Extracting the electric dipole breakup cross section of one-neutron halo nuclei from inclusive breakup observables

K. Yoshida\textsuperscript{1}, T. Fukui\textsuperscript{1}, K. Minomo\textsuperscript{1}, K. Ogata\textsuperscript{1}

\textsuperscript{1}RCNP, Osaka University
Probing halo structure

Halo nucleus can be probed with Electric Dipole (E1) break up cross section $\sigma(E1)$.

$$\sigma(E1) \propto \int_{S_n}^{\infty} n_{E1}(E) \frac{dB(E1)}{dE} dE$$

$C. A. Bertulani, G. Baur, Phys. Rep. 163, 299(1988).$

$n_{E1}(E)$: exponentially decreasing function of $E$.

\[
\begin{align*}
\text{PDR} & \quad \text{a few MeV} & \quad \text{$\sim 0.5 \text{ b}$} \\
\text{GDR} & \quad \text{$\sim 10 \text{ MeV}$} & \quad \text{$\sim 0.1 \text{ b}$}
\end{align*}
\]
Probing halo structure

One-neutron removal cross sections from $^{31}\text{Ne}$ on Pb and C, $\sigma_{\text{Pb}}^{-1n}$ and $\sigma_{\text{C}}^{-1n}$ were measured at RIBF, RIKEN. T. Nakamura et al., PRL 103, 262501 (2009).

but, $\sigma(E1)$ is not observable.

$E1$ cross section formula

$$\sigma(E1) = \sigma_{\text{Pb}}^{-1n} - \Gamma \sigma_{\text{C}}^{-1n}$$
Purpose and method

E1 cross section formula

\[ \sigma(E1) = \sigma_{Pb}^{1n} - \Gamma \sigma_{C}^{1n} \]

1. Justify the validity of E1 cross section formula
2. Find the value of the scaling factor \( \Gamma \)

We aim to establish a quantitatively reliable method of extracting the E1 breakup cross section from observables.

- CDCC (Continuum-Discretized Coupled-Channel method)
- ERT (Eikonal Reaction Theory)
- Microscopic folding model
Elastic Breakup (EB) & stripping (STR)

\[ \sigma^{-1n} = \sigma^{EB} + \sigma^{STR} \]

EB
No target excitation
\[ A(P,c+n)A \]

EXP
Calculated by CDCC

STR
Target excitation
\[ A(P,c+n)A^* \]

ERT
CDCC

Continuum-Discretized Coupled-Channels method with eikonal approximation (E-CDCC) for exclusive reaction cross sections.

M. Yahiro, K. Ogata, T. Matsumoto, K. Minomo, PTEP 2012, 01A206 (2012).

Non-perturbative, non-adiabatic description of break up reaction.

\[
\psi = \phi_0 \chi_0 + \int_0^\infty \phi_k \chi_k dk
\]

\[
\psi^{CDCC} = \sum_{i}^{i_{max}} \hat{\phi}_i \hat{\chi}_i
\]
Eikonal reaction theory (ERT) as an extension of CDCC for inclusive reaction cross section.

M. Yahiroy, K. Ogata, K. Minomo, PTP 126, 167, (2011).

in adiabatic approximation

\[ \hat{\mathcal{S}} = \hat{\mathcal{S}}_c \hat{\mathcal{S}}_n \]

solving Schrödinger equations by CDCC

\[ [T + U_c + h - E] \psi = 0 \quad \longrightarrow \quad \hat{\mathcal{S}}_c \]
\[ [T + U_n + h - E] \psi = 0 \quad \longrightarrow \quad \hat{\mathcal{S}}_n \]

\[ \sigma_{n:STR} = \int d\vec{b} \left\langle \phi_0 \left| \hat{\mathcal{S}}_c |^2 (1 - |\hat{\mathcal{S}}_n|^2) \right| \phi_0 \right\rangle \]
Microscopic reaction theory

Distorting potential

microscopic folding model for calculating the c-T and n-T potentials.

• HF density for the core and target nuclei.
• Melbourne g-matrix for NN interaction.

K. Amos et al., ANP25, 275 (2000).

Reaction systems

✓ Projectiles: $^{11}$Be, $^{15}$C, $^{19}$C, $^{31}$Ne, $^{29}$Ne, $^{33}$Mg, $^{35}$Mg, $^{37}$Mg, $^{39}$Si, $^{41}$Si

established 1n-halo candidates

✓ Targets: $^{12}$C, $^{16}$O, $^{48}$Ca, $^{58}$Ni, $^{90}$Zr, $^{208}$Pb

✓ Incident energy: 250MeV/nucleon
Validity of E1 cross section formula

Two important assumptions to establish E1 formula.

\[ \sigma(E1) = \sigma_{Pb}^{-1n} - \Gamma \sigma_{C}^{-1n} \]

- E1 dominance in Coulomb breakup
  \[ \sigma_{Pb}^{EB}(c) \simeq \sigma_{Pb}^{EB}(E1) \]
- Small interference between Coulomb and Nuclear interaction
  \[ \sigma_{Pb}^{EB} \simeq \sigma_{Pb}^{EB}(n) + \sigma_{Pb}^{EB}(c) \]

We examine
- these two assumptions
- Validity of E1 formula
- Values of \( \Gamma \) factors
Validity of E1 cross section formula

E1 dominance in Coulomb breakup

\[ \sigma_{\text{Pb}}^{\text{EB}}(c) \sim \sigma_{\text{Pb}}^{\text{EB}}(E1) \]

\[ e_{E1} = eZ_c \frac{1}{A} \quad e_{E2} = eZ_c \left( \frac{1}{A} \right)^2 \]

\[ e_{E1} \gg e_{E2} \]

\( \sigma(E2) \) damps rapidly for its \( R^{-3} \).

\[ \sigma(E1) \propto e_{E1}^2 \frac{r}{R^2} \]

\[ \sigma(E2) \propto e_{E2}^2 \frac{r^2}{R^3} \]
Validity of $E1$ cross section formula

$E1$ dominance in Coulomb breakup

$$\sigma_{Pb}^{EB}(c) \approx \sigma_{Pb}^{EB}(E1)$$

$$1 - \frac{\sigma_{Pb}^{EB}(c)}{\sigma_{Pb}^{EB}(E1)}$$

Graph showing the validity of $E1$ cross section formula with different isotopes.
Validity of E1 cross section formula

Small interference between Coulomb and Nuclear interaction

\[ \sigma_{\text{Pb}}^{\text{EB}} \approx \sigma_{\text{Pb}}^{\text{EB}}(n) + \sigma_{\text{Pb}}^{\text{EB}}(c) \]

1. Nuclear breakup at surface, while Coulomb breakup amplitude has a long tail.
2. Angular momentum \( \ell \rightarrow |\ell_0 \pm 1| \) by E1, but no such selection for the nuclear breakup.
Validity of E1 cross section formula

Two important assumptions to establish E1 formula.

- Small interference between Coulomb and Nuclear interaction
  \[ \sigma_{Pb}^{EB} \approx \sigma_{Pb}^{EB}(n) + \sigma_{Pb}^{EB}(c) \]
- E1 dominance in Coulomb breakup
  \[ \sigma_{Pb}^{EB}(c) \approx \sigma_{Pb}^{EB}(E1) \]

\[ \sigma_{Pb}^{EB}(E1) = \sigma_{Pb}^{-1n} - \sigma_{Pb}^{-1n}(n) \]

\[ = \sigma_{Pb}^{-1n} - \Gamma \sigma_{C}^{-1n}(n) \]

\[ \approx \sigma_{Pb}^{-1n} - \Gamma \sigma_{C}^{-1n} \]

\[ \Gamma = \frac{\sigma_{Pb}^{-1n}(n)}{\sigma_{C}^{-1n}(n)} \]

\[ \sigma_{C}^{-1n} \approx \sigma_{C}^{-1n}(n) \]
Validity of E1 cross section formula

Two important assumptions to establish E1 formula.

- Small interference between Coulomb and Nuclear interaction
  \[ \sigma_{\text{Pb}}^{\text{EB}} \sim \sigma_{\text{Pb}}^{\text{EB}}(n) + \sigma_{\text{Pb}}^{\text{EB}}(c) \]

- E1 dominance in Coulomb breakup
  \[ \sigma_{\text{Pb}}^{\text{EB}}(c) \sim \sigma_{\text{Pb}}^{\text{EB}}(E1) \]

**E1 cross section formula**

\[ \sigma(E1) = \sigma_{\text{Pb}}^{-1n} - \Gamma \sigma_{\text{C}}^{-1n} \]

where \( \Gamma \) is defined by

\[ \Gamma = \frac{\sigma_{\text{Pb}}^{-1n}(n)}{\sigma_{\text{C}}^{-1n}(n)} \] about 95% accuracy
Target mass number dependence of $\sigma^{-1n}(n)$

$\sigma^{-1n}(n)$ are proportional to $A^{1/3}$. 
this work

\[ \Gamma = \frac{\sigma_{\text{Pb}}(n)}{\sigma_{\text{C}}(n)} \]

previous study

T. Nakamura et al., PRL 103, 262501 (2009).

\[ \frac{A_{\text{Pb}}^{1/3} + A_{\text{pro}}^{1/3}}{A_{\text{C}}^{1/3} + A_{\text{pro}}^{1/3}} \leq \Gamma \leq \frac{A_{\text{Pb}}^{1/3}}{A_{\text{C}}^{1/3}} \]
scaling factor $\Gamma$

\[ \Gamma = \frac{\sigma_{\text{Pb}}^{-1}n(n)}{\sigma_{\text{C}}^{-1}n(n)} \]

T. Nakamura et al., PRL 103, 262501 (2009).

\[ \frac{A_{\text{Pb}}^{1/3} + A_{\text{pro}}^{1/3}}{A_{\text{C}}^{1/3} + A_{\text{pro}}^{1/3}} \leq \Gamma \leq \frac{A_{\text{Pb}}^{1/3}}{A_{\text{C}}^{1/3}} \]

$2.59$
scaling factor $\Gamma$

this work

$$\Gamma = \frac{\sigma_{Pb}^{-1} n(n)}{\sigma_{C}^{-1} n(n)}$$

previous study

T. Nakamura et al., PRL 103, 262501 (2009).

$$\frac{A_{Pb}^{1/3} + A_{pro}^{1/3}}{A_{C}^{1/3} + A_{pro}^{1/3}} \leq \Gamma \leq \frac{A_{Pb}^{1/3}}{A_{C}^{1/3}}$$

$S_n$: one-neutron separation energy [MeV]
scaling factor $\Gamma$

This work

$$\Gamma = \frac{\sigma_{Pb}^{-1}(n)}{\sigma_{C}^{-1}(n)}$$

Previous study

T. Nakamura et al., PRL 103, 262501 (2009).

$$\frac{A_{Pb}^{1/3} + A_{pro}^{1/3}}{A_{C}^{1/3} + A_{pro}^{1/3}} \leq \Gamma \leq \frac{A_{Pb}^{1/3}}{A_{C}^{1/3}}$$

$$\Gamma = (2.30 \pm 0.41)e^{-S_n} + (2.43 \pm 0.21)$$

$S_n$: one-neutron separation energy [MeV]
summary

E1 cross section formula

$$\sigma(E1) = \sigma_{Pb}^{-1n} - \Gamma \sigma_{C}^{-1n}$$

where $\Gamma$ is defined by

$$\Gamma = \frac{\sigma_{Pb}^{-1n}(n)}{\sigma_{C}^{-1n}(n)}$$

$\Gamma$ has $1n$ separation energy dependence

$$\Gamma = (2.30 \pm 0.41)e^{-S_n} + (2.43 \pm 0.21)$$

$^{31}\text{Ne}$ case

deduced $\sigma(E1)=540$ mb will become 13-20% smaller.

K. Yoshida, T. Fukui, K. Minomo, K. Ogata, PTEP 2014, 053D03
We confirmed (for 1n halo systems)

- E1 is dominant in Coulomb breakup
  \[ \sigma^{EB}_{Pb}(c) \simeq \sigma^{EB}_{Pb}(E1) \]

- Interference between Coulomb and Nuclear interaction is negligible
  \[ \sigma^{EB}_{Pb} \simeq \sigma^{EB}_{Pb}(n) + \sigma^{EB}_{Pb}(c) \]

- Stripping reaction is caused by nuclear interaction
  \[ \sigma^{STR}_{Pb} \simeq \sigma^{STR}_{Pb}(n) \]

- In case of \(^{12}\text{C}\) target, Nuclear interaction is dominant
  \[ \sigma^{-1n}_{C} \simeq \sigma^{-1n}_{C}(n) \]

E1 cross section formula

\[ \sigma(E1) = \sigma^{-1n}_{Pb} - \Gamma \sigma^{-1n}_{C} \quad \text{about 95% accuracy} \]

where \( \Gamma \) is defined by

\[ \Gamma = \frac{\sigma^{-1n}_{Pb}(n)}{\sigma^{-1n}_{C}(n)} \]
\[ \sigma_{Pb}^{-1n} = \sigma_{Pb}^{STR} + \sigma_{Pb}^{EB} \]
\[
\sigma_{Pb}^{-1n} = \sigma_{Pb}^{STR} + \sigma_{Pb}^{EB}
\]

\[
\sigma_{Pb}^{STR} \simeq \sigma_{Pb}^{STR}(n)
\]

\[
\sigma_{Pb}^{EB} \simeq \sigma_{Pb}^{EB}(n) + \sigma_{Pb}^{EB}(c)
\]

\[
\sigma_{Pb}^{-1n} = \sigma_{Pb}^{STR}(n) + \sigma_{Pb}^{EB}(n) + \sigma_{Pb}^{EB}(c)
\]
\[ \sigma_{Pb}^{-1n} = \sigma_{Pb}^{STR} + \sigma_{Pb}^{EB} \]
\[ \sigma_{Pb}^{STR} \simeq \sigma_{Pb}^{STR}(n) \]
\[ \sigma_{Pb}^{EB} \simeq \sigma_{Pb}^{EB}(n) + \sigma_{Pb}^{EB}(c) \]

\[
\sigma_{Pb}^{-1n} = \sigma_{Pb}^{STR}(n) + \sigma_{Pb}^{EB}(n) + \sigma_{Pb}^{EB}(c)
\]

\[
\sigma_{Pb}^{-1n} = \sigma_{Pb}^{-1n}(n) + \sigma_{Pb}^{EB}(c)
\]
\[ \sigma_{Pb}^{EB}(c) = \sigma_{Pb}^{1n} - \sigma_{Pb}^{1n}(n) \]

\[ \sigma_{Pb}^{1n} = \sigma_{Pb}^{STR} + \sigma_{Pb}^{EB} \]

\[ \sigma_{Pb}^{STR} \sim \sigma_{Pb}^{STR}(n) \]

\[ \sigma_{Pb}^{EB} \sim \sigma_{Pb}^{EB}(n) + \sigma_{Pb}^{EB}(c) \]

\[ \sigma_{Pb}^{1n} = \sigma_{Pb}^{1n}(n) + \sigma_{Pb}^{EB}(n) + \sigma_{Pb}^{EB}(c) \]

\[ \sigma_{Pb}^{1n} = \sigma_{Pb}^{1n}(n) + \sigma_{Pb}^{EB}(c) \]
\[ \sigma_{Pb}^{EB}(c) = \sigma_{Pb}^{-1n} - \sigma_{Pb}^{-1n}(n) \]

\[ \sigma_{Pb}^{EB}(c) \simeq \sigma_{Pb}^{EB}(E1) \]

\[ \sigma_{Pb}^{EB}(E1) = \sigma_{Pb}^{-1n} - \sigma_{Pb}^{-1n}(n) \]
\[ \sigma_{Pb}^{EB}(c) = \sigma_{Pb}^{-1n} - \sigma_{Pb}^{-1n}(n) \]
\[ \sigma_{Pb}^{EB}(E1) = \sigma_{Pb}^{-1n} - \sigma_{Pb}^{-1n}(n) \]
\[ \Gamma = \sigma_{Pb}^{-1n}(n)/\sigma_C^{-1n}(n) \]
\[ \sigma_{Pb}^{EB}(c) = \sigma_{Pb}^{-1n} - \Gamma \sigma_C^{-1n}(n) \]
\[ \sigma_{Pb}^{EB}(c) = \sigma_{Pb}^{-1n} - \sigma_{Pb}^{-1n}(n) \]

\[ \sigma_{Pb}^{EB}(c) \simeq \sigma_{Pb}^{EB}(E1) \]

\[ \sigma_{Pb}^{EB}(E1) = \sigma_{Pb}^{-1n} - \sigma_{Pb}^{-1n}(n) \]

\[ \Gamma = \sigma_{Pb}^{-1n}(n)/\sigma_{C}^{-1n}(n) \]

\[ \sigma_{Pb}^{EB}(c) = \sigma_{Pb}^{-1n} - \Gamma \sigma_{C}^{-1n}(n) \]

\[ \sigma_{C}^{-1n}(n) \simeq \sigma_{C}^{-1n} \]

\[ \sigma_{Pb}^{EB}(E1) = \sigma_{Pb}^{-1n} - \Gamma \sigma_{C}^{-1n} \]
\[
\sigma_{\text{Pb}}^{\text{EB}}(c) = \sigma_{\text{Pb}}^{-1n} - \sigma_{\text{Pb}}^{-1n}(n)
\]
\[
\sigma_{\text{Pb}}^{\text{EB}}(c) \simeq \sigma_{\text{Pb}}^{\text{EB}}(E1)
\]
\[
\sigma_{\text{Pb}}^{\text{EB}}(E1) = \sigma_{\text{Pb}}^{-1n} - \sigma_{\text{Pb}}^{-1n}(n)
\]
\[
\Gamma = \sigma_{\text{Pb}}^{-1n}(n)/\sigma_{C}^{-1n}(n)
\]
\[
\sigma_{\text{Pb}}^{\text{EB}}(c) = \sigma_{\text{Pb}}^{-1n} - \Gamma \sigma_{\text{C}}^{-1n}(n)
\]
\[
\sigma_{\text{C}}^{-1n}(n) \simeq \sigma_{\text{C}}^{-1n}
\]
\[
\sigma_{\text{Pb}}^{\text{EB}}(E1) = \sigma_{\text{Pb}}^{-1n} - \Gamma \sigma_{\text{C}}^{-1n}
\]

we want experiment
We confirmed (for 1n halo systems)

- E1 is dominant in Coulomb breakup
  \[
  \sigma^{EB}_{Pb}(c) \simeq \sigma^{EB}_{Pb}(E1)
  \]

- Interference between Coulomb and Nuclear interaction is negligible
  \[
  \sigma^{EB}_{Pb} \simeq \sigma^{EB}_{Pb}(n) + \sigma^{EB}_{Pb}(c)
  \]

- Stripping reaction is caused by nuclear interaction
  \[
  \sigma^{STR}_{Pb} \simeq \sigma^{STR}_{Pb}(n)
  \]

- In case of $^{12}$C target, Nuclear interaction is dominant
  \[
  \sigma^{1n}_{C} \simeq \sigma^{1n}_{C}(n)
  \]

E1 cross section formula

\[
\sigma(E1) = \sigma^{1n}_{Pb} - \Gamma \sigma^{1n}_{C} \quad \text{about 95% accuracy}
\]

where $\Gamma$ is defined by

\[
\Gamma = \frac{\sigma^{1n}_{Pb}(n)}{\sigma^{1n}_{C}(n)}
\]
$\sigma_C^{EB} \approx \sigma_C^{EB}(n)$ ?

$\sigma_C(c), \sigma_C(n)$

$\sigma_C(n)/\sigma_C$
\[ \sigma_{C}^{EB} \neq \sigma_{C}^{EB}(n) \]

\[ \sigma_{C}(c), \sigma_{C}(n) \]

\[ \sigma_{C}(n)/\sigma_{C} \]
Target mass number dependence of $\sigma^{EB}(n)$
$^{12}\text{C}+^{12}\text{C}$ elastic cross section at 135 MeV/nucleon

M. Yahiro, K. Ogata, T. Matsumoto, K. Minomo, PTEP 2012, 01A206 (review)
\[ \sigma_{A=1}^{-1 n} = \sigma_{A=1}^{EB} \]

\[ (\sigma_{A=1}^{STR} = 0) \]

Core-n

\((l_{\infty}, k_{\infty})\)

CDCC model space

(this work)
\[ \sigma_{A=1}^{-1n} = \sigma_{A=1}^{EB} \]

\[ (\sigma_{A=1}^{STR} = 0) \]

Core-n

\((l_{\infty}, k_{\infty})\)

\((l_{\text{max}}, k_{\text{max}})\)

CDCC model space

(this work)
\[
\sigma_{A=1}^{-1n} = \sigma_{A=1}^{EB} \\
(\sigma_{A=1}^{STR} = 0)
\]
\[
\sigma_{A=1}^{-1n} = \sigma_{A=1}^{EB} \\
\left(\sigma_{A=1}^{STR} = 0\right)
\]

Core-n

\((l_\infty, k_\infty)\)

\((l_{max}, k_{max})\)

“Tightly” bound

CDCC model space

(this work)

STR

(out of model space)
\[ \sigma_{A=1}^{-1n} = \sigma_{A=1}^{EB} \]

\[(\sigma_{A=1}^{STR} = 0)\]

Core-n

\[(l_{\infty}, k_{\infty})\]

\[(l_{\text{max}}, k_{\text{max}})\]

“Tightly” bound

CDCC model space

(this work)

STR

(out of model space)
A-R relation

Effective distance \( R \equiv (J + 1/2) / K \)

A-D relation

\[ 2\pi RD = \sigma \]
A-R relation

\[ R \propto A^{1/3} \]

\[ 1^{11}\text{Be} \]

\[ A^{1/3} = 1.79A^{1/3} + 1.12 \]

\[ 1^{15}\text{C} \]

\[ A^{1/3} = 1.41A^{1/3} + 3.40 \]

\[ 1^{19}\text{C} \]

\[ A^{1/3} = 1.69A^{1/3} + 2.82 \]

\[ 1^{31}\text{Ne} \]

\[ A^{1/3} = 1.61A^{1/3} + 3.97 \]
関係

$2\pi RD = \sigma$

図は、$A^{1/3} \cdot D$ の関係を示しています。データは次の2つの式で表されます：

- $A^{1/3} \cdot D$ relation
- $0.0078A^{1/3} - 0.0076$
A-D relation

$^1\text{Be}$

$^15\text{C}$

$^19\text{C}$

$^{31}\text{Ne}$
multipole expansion

\[ V_{1A} \propto \frac{1}{R_1} = \sum_{\lambda} \frac{r^\lambda}{R^{\lambda+1}} P_{\lambda}(\cos \theta) \]

\(\lambda\): multipolarity