Precise determination of proton magnetic radius from electron scattering data

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Radius extraction using theory-based method: Dispersively improved chiral EFT

Combines dispersion theory (analyticity, sum rules) and $\chi$EFT (dynamics, controled accuracy)
Correlates values of radii with FF behavior at larger $Q^2 \lesssim 1 \text{ GeV}^2$
Enables reliable determination of magnetic radius

**Method:** J. M. Alarcon, C. Weiss, PLB 784 (2018) 373; PRC 97, 055203 (2018);
J. M. Alarcon, A. N. Hiller Blin, M. Vicente Vacas, C. Weiss, NPA 964, 18 (2017)

**Radius extraction:** J. M. Alarcon, D. Higinbotham, C. Weiss, PRC 102 (2020) 035203
See also: J. M. Alarcon, D. Higinbotham, C. Weiss, Z. Ye, PRC 99 (2019) 044303
Motivation: Analyticity in radius extraction

- Challenges in proton radius extraction
  - Derivative at $Q^2 = 0$ from data at finite $Q^2 > 0$
  - Extrapolation $Q^2 \to 0$: Stability, functional bias?
    - Barcus, Higinbotham, this session
  - Magnetic radius: Contribution of $G^p_M$ to cross section $\propto \tau/\epsilon$, vanishes for $Q^2 \to 0$

- Analyticity
  - FFs analytic functions of $t = -Q^2$
  - Singularities at $t > 0$: Hadronic exchanges
  - Correlates functional behavior of FF at $Q^2 > 0$ with derivative at $Q^2 = 0$
  - Predicts size of higher derivatives
  - Global properties: Sum rules
    - Use in radius extraction!
**DIχEFT: Dispersively improved chiral EFT**

- **Dispersive representation**

\[
F_i(t) = \int_{t_{\text{thr}}}^{\infty} \frac{dt'}{\pi} \frac{\text{Im} F_i(t')}{t' - t - i0}
\]

Expresses analytic structure

\(\text{Im} F_i\) spectral function, constructed theoretically

- **Spectral function in \(\pi\pi\) region**

Elastic unitarity relation
Frazer, Fulco 1960; Höhler et al 1975+

Factorize \(\pi\pi\) rescattering using N/D method

\[
\Gamma_i/F_\pi: \pi\pi-N\bar{N} \text{ coupling, calculated in } \chi\text{EFT}
\]

Good convergence

\[
|F_\pi|^2: \pi\pi \text{ rescattering, taken from } e^+e^- \text{ data}
\]

Presently implemented LO + NLO + partial N2LO
Alarcon, Weiss, PLB 784 (2018) 373; PRC 97 (2018) 055203
DIχEFT: Sum rules and parameters

- Spectral function in high-mass region
  
  Parameterized by effective pole
  
  Sufficient for low-$Q^2$ form factors, uncertainty quantified
  Alarcon, Weiss PLB 784 (2018) 373

- Sum rules and parameters
  
  Sum rules for $F(0), F'(0) =$ charges, radii
  
  Express χEFT LEC in terms of radii
  
  Radii appear directly as parameters of spectral functions, control behavior
**D\chi EFT: Spectral functions**

- **Spectral functions in $\pi\pi$ region**

  Band shows variation with radii (PDG range)

  Good agreement with Roy-Steiner results

  Hoferichter et al 2017

Alarcon, Weiss, PLB 784 (2018) 373

Bands: Variation with nucleon radii (PDG range)
DIχEFT: Form factors

- Form factors from dispersion integral
  \[ G_{E,M}(t) = \int_0^\infty \frac{dt'}{4M^2} \frac{\text{Im} G_{E,M}(t')}{t' - t - i0} \]

- Family of FFs depending on radii
  Each member respects analyticity, sum rules
  Each has intrinsic theoretical uncertainty

- Radius correlated with finite-\(Q^2\) behavior
  Provided by analyticity
  \textit{Use for radius extraction!}

Alarcon, Higinbotham, Weiss, Ye PRC 99 (2019) 044303
Empirical FF: Global fit Ye et al 2017

\(G_M\) similar, dependence on \(r_M\)
Magnetic radius extraction: Procedure

- Use DIχEFT \( G_{E,M}^p(Q^2) \) with params \( r_E^p; r_M^p \)

- Fit Mainz A1 cross section data
  \[ E = 0.18 - 0.855 \text{ GeV}, \; Q^2 = 0.003 - 1.0 \text{ GeV}^2 \]
  Fit original cross secns with floating normalizations
  Alt: Fit reanalyzed cross secns of Lee Arrington Hill 2015 with recalc uncertainties: Same radii, lower \( \chi^2 \)

- Impact on magnetic radius
  Sensitivity of cross section to \( G_M^p \)
  Dependence of DIχEFT \( G_M^p \) on \( r_M^p \)
  Theoretical uncertainty from high-mass pole
  Use data up to \( Q^2 \approx 0.5 \text{ GeV}^2 \)

\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \frac{\epsilon [G_E^p]^2 + \tau [G_M^p]^2}{\epsilon (1 + \tau)}
\]

Alarcon, Higinbotham, Weiss, PRC 102 (2020) 035203
Magnetic radius extraction: Results

- Extracted radii

\[ r_E^{p} = 0.842 \pm 0.002 \text{ (fit 1}\sigma\text{) } \pm 0.005 - 0.002 \text{ (theory full-range) fm} \]

\[ r_M^{p} = 0.850 \pm 0.001 \text{ (fit 1}\sigma\text{) } \pm 0.009 - 0.004 \text{ (theory full-range) fm} \]

Magnetic radius has smaller fit uncertainty, larger theory uncorr.

Magnetic radius needs theory-based extraction method.

Consistent with results of empirical dispersive fits

Lorenz, Hammer, Meissner 2012

Alarcon, Higinbotham, Weiss, PRC 102 (2020) 035203
Summary

- **DIχEFT** describes nucleon FFs combining dispersion theory and χEFT
  
  Includes ππ rescattering and ρ resonance through unitarity
  
  Enables predictive calculations, controlled theoretical accuracy
  
  Excellent agreement with empirical FFs up to $Q^2 \sim 1 \text{ GeV}^2$ and beyond

- **DIχEFT** enables theory-based radius extraction
  
  Correlates $Q^2 = 0$ derivatives with finite-$Q^2$ behavior through analyticity + sum rules
  
  Employs radii directly as parameters ↔ LECs
  
  Enables reliable determination of magnetic radius from finite-$Q^2$ data

- **Other DIχEFT applications**

  Nucleon transverse charge/magnetization densities
  Alarcon, Weiss, in progress. APS DNP presentation KC.2 (Saturday 8:30 CDT)

  Nucleon scalar FF
  Alarcon, Weiss, PRC 96, 055206 (2017)
**DIχEFT form factors**

Evaluating using dispersion integral with spectral functions

Band shows variation with radii (PDG range).

Also quantified uncertainty from high-mass states

Excellent agreement with data. Not fit, but prediction based on dynamics