Vibrations of Slender Structures Caused by Vortices

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Abstract. Slender cylindrical structures such as overhead transmission lines, skyscrapers, chimneys, risers, and pipelines can experience flow induced vibration (FIV). The vortex vibrations are a type of FIV; they arise because of oscillating forces caused by flow separation and the detachment of vortices. The paper presents a brief overview of experimental research on vortex induced vibration - VIV of short, rigid cylinders elastically supported (with a small aspect ratio). This overview summarizes the basic results of the vortex vibration (VIV) which have been performed in the last five decades. These studies were mainly related to determining the influence of selected parameters - mass, damping and Reynolds number on the cylinder response, either in one direction only or simultaneously in the flow direction and transverse to the flow direction, and with the search for a map of vortex images in the trace (vortex wake pattern map).

1. Introduction

It is well known that the characteristics of the flows around the non-streamline bodies are mainly related to the Reynolds number, which is defined as $Re = \frac{vD}{\nu}$, where $v$ is the free jet velocity, $D$ is the characteristic length, in the case of a cylinder, it is the diameter and $\nu$ is kinematic viscosity of the liquid. When the Reynolds number exceeds a certain critical value, the flows of non-streamline bodies are characterized by a certain preferred frequency; the so-called Strouhal frequency $f_s$ ($f_s = St \frac{v}{D}$, where $St$ - Strouhal constant). At this preferred frequency, vortices are alternately broken off from both sides of the body and dropped into the track.

During this process called vortex detachment, the body experiences a large pressure drop at the back of its surface and a significant fluctuating lateral force due to the asymmetric vortex detachment. This process causes the body to vibrate with considerable amplitudes, which can lead to fatigue failure. The fatigue failure of a vibrating structure and average drag force caused by vortex induced vibrations - VIV are major problems in structural engineering where slender structures, such as bridge cables, chimneys, tall buildings, power lines, etc., are exposed to wind. The air flow causes a complex dynamic response...
of structures. There are some theoretical and experimental papers on overhead transmission lines due to the air flow [1-12].

A detailed understanding of this phenomenon is necessary to properly define the response of a structure and to successfully develop techniques to reduce or eliminate these vibrations.

VIV are an example of a medium-structure interaction. The separation of the vortices from the side surface of the structure causes its vibrations, while the movement of the structure changes the flow and thus changes the forces of the medium.

An important feature of VIV is synchronization (lock-in). For a stationary cylinder, the vortex detachment frequency \( f_s \) increases linearly with increasing flow velocity. However, when the cylinder is oscillating, its vibrations influence the separation of the vortices - in such a way that the frequency deviates from a linear relationship and synchronizes with the movement of the cylinder; it means, these frequencies become the same. It should be emphasized that the instability of synchronization is a self-limited phenomenon.

The scientific literature on VIV in the field of physical processes, structures of forming vortices or modeling is very wide. However, most of the research was conducted experimentally, and based on these experiments, researchers proposed various empirical models to describe the VIV phenomenon.

VIV models can be divided into two groups. The first group is models with one degree of freedom, characterized by the equation of motion of the structure, where the hydro/ aero elastic force component is in phase with the motion and the nonlinear component is in quadrature with the motion. The second group of models is the so-called coupled models, where the trace is a self-forcing and self-limiting oscillator, and where the lateral force is described by a non-linear equation, usually Van der Pol or Rayleigh equations. The trace oscillator is coupled to the oscillator of the structure via a force member which may be proportional to displacement, velocity, or acceleration.

The present work deals with a narrow range of possible general scientific knowledge in the field of VIV and includes a review of mainly experimental work limited to the study of a short stiff, elastically supported cylinder with a low aspect ratio \( L/D \).

2. Vibrations caused by vortices of a rigid, supported cylinder.
If a rigid cylinder, elastically supported, is immersed in the flow, the forces caused by the vortex detachment will cause the cylinder to oscillate. This movement is vibration caused by vortices. Depending on the method of support, the cylinder may vibrate only in the longitudinal or only transverse direction, or it may oscillate in both directions. The example of Authors’ research of pressure and velocity contours for cylinder for Re=10000 is shown in Figure 1.
2.1 Vibrations of the system with one degree of freedom in the transverse direction of the flow
If we assume only one degree of freedom of the system, e.g., in the crossflow direction, the equation of motion is as follows.

\[ \ddot{y} + 2\xi \omega_n \dot{y} + \omega_n^2 y = \frac{F_y}{m} \]  \hspace{1cm} (1)

where: \( \xi = \frac{c}{2m\omega_n} \), \( \omega_n = \sqrt{\frac{k}{m+m_a}} \), \( m, c, k \) - are the mass, the damping coefficient, the system stiffness, respectively and \( m_a \) is the added mass which has a large influence on the oscillation frequency and the vibration amplitude.

Excitation force can be described as [12]:

\[ F_y(t) = \frac{1}{2} \rho v_x^2 D C_{Le} \sin(v t + \delta) \]  \hspace{1cm} (2)

where \( \rho \) is air density, \( v_x \) is air velocity, \( C_{Le} \) is the lift coefficient that is calculated in the air-flow model, \( C_{Le} = \sqrt{2} C_{Le}^{RMS}, v = 2 \pi f_s, \ f_s = S_t \frac{v_x}{D} \). In the above \( S_t \) is the Strouhal number and \( f_s \) is
frequency of motion. Figure 2. presents the example of Authors’ research of variation of lift coefficient with time and spectral lift density for cylinder, for Re = 6.5 x 10^4.

![Graph (a)](image1)

![Graph (b)](image2)

**Figure 2.** Variation of lift coefficient with time (a) and spectral lift density for cylinder (b) for Re = 140 x 10^3

An important and basic, classic illustration of the VIV in the transverse direction is provided by Feng's (1968) experimental studies [13]. Feng investigated the change in the frequency of detached vortices, the frequency of vibrations and the amplitude of the cylinder's response with the mass factor $m^* \cong 250$ based on springs as a function of the reduced velocity $U^* = \frac{v}{D \cdot f_n}$ (where $f_n$ is the cylinder's
natural frequency). The mass factor can be defined as $m^* = \frac{m}{0.25\rho \pi D^2}$ and it is the ratio of the mass of the cylinder and the mass of the displaced medium (or the equivalent mass of liquid/air during oscillation). The tests were carried out in air, which meant that the added mass was negligibly small compared to the mass of the tested cylinder, which resulted in a practically constant natural frequency. Feng showed that for low flow velocities, the frequency of detached vortices follows Strouhal's law ($f_s = St \frac{D}{U}$) and the cylinder does not experience vibrations. When the reduced speed increases to about 5, the frequency of the detachable vortices reaches the natural frequency of the cylinder and the cylinder begins to oscillate. When the reduced speed exceeds 5, the frequency of the detachable vortices no longer obeys Strouhal's law, but "keep" to the cylinder natural frequency and remains constant up to $U^* \approx 7$.

When $U^*$ exceeds 7, the vortices are no longer able to "keep" to the eigen vibrations of the cylinder and the separation of the vortices occurs again according to the Strouhal's dependence and the vibrations of the cylinder disappear.

In the range $U^* \equiv 5 \div 7$, the frequency of oscillations and the frequency of detachment of vortices were equal and were close to the natural frequency of the cylinder. In the "lock-in" range, the detachment of vortices does not follow Strouhal's law ($f_s = f_n$) but is associated with large vibration amplitudes. This behavior is the result of the synchronization between the detachment of vortices and the movement of the cylinder.

In terms of the system response ($A^*$), two separate branches were observed with one branch reaching higher amplitudes than the other. These two branches are called the initial branch and the lower branch, respectively. It was also observed that the jump in the displacement between the two branches was related to a sudden change in the phase angle ($\phi$) and the vibration frequency was almost equal to the natural frequency of the cylinder vibrations in the air over the entire range, where non-zero responses occurred. In addition, a hysteresis was found between the initial and lower branch.

The extension of the timing range will occur when the rigid roller is placed in the water flow. The added mass then makes a significant contribution ($\rho_{\text{water}}/\rho_{\text{air}} \approx 800$) to the total oscillating mass.

The shapes and appearance of the branches may vary depending on the mass-damping coefficient ($m^*\xi$). For large values of $m^*$, two branches can be observed (as was the case in the Feng experiment above). On the other hand, by lowering $m^*$ to low values, an additional upper (intermediate) branch appears between the initial and lower branch. The existence of these three distinct branches has been documented by numerous experimental studies [14], [15]. The authors examined the cylinder immersed in the flow of water and showed that the synchronization range increases with the decrease in $m^*$ (with $m^*\xi = \text{constant}$) and that the peak amplitude is independent of $m^*$ but is mainly determined by the coefficient $m^*\xi$. Moreover, they observed hysteresis at the branch transition; between the initial and upper branches, while the transition between the upper and lower branches involved intermittent switching. The jumps between these three branches were related to changes in the phase angle between the motion and the exciting force [15].

In the Khalak & Williamson experiment [14] for the mass $m^*\xi = 0.013$ and for $Re = 5 \times 10^3 \div 1.6 \times 10^4$, the peak amplitude of $A^*$ was approximately 1. The synchronization phenomenon means that when the flow velocity $u^*$ increases and reaches a certain critical speed, the frequency of the vortices detaching from the stationary cylinder ($f_s$) becomes equal to the natural frequency of the structure $f_n$ and these frequencies synchronize. A further increase in speed during the timing range "pulls" the vortex detachment frequency from its "non-oscillating" value. The detachment frequency and the oscillation frequency ($f$) become equal to the natural frequency of the structure and then $f^* = f/f_n$ is close to one.
[12]. For large mass factors $m^*$, the oscillation frequencies ($f$ and $f_n$) are almost the same, while for small values of $m^*$ different responses are observed.

The upper synchronization range, for the free vibrations of the cylinder with low mass-damping coefficients, is related to the lower branch of the response, which corresponds to the constant frequency of vibrations and the level of which increases when the mass decreases.

If the mass factor is lowered below a certain critical value, $m_{kr}^*$ then the lower branch of the response disappears and joins the upper branch (for finite velocities) [13],[14].

In works of [16-18] the existence of a value limiting the mass factor was indicated, depending on the shape of the structure, at which the "lock-in" would be maintained for a given reduced speed, and desynchronization would never occur, no matter how large the reduced speed was. According to them, for a cylinder this value is $m_{kr} = 0.54$. In the sudden occurrence of large amplitudes is clearly shown when the mass has reached a value lower than 0.54. According to Prasanth [19], the two important factors determining the hysteresis behavior of a system are blockages and the laminar detachment mass factor. For a certain combination of these two factors, the hysteresis between the initial and lower branch may disappear completely. The flow field around a non-tidal body is commonly described as the observed vortex structures. Changes in vortex structures are associated with changes in branches. Govardhan & Williamson (2004) [16] identified various vortex structures in the trace of a transversely oscillating cylinder and associated them with the corresponding branches of the system response (dimensionless amplitude - reduced velocity depending on the relationship). 2S mode (2 single vortices per cycle) was tied to the initial branch and 2P (2 pairs of vortices per cycle) to the upper and lower branches. Apart from the vortex structures assigned to the appropriate branch of the response, also two jumps in the $\Phi$ phase are shown. The first jump occurs when the oscillation frequency is equal to the natural frequency of the medium, the second jump occurs when the oscillation frequency is equal to the vacuum frequency. The basis of the analysis conducted by Khalak & Williamson 1999 [12] and in works [15], [19-21] was the division of the total force $F_{total}$ into potential force (due to potential added mass) and force due to vortices ($F_{total} = F_{potential} + F_{vortex}$). The design response is strongly related to the vortex detachment mechanism and vice versa within the timing range. For roller vibrations in the air, for which $A/D$ is usually less than 0.5, a 2S structure can be expected, while for vibrations in water, for which $A/D$ is about 1 - 2S and 2P modes. The increase of the $S_q$ parameter causes the reduction of the response amplitude until the motion stops [15]. Experiments conducted by Raghavan & Bernitsas (2011) [20] in a turbulence-free surface water channel in the range of high $Re$ numbers ($2 \times 10^4 - 4 \times 10^4 < Re < 3.5 \times 10^5 - 6 \times 10^5$) indicated no hysteresis in the response and a significant difference between VIV occurring at low and high $Re$ numbers. The synchronization range of the upper branch increased as the Reynolds number increased. In contrast, the $A/D$ amplitude ratio increased with increasing $Re$ inside the upper branch. Lower branch was disappearing, overtaken by extended upper branch. In [20], two data sets are presented, the first one is for low attenuation and low $Re$ number. The second set includes high attenuation and high $Re$ (the case of Raghavan and Bernitsas [20]). These data were plotted on a Williamson and Roshko vortex structure map. In the first case, the initial branch of the response corresponds to the 2S mode vibration form. In the second case, the synchronization starts at a higher normalized speed and the response remains within the 2P mode structures, and then goes to the area of non-synchronized patterns.

In [22] the results of experimental tests of cylinders with a diameter of 2.5; 3.0 and 3.5 inch based on springs with two different stiffnesses $k = 965 \text{ kN/m and } k = 1467 \text{ kN/m}$. Increasing the natural frequency of the 2.5-inch diameter cylinder shifted the synchronization range towards higher $Re$ and increased the amplitude are shown. Increasing the diameter of the cylinder with constant stiffness had the same effect. Raghavan & Bernitsas [22] emphasize that high Reynolds numbers have a much greater effect on $A/D$ than the mass factor.
3. Conclusions

Vortex-induced vibrations of bluff bodies is of great importance in various engineering applications, such as cables, chimneys, tall buildings, power lines, etc. The periodicity of the flow due to the vortex breaking away from a non-tidal body immersed in the flow can cause it to vibrate if the body is resiliently supported or flaccid, this phenomenon is called VIV. The flow around a freely vibrating cylinder is associated with many different interesting phenomena, under certain conditions the movement of the cylinder causes the vortices to break away at the vibration frequency of the cylinder, this phenomenon is called lock-in or synchronization. It is visible in a wide range of cylinder vibration frequencies centered around the vortex break-off frequency for a stationary cylinder. During lock-in, vibrations of significant amplitudes occur which can lead to fatigue failure of the structure. Another interesting phenomenon related to the VIV is hysteresis. The oscillation amplitude for a certain range of Re number and close to the outer frequency limits for lock-in depends on whether the flow velocity increases or decreases during the experiment. Important parameters affecting the response of a rigid cylinder resiliently supported in the flow are mass ratio, damping ratio, structural stiffness, Reynolds number and aspect ratio.

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