Rescattering effects and the $\sigma$ pole in hadronic decays
I. Caprini
National Institute of Physics and Nuclear Engineering, R-077125, Bucharest, Romania

Abstract

The $\sigma$ resonance was observed as a conspicuous $\pi^+\pi^-$ peak in hadronic decays like $J/\psi \to \pi^+\pi^-\omega$ or $D^+ \to \pi^+\pi^-\pi^+$. The phase of the $\sigma \to \pi^+\pi^-$ amplitude, extracted from production data within the conventional isobar model, is assumed to coincide with that in $\pi\pi$ elastic scattering. We check the validity of this assumption by using Lehmann-Symanzik-Zimmermann (LSZ) reduction and unitarity. The rescattering effects in the final three-particle states are shown to generate a correction to the phase given by a naive application of Watson theorem. We briefly discuss the implications of this result for the pole determination from production data.

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1 Introduction

The lowest scalar resonance $\sigma$ (or $f_0(600)$) appears as a pole on the second Riemann sheet of the $I = l = 0$ partial wave amplitude of $\pi\pi$ elastic scattering (we denote this wave as $t^0_0(s)$). Although a typical resonant behaviour is not seen, because the pole is far from the real axis and is compensated in the physical region by the Adler zero, many determinations of the sigma pole are based on $\pi\pi$ scattering [1]. However, the pole was usually extracted from parametrizations valid along the physical region. The predictions are therefore affected by the large uncertainties of the analytic extrapolation to a distant point. Recently, a model-independent extrapolation into the complex plane, based on the Roy equation for $t^0_0(s)$, led to a precise prediction of the pole position [2].

The $\sigma$ resonance was also seen as a peak in BES II data on $J/\psi \to \pi^+\pi^-\omega$ [3] and in the data on $D^+ \to \pi^+\pi^-\pi^+$ reported by E791 Collaboration [4]. The conspicuous sign in production processes is explained by the absence of the Adler zero [5] [6]. It is of interest to compare the pole determinations
from production processes and $\pi\pi$ elastic scattering. In the present paper we consider some issues related to this problem.

To illustrate the discussion we consider the strong decay

$$J/\psi(p) \rightarrow \pi^+(p_1) + \pi^-(p_2) + \omega(p_3),$$

(1)

but our arguments apply also to the decay $D^+ \rightarrow \pi^+\pi^-\pi^+$, and more generally to $h \rightarrow \pi^+\pi^-h_1$, where $h$ and $h_1$ are hadrons. We define the Mandelstam variables

$$s = (p_1 + p_2)^2, \quad t = (p_1 + p_3)^2, \quad u = (p_2 + p_3)^2,$$

(2)

which satisfy $s + t + u = m_{J/\psi}^2 + 2m_\pi^2 + m_\omega^2$. The physical region of the process (1) corresponds to $s > 4m_\pi^2$, $t > (m_\pi + m_\omega)^2$ and $u > (m_\pi + m_\omega)^2$. Since some particles have nonzero spins, a decomposition in Lorentz covariants is required. For simplicity, we neglect this complication and consider an invariant amplitude $A(s, t)$ as a function of the Mandelstam variables (2).

In the conventional isobar model, the amplitude of the decay (1) is expressed as a sum of isobaric resonances in various channels$^1$. In a diagrammatic language, the three-body decay is assumed to be described by tree diagrams where the production of two-body final states proceeds via intermediate resonances. More exactly, the amplitude is written as

$$A(s, t) = A_s(s, t) + [A_t(t, s) + (t \leftrightarrow u)],$$

(3)

where the $s$-channel and $t$ ($u$)-channel amplitudes are expanded as

$$A_s(s, t) = \sum a_l(s) P_l(\cos \theta_s), \quad A_t(t, s) = \sum b_l(t) P_l(\cos \theta_t).$$

(4)

In these relations $\theta_s (= \theta_{13})$ is the angle between the three-momenta of $\pi(p_1)$ and $\omega(p_3)$ in the rest system of the two pions, and $\theta_t (= \theta_{12})$ the angle between the three-momenta of $\pi(p_1)$ and $\pi(p_2)$ in the rest system of $\pi(p_1)\omega$. The lowest partial waves in (1) are assumed to be dominated by resonances. For the process (1) the $s$-channel resonances $\sigma$, $f_0(980)$ and $f_2(1270)$ contribute to the partial waves $a_0$ and $a_2$, respectively, and $b_1^+(1235)$ appears in both $S$ and $D$-waves of the $t$ ($u$) channels. Keeping for simplicity only the contributions of $\sigma$ and $b_1$ and assuming Breit-Wigner parametrizations, one writes

$$a_0(s) = \frac{C_\sigma e^{i\Delta_0}}{m_\sigma^2 - s - im_\sigma \Gamma_\sigma(s)}, \quad b_0(t) = \frac{C_{b_1} e^{i\Delta_{b_1}}}{m_{b_1}^2 - t - im_{b_1} \Gamma_{b_1}}.$$ 

(5)

$^1$A complex constant accounting for the direct nonresonant interaction is sometimes added to the resonances (see $^4$ $^6$).
In Refs. [3, 5], \( a_0(s) \) is denoted as the \( \sigma \rightarrow \pi \pi \) amplitude.

The phases \( \Delta_{\sigma} \) and \( \Delta_{b_1} \) appearing in (5) account for the interactions of \( \sigma \omega \) and \( \pi b_1 \), respectively. In the conventional isobar model [3, 4], these phases are assumed to be independent on the Mandelstam variables. Moreover, by invoking Watson theorem [7], the phase of the \( \sigma \rightarrow \pi \pi \) amplitude \( a_0(s) \) was assumed [5, 6] to coincide, up to a constant, with the pion-pion phase shift \( \delta_0^0 \) appearing in the expression of the \( l = I = 0 \) partial wave:

\[
t^0_0(s) = \frac{1}{2i\rho(s)} \{\eta_0^0(s)e^{2i\delta_0^0(s)} - 1\},
\]

where \( \rho(s) = \sqrt{1 - 4m^2_\pi/s} \). An equivalent formulation is to assume [6] that the denominator of the function \( a_0(s) \) given in (5) coincides with the function \( D(s) \), appearing in the \( N/D \) method [8] for calculating the amplitude \( t^0_0(s) \).

The purpose of this letter is to check the validity of Watson theorem in the isobar model for decay processes. We recall that the theoretical difficulties of the three-particle decays are known since a long time. Anomalous singularities generated by rescattering effects and three-body dispersion relations were considered by several authors (see [9, 10] and older references quoted therein). In the present paper we investigate the phases of the amplitudes defined in the isobar model, using an approach based on LSZ reduction and unitarity.

2 LSZ reduction and unitarity

We start from the \( S \)-matrix element of the process [11]

\[
S_{fi} = \langle \pi(p_1) \pi(p_2) \omega(p_3); \text{out} | J/\psi(p); \text{in} \rangle.
\]

After the LSZ reduction [11, 12] of the \( \omega \) meson we obtain

\[
S_{fi} = \delta_{fi} + \frac{i}{\sqrt{2p_{3,0}}} \int dx e^{ip_{3,0}x} \langle \pi(p_1) \pi(p_2); \text{out} | \eta_\omega(x) | J/\psi(p); \text{in} \rangle,
\]

where \( p_{3,0} \) is the time component of \( p_3 \) and \( \eta_\omega = K_\omega \phi_\omega(x) \) denotes the source operator (\( K_\omega \) is the Klein-Gordon operator and \( \phi_\omega \) the interpolating field of the omega meson). In what follows we do not need the explicit expressions of the sources, but only the significance of the matrix elements involving them.
Using translational invariance \( \eta_\omega(x) = e^{iP \cdot x}\eta_\omega(0)e^{-iP \cdot x} \) where \( P \) denote
the momentum operator, we write (8) as

\[
S_{fi} = \delta_{fi} + \frac{i}{\sqrt{2p_{3,0}}} (2\pi)^{4}\delta(p_1 + p_2 + p_3 - p) \langle \pi(p_1) \pi(p_2); \text{out} | \eta_\omega(0) | J/\psi(p); \text{in} \rangle.
\]

From the general expression of the \( S \)-matrix in terms of the invariant amplitude \( A(s,t) \), it follows that

\[
A(s,t) = \frac{\mathcal{N}}{\sqrt{2p_{3,0}}} \langle \pi(p_1) | \pi(p_2); \text{out} | \eta_\omega(0) | J/\psi(p); \text{in} \rangle.
\]

where \( \mathcal{N} = 4\sqrt{p_{0}p_{1,0}p_{2,0}p_{3,0}} \) is a normalization factor. In the same way we express the invariant amplitude \( T(s,t) \) of the elastic scattering \( \pi(k_1) + \pi(k_2) \rightarrow \pi(p_1) + \pi(p_2) \) as

\[
T(s,t') = \frac{\mathcal{N}'}{\sqrt{2p_{2,0}}} \langle \pi(p_1) | \eta_{\pi_2}(0) | \pi(k_1) \pi(k_2); \text{in} \rangle,
\]

where \( \mathcal{N}' = 4\sqrt{p_{1,0}p_{2,0}k_{1,0}k_{2,0}} \), and the physical domain in the \( s \) channel is defined by \( s = (p_1 + p_2)^2 = (k_1 + k_2)^2 > 4m_\pi^2, t' = (p_1 - k_1)^2 < 0 \).

By applying once more the LSZ reduction to the matrix element (10), we obtain:

\[
A(s,t) = \frac{\mathcal{N}i}{\sqrt{2p_{3,0}2p_{2,0}}} \int dx e^{ip_2 \cdot x} \theta(x_0) \langle \pi(p_1) | [\eta_{\pi_2}(x), \eta_\omega(0)] | J/\psi(p); \text{in} \rangle,
\]

where \( \eta_{\pi_2}(x) \) is the source of the final pion \( \pi(p_2) \).

As it is known, the LSZ formalism allows the analytic continuation of the amplitude \( A(s,t) \) in the complex planes of the Mandelstam variables, where the expression (12) defines a holomorphic function (an important ingredient in the proof is causality, i.e. the fact that the retardd commutator vanishes for spacelike values \( x_2^2 < 0 \)). In what follows we only use the LSZ representation to derive the unitarity relation and explore its consequences.

By inserting a complete set of states \( |n\rangle \) in the two terms of the retarded commutator, the matrix element appearing in (12) writes as

\[
\sum_n [(\pi(p_1) | \eta_{\pi_2}(x) | n) \langle n | \eta_\omega(0) | J/\psi \rangle - \langle \pi(p_1) | \eta_\omega(0) | n \rangle \langle n | \eta_{\pi_2}(x) | J/\psi \rangle].
\]

In the two particle approximation, the lowest states which contribute in the first sum are \( |n\rangle = |\pi(k_{1})\pi(k_{2})\rangle \), where the two pions have \( I = 0 \), while
in the second sum \(|n⟩ = |π(k_1)ω(k_2)⟩\). The sum over intermediate states involves integrations upon the momenta of the on-shell particles and sums over polarizations. After imposing the translation invariance, we obtain from (12):

\[
A(s, t) = i\mathcal{N} \int dxθ(x_0) \int \frac{dk_1}{(2\pi)^3} \frac{dk_2}{(2\pi)^3} e^{i p_{12} \cdot x + ip_{13} \cdot x - ik_1 \cdot x - ik_2 \cdot x} \\
\times \frac{⟨π(p_1)|η_{π_2}(0)|π(k_1)π(k_2)⟩⟨π(k_1)π(k_2)|η_ω(0)|J/ψ(p)⟩}{\sqrt{2p_{20}}} \frac{⟨π(k_1)ω(k_2)|η_{π_2}(0)|J/ψ(p)⟩}{\sqrt{2p_{30}}} - i\mathcal{N} \int dxθ(x_0) \int \frac{dk_1}{(2\pi)^3} \frac{dk_2}{(2\pi)^3} e^{i p_{12} \cdot (x-x_0) + ik_1 \cdot x + ik_2 \cdot x} \\
\times \frac{⟨π(p_1)|η_ω(0)|π(k_1)ω(k_2)⟩⟨π(k_1)ω(k_2)|η_{π_2}(0)|J/ψ(p)⟩}{\sqrt{2p_{20}}}.
\]

(14)

The presence of \(θ(x_0)\) in the integral of Eq. (14) leads to discontinuities of the amplitude across the real axis of the Mandelstam variables [11, 12]. According to the general prescription, the discontinuity is obtained formally from the expression (14) through the replacement of \(iθ(x_0)\) by \(1/2\) [12]. Then the integral upon \(x\) gives \((2\pi)^4δ(p_1 + p_2 - k_1 - k_2)\) in the first term, and \((2\pi)^4δ(p - p_2 - k_1 - k_2)\) in the second. Moreover, in the first term of (14) we recognize the product of the amplitudes of the processes \(π(k_1) + π(k_2) → π(p_1) + π(p_2)\) and \(J/ψ(p) → π(k_1) + π(k_2) + ω(p_3)\), while in the second term appears the product of the amplitudes of the processes \(π(k_1) + ω(k_2) → π(p_1) + ω(p_3)\) and \(J/ψ(p) → π(p_2) + π(k_1) + ω(k_2)\).

From this discussion, it follows that the amplitude \(A(s, t)\) has branch cuts for \(s > 4m_π^2\) and \(t > (m_π + m_ω)^2\). By reducing the pion \(π(p_1)\) instead of \(π(p_2)\) one obtains the branch cut for \(u > (m_π + m_ω)^2\). Hence, in the physical region of the decay (1) all the variables \(s, t\) and \(u\) are above the unitarity thresholds. Singularities in both the \(s\) and \(t\) variables are expected to occur in each of the two terms in (14). We recall that in the isobar model defined in (3)-(4), the term \(A_i(s, t)\) has singularities only in the \(s\) variable, being holomorphic with respect to \(t\), while \(A_t(t, s)\) has singularities only in \(t\), being regular with respect to \(s\). This shows the limitation of the isobar model.

For definiteness we consider that the complete set of intermediate states are “out” states and recall that for one particle states the “in” and “out” sets are equivalent. Recalling the definitions (10) and (11) of the invariant amplitudes and focusing on the first term in (14), responsible for the discontinuity.
with respect to the variable \( s \), we obtain
\[
\frac{1}{2i} \{ A(s + i\epsilon, t) - A(s - i\epsilon, t) \} = \frac{1}{8\pi^2} \int \frac{dk_1}{2k_{1,0}} \frac{dk_2}{2k_{2,0}} \delta(P) T^*(s, t') A(s, t'') \quad (15)
\]
where \( P = p_1 + p_2 - k_1 - k_2 \). The amplitudes are evaluated for \( s = (p_1 + p_2)^2 = (k_1 + k_2)^2 \) and the momentum transfers \( t' = (p_1 - k_1)^2 \) and \( t'' = (k_1 + p_3)^2 \), respectively. The integral (15) is easily evaluated in the c.m.s. of the two pions \( p_1 + p_2 = k_1 + k_2 = 0 \). After the trivial integrations due to the delta functions, (15) reduces to an integral upon the angular variables:
\[
\frac{1}{2i} \{ A(s + i\epsilon, t) - A(s - i\epsilon, t) \} = \frac{1}{64\pi^2} \int d\Omega \rho(s) T^*(s, t') A(s, t'') \quad (16)
\]
where \( d\Omega = d\phi d\cos \theta'' \), \( \theta'' \) being the angle between the three momenta of \( \pi(k_1) \) and \( \omega(p_3) \) in the pion rest system. For the \( \pi\pi \) isoscalar amplitude \( T(s, t') \) we use the Legendre expansion
\[
T(s, t') = 16\pi \sum_{l=0}^{\infty} (2l + 1) P_l(\cos \theta') t^0_l(s) \quad (17)
\]
where \( \theta' \) is the angle between the three momenta \( p_1 \) and \( k_1 \). We stress that (16) is a general unitarity relation, independent of the isobar model. Let us restrict now to this model, taking for the amplitude \( A(s, t'') \) in (16) the expression given in (3)-(4). Using the well known relation \( 8 \)
\[
\cos \theta'' = \cos \theta \cos \theta' + \cos \phi \sin \theta \sin \theta' \quad (18)
\]
the integral upon the angle \( \phi \) is trivial. Recalling that only the first term in (3) has a discontinuity for \( s > 4m_{\pi}^2 \), projecting onto the \( S \)-wave and using the orthogonality of Legendre polynomials we obtain
\[
\frac{1}{2i} \{ a_0(s + i\epsilon) - a_0(s - i\epsilon) \} = \rho(s) (t^0_0(s))^* a_0(s) \\
+ \rho(s) \left[ t^0_0(s) \right]^* \int_{-1}^{1} d\cos \theta'' [A_i(t'', s) + A_u(u'', s)] . \quad (19)
\]
We recall that time reversal invariance implies the reality relation \( a_0(s - i\epsilon) = a_0^*(s + i\epsilon) \), from which it follows that the l.h.s. of (19) is real and equal to \( \text{Im} \ a_0(s) \) (if not otherwise specified, \( s \) is taken on the upper edge of the cut).
3 Watson theorem

Neglecting the four-pion channel which opens very slowly, the elastic region
extends up to the threshold for \( K \bar{K} \) creation. Below this threshold \( \eta_0^2(0) = 1 \),
therefore the amplitude (3) becomes \( t_0^0 = e^{i\delta_0^0} \sin \delta_0^0/\rho(s) \). If we neglect the
second term in the r.h.s. of (19) we obtain
\[
\frac{1}{2i} \{ a_0(s + i\epsilon) - a_0(s - i\epsilon) \} = e^{-i\delta_0^0(s)} \sin \delta_0^0(s) \ a_0(s). \tag{20}
\]
This relation implies \( a_0(s + i\epsilon) = a_0(s - i\epsilon)e^{2i\delta_0^0(s)} \), which is equivalent to
Watson theorem: the phase of \( a_0(s) \) is equal (modulo \( \pm \pi \)) to the phase shift \( \delta_0^0 \). Alternatively, writing \[8\]
\[
t_0^0(s) = \frac{N(s)}{D(s)}, \tag{21}
\]
where \( N(s) \) has only a left hand cut for \( s < 0 \) and \( D(s) \) a right hand cut for
\( s > 4m^2_\pi \), a solution of (20) has the form:
\[
a_0(s) = \frac{C(s)}{D(s)}, \tag{22}
\]
where the function \( C(s) \), real for \( s > 4m^2_\pi \), is arbitrary. By the uniqueness of
analytic continuation, this implies that \( t_0^0(s) \) and the function \( a_0(s) \) have the
same poles on the second sheet. We will come back on this point in the next
section.

If the second term in the r.h.s. of (19) is not neglected we obtain, instead
of (20), the more general relation
\[
\text{Im} \ a_0(s) = e^{-i\delta_0^0(s)} \sin \delta_0^0(s) [a_0(s) + h(s)], \tag{23}
\]
where
\[
h(s) = \frac{1}{2} \int_{-1}^{1} d \cos \theta' \left[ A_l(t', s) + [A_u(u', s)] \right]. \tag{24}
\]
We recall that the angles \( \theta_s \) and \( \theta_t \) in the expansions (4) are expressed in
terms of the Mandelstam variables, for instance:
\[
\cos \theta_s = \frac{m_\omega^2 + m_\pi^2 + \sqrt{|p|^2 + m_\omega^2} \sqrt{s - t}}{2|p|\sqrt{s/4 - m_\pi^2}}, \tag{25}
\]
where $|p| = \lambda^{1/2}(s, m_{J/\psi}^2, m_\omega^2)/(2\sqrt{s})$ is the three momentum of $J/\psi$ ($\omega$) in the rest system of the pions (here $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$). Using these relations and retaining in the expansion (4) of $A_t + A_u$ only the $S$-wave $b_0$, parametrized as in (5), we have:

$$h(s) \sim \frac{C_{b_0} e^{i\Delta b_1}}{2|p||k_2|} \ln \left[ 1 + \frac{4|p||k_2|}{m_{b_1}^2 - m_\omega^2 - m_\pi^2 - i\Gamma_{b_1}m_{b_1} - p_{3,0}\sqrt{s} - 2|p||k_2|} \right],$$

(26)

where $|k_2| = \sqrt{s/4 - m_\pi^2}$ and $p_{3,0} = \sqrt{|p|^2 + m_\pi^2}$, with $|p|$ defined below (24). If the isobar model contains also a nonresonant term [4, 6], the function $h(s)$ will include its contribution.

From Eq. (23) we can calculate a correction to Watson theorem. To this end we impose time reversal invariance, which means that the r.h.s. of (23) must be real. By requiring that the imaginary part vanishes, we obtain

$$\sin[\Phi(s) - \delta_0^0(s)] = -\frac{\text{Im} \left[ e^{-i\delta_0^0(s)}h(s) \right]}{|a_0(s)|},$$

(27)

where $\Phi(s)$ is the phase of the production amplitude:

$$a_0(s) = |a_0(s)|e^{i\Phi(s)}.$$  

(28)

The relation (27) gives a calculable correction to the phase predicted by the naive application of Watson theorem in the elastic region. Above the inelastic threshold $s = 4m_K^2$, the elasticity $\eta_0^0(s)$ in (6) drops very quickly below unity, and additional terms due to the $K\bar{K}$ intermediate states appear in the r.h.s. of (19). Since the unitarity sums for the scattering and the decay processes contain different contributions, the phase of $a_0(s)$ in the inelastic region may be quite different from $\delta_0^0(s)$.

### 4 Comments

The above analysis shows that the phase of the $\sigma \rightarrow \pi\pi$ amplitude $a_0(s)$, defined in the conventional isobar model for hadronic decays, is not exactly equal to the $\pi\pi$ phase shift, as one would think by a naive application of Watson theorem. In the isobar model, the complex constants multiplying the Breit-Wigner resonances describe the interaction of a resonance ($\sigma$) with the third hadron ($\omega$). The $s$-dependent correction $\Phi(s) - \delta_0^0(s)$ calculated above...
is generated by the individual interactions with $\omega$ of each of the outgoing pions. Actually, the rescattering effect discussed above can be visualized by a triangular diagram, given in Fig. 1 of Ref. [9]. As shown in [9], this diagram is responsible for the appearance of anomalous singularities. In the present work we emphasized the influence of the rescattering effects on the phase of the $\sigma \rightarrow \pi \pi$ amplitude defined within the isobar model.

We notice that, if the total amplitude $A(s, t'')$ appearing in the r.h.s. of the unitarity relation [16] could be expanded in a series of Legendre polynomials $P_l(\cos \theta_s)$, the standard evaluation of the integral [8] would lead to Watson theorem for each partial wave $a_l(s)$. In the case of the elastic $\pi \pi$ scattering (or in decays like $K \rightarrow \pi \pi l \nu$) such an expansion is legitimate, since in the physical region of the $s$-channel the amplitude is a holomorphic function of $t$. On the other hand, for three-body decays like (1) a similar expansion is not possible, since $P_l(\cos \theta_s)$, which are polynomials of $t$, fail to reproduce the branch cut along $t > (m_\pi + m_\omega)^2$. The isobar model attempts to take into account the singularities in all channels, but, as we discussed above, it is too simplistic. As a consequence, the phase of the $\sigma \rightarrow \pi \pi$ amplitude $a_0(s)$ defined in this model is not exactly equal to the phase-shift $\delta_0^0(s)$.

From (27) it follows that the magnitude of the phase difference $\Phi(s) - \delta_0^0(s)$ depends on the values of the parameters of the isobar model (the ratio $C_{b_1}/C_\sigma$ and the difference $\Delta_{b_1} - \Delta_\sigma$). Since an overall constant phase is irrelevant, what really matters is the variation with $s$. The difference $\Phi(s) - \delta_0^0(s)$ might be smaller than the experimental errors. However, it is important to emphasize that even a small phase difference may have an important influence on the pole determination. Indeed, an immediate consequence of our result is that the denominator of the function $a_0(s)$ in [5] should not be identical to the denominator $D(s)$ appearing in the expression (21) of $t_0^0(s)$. In Refs. [3, 5] the $\sigma$ pole is extracted from a parametrization of the denominator of $a_0(s)$ in the physical region, supposed to be valid also in the complex plane. As mentioned in the Introduction, such a method is affected by the uncertainties of analytic continuation, which are large for a distant pole. If, in addition, the denominator of $a_0(s)$ and the denominator of the $\pi \pi$ amplitude $t_0^0(s)$ differ in principle (even slightly) along the physical region, the position of the pole determined by the analytic continuation of the production data may be even

2For the decay $D^+ \rightarrow \pi^+ \pi^- \pi^+$, where the statistics is rather low, fits of equal quality were obtained both with a phase of $a_0(s)$ very different from the $\pi \pi$ phase-shift [4], and with a phase close to $\delta_0^0$ [6].
more distorted. The effect discussed above might play a role in understanding the difference between the mass and width of the lowest scalar resonance $\sigma$ extracted from the BES data for $J/\psi \rightarrow \pi^+\pi^-\omega$ decay $^{[3, 5]}$, and the values derived from Roy equation for the $I = l = 0$ elastic $\pi\pi$ amplitude $^{[2]}$.

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