Quantum Synchronization on the IBM Q System

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We report the first experimental demonstration of quantum synchronization. This is achieved by performing a digital simulation of a single spin-1 limit-cycle oscillator on the quantum processors of the IBM Q System. Applying an external signal to the oscillator, we verify typical features of quantum synchronization and demonstrate an interference-based quantum synchronization blockade. Our results show that state-of-the-art noisy intermediate-scale quantum processors are powerful enough to implement realistic open quantum systems. Finally, we discuss limitations of current quantum hardware and define requirements necessary to investigate more complex problems.

I. INTRODUCTION

Synchronization, i.e., the adjustment of the rhythm of a self-sustained oscillation to a weak perturbation, is a universal feature of many complex dynamic systems [1]. Classical synchronization has been demonstrated in a variety of very different setups ranging from electrical circuits to biological neuron systems [2–4]. Several proposals have been made to study quantum effects of synchronization in superconducting circuits [5, 6], optomechanical systems [7, 8], trapped ions [9, 10], and nanomechanical oscillators [11]. However, all the experimental demonstrations of synchronization reported to date on these platforms were operating in the classical regime [12–21], because of the challenge of sustaining a highly nonlinear oscillator in the quantum regime.

In this article, we report the first experimental demonstration of quantum synchronization. Our quantum limit-cycle oscillator is implemented in a single spin-1 system, which was recently introduced as the smallest possible system that can be synchronized [22]. We use two qubits of a quantum processor to implement the desired spin-1 system while the remaining qubits play the role of the environment sustaining the oscillation. The advantage of this approach is that the nonlinear dissipation required to study quantum synchronization corresponds to easily engineered single-qubit relaxation. With this mapping in place, we perform a digital quantum simulation [24, 25] of the synchronization dynamics on the publicly available few-qubit quantum processors at the IBM Q System [26].

The ongoing efforts to build a quantum computer have resulted in noisy intermediate-scale quantum (NISQ) devices, which are constantly improving in terms of decoherence and relaxation times, gate fidelities, and readout fidelities [27]. NISQ devices have become a highly relevant platform for simulating realistic physical problems and they have already been used to find quantum ground states [28–30] and to simulate closed-system quantum many-body dynamics [31]. Moreover, it has been shown that they can in principle be used to simulate the dynamics of open quantum systems [32–35]. Our results prove that state-of-the-art NISQ devices are indeed able to study complex open quantum systems that were not realized experimentally before.

II. SYSTEM AND MAPPING

We consider the synchronization of a single spin-1 limit-cycle oscillator to an external signal of strength \( \varepsilon \) that is described by a Hamiltonian \( \hat{H}_{\text{signal}} \). The dynamics in a frame rotating at the signal frequency and under a rotating wave approximation is given by the quantum master equation (\( \hbar = 1 \)) [23]

\[
\frac{d}{dt} \hat{\rho} = -i \left[ \Delta \hat{S}_z + \varepsilon \hat{H}_{\text{signal}}, \hat{\rho} \right] + \Gamma_{-1,0} \hat{D}[\hat{S}_+ \hat{S}_-] \hat{\rho} + \Gamma_{1,0} \hat{D}[\hat{S}_- \hat{S}_+] \hat{\rho} \quad (1)
\]

Here, \( \hat{S}_z \) is the spin-1 operator along the quantization axis and \( \Delta = \omega_0 - \omega_{\text{signal}} \) is the detuning between the spin precession frequency \( \omega_0 \) and the signal frequency \( \omega_{\text{signal}} \). By \( \hat{S}_\pm \), we denote the spin raising and lowering operators, \( \Gamma_{-1,0} \) and \( \Gamma_{1,0} \) are the decay rates towards the state \(|0\rangle\), and \( \hat{D}[\hat{O}] \hat{\rho} = \hat{O} \hat{\rho} \hat{O}^\dagger - \frac{1}{2} \{ \hat{O}^\dagger \hat{O}, \hat{\rho} \} \) is a Lindblad dissipator. The signal Hamiltonian is given by \( \hat{H}_{\text{signal}} = j_{0,1} \hat{S}_z \hat{S}_+ / \sqrt{2} - j_{0,-1} \hat{S}_- \hat{S}_z / \sqrt{2} + j_{-1,1} \hat{S}_+^2 / 2 + \text{H.c.} \) where the complex coefficients \( j_{k,l} \) determine the relative amplitude and phase of the three possible transitions in a spin-1 system, as sketched in Fig. 1(a). For instance, the combination \( j_{0,1} = j_{0,-1} \) and \( j_{-1,1} = 0 \) corresponds to a semiclassical signal, while \( j_{0,1} = j_{0,-1} = 0 \) and \( j_{-1,1} \neq 0 \) corresponds to a squeezing signal.

To simulate a quantum system on a quantum computer, its Hilbert space \( \mathcal{H}_{\text{sys}} \) needs to be mapped onto the logical Hilbert space \( \mathcal{H}_{\text{qc}} \) of the quantum computer. We choose to represent the three spin-1 states in terms of the following two-qubit states:

\[
|+1\rangle = |1\rangle_{q_1} \otimes |0\rangle_{q_0} ,
\]

\[
|0\rangle = |0\rangle_{q_1} \otimes |0\rangle_{q_0} ,
\]

\[
|-1\rangle = |0\rangle_{q_1} \otimes |1\rangle_{q_0} .
\]

Note that this encoding gives rise to a fourth state \(|X\rangle = |1\rangle_{q_1} \otimes |1\rangle_{q_0} \) outside the spin-1 Hilbert space, which needs to be isolated from the other states.
Figure 1.  (a) Energy level diagram of a spin-1 system hosting a limit-cycle oscillator. The limit cycle is stabilized by dissipative transitions towards the state $|0\rangle$ at rates $\Gamma_{\pm 1,0}$ and is subjected to an external signal which drives transitions $j_{k,l}$ between the spin-1 states. (b) Quantum-circuit implementation of the synchronization dynamics for a timestep $dt$, obtained by a Suzuki-Trotter decomposition. The gates shown in white correspond to the free evolution of the oscillator while the other circuit components correspond to the transitions of the same color in (a). Here, $R_{k}(\theta) = U_{k}(0,0,\theta)$ is the phase gate and the signals $j_{0,\pm 1}$ are mapped onto controlled gates $U_{\pm 1,0}(t) = U_{3}(j_{0,\pm 1},t,\pi,j_{\mp 1,0})$. The $U_{3}(\theta,\varphi,\lambda)$ gate, defined in the methods section, is a basis gate of the IBM quantum processor. Open (solid) circles indicate a controlled gate conditioned on the control qubit $q_{0}$ being in $|0\rangle$ ($|1\rangle$), see Eq. (2). (c) Trotter step of the $j_{1,-1}$ signal using three controlled $U_{3}(\theta,\varphi,\lambda)$ gates, where $\tau = \text{arg}(j_{-1,1})$. (d) Implementation of relaxation dynamics with $\theta_{k}(t) = 2\text{arcsin}(\sqrt{\Gamma_{k,0}})dt$. Note that the two dissipative steps in (b) could also be applied sequentially to a single ancilla qubit.

Next, the system’s continuous dynamics (1) has to be translated to the level of logical qubits, to which we can only apply a finite set of discrete unitary gates. The exact time evolution is approximated by a series of many transformations that propagate the system’s state for a small timestep $dt$. For the unitary part of Eq. (1), this is achieved by means of a Suzuki-Trotter decomposition [24]. Simulating the remaining non-unitary dissipative dynamics may seem challenging given that we can only apply unitary gates. However, this task can be achieved by simulating discrete-time unitary dynamics on an extended system where ancillary degrees of freedom mimic a dissipative environment. In fact, it has been shown that this environment can even be modeled by a single resettable qubit [32].

In our case, a single Trotter time step $dt$ that approximates the dynamics (1) up to corrections of the order $dt^{3}$ is shown in Figs. 1(b,c). This is one of the main results of this article. The signal Hamiltonian $\hat{H}_{\text{signal}}$ is implemented by controlled two-qubit rotations such that the undesired state $|X\rangle$ remains decoupled from the spin-1 system. Our mapping (2) has the benefit that the limit-cycle state $|0\rangle$ corresponds to the ground state $|q_{0}\rangle \otimes |0\rangle_{q_{0}}$ of the logical qubits. Thus, the dissipative stabilization of the limit cycle translates to energy relaxation processes on the two qubits $q_{0}$ and $q_{1}$. This allows us to implement the required nonlinear dissipation in the quantum regime with minimal complexity: The non-unitary circuit $D_{k}$ performing a measurement and subsequent reset of the ancilla qubit, shown in Fig. 1(d), implements single-qubit relaxation with a tunable relaxation rate $\Gamma_{k,0}$ [24]. As discussed in the supplemental material [36], we are effectively implementing a photon-counting quantum-trajectory simulation of the quantum master equation (1) granted that the condition $\Gamma_{k,0}dt \ll 1$ holds. Each experimental run of the circuit calculates a random quantum trajectory of a pure state and the dynamics of $\hat{\rho}$ can be recovered by an ensemble average over many quantum trajectories [37].

III. DEVICE CHARACTERIZATION

By iteratively applying $N$ Trotter steps on an ideal quantum computer, an initial state is evolved to a final state at time $T = Nd$ In a first step, we assess whether this is the case on an actual NISQ device by testing the elements of the decomposition shown in Figs. 1(b,c). We also discuss the restrictions imposed by the limited capabilities of state-of-the-art quantum processors.

Figure 2(a) shows the time evolution of the initial state $|0\rangle$ under the signal components $j_{0,\pm 1}$ on a NISQ device and the corresponding ideal noise-free result. Controlled two-qubit gates are found to induce strong depolarization errors that evolve the initial state $|0\rangle$ to a completely mixed state after only a few Trotter steps. This result is also confirmed by simulations taking into account a noise model of the IBM quantum processors provided in the python API QISKIT [39]. Given that already the signal component suffers from severe depolarization errors, it is not feasible to perform the time evolution as shown in Figs. 1(b,c) on a NISQ device. To circumvent this problem, we consider a modified circuit where we apply only uncontrolled single-qubit $U_{\pm 1,0}$ rotations.

The single-qubit-rotation error rates on the IBM quantum processors are about an order of magnitude smaller than the two-qubit controlled-not (CNOT) error rate [26]. Consequently, the uncontrolled implementation of the signal using only single-qubit rotations reproduces the ideal noise-free result almost perfectly over a much
larger range of Trotter steps. Note that the use of uncontrolled gates is only valid in the synchronisation regime, where most of the population remains in the limit-cycle state $|0\rangle$, as discussed in the next section.

Figure 2(b) demonstrates the dissipative stabilization of the limit cycle state $|0\rangle$ if no signal is applied, $j_{\pm 1,0} = j_{\pm 1,1} = 0$. Once more, the controlled two-qubit operations contained in the operations $D_k$ induce a decay of the state $|0\rangle$ towards a completely mixed state. Surprisingly, the noise induced by the dissipative stabilization is such that the limit-cycle state shows a small amount of coherence. This effect is not captured by the simple noise model provided in Qiskit. The corresponding results demonstrating that the initial states $|\pm 1\rangle$ evolve to the limit-cycle state $|0\rangle$ under the action of the dissipative terms $D_k$ are given in [36]. There, we also discuss an alternative implementation of the dissipative stabilization that requires fewer two-qubit gates and can be used to minimize noise in the coherences.

Besides the strong depolarizing effect of two-qubit gates, another limitation of IBM’s current quantum processors is that they do not allow measurement and reset operations of qubits in the middle of a quantum circuit. This means that we must use a new ancillary qubit in each timestep and measure all of them at the end of the time evolution. Therefore, the maximum number of Trotter steps we can apply is bounded by the number of available ancillary qubits on a quantum processor. Moreover, since SWAP operations are composed of three CNOT gates and suffer strong depolarizing errors, we can only use qubits that are directly connected to the system qubit $q_j$ to be relaxed, which limits us to at most four timesteps. At the moment, this is the most severe limitation for the simulation of open quantum systems on the device. We expect that it will be lifted in the near future.

IV. DEALING WITH HARDWARE CONSTRAINTS

The paradigm of quantum synchronisation allows us to adapt the quantum circuit shown in Fig. 1 to the limitations of IBM’s quantum processors. Specifically, the signal strength is linearly proportional to a small dimensionless parameter $0 \leq \eta \ll 1$ that ensures that $H_{\text{signal}}$ is only a small perturbation to the limit-cycle state [23]. Thus, the amplitudes of the coherences $\hat{\rho}_{k,X}$ are of order $\eta$ and the populations of the states $|\pm 1\rangle$ are of order $\eta^2$. That is, they are strongly suppressed as compared to the limit-cycle state $|0\rangle$ having a population of $O(1)$. Under these conditions, we can replace the controlled two-qubit gates $U_{\pm 1,0}$ by uncontrolled single-qubit rotations. In principle, the signal will now build up coherences $\hat{\rho}_{k,X}$ between the spin-1 states and the state $|X\rangle$ and it will transfer population to the state $|X\rangle$. However, both effects can be safely ignored, in particular on a noisy system, because the coherences $\hat{\rho}_{k,X}$ and the population $\hat{\rho}_{X,X}$ are only of order $\eta^3$ and $\eta^4$, respectively. Moreover, since the relaxation mechanism $D_k$ takes the state $|X\rangle$ back to $|\pm 1\rangle$, there is no risk to trap population in $|X\rangle$. Plots verifying that the coherences $\hat{\rho}_{k,X}$ are well below the limit-cycle noise threshold are shown in [36].

Having replaced controlled by uncontrolled rotations, if we additionally restrict ourselves to semiclassical signals, i.e., $j_{-1,1} = 0$, the entire unitary part of the
time evolution (1) can be simulated using only single-qubit rotations. The qubits $q_0$ and $q_1$ can now be independently assigned to physical qubits of the quantum processor, which allows us to use groups of qubits on the processor that yield high-fidelity CNOT gates between the qubits $q_{0,1}$ and their corresponding ancillary qubits, e.g., the groups $\{6, 7, 8, 9\}$ and $\{3, 10, 11, 12\}$ on the IBM Q System. On the smaller 5-qubit devices, we can evolve the qubits $q_0, q_1$ sequentially in two consecutive runs.

Given the fixed connectivity and SWAP fidelities of IBM’s current quantum processors, the limit on the available Trotter steps imposed by the device connectivity cannot be evaded. As a consequence, quantum simulation of the steady-state solution of Eq. (1) is out of reach, but we are able to demonstrate the transient buildup of synchronization, as shown in Fig. 2(c).

V. RESULTS

We now experimentally demonstrate typical features of quantum synchronization on the IBM Q System [26]. Figure 3(a) shows the phase distribution of the limit-cycle oscillator, calculated from the experimentally obtained density matrix according to the analytical formula [23]

$$S(\varphi) = \frac{3}{8\sqrt{2}} |\hat{\rho}_{1,0} + \hat{\rho}_{0,-1}| \cos[\varphi + \arg(\hat{\rho}_{1,0} + \hat{\rho}_{0,-1})]$$

+ $\frac{1}{2\pi} |\hat{\rho}_{1,-1}| \cos[2\varphi + \arg(\hat{\rho}_{1,-1})]$, (3)

as a function of the signal detuning. The solid line indicates the expected position of the peak of $S(\varphi)$ according to Eq. (1). The small differences in the positions of the maximum stem from a detuning dependence of the limit cycle stabilization mechanism due to device imperfections. Figure 3(b) confirms that the magnitude of the coherences between the spin eigenstates grows linearly with the overall signal strength $\varepsilon$, whereas the populations change only quadratically in $\varepsilon$. Therefore, the applied signal perturbs the limit-cycle state only weakly and we are in the regime of synchronization. The buildup of the coherence $\hat{\rho}_{-1,1}$ is due to higher-order effects and scales proportional to $\varepsilon^2$. Finally, a global phase applied to the signals, $j_{z1,0} \rightarrow e^{i\chi}j_{z1,0}$, rotates the phase of the coherences accordingly as demonstrated in the upper panel of Fig. 3(c). By rotating only the phase of one of the signal tones, i.e., $j_{-1,0} \rightarrow e^{i\chi}j_{-1,0}$ but $j_{1,0} = \text{const}$, the coherences $\hat{\rho}_{1,0}$ and $\hat{\rho}_{0,-1}$ in Eq. (3) can be tuned to interfere destructively, which manifests itself in an interference-based quantum synchronization blockade [23] and is demonstrated in the lower panel of Fig. 3(c). This result is the first experimental demonstration of quantum effects in synchronization.

VI. CONCLUSION

In this article, we experimentally demonstrated for the first time the synchronization of a quantum limit-cycle oscillator by a digital quantum simulation on the IBM Q System. Our results prove that state-of-the-art NISQ devices enable to study realistic open quantum systems. There are still major obstacles to the simulation of complex open quantum systems, namely, (i) low two-qubit gate fidelities, (ii) a missing qubit reset operation during the calculation, and (iii) the low effective connectivity of the devices, which is partly due to point (i). While the two-qubit gate fidelities of the quantum processors in the IBM Q System have been significantly improved [38], they still strongly restrict the maximum number of controlled operations that can be performed. Taking advantage of the paradigm of synchronization, we implemented a time evolution tailored to the hardware constraints by approximating the signals with single-qubit gates. We compensated the absence of a reset operation by using multiple ancillary qubits sequentially for the dissipative evolution, at the cost of being bounded to at most four timesteps due to the low effective connectivity of the devices.

Despite these limitations, we were able to experimentally observe a purely quantum effect in synchronization, namely a quantum interference-based synchronization blockade. Thus, state-of-the-art NISQ processors are a useful tool to study simple realistic open quantum systems.

VII. METHODS

All data presented in this article has been collected on the publicly accessible NISQ processor IBM Q System. Our results prove that state-of-the-art NISQ devices enable to study realistic open quantum systems. There are still major obstacles to the simulation of complex open quantum systems, namely, (i) low two-qubit gate fidelities, (ii) a missing qubit reset operation during the calculation, and (iii) the low effective connectivity of the devices, which is partly due to point (i). While the two-qubit gate fidelities of the quantum processors in the IBM Q System have been significantly improved [38], they still strongly restrict the maximum number of controlled operations that can be performed. Taking advantage of the paradigm of synchronization, we implemented a time evolution tailored to the hardware constraints by approximating the signals with single-qubit gates. We compensated the absence of a reset operation by using multiple ancillary qubits sequentially for the dissipative evolution, at the cost of being bounded to at most four timesteps due to the low effective connectivity of the devices.

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Figure 3. (a) Phase distribution $S(\varphi)$ of the spin-1 limit-cycle oscillator as a function of the detuning $\Delta$ between its natural frequency and the signal frequency after $N = 3$ timesteps. The solid line indicates the theoretical expectation of the position of the maximum of $S(\varphi)$, obtained by combining Eqs. (1) and (3). Parameters are $\Gamma_{1,0}/\omega_0 = 4$, $\Gamma_{-1,0}/\omega_0 = 4$, $\omega_0 dt = 0.05$, $\varepsilon/\omega_0 = 1$, $j_{-1,0} = 2 \times e^{2\pi i/6}$, $j_{1,0} = 2 \times e^{-\pi i/6}$, and $j_{-1,1} = 0$. (b) Populations and coherences as a function of the signal strength $\varepsilon/\omega_0$ for the central qubit $\varepsilon/\omega_0 = 0$. (c) Upper panel: Phase of the coherences if the overall phase $\chi$ of the signals, $j_{\pm 1,0} e^{\pm \chi}$, is varied for $\Delta/\omega = 0$ and $\varepsilon/\omega = 1$. Lower panel: Demonstration of an interference-based quantum synchronization blockade if the phase of one of the signals is varied, $j_{-1,0} = e^{i\chi} \times 2 \times e^{2\pi i/6}$ and $j_{1,0} = 2 \times e^{-2\pi i/6} = \text{const}$. Data points are the results obtained on a NISQ device, the solid line corresponds to a simulation taking into account noise, and the dashed line describes the theory results. Parameters are $\Gamma_{1,0}/\omega_0 = 4$, $\Gamma_{-1,0}/\omega_0 = 5$, $\omega_0 dt = 0.05$, $\varepsilon/\omega_0 = 1$, and $j_{-1,1} = 0$. All data of this figure has been collected on the ibmqx2 processor on qubits $q_0 = 2$ and $q_1 = 2$ in sequential runs.

The single-qubit $U_3(\theta, \varphi, \lambda)$ gate defined by

$$U_3 |0\rangle_{q_j} = \cos \frac{\theta}{2} |0\rangle_{q_j} + e^{i\varphi} \sin \frac{\theta}{2} |1\rangle_{q_j},$$

$$U_3 |1\rangle_{q_j} = -e^{i\lambda} \sin \frac{\theta}{2} |0\rangle_{q_j} + e^{i\lambda+i\varphi} \cos \frac{\theta}{2} |1\rangle_{q_j},$$

and the single-qubit gates $U_2(\varphi, \lambda) = U_3(\pi/2, \varphi, \lambda)$ and $U_1(\lambda) = U_3(0, 0, \lambda)$. The final state has been reconstructed by a single-qubit tomography of the qubits $q_0$ and $q_1$ at the end of each time evolution. The full spin-1 density matrix has been reconstructed from these tomography measurements using the built-in QISKIT functions implementing Ref. 40. Each circuit has been run with the maximum possible number of 8192 repetitions to ensure convergence.

Experiments have been performed in batches of several quantum circuits describing the time evolution for different system parameters. At the beginning of each batch, two calibration circuits have been added to measure the readout error of the qubits $q_0$ and $q_1$. The readout error for the central qubit 2 is approximately 1% [26]. Based on these calibration results, the measurement errors of all subsequent measurements in the batch have been mitigated using QISKIT methods. To validate the stability of the error mitigation procedure and to rule out drifts of the device parameters during data collection, each batch has been evaluated three times and the corresponding standard deviation is indicated by the error bars in the plots, which are smaller than the plot markers.

Simulations of the exact dynamics (1) have been performed using the python package QUTIP [41].

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Note added: After completion of this work, we became aware of Ref. 44 which studies quantum synchronization effects in an ensemble of spin-1 $^{87}$Rb atoms.

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Appendix A: Implementing dissipation

As discussed in the main text, our state representation (2) is chosen such that the dissipative stabilization of the limit-cycle state \( |0\rangle \) in Eq. (1) translates to a relaxation of the logical qubits towards the joint ground state \( |0\rangle_q \otimes |0\rangle_q \). In principle, one could take advantage of the natural energy relaxation in the quantum processor to stabilize the limit cycle at the natural relaxation rate \( \Gamma_{rel} \). However, this is not sufficient if we want to study synchronization effects for the following reason. An external signal \( H_{signal} \) creates coherences between the spin states at a certain rate \( \Gamma_{signal} \), which must be smaller than the rate \( \Gamma_{rel} \) at which the limit cycle is stabilized to satisfy the paradigm of synchronization [23]. On a physical quantum processor, noise will decrease the magnitude of the coherences at a rate \( \Gamma_{dec} \). Hence, in order to allow us to observe synchronization, the signal must overcome this decoherence, \( \Gamma_{signal} > \Gamma_{dec} \). However, this is incompatible with the requirement \( \Gamma_{rel} > \Gamma_{signal} \) since decoherence is typically stronger than energy relaxation, \( \Gamma_{dec} > \Gamma_{rel} \). Therefore, to study synchronization on a physical quantum processor, the natural energy relaxation rate \( \Gamma_{rel} \) must be artificially increased.

This can be achieved by the following circuit, also shown in Fig. 1(c) of the main text.

\[
\begin{array}{c}
\text{\( q \)} \\
\text{a} \\
\end{array}
\quad
\begin{array}{c}
U_3(\theta,0,0) \\
\end{array}
\quad
\begin{array}{c}
\text{|0\rangle} \\
\end{array}
\]

This circuit maps an initial state \( |\psi\rangle_q \otimes |0\rangle_a = (\alpha |0\rangle_q + \beta |1\rangle_q) \otimes |0\rangle_a \) to the state

\[
\left[ \alpha |0\rangle_q + \beta \cos \left( \frac{\theta}{2} \right) |1\rangle_q \right] \otimes |0\rangle_a + \beta \sin \left( \frac{\theta}{2} \right) |0\rangle_q \otimes |1\rangle_a
\]

immediately before the measurement. If we set \( \sin^2(\theta/2) = \Gamma dt \ll 1 \), the measurement projects the state of qubit \( q \) to \( |\psi_{\text{dir}}\rangle \) at a probability \( \Gamma |\beta|^2 dt \), or to

\[
|\psi_{\text{dir}}\rangle |0\rangle = \alpha \left( 1 + \frac{\Gamma}{2} |\beta|^2 dt \right) |0\rangle_q
\]

\[
+ \beta \left( 1 - \frac{\Gamma}{2} dt + \frac{\Gamma}{2} |\beta|^2 dt \right) |1\rangle_q + \mathcal{O}(dt^2)
\]

at a probability \( 1 - \Gamma |\beta|^2 dt \). This is precisely the evolution of the state vector \( |\psi\rangle \) in a stochastic Schrödinger equation of the form

\[
d|\psi\rangle = \left[ -i \left( -\frac{\Gamma}{2} \sigma_- \sigma_+ \right) + \frac{\Gamma}{2} \langle \psi | \sigma_- \sigma_+ |\psi\rangle \right] |\psi\rangle dt
\]

\[
+ \left[ \frac{\sigma_+ |\psi\rangle}{\sqrt{\langle \psi | \sigma_- \sigma_+ |\psi\rangle}} - |\psi\rangle \right] dN ,
\]

where \( dN \in \{0,1\} \) is a stochastic Poissonian increment with expectation value \( E(dN) = \Gamma \langle \psi | \sigma_- \sigma_+ |\psi\rangle dt = |\beta|^2 \Gamma dt \) [37]. The unconditional quantum master equation for the density matrix \( \hat{\rho} = \mathbb{E}[|\psi\rangle \langle \psi|] \) corresponding to Eq. (A1) describes single-qubit relaxation,

\[
\frac{d}{dt} \hat{\rho} = \Gamma D[\sigma_+] \hat{\rho} .
\]

Note that we are using the quantum-information definition of the single-qubit basis states, i.e., \( \sigma_+ |0\rangle = + |0\rangle \) and \( \sigma_+ |1\rangle = - |1\rangle \). Therefore, Eq. (A2) actually describes relaxation since \( \sigma_+ |1\rangle = |0\rangle \).

Thus, a measurement result of 1 on the ancillary qubit \( a \) represents the release of an excitation from the qubit \( q \) into the environment and resets the qubit \( q \) to its ground state.

A controlled unitary gate is implemented by at least two CNOT operations [42, 43]. Thus, the circuit given above requires at least three CNOT operations. An alternative circuit which performs exactly the same transformation of the initial state \( |\psi\rangle_q \otimes |0\rangle_a \), but requires only two CNOT gates is the following.

\[
\begin{array}{c}
q \\
\text{a} \\
\end{array}
\quad
\begin{array}{c}
U_3(\theta,0,0) \\
\end{array}
\quad
\begin{array}{c}
|0\rangle \\
\end{array}
\]

Despite the fact that both circuits ideally perform the same transformation of an initial state \( |\psi\rangle_q \otimes |0\rangle_a \), they will perform differently on a NISQ device. The parameters of the quantum processor fluctuate in time and are calibrated approximately once a day. Therefore, on each day we choose the circuit that induces the least coherences in the limit-cycle state for the given gate errors.

Appendix B: Limit-cycle stabilization

In this section, we provide additional information on the stabilization of the limit cycle.

Figure 4 displays the coherences not included in Fig. 2 of the main text. The limit-cycle state has no coherences
$\hat{\rho}_{1,X}$, $\hat{\rho}_{-1,X}$, and $\hat{\rho}_0,X$ at all. If an external signal is applied to the limit-cycle oscillator, coherences are built up due to higher-order effects, but they remain below the noise level of the limit-cycle state. This finding is in line with the paradigm of synchronization discussed in the main text.

Figure 5 shows the evolution of the populations and coherences in the absence of a signal if the initial state is different from $|0\rangle$. The data confirms that the dissipative stabilization mechanism transfers population from the initial state to the state $|0\rangle$. The coherences stay below the noise level of the limit cycle except for transient buildup dynamics associated with the state transfer.
Figure 5. Time evolution of the populations (top row) and the coherences (bottom row) if only the dissipative stabilization mechanism of the limit cycle is switched on. The initial state is (a) $|+1\rangle$, (b) $|−1\rangle$, and (c) $|X\rangle$. Parameters are the same as in Fig. 2(b) of the main text.