The tension between gauge coupling unification, the Higgs boson mass, and a gauge-breaking origin of the supersymmetric $\mu$-term

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We investigate the possibility of generating the $\mu$-term in the MSSM by the condensation of a field that is a singlet under the SM gauge group but charged under an additional family-independent $U(1)_X$ gauge symmetry. We attempt to do so while preserving the gauge coupling unification of the MSSM. For this, we find that SM non-singlet exotics must be present in the spectrum. We also prove that the pure $U(1)_X$ anomalies can always be solved with rationally charged fields, but that a large number of SM singlets are often required. For $U(1)_X$ charges that are consistent with an embedding of the MSSM in $SU(5)$ or $SO(10)$, we show that the $U(1)_X$ charges of the MSSM states can always be expressed as a linear combination of abelian subgroups of $E_6$. However, the SM exotics do not appear to have a straightforward embedding into GUT multiplets. We conclude from this study that if this approach to the $\mu$-term is correct, as experiment can probe, it will necessarily complicate the standard picture of supersymmetric grand unification.

I. INTRODUCTION

The minimal supersymmetric standard model (MSSM) has two problems associated with its Higgs boson sector. The first has to do with the $\mu$-term in the superpotential, $W_{\text{MSSM}} \supseteq \mu H_d \cdot H_u$, which must be of order the electroweak scale, $\mu \sim v = 174$ GeV, for electroweak symmetry breaking to occur at the correct scale in a natural way. This is also true of the scale of supersymmetry (SUSY) breaking. However, from the low-energy point-of-view, $\mu$ and the SUSY breaking scale are independent of each other. Even though both scales are stable under quantum corrections because of supersymmetry, it is curious that their numerical values should be so close to each other, or why $\mu$ is so much smaller than the seemingly more fundamental scales $M_{\text{GUT}}$ or $M_{\text{Pl}}$.

The second problem is related to the first. At tree-level, the mass of the lightest CP-even MSSM Higgs boson is bounded by the mass of the $Z^0$ gauge boson, $m_h^2 \leq M_Z^2 \cos^2 \beta$. This is considerably less than the current experimental bound from LEP, $m_h \gtrsim 114$ GeV, for a SM-like $h^0$. The experimental bound on the Higgs mass does not rule out the MSSM because this quantity can receive large radiative corrections, especially from a heavy scalar top, provided the SUSY breaking scale is larger than about $M_{\text{SUSY}} \gtrsim 1$ TeV. But, since the electroweak scale is related (schematically) to $\mu$ and $M_{\text{SUSY}}$ by the relation $v^2 \sim |\mu|^2 - M_{\text{SUSY}}^2$, such a large SUSY breaking scale requires a fine-tuning of $\mu$ at the percent level. Again, this fine-tuning is not disastrous, but it is not terribly appealing in a model that was motivated by naturalness in the first place.

One way to avoid both of these problems is to add a singlet chiral superfield to the MSSM. With a singlet, the $\mu$-term in the superpotential can be replaced by $W \supseteq \lambda S H_d \cdot H_u$, giving $\mu_{\text{eff}} = \lambda \langle S \rangle$. Since the vacuum expectation value (VEV) of the singlet scalar is largely determined by the associated soft SUSY breaking terms, this replacement relates $\mu_{\text{eff}}$ to $M_{\text{SUSY}}$, and thus explains the coincidence of these scales. The singlet term also helps to remove the fine-tuning required by the Higgs mass bound because it generates an additional $F$-term contribution to the Higgs mass. The tree-level bound now becomes

$$m_h^2 \leq M_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta.$$  \hspace{1cm} (1)

The value of the $\lambda$ coupling runs large in the UV, and must be less than about 0.7 if it is to remain perturbatively small up to $M_{\text{GUT}}$. Even so, the tree-level bound on the SM-like Higgs boson can be increased to about 110 GeV for $\lambda$ as large as possible and $\tan \beta \approx 2$, substantially ameliorating the Higgs mass problem of the MSSM.

Unfortunately, adding a singlet to the MSSM can create new problems. In models of softly-broken supersymmetry derived from supergravity, both the scalar and $F$-term components of the singlet develop large expectation values due to tadpole loops involving higher-dimensional, $M_{\text{Pl}}$-suppressed operators. These VEV’s are proportional to positive powers of the cutoff scale and have the effect of destabilizing the electroweak scale. This outcome may be avoided by introducing a symmetry under which the singlet is charged.

Such a symmetry may be discrete or continuous, global or gauged, but will necessarily be broken when the would-be singlet field $S$ develops a VEV on the order of the electroweak scale. Due to this breaking, ungauged symmetries are problematic: continuous global symmetries can generate troublesome axions; discrete symmetries often lead to cosmologically unacceptable domain walls in
the early universe\(^7\). Moreover, attempts to get around these problems often lead to new ones. For example, domain walls can be avoided by weakly breaking the discrete symmetry with operators of dimension greater than four, but this reintroduces the danger of destabilizing the electroweak scale by singlet tadpole loops. In particular, within the next-to-minimal supersymmetric standard model (NMSSM), a popular singlet model based on a discrete \(Z_3\) symmetry\(^6\), it is not possible to avoid the domain wall problem while maintaining a stable gauge hierarchy \(^1,1\). In light of these problems faced by global symmetries, we are led to consider gauge symmetries as a way to protect the \(S\) field from large quantum corrections.

The simplest choice for such a symmetry, and the one we shall consider in the present work, is a \(U(1)_X\) gauge symmetry in addition to the \(G_{SM} = SU(3) \times SU(2) \times U(1)_Y\) gauge symmetry of the MSSM\(^{10,11,12,13}\). The choice of a \(U(1)_X\) is also attractive because its lone gauge boson will be massive after symmetry breaking. Moreover, by protecting the \(S\) field with a gauge symmetry, there is an additional \(D\)-term contribution to the Higgs mass which further increases the tree-level mass bound,

\[
m_h^2 \leq M_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta + 2g^2 v^2 (h_u \cos^2 \beta + h_d \sin^2 \beta)^2,\tag{2}
\]

where \(g_x\) is the \(U(1)_X\) gauge coupling, and \(h_u\) and \(h_d\) are the charges of the \(H_u\) and \(H_d\) fields under this group.

In extending the MSSM to include a gauged \(U(1)_X\) and new particles, we would also like to preserve one of the particularly attractive features of the MSSM, namely the consistency of the model with grand unification. Within the MSSM, the \(G_{SM}\) gauge couplings unify at a high energy, of order \(10^{16}\) GeV\(^14\), where the deviation from exact unification is small enough that it could plausibly be explained by high-scale threshold corrections. Furthermore, the matter fields of the MSSM fit neatly into \(\mathbf{5}\) and \(\mathbf{10}\) multiplets of \(SU(5)\)\(^12\). These features suggest that the low-energy gauge structure follows from a grand-unified theory (GUT). However, in adding a \(U(1)_X\) gauge symmetry, we must ensure that the corresponding anomalies vanish, and this generally entails adding additional fields that could potentially ruin the unification of gauge couplings.

In the present work, we shall investigate whether it is possible to extend the gauge group of the MSSM to include a gauged \(U(1)_X\) under which the \(S\) field responsible for generating the \(\mu\)-term has a non-zero charge, while maintaining gauge coupling unification. In doing so we will make the following assumptions:

1. All the terms present in the MSSM superpotential appear in the superpotential of the extended model.
2. The \(U(1)_X\) charges of the MSSM matter fields are family-universal. (This is related to point 1, when assuming an unrestricted form of the Yukawa couplings in the MSSM superpotential.)
3. The exotic matter needed to cancel the \(U(1)_X\) anomalies consists either of \(G_{SM}\) singlets, or of complete \(SU(5)\) multiplets (with the usual \(G_{SM} \subset SU(5)\) embedding).
4. The full set of exotic matter is vector-like in its \(G_{SM}\) representation.

The first condition ensures that the model reproduces the correct low-energy physics, while the second prevents the emergence of possibly dangerous flavor-mixing effects. The third condition implies that the extended model will preserve gauge coupling unification. However, we will allow the states within each \(SU(5)\) multiplet to have different \(U(1)_X\) charges. We will also demand that this unification occur in a perturbative regime, the implications of which will be discussed below. The fourth condition ensures that the exotics will not induce SM anomalies, or generate overly large corrections to the precision electroweak observables.

We will begin our analysis by classifying the different possibilities for the \(U(1)_X\) symmetry, when acting on the MSSM fields, in Section \(\S\). In Section \(\S\) we shall consider the addition of pure \(G_{SM}\) singlets to the model. With only these fields, we find that the \(G_{SM}\) singlet field \(S\) responsible for generating the effective \(\mu\)-term must also be a singlet under \(U(1)_X\). We are thus forced to look at more complicated extensions involving additional \(G_{SM}\) non-singlet matter. The general implications of this matter on gauge coupling unification will be the subject of Section \(\S\). In Section \(\S\) we will examine the implications of anomaly cancellation on the possible \(U(1)_X\) charges of the MSSM fields as well as the exotics. In Section \(\S\) we shall put our results to use by constructing a concrete model. Finally, Section \(\S\) is reserved for our conclusions. Some technical details and a list of \(E_6\) charges are given in a pair of Appendices.

Finally, we note that an investigation similar to the present one has been performed in Ref.\(^12\). However, compared to this work, our starting assumptions and therefore our final conclusions are rather different.

## II. CLASSIFICATION OF POSSIBLE \(U(1)_X\)’S

Before adding exotic matter to the MSSM, let us first investigate the most general action of a \(U(1)_X\) symmetry on the fields of the MSSM. We take the superpotential to be that of the MSSM up to the replacement of the \(\mu\)-term by a \(G_{SM}\) singlet field \(S\) and possible additional terms
involving both the MSSM fields and an as yet unspecified set of exotics,
\[ W = y_u Q \cdot H_u U^c - y_d Q \cdot H_d D^c - y_e L \cdot H_d E^c + \lambda S H_d H_u + (\text{exotics}). \tag{3} \]

The family-universal gauge charges under \( G_{SM} \times U(1)_X \) are defined to be
\[ Q = (3,2,1/6, q) \quad U^c = (3,1, -2/3, u) \]
\[ D^c = (3,1,1/3, d) \quad L = (1,2, -1/2, l) \]
\[ E^c = (1,1,1, e) \quad S = (1,1, 0, s) \tag{4} \]
\[ H_u = (1,2, 1/2, h_u) \quad H_d = (1,2, -1/2, h_d) \]

where \( q, u, d, \ldots \) denote the \( U(1)_X \) charges.

For the superpotential of Eq. (3) to be gauge invariant, the \( U(1)_X \) charges must sum to zero for each allowed operator:
\[ q + u + h_u = 0 \quad q + d + h_d = 0 \]
\[ l + e + h_d = 0 \quad s + h_u + h_d = 0. \tag{5} \]

These equalities form a non-degenerate system of four equations in eight variables, and allow us to solve for \( q, u, l, \) and \( s \) in terms of \( d, e, h_u, \) and \( h_d. \) Since there are four free variables, it follows that the action of any \( U(1)_X \) on the MSSM can be expressed as a linear combination of four independent basis \( U(1)'s. \) One obvious candidate for this basis is \( U(1)_Y. \) A convenient choice for the rest of the basis is \( U(1)_{B+L} \), and the \( U(1)_X \) and \( U(1)_Y \) subgroups of \( E_8 \) with the charges of the MSSM and \( S \) fields corresponding to an embedding in the \( 27 \) representation of \( E_8. \) These charges are listed in Appendix B. For a given \( U(1)_X \) symmetry, specified by the set \{\( d, e, h_u, h_d \)}, the decomposition into the above-mentioned basis is given by
\[ Q_X^Q = \frac{2}{5}(-3d + e + 2h_u - 3h_d) Q_Y^Q \]
\[ + \frac{1}{2}(-3d - e + h_u - 3h_d) Q_{B+L}^Q \]
\[ - \frac{1}{4}(h_u + h_d) 2\sqrt{6} Q_\psi^Q \]
\[ + \frac{1}{20}(6d - 2e + h_u + h_d) 2\sqrt{10} Q_X^Q, \tag{6} \]

where \( Q_A \) is the charge of field \( \phi_i \) under gauge group \( U(1)_A. \) Notice that the singlet \( S \) is protected and the standard MSSM \( \mu \)-term is forbidden by the \( U(1)_X \) symmetry if and only if the \( U(1)_X \) contains a component of \( U(1)_Y, \) since this is the only basis-\( U(1) \) under which \( S \) can be charged. Because this conclusion follows from the gauge invariance of the required superpotential operators, it will continue to hold (for the MSSM fields) no matter what exotic fields are added to the model.

III. STANDARD MODEL EXOTICS REQUIRED

The simplest way to realize an additional \( U(1)_X \) gauge symmetry while maintaining gauge coupling unification is to augment the MSSM by fields that are singlets under \( G_{SM} \). Suppose that on top of the \( S \) field that generates the \( \mu \)-term, we include an additional set of \( G_{SM} \) singlet fields. Besides the conditions for gauge invariance of the needed superpotential operators, Eq. (3), the \( U(1)_X \) charges of all the fields are constrained by the requirement of anomaly cancellation. If the exotic matter consists only of \( G_{SM} \) singlets, the pure SM anomalies vanish automatically. The remaining anomaly conditions are due to the \( G_{SM} \) \( U(1)_X \) mixed anomalies, \( SU(3)^2 U(1)_X, \) \( SU(2)^2 U(1)_X, \) \( U(1)^2 U(1)_X, \) and \( U(1)_Y U(1)_X^2, \) as well as the gravitational-\( U(1)_X \) and \( U(1)_Y^2 \) anomalies. The four mixed anomaly conditions depend only on the charges of \( G_{SM} \) non-singlet fields, and allow only a highly restrictive choice of charge relations among the SM fields.

Consider, for instance, the constraint implied by the \( SU(3)^2 U(1)_X \) anomaly,
\[ 2q + u + d = 0. \tag{7} \]
Together with the relations in Eq. (5), this condition implies that \( h_u + h_d = 0. \) It follows that if we are to obtain a non-zero charge for the \( S \) field, we must include colored exotics in addition to \( G_{SM} \) singlets. This result may also be understood by the fact that when all exotic matter fields are \( G_{SM} \) singlets, the only allowed, anomaly-free \( U(1)_X \) gauge symmetries are linear combinations of \( U(1)_Y \) and \( U(1)_{B-L} \) when acting on the MSSM.

IV. EXOTICS AND UNIFICATION RELATIONS

The problem with colored exotics, which are necessary if the \( S \) field is to be protected by the \( U(1)_X \) symmetry, is that they interfere with the unification of gauge couplings that occurs in the MSSM. To avoid disturbing the unification relations, the exotic matter should shift the three \( G_{SM} \) gauge coupling \( \beta \)-functions by an equal amount. This is automatic provided the new matter comes in complete \( SU(3) \) multiplets, and we will focus exclusively on this possibility in the present work.

There is also a good cosmological reason for this restriction. Exotic quarks should have the same electromagnetic charges as their SM counterparts, otherwise they would be stable, and the expected thermal relic abundance of a stable heavy quark would violate the experimental bounds on the anomalously heavy baryons they would produce. If we restrict ourselves to color-triplet exotics that have the same \( SU(3) \times U(1)_{em} \) quantum numbers as their MSSM counterparts, the two smallest sets of exotics that shift the \( G_{SM} \) \( \beta \)-functions by an

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2 This conclusion may be avoided if we relax our assumptions on the form of the superpotential and family universality for the \( U(1)_X \) charges. See, for example, Ref. [13].
equal amount have the $G_{SM}$ quantum numbers of the 5 and 10 multiplets up to the signs of the hypercharges of the lepton-like states. This sign ambiguity is inconsequential if we further require that the exotics come in vector-like sets with respect to their $G_{SM}$ quantum numbers. In this sense, exotics in the form of $5 \oplus \overline{5}$ or $10 \oplus \overline{10}$ multiplets are the minimal vectorial possibilities consistent with unification.

In order to obtain constraints on which and how many $SU(5)$ multiplets are sensible to consider, we review here some general features of supersymmetric gauge coupling unification. The scale dependence of the gauge couplings at one-loop is described by

$$\frac{d\alpha_i}{dt} = -\frac{b_i}{2\pi} \alpha_i^2, \quad (8)$$

where $\alpha_i = g_i^2 / 4\pi$ and $t = \ln(Q/M_Z)$. For an $N = 1$ supersymmetric gauge theory, the coefficient $b_i$ is given by

$$b_i = 3C_2(G_i) - \sum_{r_i} S_2(r_i), \quad (9)$$

where $C_2(G_i)$ is the quadratic Casimir invariant for the adjoint of the $i$-th gauge group, and $S_2(r_i)$ is the trace invariant for the matter representation $r_i$ of $G_i$. We take $g_i$ to be normalized according to the usual embedding of $G_{SM}$ in $SU(5)$, so that the $g_1$ coupling is related to the $U(1)_{Y}$ coupling by $g_1 = \sqrt{5/3} g_Y$, and $Q_1 = \sqrt{3/5} Y$. The $\beta$-function coefficients $b_i$ for the MSSM are therefore $(b_1, b_2, b_3) = (-33/5, -1, 3)$.

The success of unification within the MSSM is usually illustrated by setting the unification point $(M_G, \alpha_G)$ by the condition $\alpha_1(M_G) = \alpha_2(M_G) := \alpha_G$, and using this point to generate a prediction for $\alpha_3(M_Z)$. Taking as inputs the values $\alpha_1^{-1}(M_Z) \simeq 59.1$ and $\alpha_2^{-1}(M_Z) \simeq 29.4$, and using the solution to Eq. (8),

$$\alpha_i^{-1}(Q) = \alpha_i^{-1}(M_Z) + \frac{b_i}{2\pi} t, \quad (10)$$

we obtain $\alpha_1^{-1} \simeq 24.1$, $M_G \simeq 2.7 \times 10^{16}$ GeV, and $\alpha_3^{-1}(M_Z) \simeq 8.2$. This value for $\alpha_3^{-1}(M_Z)$ is near the measured value of about 8.7, and is compatible with unification when higher-order corrections and reasonable high-scale thresholds are taken into account [14].

To preserve the prediction of unification, any additional matter in the model should generate an equal shift in each of the $b_i$ coefficients. This occurs automatically if the new matter comes in complete $SU(5)$ multiplets,

$$\Delta b_i = -\frac{1}{2} \left[ (N_5 + N_\overline{5}) + 3(N_{10} + N_{\overline{10}}) + 7(N_{15} + N_{\overline{15}}) + 10N_{24} + \ldots \right], \quad (11)$$

where $i = 1, 2, 3$, and $N_5$ is the number of 5’s and so on. Shifting the $b_i$ in this way does not change the unification scale, but it does increase the value of the unified gauge coupling,

$$\alpha_G^{-1} \rightarrow \alpha_G^{-1} + \frac{\Delta b}{b_2 - b_1} \left[ \alpha_1^{-1}(M_Z) - \alpha_2^{-1}(M_Z) \right] \quad (12)$$

$$\simeq \alpha_G^{-1} + (5.3) \Delta b.$$

We will take $\alpha_G^{-1} > 1$ as the condition for perturbative unification which puts an upper limit on the number of new $SU(5)$ multiplets; $N_5 \leq 8$, $N_{10} \leq 2$, $N_{15} \leq 1$, and $N_e = 0$ for any of the higher-dimensional representations. Although it is possible to have a single 15 and maintain perturbative unification, it is not possible to do so while canceling the $G_{SM}$ gauge anomalies, which are all proportional to

$$(N_5 - N_{\overline{5}}) + (N_{10} - N_{\overline{10}}) + 9(N_{15} - N_{\overline{15}}) + \ldots \quad (13)$$

Thus, we need only consider 5’s, 10’s, and their conjugates.

We may also reverse the argument in the previous paragraph to constrain the possibility of $G_{SM}$ gauge coupling unification using the number and charges of the TeV-scale exotics that are observed in experiment. For instance, the discovery of more than two 10’s or eight 5’s (or more precisely, of particles having these quantum numbers) would preclude perturbative unification. This constraint is particularly relevant for the $U(1)_X$ gauge symmetry because, as we shall see, it is frequently the case that a large number of exotics, charged under $U(1)_X$, are required to cancel the gauge anomalies. In principle, colliders could extract the mass of the $Z'$ gauge boson as well as the couplings $g_x Q_i$, where $g_x$ is the $U(1)_X$ gauge coupling and $Q_i$ is the charge of the $i$-th species, for all the SM states and exotics that are light enough to be probed. With this knowledge, we could test whether the known charges are consistent with a perturbative theory up to the GUT scale. Requiring that no Landau pole develop below the GUT scale (at one loop) puts a limit on the gauge coupling and charges of the low-scale states:

$$\sum d_i(g_x Q_i)^2 \lesssim \frac{8\pi^2}{\ln(M_{GUT}/M_G)} \simeq 2.4, \quad (14)$$

where $d_i$ is the dimensionality of the representation of the $i$-th species. One must be careful in model building that the $g_x Q_i$ charges are not too large, otherwise the $U(1)_X$ gauge coupling will go strong and mix with the SM gauge couplings at higher loop order, destroying predictable, perturbative gauge coupling unification.

V. NON-SINGLET EXOTICS

As shown in Section III exotic colored matter is needed to cancel the $U(1)_X$ anomalies and to give the $S$ field a non-zero $U(1)_X$ charge. In this section, we attempt to accomplish this by adding new matter in the form of complete $SU(5)$ multiplets.
To begin, we will assume that the exotic $SU(5)$ multiplets have universal $U(1)_X$ charges, as would be expected if these multiplets were indeed derived from $SU(5) \times U(1)_X$. On the other hand, for the sake of generality, we will not immediately impose such a condition on the charges of the MSSM matter fields, even though they do fill out complete $SU(5)$ multiplets.

We denote the $U(1)_X$ charges of the exotic $SU(5)$ multiplets by $D_a$ and $\mathbf{D}_a$, $a = 1, 2, \ldots$ for the $\mathbf{5}$'s and the $\mathbf{5}'$'s, and $Q_b$ and $\mathbf{Q}_b$ for the $\mathbf{10}$'s and $\mathbf{10}'$'s. With the exotics, the conditions for the gauge invariance of the operators in the superpotential are still given by Eq. (5), while the cancellation of $SU(3)^2U(1)_X$, $SU(2)^2U(1)_X$ and $U(1)^2_5U(1)_X$ anomalies implies

$$n_g(2q + u + d) + M = 0,$$
$$n_g(q + u + l + e) + h_+ + M = 0,$$
$$n_g(q + 6u + 2d + 3l + 6e) + 3h_+ + 5M = 0,$$

where $M = \sum_a (D_a + \mathbf{D}_a) + 3 \sum_b (Q_b + \mathbf{Q}_b)$, $h_+ = (h_u + h_d)$, and $n_g = 3$ is the number of MSSM generations. Note that these equations would be unchanged had we also included an arbitrary number of $G_{SM}$ singlets.

It is not hard to see that taken together, Eq. (5) and Eq. (15) imply that both $s = (-h_u - h_d)$ and $M$ vanish. Therefore, it is not possible to protect the $S$ field by giving it a $U(1)_X$ charge if the $G_{SM}$-charged exotics, in the form of $SU(5)$ multiplets, have a universal $U(1)_X$ charge within each multiplet. This result holds for larger $SU(5)$ multiplets as well.

### A. Non-Universal $U(1)_X$ Exotics

The main result of the above analysis is that the exotic $SU(5)$ multiplets cannot have universal $U(1)_X$ charges within each multiplet if $S$ is to be charged under this gauge symmetry. On the other hand, if we allow for non-universal $U(1)_X$ charges within the exotic $SU(5)$ multiplets, we find that it is possible to obtain a non-zero $U(1)_X$ charge for the $S$ field. A simple extension of the MSSM that illustrates this result consists of a single extra $\mathbf{5} \oplus$ $\mathbf{5}'$,

$$\mathbf{5}' = D_1^c \oplus L_1 = (\mathbf{3}, 1, 1/3, \mathbf{1}, 1/2, \mathbf{1}, 1),$$
$$\mathbf{5} = D_1 \oplus L_1 = (\mathbf{3}, 1, -1/3, \mathbf{1}, 1/2, \mathbf{1}, 1),$$

and the $G_{SM}$ singlets

$$S = (1, 1, 0, s), \quad A = (1, 1, 0, a),$$
$$B = (1, 1, 0, b) \quad Z_m = (1, 1, 0, z_m).$$

As before, the $S$ field will be responsible for generating the $\mu$-term, while $A$ and $B$ are new Higgs fields included to ensure that the $\mathbf{5} \oplus$ $\mathbf{5}'$ exotics, $D_1^c$, $D_1$, $L_1$, and $L_1^c$, obtain large tree-level masses. The $Z_m$ represent any other $G_{SM}$ singlets that might be present in the model.

If we allow no exotic fields besides the $\mathbf{5} \oplus$ $\mathbf{5}'$, $S$, $A$, and $B$, (i.e. we exclude all the $Z_m$) we find that there exist solutions that satisfy the necessary constraints, but that all of these solutions require charges that are irrational relative to each other. The reason for this is that the condition implied by the $U(1)^2_5$ anomaly is cubic in the charges. We skip the details of this derivation since such relative irrational solutions appear to us to be problematic when trying to embed them into a grand unified framework.

Instead, we focus on the full complement of exotic states, including additional $Z_m$ singlets. The conditions for the cancellation of anomalies involving $G_{SM}$ subgroups, such as $SU(3)^2U(1)_X$, depend only on the charges of the MSSM and $\mathbf{5} \oplus$ $\mathbf{5}'$ fields. On the other hand, the conditions required for the cancellation of gravitational-$U(1)_X$ and $U(1)^2_5$ anomalies depend on the charges of all the fields, including the $G_{SM}$ singlets $S$, $A$, $B$, and $Z_m$. The latter two conditions turn out to present no significant constraint at all; for any solution to the singlet-independent gauge and anomaly conditions with rational $U(1)_X$ charges, it is always possible to satisfy the remaining two $G_{SM}$ singlet-dependent equations by a judicious choice of rational $U(1)_X$ charges for the $Z_m$ fields. Our proof is given in Appendix A. Note, however, that there does not always exist a rational solution to the singlet-independent equations on account of the $U(1)_X$ condition which is quadratic in the $U(1)_X$ charges. Even so, for all the cases studied here, in which the $G_{SM}$ non-singlet exotics consist of complete $SU(5)$ multiplets, it has been possible to find a rational solution to the singlet-independent conditions.

### B. $SU(5)$-Compatible Charge Assignments

Since the quantum numbers of the MSSM matter fields fill out complete $SU(5)$ multiplets, it is natural to arrange their $U(1)_X$ charges to be consistent with an embedding in $SU(5) \times U(1)_X$. The necessary conditions for this are

$$q = u = e, \quad \text{and} \quad l = d.$$

Unfortunately, as we found above, it is not possible to apply this condition to the $SU(5)$ exotics without forcing $s = 0$ as well.

For the simple case of one set of $\mathbf{5} \oplus$ $\mathbf{5}'$ exotics, the singlet-independent gauge and anomaly conditions imply the following relations between the $U(1)_X$ charges:

$$q = -h_-/4 - L_+/(4n_g - 1)$$
$$l = 3h_-/4 - L_+/(4n_g - 1)$$
$$h_+ = L_+/(n_g - 1)$$
$$D_+ = n_g L_+/(n_g - 1)$$
$$D_- = -h_-/n_g - (n_g - 1)L_-/n_g,$$

where $h_\pm = h_u \pm h_d$, $D_\pm = \mathbf{D}_1 \pm D_1$, $L_\pm = L_1 \pm L_1$, and $n_g = 3$ is the number of generations. For the MSSM
fields, we see that their charges depend only on the two parameters $h_-$ and $L_+$. Notice as well that it is not possible to have $D_+ = L_+$ unless both they and $h_+$ vanish, illustrating the necessity of non-universal charges for the exotics.\footnote{If we include $n_g$ sets of $5 \oplus \overline{5}$'s, we find the same relationship between $D_+$ and $L_+$ as in Eq. 18.}

The relationships of Eq. 18 show that when the $SU(5)$ condition of Eq. 17 is imposed, the $U(1)_X$ charges of the MSSM fields can be expressed in terms of just two parameters, $L_+$ and $h_-$. Indeed, these charges are such that the $U(1)$'s generated by $h_- \text{ and } L_+$ coincide with the $U(1)_X$ and $U(1)_Y$ subgroups of $E_6$, respectively, with the charges of the MSSM fields corresponding to their counterparts in the $27$, as listed in Appendix B. This is in accord with what we found in Section II. In the present case, the $SU(5)$ condition of Eq. 17 forces the hypercharge and $(B+L)$ components of $U(1)_X$ to vanish, leaving behind only the two $E_6$ subgroups, $U(1)_X$ and $U(1)_Y$. Unfortunately, the charges of the $5 \oplus \overline{5}$ exotics do not follow this pattern, which makes it difficult to establish a GUT interpretation.

C. SO(10)-Compatible Charge Assignments

The MSSM matter fields, when augmented by three generations of singlet neutrinos, $N^c = (1, 1, 0, n)$, fill out complete $16$'s of SO(10). It is therefore also natural to arrange the $U(1)_X$ charges to be consistent with such an embedding, implying

$$q = u = d = l = e. \quad (19)$$

This condition is a further restriction on the one given in Eq. 17 and imposes an even stronger constraint on the $U(1)_X$ charges of the MSSM fields. By comparing with Appendix B, we see that Eq. 18 projects out the $U(1)_Y$ component of $U(1)_X$, and implies that the action of $U(1)_X$ is just that of $U(1)_Y \in E_6$, with the MSSM field charges corresponding to those of the $27$ up to normalization. Furthermore, this condition always forces $h_- = 0$.

To illustrate this result, consider the relationships between the charges with an exotic $5 \oplus \overline{5}$. Applying Eq. 19 to Eqs. 18, we find

$$q = -L_+/4(n_g - 1) \quad (20)$$

$$h_+ = L_+/n_g - 1)$$

$$h_- = 0$$

$$D_+ = n_g L_+/n_g - 1)$$

$$D_- = -(n_g - 1)L_-/n_g.$$

The charges of $q$, $h_+$, and $h_-$ coincide precisely with their values under $U(1)_Y$, up to normalization. It is fortunate that our surviving $U(1)_Y$ happens to also protect the $\mu$-term. This nice feature is implicit in the example model we present in the next section.

VI. EXAMPLE MODEL

In order to construct a concrete example, and to illustrate the technique outlined in Appendix C, we investigate the $U(1)_X$-augmented MSSM subject to the condition of Eq. 19 with a single $5 \oplus \overline{5}$ as the only exotics charged under $G_{SM}$. The $G_{SM}$ singlets we include are the $S$ field which generates the $\mu$-term, the $A$ and $B$ fields which give mass to the $5 \oplus \overline{5}$ exotics, and some number of $Z_m$ such that the $U(1)_X$ charges end up being rational. For simplicity, we will set $L_+ = 0$.

With the singlet content described above, the conditions for the cancellation of the gravitational-$U(1)_X$ anomalies reduce to

$$\sum_m z_m = \frac{9}{8} L_+ = \alpha, \quad (21)$$

$$\sum_m z_m^3 = \frac{765}{512} L_+^3 = \beta \quad (22)$$

Without loss of generality, we may set $L_+ = -b = -8$. Setting $z_0 = -9$, and noting that $(\beta - \alpha^3) = -36$, a possible choice for the additional singlet charges is therefore $\{z_0, z_1, z_2, z_3\} = \{-9, -4, 3, 1\}$. With this choice, the $U(1)_X$ charges are

$$q = u = d = l = e = 1 \quad s = 4$$

$$h_u = h_d = -2$$

$$L_1 = L_1 = -4 \quad b = 8 \quad (22)$$

For these charges, the list of gauge-invariant supersymmetric operators of lowest dimension is:

$$d = 2 \text{ Superpotential} \quad (23)$$

$$\begin{align*}
S & = Z_1 + Z_2 + Z_3 + Z_{10} + Z_{30} + Z_{10} + Z_{30} \\
D & = 3 \text{ Superpotential} \\
& = \begin{array}{c}
QU^c H_u \\
Q D^c H_d \\
L E^c H_d \\
S H_u H_d \\
AD D^c \\
B L_1 T_1 \\
Z_1 Z_2 Z_3 \\
Z_1 Z_1 B \\
B Z_0 Z_3
\end{array} \\
& = \begin{array}{c}
LL^c_1 A Z_0 \\
E^c H_d H_u Z_2 \\
H_u H_d B Z_1 \\
H_b H_d Z_{23} \\
H_u L_1 Z_2 Z_3 \\
H_d L_1 Z_2 Z_3 \\
S S L_1 Z_1 \\
S S L_1 Z_1 \\
S S L_1 Z_1 \\
A Z_1 Z_1 Z_1 \\
Z_0 Z_2 Z_3
\end{array} \\
& = \begin{array}{c}
L^1 H_d Z_2 \\
S^1 B Z_1 \\
S^1 Z_2 Z_3 \\
A^1 S B \\
B^1 S S \\
B^1 A Z_1 \\
Z_1^1 H_u H_d \\
Z_2^1 A Z_0
\end{array} \quad (26)
$$
Of these operators, the dangerous ones are $SZ_1$ and $LH_uZ_3$. The first, if allowed, would reintroduce a dimensionful coupling into the superpotential which must be of order the electroweak scale for the VEV of $S$ (and consequently $\mu_{\text{eff}} = \lambda \langle S \rangle$) to be of the right size. This is precisely the sort of $\mu$ problem we set out to avoid in the first place. The second operator would induce a mixing between the leptons and the Higgsinos of the MSSM if $Z_3$ develops a VEV, possibly leading to overly large lepton masses. Similarly, the operator $LL_i^2Z_2$ can also be problematic since it induces a mixing between the MSSM leptons and their exotic counterparts, but it may also be useful in that it facilitates the decay of the heavy leptons. All three of these operators can be forbidden by imposing an $R$-parity symmetry under which

\begin{align}
Q &= U^c = D^c = L = E^c = -1 \\
D_1 &= \bar{D}_1 = L_1 = \bar{L}_1 = Z_1 = Z_2 = -1, \\
H_u &= H_d = S = A = B = Z_0 = Z_3 = +1.
\end{align}

This set of charges still allows terms such as $Z_iZ_1B$, $Z_1Z_2Z_3$, and $BZ_0Z_3$ which help to ensure that the fermionic components of the singlets other than the $A$ get large masses provided all the $R$-even $Z_m$ fields develop VEV’s. Note also that, even without imposing an $R$-parity, the supersymmetric terms that could generate dimension five Lagrangian operators leading to proton decay are forbidden by the $U(1)_X$.

As it stands, the model still has a couple of potential problems. The first is that the $A$ field does not get a contribution to its mass from any of the renormalizable superpotential operators. Instead, the fermion component of this field only receives a mass through its mixing with the $U(1)_X$ gaugino of order $g_x \langle \phi_A \rangle$. If the low-energy $g_x$ coupling is very weak, as might be required to preserve perturbative unification in the presence of a large number of singlets, there will be a very light, mostly singlet fermion in the spectrum. If this state is the LSP, which is quite likely for a light state, it will be stable on account of $R$-parity and could overclose the universe due to the feebleness of its couplings. Thus, it would be reassuring if there was also a superpotential contribution to the $A$ fermion mass.\(^4\)

The second problem is that the mixing between the exotic quarks from the $5 \oplus \bar{5}$ and those of the MSSM is highly suppressed, being relegated to non-renormalizable operators of dimension greater than five. Such mixing induces a flavor-violating coupling between the exotic quarks, the light quarks, and the $W$ gauge boson. If these operators are suppressed by powers of $M_{\text{Pl}}$ or $M_{\text{GUT}}$, the heavy exotic quarks will be cosmologically stable. As discussed in Section [IV], this leads to a relic abundance of heavy baryons well above the experimental bounds. This conclusion regarding the stability of heavy baryons holds without question if the mixing arises from operators of dimension six or higher, as we find here. For dimension-five mixing, suppressed by a single power of $M_{\text{Pl}}$, the lifetime of the heavy baryons is of order $10^4$ s, long enough that they could potentially interfere with the predictions of Big Bang nucleosynthesis (BBN) when they decay.\(^\text{[15]}\) However, the relic abundances for the heavy baryons found in Ref.\(^\text{[17]}\) are low enough that the net effect of their decays on the light element abundances is acceptably small.

One way to resolve both problems is to include more $G_{\text{SM}}$ singlet fields. To generate a superpotential contribution to the mass of the $A$ field, we will add one set of $R$-parity even fields having $U(1)_X$ charges $\{a, 5a, -6a\}$, and a second with charges $\{-2a, -3a, 5a\}$. As discussed in Appendix [A2], these sets do not contribute to the anomalies and allow for cubic operators in the superpotential that lead to masses for the $A$ and all of the additional fields provided some of these fields develop VEV’s. These sets are also chiral, in that they do not allow any explicit bilinear couplings between themselves. In the present case, the additional fields do not induce any new bilinear couplings with the other fields in the model either. In the same way, we will induce a mixing between MSSM down quarks and their heavy counterparts, allowing the heavy quarks to decay via the $W$ gauge bosons, by introducing a SM singlet field having $U(1)_X$ charge equal to $-(d+D_1) = -(q+D_1) = 5$. If this new field is embedded in the larger set of fields having charges $\{5, 25, -30; -10, -15, 25\}$, no anomalies will be generated, all the fields within the set will obtain superpotential contributions to their masses, and no new bilinear couplings will be induced. A general method for this sort of “singlet engineering” is given in Appendix [A2].

VII. CONCLUSIONS

One of the most appealing ways to explain the $\mu$-term in supersymmetry is to promote it to a condensing scalar field charged under a new $U(1)_X$ gauge symmetry. Under the conservative assumptions stated in the introduction, we have shown that this idea leads to several interesting implications:

1. The most general $U(1)_X$ symmetry, when acting on the fields of the MSSM, is a linear combination of $U(1)_Y$, $U(1)_{B-L}$, and the $U(1)_\psi$ and $U(1)_\chi$ subgroups of $E_6$. Of these basis $U(1)$’s, only $U(1)_\psi$ is able to forbid the $\mu$-term and protect the singlet $S$ that replaces it. For an embedding of the MSSM in $SU(5) \times U(1)_X$, only the $E_6$ subgroups are allowed. For an embedding in $SO(10) \times U(1)_X$, only $U(1)_\psi$ is possible.

\(^4\) Another reason to desire this reassurance is that if the model contained a second $R$-even field with no superpotential mass term, the neutralino mass matrix would have a zero eigenvalue for any value of the gauge coupling $g_x$.\(^\text{[16]}\)
2. Anomaly cancellation requires the introduction of exotics charged under $G_{SM}$ to cancel all mixed anomalies in the theory. Such SM exotics can only be dismissed if the effective $\mu$-term is not charged under the $U(1)_X$.

3. Adding complete multiplets of $SU(5)$ according to their SM charges, which is required for “automatic gauge coupling unification”, necessitates assigning different $U(1)_X$ charges to the SM-like component states within each exotic multiplet.

4. Solutions with rational $U(1)_X$ charges for all SM-charged states generally require a large set of $Z_m$ singlet states; nevertheless, a solution to the singlet-dependent anomaly equations from these states $Z_m$ is guaranteed, and we have shown an algorithm to obtain that solution.

It is apparent from this study that if we want the $\mu$-term to originate from the spontaneous breaking of a $U(1)_X$ symmetry, it is difficult to accommodate the resulting set of states in the model within an $SU(5) \times U(1)_X$ theory. One way to embed it in this fashion, however, would be to assume that the various SM fields that can be classified as coming from $5 \oplus \overline{5}$ are really pieced together from parts of a larger set of $5 \oplus \overline{5}$ representations, each with its own $U(1)_X$ charge. Such representations arise naturally if the $SU(5)$ group is derived from a larger group such as $SO(10)$ or $E_6$, and the splitting of the multiplets may be achieved by way of an orbifold compactification of a fifth dimension $[10]$. On the other hand, from this point of view there is no obvious reason why the low-energy spectrum of $G_{SM}$-charged exotics should consist of complete $SU(5)$ multiplets. Therefore the fact that we obtain gauge coupling unification taking into account only the fields of the MSSM would appear to be a fortuitous accident.

Another attractive possibility within this scenario would be to embed the entire gauge structure, including the $U(1)_X$ gauge group, within $E_6$ $[10, 21]$. This is particularly compelling since we have shown that abelian subgroups of $E_6$ are natural candidates for the $U(1)_X$. They are the only possible candidates when $SU(5)$ or $SO(10)$ conditions are imposed on the charges of the MSSM fields. A natural special case along these lines, studied recently in Ref. [21], is to embed the MSSM fields in three $27$’s of $E_6$, and to identify the $U(1)_X$ with an unbroken linear combination of $U(1)_\psi$ and $U(1)_X$. To preserve gauge unification, an additional pair of Higgs-like doublets is needed, and this pair must be vectorial in its $U(1)_X$ charge to avoid generating anomalies. Such a model can solve the $\mu$ problem of the MSSM and increase the Higgs boson mass $[21]$; however, there is also a new (arguably less severe) $\mu$ problem associated with unification since the vectorial pair of Higgs-like doublets must survive to low energies. Our results suggest that to preserve unification while avoiding such $\mu$ problems altogether is a difficult task. As in the $SU(5) \times U(1)_X$ case, the embedding of the requisite charged exotics within $E_6$, and the splitting of their corresponding multiplets, presents an acute model-building challenge.

It is our view that a $\mu$-term originating from a charged singlet that breaks a $U(1)_X$ symmetry implies a degree of tension with supersymmetric gauge coupling unification. The picture of grand unification in the presence of such a symmetry is necessarily much more complicated than in the MSSM. In this paper we have developed some tools for constructing models of this type, and we have exhibited a particular self-consistent example. We have also pointed out some of the peculiarities required of these models that may overwhelm their attractive features. In any event, it should be possible for experiment to look for the charged exotics, the singlets, or the $Z'$ gauge boson associated with this explanation of the $\mu$-term. If confirmed, we may need to rethink unification.

APPENDIX A: SINGLET ENGINEERING

1. Solutions to non-SM Anomalies

Suppose the set of charges \{$q_i$\} of the $G_{SM}$-charged fields, with all $q_i$ rational, is a solution of the singlet-independent conditions. For any such solution, it is possible to uniformly rescale the charges such that they are integers. We will assume this has been done. Denoting the $G_{SM}$ singlet charges under $U(1)_X$ as $z_m$, $m = 0, 1, \ldots, N$, the $ggU(1)_X$ and $U(1)_X^3$ anomaly equations have the form

$$\sum_{m=0}^{N} z_m = \alpha, \quad \sum_{m=0}^{N} z_m^3 = \beta, \quad \text{(A1)}$$

where $\alpha$ and $\beta$ are linear and cubic polynomials in the $q_i$, respectively, with integer coefficients. It follows that $\alpha$ and $\beta$ are integers as well. Now suppose we choose $z_0 = \alpha$, so that

$$\sum_{m=1}^{N} z_m = 0, \quad \sum_{m=1}^{N} z_m^3 = \beta - \alpha^3. \quad \text{(A2)}$$

The remaining $z_m$ charges thus form an $N$-th partition of zero. Here is where a helpful theorem enters: The cubic sum ($\sum z_m^3$) of any integer partition of zero ($\sum z_m = 0$) is a multiple of six. Using this factor of six, we have many options to satisfy the cubic equation, including

$$36(\beta - \alpha^3) \text{ sets of } z_m = \left\{ \begin{array}{c} 2 \ 1 \ -1 \\ 6 \ 6 \ -6 \end{array} \right\}$$

$$6(\beta - \alpha^3) \text{ sets of } z_m = \left\{ \begin{array}{c} 4 \ 3 \ -1 \\ 6 \ 6 \ -6 \end{array} \right\} \quad \text{(A3)}$$

$$\beta - \alpha^3 \text{ sets of } z_m = \left\{ \begin{array}{c} 7 \ 5 \ -1 \\ 6 \ 6 \ -6 \end{array} \right\}$$
The same solutions apply if \((\beta - \alpha^3) < 0\), except the above equations are multiplied by \(-1\). If \((\beta - \alpha^3)\) happens to be a multiple of six, a solution is always possible with integer partitions of zero, the simplest of which is \((\beta - \alpha^3)/6\) copies of \(z_m = \{2, -1, -1\}\).

Thus, there is always a rational solution to the singlet-only equations as claimed. Other techniques for solving the anomaly equations can be found in [22].

2. Constructing Masses without \(\mu\)-Terms

In formulating models involving an additional gauged \(U(1)_X\), it is often the case that for a given anomaly-free set of fields, the gauge symmetries do not allow certain operators that are necessary to obtain a realistic phenomenology. The most important of these operators are usually bilinear in the fields, to generate masses for internal triplets are needed. Two chiral triplet sets that work for each of the components. By explicit calculation, we find that the corresponding fermion mass matrix is non-singular provided all three members of a triplet develop VEV’s.

As a first application, suppose the field \(Z_M\), with \(U(1)_X\) charge \(z_M\) but otherwise a singlet, is part of an anomaly-free \(U(1)_X\) model that does not allow a superpotential contribution to its mass. To generate an effective mass term involving \(Z_M\), it is sufficient to introduce a set of \(X_a\) and \(Y_b\) fields having charges \(\{z_M, 5z_M, -6z_M\}\) and \(\{-2z_M, -3z_M, 5z_M\}\). If both \(Z_M\) and some of the additional singlets get VEV’s, we find that the full mass matrix for these fields is non-singular. For a second example, suppose that we wish to induce a mixing between the fields \(P\) and \(R\) whose quantum numbers are such that the product \(PR\) is a \(G_{SM}\) singlet but has a non-zero \(U(1)_X\) charge \(Q_X\). By introducing \(X_a\) and \(Y_b\) singlets with charges \(\{-Q_X, -5Q_X, 6Q_X\}\) and \(\{2Q_X, 3Q_X, -5Q_X\}\), we obtain the desired mixing provided the \(X_i\) field with charge \(-Q_X\) condenses in the vacuum.

### APPENDIX B: BASIC \(E_6\) FACTS

Additional \(U(1)\) gauge groups may arise from the spontaneous breaking of \(E_6\). This can happen through the sequence \[E_6 \rightarrow SO(10) \times U(1)_\psi \rightarrow SU(5) \times U(1)_X \times U(1)_\psi.\]

Another attractive feature of \(E_6\) models is that all the matter fields of the MSSM can be embedded in the \([27]\) of this group. The full particle content of the \([27]\), as well as the charges under \(G_{SM}\) and \(U(1)_\psi, X\) subgroups are given in Table [II].

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| TABLE I: Particle content and subgroup charges of the \([27]\) representation of \(E_6\) |
|-----------------|-----------------|-----------------|
| \(Q\)           | \(3, 2, 1/6\)   | 1               |
| \(L\)           | \(1, 2, -1/2\)  | 1               |
| \(U\)           | \(3, 1, -2/3\)  | 1               |
| \(D\)           | \(\bar{3}, 1, 1/3\) | 1   |
| \(E\)           | \((1, 1, 1)\)   | 1               |
| \(N\)           | \((1, 1, 0)\)   | 1               |
| \(H\)           | \((1, 2, -1/2)\) | -2              |
| \(P\)           | \(\bar{3}, 1, 1/3\) | -2 |
| \(E\)           | \((1, 2, 1/2)\) | -2              |
| \(R\)           | \((3, 1, -1/3)\) | -2              |
| \(S\)           | \((1, 1, 0)\)   | 4               |

[5] In this appendix, we refer to fields that are uncharged under \(G_{SM}\) as singlets, even though we will implicitly demand that they have a non-zero \(U(1)_X\) charge.
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