Modal analysis and optimization of bus body structure

Song Deng¹,²,³, Xinghui Han¹,² and Lei Yang¹,²

¹ Hubei Key Laboratory of Advanced Technology for Automotive Components, Wuhan University of Technology, Wuhan 430070, China
² Hubei Collaborative Innovation Center for Automotive Components Technology, Wuhan University of Technology, Wuhan 430070, China
³ E-mail: guoheng0722@126.com

Abstract. To study the vibration and noise of the bus, the structural and the relevant acoustic models of the bus body are firstly established. Then, their resonant frequencies and crucial modes of vibration are analyzed. The stiffness, mass and natural frequency of the studied beams are varied to optimize the vibration frequencies and modes of the bus body. The results provide valuable guidelines for the modal analysis and optimization of the bus body structures.

1. Introduction

It is of great importance to analyze and optimize the vibration frequencies and modes of the bus body for studying the vibration and noise. However, due to a lack of theoretical guidance for reducing the vibration and noise of the public bus, many bus companies optimize bus bodies based on empirical methods in the initial design stage. Therefore, it is crucial to carry out the modal analysis and optimization of the bus body structure for the reduction of the vibration and noise.

Previous work has been carried out on the resonant frequencies and vibration modes of mechanical parts. Main studies [1, 2] mainly investigated the different vibration measurement techniques of engineering structures and the different data processing techniques, of which the application scope was analyzed based on the excitation styles. The dynamic behavior of the tyre in terms of modal parameters was studied to address various noise generation mechanisms and noise propagation phenomena [3]. The wind-induced vibrations of the front windshield at various speeds of a van-body model bus were studied using simulated and experimental methods. The deformation of the glass was investigated at different working conditions and also its dangerous frequencies and crucial modes were revealed [4, 5]. The frequency response function of the system obtained by the impact test was adopted to investigate the dynamic response of flexible parts [6, 7]. The dynamic characteristic of the large and complex structure was studied based on the eigenvalue problem, and the structural modal reanalysis was conducted for the large and multiple modifications [8, 9]. The natural frequencies of a circular plate in the absence of fluid interaction were estimated, and the pure structural natural frequencies were extracted using the method of vibration health monitoring [10, 11]. Moreover, the modal analysis is important for the predictions of the damage initiation and propagation of automotive parts. The modal damage detection techniques provides interesting opportunities for cheap and fast quality checks in the construction phase, as well as for safety evaluations and structural maintenances over the structure lifespan. The relationship of the vibration modes of beams with the damage degrees was estimated at various tensile loads [12, 13]. The influence of the fatigued adhesive joint on the modal dynamic properties of the bonded structure was studied, and it was demonstrated that modal
frequencies of single lap joints structure tend to decrease with increasing vibration fatigue cycles [14, 15]. However, it should be noted that a majority of the studies above were primarily concentrated on the modal analysis and vibration optimization of automotive parts, and the vibration and noise of vehicles were evaluated based on the general analysis methods, while little attention was paid to the application of the vibration response mechanism of the automotive parts on the complex multi-body systems. There are few effective approaches to control the resonance of acoustical-structure interaction from bus bodies. Hence, it is necessary to conduct the modal analysis and optimization of the bus body structure based on the vibration characteristics of beams.

To provide valuable guidelines for the modal analysis and optimization of the bus body structure, the structural and acoustic models of the bus body were firstly established using the Hyperworks software according to modeling procedures. Their resonant frequencies and crucial vibration modes were analyzed. Then, the significant beams of the bus body were determined for the modal analysis. The thickness and length of these beams is varied to optimize the vibration frequencies and modes of the bus body.

2. Theoretical background
Any mechanical systems can be described by the equation of motion which in matrix can be written [1]

\[ [M]\dot{\ddot{x}}(t) + [C]\dot{x}(t) + [K]x(t) = \{f(t)\] (1)

where \([M]\), \([C]\), \([K]\) are the mass, damping and stiffness matrices respectively along with the corresponding vectors of acceleration, velocity and displacement and the force was applied to the system. The equation (1) can be transformed from time domain into Laplace domain, by doing which a complicated set of coupled equations is converted into a set of simple uncoupled single degree of freedom systems

\[ s^2[M] + s[C] + [K]X(s) = \{F(s)\} \quad \text{or} \quad [B(s)]X(s) = \{F(s)\] (2)

where \(s\) is Laplace variable, and \(B(s)\) is system matrix. The system transfer function \([H(s)]\) is the inverse of the system matrix.

A general approach for tensile load estimation in beams in the case of uncertain boundary conditions was described in [16]. The method starts from the equation governing the dynamic behavior of a beam with an uniform section and subjected to a constant axial tensile force,

\[ EI \frac{\partial^4 v(x,t)}{\partial x^4} + N \frac{\partial^2 v(x,t)}{\partial x^2} + \rho \frac{\partial^2 v(x,t)}{\partial t^2} = 0 \] (3)

where \(EI\) represents the bending stiffness, \(\rho\) is the mass per unit length, \(v(x, t)\) is the transverse displacement at abscissa \(x\) and time \(t\), and \(N\) the tensile force.

3. Simulated model development
There are some main steps for the establishment of the numerical model of the bus body skeleton as follows:

1) Developing the spatial geometrical structure. According to the CAD drawing of a bus body structure (for example, the CAD drawing of the right body-side is shown in figure 1, the spatial structure of a bus body skeleton is presented in figure 2. The length of the bus body is 11600 mm, the width is 2503 mm, and the height is 2732 mm.

Figure 1. CAD drawing of the right body-side. Figure 2. Spatial structure of bus body skeleton.
(2) Dividing mesh for the bus body skeleton. The body skeleton was meshed with quadrilateral elements, and the element size was 10 mm. Beams were connected with the RBE2 elements to simulate the welds during the process of building the model.

(3) Setting the material properties and thicknesses of beams. The material used in this simulated model was Q235B steel, and the main parameters are: Young’s modulus $E=2.10\times10^5$ MPa, Poisson’s ratio $\nu=0.3$, material density $\rho=7.85\times10^3$ kg/m$^3$, and tensile strength $\sigma_t=386$ MPa. The thicknesses of beams were set according to the design drawing of the bus body skeleton. The numerical model of the body skeleton was shown in figure 3.

Figure 3. Numerical model of bus body skeleton.

During the process of establishing the acoustic model, an acoustic wavelength should pass six elements at least. The maximum edge of the element should be less than a third of the wavelength for the concerned frequencies [17]. Mesh sizes of the acoustic model can be determined by

$$L \leq \frac{\lambda_{\text{min}}}{6},$$

$$\lambda = \frac{\nu}{f},$$

where $L$ is the mesh size, $\lambda_{\text{min}}$ is the minimum wavelength, $\lambda$ is the acoustic wavelength, $\nu$ is the velocity of sound, and $f$ is the frequency of sound. In this work, the studied frequency range is 0–200Hz.

To improve the efficiency and precision of the finite element analysis, the element sizes of the acoustic cavity for the bus body and seats are 80 and 50 mm. The model is shown in figure 4. Thus, the modal analysis can be conducted to investigate the resonant frequencies and crucial vibration modes of the structural and acoustic models for the bus body.

Figure 4. Simulated model of acoustic cavity for the bus body and seats.

4. Results and discussion

Figure 5 shows the vibration modes for the acoustic cavity and bus body under various frequencies. From figure 5(a) and (b), it can be seen that the large acoustic pressure of the cavity distributes in the rear of the cavity at the frequency of 15.03 Hz. For the bus body, the major displacement occurs on the rear wall and the postmedian of the ceiling at the frequency of 15.6 Hz. According to the resonance mechanism, the frequency of the bus body gets close to that of the acoustic cavity so that the resonance occurs between the rear wall and the rear of the cavity. Similarly, figure 5(c) presents that four parts of the acoustic cavity are distributed by major acoustic pressure when the natural frequency is 43.1 Hz. Figure 5(d) shows that the large displacements appear on the beams at the rear of the body-side at the frequency of 42.4Hz. The resonance can occur between the beams at the rear of the body-side and the rear of the cavity, because the frequency (42.4 Hz) of the body gets close to that (43.1) of the cavity. Moreover, interior noise is mainly induced by the structural vibration when the concerned frequency is in the range of 20–200 Hz. The optimization of the posterior structure of the body-side should be emphasized in this work. So, it is necessary to optimize the distribution of the beams at the posterior of the body-sides for avoiding the resonance between the posterior of the body-sides and the rear of the cavity.
Figure 5. Vibration modes of acoustic cavity and bus body: (a) at 15.03Hz; (b) at 15.6Hz; (c) at 43.1Hz; (d) at 42.4Hz.

As shown in figure 6, the studied beams at the left body-side are named as $z_1$, $z_2$, and $z_3$, the studied beams at the right body-side are named as $y_1$, $y_2$, and $y_3$. The cross-section sizes of beams $z_1$, $z_2$, and $z_3$ are $40\times40\times2$ mm, $60\times50\times3$ mm, and $50\times40\times2$ mm, respectively. The cross-section sizes of beams $y_1$, $y_2$, and $y_3$ are $40\times40\times2$ mm, $60\times50\times3$ mm, and $40\times40\times2$ mm, respectively. Thus, these beams are changed to optimize the structural modes of the bus body.

Figure 6. Nomenclature of studied beams.

The thicknesses of the beams $y_2$ and $z_2$ are increased from 3 mm to 5 mm, while the other conditions remain unchanged. Figure 7 shows the structural modes of the postmedian of body-sides when the thickness of the beams $y_2$ and $z_2$ are 5mm. It can be seen that the vibration displacement, occurred on the postmedian of body-sides, has no obvious change, and the amplitude (3.517 mm) is increased with respect to the original amplitude (3.439 mm). The natural frequency of 43.0 Hz moves closer to that of the acoustic cavity. This means that the increased thickness of the beams $y_2$ and $z_2$ has little effect on the modal optimization of the bus body. Usually, the natural frequency $\omega$ of the beam depends on the stiffness and mass matrices [18].
\[
\omega = \sqrt{\frac{[K]}{M}}
\]  

here, the stiffness matrix \([K]\) is superposed by the stiffness matrix of flat shell element and the bending stiffness matrix. The relationship of the element stiffness matrix with the thickness of the beam is expressed as follows [18]

\[
[K] = [K_p] + [K_b] = E_t[K_{p.c}] + E_t^3[K_{bc}]
\]  

where \([K_p]\) is transformed from the stiffness matrix of flat shell element under the global coordinates, \([K_b]\) is transformed from the bending stiffness matrix under the global coordinates, \(t\) is the thickness of the element and has no relation with \([K_{p.c}]\) and \([K_{bc}]\). The relationship of the element mass matrix with the thickness of the beam is described as follows

\[
[M] = \rho \frac{A_e}{3} [M_c]
\]  

where \(A_e\) is the surface area of the elements, \([M_c]\) has no relation with the thickness \(t\).

According to equations (6-8), it can be deduced that the natural frequency of the beam can be increased with increasing the thickness \(t\). However, there is little change on the amplitude of the beams \(y_2\) and \(z_2\) and the natural frequency of the bus body. This suggests that the limited increase of the mass for the beams \(y_2\) and \(z_2\) cannot effectively optimize the natural frequency and vibration modes of the bus body under the real condition.

**Figure 7.** Vibration mode of postmedian of body-sides at the frequency of 43.0 Hz.

To study the effect of the stiffness of beams \(y_2\) and \(z_2\) on the natural frequency and vibration modes of the bus body, a layer of elements is increased along the axial direction of beams \(y_2\) and \(z_2\), as shown in figure 8(a). From figure 8(b) and (c), it can be seen that the vibration displacement, occurred on the postmedian of the left body-side, is reduced from 3.439 mm to 2.6 mm. Moreover, the natural frequencies of the body-sides move closer to that of the acoustic cavity to induce the resonance between the posterior of the body-sides and the rear of the cavity. This means that the limited increase of the stiffness for beams \(y_2\) and \(z_2\) cannot obviously change the natural frequency and vibration modes of the bus body under the real condition. So, it is necessary to study beams \(y_1\), \(y_3\), \(z_1\) and \(z_3\) to move the natural frequency of the bus body.

**Figure 8.** Vibration frequencies and modes of postmedian of body-sides and beam structure: (a) beam structure; (b) 43.1 Hz; (c) 43.2 Hz.
The thicknesses of beams $y_1$, $y_3$, $z_1$ and $z_3$ are increased from 2 mm to 4 mm, while the other conditions remain unchanged. The structural mode of the body is shown in figure 9. From figure 9(a), it can be seen that the amplitudes of beams $z_1$ and $z_3$ are decreased from 3.43 mm to 1.87 mm. The large amplitude appears at beam $z_5$ when the natural frequency is 41.3 Hz. This suggests that the increased stiffness of beams $z_1$ and $z_3$ suppresses their deformations despite the natural frequency of 41.3 Hz is still close to the original natural frequency of 42.4 Hz. For the right body-side (shown in figure 9(b)), the amplitude of the postmedian of the right body-side is reduced from 3.43 mm to 2.61 mm. At the moment, the natural frequency is added from 4.24 Hz to 45 Hz. This suggests that the increased thickness of beams $y_1$ and $y_3$ not only strengthens their stiffnesses to impede their deformations, but also moves the natural frequency from 42.4 Hz to 45 Hz relative to that of the acoustic cavity of 43.1 Hz based on the relationship in Equations. (6-8). Therefore, it is feasible to increase the thicknesses of beams $y_1$, $y_3$, $z_1$ and $z_3$ to optimize the vibration frequencies and modes of the bus body. However, the frequencies and modes of the bus body can be further optimized, because the large amplitude still occurs at the postmedian of the right body-side, and the frequency (45.0 Hz) does not remarkably move away from that (43.1 Hz) of the acoustic cavity.

![Figure 9](image1)

**Figure 9.** Vibration frequencies and modes of postmedian of body-sides: (a) 41.3 Hz; (b) 45.0 Hz.

![Figure 10](image2)

**Figure 10.** Structural change of beams and vibration frequencies and modes of postmedian of body-sides: (a) structural change of beams; (b) 41.2 Hz, (c) 40.0 Hz.

To further optimize the frequencies and modes of the bus body, beams $z_1$ and $z_3$ are connected into beam $z_{13}$, and beams $y_1$ and $y_3$ are connected into beam $y_{13}$. Their thickness is still 4 mm, as shown in figure 10(a). Thus, beam $z_2$ is divided into $z_{b1}$ and $z_{b3}$, and beam $y_2$ is divided into $y_{b1}$ and $y_{b3}$. From figure 10(b), it can be seen that the vibration frequency and mode of the postmedian of the left body-side has no obvious change with respect to that in figure 9(a). For this vibration and mode, these beams near beam $z_5$ should be analyzed. So, the amplitude of beam $z_5$ is not affected by beams $z_1$ and
z3 despite they are connected into beam z13. For figure 10(c), it can be seen that the amplitudes of beams y1, y2 and y3 are further reduced from 2.61 mm to 1.79 mm. The frequency (40.0 Hz) of the bus body remarkably moves away from that (43.1 Hz) of the acoustic cavity. This is because the connection of beams y1 and y3 changes the natural frequency of the postmedian of the left body-side, as follows [18]

\[ f_r = \frac{r}{2l} \sqrt{\frac{N}{\rho}} \quad r=1, 2, N_m. \]  

(9)

here, \( f_r \) is the natural frequency, \( \rho \) is the mass per unit length, \( N \) is the tensile force, \( l \) is the length of the beam. The natural frequency of the \( r \)-th mode for the beam is obtained under the tensile load because of the insignificance of the bending stiffness and end restraint. When only the rotational stiffness of the beam is negligible with respect to the bending stiffness, the natural frequency of the \( r \)-th mode is

\[ f_r = \frac{r}{2} \sqrt{\frac{2\pi^2 EI}{\rho l^4} + \frac{N}{\rho l^2}} \quad r=1, 2, N_m. \]  

(10)

According to Equations (9, 10), it is found that the natural frequency of beam y13 is decreased with respect to that of beams y1 and y3. At this moment, the stiffness and mass of beam y13 has no obvious change with respect to beams y1 and y3. The new beams y51 and y52 have higher natural frequencies relative to beam y2. This suggests that the increased length of the beam can dramatically decrease their natural frequencies to avoid the resonance occurred at the postmedian of body-sides although the stiffness and mass of the beam has no change. Therefore, to optimize the vibration frequencies and modes of the bus body, it is feasible to change the length of beams to directly alter their natural frequencies.

5. Conclusions

(1) The posterior structure of the body-side should be optimized for avoiding the resonance between the posterior of the body-sides and the rear of the cavity.

(2) The limited increase of the mass and stiffness for the beams y2 and z2 cannot effectively optimize the natural frequency and vibration modes of the bus body under the real condition.

(3) The increased thickness of beams y1, y3, z1 and z3 optimizes the vibration frequencies and modes of the bus body. But, the frequency (45.0 Hz) does not remarkably move away from that (43.1 Hz) of the acoustic cavity.

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