A COMPARATIVE ANALYSIS OF THE SUPERNOVA LEGACY SURVEY SAMPLE WITH ΛCDM AND THE $R_h = ct$ UNIVERSE*

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ABSTRACT

The use of Type Ia supernovae (SNe Ia) has thus far produced the most reliable measurement of the expansion history of the universe, suggesting that ΛCDM offers the best explanation for the redshift–luminosity distribution observed in these events. However, analysis of other kinds of sources, such as cosmic chronometers, gamma-ray bursts, and high-z quasars, conflicts with this conclusion, indicating instead that the constant expansion rate implied by the $R_h = ct$ universe is a better fit to the data. The central difficulty with the use of SNe Ia as standard candles is that one must optimize three or four nuisance parameters characterizing supernova (SN) luminosities simultaneously with the parameters of an expansion model. Hence, in comparing competing models, one must reduce the data independently for each. We carry out such a comparison of ΛCDM and the $R_h = ct$ universe using the SN Legacy Survey sample of 252 SN events, and show that each model fits its individually reduced data very well. However, since $R_h = ct$ has only one free parameter (the Hubble constant), it follows from a standard model selection technique that it is to be preferred over ΛCDM, the minimalist version of which has three (the Hubble constant, the scaled matter density, and either the spatial curvature constant or the dark energy equation-of-state parameter). We estimate using the Bayes Information Criterion that in a pairwise comparison, the likelihood of $R_h = ct$ is ∼90%, compared with only ∼10% for a minimalist form of ΛCDM, in which dark energy is simply a cosmological constant. Compared to $R_h = ct$, versions of the standard model with more elaborate parametrizations of dark energy are judged to be even less likely.

Key words: cosmic background radiation – cosmological parameters – cosmology: observations – cosmology: theory – distance scale – supernovae: general

1. INTRODUCTION

Type Ia supernovae (SNe Ia) have a well-defined luminosity, permitting their use as standard candles under the assumption that the luminosities of nearby and distant sources are similarly related to color and the light-curve shape. The use of such events has thus produced the most reliable measurement by far of the expansion history of the universe (Garnavich et al. 1998; Perlmutter et al. 1998, 1999; Riess et al. 1998; Schmidt et al. 1998), leading to the discovery of dark energy.

In recent years, several samples of SN Ia events have been assembled, and several of these have been merged to form even larger compilations, such as the Union 2.1 sample (Kowalski et al. 2008; Suzuki et al. 2012), which currently contains 580 supernova (SN) detections, including events at redshift $z > 1$ observed with the Hubble Space Telescope (see, e.g., Kuznetsova et al. 2008; Dawson et al. 2009; Riess et al. 2011). However, while such merged samples offer several advantages, the fact that each sample has its own set of systematic and intrinsic uncertainties makes it difficult to fit a cosmological model to any merged sample. As we shall discuss, and as Kim (2011) and others have already pointed out, it is questionable whether the model parameters can be estimated by simply minimizing a $\chi^2$, since parameters characterizing error dispersion(s) must be estimated simultaneously. One commonly used method estimates error dispersion(s) by constraining the reduced $\chi^2$, i.e., the $\chi^2$ per degree of freedom, to equal unity. We shall find that the method of maximum likelihood estimation (MLE) is to be preferred; however, in any method, the presence of multiple dispersion parameters is a complication.

Fortunately, some of the homogeneous samples are themselves quite large, and perfectly suited to the type of comparative analysis we wish to carry out in this paper. In fact, almost half of all SNe Ia in the Union 2.1 compilation were derived from the single, homogeneous sample assembled during the first three years of the Supernova Legacy Survey (SNLS; Guy et al. 2010). This catalog contains 252 high-redshift SNe Ia (0.15 < $z$ < 1.1). The multi-color light curves of these SNe were measured using the MegaPrime/MegaCam instrument at the Canada–France–Hawaii Telescope, using repeated imaging of four one square degree fields in four bands. The VLT, Gemini, and Keck telescopes were used to confirm the nature of the SNe and to measure their redshifts. Very importantly, since the same instruments and reduction techniques were employed for all 252 events, it is appropriate to include a single intrinsic dispersion in the analysis of the Hubble Diagram (HD) constructed from this sample. The study of catalogs such as this has led to a general consensus that ΛCDM offers the best explanation for the redshift–luminosity relationship, and observational work is now focused primarily on refining the fits to improve the precision with which the

* This work is dedicated to the memory of Prof. Tan Lu, who sadly passed away 2014 December 3. Among his many achievements, he is considered to be one of the founders of high-energy astrophysics, and a pioneer in modern cosmology, in China.
model parameters are determined. This is one of the principal motivations for attempting to merge samples to create catalogs with broader redshift coverage and better statistics. However, as successful as this program has been, several drawbacks associated with the use of SNe Ia have made it necessary to seek alternative methods of probing the cosmic spacetime. It is quite difficult to use SN measurements in unbiased, comparative studies of competing expansion histories, since at least three or four “nuisance” parameters characterizing the standard candle must be optimized simultaneously with each model’s free parameters, rendering the data compliant to the underlying theory (Kowalski et al. 2008; Suzuki et al. 2012; Melia 2012a). Several notable attempts have been made to mitigate the impact of this model dependence, e.g., through the use of kinematic variables and geometric probes that avoid parameterizing the fits in terms of pre-assumed model components (Shaferoo et al. 2012). In the end, however, even these models require the availability of measurements based on standard candles. Unfortunately, Type Ia SNe may be used for this purpose only as long as the nuisance parameters characterizing their light curves are known. The application of such methods to the Union 2.1 Type Ia SN sample uses nuisance parameters optimized for the concordance model, so the current results are not completely free of any biases.

The expansion of the universe is now being studied using several other methods, including the use of cosmic chronometers (Jimenez & Loeb 2002; Simon et al. 2005; Stern et al. 2010; Morescu et al. 2012; Melia & Maier 2013), gamma-ray bursts (GRBs; Norris et al. 2000; Amati et al. 2002; Schaefer 2003; Wei & Gao 2003; Yon et al. 2004; Ghirlanda et al. 2004; Liang & Zhang 2005; Liang et al. 2008; Wang et al. 2011; Wei et al. 2013), and high-z quasars (Kaufmann & Haehnelt 2000; Wyithe & Loeb 2003; Willott et al. 2003; Hopkins et al. 2005; Croton et al. 2006; Fan 2006; Jiang et al. 2007; Kurk et al. 2007; Tanka & Haiman 2009; Lippai et al. 2009; Melia 2013a; Hirschmann et al. 2010). In contrast to the perception based on SNe Ia that ΛCDM can best account for the observed expansion of the universe, the conclusion from these other studies is that the cosmic dynamics is better described by a cosmology we refer to as the R₀ = ct universe (Melia 2007; Melia & Abdelevader 2009; Melia & Shevchuk 2012).

For example, a comparative analysis was recently carried out with the ΛCDM and R₀ = ct cosmologies using the GRB HD (Wei et al. 2013). This study found that once the various parameters are estimated for each model individually, the R₀ = ct cosmology provides a better fit to the data. Although about 20% of the GRB events lie at least 2σ away from the best-fit curves (in both models), suggesting that some contamination by non-standard GRB luminosities is unavoidable or that the errors and intrinsic scatter associated with the data are being underestimated, various model selection techniques applied to the GRB data show that the likelihood of R₀ = ct being the correct model rather than ΛCDM is ~90% versus ~10%.

Moreover, although the redshift–distance relationship is essentially the same in ΛCDM and the R₀ = ct universe (even out to z ≥ 6–7), the redshift–age relationship is not. This motivated a recent examination of whether or not the observed growth rate of high-z quasars could be used to test the various models. Quasars at z ≥ 6 are now known to be accreting at or near their Eddington limit (see, e.g., Willott et al. 2010a, 2010b; De Rosa et al. 2011), which presents a problem for ΛCDM because this makes it difficult to understand how ~10^9 M☉ supermassive black holes could have appeared only 700–900 Myr after the big bang. Instead, in R₀ = ct, their emergence at redshift ~6 corresponds to a cosmic age of ≥ 1.6 Gyr, which was enough time for them to begin growing from ~5–20 M☉ seeds (presumably the remnants of Pop II and III SNe) at z ≤ 15 (i.e., after the onset of reionization) and still reach a billion solar masses by z ~ 6 via standard, Eddington-limited accretion (Melia 2013a).

In light of this apparent conflict between the implications of the SN Ia work and the results of other studies, we have begun to look more closely at the possibility of directly comparing how ΛCDM and R₀ = ct account for the SN measurements themselves. This is not an easy task, principally because of the enormous amount of work that goes into first establishing the SN magnitudes, and then carefully fitting the data using the comprehensive set of parameters available to ΛCDM (see, e.g., Suzuki et al. 2012). In a previous paper (Melia 2012a), the redshift–distance relationship in ΛCDM was compared with that predicted by R₀ = ct, and each with the Union 2.1 sample. It was shown that the two theories produce virtually indistinguishable profiles, though the fit with R₀ = ct had not yet been fully optimized. That is, despite the fact that this previous analysis simply used R₀ = ct to fit the data optimized with ΛCDM, the results were quite promising, suggesting that a full optimization procedure—followed separately for R₀ = ct and ΛCDM—ought to be carried out. The principal goal of this paper is to complete this study.

A direct comparison of ΛCDM with R₀ = ct using SNe Ia is also motivated by recent theoretical work suggesting that the Friedmann–Robertson–Walker (FRW) metric is more specialized than was previously thought (Melia 2013b). A close examination of the physics behind the symmetries incorporated into this well-known and often employed solution to Einstein’s equations showed that the FRW metric applies only to a fluid with zero active mass, i.e., a fluid in which p + 3ρ = 0, where p is the total pressure and ρ the total energy density. This is consistent with the equation of state used in ΛCDM, except that when one averages the pressure, ⟨p⟩, over the age of the universe, one does get ≈ – ⟨p⟩/3 for the estimated ΛCDM parameters. ΛCDM therefore appears to be an empirical approximation to R₀ = ct, asymptotically approaching the requirements of the zero active mass condition consistent with the FRW metric.

In Section 2 of this paper, we briefly summarize the contents of the SNLS sample and explain the model parameters (including those associated with the data) that are to be estimated. We present the fits to the SN data in Section 3, and a direct comparison between ΛCDM and the R₀ = ct universe is made in Section 4. We end with a discussion and conclusions in Section 5.

2. THE SNLS SN SAMPLE

The SNLS sample contains 252 high-redshift (0.15 < z < 1.1) SNe Ia discovered during the first three years of operation (Guy et al. 2010). One of the most important features of this catalog is that it constitutes a single, homogeneous sample. It also covers the very important redshift range where the standard model suggests the universe underwent a transition from decelerated to accelerated expansion.
Guy et al. (2010) used two light-curve fitters (SALT2 and SiFTO) to determine the peak magnitudes, light-curve shapes, and colors of the SNe Ia. For SiFTO, a distance modulus is defined for each SN as the linear combination

$$\mu_B = m_B + \alpha \cdot (s - 1) - \beta \cdot C - M_B,$$  

(1)

where $m_B$ is the peak rest-frame $B$-band magnitude, $s$ is the stretch (a measure of light-curve shape), $C$ is the color (peak rest-frame $B - V$), and $M_B$ is the absolute magnitude of an SN Ia. When corrected for shape and color, SN Ia luminosities have a dispersion of only ~15%. The coefficients $\alpha$, $\beta$ are thus the parameters of a luminosity model, though it is not a full statistical model since it lacks an explicit error dispersion parameter. In the present context, $\alpha$, $\beta$ and $M_B$ are “nuisance” parameters, as they cannot be estimated independently of an assumed cosmology. They must be optimized simultaneously with the cosmological parameters, as will be explained in Section 3.

The theoretical distance modulus $\mu_{th}$ is calculated for each SN from its measured redshift $z$ by

$$\mu_{th}(z) \equiv 5 \log \left( D_L(z)/10 \right),$$

(2)

where $D_L(z)$ is the model-dependent luminosity distance. A determination of $D_L$ requires the assumption of a particular expansion scenario. Both $\Lambda$CDM and $\Omega_0 = \alpha$ are FRW cosmologies, but the former assumes constant constituents in the density, written as $\rho = \rho_r + \rho_m + \rho_\Lambda$, where $\rho_r$, $\rho_m$, $\rho_\Lambda$ are, respectively, the energy densities of radiation, matter (luminous and dark), and the cosmological constant $\Lambda$. These are often expressed in terms of today’s critical density, $\rho_c \equiv 3c^2H_0^2/8\pi G$, where $H_0$ is the Hubble constant, by $\Omega_m \equiv \rho_m/\rho_c$, $\Omega_r \equiv \rho_r/\rho_c$, and $\Omega_\Lambda \equiv \rho_\Lambda/\rho_c$. In a flat universe with zero spatial curvature, the total scaled energy density is $\Omega \equiv \Omega_m + \Omega_r + \Omega_\Lambda = 1$. Since $\Omega_c \ll 1$, $\Omega_m + \Omega_\Lambda = 1$. In $\Omega_0 = c\theta$ on the other hand, whatever constituents are present in $\rho$, the principal constraint is the total equation of state $p = -\rho/3$, which as we mentioned in Section 1 is in fact required by the use of the FRW metric.

When dark energy is included with an unknown equation of state, $p = \omega \rho$, the general form of the luminosity distance in $\Lambda$CDM is given by

$$D_L^{\Lambda\text{CDM}}(z) = \frac{c}{H_0} \left( 1 + z \right) \sinh \left( \Omega_\Lambda^{1/2} \int_0^z \frac{dz'}{\Omega_m(1+z')^3 + \Omega_r(1+z')^2 + \Omega_\Lambda(1+z')^{3(1+\omega)}} \right),$$

(3)

where $c$ is the speed of light, and we have assumed that the radiation density is negligible in the local universe. $\Omega_k = 1 - \Omega_m - \Omega_\Lambda$ represents the spatial curvature of the universe—appearing as a term proportional to the spatial curvature constant $k$ in the Friedmann equation. In addition, $\sinh z$ is sinh when $\Omega_m > 0$ and sinh when $\Omega_m < 0$. For a flat universe ($\Omega_k = 0$), the right side becomes $(1+z)H_0t$ times the indefinite integral.

In the $\Omega_0 = \alpha$ universe, the luminosity distance is given by the much simpler expression

$$D_L^{\Omega_0 = \alpha}(z) = \frac{c}{H_0} \left( 1 + z \right) \ln \left( 1 + z \right).$$

(4)

The factor $cH_0$ is in fact the gravitational horizon $R_h(t_0)$ (which itself is coincident with the Hubble radius) at the present time, so we may also write the luminosity distance as

$$D_L^{\Omega_0 = \alpha}(z) = R_h(t_0)(1+z) \ln \left( 1 + z \right).$$

(5)

A more extensive description of the observational differences between $\Lambda$CDM and $\Omega_0 = \alpha$ is provided in Melia (2007, 2012a), Melia & Shevchuk (2012), Melia & Maier (2013), and Wei et al. (2013). For a pedagogical treatment, see also Melia (2012b).

3. THEORETICAL FITS

In this paper, we shall use two methods for calculating point and interval estimates of cosmological model parameters: one method commonly used in past analyses (e.g., Kowalski et al. 2008; Guy et al. 2010; Suzuki et al. 2012), and one more recently proposed, which is based on maximizing the likelihood function (e.g., Kim 2011; Melia & Maier 2013; Wei et al. 2013). As we shall see, the latter allows us to estimate all parameters, including the unknown intrinsic dispersion. However, we shall find that under some circumstances, the manner in which this latter method works offers some justification for the former, which instead minimizes $\chi^2$ subject to the condition that the reduced $\chi^2$ equal unity. In either method, however, nuisance parameters characterizing the SN luminosities and free parameters of the cosmological model must be fitted simultaneously.

Initial Remark on Free Parameters. Before we begin the actual fitting of the various models, we take a moment to summarize the free parameters available in each. As alluded to earlier, it is now understood that the symmetries incorporated into the FRW metric require a zero active mass condition (i.e., $\rho + 3\rho = 0$) in the cosmic fluid (Melia 2013b). One can understand this even without a formal proof using the following reasoning.

In spherically symmetric spacetimes, which include the special FRW case, a proper mass emerges from the introduction of the metric into Einstein’s field equations. We call this the Misner–Sharp mass (Misner & Sharp 1964), which is defined as $M(R_h) = (4\pi/3)R_h^3\rho/\varepsilon^2$, in terms of the gravitational radius $R_h = 2GM(R_h)/c^2$. This proper mass is a consequence of Birkhoff’s theorem (Birkhoff 1923), which states that none of the mass and energy beyond $R_h$ contributes to the spacetime curvature within a shell at this radius. The gravitational radius must therefore be defined as written here, although the cosmos itself may be infinite. But since $M$ is a proper mass defined in terms of the proper density $\rho$ and proper volume, $R_h$ must therefore be a proper radius as well. Thus, by Weyl’s postulate (Weyl 1923), this gravitational radius must have the form $R_h = \alpha r_h$, where $\alpha(t)$ is the universal expansion factor and $r_h$ is a constant comoving distance. One can therefore easily show from the first Friedmann equation (see, e.g., Melia & Shevchuk 2012) that $a^2 + k_0^2 = r_h^2$, where $k$ is the spatial curvature constant. In other words, the expansion rate $\dot{a}$ must be constant, which then leads, via the second Friedmann (or “acceleration”) equation, to the condition
First, \( \sigma_{\text{int}} \) is initialized to zero. Second, the \( \chi^2 \) function is minimized over the cosmological and nuisance parameters, and their values are updated. Third, with these updated values fixed, the value of \( \sigma_{\text{int}} \) for which the reduced \( \chi^2 \), i.e., \( \chi^2/\text{ dof} \equiv \chi^2/(n-k) \), equals unity is determined, and \( \sigma_{\text{int}} \) is updated. Here, \( n = 234 \), see below) is the number of SNe and \( k \) is the total number of parameters, including \( \sigma_{\text{int}} \). Steps 2 and 3 are repeated until the parameters converge within tolerance to stable values.

To find the best-fit coefficients \( \alpha, \beta \) and \( M_B \) and the cosmological parameters that define the fitted model, we use Markov chain Monte Carlo (MCMC) techniques in our calculations. Our MCMC approach generates a chain of sample points distributed in the parameter space according to the posterior probability, using the Metropolis-Hastings algorithm with uniform prior probability distributions, such that \( 0.5 < \alpha < 2.0, 2.0 < \beta < 4.0, -19.5 < M_B < -18.5, 0.0 < \sigma_{\text{int}} < 1.0, 0.0 < \Omega_m < 1.0, \) and/or \( 0.0 < \Omega_\Lambda < 1.8 \) (in the non-flat case). In the parameter space formed by the constrained nuisance parameters and cosmological parameters, a random set of initial values of the model parameters is chosen to calculate the \( \chi^2 \) (Method I), or the likelihood function (Method II). Whether the set of parameters can be accepted as an effective Markov chain or not is determined by the Metropolis-Hastings algorithm. The accepted set not only forms a Markov chain, but also provides a starting point for the next process. We then repeat this process until the established convergence accuracy can be satisfied. For each Markov chain, we generate \( 10^5 \) samples consistent with the likelihood function. Then we derive the probability distributions of the coefficients \( \alpha, \beta \) and \( M_B \) and the cosmological parameters from a statistical analysis of the samples.

To be consistent with previous work (Guy et al. 2010), we discard from the SNLS catalog all SNe with a peak rest-frame \( (B-V) > 0.2 \). Such red SNe are found only at \( z < 0.6 \) in the SNLS because they are fainter than the average, and hence are undetected (or unidentified spectroscopically) at higher redshifts. Discarding them minimizes any potential biasing of the distance modulus by an inadequate color correction. Indeed, the correction coefficient \( (\beta) \) we estimate from the bulk of the SNe may not apply to those red SNe that are more likely to be extinguished by dust in their host galaxy than bluer ones. This cut, applied to both SALT2 and SiFTO samples, discards 11 SNe. There are also three SNe whose peak magnitudes could not be obtained with SiFTO due to a lack of observations in the \( g_M \) and \( z_M \) bands. Finally, again following Guy et al. (2010), we remove SNe that are \( 3\sigma \) outliers for either of the best-fit \( \Lambda \)CDM and \( R_h = ct \) models. This removes four more SNe from both the \( \Lambda \)CDM and \( R_h = ct \) samples. Three \( (\alpha 4 D 3 dd, 04 D 1 a k, \) and \( 04 D 4 g z) \) are in common; \( 05 D 4 c x \) is removed for \( \Lambda \)CDM, while \( 05 D 2 e i \) is removed for \( R_h = ct \). In total, that leaves \( n = 234 \) SNLS SNe Ia for the analysis.

Although the number of free parameters in the dark-energy model can be as large as eight, depending on how one handles the dark energy, in this paper we take a minimalist approach and use only three of the most essential ones: these are taken from among a set that includes the Hubble constant \( H_0 \), the matter energy density \( \Omega_m \), the dark energy density \( \Omega_\Lambda \), and the dark energy equation-of-state parameter \( w_\Lambda \). In fitting, the chosen value of \( H_0 \) is not independent of \( M_B \). That is, one can
vary either $H_0$ or $M_B$, but not both. Therefore, if we take $M_B$ to be a member of the set of nuisance parameters characterizing the SN luminosities, the dark-energy model will have, at most, only two parameters: $\Omega_m$ and $\Omega_\Lambda$. For consistency with Guy et al. (2010), we use $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$ (recalling that this value does not actually affect any of the fits).

To ensure that our implementation of Method I is reliable and consistent with the results of Guy et al. (2010), we first examine a simple test case: the flat universe, in which $\Omega_\Lambda = 1 - \Omega_m$, with equation-of-state parameter $w_\Lambda = -1$. $\Omega_m$ is then the only free cosmological parameter. Guy et al. (2010) found that the assumption of flatness resulted in a value for $\Omega_m$ that we now know is inconsistent with that reported by Planck 2013 (Planck Collaboration et al. 2014). Nonetheless, it is useful to begin with this test case because it allows us to compare our results directly with theirs. Applying the above $\chi^2$-minimization procedure, we find that the fitted parameters in this case are $\alpha = 1.470 \pm 0.121$, $\beta = 3.207 \pm 0.157$, $M_B = -19.197 \pm 0.031$, and $\Omega_m = 0.235 \pm 0.035$, with $\sigma_{\text{int}} = 0.090$. In Figure 1 we show the (normalized) likelihood distribution for each parameter ({$\alpha$, $\beta$, $M_B$; $\Omega_m$}), according to the factor $\exp(-\chi^2/2)$, and 1$\sigma$, 2$\sigma$ contours for the joint distribution of each pair of parameters. For each parameter, the likelihood distribution is well approximated by a Gaussian, and the stated confidence interval is a 68% (i.e., $\pm 1\sigma$) interval for this Gaussian. Using the same method and conditions, Guy et al. (2010) obtained $\alpha = 1.487 \pm 0.097$, $\beta = 3.212 \pm 0.128$, $M_B = -19.210 \pm 0.030$, and $\Omega_m = 0.215 \pm 0.033$, with $\sigma_{\text{int}} = 0.087$. These results are quite consistent, again attesting to the reliability of our calculation.

To allow for a more realistic fit to the SNLS SN data, we removed the flatness assumption, and allowed both $\Omega_m$ and $\Omega_\Lambda$ to be free parameters. By minimizing $\chi^2$, we now obtain $\alpha = 1.469 \pm 0.123$, $\beta = 3.209 \pm 0.159$, $M_B = -19.187 \pm 0.068$, and $\Omega_m = 0.217 \pm 0.150$, $\Omega_\Lambda = 0.718 \pm 0.329$, with $\sigma_{\text{int}} = 0.090$. The (normalized) likelihood distribution for each parameter ({$\alpha$, $\beta$, $M_B$; $\Omega_m$, $\Omega_\Lambda$}), and a contour plot for each two-parameter combination, are shown in Figure 2. Dropping the flatness condition, i.e., fitting both $\Omega_m$ and $\Omega_\Lambda$, makes the best-fit $\Lambda$CDM model marginally consistent with the Planck 2013 results, because the 1$\sigma$ standard errors are now considerably larger than in the previous case.

It should be noted that the $\chi^2$ approach of Method I does not actually maximize a likelihood. Rather, it calculates a value for $\sigma_{\text{int}}$ (with no accompanying uncertainty) by requiring that $\chi^2_{\text{dof}}$ equal unity. Driving $\chi^2_{\text{dof}}$ to unity, irrespective of how well the cosmological model fits the data, makes it impossible to perform a fair comparison of competing models using Method I, especially when they have different numbers of parameters. A statistically valid analysis must estimate error dispersion parameter(s) along with all other parameters by maximizing a joint likelihood function.

**Method II.** As a superior alternative to Method I for estimating cosmological parameters and (simultaneously) the model-specific optimized nuisance parameters $\alpha$, $\beta$ and $M_B$, we employ a method described in D’Agostini (2005) and Kim (2011), which is based on the maximization of a joint likelihood function. The joint likelihood function for all these parameters and the intrinsic dispersion $\sigma_{\text{int}}$, based on a flat
Bayesian prior, is

\[
L = \prod_i \frac{1}{\sqrt{2\pi (\sigma^2_{\text{int}} + \sigma^2_{\text{int}})}} \times \exp \left( -\frac{(\mu_{\text{B},i} - \mu_{\text{th}}(z_i))^2}{2(\sigma^2_{\text{int}} + \sigma^2_{\text{int}})} \right) \times \exp \left( -\frac{\chi^2}{2} \right). \tag{8}
\]

As in Method I, each distance modulus \(\mu_{\text{B},i}\) depends on \(\alpha, \beta\) and \(M_B\), and the theoretical distance modulus \(\mu_{\text{th}}(z_i)\) depends on the cosmological parameters. MLE is, of course, a standard statistical procedure, and appeared in Method I in a limited way, as a minimization of \(\chi^2\). The new feature in Method II is that \(\sigma_{\text{int}}\) is treated on the same level as the other parameters: the likelihood \(L\) is maximized over all parameters, now including \(\sigma_{\text{int}}\). This “full MLE” provides a statistically founded method for estimating \(\sigma_{\text{int}}\) (Kim 2011). It also treats on the same level the cosmological parameter uncertainties and the potential uncertainty in \(\sigma_{\text{int}}\), which can affect each other.

The optimized \(\Lambda\)CDM parameters obtained by Method II are \(\alpha = 1.275 \pm 0.120, \beta = 2.637 \pm 0.155, M_B = -19.165 \pm 0.081, \Omega_m = 0.365 \pm 0.137, \text{and } \Omega_\Lambda = 0.846 \pm 0.353,\) with \(\sigma_{\text{int}} = 0.103 \pm 0.010\). For each parameter, the likelihood distribution is well approximated by a Gaussian, and the stated confidence interval is a 68% (i.e., \(\pm 1\sigma\)) interval for this Gaussian. As anticipated by Kim (2011), these best-fit values are not all consistent with those of Method I, though the estimated value of \(\Omega_m\) and its error are entirely consistent with the Planck 2013 results. In an analysis using mock samples, Kim concluded that differences such as these arise because the commonly used method based on the constrained minimization of \(\chi^2\) (i.e., Method I) does not include in its error propagation the covariance of \(\sigma_{\text{int}}\) with the other parameters.

The Method II values of the coefficients \(\alpha, \beta\) and \(M_B\) may now be used to calculate the distance modulus \(\mu_{\text{B}}\) of Equation (1) for each SN. From the distance moduli, we construct the HD shown in the upper panel of Figure 3. We also show the corresponding SNLS sample of 234 SNe Ia. As is now well known, the theoretical fit is excellent. The maximum value of the joint likelihood function is given by \(-2 \ln L = -238.40\), which we shall need when comparing models using the Bayes Information Criterion (BIC; see below). For completeness, Figure 3 also shows the HD residuals corresponding to the best-fit \(\Lambda\)CDM model. The likelihood distribution obtained by Method II for each parameter \((\alpha, \beta, M_B; \Omega_m, \Omega_\Lambda; \sigma_{\text{int}})\), and a contour plot of the joint distribution for each two-parameter combination, are shown in the lower panel.
Figure 3. Upper panel: Hubble diagram and Hubble diagram residuals for the SNLS sample of 234 SNe Ia in $\Lambda$CDM. Solid curve: best-fit model with $\Omega_m = 0.365$, $\Omega_{\Lambda} = 0.846$, and $p_{\Lambda} = -\rho_{\Lambda}$. Lower panel: (normalized) likelihood distributions and 2D joint distributions with 1$\sigma$, 2$\sigma$ contours, for the coefficients $\alpha$, $\beta$ and $M_B$, the intrinsic dispersion $\sigma_{\text{int}}$, and $\Omega_m$, $\Omega_{\Lambda}$. The fitting method employed is MLE (Method II).
4. A DIRECT COMPARISON BETWEEN $$\Lambda$$cdm AND THE $$R_0 = ct$$ UNIVERSE

In the $$R_0 = ct$$ universe, there is only one free parameter: the Hubble constant $$H_0$$. However, since we cannot vary $$M_0$$ and $$H_0$$ separately, there are no free parameters left to adjust the theoretical curve once we optimize $$M_0$$ as one of the nuisance parameters characterizing the SN luminosities.

Using the MLE approach (Method II), we find that for the $$R_h = ct$$ universe, the optimized nuisance parameters are $$\alpha = 1.175 \pm 0.115$$, $$\beta = 2.608 \pm 0.149$$, and $$M_0 = -18.959 \pm 0.011$$; with $$\sigma_{\text{int}} = 0.106 \pm 0.010$$. All likelihood distributions are well approximated by Gaussians, and the given confidence intervals are 68% (i.e., $$\pm 1\sigma$$) intervals for the Gaussians. As with $$\Lambda$$CDM, we plot the (normalized) likelihood distribution for each parameter ($$\alpha$$, $$\beta$$, $$M_0$$; $$\sigma_{\text{int}}$$), and 2D plots for two-parameter combinations (Figure 4). The best-fit values are quite similar to those for $$\Lambda$$CDM, but are not exactly the same, reaffirming the importance of reducing the data separately for each model being tested. The $$R_h = ct$$ distance modulus is compared to the SNLS sample in the upper panel of Figure 4. For completeness, we also show the HD residuals corresponding to the $$R_h = ct$$ universe at the bottom of this panel. The maximum value of the joint likelihood function for the optimized $$R_h = ct$$ fit corresponds to $$-2\ln L = -231.85$$. All the fits performed in this paper are summarized in Table 1 for ease of comparison.

An inspection of the Hubble diagrams in Figures 3 and 4 reveals that the distance moduli are slightly different when the nuisance parameters are optimized using different models, but both $$\Lambda$$CDM and $$R_h = ct$$ fit their respective data sets very well. One certainly gets this impression from a side-by-side comparison of the HD residuals for the SNLS sample in $$\Lambda$$CDM and $$R_h = ct$$, shown in Figure 5. However, because these models formulate their observables (such as the luminosity distance in Equations (3) and (5)) differently, and because they do not have the same number of free parameters, a decision between the models must be based on a formal model selection technique, and in this regard, the results of our analysis favor $$R_h = ct$$ over $$\Lambda$$CDM, as we shall now demonstrate quantitatively.

A companion paper (Melia & Maier 2013) discussed at length how one may use state-of-the-art model selection tools to choose the model that should be preferred in accounting for the data. We shall not reproduce that discussion here, but we do point out that to assess competing models in cosmology, a strong case for using the BIC has been made (see, e.g., Liddle 2004, 2007; Liddle et al. 2006). The BIC is applicable when data points are independent and identically distributed, which is a reasonable assumption for SN redshift–luminosity data. The method has now been used to compare several popular models against $$\Lambda$$CDM (see, e.g., Shi et al. 2012).

Despite its name, the BIC is based not on information theory, but rather on an asymptotic ($$n \to \infty$$, where $$n$$ is the number of data points) approximation to the outcome of a Bayesian inference procedure for deciding between competing models (Schwarz 1978). A comparison between the BIC values of two or more fitted models provides their relative ranks, and also a numerical measure of confidence (a likelihood or posterior probability) that each model is the best. Unlike some model selection techniques, the BIC can be applied to “non-nested” models, such as those we have here. The BIC for a fitted regression model, linear or nonlinear, captures the dominant (in the $$n \to \infty$$ limit) behavior of the associated Bayes factor, which assesses the strength of the evidence in its favor (Kass & Raftery 1995, Section 4.1). In this limit, it is only subdominant terms that are affected by such considerations as the choice of a Bayesian prior (Kuha 2004), or the extent of the model nonlinearity if any (Haughton 1988). There is accordingly a simple formula for the BIC, namely

$$\text{BIC} = -2\ln L + (\ln n)k,$$

where $$k$$ is the number of fitted parameters. The logarithmic penalty term in the BIC strongly suppresses overfitting if $$n$$ is large (the situation we have here, with $$n = 234$$, which is deep in the asymptotic regime).

A quantitative ranking of fitted models 1 and 2 is computed as follows. If BIC$$\alpha$$ comes from model $$\alpha$$, the unnormalized confidence in this model is the “Bayes weight” exp ($$-\text{BIC}\alpha$$/2).

That is, in light of the data, model $$\alpha$$ has a likelihood

$$P(\alpha) = \frac{\exp(-\text{BIC}\alpha/2)}{\exp(-\text{BIC}1/2) + \exp(-\text{BIC}2/2)}$$

of being the correct choice. The strength of the evidence for model 1 and against model 2 is quantified by $$\Delta$$BIC = BIC2 − BIC1, and the following qualitative interpretation of $$\Delta$$BIC is standard. If $$\Delta$$BIC is less than 2, the evidence is “not worth more than a bare mention” (Kass & Raftery 1995). If it is in the range 2...6, the evidence is positive; if it is in the range 6...10, the evidence is strong; and if it is greater than 10, the evidence is very strong.

With $$n = 234$$ data points and $$k = 4$$ parameters, the BIC for the optimized $$R_h = ct$$ universe is BIC1 = −210.03. For the optimized $$\Lambda$$CDM, with $$k = 6$$, the corresponding value is BIC2 = −205.67. Our analysis therefore shows that the evidence supplied by the SNLS sample for the $$R_h = ct$$ universe over $$\Lambda$$CDM is positive, and quantitatively, $$R_h = ct$$ is favored over $$\Lambda$$CDM with a likelihood of ~90% versus only ~10%.

5. DISCUSSION AND CONCLUSIONS

Our comparative analysis of $$\Lambda$$CDM and the $$R_h = ct$$ universe using the SNLS sample has shown that—contrary to earlier claims—the SNe Ia do not point to an expansion history of the universe in conflict with that implied by other kinds of sources, such as the cosmic chronometers (Jimenez & Loeb 2002; Simon et al. 2005; Stern et al. 2010; Morecos et al. 2012; Melia & Maier 2013), GRBs (Norris et al. 2000; Amati et al. 2002; Schaefer 2003; Wei & Gao 2003; Ghirlanda et al. 2004; Yonetoku et al. 2004; Liang & Zhang 2005; Liang et al. 2008; Wang et al. 2011; Wei et al. 2012), and high-$z$ quasars (Kaufmann & Haehnelt 2000; Willott et al. 2003; Wyithe & Loeb 2003; Hopkins et al. 2005; Croton et al. 2006; Fan 2006; Jiang et al. 2007; Kurk et al. 2007; Tanaka & Haiman 2009; Lippai et al. 2009; Hirschmann et al. 2010; Melia 2013a). A difficulty in using SN data has always been their dependence on the underlying cosmology. There is no question that to compare different models properly, one must optimize the nuisance parameters describing the SN luminosities separately for each expansion scenario. It is not appropriate to use data optimized for $$\Lambda$$CDM to test other models.

This has been the primary focus of our analysis in this paper. We have confirmed the argument made by Kim (2011), in particular, that one should optimize parameters by carrying out
an MLE in any situation where the parameters include an unknown intrinsic dispersion. The commonly used method, which estimates the dispersion by requiring the reduced $\chi^2$ to equal unity, does not take into account all possible covariances among the parameters. In this regard, our best-fit models for $\Lambda$CDM do not agree exactly with those of Guy et al. (2010), who optimized the model parameters using only Method I. Indeed, while their best-fit model is characterized by parameters noticeably different from those of Planck 2013, we have demonstrated that the use of Method II actually results in an optimized $\Lambda$CDM model consistent with that based on the analysis of the CMB fluctuations. Simply based on a

Figure 4. Same as Figure 3, but now for the $R_h = ct$ universe. Upper panel: Hubble diagram and Hubble diagram residuals for the data optimized for $R_h = ct$, and the corresponding theoretical curve. Lower panel: the (normalized) likelihood distributions and 2D joint distributions with 1$\sigma$, 2$\sigma$ contours, for all fitted parameters. The fitting method employed is MLE (Method II).
consideration of the standard model, therefore, our results support the proposal made by Kim (2011) that Method II should be preferred over Method I in the analysis of SN Ia samples that include unknown intrinsic dispersions.

More importantly, we have found that, when the parameter optimization is handled via the joint likelihood function, both $\Lambda$CDM and $R_h = ct$ fit their individually optimized data very well. However, the $R_h = ct$ universe has only one free parameter—the Hubble constant $H_0$—which enhances the significance of the fit over that using $\Lambda$CDM. Indeed, standard model selection techniques penalize models with a large number of free parameters. We have found that the BIC favors $R_h = ct$ over $\Lambda$CDM by a likelihood of $\sim 90\%$ versus $\sim 10\%$. The difference in likelihoods would be even greater for other variations of $\Lambda$CDM that include a larger number of free parameters, e.g., to characterize the dark-energy equation of state should $w_\Lambda$ not be constant.

This result would be quite significant on its own. However, when we consider it in concert with the analysis of all the other data sets that have been analyzed thus far, it is reasonable to conclude that $R_h = ct$ is a more accurate representation of the universe than is $\Lambda$CDM. This has far-reaching consequences that will be addressed at greater length elsewhere.

Obviously, these results call into question the conclusion that the universe is currently undergoing a period of acceleration, following an earlier period of deceleration. The fact that the fits to the data using $\Lambda$CDM often come very close to those of $R_h = ct$ (see Figure 5) lends weight to our suspicion that the standard model functions as an empirical approximation to the latter, since it has more free parameters and lacks that essential ingredient in $R_h = ct$: the equation-of-state $p = -\rho/3$. In attempting to identify the reasons why $\Lambda$CDM produces phases of acceleration and deceleration, one is reminded of trying to fit a straight line with a low-order polynomial—it is always possible to make the ends meet, but there will be inevitable wiggles in between. It appears that the early deceleration and current acceleration indicated by $\Lambda$CDM are two of these wiggles, whereas $R_h = ct$ fits the straight line perfectly.

In this work we have demonstrated that the class of SNe Ia continues to be critical to our understanding of how the universe evolves; the results reported here affirm the expectation from theory and general relativity that only a perfect fluid with zero active mass (i.e., $\rho + 3p = 0$) can be consistent with the use of an FRW metric (Melia 2013b).

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REFERENCES

Amanullah, R., Lidman, C., Rubin, D., et al. 2010, ApJ, 716, 712
Amati, L., Frontera, F., Tavani, M., et al. 2002, A&A, 390, 81
Bennett, C. L., et al. 2003, ApJS, 148, 97
Birkhoff, G. D. 1923, Relativity and Modern Physics (Cambridge, MA: Harvard Univ. Press)
Croton, D. J., Springel, V., White, S. D. M., et al. 2006, MNRAS, 365, 11
D’Agostini, G. 2005, arXiv:physics/0511112
Dawson, K. S., Aldering, G., Amanullah, R., et al. 2009, AJ, 138, 1271
De Rosa, G., Decarli, R., Walter, F., et al. 2011, ApJ, 739, 56
Fan, X. 2006, NewAR, 50, 665
Garnavich, P. M., Jha, S., Challis, P., et al. 1998, ApJ, 509, 74
Ghirlanda, G., Ghisellini, G., & Lazzati, D. 2004, ApJ, 616, 331
Guy, J., Sullivan, M., Conley, A., et al. 2010, A&A, 523, A7
Haughton, D. M. A. 1988, AnSta, 16, 342
Hirschmann, M., Khockfar, S., Burkert, A., et al. 2010, MNRAS, 407, 1016
Holz, D. E., & Linder, E. V. 2005, ApJ, 631, 678
Hopkins, P. F., Hernquist, L., Cox, T. J., et al. 2005, ApJ, 630, 705

Table 1
Best Fits for Different Cosmological Models

| Model       | $\alpha$ | $\beta$ | $M_8$ | $\sigma_{ct}$ | $\Omega_m$ | $\Omega_A$ | $-2\ln L$ | BIC       |
|-------------|----------|---------|-------|---------------|-------------|------------|-----------|-----------|
| $\Lambda$CDM ($k = 0$) | $1.470 \pm 0.121$ | $3.207 \pm 0.157$ | $-19.197 \pm 0.031$ | $0.090$ | $0.235 \pm 0.035$ | $1 - \Omega_m$ | $\cdots$ | $\cdots$ |
| $\Lambda$CDM | $1.469 \pm 0.123$ | $3.209 \pm 0.159$ | $-19.187 \pm 0.068$ | $0.090$ | $0.217 \pm 0.150$ | $0.718 \pm 0.329$ | $\cdots$ | $\cdots$ |
| Method II   | $R_h = ct$ | $1.275 \pm 0.120$ | $2.637 \pm 0.155$ | $-19.165 \pm 0.081$ | $0.103 \pm 0.010$ | $0.365 \pm 0.137$ | $0.846 \pm 0.353$ | $-238.40$ | $-205.67$ |

Figure 5. Side-by-side comparison of the Hubble diagram residuals for the SNLS sample in the standard ($\Lambda$CDM) model and the $R_h = ct$ universe.
Jiang, L., Fan, X., Vestergaard, M., et al. 2007, *AJ*, 134, 1150
Jimenez, R., & Loeb, A. 2002, *ApJ*, 574, 37
Kass, R. E., & Raftery, A. E. 1995, *J. Am. Stat. Assoc.*, 90, 773
Kauffmann, G., & Haehnelt, M. 2000, *MNRAS*, 311, 576
Kim, A. G. 2011, *PASP*, 123, 230
Kowalski, M., Rubin, D., Aldering, G., et al. 2008, *ApJ*, 686, 749
Kuha, J. 2004, *Sociol. Methods Res.*, 33, 188
Kurk, J. D., Walter, F., Fan, X., et al. 2007, *ApJ*, 669, 32
Kuznetsova, N., Barbary, K., Connolly, B., et al. 2008, *ApJ*, 673, 981
Liang, E., & Zhang, B. 2005, *ApJ*, 633, 611
Liang, N., Xiao, W. K., Liu, Y., & Zhang, S. N. 2008, *ApJ*, 685, 354
Liddle, A., Mukherjee, P., & Parkinson, D. 2006, *A&G*, 47, 040000
Liddle, A. R. 2007, *MNRAS*, 377, L74
Liddle, A. R. 2004, *MNRAS*, 351, L49
Lippai, Z., Frei, Z., & Haiman, Z. 2009, *ApJ*, 701, 360
Melia, F. 2007, *MNRAS*, 382, 1917
Melia, F. 2012a, *AJ*, 144, 110
Melia, F. 2012b, *Australian Physics*, 49, 83
Melia, F. 2013a, *ApJ*, 764, 72
Melia, F. 2013b, *AnPhy*, submitted
Melia, F., & Abdelqader, M. 2009, *IJMPD*, 18, 1889
Melia, F., & Maier, R. S. 2013, *MNRAS*, 432, 2669
Melia, F., & Shevchuk, A. S. H. 2012, *MNRAS*, 419, 2579
Misner, C. W., & Sharp, D. H. 1964, *PhRv*, 136, 571
Moresco, M., Cimatti, A., Jimenez, R., et al. 2012, *JCAP*, 8, 006
Norris, J. P., Marani, G. F., & Bonnell, J. T. 2000, *ApJ*, 534, 248
Perlmuter, S., Aldering, G., della Valle, M., et al. 1998, *Natur*, 391, 51
Perlmuter, S., Aldering, G., Goldhaber, G., et al. 1999, *ApJ*, 517, 565
Planck Collaboration, Ade, P. A. R., Aghanim, N., et al. 2014, *A&A*, 571, 16
Riess, A. G., Filippenko, A. V., Challis, P., et al. 1998, *AJ*, 116, 1009
Riess, A. G., Macri, L., Casertano, S., et al. 2011, *ApJ*, 730, 119
Schaefer, B. E. 2003, *ApJL*, 583, L71
Schmidt, B. P., Suntzeff, N. B., Phillips, M. M., et al. 1998, *ApJ*, 507, 46
Schwarz, G. 1978, *AntSta*, 6, 461
Shafieloo, A., Kim, A. G., & Linder, E. V. 2012, *PRD*, 85, 123530
Shi, K., Huang, Y. F., & Lu, T. 2012, *MNRAS*, 426, 2452
Simon, J., Verde, L., & Jimenez, R. 2005, *PhRvD*, 71, 123001
Stern, D., Jimenez, R., Verde, L., Stanford, S. A., & Kamionkowski, M. 2010, *ApJS*, 188, 280
Suzuki, N., Rubin, D., Lidman, C., et al. 2012, *ApJ*, 746, 85
Tanaka, T., & Haiman, Z. 2009, *ApJ*, 696, 1798
Wang, F.-Y., Qi, S., & Dai, Z.-G. 2011, *MNRAS*, 415, 3423
Wei, D. M., & Gao, W. H. 2003, *MNRAS*, 345, 743
Wei, J.-J., Wu, J.-F., & Melia, F. 2013, *ApJ*, 772, 43
Weyl, H. 1923, *ZPhy*, 24, 230
Willett, C. J., McLure, R. J., & Jarvis, M. J. 2003, *ApJL*, 587, L15
Willett, C. J., Albert, L., Arzoumanian, D., et al. 2010a, *AJ*, 140, 546
Willett, C. J., Delorme, P., Reiley, C., et al. 2010b, *AJ*, 139, 906
Wyithe, J. S. B., & Loeb, A. 2003, *ApJ*, 595, 614
Yonetoku, D., Murakami, T., Nakamura, T., et al. 2004, *ApJ*, 609, 935