Can the Big Bang singularity be avoided by a single scalar field?

Taotao Qiu

Physics Department, Chung-Yuan Christian University, Chung-li 320, Taiwan, Republic of China

E-mail: qiutt@mail.ihep.ac.cn

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Abstract
In this paper, we investigate the possibility of avoiding the Big Bang singularity with a single scalar field which couples non-minimally to gravity. We show that in the case when gravity couples linearly to the field, some severe conditions on the field’s potential have to be imposed. However, in the nonlinear case, it is quite generic to avoid the singularity with the single scalar field.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

The Big Bang singularity (BBS) is a notorious problem that exists in standard Big Bang theory as well as inflation theory [1]. People have made a lot of effort in solving this problem, and lots of mechanisms have been carried out, among which there are scenarios from string theory, quantum gravity theory and so on. Either reduced from these theories, or directly arising from the effective field theory, we are able to obtain a bounce solution in four-dimensional spacetime. Bouncing cosmologies, especially non-singular ones, have gained more and more development in many aspects [2]. As an alternative of the standard theory in the early universe it can not only avoid BBS, but also gives well-favored observational effects compared to the astronomical data, such as scale-invariant power spectrum and suppression of CMB low quadrupole on large scales [3]. Besides, a scenario of bouncing cosmology can give rise to large non-gaussianities [4].

If we consider our universe to be flat, then within the framework of Einstein’s general relativity in four-dimensional spacetime, a bounce is usually realized by quintom matter [5].

As has been briefly proved in [5], in order for a bounce to happen, where the scale factor of the universe contracted to some value and then expanded, the Hubble parameter has to evolve

1 Of course there are many other alternatives which lead to a bounce, such as non-scalar fields or non-perfect fluids [6]. We thank the anonymous referee for reminding us of this.
from a negative value to a positive one. It will cause the EoS of the universe to be less than \(-1\), violating the null energy condition (NEC). Generally, some phantom part will get involved that makes the physics unclear [7].

However, this dilemma can be ameliorated in many ways, by introducing non-scalar fields, or higher derivative operators and so on. One of the simplest ways is to go with the help of non-minimal coupling to gravity [8]. In quantum gravity in curved spacetime, it is argued that the existence of the non-minimal coupling term is required by the quantum corrections and renormalization [9]. This coupling includes scalar–tensor theories where the Ricci scalar couples to the field through the term \(F(\phi)R\) [10] (which will be called ‘linear coupling’ for convenience), which contains Brans–Dicke theory [11] or dilaton theory [12, 13], see [14] for a comprehensive review. Since the inclusion of the non-minimal coupling term can drive cosmic acceleration with a wider class of potential than usually considered, it can also be utilized as inflaton in the early time [15] and dark energy at current epoch [16]. With the help of non-minimal coupling, a single scalar field can also behave like systems of multi-degrees of freedom without really involving a ghost field, such as having its equation of state (EoS) cross the cosmological constant boundary as ‘Quintom’ matter [17]. Moreover, a great deal of non-minimal coupling theories can find their equivalence to modified gravity theories [20].

In this paper we study the possibility of realizing bouncing scenario with a single scalar field non-minimally coupled to gravity. We find that generally speaking it does work as expected, except that for some specific case where gravity couples linearly, some constraints have to be imposed on the field action. The paper is organized as follows: in section 2 we briefly demonstrate the general case for a non-minimal coupling field to drive a bounce; in section 3 and 4 we investigate the linear coupling case and nonlinear coupling case respectively, while conclusions come in the last section.

### 2. General conditions for a single field to avoid Big Bang singularity

To begin with, let us consider the most general action containing one scalar field and gravity, in which the two components are coupled in a very general form:

\[
S = \int d^4x \sqrt{-g} \left\{ \frac{f(R, \phi)}{2\kappa^2} + P(X, \phi) \right\},
\]

where \(\kappa^2 \equiv 8\pi G\) and the metric used here is \(g_{\mu\nu} = \text{diag}[1, -a^2(t), -a^2(t), -a^2(t)]\). \(X\) denotes the kinetic term of the field: \(X \equiv \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi\). We are working throughout this paper under the Jordan frame\(^3\). By variation of the action we can easily obtain the Friedmann equations:

\[
\frac{f(R, \phi)}{2} - \kappa^2 P(X, \phi) + \kappa^2 P_X \dot{\phi}^2 + 3(\dot{H} + H^2)f_R - 3H \dot{f}_R = 0,
\]

\[
\frac{f(R, \phi)}{2} + \kappa^2 P(X, \phi) - f_R(\dot{H} + 3H^2) + \dot{f}_R + 2H \dot{f}_R = 0,
\]

where \(f_R \equiv \frac{df}{dR}\), and the equation of motion for the field \(\phi\) is

\[
-P_X \Box \phi - P_{X\mu} \nabla_\mu \nabla^\nu \phi - P_{X\phi} \nabla_\mu \phi \nabla^\nu \phi + P_\phi + \frac{f_\phi}{2\kappa^2} = 0.
\]

\(^2\) For Quintom investigations please see [18] and the following literature; one can also see [19] for a review.

\(^3\) As is well known, the non-minimal coupling can be got rid of by performing the conformal transformation and redefining the scalar field. In that case, a bounce may not occur. This is due to the physical difference between Jordan and Einstein frames, in which the evolutions of the universe are not the same. We want to clarify this in some future work.
where $\Box \phi = \ddot{\phi} + 3H\dot{\phi}$ and similarly $f_{\phi} \equiv \frac{df}{d\phi}$ is the derivative of $f(R, \phi)$ with respect to the field $\phi$.

As mentioned before, for a flat universe with standard Einstein gravity in four-dimensional spacetime, a bounce occurs only when the NEC is violated. We can learn from Einstein equations that this leads to the vanishing of the Hubble parameter $H$ with a positive time derivative, namely $H = 0$ and $\dot{H} > 0$ at the bounce point [5]. Substituting the first condition into the above equations one gets

$$3\dot{f}_{R} + \kappa^2 (\rho + 3P) + f = 0, \quad (5)$$

$$\dot{H} = \frac{f + 2\kappa^2 \rho}{6f_{R}} > 0. \quad (6)$$

In the following sections, we will analyze in detail the application of these conditions to the non-minimal coupling scalar field. Before starting, we would like to classify the coupling term into two categories according to its properties: one is a more specific case where gravity couples linearly to the field, and the other is a more general case which refers to nonlinear coupling. Actually, these two categories are very different due to the different numbers of degrees of freedom in the background equations. Due to this, very different conclusions may be derived.

3. Category I: gravity coupling linearly to the field

3.1. General solution

Among the various possible forms the coupling terms may have, it is the simplest to study the one that Ricci scalar couples linearly to the field, for which it is easy to get a renormalizable theory without any suppression. Moreover, differing from its nonlinear counterpart, in this case the higher order derivative of $R$ is zero; thus, higher order time derivatives of $H$ will not get involved in the background equations. To study the general solution of this case, let us assume that $f(R, \phi) = R(1 - F(\phi))$, where the first term in the parentheses denotes the standard term from general relativity, and the second comes from the non-minimal coupling. From equations (2) and (3), one can get

$$-\frac{R(1 - F(\phi))}{2} + \kappa^2 \rho + 3\dot{H}(1 - F(\phi)) = 0, \quad (7)$$

$$\frac{R(1 - F(\phi))}{2} + \kappa^2 P(X, \phi) - \dot{H}(1 - F(\phi)) - F(\phi) = 0, \quad (8)$$

which can further be simplified to a very neat form:

$$-3\dot{H}^2(1 - F) + \kappa^2 \rho + 3H\dot{F} = 0, \quad (9)$$

$$2\kappa^2 X P_X + 2\dot{H}(1 - F(\phi)) - 2XF_{\phi} - F_{\phi} = 0. \quad (10)$$

In deriving these equations, we have made use of the fact $\rho = 2XP_X - P$ and $X = \dot{\phi}\ddot{\phi}$. The last of the above equations gives

$$\dot{\phi} = \frac{2\kappa^2 X P_X + 2\dot{H}(1 - F(\phi)) - 2XF_{\phi}}{F_{\phi}}. \quad (11)$$
For $H = 0$, we have

$$\rho = 0,$$

(12)

$$H = -\kappa^2 \rho_X (2k^2 XP_X - 2XF_{\phi\phi}) + \rho_\phi F_{\phi}.$$  

(13)

In deriving the last equation we also made use of equation (4). We can see from above that for the case of gravity coupling linearly to the scalar field, a bounce requires the rhs of the above equation to be larger than 0 as well as the energy density equal to 0. This can be viewed as a general condition for this case.

3.2. Single scalar field with a canonical form

First let us consider the simplest case in which the field has a standard canonical form:

$$P(X, \phi) = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi),$$

(14)

where $V(\phi)$ is its potential. Thus, the energy density of the field $\rho = \frac{1}{2} \dot{\phi}^2 + V(\phi).$ The condition of $\rho = 0$ requires that at the bounce point $\frac{1}{2} \dot{\phi}^2 + V(\phi) = 0$, or equivalently $V(\phi) = -\frac{1}{2} \dot{\phi}^2$. It can be straightforwardly seen that as long as $\dot{\phi} \neq 0$ at this point, one needs $V(\phi) < 0$, namely, a potential containing negative values at least at some region is required, while a positive-definite potential as usual does not work. But is that the case even for the possible case where $\dot{\phi} = V(\phi) = 0$, though it refers to a very specific point in $(\phi, \dot{\phi})$ space and needs strong fine-tuning? To see this, let us take a look at the other condition of $\dot{H} > 0$, which then indicates

$$\frac{V_\phi F_{\phi}}{3F_{\phi}^2 + 2\kappa^2(1 - F)} < 0,$$

(15)

where we used $\rho_\phi = V_\phi$ as well as $\rho_X = 1$. From this equation we note that $V_\phi$ can be either larger or less than 0, depending on the form of $F(\phi)$. For example, if we assume that $F(\phi) = \kappa^2 \xi \dot{\phi}^2$, then the condition is $\xi \dot{\phi} V_\phi < 0$. But anyway, as long as $V_\phi \neq 0$ at the bouncing point where $V(\phi) = 0$, there will inevitably be some region where $V(\phi) < 0$. So this will be a generic constraint on the potential of the field. In the usual case this will break down the positive definition of the energy density, which causes unphysics. However, here it is fine because of both the positive kinetic term and the effects of gravity. Thus, after some proper choice of parameters, we can still get a self-consistent system.

As a further study, we also derived the second derivative of $H$ with respect to $t$ at the bouncing point. It appears to be

$$\ddot{H} = \frac{1}{[2\kappa^2(1 - F) + 3F_{\phi}^2]^2} \{ [2\kappa^2(1 - F) + 3F_{\phi}^2] [\kappa^2 (-2\kappa^2 \phi \dot{\phi} - 4H \dot{\phi} F_{\phi} - 4H \dot{\phi} F_{\phi}]

- 4H \dot{\phi}^2 F_{\phi\phi} - V_{\phi\phi} F_{\phi} \dot{\phi} - V_{\phi\phi} F_{\phi} \dot{\phi} + F_{\phi\phi\phi} \dot{\phi}^3 + 2F_{\phi\phi\phi} \phi \dot{\phi} - 12H \dot{H} F_{\phi}^2 - 12H^2 F_{\phi} F_{\phi\phi} \}

+ (2\kappa^2 F_{\phi} \dot{\phi} - 6F_{\phi} F_{\phi\phi} \phi) \{ [\kappa^2 (-\kappa^2 \phi^2 - 4H \dot{\phi} F_{\phi} - V_{\phi} F_{\phi} + F_{\phi\phi} \phi^2) - 6H^2 F_{\phi}^2] \}.$$  

(16)

It can be straightforwardly read off that for the $\dot{\phi} = V(\phi) = 0$ case (note that $H = 0$), $\ddot{H}$ will vanish at the bouncing point. This means that in this case, the velocity of the Hubble parameter always reaches its extreme value when passing through the bouncing pivot. This is a new property that has not been pointed out by other authors in the literature.

In order to support our analytical calculation, we also try to find some regions in parameter space which can give good numerical results. In order to satisfy the constraints on the
Figure 1. The case of a canonical single scalar field and linear non-minimal coupling. The potential is chosen to be $V(\phi) = \frac{1}{2}m^2 \phi^2 - V_0$ and the coupling term is $RF(\phi) = \kappa^2 \xi R \phi^2$. The parameters are chosen to be $m = 0.1 m_{\text{pl}}$, $V_0 = 0.005 m_{\text{pl}}^4$ and $\xi = -1.0$, and the initial conditions are $\phi_i = 0.473 m_{\text{pl}}$, $\dot{\phi}_i = 0.061 m_{\text{pl}}$.

potential, we take into account two forms of potentials: (1) $V(\phi) = \frac{1}{2}m^2 \phi^2 - V_0$ and (2) $V(\phi) = V_0(e^{\lambda \phi^2} - \frac{1}{2})$. We add some negative zero-point energy to the potential to let it get some negative region. For the coupling term, we choose $F(\phi) = \kappa^2 \xi \phi^2$, where $\xi$ is some coupling constant. In figures 1 and 2 we show some numerical calculations on the Hubble parameter $H$ and the scale factor $a$. We can see that a bounce can occur naturally with $H$ crossing the zero divide line. This indicates that, in this way, the BBS can be avoided.

3.3. Single scalar field with the DBI form

Besides canonical ones, there is another form of scalar field that is very commonly used in cosmology: the Dirac–Born–Infeld (DBI) form. Actions of this form effectively describe tachyon dynamics, which can be obtained naturally in string theory [21]. Moreover, it can also drive the acceleration of the universe and can act as inflaton [22] and dark energy [23]. As inflationary models inspired by string theory, the DBI action non-minimally coupled to gravity has also been investigated in the literature, cf [24]. Here we study the last case to see whether it can avoid the Big Bang singularity.

The DBI Lagrangian containing a non-minimal coupling term is

$$P(X, \phi) = -V(\phi) \sqrt{1 - 2\alpha X},$$

(17)

where $\alpha$ is some positive constant. Its energy density can be calculated as $\rho = 2 XP_X - P = \frac{V(\phi)}{\sqrt{1 - 2\alpha X}}$. One can also obtain its EoS and sound speed square by their definitions

$$w \equiv \frac{P}{\rho} = 2\alpha X - 1, \quad c_s^2 \equiv \frac{P_X}{\rho_X} = \sqrt{1 - 2\alpha X},$$

(18)

which shows that this kind of action is stable under classical perturbations ($0 < c_s^2 < 1$) with quintessence-like behavior ($-1 < w < 0$).
The potential is chosen to be $V(\phi) = V_0(e^{\lambda \phi^2} - \frac{1}{2})$ and the coupling term is $RF(\phi) = \kappa^2 \xi R \phi^2$. The parameters are chosen to be $\lambda = -1.0m_{pl}^{-2}$, $V_0 = 0.05m_{pl}^4$ and $\xi = 1.0$, and the initial conditions are $\phi_i = 6.916 \times 10^{-4}m_{pl}$, $\dot{\phi}_i = 3.215 \times 10^{-4}m_{pl}^2$.

From (17) we obtain the equation of motion for the scalar field:

$$\square \phi = \frac{\alpha V}{2(1 - 2\alpha X)} - \frac{\alpha^2 V}{(1 - 2\alpha X)^2} \dot{X} \phi - \frac{\alpha V \dot{\phi}}{2(1 - 2\alpha X)^2} - \frac{RF\phi}{2\kappa^2} = 0,$$

where we get

$$\dot{\phi} = \frac{-2\kappa^2(3H\alpha V \phi + V_\phi + RF\phi(1 - 2\alpha X)^{\frac{1}{2}})(1 - 2\alpha X)}{2\kappa^2 - 6H^2F^2\phi(1 - 2\alpha X)^{\frac{1}{2}}}. (20)$$

and the Friedmann equation

$$3H^2(1 - F(\phi)) = \kappa^2 \frac{V(\phi)}{\sqrt{1 - 2\alpha X}} + 3HF\phi \phi.$$

From above we can see that the first condition at the bounce point $\rho = 0$ only requires $V = 0$, which is looser than that of the canonical scalar field case. Moreover, $F\phi V_\phi < 0$ is still required by the second condition $\dot{H} > 0$. Therefore, in order to have a bounce, a region of negative value of $V(\phi)$ is also inevitable.

We also make the numerical calculations, with the potential of the form $V(\phi) = V_0(e^{\lambda \phi^2} - \frac{1}{2})$ and some proper parameter choice. The bounce happens naturally from the view of the plots. As a side remark, one may note that due to different initial conditions, the model may present various behavior. From figure 3 we can see that the Hubble parameter varies very fast after the bounce, from increasing to decreasing, indicating that the universe will enter a moderate accelerating or decelerating expansion phase soon. From figure 4, one may note that the Hubble parameter experienced a period of slow variation. This indicates that it is also possible to give rise to an inflationary period after the bounce, although we will not discuss this in detail as it goes beyond the current topic.
Figure 3. The potential is chosen to be $V(\phi) = V_0 (e^{\lambda \phi^2} - \frac{1}{2})$ and the coupling term is $RF(\phi) = \kappa^2 \xi R \phi^2$. The parameters are chosen to be $V_0 = 1.0 \text{m}_{\text{pl}}^4$, $\alpha = 1.0 \text{m}_{\text{pl}}^4$, $\lambda = -1.0 \text{m}_{\text{pl}}^2$ and $\xi = 1.0$, and the initial conditions are $\phi_i = 0.407 \text{m}_{\text{pl}}$, $\dot{\phi}_i = 0.776 \text{m}_{\text{pl}}^2$.

Figure 4. The potential is chosen to be $V(\phi) = V_0 (e^{\lambda \phi^2} - \frac{1}{2})$ and the coupling term is $RF(\phi) = \kappa^2 \xi R \phi^2$. The parameters are chosen to be $V_0 = 1.0 \text{m}_{\text{pl}}^4$, $\alpha = 1.0 \text{m}_{\text{pl}}^4$, $\lambda = -1.0 \text{m}_{\text{pl}}^2$ and $\xi = 1.0$, and the initial conditions are $\phi_i = 0.500 \text{m}_{\text{pl}}$, $\dot{\phi}_i = 0.548 \text{m}_{\text{pl}}^2$.

4. Category II: gravity couples nonlinearly to the field

As a comparison, in this section, we will focus on a more general case where, in the coupling term, gravity possesses a nonlinear form. The most general form of basic equations comes from equations (2)–(4). However, for the sake of simplicity, we only discuss the canonical
The potential is chosen as \( V(\phi) = \frac{1}{2}m^2\phi^2 \) and the coupling term is chosen as \( f(R, \phi) = R - 2\kappa^2\xi \ln{(\frac{R}{R_0})}\phi^4 \). The parameters are chosen to be \( m = 1.0m_{pl}, \xi = 1.0 \) and \( R_0 = 1.0m_{pl}^2 \), and the initial conditions are \( \phi_i = 0.783m_{pl}, \dot{\phi}_i = -0.205m_{pl} \).

Therefore, we can derive the Friedmann equations and the equation of motion explicitly:

\[
\ddot{H} = \frac{1}{18Hf_{RR}} \left[ (Rf_R - f)/2 - 3Hf_{R\phi}\phi + \kappa^2 \left( \frac{\phi^2}{2} + V(\phi) \right) - 3f_RH^2 \right] - 4H\dot{H},
\]

\[
\dot{\phi} + 3H\phi + V_{\phi} - f_{\phi} = 0.
\]

Since the nonlinear term of gravity has been involved, the higher order derivatives of \( H \) appear in the equation, which makes the equations difficult to solve analytically. But as the order of derivative increases, the number of effective degrees of freedom increases, and it will be easier to violate NEC and realize the bouncing process.

Figures 5 and 6 show that a canonical field with the coupling term contains the logarithm function of \( R \). The logarithm coupling may seem strange; however, one can find a similar form in the previous phenomenological studies, see e.g. [25]. We can see from the plot that with proper choice of parameter, this kind of coupling may also give rise to a bounce scenario. Figure 7 is the plot of the universe behavior for the coupling of \( R^2 \) to some exponential potential of \( \phi \). This kind of potential looks like the dilaton potential in string theory [12]. With this kind of coupling, the universe can also pass through the bouncing point smoothly.
The potential is chosen as \( V(\phi) = \frac{1}{2}m^2 \phi^2 \) and the coupling term is chosen as \( f(R, \phi) = R - 2\xi \ln \left( \frac{R}{R_0} \right) \phi^4 \). The parameters are chosen to be \( m = 1.0m_{pl}, \xi = 1.0 \) and \( R_0 = 1.0m_{pl} \), and the initial conditions are \( \phi_i = 0.500m_{pl}, \dot{\phi}_i = 0.036m_{pl} \).}

\[ \text{Figure 6.} \]

The potential is chosen as \( V(\phi) = \frac{1}{2}m^2 \phi^2 \) and the coupling term is chosen as \( f(R, \phi) = R - 2\xi \frac{\ln \left( \frac{R}{R_0} \right)}{\xi} \phi^4 \). The parameters are chosen to be \( m = 0.1m_{pl}, \xi = 0.5 \) and \( \lambda = -5 \times 10^{-3}m_{pl}^{-1} \), and the initial conditions are \( \phi_i = 0.621m_{pl}, \dot{\phi}_i = -0.039m_{pl} \).

\[ \text{Figure 7.} \]

5. Conclusions and discussions

In this paper we investigated the possibilities of a single scalar field giving rise to a bouncing scenario in the very early universe, with some non-minimal coupling to gravity. Along with some explicit examples, we showed that it is quite possible. However, for the common case that gravity couples linearly to the field, the field will suffer from some severe conditions, such
as abandoning the positive definition of the potential. For other cases where gravity couples nonlinearly; it will be more free to get a bounce since more degrees of freedom have been evoked.

The realization of bounce with non-minimal coupling field is important in the sense that, with the involvement of gravity, one no longer needs any ghost field to violate NEC, which might cause problems. Recently there have been many works on the construction of bouncing cosmologies within the framework of non-minimal coupling theories, which have some relation to our work while focusing more on their fundamental origins [26]. Furthermore, it is expected that with the proper choice of the coupling form, it is also possible to obtain the right amount of observational signatures to meet the data, and to give rise to some new features and predictions for future experiments. For example, a recent work shows that the matter bounce scenario with non-minimal coupling will give rise to a scale-invariant spectrum and large particle production [27]. All these fancy topics are left for the forthcoming work.

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References

[1] Hawking S W and Ellis G F R 1973 The Large Scale Structure of Space-Time (Cambridge: Cambridge University Press)
Borde A and Vilenkin A 1994 Phys. Rev. Lett. 72 3305
[2] Novello M and Bergliaffa S E P 2008 Phys. Rep. 463 127
[3] Piao Y S, Feng B and Zhang X 2004 Phys. Rev. D 69 103520
Piao Y S 2005 Phys. Rev. D 71 087301
see also Cai Y F, Qiu T, Xia J Q and Zhang X 2009 Phys. Rev. D 79 021303
Cai Y F, Qiu T, Brandenberger R and Zhang X 2009 Phys. Rev. D 80 023511
[4] Cai Y F and Zhang X 2009 J. Cosmol. Astropart. Phys. JCAP06(2009)003
Cai Y F, Xue W, Brandenberger R and Zhang X 2009 J. Cosmol. Astropart. Phys. JCAP05(2009)011
[5] Cai Y F, Qiu T, Piao Y S, Li M and Zhang X 2007 J. High Energy Phys. JHEP10(2007)071
[6] Brechet S D, Hobson M P and Lasenby A N 2008 Class. Quantum Grav. 25 245016
[7] Carroll S M, Hoffman M and Trodden M 2003 Phys. Rev. D 68 023509
Cline J M, Jeon S and Moore G D 2004 Phys. Rev. D 70 043543
[8] Abreu J P, Crawford P and Mimoso J P 1994 Class. Quantum Grav. 11 1919
Bayin S S, Cooperstock F I and Faraooni V 1994 Astrophys. J. 428 439
Fakir R 1998 arXiv:gr-qc/9810054
Lee J W, Koh S, Park C, Sin S J and Lee C H 2000 Phys. Rev. D 61 027301
Biswas T, Mazumdar A and Siegel W 2006 J. Cosmol. Astropart. Phys. JCAP03(2006)009
Biswas T and Mazumdar A 2009 Phys. Rev. D 80 023519
Biswas T, Koivisto T and Mazumdar A 2010 arXiv:1005.0590 [hep-th]
Cai Y F and Saridakis E N 2010 arXiv:1007.3204 [astro-ph.CO]
For review see Mazumdar A and Rocher J 2010 arXiv:1001.0993 [hep-ph]
[9] Chernikov N A and Tagirov E A 1968 Ann. Poincaré Phys. Theor. A 9 109
Callan C G, Coleman S R and Jackiw R 1970 Ann. Phys. 59 42
Birrell N D and Davies P C W 1982 Quantum Fields in Curved Space (Cambridge: Cambridge University Press)
[10] Fuji Y and Maeda K 2003 The Scalar–Tensor Theory of Gravitation (Cambridge: Cambridge University Press)
Bouisseau B, Esposito-Farese G, Polarski D and Starobinsky A A 2000 Phys. Rev. Lett. 85 2236
[11] Brans C and Dicke R H 1961 \textit{Phys. Rev.} 124 925

[12] Gasperini M and Veneziano G 1993 \textit{Astropart. Phys.} 1 317

Gasperini M and Veneziano G 2003 \textit{Phys. Rep.} 373 1

[13] Bamba K and Yoshimura M 2006 \textit{Prog. Theor. Phys.} 115 269

[14] Sotiriou T P and Faraoni V 2010 \textit{Rev. Mod. Phys.} 82 451

De Felice A and Tsujikawa S 2010 \textit{Living Rev. Rel.} 13 3

De Felice A and Tsujikawa S 2010 arXiv:1005.0868 [astro-ph.CO]

[15] Abbott L F 1981 \textit{Nucl. Phys. B} 185 233

Futamase T and Maeda K i 1989 \textit{Phys. Rev. D} 39 399

Futamase T, Rothman T and Matzner R 1989 \textit{Phys. Rev. D} 39 405

Fakir R and Unruh W G 1990 \textit{Phys. Rev. D} 41 1783

Amendola L, Bellissai D and Occhionero F 1993 \textit{Phys. Rev. D} 47 4267

Faraoni V 1996 \textit{Phys. Rev. D} 53 6813

[16] Uzan J P 1999 \textit{Phys. Rev. D} 59 123510

Chiba T 1999 \textit{Phys. Rev. D} 60 083508

Amendola L 1999 \textit{Phys. Rev. D} 60 043501

Amendola L 2000 \textit{Phys. Rev. D} 62 043511

Faraoni V 2000 \textit{Phys. Rev. D} 62 023504

Faraoni V 2005 \textit{Class. Quantum. Grav.} 22 3235

Nojiri S and Odintsov S D 2006 eConf C 0602061 06 (arXiv:hep-th/0601213)

Nojiri S and Odintsov S D 2007 \textit{Int. J. Geom. Meth. Mod. Phys.} 4 115

Szydlowski M, Hrycyna O and Kurek A 2008 \textit{Phys. Rev. D} 77 027302

For non-minimal coupling of Gauss–Bonnet type see e.g. Bamba K, Guo Z K and Ohta N 2007 \textit{Prog. Theor. Phys.} 118 879

Nojiri S, Odintsov S D and Sasaki M 2005 \textit{Phys. Rev. D} 71 123509 and the following literature

[17] Cai R G, Zhang H S and Wang A 2005 \textit{Commun. Theor. Phys.} 44 948

Perivolaropoulos L 2005 \textit{J. Cosmol. Astropart. Phys.} JCAP(2005)001

Apostolopoulos P S and Tetradis N 2006 \textit{Phys. Rev. D} 74 064021

Gannouji R, Polarski D, Starobinsky A A 2006 \textit{J. Cosmol. Astropart. Phys.} JCAP(2006)016

Leith B M and Neupane I P 2007 \textit{Phys. Lett. B} 660 125

Chiba T 2003 \textit{Phys. Lett. B} 575 1

Flanagan E E 2004 \textit{Phys. Rev. Lett.} 92 071101

Flanagan E E 2004 \textit{Class. Quantum. Grav.} 21 417

Olmo G J 2005 \textit{Phys. Rev. D} 72 083505

Sotiriou T P 2006 \textit{Class. Quantum. Grav.} 23 5117

Faraoni V 2007 \textit{Phys. Rev. D} 75 063502

Capone M and Ruggiero M L 2010 \textit{Class. Quantum Grav.} 27 125006

[18] Feng B, Wang X L and Zhang X M 2005 \textit{Phys. Lett. B} 642 187

For quintom with higher derivatives see Li Z, Feng B and Zhang X 2005 \textit{J. Cosmol. Astropart. Phys.} JCAP(2005)002

Zhang X F and Qiu T 2006 \textit{Phys. Lett. B} 642 187

For quintom with non-scalar fields see Cai Y F and Wang J 2008 \textit{Class. Quantum Grav.} 25 165014

[19] Cai Y F, Saridakis E N, Setare M R and Xia J Q 2009 arXiv:0909.2776 [hep-th]

Qu T 2010 \textit{Mod. Phys. Lett. A} 25 909

[20] Flanagan E E 2004 \textit{Phys. Rev. Lett.} 92 071101

Chiba T 2003 \textit{Phys. Lett. B} 575 1

Flanagan E E 2004 \textit{Class. Quantum. Grav.} 21 417

Olmo G J 2005 \textit{Phys. Rev. D} 72 083505

Sotiriou T P 2006 \textit{Class. Quantum. Grav.} 23 5117

Faraoni V 2007 \textit{Phys. Rev. D} 75 063502

Capone M and Ruggiero M L 2010 \textit{Class. Quantum Grav.} 27 125006

[21] Leigh R G 1989 \textit{Mod. Phys. Lett. A} 4 2767

Aharony O, Gubser S S, Maldacena J M, Ooguri H and Oz Y 2000 \textit{Phys. Rep.} 323 183

Myers R C 1999 \textit{J. High Energy Phys.} JHEP12(1999)022

see also Sen A 1998 \textit{J. High Energy Phys.} JHEP08(1998)012

Gerasimov A A and Shatashvili S L 2000 \textit{J. High Energy Phys.} JHEP10(2000)034 and references therein

[22] Dvali G R and Tye S H H 1999 \textit{Phys. Lett. B} 450 72

Kachru S, Kallosh R, Linde A D, Maldacena J M, McAllister L P and Trivedi S P 2003 \textit{J. Cosmol. Astropart. Phys.} JCAP(2003)013

Silverstein E and Tong D 2004 \textit{Phys. Rev. D} 70 103505

see also Easson D A and Gregory R 2009 \textit{Phys. Rev. D} 80 083518
Easson D A, Mukohyama S and Powell B A 2010 Phys. Rev. D 81 023512

[23] Gibbons G W 2002 Phys. Lett. B 537 1

Mukohyama S 2002 Phys. Rev. D 66 024009

Choudhury D, Ghoshal D, Jatkar D P and Panda S 2002 Phys. Lett. B 544 231

Padmanabhan T 2002 Phys. Rev. D 66 021301

Hao J and Li X 2002 Phys. Rev. D 66 087301

Bagla J S, Jassal H K and Padmanabhan T 2003 Phys. Rev. D 67 063504

Copeland E J, Garousi M R, Sami M and Tsujikawa S 2005 Phys. Rev. D 71 043003

Saridakis E N and Ward J 2009 Phys. Rev. D 80 083003

For Quintom models whose EoS can cross −1 see Cai Y, Li M, Lu J X, Piao Y S, Qiu T and Zhang X 2007 Phys. Lett. B 651 1

Shi S G, Piao Y S and Qiao C F 2009 J. Cosmol. Astropart. Phys. JCAP04(2009)027

[24] Easson D A and Gregory R 2009 Phys. Rev. D 80 083518

Easson D A, Mukohyama S and Powell B A 2010 Phys. Rev. D 81 023512

[25] Feng B, Li H, Li M and Zhang X 2005 Phys. Lett. B 620 27

[26] Setare M R, Sadeghi J and Banijamali A 2008 Phys. Lett. B 669 9

Nozari K, Setare M R, Azizi T and Akhshabi S 2009 arXiv:0901.0090 [hep-th]

Nozari K and Sadatian S D 2009 arXiv:0904.4029 [gr-qc]

[27] Qiu T and Yang K C 2010 arXiv:1007.2571 [astro-ph.CO]