Statistical memory effects in human stride dynamics

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Abstract. In paper the Memory functions formalism (MFF) is used to analyze the dynamics of walking stride interval of healthy young adults, healthy old adults and Parkinson’s disease. Using the non-Markovian parameter, we determine the degree of manifesting the statistical memory effects in dynamics of the walking stride interval for the considered volunteers. The stride dynamics of the first two volunteer groups demonstrates the low memory effects reflected in high values of non-Markovian parameter. The Parkinsonian patients demonstrate the non-Markovian behavior of stride dynamics with strong memory effects. The method can be used to study the signal dynamics of the different nature.

1. Introduction
At present, the studying of the statistical memory effects in evolution and dynamics of natural objects is interest for theoretical and computational physics [1, 2]. At the same time, the efforts of physicists working in this field are aimed to searching the statistical and numerical indicators to optimize such measures for describing the discrete time evolution of complex systems of different nature. The solving these problems becomes especially urgent when identifying and quantitatively describing the anomalous functioning of living systems from discrete time series. The time series analysis is an important tool to study the nature of complex systems in physical research. In this paper, the Memory functions formalism (MFF) [3, 4] is used to analyze the dynamics of walking stride interval of healthy young adults, healthy old adults and Parkinson’s disease.

2. Memory function formalism
Within the framework of MFF [3, 4], we consider studied time series as the sequence \( \{x_j\} \) of a random value \( X \):

\[
X = \{x(T), x(T + \tau), x(T + 2\tau), \ldots, x(T + (N - 1)\tau)\} = \{x_0, x_1, x_2, \ldots, x_N\}.
\]

\[
x_j = x(T + j\tau), \quad \langle X \rangle = \frac{1}{N} \sum_{j=0}^{N-1} x_j, \quad \delta x_j = x_j - \langle X \rangle, \quad \sigma^2 = \frac{1}{N} \sum_{j=0}^{N-1} \delta x_j^2.
\]

Here \( T \) is the initial time point, \( \tau \) is the time interval of signal discretisation, \( \langle X \rangle \) is the mean value of \( X \), \( \delta x_j \) is fluctuation. To describe the probabilistic relations in the sequence of \( X \) we use the normalized time-dependent correlation function (TCF):

\[
a(t = m\tau) = M_0(m\tau) = \frac{1}{(N - m)\sigma^2} \sum_{j=0}^{N-m-1} \delta x_j \delta x_{j+m}, \tag{1}
\]
which is rewritten further as a scalar product of vectors of the initial state
\[ A^0_k = A^0_k(0) = \{\delta x_0, \delta x_1, \ldots, \delta x_{k-1}\} \]
and of system state in time moment \( t \)
\[ A^m_{m+k} = A^m_{m+k}(t) = \{\delta x_m, \delta x_{m+1}, \ldots, \delta x_{m+k-1}\} \]
as follows:
\[ a(t) = \frac{\langle A^0_k(0)A^m_{m+k}(t) \rangle}{\langle |A^0_k(0)|^2 \rangle}. \tag{2} \]

Using the the technique of projection operators [4] we introduce relations for the kinetic and relaxation parameters (by solving the finite-difference Liouville’s equation for the state vectors):
\[ \lambda_n = \frac{\langle W_{n-1} \hat{L} W_{n-1} \rangle}{\langle |W_{n-1}|^2 \rangle}, \quad \Lambda_n = i \frac{\langle |W_n|^2 \rangle}{\langle |W_{n-1}|^2 \rangle}, \]
where the \( \hat{L} \) is Liouville’s quasioperator, the dynamic orthogonal variables:
\[ W_0 = A^0_k(0), \quad W_1 = (i\hat{L} - \lambda_1)W_0, \quad W_2 = (i\hat{L} - \lambda_2)W_1 - \Lambda_1W_0 - \ldots, \tag{3} \]
the memory functions of the \( n \) order:
\[ M_n(t = m\tau) = \frac{\langle W_n(1 + i\tau \hat{L}_22)^mW_n \rangle}{\langle |W_n|^2 \rangle}, \]
and their power spectra:
\[ \mu_n(\nu) = \left| \frac{\tau}{N-1} \sum_{j=0}^{N-1} M_n(j\tau) \cos 2\pi \nu j\tau \right|^2. \tag{4} \]

The dynamical orthogonal variables for \( n > 1 \) are used A relaxation time of the initial TCF and memory functions of the \( n \) order are determined as follows:
\[ \tau_a = \Delta t \sum_{j=0}^{N-1} a(t_j), \quad \tau_{M_1} = \Delta t \sum_{j=0}^{N-1} M_1(t_j), \ldots, \quad \tau_{M_n} = \Delta t \sum_{j=0}^{N-1} M_n(t_j). \tag{5} \]

The set of dimensionless values will determine the statistical spectrum of non-Markovian parameter, the informational measure of memory:
\[ \{\varepsilon_i\} = \{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_{n-1}\}, \quad \varepsilon_1 = \frac{\tau_a}{\tau_{M_1}}, \quad \varepsilon_2 = \frac{\tau_{M_1}}{\tau_{M_2}}, \ldots, \quad \varepsilon_n = \frac{\tau_{M_{n-1}}}{\tau_{M_n}}. \tag{6} \]

In this paper we use the first point of the frequency-dependent case of non-Markovian parameter:
\[ \varepsilon_1 = \left\{ \frac{\mu_0(\nu)}{\mu_0(\nu)} \right\}^{\frac{1}{2}}, \right. \tag{7} \]
it is important to note that in this paper we use the values of the first order. See [4, 5] for details about the MFF.
3. Experimental data
Walking stride interval time series [6] included are from 15 subjects: 5 healthy young adults (23 – 29 years old), 5 healthy old adults (71 – 77 years old), and 5 older adults (60 – 77 years old) with Parkinson’s disease. Subjects walked continuously on level ground around an obstacle-free path. The stride interval was measured using ultra-thin, force sensitive resistors placed inside the shoe. The analog force signal was sampled at 300 Hz with a 12 bit A/D converter, using an ambulatory, ankle-worn microcomputer that also recorded the data. Subsequently, the time between foot-strikes was automatically computed. The method for determining the stride interval is a modification of a previously validated method that has been shown to agree with force-platform measures, a “gold” standard.

4. Discussion
Using the MFF toolkit, particularly the non-Markovian parameter, it was possible to determine the degree of manifesting the statistical memory effects in dynamics of the walking stride interval for the considered volunteers. Figure 1 shows the frequency spectra of the non-Markovian parameter for typical representatives of each group of subjects. The key to our analysis is the value of the non-Markovian parameter at zero frequency. If \( \varepsilon \gg 1 \), then the Markovian components dominate in dynamics of stochastic processes. The short range (low) statistical memory effects is manifested. The decrease of this information measure characterizes the expansion of memory lifetime. \( \varepsilon \sim 1 \) means that the processes are characterized by long range (high) statistical memory. When \( \varepsilon > 1 \) we can consider the studied processes to be quasi-Markovian with moderate (intermediate on time existence) statistical memory. Thus, the introduced quantitative criterion allows us to parameterize the memory effects intensity and the velocity of loss of relaxation processes. The stride dynamics of the first two volunteer groups demonstrates the low memory effects reflected in high values of non-Markovian parameter. At that the age-related alterations in stride dynamics are manifested by increasing the memory effects. The stride dynamics for the second group corresponds by quasi-Markovian case. The Parkinsonian patients demonstrate the non-Markovian behavior of stride dynamics with strong memory effects.

Figure 1. Non-Markovian parameter for considered groups of volunteers

5. Conclusion
In this paper, we have considered age- and pathological-related alterations in human locomotor activity. We have found the degree of statistical memory effects in stride dynamics at ageing and pathology. The strong memory effects has been found to reflect the pathological abnormalities
in gait dynamics. Memory function, formalizm allows to discover the alterations in studied

dynamics that are connected to different factors. The method can be used to study the signal
dynamics of the different nature.

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