Optimization of Charge Pump Based on Piecewise Modeling of Output-Voltage Ripple

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Abstract: This work proposes a piecewise modeling of output-voltage ripple for linear charge pumps. The proposed modeling can obtain a more accurate design expression of power-conversion efficiency. The relationship between flying and output capacitance, as well as switching frequency and optimize power-conversion efficiency can be calculated. The proposed modeling is applied to three charge-pump circuits: 1-stage linear charge pump, dual-branch 1-stage linear charge pump and 4 × cross-coupled charge pump. Circuit-level simulation is used to verify the accuracy of proposed modeling.

Keywords: charge pump; flying and output capacitances; output-voltage ripple; power-conversion efficiency

1. Introduction

Charge pump is an important circuit in many applications such as energy harvesting, micro-sensor, flash memory and the step-up part after rectifier of wireless power transfer. There exist many variants of charge pump, for example, linear, dual-branch linear, cross-coupled, exponential, Fibonacci and Cockcroft–Walton charge pumps [1–6]. The main purposes to develop these structures are to improve the gain and power-conversion efficiency (PCE), as well as to reduce the number of switches and flying capacitors.

Studies of PCE of charge pump is an important topic and have been carried out for many decades [7–11]. Flying and output capacitances are the considerations of optimization of PCE because smaller chip size can be achieved upon minimizing on-chip flying capacitances and production cost can be reduced when smaller and fewer off-chip flying capacitors are used. Switching frequency impacts PCE and output-voltage ripple. There are many factors towards PCE optimization. Modeling of output-voltage ripple for PCE optimization is one of the approaches [7]. However, the modeling is not accurate enough and has room to further enhance. In this paper, a piecewise modeling of output-voltage ripple of linear charge pump is proposed. Based on the proposed piecewise modeling, the expressions of PCE of 1-stage linear charge pump, dual-branch 1-stage linear charge pump and 4 × cross-coupled charge pump are found. The PCE expressions can be used to select appropriate flying and output capacitances, as well as switching frequency.

2. Methods

2.1. Proposed Piecewise Modeling of Output-Voltage Ripple for 1-Stage Linear Charge Pump

The two-phase switching operation of a 1-stage linear charge pump is shown in Figure 1. Two complementary clock signals \( \Phi_1 \) and \( \Phi_2 \), which have approximately half of a switching period \( T \), are used to control the ON and OFF of switches. Deadtime is inserted in transitions between \( \Phi_1 \) and \( \Phi_2 \) to avoid short-circuit loss of the switches. \( V_i \), \( V_o \) and \( I_o \) represent input voltage, output voltage and output current. \( C_f \) and \( C_o \) are flying and output capacitances.
capacitors. Assuming the lump resistances of switches are the same, the lump sum of the resistances from switches, routing and bond-wire could be noted as $R_l$, which is around two times of a single switch. Figure 1a shows the case for $\Phi_1 = 1$ and $\Phi_2 = 0$. $V_i$ and $C_f$ are connected in series to generate approximately two times of $V_i$ for $V_o$. The corresponding modeling is shown on the right-hand side of Figure 1a. Similarly, Figure 1b shows the case for $\Phi_1 = 0$ and $\Phi_2 = 1$. $C_f$ is charged by $V_i$, while $C_o$ maintains $V_o$ and supply current to load. The right-hand-side figure of Figure 1b shows the modeling. When $T/2$ is larger than about $6R_lC_f$, the voltage across $C_f$ is close to $V_i$. However, usually $C_f$ could not be fully charged since a large $T$ leads to large output voltage drop due to the long discharging time of $C_o$.

![Figure 1](image-url)

**Figure 1.** Two-phase switching operation of 1-stage linear charge pump (a) $\Phi_1 = 1$; $\Phi_2 = 0$ (b) $\Phi_1 = 0$; $\Phi_2 = 1$.

Figure 2 shows the output-voltage ripples of a 1-stage linear charge pump. The actual output-voltage ripple is represented by the solid black line. The green line shows the modeling proposed in [7]. There is no charging time of $C_f$ and no charge re-distribution between $C_f$ and $C_o$ in this modeling so that $V_o$ is sharply increased at $nT$ in the $n$-th switching cycle. Since the average $V_o$ (i.e., $\bar{V}_o$) is used to evaluate PCE in [7], inaccurate modeling of the output-voltage ripples results in inaccurate $\bar{V}_o$ and PCE.

![Figure 2](image-url)

**Figure 2.** Output-voltage ripples of 1-stage linear charge pump.

The proposed modeling of the output-voltage ripples is shown by the red line in Figure 2. There are three segments within a switching period, and each segment is represented by a straight line. A short period of $T_x$ is used to model the situation when $V_o$ raises at $nT$. Comparing the proposed modeling with the actual output-voltage ripple shows that the proposed piecewise method can better represent the ripple voltage at output. Thus, $\bar{V}_o$ and PCE can be predicted more accurately.

Figure 3 shows more details of the output waveform and the corresponding RC circuits (from Figure 1) of a 1-stage linear charge pump. $V_o$ starts at $V_{O3}$ at $nT$ and reaches $V_{O1}$ within $T_x$. Then, it decreases from $V_{O1}$ to $V_{O2}$ within $(T/2 - T_x)$. Finally, it further decreases
from $V_{o2}$ to $V_{o3}$ to complete a cycle. More details will be provided below to investigate the up and down of $V_o$ within a switching cycle to generate the voltage ripple.

![Figure 3. Output waveform and corresponding RC circuits of 1-stage linear charge pump.](image)

In the previous cycle, $C_f$ is previously charged to $V_{Cf}$, which is close to $V_i$, while $C_o$ is lightly discharged by the load and the voltage across $C_o$ is less than $2V_i$. At $nT$, the series-connected combination of the voltage source $V_i$ and $C_f$ has a sum of voltage of $V_i + V_{Cf}$. Thus, $C_f$ is discharged itself to provide charges to $C_o$ and the load $I_o$. As the voltage across $C_o$ is increasing, $V_o$ is increased from $V_{o3}$ to $V_{o1}$, and the required time to complete this operation is $T_x$.

From the RC circuits in Figure 3, at $nT$, the voltage of $C_f$ is $V_{Cf}$, and thus the charge of $C_f$ is $C_f V_{Cf}$. Moreover, the charge of $C_o$ is $C_o V_{o3}$. Then, at $(nT + T_x)$, the voltage across $C_f$ is dropped to $(V_{o1} - V_i - I_o R_l)$, so that the charges of $C_f$ is $C_f (V_{o1} - V_i - I_o R_l)$. $I_{R1}$ has the same value as $I_o$, since the current going into $C_o$ should be zero at $(nT + T_x)$. Meanwhile, the charge of $C_o$ is $C_o V_{o1}$. The charge supplied to the load is $I_o T_x$. By principle of conservation of charges [7], the following relationship is achieved.

$$C_f V_{Cf} + C_o V_{o3} = C_f (V_{o1} - V_i - I_o R_l) + C_o V_{o1} + I_o T_x$$  \(\text{(1)}\)

$V_{Cf}$ is the voltage obtained by capacitor $C_f$ when charging with an ideal voltage supply $V_i$ within a time period $T/2$, and the initial voltage is $V_{o2}$. Therefore, $V_{Cf}$ is given by

$$V_{Cf} = \left(1 - 2e^{-\frac{T}{2RC_f}}\right)V_i + \frac{I_o C_f R_l}{C_f + C_o} e^{-\frac{T}{2RC_f}} + V_{o2} e^{-\frac{T}{2RC_f}}$$  \(\text{(2)}\)

At $(nT + T/2)$, the voltage of $C_o$ is $V_{o2}$. Since the charge redistribution of $C_f$ and $C_o$ is complete, the current passing through $C_f$ and $C_o$ is in constant ratio. Therefore, the current of $C_f$ should be $C_f I_o / (C_f + C_o)$. As a result, the following relationship is obtained.

$$C_f (V_{o1} + I_o R_l) + C_o V_{o1} = C_f \left(V_{o2} + \frac{I_o C_f R_l}{C_f + C_o}\right) + C_o V_{o2} + I_o (T/2 - T_x)$$  \(\text{(3)}\)
Between \((nT + T/2)\) and \((n + 1)T\), the voltage across \(C_o\) drops from \(V_{o2}\) to \(V_{o3}\). The change of charges of \(C_o\) is \(C_o(V_{o2} - V_{o3})\). These charges supply current to the load to give

\[
C_o(V_{o2} - V_{o3}) = \frac{I_o T}{2} \tag{4}
\]

Assume that \(T = m_1 R_1 C_f\), \(C_o = m_2 C_f\), and \(T_x = m_3 R_1 C_f\). It should be noted that \(T/2 > 6R_1(C_f/C_o)\), i.e., \(m_1 > \frac{12m_2}{m_2 + 1}\), so that the charge redistribution between \(C_f\) and \(C_o\) is complete. By solving Equations (1)–(4), \(V_{o1}, V_{o2}\) and \(V_{o3}\) can be found, respectively.

\[
V_{o1} = 2V_i - \frac{m_1 I_o R_1}{1 - e^{-m_1/2}} - \frac{I_o R_1}{m_2 + 1} - \frac{m_2 I_o R_1}{(m_2 + 1)^2} + \frac{(m_1 - 2m_3) I_o R_1}{2(m_2 + 1)} \tag{5}
\]

\[
V_{o2} = 2V_i - \frac{m_1 I_o R_1}{1 - e^{-m_1/2}} - \frac{I_o R_1}{m_2 + 1} \tag{6}
\]

\[
V_{o3} = 2V_i - \frac{m_1 I_o R_1}{1 - e^{-m_1/2}} - \frac{I_o R_1}{m_2 + 1} - \frac{m_1 I_o R_1}{2m_2} \tag{7}
\]

From Equations (5)–(7), as well as the durations of each segment within a switching cycle, the average value of \(V_o\) is found and given by:

\[
\bar{V}_o = 2V_i - \frac{m_1 I_o R_1}{1 - e^{-m_1/2}} - \frac{I_o R_1}{m_2 + 1} - \left(\frac{1}{4} + \frac{m_3}{2m_1}\right) \frac{m_1 I_o R_1}{2m_2} - \frac{m_2 I_o R_1}{4(m_2 + 1)^2} + \frac{(m_1 - 2m_3) I_o R_1}{8(m_2 + 1)} \tag{8}
\]

For \(m_3\) in Equation (8), it can be evaluated by the following. Refer to Figure 4 for the currents and voltages of \(C_f\) and \(C_o\) during the period from \(nT\) to \((nT + T_x)\), it can be found that

\[
V_i + v_c(t) = i_R(t) R_l + v_{co}(t) \tag{9}
\]

![Figure 4](image_url)

Figure 4. Currents and voltages of \(C_f\) and \(C_o\) between \(nT\) and \((nT + T_x)\).

By considering the charges in capacitors and differentiating in Equation (9) on both sides with respect to time, it gives

\[
\frac{dV_i}{dt} + \frac{1}{C_f} \frac{dQ_{cf}(t)}{dt} = R_l \frac{di_R(t)}{dt} + \frac{1}{C_o} \frac{dQ_{co}(t)}{dt} \tag{10}
\]

where \(Q_{cf}(t)\) and \(Q_{co}(t)\) are the charges stored in \(C_f\) and \(C_o\) at \(t\). From Figure 4, it can be found that \(\frac{dQ_{cf}(t)}{dt} = -i_R(t)\) and \(\frac{dQ_{co}(t)}{dt} = i_c(t) = i_R(t) - I_o\). Based on these relationships
and substituting into Equation (9), the following expression is obtained. It is noted that $\frac{dV_i}{dt} = 0$ as $V_i$ is a dc voltage.

$$\frac{di_R(t)}{dt} = -\frac{i_R(t)}{R_f} \left( \frac{1}{C_f/C_o} \right) + \frac{I_o}{R_fC_o} \tag{11}$$

Solving the above differential equation, and determining the constant of integration by the initial condition of the circuit, the expression of $i_R(t)$ is given by

$$i_R(t) = \frac{I_oC_f}{C_f + C_o} + \left( I_o - \frac{I_oC_f}{C_f + C_o} \right) e^{\frac{t}{R_f(C_f/C_o)}} \tag{12}$$

where $C_f/C_o = \frac{C_fC_o}{C_f + C_o}$. The capacitor current of $C_o$ is given by

$$i_C(t) = i_R(t) - I_o = -\frac{I_oC_f}{C_f + C_o} + \left( I_{Co0} - \frac{I_oC_f}{C_f + C_o} \right) e^{\frac{t}{R_f(C_f/C_o)}} \tag{13}$$

where $I_{Co0} = \frac{V_i + V_Cf - V_o}{R_l}$ is the initial current of $C_o$ at $nT$.

Referring to Figure 2, the peak voltage of $V_o$ (i.e., the peak voltage across $C_o$) occurs at about $(nT + T_s)$. Therefore, $T_s$ can be found by differentiating Equation (11) with respect to time to find the maximum point. As a result, $T_s$ is given by

$$T_s = R_f \left( C_o/C_f \right) \ln \left( \frac{I_{Co0} \left( C_o + C_f \right) - I_oC_f}{I_oC_o} \right) \tag{14}$$

By solving Equation (14), we have

$$m_3 = \frac{m_2}{m_2 + 1} \left( \ln(m_1) + \ln \left( 1 + \frac{1}{m_2} \right) + \ln \left( 1 + \frac{1}{2m_2} \right) \right) \tag{15}$$

Figure 5 shows the charge transfer in both phases. In Phase 2, the total charges from $C_o$ to load is $Q_a$, and so $Q_a = I_o(T/2)$. Since the output-voltage waveform is periodic, the net charges leaving $C_o$ in Phase 2 equals to the net charges inputted into $C_o$ in Phase 1. Thus, the injected charges to $C_o$ in Phase 1 is also $Q_a$. Assuming the net charges to load in Phase 1 is $Q_b$, where $Q_b = I_o(T/2)$, the charges from $C_f$ in Phase 1 becomes $(Q_a + Q_b)$. For the series connections of $V_i$ and $C_f$ in Phase 1, the charges from $V_i$ is also $(Q_a + Q_b)$ in Phase 1. Since, again, the output-voltage waveform is periodic, the net charges leaving $C_f$ in Phase 1 equals to the net charges inputted into $C_f$ in Phase 2. As such, the charges from $V_i$ to $C_f$ in Phase 2 is $(Q_a + Q_b)$. The total charges from $V_i$ is equal to $(Q_a + Q_b)$ in Phase 1 plus $(Q_a + Q_b)$ in Phase 2, which is $2(Q_a + Q_b) = 2I_oT$. Therefore, the input current ($I_i$) from $V_i$ is given by two times of $I_o$ (i.e., $I_i = 2I_o$), which is two times the load current.

The PCE of a 1-stage linear charge pump is the ratio of output power ($P_o$) to input power ($P_i$) and is given by

$$PCE = \frac{P_o}{P_i} = \frac{\overline{V_o}I_o}{\overline{V_i}I_i} = \frac{\overline{V_o}}{2\overline{V_i}} \tag{16}$$

where $\overline{V_o}$ is the expression shown in Equation (8).
2.2. Proposed Piecewise Modeling of Output-Voltage Ripple for Dual-Branch 1-Stage Linear Charge Pump/Cross-Coupled Voltage Doubler

In this section, the proposed piecewise modeling of output-voltage ripple is applied to dual-branch 1-stage linear charge pump. It is applicable to cross-coupled voltage doubler, since the ON and OFF arrangements of switches of both dual-branch 1-stage linear charge pump and cross-coupled voltage doubler are the same. Figure 6 shows the switching of a dual-branch 1-stage linear charge pump or cross-coupled voltage doubler, where $C_{fA}$ and $C_{fB}$ are flying capacitors. Similarly, $R_l$ is used to denote the lump sum of the resistances from switches, routing, and bond-wire. The parallel structure enables the load supplied by the flying and output capacitors simultaneously when $\Phi_1 = 1; \Phi_2 = 0$ and $\Phi_1 = 0; \Phi_2 = 1$, except that only $C_o$ provides charges to the load at deadtime (i.e., $\Phi_1 = 0; \Phi_2 = 0$). In fact, $T_d$ is much shorter than $T$.

![Figure 5. Charge transfer in both phases: Phase 1 (left) and Phase 2 (right).](image)

![Figure 6. Switching of dual-branch 1-stage linear charge pump/cross-coupled voltage doubler (a) $\Phi_1 = 1; \Phi_2 = 0$ (b) $\Phi_1 = 0; \Phi_2 = 1$ (c) $\Phi_1 = 0; \Phi_2 = 0$ (i.e., deadtime).](image)
Figure 6a shows the case for $\Phi_1 = 1$ and $\Phi_2 = 0$. $V_i$ and $C_{fA}$ are connected in series to generate approximately two times of $V_i$ for $V_o$, and $C_{fB}$ is charged by $V_i$. The corresponding modeling is shown on the right-hand side of Figure 6a. $R_i$, same as before, is the lump sum of the resistances from switches, routing, and bond-wire. Similarly, Figure 6b shows the case for $\Phi_1 = 0$ and $\Phi_2 = 1$. $V_i$ and $C_{fB}$ are connected in series to provide about two times of $V_i$ for $V_o$, and $C_{fA}$ is charged by $V_i$. The right-hand-side figure of Figure 6b shows the modeling. Finally, Figure 6c shows the moment of deadtime (i.e., the case for $\Phi_1 = 0$ and $\Phi_2 = 0$), where all switches are turned off. Only $C_o$ maintains about $2V_i$ and supplies charges to the load.

Figure 7 shows more details of the output waveform of one switching cycle and the corresponding RC circuits (from Figure 6) of a dual-branch 1-stage linear charge pump and cross-coupled voltage doubler. In the previous cycle, $C_{fA}$ is previously charged to $V_{Cf}$, which is close to $V_i$, while $C_o$ is lightly discharged by the load and the voltage across $C_o$ is less than $2V_i$. At $nT$, the series-connected combination of the voltage source $V_i$ and $C_{fA}$ has a sum of voltage of $V_i + V_{Cf}$. Thus, $C_{fA}$ is discharged itself to provide charges to $C_o$ and the load. $C_{fB}$ is connected with $V_i$ for re-charging. As the voltage across $C_o$ is increasing, $V_o$ is increased from $V_{o3}$ to $V_{o1}$, and the required time to complete this operation is $T_x$. After $T_x$, where the output voltage achieves the highest value, both $C_{fA}$ and $C_o$ discharge themselves to provide charges to the load. The duration is $(T/2 - T_x - T_d)$, and $V_o$ drops to $V_{o2}$ finally. At $(nT + T/2 - T_d)$, all switches are turned off in the deadtime period. Only $C_o$ supplies charges to the load. Thus, the drop of $V_o$ is more rapid than before. At $(nT + T/2)$, $V_o$ reaches $V_{o3}$ to complete half of a cycle. Between $(nT + T/2)$ and $(n + 1)T$, the operation of the first half switching cycle repeats. The only difference is that another half of the circuit enables $C_{fB}$ to supply charges to the load. In the above analysis, it is assumed that $T/2$ is longer than $6R_i(C_f/C_o)$ (where $C_f = C_{fA} = C_{fB}$) to ensure that the redistribution of $C_f$ ($C_{fA}$ and $C_{fB}$) and $C_o$ is complete when the capacitors are connected.
As shown in Figure 7, the output-voltage waveforms in the first half and second half of a switching cycle are the same. The analysis below takes a period of $T/2$ into account. $C_{fA}$ and $C_{fB}$ are considered to have the same value, such that $C_{fA} = C_{fB} = C_f$. From the RC circuits in Figure 8, at $nT$, the voltage of $C_{fA}$ is $V_{Cf}$, and thus the charge of $C_{fA}$ is $C_f V_{Cf}$. Moreover, the charge of $C_o$ is $C_o V_{o3}$. Then, at $(nT + T_x)$, the voltage across $C_{fA}$ is dropped to $(V_{o1} - V_i - I_o R_1)$, so that the charge of $C_{fA}$ is $C_{fA} (V_{o1} - V_i - I_o R_1)$. Meanwhile, the charge of $C_o$ is $C_o V_{o1}$. The charge supplied to the load is $I_o T_x$. By the principle of conservation of charges [7], the following relationships are achieved.

**Figure 8.** Switching of 2-stage cross-coupled voltage doubler (a) $\Phi_1 = 1$; $\Phi_2 = 0$ (b) $\Phi_1 = 0$; $\Phi_2 = 1$ (c) $\Phi_1 = 0$; $\Phi_2 = 0$ (i.e., deadtime).

\[
C_f V_{Cf} + C_o V_{o3} = C_f (V_{o1} - V_i - I_o R_1) + C_o V_{o1} + I_o T_x
\]  \hspace{1cm} (17)

$V_{Cf}$ is the voltage obtained by capacitor $C_{fA}$ when charging with an ideal voltage supply $V_i$ within a time period $T/2$, and the initial voltage is $V_{o2}$. Therefore, $V_{Cf}$ is given by

\[
V_{Cf} = \left(1 - 2e^{-\frac{T - 2T_x}{2C_f R}}\right)V_i + \frac{I_o C_f R_1}{C_f + C_o} e^{-\frac{T - 2T_x}{2C_f R}} + V_{o2} e^{-\frac{T - 2T_x}{2C_f R}}
\]  \hspace{1cm} (18)

At $(nT + T/2 + T_x - T_d)$, the voltage of $C_o$ is $V_{o2}$. Since the charge redistribution of $C_{fA}$ and $C_o$ is complete, the current passing through $C_{fA}$ and $C_o$ is in constant ratio. Therefore, the current of $C_{fA}$ should be $C_f / (C_f + C_o)$. As a result, the following relationship is obtained.

\[
C_f (V_{o1} + I_o R_1) + C_o V_{o1} = C_f \left( V_{o2} + \frac{I_o C_f R_1}{C_f + C_o} \right) + C_o V_{o2} + I_o (T/2 - T_x - T_d)
\]  \hspace{1cm} (19)

Between $(nT + T/2 + T_x - T_d)$ and $(nT + T/2)$, the voltage across $C_o$ drops from $V_{o2}$ to $V_{o3}$. The change of charges of $C_o$ is $C_o (V_{o2} - V_{o3})$. These charges supply current to the load to give

\[
C_o (V_{o2} - V_{o3}) = I_o T_d
\]  \hspace{1cm} (20)

Assume that $T = m_1 R_1 C_f$, $C_o = m_2 C_f$, $T_x = m_3 R_1 C_f$, and $T_d = m_4 R_1 C_f$. It should be noted that $T/2 > 6R_1 \left( C_f / C_o \right)$, i.e., $m_1 > \frac{12m_2}{m_2 + T}$, so that the charge redistribution
between $C_f$ and $C_o$ is complete. By solving Equations (17)–(20), $V_{o1}$, $V_{o2}$ and $V_{o3}$ can be found, respectively.

$$V_{o1} = 2V_i - \frac{m_1 I_o R_1}{2(1 - e^{-((m_1)/2) + m_4})} - \frac{I_o R_1}{m_2 + 1} - \frac{m_2 I_o R_1}{(m_2 + 1)^2} + \frac{(m_1 - 2m_3 - 2m_4)I_o R_1}{2(m_2 + 1)}$$

$$V_{o2} = 2V_i - \frac{m_1 I_o R_1}{2(1 - e^{-((m_1)/2) + m_4})} - \frac{I_o R_1}{m_2 + 1}$$

$$V_{o3} = 2V_i - \frac{m_1 I_o R_1}{2(1 - e^{-((m_1)/2) + m_4})} - \frac{I_o R_1}{m_2 + 1} - \frac{m_4 I_o R_1}{m_2}$$

From Equations (21)–(23), as well as the durations of each segment within half of a switching cycle, the average value of $V_o$ is found and given by

$$\overline{V_o} = 2V_i - \frac{m_1 I_o R_1}{2(1 - e^{-((m_1)/2) + m_4})} - \frac{I_o R_1}{m_2 + 1} - \frac{m_4 (m_3 + m_4) I_o R_1}{m_1 m_2} - \frac{(m_1 - 2m_3 - 2m_4) I_o R_1}{2m_1 (m_2 + 1)^2} + \frac{(m_1 - 2m_3)(m_1 - 2m_3 - 2m_4)I_o R_1}{4m_1 (m_2 + 1)}$$

The conditions to evaluate $T_x$ is same as before, and $T_x$ has the same expression as stated in Equation (13). Thus, we have:

$$m_3 = \frac{m_2}{m_2 + 1}\left[\ln(m_1) + \ln\left(1 + \frac{1}{m_2}\right) + \ln\left(\frac{1}{2} + \frac{m_4}{m_1 m_2}\right)\right]$$

Similar to the analysis of Equation (16), the input energy is given by a simple expression, as below.

$$E_i = 2V_i I_o$$

Finally, the PCE can be easily derived by the ratio of $E_o$ to $E_i$.

$$PCE = \frac{E_o}{E_i} = \frac{\overline{V_o} I_o}{2V_i I_o} = \frac{\overline{V_o}}{2V_i}$$

where $\overline{V_o}$ is found as shown in Equation (24).

### 2.3. Proposed Piecewise Modeling of Output-Voltage Ripple for 2-Stage Cross-Coupled Voltage Doubler

To verify the application of proposed piecewise modeling, the analysis is extended to 2-stage cross-coupled voltage doubler. Figure 8 shows the switching behaviors of a 2-stage cross-coupled voltage doubler. Figure 8a shows the case for $\Phi1 = 1$ and $\Phi2 = 0$, Figure 8b illustrates the condition for $\Phi1 = 0$ and $\Phi2 = 1$ and Figure 8c reveals the situation of deadtime when $\Phi1 = 0$ and $\Phi2 = 0$. Since the number of switches in each branch is different, the lump sums of the resistances from switches, routing and bond-wire are noted as $R_{i1}$, $R_{i2}$, and $R_{i3}$. It is noted again that $T_d$ is much shorter than $T$. $C_{fA1}$ and $C_{B1}$ are the flying capacitors in the first stage, and $C_{fA2}$ and $C_{B2}$ are the flying capacitors in the second stage.

Basically, the operations for $\{\Phi1 = 1$ and $\Phi2 = 0\}$ and $\{\Phi1 = 0$ and $\Phi2 = 1\}$ are the same due to the parallel structure. Thus, the corresponding modeling of both cases are the same, except different flying capacitors are used to complete the operation of the circuit. Ideally, from the switching operations, $C_{fA1}$ and $C_{B1}$ are charged to $V_i$, while $C_{fA2}$ and $C_{B2}$ are charged to $2V_i$. Therefore, $V_o$ is $4V_i$ theoretically, which is the sum of the source voltage and the voltages across $C_{fA1}$ (or $C_{B1}$) and $C_{fA2}$ (or $C_{B2}$). During the deadtime, all switches are turned off to disconnect the output from the flying capacitors and $V_i$. Thus, the load is supplied.

Figure 9 shows the details of the output waveform of one switching cycle and the corresponding RC circuits (from Figure 8) of a 2-stage cross-coupled voltage doubler. The operations at $nT$ and $(nT + T/2)$ are the same, except another half circuit operates.
alternatively. Thus, the analysis can be conducted for half of a switching cycle. At \( nT \), \( C_{FB1} \) and \( C_{FB2} \) are previously charged to \( V_{Cj} \) and about \( 2V_i \), respectively, at \((n - 1)T + T/2\) (i.e., the same operation at \((nT + T/2)\). \( C_{FB1} \) provides charges to \( C_{FA2} \) to re-charge it to about \( 2V_i \). Similarly, they supply charges to \( C_o \) such that the output increases from \( V_{o3} \) to \( V_{o1} \). The required time to complete this operation is \( T_x \). Then, at \((nT + T_x)\), the source \( V_i \), \( C_{FA2} \), \( C_{FB1} \), \( C_{FB2} \) and \( C_o \) supply charges to the load. The discharges of flying capacitors and output capacitor decrease the output from \( V_{o1} \) to \( V_{o2} \). At \((nT + T/2 - T_x)\), all switches are turned off. Only \( C_o \) supplies charges to the load, and thus the output drops more rapidly than before from \( V_{o2} \) to \( V_{o3} \). In the above analysis, it is assumed that \( T/2 \) is longer than \( 6R_iC_f \) (where \( C_f = C_{FA1} = C_{FB1} \) and \( C_f = C_{FA2} = C_{FB2} \)) to the charge redistribution during \( T \) to \( T_x \), \( T/2 \) to \( T/2 + T_x \), is complete, while \( C_{FA2} \) and \( C_{FB2} \) are charged to about \( 2V_i \) at the end of half of a switching cycle.

**Figure 9.** Output waveform and corresponding RC circuits of 2-stage cross-coupled voltage doubler.

Based on the RC circuits in Figure 9, at \((nT - T_d)\), the charge redistribution between \( C_{FB1} \), \( C_{FA2} \), and \( C_{FB2} \) is completed. Considering the voltage at the output of the first stage of charge pump is a constant value, the current passing through \( C_{FB2} \) and \( C_o \) is in constant ratio. Hence, the current passing through \( C_{FA2} \) and \( C_o \) can be approximated as \( \frac{C_{FA2}I_o}{(C_{FA2} + C_o)} \) and \( \frac{I_o}{(C_{FA2} + C_o)} \). Similarly, the current passing through \( C_{FB2} \) and \( C_{FA1} \) is \( \frac{C_{FA2}I_o}{(C_{FA2} + C_o)} \), \( \frac{C_{FA2}I_o}{(C_{FB2} + C_o)} \), \( \frac{C_{FA2}I_o}{(C_{FA2} + C_o)} \), and \( \frac{C_{FA2}I_o}{(C_{FB2} + C_o)} \), \( \frac{I_o}{(C_{FB2} + C_o)} \). At \( nT \), the charges stored in \( C_{FA2} \) and \( C_{FB1} \) are \( \frac{C_{FA2}(V_{o2} + C_{FA2}I_oR_{i2}/(C_{FA2} + C_o)) + C_{FB1}V_{Cj}}{2} \), respectively. It is noted that the negative charges at the bottom plate of \( C_{FB2} \) should also be considered. Assuming that \( C_{F1}/C_{F2} = C_{F2}/C_o = m_2 \), the highest output voltage of the first and second stage of the charge pump achieves at \( T_{x1} \) and \( T_{x2} \). According to similar
analysis of Equations (14) and (15), it could be seen that \( T_{x1} \) and \( T_{x2} \) have close values due to the log relationship of \( T_x \) and \( nT \). Hence, it is reasonable to assume that \( V_b \) and \( V_{o1} \) both achieve at \( (nT + T_x) \). Then, at \( (nT + T_x) \), the current passing through \( C_{f2} \) and \( C_o \) is 0. Therefore, the charges remaining in \( C_{f2} \) and \( C_{R1} \) and \( C_{f2} \) are \( C_{f2}V_b, C_{f2R1}(V_b - V_l + I_oR_{11}) \) and \( C_{R2}(V_{o1} - V_b + I_oR_{12}) \), respectively. By the principle of conservation of charges [7],

\[
C_{f2}(V_{o1} + I_oR_{12}\frac{C_{f2}}{C_{f2}+C_o} - V_b) + C_{f1}V_{Cf1} - C_{f2}(V_c + I_oR_{13}\frac{C_{f2}}{C_{f2}+C_o} + \frac{C_{f1}}{C_{f2}+C_o}) = C_{f2}V_b + C_{f1}(V_b - V_l + I_oR_{11}) - C_{f2}(V_{o1} - V_b + I_oR_{12})
\]

(28)

\( T_x \) could be approximated by \( T_{x2} \), which satisfies the following equation.

\[
T_x = R_{12}\left( \frac{I_{c0}}{C_o} \right) \ln \left( \frac{I_{c0}\left( C_o + C_{f2} \right) - I_oC_o}{I_oC_o} \right)
\]

(29)

with \( I_{c0} = \frac{V_{o2} - V_{o3}}{R_o} \). For simplification of the calculation, the average output voltage value for the first stage \( V_{o2} \) is approximated as \( V_c \), which could be calculated without \( T_x \) and the related calculation of \( V_c \) is given in the following part.

\( V_{Cf1} \) is the voltage obtained by capacitor \( C_{f1} \) when charging with an ideal voltage supply \( V_c \) within a time period \( T/2 \). Therefore, \( V_{Cf1} \) is given by

\[
V_{Cf} = (1 - 2e^{-\frac{T-2T_x}{2T}}) V_c + V_c e^{-\frac{T-2T_x}{2T}} + \frac{I_c{f1}C_{f2}R_{11}}{(C_{f1} + C_{f2})(C_{f2} + C_o)} e^{-\frac{T-2T_x}{2T}}
\]

(30)

At \( nT \), the charges in \( C_{R2} \) obtained before the deadline in the previous half of a cycle is \( C_{R2}[V_c + I_oR_{13}(C_{f2} + C_o)/(C_{R2} + C_{f1} + C_o)] \), and the charges in \( C_o \) is \( C_oV_{o3} \). At \( (nT + T_x) \), the charges in \( C_{R2} \) and \( C_o \) are \( C_{R2}(V_{o1} - V_b + I_oR_{12}) \) and \( C_oV_{o1} \), respectively. The charges to the load are \( I_oT_x \). Thus, the following relationship is obtained.

\[
C_{f2}V_c + C_{f1} = C_{f2}(V_{o1} - V_b + I_oR_{12}) + C_oV_{o1} + I_oT_x
\]

(31)

Between \( (nT + T_x) \) and \( (nT + T/2 - T_d) \), some charges in \( C_{f2} \) and \( C_{B1} \) go to \( C_{R2} \). Since the overall charges in the connection of three capacitors are constant, the following equation is obtained.

\[
C_{f2}V_b + C_{f1} = (V_b - V_l + I_oR_{1}) - C_{f2}(V_{o1} - V_b + I_oR_{2})
\]

\[
= C_{f2}(V_c + I_oR_{13}\frac{C_{f2}}{C_{f2}+C_o} + \frac{C_{f1}}{C_{f2}+C_o}) + C_{f1}(V_c - V_l + I_oR_{1}1\frac{C_{f2}}{C_{f2}+C_o} + \frac{C_{f1}}{C_{f2}+C_o}) - C_{f2}(V_{o2} - V_c)
\]

(32)

Moreover, the net charges of \( C_{f2} \) and \( C_o \) will supply the load so that the following expression is obtained.

\[
C_{f2}(V_{o1} - V_b + I_oR_{2}) + C_oV_{o1} = C_{f2}((V_{o2} - V_c + I_oR_{2}\frac{C_{f2}}{C_{f2}+C_o}) + C_oV_{o2} + I_o(T/2 - T_d - T_x)
\]

(33)

Finally, during the deadline, the charges from \( C_o \) supplies to the load gives the following relationship.

\[
(V_{o2} - V_{o3})C_o = I_oT_d
\]

(34)

Solving Equations (28)–(34), \( V_{o1}, V_{o2} \) and \( V_{o3} \) could be calculated, and the average value of \( V_o \) is given by

\[
\overline{V_o} = \frac{T}{T}(V_{o1} + V_{o3}) + \frac{T_d}{T}(V_{o2} + V_{o3}) + \frac{T - 2T_x - 2T_d}{2T}(V_{o1} + V_{o2})
\]

(35)
Similar to the analysis of Equation (16), PCE can be easily derived by the ratio of \( E_o \) to \( E_i \).

\[
PCE = \frac{E_o}{E_i} = \frac{V_o I_o}{4V_i I_o} = \frac{V_o}{4V_i}
\]

(36)

where \( V_o \) is found as shown in Equation (35).

3. Results

Simple simulations were conducted to prove the analysis. Ideal switch models with non-zero on-resistance are used [12]. The supply voltage \( V_i \) is 1.6 V, and the load current \( I_o \) is 50 mA.

The values of switching frequency, flying capacitors and output capacitor are chosen according to the load, which are highly related to the ripple voltage at the output. With the condition fulfilled and the assumptions presented, the proposed analysis is applicable to different values of switching frequency, flying capacitors and output capacitor. In the simulations, for 1-stage linear charge pump and dual-branch charge pump, the switch resistance is 1 \( \Omega \) and the clock frequency is 36 kHz. The capacitance of the flying capacitor \( C_f \) and the load capacitor \( C_o \) is 4.7 \( \mu F \) and 1.5 \( \mu F \), respectively. The calculation results according to [7] and this work are shown in Tables 1 and 2. The simulation is conducted by Hspice, and the result is also given in Tables 1 and 2. The error value, which is the percentage of the difference between calculation and simulation over the simulation result, is also given for better insight.

| Parameter | \[7\] | This Work | Simulation |
|-----------|----------|------------|------------|
| \( V_o \) | \( 3.02 \) | 8.72% | \( 2.77 \) | -0.10% | \( 2.77 \) |
| \( V_o \) | \( 2.90 \) | 5.90% | \( 2.74 \) | -0.05% | \( 2.74 \) |
| \( V_o \) | \( 2.44 \) | 7.11% | \( 2.28 \) | -0.05% | \( 2.28 \) |
| \( \Delta V_o \) | \( 0.58 \) | 16.14% | \( 0.49 \) | -0.34% | \( 0.50 \) |
| \( V_o \) | \( 2.82 \) | 7.98% | \( 2.57 \) | -1.56% | \( 2.61 \) |

| Parameter | \[7\] | This Work | Simulation |
|-----------|----------|------------|------------|
| \( V_o \) | \( 3.16 \) | 5.78% | \( 2.99 \) | 0.00% | \( 2.99 \) |
| \( V_o \) | \( 3.05 \) | 4.08% | \( 2.93 \) | -0.02% | \( 2.93 \) |
| \( V_o \) | \( 3.05 \) | 4.09% | \( 2.93 \) | -0.01% | \( 2.93 \) |
| \( \Delta V_o \) | \( 0.12 \) | 85.07% | \( 0.06 \) | 0.48% | \( 0.06 \) |
| \( V_o \) | \( 3.11 \) | 4.66% | \( 2.96 \) | -0.29% | \( 2.97 \) |

For 4 \( \times \) cross-coupled charge pump, the on-resistance of each switch is 0.5 \( \Omega \) and the clock frequency is 20 kHz. The capacitance of the flying capacitor \( C_{f1}, C_{f2} \), and the load capacitor \( C_o \) is 8 \( \mu F \), 4 \( \mu F \) and 2 \( \mu F \), respectively. The simulation and calculation results are shown in Table 3.
Table 3. The simulation result for 4× cross-coupled charge pump.

| Parameter | This Work | Simulation
|-----------|-----------|-----------|
|           | [7]       | Result (V) | Error | Result (V) | Error | Result (V) |
| $V_{o1}$  |           | 5.67       | 1.58% | 5.59       | 0.04% | 5.58       |
| $V_{o2}$  |           | 5.46       | 1.28% | 5.39       | −0.08% | 5.39       |
| $V_{o3}$  |           | 4.84       | −10.26% | 5.39       | −0.08% | 5.39       |
| $\Delta V_o$ |           | 0.83       | 333.78% | 0.20       | 3.44% | 0.19       |
| $V_o$     |           | 5.36       | −2.64% | 5.49       | −0.31% | 5.50       |

According to the tables, the prediction error is reduced to below 0.1% for $V_{o1}$, $V_{o2}$ and $V_{o3}$ in all three cases, which shows a more reliable method when choosing optimal capacitor values for charge pumps.

4. Conclusions

By properly modeling the charge pump circuits, this paper demonstrates a better way for analyzing charge pumps. Hspice simulation is conducted to prove the reliability of the analysis method. According to the equation given, the PCE of charge pumps is closely related to the value of clock period, lumping resistance, flying capacitor and load capacitor. Hence, optimization of charge pump is possible based on careful choosing of the component values. Additionally, the analysis could be extended to other charge-pump circuits such as exponential, Fibonacci and Cockcroft–Walton charge pumps.

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