Y(4260) as a mixed charmonium-tetraquark state

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I. INTRODUCTION

Many of the charmonium-like states recently observed in $e^+e^-$ collisions by BaBar and Belle collaborations do not fit the quarkonia interpretation, and have stimulated an extensive discussion about exotic hadron configurations. The production mechanism, masses, decay widths, spin-parity assignments and decay modes of these states, called $X$, $Y$ and $Z$ states, have been discussed in some reviews.[1–5] Among these states, the Y(4260) is particularly interesting. It was first observed by BaBar collaboration in the $e^+e^-$ annihilation through initial state radiation.[6] and it was confirmed by CLEO and Belle collaborations.[7] The Y(4260) was also observed in the $B^+ \rightarrow Y(4260) K^- \rightarrow J/\Psi\pi^+\pi^- K^-$ decay,[8] and CLEO reported two additional decay channels: $J/\Psi \pi^0 \pi^0$ and $J/\Psi K^+ K^-$.4

Since the mass of the Y(4260) is higher than the $D^{(*)}D^{(*)}$ threshold, if it was a normal $c\bar{c}$ charmonium state, it should decay mainly to $D^{(*)} \bar{D}^{(*)}$ cross sections measured by Belle[6] and BaBar.[11] Besides, the $\Psi(3S)$, $\Psi(2D)$ and $\Psi(4S)$ $c\bar{c}$ states have been assigned to the well established $\Psi(4040)$, $\Psi(4160)$, and $\Psi(4415)$ mesons respectively, and the prediction from quark models for the $\Psi(3D)$ state is 4.52 GeV. Therefore, the mass of the Y(4260) is not consistent with any of the $1^{-}\!-\!2^{+}$ $c\bar{c}$ states.[2, 3, 12]

There are many theoretical interpretations for the Y(4260): tetraquark state[13], hadronic molecule of $D_1D$, $D_1D^*$[14], $\chi_{c1}\omega$[15], $\chi_{c1}\rho$[16], $J/\psi f_0(980)$[17], a hybrid charmonium[18], a charm baryonium[19], a cusp[20, 22], etc. Within the available experimental information, none of these suggestions can be completely ruled out. However, there are some calculations, within the QCD sum rules (QCDSR) approach,[23, 24] that can explain the mass of the Y(4260) supposing it to be a tetraquark state[26], or a $D_1 D$, $D_1 D^*$ hadronic molecule[24], or a $J/\psi f_0(980)$ molecular state[22].

In this work we use again the QCDSR approach to the Y(4260) state including a new possibility: the mixing between two and four-quark states. This will be implemented following the prescription suggested in[28] for the light scalar mesons. The mixing is done at the level of the currents and was extended to the charm sector in Ref. 29, in order to study the X(3872) as a mixed charmonium-molecular state. In particular, in Ref. 29, the mass and the decay width of the X(3872), into $2\pi$ and $3\pi$, were evaluated with good agreement with the experimental values. Agreement with the experimental results has been also obtained, applying this same approach, in the study of the X(3872) radiative decay[24], and also in the X(3872) production rate in $B$ decay[21].

In the next sections we consider a mixed charmonium-tetraquark current and use the QCDSR method to study both, mass and decay width, of the Y(4260).

II. CONSTRUCTING THE TWO-QUARK AND FOUR-QUARK OPERATOR

In order to define a mixed charmonium-tetraquark current we have to define the currents associated with charmonium and four-quarks (tetraquark) states. For the charmonium part we use the conventional vector current:

$$j_{\mu}^{(2)} = \bar{c}_a(x)\gamma_{\mu} c_a(x),$$

while the tetraquark part is interpolated by[26]

$$j_{\mu}^{(4)} = \frac{\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}} \left( (q_d^a T(x) C\gamma_5 c_b(x))(q_d(x)\gamma_{\mu}\gamma_5 C\epsilon_e^T(x)) + (q_d^a T(x) C\gamma_5 c_b(x))(q_d(x)\gamma_{\mu}\gamma_5 C\epsilon_e^T(x)) \right).$$

As in Refs. 28, 29, we define the normalized two-quark current as

$$j_{\mu}^{(2)} = \frac{1}{\sqrt{2}} (\bar{q}q) j_{\mu}^{(2)},$$


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and from these two currents we build the following mixed charmonium-tetraquark $J^{PC} = 1^{--}$ current for the $Y(4260)$ state:

$$j_\mu(x) = \sin(\theta) j^{(4)}_\mu(x) + \cos(\theta) j^{(2)}_\mu(x),$$  

(4)

### III. THE TWO-POINT CORRELATION FUNCTION

To obtain the mass of a hadronic state using the QCD SR approach, the starting point is the two-point correlation function

$$\Pi_{\mu\nu}(q) = i \int d^4x \, e^{iq\cdot x} \langle 0 | T[j_\mu(x)j^{\dagger}_\nu(0)] | 0 \rangle$$

$$= -\Pi_1(q^2) \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + \Pi_0(q^2) \frac{2 q_\mu q_\nu}{q^2},$$  

(5)

where $j_\mu(x)$ is the mixed charmonium-tetraquark interpolating current defined in Eq. (4). The functions $\Pi_1(q^2)$ and $\Pi_0(q^2)$ are two independent invariant functions associated with spin-1 and spin-0 mesons, respectively.

According to the principle of duality, Eq. (3) can be evaluated in two ways: in the OPE side, we calculate the correlation function in terms of quarks and gluons using the Wilson’s operator product expansion. The phenomenological side is evaluated by inserting, in Eq. (3), a complete set of intermediate states with $1^{--}$ quantum numbers. In this side, we parametrize the coupling of the vector state $Y$ with the current defined in Eq. (4) through the coupling parameter $\lambda_Y$

$$\langle 0 | j_\mu(x) | Y \rangle = \lambda_Y \epsilon_\mu.$$  

(6)

where $\epsilon_\mu$ is the polarization vector. Using Eq. (6), we can write the phenomenological side of Eq. (5) as

$$\Pi^{\text{phen}}_{\mu\nu}(q) = \frac{\lambda_Y^2}{M_Y^2 - q^2} \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + \ldots$$  

(7)

where $m_Y$ is the mass of the $Y$ state and the dots, in the second term in the RHS of Eq. (7), denotes the higher resonance contributions which will be parametrized, as usual, through introduction of the continuum threshold parameter $s_0$.

The OPE side can be written in terms of a dispersion relation

$$\Pi^{\text{OPE}}(q^2) = \int_{4m_c^2}^{\infty} \frac{ds \rho^{\text{OPE}}(s)}{s - q^2},$$  

(8)

where $\rho^{\text{OPE}}(s)$ is given by the imaginary part of the correlation function: $\pi \rho^{\text{OPE}}(s) = Im[\Pi^{\text{OPE}}(s)]$. In this side, we work at leading order in $a_s$, the operators and we consider the contributions from the condensates up to dimension eight. Although we will consider only a part of the of the dimension 8 condensates (related to the quark condensate times the mixed condensate), in Ref. [20] it was shown that this is the most important dimension 8 condensate contribution.

Considering the current in Eq. (11), Eq. (5) in the OPE side can be written as

$$\Pi_{\mu\nu}(q) = \frac{\langle \bar{q}q \rangle^2}{2} \cos^2(\theta) \Pi^{22}_{\mu\nu}(q) + \sin^2(\theta) \Pi^{44}_{\mu\nu}(q)$$

$$+ \frac{\langle \bar{q}q \rangle}{\sqrt{2}} \sin(\theta) \cos(\theta) \left[ \Pi^{24}_{\mu\nu}(q) + \Pi^{42}_{\mu\nu}(q) \right],$$  

(9)

where

$$\Pi^{ij}_{\mu\nu}(q) = i \int d^4x \, e^{iq\cdot x} \langle 0 | [T[j^i_\mu(x), j^{\dagger j}_{\nu}(0)] | 0 \rangle.$$  

(10)

Clearly $\Pi^{22}_{\mu\nu}(q)$ and $\Pi^{44}_{\mu\nu}(q)$ are, respectively, the correlation functions of the $J/\psi$ and $[cq][\bar{c}q]$ tetraquark state.

After making a Borel transform in both sides, and transferring the continuum contributions to the OPE side, the sum rule in the $g_{\mu\nu}$ structure for the vector meson can be written as

$$\chi_Y^2 e^{-m_Y^2/M_B^2} = \frac{\langle \bar{q}q \rangle^2}{2} \cos^2(\theta) \Pi^{22}_1(M_B^2) + \sin^2(\theta) \Pi^{44}_1(M_B^2)$$

$$+ \frac{\langle \bar{q}q \rangle}{\sqrt{2}} \sin(\theta) \cos(\theta) \left[ \Pi^{24}_1(M_B^2) + \Pi^{42}_1(M_B^2) \right],$$  

(11)

where

$$\Pi^{22}_1(M_B^2) = \int_{4m_c^2}^{s_0} ds \, e^{-s/M_B^2} \rho^{22}_{\text{pert}}(s) + \Pi^{22}_{(G_2)}(M_B^2),$$  

(12)

$$\Pi^{44}_1(M_B^2) = \int_{4m_c^2}^{s_0} ds \, e^{-s/M_B^2} \rho^{44}_{\text{pert}}(s) + \rho^{44}_{(G_2)}(s) + \rho^{44}_{(G_4)}(s) + \rho^{44}_{(\bar{c}q\bar{c}q)}(s) + \Pi^{44}_{(G_4)}(M_B^2),$$  

(13)

$$\Pi^{24}_1(M_B^2) = \int_{4m_c^2}^{s_0} ds \, e^{-s/M_B^2} \rho^{24}_{(G_2)}(s) + \Pi^{24}_{(G_4)}(M_B^2).$$  

(14)

The expressions for the spectral density $\rho(s)$ appearing in Eqs. (12) - (13) for the charmonium and tetraquark states, as well as the mixed terms are listed in Appendix A.

By taking the derivative of Eq. (11) with respect to $1/M_B^2$ and dividing the result by Eq. (11), we obtain

$$m_Y^2 = - \frac{\frac{dK(\Lambda_{\overline{MS}}^2, \theta)}{d(1/M_B^2)}}{K(\Lambda_{\overline{MS}}^2, \theta)},$$  

(15)
where

\[ K(M_B^2, \theta) \equiv \frac{(qq)^2}{2} \cos^2(\theta) \Pi^2_0(M_B^2) + \sin^2(\theta) \Pi^4_0(M_B^2) \]
\[ + \frac{(qq)^2}{2} \sin(2\theta) \cos(\theta) \left[ \Pi^2_0(M_B^2) + \Pi^4_0(M_B^2) \right]. \]

Eq. 15 will be used to extract the mass of the charmonium-tetraquark state.

A. Numerical Analysis

In Table II we list the values of the quark masses and condensates that we have used in our numerical analysis. For a consistent comparison with results obtained for the others works using QCD sum rules, these parameters values used here are the same values used in Refs. 25, 27, 31, 32.

| Parameters | Values       |
|------------|--------------|
| \( m_c(m_c) \) | (1.23 ± 0.05) GeV |
| \( \langle q\bar{q} \rangle \) | (0.23 ± 0.03) GeV |
| \( \langle \bar{q}qGq \rangle \) | \( m_0^2(\bar{q}q) \) |
| \( m_0^2 \) | (0.8 ± 0.1) GeV |
| \( (q\bar{q})^2 \) | (0.88 ± 0.25) GeV |

The continuum threshold is a physical parameter that, in the QCD-DSR approach, should be related to the first excited state with the same quantum numbers. In some known cases, like the \( \rho \) and \( J/\psi \), the first excited state has a mass approximately 0.5 GeV above the ground state mass. Since in our study we do not know the experimental spectrum for the hadrons studied, we will fix the continuum threshold range starting with the smaller value which provides a valid Borel window, as explained below. Using this criterion, we obtain \( s_0 \) in the range 4.6 ≤ \( s_0 \) ≤ 4.8 GeV.

Reliable results can be extracted from the sum rule if is possible to determine a valid Borel Window. Such Borel window is obtained by imposing a good OPE convergence, the dominance of the pole contribution and a good Borel stability. To determine the minimum value of the Borel mass we adopt the criterion for which the contribution of the higher dimension condensate should be smaller than 15% of the total contribution. Thus, \( M_{B_{\text{min}}}^2 \) is such that

\[
\left| \frac{\text{OPE summed up to dim n-1}(M_{B_{\text{min}}}^2)}{\text{total contribution}(M_{B_{\text{min}}}^2)} \right| = 0.85.
\]

(16)

In Fig. 1 we plot the relative contributions of all the terms in the OPE side. We have used \( \sqrt{s_0} = 4.70 \) GeV and \( \theta = 53^\circ \). For others \( \theta \) values outside the range \( 52.5^\circ \leq \theta \leq 53.5^\circ \), we do not have a good OPE convergence. From this figure we see that the contribution of the dimension-8 condensate is smaller than 15% of the total contribution for values of \( M_B^2 \geq 2.4 \text{ GeV}^2 \), indicating a good OPE convergence. Therefore, we fix the lower value of \( M_B^2 \) in the sum rule window as: \( M_{B_{\text{min}}}^2 = 2.4 \text{ GeV}^2 \).

To determine the maximum value of the Borel mass (\( M_{B_{\text{max}}}^2 \)) we must analyse the pole-continuum contribution. Unlike the pole contribution, the continuum contribution increases with \( M_B^2 \) due to the dominance of the perturbative contribution. Therefore, the maximum value of the Borel mass is determined in the point that the pole contribution is equal to the continuum contribution.

In Fig. 2 we see a comparison between the pole and continuum contributions. It is clear that the pole contribution is equal to the continuum contribution for \( M_B^2 = 2.90 \text{ GeV}^2 \). Therefore, for \( \sqrt{s_0} = 4.70 \text{ GeV}^2 \) and \( \theta = 53^\circ \) the Borel window is: \( 2.4 \leq M_B^2 \leq 2.90 \text{ GeV}^2 \).

After we have determined the Borel window, we can calculate the ground state mass, which is shown, as a function of \( M_B^2 \), in Fig. 3. From this figure we see that there is a very good stability in the ground state mass in the determined Borel Window, which are represented, through the crosses in Fig. 3

Varying the value of the continuum threshold in the range \( \sqrt{s_0} = 4.70 \pm 0.10 \) GeV, the mixing angle in the range \( \theta = (53.0 \pm 0.5)^\circ \), and the other parameters as indicated in Table I, we get:

\[ m_Y = (4.26 \pm 0.13) \text{ GeV}, \]

(17)
which is in a very good agreement with the experimental mass of the $Y(4260)$. Once we have determined the mass, we can use this value in Eq. (11) to estimate the meson-current coupling parameter, defined in Eq. (9). We have used the same values of the $\sqrt{s_0}$, $\theta$ and Borel Window used for the mass calculation. Thus, we get:

$$\lambda_Y = (2.00 \pm 0.23) \times 10^{-2} \text{ GeV}^5. \quad (18)$$

The parameter $\lambda_Y$ gives a measure of the strength of the coupling between the current and the state. The result in Eq. (18) has the same order of magnitude as the coupling obtained for the $X(3872)$. \cite{34}, for example.

IV. THE VERTEX FUNCTION AND THE DECAY WIDTH OF THE $Y(4260)$

The QCDSR technique can also be used to extract coupling constants and form factors. In particular, in Ref. \cite{36} the authors determined the form factors and coupling constants in many hadronic vertices containing charmed mesons, in the framework of QCD sum rules. In this section, we will use the QCDSR approach to determine the coupling constant associated with the vertex $Y J/\psi \sigma$ to estimate the decay width of the process $Y \to J/\psi \pi \pi$. We are assuming that the two pions in the final state come from the $\sigma$ meson.

To determine the coupling constant associated with the vertex $Y J/\psi \sigma$, we must evaluate the vertex function (three-point function) defined as

$$\Pi_{\mu\nu}(p, p', q) = \int d^4 x d^4 y e^{i p' \cdot x} e^{i q \cdot y} \Pi_{\mu\nu}(x, y), \quad (19)$$

with $p = p' + q$ and $\Pi_{\mu\nu}(x, y)$ given by

$$\Pi_{\mu\nu}(x, y) = \langle 0 | T \{ j^\mu_i(x) j^\nu_j(y) \} \phi^\dagger(0) | 0 \rangle. \quad (20)$$

The interpolating fields appearing in Eq. (20) are the currents for $J/\psi$, $\sigma$ and $Y(4260)$, respectively. The currents for $J/\psi$ and $Y$ were defined by Eqs. (1) and (4).

For the meson $\sigma$, we have

$$j^\sigma = \frac{1}{\sqrt{2}} \left( \bar{u}_a(x) u_a(x) + \bar{d}_a(x) d_a(x) \right). \quad (21)$$

As in the case of two-point function studied in the previous section, the three-point correlation function defined by Eq. (19) can also be described in terms of hadronic degrees of freedom (Phenomenological side) or in terms of quarks and gluons fields (OPE side). In order to evaluate the phenomenological side of the sum rule we insert, in Eq. (19), intermediate states for $Y$, $J/\psi$ and $\sigma$. Using the definitions:

$$\langle 0 | j^\mu_i | J/\psi(p') \rangle = m_{\psi} f_{\psi} \epsilon_\mu(p'),$$

$$\langle 0 | j^\sigma | \sigma(q) \rangle = A_\sigma,$$

$$\langle Y(p) | j^\psi_\sigma | 0 \rangle = \lambda_Y \epsilon^*_\sigma(p), \quad (22)$$

we obtain the following relation:

$$\Pi^{(\text{phen})}_{\mu\nu}(p, p', q) = \frac{\lambda_Y m_{\psi} f_{\psi} A_\sigma g_{\psi\sigma}(q^2)}{\left( p^2 - m_\psi^2 \right) \left( p'^2 - m_\psi^2 \right) \left( q^2 - m_\sigma^2 \right)} \times \left( p' \cdot p \right) g_{\mu\nu} - \left( p' \cdot q \right) g_{\mu\nu} - \left( p \cdot p' \right) g_{\mu\nu} + \cdots, \quad (23)$$

where the dots stand for the contribution of all possible excited states. The form factor, $g_{\psi\sigma}(q^2)$, is defined by the generalization of the on-mass-shell matrix element, $\langle J/\psi | \sigma | Y \rangle$, for an off-shell $\sigma$ meson:

$$\langle J/\psi | \sigma | Y \rangle = g_{\psi\sigma}(q^2) \left( p' \cdot p \epsilon'(p') \cdot \epsilon(p) - p' \cdot \epsilon(p) p \cdot \epsilon'(p') \right), \quad (24)$$
which can be extracted from the effective Lagrangian that describes the coupling between two vector mesons and one scalar meson:

$$\mathcal{L} = ig_Y \psi \sigma V_{\alpha \beta} A^{\alpha \beta} \sigma$$  \hspace{1cm} (25)$$

where $V_{\alpha \beta} = \partial_\alpha Y_\beta - \partial_\beta Y_\alpha$ and $A^{\alpha \beta} = \partial_\alpha \psi^\beta - \partial^\beta \psi^\alpha$, are the tensor fields of the $Y$ and $\psi$ fields respectively.

In the OPE side, we work at leading order in $\alpha_s$ and we consider the condensates up to dimension five, as shown in Fig. 3. We have chosen to work in the $p'_\nu q_\mu$ structure since it has more terms contributing for the OPE. Taking the limit $p^2 = p'^2 = -P^2$ and doing the Borel transform to $P^2 \to M^2$, we get the following expression for the sum rule in the structure $p'_\nu q_\mu$:

$$\frac{\lambda_Y A_{\sigma m_\psi} f_\psi}{(m_Y^2 - m_\psi^2)} B_{Y \psi \sigma}(Q^2) \left( e^{-m_Y^2/M^2} - e^{-m_\psi^2/M^2} \right) + B(Q^2) e^{-m_\sigma^2/M^2} = (Q^2 + m_\sigma^2) \Pi^{(\text{OPE})}(M^2, Q^2),$$

(26)

where $Q^2 = -q^2$, and $B(Q^2)$ gives the contribution to the pole-continuum transitions \cite{29, 37, 39}. $\Pi^{(\text{OPE})}(M^2, Q^2)$ is given by

$$\Pi^{(\text{OPE})}(M^2, Q^2) = \frac{\sin(\theta)}{2\sqrt{2\pi^2}} \int_0^1 d\alpha \frac{-m_\sigma^2}{(1-\alpha)M^2} \times$$

$$\times \left\{ \frac{m_\psi \langle \bar{q} q G \rangle}{Q^2} \left[ 1 - 2\alpha(1-\alpha) \right] - \frac{\langle \bar{q}^2 G^2 \rangle}{2\pi^2} \right\}.$$

(27)

The sine present in Eq. \cite{27} indicates that only the tetraquark part of current in Eq. \cite{4} contributes to the OPE side. In fact, the charmonium part of the current gives only disconnected diagrams that are not considered.

In Eq. \cite{26} $m_\psi$ and $f_\psi$ are the mass and decay constant of the $J/\psi$ and $m_\sigma$ is the mass of the $\sigma$ meson. Their values are: $m_\psi = 3.1$ GeV, $f_\psi = 0.405$ GeV \cite{40}, and $m_\sigma = 0.478$ GeV \cite{11}. The parameters $\lambda_Y$ and $A_\sigma$ represent, respectively, the coupling of the $Y$ and $\sigma$ states with the currents defined in Eq. \cite{9} and \cite{22}. The value of $\lambda_Y$ is given in Eq. \cite{13}, while $A_\sigma$ was determined in Ref. \cite{12} and its value is $A_\sigma = 0.197$ GeV$^2$.

Similarly to what was done to get $m_Y$ in Eq. \cite{15}, one can use Eq. \cite{26} and its derivative with respect to $M^2$ to eliminate $B(Q^2)$ from these equations and to isolate $g_{Y \psi \sigma}(Q^2)$. A good sum rule must be as much independent of the Borel mass as possible. Therefore, we have to determine a region in the Borel mass where the form factor is clearly stable, as a function of both $M^2$ and $Q^2$. Notice that in the region $7.0 \leq M^2 \leq 10.0$ GeV$^2$, the form factor is clearly stable, as a function of $M^2$, for all values of $Q^2$.

The squares in Fig. \cite{6} show the $Q^2$ dependence of $g_{Y \psi \sigma}(Q^2)$, obtained for $M^2 = 8.0$ GeV$^2$. For other values of the Borel mass, in the range $7.0 \leq M^2 \leq 10.0$ GeV$^2$, the results are equivalent. Since we are interested in the coupling constant, which is defined as value of the form factor at the meson pole: $Q^2 = -m_\sigma^2$, we need to extrapolate the form factor for a region of $Q^2$ where the QCDSR is not valid. This extrapolation can be done by parametrizing the QCDSR results for $g_{Y \psi \sigma}(Q^2)$ using a monopole form:
$$g_{Y\psi\sigma}(Q^2) = \frac{g_1}{g_2 + Q^2} \quad (28)$$

We do the fit for $\sqrt{s_0} = 4.74$ GeV. We notice that the results do not depend much on this parameter. The results are:

$$g_1 = (0.58 \pm 0.04) \text{ GeV}; \quad g_2 = (4.71 \pm 0.06) \text{ GeV}^2. \quad (29)$$

The solid line in Fig. 5 shows that the parametrization given by Eq. (28) reproduces very well the QCDSR results for $g_{Y\psi\sigma}(Q^2)$, in the interval $2.0 \leq Q^2 \leq 4.0$ GeV$^2$, where the QCDSR is valid.

The coupling constant, $g_{Y\psi\sigma}$ is given by using $Q^2 = -m_{\sigma}^2$ in Eq. (28). We get:

$$g_{Y\psi\sigma} = g_{Y\psi\sigma}(-m_{\sigma}^2) = (0.13 \pm 0.01) \text{ GeV}^{-1}. \quad (30)$$

The error in the coupling constant given above comes from variations in $s_0$ in the range $4.6 \leq s_0 \leq 4.8$ GeV$^2$, and in the mixing angle $52.5^\circ \leq \theta \leq 53.5^\circ$.

In Table II, we show the other values of the coupling constant corresponding to the values of $\sqrt{s_0}$ that we have considered in our calculations.

Table II: Monopole parametrization of the QCDSR results for the chosen structure, for different values of $\sqrt{s_0}$.

| $\sqrt{s_0}$ (GeV) | $g_{Y\psi\sigma}(Q^2)$ (GeV$^{-1}$) | $g_{Y\psi\sigma}(Q^2 = -m_{\sigma}^2)$ (GeV$^{-1}$) |
|---------------------|----------------------------------|-----------------------------------|
| 4.6                 | 0.14                             | 0.14                              |
| 4.7                 | 0.13                             | 0.13                              |
| 4.8                 | 0.12                             | 0.12                              |

FIG. 5. $g_{Y\psi\sigma}(Q^2)$ values obtained by varying both $Q^2$ and $M^2$.

FIG. 6. QCDSR results for $g_{Y\psi\sigma}(Q^2)$, as a function of $Q^2$, for $\sqrt{s_0} = 4.76$ GeV (squares). The solid line gives the parametrization of the QCDSR results through Eq. (28).

The decay width for the process $Y(4260) \rightarrow J/\psi \sigma \rightarrow J/\psi \pi \pi$ in the narrow width approximation is given by

$$\frac{d\Gamma}{ds}(Y \rightarrow J/\psi \pi \pi) = \frac{1}{8\pi m_{\psi}^2} |M|^2 \frac{m_{\psi}^2 - m_{\pi}^2 + s}{2m_{\psi}^2} \times \frac{\Gamma_{\sigma}(s)m_{\sigma}}{\pi} \frac{p(s)}{(s - m_{\sigma}^2)^2 + (m_{\sigma}\Gamma_{\sigma}(s))^2}, \quad (31)$$

with $p(s)$ given by

$$p(s) = \sqrt{\frac{\lambda(m_{\psi}^2, m_{\pi}^2, s)}{2m_{\psi}}}, \quad (32)$$

where $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$, and $\Gamma_{\sigma}(s)$ is the s-dependent width of an off-shell $\sigma$ meson [41].

$$\Gamma_{\sigma}(s) = \Gamma_{0\sigma} \sqrt{\frac{\lambda(s, m_{\pi}^2, m_{\sigma}^2)}{\lambda(m_{\psi}^2, m_{\pi}^2, m_{\sigma}^2)}} \frac{m_{\sigma}^2}{s}, \quad (33)$$

where $\Gamma_{0\sigma}$ is the experimental value for the decay of the $\sigma$ meson into two pions. Its value is $\Gamma_{0\sigma} = (0.324 \pm 0.042 \pm 0.021)$ GeV [41].

The invariant amplitude squared can be obtained from the matrix element in Eq. (28). We get:

$$|M|^2 = g_{Y\psi\sigma}^2(s)f(m_{\psi}, m_{\pi}, s), \quad (34)$$

where $g_{Y\psi\sigma}(s)$ is the form factor in the vertex $YJ/\psi\sigma$, given in Eq. (28) using $s = -Q^2$, and

$$f(m_{\psi}, m_{\pi}, s) = \frac{1}{3} \left( m_{\psi}^2 m_{\pi}^2 + \frac{1}{2} (m_{\psi}^2 + m_{\pi}^2 - s)^2 \right).$$
Therefore, the decay width for the process $Y(4260) \to J/\psi \pi\pi$ is given by

$$\Gamma = \frac{m_\sigma}{16\pi^2 m_Y^3} I,$$  \hspace{1cm} (35)

where we have defined

$$I = \int \frac{(m_Y - m_\psi)^2}{(2m_\pi)^2} \frac{g_\sigma^2}{gs^2} H_Y(\psi)(m_Y^2 - m_\psi^2 + s) \times f(m_Y, m_\psi, s) \frac{\rho(s)}{(s - m_\psi^2)^2 + (m_\sigma y_\sigma(s))^2}. \hspace{1cm} (36)$$

Hence, taking variations on $s_0$ and $\theta$ in the same intervals given above, we obtain from Eqs. (30)-(35) the following value for the decay width

$$\Gamma(Y \to J/\psi \pi\pi) = (1.0 \pm 0.2) \text{ MeV}, \hspace{1cm} (37)$$

which is not compatible with the experimental decay width value expected for the $Y(4260)$ state which is around $\Gamma_{\exp} \approx (88 \pm 23) \text{ MeV}$ [4].

V. SUMMARY AND CONCLUSIONS

In summary, we have used the QCDSR approach to study the two-point and three-point functions of the $Y(4260)$ state, by considering a mixed charmonium-tetraquark current. In the determination of the mass, we work with the two-point function at leading order in $\alpha_s$ and we consider the contributions from the condensates up to dimension eight. A very good agreement with the experimental value of the mass of the $Y(4260)$ is obtained for the mixing angle around $\theta \approx (53.0 \pm 0.5)^0$.

To evaluate the width of the decay $Y(4260) \to J/\psi \pi\pi$, we work with the three-point function also at leading order in $\alpha_s$ and we consider the contributions from the condensates up to dimension five. We assume that the two pions in the final state come from a $\sigma$ meson. The obtained value for width is $\Gamma_Y \approx (1.0 \pm 0.2) \text{ MeV}$, which is much smaller than the experimental value: $\Gamma_{\exp} \approx (88 \pm 23) \text{ MeV}$.

Therefore, we conclude that the $Y(4260)$ exotic state cannot be described as a mixed charmonium-tetraquark state.

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Appendix A: The spectral densities for charmonium and tetraquark

Next, we list all the spectral densities that appear in Eqs. (13)-(14) for charmonium $\Pi_1^{(G)}(M^2_B)$, tetraquark $\Pi_4^{(G)}(M^2_B)$ state as well as the mixed terms $\Pi_1^{(G)}(M^2_B)$ and $\Pi_4^{(G)}(M^2_B)$. The contributions for the last two are equal, that is, $\Pi_1^{(G)}(M^2_B) = \Pi_4^{(G)}(M^2_B)$.

For the charmonium contribution, the spectral densities are written below [24]

$$\rho_{44}^{pert}(s) = \frac{1}{3 \cdot 2^{10} \pi^6} \int \frac{\alpha}{\alpha} \frac{d\alpha}{\alpha^3} \int \frac{d\beta}{\beta^3} F^3(1 - \alpha - \beta) \times \left(2m_\pi^2(1 - \alpha - \beta)^2 - 3F(1 + \alpha + \beta)\right), \hspace{1cm} (A3)$$

$$\rho_{44}^{(G)}(s) = 0 \hspace{1cm} (A4)$$

$$\rho_{44}^{(G^2)}(s) = -\frac{(g_\sigma^2 G^2)}{3^2 \cdot 2^{11} \pi^4} \int \frac{d\alpha}{\alpha} \int \frac{d\beta}{\beta} F^3 \left\{m_c \alpha(1 - \alpha - \beta)^3 - 3m_\pi^2 F(1 - \alpha - \beta) \times \left[2m_\pi^2(1 - \alpha - \beta)^2 - 3F(1 + \alpha + \beta)\right] + 6F^2 \beta(1 - 2\alpha - 2\beta)\right\}, \hspace{1cm} (A5)$$

$$\rho_{44}^{(G^2)}(s) = -\frac{(g_\sigma^2 G^2)}{3 \cdot 2^7 \pi^4} \left\{m_c \int \frac{d\alpha}{\alpha^2} \int \frac{d\beta}{\beta} F \left[\alpha^2 - \alpha(1 + \beta - 2\beta^2)\right] + ms \int \frac{d\alpha}{\alpha^2} \left[16m_\pi^2 + 2H \left(1 - \frac{\alpha}{\alpha}\right) - \right. \right. \left. \int \frac{d\beta}{\beta} \left(m_\pi^2(9 - 3\alpha - 5\beta) + 7F\right)\right]. \hspace{1cm} (A6)$$
\[ \rho_{44}^{(q\bar{q})^2}(s) = \frac{s(q\bar{q})^2}{3^2 \cdot 2 \pi^2} (1 - 16m_c^2/s)\sqrt{1 - 4m_c^2/s} \]  
\[ \rho_{44}^{(8)}(s) = \frac{\langle q\bar{q} \rangle^2}{3^2 \cdot 2 \pi^2} \int_0^{\alpha_{max}} d\alpha \alpha (5 - 6\alpha) \]  
\[ \Pi_{44}^{(8)}(M_B^2) = -\frac{m_c^2\langle q\bar{q} \rangle \langle q\bar{q} \rangle Gq}{3 \cdot 2^4 \pi^2} \int_0^1 d\alpha \times \left[ \frac{\alpha^2 - 2m_c^2}{M_B^2 \alpha (1 - \alpha)} \right] e^{-\frac{m_c^2}{\pi \beta(1 - \alpha)}} \]  
Finally, for the mixed term we have
\[ \rho_{24}^{(q\bar{q})}(s) = -\frac{s(q\bar{q})^2}{3^2 \cdot 2 \pi^2} (1 + 2m_c^2/s)\sqrt{1 - 4m_c^2/s} \]
\[ \Pi_{24}^{(q\bar{q})Gq}(M_B^2) = -\frac{m_c^2\langle q\bar{q} \rangle \langle q\bar{q} \rangle Gq}{3 \cdot 2^4 \pi^2} \int_0^1 d\alpha \times \frac{m_c^2}{\alpha^2} e^{-\frac{m_c^2}{\pi \beta(1 - \alpha)}} \]  
In all these expressions we have used the following definitions:
\[ F = (\alpha + \beta)m_c^2 - \alpha\beta s, \]  
\[ H = m_c^2 - \alpha(1 - \alpha)s, \]
and the integration limits are:
\[ \alpha_{min} = \frac{1 - \sqrt{1 - 4m_c^2/s}}{2} \]  
\[ \alpha_{max} = \frac{1 + \sqrt{1 - 4m_c^2/s}}{2} \]  
\[ \beta_{min} = \frac{\alpha m_c^2}{(s\alpha - m_c^2)} \]  

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