Neutrino spin operator and dispersion in moving matter

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Abstract We found the spin integral of motion for neutrinos propagating in moving and polarized matter. Contrary to all previous studies this is the exact spin operator commuting with the Hamiltonian for a neutrino in matter which moves in an arbitrary direction relative to the direction of neutrino propagation. The operator obtained opens up the possibility of consistent classification of neutrino states in such a medium and, as a consequence, a systematic description of the related physical phenomena. Using the operator, we obtain a dispersion relation for neutrinos in arbitrary moving matter and consider its particular cases.

1 Introduction

The problem of describing neutrino motion in extreme external conditions (intense fields and dense matter) has achieved exceptional relevance in the multi-messenger era\cite{1}, when humanity is able to register signals of different nature coming from the Universe.

The study of the structure of neutrino signal sources – intense astrophysical processes involving compact objects (type II supernovae, gamma-ray bursts) – within the framework of the planned neutrino mega-projects\cite{2} requires a detailed understanding of the evolution of a neutrino beam inside these objects or in their environment. At that, evolution is not limited only to spatial motion, but can also include time dependence of the flavor composition and helicity (flavor and spin-flavor oscillations). In general, evolution can be very complex, since it can be influenced by many factors connected with interaction of neutrinos with their environment (dense medium and/or external fields)\cite{3}. To consistently take these factors into account, a detailed development of the theory of neutrino motion in the external environment is needed.

Propagation of a neutrino beam under external conditions in the simplest model cases can be analytically described by using the corresponding wave equation, from which a neutrino dispersion relation follows. Within the framework of this approach, it is possible to rigorously substantiate and reproduce the known results on the flavor and spin-flavor neutrinos oscillations in the medium and the electromagnetic field\cite{4,5}, and also to obtain new ones\cite{6–8}.

For instance, solutions of the so-called modified Dirac equation for neutrinos in a homogeneous medium and the corresponding expression for energy, from which, in particular, the well-known effect of neutrino spin oscillations follows, become helicity-dependent\cite{9,10}. This feature can lead to various physical effects in matter, for example, to the emission of a photon (the spin light of a neutrino\cite{9–12}), as well as to self-polarization of a neutrino beam\cite{13}.

A rigorous derivation of these effects (calculation of corresponding amplitudes) is possible only on the basis of the classification of the initial and final neutrino states according to the full set of observables, including spin quantum numbers. This example recalls that the most important point in such problems, as in the well-known case of charged particle motion in the electromagnetic field\cite{14}, is the separation of states according to the spin quantum number, i.e. within the consistent approach, the exact solving of the Dirac equation must involve determination of the independent integral of motion - the spin operator.

2 The modified Dirac equation for neutrino in matter

In this paper, we study the modified Dirac equation that describes a neutrino coherently interacting with particles of
the external matter, taking into account possible effects of matter motion and polarization. In the general case, this equation can be written as follows:

\[ i \gamma^\mu \partial_\mu - \frac{1}{2} \gamma^\mu (1 + \gamma^5) f^\mu - m \right) \Psi(x) = 0, \quad (1) \]

where \( f^\mu \) is the constant 4-vector, represented by the linear combination of the matter current vector \( j^\mu \) and the polarization vector \( \lambda^\mu \) with coefficients, determined by the type of neutrino interaction with matter particles (see, for instance, \[9,12\]), \( \gamma^5 = -i \gamma^0 \gamma^1 \gamma^2 \gamma^3 \).

For the medium consisting of identical particles, the current 4-vector is defined as \( j^\mu = n_0 v^\mu = (n, n v) \), where the matter density \( n \) is related to its velocity \( v \) and density \( n_0 \) in its rest frame by the standard kinematical relation \( n = \gamma n_0 \), where \( \gamma = 1/\sqrt{1 - v^2} \) is the Lorentz gamma factor. The polarization 4-vector in the rest frame of the medium has the components \( \lambda^\mu = (0, n_0 \xi) \), where the three-dimensional vector \( \xi \) is the average value of the matter polarization vector \[15\].

Matter polarization occurs due to the interaction with an external magnetic field (see, for example, \[16\]) and in this connection it is often assumed that equation (1) describes the scattering of neutrinos on medium particles polarized by an external field. Note that there is another physical interpretation of this phenomenon, according to which a direct interaction of the neutrino induced magnetic moment (IMM) with an external magnetic field takes place \[17\]. Both approaches are equivalent and lead to the same results.

In what follows, we will consider the problem of a massive electron neutrino propagation in moving homogeneous unpolarized medium consisting of electrons.\(^1\) Then the 4-vector \( f^\mu \) is determined only by the matter current, and within the framework of the minimally extended Standard Model we have \[9\]:

\[ \frac{1}{2} f^\mu = \tilde{n}_0 v^\mu, \quad \tilde{n}_0 = \frac{1}{2 \sqrt{2}} G_F (1 + 4 \sin^2 \theta_W) n_0, \quad (2) \]

where the dependent on the type of interaction coefficient is included in the definition of the “density with a tilde” \( \tilde{n}_0 \). Note that, the equation of the same type as (1) but in a more general form, also appears in phenomenological theories that go beyond the Standard Model (in particular, in theories with Lorentz invariance violation \[18\] and in theories with non-standard neutrino interactions \[19\]).

In the phenomenological respect, the problem of neutrino propagation in moving matter was for the first time considered in \[20\], where, on the basis of a semiclassical approach, the effect of neutrino spin flip in the transverse matter current was described and the related phenomenon of spin oscillations was discussed. This effect and its possible contribution to the evolution of neutrinos in astrophysical media were discussed later in \[21–25\]. The theory of the phenomenon was further developed in \[8\], where it was considered on the quantum level by calculating the amplitudes that enter the effective neutrino evolution equation. Neutrino spin oscillations in a supernova environment associated with the presence of IMM were considered in \[17\], see also \[26,27\].

As indicated above, in order to consistently describe neutrino processes under various external conditions and to classify solutions of the corresponding equation, it is necessary to establish the form of the spin operator, which would be the integral of motion, and also to determine the dispersion relation that would depend on the spin quantum number.

Of the works listed above, the solution of the modified Dirac equation of the considered type with the establishment of the required spin operator was carried out only in \[26,27\]. However, in these works, a particular form of the equation was investigated, and the final dispersion relation was obtained in the linear-field approximation. Note also that the particular case considered there corresponds to the null zero component \( f^0 \) of the 4-vector \( f^\mu \). In our setting (2), this component cannot vanish.

Note also that related questions about the neutrino propagator in a moving medium were studied in \[28–30\], where the exact dispersion relation for these conditions was obtained in different representations. It reduces to a trivially unsolvable fourth-order equation for energy. The dispersion relation was also obtained in \[18\] for a more general equation. In these cases, the polarization properties of the particle were either not studied at all, or were not determined in a closed form through the eigenvalues of the conserved spin operator, and thus the physical basis for the classification of the solutions remained unclear. In this study, we aimed to obtain the exact spin integral of motion for equation (1) in an explicit form and to determine the dispersion relation with the corresponding quantum number that has a clear physical meaning.

3 Neutrino spin operator in moving matter

By this means, it is necessary to find a spin operator commuting with the Hamiltonian of equation (1), which has the form:

\[ H = (\alpha p) - \tilde{n}(\alpha v) + \tilde{n}(\Sigma v) + \gamma^5 \tilde{n} + \tilde{n} + \gamma^0 \tilde{m}. \quad (3) \]

In expression (3), we have introduced the three-dimensional neutrino momentum \( p \) and used the notations \( \alpha_i = \gamma^0 \gamma^i \), \( \Sigma_i = \gamma^0 \gamma^5 \gamma^i \), \( \tilde{n} = \gamma^0 n_0 \). As a basis for the required operator, we use the 4-vector spin polarization operator \( T^\mu \) (see, for

\(^1\) The results of our work remain valid in the general case of a polarized medium also.
instance, [14,31]):
\[ T^\mu = \gamma^5 \gamma^\mu - \gamma^5 p^\mu / m. \] (4)

In this operator, let us apply the momentum “extension”
\( p^\mu \to \tilde{p}^\mu \equiv p^\mu - f^\mu = p^\mu - n_0 v^\mu \) (herewith \( T^\mu \to \tilde{T}^\mu \))
and compose a scalar product of the vector \( \tilde{T}^\mu \) with the 4-vector \( v^\mu \). It is easy to verify that commutation of the obtained quantity \( \tilde{T}^\mu v_\mu \equiv (\tilde{T}v) \) with the Hamiltonian (3) is achieved
on solutions of the Dirac equation. Then we define the spin operator by multiplying this quantity by \( m \):
\[ S = m(\tilde{T}v). \] (5)

Introducing the notation \( \tilde{H} = \tilde{p}^0 = H - \tilde{n} \), let us also present
the operator \( S \) in the expanded form:
\[ S = \gamma \left[ \gamma^5 \gamma^0 m - \gamma^5 (\tilde{H} - (\tilde{p}v)) - m \gamma^0 (\Sigma v) \right]. \] (6)

Since we obtained the expression for the conserved spin operator in the form of a Lorentz scalar, it is also suitable to
combine the rest of the kinematical characteristics - energy and momentum - into a quantity of this kind. Therefore, we
compose a scalar combination
\[ P = (\tilde{p}^\mu v_\mu) = (\tilde{p}v) = (pv) - \tilde{n}_0, \quad \tilde{n}_0 = \tilde{n}/\gamma, \] (7)

which turns out to be convenient for writing kinematic quantities in this problem. Then the block form of the spin operator
has a compact form (here we use the standard representation of the gamma matrices and take into account the implemen-
tation of the operator \( \tilde{H} \to \tilde{E} = E - \tilde{n} \) on solutions of the Dirac equation):
\[ S = \begin{pmatrix} -\gamma m(\sigma v) & \gamma m + P \\ -\gamma m + P & \gamma m(\sigma v) \end{pmatrix}, \] (8)

where \( \sigma_i \) are Pauli matrices. The eigenvalues of the operator \( S \) are
easily found and have the form:
\[ s\sqrt{p^2 - m^2} \equiv s\Lambda, \quad s = \pm 1. \] (9)

The operator obtained characterizes the longitudinal polarization of the particle with respect to the 4-vector of the
medium velocity \( v^\mu \) and determines the stationary spin states of the particle \( \Psi_s \), corresponding to the conserved quantity \( \pm \Lambda \):
\[ S\Psi_s = s\Lambda\Psi_s. \] (10)

In this regard, we note that the operator used in [29,30] to
describe neutrino polarization properties in the same problem, in terms of this article, appears to be proportional to the
scalar product \( (Tv) \) where no “extension” for the momentum is implemented in the 4-vector \( T^\mu \). This turns out to be sufficient
for this value not to be an integral of motion (as in fact was noted in [29,30]).

4 Neutrino dispersion relation in moving matter

Relation (10) together with the original equation (1) allows us
to determine the dispersion relation for neutrinos in a moving medium taking into account the spin quantum number \( s \).
To find it, one can perform a standard procedure and represent the wave function through two-component spinors
\( \Psi = (\varphi, \chi)^T \). Rewriting the original Dirac equation (1) through the system of equations for these spinors, we obtain
the compatibility condition for the equations:
\[ p^2 - m^2 = 2\tilde{n}_0(P - s\Lambda), \] (11)

where \( p^2 = E^2 - \mathbf{p}^2 \). This dispersion relation is reduced to a fourth-order algebraic equation for the particle energy.

First of all, it should be noted that the analytical expression for the dispersion law can be written in various forms. Indeed,
in [29,30] it was obtained in the form (entries are written in the notation of this work)
\[ p^2 - m^2 = 2\tilde{n}_0 \left( (pv) - s' \sqrt{(pv)^2 - p^2} \right), \] (12)

and is not reduced to (11) by simple algebraic transformation. The quantity \( s' \) also equals \( \pm 1 \), but it should have a different
physical meaning. However, when reduced to a forth-order algebraic equation with respect to \( E \), Eqs. (11) and
(12) become a general dispersion relation that does not contain the spin quantum number and that was obtained in [28]:
\[ (p^2 - m^2)^2 - 4\tilde{n}_0(pv)(p^2 - m^2) + 4\tilde{n}_0^2 p^2 = 0. \] (13)

We also note one more representation of the dispersion relation (obtained directly from (11)) - completely through the
“extended” values:
\[ \tilde{p}^2 - m^2 = -\tilde{n}_0(2s\Lambda + \tilde{n}_0). \] (14)

Let us consider some special cases of the dispersion relation solution. For a medium at rest (when \( v = 0 \), we have
\( P = E - \tilde{n}_0 = \tilde{E}, \tilde{p}^2 = \tilde{E}^2 - \mathbf{p}^2 \). Next, we will use the relation (14), rewriting it in the form
\[ (\sqrt{\tilde{E}^2 - m^2} + \tilde{n}_0)^2 = p^2, \] (15)
whence, introducing the “energy sign” \( \varepsilon = \pm 1 \), we obtain the previously known result [9,10] (\( p = |p| \)):

\[
E = \varepsilon \sqrt{(p - s\tilde{n}_0)^2 + m^2 + \tilde{n}_0}.
\]

(16)

In a similar way, a solution can be obtained for the case of the neutrino and matter parallel motion \((v|p|)\). The expression for the energy in this case has the form:

\[
E = \varepsilon \sqrt{(p \mp v\tilde{n} - s\tilde{n})^2 + m^2 \pm sp\tilde{n} + \tilde{n}},
\]

(17)

where the upper signs correspond to the motion of the medium along the direction of neutrino propagation, and the lower ones correspond to the opposite direction.

Note that in expressions (16), (17) the quantum number \( s \) is the helicity of the particle (see [9]), and they can be used for the derivation of the energy difference between ultrarelativistic left- and right-handed neutrinos \( \Delta E = E_L - E_R \) in the problem of neutrino oscillations in matter [32]. For the case of non-moving matter, from Eq. (16) the standard expression follows known from the theory of neutrino oscillations in matter [32]. For the case of non-moving matter, from Eq. (16) the standard expression follows known from the theory of neutrino oscillations in matter [32].

\[
\Delta E = 2\tilde{n}_0 \sqrt{1 - \frac{v}{1 + v}},
\]

(18)

and in the case of oncoming motion of neutrinos and matter

\[
\Delta E = 2\tilde{n}_0 \sqrt{1 + \frac{v}{1 - v}}.
\]

(19)

Both expressions agree with the general formula, which takes into account arbitrary directions of matter velocity and polarization, and which was previously found within the phenomenological approach without introduction of the neutrino spin operator [34].

In the case of transversal matter motion, \((pv) = 0\), the dispersion relation leads to a fourth-order equation with respect to \( E \). For an ultrarelativistic neutrino, with the momentum being the largest parameter, i.e., \( p \gg \tilde{n}, p \gg m \), its solutions corresponding to \( \varepsilon = 1 \) can be approximately represented as

\[
E_{s=+1} = \sqrt{(p - \tilde{n})^2 + m^2 + \tilde{n}},
\]

(20)

\[
E_{s=-1} = \sqrt{p^2 + 4\tilde{n}^2 + m^2 + 2\tilde{n}}.
\]

(21)

Under conditions \( v \to 0, m \to 0 \) these converge to (16).

Our general treatment described above can have important applications, for instance, in studies of new features in neutrino oscillations in moving matter. To illustrate, consider the \( \nu_e \leftrightarrow \nu_\mu \) neutrino oscillations in moving matter composed of electrons. Having only the expression (21) one can immediately assess the impact of the transverse matter motion on the flavor neutrino oscillations \( \nu_e \leftrightarrow \nu_\mu \) by calculating the corresponding energy difference. In the considered relativistic case, associating \( E_{s=-1} \) with left-handed neutrino energy \( E_{\nu_L} \), we get

\[
E_{\nu_L} \simeq p + 2\tilde{n} + \frac{2\tilde{n}^2v^2}{p},
\]

(22)

where terms follow in descending order of magnitude. The second term leads to the standard effective neutrino oscillation potential in matter given by \( A = 2n_{\nu_e} - 2n_{\nu_\mu} = \sqrt{2}G_F n \) (see, for instance, [32]) that enters the neutrino oscillation resonance condition. The presence of the third term provides the following shift of this value

\[
A \to A' = \sqrt{2}G_F n + \frac{8G_F^2n^2v^2}{p}\sin^2\theta_W.
\]

(23)

The corresponding oscillation probability averaged over traveled by neutrino distance is readily found to be:

\[
P = \frac{1}{2} \frac{\Delta^2 \sin^2 2\theta}{(\Delta \cos 2\theta - 2pA')^2 + \Delta^2 \sin^2 2\theta},
\]

(24)

where \( \Delta = m_2^2 - m_1^2 \) is the neutrino mass squared difference and \( \theta \) is the vacuum mixing angle. From (24) it follows that the standard flavor neutrino oscillation resonance condition shifts according to effective potential change:

\[
\frac{\Delta}{2p} \cos 2\theta = A + 8G_F^2n^2v^2\sin^2\theta_W.
\]

(25)

The extra term on the right-hand side accounts for the effect of transversal matter motion and in general is small. However it can have a substantial values for low-energy neutrinos and extra-dense matter with densities of order \( 10^{41} \text{ cm}^{-3} \) that corresponds to \( G_F n_0 \sim 10^3 \text{ eV} \) (see, for instance, [35] and references therein). For instance, under these conditions this term can be of the order of the standard one for the following set of parameters: \( v = 0.9, p \sim 10 \text{ keV} \).

5 Conclusion

An important new contribution to the theory of neutrino motion in a medium, which is presented in this article, is finding of the spin operator commuting with the Hamiltonian of a neutrino propagating in moving matter. The considered formalism, based on this spin integral of motion, makes it possible to consistently describe neutrino quantum states in matter with uniform and constant characteristics – density,
velocity, and polarization. It can be applied to problems of relativistic neutrino astrophysics, where the dynamics of the neutrino spin plays an important role. The latter aspect may be key for the interpretation of the expected data on the registration of astrophysical neutrinos by large-volume detectors such as JUNO and Hyper-Kamiokande. Thus, one of the specific areas of our formalism application may be neutrino flavor and spin oscillations, which physics is being actively studied for various types of matter within the common problem of establishing the neutrino role in supernova explosions [36].

Another important possibility of application of the stationary neutrino quantum states in matter is the consistent description of various particle interaction processes participating neutrinos in dense astrophysical media. Due to emerging difference between two neutrino spin states (as can be seen, for example, from formulas (16), (17) and (20)–(21)) such processes begin to depend on neutrino spin orientation. Given this fact, well-known processes can acquire new interesting properties, and in some cases there can take place the processes forbidden in vacuum. The relevant example is the spin light of neutrino in matter – the emission of a photon by a neutrino moving in matter upon its transition between different spin states [9, 11, 12]. The properties of this phenomenon turn out to be very sensitive to medium characteristics, including its motion. In dense astrophysical objects, the radiation can be intense enough to discuss the possibility of its experimental registration [35].

To conclude, the presented approach based on our spin integral of motion, is a new step in the consistent theoretical description of neutrino motion in a medium. It has specific areas of application in modern astrophysics and expands the possibilities of describing the elementary particles interaction processes in astrophysical conditions.

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