We study the evolution of the Universe at early stages; we discuss also preheating in the framework of hybrid braneworld inflation by setting conditions on the coupling constants \( \lambda \) and \( g \) for effective production of \( \chi \)-particles. Considering the phase between the time observable CMB scales crossed the horizon and the present time, we write reheating and preheating parameters \( N_{re} \), \( T_{re} \) and \( N_{pre} \) in terms of the scalar spectral index \( n_s \) and prove that, unlike the reheating case, the preheating duration does not depend on the values of the equation of state \( \omega^* \). We apply the slow-roll approximation in the high energy limit to constrain the parameters of D-term hybrid potential. We show also that some inflationary parameters, in particular, the spectral index \( n_s \) demand that the potential parameter \( \alpha \) is bounded as \( \alpha \geq 1 \) to be consistent with Planck’s data, while the ratio \( r \) is in agreement with observation for \( \alpha \leq 1 \) considering high inflationary e-folds. We also propose an investigation of the brane tension effect on the reheating temperature. Comparing our results to recent CMB measurements, we study preheating and reheating parameters \( N_{re} \), \( T_{re} \) and \( N_{pre} \) in the Hybrid D-term inflation model in the range \( 0.8 \leq \alpha \leq 1.1 \) and conclude that \( T_{re} \) and \( N_{re} \) require \( \alpha \leq 1 \), while for \( N_{pre} \) the condition \( \alpha \leq 0.9 \) must be satisfied, to be compatible with Planck’s results.

1 Introduction

After the brane-world inflation, preheating and reheating are considered to be the stages at which elementary particles start populating the universe that leads to matter creation at later times. In the brane-world inflationary preheating scenario, the four-dimensional Einstein equations deviate from the standard cosmology [1]. By the proposal of a certain type of compactification, the standard model particles are considered to be on the three-dimensional brane, which makes gravitons propagate in the extra dimension. This model indicates that we are living on the three-brane with a positive tension embedded in five-dimensional anti-de Sitter bulk, which appears in the Friedmann equation as an energy called brane tension. The
modifications due to additional dimension which contain the brane may have implications on the reheating process. In the early stage of reheating “preheating”[2], the slow decreasing amplitude of inflaton field coupled to $\chi$-field results a strong amplification of massless $\chi$-particles through the non-perturbative decay of the inflaton [3].

While the single-field inflation models cannot reproduce the observational data because they suffer from some fine-tuning problems on the parameters of their potentials, such as the mass and the coupling constants [4], the hybrid inflationary model produces the observed temperature fluctuations in the CMB and show a great capacity to reproduce the experimental values of all inflationary perturbation spectrum [5]. The difficulties of the single-field inflation are also overcome in the context of hybrid supersymmetric (SUSY) models [6], since they agree with the observed power spectrum of density perturbations very well [4].

In the context of supersymmetric theories, the D-term inflation looks more promising since it avoids the problem associated with the inflaton mass [7] and can be successfully implemented in the framework of supergravity theories [8]. However, this type of inflation always ends with the formation of cosmic strings [9]. The cosmic strings were analyzed in the braneworld model, and it has been shown that Fayet–Iliopoulos (FI) term $\xi$ is responsible for this phenomenon [10]. As a consequence, the choice of the right value of (FI) term will be very important to avoid the formation of the cosmic strings dominated in the conventional D-term inflation models [8].

After the supersymmetric inflation, a period that converts the stored energy density in the inflaton to the thermal bath (a plasma of relativistic particles) occurs [11]. This transition is known as reheating. During this period, ordinary matter is produced as a result of inflaton field energy loss. The scenario of reheating occurs when the inflaton oscillates around the minimum of its potential and decay into new particles at the radiation-dominated period [12]. Along with this simple canonical reheating model, a short phase includes the non-perturbative processes [12] called preheating occurs. This scenario refers to the initial stage of the reheating. Preheating is characterized by an exponential instability; this instability corresponds to an exponential growth of occupation number of quantum fluctuation, which is described by Mathieu equation. There are no direct cosmological observables that are traceable to the phases that follow inflation. However, one can consider the phase between the time observable CMB scales crossed the horizon and the present time, as a way to achieve information using indirect limits.

Since we are interested in pre-/reheating stages after SUSY brane inflation, we are concerned to study the two phases durations quantified in terms of $e$-folds numbers $N_{pre}$ and $N_{re}$, in a certain interval of the equation of state (EoS) $\omega^*$. It is obvious that the parameter $\omega^*$ has values larger than $-1/3$, which is needed to end inflation [13]. This parameter is also assumed to be smaller than +1 to not violate causality. On the other hand, its value at the beginning of the radiation-dominated era must be $+1/3$. Knowing that reheating start at $\omega^* \geq 0$ [14], therefore we will consider this parameter with the choice of different values in that interval. We also calculate preheating $e$-folds as a function of reheating temperature and analyze its effects on preheating and use the expressions for the effective D-term potential to constrain pre-/reheating parameters $(T_{re}, N_{re}, N_{pre})$ and then compare the results with the observations using the recently released Planck – 2018 data [15]. Knowing that the FI term was considered as a problem of D-term since the CMB requires $\xi \leq O(10^{15} - 10^{16})$ GeV [16]. Below this range, the formation of cosmic strings becomes more important. We conclude that the highest value possible of reheating temperature $T_{re}$ can favor a much higher $e$-folds number of preheating. Considering the D-term hybrid model, we study the pre-/reheating...
durations and take into account values of FI term that does not lead to cosmic strings formation.

Our paper is organized as follows: In the next section, we review the hybrid braneworld inflation and reheating and briefly discuss the preheating mechanism in hybrid brane inflation. In Sect. 3, we derive the reheating temperature and duration, we also derive the preheating duration in Sect. 4, finally, we constrain the parameters of D-term SUSY model and the pre-/reheating $e$-folds along with reheating temperature in Sect. 5. The last section is devoted to a conclusion.

2 Braneworld hybrid inflation and preheating

2.1 Formalism of braneworld inflation

According to the braneworld model, the $(d + 1)$–dimensional anti-de Sitter space-time, which we call the bulk, contains all the observable universe embedded in 3-brane along with gravity and particle fields. $RSII$ model is an inflation scenario of the braneworld scenario with $d = 4$. This means that 3–brane is embedded in the five-dimensional $AdS$ space [17], as a consequence the action of the field equation in $RSII$ model is written as [18]:

$$S_{RSII} = \int dx^5 \sqrt{-g_5} \left[ \frac{M_5^3}{2} R_b + \Lambda_5 \right] + \int dx^4 \sqrt{-g_4} L_{brane},$$

(1)

where

$$L_{brane} = L_{matter} + \tilde{T} = \frac{1}{2} |\partial \Phi|^2 - V + \tilde{T}.$$  

(2)

$R_b$ is the bulk spacetime scalar curvature, with the bulk metric $g_5$, $M_5$ and $\Lambda_5$ are the five-dimensional Planck mass and the cosmological constant. The second term of $RSII$ action is the Lagrangian density of ordinary matter on the brane, $V$ is the scalar potential, and $\tilde{T}$ is the brane tension. In this cosmological scenario, the metric projected onto the brane is a spatially flat Friedmann–Robertson–Walker model, the Friedmann equation on the brane has the generalized form [18,19]:

$$H^2 = \frac{8\pi}{3M_p^2} \rho \left(1 + \frac{\rho}{2\tilde{T}} \right).$$

(3)

During inflation and preheating, the term $\rho$ is expected to play important role in the evolution of the Universe. In order to have the inflationary dynamical equations leading to an accelerated expansion in the high energy limit, we approximate $\rho \gg 2\tilde{T}$, this condition holds until the end of preheating, considering the slow-roll conditions the energy density can be approximated by $\rho \approx V$, and the Friedmann equation finally takes the form:

$$H^2 = \frac{4\pi}{3M_p^2} \frac{V^2}{\tilde{T}}.$$  

(4)

The inflaton field confined on the brane satisfies the Klein–Gordon equation given by

$$\ddot{\phi} + 3H \dot{\phi} + V'(\phi) = 0,$$  

(5)
with $V'(\phi) = \partial V/\partial \phi$, we may also write the spectral index $n_s$ and the ratio of tensor to scalar perturbations $r$ as functions of the slow-roll parameters $\epsilon_k$ and $\eta_k$:

$$n_s = -6\epsilon + 2\eta + 1, \quad r \simeq 26\epsilon_k. \quad (6)$$

The previous equations are very important tools of the braneworld RSII model which have been used to solve some of the standard model problems.

2.2 Hybrid inflation and reheating

One reason to be interested in hybrid inflation is that it can be implemented in supersymmetric theories [20]. The hybrid inflation model which we study is defined by the potential [21]

$$V(\phi, \sigma) = \frac{1}{4\lambda} (M^2 - \lambda \sigma^2)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\sigma^2. \quad (7)$$

The scalar fields $\phi$ and $\sigma$ have masses $m$ and $M$, and the potential has the symmetry $\sigma \leftrightarrow -\sigma$ at large values of the field $\phi$. The potential has a maximum at $\phi = \sigma = 0$, when the field $\phi$ value is small, and a global minimum at $\phi = 0, \sigma = \sigma_0 = M/\sqrt{\lambda}$, where the symmetry is broken. The equations of motion for the homogeneous fields are

$$\ddot{\phi} + 3H\dot{\phi} = -(m^2 + g^2\sigma^2)\phi, \quad (8)$$

$$\ddot{\sigma} + 3H\dot{\sigma} = (M^2 - g^2\sigma^2 - \lambda\sigma^2)\phi, \quad (9)$$

with $\dot{\phi} \equiv \partial\phi/\partial t$ and $\ddot{\phi} \equiv \partial^2\phi/\partial t^2$, at large $\phi$ the motion begins, the effective mass of the $\sigma$ field $m_\sigma^2 = g^2\phi^2 + M^2$ get also large. After the slow-roll of the field $\phi$, and just before the end of inflation, $\phi$ acquires the critical value $\phi_c = M/g$, and the field fluctuate at the minimum of the potential $\sigma = 0$, then the symmetry breaking phase transition caused by the massless $\sigma$ field fluctuations ends inflation. This transition could either end instantaneously if the mass $M$ of the $\sigma$ field is larger than the rate of expansion $H$, or if $M$ is of the order of $H$, the transition will be very slow which will make inflation have a few more $e$-folds after the phase transition [22].

At $\sigma = 0$, the potential of inflaton field became $V(\phi) = M^4/4\lambda + m^2\phi^2/2$, considering the case where $m^2 \ll g^2 M^2/\lambda$, the vacuum energy will dominate and the Hubble constant at the time of the phase transition will be given by

$$H^2_0 = \frac{\pi}{3\lambda^2} \frac{M^8}{M^2_{\rho}} \frac{1}{T^4}, \quad (10)$$

the slow-roll condition imposes the fact that we should neglect $\ddot{\phi}$ in the inflaton equation of motion, to finally have $3H\phi \simeq -V'(\phi)$. When the scalar field $\phi$ decreases below $\phi_c = M/g$, it prepares for the process of reheating.

**Reheating Mechanism** Reheating is an important phase that describes the production of standard model particles, it occurs at the end of preheating with a decay rate $\Gamma = \Gamma(\phi \rightarrow \chi \chi) + \Gamma(\phi \rightarrow \psi \psi)$, and this means that the inflaton field $\phi$ can decay into bosons and fermions particles. Using Eq. 3 and the expression of the energy density at the reheating epoch with $\rho = \rho_{re}$ [14]:

$$\rho_{re} = \frac{\pi^2}{30} g_{re} T_{re}^4, \quad (11)$$
Fig. 1 Variation of the ratio $H_0/H$ as a function of reheating temperature, the yellow region is ruled out by $BBN$ (big bang nucleosynthesis) and the orange region represents 100$GeV$ of electroweak scale, the light green region represents the inflation at phase transition dominance, and light blue is the region of the reheating phase dominance.

one should find

$$H^2 \simeq \frac{4\pi^5}{3 \times 30^2} \frac{g_{\text{re}}^2 T^8_{\text{re}}}{M_p^2 T}.$$  \hspace{1cm} (12)

We introduce the ratio between the Hubble parameter at phase transition and the reheating phase given by

$$\left(\frac{H_0}{H}\right)^2 = \frac{30^2}{4\pi^4 g_{\text{re}}^2 \lambda^2} \frac{M^8}{g_{\text{re}}^2 T^8_{\text{re}}}.$$  \hspace{1cm} (13)

this ratio depends inversely on the reheating temperature, knowing that according to [23] the minimal possible value of this temperature is bounded by $(T_{\text{min, re}} \geq 4 MeV)$. Figure 1 shows that the ratio $H_0/H$ decreases very rapidly when we consider higher temperatures of reheating. In other words, we can say that for lower values of reheating temperature where $H_0/H > 1$, the difference between $H_0$ and $H$ is considerably high, which means that the effect of inflation at the phase transition is dominant in this region. At $H_0/H \leq 1$, we realize that reheating effects became much more important, for that reason we suppose that this condition is accompanied by a new type of expansion where the comoving Hubble scale $(aH)^{-1}$ begin to increase, which demand a certain number of $e$-foldings, that we will prove to be a total duration written as $N_{\text{pre}} + f(\omega^*) N_{\text{re}}$ in the remain sections.

In the next section, we will focus on the stage of preheating that we study by introducing a $\chi$ field coupled to the potential $V(\phi, \sigma)$.

2.3 Preheating in hybrid inflation

Particle production occurs when the fields oscillate around their minimum. The behavior of the fields after the end of inflation could describe the explosive preheating with a production of $\phi$ and $\sigma$ particles in hybrid inflation. However, we will study the particle production behavior considering an extra scalar field $\chi$, coupled to both of the previous fields [24].
\[ V(\chi) = \frac{1}{2} l_1^2 \phi^2 \chi^2 + \frac{1}{2} l_2^2 \sigma^2 \chi^2. \]

The explosive production of \( \chi \) particles occurs for certain values of \( l_1 \) and \( l_2 \). The equations of motion of quantum fluctuations for \( \chi \) field are given by

\[ \ddot{\chi}_k + 3H \dot{\chi}_k + \left( \frac{k^2}{a^2} + m_\chi^2 \right) \chi_k = 0, \]

the effective mass is written as

\[ m_\chi^2 = l_1^2 \phi^2 + l_2^2 \sigma^2. \]

The rate of expansion of the universe plays an important role in ending the parametric resonance regime, and particle production will be ended by the redshifted modes with momentum \( k/a \) that will fall out of the resonance band because of the expansion. According to the previous section, there are two fundamental frequencies near the minimum of the potential: \( \nu_\sigma = \sqrt{2} M \) and \( \nu_\phi = g M / \sqrt{\lambda} \), that corresponds to the two critical values of the fields \( \sigma \) and \( \phi \): \( \sigma_0 = M / \sqrt{\lambda} \) and \( \phi_c = M/g \). Particle production in \( \chi \) field that interacts with the fields \( \phi \) and \( \sigma \) given by Eq. 15 require an amplified fluctuations of this field around \( \phi = 0 \) while \( \sigma \simeq \sigma_0 \), this demand that the induced mass from the symmetry breaking field \( \sigma \) must be much smaller than the corresponding oscillations from the \( \phi \) field

\[ l_1^2 \phi^2 \gg l_2^2 \sigma_0^2, \]

which at the beginning of preheating corresponds to

\[ \lambda l_1^2 \gg g^2 l_2^2. \]

Using the solution of the Klein–Gordon equation given by \( \phi(t) \approx \Phi(t) \sin(\nu_\phi t) \), we can determine that the explosive production of \( \chi \)–particles will end when the amplitude of oscillations of the \( \phi \) field becomes \( \Phi(t) < (l_2 g) / l_1 \sqrt{\lambda} \).

The Mathieu equation which corresponds to the preheating process is given by

\[ \chi_k'' + \left( A(k) - 2q \cos(2z) \right) \chi_k = 0, \]

with \( \chi'' \equiv \partial^2 / \partial z^2 \) is the derivative with respect to the parameter \( z \), and

\[ A_k = \frac{k^2}{a^2 \nu_\phi^2} + 2q, \]

\[ q = \frac{l_1^2 M^2 \Phi^2}{g^2 \nu_\phi^2} / 4. \]

Taking into consideration the previous condition Eq. 17, at the initial step of preheating, Eq. 20 must satisfy \( q_0 = (\lambda l_1^2 / g^4) \Phi^2 / 4 \), for \( \lambda \gg g^2 \) we can find natural values of the parameter \( l_1 \) that ensure \( q_0 \gg 1 \). This will allow an explosive production of \( \chi \)–particles, which mean we are in the broad parametric resonance region, that corresponds to an exponential growth of quantum fluctuations occupation numbers, given by \[ 3 \]

\[ n_k(t) \simeq \exp(2\mu_k z), \]

with a frequency \( \omega_k \)

\[ \omega_k^2 = \frac{k^2}{a^2} + l_1^2 \phi^2. \]

The choice of natural values of couplings will cause an efficient particle production process, because of a large value of the growth parameter \( \mu_k \). Furthermore, for a large range of couplings we will enter the region of stochastic resonance \[ 3 \].
3 Reheating duration and temperature

The reheating duration can be extracted considering the phase between the time observable CMB scales crossed the horizon and the present time. Different eras occurred throughout this length of time that can be described by the two following equations [14]:

\[
\frac{k}{a_0 H_0} = \frac{a_k}{a_{end}} \frac{a_{re}}{a_{eq}}\frac{a_{eq} H_{eq}}{H_{eq}} H_k,
\]

(23)

the pivot scale for a specific experiment is parametrized by \( k \) [14], \( N_k \) is the \( e \)-folds of the inflation era, \( N_{pre} \), \( N_{re} \) and \( N_{RD} \), respectively, correspond to the reheating and radiation-dominance era durations. Reheating is characterized by the temperature \( T_{re} \) and a number of \( e \)-folds \( N_{re} = \ln (a_{re}/a_{end}) \) occurring before the radiation-dominated era. Using \( \rho \propto a^{-3(1+\omega^*)} \), the reheating epoch is described by:

\[
\frac{\rho_{end}}{\rho_{re}} = \left( \frac{a_{end}}{a_{re}} \right)^{-3(1+\omega^*)},
\]

(24)

where \( \rho_{end} \) and \( a_{end} \) correspond to the end of inflation and \( \rho_{re} \) and \( a_{re} \) correspond to the end of reheating. As a result:

\[
N_{re} = \frac{1}{3 (1 + \omega^*)} \ln \left( \frac{\rho_{end}}{\rho_{re}} \right),
\]

(25)

knowing that \( \rho_{end} = (6/5)V_{end} \) in the Braneworld case [25] and \( \rho_{re} = (\pi^2/30)g_re T_{re}^4 \), we replace every energy density by its expression to obtain:

\[
N_{re} = \frac{1}{3 (1 + \omega^*)} \ln \left( \frac{6}{5 \pi^2 g_re T_{re}^4} \right).
\]

(26)

The reheating temperature and the actual temperature are related as [13]:

\[
T_{re} = T_0 \left( \frac{a_0}{a_{eq}} \right)^{N_{RD}} \left( \frac{43}{11 g_{re}} \right)^{1/2},
\]

(27)

using the following expression in the previous equation:

\[
\frac{a_0}{a_{eq}} = \frac{a_0 H_k}{k} e^{-N_k} e^{-N_{re}} e^{-N_{RD}},
\]

(28)

gives the result

\[
N_{re} = \frac{4}{1 - 3\omega^*} \left[ -\frac{1}{4} \ln \left( \frac{36}{\pi^2 g_{re}} \right) - \frac{1}{3} \ln \left( \frac{11 g_{re}}{43} \right) - \ln \left( \frac{k}{a_0 T_0} \right) - \ln \left( \frac{V_{end}^{1/2}}{H_k} \right) - N_k \right],
\]

(29)

it is important to notice that the reheating duration is not defined in the value of the equation of state \( \omega^* = 1/3 \). We can simplify the previous expression considering \( g_{re} \approx 200 \) for the Braneworld case [25] and the pivot scale 0.05\( Mpc \) [14], and other numerical values From [15]: \( M_{pl} = 2.435 \times 10^{18} GeV \), \( a_0 = 1 \), \( T_0 = 2.725 K \):

\[
N_{re} = \frac{4}{1 - 3\omega^*} \left[ 6.61 - \ln \left( \frac{V_{end}^{1/2}}{H_k} \right) - N_k \right].
\]

(30)
The estimation for $N_{re}$ is not accessible for $\omega^* = 1/3$. Note that inconsistency in $N_{re}$ exists for this case because we define the beginning of radiation dominance when $\omega^*$ reaches 1/3. From Eqs. (27), (28), we obtain

$$T_{re} = \left( \frac{43}{11 g_{re}} \right)^{1/3} \frac{a_0 T_0}{k} H_k e^{-N_k e^{-N_{re}}},$$

using the previous equation with Eq. 26, we can find the reheating temperature as

$$T_{re} = \left[ \left( \frac{43}{11 g_{re}} \right)^{1/3} \frac{a_0 T_0}{k} H_k e^{-N_k} \left( \frac{36 V_{end}}{\pi^2 g_{re}} \right)^{-1/3(1+\omega^*) \frac{3(1+\omega^*)}{3\omega^* - 1}} \right].$$

To find $N_{re}$ and $T_{re}$ for a particular model, one needs to compute $N_k$, $H_k$ and $V_{end}$.

4 Preheating duration

We start our computation for the preheating case, by introducing the ratio $a_{pre}/a_{pre}$ to Eq. (23) that corresponds to preheating scale factor, as result, we obtain an equation written in terms of $e$-folds

$$\ln \frac{k}{a_0 H_0} = -N_k - N_{pre} - N_{re} + \ln \frac{a_{pre}}{a_0} + \ln \frac{H_k}{H_0},$$

where $N_{pre} = \ln \left( \frac{a_{pre}}{a_{end}} \right)$ is the number of $e$ -folds between the end of inflation and the end of preheating. Assuming that no entropy production took place after the completion of reheating, one can write [13,26]

$$\frac{a_{re}}{a_0} = \frac{T_0}{T_{re}} \left( \frac{43}{11 g_{re}} \right)^{1/3},$$

where $T_0$ is the current temperature of the Universe. Using Eq. (24) and the expression of reheating density energy $\rho_{re} = (\pi^2/30) g_{re} T_{re}^4$, we can determine an expression given by:

$$N_{tot} = \frac{1}{f(\omega^*)} N_{pre} + N_{re},$$

$$= \frac{1}{f(\omega^*)} \left[ -\ln \left( \frac{k}{a_0 T_0} \right) - \frac{1}{3} \ln \left( \frac{11 g_{re}}{43} \right) - \frac{1}{4} \ln \left( \frac{36 V_{end}}{\pi^2 g_{re}} \right) - \frac{1}{4} \ln \left( \frac{V_{end}}{H_k^4} \right) - N_k \right].$$

Eq. (35) can be reduced to

$$N_{pre} + f(\omega^*) N_{re} = \left[ 6.61 - \ln \left( \frac{V_{end}^{1/3}}{H_k} \right) - N_k \right],$$

here $f(\omega^*) = (1 - 3\omega^*) / 4$, using Eq. 26, the final form of $N_{pre}$ will be given as:

$$N_{pre} = \left[ 6.61 - \ln \left( \frac{V_{end}^{1/3}}{H_k} \right) - N_k \right] - \frac{1}{12} \frac{36 \cdot V_{end}}{(1 + \omega^*) \ln \left( \frac{36 \cdot V_{end}}{\pi^2 g_{re} T_{re}^4} \right)}.$$

According to [27,28], if reheating is instantaneous the temperature would be in the order of $10^{14} GeV$, taking into account this maximum reheating temperature, this condition favors
a much higher $e$-folds number of preheating. Next we need to find the expressions of $N_k$, $H_k$ and $V_{\text{end}}$, the tree parameters can be related to the scalar spectral index $n_s$, and they can be connected directly to a model of inflation, which means that we can use D-term model to study pre-/reheating constraints in SUSY brane-world inflation.

5 Pre-/Reheating constraints in supersymmetric braneworld inflation

Preheating and reheating are now parametrized by $e$-folds numbers $N_{\text{pre}}$ and $N_{\text{pre}}$, but we do not expect that both durations perform the same behavior. From Eqs. 30, 36, we observe that pre-/reheating are expressed as functions of tree parameters and a constant equation of state (EoS) $\omega^*$. For inflation to be ended, the value of $\omega^*$ should be larger than $-1/3$, in order to satisfy the condition of density energy dominance and preserve the causality, $\omega^*$ must be smaller than 1. During reheating, the (EoS) increased from 0 to 1/3, this will attract attention to study pre-/reheating durations as functions of (EoS) $\omega^*$, to test the effects of (EoS) parameter on both phases. Since $N_k$, $H_k$ and $V_{\text{end}}$ can be related to the scalar spectral index $n_s$, we can therefore use the observational data to place constraints on the pre-/reheating durations for the supersymmetric D-term brane inflation.

5.1 D-term hybrid inflation

D-term inflation was first proposed to solve the fine-tuning problem [29], the D-term require to introduce a U(1) symmetry with a Fayet–Iliopoulos (FI) term $\xi$, in order to have a broken supersymmetric D-term inflation [30]. The D-term Hybrid inflation was derived to the form [31]:

$$V = \frac{g^2 \xi^2}{2} \left( 1 + \alpha \ln \left( \frac{\lambda \phi^2}{\Lambda^2} \right) \right),$$

where $\alpha = g^2/16\pi^2$ and $\Lambda$ is a renormalization mass scale, considering that just before the end of inflation at $\phi_{\text{end}}$, inflation reached a critical value $\phi = \phi_c$, following the results of [5], $\phi_c$ is given as

$$\phi_c = \frac{g}{\Lambda} \sqrt{\xi}.$$

On the other hand, in the high-energy limit $\left( V \gg 2 \tilde{T} \right)$ the well-known slow-roll parameters [32] are given by $\epsilon_k = \left( M_p^2 \tilde{T} V'' \right) / (4\pi V^3)$ and $\eta_k = \left( M_p^2 \tilde{T} V''' \right) / (4\pi V^2)$, where $\epsilon_k \ll 1$, $\eta_k \ll 1$. Assuming that $g^2 \xi^2 / 4 \tilde{T} \gg 1$ in Eq.(38), therefore $g^2 \xi^2$ dominates and $V' / V = \alpha / \phi_k$. We calculate $\epsilon_k$ and $\eta_k$ as

$$\epsilon_k = \frac{\alpha^2}{4\pi} \frac{\tilde{T}}{g^2 \xi^2 / 2} \frac{M_p^2}{\phi_k^2},$$

$$\eta_k = \frac{\alpha}{4\pi} \frac{\tilde{T}}{g^2 \xi^2 / 2} \frac{M_p^2}{\phi_k^2},$$

in order to constrain the parameters of the potential in Eq.(38), the expression of the scalar field at the end of inflation is obtained by solving the equation $|\eta (\phi_{\text{end}})| = 1$ knowing that $\eta \gg \epsilon$. 
\[
\phi_{\text{end}} = \frac{M_p \sqrt{\tilde{T}}}{4\pi^{3/2} \xi}.
\] (41)

Considering the condition \( \phi_c \geq \phi_{\text{end}} \), we obtain
\[
\tilde{T} \leq \frac{16\pi^3 g^2 \xi^3}{M_p^2 \lambda^2}.
\] (42)

Based on Eq. (42) and the analysis in [33], it can be found that
\[
\lambda^2 < \frac{8\pi^2 \sqrt{3 P_s(k)}}{g^2 \xi^2},
\] (43)

\[
\xi < \frac{\sqrt{3 P_s(k)}}{8\pi} M_p^2,
\] (44)

\[
\tilde{T} < \frac{g^2 (3 P_s(k))^{3/2}}{32\lambda^2} M_p^4,
\] (45)

assuming that \( \alpha = g^2 / 16\pi^2 \) is bounded by \( 0.8 \leq \alpha \leq 1.1 \), and the power spectrum of the curvature perturbations is measured as \( P_s \simeq A_s = 2.196_{+0.05}^{-0.06} \times 10^{-9} \) from the recent Planck results [15], numerically we found
\[
\lambda^2 \leq 0.08,
\] (46)

\[
\xi \leq 1.9 \times 10^{31},
\] (47)

\[
\tilde{T} \leq 7.02 \times 10^{61} G e V^4.
\] (48)

The number of e-foldings \( N_k \) during inflation from the time when mode \( k \) leaves the horizon to the end of inflation can be obtained in the high-energy limit as
\[
N_k \simeq \frac{4\pi}{M_p^2 \tilde{T}} \int_{\phi_{\text{end}}}^{\phi_k} \frac{V^2}{V'} d\phi,
\] (49)

which we can apply to the potential Eq. (38) considering that \( g^2 \xi^2 \) dominates and \( V^2 / V' = g^2 \xi^2 \phi_k / \alpha \),
\[
N_k = \frac{4\pi}{\alpha \tilde{T} M_p^2} \frac{g^2 \xi^2}{2} \left( \frac{\phi_k^2}{\phi_{\text{end}}^2} \right),
\] (50)

considering the approximation \( \phi_k \gg \phi_{\text{end}} \), we obtain:
\[
N_k = \frac{4\pi}{\alpha \tilde{T} M_p^2} \frac{g^2 \xi^2}{2} \frac{\phi_k^2}{\phi_{\text{end}}^2},
\] (51)

from the previous equation, we can derive an expression for the inflaton field at the horizon crossing \( \phi_k \)
\[
\phi_k^2 = \frac{\alpha \tilde{T} M_p^2}{2\pi g^2 \xi^2} N_k.
\] (52)

The scalar spectral index defined as [34] \( n_s - 1 = -6\epsilon_k + 2\eta_k \) can be used to find
\[
\frac{M_p^2}{\phi_k^2} = \frac{2\pi g^2 \xi^2 / 2}{\tilde{T}} \frac{(1 - n_s)}{\alpha (3\alpha - 1)},
\] (53)
Table 1 Braneworld-inflationary parameters $n_s$ and $r$ for $50 < N_k \leq 80$ in the case of $\alpha = 0.8$

| Inflationary $e$-folds $N_k$ ($\alpha = 0.8$) | $n_s$ | $r$ |
|---------------------------------------------|-------|-----|
| 50                                          | 0.972 | 0.19 |
| 60                                          | 0.976 | 0.16 |
| 70                                          | 0.980 | 0.13 |
| 80                                          | 0.982 | 0.12 |

Table 2 Braneworld-inflationary parameters $n_s$ and $r$ for $50 < N_k \leq 80$ in the case of $\alpha = 0.9$

| Inflationary $e$-folds $N_k$ ($\alpha = 0.9$) | $n_s$ | $r$ |
|---------------------------------------------|-------|-----|
| 50                                          | 0.966 | 0.21 |
| 60                                          | 0.971 | 0.18 |
| 70                                          | 0.975 | 0.15 |
| 80                                          | 0.978 | 0.13 |

Table 3 Braneworld-inflationary parameters $n_s$ and $r$ for $50 < N_k \leq 80$ in the case of $\alpha = 1$

| Inflationary $e$-folds $N_k$ ($\alpha = 1$) | $n_s$ | $r$ |
|---------------------------------------------|-------|-----|
| 50                                          | 0.960 | 0.24 |
| 60                                          | 0.966 | 0.20 |
| 70                                          | 0.971 | 0.17 |
| 80                                          | 0.975 | 0.15 |

after replacing Eq.(53) in Eqs. (39), (40) and (49), one should get

$$\epsilon_k = \frac{\alpha \left(1 - n_s\right)}{2 \left(3\alpha - 1\right)}, \quad \eta_k = \frac{(1 - n_s)}{2 \left(3\alpha - 1\right)}, \quad N_k = \frac{2 \left(3\alpha - 1\right)}{1 - n_s}. \quad (54)$$

Since the parameter $r$ can be written as $r \simeq 24\epsilon_k$, we can explicit the dependence of parameter $\alpha$ on inflationary observables, to perform this analysis we use

$$n_s - 1 = \frac{1 - 3\alpha}{N_k}, \quad r = \frac{12\alpha}{N_k}. \quad (55)$$

considering inflation $e$-folds in the range $50 \leq N_k \leq 80$, we selected the following values $\tilde{T} \simeq 10^{58} GeV^4$, $\xi \simeq 10^{30}$, $0.8 \leq \alpha \leq 1.1$ and $\lambda \simeq 0.08$ and plotted $r$ as a function $n_s$.

Figure 2 shows that the parameter $r$ is a decreasing function with respect to $n_s$. We observe that $\alpha$ must be $1 \leq \alpha \leq 1.1$ in order to have inflation $e$-folds with scalar spectral index $n_s$ compatible with Planck’s results. We should note that according to Tables 1, 2, 3, 4 the tensor to scalar ratio is in agreement with the observations for $\alpha \leq 1$, considering higher inflation $e$-folds number $N_k \geq 70$. 

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Table 4  Braneworld-inflationary parameters $n_s$ and $r$ for $50 < N_k \leq 80$ in the case of $\alpha = 1.1$.

| $N_k$ | $n_s$ | $r$ |
|-------|-------|-----|
| 50    | 0.954 | 0.26|
| 60    | 0.961 | 0.22|
| 70    | 0.967 | 0.18|
| 80    | 0.971 | 0.16|

Fig. 2  $r$ vs. $n_s$ for D-term braneworld inflation

The quantum fluctuations of the scalar field lead also to fluctuations in the metric. In this way, one can define the amplitude of tensor perturbations as [35]

$$P_h(k) = \frac{64\pi}{M_p^2} \left( \frac{H}{2\pi} \right)^2 F^2(x),$$

(59)

where $x = H M_p \sqrt{\frac{3}{4\pi T}}$ and $F^2(x) = \left( \sqrt{1 + x^2} - x^2 \sinh^{-1} \left( \frac{1}{x} \right) \right)^{-1}$. Note that in the high-energy limit ($V \gg 2\tilde{T}$), $F^2(x) \approx \frac{3}{4}x = \frac{3}{4} \frac{V}{\tilde{T}}$. Thus, allowing

$$P_h(k) = \frac{24}{\pi M_p} H^3 \sqrt{\frac{3}{4\pi T}}.$$  

(60)

The ratio of tensor to scalar perturbations $r$ is given by

$$r = \frac{P_h(k)}{P_s(k)},$$

(61)

the inflation parameter $r$ can be written as $r \approx 24\epsilon_k$. By using Eqs. (60,61), the expression of $H_k$ is given by

$$H_k = \left( \pi M_p A_s \epsilon_k \sqrt{\frac{4\pi \tilde{T}}{3}} \right)^{\frac{1}{3}},$$

(62)
at the pivot scale \( k = 0.05 \text{Mpc}^{-1} \), the power spectrum amplitude measured by Planck is \( P_{\xi} \simeq A_s \), with \( A_s = 2.196^{+0.05}_{-0.06} \times 10^{-9} \) [15]. The brane tension is considered of the order of \( 10^{58} \text{GeV} \) [27]. Finally, for this model, we derive the following expression

\[
H_k = \left( \frac{\pi M_p A_s}{2} \frac{\alpha (1 - n_s)}{3\alpha - 1} \sqrt{\frac{4\pi T}{3}} \right)^{-\frac{1}{3}},
\]

we can determine \( V_{\text{end}} = V(\phi_{\text{end}}) \) using the expression of the scalar field at the end of inflation given by Eq.(41).

We previously constrain the value of brane tension, in that sense, we will study the effect of the brane tension \( \tilde{T} \) on reheating temperature \( T_{re} \). In the high energy limit \( \rho \gg 2\tilde{T} \), Eq.(3) can be written as

\[
H^2 = \frac{4\pi}{3M_p^2} \rho^2 T^{-\frac{5}{3}},
\]

considering \( \rho_{re} = \left( \frac{\pi^2}{30} \right) g_{re} T_{re}^4 \) we obtain the following equation

\[
T_{re} = \left( \frac{3 \times 30^2 M_p^2 H^2}{4\pi^5 g_{re} \tilde{T}} \right)^{\frac{1}{8}},
\]

the value of the scalar field at the inflation \( \phi_{\text{end}} \) leads to an expression of the Friedmann equation as:

\[
H^2(\text{end}) = \frac{4\pi}{3M_p^2} \frac{V_{\text{end}}^2}{T^{-\frac{5}{3}}},
\]

we can compute the reheating temperature \( T_{re} \) as a function of the brane tension \( \tilde{T} \) considering Eq.(65) and (66), to finally obtain

\[
T_{re} = \left( \frac{30^2 V_{\text{end}}^2}{\pi^4 g_{re}^2 \tilde{T}} \right)^{\frac{1}{8}},
\]

Figure 3 shows the evolution of the reheating temperature \( T_{re} \) as a function of brane tension \( \tilde{T} \) for different values of the parameter \( \alpha \). We remark that \( T_{re} \) has an increasing behavior as we increase the tension \( \tilde{T} \). We vary the tension \( \tilde{T} \) in the range \((2 - 10) \times 10^{58} \text{GeV}^4\), for the case \( \alpha = 0.8 \) we get \( T_{re} \sim (8.44 - 8.48) \times 10^{14} \text{GeV} \), but when we consider the case \( \alpha = 0.9 \) the temperature evolves around \( T_{re} \sim (8.69 - 8.73) \times 10^{14} \text{GeV} \), for \( \alpha = 1 \) we obtain \( T_{re} \sim (8.92 - 8.96) \times 10^{14} \text{GeV} \), while the case \( \alpha = 1.1 \) gives \( T_{re} \sim (9.13 - 9.17) \times 10^{14} \text{GeV} \).

In the following, we apply this formalism to study the variation of the reheating temperature and pre-/reheating e-folding number as a function of the perturbation spectrum.

5.2 Reheating case

The reheating phase is parametrized in terms of a duration \( N_{re} \), thermalization temperature \( T_{re} \) at the equilibrium stat of reheating, and equation of state \( \omega^* \). Note that the time evolution of the \( \omega^*(t) \) for different couplings was studied in Ref. [36] and it was shown that it varies slightly for very short times during different phases following inflation. Here, we suppose that \(-1/3 \leq \omega^* \leq 1/3\).
Fig. 3 $T_{re}$ versus $\bar{T}$ for different values of $\alpha$, the black-line corresponds to $\alpha = 0.8$, the red-line corresponds to $\alpha = 0.9$, the blue-line corresponds to $\alpha = 1$, and the green-line corresponds to $\alpha = 1.1$.

Fig. 4 Variation of $N_{re}$ as function of $n_s$ for $\alpha = 0.8$ and $\alpha = 0.9$ using different values of $\omega^*$

In Figs. 4 and 5, we have plotted the variation of reheating $e$-folding number $N_{re}$ as a function of $n_s$ for different values of $\omega^*$, in the case of the D-term hybrid potential in braneworld inflation. The vertical yellow region represents Planck – 2018 bounds on $n_s = 0.9649 \pm 0.0042$ [15]. The point where all the lines converge is called instantaneous reheating and is defined in the limit $N_{re} \rightarrow 0$. We observe that for $\alpha = 1$ all the lines are shifted toward the central value of $n_s$. The case $\alpha = 1.1$ is difficult to reconcile for all $\omega^*$ in the bounds on $n_s$.

In Figs. 6 and 7, we should note that in all cases, the $T_{re}$ converges around $10^{15} GeV$ as may be required by GUT scale baryogenesis models. Temperatures below the light gray region are ruled out by BBN, and light gray region represents $100 GeV$ of electroweak scale. The case $\alpha = 1.1$ is difficult to reconcile for $\omega^* \leq 1/4$ in the bounds on $n_s$. An instantaneous reheating $N_{re} \rightarrow 0$ leads to the maximum temperature at the end of reheating $T_{re} \sim 10^{15} GeV$.
Fig. 5  Variation of $N_{re}$ as function of $n_s$ for $\alpha = 1$ and $\alpha = 1.1$ using different values of $\omega^*$

Fig. 6  Variation of $T_{re}$ as function of $n_s$ for $\alpha = 0.8$ and $\alpha = 0.9$ using different values of $\omega^*$

Fig. 7  Variation of $T_{re}$ as function of $n_s$ for $\alpha = 1$ and $\alpha = 1.1$ using different values of $\omega^*$
5.3 Constraints on reheating temperature from gravitino abundance

The so-called “gravitino problem”\cite{37} is defined as when the gravitino decays forward to big bang nucleosynthesis (BBN), its energetic daughters would destroy the light nuclei via photo-dissociation, upsetting BBN’s successful prediction. To overcome this problem, an upper bound on the reheating temperature after inflation is required, knowing that thermal scatterings in plasma from the reheating stage following inflation cause graviton formation \cite{38}. Thus, it has been found that gravitino abundance is directly proportionate to the reheating temperature $T_{re}$ and can be approximated as \cite{39}

$$Y_{\psi} \simeq 10^{-11} \left( \frac{T_{re}}{10^{10} \text{GeV}} \right)^{10^{-11}},$$

(68)

where $Y_{\psi}$ being the entropy density. This abundance should be low so that the gravitino decay products do not destroy the light elements successfully produced during BBN; thus, we obtain the upper bound on the reheating temperature such as \cite{40}

$$T_{re} \leq 10^{6-7} \text{GeV}.$$  

(69)

This result will help us understand more about the final reheating temperature constraints since we previously concluded that reheating can be considered as instantaneous if the temperature reached $T_{re} \sim 10^{15} \text{GeV}$, we can no longer consider this case in order to avoid the gravitino abundance from reheating. Having this bound on $T_{re}$ will affect the preheating duration that we will compute in the next section, from Eq.(37) a lower value of reheating temperature can cause a decrease in the $e$-folds number of preheating.

5.4 Preheating case

As mentioned previously, the process of preheating happens in the early stages of the Universe evolution. This is considered to be necessary since as the universe expands, it cools down. Thus, right after inflation, there must be a period to make it prepare thermally for next steps. By taking this preheating period into account, we are able to find some constraints on the duration $N_{pre}$ that we show to be independent of the (EoS) $\omega^*$, this can be regarded as a possibility for gaining information about the preheating period.

In Fig. 8, we present a variation of $N_{pre}$ as functions of (EoS) $\omega^*$, the blue line is the maximum value that preheating could take ($N_{pre} \approx 10$), below this value the region with
Fig. 9  Variation of $N_{pre}$ as a function of $n_s$ for different values of parameter $\alpha$

In Fig. 9, we have computed $N_{pre}$ as a function of $n_s$, each line represents a specific value of parameter $\alpha$, we chose $\omega^* = 0$ taking into consideration the result of the work [36], that proved this value is necessary for an efficient preheating, knowing that preheating duration is independent of the choice of (EoS) $\omega^*$. The cases $\alpha = 0.8$ and $\alpha = 0.9$ are compatible with observations. However, the values $\alpha = 1$ and $\alpha = 1.1$ do not reproduce a preheating duration with compatible index spectral $n_s$ according to Planck’s data.

6 Conclusion

In this work, we have studied the pre-/reheating after supersymmetric brane-world inflation. We reviewed reheating in the context of RSII inflation by introducing the ratio $H_0/H$ and visualized the effect of reheating temperature. We have also studied the preheating mechanism in the hybrid brane inflation and set conditions on coupling constants for efficient $\chi$–particle production. By calculating the reheating temperature and pre-/reheating durations quantified in terms of e-foldings number, we show that preheating is related to the reheating temperature $T_{re}$ that affect the parameter $N_{pre}$ and prove $N_{pre}$ to be independent on the (EoS) $\omega^*$. We have set constraints on the parameters of D-term hybrid potential, for the same model we tested the compatibility of the inflationary parameters $n_s$ and $r$ with Planck’s data. We proposed as well an investigation of the brane effect on the reheating epoch. We have applied the form of the Hybrid D-term potentials presented in previous works to study preheating and reheating constraints and obtain information about pre-/reheating from the Cosmic Microwave Background. We have shown that the Hybrid D-term model shows good compatibility for $\alpha \leq 1$ in the case of reheating while the preheating case demands that $\alpha \leq 0.9$ from recent observations.

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