Shock Diffusion Analysis for a Directed Market Network Constructed with Use of the Risk Measure $\Delta\text{CoVaR}$

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Abstract. This paper studies a complex network formed as a directed graph in which nodes represent the companies traded on the NYSE or NASDAQ while directed edges represent a connectedness measure between the financial assets. The directed edge weight between any two nodes is calculated with use of the value of $\Delta\text{CoVaR}$, one of the most popular systemic risk measures proposed by M. Brunnermeier and T. Adrian in 2011. The value of $\Delta\text{CoVaR}$ measures the relationship between any two assets and is based not only on the yields of the assets, but take into account the mutual effect of its performance. In contrast with correlation coefficient, $\Delta\text{CoVaR}$ is asymmetric. The analysis is focused on the static model of the $\Delta\text{CoVaR}$ estimation. Moreover, this paper uses statistical testing procedures to assess the significance of the findings and interpretations based on this co-risk measure. We examine the intrinsic properties and regularities of stock market analyzing the directed complex network with more than 3700 stocks as nodes which have been traded on the NYSE and NASDAQ in recent years. We connect any two stock with a directed edge if the value of the corresponding $\Delta\text{CoVaR}$ is statistically significant and its normalized value is greater than a given threshold. We discuss both out-degree and in-degree distributions and find essential vertices in the network, which represent the leading stocks. We demonstrate that the network follows the power-law distribution and behaves scale-free. Moreover, we address the problem of finding influential spreaders, i.e. companies which are more likely to spread negative shocks in a large part of the network. In this paper we use three different measures (closeness centrality, betweenness centrality, PageRank) to determine the most influential stocks in the directed market graph.

1. Introduction
One of the key aspects of contemporary economic systems is that they are complex systems consisting of a large number of interdependent parts. The more complex the system, the more dependent its parts are, the more complex behavior it demonstrates. The analysis of market network properties has attracted a growing attention in the recent decade. For the first time the concept of the market graph was examined in [1] in which market network is defined as a complete weighted graph where the nodes represent stocks and weights of edges reflect similarity between stocks behavior (which can be measured e.g. by correlation). In the paper [1] an edge between two vertices is inserted in the market graph, iff the corresponding value of correlation coefficient is above a given threshold. The recent years have seen an increased interest in application and
development of the market graph approach. These research papers include empirical studies based on real market data and they investigate different structural properties and attributes of market graph such as maximum cliques, maximum independent sets, degree distribution [2–5], clustering in Pearson correlation [6], dynamics of the US market graphs [7], complexity of market graph [8]. The papers [3, 9–12] study distinctiveness for particular financial markets. Market graphs with measures of similarity diverse from correlation are studied in [9,13–17].

In this paper we use an approach to analyze the finance market data based on the representation of the interrelations between companies in the form of a directed graph. In contrast with previous research papers, to quantify the strength of connections between companies we employ the risk measure ∆CoVaR, one of the most promising systemic risk measures. The measure ∆CoVaR was introduced and analyzed by American economists Tobias Adrian and Markus K. Brunnermeier in the works [18, 19]. To highlight the systemic nature of the risk measure, its name contains prefix ’Co’ which stands for conditional, contagion, or comovement. Since then ∆CoVaR have become a powerful tool of risk management in various areas [20–25]. Theoretical properties of this measure were examined in works [26–28].

In this paper the directed market graph is constructed as follows: the vertexes represent stocks, and the two vertices are linked by directed edge if the value of corresponding ∆CoVaR is statistically significant and its normalized value is greater than a given threshold.

The main purpose of this paper is to examine the structural properties of the directed market graph. This paper presents a detailed study on the distribution of the degrees of the vertices in this graph, the edge density of this graph, and various indicators of vertex applied to find key companies in the network.

The analysis is based on a finance market data from November, 2015 to November, 2017 (500 trading days). The data used for the estimation of ∆CoVaR are daily stock return data for more than 3700 companies traded on the NYSE and NASDAQ. In the empirical part of the paper for estimation of ∆CoVaR it is used the quantile regression method. Kolmogorov-Smirnov (KS) type statistic proposed in [29] is employed to test the statistical significance of ∆CoVaR values.

2. CoVaR: definition and estimation method
2.1. Definition
Given a confidence level $q \in (0, 1)$, Value-at-Risk (VaR) of a random continuous variable $r_{i,t}$ is defined as the solution of the equation

$$\Pr (r_{i,t} \leq \text{VaR}_{q,t}^i) = q.$$ 

In other words, $\text{VaR}_{q,t}^i$ is exactly defined through the $q$-quantile of the conditional distribution of $r_{i,t}$. In this paper $r_{i,t}$ refers to the log return of the financial institution $i$ at time $t$. Practical applications of VaR can be found in the book [30]. Nowadays, VaR is one of the most well-known risk measures.

Let us now give the formal definition of CoVaR as it was proposed in [18] or [19]. $\text{CoVaR}_{q,C(r_{i,t})}^{j}$ is the value equal to the $\text{VaR}_{q,t}^j$ of institution $j$ (with log return $r_{j,t}$) conditional on some event $C(r_{i,t})$ of institution $i$. That is, $\text{CoVaR}_{q,t}^{j,C(r_{i,t})}$ is implicitly defined by the $q$-quantile of the conditional probability distribution:

$$\Pr \left( r_{j,t} \leq \text{CoVaR}_{q,t}^{j,C(r_{i,t})} \left| C(r_{i,t}) \right. \right) = q.$$ 

In the papers [18,19], $C(r_{i,t})$ refers to distress of institution $i$ and that event of distress occurs when the return of institution $i$ is equal to its VaR for some unsufficient $q$, i.e $r_{i,t} = \text{VaR}_{q,t}^i$. Works [18,19] define the event of median state of an institution as the event when an institution’s return is equal to its median, i.e. $r_{i,t} = \text{VaR}_{0.5,t}^i$. 

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Then we consider the difference between the CoVaR values estimated for institution $j$ conditional on institution $i$ being under distress and in its median state. That kind of difference is the measure of the contribution of institution $i$ to the risk of institution $j$ and is denoted as $\Delta \text{CoVaR}^{ij}$. Thus, $\Delta \text{CoVaR}^{ij}_{q,t}$ measures the influence of the institution $i$ on the institution $j$ and is defined as follows:

$$\Delta \text{CoVaR}^{ij}_{q,t} = \text{CoVaR}^{j|\text{VaR}_{i,t}}_{q,t} - \text{CoVaR}^{j|\text{Median}_{i,t}}_{q,t}.$$ 

The definition of the CoVaR$^{ij}_{q,t}$, namely the VaR of the institution $j$ conditional on the institution $i$ being at its VaR level, allows the study of the spillover effects of the whole process on the financial network. We can obtain value CoVaR$^{ij}_{\text{system}}$, which can give an answer to the following question: which institutions are most at risk during financial crises due to the fact that it reports the increase of VaR of the institution in the case of a financial crisis in the system.

One can note that $\Delta \text{CoVaR}$ are directional, i.e. $\Delta \text{CoVaR}$ of one institute conditional on other institute does not equal the $\Delta \text{CoVaR}$ after the change of order.

2.2. The static model

Estimation of CoVaR is a difficult problem, and it can be executed with the help of a great set of methods, particularly the method of quantile regression, which has been chosen for the empirical part of the study. It is the model of the regression analysis commonly used in econometrics [31,32]. There are two models of the CoVaR estimation – the static model and the dynamic model.

The static model provides an opportunity to calculate CoVaR and $\Delta \text{CoVaR}$ values that are constant over time and independent of other exogenous factors. According to this model, $\Delta \text{CoVaR}$ estimation starts with the construction of the quantile regression to find estimated coefficients for institutions $i$ and $j$.

The $q$-quantile regression describes the dependance of the predicted value of the $q$-quantile of returns of institution $j$ $\hat{X}^{i,j}_{q}$ conditional on institution $i$:

$$\hat{X}^{i,j}_{q} = \hat{\alpha}^{i}_{q} + \hat{\beta}^{i,j}_{q} X^{i}.$$ 

Then, after getting the coefficients, we can find the CoVaR and $\Delta \text{CoVaR}$ values using the following equalities:

$$\text{CoVaR}^{j|r_{i}=\text{VaR}_{i,t}}_{q,t} = \text{VaR}^{j|r_{i}=\text{VaR}_{i,t}}_{q,t} = \hat{\alpha}^{i}_{q} + \hat{\beta}^{i,j}_{q} \text{VaR}_{i,t}^{i},$$ 

$$\Delta \text{CoVaR}^{j|\text{Median}_{i,t}}_{q,t} = \text{CoVaR}^{j|r_{i} = \text{Median}_{i,t}}_{q,t} - \text{CoVaR}^{j|r_{i} = \text{VaR}_{i,t}}_{q,t} = \hat{\beta}^{i,j}_{q} (\text{VaR}_{i,t}^{i} - \text{VaR}_{0.5}^{i}).$$ 

2.3. Testing for significance

Significance can be identified if for an institution $|\Delta \text{CoVaR}|$ exceeds a given threshold level. Following the paper [29] we will assume this threshold level is equal to 0. Thus, a hypothesis test for the identification of a systemically significant institution for institution $j$ is equivalent to the following null hypothesis:

$$H_{0} : \Delta \text{CoVaR}^{j|l}_{q,t} = 0 \quad (4)$$

for a given level $q \in (0,1)$. It follows from (4), (2) and (3) that testing the significance of $\Delta \text{CoVaR}^{j|l}_{q,t}$ reduces to a joint exclusion Wald test of the $\hat{\beta}^{j|l}_{q}$ for a given $q$ (see. e.g. [33]).

The paper [29] proposes a test based on the Kolmogorov–Smirnov (KS) type statistic. It is known that the KS test gives a reasonable way to measure the discrepancy between distributions [34]. Furthermore, variants of the two-sample KS test have been widely used for inference based on a quantile process, such as those considered in [29].
Suppose we have two different values $\text{VaR}_q^i$ and $\text{VaR}_{0.5}^i$. Then $\hat{X}_q^{j,i} = \alpha_q^{i,j} + \beta_q^{j|i}\text{VaR}_q^i$ and $\hat{X}_{0.5}^{j,i} = \alpha_q^{i,j} + \beta_q^{j|i}\text{VaR}_{0.5}^i$ are the empirical quantile response functions evaluated respectively at these two values. Let $T$ be the amount of observations. Thus, we examine the following parametric empirical process:

$$v_T(q) = \sqrt{T} \left( \hat{X}_q^{j,i} - \hat{X}_{0.5}^{j,i} \right) = \sqrt{T} \left( \beta_q^{j|i}\text{VaR}_q^i - \beta_q^{j|i}\text{VaR}_{0.5}^i \right).$$

In testing the null hypothesis $H_0 : \Delta\text{CoVaR}_{q}^{j|i} = 0$ it is not difficult to notice that it is equivalent to a standard significance test $H_0 : \beta_q^{j|i} = 0$ for a given $q \in (0,1)$. The two-sided Kolmogorov–Smirnov type statistic is $K_T = |v_T(q)|$.

3. Empirical Results

3.1. The directed market graph

The daily data were collected from Thomson Reuters database, which was used to retrieve historical prices of the companies traded in the NYSE and NASDAQ for the period from November 04, 2015 to November 06, 2017 (i.e. 500 trading days). The daily closing prices have been adjusted for dividends and splits. We included in our analysis only stocks that had been traded without gaps and omissions during this period (3736 different stocks were remained, and only 15 stocks from S&P500 were eliminated).

Then we calculated $\Delta\text{CoVaR}_{q}^{j|i}$ for all possible pairs of stocks $i, j$ with $q = 0.1$, i.e we take 10%-quantile for the estimation. We used the static model of quantifying $\Delta\text{CoVaR}$ as it is described in subsection 2.2. The hypothesis of non-significance $H_0$ for ordered pairs will be checked using the Kolmogorov–Smirnov test as it is described in subsection 2.3.

Then we constructed an adjacency matrix as follows: if

- $\Delta\text{CoVaR}_{q}^{j|i}$ is statistically significant at level 5%, and
- the normalized value of $\Delta\text{CoVaR}_{q}^{j|i}$ is bigger than a given threshold level,

we put 1 as the entry on row $i$, column $j$ of the adjacency matrix (i.e. there is an edge from node $i$ to node $j$ in the directed market graph). Otherwise, we put 0.

The proportion of present edges from all possible edges in the directed network is equal to 0.00062. The network is not connected and the diameter of the connected part of our directed graph is 14, i.e. the maximum length of the shortest path between two vertices.

We constructed an undirected network as follows: if there are mutual directed edges between two companies then they are connected by undirected link. For this undirected graph the proportion of present links from all possible links is 0.00045. The undirected network is also not connected and the diameter of the connected part of our directed graph is equal to 12.

The size of maximum component, in which all vertices are reachable from others, is not very large (it is about 1100 and 600 for the directed and the undirected networks, respectively), and, for example, Apple Inc. is not present in these components.

In the next three subsections we find answers to the following questions:

- What type of the degree distribution exhibits the directed market graph?
- What type of the functional form has the clustering-degree relation for this graph?
- What are the most influential assets in the sense of spreading negative shocks to other assets?
3.2. Degree distribution analysis

A network can be characterized by its degree distribution. In this subsection we analyze the degree distribution of the directed market network. The first work on the subject was the paper of Barabasi and Albert [35]. Recent years have seen a huge amount of papers examining degree distribution of real-world complex networks arisen in sociology, physics, economics and biology. It turns out that many of them exhibit scale-free property in which the degree distribution follows a power law [35–40].

A directed graph has two separate degree distributions, namely the in- and out-degree distributions. The in(out)-degree sequence for the directed market graph is the vector \( (d_{in}^{1}, d_{in}^{2}, \ldots, d_{in}^{n}) \) with the in(out)-degree information \( d_{v}^{in} \) of every node \( v \) in the graph. Then the in(out)-degree distribution \( P_{in}^{(out)}(k) \) for the graph is calculated as the fraction of nodes in the graph with an in(out)-degree \( k \). Thus, the in(out)-degree distribution can be calculated as follows: \( P_{in}^{(out)}(k) = \frac{|\{v : d_{v}^{in(out)} = k\}|}{N} \), where \( N \) is the number of nodes in the graph. The average degree in a graph is denoted \( d = \sum k P(k) \).

It is said that a network displays a power-law degree distribution if \( P(k) \sim c k^{-\gamma} \), where the constraint \( c > 1 \) gives the proper convergence of the total probability, i.e. \( \sum k c k^{-\gamma} = 1 \). In typical market graph constructed with use of correlation coefficient, the exponent \( \gamma \) lies between 0.7 and 2.

For the undirected network, the degree distribution follows the power law with \( \gamma = 1.70 \), \( R^2 = 0.91 \). For the directed network under consideration, the distribution of in-degrees follows the power law with \( \gamma = 1.15 \), \( R^2 = 0.84 \), out-degrees follows the power law with \( \gamma = 1.83 \), \( R^2 = 0.93 \):

\[
P(k) \sim 6.70k^{-1.70}, \quad P_{in}(k) \sim 5.14k^{-1.15}, \quad P_{out}(k) \sim 7.27k^{-1.83}.
\]

The resulting models are statistically significant at 10% level of significance and is significantly better than the exponential model. The degree and in-, out- degree distributions of the directed market network is shown in Figure 1.

![Figure 1](image)

**Figure 1.** (a) – The degree distribution of the undirected market network; (b) – The in-degree distribution of the directed market network; (c) – The out-degree distribution of the directed market network

3.3. Clustering distribution analysis

Our market networks is represented by a directed graph where a link from a stock \( i \) to a stock \( j \) means that \( j \) is vulnerable with respect to a distress event of company \( i \), indicating that
negative shocks of $i$ may cause negative effect on asset $j$. Let $\Gamma_i$ denote the set of stocks which exposed to the shocks of $i$, i.e. $\Gamma_i$ includes all the stocks $j$ for which $\Delta \text{CoVaR}^j_i$ is statistically significant. The density of interactions between stocks from $\Gamma_i$ can be characterized by the local clustering coefficient of $i$. The clustering coefficient of node $i$ in a directed network is defined as follows [41]:

$$
C_{in}^i = \frac{|\{e_{jk} : j, k \in \Gamma_i\}|}{d_{in}^i (d_{in}^i - 1)},
$$

where $|\{e_{jk} : j, k \in \Gamma_i\}|$ is the set of links connecting two of $i$’s followers. We set $c_i = 0$ if $k_{in}^i \leq 1$. A bi-directional link $j \leftrightarrow k$ is counted as two distinctive links $j \rightarrow k$ and $k \rightarrow j$.

The average of $c_i^{in}$ is 0.036, and the average of $c_i^{out}$ is 0.029. The average clustering coefficient for the undirected network is equal to 0.094.

Let $C^{in(out)}(k)$ denote the average clustering coefficient of nodes with in(out)-degree $k$. It has been found that for most of real undirected networks $C(k)$ follows $C(k) \sim Bk^{-\beta}$, where the exponent $\beta$ usually lies between 1 and 2 [42–44]. It is quite remarkable that the clustering in- and out-degree distribution relations are constant functions (i.e. follow the power law with $\beta$ equal to 0).

### 3.4. The problem of identifying the influential stocks

The high spreading efficient nodes which are often called influential spreaders represent the nodes that are more likely to spread negative shocks in a large part of the network. In this subsection we consider the problem of identifying the influential assets in the sense of spreading negative shocks to other assets. These influential stocks are the important nodes which can be used in practical finance risk management applications, such as the acceleration of shocks diffusion, the control of the spread of a distress and the improvement of the robustness of economic system to negative events. Of course, to find key nodes in the network it is possible to rely on the well-known centrality measures such as degree centrality (local measure).

However, it is well established that the location of a vertex (vertex global property) may be more important than its degree (vertex local property). For example, if two companies have the same degree but different location in the network (one of them is connected with the fringe of the network and the other one with the centermost set) then they may not have equal negative shock spreading efficiency. Thus, highly connected assets with high degree may not be the best negative shock spreaders, while less connected assets connected with the center of the market graph may greatly induce the process of spreading distresses. Therefore, different global measures (such as closeness centrality or betweenness centrality) can be employed to find influential spreaders but with higher computational cost.

Various papers examine the relation between the topological properties of network nodes and their spreading efficiency in order to design efficient and effective ranking algorithms on large-scale networks. Most of these algorithms are diffusion based methods using assumption that a node is expected to be of high influence if it links to many highly influential neighbors. The well-known algorithms such as HITS [45], PageRank [46], LeaderRank [47,48], TwitterRank [49] are among these methods. A local ranking algorithm, called ClusterRank, has been recently proposed in [50]. It was shown that these methods were much more reliable than out-degree centrality and betweenness centrality in terms of ranking effectiveness.

In this paper we use three different measures (closeness centrality, betweenness centrality, PageRank) to determine the most influential stocks in the directed market graph. We calculate the closeness centrality, betweenness centrality, PageRank for every node. Then we arrange the stocks in descending order, then select top 10 stocks from each of three measures and combined it to obtain important assets presented in Table 1.
Table 1. Top stocks with higher PageRank, closeness centrality and betweenness centrality

| Company | Page Rank $\times 10^{-3}$ | Company | Betweenness Centrality $\times 10^{-3}$ | Company | Closeness Centrality $\times 10^{-1}$ |
|---------|-----------------------------|---------|-----------------------------------------|---------|---------------------------|
| rock.us | 2.83                        | spsc.us | 1.1161                                  | pfn.us  | 4.93                      |
| bxmt.us | 2.73                        | cqpp.us | 1.1148                                  | c_c.us  | 4.93                      |
| int.us  | 2.48                        | rock.us | 1.1148                                  | wgp.us  | 4.90                      |
| jrcj.us | 2.36                        | bxp.us  | 1.1144                                  | luk.us  | 4.89                      |
| exx.us  | 2.15                        | pbr-a.us| 1.1142                                  | kye.us  | 4.87                      |
| cqp.us  | 2.09                        | bxmt.us | 1.1121                                  | dsl.us  | 4.87                      |
| pbr-a.us| 1.90                        | jrcj.us | 1.1119                                  | ihd.us  | 4.87                      |
| spsc.us | 1.88                        | int.us  | 1.1118                                  | ztr.us  | 4.86                      |
| wgp.us  | 1.74                        | ovas.us | 1.1118                                  | etw.us  | 4.86                      |
| luk.us  | 1.72                        | gabc.us | 1.1115                                  | ncz.us  | 4.86                      |
| esss.us | 1.64                        | mcep.us | 1.1114                                  | gdv.us  | 4.86                      |
| afsi.us | 1.58                        | esss.us | 1.1114                                  | usa.us  | 4.86                      |
| htz.us  | 1.51                        | vsat.us | 1.1110                                  | bp.us   | 4.86                      |
| apu.us  | 1.49                        | afsi.us | 1.1109                                  | mmt.us  | 4.86                      |
| smllp.us| 1.47                        | smllp.us| 1.1108                                  | cma.us  | 4.85                      |
| murl.us | 1.45                        | pmc.us  | 1.1102                                  | flc.us  | 4.85                      |
| stdl.us | 1.40                        | nl.us   | 1.1100                                  | mplx.us | 4.85                      |
| mofg.us | 1.40                        | nbr.us  | 1.1098                                  | bgv.us  | 4.85                      |
| pmc.us | 1.39                        | cuda.us | 1.1098                                  | ftf.us  | 4.85                      |
| ihd.us | 1.27                        | mofg.us | 1.1098                                  | acp.us  | 4.85                      |

4. Conclusion
One of the key features of contemporary economic systems is that they are complex systems consisting of a large number of interdependent parts (agents). The more complex the system, the more dependent its parts are, the more complex behavior it demonstrates. In this case, complex systems can consist of a large number of parts, each of which exhibits a primitive behavior.

This paper studies a complex network formed as a directed graph in which nodes represent the companies traded on the NYSE or NASDAQ while directed edges represent a connectedness measure between the financial assets. The directed edge weight between any two nodes is calculated with use of the value of $\Delta$CoVaR, one of the most popular systemic risk measures proposed by M. Brunnermeier and T. Adrian in 2011. The value of $\Delta$CoVaR measures the relationship between any two assets and is based not only on the yields of the assets, but take into account the mutual effect of its performance. In contrast with correlation coefficient, $\Delta$CoVaR is asymmetric. The analysis is focused on the static model of the $\Delta$CoVaR estimation. Moreover, this paper uses statistical testing procedures to assess the significance of the findings and interpretations based on this co-risk measure. We examine the intrinsic properties and regularities of stock market analyzing the directed complex network with more than 3700 stocks as nodes which have been traded on the NYSE and NASDAQ in recent years. We connect any two stock with a directed edge if the value of the corresponding $\Delta$CoVaR is statistically significant and its normalized value is greater than a given threshold.

This paper studies this directed market network to identify key companies in the network based on different network analysis indicators, such as closeness centrality, betweenness centrality and PageRank. It turns out, most of the key stocks are investment funds. It was shown that
distribution of degrees of our network follows to the power law, although with non-typical indicators of degree exponent.

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