SURFACE LAYER ACCRETION IN CONVENTIONAL AND TRANSITIONAL DISKS DRIVEN BY FAR-ULTRAVIOLET IONIZATION

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ABSTRACT

Whether protoplanetary disks accrete at observationally significant rates by the magnetorotational instability (MRI) depends on how well ionized they are. Disk surface layers ionized by stellar X-rays are susceptible to charge neutralization by small condensates, ranging from ~0.01 μm sized grains to angstrom-sized polycyclic aromatic hydrocarbons (PAHs). Ion densities in X-ray-irradiated surfaces are so low that ambipolar diffusion weakens the MRI. Here we show that ionization by stellar far-ultraviolet (FUV) radiation enables full-blown MRI turbulence in disk surface layers. Far-UV ionization of atomic carbon and sulfur produces a plasma so dense that it is immune to ion recombination on grains and PAHs. The FUV-ionized layer, of thickness 0.01–0.1 g cm−2, behaves in the ideal magnetoinductive limit and can accrete at observationally significant rates at radii ≳1–10 AU. Surface layer accretion driven by FUV ionization can reproduce the trend of increasing accretion rate with increasing hole size seen in transitional disks. At radii ≲1–10 AU, FUV-ionized surface layers cannot sustain the accretion rates generated at larger distance, and unless turbulent mixing of plasma can thicken the MRI-active layer, an additional means of transport is needed. In the case of transitional disks, it could be provided by planets.

Key words: accretion, accretion disks – instabilities – magnetohydrodynamics (MHD) – protoplanetary disks – stars: pre-main sequence – ultraviolet: stars

1. INTRODUCTION

T Tauri stars, Herbig Ae/Be stars, and young brown dwarfs accrete (e.g., Hartmann et al. 2006). Perhaps the most direct evidence for accretion comes from observations of radiation at blue to ultraviolet wavelengths, emitted in excess of the Wien tail of the stellar photosphere. The ultraviolet excess arises in large part from gas that is shock heated upon free-falling onto the stellar surface (Calvet & Gullbring 1998; Johns-Krull et al. 2000; Calvet et al. 2004). From these observations, and others of optical emission lines (e.g., Hartmann et al. 1994; Herczeg & Hillenbrand 2008), it is inferred that young solar mass stars accrete at rates $\dot{M} \approx 10^{-9}$–$10^{-7}$ $M_\odot$ yr$^{-1}$ (e.g., Muzerolle et al. 2005). Any individual system may exhibit order-unity variations in $\dot{M}$ with time (Eisner et al. 2010).

Strictly speaking, these observations imply only that gas in the immediate stellar vicinity—i.e., within a few stellar radii, in and around the star’s magnetosphere—is accreting. Gas and dust orbit at much greater stellocentric distances, residing in disks that in many cases extend continuously from ~0.1 to ~100 AU (e.g., Watson et al. 2007), but there is no direct indication that any of this material is actually accreting (but see Hughes et al. 2011 for an attempt at measuring non-Keplerian motions of circumstellar gas). Nevertheless the standard assumption is that in many cases extend continuously from ~0.1 to ~100 AU (e.g., Watson et al. 2007), but there is no direct indication that any of this material is actually accreting (but see Hughes et al. 2011 for an attempt at measuring non-Keplerian motions of circumstellar gas). Nevertheless the standard assumption is that in many cases extend continuously from ~0.1 to ~100 AU (e.g., Watson et al. 2007), but there is no direct indication that any of this material is actually accreting (but see Hughes et al. 2011 for an attempt at measuring non-Keplerian motions of circumstellar gas). Nevertheless the standard assumption is that in many cases extend continuously from ~0.1 to ~100 AU (e.g., Watson et al. 2007), but there is no direct indication that any of this material is actually accreting (but see Hughes et al. 2011 for an attempt at measuring non-Keplerian motions of circumstellar gas). Nevertheless the standard assumption is that in many cases extend continuously from ~0.1 to ~100 AU (e.g., Watson et al. 2007), but there is no direct indication that any of this material is actually accreting (but see Hughes et al. 2011 for an attempt at measuring non-Keplerian motions of circumstellar gas).
even for the comparatively feeble Solar wind. At cosmic ray energies of 0.1–1 GeV, the present-day Solar wind lowers the Galactic cosmic ray flux at Earth by roughly an order of magnitude compared to estimated fluxes outside the heliosphere (see Figure 1 of Reedy 1987). Moreover, the anti-correlation between the terrestrial cosmic ray flux and Solar activity has been clearly observed over many Solar cycles (Svensmark 1998). By comparison with the Solar wind, pre-main-sequence winds have mass loss rates that are five orders of magnitude larger, and thus we expect the shielding of disk surface layers from Galactic cosmic rays to be essentially complete for young stellar systems. Nevertheless cosmic rays might still leak in from the “side,” entering the disk edge-on from the outside. The impact of “sideways” cosmic rays is difficult to assess because whether cosmic rays can penetrate radially inward depends on the radial distribution of matter and on the magnetic field distribution, both of which are unknown. Cosmic rays might spiral toward the disk along interstellar field lines only to be mirrored back. If we assume that this does not happen, and are willing to be guided by the surface density profile of the minimum-mass solar nebula, we estimate that ionization by sideways cosmic rays might be significant on the outskirts of the disk at $a \gtrsim 30$ AU. Under these generous assumptions, cosmic rays can sustain MRI turbulence in the entire outer disk—from the disk surface to the midplane, at $a \gtrsim 30$ AU. We will return briefly to this scenario in Section 4.

Bai & Goodman (2009), Turner et al. (2007), and Turner et al. (2010) studied how disk surface layers could be made MRI active by stellar X-rays. They found $\Sigma^* \sim 1–30$ g cm$^{-2}$—the range over which photons of energy 1–10 keV are stopped. In their models, the precise extent of the MRI-turbulent region depended on the abundance of charge-neutralizing grains. X-ray-ionized layers, though less deep than the layer envisioned by Gammie (1996), were considered by Turner et al. (2010) to be capable of transporting mass at observationally significant rates at $a = 1$ AU.

All of the aforementioned studies of layered accretion assumed that Ohmic dissipation limits $\Sigma^*$, as gauged by $Re$ or the related Elsasser number (Turner et al. 2007). This assumption, widespread in the literature, overestimates $\Sigma^*$ and the vigor of MRI turbulence in X-ray-ionized surface layers. In a recent attempt to estimate $\Sigma^*$, Perez-Becker & Chiang (2011, hereafter PBC11) employed an X-ray ionization model similar to those used by previous workers, but accounted for two additional factors. First, following Chiang & Murray-Clay (2007), they tested whether ambipolar diffusion—the decoupling of neutral matter from plasma—limits $\Sigma^*$. They gauged the importance of ambipolar diffusion by computing at a given depth the number of times a neutral H$_2$ molecule collides with charged species in dynamical time $\Omega^{-1}$:

$$Am = \frac{n_{\text{charge}} \beta_{\text{in}}}{\Omega} \approx 1 \left(\frac{x_{\text{charge}}}{10^{-8}}\right) \left(\frac{n_{\text{tot}}}{10^{10} \text{ cm}^{-3}}\right) \left(\frac{a}{1 \text{ AU}}\right)^{3/2}. \quad (2)$$

Here $n_{\text{charge}} \equiv x_{\text{charge}} n_{\text{tot}}$ is the total number density of singly charged species (assuming multiply charged species are negligible), $x_{\text{charge}}$ is the fractional charge density, $n_{\text{tot}}$ is the total density of hydrogen nuclei (mostly in the form of neutral H$_2$), and $\beta_{\text{in}} \approx 2 \times 10^{-9}$ cm$^3$ s$^{-1}$ is the collisional rate coefficient for singly charged species to share their momentum with neutrals (Draine et al. 1983; Millar et al. 1997). Whereas $Re$ measures how well plasma is tied to magnetic fields, $Am$ assesses how well neutral gas is coupled to plasma. No matter how large $Re$ may be, unless $Am$ is also large, the magnetic stresses felt by the plasma will not be effectively communicated to the bulk of the disk gas, which is composed overwhelmingly of neutral H$_2$ (Blaes & Balbus 1994; Kunz & Balbus 2004; Desch 2004; Wardle 2007). Numerical simulations by Hawley & Stone (1998, hereafter HS) suggested that unless $Am \gtrsim Am^* \approx 10^2$, MRI turbulence would not be sustained in neutral gas. Ambipolar diffusion is of especial concern in disk surface layers because $Am$ depends on the absolute number density of charged particles and not just the relative density as in $Re$. At a fixed ionization fraction, the rapidly decreasing gas density with increasing height above the midplane renders the MRI increasingly susceptible to ambipolar diffusion.

The second factor considered by PBC11 concerned polycyclic aromatic hydrocarbons (PAHs). Like grains, these macromolecules act as sites of ion recombination. Their abundances in disk surface layers were constrained by PBC11 using Spitzer satellite observations of PAH emission lines, detected in a large fraction of Herbig Ae stars surveyed (Geers et al. 2006). Disk PAHs were found to reduce $Am$ and $Re$ by factors $\gtrsim 10$ (PBC11). For typical stellar X-ray spectra, $Am$ reached maximum values on the order of unity at $\Sigma \sim 1$ g cm$^{-2}$. Such maximum values were attained only for the lowest plausible PAH abundances. Comparing $\max Am \approx 1$ with HS’s determination that $Am^* \sim 10^2$, PBC11 concluded that ambipolar diffusion, abetted by PAHs, reduced accretion rates in X-ray-ionized surface layers to values too small compared to observations.

The conclusion of PBC11 has since been tempered by two new developments. First, PBC11 underestimated $Am$ because they omitted the contribution from momentum-coupling collisions between H$_2$ and charged PAHs. This neglect is potentially significant because charged PAHs are as well coupled to the magnetic field as are atomic/molecular ions (HCO$^+$ and metal ions like Mg$^+$ in their simple model). They argued that because the abundance of charged PAHs is less than that of molecular ions, $Am$ should not be significantly increased. This statement applies for the lowest PAH abundance considered in PBC11 of $x_{PAH} \sim 10^{-11}$ (here measured per H), but Bai (2011) has pointed out that PAHs can dominate the charge budget at higher $x_{PAH}$. In this high-PAH limit, although PAHs decrease the free electron abundance and therefore $Re$, they can boost $Am$—and in principle $M$—in comparison to the low-PAH case. In this paper we correct for this effect by taking

$$n_{\text{charge}} = n_1 + n_{PAH(Z=-1)} + n_{PAH(Z=1)} \quad (3)$$

when computing $Am$, where the various densities refer to atomic/molecular ions, PAHs with a single negative charge, and PAHs with a single positive charge, respectively.

In another recent development, Bai & Stone (2011; hereafter BS11) undertook new simulations of the MRI with ambipolar diffusion to update those of HS. They confirmed the result of HS that when $Am \gtrsim 10–10^2$, neutrals and ions behave essentially as one fluid with a Shakura–Sunyaev transport parameter $\alpha$ that can be as high as $\sim 0.1–0.5$. Their new result is that MRI turbulence can still be sustained in the neutrals for $Am \lesssim 1$, albeit in a weakened form with max $\alpha$ decreasing with decreasing $Am$. When

\footnote{Some workers wish to ignore PAHs when studying X-ray-driven chemistry on the grounds that they are rarely detected in T Tauri stars. It is hard for us to follow this argument. Herbig Ae disks commonly evince PAHs and exhibit accretion rates similar to if not higher than those of T Tauri disks. Treating the problem of T Tauri disk accretion separately from the problem of Herbig Ae disk accretion seems unjustified. As discussed by Geers et al. (2006), PAHs usually go undetected in T Tauri stars not because they are absent but because they fluoresce less luminously in the weaker ultraviolet fields of their host stars.}
Am ≈ 1(0.1), BS11 find that max α ≈ 0.01(0.0007). Whether α attains its maximum value for a given Am depends on the assumed strength and geometry of the background magnetic field.

In this paper we consider a third, often neglected source of ionization: far-ultraviolet (FUV ≡ photon energies between ~6 and 13.6 eV) radiation emitted by the central star. As measured with the Hubble Space Telescope and Far-Ultraviolet Spectroscopic Explorer (FUSE), typical FUV luminosities from young stars are \( \gtrsim 10^{30} \text{ erg s}^{-1} \), much of it in atomic lines (e.g., Bergin et al. 2007). This radiation originates from the stellar accretion shock (Calvet & Gullbring 1998; Johns-Krull et al. 2000; Calvet et al. 2004) and from the active stellar chromosphere (Alexander et al. 2005). Photoionization/photodissociation regions generated by FUV radiation have been studied extensively (e.g., Gorti & Hollenbach 2008, and references therein; Tielens & Hollenbach 1985), but their implications for the MRI have not been well quantified. Semenov et al. (2004) carried out detailed calculations of the disk ionization fraction due to X-ray and FUV radiation in order to estimate the extent of the MRI-active layer, but they did not consider the charge-adsorbing effects of PAHs or ambipolar diffusion. We seek here to give a more comprehensive treatment, including new estimates of \( M \) driven by FUV ionization, made possible in part by the powerful new results of BS11.

High in the disk atmosphere, molecules are photodissociated and elements take their atomic form. We focus on FUV photons (having energies less than the H Lyman limit) because they are not stopped by atomic hydrogen and as such penetrate more deeply into the disk than do Lyman continuum photons. Far-UV photons ionize trace atoms. Some elements, when nearly fully ionized, may be so abundant that the criteria for MRI accretion are easily satisfied. Carbon, for example, has a first ionization energy of 11.26 eV and a cosmic number abundance relative to hydrogen of approximately \( 2.9 \times 10^{-4} \) (Lodders 2003, her Table 2). The Strömgren layer of C\( ii \) generated by FUV radiation is characterized by ionization fractions—and, by extension, values of Am and Re—up to \( 10^3 \) times larger than those reported by PBC11 for X-ray-ionized layers of H\( 2 \). Thus ambipolar diffusion may not pose the same threat in FUV-ionized layers that it does in X-ray-ionized layers. Moreover, electron densities in FUV-ionized layers may be so large that they are little impacted by PAHs or other small grains.

Of the various elements that can be ionized by FUV radiation, we model in this paper only carbon and sulfur. Their cosmic abundances are relatively high, and they are among the least likely elements to be depleted onto grains (as evidenced in the diffuse interstellar medium; e.g., Jenkins 2009; Savage & Sembach 1996). Line emission from ionized carbon ([C\( ii \)] \( 158 \mu m \)) has been detected by Herschel in protoplanetary disks (Pinte et al. 2010; Sturm et al. 2010). The observed line fluxes can be reproduced by models that assume a cosmic gas phase abundance of carbon (Woitke et al. 2010), although our own analysis shows that lower abundances are also possible because the [C\( ii \)]-emitting layer is optically thick to its own emission. The abundance of sulfur has not been so constrained because no lines from sulfur have been unambiguously detected in disks (Lahuis et al. 2007; Watson et al. 2007; Gorti & Hollenbach 2008). Nevertheless Meijerink et al. (2008) found that a sulfur abundance of up to \( 7 \times 10^{-6} \) per H, or \( \sim 2/5 \) that of cosmic (Lodders 2003), could be reconciled with the non-detection of the [S\( i \)] \( 25 \mu m \) line. The true upper limit on the sulfur abundance may be even higher because Meijerink et al. (2008) did not account for FUV ionization of S\( i \).

Our paper is structured as follows. The equations governing the ionization balance of carbon and sulfur in disk surface layers are presented in Section 2. Account is made of PAHs; sub-micron-sized grains which can attenuate FUV radiation; and H\( 2 \) which can absorb photons having energies \( >11.2 \) eV. Results for the carbon and sulfur ionization fronts, and the values for Am, Re, and \( \Sigma^* \) they imply, are given in Section 3. The impact of the Hall effect (e.g., Wardle & Salmeron 2011) on FUV-ionized layers is also considered in Section 3. A summary is supplied in Section 4, together with estimates of \( M \) as a function of disk radius, as driven by FUV radiation, X-rays, or cosmic rays. We compare our results to stellar accretion rates observed in conventional T Tauri disks and transitional disks with inner optically thin holes.

2. MODEL FOR FUV IONIZATION

At a given radius \( a \) from a star of mass \( M = 1 M_\odot \), we compute the equilibrium abundances of H\( 2 \), H\( i \), C\( i \), C\( ii \), S\( i \), and S\( ii \) as a function of the vertical column penetrated by FUV radiation. The key equations governing the ratios of H\( 2 \):H\( i \), C\( i \):C\( ii \), and S\( i \):S\( ii \) are, qualitatively,

\[
\text{Rate of dissociation of H}_2 \text{ by FUV photons} = \text{Rate of formation of H}_2 \text{ on grains},
\]

\[
\text{Rate of ionization of C}_i \text{ by FUV photons} = \text{Rate of recombination of C}_i \text{ with electrons, grains, and PAHs},
\]

and

\[
\text{Rate of ionization of S}_i \text{ by FUV photons} = \text{Rate of recombination of S}_i \text{ with electrons, grains, and PAHs}.
\]

For our standard model we take \( a = 3 \text{ AU} \). This parameter and others will be varied to explore their impact on the MRI-active column \( \Sigma^* \).

2.1. FUV Luminosity

Two ingredients of our model, H\( 2 \) and C, are dissociated and ionized, respectively, by FUV photons having practically the same wavelength range: \( \lambda \approx 912\text{–}1109 \mu m \). A third ingredient, S, is ionized by photons having \( \lambda < 1198 \mu m \). In the combined wavelength interval \( \lambda \approx 912\text{–}1198 \mu m \) we take the stellar luminosity of our standard model to be \( L_{\text{FUV}} = 10^{30} \text{ erg s}^{-1} \), in continuum photons. We take \( L_{\text{FUV}} \) to be distributed uniformly over this wavelength interval so that approximately \( (2/3) L_{\text{FUV}} \) can be absorbed by H\( 2 \), C, and S, while the remaining \( (1/3) L_{\text{FUV}} \) can be absorbed by S but not H\( 2 \) or C.

Our choice for the continuum luminosity is compatible with FUSE observations of T Tauri stars (Berger et al. 2003, see their Figure 1; admittedly their spectra are extrapolated at \( \lambda < 950 \mu m \) for TW Hydra and \( \lambda < 1150 \mu m \) for BP Tau). Our standard \( L_{\text{FUV}} \) underestimates the true ionizing luminosity because we neglect commonly observed FUV emission lines (but note that our wavelength range does not include the powerful H Ly\( \alpha \) line at 1216 \AA\;). It also neglects the large contribution from the hotter photospheres of more massive stars (e.g., Martin-Zaïdi et al. 2008). We make some account for these effects by varying \( L_{\text{FUV}} \) up to \( 10^{32} \text{ erg s}^{-1} \) in our parameter survey (Section 3.3).
2.2. Total Gas Columns and Densities

We present our results as a function of the total vertical hydrogen column density \( N_\text{tot} = N_\text{H} + 2N_\text{H}_2 \), measured perpendicular to and toward the disk midplane. An equivalent measure is the mass surface density \( \Sigma \equiv N_\text{tot}m_\text{H} \), where \( m_\text{H} = 1.7 \times 10^{-24} \text{ g} \) is the mass of the hydrogen atom. The local number density of hydrogen nuclei is approximated by

\[
n_\text{tot} \approx N_\text{tot}/h. \tag{7}\n\]

Stellar radiation enters the disk at a grazing angle \( \theta \sim 3\theta_0/\alpha \) measured from the flared disk surface (e.g., Chiang et al. 2001). We assume in this work that photons travel in straight paths, i.e., we neglect scattering of FUV radiation into directions other than that of the original beam entering the disk. Thus a beam of radiation that penetrates a vertical column \( N_\text{tot} \) has traversed a larger column parallel to the incident beam direction of \( \sim N_\text{tot}/\theta \), and is attenuated according to the latter quantity, not the former. To order of magnitude, \( \theta \sim 0.3 \).

2.3. Gas Temperature

Gas temperatures in disk surface layers are set by a host of heating and cooling processes. Models in the literature account differently for these processes and yield temperatures which disagree. At a vertical column of \( N_\text{tot} \sim 10^{22} \text{ cm}^{-2} \)—roughly where the deepest of the fronts we compute, the S/\( \text{Si} \) II ionization front, may lie—Glassgold et al. (2007, hereafter GNI07) reported a gas temperature of \( T \approx 60 \text{ K} \) at \( a \approx 5-10 \text{ AU} \) (see their Figure 2). By comparison, Gorti & Hollenbach (2008, hereafter GH08) found that at \( a = 8 \text{ AU} \), the S ionization front occurs at \( T \approx 600 \text{ K} \) (see their Figures 1 and 5). One reason for this difference seems to be that GNI07 omitted photoelectric heating from PAHs, which dominates thermal balance according to GH08. But the PAH abundance assumed by GH08 seems too high, exceeding by about an order of magnitude the highest plausible abundance inferred by PBC11. Reducing the heating rate from PAHs by more than a factor of 10 from its value in GH08 (see their Figure 4) would imply that X-ray heating dominates thermal balance—as it tends to do by default in GNI07.

In this paper we do not solve the thermal balance equations, but instead approximate the disk surface layer as vertically isothermal at a temperature drawn from GNI07. This is similar to the choice we made in PBC11, except that there we were interested in X-ray-ionized columns \( N_\text{tot} \gtrsim 10^{22} \text{ cm}^{-2} \), whereas here we are interested in FUV-ionized columns \( N_\text{tot} \lesssim 10^{22} \text{ cm}^{-2} \). As \( N_\text{tot} \) decreases from \( 10^{22} \) to \( 10^{21} \text{ cm}^{-2} \), gas temperatures in GNI07 rise by factors of \( \sim 2-3 \). At still higher altitudes for which \( N_\text{tot} < 10^{21} \text{ cm}^{-2} \), temperatures rise steeply by an order of magnitude or more. As a simple compromise, we multiply the temperature profile of PBC11 by a factor of three but otherwise keep the same scaling:

\[
T \approx 240 \left( \frac{a}{3 \text{ AU}} \right)^{-3/7} \text{ K}. \tag{8}\n\]

Our assumption that the gas is vertically isothermal ignores the steep temperature gradient at \( N_\text{tot} < 10^{21} \text{ cm}^{-2} \) where dust-gas cooling is not effective. This neglect should not be serious as the amount of mass contained in these uppermost layers is small compared to the mass in the MRI-active layer, which we will find is concentrated near the \( \text{C} \) and \( \text{S} \) ionization fronts, located within \( N_\text{tot} \sim 10^{21}-10^{23} \text{ cm}^{-2} \). Moreover, the locations of the fronts are not especially sensitive to temperature, as we have verified with a few test cases.

2.4. Very Small Grains (VSGs)

Very small grains (VSGs) having sizes of order 0.01 \( \mu \text{m} \) can attenuate FUV radiation. We parameterize this absorption with the variable \( N_{\nu \text{VSG}}^{\text{tot}} \), the total gas column for which VSGs present unit optical depth in the FUV.\footnote{Equivalent to the variable \( n_\text{eq} \) used by GH08.} We consider \( N_{\nu \text{VSG}}^{\text{tot}} \) between \( 10^{21} \) and \( 10^{22} \text{ cm}^{-2} \), taking as our standard value \( N_{\nu \text{VSG}}^{\text{tot}} = 10^{22} \text{ cm}^{-2} \). In the diffuse interstellar medium, \( N_{\nu \text{VSG}}^{\text{tot}} \approx 10^{21} \text{ cm}^{-2} \); its value in disk surface layers may be much higher—by 1–3 orders of magnitude as judged by disk infrared spectral energy distributions (see Table 3 of PBC11, and references therein)—because of grain growth and sedimentation.

Colliding with ions and faster moving electrons, VSGs carry on average a net negative charge. They therefore act as sites of ion recombination. To account for such recombination, we model VSGs as conducting spheres of radius \( s_{\text{VSG}} = 0.01 \mu \text{m} \). Their number abundance relative to hydrogen is

\[
x_{\text{VSG}} \equiv n_{\text{VSG}}/n_\text{tot} = \frac{1}{N_{\nu \text{VSG}}^{\text{tot}} \pi s_{\text{VSG}}^2}. \tag{9}\n\]

In PBC11, we solved for the detailed charge distributions of grains using a set of recurrence equations. Solving the recurrence equations to obtain self-consistent solutions for the electron density, the ion density, and the total charge carried by grains/PAHs was necessary because in the lightly ionized regions irradiated by X-rays, the charge carried by PAHs could be significant compared to the amount of charge in free electrons. By comparison, FUV-ionized regions are simpler to treat because free electrons are orders of magnitude more abundant than grains/PAHs. Thus our analysis here is simplified: we assume that every grain has an equilibrium charge \( Z_{\text{VSG}} \) such that it collides with ions as frequently as it collides with electrons. That is, \( Z_{\text{VSG}} \) is such that the rate coefficients (units of cm\(^3\) s\(^{-1}\)) \( \alpha_{\nu \text{VSG},\text{ion}} \) and \( \alpha_{\nu \text{VSG},\text{e}} \) as given by Equations (9) and (10) of PBC11—are equal (see Figures 3 and 4 of PBC11). Under this approximation, the rate at which an ion (\( \text{C} \text{II} \) or \( \text{S} \text{II} \)) neutralizes by colliding with VSGs is simply \( n_{\nu \text{VSG}} \alpha_{\nu \text{VSG},\text{ion}} \), where \( \alpha_{\nu \text{VSG}} = \alpha_{\nu \text{VSG},\text{ion}} = \alpha_{\nu \text{VSG},\text{e}} \). The values of \( \alpha_{\nu \text{VSG}} \) so derived differ by 50%–70% depending on whether the dominant ion is \( \text{C} \text{II} \) (atomic weight 12) or \( \text{S} \text{II} \) (atomic weight 32); for simplicity we take a mean value. Given our choices for \( T \) and \( s \), and assumed sticking coefficients between VSGs and electrons/ions of 1, we find that for our standard model \( Z_{\text{VSG}} \approx -1 \) in units of the electron charge,\footnote{Our \( Z \) differs slightly from the true average \( \langle Z \rangle \) calculated by PBC11.} and the corresponding \( \alpha_{\nu \text{VSG}} \approx 2.5 \times 10^{-6} \text{ cm}^3 \text{ s}^{-1} \).

A few parting comments about our treatment of VSGs: first, in deciding how VSGs attenuate FUV radiation, we do not make any explicit assumption about the grain size distribution. Our proxy for flux attenuation \( N_{\nu \text{VSG}}^{\text{tot}} \) is the column at which FUV radiation is absorbed by grains of all sizes. Flux attenuation by VSGs will play a significant role in determining the extent of MRI activity (Section 3.3). Second, in deciding how ions recombine on VSGs, we do make an explicit assumption about grain size, namely we take \( s_{\text{VSG}} = 0.01 \mu \text{m} \). Ignoring larger grain sizes maximizes the number abundance and geometric surface area of VSGs, and thus maximizes their relevance for charge balance. With this choice for grain size, ion recombination on VSGs is competitive with other recombination pathways for our standard model \( (N_{\nu \text{VSG}}^{\text{tot}} = 10^{22} \text{ cm}^{-2}) \), but is negligible for more dust-depleted models \( (N_{\nu \text{VSG}}^{\text{tot}} > 10^{22} \text{ cm}^{-2}) \).
Ion recombination on PAHs is treated analogously to ion recombination on VSGs. Each PAH has radius 6 Å, an electron (ion) sticking coefficient of 0.1 (1), and thus an average charge $Z_{\text{PAH}} \approx -0.05$ at $a = 3$ AU. The corresponding ion-PAH rate coefficient $\alpha_{\text{PAH}} \approx 1 \times 10^{-8}$ cm$^3$ s$^{-1}$.

For our standard (fiducial) model, we take the number abundance of PAHs relative to hydrogen nuclei to be $n_{\text{PAH}} = n_{\text{PAH}}/n_{\text{tot}} = 10^{-8}$. This is their maximum plausible abundance, as inferred by PBC11. For this PAH abundance, the ion recombination rate on PAHs will turn out to be only comparable to the ion recombination rate with free electrons in FUV-ionized layers (the contribution from PAHs was much more significant for the deeper and more poorly ionized layers considered in the X-ray model of PBC11). Thus we expect the thickness of the FUV-irradiated, MRI-active layer to be insensitive to PAHs, a finding we highlight at the end of Section 3.3.

Note that under our simplified treatment of recombination on condensates, the net charge of our system is not zero: VSGs and PAHs carry a net negative charge, which together with the contribution from free electrons is not balanced by the ion recombination rate with free electrons in FUV-ionized layers. However the deviation from charge neutrality (which was not present in PBC11 because that study accounted explicitly for collisional charging of condensates by ions and electrons) is small. That is, we have verified that $Z_{\text{PAH}}n_{\text{PAH}} + Z_{\text{VSG}}n_{\text{VSG}}$ is much less than the electron number density over the domain of our calculation.

### 2.5. Polycyclic Aromatic Hydrocarbons (PAHs)

Molecular hydrogen is photodissociated by FUV photons in two steps: a photon of wavelength $912$ Å < $\lambda < 1109$ Å sends the molecule into the first-excited electronic state, and then in the subsequent radiative decay, there is a $p = 23\%$ probability that the molecule lands in a vibrationally excited electronic state where it becomes unbound (Dalgarno & Stephens 1970). Our calculation of the FUV dissociation rate of H$_2$ molecules follows that of de Jong et al. (1980), according to which there are $\eta_1 = 60$ vibrationally split line transitions connecting the ground and first-excited electronic states. These lines constitute the so-called Lyman band.

The dissociation rate is

$$R_{\text{diss}} = \frac{\pi e^2 f/(m_e c)}{\nu} \frac{(2/3)L_{\text{FUV}}}{4\pi a^2 h \nu} \beta_{\text{VSG}} \beta_{\text{H}_2}.$$  \hspace{1cm} (10)

All lines of frequency $\nu$ (energy $h \nu \approx 12$ eV) in the Lyman band are assumed to be equally spaced and to have equal oscillator strengths $f = 4.6 \times 10^{-3}$. The factor $\pi e^2 f/(m_e c)$ is the frequency-integrated cross section of a single line, where $e$ and $m_e$ are the charge and mass of an electron, and $c$ is the speed of light. Thus $\eta_1[\pi e^2 f/(m_e c)]/\nu \equiv \sigma_{\text{eff}}$ approximates the effective cross section of the entire Lyman band, averaged over the incident broadband spectrum of frequency width $\sim \nu$. The factor of $2/3$ accounts approximately for the fraction of $L_{\text{FUV}}$ (as we have defined it for 912 Å < $\lambda < 1198$ Å) that covers the Lyman band (see Section 2.1).

The factor $\beta_{\text{VSG}} = \exp[-N_{\text{tot}}/(\theta N_{\text{VSG}}^{\text{vert}})]$ accounts for attenuation of FUV radiation by VSGs (recall that $N_{\text{tot}}$ is the vertical column so that $N_{\text{tot}}/\theta$ is the column parallel to the incident beam of radiation). The factor $\beta_{\text{H}_2}$ accounts for attenuation of H$_2$-dissociating photons by intervening H$_2$ (de Jong et al. 1980):

$$\beta_{\text{H}_2} = \left( \frac{1}{\tau_{\text{H}_2} \left[ \ln \left( \tau_{\text{H}_2}/\sqrt{\pi} \right) \right]^{1/2}} + \frac{b}{\tau_{\text{H}_2}} \right) \text{erfc} \left( \frac{\sqrt{\tau_{\text{H}_2} b}}{\pi \Delta^2} \right),$$  \hspace{1cm} (11)

where $\text{erfc}$ is the complementary error function. The core of each line is assumed to be Doppler broadened with frequency width $\Delta \nu_D \equiv c v/\theta$; then

$$\tau_{\text{H}_2} = \frac{\pi e^2 f}{m_e c \Delta \nu_D} N_{\text{H}_2}/\theta$$  \hspace{1cm} (12)

is a measure of the optical depth at line center along the incident beam. The Voigt parameter

$$b = \gamma/(4\pi \Delta \nu_D)$$  \hspace{1cm} (13)

compares the natural line width $\gamma = 1.16 \times 10^9$ s$^{-1}$ to the Doppler width, and the non-dimensional parameter

$$\Delta = \Delta \nu_L/(2\eta_1 \Delta \nu_D)$$  \hspace{1cm} (14)

compares the line-spacing to the Doppler width, where $\Delta \nu_L = 6 \times 10^{14}$ Hz is the frequency width of the entire Lyman band. Equation (11) applies only for $\tau_{\text{H}_2} > 1$, a condition which is valid over the domain of our calculation.

Although the photodissociating continuum for H$_2$ and the photoionizing continuum for C I nearly overlap in the FUV, we can safely neglect shielding of H$_2$ by C I because in our model the H$_2$ dissociation front occurs at a higher altitude (lower $N_{\text{tot}}$) than the C ionization front. By the same token, shielding of C I by H$_2$ cannot be neglected (see Section 2.7 where it is accounted for by the factor $\beta_{\text{C,I}}$).

Molecular hydrogen forms when two H atoms combine on a grain surface, releasing the heat of formation to the grain lattice (Gould & Salpeter 1963). The rate of formation is given by

$$R_{\text{form}} = \frac{1}{2} n_{\text{H}} n_{\text{VSG}} \pi a^2 \chi_{\text{VSG}}^2 v_{\text{H}} n_{\text{H}}.$$  \hspace{1cm} (15)

where $n_{\text{H}}$ and $v_{\text{H}}$ are the number density and mean thermal speed of atomic hydrogen, respectively, and $\eta \sim 0.2$ is the formation efficiency on olivine grains (Cazaux & Tielens 2010).

At each depth in our one-dimensional model, we solve for $n_{\text{H}}/n_{\text{H}}$ using the equilibrium condition (4):

$$R_{\text{diss}} = R_{\text{form}}.$$  \hspace{1cm} (16)

### 2.6. H$_2$: Photodissociation and Re-formation on Grains

We take the abundance of atomic carbon in the gas phase to be $x_C = n_C/n_{\text{tot}} = 10^{-4}\epsilon$, where the dimensionless factor $\epsilon$ accounts for the sequestration of carbon into grains. For our standard model, $\epsilon = 1$, which gives a gas phase carbon abundance similar to that of the diffuse interstellar medium (Jenkins 2009). By using [C II] 158 μm line fluxes from disks as measured by the Herschel satellite (Pinte et al. 2010), we estimate a lower bound on $\epsilon$ of $\sim 1/30$.

Equation (5) for photoionization equilibrium of carbon is quantified as

$$\frac{(2/3)L_{\text{FUV}}}{4\pi a^2 h \nu} n_{\text{C}} \sigma_{\text{SI}} \beta_{\text{C,I}} \beta_{\text{H}_2} \approx n_{\text{C}}(n_{\tau_{\text{H}_2}} + n_{\text{PAH}} \alpha_{\text{PAH}} + n_{\text{VSG}} \alpha_{\text{VSG}}).$$  \hspace{1cm} (17)
where \( h \nu \approx 12 \) eV is the typical FUV photon energy, \( n_{C_i} \) is the electron number density, \( \sigma_{b,e} = 10^{-17} \) cm\(^2\) is the bound-free cross section for \( e \) (Craddock et al. 1974), and \( \alpha_{rec,c} = 5.3 \times 10^{-12} (T/240 K)^{-0.6} \) cm\(^3\) s\(^{-1}\) is the radiative recombination rate coefficient for \( C_2 \) (Pequignot et al. 1991). The factor of \( 2/3 \) accounts for the fraction of \( L_{\text{FUV}} \) (as we have defined it in Section 2.1) covered by the photoionizing continuum for \( C \). The factor \( \beta_{C} = \exp(-N_{C}/\sigma_{b,e}/\theta) \) accounts for self-shielding of carbon, where \( N_{C} \) is the vertical column of \( C \). The factor \( \beta_{C,H} \) accounts for shielding of carbon by the Lyman band transitions of molecular hydrogen (de Jong et al. 1980):

\[
\beta_{C,H} = \left(1 + \frac{\tau_H b}{\pi \Delta^2}\right)^{-1} \exp \left(\frac{\tau_H \nu}{\pi \Delta^2}\right).
\]

This factor drops significantly when the \( H_2 \) column \( N_{H_2}/\theta \approx 10^{22} \) cm\(^{-2}\)—large enough for the Lorentzian wings of each Lyman line to blacken the entire Lyman band. As noted previously, all the FUV photons that can ionize \( C_i \) \((1104 \AA > \lambda > 912 \AA)\) can also dissociate \( H_2 \) \((1109 \AA > \lambda > 912 \AA)\). This is true for \( C \) but not for other elements like \( S \).

We neglect shielding of \( C \) by \( S \) because the \( S \) ionization front lies at a greater depth than that of the \( C \) ionization front (because \( S \) is a factor of 10 less abundant than \( C \) everywhere).

### 2.8. Sulfur: Abundance and Ionization Equilibrium

The abundance of gas-phase atomic sulfur is \( x_S \equiv n_S/n_{\text{tot}} = 10^{-5} \). For our standard model, \( \epsilon = 1 \), which yields a sulfur abundance similar to that of the diffuse interstellar medium (Jenkins 2009). For simplicity, we assume that the depletion abundance similar to that of the diffuse interstellar medium is proportional to \( (2/3)L_{\text{FUV}}/4\pi a^2 \) for \( S \) (Jenkins 2009). For our standard model, \( \epsilon = 1 \).

Equation (6) for photoionization equilibrium of sulfur is quantified as

\[
\frac{1}{3}L_{\text{FUV}} = 4\pi a^2 h\nu n_{S_i} \sigma_{\text{b,S}} \beta_S \phi_{\text{VSG}}
= n_{S_i}(n_e \sigma_{\text{rec,S}} + n_{PAH} \sigma_{PAH} + n_{VSG} \sigma_{VSG}),
\]

where \( h \nu \approx 11 \) eV, \( \sigma_{b,S} \approx 5.5 \times 10^{-17} \) cm\(^2\) is the photoionizing cross section for \( S \) (Craddock et al. 1974), \( \sigma_{\text{rec,S}} = 4.9 \times 10^{-12} (T/240 K)^{-0.6} \) cm\(^3\) s\(^{-1}\) is the radiative recombination rate coefficient for \( S \) (Aldrovandi \\& Pequignot 1973; Gould 1978), and \( n_{S_i} (n_S = n_{S_1} = n_{S_S}) \) is the neutral (ionized) sulfur number density. The factor \( \beta_S = \exp(-N_S/\sigma_{b,S}/\theta) \) accounts for self-shielding of sulfur, where \( N_S \) is the vertical column of \( S_1 \).

The factor of \( 1/3 \) accounts for the fraction of \( L_{\text{FUV}} \) (as we have defined it for 912 \AA < \lambda < 1198 \AA in Section 2.1) that can ionize \( S \) but not be absorbed by \( C_1 \) and \( H_2 \). Technically there should be a second term on the left-hand side of (19) that is proportional to \( 2/3 \) of \( L_{\text{FUV}} \), to account for radiation that can ionize \( S \) and be absorbed by \( C_1 \) and \( H_2 \). But this term can be dropped because it does not significantly influence the location of the \( S \) ionization front; the term is negligibly small there, at a greater depth than the \( C \) ionization front. Note that the governing equation for sulfur is coupled to the other equations for \( C \) and \( H_2 \) only via the electron density \( n_e = n_{C_1} + n_{S_1} \).

### 2.9. Numerical Method of Solution

Our calculation starts at \( N_{\text{tot}} = 10^{18} \) cm\(^{-2}\), under the assumption that the incident flux \( L_{\text{FUV}}/4\pi a^2 \) is not attenuated at this column. We take \( 10^3 \) logarithmically spaced steps to \( N_{\text{tot}} = 10^{24} \) cm\(^{-2}\). At each step, we solve for the ratio \( n_{H_2}/n_{\text{tot}} \), using the balance condition (16). The self-shielding factor \( \beta_{H_2} \) depends on \( N_{H_2} \) and so we keep a running integral of \( n_{H_2} \), i.e.,

\[
N_{H_2} = \int (n_{H_2}/n_{\text{tot}})(\Delta N_{\text{tot}}/n_{\text{tot}}).
\]

By approximating \( n_e = \max(n_{C_1}, n_{S_1}) \), we may solve analytically for the ionization states of \( C \) and \( S \). At low \( N_{\text{tot}} \), nearly all \( C \) and \( S \) are ionized and \( n_e = n_{S_1} \) with an error of \( x_C/x_S \approx 10\% \). Equation (17) becomes a quadratic for \( n_{C_1}/n_{C_1} \), and \( n_{S_1}/n_{S_1} \) is determined by substituting \( n_{C_1} \) for \( n_e \) in Equation (19).

At \( N_{\text{tot}} \) past the \( C \) ionization front, \( S \) becomes the dominant ion and we set \( n_e = n_{S_1} \). Now Equation (19) becomes a quadratic for \( n_{S_1}/n_{S_1} \). The resultant value for \( n_{S_1} \) is substituted for \( n_e \) in Equation (17) to solve for \( n_{C_1}/n_{C_1} \).

At each \( N_{\text{tot}} \), we keep track of the running columns \( N_{C_1} = \int (n_{C_1}/n_{\text{tot}})(\Delta N_{\text{tot}}/n_{\text{tot}}) \) and \( N_{S_1} = \int (n_{S_1}/n_{\text{tot}})(\Delta N_{\text{tot}}) \) to compute the self-shielding factors \( \beta_C \) and \( \beta_S \), respectively.

### 3. RESULTS FOR MRI-ACTIVE SURFACE DENSITIES

In Section 3.1 we describe how the \( H_2:H, C \) ratios vary with depth. In Section 3.2 we estimate the surface density \( \Sigma^* \) of the layer that can be MRI active. In Section 3.3 we perform a parameter study of how \( \Sigma^* \) varies with system properties. The MRI-active surface densities \( \Sigma^* \) reported in Section 3.3 are those resulting from FUV ionization only. In Section 3.4 we check that the Hall effect is not significant for FUV-ionized surface layers. All our calculations neglect turbulent mixing of plasma from disk surface layers into the disk interior. In Section 3.5, we try to assess whether this neglect is justified, by comparing the dynamical timescale with the timescale over which the FUV-irradiated layer equilibrates chemically.

### 3.1. Photodissociation and Ionization Fronts

In the top panel of Figure 1, we show the relative abundances of species as a function of vertical mass column \( \Sigma \). High in the atmosphere, hydrogen is mostly in atomic form and nearly all of the carbon and sulfur are ionized. Deeper down, starting at \( \Sigma \approx 3 \times 10^{-5} \) g cm\(^{-2}\), atomic \( H \) gives way to \( H_2 \). At the \( H_2:H \) front the Doppler broadened cores of the various \( H_2 \) Lyman transitions have become optically thick. Far-UV photons in the wings of the Lyman transitions can stream past the \( H_2:H \) front and continue to ionize \( C \) and \( S \) at greater depths.

The \( C \) \& \( C \) ionization front is located at \( \Sigma \approx 3 \times 10^{-5} \) g cm\(^{-2}\), where the optical depth to carbon ionizing photons is of order unity. At this front, both shielding of carbon by \( H_2 \) (quantified by \( \beta_{C,H} \)) and carbon self-shielding (\( \beta_C \)) are becoming significant; see the bottom panel of Figure 1.

Far-UV photons having 1198 \AA > \lambda > 1109 \AA interact with neither \( H_2 \) nor \( C \), but can ionize \( S \). These photons penetrate both \( H_2 \) and \( C \) fronts, and ionize \( S \) at greater depths. The \( S \) \& \( S \) ionization front is located at \( \Sigma \approx 1 \times 10^{-2} \) g cm\(^{-2}\). In this fiducial model, the location of the \( S \) ionization front is determined more by flux attenuation by VSGs than by self-shielding by sulfur; at the front, \( \beta_{\text{VSG}} \approx 0.2 \) and \( \beta_S \approx 1 \). Lower but still observationally realistic grain abundances are considered in Section 3.3, where we will find that the \( S \) ionization front can be as deep as \( \Sigma \approx 1 \times 10^{-1} \) g cm\(^{-2}\).
Figure 1. H$_2$ photodissociation front, and C and S ionization fronts, for our standard model at $a = 3$ AU. Results are given as functions of the vertical surface density $\Sigma$, measured perpendicular to the disk midplane (the column actually penetrated by radiation parallel to the incident beam direction is $\Sigma/\theta$). The corresponding number density of H nuclei is given on the top x-axis. Upper panel: normalized abundances of atomic H, H$_2$, C$^+$, C, S, and S$^+$. The H$_2$ photodissociation front occurs at the highest altitude above the midplane, and the S ionization front occurs at the lowest. Middle panel: optical depths to FUV radiation presented by H$_2$ (averaged over the Lyman band), C$^+$, S$^+$, and dust (VSGs). Lower panel: shielding factors by which FUV radiation is attenuated: self-shielding of H$_2$ ($\beta_{H2}$), self-shielding of C ($\beta_{C}$), shielding of C by H$_2$ ($\beta_{C,H2}$), self-shielding of S ($\beta_{S}$), and shielding by very small grains ($\beta_{VSG}$).

3.2. MRI-active Surface Density $\Sigma^*$

We measure the extent of the MRI-active column by means of the magnetic Reynolds number $Re$ and the ion–neutral collision rate $Am$. Figure 2 shows both dimensionless numbers as a function of $\Sigma$ for our standard model at $a = 3$ AU.

Above the C$^+$ ionization front, at $\Sigma \lesssim 3 \times 10^{-3}$ g cm$^{-2}$, the value of $Re$ remains constant at $\sim 10^{10}$ because $x_c$ saturates at $10^{-4}$ where nearly all the carbon is singly ionized. In this region, $Am \propto \Sigma$ because $Am \propto n_{tot}$. The value of $Am$ peaks at $\sim 3 \times 10^3$ near the C ionization front. Both $Am$ and $Re$ fall just past the C ionization front, but not by more than an order of magnitude: S, which is $10 \times$ less abundant than C, remains fully ionized out to the S ionization front at $\Sigma \sim 10^{-2}$ g cm$^{-2}$. Past the S ionization front, virtually all FUV photons are used up; the charge density, and by extension both dimensionless numbers, drop sharply.

We define the MRI-active surface density $\Sigma^*$ as follows. Bai & Stone (2011) find from an extensive series of numerical simulations that for a given $Am$, the maximum value of $\alpha$ is given by

$$\max \alpha = \frac{1}{2} \left[ \left( \frac{50}{Am^{1.3}} \right)^2 + \left( \frac{8}{Am^{0.3}} + 1 \right)^2 \right]^{-1/2}. \quad (20)$$

From the value of $\max \alpha$ so computed, we evaluate the mass accretion rate

$$\dot{M} = 2 \times 3 \pi \Sigma \nu = 6 \pi \Sigma \times \max \alpha \times \frac{kT}{\mu \Omega}. \quad (21)$$

as a function of $\Sigma$, where $\nu = \alpha c_s h$ is the viscosity, $\mu = 4 \times 10^{-24}$ g is the mean molecular weight, $k$ is Boltzmann’s constant, and the prefactor of 2 accounts for the top and bottom surfaces of the disk. Note that $\max \alpha$ is a function of $Am$, which in turn is a function of $\Sigma$ that we have computed (Figure 2). We define $\Sigma^*$ as that value of $\Sigma$ for which $\dot{M}$ peaks; see Figure 3, which plots Equation (21). According to this definition, $\Sigma^* \approx 1 \times 10^{-2}$ g cm$^{-2}$. At this column, according to Figure 2, $Re$ remains larger than $Re^*$, demonstrating that ambipolar diffusion limits MRI activity more than Ohmic dissipation does, although the effect is slight (the dominance of ambipolar diffusion over Ohmic dissipation was much more pronounced for the X-ray-ionized layers analyzed by PBC11). Referring back to Figure 1, we see that $\Sigma^*$ corresponds essentially to the S$^+$S$^+$ ionization front.
3.3. Parameter Survey, Including Sensitivity to PAHs

In Figure 4 we explore the sensitivity of $\Sigma^*$ to various parameters. For a given solid curve in a given panel, only the one parameter on the x-axis is varied while all others are held fixed. Labels on a curve indicate the values to which certain parameters are fixed when computing that curve. If a parameter is not labeled, its value was set to the standard value.

Over most of parameter space, the MRI-active surface density $\Sigma^*$ in FUV-ionized layers is of order $10^{-2} - 10^{-1}$ g cm$^{-2}$. The MRI-active surface density is set by the depth of the sulfur ionization front. VSGs reduce the depth of this front by absorbing FUV radiation (and also by enhancing the ion recombination rate, although this is not a dominant effect for FUV-ionized layers where the electron density and thus the radiative recombination rate are high). At their standard abundance—which is probably near their maximum abundance (Section 2.4)—VSGs shield sulfur from ionizing radiation more than sulfur shields itself; compare $\beta_S$ and $\beta_{VSG}$ in Figure 1.

As $N_{VSG}^{\tau=1}$ increases from $10^{22}$ to $10^{24}$ cm$^{-2}$ (i.e., as the grain abundance decreases), $\Sigma^*$ increases by a factor of three (Figure 4(b)). Now most of the shielding of sulfur from FUV radiation is provided by sulfur itself, not by VSGs. When grains are insignificant ($N_{VSG}^{\tau=1} = 10^{24}$ cm$^{-2}$), $\Sigma^*$ depends on $L_{FUV}$ and the carbon/sulfur abundance $\epsilon$ more strongly than in the case when grains are significant absorbers of FUV radiation (Figures 4(c) and (d)). The dependences in the case of low grain abundance resemble those calculated by PBC11 in their simple FUV-Strömgren model: $\Sigma^* \propto L_{FUV}^{1/2}$ and $\Sigma^* \propto 1/\epsilon$.

The MRI-active thickness $\Sigma^*$ in FUV-ionized layers hardly varies with disk radius $a$ (Figure 4(a)), because the decrease in ionizing flux with increasing $a$ is compensated by the...
lengthening dynamical time $\Omega^{-1}$ (to which $A_{m}$, the factor determining $\Sigma^*$, is proportional).

Finally, $\Sigma^*$ is not sensitive to the abundance of PAHs (Figure 4(e)). For the large electron fractions $x_e \approx 10^{-5}$–$10^{-4}$ generated by FUV ionization of carbon and sulfur, the primary channel for charge neutralization is radiative recombination with free electrons. Only at the highest PAH abundances does the ion recombination rate with PAHs ($x_{PAH}(\Sigma PAH)$) approach the ion recombination rate with electrons ($x_e\alpha_{rec.S}$). By contrast, X-ray-ionized gas has much lower electron fractions and is much more susceptible to charge recombination on small condensates like PAHs (PBC11).

The last point is echoed in Figure 5, which shows how the free electron fraction varies with PAH abundance for both FUV-ionized and X-ray-ionized surface layers. Each curve is computed at a fixed surface density characteristic of each layer: $\Sigma \approx 0.01 \text{ g cm}^{-2}$ for the FUV-irradiated layer and $\Sigma \approx 0.3 \text{ g cm}^{-2}$ for the X-ray-irradiated layer. For the FUV-ionized layer, $x_e$ remains constant over the range of plausible PAH abundances $10^{-11} \lesssim x_{PAH} \lesssim 10^{-8}$. For the X-ray-ionized layer, $x_e$ varies by about two orders of magnitude over the same range of PAH abundance; $x_e \propto 1/x_{PAH}$ for $x_{PAH} \gtrsim 5 \times 10^{-10}$ (see Section 3.2.1 of PBC11).

### 3.4. Hall Diffusion

When computing $\Sigma^*$, we have ignored the effects of Hall diffusion (Wardle 1999; Balbus & Terquem 2001; Sano & Stone 2002). When Hall diffusion dominates, only electrons are coupled to magnetic fields, and ions are de-coupled from magnetic fields by collisions with neutrals. Under these circumstances, the evolution of the MRI depends on the direction of the magnetic field $\mathbf{B}$ with respect to the angular frequency $\Omega$. Hall diffusion can increase/decrease $\Sigma^*$ by an order of magnitude or more when $\mathbf{B}$ and $\Omega$ are parallel/anti-parallel (Wardle & Salmeron 2011).

To assess the importance of Hall diffusion in FUV-ionized surface layers, we evaluate the Hall parameter

$$H_a \equiv \frac{v_A^2}{\eta_{Ha} \Omega} = \frac{e B x_i}{m_{i} c \Omega}, \quad (22)$$

Here $x_i \equiv n_i/n_{tot}$ is the fractional abundance of ions of density $n_i$, relative to hydrogen nuclei of density $n_{tot}$, $v_A = B/\sqrt{4\pi n_{tot} m_H}$ is the Alfvén velocity, $e$ is the electron charge, $c$ is the speed of light, and $\eta_{Ha} = e B/(4\pi e n_i)$ is the Hall diffusivity (e.g., Wardle & Salmeron 2011, but note that their Hall parameter is the inverse of ours, and they assume $n_1 = n_e$). The dimensionless Hall parameter is the ratio of the inductive term to the Hall term in the induction equation. If $H_a \gg 1$, then Hall diffusion is not important.

To evaluate $H_a$, we estimate $B$ from $M$ by assuming that the inequality

$$B^2 \gtrsim B_1^2 + B_2^2 \gtrsim 2B_1B_2 \quad (23)$$

saturates, and that the Maxwell stress which drives accretion is given by

$$B_1B_2 \approx \frac{M \Omega}{2h} \quad (24)$$

(see, e.g., Bai & Goodman 2009; the factor of two in Equation (24) arises because the disk has a top and bottom face).

In Figure 6 we show that $H_a \gg 1$ at $\Sigma^*$ for all $\alpha$, over the entire parameter space that we have explored. Thus we conclude that Hall diffusion is not a concern in FUV-ionized surface layers (the same may not be true for the more poorly ionized X-ray-irradiated layers; Wardle & Salmeron 2011).

### 3.5. The Possibility of Turbulent Mixing: Chemical Equilibration Timescale versus Dynamical Timescale

Throughout this paper, we have computed ionization fractions in a static atmosphere, where the MRI-active layer is not static; it is turbulent. We might have underestimated $\Sigma^*$ because turbulence can mix plasma vertically toward the midplane, deeper into the disk interior (Inutsuka & Sano 2005; Ilgner & Nelson 2006; Turner et al. 2007). Mixing would be effective if the vertical mixing time is short compared to the timescale over which ionized layers achieve chemical equilibrium. Without a full simulation of MRI turbulence, we approximate the vertical mixing time as the dynamical time $t_{dyn} = \Omega^{-1}$. For the chemical equilibration time, we substitute the radiative recombination time $t_{rec} = 1/(\alpha_{rec,S})$. The ratio $t_{rec}/t_{dyn}$ is plotted in Figure 7 (cf. Figure 11 of PBC11). Because $t_{rec}/t_{dyn} > 1$ over some portion of parameter space, mixing might well be significant in
FUV-ionized layers, and might increase $\Sigma^*$ above the values we have computed in this paper. Whether the increase would be large or small is hard to say. On the one hand, $t_{\text{rec}}/t_{\text{dyn}}$ is never far above unity, suggesting that turbulent mixing will merely introduce order-unity corrections to our estimates. On the other hand, as turbulent mixing dilutes the electron density $n_e$, the timescale ratio $t_{\text{rec}}/t_{\text{dyn}}$ might increase with increasing depth, and the process might run away.

4. SUMMARY AND IMPLICATIONS FOR DISK ACCRETION

Circumstellar disk material must be sufficiently ionized if it is to accrete by the MRI. We have considered in this paper ionization by stellar FUV radiation. Although FUV radiation cannot penetrate the disk as deeply as can X-rays, it generates ionization fractions orders of magnitude larger. Carbon and sulfur may be the principal sources of free electrons and ions, as these elements are cosmically abundant and least likely to be depleted onto grains. Far-infrared searches for carbon and sulfur emission from disks are so far consistent with gas phase abundances for both elements within a factor of $\sim 2$ of solar. Because the electron and ion abundances generated from ionized carbon and sulfur are so large, ionization fractions in FUV-irradiated layers are little impacted by PAHs. In FUV-ionized layers, fractional ion abundances $x_i \approx 10^{-3}$--$10^{-4}$, dwarfing PAH abundances of $x_{\text{PAH}} \sim 10^{-11}$--$10^{-8}$. More to the point, ion recombination on PAHs in FUV-ionized layers is at most competitive with ion recombination with free electrons. This immunity to PAHs does not apply to X-ray-ionized layers where $x_i \lesssim 10^{-9}$ and where PAHs or very small (0.01 $\mu$m sized) grains can suppress the MRI (Bai & Goodman 2009; Perez-Becker & Chiang 2011, PBC11). It is refreshing that FUV ionization is robust against the usual difficulties plaguing other, weaker sources of ionization.

In Figure 8 we compute the possible ranges of disk accretion rate $\dot{M}$ due to either FUV or X-ray ionization and compare them against the range of observed stellar accretion rates. The accretion rate $\dot{M}$ driven by FUV ionization is derived using Equations (20) and (21) for $\Sigma = \Sigma^*$ (taken from Figure 4a) and the corresponding value of $a_{\max}$. The accretion rate $\dot{M}$ driven by X-ray ionization is computed similarly, using the abundances of charged species of the standard model of PBC11, their temperature of $T = 80(a/3\text{ AU})^{-3/7}$ K, and Equations (2) and (3) of this work to compute $Am(\Sigma)$. Although our estimates of $\dot{M}$ driven by X-ray ionization may have to be revised because of the Hall effect (Wardle & Salmeron 2011), our estimates of $\dot{M}$ driven by FUV ionization should not be: FUV-ionized, MRI-active layers behave in the ideal magnetohydrodynamic limit.

Modulo the impact of the Hall effect on X-ray-driven MRI, from Figure 8 we conclude that accretion rates from FUV-ionized surface layers are of the same order of magnitude as accretion rates from X-ray-ionized layers. At large radii $a \gtrsim 10$ AU, FUV-driven accretion rates may exceed X-ray-driven rates. At $a \lesssim 10$ AU, their contributions may be more nearly equal. At $a \gtrsim 10$ AU, the FUV-irradiated surface layer can sustain accretion rates similar to those observed. At $a \lesssim 10$ AU, surface layer accretion rates, driven either by FUV or X-ray ionization, can still be observationally significant, but they diminish with decreasing radius. At $a \lesssim 1$ AU, surface layer accretion rates fall below the range typically observed for young solar-type stars.

The problem of too low an accretion rate at small radius might be alleviated by turbulent mixing of plasma from FUV-irradiated disk surface layers into the disk interior, as such mixing would enhance the thickness of the MRI-active layer.
Our crude estimate of the timescales involved (Section 3.5) suggests that this possibility is worth further consideration.

Note in Figure 8 how the X-ray-driven $\dot{M}$ actually increases from the low-PAH case to the high-PAH case. This surprising effect arises because increasing the PAH abundance (over this particular range from $x_{\text{PAH}} = 10^{-11}$ to $10^{-8}$) increases the abundances of charged PAHs themselves, more than offsets the decreased abundances of free electrons and ions. In fact, in the high-PAH case, positively and negatively charged PAHs are approximately equal in number and represent by far the most abundant charged species. The primary recombination pathway is positively charged PAHs colliding with negatively charged PAHs (reaction 14 in PBC11). Thus as the overall PAH abundance increases, charged PAHs can increase $\dot{M}$ at a given column $\Sigma$. This effect was missed by PBC11, and is highlighted by Bai (2011). The consequent enhancement in $\dot{M}$ is mitigated by the decrease in the free electron fraction and thus in $Re$. In computing the X-ray-driven $\dot{M}$ for the high-PAH case in Figure 8, we accounted for the mitigating effects of a lower $Re$ by evaluating $\dot{M}$ at the $\Sigma$ for which $Re \approx 100$, below which Ohmic dissipation would weaken the MRI.

One concern is whether gas pressures in FUV-ionized layers are lower than the magnetic field pressures required to drive our computed accretion rates. If the pressure ratio

$$\beta_{\text{plasma}} = \frac{\Sigma kT/(\mu h)}{B^2/(8\pi)}$$

is $< 1$, magnetic tension defeats the MRI. An upper bound on $\beta_{\text{plasma}}$ is obtained by substituting the lower bound on $B^2$ inferred from Equations (23) and (24). For a representative case $\Sigma^* \approx 0.1 \text{ cm}^{-2}$, Equations (23)-(25) combine to yield the topmost slanted line in Figure 8, above which max $\beta_{\text{plasma}} < 1$ and the MRI cannot operate. The range of FUV-driven accretion rates we have computed sits safely below this line.

In sum, surface layer accretion by FUV ionization can, by itself, solve the problem of protoplanetary disk accretion at large radius, but not at small radius (unless turbulent mixing of plasma can substantially thicken the MRI-active layer). This statement remains one of principle and not of fact, because MRI accretion rates depend on the strength and geometry of the background magnetic field threading the disk (e.g., Fleming et al. 2000; Pessah et al. 2007; Bai & Stone 2011), and unfortunately the field parameters are not known for actual disks.6

For completeness, we show in Figure 9 how sideways cosmic rays can, in principle, enhance accretion rates in the outermost portions of disks (see the Introduction). For an assumed radial surface density profile resembling that of the minimum-mass solar nebula, accretion rates at $a \gtrsim 30$ AU can be strongly increased by sideways cosmic rays. Although cosmic-ray-induced accretion rates at large radius can be competitive with FUV-induced rates, it is not clear that cosmic rays are not mirrored away from disks by ambient magnetic fields. Far-UV ionization does not suffer from this uncertainty.

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6 The rotation measures of magnetized FUV-ionized layers could be large, on the order of $10^7 \text{ rad m}^{-1}$ at $a \approx 30$ AU for a face-on disk (assuming an unturbulent $B = 10$ mG and an electron column density of $N_e \sim 3 \times 10^{26} \text{ cm}^{-2}$). Perhaps measurements of Faraday rotation at radio wavelengths using polarized background active galaxies, or polarized emission from the central star itself, could be used to constrain disk magnetic fields.

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Figure 8. The solid and dotted curves correspond to the cases of low and high PAH abundance increases, positively and negatively charged PAHs (reaction 14 in PBC11). Thus as the overall PAH abundance increases, charged PAHs can increase $\dot{M}$ at a given column $\Sigma$. This effect was missed by PBC11, and is highlighted by Bai (2011). The consequent enhancement in $\dot{M}$ is mitigated by the decrease in the free electron fraction and thus in $Re$. In computing the X-ray-driven $\dot{M}$ for the high-PAH case in Figure 8, we accounted for the mitigating effects of a lower $Re$ by evaluating $\dot{M}$ at the $\Sigma$ for which $Re \approx 100$, below which Ohmic dissipation would weaken the MRI.

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Figure 9. Same as Figure 8, but showing disk accretion rates driven by the combined effects of X-ray and cosmic ray ionization. Galactic cosmic rays cannot reach the disk through its top and bottom faces because of shielding by the magnetized stellar wind. They can, however, penetrate the disk "sideways" through the outermost portions of the disk, parallel to the midplane—assuming they are not magnetically mirrored away. To generate this plot we assume a total (vertical) surface density of $2200\sigma/1\text{AU}^{-3/2} \text{ cm}^{-2}$ and a cosmic ray ionization rate $\zeta_0 \exp(-\Sigma_0/\Sigma_0)$, where $\zeta_0 = 1/4 \times 10^{-17} \text{ s}^{-1}$ (Caselli et al. 1998) and $\Sigma_0 = 96 \text{ g cm}^{-2}$ is the stopping column for GeV-energy cosmic rays (Unbehydt & Nakano 1981). The factor of 1/4 in $\zeta_0$ is our estimate for the fraction of the celestial sphere (centered at the midplane) not shielded by stellar winds. The attenuating column $\Sigma$ is obtained by radially integrating the volume mass density at the midplane from $a$ to infinity. For $a \gtrsim 30$ AU, sideways cosmic rays may provide enough ionization to render the entire disk active (PBC11). Sideways cosmic rays cannot penetrate the disk at $a \lesssim 10$ AU because of intervening disk material, and so the accretion rates there are practically identical to those for the X-ray-only case (plotted in Figure 8). The solid and dotted curves correspond to the cases of low and high PAH abundances, respectively. The main assumption underlying this plot is that sideways cosmic rays are not magnetically mirrored away; this is an uncertain prospect.

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4.1. Transitional Disks

What is the relevance of MRI accretion to transitional disks, i.e., disks with inner holes (e.g., Espaillat et al. 2010; Hughes et al. 2007; Calvet et al. 2005)? Chiang & Murray-Clay (2007, hereafter CMC) proposed that MRI-active surface layers at the rim of the hole—i.e., in the rim "wall" oriented perpendicular to the disk midplane—could supply the observed accretion rates of transitional disks. Following CMC, we compute the accretion rate at the rim according to the formula

$$\dot{M}_{\text{trans}} \sim \frac{3M_{\text{lim}}}{t_{\text{diff}}} \sim 12\pi \Sigma^* \times \max \alpha \times \left(\frac{kT}{\mu}\right)^{3/2} \frac{a_{\text{rim}}^2}{GM},$$

(26)

where $M_{\text{lim}} \approx 4\pi ah \Sigma^*$ ($h$ is the disk half-thickness), $t_{\text{diff}} \sim a_{\text{rim}}^2/v$ is the time for material to diffuse from $a_{\text{rim}}$ to $a_{\text{rim}}/\sqrt{2}$, and $\Sigma^*$ is reinterpreted for transitional disk rims as the radial, not vertical, column of MRI-active material. We re-compute $\Sigma^*$ for transitional disk rims by taking the grazing angle $\theta \sim 1$, as is appropriate for radiation which penetrates the rim wall at normal incidence.
Figure 10. Same as Figure 8 but for transitional disks and showing only our FUV model. Observed rates are taken from Calvet et al. (2005, diamonds), Espaillat et al. (2007, 2008, 2010, squares), and Kim et al. (2009, circles with error bars). Theoretical rates for FUV-ionized layers are computed with Equation (26), with Σ recomputed using $θ \sim 1$. The observed trend of increasing $M_{\text{trans}}$ with increasing $a_{\text{rim}}$ is reproduced (see, however, Section 4.2 of Kim et al. 2009 for how the observed trend could reflect other correlations between stellar mass $M$ and $a_{\text{rim}}$, and $M$ and $M$).

Figure 10 shows accretion rates for transitional disk rims computed according to (26), overlaid with observations taken from Calvet et al. (2005), Espaillat et al. (2007, 2008, 2010), and Kim et al. (2009). We have chosen these references and not others because they utilize disk models similar enough to each other to yield reasonably consistent hole radii (cf. Merín et al. 2010). Rim accretion by FUV ionization seems capable of reproducing the trend of increasing $M_{\text{trans}}$ with increasing $a_{\text{rim}}$. However, there remains the problem of transporting the material that is dislodged from the rim over the decades in disk radius between the rim and the host star. Inside the rim, a conventional disk geometry applies, and we have already noted in that context that surface layer accretion rates at small radii are too small compared to those observed. For example, according to Figure 10, an accretion rate of $M_{\text{trans}} \sim 10^{-9} M_\odot \, \text{yr}^{-1}$ may be initiated at $a_{\text{rim}} = 30$ AU, but according to Figure 8, this same accretion rate cannot be sustained by the MRI inside a radius of $\sim 3$ AU. Multiple planets could solve this problem by shunting gas inward (PBC11; Zhu et al. 2011).

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