Adaptive Control With Global Exponential Stability for Parameter-Varying Nonlinear Systems Under Unknown Control Gains

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Abstract—It is nontrivial to achieve exponential stability even for time-invariant nonlinear systems with matched uncertainties and persistent excitation (PE) condition. In this article, without the need for PE condition, we address the problem of global exponential stabilization of strict-feedback systems with mismatched uncertainties and unknown yet time-varying control gains. The resultant control, embedded with time-varying feedback gains, is capable of ensuring global exponential stability of parametric-strict-feedback systems in the absence of persistence of excitation. By using the enhanced Nussbaum function, the previous results are extended to more general nonlinear systems where the sign and magnitude of the time-varying control gain are unknown. In particular, the argument of the Nussbaum function is guaranteed to be always positive with the aid of nonlinear damping design, which is critical to perform a straightforward technical analysis of the boundedness of the Nussbaum function. Finally, the global exponential stability of parameter-varying strict-feedback systems, the boundedness of the control input and the update rate, and the asymptotic constancy of the parameter estimate are established. Numerical simulations are carried out to verify the effectiveness and benefits of the proposed methods.

Index Terms—Adaptive control, exponential stability, Nussbaum function, parameter-varying nonlinear systems.

I. INTRODUCTION

THE PAST few decades have witnessed extensive developments and applications of nonlinear adaptive control for dynamic systems with unknown parameters [1], [2], [3], [4], [5], [6], [7], [8], [9]. In the early works on adaptive control [1], [2], [3], certain restrictions, such as matching condition, extended matching condition, and growth conditions on system nonlinearities, are normally imposed. The adaptive backstepping technology [4], [5], on the other hand, has completely removed these restrictive conditions, thus motivating a considerable amount of studies on adaptive control of various systems (see [8], [9]). It is noted that most existing works focus primarily on systems with constant parameters and known control directions.

To address the adaptive estimation of time-varying parameters, the pioneering work [10] proposes a method to exponentially stabilize linear time-varying systems in virtue of persistent excitation (PE) condition. Subsequently, such condition is removed in [11] and the linear model is extended to the robot system in [12]. Thereafter, the output feedback scheme is studied in [13], where the so-called projection operation is exploited to guarantee the boundedness of slow time-varying parameter estimate. For more general systems, such as strict-feedback systems, the soft sign function is introduced to cope with unknown parameters in [8] and [9]. More recently, an elegant adaptive scheme based on “congelation of variables” method is proposed in [14] and [15] to asymptotically stabilize parametric strict-feedback systems with fast time-varying parameters, opening a new venue for developing certainty equivalence controller for nonlinear time-varying systems. This method is also used to address the asymptotic/formation tracking of multiagent systems in [16], [17], and [18] and the prescribed performance control of strict-feedback systems in [19].

Interest in adaptive control of nonlinear systems with unknown control direction is stimulated by the development of Nussbaum functions [20] and the corresponding lemmas for stability analysis [21], [22]. The idea behind such control is that by specifying a controller with a Nussbaum gain, so that the controller degrades the system performance in the period with a wrong direction, but rewards the system with quicker movement to the desired state with a higher gain when the sign alternates to a correct direction in the subsequent period. In [23], [24], and [25], based upon this idea, the adaptive neural control, adaptive fuzzy control, and adaptive guaranteed performance control are proposed for nonlinear systems without a priori knowledge of the control direction, but these results need the precise information of the control gain magnitude. When the control coefficient is unknown both in sign and magnitude, some robust and adaptive results (e.g., [26] and [27]) achieving asymptotic stability of the closed-loop
system are established through a particular Nussbaum function \( \exp(x^2) \cos(\pi x/2) \). Recently, the results in [28] and [29] invent a class of enhanced Nussbaum functions to deal with time-varying and/or multivariable unknown control coefficients. The introduction of the Nussbaum functions not only presents the likelihood of realization of the adaptive scheme but also enables some practical applications. Hence, it is meaningful and necessary to address the issue of adaptive control for nonlinear systems with time-varying parameters and unknown control coefficient (including control direction and control gain magnitude).

Motivated by the above analysis, here in this work we develop an adaptive control method to guarantee global exponential stability of parameter-varying strict-feedback systems with unknown control direction. The global exponential stability of nonlinear systems is under explored even if the systems do not involve time-varying parameters and/or unknown control coefficient. The original work that employs acceleration control (with exponential convergence) for stabilizing strict-feedback systems is [30], where a time-varying scaling is used to accelerate the original system dynamics; after stabilizing the accelerated system, the convergence rate of the original system can be assigned by incorporating a suitable time-varying function into the control scheme. The contributions of this article are three-fold.

1) Different from the asymptotic results in [14] and [15] relying on the a priori knowledge of the control direction, our algorithm is effective for parameter-varying systems with unknown control direction.

2) Exponential convergence is normally realized at the rather restrictive PE condition, here we achieve exponential convergence without PE. In addition, the proposed method is capable of dealing with time-varying parameters, covering the existing one [30] on exponential stabilization as a special case.

3) Unlike some prescribed performance control results that ensure the output or the tracking error converges exponentially to near zero [31], [32], [33], [34], [35], [36], global zero-error regulation for all states can be achieved under the proposed method.

The remainder of this article is organized as follows. In Section II, we state the necessary mathematical preliminaries and a key lemma for our main results. We introduce some scalar examples in Section III, to present the main design ideas before turning to general designs. In Section IV-A, an adaptive control scheme is proposed to exponentially stabilize the parameter-varying system with a known control direction; in Section IV-B, this scheme is extended to parameter-varying systems with unknown control direction. In Section V, a comparative simulation is considered to illustrate the main results. Finally, conclusions are given in Section VI.

**Notations:** \( \mathbb{R} \) is the field of reals, \( \mathbb{R}^+ = \{ a \in \mathbb{R} : a > 0 \} \), \( \mathbb{R}^n \) denotes the \( n \)-dimensional Euclidean space, and \( \mathbb{R}^{n \times m} \) is the set of \( n \times m \) real matrices. \( \mathbb{x} = [x_1, \ldots, x_i] \in \mathbb{R}^d \) denotes a vector. \( \Gamma > 0 \) means that the symmetric matrix \( \Gamma \) with suitable dimensions is positive definite. \( |W_f| = \sqrt{\sum_{i=1}^m \sum_{j=1}^m |W_{ij}|^2} \) denotes the Frobenius norm of matrix \( W \). \( f \in \mathcal{C}^\infty \) denotes a function \( f \) has continuous derivatives of order \( \infty \), and \( f \in \mathcal{L}_\infty \) denotes a function \( f \) is bounded. In addition, \( \mathcal{L}_1 \) and \( \mathcal{L}_2 \) denote the classes of Lebesgue-integrable functions. \( \text{sgn}(\cdot) \) denotes the sign function.

**II. PRELIMINARIES**

**A. System Description and Assumptions**

Consider the following single-input single-output nonlinear systems with time-varying parameters [15]:

\[
\begin{align*}
\dot{x}_i &= \phi_{i1}(x_i)\theta(t) + x_{i+1}, \quad i = 1, \ldots, n - 1 \\
\dot{x}_n &= \phi_{n1}(x_n)\theta(t) + b(t)u
\end{align*}
\]

where \( x = [x_1, \ldots, x_n]^T \in \mathbb{R}^d \) and \( u \in \mathbb{R} \) are the state vector and the control, respectively. The regressors \( \phi_{i}(\cdot) : \mathbb{R}^d \rightarrow \mathbb{R}^q, \; i = 1, \ldots, n \), are smooth mappings and satisfy \( \phi_i(0) = 0 \).

By the mean value theorem, there exist continuous matrix-valued functions \( \Phi_i(\mathbf{x}) \in \mathbb{R}^{q \times i} \) such that \( \phi_i(x) = \Phi_i(x)\xi_i \).

The system parameters \( \theta(t) \in \mathbb{R}^q \) and \( b(t) \in \mathbb{R} \) are unknown and time varying and satisfy the following assumptions:

**Assumption 1 [15]:** The parameter \( \theta(t) \) is piecewise continuous and \( \theta(t) \in \Theta_0 \; \forall t \geq 0 \), where \( \Theta_0 \) is a completely unknown compact set. The “radius” of \( \Theta_0 \), denoted by \( \delta_{\Theta_0} \), is assumed to be known, while \( \Theta_0 \) can be unknown.

**Assumption 2 [15]:** The control direction is known and does not change. We assume that \( b(t) \) is unknown but bounded away from zero in the sense that there exists an unknown constant \( \ell_b \), such that \( 0 < |\ell_b| \leq |b(t)| \) for all \( t \geq 0 \).

**Assumption 3 [15, Sec. 6.3]:** The time-varying control coefficient \( b(t) \neq 0 \) takes values in an unknown compact set \( \Omega_b \). The control direction is unknown and does not change.

**Remark 1:** The conditions imposed on the parameters on the feedback path as in Assumption 1 and the parameter on the input path as in Assumption 2 make the considered model more general than the one considered in [4], [5], [6], [8], and [9] in terms of uncertain parameters as the latter require that \( \theta(t) \) and \( b(t) \) be time invariant. Assumption 3 is a relaxed version of Assumption 2 since it allows that the control direction is unknown.

In our later control development, we present two sets of control schemes: the first one (Theorem 1) is based on Assumption 2, in which the control gain magnitude is allowed to be unknown and time varying and the second one (Theorem 2) is built upon Assumption 3, in which both the control direction and the control gain magnitude are allowed to be unknown.

**Control Objective:** The control objective in this article is to design adaptive control schemes for system (1) to achieve globally exponential stabilization.

This problem has received a lot of attentions. But until recently, the contributions were only achieving that asymptotic stabilization and/or bounded stabilization as in [15] and [26]. The exponentially stable results can only be obtained under the persistence of excitation or under the assumption that \( \theta \) is time invariant and \( b(t) \) is 1 as in [10] and [30].
B. Enhanced Nussbaum Function and Corresponding Lemma

Definition 1 [29]: A $C^\infty$ function $N(\xi): [0, \infty) \mapsto (-\infty, \infty)$ is called an enhanced Nussbaum function if it satisfies

$$
\lim_{\xi \to \infty} \frac{1}{\xi} \int_0^\xi N^+(\tau) d\tau = \infty, \quad \lim_{\xi \to \infty} \sup_{\xi \geq 0} \frac{1}{\xi} N^-(\tau) d\tau = \infty
$$

where $N^+(\xi) = \max\{0, N(\xi)\}$ and $N^-(\xi) = \max\{0, -N(\xi)\}$. A legal fraction (with nonzero denominator) assumption is implicitly made in the above definition which excludes the trivial function $N(\xi) \equiv 0$.

Lemma 1 [29]: Consider two $C^\infty$ functions $V(t): [0, \infty) \mapsto \mathbb{R}^+, N(t): [0, \infty) \mapsto \mathbb{R}^+$. Let $b(t): [0, \infty) \mapsto [b, \bar{b}]$ for two constants $b$ and $\bar{b}$ satisfying $b > 0$. If

$$
\dot{V}(t) \leq b(t) N(\xi) + 1) \dot{\xi}(t)
$$

then $V(t)$ and $\dot{\xi}(t)$ are bounded over $[0, \infty)$.

III. BASIC DESIGN IDEAS

This section introduces the basic design ideas through three simple scalar systems.

A. Exponential Regulation for Systems With Time-Invariant Parameters

To obtain a better understanding of how time-varying scaling may be used to achieve exponential regulation, we consider

$$
\dot{x} = u + ax^2
$$

where $x \in \mathbb{R}$ and $u \in \mathbb{R}$ are the state and the control, respectively, and $a$ is an unknown constant. Let

$$
s = \phi^1 x \triangleq \mu(t)x
$$

with $\lambda > 0$ being the acceleration constant, then

$$
\dot{s} = \mu\left(\lambda x + u + \hat{a}x^2\right) + \mu(a - \hat{a})x^2.
$$

Choose the Lyapunov function as follows:

$$
V = \frac{1}{2} \dot{s}^2 + \frac{1}{2\gamma_a}\left(a - \hat{a}\right)^2
$$

where $\gamma_a > 0$ and $\hat{a}$ is the estimation of $a$, then

$$
\dot{V} = \mu s\left(\lambda x + u + \hat{a}x^2\right) + \frac{1}{\gamma_a}(a - \hat{a})\left(\gamma_a\mu sx^2 - \hat{a}\right).
$$

Denote $a(t) - \ell_a$ by $\Delta_a$. By choosing $u$ and $\dot{\hat{a}}$ as follows:

$$
\dot{\hat{a}} = \gamma_a\mu sx^2
$$

$$
u = -(k + \lambda)x - \hat{a}x^2 + v
$$

where $k > 0$, and $v$ is an auxiliary input, we obtain

$$
\dot{V} = -ks^2 + \mu sv + \mu\Delta_a sx^2.
$$

It follows from (7) that $V \in \mathcal{L}_\infty$ and therefore the boundedness of $s$ and $\hat{a}$ is guaranteed. In fact, the boundedness and the exponential convergence of $x$ are established simultaneously since $x = e^{-\lambda t}$, which further implies the boundedness of $u$. To show the asymptotic constancy of $\hat{a}(t)$, note that $\dot{\hat{a}} = \gamma_a\mu sx^2 = \gamma_a \hat{a} x^2$ and $x(t)$ is a bounded function, then there exists a number $L$ such that $|\bar{a}| \leq Ls^2$. It is seen from (7) that $s \in \mathcal{L}_2$ and hence $\dot{\hat{a}} \in \mathcal{L}_1$, then by using the argument of [37, Th. 3.1], it is concluded that $\hat{a}$ has a limit as $t \to \infty$.

B. Exponential Regulation for Systems With Time-Varying Parameters

Here, we show how to extend the aforementioned method to exponentially stabilize the systems with time-varying parameters. Consider

$$
\dot{x} = u + a(t)x^2
$$

where $x \in \mathbb{R}$ and $u \in \mathbb{R}$ are the state and the control, respectively, and $a(t)$ satisfies Assumption 1. Similar to the state scaling by a $t$-dependent function $\mu(t)$ as in Section III-A, we define $s = \mu(t)x$, then the dynamics of $s$ becomes

$$
\dot{s} = \mu\left(u + a(t)x^2\right) + \mu\lambda x
$$

$$
= \mu(\lambda x + \hat{a}x^2 + u) + \mu(a(t) - \ell_a)x^2 + \mu(\ell_a - \hat{a})x^2
$$

where $\ell_a$ is an unknown constant, $\gamma_a > 0$ and $\hat{a}$ is the estimation of $a$. Choose the Lyapunov function as follows:

$$
V = \frac{1}{2} \dot{s}^2 + \frac{1}{2\gamma_a}\left(\ell_a - \hat{a}\right)^2
$$

then

$$
\dot{V} = \mu s\left(\lambda x + \hat{a}x^2 + u\right) + \mu(a(t) - \ell_a)x^2
$$

$$
+ \frac{1}{\gamma_a}(\ell_a - \hat{a})\left(\gamma_a\mu sx^2 - \hat{a}\right).
$$

Denote $a(t) - \ell_a$ by $\Delta_a$. By choosing $u$ and $\dot{\hat{a}}$ as follows:

$$
\dot{\hat{a}} = \gamma_a\mu sx^2
$$

$$
u = -(k + \lambda)x - \hat{a}x^2 + v
$$

where $k > 0$, and $v$ is an auxiliary input, we obtain

$$
\dot{V} = -ks^2 + \mu sv + \mu\Delta_a sx^2.
$$

Note that the uncertain term $\mu\Delta_a sx^2 = \Delta_a x^2$, and satisfies the matching condition, therefore, one can design $v = -\delta_{\Delta_a} x^2/2 - \delta_{\Delta_a} x^2/2$, with $\delta_{\Delta_a} > |\Delta_a|$, to eliminate the effect of $\mu\Delta_a sx^2$. Here, $\delta_{\Delta_a}$ is assumed to be a known constant, if $\delta_{\Delta_a}$ is unknown, it is also easy to develop a classical adaptive law to build an “estimate” of $\delta_{\Delta_a}$. Therefore, the final control input is

$$
u = -(k + \lambda)x - \hat{a}x^2 - \frac{\delta_{\Delta_a}}{2}x^2 - \frac{\delta_{\Delta_a}}{2}x
$$

and the derivative of $V$ satisfies $\dot{V} \leq -ks^2$. Therefore, we can conclude boundedness of all trajectories of the closed-loop system as well as convergence of $x$ to zero using the same argument as the one used in Section III-A.
Remark 2: Note that the underlying issue becomes more difficult when the control coefficient $b(t)$ is unknown, even if the direction of control is known. To overcome this technical obstacle, we need to design an additional adaptive law to estimate $\ell_b$ (refer Assumption 2 for the meaning of $\ell_b$) while dealing with the time-varying part of $b(t) - \ell_b$ by deliberately designing a negative feedback gain. This will be shown in Section IV.

C. Exponential Regulation for Systems With Time-Varying Parameters and Unknown Control Coefficient

Now we further consider the scenario that both unknown fast time-varying parameters and unknown time-varying control coefficient are involved. We show how to integrate an enhanced Nussbaum function with the congelation of variables method as well as state scaling to design accelerated adaptive control to achieve exponential regulation without the need for PE condition. Consider

$$\dot{x} = b(t)u + a(t)x^2$$  \hspace{1cm} (15)

where $x \in \mathbb{R}$ and $u \in \mathbb{R}$ are the state and the control, respectively, $a(t)$ satisfies Assumption 1, and $b(t)$ satisfies Assumption 3. Let $s = \mu(t)x$ with $\mu(t) = e^{\xi t}$. Now choose

$$V = \frac{1}{2} s^2 + \frac{1}{2\gamma_a}(\ell_a - \hat{a})^2$$

then

$$\dot{V} = \mu s \left( \lambda x + \hat{a}x^2 + b(t)u \right) + \mu(a(t) - \ell_a)sx^2$$

$$+ \frac{1}{\gamma_a}(\ell_a - \hat{a}) \left( \gamma_a sx^2 - \hat{a} \right).$$ \hspace{1cm} (16)

Design

$$u = \mathcal{N}(\hat{\xi})\hat{u}$$

$$\hat{\xi} = \mu \hat{s} \hat{u}$$ \hspace{1cm} (17)

where $\mathcal{N}(\hat{\xi})$ is an enhanced Nussbaum function as described in Definition 1. Substituting (17) into (16), yields

$$\dot{V} = \mu s \left( \lambda x + \hat{a}x^2 + b(t)u \right) + \mu(a(t) - \ell_a)sx^2$$

$$+ \frac{1}{\gamma_a}(\ell_a - \hat{a}) \left( \gamma_a sx^2 - \hat{a} \right)$$

$$+ (b(t)\mathcal{N}(\hat{\xi}) + 1)\hat{\xi}.$$ \hspace{1cm} (18)

Note that although the first two lines of (11) and (18) have a similar structure, we cannot design $\hat{u}$ directly from formula (14), because there is a constraint in Lemma 1, namely, it requires that $\hat{\xi}(t) \geq 0$. To solve this technical obstacle, we design

$$\hat{a} = \gamma_a \mu sx^2$$

$$\hat{u} = (k + \lambda)x + \kappa(\hat{a}, x)x$$ \hspace{1cm} (19)

where $k > 0$ and $\kappa$ is a positive function, which will be designed below. As a result, (18) becomes

$$\dot{V} = -\kappa(\hat{a}, x)x^2 + \mu \hat{a}sx^2 + \mu \Delta_a sx^2$$

$$- ks^2 + (b(t)\mathcal{N}(\hat{\xi}) + 1)\hat{\xi}$$ \hspace{1cm} (20)

where $\Delta_a \triangleq a(t) - \ell_a$. Applying Young’s inequality, we have

$$\mu \hat{a}sx^2 + \mu \Delta_a sx^2 = \hat{a}s^2x + \Delta_a s^2x$$

$$\leq \frac{1}{2} ((\hat{a})^2 + 1)s^2 + \frac{\delta_{\Delta_a}}{2} (x^2 + 1)^2.$$  \hspace{1cm}  

Therefore, we select $\kappa(\hat{a}, x)$ as

$$\kappa(\hat{a}, x) = \frac{1}{2} ((\hat{a})^2 + 1) + \frac{\delta_{\Delta_a}}{2} (x^2 + 1)$$ \hspace{1cm} (21)

with $\delta_{\Delta_a} \geq |\Delta_a|$ being a known constant. Now the derivative of $V$ satisfies

$$\dot{V} \leq -ks^2 + (b(t)\mathcal{N}(\hat{\xi}) + 1)\hat{\xi} \leq (b(t)\mathcal{N}(\hat{\xi}) + 1)\hat{\xi}. \hspace{1cm} (22)$$

Recalling that $\xi = \mu \hat{s} \hat{u} = (k + \lambda + \kappa(\hat{a}, x))s^2 \geq 0$, therefore we can conclude that $V(t)$ and $\xi(t)$ are bounded by virtue of Lemma 1. Then, the boundedness of all trajectories of the closed-loop system as well as convergence of $x$ to zero can be guaranteed by using the same argument as the one used in Section III-A.

Remark 3: It is worth noting that the classical adaptive parameter update law and adaptive control law are equivalent to that designed in (6) if the acceleration constant $\lambda = 0$. This phenomenon is easy to understand since $\lambda = 0$ can be regarded as the dynamics of the original system has not been changed. In addition, the classical adaptive control law for a time-invariant parameter is equivalent to the one design in (12) for time-varying parameters if $\delta_{\Delta_a} = 0$, which is also easy to understand because $\delta_{\Delta_a} = 0$ means that $\ell_a = a(t)$, namely, the parameter will not change with time. In terms of technical realization, these findings are very useful for the controller design of higher-order systems, playing an inspiring role in the design of parameter update laws and control laws.

IV. MAIN RESULTS

Motivated by the appealing features about the time-varying scaling design method, we further explore its applicability to parametric strict-feedback systems with unknown time-varying parameters and unknown control coefficient. The high-order controller design is based upon the Backstepping technology [5]. For each step $i, (i = 1, \ldots, n)$, define the coordinate transformation as follows:

$$z_i = x_i$$

$$z_i = x_i - a_{i-1}, \quad i = 2, \ldots, n$$ \hspace{1cm} (23)

the exponential scaling

$$s_i = e^{\lambda t}z_i = \mu(t)z_i, \quad \lambda > 0$$ \hspace{1cm} (24)

the new regressor vectors

$$w_i(\xi, \theta) = W_i^T z_i = \phi_i - \sum_{j=1}^{i-1} \frac{\partial a_{i-1}}{\partial \xi_j} \phi_j, \quad a_0 = 0.$$ \hspace{1cm} (25)

Remark 4: Once we have $a_i$ is a smooth and bounded function and $a_i(0, \theta) = 0$, then from the coordinate transformation between $\bar{z}_i$ and $\xi$, we know that $\bar{z}_i = 0 \Leftrightarrow z_i = 0$. To design a low-conservative control algorithm while guaranteeing zero steady-state error, we adopt regression matrices design.
approach. Specifically, by using the mean value theorem, we can express $w_i = \phi_i - \sum_{j=1}^{i-1} [(\partial \alpha_{i-1})/\partial x_j] \phi_j$ as follows:

$$w_i = \Phi_i x_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \Phi_j x_j$$

$$= W_i x_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} W_j x_j = W_i x_i - W_j x_j = W_i x_i - W_j x_j$$

where $W_i \in \mathbb{R}^{q \times q}$, $W_j \in \mathbb{R}^{q \times q}$, and $W_i \in \mathbb{R}^{q \times q}$ are known smooth and bounded mappings, whose analytic expressions can be derived from the corresponding exact $\Phi_i$ and $\alpha_i$.

Define the tuning functions as follows:

$$\tau_i(x_i, \hat{\theta}, \mu) = \tau_{i-1} + e^{\kappa} w_i x_i = \sum_{j=1}^{i} \mu x_i x_i.$$

To clearly describe the section’s organization, we introduce some auxiliary functions before giving the main theorems

$$\xi_i = \lambda + \frac{1}{2} \left[ (\alpha_i + \gamma_i) \frac{\partial \alpha_i}{\partial \theta} \right]$$

$$\kappa(\varsigma_i, \hat{\theta}, \mu) = \kappa_n + \lambda + \frac{1}{2} \left[ \frac{\partial \varsigma_i}{\partial \theta} + \frac{\partial \varsigma_i}{\partial \mu} \right]$$

where $\delta_{\Delta_0} > 0$, $\varphi > 0$, and $\tau \in \mathbb{R}^n$ is a smooth mapping and satisfying $\psi = \psi^\top z_n$, and

$$\psi = z_n - w_i^\top x_i - \sum_{i=1}^{n} \frac{\partial \alpha_{i-1}}{\partial x_i} x_i - \frac{\partial \alpha_{i-1}}{\partial \theta} r_n$$

where $\Gamma = \Gamma^\top > 0$ is a matrix.

A. Adaptive Control for Parameter-Varying Nonlinear Systems With Known Control Direction

When the control direction is known, we add an additional adaptive law to estimate the time-varying control gain. By the conglomeration of variables method and let $u = \hat{\rho} u$, we can rewrite $b(t)u$ as follows:

$$b(t)u = \hat{\rho} u + \Delta_b \hat{\rho} u - \frac{1}{\kappa} u \left( \frac{1}{\kappa} - \hat{\rho} \right)$$

where $\Delta_b = b(t) - \kappa$, $\kappa$ is an unknown constant, and $\hat{\rho}$ is the estimate of $1/\kappa$. Based on this treatment, we state the following theorem.

Theorem 1: Consider the parameter-varying strict-feedback system (1) with unknown control gain magnitude yet known control direction. If Assumptions 1 and 2 are satisfied, then by using the control law

$$\begin{align*}
\alpha_i(x_i, \hat{\theta}, \mu) &= -(k_i + \xi_i) z_{i-1} - w_i^\top \hat{\theta} \\
\alpha_i(x_i, \hat{\theta}, \mu) &= -(k_i + \xi_i) z_{i-1} - w_i^\top \hat{\theta} + \frac{\partial \alpha_{i-1}}{\partial \theta} \tau_i + \sum_{j=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \Gamma s_i w_i \\
\xi_i &= \kappa(x_i, \hat{\theta}, \mu) z_n \\
u &= \hat{\rho} u
\end{align*}$$

where, for $i = 1, \ldots, n - 1$, the auxiliary variables $\xi_i$, $w_i$, $\xi_i$, and $\kappa$ are defined in (23)–(30), and the adaptive laws

$$\dot{\hat{\rho}} = -\gamma \frac{\text{sgn}(\ell_b)}{\mu} s_n \hat{u}$$

where the initial condition is chosen as $\hat{\rho}(0) > 0$ for $b(t) > 0$ and $\hat{\rho}(0) < 0$ for $b(t) < 0$, and

$$\dot{\hat{\theta}} = \Gamma_n \left( \frac{\tau_n}{\Delta_b} + \hat{\theta} \right), \quad \hat{\theta}(0) \geq 0$$

then the equilibrium is globally exponentially stable, that is, there exist two positive numbers $N$ and $\lambda$, such that $\|x_n(t)\| \leq Ne^{-\lambda t}$, and all signals in the system are ensured be bounded. Furthermore, $\lim_{t \to \infty} \hat{\theta}$ and $\lim_{t \to \infty} \hat{\rho}$ exist.

Proof: The closed-loop dynamics of $s_i$, $i = 1, \ldots, n$ with the control law (32) are given by

$$\dot{s}_i = -\left( k_i + n + i - 1 \right) s_i + \left( k_i + n + i - 1 \right) \frac{\delta_{\Delta_0}}{2} \left( |W_i^2| + 1 \right) s_i$$

Choosing a positive definite, radially unbounded function as a Lyapunov function candidate

$$V = \frac{1}{2} s_n + \frac{1}{2} \left( \ell_b - \hat{\theta} \right) \Gamma_{\theta} - \left( \ell_b - \hat{\theta} \right) - \frac{1}{2} \gamma \frac{\ell_b}{\ell_b - \hat{\theta}}.$$  

The derivative of $V$ is given by

$$V = -\sum_{i=1}^{n} \left( k_i + \frac{1}{2} \right) \frac{\delta_{\Delta_0}}{2} \left( |W_i^2| + n + i - 1 \right) s_i^2$$

$$+ s_n - s_n + \sum_{i=1}^{n} \frac{\partial \alpha_{i-1}}{\partial \theta} \mu s_i w_i$$

$$- \frac{1}{\gamma} \left( \ell_b - \hat{\theta} \right) \frac{\text{sgn}(\ell_b)}{\mu} s_n \hat{u} + \hat{\rho}$$

$$+ \left( \ell_b - \hat{\theta} \right) \Gamma_{\theta} - \left( \ell_b - \hat{\theta} \right) - \frac{1}{\gamma} \frac{\ell_b}{\ell_b - \hat{\theta}}.$$ 

$$- \frac{1}{\gamma} \left( \ell_b - \hat{\theta} \right) \Gamma_{\theta} - \left( \ell_b - \hat{\theta} \right) - \frac{1}{\gamma} \frac{\ell_b}{\ell_b - \hat{\theta}}.$$ 

$$- \frac{1}{\gamma} \left( \ell_b - \hat{\theta} \right) \Gamma_{\theta} - \left( \ell_b - \hat{\theta} \right) - \frac{1}{\gamma} \frac{\ell_b}{\ell_b - \hat{\theta}}.$$
The last term on the right-hand side of (41) is always negative, because \( \hat{\rho} = \text{sgn}(\ell_b)\mu_s\hat{\alpha} \), and:

1. Case I (\( \hat{b}(t) > 0 \)): \( \hat{\rho} \geq 0 \) and \( \Delta_b > 0 \), then \( \Delta_b\hat{\rho}(t) \geq 0 \) for all \( \hat{\rho}(0) > 0 \);
2. Case 2 (\( \hat{b}(t) < 0 \)): \( \hat{\rho} \leq 0 \) and \( \Delta_b < 0 \), then \( \Delta_b\hat{\rho}(t) \geq 0 \) for all \( \hat{\rho}(0) < 0 \);

where \( \hat{\rho}(0) \) is a design parameter, which can be selected according to \( \text{sgn}(\hat{b}(t)) \). According to the above analysis, it follows that \( -\kappa\Delta_b\hat{\rho}\hat{\alpha}^2 \leq 0 \), implying that \( \hat{V} \leq 0 \). Now we have

\[
\Vert \xi_r \Vert \leq \sqrt{\Vert \xi_r(0) \Vert^2 + \frac{\Vert \ell_b - \hat{\theta}(0) \Vert^2}{2\gamma} + \frac{\Vert \ell_b \Vert}{2\gamma \rho} \left( \frac{1}{\ell_b} - \hat{\rho}(0) \right)^2}.
\]  

(42)

In addition, it follows from (23) and (24) that there exists a smooth and bounded mapping \( M_{i-1} \), with \( M_{i-1}(0, \hat{\theta}, \hat{\xi}_{i-1}) = 0 \), such that:

\[
x_i = \mu^{-1} s_i + M_{i-1}(\hat{\xi}_{i-1}, \hat{\theta}, \hat{\xi}_{i-1}) \hat{\xi}_{i-1}
\]  

(43)

and from (43) that the initial condition satisfies

\[
s_i(0) = x_i(0) - M_{i-1}(\hat{\xi}_{i-1}(0), \hat{\theta}(0), \mu(0)) \hat{\xi}_{i-1}(0).
\]  

(44)

Recursively applying (44) results in that there exists a non-negative valued continuous function \( N(\chi_{ni}(0), \hat{\theta}(0), \lambda) \) such that

\[
\Vert \xi_r \Vert \leq N(\chi_{ni}(0), \hat{\theta}(0), \lambda) e^{-\lambda t}.
\]  

(45)

Therefore, the closed-loop system is globally exponentially stable.

Furthermore, it also follows from (41) that \( \hat{\theta} \in \mathcal{L}_\infty \) and \( \hat{\rho} \in \mathcal{L}_\infty \). Recall from (32) that \( \alpha_1 = -(k_1 + \zeta_1)z_1 - w_{i_1} \hat{\theta} \). Since \( z_1 = x_1 \) and \( w_1 = \phi_1 \), we see that \( \alpha_1 \) is bounded and therefore \( z_2 = x_2 - \alpha_1 \) is also bounded. The boundedness of \( \tau_1 = \mu w_1 s_1 = \Phi_1 s_1 \) is then established via (42). In addition, careful examination of (27) and (34) reveals that the quantity \( \mu \) always appears multiplied by \( z_i \), so in any instance where the \( \mu\zeta_i \) appears in the tuning functions or virtual control functions, it also can be guaranteed such functions are bounded. For example, from (32) we have \( \alpha_2 = -\zeta_2 z_2 - z_1 - w_2 \hat{\theta} - w_1 \hat{\theta} \). Continuing in the same fashion, we prove that \( \alpha_i(\ell, \ldots, n - 1) \) and \( u(t) \) are bounded. Rewrite (33) we have \( \hat{\rho} = \gamma_\rho \kappa s_1 \). Since \( s_1 \in \mathcal{L}_2 \), then \( \hat{\rho} \in \mathcal{L}_1 \), then by using the argument similar to [37, Th. 3.1], it is concluded that \( \hat{\rho} \) has a limit as \( t \rightarrow \infty \), establishing the same for \( \hat{\theta} \). This completes the proof.

Remark 5: It is crucial to ascertain that the benefits of the proposed adaptive exponential control as stated in Theorem 1 are not achieved at the price of unbounded input and/or unbounded updating rate. In fact, the design is based upon the exponential scaling (24), followed by a stabilizing adaptive control design for the scaled system. As shown in (6), the product \( e^{\lambda t}x \) in the update law is the scaled state \( \xi \), which is kept bounded by (7). The controller and adaptive laws for high-order systems inherit this feature [see (32) and (34)] and hence the boundedness of which is naturally guaranteed. In addition, after careful examination of (23)–(34), one can find that the control input, estimated parameter, and parameter updating rate are all bounded.

Remark 6: In the absence of nonvanishing uncertainties, the closed-loop signals \( \{s_i\}_{i=1}^n \) converge to zero asymptotically, while causing system states \( \{x_i\}_{i=1}^n \) converge at least \( e^{-\lambda t} \) exponentially fast to zero. Clearly, if we let \( \lambda = 0 \), then the system (1) is asymptotically stable. In this case, Theorem 1 is equivalent to [15, Proposition 1]. More importantly, since the virtual errors decrease gradually with time as the time-varying gains increase, the “peaking” phenomenon occurring in traditional high-gain feedback does not obviously exist here.

B. Adaptive Control for Parameter-Varying Nonlinear Systems With Unknown Control Direction

Significant challenge occurs in adaptive control design when the control gain is unknown and time varying. This is particularly true in the context of time-varying control design for the time-varying control coefficient in this section and the one for the time-invariant control coefficient in [22], [23], [24], and [26] is that the former requires that the independent variable of the Nussbaum function must always be non-negative, thereby
Theorem 2: Consider the parameter-varying strict-feedback system (1) with unknown control gain magnitude and unknown control direction. If Assumptions 1 and 3 are satisfied, then by using the control law

\begin{equation}
\begin{cases}
\dot{u} = \kappa (\dot{s}_n, \dot{\theta}, \mu) z_n \\
\dot{\xi} = \mu s_n \dot{u} = \kappa s_n^2 \\
u = N(\xi) \tilde{u}
\end{cases}
\end{equation}

(46)

and the virtual control laws as given in (32) and the adaptive law as given in (34), then the equilibrium is globally exponentially stable, that is there exist two positive numbers \(N\) and \(\lambda\), such that \(\|x_n(t)\| \leq Ne^{-\lambda t}\); and all signals in the system are ensured to be bounded. Furthermore, \(\lim_{t \to \infty} \dot{\theta}\) and \(\lim_{t \to \infty} \hat{\theta}\) exist.

Proof: Recalling that

\[ u = N(\xi) \tilde{u} \]

\[ \dot{\xi} = \mu s_n \dot{u} \]

(47)

The dynamic of \(s_n\) under (47) is given by

\[ \dot{s}_n = \mu \dot{z}_n + \dot{\mu} z_n \]

\[ = \mu \left( (b(t)u + \phi^\top \theta(t)) - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} (x_{j+1} + \phi^\top \theta(t)) \right) \]

\[ - \frac{\partial \alpha_{n-1}}{\partial \theta} \dot{\theta} - \frac{\partial \alpha_{n-1}}{\partial \mu} \lambda \mu \]

+ \lambda s_n. \quad (48)

\[
\text{Since} \\
\mu s_n b(t)u = \mu s_n b(t) \dot{N}(\xi) \tilde{u} = (b(t) \dot{N}(\xi) + 1) \dot{\xi} - \mu s_n \dot{u} \quad (49)
\]

then the following equation holds:

\[
s_n \dot{\xi} = (b(t) \dot{N}(\xi) + 1) \dot{\xi} + \mu s_n \left( -\dot{u} + \lambda z_n + w_n^\top \dot{\theta} \right) \\
- \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} x_{j+1} - \frac{\partial \alpha_{n-1}}{\partial \theta} \dot{\theta} - \frac{\partial \alpha_{n-1}}{\partial \mu} \lambda \mu \]

\[ + \mu s_n w_n^\top (\ell_0 - \dot{\theta}) + \mu s_n w_n^\top \Delta \theta. \quad (50)
\]

Note that the dynamics of \(s_i, i = 1, \ldots, n-1\) remain the same as in (35). Choosing the Lyapunov function candidate as follows:

\[ V = \frac{1}{2} s_n^\top s_n + \frac{1}{2} (\ell_0 - \dot{\theta})^\top \Gamma^{-1} (\ell_0 - \dot{\theta}). \quad (51)\]

According to the proof of Theorem 1 and by virtue of the control law and adaptive law as shown in Theorem 2, we obtain

\[
\dot{V} \leq -\sum_{i=1}^{n-1} k_i s_i^2 - \frac{\delta \Delta \theta}{2} s_{n-1}^\top s_{n-1} - \frac{1}{2\epsilon_{\psi}} s_{n-1}^\top s_{n-1} \\
+ \mu s_n (\lambda z_n - \tilde{u}) + \mu s_n \psi + \mu s_n w_n^\top \Delta \theta \\
+ (b(t) \dot{N}(\xi) + 1) \dot{\xi}. \quad (52)
\]

By recalling (24), (25), (29), (30), and (46), we have

\[
-\mu s_n \tilde{u} + \mu s_n \psi + \mu s_n w_n^\top \Delta \theta \\
\leq -k_n s_n^2 + \frac{\delta \Delta \theta}{2} s_{n-1}^\top s_{n-1} - \frac{1}{2\epsilon_{\psi}} s_{n-1}^\top s_{n-1} \quad (53)
\]

then (52) becomes

\[ \dot{V} \leq (b(t) \dot{N}(\xi) + 1) \dot{\xi}. \quad (54)\]

Note that \(\dot{\xi} = \kappa s_n^2\), and we deliberately design \(\kappa = k_n + \lambda + (1/2)(\delta \Delta \theta (|W_n^2| + 1) + (1/\epsilon_{\psi}) + \epsilon_{\psi} |\dot{\psi}|^2)\) as shown in (29), therefore one can conclude that \(\dot{\xi} = \kappa s_n^2 \geq 0\). By Lemma 1, it follows that \(V(t)\) and \(\xi(t)\) are bounded over \([0, \infty)\). The boundedness of \(V(t)\) leads to the boundedness of \(s_n\) and \(\hat{\theta}\). Note that the virtual control laws \(\alpha_i\) and \(\tilde{u}\) and adaptive law \(\dot{\theta}\) in Theorem 2 have exactly the same form as that of Theorem 1, hence the boundedness of these signals can be can be directly derived from the proof of Theorem 1. It is also follows that \(\dot{\rho}\) has a limit as \(t \to \infty\). The boundedness of \(\xi(t)\) yields the boundedness of \(\dot{N}(\xi)\), which further proves the boundedness of \(u(t)\). In addition, from the proofs of Theorem 1, we know that there exists a non-negative-valued continuous function \(N(x_n(0), \hat{\theta}(0), \lambda)\) such that

\[ \|x_n\| \leq N(x_n(0), \hat{\theta}(0), \lambda) e^{-\lambda t}. \quad (55)\]

Therefore, the closed-loop system is globally exponentially stable. This completes the proof.

Remark 7: In [15], the original time-invariant parameter estimation via a certainty equivalence controller was extended to a time-varying case, which shows that the system satisfying Assumptions 1 and 2 is asymptotically stable. However, it should be noted that the asymptotic results in [15] are not attractive enough for some practical applications since some applications require the system to have a rapid transient response. To study the stabilizing control of systems to achieving rapid transient response, Theorem 1 proposes an exponentially stable controller for parameter-varying nonlinear systems, and Theorem 2 extends Theorem 1 to parameter-varying nonlinear systems with unknown control directions.

Remark 8: Unlike prescribed performance control methods (see [19]), in which, the system output is guaranteed to evolve within a performance function and ultimately decay to a residual set, where the control parameters are determined according to system initial condition, our method ensures that all system states converge to zero within an exponential decay rate. On the other hand, a common technology adopted in prescribed performance control is to transform the “constrained” system into an equivalent “unconstrained” one via a coordinate transformation; however, the key idea in this article is scaling the virtual control errors by a time-varying function and the stability analysis is based on a time-varying Lyapunov function. Furthermore, we establish the global stability of the closed-loop system, without requiring an a priori knowledge of the initial condition.

V. Simulations

Consider the scenario in which a high-performance airplane flying at a high angle of attack aims at stabilizing its wing
Specifically, we set $\delta/D_1\theta$ extracted from [38], as follows:

$$
\dot{\phi} = p \\
\dot{p} = \frac{q}{I_x} S b \left(0.5 C_i \phi \sin(\alpha) + \frac{C_i p b}{2V} + C_{d_k}\right)
$$

(56)

where $\alpha$ is angle of attack in degrees, $\phi$ is the roll angle in radians, and $p$ is the roll rate in radians per second. The constants $q, S, b, I_x,$ and $V$ are the dynamic pressure, wing reference area, wing span, roll moment of inertia, and freestream air speed, respectively. The coefficients $C_i$ and $C_{d_k}$ are the rolling moment derivatives, $C_{d_k}$ is the control surface.

The parametric strict-feedback form of the wing rock model (56) by letting $\phi = x_1, x_2 = p,$ and $C_{d_k} = u$ is

$$
\dot{x}_1 = x_2 + \phi_1^T \theta(t) \\
\dot{x}_2 = b(t)u + \phi_2^T \theta(t)
$$

(57)

where $\phi_1 = 0, \phi_2 = [x_1, x_2]^T, b(t) = q S b / I_x,$ and $\theta(t) = [\theta_1(t), \theta_2(t)]^T = \left[0.5 C_i \sin(\alpha) q S b / I_x, C_i q S b^2 / (2 I_x V)\right]^T.$

Note that [38] provides the following wind-tunnel data at angle of attack $\alpha = 30^\circ$: $\theta_1 = -26.6667$ and $\theta_2 = 0.67485.$ Taking into account that the change of the attack angle will cause $\theta$ to change, therefore we assume in the simulation that $\theta_1$ and $\theta_2$ will periodically change by $\pm 2\%$ on the basis of nominal values, that is $\theta_i(t) = \theta_i + 2\% \theta_i \operatorname{sgn}(\sin(3t))$ for $i = 1, 2.$ In addition, except only knowing that $b(t) \neq 0,$ we have not obtained other information about $b(t)$ from the experimental data. In other words, the control direction and the control magnitude are unknown. Here, we set the control coefficient $b(t)$ as $b(t) = -2 + 0.2 \sin(\sin(3t)) \cos(t)$ to simulate the parameter changes at different angles of attack. Note that the parameters $\theta(t)$ and $b(t)$ comprise of a constant nominal part and a time-varying part designed to destabilize the system. For the system under consideration, it is readily verified that Assumptions 1–3 are satisfied, thus the control scheme proposed in Theorems 1 and 2 can be directly applied to stabilize (56) exponentially. Here, we consider three different controllers.

1) **Controller 1:** The adaptive asymptotic controller (AC) proposed in [15].

2) **Controller 2:** The adaptive controller with exponential convergence rate proposed in Theorem 1.

3) **Controller 3:** The adaptive Nussbaum controller with exponential convergence rate proposed in Theorem 2.

For fair comparison, we choose the same initial conditions and the same common design parameters for all controllers. Specifically, we set $\delta_{d_k} = 0.6, k_1 = k_2 = 1, [\theta_1(0), \theta_2(0)] = [0; 0], \Gamma = 0.001 I, \dot{\rho}(0) = -0.3,$ and $[x_1(0); x_2(0)] = [-1; 2.5]$ for Controllers 1–3. Also, we set an additional parameter $\lambda = 0.6$ for Controllers 2 and 3 and choose an enhanced Nussbaum function $N(\xi) = \sin(\xi) \exp(\xi^2)$ with $\xi(0) = 0$ for Controller 3.

The evolutions of the system states and control input are illustrated in Figs. 1 and 2, respectively. The evolution of adaptive parameters $\dot{\theta}_1(t)$ and $\dot{\theta}_2(t)$ are illustrated in Fig. 3; and the evolutions of $\dot{b}(t), \xi(t),$ and $N(\xi)$ are illustrated in Fig. 4. In addition, the time-varying parameters $\dot{\theta}_1(t), \dot{\theta}_2(t),$ and $b(t)$ are illustrated in Fig. 5. From simulation results, one can find that: 1) all signals are bounded and the independent variable of the Nussbaum function is always non-negative; 2) under Theorems 1 and 2, the system state converges to zero at an
have been verified with the help of suitable time-varying Lyapunov functions.

Note that if the sensor output is subject to external disturbance, that is we cannot design the controller using accurate feedback signals, then the adaptive parameters will drift because once the state converges close to the set point, the adaptive update law will be driven primarily by measurement noise and adaptive scaling gain that grows toward time. Therefore, one interesting research topic is the study of time-varying parametric adaptive algorithms that can reject parameter drift. Other further research topics include applying the method to various systems (see [39], [40]) and seeking some suitable ways to guarantee the parameters converge to their desired values (see [41], [42]).

VI. CONCLUSION

The notion of accelerating the convergence process by making use of rate function transformation and the technique of handling time-varying parameters via congelation of variables method are quite appealing in developing accelerated control for parameter-varying strict-feedback systems, which, together with the integration of the enhanced Nussbaum function, could allow new adaptive control (as simple as the traditional adaptive control) to be developed for a class of nonlinear systems with unknown time-varying parameters in the feedback path and input path, yet involving time-varying control gain that is unknown in sign and in magnitude. The stability conditions

exponential speed, while under [15], the state converges to zero at a relatively slow speed, and the overshoot is larger than the former; and 3) the “peaking” phenomenon does not appear in Controller 1, but it appears in Controller 3. As a matter of fact, how to weaken the “peaking” phenomenon produced by the Nussbaum-gain technology (Controller 3) is a challenging yet meaningful problem. In short, the above results illustrate the superiority and effectiveness of our approaches.

REFERENCES

[1] S. Sastry and A. Isidori, “Adaptive control of linearizable systems,” IEEE Trans. Autom. Control, vol. 34, no. 11, pp. 1123–1131, Nov. 1989.
[2] D. Taylor, P. V. Kokotovic, R. Marino, and I. Kanellopoulos, “Adaptive regulation of nonlinear systems with unmodeled dynamics,” IEEE Trans. Autom. Control, vol. 34, no. 4, pp. 405–412, Apr. 1989.
[3] J. Pomet and L. Praly, “Adaptive nonlinear regulation: Estimation from the Lyapunov equation,” IEEE Trans. Autom. Control, vol. 37, no. 6, pp. 729–740, Jun. 1992.
[4] I. Kanellopoulos, P. Kokotovic, and A. Morse, “Systematic design of adaptive controllers for feedback linearizable systems,” in Proc. Amer. Control Conf., 1991, pp. 649–654.
[5] M. Kristic, I. Kanellopoulos, and P. Kokotovic, Nonlinear and Adaptive Control Design. New York, NY, USA: Wiley, 1995.
[6] H. Zhang, R. Xi, Y. Wang, S. Sun, and J. Sun, “Event-triggered adaptive tracking control for random systems with coexisting parametric uncertainties and severe nonlinearities,” IEEE Trans. Autom. Control, vol. 67, no. 4, pp. 2011–2018, Apr. 2022.
[7] K. Esfandiari, F. Abdollahi, and H. Talebi, Neural Network-Based Adaptive Control of Uncertain Nonlinear Systems. Cham, Switzerland: Springer, 2022.
[8] P. Ioannou and J. Sun, Robust Adaptive Control. Englewood Cliffs, NJ, USA: Prentice-Hall, 1996.
[9] J. Zhou and C. Wen, Adaptive Backstepping Control of Uncertain Systems: Nonsmooth Nonlinearities, Interactions or Time-Variations. Berlin, Germany: Springer, 2008.
[10] D. Goodwin and E. Tseh, “Adaptive control of a class of linear time-varying systems,” IFAC Proc. Vol., vol. 16, no. 9, pp. 1–6, 1983.
[11] R. Middleton and G. Goodwin, “Adaptive control of time-varying linear systems,” IEEE Trans. Autom. Control, vol. 33, no. 2, pp. 150–155, Feb. 1988.
[12] Y. Song and R. Middleton, “Dealing with the time-varying parameter problem of robot manipulators performing path tracking tasks,” IEEE Trans. Autom. Control, vol. 37, no. 10, pp. 1597–1601, Oct. 1992.
[13] R. Marino and P. Tomei, “An adaptive output feedback control for a class of nonlinear systems with time-varying parameters,” IEEE Trans. Autom. Control, vol. 44, no. 11, pp. 2190–2194, Nov. 1999.
[14] K. Chen and A. Astolfi, “Adaptive control for nonlinear systems with time-varying parameters and control coefficient,” in Proc. IFAC World Congr., 2020, pp. 3895–3900.
[15] K. Chen and A. Astolfi, “Adaptive control for systems with time-varying parameters,” IEEE Trans. Autom. Control, vol. 66, no. 5, pp. 1986–2001, May 2021.
[16] Y. Chen, K. Chen, and A. Astolfi, “Adaptive formation tracking control of directed networked vehicles in a time-varying flowfield,” J. Guid. Control Dyn., vol. 44, no. 10, pp. 1883–1891, 2021.
[17] Y. Chen, K. Chen, and A. Astolfi, “Adaptive formation tracking control for first-order agents with a time-varying flow parameter,” IEEE Trans. Autom. Control, vol. 67, no. 3, pp. 2481–2488, May 2022.
[18] L. Wang, W. Sun, S. Su, and Y. Wu, “Adaptive prescribed performance asymptotic tracking control for nonlinear systems with time-varying parameters,” Int. J. Robust Nonlinear Control, vol. 32, no. 7, pp. 4535–4552, 2022.
M. Monahemi and M. Krstic, “Control of wing rock motion using H. Li, L. Bai, L. Wang, Q. Zhou, and H. Wang, “Adaptive neural control Q. Wang, X. Dong, G. Wen, J. Lv, and Z. Ren, “Practical out- J. Zhang and G. Yang, “Fuzzy adaptive output feedback control of non- Y. Song, K. Zhao, and M. Krstic, “Adaptive control with exponential Stabilization and Regulation of Nonlinear Z. Chen and J. Huang, “Stabilization and Regulation of Nonlinear Systems. S. Ge, F. Hong, and T. Lee, “Adaptive neural control of nonlinear time-delay systems with unknown virtual control coefficients.” IEEE Trans. Syst., Man, Cybern., B, Cybern., vol. 34, no. 1, pp. 499–516, Feb. 2004. J. Huang, W. Wang, C. Wen, and J. Zhou, “Adaptive control of a class of strict-feedback time-varying nonlinear systems with unknown control coefficients,” Automatica, vol. 93, pp. 98–105, Jul. 2018. S. Ge and J. Wang, “Robust adaptive tracking for time-varying uncertain nonlinear systems with unknown control coefficients,” IEEE Trans. Autom. Control, vol. 48, no. 8, pp. 1463–1469, Aug. 2003. Z. Chen and J. Huang, “Stabilization and Regulation of Nonlinear Systems.” Cham, Switzerland: Springer, 2015. Z. Chen, “Nussbaurn functions in adaptive control with time-varying unknown control coefficients,” Automatica, vol. 102, pp. 72–79, Apr. 2019. Y. Song, K. Zhao, and M. Krstic, “Adaptive control with exponential regulation in the absence of persistent excitation,” IEEE Trans. Autom. Control, vol. 39, no. 9, pp. 2589–2596, May 2017. C. Benichouilis and G. Rovithakis, “Robust adaptive control of feedback linearizable MIMO nonlinear systems with prescribed performance,” IEEE Trans. Autom. Control, vol. 53, no. 9, pp. 2090–2099, Oct. 2008. J. Zhang and G. Yang, “Fuzzy adaptive output feedback control of uncertain nonlinear systems with prescribed performance,” IEEE Trans. Cybern., vol. 48, no. 5, pp. 1342–1354, May 2018. Q. Wang, X. Dong, G. Wen, J. Lv, and Z. Ren, “Practical output containment of heterogeneous nonlinear multi-agent systems under external disturbances,” IEEE Trans. Cybern., early access, Jun. 21, 2022, doi: 10.1109/TCYB.2022.3175769. J. Yu, X. Dong, Q. Li, J. Lv, and Z. Ren, “Fully adaptive practical time-varying output formation tracking for high-order nonlinear stochastic multi-agent system with multiple leaders,” IEEE Trans. Cybern., vol. 51, no. 4, pp. 2265–2277, Apr. 2021. H. Li, J. Bai, L. Wang, Q. Zhou, and H. Wang, “Adaptive neural control of uncertain non-strict-feedback stochastic nonlinear systems with output constraint and unknown dead zone,” IEEE Trans. Syst., Man, Cybern., Syst., vol. 47, no. 8, pp. 2048–2059, Aug. 2017. W. Wang, J. Huang, and C. Wen, “Prescribed performance bound-based adaptive path-following control of uncertain holonomic mobile robots,” Int. J. Adapt. Control Signal Process, vol. 31, no. 5, pp. 805–822, 2017. M. Krstic, “Invariant manifolds and asymptotic properties of adaptive nonlinear stabilizers,” IEEE Trans. Autom. Control, vol. 41, no. 6, pp. 817–829, Jun. 1996. M. Monahemi and M. Krstic, “Control of wing rock motion using adaptive feedback linearization,” J. Guid. Control Dyn., vol. 19, no. 4, pp. 905–912, 1996. X. Li, C. Wen, and J. Wang, “Lyapunov-based fixed-time stabilization control of quantum systems,” J. Automat. Intell., vol. 1, no. 1, Dec. 2022, Art. no. 100005. L. Hu, H. Hu, W. Naeem, and Z. Wang, “A review on COLREGs-compliant autonomous navigation of surface vehicles: From traditional to learning-based approaches,” J. Automat. Intell., vol. 1, no. 1, Dec. 2022, Art. no. 100003. A. Glushchenko and K. Lastochkin, “Exponentially convergent direct adaptive pole placement control of plants with unmatched uncertainty under FE condition,” IEEE Control Syst. Lett., vol. 6, pp. 2527–2532, Dec. 2022. P. Tomei and R. Marino, “An enhanced feedback adaptive observer for nonlinear systems with lack of persistency of excitation,” IEEE Trans. Autom. Control, early access, Oct. 14, 2022, doi: 10.1109/TAC.2022.3214798.

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