Microscopic Calculations of Vortex-Nucleus Interaction in the Neutron Star Crust

Kazuyuki Sekizawa, Gabriel Wlazłowski, Piotr Magierski, Aurel Bulgac, and Michael McNeil Forbes

Faculty of Physics, Warsaw University of Technology, ulica Koszykowa 75, 00-662 Warsaw, Poland
Department of Physics, University of Washington, Seattle, WA 98195-1560, USA
Department of Physics & Astronomy, Washington State University, Pullman, WA 99164-2814, USA
E-mail: sekizawa@if.pw.edu.pl

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We investigate the dynamics of a quantized vortex and a nuclear impurity immersed in a neutron superfluid within a fully microscopic time-dependent three-dimensional approach. The magnitude and even the sign of the force between the quantized vortex and the nuclear impurity have been a matter of debate for over four decades. We determine that the vortex and the impurity repel at neutron densities, $0.014 \text{ fm}^{-3}$ and $0.031 \text{ fm}^{-3}$, which are relevant to the neutron star crust and the origin of glitches, while previous calculations have concluded that the force changes its sign between these two densities and predicted contradictory signs. The magnitude of the force increases with the density of neutron superfluid, while the magnitude of the pairing gap decreases in this density range.

KEYWORDS: pulsar glitch, vortex pinning, superfluid, inner crust, neutron star

1. Introduction

The origin of a neutron star glitch, a sudden spin-up of the rotational frequency, has been one of the unsolved problems in nuclear astrophysics for a long time. It has been suggested [1] that the glitch is caused by a catastrophic unpinning of a huge number of vortices from the pinning sites formed by a Coulomb lattice of nuclei immersed in a neutron superfluid in the inner crust of neutron stars. Although the vortex-nucleus interaction is undoubtedly one of the most important ingredients needed to explain the glitches, so far contradictory predictions have been made about both its magnitude and even sign. The difficulty in extracting the vortex-nucleus interaction is related to the enormous number of degrees of freedom that one has to take into account. The vortex itself represents a topological excitation of a superfluid, which may stretch and bend in various ways and the susceptibility towards vortex deformations should be derived from a microscopic equation of motion describing neutron superfluid. Moreover, the nuclear impurity may easily deform as its surface tension is significantly smaller than those of isolated nuclei. Thus various assumptions concerning the symmetry of the problem considered in the past, in order to simplify the analyses, may dramatically change results, as one can accidentally omit important degrees of freedom of either the vortex or of the impurity.

Recently, we have reported the first fully microscopic, three-dimensional and symmetry-unconstrained dynamical simulations which enabled us to extract the vortex-nucleus interaction [2].

2. Methods

2.1 TDSLDA

We have used an extension of Kohn-Sham density functional theory for superfluid systems, known as the time-dependent superfluid local density approximation (TDSLDA). It has been proved
that the TDSLDA provides very accurate description for the static and dynamic properties of strongly correlated fermionic systems such as ultracold atomic gases [3] and nuclear systems [4].

For the normal part of the functional, we used the FaNDF\textsuperscript{0} functional constructed by Fayans \textit{et al.} [5], which reproduces the infinite matter equation of state of Refs. [6,7] and various properties of finite nuclei [8,9]. In this study we have omitted the spin-orbit term, as it plays a minor role in the vortex-nucleus dynamics [2]. To the FaNDF\textsuperscript{0} functional we added a contribution describing the pairing correlations, \( E_{\text{pair}}(\mathbf{r}) = g(n_n(\mathbf{r}))|\psi_n(\mathbf{r})|^2 + g(n_p(\mathbf{r}))|\psi_p(\mathbf{r})|^2 \), where \( n_n, p \) are the anomalous densities of neutrons and protons. The coupling constant \( g \) depends on densities of neutrons \( n_n \) and protons \( n_p \), which was adjusted to reproduce the BCS-type neutron pairing gap in pure neutron matter [2].

### 2.2 Computational settings

We solved TDSLDA equations (formally equivalent to the time-dependent Hartree-Fock-Bogoliubov or time-dependent Bogoliubov-de Gennes equations) on a periodic 3D lattice without any symmetry restrictions. We used a box of 75 fm \( \times \) 75 fm \( \times \) 60 fm with a lattice spacing of 1.5 fm, which corresponds to a rather large momentum cutoff, \( p_c \approx 400 \text{ MeV}/c \). To prevent a quantum vortex from interacting with vortices in neighboring cells, due to periodic boundary conditions, we introduced a flat-bottomed external potential confining the system in a tube along the \( z \)-axis with a radius 30 fm. Spatial derivatives are evaluated using Fourier transformation. We used the Trotter-Suzuki decomposition for the time evolution operator with a predictor-corrector step with a time-step set to \( \Delta t = 0.054 \text{ fm}/c \). We generated initial states for two background neutron densities, \( n = 0.014 \text{ fm}^{-3} \) and \( 0.031 \text{ fm}^{-3} \), with a vortex line parallel to the axis of the tube and with a nuclear impurity with 50 protons [10].

### 3. Force extraction from dynamical simulations

We extracted the force exerted on the nucleus from our dynamical simulations. We performed the time integration with an external force \( F_{\text{ext}}(t) \) constant in space, which couples to protons only. This force moves the center-of-mass of protons together with bound neutrons in the nucleus. Naturally, unbound neutrons in the vicinity of the nucleus are also entrained. Assuming that there are only two forces acting on the nucleus, \( F_{\text{ext}}(t) \) and \( F(t) \), where the latter is the force arising from the interaction with the vortex, the nucleus will move with constant velocity if the relation \( F(t) + F_{\text{ext}}(t) = 0 \) holds. Thus by adjusting \( F_{\text{ext}}(t) \) we may extract the force \( F(t) \) exerted on the nucleus by the vortex. In practice we adjusted the external force during time evolution as

\[
F_{\text{ext}}(t + \Delta t) = F_{\text{ext}}(t) - \alpha [v(t) - v_0],
\]

where \( v(t) \) is the velocity of the center-of-mass of protons and \( \alpha \) is a small coefficient governing the rate of the force adjustment. In our simulations, we dragged the nucleus with a very small velocity, \( v_0 = 0.001c \), which is far below the critical velocity to ensure that no phonons are excited.

To extract the vortex-nucleus force as a function of their separation \( R \), we started simulations from a well separated configuration and dragged the nucleus along the positive-\( x \) direction towards the vortex. We displayed the results of the TDSLDA calculations for \( n = 0.014 \text{ fm}^{-3} \) in Fig. 1 (a, b, c) and for \( n = 0.031 \text{ fm}^{-3} \) in Fig. 1 (d, e, f), respectively.

Figure 1 (a, d) illustrates a typical trajectory of the motion of the vortex core in the \( xy \) plane, where the center-of-mass of the nucleus resides (\( z = 30 \text{ fm} \)). The red arrow indicates the trajectory of the center-of-mass of the nucleus. The blue dots show the positions of the vortex core at various times. As the nucleus approaches the vortex, it exerts a repulsive force \( \mathbf{F} \) on the vortex. The vortex responds it, according to the formula \( F_M \propto \mathbf{k} \times v \), where \( \mathbf{k} \) is the circulation pointing in the positive-\( z \) direction (upward perpendicular to the figure). For both densities, we found that the vortex, under the perturbation induced by the nucleus, is initially moving along the positive-\( y \) direction. It clearly indicates that the force is repulsive and initially directed along the positive-\( x \) direction away from the nucleus.
Fig. 1. The results of the TDSLDA calculations for \( n = 0.014 \text{ fm}^{-3} \) (a, b, c) and 0.031 fm\(^{-3} \) (d, e, f). The trajectory of the motion of the vortex core is displayed in subfigures (a, d). The position of the vortex core is shown in the \( xy \)-plane, where the nucleus resides. The red line shows the trajectory of the nucleus and the blue dots show the vortex-core positions at various times. The nucleus velocity is shown as a function of time in subfigures (b, e). The central (red dots) and the tangential (blue dots) components of the extracted force \( F \) are shown as a function of time in (c, f).

In Fig. 1 (b, e), we show the magnitude of the velocity of the nucleus as a function of time. The velocity is adjusted during the first few 1,000 fm/c, and subsequently is kept constant during the time evolution. It allows us to extract the force \( F(t) \) exerted on the nucleus as discussed above.

In Fig. 1 (c, f), we show the force \( F \) as a function of time. We decomposed the force into central and tangential components with respect to the vortex-core position at each time. The extracted force is, as expected, predominantly central with a negligibly small tangential component. The magnitude of the force becomes stronger for higher density. The effective range of the force increases with density, consistent with a smaller pairing gap and a larger coherence length at higher densities.

Figure 2 displays snapshots of the system \( n = 0.031 \text{ fm}^{-3} \) exhibiting shape of the nucleus and the vortex line as well as the velocity field of neutrons. While the velocity field inside the nucleus is smaller than outside (about 0.005c), which is about 5 times larger than the dragging velocity. Thus the dragging process does not influence strongly the superflow of neutrons. As the structure of both objects is affected by their interaction (the vortex bends and the nucleus deforms), the force cannot be described by a simple function of their separation only. In Ref. [2], we have deduced force per unit length to properly characterize the force for various vortex-nucleus configurations.

4. Summary

We have performed 3D, symmetry unrestricted, microscopic, dynamic simulations for a vortex-nucleus system using a time-dependent extension of DFT for superfluid systems. We have determined that the vortex-nucleus force is repulsive and increasing in magnitude with density, for the densities characteristic of the neutron star crust \( (0.014 \text{ fm}^{-3} \text{ and } 0.031 \text{ fm}^{-3}) \).

It is instructive to note that repulsive force is not ruled out by a purely hydrodynamical approach (see Supplemental Material of [2]). It is however difficult to associate unambiguously the superfluid density with the actual neutron density. Therefore hydrodynamical description can be used only to
estimate the asymptotic behavior of the vortex-nucleus interaction ($\propto 1/r^3$).

It is worth mentioning that the extracted force is at least one order of magnitude larger than those predicted in a recent phenomenological analysis [11]. The repulsive force is compatible with the so-called interstitial pinning, where vortices are trapped at positions that maximize the overall separation from the nearest nuclei [12].

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