The String Light Cone in the pp-wave Background

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Abstract

In this letter, we determine the particle and the string light cone in the pp-wave background. The result is a deformed version of the flat one. We point out the light cone exhibits an intriguing periodicity in the light cone time direction $x^+$ with a period $\sim 1/\mu$. Our results also suggest that a quantum theory in the pp-wave background can be formulated consistently only if the background is periodic in the light cone time $x^+$. 

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1 Introduction

Causality entered the realm of physics with the advent of special relativity. The upper bound on the speed of any object (namely the speed of light $c$) and the indefinite metric of the Minkowski spacetime changed the idea of causality radically. One now would speak, of events, points in spacetime. Given a present event, some future events could be influenced by the propagation of a signal, some others only by the propagation of a signal at the speed of light and yet some others could not be influenced, simply because that would require superluminal speeds. And the idea of the light cone emerged, defined simply as the hypersurface in spacetime that divides causally related events from causally unrelated. In the Minkowskian spacetime, the light cone is defined by the proper distance $ds^2$

$$ds^2 = -2dx^- dx^+ + dx^i dx^i$$  \hspace{1cm} (1)

and divides the spacetime into regions where the proper distance between events being time-like, light-like or space-like.

The marriage between quantum mechanics and special relativity proved to be a non trivial one. After many attempts, this was accomplished with the formulation of the quantum theory of fields. The question of causality in a quantum field theory is formulated in terms of the commutability of observables, or in terms of the (anti-)commutability of its fundamental fields. The light cone in point particle quantum field theory, as defined by the vanishing of the commutator of fundamental quantum fields, is the same with the classical theory, as determined by special relativity.

String, as an extended object, is intrinsically different from a particle. It is an interesting problem to study the notion of causality in string theory and to see how is it different from the particle case. This analysis has been performed for the case of a flat background \cite{1,2}, with the metric \cite{1}. There, it was found that the shape of the light cone is modified due to the extra, internal oscillatory modes of the string. For example, two open strings are causally unrelated (vanishing of the string field commutator) if

$$2\Delta x_0^- \Delta x^+ - \Delta x_0^2 - \sum_{l=1}^{\infty} |\Delta x_l|^2 < 0.$$  \hspace{1cm} (2)

Here $x^\pm = (t \pm x)/\sqrt{2}$ and $\Delta x = x - y$ for two strings $x(\sigma)$ and $y(\sigma)$. The deviation from the usual understanding of light cone is obvious and is due to the internal oscillatory modes that a string carries. This is indeed a surprising result. Note that when the stringy contribution ($l \geq 1$ terms in \cite{2}) is dropped, one recovers the usual notion of light cone of a point particle, as determined by the metric \cite{1}. Effects of string interaction on the definition of light cone has also been studied. Further discussions of the result can be found in \cite{3,4}.

Recently, string theory in pp-wave background have been studied with immense interests, largely due to the remarkable proposal \cite{5} which states that a sector of the SYM operators
with large $R$-charge is dual to the IIB string theory on a pp-wave background. The string background consists of a plane wave background,

$$ds^2 = -2dx^+dx^- - \mu^2 \sum_{i=1}^{8} (x^i)^2 dx^+ dx^+ + \sum_{i=1}^{8} dx^i dx^i,$$

(3)

together with a RR five-form. This background can be seen as a deformation of the flat background, with a curvature controlled by $\mu$. Remarkably, even in the presence of a RR 5-form flux, the string theory is exactly solvable \[8\] in the light cone gauge. It is therefore an interesting question to ask whether and how the causal structure is modified in the pp-wave case. This is the main motivation of this letter. We find that two strings in the pp-wave background are causally unrelated if

$$\Delta x_0 - \frac{\mu}{2 \sin(\mu \Delta x^+)} \sum_{i=1}^{8} \sum_{l=0}^{\infty} [(x_l^i)^2 + (y_l^i)^2] \cos(\mu \Delta x^+) - 2x_l^i y_l^i < 0.$$  (4)

The plan of this letter is as follow. In section 2 we start with a particle propagating in the pp-wave background and use the commutator algebra to extract the light cone. It is given by \[4\] by stripping out all the stringy modes ($l \geq 1$) contribution. The particle light cone is exactly the same as the one determined by the metric \[3\]. After this warm up exercise, we construct in section 3 the light cone string field in the pp-wave background. We then compute the string field commutator and use it to extract the string light cone. Discussions and further comments are found in section 4.

2 Particle light cone in the pp-wave background

Consider a free relativistic real scalar particle moving in the metric \[3\]. The light cone Hamiltonian is given by

$$H = \frac{1}{2p^+} \left( p^{i2} + m^2 + \omega^2 x^{i2} \right), \quad \omega := \mu p^+.$$  (5)

Notice that it consists of two parts. The first is the usual piece for a free massive relativistic particle and the other is an oscillator’s potential.

The field for the particle has to obey the Schrödinger equation

$$i \frac{\partial \phi}{\partial x^+} = H \phi,$$  (6)

In the coordinate representation, the equation of motion reads

$$i \frac{\partial \phi}{\partial x^+} = \frac{1}{2p^+} \left( -\frac{\partial^2}{\partial x^i x^i} + m^2 + \omega^2 x^{i2} \right) \phi := \frac{H_0}{2p^+} \phi.$$  (7)

2
\( H_0 \) is the Hamiltonian for an oscillator with mass 1/2 and a frequency \( 2\omega \). The light cone energy is

\[
H = \sum_i n^i \mu + \frac{m^2}{2p^+}
\]  

(8)

where we have dropped the zero point energy. Note that the physical mass \( m \) of the particle appears as a constant energy. We will see below that the light cone does not depend on \( m \).

The equation (7) can be solved (separate variables) and at the end, the solution one finds for the field is

\[
\phi(x^+, x^-, x^i) = \int_0^\infty \frac{dp^+}{\sqrt{2\pi p^+}} \sum_{\{n^i\}} a(p^+, \{n^i\}) e^{-i(x^+ p^- + x^- p^+)}.
\]

\[
\cdot \prod_{i=1}^8 H_{n^i} \left( \sqrt{\omega} x^i \right) \exp \left[ -\frac{1}{2} \omega (x^i)^2 \right] \sqrt{\omega/\pi} \frac{\pi e^{i\omega x^i} - \pi e^{-i\omega x^i}}{2^{n^i} (n^i)!} + h.c.
\]

(9)

Demanding the equal time commutator

\[
[\phi(x^+, x^-, x^i), \phi(x^+, y^-, y^i)] = \delta(x^- - y^-) \prod_{i=1}^8 \delta(x^i - y^i),
\]

(10)

implies the commutation relation for the creation-annihilation operators

\[
\left[ a(p^+, \{n^i\}), a^\dagger(q^+, \{m^j\}) \right] = p^+ \delta(p^+ - q^+) \delta_{\{n^i\}, \{m^j\}}
\]

(11)

and the other being zero.

We are now in position to find the propagator for the particle. It is easy to obtain

\[
[\phi(x^+, x^-, x^i), \phi(y^+, y^-, y^i)] = I_1 - I_2
\]

(12)

where \( I_1 = I_1(\Delta x^+, \Delta x^-, x^i, y^i) \) is given by

\[
I_1 = \int_0^\infty \frac{dp^+}{2\pi} e^{-i\Delta x^- p^+ - i\Delta x^+ \frac{p^2}{2p^+}} \prod_{i=1}^8 e^{-i\mu \Delta x^+ n^i} H_{n^i} \left( \sqrt{\omega} x^i \right) H_{n^i} \left( \sqrt{\omega} y^i \right) e^{-\frac{4\omega}{2^{n^i} (n^i)!}} \frac{\sqrt{\omega/\pi}}{2^{n^i} (n^i)!}
\]

and \( I_2 = I_2^* \). Here \( \Delta x^+ := x^+ - y^+ \), \( \Delta x^- := x^- - y^- \). With the propagator in our hands, we are in a position to find the light cone for the point particle. Instead of considering an oscillating phase factor, the integral can be computed by an analytical continuation of \( p^+ \). Without loss of generality, we assume \( \Delta x^+ > 0 \) and we perform an analytic continuation

\[
p^+ \rightarrow ip^+ \quad \text{for } I_1.
\]

(13)

Then

\[
I_1 = \int_0^\infty \frac{dp^+}{2\pi} e^{\Delta x^- p^+ - \Delta x^+ \frac{p^2}{2p^+}} \prod_{i=1}^8 \frac{\sqrt{\mu p^+ / \pi}}{\sqrt{1 - e^{-2\mu \Delta x^+}}}
\]

\[
\cdot \exp \left\{ \frac{\mu p^+}{2 \sin (\mu \Delta x^+)} \left[ 2x^i y^i - (x^i)^2 + y^2 \cos (\mu \Delta x^+) \right] \right\}.
\]

(14)
Performing an analytic continuation

\[ p^+ \rightarrow -ip^+ \quad \text{for } I_2, \]  

one obtains the same identical expression for \( I_2 \). The integral converges in the \( p^+ \rightarrow \infty \) limit, and hence the commutator \( \Delta x^- \) vanishes, if the exponent factor is negative, i.e.

\[ \Delta x^- = \frac{\mu}{2 \sin(\mu \Delta x^+)} \sum_{i=1}^{8} \left[ ((x^i)^2 + (y^i)^2) \cos(\mu \Delta x^+) - 2x^i y^i \right] < 0. \]  

This is the equation for the point particle light cone in a pp-wave spacetime.

A couple of remarks are in order. First we note that the light cone (16) is independent of the mass \( m \) of the particle, as one can expect on physical grounds. Secondly, we note that the particle light cone (16) is exactly the same as the light cone determined by the metric (3). See [6] for interesting discussions of the properties of light cone and causality as pertained to the pp-wave metric. Finally, we note also that the light cone is periodic in \( \Delta x^+ \sim \Delta x^+ + 2\pi/\mu \).

We will see that the same intriguing behaviour persists in the string case.

### 3 String light cone in the pp-wave background

The construction of a light cone string field theory in this new background follows the same steps with the construction in the flat background, see for example [7].

Consider a bosonic open string moving in a pp-wave background, with metric given by (3). The \( i \) index enumerates the transverse coordinates, \( i = 1, \ldots, d - 2 \). The generalization to closed string and to the supersymmetric case is straightforward. By taking a light cone gauge

\[ x^+ = p^+ \tau, \]  

the theory is exactly solvable, much like the case of a string in a flat, Minkowski, background. For a more detailed discussion, see [8,9]. In the light cone gauge, the string coordinates \( x^- \) and the transverse ones \( x^i \) are given by

\[ x^-(\sigma) = x^-_0 + \int^\sigma d\tilde{\sigma} \ x^i P^i(\tilde{\sigma}) \]  

\[ x^i(\sigma) = x^i_0 + \sqrt{2} \sum_{l=1}^\infty x^i_l \cos(l\sigma). \]  

The light cone string field \( \Phi \) depends on \( x^+, x^-_0, x^i(\sigma) \), i.e. \( \Phi = \Phi[x^+, x^-_0, x^i(\sigma)] \) and satisfies the Schrödinger equation

\[ i \frac{\partial \Phi}{\partial x^+} = H\Phi[x^+, x^-_0, x^i(\sigma)], \]  

\footnote{We thank Harald Dorn, Boris Pioline and Simon Ross for pointing out a mistaken statement made in the first version of this letter.}
where $H$ is the light cone Hamiltonian (taking $2\alpha' = 1$)

$$H = \frac{\pi}{2p^+} \int_0^\pi d\sigma \left[ -\frac{\partial^2}{\partial x^0(x_0)^2} + \frac{1}{\pi^2} \left( \frac{\partial x^i(\sigma)}{\partial \sigma} \right)^2 + \frac{\Delta^2}{\pi^2} (x^i(\sigma))^2 \right] \quad \text{and} \quad m := \mu p^+. \quad (21)$$

It is straightforward to rewrite $H$ in terms of modes and then takes the form

$$i \frac{\partial \Phi}{\partial x^+} = \frac{1}{2p^+} \sum_{l=0}^{\infty} H_l \Phi(x^+, x^-, \{x^i_l\}), \quad (22)$$

where

$$H_l := -\frac{\partial^2}{\partial x^+_l^2} + \omega_l^2 (x^i_l)^2, \quad \omega_l := \sqrt{l^2 + (\mu p^+)^2}. \quad (23)$$

Clearly, (23) is a SHO Hamiltonian with mass $1/2$ and frequency $2\omega_l$. Eq. (22) can be solved easily and the final answer for the string field is

$$\Phi = \int \frac{dp^+}{\sqrt{2\pi p^+}} \sum_{\{n_l\}} A(p^+, \{n_l\}) e^{-i(x^+ p^+ - x^- p^+)} \prod_{l=0}^{\infty} \varphi_{\{n_l\}}^l (x^i_l) + h.c. \quad (24)$$

where

$$\varphi_{\{n_l\}}^l (x^i_l) = \prod_{i=1}^{d-2} H_{n_l} \left( \sqrt{\omega_l x^i_l} \right) e^{-\omega_l (x^i_l)^2/2} \sqrt{\frac{\omega_l}{\pi^2}} \frac{\varphi_l}{2^{n_l!}}. \quad (25)$$

The light cone energy is

$$p^- = \frac{1}{p^+} \sum_{i=1}^{d-2} \sum_{l=0}^{\infty} n^i_l \omega_l, \quad (26)$$

where we have ignored the zero point energy, which is irrelevant for our analysis. The conjugate momentum field is given by, see (21),

$$\Pi[x(\sigma)] = \frac{\delta L}{\delta (\partial \Phi / \partial x^+)} \quad (27)$$

and in the present case it is

$$\Pi = i\Phi. \quad (28)$$

The Equal Time Commutation Relations for the string field are

$$[\Phi(x^+, x^-_0, \bar{x}^i), \Phi(x^+, y^-_0, \bar{y}^i)] = \delta(x^-_0 - y^-_0) \prod_{i=1}^{d-2} \delta \left[ x^i(\sigma) - y^i(\sigma) \right]. \quad (29)$$

which translate to the commutation relation

$$[A(p^+, \{n^i_l\}), A^\dagger(q^+, \{m^j_k\})] = p^+ \delta(p^+ - q^+) \delta_{\{n^i_l\}, \{m^j_k\}} \quad (30)$$
for the string creation-annihilation operators. The calculation of the general (non-equal time) commutator of the string field is not hard to perform and one finds at the end that

\[ [\Phi(x^+, x_0^-, x^i_l), \Phi(y^+, y_0^-, y^i_l)] = I_1 - I_2, \]  

(31)

where \( I_1 = I_1(\Delta x^+, \Delta x_0^-, x^i_l, y^i_l) \) is given by

\[
I_1 := \int_0^\infty \frac{dp^+}{2\pi} e^{-i\Delta x^- p^+} \prod_{l=0}^{\infty} \prod_{i=1}^{\infty} \frac{\sqrt{\omega_l/\pi}}{\sqrt{1 - e^{-\frac{2\Delta x^+ \omega_l}{p^+}}}} \cdot \exp \left\{ \frac{\omega_l}{2i \sin \left( \frac{\Delta x^+ \omega_l}{p^+} \right)} \left[ 2x^i_l y^i_l - \left( (x^i_l)^2 + (y^i_l)^2 \right) \cosh \left( \frac{\Delta x^+ \omega_l}{p^+} \right) \right] \right\}.
\]

(32)

and \( I_2(\Delta x^+, \Delta x_0^-, x^i_l, y^i_l) = I_1(-\Delta x^+, -\Delta x_0^-, x^i_l, y^i_l) = [I_1(\Delta x^+, \Delta x_0^-, x^i_l, y^i_l)]^* \). Notice that as expected, the propagator is a product of SHO propagators.

To compute \( I_1 \) and \( I_2 \), it is convenient to analytic contniuate \( p^+ \) to the complex plane. Without loss of generality, we assume \( \Delta x^+ > 0 \) and perform an analytic continuation

\[ p^+ \rightarrow ip^+ \quad \text{for} \quad I_1. \]

(33)

Then

\[
I_1 = \int_0^\infty \frac{dp^+}{2\pi} e^{\Delta x^- p^+} \prod_{l=0}^{\infty} \prod_{i=1}^{\infty} \frac{\sqrt{\omega_l/\pi}}{\sqrt{1 - e^{-\frac{2\Delta x^+ \omega_l}{p^+}}}} \cdot \exp \left\{ \frac{\omega'_l}{2 \sinh \left( \frac{\Delta x^+ \omega'_l}{p^+} \right)} \left[ 2x^i_l y^i_l - \left( (x^i_l)^2 + (y^i_l)^2 \right) \cosh \left( \frac{\Delta x^+ \omega'_l}{p^+} \right) \right] \right\},
\]

(34)

where

\[ \omega'_l = \sqrt{l^2 - (\mu p^+)^2}. \]

(35)

Performing an analytic continuation

\[ p^+ \rightarrow -ip^+ \quad \text{for} \quad I_2, \]

(36)

one obtains the same identical expression for \( I_2 \).

Therefore the commutator (31) vanishes provided that the integral (34) converges. To check when this is the case, note that the integral converges for \( p^+ \to 0 \). However problems may appear in the regime \( p^+ \to \infty \). In this regime, \( \omega'_l \) becomes imaginary

\[ \omega'_l = i \sqrt{(\mu p^+)^2 - l^2} \rightarrow i\mu p^+ \]

(37)
Therefore (34) becomes

\[ I_1 = \int_0^\infty \frac{dp^+}{2\pi} e^{-\Delta x_0^+ p^+} \prod_{i,l=0}^\infty \frac{\sqrt{\mu p^+ / \pi}}{\sqrt{1 - e^{-2\mu \Delta x^+}}} \exp \left\{ \frac{\mu p^+}{2 \sin(\mu \Delta x^+)} \left[ 2x_l^i y_l^i - ((x_i^i)^2 + (y_l^i)^2) \cos(\mu \Delta x^+) \right] \right\} \]

\[
\sim \int_0^\infty dp^+ \exp \left\{ p^+ \left[ \Delta x_0^- - \frac{\mu}{2 \sin(\mu \Delta x^+)} \sum_{i,l} \left[ ((x_i^i)^2 + (y_l^i)^2) \cos(\mu \Delta x^+) - 2x_l^i y_l^i \right] \right] \right\} 
\]

The integral converges in the \( p^+ \to \infty \) limit, and hence the commutator (31) vanishes, if the exponent factor is negative, i.e. if

\[ \Delta x_0^- - \frac{\mu}{2 \sin(\mu \Delta x^+)} \sum_{i,l} \left[ ((x_i^i)^2 + (y_l^i)^2) \cos(\mu \Delta x^+) - 2x_l^i y_l^i \right] < 0. \]  

Equation (39) determines the string light cone in the pp-wave background and is the main result of this letter.

Notice that the string light cone (39) is determined not only by the zero modes, but also by the internal oscillating modes of the string. This is something that we should expect since the same phenomenon appears in the flat background case. Keeping only the zero mode contribution, (39) reduces to (10) the light cone for the point particle in the same background.

4 Conclusions

We make a couple of remarks.

1. Unlike the flat case, the string light cone in the pp-wave case is not a function of \( x_i^j - y_l^i \) any more and hence translational invariance is lost. But this should not come as a surprise. The pp-wave metric is not translational invariant itself, so the same should apply to the light cone as well. This is consistent with what we found.

2. We note that in the limit \( \mu \to 0 \), we recover the flat background expression for the light cone, equation (2). This is expected since the metric (3) in the \( \mu \to 0 \) limit reduces to the Minkowski metric. For a nonzero \( \mu \) and in a small region of \( x^+ \) such that

\[ |\Delta x^+| \ll \frac{1}{\mu}, \]

we are probing the part of the spacetime that is very close to the original string itself. In this limit, spacetime curvature can be ignored. Physically one expects that the derivation from the flat case to be small and the light cone for the flat case (2) to be recovered. This is indeed the case. The light cone (39) is a continuous function of \( \mu \) and matches up with the flat case.
3. In this letter, we considered a free string. It would be interesting to include interactions (tree level and loops) and to study how the string light cone gets modified, see [3, 4] for a tree level computation in the flat case. String interaction in pp-wave background has been considered in [10], and reconsidered in [11] where the $Z_2$ symmetry of the background is taken into account properly. Field theory analysis of the $Z_2$ symmetry was recently performed in [12]. Our result agrees with the $Z_2$ symmetry of the background. It will be interesting to examine this issue by including the $Z_2$ invariant interaction.

4. Finally we note that the light cone (39) is periodic with $\Delta x^+ \sim \Delta x^+ + 2\pi/\mu$. The follows from a periodicity of the light cone time

$$x^+ \sim x^+ + 2\pi/\mu.$$ (41)

However we did not impose any periodic boundary condition on the metric to start with. Our result suggests that to have a consistent quantum theory on the pp-wave background, the background should have this periodicity (41). See [14] for related works on discrete light cone quantization (DLCQ) of string in pp-wave background. Also we do not understand yet the physical significance of the periodicity of the causal structure of spacetime. We expect this to lead to extremely interesting physics. We leave these issues for further studies.

Acknowledgements

CSC would like to thank Bin Chen, Harald Dorn, Valya Khoze, Feng-Li Lin, Boris Pioline, Simon Ross, Rodolfo Russo and Gabriele Travaglini for interesting discussions and useful comments. KK would like to acknowledge George Georgiou and Apostolos Dimitriadis for interesting discussions and useful comments. We acknowledge grants from EPSRC, Nuffield foundation of UK, NSC of Taiwan and University of Durham.

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3Note that the if one think of the pp-wave metric as arised from the Pernose limit, then by definition $x^+$ is automatically periodic with periodicity $2\pi/\mu$. CSC thanks Rodolfo Russo for bringing this very interesting observation to his attention.

4Simon Ross suggests to us that, apart from the periodicity interpretation, since (16) and (39) is singular at $\Delta x^+ = \pi/\mu$, one may argue that there is a relationship between pasts and futures shifted by $\pi/\mu$. See the first paper in [6] for more detailed discussions.
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