Odd-parity pairing correlations in a d-wave superconductor

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We theoretically study the effects of spin-orbit interactions on symmetry of a Cooper pair in a spin-singlet d-wave superconductor in two-dimension. The pairing symmetry is analyzed in terms of the anomalous Green’s function which is obtained by solving the Gor’kov equation analytically. A spin-orbit interaction induces a spin-triplet p-wave pairing correlation in a uniform superconductor. An odd-frequency spin-triplet s-wave pairing correlation appears at a surface of such superconductor as a result of breaking inversion symmetry locally. We also discuss a close relationship among the odd-frequency pairing correlation, chirality of surface bound states at the zero energy, and the anomalous proximity effect. The obtained results enable us to design a superconductor which causes the strong anomalous proximity effect.

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I. INTRODUCTION

Proximity structures consisting of spin-triplet superconductors indicate unusual electric transport properties such as the quantization of zero-bias conductance in a normal-metal/superconductor (NS) junction and the fractional current-phase relationship of the Josephson current in a superconductor/normal-metal/superconductor (SNS) junction.\textsuperscript{1,2} Such phenomena are called anomalous proximity effect and have been considered as the phenomena unique to Majorana Fermions.\textsuperscript{3–7} The topologically protected surface states at the zero energy penetrate into a dirty normal metal and form the resonant transmission channels at the Fermi level, which causes the perfect electron transmission through a dirty normal metal.\textsuperscript{8–10} The enhancement of the local density of states (LDOS) at the zero energy\textsuperscript{1,2} is a direct result of the penetration of the zero-energy states (ZESs) into a normal metal. Unfortunately, well-established spin-triplet superconductors have never been discovered yet. To observe the anomalous proximity effect in experiments, a recipe for an artificial spin-triplet superconductor is necessary.

Tamura and Tanaka\textsuperscript{11} have studied theoretically the LDOS at a dirty normal metal attached to a $d_{xy}$-wave superconducting film with Rashba spin-orbit interaction, where the NS interface is parallel to the $y$ direction. The proximity effect of a $d_{xy}$-wave superconductor is absent in this configuration.\textsuperscript{12–14} They found the modest enhancement of the LDOS at the zero energy, which can be an evidence of the anomalous proximity effect. Although the signal of the proximity effect is very weak, the results are highly nontrivial. Their results imply that the Rashba spin-orbit interaction would modify symmetry of Cooper pairs in a superconductor\textsuperscript{4,15–22} and that in a normal metal. Although they found an odd-frequency spin-triplet $s$-wave Cooper pair in the normal metal, mechanisms of the symmetry conversion are still unclear. Thus we have never known how to synthesize a superconductor which exhibits strong anomalous proximity effect.

We address these issues in the present paper.

In this paper, we theoretically study symmetry of a Cooper pair in a $d_{xy}$-wave superconductor in the presence of spin-orbit interactions. We first calculate the Green’s function in an infinitely long wire of $d_{xy}$-wave superconductor with a spin-orbit interaction $\lambda k_y \sigma_1$ and that with $-\lambda k_x \sigma_2$, where $k_x(k_y)$ is the wave number in the $x$ ($y$) direction and $\sigma_j$ for $j = 1 - 3$ is the Pauli’s matrix in spin space. The Green’s function at a surface along the $y$ direction is obtained by solving the Lippmann-Schwinger equation exactly. The first term $\lambda k_y \sigma_1$ generates a spin-triplet $p_x$-wave pairing correlation in the bulk of a superconductor and an odd-frequency spin-triplet $s$-wave pairing correlation at its surface. In addition, we will show that the surface bound states at the zero energy retain their high degeneracy owing to a chiral symmetry of the Hamiltonian. These conclusions explain the enhancement of the LDOS in the previous paper\textsuperscript{11}. On the other hand, the second term $-\lambda k_x \sigma_2$ generates a spin-triplet $p_y$-wave pairing correlation in a uniform state. When the two terms coexist, however, the second term suppresses the anomalous proximity effect seriously because it breaks the chiral symmetry. This conclusion explains why the enhancement of the LDOS is weak in the previous paper\textsuperscript{11}. On the basis of the obtained result, we will discuss a theoretical design for a superconductor which causes the strong anomalous proximity effect.

This paper is organized as follows. In Sec. II, we analyze the chiral property of the surface bound states at the zero energy. In Secs. III and IV, symmetry of a Cooper pair appearing at a surface of a $d_{xy}$-wave superconductor with spin-orbit interactions are analyzed by using the anomalous Green’s function. In Sec. V, we discuss effects of the two types of spin-orbit interaction on the anomalous proximity effect. The conclusion is given in Sec. VI. Throughout this paper, we use units of $\hbar = k_B = c = 1$, where $k_B$ is the Boltzmann constant and $c$ is speed of light.
II. TWO TYPES OF SUPERCONDUCTOR

In the previous paper, the authors analyzed the Bogoliubov-de Gennes (BdG) Hamiltonian given by

$$\hat{H}_{\text{BdG}}(k) = \begin{bmatrix} \hat{H}_N(k) & \hat{\Delta}(k) \\ -\hat{\Delta}^*(k) & -\hat{H}_N(-k) \end{bmatrix},$$

(1)

where the symbols $\cdots$ and $\cdots$ represent $4 \times 4$ and $2 \times 2$ matrices, respectively. A spin-singlet $d_{xy}$-wave pair potential is defined by

$$\hat{\Delta}(k) = \Delta_k \hat{\sigma}_2, \quad \Delta_k = \Delta \hat{\sigma}_y \hat{k}_y,$$

(2)

where $\hat{\sigma}_j$ for $j = 1 - 3$ is the Pauli’s matrix in spin space and $k_x(k_y)$ is the wavenumber in the $x(y)$ direction. The wave numbers in the pair potential are normalized to the Fermi wavenumber $k_F$ as $\hat{k}_x = k_x/k_F$ and $\hat{k}_y = k_y/k_F$. The normal state Hamiltonian is described as

$$\hat{H}_N(k) = \xi_k + \lambda k_y \hat{\sigma}_1 - \lambda k_x \hat{\sigma}_2,$$

(3)

$$\xi_k = \frac{\hbar^2 k^2}{2m} - \epsilon_F,$$

(4)

and $\epsilon_F = k_F^2/(2m)$ is the Fermi energy. When a thin film of $d$-wave superconductor stays on a substrate, breaking inversion symmetry in the $z$ direction indices the Rashba spin-orbit interaction.

In this paper, however, we divide the Rashba spin-orbit interaction into two parts: $\lambda k_y \hat{\sigma}_1$ and $-\lambda k_x \hat{\sigma}_2$, and study how $\lambda k_y \hat{\sigma}_1$ and $-\lambda k_x \hat{\sigma}_2$ modify the symmetry of Cooper pairs independently. Fig. 1 shows the schematic pictures of a superconductor under consideration. A superconductor in Fig. 1(a) is infinitely long in the $x$ direction and the width of the superconductor is $W$ in the $y$ direction. We apply the periodic boundary condition in the $y$ direction. In what follows, we analyze the two BdG Hamiltonians given by

$$\hat{H}_1 = \begin{bmatrix} \xi_k & \lambda k_y & 0 & \Delta_k \\ \lambda k_y & \xi_k & -\Delta_k & 0 \\ 0 & -\Delta_k & -\xi_k & \lambda k_y \\ \Delta_k & 0 & \lambda k_y & -\xi_k \end{bmatrix},$$

(5)

and

$$\hat{H}_2 = \begin{bmatrix} \xi_k & i\lambda k_x & 0 & \Delta_k \\ -i\lambda k_x & \xi_k & -\Delta_k & 0 \\ 0 & -\Delta_k & -\xi_k & -i\lambda k_x \\ \Delta_k & 0 & i\lambda k_x & -\xi_k \end{bmatrix},$$

(6)

and

$$\hat{H}_2 = \begin{bmatrix} \xi_k & \lambda k_y & 0 & \Delta_k \\ \lambda k_y & \xi_k & -\Delta_k & 0 \\ 0 & -\Delta_k & -\xi_k & \lambda k_y \\ \Delta_k & 0 & \lambda k_y & -\xi_k \end{bmatrix},$$

(7)

and

$$\hat{H}_2 = \begin{bmatrix} \xi_k & i\lambda k_x & 0 & \Delta_k \\ -i\lambda k_x & \xi_k & -\Delta_k & 0 \\ 0 & -\Delta_k & -\xi_k & -i\lambda k_x \\ \Delta_k & 0 & i\lambda k_x & -\xi_k \end{bmatrix},$$

(8)

where $\hat{\sigma}_j$ for $j = 1 - 3$ is the Pauli’s matrix in particle-hole space. Such types of spin-orbit interaction in a superconductor have been discussed in the context of Majorana fermions and play a key role in stabilizing surface bound states with flat dispersion. In this paper, we assume the relation

$$\Delta \ll \lambda k_F \ll \epsilon_F,$$

(9)

and discuss the effects of the spin-orbit interaction on the superconducting states within the first order of

$$\alpha = \frac{\lambda k_F}{2\epsilon_F} \ll 1.$$
The wavenumber in superconducting state is given by
\[ p_{s}^{2} = m \Delta \left| \bar{p}_{s} \right| k_{y} \] as a function of \( k_{y} \). A transport channel at \( k_{y} \) on the Fermi surface of ± branch is propagating for \( p_{s}^{2} > 0 \) and is evanescent for \( p_{s}^{2} < 0 \). Therefore,
\[ n_{+}(k_{y}) \equiv \Theta(p_{1+}^{2}) + \Theta(p_{1-}^{2}), \] represents the number of the propagating channels at \( k_{y} \), where
\[ \Theta(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0 \end{cases} , \] is the step function. We also define
\[ n_{-}(k_{y}) \equiv \Theta(p_{1+}^{2}) - \Theta(p_{1-}^{2}), \] for the latter use. In (I), the transport channels in the + branch are propagating and those in the − branch are evanescent. In (II), the transport channels in the two branches are propagating. Finally, in (III), the transport channels in the + branch are evanescent and those in the − branch are propagating. In Figs. 3(a) and (b), we plot \( n_{\pm} \) as a function of \( k_{y} \). We note that \( n_{-} \) is an odd function of \( k_{y} \). A superconductor described by \( H_{1} \) hosts highly degenerate ZESs at its surface around \( x \geq 0 \) as shown in Fig. 1(b). A ZES appears for each propagating channel. The wave functions of such surface bound states are described by
\[ \psi_{1+}(x) = \begin{bmatrix} 1 \\ \frac{1}{i s_{k}} \\ -i s_{k} \end{bmatrix} e^{-q_{x} x} \sin p_{1+} x, \] \[ \psi_{1-}(x) = \begin{bmatrix} -1 \\ \frac{1}{i s_{k}} \\ i s_{k} \end{bmatrix} e^{-q_{x} x} \sin p_{1-} x, \] \[ s_{k} = \text{sgn}(k_{y}). \] The wavenumber in superconducting state is given by
\[ [p_{s}^{2} \pm 2m\Delta |\bar{p}_{s}| k_{y}]^{1/2} \approx p_{s} \pm iq_{x}, \] for \( s = \pm \). The imaginary part of the wave number is estimated at \( \epsilon = 0 \) as \( q_{x} = (\Delta/\nu_{F})|\bar{k}_{y}| \) with \( \nu_{F} = k_{F}/m \) being the Fermi velocity. As shown in Eqs. (17) and (18), \( q_{x} \) characterizes the spatial area of the surface bound states.

The Hamiltonian Eq. (6) preserves chiral symmetry as
\[ \{ \hat{H}_{1}, \hat{\Gamma} \} = 0, \quad \hat{\Gamma} = \hat{\tau}_{1} \hat{\sigma}_{2}. \] The eigenvalues of \( \hat{\Gamma} \) are \( \gamma = \pm 1 \) because of \( \hat{\Gamma}^{2} = 1 \). It is mathematically true that a zero-energy state is an eigenstate of \( \hat{\Gamma} \) and belongs to a chiral eigenvalue either \( \gamma = 1 \) or \( \gamma = -1 \). Indeed, the wave functions in Eqs. (17) and (18) satisfy
\[ \hat{\Gamma} \psi_{1+} = -s_{k} \psi_{1+}, \quad \hat{\Gamma} \psi_{1-} = -s_{k} \psi_{1-}. \] The chiral eigenvalues \( g_{1\pm} = -s_{k} \) depends on \( k_{y} \) as shown in Figs. 3(c) and (d), where we input \( g_{1\pm} = 0 \) for an evanescent channel.

The number of propagating channels are calculated as
\[ N_{c} = \sum_{k_{y}} n_{+}(k_{y}) = \frac{W}{2\pi} \int_{-\infty}^{\infty} n_{+}(k_{y}) = \left[ \frac{2Wk_{F}}{\pi} \right]_{G}, \] where \( [\cdots]_{G} \) is the Gauss’s symbol meaning the integer part of the argument. Since a propagating channel hosts a ZES, the number of the ZESs at a surface is \( N_{c} \).
B. Surface bound states of $\hat{H}_2$

The positive eigenvalues of the Hamiltonian in Eq. (8) are calculated to be

$$E_{2,\pm} = \sqrt{\xi_{2,\pm}^2 + \Delta_{\pm}^2}, \quad \xi_{2,\pm}(k_y) = \xi_k \pm \lambda k.$$  \hfill (24)

In Fig. 2(b), we illustrate the two splitting Fermi surfaces characterized by $\xi_{2,\pm} = 0$. The wave numbers in the $x$ direction on the Fermi are calculated as

$$p_{2,\pm} = \sqrt{k_F^2 - k_y^2} \mp \alpha k_F.$$  \hfill (25)

For $|k_y| \leq k_F$, the transport channels are propagating. A topologically protected ZES appears for each propagating channel. The wave functions of such surface bound states are calculated as

$$\psi_{2,+}(x) = \begin{bmatrix} s_k \\ -is_k \\ 1 \\ -i \end{bmatrix} e^{-q_\pm x} \sin p_{2,+} x,$$

$$\psi_{2,-}(x) = \begin{bmatrix} s_k \\ is_k \\ 1 \\ -i \end{bmatrix} e^{-q_\pm x} \sin p_{2,-} x.$$  \hfill (26)-(27)

The Hamiltonian $\hat{H}_2$ preserves chiral symmetry

$$\{\hat{H}_2, \hat{\Gamma}\} = 0,$$  \hfill (28)

where $\hat{\Gamma}$ is defined in Eq. (21). We also find the relations,

$$\hat{\Gamma} \psi_{2,+} = -s_k \psi_{2,+}, \quad \hat{\Gamma} \psi_{2,-} = -s_k \psi_{2,-}.$$  \hfill (29)

The chiral eigenvalues $g_{2,\pm} = -s_k$ coincide with $g_{1,\pm}$ in $H_1$.

C. An extra chiral symmetry of $\hat{H}_1$

The operator $\hat{\Gamma}$ in Eq. (21) is derived from the combined operations of time-reversal $T = i\sigma_3 K$ and particle-hole $\hat{C} = \tau_1 K$, where $K$ means the complex conjugation plus $k \rightarrow -k$. Thus the Hamiltonian of any time-reversal invariant superconductor satisfies Eq. (21). We find that

$$\mathcal{N} = \sum_{k_y} [g_{1,+} + g_{1,-}] = 0,$$  \hfill (30)

because $g_{1,+} + g_{1,-}$ is an odd function of $k_y$ as shown in Fig. 3(e). This conclusion is valid for all the Hamiltonian belonging to class DIII.\textsuperscript{25}

To proceed the argument further, we explain an important relation between $\mathcal{N}$ and the anomalous proximity effect. The presence of a ZESe at a surface is characterized topologically by a winding number $W_{\pm}(k_y)$ which is defined in a one-dimensional Brillouin zone by fixing $k_y$ at a certain value.\textsuperscript{26,27} According to a standard definition, $W_{\pm} = -g_{1,\pm}$ holds.\textsuperscript{26} Thus the number of surface bound states at the zero energy is $N_\gamma$ for both $H_1$ and $H_2$. The large degree of degeneracy at the zero energy is a direct consequence of translational symmetry in the $y$ direction. Generally speaking, the potential disorder would lift such a high degeneracy because it breaks translational symmetry. A ZES with a chiral eigenvalue of $\gamma = 1$ and another ZES with the opposite chiral eigenvalue of $\gamma = -1$ are couple to each other one-by-one in the presence of potential disorder and form two nonzero-energy states. Therefore the degeneracy at the zero energy is fragile under the potential disorder when ZESs with $\gamma = 1$ and those with $\gamma = -1$ coexist. It is also true that the degeneracy at the zero energy is robust when all the ZESs belong to the same chiral eigenvalue.\textsuperscript{8-10} Eq. (30) implies that the number of the zero-energy states with $\gamma = 1$ and that with $\gamma = -1$ ($N_\mp$) is equal to each other. As a consequence, no ZES remains under the potential disorder near a surface. Namely Eq. (30) implies the anomalous proximity effect is absent in a normal metal attached to such a superconductor. The schematic figure of an NS junction is illustrated Fig. 1(c).

The anomalous proximity effect is characterized by a nonzero integer number $N_\mp - N_\pm = \mathcal{N}$. In this case, ZESs at a surface retain $\mathcal{N}$-fold degeneracy even in the presence of potential disorder. In other words, $\mathcal{N}$ zero-energy states with $\gamma = 1$ cannot find their chiral partner. The penetration of the remaining ZESs into a normal metal attached to the surface causes the enhancement of the LDOS at the zero energy in the normal metal.\textsuperscript{11} Moreover, the index $\mathcal{N}$ characterizes the quantized value of the zero-bias differential conductance of such a NS junction as\textsuperscript{10},

$$G_{\text{NS}} = \frac{2e^2}{h} |\mathcal{N}|,$$  \hfill (31)

in the limit of strong potential disorder in the normal metal. Therefore the index $\mathcal{N}$ measures the strength of the anomalous proximity effect. For instance, $\mathcal{N} = 0$ indicates the absence of the anomalous proximity effect.

Finally we discuss an extra chiral symmetry of $H_1$. The Hamiltonian Eq. (6) anticommutes to

$$\hat{\Gamma}_A = \hat{\tau}_2 \hat{\sigma}_3.$$  \hfill (32)

It is easy to show

$$\hat{\Gamma}_A \psi_{1,+} = s_k \psi_{1,+}, \quad \hat{\Gamma}_A \psi_{1,-} = -s_k \psi_{1,-}.$$  \hfill (33)

The chiral eigenvalues $g_{A,+} = s_k$, $g_{A,-} = -s_k$ and $g_{A,+} + g_{A,-}$ are plotted as a function of $k_y$ in Figs. 3(c), (d) and (f), respectively. The index in terms of the chiral eigenvalue of $\hat{\Gamma}_A$ can be calculated as

$$\mathcal{N}_{\text{ZES}} = \sum_{k_y} [g_{A,+} + g_{A,-}] = \left[ -\frac{2W_{k_F}}{\pi} \right] G.$$  \hfill (34)

The index is a finite value in the presence of spin-orbit interaction. The results implies the minimum value of
the zero-bias conductance would be quantized as

\[ G_{NS} = \frac{2e^2}{h} |N_{ZES}|, \tag{35} \]

due to the anomalous proximity effect. The integrand of Eq. (34) is nonzero at (I) and (III) in Eq. (12). Therefore we will make clear what happens on the anomalous Green’s function in these regions in the next section.

III. PAIRING CORRELATIONS IN \( H_1 \)

We first calculate the Green’s function in an infinitely long superconductor as shown in Fig. 1(a). When the Hamiltonian is given in Eq. (1), the Gor’kov equation is given by

\[ \begin{bmatrix} \hat{G}_r(k) \\ -\hat{F}_r(k) \end{bmatrix} = \tilde{\tau}_0 \tilde{\sigma}_0, \tag{36} \]

where

\[ X'_r(k) = X'_s(-k), \tag{37} \]

means the particle-hole conjugation. The solution of the Gor’kov equation is obtained as

\[ \hat{G}_r(k) = \left[ (\epsilon - \hat{H}_N) + \hat{\Delta} (\epsilon + \hat{H}_N)^{-1} \hat{\Delta} \right]^{-1}, \tag{38} \]

\[ \hat{F}_r(k) = \left[ \hat{\Delta} + (\epsilon + \hat{H}_N) \hat{\Delta}^{-1} (\epsilon - \hat{H}_N) \right]^{-1}. \tag{39} \]

The retarded Green’s function for \( H_1 \) is represented as

\[ \hat{G}_r(x - x') = \frac{1}{W} \sum_{k_y} e^{ik_y(y - y')} \hat{G}_r(x - x'), \tag{40} \]

\[ \hat{F}_r(x - x') = \frac{1}{W} \sum_{k_y} e^{ik_y(y - y')} \hat{F}_r(x - x'), \tag{41} \]

with

\[ \hat{G}_r(x - x') = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk e^{ik(x-x')} \frac{1}{2} \]

\[ \times \left[ \frac{(\epsilon + \xi_{1,+})(1 + \hat{\sigma}_1)}{(\epsilon + i\delta)^2 - E_{1,+}^2} + \frac{(\epsilon + \xi_{1,-})(1 - \hat{\sigma}_1)}{(\epsilon + i\delta)^2 - E_{1,-}^2} \right]. \tag{42} \]

\[ \hat{F}_r(x - x') = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk e^{ik(x-x')} \frac{\Delta_k}{2} \]

\[ \times \left[ \frac{1 - \hat{\sigma}_1}{(\epsilon + i\delta)^2 - E_{1,+}^2} + \frac{1 + \hat{\sigma}_1}{(\epsilon + i\delta)^2 - E_{1,-}^2} \right] i\hat{\sigma}_2. \tag{43} \]

By carrying out the integration, we obtain the normal Green’s function as

\[ \hat{G}_e(x - x') \approx -\frac{i e^{-q_\pi|x-x'|}}{2\Omega v_p} \left[ n_+(k_y) + n_-(k_y) \hat{\sigma}_1 \right] \]

\[ \times \left[ \epsilon \cos(p_x|x - x'|) + i \Omega \sin(p_x|x - x'|) \right], \tag{44} \]

with \( v_{pe} = p_e/m, \ \Omega = \sqrt{(\epsilon + i\delta)^2 - \Delta_k^2}, \) and \( p_e = \sqrt{k_x^2 - k_y^2 + 2\delta |k_y| k_F}. \) The anomalous Green’s function is also calculated as

\[ \hat{F}_e(x - x') \approx -\frac{i e^{-q_\pi|x-x'|}}{2\Omega v_F} \Delta_k \sin(p_x(x - x')) \times [n_+(k_y) + n_-(k_y) \hat{\sigma}_1] i\hat{\sigma}_2. \tag{45} \]

Here we consider the effects of the spin-orbit interaction within the first order of \( \alpha. \) The details of the derivation of these Green’s functions from Eqs. (42) and (43) are shown in Appendix A. As we mentioned at the end of Sec. II, the factor of \( \Theta(p_{\pi \pm}^2) \) and \( n_\pm(k_y) \) play a crucial role in the following argument. To check symmetry of the pairing correlations, we apply the analytic continuation \( \epsilon + i\delta \rightarrow i\omega_n \) to the anomalous Green’s function,

\[ \hat{F}_{\omega_n}(r - r') = -\hat{F}_{\omega_n}(r - r'), \tag{47} \]

where \( T \) means the transpose of the spin matrix meaning the interchanging spin of two electrons. The relation is derived from a fact that electrons obey the Fermi-Dirac statistics. The first term in Eq. (46) represents spin-singlet \( d_{xy\pm} \)-wave pairing correlation because it changes sign under \( x \leftrightarrow x' \) and \( y \leftrightarrow y' \) independently. Such pairing correlation is linked to the pair potential in the presence of attractive interactions between two electrons. The second term, on the other hand, remains unchanged under \( y \leftrightarrow y' \) and changes its sign under \( x \leftrightarrow x' \). The spin-orbit interaction induces a spin-triplet \( p_{\pm} \)-wave pairing correlation in uniform ground state. In Appendix A, we also show the four parts of the Green’s function in a uniform superconductor in Eqs. (A12)-(A15).

A. Pairing correlation at a surface

The purpose of this paper is to study symmetry of a Cooper pair appearing at a surface of a superconductor as shown in Fig. 1(b). To cut an infinitely long superconductor at \( x = 0 \), we introduce a high potential barrier \( V \delta(x)\hat{\tau}_3 \). Details of the method based on the Lippmann-Schwinger equation are explained in Appendix B. Eqs. (A12)-(A15) correspond to the elements of unperturbed Green’s function \( \hat{G}^{(0)} \) in Appendix B. The Green’s function near the surface can be calculated in Eq. (B7). We supply the results of \( \hat{Q} \)

\[ \lim_{V \rightarrow \infty} \hat{Q}(i\omega_n) = \frac{\Omega_n v_p}{2i\omega_n} \left[ n_+(k_y) + n_-(k_y) \hat{\sigma}_1 \hat{\tau}_3 \right]. \tag{48} \]

The anomalous Green’s function at a surface results in,
This is the central result of this paper. The first component represents pairing correlation belonging to even-frequency spin-singlet $d_{xy}$-wave pairing symmetry and is linked to the pair potential. The second component represents a pairing correlation belonging to even-frequency spin-triplet $p_x$-wave symmetry. We have already discussed symmetry of these components because they exist in a uniform superconductor as shown in Eq. (46).

The relation between these components are illustrated schematically in Fig. 4(a). The third correlation belongs to odd-frequency spin-singlet $p_y$-wave symmetry because it changes the sign under $y \leftrightarrow y'$, whereas it preserves the sign under $x \leftrightarrow x'$. The spin-singlet $d_{xy}$-wave component in the anomalous Green’s function is an odd function of $x - x'$, which is responsible for two phenomena at a surface: the appearance of highly degenerate surface bound states at the zero energy and the appearance of an odd-frequency $p_y$-wave pairing correlation. The last component in Eq. (49) belongs to odd-frequency spin-triplet $s$-wave symmetry. It is easy to check that this component does not change its sign in $x \leftrightarrow x'$ and $y \leftrightarrow y'$ independently. As illustrated in Fig. 4(a), this component is generated because the even-frequency spin-triplet $p_x$-wave component in the anomalous Green’s function is an odd function of $x - x'$. We conclude that the odd-frequency spin-triplet $s$-wave pairing correlation appears at a surface even through the spin-triplet $p_x$-wave pair potential is absent.

Here we discuss roles of these pairing correlations in the proximity effect in a dirty normal metal attached to the superconductor as shown in Fig. 1(c). The first three components in Eq. (49) cannot penetrate into a dirty normal metal because all of them are an odd-function of $y - y'$.[12-14] Only an $s$-wave component penetrates into a normal metal and modifies electromagnetic properties of the normal metal. The resulting proximity effect becomes strong at a low temperature because the last component is singular at $\omega_n = 0$. As the last term is proportional to $n_-(k_y)$, an odd-frequency spin-triplet $s$-wave correlation exists only region I and III in $k_y$. This property is closely related to the fact that $g_A+$ and $g_A-$ in Eq. (34) is a nonzero value of -1 at these regions as shown in Fig. 3(f). The anomalous proximity effect is characterized by the penetration of two objects into a normal metal. One is $s$-wave Cooper pairs whose pairing correlation is proportional to $1/\omega_n$. The other is zero-energy quasiparticles who belong to the same chiral eigenvalue.[8,10]

\[
\hat{F}_{\omega_n}^S (r, r') = \frac{1}{W} \sum_{k_y} e^{ik_y (y-y')} \frac{e^{-q_x (x+x')}}{2v_F} \left[ \frac{i}{\Omega_n} \sin p_x (x - x') \Delta \hat{k}_y n_+ (k_y) i\hat{\sigma}_2 - \frac{i}{\Omega_n} \sin p_x (x - x') \Delta \hat{k}_y n_- (k_y) \hat{\sigma}_1 i\hat{\sigma}_2 \right] + \frac{2}{\omega_n} \sin p_x x \sin p_x x' \Delta \hat{k}_y n_+ (k_y) i\hat{\sigma}_2 - \frac{2}{\omega_n} \sin p_x x \sin p_x x' \Delta \hat{k}_y n_- (k_y) \hat{\sigma}_1 i\hat{\sigma}_2 \right]. \tag{49}
\]

**B. Local density of states at a surface**

The local density of states at a surface is defined by

\[
N_S (x, \epsilon) = -\frac{1}{2\pi} \text{Im} \int_{-W/2}^{W/2} dy \text{Tr} \left[ \hat{G}_S^0 (r, r') \right], \tag{50}
\]

where the normal Green’s function at a surface is calculated to be

\[
\hat{G}_S (r, r') = \sum_{k_y} \frac{i e^{ik_y (y-y')} e^{-q_x (x+x')}}{2Wv_F \Omega (\epsilon + i\delta)} \left[ \epsilon^2 \cos p_x (x + x') + i \Omega \epsilon \sin p_x (x + x') + 2\Delta_k^2 \sin p_x x \sin p_x x' \right] \times [n_+ (k_y) + n_- (k_y) \hat{\sigma}_1] \cdot \hat{\sigma}_3 \tag{51}
\]

**FIG. 4.** The pairing correlations appearing in a $d_{xy}$-wave superconductor. At a surface of $d_{xy}$-wave superconductor, an odd-frequency spin-singlet $p_y$-wave pairing correlation always appear as a result of the sign change in the $d_{xy}$-wave pairing correlation under $x \leftrightarrow x'$. In (a), a spin-orbit interaction $\lambda k_y \hat{\sigma}_1$ induces spin-triplet $p_x$-wave pairing correlation in bulk. An odd-frequency spin-triplet $s$-wave component appears at a surface as a result of the sign change of the $p_y$-wave pairing correlation. In (b), $-\lambda k_x \hat{\sigma}_2 \hat{\sigma}_3$ induces spin-triplet $p_y$-wave pairing correlation in bulk.
The LDOS at a surface results in
\[ N_S(x, \epsilon) = \delta(\epsilon) \sum_{k_y} \frac{2|\Delta_k|}{v_{F_x}} \left\{ \sin(p_{xy}x) e^{-q_{xy}x} \right\}^2 \]
\[ \times n_+(k_y), \tag{52} \]
for \( \epsilon \ll \Delta \). The local density of states at a surface has a peak at the zero energy, which reflects the presence of highly degenerate surface bound states at the zero energy. The appearance of an odd-frequency pairing correlation and that of surface bound states at the zero energy are kinked to each other directly. In Eq. (48), \( 1/i\omega_n \) in \( Q \) characterizes odd-frequency symmetry of pairing correlations appearing at a surface. Simultaneously, the analytic continuation of \( Q(i\omega_n) \rightarrow Q(\epsilon + i\delta) \) in Eq. (48) describes the peak in the density of state at the zero energy in Eq. (52) through the transformation of
\[ \frac{1}{\epsilon + i\delta} = \frac{P}{\epsilon} - i\pi\delta(\epsilon). \tag{53} \]

IV. PAIRING CORRELATIONS IN \( \hat{H}_2 \)

The solution of the Gor’kov equation for \( \hat{H}_2 \) is represented as
\[ \hat{G}_r(x - x') = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk e^{ik(x-x')} \frac{1}{2} \]
\[ \times \left[ \frac{\epsilon + \xi_{2+}(1 + \hat{\sigma}_2)}{(\epsilon + i\delta)^2 - E_{2+}^2} + \frac{\epsilon + \xi_{2-}(1 - \hat{\sigma}_2)}{(\epsilon + i\delta)^2 - E_{2-}^2} \right], \tag{54} \]
\[ \hat{F}_r(x - x') = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk e^{ik(x-x')} \Delta_k \frac{1}{2} \]
\[ \times \left[ \frac{1 - \hat{\sigma}_2}{(\epsilon + i\delta)^2 - E_{2-}^2} + \frac{1 + \hat{\sigma}_2}{(\epsilon + i\delta)^2 - E_{2+}^2} \right] i\hat{\sigma}_2. \tag{55} \]
Within the first order \( \alpha \), the retarded Green’s functions in an infinitely long superconductor are calculated as,
\[ G_r(x - x') = \frac{-i}{\Omega v_{k_x}} e^{-q_y|x-x'|} \]
\[ \times \left| \epsilon \cos k_y |x-x'| + i\Omega \sin k_y |x-x'| \right|, \tag{56} \]
\[ F_r(x - x') = \frac{-i}{\Omega v_{k_x}} e^{-q_y|x-x'|} \left[ i\Delta_k \sin k_y |x-x'| \right. \]
\[ + \alpha \Delta k_y \cos k_y (x - x') \hat{\sigma}_2 \left. \right| i\hat{\sigma}_2. \tag{57} \]
The derivation of these equations are explained in Appendix A. The spin-orbit interaction does not modify the normal Green’s function within the first order of \( \alpha \) but it induces the second term in the anomalous Green’s function. To check the symmetry of pairing correlation, we apply the analytic continuation to the Matsubara representation,
\[ F_{i\omega_n}(x - x') = \frac{-1}{\Omega v_{k_x}} e^{-q_y|x-x'|} \left[ i\Delta_k \sin k_y (x - x') \right. \]
\[ + \Delta \Delta k_y \cos k_y (x - x') \hat{\sigma}_2 \left. \right| i\hat{\sigma}_2. \tag{58} \]

The first term belongs to spin-singlet \( d_{xy} \)-wave symmetry and is linked to the pair potential. The second term belongs to spin-triplet \( p_y \)-wave symmetry because it changes sign under \( k_y \rightarrow -k_y \) but retain its sign under \( x \leftrightarrow x' \). The spin-orbit interaction induces a spin-triplet \( p_y \)-wave pairing correlation in bulk.

The Green’s function at a surface is also calculated by solving the Lippmann-Schwinger equation. It is easy to have an expression
\[ \lim_{\nu \rightarrow \infty} Q(i\omega_n) = \frac{\Omega_n}{\omega_n} \frac{h v_{k_x}}{\omega_n^2} \left[ -i\omega_n - i\Delta \Delta k_y \right. \]
\[ - i\Delta k_y - i\omega_n \], \tag{59} \]
within the first order of \( \alpha \). The anomalous Green’s function at a surface is calculated to be
\[ F_{i\omega_n}^{\alpha}(r, r') = \frac{1}{W} \sum_{k_y} e^{ik_y(y-y')} \frac{e^{-q_y(x+x')}}{v_x} \]
\[ \times \left[ i\Delta_k \frac{\sin k_y |x-x'| + 2\Delta_k \sin k_x \sin k_x'}{\Omega_n} \right. \]
\[ + \alpha \Delta k_y \cos k_y (x - x') \hat{\sigma}_2 \left. \right| i\hat{\sigma}_2. \tag{60} \]
The second term is induced at a surface and belongs to odd-frequency spin-singlet \( p_y \)-wave symmetry. The relations among the pairing correlations are illustrated in Fig. 4(b). The proximity effect is absent in a normal metal attached to a superconductor described by \( \hat{H}_2 \) because all of the pairing correlations in Eq. (60) are an odd-function of \( y - y' \). Thus the spin-orbit interaction \( \lambda k_x \hat{\sigma}_2 \) does not contribute to the proximity effect. Finally, we supply the local density of states at a surface,
\[ N_S(x, \epsilon) = \sum_{k_y} \frac{4|\Delta_k|}{v_{k_x}} \left\{ \sin(k_y x) e^{-q_y x} \right\}^2 \delta(\epsilon). \tag{61} \]

The spin-orbit interaction in \( \hat{H}_2 \) does not modify the local density of states at a surface within the first order of \( \alpha \). As we have discussed in Sec. II C, the potential disorder near a surface lifts the degeneracy at the zero energy. As a result, the zero-energy peak in the local density of states vanishes at such a dirty surface.

V. DISCUSSION

A spin-orbit interaction \( \lambda k_y \hat{\sigma}_2 \) induces an odd-frequency spin-triplet \( s \)-wave pairing correlation at a surface of a \( d_{xy} \)-wave superconductor. The spectra of local density of states at a dirty surface would be roughly estimated as
\[ N_S(\epsilon) \approx \delta(\epsilon) \frac{\Delta}{v_F} |N_{ZES}|, \tag{62} \]
because of the extra chiral symmetry in \( \hat{H}_1 \). The close relationship between the odd-frequency \( s \)-wave pairing
correlation and the zero-energy states at a surface suggests the anomalous proximity effect in a normal metal. A spin-orbit interaction $-\lambda k_s \sigma_2$, on the other hand, does not contribute to the proximity effect. Here we discuss briefly the proximity effect of a $d_{xy}$-wave superconductor where the two spin-orbit interaction terms coexist and form the Rashba spin-orbit interaction $\lambda k_y \sigma_1 - \lambda k_x \sigma_2$. Unfortunately, the coexistence of the two interaction terms weakens the anomalous proximity effect because the interaction $-\lambda k_x \sigma_2$ breaks the extra chirality of $H_1$. It is easy to confirm that $-\lambda k_x \sigma_2 \gamma_3$ does not anticomute to $\Gamma_A$. Therefore, $|N_{ZES}|$-fold degeneracy at the zero-energy is not protected in the absence of the extra chiral symmetry. This explains why the signal of the anomalous proximity effect is very weak in the previous paper. To observe strong anomalous proximity effect, a $d_{xy}$-wave superconductor described by $H_1$ is necessary.

The controle of spin-orbit interactions has been an important issue also in spintronics. The spin-orbit interaction in $H_1$ can be realized by tuning the Rashba spin-orbit interaction and the Dresselhaus spin-orbit interaction. It has been known that such type of interaction stabilizes so called persistent spin helix and generates the flat Majorana band at the edge of a superconductor. Thus it would be possible to fabricate a superconductor exhibits the strong anomalous proximity effect by combining existing technologies.

VI. CONCLUSION

We theoretically study the effects of two types of spin-orbit interaction on the symmetry of a Cooper pair in a spin-singlet $d_{xy}$-wave superconductor. To analyze the pairing symmetry in an infinitely long superconductor, we solve the Gor’kov equation and obtain the anomalous Green’s function analytically. The anomalous Green’s function at surface of a superconductor is is calculated by solving the Lippmann-Schwinger equation exactly. The results show that a spin-orbit interaction term generates spin-triplet $p_x$-wave pairing correlation in bulk and an odd-frequency spin-triplet $s$-wave pairing correlation at a surface. By considering chiral symmetry of Hamiltonian, we discussed a close relationship among an odd-frequency $s$-wave pairing correlation, the degenerate zero-energy states and the anomalous proximity effect. We also discuss that one interaction term in the Rashba-type spin-orbit interaction enhances the anomalous proximity effect, whereas the other interaction term suppresses the anomalous proximity effect seriously. On the basis of the obtained results, we propose a design for a superconductor which exhibits the strong anomalous proximity effect.

To assist these conclusions, we should study low-energy transport properties through a dirty normal metal such as the conductance in a NS junction and the Josephson current in a SNS junction. The investigation by using numerical simulation is under way. Results will be presented in somewhere else.

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Appendix A: Green’s function in real space

To calculate the Green’s function in an infinitely long superconductor $\tilde{G}_s(x - x')$, it is necessary to carry out the integration

$$I_{1, \pm} = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk e^{ik(x - x')} \frac{f(k, \xi_{1, \pm})}{(\epsilon + i\delta - E_{1, \pm})(\epsilon + E_{1, \pm})},$$

where $f$ is an analytic function. For $\epsilon > 0$, $(\epsilon + E_{1, \pm})^{-1}$ is analytic, whereas $(\epsilon + i\delta - E_{1, \pm})^{-1}$ has two poles at $k = k_h^\pm = -k_h^\pm + i\delta^+$ and $k = k_h^\pm = k_h^\pm + i\delta^+$ with

$$k_h^\pm = \left[p_{1, \pm}^2 + \frac{2m}{h^2} \Omega \right]^{1/2}, \quad k_h^\pm = \left[p_{1, \pm}^2 - \frac{2m}{h^2} \Omega \right]^{1/2}, \quad \Omega = \sqrt{(\epsilon + i\delta)^2 - \Delta_k^2}$$

where $p_{1, \pm}$ is defined in Eq. (13). Paying attention to the relations $\xi_{1, \pm} \rightarrow \Omega$ at $k = k_h^\pm$ and $\xi_{1, \pm} \rightarrow -\Omega$ at $k = -k_h^\pm$, $\epsilon + i\delta - E_{1, \pm}$ at the denominator of Eq. (A1) can be expanded around $k = k_h^\pm$ as

$$\epsilon + i\delta - E_{1, \pm}(k) \approx \epsilon - E_{1, \pm}(k_h^\pm) + i\delta - \frac{\xi_{1, \pm}}{E_{1, \pm}(k_h^\pm)} \frac{\hbar^2 k_h^\pm}{m} (k - k_h^\pm) = -\frac{\Omega \hbar^2 k_h^\pm}{\epsilon} (k - k_h^\pm - i\delta^+)$$

(A3)
Around a pole of \( k = -k_h^s \), the \( \epsilon + i\delta - E_{1,\pm} \) becomes

\[
\epsilon + i\delta - E_{1,\pm}(k) \approx \epsilon - E_{1,\pm}(-k_h^s) + i\delta - \frac{\xi_{1,\pm}}{E_{1,\pm}(-k_h^s)} \frac{-h^2k_h^s}{m}(k + k_h^s) = -\frac{\Omega}{\epsilon} \left( -\frac{h^2k_h^s}{m} \right) (k + k_h^s - i\delta').
\] (A4)

By picking up the residues of these poles, the integral in Eq. (A1) is calculated as

\[
I_{1,\pm} = -i \frac{m}{2\Omega h^2} e^{-q_x|x-x'|} \left[ \frac{f(s_x p_x, \Omega)}{p_x} e^{ip_x|x-x'|} + \frac{f(-s_x p_x, -\Omega)}{p_x} e^{-ip_x|x-x'|} \right] \Theta(p_{1,\pm}^2),
\] (A5)

\[
s_x = \text{sgn}(x-x').
\] (A6)

where we consider the contribution from the propagating channels as indicated by a factor \( \Theta(p_{1,\pm}^2) \). The integral on the upper-half complex plane converges for \( x - x' > 0 \). On the other hand for \( x - x' < 0 \), the transformation of \( k \to -k \) is necessary, which produces a factor of \( s_x \). In the text, we mainly discuss the Green’s function for \( 0 < \epsilon \ll \Delta \).

In such case, it is possible to apply an approximation for the wavenumber \( k_h^s = p_{1,\pm} + iq_x \) and \( k_h^b = p_{1,\pm} - iq_x \) with \( q_x = (\Delta/\hbar v_F)\hat{k}_y \). We also apply \( p_{1,\pm} \approx p_{1,\pm} \approx p_x = \sqrt{k_h^2 - k_y^2 + 2\alpha k_F|k_y|} \) because the amplitude of the wavenumber is not important for analyzing the pairing symmetry. By the same reason, we consider \( q_x \) only in the exponential function. The integral under these approximations is represented as

\[
I_{1,\pm} \approx -i \frac{m}{2\Omega h^2} e^{-q_x|x-x'|} \left[ \frac{f(s_x p_x, \Omega)}{p_x} e^{ip_x|x-x'|} + \frac{f(-s_x p_x, -\Omega)}{p_x} e^{-ip_x|x-x'|} \right] \Theta(p_{1,\pm}^2).
\] (A7)

The normal Green’s function in Eq. (42) is calculated as

\[
G_\epsilon(x - x') = -i \frac{m}{4\Omega h^2} e^{-q_x|x-x'|} \left[ \frac{\epsilon + \Omega}{p_x} e^{ip_x|x-x'|} + \frac{\epsilon - \Omega}{p_x} e^{-ip_x|x-x'|} \right] (1 + \hat{\sigma}_1) \Theta(p_{1,\pm}^2)
\] (A8)

\[
-\frac{m}{4\Omega h^2} e^{-q_x|x-x'|} \left[ \frac{\epsilon + \Omega}{p_x} e^{ip_x|x-x'|} + \frac{\epsilon - \Omega}{p_x} e^{-ip_x|x-x'|} \right] (1 - \hat{\sigma}_1) \Theta(p_{1,\pm}^2),
\] (A9)

The anomalous Green’s function results in

\[
F_\epsilon(x - x') = -i \frac{m s_x}{4\Omega h^2} e^{-q_x|x-x'|} \left[ \frac{\Delta \hat{k}_y \hat{p}_x}{p_x} e^{ip_x|x-x'|} - \frac{\Delta \hat{k}_y \hat{p}_x}{p_x} e^{-ip_x|x-x'|} \right] (1 + \hat{\sigma}_1) \Theta(p_{1,\pm}^2)
\] (A10)

\[
-\frac{m s_x}{4\Omega h^2} e^{-q_x|x-x'|} \left[ \frac{\Delta \hat{k}_y \hat{p}_x}{p_x} e^{ip_x|x-x'|} - \frac{\Delta \hat{k}_y \hat{p}_x}{p_x} e^{-ip_x|x-x'|} \right] (1 - \hat{\sigma}_1) \Theta(p_{1,\pm}^2),
\] (A11)

To obtain the Green’s function at a surface, the four parts of the Green’s in a uniform superconductor are necessary. We supply them as follows,

\[
\hat{G}_{i\omega_n}(x - x') = -\frac{m}{2\Omega_B h^2} e^{-q_x|x-x'|} \left[ i\omega_n \cos(p_x|x-x'|) - \Omega_n \sin(p_x|x-x'|) \right] [n_+(k_y) + n_-(k_y)] \hat{\sigma}_1,
\] (A12)

\[
\hat{F}_{i\omega_n}(x - x') = -\frac{m}{2\Omega_B h^2} e^{-q_x|x-x'|} \Delta \hat{p}_y \hat{k}_y i \sin(p_x|x-x'|) [n_+(k_y) + n_-(k_y)] \hat{\sigma}_2.
\] (A13)

\[
\hat{F}_{i\omega_n}(x - x') = -\frac{m}{2\Omega_B h^2} \hat{p}_y \hat{k}_y i \sin(p_x|x-x'|) [n_+(k_y) - n_-(k_y)] \hat{\sigma}_1,
\] (A14)

\[
\hat{F}_{i\omega_n}(x - x') = -\frac{m}{2\Omega_B h^2} \Delta \hat{p}_y \hat{k}_y i \sin(p_x|x-x'|) [-n_+(k_y) + n_-(k_y)] \hat{\sigma}_2.
\] (A15)

where \( \Omega_n = \sqrt{(\omega_n)^2 + \Delta_k^2} \) and the particle-hole transformation is expressed by

\[
X_{i\omega_n}(x, k_y) = X_{i\omega_n}^*(x, -k_y),
\] (A16)
in this representation.

We supply only the results for $I_2$,

$$I_{2,\pm} = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk \, e^{ik(x-x')} \frac{f(k, \xi_{2,\pm})}{(\epsilon + i\delta - E_{2,\pm})(\epsilon + E_{2,\pm})},$$

(A17)

$$= -i e^{-q_0|x-x'|} \left[ f(\mp\alpha k_F + s_x k_x, \Omega) e^{i k_x |x-x'|} + f(\mp\alpha k_F - s_x k_x, -\Omega) e^{-i k_x |x-x'|} \right],$$

(A18)

where $v_x = \hbar k_x/m$ and $k_x = \sqrt{k_F^2 - k_y^2}$ is a real wavenumber.

Appendix B: Lippmann-Schwinger equation

The Lippmann-Schwinger equation relates the Green’s function in the presence of a perturbation to that in the absence of the perturbation as

$$\hat{G}_{i\omega_n}(r, r') = \hat{G}_{i\omega_n}^{(0)}(r, r') + \int dr_1 \hat{G}_{i\omega_n}^{(0)}(r, r_1) \hat{V}(r_1) \hat{G}_{i\omega_n}(r_1, r'),$$

(B1)

where $\hat{V}(r)$ is the perturbation potential. In this paper, we introduce a wall at $x = x_0$

$$\hat{V}(r) = V \delta(x - x_0) \hat{T}_3,$$

(B2)

to divide an infinitely long superconductor into two semi-infinite superconductors. Thus $G_{i\omega_n}^{(0)}$ is the Green’s function in a uniform superconductor which is infinitely long in the $x$ direction. Although the wall breaks the translational symmetry in the $x$ direction, the superconductor is translational invariant in the $y$ direction. Therefore it is possible to represent the Green’s function as

$$\hat{G}_{i\omega_n}(r, r') = \frac{1}{W} \sum_{k_y} \hat{G}_{i\omega_n}(x, x') e^{ik_y(y-y')}.$$  

(B3)

By substituting the expression into the equation, we find

$$\hat{G}_{i\omega_n}(x, x') = \hat{G}_{i\omega_n}^{(0)}(x, x') + \hat{G}_{i\omega_n}^{(0)}(x, x_0) V \hat{T}_3 \hat{G}_{i\omega_n}(x_0, x').$$

(B4)

By putting $x = x_0$, we obtain the Green’s function,

$$\hat{G}_{i\omega_n}(x_0, x') = \left[ 1 - \hat{G}_{i\omega_n}^{(0)}(x_0, x_0) V \hat{T}_3 \right]^{-1} \hat{G}_{i\omega_n}^{(0)}(x_0, x').$$

(B5)

Finally, we reach an relation,

$$\hat{G}_{i\omega_n}(x, x') = \hat{G}_{i\omega_n}^{(0)}(x, x') + \hat{G}_{i\omega_n}^{(0)}(x, x_0) Q(i\omega_n) \hat{G}_{i\omega_n}^{(0)}(x_0, x'),$$

(B6)

$$\hat{G}_{i\omega_n}^{(0)}(x, x_0) = \hat{G}_{i\omega_n}(x, x') Q(i\omega_n) \hat{G}_{i\omega_n}^{(0)}(x_0, x'),$$

(B7)

$$Q(i\omega_n) = V \hat{T}_3 \left[ 1 - \hat{G}_{i\omega_n}^{(0)}(x_0, x_0) V \hat{T}_3 \right]^{-1}.$$  

(B8)

The Green’s function at a surface of a superconductor is calculated by taking a limit of $V \rightarrow \infty$. In the text, we input $x_0 = 0$ and analyze Eq. (B7) near the surface $0 < x < \xi_0$ and $0 < x' < \xi_0$ with $\xi_0 = \hbar v_F/\pi \Delta$ being the coherence length.

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