On the Sensitivity of L/E Analysis of Super-Kamiokande Atmospheric Neutrino Data to Neutrino Oscillation Part 2

Four Possible L/E Analyses for the Maximum Oscillation by the Numerical Computer Experiment

E. Konishi1, Y. Minorikawa2, V.I. Galkin3, M. Ishiwata4, I. Nakamura4, N. Takahashi1, M. Kato5 and A. Misaki6

1 Graduate School of Science and Technology, Hirosaki University, Hirosaki, 036-8561, Japan
2 Department of Science, School of Science and Engineering, Kinki University, Higashi-Osaka, 577-8502, Japan
3 Department of Physics, Moscow State University, Moscow, 119992, Russia
4 Department of Physics, Saitama University, Saitama, 338-8570, Japan
5 Kyowa Interface Science Co.,Ltd., Saitama, 351-0033, Japan
6 Innovative Research Organization, Saitama University, Saitama, 338-8570, Japan
7 Research Institute for Science and Engineering, Waseda University, Tokyo, 169-0092, Japan

e-mail: konishi@si.hirosaki-u.ac.jp

Abstract. In the previous paper (Part 1), we have verified that the SK assumption on the direction does not hold in the analysis of neutrino events occurred inside the SK detector. Based on the correlation between $L_\nu$ and $L_\mu$ (Figures 12 and 13 in Part 1) and the correlation between $E_\nu$ and $E_\mu$ (Figure 14 in Part 1), we have made four possible $L/E$ analyses, namely $L_\nu/E_\nu$, $L_\nu/E_\mu$, $L_\mu/E_\nu$ and $L_\mu/E_\mu$. Among four kinds of $L/E$ analyses, we have shown that only $L_\nu/E_\nu$ analysis can give the signature of maximum oscillations clearly, not only the first maximum oscillation but also the second and third maximum oscillation, while the $L_\mu/E_\mu$ analysis which are really done by Super-Kamiokande Collaboration cannot give the maximum oscillation at all. It is thus concluded from those results that the experiments with the use of the cosmic-ray beam for neutrino oscillation, such as Super-Kamiokande type experiment, cannot find the maximum oscillation from $L/E$ analysis, because the incident neutrino cannot be observed due to its neutrality. Therefore, we would suggest Super-Kamiokande Collaboration to re-analyze the zenith angle distribution of the neutrino events which occur inside the detector carefully, because $L_\nu$ and $L_\mu$ are alternative expressions of the cosine of the zenith angle for the incident neutrino and that for the emitted muon, respectively.

PACS. 1 3.15.+g, 14.60.-z

1 Introduction

In Figures 12 and 13 of the preceding paper[1], we have shown that the SK assumption on the direction that the directions of the incident neutrinos are the same as those of the emitted muons does not hold. Also, in Figure 14 of the same paper, we have shown that the energies of the incident neutrinos cannot be determined from the those of the emitted muons, uniquely. However, the discrepancies between two variables in Figures 12 and 13 are distinctively large compared with those in Figure 14. Therefore, non-holding of the SK assumption on the direction plays an essential role in the $L/E$ analysis for finding the maximum oscillation (oscillation pattern in neutrino oscillation).

The survival probability of a given flavor is given in Eq.(1), in the case of Super-Kamiokande Collaboration. The variables for the $L/E$ analysis are $L_\nu$ and $E_\nu$, where $L_\nu$ denotes the flight length for the incident neutrino between the generation point of the incident neutrino and the interaction point of the neutrino concerned in the detector, and $E_\nu$ is the energy of the incident neutrino.

2 $L/E$ Distributions in Our Numerical Computer Experiment

Our computer numerical experiments are carried out in the unit of 1489.2 days. Hereafter, we call 1489.2 live days as one SK live day. The live days of 1489.2 is the total live days for the analysis of the neutrino events generated inside the detector by Super-Kamiokande Collaboration[2]. We repeat one SK live day experiment as much as 25 times, namely, the total live days for our computer numerical experiments is 37230 live days (25 SK live days). In Figure[3] we show $L_\nu/E_\nu$ distribution without oscillation for one experiment (1489.2 live days) among twenty five computer numerical experiments. In those numerical
experiments, there are statistical uncertainties only which are due to both the stochastic character in the physical processes concerned and the geometry of the detectors. Therefore we add the standard deviation as for the statistical uncertainty around their average in the forthcoming graphs, if necessary. In Figure 2 we show the statistical uncertainty, the standard deviations around their average values through twenty five experiments. Similarly for other possible combinations of \( L \) and \( E \) (\( L_{\nu}/E_\nu \), \( L_\mu/E_\nu \) and \( L_\mu/E_\mu \)) for 37230 live days (25 SK live days) we did so.

2.1 \( L_{\nu}/E_\nu \) distribution

2.1.1 For null oscillation

In Figures 1 and 2 both distributions show the sinusoidal-like character for \( L_{\nu}/E_\nu \) distribution, namely, the appearance of the top and the bottom, even for null oscillation. The uneven histograms in Figure 1 comparing with those in Figure 2 show that the statistics of Figure 1 is not enough compared with that of Figure 2. Roughly speaking, smaller \( L_{\nu}/E_\nu \) correspond to the contribution from downward neutrinos, larger \( L_{\nu}/E_\nu \) correspond to that from upward neutrinos and \( L_{\nu}/E_\nu \) near the minimum correspond to the horizontal neutrinos, although the real situation is more complicated, because the backscattering effect in QEL as well as the azimuthal angle effect in QEL could not be neglected. From Figure 2 we understand that the bottom around 70 km/GeV denotes the contribution from the horizontal direction and has no relation with neutrino oscillation in any sense.

2.1.2 For oscillation (SK oscillation parameters)

The survival probability of a given flavor, such as \( \nu_\mu \), is given by

\[
P(\nu_\mu \to \nu_\mu) = 1 - \sin^22\theta \cdot \sin^2(1.27\Delta m^2 L_{\nu}/E_\nu).\]  

Then, for maximum oscillations under SK neutrino oscillation parameters, we have

\[
1.27\Delta m^2 L_{\nu}/E_\nu = (2n + 1) \times \frac{\pi}{2}, \]  

where \( \Delta m^2 = 2.4 \times 10^{-3}\text{eV}^2 \). From Eq.(2), we have the following values of \( L_{\nu}/E_\nu \) for maximum oscillations.

\[
L_{\nu}/E_{\nu} = 515\text{km/GeV} \quad \text{for } n = 0 \quad (3 - 1)
\]

\[
= 1540\text{km/GeV} \quad \text{for } n = 1 \quad (3 - 2)
\]

\[
= 2575\text{km/GeV} \quad \text{for } n = 2 \quad (3 - 3)
\]

and so on.

In Figure 3 we give the survival probability \( P(\nu_\mu \to \nu_\mu) \) as a function of \( L_{\nu}/E_\nu \) under the neutrino oscillation parameters obtained by Super-Kamiokande Collaboration.
In cosmic ray experiments, the energy spectrum of the incident neutrino, is convoluted into the survival probability.

In Figure 4, we give one example of the $L_\nu/E_\nu$ distribution for one SK live day (1489.2 live days) among twenty five sets of the computer numerical experiments in the unit of one SK live day. In Figure 5, we give another example for one SK live day. Arrows A, B and C represent the first, the second and the third maximum oscillation which are given in Eq. (3-1), (3-2) and (3-3), respectively. By the definition of our computer numerical experiments, there are no experimental error bars in $L_\nu/E_\nu$ distributions in Figures 4 and 5.

In Figure 6, we show the $L_\nu/E_\nu$ distribution for 14892 live days (10 SK live days). Compared Figure 6 with Figures 4 and 5, it is clear that $L_\nu/E_\nu$ distribution in Figure 6 becomes smoother due to larger statistics.

In Figure 7, we can add the statistical uncertainty (standard deviation in this case) around their average, because every one SK live day experiment among twenty five sets of the experiments fluctuates one by one due to their stochastic character in their physical processes and geometrical conditions of the detectors concerned. In order to make the image of the maximum oscillations in $L_\nu/E_\nu$ distributions clearer, we show the correlations between $L_\nu$ and $E_\nu$ in Figures 8 and 9, which correspond to Figures 4 and 6, respectively. In Figure 8 for one SK live day, we can observe vacant regions for the events concerned assigned by A, B and C. In Figure 9 for ten SK live days, the existence of the vacant regions for the events concerned becomes clearer due to larger statistics.

In Figure 10, we give $L_\nu/E_\nu$ distribution with 14892 live days (10 SK live days) in the linear scale which is another expression of the same content as in Figure 9. Also, it is the survival probability convoluted with the incident neutrino energy spectrum. If we compare Figure 10 with Figure 3, then, we clearly see the series of maximum oscillations characterized with $n=0$ (A), 1(B), 2(C) and so on which are given by Eq.(2). It is clear from Figure 10 that the maximum oscillations with $n=0,1$ and 2 have the almost same frequencies under the incident neutrino energy spectrum utilized by Super-Kamiokande Collaboration [3] (see footnote 1). The situation shown in Figures 8 to 10 shows definitely that our computer numerical exper-
We have repeated the computer numerical experiment for one SK live day as much as twenty five times independently, in both cases with oscillation and without oscillation. Consequently, there are 625 (= $25 \times 25$) sets of ratios of $(L_\nu/E_\nu)_{osc}/(L_\nu/E_\nu)_{null}$ for one SK live day which correspond to Eq.(1). In Figure 11 we show one example among 625 combinations. In Figure 12 we show the same ratio for 14892 live days (10 SK live days). In conclusion, from Figures 4 to 12, we can reproduce the minimum extrema for neutrino oscillation in our $L_\nu/E_\nu$ analysis. This fact shows doubtlessly that our computer numerical experiments are done in the correct manner.

2.2 $L_\mu/E_\mu$ distribution

As physical quantities which can really be observed are $L_\mu$ and $E_\mu$ instead of $L_\nu$ and $E_\nu$, therefore we examine $L_\mu/E_\mu$ distribution focusing the existence of the maximum oscillation.
2.2.1 For null oscillation

In Figure 13 we give one sample for one SK live day (1489.2 live days) from the totally 37230 live days (25 SK live days) events, each of which has 1489.2 live days. Figure 14 shows the average distribution accompanied by the statistical uncertainty bar (not experimental error bar). It is clear from these figures that the existence of the dip or bottom, namely the sinusoidal character, means the contribution merely from horizontal contribution, having no relation with any neutrino oscillation character.

2.2.2 For oscillation (SK oscillation parameters)

In Figures 15 and 16 we give the $L/\mu$ distributions with oscillation for 1489.2 live days (one SK live day) and 37230 live days (25 SK live days), respectively. In Figure 15, we may observe the uneven histogram, something like curious bottoms coming from neutrino oscillation. However, in Figure 16 where the statistics is 25 times as much as that of Figure 15, the histogram becomes smoother and such bottoms disappear, which turns out finally for the bottoms to be pseudo. It is impossible to extract the neutrino oscillation parameters from the comparison of Figure 16 with Figure 14.

In Figures 17 and 18 correspondingly, we give the correlation between $L/\mu$ and $E/\mu$ for 1489.2 live days (one SK live day) and 14892 live days (10 SK live days), respectively. In Figure 19, we give the $L/\mu$ distribution for 14892 live days (10 SK live days) in the linear scale which is another expression of the same content as in Figure 18. As in Figure 18, we cannot find any maximum oscillation-like phenomena in Figure 19, which is contrast to Figure 10.

It is clear from the figures that we can not observe the maximum oscillation in $L/\mu$, on the contrary to Figures 4 to 10 which give the maximum oscillations. Namely, we may conclude that we can not observe the sinusoidal flavor transition probability of neutrino oscillation against the claim by Super-Kamiokande Collaboration[4] when we adopt physically observable quantities, such as $L/\mu$ and $E/\mu$.

In order to confirm the disappearance of the psuedo maximum oscillations, in Figures 20 and 21 we give the survival probability of a given flavor for $L/\mu$ distribution, namely, $(L/\mu/E)_{osc}/(L/\mu/E)_{null}$, for 1489.2 live days.
Fig. 17. The correlation diagram between $L_\mu$ and $E_\mu$ with oscillation for 1489.2 live days (one SK live day).

Fig. 18. The correlation diagram between $L_\mu$ and $E_\mu$ with oscillation for 14892 live days (10 SK live days).

Fig. 19. $L_\mu/E_\nu$ distribution with and without oscillation for 14892 live days (10 SK live days).

Fig. 20. The ratio of $(L_\mu/E_\mu)_{osc}/(L_\mu/E_\mu)_{null}$ for 1489.2 live days (one SK live day).

Fig. 21. The ratio of $(L_\mu/E_\mu)_{osc}/(L_\mu/E_\mu)_{null}$ for 14892 live days (10 SK live days).

2.3 $L_\mu/E_\nu$ distribution

Now, we examine the $L_\mu/E_\nu$ distribution which Super-Kamiokande Collaboration treat in the their paper, ex-pecting the evidence for the oscillatory signature in at-mospheric neutrino oscillations.

2.3.1 For null oscillation

In Figures 22 and 23, we give the $L_\mu/E_\nu$ distribution without oscillation for 1489.2 live days (one SK live day) and 37230 live days (25 SK live days), respectively. Comparing Figure 22 with Figure 23, the larger statistics makes the distribution smoother. Also, there is a sinusoidal-like bottom which has no relation with neutrino oscillation.
2.3.2 For oscillation (SK oscillation parameters)

In Figures 24 and 25, we give the $L_\mu/E_\nu$ distribution with oscillation for 1489.2 live days (one SK live day) and 37230 live days (25 SK live days), respectively. In Figure 24 we may find something like a bottom which corresponds to the first maximum oscillation near $\sim 200$ (km/GeV). However, such the dip disappears, by making the statistics larger as shown in Figure 25.

2.3.3 $L_\mu/E_{\nu,SK}$ distribution for the oscillation

Instead of $E_\nu$ which is correctly sampled from the corresponding probability functions, let us utilize $E_{\nu,SK}$ which is obtained from the "approximate" formula (Eq.(6) in the preceding paper[1]).

We express $E_\nu$ described in Eq.(6) of the preceding paper[1] utilized by Super-Kamiokande Collaboration as $E_{\nu,SK}$ to discriminate our $E_\nu$ obtained in the stochastic manner correctly.

In Figure 26 we give $L_\mu/E_{\nu,SK}$ distribution for 14892 live days (10 SK live days), comparing with $L_\mu/E_\nu$ distribution. It is understood from the comparison that there
is no significant difference between $L_\mu/E_\mu$ distribution and $L_\mu/E_\nu$ one. This fact tells us that the "approximate" formula for $E_\nu$ by Super-Kamiokande Collaboration does not produce so significant error practically. Although this kind of formula is not suitable for the treatment of stochastic quantities, the result is understandable from Figure 14 in the preceding paper [1]. Also, we can conclude that we do not find any hole corresponding to the maximum oscillation in $L_\mu/E_\nu$ or $L_\mu/E_{\nu,SK}$ distributions. The reason why the Figure 25 cannot show such dip structure as shown in Figures 4 and 6 comes from the situation that the role of $L_\nu$ is much more crucial than that of $E_\nu$ in the $L/E$ analysis. Namely, $L_\nu$ cannot be replaced by $L_\mu$ at all. Also, see the discussion in the following subsection 4.4.

### 2.4 $L_\nu/E_\mu$ distribution

#### 2.4.1 For null oscillation

In Figure 27 we give $L_\nu/E_\mu$ distribution without oscillation for 37230 live days (25 SK live days) of Super-Kamiokande Experiment to consider statistical fluctuation effect as precisely as possible. It is clear from the figure that there is not any dip corresponding to the maximum oscillation which is expected to appear in presence of neutrino oscillation, as it must be.

#### 2.4.2 For oscillation (SK oscillation parameters)

In Figure 28 we give the corresponding distribution with the oscillation. In Figure 29 we give the correlation diagram between $L_\nu$ and $E_\mu$ for 14892 days (10 SK live days). On the contrary to Figure 25, there are surely some kinds of holes in Figure 28 and furthermore we can discriminate the strip pattern in Figure 29 similarly as in Figure 3.

Therefore, we surmise from Figures 28 and 29 that we may observe some "maximum oscillation like" quantities which are related to the maximum oscillations in the $L_\nu/E_\mu$ distribution through the correlation between $E_\mu$ and $E_\nu$ shown in Figure 14 in the preceding paper [1]. However, it seems to be difficult to extract a pair of concrete values of $L_\nu$ and $E_\nu$ through the analysis of the $L_\nu/E_\mu$ distribution. In Figure 30, we make a comparison between $L_\nu/E_\nu$ distribution and $L_\nu/E_\mu$ distribution where the correlation between $E_\mu$ and $E_\mu$ is shown in Figure 14 in the preceding paper [1]. It is clear from the figure that the $L_\nu/E_\nu$ distribution demonstrates the maximum oscillation as already shown in Figures 4 to 10 and the $L_\nu/E_\mu$ distribution also demonstrates the maximum oscillation-like as already shown in Figure 28 and 29. In Figure 31 we give the relation between $L_\nu/E_\nu$ distribution and $L_\nu/E_\mu$ distribution where the same correlation between $E_\nu$ and $E_\mu$ holds in the case of Figure 29. It is also clear from the figures that both the distributions demonstrate neither the maximum oscillation nor the maximum oscillation-like, which is also clear from Figures 11 to 19 and Figures 24 to 25. Thus, it can be concluded from Figures 13 and 14 in the preceding paper [1] and Figure 30 and Figure 31 in the present paper that $L_\nu$ plays an essential role compared with others $L_\mu$, $E_\nu$ or $E_\mu$. In other words, it should be noticed that $L_\nu$ cannot be approximated by $L_\mu$, while $E_\nu$ can be obtained approximately from $E_\mu$ through some procedure. Also, such a serious discrepancy between
3 Comparison of $L/E$ Distribution in the Super-Kamiokande Experiment with our Results

In our classification, the $L/E$ distribution by Super-Kamiokande Collaboration [2] [4] should be compared directly with our $L_E/E$ distribution. Taking account of their assertion of existence of the maximum oscillation we compare their results with our results on $L_E/E$ in Figure 30, 31. It is clear from the figure that there are two big differences between them.

One is that we observe the first maximum oscillation ($L_E/E = 515 \text{ km/GeV}$ under the SK oscillation parameters) sharply, while SK observe it in the wider range of $L_E/E = 100 \sim 800 \text{ km/GeV}$.

Such the lack of the neutrino events over the wide range may be due to the measurement of $L_E$, but not $L_\nu$, because the given definite $L_\nu$ corresponds to $L_\mu$ over a wide range and vice versa (See also the correlation between $L_\nu$ and $L_\mu$ in Figure 34 and 37)

The other is that there is big difference between them as for the position which give the maximum frequency for the events concerned. Here, we do not mention to the existence of the maximum oscillation which is derived from the measurement of $L_\mu$ utilized in Super-Kamiokande Collaboration, because one cannot observe the maximum oscillation, if we utilize $L_\nu$ (see Figures 34 and 37). Consequently, we examine the second point as for the maximum frequency for the events concerned. Our computer numerical experiment gives the maximum frequency for interval $1.0 < L_\nu/E_\nu < 1.26 \text{ (km/GeV)}$ as shown in Figure 32.

In Figure 32, we give the correlation between $L_\nu$ and $E_\nu$ for interval $1.0 < L_\nu/E_\nu < 1.26 \text{ (km/GeV)}$. It is clear from the figure that the larger part of the incident neutrino events is occupied by the vertically downward ones and the smaller part is occupied by the horizontally downward neutrino events. This is quite reasonable, because more intensive downward flux contribute to the maximum frequency for the events concerned, compared with weaker upward flux under the Super-Kamiokande neutrino oscillation parameters.

In Figure 33, we give the correlation diagram between $L_\nu$ and $E_\mu$ for the same intervals as in Figure 32. It is clear from Figure 33 that the majority of the events is concentrated into the squared regions with $L_\nu < 10 \text{ km}$ and $L_\mu < 10 \text{ km}$. This denotes that the downward incident neutrinos produce muons toward the forward direction with either smaller or larger angles and only the smaller part of the downward incident neutrino events produce the upward muons due to backscattering (1000 to 10,000 km in $L_\nu$) as well as the azimuthal angle effect in QEL. In Figure 35, we give the correlation diagram between $L_\mu$ and $E_\mu$ for the same intervals as in Figure 33. It is clear from this figure that the produced muons with higher energies are ejected toward the forward and vertical-like directions, while the produced muons with lower energies may be ejected toward the backward or horizontal-like di-

---

Fig. 30. Comparison between $L_\nu/E_\nu$ distribution and $L_\mu/E_\mu$ distribution with oscillation for 37230 days (25 SK live days).

Fig. 31. Comparison between $L_\nu/E_\nu$ distribution and $L_\mu/E_\mu$ distribution with oscillation for 37230 days (25 SK live days).

Fig. 32. The comparison of $L/E$ distribution for single-ring muon events due to QEL among Fully Contained Events with the corresponding one by the Super-Kamiokande Experiment.
Fig. 33. Correlation diagram between $L_\nu$ and $E_\nu$ for $1.0 < L_\nu/E_\nu < 1.26$ (km/GeV) which corresponds to the maximum frequency of the neutrino events for $L_\nu/E_\nu$ distribution in our computer numerical experiment for 14892 live days (10 SK live days).

Fig. 34. Correlation diagram between $L_\nu$ and $L_\mu$ for $1.0 < L_\nu/E_\nu < 1.26$ (km/GeV) under the neutrino oscillation parameters obtained by Super-Kamiokande Collaboration for 14892 live days (10 SK live days).

Fig. 35. Correlation diagram between $L_\mu$ and $E_\mu$ for $1.0 < L_\nu/E_\nu < 1.26$ (km/GeV) which corresponds to the maximum frequency of the neutrino events for $L_\mu/E_\nu$ distribution in our computer numerical experiment for 14892 live days (10 SK live days).

Fig. 36. Correlation diagram between $L_\nu$ and $E_\nu$ for $20 < L_\nu/E_\nu < 25$ (km/GeV) which corresponds to the maximum frequency of the neutrino events for $L_\mu/E_\nu$ distribution in SK experiment for 14892 live days (10 SK live days).

Fig. 37. Correlation diagram between $L_\nu$ and $L_\mu$ for $20 < L_\nu/E_\nu < 25$ (km/GeV) under the neutrino oscillation parameters obtained by Super-Kamiokande Collaboration for 14892 live days (10 SK live days).

Fig. 38. Correlation diagram between $L_\mu$ and $E_\mu$ for $20 < L_\nu/E_\nu < 25$ (km/GeV) which corresponds to the maximum frequency of the neutrino events for $L_\mu/E_\nu$ distribution in SK experiment for 14892 live days (10 SK live days).
frequency of the neutrino events for L

For the same intervals as in Figure 36, we can find the following interesting situation. As it is clearly understandable from Figure 36, horizontal-like downward neutrinos produce the muons in the three different regions, namely, vertical-like downward muons, horizontal-like downward muons and upward muons. From horizontal-like downward neutrinos with rather low energies, the vertically downward muons are ejected with rather large scattering angles. On the other hand, the horizontal-like downward muons are ejected with rather small angles whose energies close to the incident neutrinos energies. Furthermore, the upward muons are produced either due to backscattering or due to the azimuthal effect in QEL for horizontal-like incident neutrinos (see Figures 8 and 9 in the preceding paper[1]). Thus, from the comparison of Figures 35 and 36 with Figures 39 and 38, it is reasonable for the maximum frequency of the Lν/Eν events to occur for 1.0 < Lν/Eν < 1.26 (km/GeV), and not to occur for 20 < Lν/Eν < 25 (km/GeV) where Super-Kamiokande Collaboration “assert”.

Finally, we examine the correlation between Lµ and Eν for 15.8 < Lµ/Eν < 31.6 (km/GeV) where Super-Kamiokande Collaboration give the maximum frequency of L/E neutrino events as shown in Figure 24. Although we compare their frequency with that of our Lµ/Eν in Figure 29 we can compare their frequency with that of our Lµ/Eν in Figure 31 which shows big difference between them. In Figure 26 we give the correlation diagram between Lµ and Eν for 15.8 < Lµ/Eν < 31.6 (km/GeV). In

E.Konishi et al.: On the Sensitivity of L/E Analysis of SK Neutrino Oscillation

Correlation diagram between Lµ and Eν for interval 20 < Lµ/Eν < 25 (km/GeV).

It is clear from Figure 36 that Lν distribute over 27 ~ 120 km, corresponding to cosθν ∼ 0.1 ~ 0, which denotes the horizontal-like downward neutrino events. The frequency of the horizontal-like downward neutrino events in Figure 36 are pretty smaller than that of the vertical-like downward neutrino events in Figure 33 due to smaller solid angles. In Figure 36, we give the correlation diagram between Lν and Lµ for 20 < Lν/Eν < 25 (km/GeV). It is impressive from the figure that Lµ distribute over four orders of magnitude (2 km to 1.2 × 10^4 km), while Lν cover within one order of magnitude (20 ~ 120 km). This fact denotes that the effect of the azimuthal angles in QEL is pretty strong even in the horizontal-like downward neutrino events in which the produced muons are apparently judged to come from the upward direction (see Figure 3-c and Figures 8 to 10 in the preceding paper[1]).

In Figure 38, we give the correlation diagram between Lµ and Eν for the same intervals as in Figure 36. If we compare Figure 39 with Figure 38 then we can find the following interesting situation. As it is clearly understandable from Figure 39, horizontal-like downward neutrinos produce the muons in the three different regions, namely, vertical-like downward muons, horizontal-like downward muons and upward muons. From horizontal-like downward neutrinos with rather low energies, the vertically downward muons are ejected with rather large scattering angles. On the other hand, the horizontal-like downward muons are ejected with rather small angles whose energies close to the incident neutrinos energies. Furthermore, the upward muons are produced either due to backscattering or due to the azimuthal effect in QEL for horizontal-like incident neutrinos (see Figures 8 and 9 in the preceding paper[1]). Thus, from the comparison of Figures 35 and 36 with Figures 39 and 38, it is reasonable for the maximum frequency of the Lν/Eν events to occur for 1.0 < Lν/Eν < 1.26 (km/GeV), and not to occur for 20 < Lν/Eν < 25 (km/GeV) where Super-Kamiokande Collaboration “assert”.

Finally, we examine the correlation between Lµ and Eν for 15.8 < Lµ/Eν < 31.6 (km/GeV) where Super-Kamiokande Collaboration give the maximum frequency of L/E neutrino events as shown in Figure 24. Although we compare their frequency with that of our Lµ/Eν in Figure 29 we can compare their frequency with that of our Lµ/Eν in Figure 31 which shows big difference between them. In Figure 26 we give the correlation diagram between Lµ and Eν for 15.8 < Lµ/Eν < 31.6 (km/GeV). In
Figures 10 and 11 give the corresponding correlation diagrams between $L_\mu$ and $L_\nu$, and $L_\mu$ and $E_\nu$, respectively.

It is clear from Figure 13 that Super-Kamiokande Collaboration measure the vertical-like downward muons. It is also clear from Figures 10 and 11 that these vertical-like downward muons are produced by the incident neutrinos whose $L_\nu$ are distributed over four orders of magnitude. These incident neutrinos are classified into two parts. One is the downward incident neutrinos $(1.0 < L_\nu < 100 \text{ km})$ and the other $(L_\nu > 100 \text{ km})$ is the upward incident neutrinos. The majority of the incident neutrino is occupied by the vertical-like downward. However, the frequency of the upward neutrinos is in the same order of the magnitude as the horizontal-like downward. The upward incident neutrinos may produce downward muons due to both backscattering and the azimuthal angle effect in QEL. At any rate, for the measured muons in the case of the maximum frequency of the events, $L_\mu$ of the corresponding incident neutrinos distribute over four orders of magnitude. Shortely speaking, for the maximum frequency of the neutrino events $15.8 < L_\nu/E_\nu < 31.6 \text{ (km/GeV)}$, the magnitude of the $L_\nu$ of the produced muons lie within one order of magnitude (see Figure 13), although the $L_\nu$ of the incident neutrinos which produce these muons distribute over four orders of magnitude. In other words, it is concluded that Super-Kamiokande Collaboration do not measure the definite direction of the incident neutrinos as far as they measure $L_\mu$. It is furthermore noticed from the comparison of Figure 11 with Figure 29 that Figure 11 is obtained from Figure 29 by cutting off the stripe of $15.8 < L_\nu/E_\nu < 31.6$ (km/GeV). Therefore, we can recognize the vacant region of the neutrino events faintly in the part between 100 and 1000 (km/GeV) in Figure 11 which is clearly shown in Figure 29. The vacant region of the events shows indication of neutrino oscillation.

The summary on Figures from 33 to 41 are as follows; Figures from 33 to 35 represent the mutual relations among $L_\nu$, $L_\mu$, $E_\nu$, and $E_\mu$ near our maximum frequency of $L_\nu/E_\nu$ distribution. Here, all the incident neutrinos are occupied by the downward vertical-like neutrinos, while the majority of the emitted muons is occupied by the downward muon and the minority is occupied by the upward muon. Figures from 39 to 41 represent the similar mutual relations for our $L_\mu/E_\nu$ distribution which correspond to the near the maximum frequency of $L_\mu/E_\nu$ distribution obtained by Super-Kamiokande Collaboration. Here, almost the incident neutrinos are occupied by the downward horizontal-like neutrinos, while about half of emitted muons is recognized as the downward muon and the other half is done as the upward muon. Figures from 35 to 37 represent the mutual similar relations, assuming the numerical values of the maximum frequency of $L_\mu/E_\nu$ distribution obtained by Super-Kamiokande Collaboration. Here, the majority of the emitted muons is occupied by the horizontal like muons, while their parent neutrinos come from both the downward neutrinos and the upward ones. The common characters through Figures from 33 to 41 is that for given definite $L_\nu$($L_\mu$) we find $L_\mu$($L_\nu$) which distribute over the four order of magnitudes.

4 Conclusion

The assumption made by Super-Kamiokande Collaboration that the direction of the reconstructed lepton approximately represents the direction of the original neutrino does not hold even approximately [3]. This is logically equivalent to the statement that $L_\nu$ cannot be replaced by $L_\mu$ even if approximately. This is really clarified in Figures 12 and 13 in the preceding paper[1].

Although the derivation of $E_\nu$ from $E_\mu$ (Eq.(6) of the preceding paper[1]) is theoretically, irrelevant to the stochastic plobrem, because of the neglect of the stochastic character in physical processes concerned, such the approximation does not induce so practically serious error compared with the assumption of $L_\nu \approx L_\mu$. As clarified in Figures 12 to 14, the maximum oscillation in $L/E$ analysis can be observed only in the $L_\nu/E_\nu$ distribution and it is quite natural by the definition of the probability for a given favor whose argument is $L_\nu/E_\nu$ (Eq.(1)). As clarified in Figures 15 to 19 and Figures 24 to 26, the maximum oscillation for the presence of neutrino oscillation cannot be observed from both $L_\nu/E_\nu$ and $L_\mu/E_\mu$. The relation between $L_\nu$ and $L_\mu$ is too complicate to extract similar expression to Eq.(6) of the preceding paper[1] for the argument on $L_\nu/E_\nu$ and $L_\mu/E_\mu$. Similarly in the case of argument of $L_\nu/E_\nu$, we can indicate something like the maximum oscillation in $L_\nu/E_\nu$ distribution which are shown in Figures 27 to 29. The situation is derived from the fact that what plays a decisive role in $L/E$ analysis is $L_\nu$, but not $E_\nu$, which are clearly shown by comparing Figures 12 and 13 with Figure 14 in the preceding paper[1].

As for $L/E$ distribution obtained by Super-Kamiokande Collaboration, we definitely indicate that the maximum oscillation cannot be observed through the measurement of $L_\mu$. Consequently, we cannot observe the maximum oscillation in $L/E$ analysis which is carried out in Super-Kamiokande Collaboration. Furthermore, one cannot find the maximum frequency of $L/E$ events at the position where Super-Kamiokande Collaboration observe, even if one can observe $L_\mu$.

In conclusion, the maximum oscillation in $L/E$ analysis can be observed only in $L_\nu/E_\nu$, but not in any other combinations of $L$ with $E$. However, $L_\nu$ is physically unobservable quantities and it cannot be approximated by $L_\mu$, because the assumption between $L_\nu$ and $L_\mu$ does not hold even if statistically. Consequently, it should be concluded that Super-Kamiokande cannot observe the maximum oscillation in their $L_\nu/E_\nu$SK analysis.

Finally, our conclusion that $L_\nu$ cannot be approximate by $L_\mu$ is logically equivalent to the statement that $\cos^2 \theta$ cannot be approximated by $\cos^2 \theta_\mu$, where $\cos^2 \theta$ denotes cosine of the zenith angle of the incident neutrino and $\cos^2 \theta_\mu$ denotes that of the produced muon, respectively [3]. In Super-Kamiokande Collaboration, they approximate $\cos^2 \theta_\nu$ as $\cos^2 \theta_\mu$. (See the reproduction of their statements in the 2 page in the present paper). The analysis of the zenith angle distribution of the atmospheric neutrino events Super-Kamiokande Collaboration will be re-examined in our subsequent papers.
References

1. Konishi, E et al., arXiv hep-ex/1007.3812v1
2. Ashie, Y et al., Phys. Rev. D 71 (2005) 112005.
3. Honda, M., et al., Phys. Rev. D 52 (1996) 4985.
   Honda, M., et al., Phys. Rev. D 70 (2004) 043008-1.
4. Ashie, Y et al., Phys. Rev. Lett. 93 (2004) 101801-1.
5. Konishi, E et al., arXiv hep-ex/0808.0664v2