A Three Higgs Doublet Model for the Fermion Mass Hierarchy Problem

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Abstract

In this paper we propose an explanation to the Fermion mass hierarchy problem by fitting the type-II seesaw mechanism into the Higgs doublet sector, such that their vacuum expectation values are hierarchal. We extend the Standard Model with two extra Higgs doublets as well as a spontaneously broken $U_X(1)$ gauge symmetry. All fermion Yukawa couplings except that of top quark are of $\mathcal{O}(10^{-2})$ in our model. Constraints on the parameter space from Electroweak precision measurements are studied. Besides, the neutral component of the new fields, which are introduced to cancel the anomalies of the $U(1)_X$ gauge symmetry can be dark matter candidate. We investigate its signature in the dark matter direct detection.

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I. INTRODUCTION

In the Standard Model (SM) of particle interactions, charged fermions get masses through the spontaneously broken of the electroweak symmetry and the Higgs mechanism, while neutrinos are massless. At $M_Z$, the charged lepton masses and the current masses of quarks are given by

$$
\begin{align*}
  m_e & \sim 0.51 \text{ MeV} & m_\mu & \sim 0.105 \text{ GeV} & m_\tau & \sim 1.7 \text{ GeV} \\
  m_u & \sim 1 \text{ MeV} & m_c & \sim 1.3 \text{ GeV} & m_t & \sim 174 \text{ GeV} \\
  m_d & \sim 5 \text{ MeV} & m_s & \sim 0.13 \text{ GeV} & m_b & \sim 4 \text{ GeV},
\end{align*}
$$

which shows an enormous hierarchy among the Yukawa couplings $y_\psi$. For example, we have

$$
y_u/y_t \sim 10^{-5}
$$

for the quark sector.

For the neutrino sector, recent results from solar, atmosphere, accelerator and reactor neutrino oscillation experiments show that neutrinos have small but non-zero masses at the sub-eV scale and different lepton flavors are mixed. If neutrinos are Dirac particles, their masses may come from the Higgs mechanism, then we have $y_\nu/y_t \sim 10^{-12}$, which seems even unnatural. For the case neutrinos being Majorana particles, the most popular way to explain neutrino masses are the seesaw mechanism. If we assume the Yukawa couplings between left-handed lepton doublet and right-handed neutrinos are of order 1, then we have $m_t/m_N \sim 10^{-12}$, which is also unnatural.

In this paper, we attempt to solve or explain the charged fermion and neutrino mass hierarchy problem in the three Higgs doublet model. There are already many excellent literatures focusing on this issue. In our model, one Higgs doublet get its vacuum expectation value (VEV) in the same way as that of the SM Higgs boson, while the other two Higgs fields get their VEVs through the mechanism similar to type-II seesaw model, i.e., they get their VEVs through their mixings with the SM Higgs. Such that the VEVs can be normal hierarchal, which is guaranteed by the spontaneously broken $U(1)$ gauge symmetry. We set them to be $v_1 = 100 \text{ MeV}$, $v_2 = 10 \text{ GeV}$ and $v_3 = 173 \text{ GeV}$ in our paper. For each generation of charged fermions, there is one Higgs field responsible the origin of their masses. For the neutrino sector, there are only Yukawa couplings with the

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1 For similar ideas on the VEVs of Higgs doublet, see the private Higgs model, the two Higgs doublet model with softly breaking $U(1)$ symmetry and for neutrino masses.
first generation Higgs field. Such that Dirac neutrino mass matrix is naturally small without requiring small Yukawa coupling constants. Then active neutrinos may get small but non-zero masses through the TeV-scale seesaw mechanism. We introduce some new fields to cancel anomalies of the $U(1)_X$ gauge symmetry, and the neutral component of them can be cold dark matter candidate. We will study its signatures in dark matter direct detection experiments.

The note is organized as follows: In section II we give a brief introduction to the model, including particle contents, Higgs potential and scalar mass spectrum. Section III is devoted to study the fermion masses. We investigate constraints on the model from Electroweak precision measurements and dark matter phenomenology in section IV and V. The last part is concluding and remarks.

II. THE MODEL

| Fields | $q_L^0$ | $q_L^{d}$ | $q_L^{u}$ | $u_R$ | $e_R$ | $e_R$ | $t_R$ | $s_R$ | $b_R$ | $c_R$ | $\ell_R$ | $\nu_{R i}$ | $\psi_L^i$ | $\eta^k_L$ | $\eta^k_R$ | $\xi_k^L$ | $\xi_k^R$ | $H_1$ | $H_2$ | $H_3$ | $\Phi$ |
|--------|--------|----------|----------|------|------|------|------|------|------|------|--------|---------|----------|----------|----------|----------|--------|--------|--------|--------|
| $U_X(1)$ | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | 1 | 1 | -1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |

TABLE I: Particle contents and their quantum numbers under $U_X(1)$ gauge symmetry. $i = 1, 2, 3$ and $k = 1, \cdots, 6$. $q_L^u = (u_L, d_L)^T$, $q_L^d = (c_L, s_L)^T$, $q_L^c = (t_L, b_L)^T$, $\ell_L$ denotes left-handed lepton doublets.

We extend the SM with three right-handed neutrinos, two extra Higgs doublet, one Higgs singlet as well as a flavor dependent $U(1)_X$ gauge symmetry. Six generation fermion singlets $\eta(\xi)$ with $U(1)_X$ hypecharge $(-)1$ as well as three generation fermion singlets $\psi_L$ with $U(1)_X$ hypecharge 0 are introduced to cancel the anomalies. The particle contents and their representation under the $U(1)$ gauge symmetry are listed in table II. We apply the type-II seesaw mechanism to the Higgs doublet sector. The most general Higgs potential can be written as

$$
\mathcal{L}_{\text{Higgs}} = +m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 - m_3^2 H_3^\dagger H_3 - m_0^2 \Phi^\dagger \Phi + \lambda_0 (\Phi^\dagger \Phi)^2 + \lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2 + \lambda_3 (H_3^\dagger H_3)^2 + \lambda_4 (H_1^\dagger H_1) (H_2^\dagger H_2) + \lambda_5 (H_1^\dagger H_1) (H_3^\dagger H_3) + \lambda_6 (H_2^\dagger H_2) (H_3^\dagger H_3) + \lambda_7 (H_1^\dagger H_2) (H_1^\dagger H_1) + \lambda_8 (H_1^\dagger H_3) (H_3^\dagger H_1) + \lambda_9 (H_2^\dagger H_3) (H_3^\dagger H_2) + \lambda_{10} (\Phi^\dagger \Phi) (H_1^\dagger H_1) + \lambda_{11} (\Phi^\dagger \Phi) (H_2^\dagger H_2) + \lambda_{12} \Phi^\dagger \Phi H_3^\dagger H_3
$$
It is obviously that $H_1$ and $H_2$ shall develop no VEVs without terms in the bracket of Eq. 2.
The conditions for $L_{\text{Higgs}}$ develops minimum involve four constraint equations. By assuming
\[ \langle H \rangle = v_1/\sqrt{2}, \langle \eta \rangle = v_2/\sqrt{2}, \langle \varphi \rangle = v_3/\sqrt{2} \text{ and } \langle \Phi \rangle = v_4/\sqrt{2}, \]
we have
\begin{align*}
+ & m_1^2 v_1 + \lambda_1 v_1^2 + \frac{1}{2} v_1 [ (\lambda_1 + \lambda_7) v_2^2 + (\lambda_5 + \lambda_8) v_3^2 + \lambda_10 v_4^2 ] + \frac{1}{2} \lambda_13 v_2 v_3^2 + \mu_1 v_3 v_4 = 0, \\
+ & m_2^2 v_2 + \lambda_2 v_2^2 + \frac{1}{2} v_2 [ (\lambda_1 + \lambda_7) v_3^2 + (\lambda_6 + \lambda_9) v_3^2 + \lambda_11 v_4^2 ] + \frac{1}{2} \lambda_13 v_2 v_3^2 + \mu_2 v_3 v_4 = 0, \\
- & m_3^2 v_3 + \lambda_3 v_3^2 + \frac{1}{2} v_3 [ (\lambda_5 + \lambda_8) v_2^2 + (\lambda_6 + \lambda_9) v_2^2 + \lambda_12 v_4^2 ] + \lambda_13 v_1 v_2 v_3 + \mu_1 v_1 v_4 + \mu_2 v_2 v_4 = 0, \\
- & m_0^2 v_4 + \lambda_0 v_4^2 + \frac{1}{2} v_4 [ \lambda_{10} v_1^2 + \lambda_{11} v_2^2 + \lambda_{12} v_3^2 ] + \mu_1 v_3 + \mu_2 v_2 v_3 = 0.
\end{align*}

Let $m_i^2, \lambda_i > 0, \lambda_{13} = 0$ (for simplificity) and $|\mu_i| \ll m_i$, then we have
\[ v_1 \approx \frac{\mu_1 v_3 v_4}{m_1^2}, \quad v_2 \approx \frac{\mu_2 v_3 v_4}{m_2^2}, \quad v_3 \approx \frac{m_3^2}{\lambda_3}, \quad v_4 \approx \frac{m_0^2}{\lambda_0}. \]

Notice that $v_1$ and $v_2$ are suppressed by their masses, which is quite similar to that in the
type-II seesaw mechanism. So we can get relatively small $v_1$ and $v_2$ without conflicting with
any electroweak precision measurements. By setting $m_1 \sim 10 m_2$ and $\mu_1 \sim \mu_2$ we get the
normal hierarchal VEVs for the Higgs sector. We set $O(v_1) \sim 0.1 \text{ GeV}, O(v_2) \sim 1 \text{ GeV}
and $O(v_3) \sim 100 \text{ GeV}$ in our following calculation. In this way the fermion mass hierarchy
problem will be fixed, as will be shown in the next section.

After all the symmetries are broken, there are four goldstone particles eaten by $W^\pm, Z
and Z'$. The mass matrix for the CP-even Higgs bosons can be written as
\[ M_{\text{even}}^2 \approx \begin{pmatrix} m_1^2 + v_1^2 \lambda_1 & \frac{1}{2} v_1 v_2 (\lambda_1 + \lambda_7) & \frac{1}{2} v_1 v_3 (\lambda_5 + \lambda_8) - \mu_1 v_4 & \frac{1}{2} v_1 v_4 \lambda_{10} - v_3 \mu_1 \\ \frac{1}{2} v_2 v_3 (\lambda_6 + \lambda_9) - \mu_2 v_4 & m_2^2 + v_2^2 \lambda_2 & \frac{1}{2} v_2 v_4 (\lambda_6 + \lambda_9) - v_3 \mu_2 \\ v_3^2 \lambda_3 & \frac{1}{2} v_3 v_4 \lambda_{12} - v_2 \mu_2 & v_4^2 \lambda_4 \end{pmatrix}. \]

It can be block diagonalized and the mapping matrix can be written as
\[ V \approx \begin{pmatrix} \Psi_1 & 0 \\ -T^T \mathcal{Z}^{-1} & \Psi_2 \end{pmatrix}, \]
where $\Psi_i$ is the $2 \times 2$ unitary matrix and the expressions of $\mathcal{T}$ and $\mathcal{Z}$ are listed in the
appendix. The corresponding mass eigenvalues are then
\[ M_1^2 \approx c^2 (m_1^2 + v_1^2 \lambda_1) + s^2 (m_2^2 + v_2^2 \lambda_2) + c s v_1 v_2 (\lambda_4 + \lambda_7), \]
\[ M_2^2 \approx s^2(m_1^2 + v_1^2 \lambda_1) + c^2(m_2^2 + v_2^2 \lambda_2) - c s v_1 v_2 (\lambda_4 + \lambda_7), \]
\[ M_3^2 \approx c^2(v_3^2 \lambda_3 - v_4^2 \alpha) + s^2(v_4^2 \lambda_4 - v_3^2 \alpha) - c' s' v_3 v_4 (\lambda_{12} - 2 \alpha), \]
\[ M_4^2 \approx s^2(v_3^2 \lambda_3 - v_4^2 \alpha) + c^2(v_4^2 \lambda_4 - v_3^2 \alpha) + c' s' v_3 v_4 (\lambda_{12} - 2 \alpha), \]
where \( \alpha = \mu^2 m_1^{-2} + \mu_2 m_2^{-2} \), \( c(\alpha), s(\alpha) = \cos \theta(\alpha), \sin \theta(\alpha) \) with
\[ \theta = \arctan \frac{v_1 v_2 (\lambda_4 + \lambda_7)}{m_2^2 + v_2^2 \lambda_2 - m_1^2 - v_1^2 \lambda_1}, \quad \theta' = \arctan \frac{v_3 v_4 (\lambda_{12} - 2 \alpha)}{v_3^2 \lambda_4 - v_4^2 \lambda_3 + \alpha(v_4^2 - v_3^2)}. \]
The mass matrix for the CP-odd Higgs fields is
\[ M_{odd}^2 \approx \begin{pmatrix} m_1^2 & 0 & -v_4 \mu_1 & -v_3 \mu_1 \\ * & m_2^2 & -v_4 \mu_2 & -v_3 \mu_2 \\ * & * & \mu_1 v_1 v_3^{-1} v_4 + \mu_2 v_2 v_3^{-1} v_4 & -v_1 \mu_1 + v_2 \mu_2 \\ * & * & * & \mu_1 v_1 v_3^{-1} v_4 + \mu_2 v_2 v_3^{-1} v_4 \end{pmatrix}, \]
which has two non-zero eigenvalues
\[ M^2 = \frac{1}{2v_1 v_2 v_3 v_4} \left( v_2 \mu_1 [v_3^2 v_4^2 + v_1^2 (v_3^2 + v_4^2)] + v_1 \mu_2 [v_3^2 v_4^2 + v_2^2 (v_3^2 + v_4^2)] \right) \pm \sqrt{Q - P}, \]
where
\[ P = 4 \frac{\mu_1 \mu_2}{v_1 v_2} \prod_{i=1}^{4} v_i \left[ v_3^2 v_4^2 + v_2^2 (v_3^2 + v_4^2) + v_1^2 (4v_2^2 + v_3^2 + v_4^2) \right], \]
\[ Q = \left\{ v_2 [v_3^2 v_4^2 + v_1^2 (v_3^2 + v_4^2)] \mu_1 + v_1 [v_3^2 v_4^2 + v_2^2 (v_3^2 + v_4^2)] \mu_2 \right\}^2. \]
The other two are Goldstone bosons eaten by \( Z \) and \( Z' \), separately.

Let’s give some comments on the \( Z - Z' \) mixing. Phenomenological constraints typically require the mixing angle to be less than \((1 \sim 2) \times 10^{-3}\) \[26\] and the mass of extra neutral gauge boson to be heavier than 860 GeV \[27\]. The multi-Higgs contributions to \( Z - Z' \) mixing from both tree-level and one-loop level corrections are studied in Ref \[28\]. A suitable mass hierarchy and mixing between \( Z \) and \( Z' \) are maintained by setting \( v_1, v_2 < 10 \) GeV, \( v_4 \sim 1 \) TeV and \( g \sim g_X \).

### III. FERMION MASSES

Due to the flavor-dependent \( U(1)_X \) symmetry, the Yukawa interaction of our model can be written as
\[ -L_{\text{Yukawa}} = +q_L Y_{wu} \tilde{H}_1 u_R + q_L Y_{cc} \tilde{H}_2 c_R + q_L Y_{tt} \tilde{H}_3 t_R + q_L Y_{ut} \tilde{H}_2 t_R + q_L Y_{ct} \tilde{H}_1 t_R \]
The charged lepton mass matrix is quite similar to that in the car$\overline{\text{e}}$t section. Constraint on the Yukawa couplings from electroweak precision measurements, which will be Eq. 14, Yukawa coupling constant can be nearly at the same order. But we need to study $i$ constants, except that of top quark, are of order scale without requiring relatively small $\eta$ and $\xi$ get masses after the $U(1)_X$ symmetry spontaneously broken. Besides they mix with the charged leptons through the Yukawa interactions. To be consistent with the EW precision measurements, we assume the mixing is relatively small. $\psi_L$ may get the mass in.

$$
\begin{align*}
+q_L^d Y_{d\alpha}^d H_1 D_{R\alpha} + q_L^e Y_{e\alpha}^d H_2 D_{R\alpha} + q_L^d Y_{d\alpha}^d H_3 D_{R\alpha} \\
+\xi e Y_{\alpha}^e H_1 e_R + c_L^e Y_{\alpha}^e H_2 e_R + c_L^e Y_{\alpha}^e H_3 e_R + c_L Y_{\alpha\beta} H \nu R \beta \\
+\eta L Y_{ij} \Phi_1 + \xi L Y_{ij} \Phi_2 + c_L Y_{\alpha}^e H_3 \eta_R k + c_L Y_{\alpha}^e H_3 \eta R_k + h.c.
\end{align*}
$$

After $U(1)_X$ and electroweak symmetry spontaneously broken, we may get the mass matrix for the upper quarks and down quarks:

$$
M_u = \begin{pmatrix}
Y_{11}^u v_1 & 0 & Y_{13}^u v_2 \\
0 & Y_{22}^u v_2 & Y_{23}^u v_1 \\
0 & 0 & Y_{33}^u v_3
\end{pmatrix}, \quad
M_d = \begin{pmatrix}
Y_{11}^d v_1 & Y_{12}^d v_1 & Y_{13}^d v_1 \\
Y_{21}^d v_2 & Y_{22}^d v_2 & Y_{23}^d v_2 \\
Y_{31}^d v_3 & Y_{32}^d v_3 & Y_{33}^d v_3
\end{pmatrix}.
$$

As we showed in the last section, $v_i$ is hierarchal and we set $v_1 = 0.1 \text{ GeV}, v_2 = 10 \text{ GeV} \ and \ v_3 = 173 \text{ GeV}$ in our calculation. For simplification we may also set $M_u, M_d$ to be nearly diagonal matrices using discrete flavor symmetry, such as $Z_2^3$. Then $v_i$ is only responsible for the origin of the $i$th generation quark masses. In that case all the Yukawa coupling constants, except that of top quark, are of $\mathcal{O}(10^{-2})$. Even for the most general case of Eq. 14, Yukawa coupling constant can be nearly at the same order. But we need to study constraint on the Yukawa couplings from electroweak precision measurements, which will be carried out in the next section.

The most general charged lepton mass matrix and Dirac neutrino mass matrix are

$$
M_e = \begin{pmatrix}
Y_{e1}^e v_1 & Y_{e2}^e v_1 & Y_{e3}^e v_1 \\
Y_{e2}^e v_2 & Y_{e2}^e v_2 & Y_{e3}^e v_2 \\
Y_{e3}^e v_3 & Y_{e3}^e v_3 & Y_{e3}^e v_3
\end{pmatrix}, \quad
M_D = v_1 \begin{pmatrix}
Y_{11}^\nu & Y_{12}^\nu & Y_{13}^\nu \\
Y_{21}^\nu & Y_{22}^\nu & Y_{23}^\nu \\
Y_{31}^\nu & Y_{32}^\nu & Y_{33}^\nu
\end{pmatrix}.
$$

The charged lepton mass matrix is quite similar to that in the $A_4$ model [28, 29]. We set it to be diagonal using $Z_2 \times Z_2 \times Z_2$ flavor symmetry, which is explicitly broken by neutrino Yukawa interactions. In this case $Y_{e1}^e$ is of order $\mathcal{O}(10^{-2})$. The Dirac neutrino mass matrix is proportional to $v_1$, thus it can be at the $M_e \text{V}$ scale without requiring relatively small neutrino Yukawa couplings. The right handed neutrino masses may come from the effective operator $\alpha \tilde{\Lambda}^{-1} \Phi^2 \nu_R \nu_R + h.c.$ Integrating out heavy neutrinos, we derive the mass matrix of active neutrinos: $M_\nu = v_2^2 Y^\nu M_R^{-1} Y^{\nu T}$. Setting $\mathcal{O}(Y^{\nu}) \sim 10^{-2}$ and $M_R \sim 100 \text{ GeV}$, we derive electron-volt scale active neutrino masses.
the same way as that of right-handed neutrinos. It can be stable particle with the help of \(Z_2\) flavor symmetry, thus it can be dark matter candidate. It’s phenomenology will be studied in section V.

IV. CONSTRAINTS

There are two major constraints on any extension of the Higgs sector of the SM.: the \(\rho\) parameter and the flavor changing neutral currents(FCNC). Notice that in a model with only Higgs doublet, the tree level of \(\rho = 1\) is automatic without adjustment to any parameters in the model. For our model \(\rho\) is maintained as the constraint on the \(Z - Z'\) mixing is fulfilled. Our model doesn’t obey the the theorem called Natural Flavor Conservation by Glashow and Weinberg, such that there are tree level FCNC’s mediated by the Higgs boson. In the basis where \(M_u\) is diagonalized, \(M_D\) can be written as

\[
M_d = U_{CKM} \cdot \hat{D} \cdot U_R^\dagger \Rightarrow Y_D = \begin{pmatrix}
  v_1^{-1} & 0 & 0 \\
  0 & v_2^{-1} & 0 \\
  0 & 0 & v_3^{-1}
\end{pmatrix} U_{CKM} \hat{D} U_R^\dagger, \tag{17}
\]

where \(\hat{D} = \text{diag}\{m_d, m_s, m_b\}\). and \(U_{CKM}\) is the CKM matrix. Then the flavor changing neutral current can be written as

\[
(q^u_L \ q^c_L \ q^t_L) \bar{U}_{CKM} \text{Diag}\{v_1^{-1}H_1, v_2^{-1}H_2, v_3^{-1}H_3\} U_{CKM} \hat{M}_D \begin{pmatrix}
  d_R \\
  s_R \\
  b_R
\end{pmatrix} + \text{h.c.} \tag{18}
\]

In this section, we consider various processes where FCNC may contribute significantly. Taking into account the experimental results of these processes, we may constrain the parameter spaces of the model.

A. \(K - \bar{K}\) mixing

There are two well measured quantities related to \(K - \bar{K}\) mixing: the mass difference and the CP violating observable. In this paper, we only focus on the contribution to the mass difference \(\Delta M_K\), which get its main contribution from the tree level exchange of \(h_i^0\) (We assume CP-odd Higgs bosons being much heavier than CP-even ones, which dominate
the contributions to the $K - \bar{K}$ mixing). The relevant vertices can be read from Eq. 18:

$$
\begin{align*}
\bar{d}_L s_R h_0^0 & \quad m_s v_i^{-1} U_{i1}^* U_{i2}^* , \\
\bar{s}_L d_R h_0^0 & \quad m_d v_i^{-1} U_{i2}^* U_{i1}^* ,
\end{align*}
$$

Thus the mass difference can be derived through the mass insertion method:

$$
\Delta M^{S}_{12} = \sum_i^{f_K} \frac{f_K^2 m_K^2}{2 M_i^2} \left\{ A_i^2 \left[ -1 + \frac{11 m_K^2}{(m_s + m_d^2)} \right] + B_i^2 \left[ 1 - \frac{m_K^2}{(m_s + m_d^2)} \right] \right\},
$$

where

$$
A_i = \frac{1}{2} (m_s - m_d) v_i^{-1} U_{i2}^* U_{i1}^* ,
$$

$$
B_i = \frac{1}{2} (m_s + m_d) v_i^{-1} U_{i2}^* U_{i1}^* .
$$

Using $f_K = 114$ MeV, $m_K = 497.6$ MeV and values of CKM matrix listed in PDG, We plot in the left panel of the Fig. 1 $\Delta M_K$ as the function of $m_2$, the mass of the neutral component of the second Higgs doublet $H_2$. In plotting the figure we set $v_1 = 0.1$ GeV, $v_2 = 10$ GeV, $v_3 = 173$ GeV as well as $m_1 = 20 m_2$, which is natural because $v_i (i = 1, 2)$ is inverse proportional to the $m_i^2$. The horizontal line in the figure represents the experimental value. To fulfill the experimental constraint, $m_2$ should be no smaller than $8.66$ TeV in our model. This value might be accessible at the future LHC.

FIG. 1: $\Delta M_K$ (the left panel of the figure) and $\Delta M_D$ (the right panel of the figure) as the function of $m_2$ the mass eigenvalue of the $h_0^0$.
B. $D - \bar{D}$ mixing

The $D - \bar{D}$ mixing in our model is a little different form that of $K - \bar{K}$ mixing. The contributions to the $D - \bar{D}$ mixing come from box diagrams, which include the SM $W$ boson diagram, the two Higgs diagrams and the mixed diagrams. We assume the two Higgs diagrams dominant the contribution. The following are relevant vertices:

\[
\begin{align*}
&c_L d_R h_i^+: m_d v_i^{-1} U_{i2}^* U_{i1}, \\
&c_L s_R h_i^+: m_s v_i^{-1} U_{i2}^* U_{i2}, \\
&c_L b_R h_i^+: m_b v_i^{-1} U_{i3}^* U_{i3}, \\
&\bar{u}_L d_R h_i^+: m_d v_i^{-1} U_{i1}^* U_{i1}, \\
&\bar{u}_L s_R h_i^+: m_s v_i^{-1} U_{i1}^* U_{i2}, \\
&\bar{u}_L b_R h_i^+: m_b v_i^{-1} U_{i1}^* U_{i3}.
\end{align*}
\]

Then we have

\[
M_{12}^D = \frac{1}{384\pi^2} \Lambda^2 f_D^2 m_D \sum_m \sum_n y_m y_n \sum_{ij} \mathcal{Y}_{im}^i \mathcal{Y}_{cm}^j \mathcal{Y}_{un}^i \mathcal{Y}_{cn}^j I(y_m, y_n, y_i, y_j),
\]

where $y_\alpha, y_\beta = m_\alpha, m_\beta / \Lambda^2$ and $\mathcal{Y}_{mn}^i = v_i^{-1} U_{im}^* U_{in}$. The explicit expression of integration $I(a, b, c, d)$ can be found in Ref. [18].

Using $f_D = 170$ MeV and $M_D = 1864$ MeV, we plotting in the right panel of Fig. 1 $\Delta M_D$ as a function of $m_2$. Our parameter settings are the same as that of the $K - \bar{K}$ mixing. the horizontal line in the figure represent the experimental value. We can read from the figure that the data of $D - \bar{D}$ mixing constraints the mass of $h_2^+$ to be no smaller than 4.2 TeV.

C. $B - \bar{B}$ mixing

The mass difference in the neutral B meson system has been well measured by the D0 Collaboration and the CDF Collaboration at the Fermilab Tevatron. Similar to that of $K - \bar{K}$ mixing, there are also tree-level contributions to the $\Delta M_{B_a}$. The following are relevant vertices that might lead to $B_{\alpha} - \bar{B}_{\alpha}$ mixing:

\[
\begin{align*}
&d_L b_R h_0^i m_b v_i^{-1} U_{i2}^* U_{i3}, \\
&\bar{d}_L d_R h_0^i m_d v_i^{-1} U_{i3}^* U_{i1}, \\
&\bar{b}_L b_R h_0^i m_b v_i^{-1} U_{i3}^* U_{i2}, \\
&\bar{b}_L s_R h_0^i m_s v_i^{-1} U_{i2}^* U_{i2}.
\end{align*}
\]

Direct calculation gives

\[
\Delta M_{12}^{B_a} = \sum_i f_i^2 m_{B_a} \left\{ C_{\alpha i} \left[-1 + \frac{11 m_K^2}{(m_s + m_d)^2} \right] + D_{\alpha i} \left[1 - \frac{m_K^2}{(m_s + m_d)^2} \right] \right\},
\]

where

\[
C_{\alpha i} = \frac{1}{2} (m_b - m_\alpha) v_i^{-1} U_{i3}^* U_{j\alpha},
\]

\[
D_{\alpha i} = \frac{1}{2} (m_b + m_\alpha) v_i^{-1} U_{i3}^* U_{j\alpha}.
\]
FIG. 2: $\Delta M_{BS}$ (the left panel of the figure) and $\Delta M_{BD}$ as the function of $m_2$ the mass eigenvalue of the $\phi^0_2$.

and $m_{B_s} = 5367.5$ MeV, $m_{B_0} = 5279.4$ MeV. Using the same input as that of the $K - \bar{K}$ mixing case, we plot in the left panel of Fig. 2 $\Delta M_{B_0}$ and in the right panel $\Delta M_{B_s}$ as the function of $m_2$, where the horizontal lines in both cases represent the corresponding experimental data. Our results show that $\Delta M_{B_0}$ is not so sensitive to $m_2$, which is because $H_2s$' contribution is heavily suppressed by the CKM. Our numerical results shows that $m_2$ should be no smaller than 0.8 TeV.

D. $\mu \rightarrow e\gamma$

Now we come the lepton sector and discuss constraint on the model from lepton flavor violating decays. Among the current available experimental data, $\mu \rightarrow e\gamma$ gives the strongest constraint. We assume the Yukawa matrix for the charged leptons is diagonal such that the only relevant Yukawa interactions are $\ell_L Y^\nu \tilde{H}_1 N_R + \text{h.c.}$. Their contribution to the $\mu \rightarrow e\gamma$ can be written as

$$BR(\mu \rightarrow e + \gamma) = \frac{3e^2}{64\pi^2 G_F} |F|^2 \left(1 - \frac{m_e^2}{m_\mu^2}\right)^3,$$

with

$$F = \frac{Y^\nu_{\mu i} Y^\nu_{\mu i}}{12(m_1^2 - m_{N_i}^2)} \left\{-2 + \frac{9m_1^2}{m_1^2 - m_{N_i}^2} - 6 \left(m_1^2 - m_{N_i}^2\right)^2 + \frac{6m_1^4 m_{N_i}^2}{(m_1^2 - m_{N_i}^2)^3} \ln \left(\frac{m_1^2}{m_{N_i}^2}\right)\right\}.$$
where $m'_1$ is the mass eigenvalue of $h_1^+$ and $m_{N_i}$ is the mass eigenvalues of right handed neutrinos. In deriving the upper results we have assumed $m_{N_i} < m'_1$.

The current experimental upper bounds for the $BR(\mu \to e\gamma)$ is $1.2 \times 10^{-11}$. By assuming $m'_1 \sim 4.5$ TeV and $m_{N_i} \sim 500$ GeV, we can get the upper bound for the $Y_{ei}Y_{\mu i}^*$ which is about of order 1, i.e., there are no severe constraint on the neutrino Yukawa couplings from lepton flavor violations.

V. DARK MATTER

In our model the neutral fermions $\psi_L$ (introduced to cancel the anomalies of $N_{R_i}$) is stable and thus can be dark matter candidate. Its relic density can be written as

$$\Omega h^2 \simeq \frac{1.07 \times 10^9 \text{ GeV}^{-1}}{M_{Pl}} \frac{x_f}{\sqrt{g_*}} \left( \frac{19M_{\chi}^2g_{\chi}^4}{4\pi \left[ (4M_{\chi}^2 - M_{Z'}^2)^2 + M_{Z'}^2\Gamma_{Z'}^2 \right]} \right)^{-1} x_f$$

(27)

where $h$ is the Hubble constant in units of $100$ km/s·Mpc, $M_{Pl} = 1.22 \times 10^{19}$ GeV is the Planck mass, $g_*$ accounts the number of relativistic degrees of freedom at the freeze-out temperature and $M_{Z'}$ is the mass of $Z'$ with $\Gamma_{Z'}$ its decay width. We set $x_f$ equals to 20 in our calculation, a typical value at the freeze-out for weakly interacting particles.

The elastic scattering cross section of Dark matter off the nucleon can be written as

$$\sigma_n^{SD}(\chi + n \to \chi + n) = \frac{6}{\pi} \left( \frac{M_n M_{\chi}}{M_n + M_{\chi}} \right)^2 \left( \sum_{q=u,d,s} d_q \Delta q^{(n)} \right)^2$$

(28)

We follow the DARKSUSY[30] and use the following inputs for the spin-dependent calculations:

$$\Delta_p^u = +0.77 , \quad \Delta_p^d = -0.40 , \quad \Delta_p^s = -0.12 ,$$

$$\Delta_n^u = -0.40 , \quad \Delta_n^d = +0.77 , \quad \Delta_n^s = -0.12 .$$

(29)

For our model, the coefficient $d_q$ can be written as

$$d_q = \frac{1}{4} a_q g^2 M_{Z'}^2 ,$$

(30)

where $a_q$ is the hypercharge of quarks under the new $U(1)$ gauge symmetry.

The cosmological experiments have precisely measured the relic density of the nonbaryonic cold dark matter: $\Omega_D h^2 = 0.1123 \pm 0.0035$ [31]. Taking this result into Eq. 27 we
FIG. 3: $\sigma(\chi + n \rightarrow \chi + n)$ as function of dark matter mass $M_{DM}$ constrained dark matter relic density.

may derive $g_X$ as implicit function of $M_{DM}$ and $M_{Z'}$. Then one free parameter is reduced. We plot in Fig. 3 $\sigma(\chi n \rightarrow \chi n)$ as the function of the mass of the dark matter constrained by the dark matter relic density. The solid and dotted lines correspond to $M_{Z'} = 600$ and 800 GeV, separately. The Xenon-100 [32] gives the strongest constraint on the dark matter-nucleon scattering cross section in the region, which is about $[1 \times 10^{-44}, 4 \times 10^{-44}]$. It constrains $M_{DM}$ lying near $1/2M_{Z'}$ for our model, around which all the experimental constraints may be fulfilled.

VI. CONCLUSION

In this paper, we proposed a possible solution to the fermion mass hierarchy problem by fitting the type-II seesaw mechanism into the Higgs doublet sector. We extended the Standard Model with two extra Higgs doublets as well as a spontaneously broken $U_X(1)$ gauge symmetry. The VEVs of Higgs doublets are normal hierarchal due to the $U(1)_X$ symmetry. In our model all the Yukawa couplings of quarks and leptons except that of top quark, are of order $\mathcal{O}(10^{-2})$. Constraints on the model from meson mixings, lepton flavor violations as well as dark matter direct detection were studied. The masses of new Higgs
fields can be several TeV, the collider signatures of which are important but beyond the scope of this paper will be shown in somewhere else.

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Appendix A: Diagonalization of $4 \times 4$ Higgs mass matrix

The CP-even Higgs matrix can only be blog diagonalized. We first write it as

$$M_{CP-even}^2 = \begin{pmatrix} Z & T \\ T^T & Z' \end{pmatrix}$$

where $Z$, $T$ and $Z'$ are $2 \times 2$ sub-matrix with

$$Z = \begin{pmatrix} m_1^2 + v_1^2 \lambda_1 & \frac{1}{2} v_1 v_2 (\lambda_4 + \lambda_7) \\ * & m_2^2 + v_2^2 \lambda_2 \end{pmatrix},$$

$$T = \begin{pmatrix} \frac{1}{2} v_1 v_3 (\lambda_5 + \lambda_8) - \mu_1 v_4 & \frac{1}{2} v_1 v_4 \lambda_{10} - v_3 \mu_1 \\ \frac{1}{2} v_1 v_3 (\lambda_6 + \lambda_9) - \mu_2 v_4 & \frac{1}{2} v_1 v_4 \lambda_{11} - v_3 \mu_2 \end{pmatrix}.$$
(1981); J. Schechter and J. W. F. Valle, Phys. Rev. D 22, 2227 (1980); R. N. Mohapatra
and G. Senjanovic, Phys. Rev. D 23, 165 (1981).
[4] R. Foot, H. Lew, X. G. He and G. C. Joshi, Z. Phys. C 44, 441 (1989).
[5] C. D. Froggatt and Nielsen, Nucl. Phys. B 147, 277(1979).
[6] K. R. Dienes, E. Dudas and T. Gherghetta, Phys. LettB 436, 55 (1998).
[7] I. Gogoladze, C. A. Lee, T. Li and Q. Shafi, Phys. Rev. D 78, 015024 (2008).
[8] H. Fritzsch and Z. Z, Xing, Prog. Par. Nucl. Phys. 45, 1 (2000).
[9] Y. Buchmuller and T. Yanagida, Phys. Lett. B 445, 399 (1999).
[10] Y. Nir, Phys. Lett. B 354, 107 (1995).
[11] J. J. Heckman and C. Vefa, Nucl. Phys. B 837, 137 (2010).
[12] F. Bazzocchi, M. Frigerio and S. Morisi, Phys. Rev. D 78, 116018 (2008).
[13] K. Koshioka, Mod. Phys. Lett. A 15, 29 (2000).
[14] S. Davidson, G. Isidori and S. Uhlig, Phys. Lett. B 63, 73 (2008).
[15] G. J. Ding, Phys. Rev. D 78, 036011 (2008).
[16] C. D. Froggatt, G. Lowe and H. B. Nielsen, Nucl. Phys. B 414, 579 (1994).
[17] F. Feruglio and Y. Lin, Nucl. Phys. B 800, 77 (2008).
[18] Y. Grossman, Nucl. Phys. B 426, 355 (1994).
[19] R. A. Porto and A. Zee, Phys. Lett. B 666, 491 (2008); Phys. Rev. D 79, 013003 (2009).
[20] E. Ma, Phys. Rev. Lett. 86 2502 (2001); Phys. Lett. B 516, 165 (2001).
[21] S. M. Davidson, H. E. Logan, Phys. Rev. D 80, 095008 (2009); T. Morozumi, H. Takata and
K. Tamai, arXiv: 1009.1026[hep-ph].
[22] F. Josse-Michaux and E. Molinaro, [arXiv:1109.0482[hep-ph]].
[23] N. Haba and O. Seto, [arXiv:1106.5353[hrp-ph]]; Prog. Theor. Phys. 125, 1155 (2011); N. Haba
and K. Tsumura, JHEP 1106, 068 (2011); N. Haba and M. Hirotsu, Eur. Phys. J. C 69, 481
(2010).
[24] W. Grimus and L. Lavoura, Phys. Lett. B 687, 188 (2010).
[25] W. Chao and M. Ramsey-Musolf, to appear.
[26] P. Abreu, et al., (DELPHI Collaboration), Phys. Lett. B 485, 45 (2000); R. Barate et al.,
(ALEPH Collaboration), Eur. Phys. J. C. 12, 183 (2000); J. Erler, P. Langacker, S. Munir
and E. R. Pena, arXiv: 0906.2345.
[27] J. F. Grivaz, Int. J. Mod. Phys. A 23, 3849 (2008) and reference therein.
[28] K. S. Babu, E. Ma and J. W. F. Valle, Phys. Lett. B 552, 207 (2003).

[29] X. G. He, Y. Y. Keum and R. R. Volkas, JHEP 0604, 039 (2006).

[30] P. Gondolo, J. Edsjo, P. Ullio, L. Bergstrom, M. Schelke and E. A. Baltz, JCAP 0407, 008 (2004).

[31] E. Komatsu, et al., arXiv: 1001.4538[astro-ph.CO]

[32] E. Aprile et al., XENON 100 Collaboration, Phys. Rev. Lett. 107, 131302, (2011).