Weighing in on the Higgs

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(Dated: December 21, 2013)

Abstract

Assuming the validity of the Standard Model, or more generally that possible physics beyond it would have only small effects on production cross sections, branching ratios and electroweak radiative corrections, I determine the mass of the Higgs boson to \( M_H = 124.5 \pm 0.8 \) GeV at the 68% CL. This is arrived at by combining electroweak precision data with the results of Higgs boson searches at LEP 2, the Tevatron, and the LHC, as of december of 2011. The statistical interpretation of the method does not require a look-elsewhere effect correction. The method is then applied to the data available at the time of the 2012 summer conferences. In this case, a remarkable bell-shaped \( M_H \) distribution is observed, and \( M_H = 125.5 \pm 0.5 \) GeV is extracted. The significance of the bulk (signal) region of the distribution of neither experiment actually exceeds five standard deviations, but the combination implies a 6.8 \( \sigma \) effect.
I. INTRODUCTION

The LHC Collaborations ATLAS [1] and CMS [2] have presented preliminary combinations of their Standard Model (SM) Higgs boson searches in data sets which correspond in the most sensitive channels to integrated luminosities of 4.6 to 4.9 fb$^{-1}$ of $pp$ collisions at $\sqrt{s} = 7$ TeV. In addition, the CDF and DØ Collaborations combined results on searches in $p\bar{p}$ collisions at the Tevatron [3] at $\sqrt{s} = 1.96$ TeV, based on luminosities ranging from 4.0 to 8.6 fb$^{-1}$. In this brief communication, I analyze their findings simultaneously with earlier results from LEP 2 [4] and with constraints from electroweak precision data. The goal is to obtain the most likely values for the mass, $M_H$, of the SM Higgs boson by taking all experimental information at face value and by explicitly accounting for any tensions or inconsistencies between data and background hypotheses, data and signal hypotheses, as well as (implicitly) between different data sets. The statistical interpretation is unambiguous within Bayesian data analysis, the natural framework [5, 6] for parameter estimation. (For an alternative approach, see Ref. [7].) Thus, I give an answer to the question: “Assuming the approximate validity of the SM and allowing all experimental information, what is $M_H$?”.

This article is organized as follows: Sec. II describes both the method and the data used in this work. The results are presented in Sec. III, while Sec. IV gives conclusions and an outlook. Finally, Appendix A incorporates significant experimental updates including first results from the LHC operating at $\sqrt{s} = 8$ TeV.

II. DATA

A. Data treatment

The master equation used for this is given by,

$$p(M_H) = e^{-\chi^2_{EW}(M_H)/2} \frac{Q_{\text{LEP}}}{Q_{\text{Tevatron}}} \frac{Q_{\text{LHC}}}{M_H^{-1}},$$

(1)

where the first factor is from the precision data, and $Q_{\text{LEP}}(M_H)$ and $Q_{\text{Tevatron}}(M_H)$ are the ratios of the likelihood for the signal of a particular $M_H$ hypothesis plus the background (H+B) to that of the background (B) alone [8]. Similarly, $Q_{\text{LHC}} = Q_{\text{ATLAS}}(M_H) Q_{\text{CMS}}(M_H)$. Unfortunately, for the latest ATLAS and CMS data these quantities have not been made
publicly available. I construct $Q_{\text{ATLAS}}$ and $Q_{\text{CMS}}$ through the relation,

$$2 \ln Q \equiv \chi^2_{H+B} - \chi^2_B \equiv \left( \frac{1 - \bar{\sigma}_{\text{obs}}}{\Delta \bar{\sigma}_+} \right)^2 - \left( \frac{\bar{\sigma}_{\text{obs}}}{\Delta \bar{\sigma}_-} \right)^2,$$

(2)

where $\bar{\sigma}_{\text{obs}}$ can be thought of as an effective observed cross section combining the various channels considered by the LHC Collaborations. It is normalized to the corresponding Higgs boson cross-section at the reference $M_H$, i.e. $\bar{\sigma}_S(M_H) \equiv 1$. The errors are in general asymmetric, with $\Delta \bar{\sigma}_+$ ($\Delta \bar{\sigma}_-$) pointing in the signal (background) direction. One expects $\Delta \bar{\sigma}_+ > \Delta \bar{\sigma}_-$ from Poisson statistics, but cases with $\Delta \bar{\sigma}_+ < \Delta \bar{\sigma}_-$ also occur frequently. If a fluctuation below the background is seen, $\bar{\sigma}_{\text{obs}} < 0$, then $\Delta \bar{\sigma}_+$ is used in both terms in Eq. (2), and conversely, whenever $\bar{\sigma}_{\text{obs}} > 0$ then only $\Delta \bar{\sigma}_-$ enters. The factorized form (1) is a reflection of the fact that mutual correlations between ATLAS and CMS, and between the LHC and the Tevatron, are ignored. This is a justifiable approximation, since in the most important regions counting rates are low and therefore statistical uncertainties expected to dominate.

Consider as an example the excess at 126 GeV seen by ATLAS (Sec. II B 1). In this case, $\chi^2_B = 9.8$ and since the H+B hypothesis itself is not perfectly matched either, $\chi^2_{H+B} = 1.1$, Eq. (2) gives,

$$2 \ln Q_{\text{ATLAS}}(126 \text{ GeV}) = -8.7$$

Note, that $\sqrt{\chi^2_B} = 3.1$ is 0.5σ lower than the quoted local significance of 3.6σ. This can be traced to the signal cross section which introduces an additional uncertainty (dominated by $\sim \pm 20\%$ from the gluon-fusion production [9]). While it does not affect the $p$-value for an excess over background it does enter this analysis which is based directly on a determination of the signal strength[1] and thus works to reduce the significance. Conversely, the significance for background-like outcomes is enhanced. Thus, $Q_{\text{LHC}}$ tends to be on the conservative side in the most interesting mass region, which compensates for the neglected correlations between ATLAS and CMS.

This completes the definition of the likelihood model used for this analysis. The last factor in Eq. (2) is the (improper) non-informative prior density chosen such that the variable ln $M_H$ has a flat prior which one can argue is the most conservative (least informative) one for a

1 Alternatively, one could compute $Q(M_H)$ directly from the $p$-values for H+B and B hypotheses, but the former have not been made available for CMS, and it is preferable to treat both LHC experiments in identical ways.
FIG. 1. Combination of all direct SM Higgs boson search results (see text). Even at the 5 \( \sigma \) level of confidence (in the loose frequentist sense), there are only two remaining \( M_H \) ranges (ignoring another local minimum near 540 GeV), namely \( 115.6 \text{ GeV} < M_H < 128.1 \text{ GeV} \) and \( M_H > 584 \text{ GeV} \).

variable defined over the real numbers. The numerical significance of changing to a prior which is flat in \( M_H \) itself does not exceed the 0.1 GeV level in the determination of \( M_H \).

Before discussing the results for \( p(M_H) \) in Section III, I summarize some of the individual findings, and how they enter into this analysis (see also, the recent historical account in Ref. [10]).

B. Input Data

1. ATLAS

ATLAS excludes the Higgs boson mass ranges from 112.7 to 115.5 GeV, from 131 to 237 GeV, and from 251 to 453 GeV at the 95% CL. An excess of events is observed for a Higgs boson mass close to \( M_H = 126 \) GeV. The maximum local significance of this excess is 3.6 \( \sigma \) above the expected background, while the probability of such a fluctuation to happen anywhere in the full explored Higgs mass domain corresponds to a global significance of 2.3 \( \sigma \). The three most sensitive channels in this mass range, \( H \to \gamma\gamma, H \to ZZ^{(*)} \to \ell^+\ell^-\ell^+\ell^-, \) and \( H \to WW^{(*)} \to \ell^+\nu\ell^-\bar{\nu}, \) contribute individual local significances of 2.8 \( \sigma \), 2.1 \( \sigma \), and 1.4 \( \sigma \), respectively, to the excess [1].
FIG. 2. Combination of all direct SM Higgs boson search results with the indirect precision data. Compared to Fig. [1] only the low mass window remains.

There is also an excess number of $H \rightarrow ZZ$ candidates around $M_H = 244$ GeV and towards the upper end of the search window (600 GeV). They are of lower significance but describe the H+B hypothesis better than the background, given that

$$2 \ln Q_{ATLAS}(244 \text{ GeV}) \approx 2 \ln Q_{ATLAS}(560 \text{ GeV}) \approx -3$$

are negative.

2. CMS

Based on the $\gamma\gamma$, $b\bar{b}$, $\tau^{+}\tau^{-}$, $W^{+}W^{-}$, and $ZZ$ decay channels, CMS excludes the Higgs mass range from 127 GeV to the upper end of the search interval of 600 GeV (95% CL). In the remaining search interval between 110 and 127 GeV two excesses are observed: three candidate $H \rightarrow ZZ^{(*)} \rightarrow \ell^{+}\ell^{-}\ell^{+}\ell^{-}$ events were reconstructed consistent with $M_H = 119.5$ GeV, compared to 1.7 (0.7) expected events for the H+B (B) hypothesis. While this is corroborated by an excess in $H \rightarrow WW^{(*)}$ and also in the less significant $b\bar{b}$ and $\tau^{+}\tau^{-}$ channels, the more sensitive $H \rightarrow \gamma\gamma$ channel shows a deficit below background. When combined, Eq. (2) yields,

$$2 \ln Q_{CMS}(119.5 \text{ GeV}) = -5.6$$
On the other hand, there is an excess in $H \to \gamma\gamma$ corresponding to $M_H = 123.5$ GeV. The signal strength is $1.7 \pm 0.8$ times the expected one which amounts to a local significance of $2.3 \sigma$. Including the other channels — which are consistent with both the B and H+B hypotheses — gives a local (global) significance of $2.6 \ (1.9) \sigma \ [2]$. When combined these data match perfectly with $M_H = 124$ GeV ($\chi^2_{H+B} = 0$), and Eq. (2) gives,

$$2 \ln Q_{CMS}(124 \text{ GeV}) = -6.6,$$

where in this case the value of $\sqrt{\chi^2_B} = 2.6$ agrees exactly with the quoted local significance.

3. **Tevatron**

The most recent combination from the Tevatron is based on 71 mutually exclusive final states from CDF and 94 from DØ. A small excess of data events is found in the mass range between 125 GeV and 155 GeV with

$$2 \ln Q_{Tevatron}(130 \text{ GeV}) = -1.9,$$

while the region between 156 GeV and 177 GeV is excluded at the 95% CL \[3\].

4. **LEP 2**

The input from LEP 2 is unchanged with respect to Ref. \[8\] which was the last analysis of the type presented here before the LHC started data taking in earnest. At LEP 2 with energies up to $\sqrt{s} \approx 209$ GeV, the Higgs boson was searched for in the dominant ($\approx 74\%$) $b\bar{b}$ decay channel, produced in the Higgsstrahlung process, $e^+e^- \to ZH$. In addition, the $H \to \tau^+\tau^-$ channel ($\approx 7\%$) was studied for the $Z$ boson decaying into two jets. The combination \[4\] of the four experiments, all channels and all $\sqrt{s}$ values, resulted in the nominal lower bound, $M_H \geq 114.4$ GeV. However, the combined data are neither particularly compatible with the hypothesis $M_H = 115$ GeV (15% CL), nor with background only (9% CL). The reason is that the results by ALEPH are by themselves in very good agreement with $M_H \approx 114$ GeV (due to an excess in the 4-jet channel) thereby strongly rejecting the background only hypothesis, while the results based on the other channels and experiments (especially DELPHI) are incompatible with any signal. Overall, a signal for $115 \text{ GeV} \leq$
$M_H \leq 119.5$ GeV is favored by the data, but not with high significance,

$$2 \ln Q_{\text{LEP}}(117 \text{ GeV}) = -1.7$$

The combination of all direct search results are illustrated in Fig. 1. Shown is the $\chi^2$ difference relative to the most signal-like Higgs mass of 125 GeV,

$$\Delta \chi^2 \equiv -2 \ln \frac{p(M_H)}{p(125 \text{ GeV})},$$

where $-2 \ln p(125 \text{ GeV}) = 13.2$. This value is indicated by the red line in the figure and corresponds to vanishing reach or else to cases where the overall search results are equally well (or poorly) described by the H+B and B hypotheses.

5. **Precision Data**

The input electroweak precision data are dominated by the results of the $Z$-pole experiments at LEP and the SLC [11] and correspond basically to those described in detail in Ref. [12]. Despite a few discrepancies, the fit describes well the data with a $\chi^2$/d.o.f. = 45.6/42. The probability of a larger $\chi^2$/d.o.f. is 33%. Only the muon magnetic moment anomaly from BNL [14] and the $b$-quark forward-backward asymmetry from LEP 1 are currently showing large (3.1σ and 2.7σ) deviations. In addition, the polarization asymmetry from SLD differs by 1.7σ. The effective $\nu$-quark coupling $g_\nu^2$ from NuTeV [15] is nominally in conflict with the SM, as well, but the precise status is under investigation by the Collaboration.

By themselves, the precision data give the 1σ result,

$$M_H = 99^{+28}_{-23} \text{ GeV},$$

which covers exactly the low mass range not yet excluded by CMS. Compared to previous analyses [5,8] the indirect precision data now play a less pronounced rôle as they do not have much discriminatory power within the remaining low mass window, 115.5 GeV $< M_H < 127$ GeV, since

$$\chi^2_{\text{EW}}(127 \text{ GeV}) - \chi^2_{\text{EW}}(115.5 \text{ GeV}) = 0.63$$

However, they are the only source of information in the high mass region, $M_H \gtrsim 600$ GeV, which is currently beyond the reach of the LHC. And they are crucial to guarantee a normalizable posterior density.
FIG. 3. The normalized probability distribution of $M_H$ in the low mass region based on all data. Shown in green (blue) is the 68% (98.2%) CL highest probability density region.

The combination of all direct search results with the indirect precision data is shown in Fig. 2. Compared to Fig. 1 the high mass region is now also ruled out at the 8 $\sigma$ level.\(^2\)

III. RESULTS

The main result of this communication is the normalized probability distribution of $M_H$ displayed in Fig. 3 which is based on all available data as summarized in Sec. II B (see Fig. 7 below for the most recent data). Indicated in green is the 68% CL allowed highest probability density region. It is given by the range, $123.7 \text{ GeV} \leq M_H \leq 125.3 \text{ GeV}$, which I write in a more colloquial form as,

$$M_H = 124.5 \pm 0.8 \text{ GeV},$$

even though the central value is close to the minimum within this range rather than representing the mode. Eq. (3) does contain, however, both modes at 124 and 125 GeV which originate from CMS and ATLAS, respectively. Nominally (as reviewed in Sec. II B 1) the latter would be expected to be closer to 126 GeV (see Fig. 4). However, CMS does not see a signal there, and the peak is effectively cut, lowered, and shifted. On the other hand,\(^2\) This is ignoring the fact that perturbation theory becomes unreliable for Higgs masses near the unitarity bound of about 800 GeV and beyond, so that the exclusion in those regions is rather of qualitative nature.
FIG. 4. The normalized probability distribution of $M_H$ in the low mass region based on all data except for CMS. Shown in green is the 68% CL highest probability density region, which is given by $M_H = 126.4^{+0.6}_{-1.5}$ GeV.

the CMS peak at 124 GeV (see Fig. 5) is perfectly consistent with ATLAS, so its peak gets enhanced in the combination. The second CMS peak at 119.5 GeV, however, is clearly disfavored by ATLAS.

Eq. (3) also contains the mean of the distribution given by $M_H = 124.4 \pm 1.0$ GeV, and is almost identical to the 68% CL central interval, where the median happens to coincide with the central value of Eq. (3).

When taken together the twin peak structure in Fig. 3 contains the highest probability density at the 98.2% CL corresponding to 2.4 $\sigma$. This is obtained by summing the probability under each mass bin that is higher than the highest probability mass bin under the subleading peak near 119.5 GeV. This may be set against the 3.6 and 2.6 $\sigma$ maximal local $p$-factors for background fluctuations quoted by ATLAS and CMS, respectively, or to the “de-rated” significances of 2.2 and 0.6 $\sigma$ after accounting for the so-called look elsewhere effect (LEE) [16] which is applicable if the location for a hypothetical excess is a priori unknown. The necessity for the LEE adjustment (i.e., accounting for trial factors) arises from the frequentist set-up. ATLAS and CMS estimate their trial factors based on observed local data fluctuations but the followed procedure [17] is bound to be somewhat arbitrary. Among other things it depends on what one considers a priori excluded by previous experiments.
FIG. 5. The normalized probability distribution of $M_H$ in the low mass region based on all data except for ATLAS. Shown in green are the 68% CL highest probability density regions, which are given by the two ranges, $M_H = 118.8^{+1.0}_{-0.6}$ GeV and $M_H = 123.9^{+1.2}_{-1.8}$ GeV.

or data sets. For example, if “elsewhere” is restricted to the low mass region up to about 145 GeV, the de-rated significances read 2.5 and 1.9 $\sigma$, respectively.

It is amusing that in this way considerations of prior knowledge re-enter the frequentist framework, which is sometimes chosen over the Bayesian one specifically to avoid prior densities\footnote{Any reasonable \textit{probabilistic} model is necessarily equivalent to a Bayesian model with some prior even though the choice of prior may be highly implicit. This is so, because the proof of Bayes' theorem needs no assumptions other than the axioms of probability theory.}. But as can be seen from the discussion in the previous paragraph, the dependence on prior knowledge of the LEE is very strong in the case of CMS, and still 0.3 $\sigma$ for ATLAS\footnote{[1]}. In contrast, it is negligible for the results presented in this work.

IV. CONCLUSIONS AND OUTLOOK

I have collected all experimental information relevant to determine the Higgs boson mass, and performed a simultaneous analysis. The result, $M_H = 124.5 \pm 0.8$ GeV, is remarkably precise and, of course, driven in large parts by the LHC (see Fig. 6 for the probability density when the LHC data are removed). Incidentally, $M_H$ is determined to slightly higher absolute and slightly lower relative accuracy than the top quark mass\footnote{[18]}, and if the SM is correct
FIG. 6. The normalized probability distribution of $M_H$ in the low and intermediate mass regions based on all data except for the LHC. Shown in green is the 68% CL highest probability density.

then the mass of the Higgs boson would have been measured accurately before its existence is indisputably confirmed.

In addition to providing a well-defined determination of $M_H$, the Bayesian statistical model employed here also establishes an unambiguous measure of significance. The highest probability density under the twin peaks in Fig. 3 integrates to 98.2% corresponding to 2.4 $\sigma$. Assuming the two LHC experiments see identical results with the next two or three data sets of the same size, this would increase to 4.6 $\sigma$ or 5.4 $\sigma$, respectively. Thus, the conventional 5 $\sigma$ should be reached roughly with an additional 12 fb$^{-1}$ per experiment of data. This can be achieved in 2012 with a luminosity corresponding to a one third increase relative to a good LHC week in October of 2011 (for example, by decreasing the effective beam size, $\beta^*$, by about 25%).

Appendix A: Updated notes and figures

While this article was under consideration for publication, both the ATLAS [19] and CMS [20] Collaborations announced the observation of a new boson with mass, respectively, given by $M = 126.0 \pm 0.4 \pm 0.4$ GeV and $M = 125.3 \pm 0.4 \pm 0.5$ GeV. A simple weighted average would give, $M = 125.7 \pm 0.4$ GeV. The datasets used correspond to integrated
FIG. 7. The normalized probability distribution of $M_H$ in the low mass region based on all data available in the summer of 2012, with the 68% CL highest probability density region highlighted (in green). Also shown are two reference Gaussian: (i) the dashed one (in black) is centered around the median, $M_H = 125.50$ GeV, and has the same width as the 68% CL central probability interval, \( \pm 0.46 \) GeV; (ii) the solid one (in red) is based on mean and variance, $M_H = 125.49 \pm 0.50$ GeV.

Luminosities of up to approximately 5.1 fb$^{-1}$ collected at $\sqrt{s} = 7$ TeV in 2011 and up to 5.8 fb$^{-1}$ at $\sqrt{s} = 8$ TeV in 2012, and the local significances are quoted at 5.9 and 5.0 $\sigma$ for the two experiments, respectively. The CDF and DØ Collaborations [21] also released a much improved analysis of their data, revealing an excess in the 115 to 140 GeV mass range with a local significance of 3.0 $\sigma$.

Here I update the results reflecting these developments. The 68% CL allowed highest probability density range is now $125.02$ GeV $\leq M_H \leq 125.95$ GeV, or in short,

$$M_H = 125.5 \pm 0.5 \text{ GeV}.$$  \hspace{1cm} (A1)

Unlike the remarks following Eq. (3), the latest data combine to a nearly bell-shaped curve as shown in Fig. 7 with coinciding mean, median, and mode. The significance of the bulk region of the probability distribution, 121.1 GeV $\leq M_H \leq 130.0$ GeV — according to the method introduced here and illustrated in Fig. 8 — is 6.8 $\sigma$, i.e., the tail regions contain a probability of $9 \times 10^{-12}$. Given the greater effectiveness of the 8 TeV LHC for Higgs searches, this is is consistent with but exceeds somewhat the expectation expressed in Sec. IV.
FIG. 8. Combination of all direct SM Higgs boson search results with the indirect precision data, following the updates for the summer conferences of 2012. This is to be compared with Fig. 2, but here the focus is on the low mass region. It is illustrated how the bulk (signal) region can be defined unambiguously, permitting the extraction of the total signal probability. Using the inverse error function, this can then be translated back into the number of standard deviations of the signal without reference to the LEE.

Similarly, Fig. 9 (Fig. 10) shows the corresponding distribution for the combination of the ATLAS (CMS) and LEP 2 Higgs search results with the electroweak precision data. The significance of the ATLAS bulk region is $4.9 \sigma$ and larger that the one from CMS ($4.2 \sigma$), but the CMS data are slightly sharper peaked as can be seen from the reference Gaussian densities also shown in the figures. The probability that the true $M_H$ resides in one of the tails is close to $10^{-6}$ for ATLAS and $3 \times 10^{-5}$ for CMS. These numbers are several orders of magnitude larger than the background fluctuation probabilities ($p$-values) of $1.7 \times 10^{-9}$ and $3 \times 10^{-7}$, respectively.

Finally, Fig. 11 shows the case for the combination of the CDF, DØ and LEP 2 Higgs search results with the electroweak precision data, with a bulk region significance of $3.4 \sigma$. The tail probability of $6 \times 10^{-4}$ is in this case lower than the background $p$-value of $1.5 \times 10^{-3}$. 
FIG. 9. The normalized probability distribution of $M_H$ in the low mass region based on Higgs search results from LEP 2 and ATLAS, as well as the electroweak precision data, with the 68% CL highest probability density region highlighted (in green). Also shown are the median (in black) and mean (in red) motivated reference Gaussian densities (cf. Fig. 7), with $M_H = 126.30 \pm 0.72$ GeV and $M_H = 126.28 \pm 0.71$ GeV, respectively, which happen to be almost identical in this case.

ACKNOWLEDGMENTS

It is a pleasure to thank Nima Arkani-Hamed, Paul Langacker, and Edward Witten for stimulating discussions and encouragement. This work was supported by CONACyT (México) project 82291–F and by PASPA (DGAPA–UNAM).

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FIG. 10. The normalized probability distribution of $M_H$ in the low mass region based on Higgs search results from LEP 2 and CMS, as well as the electroweak precision data, with the 68% CL highest probability density region highlighted (in green). Also shown are the median (in black) and mean (in red) motivated reference Gaussian densities (cf. Fig. 7), with $M_H = 124.69 \pm 0.69$ GeV and $M_H = 124.79 \pm 0.69$ GeV, respectively.

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FIG. 11. The normalized probability distribution of $M_H$ in the low mass region based on Higgs search results from LEP 2 and the Tevatron, as well as the electroweak precision data, with the 68\% CL highest probability density region highlighted (in green). Also shown are the median (in black) and mean (in red) motivated reference Gaussian densities, with $M_H = 119.6 \pm 5.1$ GeV and $M_H = 121.7 \pm 5.5$ GeV, respectively (cf. Fig. 7).

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