Photoproduction of Pentaquark $\Theta^+$ and Chiral Symmetry Restoration in Hot and Dense Medium

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The photoproduction rate of pentaquark $\Theta^+$ is calculated in a hot and dense medium. At high temperature and density, due to the restoration of chiral symmetry, photoproduction energy threshold is increased. Above the threshold the production cross section is strongly enhanced.

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I. INTRODUCTION

The pentaquark baryon $\Theta^+ (uudd\bar{s})$ candidate was first observed by the LEPS group at Spring-8 in the $\gamma n \rightarrow K^+ K^- n$ reaction\cite{1}. However, the existence of the pentaquark states is remaining an open question as some experimental groups confirmed the observation\cite{2} and some reported negative results\cite{3}. Since Diakonov et al. predicted the possible existence of the exotic baryon in their chiral soliton model\cite{4}, lots of theoretical calculations for $\Theta^+$ production and related dynamics have been carried out, see refs\cite{5,6,7}.

At hadron level the process $\gamma n \rightarrow \Theta^+ K^-$ has been analyzed with different couplings\cite{8,9}, namely the pseudoscalar(PS), pseudovector(PV) couplings, and the hybrid model(HM) which incorporates both PS and PV couplings where PS dominates the interactions at low energy and PV is responsible for the high energy interactions. Taking into account the finite hadron size effects, the cross sections for the process are 120 nb and 15 nb for positive and negative parity $\Theta^+$, respectively.

More recently the pentaquark production was investigated with medium effects\cite{7,12}. Including the effects of Quark-Gluon plasma(QGP) formation and hadronic interactions the authors reported their results of pentaquark production in central Au+Au collisions at RHIC\cite{12}. They found that the yields of $\Theta^+$ is determined by the initial formation from QGP phase and the hadronic rescatterings do not affect the total cross sections remarkably.

It is well known that the spontaneous breaking of chiral symmetry controls the vacuum properties of hadrons. The restoration of the symmetry at finite temperature and density is one of the most important predictions of the lattice gauge theory(LGT) calculation. It is found that the effect of the symmetry restoration on the pentaquark production is significant and sensitive to the parity of the pentaquark. In the hot and dense medium, the shift in mass $\Delta M_\Theta$, and the shift in width $\Delta \Gamma_\Theta$ are found to be much larger for positive parity $\Theta^+$ than that for negative parity. While the magnitude of the mass shift is much smaller than the mass $M_\Theta = 1540$ MeV in vacuum, the shift in width is several times larger than the vacuum width. On the other hand, in the absence of the chiral restoration, the shifts in mass and width can be neglected.

In this paper, we present the results of photoproduction rate of $\Theta^+$ in medium. The effects of the chiral symmetry restoration at high temperature and density on the production will be analyzed. The formula for $\Theta^+$ production in medium, including the PS, PV and HM couplings, will be presented in Section II. Results and discussions are in Section III and the summaries will be given in the last Section.

II. FORMULAS

We take the effective pseudovector and pseudoscalar $NN\Theta^+ K$ couplings\cite{3,8,9,12},

$$\mathcal{L}_{PV} = -\frac{g_\gamma^2}{2f_\pi} \Theta_{\mu\nu\gamma\delta} \left( \partial^\mu K^+ n - \partial^\mu K^0 p \right),$$

$$\mathcal{L}_{PS} = ig\Theta_{\gamma5} \left( K^+ n - K^0 p \right),$$

(1)

where, the positive parity of $\Theta^+$ is assumed. Since the parity of $\Theta^+$ has not yet been determined experimentally, we will consider in the following the interactions with negative parity of $\Theta^+$ by removing the factor $i\gamma_5$ from the above vertexes. The pseudovector and pseudoscalar coupling constants $g_\gamma^2$ and $g$ can be fixed by reproducing the mass $M_\Theta = 1540$ MeV and decay width $\Gamma_\Theta = 15$ MeV in the vacuum. Through the calculation at tree level one has $g_\gamma^2 = 0.28$ and $g = 3.8$ for positive parity and $g_\gamma^2 = 0.16$ and $g = 0.53$ for negative parity\cite{12}.

The in-medium static properties, especially the particle masses, are different from that in vacuum. It is well known that the in-medium behavior of nucleons can be well described by the chiral symmetry restoration at finite temperature and density. While the quantitative result depends on the models used, the qualitative temperature and density dependence of the nucleon mass $M_N$ is not sensitive to the details of different chiral models\cite{13}. A simple model to describe chiral symmetry breaking in vacuum and symmetry restoration in medium is the Nambu-Jona-Lasinio (NJL) model\cite{14}.

Within this model, one can obtain the hadronic mass spectrum and the static properties of mesons remarkably well. In particular, one can recover the Goldstone mode, and some important low-energy properties of current algebra such as the Goldberger-Treiman and Gell-Mann-Oakes-Renner relations\cite{15}. In mean field approximation
where $\mu$ is the baryon chemical potential, $T$ is the temperature, $N_c$ and $N_f$ are the color and flavor degrees of freedom, $m_q$ and $E_p = \sqrt{p^2 + m_q^2}$ are the effective quark mass and energy, and $f_f$ is the Fermi-Dirac distribution function $f_f(x) = 1/(e^{x/T} + 1)$. The three parameters in the model, the current quark mass $m_0$ which breaks explicitly the chiral symmetry in vacuum, the coupling constant $G$, and the momentum cutoff $\Lambda$, can be fixed by fitting the chiral condensate, pion mass and pion decay constant in vacuum [12, 13]. The temperature and density dependence of the nucleon mass calculated through [4] is very close to that obtained from the other often used models describing nuclear matter [10].

Since the effects of the attractive scalar and the repulsive vector potentials cancel each other, the mass of $K^+$ only increase slightly at finite temperature and density. Although the $K^-$ mass drops much larger in the medium, it is negligible compared to the changes of in-medium nucleon mass. For these reasons, we take the kaon mass as a constant through out our calculations [8, 9].

The temperature and density effect on the pentaquark $\Theta^+$ mass and width can be determined through calculating the lowest order $\Theta^+$ self-energy in the medium [8].

Now we come to the in-medium calculation of the cross section for the process $\gamma n \rightarrow \Theta^+ K^-$ shown diagrammatically in Fig. 1. To have a comparison with the calculations in vacuum [6, 14], we take only the diagrams at tree level. For the channel PS, we consider the Born terms Fig.1b, c and d, while for the channel PV, we should include the contact term Fig.1e because of the replacement $\partial_\mu \rightarrow \partial_\mu + ieA_\mu$ in the lagrangian [11].

The electromagntical vertexes in Fig. 1 are defined through the Lagrangian density [9].

\[
\mathcal{L}_{\gamma KK} = \frac{ie}{2} \left[ \left( \bar{K}^+ K^- \right) K^+ - \left( \bar{K}^- K^+ \right) K^- \right] A_\mu ,
\]

\[
\mathcal{L}_{\gamma nn} = -\frac{ie\kappa_n}{M_n} \bar{n}\sigma_{\mu\nu} k^\nu n A_\mu ,
\]

\[
\mathcal{L}_{\gamma \Theta \Theta} = -e\bar{\Theta} \left( \gamma_\mu + i\frac{\kappa_\Theta}{2M_\Theta} \sigma_{\mu\nu} k^\nu \right) \Theta A_\mu ,
\]

where $\kappa_n$ and $\kappa_\Theta$ are the neutron and pentaquark magnetic moments in unit of nuclear magneton, we take $\kappa_n = -1.91$ and $\kappa_\Theta = 0$ [10] through out the calculation.

In the initial nucleon rest frame, the cross section at tree level in the medium can be written as

\[
\sigma(k) = \int_0^{\theta_{\text{max}}} d\theta \sin \theta \frac{1}{32\pi k M_N \sqrt{(k + M_N)(k') - E_{K^-} k \cos \theta}} \times \sum |M|^2 (1 - f_f(E_\Theta - \mu)) (1 + f_f(E_{K^-})) ,
\]

where $M = M_{\gamma n \rightarrow \Theta^+ K^-}$ is the amplitude of the process, $\theta$ is the angle between the initial photon and final kaon momenta, $\theta_{\text{max}}$ is the maximum angle restricted by energy and momentum conservations, and $E_{K^-} = \sqrt{k^2 + M_{K^-}^2}$ and $E_\Theta = \sqrt{p^2 + M_{\Theta}^2}$ are $K^-$ and $\Theta^+$ energies. The summation symbol in (3) means the average over the initial states (the photon polarization and neutron spin) and the summation over the final states (the pentaquark spin). The temperature and density effect in the cross section is hidden in the nucleon and pentaquark masses which are governed by the chiral symmetry restoration and shown explicitly in the final state statistics factor $(1 - f_f)(1 + f_f)$ with the Bose-Einstein distribution function $f_f(x) = 1/(e^{x/T} - 1)$.

For the hybrid model (HM), which combines the PS and PV channels, can better reproduce the experimental result than the individual PS or PV channel in vacuum, we use the following amplitude [11].

\[
M_{\text{HM}} = M_{\text{PS}} + \Delta M \cdot F_{\text{HM}} ,
\]

\[
\Delta M = M_{\text{PV}} - M_{\text{PS}} ,
\]
where $F_{HM} = \Lambda_{HM}^2 / \left( \Lambda_{HM}^2 + k^2 \right)$ is the mixing parameter depending on the kaon momentum $k'$ and the cutoff $\Lambda_{HM} = 450 \text{ MeV}$ [6, 15].

III. NUMERICAL RESULTS

The results of the in-medium pentaquark production cross section, as a function of photon energy $E_\gamma = k$, are shown in Fig. 2. Both positive and negative parities of the pseudoscalar (PS), pseudovector (PV) and hybrid couplings (HM) are shown in left- and right-plots, respectively. In each case, we consider three different properties of the medium characterized by temperature $T$ and baryon chemical potential $\mu$. The values of $(T, \mu) = (0, 0), (0.2, 0.05) \text{ GeV}, (0.15, 0.3) \text{ GeV}, (0.1, 0.8) \text{ GeV}$ (dashed lines) and $(0.1, 0.8) \text{ GeV}$ (dot-dashed lines) are approximately corresponding to the fireballs created in heavy ion collisions at RHIC, SPS, and AGS, respectively. The results corresponding to the vacuum are also shown as solid-lines in the plots [6, 14]. In all calculations only baryon chemical potential is considered and the strangeness chemical potential is ignored. Therefore the pentaquark chemical potential is the same as the nucleon chemical potential. Since the photon energy is in the range of 10 GeV in heavy ion collisions [16], we limited our calculation at 12 GeV.

We first discuss the production threshold:

$$k_{th} = \frac{(M_\Theta + M_K)^2 - M_N^2}{2M_N}.$$  (6)

It is found that the dependence on the temperature and potential of the nucleon mass [12, 13] is much stronger than that of the pentaquark mass [1]. At finite temperature and potential, the decreased nucleon mass leads to the increase of the $k_{th}$. From the equation (6), we have $M_N(T, \mu)/M_N(0, 0) = 0.32, 0.71, 0.49$ correspond-
ing to the mediums with \((T, \mu) = (0.2, 0.05)\) GeV, 
\((0.15, 0.3)\) GeV, and \((0.1, 0.8)\) GeV, the maximum shift is for \((T, \mu) = (0.2, 0.05)\) GeV and the minimum shift is for \((T, \mu) = (0.15, 0.3)\) GeV. Since the nucleon mass is independent of either the coupling or the parity, the shifts are the same for all couplings and parities.

As one can see in Fig. 2 when above the threshold, the production cross section in the medium is enhanced compared to the vacuum production. In the cases of PS and HM, the enhancement is universal at any temperature and density. Similar to the threshold shift, the enhancement strongly depends on the nucleon mass drop, i.e. the degree of chiral symmetry restoration.

As we mentioned above, the temperature and density are introduced into our calculation through two ways. The first one is through the effective nucleon mass which is dominated by the degree of chiral symmetry restoration. The second one, independent of the chiral properties, is the statistical factor \((1 - f_f)(1 + f_b)\) and the sum-mation of the loop frequency when the in-medium pentaquark mass is evaluated. In order to make sure that the chiral symmetry restoration is the driving force for the threshold shift and the enhancement in the production cross section, we now repeat the cross section calculations, for PS\(^+\) coupling, without the chiral transition and forcing the nucleon mass at its vacuum value. The result is shown in Fig. 3. For the other channels the results are similar. Qualitatively different from the case with chiral symmetry restoration shown in Fig. 2 where the threshold shifts to the right significantly, it shifts now to the left slightly due to the small decrease of the in-medium pentaquark mass resulted from the loop frequency summation. From the ratio of the cross section without chiral restoration to the one in the vacuum, shown in the right panel of Fig. 3 the in-medium effect on the cross section can be neglected, especially in the high energy region, if the chiral phase transition is turned off.

We also analyzed the dependence of the production cross section on magnetic moment \(\kappa_{\Theta}\) and width \(\Gamma_{\Theta}\). The hadron masses and the production threshold \(k_{th}\) are independent of \(\kappa_{\Theta}\), but the amplitude \(M\) is related to \(\kappa_{\Theta}\) due to the vertex \(\gamma \Theta \Theta\). The estimated range of the magnetic moment is \(0 < \kappa_{\Theta} < 0.7\) \[5, 6, 14\]. A 50\% increase in the production cross section is found in our calculation changing \(\kappa_{\Theta}\) from zero to 0.7 at \(E_\gamma = 12\) GeV with PV\(^-\) coupling. In general, we find that the production cross section is increased with \(\kappa_{\Theta}\) for negative parity and decreased with \(\kappa_{\Theta}\) for positive parity \(\Theta^+\). In all above calculations, a fixed width \(\Gamma_{\Theta} = 15\) MeV has been used \[8\]. When we change \(\Gamma_{\Theta}\), the nucleon mass does not change and the pentaquark mass shift in the medium is always small compared with its vacuum value \(M_{\Theta} = 1540\) MeV. When we neglect the in-medium mass shift we reach the following relationship between production cross sections:

\[
\frac{\sigma_{\Gamma_{\Theta}}}{\sigma_{\Gamma_{\Theta}}} \sim \frac{\Gamma_{\Theta}^1}{\Gamma_{\Theta}^2}.
\]

IV. CONCLUSIONS

We reported our investigation on the production of pentaquark \(\Theta^+\) through the process \(\gamma n \rightarrow \Theta^+ K^-\). Medium effects via PS, PV, HM couplings were included in the study. The NJL model was used to describe the mean field with chiral degrees of freedom. At high temperature and density, we found that the restoration of chiral symmetry has two significant effects on the pentaquark production rates: (1) The threshold shifts towards higher energy; (2) Beyond the threshold, the total

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**Fig. 3**: The cross section without considering the chiral symmetry restoration for the channel PS\(^+\). The left panel is the cross section around the threshold, and the right panel is the ratio of the cross section in the medium to the one in the vacuum. The solid, dotted, dashed and dot-dashed lines correspond, respectively, to the mediums with \((T, \mu) = (0, 0), (0.2, 0.05)\) GeV, 
\((0.15, 0.3)\) GeV, and \((0.1, 0.8)\) GeV.
cross section is strongly enhanced. Without the chiral phase transition, the medium effect on the pentaquark production is found to be negligible.

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