A novel robust generalized backstepping controlling method for a class of nonlinear systems

Malek Ghanavati, Karim Salahshoor, Mohammad Reza Jahed Motlagh, Amin Ramezani and Ali Moarefianpour

Abstract: This study is aimed at introducing a new robust control strategy in developed backstepping method designed for a special class of nonlinear systems, which does not require any information on the upper bound of parametric uncertainties. A robust generalized backstepping method (RGBM) is introduced by using of both generalized backstepping method and nonlinear damping of Lyapunov redesign. The RGBM is more efficient than the standard backstepping method because the standard backstepping method can only be applied to strictly feedback systems while RGBM expands this class. In addition, RGBM has could be applied to a special class of nonlinear systems with unmatched uncertainty and unknown upper bound. Numerical simulations demonstrate the feasibility and advantages of the proposed algorithm. Finally, the compressor surge control system is reviewed to show the robustness of the proposed scheme.

ABOUT THE AUTHORS
Malek Ghanavati received the BSc in Electrical Power, 2001 and the MSc degree in 2006, Iran. Currently, he is a PhD student in Control and System Engineering, Science and Research branch, Islamic Azad University, Iran. His research interests include nonlinear control, Process Control.

Karim Salahshoor is a professor at the Department of Automation and Instrumentation, Petroleum University of Technology (PUT), Tehran, Iran. His research interests include Intelligent Control and Advanced Process Control and Intelligent Monitoring.

Mohammad Reza Jahed Motlagh is a professor at the Department of Electrical Engineering, Iran University of Science and Technology, Iran. His research interests include Control Engineering, Nonlinear System and Artificial Intelligence.

Amin Ramezani is an assistant professor at the Department of Electrical Engineering, Tarbiat Modares University, Iran. His research interests are Process Control and Automation, Discrete Event Systems and Stochastic Process Control.

Ali Moarefianpour is an assistant professor of Electrical Engineering Department, Science and Research branch, Islamic Azad University, Iran. His research interests are Convex Optimization, Industrial Control and Artificial Intelligence.

PUBLIC INTEREST STATEMENT
This paper focuses on using robust control to develop a backstepping method for a special class of nonlinear systems, which does not require any information about upper bound of parametric uncertainty. In this research, two methods (developed backstepping method and nonlinear damping method) are combined to produce the robust generalized backstepping method (RGBM). Compared to the standard backstepping method, RGBM is far more efficient, because standard method is applicable in strictly feedback systems, but RGBM expands this class of systems. Another advantage of RGBM is that it can be applied to a special class of nonlinear systems. This control method, tend to stabilize the compressor system under surge instability.
1. Introduction

The backstepping method was first introduced in 1990 as a recursive Lyapunov method (Astrom & Wittenmark, 1995; Kokotović & Arcak, 2001; Krstic, Kanellakopoulos, & Kokotovic, 1995; Zhou & Wen, 2008). Recently, backstepping method and its application have been thoroughly discussed in nonlinear control (Krstic et al., 1995; Niu, Ahn, Li, & Liu, 2017; Niu & Li, 2017; Sepulchre, Janković, & Kokotović, 1997). This method can be kept away from unnecessary high gains and obtain overall stabilization in strict-feedback systems. Unlike feedback linearization methods (Isidori, 1989), backstepping method is more flexible in separating useless nonlinearities from useful nonlinearities and it can be adjusted with nonlinear parametric uncertainties, as well (Marino & Tomei, 1993; Qu, 1993).

As a recursive method, the robust backstepping system needs to overcome the nonlinear uncertainties in each stage. Some researchers (Freeman & Kokotović, 1996; Sepulchre et al., 1997) tried to design a system based on some calculations to create dominant functions in robust backstepping system. For that class of systems with strict feedback, the standard robust backstepping method can reach an overall stability in the presence of static uncertainties (Freeman & Kokotović, 1996). Nevertheless, standard small gain and relative differential equations should be referred (Battilotti, 1999; van der Schaft, 1996) or nonlinear small-gain theory should be applied to have a robust system against dynamic uncertainties, (Ghanavati & Chakravarthy, 2017; Jiang, Teel, & Praly, 1994; Praly & Wang, 1996). Generally, a stability constraint in optimal backstepping control does not necessarily lead to a pre-defined dynamic uncertainty, but it can change the static uncertainty to a robust uncertainty. As defined in Freeman and Kokotović (1996), the concept of Lyapunov equations cannot be applied to dynamic uncertainty.

An adaptive method was presented in Ghanavati and Chakravarthy (2015) and Lu and Xia (2012) using backstepping and sliding mode techniques to control the exact state as well as robustness against external disturbances and inertia uncertainties. Controller proposed by Lu and Xia (2012) is contiguous, in which no chattering phenomenon occurs. In addition, although no information on uncertain inertia matrix is included in this controller, its performance is desirable. The authors (Li, Li, & Tong, 2013; Queiroz, Arau, Fernandes, Dias, & Oliveira, 2010) used the modular backstepping indirect adaptive control method to control nonlinear systems; Van Oort, Sonneveldt, Chu, and Mulder (2007) used this method to control a nonlinear model, as well. The command filtering technique is used in modular adaptive backstepping method, but the natural frequency and filter damping coefficient are defined in continuous filter mode (Trigo, 2011). An adaptive robust controller using backstepping method is presented by Sheng, Huang, Zhang, and Huang (2014) in which fuzzy system was used to model unknown system characteristics. However, the main disadvantage of all above-mentioned controllers is that the designer needs to have information on system characteristics and/or the disturbance bounds. In practical systems, information on system characteristics, uncertainty and unknown upper bound sometimes incomplete, and there is no detailed information about them to design controller. Thus, it has been shown on robustness of the controlled system to cope with mentioned problems.

Inspired by generalized backstepping method, Ghanavati, Chakravarthy, and Menon (2017) and Sahab and Zarif (2009) tried to stabilize and track a special class of autonomous nonlinear control systems. However there is no constraint on the input and state variables so the present paper tried to address this issue. Consequently, in this research, results obtained from Sahab and Zarif (2009) are improved to design a special class of nonlinear systems, in which information on the upper bound of uncertainties is not needed. The robust generalized backstepping method (RGBM) enjoys two advantages over adaptive robust backstepping method with regulation functions presented in Yao (2003) and Zhang, Lu, and Xu (2012). Theoretically, asymptotic stability of state variables is guaranteed in
the mentioned method. Scientifically, RGBM is more efficient than other methods and can be applied in a special class of nonlinear systems with unmatched uncertainty as well as unknown upper bound. It can be observed in the conclusion section that this method provides a higher performance than the improved backstepping method used in Sahab and Zarif (2009) and Sheng et al. (2014).

Compared to the above mentioned research results, the content obtained of this paper indicates that RGBM has several advantages such as guaranteeing the asymptotic stability of the state variables and capability of being used in a special class of nonlinear systems with unmatched uncertainty and unknown upper bound. A numerical simulation was conducted to demonstrate the advantages and achievability of the presented algorithm. Results of the simulation show that RGBM controller guarantees robustness and, at the same time, it is able to control surge in serial compressors with disturbance and uncertainty.

This paper is organized as follows. The robust backstepping method is generalized for a special class of nonlinear systems with unmatched uncertainty in Section 2. Some assumptions are also provided in this section. In Section 3, RGBM is used to design a surge controller in serial compressors.

| Variable                          | Dim            | Non-dim      |
|-----------------------------------|----------------|--------------|
| Pressure rise coefficient         | \( p \)        | \( \psi = \frac{p}{\frac{1}{2} \rho U^2} \)          |
| Compressor pressure rise          | \( p_c \)      | \( \psi_c \)  |
| Mass flow coefficient             | \( m \)        | \( \varphi = \frac{m}{\rho U A_c} \)              |
| Valve gain                        | \( K \)        |              |
| Inlet duct length                 | \( l_i \)      |              |
| Compressor length                 | \( L_c \)      |              |
| Compressor and ducts length       | \( l_c \)      |              |
| Exit duct length                  | \( l_e \)      |              |
| Compressor duct area              | \( A_c \)      |              |
| Greitzer’s parameter              | \( \beta \)    | \( \beta = \frac{\nu}{a c} \sqrt{\frac{V_p}{A_c}} \) |
| Fluid density                     | \( \rho \)     |              |
| Speed of sound                    | \( a_s \)      |              |
| Mean rotor speed                  | \( U \)        |              |
| Plenum volume                     | \( V_p \)      |              |
| Time                              | \( t \)        | \( \xi = \frac{U}{\beta} t \)                      |
| Reciprocal time lag of compressor passage | \( \alpha \) |              |
| Compressor characteristic semi width | \( W \) |              |
| Compressor characteristic semi height | \( H \) |              |
| Rotor mean radius                 | \( R \)        |              |

**Subscripts**

- Compressor: \( c \)
- Throttle: \( T \)
- Plenum: \( P \)
- Inlet: \( I \)
- Close-coupled valve: \( V \)
- Exit: \( E \)
- Initial: \( 0 \)
- Throttle control valve: \( TCV \)
- Inlet Guide Vanes: \( IGV \)

| Table 1. Dimensional and non-dimensional variables (Uddin & Gravdahl, 2012) |   |
|-----------------------------------|----------------|--------------|
| Variable                          | Dim            | Non-dim      |
The results of simulation and discussions are presented in Section 4. Finally, Section 5 deals with conclusion of the paper. The utilized notations in this paper are summarized in Table 1.

2. Problem formulation by robust generalized backstepping

The extended backstepping method is applied in a bigger class of systems than the normal backstepping method (Sahab & Zarif, 2009). This class of nonlinear systems is as follows:

\[
\dot{X} = F(X) + G(X)\eta
\]

\[
\dot{\eta} = f(X, \eta) + g(X, \eta)u
\]

(1)

where \(f, g, F, G\) are known functions and \(u \in R\) is the control input. As such, \(X = [x_1, x_2, \ldots, x_n] \in R^n, \eta \in R\).

Using Lyapunov redesign, (Khalil, 2002) presented nonlinear damping controller, which can help in making the robust design against a class of nonlinear uncertain systems under matching conditions and unknown upper bound. This method can be applied on systems with the following form

\[
\dot{x} = f(t, x) + g(t, x)[u + \Gamma(t, x)\delta(t, x, u)]
\]

(2)

where \(f, g\) and \(\Gamma\) are known functions, while \(\delta\) is an unknown function and \(u\) is the control input. By using of two approaches mentioned in Khalil (2002) and Sahab and Zarif (2009), a new method presented which can be applied to a special class of nonlinear systems with unmatched uncertainty and unknown upper bound. This class of nonlinear systems is considered as follows:

\[
\dot{x}_i = f_i(x) + g_i(x)\eta_i + h_i(\delta_i(x), x), \quad 1 \leq i \leq n
\]

\[
\dot{\eta}_i = f_{n+i}(x, \eta_i) + g_{n+i}(X, \eta_i)u_i + h_{n+i}(X, \eta_i)\delta_{n+i}(X, \eta_i)
\]

(3)

where \(f_i, g_i, h_i, i = 1, 2, \ldots, 2n\) are known functions and \(\delta_i, i = 1, 2, \ldots, 2n\) are unknown functions. \(u_i \in R, i = 1, 2, \ldots, n\) are the control inputs. As such we have: \(X = [x_1, x_2, \ldots, x_n] \in R^n\) and \(\eta_i \in R, i = 1, 2, \ldots, n\). It is also assumed that \(f_i, g_i, h_i, \delta_i, i = 1, 2, \ldots, 2n\) for all \((t, X, \eta_i) \in [0, \infty) \times R^n \times R\), \(i = 1, 2, \ldots, n\), in relation to \(t\) are piecewise continuous and in relation to \(X, \eta_i\) are locally Lipschitz. In addition, \(\delta_i, i = 1, 2, \ldots, n\) is uniformly bounded for all \((t, X)\) and \(\delta_{n+i}, i = 1, 2, \ldots, n\) is uniformly bonded for all \((t, X, \eta_i)\).

This assumption ensures that the positive definite function \(V(x)\) with a negative definite derivative is as follows:

\[
V(x) = \frac{1}{2} \sum_{i=1}^{n} x_i^2
\]

\[
\dot{V}(x) = \sum_{i=1}^{n} x_i(f_i + g_i(\varphi_i + \psi_i) + h_i \delta_i) \leq -w(x)
\]

(4)

where \(w(x)\) is a positive definite function and \(\varphi_i, \psi_i\) are states feedback control law. Then system (3) can be rewritten as:

\[
\dot{x}_i = f_i + g_i(\varphi_i + \psi_i) + g_i(\eta_i - \varphi_i - \psi_i) + h_i \delta_i, \quad 1 \leq i \leq n
\]

\[
\dot{\eta}_i = f_{n+i} + g_{n+i}u_i + h_{n+i}(\delta_{n+i})
\]

(5)

Now, using the following inverse variation we have:

\[
z_i = \eta_i - \varphi_i - \psi_i, \quad 1 \leq i \leq n
\]

(6)
\[ z_i = f_{n+i} + g_{n+i}u_i + h_{n+i} \delta_{n+i} - \sum_{j=1}^{n} \left( \frac{\partial \varphi_i}{\partial x_j} + \frac{\partial \psi_i}{\partial x_j} \right) (f_j + g_j \eta_j + h_j \delta_j) \]  

(7)

After rewriting (5) we get:

\[ \dot{x}_i = f_i + g_i(\varphi_i + \psi_i) + g_i z_i + h_i \delta_i, \quad 1 \leq i \leq n \]

\[ \dot{z}_i = f_{n+i} + g_{n+i}u_i + h_{n+i} \delta_{n+i} - \sum_{j=1}^{n} \left( \frac{\partial \varphi_i}{\partial x_j} + \frac{\partial \psi_i}{\partial x_j} \right) (f_j + g_j \eta_j + h_j \delta_j) \]  

(8)

Now, the Lyapunov function in (3) is selected as:

\[ V_f(x, \eta) = \frac{1}{2} \sum_{i=1}^{n} x_i^2 + \frac{1}{2} \sum_{i=1}^{n} z_i^2 = V(x) + \frac{1}{2} \sum_{i=1}^{n} z_i^2 \]

(9)

which is a positive definite function. So, it is sufficient to specify if its derivative is negative definite. By differentiating (9) in relation to time, the following function is obtained

\[ \dot{V}_f(x, \eta) = \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} (f_i + g_i(\varphi_i + \psi_i) + h_i \delta_i) + \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} g_i z_i + \sum_{i=1}^{n} z_i \dot{z}_i \]

(10)

Considering (4), (10) can be simplified as

\[ \dot{V}_f(x, \eta) = -w(x) + \sum_{i=1}^{n} x_i g_i z_i + \sum_{i=1}^{n} z_i \left( f_{n+i} + g_{n+i}u_i + h_{n+i} \delta_{n+i} - \sum_{j=1}^{n} \left( \frac{\partial \varphi_i}{\partial x_j} + \frac{\partial \psi_i}{\partial x_j} \right) (f_j + g_j \eta_j) - \sum_{j=1}^{n} \left( \frac{\partial \varphi_i}{\partial x_j} + \frac{\partial \psi_i}{\partial x_j} \right) h_j \delta_j \right) \]

(11)

Now, the control inputs \( u_i \) are considered as:

\[ u_i = \frac{1}{g_{n+i}} \left( -f_{n+i} + \sum_{j=1}^{n} \left( \frac{\partial \varphi_i}{\partial x_j} + \frac{\partial \psi_i}{\partial x_j} \right) (f_j + g_j \eta_j) - x_i g_i z_i - k_i (\eta_i - \varphi_i - \psi_i) + v_i \right), k_i > 0 \]

(12)

where \( v_i \) is the virtual control input and \( g_{n+i}(X) \neq 0 \). Equation (11) can be rewritten after applying the control inputs from (12)

\[ \dot{V}_f(x, \eta) = -w(x) - \sum_{i=1}^{n} k_i z_i^2 + \sum_{i=1}^{n} z_i \left( v_i + h_{n+i} \delta_{n+i} - \sum_{j=1}^{n} \left( \frac{\partial \varphi_i}{\partial x_j} + \frac{\partial \psi_i}{\partial x_j} \right) h_j \delta_j \right) \]

(13)

Now, considering the following equation, we look for the virtual control variables \( v_i \)

\[ \sum_{i=1}^{n} z_i \left( v_i + h_{n+i} \delta_{n+i} - \sum_{j=1}^{n} \left( \frac{\partial \varphi_i}{\partial x_j} + \frac{\partial \psi_i}{\partial x_j} \right) h_j \delta_j \right) \]

(14)

So, it is assumed that \( v_i = v_{i1} + v_{i2} \) where \( v_{i1} \) and \( v_{i2} \) are used to make (15) and (16) negative, respectively

\[ \sum_{i=1}^{n} z_i (v_{i1} + h_{n+i} \delta_{n+i}) \]

(15)

\[ \sum_{i=1}^{n} z_i \left( v_{i2} - \sum_{j=1}^{n} \left( \frac{\partial \varphi_i}{\partial x_j} + \frac{\partial \psi_i}{\partial x_j} \right) h_j \delta_j \right) \]

(16)

Selecting
\( v_{ji} = -\gamma_{Ai} z_i \| h_{ni} \|_2^2, \quad \gamma_{Ai} > 0 \) \hfill (17)

The following is obtained
\[
\sum_{i=1}^{n} \left( -\gamma_{Ai} z_i \| h_{ni} \|_2^2 + z_i \| h_{ni} \|_2 K_{Ai} \right)
\]
where \( K_{Ai} \) is the (unknown) upper bound on \( \| z_i \|_2 h_{ni} \|_2 \). In Equation (18) reaches the following maximum
\[
\sum_{i=1}^{n} \frac{K_{Ai}^2}{\gamma_{Ai}^2}
\]

Selecting \( v_{ji} = -\gamma_{Bi} z_i \| H_i \|_2^2, \quad \gamma_{Bi} > 0 \) \hfill (21)
where
\[
H_i = -\sum_{j=1}^{n} \left( \frac{\partial \varphi_j}{\partial x_j} + \frac{\partial \psi_j}{\partial x_j} \right) h_j
\]

The following is obtained
\[
\sum_{i=1}^{n} \left( -\gamma_{Bi} \| z_i \|_2^2 H_i \|_2^2 + \| z_i \|_2 H_i \|_2 K_{Bi} \right)
\]
Where, \( K_{Bi}, i = 1, 2, \ldots, n \) are the (unknown) upper bounds on \( \| \tilde{z}_i \|_2 \) \hfill (23)

In Equation (23) reaches the following maximum
\[
\sum_{i=1}^{n} \frac{K_{Bi}^2}{\gamma_{Bi}^2}
\]

So, we have:
\[
\dot{V}_T(X, \eta) = -w(X) - \sum_{i=1}^{n} k_i z_i^2 + \sum_{i=1}^{n} \frac{K_{Ai}^2}{\gamma_{Ai}^2} + \sum_{i=1}^{n} \frac{K_{Bi}^2}{\gamma_{Bi}^2}
\]
(26)

Since \( w(X) \) belongs to class \( \mathcal{K}_\infty \), it is always true that \( \dot{V} \) is negative outside some ball and according to Lemma 14.1 from (Khalil, 2002), for each initial state \( x(t_0) \) the solution of the closed-loop system is uniformly bounded. Using (Khalil, 2002), the additional control \( ps_i(X), i = 1, 2, \ldots, n \) can be considered as:
\[
\psi_i = -\frac{\gamma_i}{g_i} x_i \| h_i \|_2^2, \quad \gamma_i > 0 \hfill (27)
\]
where, \( g_i(X) \neq 0, i = 1, 2, \ldots, n \).
3. An illustrated example

In this section, a generalized backstepping controller is designed, which does not require any information about disturbance upper bound or parametric uncertainty and throttle gain and throttle valve feature. Thus, we have shown the robustness of the controlled system to cope with the time varying flow and pressure disturbances. The dynamic model (MG-Model) for a compressor was presented by Moor and Greitzer (1986), which includes a gas compressor, pressure tank, a close coupled valve (CCV), a throttle valve and the connective ducts, as illustrated in Figure 1 (Gravdahl & Egeland, 1997).

To this end, equations in Figure 1 describing the modes of three serial compressors are stated as:

\[
\dot{p}_1 = \frac{1}{4B_1^2 c_1} \left( m_1 - \varphi_{T_1}(p_1) + \delta_1 h_1(p_1, p_2, p_3) \right)
\]

\[
m_1 = \frac{1}{l_{c_1}} (\psi_{c_1}(m_1) - p_1 - u_1 + \delta_4 h_4(m_1, m_2, m_3))
\]

\[
\dot{p}_2 = \frac{1}{4B_2^2 c_2} (m_2 - \varphi_{T_2}(p_2) + \delta_2 p_2)
\]

\[
m_2 = \frac{1}{l_{c_2}} (\psi_{c_2}(m_2) - p_2 - u_2 + \delta_5 m_1)
\]

\[
\dot{p}_3 = \frac{1}{4B_3^2 c_3} (m_3 - \varphi_{T_3}(p_3) + \delta_3 p_3)
\]

\[
m_3 = \frac{1}{l_{c_3}} (\psi_{c_3}(m_3) - p_3 - u_3 + \delta_6 m_2)
\]

(28)

Selecting

\[
x_1 = p_1, x_2 = p_2, x_3 = p_3, \eta_1 = m_1, \eta_2 = m_2, \eta_3 = m_3
\]

(29)

and according to Gravdahl and Egeland (1999), we have:

\[
h_1(p_1, p_2, p_3) = 0, h_4(m_1, m_2, m_3) = 0
\]

(30)

Equation (28) is rewritten as:
\[
\begin{align*}
\dot{x}_1 &= \frac{1}{4B_1^2l_c}(\eta_1 - \varphi_1(x_1)) \\
\dot{x}_2 &= \frac{1}{4B_2^2l_c}(\eta_2 - \varphi_2(x_2) + \delta_2x_1) \\
\dot{x}_3 &= \frac{1}{4B_3^2l_c}(\eta_3 - \varphi_3(x_3) + \delta_3x_2) \\
\dot{\eta}_1 &= \frac{1}{l_c}(\psi_{c_1}(\eta_1) - x_1 - u_1) \\
\dot{\eta}_2 &= \frac{1}{l_c}(\psi_{c_2}(\eta_2) - x_2 - u_2 + \delta_2\eta_1) \\
\dot{\eta}_3 &= \frac{1}{l_c}(\psi_{c_3}(\eta_3) - x_3 - u_3 + \delta_3\eta_2)
\end{align*}
\] (31)

Now, selecting
\[
\begin{align*}
Z_1 &= \eta_1 - \varphi_1 - \psi_1 \\
Z_2 &= \eta_2 - \varphi_2 - \psi_2 \\
Z_3 &= \eta_3 - \varphi_3 - \psi_3
\end{align*}
\] (32)

According to (9) the overall Lyapunov function is selected as:
\[
V(x,z) = 2B_1^2l_c x_1^2 + 2B_2^2l_c x_2^2 + 2B_3^2l_c x_3^2 + \frac{l_c}{2}z_1^2 + \frac{l_c}{2}z_2^2 + \frac{l_c}{2}z_3^2
\] (33)

which is a positive definite function. To make stable the overall system, it is sufficient to have a negative Lyapunov equation derivative.
\[
\dot{V}(x,z) = x_1(z_1 + \varphi_1 + \psi_1 - \varphi_{c_1}(x_1)) + x_2(z_2 + \varphi_2 + \psi_2 - \varphi_{c_2}(x_2) + \delta_2x_1) + x_3(z_3 + \varphi_3 + \psi_3 - \varphi_{c_3}(x_3) + \delta_3x_2)
\]
\[
+ Z_1(\psi_{c_1}(\eta_1) - x_1 - u_1 - l_c(\varphi_1 + \psi_1)) + Z_2(\psi_{c_2}(\eta_2) - x_2 - u_2 + \delta_2\eta_1 - l_c(\varphi_2 + \psi_2)) + Z_3(\psi_{c_3}(\eta_3) - x_3 - u_3 + \delta_3\eta_2)
\] (34)

The virtual control vectors \(\varphi_i\) and \(i = 1, 2, 3\) are selected as:
\[
\varphi_1 = \varphi_2 = \varphi_3 = 0
\] (35)

We have:
\[
\dot{V}(x,z) = -x_1\varphi_{c_1}(x_1) - x_2\varphi_{c_2}(x_2) - x_3\varphi_{c_3}(x_3) + x_1\psi_1 + x_2(\psi_2 + \delta_2x_1) + x_3(\psi_3 + \delta_3x_2)
\]
\[
+ Z_1(\psi_{c_1}(\eta_1) - u_1 - l_c\psi_1) + Z_2(\psi_{c_2}(\eta_2) - u_2 - l_c\psi_2 + \delta_2\eta_1) + Z_3(\psi_{c_3}(\eta_3) - u_3 - l_c\psi_3 + \delta_3\eta_2)
\] (36)

Selecting
\[
\psi_1 = 0, \psi_2 = -\gamma_2x_2 x_1^2, \psi_3 = -\gamma_3x_3 x_2^2, \gamma_2, \gamma_3 > 0
\] (37)

and selecting \(u_i\) as follows
\( u_1 = \psi_c (\eta_1) + k_1z_1k_1 > 0 \)  

(38)

We have:

\[
\dot{V}(x, z) = -x_1 \varphi_T(x_1) - x_2 \varphi_T(x_2) - x_3 \varphi_T(x_3) - \eta_2(x_3) - \frac{\dot{\eta}_2^2}{2\eta_2} + \frac{\dot{\eta}_2^2}{4\eta_2} - \eta_3(x_2 \dot{x}_3 - \frac{\dot{x}_3}{2\eta_3})^2 \\
+ \frac{\dot{\eta}_2^2}{4\eta_3} - k_1 z_1^2 + z_2(\psi_c(\eta_2) - u_2 + \delta_2 \eta_1 - l_c \psi_2) + z_3(\psi_c(\eta_3) - u_3 + \delta_c \eta_2 - l_c \psi_3)
\]

(39)

Now, in order to obtain \( u_2 \), the following equation is considered

\[
z_2(\psi_c(\eta_2) - u_2 + \delta_2 \eta_1 - l_c \frac{1}{4B_1^2 l_c} \frac{\partial \psi_c}{\partial x_1} (\eta_1 - \varphi_T(x_1)) - l_c \frac{1}{4B_2^2 l_c} \frac{\partial \psi_c}{\partial x_2} (\eta_2 - \varphi_T(x_2)) - l_c \frac{1}{4B_3^2 l_c} \frac{\partial \psi_c}{\partial x_3} (\eta_3 - \varphi_T(x_3))) + k_2 z_2 - V_2, k_2 > 0
\]

(40)

Selecting \( u_2 \) as follows

\[
u_2 = \psi_c(\eta_2) - l_c \left( \frac{1}{4B_1^2 l_c} \frac{\partial \psi_c}{\partial x_1} (\eta_1 - \varphi_T(x_1)) + \frac{1}{4B_2^2 l_c} \frac{\partial \psi_c}{\partial x_2} (\eta_2 - \varphi_T(x_2)) + k_2 z_2 - V_2, k_2 > 0
\]

(41)

We have:

\[-k_2 z_2^2 + z_2 \left( V_2 + \delta_5 \eta_1 - l_c \frac{1}{4B_2^2 l_c} \frac{\partial \psi_c}{\partial x_2} (\eta_2 - \varphi_T(x_2)) \right) \]

(42)

Defining

\[V_2 = V_{21} + V_{22}, H_2 = -\frac{1}{4B_2^2} \frac{\partial \psi_c}{\partial x_2} x_2 \]

(43)

We have:

\[-k_2 z_2^2 + z_2 (V_{21} + \delta_5 \eta_1) + z_3 (V_{22} + H_2, \delta_1) \]

(44)

Selecting

\[V_{21} = -c_{21} z_2 \eta_1, V_{22} = -c_{22} z_2 H_2, c_{21}, c_{22} > 0 \]

(45)

We have:

\[-k_2 z_2^2 - c_{21} \left( z_2 \eta_1 - \frac{\delta_5}{2c_{21}} \right)^2 + c_{22} \left( z_2 H_2 - \frac{\delta_2}{2c_{22}} \right)^2 + \delta_2^2 \]

(46)

Now, in order to obtain \( u_3 \), the following equation is considered:

\[
z_3 \left( \psi_c(\eta_3) - u_3 + \delta_6 \eta_2 - l_c \frac{\partial \psi_c}{\partial x_2} \left( \frac{1}{4B_2^2 l_c} \right) (\eta_2 - \varphi_T(x_2)) - l_c \frac{\partial \psi_c}{\partial x_2} \left( \frac{1}{4B_2^2 l_c} \right) x_2 \delta_2 \right) \]

(47)

Selecting \( u_3 \) as follows:
\[ u_3 = \psi_c(\eta_3) - l_c \left( \frac{1}{4B^2_{3,1}} \frac{\partial \psi_3}{\partial x_2} (\eta_2 - \varphi_{T_1}(x_2)) + \frac{1}{4B^2_{3,1}} \frac{\partial \psi_3}{\partial x_3} (\eta_3 - \varphi_{T_1}(x_3)) \right) + k_3 z_3 - V_3, k_3 > 0 \] (48)

We have:

\[ -k_3 z_3^2 + z_3 \left( V_3 + \delta_6 \eta_2 - \frac{l_c}{4B^2_{2,1}} \frac{\partial \psi_3}{\partial x_2} x_1 \delta_2 - \frac{1}{4B^2_{3,1}} \frac{\partial \psi_3}{\partial x_3} x_2 \delta_3 \right) \] (49)

Defining

\[ V_3 = V_{31} + V_{32} + V_{33}, H_{31} = - \frac{l_c}{4B^2_{2,1}} \frac{\partial \psi_3}{\partial x_2} x_1, H_{32} = - \frac{1}{4B^2_{3,1}} \frac{\partial \psi_3}{\partial x_3} x_2 \] (50)

We have:

\[ -k_3 z_3^2 + z_3 \left( V_{31} + \delta_6 \eta_2 \right) + z_3 \left( V_{32} + H_{31} \delta_2 \right) + z_3 \left( V_{33} + H_{32} \delta_3 \right) \] (51)

Selecting

\[ V_{31} = -c_{31} z_3^2 \eta_2, V_{32} = -c_{32} z_3^2 H_{31}, V_{33} = -c_{33} z_3^2 H_{32}, c_{31}, c_{32}, c_{33} > 0 \] (52)

We have:

\[ -k_3 z_3^2 = c_{31} \left( z_3 \eta_2 - \frac{\delta_6}{2c_{31}} \right)^2 + \frac{\delta_6^2}{4c_{31}} - c_{32} \left( -\frac{\delta_3}{2c_{32}} - \frac{\delta_3}{2c_{32}} \right)^2 + \frac{\delta_3^2}{4c_{32}} - c_{33} \left( -\frac{\delta_3}{2c_{33}} - \frac{\delta_3}{2c_{33}} \right)^2 + \frac{\delta_3^2}{4c_{33}} \] (53)

Finally, substituting \( u_2 \) and \( u_3 \) in (39) we have:

\[ V(x, z) = -x_i \varphi_{T_1}(x_i) - x_i \varphi_{T_2}(x_i) - x_i \varphi_{T_3}(x_i) - k_3 z_3^2 - k_3 z_3^2 - k_3 z_3^2 - k_3 z_3^2 - k_3 z_3^2 - k_3 z_3^2 - r_3 \left( x_1 x_1 - \frac{\delta_1}{2c_{11}} \right)^2 - r_3 \left( x_2 x_2 - \frac{\delta_1}{2c_{21}} \right)^2 - c_{31} \left( z_3 \eta_1 - \frac{\delta_3}{2c_{31}} \right)^2 - c_{32} \left( z_3 \eta_1 - \frac{\delta_3}{2c_{32}} \right)^2 - c_{33} \left( z_3 \eta_1 - \frac{\delta_3}{2c_{33}} \right)^2 \] (54)

According to the Equation (2.45) from Gravdahl and Egeland (1999), the throttle is assumed passive, that is \( \dot{\psi} \dot{\Phi}_i(\dot{\psi}) \geq 0 \) \( \forall \dot{\psi} \). We have:

\[ \dot{\psi} \dot{\Phi}_i(\dot{\psi}) \geq -x_i \varphi_{T_i}(x_i), \quad i = 1, 2, 3 \] (55)

and according to the Lemma 2.26 from Krstic et al. (1995), it follows that \( V \) is negative whenever \( W(X) > \frac{\delta_3}{4c_{33}} \)

4. Numerical simulation and discussions

In this section, RGBM controller presented in the previous sections is applied in three serial compressors using MATLAB software, as shown in Figure 2. Compressors equipped with CCV are modeled based on MG-Model (Gravdahl & Egeland, 1997).

Parameters of compressors are the same and according to Greitzer (1976), they are considered as:
According to Gravdahl and Egeland (1999), flow disturbance can be a result of the compressor upstream processes or their serial pattern so in the conducted simulations, impact of the compressors

\[
\begin{align*}
B_1 &= 1.8, \quad l_{c1} = 3, \quad H_1 = 0.18, \quad W_1 = 0.25, \quad \Psi_{c01} = 0.3 \\
B_2 &= 1.8, \quad l_{c2} = 3, \quad H_2 = 0.18, \quad W_2 = 0.25, \quad \Psi_{c02} = 0.3 \\
B_3 &= 1.8, \quad l_{c3} = 3, \quad H_3 = 0.18, \quad W_3 = 0.25, \quad \Psi_{c03} = 0.3
\end{align*}
\]

Figure 2. Arrangement of three serial compressors.
Source: Menon (2005).

Figure 3. Output flow and pressure for compressor 1, 2 and 3.

Figure 4. Control signal for compressor 1.
serial pattern is shown as flow disturbance in the equations. In order to have a more precise investigation on the controller mentioned in this research and to have a better understanding of the controller performance in different states, compressor output pressure and flow diagrams are compared before and after the application of controlling force. In all examined states, despite the fact that there is no information on the upper bound of parametric uncertainties, it can be observed that the performance of the designed controller with RGBM is robust and more effective compared with extended backstepping method presented in Sahab and Zarif (2009). Although the method presented in Sahab and Zarif (2009) requires information on nonlinear uncertain systems under matching conditions, its lack of robustness to bounded uncertainties is clearly observed.

There are several applications such as ones provided by Yao, Jiao, and Ma, (2015), Yao, Deng, and Jiao (2015) and Sheng et al. (2014), which we can apply the controller to demonstrate the effectiveness of the method. In this section we consider the system in Sheng et al. (2014) and our goal is to compare the method presented in this paper with the existing backstepping controllers. In the first
step the system is simulated with no control input. Then, in compressor 1, at \( t = 100 \) s, in compressor 2, at \( t = 120 \) s and in compressor 3, at \( t = 150 \) s, the throttle valve gain coefficient \( \gamma_T \) is reduced from 0.65 to 0.6. The flow and pressure outputs for compressors 1–3 is shown in Figure 3.

Results of simulation on the uncontrolled system are presented in Figure 3. It can be observed in the figures that when the throttle gain is reduced from 0.65 to 0.6 so the compression system is unstable. The simulation results clearly show the transfluence of the compressor operating range and its entrance into the surge instability area, which creates a limit cycle. Figures 4–6, show the control signals. As can be seen, signals indicated in this paper are positive and follow the actuator restriction (valve capacity requires the constraint \( 0 \leq u(t) \leq 1 \) to hold), while the control signal of Sheng et al. (2014) is negative. As illustrated in Figures 7–9, the status of output flow was studied by applying the control signal through RGBM controller. The pressure outputs for compressors 1–3 used RGBM controller is shown in Figures 10–12, respectively.
As it can be seen in Figures 7–12, the flow and pressure for compressors 1, 2 and 3 are stabilized by using the designed RGBM controller and the system is controlled in the presence of disturbance, while the equilibrium point is located in the unstable area. As derived in Section 2, RGBM controller can damp the unstable modes as the operation of the compression system pushed into the surge area by restricting the flow through the throttle valve. Given the control signal is applied to CCV actuator, it cannot take a negative value and this is the advantage of the present research and designed controller while the control signal for Sheng et al. (2014) is negative and physically impractical. In addition, the controller presented in Sahab and Zarif (2009) cannot control any of these compressors by applying the control signal, which uses the method of Sahab and Zarif (2009), all three compressors studied in this research enter the surge area and become unstable.
5. Conclusion

Results obtained from the simulation show that RGBM enjoys a great accuracy. Regarding to the diagrams presented in this paper, it can be observed that RGBM controller has stabilized all the three compressors with great accuracy. RGBM controller provides a good performance in the presence of parametric uncertainty under unmatched condition. Since this method has a lower level of error and a better tracking compared with extended backstepping method, it is more suitable for a special class of nonlinear systems than for a standard and extended backstepping method, in the case of unmatched uncertainty and unknown upper bound. In addition, it was observed that there is no instability in the simulation results and, unlike the standard backstepping method RGBM presented in this paper is more robust against the bounded uncertainties, which is one of its advantages of RGBM over other methods. Further, RGBM provides a high accuracy for nonlinear systems with strictly feedback form even when a bounded uncertainty exists.
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Author details
Malek Ghanavati1
E-mail: m.ghanavati@srbiau.ac.ir
ORCID ID: http://orcid.org/0000-0001-9794-6377
Karim Salashoor2
E-mail: salashoor@put.ac.ir
ORCID ID: http://orcid.org/0000-0002-9751-9540
Mohammad Reza Jahed Motlaghi3
E-mail: jahedmr@iust.ac.ir
ORCID ID: http://orcid.org/0000-0001-8662-1155
Amin Ramezani4
E-mail: ramezani@modares.ac.ir
ORCID ID: http://orcid.org/0000-0002-9751-9540
E-mail: salahshoor@put.ac.ir
ORCID ID: http://orcid.org/0000-0001-9794-6377
E-mail: m.ghanavati@srbiau.ac.ir

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