Enhanced Adaptive Successive-Cancellation List Decoder

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ABSTRACT. Adaptive successive cancellation list decoder (ASCLD) with upsizing scheme as a well-known approach can significantly reduce the average list size of SCL algorithm for polar codes with cyclic redundancy check (CRC). Since it needs to re-decode the whole received bits if no survival path verifies CRC, it dramatically increases the decoding latency. In this paper, a segmented ASCLD is proposed to reduce the decoding delay of ASCLD, which deals with the multi-CRCs concatenated polar codes. Unlike the normal ASCLD, once all the survived paths of one decoding partition cannot pass its corresponding CRC, a new decoding attempt is immediately initiated with larger list size. Moreover, based on analyzing the performance of the proposed system, a series of optimization strategies are used to improve the performance. Simulation results show that the proposed scheme can greatly reduce the delay of upsizing ASCLD without performance loss.

1. INTRODUCTION
Polar codes can achieve the symmetric capacity of binary-input discrete memoryless channel as the code length $N \to \infty$ using successive-cancellation (SC) decoding algorithm \cite{1}. For finite code length, however, the error-correcting performance under SC decoding is worse than that of the state-of-the-art codes such as low-density parity-check (LDPC) codes. To improve the performance of polar codes, SC-list (SCL) decoding algorithm is proposed in \cite{2}\cite{3} and cyclic redundant code (CRC) is concatenated as an outer code to improve the accuracy of the final path selection. Such a decoding scheme, which is called Ca-SCL, enables polar codes to compete with the state-of-the-art codes.

However, the downside of SCL is apparent: the requirement of exploring various decoding paths causes as high as $L$ times of the decoding complexity of SC decoder, where $L$ is the list size. The adaptive algorithm is one of the method to solve this problem. \cite{4} proposed an adaptive SCL (ASCL) decoder with list upsizing scheme (upsizing ASCL) in which the list size is not constant but adaptive based on the check results of CRC. This reduces the average list size compared with conventional SCL algorithm while tends to increase the decoding delay. Another downsizing ASCL is proposed in \cite{5} to reduce latency of upsizing ASCL and average list size of Ca-SCL. The list size can be downsized if the decision criterion is met. The criterion, however, is based on empirical evidence and therefore would cause performance degradation.

After that, \cite{6}-\cite{8} each proposed a structure which involves partitioned information vector and CRC. Compared with the conventional single Ca-SCL, these systems can reduce the memory space but the their performance remains to be improved.

In this work, we first propose a segmented upsizing ASCL scheme to reduce the decoding delay, which deals with the coding scheme that the source vector is split into several partitions and each partition is protected by one short CRC. Once none of the survival decoding paths of one partition can
pass the corresponding CRC check, the received codeword is immediately re-decoded, or rather, decoded again with larger list size.

The remainder of this paper is organized as follows: The background on polar codes and the structure of CA-SCL, ASCL and multi-CRC are reviewed in Section II. In Section III, the enhanced ASCL decoding algorithm is proposed to reduce decoding delay. Then, based on analyzing error correcting and detecting features of the proposed system, we give some strategies to improve the performance in Section IV. The complexity of the proposed system is compared with the conventional ASCL and SCL decoder in Section V. The simulation results are given in Section VI. We conclude this paper in Section VII.

2. Notation Conventions

In this paper, the set of binary, positive integer and natural number is denoted by $\mathbb{B}$, $\mathbb{Z}^+$ and $\mathbb{N}$, respectively. Let the calligraphic characters (e.g., $\mathcal{A}$) denote sets, where $|\mathcal{A}|$ is the cardinality of $\mathcal{A}$. Let $\mathcal{A}-\mathcal{B}$ denote the difference set of $\mathcal{A}$ and $\mathcal{B}$, and $\mathcal{A}^c$ denote the complement set of $\mathcal{A}$. Furthermore, we use notation $\mathcal{u}_i^\mathcal{r}$ to denote a vector $(u_i, u_{i+1}, \ldots, u_{i+r})$ where $u_i$ is the $i$-th element. Besides, $\mathcal{u}_i$ is another notation to denote a vector, where the subscript is the index set of the elements. We use $\mathcal{u} = 0$ to denote the $\mathcal{u}$ is all-0 vector. We write bold letters, such as $\mathbf{G}$, to denote the matrices.

Throughout this paper, $\log(.)$ means “logarithm to base $2$” and $\phi$ denote null set. $\psi(.)$ stands for the decoding operation where the subscript is the type of decoder and the parameter in the bracket is the bits to be decoded and $\hat{u}_i^\mathcal{r}$ and $x_\mathcal{b}$ denote the estimated sequence of $u_i^\mathcal{r}$ and $x_\mathcal{b}$, respectively. Especially, $\psi_{L,SCL}(.)$ is used to denote SCL decoding with list size $L$, where $\hat{u}_i^\mathcal{r}(t)$ and $x_\mathcal{b}(t)$ denote the $t$-th decoding path of $u_i^\mathcal{r}$ and $x_\mathcal{b}$, respectively. And $\hat{x}_\mathcal{b}(\Xi)$ denote all the $L$ decoding paths for $x_\mathcal{b}$. The $\varphi(.)$ denotes the CRC encoding operation of the bits in the bracket. We use to $\otimes$ denote the Kronecker product.

3. CRC Concatenated Polar Codes

A polar code with code length $N = 2^r$ and message length $K$ can be denoted by PC($N,K$). The information set and frozen set are denoted as $\mathcal{A}$ and $\mathcal{A}^c$, and the information and frozen bits vector are denoted as $x_\mathcal{b}$ and $x_\mathcal{b} = 0$. In CA-SCL, CRC is concatenated with polar code as an outer code. Let $u_i^\mathcal{r} = 2^x$ and $x_\mathcal{b}$ be the input sequence of CRC and polar code, respectively. The length of CRC bits is $\$S$ and its generator polynomial is $p(x) = p_0 x^r + p_1 x + p_2$. $u_i^\mathcal{r}$ is first encoded by the CRC to generate information vector of polar code:

$$u_i^{x+c} = x_\mathcal{b} = \varphi(u_i^\mathcal{r})$$

(1)

where $u_i^{x+c}$ are the check bits and $|\mathcal{A}|=r+k$. The input vector of polar code $x_\mathcal{b}$ consists of $x_\mathcal{b}$ and $x_\mathcal{b}$. Then, the codeword of polar code can be obtained as:

$$c_i^\mathcal{b} = x_\mathcal{b}G_\mathcal{b} = x_\mathcal{b}G_\mathcal{b}$$

(2)

with $G_\mathcal{b} = \begin{bmatrix} 10^{\mathcal{b}a} \\ 11 \end{bmatrix}$ is the generation matrix of polar code composed of rows in $G_\mathcal{b}$ with indexes in $\mathcal{A}$. Moreover, we use $H_r(\mathcal{C})$ to denote the MHW of the codeword set $\mathcal{C}$. Let $d_{\min}(\mathcal{A})$ and $d_{\min}(\mathcal{A})$ to denote the MHW and subminimal Hamming weight of the codewords with $\mathcal{A}$, respectively.

4. Ca-SCL Decoder

In Ca-SCL, codeword sequence $\chi_i$ is first decoded by SCL decoder. Let $\hat{\chi}_i$ denote an estimate of the information bit $\chi_i$. When estimating the $i$-th ($i \in \mathcal{A}$) bit, SCL decoder considers both the probability of $\hat{\chi}_i = 1$ and $\hat{\chi}_i = 0$, and reserves the $L$ most-likely decoding paths, rather than just one in SC decoder. Thus, every path has a path metric (PM) which is used as a cost function and updated at every estimate.
After estimating a bit, many sorters are required to choose $L$ paths with the lowest PM. The sorting process is time-consuming and the most common algorithm is red-black (R-B) tree sorting with $O(\log L)$. At the end of the SCL decoder, CRC is conducted to select the verify path as the output. We use $\hat{l}_k^i(l)$ to denote the $l$-th candidates decoded sequence. Let $P_l$ denote the event that the $l$-th path can verify CRC, which can be express as:

$$P_l = \{ \varphi^{-1}(\hat{l}_k^i(l)) = \hat{l}_m^i(l), l \in \{1, \ldots, L\} \}$$

(3)

As a contrast, $P_l = \emptyset$ or $\overline{P}$ means that the $l$-th path cannot verify CRC. If more than one path can verify CRC, the most reliable one should be the decoded information source. If the path can not be obtained, the most reliable path will be the decoding outcome. The source vector can be obtained by removing the $r$ bits at the tail. Owning the bijective relation of the codeword set and information set, in this paper the codeword and information vector are deemed to be decoded simultaneously.

5. **Adaptive SCL**

In [4], authors observed that SCL with very small $L$ can decode the received bits correctly in most cases. Based on this observation, they proposed an upsizing ASCL decoding scheme which made several decoding attempts by increasing the list size gradually until at least one decoding path can pass the CRC or the maximum list size ($L_{\text{max}}$) is reached. Compared with conventional SCL decoder, such scheme can achieve similar decoding performance and greatly reduce the average list size. The main problem of the upsizing ASCL is the long decoding delay in that the received codeword is needed to be decoded more than once, or re-decoded, if no verified decoding path. Meanwhile, a downsizing ASCL decoder is proposed in [5] which can reduce the list size if part of decoding paths are much more reliable than others. In this paper, we focus on the optimization of upsizing ASCL. Hence, we simply call it as ASCL if not otherwise specified.

6. **Multiple CRC**

A multiple CRC structure was proposed in [6] to aid SCL decoding. Source vector is split equally into several partitions which are protected and appended by their corresponding CRC sequence. In the following paper, information bits and CRC bits in the same partition are collectively referred to as one partition. At the decoder, on completion of the decoding for one partition, the CRC detection is immediately conducted. Among the paths which can pass the CRC, the most reliable one is selected as the input of the next partition. This decoding scheme uses less memory space but causes performance degradation. That is partly due to the error propagation caused by undetected error, and partly to the number of the codewords with MHW can not be effectively reduced.

**Note.** In the following paper, a Ca-SCL decoder with $L$ paths and CRC length $r$ can be denoted as Ca-SCL[$L, r$]. For an ASCL decoder with maximum list size $L_{\text{max}}$ and CRC length $r$ can be denoted as ASCL[$L_{\text{max}}, r$]. We use MC[$M, r$] to denote a multi-CRC with $M$ partitions and the total CRC length $r$. In addition, for the concatenated coding scheme, codeword vector $c^N$ is also called as standalone polar codeword.

7. **ASCL for Multi-CRC Polar Codes**

In this section, a new polar coding structure is proposed to reduce latency of the original ASCL decoder.

Concretely, it can be seen as a straightforward combination of multi-CRC and ASCL. Compared with the original A-SCL, it can reduce the number of bits that need to be re-decoded and detect errors in a timely manner. We will introduce encoding and decoding process of the scheme respectively.

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**Algorithm 1. LSCD-based Multilevel searching algorithm**

**Input:** $y^N$, $r_c$, $L_{\text{max}}$
Output: Estimated vector $\hat{u}^e$

Initialization: $L \leftarrow 1$

For $m = 0, 1, \ldots, M - 1$

For $j = 1, 2, \ldots, |x_u|$

$\delta \leftarrow \sum_{j=0}^{|x_u|} x_u + j$

$\hat{u}_e \leftarrow \psi_{SCL}^e (u_e)$

$\phi^+(u_e(\Xi))$, $P_2 = \bigcup_{j=1}^L P_j$

If $\{ P_2 = \bigcup_{j=1}^L P_j \}$

$L \leftarrow 2L$

If $\{ L > L_{\text{max}} \}$

Break;

Declare failure;

else

$M \leftarrow 1$, $j \leftarrow 1$

Re-decode

Store the verified path with the lowest PM for the next stage

Return $\hat{u}^e$

7.1 Encoding Procedure

In the encoder, the proposed scheme splits the source vector $u^e_1$ equally into $M$ partitions, i.e. $u^e_1 = \{u, \ldots, u_M\}$, with $u_m = \left\{ u_{i_{m-k}}, \ldots, u_{i_m + \mu} \right\} \in B^k$, $m \in \{1, \ldots, M\}$. Each partition is protected and appended by one CRC sequence. Let $r_m$ be CRC code length for the $m$-th partition, where $\sum_{m=1}^{M} r_m = r$ to maintain the same code rate with conventional Ca-SCL scheme's. The information vector of polar code can be juxtaposed by: $x_u = \{x_1, \ldots, x_u\}$, with $x_u = \phi(u_e)$. After inserting the frozen bits, the input vector of polar code is denoted as: $x^f = \{x_1, \ldots, x_u\}$. In brief, the encoding process of the proposed scheme is similar with multi-CRC's except that the length and polynomial of CRC for each partition can be unequal.

7.2 Decoding Procedure

In the decoding architecture, $L$ is initialized by 1. On the completion of the decoding process for $x_u$, the decoder will conduct CRC detecting and make decision on whether the decoding will continue or re-decode with larger $L$. If there exists any decoding path verifies the current CRC, the decoding will continue and only the most reliable one could be reserved to the next decoding stage. Otherwise, the decoder re-decode the received codeword with doubling-$L$ until $L$ reaches the predefined $L_{\text{max}}$. If $L$ has been increased, it will not be changed back and continue to be used in the next stage. The specific proposed decoder can be described as MC-ASCL $[L_{\text{max}} r_1, r_{\text{max}}]$ and the whole decoding process is summarized in the Algorithm 1.
7.3 Improvements for Proposed System

**Definition 1.** For a transmitted vector \( x \), we define \( e(x) = x \oplus x \) as the error pattern of \( x \). Obviously, \( e(x) = \theta \) stands for decoding correct.

The error pattern can be used for both coded and information bits. CRC cannot detect all the error patterns of the decoded paths from SCL decoder, yet it can be used to reduce the number of concatenated codewords with MHW. For this purpose, we propose a priori protection scheme that the source bits which make contribution to \( A_{\text{min}} \) are given priority to be protected. We first define four types of codewords set which will be used in the subsequent demonstration:

1. \( C^*_\nu \): The set of the standalone polar codewords with index set \( \nu \) except the all zero one, i.e., \( |\nu| = 2^k - 1 \).
2. \( C^*_\nu \): The set of the standalone polar codewords with MHW \( d_{\text{min}} \), which can be obtained by exhaustive search methods based on SCL decoder [9][10].
3. \( C_\nu \): The set of the concatenated codewords with MHW \( d_{\text{min}} \), which consists of the codewords in \( C^*_\nu \) whose information sequence can pass the CRC.
4. \( C_\nu \): The difference set of \( C^*_\nu \) and \( C_\nu \).

**Remark. 1** For all-0 transmitted vector, \( C^*_\nu \) is the set of all the potential error patterns of standalone polar codewords. \( C^*_0 \) is the set of error patterns with weight \( d_{\text{min}} \).\( C^*_\nu \) means all the potential erroneous codewords except the elements in \( C^*_\nu \). Obviously, \( H_M(C^*_\nu) = d_{\text{min}}(\nu) \) and \( |C_\nu| = A_{\text{min}} \).

**Lemma 1.** If a codeword set \( \nu \subset C^*_\nu \) and \( H_M(C_\nu) = H_M(C^*_\nu) \), the codeword set is the subset of \( C^*_\nu \), i.e., \( C_\nu \subset C^*_\nu \).

**Proof:** This lemma can be proved by contradiction. Assuming that \( C_\nu \not\subset C^*_\nu \), then let \( e_\nu \) denote the codeword in \( C_\nu \) but not in \( C^*_\nu \). According to the definition of \( C^*_\nu \), one can obtain \( e_\nu \subset C^*_\nu \) and \( H_M(C_\nu) = d_{\text{min}}(\nu) \). That is contradicted with the condition and the lemma get proved.

Based on the assumption of all-0 sequence, one can learn that for standalone polar codes, the \( C^*_\nu \) influence on the performance most compared with codewords in \( C^*_\nu \) at high \( E_b/N_0 \). Thus, for the concatenated codes, the CRC should be used to minimize \( |C_\nu| \).

In fact, the previous work [11] has proposed a partial protection scheme that CRC only protect the crucial bits to improve the performance of single CA-SCL. The index set of the crucial bits were obtained by empirical evidence. This part will theoretically prove the conjecture. The definition of crucial bits is first introduced.

**Definition:** For polar codes with \( d_{\text{min}}(\nu) \), if the information bits with index in set \( \nu \subset A \), which is denoted as \( \nu_{\text{min}} \), can satisfy two following conditions:

1. When \( H_M(e(\nu_{\text{min}})) = \theta \), the set of all the other potential erroneous estimated codewords \( C^*_\nu \) is the subset of \( C^*_\nu \), i.e. \( C^*_\nu \subset C_{\text{min}} \).
2. \( \nu_{\text{min}} \) is the minimum set and cannot be punctured to satisfy condition 1.

We define \( \nu_{\text{min}} \) as crucial bits and \( \nu_{\text{min}} \) as crucial indices.

A main characteristic of crucial bits can be easily obtained from he definition:

Let \( e_\nu \) and \( \hat{e}_\nu \) denote the transmitted codeword and decoded codeword, respectively. If \( e_\nu \in C_{\text{min}} \), the error pattern of the crucial bits \( e(\nu_{\text{min}}) \neq \theta \).

**Proposition 1.** Given polar codes with information set \( A \), let \( w_i(\nu) \) denote the weight of the \( i \)-th row of \( G^\nu \). The information bits \( \nu_{\text{min}} \) whose index can be described as: \( \nu_{\text{min}} = \{ i \in A | w_i(\nu) = d_{\text{min}}(A) \} \) is crucial bits.
**Proof:** For convenience of notation, in the proof we also use $\mathcal{C}_i$ to denote the set of all the potential erroneous estimated codewords when $e(u_{A_{\min}}) = 0$. We first prove that $A_{\min}$ satisfies the condition (1), i.e. $\mathcal{C}_i \subset \mathcal{C}_i^r$. Based on the all-0 assumption, bits $u_{A_{\min}}$ are ensured to be decoded correctly means that the bits $u_{A_{\min}}$ are fixed to 0. Therefore $u_{A_{\min}}$ can be seen as frozen bits. Thus we can achieve a series equations as following:

$$H_j(C_i^r) = H_j(C_i^{r_{A_{\min}}}) = d_{\min}(A) = H_j(C_i)$$

Eq. (4) includes three steps. It is easy to get step (a) based on above analysis. Step (b) is obtained from the conclusion of [18, Theorem 4] that $H_j(C_i^r)$ is equal to the minimum row weight of $G_i^r$. Step (c) is based on the definition of $\mathcal{C}_i^r$. Meanwhile, it’s obviously that $\mathcal{C}_i^{r_{A_{\min}}} \subset \mathcal{C}_i^r$. From Lemma 1, we can obtain: $\mathcal{C}_i \subset \mathcal{C}_i^r$. Thus the condition (1) holds.

Then we need to prove $u_{A_{\min}}$ cannot be punctured. The proof is straightforward by contradiction. We denote the set of crucial bits as $u_c$ and the redundant bit in $u_{A_{\min}}$ as $u_r$. For decoded sequence with only $u_r$ decoded wrong, the Hamming weight of the corresponding codeword is equal to $d_{\min}(A)$ which contradicts condition (a). Thus, $u_{A_{\min}}$ satisfies two conditions of crucial bits, then the proposition follows.

The above proposition offer a method to identify the bits which make contribution to $C_i^{r_{A_{\min}}}$. Following by this method, we propose a priori protection scheme to reduce $A_{\min}$. For the $m$th partition, if there exist crucial bits in it, the corresponding CRC only protect the crucial bits. Otherwise, CRC still protect all the bits.

8. Results Discussion

The advantages of adaptive SCL and multi-CRC are retained. For $|L_{\min}| = 32$, the mean of $L$ of enhanced MASCL is $(25.43, 19.04, 13.24, 6.84, 4.23, 2.23, 2.03)$ for SNR between 0.5dB and 2.25dB in steps of 0.25dB. And the maximum memory required of the baseline SCL is reduced to $\frac{s + l_r + r_m}{K + r}$. For CRC polynomial designing, the proposed decoder only exhaustive search for $2^r$ values compared with $2^r$ in baseline SCL. Owing to the designed CRC polynomial to eliminate their corresponding codewords with MHW, the re-decoding rates for $M$ blocks are roughly equal as shown in Table.1. Without considering the undetect error, if re-decoding occur in MASCL, it is also inevitable in adaptive SCL. Thus, the re-decoding efficiency of adaptive SCL is improved by about

$$\sum_{i=m}^{K-s} \left( \frac{K - (s + (l_r + r_m))}{M} \right).$$

Table 1. Table captions should be placed above the table

| $L$  | $E_0/N_0$ | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ | $\gamma_4$ | $\gamma$ |
|-----|-----------|------------|------------|------------|------------|---------|
| 32  | 1.50      | 0.050      | 0.056      | 0.082      | 0.003      | 0.191   |
| 32  | 1.75      | 0.052      | 0.081      | 0.123      | 0.005      | 0.261   |
| 32  | 2.00      | 0.113      | 0.103      | 0.219      | 0.006      | 0.441   |
| 32  | 2.25      | 0.046      | 0.164      | 0.555      | 0.012      | 0.777   |
| 128 | 1.50      | 0.162      | 0.142      | 0.104      | 0.002      | 0.410   |
| 128 | 1.75      | 0.264      | 0.242      | 0.133      | 0.003      | 0.642   |
| 128 | 2.00      | 0.082      | 0.152      | 0.494      | 0.005      | 0.733   |
The simulation results for the FER comparison between different schemes are proposed in Fig. 1 and 2. The parameters of the simulation set-up are: \(N = 1024\), \(K = 512\), and \(r = \frac{K}{N}\). For compactness, we use decimal representation for the coefficients of CRC generator polynomial excluding the highest and the lowest power. For instance, \(P = 16386\) stands for \(P(x) = x^{16} + x^{15} + x^{2} + 1\). In baseline CA-SCL decoding, the CRC length \(r\) is considered in \(\{16(16386),24(x^{23} + x^{22} + x^{6} + x^{5} + x + 1)\}\). In Fig. 1 the total length of CRC is 24 and in Fig. 2 the length is 16. In MASCL and its enhanced version, \(M\) is fixed to 4 and \(r_{m} = 4\) or 6. For MASCL the polynomial is \(P = 7\) and \(P = 1\) when \(r_{m} = 5\) or \(r_{m} = 6\), respectively. In the enhanced scheme, the polynomial is searched by Algorithm 4. For \(r_{m} = 4\), \(P = \{1,2,7,7\}\); for \(r_{m} = 6\), \(P = \{1,13,7,1\}\). It can be seen that adaptive SCL have similar performance with original SCL decoding at high SNR. One can observe in Fig. 2 that for 24-bits CRC, MASCL have about 0.15 dB gap from baseline CA-SCL at high SNR, while can be improved 0.2dB by the enhanced version. In Fig. 3, we compared with the performance of different schemes with 16-bits CRC. The performance of enhanced MASCL is slightly better than CA-SCL’s while both outperform MASCL about 0.15 dB. In addition, enhanced MASCL can be more flexible in \(r_{m}\) and \(M\). We empirically adjust the CRC length according to different \(r_{\text{min}}\) for each sub-blocks, the performance has a potential for further promotion. In Fig. 3, we give a empirically scheme, in which \(M = 3\), \(r = \{5,6,5\}\) and the polynomial. The performance of MASCL can be improved by about 0.2 dB. \(P = \{3,1,2\}\)

![Figure 1. Performance comparison of difference schemes with 24-bits CRC.](image1)

![Figure 2. Performance comparison of difference schemes with 16-bits CRC.](image2)

### 8.1 Conclusion

We proposed MASCL to improve the re-decoding efficiency of adaptive SCL. By analyzing the CRC is the crux to the performance loss of previous works. We proposed an enhanced MASCL decoder by designing CRC. The simulation results show that the enhanced MASCL can outperform original CA-SCL while reducing the decoding complexity and memory.

### REFERENCES

[1] E. Arikan, "Channel polarization: A method for constructing capacity-achieving codes for symmetric binary-input memoryless channels," IEEE Trans. Inf. Theory, vol. 55, no. 7, pp. 3051-3073, Jul. 2009.
[2] I. Tal and A. Vardy, “List decoding of polar codes,” IEEE Trans. Inf. Theory, vol. 61, no. 5, pp. 2213-2226, May 2015.
[3] K. Niu and K. Chen, “CRC-aided decoding of polar codes,” IEEE Commun. Lett, vol. 16, no. 10, pp. 1668-1671, Oct. 2012.
[4] B. Li, H. Shen and D. Tse, “An adaptive successive cancellation list decoder for polar codes with cyclic redundancy check,” IEEE Commun. Lett., vol. 16, no. 12, pp. 2044-2047, Dec. 2012.
[5] C. Zhang, Z. Wang, X. You and B. Yuan, “Efficient adaptive list successive cancellation decoder for polar codes,” in Proc. Asilomar Conference on Signals, Systems and Computers (Asilomar), Nov 2014, pp. 126–130.
[6] J. Guo, Z. Shi, Z. Liu, Z. Zhang and Q. Liu, “Multi-CRC polar codes and their applications,” IEEE Commun. Lett., vol. 20, issue. 2, pp. 212-215, Dec. 2015.
[7] H. Zhou, C. Zhang, S. Xu and X. You, “Segmented CRC-aided SC list polar decoding,” in Proc. IEEE Vehicular Technology Conference-Spring (VTC-Spring), Nanjing, China, Nov. 2016, pp. 1-5.
[8] S. A. Hashemi, M. Mondelli, S. H. Hassani, R. Urbanke and W. J. Gross, “Partitioned list decoding of polar codes: analysis and improvement of finite length,” in Proc. of the IEEE Global Commun. Conference (GLOBECOM), Singapore, Singapore, Dec. 2017, pp. 1-7.
[9] Z. Z. Liu, K. chen, J. Niu and Z. Q. He, “Distance spectrum analysis of polar codes,” in Proc. IEEE WCNC, Apr. 2014, pp. 6-9.
[10] Q. S. Zhang, A. J. Liu, X. F. Pan and K. G. Pan, “CRC code design for list decoding of polar codes,” IEEE Commun. Lett., vol. 21, no. 6, Jun. 2017, pp. 1229 - 1232.
[11] Q. S. Zhang, A. J. Liu and X. F. Pan, “Efficient CRC concatenation scheme for polar codes,” IET Electr. Lett., vol.53, no.13, June. 2017, pp. 860–862.