Electrically modulated SQUID with single Josephson junction coupled by a time-reversal breaking Weyl semimetal thin film

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I. INTRODUCTION

As the host to Weyl fermions in condensed matter, the Weyl semimetal (WSM) is a topological semimetal where three-dimensional linearly dispersed Weyl cones appear in pairs in momentum space1–6. Two paired Weyl nodes have opposite chiralities and are connected by Fermi arc surface states1,7,8. The essential property of Weyl fermions is the apparent violation of charge conservation known as the chiral anomaly, which leads to the unusual transport properties of WSMs, such as negative magnetoresistance8, chiral magnetic effect9, anomalous hall effect10, and non-local transport11. In these transport signatures, either bulk states or surface states dominate the transport. Nevertheless, unlike the fully gapped topological insulator, the gapless WSM hosts both bulk states and surface states to support the transport, especially in the thin film geometry. The investigation in the quantum interference between the bulk channel and the surface channel in WSMs is very desirable.

On the other hand, since the recent experimental realization of Josephson φ0-junction based on a nanowire quantum dot12, the interest in φ0-junctions has been revived13–22. The so-called φ0-junction, namely, the anomalous Josephson effect23–35, has an unconventional current-phase relation (CPR) \( I(\phi) = I_c \sin(\phi - \phi_0) \), with an arbitrary ground-state phase difference \( \phi_0 \) which is directly proportional to the product of the transverse electric field and the cross section area of the junction. For a suitable Fermi energy, the bulk states make comparable contributions to the Josephson current with the current-phase relation of a 0-junction. The interference between the surface channel and the bulk channel results in an electrically modulated SQUID with single Josephson junction, which provides an experimental proposal to identify magnetic Weyl semimetals and may have potential applications in superconducting quantum computation.

Usually, the superconducting quantum interference device (SQUID) consists of two Josephson junctions and the interference therein is modulated by a magnetic flux. In this work, we propose an electrically modulated SQUID consisting of single Josephson junction coupled by a time-reversal breaking Weyl semimetal thin film. For a low Fermi energy, the Josephson current is only mediated by Fermi arc surface states, and has an arbitrary ground-state phase difference \( \phi_0 \) which is directly proportional to the product of the transverse electric field and the cross section area of the junction. For a suitable Fermi energy, the bulk states make comparable contributions to the Josephson current with the current-phase relation of a 0-junction. The interference between the surface channel and the bulk channel results in an electrically modulated SQUID with single Josephson junction, which provides an experimental proposal to identify magnetic Weyl semimetals and may have potential applications in superconducting quantum computation.
TR breaking WSM is described by a minimal two-node model that the translational symmetry is preserved along the parameters $\Delta = 0$.

The red curve is the $k$ potential ($\mu$ lines contribute to the Josephson current for a fixed chemical potential $\mu_W$) of the WSM thin film sandwiched between two general bulk states. Finally, a brief summary is given in Sec. V.

**II. MODEL AND FORMALISM**

We consider a Josephson junction that consists of a TR breaking WSM thin film sandwiched between two general $s$-wave superconductors. As shown in Fig. 1, the hybrid junction lies along the $x$ direction and has a quantum constriction in the $y$ direction. For simplicity, we assume that the translational symmetry is preserved along the $z$ direction and thus the corresponding wave vector $k_z$ is a good quantum number. In the normal state, the TR breaking WSM is described by a minimal two-node model

$$H_W = \sum_{r,k_z} \left( \begin{array}{cc} h_w(k_z) & 0 \\ 0 & -h_w^*(-k_z) \end{array} \right) \Phi_{r,k_z} \quad (2)$$

where $r = (x,y)$ is the site index, $r_0 = x$ or $y$ represents the unit vector along $x$ or $y$ direction, $\Phi_{r,k_z} = [v^r_{\uparrow,k_z}, v^r_{\downarrow,k_z}, c^r_{\uparrow,-k_z}, c^r_{\downarrow,-k_z}]^T$ is the field operator with $v^r_{\uparrow,k_z}$ the annihilation operator of an electron at site $r$ with spin $\uparrow$ ($\downarrow$) and momentum $\pm k_z$. The components included in the Hamiltonian are

$$h_w(k_z) = (M - 2t \cos k_z) \sigma_z - \mu_W,$$

$$h_\infty = -i \sigma_z - \frac{1}{2} i \lambda \sigma_y, h_y = -i \sigma_z - \frac{1}{2} i \lambda \sigma_y. \quad (3)$$

Moreover, a transverse electric field $E_y$ has also been considered and modelled by linearly increasing on-site energies along the $y$ direction. It can be equivalently modelled by the modification of the chemical potential $\mu_W \rightarrow \mu_W - e E_y y$ with $e$ the unit charge.

For the two superconducting leads, we consider two general $s$-wave superconductors described by

$$H_S = \sum_{\gamma,r,k_z} \left( \begin{array}{cc} h_\gamma(k_z) & \Delta e^{i\varphi_\gamma} i \sigma_y \\ \Delta e^{-i\varphi_\gamma} i \sigma_y & -h_\gamma(k_z) \end{array} \right) \Phi_{\gamma,r,k_z} \quad (4)$$

where $h_\gamma(k_z) = -2t \cos k_z - \mu_S$ with $\mu_S$ being the chemical potential in superconducting leads, the sum over $\gamma$ refers to the left and right superconducting leads which are assumed to have the same nearest-neighbor hopping energy $t$ as that in the WSM, $\Delta$ is the superconducting gap, $\varphi_\gamma = \pm \frac{\pi}{2}$ for the left and right superconductor respectively with $\varphi$ the macroscopic phase difference between two superconducting leads. The coupling between the WSM and two superconducting leads is described by

$$H_C = \sum_{r,k_z} \left( \begin{array}{cc} t & 0 \\ 0 & t \end{array} \right) \Phi_{r+\infty,k_z} + H.c. \quad (5)$$

where the sum over $r$ refers to the left sites at the two interfaces and two interfaces are assumed to be transparent for simplicity. Thus, the whole Josephson junction is described by the Hamiltonian $H = H_W + H_S + H_C$. By using nonequilibrium Green’s functions, the Josephson current through column $l$ in the central WSM region...
for a given $k_z$ is calculated by

$$
I(k_z) = \frac{1}{\hbar} \int_{-\infty}^{\infty} \text{Tr} \left[ \hat{t} \hat{e} \tilde{G}_{L,1}^{\leq}(k_z) - \hat{e} \hat{G}_{L,1}^{\leq}(k_z) \right] dE,
$$

where $\hat{t} = -t_\tau \otimes \sigma_z + \frac{1}{2} i \lambda_{\tau_0} \otimes \sigma_x$ and $\hat{e} = -e \tau_0 \otimes \sigma_y$ denote the hopping matrix and the charge matrix respectively. $\tau_\alpha \otimes \sigma_\beta$ is the Pauli (unit) matrix in Nambu space. In equilibrium, the lesser-than Green’s function is calculated by $G^< = f(E) [G^0 - G^x]$, where $f(E)$ is the Fermi-Dirac distribution function. The retarded and advanced Green’s functions read

$$
G^+(E) = \left[ G^0(E) \right]^\dagger = \frac{1}{E - H_D - \Sigma^r_{L}(E) - \Sigma^r_{R}(E)},
$$

where $H_D$ is the Hamiltonian of the WSM region. The retarded self-energy $\Sigma^r_{L(R)}(E)$ due to coupling with the superconducting leads $L(R)$ can be calculated numerically by the recursive method. Finally, the total Josephson current is given by $J = \frac{\delta c}{2\pi} \int_{-\pi/a}^{\pi/a} I(k_z) dk_z$.

In addition, the Andreev bound state (ABS) spectra can also be numerically calculated through the Green’s function technique. The ABSs result in peaks of particle density within the superconducting gap. By searching the peaks of particle density in column $l$ ($L_x \geq l \geq 1$)

$$
\rho_l = -\frac{1}{\pi} \text{Im} \left[ \text{Tr} \{ G^+ (l, l) \} \right]
$$

at a given phase difference $\varphi$, the energies of ABS levels can be located. Then the ABS spectra can be obtained by scanning $\varphi$, which is helpful for understanding the behavior of Josephson current.

### III. ANOMALOUS JOSEPHSON EFFECT

Next, we present the numerical results for the Josephson current. In our numerical calculations, $t = 1$ is the unit of energy, $\lambda = 2$ and $\Delta = 0.01$. $a = 1$ is the unit of length, $1/a$ is the unit of the wave vector and $k_0 = 0.5\pi$. The geometric parameters of the junction are set to be $L_x = 100$, $L_y = 50$, $L_z = 1000$, and $W = 100$. The unit of transverse electric field $E_y$ is set to be $t/ea$ while a constant chemical potential ($\mu_S = -4.4t$) is used for the two superconductors. The range of $k_z$ of the electronic states in the Fermi surfaces is determined by $\mu_S$, i.e., $|k_z| < 0.43\pi$, is consistent with the $k_z$ range in which the Josephson current is nonzero (as shown in Fig. 1 (c)).

First, we consider the situation where the chemical potential in the WSM is low, for example, $\mu_W = 0.1t$. For such a low $\mu_W$, there exist only Fermi arc states in the range $|k_z| < 0.43\pi$ (see Fig. 1 (b)). For each given $k_z$, the WSM is mapped to a two-dimensional quantum anomalous Hall (QAH) insulator. The QAH edge states are responsible for the so-called Fermi arc surface states. The spin texture of the QAH edge states stemmed from this WSM model (Eq. (1)) is shown to permit the Andreev reflections between the edge states at the upper and the bottom edges. It means that the Fermi arc surface states can form ABSs. As sketched in Fig. 1 (a), in such ABSs, electrons are localized in one surface while holes in the other surface. The separation of electrons and holes in space makes it possible that a transversal electric field $E_y$ endow two paired electrons with different energies. Thus electrons and holes have different wave vectors.

Fig. 2 (a) shows the $E_y$ induced wave vector difference $\delta k_x = k_x^c - k_x^d = -E_y L_y / \lambda$. In the formation of ABS, this difference in wave vector leads to an additional phase accumulation $\delta k_x L_z$ (for the right-going ABS) due to the travelling of electrons and holes. This additional phase should be offset by the phase difference of two superconductors $\varphi$. Therefore, the phase shift, or the ground-state phase difference will be $\varphi_0 = \delta k_x L_z = -E_y L_y / \lambda$ with $S = L_x L_y$, which is consistent with the CPRs shown in Fig. 2 (b) for $\lambda = 2$. The temperature is taken to be $T = 0.5T_c$, which ensures that the first harmonic dom-
(b) clearly shows that the surface supercurrent is peri-
verify the same

The small deviation is due to the slight

modulated SQUID. For a suitably chosen

the bulk channel. These two channels form an electrically

result there are two channels available to carry the Joseph-

pate in the transport in the range
(shown in Fig. 3)

4 (a) shows the total Josephson current as a function of

μW when the phase difference is fixed to φ = π/2. The

Josephson current first decreases gradually with increasing

μW because the supercurrent in the surface channel

is greatly enhanced due to the larger penetration depth

of the surface states. The penetration depth sensitively
determines the coupling of electron and hole, thus the
amplitude of Andreev reflection and Josephson current.
For a higher μW, the bulk channel also participate in the
transport, the Josephson current goes up sharply,
and approaches to 0 nearly at μW = 0.69t. In addition,
the oscillations in the supercurrent come from the multi-
reflection in the normal reflection at interfaces. Fig. 4
(b) shows the k_z resolved supercurrent I(k_z, φ = π/2) as
a function of μW. It is clearly shown that the bulk chan-
nel is open at lower μW for larger k_z, which is consistent
with the energy dispersion of electrons in the WSM.

At μW = 0.69t, the two channels almost contribute the
same amplitude of the supercurrent. Since the surface
channel is a φ_0-junction and the bulk channel remains a
0-junction, the total Josephson current in the first har-
monic approximation is expected to be

\[ J = J_0 \left[ \sin(\varphi - \varphi_0) + \sin \varphi \right] = 2J_0 \cos \left( \frac{\varphi_0}{2} \right) \sin \left( \varphi - \frac{\varphi_0}{2} \right), \]  \tag{9} \]

where \( \varphi_0 = -E_y S/\lambda \). The critical current is defined to
be \( J_c = 2J_0 \left| \cos \frac{\varphi_0}{2} \right| = 2J_0 \left| \cos \frac{E_y S}{2\lambda} \right| \). In particular, the
Josephson current at \( \varphi = \frac{\pi}{2} \) is \( J(\frac{\pi}{2}) = J_0 [1 + \cos \varphi_0] \),
which gives a good fitting of our numerical results shown
in Fig. 5 (a). The small deviation is due to the slight
decrease of bulk supercurrent with increasing \( E_y \).
Fig. 5 (b) clearly shows that the surface supercurrent is peri-
odically modulated by \( E_y \) while the bulk supercurrent is
not sensitive to \( E_y \).

FIG. 3. ABS spectra with fixed \( k_z = 0.42\pi \) and various trans-
verse electric fields (a) \( E_y = 0 \), (b) \( E_y = \pi/S \), (c) \( E_y = 2\pi/S \),
and (d) \( E_y = 3\pi/S \). Other parameters are the same as those in Fig. 2.

FIG. 4. The Josephson current as a function of \( \mu_W \) with fixed
\( E_y = 2\pi/S \). (a) \( J(\varphi = \pi/2) \) versus \( \mu_W \). (b) Contour plot of
\( I(\varphi = \pi/2) \) versus \( \mu_W \) and \( k_z \). The dashed curves are the
energy dispersions of electrons in the WSM as a reference.
Other parameters are the same as those in Fig. 2.
As shown in Eq. 9, the phase shift $\varphi_0$ in the surface supercurrent is directly proportional to the transverse electric field $E_y$ and the cross section area $S = L_x L_y$, which is similar to the situation in the usual magnetically modulated SQUID. Now we comment on the conditions in which this simple relation is valid. First, the surface states should be localized enough to the surfaces. Otherwise, the effect of $E_y$ will be weaker. It means that the $k_z$ range should keep away enough from the Weyl nodes. The key parameter to make this condition satisfied is $\mu_S$ which determines the $k_z$ range. The second condition is $\mu_W \ll 2t(1 - \cos k_0)$ which makes the dispersion $E(k_x)$ of surface states linear and the Fermi velocity remains $\lambda$.

Finally, we comment on the experimental realization of the modulation of the transversal electric field. First, two gate voltages at two surfaces of WSM can induce an exactly transverse electric field. Second, even in the presence of longitudinal component of the electric field, the Josephson current will not change much based on the following considerations. The ABSs formed by Fermi-arc surface states separate electron and hole in space only along the $y$ direction. Therefore only the $y$ component of electric field $E_y$ can endow two paired electrons with different energies, thus endow electron and hole with different wave vectors. It is just this wave vector difference between electron and hole that leads to an anomalous phase shift, and finally results in the oscillation of the critical current from the interference with the bulk ABSs. The numerical results also verify that the other components of electric field do not affect the Josephson current much.

V. CONCLUSION

In conclusion, we propose an electrically modulated SQUID with single Josephson junction coupled by a TR breaking Weyl semimetal thin film. There exist two channels, the surface channel and the bulk channel, to carry the supercurrent. The surface channel serves as a $\varphi_0$-junction where the ground-state phase difference is simply modulated by a transverse electric field as $\varphi_0 = -E_y S/\lambda$. The bulk channel remains always a 0-junction. The quantum interference between the two channels results in an electrically modulated SQUID. This proposed Josephson junction with arbitrarily tunable critical current and ground-state phase difference may have potential applications in the fields of superconducting electronics and superconducting quantum computation.

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