Applying the scalability apparatus
to estimate the thermal efficiency of a single finned tube

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Abstract. The paper explores the possibility of scaling the integral parameters of the cooling system operation using the working elements with the length from 0.01 to 0.5 m. Numerical solution of the conjugate problem of external aeromechanics, internal hydrodynamics and heat exchange is carried out. Parametric studies of the cooling process and aerodynamic resistance of finned tubular elements of various lengths are performed. As a result of generalization and unification of the computational experimental data, a numerical coefficient has been obtained to calculate the necessary integral characteristics for a cooling element of the assigned length.

1. Introduction

The main task in developing the advanced air-cooled heat exchangers (HE) is to increase their energy efficiency, which is significantly affected by both the heat transfer fluids used and the material of working parts. The use of various types of heat exchangers for cooling the working substance flow is widespread [1-12]. In some cases, the designed heat exchangers have specific property requirements [5]. For example, to cool the working medium in a closed system with minimum losses of pressure characteristics of the cooled flow, it is advisable to use convective HE, including those containing packages of finned thin-walled tubes of different configuration [2,3,5,6,8-12]. In this case, linear dimensions of such bundles, as well as the number of tubes in the bundle, will be determined by the geometry of the coolant supply channel. In this regard, for a correct and effective study of cooling processes, it is necessary to know about the scalability of both dimensionless heat transfer coefficients and aerodynamic drag coefficients for various linear sizes of tubes.

A wide range of experimental and numerical works devoted to the analysis of heat exchange processes in various kinds of heat exchangers and technical devices is presented in the literature [1-12]. So, the publication by A. A. Zhukauskas [1] provides a generalization of results of a series of experimental studies of the types of single-phase heat exchangers the most demanded at that time, and considers the criterial dependences for calculation of Nusselt numbers in various HE. However, an assessment of the correctness of the obtained laws for convective HE, in which the working and cooled media are different substances in different aggregate states, is not presented in the literature. Numerical modeling of heat exchange processes, turbulence and vorticity phenomena in devices with tubular elements of various geometry, including finned tubes, is described in [2-8]. In [9-12] the research results for heat transfer and hydraulic resistance of the finned tube bundle in air condensers and similar heat exchangers are given. However, the conjugate statement of research problems, including heat exchange between the working fluid and the wall, heat conduction in the wall, and heat exchange between the wall and the oncoming flow of the cooled substance are not discussed in these
works. The influence of linear dimensions of finned tubular elements on the process of cooling the working medium has not been investigated either.

This work studies possibilities of applying the scalability apparatus to estimate the thermal efficiency of a single finned tube.

2. Mathematical simulation

The conjugate problem of heat exchange between the flow of the viscous heat conductive gas and the finned tubular element was considered (Figure 1). The tubular element was made as an aluminum tube with the diameter 0.03 m. Tube ribbing was ring-shaped and transverse; ribs were similar and uniformly arranged along the carrying tube with the pitch of 0.005 m, the height of ribs was 0.013 m and the thickness of ribs was 0.001 m. Dimensions of the computational area along the flow motion was chosen by the condition of minimizing the influence of the input and output boundaries on the tube streamlining.

Figure 1. Computational domain.

The system of equations was solved for mathematical simulation of the viscous compressed gas motion and included the additional condition equation:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0
\]

\[
\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \mu \frac{\partial u_i}{\partial x_j} \right] + F_i
\]

\[
\frac{\partial \rho E}{\partial t} + \frac{\partial \rho E u_i}{\partial x_i} = -\frac{\partial \rho u_j}{\partial x_j} + \frac{\partial \rho u_j u_j}{\partial x_j} + \frac{\partial q_i}{\partial x_j} + F_i u_j
\]

\[
\rho c_v \frac{\partial T}{\partial t} = \lambda \frac{\partial^2 T}{\partial x_j^2}
\]

\[
p = \rho RT
\]

The following designations are used in the above formulas (1)-(5): \(\rho\) is the gas density, \(p\) is the pressure, \(u_i\) is the velocity component, \(F_i\) is the external volume force, \(T\) is the temperature, \(q_i\) is the component of the heat flux density vector, \(R = 287 \ J / (kg \cdot K)\) is the specific gas constant; \(\mu\) is the dynamic viscosity, \(\lambda\) is the heat conductivity factor, \(E = C_i T + 0.5u_i^2\) is the total specific energy,
\[ H = E + p / \rho = C_v T + 0.5u_i^2 = h + 0.5u_i^2 \]

is the total specific enthalpy; \( \tau_{ij} = 2\mu S_{ij} - \frac{2}{3}H \frac{\partial u_k}{\partial x_i} S_{ij} \)

is the viscous stress tensor; and \( S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \)

is the strain velocity tensor. Coefficients of the molecular viscosity \( \mu \) and heat conductivity \( \lambda \) depend on the temperature in accordance with \[1\].

Initial equations are averaged in accordance with \[13-15\]. The Navier-Stocks averaged system is closed by the model of the Reynolds stress component transfer within the Mentor model \[13, 14\]. All constants of the Mentor model \[14\] are calculated in accordance with \[13-16\] with values from standard models \( k - \varepsilon \) and \( k - \omega \) as \( \alpha = \alpha_F + \alpha_z (1 - F) \), etc. The constants of this model are \( \beta = 0.09 \), \( \alpha_i = 5/9 \), \( \alpha_z = 0.44 \), \( \beta_i = 3/40 \), \( \beta_z = 0.0828 \), \( \sigma_{11} = 0.85 \), \( \sigma_{12} = 1 \), \( \sigma_{ao} = 0.5 \), \( \sigma_{ao2} = 0.856 \), \( a_i = 0.31 \).

The boundary conditions of the problem are set as follows: at the inlet section and on the upper boundary the average velocity of the gas incoming flow is 8.5 m/s and the total gas temperature is 342 K; The liquid coolant velocity at the tube inlet is equal to 1 m/s and the coolant liquid temperature is 213 K; at the outlet section the zero gradient conditions for the velocity and temperature and the fixed pressure value; on solid surfaces not involved in the heat transfer the adhesion conditions for the velocity and zero gradient conditions for the temperature and pressure; at conjugation boundaries the boundary conditions of the 4th kind including the temperature equivalence for walls and heat fluxes.

3. Calculation results

Numerical modeling of the conjugated problem was carried out for tube lengths \( L \) 0.03, 0.06, 0.12, 0.15, 0.18, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45 and 0.5 m.

As a result of these calculations, distributions of the fields of physical values were received at streamlining of finned tubes of various lengths with an internal stream of the heat transfer fluid by a cooled air flow. The liquid was fed into the tube from top to bottom. The cooled air was flowing from left to right. Figure 2 shows the distribution of the temperature (Fig. 2,a) and velocity (Fig. 2,b) of the coolant (ethylene glycol) in a tube with the length of 0.2 m.

Figure 2. Distribution field inside the 0.2 m tube: a) temperature; b) velocity.

Figure 2 shows that the core temperature of the ethylene glycol flow increases by 0.7K during a single pass through the tube with 0.2 m length. The influence of gravity is also observed when liquid is fed from top to bottom, resulting in the increase of the flow velocity in the outlet section.
Figure 3 shows temperature fields in the cooler and the tube for tubes with the length of 0.03, 0.06, 0.12, 0.15, 0.18, 0.2, 0.25, 0.3, 0.35, 0.4 and 0.5 m. Taking into account the temperature field shown in Figure 2a, it may be seen that starting from the tube length of 0.12 m the temperature profile is self-similar. Unmeasured temperature profiles along the channel also confirm this property (Figure 4).

![Figure 3. Distribution of the coolant temperature for various tube lengths.](image_url)

A similar trend is observed in the external flow. Figure 5 shows the temperature gradient of the cooled gas flow against the length of the tube. The maximum divergence agrees with the initial section of the flow corresponding to the area of flow stabilization. At this section, there is an intensive restructuring of the flow from the shock profile at the channel inlet to the parabolic Poiseuille profile specific for laminar flows.

The joint analysis of temperature and velocity fields has shown that there is a single scale factor, which allows determining the pressure and velocity in the core of the coolant flow by a simple recalculation. According to the obtained calculated data, this coefficient $C_m = -1.15 \cdot L$.

Comparison of the expected values of the temperature drop and velocity head obtained using the scaling factor with the test sample of calculation data shows that the discrepancy of values does not exceed 15%.
Figure 5. Variation of the temperature drop as a function of the tube length along the tube axis.

The external gas dynamics allowed not only estimating the gas cooling behind the finned tube, but also determining the aerodynamic drag coefficient of the tube. The diagram of the resistance vs. the length of the tube is shown in Figure 6. The obtained data are well approximated by the power dependence of the drag coefficient on the tube length. A part of the calculated data was used as a test sample. The rest of the data were the verifying ones for the plotted trend line. The significance level was 0.95%.

Figure 6. Dependence of the drag coefficient

Conclusions
As a result of the performed work, the single numerical coefficient has been obtained to calculate the necessary integral characteristics for the tube of the preset length. In the future, the use of the scaling factor will allow significantly reducing the computational costs when modeling the finned tube package, since they are proportional to the tube length. Studies have confirmed that 0.03 m and 0.06 m tubes cannot be used as a basis for scaling results since the stabilization zone occupies more than 50% of the element length. Results for 0.12 m, 0.2 m, 0.35 m and 0.5 m tubes are shown to be self-similar, which allows using the results for the 0.12 m tube as a reference and recalculating the results for the required tube length using the scaling factor $C_m$.

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References
[1] Zhukauskas A A 1982 *Convective transfer in heat exchange devices* (Moscow: Science)
[2] Liu S and Sakr M 2013 *Renew. Sustain. Energy Rev.* 19 64–81
[3] Rahai H R and Wong T W 2002 Appl. Therm. Eng. 22 1037–45
[4] Yakut K and Sahin B 2004 Appl. Therm. Eng. 24 2427–38
[5] Koroleva M R, Saburova E A and Chernova A A 2020 Journal of Physics: Conference Series 1675(1) 012009
[6] Kalibasha J, Karthikeyan G and Karuppusamy S 2015 Journal of Recent Research in Engineering and Technology vol. 2 5 40–3
[7] Shoji Y, Sato K and Oliver D R 2003 Heat Transf. Asian Res 32 99–107
[8] Gizzatullina A, Koroleva M, Mishchenkova O and Chernova A 2020 Proceedings-2020 Ivannikov Ispras Open Conference, ISPRAS 2020 142–9, 9394095
[9] Fedorov V A, Milman O O, Ananyev P A, Ptachyn A V, Zhinov A A, Karushev A K and Shevelev D V 2015 Herald of the Bauman Moscow state technical university. Ser. Mechanical Engineering 5 87–105
[10] Xing Xuea, Xianming Fenga, Junmin Wanga and Fang Liu 2012 Procedia engineering 31 817–22
[11] Romanova E V, Koliukh A N and Lebedev E A 2017 Transactions TSTU 23(3) 420–7 doi: 10.17277/vestnik.2017.03.pp.420-427
[12] Zhinov A A, Shevelev D V and Ananiev P A 2013 Science and education 3 105–16
[13] Volkov K N and Emelyanov V N 2008 Modeling of large vortices in calculations of turbulent flows
[14] Menter F R, Kuntz M and Langtry R 2003 Proc. 4th. Int. Symp. on Turbulence, Heat and Mass Transfer
[15] Isaev S, Popov I, Gritckevich M and Leontiev A 2019. Acta Astronautica, 163 A
[16] Volkov K and Emelyanov V. 2011 Mass syply gas flows in channels and paths of power plants
[17] OpenFoam. Free CFD Software. Available online: http://openfoam.org/ (accessed on 22 March 2019)