Private Rank Aggregation under Local Differential Privacy

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Abstract—In typical collective decision making scenarios, rank aggregation aims to combine different agents’ preferences over the given alternatives into an aggregated ranking that agrees the most with all the preferences. However, since the aggregation procedure relies on a data curator, the privacy within the agents’ preference data could be compromised when the curator is untrusted. All existing works that guarantee differential privacy in rank aggregation assume that the data curator is trusted. In this paper, we first formulate and address the problem of locally differentially private rank aggregation, in which the agents have no trust in the data curator. We propose an effective and efficient protocol LDP-KwikSort, with the appealing property that each agent only needs to answer a small number of pairwise comparison queries from the untrusted curator with controllable noise, and the aggregated ranking could maintain an acceptable utility compared with that of the non-private protocol. Theoretical and empirical results demonstrate that the proposed solution can achieve the practical trade-off between the utility of aggregated ranking and the privacy preserving of agents’ pairwise preferences.

Index Terms—Rank Aggregation, KwikSort Algorithm, Local Differential Privacy.

I. INTRODUCTION

Aggregation is the process of combining multiple inputs into a single output which represents all of the inputs in some sense. In the research of computational social choice, the main research issue is on how to better aggregate the preferences of individual agents, or the participating decision makers [1]. In those areas, aggregation provides solutions to problems which often involve multiple individual preferences that could be conflicting. In the realistic scenarios such as recommender system, crowdsourcing or electoral system, the preferences of individual agents are often represented as rank data where the alternatives are ranked in order. Accordingly, rank aggregation has been a topic with broad interests in various applications.

Since most preferences data are inevitably involved with sensitive information of individual agents, the collecting, analysing and publishing of these data would be a potential threat to individual’s privacy. Even though the disclosure of an individual’s preferences is not always embarrassing, the ability to deduce them may make those agents susceptible to coercion. These factors prevent the contributions of accurate preferences from individual agents, and inhibit the performance of the aggregation from being fully realized. However, this concern cannot be comprehensively addressed by traditional privacy-preserving methods such as anonymization, as evidenced by the Hugo Awards 2015 incident [2]. This incident can be attributed to linkage attack, which is possible when the adversary has gained unexpected background knowledge, e.g., another non-anonymous database with some information on the victims.

Considering the above weakness of anonymization techniques and especially in the scenario of aggregated ranking release, two recent works [3], [4] adopted the rigorous differential privacy (DP) framework [5]–[7], and proposed several differentially private rank aggregation algorithms. Based on the properties of the curator model of DP, the data curator is assumed to be fully trusted and can access all agents’ ranking preference profiles, while the adversary with any background knowledge could not confidently infer the existence of an agent’s profile from the aggregated ranking released by the curator.

However, with the increased awareness of privacy preserving in data collection, both the academic and industrial communities are getting more interested in the local model of DP (LDP) [8]–[10] where the curator is assumed to be untrusted. Within the LDP model, the agents could add noise using the designed LDP protocol into their preference profile before reporting to the curator, who could also estimate the population statistics from the noisy data. More specifically, in this paper, we are addressing the following problem called locally differentially private rank aggregation (LDP-RA): the agents have their ranking preferences over the given alternatives, and the curator with the authority and computing capability will collect and aggregate those preference profiles into a final overall ranking list. When the agents are not trusting the curator’s capability of preventing their privacy from potential attacks, the challenges for a private rank aggregation are then on how to enable the agents to avoid sharing their original ranking preference profiles with the untrusted curator, and how to enable the curator to approximately aggregate those rankings with an acceptable utility.

The main contribution of our work is LDP-KwikSort, an effective and efficient LDP protocol for the LDP-RA problem. In the design of such a protocol, we encountered and tackled two technical issues as follows:

- When considering to adopt the widely used randomized response (RR) technique for the LDP protocol design, it is found that RR cannot be directly used in our scenario.
– Instead of adding noise into the whole ranking list on each agent’s side, we focus on protecting the pairwise comparison preferences within the ranking list. To achieve that, we leverage on the approximate rank aggregation algorithm KwikSort [11], [12], which only requires the input as agents’ pairwise preferences over m alternatives. Furthermore, in the proposed protocol, we allow the untrusted curator to ask the agents with pairwise comparison queries, and we let the agents report their answers with the RR technique. Thus, on the one hand, this pairwise comparison preference perturbation phase could satisfy LDP; on the other hand, by the estimation phase, the untrusted data curator could approximately estimate the useful frequencies for rank aggregation. Finally, the proposed protocol could output an aggregated ranking, which preserves the pairwise comparison preferences of the agents.

• When adopting RR in the way as described above, it comes up with the question of how could we choose an appropriate number of rounds to invoke RR, i.e., the query number $K$.

– Although the KwikSort algorithm with full pairwise comparison preferences ($K = \binom{m}{2}$) always outperforms that with incomplete information, it is noted that the situation of the LDP-KwikSort protocol is different due to the fundamental contradiction between the privacy budget and the utility in designing DP algorithms. More specifically, based on the composition theorem in DP theory, given a specific privacy budget $\epsilon$ for each agent, he/she would divide this budget into pieces $\epsilon_K$ for answering each query and a smaller $\epsilon_K$ means more noise in the reported answer. Thus a larger $K$ would lead to a larger RR noise for answering each query, which may distort the overall utility. For better investigating the trade-off between $\epsilon$ and $K$, we provide the error bound of our protocol, which indicates that LDP-KwikSort with $K = 1$ query could outperform that with $K > 1$ queries.

For performance evaluation, we conduct experiments on three real-world datasets (TurkDots, TurkPuzzle and SUSHI) and several synthetic datasets generated from the Mallows model. By observing the average Kendall tau distance resulted from LDP-KwikSort, the state-of-the-art solution DP-KwikSort [4] for the curator model and the non-private KwikSort, it shows that our solution achieves strong privacy protection while maintaining an acceptable utility.

The rest of this paper is organised as follows. Section II provides the background on non-private and private rank aggregation, local differential privacy and related work. Section III formalises the LDP-RA problem and proposes the LDP-KwikSort protocol, followed by theoretical analysis in Section IV, empirical analysis in Section V, and conclusions in Section VI.

II. PRELIMINARIES AND RELATED WORK

In this section, we introduce the relevant concepts of rank aggregation, private rank aggregation and the building blocks of LDP protocol, as well as the related work. Table I lists the notations used in this paper.

| Symbol | Description |
|--------|-------------|
| $N$    | set of n agents, $N = \{1, \ldots, n\}$ |
| $A$    | set of m alternatives, $A = \{a_1, \ldots, a_m\}$ |
| $\mathcal{L}(A)$ | all the possible permutations of alternatives in $A$ |
| $L_i$ | ranking preference profile of agent $i$, where $L_i \in \mathcal{L}(A)$ and $i \in N$ |
| $L_i^{-1}(a_j)$ | ranking index of alternative $a_j$ in $L_i$ |
| $L$ | combined profile that contains all the agents’ ranking preference profiles |
| $L_i(k)$ | aggregated ranking over the given combined profile $L$ |
| $\mathbf{K}(L_i, L_k)$ | Kendall tau distance between $L_i$ and $L_k$ |
| $\mathbf{R}(L_i, L_k)$ | average Kendall tau distance among $L_i$ and all the profiles in $L$ |
| $C_{a_ja_k}(L)$ | count of the times that $L_i^{-1}(a_j) < L_i^{-1}(a_k)$ in $L$ |
| $\text{comp}_L(a_j, a_k)$ | result of comparison function over $(a_j, a_k)$ |
| $\text{comp}_P(L)$ | aggregated pairwise comparison profile |
| $L_{min}$ | aggregated ranking by the proposed protocol |
| $\epsilon$ | overall privacy budget for each agent |
| $K$ | number of queries from the curator to each agent |
| $\theta$ | dispersion parameter of the Mallows model |

A. Rank Aggregation

Given a set of $m$ alternatives that $A = \{a_1, \ldots, a_m\}$ and $n$ agents participated in a preference aggregation procedure, the preference profile of an agent $i \in N$ is represented as a permutation or a ranking $L_i \in \mathcal{L}(A)$ of those $m$ alternatives. Hence, the ranking index of alternative $a_j \in A$ in $L_i$ is denoted by $L_i^{-1}(a_j)$ with a value between 0 (the best) to $m - 1$ (the worst). Then a combined profile of these ranking preference profiles is denoted by $L = (L_1, \ldots, L_n) \in \mathcal{L}(A)^n$, based on which, the rank aggregation algorithm generates a representative ranking $L_{min}$ that sufficiently summarizes $L$.

To evaluate the quality of the aggregated ranking $L_{min}$, Kendall tau distance is commonly used to count the number of pairwise disagreements between two rankings: $\mathbf{K}(L_i, L_j) = |\{(a_j, a_k) : a_j < a_k, L_i^{-1}(a_j) < L_j^{-1}(a_k) \text{ but } L_i^{-1}(a_j) > L_j^{-1}(a_k)\}|$. Then given the vote profile $L = (L_1, \ldots, L_n) \in \mathcal{L}(A)^n$, its average Kendall tau distance with an aggregated ranking $L_{min}$ is defined as: $\mathbf{K}(L_{min}, L) = \frac{1}{n} \sum_{i \in N} \mathbf{K}(L_i, L_i)$. When the above distance achieves the minimum, the relevant $L_{min}$ is referred to as the Kemeny optimal aggregate ranking. However, it is NP-hard to compute this kind of ranking when $n > 3$, and various approximate algorithms have been proposed as summarized in [13].

In this paper, we leverage on a Kendall tau distance based algorithm, KwikSort, which could achieve 11/7-approximation by adopting the QuickSort strategy [12]. Specifically, the sorting of any two alternatives $a_j, a_k \in A$ is based on the counts of how many times $a_j$ (resp. $a_k$) is preferred over $a_k$ (resp. $a_j$) among the rankings in $L$, which is formally defined as $C_{a_ja_k}(L) = |\{\text{for all } L_i \in L | L_i^{-1}(a_j) < L_i^{-1}(a_k)\}|$ and $C_{a_ka_j}(L) = |\{\text{for all } L_i \in L | L_i^{-1}(a_k) < L_i^{-1}(a_j)\}|$. Upon execution, KwikSort would
first randomly pick an alternative \( a_p \in A \) as the pivot, then classify all alternatives \( a_q \in A \) using the comparison function 
\[ \text{cmp}_{L}(a_p, a_q) = (C_{a_q a_p}(L) - C_{a_p a_q}(L)) \in \text{cmp}(L). \]
That is, if \( \text{cmp}_{L}(a_p, a_q) < 0 \) (resp. \( \geq 0 \)), the alternative \( a_q \) would be classified into the left (resp. right) side of the pivot \( a_p \). This procedure repeats until all alternatives have been sorted into a ranking \( L_{KS} \).

B. Private Rank Aggregation

Rank aggregation under the differential privacy (DP) framework is a relatively new topic in the relevant community. [3], [4] considered the curator model of DP in which the trusted data curator has access to all agents’ ranking preference profiles and would release the aggregated ranking by applying a differentially private algorithm \( M \) on them:

**Definition 1 (Differential Privacy).** A randomized algorithm \( M \) satisfies \( \epsilon \)-differential privacy if for all \( \mathcal{O} \subseteq \text{Range}(M) \) and for all neighbouring datasets \( L \) and \( L' \) differing on at most one record \( L_i \) (i.e., the ranking preference profile of agent \( i \)), we have

\[
Pr[M(L) \in \mathcal{O}] \leq e^\epsilon \cdot Pr[M(L') \in \mathcal{O}].
\]

The intuition behind above definition is that the adversary cannot confidently distinguish two outputs (aggregated rankings) of the differentially private algorithm \( M \) on a dataset \( L \) and its neighbouring dataset \( L' \). Then the present or absent status of any agent’s ranking as a record within the input dataset is rigorously protected with the uncertainty of the algorithm’s outputs, which is measured by the privacy budget \( \epsilon \).

To satisfy the definition of differential privacy and for those query functions with numeric output, the Laplace mechanism is usually utilized. Relying on the strategy of adding the query functions with numeric output, the Laplace algorithm’s outputs, which is measured by the privacy budget \( \epsilon \), can be certain about whether the received answer is true or false in terms of a single agent. However, the investigator could still apply a reconstruction procedure to obtain useful statistics, such as how many agents are actually with the answer ‘Yes’.

C. Local Differential Privacy

Unlike the curator model, in the local model [8]–[10] of differential privacy (LDP), each agent would first locally perturb his/her data by adopting a randomized algorithm, referred to as the local randomizer \( M_R \), which satisfies \( \epsilon \)-differential privacy. Then, the agent would upload the perturbed data to the untrusted curator, who should not infer the sensitive information of each agent but can post-process those data to obtain the population statistics for further analysis. LDP is formally defined as:

**Definition 3 (Local Differential Privacy) [5], [14].** A protocol \( \mathcal{P} \) satisfies \( \epsilon \)-local differential privacy (\( \epsilon \)-LDP) if it accesses a dataset \( D = (D_1, ..., D_n) \in D^n \) only via \( K \) invocations of a local randomizer \( M_R \), in which each invocation is \( \epsilon_k \)-differentially private and \( \sum_{k=1}^{K} \epsilon_k \leq \epsilon \). That means for each agent \( i \in [n] \), let \( M_R^{(1)}, ..., M_R^{(K)} \) denote the protocol’s invocations of \( M_R \) on the dataset \( D_i \) (agent \( i \)’s preference profile), the protocol \( \mathcal{P}() \) \( \triangleq (M_R^{(1)}(), M_R^{(2)}(), ..., M_R^{(K)}()) \) is \( \epsilon \)-locally differentially private if for any neighbouring pair of datasets \( D_i, D'_i \in D \) and \( \mathcal{O} \subseteq \text{Range}(M_R) \), we have

\[
Pr[M_R(D_i) \in \mathcal{O}] \leq e^\epsilon \cdot Pr[M_R(D'_i) \in \mathcal{O}].
\]

To design LDP protocols, randomized response (RR) technique [15]–[17] has been widely adopted. As an indirect questioning technique for sensitive questionnaires, RR allows the participating agents to answer questions with plausible deniability. Specifically, when receiving a question in forms of predicate query whose answer can be either ‘Yes’ or ‘No’, the agent is allowed first to flip a biased coin and then gives his/her true answer to the investigator if the coin turns head with a probability \( p \), or otherwise reports the false answer. In this way, the investigator (e.g., an untrusted data curator) cannot be certain about whether the received answer is true or false in terms of a single agent. However, the investigator could still apply a reconstruction procedure to obtain useful statistics, such as how many agents are actually with the answer ‘Yes’.

Formally, a transformation matrix \( M \) is used to include the above coin flipping probabilities \( p_{u,v} \), of a true answer \( v \) being transformed into the provided answer \( u \). For example, in the following matrix, the subscript ‘0’ (or ‘1’) denotes the answer ‘Yes’ (or ‘No’):

\[
M = \begin{pmatrix}
  p_{0,0} & p_{0,1} \\
  p_{1,0} & p_{1,1}
\end{pmatrix} = \begin{pmatrix}
  p & 1-p \\
  1-p & p
\end{pmatrix}.
\]

Then based on the knowledge of \( M \), once obtaining \( x_0 \) (or \( x_1 \)), the number of agents who answer ‘Yes’ (or ‘No’), the curator could use the unbiased maximum likelihood estimate (MLE) [18] to obtain an estimation \( \hat{x}_0 \) (or \( \hat{x}_1 \)), which approximates the number of agents whose true answer are ‘Yes’ (or ‘No’).

\[
\hat{x}_i = \mathbf{M}^{-1} \mathbf{y}, \tag{II.1}
\]
where \( \mathbf{M}^{-1} = (x_0, x_1)^T \), \( \mathbf{y} = (y_0, y_1)^T \), and \( \mathbf{M}^{-1} \) is the inverse matrix of \( M \).

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1 We follow the setting of KwikSort in Python package pwlisorder that directly places \( a_{q} \) after \( a_{p} \) if \( \text{cmp}_{L}(a_{p}, a_{q}) = 0 \). In some other settings, this placement may be done arbitrarily.
Moreover, it is important that RR for binary attribute survey has been proved to satisfy $\epsilon$-differential privacy when $p$ has the following relationship with the privacy budget $\epsilon$ [19]–[21]:

$$p = \frac{e^\epsilon}{1 + e^\epsilon}. \quad (II.2)$$

For the LDP protocols in the voting scenario, the LDP-WeVote protocol in [22] is the closest to our solution. However, LDP-WeVote is only designed to the weighted voting game in which the inputs are the binary data and the output is the winner alternative instead of the ranking over all alternatives. To the best of our knowledge, no existing work has been devoted to locally differentially private rank aggregation (LDP-RA) as defined in Section III.

III. LOCALLY PRIVATE RANK AGGREGATION

In this section, we formalise the problem of locally differentially private rank aggregation (LDP-RA), and then propose a solution called LDP-KwikSort protocol.

A. Problem Formalization

In the context of LDP-RA, given $m$ alternatives, the associated combined profile $L = (L_1, ..., L_n)$ can be considered as an instantiation of $D = (D_1, ..., D_n)$ in Definition 3. The task is then to design a local randomizer $M_R$ which locally perturbs each agent’s ranking preference profile $L_i$ before reporting to the untrusted curator. On the other hand, we expect that the constituted $\epsilon$-LDP protocol $\mathcal{P}$ outputs the aggregated ranking $\hat{L}_\mathcal{P}$ with an acceptable utility as measured by the average Kendall tau distance $\mathbf{K}(\hat{L}_\mathcal{P}, L) = \frac{1}{n} \sum_{i \in N} \mathbf{K}(\hat{L}_\mathcal{P}, L_i)$.

B. Protocol Overview

By leveraging the KwikSort algorithm and RR technique, we propose the locally differentially private KwikSort (LDP-KwikSort) protocol, for which the rationale can be summarized by Figure 1.

Algorithm 1 Agent $i$’s operation in LDP-KwikSort

Input: Agent $i$’s ranking over $m$ alternatives, overall privacy budget $\epsilon$ for each agent, number of queries $K$, coin flipping probability $p$.

Output: Perturbed answers $\tilde{a}_i$ regarding the queries.

1: Receives $K$ queries (e.g., do you prefer alternative $a_j$ to $a_k$?) from curator.
2: for each query $k \in [K]$ do
3: Randomly generates a number $r$ from $[0, 1]$. ▶ Local Randomizer
4: if $r \leq p$ then
5: Use the true answer.
6: else
7: Use the false answer.
8: end if
9: end for
10: Reports the perturbed answers $\tilde{a}_i$ to curator.

D. Curator’s Operation

As shown in Algorithm 2, once collected the noisy answers from the agents, the curator would activate several procedures to obtain the final ranking. Firstly (Line 2), for each possible pair of alternatives $a_j$ and $a_k$, the curator would count the numbers of how many agents preferred alternative $a_j$ to alternative $a_k$ as $\tilde{C}_{a_j,a_k}(L)$, and how many agents reported the opposite preference as $\tilde{C}_{a_k,a_j}(L)$. Secondly (Line 3 – 6), the curator would adopt the estimation procedure by Eq. (II.1), where $\tilde{X} = (\tilde{C}_{a_j,a_k}(L), \tilde{C}_{a_k,a_j}(L))$, $\tilde{Y} = (\tilde{C}_{a_j,a_k}(L), \tilde{C}_{a_k,a_j}(L))^T$. Then the results from the comparison function $\tilde{cmp}(a_j, a_k)$ could be computed. Furthermore, after the pairwise comparison results of all the possible pairs of alternatives have been computed, the curator would assemble them into the estimated version of aggregated pairwise comparison profile $\tilde{cmp}(L)$. Thirdly (Line 7 – 8), by taking $\tilde{cmp}(L)$ as input, the standard KwikSort algorithm would be executed to generate the aggregated ranking $\hat{L}_\mathcal{P}$.

Fig. 1. Rationale for LDP-KwikSort Protocol
Algorithm 2 Curator’s operation in LDP-KwikSort

Input: Agents’ perturbed answers \( a \), overall privacy budget \( \epsilon \) for each agent, number of queries \( K \), coin flipping probability \( p \).

Output: Aggregated ranking \( L \).

1: for each possible pair \( a_j \) and \( a_k \) do
2: \( \triangleright \) Aggregation, estimation and computation
3: Aggregates the combined profile as counts \( \tilde{C}_{a_j a_k}(L) \) and \( \tilde{C}_{a_k a_j}(L) \).
4: Estimates the accurate numbers \( \hat{C}_{a_j a_k}(L) \) and \( \hat{C}_{a_k a_j}(L) \) by Eq. (II.1).
5: \( \triangleright \) KwikSort
6: Obtains the estimated version of aggregated pairwise comparison profile \( \hat{C}^{\text{cmp}}(L) \).
7: Executes the KwikSort algorithm. \( \triangleright \) KwikSort
8: return \( L \).

IV. THEORETICAL ANALYSIS

In this section, we provide the privacy and utility guarantees, as well as the computational complexity of the proposed LDP-KwikSort protocol.

A. Privacy Guarantee of LDP-KwikSort

We have the following theorem on the privacy guarantee of the LDP-KwikSort protocol.

Theorem 1. The LDP-KwikSort protocol satisfies the \( \epsilon \)-local differential privacy.

Proof. Recall that in Algorithm 1, for each participating agent \( i \in N \), the RR technique is adopted as the local randomizer \( M_R \) to perturb the original answer \( o_i \) into \( \tilde{o}_i \) for \( K \) invocations. In each invocation, since the answer could be ‘Yes’ (denoted as \( v_0 \)) or ‘No’ (denoted as \( v_1 \)), for any input \( v_0, v_1 \) and output \( y \), we have

\[
Pr[M_R(v_0) = y] \leq \frac{p}{1 - p} = \frac{e^{\epsilon/K}/(e^{\epsilon/K} + 1)}{1/(e^{\epsilon/K} + 1)} = e^{\epsilon/K}.
\]

Consequently, according to Definition 3, we can conclude that the LDP-KwikSort protocol satisfies the \( \epsilon \)-local differential privacy. \( \square \)

B. Utility Guarantee of LDP-KwikSort

As described in Algorithm 2, the proposed LDP-KwikSort protocol mainly contains two parts: the aggregated pairwise comparison profile estimation and the KwikSort algorithm. It is known that KwikSort could output \( 11/7 \)-optimal aggregated ranking with any given aggregated pairwise comparison profile \( \hat{C}^{\text{cmp}}(L) \). Then it is important to measure the error of \( \hat{C}^{\text{cmp}}(L) \) from the first part, which indicates the overall performance of LDP-KwikSort. When investigating the internal disagreements of the original aggregated pairwise comparison profile \( Cmp(L) \) and its estimated version \( \hat{C}^{\text{cmp}}(L) \), we use the following Eq. (II.1):

\[
\hat{C}^{\text{cmp}}(a_j, a_k) = \hat{C}_{a_j a_k}(L) - \hat{C}_{a_k a_j}(L) = \frac{\hat{C}_{a_j a_k}(L) - \hat{C}_{a_k a_j}(L)}{p - (1 - p)} = \frac{\hat{C}^{\text{cmp}}(a_j, a_k)}{2p - 1},
\]

where \( \hat{C}_{a_j a_k}(L) \) and \( \hat{C}_{a_k a_j}(L) \) represent the observed counts of the answers \( L_i^{-1}(a_j) < L_i^{-1}(a_k) \) and \( L_i^{-1}(a_k) < L_i^{-1}(a_j) \) on the curator’s side.

Since the denominator \((2p - 1)\) is always positive, we conclude that the sign of \( \hat{C}^{\text{cmp}}(a_j, a_k) \) is same as that of \( \hat{C}^{\text{cmp}}(a_j, a_k) \). As the KwikSort algorithm determines the order of two alternatives only by checking the sign of \( \hat{C}^{\text{cmp}}(a_j, a_k) \), it is the key to measure the probability that the sign of \( \hat{C}^{\text{cmp}}(a_j, a_k) \) is different with that of \( Cmp(a_j, a_k) \). That is to say, the achieved utility of LDP-KwikSort would rely on the condition that for the ground truth \( L_{(k)}^{-1}(a_j) < L_{(k)}^{-1}(a_k) \) whether the curator could directly obtain it by just observing the noisy data from the agents.

Now we present an error bound and the associated proof which are based on the assumption that the agents’ ranking preference profiles are generated by the Mallows model [23]. The Mallows model is a probabilistic model for ranking generation in which the probability of generating a ranking is based on the dispersion parameter \( \theta \in [0, 1] \) and the distance between this ranking and a ground truth ranking. Specially, \( \theta = 1 - \frac{q_M}{p_M} \) where \( p_M \in [1/2, 1] \) is the probability for generating the relative ranks of every pair of alternatives \( a_j \) and \( a_k \) which is consistent with the ground truth, and \( q_M = 1 - p_M \) is the false probability.

Theorem 2. If all the ranking preference profiles are generated by the Mallows model with the dispersion parameter \( \theta \) and a ground truth ranking, the estimation part of LDP-KwikSort produces the error \( < 6\mu \) with probability at least 1 - \( 2 - 2^\mu \), where \( \mu = 2 \mu \exp(\frac{-\mu^2 \ln^2(\frac{1}{\sqrt{2}})}{\epsilon^2/4 + 1}) \) and \( \theta = \frac{\mu}{\sqrt{\mu}} \).

Proof. In our scenario, for a certain pairwise comparison query ‘whether \( L_i^{-1}(a_j) < L_i^{-1}(a_k) \) or not’, there would be \( n^* = \frac{k}{\binom{2}{\frac{k}{2}}} \) agents involved, and each agent reports his/her true answer (resp. false answer) with probability \( p = \frac{e^{\epsilon/K}}{e^{\epsilon/K} + 1} \) (resp. \( q = 1 - p = \frac{1}{e^{\epsilon/K} + 1} \)). When considering the assumption regarding the Mallows model, an agent finally reports the ground truth answer (i.e., \( L_i^{-1}(a_j) < L_i^{-1}(a_k) \) for all \( j < k \) with probability \( p = p_M \cdot q + q_M \cdot q \) and reports the opposite answer with probability \( q = p_M \cdot q + q_M \cdot p \). Since the above procedure could be seen as a Bernoulli trial, herein we adopt the variation of Hoeffding’s inequality [24] for the Bernoulli trial as follows:

Lemma 1. For a Bernoulli trial in which \( n \) times of experiment have been conducted and each experiment outputs outcome \( A \)
with probability $p$ and outcome $B$ with probability $q = 1 - p$, we have the following probability bound for some $\delta > 0$:
$$Pr[(p - \delta)n \leq H_1(n) \leq (p + \delta)n] \geq 1 - 2\exp(-2\delta^2 n),$$
where $H_1(n)$ is the number of outcome $A$ in $n$ experiments.

We then have the following probability bounds by Lemma 1:
$$Pr[(p - \delta)n^* \leq \tilde{C}_{a,ak}(L) \leq (p + \delta)n^*] \geq 1 - 2\exp(-2\delta^2 n^*),$$
$$Pr[(q - \delta)n^* \leq \tilde{C}_{a,a_j}(L) \leq (q + \delta)n^*] \geq 1 - 2\exp(-2\delta^2 n^*),$$
and hence
$$Pr[(p - q - 2\delta)n^* \leq \tilde{C}_{a,ak}(L) - \tilde{C}_{a,a_j}(L) \leq (p - q + 2\delta)n^*] \geq 1 - 2\exp(-2\delta^2 n^*).$$

Next, when we set $p - q = 2\delta$, the following probability bounds are obtained:
$$Pr[0 \leq \tilde{C}_{a,ak}(L) - \tilde{C}_{a,a_j}(L) \leq 4\delta n^*] \geq 1 - 2\exp(-2\delta^2 n^*)$$
and
$$Pr[\bar{c}_{cm}PI(a_j, a_k) = \tilde{C}_{a,ak}(L) - \tilde{C}_{a,a_j}(L) < 0]
\leq 2\exp(-2\delta^2 n^*)
= 2\exp(-2(pM \cdot p + qM \cdot q + pM \cdot q + qM \cdot p)^2 n^*)
= 2\exp(-2\theta^2 nK
(\epsilon + 2K)^2 m(m - 1))
= P_1, \quad \text{(IV.2)}$$
where $\theta^* = \frac{\theta}{2 - \theta}$ and we adopt the inequality $\epsilon^2 \geq x + 1$ for simplifying the result in the last derivation.

After obtaining the error probability for a certain pairwise comparison query, we now apply the following Chernoff bound for all possible queries:

Lemma 2. Let $X_1, \ldots, X_m$ be independent Poisson trials such that $Pr(X_i = 1) = p_i$. Let $X = \sum_{i=1}^m X_i$ and $\mu = E[X]$. Then for $R \geq 6\mu$,
$$Pr(X \geq R) \leq 2^{-R}.$$

Thus, we finish the proof with obtaining
$$\mu = E[X] = E[\sum_{i=1}^m X_i] = \sum_{i=1}^m E[X_i] = \sum_{i=1}^m P_i
\leq 2 \left( \frac{m}{2} \right) \exp(-\theta^2 \epsilon^2 nK
(\epsilon + 2K)^2 m(m - 1)). \quad \text{(IV.3)}$$

As a summary of above analysis, we observe that 1) when the number of alternatives $m$, the number of agents $n$ and the privacy budget $\epsilon$ are fixed, the error probability would increase as the increasing number of queries $K$; 2) when more agents are involved or given a large privacy budget, i.e., $n \to \infty$ or $\epsilon \to \infty$, the error probability could be reduced to zero; and 3) when the dispersion parameter $\theta$ is relatively large, i.e., the generated rankings are closer to the ground truth ranking, we could obtain a high probability for achieving a lower error. In Section V, we conduct a series of experiments to verify these theoretical results.

C. Computational Complexity of LDP-KwikSort

We analyse the computational complexity of the proposed protocol with $K$ queries in terms of the following aspects.

Running time. For each agent, since only one query is required, the running time of Algorithm 1 is $O(K)$. On the curator’s side, before the execution of Algorithm 2, the querying phase consumes the running time $O(Kn)$, and then the aggregation, the estimation and the computation phase in Algorithm 2 consume $O((\frac{m}{2})^2) \approx O(m^2)$ while the followed KwikSort procedure consumes $O(m \log m)$. Thus, the total running time of the curator’s operation is $O(Kn) + O(m^2) + O(m \log m)$.

Processing memory. As LDP-KwikSort maintains $n$ times the sums of at most $K$ bit for each agent, the processing memory is $O(Kn)$.

Communication cost. The number of queries directly impacts the communication cost, which includes $O(K)$ for each agent and $O(Kn)$ for the curator.

V. EMPIRICAL ANALYSIS

In this section, we present the performance evaluation of the proposed LDP-KwikSort protocol and the competitors on both real and synthetic datasets, investigate how will the parameters ($\epsilon$, $K$, $m$ and $n$) impact on performance.

A. Experiment Settings

1) Competitors: We consider KwikSort, DP-KwikSort and Baseline as the competitors of the LDP-KwikSort protocol.

KwikSort. The non-private approximate rank aggregation algorithm KwikSort described in [12] would show an empirical error lower bound. We adopt this algorithm in experiments as the non-private protocol, in which each agent responds to each query with the true answer to the curator, and the latter releases the aggregated ranking without adding any noise. We provide the performance of KwikSort with $K = 1$ query and $K = \frac{m}{2}$ queries, respectively.

DP-KwikSort. The state-of-the-art differentially private algorithm DP-KwikSort [4] could be seen as a private protocol under the curator model of DP, in which each agent responds to each query with the true answer to the curator, and the latter adopts the Laplace mechanism to introduce the noise into the comparison function during the rank aggregation. The noisy comparison function is denoted as $\bar{c}_{cm}PI(a_j, a_k) = \bar{c}_{cm}PI(a_j, a_k) + Z$ where $Z \sim \text{Lap}(1/\epsilon')$ and $\epsilon' = \epsilon/(m - 1) \log m$. The number of queries in DP-KwikSort is default as $K = \frac{m}{2}$.

Baseline. We consider the Baseline as the proposed protocol LDP-KwikSort with the maximum number of queries $K = \frac{m}{2}$. When being executed, each agent reports
his true answer of each query to the curator with probability \( p = \frac{\epsilon}{1 + \epsilon^2} \), and the curator proceeds to reconstruct statistics and then releases the aggregated ranking. In the following experiments, we also provide the performance of LDP-KwikSort with \( K = m \) queries for better observing the change in trend.

2) Datasets and Configuration: The involved four protocols are implemented in Python 2.7 based on the package pwl distortor 0.1 [25], and executed on an Intel Core i5 – 3210M 2.50GHz machine with 6GB memory. In each experiment, each protocol was tested 30 times, and its mean score of the adopted utility metrics was reported.

The experiments are conducted on three real-world datasets (TurkDots, TurkPuzzle and SUSHI) and several synthetic datasets. Among them, TurkDots and TurkPuzzle datasets [26] were collected from the crowdsourcing marketplace Amazon Mechanical Turk, which respectively contain \( n = 795 \) and \( n = 793 \) agents’ full ranking preference profiles over \( m = 4 \) alternatives. SUSHI dataset [27] contains the full rankings of \( m = 10 \) types of sushi from \( n = 5000 \) agents in a questionnaire survey. The synthetic datasets were generated by the Mallows model with R package PerMallows 1.13 [28]. We set \( \theta = 0.25, \theta = 0.5 \) and \( 0.75 \) in experiments, which indicates that the generated rankings are relatively far from or near the ground truth ranking, while \( m \) and \( n \) could be varied on demand.

3) Utility Metrics: Error Rate. We coin the measurement error rate to reflect how accurate the estimated version of aggregated pairwise comparison profile \( \hat{\text{cmp}}(L) \) agrees with the original profile \( \text{cmp}(L) \). Specifically, for all the possible alternative pairs \( a_j \) and \( a_k \) where \( j < k \),

\[
\text{Error Rate} = \frac{|\{\text{cmp}_L(a_j, a_k) \geq 0 \text{ but } \hat{\text{cmp}}_L(a_j, a_k) < 0\}|}{\binom{m}{2}} + \frac{|\{\text{cmp}_L(a_j, a_k) < 0 \text{ but } \hat{\text{cmp}}_L(a_j, a_k) \geq 0\}|}{\binom{m}{2}}.
\]

(V.1)

Average Kendall Tau Distance. As mentioned before, we measure the achieved accuracy of the aggregated ranking \( L_P \) by adopting the average Kendall tau distance \( K(L_P, L) = \frac{1}{n} \sum_{i \in N} K(L_P, L_i) \). Furthermore, for the convenience of comparison with different \( m \), we normalise the values by \( m(m-1)/2 \).

B. Results of Error Rate

In this experiment, we adopt the Mallows datasets with dispersion parameter \( \theta = 0.5 \), set the number of agents \( n = 5000 \) and vary the number of alternatives \( m \in \{15, 30, 45\} \) as well as the privacy budget \( \epsilon \in \{0.01, 0.1, 0.5, 1.0, 2.0, 3.0\} \). When given the original aggregated pairwise comparison profile \( \text{cmp}(L) \), we compare the involved four protocols in terms of their error rate of the noisy aggregated pairwise comparison profiles \( \hat{\text{cmp}}(L) \) and \( \hat{\text{cmp}}(L) \), respectively.

Table II shows that the LDP-KwikSort protocol with \( K = 1 \) consistently outperforms the ones with the maximum and the smaller number of queries (e.g., \( K = \binom{m}{2} \) and \( K = m \)), and its performance is close to that of DP-KwikSort. As the increase of the privacy budget \( \epsilon \), all protocols have lower error rates. Given a large number of alternatives \( m \), say 45, all the protocols have higher error rates compared with the situations with smaller \( m = 15 \) or 30. The results indicate that when we only focus on the estimate accuracy of the sign of \( \hat{\text{cmp}}_L(a_j, a_k) \) instead of its absolute value in \( \hat{\text{cmp}}(L) \), asking

| Protocols                  | \( \epsilon = 0.01 \) | \( \epsilon = 0.1 \) | \( \epsilon = 0.5 \) | \( \epsilon = 1.0 \) | \( \epsilon = 2.0 \) | \( \epsilon = 3.0 \) |
|---------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| DP-KwikSort(\( K_{\text{Max}} \)) | 0.2768                 | 0.0102                 | 0                      | 0                      | 0                      | 0                      |
| LDP-KwikSort(\( K_{\text{Max}} \)) | 0.4987                 | 0.4737                 | 0.4692                 | 0.3994                 | 0.3244                 | 0.2594                 |
| LDP-KwikSort(\( K = m \)) | 0.4927                 | 0.4879                 | 0.3714                 | 0.2860                 | 0.1324                 | 0.0600                 |
| LDP-KwikSort(\( K = 1 \)) | 0.4654                 | 0.3654                 | 0.1349                 | 0.0435                 | 0.0152                 | 0.0073                 |

\( \theta = 0.5, n = 5000, m = 15 \)

| Protocols                  | \( \epsilon = 0.01 \) | \( \epsilon = 0.1 \) | \( \epsilon = 0.5 \) | \( \epsilon = 1.0 \) | \( \epsilon = 2.0 \) | \( \epsilon = 3.0 \) |
|---------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| DP-KwikSort(\( K_{\text{Max}} \)) | 0.3808                 | 0.0483                 | 0.0005                 | 0                      | 0                      | 0                      |
| LDP-KwikSort(\( K_{\text{Max}} \)) | 0.4905                 | 0.4886                 | 0.4755                 | 0.4551                 | 0.4375                 | 0.4128                 |
| LDP-KwikSort(\( K = m \)) | 0.4900                 | 0.4740                 | 0.4309                 | 0.3897                 | 0.2968                 | 0.2295                 |
| LDP-KwikSort(\( K = 1 \)) | 0.4339                 | 0.3882                 | 0.2076                 | 0.0841                 | 0.0325                 | 0.0192                 |

\( \theta = 0.5, n = 5000, m = 30 \)

| Protocols                  | \( \epsilon = 0.01 \) | \( \epsilon = 0.1 \) | \( \epsilon = 0.5 \) | \( \epsilon = 1.0 \) | \( \epsilon = 2.0 \) | \( \epsilon = 3.0 \) |
|---------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| DP-KwikSort(\( K_{\text{Max}} \)) | 0.4186                 | 0.0937                 | 0.0005                 | 0                      | 0                      | 0                      |
| LDP-KwikSort(\( K_{\text{Max}} \)) | 0.4950                 | 0.4925                 | 0.4923                 | 0.4885                 | 0.4704                 | 0.4581                 |
| LDP-KwikSort(\( K = m \)) | 0.4900                 | 0.4925                 | 0.4923                 | 0.4885                 | 0.4704                 | 0.4581                 |
| LDP-KwikSort(\( K = 1 \)) | 0.4060                 | 0.3697                 | 0.2380                 | 0.1260                 | 0.0454                 | 0.0273                 |

\( \theta = 0.5, n = 5000, m = 45 \)
one query from the curator would be much better than asking multiple queries. As shown in Algorithm 2, the KwikSort algorithm would take as input $\text{cmp}_L(a_j, a_k)$, we investigate the utilities of the aggregated ranking in the following experiments.

C. Results of Average Kendall Tau Distance

1) The Impact of Privacy Budget: In this experiment, we ran LDP-KwikSort and its competitors on three real-world datasets and Mallows datasets with dispersion parameter $\theta =$ 0.25, 0.5, 0.75. For the TurkDots, TurkPuzzle and SUSHI datasets, the numbers of agents and alternatives are fixed by default. And for the Mallows datasets, we set $n =$ 5000 and $m =$ 15. With varying the privacy budget $\epsilon \in \{0.01, ..., 5.0\}$, we investigate how does this parameter impact the average Kendall tau distance of the aggregated rankings. Firstly, the results in Figure 2 show that the non-private KwikSort ($K$ Max) determines the overall lower error bound and DP-KwikSort ($K$ Max) approaches it rapidly although with a small privacy budget, say $\epsilon = 0.1$. That is because the Laplacian noise could be well controlled in the curator model. Secondly, for the local model protocols, LDP-KwikSort ($K =$ 1) consistently outperforms other ones, and it gets close to the curve of DP-KwikSort ($K$ Max) more quickly with a larger $\epsilon$. It is also noted that KwikSort ($K =$ 1) determines the lower error bound of LDP-KwikSort ($K =$ 1). Thirdly, different distributions of preference profiles would also impact the performances. For example, in Figures 2(a) and 2(d), when $\epsilon = 1.0$, LDP-KwikSort ($K =$ 1) achieves the average Kendall tau distance of 0.4343 and 0.1212, and it underperforms DP-KwikSort ($K$ Max) by 6.6% and 13.4%, respectively. Moreover, all protocols achieve a better performance on the Mallows dataset with $\theta = 0.75$, and this is due to the fact that a larger $\theta$ means the generated rankings are closer to the ground truth ranking, which makes it easy for the protocols to output an approximate optimal aggregated ranking. Finally, it shows that KwikSort ($K =$ 1) always underperforms KwikSort ($K$ Max), and their gap would be relatively large when given a small $\theta$, and DP-KwikSort ($K$ Max) could outperform KwikSort ($K =$ 1) by increasing $\epsilon$.

2) The Impact of Agents and Alternatives Amounts: We then investigate the effect of the numbers of agents $n$ and alternatives $m$. In Figure 3(a)-3(c), all protocols are executed on the real-world datasets in which the number of alternatives are fixed by default and the privacy budget is set to $\epsilon = 1.0$. We can see that KwikSort ($K$ Max) still shows the lower error bound among the involved protocols, and with increasing the number of agents, say from $n = 10$ to $n = 50$ in Figure 3(c), it can achieve an error below 0.34. That is because when the agent number is less than the number of possible pairs of alternatives, e.g., $m = 45$, KwikSort ($K$ Max) could not aggregate a sufficient pairwise comparison profile in which each pairwise comparison $\text{cmp}_L(a_j, a_k)$ should be non-zero. For the LDP-KwikSort protocol, we observe that the case of $K =$ 1 generally outperforms that of maximum $K$. For example, when having $n =$ 500 agents participated in crowdsourcing, $K =$ 1 case outperforms the case with maximum $K$ by 6.5%, 7.6% and 7.3% improvement on these three datasets, respectively. Basically, the involved protocols show the tendency of convergence as the increasing of the

![Fig. 2. Comparison of protocols in terms of average Kendall tau distance on the adopted datasets, across varying the privacy budget $\epsilon \in \{0.01, ..., 5.0\}$.

![Fig. 3. Results of Average Kendall Tau Distance on the adopted datasets, across varying the number of agents $n$ and alternatives $m$.](attachment:image.png)
Fig. 3. Comparison of protocols in terms of average Kendall tau distance on the adopted datasets, across varying number of agents $n$. Privacy budget $\epsilon$ is fixed at 1.0.
number of agents.

In Figure 3(d)-3(l), we run the involved protocols on the Mallows datasets with \( \theta \in \{0.25, 0.5, 0.75\} \), vary the number of agents in a much larger range from \( n = 10 \) to \( n = 10000 \), and observe the performance under \( m \in \{15, 30, 45\} \) and \( \epsilon = 1.0 \). The results show that with an increasing number of alternatives from \( m = 15 \) to \( m = 45 \), the lower error bound by KwikSort (\( K \) Max) is decreasing, and it is much harder for the other protocols especially LDP-KwikSort (\( K \) Max) to achieve this bound, which is consistent with the similar empirical results in [4]. Given a large number of agents, LDP-KwikSort with \( K = 1 \) shows a significant improvement. However, even with a large \( n \), LDP-KwikSort (\( K \) Max) always shows an unstable performance, especially when the number of alternatives is relatively large.

D. Summary and Recommendations

Based on above empirical results, we can conclude that the proposed LDP-KwikSort protocol with one query can achieve the closest performance of DP-KwikSort in terms of error rate and the average Kendall tau distance when compared with the protocols with multiple queries. After all, our proposed LDP-KwikSort protocol is based on the assumption of untrusted data curator, and could satisfy a more realistic scenario of protecting agents’ privacy compared with DP-KwikSort. When using the proposed protocol, we recommend \( \epsilon = 1.0 \) for the situations with fewer alternatives such as \( m \leq 15 \), and \( \epsilon = 3.0 \) when considering a relatively large number of alternatives (e.g., \( m = 45 \).

VI. Conclusion

Rank aggregation aims to combine different agents’ preferences over the given alternatives into an aggregated ranking that agrees the most with all the preferences. In the scenario of collective decision making, since the aggregation procedure relies on a data curator, the privacy within the agents’ preference data could be compromised when the curator is untrusted. All existing works that guarantee differential privacy in rank aggregation assume that the data curator is trusted. This chapter first formalises and studies the locally differentially private rank aggregation (LDP-RA) problem. Specifically, we design the LDP-KwikSort protocol which could protect the pairwise comparison within the ranking list and show a combination of the properties from the approximate rank aggregation algorithm KwikSort and the RR technique. Theoretical results and empirical results on the real-world and synthetic datasets demonstrate that each agent only needs to answer a small number of pairwise comparison queries from the untrusted curator with controllable noise, and the aggregated ranking could maintain an acceptable utility compared with that of the non-private protocol.

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