Weak Phases from Topological-Amplitude Parametrization

- Introduction
  - Theoretically Clean Cases
  - Cases Which Need Theoretical Inputs

- Amplitude Parametrization
  - $B \to K\pi$ case
  - $B \to \pi\pi$ case

- Conclusions
I. Introduction

One of the major missions in $B$ physics is to determine the weak phases in the CKM matrix for CP violation

**Unitarity Test:** Search for deviation from SM

- Using direct measurements of angles check:
  \[ \alpha + \gamma + \beta = \pi \]

- Check if $B$ decays are consistent with the range of angles from the CKM fit?

To determine the weak phase angles, one has two different methods:

- Theoretically Clean Cases
- Cases Which Need Theoretical Inputs
Theoretically Clean Cases

Tree dominated process $\rightarrow$ Penguins Free or Small Penguins contributions

Problem: Some of the cases have very small branching ratio $\rightarrow$ difficult to measure.

Cases Which Need Theoretical Inputs

Have both Tree(T) and Penguins(P) contributions. The interference between T and P causes the uncertainties up to $\sim 30\%$

Problem: One needs theoretical inputs to constrain the uncertainties. For example, impose Isosping Symmetry, SU(3), U-Spin....
II. Amplitude Parametrization

We impose counting rules for the various amplitudes in terms of power of Wolfenstein parameter $\lambda \sim 0.22$.

- Assign an explicit power of $\lambda$ to each topology according to PQCD.

- Drop the topologies with higher power of $\lambda$ until the number of free parameters are equal to the number of measurements.
  - To $O(\lambda^2)$, the error is $\sim 5\%$
  - To $O(\lambda)$, the error is $\sim 20\%$

- Solve the simultaneous equations to get weak phase and amplitudes.

- Check the solved amplitudes see if they satisfy the power counting rules from PQCD.
The Branching ratio:

\[ B(B \rightarrow M_1M_2) = \frac{\tau_B}{16\pi m_B^3} |A(B \rightarrow M_1M_2)|^2 \]

where

\[ m_B = 5.28 \text{ GeV}, \quad \tau_{B^\pm} = 1.674 \times 10^{-12} \text{s}, \]

\[ \tau_{B^0} = 1.542 \times 10^{-12} \text{s} \]

The effective Hamiltonian:

\[ H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qs}^* V_{qb} \left[ C_1(\mu)O_1^{(q)}(\mu) + C_2(\mu)O_2^{(q)}(\mu) \right. \]

\[ + \left. \sum_{i=3}^{10} C_i(\mu)O_i(\mu) \right] \]

Where \( V_{qq} \) is the elements of CKM matrix.

The operators
\begin{align*}
O_1^{(q)} &= (s_i q_j)_{V-A} (\bar{q}_j b_i)_{V-A}, \\
O_2^{(q)} &= (s_i q_i)_{V-A} (\bar{q}_j b_j)_{V-A}, \\
O_3 &= (s_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V-A}, \\
O_4 &= (s_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A}, \\
O_5 &= (s_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V+A}, \\
O_6 &= (s_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A}, \\
O_7 &= \frac{3}{2} (s_i b_i)_{V-A} \sum_q e_q (\bar{q}_j q_j)_{V+A}, \\
O_8 &= \frac{3}{2} (s_i b_j)_{V-A} \sum_q e_q (\bar{q}_j q_i)_{V+A}, \\
O_9 &= \frac{3}{2} (s_i b_i)_{V-A} \sum_q e_q (\bar{q}_j q_j)_{V-A}, \\
O_{10} &= \frac{3}{2} (s_i b_j)_{V-A} \sum_q e_q (\bar{q}_j q_i)_{V-A},
\end{align*}

The $i, j$ is the color indices.

The caracteristic scale is

$$\mu \sim \sqrt{m_b \bar{\Lambda}} \sim 1.5 \text{Gev}$$

and $\bar{\Lambda} = m_B - m_b$. 
The Wilson coefficients are

\begin{align*}
C_1 &= -0.510, & C_2 &= 1.268, \\
C_3 &= 2.7 \times 10^{-2}, & C_4 &= -5.0 \times 10^{-2}, \\
C_5 &= 1.3 \times 10^{-2}, & C_6 &= -7.4 \times 10^{-2}, \\
C_7 &= 2.6 \times 10^{-4}, & C_8 &= 6.6 \times 10^{-4}, \\
C_9 &= -1.0 \times 10^{-2}, & C_{10} &= 4.0 \times 10^{-3}.
\end{align*}

The Wolfenstein parametrization for the CKM matrix is

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]

\[
= \begin{pmatrix}
1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix},
\]

where \(\lambda = 0.2196 \pm 0.0023\), \(A = 0.819 \pm 0.035\), and \(R_b \equiv \sqrt{\rho^2 + \eta^2} = 0.41 \pm 0.07\).

The phases \(\phi_1\) and \(\phi_3\) are defined via \(V_{td} = |V_{td}| \exp(-i\phi_1)\) and \(V_{ub} = |V_{ub}| \exp(-i\phi_3)\), respectively.
The most general parametrization of the $B \to K\pi$ decay amplitudes are given by

$$A(B^+ \to K^0\pi^+) = P \left(1 - \frac{P_{ew}}{P} + \frac{T^a}{P}e^{i\phi_3}\right),$$

$$A(B^0_d \to K^+\pi^-) = -P \left(1 + \frac{T}{P}e^{i\phi_3}\right),$$

$$\sqrt{2}A(B^+ \to K^+\pi^0) = -P \left[1 + \frac{P_{ew}}{P}ight.$$  
$$+ \left(\frac{T}{P} + \frac{C}{P} + \frac{T^a}{P}\right)e^{i\phi_3}\left.\right],$$

$$\sqrt{2}A(B^0_d \to K^0\pi^0) = P \left(1 - \frac{P_{ew}}{P} - \frac{P_{ew}^c}{P} - \frac{C}{P}e^{i\phi_3}\right),$$

where $P = P_{QCD} + e_uP_{ew}^c + e_uP_{ew}^a$

We have 12 unknowns (6 amplitudes (11 unknowns) + $\phi_3$) with 9 experimental inputs.

From PQCD, one have

$$\frac{M^{nf}}{F^e} \sim \left[\ln \frac{m_B}{\Lambda_{QCD}}\right]^{-1} \sim \lambda,$$

$$\frac{F^a_{(V-A)}}{F^e} \sim \frac{\Lambda_{QCD}}{m_B} \sim \lambda^2,$$

$$\frac{F^a_{(V+A)}}{F^e} \sim \frac{2m_0}{m_B} \sim \lambda^0,$$
Assign the power counting rule to the Wilson coefficients

\[
\begin{align*}
O(1) & : a_1 , \\
O(\lambda) & : a_2 , 1/N_c \\
O(\lambda^2) & : C_4 , C_6 , a_4 , a_6 \\
O(\lambda^3) & : C_3 , C_5 , C_9 , a_3 , a_5 , a_9 \\
O(\lambda^4) & : C_{10} , \\
O(\lambda^5) & : C_7 , C_8 , a_7 , a_8 , a_{10}
\end{align*}
\]

with \( a_1 = C_2 + C_1/N_c \), \( a_2 = C_1 + C_2/N_c \), and \( a_i = C_i + C_{i-1}/N_c \) for \( i = 4, 6, 8, 10 \).

Combine all above, one have

\[
\begin{align*}
\frac{T}{P} & \sim \frac{V_{us}V_{ub}^* a_1}{V_{ts}V_{tb}^* a_{4,6}} \sim \lambda , \\
\frac{P_{ew}}{P} & \sim \frac{a_9}{a_{4,6}} \sim \lambda , \\
\frac{C}{T} & \sim \frac{a_2}{a_1} \sim \lambda , \\
\frac{T^a}{T} & \sim \frac{F^a_{(V-A)}}{F^e} \sim \frac{M^{nf}}{F^e} \frac{C_1}{a_1N_c} \sim \lambda^2 , \\
\frac{P^{cw}}{P} & \sim \frac{a_{8,10}}{a_{4,6}} \sim \frac{M^{nf}}{F^e} \frac{C_9}{a_{4,6}N_c} \sim \lambda^3 , \\
\frac{P^{aw}}{P} & \sim \frac{F^a_{(V+A)} a_{8,10}}{F^e} \sim \frac{M^{nf}}{F^e} \frac{C_9}{C_{4,6}N_c} \sim \lambda^3 .
\end{align*}
\]
The power of $P_{ew}^c/P \sim \lambda^3$, is different from the parametrization of Gronau and Lendon($\lambda^2$).

Drop $O(\lambda^3)$ term($T^a/P, P_{ew}^c/P, P_{ew}^a/P$), and $A_{CP}(B^\pm \to K^0\pi^\pm)$, 
$\Rightarrow$ 8 equations with 8 unknowns. 
$\Rightarrow$ Error is $\sim 5\%$

Since the time-dependent asymmetry in the $B_d^0 \to K_S\pi^0$ decay still has big uncertainty, drop futher $O(\lambda^2)$ term. We have

\[
A(B^+ \to K^0\pi^+) = P, \\
A(B_d^0 \to K^+\pi^-) = -P \left(1 + \frac{|T|}{P} e^{i\phi_3} e^{i\delta_T}\right), \\
\sqrt{2}A(B^+ \to K^+\pi^0) = -P \left(1 + \frac{|P_{ew}|}{P} e^{i\delta_{ew}} \right. \\
\left. + \frac{|T|}{P} e^{i\phi_3} e^{i\delta_T}\right), \\
\sqrt{2}A(B_d^0 \to K^0\pi^0) = P \left(1 - \frac{|P_{ew}|}{P} e^{i\delta_{ew}}\right),
\]
and 6 experimental data

\[ \text{Br}(B^\pm \to K^0 \pi^\pm) = (20.6 \pm 1.4) \times 10^{-6} , \]
\[ \text{Br}(B^0_d \to K^{\pm} \pi^{\mp}) = (18.2 \pm 0.8) \times 10^{-6} , \]
\[ \text{Br}(B^\pm \to K^{\pm} \pi^0) = (12.8 \pm 1.1 \times 10^{-6} , \]
\[ \text{Br}(B^0_d \to K^0 \pi^0) = (11.5 \pm 1.7) \times 10^{-6} , \]
\[ \mathcal{A}(B^0_d \to K^{\pm} \pi^{\pm}) = -(10.2 \pm 5.0)\% , \]
\[ \mathcal{A}(B^\pm \to K^{\pm} \pi^0) = -(9.0 \pm 9.0)\% . \]

Solve the simutaneous equations, We have center values

\[ \frac{|T|}{P} = 0.23 , \quad \delta_T = -13^o , \quad 0.06 < \frac{|T|}{P} < 0.72 \]
\[ \frac{|P_{ew}|}{P} = 0.50 , \quad \delta_{ew} = -88^o , \quad 0.22 < \frac{P_{ew}}{P} < 0.70 \]
\[ \phi_3 = 102^o , \quad 26^o < \phi_3 < 151^o , \]

Agree with the results from PQCD, QCDF and Rosner and Gronau(hep-ph/0307095)

Large electroweak penguin? ⇒ New physics?
\[ B \to K\pi \text{ to } O(\lambda^2) \]

Consider upto \( O(\lambda^2) \), we have the amplitudes

\[
A(B^+ \to K^0\pi^+) = P, \\
A(B_d^0 \to K^+\pi^-) = -P \left( 1 + \frac{|T|}{P} e^{i\phi_3} e^{i\delta_T} \right), \\
\sqrt{2}A(B^+ \to K^+\pi^0) = -P \left[ 1 + \frac{|P_{ew}|}{P} e^{i\delta_{ew}} + \left( \frac{|T|}{P} e^{i\delta_T} + \frac{|C|}{P} e^{i\delta_C} \right) e^{i\phi_3} \right], \\
\sqrt{2}A(B_d^0 \to K^0\pi^0) = P \left( 1 - \frac{|P_{ew}|}{P} e^{i\delta_{ew}} - \frac{|C|}{P} e^{i\delta_C} \right),
\]

with two new experimental data:

\[
A(B_d^0(t) \to K_S\pi^0) = -C_{K\pi} \cos(\Delta M_d t) + S_{K\pi} \sin(\Delta M_d t),
\]

where

\[
C_{K\pi} = \frac{1 - |\lambda_{K\pi}|^2}{1 + |\lambda_{K\pi}|^2} = 0.4 \pm 0.28 \pm 0.1 \\
S_{K\pi} = \frac{2 \text{ Im}(\lambda_{K\pi})}{1 + |\lambda_{K\pi}|^2} = 0.48 \pm 0.47 \pm 0.11
\]

and

\[
\lambda_{K\pi} = e^{2i\phi_1} \frac{P + P_{ew} + C e^{-i\phi_3}}{P + P_{ew} + C e^{i\phi_3}}.
\]
Found possible solutions for center value:

\[
\frac{|T|}{P} = 0.33 \ , \ \delta_T = -8.9^\circ ,
\]
\[
\frac{|P_{ew}|}{P} = 0.50 \ , \ \delta_{ew} = 94.2^\circ ,
\]
\[
\frac{|C|}{P} = 0.26 \ , \ \delta_C = 94^\circ ,
\]
\[\phi_3 = 99^\circ , \]

or

\[
\frac{|T|}{P} = 0.15 \ , \ \delta_T = -19.8^\circ ,
\]
\[
\frac{|P_{ew}|}{P} = 0.48 \ , \ \delta_{ew} = -87.2^\circ ,
\]
\[
\frac{|C|}{P} = 0.26 \ , \ \delta_C = 146^\circ ,
\]
\[\phi_3 = 93^\circ , \]
\[ B \to \pi\pi \text{ Case} \]

The general parametrizations are:

\[
\sqrt{2}A(B^+ \to \pi^+\pi^0) = -T \left[ 1 + \frac{C}{T} + \frac{P_{\text{ew}}}{T} e^{i\phi_2} \right],
\]

\[
A(B^0_d \to \pi^+\pi^-) = -T \left( 1 + \frac{P}{T} e^{i\phi_2} \right),
\]

\[
\sqrt{2}A(B^0_d \to \pi^0\pi^0) = T \left[ \left( \frac{P}{T} - \frac{P_{\text{ew}}}{T} \right) e^{i\phi_2} - \frac{C}{T} \right],
\]

The power counting rules are

\[
\frac{P}{T} \sim \frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \frac{a_{4,6}}{a_1} \sim \lambda,
\]

\[
\frac{C}{T} \sim \lambda,
\]

\[
\frac{P_{\text{ew}}}{T} \sim \lambda^2,
\]

\[
\frac{T^a}{T} \sim \frac{M_{nf}}{M_e} \frac{C_2}{a_1 N_c} \sim \lambda^2,
\]

\[
\frac{P_{\text{ew}}^c}{T} \sim \frac{P_{\text{ew}}^a}{T} \sim \lambda^4.
\]
Drop the $O(\lambda^2)$ and higher terms, we have

\[
\sqrt{2}A(B^+ \to \pi^+\pi^0) = -T \left(1 + \frac{|C|}{T} e^{i\delta_C}\right),
\]

\[
A(B_{d}^{0} \to \pi^+\pi^-) = -T \left(1 + \frac{|P|}{T} e^{i\phi_2 e^{i\delta_P}}\right),
\]

\[
\sqrt{2}A(B_{d}^{0} \to \pi^0\pi^0) = T \left(\frac{|P|}{T} e^{i\phi_2 e^{i\delta_P}} - \frac{|C|}{T} e^{i\delta_C}\right),
\]

and the time dependent CP asymmetry

\[
A(B_{d}^{0}(t) \to \pi^+\pi^-) = \frac{B(\bar{B}_d^{0}(t) \to \pi^+\pi^-) - B(B_{d}^{0}(t) \to \pi^+\pi^-)}{B(\bar{B}_d^{0}(t) \to \pi^+\pi^-) + B(B_{d}^{0}(t) \to \pi^+\pi^-)}
= -C_{\pi\pi} \cos(\Delta M_d t) + S_{\pi\pi} \sin(\Delta M_d t),
\]

where

\[
C_{\pi\pi} = \frac{1 - |\lambda_{\pi\pi}|^2}{1 + |\lambda_{\pi\pi}|^2}, \quad S_{\pi\pi} = \frac{2 \text{Im}(\lambda_{\pi\pi})}{1 + |\lambda_{\pi\pi}|^2},
\]

and

\[
\lambda_{\pi\pi} = e^{2i\phi_2} \frac{1 + e^{-i\phi_2 P/T}}{1 + e^{i\phi_2 P/T}}.
\]
The 5 experimental data are

\[
\begin{align*}
\text{Br}(B^\pm \to \pi^\pm \pi^0) &= (5.2 \pm 0.8) \times 10^{-6}, \\
\text{Br}(B_d^0 \to \pi^\pm \pi^\mp) &= (4.6 \pm 0.4) \times 10^{-6}, \\
\text{Br}(B_d^0 \to \pi^0 \pi^0) &= (1.97 \pm 0.47) \times 10^{-6}, \\
C_{\pi\pi} &= -(38 \pm 16)\%, \\
S_{\pi\pi} &= -(58 \pm 20)\%.
\end{align*}
\]

and assume

\[
A(B_d^0 \to \pi^0 \pi^0) = (0 \pm 50)\%.
\]

We have four available solutions

\[
\begin{align*}
\frac{P}{T} &= 0.21e^{96^\circ i}, & C &= 0.99e^{-84^\circ i}, & \phi_2 &= 107^\circ, \\
\frac{P}{T} &= 0.21e^{76^\circ i}, & C &= 0.83e^{76^\circ i}, & \phi_2 &= 111^\circ, \\
\frac{P}{T} &= 0.67e^{162^\circ i}, & C &= 0.49e^{-18^\circ i}, & \phi_2 &= 72^\circ, \\
\frac{P}{T} &= 0.67e^{10^\circ i}, & C &= 0.16e^{170^\circ i}, & \phi_2 &= 147^\circ.
\end{align*}
\]

with the range of \(\phi_2\) is

\[51^\circ < \phi_2 < 176^\circ\]
III. Conclusions

- In $B \rightarrow K\pi$ case, the uncertainty can be as small as $\sim 5\%$.

- To $O(\lambda)$, we get $\phi_3 = 102^\circ$. The results satisfy the power counting and agree with PQCD.

- Large electroweak penguin ($P_{ew}/P = 0.5$) ⇒ New Physics?
  Since available range is still wide ($0.2 \sim 0.7$) ⇒ Might not be an evidence for new physics!

- In $B \rightarrow \pi\pi$ mode, the $P_{ew}$ might not be small ($O(\lambda^2)$). One need to re-exame the calculation from PQCD, as well as QCDF, analysis of $B \rightarrow \pi\pi$.

- Need to put more theoretical efforts to extract $\phi_2$ from $B \rightarrow \pi\pi$ data.