A general relativistic estimation of the black hole mass-to-distance ratio at the core of TXS 2226–184

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Received 28 March 2022 / Accepted 7 June 2022

ABSTRACT

In this work we make use of a general relativistic method to estimate the mass-to-distance ratio $M/D = 3.54^{+0.20}_{-0.12} \times 10^4 M_\odot/Mpc^{-1}$ of the black hole hosted at the core of the active galactic nucleus of TXS 2226–184, along with its right ascension offset and the recession redshift (velocity) of the galaxy. Our statistical fit is based on the frequency shift of photons emitted by water masers and their orbital positions when circularly revolving around the black hole center within the accretion disk of the active galactic nucleus. By taking a previously reported distance to the galaxy into account, we compare the result of the black hole mass fit to an estimate based on a mass-luminosity correlation. We find that the black hole mass at the core of TXS 2226–184 obtained with the aid of the statistical fit using the general relativistic method, $M = 3.67^{+0.20}_{-0.18} \times 10^6 M_\odot$, is approximately 0.6 times the black hole mass, $M_{BH} = 6.24^{+1.22}_{-1.77} \times 10^6 M_\odot$, which was computed with the mass–luminosity correlation.

Key words. black hole physics – masers – galaxies: high-redshift – galaxies: kinematics and dynamics – galaxies: nuclei – galaxies: luminosity function, mass function

1. Introduction

Over 100 years ago, Albert Einstein published his theory of general relativity (Einstein 1915). Two months later, Karl Schwarzschild published a solution to Einstein’s field equations (Schwarzschild 1916), describing the gravitational field outside a spherically symmetric and static body. This solution is useful to describe the spacetime curvature generated by astrophysical objects such as stars and it was later understood to describe a black hole (BH). A BH is a region of spacetime where the gravitational field is so strong that not even light can escape beyond the event horizon. A Schwarzschild BH is fully characterized by its mass. In 1963, Kerr constructed a solution that describes the gravitational field of a rotating BH (Kerr 1963), which is completely characterized by its mass and angular momentum.

In recent years, there has been important observational evidence of the existence of BHs. For instance, the observation of the stars orbiting with a very short period around Sgr A* in the center of our galaxy indicates the presence of a supermassive BH Ghez et al. (1998, 2008), Genzel et al. (1997, 2010), Do et al. (2019), and Abuter et al. (2020), the detection of gravitational waves produced by the merger of BHs by LIGO-Virgo Collaborations Abbott et al. (2016, 2020a,b), and the imaging of the M87 and SgrA* BH shadows by the EHT collaboration Event Horizon Telescope Collaboration (2019, 2022).

Since BHs do not emit electromagnetic radiation, one way to study these enigmatic entities consists in observing their influence on stars, accretion disks, gas particles, etc. that orbit them. For certain astrophysical systems, the positions of the orbiting bodies and the redshift and blueshift of the photons they emit are available and can be measured. For this reason, several models that relate these observational quantities of the orbiting objects to the mass and the mass-to-distance ratio of the BH have been developed.

Water megamasers, emitting at 22 GHz, are astrophysical objects that have been found within active galactic nuclei (AGNs), where they orbit central BHs (Claussen et al. 1984). The prefix “mega” refers to the intense luminosity emitted by the water maser in the AGN $(L > 10L_c)$ compared to the luminosity of galactic masers $(L < L_c)$, which are associated with star-forming regions (Genzel & Downes 1977).

Very long baseline interferometry (VLBI) is an accurate technique for observing the positions and displacements of these maser features, as it gives us appropriate submilliarcsecond resolution for objects at (sub)parsec distances from the center of active galaxies. Telescopes, such as the Green Bank Telescope, the NRAO Very Long Baseline Array (VLBA)1, the European VLBI Network (EVN)2, and the Effelsberg 100 m one Reid et al. (2008), Braatz et al. (2010), and Kuo et al. (2011), provide us with observational data of positions and redshifts associated with

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1 The VLBA is operated by Associated Universities, Inc., under a cooperative agreement with the National Science Foundation which is a facility of the National Radio Astronomy Observatory (NRAO).

2 The European VLBI Network (EVN) is a network of radio telescopes located primarily in Europe and Asia, with additional antennas in South Africa and Puerto Rico. Support for proposal preparation, scheduling, and correlation of EVN projects is provided by the Joint Institute for VLBI ERIC (JIVE); JIVE stands for the European Research Infrastructure Consortium.
the megamasers that allow us to estimate the mass of their central
BH within an appropriate model.

In Herrnstein et al. (2005), the authors used a dynamic
Keplerian model with relativistic corrections for maser features
to fit the mass of the BH hosted at the core of the NGC 4258
galaxy. This modeling related the radial velocity of water mega-
masers to the observed redshift of the emitted photons by mak-
ning use of the optical definition of the redshift. The Newtonian
approach works accurately when the object is far enough from
the gravitational source, but when the orbiting objects get close
enough to the BH event horizon, the general relativistic effects
become stronger and relevant. Thus, when this happens, the red-
shift of the photons begins to have important general and special
relativistic contributions.

A model for test particles orbiting a Kerr BH was presented
in Herrera-Aguilar & Nucamendi (2015). In this formalism, the
influence of the BH on the curvature of spacetime was taken
into account, and therefore the so-called gravitational redshift
was included in the total redshift of photons emitted by particles
orbiting the BH.

In Nucamendi et al. (2021), the authors used a simplified ver-
sion of the latter model and considered a Schwarzschild BH with
water megamasers orbiting the AGN of NGC 4258. The authors
estimated the mass-to-distance ratio of the BH at the galactic
core of this galaxy as well as its peculiar redshift using a general
relativistic approach. The peculiar redshift is related to the pecu-
liar velocity of the galaxy with respect to the distant observer
using the same optical definition.

Another way to estimate the BH mass without using a model
based on gravity relies on a correlation between the mass of
the BH and the bulge luminosity $M_{BH}-L_{bulge}$ of the host galaxy
(Kormendy & Ho 2013). It is important to note that this cor-
relation uses the $K$ band in the near-infrared instead of the
visible spectrum. This correlation was initially based on the
study of disk galaxies, but it also works for elliptical galaxies
because they are morphologically equivalent to the bulge of disk
galaxies.

One of the brightest known H$_2$O maser sources was dis-
covered in the AGN of the TXS 2226–184 galaxy using the
Effelsberg telescope (Koekemoer et al. 1995). The isotropic
luminosity associated with this water maser is so large, $L = 6100 \pm 900 L_\odot$ (Koekemoer et al. 1995), that it is called a “giga-
maser”. The distance to TXS 2226–184 is $D = 103.8571 \pm 0.2606 Mpc$\footnote{In Kuo et al. (2018), the reported
distance to TXS 2226–184 is $D = 107.1 Mpc$; however,
since this distance has no associated uncertainties,
we cannot use this result to estimate the value of the BH mass
with properly propagated errors.} as a result of applying Hubble’s law to the
reported systemic velocity $V_{0,\text{Effelsberg}} = 7270 \pm 18.24 \text{ km s}^{-1}$
in Taylor et al. (2002) and Surcis et al. (2020) and assuming
$H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ Kuo et al. (2018), these authors did not
report uncertainties taken into account for the Hubble constant
value). TXS 2226–184 has been optically classified as an ellipti-
cal galaxy (Koekemoer et al. 1995).

In Ball et al. (2005), the authors observed with the VLBA
seven H$_2$O maser emission clusters in TXS 2226–184. The clusters
were linearly distributed from the northeast to the south-
west with the position angle $= 25^\circ$. The observational maser
data have five redshifted maser features and two blueshifted
maser features, where only one blueshifted maser was along
the linear distribution (the most southwest maser feature),
and the other one was about 7 mas southeast of the linear
distribution. Given these features, the authors associated the

distribution of the maser clusters with a parsec-scale, rotat-
ing disk, where the farthest blueshifted maser was situated
completely outside the disk. However, the data reported in
Ball et al. (2005) did not provide absolute positions of the maser
features.

In Surcis et al. (2020), the authors made new observations
of the H$_2$O gigamaser in TXS 2226–184 with the VLBA (one
epoch) and the EVN (two epochs). The authors detected six
maser features in epoch 2017.45 (VLBA), one in epoch 2017.83
(EVN), and two in epoch 2018.44 (EVN). In the data corre-
spanding to epoch 2017.45 (VLBA), only one blueshifted maser
feature was detected, while the other five maser features were
redshifted with respect to the systemic velocity of TXS 2226–
184. In addition, the authors provided absolute positions of the
maser features.

Most of the masers located in the accretion disks of super-
massive BHs are megamasers. Estimates of BH masses using
megamaser dynamics are on the order of $10^9$–$10^9 M_\odot$. So far,
no estimate of the mass of central BHs in galaxies hosting a
gigamaser has been made, leaving the question open of whether
the high luminosity of the maser is related to a central BH
mass of magnitude greater than $10^9$–$10^9 M_\odot$. New observa-
tions of the gigamaser made with the VLBA have been pub-
lished in Surcis et al. (2020) and, in principle, they give us
the opportunity to investigate whether there is a connection
between its intense luminosity and the mass of the central
BH.

2. General relativistic model

Here we assume a static and spherically symmetric spacetime.
Thus, we used the Schwarzschild metric (in natural units):

$$ds^2 = \frac{dr^2}{f} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) - f dr^2, \quad f = 1 - \frac{2m}{r}, \quad (1)$$

where $m = GM/c^2$ and $M$ is the BH mass.

The general relativistic model which we used was developed in
Nucamendi et al. (2021). We consider that massive test par-
ticles (photon sources such as stars, masers, and other bodies)
follow a geodesic path. The geodesic motion of massive test par-
ticles is described by the four-velocity $U^\mu = (U^t, U^r, U^\theta, U^\phi)$
normalized to unity $U^\mu U_\mu = -1$.

From this relation, we obtain an equation similar to the con-
servation law of energy for a nonrelativistic particle with energy
$E/2$. For the special case of equatorial and circular orbits,
the expression for the four-velocity components simplifies since
$U^\theta = 0 = U^\phi$. Therefore,

$$U^t = \pm \frac{r}{\sqrt{r^2 - 3m}}, \quad U^r = \pm \frac{1}{r} \sqrt{\frac{m}{r^2 - 3m}}, \quad (2)$$

where the $\pm$ signs correspond to the angular velocity direction of
the orbiting object.

The photons emitted by the test particles have a four-
momentum $k^\mu = (k^t, k^r, k^\theta, k^\phi)$. These photons move along null
and equatorial geodesics ($k^\theta k_\theta = 0$), so that the emitted and
detected frequencies can be written in terms of the parameters
of the metric.

Emitted photons experience a Schwarzschild redshift or
blueshift on their way from the source bodies toward a
static observer, which stands far away from the BH ($U^\mu |_a = \text{null}$. \quad (3)
(1, 0, 0, 0)|d; r_d → ∞), and read as follows:

\[ 1 + z_{\text{Schw};2} = 1 + z_g + z_{\text{kin}} = \frac{\omega_e}{\omega_d} = \frac{(k_\lambda U^p)_e}{(k_\lambda U^p)_d} = \frac{(U^g - b_x U^p)_e}{(U^g - b_x U^p)_d} \equiv (U^g - b_x U^p)_e, \]

where the subscripts \((1/2)\) correspond to \((t/z)\), the subscript \((e)\) refers to the emitter, \((d)\) refers to the detector, and \(\omega\) is the photon frequency. Here, \(b_x\) is the light bending (deflection) parameter to the right and left of the line of sight; \(z_g\) and \(z_{\text{kin}}\) represent the gravitational and kinematic redshifts, respectively, and are expressed as follows:

\[ b_x = \mp \sqrt{\frac{g_{ee}}{g_{ds}}}; \quad z_g = \sqrt{\frac{r_e}{r_e - 3m}} - 1, \quad z_{\text{kin}} = \pm \sqrt{\frac{m r_e}{(r_e - 2m)(r_e - 3m)}}, \]

where the \(\pm\) signs in Eq. (5) correspond to an approaching and receding object with respect to a far away observer, yielding the kinematic redshift \(z_{\text{kin}}\) and blueshift \(z_{\text{kin}}\).

Hereinafter we use the approximation \(\Theta \approx r_e/D\) for the angular distance between a given maser and the BH position, where \(r_e = D\) is the distance from the detector to the BH. Then the gravitational and kinematic redshift become the following:

\[ z_g = \sqrt{\frac{1}{1 - 3m/2r_e}} - 1, \quad z_{\text{kin}} = \pm \sqrt{\frac{m r_e}{(1 - 2m/3r_e)(1 - 3m/2r_e)}}. \]

For a full description of realistic systems, we must take into account the recession redshift \(z_{\text{rec}}\) given by the composition of the peculiar redshift, \(z_p\), related to the peculiar velocity of the galaxy with respect to the observer, and the cosmological redshift, \(z_{\text{cosm}}\), associated with the expansion of the universe when the galaxy is within the *Hubble* flow. Both of these redshifts have a different nature. Thus the total redshift is given by the following composition (Davis & Scrimgeour 2014):

\[ 1 + z_{\text{tot};2} = (1 + z_{\text{Schw};2})(1 + z_{\text{rec}}); \quad (1 + z_{\text{rec}}) = (1 + z_{\text{boost}})(1 + z_{\text{cosm}}). \]

Strictly speaking, one should consider a Schwarzschild-Friedmann-Robertson-Walker metric in order to take into account the expansion of the universe in the BH geometry that generates the cosmological redshift. However, we do not have the expression for the redshift for such a metric at hand, and as a first approximation we made use of the Schwarzschild background.

The peculiar redshift is defined through the special relativistic boost (Rindler 1982)

\[ 1 + z_{\text{boost}} = \gamma (1 + \beta \cos \alpha), \quad \gamma \equiv (1 - \beta^2)^{-1/2}, \quad \beta \equiv \frac{v_p}{c}, \]

where \(v_p \equiv c z_p\) and \(v_p \cos \alpha\) is the radial component of the peculiar velocity of the galaxy with respect to the observer (see Fig. 1). Thus, in principle the \(\alpha\) angle encodes the transversal motion of the galaxy with respect to the line of sight.

3. Mass–luminosity correlation

Increased availability in BH mass demography has led to the observation that there appears to be a close correlation between the BH mass and the bulge properties of the host galaxy. The most studied correlations of bulge properties with BH masses are given by the mass-velocity dispersion correlation \(M_\text{BH} - \sigma^*\), Gebhardt et al. (2000) and Ferrarese & Merritt (2000) and the mass–luminosity correlation \(M_\text{BH} - L_\text{bulge}\), Dressler et al. (1989), Kormendy & Richstone (1995), and Marconi & Hunt (2003). In Kormendy & Richstone (1995), the authors found that the BH masses show a correlation with the absolute blue luminosity of the host bulge for eight galaxies. In Marconi & Hunt (2003), luminosity was taken in the \(K\) band centered on 2.2 \(\mu m\) (in the near-infrared 136 THz range) instead of the blue luminosity because the scatter in the first band is smaller than in the second one.

For an overview of the \(M_\text{BH} - L_\text{bulge}\) correlation, followed by a larger sample of galaxies of a different type and distinct BH masses hosted at their center, readers can refer to Sect. 6 in Kormendy & Ho (2013). In the case of elliptical galaxies, we must consider their total luminosity because these galaxies are morphologically equivalent to the bulge component of disk galaxies. The next equation shows the correlations of \(M_\text{BH}\) with \(L_K\_\text{bulge}\) (Kormendy & Ho 2013):

\[ M_\text{BH} = \left(0.542^{+0.060}_{-0.061}\right) \left(\frac{L_K\_\text{bulge}}{10^{10} L_K}\right)^{1.21 \pm 0.09}, \]

where \(L_K\_\text{bulge}\) is the bulge luminosity and \(L_K\) is the solar luminosity in the \(K\) band.

4. Observation of the \(H_2O\) gigamaser in TXS 2226–184

In this Letter, we consider the VLBI observations of gigamaser features in the AGN of TXS 2226–184. We use the data reported by Taylor et al. (2002) and Surcis et al. (2020). The latter authors measured the redshift of photons emitted at the points of maximum emission and their absolute positions with errors lesser than 1 milliarcsecond. According to
these authors, the TXS 2226–184 galaxy is located at a distance \( D = 103.85 \pm 0.26 \) Mpc with an adopted center at \( \alpha_{2000} = 22^h 29^m 12^s 49\text{.}660 \pm 0.000291, \delta_{2000} = -18^\circ 47^\prime 24^\prime\prime 00 \pm 0.000409 \), with peculiar velocity \( V_{\text{pec}} \) of \( 7270 \pm 18.24 \) km s\(^{-1}\). We used the observations of redshift and masers positions corresponding to VLBA data epoch 2017.45 on June 12, 2017. The VLBA provides an angular resolution of 0.2 milliarcsec (mas) and spectral resolution of 1 km s\(^{-1}\) at 22 GHz. The observational data of the maser were sparse, with only six maser features reported, five redshifted, and only one blueshifted. Despite the minimal quantity of data, we still can perform a statistical fit to estimate the mass-to-distance ratio of the central BH of this galaxy.

5. Statistical fit with our general relativistic model

The observations indicate that a set of water maser clouds is allocated on the accretion disk of a central BH hosted at the AGN of the galaxy TXS 2226–184. These features lie on the equatorial plane since we see the disk edge-on, and we shall assume that their motion is circular around the BH. Therefore we can make use of Eq. (7) to model their total redshifts and blueshifts, which are directly observed.

To fit the parameters, we used the least-squares estimation \( \chi^2 \) by a Bayesian statistical fit based on the Markov–chain Monte Carlo scheme applied to the maser data using the general relativistic formalism. We emphasize that we applied our fits to directly measured general relativistic invariant quantities.

The parameters we fit are the mass-to-distance ratio \( M/D \), the right ascension (RA) offset \( x_0 \) of the BH, and the recession redshift of the galaxy \( z_{\text{rec}} \). The position data of the maser features reported in Surcis et al. (2020) are presented by considering the brightest maser as a reference. Instead of using a reference maser, we propose a new reference origin at the geometric center of the maser system (see Fig. 2), so that the fitted BH position will be estimated with a reference to that point. However, by varying \( y_0 \) within the observed height of the disk, we see that the estimate of \( M/D \) changes at the third significant figure after the decimal point in comparison to the estimation with \( y_0 = 0 \) mas. This change is well beyond the \( M/D \) uncertainty and reveals the thin character of the disk, implying that \( y_0 \) does not influence the estimation of this quantity. Indeed we shall assume that masers do not lie completely along the midline, but that they are uniformly scattered about it with a scattering angle \( \delta \phi \) and that the disk inclination \( \theta_i \) is parameterized by the polar angle toward the equatorial plane.

Now, we present the \( \chi^2 \) of the general relativistic model based on Herrnstein et al. (2005) and Nucamendi et al. (2021):

\[
\chi^2 = \sum_{k=1}^{n} \frac{\left[ \frac{\Delta v_{\text{obs}}}{\sigma_{\text{obs}}} - (1 + z_g + \epsilon \sin \theta_i \cdot z_{\text{rec})} \right] (1 + z_{\text{rec}}) + 1 \right]^2}{\sigma_{\text{obs}}^2 + \kappa^2 \sin^2 \theta_i (1 + z_{\text{rec}})^2},
\]

where the first term in the numerator refers to the observed redshift and the remaining terms are related to our model. In the denominator, \( \sigma_{\text{obs}}^2_{\text{tot}} \) is the error associated with the total redshift.

This quantity is \( \delta z_{\text{tot}} \) and means the variation of the total redshift as shown in Nucamendi et al. (2021) is as follows:

\[
\delta z_{\text{tot}} = (\delta z_g + \delta z_{\text{kin}})(1 + z_{\text{rec}}).
\]

Following the latter work, we considered the redshift errors caused by the errors in the positions so that \( \delta z_g = \left\{1 + z_g \right\} \left(\frac{1 + z_{\text{rec}}}{2z_{\text{rec}}} \right) \Delta \alpha \).
luminosity is \( L_K = 2.5 \times 10^9 L_\odot \) with no reported uncertainties. In this framework, we accordingly substituted this reported luminosity into Eq. (8), thus, the BH mass estimate reads as follows:

\[
M_{\text{BH}} = 6.24^{+3.63}_{-2.27} \times 10^6 M_\odot.
\] (13)

Using the mass–luminosity method, we get a mass of the same order of magnitude as the estimate using the method of general relativity. However, we get a substantial uncertainty compared to that estimate, which is in fact an order of magnitude larger. Thus, by comparing these results, we see that a fit based on the redshift of the particles is more precise and reliable than using estimates based on the galaxy properties such as luminosity.

7. Conclusions and discussion

Our general relativistic approach provides estimates for the mass-to-distance ratio of the BH hosted at the AGN of TXS 2226–184 \( (M/D = 3.54^{+0.20}_{-0.18} \times 10^4 M_\odot \text{Mpc}^{-1}) \) as well as for its RA offset, the recession redshift of the host galaxy, and its associated velocity. Furthermore, this model also allowed us to quantify the gravitational redshift of each of the maser features; we calculated it for the two closest masers to the central BH of this galaxy. The gravitational redshift obtained for each of the gigamasers is one order of magnitude smaller than the detector sensitivity, implying that this quantity cannot be currently detected in an unambiguous manner in this astrophysical system.

Starting from our estimate of the mass-to-distance ratio of the BH located at the core of TXS 2226–184 and the distance to this galaxy based on a previous work (Taylor et al. 2002), we obtain \( M = 3.67^{+0.20}_{-0.18} \times 10^6 M_\odot \). Therefore, TXS 2226–184 hosts a BH with a mass of the same order expected for BHs hosted in galaxies associated with megamasers emission. This result allows us to conclude that the high luminosity of the gigamasers is not related to a more massive central BH.

By comparing the results obtained for the mass of the BH hosted in TXS 2226–184 (see Eqs. (12) and (13)), we find that the mass obtained from the \( M_{\text{BH}} = L_{\text{K, bulge}} \) correlation is approximately 1.6 times the mass obtained from the statistical fit using the general relativistic method. Finally, we note that the accuracy of the results differs by an order of magnitude, with the relativistic fit being the most accurate. However, these results are not mutually exclusive due to the uncertainties of the estimate based on the mass–luminosity correlation. Although less accurate, the \( M_{\text{BH}} = L_{\text{bulge}} \) correlation is a good first approximation for systems for which there is no relevant data to make use of the general relativistic model.

Among the possible systematic errors of our modeling, one could consider modifications of the edge-on view and a warped disk, in particular. By performing small variations to the inclination disk parameter \( \theta_0 \) (up to 5°)\(^4\), the \( M/D \) estimation changes on the order of 1%, which is well behind the uncertainty of this ratio. According to Eq. (9), the warping of a disk is correlated with the \( M/D \) parameter. By considering a linear inclination warping along the radius of the disk, and performing variations in the inclination gradient (up to 0.04 rad/mas based on the masers distribution), the \( M/D \) estimation is altered around 5% and the corresponding reduced \( \chi^2_{\text{red}} = 1.43 \).

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Footnote 4: According to Kuo et al. (2011) and Darling (2017), if a thin disk is inclined by more than \( \sim 5° \) from an edge-on view, the masers will be no longer beamed toward us.

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Table 1. Posterior parameters for the BH located at the core of TXS 2226–184.

| TXS 2226–184 | Relativistic estimation |
|--------------|-------------------------|
| \( M/D \) \( (10^4 M_\odot \text{Mpc}^{-1}) \) | 3.54^{+0.20}_{-0.18} |
| \( z_{\text{rec}} \) \( (10^{-2}) \) | 2.4316^{+0.0011}_{-0.0011} |
| \( \theta_{\text{rec}} \) \( (\text{km s}^{-1}) \) | 7289.8^{+1.4}_{-3.3} |
| \( x_0 \) \( (\text{mas}) \) | 0.69^{+0.21}_{-0.22} |
| \( \chi^2_{\text{red}} \) | 1.512 |

Table 2. Gravitational redshift for the masers closest to the central BH.

| Maser | \( z_g \) | Velocity \( (\text{km s}^{-1}) \) |
|-------|----------|------------------|
| Red   | \( 2.74 \times 10^{-7} \) | 8.21 \times 10^{-2} |
| Blue  | \( 1.59 \times 10^{-7} \) | 4.77 \times 10^{-2} |

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Fact that these quantities have a very different nature. Since both of these redshifts do not depend on the position of the masers, they are degenerate and cannot be estimated separately.

If we use the distance to the BH reported in Taylor et al. (2002) \( D = 103.85 \pm 0.26 \text{ Mpc} \), we get the following BH mass estimate:

\[
M = 3.67^{+0.20}_{-0.18} \times 10^6 M_\odot.
\] (12)

We can also use this model to calculate the gravitational redshift of each maser. Below in Table 2 we display the gravitational redshift for the two closest masers to the BH.

6.2. BH mass–luminosity correlation

In Koekemoer et al. (1995), the authors measured the luminosity of the TXS 2226–184 galaxy in the \( K \) band; the adopted
Acknowledgements. The authors are grateful to D.E. Villaraos-Serés for fruitful discussions and to FORDECYT-PRONACES-CONACYT for support under grant No. CF-MG-2558591; U.N. also was supported under grant CF-140630. A.H.-A. and U.N. thank SNI and PROMEP-SEP and were supported by grants VIEP-BUAP No. 122 and CIC-UMSNH, respectively. O.G.-R. and A.V.-R. acknowledge financial assistance from CONACyT through PhD grants No. 885032 and No. 1007718, respectively.

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