The Effect of Splayed Pins on Vortex Creep and Critical Currents

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We study the effects of splayed columnar pins on the vortex motion using realistic London Langevin simulations. At low currents vortex creep is strongly suppressed, whereas the critical current \( j_c \) is enhanced only moderately. Splaying the pins generates an increasing energy barrier against vortex hopping, and leads to the forced entanglement of vortices, both of which suppress creep efficiently. On the other hand splaying enhances kink nucleation and introduces intersecting pins, which cut off the energy barriers. Thus the \( j_c \) enhancement is strongly parameter sensitive. We also characterize the angle dependence of \( j_c \), and the effect of different splaying geometries.

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The enhancement of critical currents in superconductors through irradiation by heavy ions is well established. The ions create extended columnar defects that localize individual vortices much more effectively than naturally occurring point defects. It was suggested by T. Hwa et al. that pinning could be further improved by splaying these columnar defects. In recent experiments, splayed pinning has been created directly through multiple-step irradiation of the sample, or indirectly by irradiation through thin foils. Up to an order of magnitude enhancement of the critical current relative to parallel columnar pins has been reported.

Columnar pins which are aligned with the field direction allow a flux line to sit in a potential minimum in every layer without having to pay any elastic energy. There are no competing tendencies which favor point over columnar pinning, and hence there is a clear expectation that columnar pinning will greatly enhance the critical current, as is universally born out experimentally.

That splay should further enhance \( j_c \) seems much less compelling. It has been argued that a vortex hopping between two splayed pins experiences a linearly increasing energy cost, whereas once a hop originates between columnar pins there is no further cost for completion of the jump. This leads to a suppression of motion for splayed pins. An additional mechanism for enhancement of \( j_c \) that we suggest is an increase in collective pinning effects due to greater vortex entanglement in the presence of splayed pins. This second mechanism may be more effective than the first since, although the distance between two splayed defects increases as one moves away from their point of closest approach, the separation between the pins may be very small at this closest point. Thus one could argue that splaying leads to an increased nucleation of hopping. Furthermore, the increasing energy cost which exists for two splayed pins can be cut off by the additional pins that are present.

Set against the suggestions that splay enhances pinning is also the fact that splaying undermines the key feature which made columnar pinning so effective in the first place. Vortices must elongate in order to take advantage of the pins, though this wandering is not random, as for point pinning. Even accepting the increasing energy cost for hopping and entanglement arguments, it is not obvious that they will dominate over the very special topological effectiveness of columnar pinning. The uncertain consequences of this set of competing effects is reflected experimentally in the fact that the enhancement of pinning by splay is sensitive to the details of the splay, including whether the angular distribution is Gaussian or bimodal, the splay angle, and the material used.

In spite of the importance of splay for critical current enhancement, no realistic numerical simulations of vortices interacting with splayed pinning have been reported up until this time. In this paper we present the first analysis of the effects of splayed pinning using overdamped molecular dynamics simulations. By examining the distribution of lengths of pinned vortex segments between kinks, we explicitly verify the conjecture that the energy cost for vortex motion increases for intermediate length segments in the presence of splay, as a two pin argument suggests. We refine this picture in a crucial way by demonstrating how this growth is cut off by intersections with additional pins. This measure at the same time shows an enhancement of short length segments (nucleation events). By comparing splay which is in the plane of the Lorentz force with splay that is orthogonal to it, we can also address the effect of entanglement on pinning. We find that the combination of entanglement and confinement can lead to a suppression of creep, but only in a somewhat limited parameter range. Finally, we emphasize the need to distinguish suppression of vortex creep and suppression of the critical current when determining the efficacy of different pinning configurations.

We conduct overdamped molecular dynamics simulations using a London-Langevin model. We refer the reader to [1] for details. The key feature of the approach is the incorporation of forms for the vortex-vortex in-
teraction, elastic bending, and thermal Langevin forces, which use experimental values for the coherence and penetration lengths, and anisotropy parameter, and hence allows us to simulate the system without a host of adjustable parameters. In [1], we verified that our choices accurately reproduce the experimental phase diagram. The current simulations are performed at \( T = 77 \, \text{K} \), inside the glassy phase as evidenced by the creep-like E-J curves below. The samples are of size \( 106 \times 106 \), containing 49 vortices extending through 80 layers.

The pinning potential representing irradiated defects is modeled by short-range attractive parabolic wells of radius \( r_p = 0.5 \lambda \) which are spatially correlated on neighboring layers to form columnar pins. The wells in different layers belonging to a single column have the same pinning energy \( U_p^i \), with \( U_p^i \), selected from a Gaussian distribution with mean \( U_p^i = 0.08 \) and standard deviation \( \sigma_p = 0.012 \). With this choice the vortex depinning transition falls near \( j/j_0 \approx 0.08 \) (where \( j_0 \) is the BCS depairing current), and the magnitude of the pinning force is on average of the same order as the elastic and Lorentz forces. We consider several different pinning geometries: parallel columnar pins, in which the columns are aligned with the \( z \) axis; transverse bimodal splay, in which each pin is tilted at angles \( \pm \theta \) from the \( z \) axis in the plane transverse to the direction of vortex motion; longitudinal bimodal splay, in which the tilt plane is spanned instead by the direction of vortex motion and the \( z \) axis; Gaussian splay, in which \( \theta \) is selected from a Gaussian distribution centered about zero and the tilt direction is uniformly distributed in the \( x-y \) plane; and point pins, which are not spatially correlated between layers.

We first study single vortex phenomena by performing simulations in which the number of vortices is less than the number of pins, \( N_v < N_p \). In Fig. 1, we compare two samples with an equal density of columnar pins. Here the dimensionless resistivity, \( \rho = (E/j)/\rho_{BS} \) is defined in units of \( \rho_{BS} \), the Bardeen-Stephen resistivity. \( E \) is the electric field and \( j \) is the current density. In the first sample the pins are aligned parallel to the \( z \) axis, and in the second the pins are splayed at \( \theta = \pm 5.7^\circ \) from the \( z \) axis, transverse to the direction of vortex motion. We find a strong suppression of the creep of vortices by splay in the low current regime. While the dynamic range is limited, when a creep type exponential fit, \( E \sim \exp(-1/j^\mu) \), is performed, it is consistent with an increased value of \( \mu \).

The effect of splaying decreases with increasing current. The critical current \( j_c \) is typically defined via a threshold criterion \( \rho(j_c) = \rho^i \). Choosing low thresholds we clearly observe an enhancement of the critical current. For example, using \( \rho^i = 10^{-4} \), \( j_c \) is enhanced by 20%. At higher thresholds this enhancement is reduced, however, finally disappearing for \( \rho^i > 0.05 \).

Now we analyze the physical mechanisms at work in the presence of splay. Single vortex phenomena dominate in the dilute vortex limit, considered here. At low applied currents, \( j/j_0 < 0.1 \), vortices move between pins by thermally activated double kinks [1, 3]. For parallel columnar pins, extending an already formed double kink does not cost extra energy [Fig. 1(a), simulation image]. For splayed pins with a bimodal distribution, double kinks between pairs of pins tilted in opposite directions [Fig. 1(b), simulation image] experience an increasing, or confining energy barrier, since after the nucleation of the kink the unpinned vortex segment must keep growing longer in the high energy region between the pins. On the other hand, pairs of pins tilted in the same direction with respect to the \( z \) axis are twice as far apart on average than in the columnar case, so the nucleation of double kinks bridging parallel pins must span twice as long a distance, and hence costs more energy.

We extract the energy barrier against the spreading of kinks by measuring the distribution function, \( P(l_k) \), of the lengths \( l_k \) of the vortex segments between kinks. If the kink energy does not depend on its length, as is expected for columnar pins, \( P(l_k) \) should be roughly uniform for pins of equal depth, and exhibit slow decay with length for pins with differing depths. If the kink energy grows linearly with length, as expected for splayed pins, \( P(l_k) \) should fall off at large \( l_k \) exponentially.

In Fig. 2 we show \( P(l_k) \) for \( j/j_0 = 0.065 \). The columnar distribution is rather uniform, whereas the splayed one exhibits a significant enhancement at small \( l_k \) relative to the columnar case, followed by a rapid fall in the intermediate region, and a slow decay at the large \( l_k \) regime. The enhancement at small \( l_k \)’s has a simple explanation: because of the splaying, the minimal distance between the pins is much smaller than for the columnar case. Therefore the nucleation of the double kinks which takes place here, and the formation of short segments, is much enhanced. The subsequent fall of \( P(l_k) \) at intermediate \( l_k \) is consistent with an exponential, supporting the picture of a linearly increasing potential barrier. At large values \( l_k \gtrsim 13 \xi \), however, the decay of \( P(l_k) \) is slowed down, and \( P(l_k) \) tracks the columnar distribution. This can be attributed to the interference of additional pins. The high energy segment between two tilted pins increases only until one of the kinks reaches a third pin intersecting the second pin onto which the vortex is hopping. In the present bimodal distribution this third pin is parallel to the first one. Thus, as the kinks slide further, the length of the high energy segment remains unchanged, in complete analogy to the columnar case.

To consider many-vortex phenomena, we move to the high density limit, \( N_v > N_p \). For \( N_v = 2N_p \), half of the vortices are directly pinned, and the rest are pinned only by the repulsion of their pinned neighbors. At low driving currents, only the latter, interstitial vortices move. The flowing interstitial vortices are roughly aligned with the applied magnetic field, whereas the pinned vortices are tilted. Thus the interaction between these two types of vortices results in their entanglement.
Fig. 3 shows the result of our simulations. First, the critical current is much reduced compared to the case $N_v < N_p$, due to the fact that the interstitial vortices de-pin much more easily than their pinned neighbors. Second, when $j_c$ is defined using the same threshold resistivity as before, $\rho' = 10^{-4}$, we observe a factor of 2 enhancement in $j_c$ by splay, compared to the 20% seen in Fig. 2. This strongly suggests that the forced entanglement of vortices is capable of impressive additional enhancements of the critical current.

Here we are in the position to check whether the physics of splayed pins is “adiabatically connected” to the columnar case. It is expected that either a single vortex is still sticking to a single pin, after it has been splayed, or that several vortices will pin to a column in such a way that columns are either (nearly) fully occupied, or (nearly) completely empty, forming vortex-like “quasi lines” [3]. We tested these expectations by determining the distribution function of percentage-wise occupation of the splayed pins. Remarkably, this distribution function shows sharp peaks around zero and 100% occupancy, with very suppressed values in between, verifying the expectations. At small angles this is mostly due to vortices still sticking to a single pin.

We close this section by mentioning that the enhancement of $j_c$ occurred only for a rather limited range of the model’s parameters; for large regions of the parameter space we found either minimal effects, or the reduction of $j_c$ upon splaying as the magnetic field was further increased. This shows the powerful influence of the enhanced kink nucleation and cutoff of the confining potential by third pins.

Experimentally up to tenfold $j_c$ enhancements were reported [4,5], exceeding but comparable to our results. This enhancement is strongly dependent on the magnetic field and temperature, however, often diminishing to small values, or even turning into a reduction instead. This is consistent with our finding of the importance of the magnetic field and temperature, however, often diminishing to small values, or even turning into a reduction instead.

As shown in Fig. 4, we find the highest resistivity for point pins, lower for columnar, and the lowest for bimodal transverse splay. We find that the Gaussian splay produces an enhancement of the creep relative to columnar pins, which is consistent with experiments [6,7]. Among the bimodal splay configurations, transverse splay suppresses creep more than longitudinal splay, again in agreement with experiments [4]. Transverse splay is more effective than longitudinal, because it forces the entanglement of vortices more effectively. Also, longitudinal splay brings the pins closer in the direction of vortex motion, thus helping the nucleation of the double kinks.

Finally we return to the current dependence. As the applied current increases to values approaching the depinning regime, the vortices are pinned less effectively and spend more time between pins. The vortex motion is no longer dominated by kinks, weakening the single vortex arguments. Also, forced entanglement is no longer effective at preventing vortex motion near the depinning regime, since when a vortex begins to move, it is increasingly likely that its forward neighbor is already moving, or that the push from behind is sufficient to start its motion. Thus instead of a forced entanglement, both vortices move together. For all these reasons, the creep suppression by splay decreases with increasing currents, and completely disappears in the depinning regime.

In conclusion, we have used realistic London Langevin simulations to study the motion of vortices in the presence of splayed columnar defects. We found that splaying introduces a confining potential against vortex hopping. It also enhances kink nucleation and introduces third pins, however, cutting off this linear potential. We also established the importance of forced entanglement. Finally we analyzed the angle dependence of the critical current, and compared different splaying geometries. Several of our results compare favorably to experiments.

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FIG. 1. Resistivity $\rho$ as a function of applied current $j/j_0$ for columnar (filled circles) and transverse bimodal splayed (open squares) defects, in samples containing 49 vortices, 225 pins, and 80 layers. The splay angle $\theta = \pm 5.7^\circ$. A clear reduction of the resistivity is seen for splayed pinning at small applied currents. Inset: Simulation images of individual vortices forming double kinks (a): between two columnar pins, (b): between two splayed pins.

FIG. 2. Distribution of the lengths of vortex segments between kinks, $P(l_k)$, for samples with columnar (filled circles) and splayed (open squares) pinning, with the same parameters shown in Fig. 1 at $j/j_0 = 0.065$ (in the vortex creep regime). For the columnar pinning, $P(l_k)$ depends weakly on length, but $P(l_k)$ for splayed pinning falls off more rapidly, indicating that the kink energy grows with length $l_k$. Inset: Critical current $j_c$ as a function of the angle $\pm \theta$ between the $z$ axis and the splayed pins for the system of Fig. 1. The initial increase of $j_c$ with $\theta$ stops above $\theta \sim 10^\circ$ when the pins become so tilted that the vortices no longer accommodate to them.

FIG. 3. Resistivity $\rho$ versus driving current $j/j_0$ for a sample containing 49 vortices and 25 pins. Filled circles: parallel columnar pinning; open squares: transverse bimodal splayed pinning.
FIG. 4. Resistivity $\rho$ versus driving current $j/j_0$ for several different pinning geometries: point pinning (plus signs), parallel columnar pinning (filled circles), transverse bimodal (open squares) and longitudinal bimodal (open right triangles) splay pinning at $\theta = \pm 5^\circ$, and Gaussian splay pinning with $\sigma_\theta = 5^\circ$ (open up triangles). All correlated pinning produces lower resistivities than point pinning. Transverse bimodal splay produces a lower $\rho$ than columnar pinning, but Gaussian splay produces a higher $\rho$. 