Chi**2 analysis of the minimal supergravity model including WMAP, g(mu)-2 and b→ s gamma constraints

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Abstract: Recent results from the WMAP measurements of the cosmic background radiation yield very tight constraints on the relic density of supersymmetric cold dark matter. We combine the WMAP constraint with those from the anomalous magnetic moment of the muon and the b → sγ branching fraction in a χ^2 determination over the minimal supergravity model (mSUGRA) parameter space. The most favored region of mSUGRA parameter space for almost all tan β values is the hyperbolic branch/focus point (HB/FP) region, with moderate to small values of superpotential Higgs mass |µ| and large GUT scale scalar mass m_0. These favored regions of mSUGRA parameter space can be probed by direct search experiments for supersymmetric dark matter. An exception to the HB/FP region can occur at very large tan β with positive µ values, where wide regions allow resonance annihilation of neutralinos in the early universe.

Keywords: Supersymmetry Phenomenology, Supersymmetric Standard Model, Dark Matter, Rare Decays.
The past decade has witnessed increasingly precise measurements of the anisotropies of the cosmic microwave background radiation left over from the Big Bang\[1\]. The most recent results come from the Wilkinson Microwave Anisotropy Probe (WMAP) satellite measurements. Astonishingly, an analysis of their results pinpoints the age of the universe to be $13.7 \pm 0.2$ Gyr\[2\]. In addition, the geometry of the universe is flat, consistent with simple inflationary models. The dark energy content of the universe is found to be about 73%, while the matter content is about 27%. A best fit of WMAP and other data sets to cosmological parameters in the $\Lambda CDM$ cosmological model yields a determination of baryonic matter density $\Omega_b h^2 = 0.0224 \pm 0.0009$, a total matter density of $\Omega_m h^2 = 0.135^{+0.008}_{-0.009}$, and a very low density of hot dark matter (relic neutrinos). From these values the cold dark matter (CDM) density of $\Omega_{CDM} h^2 = 0.1126^{+0.0161}_{-0.0181}$ (at 2\(\sigma\)) can be inferred. The new WMAP results can thus be used to obtain more severe constraints on particle physics models that include candidates for cold dark matter, such as supersymmetric theories\[3\].

It is well known that the lightest supersymmetric particle (LSP) of many supersymmetric models has the necessary attributes to make up the bulk of cold dark matter in the universe\[4\]. This holds true especially in supergravity models where supersymmetry breaking occurs in a hidden sector of the model\[5\]. SUSY breaking is communicated to the observable sector via gravitational interactions, leading to soft SUSY breaking mass terms which can be of order \(\sim 1\) TeV, so that the hierarchy between the weak scale and any other high scale such as $M_{GUT}$ or $M_{Pl}$ can be stabilized. The simplest of these models assumes a flat Kahler metric and a simple form for the gauge kinetic function at the high scale. Here, motivated by gauge coupling unification at $M_{GUT} \simeq 2 \times 10^{16}$ GeV, we thus assume common scalar masses $m_0$, common gaugino masses $m_{1/2}$, and common trilinear terms $A_0$ all valid at scale $Q = M_{GUT}$. Below $M_{GUT}$, the effective theory is assumed to be the minimal supersymmetric standard model (MSSM). The weak scale sparticle masses and couplings are determined by renormalization group running between $M_{GUT}$ and $M_{\text{weak}}$, which leads to radiative electroweak symmetry breaking (REWSB). The mSUGRA model parameters

\[
m_0, m_{1/2}, A_0, \tan \beta \text{ and } \text{sign}(\mu)
\]

then determine all superparticle and Higgs boson masses and mixings. Here, $\tan \beta$ is as usual the ratio of Higgs field vevs. We use the program ISAJET v7.64p\footnote{Isajet 7.64p is Isajet 7.64 modified to gain access to low $\mu$ sparticle mass solutions.} for our sparticle mass calculations\[\text{[6]}\].

Once the sparticle masses and mixings are determined, a variety of observable quantities can be calculated. In this letter, we focus especially on the neutralino relic density $\Omega_{\tilde{\chi}} h^2$. The relic abundance of neutralinos can be calculated by solving the Boltzmann equation as formulated for a Friedmann-Robertson-Walker universe. Central to this calculation is the evaluation of the neutralino annihilation and co-annihilation cross sections, which must then be convoluted with the thermal distribution of neutralinos (and possibly other co-annihilating particles) present in the early universe. We adopt the calculation of Ref.\[\text{[7]}\], wherein all relevant annihilation and co-annihilation reactions are included along with relativistic thermal averaging\[\text{[8]}\] (see also Ref.\[\text{[9]}\] for a recent relic density calcu-
lation). Four regions of mSUGRA model parameter space emerge where the CDM relic density is consistent with measurements. These include A.) a bulk region at low $m_0$ and low $m_{1/2}$, where neutralino annihilation occurs mainly via $t$-channel slepton exchange, B.) the stau co-annihilation region where $\tilde{Z}_1 - \tilde{\tau}_1$ and $\tilde{\tau}_1 - \tilde{\tau}_1$ annihilations contribute, C.) a region where $2m_{\tilde{Z}} \sim m_{A,H}$, where neutralinos can annihilate via $s$-channel pseudoscalar ($A$) and heavy scalar ($H$) Higgs bosons, and D.) the region at large $m_0$ where $|\mu|$ becomes small (known as the hyperbolic branch/focus point (HB/FP) region), and the growing higgsino component of $\tilde{Z}_1$ allows for efficient neutralino annihilation and co-annihilation.

Another important constraint on the mSUGRA model comes from comparison of the predicted rate for $b \to s\gamma$ decay against experimental measurements. Here, we adopt the branching fraction $BF(b \to s\gamma) = (3.25 \pm 0.54) \times 10^{-4}$ for our analysis, and use the theoretical evaluation given in Ref. $[17, 18]$. Generally, the value of $BF(b \to s\gamma)$ calculated in the mSUGRA model differs most from the SM prediction in the region of low $m_0$ and $m_{1/2}$, where sparticle masses are light, and SUSY loop contributions can be large.

The recently improved measurement of the muon anomalous magnetic moment $a_\mu = (g - 2)_\mu$ also provides an important constraint on supersymmetric models. A recent determination of the deviation between the measured value of $a_\mu$ and the SM prediction has been made by Narison, including additional scalar meson loops. His determination using $e^+e^- \to hadrons$ data to evaluate hadronic vacuum polarization contributions yields $\Delta a_\mu = (24.1 \pm 14.0) \times 10^{-10}$, which we adopt for this analysis. An alternative determination using $\tau$-decay data may include additional systematic uncertainties, and is usually considered less reliable.

Finally, we include in our determination of allowed parameter space direct superparticle and Higgs boson search results from the LEP2 experiments. The most important of these is that $m_{\tilde{W}_1} > 103.5$ GeV on the lightest chargino, and $m_h > 114.1$ GeV when the lightest Higgs boson is SM-like. In regions where $m_A$ is small, this bound may be considerably reduced to $m_h \gtrsim 90$ GeV, depending as well on the value of $\tan\beta$.

In this analysis, we compute a $\chi^2$ value constructed from the mSUGRA model calculated values of $\Omega_{\tilde{Z}} h^2$, $BF(b \to s\gamma)$ and $a_\mu^{SUSY}$, along with the above mentioned central values and error bars. If the relic density $\Omega_{\tilde{Z}} h^2$ falls below the WMAP central value, then we do not include it in our $\chi^2$ determination since other forms of CDM may be present. Thus, at each point in model parameter space, we determine the value of $\chi^2$, and represent the value $\log(\chi^2/3)$ by various colors in the $m_0$ vs. $m_{1/2}$ plane for different values of $\tan\beta$ and $\text{sign}(\mu)$. The green regions generally correspond to a $\chi^2/dof$ value less than 4/3, while yellow regions have $4/3 \lesssim \chi^2/dof \lesssim 25/3$. The yellow regions shade into red for larger $\chi^2/dof$ values. We adopt $A_0 = 0$ throughout our analysis. In general, our conclusions do not change qualitatively upon variation of $A_0$, unless extreme values of the parameter are adopted.

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2 In this paper, we consider mSUGRA solutions with $A_0 = 0$ only. For particular $A_0$ values, the value of $m_{\tilde{t}_1}$ can be dialed to near degeneracy with $m_{\tilde{Z}_1}$, so that a fifth region of stop-neutralino co-annihilation occurs.
In Fig. 1, we show the $m_0$ vs. $m_{1/2}$ plane for $\mu < 0$ and values of $\tan \beta = a.)$ 10, $b.)$ 30, $c.)$ 45 and $d.)$ 52. The gray regions are excluded either by a non-neutralino LSP, or by a breakdown in the REWSB mechanism, signaled by $\mu^2 < 0$ or by $m_A^2 < 0$. The blue shaded region denotes points excluded by the LEP2 bound $m_{\tilde{W}_1} > 103.5$ GeV, and the region below the blue contour is where $m_h < 114.1$ GeV. We see from frame $a.)$ that only tiny green regions occur along the excluded region at low $m_0$ where stau co-annihilation occurs, or at the boundary of “No REWSB”, where $|\mu|$ becomes small, the HB/FP region. The bulk region at low $m_0$ and $m_{1/2}$ is excluded by the LEP2 bound on $m_h$, but also has a large negative value of $a_{\mu}^{SUSY}$ and a large value of $BF(b \to s\gamma)$. As $\tan \beta$ increases to 30, the HB/FP region stands out as the main green region with low $\chi^2/dof$. Increasing $\tan \beta$ to 45 as in frame $c.)$, the HB/FP region remains the most viable, while a yellow corridor of neutralino annihilation via $s$ channel $A$ and $H$ appears as splitting the plot. In addition, a tiny gray region at low $m_0$ and $m_{1/2}$ has emerged, where $m_A^2 < 0$. As $\tan \beta$ increases to 52 as in frame $d.)$, the $m_A^2 < 0$ constraint has begun to overwhelm the plot at low $m_0$, pushing the Higgs annihilation region to larger parameter values, where a few green points emerge. The HB/FP region remains robust. The parameter space becomes completely excluded at $\tan \beta$ values of 55 and higher.

The lesson from the $\mu < 0$ plots is that the most robust region of mSUGRA model parameter space is the HB/FP region, where the neutralino has a significant higgsino component, so that efficient annihilation (and co-annihilation) of neutralinos can occur in the early universe, in spite of quite heavy, multi-TeV values of scalar masses. In fact, these scalar masses are sufficiently heavy to suppress possible SUSY CP and flavor violating processes, while maintaining naturalness[14]. They thus provide at least a partial solution to the SUSY flavor and CP problems.

If in fact the relic cold dark matter is made of HB/FP neutralinos, can these DM particles be detected by direct search experiments? We show the reach of several direct detection experiments (CDMS, CDMS2[25]/CRESST[26] and Genius[27]) for SUSY CDM by the black contours, via the spin-independent neutralino-proton scattering rates as calculated in Ref. [28]. Similar results are given in Feng, Matchev and Wilczek[15], although their HB/FP region occurs at lower values of $m_0$ than ours.\footnote{These contours emerge from digitizing the $\sigma$ vs. $m_{\tilde{Z}}$ reach contours presented by the various experimental groups. We have not scaled the reach contours according to the value of the neutralino relic density.} It is gratifying to note that the most favored regions of parameter space are also accessible to direct search experiments, especially large scale experiments such as Genius and Zeplin 4[30].

In Fig. 2, we show the same mSUGRA model plane plots, except for $\mu > 0$. In this case, the values of $BF(b \to s\gamma)$ and especially $a_{\mu}^{SUSY}$ are more easily accommodated by the data[18]. The frame $a.)$ for $\tan \beta = 10$ is qualitatively similar to the $\mu < 0$ case, with the most favorable region again being the HB/FP region. As $\tan \beta$ increases to 30, the HB/FP region becomes even more promising. In addition, there are some tiny regions along the stau co-annihilation border where a low $\chi^2/dof$ can be found. The HB/FP region remains most promising for $\tan \beta$ values of 45 and 52. As in the $\mu < 0$ case, it should be possible
Figure 1: Plot of $\chi^2$/dof for the mSUGRA model in the $m_0$ vs. $m_{1/2}$ plane for $\mu < 0$, $A_0 = 0$ and $\tan \beta = 10, 30, 45$ and 52.

Finally, we show in Fig. 3 $\mu > 0$ planes for very large values of $\tan \beta = 54, 56, 58$ and 60.\(^5\) In frame a.) for $\tan \beta = 54$, the HB/FP region is even more pronounced than in Fig. 2. Meanwhile, a corridor of $s$-channel Higgs annihilation is opening up at lower values of $m_0$, as shown by the green and yellow region. For $\tan \beta = 56$ in frame b.), the $m_A^2 < 0$ constraint

\(^{5}\)Such large values of $\tan \beta$ fulfill naturalness conditions if one loop corrections are included in evaluating the scalar potential.\(^{[31]}\)
has begun to usurp the HB/FP region. In this case, now a broad region of rapid neutralino annihilation has opened up where $2m_{\tilde{z}} \sim m_{A,H}$. The region intermediate between these which is shaded yellow has $\Omega_{\tilde{z}_1} h^2$ just beyond the WMAP constraint. The low $m_0$ and $m_{1/2}$ region has a somewhat higher $\chi^2$ value due to the value of $BF(b \to s\gamma)$ dropping below $2 \times 10^{-4}$. As $\tan \beta$ increases to 58 in frame c.), a significant region of resonance annihilation is evident. Much of it is accessible to direct dark matter search experiments. Finally, in frame d.), only a fraction of parameter space remains viable, but none of it with a low $\chi^2/dof$ value. The entire $m_0$ vs. $m_{1/2}$ plane is excluded for $\tan \beta \geq 62$. 

Figure 2: Plot of $\chi^2/dof$ for the mSUGRA model in the $m_0$ vs. $m_{1/2}$ plane for $\mu > 0$, $A_0 = 0$ and $\tan \beta = 10, 30, 45$ and 52.
To summarize, we have combined the constraints from WMAP on the neutralino relic density with constraints from $BF(b \rightarrow s\gamma)$ and $(g-2)_\mu$ in a $\chi^2$ analysis which determines favorable and unfavorable regions of mSUGRA model parameter space. We find the bulk neutralino annihilation region (A.) at low $m_0$ and $m_{1/2}$ essentially ruled out by constraints from LEP2, $BF(b \rightarrow s\gamma)$ and $(g-2)_\mu$. In addition, the stau co-annihilation region (B.) has only tiny favorable regions, which would require fine-tuning of parameters to satisfy all constraints. The HB/FP region (D.) emerges as a significant region satisfying all constraints over a wide range of tan $\beta$ values, and also offers at least a partial solution to the SUSY
flavor and $CP$ problems. In addition, the neutralino resonance annihilation regions (C.) for $\mu > 0$ and large $\tan \beta$ can satisfy all constraints. The favorable HB/FP regions are all accessible to direct dark matter search experiments, and much of it should be accessible to TeV scale linear colliders\[32\], since $|\mu|$ is small, and the lightest chargino frequently lighter than $\sim 500$ GeV. The HB/FP region should also be accessible at LHC searches as long as $m_{1/2}$ is not too large, so that gluino pair production occurs at a high enough rate\[33\].

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References

[1] A. T. Lee et al., Astrophys. J. 561, L1 (2001) [arXiv:astro-ph/0104459]; C. B. Netterfield et al. [Boomerang Collaboration], Astrophys. J. 571, 604 (2002) [arXiv:astro-ph/0104460]; C. Pryke, N. W. Halverson, E. M. Leitch, J. Kovac, J. E. Carlstrom, W. L. Holzapfel and M. Dragovan, Astrophys. J. 568, 46 (2002) [arXiv:astro-ph/0104490].

[2] C. L. Bennett et al., arXiv:astro-ph/0302207; D. N. Spergel et al., arXiv:astro-ph/0302209.

[3] J. Ellis, K. A. Olive, Y. Santoso and V. C. Spanos, [hep-ph/0303043]. U. Chattopadhyay, A. Corsetti and P. Nath, [hep-ph/0303201].

[4] For reviews, see e.g. G. Jungman, M. Kamionkowski and K. Griest, [Phys. Rept. 267 (1996) 197]; K. A. Olive, [arXiv:astro-ph/0301505].

[5] A. Chamseddine, R. Arnowitt and P. Nath, [Phys. Rev. Lett. 49 (1982) 976]; R. Barbieri, S. Ferrara and C. Savoy, [Phys. Lett. B 119 (1982) 343]. L. J. Hall, J. Lykken and S. Weinberg, [Phys. Rev. D 27 (1983) 2359]; for a review, see H. P. Nilles, [Phys. Rept. 110 (1984) 1].

[6] H. Baer, F. Paige, S. Protopopescu and X. Tata, [hep-ph/0001088].

[7] H. Baer, C. Balázs and A. Belyaev, [J. High Energy Phys. 0203 (2002) 042] [arXiv:hep-ph/0202076] and arXiv:hep-ph/0211213.

[8] P. Gondolo and G. Gelmini, [Nucl. Phys. B 360 (1991) 145]. J. Edsjö and P. Gondolo, [Phys. Rev. D 56 (1997) 1873].

[9] J. Edsjo, M. Schelke, P. Ullio and P. Gondolo, [JCAP 0304, 001 (2003) [arXiv:hep-ph/0301106]].

[10] J. Ellis, T. Falk and K. Olive, [Phys. Lett. B 444 (1998) 367]; J. Ellis, T. Falk, K. Olive and M. Srednicki, [Astropart. Phys. 13 (2000) 181].

[11] M. Drees and M. M. Nojiri, [Phys. Rev. D 47 (1993) 376]; P. Nath and R. Arnowitt, [Phys. Rev. Lett. 70 (1993) 3696]. H. Baer and M. Brhlik, [Phys. Rev. D 53 (1996) 597] and [Phys. Rev. D 57 (1998) 576]; V. Barger and C. Kao, [Phys. Rev. D 57 (1998) 3131] and [Phys. Lett. B 518 (2001) 117]. A. Lahanas, D. Nanopoulos and V. Spanos, [Phys. Rev. D 62 (2000) 023513] and [Mod. Phys. Lett. A 16 (2001) 1221]. H. Baer, M. Brhlik, M. A. Diaz, J. Ferrandis, P. Mercadante, P. Quintana and X. Tata, [Phys. Rev. D 63 (2001) 015007]. J. R. Ellis, T. Falk, G. Ganis, K. A. Olive and M. Srednicki, [Phys. Lett. B 510 (2001) 236]. L. Roszkowski, R. Ruiz de Austri and T. Nihei, [J. High Energy Phys. 0108 (2001) 024].
A. Djouadi, M. Drees and J. L. Kneur, *J. High Energy Phys.* **0108** (2001) 053; A. Lahanas and V. Spanos, *Eur. Phys. J.* **C 23** (2002) 183.

[12] H. Baer, C. H. Chen, F. Paige and X. Tata, *Phys. Rev. D* **52** (1995) 2746 and *Phys. Rev. D* **53** (1996) 6241.

[13] K. Chan, U. Chattopadhyay and P. Nath, *Phys. Rev. D* **58** (1998) 096004.

[14] J. Feng, K. Matchev and T. Moroi, *Phys. Rev. D* **61** (2000) 075005.

[15] J. Feng, K. Matchev and F. Wilczek, *Phys. Lett. B* **482** (2000) 388 and *Phys. Rev. D* **63** (2001) 045024. The diminished relic density and enhanced neutralino direct detection rates in the HB/FP region can already be seen from plots at large $\tan\beta$ in H. Baer and M. Brhlik, *Phys. Rev. D* **57** (1998) 570.

[16] C. Boehm, A. Djouadi and M. Drees, *Phys. Rev. D* **62** (2000) 035012; J. Ellis, K. Olive and Y. Santoso, *Astropart. Phys.* **18** (2003) 393; G. Belanger, F. Boudjema, A. Pukhov and A. Semenov, *Comput. Phys. Commun.* **149** (2002) 103; see also Ref. [9].

[17] H. Baer and M. Brhlik, *Phys. Rev. D* **55** (1997) 3201; H. Baer, M. Brhlik, D. Castaño and X. Tata, *Phys. Rev. D* **58** (1998) 015007.

[18] H. Baer, C. Balázs, A. Belyaev, J. K. Mizukoshi, X. Tata and Y. Wang, *J. High Energy Phys.* **0207** (2002) 050 and hep-ph/0210441.

[19] G. W. Bennett *et al.* [Muon g-2 Collaboration], Phys. Rev. Lett. **89**, 101804 (2002) [Erratum-ibid. **89**, 129903 (2002)] [arXiv:hep-ex/0208001].

[20] S. Narison, hep-ph/0303004.

[21] H. Baer, C. Balázs, J. Ferrandis and X. Tata, *Phys. Rev. D* **64** (2001) 035004.

[22] Joint HEP2 Supersymmetry Working Group, Combined Chargino Results, up to 208 GeV, http://alephwww.cern.ch/lepsusy/www/inos_moriond01/charginos.pub.html.

[23] LEP Higgs Working Group Collaboration, hep-ex/0107030.

[24] W. de Boer, M. Huber, C. Sander, A. V. Gladyshev and D. I. Kazakov, *Phys. Lett. B* **515** (2001) 283.

[25] D. Abrams *et al.* [CDMS Collaboration], Phys. Rev. D **66**, 122003 (2002) [arXiv:astro-ph/0203500]; G. Eigen, R. Gaitskell, G. D. Kribs and K. T. Matchev, in *Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001)* ed. N. Graf, eConf **C010630**, P342 (2001) [arXiv:hep-ph/0112312].

[26] M. Bravin *et al.* [CRESST-Collaboration], Astropart. Phys. **12**, 107 (1999) [arXiv:hep-ex/9904005].

[27] H. V. Klapdor-Kleingrothaus, Nucl. Phys. Proc. Suppl. **110**, 364 (2002) [arXiv:hep-ph/0206249].

[28] H. Baer, C. Balázs, A. Belyaev and J. O’Farrill, to appear.

[29] B. C. Allanach, S. Kraml and W. Porod, *J. High Energy Phys.* **0303** (2003) 016.

[30] D. B. Cline, H. g. Wang and Y. Seo, in *Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001)* ed. N. Graf, eConf **C010630**, E108 (2001) [arXiv:astro-ph/0108147].
[31] H. Baer, J. Ferrandis and X. Tata, arXiv:hep-ph/0211418; see also A. Nelson and L. Randall, 
*Phys. Lett.* B **316** (1993) 516.

[32] H. Baer, R. Munroe and X. Tata, *Phys. Rev.* D **54** (1996) 6735; for a review, see T. Abe et al. (American Linear Collider Working Group Collaboration), hep-ex/0106056; J. A. Aguilar-Saavedra et al. (ECFA/DESY LC Physics Working Group Collaboration), hep-ph/0106315.

[33] See Ref. [12]; H. Baer, C. H. Chen, M. Drees, F. Paige and X. Tata, *Phys. Rev.* D **59** (1998) 055014; S. Abdullin et al. (CMS Collaboration), hep-ph/9806366; H. Baer, C. Balázs, A. Belyaev, T. Krupovnickas and X. Tata, FSU-HEP/030416 (2003).