \( \lambda_c = e^{-ikcL} = e^{-i\alpha_c} \) are the phase shifting factors of the SCs and EC, respectively. Estimating Eq. (8) at the ends of the channels with the aid of Eq. (9) and comparing with Eq. (5) leads to the following to determine the normalized volume velocities (\( \tilde{v} \)) at the ends of the channels:

\[
\begin{bmatrix}
1 - i \left( \frac{1}{\sigma_c \tan \alpha_c - \delta_{cc}} \right) & \frac{i}{\sigma_c \sin \alpha_c} & 1 + i \delta_{cs} & 0 \\
\frac{i}{\sigma_c \sin \alpha_c} & 1 - i \left( \frac{1}{\sigma_c \tan \alpha_c - \delta_{cc}} \right) & 0 & 1 + i \delta_{cs} \\
1 + i \delta_{cs} & 0 & 1 - i \left( \frac{1}{\sigma_s \tan \alpha_c - \delta_{ss}} \right) & \frac{i}{\sigma_s \sin \alpha_s} \\
0 & 1 + i \delta_{cs} & \frac{1}{\sigma_s \sin \alpha_s} & 1 - i \left( \frac{1}{\sigma_s \tan \alpha_c - \delta_{ss}} \right)
\end{bmatrix}
\begin{bmatrix}
\tilde{v}_{c1} \\
\tilde{v}_{c2} \\
\tilde{v}_{s1} \\
\tilde{v}_{s2}
\end{bmatrix} =
\begin{bmatrix}
Z \\
0 \\
2 \\
0
\end{bmatrix}
\tag{10}
\]

where \( \tilde{v}_{c1}, \tilde{v}_{c2}, \tilde{v}_{s1} \) and \( \tilde{v}_{s2} \) are defined by

\[ \tilde{v} = \rho c \frac{\sigma}{B_1} \tag{11} \]

Considering \( |x| \to \infty \), Eq. (2) results in the transmission and reflection coefficients estimated in the far-field,

\[ p_t(x) = P_t (\tilde{v}_{c2} + \tilde{v}_{s2}) e^{-i k(x-L)} \quad \text{and} \quad p_r(x) = P_t (1 - \tilde{v}_{c1} - \tilde{v}_{s1}) e^{i k x} \tag{12} \]

respectively. Then the transmission and the reflection coefficients are readily obtained as

\[ T = \tilde{v}_{c2} + \tilde{v}_{s2} \]
\[ R = 1 - \tilde{v}_{c1} - \tilde{v}_{s1} \tag{13} \]

Theoretical model of unit cell containing either only SCs or only EC can be derived by simplifying Eq. (10). Now, we consider an effective homogeneous medium (EHM) representing the unit cell, as illustrated in Fig. 1(c) and 1(d). The EHM is acoustically characterized by effective mass density \( \rho_e \) and effective sound velocity \( c_e \), which can be retrieved using scattering matrix method [33].

\[
\begin{bmatrix}
P_1 \\
P_2
\end{bmatrix} =
\begin{bmatrix}
\cos(k_eH) & iZ_e \sin(k_eH) \\
iZ_e^{-1} \sin(k_eH) & \cos(k_eH)
\end{bmatrix}
\begin{bmatrix}
P_2 \\
P_2
\end{bmatrix}
\tag{14}
\]

Solutions to Eq. (14) lead to the effective velocity, impedance, mass density and bulk modulus reading, respectively,

\[ c_e = \frac{\omega}{k_e} = \frac{\omega H}{\cos^{-1} \left( \frac{P_1 U_1 + P_2 U_2}{P_2 U_2 + P_1 U_1} \right)} \]
\[ Z_e = i \frac{P_2^2 - P_1^2}{P_2 U_2 + P_1 U_1} \sqrt{\frac{(P_1 U_2 + P_2 U_1)^2}{(P_2^2 - P_1^2)(U_1^2 - U_2^2)}} \tag{15} \]
\[ \rho_e = \frac{Z_e}{c_e} \quad \text{and} \quad B_e = Z_e c_e. \]

The total pressures and velocities at the left (\( x = x_0 \)) and right (\( x = x_0 + a \)) ends can be further evaluated using Euler’s equation and Eq. (12) as
\[ P_1 = P_i \left\{ 2 \cos(kx_0) - (\vec{\sigma}_c + \vec{\sigma}_s)e^{ikx_0} \right\} \]
\[ U_1 = \frac{P_i}{\rho c} \left\{ (\vec{\sigma}_c + \vec{\sigma}_s)e^{ikx_0} - 2i \sin(kx_0) \right\} \]
\[ P_2 = P_i (\vec{\sigma}_c + \vec{\sigma}_s) e^{-ik(x_0 + H)} \]
\[ U_2 = \frac{P_i}{\rho c} (\vec{\sigma}_c + \vec{\sigma}_s) e^{-ik(x_0 + H)} \]

(16)

To validate the aforementioned analysis, the unit cell with \( a = 88.89 \text{ mm}, b = 11 \text{ mm}, d = 26 \text{ mm}, L = 63 \text{ mm} \) and \( \rho_c/\rho = c/c_c = 4.375 \) is investigated numerically and theoretically. To investigate the Fano resonance involved and its associated sound insulation application, transmission loss (TL), defined as \( 20 \times \log_{10} |p_i/p_t| \), is assigned as a measure of the sound insulation performance. The existence of Fano resonances results in Fano profiles in the TL spectrum. Excellent agreement between the analytically and the numerically obtained TLs is observed in Fig. 2, which proves the correctness of our theoretical model. Since the space opening fraction, \( \sigma_s = 29.25\% \), is quite large, the unit cell composed of only SCs possesses low TL. In the case of only EC, the TL gets significantly enhanced thanks to a much smaller opening fraction of 12.38\%. Moreover, the EC, at its resonances, acts as either a monopole or a dipole to pump the acoustic pressure from the upstream to downstream, due to impedance matching at the inlet of the channel. As a result, the TL reaches dips at the resonance frequencies (595 Hz for the monopolar mode and 1190 Hz for the dipolar mode); see Fig. 2. With both SCs and EC included, the performance is greatly optimized, compared to the other two scenarios. Specifically, two sharp peaks at 658 Hz (monopole) and 1089 Hz (dipole) appear. The interaction between the EC and SCs, represented by \( 1 + i\delta_{cs} \) in Eq. (10), shifts the EC resonances. Specifically, the dip corresponding to the monopole mode slightly redshifts to 560 Hz, while the one corresponding to the dipole mode blueshifts to 1210 Hz. Enhancement of the TL is visible within the range between the two resonant frequencies. Due to short acoustic lengths of SCs, at low frequencies, the upper stream, SCs and downstream operate collectively as an effective waveguide, which supports an acoustic linear flow in term of phase change (background mode). The EC acts as a resonator supporting both monopolar and dipolar modes of significantly larger Q-factors than the background mode. When interfering with the acoustic background mode, the sharp resonances created by the interference exhibit asymmetric TL profiles, corresponding to Fano resonances.
Examining Eq. (13) reveals the condition for the occurrence of TL dips: \( \bar{\nu}_{s1} + \bar{\nu}_{c1} = 1 \), meaning that \( \bar{\nu} \) at the inlets of the SCs and EC are complementary. In other words, the total volume velocity passing through the inlets is equal to the incident volume velocity, which obviously refers to the acoustic impedance matching condition. The TL, on the other hand, reaches maximums when the transmission coefficient vanishes. The condition for the TL peaks reads \( \bar{\nu}_{c2} + \bar{\nu}_{s2} = 0 \), which indicates the normalized volume velocities at the outlets of the SCs and EC are identical in amplitude, but out-of-phase. From another perspective, the normalized acoustic admittance at the outlet of the coupling unit, \( 1/Z_2 = 1/Z_{s2} + 1/Z_{c2} = \bar{\nu}_{s2}/p + \bar{\nu}_{c2}/p \), is zero, indicating that the normalized acoustic impedance \( Z_2 \to \infty \), and the outlet now behaves as an acoustically rigid wall. In this case, the unit cell blocks almost all the incoming pressure wave, leading to nearly zero transmission coefficients.
The TL dips and peaks are intrinsically in consequence of the formation of Fano resonances. To deepen the understanding of the physical mechanism, we then plot the phase and amplitude of $\mathbf{\bar{\sigma}}$ at the channel outlet in Fig. 3(a) and 3(b), respectively. Theoretically, for lossless resonant structures, an abrupt phase jump $\pi$ occurs when experiencing a resonance [24]. Due to large damping effect of the EC, defined by the radiation factor $b/a = \sigma_c = 12.4\%$, the phase jump $\arg(\mathbf{\bar{\sigma}_{c2}})$ only reaches about $0.8\pi$ following two low-slope phase profiles corresponding to the monopolar and dipolar modes. Since the acoustic length of SC is shorter than that of the EC, the phase $\arg(\mathbf{\bar{\sigma}_{c2}})$ decreases slower than $\arg(\mathbf{\bar{\sigma}_{s2}})$. As for the phase delay between EC and SCs, i.e. $\arg(\mathbf{\bar{\sigma}_{c2}})-\arg(\mathbf{\bar{\sigma}_{s2}})$, it reaches $\pi$ at points A, B and C right after monopole, before dipole and in the middle, respectively. Now, let us investigate the amplitudes of $\mathbf{\bar{\sigma}}$. Due to zero reactance at the ends of the EC at its resonances, while $|\mathbf{\bar{\sigma}_{c2}}|$ reaches the local maximums at the monopolar and dipolar resonances, $|\mathbf{\bar{\sigma}_{s2}}|$ exhibits correspondingly two local minimums. The amplitude of the downstream normalized volume velocity, $|\mathbf{\bar{\sigma}_{c2}} + \mathbf{\bar{\sigma}_{s2}}|$, takes maximum values (unity) at points D and E. These two points correspond to the TL dips discussed previously. Furthermore, $|\mathbf{\bar{\sigma}_{s2}}|$ crosses $|\mathbf{\bar{\sigma}_{c2}}|$ at points A and B featuring an out-of-phase pair of $\mathbf{\bar{\sigma}_{c2}}$ and $\mathbf{\bar{\sigma}_{s2}}$. As a result, $|\mathbf{\bar{\sigma}_{c2}} + \mathbf{\bar{\sigma}_{s2}}|$ drops to zero at these points, and maintains of small values between points A and B, where $|\mathbf{\bar{\sigma}_{c2}}|$ and $|\mathbf{\bar{\sigma}_{s2}}|$ are nearly out-of-phase as well. Consequently, the TL spectrum shows two sharp peaks at these two points. The TL peaks at the Fano resonances can also be explained using the effective medium representation. Following Eq. (14) and (15), we plot the normalized effective density $\rho_e/\rho$ and bulk modulus $B_e/B$ in Fig. 3(c). Two separate single-negative stop bands (SNBs), corresponding to a negative density region (590 - 733 Hz) and a negative bulk modulus region (936 - 1178 Hz), can be witnessed. The normalized decaying factor (NDF) $-\text{Im}\{k_e\}/2\pi$, where $k_e$ denotes the effective wave number, is also presented in Fig. 3(c). It is realized that $\text{Im}\{k_e\}$ is zero everywhere except within the SNBs, whereas $\text{Re}\{k_e\}$ (not shown here) is zero within SNBs and positive at other frequency. Within the SNB regions, the NDF takes large values, meaning that the propagating wave exponentially decays. In brief, the Fano resonances results in two SNBs providing considerably high TL, efficiently blocking sound fields.

Graphical illustration for the analysis above can be found in the pressure and velocity distribution fields at the two Fano resonances [see Fig. 3(d)-(g)]. It is clearly seen that the SCs and EC operate as two acoustic pumps with out-of-phase profiles. While the latter supports monopolar and dipolar modes, the former shows opposite responses accordingly. Therefore, with an excitation, the downstream radiation fields from SCs and EC are out-of-phase, and thus experience destructive interference. More specifically, the acoustic field of the channels are in-phase at the inlet, but out-of-phase at the outlet, as observed from Fig. 3(f)-(g). The out-of-phase velocity fields create a near-field oscillating acoustic flow between the outlets of the EC and SCs which constructs an acoustic shield at the outlet, blocking all the incoming pressure.

3. Dampening Fano resonances

The Fano resonances exhibits asymmetric profiles featuring TL dips and TL peaks. For the practical noise isolation applications, it is favored to have overall optimized TL profile rather than the fluctuating one. Since the relation between the dip and peak is causality, mitigating dips means dampening the local resonance of EC, which consequently lowers the TL peaks. With a certain amount of damping in the EC, an optimal TL profile is expected. Here, acoustic foams are placed inside the EC to introduce damping so as to achieve a less fluctuating TL profile. The acoustic foam is an open-cell, two-phase, porous material,
comprising of elastic matrix (solid phase) and the fluid phase inside the matrix. The well-known Biot’s model \([34, 35]\) provides the most comprehensive model to describe the vibro-acoustics of the coupling between two phases. It requires several experimental inputs, and the corresponding calculation is quite time-consuming. Given this reason, the effective medium theory is appropriately used to model equivalently the porous acoustic material. Among all the studied models, the five-parameter model proposed by Johnson-Champoux-Allard (JCA) is the most widely used as it provides simplicity and high accuracy. In the JCA model, the mass density characterizing the visco-inertial effects was proposed by Johnson et al \([36]\) as

\[
\tilde{\rho}(\omega) = \frac{\tau_\infty \rho}{\varepsilon_p} \left[ 1 + \frac{\varepsilon_p \phi_f}{i \omega \rho \tau_\infty} \sqrt{1 + i \frac{4 \rho \omega \eta \tau_\infty^2}{\varepsilon_p^2 \phi_f^2 \Lambda_v^2}} \right]
\]  (17)

The bulk modulus reads, by Champoux and Allard \([37]\),

\[
\tilde{B}(\omega) = \frac{\gamma P_0 / \phi_f}{\gamma - (\gamma - 1)} \left[ 1 - i \frac{8 k}{\rho \omega C_p \Lambda_v^2} \right]^{1/2} \left[ 1 + i \frac{\rho \omega C_p \Lambda_t^2}{16 k} \right]^{1/2}
\]  (18)

where, \(\rho, C_p, \gamma, \eta\) and \(k\) denote the mass density, heat capacity at constant pressure, ratio of specific heats, dynamic viscosity and thermal conductivity of the fluid phase, respectively. Other parameters of the JCA model used to describe the acoustic foam employed in this research are referred from Ref.\([38]\) and listed in Table 1.

| Properties                  | Symbol | Value      |
|-----------------------------|--------|------------|
| Porosity                    | \(\varepsilon_p\) | 95%        |
| Flow resistivity            | \(\phi_f\) | 25000 N \cdot s/m^4 |
| Viscous characteristic length | \(\Lambda_v\) | 93 \(\mu\)m |
| Thermal characteristic length | \(\Lambda_t\) | 93 \(\mu\)m |
| Tortuosity factor           | \(\tau_\infty\) | 1.4        |

Table 1: Porous matrix properties of the acoustic porous material

Thanks to the large porosity of acoustic foams, they are weak at blocking sound waves, meaning that they are suitable for dampening or absorption applications. To optimize the damping effect, we deploy the acoustic foams at the velocity antinodes inside the EC. From Fig. 3(d)-(e), the velocity antinodes are located at the nodes of pressure mode shapes (the inlet and outlet for the monopolar mode, and the inlet, outlet and center for the first dipolar mode). With the foams deployed, the velocity fields inside the channel decay, and the resonances therefore get weakened. Consequently, the dampened Fano resonances will inevitably lower the TL peaks but simultaneously enhance the TL dips, producing overall strong TL profile. The thickness of the foam plays an important role in optimizing the TL profile.
Fig. 4: The effect of acoustic foam on the performance of the coupling structure.

From Fig. 4, it is clearly seen that employing the acoustic foam does make the TL profile less severe. Increasing the foam thickness ($t_f$), which intuitively makes the damping effect heavier, leads to relatively uniform TL profile. This is the evidence of the metadamping in the proposed unit cell. When $t_f = 3$ mm, the peaks and dips are nearly unrecognizable due to the over dampened EC. Further increasing the foam thickness will eventually make the TL profile approach the case “S-S” (unit cell with only SCs) presented in Fig. 2. This is simply because the foam filling the EC results in high impedance, driving the EC to perform as a solid part rather than an acoustic channel.
Before studying the partition wall, decoding the discussed unit cell [see Fig. 5(a)] into a real structure is necessary. To achieve this goal, the space coiling cell (SCC) design is employed in this research. As shown in Fig. 5(b), the unit cell size is kept unchanged. The EC is transformed into a space-coiling channel such that the acoustic length remains unchanged. To validate the decoded model, the TL spectra of both models are presented in Fig. 5(c). The results, which exhibit great agreement between both models, well proves the correctness and accuracy of the decoding method and the resulting SCC.

Next, we employ the SCC to form the partition wall possessing both efficient ventilation and sound isolation function across a broad band 0.25-4 kHz. It covers most of the typical airborne noise frequencies in living and working environments. The proposed design of the partition wall, as shown in Fig. 6(a), consists of supercells detailed in Fig. 6(b). The supercell comprises of three SCCs with the same open faction, i.e. $\sigma_c = 1 - L/a = 29.25\%$, but different coiling channel widths (5.9 mm, 7.9 mm and 11 mm). The reason for doing so is to generate multiple Fano frequencies distributed nearly uniformly over the frequency range of interest. To get rid of the sharp dips caused by the resonances, 2 mm foam layers are employed [see Fig. 6(c)] to dampen the resonances. COMSOL Multiphysics is utilized to simulate the soundproof wall for both undampened and dampened cases. The results for normal incidence are plotted in Fig. 6(d). In the undampened scenario, plenty of TL peaks and dips exist due to the excitation of Fabry-Perot resonances/antiresonances in the horizontal direction. Whereas the dampened case presents overall much better performance with TL dips mitigated and removed, although most of the peaks are dampened and even disappear. Some dips are even converted into peaks (at 1234 Hz corresponding to the monopolar mode of the second SCC and at 1590 Hz corresponding to the monopolar mode of the first SCC. After all, the SCC design, together with the dampening operation, provides moderate yet optimal TL profile within the

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**Fig. 5**: (a) – the effective coupling channel, (b) - the acoustically equivalent real space-coiling structure, and (c) – the comparison between the effective and real structure models.
2200 Hz wide frequency range, leading to acceptable sound insulation performance while preserving a certain ventilation efficiency.

**Fig. 6:** (a) - Soundproof wall comprise three rows of resonators, (b) – A detail on a unit-cell of the wall, (c) – The unit-cell is dampened, and (d) - The performance of the wall and damped wall.

4. **Double negativity from couple of Fano resonances**
As analyzed previously, the Fano resonances produce single-negative bands. By overlapping a negative density band on a negative bulk modulus band, a double negative band (DNB) can be derived. Inspired by this, a coupled-channel SCC formed by combining two single-channel SCCs is proposed in Fig. 7(a). The dimensions of the coupled-channel SCC, H and L, are the same as those in Fig. 2. The opening, s, is the tuning parameter to adjust the Fano resonance of channel 2 in order to overlap the two SNBs generated by the two channels. Specifically, channel 1 is a uniform coiling channel with \( h_1 = 4.17 \) mm, while channel 2 is a non-uniform coiling channel with \( h_2 = 6.75 \) mm and \( s = 11.25 \) mm. To investigate the possible DNB, the effective density and bulk modulus of the coupled-channel SCC are then semi-analytically calculated using parameter retrieved method. From Fig. 7(b), other than the SNB possessing single negative bulk modulus, we observe an overlap of two SNBs representing both negative density and negative bulk modulus. In principle, these two SNBs together contribute to the formation of a DNB (highlighted in grey) which features a pass band rather than stop bands. In order to examine its pass band property, we array 8 coupled-channel SCCs and send in plane wave excitations to obtain the transmission spectrum shown in Fig. 7(c). As can been readily seen, the highlighted region shows stop bands for solely channel 1 or 2, since either of them can only produce a SNB of single negative parameter when operating alone. When they cooperate, i.e., in the presence of coupled-channel SCC, the stop bands are converted into a pass band with almost unitary transmission. This well prove the existence of the DNB. Lastly, we choose one of the notable examples of double negative materials, namely the negative refraction, to graphically demonstrate the DNB. Fig. 7(d) and 7(e) show the pressure field distribution at 950 Hz for the effective medium and the real SCC structures, respectively. The pressure field gets refracted negatively twice at the interfaces between air and the SCC wall, apparently ensuring the existence of the DNB. Overall, good agreement can be found between the two models, although some minor discrepancies can be observed. This can be greatly improved through shifting the DNB to lower frequencies by carefully engineering the geometrical parameters of the coupled-channel SCCs.
V. Conclusion

The Fano resonances resulting from the coupling between SCs and EC have been fully investigated. At resonances, SCs and EC operate as out-of-phase acoustic pumps, which produce destructive interference of the velocity fields at the channel outlets. It is well explained using the developed theoretical model. Each Fano resonance corresponds to a SNB, i.e. negative density and bulk modulus match to dipolar and monopolar resonances respectively, where the imaginary part of the wavevector is negative and reaches to maximum absolute values at the Fano resonant frequencies. In addition, acoustic metadamping has been introduced by deploying very thin acoustic foam layers at the velocity antinodes inside EC. Effective unit-cell study shows the metadamping not only blunts the peaks, but also uplifts the dips and zero region of NDF. Consequently, an ultra-broadband TL has been observed in the proposed soundproof wall based on the metadamping design. All in all, the proposed the Fano resonances, acoustic metadamping and associated theoretical model lay a strong foundation for developing partially open and high-performance soundproof wall, and sound absorbers.

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