The Sound of the Little Bangs

Ágnes Mócsy
Pratt Institute, Department of Math and Science, Brooklyn, NY 11205, USA
Frankfurt Institute for Advanced Studies, Ruth-Moufang-Str.1, D-60438 Frankfurt am Main, Germany

Paul Sorensen
Physics Department, Brookhaven National Laboratory, Upton NY 11973, USA

Data from ultrarelativistic heavy-ions collisions show evidence for temperature-fluctuations on the freeze-out surface of the expanding fireball. These may be remnants of density inhomogeneities in the initial collision overlap region. We present a power-spectrum analysis for heavy-ion collisions analogous to the analysis of the cosmic microwave background radiation. We use a Glauber model for eccentricity to extract the transfer-function needed to produce the observed spectrum and discuss its relation to the mean-free-path of the matter created in the collisions.

PACS numbers:

A commonly quoted goal of the heavy-ion programs at Brookhaven Lab (BNL) and CERN, is to recreate conditions similar to those shortly after the Big Bang when the universe was filled with a quark-gluon plasma (QGP). QGP can be created by smashing heavy nuclei together at relativistic speeds in collisions called "little bangs". In the past few decades many theoretical and experimental advances have been made in the study of heavy-ion collisions. Comparisons to the early universe, however, have been scarce [1, 2]. In this paper we explore an analogy between heavy-ion collisions and Big Bang cosmology. Using the heavy-ion equivalent of the map of the cosmic microwave background radiation (CMB), we determine the power-spectrum for the little bangs and we estimate the transfer-function necessary to produce the spectrum from the initial conditions.

The hot fireball created in little bangs rapidly expands and cools and when cold enough forms hadrons. Eventually, the system spreads out enough that hadrons stop interacting. This is called the surface of last scattering or freeze-out. The particles then free-stream to the detectors. From measurements of the number, mass, and momentum of these particles, we must infer the properties of the matter that emitted them and extract the essential physics of the QGP. One of the most important discoveries at the Relativistic Heavy Ion Collider (RHIC) at BNL is that the tiny speck of QGP matter produced in the little bangs behaves much like a liquid [3]. This finding is based on the observation that the spatial asymmetries in the initial overlap zone show up as asymmetries in the momentum distributions of final state particles. The observed anisotropy is typically represented by the second Fourier component ($v_2$) of the azimuthal distribution of observed particles relative to the reaction-plane [4]. $v_2$ most strongly reflects the almond shape of the initial nuclear overlap region for non central collisions. The magnitude of $v_2$ can be described surprisingly well with ideal relativistic hydrodynamic models suggesting a liquid-like behavior with minimal viscosity. The success of these models seems to indicate that the mean-free-path of interactions for the systems constituents is significantly smaller than the size of the system.

Experiments at RHIC have also discovered correlations between particles that extend over a broad range in the longitudinal direction but are narrow in the azimuthal (transverse) direction forming a ridge [5]. A number of different scenarios have been proposed to explain the ridge [6,13]. These include minimum-bias (soft) jets in Au+Au collisions [6], soft gluons radiated by hard partons traversing the overlap region [7], beam-jets boosted by the radial expansion [8], viscous broadening [9], and flux-tube like structures boosted by the radial expansion [11, 12]. The extent of the correlation in the longitudinal direction requires by causality that it must be established very early in the collisions [12]. One of us (PS) proposed that the correlation structures may be understood in terms of fluctuations of higher Fourier components of $v_n$, particularly $\sqrt{\langle v_n^2 \rangle}$, that arise from anisotropies in the initial energy density converted into momentum space during the expansion [13]. It was subsequently shown with the NEXSPHERIO hydrodynamic model, that indeed, lumpy initial conditions lead to structures similar to those observed in the two-particle correlation measurements [14]. Alver and Roland [15] used the RQMD model to show that lumpiness in the initial collision geometry can lead to a $\sqrt{\langle v_n^2 \rangle}$ in the azimuthal particle production. Petersen et al. [16] carried out a similar analysis using an event-by-event hydrodynamic model.

An analogy between the expansion of heavy-ion collisions starting from a lumpy initial energy density and the expansion of the universe starting with quantum fluctuations stretched to cosmological sizes was first pointed out by Mishra et al. [2]. They also proposed that the RMS values of $v_n (\sqrt{\langle v_n^2 \rangle})$ could be measured in heavy ion collisions analogous to the power-spectrum extracted from the CMB. They didn’t however make a connection between $\sqrt{\langle v_n^2 \rangle}$ and the already existing two-particle correlation measurements. In this work we use transverse...
momen tum ($p_T$) correlations published by the STAR collaboration \citen{17} to extract the power-spectrum for heavy-ion collisions. Since the $p_T$ spectra reflects the temperature, $p_T$ correlations are sensitive to local temperature fluctuations. These measurements are directly analogous therefore, to the maps of the CMB. We use a Monte Carlo Glauber model \cite{18} for initial eccentricities to extract the transfer-function required to convert the initial coordinate-space anisotropy into the anisotropy seen in momentum-space. This analysis facilitates a more direct comparison between relativistic heavy-ion collisions and the early universe.

Analogy with Big Bang Cosmology: Measurements of the CMB reveal temperature fluctuations corresponding to over- or under-densities present at the surface of last scattering at about 300,000 years after the Big Bang \cite{19}. These density fluctuations ultimately explain the structure in our universe (Fig. 1 left). Just as quantum fluctuations stretched to cosmic sizes by inflation show up in the CMB, we expect fluctuations from the beginning of the little bangs to show up in heavy-ion data (Fig. 1 right). Measuring temperature-fluctuations in the CMB required precise measurements at more than two million points in the sky. Enough photons are detected at each point to reconstruct the black-body spectrum from which the temperature is determined. In a heavy-ion collision, a few thousand particles are created at most, so a similar map cannot be made for each collision. But whereas we only observe one universe, billions of collisions are created in the lab. By studying $p_T$ correlation data (sensitive to local changes in the $p_T$-spectra and thus the temperature) accumulated from millions of these collisions, we can search for evidence of hotspots on the surface of last scattering.

![FIG. 1: Schematic of the expansion of the universe after the Big Bang (left) and the expansion of a fireball after little bangs (right). The illustration is by Alexander Doig.](image)

Survival of Density Fluctuations and Various Scales: In Fig. 2 we show the temperature fluctuations calculated from a heavy-ion event generator near the beginning of the expansion and 4 fm/c later. We determined these temperature profiles in the transverse plane at mid-rapidity by translating the energy-density profiles of Werner et al. \cite{20} into temperature, using the parametrized lattice QCD results for energy-density vs temperature from \cite{21}. The simulations indicate that collisions of Au nuclei (12 fm across), may contain hotspots of size $l_{spot} \approx 1.5$ fm and that remnants of those hotspots persist during the collisions evolution. We also consider the lengths of the acoustic horizon $H$ and the mean-free-path $l_{mfp}$ of the systems constituents to be important.

The acoustic horizon defines how far mechanical information can have propagated through the medium at time $\tau$: $H(\tau) = \int_0^{\tau_{fo}} c_s(\tau)d\tau$, where $\tau_{fo}$ is the freeze-out time. This relates to the growth rate of $l_{spot}$. We determined $H(\tau)$ from lattice data on the speed of sound ($c_s$) vs energy density \cite{22} and a hydrodynamic model to specify the energy density vs $\tau$ \cite{23}. Fig. 3 shows the acoustic horizon for QCD matter. The phase-transition from QGP to hadron-gas can be seen as a flattening in the slope of $H$ at $\tau \approx 10$ fm/c when $H$ is about 5 fm. The acoustic horizon also dominates the time dependence of the sound that an observer inside the medium would hear (see \cite{21} and \cite{24}). The sound at freeze-out is composed of a superposition of different waves with different frequencies that can be determined from the two-particle momentum correlation data. The horizon defines when frequencies can be heard: Only after half a wavelength fits inside the horizon would that wavelength become “audible”. This is the same effect that leads to the lack of large scale fluctuations in the CMB.

The fact that hydrodynamic models do a reasonable job of predicting the value of $v_2$ suggests that $l_{mfp}$ can be considered small compared to the size of the system. By examining the power-spectrum of heavy-ion collisions which includes information for all values of $n$ (beyond just $n = 2$ or $n = 3$), we hope to better constrain $l_{mfp}$. As we increase $n$, we reduce the length scale probed. We only expect an efficient conversion of coordinate-space anisotropies into momentum space when $l_{mfp} \ll 2\pi \langle R \rangle /n$ where $\langle R \rangle$ is the average radius of the systems constituents \cite{25}.

Power-Spectrum and Transfer-Function: We determine the power-spectrum from two-particle momentum correlations (related to $\langle p_1 \rangle$ vs $\langle p_2 \rangle$) vs the relative azimuthal angles between the particles $\Delta \phi$ \cite{17}.

![FIG. 2: Temperature profile in the transverse plane for mid-rapidity at proper time $\tau = 0.6$ (left) and 4.6 fm/c (right).](image)
A narrow peak positioned around small angle separation is observed in the data. This tells us that if a particle comes out with above average $p_T$, then the nearby particles also tend to have large $p_T$. This is consistent with expectations from hotspots on the surface of last scattering. The correlation of these fast particles suggests that they are born out of the same high-density, high-temperature lump. We will not attempt to decompose the correlation into different components, i.e. jets and resonances and background. The power-spectrum we extract from data should and does contain all these contributions. We argue that jets do not dominate the observed correlation because the correlations are too large in magnitude, too narrow in $\phi$, and too broad in $\eta$; there will likely be some contribution but it is suppressed by $1/m$ multiplicity. As for resonances, if a hotspot is there, it will emit massive and/or high momentum particles. The decay of a hotspot can proceed through decays into resonances. The power-spectrum reflects all these contributions. Our interpretation of the correlations data in terms of hotspots is supported by several pieces of ancillary evidence. 1) $v_n$-fluctuations are close to what we expect from density fluctuations from several models and the two-particle correlations data also match what we expect from these models [14][16][27]. This gives us confidence that the correlations are dominated by the over- and under-densities at the start of the expansion phase. 2) An improved description of particle $p_T$ spectra is obtained when temperature fluctuations are considered [28].

The $p_T$ correlations vs relative angles between the emitted particles ($\Delta \phi$ and $\Delta \eta$) where parametrized in the STAR paper [17]. To extract the power-spectrum, we use that parametrization with $\Delta \eta = 0$ and Fourier-transform the correlation function versus $\Delta \phi$. The coefficients

$$a_n = \frac{2}{\pi} \int_0^\infty f(\Delta \phi) \cos(n \Delta \phi) d(\Delta \phi).$$

(1)

vs harmonic $n$ make up the power-spectrum. If we had used number correlations instead of $p_T$ correlations $a_n \approx v_n^2$ [29]. The power-spectrum for little bangs is shown in Figure 4 (left).

Having extracted the power-spectrum from the azimuthal correlations of observed particles, we can compare that to the azimuthal distribution of matter in the initial overlap zone. This is calculated using the participant eccentricity ($\varepsilon_{n,part}$) for all harmonics $n$ as in Ref. [15]. The fluctuations in the initial geometry cause the major axis of the eccentricity to fluctuate away from the reaction-plane direction. $\varepsilon_{n,part}$ is the eccentricity calculated along the major-axis. Fig. 4 (right) shows $\langle \varepsilon_{n,part}^2 \rangle$ from a Monte Carlo Glauber model [13] for perfectly central collision ($b=0$ fm) and for the 5% most central collisions. The large $n = 2$ term persists even for central collisions because we include the intrinsic deformation of the Au nucleus in our Monte Carlo. The $n = 1$ term is small because the participants are re-centered so that $\langle x \rangle = \langle y \rangle = 0$. For $b = 0$ collisions we note that the eccentricity is nearly independent of $n$ for $n > 2$. This is because for symmetric collisions and point-like participants, $\varepsilon_{part}$ depends only on the number of participants, independent of $n$ [30]. In this case, if all harmonics were converted into momentum space equally well, the final correlation function tend to a Dirac delta function at $\Delta \phi = 0$. We expect however, that the conversion of higher harmonic eccentricity will be damped due to the existence of the length scale $l_{mfp}$. The conversion will be efficient only when $l_{mfp} < 2\pi \langle R \rangle / n$. We can investigate the damping of the higher modes by plotting the transfer-function which is the ratio of the power-spectrum in Fig. 4 (left) to $\langle \varepsilon_{part,n}^2 \rangle$ in Fig. 4 (right).
when the collision of coordinate space and momentum space can be easily understood in terms of an inefficiency in the transfer-function required to describe the RHIC data can be easily understood in terms of an inefficiency in the transfer-function.

Acknowledgments: While preparing this draft, a paper on a similar topic was posted [32]. We thank Marcus Bleicher and Klaus Werner for providing the data for the energy density profile and Alex Doig for preparing several illustrations. AM thanks FIAS for the kind hospitality.

[1] K. Yagi, T. Hatsuda and Y. Miake, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 23, 1 (2005); H. Schade and B. Kampfer, Phys. Rev. C 79, 044909 (2009); D. Boyanovsky, H. J. de Vega, D. J. Schwarz, Ann. Rev. Nucl. Part. Sci. 56, 441-500 (2006).

[2] A. P. Mishra et al., Phys. Rev. C 77, 064902 (2008).
[3] M. Gyulassy, L. McLerran, Nucl. Phys. A750, 30-63 (2005);
[4] The reaction-plane is defined by the beam axis and the vector connecting the centers of the colliding nuclei.
[5] M. Daugherty [STAR Collaboration], J. Phys. G 35, 104090 (2008); J. Adams et al. [STAR Collaboration], Phys. Rev. Lett. 95:152301, (2005); J. Adams et al. [STAR Collaboration], Phys. Rev. C 73 (2006) 064907; B. I. Abelev et al. [STAR Collaboration], Phys. Rev. C 80, 064912 (2009); J. Adams et al. [STAR Collaboration], Phys. Rev. C 75 (2007) 034901; B. Alver et al. [PHOBOS Collaboration], J. Phys. G 35, 104080 (2008).
[6] T. A. Trainor [STAR Collaboration], AIP Conf. Proc. 828, 238 (2006).
[7] A. Majumder, B. Muller, S. A. Bass, Phys. Rev. Lett. 99, 042301 (2007).
[8] E. V. Shuryak, Phys. Rev. C 76, 047901 (2007).
[9] S. Gavin, M. Moschelli, J. Phys. G G35, 104084 (2008).
[10] S. A. Voloshin, Phys. Rev. B 632, 490 (2006). Nucl. Phys. A 802, 107 (2008).
[11] S. A. Voloshin, Nucl. Phys. A749, 287-290 (2005).
[12] A. Dumitru et al., Nucl. Phys. A810, 91 (2008); S. Gavin, L. McLerran, G. Moschelli, Phys. Rev. C79, 051902 (2009).
[13] P. Sorensen, J. Phys. G: Nucl. Part. Phys. 37 094011 (2010).
[14] J. Takahashi et al., Phys. Rev. Lett. 103, 242301 (2009).
[15] B. Alver, G. Roland, Phys. Rev. C 81, 054905 (2010).
[16] H. Petersen et al., arXiv:1008.0625 [nucl-th].
[17] J. Adams et al. [STAR Collaboration], J. Phys. G 32, L37 (2006).
[18] M. L. Miller et al., Ann. Rev. Nucl. Part. Sci. 57, 205 (2007).
[19] http://map.gsfc.nasa.gov/
[20] K. Werner et al., arXiv:1004.0805 [nucl-th].
[21] A. Bazavov et al., Phys. Rev. D 80, 014504 (2009); S. Borsanyi et al., arXiv:1007.2580 [hep-lat].
[22] P. Huovinen, P. Petreczky, Nucl. Phys. A837, 26-53 (2010).
[23] P. F. Kolb, Heavy Ion Phys. 21, 243 (2004).
[24] http://soundofthelittlebang.com/
[25] http://www.youtube.com/watch?v=jFSQ039Com-Q
[26] n = 1 is suppressed by momentum conservation.
[27] P. Sorensen, Proceedings of 24th Winter Workshop on Nuclear Dynamics, South Padre Island, Texas, 5-12 Apr 2008.
[28] Z. Tang et al., Phys. Rev. C 79, 051901 (R) (2009).
[29] T. A. Trainor and D. T. Kettler, Int. J. Mod. Phys. E 17, 1219 (2008). T arXiv:0704.1674 [hep-ph].
[30] Giving the participants a size causes \( \langle n_{part} \rangle \) to drop as a function of \( n \). The drop occurs at \( n \approx \pi \langle R \rangle / r_{part} \) where \( r_{part} \) is the radial size of the participant and \( \pi \langle R \rangle \approx 3 \text{fm} \) is the average radial size of a participant. This drop is similar to the drop we expect in the transfer-function due to \( l_{mfp} \) and the effects could be confused in the data.
[31] H. J. Drescher et al., Phys. Rev. C 76, 024905 (2007).
[32] S. Gavin and M. Abdel-Aziz, Phys. Rev. Lett. 97, 162302 (2006).
[33] P. Staig and E. Shuryak, arXiv:1008.3139 [nucl-th].