Rendezvous maneuvers using Genetic Algorithm

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Abstract. The present paper has the goal of studying orbital maneuvers of Rendezvous, that is an orbital transfer where a spacecraft has to change its orbit to meet with another spacecraft that is travelling in another orbit. This transfer will be accomplished by using a multi-impulsive control. A genetic algorithm is used to find the transfers that have minimum fuel consumption.

1. Introduction
To study space maneuvers that search for the minimum fuel consumption in interplanetary missions is an important field of research for the development of space technologies. The problem of transferring a spacecraft between two coplanar circular orbits with free time in a central force field was studied in the present research looking for multi-impulsive transfer using a genetic algorithm. It is obtained as a solution a four-impulsive elliptical transfer orbit [1 – 4]. In this particular case the transfer of the spacecraft is made from one body back to the same body [1]. The literature is extensive with respect to the problems involving transfer orbits and optimal spacecraft maneuvers [1 - 12].

2. Genetic Algorithm
The genetic algorithm is a stochastic global search method inspired on the natural genetic and biological evolution. The genetic algorithm operates on a population of potential solutions by applying the principle of survival of the fitness to produce better and better approximations to find the best solution.

At each generation, it is created new individuals by the process of selecting individuals according to their level of fitness in the problem domain and breeding them together using operators inspired in natural genetics. So, the idea is to create individuals that are better suited to their environment than the individuals that existed in the previous generation.

In this case, a generation is one iteration with a certain quantity of evaluation of the objective function. In each evaluation, the genetic operators are applied, inspired in natural genetics. For the representation, the individuals are created as a chromosome where each one is considered a possible solution for the problem.

Broadly speaking, genetic algorithms differ from traditional techniques by using probabilistic rules, not deterministic, and by working with population of individuals, different of other methods that...
work from a single point. This is a great advantage, because by maintaining a population with good individuals, the probability of reaching a false peak is reduced.

This algorithm introduces many new concepts, including the fitness of a chromosome that represents the quality of the individual at the environment, the selection probability of an individual, the crossover operator, the mutation and the epidemic operators to introduce random perturbations in the search. In Figure 1 it is demonstrated an analogy to help the understanding of the link between genetic and computer languages.

2.1. Crossing-Over
The crossing-over operator is used to create new individuals from parents. This new individual is created from a random cut-off dividing the parents in two parts, having the same or different sizes. The first son consists on the left part of the father with the right part of the mother and the second son consists on the right part of the father with the left part of the mother. Figure 2 demonstrates this schema.

2.2. Mutation
The mutation operator is used to introduce random perturbations in the search, having a low occurrence probability, around 0.5%. This perturbation is created by generating a random number between 0 and 1. If this number is smaller than the occurrence probability, the mutation operator works, flipping randomly a bit in the chromosome, if not, the mutation operator is ignored.

2.3. Epidemic
The epidemic is another operator used to introduce random in the search, having a very low occurrence probability, around 0.01%. This perturbations is created by generating a random number between 0 and 1, if this number is smaller than the occurrence probability, the epidemic operator works, killing a random part of the population and creating new individuals to occupy the environment.

2.4. Objective Function
Comparisons are made for the fitness of the solutions to decide which individual should be propagated to the next generation. Normally, fitness is directly related to the value of the objective solution, with a better objective value indicating the higher fitness. When the Genetics Algorithm (GA) procedure calls its function objective, it passes an array in the first parameter that specifies the selected solution, which is referred as the selection parameter. The selection parameter must not be altered in any way by the function objective.
The fitness of each individual should be computed using the five data that define the problem \((a_1, e_1, a_2, e_2, \Delta \omega)\), the first being unit because of the normalization) and the three genes \((\nu_1, \Delta \nu, y)\) that characterize the individual. One obtains, in sequence [13]:

the true anomaly of the arrival point

\[ \nu_i = \nu_{i-1} + \Delta \nu \]  

(1)

the radii of the departure and arrival point are given by

\[ r_1 = \frac{a_1 (1 - e_1^2)}{1 + e_1 \cos \nu_1} \]  

(2)

\[ r_2 = \frac{a_2 (1 - e_2^2)}{1 + e_2 \cos \nu_2} \]  

(3)

the distance between \(P_1\) and \(P_2\), on each arc, is

\[ c = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos \Delta \nu} \]  

(4)

the semi-major axis of the transfer orbit is

\[ a_{\text{min}} = \frac{r_1 + r_2 + c}{4} \]  

(5)

the distances \(c_1\) and \(c_2\) of \(P_1\) and \(P_2\) from the vacant focus \(F_2\) can be specified by the equations

\[ c_i = 2a - r_i \]  

(6)

The Figure 3 shows a description of several important variables.

**Figure 1 - Geometry of the Problem and the angles involved in the problem.**

The angles can be calculated by

\[ y = \arccos \left( \frac{r_1^2 - r_2^2 + c^2}{2r_1 c} \right) \]  

(7)
the eccentricity of the transfer orbit is given by
\[ e_t = \sqrt{c_1^2 + r_1^2 - 2c_1r_1 \cos y_2} / 2a_t \]  \hspace{1cm} (9)

the true anomaly \( \theta_1 \) of the P₁ on the transfer orbit is
\[ \theta_1 = \arccos \left( \frac{a_1(1-e_1^2)-r_1}{r_1 e_t} \right) \]  \hspace{1cm} (10)

the argument of perigee for the transfer orbit is
\[ \omega = v_1 - \theta_1 \]  \hspace{1cm} (11)

which is the angle between the perigees of the transfer and the initial orbits.

The geometry of the manoeuvre has been measured. One calculates the radial and the tangential components of the spacecraft velocity before and after both impulses, what permits the computation of the total \( \Delta V \), which has been assumed as the measurement of the individual fitness. Non-dimensional variables are used in the code [3]. They are shown below.
\[ r = \frac{\rho}{a_1} \]  \hspace{1cm} (12)
\[ v = \frac{\rho}{\sqrt{a_1}} \]  \hspace{1cm} (13)

The distances and velocities references are the semi-major axis of the initial orbit and the velocity of a spacecraft in a circular orbit with the same energy as the initial one. So, the reference time is \( \sqrt{a_1^2 / \mu} \).

3. Numerical Solutions
Several transfers were simulated with the procedure developed using the genetic algorithm [2 - 4]. The present paper shows one of them, with initial radius \( r_o = 2 \) and final radius \( r_f = 2 \). The genetic algorithm provided satisfactory solutions when compared to solutions found in the literature [3 - 5]. The population comprises of 800 individuals and up to 400 generations of individuals (Figure 4).
Figure 4 - The variables of the problem using the genetic algorithm method.

Figure 5 - Transfer Maneuver from one orbit back to the same orbit.

Figure 5 shows a simulation with four-impulses and the total consumption is $\Delta V = 0.688651$ u.n., for a mission leaving and coming back to the same orbit.

4. Conclusion

The rendezvous problem was studied using four-impulsive maneuvers to complete the transfer with the use of genetic algorithm methods. The solutions obtained by the procedure are satisfactory when compared to the classic transfer maneuvers like, for example, Hohmann maneuvers [2].
The results indicate that the maneuvers using the genetic algorithm does not provide better fuel consumption in all cases simulated, but the method proved to find efficient solutions, close to the classical methods for two-impulses, and effective in terms of fuel consumption.

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