Investigation of strengths distributions of Gamow-Teller in sd and fp shell nuclei
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Abstract: The strength distribution of Gamow-Teller (GT) in the 26Si→26Al, 28Si→28P, 42Sc→42Ca and 44Sc→44Ca transitions are calculated by using shell model with full basis which assume the valence nucleons in the sd and fp shells. The calculation of the shell model were conducted in the sd and fp shells without any restriction by using USDB and USDA interactions for sd shell and GXPF1A, KB3G and FPD6 effective interactions for fp shell. Our theoretical results of shell model calculations compared with the data obtained from (3He, t) and (p, n) charge-exchange reactions. Calculated B(GT) strength distributions agree well with the recently available experimental data and with the summed transition strengths B(GT).

Keywords: Gamow-Teller transitions, Charge-exchange reactions, Isospin symmetry, Shell Model.

1. Introduction

The spin-isospin reaction in nuclei has long been observed using charge-exchange reactions. The neutron (proton) is converted into a proton to neutron with change of isospin T=1, with and without spin transition ΔS = 1 or ΔS = 0. Furthermore, extensive studies have been conducted of Gamow-Teller (GT) transformations ΔT = 1, ΔS = 1 and (T=1, T=1, TS=1 and ΔT = 1. Through the στ + operator these transformations are mediated, combining the same original and last states as β+ . The late evolution of stars is defined by weak-interaction levels via capture of electron (EC) and β-decay [1-8]. Since β-decay has only access to states within a very restricted energy range, and because of the lack of contact direct measurements with neutrinos are difficult to complete, reactions of charging-exchange with hadronic samples are a preferred tool to chart the response of Gamow-Teller.

Gamow–Teller transformation properties have been observed by Yu-Mei Zhang [9] to provide energy for combined neutron-rich fluorine, oxygen and nitrogen isotopes. The configurations of the nuclei are represented via WBT interactions in the space of p–sd shell using the nuclear-shell model. The distribution strength of Gamow-Teller equations reproduces the measured data in the region with low-energy accurately. For shell nuclei, the strengths of Gamow-Teller, which is used to estimate weak rates of interaction in the stellar surroundings, have been estimated by a large-scale shell model calculation [10,11]. Instead of those determined by the FFN, use of these new rates has a significant influence on later predictions stages of stars [6-8].

The theoretical strength distributions should therefore be accurate and in line with experimental data. Gamow-Teller’s transition forces were obtained between $^{56}$Ni and $^{56}$Cu, and the predictions of shell model in the p-shell utilizing interactions KB3G and GXPF1A were compared b M Sasano et al. [12]. The GXPF1A interaction estimates are much higher than those that use the KB3 G interaction [12], replicated by the GT experimental power distribution. Obaid and Majeed [13] have examined the nuclear Gamow-Teller (GT) transitional force distribution B(GT), for certain sd-shell nodes in reaction (3He, t).

It is therefore essential that the distributions of theoretical strength are accurate and in accordance with experimental data. M Sasano et al., obtained Gamow-Teller transition strengths from $^{56}$Ni to $^{56}$Cu and...
compared with shell-model predictions in the pf -shell using the KB3G and GXPF1A interactions by M Sasano et al. The measured distribution of GT strengths are reproduced by GXPF1A interaction much better than the calculations that employed the KB3G interaction [12]. The nuclear Gamow-Teller (GT) transition strength distributions B(GT) have been studied by Obaid and Majeed [13], for selected sd shell nuclei in the reaction of type charge-exchange denoted as (³He, t).

The effective USDA and USDB interactions designed for the sd model region were employed and the calculated results were fairly accepted. In their report. For understanding the mechanisms for neutron star and black formation, the transitions from Gamow-Teller (GT) to ⁴⁶Ti→⁴⁶V, ⁴⁷Ti→⁴⁷V, ⁴⁸Ti→⁴⁸V and ⁴⁰Cr→⁴⁰Mn charge-exchange-reaction were studied by Obaid and Majeed [14] in nuclear structure and astrophysical processes.

In this paper, we describe our calculations of Gamow-Teller strength distributions in sd and fp shells. The shell model calculations carried out using the shell model code NushellX@MSU[19] , to obtain the B(GT) strength for ²⁶Si→²⁶Al, ²⁶Si→²⁶P, ⁴⁲Sc→⁴⁲Ca and ⁴⁴Sc→⁴⁴Ca in the sd and fp shells without any restriction by using USDB and USDA [15] interactions designed for sd shell region and GXFP1A, KB3G and FPD6 effective interactions designed for fp shell. The results B(GT) predictions and their accumulated B(GT) will be compared with the corresponding experimental data.

2. Theoretical framework

The operator connecting the initial and the final states to the GT transition may be written as [20]

\[ \langle \sigma \tau \rangle = \frac{\langle f | \sum_k \sigma k \tau_k^* | i \rangle}{\sqrt{2J_f + 1}} \]  

(1)

with

\[ \tau_\pm = \frac{1}{2} (\tau_x + i \tau_y), \]  

(2)

where \( \sigma \) is the operator of the Pauli and \( \tau \) the isospin, \( |f\rangle \) and \( |i\rangle \) and represents the final and initial transition states, respectively.

The B(GT) is the probability reduced for the transition from GT, commonly used to indicate the strength of GT [20].

\[ B(GT) = \left( \frac{g_A}{g_V} \right)^2 \langle \sigma \tau \rangle^2 \]  

(3)

where \( |g_A/g_V| = 1.26 \), is the axial-vector ratio of constants to the coupling vector. For the GT reduced elements matrix, the sum rule is [21]

\[ \sum_f [ B_{i,f}(GT_-) - B_{i,f}(GT_+) ] = 3 \left( \frac{g_A}{g_V} \right)^2 (N_i - Z_i) \]  

(4)

3. Results and Discussion

3.1 ²⁶Si→²⁶Al

Figure 1 displays the measured and calculated strength distribution of B(GT) to the transition ²⁶Si→²⁶Al. The B(GT) have been calculated from the ground state of ²⁶Si (0⁺)→²⁶Al (1⁺) without imposing any restriction in the sd shell by adopting the effective interactions USDA and USDB. The GT strength distribution has been measured in a ²⁶Si (³He, t) experiment [21,22] and ²⁶Si (p, n) experiment [23]. Figure 2 illustrates the accumulated sum of B(GT) versus the excited energy. The shell model theoretical calculations conducted using the effective interactions USDB and USDA interactions reproduce the data very well for both B(GT) and their accumulated sum of B(GT).
3.2 $^{28}\text{Si} \rightarrow ^{28}\text{P}$

Figure 3 displays the shell model predictions and their comparison with the measured data of $B\,(GT)$ distribution strength for $^{28}\text{Si} \rightarrow ^{28}\text{P}$ transition. The $B\,(GT)$ transition strengths were predicted from ground level of $^{28}\text{Si} \,(0^+)$ to $^{28}\text{P} \,(1^+)$ states with no restriction imposed on the model space using USDA and UDSB interactions. The strength distribution of $B\,(GT)$ has been measured in $^{28}\text{Si} \,(^3\text{He}, t)$ experiment up to excitation energies below 6.5 MeV [24]. Figure 4 presented the accumulated sums of $B\,(GT)$ versus excited energy $E_x(^{28}\text{P})$. It is found that the USDA effective interaction produced an excitation energy and $B\,(GT)$ strengths reproduced better by the USDA than USDB interaction.

![Figure 1](image1.png)
**Figure 1:** Shows the theoretical values of $B\,(GT)$ compared to the corresponding experimental data [21,22,23] for $^{26}\text{Si} \rightarrow ^{26}\text{Al}$ transition.

![Figure 2](image2.png)
**Figure 2:** Shows the $\sum B\,(GT)$ distributions compared to measured data [21,22,23] for $^{26}\text{Si} \rightarrow ^{26}\text{Al}$ transition.

![Figure 3](image3.png)
**Figure 3:** Shows the theoretical values of $B\,(GT)$ compared to the corresponding experimental data [24] for $^{28}\text{Si} \rightarrow ^{28}\text{P}$ transition.

![Figure 4](image4.png)
**Figure 4:** Shows the $\sum B\,(GT)$ distributions compared to measured data [24] for $^{28}\text{Si} \rightarrow ^{28}\text{P}$ transition.
3.3 $^{42}\text{Sc} \rightarrow ^{42}\text{Ca}$

The calculated strength distributions of B(GT) utilizing the shell model in full fp model space without any restriction along with their comparison to the measured data for the transition $^{42}\text{Sc} \rightarrow ^{42}\text{Ca}$ is displayed in Fig.5. The B (GT) strength distribution is determined form the ground state of $^{42}\text{Sc}$ ($0^+$) to $^{42}\text{Ca}$ ($1^+$) states using the GXFP1A, KB3G and FPD6 effective interactions designed for the fp model space. The measured of reaction of type charge-exchange of is observed via $^{42}\text{Sc} (^3\text{He}, t )^{42}\text{Ca}$ [25]. Following many previous works performed by many authors the shell model prediction for the B(GT) strengths were quenched by a quenching factor (0.74)$^2$ to fit the observed data. Figure 6 displays the accumulating sums of B(GT) versus excitation energy. The predicted shell model calculations utilizing the three effective interactions agreed very well for the total GT strength till 12 MeV of excitation energy extracted from the $^{42}\text{Sc} (^3\text{He}, t )$ data.

![Figure 5](image5.png)

**Figure 5:** Shows the theoretical values of B(GT) compared to the corresponding experimental data [25] for $^{42}\text{Sc} \rightarrow ^{42}\text{Ca}$ transition.

![Figure 6](image6.png)

**Figure 6:** Shows the $\sum B(\text{GT})$ distributions compared to measured data [25] for $^{42}\text{Sc} \rightarrow ^{42}\text{Ca}$ transition.

3.4 $^{44}\text{Sc} \rightarrow ^{44}\text{Ca}$

The theoretical distributions of GT strength estimated by the shell model along with the comparison with the corresponding measured data for the transition $^{44}\text{Sc} \rightarrow ^{44}\text{Ca}$ are depicted in the Fig. 7. The measured data are extracted from the β-decay $^{44}\text{Sc}(0^+) \rightarrow ^{44}\text{Ca}(1^+)$ till the excitation energy 6.818MeV [26]. A quenching factor of (0.74)$^2$ were used to be multiplied our theoretical predictions following the work of other researchers in this mass region. The effective interactions GXFP1A, KB3G and FPD6 designed in this mass region were utilized to perform the calculations. The calculations of the accumulated B(GT) values are shown in Fig.8 versus the excited energy of $^{44}\text{Ca}$. The results show clearly that the shell model with these effective interactions are able to reproduce the measured data using the quenching factor of (0.74)$^2$.
4. Conclusion

The present study conducted by utilizing the calculations of shell model in large basis using the full sd and fp model space to study the transition strengths of and their accumulated sums for the transitions $^{26}\text{Si} \rightarrow ^{26}\text{Al}$, $^{28}\text{Si} \rightarrow ^{28}\text{P}$, $^{42}\text{Sc} \rightarrow ^{42}\text{Ca}$ and $^{44}\text{Sc} \rightarrow ^{44}\text{Ca}$. The choice of $(0.74)^2$ as quenching factor to multiply our theoretical predictions is adequate and we were able to reproduce the measured data well for all the studied nuclei. This work can be repeated for more sd and fp shell nuclei to trace the success or shortfall of the effective interactions derived for this mass region and to have better test ground for the ability of the shell model with large scale in predicting the Gamow-Teller $B(\text{GT})$ strengths. The present study might be very important step toward better understanding of the evolution of supernova and star formation.

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