Low-cycle fatigue life prediction of a variable thickness disk

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Abstract. The turbomachines are characterized by complex geometry of the main parts and extreme operation conditions. The review of the testing results indicates that in most cases the failure of turbine disks and blades is caused by low- and high-cycle fatigue. We included the experimental dependences of the material properties of temperature, time and number of loading cycles in the fatigue life calculation. This enabled us to observe the kinetics of the part stress-strain state which is present even under constant external load. The observed increase of strain (decrease of stress) at the constant external load indicates the process of cyclic damage accumulation in the disk material. The plastic strains determine the damage fields (distribution of damage in the part material) which in turn allow predicting the area of initiation and probable growth direction of a fatigue crack in a part. We chose a material damage model, and numerically calculated the damage field by taking into account the change of the part stress-strain state with the number of cycles. The damage fields allow determining the point life (time or number of cycles to failure since the moment when a crack passes through a given point), the number of cycles before crack initiation, number of cycles to failure with a crack, crack propagation speed in the damaged and not damaged material. Finally, we determined the permissible number of cycles, time to failure and crack length with a given safety factor. The numerical investigations of the cyclic failure of machine parts should be conducted taking into account the kinetics of the accumulated damage fields.

1. Introduction

The improvement of the service life of gas-turbine engines and their reliability is an important issue of ensuring the safety, reliability, and efficiency of transportation. The life prediction of a machine part has to take into account the real operating conditions, the kinetics of the mechanical properties and damage of the part material. Taking into account the cumulative damage allows correcting the predicted number of cycles to failure which creates prospects of reducing the construction safety factor. The numerical solution also allows estimating the remaining service life of the part after crack initiation, which will allow extending the operating time for the part [1-6].

The low-cycle fatigue of machine parts is mostly determined by material properties, part geometry, and operation conditions [1-11].

The most common and tested method of predicting low-cycle fatigue life is based on stress-kinetic criterion taking into account the kinetics of the material mechanical properties, creep and relaxation. The low-cycle fatigue life is calculated based on the analysis of the damage determined by linear accumulation of quasi-static and fatigue damage. There is considerable amount of research on fatigue analysis based on damage fields done by I. A. Birger, N. A. Makhutov, M.A. Daunis, M. D. Novopashin, V. V. Zatsarinnii [1, 2, 5, 6].
2. Kinetics of stresses and strains of a variable thickness model disk

The flowchart of the algorithm for analyzing the stress-strain state (SSS) and its kinetics is presented in figure 1.

![Flowchart of the algorithm for analyzing the stress-strain state and its kinetics.](image)

**Figure 1.** Flowchart of the analysis of the stress-strain state and its kinetics.

We used the proposed algorithm to analyze the SSS kinetics for calculating the fatigue life of a model disk (See figure 2) with blades on its periphery [1-3]. The disk rotates with angular speed \( \omega \). It is non-uniformly heated to temperature \( t(r) \) in the radial direction.

![Variable thickness model disk with blades.](image)

**Figure 2.** A variable thickness model disk with blades.

The variable thickness disk with a central cylindrical hole with radius \( r_0=0.025 \) m is mounted on a shaft without interference fit. The radial dimensions are \( r_1=0.05 \) m, \( r_2=0.2 \) m, \( r_d=0.21 \) m; the thickness dimensions are \( h_1 = 0.052 \) m, \( h_2 = 0.035 \) m, \( h_3 = 0.0177 \) m, \( h_4 = 0.022 \) m. The disk and the blades are made of EI437b alloy. At \( \omega=0 \) the disk temperature is uniform \( (t|_{\omega=0}=t_0=20^\circ C) \). The startup time is \( \tau=4.5 \) s. At rated mode the speed is \( n=15000 \) rpm and the temperature is \( t(r_0)=-150^\circ C \) at inner radius, \( t(r_d)=520^\circ C \) at outer radius. The disk temperature field is axis-symmetrical and depends only on the radius \( r \). It is constant throughout the disk thickness. We assume that the coefficient of thermal expansion \( \alpha \) is constant; the disk temperature at steady state is a linear function of \( r \); the angular
velocity $\omega$ is a linear function of time during startup and slow down stages. The number of blades is $Z=47$.

The first stage of predicting the part fatigue life is calculation of its stress-strain state. The stress-strain state is either determined experimentally or numerically (using an elastic solution as a first approximation). The common numerical methods include the Finite Element Method and Finite Difference Method. The Boundary Element Method is also used. It allows reducing the dimensionality of a problem.

The next stage is identifying plastic strain zones in the part for the given thermal-mechanical load using different criteria. If a criterion indicates the presence of plastic strain in the whole part on in certain area of it, then an elasto-plastic analysis is conducted. There are several ways to conduct the elasto-plastic analysis:

1. Using intensity coefficients to calculate the corresponding stresses and strain in the elasto-plastic region;
2. If the stress-strain state within elastic limits is known, then the method of variable elasticity parameters can be used to conduct elasto-plastic analysis of the part. However, the material stress-strain curve and the temperature dependence of yield strength $\sigma_{0.2}$ and ultimate strength $\sigma_u$ must be known in order to use this method;
3. Using plasticity theory for explicit calculation of the elasto-plastic stresses and strains in the part.

The elasto-plastic stress-strain state calculated using the method of variable elasticity parameters is shown in figure 3. The plots indicate that the stress intensity $\sigma_i$ and strain intensity $\varepsilon_i$ are maximum for $r = 0.025$ m. It is safe to assume that the crack will be initiated in this region.

![Figure 3](image)

**Figure 3.** Elasto-plastic stress-strain state of the disk, $n = 15000$ rpm, $t = t(r)$: stress distribution (a) and strain distribution (b) along radius $r$.

We can determine the kinetics of stresses and strains if we know the relationships between material properties (consequently, stresses and strains) and the number of loading cycles (half-cycles) [6]. The other way to do this is to use experimental data on the kinetics of material properties and embed this data in the elasto-plastic analysis.

We determined the kinetics of the strain and stress intensity in the most loaded region at $r = 0.025$ m (see figure 4) by taking into account the fatigue kinetics of the main mechanical properties (consequently, stress-strain curve) with the number of cycles $N$. The analytic relations were acquired by approximating the openly published experimental data. This data allows determining the main mechanical properties as a function of stress intensity amplitude $\sigma_a = f(N)$. 

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\sigma_a = f(N)
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Figure 4. Kinetics of stresses (a) and strains (b) of the disk at $r = 0.025$ m.

The strain kinetics analysis based on the relationship between mechanical properties and number of cycles and stress intensity amplitude allowed concluding that the strain intensity increased by a factor of 1.5 if the number of cycles $N$ is less than 20.

3. Crack initiation life and fatigue life

The analysis of stress and strain intensity will serve as basis as we pass over to predicting the strength and fatigue life under low-cycle thermomechanical loading (figure 5).

![Flowchart]

Figure 5. Flowchart of the fatigue life prediction algorithm.

In order to predict the crack initiation life we first need to identify and calculate the damage fields in the material and determine the kinetics of the damage fields based on the analysis of stress-strain...
state kinetics. The damage is given by linear accumulation of cyclic and static damage components for non-isothermal low-cycle loading [1, 5, 6]:

\[ d = d_f + d_s \]  \hspace{1cm} (1)

If the material is loaded at elevated temperature the damage is given by in terms of strain:

\[ d = \int_0^{N_f} \left( \frac{2e_a^{(k)}}{e_f^{(k)}} \right)^2 dN + \int_0^{e_f} \frac{d e}{e_f^{(k)}} \]  \hspace{1cm} (2)

where \( e_f \) - failure strain, \( e_a^{(k)} \) - amplitude of cyclic strain intensities.

We should note the equation (2) allows calculating damage for both low- and high-cycle fatigue. The crack initiation life is the life when the cumulative damage is equal to one. The other way to determine the crack initiation life is determining crack initiation life scatter using hard and soft loading schemes.

Based on the strain kinetics, we determine the damage in the calculation points and the damage kinetics in them (see figure 6).

**Figure 6.** Damage \( d \) and strain \( \varepsilon_i \) vs radius \( r \).

We determined the crack initiation life \( N_0 = 168 \) when the cumulative damage was equal to one [5, 6]. The figure 6 shows the plots of cumulative damage \( d \) and strain \( \varepsilon_i \) vs disk radius \( r \) when a crack is initiated at the edge of the central hole.

The final stage of the calculation is determination of the crack length, crack growth rate, and the number of cycles to failure. The crack growth rate and crack length can be determined for the given number of cycles by taking into account the decrease of the failure strain \( e_f \) to the value of \( e_{fd} \) because of cumulative damage \( d \) [5, 6].

We used the model presented in [6] where we determined the stress-strain state after crack initiation and neglect the discontinuity in the material. Then we took into account the change of material properties by changing the failure strain (ultimate plastic strain) \( e_f \) according to the cumulative damage in each calculation point of the part. We continued taking into account the damage accumulation in the other calculation points by reducing \( e_f \) to \( e_{fd} \). At the same time, we determined the crack length \( l \) corresponding to the given number of cycles \( N \).

If we observe the crack growth in the model disk of variable thickness, we should note the following. If there are two calculation points on the same radius, then the cumulative damage is going to be greater (the corresponding fatigue life is going to be less) for the point with the least disk thickness. Then we set the equality of the cumulative damage to zero as a failure criterion and calculated the number of cycles to failure \( N_f \). It is equal to 1960 cycles (see figure 7). The crack is propagating from the edge of the central hole to the disk rim. We should note that the prediction of the
crack initiation life $N_0$ and fatigue life $N_f$ was conducted taking into account the cumulative damage fields.

![Figure 7](image)

**Figure 7.** Crack length $l$ vs number of cycles $N$ for the model variable thickness disk.

The plot in figure 7 clearly indicates that the crack length starts increasing rapidly when the number of cycles approaches the number of cycles to failure $N_f$. This relationship was obtained taking into account the damage fields.

4. **Conclusions**

Thus, prediction of low-cycle fatigue requires data on the kinetics of the stress-strain state in the areas of the analyzed part having maximum stress. In real or model conditions this data can only be acquired experimentally which takes funding. Therefore, numerical and numerical-experimental methods mentioned above are more promising ways of estimating the strain fields that determine the damage fields.

The fatigue life of machine parts was determined with an algorithm, each stage of which can be implemented numerically or based on the experimental data. Based on the method proposed in [5, 6], we conducted a numerical calculation of strength and life characteristics of the variable thickness model disk under thermal-mechanical loading. The calculation is based on the analysis of cumulative disk damage kinetics. The numerical experiment yielded not only the values of crack initiation life and fatigue life, but also the crack initiation region and the crack growth directions.

**References**

[1] Makhutov N A 2004 Fatigue of metals in a wide range of cycles *Industrial Laboratory. Diagnostics of Materials* **70**(4) 37-41 (in Russian)

[2] Makhutov N A and Gadenin M M 2005 Investigation of nonlinear effects of deformation and failure criteria *Industrial Laboratory. Diagnostics of Materials* **71**(8) 57-67 (in Russian)

[3] Sosnovsky L A and Makhutov N A 2005 General approach to estimating the damage intensity at cyclic loading, friction and complex loading *Industrial Laboratory. Diagnostics of Materials* **71**(2) 38-40 (in Russian)

[4] Makhutov N A, Kossov V S and Oganyan E S 2007 On the issue of predicting operating life and safety of rolling stock constructions *Industrial Laboratory. Diagnostics of Materials* **73**(11) 43-47 (in Russian)

[5] Veretimus N K and Veretimus D K 2013 Numerical and experimental estimation of the damage accumulation under non-isothermal loading *Industrial Laboratory. Diagnostics of Materials* **79**(2) 51-55 (in Russian)

[6] Veretimus N K and Veretimus D K 2015 Effect of the cumulative damage on the designed rate of the low-cycle crack *Engineering Journal: Science and Innovation* **11**(47) (in Russian) http://engjournal.ru/articles/1425/1425.pdf

[7] Makhutov N A, Permyakov V N, Kravtsova Yu A and Botvina L R 2007 Estimation of the
product pipeline condition after long-term operation Industrial Laboratory. Diagnostics of Materials 73(2) 54-57 (in Russian)

[8] Makhutov N A, Petrov V P, Kuksova V I and Moskvitin G V 2008 Modern trends of scientific research on the issue of mechanical engineering and machine science Engineering and Automation Problems 3 3-19 (in Russian)

[9] Makhutov N A and Reznikov D O 2008 Estimation of vulnerability of technical systems and its place in the risk analysis procedure Issues of Risk Analysis 5(3) 76-89 (in Russian)

[10] Makhutov N A, Lisin Yu V, Gadenin M M, Permyakov V N, Fedota V I and Aladinsky V V 2012 Provision of security of main pipelines by risk criteria Science & Technologies: Oil and Oil Products Pipeline Transportation 3 10-16 (in Russian)

[11] Kulagin V A, Moskvichev V V, Makhutov N A, Markovich D M and Shokin Yu I 2016 Physical and mathematical modelling of high speed hydrodynamics on the experimental base of the Krasnoyarsk hydroelectric power station Herand of the Russian Academy of Sciences 86(11) 978-990 (in Russian)