Quantum Simulation of the Agassi Model in Trapped Ions

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A digital quantum simulation of the Agassi model from nuclear physics with a trapped-ion quantum platform is proposed and analyzed. The proposal is worked out for the case with four different sites, to be implemented in a four-ion system. Numerical simulations and analytical estimations are presented to illustrate the feasibility of this proposal with current technology. The proposed approach is fully scalable to a larger number of sites. The use of a quantum correlation function as a probe to explore the quantum phases by quantum simulating the time dynamics, with no need of computing the ground state, is also studied. Evidence that the amplitude of the quantum Rabi oscillations in this quantum simulation is correlated with the different quantum phases of the system is given. This approach establishes an avenue for the digital quantum simulation of useful models in nuclear physics with trapped-ion systems.

Introduction.— During the past few decades the possibility of using controllable quantum systems to simulate other quantum systems has been explored extensively [1]. Different quantum platforms have been proposed to reproduce quantum models experimentally, including superconducting circuits, ion traps, cold atoms, quantum dots, as well as quantum photonics [1]. One of the emerging fields proposed for quantum simulations is the analysis of nuclear physics models. In particular, a cloud quantum computing of an atomic nucleus [2], a quantum-classical simulation of Schwinger-model dynamics [3], and quantum simulations of quantum field theories with trapped ions and superconducting circuits [4–7] have been proposed and sometimes experimentally realized. For a thorough review of this research field with updated references see Ref. [8]. However, a paradigmatic quantum nuclear system such as the Agassi model [9] has not been analyzed in the context of quantum simulations. Its relevance for a wide variety of fields, including many-body quantum systems and quantum phase transitions, as well as the difficulty to numerically compute the dynamics and static properties of large quantum systems, motivates the quantum simulation of the Agassi model.

The Agassi model [9] is a simple but far from trivial quantum model that includes a combination of long range monopole-monopole and short range pairing interactions. It was first proposed in nuclear physics since it is an exactly solvable model that provides a schematic version of the pairing-plus-quadrupole model that has been extensively used in nuclear structure [10]. This model presents a rich quantum-phase diagram for the ground state, containing several phase transition lines [11–13], and has been widely studied in a variety of fields. Apart from the symmetric phase, the model has two broken-symmetry phases: one superconducting, linked to the pairing interaction, and another parity-broken phase linked to the monopole-monopole interaction. The phase diagram of the Agassi model has been studied within a mean field formalism. As known, this kind of formalism is valid for finite-N effects are important, the corresponding phases and transitions are blurred and more detailed studies are needed for a clear understanding. For this purpose, quantum platforms could be used to mimic the Agassi model with mesoscopic number of particles, enabling more insights into the different phases and phase transitions.

In this paper, we propose and analyze the digital quantum simulation of the Agassi model [9] in a trapped-ion quantum platform [14, 15]. A quantum simulation of the Agassi model may enable one to carry out a full-fledged analysis of this model for a mesoscopic number of sites, in such situations where all classical methods will fail. Moreover, with our approach, the extension to possible scenarios with inhomogeneous couplings where mean field methods will fail is direct, allowing one for the scalable quantum simulation of nuclear physics models inaccessible to classical supercomputers. We also study how to employ quantum correlation functions as a probe to explore the quantum phases in the system via a quantum simulation of the time dynamics, without needing to compute the ground state. Indeed, we give evidence that the amplitude of the quantum Rabi oscillations [16] in this quantum dynamics is correlated with the different quantum phases of the model.

The Agassi model of nuclear physics.— The Agassi model [9] consists of N interacting fermions which occupy two levels, each of degeneracy Ω, where Ω is even, and j = Ω/2. The lower level σ = −1 has negative parity, and the upper level σ = 1 has positive parity. The magnetic quantum number takes the values m = ±1, . . . , ±j.
Thus, a single-particle state is labeled by \((\sigma = \pm, m)\).

The Hamiltonian is

\[
H = \varepsilon J^0 - g \sum_{\sigma, \sigma' = -1, 1} A^\dagger_{\sigma} A_{\sigma'} - \frac{V}{2} \left[ (J^+)^2 + (J^-)^2 \right],
\]  

(1)

where, implicitly, positive (or null) coefficients are assumed. The operators in \(H\) are

\[
J^+ = \sum_{m} c_{1m}^\dagger c_{-1m} = (J^-)^\dagger, \\
J^0 = \frac{1}{2} \sum_{m} \left( c_{1m}^\dagger c_{1m} - c_{-1m}^\dagger c_{-1m} \right), \\
A^\dagger_i = \sum_{m=1}^j c_{1m}^\dagger c_{i-1m} = (A_1)^\dagger, \\
A^\dagger_{i-1} = \sum_{m=1}^j c_{-1m}^\dagger c_{i-1m} = (A_{-1})^\dagger, \\
N_\sigma = \sum_{m=-j}^j c_{\sigma m}^\dagger c_{\sigma m}, \quad N = N_1 + N_{-1},
\]

(2)-(6)

being \(c_{\sigma m}^\dagger (\sigma_{\sigma m})\) fermion operators that create (annihilate) a single fermion in the state \(|\sigma, m\rangle\).

**Quantum simulation of Agassi model in trapped ions.**—We consider a system with \(j = 1\), which contains four different sites, to analyze a case that may be experimentally realized with current technology. To simplify the notation we relabel the fermions as

\[
c_{1,1} \rightarrow c_1, \quad c_{1,-1} \rightarrow c_2, \quad c_{-1,1} \rightarrow c_3, \quad c_{-1,-1} \rightarrow c_4,
\]

(7)

and the corresponding relationships for the creation operators. The Jordan-Wigner image of the above fermions is

\[
c_i = I_1 \otimes \cdots \otimes I_{i-1} \otimes \sigma_i^- \otimes \sigma_{i+1}^+ \otimes \cdots \otimes \sigma_N^-,
\]

(8)

and the corresponding Hermitian conjugate one for the creation operator. We consider \(N = 4\) in this example and \(\sigma_i^\pm = \frac{1}{2}(\sigma_i^x \pm i\sigma_i^y)\), being \(\sigma_i^x, \sigma_i^y, \sigma_i^z\) the Pauli matrices at position \(i\). We point out that our space is of dimension \(2^4 = 16\) and, therefore, each operator is given by a \(16 \times 16\) matrix.

The spin image of the building block operators is

\[
J^+ = -\sigma_2^- \otimes \sigma_4^- \otimes \sigma_3^- - \sigma_1^- \otimes \sigma_2^- \otimes \sigma_3^-, \\
J^0 = (1/4) (\sigma_1^+ + \sigma_2^+ - \sigma_3^+ - \sigma_4^+), \\
J^- = (J^+)^\dagger = -\sigma_2^- \otimes \sigma_4^- \otimes \sigma_3^- - \sigma_1^- \otimes \sigma_2^- \otimes \sigma_3^-, \\
A_1^\dagger = \sigma_1^+ \otimes \sigma_2^+, \quad A_{-1}^\dagger = \sigma_1^+ \otimes \sigma_2^-, \\
A_1 = \sigma_1^- \otimes \sigma_2^+, \quad A_{-1} = \sigma_3^- \otimes \sigma_4^-.
\]

(9)-(13)

Finally, it is possible to write down the Agassi Hamiltonian for the case of \(j = 1\) as

\[
H = H_1 + H_2 + H_3,
\]

(14)

where

\[
H_1 = \frac{\epsilon - g}{4} (\sigma_1^+ + \sigma_2^+) - \frac{\epsilon + g}{4} (\sigma_3^+ + \sigma_4^+), \\
H_2 = -\frac{g}{4} (\sigma_1^+ \otimes \sigma_2^+ + \sigma_3^- \otimes \sigma_4^-), \\
H_3 = -(g + V) (\sigma_1^+ \otimes \sigma_2^- \otimes \sigma_3^- \otimes \sigma_4^+ + \sigma_1^- \otimes \sigma_2^- \otimes \sigma_3^+ \otimes \sigma_4^-).
\]

(15)-(17)

Note that \(H_1\) and \(H_2\) only depend on \(\sigma^z\) and, therefore, any state with well defined \(\sigma^z\) components will be its eigenstate. \(H_3\) depends on \(g\) and \(V\) and it vanishes for \(g = -V\). Moreover, one should consider that,

\[
[H_1, H_2] = 0, \quad [H_2, H_3] = 0, \quad [H_1, H_3] \neq 0.
\]

(18)

The term \(H_3\) can be further decomposed in terms of tensor products of Pauli matrices,

\[
H_3 = -\frac{g + V}{8} (\sigma_1^+ \sigma_2^- \sigma_3^- \sigma_4^+ + \sigma_1^+ \sigma_2^+ \sigma_3^+ \sigma_4^- + \sigma_1^- \sigma_2^- \sigma_3^+ \sigma_4^+ + \sigma_1^- \sigma_2^+ \sigma_3^- \sigma_4^- + \sigma_4^+ \sigma_2^- \sigma_3^+ \sigma_1^- + \sigma_4^- \sigma_2^+ \sigma_3^- \sigma_1^-, \\
- \sigma_4^+ \sigma_2^- \sigma_3^- \sigma_1^+ - \sigma_4^- \sigma_2^+ \sigma_3^+ \sigma_1^-)
\]

(19)

It is worth noting that for this simple case, \(j = 1\), the ion-mapped Hamiltonian \([13]\) depends on just one effective control parameter, \(g + V\) (see Eq. \([19]\)), and not on \(g\) and \(V\) separately, as in thermodynamic limit of the model \([11, 13]\). Therefore, it is only possible to distinguish for \(j = 1\) between a symmetric and a broken-symmetry phase. The critical point in the transitional path between these two phases is \(g + V = 1\).

**Theoretical model for the implementation.**—In order to carry out a quantum simulation with the Agassi model, we suggest to employ a digital protocol, via a Lie-Trotter-Suzuki decomposition \([1]\). The protocol will rely on expressing the quantum evolution operator \(U(t) = \exp(-iHt)\) for the Hamiltonian \(H\) in Eq. \([14]\) by means of a Trotterized dynamics, in terms of \(H_{1,2,3}\) of Eqs. \([15]\), \([16]\), and \([17]\),

\[
U(t) \simeq \{\exp[-i(H_1 + H_2)(t/\eta_T)] \exp[-iH_3(t/\eta_T)]\}^{\eta_T}
\]

(20)

where \(\epsilon\) the error produced will depend on the commutator \(([H_1 + H_2, H_3])\) and scale as \(1/\eta_T\), where \(\eta_T\) denotes the number of Trotter steps.

Once the dynamics has been decomposed into the previous blocks, each of these can be implemented efficiently with trapped-ion systems. The operator \(\exp(-iH_1t)\) consists of single-qubit gates which are customary with trapped ions, to fidelities often above 99.99\% \([17]\). The operator \(\exp(-iH_2t)\) is composed of two two-qubit gates which can be carried out via Mølmer-Sørensen gates with fidelities above 99.9\% in some experimental setups \([18]\), in addition to single-qubit gates to rotate the basis from \(x\) to \(z\). And finally, \(\exp(-iH_3t)\) consists of the exponential of sum of tensor products of four Pauli matrices, which can be carried out efficiently with trapped
ions \[19, 20\]. Namely, each exponential of a tensor product of four Pauli operators can be implemented via two Mølmer-Sørensen gates and a local gate, together with the necessary single qubit gates to rotate the bases of the Pauli operators in the tensor product to those needed. Given that all the 4-body terms in Eq. \[19\] commute, they may be carried out sequentially without significant error, namely, with one Trotter step for the whole \(H_T\) term.

The scaling in our protocol is efficient, given that the number of necessary elementary gates in trapped ions, i.e., single and two-qubit gates, is polynomial in the number of interacting fermions, \(N\), in the Agassi model. On the other hand, with a classical computer the scaling would be inefficient given that the Hilbert space dimension would grow exponentially in \(N\). Of course, under highly symmetric configurations one may obtain a solution, in some cases, in terms of polynomial resources. However, in general terms, with a generalized Agassi model with, e.g., inhomogeneous couplings, this will not be possible and a quantum simulator such as the one proposed here will provide an exponential gain in resources with respect to a classical computer.

In Fig. 2 we plot the survival probability \(|\langle \phi(t)|\phi(0)\rangle|^2\) as a function of \((g + V)t\) for \(\epsilon = 1\), \(g = V = 0.5\) in panel a) and \(\epsilon = 1\), \(g = V = 1\) in panel b). The considered initial state is \(|\downarrow_1 \otimes \downarrow_2 \otimes \uparrow_3 \otimes \uparrow_4\rangle\).

In Fig. 2 we plot the survival probability \(|\langle \phi(t)|\phi(0)\rangle|^2\) as a function of \((g + V)t\) to show that the dynamics significantly changes in the time interval considered in Fig. 1.

Finally, in Fig. 3 we depict the correlation function \(\sigma_z(12) = \langle \sigma_1^z\sigma_2^z \rangle - \langle \sigma_1^z \rangle \langle \sigma_2^z \rangle\), showing that the time dynamics alone can be used as a probe to explore the different quantum phases of the system via this correlation function. As mentioned above, the critical point in the Agassi model for \(j = 1\) is given by \(g + V = 1\). Fig. 3 shows that at one side of the phase transition the correlation amplitude is smaller than one (symmetric phase), while it is 1 at the other side (broken phase), and extra oscillations appear which amplitudes depend on the control parameter value. This is meaningful as \(\sigma_z(12)\) is a measure of the entanglement, which typically can change undergoing a quantum phase transition. This is more clearly shown in Fig. 3b), in which the amplitude of the oscillation is plotted as a function of the control parameter \(g = V\), where the critical point is at \(g = V = 0.5\). The figure shows that this amplitude reaches the value 1 at the critical point and keeps this value in the broken-symmetry phase \((g + V \geq 1)\). This is an evidence that one does not need to compute the ground state to distinguish the different quantum phases in the system. The more direct time dynamics for a typical initial state allows one to obtain signatures of these quantum phases in the amplitude of the quantum Rabi oscillations. We depict the case with \(n_T = 5\), although we point out that, for our example with \(N = 4\), the case with \(n_T = 3\) already provides accurate results.

Moreover, we can observe in Fig. 4 that the digital error remains negligible for a sizable time evolution with a nontrivial dynamics and a sufficiently large \(n_T\). With respect to the total gate error in a plausible implementa-

![FIG. 1. Fidelity |\langle \phi(t)|\phi(t_F)\rangle|^2 as a function of (g + V)t for n_T = 10 in panels a) and c) and as a function of n_T for t_F = 10 in panel b) and t_F = 2 in panel d). In all cases the parameters of the Hamiltonian are \(\epsilon = 1\) and \(g = V = 1\).](image-url)
Eugene F. Dumitrescu, Alex J. McCaskey, Gaute Hagen, I.M. Georgescu, S. Ashhab, and Franco Nori, “Quantum simulation”, Rev. Mod. Phys. 86, 153 (2014).

FIG. 3. Correlation function \( \sigma_z(12) \equiv \langle \sigma_z^1 \sigma_z^2 \rangle - \langle \sigma_z^1 \rangle \langle \sigma_z^2 \rangle \) for an initial state \( | \downarrow_1 \otimes \downarrow_2 \otimes \uparrow_3 \otimes \uparrow_4 \rangle \) and Hamiltonian parameters \( \epsilon = 1, g = 0.5, \) and \( V = 0 \) in a), \( \epsilon = 1, g = 0.4, \) and \( V = 0.4 \) in b), and \( \epsilon = 1, g = 0.5, \) and \( V = 1 \) in c). Lines correspond to exact calculations while dots refer to a Trotter expansion with \( n_T = 5 \). In panel d) it is depicted the amplitude of \( \sigma_z(12) \) as function of \( g = V \) and the phase diagram, SP for the symmetric phase and BSP for the broken-symmetry phase. The critical point is \( g = V = 0.5 \).

Conclusions.— We have proposed and analyzed the quantum simulation of the Agassi model with a trapped-ion quantum platform. Our numerical simulations and analytical estimations show that this protocol is feasible with current technology. The proposal has been exemplified with four sites to be implemented with four trapped ions, while it is scalable to many sites with polynomial resources. We also give evidence that the time dynamics of a quantum correlation function for typical initial states can serve as a probe to explore the different quantum phases of the model, with no need of computing specifically the ground state. Indeed, the different phases of the system can be matched to the quantum Rabi amplitudes of the dynamics. With recent advances in trapped-ion quantum platforms approaching a few tens of ions in a quantum processor [21], we are already going through the crossover for outperforming the fastest classical supercomputers for useful scientific problems. Our approach is a step in this direction, for the efficient quantum simulation of the Agassi model and related nuclear physics systems with trapped-ion quantum platforms.

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