Exact Analytic Solution for a Ballistic Orbiting Wind

Francis P. Wilkin and Harry Hausner
Union College, Schenectady, NY 12308, USA

Received 2016 December 31; revised 2017 June 7; accepted 2017 June 8; published 2017 July 19

Abstract

Much theoretical and observational work has been done on stellar winds within binary systems. We present a new solution for a ballistic wind launched from a source in a circular orbit. The solution is that of a single wind—no second wind is included in the system and the shocks that arise are those due to the orbiting wind interacting with itself. Our method emphasizes the curved streamlines in the corotating frame, where the flow is steady-state, allowing us to obtain an exact solution for the mass density at all pre-shock locations. Assuming an initially isotropic wind, fluid elements launched from the interior hemisphere of the wind will be the first to cross other streamlines, resulting in a spiral structure bounded by two shock surfaces. Streamlines from the outer wind hemisphere later intersect these shocks as well. An analytic solution is obtained for the geometry of the two shock surfaces. Although the inner and outer shock surfaces asymptotically trace Archimedean spirals, our tail solution suggests many crossings where the shocks overlap, beyond which the analytic solution cannot be continued. Our solution can be readily extended to an initially anisotropic wind.

Key words: binaries – general – hydrodynamics – ISM: bubbles – planetary nebulae: general – shock waves – stars: winds, outflows

1. Introduction

Virtually all stars have a wind, and a large fraction of stars are in binary or multiple systems (Duchêne & Kraus 2013), where the orbital motion will influence the flow of the wind. The motion of the driving source results in a “reflex” perturbation somewhat similar to parallax, whereby the flow is altered due to the changing location and velocity of the source (Soker 1994; Cantó et al. 1999; He 2007; Raga et al. 2011).

Binarity has been invoked as the cause of a wealth of structures in planetary nebulae and their asymptotic giant branch (AGB) precursors (Morris 1981; Livio 1982; Soker 1994; Mastrodemos & Morris 1999). Dramatic examples include spiral nebulae such as AFGL 3068 (Mauron & Huggins 2006; Morris et al. 2006) and CIT 6 (Kim et al. 2013). Detailed analyses have been presented to use the nebular morphology and kinematics to diagnose unresolved features of the drivers, both to infer binarity and binary separation, as well as to determine wind properties (Kim & Taam 2012c; Homan et al. 2015). The gravitational wake mechanism (Kim 2011; Kim & Taam 2012a) can also produce a spiral pattern, but has a more flattened morphology, allowing one to, in principle, distinguish the presence of two separate, spiral morphologies. Additionally, the presence of two winds in a binary system leads to a collision region bounded by two shocks, which may develop mixing in the presence of a wealth of instabilities (Myasnikov & Zhekov 1991; Monnier et al. 1999; Lamberts et al. 2012; Hendrix et al. 2016; Rauw & Nazé 2016).

We present a new, exact solution to the problem of an initially isotropic wind launched ballistically from an orbiting point source in a circular orbit. Although the collision of two such winds could be considered and was indeed our original motivation, the present work considers only a single wind source, as if the second mass did not launch a wind at all. In Section 2, we describe the streamlines and kinematics; in Section 3, we derive the density structure, and Section 4 discusses the shock criterion and the implications of this work.

2. Mathematical Formulation

We consider a wind launched from an orbiting star. Although the motion will clearly be time-dependent, a steady-state flow pattern should exist in the reference frame corotating with the star’s orbital motion about the center of mass, provided the orbit is circular and the wind launch conditions are unchanging in that frame. Let the star orbit a distance $R_o$ from the origin in the $X$-$Y$ plane, at angular frequency $\omega$ and counterclockwise as viewed from above. In the inertial frame, the star’s position and velocity as a function of time are

$$
R_\omega(t) = R_o(\dot{x} \cos \omega t + \dot{y} \sin \omega t),
$$

$$
V_\omega(t) = \omega R_o(\dot{x} \sin \omega t + \dot{y} \cos \omega t).
$$

Here $\dot{x}$ and $\dot{y}$ are the usual unit vectors along the $X$ and $Y$ axes. The Cartesian coordinates in the inertial and corotating (primed) frames are related by $Z = Z'$ and

$$
X = X' \cos \omega t - Y' \sin \omega t,
$$

$$
Y = X' \sin \omega t + Y' \cos \omega t.
$$

Now consider a fluid element launched from the star’s location, taken as a point source. Neglecting gravitational, radiative, pressure, and other forces, the fluid element has unchanging velocity in the inertial frame, provided that streamlines do not cross. Our neglect of gravitational force on the ballistically moving fluid element is most valid for wide binaries, where the launch speed is assumed to be characteristic of the depth of the gravitational well of the launching source, and much faster than the escape speed from the binary system. By assuming the wind launches impulsively from a point source at time $\tau$, we can write a fluid trajectory equation

$$
R(t) = R_\omega(\tau) + (t - \tau)[V_\omega(\alpha, \delta, \tau) + V_w(\alpha, \delta, \tau)],
$$

---

1 Present address: Department of Physics, University of Wisconsin-Madison.
where \( t > \tau \). The velocity of the fluid element in the inertial frame is the star’s velocity at the time of launch \( \mathbf{V}_w(\tau) \), plus the launch velocity \( \mathbf{V}_w'(\alpha, \delta, \tau) \) in the comoving frame of the star for the appropriate launch direction. We define launch angles in the corotating frame, with \( \alpha \) being azimuthal angle counterclockwise relative to the \( X' \) axis, and \( \delta \) being the latitude above the \( X'Y' \) plane (see Figure 1).

The inverse transformation is used to obtain the unit vectors at the launch time:

\[
\hat{x}'(\tau) = \hat{x} \cos \omega \tau + \hat{y} \sin \omega \tau, \\
\hat{y}'(\tau) = -\hat{x} \sin \omega \tau + \hat{y} \cos \omega \tau.
\]  

(5)

If we take the initial launch of the wind in the corotating frame to be steady and isotropic, we have simply \( \mathbf{V}_w'(\alpha, \delta, \tau) = \mathbf{V}_w \hat{v}_c \). Here \( \hat{v}_c \) is a unit vector pointing radially away from the star and given in terms of the launch angles by

\[
\hat{v}_c = \hat{x}'(\tau) \cos \delta \cos \alpha + \hat{y}'(\tau) \cos \delta \sin \alpha + \hat{z}(\tau) \sin \delta.
\]  

(6)

Using Equations (1), (2), and (4)–(6), the trajectory in the inertial frame in terms of the launch angles is given by

\[
X = R_o \cos \omega \tau + (t - \tau)[V_w \cos \delta \cos \beta - R_o \omega \sin \omega \tau], \\
Y = R_o \sin \omega \tau + (t - \tau)[V_w \cos \delta \sin \beta + R_o \omega \cos \delta \omega \tau], \\
Z = (t - \tau) V_w \sin \delta,
\]  

(7)

where, for brevity, we have introduced \( \beta \equiv \alpha + \omega \tau \). We nondimensionalize the equations by using length unit \( R_o \) and time unit \( R_o/V_w \), so dimensionless position is \( \mathbf{r} = \mathbf{R}/R_o \) with dimensionless components \( x, y, \) and \( z \), and dimensionless velocity is \( \mathbf{v} = \mathbf{V}/V_w \). We introduce the dimensionless parameter

\[
p = \frac{\omega R_o}{V_w},
\]  

(8)

as the ratio of orbital speed to wind launch speed. Using the inverse transformation from Equation (5) (applied at time \( t \) rather than \( \tau \)), and defining a time-like coordinate along the streamline as

\[
s = \frac{V_w}{R_o}(t - \tau),
\]  

(9)

we obtain the (dimensionless) trajectory in the corotating frame:

\[
x' = \cos \delta \cos \gamma + \sin \delta \cos \alpha - s \cos \delta \sin (\alpha - p s), \\
y' = \sin \delta \cos \gamma - \sin \delta \sin (\alpha - p s), \\
z' = s \sin \delta.
\]  

(10)

2 In spherical coordinates,

\[
r' = \sqrt{1 + s^2 + p^2 s^2 + 2 s \cos \delta (\alpha + ps \sin \alpha)}.
\]  

(11)

The (dimensional) velocity in the corotating frame can be obtained by taking the derivative of \( \mathbf{R}' = R_o \mathbf{r}' \) with respect to time \( t \). However, since \( \tau \) is constant as the fluid element moves, we can equivalently differentiate the dimensionless position \( \mathbf{r}' \) with respect to \( s \). This yields the primed, dimensionless velocity components

\[
\mathbf{v}_c' = \cos \delta \cos \gamma + ps [p \cos \delta \cos \gamma] \\
\mathbf{v}_c' = \cos \delta \sin \gamma - ps [p \sin \delta \cos \gamma] \\
\mathbf{v}_c' = \sin \delta
\]  

(12)

where \( \gamma \equiv \alpha - p s \). Equations (10) and (12) give a complete description of the wind kinematics in the corotating frame in parametric form. By taking the limit of no orbital motion, which corresponds to vanishing \( p \), we can see that these equations yield free streaming at the wind speed in the initial launch direction. We also require the magnitude of the velocity, whose square is given by

\[
\mathbf{v}'^2 = 1 + p^2 s^2 (\cos^2 \delta + p^2) + 2 (\cos \alpha + ps \sin \alpha) \cos \delta.
\]  

(13)

Figure 2 shows the streamline shape in the rotating frame for the value \( p = 0.5 \). Note the characteristic trailing spiral pattern. As will be described in Section 4, streamline crossing begins at the location of the inner and outer solid black curves, and is confined to the region between them. Figure 3 shows the streamline shape in the rotating frame for four launch angles and three values of \( p \). The streamlines are nearly straight for small \( p \), and become increasingly distorted in a trailing spiral pattern at larger \( p \) and \( s \) values. For sufficiently large values of \( s \) (for any \( p > 0 \)), streamlines will eventually cross, indicating the formation of a shock.

### 3. Derivation of the Fluid Density

The behavior of the density \( \rho \) as a function of position can be determined by solving the mass conservation equation in steady-state in the rotating frame. We introduce the density unit

\[
\rho_o = \frac{M_w}{4\pi R_o^2 V_w},
\]  

(14)

\(^2\) The coordinate \( s \) does not, however, represent arc length along a streamline. In fact, \( ps \) is simply the angle in radians that the launching star has moved about the center of mass since launch of the fluid element at time \( \tau \).
where \( \dot{\mathcal{M}_w} \) is the wind mass-loss rate, so the dimensionless density is \( \varrho = \rho / \rho_b \). Using \( \mathcal{V} \), for regions where streamlines have not crossed, the mass conservation equation is

\[
\nabla \cdot (\varrho \mathcal{V}) = 0. \tag{15}
\]

Our derivation will involve elements of tensor calculus because the most appropriate set of curvilinear coordinates will not result in an orthonormal basis. The reader unconcerned with the details may safely jump to Equation (23), our result for the density.

We choose a set of coordinates \( \{ \mathcal{X} \} = \{ \alpha, \delta, s \} \), where the superscript is not an exponent but an index that takes values \( \alpha, \delta, s \). The natural (covariant) basis vectors with respect to these coordinates are given by

\[
\mathbf{b}_i = \frac{\partial \mathbf{r}'}{\partial \mathcal{X}^i}.
\]  

where the position vector \( \mathbf{r}' \) is evaluated using Equation (10). The first two basis vectors are

\[
\mathbf{b}_a = -\mathcal{X}' \cos \delta \sin \gamma + \mathcal{Y}' \cos \delta \cos \gamma, \quad \mathbf{b}_b = -\mathcal{X}' \sin \delta \sin \gamma + \mathcal{Y}' \sin \delta \cos \gamma. \tag{17}
\]

while the third is \( \mathbf{b}_c = \mathbf{v}' \), whose Cartesian components are given by Equation (12). Because the velocity vector is entirely in the direction of \( \mathbf{b}_c \), the contravariant components of the velocity vector are \( \mathbf{v}' = \mathbf{v}^c = 0 \), and \( \mathbf{v}^2 \) is given by Equation (13). The determinant \( g \) of the metric tensor is

\[
g = (1 + p^2 s \cos \alpha \cos \delta)^2 s^4 \cos^2 \delta. \tag{19}
\]

This allows us to take a divergence using our coordinate system. For a vector \( \mathbf{A} = A^a \mathbf{b}_a + A^b \mathbf{b}_b + A^c \mathbf{b}_c \), the divergence is (Aris 1962)

\[
\nabla \cdot \mathbf{A} = \frac{1}{\sqrt{g}} \left[ \frac{\partial}{\partial \alpha} (\sqrt{g} A^\alpha) + \frac{\partial}{\partial \delta} (\sqrt{g} A^\delta) + \frac{\partial}{\partial s} (\sqrt{g} A^s) \right]. \tag{20}
\]

If we apply this formula for divergence to the conservation of \( \varrho \mathbf{v}' = \varrho \mathbf{v} \), according to Equation (15), only the third term of the divergence is needed since the first two components vanish. We obtain upon integration

\[
\sqrt{g} \varrho = f (\alpha, \delta). \tag{21}
\]

The function \( f (\alpha, \delta) \) is determined by matching the above formula to the behavior close to the wind source where the flow is isotropic in the limit of small \( s \). Considering the mass density at a small radius \( R_s = R_s \delta \) centered on the source, \( \rho = \frac{\dot{\mathcal{M}_w}}{4\pi R_s^2 V_w} = \rho_b f (\alpha, \delta) / \sqrt{g_1} \). \tag{22}

For this case, we have the limit \( \sqrt{g_1} \to s_1^2 \cos \delta \), implying \( f = \cos \delta \). Equation (21) then gives the density as

\[
\varrho = \frac{1}{s_1^2 (1 + p^2 s \cos \alpha \cos \delta)}. \tag{23}
\]

On more intuitive grounds, mass conservation may be enforced in an integral sense by imagining a narrow streamtub e, bounded along its length by streamlines. Mass may enter or leave the streamtube only at its ends: one located...
on a small spherical surface centered on the wind source, and
the other at an arbitrary (but larger) constant value of \( s \). Let the
foot of the streamtube lie on a small sphere of radius \( R_f = R_0 \sin \alpha \), and have two sides of constant \( \alpha \), and \( \alpha + d\alpha \),
and two sides of constant \( \delta \) and \( \delta + d\delta \). The area of the
enclosed surface patch is \( R_f^2 \sin^2 \alpha \cos \delta \sin \alpha \sin \delta \). We generate a thin
volume by including a thickness \( dR = V_w \, dt = R_s \, ds \). The
amount of mass in this volume must equal that at the far end of
the streamtube in a volume of matching \( d\alpha \), \( d\delta \), and \( ds \). The
volume element at either end of the streamtube is described by
the scalar triple product

\[
\mathcal{V} \, d\alpha \, d\delta \, ds = \mathbf{b}_f \cdot (\mathbf{b}_a \times \mathbf{b}_b) \, d\alpha \, d\delta \, ds.
\]  

(24)

The cross product \( \mathbf{b}_a \times \mathbf{b}_b \) \( d\alpha \, d\delta \equiv \mathbf{a}, d\alpha \, d\delta \) defines a vector
area element on a surface of constant \( s \), where

\[
\mathbf{a} = s^2 \cos \delta [\hat{x}' \cos \delta \cos \gamma + \hat{y}' \cos \delta \sin \gamma + \hat{z}' \sin \delta].
\]

(25)

Dotting this with the velocity vector \( \mathbf{v}' = \mathbf{b}_f \) shows that the
"volume factor" \( \mathcal{V} \) is identical to \( \sqrt{g} \),

\[
\mathcal{V} = \sqrt{g} = s^2 \cos \delta (1 + p^2 \, s \, \cos \alpha \cos \delta).
\]

(26)

Thus, in dimensional units, and using subscript "1" at the inner
point, without subscript at the outer, \( \rho = \rho_1 \, \mathcal{V}_1 / \mathcal{V} = \rho_1 \sqrt{g} / \sqrt{g} \),
which gives

\[
\rho = \frac{M_w}{4\pi R_f^2 V_w} \frac{1}{s^2 (1 + p^2 \, s \, \cos \alpha \cos \delta)}.
\]

(27)

Equation (27) is the dimensional form of our previous result,
Equation (23). Figure 4 shows the resulting density structure in
the orbital plane, which has a one-armed spiral with density
cusp, in addition to the singularity associated with the wind
source.

4. Discussion

4.1. Streamline Crossing and Shocks

The expression for density contains a factor of \( 1/s^2 \), which
for an isotropic wind from a stationary source (e.g., taking
\( p = 0 \)) results in an inverse distance-squared dependence.
However, for a moving wind, this is more directly interpreted
as an inverse square dependence on time since launch because,
for this scenario, the distance from the source is not
proportional to time. The additional factor in parentheses results in a density enhancement when \( \cos \alpha < 0 \), and a density
deficit when \( \cos \alpha > 0 \). We will call these the inner (pointing
toward the origin) and outer launch hemispheres, respectively.
In fact, the density diverges when \( \sqrt{g} \, \sec \delta = 0 \). Because we
are not interested in the trivial divergence at the wind source,
the vanishing of the factor in parentheses in Equation (26)
yields the condition along a given inner streamline (one for
which \( \cos \alpha < 0 \)) that

\[
s_{\text{crit}} = -\frac{1}{p^2 \cos \alpha \cos \delta}.
\]

(28)

Figure 5 shows the outer branch of the predicted shock location
for the more moderate \(^3\) value \( p = 0.1 \). For outer streamlines,
defined by \( \cos \alpha > 0 \), the quantity \( g \) never vanishes and the
density remains finite. The surface where \( s = s_{\text{crit}} \) therefore
represents a shock surface, and the formal streamline solutions
cannot be applied to collisional fluids for \( s > s_{\text{crit}} \). \(^4\) The
condition for local streamline crossing can also be derived by
assuming an arbitrary set of values for \( p, \alpha, \delta, s \) and demanding
that two neighboring, but initially distinct fluid elements (the
second having parameter values \( p, \alpha + d\alpha, \delta + d\delta, s + ds \))
have the same position using Equation (10). For brevity, we
will not present this lengthy calculation here, but note that it
yields \( \sqrt{g} = 0 \), reproducing our shock condition. Evidently,

\(^3\) Soker (1994) estimates a range of \( p \)-values from 0.01–0.3 for wide binaries
within planetary nebulae.

\(^4\) For collisionless systems such as neutrino winds, the methods described
here could be used to obtain the density of particles between the caustics by
summing overlapping families of streamlines.
streamlines that cross their nearest neighbors, that is, those launched with only a differentially distinct angle. These overlaps are properly termed caustics, and can be located with our shock criterion. The second type of streamline crossing we shall term “non-local,” because initially widely spaced streamlines may cross. This includes, but is not limited to, all the streamline crossings involving outer streamlines. Outer streamlines never have self-shocking, but by crossing the caustic boundaries they do cross inner streamlines. Additionally, within the caustic zone, inner streamlines cross an infinite number of other inner streamlines in a continuous fashion. We do not have a general solution for such non-local crossings, other than the direct solution of Equation (10), which proves to be very cumbersome. Nonetheless, Figures 6–7 show this behavior for the chosen value of \( p \).

4.2. The Cusp

The innermost point on the shock surface is a needle-like cusp of vanishing opening angle in the orbital plane. The location of the point is found from Equations (11) and (28) by solving for the launch angle resulting in minimal radius for the shock. The critical streamline that enters the cusp tangentially in the orbital plane has launch angle and \( s \)-value

\[
\alpha_{cusp} = \pi + \sin^{-1} p, \tag{29}
\]

\[
s_{cusp} = \frac{1}{p^2 \sqrt{1 - p^2}}. \tag{30}
\]

The location of the cusp is then found by using these values in the trajectory Equation (11) as

\[
r_{cusp} = \frac{1 - p^2}{p^2}. \tag{31}
\]

We note that in the limit of small \( p \), Equation (31) is consistent with the results of Raga et al. (2011) and Cantó et al. (1999), who assumed a purely radial wind of time-dependent speed, launched from the center of mass, obtaining in dimensional units \( R_{cusp} = v_{w}^2 / R_{\odot} \omega^2 \). Our result generalizes this with an exact treatment of centrifugal and coriolis effects. Kim & Taam (2012b) also give an empirical formula predicting a smaller standoff location based upon numerical simulations, although these simulations also incorporated gravitational perturbations. The discrepancy may be associated with the difficulty of determining the innermost location along the shock, because the convergence of streamlines prior to reaching the cusp results in a density enhancement, which may have been interpreted as a cusp in their simulations (see Figure 4). For \( p > 1 \), no cusp occurs, and there will be regions unreached by the flow due to a centrifugal barrier. In particular, note that the same streamline that intersects the cusp point also goes through the origin. For \( p > 1 \), the flow does not reach the origin, with implications for identical colliding winds: no stagnation point region would be possible. Because our neglect of gravitational forces was justified by application to wide binaries only (small \( p \)), we will not discuss the case \( p > 1 \).

4.3. The Shocked Region

Our streamline analysis does not provide a description of the region between the two shock surfaces. However, an asymptotic, “tail” solution for the shock may be developed for the outer part of the shock spiral by noting that the

![Figure 6. Inner streamlines in the orbital plane for \( p = 0.5 \), with \( \alpha \)-spacings of \( \pi/16 \). All inner streamlines eventually shock. They may enter the shock region by crossing the inner or outer shock, depending on their \( \alpha \)-value.](image1)

![Figure 7. Outer streamlines in the orbital plane for \( p = 0.5 \), with \( \alpha \)-spacings of \( \pi/16 \). All outer streamlines eventually cross either the inner or outer shock.](image2)
azimuthal range \(\pi/2 < \alpha < 3\pi/2\) is mapped to an infinite range of azimuthal angle in the outer region, so that most of the spiral corresponds to a small \(\epsilon \gg 0\) region of either \(\alpha = \pi/2 + \epsilon\) or \(\alpha = 3\pi/2 - \epsilon\). Physically, this means most of the wind mass has reached the shock before many windings occur. Either limit yields \(s_{\text{crit}} \approx 1/p^2 \cos \delta \approx \zeta/p\) by direct substitution into Equation (28) and expansion to leading order. Substitution into Equation (10) gives outer solution from \(3\pi/2 - \epsilon\), and inner solution from \(\pi/2 + \epsilon\), as

\[
r'_{\text{tail}} \approx (\zeta/p)(\mp \cos \delta \sin \zeta + p \sin \zeta) + \dot{y}'(\mp \cos \delta \cos \zeta + p \cos \zeta) + \dot{z}' \sin \delta),
\]

which yields radii for the two tails

\[
r_{\text{tail}} = (\zeta/p) \sqrt{1 \pm 2p \cos \delta + p^2}.
\]

Both tail solutions take the form of an Archimedean spiral, where \(\zeta\) plays the role of azimuthal angle.

The radial spacing for a given curve is found by adding argument \(2\pi\) to \(\zeta\), which gives

\[
\Delta R = 2\pi R_o \sqrt{1 - 2p \cos \delta + p^2} / p.
\]

This radial step is a generalization of the expression given by Mauron & Huggins (2006) and Raga et al. (2011) and agrees with both for vanishing \(p\). On the other hand, the radial spacing (fixed \(\zeta\) and \(\delta\)) between the two shock curves provides an estimate of the thickness of the shell of shocked gas. When evaluated for small \(p\), this radial thickness is \(\Delta R \approx R_o \zeta \cos \delta\), and using \(R \approx R_o \zeta / p\), the fractional thickness is \(\Delta R / R \approx p \cos \delta\). Because the radial step for the inner shock is smaller than that for the outer shock, and these steps are approximately constant, the mis-match implies that eventually the outer shock for, say, \(n\) orbits, should cross the inner shock for \(n + 1\) orbits, resulting in a pinching-off of all unshocked streamlines. The intersection of two shocks may produce filamentary or spoke-like features, though for a large enough intersection angle, a Mach stem could result (e.g., Hartigan et al. 2016). Given the complex spiral geometry, we cannot anticipate the details of such Mach structures here, and leave them for further study, which could be done with 3D hydrodynamics simulations.  

4.4. Summary

We have considered the flow structure of an initially isotropic wind from a point source in a circular orbit. Assuming ballistic flow, the pre-shock particle trajectories are straight lines in the inertial frame of the center of mass, and trailing spirals in the frame rotating with the source. In the corotating frame, the flow is steady-state and trajectories are initially non-intersecting streamlines. By solving the mass conservation condition in a curvilinear coordinate system based upon these streamlines, we obtain an exact solution for the density structure. This density has a latitude and initial launch angle dependence. Interestingly, the usual \(1/p^2\) dependence is modified as an inverse square dependence on time of flight, modified by a geometric factor.

The flow is separated into two behaviors. The inner launch hemisphere generates a converging flow so that streamlines eventually intersect with their neighbors, resulting in a shock. The outer hemisphere of launch has non-intersecting, diverging streamlines. However, outer streamlines do eventually run into the shock region. A local criterion for streamline crossing gives the region beyond which the description is incomplete. A full description of the post-shock flow as a spiral shell is beyond the scope of this study.

The ballistic approximation we have adopted is appropriate beyond the acceleration zone for the stellar wind. For most types of stellar winds, the acceleration zone is relatively localized either close to the star (e.g., those of coronal winds) or near the dust condensation radius (for late stages of low-mass stars such as red giants and AGB stars). However, for winds of O and Wolf–Rayet stars driven by photon scattering on spectral lines, the acceleration zone may be much more extended spatially (Castor et al. 1975; Owocki 1990). The applicability of the current solutions would have to be determined on a case by case basis depending on the type of wind acceleration mechanism and the binary separation.

The generic nature of our model and the ubiquity of winds in widely separated binaries imply that our solution should be applicable to modeling a wide variety of observed sources such as spiral nebulae of AGB stars, structures within planetary nebulae, and colliding winds in binary systems.

H.H. acknowledges financial support from the Lee L. Davenport Fellowship.

References

Aris, R. 1962, Vectors, Tensors, and the Basic Equations of Fluid Mechanics (New York: Dover)  
Cantó, J., Raga, A. C., Koenigsberger, G., & Moreno, E. 1999, in IAU Symp. 183, Wolf–Rayet Phenomena in Massive Stars and Starburst Galaxies, ed. K. A. van der Hucht, G. Koenigsberger, & P. R. J. Eenens (Cambridge: Cambridge Univ. Press), 338  
Castor, J. I., Abbott, D. C., & Klein, R. I. 1975, ApJ, 195, 157  
Duchêne, G., & Kraus, A. 2013, ARA&A, 51, 269  
Hartigan, P., Foster, J., Frank, A., et al. 2016, ApJ, 823, 148  
He, J. H. 2007, A&A, 467, 1081  
Hendrix, T., Keppens, R., van Marle, A. J., et al. 2016, MNRAS, 460, 3975  
Homan, W., Decin, L., de Koter, A., et al., 2015, A&A, 579, A118  
Kim, H. 2011, ApJ, 739, 102  
Kim, H., Hsieh, I.-T., Liu, S.-Y., & Taam, R. E. 2013, ApJ, 776, 86  
Kim, H., & Taam, R. E. 2012a, ApJ, 744, 136  
Kim, H., & Taam, R. E. 2012b, ApJ, 759, 59  
Kim, H., & Taam, R. E. 2012c, ApJL, 759, L22  
Lamberts, A., Dubus, G., Lesar, G., & Fromang, S. 2012, A&A, 546, A60  
Livio, M.int 1982, A&A, 105, 37  
Mastrodemos, N., & Morris, M. 1999, ApJ, 523, 357  
Mauron, N., & Huggins, P. J. 2006, A&A, 452, 257  
Monnier, J. D., Tuthill, P. G., & Danchi, W. C. 1999, ApJL, 525, L97  
Morris, M. 1981, ApJ, 249, 572  
Morris, M., Sahai, R., Matthews, K., et al., 2006, in IAU Symp. 234, Planetary Nebulae in our Galaxy and Beyond, ed. M. J. Barlow & R. H. Mendez (Cambridge: Cambridge Univ. Press), 469  
Myasnikov, A. V., & Zhekov, S. A. 1991, Ap&SS, 184, 287  
Owocki, S. P. 1990, RVMA, 3, 98  
Raga, A. C., Cantó, J., Esquivel, A., Huggins, P. J., & Mauron, N. 2011, in Asymmetric Planetary Nebulae V, ed. A. A. Zijlstra et al. (Jodrell: Jodrell Bank Center for Astrophysics), 185  
Rauw, G., & Nazé, Y. 2016, AdSpR, 58, 761  
Soker, N. 1994, MNRAS, 270, 774