Skyrmions and pentaquarks in the quark-hadron continuity perspective

R. Casalbuoni

Department of Physics, University of Florence, and INFN-Florence, Italy

G. Nardulli

Department of Physics, University of Bari and INFN-Bari, Italy

We argue that in the color-flavor-locking (CFL) superconducting phase classical soliton solutions can exist, whose excitations should be interpreted as states formed by a quark (or an antiquark) and condensed diquarks. This finding extends the picture of quark-hadron-continuity showing the existence of a region, intermediate between the CFL and the hypernuclear phase, where chiral solitons and Nambu Goldstone bosons can exist. We derive an expression of the soliton mass in terms of the QCD coupling, $g_s$, and the Nambu Goldstone boson parameters. From the quark-hadron continuity we can draw an argument in favor of the interpretation of the Θ$^+$ (1540) particle in terms of a strange antiquark and two highly correlated ud pairs (diquarks).

I. INTRODUCTION

The observation of the baryon resonance Θ$^+$ (1540) has been recently reported by several groups. The results of the LEPS [1], DIANA [2], CLAS [3, 4], SAPHIR [5], SVD [6], COSY-TOF [7], ZEUS [8], and HERMES [9] experiments as well as analyses of old bubble chamber experiments [10] show the existence of this narrow state ($\Gamma \sim$ a few MeV), decaying into $K^+n$ or $K^0_s p$. The simplest quark model interpretation is that of a pentaquark, i.e. an exotic state formed by five quarks: $udud\bar{s}$. Other narrow exotic cascade states, e.g. a $\Xi^{--}$ state with quantum numbers $B = 1, Q = S = -2$, and also a $\Xi^{-}$ and $\Xi^{0}$ state have been

*Electronic address: casalbuoni@fi.infn.it
†Electronic address: giuseppe.nardulli@ba.infn.it
reported by the NA49 Collaboration, see [11]. Also these signals can be interpreted as pentaquark states, e.g. for $\Xi^{-}$, $\bar{d}s\bar{d}s\bar{u}$. Much experimental effort is expected in the near future to consolidate these findings and clarify the experimental problems. In any event the appearance of exotic states, coming after years of fruitless experimental researches of exotica, has revived theoretical interest in QCD spectroscopy and its low energy models. Pentaquark states were indeed predicted long ago in the framework of the chiral soliton model [12, 13], which is an extension to three flavors [14, 15, 16] of the Skyrme model [17, 18]. Its mass was also correctly predicted by [19] and [20]. In the chiral quark soliton model [21, 22] all baryonic states are interpreted as arising from quantizing the chiral nucleon soliton and the pentaquark emerges as the third rotational excitation with states belonging to an antidecuplet with spin $s = 1/2$. Other interpretations have been proposed after the discovery of the $\Theta^+(1540)$, most notably the one of Jaffe and Wilczek [23, 24] who propose that the $\Theta^+$ comprises two highly correlated $ud$ pairs (diquarks: $Q$) and an $\bar{s}$. Diquark properties are similar to those of the diquark condensates of QCD in the high density color-flavor-locking (CFL) phase [25]. The two diquarks are in spin 0 state, antisymmetric in color and flavor. Together they produce a $Q\bar{Q}$ state in the flavor-symmetric $6_f$ that must be antisymmetric in color and in $p-$wave to satisfy Bose statistics. When combined with the antiquark the diquarks produce a $T\bar{Q}Q$ with spin 1/2 and positive parity (they can also produce a $8_f$, and mixing is possible).

The hypothesis that the attractive interaction in the antisymmetric color channel may play a role both at low and high density quark matter is especially interesting in the light of the quark-hadron continuity which has been suggested [26] to exist between the CFL and the hypernuclear phase. Due to the formation of the CFL condensate that breaks color, flavor and the electric charge, though conserving a combination of the electric charge and of the color generator $T_8$, the physical states are obtained by dressing the quarks by diquarks. The result is that in this phase eight quarks have exactly the same quantum numbers of baryons. Also the ninth quark corresponds to a singlet with a gap which is twice the gap of the octet. The same phenomenon takes place for the other states, as for instance, the gluons in the CFL and the vector mesons in the low density phase. In fact, the gluons are dressed by a pair $\bar{Q}Q$ giving rise to vector states with the same quantum number of the octet of vector resonances ($\rho$, etc.). Also, the NG field $\phi$ associated with the breaking of $U(1)_V$ can be related to a possible light meson $H$ of the hypernuclear phase [26]. The state $H$ which
is a six-quark singlet of the type \textit{udsuds} was introduced by \cite{27} in the context of the bag model. A more detailed discussion of the quark-hadron continuity can be found in \cite{26}.

Quark-hadron continuity plays a role in relating quark and baryons in the low-lying octet. Apparently it also matters in assigning a role to diquark attraction at zero baryonic densities. In this Letter we suggest that another sign of it is the possible existence of baryon chiral solitons also at finite density. We show below that they could arise in QCD at finite density by the existence of a Skyrme term in the effective lagrangian for the Nambu-Goldstone bosons of the CFL phase. The static solution of the classical equations of motion has the same form of the chiral soliton model of refs. \cite{15,16} and \cite{12,13}. Therefore its quantization will eventually produce baryonic states with properties similar to those of the low-lying baryonic octet as well as of its excitations, and in particular the pentaquark.

In Section II we discuss the effective lagrangian describing the light modes of the CFL phase \cite{28} and we show that the decoupling of the gluons generate a Skyrme term. In Section III we evaluate the soliton mass by extrapolating the parameters of the effective lagrangian down to chemical potentials of order $400 \div 500$ $MeV$. We find a value of about $1200$ $MeV$ which is in the right ball-park. Also we evaluate the size of the soliton and we discuss the validity of our calculation. In Section III we discuss our results with a particular emphasis about the implications of the quark-hadron continuity idea on the pentaquark states.

II. THE EFFECTIVE LAGRANGIAN FOR THE GOLDSTONE BOSONS

We recall briefly the form of the effective lagrangian for the light modes of the CFL phase. At this level the gluons should be already decoupled since $p \ll \Delta$ and we know from \cite{29} that the gluons in the CFL phase have physical masses of order $\Delta$. However, since we want to show that precisely the process of decoupling the gluon fields produces the Skyrme term, we will write the effective action for the full set of 18 Goldstone bosons from the breaking of $U(3)_L \otimes U(3)_R \otimes SU(3)_c$ to $SU(3) \otimes Z_2 \otimes Z_2$. The set includes also the Goldstones to be eaten up by the gluon fields. The effective lagrangian in this form has been discussed in \cite{28} (see also \cite{30}). We can associate the Goldstone fields to the left(right)-handed spin 0 diquark condensates according to

\begin{equation}
\hat{X}^i_a \approx \epsilon^{ijk} \epsilon_{\alpha \beta \gamma} \langle \psi^j_{\beta L} \psi^k_{\gamma L} \rangle^*, \quad \hat{Y}^i_a \approx \epsilon^{ijk} \epsilon_{\alpha \beta \gamma} \langle \psi^j_{\beta R} \psi^k_{\gamma R} \rangle^*,
\end{equation}

(1)
with $\hat{X}$ and $\hat{Y}$ $3 \times 3$ unitary matrices. For the following it will be more convenient to separate the $U(1)$ factors from $\hat{X}$ and $\hat{Y}$ by defining $U(1)$ and $SU(3)$ fields

$$\hat{X} = X e^{2i(\phi + \theta)}, \quad \hat{Y} = Y e^{2i(\phi - \theta)}, \quad X, Y \in SU(3).$$

The transformation properties of these fields under the full symmetry group are

$$X \rightarrow g_c X g_L^T, \quad Y \rightarrow g_c Y g_R^T, \quad \phi \rightarrow \phi - \alpha, \quad \theta \rightarrow \theta - \beta,$$

with $\alpha$ and $\beta$ the parameters of the groups $U(1)_V$ and $U(1)_A$ respectively and $g_c \in SU(3)_c$, $g_{L,R} \in SU(3)_{L,R}$.

The breaking of the global symmetry can be discussed also using the gauge invariant fields

$$\Sigma^i \Sigma^j = \sum_\alpha (Y^\dagger \alpha)^X \Sigma_i^j \rightarrow \Sigma = Y^\dagger X.$$

The $\Sigma$ field describes the 8 Goldstone bosons corresponding to the breaking of the chiral symmetry $SU(3)_L \otimes SU(3)_R$, as it is made clear by the transformation properties of $\Sigma^T$, $\Sigma^T \rightarrow g_L \Sigma^T g_R^\dagger$. That is $\Sigma^T$ transforms as the usual chiral field. The other two fields $\phi$ and $\theta$ provide the remaining Goldstone bosons related to the breaking of the $U(1)$ factors. However, since the $U(1)_A$ symmetry is anomalous although, asymptotically in $\mu$, gets restored, we will omit this field in the following discussion.

In order to build up an invariant lagrangian, it is convenient to define the following currents

$$J^\mu_X = XD^\mu X^\dagger = X (\partial^\mu X^\dagger + X^\dagger g^\mu) = X \partial^\mu X^\dagger + g^\mu,$$

$$J^\mu_Y = Y D^\mu Y^\dagger = Y (\partial^\mu Y^\dagger + Y^\dagger g^\mu) = Y \partial^\mu Y^\dagger + g^\mu,$$

with

$$g^\mu = ig_s g^a_{\mu} T^a$$

the gluon field and

$$T^a = \frac{\lambda_a}{2}$$

the $SU(3)_c$ generators. These currents have simple transformation properties under the full symmetry group $G$:

$$J^\mu_{X,Y} \rightarrow g_c J^\mu_{X,Y} g_c^\dagger.$$
The most general lagrangian, up to two derivative terms, invariant under $G$, the rotation group $O(3)$ (Lorentz invariance is broken by the chemical potential term) and the parity transformation, defined as:

$$P : \ X \leftrightarrow Y, \ \phi \to \phi,$$

is

$$L = -\frac{F_T^2}{4} \text{Tr} \left[ (J^0_X - J^0_Y)^2 \right] - \alpha_T \frac{F_T^2}{4} \text{Tr} \left[ (J^0_X + J^0_Y)^2 \right] + \frac{1}{2} (\partial_0 \phi)^2 +$$

$$+ \frac{F_S^2}{4} \text{Tr} \left[ |J_X - J_Y|^2 \right] + \alpha_S \frac{F_S^2}{4} \text{Tr} \left[ |J_X + J_Y|^2 \right] - \frac{v^2}{2} |\nabla \phi|^2 - \frac{1}{g_s^2} T \text{Tr}[\epsilon E^2 - \frac{1}{\lambda} B^2],$$

or, in terms of the fields $X$ and $Y$

$$L = -\frac{F_T^2}{4} \text{Tr} \left[ (X \partial_0 Y^\dagger - Y \partial_0 Y^\dagger)^2 \right] - \alpha_T \frac{F_T^2}{4} \text{Tr} \left[ (X \partial_0 X^\dagger + Y \partial_0 Y^\dagger + 2g_0)^2 \right]$$

$$+ \frac{F_S^2}{4} \text{Tr} \left[ |X \nabla X^\dagger - Y \nabla Y^\dagger|^2 \right] + \alpha_S \frac{F_S^2}{4} \text{Tr} \left[ |X \nabla X^\dagger + Y \nabla Y^\dagger + 2g|^2 \right]$$

$$+ \frac{1}{2} (\partial_0 \phi)^2 - \frac{v^2}{2} |\nabla \phi|^2 - \frac{1}{g_s^2} T \text{Tr}[\epsilon E^2 - \frac{1}{\lambda} B^2],$$

(10)

where

$$F_{\mu\nu} = \partial_\mu g_\nu - \partial_\nu g_\mu - [g_\mu, g_\nu],$$

(11)

and

$$E_i = F_{0i}, \quad B_i = \frac{1}{2} \epsilon_{ijk} F_{jk}. \quad (12)$$

The parameters $\epsilon$ and $\lambda$ are the dielectric constant and the magnetic permeability of the dense condensed medium.

Notice that the gluons $g_0^a$ and $g_i^a$ in the CFL vacuum acquire Debye and Meissner masses given by

$$m_D^2 = \alpha_T g_s^2 F_T^2, \quad m_M^2 = \alpha_S g_s^2 F_S^2 = \alpha_S g_s^2 v^2 F_T^2,$$

(13)

where we have introduced

$$v^2 = \frac{F_S^2}{F_T^2}. \quad (14)$$

It should be stressed that these are not the true rest masses of the gluons, since there is a large wave function renormalization effect making the gluon masses of the order of the gap $\Delta$, rather than $\mu$ \[29\]. Since this description is supposed to be valid at low energies below the gap $\Delta$, we can decouple the gluons solving their classical equations of motion neglecting the kinetic term. The result from Eq. (10) is

$$g_\mu = -\frac{1}{2} \left( X \partial_\mu X^\dagger + Y \partial_\mu Y^\dagger \right).$$

(15)
By substituting this expression in Eq. (10), and performing a gauge rotation to get \( Y = 1 \), one obtains
\[
\mathcal{L} = \frac{F_T^2}{4} \left( \text{Tr}[\tilde{\Sigma} \tilde{\Sigma}^\dagger] - v^2 \text{Tr}[\tilde{\nabla} \Sigma \cdot \tilde{\nabla} \Sigma^\dagger] \right) + \frac{1}{2} \left( \phi^2 - v_\phi^2 |\nabla \phi|^2 \right) - \frac{1}{g_s^2} T_r [\epsilon E^2 - \frac{1}{\lambda} B^2],
\]
(16)
where now
\[
E_i = \frac{1}{4} [\Sigma \partial_0 \Sigma^\dagger, \Sigma \partial_i \Sigma^\dagger], \quad B_i = \frac{1}{8} \epsilon_{ijk} [\Sigma \partial_j \Sigma^\dagger, \Sigma \partial_k \Sigma^\dagger].
\]
(17)
Therefore, except for the breaking of the Lorentz symmetry, we recognize in the first term the lowest order chiral lagrangian and, in the last one, the Skyrme term [18].

It is interesting to notice that the idea of the Skyrme term generated by decoupling the gauge boson of a hidden symmetry [31] is realized here by decoupling the gluons (see also [32]). Notice also that one has to add to this effective lagrangian the Wess-Zumino term as discussed in [33, 34]. It turns out that the Wess-Zumino contribution coincides with the one at zero density (see also [35]). The addition of this term is vital for getting the right quantum numbers for the baryons once the classical soliton solution is quantized [14, 16].

### III. NUMERICAL ESTIMATES

Static solutions minimizing the energy are found assuming a constant field \( \phi \); for \( \Sigma \) we make the usual choice [14] incorporating the hedgehog ansatz for the SU(2) chiral subgroup
\[
\Sigma(x) = \begin{pmatrix} \exp[i(x \cdot \tau F(r)/r] & 0 \\ 0 & 1 \end{pmatrix},
\]
(18)
with \( r = |x|, F(0) = \pi, F(r) \to 0 \) when \( r \to \infty \). The soliton mass is a functional of \( F(r) \) subject to the boundary conditions given above. Minimization of the energy gives as a result the usual relation between the parameters of the lagrangian and the soliton mass [18, 15]. In our case one should take into account a different normalization of the pion decay constant and the pion velocity \( v \), producing \( F_\pi \to 2F_T v \). Besides, the chromo-magnetic permeability changes the coupling \( g_s \) to \( g_s \sqrt{\lambda} \). As a result we get
\[
M = 36.5 \frac{2F_T v}{g_s \sqrt{\lambda}}.
\]
(19)
It should be stressed that the soliton mass is given here in terms of the parameters of the low-energy theory, \( F_T, v \), the magnetic permeability of the dense medium \( \lambda \) and in terms of the strong coupling constant \( g_s \). Therefore, at least in principle, there are no free parameters...
and everything could be determined by the fundamental theory. In fact, if we use the results of the calculations at high density (see e.g. [29, 36]) we get

\[ F_T^2 = \frac{\mu^2(21 - 8 \ln 2)}{36\pi^2}, \quad v = \frac{1}{\sqrt{3}}, \]

and

\[ \frac{1}{\lambda} = 1 + \frac{\mu^2 g_s^2}{30\pi \Delta^2} (a + b), \]

\[ a + b = \frac{1}{108\pi} \left( 41 - \frac{112}{3} \ln 2 \right). \quad (21) \]

The gap is also determined by QCD at high density [37]. We can now extrapolate this high-density prediction to values of \( \mu \) of order 400 ÷ 600 MeV to get an idea of the order of magnitude in a region that should be not too far from the hypernuclear phase. Also we consider \( \Delta = 40 \) and 80 MeV. For \( g_s \) we take \( \sim 3.5 \) corresponding to \( g_s^2/4\pi \sim 1 \). The

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{The soliton mass at finite density in the CFL phase as a function of the baryonic chemical potential \( \mu \), for two values of the gap \( \Delta \).}
\end{figure}

soliton mass \( M \) reported in Fig. corresponds to the classical solution, and does not take into account \( SU(3)_f \) breaking corrections or excitation energies such as, for example, those corresponding to the pentaquark states.

Fig. shows that around 400 MeV the soliton mass is about 1200 MeV, which, in the light of the quark-hadron continuity, is in the right ball-park, see the discussion in the next Section. It also shows that at weak coupling, i.e. at larger values of the chemical potential
the mass increases and the soliton effectively decouples. Notice that the dependence of the soliton mass on the gap, at least in this range of $\mu$ and $\Delta$, is rather weak due the very small coefficient in front of $\mu^2/\Delta^2$. In [31] corrections due to higher derivative terms have been discussed arguing that they should be small, however, we would like to discuss here the validity of our approximation by looking at the size of the soliton. Using the results obtained in [15] we find for the isoscalar mean radius (which can be roughly assumed as the size of the instanton)

$$r_0 \approx \frac{2.11}{2vF_T g_s \sqrt{\lambda}}$$

(22)

On the other hand our effective lagrangian is valid up to energies lower than the gap $\Delta$. Therefore, in order to describe correctly the soliton by means of our effective lagrangian, one should have $2vF_T g_s \sqrt{\lambda} \ll 2.11\Delta$. We have studied this condition by varying $\mu$. Since the mass of the soliton increases with $\mu$ it decouples at high values of the chemical potential and we do not expect to get a good description of the soliton in this regime. Let us now consider smaller values of $\mu$ for which $\lambda \approx 1$ (in practice this means $\mu \lesssim 10\Delta$). We get the condition $1/r_0 \approx 0.4\mu$. If we use $\mu \approx 400 \text{ MeV}$ and $\Delta \approx 80 \text{ MeV}$, the result is $1/r_0 \approx 2\Delta$. Strictly speaking our description is not valid up to this energy. However, in reference [29] we have shown that within the same approximation one can evaluate the mass of the gluons (of order $2\Delta$) in the CFL phase within a 30% with respect to the exact value. Therefore we can hope that the same approximation holds at the same level also in the present case. Clearly a better approximation would be obtained by introducing higher derivatives in the expansion. Let us estimate the error we are doing neglecting them. To this end we will vary the function $F(r)$, taking into account that also for the varied function, $F(0) = \pi$, which gives the right topological number, and that, for $r >> r_0$, the new terms are negligible. Therefore we have chosen to vary $F(r)$ in two ways. In both cases we vary continuously $F(r)$ within the interval $(0, 2r_0)$ by keeping $F(0)$ and $F(r)$ fixed, for $r > 2r_0$. In the first case we increases the value of $F(r)$ at $r_0$ by 50%, whereas in the second case we take it 1/2 of the original value. The results are the following: the mean radius increases of about 30% in the first case, whereas it is reduced by 50% in the second one. On the other hand in both cases the mass of the soliton increases of about 30%. This result follows from our estimate which is a lower bound for the soliton mass since it is obtained by a variational procedure. Therefore, our estimate is that the error we are performing should not be higher than 30-50% and that our results should be qualitatively robust.
Again we remark that, within our approximation, we have obtained a very well defined expression for the soliton mass in the CFL phase, containing no arbitrary parameters.

IV. DISCUSSION AND CONCLUSIONS

In Section II we have shown that at energies close to the Fermi energy $E_F$ and much smaller than $E_F + \Delta$ the fermions decouple and the relevant degrees of freedom are the Nambu-Goldstone bosons that can be thought of as $Q\bar{Q}$ bound states. As discussed in the introduction we expect that in the CFL phase, besides these states, both quarks and gluons become dressed, thus producing states such as $qQ, \bar{Q}gQ$, etc. The very high density CFL case is depicted on the right-hand corner of the cartoon in Fig. 2. In Section III we have shown that by taking the next-to-leading order in the gluon decoupling process we get a Skyrme term in the low energy lagrangian. Therefore the theory predicts soliton states with the same quantum numbers of baryons. However, being at weak coupling, the solitons have large masses ($M \approx 1/g_s$). As a consequence we expect the solitons to decouple at the CFL densities. At lower densities QCD coupling gets stronger and the soliton mass decreases. At these intermediate densities we expect the low energy physics to be still described by the chiral lagrangian, but with the soliton states entering in to play. This correspond to the central part of Fig. 2. As discussed in the previous Section, this is also the region where we expect that our approximation is valid.

By decreasing the density one should go smoothly to the hypernuclear phase where the physical states are pions, vector mesons and baryons (with the further singlet state $H \approx udsuds$ corresponding to the Goldstone boson $\phi$), as shown in the left corner of Fig. 2.

![FIG. 2: A cartoon depicting the transition from the CFL to the hypernuclear phase.](image-url)
Therefore the transition from CFL to the hypernuclear phase appears completely smooth and without phase transitions. From this point of view the existence of pentaquark states seems completely natural. In fact, in the high-density limit, as we have seen, quarks live in a dense medium made of diquark condensates. Therefore a quark can bind a given number of diquarks. In particular, one can form a bound state of the type $\bar{q}QQ$, that is a pentaquark. This same object is naturally described as a soliton, and therefore it is expected to exist also in the intermediate region and, by the quark-hadron continuity argument, in the hypernuclear phase. Strictly speaking we cannot say that this state persists also through the transition from the hypernuclear to the nuclear phase, but this hypothesis appears to be very natural. Some support to these qualitative ideas comes from the numerical results of Section III. Using the low energy parameters, as derived from the high-density limit, in the intermediate density region, $\mu \approx 400 \div 500$ MeV we get the right order of magnitude (see Fig. I), that is $1.1 \div 1.7$ GeV varying the gap between 40 and 80 MeV. In the same vein we can comment briefly about the expected width for the lowest lying pentaquark state. In order for it to decay into a baryon and a kaon a breaking of diquark condensates should be produced. Of course this is very unlikely to happen at high density. One may expect that this feature survives going all the way down to the nuclear phase.

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[1] T. Nakano et al. (LEPS), Phys. Rev. Lett. 91, 012002 (2003), hep-ex/0301020.
[2] V. V. Barmin et al. (DIANA), Phys. Atom. Nucl. 66, 1715 (2003), hep-ex/0304040.
[3] S. Stepanyan et al. (CLAS), Phys. Rev. Lett. 91, 252001 (2003), hep-ex/0307018.
[4] V. Kubarovsky et al. (CLAS), Phys. Rev. Lett. 92, 032001 (2004), hep-ex/0311046.
[5] J. Barth et al. (SAPHIR) (2003), hep-ex/0307083.
[6] A. Aleev et al. (SVD) (2004), hep-ex/0401024.
[7] M. Abdel-Bary et al. (COSY-TOF) (2004), hep-ex/0403011.
[8] S. Chekanov et al. (ZEUS) (2004), hep-ex/0403051.
[9] A. Airapetian et al. (HERMES), Phys. Lett. B585, 213 (2004), hep-ex/0312044.

[10] A. E. Asratyan, A. G. Dolgolenko, and M. A. Kubantsev, Phys. Atom. Nucl. 67, 682 (2004), hep-ex/0309042.

[11] C. Alt et al. (NA49), Phys. Rev. Lett. 92, 042003 (2004), hep-ex/0310014.

[12] A. V. Manohar, Nucl. Phys. B248, 19 (1984).

[13] M. Chemtob, Nucl. Phys. B256, 600 (1985).

[14] E. Witten, Nucl. Phys. B223, 433 (1983).

[15] G. S. Adkins, C. R. Nappi, and E. Witten, Nucl. Phys. B228, 552 (1983).

[16] E. Guadagnini, Nucl. Phys. B236, 35 (1984).

[17] T. H. R. Skyrme, Proc. Roy. Soc. Lond. A260, 127 (1961).

[18] T. H. R. Skyrme, Nucl. Phys. 31, 556 (1962).

[19] L. C. Biedenharn and Y. Dothan, From SU(3) to gravity (Ne’eman Festschrift) (Cambridge Univ. Press, 1986).

[20] M. Praszalowicz, in M. Jezabek and M. Praszalowicz eds., Skyrmions and Anomalies (World Scientific, Singapore, 1987).

[21] H. Walliser, Nucl. Phys. A548, 649 (1992).

[22] D. Diakonov, V. Petrov, and M. V. Polyakov, Z. Phys. A359, 305 (1997), hep-ph/9703373.

[23] R. L. Jaffe and F. Wilczek, Phys. Rev. Lett. 91, 232003 (2003), hep-ph/0307341.

[24] R. Jaffe and F. Wilczek (2004), hep-ph/0401034.

[25] M. G. Alford, K. Rajagopal, and F. Wilczek, Nucl. Phys. B537, 443 (1999), hep-ph/9804403.

[26] T. Schafer and F. Wilczek, Phys. Rev. Lett. 82, 3956 (1999), hep-ph/9811473.

[27] R. L. Jaffe, Phys. Rev. Lett. 38, 195 (1977).

[28] R. Casalbuoni and R. Gatto, Phys. Lett. B464, 111 (1999), hep-ph/9908227.

[29] R. Casalbuoni, R. Gatto, and G. Nardulli, Phys. Lett. B498, 179 (2001), hep-ph/0010321.

[30] D. K. Hong, M. Rho, and I. Zahed, Phys. Lett. B468, 261 (1999), hep-ph/9906551.

[31] M. Abud, G. Maiella, F. Nicodemi, R. Pettorino, and K. Yoshida, Phys. Lett. B159, 155 (1985).

[32] A. D. Jackson and F. Sannino, Phys. Lett. B578, 133 (2004), hep-ph/0308182.

[33] F. Sannino, Phys. Lett. B480, 280 (2000), hep-ph/0002277.

[34] R. Casalbuoni, Z. Duan, and F. Sannino, Phys. Rev. D63, 114026 (2001), hep-ph/0011394.

[35] S. D. H. Hsu, F. Sannino, and M. Schwetz, Mod. Phys. Lett. A16, 1871 (2001), hep-
ph/0006059.

[36] D. T. Son and M. A. Stephanov, Phys. Rev. D61, 074012 (2000), hep-ph/9910491.

[37] D. T. Son, Phys. Rev. D59, 094019 (1999), hep-ph/9812287.