Finite-time disturbance observer-based trajectory tracking control for flexible-joint robots

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Abstract This paper proposes a robust finite-time control scheme for the high-precision tracking problem of (FJRs) with various types of unpredictable disturbances. Specifically, based on a flatness dynamic model, a finite-time disturbance observer (FTDO) with only link-side position measurements is firstly developed to estimate the lumped unknown time-varying disturbance and unmeasurable states. Then, through the information of the states and disturbances provided by the FTDO, a robust output feedback controller is constructed, which can accomplish the tasks of disturbance suppression and trajectory tracking in finite time. Moreover, a rigorous stability analysis of the closed-loop system based on a finite-time bounded (FTB) function is conducted. Finally, the simulation results validate the feasibility and superiority of the proposed control scheme against other existing control results.

Keywords Finite-time control · Finite-time disturbance observer (FTDO) · Time-varying disturbance · Flexible-joint robots (FJRs)

1 Introduction

In recent decades, robots with flexible joints have been abundantly used in various fields due to various attractive properties, such as smooth force transmission, back driveability, and shock tolerance [1]. Unlike traditional
robots, flexible-joint robots (FJRs) are constructed by introducing elastic components between the links and actuators. Such robots that possess compliant behaviors are suitable for physical human–robot interaction tasks, such as rehabilitation, collaboration, and assistance, where a mechanism to guarantee human safety is required when the robots operate within an uncertain environment [2–4].

In general, however, a flexible-joint (FJ) robotic system is nonlinear, implying that introducing the FJ will increase the order of the underlying dynamics of the robot. Under this circumstance, the new system has twice higher number of orders than the old system, and the degrees of freedom of the new system is larger than the number of control inputs [2,5]. Moreover, in real applications, FJRs are commonly disturbed by various unknown time-varying disturbances (matched or mismatched), covering internal parameter uncertainties, modeling errors, couplings, and external load disturbances or variable operating environments. These factors bring difficulties to the high-performance control of FJRs and motivate the design of robust tracking control in this work.

To address these issues, a number of control schemes for FJRs have been explored, such as feedback linearization control [6], sliding mode control [7], singular perturbation control [8,9], adaptive control [10,11], backstepping control [12,13], predictive control [14], passivity-based control [15], disturbance rejection control [16–18], and learning-based control [19–21]. These control strategies above-mentioned can improve the control performance of FJRs from different aspects, but at least speed measurements are required in the methods. Nevertheless, speed measurements obtained by the tachometers are liable to be contaminated by noises. In addition, considering the reductions in the equipment cost and related maintenance, as well as the size and weight of the drive system, speed sensors are frequently excluded in the robotic systems [22]. In the views of these, it is novel to design a practical control scheme for FJR robot systems without speed measurements.

For the proposed control problem, some related studies also provide solutions. Melhem et al. [23] proposed a global output feedback tracking control scheme by using a partial state-feedback linearization technique and a backstepping control design, and many of the studies with similar control schemes have been designed for various nonlinear systems [24–27]. In [24], an adaptive output feedback control via a dynamic surface design approach was proposed. Besides, Talole et al. [25] reported a robust output feedback controller based on feedback linearization for a single-link FJR. The aforementioned studies evidence that the output feedback control structure is an appropriate choice for designing a robotic control system without using speed sensors. However, the systems under these control schemes can only achieve asymptotic tracking results.

When it comes to improving the control performance of the system, finite-time control is an effective control method and has been widely addressed in many applications. For instance, Bhat et al. [28] proposed a finite-time feedback control (FTFBC) method for the systems without uncertainties/disturbances. By employing a barrier Lyapunov function and adding a power integrator technique, a finite-time control scheme was designed in [29]. Base on the terminal sliding mode control (TSMC) technique, a continuous finite-time control law was developed for the disturbed FJRs in [30]. It is worth noting that all the methods mentioned above and some references therein show that the finite-time technique can effectively upgrade the control performance of the systems.

Furthermore, considering that the control performance of FJRs is commonly influenced by unknown time-varying disturbances, many observation techniques have been introduced. In [25], an extended state observer (ESO) was designed for observing the lumped disturbance under the assumption that the disturbances change slowly. To acquire the information about the time-varying disturbances, reference [2] developed two generalized proportional integral observers (GPIO) to estimate the link- and motor-side disturbances exerted on an FJR, respectively. Nevertheless, the disturbances can only be estimated in an asymptotic manner when DPIO is employed. A finite-time disturbance observer (FTDO) was proposed in [31] for a nonlinear system to observe the states and disturbances in finite time, and this observer scheme has been used effectively in other practical systems [32–34].

Inspired by the aforementioned literature review, this work aims to propose a robust finite-time control scheme on the tracking problem of FJ robotic systems via output feedback. Firstly, a flatness dynamic model of a general FJRs is proposed, which regards the unknown matched and mismatched disturbances of the system as a lumped time-varying disturbance. This lumped disturbance covers internal parameter uncertainties, modeling errors, couplings, and external load...
disturbances or the changing environments. Next on, an FTDO that only uses link-side position measurements is constructed for estimating the unmeasurable system states and unknown time-varying disturbances. Based on the feedback of states and disturbances observations, a robust finite-time output feedback controller is constructed, which can not only cope with both the matched and mismatched time-varying disturbances in finite time but also endow the FJRs with the capability of finite-time tracking.

The contributions of this work are: (1) A general robust output feedback control scheme is developed to cope with the matched and mismatched time-varying disturbances on the motor side and link side simultaneously. (2) The proposed scheme is devised based on link-side position measurements and only needs a few mechanical parameters that can be obtained easily, which greatly reduces the modeling burden and hardware cost. (3) The coefficients of the proposed controller and FTDO can be simply configured through the pole placement method and the given formulas. (4) The finite-time stability of the constructed closed-loop system is ensured based on a finite-time bounded (FTB) function. (5) The performances of the proposed scheme on the control accuracy and robustness are validated by the comparative studies of the simulation results.

The remainder of this paper is structured: Section 2 describes the dynamics of FJRs and its flatness description. Section 3 reveals the details of the presentation is used throughout this paper: Section 4 validates the feasibility of the proposed method on a two-link FJRs. Section 5 makes the conclusion of this work. The following represents the conclusion of this work. The following representation is used throughout this paper: \[ [x]^γ = [x_1]^γ , [x_2]^γ , \ldots , [x_n]^γ ]^T \in \mathbb{R}^{n \times n} \]

where \( q = [q_1, q_2, \ldots, q_n]^T \in \mathbb{R}^n \) represents joint positions; \( \theta = [\theta_1, \theta_2, \ldots, \theta_n]^T \in \mathbb{R}^n \) represents motor positions; \( M(q) \in \mathbb{R}^{n \times n} \) is an inertia matrix (symmetrical matrix); \( C(q, \dot{q}) \in \mathbb{R}^{n \times n} \) is a centripetal-Coriolis matrix; \( G(q) \in \mathbb{R}^n \) is the exerted gravitational torques; \( J \in \mathbb{R}^{n \times n} \), \( B \in \mathbb{R}^{n \times n} \) and \( K \in \mathbb{R}^{n \times n} \) refer to the motor inertia (diagonal matrix), motor damping, and elastic coefficient matrix (diagonal matrix), respectively; \( w_1 \) and \( w_2 \) are external disturbances or/unmodeled dynamics of the link- and motor-side, respectively; \( \tau \in \mathbb{R}^n \) is a vector of control torques. Note that \( n \) is the total joint number and \( q \) can be measured in this work.

This work aims to propose an output feedback control scheme that can cope with the unknown matched and mismatched time-varying disturbances of FJRs in finite-time so that the given trajectory \( \dot{q}_r \) can be followed by the system output \( q \) as fast and accurate as possible.

2.2 Flatness description of dynamics model

Notice that system (1) is differentially flat with a flat output \( q \). In fact, all uncertain system variables can be expressed in terms of the flat output \( q \) and the unknown external disturbances \( w_1 \) as well as \( w_2 \) [25]. For more details on the introduction of the differential flat-based system, one can refer to [36,37].

Let \( x_1 = q, x_2 = \dot{q}, x_3 = \theta, \) and \( x_4 = \dot{\theta} \), system (1) can be formulated as the following state space:

\[
\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= M^{-1}(x_1)(K(x_3 - x_1)) - C(x_1, x_2)x_2 - G(x_1) + w_1, \\
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= J^{-1}(\tau - Bx_4 - K(x_3 - x_1)) + w_2.
\end{aligned}
\]
Define $z = x_1$, the following form can be derived:

$$x_1 = z, \quad x_2 = \dot{z},$$

$$x_3 = K^{-1}(M(z)\dot{z} + C(z, \dot{z})\dot{z} + Kz + G(z) - w_1),$$

$$x_4 = K^{-1}\left(M(z)\dot{z} + \dot{M}(z)\dot{z} + C(z, \dot{z})\dot{z} + \dot{C}(z, \dot{z})z + \dot{K}z + \dot{G}z - \dot{w}_1\right),$$

where $\delta(\cdot)$ is a function of $z$, $M(z)$, $C(z, \dot{z})$, $G(z)$, $M$, $J$, $B$, $w_1$, $w_2$ and their derivatives, a detailed expression of $\tau$ can be found in Appendix.

System (3) can be modeled by the following canonical form:

$$z^{(4)} = b_0\tau + \xi,$$  \hspace{1cm} (4)

where $b_0 = b - \Delta b$, $b_0 = K_0/(M_0(z)J_0)$ is the nominal value of $b = K/(M(z)J)$ and $\Delta b$ is the uncertainty of $b$; $\xi = \Delta b\tau - \Delta b\delta(\cdot)$.

**Assumption 1** For system (4), the following assumptions are made:

1. The disturbances, $\omega_1$ and $\omega_2$, are time-varying signals, which are known to be uniformly and absolutely bounded, separately. Besides, the disturbances are completely unknown.

2. Let the system parameters $M(z) = M_0(z) + \Delta M(z)$, $K = K_0 + \Delta K$, $J = J_0 + \Delta J$, where $M_0(z)$, $K_0$, and $J_0$ are the nominal values of $M(z)$, $K$, and $J$, respectively; $\Delta M(z)$, $\Delta K$, and $\Delta J$ are the uncertainties of $M(z)$, $K$, and $J$, respectively. In addition, the nominal parameters $M_0(z)$, $K_0$, and $J_0$ are perfectly known. Plus, other time-varying parameters, including $C(q, \dot{q})$, $G(q)$, and $B$, are unknown.

3. At least the first $m \in \mathbb{R}^+$ times derivatives of $\xi = [\xi_1, \xi_2, \ldots, \xi_n]^T$ exist, and $\xi^{(m)}_i$ has a Lipschitz constant $L_i$ [31]. This treatment implies that the rate of change of the disturbance is always limited in the physical world, which is a general internal model solution given in the observer-based control design [38,39].

### 3 Robust finite-time control design

In this section, the design of the proposed control scheme for FJR and its stability analysis is provided.

#### 3.1 Controller design

System (4) can be formulated based on the following extended state space:

$$\begin{align*}
\dot{z}_i &= z_{i+1}, \quad i = 0, 1, 2, \\
\dot{z}_3 &= \varphi_0 + b_0\tau, \\
\dot{\varphi}_j &= \varphi_{j+1}, \quad j = 0, 1, \ldots, m - 2, \\
\dot{\varphi}_{m-1} &= \xi^{(m)},
\end{align*}$$

where $z_0 = z$, $z_1 = \dot{z}$, $z_2 = \ddot{z}$, $z_3 = z^{(3)}$, $\varphi_0 = \xi$, $\varphi_1 = \dot{\xi}$, $\ldots$, $\varphi_{m-1} = \xi^{(m-1)}$.

For system (5), an FTDO can be constructed by

$$\begin{align*}
\dot{v}_i &= v_{i+1}, \quad i = 0, 1, 2, \\
\dot{v}_3 &= b_0\tau + v_3, \\
\dot{\varphi}_j &= v_{4+j}, \quad j = 0, 1, \ldots, m - 2, \\
\dot{\varphi}_{m-1} &= -\mu_1 \text{sign}(\varphi_{m-1} - v_{m+2}), \\
v_0 &= \hat{z}_1 - \mu_4 L^{\frac{1}{m+4}} [\hat{z}_0 - z_0]_{\frac{m+3}{m+4}}, \\
v_3 &= \tilde{z}_{i+1} - \mu_4 L^{\frac{1}{m+4}} [\tilde{z}_i - v_{i+1}]_{\frac{m+3}{m+4}}, \\
v_3 &= \hat{\varphi}_0 - \mu_4 L^{\frac{1}{m+4}} [\hat{\varphi}_j - v_{3+j}]_{\frac{m+1}{m+4}}, \\
v_{4+j} &= \hat{\varphi}_{j+1} - \mu_4 L^{\frac{1}{m+4}} [\hat{\varphi}_j - v_{3+j}]_{\frac{m+1}{m+4}},
\end{align*}$$

where $z_0$ is the output of system (5); $L = \text{diag}(L_1, L_2, \ldots, L_n)$. It can be inferred from [31,39] that $\hat{z}_0, \hat{z}_1, \hat{z}_2, \hat{z}_3, \varphi_0, \varphi_1, \ldots, \varphi_{m-1}$ will converge to $z_0, z_1, z_2, z_3, \varphi_0, \varphi_1, \ldots, \varphi_{(m-1)}$ in finite-time, if the gains $\mu_i$ ($i = 1, 2, \ldots, m + 4$) are selected properly.

According to the estimations of (6), the proposed robust finite-time output feedback controller is thus constructed for system (4):

$$\tau = b_0^{-1} \left( z^{(4)}_r - \sum_{i=0}^{3} (c_{i+1}[\hat{z}_i - z^{(i)}_r]^{\alpha_{i+1}} - \hat{\varphi}_0) \right),$$  \hspace{1cm} (7)

where $z_r = q_r = [q_{r1}, q_{r2}, \ldots, q_{rn}]^T$; $c_i = \text{diag}(c_{i1}, c_{i2}, \ldots, c_{in})$, $c_{i1}, c_{i2}, \ldots, c_{in} > 0$, ($i = 1, 2, 3, 4$), $c_{ij}$ can be selected such that all of the roots in $p_c(s) = [p_1^c(s), p_2^c(s), \ldots, p_n^c(s)]^T$

$\begin{align*}
p_c(s) &= L^3 + c_1^3s^3 + c_2s + c_1,
\end{align*}$

are within the left-half side of the $s$-plane. $\alpha_i$ ($i = 1, 2, 3, 4$) is the controller’s coefficient, and $\alpha_1, \alpha_2,$
\( \alpha_3, \alpha_4 \) satisfy the following conditions: \( \alpha_i - 1 = \frac{\alpha_i + 1}{2 \alpha_i + 1 - \alpha_i}, (i = 2, 3, 4) \), with \( \alpha_5 = 1, \alpha_4 = \alpha, \alpha \in (1 - \delta, 1), \delta \in (0, 1) \).

Remark 1 It is noted that the proposed control scheme consists of observer (6) and controller (7). For the selection of parameters \( m, \mu_i, \) and \( L \) in observer (6), the tradeoffs are as follows: the larger parameter \( m \), the higher observer accuracy and the greater the computational burden [2]; the larger parameters \( \mu_i \) and \( L \), the faster convergence performance and the higher sensitivity to input noises and the sampling period [31]. For the parameters \( e_i \) and \( c_i \) in controller (7), they are related to the tracking performance and disturbance rejection performance of the system, where the configuration of parameter \( e_i \) must meet the following tradeoff: the larger parameter \( e_i \), the faster tracking speed and stronger anti-disturbance capability and the greater the control input [28].

3.2 Stability analysis

To discuss the stability of the designed scheme, the observer is formulated by (6) and the controller is formulated by (7).

By defining \( e_{\tilde{z}} = \hat{\tilde{z}} - z_i \) and \( e_{\phi_j} = \hat{\phi}_j - \phi_j \) and subtracting (5) from (6), the error dynamics of FTDO (6) can be derived:

\[
\begin{align*}
\dot{\tilde{z}}_0 &= e_{\tilde{z}} - \mu_{m+4} L^{\frac{1}{m+4}} [e_{\tilde{z}0}]^{\frac{m+3}{m+4}}, \\
\dot{\tilde{z}}_i &= e_{\tilde{z}i+1} - \mu_{m+4-i} L^{\frac{1}{m+4-i}} [e_{\tilde{z}i} - \dot{\tilde{z}}_{i-1}]^{\frac{m+3-i}{m+4-i}}, \\
\dot{\tilde{z}}_3 &= e_{\phi_0} - \mu_{m+1} L^{\frac{1}{m+1}} [e_{\tilde{z}3} - \dot{\tilde{z}}_{2}]^{\frac{m}{m+1}}, \\
\dot{\tilde{z}}_j &= e_{\phi_j} - \mu_m L^{\frac{1}{m}} [e_{\phi_j} - \dot{\tilde{z}}_{j-1}]^{\frac{m-1-j}{m-j}}, \\
\dot{\tilde{z}}_{j-1} &= -\mu_1 L \text{sign}(e_{\phi_{j-1}} - \dot{\tilde{z}}_{j-2}) + [-\bar{L}, \bar{L}],
\end{align*}
\]

where \( \bar{L} = [L_1, L_2, \ldots, L_n]^T \).

Let \( e = e_0 = z_0 - z_r \) and \( e_i = z_i - z_r^{(i)}, \) \( (i = 1, 2, 3) \), the error dynamics of system (5) can be obtained through plugging (7) into (5):

\[
\begin{align*}
\dot{e}_i &= e_{i+1}, \quad i = 0, 1, 2, \\
\dot{e}_3 &= -\sum_{i=0}^{3} (c_{i+1} [e_i + e_{\tilde{z}i}]^{\alpha_{i+1}}) - e_{\phi_0},
\end{align*}
\]

The closed-loop system based on the error dynamics of system (9) and observer (8) is then governed by

\[
\begin{align*}
\text{System (9)}, \\
\text{System (8)}.
\end{align*}
\]

Lemma 1 [40] Considering the time-varying cascade system

\[
\begin{align*}
\dot{\tilde{z}}_1 &= f_1(t, \chi_1) + g(t, \chi_2), \\
\dot{\tilde{z}}_2 &= f_2(t, \chi_2),
\end{align*}
\]

and the conditions: (1) The subsystem \( \dot{\tilde{z}}_1 = f_1(t, \chi_1) \) and \( \dot{\tilde{z}}_2 = f_2(t, \chi_2) \) reach the equilibrium point in finite time; (2) For any fixed and bounded \( \chi_2 \), there exists a positive definite and radially unbounded FTB function \( B(t, \chi_1) \) that satisfies

\[
\dot{B}(t, \chi_1) \leq AB(t, \chi_1) + L_B,
\]

where \( A \) and \( L_B \) are positive constants. Then, the system (11) is called finite-time stable if both the conditions (1) and (2) are satisfied.

Theorem 1 Considering the unpredictable time-varying disturbances that satisfy Assumption 1 are acting on system (4), and a robust finite-time control method based on the combination of FTDO (6) and controller (7) is employed. If the gains \( \mu_i \) of FTDO (6) and \( c_i \) of controller (7) satisfy \( \mu_i > 0 \) and \( c_{i1}, c_{i2}, \ldots, c_{in} > 0, (i = 1, 2, 3, 4) \), respectively, then the desired equilibrium point can be reached by the error system (10) in finite-time.

Proof The closed-loop system (10) can be derived by

\[
\begin{align*}
\dot{\tilde{z}}_1 &= f_1(t, \chi_1) + g(t, \chi_2), \\
\dot{\tilde{z}}_2 &= f_2(t, \chi_2),
\end{align*}
\]

where \( f_1(t, \chi_1) = f_1(t, e_i), f_2(t, \chi_2) = f_2(t, [e_{\tilde{z}i}, e_{\phi_j}]^T) \).

The proof is divided into three steps. Firstly, the subsystem \( \dot{\tilde{z}}_2 = f_2(t, \chi_2) \) is finite-time stable when \( t > T_0 > 0 \). Secondly, the system variables \( \chi_1 \) will not diverge to infinity in any given time interval \([0, T_0]\). Thirdly, the system \( \dot{\tilde{z}}_1 = f_1(t, \chi_1) \) is finite-time stable, i.e., the tracking error \( e_0 = q - q_r \) will converge to zero in finite-time.

Step 1 (Finite-Time Stability of Observer Variables) Based on system (5) and the designed observer (6), dynamic equation (8) including observer errors \( e_{\tilde{z}i} \) and \( e_{\phi_j} \) can be arranged. According to Theorem 5 in [31], if
the coefficients $\mu_i$ in FTDO (6) are selected appropriately, the observer error system (8) is finite-time stable, that is, the subsystem $\dot{x}_2 = f_2(t, x_2)$ is finite-time stable when $t > T_0 > 0$, satisfying (1) $\forall t \leq T_0$, $e_{z\ell}$ and $e_{\varphi j}$ are bounded by positive constants $\|e_{z\ell}\| \leq E_{z\ell}$, $\|e_{\varphi j}\| \leq E_{\varphi j}$. (2) $\forall t > T_0$, $e_{z\ell} = e_{\varphi j} = 0$.

**Step 2 (Finite-Time Boundness of System States)** Defining an FTB function $V(e_i, t) = \sum_{i=0}^{3} (1/2) e_i^T e_i$ [40], the derivative of $V(e_i, t)$ can be represented as follows:

$$\dot{V} = e_0^T e_1 + e_1^T e_2 + e_2^T e_3 + e_3^T e_3$$

$$\leq \frac{2}{2} \sum_{i=0}^{3} \|e_i\|^2 + \|e_{i+1}\|^2 + e_3^T e_3$$

$$\leq 2V + e_3^T e_3.$$  (13)

According to the error dynamic (9),

$$\|\dot{e}_3\| = \left\| \sum_{i=0}^{3} \left( e_{i+1} \left[ e_i + e_{z\ell} \right] \right)^{\alpha_i + 1} + e_{\varphi 0} \right\|$$

$$\leq \bar{\tau} \sum_{i=0}^{3} \left( \|e_i + e_{z\ell}\|^{\alpha_i + 1} + \|e_{\varphi 0}\| \right),$$

with $\bar{\tau} = \max\{c_{ij}\}$. For the vector $x \in \mathbb{R}^n$, $\|x\|_{\gamma} < n + \sum_{i=1}^{n} \gamma |x_i| < n + \|x\|$ holds, where $\gamma \in (0, 1)$.

With this property in mind, it can be verified that

$$\|e_i + e_{z\ell}\|^{\alpha_i + 1} < n + \|e_i + e_{z\ell}\|.$$  (14)

Substituting above equations into (13) yields

$$\dot{V} < 2V + \|e_3\| \left( \bar{\tau} \sum_{i=0}^{3} \left( (n + \|e_i\| + \|e_{z\ell}\|) + \|e_{\varphi 0}\| \right) \right)$$

$$= 2V + n\bar{\tau} \sum_{i=0}^{3} \|e_3\| + \bar{\tau} \sum_{i=0}^{3} \|e_3\| \|e_i\|$$

$$+ \bar{\tau} \sum_{i=0}^{3} \|e_3\| \|e_{z\ell}\| + \|e_3\| \|e_{\varphi 0}\|$$

$$\leq 2V + n\bar{\tau} \sum_{i=0}^{3} \left( \frac{1}{2} \|e_3\|^2 + \bar{\tau} \sum_{i=0}^{3} \|e_3\|^2 \|e_i\|^2 \right)$$

$$+ \bar{\tau} \sum_{i=0}^{3} \|e_3\| \|e_{z\ell}\| + \|e_3\| \|e_{\varphi 0}\|$$

$$\leq N_1 V + N_2.$$  (16)

where

$$N_1 = 2 + 4(n + 2)\bar{\tau} + \bar{\tau} \sum_{i=0}^{3} E_{z\ell} + E_{\varphi 0},$$

$$N_2 = \frac{1}{2} \left( 4n\bar{\tau} + \bar{\tau} \sum_{i=0}^{3} E_{z\ell} + E_{\varphi 0} \right).$$

It yields from (16) that

$$V \leq -N_2/N_1 + (V(0) + N_2/N_1)e^{N_1t}.$$  (17)

Therefore, for any bounded time $T_e$, $V$ is bounded, i.e., $V$ and the errors $e_0, e_1, e_2, e_3$ do not diverge to infinity when $t \leq T_e$.

**Step 3 (Finite-Time Convergence of Output Tracking Error)** Defining a bounded time constant $T = \max\{T_o, T_e\}$. When $t > T$, $e_{z\ell} = 0$ and $e_{\varphi 0} = 0$ are achieved.

With this in mind, the subsystem $\dot{x}_1 = f_1(t, x_1) + g(t, x_2)$ is reduced to $\dot{x}_1 = f_1(t, x_1)$, meaning that according to system (9), the following equations can be obtained:

$$\begin{cases}
\dot{e}_i = e_{i+1}, & i = 0, 1, 2, \\
\dot{e}_3 = - \sum_{i=0}^{3} \left( c_i \left[ e_i \right]^{\alpha_i + 1} \right).
\end{cases}$$  (18)

or written as

$$e^{(4)}_0 + c_4 [e^{(3)}_0]^{\alpha_4} + c_3 [e^{(2)}_0]^{\alpha_3} + c_2 [e^{(1)}_0]^{\alpha_2} + c_1 [e^{(0)}_0]^{\alpha_1} = 0.$$  (19)

If the coefficients $c_i$ and $\alpha_i$ are appropriately configured based on the principles given earlier, it can be concluded from Proposition 8.1 in [28] that the states of the system (18) or (19) can converge to zero in finite time, that is, the tracking error $e_0 = q - q_r$ will slide to zero in finite time.

With Steps 1–3, the closed-loop system (10) is proved to be finite-time stable from Lemma 1, indicating that the system output $q$ can track the desired trajectory $q_r$ in finite time in spite of unknown matched and mismatched time-varying disturbances.

**Remark 2** The proposed control scheme is devised by merging the FTFBC method with the estimations of the developed FTDO. Notice that noncompliance with the separation principle of the nonlinear systems such as FJR, causing that the proposed scheme is not merely a simple combination of the two methods. Hence, for an FJ robotic system, not only should the observer and feedback controller be designed with efforts, but also the finite-time stability needs to be ensured.

**Remark 3** There are mainly three types of finite-time control techniques: the FTFBC scheme [28], finite-time
control with adding power integrator \cite{29,41,42}, and TSMC scheme \cite{30,43}. However, the FTFBC scheme is to be designed for the systems without uncertainties, the introduction of adding power integrator will raise the computational costs, and the TSMC approach can cause a chattering phenomenon due to its design structure. The proposed control strategy includes the feedforward and finite-time feedback terms that can realize disturbance compensation. The performance on disturbance rejection of this controller is better than the FTFBC scheme. Besides, compared with the finite-time control with adding power integrator, the proposed control methodology can significantly reduce the computational costs. Moreover, compared with the TSMC method, this work designs a continuous control law. Therefore, due to the inherent structural advantages, the proposed control scheme endows the FJ robotic systems with high-precision tracking and anti-disturbance performance.

4 Simulation results

In this section, several control approaches are applied to a two-link FJRs. To demonstrate the superiority of the proposed FTDOBC scheme, the results are compared with the other three control schemes: the LFBC method, the proposed FTDOBC scheme, and the GPIOBC scheme, where the GPIOBC scheme is formulated as follows:

\[
\tau = b_0^{-1} \left( z_r^{(4)} - \sum_{i=0}^{3} \left( c_{i+1}(\hat{z}_i - z_r^{(i)}) \right) - \hat{\varphi}_0 \right),
\]

with the corresponding GPIO:

\[
\begin{align*}
\dot{\hat{z}}_0 &= \hat{z}_{i+1} - \lambda_{m+4-i}(\hat{z}_0 - z_0), \\
\dot{\hat{z}}_3 &= \hat{\varphi}_0 + b_0\tau - \lambda_{m+1}(\hat{z}_0 - z_0), \\
\dot{\hat{\varphi}}_j &= \hat{\varphi}_{j+1} - \lambda_{m-j}(\hat{z}_0 - z_0), \\
\dot{\hat{\varphi}}_{m-1} &= -\lambda_1(\hat{z}_0 - z_0).
\end{align*}
\]

(21)

It should be pointed out that when the term \( \hat{\varphi}_0 \) in (7) and (20) are all set as zero, the GPIOBC and FTDOBC methods will be turned into the traditional LFBC and FTBFC methods, respectively.

Remark 4 Notice that both the proposed control approach and the existing GPIOBC method can handle the time-varying disturbances of the systems \cite{2,25,37}. The difference is that the proposed method can enable the system to achieve the performance of tracking and disturbance rejection in finite time.

Consider the dynamics of two-link FJRs (1) with

\[
M(q) = \begin{bmatrix}
(m_1 + m_2)l_1^2 & m_1 l_1 l_2(s_1 s_2 + c_1 c_2) \\
 m_1 l_1 l_2(s_1 s_2 + c_1 c_2) & m_2 l_2^2
\end{bmatrix},
\]

\[
C(q, \dot{q}) = m_2 l_1 l_2(c_1 s_2 - s_1 c_2) \begin{bmatrix} 0 & -q_2 \\
 -q_1 & 0 \end{bmatrix},
\]

\[
G(q) = \begin{bmatrix}
-(m_1 + m_2)l_1^2 \\
- m_2 l_2 g s_2
\end{bmatrix}, J = \begin{bmatrix} J_1 & 0 \\
 0 & J_2 \end{bmatrix},
\]

\[
B = \begin{bmatrix} B_1 & 0 \\
 0 & B_2 \end{bmatrix}, K = \begin{bmatrix} K_1 & 0 \\
 0 & K_2 \end{bmatrix}, \tau = \begin{bmatrix} \tau_1 \\
 \tau_2 \end{bmatrix},
\]

\[w_1 = [w_{11}, w_{12}]^T, w_2 = [w_{21}, w_{22}]^T, \text{ and } c_1 = \cos(q_1), s_1 = \sin(q_1), c_2 = \cos(q_2), s_2 = \sin(q_2).\]

The nominal parameters of the two-link FJRs are given in Table 1.

The parameters \( c_i \) in both FTDOBC (7) and GPIOBC (20) schemes satisfy the following polynomial form:

\[
p_c(s) = [(s^2 + 2\zeta_c \omega_c + \omega_c^2)^2, (s^2 + 2\zeta_c \omega_c + \omega_c^2)^2]^T,
\]

with

\[
c_1 = \text{diag}(c_{11}, c_{12}) = \text{diag}(\omega_c^4, \omega_c^4),
\]

\[
c_2 = \text{diag}(c_{21}, c_{22}) = \text{diag}(4\zeta_c \omega_c^3, 4\zeta_c \omega_c^3),
\]

\[
c_3 = \text{diag}(c_{31}, c_{32}) = \text{diag}(2\omega_c^2 + 4\zeta_c^2 \omega_c^2, 2\omega_c^2 + 4\zeta_c^2 \omega_c^2),
\]

\[
c_4 = \text{diag}(c_{41}, c_{42}) = \text{diag}(4\zeta_c \omega_c, 4\zeta_c \omega_c).
\]

where \( \omega_c = 6, \zeta_c = 1, \alpha_1 = 9/13, \alpha_2 = 9/12, \alpha_3 = 9/11, \alpha_4 = 9/10. \) Let \( m = 2, \) the parameters \( \mu_i \) in FTDO (6) are set as: \( [\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6] = [1.1, 1.5, 2, 3, 5, 8], \) the parameter \( L \) in FTDO (6) is selected as \( L = \text{diag}(L_1, L_2) = \text{diag}(8^6, 8^6). \) The parameters \( \lambda_i = \text{diag}(\lambda_{i1}, \lambda_{i2}) \) in GPIO (21) satisfy the sixth-order polynomial form:

\[
p_o(s) = \left[(s^2 + 2\zeta_o \omega_o s + \omega_o^2)^3, (s^2 + 2\zeta_o \omega_o s + \omega_o^2)^3 \right]^T,
\]

with

\[
\lambda_1 = \text{diag}(\lambda_{11}, \lambda_{12}) = \text{diag}(\omega_o^6, \omega_o^6),
\]

\[
\lambda_2 = \text{diag}(\lambda_{21}, \lambda_{22}) = \text{diag}(6\zeta_o \omega_o^5, 6\zeta_o \omega_o^5),
\]

\[
\lambda_3 = \text{diag}(\lambda_{31}, \lambda_{32}) = \text{diag}(2\omega_o^4 + 12\zeta_o^2 \omega_o^4, 2\omega_o^4 + 12\zeta_o^2 \omega_o^4).
\]
Fig. 1 Tracking trajectories of two-link FJRs under the LFBC \([a(1)–c(1)]\) and FTFBC \([a(2)–c(2)]\) schemes, in the case of \(\phi_0 = 0\) in (20) and (7), respectively. \(a(1)–(2)\) Joint positions; \(b(1)–(2)\) Tracking errors; \(c(1)–(2)\) Control torques.

Fig. 2 Tracking trajectories of two-link FJRs under the GPI-OBC \([a(1)–f(1)]\) and FTDOBC \([a(2)–f(2)]\) schemes, in the absence of matched and mismatched time-varying disturbances \((w_1 = w_2 = 0)\), respectively. \(a(1)–(2)\) Joint positions; \(b(1)–(2)\) Tracking errors; \(c(1)–(2)\) Control torques; \(d(1)–(2)\) Estimated positions; \(e(1)–(2)\) Estimation errors; \(f(1)–(2)\) Estimated disturbances.
Table 1  Nominal parameters of two-link FJRs

| Parameter | Description | Values | Unit |
|-----------|-------------|--------|------|
| $m_1$     | Mass of link 1 | 1      | kg   |
| $m_2$     | Mass of link 2 | 1      | kg   |
| $l_1$     | Length of link 1 | 2      | m    |
| $l_2$     | Length of link 2 | 2      | m    |
| $J_1$     | Joint flexibility | 1      | kg·m² |
| $J_2$     | Joint flexibility | 1      | kg·m² |
| $B_1$     | Damping coefficient | 0.9  | N·m·s/rad |
| $B_2$     | Damping coefficient | 0.9  | N·m·s/rad |
| $K_1$     | Joint stiffness  | 100    | N·m/rad |
| $K_2$     | Joint stiffness  | 100    | N·m/rad |
| $g$       | Gravitational constant | 9.8   | m/s² |

Fig. 3  Tracking trajectories of two-link FJRs under the GPI-OBC [a(1)–f(1)] and FTDOBC [a(2)–f(2)] schemes, in the presence of mismatched time-varying disturbance $w_1$, respectively.

a(1)–(2) Joint positions; b(1)–(2) Tracking errors; c(1)–(2) Control torques; d(1)–(2) Estimated positions; e(1)–(2) Estimation errors; f(1)–(2) Estimated disturbances
Fig. 4 Tracking trajectories of two-link FJRs under the GPI-OBC [a(1)–f(1)] and FTDOBC [a(2)–f(2)] schemes, in the presence of matched time-varying disturbance $w_2$, respectively.

\begin{align*}
\lambda_4 &= \text{diag}(\lambda_{41}, \lambda_{42}) = \text{diag}(12\zeta_\omega \omega_\omega^3 + 8\zeta_\omega^3 \omega_\omega^3, 12\zeta_\omega \omega_\omega^3 + 8\zeta_\omega^3 \omega_\omega^3), \\
\lambda_5 &= \text{diag}(\lambda_{51}, \lambda_{52}) = \text{diag}(3\omega_\omega^2 + 12\zeta_\omega^2 \omega_\omega^2, 3\omega_\omega^2 + 12\zeta_\omega^2 \omega_\omega^2), \\
\lambda_6 &= \text{diag}(\lambda_{61}, \lambda_{62}) = \text{diag}(6\zeta_\omega \omega_\omega, 6\zeta_\omega \omega_\omega),
\end{align*}

where $\omega_\omega = 38$, $\zeta_\omega = 1$. In this case, the desired trajectories for the two-link FJRs are set as $q_r = [q_{r1}, q_{r2}]^T = [0.6 \sin (0.5 \pi t), 0.5 \sin (0.2 \pi t) + 0.5 \sin (0.4 \pi t)]^T \text{ rad}$, the time-varying variable $d = 5e^{-5\sin (10t^2)} \cos (5t^2)^2$ will be used later. To keep the consistency in the comparison, the saturation limit of control inputs $\tau_i$ is $\pm 100 \text{ N-m}$ and the sampling periods are all set as 0.1 ms.

**Test 1:** LFBC v.s. FTFBC for two-link FJRs (namely $\hat{\varphi}_0 = 0$ in (20) and (7)) In this comparative test, the tracking performance of the LFBC and FTFBC schemes on the two-link FJRs, which are equivalent to the special cases ($\hat{\varphi}_0 = 0$) of the GPI-OBC and FTDOBC schemes, respectively, are given. Figure 1 shows the performance of the LFBC and FTFBC strategies. In Fig. 1b(1) and b(2), when the two control schemes are applied to the two-link FJRs, these two controllers cannot guarantee good tracking performance for the system because there is a nonzero term $\xi(t)$ in the system (4) that contains model parameters even if there is no external disturbances ($w_1 = w_2 = 0$). Besides, it is obvious that the tracking accuracy of the FTFBC approach is better than that of the LFBC approach.

Test 2: GPIOBC v.s. FTDOBC for two-link FJRs in the absence of matched/mismatched time-varying disturbances (namely $w_1 = w_2 = 0$) In the second test, the
tracking performance of the GPIOBC and FTDOBC schemes are given under \( w_1 = w_2 = 0 \). Figure 2 shows the results of their joint positions, tracking errors, control torques, estimated positions, estimation errors, and estimated disturbances. It can be observed that these two control algorithms can achieve satisfied control performance compared with the aforementioned LFBC and FTFBC algorithms. Notice that in Fig. 2b(2), where the FTDOBC method is employed, the steady-state tracking errors are smaller than that in Fig. 2b(1) of the GPIOBC method. Meanwhile, it can be observed in Fig. 2e(1) and e(2) that estimation errors of the FTDOBC scheme are much less than that of the GPIOBC method. These results conclude that compared with the GPIOBC scheme, the two-link FJRs under the FTDOBC scheme obtains a higher precision tracking performance.

**Test 3: GPIOBC v.s. FTDOBC for two-link FJRs in the presence of mismatched time-varying disturbance (namely \( w_2 \neq 0 \))** In the third test, the robustness performance of the GPIOBC and FTDOBC schemes against the mismatched time-varying disturbance is investigated. The time-varying disturbance \( w_1 = [d, d]^T \) N-m is applied to the link-side of FJRs when the system runs at 10 s. Figure 3 provides the tracking trajectories of their joint positions, tracking errors, control torques, estimated positions, estimation errors, and estimated disturbances. In Fig. 3b(1), the steady-state tracking errors of the system by using the GPIOBC strategy are within the ranges \([\pm 0.05, \pm 0.05]^T\) rad, which are much wider than that in Fig. 3b(2) (under the FTDOBC scheme). These results demonstrate that the system under the proposed control approach can obtain more robust performance while facing the mismatched time-varying disturbance.

**Test 4: GPIOBC v.s. FTDOBC for two-link FJRs in the presence of matched time-varying disturbance (namely \( w_2 \neq 0 \))** Finally, in the fourth comparative test, the results of robustness against matched time-varying disturbance are given through the GPIOBC and FTDOBC methods, respectively. The time-varying disturbance \( w_2 = [d, d]^T \) N-m is added through the input channel of the FJRs at 10 s. Figure 4 shows the results of joint positions, tracking errors, control torques, estimated positions, estimation errors, and estimated disturbances under the two control schemes. It is clear from Fig. 4b(1) and b(2) that the proposed approach has better tracking performance. To demonstrate quantitatively the advantages of the FTDOBC method, we also provide the mean absolute error (MAE) of tracking error in steady-state (after 10 s), as shown in Table 2, where \( e_{01} = q_1 - q_{1r}, e_{02} = q_2 - q_{2r} \). These results validate the proposed control methodology on the matched disturbance rejection and the capability of trajectory tracking.

**Remark 5** According to the results of the two-link FJRs given above, the performances of the devised FTDOBC scheme in terms of high-precision trajectory tracking and disturbance suppression are obviously superior to the existing LFBC, FTFBC, and GPIOBC schemes. Besides, due to its simple design and easy parameter tuning processes, the proposed control strategy can be applied to other similar systems effortlessly.

## 5 Conclusion

In this work, to achieve high-precision tracking control performance, a robust finite-time output feedback control scheme is proposed for the FJ robots that suffer...
from unknown various disturbances. Compared with other schemes, the proposed scheme only requires a few mechanical parameters that can be easily obtained and the measurements of the link-side positions, which greatly reduces the modeling burden and hardware costs. Besides, the parameters of the controller and observer can be simply calculated through the pole placement method and the given formula. The finite-time stability of the closed-loop system is validated by a detailed proof. Finally, the superior performance of the proposed control scheme in terms of control accuracy and robustness has been demonstrated by a series of comparative studies based on numerical simulation.

Data availability The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Appendix

\( \tau \) is the following

\[
\tau = J \dot{x}_4 + B x_4 + K (x_3 - x_1) - w_2
\]

\[
= \frac{J}{K} (M (z) \dot{z}^{(4)} + 2 \dot{M} (z) \dot{z}^{(3)} + \dot{M} (z) \ddot{z} + C (z, \dot{z}) \dot{z}^{(3)} + 2 \dot{C} (z, \dot{z}) \dot{z} + \ddot{C} (z, \dot{z}) \dot{z} + K \ddot{z} + \dot{G} (z) - \omega_1) + \frac{B}{K} (M (z) \ddot{z} + \dot{C} (z, \dot{z}) \ddot{z} + \dot{C} (z, \dot{z}) \ddot{z} + K \ddot{z} + \dot{G} (z) - \omega_1)
\]

\[
= \frac{J M (z)}{K} \dot{z}^{(4)} + \frac{2 J M (z)}{K} \dot{M} (z) \dot{z}^{(3)} + \frac{B M (z)}{K} + \frac{J C (z, \dot{z})}{K} \dot{z}^{(3)} + \frac{J \dot{M} (z)}{K} \ddot{z} + \frac{2 J \dot{C} (z, \dot{z})}{K} \dot{z}^{(3)} + \frac{B C (z, \dot{z})}{K} + \frac{C (z, \dot{z})}{K} B \dot{z} + \frac{M (z)}{K} \ddot{z} - \omega_2.
\]

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