Muonic atoms with extreme nuclear charge

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Abstract

Bound muons (also pions, kaons, etc) increase the fission barrier and produce some stabilizing effects for highly charged nuclei. If the binding energy of the muon exceeds $mc^2$, it becomes stable. The $1s$ state of a muon inside an exotic nucleus with atomic number $A = 5Z/2$ and such large charge $Z$ that the $1s$ energy $E$ is in the range $0 \leq E < -mc^2$ is considered.

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As well known, all atoms with nuclear charge $Z > 100$ have very short lifetimes and atoms with $Z > 118$ have not as yet been found. However, the behavior of orbitals in the field of a highly charged “nucleus” remains an interesting theoretical problem. There are various motivations for the study of such systems. Firstly, in collisions of heavy nuclei (e.g. uranium with charge $Z_i = 92$) a compound nucleus with the charge $Z = 2Z_i$ is formed for a short time. Electron-positron pair creation due to “vacuum breakdown” is expected in this situation, provided the total compound nucleus charge $Z$ exceeds a critical charge $Z_c \approx 170$ (see e.g. review \cite{1} and the book \cite{2}). An experimental investigation of this phenomenon has been done in GSI, with new experiments in preparation \cite{3}. Then, there are speculations about exotic forms of matter such as “strange matter” where a nucleus contains strange quarks in addition to “normal” $u$ and $d$ quarks \cite{4}; there may be highly charged small primordial black holes \cite{5} and the associated black hole “atoms” \cite{6}.

In this note, we would like to consider the purely theoretical problem of a charged particle in the superstrong Coulomb field. The electron problem is actually very well studied (see e.g. \cite{1}). However, the Coulomb problem for a heavier particle has some new and interesting features. Consider, for example, a muon. Normally, it decays into electron and neutrinos. However, the bound muon is stable if the binding energy exceeds $mc^2$ so the total relativistic energy $E < 0$. In this case the bound muon has a significant stabilizing effect on the “nucleus”, it contributes $\sim mc^2 \approx 100$ MeV to the nuclear fission barrier. In the case of bosonic particles like pions or kaons the effect can be even more significant since many particles may be placed in $1s$ state and the critical charge for spin $S = 0$ and $S = 1$ is much smaller than for $S = 1/2$ ($Z_c=68$ for a very small nucleus, i.e. a factor of two smaller than for Dirac particles). Moreover, the strong interaction may produce an additional attraction and reduce the critical charge.

In our numerical calculations, we assume a standard nuclear charge density with radius $R = 1.15A^{1/3}$ fm, and atomic number $A = 5Z/2$. Let us start our consideration from the conventional electron case. The electron density is localized mostly outside the nucleus even as the nuclear charge approaches the critical value $Z_c \approx 170$; thus, there is no collapse of the electron wave function inside the nucleus. In Fig.\cite{11} we plot the large and small components of $1s$ wave function and the radial density for charge $Z=166$, which is close to the critical charge. The electron energy in this case is $E = -0.796 mc^2$ (the energy for the critical charge is $E_c = -mc^2$). Thus, the electron wave function for the finite nucleus in the main area
FIG. 1: Radial wave functions $P(r)$ and $Q(r)$ and the radial charge density $4\pi r^2 \rho(r)$ of a 1$s$ electron ($E = -0.796 mc^2$) in the field of a nucleus of charge $Z = 166$ are shown.

is not very different from the point-nucleus case. Consequently, the critical charge for the finite nucleus $Z_c \approx 170$ is relatively close to the case of a very small nucleus where $Z_c = 138$.

Now consider the muon case. The non-relativistic muonic atom is $m_\mu/m_e = 200$ times smaller than the usual electronic atom. In the relativistic case $E < 0$, the muon wave function for the 1$s$ state is localized inside the nucleus where the electrostatic potential has the oscillator shape

$$V(r) = -\frac{Ze^2}{R} \left(\frac{3}{2} - \frac{r^2}{2R^2}\right).$$

The critical charge for this oscillator potential is very different from the Coulomb potential. The energy reaches $E = 0$ for a nuclear charge $Z \approx 840$ and the critical value $E = -mc^2$ for $Z_c \approx 2000$.

In the non-relativistic case, the radius of 1$s$ state is inversely proportional to the nuclear charge $Z$. Surprisingly, in the relativistic case $E < 0$, the muon wave function is practically independent of $Z$ (see Fig. 2, upper panel). The explanation is as follows: In the main area, the electrostatic potential $V(r)$ is approximately constant. A constant potential $V$
shifts the energy (by \( V \)), however, it does not change the wave function since the Dirac (and Schrödinger) equations contain the combination \( E - V \) which does not change. To illustrate this explanation, we plot \( E - mc^2 - V(r) \) for \( Z = 840 \) \( (E \approx 0) \) and for \( Z = 2000 \) \( (E \approx -mc^2) \) in the lower panel of Fig. 2. The corresponding wave functions practically coincide (see the upper panel). In Fig. 3 we present the radial densities of 1s, 2s, 3s and 4s states for critical charge \( Z_c \approx 2000 \). All of these states are localized inside the nucleus.

Note that the “critical charges” for muons are so large that there is hardly any chance of actual stabilization of a “normal” nuclear matter by muons. Therefore, in the muon case this problem is only an exercise in the relativistic quantum mechanics (critical charges in the oscillator-like potential of the finite nucleus). However, this solution may give us some hints as to what one should expect in more interesting cases of pions and kaons. We do not consider these cases in the present note since they require more sophisticated calculations including the strong interaction.

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[1] Ya. B. Zel'dovich and V.S. Popov, Usp. Fiz. Nauk 14, 403 (1972) [Sov. Phys. Usp. 14, 673 (1972)].
[2] W. Greiner, B. Muller, and J. Rafelski, Quantum Electrodynamics of Strong Fields, Texts and Monographs in Physics, Springer-Verlag, Berlin, 1985.
[3] www.gsi.de/fair
[4] E. Witten, Phys. Rev. D 30, 272 (1984). R.L. Jaffe et al Rev. Mod. Phys. 72, 1125 (2000).
[5] S. Hawking, Mon. Not. R. Astron. Soc. 152, 75 (1971).
[6] V.V. Flambaum, J.C. Berengut. Phys. Rev. D 63, 084010 (2001).
FIG. 2: Upper panel: The $1s$ radial density for a muon (energy slightly above $-mc^2$) in the field of a nucleus with charge $Z = 2000$ and radius $R = 19.2$ fm (red curve) is seen to be essentially identical to the $1s$ radial density of a muon (energy 0) in the field of a nucleus with $Z = 840$, $R = 14.6$ fm (black curve). Lower panel: The functions $E - mc^2 - V(r)$ for the $E = -mc^2$ case (red curve) and $E = 0$ case (black curve) are essentially identical in the region where the densities are nonvanishing, explaining the fact that the densities agree in the two cases.
FIG. 3: Upper panel: Radial densities for the four lowest $ns$ states of a muon in the field of a nucleus with charge $Z = 2000$ and radius $R = 19.2$ fm. Lower panel: Energies of the lowest four $ns$ states are shown in comparison with the potential energy $V(r)$. 