Neutrino tomography of Earth

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Cosmic-ray interactions with the atmosphere produce a flux of neutrinos in all directions with energies extending above the TeV scale. The Earth is not a fully transparent medium for neutrinos with energies above a few TeV, as the neutrino–nucleon cross-section is large enough to make the absorption probability non-negligible. Since absorption depends on energy and distance, studying the distribution of the TeV atmospheric neutrinos passing through the Earth offers an opportunity to infer its density profile. This has never been done, however, due to the lack of relevant data.

Here we perform a neutrino-based tomography of the Earth using actual data—one year of through-going muon atmospheric neutrino data collected by the IceCube telescope. Using only weak interactions, in a way that is completely independent of gravitational measurements, we are able to determine the mass of the Earth and its core, its moment of inertia, and to establish that the core is denser than the mantle. Our results demonstrate the feasibility of this approach to study the Earth’s internal structure, which is complementary to traditional geophysics methods. Neutrino tomography could become more competitive as soon as more statistics is available, provided that the sources of systematic uncertainties are fully under control.

A reliable estimate of the density profile of the Earth is essential to solve a number of important problems in geophysics, such as the dynamics of the core and mantle, the mechanism of the geomagnetic dynamo or the bulk composition of the Earth. Most of our knowledge about the internal structure of the Earth and the physical properties of its different layers come from geophysics and, in particular, from seismological data. Moreover, information from geodesy, geomagnetic and geodynamical data, solid state theory and high-temperature/high-pressure experimental results is also used.

The determination of the density distribution of the Earth from the bulk sound velocity of seismic waves in combination with normal modes is a well-established method with statistical uncertainties in the lower mantle and the outer core at the percent level and below for 250–300 km resolving intervals, with larger errors as radial resolution increases. The reconstruction of a three-dimensional profile is, however, a very demanding nonlinear inversion problem of different seismic data. Moreover, as wave velocities also depend on composition, temperature, pressure and elastic properties, this necessarily introduces uncertainties in the density estimate. Most studies of Earth’s radial structure are based on empirical relations between seismic wave velocities and density, such as Birch’s law, which may fail at the higher densities of Earth’s core, and the Adams–Williamson equation. A good understanding of the Earth’s interior, aiming at simultaneously determining the density variations and the origin of such waves in terms of temperature and composition variations, cannot be done from seismic wave velocity variations alone, and another, independent piece of information is needed. Therefore, a precise modelling of the different layer compositions which are crossed by seismic waves is required. Even though several million earthquakes occur in the Earth every year, only of the order of hundred of them have magnitudes larger than 6 (ref. 11). Most of them do not occur on the surface, and the origin of the wave must be inferred by comparing time delays from different seismographs. Finally, only a small fraction of the registered seismic waves cross the Earth’s core. For all these reasons, using other complementary and independent methods to infer the density profile of the Earth is important.

Neutrinos can be used to study the Earth’s interior in several ways. First, experiments such as KamLAND and Borexino are currently measuring the so-called geo-neutrino flux (that is, neutrinos produced by the decay of radioactive elements in the Earth’s interior), which provides information that can be used to understand its composition. On the other hand, a good knowledge of neutrino propagation through the Earth may give relevant information about the Earth’s density profile. Neutrino propagation does, indeed, depend on the details of the matter structure between the source and the detector. For neutrinos with energies below 1 TeV, the matter profile affects the neutrino oscillation pattern, whereas for neutrinos with energies in the multi-TeV range, the neutrino flux observed at the detector depends on the number of nucleons along its path, as neutrinos can undergo inelastic scattering and become absorbed. Indeed, the idea of performing absorption radiographs of the Earth with neutrinos dates back to more than four decades ago. To our knowledge, the first mention of this possibility was advanced in an unpublished CERN preprint in October 1973 by Placci and Zavattini and by Volkova and Zatsepin in a talk in 1974, considering man-made neutrinos. The idea of combining Earth’s neutrino radiographies (that is, performing a neutrino tomography) is based on studying the attenuation of neutrinos crossing the Earth from different angles with respect to the position of the detector. The column depth traversed by a neutrino that has passed through the entire Earth’s diameter is 11 kton cm$^{-2}$ (1.1 x 10$^{10}$ cm water equivalent). For neutrinos with an energy of ~40 TeV, the absorption length in the Earth becomes comparable to its diameter. (nσ)$^{-1}$ ~ 2R$_{\text{eq}}$, where $n$ is the average nucleon number density, $\sigma$ the neutrino–nucleon total cross-section and $R_{\text{eq}}$ = 6,371 km the mean radius of the Earth. Therefore, for few-TeV neutrinos there is a non-negligible probability for the incoming neutrino flux to be suppressed, $e^{-\sigma\tau} < 1$, where $L = 2R_{\text{eq}}$cos $\theta_i$ is the path length in the Earth as a function of the zenith angle $\theta_i$ (Fig. 1a).

Atmospheric neutrinos offer a large range of baselines (from a few to thousands of kilometres) and energies (from MeV to tens of TeV), with an energy spectrum that falls as $E^{-2.7}$. Therefore, they represent a suitable source for neutrino tomography. Although neutrino interactions are rare, with the operation of kilometre-cube detectors such as IceCube, a large event sample can be harvested. In this work we use the publicly available IceCube one-year up-going muon sample, collected during 2011–2012 and referred to as IC86 (IceCube 86-string configuration), which contains 20,145 muons.
detected over a live time of 343.7 days\(^6\) (a preliminary attempt using IceCube data with very limited event statistics was presented in 2012\(^2\)). These muons are produced by up-going neutrinos and antineutrinos which, after crossing the Earth, interact via charged-current processes in the bedrock or ice surrounding the detector. While propagating inside the detector at a speed higher than the speed of light in ice, these muons emit Čerenkov light, which is detected by the digital optical modules of the IceCube array. Details about the data sample and the modelling of the predicted event rate are provided in Methods.

The energy and zenith distributions of the IC86 sample are shown in Fig. 1b. Since the atmospheric neutrino spectrum is a steeply falling function of the energy and, for the lowest energies, the neutrino absorption length is much larger than the Earth’s diameter, most of the neutrinos in the sample do not undergo significant absorption. Therefore, the distribution of the full sample is very similar to the atmospheric neutrino distribution at the Earth’s surface, which is more peaked towards the horizon\(^7\). For higher energies, however, the observed event spectrum corresponding to up-going neutrinos with the longest trajectories through the Earth (\(\cos \theta_z^{\text{rec}} \sim -1\)) is suppressed with respect to the zenith-symmetric flux corresponding to downward-neutrinos that propagate only a few tens of kilometres without crossing the Earth (\(\cos \theta_z^{\text{rec}} \sim 1\)). The effect is more pronounced for neutrinos with higher energies and for those with longer propagation paths in the Earth, as they have a larger probability of interaction. Hence, by studying the zenith and energy distributions of the atmospheric neutrino flux and comparing them with the flux without attenuation, information on the Earth’s density profile can be extracted. However, all events are useful: the events with the lowest energies or more horizontal trajectories serve to fix the overall normalization and attenuation paths in the Earth, as they have a larger probability of interaction. As an indication, we also show the expectations for the central value and the 1\(\sigma\) statistical error of this ratio using the one-dimensional Preliminary Reference Earth Model (PREM)\(^2\).

We have parametrized the Earth’s density with a one-dimensional five-layer profile with constant density in each of the layers (Fig. 1a). One of the edges is chosen at the core/mantle boundary and another one at the inner core/outer core boundary, so that we select three layers in the core (one for the inner core and two for the outer core) and two layers in the mantle. We have checked that, with this number of layers, current data are not yet sensitive to the particular profile within a given layer (see Supplementary Figs. 3, 4 and Supplementary Table 1) and, therefore, there is no expected gain when using more layers or a more realistic density profile. We fit the average density of each of the layers, which is allowed to vary freely, and obtain our main result, a one-dimensional density profile of Earth measured by means of weak interactions (Fig. 3). With one-year statistics the uncertainties are large, but still compatible with results from geophysical methods within a 68\% credible interval. Notice that these results are obtained from one-dimensional marginalized posterior probability distributions and correlations among all the parameters in the fit (five densities and four nuisance parameters) are not shown here. Therefore, they give a conservative representation of the allowed ranges for the density of individual layers. For the interested reader, technical details regarding the fit procedure are given in Methods.

From the results of the fit, we compute the mass of the Earth as weighted by neutrinos and obtain \(M_E = (6.0_{-1.3}^{+1.1}) \times 10^{24}\) kg (Fig. 4a), to be compared to the most precise gravitational measurement to date\(^2\) of \(M_E^{\text{grav}} = (5.9722 \pm 0.0006) \times 10^{24}\) kg. Clearly, albeit within large uncertainties, both results are in very good agreement.
We can also estimate the mass of the Earth's core, a parameter that may be useful (as soon as statistical errors decrease) as an input for geophysical measurements of the Earth's density profile. The result for this quantity is $M_{\text{core}}^\text{grav} = (2.72^{+0.09}_{-0.08}) \times 10^{24}$ kg, which is slightly larger than the result from geophysical density models that estimate the mass of the core to be ~33% of the total mass of the Earth (see Fig. 4b).

From our measurement of the one-dimensional density profile we can determine the Earth's moment of inertia, for which we get $I_0^\text{grav} = (6.9 \pm 2.4) \times 10^{37}$ kg m$^2$ (Fig. 4c), in agreement with the current (gravitationally inferred) measurement of the mean moment of inertia, $I_0^{\text{grav rk}} = (8.01736 \pm 0.00097) \times 10^{37}$ kg m$^2$. The smaller moment of inertia from neutrino data, as compared to gravitational measurements, implies a central value with a larger departure from homogeneity, as shown in Fig. 4c (even though they are fully compatible between each other due to the still large uncertainties).

Another piece of information regarding the Earth's interior that we can extract from the currently available data is to detect that the core is denser than the mantle. Notice that, implicitly, this is a strong assessment in favour of a non-homogeneous Earth (something that...
was expected to be possible to prove at 3σ after ten years of IceCube data and seems to be already established at more than 2σ using IC86 alone). We determine the difference between the average density within the two layers we divide the mantle into, \( \rho_{\text{mantle}} \) and the average density within the three layers corresponding to the core, \( \rho_{\text{core}} \). The result for this difference, measured by weak interactions, is \( (\rho_{\text{core}} - \rho_{\text{mantle}}) = 1.1^{+0.3}_{-0.2} \text{ g cm}^{-3} \) (Fig. 4d). From this result, a denser Earth’s mantle has a p-value of 0.011 for our default model of the atmospheric neutrino flux.

As a test of consistency and as a matter of accounting for further systematic uncertainties, all observables have also been computed for other atmospheric neutrino fluxes, as well as using different modelling of the inner structure of the Earth. In all cases, the results are compatible with the ones presented here (Supplementary Figs. 1–4 and Supplementary Table 1).

At high enough energies (few TeV), the passing of neutrinos through the Earth is sensitive to the number density of nucleons and, therefore, this test represents an effective counting of nucleons in the Earth. Unlike gravitational methods, the estimation of the Earth’s mass with neutrinos relies purely on weak interactions and on the nucleon masses. Conceptually, this is a completely different method from gravitational ones. We have shown that, using the publicly available data from the IceCube neutrino telescope, this method starts being feasible. Future data will greatly improve the measurements presented here (we remind the reader that more data already collected by IceCube in the same energy range are not yet publicly available in the format required to perform this analysis, but hopefully will be released soon), including data from the future KM3NeT detector in the Mediterranean Sea. For this reason, we have also estimated the projected sensitivity with future IceCube data (Supplementary Figs. 5,6).

As a final comment, it is important to stress that a non-gravitational measurement of Earth’s mass, as is the one presented here, could also probe that all the matter that contributes to Earth’s gravitational field is baryonic matter (protons, neutrons and electrons). With current neutrino data, however, a small fraction in the form of (non weakly-interacting) dark matter, which would not attenuate the passage of neutrinos, cannot yet be fully excluded.

Online content
Any methods, additional references, Nature Research reporting summaries, source data, statements of data availability and associated accession codes are available at https://doi.org/10.1038/s41567-018-0319-1.

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Author contributions
The idea was conceived by A.D. The approach of the study was discussed by all authors. J.S. performed all the numerical calculations and prepared the figures. S.P.-R. wrote the text, with inputs from A.D. Bibliography selection was performed by A.D. and S.P.-R. All authors discussed the results and commented on the manuscript.

Competing interests
The authors declare no competing interests.

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Methods
IC86 atmospheric neutrino sample. The 2011–2012 up-going muon sample of the IceCube detector (IC86) was selected because of its very good angular information (better than a degree for TeV energies) and its good statistics around TeV energies. These 201.145 muons, detected over 343.7 days, were produced by atmospheric muon neutrino and antineutrino interactions in the medium surrounding the detector. In turn, these neutrinos originated from decays of atmospheric pions and kaons (and with a contamination from other sources below 0.1%) in the Northern Hemisphere, and have all traversed the Earth. The sample covers a solid angle of 2π, making it particularly suitable for the kind of study performed here. The energy of the muons in the IC86 sample lies between ~400 GeV and ~20 TeV and is reconstructed, based on energy losses along the track, with a resolution of $\sigma_{E}/E$ = 0.5. Since the median opening angle with respect to the parent neutrino direction is $0.7E_{\nu}/E_{\mathrm{IC86}}$ degrees, the muon direction is a very good proxy for the original neutrino direction. The muon zenith angle can be reconstructed with a resolution in cos$\theta_{z}$ between 0.005 and 0.015.

Models of atmospheric neutrino fluxes. The atmospheric neutrino flux is characterized in terms of the cosmic-ray primary spectrum entering the atmosphere and the hadronic-interaction model that controls the development of the shower that finally produces the flux of neutrinos. Several choices for the model of the atmospheric neutrino flux are currently compatible with all available data. In this Letter, our default choice for the primary cosmic-ray spectrum is the Honda–Gaisser primary cosmic-ray spectrum with the Gaisser–Hillas H3a atmospheric neutrino flux, which are currently compatible with all available data. In this work, we have not imposed any external constraint on the Earth’s mass or moment of inertia, as the propagation of neutrinos through the Earth is not sensitive to these parameters. On the other hand, these priors have a rather small impact on the inner and outer core densities. The 68% credible interval for the average core density from an analysis of the full IceCube Monte Carlo sample, core mass constraint on inertia and the difference in average density of the core and mantle by comparing our results with the outcome of a fit where the four nuisance parameters have been fixed to their corresponding best-fit values. We have found that these systematic errors contribute to approximately 30% of the error on the derived quantities, $\Delta\rho_{\text{neu}}$, $\Delta\rho_{\text{core}}$, $\rho_{\text{Earth}}$, and $\rho_{\text{Earth}} - \rho_{\text{core}}$. Finally, note that the optical properties of the ice surrounding the detector also represent an important source of systematic uncertainties, which are comparable to current uncertainties. We may limit the potential of the method once statistical uncertainties decrease. Nevertheless, with the available information we cannot further investigate this issue further. This should be done with results from the full IceCube Monte Carlo.

Neutrino–nucleon cross-sections. Overall, in the energy range relevant for this analysis (that is, neutrino energies between a few hundred GeV and a few tens of TeV), the uncertainty on the neutrino–nucleon and antineutrino–nucleon cross-sections are about 2–3% and 4–10%, respectively. This can be parametrized by using different parton distribution functions (PDFs). In this work, our default PDFs are the ones from the HERAPDF set. Given the degeneracy between the interaction cross-section and the Earth’s density, we expect the propagated uncertainties in the derived quantities to be of the same order. We have checked that this is so and given the much larger statistical uncertainties in the data, the errors in the cross-section have no significant effect on our results.

Note that taking a complementary approach to the one presented in this Letter, one could try to confirm the value of the neutrino–nucleon cross-section at these energies, assuming the Earth’s density profile to follow the PREM—21. Propagation of neutrinos through the Earth. The transport equations for neutrinos traversing the Earth, which we solve using the s-SQuIDs package, consist of four main ingredients (see, for example 21): the standard evolution Hamiltonian in mass, which includes the vacuum mass-mixing terms and the effect of coherent forward scattering off electrons of the medium, given by the matter interaction potential; the attenuation effect caused by neutrino inelastic interactions with matter, either via charged-current or neutral-current processes; the redistribution of neutrinos from higher to lower energies after neutral-current interactions; and, finally, the neutrino regeneration term from tau lepton decays. Neutrino flavour oscillations in matter, given by the first term, represent the dominant effect for neutrino energies below a few hundred GeV. On the other hand, the other terms become dominant for neutrinos with higher energies. Since the neutrino–nucleon cross-section increases with energy, at these energies the neutrino flux gets attenuated. In the case of neutral-current interactions neutrinos are degraded in energy. In the case of charged-current interactions, neutrinos are absorbed and a lepton of the same flavour is produced. Whereas in the case of electron and muon neutrinos (and antineutrinos), the associated lepton (electron or muon) is rapidly absorbed in the Earth and does not contribute to the high-energy neutrino flux, the tau leptons produced after tau neutrino charged-current interactions decay before losing too much energy. In these decays, a new tau neutrino (or antineutrino) with lower energy is produced and thus, they get regenerated. Moreover, secondary electron and muon neutrinos (and antineutrinos) are also produced after tau lepton decays. For the energies we consider here, neutrino oscillations are suppressed and the effects of tau neutrino regeneration and secondary production of electron and muon neutrinos are negligible. On one hand, these neutrinos are rarely produced in the atmosphere, and on the other hand, this effect is important only for spectra much higher than the atmospheric neutrino one. Therefore, for the sake of saving computational time, we have not included the regeneration or secondary production terms in this work. We stress the corrections are much smaller than the precision on the determination of the Earth’s profile achieved with current data.

Earth modelling. We have considered two models for the Earth, fixing the position of the core/mantle boundary and the transition from the inner to the outer core. On one hand, we have parametrized the Earth’s density with a one-dimensional five-layer model, $R_{\oplus} \times R_{\oplus}$, with constant density in each of the layers (Fig. 1a). The layers are defined as follows: $R_{\oplus}$, $R_{\oplus}$, $[0.195,0.3725]$ or $[0.195,0.3725]$, $R_{\oplus}$, $[0.3725,0.555]$, $R_{\oplus}$, $[0.55775,0.775]$, $R_{\oplus}$, $[0.775,1]$, $R_{\oplus}$, earth’s mean radius, $R_{\oplus} = 6.371$ km. The first layer corresponds to the inner core, the second and third layers (of equal thickness) to the outer core, and the fourth and fifth layers (of equal thickness) to the mantle. The density of each of these layers is allowed to float freely and independently. On the other hand, we have also considered a model with five layers, again, but with a density profile within each layer that follows that of the PREM. The density in each of these layers is multiplied by a factor which is also allowed to vary freely and independently of the others. For the current data set, we do not expect that a larger number of layers would change the results presented here. Indeed, this is partly explained by the similarity of the results of the flat-layer model and the PREM-based model with five layers. Since our aim in this work is to evaluate the sensitivity of the neutrino attenuation effect on the Earth’s density profile, throughout this work, we have not imposed any external constraint on the Earth’s mass or moment of inertia, which are (gravitationally) known much more precisely than the Earth’s density profile. The results for the different cases are shown in Supplementary Figs. 3a and Supplementary Table 1. In order to understand the importance of these external constraints, we have also performed an analysis of the present statistical sample, including the total mass of the Earth and its moment of inertia as external priors. Since approximately 70% of the Earth’s mass is located in the mantle, this procedure, in practice, corresponds to fixing the mantle density to the PREM value within small errors. The 68% credible interval for the upper mantle density changes from $\rho_{\text{Earth}}[1.22,4.78]$ g cm$^{-3}$ for the unconstrained fit to $\rho_{\text{Earth}} [1.43,4.79]$ g cm$^{-3}$ for the constrained one. On the other hand, these priors have a rather small impact on the inner and outer core densities. The 68% credible interval for the average core density from an unconstrained fit is $\rho_{\text{core}}[8.65,18.6]$ g cm$^{-3}$, whereas for the constrained fit we get $\rho_{\text{core}}[9.65,18.6]$ g cm$^{-3}$.

Parameter estimation. To quantitatively assess the power of the one-year up-going muon IC86 sample to determine the Earth’s density profile, we performed a likelihood analysis using all the events in the data sample and characterizing each event by its reconstructed muon energy and zenith angle. The full likelihood is defined as the bin product of the Poisson probability of measuring $N_{\text{bin}}$ for the expected value $N_{\text{bin}}$ times the product of Gaussian probabilities for the full nuisance parameters. The log-likelihood (up to a constant) is given by

$$\ln L(D; p) = \sum_{i \in \text{bins}} \left( N_{\text{bin}}^{\text{obs}} \ln N_{\text{bin}}^{\text{obs}} - N_{\text{bin}}^{\text{obs}} \ln N_{\text{bin}}^{\text{obs}} - \frac{(N_{\text{bin}}^{\text{obs}} - N_{\text{bin}}^{\text{pred}})^{2}}{2\sigma_{\ln N_{\text{bin}}}} \right)$$

where the subindex $i$ refers to a bin in reconstructed muon energy ($E_{\mu}$) and the reconstructed zenith angle ($\cos \theta_{z}$); $N_{\text{bin}}^{\text{obs}}(p; \eta)$ is the expected
number of events for a given value of the densities in each layer (parameterized by \( \rho \)) and the nuisance (\( \theta \equiv (N, \pi/K, \Delta \nu, \text{DOM}_\nu) \)) parameters in the binned and the nuisance parameters with Gaussian priors through the MultiNest nested distribution library (https://getdist.readthedocs.io/en/latest/), which is one of the most efficient and robust tools for Bayesian analyses for problems with a large number of parameters, as in our case.

Data availability

The IceCube data we consider in this paper are the same sample used by the collaboration to search for resonant matter effects induced by light sterile neutrinos. The Monte Carlo results used to simulate the detector characteristics and all data are publicly available and can be downloaded from https://icecube.wisc.edu/science/data/IC86-sterile-neutrino.

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