Formation of $\phi$ Mesic Nuclei

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We consider the structure and formation of the $\phi$ mesic nuclei to investigate the experimental feasibility of the observation of signals of the $\phi$ mesic nucleus formation. $\phi$ mesic nuclei are considered to be very important objects to study the in-medium modification of the $\phi$-meson spectral function at finite density. We consider ($\bar{p}, \phi$), ($\gamma, p$) and ($\pi^-, n$) reactions to produce a $\phi$-meson inside the nucleus and evaluate the effects of its medium modifications to the reaction cross sections. We also estimate the consequences of the uncertainties of the in-medium $\bar{K}$ self-energy to the $\phi$-nucleus interaction. We find that it may be possible to see peak structures in the reaction spectra for the strong attractive potential cases. On the other hand, for strong absorptive interaction cases with relatively weak attraction, it is very difficult to observe clear peaks and we may need to know the spectrum shape in a wide energy region to deduce the properties of $\phi$.

Subject Index: 215, 233

§1. Introduction

Mesic atoms such as pionic- and kaonic-atoms are Coulomb assisted meson-nucleus bound systems and have been studied systematically for a long time.1) The binding energies and widths of these bound states provide us unique and valuable information on the meson-nucleus interactions. Mesic nuclei, another kind of meson-nucleus systems, are meson-nucleus bound states mainly due to attractive strong interactions and have also been studied intensively these days.2)–10) In the contemporary hadron-nuclear physics, mesic atoms and mesic nuclei are considered to be very interesting objects in the following two aspects. First, they are strongly interacting exotic many body systems which should be explored by nuclear physicists. Second, mesic atoms and mesic nuclei provide unique laboratories for the studies of hadron properties at finite density which are significantly important to explore various aspects of the symmetries of the strong interaction.11),12)

In this paper, we report our studies on structure and formation of $\phi$ mesic nuclei. Especially, we study in detail the experimental feasibility of the formation

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of $\phi$ mesic nuclei, which are considered to be very interesting to deduce $\phi$ meson properties at finite density. We can intuitively expect to obtain new information on the OZI rule\textsuperscript{13} and the $\bar{s}s$ component of the nucleon at finite density from the study of the $\phi$ meson properties in nucleus. The analysis of QCD sum rules\textsuperscript{14} and the data taken at KEK\textsuperscript{15} suggested a 3\% mass reduction of $\phi$ at normal nuclear density. On the other hand, in-medium properties of the $\phi$ meson have also been studied theoretically with a close relation to $K$ and $\bar{K}$ meson properties in-medium because of the strong $\phi \to K\bar{K}$ coupling. The $\phi$ meson self-energy calculated in Refs. 16) and 17) indicates a significantly smaller attractive potential for $\phi$. Since we can expect to obtain new information from the $\phi$ meson bound states, which is complementary to the invariant mass measurements,\textsuperscript{15} we study the experimental feasibility of the formation of the $\phi$ mesic nuclear states to know the in-medium $\phi$ properties. Especially, in the experiment in Ref. 15), only a small fraction of the produced $\phi$ meson decays inside the nucleus because of the long lifetime of the $\phi$ meson. In the $\phi$ mesic nucleus, all $\phi$ mesons must stay inside the nucleus until their decay or absorption. Thus, we may have possibilities to get better information on $\phi$ properties in nucleus.

We show the calculated results of the bound states and formation spectra for some reactions in the following sections which will help to consider the actual design of the experiments.

§2. Optical potential and bound states

We study the properties of the $\phi$ mesic nuclei by solving the Klein-Gordon equation,

\begin{equation}
[-\vec{\nabla}^2 + \mu^2 + \Pi_\phi(E, \rho(r))]\phi(\vec{r}) = \omega^2 \phi(\vec{r}).
\end{equation}

Here, $\mu$ is the $\phi$-nucleus reduced mass. We employ the empirical Woods-Saxon form for the nuclear matter density as

\begin{equation}
\rho_m(r) = \rho_p(r) + \rho_n(r) = \frac{\rho_0}{1 + \exp[(r - R)/a]},
\end{equation}

where $R$ and $a$ indicate radius and diffuseness parameters for the nucleus, and are fixed to be $R = 1.18A^{1/3} - 0.48$ (fm) and $a = 0.5$ (fm), respectively. The density distributions for the center of protons or neutrons, which are used in the calculations, are given by the same prescription as in Ref. 18). We solve the Klein-Gordon equation numerically following the method of Oset and Salcedo.\textsuperscript{19}

In Eq. (2.1) $\Pi_\phi(E, \rho(r))$ stands for the $\phi$ meson in-medium self-energy, which we evaluate in a hadronic many-body framework\textsuperscript{17} which we briefly discuss below. In vacuum, the $\phi$ spectrum is dominated by its coupling to $KK$ pairs, which accounts for roughly 85\% of the $\phi$ decay width. It is thus expectable that the $\phi$ meson properties in the medium are closely related to the renormalization of $\bar{K}$ and $K$ through the kaon nucleon interaction. Moreover, despite the strong coupling to kaons, the $\phi$ decay width in vacuum is as small as $\sim 4$ MeV, which reflects the fact that the kaon pair has little available phase space ($M_{\phi} \gtrsim 2m_K$) and the $\phi \to K\bar{K}$ decay takes place barely above threshold. Therefore, in-medium modifications of the properties
of kaons (e.g., an attractive $\bar{K}$ nuclear potential) will have a strong effect in the $\phi$ meson reactivity at finite nuclear density. In our calculation of the in-medium $\phi$ self-energy we have neglected the contribution from the $\phi \rightarrow 3\pi$ decay channel. The reason is two-fold: first, the contribution of the $3\pi$ decay mode to the $\phi$ vacuum width is suppressed as compared to the kaon channel (which can be understood as the process taking place via $\phi - \omega$ mixing); and second, one does not expect sizable effects in the $\phi$ spectrum (at least at moderate densities) from the relative change of the three-pion phase space in the medium with respect to the vacuum case.

Following Ref. 20), the $\phi\bar{K}K$ coupling is described in a gauge vector representation with minimal coupling to the pseudoscalar meson octet chiral $SU(3)$ Lagrangian. The $\phi$ self-energy at one loop is built from $\bar{K}K$ loop and kaon tadpole diagrams. The only coupling constant, $g_\phi$, is straightforwardly determined from the $\phi \rightarrow \bar{K}K$ vacuum decay width. Nuclear effects are accounted for in our model by using the in-medium $\bar{K}$ and $K$ propagators, which are dressed by pertinent self-energies obtained in a self-consistent, many-body framework from an evaluation of the kaon nucleon interaction within an $SU(3)$ chiral unitary approach. The model incorporates the $S$- and $P$-waves of the kaon-nucleon interaction. A direct coupling of the $\phi$ meson field to the nucleons ($\phi NN$) is OZI forbidden and has been neglected in our approach. At tree level, the $S$-wave $\bar{K}N$ amplitude arises from the Weinberg-Tomozawa term of the meson baryon octet chiral Lagrangian. Unitarization in coupled channels is imposed by solving the Bethe-Salpeter equation with on-shell amplitudes. Remarkably, with a single regularization parameter, the unitarized $\bar{K}N$ amplitude generates dynamically the $\Lambda(1405)$ resonance in the $I = 0$ channel and provides a satisfactory description of low-energy scattering observables. The in-medium solution of the $S$-wave $\bar{K}N$ amplitude accounts for Pauli-blocking effects, mean-field binding on all the baryons, and the dressing of the pion and kaon propagators through their corresponding self-energies, which leads to a self-consistent calculation. The model incorporates, in addition, a $P$-wave contribution to the $\bar{K}$ self-energy from hyperon-hole ($Yh$) subthreshold excitations, including $\Lambda$, $\Sigma$ and $\Sigma^*$ components. These excitation modes contribute at finite kaon momentum and provide considerable strength in the $\bar{K}$ spectral function at low energies, below the quasi-particle peak. The $S$-wave self-energy is mostly responsible for an attractive $\bar{K}$ nuclear optical potential, which amounts to about $40 - 60$ MeV at normal nuclear matter density. In addition, the $\bar{K}$ spectral function exhibits a rich, wide structure as a result of the $\Lambda(1405)/h$ excitation, which mixes with the kaon quasi-particle mode and the $P$-wave modes at low energies. Regarding the $K$ meson, due to the absence of $S = +1$ baryon resonances in the $KN$ channel, its in-medium self-energy can be reasonably described by a low-density $T\rho$ approximation which results in a mildly repulsive nuclear potential. Recent improvements over this approximation, taking into account the $K$ self-energy into the full self-consistent scheme both in cold nuclear matter and at finite temperature, have been reported in 24) and 25). Within this model, the $\phi$ meson decay probability at finite nuclear density is notably increased, as the $\phi$ self-energy accounts for additional in-medium decay channels such as $\phi N \rightarrow KY$ (with $Y = \Lambda, \Sigma, \Sigma^*$) and $\phi N \rightarrow [K\Lambda(1405)] \rightarrow KMY$ (with $MY = \pi\Lambda, \pi\Sigma$). The $\phi$ decay width at normal nuclear matter density is predicted to be about 6 times the...
vacuum width. A theoretical analysis of the \( \phi \) nuclear photoproduction reaction in connection with the in-medium \( \phi \) inelastic decay width was elaborated in \( 26) \).

In addition to the renormalization of the kaon propagators with realistic in-medium self-energies, our model of the \( \phi \) self-energy accounts for pertinent vertex corrections \( (\phi KY N^{-1}, \phi KMY N^{-1}) \) in order to guarantee the transversality of the \( \phi \) self-energy tensor, which then fulfills the associated Ward-Takahashi identities. Further details about transversality of vector mesons in hadronic many-body approaches can be found, for instance, in Refs. \( 16), 17), \) and \( 22) \).

We show the \( \phi \) meson optical potential \( V_{\phi}^{\text{opt}}(r, E) \) in Fig. 1, which is obtained from the \( \phi \) meson self-energy \( \Pi_{\phi} \) in Ref. \( 17) \) as

\[
V_{\phi}^{\text{opt}}(r, E) = \frac{1}{2\mu} \left[ \Pi_{\phi}(E, \rho(r)) - \Pi_{\phi}(E, 0) \right],
\]

within the local density approximation. The self-energy \( \Pi_{\phi} \) depends on the energy \( E \) of the \( \phi \) meson and on the density \( \rho \). Here, we have defined the \( \phi \) meson optical potential by the in-medium \( \phi \) meson self-energy \( \Pi_{\phi}(E, \rho(r)) \) subtracted by the vacuum self-energy \( \Pi_{\phi}(E, 0) \), and we use the observed \( \phi \) meson mass \( 27) \) to evaluate the reduced mass \( \mu \). Since the \( V_{\phi}^{\text{opt}} \) does not include the vacuum self-energy, the calculated widths should be considered as the variations from the width in free space.

As shown in Fig. 1, the theoretical potential based on \( \Pi_{\phi}^{17) \) is a weak attractive potential, \( \text{Re} V_{\phi}^{\text{opt}}(0, E = m_{\phi}) \sim -7.5 \text{ MeV} \). The absorptive part of the potential \( \text{Im} V_{\phi}^{\text{opt}}(r, E) \) is also weak and has a similar strength as the real part. The theoretical model of Ref. \( 17) \) is based on the OZI rule,\( 13) \) where the direct coupling of the \( \phi \) meson optical potential by the in-medium \( \phi \) meson self-energy \( \Pi_{\phi}(E, \rho(r)) \) subtracted by the vacuum self-energy \( \Pi_{\phi}(E, 0) \), and we use the observed \( \phi \) meson mass\( 27) \) to evaluate the reduced mass \( \mu \). Since the \( V_{\phi}^{\text{opt}} \) does not include the vacuum self-energy, the calculated widths should be considered as the variations from the width in free space.

We calculate the binding energies B.E. and widths \( \Gamma \) of the bound states, which are related to the eigenenergy \( \omega \) in Eq. \( (2.1) \) as \( \omega = \mu - \text{B.E.} - i\Gamma/2 \), by solving the Klein-Gordon equation to know the resonance energies in the reaction spectra observed in experiments. Here, we solve the Klein-Gordon equation to obtain the B.E. consistent to the \( \phi \) meson energy \( E \) used to evaluate the \( \phi \) meson optical potential \( V_{\phi}^{\text{opt}}(r, E) \). The calculated results are shown in Table I for four nuclei which are expected to be in the final states in the one proton pick-up reactions considered in the next section. We see that there exist several bound states of \( \phi \) in nuclei, however, the large widths of the states suggest the difficulties of the observation of
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Fig. 1. The $\phi$ meson optical potentials as a function of the radial coordinate $r$ for $\phi-^{11}\text{B}$ system obtained from $I_{\phi}$ in Ref. 17). The left and right panels show the real and imaginary parts, respectively. The solid, dashed, dotted, and dotted-dashed lines indicate the potential strength for the $\phi$ meson energies $E - m_{\phi} = 0\text{ MeV}$, $-10\text{ MeV}$, $-20\text{ MeV}$, and $-30\text{ MeV}$, respectively.

Table I. Calculated binding energies and widths (FWHM) of $\phi$ meson bound states in $^{11}\text{B}$, $^{39}\text{K}$, $^{123}\text{In}$, and $^{207}\text{Tl}$. Width shows the variation of the in-medium $\phi$ decay width from that in vacuum $\Gamma_{\phi}^{\text{free}} = 4.26\text{ MeV}^{27}$ in this table (see details in text).

|         | Shallow potential | Deep potential |
|---------|------------------|----------------|
|         | B.E.$(\Gamma)$ [MeV] | B.E.$(\Gamma)$ [MeV] |
| $^{11}\text{B}$ | none 1.0 (17.9) | none 1.0 (17.9) |
| $^{39}\text{K}$ | none 1.0 (20.0) | none 1.0 (20.0) |
| $^{123}\text{In}$ | $^1s$ 2.34(21.6) | $^1s$ 2.34(21.6) |
|         | $^2p$ 11.8 (19.8) | $^2p$ 11.8 (19.8) |
| $^{207}\text{Tl}$ | $^1s$ 3.73(22.5) | $^1s$ 3.73(22.5) |
|         | $^2s$ 12.2 (19.9) | $^2s$ 12.2 (19.9) |
|         | $^2p$ 26.3 (19.9) | $^2p$ 26.3 (19.9) |
|         | $^3p$ 13.1 (20.4) | $^3p$ 13.1 (20.4) |
|         | $^3d$ 27.5 (19.8) | $^3d$ 27.5 (19.8) |

peak structures in the missing mass spectra.

§3. Formation spectra of $\phi$ mesic nuclei

We consider three types of proton pick-up reactions to form the $\phi$-mesic nucleus which are $(\gamma,p)$, $(\pi^-,n)$, and $(\bar{p},\phi)$. The elementary processes of these three reactions are $\gamma + p \rightarrow \phi + p^{28}$, $\pi^- + p \rightarrow \phi + n^{29}$, and $\bar{p} + p \rightarrow \phi + \phi^{30}$ respectively. We would like to make a few comments on the data in Ref. 28), which have not been published yet. In the present calculation, we need to use the backward cross section of $\phi$ production in the elementary process, which was evaluated here by a functional fit to the data shown in Ref. 28). However there are large uncertainties in the value.
Fig. 2. Momentum transfer of the ($\gamma, p$) (solid curve), ($\pi^-, n$) (dashed curve) and ($\bar{p}, \phi$) (dotted curve) reactions for the $\phi$ mesic nucleus formation. Protons in the initial state are assumed to be in $1p_{3/2}$ state in $^{12}\text{C}$.

Fig. 3. Calculated spectra for the $\phi$-nucleus systems formation plotted as a function of the emitted $\phi$ meson energy in the $^{12}\text{C}(\bar{p}, \phi)$ reaction at $p_{\bar{p}} = 1.3$ GeV/c. Total cross sections (upper panels) and conversion parts (lower panels) are calculated with the theoretical (shallow) optical potential (left panels) and the scaled (deep) optical potential (right panels). The dashed and dotted lines show the contributions from different proton hole states coupled to the sufficient numbers of partial waves of $\phi$ meson. The vertical dashed line indicates the $\phi$ meson production threshold.

We first show in Fig. 2 the momentum transfer of the reactions as it is an important information of meson bound state formation. We found that the momentum transfer of the ($\bar{p}, \phi$) reaction is relatively smaller than the other two reactions. However, we also found that the momentum transfer is always larger than 100 MeV/c in the energy range plotted in Fig. 2.

We use the Green’s function method to calculate the formation spectra of the.
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$\phi$ meson from nucleus\textsuperscript{34) as in the cases of other mesic nucleus formation.\textsuperscript{3)–7,9,10)} We briefly explain the formalism for the $(\bar{p}, \phi)$ reaction as an example. The expected spectra of the $(\bar{p}, \phi)$ reactions \(\frac{d^2\sigma}{d\Omega dE_\phi}\) are evaluated from the nuclear response function \(S(E)\) and the elementary cross section \(\frac{d\sigma}{d\Omega}\)\textsuperscript{ele} in the impulse approximation as

\[
\left(\frac{d^2\sigma}{d\Omega dE_\phi}\right) = \left(\frac{d\sigma}{d\Omega}\right)\textsuperscript{ele} \times S(E) .
\]

The calculation of the nuclear response function with a complex potential is formulated by Morimatsu and Yazaki\textsuperscript{34} as

\[
S(E) = -\frac{1}{\pi} \text{Im} \sum_f \int \! d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 \left\{ \phi(\vec{r}) \frac{1}{E - H_\phi + i\epsilon} \phi^+(\vec{r}') |\alpha\rangle \right\} G(E; \vec{r}, \vec{r}') \tau_f ,
\]

where the summation is taken over all possible final states. \(G(E; \vec{r}, \vec{r}')\) is the Green’s function of \(\phi\) meson interacting in the nucleus and defined as

\[
G(E; \vec{r}, \vec{r}') = \langle \alpha | \phi(\vec{r}) \frac{1}{E - H_\phi + i\epsilon} \phi^+(\vec{r}') |\alpha\rangle ,
\]

where \(\alpha\) indicates the proton hole state and \(H_\phi\) indicates the Hamiltonian of the \(\phi\) meson-nucleus system. The amplitude \(\tau_f\) denotes the transition of the incident particle (\(\bar{p}\)) to the nucleon-hole and the outgoing \(\phi\) meson, involving the nucleon-hole wavefunction \(\psi_{jN}\) and the distorted waves \(\chi_i\) and \(\chi_f\) of the projectile and ejectile. By taking the appropriate spin sum, the amplitude \(\tau_f\) can be written as

\[
\tau_f(\vec{r}) = \chi_f^*(\vec{r}) \xi_{1/2, m_s}^* [Y_{l_\phi}^*(\hat{\vec{r}}) \otimes \psi_{jN}(\vec{r})] J_M \chi_i(\vec{r}) ,
\]

with the meson angular wavefunction \(Y_{l_\phi}^*(\hat{\vec{r}})\) and the spin wavefunction \(\xi_{1/2, m_s}\) of the ejectile. We assume the harmonic oscillator wavefunctions for \(\psi_{jN}\) with the empirical range parameter.

The semi-exclusive spectra can be calculated by decomposing the response function (3.2) into the escape and conversion parts: \(S = S_{\text{esc}} + S_{\text{con}}\). This decomposition can be done exactly by

\[
S_{\text{con}}(E) = -\frac{1}{\pi} \sum_f \int \! d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 \left\{ \phi(\vec{r}_1) \frac{1}{E - H_\phi + i\epsilon} \phi^+(\vec{r}_2) \right\} G(E; \vec{r}_1, \vec{r}_2) \text{Im} V_{\text{opt}}^{\phi}(r_2, E) G(E; \vec{r}_2, \vec{r}_3) \tau_f(\vec{r}_3) \]

and

\[
S_{\text{esc}}(E) = -\frac{1}{\pi} \sum_f \int \! d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 d\vec{r}_4 \tau_f(\vec{r}_1) \left\{ \delta(\vec{r}_1 - \vec{r}_2) + G(E; \vec{r}_1, \vec{r}_2) V_{\text{opt}}^{\phi}(r_2, E) \right\}
\times \text{Im} G_0(E; \vec{r}_2, \vec{r}_3) \left\{ \delta(\vec{r}_3 - \vec{r}_4) + V_{\text{opt}}^{\phi}(r_3, E) G(E; \vec{r}_3, \vec{r}_4) \right\} \tau_f(\vec{r}_4) ,
\]
where $V_{\phi}^{\text{opt}}(r, E)$ is the $\phi$ meson-nucleus optical potential given in the Hamiltonian. The conversion part is known to express the contributions of the $\phi$ meson absorption to the $(\bar{p}, \phi)$ spectra.\(^{34}\)

We show the calculated cross sections of the $^{12}\text{C}(\bar{p}, \phi)$ reaction proposed in Ref. 35) for the formation of a $\phi$ - $^{11}\text{B}$ system in Fig. 3 for the theoretical (shallow) optical potential case in the left panels and for the scaled (deep) optical potential case in the right panels. The conversion parts of the spectra are also shown in the lower panels in addition to the total spectra in the upper panels. Since the imaginary part of the $\phi$ meson optical potential does not include the contribution of the $\phi$ decay in free space as explained in §2, we implement a smearing by a Gaussian distribution with the $\phi$ meson decay width in vacuum to the spectra shown in this article to simulate the effects of the $\phi$ decay in free space. In these spectra, the vertical dashed lines show the $\phi$ meson production threshold with the daughter nuclei $^{11}\text{B}$ in ground state, which is considered to be $p_{3/2}^{-1}$ proton hole state of $^{12}\text{C}$ here. The separation energies of protons $S_p$ from the single particle levels in target $^{12}\text{C}$ are taken into account to calculate the spectra and we get $S_p(p_{3/2}^{-1}) = 15.957$ MeV and $S_p(s_{1/2}^{-1}) = 33.857$ MeV, respectively.\(^{36}\) Thus, it should be noted that the $\phi$ meson production threshold with the proton $s_{1/2}^{-1}$ hole state, which corresponds to the excited state of the daughter nuclei, is 17.9 MeV ($= S_p(s_{1/2}^{-1}) - S_p(p_{3/2}^{-1})$) shifted to the right direction from the vertical line in the figure. The contributions from the $\phi$ meson bound states should appear in the subthreshold region of the reaction spectra which corresponds to the left side of the threshold in the figure. Especially, the bound states with sufficiently narrow widths are expected to be seen as resonance peaks in the subthreshold region as in the case of the observation of the deeply bound pionic atoms.\(^{31-33}\) On the right side of the threshold in the spectra, we expect to see the contributions from the quasi-free (unbound) $\phi$ meson production in the reaction.

For the shallow optical potential case, since no bound state exists as shown in Table I, the spectrum has a smooth shape due to the quasi-free $\phi$ production without any indications of the bound states in the subthreshold region. For the deep potential case, we find some enhancement of the spectra in the bound energy region. However, we do not observe any clear structures in the spectrum due to the bound states again for the deeper potential case even when there exists one bound state. The main reasons why the spectrum obtained with the bound state shows no clear indications of the state are the existence of large contributions from the unbound $\phi$ meson production with higher partial waves, and the large decay width of the bound state as discussed in the next section. The large contributions of the unbound $\phi$ meson mask the signals of the bound state and the width makes the signal peak unclear.

We also find that the conversion contributions shown in lower panels, which are defined in Eq. (3-6) as a contribution of the absorptive interaction $\text{Im} V_{\phi}^{\text{opt}}(r, E)$ and are expected to correspond to the $(\bar{p}, \phi)$ spectra coincident with the particle emissions from $\phi$ meson absorption in nucleus, resemble each other for both the shallow and deep potential cases, even though they have different constitutions of subcomponents. One of the advantages of the $(\bar{p}, \phi)$ reaction is a possible background
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Fig. 4. Calculated spectra for the $\phi$-nucleus systems formation plotted as a function of the emitted $\phi$ meson energy in the $^{124}\text{Sn}(\bar{p}, \phi)$ reaction at $p_\bar{p} = 1.3$ GeV/c. We include the contributions from outer proton orbits, $1g_9/2$ (dashed lines), $2p_1/2$ (dotted lines), and $2p_3/2$ (dash-dotted lines). Total cross sections are calculated with the theoretical (shallow) optical potential (left panel) and the scaled (deep) optical potential (right panel). The vertical dashed line indicates the $\phi$ meson production threshold.

reduction by the conversion spectrum as described in Ref. 35). Thus, the conversion spectrum in Fig. 3 will be an important piece of information to observe the $\phi$ meson properties in nucleus.

To investigate the mass number dependence of the formation spectra, we show the calculated spectra in Fig. 4 for $^{124}\text{Sn}$ target with the shallow and deep potential cases. We find that there appear some enhancements in the $\phi$ meson bound energy region for the deep potential case. However, again we do not see any clear structures corresponding to the bound states. As we have expected from the widths of the bound states, it is difficult to observe any peak structure due to bound states formation in the standard missing mass spectra.

In Figs. 5 and 6, we show the calculated results for different formation reactions, namely, $^{12}\text{C}(\gamma, p)$ (Fig. 5) and $^{12}\text{C}(\pi^-, n)$ (Fig. 6). In both cases, the momentum transfers of the reactions are larger than for the $\bar{p}$ induced case and, hence, the spectra around threshold are suppressed in these reactions.

§4. Discussion

In the previous section, we have calculated the formation spectra of $\phi$ mesic nuclei using the optical potential based on the theoretical $\phi$ self-energy\cite{17} and the scaled potential with a deeper real part than the theoretical one to consider the experimental feasibility. We have found that it is very difficult to observe clear signals even for the scaled potential case with a bound state because of the large contributions from unbound $\phi$ meson production process and the large decay width of the bound state. In this section, we study the experimental feasibility of the $^{12}\text{C}(\bar{p}, \phi)$ reaction using the optical potential with various depths to know the conditions to observe clear signals of the existence of the bound states. We also consider the effects
Fig. 5. Calculated spectra for the $\phi$-nucleus systems formation plotted as a function of the emitted proton energy in the $^{12}\text{C}(\gamma,p)$ reaction at $p_\gamma = 2.7$ GeV/c. The dashed and dotted lines show the contributions from $s^{-1/2}_{1/2}$ and $p^{-3/2}_{3/2}$ proton hole states, respectively. The vertical dashed line indicates the $\phi$ meson production threshold. The theoretical (shallow) optical potential is used.

Table II. Calculated binding energies and widths (FWHM) of $\phi$ meson bound states in $^{11}\text{B}$ with the energy independent optical potential. The width shows the variation of the in-medium $\phi$ decay width from that in vacuum $\Gamma_{\phi}^{\text{free}} = 4.26$ MeV$^{27}$ as in Table I.

| $V_{\phi}^{\text{opt}}(0)$ | B.E.$\Gamma$ [MeV] | B.E.$\Gamma$ [MeV] |
|--------------------------|-------------------|-------------------|
| $(-70, -10)$ MeV         | 36.6(16.0)        | 36.2(32.1)        |
| $(-70, -20)$ MeV         | 12.3(11.4)        | 11.5(23.1)        |

of the ambiguities of the in-medium $\bar{K}$ self-energy to the $\phi$ mesic nucleus formation.

First, we show the spectra calculated with an energy independent potential in Woods-Saxon form in Fig. 7. We consider here two different imaginary potential strengths at the origin as $V_{\phi}^{\text{opt}}(r = 0) = (-70, -10)$ and $(-70, -20)$ MeV. These potentials provide the bound states in $^{11}\text{B}$ as compiled in Table II. In Fig. 7, we observe that the $2p$ bound states of $\phi$ can be seen in the spectrum for the weaker absorptive potential case. In this case, the level width is 11.4 MeV and is almost the same size as the binding energy. As for the stronger absorptive potential case, the signal of the same state is uncleaner but is still seen as a shoulder structure in the spectrum. In this case, the binding energy and width are 11.5 MeV and 23.1 MeV, respectively. Hence, we can say semi-quantitatively that one criterion for observing the bound state signal in the missing mass spectrum is B.E. $\geq \Gamma/2$. The criterion guarantees that the signal appears in the spectra separately enough from the quasi-elastic contributions. Another important point is the strength of the signals. As we can see in Table II, the $1s$ states of $\phi$ in both potential cases satisfy the criterion
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Fig. 6. Calculated spectra for the $\phi$-nucleus systems formation plotted as a function of the emitted neutron energy in the $^{12}$C($\pi^-$, $n$) reaction at $p_{\pi^-} = 2.0$ GeV/c. The dashed and dotted lines show the contributions from $s_{1/2}^{-1}$ and $p_{3/2}^{-1}$ proton hole states, respectively. The vertical dashed line indicates the $\phi$ meson production threshold. The theoretical (shallow) optical potential is used.

Fig. 7. Calculated spectra for the $\phi$-nucleus systems formation plotted as functions of the emitted $\phi$-meson energy in the $^{12}$C($\bar{p}$, $\phi$) reaction at $p_{\bar{p}} = 1.3$ GeV/c. We used an energy independent optical potential of Woods-Saxon type. The left panel shows the results with $V_{\phi \text{opt}}(r = 0) = (-70, -10)$ MeV, and the right panel shows those with $V_{\phi \text{opt}}(r = 0) = (-70, -20)$ MeV. The dashed and dotted lines indicate the contributions from different subcomponents $[l_j^{-1} \otimes l_o]$ as shown in the figure.

B.E. $\geq \Gamma/2$ and they are seen as peak structures in the plot of subcomponents in Fig. 7. However, they do not appear in the total spectra because their cross sections are significantly smaller than those of the 2p states. As we have explained, the large background is one of the reasons why the 1s state of $\phi$ meson in $^{11}$B shown in Table I does not appear in the right panels in Fig. 3.
Similarly, we need to impose an additional condition such as (Level Spacing) $\geq \Gamma/2$ for heavier nucleus cases, where we have several subcomponents due to several nucleon holes and $\phi$ bound states, to avoid the overlapping of many contributions of the subcomponents smearing out the signals. We see that this condition is not satisfied for $\phi$ states in $^{123}\text{In}$ shown in Table I and Fig. 4.

We mention another semi-quantitative feature of the $\phi$ mesic nucleus shown in Tables I and II. The values of the widths of the bound states are distributed within a relatively narrow range as

$$|\text{Im } V_{\phi}^{\text{opt}}(0)| \leq \Gamma \leq 2|\text{Im } V_{\phi}^{\text{opt}}(0)|.$$  \hspace{1cm} (4.1)

This feature is completely different from the Coulomb assisted bound states, and seems to be quite natural for the mesic nuclei by considering the structure of them. Thus, we can say that by knowing the imaginary potential depth we can estimate the widths of the states by Eq. (4.1) and, then, can estimate the necessary binding energies and/or the level spacings to observe the clear signals in the spectrum as described in the previous paragraphs.

Next we consider the energy dependent deep potentials to check the effects of the energy dependence. We scale again the theoretical potential in Ref. 17) and consider deep real potential cases as $\text{Re } V_{\phi}^{\text{opt}}(0, E = m_{\phi}) = -40, -70, -100$ MeV keeping the $\text{Im } V_{\phi}^{\text{opt}}(r, E)$ to the theoretical value. We mention here again that the scaling of the real part does not have any theoretical foundations. The calculated results are shown in Fig. 8. We find again that we can observe a clear peak structure due to bound states formation if the optical potential is attractive enough. In Fig. 8, the contribution from bound $\phi$ meson formation in $2p$ state is observed as a peak between $T_{\phi} = 500$ MeV and 600 MeV in the middle and right panels. As shown in Fig. 1, the imaginary potential is weaker than $-13$ MeV in this energy region and the criterion described above is confirmed to be satisfied for the strong attractive potential cases shown in the middle and right panels in Fig. 8. In the left panel, the tiny cusp around $T_{\phi} = 500$ MeV is not due to the bound state but to the threshold effects.

We discuss the effects of the energy dependence of the potential by comparing the middle panel of Fig. 8 with Fig. 7. The potential has an energy dependence around the threshold as shown in Fig. 1. For smaller $\phi$ energies the potential becomes more attractive and less absorptive. In the case of the middle panel of Fig. 8, the $\phi$ mesic $2p$ state has B.E. = 33.3 MeV and $\Gamma=15.6$ MeV. Because of the energy dependence of the real part of the potential, the B.E. is significantly larger than the energy independent cases listed in Table II.* The position of the peak due to this $2p$ state formation is shifted to left in Fig. 8 middle panel from those in Fig. 7. As for the imaginary part, no significant effect to the spectrum is found due to the energy dependence in these cases.

Finally, we have checked the sensitivity of the optical potential $V_{\phi}^{\text{opt}}(r, E)$ to the $K^-$ self-energy $\Pi_{K^-}$ in the nucleus. Since in the theoretical model of Ref. 17),

* Since we use the potential with scaled real part in Fig. 8, the energy dependence is also scaled and the effects of it somehow enhanced.
the medium modification of the $K^+$ and $K^-$ cloud around the $\phi$ meson dominantly causes the modifications of the $\phi$ properties as explained §2, and since the $K^-$ self-energy $\Pi_{K^-}$ has a large uncertainty and the actual strength of the $K^-$ optical potential is not yet fully constrained by experiment, it is interesting to study its effects in $\Pi_{\phi}$ and in the properties of $\phi$ bound states. For this purpose, we simply parametrize the modified $K^-$ self-energy $\tilde{\Pi}_{K^-}$ as
\[
\tilde{\Pi}_{K^-} = a\Pi_{K^-},
\]  
where the $\Pi_{K^-}$ on the right-hand side is obtained by the chiral unitary model in Ref. 17). We consider the constant parameter $a$ to scale the real and imaginary parts of $\Pi_{K^-}$. We show the calculated $\Pi_{\phi}$ for two cases with $a = 1$ and 2 in Fig. 9. We find that a certain ambiguity of $\Pi_{\phi}$ due to that of $\Pi_{K^-}$ exists as we expected. However, the strengths of the real and imaginary parts of $\Pi_{\phi}$ are correlated, namely, the strong attractive real part always appears with the strong absorptive imaginary part and so on. Around the threshold $E_{\phi} \sim 1 \text{ GeV}$, we have $|\text{Re} \ V_{\text{opt}}^{\phi}\rangle \approx |\text{Im} \ V_{\text{opt}}^{\phi}\rangle/2$ for both cases and thus we do not expect improvements to observe a clear structure in the spectrum. Thus, in any case, it will be difficult to see clear structures well separated from the threshold and from the quasi-elastic contribution in the formation cross section according to the criterion discussed above, since the binding energy B.E. is always smaller than $|\text{Re} \ V_{\text{opt}}^{\phi}\rangle$ at $\rho_0$ in the nucleus.

§5. Summary

In this article we have studied the structure and formation of the $\phi$ mesic nuclei to know the experimental feasibility of the observation of the $\phi$ meson bound states, which are expected to be very interesting to get new information on the in-medium modifications of the $\phi$ meson properties. We have calculated the binding energies and the absorption widths of the $\phi$ meson bound states. We have also studied the production of the $\phi$ meson in the nucleus and obtained numerical results for the formation spectra of the ($\bar{p}, \phi$), ($\gamma, p$), and ($\pi^-, n$) reactions. Due to the large width of the $\phi$ meson in nucleus, we found that the observation of a clear peak in the missing mass spectra will be difficult if the $\phi$-nucleus potential is weakly attractive as reported in Refs. 16) and 17). A deep potential ($\text{Re} \ V_{\text{opt}}^{\phi}(0, E = m_\phi) \sim -30 \text{ MeV}$) corresponding to the 3% mass reduction reported in Refs. 14) and 15) is not enough to produce clear peaks in the reaction spectra for the imaginary potential strength evaluated in Ref. 17). By considering various cases with different potential strengths, we find that the condition $\text{B.E.} \geq \Gamma/2$ must be satisfied for the light nucleus cases to observe a clear peak structure due to the bound state formation. For the widths of the mesic nucleus states, we can make a rough estimation as $|\text{Im} \ V_{\text{opt}}^{\phi}(0)| \leq \Gamma \leq 2|\text{Im} \ V_{\text{opt}}^{\phi}(0)|$, which is not valid for Coulomb assisted atomic bound states but only to be applied to the mesic nuclear states. These relations can be used as criteria to have clear signals of the bound state formation in the formation spectra. We have also considered the sensitivity of the $\phi$-nucleus interaction to the uncertainties of the anti-kaon self-energy in nucleus.
Fig. 8. Calculated spectra for the $\phi$-nucleus systems formation plotted as functions of the emitted $\phi$-meson energy in the $^{12}\text{C}(\bar{p}, \phi)$ reaction at $p_\bar{p} = 1.3$ GeV/c. Total cross sections (upper panels) and conversion parts (lower panels) are calculated with deep potentials obtained by scaling the theoretical potential\cite{17} as $\text{Re} V_{\phi}^{\text{opt}}(0, E = m_\phi) = -40$, $-70$, and $-100$ MeV with keeping the imaginary part unmodified. The dashed and dotted lines indicate the contributions from different subcomponents $[l_j^{-1} \otimes l_\phi]$ as shown in the figure.

As a conclusion, we find useful relations which must be satisfied to observe clear signals in the formation cross sections, and we find that it is not easy to observe clear signals of the $\phi$-mesic nuclei in the standard missing mass observation for the case of 3% mass reduction in nucleus with the imaginary potential in Ref. 17), for example. However, the subthreshold meson production in nucleus has the definite advantage to observe the meson properties at finite density since all the populated mesons stay inside the nucleus until the absorption by the nucleus. Thus, we think that further theoretical investigations are very important to find a better way to observe subthreshold $\phi$ mesons in nucleus.

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Fig. 9. Real part (left) and imaginary part (right) of the calculated $\phi$-nucleus optical potential $V_{\phi}^{\text{opt}}$ are plotted as a function of the $\phi$ meson energy $E_{\phi}$ at $\rho = \rho_0$. Two lines show the $V_{\phi}$ obtained with different choices of $\Pi_K^-$ in the model of Ref. 17) (see details in the text).

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