Temporal Correlation of Interference in Bounded Mobile Ad Hoc Networks with Blockage

Abstract—In mobile wireless networks with blockage, different users, and/or a single user at different time slots, may be blocked by some common obstacles. Therefore the temporal correlation of interference does not depend only on the user displacement law but also on the spatial correlation introduced by the obstacles. In this letter, we show that in mobile networks with a high density of users, blockage increases the temporal correlation of interference, while in sparse networks blockage has the opposite effect.

Index Terms—Blockage, Correlation, Interference, Mobility.

I. INTRODUCTION

The temporal correlation of interference affects the temporal correlation of outage, and subsequently, it impacts many network performance metrics, e.g., end-to-end throughput, multi-hop delay, etc. Assuming uncorrelated user activity and fading over time, the user mobility is the main factor reducing the temporal correlation of interference [1].

In areas with blockage, different users as well as a single user at different time slots may be blocked by some common obstacles. In general, interference is dominated by the Line-of-Sight (LoS) transmissions, and the transitions between LoS and Non-Line-of-Sight (NLoS) propagation conditions due to mobility will reduce the temporal correlation of interference. However, at the same time, blockage increases the spatial correlation among the users, which has an adverse effect on the temporal interference statistics, e.g., when different users, despite the mobility, are still under correlated penetration losses, the interference level will not vary significantly. Studying the impact of blockage on the moments of interference is a topic of growing interest [2]–[4], considering the ongoing standardization activities for commercial wireless networks in millimeter-wave bands. Nevertheless, interference correlation with blockage is yet to be studied.

In [2], blockage is modeled by a Boolean scheme of rectangles, and the ratio of penetration power loss due to blockage is incorporated into the interference model. The model neglects the correlation between different links. Therefore, issues related to interference correlation, in space and time, are not addressed. In [3], [4], the impact of blockage is incorporated into the performance analysis of millimeter-wave networks by determining an effective LoS region and assigning different channel models to LoS and NLoS users.

In this letter, we illustrate that the impact of blockage on the temporal correlation of interference depends on the user density. In sparse networks, where the spatial correlation among the users is negligible, the transitions in the propagation conditions from LoS to NLoS due to mobility dominate the temporal statistics of interference. As a result, blockage reduces the temporal correlation of interference. In dense networks, the correlation among the users dominates the temporal correlation of interference and mobility may not help much in reducing it. As a result, the temporal correlation increases as compared to the case without blockage.

In our analysis, we use the Random Waypoint Mobility (RWPM) model, see for instance [5], as an example model because it has some desirable features for our problem: It is defined over a finite area, it results in a non-uniform distribution of users, i.e., the network is more sparse close to the boundaries than near the center, and it allows studying different levels of mobility by varying the think time. We use the RWPM model over a one-dimensional lattice because in that case the user displacement law is known for time-lags equal to one and two time slots [6].

II. SYSTEM MODEL

We consider a Poisson number of users, with mean $K$, which are moving across a one-dimensional lattice of size $N$, according to the model described in [6]. According to it, each user selects uniformly at random a destination, and travels with a constant speed $v = 1$ lattice point per time slot. When it reaches the destination, it stops and thinks for a number of time slots selected from the discrete uniform distribution on $\{0, 1, \ldots, M\}$. Let us denote the Random Variable (RV) of the $i$-th user location by $x_i$. Its Probability Distribution Function (PDF) in the steady state is [6]

$$f_{x_i}(n) = \frac{p}{N} + (1-p) \frac{3N (2n-1) - 6n (n-1) - 3}{N (N^2 - 1)}, n \leq N$$  

(1)

where $p = \frac{M/2}{N^2 (N+1)/2}$ is the average think time for a randomly selected user.

The calculation of temporal interference correlation requires the user displacement law [6]. Given the location $n$, this is the probability $P(n+k, \tau)$ that the user is located at the lattice point $(n+k)$ after $\tau$ time slots. The RWPM model introduces different levels of mobility at different locations. For instance, the probability that a user thinks at the lattice point $n$ is $P(n, 1) = \frac{1}{N} f_{x_i}(n)$, which means that the users close to the center tend to move with higher probability than the users near the boundaries [6]. We compute the interference at the locations, $y_p = n + c$, $n = 1, 2, \ldots, \frac{N}{2}$ and $c \in (0, 1)$, in-between the lattice points, from the border to the center.

Let consider a Poisson number of obstacles, with mean $N_o$, distributed uniformly at random in the continuous space $[1, N]$. The obstacles do not hinder the user moves, but they attenuate the user signal. The number of obstacles $n_o$ on the link $x_i \rightarrow y_p$, between the $i$-th user and the location $y_p$, is a Poisson RV with parameter $q_i N_o$, $\text{Po}(q_i N_o)$, where $q_i = \frac{d_i}{\gamma d_i}$, $d_i = |x_i - y_p|$. The fraction of penetration power loss per obstacle follows the uniform distribution in the interval $[0, \gamma]$, $\gamma \leq 1$. The fraction
of penetration loss, $\beta_i$, over the link $x_i \rightarrow y_p$ is equal to the product of the power loss fractions from all obstacles on that link. Note that the RVs $\beta_i$ and $x_i$ are dependent, e.g., the longer the link $x_i \rightarrow y_p$ is, the higher the penetration loss should be, because more obstacles are likely to block the user.

Assuming common transmit power level $P_t$ for all users, the interference at an arbitrarily selected time slot $t$ is

$$I(t) = P_t \sum_i \xi_i(t) h_i(t) \beta_i(t) g(x_i(t) - y_p)$$

where $\xi_i$ is a Bernoulli RV describing the $i$-th user activity, $E\{\xi_i\} = \xi_i \forall i$, $h_i$ is an exponential RV with unit mean modeling Rayleigh fading, $x_i \in \{1, 2, \ldots, N\}$ is the RV for the $i$-th user location with PDF given in (1), and $g(x) = \frac{\epsilon}{\pi(x^2 + \epsilon^2)}$ is the distance-based propagation pathloss function, where $\epsilon$ is used to avoid singularity at $x = 0$.

It is assumed that the user activity and fading are independent and identically distributed (i.i.d.) over time slots and users. On the other hand, with the RWPM model, the locations of each user are correlated in time. Different users move independently of each other but their penetration losses are in general correlated because they may be blocked by some common obstacles. The Moment Generating Function (MGF) of the interference at two time slots $t$ and $\tau$ is

$$\Phi_2 = \int_{h \beta x} \sum \xi^{(t)+s2}\xi^{(t)} f_x f_h \Phi(K) \text{d}h \text{d}x \text{d}h \text{d}\beta,$$

where $\xi$, $h$, $x$ and $\beta$ are vectors of RVs with elements, $\xi_i, h_i, x_i$ and $\beta_i \forall i$ at time slots $t$ and $\tau$, and the arguments in the PDFs are omitted for brevity.

In the steady state, the moments of the interference become independent of the time we take the measurements, and the Pearson correlation coefficient at time-lag $l = |t - \tau|$ becomes

$$\rho_l = \frac{E\{I(t)I(\tau)\} - E\{I(t)\}^2}{E\{I(t)^2\} - E\{I(t)^2\}}.$$  \hspace{1cm} (2)

### III. INTERFERENCE MEAN AND VARIANCE

Conditioned on the number of obstacles $n_o \geq 1$ over the link $x_i \rightarrow y_p$, the PDF of the fraction of penetration power loss $f_{\beta_i | n_o} \triangleq h(\beta_i)$ is equal to the PDF of the product of $n_o$ i.i.d. uniform RVs with support $[0, \gamma]$. This PDF is defined over the interval $[0, \gamma^{n_o}]$ and it is equal to [7]

$$h(\beta_i) = \frac{1}{\gamma^{n_o}(n_o - 1)!} \left( \log \left( \frac{\gamma^{n_o}}{\beta_i} \right) \right)^{n_o - 1}.$$  \hspace{1cm} (3)

The PDF $f_\beta$ can be computed by averaging the PDF $h(\beta_i)$ over the Poisson RV $n_o$. For $\gamma = 1$, it can be written in terms of the modified Bessel function of the first kind. For $\gamma < 1$, the PDF $f_\beta$ is more difficult to obtain. Nevertheless, we are only interested in the moments of $\beta_i$ which can be computed as follows

$$E\{\beta_i^s\} = \int_{\beta_i} \sum_{n_o=1}^\infty h(\beta_i)n_oPo(q_i,N_o)d\beta_i + e^{-q_iN_o},$$

$$= \sum_{n_o=1}^\infty Po(q_i,N_o) \int_{\beta_i} h(\beta_i)n_o d\beta_i + e^{-q_iN_o},$$

$$= e^{-q_iN_o(1-\gamma^2)} = e^{-\alpha d_2(1-\gamma^2)}.$$  \hspace{1cm} (4)

In (a), the rightmost term $e^{-q_iN_o}$ corresponds to the LoS probability, i.e., $n_o = 0$, where there is no penetration loss at all, in (b) we can reverse the orders of integration and summation because the RVs $\beta_i$ are independent of each other, and in (c) we used equation (3) to compute $E\{\beta_i^s\} = \gamma^{n_o}(1+\gamma)^{-n_o}$, before averaging over the Poisson distribution $Po(q_i,N_o)$. The term $\alpha = \frac{\gamma^{n_o}}{N_o}$ can be seen as an indicator of the density of obstacles in the area.

With the moments of the RV $\beta_i$ at hand, one can now proceed with the computation of interference moments. While doing so, one has to average over the distributions of fading, penetration loss, number of users, user activity and location.

$$E\{I^2\} = \sum_i \int_h \int_x \sum \xi_i h_i \beta_i g(d) f_x f_h \Phi(K) \text{d}h \text{d}x \text{d}h \text{d}x,$$

$$= \sum_i E\{h_i\} E\{\xi_i\} \sum x_i^{(t)} \sum n_o \Phi(K)$$

$$= \sum_i E\{h_i\} E\{\xi_i\} \sum \sum \Phi(K)$$

$$= K \sum x_i^{(t)} e^{-\alpha d_2(1-\gamma^2)} g(d) f_x(n) \text{d}x.$$

Here, (a) follows from the fact that the RVs $x_i, \beta_i$, are independent, and (b) follows after evaluating equation (4) for $s = 1$, using that the users are indistinct, and taking the average in terms of the Poisson distribution $Po(K)$. Also, the transmit power level has been taken equal to $P_t = 1$, $d_{n_o} = n_o - y_p$, and $E\{h_i\}$ describes the mean penetration loss over the distance $d_{n_o}$. One may see that the impact of blockage on the mean is captured by scaling the link budget $g(d_n)$ with $e^{-\alpha d_2(1-\gamma^2)}$. Following the same assumptions, the second moment of interference is

$$E\{I^2\} = 2K \sum e^{-\alpha d_2(1-\gamma^2)} g^2(d) f_x(n) + \sigma$$

where it has been used that $E\{h_i^2\} = 2$, $E\{\xi_i^2\} = 1$, $E\{\beta_i^2\} = e^{-\alpha d_2(1-\gamma^2)}$, and the term $\sigma$ captures the correlation between different users

$$\sigma = K^2 \sum e^{-\alpha d_2(1-\gamma^2)} g(d_n)g(d_m)f_x(n)f_x(m).$$

In order to compute the cross-correlation of penetration loss at different locations we separate between the following cases:

- $n > y_p$ and $m < y_p$ or $n < y_p$ and $m > y_p$. In that case, the links $n \rightarrow y_p$ and $m \rightarrow y_p$ do not share common obstacles and the penetration losses become uncorrelated. Thus, $E\{\beta_i\beta_m\} = e^{-\alpha d_2(1-\gamma^2)}$.
- $n > y_p$ and $m > y_p$ or $n < y_p$ and $m < y_p$. In that case, the links $n \rightarrow y_p$ and $m \rightarrow y_p$ may share common obstacles. Let assume that $d_{n_o} > d_{m_o}$. Then, $E\{\beta_i\beta_m\} = E\{\beta_i^2\beta_m\}$, where $\beta_k$ is the penetration loss over the
of Standard Deviation of Interference at the Locations $y_k$ for $c = \frac{1}{2}$.

The model is validated at seven locations for mean number of obstacles $N_c = 10$ and $N_c = 40$. The minimum attenuation per obstacle is $3 \text{ dB}$ if $\gamma = 0.5$. Lattice size $N = 50$, $K = 50$ users, continuous user activity $\xi = 1$, pathloss exponent $\alpha = 2$, $\epsilon = 0.5$, and maximum think time $M = 5$ time slots.

distance $d_k = d_m - d_n$. Due to the fact that the penetration losses over the distances $d_n$ and $d_k$ are uncorrelated, $E\{\beta_n \beta_k\} = e^{-\alpha \left( (1 - \frac{1}{\gamma^2}) (d_m - d_n)(1 - \frac{1}{\gamma^2}) \right)}$. In a similar manner one can compute the correlation for $d_m \leq d_n$. Finally, $E\{\beta_n \beta_k\} = e^{-\alpha \left( \min(d_n, d_m) \left(1 - \frac{1}{\gamma^2}\right) \right) (d_m - d_n)} (1 - \frac{1}{\gamma^2})$.

**Remark 1.** For impenetrable obstacles, $\gamma = 0$, when $n > y_p$ and $m < y_p$ or vice-versa, $E\{\beta_n \beta_k\} = e^{-\alpha (d_m + d_n)}$. Otherwise, $E\{\beta_n \beta_k\} = e^{-\alpha \max(d_n, d_m)}$.

The calculation of the mean and standard deviation of interference distribution are validated in Fig. 1. The impact of blockage on the mean is more prominent close to the center because over there, the kernel $e^{-\alpha d_n (1 - \frac{1}{\gamma^2})}$ filters out interference from both sides of location $y_k$. On the other hand, near the boundaries, fewer users are located and the interference is practically generated from one direction. The standard deviation of the generated interference is affected less from blockage due to the following reasons: (i) The kernels $e^{-\alpha d_n (1 - \frac{1}{\gamma^2})}$ and $e^{-\alpha d_n (1 - \frac{1}{\gamma^2})}$, which are less than unity, are under the square root in the computation of the standard deviation. Essentially, they filter out interference less aggressively. (ii) The spatial correlation of the generated interference increases the standard deviation. Actually, in Fig. 1, one may see that by ignoring the spatial correlation, $\sigma = E\{I\}^2$, the underestimation error may become non-negligible.

**IV. TEMPORAL INTERFERENCE CORRELATION**

Even if the user mobility does not induce correlation, the penetration losses for a single user at two different time slots $t$ and $\tau$ can still be correlated provided that the links $x_i(t) \rightarrow y_p$ and $x_i(\tau) \rightarrow y_p$ share some obstacles. In general, the cross-correlation of interference $E\{I(t) \cdot I(\tau)\}$ depends on the user displacement law and the correlation of the RVs $\beta_i(t)$ and $\beta_i(\tau)$. After taking the first-order cross-derivative of the MGF $\frac{\partial^2}{\partial x_i \partial x_j} \Phi_Z(s_1 = 0, s_2 = 0)$, the interference cross-correlation at time-lag $l$ can be read as $E\{I(t) \cdot I(\tau)\} = K \zeta^2 \sigma_l + \sigma_\tau$, where

$$
\sigma_l = \sum_{n=1}^{N} E\{\beta_n \beta_{n+k}\} g(d_n) g(d_{n+k}) P(n+k, \tau) f_z(n).
$$

Equation (5) can be used to calculate the cross-correlation of interference with blockage for any mobility model. The user displacement probabilities, $P(n+k, \tau)$, for the mobility model considered in this paper are available in [6]. Next, we show how to compute the cross-correlation, $E\{\beta_n \beta_{n+k}\}$, for time-lag $l = 1$. Keeping in mind that the number of obstacles over the link $n \rightarrow y_p$ follows the Poisson distribution $Po(\alpha d_n)$, it remains to identify the distribution of obstacles over the link $(n+k) \rightarrow y_p$ for all possible user displacements $k \in \{-1, 0, 1\}$.

**Case 1:** $n < \lfloor y_p \rfloor$. (i) If the user thinks with probability $P(n+1, 1)$, the RVs $\beta_n$ and $\beta_{n+k}$ are fully correlated. Hence, $E\{\beta_n \beta_{n+k}\} = e^{-\alpha d_n (1 - \frac{1}{\gamma^2})} e^{-\alpha (1 - \frac{1}{\gamma^2})}$. (ii) If the user moves to the right with probability $P(n+1, 1)$, the number of obstacles that the user bypasses follows the Poisson distribution $Po(\alpha d_n)$. Hence, $E\{\beta_n \beta_{n+1}\} = e^{-\alpha d_n (1 - \frac{1}{\gamma^2})} e^{-\alpha (1 - \frac{1}{\gamma^2})}$. (iii) If the user moves to the left with probability $P(n+1, 1)$, the extra number of obstacles blocking the user signal follows the Poisson distribution $Po(\alpha)$ and, $E\{\beta_n \beta_{n-1}\} = e^{-\alpha d_n (1 - \frac{1}{\gamma^2})} e^{-\alpha (1 - \frac{1}{\gamma^2})}$. Therefore for $n < n_t$, $n_t = \lfloor y_p \rfloor$, the mean product of vectors $\beta_i$ and $x_i$, $\sigma_{12} \triangleq \sigma_{12} \lfloor n_t \atu n_t \atu n_t \lfloor n_t = \lfloor y_p \rfloor$, can be computed as

$$
\sigma_{12} = \sum_{n=1}^{N} g(d_n) f_z(n) e^{-\alpha (1 - \frac{1}{\gamma^2})} P(n, 1) g(d_n) \left( P(n+1, 1) g(d_n) + e^{\alpha (1 - \frac{1}{\gamma^2})} P(n, 1) g(d_{n+1}) \right) + \left( \varphi (1 - \frac{1}{\gamma^2}) \right) \left( P(n-1, 1) g(d_{n-1}) + e^{-\alpha (1 - \frac{1}{\gamma^2})} P(n+1, 1) g(d_{n+1}) \right).
$$

**For impenetrable obstacles, $\gamma = 0$, which is a reasonable approximation for propagation in the millimeter-wave bands, the above equation can be further simplified.**

**Case 2:** $n > \lfloor y_p \rfloor$. Following the same line of reasoning described under the Case 1, we may compute $\sigma_{12} \triangleq \sigma_{12} \lfloor n > n_t \atu n_t \lfloor n_t = \lfloor y_p \rfloor$.

$$
\sigma_{12} = \sum_{n=1}^{N} g(d_n) f_z(n) e^{-\alpha (1 - \frac{1}{\gamma^2})} P(n, 1) g(d_n) \left( P(n+1, 1) g(d_n) + e^{\alpha (1 - \frac{1}{\gamma^2})} P(n+1, 1) g(d_{n+1}) \right).
$$

**Case 3:** $n = n_t$. When the user is located at $n_t = \lfloor y_p \rfloor$, $d_{n_t} = c$, and it moves to the left, $E\{\beta_n \beta_{n+1}\} = e^{-\alpha c (1 - \frac{1}{\gamma^2})} e^{-\alpha (1 - \frac{1}{\gamma^2})} e^{-\alpha (1 - \frac{1}{\gamma^2})}$. When it moves to the right, it passes over the location $y_p$ and the number of obstacles it sees at the two time slots are i.i.d. Poisson RVs. Therefore $E\{\beta_n \beta_{n+1}\} = e^{-\alpha c (1 - \frac{1}{\gamma^2})} e^{-\alpha (1 - \frac{1}{\gamma^2})} e^{-\alpha (1 - \frac{1}{\gamma^2})}$. Where $c=1-e$. The term $\sigma_{13} \triangleq \sigma_{13} \lfloor n_t = n_t$, can be written as

$$
\sigma_{13} = g(c) f_z(n_t) e^{-\alpha (1 - \frac{1}{\gamma^2})} \left( P(n_t, 1) g(c) + e^{-\alpha (1 - \frac{1}{\gamma^2})} P(n_t+1, 1) g(c) + e^{-\alpha (1 - \frac{1}{\gamma^2})} P(n_t-1, 1) g(1+c) \right).
$$
Case 3: one can do the computation for
and the term
for a mobile network and a static network with user dis-
static case and also makes it location-dependent.
increases further the temporal correlation of interference in the
The calculations for
out in a similar manner.
Finally, one has to sum up the terms \( \sigma_{1j} \), \( j = 1, \ldots, 4 \), and the calculation of \( \sigma_l \) for \( l = 1 \) is complete. After using \( \sigma_l \) to compute the cross-correlation of interference, and substituting this back in equation (2), one can calculate the correlation coefficient for \( l = 1 \). The calculations for \( l > 1 \) can be carried out in a similar manner.
The correlation coefficients for \( l = 1 \) and \( l = 2 \) are depicted in Fig. 2. Without blockage, the temporal correlation of interference is higher close to the border because over there the level of mobility is lower. The impact of blockage on the temporal correlation depends on the location and the density of users. Close to the boundaries, where the user density is low, the transitions from LoS to NLoS and vice versa dominate, and the interference correlation becomes less as compared to the case without obstacles. On the other hand, close to the center, where the user density increases, the spatial correlation among the users dominates over the randomness introduced by the mobility, and the correlation coefficient becomes higher. In the limit of infinite think time, \( M \to \infty \), the network becomes static and the user distribution uniform. Without blockage, the correlation coefficient under Rayleigh fading and continuous user activity is equal to \( \frac{1}{2} \) [1]. In Fig. 2, we see that blockage increases further the temporal correlation of interference in the static case and also makes it location-dependent.
In Fig. 3, we compare the temporal correlation coefficients for a mobile network and a static network with user dis-

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