Hydrogen and muonic-Hydrogen Atomic Spectra in Non-commutative Space-Time

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Abstract

Comparing electronic Hydrogen with muonic Hydrogen shows that the discrepancy in measurement of the Lamb shift in the both systems are relatively of order of $\left(\frac{m_{\mu}}{m_e}\right)^4 - 5$. We explore the spectrum of Hydrogen atom in noncommutative QED to compare the noncommutative effects on the both bound states. We show that in the Lorentz violating noncommutative QED the ratio of NC-corrections is $\left(\frac{m_{\mu}}{m_e}\right)^3$ while in the Lorentz conserving NCQED is $\left(\frac{m_{\mu}}{m_e}\right)^5$. An uncertainty about $1\,Hz \ll 3\,kHz$ in the Lamb shift of Hydrogen atom leads to an NC correction about $10\,MHz$ in the Lorentz violating noncommutative QED and about $400\,GHz$ in the Lorentz conserving noncommutative QED.

1 Introduction

As a simple system, hydrogen atom with high precision measurements in atomic transitions is one of the best laboratories to test QED and new physics as well. Meanwhile,

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there is some discrepancy between the recent measurement of the muonic hydrogen Lamb shift and the corresponding proton radius and the CODATA value which is obtained from the spectroscopy of atomic hydrogen and electron-proton scattering \[1\]. There are two possibilities to explain the discrepancy: 1- the theoretical calculation within the standard model is incomplete. 2- existence of new physics beyond the standard model. There are attempts to explain the new physics by considering a new particle in MeV range where many stringent limits suppress its existence \[2\]. However, the effective new interactions do not need necessarily new particles to mediate the new interactions. For instance, noncommutative (NC) space can induce new interactions in QED without adding new particles in the theory. Theoretical aspects of the noncommutative space have been extensively studied by many physicists \[3\]. Meanwhile, noncommutative standard model (NCSM) via two different approaches is introduced in \[4, 5, 6\] and its phenomenological aspects are explored in \[7\]. Here we would like to study two body bound state in noncommutative space to explore the differences in the electronic and muonic hydrogen spectrum. There are many studies on the hydrogen atom in the NC space-time \[8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\]. However, the effect of NC-space on the hydrogen atom at the lowest order is doubtful. P. M. Ho and H. C. Kao \[8\] have shown that there is not any correction on the space-space NC-parameter in this system. In fact, for the proton as a point particle $\theta_p = -\theta_e$ and the corrections on the Coulomb potential coming from both particles cancel out each other. However, the proton is not a point particle and the parameter of noncommutativity is an effective parameter and is not equal to $-\theta_e$ \[10\]. Furthermore, even if the proton can be considered as a point particle, the space time noncommutativity has some impact on the spectrum of the atom. Meanwhile, in the Lorentz conserving NCQED the NC-parameter appears as $\theta^2$ which is equal for a point particle and its antiparticle. Therefore, it is reasonable to examine the hydrogen atom in the NC space. As $\theta_{\mu\nu}$ has dimension $-2$ one expects an energy shift proportional to $(\theta m^2)^2 mc^2 \alpha^2$. In fact, in the NC-space a larger energy shift for the muonic Hydrogen is expected in comparison with the ordinary Hydrogen which is in agreement with the experimental data.

In Section II we explore two-body bound state in NC-space. In section III we examine the 1S-2S transition and the Lamb-shift for Hydrogen and muonic-Hydrogen and the $g$-factor for electron and muon. In section IV we give a brief review on the Lorentz conserving NCQED and calculate the $g$-factor for electron and muon, the 1S-2S transition and the Lamb-shift for Hydrogen and muonic-Hydrogen. In section V we summarize our results.
2 2-body bound state in Noncommutative Space-Time

Since the NC-parameters for a point particle and its antiparticle are opposite, the NC-correction on the potential for the space-space part of NC-parameter is zero at the lowest order. It can be shown that the Coulomb potential in the Schrodinger equation is proportional to \((\theta_p + \theta_e)_{ij}\) that is zero in the point particle limit of the proton [19] [8].

In the both references the starting point is the Schrodinger equation in the NC space. However, one can show how the NC field theory through the BS-equation leads to the Schrodinger equation with a modified potential[20]. In fact, it is better to start from the NC-field theory (NCFT) to avoid the mistake on the NC-contributions from each particle in the bound state. For instance, the correct potential in the NC-Schrodinger equation can be explored in studying the kernel in the BS-equation for the corresponding NCFT. For this purpose examining the electron-proton scattering amplitude in the NCQED is adequate to derive the appropriate potential for the NCQM.

In a canonical noncommutative space, space-time coordinates are not numbers but operators which do not commute

\[ [\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} = \frac{C^{\mu\nu}}{\Lambda_{NC}^2}, \tag{1} \]

where \(\theta^{\mu\nu}\) is the parameter of noncommutativity, \(C^{\mu\nu}\) is a constant and dimensionless antisymmetric tensor and \(\Lambda_{NC}\) is the noncommutative scale. Since noncommutative parameter, \(\theta^{\mu\nu}\), is constant and identifies a preferred direction in space, canonical version of non-commutative space-time leads to the Lorentz symmetry violation. There are two versions to construct the NCQED [4, 5, 6]. In the first one in contrast with the ordinary QED a momentum dependent phase factor appears in the charged fermion-photon vertex as follows [21]

\[ i e Q \gamma_\mu \exp(ip_\mu \theta^{\mu\nu} p'_\nu), \tag{2} \]

where \(p_\mu\) and \(p'_\nu\) are the incoming and outgoing momenta and \(\theta\) is the NC-parameter. Therefore, the electron-proton amplitude can be written as

\[ \mathcal{M}_{NC} = \mathcal{M} \exp(i p_\mu (\theta^e)_{\mu\nu} p'_\nu) \exp(i k_\mu (\theta^p)_{\mu\nu} k'_\nu), \tag{3} \]

where \(p\) (\(p'\)) and \(k\) (\(k'\)) are the incoming (outgoing) momenta of the electron and proton, respectively. One can easily see that the exponent in terms of the momentum transfer \(q\) in the center of mass is

\[ - i (\bar{\theta}_e + \bar{\theta}_p) \cdot \vec{p} \times \vec{q} - i p_i (\theta^{i0}_e + \theta^{i0}_p) q_0 - i (p_0 \theta^0_i - k_0 \theta^{0i}_p) q_i, \tag{4} \]
where for $\theta_p = -\theta_e = \theta$ results in

$$i(\sqrt{m_e^2 + p^2} + \sqrt{m_p^2 + p^2})\theta^0 q_i. \quad (5)$$

Therefore, for $m_e^2$ and $m_p^2 \gg p^2$, the non-relativistic potential in the NCQM is the Fourier transform of

$$\frac{e^2 \exp[i(m_e + m_p)\theta^0 q_i]}{q^2}, \quad (6)$$

that leads to $V_{NC} = V(\vec{r} - (m_e + m_p)\vec{\theta}_t)$ where $\vec{\theta}_t = (\theta_{10}, \theta_{20}, \theta_{30})$. In fact, to the lowest order of both $\theta$ and $\alpha$ the hydrogen atom only receives some contribution from the temporal part of the NC-parameter. Meanwhile, in [11] and [12] a new correction due to the non commutativity of the source at the lowest order of the space part of the NC-parameter is found. Nonetheless, it is of the order $\alpha^6$ and is not at the lowest order of $\alpha$ too.

In the second approach, via Seiberg-Witten maps, the fields also depend on the noncommutative parameter. Using Seiberg-Witten maps, noncommutative standard model and Feynman rules are fully provided in references [4, 5]. In this approach, two fermion-photon vertex is [5]

$$ie Q_f \gamma_{\mu} + \frac{1}{2} e Q_f [(p_{out} \theta p_{in})\gamma_{\mu} - (p_{out} \theta)(\hat{p}_{in} - m_f) - (\hat{p}_{out} - m_f)(\theta p_{in})_{\mu}], \quad (7)$$

Where $Q$ is the fermion charge and $p_{in}$ and $p_{out}$ are incoming and outgoing momenta, respectively. Considering proton as a point particle, scattering amplitude of electron-proton in the on-shell limit can be given as follows

$$iM = \bar{u}(p')[-ie\gamma^\mu - \frac{1}{2} e(p' \theta e p)\gamma^\mu]u(p)(-ig_{\mu\nu}/q^2)\bar{\nu}(k)[ie\gamma^\nu + \frac{1}{2} e(k' \theta_p k)\gamma^\nu]\nu(k'), \quad (8)$$

where $p$ ($k$) and $p'$ ($k'$) are incoming and outgoing momenta of electron (proton), respectively. At the lowest order of $\theta_{\mu\nu}$, (8) is the same as (3) that means in the second approach one has the same result as is given in (6). In fact, for point particles in the QED bound states such as positronium, there is no NC-correction at the lowest order of $\alpha$ and $\theta_{ij}$.

3 Hydrogen Atom in Noncommutative Space-Time

In a two body bound state the point particles satisfy $\theta_p = -\theta_e = \theta$ and the interaction potential only depends on the time part of the NC-parameter as is shown in (6). Nevertheless, proton in the Hydrogen atom is not a point particle and has an effective
NC-parameter in terms of the NC-parameters of its contents \[10\] which is not equal to \(-\theta_e\). In fact, for the NC-parameter of the order of 1TeV the electron in the Hydrogen cannot probe inside the proton to see its noncommutativity. Therefore, at the low energy limit from the noncommutative point of view proton is a macroscopic particle and \(\theta_{proton} \simeq 0\). Therefore, in the non-relativistic limit and for the energy scale of atom, equation (4) leads to

\[-i\vec{\theta}_e \cdot \vec{p} \times \vec{q} - ip_e\theta^0_0 q_0 - i\theta^0_0 q_i.\]  

(9)

Equation (9) for \(\theta^0_0 = 0\) and \(\theta^{ij} \neq 0\), has been already considered to find bound on the NC-parameter in Hydrogen atom \[9,11,12,13\]. Here we examine the temporal part \((\theta^0_0 \neq 0 \text{ and } \theta^{ij} = 0)\) where (6) leads to

\[e^2 \exp[i m_e \theta^0_0 q_i],\]  

(10)

or

\[V_{nc}(\vec{r}) = V(\vec{r} - m_e \vec{\theta}_t) \simeq -\alpha m_e \frac{\vec{\theta} \cdot \vec{r}^2}{r^4}.\]  

(11)

It should be noted that the quantum mechanics is unitary for the temporal part of NC-parameter \[22,23\]. In (11) only the parallel part of \(\vec{\theta}\) with \(\vec{r}\) has some contribution on the NC-potential that is

\[V_{nc}(r) = -m_e \alpha \theta_{st} \frac{1}{r^2},\]  

(12)

where \(\theta_{st} = \theta_{||} = |\vec{\theta}| \cos \phi\) (\(\cos \phi\) is the angle between \(\vec{\theta}\) and \(\vec{r}\)) and \(\alpha\) is the fine structure constant. Using the perturbation theory, one can easily find the energy level shift as

\[\Delta E_{NC}^{H-atom} = -\frac{m^3_e \alpha^3 \theta_{st}}{n^3(2l + 1)}.\]  

(13)

Here we fix the NC-parameter by the most precise experimental value in the Hydrogen atom (i.e. \(1S\rightarrow 2S\) transition) then we find the effects of the NC-space-time on the other physical quantities such as Lamb-shift in atom and the anomalous magnetic moment.

**1S – 2S Transition:** The obtained energy shift (13), from the temporal part of noncommutativity, leads to an additional contribution on the theoretical value of 1S-2S transition in Hydrogen atom as follows

\[\Delta E_{NC}^{1S-2S} = \frac{7}{8} m^3_e \alpha^3 \theta_{st}.\]  

(14)

The experimental value for 1S-2S transition in Hydrogen atom is \[24\]

\[f_{1S-2S} = (2466061102474581 \pm 34) Hz.\]  

(15)
Now comparing the given uncertainty on the experimental value in (15) with (14), gives an upper bound on $\theta_{st}$ as

$$\theta_{st} \leq (1.4 \text{ TeV})^{-2}. \quad (16)$$

**Lamb Shift:** For $\Lambda = 1.5 TeV$ the NC-correction on the lamb shift for electronic hydrogen is

$$\Delta E_{NC}^{H_e} = -\frac{m_e^3 \alpha^3}{12 \Lambda^2} \sim 2Hz \ll 3kHz,$$

where $3kHz$ is the current experimental accuracy on the lamb shifts $2s_{1/2} - 2p_{1/2}$ in the Hydrogen atom[25]. Meanwhile, for the muonic hydrogen one has

$$\Delta E_{NC}^{H_\mu} = \left(\frac{m_\mu}{m_e}\right)^3 \Delta E_{NC}^{H_e} = 20MHz \sim 2 \times 10^{-5} \text{meV}. \quad (18)$$

**g-2 for electron and muon:** Since the NC-parameter has dimension -2 the dimensionless quantity $a = \frac{g-2}{2}$ should be corrected, at the lowest order in NC-space, as $C(\frac{\alpha}{2\pi}) p_\mu \theta^{\mu\nu} p_\nu$, where $C$ is a constant which is obtained in [26][21]. Therefore, for $\Lambda = 1.5 TeV$ the NC-correction on $a$ for electron is

$$a_e = \frac{5 \alpha}{6 \pi} \frac{p^2}{\Lambda^2} \simeq \frac{5 \alpha}{6 \pi} \frac{m_e^2}{\Lambda^2} \sim 10^{-16}, \quad (19)$$

and for the muon where in E286 experiment has a momentum about 3 GeV is

$$a_\mu = \frac{5 \alpha}{6 \pi} \frac{p^2}{\Lambda^2} \simeq \frac{5 \alpha}{6 \pi} \left(\frac{3}{1500}\right)^2 \sim 3 \times 10^{-9}. \quad (20)$$

4 **Hydrogen atom in Lorentz Conserving Noncommutative Space-Time**

As the NC-parameter is a real and constant Lorentz tensor, there is, obviously, a preferred direction in a given particle Lorentz frame which leads to the Lorentz symmetry violation. On the other hand, experimental inspections for Lorentz violation, including clock comparison tests, polarization measurements on the light from distant galaxies, analyses of the radiation emitted by energetic astrophysical sources, studies of matter-antimatter asymmetries for trapped charged particles and bound state systems [27] and so on, have thus far failed to produce any positive results. These experiments strictly bound the Lorentz-violating parameters, therefore, in the lower energy limit, the Lorentz symmetry is an almost exact symmetry of the nature [28]. However, Carlson, Carone, and Zobin (CCZ) have constructed Lorentz-conserving noncommutative quantum electrodynamics based on a contracted Snyder algebra [29].
the parameter of noncommutativity is not a constant but an operator which transforms as a Lorentz tensor. In fact, (11) should be extended to

\[ [\hat{x}^\mu, \hat{x}^\nu] = i\hat{\theta}^{\mu\nu}, \quad [\hat{\theta}^{\alpha\beta}, \hat{\theta}^{\mu\nu}] = 0, \quad [\hat{\theta}^{\mu\nu}, \hat{x}^\nu] = 0, \]

(21)

where \( \hat{\theta}^{\mu\nu} \) is an operator. Consequently, according to the Weyl-Moyal correspondence, to construct the LCNC action the ordinary product should be replaced with the star product as follows

\[ f \ast g(x, \hat{\theta}) = f(x, \theta) \exp\left(i/2 \vec{\partial}_\mu \theta^{\mu\nu} \vec{\partial}_\nu\right) g(x, \theta). \]

(22)

In this formalism a sufficiently fast falling weight function \( W(\theta) \) has been used to construct the Lorentz invariant lagrangian in a non-commutative space as

\[ \mathcal{L}(x) = \int d^6 \theta W(\theta) \mathcal{L}(\phi, \partial \phi), \]

(23)

where the Lorentz invariant weight function \( W(\theta) \) is introduced to suppresses the NC-cross section for energies beyond the NC-energy scale. In reference [30] on the existence of an invariant normalized weight function is discussed and an explicit form for \( W(\theta) \) is given in terms of Lorentz invariant combinations of \( \theta^{\mu\nu} \)'s. The function \( W(\theta) \) can be used to define an operator trace as

\[ Tr \hat{f} = \int d^4 x d^6 \theta W(\theta) f(x, \theta), \]

(24)

in which \( W(\theta) \) has the following properties

\[ \int d^6 \theta W(\theta) = 1, \]

(25)

\[ \int d^6 \theta W(\theta) \theta^{\mu\nu} = 0, \]

(26)

\[ \int d^6 \theta W(\theta) \theta^{\mu\nu} \theta^{\nu\kappa} = \left\langle \theta^2 \right\rangle (g^{\mu\nu} g^{\kappa\lambda} - g^{\mu\kappa} g^{\nu\lambda}), \]

(27)

where

\[ \left\langle \theta^2 \right\rangle = \int d^6 \theta W(\theta) \theta^{\mu\nu} \theta_{\mu\nu}. \]

(28)

As (26) shows in the expansion of the Lagrangian (23) in terms of the NC-parameter the odd powers of \( \theta_{\mu\nu} \) vanishes. In fact, to obtain the nonvanishing \( \theta \)-dependence terms, all fields should be expanded at least up to the second order of the NC-parameter. The
Lorentz conserving NCSM is fully introduced in [31] and its fermionic part where we are interested to explore is given as follows

\[
S_{\text{fermion}} = \int d^4x (\mathcal{L}^D L + \overline{R}^D R) + \int d^6\theta \int d^4x W(\theta) \theta^{\mu\nu} \theta^{\rho\lambda} \left( -\frac{i}{8} \mathcal{L}^\rho F^0_{\mu\nu} F^0_{\chi\lambda} D^0_{\nu L} 
- \frac{i}{4} \mathcal{L}^\rho F^0_{\mu\nu} F^0_{\nu\kappa} \mathcal{D}_\lambda L - \frac{1}{8} \mathcal{T}^\rho (F^0_{\mu\nu} D^0_{\nu L} D^0_{\lambda L}) \right) + \int d^6\theta \int d^4x W(\theta) \theta^{\mu\nu} \theta^{\rho\lambda} \left( -\frac{i}{8} \overline{R}^\rho F^0_{\mu\nu} F^0_{\chi\lambda} D^0_{\nu R} 
- \frac{i}{4} \overline{R}^\rho F^0_{\mu\nu} F^0_{\nu\kappa} \mathcal{D}_\lambda R - \frac{1}{8} \overline{R}^\rho (F^0_{\mu\nu} D^0_{\nu R} D^0_{\lambda R}) \right),
\]

(29)

where \( L \) and \( R \) stand, respectively, for left and right handed fermions and \( F^0_{\mu\nu} \) is the ordinary field strength in the standard model. To find the LCNC-effects, at the lowest order, on the hydrogen atom we only consider the QED part of the NC-action (29) as follows

\[
\int d^6\theta \int d^4x eQ_f (\mathcal{L}^0 A_0^\mu \nu L - \frac{1}{8} \theta^{\mu\nu} \theta^{\rho\lambda} \mathcal{L}^0 \gamma^\rho \partial_\mu A_{0\kappa\rho} \partial_\nu \partial_\lambda \nu L + \overline{e} A^\mu_0 e_L - \frac{1}{8} \theta^{\mu\nu} \theta^{\rho\lambda} \mathcal{L}^0 \gamma^\rho \partial_\mu A_{0\kappa\rho} \partial_\nu \partial_\lambda e),
\]

(30)

Where \( A_{0\mu\nu} = \partial_\mu A_{0\nu} - \partial_\nu A_{0\mu} \) and the charged fermions interact with photon via the following vertex

\[
i e Q_f \gamma^\mu (1 + \frac{\langle \theta^2 \rangle}{96} (\frac{q^4}{4} - m^2_f q^2)).
\]

(31)

In [31] the \( \theta \)-dependence is appeared as \( \langle \theta^2 \rangle \) which is similar for both particle and its antiparticle. In fact, in LCNC-QED in contrast with NCQED according to \( \theta_f^- = -\theta_f^+ \), particle vertex doesn’t cancel antiparticle vertex. Therefore, in \( f^- f^+ \) bound state the LCNC effect via the \( f^- f^+ \) scattering amplitude, at low energy limit, leads to a potential in momentum space as

\[
\tilde{V}(q) = -\frac{e^2}{q^2} - \frac{e^2 (m_{f^-}^2 + m_{f^+}^2) \langle \theta^2 \rangle}{96},
\]

(32)

Where to obtain (32), at low momentum transfer, the second term in (31) is ignored in comparison with the third one. However, proton as a particle with internal structure doesn’t see the NC-space in those systems which the momentum transfer is small such as Hydrogen like atom. Therefore, the NC-potential in the Hydrogen atom is

\[
\tilde{V}(q) = -\frac{e^2}{q^2} - \frac{e^2 m_{f^-}^2 \langle \theta^2 \rangle}{96},
\]

(33)
or
\[
V(r) = -\frac{e^2}{4\pi r} - \frac{e^2m_e^2\langle\theta^2\rangle}{96}\delta(r). \quad (34)
\]
The NC-correction on the Coulomb potential in (34) is small and its expectation value directly gives the energy shift on the energy levels of the Hydrogen atom as follows
\[
\Delta E_{H-atom}^{\text{LCNC}} = -\langle\psi|\frac{e^2m_e^2\langle\theta^2\rangle}{96}\delta(r)|\psi\rangle = -\frac{e^2m_e^2\langle\theta^2\rangle}{96}|\psi_{nl}(r = 0)|^2, \quad (35)
\]
Where \( |\psi_{nl}(r = 0)|^2 = \frac{\alpha^3m^3}{\pi n^3}\delta_{l0} \) leads to
\[
\Delta E_{H-atom}^{\text{LCNC}} = -\frac{m_e^5\alpha^4(\theta^2)}{24n^3}\delta_{l0}, \quad (36)
\]
or with \( \Lambda_{\text{LCNC}} = \left(\frac{12}{\alpha^2}\right)^4 \), (36) can be rewritten as
\[
\Delta E_{H-atom}^{\text{LCNC}} = -\frac{m_e^5\alpha^4}{2n^3\Lambda_{\text{LCNC}}^4}\delta_{l0}. \quad (37)
\]

1S – 2S Transition: The experimental value for 1S-2S transition in the Hydrogen atom can fix the upper bound on the parameter \( \Lambda_{\text{LCNC}} \) as follows
\[
\Delta E_{1S-2S}^{\text{LCNC}} = \frac{7m_e^5\alpha^4}{16}\frac{1}{\Lambda_{\text{LCNC}}^4} \sim 34Hz, \quad (38)
\]
leads to
\[
\Lambda_{\text{LCNC}} \sim 0.2\text{GeV}. \quad (39)
\]

Lamb Shift: For \( \Lambda = 0.5\text{GeV} \) the NC-correction on the lamb shift for electronic hydrogen is
\[
\Delta E_{NC}^{H_e} = \frac{m_e^5\alpha^4}{16}\frac{1}{\Lambda_{\text{LCNC}}^4} \sim 0.1Hz \ll 3kHz, \quad (40)
\]
while for the muonic hydrogen one has
\[
\Delta E_{NC}^{H\mu} = \left(\frac{m_\mu}{m_e}\right)^5\Delta E_{NC}^{H_e} \simeq 40GHz \sim 0.03\text{meV}. \quad (41)
\]

\( g-2 \) for electron and muon: As Eq.(31) shows the NC-correction on \( a = \frac{g-2}{2} \) should be proportional to \( q^2 \) which leads to zero NC-correction on \( a \) at zero momentum transfer.
In this paper two body bound state has been studied by examining the scattering amplitude in the Lorenz violated (LV) and Lorentz conserving (LC) NCQED as given in (3) and (32), respectively. For a bound state of a particle with its antiparticle the NC potential in LVNCQED, in contrast with LCNCQED, only depends on the space-time part of the NC-parameter, see (4). As the proton in Hydrogen atom is not a point particle, $\theta_p \neq \theta_e$. In fact in $ep$-scattering, in the low energy limit, the electron cannot probe inside the proton to see its NC-effects. In the Lorentz violated NCQED for Hydrogen and muonic Hydrogen atom we have found:

1- In $1S - 2S$ transition in the Hydrogen atom the NC-parameter has been fixed of the order of $\Lambda_{NC} = 1.5 \text{ TeV}$.

2- $\Lambda_{NC} = 1.5 \text{ TeV}$ leads to an NC-shift on the $2s_{1/2} - 2p_{1/2}$ transition in hydrogen atom about $2 \text{ Hz}$ which is far from the $3 \text{ kHz}$ current uncertainty on the Lamb shift in Hydrogen atom. As (18) shows an uncertainty of order of $3 \text{ kHz}$ in the Lamb shift of Hydrogen leads to $(3 \text{ kHz})(\frac{m_e}{m_\mu})^3 = 26 \text{ GHz}$ which can only explain a small part of the current deviation between the experimental measurement and the theoretical prediction.

3- The NC-effect on the g factors of electron and muon are $a_e = 10^{-16}$ and $a_\mu = 10^{-9}$, respectively. The obtained values for $a_e$ and $a_\mu$ are in agreement with the experimental measurements.

In the Lorentz conserving NCQED for Hydrogen and muonic Hydrogen atom we have found:

1- In $1S - 2S$ transition in the Hydrogen atom the NC-parameter has been fixed about $\Lambda_{NC} = 0.5 \text{ GeV}$.

2- $\Lambda_{NC} = 0.5 \text{ GeV}$ leads to an NC-shift on the $2s_{1/2} - 2p_{1/2}$ transition in hydrogen atom about $0.1 \text{ Hz}$ which is far from the $3 \text{ kHz}$ current uncertainty on the Lamb shift in Hydrogen atom. As (11) shows an uncertainty of order of $3 \text{ Hz} \ll 3 \text{ kHz}$ in the Lamb shift of Hydrogen leads to $(3 \text{ Hz})(\frac{m_e}{m_\mu})^5 = 1000 \text{ GHz}$ which can explain the current difference between the experimental measurement and the theoretical prediction about $0.3 \text{ meV}$ [1].

3- The NC-correction on $a = \frac{g-2}{2}$ is proportional to $q^2$ which is zero at the zero momentum transfer.
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