Possible persistence of the metal-insulator transition in two-dimensional systems at finite temperatures

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For the immediate vicinity of the metal-insulator transition (MIT), data on the dependence of the resistivity \(\rho\) on the charge carrier concentration \(n\) from an Si MOSFET experiment by Kravchenko et al. and from an AlAs quantum well study by Papadakis and Shayegan are reanalyzed. In both cases, the \(\rho(T = \text{const.}, n)\) curves for various values of the temperature \(T\) seem to exhibit an offset concerning \(n\), where the related resistivity is close to \(h/e^2\). This offset may result from a peculiarity in \(\rho(T = \text{const.}, n)\) indicating the MIT to be present also at finite \(T\). More detailed experiments are imperative.

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The metal-insulator transition in two-dimensional (2D) systems has been under controversial debate for the last years. Its existence was denied by the localization theory by Anderson et al.\(^1\) Thus it came as a big surprise when Kravchenko et al. first reported on a strong decrease of \(\sigma\) with decreasing \(T\) in high mobility MOSFET samples.\(^2\) They considered the conduction in the related \((T, n)\) region as metallic, an interpretation which was called in question in particular by Altshuler and Maslov.\(^3\) Compare also Refs. 4,5. A current review on this field is given in Ref. 6.

In the context of this debate, it is instructive to compare the usual approaches to data interpretation in experimental studies of the MIT in 2D and 3D disordered systems. In the 2D case, the common intersection point of resistivity curves, \(\rho(T = \text{const.}, n)\), measured at various fixed temperatures, is often considered as indication of the MIT.\(^6\) In this way, the critical concentration \(n_c\) is defined by \(d\rho(T, n_c)/dT\big|_{T \to 0} = 0\).

In the 3D case, however, most publications do not pay special attention to the corresponding concentration value. Instead, the MIT usually is identified with the vanishing of the parameter \(a\) in fits of the conductivity \(\sigma(T, n = \text{const.})\) by means of the ansatz \(a(n) + b(n) \cdot T^p\), mostly with \(p = 1/2\) or 1/3 according to Refs. 8 and 9, respectively. For a typical such work see Ref. 10. However, this approach to locating the MIT suffers from inconsistencies.\(^11\)

An alternative, phenomenological approach to the data analysis for 3D systems is based on universal features of \(\sigma(T, n)\), in particular on a scaling law for the \(T\) dependence in the hopping region,\(^11\) see also Ref. 13. It yields several independent arguments for the MIT occurring when \(d\rho(T, n_c)/dT\big|_{T \to 0} = 0\), in analogy to the 2D case. These investigations finally lead to a phenomenological model for amorphous Si\(_{1-x}\)Cr\(_x\) simultaneously describing both sides of the MIT, see Ref. 14. Certain features of the model are observed also at other substances.\(^15\)

This phenomenological model has an unusual feature: The MIT is expected to persist at finite \(T\), that means the \(\rho(T = \text{const.}, n)\) curves should exhibit peculiarities as charted in Fig. 1. Surely, such a sharp, but continuous MIT at finite \(T\) would contradict common expectations. However, some indirect evidence for this hypothesis comes from the inspection of \(d\sigma/dT\) versus \(\sigma\) plots.\(^15\) Nevertheless, the situation is far from being clear: Detailed studies of \(\rho(T = \text{const.}, n)\) in the \(n\) region where \(d\sigma/dT\) changes its sign are missing yet. Unfortunately, appropriate experiments as stress tuning.\(^15\) have focused on another concentration / conductivity region.

Here I examine the question whether an analogous peculiarity of \(\rho(T = \text{const.}, n)\) might exist in 2D systems — of course, the related critical exponents would be expected to have other values than in the 3D case —. For that I reconsider experimental data from the literature with a "magnifying glass".

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Results on $\rho(T = \text{const.}, n)$ from the MOSFET experiment by Kravchenko et al. are replotted in Fig. 2 where only the lowest four $T$ values are taken into account. The data were reconstructed from Fig. 1 of Ref. [7] by “retranslating” the Postscript file of the preprint version of this work, Ref. [21]. Note that I consider here only a small part of the original data: In Fig. 2, $n$ and $\rho$ vary by factors of about 1.2 and 12, respectively, whereas the original Fig. 1 of Ref. [7] presents variations by factors of 2 and 4000, respectively.

In analyzing these data, one notes strikingly large fluctuations in the region around $n = 1.02 \times 10^{11} \text{cm}^{-2}$ for all $T$ values considered here. Detailed inspection uncovers a more important feature: $\rho(T = \text{const.}, n)$ seems to have a special structure there, causing an offset with respect to $n$ in the global variation of these curves. This is connected with a reduction of the slope in the offset region.

Remarkably, at the lower end of the related $n$ interval, $\rho$ is close to $\hbar/e^2$. The accuracy of the data in Fig. 2 is not sufficient to determine a common intersection point of the $\rho(T = \text{const.}, n)$ curves. However, it can be stated that, if such a point exists, it is located within the interval $[0.95, 1.05]$ which includes also the “offset region”. (Kravchenko et al. regard $0.96 \times 10^{11} \text{cm}^{-2}$ as critical concentration, and claim $\rho$ to have a value of about $2 \hbar/e^2$ there. The difference between their result and my cautious estimate has two reasons: Here I focus only at the low-$T$ region. Kravchenko et al. treated the possible peculiarity, discussed here, as random fluctuation of smooth $\rho(T = \text{const.}, n).$

Fig. 3 shows $\rho(T, n)$ data measured at an AlAs quantum well by Papadakis and Shayegan [5]. These data were reconstructed by digitizing the $\rho(T, n = \text{const.})$ dependences presented in Fig. 2 of Ref. [22] for $T = 0.30, 0.45,$ and $0.60 \text{ K}$.

All $\rho(T = \text{const.}, n)$ curves given in Fig. 3 seem to exhibit a special structure between $n = 0.65 \times 10^{11} \text{cm}^{-2}$ and $0.70 \times 10^{11} \text{cm}^{-2}$, causing an offset in $\rho(T = \text{const.}, n)$ with respect to $n$. Such an offset, provided it is not an artifact of random errors of the $n$ values, can only be understood in terms of a peculiarity in $\rho(T = \text{const.}, n)$ there, and/or as result of the curvature of $\rho(T = \text{const.}, n)$ changing the sign in this region.

Moreover, two details of Fig. 3 are remarkable: At $n = 0.65 \times 10^{11} \text{cm}^{-2}$, the lower concentration boundary of the offset region, the values of $\rho$ are close to $\hbar/e^2$. The three $\rho(T = \text{const.}, n)$ curves intersect each other almost at the same point, just in the region where this special structure seems to be located.

For both experiments, one could of course object that the emphasized features also might arise from random deviations in the measuring values of $n$ and $\rho$. However, it seems to be very unlikely that all the following coincidences occur only by chance:

(i) These features seem to be present for all temperatures considered.

(ii) They are observed at data from two independent experiments at different kinds of samples, made up of different materials.

(iii) The peculiarities have qualitatively the same form in both data sets.

(iv) They occur in both cases in the same resistivity region, just below $\hbar/e^2$.

(v) They are observed in both cases close to the common intersection point of the $\rho(T = \text{const.}, n)$ curve set.

It has to be mentioned that similar hints to a possible peculiarity could not be found in data from other related publications, e.g. Ref. [23]. However, this is not a strong objection to the above interpretation since various kinds of inhomogeneities may wash out the comparably small effect considered here.

FIG. 2: log-log representation of $\rho(T = \text{const.}, n)$ for the intermediate vicinity of the MIT in a high-mobility MOSFET. Data were obtained from Fig. 1 of Ref. [7]. The dashed lines serve only as guide to the eye.

FIG. 3: log-log representation of $\rho(T = \text{const.}, n)$ for the vicinity of the MIT in an AlAs quantum well. Data were obtained from Fig. 2 of Ref. [22]. The dashed lines are included as guide to the eye.
Concluding, Figs. 2 and 3 together, in particular the offsets in both curve sets, suggest that, for a certain range of finite $T$, $\rho(T = \text{const.}, n)$ may exhibit a peculiarity close to $\rho = h/e^2$. On the one hand, this would be a direct fingerprint of the still nowadays controversial MIT in 2D systems. On the other hand, it would indicate the persistence of the $T = 0$ phenomenon MIT at finite $T$, in contradiction to common expectations. Further, more detailed measurements are urgently needed to clarify this fundamental problem.

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1. E. Abrahams, P.W. Anderson, D.C. Licciardello, and T.V. Ramakrishnan, Phys. Rev. Lett. 42, 673 (1979).
2. S.V. Kravchenko, G.V. Kravchenko, J.E. Furneaux, V.M. Pudalov, and M. D’Iorio, Phys. Rev. B 50, 8039 (1994).
3. B.L. Altshuler and D.L. Maslov, Phys. Rev. Lett. 82, 145 (1999).
4. S.V. Kravchenko, M.P. Sarachik, and D. Simonian, Phys. Rev. Lett. 83, 2091 (1999).
5. B.L. Altshuler and D.L. Maslov, Phys. Rev. Lett. 83, 2092 (1999).
6. E. Abrahams, S.V. Kravchenko, and M.P. Sarachik, Rev. Mod. Phys. 73, 251 (2001).
7. S.V. Kravchenko, W.E. Mason, G.E. Bowker, J.E. Furneaux, V.M. Pudalov, and M. D’Iorio, Phys. Rev. B 51, 7038 (1995).
8. B.L. Altshuler and A.G. Aronov, Zh. Eksp. Teor. Fiz. 77, 2028 (1979) [Sov. Phys. JETP 50, 968 (1979)].
9. J. Newson and M. Pepper, J. Phys. C 19, 3983 (1986).
10. S. Waffenschmidt, C. Pfleiderer, and H. v. Löhneysen, Phys. Rev. Lett. 83, 3005 (1999).
11. A. Möbius, Phys. Rev. B 40, 4194 (1989).
12. A. Möbius, C. Frenzel, R. Thielisch, R. Rosenbaum, C.J. Adkins, M. Schreiber, H.-D. Bauer, R. Grötzschel, V. Hoffmann, T. Krieg, N. Matz, H. Vinzelberg, and M. Witcomb, Phys. Rev. B 60, 14209 (1999), and references therein.
13. A. Möbius, D. Elefant, A. Heinrich, R. Müller, J. Schumann, H. Vinzelberg, and G. Zies, J. Phys. C 16, 6491 (1983).
14. A. Möbius, H. Vinzelberg, C. Gladun, A. Heinrich, D. Elefant, J. Schumann, and G. Zies, J. Phys. C 18, 3337 (1985).
15. A. Möbius, Z. Phys. B 79, 265 (1990).
16. A. Möbius, Z. Phys. B 80, 213 (1990).
17. A. Möbius, J. Phys. C 18, 4639 (1985), and references therein.
18. M.P. Sarachik and P. Dai, Europhys. Lett. 59, 100 (2002).
19. A. Möbius, phys. stat. sol. (b) 144, 759 (1987).
20. R.L. Rosenbaum, M. Slutzky, A. Möbius, and D.S. McLachlan, J. Phys.: Cond. Matt. 6, 7977 (1994).
21. S.V. Kravchenko, W.E. Mason, G.E. Bowker, J.E. Furneaux, V.M. Pudalov, and M. D’Iorio, cond-mat/9412103.
22. S.J. Papadakis and M. Shayegan, Phys. Rev. B 57, R15068 (1998).
23. V.M. Pudalov, G. Brunthaler, A. Prinz, and G. Bauer, cond-mat/0103087.