Reduction of a noise influence with sinusoidal signal meters

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Abstract. The purpose of the article is to suggest digital processing methods of sinusoidal
signal measurements data as well as compensation of sinusoidal noises. This kind of task
arises, for example, in implementation of feeder control of electrical power networks,
in which an additional ELF voltage source is included between the controlled network and the ground. A
leakage current measuring device for this frequency should be installed on each feeder. The
suggestion is to use averaging of the sinusoidal signal over several periods to reduce the effect
of random interference. For this purpose, several periods of the signal are stored in digital
form, measured after equidistant moments of time. Time sampling frequency is selected at least
an order of magnitude higher than the frequency of the signal being measured. Then the values
"bound" to a particular signal phase are averaged. In addition, it was proposed to use the
approximation of the obtained points by a trigonometric polynomial. A series of averaged
values corresponding to one period are approximated for this purpose by the sum of the
constant component as well as the cosine and sinusoidal components. For approximation, the
least squares method is used, which provides the minimum discrepancy between the measured
values and the analytical function. The measurement result is presented in a complex form, i.e.
as a sum of cosine (actual) and sinusoidal (imaginary) components. As a result, it is possible to
isolate oneself from noise and measure the sinusoidal signal with high precision.

1. Introduction
It is quite often necessary to reduce the influence of noise when measuring low amplitude
sinusoidal signals. Such a task arises at the feeder control of insulation resistance of electrical power networks [1
- 5]. One of the common methods of per-feeder control is that an additional ELF voltage source, e.g.
1 Hz, is included between the monitored network and the ground. Each outgoing feeder is equipped
with a current transformer, which is configured to measure the signal at this frequency [6, 7]. If the
resistance of the feeder insulation decreases, the leakage current measured by the current transformer
increases [8]. This method can be used for both DC and AC networks [9, 10].

The lower the frequency of the additional voltage, the weaker the capacitive current associated with
the presence of capacitance between the cable veins and the ground, therefore justify use the infra-low
frequency. For example, Schneider Electric as one of the leading companies in this field uses a
frequency of 2.5 Hz. Low frequency induces low EMF in the current transformer, which also leads to
a deterioration in the signal-to-noise ratio. In existing feeder control systems, the sensitivity limit of the meters allows to detect a decrease in insulation resistance of an order of 200 kOhm.

However, there is a need to detect feeders with an insulation resistance of 1 MOhm to 10 MOhm [11], that can be ensured by increasing the sensitivity of the current meter and reducing the frequency of the additional voltage source. Modern measuring tools allow you to significantly increase the weakness of the signals. As a rule, the remaining problem consists in the need to reduce the impact of noise.

2. Synchronous averaging method

The proposed synchronous averaging method can be used to measure the low-amplitude sinusoidal signal in the presence of noise. The essence of the synchronous averaging method is as follows. The noise-containing measured signal is sampled in time and by level using an ADC. The result is a series of values \( b(t_i) \), where \( i \) is the measurement number and \( t_i \) is the measurement time points. According to Kotelnikov's theory, the sampling frequency should be at least twice as high as the frequency of the signal being measured. Further we will consider an example, if the sampling frequency is 16 times higher than the frequency of the signal being measured, i.e. for single (1) period of the frequency of the signal being measured 16 values are obtained, let's designate this number as \( L \). In accordance with the proposed method, the obtained values are grouped into groups of \( L \) values, for instance, 16 values. In each group the numbers of elements (values) are "bound" to a certain phase of the signal, for example, the first one is always at the beginning of the period and so on. By digitizing several periods of the signal being measured and splitting the data into groups, we get the result, which is illustrated in Figure 1.

![Figure 1. Two digitised periods](image)

Figure 1 as an example presents the results of digitization of two periods, however tens of periods can be used in practice. Let us designate the number of periods used for further processing as \( M \). As a result, we produce a data matrix with the dimension \( L \times M \).

Then it is necessary to average all \( M \) values of the first line, as well as all \( M \) values of the second line and so on. As a result, we get \( L \) (e.g., 16) of the average values bound to the phase of the signal being measured, i.e. we get the column \( b \) with \( L \) elements. Since the interference has a random character, the influence of the interference on the result decreases due to averaging. With the use of \( M \) periods, the RMS deviation decreases by a factor of \( \sqrt{M} \) [12, 13]. So, for example, to reduce random noise by 10 times it is necessary to perform averaging of 100 periods.

The measurement process must be synchronized by a separate signal. The measured signal itself cannot be used to obtain synchronizing time stamps because it contains noise. The signal to be measured may be very small and it is difficult to obtain synchronizing time stamps. In addition, in many cases it is not only necessary to measure the amplitude of the sinusoidal signal, but also to
determine the phase. To determine the phase, it is necessary to have a synchronizing signal with reference to which the phase is defined.

If the measured signal has a power grid voltage frequency of 50 Hz, the synchronizing signal can be obtained using the power grid voltage, e.g. using the power supply circuitry. If the measured signal is connected to a voltage source of infra-low frequency, which is connected for feeder control, the synchronizing signal must be generated by this voltage source.

3. Application of the least squares method

Once a series of averaged values has been obtained, which corresponds to one period of time for the signal being measured, it is possible to proceed to obtain its numerical characteristics. As a rule, interest is aroused by the amplitude value, effective (RMS) value. However, according to the authors, it is necessary first to obtain the cosine and sinusoidal components of the signal, which corresponds to the actual and imaginary component of the corresponding vector. Approximation by the method of least squares is very suitable for this purpose. With the help of approximation it is possible to select a function with cosine and sinusoidal (real and imaginary) components, which passes through the obtained points as closely as possible.

It is quite necessary to take into account the presence of some constant displacement of the signal being measured before proceeding to form an analytical expression for the approximating function. Even when using modern operating amplifiers, it is necessary to take into account the presence of zero shift and temperature drift in them. In addition, ADC microcontrollers can usually measure positive polarity voltages. The obtained values are also positive, e.g. from 0 to 4095. If the signal to be measured has both polarities, it should be shifted to the positive side, and then the zero value of the analog signal will correspond to the positive value of the result of analog-to-digital conversion, for example, 2048. Of course, it is possible to subtract 2048 from each measured value and operate with variables that have a sign. Anyway, some offset will remain due to the influence of different errors.

In connection with the above, the most suitable approximation function would be a trigonometric polynomial of the form [14]:

$$ b(t) = \frac{A_0}{2} + \sum_{k=1}^{n} (A_k \cdot \cos(k \cdot \omega \cdot t) + B_k \cdot \sin(k \cdot \omega \cdot t)) $$

(1)

where $k \cdot \omega$ is the angular frequency of the $k$-th harmonic oscillation.

This trigonometric polynomial is a partial sum of the Fourier series. To obtain a signal offset, as well as to obtain a real and imaginary component of the signal, it is sufficient to use only the first harmonic. In this case, it will be necessary to find three unknowns, denote them $x_1, x_2, x_3$, and in accordance with this, the expression (1) will appear as:

$$ b(t) = x_1 + x_2 \cdot \cos(\omega \cdot t) + x_3 \cdot \sin(\omega \cdot t). $$

Unknown coefficients $x_1, x_2, x_3$ can be found by solving the system of linear algebraic equations (SLAE) using the method of least squares. Let us compile a SLAE of the form:

$$
\begin{align*}
  x_1 + x_2 \cdot \cos(\omega \cdot t_1) + x_3 \cdot \sin(\omega \cdot t_1) &= b(t_1) \\
  x_1 + x_2 \cdot \cos(\omega \cdot t_2) + x_3 \cdot \sin(\omega \cdot t_2) &= b(t_2) \\
  &\vdots \\
  x_1 + x_2 \cdot \cos(\omega \cdot t_N) + x_3 \cdot \sin(\omega \cdot t_N) &= b(t_N)
\end{align*}
$$

Record the system in matrix form:
Let us designate the left matrix as $A$, the column of unknowns as $x$, the column of free members as $b$. Then the equation has appeared as:

$$ A \cdot x = b. $$

There are three unknowns and $N$ equations in this system of equations. The obtained system is redefined, as the number of equations exceeds the number of unknown ones. Based on the least squares method, the solution to the overridden SLAE of $A \cdot x = b$ type, with which there will be the minimum sum of the difference squares of the left and right parts of the system equations, is the solution of the following system:

$$ A^T \cdot A \cdot x = A^T \cdot b. $$

The following expression is the solution to this SLAE:

$$ x = (A^T \cdot A)^{-1} \cdot A^T \cdot b. $$

(2)

Such an SLAE can be solved by one of the known methods, for example, by excluding Gauss method, by Jordan method or by another one. After finding the values of $x_1$, $x_2$ and $x_3$, there are essentially real and imaginary components of the measured signal that correspond to the values of $x_2$ and $x_3$ respectively.

If you assume that the number of measured values corresponds to one period of the measured signal, then expression (2) can be simplified considerably. In this case, the series of obtained values, i.e. column $b$ with $L$ elements, corresponds to one period. The result looks like this [14]:

$$ x_1 = \frac{1}{L} \cdot \sum_{i=1}^{L} b(t_i), $$

$$ x_2 = \frac{2}{L} \cdot \sum_{i=1}^{L} (b(t_i) \cdot \cos(\omega \cdot t_i)), $$

(3)

$$ x_3 = \frac{2}{L} \cdot \sum_{i=1}^{L} (b(t_i) \cdot \sin(\omega \cdot t_i)). $$

(4)

The following conclusions can be made. If the number of processed values does not correspond to or does not correspond exactly to one period of the measured signal, the SLAE solution (2) should be used to calculate the values $x_2$ and $x_3$. If the number of processed values corresponds to one measurement period, use expressions (3) and (4) to calculate the values $x_2$ and $x_3$.

By calculating the values $x_2$ and $x_3$, we obtain the corresponding actual and imaginary components of the measured signal. The amplitude value of the $I_a$ signal is determined by the equation:

$$ I_a = \sqrt{x_1^2 + x_2^2}. $$

The effective (RMS) value of $I_d$ is determined by the equation:

$$ I_d = \frac{I_a}{\sqrt{2}}. $$

4. Conclusion

Implementation of the proposed methods of digital data processing for the measurement of sinusoidal signals allowed producing a non-contact current meter of high sensitivity. With an internal diameter of the core ring of the current meter of 30 mm and a frequency of 1 Hz, this meter can measure current...
with an amplitude of 50 µA, while the error of measurement does not exceed 10%. For currents with an amplitude of 1 mA, the error does not exceed 1%.

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