STOCHASTIC VOLterra INTEGRAL EQUATIONS WITH JUMPS AND NON-LIPSCHITZ COEFFICIENTS

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Abstract. Stochastic Volterra integral equations with jumps (SVIEs) have become very common and widely used in numerous branches of science, due to their connections with mathematical finance, biology, engineering and so on. In this paper, we apply the successive approximation method to investigate the existence and uniqueness of solutions to the SVIEs driven by Brownian motion and compensated Poisson random measure under non-Lipschitz condition.

1. Introduction

Stochastic differential equations (SDEs) with jumps offer the most flexible, numerically accessible mathematical framework to help model the evolution of financial and other random quantities through time. In particular, feedback effects can be easily modeled and jumps enable us to frame events. It is important to be able to incorporate event driven uncertainty into a model, and this can be expressed by jumps. This arises, for instance, when one works on credit risk, insurance risk, or operational risk. SDEs enable us to model the feedback effects in the presence of jumps and independence on the level of the state variable itself [6, 10, 11, 22].

The Poisson process is usually used When one needs to handle the mathematical description of jumps, due to as its advantage of counting events and generating an increasing sequence of jump times related to each event that it counts, and consequently, gives the number of jumps that have occurred up to any point in time. However, it is more appropriate to use Poisson measures driven-SDEs when events that have randomly distributed jumps are modeled, instead of Poisson processes, for complete exposition on this topic, we refer to the monographs [9, 21, 16, 20, 2, 19] for discussion of recent developments.

In real life, many phenomena can be mathematically formulated by SVIEs. Therefore, in past decades, SVIEs have attracted the attention of many scholars, who have investigated the analytical and numerical solutions of such kind of equations, the main references are [4, 7, 8, 24, 15, 23, 12].

Recently, [1] studied the controlled stochastic Volterra integral equations with jumps [1], under a Lipschitz condition, they proved the existence and uniqueness of solutions to SVIEs with jumps under Lipschitz condition. Moreover, in [13] the authors of this paper joined Abouagwa M. and Almushaira M. and established the existence and uniqueness of solutions for the SVIEs under Lipschitz condition, and they provided numerical solutions for such equations by applying the Euler–Maruyama scheme.

Key words and phrases. Stochastic differential equations with jumps; Lévy process; Poisson random measure; Stochastic Volterra integral equations; Non-Lipschitz condition.

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Motivated by the aforementioned works, in the present paper, we apply the method of successive approximation, Bihari’s inequality and Doob’s martingale inequality to investigate the existence and uniqueness of the solutions to the SVIEs driven by Brownian motion and pure jump Lévy motion with non-Lipschitz coefficients.

To finish this introduction, we note the general structure of this paper. In Section 2, we introduce some preliminaries and give a brief insight about the SVIEs, moreover we include some requisite lemmas. Section 3 is devoted to prove the existence and uniqueness theorem, lastly we give an example of the SVIE which has a unique solution.

2. SOME PREPARATIONS

Throughout this paper, unless otherwise specified, we let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ to be a filtered probability space satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all $P$-null sets). Let $|\cdot|$ be the Euclidean norm on $\mathbb{R}^n$.

Consider the stochastic Volterra integral equation with jumps of the form

$$x(t) = \varphi(t) + \int_0^t f(t, s, x(s))ds + \int_0^t g(t, s, x(s))dW_s + \int_0^t \int_{R_0} h(t, s, x(s), \xi)\tilde{N}(ds, d\xi),$$

where the initial state $\varphi(t)$ is a given $\mathcal{F}_t$-adapted càdlàg process, $E|\varphi(t)|^2 < \infty$ for $t \in [0, T]$, $0 < T < \infty$. $W(t)$ is Brownian motion, $\tilde{N}(dt, d\xi) := N(dt, d\xi) - \nu(d\xi)dt$ is a compensated Poisson random measure, $\nu$ represents the Lévy measure of the jump counting measure $N$, and $R_0 = \mathbb{R}^n - \{0\}$. We impose that $P(\int_0^T \int_{R_0} |c(t, s, \xi)|X(s)^2\nu(d\xi)ds < \infty) = 1$.

Next, we give some requisite lemmas. The following lemma is taken from [5].

**Lemma 2.1 (Bihari’s inequality).** Let $T > 0$, $y_0 \geq 0$, and $y(t), z(t)$ be continuous functions on $[0, T]$. Assume that $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ ($\mathbb{R}_+$ is the set of of all nonnegative real numbers) is a concave continuous non-decreasing function such that $\alpha(v) > 0$ for $v > 0$. If

$$y(t) \leq y_0 + \int_0^t z(s)\alpha(y(s))ds \quad \forall t \in [0, T],$$

then

$$y(t) \leq G^{-1}(G(y_0) + \int_0^t z(s)ds) \quad \forall t \in [0, T],$$

so that $(G(y_0) + \int_0^t z(s)ds) \in \text{Dom}(G^{-1})$, where $G(v) = \int_0^v \frac{ds}{\alpha(s)}$, $v > 0$. Moreover, if $y_0 = 0$ and $\int_0^v \frac{ds}{\alpha(s)} = \infty$, then $y(t) = 0$ for all $t \in [0, T]$.

**Lemma 2.2 (Doob’s martingale inequality).** If $\{X(t)\}_{t \geq 0}$ is a positive submartingale, then for any $p > 1$ and for all $t > 0$,

$$E \left[ \sup_{0 \leq s \leq t} |X(s)|^p \right] \leq \left( \frac{p}{p - 1} \right)^p E[|X(t)|^p].$$

We refer to [3], theorem 2.1.5, to obtain the proof.

In the rest of this work, the following assumptions are imposed on the coefficients of Eq. (I).
$H_1$ - There exists a positive constant $C$ such that for all $0 \leq s \leq t \leq T$, $x \in \mathbb{R}^n$

$$|a(t, s)x|^2 + |b(t, s)x|^2 \vee \int_{R_0} |c(t, s, \xi)x|^2 v(d\xi) \leq C(1 + |x|^2)$$

$H_2$ - Let $a$, $b$ and $c$ be measurable real–valued functions, for each fixed $t \in [0, T]$ and for all $x, y \in \mathbb{R}^n$, there exist two functions $\lambda$ and $\kappa$, such that

$$|f(t, s, x) - f(t, s, y)|^2 \vee |g(t, s, x) - g(t, s, y)|^2$$

$$\vee \int_{R_0} |h(t, s, x, \xi) - c(t, s, y, \xi)|^2 v(d\xi)$$

$$\leq \lambda(t)[\kappa(|x - y|^2)].$$

where $\lambda \in L^2([0, T]; \mathbb{R})$, and $\kappa(.)$ is monotone non-decreasing, continuous and is a concave function, satisfying $\kappa(0) = 0$ and $\kappa(v) > 0$, such that

$$\int_{0^+} \frac{dv}{\kappa(v)} = \infty.$$

3. The main result

In this section, we apply some technical tools from Stochastic integration with respect to compensated Poisson random measures and Lévy noise (see [9, 21, 12, 19], for complete picture on this point) together with Lemma 1 and Lemma 2, to provide a systematic proof for the existence and uniqueness solution to Eq. (1) under non–Lipschitz condition. This is the massage of the next theorem.

**Theorem 3.1.** Suppose that Assumptions $H_1$–$H_2$ hold. If $\varphi$ is a monotonically increasing, then Eq. (1) has a unique solution.

In order to prove Theorem 3.1, we define a sequence of successive approximations \{x^k, k = 1, 2,...\} with $x^0(t) = \varphi(t)$, as follows:

$$x^k(t) = \varphi(t) + \int_0^t f(t, s, x^{k-1}(s))ds + \int_0^t g(t, s, x^{k-1}(s))dW_s$$

$$+ \int_0^t \int_{R_0} h(t, s, x^{k-1}(s), \xi)\tilde{N}(ds, d\xi), \quad t \in [0, T], \quad k = 1, 2,...$$

(2)

**Lemma 3.2.** With assumptions as in Theorem 3.1, for all $t \in [0, T]$, there exists a positive constant $C_1$, such that

$$E|x^k(t)|^2 \leq C_1, \quad k = 1, 2,...$$

**Proof.** Let $\overline{T} := \max\{T, 1\}$. Here, for $k = 1, 2, ..., \$ we shall show that

$$E|x^k(t)|^2 \leq 4E|\varphi(T)|^2 \sum_{\ell=0}^k \frac{(4\overline{T})^\ell}{\ell!} t^\ell + \sum_{\ell=1}^k \frac{(4\overline{T})^\ell}{\ell!} 4^\ell. \quad (3)$$

First, for $k = 1$, from Eq. (2) and using the following simple inequality

$$|x_1 + x_2 + ... + x_m|^2 \leq m(|x_1|^2 + |x_2|^2 + ... + |x_m|^2), \quad (4)$$

we get

$$E|x^1(t)|^2 \leq 4E|x^0(t)|^2 + 4E \int_0^t f(t, s, x^0(s))ds.$$

This proves that the sequence \( \{ x^k(t), k = 1, 2, \ldots \} \) has a uniform bound on \([0, T]\). And this completes the proof of Lemma 3.2.  

□
Lemma 3.3. Let Assumptions $H_1 - H_2$ are fulfilled, there exists a positive constant $C_3$ such that for all $0 \leq t \leq T$, $k, m \geq 1$,

$$E \left[ \sup_{0 \leq s \leq t} |x^{k+m}(s) - x^k(s)|^2 \right] \leq C_3 t.$$ 

Proof. By Eq. (2) and the basic inequality (1), we have

$$E \left( \sup_{0 \leq s \leq t} |x^{k+m}(s) - x^k(s)|^2 \right) \leq 3E \left[ \sup_{0 \leq s \leq t} \int_0^s |f(t, u, x^{k+m-1}(u)) - f(t, u, x^{k-1}(u))|du|^2 \right]$$

$$+ 3E \left[ \sup_{0 \leq s \leq t} \int_0^s |g(t, u, x^{k+m-1}(u)) - g(t, u, x^{k-1}(u))|dW_u|^2 \right]$$

$$+ 3E \left[ \sup_{0 \leq s \leq t} \int_0^s h(t, u, x^{k+m-1}(u), \xi) - h(t, u, x^{k-1}(u), \xi) |\tilde{N}(du, d\xi)|^2 \right].$$

Thanks to Cauchy–Schwarz inequality, Lemma 2.2 and Assumption $H_2$,

$$E \left( \sup_{0 \leq s \leq t} |x^{k+m}(s) - x^k(s)|^2 \right) \leq 3TE \int_0^t |f(t, s, x^{k+m-1}(s)) - f(t, s, x^{k-1}(s))|^2 ds$$

$$+ 12E \int_0^t |g(t, s, x^{k+m-1}(s)) - g(t, s, x^{k-1}(s))|^2 ds$$

$$+ 12E \int_0^t |h(t, s, x^{k+m-1}(s), \xi) - h(t, s, x^{k-1}(s), \xi)|^2 \nu(d\xi) ds$$

$$\leq 12\tilde{T}\lambda(t) E \int_0^t \kappa(\nu)|x^{k+m-1}(s) - x^{k-1}(s)|^2 ds,$$

which, with the help of Jensen’s inequality and Lemma 3.2, gives

$$E \left( \sup_{0 \leq s \leq t} |x^{k+m}(s) - x^k(s)|^2 \right) \leq 12\tilde{T}\lambda(t)$$

$$\times \int_0^t \kappa(\nu)|x^{k+m-1}(s) - x^{k-1}(s)|^2 ds$$

$$\leq C_2 \int_0^t \kappa(4C_1) ds \leq C_3 t,$$ (6)

where $C_2 := 12\tilde{T}\lambda(t)$. Hence Lemma 3.3 is obtained. □

We now choose $v \in [0, T]$, $0 \leq t \leq v$, such that $\kappa(C_3 t) \leq C_3$, where $\kappa(\eta) = C_2\eta$. Then, define the following sequences.

$$\psi_1(t) = C_3 t$$

$$\psi_{k+1}(t) = \int_0^t \kappa(\psi_k(s)) ds, \ k \geq 1$$ (7)

$$\psi_{k,m}(t) = E \left( \sup_{0 \leq s \leq t} |x^{k+m}(s) - x^k(s)|^2 \right), \ k, m = 1, 2, ...$$ (8)

Lemma 3.4. There exists a positive $t \in [0, v]$ such that, for all $k, m \geq 1$, we have

$$0 \leq \psi_{k,m}(t) \leq \psi_k(t) \leq \psi_{k-1}(t) \leq ... \leq \psi_1(t), \ \forall t \in [0, v].$$ (9)
Proof. By Lemma 3.3, we have

\[ \psi_{1,m}(t) = E(\sup_{0 \leq s \leq t} |x^{1+m}(s) - x^{1}(s)|^2) \leq C_3 t = \psi_1(t). \]

By the definition of \( \kappa \) and equations (7)-(8), we have

\[ \psi_{2,m}(t) = E(\sup_{0 \leq s \leq t} |x^{2+m}(s) - x^{2}(s)|^2) \]

\[ \leq C_2 \int_0^t E(\sup_{0 \leq u \leq s} |x^{1+m}(u) - x^{1}(u)|^2) \, ds \]

\[ \leq \int_0^t \kappa(\psi_{1,m}(s)) \, ds \]

\[ \leq \int_0^t \kappa(\psi_1(s)) \, ds := \psi_2(t). \]

Then, we also have

\[ \psi_2(t) = \int_0^t \kappa(\psi_1(s)) \, ds \leq \int_0^t \kappa(C_3 s) \, ds \leq \int_0^t C_3 ds = C_3 t = \psi_1(t). \]

It has been shown that

\[ 0 \leq \psi_{2,m}(t) \leq \psi_2(t) \leq \psi_1(t), \quad \forall t \in [0,v]. \]

Now, we suppose that (3) holds for some \( k \). Therefore, using the same inequalities above, yields

\[ \psi_{k+1,m}(t) \leq C_2 \int_0^t E(\sup_{0 \leq u \leq s} |x^{k+m}(u) - x^{k}(u)|^2) \, ds \]

\[ \leq \int_0^t \kappa(\psi_{k,m}(s)) \, ds \leq \int_0^t \kappa(\psi_k(s)) \, ds := \psi_{k+1}(t), \quad 0 \leq t \leq v. \]

On the other hand, we have

\[ \psi_{k+1}(t) = \int_0^t \kappa(\psi_k(s)) \, ds \leq \int_0^t \kappa(\psi_{k-1}(s)) \, ds := \psi_k(t), \quad \forall t \in [0,v]. \]

This completes the proof. \( \square \)

**Proof of Theorem 3.1. Existence.** We now argue that

\[ E(\sup_{0 \leq s \leq t} |x^{k+m}(s) - x^{k}(s)|^2) \to 0, \quad t \in [0,v], \]

as \( k, m \to \infty \), noticing that \( \psi_n \) is continuous on \( [0,v] \) and for every \( k \geq 1 \), \( \psi_n(.) \) is decreasing on \( [0,v] \). Furthermore, for each \( t \), \( \psi_n(t) \) is a decreasing sequence. Hence, we can define the function \( \psi(t) \) by

\[ \psi(t) = \lim_{k \to \infty} \psi_k(t) = \lim_{k \to \infty} C_2 \int_0^t \psi_{k-1}(s) \, ds = C_2 \int_0^t \psi(s) \, ds, \tag{10} \]

for all \( 0 \leq t \leq v \). Consequently, since \( \psi(t) \) is a continuous function on \( [0,v] \), \( \psi(0) = 0 \), then by (11) and conditions \( H_1 - H_2 \), all the conditions of Lemma 2.1 are satisfied, hence \( \psi(t) = 0 \) for every \( t \in [0,v] \). Now, from Lemma 3.4, we get

\[ \psi_{k,m}(t) \leq \sup_{0 \leq s \leq v} \psi_k(s) \leq \psi_k(v) \to 0, \quad t \in [0,v] \]
as \( k \to \infty \), thus, \( \{x^k(t)\}_{k=1}^\infty \) is a Cauchy sequence on \( L^2[0,T] \). Then, by Lemma 3.2, we have
\[
E|\{x(t)\}|^2 \leq C_1,
\]
where \( C_1 \) is a positive constant.

By the above discussion, it is easy to conclude that, for all \( t \in [0,v] \),
\[
\begin{align*}
E|\int_0^t [f(t,s,x^k(s)) - f(t,s,x(s))]ds|^2 &\to 0 \\
E|\int_0^t [g(t,s,x^k(s)) - g(t,s,x(s))]dW_s|^2 &\to 0 \\
E|\int_0^t \int_{R_0} h(t,s,x^k(s),\xi) - h(t,s,x(s),\xi)|N(ds,d\xi)|^2 &\to 0,
\end{align*}
\]
as \( k \to \infty \). Taking limits on both sides of (2), we get
\[
x(t) = \varphi(t) + \int_0^t f(t,s,x(s))ds + \int_0^t g(t,s,x(s))dW_s \\
+ \int_0^t \int_{R_0} h(t,s,x(s),\xi) \tilde{N}(ds,d\xi),
\]
and consequently \( x(t) \) is a solution for Eq. (1) on \([0, T]\). By iteration, the existence of solutions to Eq. (1) can be obtained on \([0, T]\).

**Uniqueness:** Assume that we have two solutions \( x(t) \) and \( y(t) \) to Eq. (1) with \( x(0) = y(0) \), and consider the setup from the previous part, we get
\[
E|\{x(t) - y(t)\}|^2 \leq 3E \left[ t \int_0^t |f(t,s,x(s)) - f(t,s,y(s))|^2ds \right] \\
+ 3E \left[ \int_0^t |g(t,s,x(s)) - g(t,s,y(s))|^2ds \right] \\
+ 3E \left[ \int_0^t \int_{R_0} |h(t,s,x(s),\xi) - h(t,s,y(s),\xi)|^2v(d\xi)ds \right]
\leq 3T^2E \int_0^t \left[ |f(t,s,x(s)) - f(t,s,y(s))|^2 + |g(t,s,x(s)) - g(t,s,y(s))|^2 \right] \]
\[
+ \int_{R_0} |h(t,s,x(s),\xi) - h(t,s,y(s),\xi)|^2v(d\xi)ds.
\]
Next, applying Jensen’s inequality and Assumption \( H_2 \), yields
\[
E|\{x(t) - y(t)\}|^2 \leq 3T^2 \lambda(t) \int_0^t \kappa(E|\{x(s) - y(s)\}|^2ds,
\]
since we have \( E|\{x(t) - y(t)\}|^2 = 0 \) at \( t = 0 \), and \( \int_0^\infty \frac{ds}{s} = \infty \). Hence, by Lemma 2.1, we obtain
\[
E(\sup_{0 \leq s \leq t} |x(s) - y(s)|^2) = 0, \quad \forall \ t \in [0,T].
\]
Therefore, \( x(t) = y(t) \), for all \( t \in [0,T] \), which proves uniqueness.

Thus, the proof of Theorem 3.1 is completed.
Remark 3.5. In Theorem 3.1, under non–Lipschitz condition, we prove that the SVIEs have unique solutions. Moreover, in view of assumption $H_2$, if we set $\lambda(t)\kappa(|x|) = L|x|$, for some positive constant $L$, then in particular, we see that the Lipschitz condition is a special case of our proposed condition. In other words, we have generalised the existence and uniqueness results for SVIEs in $f$.

Example 3.6. Let us suppose that $W_t$ is a scalar Brownian motion, and $\tilde{N}(dt, d\xi)$ is a Poisson random measure with $\sigma$–finite measure $\nu(d\xi) = \lambda f(\xi)d\xi$, where $\lambda = 2$ is the jump rate and $f(\xi) = \frac{1}{\sqrt{2\pi}}\exp(-\frac{(\ln\xi)^2}{2})$, $0 < \xi < \infty$. Note that, $W_t$ and $\tilde{N}(dt, d\xi)$ are assumed to be independent.

Consider the following SIVE

$$x(t) = \varphi(t) + \frac{1}{2} \int_0^t x(s)ds + 4 \int_0^t \cos^2(t-s)x(s)dW_s$$

$$+ c \int_0^t \int_{R_0} \xi^2 x(s)\tilde{N}(ds, d\xi), \quad \varphi(t) = 1, \quad t \geq 0. \quad (11)$$

Here $f(t, s, x) = \frac{1}{2}x$, $g(t, s, x) = 4\cos^2(t-s)x$ and $h(t, s, x, \xi) = c\xi^2 x$, $c > 0$.

Obviously, the conditions of Theorem 3.1 is satisfied. Then, the SVIE (11) has a unique solution.

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