Holography&Transplantation and All That
(The tip of an iceberg for a paradigmatic change in QFT?)

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Abstract
Recent developments in local quantum physics have led to revolutionary conceptual changes in the thinking about a more intrinsic formulation and in particular about unexpected aspects of localized degrees of freedom. This paradigmatic change is most spectacular in a new rigorous form of “holography” and “transplantation” as generic properties in QFT beyond the rather special geometric black hole setting in which the geometric manifestations of these properties were first noted. This new setting is also the natural arena for understanding the rich world of “black hole analogs” ("dumb holes" for phonons).

The mathematical basis for all this is the extremely powerful Tomita-Takesaki modular theory in operator algebras. The rich consequences of the impressive blend of this theory with physical localization entails among other things the presence of “fuzzy” acting infinite dimensional symmetry groups, a spacetime interpretation and derivation of the $d=1+1$ Zamolodchikov-Faddeev algebra (i.e. a better understanding of the bootstrap-formfactor approach) and the noncommutative multiparticle structure of “free” anyons based on the use of Wigner representation theory.

1 Introduction: important crossroads in QFT

Genuine revolutions in theoretical physics often come in a conservative veil, and as a result are not immediately noticed unless they lead rapidly to new experimentally accessible predictions. In the case of Einstein’s special relativity the conservative element was the fact that not only was the Lorentz transformation formula known before, but its incorrect interpretation in terms of an ether-caused material contraction and dilation effect would have created havoc with the Maxwell theory. The revolutionary aspect of Einstein’s contribution consisted in discovering a new principle by successfully synthesizing existing
principles according to their intrinsic logic rather than cutting historical links by inventing (as opposed to discovering) one.

The renormalized perturbation theory for quantum fields which is inexorably linked with the names Tomonaga, Feynman Schwinger and Dyson also illustrates this point. Apart from some modern technology (as compared e.g. to that in Wentzel’s and Heitler’s prior textbooks) it is based on the old Heisenberg-Pauli canonical quantization supplemented with the elaboration of a remark of Kramers who reminded his colleagues of a lesson on selfmasses (and the difference between formal Lagrangian and genuine physical parameters in general) he learned from the Poincaré and Lorentz treatment of classical particles within the setting of classical fields. To be mathematically correct, Kramers remark was indispensable in removing infinities relative to the quantization methods by which physicists in those days entered QFT. But as the uniqueness and generic aspect of the final finite physical (i.e. physically parametrized) results already preempted, the intermediate infinities and the need for their removal were only caused by the necessary repair of a slightly incorrect starting point which warranted some intermediate “artistic” steps. The conceptual and mathematical refined satisfactory treatment which removed the necessity to deal with infinities\(^1\) in terms of a totally finite procedure was later presented in the work of Lehmann, Symanzik and Zimmermann \(^1\).

The revolution which is the subject of the present paper also has roots in the history of theoretical physics, but its most recent limelight is connected with words like holography, transmutation and scanning of data of local quantum physics.

It is very instructive to pause and take a brief look at its historical roots before presenting the actual setting. The before mentioned LSZ theory combined with other developments in the early 60\(^{th}\)s clarified to some degree the relation between particles and fields and showed that the distinction between elementary and bound/composite (even after having specified a concrete model) is not a property which one should attribute to particles\(^2\) but to (generalized, superselected) charges which can be transferred between them.

The LSZ theory did however not clarify the inverse problem (S-matrix→QFT) nor did it answer the closely related question of which field plays the role of what was called the ”interpolating” field; must it be a distinguished Lagrangian field or can it be something else? This question was partially answered by the realization that local fields come in equivalence classes (Borchers classes) \(^2\) and the S-matrix is to be associated with a whole class and not with an individual member (it turns out that there exist no physical principles which would be able to distinguish a preferred field).

\(^{1}\)Schwinger also realized later that a careful definition of polynomial functions of fields in terms of point split methods can avoid intermediate infinities. Whereas these finite methods are important as a matter of principle, they do not have the calculational efficiency of the somewhat more artistic Feynman method.

\(^{2}\)We mean particles in the conceptually precise sense of Wigner’s description, namely in terms of irreducible positive energy representations with finite spin/helicity of the Poincaré group.
This observation was one of the theoretical pillars of algebraic QFT, the other one was the superselection idea introduced by Wick, Wigner and Wightman in a very special geometric context and generalized to “charges” by Haag and Kastler. The somewhat radical message was that the utmost conceptual simplicity between off- and on-shell local quantum physics is obtained by abandoning fields in favor of (nets of) local operator algebras. This step was similar but somewhat more radical than that from old-fashioned coordinatized geometry to intrinsic modern coordinate-free differential geometry.

As was to be expected, the first success of this different viewpoint was a purely theoretical achievement namely a profound understanding of the internal symmetry concept associated with the action of compact groups which arose as a generalization of Heisenberg’s “isospin”. What was at first a strongly motivated conjecture (and an everyday experience in Lagrangian quantization) finally turned into a deep theory. Namely it follows from the causality and spectral principles (without using Lagrangian quantization) in the absence of zero mass that local observables in QFTs of spacetime dimension \(d \geq 1+3\) lead to Wigner particles and their interpolating fields which obey the spin-statistics theorem including the existence of a computable (from the structure of observables) compact symmetry group acting on multiplicity indices of fields. Like in Marc Kac famous aphorism about Weyl’s inverse problem: “how to hear the shape of a drum”, the observable “shadow” (which obeys the physical causality, localization and stability properties) determines uniquely the charged fields with their statistics (including possible braid group statistics in low spacetime dimensions) and their multiplicity structure including the concrete internal symmetry group (even though the observables consisted of neutral operators which were invariant under the group action!). This explained in particular why in Lagrangian quantization one was never able to find any other realization of inner symmetries and why the low-dimensional braid group structure which leads to another symmetry concept is out of reach for the Lagrangian formalism. The proof is mathematically as well as conceptually very deep and a far shot removed from the tautologies by which the Bose/Fermi alternative is “proven” in quantum mechanics textbooks.

In recent times more surprising findings have been added which have no natural description in the standard framework. But the observation which perhaps attracts most attention to an ongoing change of paradigm in QFT is associated with new concepts which have become known under the names holography and transplantation. These are unexpected relation between QFTs in different spacetime dimensions (or in the same spacetime dimension but with different curved spacetime metric aspects), which, even if expressible on each side in terms of pointlike fields nevertheless require a mediating algebraic concept, which go far beyond the standard field point of view and the Lagrangian formalism 3.

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3. Whereas in differential geometry coordinate transformations between charts are still part of the definitions, the algebraic formulation of QFT contains no trace of field-coordinatization.

4. In higher dimensional conformal theories there is also a conformal decomposition theory in the timelike region leading to a braided structure which in turn sets the spectrum of anomalous scale dimensions 4.
particular. As many properties of QFT they have been seen first through their geometric manifestations which in this case means in the curved spacetime setting. They where so startling that their protagonist \textcite{6} proposed them as a characteristic manifestation of the elusive Quantum Gravity. In analogy with the Maldacena versus Rehren AdS-CQFT controversy \textcite{7}, we will show that many of these properties are preempted in the setting of locality and spectral principles usual QFT.

This in my view a quite revolutionary change of paradigm in QFT (which shares with the above mentioned discovery of renormalized perturbation and the LSZ enrichment of the particle-field relation its strong historical ties with previously known principles) will be the main subject of these notes. The reason why I follow Borchers \textcite{8} in using the adjective “revolutionary” can be best understood in a historical context by citing examples for which I would not use this terminology.

Let me illustrate this by recalling an anecdotal episode in the early 60s. Although at that time it was already known that the a one-to-one particle-field relation was an illusion of the perturbative use of the Lagrangian formalism, this knowledge was not yet part of the general consciousness of particle physicist. Rather the idea that field operators come like classical fields with an distinct intrinsic meaning was the prevalent mode of thinking. This required to deal with rather involved Lagrangians in which these fields (mesons, baryons or their constituents) feature as fundamental Lagrangian fields. J. J. Sakurai \textcite{9}, one of the leading particle physicists of those years, emphasized the importance to incorporate vectormesons and addressed but did not solve the problems of renormalizability. At that time I was a junior collaborator of Rudolf Haag and as such I already had acquired some familiarity with the idea of local equivalence classes of interpolating fields associated with one particle. The origin of this difference to classical Lagrangian fields was that whereas a classical particle was a separate entity and had to be imposed “by hand” into the classical field theory (and the prize to pay were those infinite selfenergies observed at the beginning of last century by Poincaré and Lorentz), the concept of a Wigner particle was already preempted by quantum fields as a result of the discrete irreducible component of the positive energy representation of the Poincaré group which is democratically attached to every individual field in the local equivalence class of all fields carrying the same superselected charge. When Murray Gell-Mann in 1961 was passing through the University of Illinois, there was an interesting discussion with Haag on the partially conserved axial current problem (PCAC) and its relation to the pion field. In the course of it Murray Gell-Mann suddenly looked straight at us and in his matchless manner of condensing complex ideas into just one short phrase he said “you mean we can shoot Sakurai?” , using Sakurai’s name for the whole “one particle – one Lagrangian field” mode of thinking.

Another crossroad also located at the particle-field joint, but this time with a greater richness of physical consequences, is the seminal idea of gauge theory which not only made the enigma of confined quarks more respectable, but also helped to incorporate vectormesons into the family of particles which possess
renormalizable interactions. If one wants to call this a revolution one should perhaps add the adjective “unfinished”. The reason for this suggestion becomes clear if one analyses the present situation from slightly more local quantum physical (instead of a differential geometric) viewpoint. After all the physical problem to reconcile massive vector mesons with the power counting of renormalizability was to overcome the obvious obstacle that \((m, s = 1)\) physical particles have an operator dimension 2 (and not 1 as their classical counterpart) which makes any local coupling (which must be at least trilinear in order to serve as an interaction) of operator dimension \(\geq 5\) nonrenormalizable. Such couplings in the causal perturbation theory lead to an ever increasing number of parameters (with the perturbative order) which renders them practically useless\(^5\). It is a characteristic limitation of the Lagrangian quantization approach that Lagrangian (i.e. non-composite) fields \(\psi\) with higher operator dimensions than \(d_\psi = 1\) lead to such a situation whereas the underlying principles of QFT do not indicate such a limitation to low spin fields.

It is well-known that the adaptation of the classical gauge idea together with the Higgs mechanism finally suggested a way to come to a renormalizable theory where at the end physical vector mesons really do obtain their physical dimension 2 (ignoring logarithmic corrections as usual in power counting). Since there is however only one renormalizable theory (i.e. one coupling strength) in which vector mesons interact with themselves and with other fields (and not several competing couplings as in the classical setting), it is somewhat misleading to talk about a gauge principle outside the classical setting. A principle after all serves to select between a number of possible local interactions and this is only needed in the classical theory where the number of Lorentz invariant couplings increase with increasing number of Lorentz indices, whereas in the quantum field theory one usually considers the renormalizability as the overriding principle (simply because anything “quantum”, even if presently poorly understood, is always considered as a potentially more fundamental than a classical structure). Thus one obtains the gauge structure (uniqueness+semiclassical limit) as a result of renormalizability instead of the other way around. Indeed there exists a mathematical implementation of this idea which compared with the gauge approach has the advantage that the Higgs degrees of freedom are not put in by hand but the necessity to introduce additional bosonic physical degrees of freedom is a consequence of the causal perturbation theory. The mathematical trick to lower the dimension of the vectormeson is to use a (BRS inspired) cohomological representation on the level of the Wigner one particle theory (this is only possible for nonzero mass of the incoming vectormesons) which then guarantees that the BRS-like Fock space ghosts do not really participate in the interaction even though they do their job as a renormalization “catalyzer” \(^1\).

\(^5\)It has been suggested that they may serve as effective Lagrangians. Whereas this may be true as a post factum phenomenological observation, it is incorrect to think that there are Lagrangians which describe the situation below a certain energy with a uniform \(\varepsilon\)-precision. For this one would have to limit the local quantum physical “phase space” (i.e. roughly speaking energy and localization) or name those observables for which one can really establish such a control.
With other words the fact that the ghosts do not interact among themselves and that they enter through a cohomological argument facilitates the local descend to the massive physical fields at the end which then form an operator algebra in a Hilbert space ("unitarity") like any other algebra (i.e. in which the catalyzer left no noticable trace[1]). The intrinsic physical characteristics is not the Higgs mechanism but rather the Schwinger-Swieca screening which is a consequence of the presence of the new physical degrees of freedom (without the Higgs condensates, i.e. just ordinary bosonic matter fields). Such theories can be distinguished from massive QED-like models in which it is not possible to have both renormalizable (polynomially bounded) and physical (electron) fields.

Massless theories in this approach are defined by a limiting procedure in which the Higgs-like degrees of freedom decouple and the matter particles become "infraparticles" (particles inexorably linked to their photon clouds) or leave the physical spectrum which can be in principle studied in perturbation theory (but this has not been done).

It is presently not known whether for higher spins there exist more general (cohomological or other) tricks which help to go beyond the renormalizable/nonrenormalizable frontier set by power counting of the physical degrees of freedom and maintain the free field dimensions (necessarily \( \geq 2 \)) apart from logarithmic corrections. With other words it is not known whether these frontiers set by the Lagrangian formalism are really natural as far as the principles are concerned. Note that by emphasizing the uniqueness of vectormeson couplings within the renormalizability requirement we have not changed any of the physical correlation functions obtained in the standard Higgs/gauge approach (apart from the fact that the addition of new physical degrees of freedom was required by quantum consistency and not by symmetry breaking a la Higgs of something which never was a physical symmetry to start with[1]). The main advantage is primarily conceptual: the quantum uniqueness explains (in the quasiclassical limit) the classical gauge structure.

Our viewpoint on vectormesons is similar to that in [12], apart from the emphasis on a "quantum gauge principle" (which obviously we do not share) in the latter work. We think that our particle physics viewpoint is more suited to highlight the incomplete aspect of the "gauge revolution" i.e. the question of whether the "cohomological catalyzer trick" to overcome the power counting barrier of Lagrangian quantization is the tip of an iceberg of similar tricks for higher spins or more generally whether the present Lagrangian frontier is the true frontier of local quantum physics. It is hard to imagine that one is able to solve these fundamental questions inside the standard Lagrangian framework.

This brief historical exposition of important cross roads in QFT is meant to emphasize the fact that apart from an increase in computational sophistication and elegance the basic formulation of using pointlike fields for the description of interacting particles did not suffer any violent changes through 70 years of QFT. On the other hand the algebraic framework of local quantum physics

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6Without the "cohomological catalyzer", the operator dimension of the vectormeson field would increase with the perturbative order.
offers not only new concepts but also demands a different mathematical formalism. Compared with the mentioned historical illustrations the change is indeed revolutionary especially to those for who QFT has become synonymous with (Euclidean) functional integrals.

The ideas which have been denoted in a placative manner in the title of this paper as "holography, transplantation and all that" however really do amount to a change of paradigm or a revolution even though they have been developed on the basis of the same physical principles as the standard textbook formulation of QFT. Their realization does not only require new concepts but (different from the mentioned previous crossroads) also a significant change of the mathematical formalism. The main novel point is the conversion of the standard formulation into nets of operator algebras indexed by spacetime regions or to start directly from the latter. By this elimination of field coordinatization in terms of a more intrinsic description (similar to the modern coordinate free way of doing differential geometry) one gains an unexpected flexibility of reprocessing degrees of freedom and changing their spacetime affiliation; this has no known counterpart in the standard setting of pointlike fields. At the end one then may, if desired, return to a pointlike description, but the field generators for the reprocessed (holographic projection, transplantation) net have no simple local relation to the original generating fields; here the analogy to a coordinate transformation in differential geometry breaks down.

This new point of view is not only useful as a preparatory step towards a better quantum understanding of Bekenstein-Hawking like black hole problems⁷ but it also leads to a better access to more mundane problems possibly related to laboratory physics as the description of “free” d=1+2 anyons and plektons i.e. charge-carrying semiinfinitely localized operators associated with braid group representations. Last not least it leads to a vast generalization of the framework of Wigner symmetries: besides the visible Poincaré or conformal symmetries there exists an infinite group of fuzzy (non pointlike, not geometric) automorphisms of the net of operator algebras which have other invariant states than the vacuum and are unitarily implemented [23]. These new Wigner symmetries owe their existence to the noncommutative structure and therefore cannot be seen by a Noether type argument, but are nevertheless in complete harmony with the causality and localization structure of the theory.

Whenever we want the reader to not think about QFT within the standard textbook framework but to be more open-minded, we will add the adjective algebraic (AQFT) or simply use “local quantum physics” (LQP).

⁷In the curved spacetime context some of the more abstract ideas of reprocessing operator algebras have classical geometric manifestations by which they have been seen. Degrees of freedom conversion on causal horizons in Minkowski spacetime (horizons of wedges or double cones) remain more hidden behind the noncommutative aspects of real time local quantum physics.
2 Limitations of standard QFT, transition to field-coordinatization-free formulation

The only structure in the Lagrangian quantization approach to which holography has a remote connection is the light-front or $p \to \infty$ frame quantization method [13]. For brevity let us look at the simplest case: the $d=1+1$ light-ray “quantization”. If we start from a massive local free field $A(x)$ and restrict it to the lightray from spacelike $x$ (say from inside the right spacelike wedge $W$)

$$A(x) = \frac{1}{\sqrt{2\pi}} \int (e^{-ipx} a(\theta) + h.c.) \ d\theta, \ p = m(ch\theta, sh\theta)$$  \hspace{1cm} (1)

where the limit of lightray restriction is taken in such a way that the inner product $px$ in the plane wave factor remains finite i.e. as $\chi = \hat{\chi} + lnr$ where $\hat{\chi}$ stays finite and the diverging $lnr$ term is compensated by the vanishing $r$-factor.

Of the two possibilities we select the upper horizon limit $x_+ \to 0$ where $x_\pm$ are the lightray combinations. The resulting field $A(x_+)$ has the usual infrared divergence of a massless scalar field (better visible by returning to $p_- = me^{-\theta}$) which can be controlled either by restricting the test functions for $A(x_+)$ by imposing $\int f(x_+) dx_+ = 0$ or by using as the field $j = \partial_x A(x_+)$ instead of $A(x_+)$ for the generation of observables. The restriction process on the lower horizon leads to $A(x_-)$ which has the same Fock space creation and annihilation operators $a^\#$ as $A(x_+)$.

The massless limit $m \to 0, \ \theta = \hat{\theta} + logm$ on the other hand cannot be carried out on the level of operators in Fock space. Instead one has to perform it on the level of correlation function ($\theta \to \infty$ in the $a^\#(\theta)$ operators is meaningless) and regain a Hilbert space and operators therein by (GNS-) reconstruction. The result are the two chiral fields $A_\pm(x_\pm)$ of chiral conformal field theory (with the same infrared proviso as in the previous case) apart from a doubling of degrees of freedom as compared to the previous case we encounter the same fields.

The difference in the number of degrees of freedom is expected on classical grounds; whereas for massless $d=1+1$ fields one needs both upper and lower horizons of a wedge in order to specify the data inside the wedge, the massive case requires the characteristic data on only one of the horizons $R_u,l$. In terms of the algebras generated by the fields the quantum version reads

$$\mathcal{A}(W) = \begin{cases} \mathcal{A}_+(R_u) \otimes \mathcal{A}_-(R_l), & m = 0 \\ \mathcal{A}(R_u) = \mathcal{A}(R_l), & m \neq 0 \end{cases}$$  \hspace{1cm} (2)

but it is very important to understand the precise mathematical meaning of these operator relations and for this reason we will present the mathematical concepts below.

The presence of interactions lead to some significant changes which essentially render the lightray restriction in terms of fields ill-defined and useless. The lightray quantization actually almost never worked for interacting theories;
in those cases where it was made to work be decree/brute force, its connection with the original local theory became completely obscure. The reason is two-fold. On the one hand the lightfront fields suffer the same restrictions as those of the canonical quantization in that the Kallen-Lehmann spectral functions must decrease so that \( \int \rho(\kappa^2) d\kappa^2 < \infty \). This leaves only a few not very interesting superrenormalizable \( \phi_2^{2n} \) couplings whereas all really interesting renormalizable interactions do not allow a lightray approach. To be more specific, the lightfront formulation based on canonical structures becomes as “artistic” as the spatial equal time canonical formulation or the functional integral approach. It is well known that neither canonical commutation relations nor functional integral representations are true properties for the physical fields (after renormalization); the only surviving property are the (bosonic, fermionic, plektonic) spacelike commutations.

The second reason is even more serious and problematic even in the superrenormalizable theories with decreasing spectral functions. The lightfront approach should not change the physical content of the theory and therefore one should be able to return to the standard formulation in which causality and localization are manifest. Mass spectra without a localization concept are not of much physical use; since all our interpretation goes through locality; a direct interpretation in momentum space without localization is an illusion. There is a good reason why despite more than 30 years writing about lightfront/p \( \to \infty \) calculations one looks in vain for a hint in this direction. The point is that in this particular kind of problem the otherwise successful standard approach (based on field coordinatizations i.e. short distance singular pointlike fields) has been stretched beyond its limits of validity.

In order to maintain conceptual clarity and to keep things under mathematical control, one must use a coordinate-free intrinsic formulation of QFT (which often is referred to as AQFT or LQP). This will give the right framework for making holographic mappings of degrees of freedom and in this sense saves the intuitive content of the old lightfront approach. Before we explain the modular inclusion concept which gives the mathematical underpinning of the holographic reprocessing, we present a brief rundown on the requirements of AQFT.

- (i) There is a map of compact regions \( \mathcal{O} \) in Minkowski space into von Neumann operator algebras \( \mathcal{A}(\mathcal{O}) \) which are subalgebras of all operators \( \mathcal{B}(\mathcal{H}) \) in some Hilbert space \( \mathcal{H} \):

\[
\mathcal{A} : \mathcal{O} \to \mathcal{A}(\mathcal{O}) \tag{3}
\]

It is sufficient to fix the map on the Poincaré invariant family of double cone regions \( (V_\pm : \text{forward/backward lightcone}) \)

\[
\mathcal{O} = (V_+ + x) \cap (V_- + y) , \ y - x \in V_+ \tag{4}
\]

The \( C^* \)-completion of this family yields the global \( C^* \)-algebra \( \mathcal{A}_{\text{quasi}} \):

\[
\mathcal{A}_{\text{quasi}} = \bigcup_{\mathcal{O} \in \mathcal{M}} \mathcal{A}(\mathcal{O}) \tag{5}
\]
The $C^*$-algebras for noncompact regions are analogously defined by inner approximation with double cones $O$. Since they are concrete operator algebras in a common Hilbert space they have a natural von Neumann closure $M = M''$. A closely related (but independent) assumption

$$\left\{ \bigcup_a M(O + a) \right\}'' = M(M), \; M = \text{Minkowski spacetime} \quad (6)$$

is called weak additivity.

- (ii) The family $A$ forms a “net” i.e. a coherent (isotonic) family of local algebras:

$$O_1 \subset O_2 \implies A(O_1) \subset A(O_2) \quad (7)$$

In case the local algebras represent observables one requires another physically motivated coherence property namely Einstein causality or its strengthened form called Haag duality

$$Einstein \ causality \ : \quad A(O) \subset A(O')' \quad (8)$$

$$Haag \ duality \ : \quad A(O) = A(O')' \quad (9)$$

- (iii) Covariance and stability (positive energy condition) with respect to the Poincaré group $P$. For observable nets:

$$\alpha(a, \Lambda)(A(O)) = A(\Lambda O + a)$$

$$U(a, 1) = e^{iPa}, \ specP \in V_+$$

where the unitaries represent the covering group $\tilde{P}$ in $H$. A particular case is that the $P$-spectrum contains the vacuum state $P|0\rangle = 0$. We will call this net of algebras the vacuum net and the Hilbert space $A|0\rangle$ the representation space of the vacuum sector. For thermal states this stability requirement has to be changed \[3\].

- (iv) Time slice property (causal shadow property): Let $O$ be the causal shadow region associated with a subregion $C(O)$ of a Cauchy surface $C$ and let $U$ be a (timeslice) neighborhood of $C(O)$ in $O$, then

$$A(O) = A(U) \quad (10)$$

- (v) Phase space structure of LQP

$$the \ map \ \Theta : A(O) \to e^{-\beta P_0}A(O)|\Omega \rangle \quad (11)$$

A more detailed account of this intricate property together with some physical background will be given in the discussion of the “degree of freedom” issue in the next two sections.
Some comments on the physical ideas behind the requirements are in order.

In QM as formulated by von Neumann, the commutant of a collection of Hermitian operators is a weakly closed algebra formed from all operators which have a compatible measurement relation (physical interpretation of commutant) with the given collection. In LQP the Einstein causality property (ii) tells us that if the original collection generates all observables which can be measured in a given spacetime region \( \mathcal{O} \), then the commensurable measurements are associated with observables in the spacelike complement. There are two very important stronger versions of Einstein causality: Haag duality and statistical independence. Haag duality is the case of equality in (\( \mathfrak{g} \)) i.e. the totality of all commensurable measurements is exhausted by the spacelike disjoint localized observables (in QM such a characterization does not exist). One can show (if necessary by suitably enlarging the local net within the same vacuum Hilbert space) that Haag duality can always be achieved. It turns out that the inclusion of the original in the Haag dualized net contains profound information on “spontaneous symmetry breaking” [16], an issue which will not be treated in this survey. The magnitude of violation of Haag duality in other non-vacuum sectors is related to properties of their nontrivial superselection charges whose mathematical description is done in terms of endomorphisms \( \rho \) of the net (the Jones index of the inclusion \( \rho(A) \subset A \) is a quantitative measure). Neither Einstein causality nor Haag duality guaranty “statistical independence” i.e. a tensor product structure between two spacelike separated algebras analogous to the factorization for the inside/outside region of a quantum mechanical quantization box. This kind of strengthening of causality cannot be formulated in pure algebraic terms but needs properties of states as well. It turns out that the nuclearity of the QFT phase space in (v) is sufficient for the statistical independence property. More details in a not too heavy mathematical setting can be found in [17].

The positivity of energy is a specific formulation of stability adapted to particle physics which deals with local excitations of a Poincaré invariant vacuum. It goes back to Dirac’s observation that if one does not fill the bottom of the negative energy sea associated with the formal energy-momentum spectrum of the Dirac equation, an external electromagnetic interaction will create havoc. In case of thermal states it is the so-called KMS condition which secures stability; this is the only change in the adaptation to thermal physics [3].

The energy positivity leads via analytic properties of vacuum expectation values to the cyclicity of the vacuum with respect to the action of \( \mathcal{A}(\mathcal{O}) \) i.e. \( \mathcal{A}(\mathcal{O})\Omega = H \) and for \( \mathcal{O}'s \) with a nontrivial causal complement the use of cyclicity also yields the absence of local annihilators i.e. \( \mathcal{A}\Omega = 0, A \in \mathcal{A}(\mathcal{O}) \triangleright A = 0 \). Both properties together are known under the name of Reeh-Schlieder property [4]. This property is very different from what one is accustomed to in QM since it permits a creation of a particle “behind the moon” (together with an antiparticle in some other far remote region) by only executing local operations of short-lived duration on the earth. Mathematically this is the starting point for the Tomita-Takesaki modular theory which we will return to below. On the physical side the attempts to make this exotic mathematical presence of a
dense set of state vectors by local operations physically more palatable has led to insights into the profound role of the phase space structure \(\nu\) \[3\].

Intuitively the connection with the formulation in terms of pointlike fields is that the latter (smeared with \(\mathcal{O}\)-supported test functions in order to obtain unbounded operators) are generators of an operator algebra \(\mathcal{A}(\mathcal{O})\), but (as already known from the simpler case of generators of noncompact Lie-groups) the devil lies in the details about domains which will not concern us here since we would like to present the new ideas with the minimum amount of technicalities. In particular the problem whether each net of local observables fulfilling the above requirements possesses generating pointlike fields is an open question. For chiral conformal theories this has been shown \[14\].

Now we come to modular theory, which in a recent paper was referred to as a revolutionizing tool \[8\]. Since even in the setting of QFT “modular” occurs with different meanings, we will briefly define its present use. As a side remark we mention that the more common use is that of modular invariance in chiral conformal field theory (although this is not its present meaning, a future connection to this causality-related classification tool for certain families of 2-dimensional local models to the present also locality-based use of the Tomita-Takesaki modular theory in local quantum physics is by no means ruled out).

The Tomita-Takesaki modular theory describes a structural property of a single operator algebra \(\mathcal{A}\) in “standard form” i.e. under the assumption that the Hilbert space contains a vector \(\Omega\) with respect to which the \(\mathcal{A}\) acts in a cyclic and separating manner which means that \(\mathcal{A}\Omega\) is dense in \(\mathcal{H}\) and that \(\mathcal{A}\) contains no annihilators of \(\Omega\) \((\simeq\) to the denseness of \(\mathcal{A}'\Omega\), \(\mathcal{A}'=\text{commutant of }\mathcal{A}\)). If the Hilbert space is separable, there exist always plenty of such vectors. Operator algebras in standard form permit the definition of the involutive antilinear \(\text{generally unbounded }\)Tomita \(S\)-operator which without loss of generality can be assumed to be closed

\[
SA\Omega = A^*\Omega, \quad S = J\Delta^{\frac{1}{2}} \tag{12}
\]

\[
S^2 \subset 1
\]

This operator relates the dense set \(A\Omega\) to the dense set \(A^*\Omega\) for \(A \in \mathcal{A}\) and gives an antiunitary \(J\) and an unbounded positive operator \(\Delta^{\frac{1}{2}}\) by polar decomposition \(S = J\Delta^{\frac{1}{2}}\) which have the following relation with the algebra

\[
\text{Ad}\Delta^{it}\mathcal{A} = \mathcal{A} \tag{13}
\]

\[
\text{Ad}J\mathcal{A} = \mathcal{A}'
\]

The nontrivial miraculous properties of this decomposition are the existence of an automorphism \(\sigma_\omega(t) = \text{Ad}\Delta^{it}\) which propagates operators within \(\mathcal{A}\) (the first relation) and only depends on the state \(\omega\) (and not on the implementing vector \(\Omega\)) and a that of an antiunitary involution \(J\) which maps \(\mathcal{A}\) onto
its commutant $\mathcal{A}'$ (the second relation). An important thermal aspect of the Tomita-Takesaki modular theory is the validity of the Kubo-Martin-Schwinger (KMS) boundary condition

$$\omega(\sigma_{t-i}(A)B) = \omega(B\sigma_t(A)), \quad A, B \in \mathcal{A}$$

(14)
i.e. the existence of an analytic function $F(z) \equiv \omega(\sigma_z(A)B)$ holomorphic in the strip $-1 < \text{Im} z < 0$ and continuous on the boundary with $F(t-i) = \omega(B\sigma_t(A))$. The fact that the modular theory applied to the wedge algebra has a geometric aspect (with $J$ equal to the TCP operator times a spatial rotation and $\Delta^\mu = U(\Lambda W(2\pi t)))$ is not limited to the interaction-free theory [3]. These formulas are identical to the standard thermal KMS property of a temperature state $\omega$ in the thermodynamic limit if one formally sets the inverse temperature $\beta = \frac{1}{kT}$ equal to $\beta = -1$. This thermal aspect (including Unruh’s detector Gedankenexperiment for the presence of radiation) is the Unruh-Hawking effect of quantum matter enclosed behind event/causal horizons.

Our special case at hand, in which the algebras and the modular objects are constructed functorially from the Wigner theory, suggest that the modular structure for wedge algebras may always have a geometrical significance with a fundamental physical interpretation in any QFT. This is indeed true, and within the Wightman framework this was established by Bisognano and Wichmann [3].

2.1 Connection with black hole analogs

It cannot be emphasized enough that the thermal aspects of localization including Hawking-like radiation are (contrary to widespread opinion) not an exclusive attribute of curved spacetime physics or Poincaré invariance. Rather they represent a very generic properties of quantum systems with infinite degrees of freedom. They are related to the vacuum polarization structure and the possibility of spatially localizing quantum matter at a fixed time in such a way that there is a causal disjoint open region and that none of the two regions contains annihilation operators of the ground state $\Omega$ (a sufficient condition is a finite propagation speed in LQP). In other words the field localization with respect to the ground state should exhibit a unique field − state vector relation

$$A(x) \leftrightarrow A(x)\Omega$$

(15)
whose precise mathematical formulation is known under the name Reeh-Schlieder theorem. In other words Hawking-like thermal behavior is present whenever the ground state of an infinite degree of freedom problem upon restriction to localized quantum matter behaves “as if it would be the vacuum in QFT”.

The phenomenon is an extremely generic one and it is somehow easier to characterize the non-thermal exceptions. Take for example Schrödinger QM in the Fock space formulation. The restriction to $\psi$-operators localized in a bounded spatial region at a fixed time does not lead to virtual polarization effects since there is no difference between the virtual (off-shell) particle number conservation and that of the real (on-shell) one, so that the effect of vacuum
polarization is ruled out. In that case the vacuum in disjoint spacial regions simply factorizes, whereas the relation requires the virtual polarization property of the vacuum. This is of course consistent with the absence of any finite propagation speed and of causal horizons.

On the other hand phonons and many other nonrelativistic systems with many degree of freedom systems do show the Hawking analog in the formation of “dumb holes” and alike traps for outgoing signals. In fact one finds a whole zoo of black hole analog system in various different areas of physics. Whereas the structural presence of these thermal properties is required by consistency of the theory, their experimentally accessibility remains a matter of (dis)belief. Our present modular approach makes it easy to agree with the highlighted generality of this thermal/radiation phenomenon in the cited article, although the emphasis on the Lorentz group and the metric structure as the reason instead of the local quantum physical structure may be a bit self-defeating. It certainly goes against the initially proclaimed spirit of that article that in order to understand the universality of the phenomenon one should get away from differential geometric black hole type arguments.

With the localization temperature understood in terms the KMS aspect of modular theory, it is natural to inquire about localization entropy. Since local operator algebras turn out to be of hyperfinite von Neumann type (i.e. algebras which unlike quantum mechanical algebras do not tensor factorize), the von Neumann entropy is a priori not defined. However with the so-called split property which follows from the above phase space nuclearity requirement it is possible to find an eminent physical substitute. The split property says that by surrounding a causal horizon of a double cone\( D \) with a “collar” of size\( \delta \) one obtains a canonical way to construct a quantum mechanical tensor-factorizing type I operator algebra\( B_\delta \) which contains the given double cone algebra and is contained in the larger double cone algebra\( \hat{D} \) whose localization region is\( D + \delta \). In this way the vacuum fluctuation at the horizon can be controlled.

The algebra\( B_\delta \) does not have a precise localization region, its localization boundary within the collar of size\( \delta \) is totally fuzzy. Using factor\( B_\delta \), algebra of all operators factorizes in the manner well-known from the inside/outside quantization box tensor products in quantum mechanics (in the 2nd quantized formulation)

\[
\begin{align*}
B(H) &= B_\delta \otimes B_{\delta}' \\
H &= H_i \otimes H_o \\
\Omega &= \Omega_i \otimes \Omega_o \\
i, o : inside, outside
\end{align*}
\]

where outside here means the causal disjoint of\( D + \delta \). Different to quantum mechanics, the global physical vacuum does not factorize into the (analog of the) split vacua; rather it remains a highly entangled thermal (with the same Hawking temperature) state with respect to the split decomposition. The magnitude of the entanglement of the vacuum is measured in terms of the entropy\( S(\Omega, B_\delta) \) of\( B_\delta \) relative to\( \Omega \). This entropy diverges in the limit\( \delta \rightarrow 0 \) which just
expresses the fact that the entropy for the original type III double cone algebra is ill-defined. In order to get an intuitive feeling for the expected properties of this localization entropy $S(\Omega, B_\delta)$ we look at similar but already well understood vacuum polarization phenomena.

As first noticed by Heisenberg soon after the discovery of QFT (and later elaborated and used by Euler, Weisskopf and many others), the partial charge:

$$Q_V \Omega = \int_V j_0(x) d^3x \Omega = \infty$$

(17)

diverges as a result of uncontrolled vacuum particle/antiparticle fluctuations at the boundary. In order to quantify this divergence one should act with a more carefully defined “partial charge” on the vacuum ($s=$-dimension of space). This is an unbounded operator defined by smearing with a test function

$$Q_{R,\delta} = \int j_0(x) f(x_0) g_\delta(x/R) d^3x$$

(18)

$$g_\delta(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 + \delta \end{cases}$$

$$f(x_0) \geq 0, \supp f = \{x_0 \mid |x_0| < \varepsilon\} , \int f dx_0 = 1$$

The vectors $Q_{R,\delta} \Omega$ only converge weakly for $R \to \infty$ on a dense domain. Their norms diverge as

$$(Q_{R,\delta} \Omega, Q_{R,\delta} \Omega) \leq c(\delta) R^{s-1}$$

\sim area

(19)

The surface character of this vacuum polarization is reflected in the area behavior. The proportionality factor $c(\delta)$ depends on the collar size $\delta$ and diverges for $\delta \to 0$. In a conformal field theory $\delta$ and $R$ become related.

Since the localization entropy is also a vacuum polarization phenomenon, one expects a similar behavior: an area law (at least for large double cone radius $\delta$) with a coefficient which diverges with shrinking size of the collar (the region of spatial change of the test function g in the present analogy). An analogy is of course no replacement for a direct calculation. It is typical for many properties related to modular localization that they are easy to define and their existence is beyond doubt, but they resist explicit calculation; perhaps because there are presently no efficient calculational techniques.

Contrary to [15], I believe that an area proportionality of entropy is inexorably related with a Hawking like thermal behavior i.e. one cannot have one without the other. However Bekenstein’s geometric-classical derivation of the area law which gives a completely universal finite result does not depend on a collar size and the particular kind of quantum matter. This could find its explanation that the geometrical classical curved spacetime does not encode the diverging leading term but only the contribution to the area law which remains finite in the limit of vanishing collar size. After all the quantum version
of the classical equations from which the Bekenstein law has been derived has to be renormalized and this could affect equally all terms in that relation, not just the area term. Although the concept and the definition of localization entropy as a measure for the entanglement of the vacuum with respect to the splitting situation is quite clear and unambiguous, it has up to now resisted explicit calculation. In addition the interface between local quantum physics and geometrodynamics i.e. the connection of the quantum localization entropy with the classical Bekenstein entropy remains obscure. But the well-understood connection between Hawking temperature and modular localization leaves no doubt that there is a relation. It is encouraging for somebody who does not believe in accidents that the same modular ideas which are relevant for localization entropy also lead to the presence of infinite dimensional fuzzy symmetry groups whose action on the holographic image is that of chiral diffeomorphisms.

This is not far removed from Carlip’s use of the Virasoro algebra in his attempt to isolate the relevant dynamical variables for the entropy on the horizon. The universality of the Bekenstein law fits nicely the idea that the holographic projection onto the horizon “kinematizes” the degrees of freedom in that it reprocesses the original degrees of freedom into a structureless chiral theory with only (half)integer valued scale dimensions and that the original dynamical richness has been transferred into the structure of the action of automorphism (notably the one along the opposite lightray) on that kinematical algebra.

3 Reprocessing localization: holography & transplantation

In this section we will present the holography method which reformulates the intuitive content of the lightfront quantization idea in such a way that becomes conceptually acceptable and mathematically controllable. The mathematical basis is a rather deep property of a special class of inclusions of two (weakly closed) operator algebras \( \mathcal{N} \subset \mathcal{M} \) which act in the same Hilbert space. If both of them are commutative i.e. for algebras of functions on a topological space as the integration spaces in Euclidean functional integrals, then the projection \( E : \mathcal{M} \to \mathcal{N} \) corresponds physically to the well-known “integrating out degrees of freedom” (Wilson) or “decimation” (Kadanoff) procedure of the renormalization group formalism. If however the algebras are noncommutative, then not every inclusion leads to such a conditional expectation

\[
E(n_1 m n_2) = n_1 E(m) n_2
\]  

The precise condition which is equivalent to the existence of an \( E \) has been found by Takesaki and simply states that \( E(\mathcal{M}) \) exists iff the restriction of the modular group \( \sigma^t_\mathcal{M} \) to \( \mathcal{N} \) is the modular group of \( \mathcal{N} \)

\[
\sigma^t_\mathcal{M}|_\mathcal{N} = Ad\Delta^\mu_\mathcal{M}|_\mathcal{N} = \sigma^t_\mathcal{N} = Ad\Delta^\mu_\mathcal{N}|_\mathcal{N}
\]  

This is a quite severe restriction which leads to the Jones subfactor theory i.e. a framework which extends compact group symmetries. In QFT the superselection theory of DHR leads to this situation.
The inclusions we need for the mathematical formulation of holography go one step beyond this Takesaki situation in that the restriction leads only to a one-sided compression

\[ Ad\Delta^it \mathcal{N} \subset \mathcal{N} \quad (22) \]
\[ t \leq 0, \pm halfsided \ \text{modular} \quad (23) \]

(when we simply say modular, we mean by convention \( t < 0 \)) We assume that \( \cup_t Ad\Delta^it \mathcal{N} \text{ is dense in } \mathcal{M} \) or equivalently that \( \cap_t \Delta^it \mathcal{N} = \mathbb{C} \cdot 1. \)

The above modular inclusion situation has in particular the consequence that the two modular groups \( \Delta^it \mathcal{M} \) and \( \Delta^it \mathcal{N} \) generate a two-parametric group of translations and dilations in which the translations have positive energy [20]. Let us now look at the relative commutant (see appendix of [21]). Let \( (\mathcal{N} \subset \mathcal{M}, \Omega) \) be modular with nontrivial relative commutant. Then consider the subspace \( \text{generated by relative commutant } H_{\text{red}} \equiv (\mathcal{N} \cap \mathcal{M})\Omega \subset H. \) The modular unitary group of \( \mathcal{M} \) leaves this subspace invariant since \( \Delta^it \mathcal{M}, t > 0 \) maps \( \mathcal{N} \cap \mathcal{M} \) into itself by the inclusion being modular. Now consider the orthogonal complement of \( H_{\text{red}} \) in \( H. \) This orthogonal complement is mapped into itself by \( \Delta^it \mathcal{M} \) for positive \( t \) since for \( \psi \text{ be in that subspace, then} \)

\[ \langle \psi, \Delta^it (\mathcal{N} \cap \mathcal{M})\Omega \rangle = 0 \ \text{for } t > 0. \quad (24) \]

Analyticity in \( t \) then gives the vanishing for all \( t, \) i.e. invariance of \( H_{\text{red}}. \)

Due to Takesaki’s theorem [19], we can then restrict \( \mathcal{M} \) to \( H_{\text{red}} \) using a conditional expectation to this subspace defined in terms of the projector \( P \) onto \( H_{\text{red}}. \) Then

\[ E(\mathcal{N} \cap \mathcal{M}) \subset \mathcal{M}[\mathcal{N} \cap \mathcal{M}] = E(\mathcal{M}) \quad (25) \]
\[ E(\cdot) = P \cdot P \quad (26) \]

is a modular inclusion on the subspace \( H_{\text{red}}. \) \( \mathcal{N} \) also restricts to that subspace, and this restriction \( E(\mathcal{N}) \) is obviously in the relative commutant of \( E(\mathcal{N} \cap \mathcal{M}) \subset E(\mathcal{M}). \) Moreover using arguments as above it is easy to see that the restriction is cyclic with respect to \( \Omega \) on this subspace. Therefore we arrive at a reduced modular “standard inclusion”

\[ (E(\mathcal{N}) \subset E(\mathcal{M}), \Omega) \quad (27) \]

Standard modular inclusions are known to be isomorphic to chiral conformal field theories [22] i.e. they lead to the canonical construction of a net \( \{ \mathcal{A}(I) \}_{I \in \mathcal{K}} \) indexed by intervals on the circle with the Möbius group \( \text{PL}(2,\mathbb{R}) \) acting in correct manner, including positive energy condition.

Let us now apply this to wedge algebras which are known to satisfy the modular prerequisites. As before, we take the simplest case of a massive \( d=1+1 \) theory

\[ \mathcal{M} = \mathcal{A}(W) \quad (28) \]
\[ \mathcal{N} = AdU(1)\mathcal{A}(W) \]

17
where \( U(a) \) stands for the lightlike translations which slides \( W \) along the upper right lightray into itself i.e. \( W_a \equiv W + a \subset W \). The positivity of the translational spectrum makes \( \mathcal{A}(W_a) \subset \mathcal{A}(W) \) a modular inclusion. The standardness of this inclusion i.e. the triviality \( (E(\mathcal{M}) = \mathcal{M}) \) of the above conditional expectation is the quantum counterpart of the classical characteristic property mentioned before \( [1] \). The relative commutant is obviously localized on the upper lightray. In fact it becomes a subalgebra of a conformal net \( \mathcal{A}(R) \). It is very important here to avoid equating chiral conformal theories with zero mass particles. The lightray momenta \( P_{\pm} \) always have a gapless spectrum even though the \( d=1+1 \) mass operator \( M^2 = P_+ P_- \) may possess a gap.

The surprising result of the existence of a chiral net on the upper horizon is the presence of the new rotational symmetry which is directly related to the possibility of compactifying the real line of chiral theories. Since the chiral net consists of operator algebras which act in the same Hilbert space, the associated unitary rotation operator acts also on the operators of the original \( d=1+1 \) massive theory. Of course it does not belong to the geometric symmetry operations which are exhausted by Poincaré transformations. It is a fuzzy symmetry of the original net in the sense we have recently introduced this concept \( [2] \). On the other hand the opposite lightray translation \( U_- (a) = e^{iP_- a} \) which is a Poincaré transformation in the original \( d=1+1 \) net becomes fuzzy on the chiral net. Therefore diffeomorphisms of QFTs may become fuzzy in the holographic processing and vice versa diffeomorphisms in the holographic image may become fuzzy in the original spacetime indexing. There is absolutely no possibility of understanding this “scrambling up” process in the presence of interactions in the setting of pointlike fields, even though each side of the holographic imaging may have a perfectly well-defined description in terms of such fields (as generators of the respective net of algebras).

We mention in passing that within the same modular setting one can show that each chiral theory (in our extended sense) has an infinite group of diffeomorphisms of modular origin \( [8] \). Higher dimensional (not necessarily massive) theories as well as \( d=1+1 \) massive models have a hidden infinite dimensional fuzzy symmetry group which is the fuzzy analog of the chiral diffeomorphisms whose infinitesimal generators form the centrally extended Virasoro algebra. In the holographic image this fuzzy symmetry becomes a bona fide diffeomorphism group. It is an interesting question whether the existence of the fuzzy translation \( U_- (a) \) is the only distinction between the chiral nets in the sense of this paper and the standard chiral field theory with a chiral energy-momentum tensor.

Whereas in the free field case of the previous section the zero mass conformal limit is (apart from multiplicity) the same as the lightray limit, the interaction forces both chiral theory to be significantly different; this can be made explicit within the setting of \( d=1+1 \) factorizing models \( [23] \).

---

8If one defines chiral theories in the more limited context of chiral decomposition of a massless \( d=1+1 \) theory with a traceless energy momentum tensor these diffeomorphisms of the circle are there from the very start and the modular theory only serves to show their modular origin.
In $d>1+1$ the presence of transversal directions to the say $x$-$t$ wedge complicates the problem because the modular inclusion cannot resolve the transversal local net structure. It is not difficult to see that one needs $d-1$ Lorentz transformation which tilt the standard wedge into $d-1$ different positions. The $d-1$ wedge algebras are reprocessed by the above holography into $d-1$ copies of one chiral theory. It is not clear at this stage of the development whether one should process these chiral copies into the net structure of the $d-1$ light front in order to formulate a holographic map onto the horizon of the $d$-dimensional wedge, or whether it is more practical to use these $d-1$ chiral theories in the spirit of scanning the original theory directly in terms of the relative position of $d-1$ chiral theories.

An inclusion preserving map of a net in a given curved spacetime onto a net of another spacetime in the same dimension is called a transplantation. An interesting recent example is the transplantation of local quantum matter from the $SO(4,1)$ symmetric deSitter spacetime (dS) to a net on the only $SO(4)$-symmetric Robertson-Walker world (RW). This transplantation is achieved in terms of a bijective inclusion preserving map $\Xi$ of the net of dS wedge algebras $\{A(W)\}_{W \in W_{dS}}$ onto that of the RW wedge algebras $\{A(W)\}_{W \in W_{RW}}$. Instead of explaining details, we refer to the beautifully written paper [24]. The construction also illustrates two more concepts of modular origin: the Condition of Geometric Modular Action (CGMA) and the Modular Stability Condition.

Compared with our previous 2-dimensional illustration of holography one notices two interesting differences. On the one hand the transplantation of dS to RW retains more geometric features in that the symmetry group $SO(4)$ of the “Transplant” is fully contained in $SO(4,1)$, only the action of $SO(4) \backslash SO(4,1)$ is fuzzy. On the other hand the double-cone algebras which are defined by intersecting wedges are trivial in $\mathcal{A}_{RW}$ as soon as the double-cone size becomes smaller than a certain parameter [24]. This of course means that the RW models constructed in this way do not possess pointlike field generators. This raises the interesting question whether there also exist physically admissible nets which are not the transplants (or holographic images) of standard QFT. I do not know such a model.

4 An exceptional case: the AdS-CQFT isomorphism

There has been hardly any problem in particle physics which has attracted as much attention as the problem if and in what way quantum matter in the Anti deSitter spacetime and the one dimension lower conformal field theories are related and whether this could possibly contain clues about the meaning of quantum gravity. One reason why this historically first example enjoyed such a widespread popularity as a test case for new ideas about holography was that with some physical hindsight and artistic abilities to read and interpret
euclidean functional integrals it can be seen within the standard setting of using field coordinatization. It is not too difficult to see that univalued correlation functions of some field on AdS spacetime upon suitable scaling adjustments (in the limit approaching the boundary at spatial infinity) correspond to fields on compactified Minkowski spacetime \( \hat{M} \). The latter property is synonymous with the validity of Huygens principle and hence with the absence of anomalous scale dimensions so that apart from different normalization constants the correlation functions have a similar decomposition into covariant rational functions on the complexified \( \hat{M} \) as those for composite free fields. In order to incorporate anomalous dimensions one has to start with AdS objects which live on the covering of AdS. So conventional field theoretic methods allow to understand the isomorphism 25 in the direction

\[
\text{AdS}_{d+1} \xrightarrow{\text{rescaled}} \text{CQFT}_d
\]

apart from a fine point which is related to causal propagation and which will be mentioned later. The other direction of the arrow from the lower to the higher dimensional theory is more subtle and cannot be done on correlation functions or in terms of pointlike fields 26. The reason is that there is a certain amount of “nonlocal scrambling” of degrees of freedom going on. A useful intuitive picture is obtained by imagining the spacelike boundary of \( \text{AdS}_{d+1} \) to form the wall of a \( d+1 \) dimensional cylinder with identified top-bottom (corresponding to the periodic AdS-time) and the \( \text{AdS}_{d+1} \) bulk filling its inside. For the conformal theory on the boundary the natural causal building blocks are the double cones which are conformally equivalent to wedges. The AdS wedges may be viewed as the natural wedge-like prolongation of the conformal double cones into the bulk; this purely geometric relation is of course a reflection of the fact that the two worlds share despite their different dimension the highest possible symmetry group. This intuitive picture gives the correct idea about the isomorphism namely as a sur- and injective map of the AdS wedge algebras and their double cone shadow algebras on the boundary. The two theories would share the same Hilbert space and a common algebraic structure and their only, but physically significant difference would be the different spacetime indexing. This is indeed the content of a rigorous mathematical theorem 25.

There is another interesting lesson to be learned concerning the degrees of freedom in this holographic reprocessing of higher to lower spacetime dimensions. Intuitively one of course expects that a pointlike AdS theory has too many degree of freedoms in order to be physically acceptable on the conformal side 28. This would show up in the breakdown of the causal propagation. The algebraic formulation of causal propagation is that the operator algebra localized in a piece of timeslice \( T = \mathcal{O}^{(d-1)} \times \delta \) of thickness \( \delta \) is equal to that of the causal shadow \( D_T \) (for \( \mathcal{O}^{(d-1)} \) a sphere, \( D_T \) would be a double cone) cast by \( \mathcal{O}^{(d-1)} \)

\[
\mathcal{A}(T) = \mathcal{A}(D_T)
\]
This is evidently the local quantum algebraic adaptation of the classical Cauchy propagation. Since our basic data consists of the net of double cones, we should use the weak additivity property of AQFT and cover the time slice $T$ by a family of small double cones. It is geometrically obvious that each small double cone is the boundary projection of a AdS wedge which contrary to the large wedge associated to the original double cone $D_T$ only modestly enters the bulk. So if we start from $T$ and move upward into $D_T$ we expect more and more degrees of freedom entering “sideways” and spoil the causal propagation. Such a “poltergeist QFT” is of course unacceptable as long as particle physics remains a science of de-mystification (whether this is still a characteristic property of what some physicist are presently doing is another matter). A nice explicit illustration of this phenomenon is obtained by computing the conformal field theory corresponding to a 5-dimensional zero mass free AdS $\Phi_0$ field. It is a generalized free field $\varphi$ with scale dimension $d_\varphi = 2$

$$
\langle \varphi(x)\varphi(y) \rangle = c \left[ \frac{1}{(x-y)^2} \right]^2 \quad (31)
$$

$$
\langle \varphi(x_1)\ldots\varphi(x_n) \rangle = \begin{cases} 
\prod_{\text{pairings}} \langle \varphi(x_i)\varphi(x_j) \rangle, & n \text{ even} \\
0, & n \text{ odd}
\end{cases}
$$

It is known that generalized free fields with certain increasing Kallen-Lehmann spectral weights $\rho(\kappa^2)$ violate the timeslice property and one checks that those homogeneous $\rho(\kappa^2)$ which correspond to scale invariant theories with dimensions larger than the canonical one all belong to this unphysical kind. This destroys the nice dream of enlarging the extremely scarce set of 4-dimensional candidates of conformal models by doing Lagrangian field theory on the AdS side. On the other hand the Wigner representation theory for particles $(0,s)$ of zero mass is automatically (i.e. without enlargement of the irreducible representation space of the Poincaré group) conformally invariant. These degrees of freedom are too few in order to lead to pointlike fields on AdS [25]. Rather the pointlike conformal boundary fields stretch as (Nielsen-Olsen like) strings into the bulk i.e. the configuration is constant along the string direction. There is no way to escape the mathematical theorem that the isomorphism is inconsistent with an imagined relation between two Lagrangian field theories as suggested in the work of Maldacena Witten and others [25] which is based on nonrigorous arguments about converting functional integrals. As far as I know, there has been no mathematical explanation how such an argument concerning a relation between special models which is so close to a structural theorem [26] and yet differs from it can be upheld. Inconvenient mathematical theorems simply do not disappear by ignoring them.

\footnote{For massive free fields one has to use the covering of AdS, and the resulting generalized free field have anomalous dimension and live on the covering $\tilde{M}$ of $M$.}

\footnote{These “kinematical” localization strings do not have internal dynamical degrees of freedom as those of string theory where the word string refers to the dynamical spectrum and not to localization.}
These critical remarks about Lagrangian interpretations of relations between models in different spacetime models can be extended to the idea of branes. To be more specific, branes are pictured as a spacetime submanifolds with a physical interpretation in an ambient spacetime which is also required to have physical properties and not be just an auxiliary construct at the service of the branes. The argument is analogous to the previous one; if one starts with an ambient theory of pointlike fields then the causal propagation is wrecked as a result of transversal degrees entering continuously from the side, and if one starts on the brane side then one can only maintain the ambient causal propagation via transversal kinematical string-like degrees of freedom. Of course one can use the brane idea (quasi)classically in a technical sense in order to produce lower dimensional configurations from higher dimensional ones.

The closely related Klein-Kaluza idea for dimensional reduction is only consistent in quasiclassical simplification of QFT which ignore the characteristic phenomenon of vacuum polarizations. Most of the discussions in the literature are either quasiclassical from the start or the removal of the higher Fourier modes associated with the dimensions to be converted into internal symmetries is done before the local quantum aspects are investigated. If one would not have interchanged the “small dimension limit” (the “curling up” procedure) with quantization but really have taken this limit in the full correlations functions including the vacuum polarization, then for most operators the latter would have diverged in an uncontrollable way. In those few cases where authors tried to do it the correct way, such divergent fluctuation prevented meaningful limits \[27\]. Besides this, the DR theory tells us that inner symmetry is nothing else as geometrically encoded para-statistics \[3\], and particle/field statistics is really a far cry away from transversal spacial dimensions.

It is interesting to contrast the holographic property with those other ideas of dimensional reductions. Whereas the first one needs the full power of non-commutative local quantum aspects (to be able to apply modular theory), the latter seem to make only sense (quasi)classically.

5 An area of application: modular constructed anyons

Revolutionary changes in theoretical physics should sooner or later lead to experimentally verifiable consequences. Holography&transplantation and the other other new concepts in sections 2 and 3 as important for the structure of particle physics as they may be can hardly be expected to have laboratory manifestations in the near future, and even the nice black hole analogs from acustics (dumb holes), hydrodynamics and other areas are presently more a part of science fiction than of laboratory physics.

It is not necessary that the contact with the real world takes place in the same area which highlighted the paradigmatic change. The only requirement which an application of the new approach has to fulfill is that the field-coordinate-free
modular localization approach should be the essential tool for its construction; with other words it should either be inaccessible by Lagrangian quantization methods or a forced attempt to derive it with standard methods should necessitate the use of additional unnatural recipes and auxiliary inventions.

5.1 weakness of the standard approach

Such an area is the would-be theory of \(d=1+2\) braid group statistics fields/particles. Here we will accept the claim that braid group statistics is the basis of some condensed matter effects (in particular the fractional Hall effect) on face value. The difficulties one encounters with braid group statistics in the standard setting are the following

- It is not possible to maintain the plektonic (braided) spin-statistics connection in quantum mechanics, it rather requires the presence of (virtual) vacuum polarization

- The prerequisites for a euclidean functional integral representation of correlation functions of plektonic operators are violated and hence a definition of plektonic correlation functions on the basis of such representations is not reliable.

These two statements need some explanatory comments.

It is well-known that spin-statistics matters can only be meaningfully investigated in relativistic QFT. Their use in the setting of quantum mechanics is based on the observation that in the nonrelativistic limit one maintains the spin-statistics connection and obtains strict particle number conservation (real and virtual). In this form the statement only applies to \(d=1+3\) space-time dimensions where the only admissible statistics is Bose/Fermi statistics. In \(d=1+2\) braid group statistics is possible, but in order to sustain it into the nonrelativistic limit, one has to abandon the conservation of virtual particle number (no vacuum polarization) and retain only the real particle number conservation. But quantum mechanics is characterized by the absence of vacuum polarization and therefore the nonrelativistic limit of a plektonic QFT will be a nonrelativistic QFT and not a QM. The many quantum mechanical descriptions of anyons in terms of Aharonov-Bohm potentials are misleading; whereas it is possible to deform the spin-value by auxiliary vector potentials, it is not possible to maintain the relativistically plektonic spin-statistics connection (as the usual spin-statistics relation the derivation has to be done in the relativistic setting no matter if the physical application is relativistic or nonrelativistic) since QM does not allow for virtual vacuum polarization. The rigorous proof consists in showing that in the relativistic setting of “plektons” (the abelian braid group representations are better known as anyons) there can be no polarization-free-generators (PFG) which applied to the vacuum generate a one particle state without admixture of particle/antiparticle polarization clouds [31]. Another way of interpreting this result is to say that the mere sustention of spin-statistics (even without thinking about genuine interparticle
interactions) already requires the presence of these virtual clouds; in the standard formalism one would have to blame this on some mysterious interaction (which only exists to get the spin&statistics going) whereas the new formalism would automatically shift the cut between kinematics (statistics) and dynamics (interactions). A third very provocative way of presenting these observations is to say that quantum mechanics owes its physical relevance to the existence of PFG Bose/Fermi fields which create states which are free of vacuum polarization for any localization size (the usual free fields).

The second above listed difficulty is not independent of the first one. Quantum field theories which lead to “nonlocal” fields i.e. fields which neither commute nor anticommute for spacelike distances, are necessarily more “noncommutative”. The conceptual framework of Nelson-Symanzik-Guerra for the derivation of Euclidean Feynman-Kac functional representations was quite subtle (before it degenerated into the present “path-integral cult”) and limited to a small family of superrenormalizable interactions. Fermions may be formally incorporated by first reading them back into classical physics as Grassmann algebra valued objects and then subjecting them to an appropriately adapted Euclidean machinery. Outside that narrow setting this formalism is at best an artistic tool of exploration. Even if we take a very permissive attitude and ignore the fact that in strictly renormalizable theories (operator dimensions of interaction Lagrangians equal to spacetime dimensions) the renormalized correlations are only Einstein causal but do not fulfill canonical commutation relation nor euclidean functional integral representation, there is the question whether this Euclidean setting applied to Lagrangians containing e.g. Chern-Simons terms defines (a) a quantum field theory at all (i.e. a theory of operators in a Hilbert space) and in case it does (b) if the correlation functions obey the spin&statistics property. One can of course simply go ahead and with hindsight and ingenuity arrive at some quasiclassical consistency on the level of Berry phases. But a quasiclassical consistency is no replacement for an understanding of the spin&statistics issue which requires a very noncommutative structure. In fact as our above criticism of the quantum mechanical approach based on the Aharonov-Bohm effect to the spin&statistics issue shows, there is all reason to mistrust quasiclassical topological arguments in which the vacuum polarization structure does not play an essential role. Whatever kind of quantum theory a Euclidean functional integral containing a Chern-Simons term represents, it is not sufficiently noncommutative in order to represent operators consistent with braid group spin&statistics theorem. This has led to adding noncommutativity by hand or by taking some ideas from string theory. The attempt to enforce this noncommutativity into condensed matter physics by making spacetime noncommutative really seems to be very farfetched, even if it is only for the purpose of obtaining an “effective” interaction. In the following I will sketch a very conservative constructive approach which starts from Wigner one-particle theory and at the end leads to operators which have the minimal vacuum polarization structure which is necessary to maintain the spin&statistics connection.
5.2 The modular approach to “free” anyons

The approach to plektons based on the new modular ideas leads to a well-defined canonical procedure which is presently in progress and will be published in the near future. We will make no attempt here to explain all the mathematical steps in such a constructive approach, but in order to underline the “revolutionary” aspects of the modular localization approach it is interesting to mention some of the physical concepts which are used to construct “free” abelian braid group fields i.e. anyons (leaving aside the more involved construction of general plektons). Here the word “free” means that we start from Wigner’s one particle representation theory in d=1+2 with an abelian little group and that in constructing the associated multiparticle spaces and associated fields we seek that realization of the spin&statistics connection which contains no interactions i.e. is uniquely determined by combining the Wigner representation theory with modular localization theory.

If the spin is (half)integer then we would immediately go to the (anti)symmetrized tensor Fock spaces and verify that the resulting free fields solve the problem. In case of $s \neq \text{halfinteger}$ such a procedure is not only ill-motivated but even wrong (it would lead to generalized free fields which violate spin&statistics). The big surprise is now that this innocent looking Wigner one-particle theory already preempts in an extremely subtle way all the modular structural properties which we need in order to construct “free” anyons. The irony is that this undeserved recent gift has its origin in the very same properties which led to the ill-fated Newton-Wigner localization (i.e. the impossibility to impose a relativistically invariant localization concept modeled after the Born x-space probability interpretation of wave functions) and the various paradoxes under the name of Klein: the Wigner one-particle theory is conceptually different from what one expects from a relativistic made Schrödinger quantum mechanics. This somewhat hidden feature has its strongest outing in the existence of a pre-modular interpretation of the Wigner theory: the correct relativistic way of localization is a spatial version of the Tomita-Takesaki theory.

This pre-modular theory encodes relativistic localization into the position of real Hilbert spaces within the complex Wigner representation space $[32] [33]$. One starts with the boost transformation associated with a wedge and its reflection transformation along the rim of the wedge. For the standard $x$-$t$ wedge $W_0$ these are the $\Lambda_x^{-t}(\chi)$ Lorentz boost and the $x$-$t$ reflection $r_{x-t} : (x,t) \rightarrow (-x,-t)$ which according to well-known theorems is represented antiunitarily in the Wigner theory. One then starts from the unitary boost group $u(\Lambda(\chi))$ and forms by the standard functional calculus the unbounded “analytic

\[ \text{In case of charged particles the Wigner theory should be suitably extended by a particle/antiparticle doubling.} \]
continuation”. In terms of modular notation we define

\[
\begin{align*}
\mathfrak{s} &= j_\delta^\dagger \\
\mathfrak{j} &= u(r) \\
\delta^{\dagger t} &= u(\Lambda(-2\pi t))
\end{align*}
\]

where \(u(\Lambda(\chi))\) and \(u(r)\) are the unitary/antiunitary representations of these geometric transformations in the (doubled, if particles are not selfconjugate) Wigner theory. Note that \(u(r)\) is apart from a \(\pi\)-rotation around the x-axis the one-particle version of the TCP operator. On the other hand \(\mathfrak{s}\) is a very unusual object namely an unbounded antilinear operator which on its domain is involutive \(\mathfrak{s}^2 = 1\). The real subspace \(H_{R}(W_0)\)

\[
H_{R}(W_0) = \{ \psi \in H \mid \mathfrak{s}\psi = \psi \}
\]

consists of momentum space wave functions which are boundary values of analytic functions in the lower \(i\pi\)–strip of the rapidity variable \(\theta\) and whose boundary value on one rim is the complex conjugate of that on the other. The -1 eigenvalues of \(\mathfrak{s}\) do not give rise to a new problem since multiplication of the +1 eigenfunctions with \(i\) convert them into the -1 eigenfunctions. The real subspace \(H_{R}(W_0)\) is closed in the complex Hilbert space topology and the complexification \(H_{R}(W_0) + iH_{R}(W_0)\) gives a space which is dense in the complex Wigner space \(H\). This surprising fact (which is the Wigner one-particle analog of the Reeh-Schlieder denseness of local field states in full quantum field theory) has no parallel in any other area of quantum physics. It suggests that the above mentioned unusual property of the \(\mathfrak{s}\)-operator may be the vehicle by which geometric physical properties of spacetime localization are encoded into the abstract domain properties of unbounded operators. Indeed the application of the Poincaré group to the subspace of the standard wedge generates a coherent net of subspaces \(\{H_{R}(W)\}_{W \in \mathcal{W}},\mathcal{W}\) the family of all wedges. Some rather straightforward checks reveal that this interpretation is consistent, namely in the present setting this localization interpretation gives consistency with the net properties of the spaces \(H_{R}(\mathcal{O})\)’s

\[
H_{R}(\mathcal{O}) \equiv \cap_{W \supseteq \mathcal{O}} H_{R}(W)
\]

as well as with the conventional field theoretic construction using pointlike fields where it agrees with localized covariant functions defined in terms of support properties of Cauchy initial data (the compact localized spaces \(H_{R}(\mathcal{O})\) are however only nontrivial for \(s=(\text{half})\text{integer}\) in which case the Wigner rotation \(R(\Lambda, p)\) is free of cuts in the modular strip). In the case of \(d=3+1\) Wigner’s “continuous spin” representation \([34]\) and for \(d=2+1\) \(s\neq \text{halfinteger}\) these double cone localization spaces turn out to be trivial and we will return to this last case below.

In the halfinteger spin case the relation of Wigner subspaces and localized subalgebras is accomplished with the help of the CCR or CAR functors which
map real subspaces $H_R(O)$ into von Neumann $A(H_R(O))$ subalgebras and which define a limited but rigorous meaning of the word “quantization”

$$J, \Delta, S : = \Gamma(j, \delta, s)$$  \hspace{1cm} (35)

where the functorial map $\Gamma$ carries the functions of the Wigner theory into the Weyl operators in Fock space (for the fermionic CAR-algebras there is an additional modification). Whereas as previously explained the “pre-modular” operators denoted by small letters act on the Wigner space, the modular operators $J, \Delta$ have an $Ad$ action on the von Neumann algebras which are functorially related to the subspaces and which makes them objects of the previously presented Tomita-Takesaki modular theory

$$A(O) = \text{alg} \{ W(f) = e^{i(a(f) + h.a.)} | f \in H_R(O) \}$$  \hspace{1cm} (36)

where the first map of real subspaces into operator algebras is in terms of the Weyl functor whereas the second map denoted by $\psi$ is based on the CAR functor.

In the case of $d=1+2$ Wigner particles with spin $s \neq$ halfinteger there are some important changes. As in the case of $s=\frac{1}{2}$, the spin-statistics phase is already preempted in the Wigner theory through the appearance of a mismatch between the spatial opposite which is described by the symplectic complement $H_R(W)'$ with the geometric complement $H_R(W') = u(R(\pi))H_R(W)$. One can show that this mismatch is described by a phase factor $t$ (the statistics phase)

$$H_R(W') = tH_R(W)'$$  \hspace{1cm} (37)

This suggests that the Tomita $S$-operator of the associated full QFT is of the form

$$S = TJ\Delta^\frac{1}{2}$$  \hspace{1cm} (38)

with $T$ a twist operator whose necessity was preempted by $t$ in the Wigner theory.

The compactly localized subspaces $H_R(O)$ turn out to be trivial but the intersection of two (not oppositely localized) wedges which defines a noncompact spacelike cone is nontrivial

$$H_R(C) = H_R(W) \cap \{ u(R(\pi))H_R(W) \}$$  \hspace{1cm} (39)

where $R(\pi)$ a $\pi$-rotation leading to a 90 degree rotated wedge. There is a tricky point in the calculation of intersections of wedges which is related to the appearance of cuts for $s \neq$ halfinteger in the Wigner rotation $R(\Lambda, p)$ which in the present $d=1+2$ anyon case is really a nonlocal phase $\Phi(\Lambda, p)$. This has been successfully treated in [33]. In that work the spin&statistics derivation for spacelike cone localized fields of [35] was adapted to the braid group situation.

The presence of the anyonic twist $t \neq \pm 1$ is an obstruction against the existence of an operator $A(C)$ such that $A(C) \Omega \in H_R(O) + iH_R(O)$. Such PFGs i.e.
operators which upon application to the vacuum create polarization cloud-free one particle states only exist for wedges whereas any stronger localization in the presence of interactions or genuine braid group statistics (instead of permutation group statistics) necessarily causes polarization clouds which accompany the one-particle state. The situation is somewhat similar to that of \( d=1+1 \) factorizing models for which the Fourier transforms of wedge-localized PFGs turn out to fulfill a Zamolodchikov-Faddeev algebra \(^{[32]}\). Since the mass shell inherits the ordering structure \( p > q \) from the forward light cone, it is possible to incorporate the anyonic phase \( t \) into (Zamolodchikov-Faddeev like) mass shell commutation relations

\[
Z(p)Z(q) = tZ(q)Z(p), \quad q > p
\]

\[
Z(p)Z^*(q) = 2p_0\delta(\vec{p} - \vec{q}) + tZ^*(q)Z(p), \quad q > p
\]  

(40)

As in the \( d=1+1 \) factorizing case the \( Z \)'s are the candidates for the Fourier transforms of wedge-localized PFGs and analog to that case one also expects here that the desired spacelike cone localized free \(^{12}\) anyon operators \( A(C) \) come about by taking intersections of wedge algebras which will determine the polarization clouds in terms of infinite power series in the \( Z \)-operators. We hope to return to this interesting construction in a different context.

6 Concluding remarks

Apparently QFT only attains its maximal naturalness and flexibility if one passes from fields to nets of algebras since the radical reprocessing of degrees of freedom in holography and transplantation does not seem to be expressible in terms of field coordinatizations. Such a standpoint seems to be important in the quest of definition of a localization entropy which describes the entanglement of the vacuum state in the “split situation”. This may generate the desired generic local quantum physical prerequisite for the universal nature of a quantum Bekenstein area law.

Furthermore there exist certain physical problems in which pointlike fields are not the proper tool, e.g. the charge-carrying operators whose multiple application to the vacuum creates state vectors with braid group statistics necessarily have a noncompact extension (semiinfinite stringlike). Finally there is the Holy Grail of a constructive approach in which the ultraviolet problems are sidestepped by avoiding the use of pointlike fields. Although the first attempt, the \( S \)-matrix bootstrap approach of the 60s failed, its two dimensional version for factorizing \( S \)-matrices worked with the help of special recipes. Recently it was shown that modular theory allows to justify these recipes in terms of modular localization properties and to obtain in particular a spacetime interpretation of the Zamolodchikov-Faddeev algebra. Since only general principles are used, there is the hope that the method can also be extended to more general cases.

\(^{12}\)Free in the sense of triviality of scattering. The constant anyonic phase factor \( t \) does not have the interpretation of a scattering matrix.
The field-coordinate free approach is based on modular Tomita-Takesaki theory applied to nets of operator algebras. It is very surprising that quantum field theory after 70 years of existence is capable to take such a revolutionary turn while being totally faithful with its (in recent decades unfortunately almost forgotten) causality and stability (spectral) principles. But this conservative aspect is also the source of problems which the present approach has with the prevalent Zeitgeist which seems to follow more the desire for good entertainment than to drive forward particle theory. For the large decades progress in particle theory has been looked for by inviting nature to follow inventions like supersymmetry, string theory, M-theory etc. instead of paying more conceptual attention to the secrets which nature has hidden behind the already unravelled principles.

The situation became aggravated by the unclear experimental situation and the increasing loss of independence of experimentalizers on what their theory friends tell them (see the “small extra dimensions” search experiments). I believe that in such a situation an approach which stays reasonably close to the historical achievements in particle physics is the most reasonable one. The difficulty of course is that with the accelerating process of loss of knowledge relevant for particle theory (in some aspects the knowledge of many especially young particle physicists has already fallen behind the LSZ theory in which to some extend the relation between fields and particles was already clarified), there is an increasing gap of communication which makes reading of articles dealing with conceptual problems of QFT difficult. Last not least progress on conceptual-mathematical problems of local quantum physics demands a lot of patience and strong links with history. This more contemplative spirit is certainly not compatible with the speed which is characteristic for the dominating areas of particle theory which, not unlike a French concord jet, would crash if the velocity is not maintained above a certain level.

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Note added: Meanwhile I have placed some lecture notes on the server which (on certain aspects) are more detailed [37].

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