Quantum computing with an electron spin ensemble

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We propose to encode a register of quantum bits in different collective electron spin wave excitations in a solid medium. Coupling to spins is enabled by locating them in the vicinity of a superconducting transmission line cavity, and making use of their strong collective coupling to the quantized radiation field. The transformation between different spin waves is achieved by applying gradient magnetic fields across the sample, while a Cooper Pair Box, resonant with the cavity field, may be used to carry out one- and two-qubit gate operations.

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The construction of a large quantum computer is a challenge for current research. The overarching problem is to develop physical systems which can reliably store thousands of qubits and which allow addressability of individual bits and pairs of bits in gate operations. Proposals in which single trapped ions or atoms encode qubits in their internal state have successfully demonstrated the building blocks for few-bit devices, while scaling of these systems to larger register sizes is believed to require interconnects, e.g., with optical transmission. A novel collective encoding scheme for qubits proposes to use many identical quantum systems to encode each qubit, either in the collective population of different internal states [1, 2, 3] or in different spatial modes of excitation of the entire system [4, 5].

In this Letter we propose a hybrid approach to quantum computing making use of an ensemble of $10^{10} - 10^{12}$ electron spins coupled to a superconducting transmission line cavity. We will describe how a large number of spatial modes can be addressed in the spin ensemble, and how a transmon Cooper Pair Box (CPB) [6] integrated in the cavity can provide one- and two-bit gates for quantum computing in the spin ensemble [7, 8]. Our scheme enables materials for which large spin coherence times have been demonstrated in ensemble measurements to be incorporated into a solid state device. In this way, without requiring single spin measurement or strong coupling to a cavity, full use can be made of the sophisticated techniques which are now well established for control of large numbers of spins.

The proposed physical setup, as illustrated in Fig. 1, consists of a superconducting transmission line cavity coupled to a large number of solid-state electron spins doped into or deposited on the surface of the substrate. Two interesting choices for the electron spins would be P-doped Si and endohedral fullerene molecules, e.g., N@C$_{60}$, which would offer spin coherence times up to tens of milliseconds [9, 10, 11]. Hahn-echo techniques may be applied to counter inhomogeneous broadening mechanisms, and the coherence time scale may even be further extended by transferring the electron spin state to nuclear spin degrees of freedom where coherence times exceeding seconds have been demonstrated [12]. The spins are biased with a homogeneous magnetic field $B$ in the plane of the cavity, causing Larmor precession at an angular frequency of $\omega_a = m_0 B/\hbar$, where $m_0$ is the magnetic dipole moment of the spins. With a cavity resonance frequency of $\omega_c \approx 2\pi \times 5$ GHz a bias field of $B = 180$ mT is required to bring the spin precession into resonance. Even in the presence of the bias field, cavity linewidths as low as $\kappa \approx 2\pi \times 250$ kHz are possible [13].

The $q^{th}$ spin located at $r_q$ will couple to the cavity annihilation and creation operators, $\hat{a}$ and $\hat{a}^\dagger$, with a
strength \( g_q \equiv m_0 B_0(\mathbf{r}_q)/2 \hbar \) where \( B_0(\mathbf{r}) \) is the zero-point magnetic field of the cavity mode. We will use \( \bar{g} = \sqrt{\sum_q g(\mathbf{r}_q)^2/N} \) to denote the average coupling strength.

We now consider the collective coupling between the spin ensemble and the cavity. Assuming the spin-cavity detuning \( \Delta_c \equiv \omega_s - \omega_c \) to be small compared to \( \omega_s, \), we can apply the rotating wave approximation and describe the coupling by the interaction Hamiltonian

\[
\hat{H}_{sc} = \sum_{q=1}^{N} \hbar g_q \left( \hat{a} \hat{\sigma}_-^{(q)} + \hat{a}^{\dagger} \hat{\sigma}_+^{(q)} \right) - \hbar \Delta_c \hat{a}^{\dagger} \hat{a} \\
= \hbar \sqrt{N} \bar{g} \left( \hat{a}^{\dagger} \hat{b} + \hat{a} \hat{b}^{\dagger} \right) - \hbar \Delta_c \hat{a}^{\dagger} \hat{a}, \quad (1)
\]

where \( \hat{\sigma}_-^{(q)} (\hat{\sigma}_+^{(q)}) \) is the Pauli spin lowering (raising) operator of the \( q^{th} \) spin, and we introduce the collective spin lowering operator \( \hat{b} = \frac{1}{\sqrt{N}} \sum_{q=1}^{N} \hat{\sigma}_-^{(q)} g_q/\bar{g} \) and its Hermitian adjoint \( \hat{b}^{\dagger} \) in the second line. Assuming that the sample is strongly polarized, the collective lowering and raising operators obey the commutator relation \( [\hat{b}, \hat{b}^{\dagger}] = 1 \), so that the collective excitation behaves like a harmonic oscillator degree of freedom. The lowest two spin oscillator states are the state with all spins pointing down, \( |\text{vac} \rangle \equiv |0\rangle^\otimes N \), and the state with a single collective excitation

\[
|\psi_1(0) \rangle \equiv \hat{b}^{\dagger} |\text{vac} \rangle = \frac{1}{\sqrt{N}} \sum_q \frac{g_q}{\bar{g}} |0\rangle_1 \cdots |1_q \cdots 0_N \rangle. \quad (2)
\]

According to the second line of Eq. (1) the cavity field and the collective spin oscillator behave as two coupled oscillators with a coupling strength that is enhanced with a factor of \( \sqrt{N} \) relative to the single spin coupling strength, so that any states of the two systems are interchangeably mapped between them with the collective Rabi frequency \( \sqrt{N} \bar{g} \). For the parameters given in Fig. 1, the effective coupling \( \sqrt{N} \bar{g} \) is \( \sim 2\pi \times 6 \text{ MHz} \), and, thus, exceeds by orders of magnitude the decay rates of collective spin excitations (governed by the single spin decay rate [14]) as well as the cavity decay rate \( \kappa \).

The long coherence lifetimes in trapped atomic and molecular systems have inspired recent proposals to transfer and store the cavity field excitation in a single rotationally excited polar molecule [15], and, to benefit from the collectively enhanced coupling, in collectively excited states of many molecules [16]. In these proposals, molecules would have to be cooled and trapped in the close vicinity of the cavity transmission line, while the electron spins in the present proposal are being held in a host material - at the price of shorter achievable coherence times. In a physical set-up similar to ours [17], the energy splitting of the two lower eigenstates of a coupled CPB-cavity system differs from the excitation energies to higher excited states, and thus they form an effectively closed two-level system which may be resonantly coupled to a collective spin oscillator.

Our objective is to store qubits for quantum computing, and in particular to store many qubits in the same medium. To this end, we consider the application of a magnetic field gradient to the sample for a duration \( \tau \) as described in Fig. 1. The field provides a linearly varying Zeeman energy shift across the sample so that after a certain interaction time, a linearly varying spatial phase \( \exp(ikz) \), with \( k = -m_0 \Delta B \tau/L \hbar \) is encoded in the excited state component of the individual spins at position \( z \) along the sample, and the collective state \( |\psi_1(0) \rangle \) is transferred to

\[
|\psi_1(k) \rangle \equiv \frac{1}{\sqrt{N}} \sum_q \frac{g_q}{\bar{g}} e^{ikz_q} |0\rangle_1 \cdots |1_q \cdots 0_N \rangle. \quad (3)
\]

Different positive and negative values of the collective spin wave number, \( k \), can be chosen and we will in the following refer to the magnetic gradient pulse, accomplishing a specific value of the phase gradient, as a \( (k) \)-pulse applied to the system. The key idea of the holographic quantum register is that the gradient pulses let us successively access a number of collective excitation modes of the same spin ensemble, such that the read-in of each new qubit does not disturb the previously stored qubits, because only the \( k = 0 \) spin mode of the new qubit interacts with the cavity field. These modes are 1-dimensional \( k \)-space voxels as used in magnetic resonance imaging [18]; similar orthogonal collective atomic excitation modes are being studied for storage of multiple modes of light [5, 19, 20, 21, 22].

To identify modes which are truly independent, and which can hence be used for storage of different qubits, we introduce the creation operator for the \( i^{th} \) spin wave mode as

\[
\hat{b}^{\dagger}(k_i) \equiv \frac{1}{\sqrt{N}} \sum_{q=1}^{N} \frac{g_q}{\bar{g}} e^{ik_i z_q} \hat{\sigma}_+^{(q)}, \quad (4)
\]

and consider the commutator \( [\hat{b}(k_i), \hat{b}^{\dagger}(k_j)] \) in the fully polarized limit. If the spins are arranged on a Bravais lattice and have equal coupling strengths, the commutator vanishes for any pair of members of the reciprocal lattice, so that the corresponding modes are perfectly independent [23]. For a general geometry, we find that \( [\hat{b}(k_i), \hat{b}^{\dagger}(k_j)] = M(k_j - k_i) \), where we have introduced the mode overlap \( M(\Delta k) \equiv \sum_q e^{i\Delta k z_q} |g_q|^2/N \bar{g}^2 \).

For the uniformly doped \( L = \lambda \) transmission line cavity illustrated in Fig. 1, \( M(\Delta w/2\pi/\lambda) = \sin(\pi \Delta w)/(1 - (\Delta w/2)^2) \) in the continuous \( (N \rightarrow \infty) \) limit, so that in this case, the mode overlap vanishes when the difference in winding number \( \Delta w \equiv \Delta k L/2\pi \) is equal to any integer except 0 or \( \pm 2 \). It follows that choosing \( k_n = 3n \times 2\pi/L \), or even \( \{k_n\} = \{0, 3, 4, 7, 8, \ldots\} \times 2\pi/L \), for the register modes will ensure that \( M(k_j - k_i) = 0 \) for all pairs.
of modes. The duration of a $k = 2\pi/L$ gradient pulse is $2\pi\hbar/m_0 \Delta B$, and to implement such a pulse in 100 ns for the system of Fig. 1, a field gradient of 13 mT/m is required across the sample. A gradient of this strength allows us to sequentially address hundreds of modes, while correspondingly stronger gradients or longer pulse interaction times are needed to switch between any pair of modes.

We have now established that the $k$-modes of the spin ensemble in the strongly polarized limit behave as a large number of independent harmonic oscillators. Using gradient pulses, these spatial modes can be mapped to the $k = 0$ mode which is the only one that can be brought to interact strongly with the cavity field. When the spin ensemble is brought into resonance by adjusting the bias field $B$, the collective coupling [Eq. (1)], swaps the states of the $k = 0$ mode and the cavity at the effective coupling rate $\sqrt{N}g$. We can thus selectively swap the state of the cavity with any $k_i$ mode of the spin ensemble without disturbing any other mode by applying first a $(-k_i)$ pulse followed by a sweep through resonance and finally a $(k_i)$ pulse. To extend our addressable qubit register into a full quantum information processing system, we need to further include a physical qubit system which can be initialized, manipulated and read out, and which can facilitate a suitably non-linear interaction between the oscillator modes as outlined in Fig. 2. Our candidate for this qubit is the transmon CPB [6] which has already been integrated experimentally with the system geometry illustrated in Fig. 1. Single qubit unitary operations can be achieved by applying classical resonant fields to the CPB system, and by swapping the states between any selected $k_i$ and the CPB degree of freedom, and one-qubit gates as well as readout [24] can hence be applied to any qubit in our register. By swapping a control qubit to the CPB and subsequently swapping a target qubit to the cavity field, the controlled interaction between these two system effectively implements a two-qubit gate on any pair of qubits [4].

So far, we have only discussed the coupling of the $k = 0$ mode to a quantum field in a cavity, but the individual addressability and independence of the different $k$-modes, can also be demonstrated with magnetic field gradients and classical resonant field pulses. A short duration classical pulse “tilts” the macroscopic magnetization vector of the sample by a small angle, generating a component of the magnetization perpendicular to $z$ proportional to the tilt angle; the component parallel to $z$ changes only to second order. During a magnetic field gradient pulse, the perpendicular component of the magnetization precesses non-uniformly across the sample, encoding the excitation in a spin wave and reducing the total perpendicular component of the magnetization effectively to zero. Subsequent excitations may be stored in spin wave modes in a similar way. Excitations stored in this way may be detected by applying a gradient pulse that converts a particular spin wave back into a uniform transverse magnetization. In the uniform bias field, the uniform transverse magnetization precesses coherently and its inductive signal may be detected using a standard pulsed magnetic resonance spectrometer.

Our proposal relies on several key assumptions, and we now turn to a discussion of potential errors resulting from mechanisms bringing their validity into question.

In order for the collective spin wave excitations to act as independent oscillator modes, the density of excited spins must be low. This would be adequately fulfilled at an operating temperature of 20 mK where the thermal excitation probability of spins is $p \approx \pi \times 10^{-5}$. Even at such low temperature, a large total number $pN$ of thermally excited spins will be present in the sample. The reason that we expect to be able to recognize a single collective spin excitation on the background of, possibly, millions of excited spins is that these spin excitations are distributed over all spins and hence over all collective modes of the system, while the excitation in the small number of modes that we interact with is negligible. For each collective mode, the population outside the ground state will only be $\langle \hat{b}^\dagger(k_i)\hat{b}(k_i) \rangle \approx p$. The fact that most of the excitation resides in spectator modes, which are not coupled to the cavity field, can, in the case of homogeneous coupling, be argued more directly in the collective spin representation of the system [25]. Further, the independence of the modes ensures that we may actively cool the $k_i$ modes used in the register by transferring their excitation to the cavity field, allowing an efficient preparation of the ground state of the register, even at finite temperatures.

The dipole-dipole interactions must be weak enough that the spin-waves are eigenstates of the system Hamiltonian. This will not be strictly fulfilled, and the dipole-dipole interaction will lead to decoherence of the register. For random doping as considered here, we expect the decoherence rate to be on the order of the line broadening induced by random dipole-dipole coupling [26, 27], while for a regular lattice of spins, the dominant decoher-
ence mechanism is expected to be dipole-mediated Raman scattering with thermal spin-waves [28, 29]. With the spins distributed uniformly in a layer of 1 μm in the setup of Fig. 1, the dipole-induced line broadening would be ∼ 2π × 50 kHz [30]. Also, in a real experimental apparatus, it is difficult to make the bias field sufficiently homogeneous that different electrons experience the same precession, so that the collective excitation of the ensemble remains coherent for times comparable to the intrinsic dephasing time.

Errors due to static inhomogeneities in the external fields, e.g. field distortions due to the Meissner effect of the superconducting cavity electrodes, are well known in nuclear and electronic spin resonance studies and can be countered by applying a classical refocusing pulse which stimulates a Hahn-echo. The superconducting surfaces may also contribute a fluctuating external field [31], the effect of which can be minimized by not positioning spins close to electrode surfaces. As √Nφ is independent of mode volume for constant spin density, this can be achieved without loss of effective coupling strength. For the system illustrated in Fig. 1, the refocusing could be performed by introducing a strong microwave field at the Larmor frequency, either through the cavity or by an alternative coupling mechanism. Ideally, the refocusing pulses rotate all members of the ensemble by exactly π about an axis perpendicular to z. In practice, however, it is not possible to apply perfect π-pulses and this strategy is expected to introduce a very large number of excitations into the system. It might be expected that a significant fraction of such excitations would enter and swamp the register modes, but on closer inspection this is not the case. To argue this, let us for simplicity assume that the classical driving field shares the amplitude and is not the case. To argue this, let us for simplicity assume that the classical driving field shares the amplitude and r

\[ B_0(r) \]

In this case, any deviation \[ ε_1 \] (\[ ε_2 \]) of the area of the first (second) echo pulse from π would introduce \( \sim \sqrt{N} \) excitation in the \( k = 0 \) mode and similarly \( N \sqrt{\frac{\epsilon_1^2}{\epsilon_2^2}} \) excitations into an “inhomogeneous” mode where the relative phase of the excited spins are given by the inhomogeneous precession over the Hahn pulse interval. In general, the “inhomogeneous” mode is not a k-mode, although in the absence of inhomogeneities it corresponds to the \( k = 0 \) mode. In the discrete spin case, the mode overlap between any pair of modes is \( \sim 1/\sqrt{N} \), so that the Hahn-echo errors will introduce \( \sqrt{\epsilon_2^2} \) excitations into each register mode. The echo sequence can be applied to free spin ensembles without a cavity and CPB and the effect of the refocusing can thus be demonstrated for classical spin waves. The functioning of the cavity and the CPB will not be seriously impeded, but special means may be needed to avoid excessive direct excitation of these components of the hybrid system during the application of the strong echo pulses.

In summary, we have proposed a quantum register capable of holding hundreds of physical qubits in collective excitations of a spin ensemble, and we have indicated how to perform qubit encoding, one- and two-bit gates, and read-out in this system. Further, we have identified mechanisms and properties of the collective states that make the register resilient to the effects of finite polarization and errors introduced by echo sequences used to counter the effects of bias field inhomogeneities.

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[1] E. Brion, K. Mølmer, and M. Saffman, “Quantum computing with collective ensembles of multilevel systems,” Phys. Rev. Lett. 99, 200501 (2007).
[2] M. Saffman and K. Mølmer, “Scaling the neutral-atom rydberg gate quantum computer by collective encoding in holmium atoms,” Phys. Rev. A 78, 012336 (2008).
[3] K. Tordrup and K. Mølmer, “Quantum computing with a single molecular ensemble and a Cooper-pair box,” Phys. Rev. A 77, 020301(R) (2008).
[4] K. Tordrup, A. Negretti, and K. Mølmer, “Holographic quantum computing,” Phys. Rev. Lett. 101, 040501 (2008).
[5] K. Surnacz, J. Nunn, K. Reim, K. C. Lee, V. O. Lorenz, B. Sussman, I. A. Walmsley, and D. Jaksch, “Efficient spatially resolved multimode quantum memory,” Phys. Rev. A 78, 033806 (2008).
[6] J. Koch, T. M. Yu, J. Gambetta, A. A. Houck, D. I. Schuster, J. Majer, A. Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, “Charge-insensitive qubit design derived from the Cooper pair box,” Phys. Rev. A 76, 042319 (2007).
[7] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. Kumar, S. M. Girvin, and R. J. Schoelkopf, “Strong coupling of a single photon to a superconducting qubit using circuit quantum electrodynamics,” Nature 431, 162 (2004).
[8] R. J. Schoelkopf and S. M. Girvin, “Wiring up quantum systems,” Nature 451, 664 (2008).
[9] A. M. Tyryshkin, S. A. Lyon, A. V. Astashkin, and A. M. Raitsimring, “Electron spin relaxation times of phosphorus donors in silicon,” Phys. Rev. B 68, 193207 (2003).
[10] J. J. L. Morton, A. M. Tyryshkin, A. Ardavan, K. Poryfrakis, S. A. Lyon, and G. A. D. Briggs, “Electron spin relaxation of N@C60 in Cs2,” J. Chem. Phys. 124, 014508 (2006).
[11] J. J. L. Morton, A. M. Tyryshkin, A. Ardavan, K. Poryfrakis, S. A. Lyon, and G. A. D. Briggs, “Environmental effects on electron spin relaxation in N@C60,” Phys. Rev. B 76, 085418 (2007).
[12] J. J. L. Morton, A. M. Tyryshkin, R. M. Brown, S. Shankar, B. W. Lovett, A. Ardavan, T. Schenkel,
E. E. Haller, J. W. Ager, and S. A. Lyon, “Solid-state quantum memory using the $^{31}$P nuclear spin,” Nature 455, 1085 (2008).

[13] D. Schuster et al. In preparation.

[14] B. A. Chase and J. M. Geremia, “Collective processes of an ensemble of spin-1/2 particles,” Phys. Rev. A 78, 052101 (2008).

[15] A. Andre, D. DeMille, J. M. Doyle, M. D. Lukin, S. E. Maxwell, P. Rabl, R. J. Schoelkopf, and P. Zoller, “A coherent all-electrical interface between polar molecules and mesoscopic superconducting resonators,” Nat. Phys. 2, 636 (2006).

[16] P. Rabl, D. DeMille, J. M. Doyle, M. D. Lukin, R. J. Schoelkopf, and P. Zoller, “Hybrid quantum processors: Molecular ensembles as quantum memory for solid state circuits,” Phys. Rev. Lett. 97, 033003 (2006).

[17] A. Imamoglu, “Cavity-QED based on collective magnetic dipole coupling: spin ensembles as hybrid two-level systems,” Phys. Rev. Lett. 102, 083602 (2009).

[18] P. T. Callaghan, Principles of nuclear magnetic resonance microscopy. Clarendon Press, Oxford, 1993.

[19] S. A. Moiseev and S. Kröll, “Complete reconstruction of the quantum state of a single-photon wave packet absorbed by a doppler-broadened transition,” Phys. Rev. Lett. 87, 173601 (2001).

[20] G. Hété, M. Hosseini, B. M. Sparkes, D. Oblak, P. K. Lam, and B. C. Buchler, “Photon echoes generated by reversing magnetic field gradients in a rubidium vapor,” Opt. Lett. 33, 2323 (2008).

[21] B. Zhao, Y.-A. Chen, X.-H. Bao, T. Strassel, C.-S. Chuu, X.-M. Jin, J. Schmiedmayer, Z.-S. Yuan, S. Chen, and J.-W. Pan, “A millisecond quantum memory for scalable quantum networks,” Nat. Phys. 5, 95 (2009).

[22] R. Lauro, T. Chanelière, and J.-L. Le Gouët, “Spectral hole burning for stopping light,” Phys. Rev. A 79, 053801 (2009).

[23] F. J. Dyson, “General theory of spin-wave interactions,” Phys. Rev. 102, 1217 (1956).

[24] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, J. Majer, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, “Approaching unit visibility for control of a superconducting qubit with dispersive readout,” Phys. Rev. Lett. 95, 060501 (2005).

[25] J. Wesenberg and K. Mølmer, “Mixed collective states of many spins,” Phys. Rev. A 65, 062304 (2002).

[26] P. W. Anderson, “Theory of paramagnetic resonance line breadths in diluted crystals,” Phys. Rev. 82, 291 (1951).

[27] C. Kittel and E. Abrahams, “Dipolar broadening of magnetic resonance lines in magnetically diluted crystals,” Phys. Rev. 90, 238 (1953).

[28] M. Sparks and C. Kittel, “Ferromagnetic relaxation mechanism for $M_1$ in yttrium iron garnet,” Phys. Rev. Lett. 4, 232 (1960).

[29] M. Sparks, “Theory of three-magnon ferromagnetic relaxation frequency for low temperatures and small wave vectors,” Phys. Rev. 160, 364 (1967).

[30] J. H. Wesenberg and K. Mølmer, “The field inside a random distribution of parallel dipoles,” Phys. Rev. Lett. 93, 143903 (2004).

[31] S. Sendelbach, D. Hover, A. Kittel, M. Muck, J. M. Martinis, and R. McDermott, “Magnetism in SQUIDs at millikelvin temperatures,” Phys. Rev. Lett. 100, 227006 (2008).