Abstract
In models where the suppression of the tree level couplings of fermions with Goldstone bosons is not protected by symmetry, the loop-induced couplings can in general dominate over the tree couplings. We demonstrate this by calculating the decay $\nu \rightarrow \nu' + \text{Majoron}$ in the simplest singlet Majoron model. It is shown that 1-loop contributions can be orders of magnitude bigger than the tree-level result, and hence allows a wide range of interesting values of neutrino mass and mixing to be compatible with cosmological constraints.
Theoretical models with $B - L$ as a spontaneously broken global symmetry have a Nambu-Goldstone boson called Majoron ($J$), where $B$ and $L$ are respectively the baryon and lepton number. In these models a neutrino can decay to a Majoron, $\nu \to \nu' + J$. Such an invisible decay can be of cosmological interest. In particular, a neutrino with a mass lying in the cosmologically excluded region between 30 eV and 2 GeV for stable neutrinos [1] would still be allowed if it decays sufficiently fast. In that case the standard cosmological constraint derived from the mass density of the universe is [2]

$$m_\nu \sqrt{\tau_\nu / t_0} < 30 \text{ eV},$$

(1)

where $m_\nu$ and $\tau_\nu$ are the mass and the lifetime of $\nu$, whereas $t_0 \approx 5 \times 10^{17} \text{ s}$ is the present age of the universe. The bound can be strengthened to [3]

$$m_\nu \sqrt{\tau_\nu / 10^4 s} < 1 \text{ MeV},$$

(2)

provided one assumes the canonical picture of galaxy formation. However, since the physics of galaxy formation is not fully understood, this bound should be taken less seriously than the previous one.

In many simple versions of Majoron Models the decay process occurs with a negligibly small rate [4] at the tree level. This follows because to a first approximation the neutrino-Majoron coupling matrix often turns out to be proportional to the neutrino mass matrix [5]. Thus, in the mass eigenstate basis the neutrino-Majoron interaction is diagonal and the decay wouldn’t occur at this level. To have a nonzero tree-level off-diagonal coupling it is necessary to consider non-leading terms, which are related to the mixings of the light neutrinos $\nu$ and $\nu'$ with some other very heavy species and hence highly suppressed.

This feature is generally unprotected by symmetries, however. Modifications introduced by loop corrections can therefore be potentially significant, leading to a rare situation in which the one loop correction is larger than the tree-level result. Consider the earliest Majoron model of Chikashige, Mohapatra and Peccei [6] as an example. The model is simple in the sense that it requires only minor modifications over the standard model. Therefore the calculation is relatively simple, and the result is dramatic. The model differs from the standard model by a right-handed neutrino $N_R$ for each generation, and a Higgs singlet

$$\Phi(x) = \frac{1}{\sqrt{2}}(v_1 + s(x) + iJ(x)),$$

(3)

which carries 2 units of $B - L$ quantum number. The vacuum expectation value of $\Phi$, $v_1$, breaks $B - L$ spontaneously and gives rise to a Majoron $J$. As a gauge singlet, $J$ does not interact directly with the $Z$ boson, and hence unlike many other later versions this model has not been ruled out by the measurements of $Z$-width.

At the tree level, the majoronic decay of neutrino is determined by the Yukawa interaction

$$- \mathcal{L}_Y = \sum_{a,A} \frac{\sqrt{2}}{v_2} D_{aA} \overline{\psi}_L \phi N_{RA} + \sum_{A,B} \frac{1}{\sqrt{2}v_1} M_{AB} \overline{N}_{LA} \Phi N_{RB} + \text{h.c.},$$

(4)

where $\psi_L$ is the usual lepton doublet and $\tilde{N} = C\gamma_0 N^\ast$. $\phi$ is the standard Higgs doublet with its neutral component $\phi^0$ given by

$$\phi^0(x) = \frac{1}{\sqrt{2}}(v_2 + H(x) + i\varphi(x)).$$

(5)
\( D \) and \( M \) are coupling matrices in flavor space, \( v_2 \) is the vacuum expectation value which breaks the gauge symmetry. The mass terms of the neutrinos are given by

\[
- \mathcal{L}_{\text{mass}} = \frac{1}{2} \left( \overline{\nu_L} \overline{\mathcal{D}} N_L \right) \left( \begin{array}{cc} 0 & D \\ DT M \end{array} \right) \left( \begin{array}{c} \tilde{\nu}_R \\ N_R \end{array} \right) + \text{h.c.},
\]

(6)

where each entry in the mass matrix represents a 3 \( \times \) 3 matrix for 3 generations of fermions.

In the present model the elements of \( D \) arise from the same physics that gives rise to the masses of charged leptons. As a result, we assume

\[
D_{aA} \ll g v_2 \quad \forall a, A.
\]

(7)

where \( g \) is the SU(2) gauge coupling. To proceed, we further assume that CP is conserved and that

\[
r \equiv v_2/v_1 \ll 1.
\]

(8)

This implements the well-known see-saw mechanism \([5]\) for the generation of small neutrino masses. In addition, we assume for simplicity that

\[
M_{AB} \sim v_1 \quad \forall A, B.
\]

(9)

These assumptions are not crucial. However they do simplify the calculation.

Calling the doublet Yukawa couplings collectively as \( y \), we can then diagonalize the mass matrix perturbatively in powers of \( r \) and \( y \). To order \( r^2y^2 \) one obtains the following relations between the mass eigenstates and the weak eigenstates \([8]\):

\[
\left( \begin{array}{c} \nu_L \\ \tilde{\nu}_R \\ N_L \\ N_R \end{array} \right) = \left( \begin{array}{cccc} 1 - \frac{1}{2} \rho \rho^T & \rho & f^* & 0 \\ -\rho^T & 1 - \frac{1}{2} \rho \rho^T & 0 & F^* \\ \rho & 0 & \chi_L & X_L \\ f & 0 & F & \chi_R \end{array} \right),
\]

(10)

where \( \rho \equiv DM^{-1} \sim ry \). To order \( \rho^2 \), the mass terms are

\[
- \mathcal{L}_{\text{mass}} = \frac{1}{2} \left[ \overline{\chi}_L m \chi_R + \overline{\chi}_L MX_R \right] + \text{h.c.},
\]

(11)

with diagonal mass matrices

\[
m = f^T \left( -\rho \mathcal{M} \rho^T \right) f,
\]

(12)

\[
M = F^T \left( \mathcal{M} + 1/2 \rho^T \rho \mathcal{M} + 1/2 \rho \mathcal{M} \rho^T \right) F.
\]

(13)

The diagonalization into a light block and a heavy block is performed by the first matrix on the right hand side of Eq. (10). The two unitary matrices \( f \) and \( F \) determine the mixings among the light-light and the heavy-heavy neutrinos, respectively. Notice \( f \) and \( F \) can be complex, even if CP is conserved, unless the Majorana eigenstates all have the same CP eigenvalue \([9]\).

From Eqs. (4), (10) and (12) one can easily show that, to \( \mathcal{O}(\rho^2) \sim \mathcal{O}(r^2y^2) \), the \( \chi\chi J \) coupling matrix is proportional to \( m \) which is diagonal \([3]\). Although the next order terms are not diagonal \([1]\), they are
\( O(\rho^4) \sim O(r^4 y^4) \), rendering their contributions to the neutrino decay too small to be of practical interest. We shall show that this feature is modified drastically by radiative corrections. To discuss this effect, let us parameterize the effective interaction for the decay \( \chi_a \rightarrow \chi_b + J \), keeping terms up to \( O(r^2 y^2) \), as

\[
\mathcal{L}_{\text{eff}} = -i\bar{\chi}_b \left( \frac{m_a}{v_1} \gamma_5 \delta_{ba} + T_{ba} R - T_{ba}^* L \right) \chi_a J + \text{h.c.},
\]

(14)
in which the first term is the tree-level result and \( T_{ba} \) comes from loop contributions to be calculated below. The 1-loop diagrams for the decay in the Feynman gauge are shown in Figs. 1, 2, and 3. Graphs with an \( H-s \) mixing are negligible due to (8). Charged-current interactions do not make any contribution at this level. The Feynman rules for relevant couplings are summarized in Fig. 4.

The contributions of various diagrams are organized in accordance with the virtual bosons propagating in the loop. For example, \( T^{(Z)}_{ba} \) represents the contribution from all diagrams containing a virtual \( Z \).

Neglecting the mass of the final state neutrino and using the shorthand

\[
P = \tilde{f}^T \tilde{D} \tilde{M}^{-1} F^*,
\]

(15)
we find, up to terms of order \( O(r^2 y^2) \) times a potential logarithmic factor,

\[
T_{ba}^{(Z,H,\phi)} = 0,
\]

(16)

\[
T_{ba}^{(s)} = -\frac{1}{8\pi^2} \sum_A \tilde{P}_{ba} P_{aA} \left( \frac{M_A}{v_1} \right)^3 I(M_A^2, M_s^2),
\]

(17)

\[
T_{ba}^{(J)} = \frac{1}{8\pi^2} \sum_A \tilde{P}_{ba} P_{aA} \left( \frac{M_A}{v_1} \right)^3 I(M_A^2, 0),
\]

(18)

\[
T_{ba}^{(Js)} = \frac{1}{8\pi^2} \sum_A \tilde{P}_{ba} P_{aA} \left( \frac{M_A}{v_1} \right)^3 \frac{M_s^2}{M_A^2 - M_s^2} \ln \frac{M_A^2}{M_s^2},
\]

(19)

where \( M_s \) is the mass of the singlet scalar \( s \) and

\[
I(x,y) = \frac{1}{\zeta^2} \frac{\zeta}{(\zeta + x)(\zeta + y)}
\]

(20)
which is divergent. However, the sum of Eqs. (16-19) is finite:

\[
T_{ba} = 2T_{ba}^{(Js)}.
\]

(21)

Using the definition of \( P \) from Eq. (15), one sees that \( T_{ba} \) is of order \( m_a/4\pi^2 v_1 \) times an interfamilly mixing for \( M_A \sim v_1 \) and reasonable choices of \( M_s/M_A \). Despite the loop suppression factor \( 1/4\pi^2 \), for a wide range of interesting values of \( m \) and \( M \) this result is orders of magnitude bigger than the tree-level result, which is \( O(r^4 y^4) \), as pointed out earlier.

There is a technical subtlety associated with the choice of the representation of Majoron fields. Instead of using Eq. (3) as the definition of \( J \), one can use a non-linear representation:

\[
\Phi(x) = \frac{1}{\sqrt{2}} (v_1 + s(x)) e^{iJ(x)/v_1}.
\]

(22)
The interaction Hamiltonian now has momentum-dependent terms. As a result, the formulation of Feynman rules is different from the usual situation. The two representations, Eqs. (3) and (22), give identical results for tree-level processes in which momentum-dependent interactions are negligible. For the
case at hand, a naive calculation following (22) without taking into account the associated Feynman rule difference would incorrectly give $T_{ba}^{(J_s)} = 0$, and as a result the final answer would be a factor of 2 smaller.

Let us now examine the significance of the loop result. With our assumption of CP invariance, the mixing matrices satisfy the constraints [3, 12] $f'_{ab} = \eta_b f_{ab}$, and a similar one for $F_{AB}$, where the CP eigenvalue of $\chi_b$ is $i\eta_b$, with $\eta_b = \pm 1$. Thus,

$$P_{aA}^* = \eta_a \eta_A P_{aA}. \quad (23)$$

Due to the Majorana nature of the neutrinos, the hermitian conjugate term in Eq. (14) contributes equally as the other one. The amplitude of the decay can therefore be written as

$$\mathcal{A}(\chi_a \to \chi_b + J) = \frac{1}{2\pi^2} \theta_{ba} \bar{u}_b (R - \eta_a \eta_b L) u_a, \quad (24)$$

where $u_{a,b}$ are the standard spinors, and for convenience we have defined a parameter $\theta_{ba}$:

$$\theta_{ba} = \sum_A P_{bA} P_{aA} \left( \frac{M_A}{v_1} \right)^3 \frac{M_s^2}{M_A^2 - M_s^2} \ln \left( \frac{M_A^2}{M_s^2} \right), \quad (25)$$

which plays the role of $m_a / v_1$ times an effective light-light interfamily mixing. Note that $\theta$ vanishes when $v_1 \to \infty$, since the heavy neutrinos decouple in that limit. Eq. (24) demonstrates that the transition amplitude is purely pseudoscalar if the two Majorana states have the same CP property, and purely scalar if their CP eigenvalues are opposite. Intuitively, this is obvious since the CP eigenvalue of the Majoron is negative.

From Eq. (24), the rate of the decay for either sign of $\eta_a \eta_b$ is

$$\tau^{-1}(\chi_a \to \chi_b J) = \frac{\theta_{ba}^2 m_a}{32\pi^5}. \quad (26)$$

Substituting (25) into (1) yields

$$\theta_{ba}^2 (1 \text{eV}/m_a) > 1.5 \times 10^{-32}. \quad (27)$$

This result implies that, for a wide range of $v_1$ and neutrino masses and mixings, the radiatively-induced neutrino-Majoron decay is fast enough to allow the neutrino be compatible with the big bang cosmology.

To have a feeling for our result, we take from [10], [12] and (23) our order-of-magnitude estimates $m_a \sim v_1 \rho^2$ and $\theta \sim \rho^2$. Replacing $\rho$ by $m_D / v_1$, where $m_D$ is the relevant Dirac mass, we obtain $m_a \sim 10^4 \text{eV} \times (1 \text{TeV}/v_1) \times (m_D/100 \text{MeV})^2$. Thus for $v_1 = 1 \text{TeV}$ and $m_D = m_\mu$, $m_a \approx 10 \text{keV}$. Such a neutrino can easily satisfy the cosmological constraint (1) which only requires from (27)

$$m_a > 1.5 \times 10^{-8} \text{eV} \times (v_1/1 \text{TeV})^2. \quad (28)$$

For $v_1$ of order TeV, even after taking all the uncertainties of an order-of-magnitude estimate, this shows that the mechanism can provide a fast decay for neutrinos of any mass. One can also express Eq. (28) as a bound on $v_1$, i.e., $v_1 < 10^4 \text{TeV} \times (m_D/100 \text{MeV})^{2/3}$. If one uses the bound of galaxy formation from Eq. (2), the constraint is similar to Eq. (27) with the right hand side replaced by $6.6 \times 10^{-28}$. The corresponding bound on the neutrino mass is $m_a > 6.6 \times 10^{-4} \text{eV} \times (v_1/1 \text{TeV})^2$ with the same phenomenological conclusion.

On the other hand, if one has to rely on the tree level contribution only [10], instead of $\theta / 2\pi^2$ we obtain $2\rho^4$ as the order-of-magnitude estimate for the coefficient in the amplitude of Eq. (24), the factor 2 occurring due to the Majorana nature of the neutrinos. The bound in Eq. (1) can then be satisfied if

$$m_a > 20 \text{keV} \times (v_1/1 \text{TeV})^{4/3}. \quad (29)$$
This can be satisfied for a narrow range of values of the mass of $\nu_{\mu}$ provided $v_1$ is not much larger than a TeV. However, for the bound in Eq. (2), the factor 20 keV from this equation should be replaced by 750 keV, which cannot be satisfied for any $\nu_{\mu}$ mass allowed by experiments.

In conclusion, we have calculated 1-loop radiative corrections to the neutrino-Majoron decay in a singlet Majoron model. The result shows that loop contributions can decisively dominate the tree-level contribution, allowing a wide range of interesting values of neutrino mass and mixing be compatible with the big bang cosmology. Note that the sole reason that the 1-loop result dominates over the tree level result is just because the tree level suppression is accidental in nature, that is, it is not enforced by any symmetry. It has little to do with the other details of the model. Therefore, one expects that the same conclusion can be drawn in a large class of models in which there is such accidental tree level suppression of off-diagonal majoron couplings. In addition, since the accidental suppression disappears at the 1-loop, one does not expect that including higher loop contributions would change our result qualitatively.

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Figure Captions

Figure 1: One-loop graphs for the leading terms of the decay $\chi_a \to \chi_b + J$ involving a $Z$, $\varphi$ or $H$ internal line. The contribution of these diagrams appear in Eq. (16).

Figure 2: Same as Fig. 1 except the internal boson lines are either $s$ or $J$. The contribution of these diagrams are summarized in Eqs. (17) and (18).

Figure 3: Diagrams involving the trilinear scalar vertex $JJs$ which contribute at the leading order. Their contribution is summarized in Eq. (19).

Figure 4: Feynman rules relevant for the calculation of 1-loop corrections to the decay $\chi_a \to \chi_b + J$. The Feynman rules for fermion vertices with $H(s)$ can be obtained by multiplying the corresponding $\varphi(J)$ vertex by $-i\gamma_5$. 

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