On the energy-shell contributions of the three-particle - three-hole excitations

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Abstract

The response functions for the extended second and third random phase approximation are compared. A second order perturbation calculation shows that the first-order amplitude for the direct $3p3h$ excitation from the ground state cancels with those that are engendered by the $1p1h$-$3p3h$ coupling. As a consequence nonvanishing $3p3h$ effects to the $1p1h$ response involve off energy shell renormalization only. On shell $3p3h$ processes are absent.

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Many efforts have been devoted during the last few years in developing generalized random phase approximations (RPA), which go beyond the standard one-particle - one-hole (1p1h) approach [1]. This has been accomplished by including additional correlation effects in both the ground state and the excited states [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. The reasons for that were mainly: i) the problem of the missing strength in the Gamow-Teller (GT) resonances, induced by \((p, n)\) reactions [17, 18], and ii) the issue of the missing charge and missing dip-strength in quasielastic electron scattering [19]. In particular, the extended second RPA (ESRPA), which explicitly includes the 2\(p\)2\(h\) ground state correlations (GSC), was extensively used to describe the above mentioned nuclear excitations [2, 4, 5, 6, 10, 11, 12, 13, 15]. Yet, it is self evident that when the 2\(p\)2\(h\) admixtures are present in the ground state, the external excitation field can lead, not only to the 1\(p\)1\(h\) and 2\(p\)2\(h\) states in the final nucleus, but also to the 3\(p\)3\(h\) states. However, as the ESRPA does not involve the 3\(p\)3\(h\) propagator these excitations cannot appear within the response function as real on the energy-shell processes. Recently the 3\(p\)3\(h\) degrees of freedom were explicitly included within a Tamm-Dancoff approach (TDA), and their effects on the non-energy-weighted GT sum-rule were discussed [14]. Also an extended third RPA (ETRPA), which possesses as the TDA limit the formalism developed in ref. [14], has been used to study the effects of 3\(p\)3\(h\) excitations on the static strength function for quasielastic electron scattering [16].

The purpose of this paper is to present some results for on the energy-shell 3\(p\)3\(h\) effects in the response function. This is done in the context of the full ETRPA approach which is therefore reviewed below. The nature of the resulting response function is then confronted to what one obtains using the ESRPA by performing a perturbative expansion of the responses in each case. The possibility of having a three nucleon ejection process is finally analyzed in this framework.

Let us start with the linear response to an external field \(\hat{F}\) defined as

\[
R(E) = -i \int_{-\infty}^{\infty} \langle 0 | T \left[ \hat{F}^{H\dagger}(t) \hat{F}^{H}(0) \right] | 0 \rangle e^{iEt} dt,
\]

where \(\hat{F}^{H}(t) \equiv e^{i\hat{H}t} \hat{F} e^{-i\hat{H}t}\), \(\hat{H} = \hat{H}_0 + \hat{V}\), with \(\hat{H}_0\) and \(\hat{V}\) being, respectively, the Hartree-Fock (HF) mean field and the residual interaction. The spectral representation of the response
function, in terms of a set \{|\nu\rangle\} of eigenstates of the hamiltonian \(\hat{H}\), reads

\[
R(E) = \sum_{\nu} \left[ \frac{\langle \tilde{0} | \hat{F}|\nu\rangle \langle \nu | \hat{F}^\dagger |\tilde{0}\rangle}{E - E_{\nu} + i\eta} - \frac{\langle \tilde{0} | \hat{F}^\dagger |\nu\rangle \langle \nu | \hat{F} |\tilde{0}\rangle}{E + E_{\nu} - i\eta} \right],
\]

(2)

where \(\eta\) is an infinitesimal positive number.

Within the equation of motion method [1], the set \{\nu\} is generated as

\[|\nu\rangle = \Omega_{\nu}^\dagger |\tilde{0}\rangle; \quad \Omega_{\nu}^\dagger = \sum_i X_{\nu i}^\prime C_i^\dagger - \sum_j Y_{\nu j} C_j,\]

(3)

and

\[\Omega_{\nu} |\tilde{0}\rangle = 0, \quad \text{for all} \ \nu.\]

(4)

The operators \(C_i^\dagger\) and \(C_i\) (with \(C_i^\dagger = a_{p1}^\dagger \cdots a_{p1}^\dagger a_{h1} \cdots a_{h1}\)) create and annihilate \(i\) particle-hole pairs on the HF vacuum \(|0\rangle \equiv |0p0h\rangle\), respectively.

The equation of motion for \(\Omega_{\nu}^\dagger\)

\[\langle \tilde{0} | [\Omega_{\nu}, [H, \Omega_{\mu}^\dagger]] |\tilde{0}\rangle = E_\nu \langle \tilde{0} | [\Omega_{\nu}, \Omega_{\mu}^\dagger] |\tilde{0}\rangle \delta_{\nu,\mu},\]

(5)

where \(E_\nu\) stands for the excitation energy of the state \(|\nu\rangle\), leads to the RPA-like eigenvalue problem

\[AX_{\nu} = E_{\nu} N X_{\nu},\]

(6)

with

\[A = \begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix}, \quad X_{\nu} = \begin{pmatrix} X_{\nu}^\prime \\ Y_{\nu} \end{pmatrix}, \quad N = \begin{pmatrix} N & 0 \\ 0 & -N^* \end{pmatrix}.\]

(7)

The submatrices \(A, B\) and \(N\) given by

\[A_{i,j} = \langle \tilde{0} | [C_i, [H, C_j^\dagger]] |\tilde{0}\rangle, \quad B_{i,j} = \langle \tilde{0} | [C_i, [H, C_j]] |\tilde{0}\rangle, \quad N_{i,j} = \langle \tilde{0} | [C_i, C_j] |\tilde{0}\rangle,\]

(8)

and using eqs. (2-6) it is possible to write the response in representation independent form as

\[R(E) = \mathcal{F}^\dagger (EN - A + i\eta \mathcal{L})^{-1} \mathcal{F},\]

(9)

where \(\mathcal{F}\) is defined as

\[\mathcal{F} \equiv \begin{pmatrix} F_A^A \\ F_B^B \end{pmatrix}, \quad \text{with} \quad \begin{cases} F_A^A_{i,j} = \langle \tilde{0} | [C_i, \hat{F}] |\tilde{0}\rangle, \\ F_B^B = F_A^A_{i,j} (F \rightarrow \hat{F}^\dagger). \end{cases}\]

(10)
After splitting the Hilbert space of $ipih$ states into a P-space that includes only the $1p1h$ states and the Q-space that spans on the rest of the states, the response function can be written as

$$R(E) = \tilde{F}_P(E)G_P(E)\tilde{F}_P(E) + \mathcal{F}_Q^\dagger G_Q(E)\mathcal{F}_Q,$$

where

$$G_P(E) = [EN_P + i\eta I_P - \mathcal{A}_P - (\mathcal{A}_{PQ} - N_{PQ}E) G_Q(E) (\mathcal{A}_{QP} - N_{QP}E)]^{-1},$$

with

$$G_Q(E) = [EN_Q + i\eta I_Q - \mathcal{A}_Q]^{-1},$$

and

$$\tilde{F}_P(E) = \mathcal{F}_P - N_{PQ}\mathcal{F}_Q + \mathcal{A}_{PQ}G_Q(E)\mathcal{F}_Q.$$

In standard RPA the state $|\tilde{0}\rangle$ is approximated by the HF ground state and the Q-space is absent, while the so called extended RPA incorporates perturbative ground state $2p2h$ admixtures and a perturbatively suggested truncation of the dynamical matrices and excitation operator. It is obtained by:

i) evaluating the matrix elements (8) and (10) for

$$|\tilde{0}\rangle = c_0|0\rangle + \sum_{2_0} c_{2_0}|2_0\rangle,$$

where

$$c_0 \simeq 1 - \frac{1}{2} \sum_{2_0} |c_{2_0}|^2, \quad c_{2_0} \simeq -\frac{V_{2_0,0}}{E_{2_0}},$$

$2_0 \equiv (p_1p_2h_1h_2)\rangle_0$ represents the $2p2h$ ground state admixtures, $E_{2_0}$ the corresponding unperturbed energy and $V_{2_0,0} \equiv \langle 2_0|V|0\rangle$, and

ii) keeping terms up to second order in $\hat{V}$ for the forward sector within the P space, terms linear in $\hat{V}$ for the backward sector within the P space and for the coupling between the P and Q spaces, and only terms of zeroth order within the Q space. Under these conditions the norm matrix elements read

$$N_{ij} = \delta_{ij} + \Delta N_{ij}$$

3
where $i \equiv ipih$ and the nonzero $\Delta N_{ij}$ are

$$
\Delta N_{11'} = \sum_{2_0, 2_0'} c_{2_0}^* c_{2_0'} \langle 2_0 | \hat{D}_{11'} | 2_0' \rangle, \quad \Delta N_{13} = \sum_{2_0} c_{2_0}^* \langle 1; 2_0 | 3 \rangle,
$$

where $\hat{D}_{11'} = [\hat{C}_1, \hat{C}_1^d] - \delta_{11'}$ and $\langle 1; 2_0 | 3 \rangle$ is the overlap between the $1p1h \otimes (2p2h)_0$ and $3p3h$ final state configurations. (Note that within the quasi-boson approximation $\hat{D}_{11'} \equiv 0$.) The explicit result for the matrix element $\langle 2_0 | \hat{D}_{11'} | 2_0' \rangle$ is

$$
\langle 2_0 | \hat{D}_{11'} | 2_0' \rangle = (1 + P(h_1, h_2) P(h_1', h_2')) \left[ \delta_{p, p'} \delta_{h_1, h_1'} \delta_{h_2, h_2'} \right] + p \leftrightarrow h,
$$

where $P(i, j) \equiv [1 - P(i, j)]$, while the operator $P(i, j)$ exchanges the arguments $i$ and $j$.

The forward going energy matrix elements are evaluated in the same way and one gets

$$
A_{ij} = \delta_{ij} E_j + V_{ij} + \Delta A_{ij},
$$

where $V_{ij} \equiv \langle i | \hat{V} | j \rangle$ and the nonzero matrix elements $\Delta A_{ij}$ are:

$$
\Delta A_{11'} = \sum_{2_0, 2_0'} (E_1 - E_{2_0}) c_{2_0}^* c_{2_0'} \langle 2_0 | \hat{D}_{11'} | 2_0' \rangle, \quad \Delta A_{13} = \Delta N_{13} E_3.
$$

The one-body matrix elements are:

$$
F_i^A = \begin{cases} 
f_1 + \sum_{1'} \Delta N_{11'} f_{1'} & \text{for } i = 1 \\
\sum_{2_0} c_{2_0}^* f_{2_0} & \text{for } i > 1,
\end{cases}
$$

where

$$
f_1 \equiv \langle 1 | \hat{F} | 0 \rangle, \quad \text{and} \quad f_{2_0} \equiv \langle i | \hat{F} | 2_0 \rangle.
$$

Before proceeding it is convenient to introduce the unperturbed Green’s function:

$$
G^0(E) \equiv \begin{pmatrix} G^0(E) & 0 \\ 0 & G^0(-E) \end{pmatrix},
$$

4
where \( G^0(E) \equiv [E^+ - A(\hat{H} = \hat{H}_0)]^{-1} \) (with \( E^+ \equiv E + i\eta \)) and rewrite the perturbed Green function within the space \( P \) in the form

\[
G_P(E) = \left[ (G^0_P(E))^{-1} - \mathcal{K}_P(E) \right]^{-1},
\]

(25)

where

\[
\mathcal{K}_P(E) \equiv \mathcal{K}_{11'}(E) = \left( \begin{array}{cc}
V_{11'} + \Sigma_{11'}(E) & B_{11'} \\
B_{11'}^* & V_{11'} + \Sigma_{11'}(-E)
\end{array} \right),
\]

(26)

with

\[
\Sigma_{11'}(E) = \Delta \Sigma_{11'}^{(2)}(E) + \Delta \Sigma_{11'}^{(3)}(E) + \sum_{i=2,3} V_{i1} G^0_{ii}(E) V_{i1'},
\]

(27)

and

\[
\begin{align*}
\Delta \Sigma_{11'}^{(2)}(E) &= \Delta A_{11'} - \Delta N_{11'} E, \\
\Delta \Sigma_{11'}^{(3)}(E) &= - \left( 2V_{13} - \Delta N_{13} (G^0_{33}(E))^{-1} \right) \Delta N_{31'}.
\end{align*}
\]

(28)

In the above equations \( V_{ij} \) stands for the matrix representation of the residual interaction within the \( ipih \otimes jpjh \) subspace.

The response function now reads

\[
R(E) = \tilde{F}_{1'}(E) G_{11'}(E) \tilde{F}_{1'}(E) + \sum_{i=2,3} \mathcal{F}_{i}^1 G^0_{ii}(E) \mathcal{F}_{i},
\]

(29)

where

\[
\tilde{F}_1(E) \equiv \left( \begin{array}{c}
\tilde{F}_1^A(E) \\
\tilde{F}_1^B(E)
\end{array} \right), \text{ with }
\]

\[
\begin{align*}
\tilde{F}_1^A(E) &= f_1 + \Delta \tilde{F}_1(E), \\
\Delta \tilde{F}_1(E) &= \Delta F_1^{(2)} + \Delta F_1^{(3)} + \sum_{i=2,3} V_{i1} G^0_{ii}(E) F_i, \\
\Delta F_1^{(2)} &= \Delta N_{11'} f_1', \quad \Delta F_1^{(3)} = - \Delta N_{13} F_3.
\end{align*}
\]

(30)

From the expressions for \( \Delta A_{11'} \) and \( \Delta N_{11'} \), given by Eqs. (18) and (21) respectively, the matrix elements \( \Delta \Sigma_{11'}^{(2)}(E) \) and \( \Delta \tilde{F}_1^{(2)} \) can be expressed as:

\[
\begin{align*}
\Delta \Sigma_{11'}^{(2)}(E) &= - \sum_{2_0,2_0'} c_{2_0}^* c_{2_0'} (\hat{D}_{11'} | 2_0') (E - E_1 + E_{2_0'}), \\
\Delta \tilde{F}_1^{(2)} &= \sum_{2_0,2_0'} c_{2_0}^* c_{2_0'} (\hat{D}_{11'} | 2_0') f_1'.
\end{align*}
\]

(31) \hspace{1cm} (32)
Moreover, from the relationships

$$V_{13} = - \sum_{2_0} c_{2_0}^* E_{2_0} \langle 1; 2_0 | 3 \rangle; \quad f_{3,2_0} = \sum_1 \langle 3 | 1; 2_0 \rangle f_1,$$

one obtains

$$\Delta \Sigma_{11'}^{(3)} = \sum_{2_0,2'_0} c_{2_0}^* c_{2'_0} \langle 1; 2_0 | 1'; 2'_0 \rangle (E - E_1 + E_{2'_0}),$$

$$\Delta \tilde{F}_1^{(3)} = - \sum_{2_0,2'_0} c_{2_0}^* c_{2'_0} \langle 1; 2_0 | 1'; 2'_0 \rangle f_{1'}.$$

We can note here that

$$\langle 1; 2_0 | 1'; 2'_0 \rangle = \langle 2_0 | \hat{D}_{11'} + \hat{d}_{11'} | 2'_0 \rangle, \quad \text{with} \quad \hat{d}_{11'} = \delta_{11'} + C_{1'}^t C_1,$$

and thus in summary we get:

i) in the ESRPA (where the $Q$ space includes only the $2p2h$ excitations)

$$\Sigma_{11'}(E) = - \sum_{2_0,2'_0} c_{2_0}^* c_{2'_0} \langle 2_0 | \hat{D}_{11'} | 2'_0 \rangle (E - E_1 + E_{2'_0}) + \sum_{2} V_{12} V_{21'},$$

$$\tilde{F}_1(E) = f_1 + \sum_{2_0,2'_0,1'} c_{2_0}^* c_{2'_0} \langle 2_0 | \hat{D}_{11'} | 2'_0 \rangle f_{1'} + \sum_{2,2_0} V_{12} f_{22_0} c_{2_0}^*,$$

ii) in the ETRPA (where the $Q$ space includes both the $2p2h$ and $3p3h$ excitations)

$$\Sigma_{11'}(E) = \sum_{2_0,2'_0} c_{2_0}^* c_{2'_0} \langle 2_0 | \hat{d}_{11'} | 2'_0 \rangle (E - E_1 + E_{2'_0}) + \sum_{i=2,3,2_0} V_{1i} V_{i1'},$$

$$\tilde{F}_1(E) = f_1 - \sum_{2_0,2'_0} c_{2_0}^* c_{2'_0} \langle 2_0 | \hat{d}_{11'} | 2'_0 \rangle f_{1'} + \sum_{i=2,3,2_0} V_{1i} f_{22_0} c_{2_0}. $$

The results (37) and (38) are in essence those obtained previously by Arima and collaborators \cite{5,11} and by the Jülich group \cite{6,12}. On the other hand, when terms containing the matrix elements $\langle 2_0 | \hat{d}_{11'} | 2'_0 \rangle$ are neglected in eqs. (39) and (40), one finds the results derived in our previous works \cite{16}.

These terms give rise to disconnected graphs, which are nonphysical, as well as to double connected graphs represented in fig. 1d and 1e, respectively. As seen from relations (43) and (47) below, they do not contribute to the response function.
In order to elucidate some of the content of these equations we turn next to a perturbative expansion of the response function and examine the leading corrections to the unperturbed 1p1h response \( R^0(E) = \sum_1 |f_i|^2/(E^+ - E_1) \). To achieve maximum simplicity we first omit the residual interaction within the 1p1h sector and backward contributions, so that to second order the Bethe-Salpeter equation Eq. (23) reads

\[
G_{11'}(E) \approx G^0_{11}(E) + G^0_{11}(E)\Sigma_{11'}(E)G^0_{11'}(E),
\]

which substituted in Eq. (11) leads to the desired approximation for the response function. Within the ESRPA one gets:

\[
R(E) \approx R^0(E) + \sum_{2,2_0,2'_0} c^*_2 f_{22_0}^* f_{22'_0} \frac{2 \sum_{1,2,2_0} \Re(f_{12_0}^* f_{22_0} c_{2_0})}{E^+ - E_1} \frac{V_{12}}{E^+ - E_2},
\]

\[
+ \sum_{1,1'} \frac{f_1^*}{E^+ - E_1} \left[ \sum_{2_0,2'_0} c^*_2 c_{2_0} \langle 2_0 | \hat{D}_{11'} | 2'_0 \rangle (E - E_1 - E_{2_0}) + \sum_{2} \frac{V_{12} V_{21'}}{E^+ - E_2} \right] \frac{f_{1'}}{E^+ - E_{1'}},
\]

and in the ETRPA:

\[
R(E) \approx R^0(E) + \sum_{i=2,3,2_0,2'_0} c^*_2 f_{i2_0}^* f_{i2'_0} \frac{2 \sum_{i=2,3,1,2_0} \Re(f_{i2_0}^* f_{22_0} c_{2_0})}{E^+ - E_1} \frac{V_{1i}}{E^+ - E_i},
\]

\[
- \sum_{1,1'} \frac{f_1^*}{E^+ - E_1} \left[ \sum_{2_0,2'_0} c^*_2 c_{2_0} \langle 2_0 | \hat{d}_{11'} | 2'_0 \rangle (E - E_1 - E_{2_0}) - \sum_{i=2,3} \frac{V_{13} V_{31'}}{E^+ - E_i} \right] \frac{f_{1'}}{E^+ - E_{1'}},
\]

Now the two expressions (42) and (43) can be shown to be equivalent. This results in fact from explicitly performing the sums over 3p3h states in Eq. (43). To do that one first rewrites these sums making use of relations (33) and (36) as:

\[
\sum_{3,2_0,2'_0} c^*_2 f_{32_0}^* f_{32'_0} \frac{1}{E^+ - E_3} c_{2_0} = \sum_{3,2_0,2'_0} c^*_2 f_1^* \frac{\langle 2_0 | (\hat{D}_{11'} + \hat{d}_{11'}) | 2'_0 \rangle}{E^+ - E_1 - E_{2_0}} f_{1'} c_{2_0},
\]

\[
2 \sum_{1,3,2_0} \frac{\Re(f_{1}^* f_{32_0} c_{2_0})}{E^+ - E_1} \frac{V_{13}}{E^+ - E_3} = -2 \sum_{1,2_0,2'_0} c^*_2 f_1^* \frac{E_{2_0} \langle 2_0 | (\hat{D}_{11'} + \hat{d}_{11'}) | 2'_0 \rangle}{(E^+ - E_1)(E^+ - E_1 - E_{2_0})} f_{1'} c_{2_0}.
\]
and

\[ \sum_{1,1',3} \frac{f_1^*}{E_1'} \frac{f_1'}{E_3'} \sum_{1,1',2,2'} \frac{f_1^* c_2 E_2 (2_0|D_1 + D_1'|2_0') E_2 f_1' c_2'}{(E_1' E_1 E_2 (E_1' E_1 E_2)} \]

The result of performing the sum is:

\[ \sum_{1,1',2,2'} \frac{f_1^*}{E_1'} \frac{f_1'}{E_3'} \sum_{1,1',2,2'} \frac{f_1^* c_2 E_2 (2_0|D_1 + D_1'|2_0') E_2 f_1' c_2'}{(E_1' E_1 E_2 (E_1' E_1 E_2)} \]

which substituted in Eq. (43) gives the expression (42) also for the ETRPA response.

The cancellation among the \(3p3h\) on the energy-shell contributions can be exhibited also making use of the Rayleigh-Schrödinger perturbation expansion, i.e.,

\[ |\tilde{i}\rangle = |i\rangle + |i\rangle^{(1)} + \cdots \quad \text{and} \quad \tilde{E}_i = E_i + E_i^{(1)} + \cdots, \quad i = ipih, \]

where the perturbed wave functions and energies are indicated by the symbol \(\sim\) and the superscript points the order of the correction introduced by the residual interaction \(\tilde{V}\) on the unperturbed quantities \(|i\rangle\) and \(E_i\). The amplitude for the \(\tilde{F}\)-excitation from the correlated ground state to the perturbed \(3p3h\) states reads

\[ \langle 3|\tilde{F}|0\rangle = \frac{\langle 3|[\hat{H}, \tilde{F}]|0\rangle}{E_3 - E_0} = \frac{\langle 3|[\hat{H}, \tilde{F}]|0\rangle}{E_3 - E_0} + O(\tilde{V}^2), \]

with

\[ \langle 3|[\hat{H}, \tilde{F}]|0\rangle = \sum_{1} \langle 3|\tilde{V}|1\rangle \langle 1|\tilde{F}|0\rangle - \sum_{2_0} \langle 3|\tilde{F}|2_0\rangle \langle 2_0|\tilde{V}|0\rangle \equiv 0, \]

where the last equivalence is a direct consequence of the relations (33), i.e.,

\[ \sum_{1} V_{31} f_1 = - \sum_{1,2_0} c_{2_0} E_2 (1_2|2_0) f_1 = \sum_{2_0} f_{3,2_0} V_{2_0} \]

Thus we see once more that, up to the second order in \(\tilde{V}\), the \(3p3h\) final states do not contribute to the response function and that \(|\langle 3|\tilde{F}|0\rangle|^2 \approx O(\tilde{V}^4)\). The Goldstone diagrams for

\footnote{Note that \(\hat{H}_0\) does not contribute since \(\langle 3|\tilde{H}_0, \tilde{F}|0\rangle = 0\).}
the fourth order $3p3h$ on the mass-shell contributions to the response function are shown in figs. 1e and 1f.

At first glance it might look as if the connected Goldstone diagrams associated with the terms (44), (45) and (46) of the ETRPA response (illustrated in Figs. 1a, 1b and 1c, respectively) should give rise to on the mass-shell $3p3h$ contributions, through the imaginary part of the propagator $(E^+ - E_3)^{-1}$. However, Eq. (47) shows that these contributions in fact cancel out so that the $3p3h$ sector only affects the $1p1h$ excitations by coupling them with the virtual intermediate states $|1;2_0\rangle$. Thus in spite of including the $3p3h$ propagator in the Green’s function, three nucleon ejection does not occur in the leading order processes. The above mentioned diagrams also explain the physical meaning of the fourth term in the expression (42). The cancellation of on shell $3p3h$ contributions results from the destructive interference between amplitudes involving creation of the $3p3h$ state from a ground state correlation and from $V_{31}$ coupling respectively. A similar calculation in which the backward part of Eq. (25) and/or the residual interaction within the $1p1h$ space are kept up to the relevant order leads again to the same result. It is worth stressing that this does not depend on the form of the two-body force used as residual interaction or on the size of single particle space.
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Figure 1: Graphical representation of the second and fourth order contributions to the response function. The dotted circles (⊙) denote the one-body vertices and the filled ones (•) indicate the two-body matrix elements. The diagrams (a), (b) and (c) correspond, respectively, to the terms given by eqs. (44), (45) and (46). Second order unlinked and double-linked graphs analogous to the diagram (c) are shown in figures (d) and (e), respectively. The last ones, although contained in eqs. (39) and (40), do not contribute to the response function. Finally, figure (f) illustrates the fourth order on the energy-shell $3p3h$ processes.