Comment on “Using Fermi Statistics to Create Strongly Coupled Ion Plasmas in Atom Traps”

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The fermionic exchange energy of ultracold ions is computed. It is shown that this effect allows to increase the ion coupling in traps by about an order of magnitude compared to the classical case.

In this Comment we analyze the prospects of producing a strongly coupled ion plasma by laser ionization of an ultracold gas. A recent Letter [1] suggested that the Fermionic exchange of the ensemble of atoms (before ionization) would allow to increase the ionic coupling drastically. While this general idea is correct, we show that the predicted “orders of magnitude” effect may only appear in a transient regime. For the finite state coupling an upper bound is derived which is much lower.

Ref. [1] assumes that after ionization the electrons equilibrate instantaneously whereas the ions form a one-component plasma with a statically screened Coulomb interaction (screening parameter \(\kappa = 1/rd = \sqrt{4\pi n_a e^2/kT_e}\) determined by the electrons). Initially ion equilibration proceeds under isolated conditions, i.e. total energy \(E\) remains constant

\[ E(t) = K(t) + U_F(t) + U(t) = K_0 + U_{F0} + U_0 = E_0. \]

Here, \(K\), \(U_F\) and \(U\) are, respectively, the kinetic, exchange (mean field or Fock) and correlation energy of the ions, and “0” denotes the initial state.

If the initial state is uncorrelated, \(U_{F0} = U_0 = 0\), the system is heating up \(K\) and \(U_F(t) \geq K_0\), as a consequence of growing attractive (negative) correlation energy, \(U(t) < 0\). Ref. [1] suggested to use ionic exchange, \(U_{F0} < 0\) to reduce the heating. This is reasonable since atom cooling below the Fermi temperature has already been achieved experimentally [3].

For a homogeneous plasma, the final state temperature \(T\) or coupling \(\Gamma \equiv U_c/k_BT\) (\(U_c = e^2/a\) is the mean Coulomb energy and \(a\) the mean interparticle distance), can be estimated from Eq. (1) by setting \(U_0 \to 0\),

\[ \frac{1}{\Gamma} + \frac{|U_{F0}(T_0)| - |U_F(T)| - k_BT_0}{U_c} = \frac{2}{3} \left( \frac{|u|}{\Gamma} - \frac{\kappa}{2} \right), \]

where the r.h.s. is the final state correlation energy \(U\) (in units of \(U_c\)), cf. [3]. Solutions of Eq. (2) are shown in Fig. 1 for potassium ions with density \(n = 1.1 \times 10^{11}\) cm\(^{-3}\) and \(\kappa = 4.5/a\). Note that the Fock energy \(U_{F0}\) differs from the familiar exchange energy as it involves the screened interaction instead of the Coulomb potential and is computed numerically. For the initial temperature \(T_0\) we choose 300nK (corresponding to \(T = 0.5T_F\) of ions in a trap [3]) and 6.6nK \((T = 0.5T_F\) in a homogeneous system studied in [1]). The highest possible effect can be estimated from the limit \(T_0 \to 0\) for which the exchange energy can be found analytically,

\[ \frac{U_F^e(0, \kappa)}{\nu \cdot U_c} = \frac{1}{4} \left( \frac{3}{2\pi} \right)^{2/3} \left\{ 3 - 2\alpha^2 - 8\alpha \arctan \frac{1}{\alpha} + 2\alpha^2(3 + \alpha^2) \ln \left(1 + \frac{1}{\alpha^2}\right) \right\}, \quad \text{with} \quad \alpha = \frac{\kappa}{2k_F}, \]

where \(k_F\) is the Fermi wave number and \(\nu = 2(1)\) for spin polarized (un-polarized) ions.

Our analysis reveals that the final state \(\Gamma\) indeed increases due to exchange effects, from 17 (without correlations) up to \(\Gamma \simeq 140\), for \(T_0 = 0\) and spin polarized ions, cf. Fig. 1. To achieve this, in homogeneous systems, requires temperatures below 100nK whereas, in traps, several 100nK should be sufficient.

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