Dual-density waves with neutral and charged dipolar excitons of GaAs bilayers

Strongly correlated quantum particles in lattice potentials are the building blocks for a wide variety of quantum insulators—for instance, Mott phases and density waves breaking lattice symmetry\(^1\). Such collective states are accessible to bosonic and fermionic systems\(^2\). To expand further the spectrum of accessible quantum matter phases, mixing both species is theoretically appealing because density order then competes with phase separation\(^3\). Here we manipulate such a Bose–Fermi mixture by confining neutral (boson-like) and charged (fermion-like) dipolar excitons in an artificial square lattice of a GaAs bilayer. At unitary lattice filling, strong inter- and intraspecies interactions stabilize insulating phases when the fraction of charged excitons is around (1/3, 1/2, 2/3). We evidence that dual Bose–Fermi density waves are then realized, with species ordered in alternating stripes. Our observations highlight that dipolar excitons allow for controlled implementations of Bose–Fermi Hubbard models extended by off-site interactions.

Strongly interacting quantum particles confined in periodic potentials are specifically described by the Hubbard model (HM)\(^1\). This Hamiltonian constitutes one of the most celebrated models of condensed matter physics. For fermions the HM has been extensively studied, underlying the current theoretical description of high-temperature superconductivity\(^2\). For bosons it has also raised considerable attention, triggered by pioneering experiments with ultracold atoms\(^4\). The physics of the HM has recently been fostered by two-dimensional semiconductors. These offer new opportunities to study bosonic and fermionic realizations of Hubbard Hamiltonians. Triangular moiré lattices obtained by interfacing atomic monolayers provide a successful example for Fermi Hubbard physics. Indeed, moiré lattices have enabled electronic Wigner-like crystals\(^6\), as well as a wide range of insulating density waves (DWs) at fractional lattice fillings\(^8\). For bosons, the HM has been implemented with dipolar excitons that are Coulomb-bound electrons and holes spatially separated in a GaAs bilayer\(^7\). These are efficiently confined in gate-defined electrostatic lattices\(^9\), and characterized by their photoluminescence (PL) emission. Mott insulator and chequerboard solid were then reported at integer and half-fillings, respectively\(^11\), the latter phase evidencing the extended Bose–HM\(^16\),

Here we implement the Bose–Fermi Hubbard Hamiltonian extended by off-site interactions, by confining dipolar excitons and holes in an electrostatic lattice of a GaAs bilayer. These two species experience strong attractions. Holes are thus captured in lattice sites occupied by excitons, yielding fermion-like charged excitons (CX). With a charge control down to an ultralow residual doping level, we implement the Bose–Fermi HM by controlling neutral and charged excitons fillings, \(\nu_X\) and \(\nu_{CX}\), respectively. At \((\nu_X + \nu_{CX}) = 1\) we uncover incompressible states for fractional values of \(\nu_{CX}\) around (1/3, 1/2, 2/3). We find spectroscopic evidence that neutral and charged excitons then implement alternating striped DWs. Thus, we confirm the rich variety of quantum insulators accessible to Bose–Fermi mixtures\(^13\).

As illustrated in Fig. 1a, we study a field-effect device where an array of surface gate electrodes, polarized at \(V_g\), applies a transverse electric field sinusoidally varying in the plane of a GaAs double-quantum well. The latter confines excitons and holes, spatially separated in each layer and Coulomb bound, to realize dipolar excitons. These are

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Fig. 1 | Neutral and charged excitons’ Bose–Fermi mixture. a. An array of gate electrodes polarized at $V_g$ imprints an artificial electrostatic two-dimensional lattice (sinusoidal white wave) for neutral (blue) and charged (red) excitons confined in a GaAs bilayer (grey lines) embedded in an AlGaAs heterostructure (black). b. In the lattice, on-site interactions exceed confinement depth so that neutral and charged excitons experience NN interactions only—$V_{X,X}$, $V_{X,CX}$ and $V_{CX,CX}$. c. PL spectra for $V_g = (−1.02, −0.9935, −0.96)$ V at unitary filling, from top to bottom. Lines represent model spectra obtained by adjustment of PL lines with a lorentzian-like profile limited by our spectral resolution; blue and red areas mark neutral and charged excitons PL, respectively. d. CX PL spectrum around half-filling reproduced by setting 44 and 56% occupation for the lower and upper WS (yellow and red, respectively). WS energy separation is set to 100 μeV while the black line represents the modelled profile. e. Scaling of charged and neutral exciton energies (red and blue, respectively) as a function of $V_g$. Points are wider than the horizontal error while the vertical error provides spectral precision (±15 μeV). Solid lines represent theoretical variations given by (−e×d$V_g$/L) (Methods). All experiments were realized at $T = 330$ mK. c,d. Error bars are given by poissonian noise.

characterized by their permanent electric dipole, interacting with the applied field. Hence, we confine excitons in a 250 nm period square lattice, with around 250 μeV depth. Excitons then have two accessible confined states (Wannier states, WS) separated by 150 μeV (ref. 12 and Methods). Moreover, note that excitons are optically injected, using 100 ns laser excitation repeated at 1 MHz. The laser excitation power $P$ controls the lattice filling, and we study the PL radiated 300 ns after extinction of the laser pulse (Methods). Because this delay is around half of the excitons’ effective lifetime, carriers are efficiently thermalized down to our lowest bath temperature ($T = 330$ mK).

Figure 1c shows the PL spectra radiated by our device for different gate voltages and for a lattice filling around 1. Let us first note that here and, in the following, PL is averaged over a $3.0 \times 2.5$ μm$^2$ region thereby including around 120 lattice sites (Methods). While at $V_g = −1.02$ V the top panel of Fig. 1c shows that PL is mostly due to the recombination of dipolar excitons (high energy peak), we observe that a second emission grows at lower energies when $V_g$ is reduced to about $−0.9935$ V (middle). By further decreasing $V_g$ to $−0.96$ V, the bottom panel shows that PL is mostly given by the low energy contribution. Such dual PL emission has previously been reported for GaAs bilayers13,24. It was shown that the low-energy line marks the strong attraction between excitons and excess carriers, yielding (composite) fermionic complexes usually referred to as CX. On the other hand, the higher-energy PL line signals the radiative recombination of neutral dipolar excitons. The energy separation between these two contributions, around 1 meV, well matches theoretical expectations25,26. Moreover, we note that excess carriers can be attributed to holes only for our heterostructure characterized by ultrahigh electronic mobilities27. Indeed, holes are trapped in the regions of the bilayer where a perpendicular field is applied—that is, inside the lattice—whereas electrons are expelled out of this zone28.

Neutral dipolar excitons are characterized by a linear dependence of their PL energy with $V_g$ (ref. 24), scaling like ($e × dV_g/L$). Here $e$ denotes electron charge, $d$ spatial separation between the centre of the two quantum wells and $L$ the thickness of our field-effect device (Methods). In Fig. 1e we see this signature (blue), for $−1.02 \leq V_g \leq −0.92$ such that lattice depth remains essentially unchanged (Methods). Furthermore, we note that the PL energy of CX follows the same scaling (red). We then deduce that holes are captured in the lattice sites occupied by dipolar excitons. As a result, our device efficiently confines both neutral and charged dipolar excitons in the same lattice potential. This conclusion is further confirmed by scrutinizing the spectrum of the PL emitted by CX. Indeed Fig. 1d underlines that, for low lattice fillings, the PL
Compressibility, \( \kappa_{\text{CX}} \), normalized to the level set by poissonian fluctuations, around unitary (a) and half-fillings (b). Bottom, variation in \( \kappa_{\text{CX}} \) as a function of \( V_g \) for \( P = 14 \, \text{nW} \) (a) and 7.6 nW (b). Top, \( \kappa_{\text{CX}} \) as a function of \( P \) for \( V_g = -0.945 \, \text{V} \) (a) and \(-0.930 \, \text{V} \) (b). The white area shows the regime where hole and exciton densities are equal so that the lattice confines only CXs. Blue and red areas denote regimes where excitons and holes, respectively, are in excess. Black lines are a visual guide only. c,d, PL spectrum for \( \nu_{\text{CX}} = 1 \) (14 nW, \(-0.945 \, \text{V}\)) (c) and \( \nu_{\text{CX}} = 1/2 \) (7.6 nW, \(-0.930 \, \text{V}\)) (d), reproduced by the lorentzian-like profile given by our spectral resolution (red). All experiments were realized at \( T = 330 \, \text{mK} \), and we statistically analysed ten repetitions for every experimental setting. a,b, Vertical error bars provide \( \pm 0.03 \) precision when measuring compressibility, while horizontal error bars are smaller than point size. c,d, Error bars show poissonian noise.

The spectrum consists of two emission lines separated by around 100 \( \mu \text{eV} \). This magnitude agrees reasonably with the theoretically expected energy separation between the two WS accessible to an exciton complex consisting of two holes and one electron, and for around 250 \( \mu \text{eV} \) lattice depth (Methods).

Figure 1c and Extended Data Fig. 1 show that the fractions of neutral and charged excitons are efficiently varied by gate voltage. On the other hand, we verified that the overall lattice filling (\( \nu_X + \nu_{\text{CX}} \)) is mostly given by the laser excitation power \( P \) (Extended Data Fig. 1). Experimentally, to set \( \nu_X \) and \( \nu_{\text{CX}} \) we first adjusted \( V_g \) so that \( \nu_{\text{CX}} = 0 \). The neutral excitons filling \( \nu_X \) is then controlled by \( P \), set for instance such that \( \nu_X = 1 \) by realizing a Mott insulator. For the chosen laser excitation power, \( \nu_{\text{CX}} \) is tuned by the gate voltage while \( (\nu_X + \nu_{\text{CX}}) \) remains mostly unchanged (Extended Data Fig. 1). Note that neutral and charged exciton fillings are quantified by computing their PL integrated intensities (Methods). Thus we assess the purity of our device. Indeed, Extended Data Fig. 2 shows that, by suitable adjustment of \( V_g \), the PL due to CX cannot be distinguished from the spectral noise for \( \nu_X = 0.5 \). This shows that the concentration of residual holes is bound to \( 4 \times 10^{7} \, \text{cm}^{-2} \) (Methods). Remarkably, this value is comparable to the record residual doping measured for our heterostructure\(^{11} \), a regime previously inaccessible to optical techniques.

By confining tuneable concentrations of neutral and charged excitons, our lattice device can emulate Hubbard Hamiltonians continuously from the bosonic to the fermionic regime; we recently quantified the former\(^{11,12} \). Below, we first implement the Fermi HM by equalization of the fraction of excess holes and excitons. Combined interspecies attractions and fast carrier tunnelling (Methods) then ensure that the lattice is filled only with CX.

Exploring the \((P, V_g)\) parameter space we found two specific combinations, namely (7.6 nW, \(-0.93 \, \text{V}\)) and (14 nW, \(-0.945 \, \text{V}\)) for which CX compressibility \((\kappa_{\text{CX}})\) is minimized (white areas in Fig. 2a,b). In fact, varying either \( V_g \) or \( P \) from these coupled values leads to a twofold increase in \( \kappa_{\text{CX}} \) towards the level given by poissonian fluctuations, since excitons (blue area) or holes (red area) are then in excess. Furthermore, the two incompressible phases are characterized by a lorentzian-like PL spectrum given by our spectral resolution (Fig. 2b–d). Hence, we deduce that CXs mostly occupy the same state in the lattice. In the charge-neutral regime we verified that \( \nu_X = 1/2 \) for \( P = 7.6 \, \text{nW} \) and 1 for \( P = 14 \, \text{nW} \). The spectra displayed in Fig. 2a,b then evidence two insulating states, at \( \nu_{\text{CX}} = (1/2, 1) \), theoretically corresponding to MI and chequerboard solid, respectively\(^{4} \).

In regard to neutral excitons\(^{11} \), from the difference \( \Delta \) between PL energy at unitary and half-fillings we deduce the nearest-neighbour (NN) interaction strength \( V_{\text{CX,CX}} \) (Fig. 1b). Indeed, at \( \nu_{\text{CX}} = 1 \) the energy of a MI is enhanced by \( 4 \, V_{\text{CX,CX}} \) compared with a chequerboard phase, because NN couplings are fully avoided for the latter. Taking into account the slight difference in gate voltage necessary to realize the two insulating states, we obtain \( \Delta = (-0.015 \times e \times d/L + 4 \times V_{\text{CX,CX}}) \), leading to \( V_{\text{CX,CX}} = 140 \pm 15 \, \mu \text{eV} \). Importantly, this magnitude is confirmed by studying thermal melting of the insulating state at \( \nu_{\text{CX}} = 1/2 \). Extended Data Fig. 3 shows that \( \kappa_{\text{CX}} \) is minimized up to a critical temperature...
Eseparated insulators of neutral and charged excitons yield (line is a visual guide only. Bottom horizontal dashed line shows that spatially
ν
by the fraction of fermionic defects—that is, by the fraction of charged compressibility,
the gate voltage, and we monitor simultaneously the corresponding
tions prevent double occupancies in our lattice (Methods). We continu-
ν
of
namely that a chequerboard solid melts when
E
PL energy of neutral and charged excitons, respectively. Δ
E
expectation, we studied Δ
E
X,CX(0.1) ≈ 4(Δ
E
X,X) – Δ
E
X,CX(0.9)) ≈ 220 μeV (dotted
around 1 K. Thus our measurements follow theoretical expectations, namely that a chequerboard solid melts when
κ
7 becomes of the order of
V
X,CX, (refs. 12,13).
In the following we turn to the Bose–Fermi HM in the regime where the overall filling (νX + νCX) is set to unity, because strong on-site interactions prevent double occupancies in our lattice (Methods). We continuously vary neutral and charged exciton filling fractions by sweeping the gate voltage, and we monitor simultaneously the corresponding compressibility, χX and χCX, respectively, by statistically computing PL intensity fluctuations (Methods).
Figure 3a presents χX (blue) and χCX (red), normalized by the level set by poissonian fluctuations, as a function of
νCX. For νCX ≤ 0.15, the PL spectrum is dominated by the excitonic component and χX is mini-
mized. Excitons then realize a boson-like MI that is weakly perturbed by the fraction of fermionic defects—that is, by the fraction of charged excitons νCX. Similarly, χCX is minimized for νCX ≥ 0.85 so that charged excitons realize a fermion-like MI at unitary filling, weakly perturbed by the fraction of (bosonic defects) excitons νX. By contrast, χX and χCX are mostly of the order of the level given by poissonian fluctuations for 0.2 ≤ νCX ≤ 0.8. The system evolves therefore in a normal fluid phase. Nevertheless, for νCX = (0.35, 0.52, 0.61) ± 0.03 we strikingly observe that χX and χCX simultaneously drop, manifesting correlated incompressible states.
Combined insulating phases at νCX = (1/3, 1/2, 2/3) suggest that neutral and charged excitons realize dual DWS\textsuperscript{12–15}, namely, DWs and spatially separated Mott phases of neutral and charged excitons (Methods).
We first extract interspecies NN interaction strength
V
X,CX (Fig. 1b) by comparing Δ
E
X,CX between the two MI regimes (Extended Data Fig. 4). At νCX = 0.1, neutral excitons realize a Mott insulator with several charged exciton impurities so that Δ
E
X,CX(0.1) = 4(V
X,X − V
X,CX). In the same way, at νCX = 0.9, we obtain Δ
E
X,CX(0.9) = 4(V
X,X − V
X,CX). Accordingly we find that
V
X,CX = (V
X,X + V
X,CX)/2 − (Δ
E
X,CX(0.1) − Δ
E
X,CX(0.9))/8. From the measurements shown in Fig. 3b (black points), we conclude that
V
X,X = (75 ± 10) μeV because
V
X,X = (30 ± 5) μeV\textsuperscript{12} and
V
X,CX = (140 ± 15) μeV.
As shown above, repulsive NN interactions between charged excitons greatly exceed other interaction strengths. Accordingly, Fig. 3b and Extended Data Fig. 4 show that Δ
E
X,CX (dashed line in Fig. 3b) is smallest for spatially separated Mott phases, notably at
νCX = (1/3, 1/2, 2/3). By contrast, the greatest amplitude is obtained for a double-chequerboard DW at
νCX = 1/2 while intermediate values are found for striped DWs (Fig. 3b and Extended Data Fig. 4). For this latter ordering one would expect that, at νCX = 2/3, Δ
E
X,CX = 2(V
X,X − V
X,CX), corresponding to an increase by 2(V
X,X − V
X,CX) = 130 μeV compared with the exciton MI regime. Remarkably, this enhancement is confirmed experimentally in Fig. 3b, since (Δ
E
X,CX(0.61) − Δ
E
X,CX(0.9)) = (100 ± 15) μeV. Hence, we deduce that alternating stripe phases are realized at
νCX = 0.61 ± 0.03, relatively close to the theoretical 2/3 filling. Similarly, we verify in Fig. 3b that (Δ
E
X,CX(0.36) − Δ
E
X,CX(0.12)) = 2(V
X,X − V
X,CX). This matching again reveals that alternating stripes are favoured, with a pattern symmetrical with that for
νCX = 2/3. Finally, at νCX = 1/2, Fig. 3b shows that (Δ
E
X,CX(0.52) − Δ
E
X,CX(0.12)) = (0 ± 15) μeV, as expected for horizontal or vertical stripes; otherwise, (Δ
E
X,CX(0.52) − Δ
E
X,CX(0.1)) would amount to around 220 μeV (dotted line) for alternating chequerboards (Fig. 3b and Extended Data Fig. 4).
To conclude, we have characterized the insulating part of the phase diagram for the Bose–Fermi HM. Dipolar excitons naturally explore the regime extended by off-site interactions so that DWs emerge, even at half boson/fermion filling, unlike for mixtures bound to short-range interactions\cite{5, 6}. Further reduction of the carrier temperature to a few tens of milli-Kelvins, which is within experimental reach, would extend our studies to the situation where excitons would exhibit extended phase coherence. Mixtures of neutral and charged excitons could then serve as a platform to engineer supersolid-like states stabilized by ordered fermionic phases. Also, the low (boson)-doping regime relevant to superconducting electronic systems may be accessed, by doping Mott insulators of charged excitons with a coherent fluid of neutral ones.

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Methods
Sample structure and experimental procedure
The 250-nm-period electrostatic lattice is identical to that studied in ref. 21. It is based on two 8-nm-wide GaAs quantum wells separated by a 4 nm AlGaAs barrier. Dipolar excitons are made by spatially separated electrons and holes then carry a permanent electric dipole with a moment of around 12 e.nm, where e denotes the electron charge. Also, note that the quantum wells are positioned 200 nm below the surface of the field-effect device in which they are embedded, and 150 nm above a conductive layer that serves as electrical ground.

Electronic carriers are injected into the lattice potential using a laser excitation at resonance with direct exciton absorption of the quantum wells. Hence we ensure that the concentration of photo-injected free carriers is minimized, although not necessarily vanishingly small. This regime is found only by suitable adjustment of $V_g$. Indeed, charge control is accessed for correlated values of $P$ and $V_g$. This behavior is illustrated in Fig. 2, with the optimal values of $V_g$ to engineer CX insulators at $V_{CX} = 1$ and 1/2 differing by 0.015 V.

For spectral analysis of the PL radiated by our device we rely on a grating of 1,800 lines mm$^{-1}$. The PL is then sample with 15 μeV precision so that the narrowest profile possibly measured — for example, a laser line — is lorentzian-like with around 150 μeV full-width at half-maximum. To model PL spectra we assign this profile to the emission from the WS for neutral and charged excitons.

As stated above, we extract $V_{CX}$ and $V_X$ by computing the PL integrated intensities of neutral and charged excitons, these being directly proportional to the concentration of each species. Importantly, we verified that the overall lattice filling ($V_X + V_CX$) weakly varies in the range of gate voltages explored. Extended Data Fig. 1 shows that, for fixed $P$, the overall PL integrated intensity is rather constant. Accordingly, we assumed that only $V_X$ and $V_{CX}$ vary with $V_g$ and, given the signal-to-noise ratio of our experiments, we deduced that $V_{CX}$ is obtained with ±0.03 precision.

Excitonic compressibility
To quantify the compressibility of neutral and charged excitons, we statistically analyse intensity fluctuations of their PL maxima, $A_{X,max}$ and $A_{CX,max}$, respectively. In practice, for every experimental setting we rely on samples of ten measurements, all performed under fixed conditions. Note that sample size is limited by the time (6 h) available at our lowest bath temperature, because acquisition of a single ten-measurement sample requires around 20 min.

By monitoring PL intensity fluctuations, we directly compute compressibility, because $(dX/dT)$ is proportional to $\sigma_{X,max}(A_{X,max})/A_{X,max}$ according to the fluctuation-dissipation theorem. Here $\sigma_{X,max}$ and $A_{X,max}$ denote the variance and mean value of the PL maximum $A_{X,max}$, respectively. Moreover, note that for every experimental setting we systematically rescale neutral and charged exciton compressibility to the value computed for the level of poissonian fluctuations; thereby, experimental observations are more easily confronted. Finally, for our sample size—that is, ten realizations for each experimental condition—we estimated the precision with which compressibility is deduced. For that, we computed $k_{X,CX}$ by random selection of seven measurements out of ten, from which we observed that $k_{X,CX}$ varies by around ±0.03. We then assigned this error bar to measured compressibility.

Neutral and charged excitons in the lattice
Relying on finite element simulations, we have computed the lattice profile confining neutral dipolar excitons. We thus found that the lattice depth, $V_g$, is about 250 μeV when $V_X$ is set to −1 V. Excitons are then confined in two WS, separated by around 150 μeV. Furthermore, we computed the variation in lattice depth when $V_g$ is varied between −0.92 and −1.02 V, as in the experiments shown in Figs. 1−3. We concluded that $V_g$ varies from 250 to 230 μeV in this range, and that energy separation between the two accessible WS is then between 140 and 150 μeV. This variation is small compared with other energy scales, such as thermal energy, and thus plays a minimal role in our studies.

We also simulated the nature of the lattice confinement for CX. Noting that holes have their minimum potential energy at the position of the lattice sites, we first considered that holes are captured in lattice sites occupied by dipolar excitons, favoured by strong interspecies attraction. For electron and hole effective masses, $m_e = 0.07 m_0$ and $m_h = 0.12 m_0$ (ref. 22), respectively and, considering the same 250-μeV-deep lattice, we find that CX are confined in two WS separated by around 120 μeV, in reasonable agreement with experimental findings (Fig. 1).

Alternatively, one may consider that CX are confined in the lattice due to interaction between their positive charge and the electric field applied in the plane of the double-GaAs quantum wells. To explore this scenario, we computed the lattice depth for a point-like and positively charged exciton. We deduced that $V_{CX}$ would then be around 1 μeV but, more importantly, that the point-like charged particle would have access to three WS, separated by around 250 μeV. This conclusion is in contradiction to the measurements shown in Fig. 1. The hole confinement in the lattice is then better captured by considering its strong Coulomb attraction with dipolar excitons.

Hubbard parameters for neutral and charged excitons
We recently calculated and measured on-site and NN interactions between dipolar excitons, $U_{X,X}$ and $U_{X,CX}$, respectively. We thus found that $U_{XX}$ exceeds lattice depth and can be as large as 1 meV for the lowest WS, whereas $U_{X,CX} = (30 ± 5) μeV$. On the other hand, the measurements shown in Figs. 2 and 3 yield $V_{CX,XX} = (140 ± 15) μeV$ and $V_{CX,CX} = (75 ± 10) μeV$. This demonstrates that repulsions between neutral excitons constitute the weakest interaction channel. Also, we note that on-site intra- and interspecies scatterings largely exceed lattice depth so that lattice sites cannot be doubly occupied.

In ref. 21 we have shown that the tunnelling strength between lattice sites amounts to a few micro-electronvolts for neutral excitons. Given that the hole effective mass is around 2/3 that of the exciton, we deduced that holes also benefit from tunnelling strength in the micro-electronvolt range. Thus we ensure that the lattice confines CX only when the hole and neutral exciton concentration are matched (Fig. 2), since our experiments are acquired during a 100-ns-long time interval allowing for hundreds of particles tunnelling between lattice sites.

Minimum hole doping level
Extended data Fig. 2 reports a PL spectrum measured for $V_g = 1/2$ and for a minimized concentration of holes. The spectrum is obtained by averaging ten realizations; its maximum is $34 ± 1.8$ counts min$^{-1}$ while the level of spectral white noise is $3.5 ± 0.6$ counts min$^{-1}$. The signal-to-noise ratio (SN) is then about ten in these experiments. Furthermore, we deduce that $V_{CX} = 1/2$ translates to a mean concentration of neutral excitons, $n_X = 4 × 10^8$ cm$^{-2}$ for our 250-nm-period lattice. Since we do not distinguish PL due to charged excitons from the spectral noise level, we deduce an upper limit for the mean concentration of holes in these measurements, namely $n_h = n_h/SN = 4 × 10^5$ cm$^{-2}$. This value, limited by SN, is then of the same order as that obtained from transport techniques for our GaAs quantum wells ($1.5 × 10^7$ cm$^{-2}$ (ref. 23)).

Spatial order of incompressible phases
Within the framework of the HM, we expect theoretically that density waves will emerge for fractional fillings, mostly around (1/3, 1/2, 2/3) where stripes or checkerboard orders are possibly realized. In our lattice, neutral and charged excitons interact through NN couplings, $V_{XX} = V_{X,CX}/2 - V_{CX,XX}/5$. We then directly deduce mean single-particle interaction energy, $\epsilon = (V_{XX} + V_{X,CX})$, with $\epsilon_X$ and $\epsilon_{CX}$ denoting mean neutral and charged exciton interaction energy, respectively.
At $\nu_{\text{CX}} \sim \frac{1}{3}, \frac{2}{3}$, DWs necessarily correspond to alternating diagonal stripes of neutral and charged excitons. We find that these are characterized by $\epsilon = \frac{4}{3}(2V_{\text{X,CX}} + V_{\text{CX,CX}}) = 240$ μeV and $4/3(2V_{\text{X,CX}} + V_{\text{CX,CX}}) = 387$ μeV at $\nu_{\text{CX}} = 1/3$ and $2/3$, respectively (Extended Data Fig. 4). On the other hand, phase-separated Mott insulators exhibit mean interaction energy of around 267 and 413 μeV, respectively. These are thereby unfavourable. The explicit evaluations shown above illustrate that alternating stripe phases exhibit lower energy than separated Mott phases when $V_{\text{X,CX}} < \frac{1}{2}(V_{\text{X,X}} + V_{\text{CX,CX}})$. Importantly, this condition is satisfied in our experiments because $V_{\text{X,CX}} = \frac{1}{2}(V_{\text{X,X}} + V_{\text{CX,CX}})(0.1) - \Delta E_{\text{X,CX}}(0.9)/8$, as detailed in the main text.

At $\nu_{\text{CX}} \sim \frac{1}{2}$, we deduce that $\epsilon = 4V_{\text{X,CX}} = 300$ μeV for a double-chequerboard DW, and $(2V_{\text{X,CX}} + V_{\text{X,X}} + V_{\text{CX,CX}}) = 320$ μeV for a striped DW. On the other hand $\epsilon = 2(V_{\text{X,X}} + V_{\text{CX,CX}}) = 340$ μeV for separated Mott phases of neutral and charged excitons. Again, we find that DW arrangements are energetically favoured over phase separation. However, we also find that the alternating stripe phase exhibits a larger mean interaction energy than the chequerboard phase. This conclusion contrasts with our observations, which may at first suggest that the precision with which we extract NN interaction strength is not sufficient for accurate determination of DW ground-state energy. Alternatively, one cannot exclude the fact that the alternating stripe phase at $\nu_{\text{CX}} = \frac{1}{2}$ is of metastable nature. Indeed, insulating states with Bose–Fermi mixtures are characterized by instabilities, triggered by a sensitive competition between inter- and intraspecies interaction strength.

Data availability
Source data are available for download, together with the published manuscript.

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Author contributions
K.B. and L.P. realized the GaAs bilayer while C.L., S.S. and F.D. fabricated the gate electrodes imprinting the 250-nm-period electrostatic lattice. C.L. and F.D. performed all experiments and data analysis and wrote the manuscript. F.D. designed the project.

Competing interests
The authors declare no competing interests.

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Extended Data Fig. 1 | Lattice filling vs. gate voltage. a Total integrated intensities of the PL radiated by neutral and charged excitons, as a function of $V_g$ and at unitary filling ($P=14$ nW). b Same experimental results as in a but expressed as a function of $\nu_{CX}$. c Scaling of $\nu_{CX}$ as a function of $V_g$ deduced from the measurements shown in a and b. The line provides a guide for the eyes.

Experiments were all realised at 330 mK and acquired during four different experimental runs so that detection efficiencies are close but not identical. Vertical error bars display the poissonian precision in a-b and the ± 0.03 precision on $\nu_{CX}$ in c. In a-c, the horizontal error is smaller than the points size while in b it corresponds to the precision when extracting $\nu_{CX}$. 
Extended Data Fig. 2 | Evaluation of the residual doping level. PL spectrum radiated by neutral dipolar excitons for νx = 1/2 (at νcx = 0). The spectrum is measured by averaging 10 realisations performed under unchanged conditions. The profile is given by our spectral resolution, that is reproduced by a single lorentzian-like line with around 150 μeV full-width-at-half-maximum (blue area and black line). Measurements were performed at 330 mK, error bars displaying the level of poissonian fluctuations.
Extended Data Fig. 3 | Thermal melting of CX insulators at $\nu_{CX} = 1/2$ and 1. a Compressibility $\kappa_{CX}$ normalised to the level given by poissonian noise for $(\nu_{CX} = 1/2, \nu_X \approx 0)$ as a function of the bath temperature. b Identical measurements for $(\nu_{CX} = 1, \nu_X \approx 0)$. While in a the thermal melting of the insulating phase occurs around 1K, as expected for the magnitude measured for $V_{CX,CO}$, a similar critical temperature is found in b for the Mott phase. This possibly reflects fluctuations of the density of injected holes while the bath temperature is increased. For all measurements error bars mark our statistical precision when computing the compressibility ($\pm 0.03$).
Extended Data Fig. 4 | Interaction energies and spatial ordering. Possible configurations of incompressible phases made by neutral (blue) and charged (red) excitons. The respective energy shifts of PL energies, $E_x$ and $E_{cx}$, are indicated below each configuration together with the resulting magnitude of $\Delta E_{x,cx}$ by only taking into account NN interactions.