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Local Banach-space dichotomies and ergodic spaces. (English) Zbl 07755542
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Summary: We prove a local version of Gowers’ Ramsey-type theorem [Ann. of Math. 156 (2002)], as well as local versions both of the Banach space first dichotomy (the “unconditional/HI” dichotomy) of Gowers [Ann. of Math. 156 (2002)] and of the third dichotomy (the “minimal/tight” dichotomy) due to Ferenczi-Rosendal [J. Funct. Anal. 257 (2009)]. This means that we obtain versions of these dichotomies restricted to certain families of subspaces called D-families, of which several concrete examples are given. As a main example, non-Hilbertian spaces form D-families; therefore versions of the above properties for non-Hilbertian spaces appear in new Banach space dichotomies. As a consequence we obtain new information on the number of subspaces of non-Hilbertian Banach spaces, making some progress towards the “ergodic” conjecture of Ferenczi-Rosendal and towards a question of Johnson.

MSC:
46B20 Geometry and structure of normed linear spaces
46B03 Isomorphic theory (including renorming) of Banach spaces
03E15 Descriptive set theory
03E60 Determinacy principles
05D10 Ramsey theory

Keywords:
ergodic Banach spaces; Ramsey theory; Banach-space dichotomies; non-Hilbertian spaces; minimal Banach spaces; hereditarily indecomposable Banach spaces

Full Text: DOI arXiv

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