Isolating the Penguin-diagram Contribution to \( CP \) Violation in \( B_{d}^{0} \) vs \( B_{d}^{0} \rightarrow \pi^{+}\pi^{-} \)

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Abstract

A reliable prediction for \( CP \) violation in \( B_{d}^{0} \) vs \( B_{d}^{0} \rightarrow \pi^{+}\pi^{-} \) suffers from the penguin-diagram induced uncertainty. With the help of SU(3) relations, we show that both the magnitude and strong phase shift of the penguin amplitude can be approximately determined only from the branching ratios of \( B_{d}^{0} \rightarrow \pi^{+}\pi^{-} \), \( B_{d}^{0} \rightarrow K^{+}\pi^{-} \), and \( B_{u}^{+} \rightarrow K^{0}\pi^{+} \).

PACS numbers: 11.30.Er, 12.15.Ff, 13.25.Hw

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In addition to $B^0_d \to J/\psi K_S$, $B^0_d \to \pi^+\pi^-$ is another very promising process for the study of $CP$ violation in neutral $B$ decays [1]. However, the penguin-diagram induced effect on $B^0_d \to \pi^+\pi^-$ vs $\bar{B}^0_d \to \pi^+\pi^-$ may be quite large so that a reliable prediction for the $CP$-violating asymmetry between them is difficult [2,3]. It has been shown that the penguin contributions to neutral $B$ decays into $CP$ eigenstates can in principle be rigorously determined with detailed studies of the time dependence of several correlative decay modes [4,5]. Long before such measurements are carried out at asymmetric $e^+e^-$ $B$ factories or hadron machines, it is very useful at present to pursue an approximate way to probe the penguin effect on $CP$ violation by using the accessible (time-independent) data of $B$-meson decays.

In Ref. [6] Silva and Wolfenstein have applied SU(3) symmetry to $B^0_d \to \pi^+\pi^-$ and $B^0_d \to K^+\pi^-$ as a probe of the penguin contribution. Neglecting the final-state interactions which generally produce a strong phase difference between the penguin and tree amplitudes, they found that the magnitude of the penguin amplitude could be approximately determined from the ratio of the decay rates of these two processes. A more general and complete application of SU(3) symmetry to $B$ decays into two pseudoscalar mesons has recently been given by Gronau et al [7]. With some triangle relations among the decay amplitudes of $B \to \pi\pi, \pi K$, and $KK$, they explored various possibilities to extract the weak Cabibbo-Kobayashi-Maskawa (CKM) phases and the strong final-state phase shifts only through the time-independent measurements of decay rates. In this letter, we shall concentrate on the penguin-induced strong interactions in the SU(3)-related transitions $B^0_d \to \pi^+\pi^-$, $B^0_d \to K^+\pi^-$, and $B^+_u \to K^0\pi^+$. We show that both the relative magnitude and strong phase shift between the penguin and tree amplitudes are determinable only if the branching ratios of these decays are measured. The signals of direct and indirect $CP$ violation in $B^0_d$ vs $\bar{B}^0_d \to \pi^+\pi^-$ can be consequently determined without observing the time dependence of decay rates. Using the effective weak Hamiltonian and factorization approximation, we estimate SU(3) breaking in the tree and penguin amplitudes for decays of the types $B \to \pi\pi$ and $B \to K\pi$.

We begin with a graphical description for the decay modes in question. To the lowest-order weak interactions in the standard model, any $B$ decays into two light mesons can be described using a set of ten topologically distinct quark diagrams [8]. As shown in Fig. 1, the diagrams $n$ and $n'$ (with $n = 1, \cdots, 5$) are different from each other in the final-state
hadronization of the valence quarks. The overall amplitude of a decay mode \( B \to f \) is expressed in terms of the quark-diagram amplitudes, \( A_n \) and \( A_{n'} \), as

\[
A(B \to f) = \sum_{n=1}^{5} C_n A_n(f) + \sum_{n'=1}^{5} C_{n'} A_{n'}(f),
\]

where \( C_n \) and \( C_{n'} \) are real coefficients. Adopting the usual valence-quark conventions for the SU(3) mesons [9], we obtain from Fig. 1 that

\[
A(B^0_d \to \pi^+\pi^-) = A_1(\pi^+\pi^-) + A_2(\pi^+\pi^-) + A_4(\pi^+\pi^-) + A_5(\pi^+\pi^-),
\]

\[
A(B^+_u \to K^0\pi^+) = A_3(K^0\pi^+) + A_4(K^0\pi^+),
\]

\[
A(B^0_d \to K^+\pi^-) = A_1(K^+\pi^-) + A_4(K^+\pi^-).
\]

Subsequently we neglect the amplitudes \( A_2(\pi^+\pi^-) \), \( A_5(\pi^+\pi^-) \), and \( A_3(K^0\pi^+) \), which have been argued to be formfactor- or helicity-suppressed in comparison with those from the spectator-type diagrams 1 and 4 [10]. In addition, we assume that the penguin amplitudes \( A_4(\pi^+\pi^-) \) and \( A_4(K^0\pi^+) \) (or \( A_4(K^+\pi^-) \)) are dominated by the top-quark loop and therefore have the weak phases \( \text{arg}(V_{td}^* V_{tb}) \) and \( \text{arg}(V_{ts}^* V_{tb}) \), respectively. The approach to test these two approximations has been suggested in Ref. [7]. Factoring out the weak-interaction part (i.e., the CKM matrix elements), the decay amplitudes in Eq. (2) are approximately given by

\[
A(B^0_d \to \pi^+\pi^-) = A \lambda \left[ r e^{i\gamma} \tilde{A}_1(\pi^+\pi^-) + s e^{-i\beta} \tilde{A}_4(\pi^+\pi^-) \right],
\]

\[
A(B^+_u \to K^0\pi^+) = -A \lambda^2 \tilde{A}_4(K^0\pi^+),
\]

\[
A(B^0_d \to K^+\pi^-) = A \lambda^2 \left[ r e^{i\gamma} \tilde{A}_1(K^+\pi^-) - \tilde{A}_4(K^+\pi^-) \right],
\]

where

\[
r = \lambda^2 \sqrt{\rho^2 + \eta^2}, \quad s = \lambda^2 \sqrt{(1-\rho)^2 + \eta^2},
\]

\[
\beta = \text{arctan} \left( \frac{\eta}{\rho} \right), \quad \gamma = \text{arctan} \left( \frac{\eta}{1-\rho} \right).
\]

In Eqs. (3) and (4), \( A, \lambda, \rho, \) and \( \eta \) are the Wolfenstein parameters of the CKM matrix. The phases \( \beta \) and \( \gamma \) correspond to two angles of the CKM unitarity triangle.

With isospin symmetry, one obtains \( \tilde{A}_4(K^0\pi^+) = \tilde{A}_4(K^+\pi^-) \). The SU(3) invariance indicates \( \tilde{A}_1(\pi^+\pi^-) = \tilde{A}_1(K^+\pi^-) \) and \( \tilde{A}_4(\pi^+\pi^-) = \tilde{A}_4(K^+\pi^-) \). In practice, one should take into account SU(3) violation for the \( B \to \pi\pi \) and \( B \to K\pi \) transitions. Let us estimate the
SU(3) breaking effect on the above reduced amplitudes using the factorization approximation. The effective weak Hamiltonian responsible for the transitions $\bar{b} \to (\bar{u}u)\bar{q}$ (with $q = d$ or $s$) is [11]

$$\mathcal{H}_{\text{eff}}(\Delta B = +1) = \frac{G_F}{\sqrt{2}} \left[ V_{ub}^* V_{us} (\bar{c}_1 Q_1^d + \bar{c}_2 Q_2^d) - V_{td}^* V_{ts} \left( \sum_{i=3}^{6} \bar{c}_i Q_i^d \right) \right], \quad (5)$$

where $\bar{c}_i$ ($i = 1, \cdots, 6$) are the Wilson coefficients, and $Q_i^d$ form an operator basis defined by

$$Q_1^d = (\bar{b}_u u_\beta)_{V-A} (\bar{u}_\alpha q_\alpha)_{V-A}, \quad Q_2^d = (\bar{b}_u u_\alpha)_{V-A} (\bar{u}_\beta q_\beta)_{V-A},$$

$$Q_3^d = (\bar{b}_q q_\alpha)_{V-A} \sum_{q'} (\bar{q'}_\alpha q')_{V-A}, \quad Q_4^d = (\bar{b}_\alpha q_\beta)_{V-A} \sum_{q'} (\bar{q'}_\beta q')_{V-A},$$

$$Q_5^d = (\bar{b}_q q_\alpha)_{V-A} \sum_{q'} (\bar{q'}_\beta q')_{V-A}, \quad Q_6^d = (\bar{b}_\alpha q_\beta)_{V-A} \sum_{q'} (\bar{q'}_\beta q')_{V-A}. \quad (6)$$

In the factorization approximation, we obtain

$$\frac{\tilde{A}_1(K^+\pi^-)}{A_1(\pi^+\pi^-)} = \frac{\langle K^+\pi^- | \bar{c}_1 Q_1^d + \bar{c}_2 Q_2^d | B_d^0 \rangle}{\langle \pi^+\pi^- | \bar{c}_1 Q_1^d + \bar{c}_2 Q_2^d | B_d^0 \rangle} = \frac{f_K}{f_\pi}, \quad (7)$$

$$\frac{\tilde{A}_4(K^+\pi^-)}{A_4(\pi^+\pi^-)} = \frac{\langle K^+\pi^- | \sum_{i=3}^{6} \bar{c}_i Q_i^d | B_d^0 \rangle}{\langle \pi^+\pi^- | \sum_{i=3}^{6} \bar{c}_i Q_i^d | B_d^0 \rangle} = \mathcal{C} \frac{f_K}{f_\pi},$$

where the factor

$$\mathcal{C} = \left( \bar{c}_3 N_c + \bar{c}_4 \right) + \left( \bar{c}_5 N_c + \bar{c}_6 \right) \frac{2m_K^2}{(m_u + m_s)m_b}$$

$$\left( \bar{c}_3 N_c + \bar{c}_4 \right) + \left( \bar{c}_5 N_c + \bar{c}_6 \right) \frac{2m_\pi^2}{(m_u + m_s)m_b} \quad (8)$$

arises from transforming the $(V - A)(V + A)$ currents of $Q_{5,6}^g$ into the $(V - A)(V - A)$ ones. Following the approximate rule of discarding $1/N_c$ corrections in nonleptonic exclusive $B$ decays [12], we estimate $\mathcal{C}$ using $m_u = 5$ MeV, $m_d = 10$ MeV, $m_s = 175$ MeV, $m_b = 4.8$ GeV, $\bar{c}_4 = -0.034$, and $\bar{c}_6 = -0.042$ [11]. It turns out that $\mathcal{C} \approx 1.0$. In addition, we have $f_K/f_\pi \approx 1.2$ from the existing experimental data [9]. These two numbers serve as an illustration of the magnitude of SU(3) violation in $B_d^0 \to \pi^+\pi^-$ and $B_d^0 \to K^+\pi^-$. The penguin effect on $B_d^0 \to \pi^+\pi^-$ can be described by the modulus and phase of the ratio of $\tilde{A}_4(\pi^+\pi^-)$ and $\tilde{A}_1(\pi^+\pi^-)$:

$$\chi = \left| \frac{\tilde{A}_4(\pi^+\pi^-)}{\tilde{A}_1(\pi^+\pi^-)} \right|, \quad \theta = \arg \left[ \frac{\tilde{A}_4(\pi^+\pi^-)}{\tilde{A}_1(\pi^+\pi^-)} \right]. \quad (9)$$
Using the SU(3) relations given in Eq. (7), we connect $\chi$ and $\theta$ to the following two observables:

$$R_1 = \frac{|A(B_d^0 \to K^+\pi^-)|^2}{|A(B_d^+ \to K^0\pi^+)|^2}, \quad R_2 = \frac{|A(B_d^0 \to \pi^+\pi^-)|^2}{|A(B_d^0 \to K^0\pi^+)|^2}. \quad (10)$$

$R_1$ and $R_2$ can be determined from the branching ratios of the three decay modes in question. We find

$$R_1 = 1 + \frac{r^2}{C^2\chi^2} - \frac{2r}{C\chi} \cos(\gamma - \theta),$$

$$R_2 = \frac{s^2}{\chi^2C^2} \left( \frac{f_\pi}{f_K} \right)^2 \left[ 1 + \frac{r^2}{s^2\chi^2} + \frac{2r}{s\chi} \cos(\beta + \gamma - \theta) \right]. \quad (11)$$

The weak phases $\beta$ and $\gamma$ are related to the real parameters $r$ and $s$ (or $\rho$ and $\eta$) through Eq. (4). A precise measurement of $\epsilon_K$ (the $CP$-violating parameter in the $K^0 - \bar{K}^0$ system) and $B_d^0 - \bar{B}_d^0$ mixing may fix the values of $\beta$ and $\gamma$. Independently these two phases can be measured in some distinct nonleptonic decays of $B$ mesons. For instance, it is possible to extract $\beta$ from the $CP$ asymmetry in $B_d^0$ vs $\bar{B}_d^0 \to J/\psi K_S$ [1] and $\gamma$ from the decay rates of $B_u^\pm \to K^\pm D^0, K^\pm \bar{D}^0$, and $K^\pm D_{CP}$ [13]. Assuming $\beta$ and $\gamma$ are determined and $R_1$ and $R_2$ are measured, we may use Eq. (11) to probe the strong-interaction parameters $\chi$ and $\theta$. Keeping only the positive $\chi$, the value of $\theta$ will be determinable with only a two-fold discrete ambiguity.

We proceed to discuss the penguin effect on the $CP$-violating asymmetry in $B_d^0$ vs $\bar{B}_d^0 \to \pi^+\pi^-$. For either time-dependent or time-integrated measurements, $CP$ violation is signified by the following two observables [5]:

$$T_{\pi^+\pi^-} = \frac{1 - |\zeta_{\pi^+\pi^-}|^2}{1 + |\zeta_{\pi^+\pi^-}|^2}, \quad T'_{\pi^+\pi^-} = \frac{-2\text{Im}(e^{-2i\beta}\zeta_{\pi^+\pi^-})}{1 + |\zeta_{\pi^+\pi^-}|^2}, \quad (12)$$

where $\zeta_{\pi^+\pi^-} = A(B_d^0 \to \pi^+\pi^-)/A(B_d^0 \to \pi^+\pi^-)$. The nonvanishing $T_{\pi^+\pi^-}$ and $T'_{\pi^+\pi^-}$ imply direct $CP$ violation in the decay amplitude and indirect $CP$ violation from the interference between decay and $B_d^0 - \bar{B}_d^0$ mixing, respectively. With the help of Eqs. (7) and (11), we obtain

$$|\zeta_{\pi^+\pi^-}|^2 = \frac{s^2}{\chi^2C^2R_2} \left( \frac{f_\pi}{f_K} \right)^2 \left[ 1 + \frac{r^2}{s^2\chi^2} + \frac{2r}{s\chi} \cos(\beta + \gamma + \theta) \right],$$

$$\text{Im}(e^{-2i\beta}\zeta_{\pi^+\pi^-}) = -\frac{r^2}{\chi^2C^2s\chi^2R_2} \left( \frac{f_\pi}{f_K} \right)^2 \left[ \sin 2(\beta + \gamma) + \frac{2s\chi}{r} \sin(\beta + \gamma) \cos \theta \right]. \quad (13)$$

One can observe that the $\sin 2(\beta + \gamma)$ term in $T'_{\pi^+\pi^-}$ is maximally corrected by the penguin contribution in the case of $\theta = 0$ or $\pm \pi$. 5
For illustration, we give a numerical estimate of the decay-rate ratios \( R_1 \) and \( R_2 \) and the \( CP \)-violating observables \( (T_{\pi^+\pi^-} \) and \( T'_{\pi^+\pi^-} \) as functions of the penguin-amplitude parameters \( \chi \) and \( \theta \). The Wolfenstein parameters are taken as \( A = 0.86, \lambda = 0.22, \rho = 0.14, \) and \( \eta = 0.36 \), which are consistent with the current data on \( \epsilon_K \) and \( B^0_d - \bar{B}^0_d \) mixing [14]. The results are shown in Figs. 2 and 3. One observes that \( \chi \) and \( \theta \) can be well determined from \( R_1 \) and \( R_2 \) if \( \chi \) is in the range \( 0.05 \leq \chi \leq 0.3 \). On the other hand, a nonvanishing \( \theta \) is crucial for direct \( CP \) violation \( (T_{\pi^+\pi^-}) \) and has significant influence on indirect \( CP \) violation \( (T'_{\pi^+\pi^-}) \). It should be noted that in Fig. 3 the values of \( T_{\pi^+\pi^-} \) and \( T'_{\pi^+\pi^-} \) depend sensitively upon the input of the CKM matrix parameters.

The combined branching ratio of \( B^0_d \to \pi^+\pi^- \) and \( B^0_d \to K^+\pi^- \) has been measured to be \( (2.4^{+0.8}_{-0.7} \pm 0.2) \times 10^{-5} \) [15]. It is most likely that the decay rates of these two processes are of the same order [7]. Since the tree amplitude of \( B^0_d \to K^+\pi^- \) is \( \lambda^2 \)-suppressed in comparison with the penguin amplitude, one expects that the latter dominates the decay. Accordingly the branching ratio of \( B^+_u \to K^0\pi^+ \) should also be of the order \( 10^{-5} \). With this level of decay rates the above three transitions will soon be measurable at current \( e^+e^- \) colliders or hadron machines. It is therefore possible to obtain some useful information on \( CP \) violation in \( B^0_d \) vs \( \bar{B}^0_d \to \pi^+\pi^- \), long before the time-dependent measurements are carried out at the future asymmetric \( B \) factories.

One can in principle apply SU(3) symmetry to other \( B \to \pi\pi \) and \( B \to K\pi \) (or \( B \to KK \) transitions in order to probe the weak and strong phases only from measurements of the decay rates [7]. For example, the penguin effect in \( B^0_d \) vs \( \bar{B}^0_d \to \pi^+\pi^- \) may also be determined by studying \( B^0_d \to \pi^+\pi^- \), \( B^+_u \to \pi^+\pi^0 \), and \( B^+_u \to K^+\pi^0 \). The tree amplitudes of the latter two modes get non-negligible contributions from the color-suppressed diagrams 1', hence we should take account of SU(3) breaking in the reduced amplitudes \( \tilde{A}_{1'}(\pi^+\pi^0) \) and \( \tilde{A}_{1'}(K^+\pi^0) \). In the factorization approximation, one finds

\[
\frac{\tilde{A}_{1'}(K^+\pi^0)}{A_{1'}(\pi^+\pi^0)} = \frac{\langle K^+\pi^0|\bar{c}_1Q_1^+ + \bar{c}_2Q_2^+|B_u^+\rangle}{\langle \pi^+\pi^0|\bar{c}_1Q_1^+ + \bar{c}_2Q_2^+|B_u^+\rangle} \approx \frac{F^{BK}_0(m^2)}{F^{B\pi}_0(m^2)} \approx \frac{F^{BK}_0(m^2)}{F^{B\pi}_0(m^2)} ,
\]

\[
\frac{\tilde{A}_{1'}(\pi^+\pi^0)}{A_{1'}(\pi^+\pi^0)} = \frac{\bar{c}_1}{\bar{c}_2} \ \text{or} \ \frac{\bar{c}_2}{\bar{c}_1} ,
\]

where the 1/\( N_c \) corrections have been discarded [12]. Using \( F^{BK}_0(0) = 0.379 \) and \( F^{B\pi}_0(0) = \)

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2 Within the standard model, several rough estimates have given \( \chi \sim 0.1 \) (see, e.g., Refs. [2,3]).
as well as $\bar{c}_1 = -0.291$ and $\bar{c}_2 = 1.133$, we obtain $F_{BK}^B(m^2_{\pi})/F_{0}^B(m^2_{\pi}) \approx 1.1$ and $\bar{c}_1/\bar{c}_2 \approx -0.26$. It is expected that the magnitudes of $\tilde{A}_1$ and $\tilde{A}_4$ are comparable in either $B_u^+ \to \pi^+\pi^0$ or $B_u^+ \to K^+\pi^0$. Our estimates in Eqs. (7) and (14) indicate that SU(3) breaking in the factorizable amplitudes $\tilde{A}_1$, $\tilde{A}_1'$, and $\tilde{A}_4$ seems to be comparable. If this is true, the SU(3) approach used here and in Refs. [6,7] should be quite practical for investigating $CP$ violation and final-state interactions in decays of the types $B \to \pi\pi, K\pi$, and $KK$.

With the help of SU(3) relations, we have shown that the penguin effect on $CP$ violation in $B_d^0$ vs $\bar{B}_d^0 \to \pi^+\pi^-$ can be approximately determined only from measurements of the decay rates of $B_d^0 \to \pi^+\pi^-$, $B_d^0 \to K^+\pi^-$, and $B_u^+ \to K^0\pi^+$. These three transitions are becoming accessible in the experiments of $B$ physics. Some other charmless two-body decays of $B$ mesons, if they are favored for experimental observation, are worth studying in a similar way.

I would like to thank Professor H. Fritzsch for his hospitality and encouragements. I am also grateful to Professor D. Wyler for calling my attention to the topic under discussion. I finally acknowledge the financial support from the Alexander-von-Humboldt Foundation.
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Figure 1: Possible quark diagrams for a $B$ meson decaying into two light mesons.
Figure 2: The ratios of decay rates $R_1$ and $R_2$ as functions of the penguin-amplitude parameters $\chi$ and $\theta$. 
Figure 3: The $CP$-violating observables $T_{\pi^+\pi^-}$ and $T'_{\pi^+\pi^-}$ as functions of the penguin-amplitude parameters $\chi$ and $\theta$. 