Constant Curvature Black Hole and Dual Field Theory

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Abstract

We consider a five-dimensional constant curvature black hole, which is constructed by identifying some points along a Killing vector in a five-dimensional AdS space. The black hole has the topology $\mathcal{M}_4 \times S^1$, its exterior is time-dependent and its boundary metric is of the form of a three-dimensional de Sitter space times a circle, which means that the dual conformal field theory resides on a dynamical spacetime. We calculate the quasilocal stress-energy tensor of the gravitational background and then the stress-energy tensor of the dual conformal field theory. It is found that the trace of the tensor does indeed vanish, as expected. Further we find that the constant curvature black hole spacetime is just the “bubble of nothing” resulting from Schwarzschild-AdS black holes when the mass parameter of the latter vanishes.
1 Introduction

Recently there has been much interest in studying string theory in time-dependent spacetimes. The authors of papers [1] discussed orbifold constructions giving solutions with tractable string descriptions. Aharony et al [2] considered spacetimes of so-called “bubbles of nothing”, which are double analytic continuations of Schwarzschild or Kerr black hole spacetimes. These spacetimes are interesting examples of smooth time-dependent solutions because they are consistent backgrounds for string theory at least to leading order since they are vacuum solutions to Einstein’s equations. More recently, Balasubramanian and Ross [3] have extended the discussions of [2] to the cases of asymptotically anti-de Sitter (AdS) spacetimes. They considered “bubbles of nothing” constructed by analytically continuing (Schwarzschild, Reissner-Nordström, and Kerr) black holes in AdS spaces. Since these spacetimes are asymptotically AdS, so it is possible to study them through the AdS/CFT correspondence [4]. Further since these spacetimes are time-dependent and then the boundary metric is also time-dependent, in some sense it therefore opens a window to study dual strong coupling field theory in time-dependent backgrounds. For the “bubbles of nothing” resulted from double analytic continuation of the Schwarzschild-AdS black hole spacetime, its boundary metric is of the form of de Sitter (dS) space times a circle. Balasubramanian and Ross calculated stress-energy tensor of dual field theory to “bubbles of nothing”. The bubble solutions in AdS spaces are first constructed in [5].

The work of Balasubramanian and Ross is reminiscent of the constant curvature black holes constructed by Banados [6] by identifying some points along a Killing vector in an AdS space. The so-called constant curvature black holes are analogues of BTZ black holes in higher dimensions. The topology structure of the black hole are quite different from that of usual black holes. For example, in $D$ dimensions the usual black holes have the topology $\mathcal{M}_2 \times S^{D-2}$, while the constant curvature black holes have the topology $\mathcal{M}_{D-1} \times S^1$. Here $M_n$ denotes a conformal Minkowski space in $n$ dimensions. Of course, in higher dimensions ($D > 4$) it is possible to have black hole structure if one replaces the factor $S^{D-2}$ by other topologies in $\mathcal{M}_2 \times S^{D-2}$. In the asymptotically AdS space, even in four dimensions one can have other topological even horizons by replacing $S^2$ by other forms, resulting in so-called topological black holes (for a more or less complete list of references see [7]). The causal structure of those black holes is still determined by the two-dimensional manifold $\mathcal{M}_2$ for these cases. For the constant curvature black holes, however, the causal structure is determined by a $(D-1)$-dimensional manifold $\mathcal{M}_{D-1}$. Of special interest is that the exterior of these black holes are time-dependent. So it is possible to discuss dual field theory in these dynamical spacetimes through the AdS/CFT
In this note we consider a five-dimensional constant curvature black hole. This is of particular interest because its dual is $N = 4$ supersymmetric Yang-Mills theory if the AdS/CFT correspondence holds in the case we are discussing. In Sec. 2 we introduce the construction of the black hole and discuss some salient features of the black hole [6, 8]. Although the black hole has the strange topology and its exterior is not static, we find that the surface counterterm approach [9] still works well. In Sec. 3 using the surface counterterm approach we calculate the quasilocal stress-energy tensor of gravitational field and identify it to the stress-energy tensor of the dual field theory. Sec. 4 is devoted to comparing with related investigations to the black hole by other authors, and to discussing the relation between our results and those in [3]. In particular we point out that the constant curvature black hole spacetime is just the “bubble of nothing” resulting from the Schwarzschild-AdS black hole if $r_0 = 0$ in [3].

2 Constant curvature black holes

A five dimensional AdS space is defined as the universal covering space of a surface obeying

\[-x_0^2 + x_1^2 + x_2^2 + x_3^2 + x_4^2 - x_5^2 = -l^2,\]

where $l$ is the AdS radius. This surface has fifteen Killing vectors, seven rotations and eight boosts. Consider the boost $\xi = (r_+/l)(x_4 \partial_5 + x_5 \partial_4)$ with norm $\xi^2 = r_+^2(-x_4^2 + x_5^2)/l^2$, where $r_+$ is an arbitrary real constant. The norm can be negative, zero, or positive. In terms of the norm, the surface (2.1) can be re-expressed as

\[x_0^2 = x_1^2 + x_2^2 + x_3^2 + l^2(1 - \xi^2/r_+^2).\]

When $\xi^2 = r_+^2$, the surface (2.2) reduces to a null one

\[x_0^2 = x_1^2 + x_2^2 + x_3^2,\]

while $\xi^2 = 0$, it becomes a hyperboloid

\[x_0^2 = x_1^2 + x_2^2 + x_3^2 + l^2.\]

The cone (2.3) has two pointwise connected branches, called $H_f$ and $H_p$ [3], defined by

\[
\begin{align*}
H_f & : \quad x^0 = +\sqrt{x_1^2 + x_2^2 + x_3^2}, \\
H_p & : \quad x^0 = -\sqrt{x_1^2 + x_2^2 + x_3^2}.
\end{align*}
\]

1Ref. [3] discussed the four dimensional case, it is straightforward to generalize to the higher dimensional cases.
Similarly, the hyperboloid (2.4) has two disconnected branches, named $S_f$ and $S_p$,

\begin{align*}
S_f &: \quad x^0 = +\sqrt{x_1^2 + x_2^2 + x_3^2 + l^2}, \\
S_p &: \quad x^0 = -\sqrt{x_1^2 + x_2^2 + x_3^2 + l^2}.
\end{align*}

(2.6)

The Killing vector $\xi$ is spacelike in the region contained in-between $S_f$ and $S_p$, is null at $S_f$ and $S_p$ and is timelike in the causal future of $S_f$ and in the causal past of $S_p$.

Identifying the points along the orbit of $\xi$, another one-dimensional manifold becomes compact and isomorphic to $S^1$. The region where $\xi^2 < 0$ has a pathological chronological structure and therefore it must be cut off from physics spacetime. In this sense, the surface $\xi^2 = 0$ is a singularity, $S_f$ is the future one and $S_p$ the past one. The surface $\xi^2 = r_+^2$ is a horizon, $H_f$ is the future one and $H_p$ the past. Through the above analysis, it turns out that one can construct a black hole by identifying point along the orbit of the Killing vector $\xi$. Since the starting point is the AdS, the resulting black hole therefore has a constant curvature as the AdS. The topology of the black holes is $\mathcal{M}_4 \times S^1$, which is quite different from the usual topology, $\mathcal{M}_2 \times S^3$, of five-dimensional black holes, where $\mathcal{M}_n$ denotes a conformal Minkowski space in $n$ dimensions. The Penrose diagram of the constant curvature black holes has been drawn in Refs. [6, 8].

The constant curvature black holes can be best described by using Kruskal coordinates. In Ref. [1] a set of coordinates on the AdS for the region $\xi^2 > 0$ has been introduced. The six dimensionless local coordinates $(y_i, \varphi)$ are

\begin{align*}
x_i &= \frac{2ly_i}{1 - y^2}, \quad i = 0, 1, 2, 3, \\
x_4 &= \frac{lr}{r_+} \sinh \left( \frac{r_+ \varphi}{l} \right), \\
x_5 &= \frac{lr}{r_+} \cosh \left( \frac{r_+ \varphi}{l} \right),
\end{align*}

(2.7)

with

\begin{align*}
r &= r_+ \frac{1 + y^2}{1 - y^2}, \quad y^2 = -y_0^2 + y_1^2 + y_2^2 + y_3^2.
\end{align*}

(2.8)

Here $-\infty < y_i < \infty$ and $-\infty < \varphi < \infty$ with the restriction $-1 < y^2 < 1$. In these coordinates the boundary $r \to \infty$ corresponds to the hyperbolic “ball” $y^2 = 1$, and the induced metric can be written down

\begin{align*}
ds^2 &= \frac{l^2(r + r_+)^2}{r_+^2}(-dy_0^2 + dy_1^2 + dy_2^2 + dy_3^2) + r^2d\varphi^2.
\end{align*}

(2.9)

Obviously the Killing vector is $\xi = \partial_\varphi$ with norm $\xi^2 = r^2$. The black hole spacetime is thus simply obtained by identifying $\varphi \sim \varphi + 2\pi n$ and the topology of the black hole clearly is $\mathcal{M}_4 \times S^1$. 

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The constant curvature black holes can also be described by using Schwarzschild coordinates. Introducing local “spherical” coordinates \((t, r, \theta, \chi)\) in the hyperplane \(y_i\):

\[
\begin{align*}
y_0 &= f \cos \theta \sinh(r_+ t/l), \quad y_1 = f \cos \theta \cosh(r_+ t/l), \\
y_2 &= f \sin \theta \sin \chi, \quad y_3 = f \sin \theta \cos \chi,
\end{align*}
\]

where \(f = [(r - r_+)/(r + r_+)]^{1/2}\), \(0 \leq \theta \leq \pi/2\), \(0 \leq \chi \leq 2\pi\) and \(r_+ \leq r < \infty\), one can find that the solution (2.9) becomes

\[
ds^2 = l^2 N_2^2 d\Omega_3 + N^{-2} dr^2 + r^2 d\varphi^2,
\]

(2.11)

where

\[
N_2^2 = \frac{r^2 - r_+^2}{l^2}, \quad d\Omega_3 = -\sin^2 \theta dt^2 + \frac{l^2}{r_+^2} (d\theta^2 + \cos^2 \theta d\chi^2).
\]

(2.12)

This is the black hole solution in the Schwarzschild coordinates. Here \(r = r_+\) is the black hole horizon location. In these coordinates the solution looks static. But we can see from (2.10) that the form (2.11) does not cover the full outer region of black hole since the difference \(y^2_1 - y^2_0\) is constrained to be positive in the region covered by these coordinates. Indeed, it has been proved that there is no globally timelike Killing vector in this geometry [10].

Similar to the case in four dimensions [8], we find that there is another set of coordinates, which has the advantage of covering the entire exterior of the Minkowskian black hole geometry:

\[
\begin{align*}
y_0 &= f \sinh(r_+ t/l), \quad y_1 = f \cos \theta \cosh(r_+ t/l), \\
y_2 &= f \sin \theta \cos \chi \cosh(r_+ t/l), \quad y_3 = f \sin \theta \sin \chi \cosh(r_+ t/l),
\end{align*}
\]

(2.13)

where \(f\) is given as before, \(0 \leq \theta \leq \pi\), \(r_+ \leq r < \infty\) and \(0 \leq \chi \leq 2\pi\). In terms of these coordinates, the solution can be expressed as

\[
ds^2 = N^2 l^2 d\Omega_3 + N^{-2} dr^2 + r^2 d\varphi^2,
\]

(2.14)

where \(N^2 = (r^2 - r_+^2)/l^2\) and

\[
d\Omega_3 = -dt^2 + \frac{l^2}{r_+^2} \cosh^2(r_+ t/l) (d\theta^2 + \sin^2 \theta d\chi^2).
\]

(2.15)

In these coordinates, the time-dependence of the solution is obvious. This situation is quite similar to the case of de Sitter spacetimes, where in the static coordinates the de Sitter space looks static within the cosmological horizon, but not cover the whole de Sitter
space, while in the global coordinates the solution covers the whole space, but is obviously

time-dependent. A different point is that here both sets of coordinates (2.10) and (2.13)

only describe the exterior of the black hole.

The Euclidean black hole solution can be obtained by replacing \( t \) by \(-i(\tau + \pi l/(2r_+))\) in (2.15). In this case, \( d\Omega_3 \) becomes

\[
d\Omega_3 = d\tau^2 + \frac{l^2}{r_+^2} \sin^2(r_+\tau/l)(d\theta^2 + \sin^2 \theta d\chi^2) .
\]

(2.16)

In order the \( \Omega_3 \) to be a regular three-sphere, the \( \tau \) must have the range, \( 0 \leq \tau \leq \beta \) with

\[
\beta = \frac{\pi l}{r_+}.
\]

(2.17)

Regarding this as the inverse Hawking temperature of the black hole seems problematic

since the surface gravity of the black hole is \( \kappa = r_+/l \) and \( \beta \) does not obey the usual

relation of black hole thermodynamics, \( \beta = 2\pi/\kappa \). Needless to say, it is of great interest

to further study the Hawking evaporation of the black hole.

Defining \( r^2 = r_+^2(1 + \tilde{r}^2/l^2) \), and \( \tau = l\phi/r_+ \), we find that the Euclidean black hole

becomes

\[
ds^2 = \left(1 + \frac{\tilde{r}^2}{l^2}\right) dr_+^2 + \left(1 + \frac{\tilde{r}^2}{l^2}\right)^{-1} d\tilde{r}^2 + \tilde{r}^2(d\phi^2 + \sin^2 \phi(d\theta^2 + \sin^2 \theta d\chi^2)).
\]

(2.18)

Since the \( \tau \) has the range from zero to \( \beta \), so we have \( 0 \leq \phi \leq \pi \). The solution (2.18)

is a Euclidean AdS space if one regards \( r_+\varphi \) as the Euclidean time. Because the \( \varphi \) has

the period \( 2\pi \), the solution (2.13) can be viewed as a thermal AdS space [8]. This is the

Euclidean version of the black hole.

The vacuum state is subtle for the black hole solution (2.14) because it has not a

smooth limit as \( r_+ \to 0 \). Redefining \( \theta = r_+\tilde{\theta}/l \), one can obtain a well-defined limit of the

solution (2.14):

\[
ds^2 = \frac{l^2}{\rho^2} \left(-dt^2 + d\rho^2 + \tilde{\theta}^2 d\chi^2 + l^2 d\varphi^2\right),
\]

where \( \rho = l/r \) and the range of \( \tilde{\theta} \) is \( 0 \leq \tilde{\theta} < \infty \). The form of the solution (2.19) looks like

the one of AdS space in the Poincare coordinates, however, the coordinate \( \varphi \) has a period

\( 2\pi \). It implies that the form (2.19) is an AdS space with identified points. It is of course

right because the constant curvature black hole is obtained by identifying some points in

an AdS space.
3 Dual field theory: Stress-energy tensor

Since the constant curvature black hole (2.14) is obtained by identifying some points along a Killing vector in an AdS space, so one can regard the black hole spacetime as a solution of Einstein’s equations with a negative cosmological constant, namely a solution of the following action:

\[ S = \frac{1}{16\pi G} \int_{\mathcal{M}} d^5x \sqrt{-g} \left( R + \frac{12}{l^2} \right) - \frac{1}{8\pi G} \int_{\partial\mathcal{M}} d^4x \sqrt{-h} K, \]  

(3.1)

where the second term is the Hawking-Gibbons surface term, \( K \) is the trace of the extrinsic curvature for the boundary \( \partial\mathcal{M} \) and \( h \) is the induced metric of the boundary. For a five-dimensional asymptotic AdS space, the suitable surface counterterm is

\[ S_{ct} = -\frac{1}{8\pi G} \int_{\partial\mathcal{M}} d^4x \sqrt{-h} \left( \frac{3}{l} + \frac{l}{4}R \right), \]  

(3.2)

where \( R \) is the curvature scalar for the induced spacetime \( h \) and \( R \) for the bulk spacetime \( g \). With this counterterm, the quasilocal stress-energy tensor of the gravitational field is

\[ T_{ab} = \frac{1}{8\pi G} \left( K_{ab} - Kh_{ab} - \frac{3}{l}h_{ab} + \frac{l}{2}G_{ab} \right), \]  

(3.3)

where \( G_{ab} \) is the Einstein tensor for the induced metric, \( K_{ab} = -h_\mu^a \nabla_\mu n_b \), and \( n_a \) is the unit normal to the boundary surface.

For the black hole spacetime (2.14), we choose a timelike hypersurface with a fixed \( r(> r_+) \). Calculating the extrinsic curvature and the Einstein tensor for the induced metric \( h \), we obtain

\[ T_{tt} = -\frac{1}{8\pi Gl} \frac{r_+^4}{8r^2} + \cdots, \]
\[ T_{\theta\theta} = \frac{l\cosh^2(r_+/l)}{8\pi G} \frac{r_+^2}{8r^2} + \cdots, \]
\[ T_{xx} = \frac{l\cosh^2(r_+/l)\sin^2\theta}{8\pi G} \frac{r_+^2}{8r^2} + \cdots, \]
\[ T_{\varphi\varphi} = -\frac{1}{8\pi Gl} \frac{3r_+^4}{8r^2} + \cdots, \]  

(3.4)

where \( \cdots \) denote higher correction terms, which will vanish when we take the limit \( r \to \infty \).

For the black hole spacetime, up to a conformal factor the boundary metric, on which the dual field theory resides, is

\[ ds_{\text{CFT}}^2 = \gamma_{ab} dx^a dx^b = -r_+^2 dt^2 + l^2 \cosh^2(r_+/l)(d\theta^2 + \sin^2\theta d\chi^2) + r_+^2 d\varphi^2. \]  

(3.5)
Note that here the coordinate $t$ is dimensionless. This spacetime is obviously a $dS_3 \times S^1$, which is a time-dependent background. Corresponding to the boundary metric (3.5), the stress-energy tensor for the dual field theory can be calculated using the following relation [13]

$$\sqrt{-\gamma} \gamma^{ab} \tau_{bc} = \lim_{r \to \infty} \sqrt{-h} h^{ab} T_{bc}. \quad (3.6)$$

Substituting (3.4) into (3.6), we obtain

$$\tau_{tt} = \frac{1}{8\pi G} \frac{1}{8l}, \quad \tau_{\theta\theta} = \frac{1}{8\pi G} \frac{1}{8l}, \quad \tau_{\chi\chi} = \frac{1}{8\pi G} \frac{1}{8l}, \quad \tau_{\varphi\varphi} = -\frac{1}{8\pi G} \frac{3}{8l}. \quad (3.7)$$

Some points about this stress-energy tensor are worthwhile to mention here. First of all, we notice that the tensor is independent of the parameter $r_+$. This can be understood since the black hole solution is locally equivalent to the AdS space. That is, after a rescaling of coordinates the dependence of the metric on $r_+$ can disappear. Second, although the tensor is the function of the radius $l$ of AdS space only, it is not the contribution of the Casimir effect (in the case of five-dimensional Schwarzschild-AdS black holes the stress-energy tensor of the dual CFT has a Casimir term [9]). There is no Casimir energy associated with the black hole solution (2.14). This can be understood from the vacuum state (2.13), an AdS space in the Poincare coordinates with some identification. It is well-known that there is no Casimir energy for the AdS black holes with Ricci flat horizons [11], which also holds even for asymptotically de Sitter spaces [12]. Third, the trace of the tensor does vanish, as expected, since dual field theory is the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, whose conformal symmetry is not broken even if quantum corrections are taken into account. Further, for the conformal field theory in four dimensions, in general there is a conformal anomaly proportional to $(R_{ab} R^{ab} - R^2 / 3)$ [3], where $R_{ab}$ and $R$ are the Ricci tensor and curvature scalar of the boundary metric, respectively. In our case, it is easy to check that this term vanishes for the background (3.3). Finally, the positive sign of the quantity $\tau_{tt}$ indicates that the black hole has a negative mass and the dual field theory has a negative energy density.

4 Discussions

A $D$-dimensional constant curvature black hole has unusual topological structure $\mathcal{M}_{D-1} \times S^1$. So it is quite difficult to calculate some conserved quantities associated with the black hole spacetime, in particular due to that there is no globally timelike Killing vector in the geometry of the black hole, which is quite similar to the case of asymptotically de
Sitter spaces. In [3] Banados considered a five dimensional rotating constant curvature black hole and embedded it to a Chern-Simons supergravity theory. By computing related conserved charges it was found that the black hole mass is proportional to the product of outer horizon $r_+$ and inner horizon $r_-$, while the angle momentum proportional to the sum of two horizons. Further the entropy of black hole is found to be proportional to the inner horizon $r_-$. This approach has two obvious drawbacks. Firstly the result cannot be degenerated to the non-rotating case. Secondly it cannot be generalized to other dimensional cases.

In [14] Creighton and Mann considered the quasilocal thermodynamics of a four dimensional constant curvature black hole in general relativity by calculating some thermodynamic quantities at a finite boundary which encloses the black hole. In this context it was found that the entropy is not associated with the event horizon, but the Killing horizon of a static observer, which is tangent to the event horizon of the black hole. The quasilocal energy density (see (11) in [14]) is found to be negative.

In this paper we considered a five-dimensional constant curvature black hole. The exterior of the black hole is time-dependent and the boundary spacetime is of the form of a $dS_3$ times a circle. By employing the surface counterterm approach, we obtained quasilocal stress-energy tensor of gravitational field and then that of the dual conformal field theory which resides on the dynamical boundary spacetime. As expected, the trace of the tensor vanishes since the dual field theory is $\mathcal{N} = 4$ super Yang-Mills theory. It is found that the field theory has a negative energy density on the time-dependent background. In addition, we would like to emphasize that the surface counterterm approach does not apply in four or six dimensions, but we do not know whether it applies in seven dimensions or more higher dimensions at the moment $^2$.

Since our boundary metric is a $dS_3 \times S^1$ spacetime, the same as the one for the “bubbles of nothing” in [3], a natural question then arises: what is the relation between our discussion and [3]? We notice that the spacetime we considered is a black hole solution, the dual field theory is then at finite temperature and the parameter $r_+$ is related to the Hawking temperature of the black hole, while the “bubble of nothing” is not a black hole solution and the physical meaning of the parameter $r_0$ in [3] is not very clear. However, we

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$^2$ In the earlier version of this paper, we also discussed the thermodynamics of the black hole through calculating the Euclidean action of black hole with the surface counterterm approach. Both the mass and entropy were found to be negative. However, as pointed out to us by the referee, the interpretation of the $\beta$ as the inverse Hawking temperature of the black hole seems problematic. As a result, the resulting calculations concerning the mass and entropy become unbelievable. We thank the referee for pointing this out.
find that when $r_0 = 0$ the solution (2) in [3] can be identified with the constant curvature black hole (2.14) if $r_+ = l$ through a suitable coordinate transformation. Indeed, in this case it can be seen that the stress-energy tensor (18) of dual field theory in [3] is completely the same as ours [see (3.7)]. Here it should be pointed out that when $r_0 \neq 0$ the “bubble of nothing” solution in [3] cannot be explained as a black hole. Further we would like to mention that the vacuum solutions are different in both cases, in our case the vacuum solution is (2.19), an AdS space in Poincare coordinates with some identifying points, while the vacuum solution is the one (2) with $r_0 = 0$ in [3]. This might give rise to some differences in explaining the stress-tensor tensor of dual field theory.

It would be interesting to further study some properties of dual field theory to the constant curvature black hole through the AdS/CFT correspondence, for example to calculate the Wilson loop.

### Acknowledgments

The author thanks J.X. Lu and Y.S. Wu for quite helpful discussions, Z.Y. Zhu for a reading of manuscript, Balasubramanian and Ross for useful comments and for pointing out a calculation error in the earlier version. This work was supported in part by a grant from Chinese Academy of Sciences.

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