Effect of the heat transfer coefficient on modelling the energy deposition of destructed meteoroid

I G Brykina¹ and M D Bragin²,³

¹ Institute of Mechanics of Lomonosov Moscow State University, Moscow, 119192, Russia
² Keldysh Institute of Applied Mathematics, Moscow, 125047, Russia
³ Moscow Institute of Physics and Technology, Dolgoprudny, Moscow Region, 141701, Russia

E-mail: shantii@mail.ru

Abstract. Models of meteoroid destruction into the cloud of fragments moving with a common shock wave are considered: the two-parameter model that takes into account changes of the shape and density of the cloud, and simple models that leave these factors out of account, including those used in the literature. Models differ in equations governing the lateral expansion of the cloud. We numerically simulate the interaction of the Chelyabinsk asteroid with the Earth’s atmosphere by solving the meteor physics equations using fragment cloud models, and study the models abilities to reproduce the observational energy deposition curve. Heat transfer coefficient effect on simulating energy deposition using different models is estimated. For simple fragment cloud models, the optimal coefficient in the cloud lateral expansion equation is proposed as a function of the heat transfer coefficient (ablation parameter) to match the observational altitude of the bolide peak brightness. The optimal value of the uncertainty parameter in the expression for the heat transfer coefficient is found to match also a shape of the observational energy deposition curve. It is shown that the optimal simple and the two-parameter models give results of modelling the energy deposition of the Chelyabinsk asteroid and estimates of its entry mass, which are very close to each other and are consistent with the observational data.

1. Introduction

Destruction of cosmic bodies due to aerodynamic loads that increase during their penetration into denser layers of the Earth's atmosphere is one of the main processes, along with ablation, that have a significant influence on the interaction with the atmosphere. Fragmentation of cosmic objects entering the atmosphere with different velocities, compositions, structures, sizes and strengths can occur in different ways, so there are different approaches to modelling their destruction. When a meteoroid breaks up into a large number of fragments, at the first stage they move with a common shock wave as a single body. Then, fragments are scattered far enough to have their own shock waves that interact with each other. At the third stage, after complete separation, fragments move independently. To simulate meteoroid disruption at the first stage, models of a cloud of fragments united by a common shock wave are usually used. Such a cloud of fragments under the action of aerodynamic forces is deformed (flattened): it is compressed in the direction of flight and expands in the lateral direction. The proposed models differ in the equation for the rate of increase of the midsection radius of a
fragmented meteoroid, which describes its lateral expansion. Grigoryan [1] was the first to develop such a model for the meteoroid break-up into a cloud of small pieces, moving as a single body. Later, similar models of the flattened cloud of fragments were developed [2–5] and used [6–10] in various studies. These models could be applied to simulate the fragmentation of cosmic objects larger than a few meters, which are intensively destroyed.

Here we study modelling the energy deposition of a destructed meteoroid in the atmosphere using models of a cloud of fragments moving with a common shock wave: the two-parameter model developed by the authors [5, 10], which takes into account changes in the shape and density of the cloud, and simple models that leave these factors out of account, including models used in the literature [1, 6] and [4]. The models are compared in terms of the possibility to simulate the energy deposition of the Chelyabinsk bolide with their use. For this purpose, the meteor physics equations are numerically solved for each model. The radiative heat transfer coefficient, which is a key parameter in the equations, is approximated as a function of the body velocity and size and the atmospheric density; its constant value, accepted in the literature, is also used. By varying an uncertainty factor introduced in the expression for the heat transfer coefficient, we study its effect on modelling the bolide energy deposition when using different fragment cloud models, and on the ability of these models to reproduce observational data.

2. Fragmentation models

The entry of a meteoroid into the Earth’s atmosphere at an angle $\theta$, relative to the horizon is considered. It is assumed that before a destruction begins, the meteoroid has a sphere shape, and then it continues its flight as a cloud of fragments and vapor united by a common shock wave. Under the action of pressure forces, a sphere is transformed into a spheroid with the half-axis ratio $b/a = k \geq 1$. In the two-parameter model [5], two parameters are introduced: the flattening parameter $k$ and the parameter $\gamma (\gamma \leq 3)$, which characterizes decrease of the density of the fragmented meteoroid $\delta$ due to increase of gaps between fragments: $\delta = \delta_i / \gamma^2$, where $\delta_i$ is the initial density.

In the two-parameter model, the equation for the rate of increase of the fragment cloud midsection radius $R_s$, which characterizes the lateral expansion of the cloud, is as follows

$$\frac{dR_s}{dt} = \left(\frac{\gamma^3}{k}\right)^{1/2} \left(\frac{\rho}{\delta_e}\right)^{1/2} V, \quad k = \frac{4\pi \delta_e}{3M} \frac{R_s^3}{\gamma}, \quad \gamma = 1 + \frac{\rho_s^{1/2} - \rho_i^{1/2}}{\rho_m^{1/2} - \rho_i^{1/2}} (\gamma_m - 1).$$

(1)

Here $\rho$ is the atmospheric density, $t$ is the time, $V$ and $M$ are the meteoroid velocity and mass; subscripts $f$ and $m$ correspond to values at altitudes of fragmentation start $h_f$ and of maximum bolide brightness $h_m$. Parameter $\gamma_m$ is adjusted to match the observed height $h_m$. A detailed derivation of equations (1) and a description of the model are given in [5, 10].

In simple models, equations for the midsection radius $R_s$ have the same structure and differ only in the value of the coefficient $c$, which is assumed to be constant

$$\frac{dR_s}{dt} = c \left(\frac{\rho}{\delta_e}\right)^{1/2} V, \quad c = \text{const}.$$  

(2)

In the Grigoryan’s model [1, 6] $c = 1$, and in the Hills and Goda’s model [4] $c = (7/2)^{1/2}$. Simple fragment cloud models, in contrast to the two-parameter model, do not take into account decrease of density of a disrupted meteoroid and change of its shape. Equation (2) has an analytical solution in the case of rectilinear trajectory and isothermal atmosphere, which shows that the midsection radius is determined only by initial parameters at the altitude of break up start, ablation does not affect it. So, fragmentation problem is separated from ablation and motion problem for simple models. There is some contradiction in this: ablation affects the meteoroid mass and does not affect its midsection radius. For the two-parameter model, problems of fragmentation, ablation and motion are coupled.
3. Equations

For numerical simulation of the interaction between a cosmic body and the atmosphere, the generalized meteor physics equations [11] are used. We use the equations of motion, ablation (mass loss) and trajectory, and relation for the isothermal atmosphere in the form

\[
M \frac{dV}{dt} = - \frac{\pi}{2} R^2 C_D \rho V^2 + M g \sin \theta, \quad Q \frac{dM}{dt} = - \frac{\pi}{2} R^2 C_H \rho V^3, \\
\frac{dh}{dt} = - V \sin \theta, \quad \frac{d\theta}{dt} = \frac{g \cos \theta - V \cos \theta}{V} - \frac{1}{R_0 + h}, \quad \rho = \rho_0 \exp \left( - \frac{h}{h_*} \right) .
\]

(3)

Here \( C_D \) is the drag coefficient, \( C_H \) is the heat transfer coefficient per unit midsection area, \( Q \) is the effective heat of mass loss, \( \theta \) is the angle of inclination of the trajectory with respect to the horizon (to the tangent to the Earth's surface), \( g \) is the gravitational acceleration, \( \rho_0 = 1.29 \text{ kg/m}^3 \), \( h_* = 7 \text{ km} \), \( R_0 \) is a radius of the Earth. Equations (3) differ from the equations of the simple physical meteor theory [12], which were used in our previous studies [5, 10], in that they take into account the gravity force and the curvilinearity of the trajectory, i.e. change of the angle \( \theta \).

The bolide kinetic energy \( E \) deposited per unit height along its trajectory is determined by the formula

\[
\frac{dE}{dh} = - \frac{1}{V \sin \theta} \frac{d}{dt} \left( \frac{MV^2}{2} \right) = - \frac{1}{V \sin \theta} \left( \frac{V^2}{2} \frac{dM}{dt} + MV \frac{dV}{dt} \right) .
\]

(4)

To calculate the mass loss, velocity, energy deposition and fragment cloud expansion of a disrupted meteoroid, equations (3), (4) are solved together with equation (2) when using simple models, and together with equations (1) when using the two-parameter model; before the start of break-up, \( \gamma = 1 \) and \( k = 1 \) are assumed.

Key parameters in equations (3), (4) are the drag coefficient \( C_D \) and the heat transfer coefficient \( C_H \). In the literature, the drag coefficient is usually assumed to be constant, and when using simple fragment cloud models, it is assumed to be 1 or 1.5. We use for the drag coefficient \( C_D \) of a spheroidal body:

\[
C_D = 2 - 1/k \quad \text{or} \quad C_D = 1.78 - 0.85/k .
\]

(5)

The first formula (5) is an approximation of the exact analytical solution obtained for a spheroid when using the Newton formula for the surface pressure [5]. The second formula has been obtained by approximating the results of numerical calculations [13] of hypersonic flow of dissociated air over spacecraft with a spheroidal front surface. The formulas agree with up to 12% accuracy.

The meteoroid mass loss is caused by intensive heating, which in the case of large bodies is mainly radiative. The radiative heat transfer coefficient \( C_H \) as a function of the body velocity \( V \), its radius of curvature at a stagnation point \( R \) and the atmospheric density \( \rho \) is calculated by the approximate formula

\[
C_H = \psi \phi C_{H0} (V, R, \rho) .
\]

(6)

The heat transfer coefficient at a stagnation point of sphere with indestructible surface \( C_{H0} (V, R, \rho) \) is calculated by the formula (8) of paper [10], which is some modification of the correlation obtained in study [14]. Parameter \( \phi \) characterizes change of the heat flux along the surface and we set it here equal to 0.7. Note that there is uncertainty in the value of the radiative heat flux to the body due to the unaccounted influence of the precursor absorption, turbulence, absorption by a meteoroid vapor layer, uncertainty in the optical properties of hot air and vapors, in the radiation transport and flow field models, and other factors. Therefore, the uncertainty factor \( \psi \) is introduced into the expression for the heat transfer coefficient, which is varied in calculations in order to estimate the influence of the heat transfer coefficient's uncertainty on modelling the meteoroid energy deposition and on ability of the
considered fragmentation models to reproduce the observational data for the Chelyabinsk event. We also use in calculations the constant value of the heat transfer coefficient accepted in the literature.

4. Results and discussion

The models of a cloud of fragments are used to simulate the interaction with the atmosphere of the Chelyabinsk asteroid, which entered the Earth's atmosphere on February 15, 2013, in order to compare their abilities to reproduce the observational curve of energy deposition [15]. Another purpose was to evaluate the effect of the heat transfer coefficient on the results of modelling the bolide energy deposition, the lateral expansion of the fragment cloud, and the applicability of models.

For each fragmentation model, the meteor physics equations (3), (4) with coefficients (5), (6) are solved together with equations (1) or (2) using the Runge–Kutta method. Results of processing of observational data [16] are used as initial parameters: atmospheric entry velocity $V_e = 19$ km/s, entry angle $\theta_e = 18^\circ$, entry density $\delta_e = 3.3$ g/cm$^3$, altitude of fragmentation start $h_f = 45$ km, which corresponds to the initial bulk strength $\sigma$ of about 0.75 MPa; the effective heat of mass loss $Q$ is set equal to 6 km$^2$/s$^2$. Unknown entry mass $M_e$ in each calculation case is adjusted to match the observed maximum energy deposition of the bolide of about 81 kt TNT km$^1$ [15]. When using the two-parameter model, parameter $\gamma_m$ is adjusted to match the altitude of the observed peak brightness.

Influence of the heat transfer coefficient $C_H$ on the results of calculating the meteoroid energy deposition per unit height $dE/dh$ and its midsection radius $R_e$ normalized to the entry value $R_e$ is shown in Figure 1 versus the flight altitude $h$ in the case of using the two-parameter model.

![Figure 1. Heat transfer coefficient effect on the meteoroid energy deposition and midsection radius for the two-parameter model; $h_f = 45$ km. Black curve is observational [15].](image)

For the two-parameter model, with decreasing $C_H$ (decreasing parameter $\psi$), the meteoroid ablation (mass loss) decreases and, accordingly, the midsection radius and lateral expansion of the fragment cloud increase. The heat transfer coefficient has less effect on the bolide energy deposition, especially before the peak brightness. Adjusting the parameter $\gamma_m$ makes it possible to match the altitude of the observed peak brightness, therefore, varying the parameter $\psi$ only leads to a change in the shape of the energy deposition curve, making it slightly wider when decreasing $\psi$. The best agreement with the observational energy deposition curve [15] is attained at $\psi = 1$ ($\psi$ from 0.8 to 1.2) and at $C_H = 0.1$. The $C_H$ value has little effect on the estimate of the meteoroid entry mass $M_e$: at $\psi = 1.2$, 1, 0.8, 0.5, $M_e$ is equal to 1.318, 1.325, 1.337, 1.355×10$^7$ kg; at $C_H = 0.1$, $M_e = 1.325×10^7$ kg. The two-parameter model gives values of the entry mass close to most probable values according to estimates of [16] and [17].

In simple models, varying the parameter $\psi$ has almost no effect on the midsection radius, because in these models, as mentioned above, the fragmentation and ablation problems are separated (a very small effect occurs through the radius $R_f$ at the height $h_f$). However, the heat transfer coefficient has a significant effect on the calculated altitude, where the maximum brightness of the bolide is attained, in contrast to the two-parameter model. This is shown in Figures 2 and 3 (a). At $\psi = 1$, the models [1, 6]
(c = 1) and [4] (c = 1.87) give the altitude of the bolide peak brightness much higher than the observed one (Figure 2 (a)). This is due to the very rapid lateral expansion of the fragment cloud in these models, especially in the model [4] (Figure 3 (c), red line), and, as a result, too early growth of the energy deposition curve. By reducing the heat transfer coefficient, one can move down the altitude of the peak brightness (Figures 2 (b, c) and 3 (a)). Thus, for the model [1, 6] with coefficient c in equation (2) equal to 1, a satisfactory agreement with the observational curve of the bolide energy deposition is attained at $\psi = 0.3$ and at constant $C_H = 0.03$.

**Figure 2.** Heat transfer coefficient effect on the bolide energy deposition for simple models: [1, 6] (c = 1, red lines) and [4] (c = 1.87, green lines); $h_f = 45$ km. Black curve is observational [15].

**Figure 3.** Effect of heat transfer coefficient (a) and effect of midsection radius cutoff (b, c, green lines) on the bolide energy deposition for model [4] (c = 1.87); $h_f = 45$ km. Black curve is observational [15].

For the model [4] with parameter $c = 1.87$, reducing $C_H$ down to 0 (no ablation) does not allow us to attain agreement with observational data, as shown in Figure 3 (a). At any value of the parameter $\psi$, the altitude of the peak brightness remains higher than the observed one, which is explained by very strong growth of the midsection radius of the fragment cloud (red line in Figure 3 (c)) due to the large parameter $c$, which is almost twice the parameter $c$ in the model [1, 6]. Unrealistic increase of the midsection radius in the model [4] is usually limited in the literature to a value of $R_S/R_e$ not exceeding 7–8 [7, 9]. As Figure 3 (b) shows, cutoff of the midsection radius (Figure 3 (c)) at $R_S/R_e = 7.5$ makes it possible at $\psi = 0.3$ to shift an altitude of the peak brightness downward so that it coincides with the observed one. However, the calculated energy deposition curve in this case is much wider than the observational one, and value of the entry mass $M_e = 2.5 \times 10^7$ kg is about twice the most probable values [16, 17].

Based on computations performed for simple models at different values of parameters $c$ and $\psi$, as well as at constant values of $C_H$, the optimal parameter $c$ is found as a function of the parameter $\psi$ (or constant $C_H$), at which the calculated altitude of the peak brightness of the Chelyabinsk bolide coincides with the observed one [15]. The dependence of the optimal parameter $c$ on the parameter $\psi$ when using formula (6) for the heat transfer coefficient and on the constant $C_H$ value when using it can be represented as

$$c(\psi) = (\psi + 0.7)^{-0.92}, \quad c(C_H) = (10C_H + 0.7)^{-0.92}.$$  

(7)
In the ablation equation, the heat transfer coefficient $C_H$ is included in the combination $C_H/Q = C_{ab}$, which we call the ablation parameter as in [9]. Therefore, we present the optimal parameter $c$ in simple fragmentation models as a function of the ablation parameter. Note that in the literature, the combination $C_H/QC_D$ is usually called the ablation coefficient and is assumed, like the drag coefficient, to be constant. Taking into account that in the calculations the effective heat of mass loss $Q$ was set equal to 6 km$^2$/s$^2$, we obtain the following dependence of the optimal parameter $c$ on the parameter $C_{ab}$ when using a constant value for $C_H$ and on the parameter $\psi_{ab} = \psi/Q$ when using formula (6)

$$c(\psi_{ab}) = (6\psi_{ab} + 0.7)^{-0.92} (\psi_{ab} = \psi/Q), \quad c(C_{ab}) = (60C_{ab} + 0.7)^{-0.92} (C_{ab} = C_H/Q = \text{const}) .$$

Figure 4. The bolide energy deposition for simple models with optimal parameter $c$ at various $C_H$ (a) and for simple (blue line) and two-parameter (red) models at $\psi = 1$ (b); $h_f = 45$ km. Black curve is observational [15].

The results of modelling energy deposition using simple fragment cloud models with the optimal parameter $c$ in the midsection radius equation are shown in Figure 4 (a) for different settings of the heat transfer coefficient. When the parameter $\psi$ changes from 1 to 0 (no ablation), or a constant value of $C_H$ changes from 0.1 to 0, the optimal parameter $c$ changes from 0.615 to 1.39. In this connection, the shape of the energy deposition curve becomes slightly wider and lateral expansion of the cloud of fragments increases: at an altitude of 25 km, the growth of the midsection radius at $\psi = 0$ ($c = 1.39$) $R_S/R_e \approx 16$ is twice as high as $R_S/R_e \approx 8$ at $\psi = 1$ ($c = 0.615$). When using simple models with parameter $c$ in the equation for midsection radius (2) of less than 1.39, the calculated altitude of the bolide peak brightness is higher than the observed one, so in this case it is not possible to obtain agreement with the observational data for the Chelyabinsk event.

The best agreement with the observational data for simple fragmentation models, taking into account not only an altitude of the bolide peak brightness, but also a shape of the energy deposition curve, an estimate of the entry mass and a degree of the fragment cloud lateral expansion, is attained at the optimal parameter $c = 0.615$ and ablation parameters: $C_{ab} = 0.0167$ s$^2$/km$^2$ ($C_H = 0.1$, $Q = 6$ km$^2$/s$^2$) when using a constant value for $C_H$ or $\psi_{ab} = 0.167$ s$^2$/km$^2$ ($\psi = 1$, $Q = 6$ km$^2$/s$^2$) when using formula (6). The calculated energy deposition curves for this optimal simple model and for the two-parameter model at the same value of ablation parameter are shown in Figure 4 (b). It can be seen that the results obtained for both models are very close to each other and are in good agreement with the observational curve. The midsection radius in both models takes quite acceptable values along the trajectory: $R_S/R_e$ does not exceed 5.5 down to an altitude of 29.5 km of the bolide peak brightness and 8.2 to an altitude of 25 km. These two fragmentation models give estimates of the meteoroid mass at atmospheric entry of $1.285 \times 10^7$ and $1.325 \times 10^7$ kg, which are very close to the estimates of 1.2 and $1.3 \times 10^7$ kg [16, 17] based on the observational data.
5. Conclusions
The heat transfer coefficient effect on modelling the energy deposition of the Chelyabinsk asteroid using the two-parameter and simple fragment cloud models of meteoroid destruction is evaluated. For simple fragmentation models, the optimal coefficient $c$ in the equation, governing a lateral expansion of the cloud, is found as a function of the heat transfer coefficient (ablation parameter), which gives the match of the calculated altitude of the bolide peak brightness with the observational one. The optimal value of the ablation parameter is found to match also the shape of the observational energy deposition curve. Two fragmentation models: the optimal simple model and the two-parameter model give the results of modeling the energy deposition of the Chelyabinsk asteroid and estimates of its atmospheric entry mass, which are very close to each other and are in agreement with the observational data.

References
[1] Grigoryan S S 1979 Meteorites motion and destruction in planet atmospheres Cosmic Res. 17 724
[2] Melosh H J 1981 Atmospheric breakup of terrestrial impactors Proc. Lunar Planet. Sci. 12A 29
[3] Chyba C F, Thomas P J and Zahnle K J 1993 The 1908 Tunguska explosion – Atmospheric disruption of a stony asteroid Nature 361 40
[4] Hills J G and Goda M P 1993 The fragmentation of small asteroids in the atmosphere Astron. J. 105 1114
[5] Brykina I G 2018 Large meteoroid fragmentation: modeling the interaction of the Chelyabinsk meteoroid with the atmosphere Solar Syst. Res. 52 426
[6] Grigoryan S S, Ibodov F S and Ibadov S I 2013 Physical mechanism of Chelyabinsk superbolide explosion Solar Syst. Res. 47 268
[7] Collins G S, Lynch E, McAdam and R Davison T M 2017 A numerical assessment of simple airblast models of impact airbursts Meteorit. & Planet. Sci. 52 1542
[8] Register P J, Mathias D L and Wheeler L F 2017 Asteroid fragmentation approaches for modeling atmospheric energy deposition Icarus 284 157
[9] McMullan S and Collins G S 2019 Uncertainty quantification in continuous fragmentation airburst models Icarus 327 19
[10] Brykina I G and Bragin M D 2020 On models of meteoroid disruption into the cloud of fragments Planetary & Space Sci. 187 104942
[11] Brykina I G and Egorova L A 2020 Modeling motion, ablation and energy deposition of meteoroid in the atmosphere taking account of the curved trajectory Physical-Chemical Kinetics in Gas Dynamics 21 (2) 903
[12] Bronshhten V A 1983 Physics of Meteoric Phenomena (Dordrecht: Springer)
[13] Golomazov M M, Litvinov I A, Litvinov L A, Ivankov A A and Finchenko V S 2011 Numerical simulation of flow past descent vehicles during planetary entry Herald of the Bauman Moscow state technical university, Mechanical engineering 4(85) 42
[14] Brykina I G and Egorova L A 2019 Approximation formulas for the radiative heat flux at high velocities Fluid Dyn. 54 562
[15] Brown P G et al. 2013 A 500-kiloton airburst over Chelyabinsk and an enhanced hazard from small impactors Nature 503 238
[16] Borovička J, Spurný P, Brown P, Wiegert P, Kalenda P, Clark D and Shrbený L 2013 The trajectory, structure and origin of the Chelyabinsk asteroidal impactor Nature 503 235
[17] Popova O P et al 2013 Chelyabinsk airburst, damage assessment, meteorite recovery, and characterization Science 342 1069