ΛNN three-body problem within s-wave inverse scattering on theoretical data

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Abstract

ΛN potentials recovered through the application of s-wave inverse scattering on theoretical data are demonstrated on the ΛNN three-body problem. The spin-dependent Malfliet-Tjon I/III potential, with benchmark parameters that bind the deuteron at -2.2307 MeV, represents the NN interaction. The three-body problem is solved through the hyperspherical method. From spin-averaged effective ΛN potentials with one-quarter spin singlet and three-quarters spin triplet contributions, the binding energy and root-mean-square radius of Λnp ($J^\pi = 1/2^+$) computed is found to be -3.0759 MeV and 7.7 fm, respectively. This is higher than the current experimental binding energy for Λnp ($J^\pi = 1/2^+$), but consistent with recent trends in high-precision measurements on the lifetime of the same hypernucleus. With charge symmetry breaking in the ΛN potentials, this Λnp ($J^\pi = 1/2^+$) binding energy was found to be consistent with a bound Λnn ($J^\pi = 1/2^+$) state.

Keywords: lambda nucleon nucleon, lambda hypertriton, inverse scattering theory, Gel’fand-Levitan-Marchenko method, hyperspherical method

1. Introduction

In hypernuclear physics the lambda-nucleon-nucleon (ΛNN) three-body problem plays a very important role, akin to that of the NN and NNN systems in the non-strange sector of nuclear physics. In particular, the spin-isospin channel Λnp ($J^\pi = 1/2^+$) is traditionally used for constraining lambda-nucleon forces. After a bound state (or resonance?) was inferred for Λnn ($J^\pi = 1/2^+$) [1], these two systems together now serve to provide more constraints on lambda-nucleon forces. The lambda hypertriton (Λnp) is important because it can be used to constrain both the Λp and Λn forces, while Λnn is crucial for constraining the Λn force. It is therefore of uppermost importance that the same set of Λp and Λn forces, with charge symmetry breaking, is used to study both three-body problems. Through this approach, the following important double question can be answered: what Λnp binding energy is consistent with a Λnn bound state or resonance?

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More than half a century after the first observation of a hypernucleus in nuclear emulsions, there is still no published data for $\Lambda n$ scattering. This is because there are no lambda hyperon targets and, in the case of inverse kinematics, lambda hyperon beams are very difficult to prepare. The lack of $\Lambda n$ data is a serious setback for all methods used in the development of $\Lambda N$ potentials: Meson-Exchange theory, Chiral Effective Field theory, quantum chromodynamics and, recently, inverse scattering theory. As a result, basic structural properties like binding energy, size and lifetime of the simplest hypernucleus cannot be accurately computed.

Since 1958, a number of calculations on $\Lambda nn$ predicted that a bound state was not possible \[2, 3, 4, 5, 6\]. After evidence for a possible bound state was inferred from heavy-ion experiments by the HypHI Collaboration at GSI, a number of recent theoretical studies based on improved $\Lambda N$ forces from Meson-Exchange theory, Chiral Effective Field theory and quantum chromodynamics have also argued against the existence of such a bound state \[7, 8, 9, 10, 11, 12, 13\]. In order to provide a new perspective on the $\Lambda N$ force, inverse scattering theory has recently been applied on theoretical $\Lambda p$ and $\Lambda n$ scattering data \[14\]. Spin averages of the recovered $\Lambda p$ and $\Lambda n$ potentials (GLM-YN0) were tested in \[15\] by computing the binding energy and root-mean-square radius of $\Lambda np (J^\pi = 1/2^+)$, using Malfliet-Tjon V potential for the NN interaction. The aim of this paper is to use GLM-YN0 potentials with the semi-realistic spin-dependent Malfliet-Tjon I/III NN potential that correctly binds the deuteron, in exploring the $\Lambda np (J^\pi = 1/2^+)$ and $\Lambda nn (J^\pi = 1/2^+)$ three-body problems.

2. Lambda-neutron and lambda-proton interactions: GLM-YN0 potentials

The charge-asymmetric pair of $\Lambda p$ and $\Lambda n$ potentials used in this paper were developed through Gel’fand-Levitan-Marchenko theory \[14\]. These potentials were restored through the application of this method on sub-threshold theoretical $s$-wave scattering phases. Based on the spin multiplicities of the $^1S_0$ and $^3S_1$ $\Lambda N$ states, effective potentials are constructed from one-quarter spin singlet and three-quarters spin triplet potentials from inverse scattering theory \[15\]:

\[
V_{\Lambda p} = \frac{1}{4}V_{\Lambda p}(^1S_0) + \frac{3}{4}V_{\Lambda p}(^3S_1)
\]

\[
V_{\Lambda n} = \frac{1}{4}V_{\Lambda n}(^1S_0) + \frac{3}{4}V_{\Lambda n}(^3S_1)
\]

The data obtained from these spin-averages, which will henceforth be referred to as GLM-YN0 potentials, was fitted with the following three-range Gaussians:

\[
V_{\Lambda N}(r) = \sum_{i=1}^{3} V_i \exp \left\{ -\frac{(r - \mu_i)^2}{\sigma_i^2} \right\}.
\]
Figure 1 illustrates the features of GLM-YN0 potentials, especially a noticeable charge symmetry breaking. The theoretical scattering phases used for inverse scattering theory in [14] carry in them not only this feature of charge asymmetry, but also $\Lambda - \Sigma$ coupling. Therefore, it can be stated that even though GLM-YN0 potentials only explicitly have a central term, the effects of charge symmetry breaking and $\Lambda - \Sigma$ coupling are built into this central term, since phase equivalence with NSC1997f was verified in [14]. The parameters $V_i$, $\mu_i$ and $\sigma_i$ in Equation (3) were determined through a nonlinear least squares fit. Estimated values are reported in Table I.

![Figure 1: GLM-YN0 effective $\Lambda p$ and $\Lambda n$ potentials: these potentials are constructed from one-quarter of the spin singlet state and three-quarters of the triplet state potential from inverse scattering theory.](image)

3. Neutron-proton and neutron-neutron interactions: Malfliet-Tjon I/III potential

Malfliet-Tjon I/III potential is used to represent the NN interaction. In the spin-triplet states, the $(S, I) = (1, 0)$ potential is

$$V_{np}(r) = [V_A^t \exp(-\mu_a r) + V_R^t \exp(-\mu_r r)]/r \quad (4)$$

In the spin-singlet state, $(S, I) = (0, 1)$, the potential is given by

$$V_{nn}(r) = [V_A^s \exp(-\mu_a r) + V_R^s \exp(-\mu_r r)]/r \quad (5)$$
Table 1: Parameters for GLM-YN0 effective $\Lambda p$ and $\Lambda n$ potentials.

| $i$  | $V_i$/MeV | $\mu_i$/fm | $\sigma_i$/fm |
|------|-----------|------------|--------------|
| 1    | 45.88     | $0.1148 \pm 0.0006601$ | $-0.3932 \pm 0.0008502$ |
| 2    | $8.106e + 07$ | $-1.193 \pm 0.001948$ | $0.3575 \pm 0.0005306$ |
| 3    | $-47.04$  | $0.3748 \pm 0.0001386$ | $0.1667 \pm 0.0002179$ |

The parameters used for these NN potentials, shown in Table 2, are taken from [16]. These parameters were established so that the Malfliet-Tjon I/III potential can be used as a tool for benchmarking few-body computations in nuclear physics. With these parameters, the binding energy of the deuteron was found to be $-2.2307$ MeV.

Table 2: Parameters for Malfliet-Tjon I/III NN potentials, from [16].

| $V_A''$ | $V_R''$ | $\mu_a$ | $\mu_r$ |
|---------|---------|---------|---------|
| $-626.885$ | $1438.72$ | $1.55$ | $3.11$ |
| $-513.968$ | $1438.72$ | | |

4. Hyperspherical method

Let $\{x_i, y_i\}$ be Jacobi coordinates, after elimination of centre-of-mass motion [17]. These coordinates are transformed into six-dimensional hyperspherical coordinates $\{\rho, \Omega_5\}$, where $\rho = \arctan(y_i/x_i)$ is the hyperradius and $\Omega_5$ represents one hyperangle ($\theta_i$) and two sets of polar angles ($\nu_x, \nu_y, \omega_x, \omega_y$). Faddeev amplitudes are given by

$$\psi_{\alpha_i}^{i,J}(x_i, y_i) = \sum_{K_i=K_{\min}}^{K_{\max}} \rho^{-5/2} X_{\alpha_i,K_i}(\rho) \Phi_{K_i}^{\ell_x, \ell_y}(\theta_i),$$

where $K_i$ are hyperangular momenta and $\alpha_i = \{(\ell_x, \ell_y)L_i, (s_j, s_k)S_{z_i}\}J_i$; $s_i$ is the coupling order for Faddeev amplitudes. The $\Lambda NN$ three-body problem is simulated through the following system of coupled hyperradial equations [18]:
\[
\left\{ -\frac{\hbar^2}{2m} \frac{d^2}{d\rho^2} + \frac{\hbar^2}{2m \rho^2} \mathcal{L}_{K_i}(\mathcal{L}_{K_i} + 1) - E \right\} \chi_{\alpha_i, K_i}(\rho) = -\sum_{j\alpha_j K_j} V^{ij}_{\alpha_i K_i, \alpha_j K_j}(\rho) \chi_{\alpha_j, K_j}(\rho),
\]
where \( m \) is a unit mass (taken here to be the mass of a nucleon, 939.0 MeV), \( \mathcal{L}_{K_i} = K_i + 3/2 \) and \( V^{ij}_{\alpha_i K_i, \alpha_j K_j}(\rho) \) are coupling potentials. This coupled system of hyperradial equations is converted to the following eigenvalue problem [17]:

\[
H a = E a
\]
where \( a \) is the eigenvector and \( E \) the eigenvalue.

5. Results and discussion

5.1. \( \Lambda np \) (\( J^\pi = 1/2^+ \))

The eigenvalue problem in Equation [8] was solved using the computer code FaCE [17]. For \( \Lambda np \) (\( J^\pi = 1/2^+ \)), the \( np \) potential in Equation [4] was used, together with GLM-YN0 \( Ap \) and \( \Lambda n \) potentials presented in this paper. The binding energy and rms matter radius computed are found to be (-3.0759 MeV, 7.7 fm). This gives a lambda separation energy of \( B_{\Lambda} = 0.85 \) MeV, relative to the deuteron binding energy for the \( np \) potential used in the calculation (-2.2307 MeV). In a recent experiment it was found that \( B_{\Lambda} = 0.41 \) MeV [19], which is a consequential increase from the widely used value \( B_{\Lambda} = 0.13 \) MeV [20]. A more accurate experiment, that is expected to measure an even higher \( \Lambda np \) binding energy, is in the planning phase at JLab [21]. A higher lambda separation energy in \( \Lambda np \) has many consequences in hypernuclear physics, some of which are discussed within chiral effective field theory in [22] for \( B_{\Lambda} = 0.23 \) to 0.51 MeV.

The rms radius of \( \Lambda np \) is another property that is dependent on the \( \Lambda np \) wavefunction, even though it has never been measured in any experiment. As expected, the value computed here is smaller than that from models in which \( \Lambda np \) has a lower binding energy. For example, a \( \Lambda \)-deuteron rms radius of 10.3 fm was recently computed in pionless effective field theory [23] while a value of 10.6 fm is reported in [24].

A recent high-precision measurement by the STAR Collaboration at RHIC-BNL [25] found the lifetime of \( \Lambda np \) (\( J^\pi = 1/2^+ \)) to be about 50% shorter than that of a free \( \Lambda \) hyperon: this clearly implies that \( Ap \) and \( \Lambda n \) forces are much stronger than currently used forces that compute lambda hypertriton lifetimes nearly equal to that of a free \( \Lambda \) hyperon. Our potentials are therefore in accord with the STAR Collaboration lifetime measurements that challenge the notion of \( \Lambda np \) being a loose \( \Lambda \) hyperon around a deuteron core. Therefore, an important test for GLM-YN0 \( Ap \) and \( \Lambda n \) potentials lies in the computation of the lifetime of \( \Lambda np \) (\( J^\pi = 1/2^+ \)).
5.2. \( \Lambda nn (J^\pi = 1/2^+) \)

For \( \Lambda nn (J^\pi = 1/2^+) \), the \( nn \) potential in Equation 5 was used, together with GLM-YN0 \( \Lambda n \) potential. By examining all the computed eigenvalues of Equation 5, two possible sets of values for the binding energy and rms matter radius are found: (-0.0695 MeV, 12.3 fm) and (-0.6112 MeV, 10.1 fm). The HypHI Collaboration, in their announcement of a possible \( \Lambda nn \) bound state, did not state any value for its binding energy. Neither was a spin and parity assigned. There is therefore no experimental results to compare with. Notwithstanding, one can immediately suspect that the second value is too high because there are no \( \Lambda n \) or \( nn \) bound states, compared to \( \Lambda np \) for which there is a \( np \) bound state.

The most likely binding energy from our calculation (-0.0695 MeV) is close to that reported in [10, 13] as -0.069 MeV. The result from [10, 13] is obtained by scaling up meson theory \( \Lambda N \) potentials to about 35\% in a few-body calculation carried out through the Complex Energy Method. In [6], a bound state was found by scaling up their \( \Lambda N \) potential by 50\%, though the value of the binding energy computed was not stated. Motivated by the charge asymmetry relation between \( \Lambda p \) and \( \Lambda n \) forces, an important question arises from the studies in [6, 10, 13], and all other studies that computed a \( \Lambda nn \) bound state by scaling up the attraction in the \( \Lambda n \) force: what is the binding energy of \( \Lambda np \) after both \( \Lambda p \) and \( \Lambda n \) are scaled up by the same amount? In [9], this question was explored using a lambda-nucleon force which is phase-equivalent with the NSC1997f force: by scaling up the tensor component in this force (which acts in the \( 3S_1 \) state) by 1.20, a bound state is found for \( \Lambda nn \) at -0.054 MeV, and the corresponding lambda separation energy for \( \Lambda np \) computed is \( B_\Lambda = 0.83 \) MeV. This value of \( B_\Lambda \) is nearly the same as that presented in this paper for GLM-YN0 potentials (\( B_\Lambda = 0.85 \) MeV).

Recently, a new experiment to search for a bound state or resonance in \( \Lambda nn (J^\pi = 1/2^+) \) was carried out at JLab [26]. At the time of writing this paper, data from this experiment was still being analysed.

6. Conclusions

In this paper, charge asymmetric lambda-nucleon potentials have been proposed. These potentials were obtained by averaging spin-dependent lambda-nucleon potentials recovered through the Gel'fand-Levitan-Marchenko method on theoretical scattering data. The potentials (GLM-YN0) are constructed from one-quarter spin singlet plus three-quarters spin triplet potentials. GLM-YN0 potentials were used in solving the \( \Lambda NN \) three-body problem, together with Malfliet-Tjon I/III nucleon-nucleon potential. \( \Lambda np (J^\pi = 1/2^+) \) binding energy and root-mean-square matter radius computed were found to be (-3.0759 MeV, 7.7 fm). These two results reveal that \( \Lambda p \) and \( \Lambda n \) forces are much stronger, a fact that is strongly evident from the 2018 \( \Lambda np (J^\pi = 1/2^+) \) lifetime measurements by the STAR Collaboration. GLM-YN0 \( \Lambda n \) forces gave rise to a bound \( \Lambda nn (J^\pi = 1/2^+) \) state, whose binding energy and root-mean-square radius are reported as (-0.0695 MeV, 12.3 fm) or (-0.6112 MeV, 10.1 fm). This \( \Lambda nn (J^\pi = 1/2^+) \) bound state
arises naturally within GLM-YN0 potentials. Computing $\Lambda np$ ($J^\pi = 1/2^+$) lifetime and the size of charge symmetry breaking in isospin multiplets are decisive tests on the accuracy of our lambda-nucleon forces from inverse scattering theory.

7. References

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