Survey article

A new perspective on the role of mathematics in medicine

Ahmed I. Zayed

Department of Mathematical Sciences, DePaul University, Chicago, IL 60614, USA

HIGHLIGHTS

- The article gives a brief account of the development of mathematics and its relationship with practical applications.
- This is an expository article that sheds light on the role of mathematics in medical imaging.
- It traces the development of CT scan from infancy to the present.
- It reports on new advances in MRI technology.
- Mathematical concepts explained in non-technical terms.

GRAPHICAL ABSTRACT

The Radon transform is the mathematical basis of computer tomography.

ARTICLE INFO

Article history:
Received 22 October 2018
Revised 25 January 2019
Accepted 26 January 2019
Available online 6 February 2019

Keywords:
Computed tomography
CT scan
Radon transform
Magnetic Resonance Imaging (MRI)
Compressed sensing

ABSTRACT

The aim of this expository article is to shed light on the role that mathematics plays in the advancement of medicine. Many of the technological advances that physicians use every day are products of concerted efforts of scientists, engineers, and mathematicians. One of the ubiquitous applications of mathematics in medicine is the use of probability and statistics in validating the effectiveness of new drugs, or procedures, or estimating the survival rate of cancer patients undergoing certain treatments. Setting this aside, there are important but less known applications of mathematics in medicine. The goal of the article is to highlight some of these applications using as simple mathematical formulations as possible. The focus is on the role of mathematics in medical imaging, in particular, in CT scans and MRI.

Introduction

The subject of this expository article was motivated by discussions I have had with some of my colleagues who are renowned professors at medical and engineering schools in Egypt. I was intrigued but not totally surprised to know that most of them were not aware of the role that mathematics has played in their fields whether in statistical analysis or even more importantly in the advancement of the technologies that they use every day. Mathematics for them is just an abstract and dry subject that you study to become a teacher, or if you are lucky, you become a university professor in the faculty of science or engineering.
My target audience is physicians who do not have advanced background in mathematics but are interested in learning more about the role that mathematics plays in medical imaging. To this end, I have intentionally written the article as an expository article and not as a research one. For those who would like to learn more about the mathematical formulation involved, they may consult the references at the end of the article for details.

Mathematics, which comes from the Greek Word "μαθήματα", meaning subject of study, is one of the oldest subjects known to mankind. Its history goes back thousands of years. Archaeological discoveries indicated that people of the Old Stone Age as early as 30,000 B.C could count. Mathematics, which, in the early days meant arithmetic and geometry, was invented to solve practical problems. Arithmetic was needed to count livestock, compute transactions in trading and bartering, and in making calendars, while geometry was needed in setting boundaries of fields and properties and in the construction of buildings and temples.

The Ancient Egyptians, Babylonians, and Mayan Indians of Central America developed their own number systems and were able to solve simple equations. While the Ancient Egyptians' number system was decimal, i.e., counting by powers of 10, the Babylonian's system used powers of 60, and the Mayans' system used powers of 20. The decimal system is the most commonly used system nowadays.

In those early civilizations deriving formulas and proving results were not common. For example, the Ancient Egyptians knew of and used the famous Pythagorean Theorem for right-angle triangles, but did not provide proof of it. A more striking example is the formula for the volume of a truncated pyramid, which was inscribed on the Moscow Papyrus but without proof, so how the Ancient Egyptians obtained that formula is still a mystery.

The nature of mathematics changed with the rise of the Greek civilization and the emergence of the Library of Alexandria where Greek scholars and philosophers came to pursue their study and contribute to the intellectual atmosphere that prevailed in Alexandria. The master and one of the most genius minds of all times was Euclid who taught and founded a school in Alexandria (circa 300 B.C.). He wrote a book "Elements of Geometry," also known as the "Elements," which consisted of 13 volumes and in which he laid the foundation of mathematics as we know it today.

In Euclid's view, mathematics is based on three components: definitions, postulates, and rules of logic, and everything else, lemmas, propositions, and theorems are derived from these components. The notion of seeking after knowledge for its own sake, which was completely alien to older civilizations, began to emerge. As a result, the Greeks transformed mathematics and viewed it as an intellectual subject to be pursued regardless of its utility.

This new way of thinking about mathematics has become the norm and continued until now. The nineteenth and twentieth centuries witnessed the rise of abstract fields of mathematics, such as abstract algebra, topology, category theory, differential geometry, etc. Mathematicians focused on advancing the knowledge in their fields regardless of whether their work had any applications. In fact, some zealot mathematicians bragged that their work was intellectually beautiful but had no applications. A prominent representative of this group was the British mathematician, Godfrey Harold Hardy (1877–1947), one of the most renowned mathematicians of the twentieth century, who once said [2].

"I have never done anything "useful". No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world."

Ironically, in 1908 one of Hardy’s contributions to mathematics turned out to be useful in genetics and had a law named after him “Hardy’s Law.” It dealt with the proportions in which dominant and recessive Mendelian characters would be transmitted in a large mixed population. The law proved to be of central importance in the study of Rh-blood-groups and the treatment of haemolytic disease of the newborn.

Here we should distinguish between two different but closely related branches of mathematics: applied mathematics and pure mathematics. Applied mathematics deals with real-world problems and phenomena and try to model them by equations and formulas to better understand them and manage or predict them more efficiently. Pure mathematics, on the other hand, and contrary to the common belief, does not only deal with numbers. It deals with abstract entities and tries to find relations between them and patterns and structures for them, and generalize them whenever possible. The British philosopher, logician, and mathematician, Bertrand Russell (1872–1970) described mathematics in a philosophical and somewhat sarcastic way as.

"Mathematics maybe defined as the subject in which we never know what we are talking about, nor whether what we are saying is true."

Nevertheless, there is a plethora of examples of useful and practical applications that came out of the clouds of abstract mathematics, such as Hardy’s work mentioned above and the work of John Nash (1928–2015) on Game Theory which earned him the Nobel Prize in Economics in 1994.

In the next sections we will see two other examples of ideas from pure mathematics that turned out to be useful in medicine. I will try not to delve into technicality and keep the presentation as simple and non-technical as possible, but some mathematical formulations will be introduced for those who are interested, but which the non-expert can skip. This approach may lead me to give loose interpretations or explanation of some facts for which I apologize. The focus will be on two techniques in medical imaging.

Mathematics and Computed Tomography (CT) scan

The most common application of mathematics in medicine and pharmacology is probability and statistics where, for example, the effectiveness of new drugs or medical procedures is validated by statistical analysis before they are approved, for example, in the United States by the Food and Drug Administration (FDA). But in this short article I will try to shed light on other applications of pure mathematics, in particular, on two recent technological advances in the medical field that probably would not have existed without the help of mathematics.

The first story is about CT scan, known as computed tomography scan, or sometimes is also called CAT scan, for computerized axial tomography or computer aided tomography. The word tomography is derived from the old Greek word “τομή = tomos”, meaning “slice or section” and “γραφή = grapho”, meaning “to write.”. Medical imaging is about taking pictures and seeing inside of the human body without incisions or having to cut it to see what is inside. What is an image? or more precisely what is a black

---

1 It is called the Moscow Papyrus because it repos in the Pushkin State Museum of Fine Arts in Moscow.

2 The branch of mathematics that deals with numbers and their properties is called Number Theory.

3 Bertrand Russell was a writer, philosopher, logician, and an anti-war activist. He discovered a paradox in set theory which was named after him as Russell’s Paradox. He received the Nobel Prize in Literature in 1950.

4 John Nash was considered a mathematical genius. He received his Ph.D. from Princeton University in 1950 and later became a professor of mathematics at MIT. He suffered from mental illness in midst of his career and was the subject of the movie “A Beautiful Mind”.

5 Harold’s law was first named the Hardy’s Law.
and white digital image? A digital image or a picture is a collection of points, called pixels and is usually denoted by two coordinates \((x, y)\), and each pixel has light intensity, called gray level, ranging from white to black.

Mathematically speaking, a black and white picture is a function \(f(x, y)\) that assigns to each pixel some number corresponding to its gray level. In the 1920s, pictures were coded using five distinct levels of gray resulting in low quality pictures. Nowadays, the number of gray levels is an integer power of 2, that is \(2^k\) for some positive integer \(k\). The standard now is 8-bit images, that is \(2^8 = 256\) levels of gray, with 0 for white and 255 levels or shades of gray. An image with many variations in the gray levels tends to be sharper than an image with small variations in the gray scale. The latter tends to be dull and washed out [3].

One of the oldest techniques used in medical imaging is X-rays where the patient is placed between an X-ray source and a film sensitive to X-ray energy, but in digital radiography the film is digitized or the X-rays after passing through the patient are captured by a digital devise. The intensity of the X-rays changes as they pass through the patient and fall on the film or the devise. Another medical application of X-ray technology is in Angiography where an X-ray contrast medium is injected into the patient through a catheter which enhances the image of the blood vessels and enables the radiologist to see any blockage. X-rays are also used in industry and in screening passengers and luggage at airports.

But more modern and sophisticated machines than X-ray machines are the CT scanners which produce 3-dimensional images of organs inside the human body. How do they work and what is the story behind them?

The first CT scanner invented by Allen Cormack and Godfrey Hounsfield in 1963 had a single X-ray source and a detector which moved in parallel and rotated during the scanning process. This technique has been replaced by what is called a fan-beam scanner in which the source runs on a circle around the body firing a fan (or a cone) of X-rays which are received after they pass through the body by an array of detectors in the form of a ring encircling the patient and concentric with the source ring. The process is repeated and the data is collected and processed by a computer to construct an image that represents a slice of the object. The object is slowly moved in a direction perpendicular to the ring of detectors producing a set of slices of the object which, when put together, constitute a three-dimensional image of the object.

Recall that a black and white image is just a function \(f(x, y)\) defined on pixels. In a standard college calculus course, students are taught how to integrate functions and, with little effort, they can integrate functions along straight lines. The integral of a function along a straight line in some sense measures a weighted average of the function along that line. But a more interesting and much more challenging mathematical problem is the inverse problem, that is suppose that we know the line integrals of a function \(f(x, y)\) along all possible straight lines, can we construct \(f(x, y)\)?

When an X-ray beam passes through an object lying perpendicularly to the beam path, the detector records the attenuation of the beam through the ray path which is caused by the tissues’ absorption of the X-rays. What the detector records is proportional to a line integral along that path of the function \(f(x, y)\) that represents the X-ray attenuation coefficient of the tissue at the point \((x, y)\). If we rotate the beam around the object, the detector will measure the line integrals of the function \(f(x, y)\) from all possible directions. Now the following question immediately arises: can we construct \(f(x, y)\) from its line integrals? Since \(f(x, y)\) represents, in some sense, the image of the cross section of the object, that question is equivalent to asking whether we can construct the image of the cross section of the object from the data that the detector compiled. This question was the starting point for Allen Cormack, one of the inventors of the CT scanner.

In 1956, Allen Cormack, a young South African physicist, was appointed at the Radiology Department at the Groote Schuur hospital, the teaching hospital for the University of Cape Town’s medical school. This hospital later became the site of the world’s first heart transplant. Cormack took on himself, as one of his first duties at the new job, the task of finding a set of maps of absorption coefficients for different sections of the human body.

The results of the task would make X-ray radiotherapy treatments more efficient. He soon realized that what he needed to complete his task was measurements of the absorption of X-rays along lines in thin sections of the body. Since the logarithm of the ratio of incident to emergent X-ray intensities along a given line is just the line integral of the absorption coefficient along that line, the problem mathematically was equivalent to finding a function \(f(x, y)\) from the values of its integrals along all or some lines in the plane [4].

“This struck me as a typical nineteenth century piece of mathematics which a Cauchy or a Riemann might have dished off in a light moment, but a diligent search of standard texts on analysis failed to reveal it, so I had to solve the problem myself,” says Cormack. “I still felt that the problem must have been solved, so I contacted mathematicians on three continents to see if they knew about it, but to no avail” adds Cormack [4].

A few years later, Cormack immigrated to the United States and became a naturalized citizen. Because of the demands of his new position, he had to pursue his problem part time as a hobby. But by 1963 he had already found three alternative forms of solutions to the problem and published his results. He contacted some research hospitals and groups, like NASA, to see if his work would be useful to them but received little or no response.

Cormack continued working on some generalizations of his problem, such as recovering a function from its line integrals along circles through the origin. Because there was almost no response to his publications, or at least that was what he thought, Cormack felt somewhat disappointed and forgot about the problem for a while. By a mere accident, Cormack discovered that his mathematical results were a special case of a more general result by Johann Radon, in which Radon introduced an integral transform and its inverse and showed how one could construct a two-dimensional function \(f(x, y)\) from its line integrals. Even more, he showed how one can reconstruct an \(n\)-dimensional function from its integrals over hyper-planes of dimension \(n - 1\). That integral transform is now called the Radon transform. The transform and its inverse are the essence of the mathematical theory behind CT scans.

As is often the case with many beautiful and significant mathematical discoveries, the Radon transform was discovered and went unnoticed for very many years. And when it was rediscovered, it was rediscovered independently by several people in different fields. The Radon transformation without doubt is one of the most versatile function transformations. Its applications are numerous and its scope is immense. Chief among its applications are computed tomography (CT) and nuclear magnetic resonance (NMR). Not only the transform, but also its history deserves a great deal of attention.

---

1. Augustin Louis Cauchy (1789–1857) was one of the leading French mathematicians of the 19th century who contributed significantly to several branches of mathematics, in particular, to mathematical analysis.
2. Bernhard Riemann (1826–1866) was one of the best German mathematicians of his era. Many mathematical concepts and results were named after him, such as Riemann integrals, Riemann surfaces, and the famous Riemann Hypothesis which is still an open question.
So, who is Radon? and what is the significance of his work? Johann Radon was born on December 16th, 1887 and died on May 25th, 1956. He was an Austrian professor of mathematics who worked at different universities in Austria and Germany but his final destination was at the Institute of Mathematics at the University of Vienna where he was appointed professor in 1946. He later became a dean and the rector at the University of Vienna.

The saga of the Radon transform began in 1917 with the publication of Johann Radon’s seminal paper [5] “Über die Bestimmung Von Funktionen durch ihre Integralwerte längs gewisser Mannigfaltigkeiten,” or “On the determination of functions from their integrals along certain manifolds.” At that time, Radon was an assistant at the University of Vienna where he was appointed professor in 1946. His final destination was at the Institute of Mathematics at the University of Vienna where he worked at different universities in Austria and Germany but his last destination was at the Institute of Mathematics at the University of Vienna.

In that paper Radon demonstrated how one could reconstruct a function of two variables from its integrals over all straight lines in the plane. He also discussed other generalizations of this problem, for example, reconstructing a function from its integrals over other smooth curves, as well as, reconstructing a function of n variables from its integrals over all hyper-planes.

One of the beauties and strength of mathematics may be gleaned from the following examples. We cannot visualize objects in dimensions higher than three, nevertheless, Radon’s result shows that we can theoretically construct images of n dimensional objects, which are functions of n variables, if we know their integrals over hyper-planes of dimension n−1.

Although his paper had some direct ramifications on solutions of hyperbolic partial differential equations with constant coefficients, it did not receive much attention even from Radon’s colleagues at the University of Vienna. This may be attributed to World War I and the turmoil that permeated the political atmosphere in Europe during that period. It should be emphasized that Radon did not have any applications in mind and probably never imagined that his work would be used in saving lives 50 years later.

In the late 1960s, at the Central Research Laboratories of a company called Electrical and Musical Industries (EMI), best known as publisher of the Beatles records, Godfrey Hounsfield, a British engineer, used some of Cormack’s ideas to develop a new X-ray machine that revolutionized the field of medical imaging. Soon after that Cormack and Hounsfield joined forces and collaborated in refining the invention and developing the CT-scanning technique. Although the first image obtained by CT scan took hours to process, it was the beginning of a new and remarkable invention.

The work of Hounsfield and Cormack culminated in their receiving the 1979 Nobel Prize in Physiology or Medicine. In their Nobel Prize addresses they acknowledged the work of other pioneers in the field, in particular, the work of Radon in 1917.

The next few paragraphs are written for those who have some mathematical knowledge and interested in knowing some of the mathematical formulation of the Radon transform, but the non-experts can skip this part and go to the next section.

The inversion of the Radon transform is clearly equivalent to the problem of reconstructing a function f from the values of its line integrals. If we write the equation of a straight line $L$ in the form $p = x \cos \phi + y \sin \phi$, where $p$ is the length of the normal from the origin to $L$ and $\phi$ is the positive angle that the normal makes with the positive x-axis (see Fig. 1), then the Radon transform of $f$ can be written in the form

$$ R[f](p, \phi) = \int_{L} f(x,y) \, ds, $$

where $ds$ is the arc length along $L$.

If we rotate the coordinate system by an angle $\phi$, and label the new axes by $p$ and $s$, then $x = p \cos \phi - s \sin \phi$, $y = p \sin \phi + s \cos \phi$, and $f^\dagger(p, \phi)$ takes the form

$$ f^\dagger(p, \phi) = \int_{-\infty}^{\infty} f(p \cos \phi - s \sin \phi, p \sin \phi + s \cos \phi) \, ds. \quad (1) $$

Formula (1) is practical to use in two dimensions; however, it does not lend itself easily to higher dimensions. To generalize it to higher dimensions, let us introduce the unit vectors $\xi = (\cos \phi, \sin \phi)$ and $\xi^\perp = (-\sin \phi, \cos \phi)$, so that $x = (x,y) = (r, \theta) = p_\xi + t\xi^\perp$, for some scalar parameter $t$, where $r$ and $\theta$ are the polar coordinates. The equation of the line $L$ can now be written in terms of the unit vector $\xi$ as $p = \xi \cdot x$, where the · denotes the scalar product of vectors. We may write

$$ R[f^\dagger](p, \xi) = \int_{-\infty}^{\infty} f p_\xi + t\xi^\perp \, dt = \int_{-\infty}^{\infty} f(x) \delta(p - \xi \cdot x) \, dx, $$

where $dx = dx dy$, and $\delta$ is the delta function. Using the last representation of the Radon transform in two-dimensions, we can now extend it to $n$ dimensions as

$$ R[f^\dagger](p, \xi) = \int_{-\infty}^{\infty} f(x) \delta(p - \xi \cdot x) \, dx, $$

where $p = x \cdot \xi = x_1 \xi_1 + \cdots + x_n \xi_n$ is a hyperplane and $x = (x_1, \ldots, x_n) \in \mathbb{R}^n, n \geq 2, dx = dx_1 \cdots dx_n$, and $\xi$ is a unit vector in $\mathbb{R}^n$.

What is more important for applications is the inverse Radon transform which unfortunately is too complicated to be stated here, but the interested reader can consult [6,7] for details. It is worth noting that the inversion formula depends on the dimension of the space; there is one formula for even dimensions and another for odd dimensions.

What is new in MRI?

Magnetic Resonance Imaging (MRI) is another technology used in medical imaging to do different tasks, such as angiography and dynamic heart imaging. It is based on the interaction of a strong magnetic field with the hydrogen nuclei contained in the body’s water molecules. It uses strong magnetic filed and radio waves to construct images of the body from signals that are detected by sensors.

One of the advantages of MRI over CT scanning is that it does not involve X-rays or radiation and it produces images of soft tissues that X-ray CT cannot resolve without radiation. But on the other hand, some of its disadvantages are: it takes longer to perform, it is louder, and it subjects the patient to be confined into a narrow tube which is problematic for patients who are claustrophobic. MRI researchers always wanted to speed up the scanning process but did not know how. Mathematics of compressed sensing and high-dimensional geometry showed them how.

Recent advances in mathematics research have led to great improvements in the design of MRI scanners. In the next few paragraphs I will try to explain in non-technical terms, as much as possible, the mathematics behind these advances.

Suppose we have an image or an audio signal represented by measured numerical data. These data can be arranged in a row or a column consisting of $n$ entries, which in mathematics is called a vector with $n$ components, where $n$ is usually a large number. Let us denote the original (input) signal by $x$. After the signal is processed by a machine, the output signal, which will be denoted by $y$, is received by the detector. In general, $y$ may not have the same number of components as $x$; therefore, let us assume that $y$ has $m$ components. This operation is presented mathematically by the equation $y = Ax$, where $A$ is an $m \times n$ rectangular array of numbers.
calculated matrix. An important problem is the following: can we recover $x$ from the received (output) data $y$?

In a standard college course on linear algebra students are taught that if $m$ is bigger than or equal to $n$, i.e. $n \leq m$, $x$ can be recovered but sometimes the answer is not unique. In other words, if the received signal contains more data (some of which maybe redundant) than the input signal, the original signal may be recovered. However, if $m$ is smaller than $n$, i.e. $m < n$, it is impossible to recover $x$.

In the last few years, a new field of research in mathematics, called compressed sensing, gained much popularity and led to surprising results. In essence, it shows that, under certain conditions, and in some cases even if $m$ is up to about 12% of $n$, one can recover the input signal $x$ or a very good approximation thereof. The underlying condition is the sparsity of $x$. A signal is called sparse or compressible if most of its components are zeros. Many real-world signals are compressible and several techniques used in computer technology depend on this assumption, such as JPEG and MP3. JPEG is the most commonly used image format that is used by digital cameras and used to upload or download images to and from the internet. Likewise, MP3 is the audio coding format used for storing and transmitting digital audio signals.

If we denote the number of non-zero components of $x$ by $s$, then clearly $s$ is smaller than $n$, but the real difficulty is we do not know off-hand the location of the non-zero components of $x$. The non-zero components are the components that carry the essential information in $x$. The real challenge lies in the construction of the matrix $A$ and the algorithms used to reconstruct $x$ or a good approximation thereof from the received data $y$. Because $y$ has fewer components than $x$, this process is called data compression.

With the help of the theory of random matrices, it has been shown that using certain type of random matrices $A$, all $s$-sparse vectors $x$ can be reconstructed with high accuracy, provided that $m$ is chosen such that

$$Cs \ln \left( \frac{n}{s} \right) \leq m,$$

where $C$ is a constant independent of $s, m, n$, and “$\ln$” stands for the natural logarithm [8].

The reader may wonder how a small amount of data can be useful in signal recovery. An example of that can be seen in DNA profiling. Although 99.9% of human DNA sequences are the same in every person, enough of the DNA is different that it is possible to distinguish one individual from another. DNA profiling is commonly used in parentage testing and as a forensic technique in criminal investigations. For example, comparing one individuals’ DNA profile to DNA found at a crime scene ascertains the likelihood of that individual being involved in the crime.

Around 2004, four mathematicians David Donoho and his former Ph.D. student, Emmanuel Candès, Stanford University, together with Justin Romberg, Georgia Institute of Technology, and Terence Tao, University of California, Los Angeles, laid the foundations of this interesting topic, compressed sensing. Compressed sensing, which is at the intersection of mathematics, engineering, and computer science, has revolutionized signal acquisition by enabling complex signals and images to be recovered with very good precision using a small number of data points. Realizing the potential of the utility of compressed sensing, MRI researchers worked diligently to derive new algorithms to speed up the scanning time.

In 2009, a group of researchers at Lucille Packard Children’s Hospital in Stanford, California, showed that pediatric MRI scan times could be reduced in certain tasks from 8 min to 70 s. This promising result showed the potential impact of compressed sensing on MRI technology. Compressed sensing accelerates the scanning time which saves time and cost by allowing health care providers to deliver the same service to more patients in the same amount of time. In addition, children can now undergo MR imaging without sedation – they need to sit still for 1 min rather than 10 min. Cardiologists can see in detail the motions of muscle tissue in the beating heart [9].

In June 28, 2017, Donoho gave a congressional briefing on Capital Hill. In his presentation before a subcommittee of the United States Congress Donoho explained the impact of compressed sensing on MRI research that resulted in accelerating the scanning time. He then argued that increasing federal support for basic scientific research is a good investment for the federal government because it will lead to better use of tax-payer money by reducing the medical expenses in different programs paid for by the government.

It is estimated that 40 million scans are performed yearly in the United States. Diagnostic imaging costs US$ 100 billion yearly and MR imaging makes up a big share of that. Tens of millions of MRI scans that are performed annually can soon be sped up dramatically. Recently, FDA approved technologies that would accelerate 3-dimensional imaging by eight times and dynamic heart imaging by 16 times [9].

The work of Donoho, Candès, Romberg and Tao, was refined by Donoho and culminated in Donoho’s receiving the prestigious Gaus Prize at the International Congress of Mathematicians, Rio de Janeiro, Brazil, August 1–10, 2018, for his contribution to compressed sensors. In his award citation [10], it was noted that his research revolutionized MRI scanning through the application of his findings. MRI scans can now be effectively carried out in a fraction of the time they had previously taken. Precision MRI scans that once took 6 min can now be carried out in 25 s. This is particularly significant for elderly patients with respiratory issues who may have difficulty holding their breath during scans, and for children, who have a tendency to fidget, and are often unable to stay still for long.

Three of the biggest scanner manufacturers, GE, Siemens, and Philips, now use technologies based on his work. Siemens’s new technology allows movies of the beating heart and GE’s technology allows rapid 3-dimensional imaging of the brain. Both companies claim that their products use compressed sensing.

**Conclusions**

The purpose of this expository article was to shed light on the role that mathematics plays in the advancement of medicine. The focus was on medical imaging, in particular, on CT scan and MRI. It was shown that the mathematical theory of the CT scan was founded on the Radon transform which was introduced by the Austrian mathematician Johann Radon in 1917 and who apparently had no particular application in mind. We also briefly discussed recent results in mathematics, such as compressed sensing that
has led to speeding up the MRI scanning time significantly. In conclusion, I hope that I was able to convince the reader that sometimes pure mathematics research may produce useful and practical applications, some of which may lead to new innovations and even life-saving technologies.

Future perspectives

The future of medical imaging is very promising. Many technological and theoretical techniques are being developed to revolutionize the field. Numerical algorithms are being developed to speed up the scanning process and newer machines are designed to implement them and produce more efficient and better images. Because the subject of medical imaging is so rich and wide, it is hard for a short article like this one to cover all its aspects. For example, we have not discussed helical computed tomography in which X-ray machines scan the body in a spiral path which allows more images to be made in shorter time than in parallel scanning, nor have we delved into the subject of positron emission tomography, also known as a PET scan, which when combined with a CT or MRI scan, it can produce 3-D multidimensional, color images of the inside of the human body. We hope that these topics will be the subject of a future survey article.

Conflict of interest

The authors have declared no conflict of interest.

Compliance with Ethics Requirements

This article does not contain any studies with human or animal subjects.

References

[1] Burton D. The history of mathematics: an introduction. New York: McGraw Hill; 2007.
[2] Titchmarsh EC, Godfrey Harold Hardy. J. Lond. Math. Soc. 1995;25:81–101.
[3] Gonzalez R, Woods R. Digital image processing. New Jersey: Prentice Hall; 2002.
[4] Gindikin S, Michor P. 75 years of radon transform. Cambridge, MA: International Press Publications; 1994.
[5] Radon J. Über die Bestimmung Von Funktionen durch ihre Integralwerte langs gewisser Mannigfaltigkeiten. Berichte Sächsische Akademie der Wissenschaften, Leipzig, Math.- Phys. Kl. 1917;69:262–7.
[6] Natterer F. The mathematics of computerized tomography. New York: John Wiley & Sons; 1986.
[7] Zayed A. Handbook of function and generalized function transformations. Boca Raton, FL: CRC Press; 1996.
[8] Foucart S, Rauhut H. A mathematical introduction to compressive sensing. New York: Birkhäuser; 2013.
[9] Donoho D. From blackboard to bedside: high-dimensional geometry is transforming the MRI industry. Notices Am Math Soc 2018(January):40–4.
[10] https://www.mathunion.org/fileadmin/IMU/Prizes/Gauss/David%20Donoho-20180825-DLD-c-names.pdf.