The dynamical stability of W Ursae Majoris-type systems

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ABSTRACT

Theoretical study indicates that a contact binary system would merge into a rapidly rotating single star due to tidal instability when the spin angular momentum of the system is more than a third of its orbital angular momentum. Assuming that W Ursae Majoris (W UMa) contact binary systems rigorously comply with the Roche geometry and the dynamical stability limit is at a contact degree of about 70 per cent, we obtain that W UMa systems might suffer Darwin’s instability when their mass ratios are in a region of about 0.076–0.078 and merge into the fast-rotating stars. This suggests that the W UMa systems with mass ratio \( q \leq 0.076 \) cannot be observed. Meanwhile, we find that the observed W UMa systems with a mass ratio of about 0.077, corresponding to a contact degree of about 86 per cent, would suffer tidal instability and merge into the single fast-rotating stars. This suggests that the dynamical stability limit for the observed W UMa systems is higher than the theoretical value, implying that the observed systems have probably suffered the loss of angular momentum due to gravitational wave radiation (GR) or magnetic stellar wind (MSW).

Key words: instabilities – binaries: close – blue stragglers – stars: evolution – stars: rotation.

1 INTRODUCTION

Interest in W Ursae Majoris (W UMa) binaries was revived recently with the discovery of large number of new ones among blue stragglers in open and globular clusters (Kaluzny & Shara 1988; Kaluzny 1990; Mateo et al. 1990; Yan & Mateo 1994; Mateo 1996). It appears likely that at least some of blue stragglers are formed by the merging of contact binaries. The models of W UMa systems (Li, Han & Zhang 2004, 2005) had shown that W UMa systems evolve into contact binaries with extreme mass ratios and then evolve into single, fast-rotating stars (FK Com stars) or blue stragglers due to Darwin’s instability when the spin angular momentum of the system is more than a third of its orbital angular momentum (Hut 1980; Eggleton & Kiseleva-Eggleton 2001).

The spin angular momentum has a significant influence on the evolution of W UMa contact binaries, especially for those with extreme mass ratios. Neglecting the spin angular momentum of the small secondary, Rasio (1995) first derived a cut-off mass ratio for contact binary systems and the cut-off mass ratio depends on the structure and the contact degree. Meanwhile, Rasio & Shapiro (1995) identified the dynamical stability limit at a contact degree of about 70 per cent for W UMa systems. Yang (2005b) derived a cut-off mass ratio of about 0.021 from the observed W UMa systems with mass ratios \( q < 0.25 \). In this work, using the observational data and the theoretical results reported recently, we investigate the dynamical stability limit of W UMa systems, and find that W UMa systems begin merging when their mass ratios are in a region of about 0.076–0.078. This suggests that W UMa systems with mass ratio \( q \leq 0.076 \) cannot be observed. Meanwhile, we estimate that the observed W UMa systems with a minimum mass ratio of about 0.077, corresponding to a contact degree of about 86 per cent, would suffer Darwin’s instability and begin merging. This suggests that the minimum mass ratio for the observed W UMa systems is larger than the theoretical value, implying that the observed W UMa systems have suffered the loss of angular momentum owing to gravitational wave radiation (GR) or magnetic stellar wind (MSW).

2 DYNAMICAL STABILITY LIMIT

It is well known that a contact binary would suffer Darwin’s instability and then coalesce into a fast-rotating single star (including FK Com-type stars and blue stragglers) when the spin angular momentum of the system is more than a third of its orbital angular momentum. The spin angular momentum of a binary star can be expressed as

\[
J_{\text{spin}} = \left( k_1^2 M_1 R_1^2 + k_2^2 M_2 R_2^2 \right) \omega_s,
\]

where \( M_{1,2} \) and \( R_{1,2} \) are the masses and radii of the primary and the secondary in solar units, \( k_{1,2} \) are the ratios of the gyration radii to the stellar radii for both components, and \( \omega_s \) is the spin angular velocity. The orbital angular momentum of the binary system reads

\[
J_{\text{orb}} = \frac{M_1 M_2}{M_1 + M_2} A^2 \omega_s,
\]

where \( \omega_s \) is the orbital angular velocity and \( A \) the orbital radius of the binary. We assume that the W UMa systems are in synchronous...
rotation (i.e. $\omega_1 = \omega_2$) and the ratios of the gyration radii to the stellar radii for both components are equal (i.e. $k_1^2 = k_2^2 = k^2$); the ratio of the spin angular momentum to the orbital angular momentum can be written as

$$R = \frac{J_{\text{spin}}}{J_{\text{orb}}} = \frac{k^2 (M_1 R_1^2 + M_2 R_2^2)}{A^2 M_1 M_2 (M_1 + M_2)} = k^2 \left( 1 + q \left( \frac{R_1}{A} \right)^2 \right) \left( 1 + q \left( \frac{R_2}{R_1} \right)^2 \right),$$

(3)

where $q = M_2/M_1 < 1$ is the mass ratio of the binary system. If a contact binary system is a marginal contact one in which two components have just filled the inner Roche lobes, following Eggleton (1983), the relative radii of both components of the contact binary read

$$R_i = \frac{0.49 q^{2/3}}{A} \frac{q^{2/3}}{0.6 q^{2/3} + \ln(1 + q^{1/3})},$$

(4)

$$R_i = \frac{0.49 q^{2/3}}{A} \frac{q^{2/3}}{0.6 q^{2/3} + \ln(1 + q^{1/3})},$$

(5)

If the two components of a contact binary system have filled the outer Roche lobes, the relative radii of the components can be expressed by the following equations (Yakut & Eggleton 2005):

$$R_i = \frac{0.49 q^{2/3} + 0.27 q - 0.12 q^{3}}{0.6 q^{2/3} + \ln(1 + q^{1/3})},$$

(6)

$$R_i = \frac{0.49 q^{2/3} + 0.15}{0.6 q^{2/3} + \ln(1 + q^{1/3})},$$

(7)

Using equations (3), (4) and (5) and taking $k^2$ to be the values of main-sequence stars (from 0.05 for no-convection stars to 0.21 for fully-convective stars), we can obtain a relationship between the angular momentum ratio $R_o$ and the mass ratio $q$ for marginal contact systems:

$$R_o = k^2 \frac{1 + q}{q} \left( \frac{0.49 q^{2/3}}{0.6 q^{2/3} + \ln(1 + q^{1/3})} \right)^2 \left( 1 + q \left( \frac{0.6 q^{2/3} + \ln(1 + q^{1/3})}{0.6 q^{2/3} + \ln(1 + q^{1/3})} \right)^2 \right).$$

(8)

Setting equation $R_o = 1/3$, and solving (numerically) for $q$, we obtain the minimum mass ratio $q_{\text{min,in}}$ for stability of marginal contact binaries with the different values of $k^2$ of main-sequence stars and the relation between the minimum mass ratio $q_{\text{min,in}}$ and $k^2$ is shown in Fig. 1. Using equations (3), (6) and (7), we obtain a relationship between the angular momentum ratio $R_o$ and the mass ratio $q$ for contact systems in which both components have filled the outer Roche lobes:

$$R_o = k^2 \frac{1 + q}{q} \left( \frac{0.49 + 0.15 q^{2/3}}{0.6 + q^{2/3} \ln(1 + q^{1/3})} \right)^2 \left( 1 + q \left( \frac{0.6 q^{2/3} + \ln(1 + q^{1/3})}{0.6 q^{2/3} + \ln(1 + q^{1/3})} \right)^2 \right).$$

(9)

Setting equation $R_o = 1/3$, and solving (numerically) for $q$, we obtain the minimum mass ratio, $q_{\text{min,out}}$, for stability of W UMa systems in which two components have filled the outer Roche lobes with the different values of $k^2$ and the relation between $q_{\text{min,out}}$ and $k^2$ is also shown in Fig. 1. As seen from Fig. 1, the value of $q_{\text{min,in}}$ is smaller than that of $q_{\text{min,out}}$ at any value of $k^2$. This suggests that the stability of a contact binary with given masses indeed depends on the degree of contact $f$, or filling factor, $0 < F = 1 - f < 1$. Meanwhile, the more is the value of $k^2$, the larger are the minimum mass ratios ($q_{\text{min,in}}$ and $q_{\text{min,out}}$) and the difference between $q_{\text{min,in}}$ and $q_{\text{min,out}}$.

Taking $k^2 = 0.06$ as Rasio (1995), the ratio of the spin angular momentum to the orbital angular momentum against the mass ratio is plotted in Fig. 2. Meanwhile, we obtain $q_{\text{min,in}} = 0.071$ and $q_{\text{min,out}} = 0.078$ for $k^2 = 0.06$. In general, the dynamical instability of W UMa contact binary systems should occur at an overcontact stage with a higher degree of contact rather than at a marginal contact one. The numerical study of the equilibrium and stability of close binary systems (Rasio & Shaprio 1995) had identified the dynamical stability limit at a contact degree of about 70 per cent, corresponding to a minimum mass ratio of about 0.076. This suggests that the W UMa systems with mass ratios in a region of 0.076–0.078 would suffer Darwin’s instability and then coalesce into the fast-rotating single stars (including FK Com stars and blue stragglers), implying that the W UMa systems with mass ratios $q \leq 0.076$ cannot be observed.

We collect the absolute parameters of some of W UMa systems (listed in Table 1) reported recently, together with those compiled by Maceroni & van’t Veer (1996), Gazeas et al. (2005), and by Yakut

![Figure 1](https://i.imgur.com/1Q5Q5Q5.png)

**Figure 1.** The minimum mass ratio for stability $q_{\text{min}}$ in the Roche approximation. The parameter $k^2$ is the dimensionless gyration radius of the stars.

![Figure 2](https://i.imgur.com/2Q5Q5Q5.png)

**Figure 2.** The angular momentum ratio $R$ against the mass ratio, $q$. The solid lines represent theoretical results corresponding to the systems filling the inner Roche lobe and the outer Roche lobe, respectively. The solid dots represent the observed W UMa systems.
and solving (numerically) for \( \log q \) the loss of angular momentum. Since the loss of angular momentum suggests that the observed W UMa systems have probably suffered the loss of angular momentum via MSW, the contact degrees of them have exceeded 0.076, for the observed systems. It is larger than 0.076 for the observed systems. It is larger than 0.076 in and contact degree \( f \) to increase, so that leads \( f \) to have exceeded 70 per cent before the observed systems reaching the minimum mass ratio \( q = 0.076 \).

### 3 DISCUSSIONS AND CONCLUSIONS

The models of low-mass W UMa-type systems (Li et al. 2004, 2005) have shown that W UMa systems would suffer Darwin’s instability and evolve into fast-rotating single stars (i.e. FK Com stars or blue stragglers). Rasio (1995) predicted a cut-off of the mass ratio for W UMa systems at about 0.09. Considering the rotation of the secondary, we obtain that the minimum mass ratio for stability of the marginal contact binaries is of about \( q_{\text{min,obs}} = 0.071 \), suggesting that the rotation of the secondary still definitely has an influence on the determination of the dynamical stability limit of W UMa systems. Rasio (1995) had predicted that the dynamical stability limit should depend on the fill-factor of W UMa systems. Using the new results in studies of the W UMa systems, we investigate the dynamical stability limit of W UMa systems in which both components fill their outer Roche lobes and obtain a minimum mass ratio \( q_{\text{min,out}} \) of about 0.078 for this kind of systems. This suggests that the dynamical stability limit of W UMa systems indeed depends on their fill-factor.

The difference between \( q_{\text{min,obs}} \) and \( q_{\text{min,out}} \) suggests that the dynamical stability of the W UMa contact binaries with given mass depends on the degree of contact \( f = (\Omega_2 - \Omega_{\text{in}})/\Omega_{\text{out}} \), which is consistent with the result predicted by Rasio (1995). Rasio & Shapiro (1995) had identified the dynamical stability limit of W UMa systems at about \( f = 70 \) per cent, which corresponds to a minimum mass ratio of about 0.076. We obtain a minimum mass ratio of about 0.077, corresponding to a contact degree \( f = (q_{\text{min,obs}} - q_{\text{min,in}})/(q_{\text{min,out}} - q_{\text{min,in}}) \approx 86 \) per cent, for the observed systems. The difference between Rasio & Shapiro’s result and ours derived from the observed systems suggests that the observed W UMa systems have probably suffered the loss of angular momentum owing to GR or MSW during the evolution of the observed W UMa systems. In fact, Einstein soft X-ray observations and International Ultraviolet Explorer (IUE) ultraviolet observations (Vaiana 1980; Eaton 1983; Rucinski & Vilhu 1983; Vilhu 1983) have shown that W UMa systems are strong sources. This suggests surface activity of the kind observed on our Sun, and so the presence of the magnetic field. Since the observed W UMa systems rapidly lose the angular momentum via MSW, the contact degrees of them have exceeded 70 per cent before their mass ratios decrease to the minimum mass ratio \( q = 0.076 \).

Among about 200 contact binaries with reliable photometric data, the system V857 Her has an extremely small mass ratio, \( q = 0.0653 \) (Qian et al. 2005) for observed W UMa systems at present. This is a new ‘record’ for a contact binary, surpassing the well-known AW UMa with \( q = 0.075 \) (Rucinski 1992) and SV Crv with \( q = 0.066 \) (Rucinski et al. 2001). It is a tremendous contribution for theory that W UMa systems with the extremely small mass ratios are found. Only a slight difference between observations and theory not only indicates that theory has come close to a decent prediction, but also indicates that some physical processes taking place in the observed W UMa systems have not been considered in the present theory. At first, the dynamical stability limit of W UMa systems may also depend on the structure (i.e. the value of \( k^2 \)) and the value of \( k^2 \) may decrease with the evolution. In addition, the differential rotation of the observed systems would lead the ratios of the gyration radii to the stellar radii of the components of the W UMa systems to decrease. Therefore, the differential rotation of the components of

### Table 1. The physical parameters of some W UMa systems.

| Stars     | \( P \) (d) | \( M_1 \) (\( M_\odot \)) | \( q \) | \( R_1 \) (\( R_\odot \)) | \( R_2 \) (\( R_\odot \)) | References |
|-----------|-------------|-----------------|------|-----------------|-----------------|------------|
| V410 Aur  | 0.3663      | 1.30             | 0.146| 1.40             | 0.61             | (1)        |
| XY Boo    | 0.3705      | 0.91             | 0.186| 1.23             | 0.61             | (1)        |
| AH Cnc    | 0.3605      | 1.21             | 0.149| 1.36             | 0.62             | (2)        |
| CW Cas    | 0.3188      | 1.06             | 0.547| 1.01             | 0.76             | (3)        |
| V776 Cas  | 0.4404      | 1.63             | 0.13 | 1.71             | 0.71             | (4)        |
| V899 Her  | 0.4212      | 2.10             | 0.566| 1.57             | 1.22             | (5)        |
| BB Peg    | 0.3615      | 1.38             | 0.362| 1.26             | 0.76             | (6)        |
| V351 Peg  | 0.5933      | 1.63             | 0.361| 1.87             | 1.19             | (7)        |
| AU Ser    | 0.3865      | 0.90             | 0.710| 1.10             | 0.94             | (8)        |
| VZ Psc    | 0.2613      | 0.81             | 0.800| 0.78             | 0.70             | (9)        |

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2. Zhang, Zhang & Deng (2005);
3. Barone et al. (1988);
4. Djurasevic et al. (2004);
5. Ozdemir et al. (2002);
6. Lu & Rucinski (1999);
7. Albayrak et al. (2005);
8. Gurol (2005);
9. Hrivnak, Guinan & Lu (1995).

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**Figure 3.** The angular momentum ratio \( R \) against the mass ratio, \( q \). The solid line represents the fitting result. The solid dots represent the observed W UMa systems.
the W UMa systems is another physical reason why some W UMa systems with the mass ratios smaller than the minimum mass ratio 0.076 still can be observed. However, contact binaries with mass ratios significantly below those of SX Crv and V857 Her cannot be observed, since they are tidally unstable and quickly merge into a single, fast-rotating object, on a tidal time-scale of about $10^3$–$10^4$ yr (Rasio 1995).

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