Transiting Planets Orbiting Source Stars in Microlensing Events

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ABSTRACT

The phenomenon of microlensing has successfully been used to detect extrasolar planets. By observing characteristic, rare deviations in the gravitational microlensing light curve one can discover that a lens is a star–planet system. In this paper we consider an opposite case where the lens is a single star and the source has a transiting planetary companion. We have studied the light curve of a source star with transiting companion magnified during microlensing event. Our model shows that in dense stellar fields, in which blending is significant, the light drop generated by transits is greater near the maximum of microlensing, which makes it easier to detect. We derive the probability for the detection of a planetary transit in a microlensed source to be of $2 \times 10^{-6}$ for an individual microlensing event.

Key words: planetary systems – Gravitational lensing: micro

1. Introduction

The search for extrasolar planets is a very dynamically developing branch of modern astrophysics. After the discovery of the first planet (Wolszczan and Frail 1992), and the detection of the first planet orbiting a solar-type, main sequence star (Mayor and Queloz 1995) several planet detecting methods have been developed. There are two natural phenomena, which can be used for planetary detection, namely gravitational microlensing and transits. In this work we consider the advantages of both phenomena occurring together – the case of gravitational microlensing of a transiting planetary system (hereafter MiTr).

First planetary transit was detected in 1999 (Charbonneau et al. 2000), but it was only a confirmation of the existence of a planet – HD 209458b had already been detected by the means of the radial velocity measurements. First planet ever discovered with the transit method was OGLE-TR-56b (Udalski et al. 2002b, Konacki
et al. 2003). Since then, the transit method has been very successful and today it is one of the most effective methods of discovering extrasolar planets.

The most problematic issue in planetary transits detection are the objects mimicking such transits while being of completely different origin. Many false-positives are observed in very crowded fields, in which blending with neighboring stars is very common. If there is a star behind or in front of an eclipsing binary star, eclipses are shallowed and hence the main eclipse can look similar to a planetary transit (secondary eclipse is then lost in the noise). Another example of false-positives are binaries generating grazing eclipses (which can be as shallow as planetary transits) and binary star systems in which one component is much smaller than the other. In this work we consider the first one, the most common case of false-positives: blended binary stars. If such object is microlensed, it is possible to derive blending parameter and to answer the question whether the drop of light during a transit is caused by a planet.

Gravitational microlensing phenomena can be directly used for exoplanets search (Mao and Paczyński 1991). When a lens consists of two components (for example a star and a planet), it is possible to detect specific peaks on regular microlensing light curves. Observing such phenomena can provide the mass ratio of the components, which is crucial to determine whether it is a planetary system or a binary star. Microlensing phenomena are incredibly rare, because almost perfect alignment of three objects has to occur. Therefore observations must be conducted in very dense stellar fields, most of all toward the Galactic bulge, to increase chances of detecting microlensing events. For example, the OGLE-IV survey currently detects about 2000 microlensing events every year, among which at least a dozen shows planetary signatures (e.g., Poleski et al. 2014).

In this paper we simulate the case, where a magnified source hosts a planet. We consider a single point-like lens and derive the probability of such configuration to happen and be detectable in currently on-going microlensing surveys. First study of the microlensed planetary transit was conducted by Lewis (2001), however they only considered an influence of transits on the caustic crossing binary lens events.

The paper is organized as follows. In Section 2 we describe the model of planetary transit and its microlensing as well as the model for the accuracy of the photometry. Then in Section 3 we describe the results of our simulations and derive the probability of the microlensed transit source. We summarize the results in Section 4.

2. Model

We consider a situation in which the light from a star (hence a source star), which is being transited by a planet, is amplified due to microlensing by a single lensing object.
2.1. Planetary Transit

Planetary transit is a straightforward geometrical problem, however, a few simplifying approximations are needed. First of all, we assume that planet moves along a straight line. In fact, its path is an ellipse, but compared to the whole period, a transit usually lasts short enough for this assumption to be reasonable. We also consider only circular orbits of planets, because transit method is sensitive to planets orbiting very close to their parent stars. Such configuration causes strong tidal effects and hence, circularization of the orbit.

We then consider the following surface brightness $I(r)$ model, which includes simple approximation of limb darkening (Heyrovsky 2007):

$$I(r) = I_0 \left( 1 - \Gamma \left( 1 - \sqrt{\frac{R_\star^2 - r^2}{R_\star^2}} \right) \right).$$ (1)

Intensity of radiation $I$ depends on the distance $r$ from the star center, stellar radius $R_\star$ and limb darkening coefficient $\Gamma$. The latter is defined by the surface luminosity values on the center and on the limb:

$$\Gamma = \frac{(I(0) - I(R_\star))}{I(0)}.$$ (2)

Value of this coefficient varies from 0 (for the star disk equally luminous from center to the edge) to 1 (for the star disk which surface brightness is zero on the edge). We assume that stars and planets are perfectly spherical – we do not take into account the fact that rotating spherical bodies are typically flattened on their poles.

It is obvious that the apparent luminosity drop during a transit depends on the ratio of angular sizes of the planet and the star, which does not depend on the distance to the system. Thus, it does not matter whether a planet is orbiting 0.01 a.u., 5 a.u., or 10 a.u. from its parent star – depth of transit will be always the same (although semi-major axis has an indirect influence on transit’s duration).

Our model of a transit has four main parameters: stellar radius $R_\star$, planetary radius $R_p$, orbital period $P$ and semi-major axis $a$. Apart from those there are also other parameters like blending parameter $f$, limb darkening coefficient $\Gamma$ and orbital inclination $i$. The flux of radiation, when neglecting transits, is given by:

$$F_{\text{max}} = F_1 + F_{\text{bl}}$$ (3)

where $F_1$ is the flux of the star, while $F_{\text{bl}}$ is the flux of the third light (blend). Now we can define time-dependent flux of the system during the transit:

$$F_u(t) = F_{\text{max}} - \Delta F(t),$$ (4)

$\Delta F(t)$ is the flux deficit caused by the passage of a planet in front of the disk of the star. If surface brightness of the star $I_1$ were constant, we would have the following
formula for the flux deficit:

\[ \Delta F(t) = I_1 \pi R_p^2 \alpha(t) = F_1 \left( \frac{R_p}{R_*} \right)^2 \alpha(t), \] (5)

\( \alpha(t) \) is the ratio between the overlapping areas of the stellar and planetary disks to the whole planetary disk area. Including the limb darkening from (1) gives:

\[ \Delta F(t) = \int \int_{S(t)} I(r) \, dS. \] (6)

Integration is done over the surface \( S(t) = \pi R_p^2 \alpha(t) \). The calculations of \( \Delta F(t) \) are time consuming, since Monte Carlo method is used in the surface integrals.

Now, we convert the depth of a transit to magnitudes, as a function of time:

\[ \Delta m_{\text{tr}}(t) = m_{\text{tr}}(t) - m_{\text{max}} = -2.5 \log \left( \frac{F_{\text{tr}}}{F_{\text{max}}} \right) = -2.5 \log \left( 1 - f \frac{\Delta F(t)}{F_1} \right) \] (7)

where \( f \) is the blending parameter, defined as

\[ f = \frac{F_1}{F_1 + F_{\text{bl}}}. \] (8)

Blending parameter varies from 0 to 1. For \( f = 1 \) the contribution of blending light to the flux is negligible, while \( f \to 0 \) means that the blend is dominating. Simulation of a transit is done by calculating \( \Delta m \) for many consecutive instants of time, corresponding to changing positions of the planet.

Eventually, we obtain the depth of a transit as a function of time and seven parameters \((R_*, R_p, P, a, f, \Gamma, i)\). We can now create synthetic light curves showing the apparent brightness decrease during the transit. Fig. 1 shows examples of synthetic light curves for different sets of initial parameters. One can notice the influence of the blending parameter on the shape of the light curve. For \( f < 1 \) (\( f = 0.5 \) here) the transit is shallowed, which is crucial for our further considerations. If we observed a blended eclipsing binary stars system, which generates light drop much greater than planetary transit, such shallowing can make this binary star to mimic star–planet system.

2.2. Microlensing of a Transit

We have derived the formula for the flux \( F_{\text{tr}} \) of the system during planetary passage in front of the stellar disk (4), and its flux \( F_{\text{max}} \) outside the transit (3). Let us now add microlensing to the picture:

\[ F_{\text{max}}(t) = A(t) F_1 + F_{\text{bl}} = F_1 \left( A(t) + \frac{1}{f} - 1 \right), \] (9)

\[ F_{\text{tr}}(t) = F_{\text{max}} - A(t) \Delta F(t). \] (10)
Fig. 1. Shape of a planetary transit. Red curve (solid) presents the simplest simulation: inclination \( i = 90^\circ \), without limb darkening and blending. The green one (dashed) includes limb darkening, while the blue curve (dash-dotted) shows the influence of blending. The black double-dotted line presents all the effects together for a case of inclination of \( 85^\circ \).

Function \( A(t) \) describes amplification due to gravitational microlensing.

The amplification in the parametrization of Paczyński (1996) is given by:

\[
A(t) = \frac{u^2(t) + 2}{u(t)\sqrt{u^2(t) + 4}}
\]

(11)

where \( u \) is the source-lens distance in units of the Einstein radius, projected on the lens plane. The value of \( u \) varies due to the relative motion of the source and the lens, with \( u = u_0 \) at the closest approach of these two objects. We can describe parameter \( u \) using Einstein time \( t_E \) and \( u_0 \):

\[
u(t) = \sqrt{u_0^2 + \left(\frac{t - t_0}{t_E}\right)^2}.
\]

(12)

Parameter \( t_0 \) denotes the moment of the highest amplification, when \( u = u_0 \). We choose \( t_0 = 0 \) in the following analysis.

Microlensing alters the expression for the depth of transit:

\[
\Delta m(t) = -2.5 \log \left( \frac{F_{\text{max}}(t)}{F_{\text{tr}}(t)} \right) = -2.5 \log \left( \frac{A(t)F_1 + F_{\text{bl}}}{A(t)F_1 + F_{\text{bl}} - A(t)\Delta F(t)} \right).
\]

(13)
After rewriting $F_{bd}$ using the blending parameter $f$ we obtain the final formula for the depth of the transit:

$$\Delta m(t) = -2.5\log \left( \frac{A(t) + \frac{1}{f} - 1}{A(t) + \frac{1}{f} - 1 - A(t) \frac{\Delta F(t)}{F_1}} \right). \quad (14)$$

Finally, the light curve including effects of microlensing and transits is given by:

$$m(t) = m_{\text{max}} + \Delta m(t)$$

while for transits alone it is $m_\text{tr}(t) = m_{\text{max}} + \Delta m_\text{tr}(t)$ (cf. Eq. 7). Fig. 2 shows an example of a light curve generated by our model.

![Fig. 2. Exemplary MiTr phenomenon. Light curve was generated for the following set of initial parameters: $u_0 = 0.1$, $f = 1$, $\Gamma = 0.3$, $R_* = 1 \, R_\odot$, $R_p = 0.6 \, R_\odot$, $T = 15 \, \text{d}$, $a = 0.02 \, \text{a.u.}$, $I_0 = 15 \, \text{mag}$, $t_E = 28 \, \text{d}$. This set was selected so that the characteristic shape of the MiTr light curve was clearly seen on the plot. Therefore the drop of brightness is much greater than typical caused by a planetary transit.](image)

2.3. Accuracy of the Photometry

To derive constraints on the detection of microlensed transiting sources, we need to know how precisely we can measure depth of transits. In this work we assume that uncertainties of the photometry are similar to those in the Galactic bulge fields data of the OGLE-III project (Udalski et al. 2002a). In that survey the
typical uncertainty for a 15 mag source was about 0.005 mag. In order to scale the error-bar with the brightness during its change in a microlensing event we use the empirical formula from Wyrzykowski (2005) (see also Wyrzykowski et al. 2009):

\[
\Delta I = \Delta I_0 10^{0.33875(I-I_0)}
\]  

(16)

where \( \Delta I \) is the uncertainty of the brightness \( I \) and \( I_0 \) is a normalizing brightness for which \( \Delta I_0 \) is known. Using the fact that for \( I_0 = 15 \) mag \( \Delta I_0 = 0.005 \) mag, we can compute the error-bars for any simulated magnitude. Top panel of Fig. 3 shows how \( \Delta I \) changes in a microlensing event with maximum amplification at \( t = 0 \).

![Graph showing changes in brightness uncertainty with time](image)

**Fig. 3.** Top: Changes of the uncertainty of photometric measurements during the MiTr phenomenon. Initial parameters describing microlensing events were \( u_0 = 0.1, t_0 = 0 \) d, \( t_E = 28 \) d, \( I_0 = 15 \) mag. Significant blending has been added (\( f = 0.1 \)). Bottom: Changes in transits depths during the microlensing event. This light curve was generated for the same parameters as above.

In the case of a blended event with the source exhibiting variability in the form of transits, the amplitude of that variability will increase with the amplification (Wyrzykowski et al. 2006). In the limit of infinite amplification, the amplitude of the variability reaches its completely de-blended value. In other words, during the microlensing we can measure the depth of the transit as if we used a much larger telescope with much higher spatial resolution and no blending from nearby stars nor the lens. In reality, the amplitude changes with amplification, depending on the amount of blending. Bottom panel of Fig. 3 shows how the amplitude of transit varies with amplification for maximum amplification of \( A \approx 10 \) (\( u_0 = 0.1 \)) and
blending parameter $f = 0.1$. The change in amplitude combined with the increase of accuracy of the photometry during a microlensing event are the basis of our argument for a feasibility of the detection of a planet transiting the source during microlensing event.

3. Results

3.1. Simulations of MiTrs

We simulated microlensed transits for a range of parameters of the planetary systems and microlensing events. The values for microlensing parameters were drawn in each simulation from the distributions obtained for the 3500 standard microlensing events found in the OGLE-III data (Wyrzykowski et al. in preparation), providing realistic statistics of parameters for microlensing events toward the Galactic bulge.

Fig. 4 shows examples of simulated microlensing events with planetary transits for a selection of interesting combinations of their parameters. The planet radius is arbitrarily set to $R_p = 1 R_{\text{Jup}}$. We show the synthetic light curve of the event in each

![Fig. 4. Three simulations of interesting cases of MiTr phenomena for Hot Jupiter orbiting a small star. The following initial parameters were applied here: $R_\ast = 0.75 R_\odot$, $R_p = 1 R_{\text{Jup}}$, $T = 4$ d, $a = 0.05$ a.u., $I_0 = 15$ mag, $t_E = 28$ d, $i = 90^\circ$. Top panels show synthetic light curves for three different sets of blending and microlensing parameters. On the left both microlensing magnification and blending are negligible ($u_0 = 1$, $f = 1$), in the middle panels, microlensing magnification is small ($u_0 = 1$), but blending is significant ($f = 0.15$). Right panel presents the most interesting case: both blending and microlensing magnification are significant ($u_0 = 0.1$, $f = 0.15$). Bottom panels show depth of transits (green solid lines) and the level of three times uncertainty of the photometry (blue dashed lines), calculated for the same values of $f$ and $u_0$ as for light curve above. We are unable to detect a transit in the case of a strong blending (middle panels), but if the magnification is strong enough the transit can be detectable near the maximum (bottom right panel). Note that there are different vertical scales in the upper panels.](image-url)
top panel. The comparison of the depth of microlensed transits to the photometric uncertainty, $3\Delta I$, is presented in the bottom panel. One can easily see when the transits become detectable – once their depth exceeds the typical error-bar of the measurement (set as three standard deviations).

Left panels of Fig. 4 show the most common situation in sparse fields: in case of very little blending (i.e., when in the baseline light is composed only of the source and its transits) the microlensing amplification only shifts the brightness to a higher level of brightness, increasing slightly the precision of the photometry. Middle panels of Fig. 4. show the case when the source star with transits is severely blended and the transits are shallowed to the limit beyond detectability. However, once microlensed, the transits become significantly deeper due to the fact that the source star becomes much brighter and starts to dominate over the blending objects (right panels of Fig. 4).

### 3.2. Probability of Detection in Microlensing Surveys

It is obvious that microlensing of a planetary transit is an extremely rare phenomenon. Here we estimate the probability $P_{\text{mitr}}$ for a detection of the microlensed source with planetary transits among all detected microlensing events. Again, we only consider here observations toward the Galactic bulge (and in particular the Baade’s Window), because only such dense fields provide a significant number of microlensing events.

First component of the overall probability is the probability that a source hosts a transiting planet ($P_{\text{tr}}$). For Hot Jupiters (HJs) considered here we assumed $P_{\text{tr}} = 1/310$ as calculated by Gould et al. (2006), based on the observational data. This is the probability derived for Galactic disk stars, while our simulations are performed for the Galactic bulge. The rate of transiting HJs for the central part of our Galaxy is probably different than calculated by Gould et al. (2006). Even though, we use those calculations as a reasonable approximation of $P_{\text{tr}}$ in the Galactic bulge.

Second component is $P_{\text{det}}$ – a probability that a MiTr event will have at least two detectable transits. We used our model of the MiTr to derive this probability in the following way. We drew the brightness in the baseline $I_0$, blending parameter $f$, impact parameter $u_0$ and the event time scale $t_E$ from the observational distribution of microlensing events as found in the OGLE-III data (Wyrzykowski et al. in preparation). Then, from the Besançon model of the Galaxy (Robin et al. 2004), we obtained the relation between the radius of the star $R_*$ and its brightness $I_s$, where $I_s$ is the source brightness in the microlensing event derived from $I_0$ and $f$. We only selected stars belonging to the dominant bulge population, to assure we probe the most likely population of the source stars. Combing all those parameters allowed us to simulate a MiTr event. The simulation was performed with fixed period $P = 4$ d, $a = 0.05$ a.u. and $R_p = 1 R_{\text{Jup}}$, as common parameters for HJ planet population. For each set of parameters, probability was calculated for different phases of transits and averaged. Using the MiTr model we generated
10,000 microlensing events with varying microlensing and stellar radii parameters. In each event we computed the depth of the transit for the maximum amplification and compared it to the expected photometric error-bars ($3\Delta I$), thus obtaining a probability of detection of each case.

Last point to take into account is the fact that in a blended microlensing event in the Galactic bulge there are on average 2.3 stars which could potentially be microlensed in a typical OGLE-like resolution image and with OGLE-like sensitivity (Wyrzykowski 2005, Kozłowski 2007, Kozłowski private communication 2013). Every time we simulate a microlensing of an object which consists of a few blended stars, we assume, that microlensing is related to the one with a transiting companion. In reality, each component of blended light source has the same probability to be magnified by means of microlensing. Thus the $P_{bl} = 1/2.3$ factor has to be included in the overall probability. We need to account for all those sources with detectable transits within the blend which do not end up to be microlensed.

The final probability for occurrence of a detectable planetary transit of HJ in a microlensing event is given by the product $P_{mitr} = P_{det} \times P_{tr} \times P_{bl}$. For HJs, considered in our simulations, we obtain the probability of $P_{mitr} = 2 \cdot 10^{-6}$. Hence, we should detect approximately two MiTr of HJs among every million microlensing events. With the current rate of microlensing events detections of about 2000 per year (OGLE), it is very unlikely that MiTr event will be found soon. OGLE survey so far, during all its phases, have found about 10,000 microlensing events (with nearly 6000 during OGLE-IV alone), hence there is probably no microlensed transit present in the archival data. Nevertheless, a possibility of seeing MiTr in the well sampled future microlensing events, should be considered when, for example, analyzing an anomaly at the top of a microlensing event.

4. Summary and Conclusions

In this paper we presented theoretical predictions about the microlensing of the source hosting a transiting planet. One of the main properties of this novel method for finding planets is a capability of breaking the common degeneracy in the planetary radius estimate due to the third light in eclipsing binaries. We show that microlensing in crowded fields can bring up the transits which otherwise remain buried in the photometric noise thanks to two combined effects: increase in brightness, hence improvement in measurement accuracy, and increase in the transit depth due to de-blending of the lensed source. We estimated the probability of such a phenomenon to $2 \cdot 10^{-6}$ in a survey with properties similar to the OGLE project. This result means that most likely such an interesting phenomenon has not yet been observed, and will not be observed in the near future. Though our simulations yield somewhat negative result, information that MiTr signal is most likely not present in the data could be useful, i.e., for eliminating potential sources of unknown irregularities in light curves of microlensing events.
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