Nearly-degenerate $p_x + ip_y$ and $d_{x^2−y^2}$ pairing symmetry in the heavy fermion superconductor YbRh$_2$Si$_2$

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Recent discovery of superconductivity in YbRh$_2$Si$_2$ has raised particular interest in its pairing mechanism and gap symmetry. Here we propose a phenomenological theory of its superconductivity and investigate possible gap structures by solving the multiband Eliashberg equations combining realistic Fermi surfaces from first-principles calculations and a quantum critical form of magnetic pairing interactions. The resulting gap symmetry shows sensitive dependence on the in-plane propagation wave vector of the quantum critical fluctuations, suggesting that superconductivity in YbRh$_2$Si$_2$ is located on the border of $(p_x + ip_y)$ and $d_{x^2−y^2}$-wave solutions. Comparison with latest experiment points to multiple superconducting phases with spin-singlet $d_{x^2−y^2}$-wave pairing at zero field and a field-induced spin-triplet $(p_x + ip_y)$-wave state. In addition, the electron pairing is found to be dominated by the ‘jungle-gym’ Fermi surface rather than the ‘doughnut’-like one, in contrast to previous thought. This requests a more elaborate and renewed understanding of the electronic properties of YbRh$_2$Si$_2$.

Recent discovery of superconductivity in YbRh$_2$Si$_2$ has doubled the total number of Yb-based heavy fermion superconductors [1]. While YbRh$_2$Si$_2$ has been a subject of decade-long studies due to its peculiar quantum critical properties [2, 3], this latest discovery has stimulated new interest concerning the nature of its pairing symmetry. Upon applying magnetic field, further exploration shows that the superconductivity ($T_c \approx 6 \text{ mK at zero field}$) is initially suppressed and then replaced by a new superconducting phase ($T_c \approx 2 \text{ mK}$) above 4 mT [4], pointing towards the possibility of multiple superconducting phases tuned by the magnetic field. At higher temperatures, the angle-resolved photoemission spectroscopy (ARPES) has observed large Fermi surfaces of dominant $f$-orbital characters down to 1 K [5], implying the existence of itinerant Yb-4$f$ electrons for superconducting pairing. Indeed, it is currently believed that superconductivity in YbRh$_2$Si$_2$ is formed of heavy-electron pairs. Still, question remains concerning the origin of potential pairing glue and symmetry of the gap structure. A satisfactory understanding of the pairing mechanism is still lacking.

A probable candidate for the pairing glue might come from magnetic quantum critical fluctuations. Although superconductivity was so far only explored in the antiferromagnetic (AFM) phase below $T_N = 70 \text{ mK}$ [6], microscopic coexistence of the two phases has been excluded [7]. The magnetically ordered phase is believed to contain significant fluctuations. It has a tiny ordered moment ($< 0.1\mu_B/\text{Yb}^{3+}$) compared to the effective moment, $\mu_{\text{eff}} \approx 1.4\mu_B$ per Yb$^{3+}$, derived from a Curie-Weiss fit of the susceptibility right above $T_N$ [8, 9]. Nuclear magnetic resonance has revealed strong AFM fluctuations near the quantum critical point (QCP) [10]. By contrast, neutron scattering experiments have detected significant ferromagnetic (FM) fluctuations below 30 K, which evolve into incommensurate in-plane AFM correlations with a propagation wave vector $Q_{\perp} = \pm (0.14 \pm 0.04, 0.14 \pm 0.04)$ at 0.1 K [11]. Thus, superconductivity in YbRh$_2$Si$_2$ might also be mediated by magnetic quantum critical fluctuations, similar to those in many other heavy fermion superconductors [12, 13].

From the theoretical perspective, the phase-separated coexistence of a long-range magnetic order should play no major role in determining the superconducting gap symmetry. For simplicity, one might ignore first the presence of antiferromagnetism and consider in theory solely the superconducting instability. This allows us to calculate the pairing symmetry based on realistic heavy electron band structures derived from first-principles calculations and a phenomenological form of magnetic quantum critical pairing interactions. We find that YbRh$_2$Si$_2$ is located on the border of a $d_{x^2−y^2}$-wave spin-singlet state and a $(p_x + ip_y)$-wave spin-triplet state. The exact ground state depends sensitively on the in-plane $(h)$ component of the vector $Q \equiv (h, h, l)$ of the pairing interactions. Comparison with latest experiment suggests a spin-singlet $d_{x^2−y^2}$-wave pairing state at zero field and a field-induced spin-triplet $(p_x + ip_y)$-wave state, although experiments are still inconclusive. We also find that the electron pairing is most influenced by the so-called ‘jungle-gym’ Fermi surface, rather than the ‘doughnut’-like one often considered in the literature.

The electronic structures of YbRh$_2$Si$_2$ were obtained using the density functional theory (DFT) taking into
consideration both the spin-orbit coupling and an effective Coulomb interaction $U = 8 \text{ eV}$ [10]. As shown in Fig. 1 we find two flat bands that cross the Fermi energy and exhibit strong hybridization between Yb-4$f$ and Rh-4$d$ orbitals. The electron band along the $\Gamma$-X-P path produces the so-called ‘jungle-gym’ Fermi surface, and the hole band around Z point yields the ‘doughnut’-like Fermi surface. The results are plotted in Fig. 1(b) and the value of $U$ was chosen to yield the same topological structures as in previous calculations [20, 21]. Experimentally, the ‘doughnut’-like Fermi surface has been observed by ARPES [7, 22–28], in agreement with theoretical predictions [20, 21], while the ‘jungle-gym’ Fermi surface was missing but argued to be covered up by surface states [7]. In de Haas-van Alphen measurements, a high-frequency mode has been detected and attributed to the ‘jungle-gym’ Fermi surface [29].

The pairing symmetry can be investigated by solving the linearized Eliashberg equations [30–33],

$$Z_{\mu} (k, i\omega_n) = 1 + \frac{\pi T}{\omega_n} \sum_{\nu, m} \oint_{\text{FS}_{\nu}} \frac{dk_{\parallel}}{(2\pi)^3 v_{\nu, k_{\parallel}'}} \text{sgn} (\omega_m) \times V^{\mu\nu} (k - k', i\omega_n - i\omega_m) \lambda \phi_\mu (k, i\omega_n) = -C \pi T \sum_{\nu, m} \oint_{\text{FS}_{\nu}} \frac{dk_{\parallel}}{(2\pi)^3 v_{\nu, k_{\parallel}'}} \times \frac{V^{\mu\nu} (k - k', i\omega_n - i\omega_m)}{|\omega_m Z_\nu (k', i\omega_m)|} \phi_\nu (k', i\omega_m) ,$$

where $\mu$ and $\nu$ are the band indices, $\text{FS}_\nu$ denotes the Fermi surface of band $\nu$, $v_{\nu, k_{\parallel}'}$ is the corresponding Fermi velocity, $V^{\mu\nu}$ is the intraband ($\mu = \nu$) or interband ($\mu \neq \nu$) interactions, $\omega_n/m$ is the fermionic Matsubara frequency, $Z_{\mu}$ is the renormalization function, and $\phi_\mu$ is the anomalous self-energy related to the gap function, $\Delta_\mu = \phi_\mu/Z_{\mu}$. The prefactor $C$ is unity for spin-singlet pairing and $-1/3$ for spin-triplet pairing. $\lambda$ is the eigenvalue of the kernel matrix for each pairing channel and its largest value determines the dominant pairing state at $T_c$. Unlike iron pnictides, where the Fermi surfaces are mostly quasi-two-dimensional and nearly isotropic, the Fermi surfaces here are highly anisotropic and three-dimensional, so the superconducting gap structures cannot be easily captured by the low-order trigonometric harmonics near the high-symmetric points [34, 35]. It is therefore necessary to derive the detailed gap structures by solving the Eliashberg equations numerically.

However, there are still two obstacles before we can proceed to do the calculations. First, controversial still remains regarding the exact form of the magnetic quantum critical fluctuations. While different theories have been proposed based on local quantum criticality [36, 37] or critical quasiparticles [38, 41], neutron scattering experiments seem to have detected simple spin-density-wave (SDW) type fluctuations [11]. We will not try to judge these different scenarios. Rather, we adopt a generic and phenomenological form for the pairing interactions [30, 33, 11, 42],

$$V^{\mu\nu} (Q, \nu n) = \frac{V_0^{\mu\nu}}{1 + \xi^2 (Q - Q)^2 + |\nu n/\Lambda_{sd}|^\alpha} ,$$

where $V_0^{\mu\nu}$ are free parameters controlling the relative strength of intra- and interband pairing forces. The exponent $\alpha$ defines different quantum critical scenarios and takes the value of 1 for SDW [11], 0.75 for local quantum criticality [36, 37] and 0.5 for critical quasiparticle theory [38, 40]. We estimated the correlation length $\xi \approx 6 \text{ Å}$ very crudely from neutron scattering experiments [11] and chose the characteristic spin-fluctuation frequency $\Lambda_{sd} \approx 1 \text{ meV}$ such that the magnetic Fermi energy $\Gamma_{sd} = \Lambda_{sd} (\xi/a)^2 \approx 2.2 \text{ meV}$ equals roughly the Kondo energy scale [1]. For numerical calculations, we discretize the whole Brillouin zone into $70 \times 70 \times 70$ k-meshes and take 8192 Matsubara frequencies for the $\omega_n$-summation to be cut off at around $\Gamma_{sd}$. The gap structure in the momentum space is then solved with the approximation $g_{\mu, k} = \Delta_{\mu} (k, i\omega_n) \approx \Delta_{\mu} (i\pi T_c)$. Interestingly, our calculations show that the gap symmetry is independent of $\alpha$ but mainly determined by the momentum structure of the pairing interactions. Here comes the second obstacle that concerns the vector $Q = (h, h, l)$. Experimentally, it was found to evolve with temperature from $h = 1 \text{ d} = 0 \text{ (FM)}$ below 30 K to $h = 0.14 \pm 0.04 \text{ (AFM)}$ at 0.1 K [11]. Since its exact value for the electron pairing at $T_c$ is yet to be measured, we are forced to consider a wide range of possibilities around these experimental observa-
tions. Such a strategy turns out to be helpful and reveals the nearly degenerate nature of the superconductivity in YbRh$_2$Si$_2$.

Figure 2 plots the eigenvalues of three major pairing channels for different choices of $Q$. For simplicity, we only present the data for $\alpha = 1$ and assume a band-independent $V_0^{\mu \nu}$. We have examined other choices in a reasonable range of variation and found no qualitative influence on our main conclusions. Figures 2(a) and 2(b) compare the eigenvalues as a function of $l$ for fixed $h = 0.1$ and 0.2, revealing a solution of either $(p_x + ip_y)$ or $d_{x^2-y^2}$-wave over a wide parameter range of $l$. Thus the electron pairing is insensitive to magnetic fluctuations along $c$-axis. We also plot the $l$-dependence of the eigenvalues for a typical $l = 0.25$ in Fig. 2(c), where we could see clear transitions of the leading pairing channel from $(p_x + ip_y)$ to $d_{x^2-y^2}$ at $h \approx 0.13$ and then to a nodal $s$-wave solution at $h \approx 0.35$, indicating that in-plane magnetic fluctuations play a crucial role in determining the pairing symmetry. For clarity, typical gap structures of above solutions are plotted in Fig. 3 for different values of $h$ at fixed $l = 0.25$. For $h = 0.1$, we derive a two-fold degenerate solution with $p_x$ and $p_y$ symmetry as shown in their dependence on the azimuthal angle ($\phi$). Their mixture gives the chiral $(p_x + ip_y)$-wave gap to minimize the pairing energy, $E = -\frac{4}{3} \sum_{k', \mu, \nu} V_{k'k}^{\mu \nu} (c_{\nu, k' \mu}^\dagger c_{\nu, k \mu}) (c_{\nu', k' \mu}^\dagger c_{\nu', k \mu})$, where $\nu, \alpha, \beta$ are spin indices. For $h = 0.2$, a $d_{x^2-y^2}$-wave gap is obtained which changes sign when $\phi$ rotates by $\pi/2$ and contains nodes on the $k_x = \pm k_y$ plane. For $h = 0.4$, we identify a nodal $s$-wave solution with accidental nodes on the 'doughnut'-like Fermi surface.

To extract key factors that determine the pairing symmetry, we separate out contributions from each Fermi surface and define the band-resolved eigenvalues

$$\lambda_{\mu \nu} = \frac{\int_{FS_{\mu}} \frac{dk}{(2\pi)^3} \int_{FS_{\nu}} \frac{dk}{(2\pi)^3} K_{k,k'}^{\mu \nu} g_{k,k'}^\dagger g_{k',k}}{\int_{FS_{\mu}} \frac{dk}{(2\pi)^3} |g_{k,k}|^2}, \quad (3)$$

where $K_{k,k'}^{\mu \nu} = -C_\pi T_c \sum_m V_{k,k'}^{\mu \nu} (i\pi T_c - i\omega_m)/|\omega_m|$ and $V_{k,k'}^{\mu \nu} (i\omega_n) = [V^{\mu \nu}(k - k', i\omega_n) \pm V^{\mu \nu}(k + k', i\omega_n)]/2$ for spin-singlet (+) and triplet (−) pairings, respectively. $\lambda_{\mu \nu}$ represents the effective pairing strength between the $\mu$ and $\nu$ Fermi surfaces. For $\mu = \nu$, it denotes the interband contribution within each Fermi surface, while for $\mu \neq \nu$, it accounts for the contribution from interband pair scattering. The true eigenvalue is a sum of all terms.
\[ \lambda = \sum_{\mu, \nu} \lambda_{\mu \nu} \]  

Figure 2(d) plots the band-resolved \( \lambda_{\mu \nu} \) for the leading solutions in each regime as a function of \( h \). In all three regimes, \( \lambda_{11} \) is always the largest, implying that the ‘jungle-gym’ Fermi surface is the major player in forming superconductivity. To understand this, we consider the electron pairing on each single Fermi surface alone and solve the one band Eliashberg equations with the same parameters. The results are compared in Figs. 2(e) and 2(f). For small \( h \), both Fermi surfaces have the same leading \((p_x + ip_y)\)-wave solution owing to the ferromagnetic-like pairing interaction; while for intermediate \( h \), the ‘jungle-gym’ Fermi surface favors a \( d_{x^2-y^2} \)-wave gap but the ‘doughnut’-like Fermi surface yields a nodal \( s \)-wave gap. Thus for the two-band model, the ‘jungle-gym’ Fermi surface dominates the leading pairing channel and gives rise to the \( d_{x^2-y^2} \)-wave gap for intermediate \( h \). We attribute this to the special topology of the ‘jungle-gym’ Fermi surface which is more strongly renormalized and matches better the momentum structure of the pairing glue than the ‘doughnut’-like one. The fact that \( \lambda_{22} \) is suppressed to almost zero in the two-band calculations compared to its value in the single-band calculations reflects microscopic competition of the pair formation on two Fermi surfaces. We would like to note that the ‘doughnut’-like Fermi surface was often treated as the major or only player in previous literatures. Our results suggest that this might be an oversimplified picture.

Figure 3 summarizes all the leading solutions on a global phase diagram of the superconductivity with varying \( Q \) for YbRh\(_2\)Si\(_2\). Among them, \((p_x + ip_y)\) dominates the lower part of the phase diagram with small \( h \), \( d_{x^2-y^2} \) governs most of the upper part, while the nodal \( s \)-wave solution only occurs at the corners. These are not unexpected, as the \((p_x + ip_y)\)-wave solution is a spin-triplet state favored by FM-like fluctuations with small \( h \). \( d_{x^2-y^2} \) originates from the nested ‘jungle-gym’ Fermi surface and associated AFM fluctuations, and the nodal \( s \)-wave solution, which is not crucial, might appear when large-momentum transfers start to correlate Cooper pairs on different portions of the Fermi surfaces. The true ground state of the superconductivity in YbRh\(_2\)Si\(_2\) can then be determined if the exact wave vector responsible for the pairing below \( T_c \) is known. Unfortunately, this requires very challenging experiment and is always the largest, implying that the ‘jungle-gym’ Fermi surface is the major player in forming superconductivity. To understand this, we consider the electron pairing on each single Fermi surface alone and solve the one band Eliashberg equations with the same parameters. The results are compared in Figs. 2(e) and 2(f). For small \( h \), both Fermi surfaces have the same leading \((p_x + ip_y)\)-wave solution owing to the ferromagnetic-like pairing interaction; while for intermediate \( h \), the ‘jungle-gym’ Fermi surface favors a \( d_{x^2-y^2} \)-wave gap but the ‘doughnut’-like Fermi surface yields a nodal \( s \)-wave gap. Thus for the two-band model, the ‘jungle-gym’ Fermi surface dominates the leading pairing channel and gives rise to the \( d_{x^2-y^2} \)-wave gap for intermediate \( h \). We attribute this to the special topology of the ‘jungle-gym’ Fermi surface which is more strongly renormalized and matches better the momentum structure of the pairing glue than the ‘doughnut’-like one. The fact that \( \lambda_{22} \) is suppressed to almost zero in the two-band calculations compared to its value in the single-band calculations reflects microscopic competition of the pair formation on two Fermi surfaces. We would like to note that the ‘doughnut’-like Fermi surface was often treated as the major or only player in previous literatures. Our results suggest that this might be an oversimplified picture.

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Yet experiments so far are inconclusive. In the original work, only one superconducting phase was reported below about \( 2 \) mK \[4\]. It has an extrapolated upper critical field, \( H_{c2}(T \to 0) \approx 30 - 50 \) mT, comparable to its orbital limiting field, \( H_{c2,\text{orb}} = 0.693(-dH_{c2}/dT)|_T, T_c \approx 35 \) mT \[45\] but well beyond the Pauli limiting field, \( H_{c2,\text{P}} = 1.84 T_c \approx 3.7 \) mT \[46\]. Since the Pauli limit is generally associated with pair breaking of the spin-singlet, the fact that \( H_{c2,\text{P}} \ll H_{c2,\text{orb}} \approx H_{c2} \) manifests dominant orbital effects and suggests that this single super-
conducting phase should be of spin-triplet pairing, in agreement with the first scenario in Fig. 4(b). However, latest experiment reported a different zero-field superconducting phase with $T_c \approx 6\text{ mK}$ and its transition to a field-induced phase with $T_c \approx 2\text{ mK}$ at about 4 mT [6]. The two phases show very different field dependence of $T_c$. While the field-induced phase is very similar to the originally observed (spin-triplet) one [1], the zero-field phase has an extrapolated upper critical field, $H_{c2}(T \to 0) \approx 4\text{ mT}$, which is below its Pauli limiting field, $H_{c2,P} = 1.84T_c \approx 11\text{ mT}$, and much smaller than the estimated orbital limiting field, $H_{c2,orb} \approx 27\text{ mT}$. Since $H_{c2} < H_{c2,P} < H_{c2,orb}$, the zero-field phase is most probably spin-singlet. Thus the latest experiment seems to support the second scenario proposed in Fig. 4(b). If this is the case, our theory predicts that the zero-field phase should be a $d_{x^2-y^2}$-wave spin-singlet state, and the field-induced phase would then be a $(p_x + ip_y)$-wave spin-triplet state. This further implies the existence of multiple superconducting phases under magnetic field being an intrinsic electronic property of YbRh$_2$Si$_2$, although the presence of nuclear order might play a role in the phase diagram. The seeming “inconsistency” of two experiments, possibly influenced by some yet-to-be-identified factors in the experimental setup, might actually be a supporting evidence for our proposal of two nearly-degenerate pairing states. More measurements are needed to clarify this uncertainty.

To summarize, we have proposed a quantum critical pairing mechanism for the newly-discovered superconductivity in YbRh$_2$Si$_2$ and explored numerically its possible gap symmetry using phenomenological pairing interactions with realistic band structures from first-principles calculations. For both theoretical parameters, we obtain nearly-degenerate $d_{x^2-y^2}$ and $(p_x + ip_y)$-wave solutions. This leads to two candidate temperature-magnetic field phase diagrams. While the original experiment seems to support a single $(p_x + ip_y)$-wave superconducting phase, the latest experiment supports the scenario of two superconducting phases. In the latter case, our result implies a spin-singlet $d_{x^2-y^2}$-wave pairing state at zero field and a field-induced spin-triplet $(p_x + ip_y)$-wave state. Further, our calculations show that the ‘jungle-gym’ Fermi surface plays the major role for electron pairing rather than the ‘doughnut’-like one. This differs from the conventional picture and requests more elaborate investigations of the Fermi surfaces in pursuit of a concrete and thorough understanding of the electronic properties of YbRh$_2$Si$_2$.

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[1] E. Schubert, M. Tippmann, L. Steinke, S. Laubsg, A. Steppke, M. Brando, C. Krellner, C. Geibel, R. Yu, Q. Si, and F. Steglich, Science 351, 485 (2016).
[2] J. Custers, P. Gegenwart, H. Wilhelm, K. Neuhauser, Y. Tokiwa, O. Trovarelli, C. Geibel, F. Steglich, C. Pépin, and P. Coleman, Nature 424, 524 (2003).
[3] S. Paschen, T. Lühmann, S. Wirth, P. Gegenwart, O. Trovarelli, C. Geibel, F. Steglich, P. Coleman, and Q. Si, Nature 432, 881 (2004).
[4] S. Friedemann, T. Westerkamp, M. Brando, N. Oeschler, S. Wirth, P. Gegenwart, C. Krellner, C. Geibel, and F. Steglich, Nat. Phys. 5, 465 (2009).
[5] O. Stockert and F. Steglich, Ann. Rev. Condens. Matter Phys. 2, 79 (2011).
[6] J. Saunders, Quantum materials into the microkelvin regime, (Advanced School and Workshop on Correlations in Electron Systems: from Quantum Criticality to Topology, 2018),
[7] K. Kummer, S. Patil, A. Chikina, M. Gütter, M. Hoppner, A. Generalov, S. Danzenbächer, S. Seiro, A. Hannaske, C. Krellner, Y. Kucherenko, M. Shi, M. Radovic, E. Rienks, G. Zwickyagl, K. Matho, J. W. Allen, C. Laubschat, C. Geibel, and D. V. Vyalikh, Phys. Rev. X 5, 011028 (2015).
[8] O. Trovarelli, C. Geibel, S. Mederle, C. Langhammer, F. M. Grosche, P. Gegenwart, M. Lang, G. Sparn, and F. Steglich, Phys. Rev. Lett. 85, 626 (2000).
[9] P. Gegenwart, J. Custers, C. Geibel, K. Neuhauser, T. Tayama, K. Tenya, O. Trovarelli, and F. Steglich, Phys. Rev. Lett. 89, 056402 (2002).
[10] K. Ishida, K. Okamoto, Y. Kawasaki, Y. Kitaoka, O. Trovarelli, C. Geibel, and F. Steglich, Phys. Rev. Lett. 89, 107202 (2002).
[11] C. Stock, C. Broholm, F. Demmel, J. Van Duijn, J. W. Taylor, H. J. Kang, R. Hu, and C. Petrovic, Phys. Rev. Lett. 109, 127201 (2012).
[12] M. P. Allan, F. Massee, D. K. Morr, J. Van Dyke, A. W. Rost, A. P. Mackenzie, C. Petrovic, and J. C. Davis, Nat. Phys. 9, 468 (2013).
[13] J. S. Van Dyke, F. Massee, M. P. Allan, J. C. Séamus Davis, C. Petrovic, and D. K. Morr, Proc. Natl. Acad. Sci. USA 111, 11663 (2014).
[14] D. J. Scalapino, Rev. Mod. Phys. 84, 1383 (2012).
[15] Y.-F. Yang, D. Pines, and N. J. Curro, Phys. Rev. B 92, 195131 (2015).
[16] J. P. Perdew, K. Burke, and M. Ernzerhof, Phys. Rev. Lett. 77, 3865 (1996).
[17] V. I. Anisimov, F. Aryasetiawan, and A. Lichtenstein, J. Phys.: Condens. Matt. 9, 767 (1997).
[18] M.-T. Suzuki and H. Harima, J. Phys. Soc. Jpn. 79, 024705 (2010).
[19] P. Blaha, K. Schwarz, G K H Madsen, D. Kvasnicka and J. Luitz, Wien2k: An Augmented Plane Wave plus Local orbital Program for Calculating the Crystal Properties.
