Sequential attacks against differential-phase-shift quantum key distribution with weak coherent states

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We investigate limitations imposed by sequential attacks on the performance of differential-phase-shift quantum key distribution protocols that use pulsed coherent light. In particular, we analyze two sequential attacks based on unambiguous state discrimination and minimum error discrimination, respectively, of the signal states emitted by the source. Sequential attacks represent a special type of intercept-resend attacks and, therefore, they do not allow the distribution of a secret key.

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I. INTRODUCTION

Quantum key distribution (QKD) [1] is a technique that exploits quantum effects to establish a secure secret key between two parties (usually called Alice and Bob). This secret key is the essential ingredient of the one-time-pad or Vernam cipher [2], the only known encryption method that can provide information-theoretic secure communications.

The first complete scheme for QKD is that introduced by Bennett and Brassard in 1984 (BB84 for short) [3]. A full proof of the security for the whole protocol has been given in Ref. [4]. After the first demonstration of the feasibility of this scheme [5], several long-distance implementations have been realized in the last years (see, for instance, Ref. [6] and references therein). However, these practical approaches differ in many important aspects from the original theoretical proposal, since that demands technologies that are beyond our present experimental capability. Specially, the signals emitted by the source, instead of being single-photons, are usually weak coherent pulses (WCP) with typical average photon numbers of 0.1 or higher. Moreover, the detectors employed by the receiver have a low detection efficiency and are noisy due to dark counts. These facts, together with the loss and the noise introduced by the quantum channel, jeopardize the security of the protocol, and leads to limitations of rate and distance that can be covered by these techniques [7, 8].

The main security threat of QKD based on WCP arises from the fact that some pulses contain more than one photon prepared in the same polarization state. Now, an eavesdropper (Eve) can perform, for instance, the so-called Photon Number Splitting (PNS) attack on the multi-photon pulses [7]. This attack provides Eve with full information about the part of the key generated from the multi-photon signals, without causing any disturbance in the signal polarization. As a result, it turns out that the BB84 protocol with WCP can give a key generation rate of order $O(\eta^2)$, where $\eta$ denotes the transmission efficiency of only the quantum channel [8].

To obtain higher secure key rates over longer distances, different QKD schemes robust against the PNS attack have been proposed in recent years. One of these schemes is the so-called decoy-states [10, 11], where Alice randomly varies the mean photon number of the signal states sent to Bob by using different intensity settings. This technique delivers a key generation rate of order $O(\eta)$ [10, 11]. Other possibility is based on the transmission of two non-orthogonal coherent states together with a strong reference pulse [12]. This scheme has been analyzed in Ref. [13], where it was confirmed that also in this scenario the secure key rate is of order $O(\eta)$. Finally, another possible approach is the use of differential-phase-shift (DPS) QKD protocols [14–17]. In this kind of schemes, Alice sends to Bob a train of WCP whose phases are randomly modulated by 0 or $\pi$. On the receiving side, Bob measures out each incoming signal by means of an interferometer whose path-length difference is set equal to the time difference between two pulses. In this case, however, a secure key rate of order $O(\eta)$ has only been proven so far against a particular type of individual attacks where Eve acts on photons individually, rather than signals [13]. Whether DPS QKD is secure against the most general attack remains an important open question.

In this paper, we investigate limitations imposed by sequential attacks on the performance of DPS QKD protocols. In this kind of attacks, Eve measures out every coherent state emitted by Alice and prepares new signal states, depending on the results obtained, that are given to Bob. Whenever Eve obtains a predetermined number of consecutive successful measurement outcomes, then she prepares a train of WCP that is forwarded to Bob. Otherwise, Eve sends vacuum signals to Bob to avoid errors. Sequential attacks constitute a special type
of intercept-resend attacks\textsuperscript{18,20} and, therefore, they do not allow the distribution of a secret key\textsuperscript{21}. Here we shall consider a conservative definition of security, \textit{i.e.}, we assume that Eve can control some flaws in Alice’s and Bob’s devices (\textit{e.g.}, the detection efficiency and the dark count probability of the detectors), together with the losses in the channel, and she exploits them to obtain maximal information about the shared key.

We analyze two possible sequential attacks. In the first one, Eve realizes unambiguous state discrimination (USD) of Alice’s signal states\textsuperscript{18,22,23}. When Eve identifies unambiguously a signal state sent by Alice, then she considers this result as successful. Otherwise, she considers it a failure. In the second attack, Eve performs first a filtering operation on each signal emitted by Alice and, afterwards, she measures out each successful filtered state following the approach of minimum error discrimination (MED)\textsuperscript{24,25}, \textit{i.e.}, she guesses the identity of the filtered state with the minimum probability of making an error. (See also Ref.\textsuperscript{20}.) As a result, we obtain upper bounds on the maximal distance achievable by DPS QKD schemes as a function of the error rate in the sifted key, the double click rate at Bob’s side, and the mean photon-number of the signals sent by Alice.

Instead of using an USD measurement on each signal state sent by Alice, like in the first sequential attack that we consider, Eve could as well employ the same detection device like Bob. This sequential attack was very briefly introduced in Ref.\textsuperscript{15}. A successful result is now associated with obtaining a click in Eve’s apparatus, while a failure corresponds to the absence of a click. However, since Alice’s signal states are typically coherent pulses with small average photon number, the probability of obtaining a successful result in this scenario is always smaller than the one of a sequential USD attack. Therefore, a sequential USD attack can provide tighter upper bounds on the performance of DPS QKD protocols than those derived from an eavesdropping strategy where Eve uses the same measurement apparatus like Bob.

A different QKD scheme, but also related to DPS QKD protocols, has been proposed recently in Ref.\textsuperscript{27}. (See also Ref.\textsuperscript{28}.) However, since the abstract signal structure of this protocol is different from the one of DPS QKD schemes, the analysis contained in this paper does not apply to that scenario. Sequential attacks against the QKD protocol introduced in Ref.\textsuperscript{27} have been investigated in Ref.\textsuperscript{23} following a similar approach like in this paper.

The paper is organized as follows. In Sec.\textsuperscript{III} we describe in more detail DPS QKD protocols. Then, in Sec.\textsuperscript{IV} we present sequential attacks against DPS QKD schemes. Sec.\textsuperscript{V} includes the analysis for a sequential USD attack. Here we obtain an upper bound on the maximal distance achievable by DPS QKD schemes as a function of the error rate, the double click rate at Bob’s side, and the mean photon-number of Alice’s signal states. Similar results are derived in Sec.\textsuperscript{VI} now for the case of sequential attacks based on MED of the signals sent by Alice. Finally, Sec.\textsuperscript{VI} concludes the paper with a summary.

\section{II. Differential-Phase-Shift QKD}

The setup is illustrated in Fig.\textsuperscript{1}\textsuperscript{14–17}. Alice prepares first a train of coherent states $|\alpha\rangle$ and, afterwards, she modulates, at random and independently every time, the phase of each pulse to be 0 or $\pi$. As a result, she produces a random train of signal states $|\alpha\rangle$ or $|-\alpha\rangle$ that are sent to Bob through the quantum channel. On the receiving side, Bob uses a 50 : 50 beam-splitter to divide the incoming pulses into two possible paths and then he recombines them again using another 50 : 50 beam-splitter. The time delay introduced by Bob’s interferometer is set equal to the time difference $\Delta t$ between two pulses. Whenever the relative phase between two consecutive pulses is 0 ($\pm\pi$) only the photon detector $D_0$ ($D_1$) may produce a “click” (at least one photon is detected). For each detected event, Bob records the exact time where he obtained a click and the actual detector that fired.

Once the quantum communication phase is completed, Bob uses a classical authenticated channel to announce the time instances where he detected at least one photon. From this information, together with the knowledge of the phase value used to modulate each pulse, Alice can infer which photon detector fired at Bob’s side each given time. Then, Alice and Bob can agree, for instance, to select a bit value “0” whenever the photon detector $D_0$ clicked, and a bit value “1” if the detector $D_1$ fired. In an ideal scenario, Alice and Bob end up with an identical string of bits representing the \textit{sifted key}. Due to the noise introduced by the quantum channel together with possible imperfections of Alice’s and Bob’s devices, however, the sifted key typically contains some errors. Then, Alice and Bob perform error-correction to reconcile the data, and privacy amplification to decouple the data from Eve. (See, for instance, Ref.\textsuperscript{1}.)

In the next section we analyze simple sequential attacks against the DPS QKD protocol introduced above that are particularly suited for the signal states and detection methods employed by Alice and Bob, together with the attenuation introduced by the channel. Let us
emphasize here that these attacks might not be optimal, but, as we will show below, they already impose strong restrictions on the performance of DPS QKD schemes with weak coherent pulses.

III. SEQUENTIAL ATTACKS AGAINST DIFFERENTIAL-PHASE-SHIFT QKD

A sequential attack can be seen as a special type of intercept-resend attack. First, Eve measures every coherent state emitted by Alice with a detection apparatus located very close to the sender. Afterwards, she transmits each measurement result through a lossless classical channel to a source close to Bob. Whenever Eve obtains a predetermined number of consecutive successful measurement outcomes, this source prepares a train of new signal states that is forwarded to Bob. Otherwise, Eve sends vacuum signals to Bob to avoid errors. Whether a measurement is considered to be successful or not and which type of non-vacuum states Eve sends to Bob depends on Eve’s particular eavesdropping strategy and on her measurement device. Sequential attacks transform the original quantum channel between Alice and Bob into an entanglement breaking channel [26] and, therefore, they do not allow the distribution of a secret key [21].

We begin by introducing Eve’s measurement apparatus. As mentioned previously, we shall consider two possible alternatives. Each alternative provides a different sequential attack. In the first one, Eve realizes USD [22, 23] of Alice’s signal states. Whenever Eve identifies unambiguously a signal state sent by Alice, i.e., she determines without error whether it is $\ket{\alpha}$ or $\ket{-\alpha}$, she considers this result as successful. If the measurement outcome corresponds to an inconclusive result then she considers it a failure. The second eavesdropping strategy can be decomposed into two steps: first, Eve performs a filtering operation on each signal state sent by Alice with the intention to make them, with some finite probability, more “distinguishable”. A failure refers now to those signal states for which the filtering operation does not succeed. Afterwards, Eve measures out each successful filtered state following the approach of MED [24, 25]. Her goal is to guess the identity of the filtered states with the minimum probability of making an error. Notice that the first eavesdropping strategy can be considered as a special case of the second eavesdropping strategy where the probability that Eve makes an error in distinguishing a state $\ket{\alpha}$ and $\ket{-\alpha}$ is exactly zero. We shall denote as $p_{\text{succ}}$ the probability that Eve obtains a successful result whatever the measurement device she employs.

In order to evaluate her measurement outcomes, we shall consider that Eve divides her data into different blocks of length $M$, where each block contains $M$ consecutive measurement results. Moreover, we assume that Eve analyzes each block of data independently, i.e., without considering the data included in other blocks. As we will show later on, this eavesdropping strategy will necessarily create some error rate that decreases when incrementing the block length $M$. In this scenario, we define the integer parameter $M_{\text{min}}$, with $[M/2 + 1] \leq M_{\text{min}} < M$, as the minimum number of consecutive successful results within a block that Eve needs to obtain in order to send Bob a new train of coherent states $|\beta e^{i\theta}|$. This definition of $M_{\text{min}}$ arises from the particular eavesdropping strategies that we consider here, and the role of this parameter $M_{\text{min}}$ will become clear later on. More precisely, if $m$ denotes the total number of consecutive successful outcomes obtained by Eve within a block, then, whenever $m$ is bigger than $M_{\text{min}}$, Eve prepares $m$ consecutive coherent states $|\beta e^{i\theta_1}|, |\beta e^{i\theta_2}|, \ldots, |\beta e^{i\theta_m}|$, together with $M-m$ vacuum states for those unsuccessful results within the block and sends these signals to Bob. On the other hand, if $m < M_{\text{min}}$ Eve sends to Bob $M$ vacuum states. The case $m = M_{\text{min}}$ deserves a special attention. We shall consider that in this case Eve employs a probabilistic strategy that combines the two previous ones. In particular, we assume that Eve sends to Bob $M_{\text{min}}$ consecutive coherent states $|\beta e^{i\theta_1}|, |\beta e^{i\theta_2}|, \ldots, |\beta e^{i\theta_{M_{\text{min}}}}|$ with probability $q$ and, with probability $1 - q$, she sends to Bob $M$ vacuum states. That is, the parameter $q$ allows Eve to smoothly fit her eavesdropping strategy to the observed data.

The angle $\theta_j$ of a coherent state $|\beta e^{i\theta_j}|$ prepared by Eve depends on her particular measurement strategy. When she utilizes an USD measurement, then $\theta_j = 0$ if the state identified by her measurement is $\ket{\alpha}$, and $\theta_j = \pi$ if the state identified is $\ket{-\alpha}$. A similar criterion can also be applied to the case where Eve performs a filtering operation followed by a MED measurement on the successful filtered states: If the result obtained is associated with the state $\ket{\alpha}$ then $\theta_j = 0$, otherwise $\theta_j = \pi$. Fig. 2 shows a graphical representation of such a sequential attack for the case $M = 5$ and $M_{\text{min}} = 3$. In this example, moreover, we assume that Eve obtains $m = 4$ consecutive successful results within a block.

Next, we obtain an expression for the Gain of a sequential attack, i.e., the probability that Bob obtains
a click per signal state sent by Alice, as a function of the parameters $M$, $M_{\text{min}}$, $q$, the probability $p_{\text{succ}}$ of obtaining a successful result, and the mean photon-number $\mu_s = |\beta|^2$ of the coherent states sent by Eve. Afterwards, we study the two sequential attacks introduced above in more detail. The objective is to find an expression for the quantum bit error rate (QBER) introduced by Eve, and for the resulting double click rate at Bob’s side in each of these two attacks.

### A. Gain of a sequential attack

The Gain of a sequential attack is defined as $N_{\text{clicks}}/N$, where $N_{\text{clicks}}$ represents the average total number of clicks obtained by Bob, and $N$ is the total number of signal states sent by Alice. In this definition, we consider that double clicks contribute to $N_{\text{clicks}}$ like single clicks. The parameter $N_{\text{clicks}}$ can be expressed as $N_{\text{clicks}} = (N/M)N_{\text{clicks}}^M$, with $N_{\text{clicks}}^M$ denoting the average total number of clicks per block of length $M$ at Bob’s side. With this notation, the Gain of a sequential attack, that we shall denote as $G$, can then be written as

$$G = \frac{1}{M}N_{\text{clicks}}^M. \tag{1}$$

Next, we obtain an expression for $N_{\text{clicks}}^M$. We shall distinguish several cases, depending on the number $m$ of coherent states $|\beta e^{i\theta_1}\rangle, |\beta e^{i\theta_2}\rangle, \ldots, |\beta e^{i\theta_m}\rangle$ that Eve sends to Bob inside a given block and the position of these coherent states in the block [30]. These cases are illustrated in Fig. 3 where we also include the a priori probabilities to be in each of these scenarios. Note, however, that the average total number of clicks in each of these cases will depend on whether the last signal state of a previous block is actually a coherent state or not. To include this boundary effect between blocks in our analysis, we shall always distinguish two possible alternatives for each case included in Fig. 3 depending on the identity of the last signal state contained in the previous block. The probability of this last signal being a coherent state, that we shall denote as $p$, is calculated in Appendix A and it is given by

$$p = \left[p_{\text{succ}} + (1 - p_{\text{succ}})q\right]p_{\text{succ}}^M. \tag{2}$$

Similarly, $1 - p$ represents the probability that the last signal in a block is a vacuum state. Fig. 4 illustrates these two alternatives for the case where Eve sends to Bob a block of signals containing $M$ coherent states.

Let us now analyze the different scenarios included in Fig. 3 in more detail. When Eve sends to Bob a block of signals containing $M$ coherent states (Case A in Fig. 3) then: If the last signal state of the previous block is a coherent state, then it turns out that the average total number of clicks obtained by Bob is given by $Ms$, where the parameter $s$ has the form

$$s = 1 - \exp(-\mu_s), \tag{3}$$

![FIG. 3: Possible blocks of $M$ signals that Eve sends to Bob together with their a priori probabilities. Case A: The block contains $M$ coherent states. Case B: The first $m \in (M_{\text{min}}, M)$ signals of the block are coherent states, while the last $M - m$ signals are vacuum states. Case C: The block contains first $M - m$ vacuum states followed by $m \in (M_{\text{min}}, M)$ coherent states. Case D: The block has $m \in (M_{\text{min}}, M)$ coherent states and, at least, the first and the last signal of the block are vacuum states. Case E: The first $M_{\text{min}}$ signals of the block are coherent states, while the last $M - M_{\text{min}}$ signals are vacuum states. Case F: The block contains first $M - M_{\text{min}}$ vacuum states together with $M_{\text{min}}$ coherent states. Case G: The block has $M_{\text{min}}$ coherent states and, at least, the first and the last signal of the block are vacuum states. Case H: The block contains only vacuum states. The a priori probability of this last scenario is given by $1 - \sum p_i$, with $p_i$ representing the a priori probabilities of each of the previous cases.]

![FIG. 4: Eve sends to Bob a block of signals containing $M$ coherent states (Block $n$ in the Figure). Case A: with probability $p$, where $p$ is given by Eq. (2), the last signal state of the previous block is a coherent state. Case B: with probability $1 - p$ the last signal state of the previous block is a vacuum state.]

with $\mu_\beta$ being again the mean photon-number of the coherent states $|\beta e^{i\delta_0}\rangle$ sent by Eve. Otherwise, the average total number of clicks at Bob’s side can be written as $t + (M - 1)s$, where the parameter $t$ is given by

$$t = 1 - \exp\left(-\frac{\mu_\beta}{2}\right). \quad (4)$$

The analysis of the remaining cases is similar. If the first $m \in (M_{\text{min}}, M)$ signal states of the block are coherent states, while the last $M - m$ signals are vacuum states (Case B in Fig. 3) then: If the last state of the previous block is a coherent state, the average total number of clicks obtained by Bob is given by $t + m_s$. Otherwise, the average total number of clicks at Bob’s side can be written as $2t + (m - 1)s$. Eve can as well send to Bob a block containing first $M - m$ vacuum states followed by $m \in (M_{\text{min}}, M)$ coherent states (Case C in Fig. 3). In this situation, if the last state of the previous block is a coherent state, the average total number of clicks obtained by Bob is given by $2t + (m - 1)s$. Otherwise, the average total number of clicks has the form $t + (m - 1)s$. When Eve sends to Bob a block of signals, at least, the first and the last signals of the block are vacuum states (Case D in Fig. 3) then: If the last state of the previous block is a coherent state, the average total number of clicks obtained by Bob is given by $3t + (m - 1)s$. Otherwise, the average total number of clicks has the form $2t + (m - 1)s$. The cases E, F, and G, in Fig. 3 are completely analogous to the the cases B, C, and D, respectively. The only difference arises in the a priori probabilities to be in each of these scenarios. Now, these a priori probabilities need to be multiplied by the factor $q$ introduced in Sec. III, i.e., by the probability that Eve actually decides to send $M_{\text{min}}$ coherent states in the block. Finally, when the block contains only vacuum states (Case H in Fig. 3) then: If the last state of the previous block is a coherent state the average total number of clicks obtained by Bob is given by $t$. Otherwise, the average total number of clicks is zero.

After adding all these terms, together with their a priori probabilities, we obtain that the average total number of clicks per block of length $M$ at Bob’s side in a sequential attack can be expressed as

$$N_{\text{clicks}}^M = pt + p_{\text{suc}}^M u_M + \sum_{M_{\text{min}} \leq m < M} q^{\delta_{mM_{\text{min}}}}(1 - p_{\text{suc}})p_{\text{suc}}^m v_m + (M - m - 1)(1 - p_{\text{suc}})w_m, \quad (5)$$

where $\delta_{mM_{\text{min}}}$ is equal to one if $m = M_{\text{min}}$ and it is zero otherwise, and the parameters $u_M$, $v_m$, and $w_m$, are given by

$$u_M = (1 - 2p)t + (M - 1 + p)s,$$

$$v_m = (3 - 2p)t + (2m + p - 2)s,$$

$$w_m = 2t + (m - 1)s. \quad (6)$$

**IV. SEQUENTIAL UNAMBIGUOUS STATE DISCRIMINATION ATTACK**

As already mentioned in the previous section, in this attack Eve performs unambiguous state discrimination (USD) $^{22, 23}$ of Alice’s signal states. Whenever Eve identifies without error a signal state sent by Alice then she considers this result as successful. If the identification process does not succeed, then she considers it a failure. The probability of obtaining a successful result per signal state sent by Alice has the form $^{22}$

$$p_{\text{suc}} = 1 - |\langle \alpha | - \alpha \rangle| = 1 - \exp(-2\mu_\alpha), \quad (7)$$

where $\mu_\alpha$ is the mean photon-number of the signal states sent by Alice, i.e., $\mu_\alpha = |\alpha|^2$.

Next, we obtain an expression for the quantum bit error rate (QBER) introduced by Eve with this attack, and also for the resulting double click rate at Bob’s side.

**A. Quantum bit error rate**

The QBER is defined as $N_{\text{errors}}/N_{\text{clicks}}$, where $N_{\text{errors}}$ represents the average total number of errors obtained by Bob, and $N_{\text{clicks}}$ is again the average total number of clicks at Bob’s side. The parameter $N_{\text{errors}}$ can be expressed as $N_{\text{errors}} = (N/M)N_{\text{errors}}^M$, with $N_{\text{errors}}^M$ denoting the average total number of errors per block of length $M$. With this notation, the QBER of a sequential attack, that we shall denote as $Q$, can then be expressed as

$$Q = \frac{1}{M} \frac{N_{\text{errors}}^M}{N_{\text{errors}}^M}. \quad (8)$$

Next, we obtain an expression for $N_{\text{errors}}^M$. We shall distinguish the same cases as in the previous section, depending on the number $m$ of coherent states $|\beta e^{i\delta_1}\rangle, |\beta e^{i\delta_2}\rangle, \ldots, |\beta e^{i\delta_m}\rangle$ inside a block and their position in the block.

Whenever the previous signal of a coherent state inside the block is a coherent state, then no errors occur since both signals have the proper relative phase between them. On the contrary, if the previous signal of a coherent state is a vacuum state or if the previous signal of a vacuum state is a coherent state then it turns out that an error can happen with probability $\exp(-\mu_\beta/4)[1 - \exp(-\mu_\beta/4)] + [1 - \exp(-\mu_\beta/4)]^2/2 =$
the last signals of the block are vacuum states (Case D in Fig. 3). Then, if the last state of the previous block is a coherent state, the average total number of errors obtained by Bob is given by $3t/2$. Otherwise, the average total number of errors is $t$. Like in the previous section, the results for the cases E, F, and G, in Fig. 3 can be obtained directly from the cases B, C, and D, respectively. One only needs to multiply the a priori probabilities to be in each of these last three scenarios by the factor $q$. Finally, if the block contains only vacuum states (Case H in Fig. 3) and the last state of the previous block is a coherent state, then the average total number of errors is given by $t/2$. Otherwise, the average total number of errors is zero.

After adding all the terms together, and taking into account the a priori probabilities of each case, we obtain that the average total number of errors per block of length $M$ in a sequential USD attack has the following form

$$N^M_{\text{errors}} = tS,$$

where the parameter $S$ is given by

$$S = \frac{p}{2} + p_{\text{succ}}^M \left( \frac{1}{2} - p \right) + \sum_{M_{\text{min}} \leq m < M} q^{mM_{\text{min}}} (1 - p_{\text{succ}})^m \left[ \left( \frac{3}{2} - p \right) + (M - m - 1)(1 - p_{\text{succ}}) \right].$$

B. Double click rate

The double click rate at Bob’s side, that we shall denote as $D_c$, is typically defined as $D_c = N_{D_c}/N$, where $N_{D_c}$ refers to the average total number of double clicks obtained by Bob, and $N$ is again the total number of signal states sent by Alice. $N_{D_c}$ is given by $N_{D_c} = (N/M)N^M_{D_c}$, with $N^M_{D_c}$ denoting the average total number of double clicks per block sent by Eve at Bob’s side. The $D_c$ can be written as

$$D_c = \frac{1}{M}N^M_{D_c}.$$

In order to obtain an expression for $N^M_{D_c}$, we can again distinguish the same different cases included in Fig. 3. Double clicks can only occur when the previous signal of a coherent state is a vacuum state or when the previous signal of a vacuum state is a coherent state. The probability to obtain a double click in each of these two scenarios, that we shall denote as $d$, is given by

$$d = \left[ 1 - \exp \left( -\frac{\mu_3}{4} \right) \right]^2.$$  

Otherwise, the probability to have a double click is always zero. The analysis is then completely equivalent to the one included in Sec. [IVA] one only needs to substitute the parameter $t/2$ by $d$. We obtain, therefore, that the average total number of double clicks per block sent by Eve in a sequential USD attack can be written as

$$N^M_{D_e} = 2dS,$$

with $S$ given by Eq. (10).

C. Evaluation

We have seen above that a sequential USD attack can be parameterized by the block size $M$, the minimum number $M_{\text{min}}$ of consecutive successful results within a block that Eve needs to obtain in order to send Bob a new train of coherent states, the mean photon-number $\mu_3$ of these coherent states sent by Eve, and the value of the probability $q$, i.e., the probability that Eve actually decides to send $M_{\text{min}}$ coherent states in a block instead of only vacuum states.

Fig. 5 shows a graphical representation of the Gain versus the QBER in this attack for different values of the maximum tolerable double click rate at Bob’s side. In this example we consider that the mean photon number of Alice’s signal states is given by $\mu_3 = 0.16$. Moreover, we fix the value of $M_{\text{min}}$ as $M_{\text{min}} = \lfloor M/2 + 1 \rfloor$ and,
for each given values of the parameters $M$, $q \in [0, 1]$, and the maximum tolerable double click rate obtained by Bob, we perform a numerical optimization to find the optimal mean photon number $\mu_2$ for each case, i.e., the one that provides a lower QBER for a given value of the Gain. Fig. 5 also includes experimental data from Ref. [16]. According to these results we find that, unless Alice and Bob reject a double click rate as low as $10^{-8}$, the DPS QKD experiment reported in Ref. [16] would be insecure against a sequential USD attack. More precisely, our analysis suggest that in this kind of QKD protocols is not enough for Alice and Bob to include the effect of the double clicks obtained by Bob in the QBER [31], but it might be very useful for the legitimate users to monitor also the double click rate to guarantee security against a sequential attack. The authors of Ref. [16] already noticed in Ref. [17] that their experiment is not covered by the existing initial security analysis provided in Ref. [15]. Our result is strong as it also shows that when the double click rate at Bob’s side is above $10^{-8}$ no improved classical communication protocol or improved security analysis might allow the data of Ref. [16] to be turned into secret key.

Fig. 5 shows a graphical representation for the case where Alice and Bob do not monitor separately the double click rate and Eve can optimize the mean photon number $\mu_2$ for each given values of $M$, $M_{min} = \lfloor M/2 + 1 \rfloor$, and the parameter $q$, without any restriction on the maximum tolerable double click rate at Bob’s side.

A similar representation is plotted in Fig. 6, but now for the case $\mu_a = 0.2$ and for different values of the maximum double click rate at Bob’s side. In this figure we also include data from a recent experiment reported in Ref. [17], where the QBER was reduced to a value of only $0.03$. The scenario where Alice and Bob do not monitor separately the double click rate obtained by Bob is illustrated in Fig. 8. In both cases, our results are consistent with the possibility to create secret keys.

According to the figures presented in this section, whenever Eve tries to increase the Gain of this attack by reducing, for instance, the size $M$ of her blocks, she also increases the resulting QBER obtained by Bob. The maximum value of the Gain that Eve can achieve, however, is actually limited by the probability $p_{succ} = 1 - \exp(-2\mu_a)$ of obtaining a successful result when distinguishing unambiguously the states $|\pm\alpha\rangle$. Since,
by definition, \( |M/2 + 1| \leq M_{\text{min}} < M \), the minimum value of a valid block size \( M \) is given by \( M = 3 \). This means, in particular, that in order to maximize the Gain of a sequential USD attack the best choice for Eve is to select \( M = 3 \) and \( M_{\text{min}} = 2 \). Moreover, we can assume that Eve always sends to Bob \( M_{\text{min}} \) coherent states \( |\beta e^{i\theta_1} \rangle, |\beta e^{i\theta_2} \rangle, \ldots, |\beta e^{i\theta_{M_{\text{min}}}} \rangle \) when she obtains \( M_{\text{min}} \) successful results (i.e., \( q = 1 \)), and that these coherent states have a really high mean photon number such as she increases Bob’s probability of obtaining a click (i.e., \( \mu_\beta \gg 1 \) and, therefore, \( s \approx 1, t \approx 1, \) and \( d \approx 1 \)). Using these values in Eq. (11) and Eq. (12) we obtain that the maximum value of the Gain in this attack is given by

\[
G_{\text{max}} \approx \frac{1}{3} (6 - 2p_{\text{suc}}^2 - p_{\text{suc}}^2) p_{\text{suc}}^2. \tag{14}
\]

In this case the QBER, and the double click rate at Bob’s side are, respectively, given by \( Q \approx (2 - 2p_{\text{suc}} - p_{\text{suc}}^2)/(6 - 2p_{\text{suc}} - p_{\text{suc}}^2) \) and \( D_c \approx 2(2 - p_{\text{suc}} - p_{\text{suc}}^2) p_{\text{suc}}^2/3 \).

On the contrary, the minimum value of the Gain occurs when Eve treats the total number of signals \( N \) sent by Alice as a single block, i.e., \( M = N \), and she further imposes \( M_{\text{min}} = M - 1, q = 0, \) and \( s \approx 1 \). In this case, the minimum Gain is given by \( p_{\text{suc}}^N \), and the QBER and double click rate at Bob’s side are both zero. This scenario corresponds to the situation where Eve only sends \( N \) coherent states \( |\beta e^{i\theta_1} \rangle, |\beta e^{i\theta_2} \rangle, \ldots, |\beta e^{i\theta_N} \rangle \) to Bob when she succeeds discriminating without error all the signal states sent by Alice.

Finally, let us mention that, instead of using an USD measurement on each signal state sent by Alice, Eve could as well employ the same detection device like Bob. This sequential attack was very briefly introduced in Ref. [15]. In this case, a successful result is associated with obtaining a click in Eve’s apparatus, while a failure corresponds to the absence of a click. The train of coherent states \( |\beta e^{i\theta_1} \rangle, |\beta e^{i\theta_2} \rangle, \ldots, |\beta e^{i\theta_{M_{\text{min}}}} \rangle \) that Eve sends to Bob is now selected such as the relative phase between consecutive signals agree with Eve’s measurement results. If we assume that Eve does not analyze each block of data independently, but she also includes a proper relative phase between blocks when the last signal of a previous block and the first signal of the following one are coherent states, then the results included in this section also apply to that case. Otherwise, the QBER in such kind of attack will be always higher than in a sequential USD attack. However, since Alice’s signal states are typically coherent pulses with small average photon number (i.e., \( |\alpha|^2 \ll 1 \), Eve observes click events only occasionally. In particular, when she uses the same detection apparatus like Bob then the probability of obtaining a successful result will be always smaller than the one of a sequential USD attack. More precisely, this success probability has now the form \( p_{\text{suc}} = 1 - \exp(-\mu_\alpha) \), and is smaller than the success probability given by Eq. (11).

Fig. 9 shows a graphical representation of the Gain versus the QBER for a sequential USD attack together with a sequential attack where Eve employs the same detection apparatus like Bob. In this example the maximum tolerable double click rate at Bob’s side is \( D_{c} < 10^{-8} \) and the mean photon number of Alice’s signal states is \( \mu_\alpha = 0.16 \). Moreover, we fix again the value of \( M_{\text{min}} \) as \( M_{\text{min}} = |M/2 + 1| \) and, for each given values of the parameters \( M \) and \( q \in [0,1] \), we perform a numerical optimization to find the optimal \( \mu_\beta \) for each case like before. From the results included in Fig. 9 we see that a sequential USD attack can provide tighter upper bounds on the performance of DPS QKD schemes than a sequential attack with Eve employing the same detection device like Bob.
V. SEQUENTIAL MINIMUM ERROR DISCRIMINATION ATTACK

In this eavesdropping strategy Eve performs first a filtering operation on each signal state sent by Alice with the intention to make them, with some finite probability, more “distinguishable”. Afterwards, Eve measures out each successful filtered state with a measurement device that gives her the minimum value of the error probability when identifying the states \[24, 25\]. Her goal is to try to determine whether the filtered states originate from \(|\alpha\rangle\) or from \(|-\alpha\rangle\).

The coherent states sent by Alice can be expressed in some orthogonal basis \(\{|0\rangle, |1\rangle\}\) as follows:

\[
|\pm \alpha\rangle = a|0\rangle \pm b|1\rangle,
\]
where we assume, without of generality, that the coefficients \(a\) and \(b\) are given by

\[
a = \sqrt{\frac{1}{2}}[1 + \exp(-2\mu_\alpha)] \quad (16)
\]
\[
b = \sqrt{\frac{1}{2}}[1 - \exp(-2\mu_\alpha)],
\]

that is, they satisfy, \(a \in \mathbb{R}, b \in \mathbb{R}, a^2 + b^2 = 1,\) and \(a > b\) when \(\mu_\alpha \neq 0\).

We shall consider that Eve uses a filtering operation defined by the following two Kraus operators \[32\]:

\[
A_{\text{suc}}(\lambda) = \lambda|0\rangle\langle 0| + |1\rangle\langle 1|,
\]
\[
A_{\text{fail}}(\lambda) = \sqrt{1 - \lambda^2}|0\rangle\langle 0|,
\]

where the coefficient \(\lambda\) satisfies \(\lambda \in [b/a, 1]\). This parameter allows Eve to increase the probability of obtaining a successful result and, therefore, she can increase the Gain of her attack. On the other hand, Eve can introduce also more errors at Bob’s side.

Suppose that the filtering operation receives as input the state \(|\pm \alpha\rangle\). The probability of getting a successful result can be calculated as

\[
p_{\text{suc}}^\lambda \equiv p_{\text{suc}} = \text{Tr}(|\pm \alpha\rangle\langle \pm \alpha| A_{\text{suc}}^\lambda A_{\text{suc}}(\lambda)|\pm \alpha\rangle\langle 0|)
\]

This quantity is given by

\[
p_{\text{suc}}^\lambda = a^2\lambda^2 + b^2. \quad (21)
\]

If the filtering operation succeeded, the resulting normalized filtered state, that we shall denote as \(|\pm \alpha_{\text{suc}}\rangle\), can be calculated as

\[
|\pm \alpha_{\text{suc}}\rangle = \frac{1}{\sqrt{p_{\text{suc}}^\lambda}}(\lambda a|0\rangle \pm b|1\rangle).
\]

As already mentioned previously, in order to decide which signal state was used by Alice, we consider that Eve follows the approach of MED. That is, she employs a measurement strategy that guesses the identity of the signals \(|\pm \alpha_{\text{suc}}\rangle\) with the minimum probability of making an error. For the case of two pure states with equal a priori probabilities, like it is the case that we have here, the optimal value of the error probability, that we shall denote as \(p_{\text{err}}\), is given by

\[
p_{\text{err}} = \frac{1}{2} \left[a\lambda - b\right]^2 + \frac{2}{a^2\lambda^2 + b^2} \quad (23)
\]

The von Neumann measurement which can be used to attain this error probability is given by the optimum detector states \(|\pm\rangle = 1/\sqrt{2}|0\rangle \pm |1\rangle\).

Note that the sequential USD attack introduced in Sec. IV can then be seen as a special case of this sequential MED attack. When \(\lambda = b/a\), the success probability in a sequential MED attack is given by

\[
p_{\text{suc}} = 2b^2 = 1 - \exp(-2\mu_\alpha),
\]

which coincides with the success probability given by Eq. (7). Moreover, in this case the error probability \(p_{\text{err}}\) is zero.

Next, we obtain an expression for the QBER introduced by Eve with this attack, and also for the resulting double click rate at Bob’s side.

A. Quantum bit error rate

From Eq. (3) we learn that in order to obtain an expression for the QBER in a sequential attack we only need to find the average total number of errors \(N_{\text{err}}^M\) per block of length \(M\).

Now, however, the analysis is slightly different from that considered in Sec. IV A since two consecutive coherent states in a block can also produce errors. This arises from the fact that sometimes Eve does not identify correctly the signal states \(|\pm \alpha\rangle\) sent by Alice. In particular, whenever the previous signal of a coherent state inside a block is also a coherent state, then an error can occur with probability

\[
p_{\text{err}} = (1 - p_{\text{err}} + p_{\text{err}}(1 - p_{\text{err}}))\text{\ s},
\]

where \(p_{\text{err}}\) is given by Eq. (23) and \(s\) is given by Eq. (3). This is the probability that only one of the two coherent states is wrongly identify by Eve and Bob detects the error by means of a click in his apparatus. We shall denote this error probability as \(\tilde{p}_{\text{err}}\). Using Eq. (23), we can write \(\tilde{p}_{\text{err}}\) as

\[
\tilde{p}_{\text{err}} = \frac{1}{2} \left(a^2\lambda^2 - b^2\right)^2.
\]

If the previous signal of a coherent state is a vacuum state or if the previous signal of a vacuum state is a coherent state then the error probability is the same as in Sec. IV A i.e., it has the form \(t/2\) with \(t\) given by Eq. (4).

We can now address the different cases contained in Fig. 3 like in the previous sections and obtain an expression for \(N_{\text{err}}^M\) as a function of these two error proba-
bilities. The analysis is included in Appendix B. We find that \( N_{errors}^M \) can be written as

\[
N_{errors}^M = \frac{pt}{2} + p_{suc}^M \hat{w}_M + \sum_{M_{min} \leq m < M} q^{m}_{M_{min}} (1 - p_{suc}) p_{suc}^m \left[ \hat{v}_m + (M - m - 1)(1 - p_{suc}) \hat{w}_m \right],
\]

(25)

where the parameters \( \hat{u}_M, \hat{v}_m, \) and \( \hat{w}_m \), are given by

\[
\begin{align*}
\hat{u}_M &= \frac{1 - 2p}{2} t + (M - 1 + p) \hat{p}_{err}, \\
\hat{v}_m &= \frac{3 - 2p}{2} t + (2m + p - 2) \hat{p}_{err}, \\
\hat{w}_m &= t + (m - 1) \hat{p}_{err},
\end{align*}
\]

(26)

and with \( p \) given by Eq. (2).

**B. Double click rate**

Like in the case of a sequential USD attack, also in this attack double clicks can happen only when the previous signal of a coherent state is a vacuum state or when the previous signal of a vacuum state is a coherent state. The probability to obtain a double click in each of these two scenarios does not depend on the value of the phase \( \theta_i \) of the coherent state \( |\beta e^{i\theta_i}\rangle \) involved, but it depends only on the mean photon-number \( \mu_3 \). This means that the analysis included in Sec. [IVB] also applies here, and the average total number of double clicks per block sent by Eve in a sequential MED attack is also given by Eq. (13).

**C. Evaluation**

In Fig. 10 we plot the Gain versus the QBER in a sequential MED attack for a fix value of the maximum tolerable double click rate at Bob’s side \( (D_c < 10^{-8}) \) and for different values of the parameter \( \lambda \). Like in Sec. [IVC] we fix the value of \( M_{min} \) as \( M_{min} = \lfloor M/2 + 1 \rfloor \), and we perform a numerical optimization to find the optimal mean photon number \( \mu_3 \) for each given values of the parameters \( M, q, \) and \( \lambda \). Moreover, in this example, we consider that the mean photon number of Alice’s signal states is given by \( \mu_\alpha = 0.16 \) and we also include the experimental data obtained in Ref. [10].

A similar graphical representation is included in Fig. 11 but now for the case where Alice and Bob do not monitor separately the double click rate and Eve can optimize the mean photon number \( \mu_3 \) for each given values of \( M, M_{min} = \lfloor M/2 + 1 \rfloor, q, \) and the parameter \( \lambda \), without any restriction on the maximum tolerable double click rate at Bob’s side.

While in a sequential USD attack the maximum value of the Gain is given by Eq. (13), in a sequential MED attack Eve can always increase the value of the Gain at the expense of also increasing the resulting QBER at Bob’s side, just by incrementing the parameter \( \lambda \). In particular, in the limit case of \( \lambda = 1 \), i.e., the filtering operation is just the identity operation, we have that \( p = 1 \) and \( p_{\lambda = 1}^{suc} = 1 \). In this situation, the Gain, the QBER, and the double clock rate at Bob’s side are, respectively, given by \( G = 1 - \exp(-\mu_3), Q = \exp(-4\mu_\alpha)/2, \) and \( D_c = 0 \). That is, by selecting a proper mean photon number \( \mu_3 \) Eve can always access any high value of the Gain.

**VI. CONCLUSION**

In this paper we have quantitatively analyzed limitations on the performance of differential-phase-shift (DPS) quantum key distribution (QKD) protocols based on weak coherent pulses. For that, we have investigated simple eavesdropping strategies based on sequential attacks: Eve measures out every coherent state emitted by Alice and prepares new signal states, depending on the results obtained, that are given to Bob. Whenever Eve
obtains a predetermined number of consecutive successful measurement outcomes, then she prepares a train of new coherent pulses that is forwarded to Bob. Otherwise, Eve sends vacuum signals to Bob to avoid errors. Sequential attacks transform the original quantum channel between Alice and Bob into an entanglement breaking channel and, therefore, they do not allow the distribution of a secret key.

Specifically, we have considered two possible sequential attacks. In the first one, Eve realizes unambiguous state discrimination (USD) of Alice’s signal states. When Eve identifies unambiguously a signal state sent by Alice, then she considers this result as successful. Otherwise, she considers it a failure. In the second attack, Eve performs first a filtering operation on each signal emitted by Alice and, afterwards, she measures out each successful filtered state following the approach of minimum error discrimination, i.e., she guesses the identity of the filtered state with the minimum probability of making an error. As a result, we obtained upper bounds on the maximal distance achievable by differential-phase-shift quantum key distribution schemes as a function of the error rate in the sifted key, the double click rate at Bob’s side, and for different values of the parameter $\lambda$: $\lambda_1 = b/a$ (solid), $\lambda_2 = b/a + (1 - b/a)/5$ (dashed), $\lambda_3 = b/a + 2(1 - b/a)/5$ (dotted), $\lambda_4 = b/a + 3(1 - b/a)/5$ (dashed-dotted), $\lambda_5 = b/a + 4(1 - b/a)/5$ (thick solid), and $\lambda_6 = 1$ (thick dashed). The mean photon number of Alice’s signal states is $\mu_a = 0.16$. The triangle represents experimental data from Ref. [16].

![FIG. 11: Gain versus QBER in a sequential MED attack for the case where Alice and Bob do not monitor separately the double click rate obtained by Bob, and for different values of the parameter $\lambda$: $\lambda_1 = b/a$ (solid), $\lambda_2 = b/a + (1 - b/a)/5$ (dashed), $\lambda_3 = b/a + 2(1 - b/a)/5$ (dotted), $\lambda_4 = b/a + 3(1 - b/a)/5$ (dashed-dotted), $\lambda_5 = b/a + 4(1 - b/a)/5$ (thick solid), and $\lambda_6 = 1$ (thick dashed). The mean photon number of Alice’s signal states is $\mu_a = 0.16$. The triangle represents experimental data from Ref. [16].](image-url)

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Appendix A: Probability $p$

In this Appendix we obtain an expression for the probability $p$ that the last signal in a given block is a coherent state $|\beta e^{i\theta}\rangle$.

Let $p_m$ be the probability of Eve sending to Bob $m$ consecutive coherent states within a block of length $M$ such that the last signal of the block is a coherent state. This probability is given by

$$p_m = \begin{cases} 0 & \text{if } m < M_{\text{min}} \\ q (1 - p_{\text{suc}}) p_{\text{suc}}^M & \text{if } m = M_{\text{min}} \\ (1 - p_{\text{suc}}) p_{\text{suc}}^m & \text{if } M_{\text{min}} < m < M \\ p_{\text{suc}}^m & \text{if } m = M. \end{cases}$$ (A1)

For each given block of signals that Eve sends to Bob we
have, therefore, that $p$ can be written as

$$
P = \sum_{m=M_{\text{min}}}^{M} p_m = [p_{\text{succ}} + (1 - p_{\text{succ}})q]p_{M_{\text{min}}}^{M_{\text{min}}}. \tag{A2}
$$

Similarly, $1 - p$ represents the probability that the last signal in a block is a vacuum state.

**Appendix B:** $N_{\text{errors}}^M$ in a sequential minimum error discrimination attack

In this Appendix we obtain an expression for the average total number of errors $N_{\text{errors}}^M$ per block of length $M$ sent by Eve in a sequential MED attack.

We shall distinguish the different cases included in Fig. 3, i.e., as a function of the number $m$ of coherent states inside a block and their position in the block.

Let us begin with Case A in Fig. 3. According to Sec. 4A whenever the last signal state of the previous block is a coherent state then the average total number of errors obtained by Bob is given by $M\tilde{p}_{\text{err}}$. Otherwise, it is given by $(M - 1)\tilde{p}_{\text{err}} + t/2$. If the first $m \in (M_{\text{min}}, M)$ signal states of the block are coherent states (Case B in Fig. 3) and the last state of the previous block is also a coherent state, then the average total number of errors obtained by Bob is given by $m\tilde{p}_{\text{err}} + t/2$. Otherwise, the average total number of errors is $(m - 1)\tilde{p}_{\text{err}} + t$.

The results for the cases E, F, and G, in Fig. 3 can be obtained directly from the cases B, C, and D, respectively. One only needs to multiply the a priori probabilities to be in each of these last three scenarios by the factor $q$.

Finally, whenever the block that Eve sends to Bob contains only vacuum states (Case H in Fig. 3) and the last signal of the previous block is a coherent state, then the average total number of errors is given by $t/2$. Otherwise, the average total number of clicks is zero.

After including all the a priori probabilities to be in each of the different cases discussed above, we obtain that the average total number of errors per block of length $M$ in a sequential MED attack is given by Eq. (25).

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