QUALITATIVE ASPECTS OF QUASAR MICROLENSING WITH TWO MASS COMPONENTS: MAGNIFICATION PATTERNS AND PROBABILITY DISTRIBUTIONS

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ABSTRACT

It has been conjectured that the distribution of magnifications of a point source microlensed by a randomly distributed population of intervening point masses is independent of its mass spectrum. We present gedanken experiments that cast doubt on this conjecture and numerical simulations that show it to be false.

Subject headings: dark matter — gravitational lensing — quasars: general

Online material: color figures

1. INTRODUCTION

Every investigation of microlensing at high optical depth that has explored the effect of multiple microlens mass components has led to the conclusion that the magnification probability distribution is independent of the spectrum of microlens masses. The recent effort by Wyithe & Turner (2001) is typical. While it was not their principal result, they comment in passing, “...we confirm the finding of Wambsganss (1992) and Lewis & Irwin (1995) that the magnification distribution is independent of the mass function.”

This conjecture has important consequences regarding the more general applicability of microlensing studies that are limited to a single mass component. Although galaxies have stars with a range of masses, restricting to a single component makes analytic calculations more tractable (e.g., Peacock 1986; Schneider 1987; Kofman et al. 1997) and greatly decreases the number of cases that must be simulated numerically (e.g., Wambsganss 1992; Lewis & Irwin 1995; Wyithe & Turner 2001). If true, the conjecture simplifies things considerably.

Both theoretical and experimental lines of evidence lead to this conclusion, which has struck many investigators as obvious. On the experimental side, simulations like those carried out by Wyithe & Turner (2001) and their predecessors produce magnification histograms for different mass distributions that appear to be indistinguishable for fixed surface mass density and shear.

On the theoretical side, the high-magnification tail of the magnification probability distribution has been shown to be independent of the microlens mass spectrum (Schneider 1987). Moreover, Wambsganss et al. (1992) showed that the average number of positive-parity micromages depends only upon the surface mass density (or equivalently the convergence) and the shear. Since the scale-free nature of gravity requires that the magnification probability distribution for a point source be the same for microlensing by a single mass of any size, it would appear strange if a mixture of two masses (at constant convergence and shear) produced a different magnification probability distribution.

There is, however, at least one argument against this apparently obvious conclusion, which we detail in §2 below. It suggests that the magnification probability distribution does depend upon the mass spectrum. The argument suggests that the dependence would show up in a highly magnified negative-parity macroimage, typically one of a close pair of images in a quadruply imaged quasar such as PG 1115+080.

We have carried out lensing simulations of such an image (at constant convergence and shear) for a variety of different cases. In Figure 1 we show simulations with two populations of point masses. The first component is composed of 1.000 $M_\odot$ objects hereafter referred to as “microlenses.” The second component is composed of 0.005 $M_\odot$ objects hereafter referred to as “nanolenses.” The designations and mass scale are arbitrary but are intended to convey the sense that the microlenses are very much smaller than the lensing galaxy and that the nanolenses are very much smaller than the microlenses.

The eight panels of Figure 1 show magnification histograms obtained by varying the mass fractions in the microlensing component, with the remaining fraction in the nanolensing component. For the sake of comparison, we reproduce in each panel the result for a pure microlensing component. As the fraction contributed by microlenses decreases to 20% and 10%, the histogram broadens out and develops a second peak. But as it decreases further to 0%, the magnification distribution narrows and ends up looking like the 100% case (modulo finite source effects and sample variance). Unless our simulations are faulty, the conjecture is false.

In §2 we put forward a qualitative argument for the dependence of the microlensing probability distribution on the mass spectrum. In §3 we give details of the numerical simulations that confirm the effect. In §4 we offer a qualitative
interpretation of our results. In § 5 we discuss some astrophysical consequences.

2. AN ARGUMENT FOR THE DEPENDENCE OF THE MAGNIFICATION PROBABILITY DISTRIBUTION ON THE MASS SPECTRUM

In Figure 2a we show the magnification probability distribution for a simulation of a negative-parity macroimage with convergence $\kappa = 0.55$ and shear $\gamma = 0.55$. In this simulation all of the mass is in microlenses of a single mass. In Figure 2b we again show the magnification probability distribution for a simulation of a negative-parity macroimage with convergence $\kappa = 0.55$ and shear $\gamma = 0.55$, but in this case 20% of the mass is in microlenses of a single mass and 80% of the mass is in a smooth mass sheet. The two histograms look quite different, with the first showing a single peak and the second being significantly broader and showing two peaks.¹

Now suppose that the smooth mass sheet of Figure 2b is divided into randomly distributed point masses that are very much smaller than the microlenses. We then have a microlensing component with $\kappa_{\text{micro}} = 0.11$ and a nanolensing component with $\kappa_{\text{nano}} = 0.44$. If the hypothesis that the magnification distribution is independent of the mass spectrum were correct, the magnification probability distribution would look the same as that of Figure 2a.²

Finally, suppose we take our source to be extended rather than pointlike. In particular, we imagine our source is much larger than the Einstein rings of our nanolenses but much smaller than the Einstein rings of our microlenses. The nanolenses should behave like a smooth component and the magnification probability distribution should look like Figure 2b.

Alternatively, we can compute the magnification probability distribution for our extended source by taking the magnification map for a point source and convolving it with the surface brightness distribution of the extended source. Such a convolution will inevitably smooth the map out, increasing the values of low-magnification pixels and decreasing the values of high

¹ The bi- and even tri-modality of magnification histograms has frequently been noted (Rauch et al. 1992; Wambsganss 1992; Lewis & Irwin 1995; Schechter & Wambsganss 2002). The peaks can be indexed by the number of “extra” positive-parity microimages (Rauch et al. 1992; Granot et al. 2003).

² An anonymous referee has argued that breaking up the smooth sheet into small clumps can only broaden the magnification histogram and that the conjecture must therefore be incorrect. This is borne out by the simulations presented in the following section.
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Figure 2

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Our two alternative schemes for computing the magnification

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ASPECTS OF MICROLENSING: MAGNIFICATION DISTRIBUTIONS 79

The magnification distributions for these simulations are

and external shear, corresponding to an (average) magnification

m

nano masses

m

micro objects is unimodal, with the smooth-

m

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m

nano rather than in smooth matter.

The magnification maps for these simulations are displayed

in Figure 3, with the left panels presenting the full 20R_E, while

the right panels focus upon a 1R_E part of the map. For the

smooth-matter case (middle panels) and the m

nano scenario (bottom panels), the location of the m

micro objects are the

same.

In comparing the panels, it is apparent that the smooth matter

and m

nano simulations possess similar large-scale structure in their

magnification maps, structure that is somewhat different from

the case where all the mass is in m

micro objects. On smaller

scales, however, the magnification patterns for the smooth

mass and m

nano cases are quite different, with the presence of the

smaller-mass nanolenses breaking up the magnification

structure into smaller-scale caustics.

The magnification distributions for these simulations are

presented in Figure 4. As discussed previously, the case where

all the mass is in m

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matter case being bimodal. The case containing m

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m

micro = 1 M

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and external shear, corresponding to an (average) magnification

of |μ| = 10.0 (negative parity). The positions of the lenses

were distributed randomly in a circle significantly larger than

the shooting region. The total number of rays per frame was

typically about n_rays ≈ 10^{10}, resulting in over 200 rays per

pixel on average (the shooting region was larger than the re-

ceiving region so that a significant number of rays landed

outside the latter).

We performed a series of simulations with changing mass

components. For the first series, we used two mass compo-
nents with a mass ratio of m

micro/m

nano = 200. For specificity

we adopted m

micro = 1 M

s, appropriate to stars, and m

nano = 0.005 M

s, as might apply to very massive planets.

We started with three cases: in the first case, 100% of the

mass was in microlenses with mass m

micro; in the second, 50%

of the microlenses were replaced with smooth matter; in the

third case, 50% of the matter was in nanolenses with mass

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matter case being bimodal. The case containing m

nano masses
clearly differs from the solely $m_{\text{micro}}$ case, also appearing bimodal and similar in form to the smooth-matter case, which is at odds with the conjecture.

An examination of the magnification maps in Figure 3 illuminates the differences between the magnification distributions in Figure 4. Compared with the smooth-matter map, the map for 100% $m_{\text{micro}}$ has a higher density of caustics and fewer regions of demagnification (light gray). These regions of the source plane produce no positive-parity images. Crossing caustics produces extra positive-parity images and additional
magnification. These regions dominate the magnification histogram. In the smooth-matter case, the magnification map has been “opened up,” revealing more extended regions with no positive-parity image and enhancing the low-magnification peak seen in the magnification distribution. On large scales the magnification distribution for the 50% \( m_{\text{nano}} \) case resembles that of the smooth-matter case, again with larger regions without positive-parity images. Thus, its magnification probability distribution looks more like that of the smooth-matter case than that of the 100% \( m_{\text{micro}} \) case. Indeed, the 50% \( m_{\text{nano}} \) case is even broader than the smooth case, because of the additional corrugation of the large-scale map by the small-scale lenses.

Further simulations were undertaken in an attempt to understand this difference. Again, we started with 100% microlenses, \( m_{\text{micro}} \). Then we put 1% of the total mass in nanolenses, \( m_{\text{nano}} \), (re)distributing them randomly over the lens plane. We increased the nanolens mass fraction to 2%, 5%, and 10%, and then proceeded in steps of 10% to 90%. We ended symmetrically with 95%, 98%, 99%, and 100% nanolenses, for a total of 17 different cases. The number of lenses ranged from 25,000 (for 100% \( m_{\text{micro}} \)) to 2,600,000 (for 100% \( m_{\text{nano}} \)).

A selection of the resulting two-dimensional magnification patterns is shown in Figure 5. The top six panels show the full simulation, while the bottom six panels show an expanded inset. Particularly notable is the similarity between the top left panel (with all the mass in microlenses) and the bottom right panel (the blowup of the map when all the mass is in nanolenses). Simulations of this series were used to produce the histograms in Figure 1.

For the final series, we took 50% of the mass to be in microlenses and 50% in nanolenses but let the masses of the nanolenses vary with \( m_{\text{nano}}/m_{\text{micro}} = 0.32, 0.10, 0.032, 0.01, \) and 0.0032. As bracketing cases we considered \( m_{\text{nano}}/m_{\text{micro}} = 1 \) and the 50% smooth case, corresponding to \( m_{\text{nano}}/m_{\text{micro}} \to 0 \), making seven cases altogether. The magnification patterns for all but the smooth case are displayed in Figure 6. The magnification distributions are seen in Figure 7.

This last series shows that the conjecture fails only gradually. The presence of the second component becomes significant (for our simulation) only when the nanolens masses are one-tenth those of the microlenses. By the time the nanolenses are one-hundredth those of the microlenses, the effect is as large as we can measure. In hindsight this onset would have been more appreciable had we put 80% of the mass into nanolenses (as in the third panel of Fig. 1), but the qualitative effects would have been the same.

Our lensing simulations have two limitations:

1. With the limited size of \( L = 20R_E \), the magnification maps exhibit features that are correlated on scales uncomfortably close to the size of the simulation volume. An ensemble of simulations with the same parameters will exhibit differences due to sample variance. We checked this for a few cases and found this sample variance to be small compared to the observed differences needed to make our case. Moreover, in the series of simulations described above, we kept the positions of the stars fixed to the extent possible, so that we could study the differential changes from one case to the next, and hence to minimize the effects of sample variance.

2. The finite pixel size corresponds to a minimum source size, i.e., our results are not quite applicable to a point source. However, a size of \( 0.004 R_E \) per 5000 pixel is small enough for the effects we want to study and explore. This finite size unavoidably cuts off the magnification distribution at very high magnifications and leads to deviations from the power-law behavior, but the low- and intermediate-magnification region we are interested in (see next section) is not strongly affected by that.

Despite the inevitable limited dynamic range for such simulations, we have tried to choose parameters such that we can demonstrate the effect most convincingly.

4. INTERPRETATION

In the previous section we simulated cuts through the \( \kappa_{\text{nano}}/\kappa_{\text{micro}}, m_{\text{nano}}/m_{\text{micro}} \) plane at fixed \( \kappa_{\text{tot}} \) and \( \gamma \). Somewhat counterintuitively, we find that at fixed mass ratio \( m_{\text{nano}}/m_{\text{micro}} = 1/200 \), the magnification probability histogram is broader for comparable amounts of two very different masses than it is for a single component of either one mass or the other.

The scale invariance of gravity demands that, for a point source, the magnification histograms of single components of very different masses should be identical. But our experiments show that for two very different mass components, the magnification map looks very much like that of the higher-mass component immersed in a perfectly smooth component. Only on small scales are there differences. This can be seen in Figure 3.

Suppose one grants that the magnification probability distribution for two very disparate components looks like that for a single component and a smooth component. The arguments set forth in Rauch et al. (1992), Schechter & Wambsganss (2002), and Granot et al. (2003) would come into play: the magnification histogram tends to be broadest when the effective magnification computed from the effective convergence and the effective shear is of order \(|\mu| \approx 3–4\). Alternatively, the fluctuations are largest when the number of extra positive-parity images is roughly unity. In such cases one tends to get two peaks.
Fig. 5.—Magnification maps for the case $\kappa_{\text{tot}} = \gamma = 0.55$. In the top six panels, the proportion of microlenses with a particular mass is changed such that (from left to right and top to bottom) the percentage of $\kappa_{\text{tot}}$ in 0.005 $M_\odot$ objects is 0%, 20%, 60%, 90%, 98%, and 100%; the remainder of $\kappa_{\text{tot}}$ is in 1 $M_\odot$ masses in each case. Each magnification panel is $L = 20R_E$ in extent. The six panels at the bottom show close-ups of the lower left corners of the same sequence, respectively; side lengths here are $L = 1R_E$ (defined for a 1 $M_\odot$ lens).
Fig. 6.—Magnification maps for the case $\kappa_{\text{tot}} = \gamma = 0.55$. In each of the top six panels, 50% of $\kappa_{\text{tot}}$ is composed of $1 M_\odot$ objects, while the remainder is composed of (top row, from left to right) $1 M_\odot$, $\sqrt{0.1} M_\odot$, $0.1 M_\odot$, (second row) $\sqrt{0.001} M_\odot$, $0.01 M_\odot$, and $\sqrt{0.00001} M_\odot$. Each panel is $20R_E$ in extent. The bottom six panels present close-ups of the lower left corner of the same magnification maps (extent is $L = 1R_E$).
However, the magnification probability distribution for two disparate components is not exactly that of a single component and a smooth component. The low-mass component produces additional structure in the magnification map, further broadening the magnification histogram, rounding off its peaks and filling in its valleys. There is evidence for this in Figures 2 and 3. Once one substitutes the low-mass component for a smooth component, the number of extra positive-parity images increases from roughly unity to something significantly larger. While this tends to round off the two peaks, it does not narrow the magnification histogram. The arguments of Schechter & Wambsganss (2002) and Granot et al. (2003) that the magnification histogram is broadest when there is, on average, one extra positive-parity image does not hold for two disparate mass components. The reason is that the extra positive-parity images cluster around the images produced by the single mass component, breaking them into pieces but only slightly changing the combined contribution to the flux.

5. ASTROPHYSICAL CONSEQUENCES

In gravitational lensing, magnifications depend upon second derivatives (with respect to position) of the time delay function (e.g., Blandford & Narayan 1986). Deflections depend upon first derivatives. And time delays depend upon the function itself. The second derivatives are dimensionless, with the consequence that the magnification of an image (unlike its deflection and time delay) contains no information about the mass of the intervening lens.

Image position fluctuations due to microlensing manifestly do contain information about the mass scale of the intervening microlenses (Lewis & Ibata 1998; Treyer & Wambsganss 2004, and references therein). Moreover, the timescale over which brightness fluctuations occur likewise contains information about the mass scale of the intervening microlenses (and on the distribution of microlens masses) if one knows the relative velocities of the microlenses and the source (Wyithe & Turner 2001). However, the amplitude of those brightness fluctuations is independent of mass scale.

In this paper we consider the dependence of brightness fluctuations not on mass scale, the first moment of the microlens mass distribution function, but on higher-order dimensionless moments of that mass distribution. We have demonstrated (through our simulations using two mass components) that the magnification probability does depend upon

![Magnification distributions for a scenario with $\kappa_{\text{tot}} = \gamma = 0.55$ and the matter being split evenly between two mass components (50% $m_{\text{micro}}$, 50% $m_{\text{nano}}$). The mass ratios are $m_{\text{micro}}/m_{\text{nano}} = 0.316, 0.100, 0.032, 0.01, 0.003$ for the first five panels, and $m_{\text{nano}}$ is assumed to be entirely smoothly distributed in the last panel. The dotted histogram is the respective panel for the case with 100% of the matter in $m_{\text{micro}}$. [See the electronic edition of the Journal for a color version of this figure.]](image)
those higher moments. We have not, however, explored the full range of astrophysically interesting mass distributions.

We have chosen to explore in detail the specific case of two mass components with $\kappa_{\text{tot}} = \gamma = 0.55$, appropriate to one of two images in a highly magnified pair, as in the case of PG 1115+080 (Young et al. 1981) or SDSS 0924+0219 (Inada et al. 2003). The argument of § 2 led us to believe that the effects of using two mass components rather than a single mass component would be appreciable in this case. But what about other values of the convergence and shear? How much does the mass spectrum matter for images of a quasar that are not saddle points or not highly magnified?

A thorough treatment of this question would explore a substantial fraction of the $\kappa, \gamma$ plane and would quantify with some statistic the differences between a single mass component and a range of masses. Such a treatment lies beyond the scope of this paper.

The fact that most previous investigators have failed to detect the effects of a range of microlens masses would argue that, to first order, such effects can be ignored. Even in the present case, where the convergence and shear have been chosen to maximize the effects, they are not large. Most mass distributions tend to put most of the mass at one or the other end of the mass distribution. The present simulations would seem to indicate that only if appreciable mass fractions are in components that differ by more than a factor of ten in mass will the effects of a range of masses be substantial.

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