Spin-$\frac{5}{2}$ resonance contributions to the pion-induced reactions for energies $\sqrt{s} \leq 2.0$ GeV.

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Abstract. The spin-$\frac{5}{2}$ resonance effects are studied within the coupled channel effective Lagrangian model for baryon resonance analysis. We extend our previous hadronic calculations to incorporate the $D_{15}$, $F_{15}$, $D_{35}$, $F_{35}$ states. While the effect of the spin-$\frac{5}{2}$ resonances to the $\eta N$, $K \Lambda$, and $K \Sigma$ reactions are small, the contribution to the $\omega N$ is found to be important. The results for the 'conventional' and Pascalutsa-like spin-$\frac{5}{2}$ descriptions are discussed.

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1 Introduction

The extraction of baryon-resonance properties is one of the important tasks of modern hadron physics. Great efforts have been made in the past to obtain this information from the analysis of pion- and photon-induced reaction data. The precise knowledge of these properties is an important step towards understanding the hadron structure finally and the strong interactions.

Some quark models (see \cite{1} and Refs. therein) predict that the baryon resonance spectrum may be richer than discovered so far. This is the so-called problem of 'missing' nucleon resonances. One assumes that these states are weakly coupled to pion channels and are consequently not clearly seen in $\pi N$, $2\pi N$ and $\eta N$ reactions from which experimental data are most often used for baryon-resonance analyses. To incorporate other possible final states a unitary coupled-channel model (Giessen model) has been developed which includes $\gamma N$, $\pi N$, $2\pi N$, $\eta N$, $K \Lambda$ final states and deals with all available experimental data on pion- and photon-induced reactions \cite{23}. The most recent extensions of this model include $K \Sigma$ and $\omega N$ final states \cite{4,5,6} as well, which allows for the simultaneous analysis of all hadronic and photoproduction data up to $\sqrt{s} = 2$ GeV.

A shortcoming of this study is the missing of higher-spin resonances with spin $J > \frac{3}{2}$. Since the spin-$\frac{3}{2}$ resonances have large electromagnetic couplings, this limited the previous analysis of the Compton scattering data to the energy region $\sqrt{s} \leq 1.6$ GeV. Moreover, the extension to higher-spin baryon spectra becomes unavoidable for investigation of 'hidden' or 'missing' nucleon resonances. In particular, a study of the spin-$\frac{5}{2}$ part of the baryon spectra can shed light on the dynamics of the vector ($\omega$ and $\rho$) meson production mechanisms which is itself a very intriguing question (see \cite{10} and references therein).

In the present paper we study the effect of spin-$\frac{5}{2}$ resonance contributions to $\pi N$, $2\pi N$, $\eta N$, $K \Lambda$, $K \Sigma$, and $\omega N$ final states. Starting from the effective Lagrangian coupled-channel model \cite{5} we extend our previous hadronic calculation \cite{5} by including the $D_{15}$, $F_{15}$, $D_{35}$, $F_{35}$ resonances and simultaneously analysing all available pion-induced reaction data in up to 2 GeV energy region. Due to the coupled-channel calculations this model provides a stringent test for the resonance contributions to the all open final states. Similar to the spin-$\frac{3}{2}$ case in \cite{5,6}, the contributions from spin-$\frac{5}{2}$ states are investigated for two different types of the spin-$\frac{5}{2}$ couplings: for the 'conventional' ('C') and Pascalutsa ('P') prescriptions. While the first approach dates back to the original work of Rarita and Schwinger \cite{11} and is widely used in the literature, the latter one assumes the gauge-invariant resonance coupling. Although the data quality is not good enough to distinguish between these two pictures now, this question is challenging for an understanding of the meson-baryon interactions. With this aim in mind, the present work extends our earlier multi-channel analysis based on an effective Lagrangian approach by including also the spin-$\frac{5}{2}$ resonances.

The paper is organized as follows. We start in Sec. \cite{2} with a description of the formalism concentrating mainly on the spin-$\frac{5}{2}$ couplings; the complete discussion of our model including all other couplings can be found in \cite{3,4}. In Sec. \cite{5} we discuss the results of our calculations in comparison with the previous studies \cite{5} and finish with a summary.
2 The Giessen model

We solve the Bethe-Salpeter coupled-channel equation in the K-matrix approximation to extract scattering amplitudes for the final states under consideration. The validity of the K-matrix approximation has been tested by Pearce and Jennings who have performed a fit to the elastic πN phase shifts up to 1.38 GeV with the ‘smooth’, Blankenbecler-Sugar and the K-matrix propagators [14]. They have found no significant differences in the parameters extracted in the three cases. Also, a successful description of the pion- and photon-induced reaction data [6] and η-production [14] points to the applicability of this approximation for investigation of the baryon resonance spectra.

In order to decouple the equations we perform a partial-wave decomposition of the $T$ matrix into total spin $J$, isospin $I$, and parity $P = (-1)^{J±±}$. Then the partial-wave amplitudes can be expressed in terms of an interaction potential $K$ via the matrix equation

$$T^{I,J±} = \left[ \frac{K^{I,J±}}{1 - iK^{I,J±}} \right],$$  

(1)

where each element of the matrices $T^{I,J±}_{fi}$ and $K^{I,J±}_{fi}$ corresponds to a given initial and final state ($i, f = \pi N, 2\pi N, \eta N, K A, K S, \omega N$). The interaction potential is approximated by tree-level Feynman diagrams which in turn are obtained from effective Lagrangians [12]. The $T$-matrix fulfils unitarity as long as the $K$ matrix is hermitian.

In our model the following 19 resonances are included $P_{32}(1323), P_{13}(1440), D_{13}(1520), S_{11}(1535), P_{32}(1600), S_{11}(1620), S_{11}(1650), D_{15}(1675), F_{15}(1680), D_{33}(1700), P_{31}(1710), P_{13}(1720), P_{33}(1750), P_{33}(1900), P_{33}(1920), F_{33}(1905), D_{33}(1930), F_{15}(2000)$, and $D_{13}(2050)$, which is denoted as $D_{13}(2080)$ by the PDG [11].

The Lagrangian for the spin-$\frac{5}{2}$ resonance decay to a final baryon $B$ and a (pseudo)scalar meson $\varphi$ is chosen in the form

$$\mathcal{L}_{BR}^{\frac{5}{2}} = \frac{g_{BR}^{2}}{m_{BR}^{2}} \bar{u}_{R}^{\nu} \Theta_{\mu \nu}(a) \Theta_{\nu \lambda}(a') \Gamma_{S} u_{B} \partial^{\lambda} \partial^{\varphi} + h.c. \tag{2}$$

with the matrix $\Gamma_{S} = \mathbb{1}$ if resonance and final meson have identical parity and $\Gamma_{S} = i\gamma_{5}$ otherwise. The off-shell projector $\Theta_{\mu \nu}(a)$ is defined by

$$\Theta_{\mu \nu}(a) = g_{\mu \nu} - a \gamma_{\mu} \gamma_{\nu}, \tag{3}$$

where $a$ is a free off-shell parameter. Since the on-shell symmetric spin-$\frac{5}{2}$ field $u_{R}^{\mu}$ has to obey the Dirac equation and satisfies the conditions $\gamma_{\mu} u_{R}^{\mu} = \partial_{\mu} u_{R}^{\mu} = g_{\mu \nu} u_{R}^{\mu} = 0$ the second part in (3) only contributes for off-shell particles, giving rise to lower off-shell spin components in (2). In general the interaction Lagrangian (2) can have two off-shell projectors matched with both vector indices of the resonance field tensor. However, as we will see later, a good description of the experimental data can be achieved already with a single parameter $a$ keeping the second one equal to zero. Thus, to keep our model as simple as possible we use only one off-shell projector in (2).

The widths of the hadronic resonance decays as extracted from the Lagrangian (2) are

$$\Gamma_{\pm}(R_{\frac{5}{2}} \to \varphi B) = I \frac{g_{BR}^{2}}{30 m_{R} m_{\varphi}} k_{\varphi}^{5} E_{B} \pm m_{B}, \tag{4}$$

The upper sign corresponds to the decay of a resonance into a meson with the identical parity and vice versa. $I$ is the isospin factor and $k_{\varphi}$, $E_{B}$, and $m_{B}$ are the meson momentum, energy and mass of the final baryon, respectively.

The coupling of the spin-$\frac{5}{2}$ resonances to the $\omega N$ final state is chosen to be

$$L_{BR}^{\omega} = \bar{u}_{R}^{\mu} \Gamma_{V} \left( \frac{g_{1}}{4 m_{N}^{2}} - \frac{g_{2}}{8 m_{N}^{2}} \partial_{\mu}^{\nu} \right) \omega_{\nu} + h.c., \tag{5}$$

where the matrix $\Gamma_{V}$ is $\mathbb{1}$ ($i\gamma_{5}$) for positive (negative) resonance parity and $\partial^{\nu}_{\mu}$ ($\partial^{\nu}_{\mu}$) denotes the partial derivative of the nucleon and the $\omega$-meson fields, respectively. The above Lagrangian is constructed in the same manner as the one for spin-$\frac{1}{2}$ in [6]. Similar couplings were also used to describe electromagnetic processes [10, 15, 16, 17]. Since the different parts of (5) contribute at different kinematical conditions we keep all three couplings as free parameters and vary them during the fit. The helicity amplitudes for the decay $R \to \omega N$ are given by

$$A_{\omega}^{\frac{5}{2}} = \sqrt{E_{\omega} \pm m_{\omega}} \frac{k_{\varphi}}{\sqrt{5 m_{N}^{2}}} \left( -g_{1} (m_{N} \pm m_{R}) \right. + g_{2} \left. (m_{R} E_{\omega} - m_{N}^{2}) \right) \frac{1}{2 m_{N}},$$

$$A_{\omega}^{\frac{1}{2}} = \sqrt{E_{\omega} \pm m_{\omega}} \frac{k_{\varphi}}{4 m_{N}^{2}} \left( g_{1} (m_{N} \pm (m_{R} - 2 E_{\omega})) \right. + g_{2} \left. (m_{R} E_{\omega} - m_{N}^{2}) \right) \frac{1}{2 m_{N}},$$

$$A_{\omega}^{0} = \sqrt{(E_{\omega} \pm m_{\omega})} \frac{k_{\varphi} m_{\omega}}{4 m_{N}^{2}} \left( g_{1} \pm g_{2} \frac{E_{\omega}}{2 m_{N}} \right. \pm g_{3} \left. (m_{R} - E_{\omega}) \right) \frac{1}{2 m_{N}}, \tag{6}$$

with upper (lower) signs corresponding to positive (negative) resonance parity. The lower indices stand for the helicity $\lambda$ of the final $\omega N$ state $\lambda = \lambda_{V} - \lambda_{N}$, where we use an abbreviation as follows: $\lambda = 0 : 0 + \frac{1}{2}, \frac{1}{2} : 1 - \frac{1}{2}, \frac{3}{2} : 1 + \frac{1}{2}$. The resonance $\omega N$ decay width $\Gamma_{\omega N}$ can be written as the sum over the three helicity amplitudes given above:

$$\Gamma_{\omega N} = \frac{2}{(2J + 1)} \frac{k_{\varphi} m_{N}}{2 \pi m_{R}} \sum_{\lambda = 0}^{3/2} |A_{\lambda}^{\omega N}|^{2}, \tag{7}$$

where $J = \frac{5}{2}$ for the spin-$\frac{5}{2}$ resonance decay.

For practical calculations we adopt the spin-$\frac{5}{2}$ projector in the form

$$P_{\frac{5}{2}}^{\mu \nu, \rho \sigma}(q) = \frac{1}{2} (T^{\mu \rho} T^{\nu \sigma} + T^{\mu \sigma} T^{\nu \rho}) - \frac{1}{5} T^{\mu \nu} T^{\rho \sigma}$$
where \( \varepsilon \) have been discussed in \([5,23,24]\) and here we perform couplings are used. The differences between these descriptions have in account when the Pascalutsa\( ^{\text{tive}} \) Lagrangian approach as long as additional contact \( \text{D} \) , that both prescriptions are equivalent in the effective \([22]\) , which has also been used in an analysis of \( K \Lambda \) photoproduction \([15]\)

As is well known the description of particles with spin \( J > \frac{1}{2} \) leads to a number of different propagators which have non-zero off-shell lower-spin components. To control these components the off-shell projectors \([3]\) are usually introduced. There were attempts to fix the off-shell parameters and remove the spin-$\frac{1}{2}$ contribution in the case of spin-$\frac{3}{2}$ particles \([18]\). However, it has been shown \([19]\) that these contributions cannot be suppressed for any value of \( a \). Indeed, Read \([20]\) has demonstrated that the choice of the off-shell parameter in the coupling is closely linked to the off-shell behavior of the propagator.

To overcome this problem Pascalutsa suggested gauge invariance as an additional constraint to fix the interaction Lagrangians for higher spins and remove the lower-spin components \([21]\). Constructing the spin-$\frac{3}{2}$ interaction for a Rarita-Schwinger field \( u_{\frac{3}{2}} \) by only allowing couplings to the gauge-invariant field tensor \( U_{\mu \nu} = \partial^\mu u^\nu - \partial^\nu u^\mu \) Pascalutsa derived an interaction which (for example) for the \( \pi N \Delta \) coupling is

\[
\mathcal{L}_{\pi N \Delta} = \int u_{\pi \gamma} \gamma_5 \gamma_\mu \bar{u}_{\pi} \gamma_\rho \partial_\nu \varphi + \text{h.c.},
\]

where \( \bar{u}_{\pi} \) is the tensor dual to \( U_{\pi \nu} \). To this end the inelastic \( 2\pi N \) channel is parameterized by an effective \( \zeta N \) channel where \( \zeta \) is an effective isovector meson with mass \( m_\zeta = 2m_\pi \). Thus \( \zeta N \) is considered as a sum of different \( (\pi \Delta, \rho N, \text{etc.}) \) contributions to the total \( 2\pi N \) flux. We allow only resonance \( \zeta N \)-couplings since each background diagram would introduce a meaningless coupling parameter. Despite this approximation the studies \([23,14]\) have achieved a good description of the total partial-wave cross sections \([23]\) and we proceed in our calculations by using the above prescription. For the \( R \to \zeta N \) interaction the same Lagrangians are used as for the \( R \to \pi N \) couplings taking into account the positive parity of the \( \zeta \) meson.

### 3 Results and discussion

We use the same database as in \([4]\) with additional elastic \( \pi N \) data for the spin-$\frac{3}{2}$ partial wave amplitudes taken from the VPI group analysis \([25]\). To the end the inelastic \( 2\pi N \) channel is parameterized by an effective \( \zeta N \) channel where \( \zeta \) is an effective isovector meson with mass \( m_\zeta = 2m_\pi \). Thus \( \zeta N \) is considered as a sum of different \( (\pi \Delta, \rho N, \text{etc.}) \) contributions to the total \( 2\pi N \) flux. We allow only resonance \( \zeta N \)-couplings since each background diagram would introduce a meaningless coupling parameter. Despite this approximation the studies \([23,14]\) have achieved a good description of the total partial-wave cross sections \([23]\) and we proceed in our calculations by using the above prescription. For the \( R \to \zeta N \) interaction the same Lagrangians are used as for the \( R \to \pi N \) couplings taking into account the positive parity of the \( \zeta \) meson.

\[
\sum_{\gamma} q_{\mu} q_{\nu} / m_R^2 \to q_{\mu} q_{\nu} / q^2.
\]

This procedure is similar to that which has been used in the spin-$\frac{1}{2}$ case \([21]\). It has been shown for the spin-$\frac{3}{2}$ case \([22]\) , that both prescriptions are equivalent in the effective Lagrangian approach as long as additional contact interactions are taken into account when the Pascalutsa couplings are used. The differences between these descriptions have been discussed in \([5,23,24]\) and here we perform calculations by using both ‘conventional’ (C) and Pascalutsa (P) approaches. Similar to the spin-$\frac{1}{2}$ case \([21]\), the off-shell parameters \( a \) in \([3]\) can be linked to the coupling strengths extracted through \([6]\).

In order to take into account the internal structure of mesons and baryons each vertex is dressed by a corresponding formfactor:

\[
F_p(q^2, m^2) = \frac{A^4}{A^4 + (q^2 - m^2)^2}.
\]

Here \( q \) is the four momentum of the intermediate particle and \( A \) is a cutoff parameter. In \([5]\) it has been shown that the formfactor \([12]\) gives systematically better results as compared to other ones, therefore we do not use any other forms for \( F(q^2) \). The cutoffs \( A \) in \([12]\) are treated as free parameters and allowed to be varied during the calculation. However we demand the same cutoffs in all channels for a given resonance spin \( J : A^{I}_{\pi N} = A^{I}_{\pi N} = A^{I}_{\eta N} = \ldots \) etc., \( (J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}) \). This greatly reduces the number of free parameters; i.e. for all spin-$\frac{5}{2}$ resonances there is only one cutoff \( A^{\frac{5}{2}} \) for all decay channels.

To take into account contributions of the \( 2\pi N \) channel in our calculations we use the inelastic partial-wave cross section \( \sigma_{2\pi N}^I \) data extracted in \([25]\). To this end the inelastic \( 2\pi N \) channel is parameterized by an effective \( \zeta N \) channel where \( \zeta \) is an effective isovector meson with mass \( m_\zeta = 2m_\pi \). Thus \( \zeta N \) is considered as a sum of different \( (\pi \Delta, \rho N, \text{etc.}) \) contributions to the total \( 2\pi N \) flux. We allow only resonance \( \zeta N \)-couplings since each background diagram would introduce a meaningless coupling parameter. Despite this approximation the studies \([23,14]\) have achieved a good description of the total partial-wave cross sections \([23]\) and we proceed in our calculations by using the above prescription. For the \( R \to \zeta N \) interaction the same Lagrangians are used as for the \( R \to \pi N \) couplings taking into account the positive parity of the \( \zeta \) meson.

The results presented in the following are from ongoing calculations to describe the data in all channels simultaneously. The resulting \( \chi^2 \) of our best overall hadronic fits are given in Table 1. The obtained \( \chi^2_{\pi \pi} \) and \( \chi^2_{\pi \eta} \) are calculated using experimental data from all \( \pi N \) and \( 2\pi N \) partial waves up to spin-$\frac{5}{2}$ except the \( D_{25} \) wave. We find a problem with the description of the \( D_{25} \) partial wave so the resulting \( \chi^2_{\pi \pi} \) turns out to be very large. Hence the \( \chi^2_{\pi \pi} \) values given in Table 1 are calculated by neglecting the latter wave.
the $\pi N$ datapoints for the $D_{35}$ partial wave. From Table 1 one can conclude that the conventional $C$-prescription leads to a better description of the data in all partial waves. Note, that since the $P$ coupling does not have ‘off-shell’ background we also include additional $D_{13}(1700)$ and $S_{31}(1900)$ resonances in the $P$-calculations.

Compared to the previous best hadronic fits $C$-$p$-$\pi^+$ and $P$-$p$-$\pi^+$ from [5], we obtain the same values for cut-offs and non-resonant couplings. The only exception is the $A_{1^-} = 2.79$ for the $C$-coupling which is less than that of $C$-$p$-$\pi^+$. In addition we find $g_{NN\pi} = 4.59(4.20)$ and $\kappa_{N\pi\pi} = 0.12(0.06)$ for $C(P)$ coupling calculations which slightly differ from [5]. The results for the $\pi N$ partial wave amplitudes are shown on the Figs. 1-2 in comparison with $C$-$p$-$\pi^+$ result from [5]. We do not show here the corresponding $P$-$p$-$\pi^+$ result since it almost coincides with the new $P$-calculations. The main differences are found for the conventional coupling calculations in comparison with the previous study. A substantially better description in the $P_{13}$ partial wave is due to the additional off-shell background generated by spin-$\frac{5}{2}$ resonances. The same effect also improves the description of the real and imaginary high energy tails of the $P_{31}$ and $S_{31}$ amplitudes, respectively. The contribution from the spin-$\frac{5}{2}$ resonances can also be seen in the $D_{33}$ amplitude which is also affected by spin-$\frac{5}{2}$ off-shell components. This leads to a worsening in the imaginary part of $D_{33}$ above 1.8 GeV, giving however improvement in the corresponding real part.

The $D_{15}(1675)$, $F_{15}(1680)$, and $F_{15}(1900)$ resonances were included in our calculations. We have also found evidence for a second $F_{15}$ state around 1.98 GeV which is rated two-star by [7]. The results for $\pi N \to 2\pi N$ partial wave cross sections are shown in Fig. 3 We stress that the $\pi N$ partial wave inelasticities are not fitted but obtained as a sum of the individual contributions from all open channels.

In the following each spin-$\frac{5}{2}$ wave is discussed separately. The extensive discussion of the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ partial waves can be found in [5]. The parameters of the corresponding baryon resonances are listed in [27].

$D_{15}$. The elastic VPI data show a single resonant peak which corresponds to the well established $D_{15}(1675)$ state. We find a good description of the elastic amplitude in both the $C$- and $P$-calculations.

The $2\pi N$ data [25] are systematically below the total inelasticity of the VPI group [26]. This can be an indication that apart from $2\pi N$ there are additional contributions from other inelastic channels. However, in the analysis of Manley and Saleski [28] as well as in the most recent study of Vrana et al. [29] the total inelasticity in the $D_{15}$ wave is entirely explained by the resonance decay to the $\pi\Delta$ channel. We also find no significant contributions from the $\eta N$, $K\Lambda$, $K\Sigma$, and $\omega N$ channels to the total $\pi N$ inelasticity in the present hadronic calculations. The calculated $2\pi N$ cross sections are found to be substantially above the data from [24] in all fits. Indeed, the difference between the $2\pi N$ and inelasticity data runs into 2 mb at 1.67 GeV. This flux can be absorbed by neither $\eta N$, $K\Lambda$, $K\Sigma$, $\omega N$ channels, see Fig. 4. Thus we conclude that either the $\pi N$ and $2\pi N$ data are inconsistent with each other or other open channels (as $3\pi N$) must be taken into account. To overcome this problem and to describe the $\pi N$ and $2\pi N$ data in the $D_{13}$ partial wave the original $2\pi N$ data error bars [25] were enlarged by a factor 3. The same procedure was also used by Vrana et al. [29] and Cutkosky et al. [30] to fit the inelastic data.

In the both $C$ and $P$ coupling calculations the total inelasticities in the $D_{13}$ wave almost coincide with the partial-wave cross sections and therefore are not shown in Fig. 5 (left top). A good description of the inelasticity in the $D_{13}$ wave is achieved and the extracted resonance parameters are also in agreement with other findings (see next section).

$F_{15}$. The $F_{15}(1680)$ and $F_{15}(2000)$ resonances are identified in this partial wave. The inclusion of the second resonance significantly improves the description of the $\pi N$ and $2\pi N$ experimental data in the higher-energy region. Some evidence for this state was also found in earlier works [25]. A visible inconsistency between the inelastic VPI data and the $2\pi N$ cross section from [25] above 1.7 GeV can be seen in Fig. 4 (left bottom). The three data points at 1.7, 1.725, and 1.755 GeV have, therefore, not been included in our calculations. Finally we achieve a reasonable description for both $\pi N$ and $2\pi N$ data. The $C$ and $P$ coupling calculations give approximately the same results.

$F_{35}$. A single resonance state $F_{35}(1905)$ was taken into account. Some other models find an additional lower-lying resonance with a mass of about 1.75 GeV [28, 31]. However, we already find a good description of the elastic $\pi N$ amplitudes and the $2\pi N$ cross sections by only including the single $F_{35}(1905)$ state. The inclusion of a second state with somewhat lower mass leads to a worse description of the $\pi N$ and $2\pi N$ data due to the strong interference between the two nearby states. The two $2\pi N$ data points at 1.87 and 1.91 GeV, which are apparently above the total inelasticity, have not been included in the calculations.

The total inelasticity in the $F_{35}$ partial wave almost coincides with the calculated $2\pi N$ cross section and is not shown in Fig. 5. Note, that the $2\pi N$ data at 1.7 GeV are slightly below the total inelasticity from [26]. This could indicate that other inelastic channels (as $3\pi N$) give additional contributions to this partial wave.

There are also difficulties in the description of the $2\pi N$ low-energy tails of the $D_{15}$ and $F_{15}$ partial waves below 1.6 GeV, where the calculated cross sections are slightly below the $2\pi N$ data. The discrepancy leads to a significant rise in $\chi_{\pi2\pi}^2$ (cf. Table 1). The same behavior has been found in our previous calculations for the $D_{13}$ partial waves [5].

| Fit | Total $\pi$ | $\chi_{\pi\pi}^2$ | $\chi_{\pi\Delta}^2$ | $\chi_{\pi\Sigma}^2$ | $\chi_{\pi\Omega}^2$ | $\chi_{\Omega\Omega}^2$ |
|-----|-------------|----------------|----------------|----------------|----------------|----------------|
| $C$ | 2.60        | 1.83           | 1.93           | 1.83           | 2.14           | 1.37           |
| $P$ | 3.65        | 10.06          | 2.93           | 2.93           | 2.54           | 1.83           |

Table 1. $\chi^2$ of the $C$ (first line) and $P$ fits (second line). The $D_{35} \pi N$ and $2\pi N$ data have not been taken into account (see text).
Fig. 1. The $\pi N \rightarrow \pi N$ partial waves for $I = \frac{1}{2}$. The solid (dashed) line corresponds $C(P)$-calculations. The dash-dotted line is the best hadronic fit $C_p\pi^+$ from [5]. The data are taken from [26].
Fig. 2. The $\pi N \rightarrow \pi N$ partial waves for $I=\frac{1}{2}$. The solid (dashed) line corresponds $C(P)$-calculations. The dash-dotted line is the best hadronic fit $C-P-\pi^+$ from [5]. The dotted line is the result for the $D_{35}$-wave obtained with reduced nucleon cutoff (see text). The data are taken from [26].
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Fig. 3. The inelastic $D_{15}$, $F_{15}$, $F_{35}$, and $D_{35}$ waves. The solid (dashed) line corresponds to calculation $C$ ($P$) for the $2\pi N$ channel. Open and filled circles represent the total inelasticity from the VPI group [26] and the $2\pi N$ data [25], respectively. The calculated inelasticities almost coincide with the calculated $2\pi N$ cross sections and are not shown here. Calculation with a reduced nucleon cutoff is shown by the dotted line.

| $L_{21,2S}$ | mass | $\Gamma_{\text{tot}}$ | $R_{\pi N}$ | $R_{2\pi N}$ | $R_{3\pi N}$ | $R_{K\Lambda}$ | $R_{K\Sigma}$ | $R_{\omega N}$ |
|-------------|------|-----------------|----------|------------|---------|-----------|-----------|-----------|
| $D_{15}(1675)$ | 1665 | 144 | 40.2 | 59.1$(-)$ | 0.6$(-)$ | 0.0$($+) | $-0.04$ | $-0.04$ |
| | 1662 | 138 | 41.2 | 58.4$($+) | 0.4$(-)$ | 0.0$(-)$ | 0.02$($+) | $-0.02$ |

| $F_{15}(1680)$ | 1674 | 120 | 68.5 | 31.5$(-)$ | 0.1$($+) | 0.0$($+) | 0.07$($+) | $-0.07$ |
| | 1669 | 122 | 65.8 | 34.2$($+) | 0.0$(-)$ | 0.0$($+) | 0.13$($+) | $-0.13$ |

| $F_{15}(2000)$ | 1981 | 361 | 9.0 | 84.0$($+) | 4.3$(-)$ | 0.5$($-) | 0.4$(-)$ | 2.2 |
| | 1986 | 488 | 9.5 | 88.2$(-)$ | 0.3$(-)$ | 0.1$($+) | 0.2$(-)$ | 1.7 |

| $F_{35}(1905)$ | 1859 | 400 | 11.3 | 88.7$($+) | $-$ | $-$ | 0.7$($+) | $-$ |
| | 1830 | 457 | 10.3 | 89.7$(-)$ | $-$ | $-$ | 0.0$($+) | $-$ |

Table 2. Properties of the spin-$\frac{3}{2}$ resonances considered in the present calculation. Masses and total widths $\Gamma_{\text{tot}}$ are given in MeV, the decay ratios $R$ in percent of the total width. In brackets, the sign of the coupling is given (all $\pi N$ couplings are chosen to be positive). a: The coupling is presented since the resonance is below threshold. b: Decay ratio in 0.1 permil. The first line corresponds to $C$-calculation and the second one to $P$.

There, it has been suggested that the problem might be caused by the description of the $2\pi N$ channel in terms of an effective $\zeta N$ state. Indeed, the findings of [28,29] show strong $\pi\Delta$ decay ratios in all three $D_{15}$, $F_{15}$, and $F_{35}$ partial waves. The description of the $2\pi N$ channel in terms of the $\rho N$ and $\pi\Delta$ channels may change the situation when taking into account the $\rho N$ and $\pi\Delta$ phase spaces and corresponding spectral functions. Upcoming calculations will address this question.

$D_{35}$. A single $D_{35}(1930)$ resonance is taken into account. However, there is no clear resonance structure in the $\pi N$ data for this partial wave. The data [29] also show a total inelasticity at the 2 mb level whereas the $2\pi N$ channel was found to be negligible [24]. It has been suggested [25] that this channel could have an important inelastic $3\pi N$ contribution. Since the measured $2\pi N$ cross section is zero we have used the inelastic $\pi N$ data with enlarged error bars instead of the $2\pi N$ data to pin down the $2\pi N$ $D_{35}$ contributions. Even in this case we have found difficulties in the description of the $D_{35}$ partial wave. The $\pi N$ channel turns out to be strongly influenced by the $u$-channel nucleon and resonance contributions which give
significant contributions to the real part of $D_{35}$. As can be seen in Fig. 2 the $C$- and $P$-coupling calculations cannot give even a rough description of the experimental data [26]. The situation can be improved by either using a reduced nucleon cutoff $A_N$ or by negleecting the nucleon $u$-channel contribution in the interaction kernel. The latter approximation has been used in the coupled-channel approach of Lutz et al. [33]. To illustrate this point we have carried out an additional fit for the $C$-coupling with the reduced cutoff $A_N=0.91$ taking only the $\pi N$ and $2\pi N$ data into account. The calculated $\chi^2$ are $\chi^2_{\pi\pi}=3.63$ and $\chi^2_{2\pi\pi}=7.87$ where the $D_{35}$ data are also taken into account (note, that all values in Table II are calculated by neglecting these datapoints ). The results for the $D_{35}$ partial wave...
are shown in Fig. 2 by the dotted line. In all calculations for \( D_{35} \) presented in Fig. 2 the \( D_{35}(1930) \) mass was found to be about 2050 MeV. One sees that the calculations with a reduced nucleon cutoff lead to a better description of the \( D_{35} \) data giving, however, a worse description of other \( \pi N \) partial-wave data. Note, that a reduction of the nucleon cutoff is required for a successful description of the lower-spin photoproduction multipoles \( [34,31,29] \) which also leads to a worsening in \( \chi^2 \) for the \( \pi N \) elastic channel.

Finally, we conclude that the main features of the considered spin-\( \frac{3}{2} \) partial waves except for \( D_{35} \) are well reproduced. From Figs. 1 \& 2 one can see that there is no significant difference between the conventional [5] and the Pascalutsa [11] spin-\( \frac{5}{2} \) couplings.

The parameters of the spin-\( \frac{3}{2} \) resonances are presented in Table 2. We note that the total resonance widths calculated here do not necessarily coincide with the full widths at half maximum because of the energy dependence of the decay widths \( [11] \) and the formfactors used \( [5] \). We do not show here the parameters of the \( D_{35}(1930) \) resonance because of the problems in the \( D_{35}(1930) \) partial wave. Although a good description of the experimental data is achieved some differences in the extracted resonance parameters for the \( C \)- and the \( P \)-couplings calculations exist.

We obtain a little lower mass for the \( F_{15}(1675) \) as compared to that obtained by Manley and Saleski [28] and Vrana et al. [29], but in agreement with other findings [33,34]. The total width is found to be consistent with the results from [33,34,29]. In the \( \eta N \) channel our calculations show a small (\( \approx 0.6\% \)) decay fraction which is somewhat higher than the value obtained by Batinić et al.: 0.1\% [34], whereas Vrana et al. give another bound: \( \pm 1\% \). We conclude that both fits give approximately the same results for the resonance masses and branching ratios.

The properties of the \( F_{15}(1680) \) state are found to be in good agreement with the values recommended by [7]. We find a somewhat smaller branching ratio in the \( \eta N \) channel as compared to that of [33]. However, the obtained value \( R_{\eta N} = 0.1\% \) is again in agreement with the findings of Vrana et al. [29]: \( \pm 1\% \). The parameters of the second \( F_{15}(2000) \) resonance differ strongly in various analyses: Manley and Saleski [28] give 490 \pm 310 MeV for the total decay width while other studies [30,31] find it at the level of 95 – 170 MeV. Moreover, this state has not been identified in the investigations of [29,33]. Although we find different results for \( T_{tot} \) in the two independent calculations, the branching ratios are close to each other. A small decay width of about 4.3\% is found for the \( \eta N \) channel (C). However, since the \( F_{15}(2000) \) resonance is found to be strongly inelastic with 84-88\% of inelasticity absorbed by the \( 2\pi N \) channel, more \( 2\pi N \) data above 1.8 GeV (cf. Fig. 3) are needed for a reliable determination of the properties of this state.

The properties of the \( F_{15}(1905) \) state are in good agreement with [7]. Both fits give approximately the same result for the decay branching ratios.

All considered resonances have a rather small decay ratios to the \( \eta N, K\Lambda, K\Sigma \), and \( \omega N \) channels. The only exception is the \( F_{15}(2000) \) resonance where a small decay width to \( \eta N \) has been found for the conventional coupling calculations.

In Fig. 4 the results for \( \eta N, K\Lambda, K\Sigma \), and \( \omega N \) total cross sections are shown in comparison with best hadronic fit C-p-\( \pi \) from [4]. The main difference from the previous result is found in the \( \omega N \) final state where a visible effect from the inclusion of the spin-\( \frac{3}{2} \) resonances is found in the \( C \)-calculations. Although the \( D_{15}(1675) \) and \( F_{15}(1680) \) states are below the \( \omega \) production threshold, they give noticeable contributions in the \( C \)-coupling calculations. This effect is, however, less pronounced in the \( P \)-calculations where the role of \( D_{15}(1675) \) and \( F_{15}(1680) \) are found to be less important.

Since the hadronic \( \omega N \) data include about 115 datapoints the couplings to the \( \omega N \) channel are not well constrained and inclusion of photoproduction data may change the situation [5]. Looking to the \( \omega \)-photoproduction reaction the new SAPHIR data may give an opportunity to distinguish between various reaction mechanisms. We are presently working on this [37].

4 Summary and outlook

We have performed a first investigation of the pion-induced reactions on the nucleon within the effective Lagrangian coupled-channels approach including spin-\( \frac{3}{2} \) resonances. To investigate the influence of additional background from the spin-\( \frac{3}{2} \) and -\( \frac{3}{2} \) resonances calculations using both the conventional and the Pascalutsa higher-spin couplings have been carried out. A good description of the available experimental data has been achieved in all \( \pi N, 2\pi N, \eta N, K\Lambda, K\Sigma \), and \( \omega N \) final states within both frameworks. The \( \chi^2 \) is somewhat worse for the Pascalutsa prescription, but this is at least partly due to the absence of additional off-shell parameters in these calculations. In view of this ambiguity in the coupling it is gratifying to see that both coupling schemes lead to similar physical results for the baryon properties. The effective Lagrangian model used in our calculations imposes stringent physical constraints on the various channels and, in particular, on the interplay of the resonance and background contributions. The latter are generated by the same Lagrangian without any new unphysical parameters. Thus any remaining discrepancy between the data and the calculation points to the necessity to improve our understanding of the meson-baryon interactions further, for example, by including additional \( t \)-channel exchanges.

Apart from \( 2\pi N \) we find no significant contributions from other channels to the total \( \pi N \) inelasticities in the spin-\( \frac{3}{2} \) waves. Nevertheless, the contributions from higher-spin resonances can be important in the \( \omega \)-production channel. More data on this reaction are highly desirable to establish the role of different reaction mechanisms.

We have found evidence for \( F_{15}(2000) \) resonance which is rated two-star by [7] and has not been included in the most recent resonance analysis by Vrana et al. [29]. However, more precise \( \pi N \) and \( 2\pi N \) data are necessary to identify this state more reliably in purely hadronic calculations.
For a complete description of $\pi N$ scattering up to higher energies the $J=\frac{3}{2}$ resonances are obviously needed. Compared to our previous study we arrive at a better description in the $\pi N$, $\eta N$, and $\pi\Sigma$ channels for the conventional coupling calculations. Looking only at the lower partial-waves, the improvement in $\pi N$ is only possible due to the additional off-shell background from the spin-$\frac{1}{2}$ resonances. On the other hand, the missing background in the Pascalutsa prescription is compensated by contributions from the $D_{15}$ and $F_{15}$ resonances allowing for a better description in the $\eta N$ and $\omega N$ final states.

We are proceeding with the extension of our model by performing a combined analysis of pion- and photon-induced reactions taking into account spin-$\frac{3}{2}$ states. Moreover, the decomposition of the $2\pi N$ channel into $\rho N$, $\pi\Delta$ etc. states will be the subject of further investigations.

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