Rule Extraction of Generalized Incomplete Variable Multi-granularity Rough Set

MO Jing-lan
Mathematics Teaching Department, Lushan College of Guangxi University of Science and Technology, Liuzhou 545616, Guangxi, China
e-mail: jinglan022@163.com

Abstract—According to the advantages and disadvantages of feature relation and multigranularity decision rough set in generalized incomplete information system, a multigranularity rough set model based on feature relation is proposed. Firstly, a variable multi-granularity rough set model is proposed, and an optimistic and pessimistic variable multi-granularity decision rough model based on feature relation is constructed. By combining variable multi-granularity and approximate distribution reduction, the variable multi-granularity approximate distribution reduction is proposed, and all the decision rules and possible decision rules are obtained. The results show that the method of variable multi-granularity approximate distribution reduction and rule acquisition is more reasonable and effective than the original method of single granularity approximate distribution reduction and decision acquisition.

1. INTRODUCTION
The rough set theory proposed by the famous polish scholar Z. Pawlak [1-2] is a mathematical tool to deal with fuzzy and uncertain data, mainly dealing with complete information system. However, due to various reasons such as limited means of obtaining information and loss of recording information, the information system is incomplete. Kryszkiewicz presented tolerance relation [3] and his binary relation is able to deal with incomplete information system. In order to be able to use the rough set theory to deal with more complex generalized incomplete information systems, J.G. Rzymala. Busse’s feature relation [4] are put forward. Granular computing theory [5-7] is a core part of the study of the current field of artificial intelligence, multi-granularity is a core concept of granular computing theory, said in two or more different granularity constitute a multiple granularity spatially analyzed problem solving research, based on this perspective, Qian Yuhua et al. [8], puts forward the concept of multi-granularity rough set are given more granularity and pessimistic optimistic granularity rough set model, in 2010, Qian Yuhua et al. [9] and presented based on tolerance relation in incomplete multi-granularity rough set model. In the literature [10], Wang Lijuan et al proposed an incomplete multi-degree rough set model based on similarity relation. Literature [11-12] studied the granularity reduction of pessimistic multi-granularity rough sets and proposed different granularity reduction algorithms. Literature [13] studies the granularity reduction of variable multigranularity rough sets, and literature [14] proposes a multigranularity rough decision analysis method based on the possibility degree tolerance relationship in incomplete information system. In the incomplete multi-granularity rough set model proposed in literature [14], any known attribute value can be compared with any...
unknown attribute value, and it is considered that they are equal. However, in reality, many unknown attribute values cannot be compared with known attribute values.

The incomplete multi-granularity rough set model proposed in literature [14] is based on the same assumption: the unknown attribute values in the information system are of omission type, and the incomplete information system cannot process noisy data. There is another interpretation of unknown attribute value in incomplete information system, that is, missing unknown attribute values are not allowed to be compared. In this paper we discuss omissions and lost unknown attribute values at the same time, the existence of generalized incomplete information system, introducing the ideas of multi-granularity generalized incomplete information system, put forward based on the characteristics of the relationship of multi-granularity rough set model and variable multiple granularity rough set model, and puts forward the variable multiple granularity (under) the concept of approximate distribution reduction, thus the most Jane determine the decision rules and to obtain all the minimalist may decision rules.

2. EASE OF USE

2.1. generalized incomplete information system

**Definition 1**[4] Let $S=(U, AT, V, f)$ is an information system, Where $U$ is a non-empty finite set of objects, called on the field; $AT$ is the set of non-empty finite attributes; $V = \bigcup_{a \in AT} V_a$ is the set of property values, and $V_a$ is the range of the attribute $a \in AT$ ; $f : U \times AT \rightarrow V$ is an information function. For a given object $x$, assign $a$ value to the object $x$ under the attribute $a$.If $AT = C \cup D, C \cap D = \emptyset , C$ and $D$ are condition attribute set and decision attribute set respectively, and $V$ is attribute value, $V_c = \{V_q | q \in C\}$ and $V_d = \{V_d | d \in D\}$ are conditional attribute value set and decision attribute value set respectively, then $S$ is called decision system. Unless otherwise stipulated in the $S, \exists x \in U, a \in C, a(x) = *$ or $a(x) = ?$ (“*” represents the missing unknown value, “?” stands for missing type unknown value). If two types of unknown attribute values exist simultaneously, $S$ is called generalized incomplete information system.

2.2. Characteristics of the relationship

**Definition 2**[4] Let $S=(U, AT, V, f)$ is a generalized incomplete information system, for $\forall A \subseteq AT$, the characteristic relationship determined by $a$ is defined as:

$$K(A) = \{(x, y) \in U \times U | \forall a \in A \land f(x,a) \neq f(y,a) \Rightarrow f(x,a) \neq * \vee f(y,a) = * \}.$$  

Feature relation $K(A)$ is regarded as A mixed manifestation of tolerance relation and asymmetric similarity relation, which retains both the relevant properties of tolerance relation and asymmetric similarity relation. Feature relation can deal with incomplete information system with missing type or missing type unknown attribute value. It inherits tolerances and nought.

The advantage of symmetric similarity relation, has more reasonable classification ability, but also has some problems.

3. Variable multigranularity rough set based on feature relation

3.1. Multi-granularity rough set based on feature relation

This section introduced the ideas of multi-granularity generalized incomplete decision system, based on the characteristics of the relationship between the granularity of the rough set is from the perspective of granular computing, there is a binary relation from the export of a single grain structure extended to multiple granularity structure composed of multiple binary relations export, so as to build more granularity rough set model.
**Definition 3** \( IIS = (U, AT \cup \{ d \}, V, f) \) is a generalized incomplete decision system, let \( A_1, A_2, \ldots, A_m \) is subset of \( m \) properties of \( AT \), for \( x \in U \), the upper and lower approximations of optimistic multi-granularity rough sets under feature relation family \( K(A_1), K(A_2), \ldots, K(A_m) \) are as follows:

\[
\sum_{i=1}^{m} A_{x}^{+} (X) = \{ x \in U : K_{A_1}(x) \subseteq X \vee \cdots \vee K_{A_m}(x) \subseteq X \} ;
\]

\[
\sum_{i=1}^{m} A_{x}^{-} (X) = \sum_{i=1}^{m} A_{x}^{+} (-X).
\]

**Theorem 1** \( IIS = (U, AT \cup \{ d \}, V, f) \) is a generalized incomplete decision system, let \( A_1, A_2, \ldots, A_m \) is subset of \( m \) properties of \( AT \), for \( x \in U \), under the characteristic relation, the following equation is true:

\[
\sum_{i=1}^{m} A_{x}^{c} (X) = \{ x \in U : K_{A_1}(x) \nsubseteq X \setminus \emptyset \wedge \cdots \wedge K_{A_m}(x) \nsubseteq X \setminus \emptyset \}.
\]

Proof: we know from definition 3,

\[
x \in \sum_{i=1}^{m} A_{x}^{c} (X) \iff x \notin \sum_{i=1}^{m} A_{x}^{+} (-X),
\]

\[
\Rightarrow K_{A_1}(x) \nsubseteq (-X) \wedge \cdots \wedge K_{A_m}(x) \nsubseteq (-X),
\]

\[
\Rightarrow K_{A_1}(x) \nsubseteq X \setminus \emptyset \wedge \cdots \wedge K_{A_m}(x) \nsubseteq X \setminus \emptyset.
\]

**Definition 4** \( IIS = (U, AT \cup \{ d \}, V, f) \) is a generalized incomplete decision system, let \( A_1, A_2, \ldots, A_m \) is subset of \( m \) properties of \( AT \), for \( x \in U \), the upper and lower approximations of pessimistic multi-granularity rough sets under feature relation family \( K(A_1), K(A_2), \ldots, K(A_m) \) are as follows:

\[
\sum_{i=1}^{m} A_{x}^{+} (X) = \{ x \in U : K_{A_1}(x) \subseteq X \wedge \cdots \wedge K_{A_m}(x) \subseteq X \} ;
\]

\[
\sum_{i=1}^{m} A_{x}^{-} (X) = \sum_{i=1}^{m} A_{x}^{+} (-X).
\]

**Theorem 2** \( IIS = (U, AT \cup \{ d \}, V, f) \) is a generalized incomplete decision system, let \( A_1, A_2, \ldots, A_m \) is subset of \( m \) properties of \( AT \), for \( x \in U \), the following equation is true:

\[
\sum_{i=1}^{m} A_{x}^{c} (X) = \{ x \in U : K_{A_1}(x) \nsubseteq X \setminus \emptyset \vee \cdots \vee K_{A_m}(x) \nsubseteq X \setminus \emptyset \}.
\]

Proof: we know from definition 4,

\[
x \in \sum_{i=1}^{m} A_{x}^{c} (X) \iff x \notin \sum_{i=1}^{m} A_{x}^{+} (-X) \iff K_{A_1}(x) \nsubseteq (-X) \vee \cdots \vee K_{A_m}(x) \nsubseteq (-X),
\]

\[
\Rightarrow K_{A_1}(x) \nsubseteq X \setminus \emptyset \vee \cdots \vee K_{A_m}(x) \nsubseteq X \setminus \emptyset.
\]

3.2. \( \gamma \) variable multi-granularity rough set based on feature relation

**Definition 5** \( IIS = (U, AT \cup \{ d \}, V, f) \) is a generalized incomplete decision system, let \( A_1, A_2, \ldots, A_m \) is subset of \( m \) properties of \( AT \), and \( A \subseteq AT, i = \{ 1, 2, \ldots, m \} \), let \( A \in \{ A_1, A_2, \ldots, A_m \} = \{ a_1, a_2, \ldots, a_i \} \), \( 0 < \gamma \leq 1 \), for \( X \subseteq U \). The upper and lower approximations of R variable multi-granularity rough set of attribute A are defined as follows:

\[
\sum_{i=1}^{m} A_{x}^{+} (X) = \{ x \in U : (K_{A_1}(x) \subseteq X \vee \cdots \vee K_{A_m}(x) \subseteq X \vee (\gamma) \}
\]

\[
\sum_{i=1}^{m} A_{x}^{-} (X) = \sum_{i=1}^{m} A_{x}^{+} (-X).
\]
By defining 5, when $0 < \frac{1}{m} \leq \gamma \leq 1$, $\gamma$ variable multiple granularity for multi-granularity rough set, rough set degradation when $\gamma = 1$, $\gamma$ variable multiple granularity for the pessimistic multi-granularity rough set, rough set degradation accordingly under the characteristic relations of optimism multi-granularity rough set and rough set is $\gamma$ variable granularity more pessimistic multi-granularity two special forms of rough set.

**Theorem 3** \( \text{IIS} = (U, AT \cup \{d\}, V, f) \) is a generalized incomplete decision system, let \( A_1, A_2, \ldots, A_n \) is subset of \( m \) properties of \( AT \), and \( A_i \subseteq AT, i = [1,2,\ldots,m]\), let \( A = \{A_1,A_2,\ldots,A_n\} = \{a_1,a_2,\ldots,a_n\} \), \( 0 < \gamma \leq 1 \), for \( X \subseteq U \), acquire

\[
\sum_{i=1}^{m} A_{i,x}^\epsilon (X) = \left\{ x \in U : \left( K_a(x) \cap X \neq \emptyset \right) \wedge K_a(x) \cap X \neq \emptyset \wedge \cdots \wedge K_a(x) \cap X \neq \emptyset \right\}.
\]

Proof: we know from definition 5

\[
x \in \sum_{i=1}^{m} A_{i,x}^\epsilon (X) \iff x \notin \sum_{i=1}^{m} A_{i,x}^\epsilon (-X)
\]

\[
\Rightarrow \left| K_a(x) \bigcap (-X) \wedge K_a(x) \bigcap (-X) \wedge \cdots \wedge K_a(x) \bigcap (-X) \right| \geq \gamma
\]

\[
\Rightarrow \left| K_a(x) \bigcap X \neq \emptyset \wedge K_a(x) \bigcap X \neq \emptyset \wedge \cdots \wedge K_a(x) \bigcap X \neq \emptyset \right| \geq \gamma.
\]

**Theorem 4** \( \text{IIS} = (U, AT \cup \{d\}, V, f) \) is a generalized incomplete decision system, let \( A_1, A_2, \ldots, A_n \) is subset of \( m \) properties of \( AT \), and \( A_i \subseteq AT, i = [1,2,\ldots,m]\), let \( A = \{A_1,A_2,\ldots,A_n\} = \{a_1,a_2,\ldots,a_n\} \), \( 0 < \gamma \leq 1 \), for \( X \subseteq U \). Under the characteristic relation, the $\gamma$ variable optimistic multi-granularity rough set has the following properties:

1. \( \sum_{i=1}^{m} A_{i,x}^\epsilon (X) \subseteq X \subseteq \sum_{i=1}^{m} A_{i,x}^\epsilon (X) ; \)
2. \( \sum_{i=1}^{m} A_{i,x}^\epsilon (U) = U = \sum_{i=1}^{m} A_{i,x}^\epsilon (U) ; \)
3. \( \sum_{i=1}^{m} A_{i,x}^\epsilon (-X) = - \sum_{i=1}^{m} A_{i,x}^\epsilon (X) ; \)
4. \( \gamma_1 \leq \gamma_2 \Rightarrow \sum_{i=1}^{m} A_{i,x}^\epsilon (X) \supseteq \sum_{i=1}^{m} A_{i,x}^\epsilon (X) ; \)
5. \( \sum_{i=1}^{m} A_{i,x}^\epsilon (-X) = - \sum_{i=1}^{m} A_{i,x}^\epsilon (X) ; \)

Proof: (1), (2) can be obtained by definition 3.

(3) We know \( \sum_{i=1}^{m} A_{i,x}^\epsilon (-X) = - \sum_{i=1}^{m} A_{i,x}^\epsilon (X) \) from definition 3, for \( X = -X \). There are \( \sum_{i=1}^{m} A_{i,x}^\epsilon (X) = - \sum_{i=1}^{m} A_{i,x}^\epsilon (X) \), so, to prove to get \( \sum_{i=1}^{m} A_{i,x}^\epsilon (-X) = - \sum_{i=1}^{m} A_{i,x}^\epsilon (X) \).
(4) For any \( x \in \sum_{i=1}^{m} A_{r_i}^\omega (X) \), We know from definition 3 that there is \( A \subseteq AT, i = \{1, 2, \ldots, m\} \), and \( A \in \{A_1, A_2, \ldots, A_m\} \), make \( \frac{|N_{\omega A}(X)|}{m} \geq \gamma_2 \), Because we know what we have \( \gamma_1 \leq \gamma_2 \), So there are \( |N_{\omega A}(X)| \geq \gamma_1 \), That is \( x \in \sum_{i=1}^{m} A_{r_i}^\omega (X) \), Therefore, it can be proved \( \sum_{i=1}^{m} A_{r_i}^\omega (X) \subseteq \sum_{i=1}^{m} A_{r_i}^\omega (X) \). Something similar can be done \( \sum_{i=1}^{m} A_{r_i}^\omega (X) \equiv \sum_{i=1}^{m} A_{r_i}^\omega (X) \).

3.3. \( \gamma \) variable multi-granularity rough set reduction and rule acquisition based on feature relation

Reduction and rule extraction are the core contents of rough set theory. Reduction is to remove unnecessary attributes and attribute values without changing the classification ability of information system, so as to extract rules for decision makers. This paper introduces the theory of multi-granularity and puts forward the concept of \( \gamma \) variable multi-granularity upper and lower approximate distribution reduction based on feature relation, so as to obtain the rules of generalized incomplete information system.

Definition 6 IIS = \((U, AT \cup \{d\}, V, f)\) is a generalized incomplete decision system, let \( A_1, A_2, \ldots, A_m \) is subset of \( m \) properties of \( AT \), and \( A \subseteq AT, i = \{1, 2, \ldots, m\} \), let \( A \in \{A_1, A_2, \ldots, A_m\} = \{a_1, a_2, \ldots, a_m\} \), \( 0 < \gamma \leq 1 \), for \( X \subseteq U \), \( U / \text{ind}(D) = \{D_1, D_2, \ldots, D_o\} \), remember to

\[
\begin{align*}
AT^\omega_r (D) & = \left\{ \sum_{i=1}^{m} A_{r_i}^\omega (D_1), \sum_{i=1}^{m} A_{r_i}^\omega (D_2), \ldots, \sum_{i=1}^{m} A_{r_i}^\omega (D_o) \right\} \\
\overline{AT^\omega_r} (D) & = \left\{ \sum_{i=1}^{m} \overline{A_{r_i}^\omega} (D_1), \sum_{i=1}^{m} \overline{A_{r_i}^\omega} (D_2), \ldots, \sum_{i=1}^{m} \overline{A_{r_i}^\omega} (D_o) \right\}.
\end{align*}
\]

(1) If \( A_{r_i}^\omega (D) = \overline{AT^\omega_r} (D) \), then, it is said that attribute set \( A \) is the under approximate distributed coordination set of conditional attribute set \( AT \) \( \gamma \) variable multi-granularity of decision attribute set \( D \), if \( A_{r_i}^\omega (D) = \overline{AT^\omega_r} (D) \) and only if \( \forall B \subseteq A \), \( B^\omega_r (D) \neq \overline{A_{r_i}^\omega} (D) \), then attribute set \( A \) is the under approximate distribution reduction \( \gamma \) variable multi-granularity of the conditional attribute set \( AT \) about the decision attribute set \( D \).

(2) If \( \overline{A}_{r_i}^\omega (D) = \overline{AT^\omega_r} (D) \), then, it is said that attribute set \( A \) is the upper approximate distributed coordination set of conditional attribute set \( AT \) \( \gamma \) variable multi-granularity of decision attribute set \( D \), if \( \overline{A}_{r_i}^\omega (D) = \overline{AT^\omega_r} (D) \) and only if \( \forall B \subseteq A \), \( B^\omega_r (D) \neq \overline{A_{r_i}^\omega} (D) \), then attribute set \( A \) is the upper approximate distribution reduction \( \gamma \) variable multi-granularity of the conditional attribute set \( AT \) about the decision attribute set \( D \).

Theorem 5 IIS = \((U, AT \cup \{d\}, V, f)\) is a generalized incomplete decision system, let \( A_1, A_2, \ldots, A_m \) is subset of \( m \) properties of \( AT \), and \( A \subseteq AT, i = \{1, 2, \ldots, m\} \), let \( A \in \{A_1, A_2, \ldots, A_m\} = \{a_1, a_2, \ldots, a_m\} \), \( 0 < \gamma \leq 1 \), for \( X \subseteq U \), if

\[
\begin{align*}
AT^\omega_r (x) & = \{X_j \in U / \text{IND}(\{d\}): x \in \sum_{i=1}^{m} A_{r_i}^\omega (X_j) \} \\
\overline{AT^\omega_r} (x) & = \{X_j \in U / \text{IND}(\{d\}): x \in \sum_{i=1}^{m} \overline{A_{r_i}^\omega} (X_j) \}.
\end{align*}
\]

Then

(1) \( A^\omega_r (\{d\}) = AT^\omega_r (\{d\}) \Leftrightarrow \forall x \in U, A^\omega_r (x) = AT^\omega_r (x) \);
(2) \( \overline{A}_{r_i}^\omega (\{d\}) = \overline{AT^\omega_r} (\{d\}) \Leftrightarrow \forall x \in U, A_{r_i}^\omega (x) = \overline{AT^\omega_r} (x) \).
\textbf{Theorem 6} \(\text{IS} = (U, AT \cup \{d\}, V, f)\) is a generalized incomplete decision system, let \(A_1, A_2, \cdots, A_n\) is subset of \(m\) properties of \(AT\), and \(A \subseteq AT, i = [1,2,\cdots,m]\), let \(A = \{A_1, A_2, \cdots, A_n\} = \{a_1, a_2, \cdots, a_n\}, 0 < \gamma \leq 1,\) then

(1) \(\forall x \in U, A_i^\prime (x) = A_i^\prime \gamma (x) \iff \exists A \in A, \text{ when } A_i^\prime \gamma (x) \nless A_i^\prime (y), K_A (y) \nless K_A (x) \) set up:

(2) \(\forall x \in U, A_i^\prime (x) = A_i^\prime \gamma (x) \iff \exists A \in A, \text{ when } A_i^\prime \gamma (y) \nless A_i^\prime (x), K_A (y) \nless K_A (x) \) set up.

\textbf{Definition 7} \(\text{IS} = (U, AT \cup \{d\}, V, f)\) is a generalized incomplete decision system, \(0 < \gamma \leq 1,\) if

\[D_{\text{el}}^\gamma \cap U = \{(x, y) \in U^2 : A_i^\prime \gamma (x) \nless A_i^\prime (y)\},\]

\[D_{\text{ul}}^\gamma \cap U = \{(x, y) \in U^2 : A_i^\prime \gamma (y) \nless A_i^\prime (x)\},\]

We can be defined

\[D_{\text{el}}^\gamma (x, y) = \begin{cases} \quad a \in AT : (x, y) \in D_{\text{el}}^\gamma, \quad (x, y) \in D_{\text{el}}^\gamma, \\ \quad \therefore \quad \therefore \quad \therefore \quad \therefore\end{cases},\]

\[M_{\text{el}}^\gamma = \{D_{\text{el}}^\gamma (x, y) : x, y \in U\}, M_{\text{ul}}^\gamma = \{D_{\text{ul}}^\gamma (x, y) : x, y \in U\},\]

then \(M_{\text{el}}^\gamma\) and \(M_{\text{ul}}^\gamma\) are respectively called the approximate distribution identification matrix under \(\gamma\) variable multi-granularity for generalized incomplete decision system and the approximate distribution identification matrix under \(\gamma\) variable multi-granularity for generalized incomplete decision system.

\textbf{Theorem 7} \(\text{IS} = (U, AT \cup \{d\}, V, f)\) is a generalized incomplete decision system, let \(A_1, A_2, \cdots, A_n\) is subset of \(m\) properties of \(AT\), and \(A \subseteq AT, i = [1,2,\cdots,m]\), let \(A = \{A_1, A_2, \cdots, A_n\} = \{a_1, a_2, \cdots, a_n\}, 0 < \gamma \leq 1,\) then

(1) \(A_i^\prime (d) = A_i^\prime \gamma (d) \iff \forall D_{\text{el}}^\gamma (x, y) \neq \emptyset, A \cap D_{\text{el}}^\gamma (x, y) \neq \emptyset;\)

(2) \(A_i^\prime (d) = A_i^\prime \gamma (d) \iff \forall D_{\text{ul}}^\gamma (x, y) \neq \emptyset, A \cap D_{\text{ul}}^\gamma (x, y) \neq \emptyset;\)

\textbf{Definition 8} \(\text{IS} = (U, AT \cup \{d\}, V, f)\) is a generalized incomplete decision system, \(0 < \gamma \leq 1,\) if

\[\Delta_{\text{el}}^\gamma = \bigwedge_{(x, y) \in D_{\text{el}}^\gamma} D_{\text{el}}^\gamma (x, y), \quad \Delta_{\text{ul}}^\gamma = \bigwedge_{(x, y) \in D_{\text{ul}}^\gamma} D_{\text{ul}}^\gamma (x, y),\]

then it is said that \(\Delta_{\text{el}}^\gamma\) and \(\Delta_{\text{ul}}^\gamma\) of the approximate distribution identification function of \(\gamma\) variable multi-granularity and the approximate distribution identification function of \(\gamma\) variable multi-granularity. Where \& and \lor represent conjunctive and disjunctive operations respectively.

\textbf{Theorem 8} \(\text{IS} = (U, AT \cup \{d\}, V, f)\) is a generalized incomplete decision system, let \(A_1, A_2, \cdots, A_n\) is subset of \(m\) properties of \(AT\), and \(A \subseteq AT, i = [1,2,\cdots,m]\), let \(A = \{A_1, A_2, \cdots, A_n\} = \{a_1, a_2, \cdots, a_n\}, 0 < \gamma \leq 1,\) then

(1) If \(A\) is an under approximate reduction for \(\gamma\) variable multi-granularity,then \(\forall x \in U, a \in A, j \in V_{\text{u}}, r_{j} : \land (a, v_{h}) \lor (d, j)\) is the simplest deterministic decision rule;

(2) If \(A\) is an upper approximate reduction for \(\gamma\) variable multi-granularity,then \(\forall x \in U, a \in A, j \in V_{\text{u}}, r_{j} : \land (a, v_{h}) \lor (d, j)\) is the simplest possible decision rule.

Proof: the definition of under (upper) approximate reduction under \(\gamma\) variable multi-granularity is obviously obtained.

4. Case analysis
\(\text{IS} = (U, AT \cup \{d\}, V, f)\) is a generalized incomplete decision system, as shown in table 1, Where, \(U = (x_1, x_2, x_3, x_4, x_5, x_6)\) is the object set, \(AT = \{a_1, a_2, a_3, a_4\}\) is the conditional attribute set, \(\gamma = 0.4\) is the threshold value, \(d\) is decision attribute set, the unknown value can be compared with any other known
attribute value. “*” represents the missing unknown value, which cannot be compared with any other known attribute value. “?” represents a lost type unknown value that cannot be compared with any other known attribute value.

Table 1. Generalized incomplete decision system

| U/A  | a1 | a2 | a3 | a4 | d   |
|------|----|----|----|----|-----|
| x₁   | 1  | 1  | 1  | 1  | 1   |
| x₂   | 2  | *  | 1  | 1  | 2   |
| x₃   | *  | ?  | *  | 2  | 2   |
| x₄   | 1  | 1  | 1  | 1  | 2   |
| x₅   | ?  | *  | 1  | 2  | 3   |
| x₆   | 2  | 1  | 1  | *  | 2   |

From Table 1, we get \( U / \text{IND}(d') = (D_1, D_2, D_3) \), \( D_1 = \{x_1, x_2, x_3, x_4, x_5, x_6\} \). Suppose the attribute subset family of the generalized incomplete decision system is as follows: \( A = \{a_1, a_2, a_3, a_4, a_5\} \), take the threshold \( \gamma = 0.4 \).

(1) In the case of single particle size, the attribute reduction method of the generalized incomplete decision system based on feature relation is proposed based on the general rough set theory, and an approximate reduction \( \text{red}_1 = \{a_4, a_5\} \) at single particle size and an approximate reduction \( \text{red}_0 = \{a_1, a_2\} \) at single particle size are obtained. According to the lower approximation reduction of the generalized incomplete decision system, four decision rules of the system can be obtained, and the same is true for the upper approximation reduction of the generalized incomplete decision system. Therefore, in the case of single granularity, the upper and lower approximate sets in Table 1 are completely consistent, which cannot reflect the advantages of upper and lower approximate distribution reduction.

(2) In the case of variable multi-granularity of threshold \( \gamma = 0.4 \), the generalized incomplete decision system can obtain approximate reduction \( \text{red}_{1,1} = \{a_4, a_5\} \), \( \text{red}_{2,1} = \{a_4, a_5\} \), and \( \text{red}_{3,1} = \{a_1, a_5\} \) under three variable multi-granularity. It can be seen that in the case of variable multi-granularity of threshold \( \gamma = 0.4 \), the generalized incomplete decision system not only obtains a lower approximation reduction under 3 single granularities, but also obtains another two approximate reductions \( \text{red}_{2,2} = \{a_4, a_5\} \) and \( \text{red}_{3,2} = \{a_1, a_5\} \) under variable multi-granularity.

In the same way, three approximate reduction \( \text{red}_{1,2} = \{a_4\} \), \( \text{red}_{2,2} = \{a_4, a_5\} \) and \( \text{red}_{3,2} = \{a_1, a_5\} \) on variable multi-granularity can be obtained. Compared with the upper approximation reduction of single particle size, it is found that \( \text{red}_{1,1} = \{a_4, a_5\} \) is simpler than \( \text{red}_0 = \{a_1, a_5\} \), and another upper approximation reduction \( \text{red}_{3,2} = \{a_1, a_5\} \) can be obtained under variable multi-particle-size.

Taking \( \text{red}_{1,2} = \{a_4\} \) as an example, the following 5 decision rules with variable multi-granularity can be obtained:

1. \((a_4) \rightarrow (d.1)\);
2. \((a_4) \rightarrow (d.2)\);
3. \((a_4) \rightarrow (d.2)\);
4. \((a_4) \rightarrow (d.3)\);
5. \((a_4) \rightarrow (d.2)\).

By the case 1, the multi-granularity approximate distribution reduction method can not only obtain approximate single particle size distribution reduction method of approximate distribution reduction, and can get more approximate distribution reduction, and the reduction to further eliminate the redundant information, get more qualified reduction, thus can get more Jane's decision rules. The upper and lower approximate distributions obtained under the theory of multi-particle size are inconsistent, which can be approximated better and meet the needs of real life.
5. Conclusion
This article employed multi-granularity theory to deal with the decision rules of generalized incomplete decision system and proposed multiple granularity rough set model and Variable multi-granularity rough set model based on feature relationships. Based on that, combining Variable multi-granularity theory with the approximate distribution reduction methods, a decision theorem for upper and lower approximate distribution reduction of generalized incomplete variable multi-granularity, Variable multi-granularity upper and lower approximate distribution resolution matrix and Resolution function and reduction formula were put forward. As the simplest rule of acquisition tactic is given, variable multi-granularity minimization rules and variable multi-granularity minimization rules can be obtained. Combined with case studies, it is found that the variable multi-granularity approximate distribution reduction method can obtain more reasonable upper and lower approximate distribution reduction than the original approximate distribution reduction method in the case of single granularity, so as to obtain more effective decision rules. It can be seen that the variable multi-granularity approximate distribution reduction and decision acquisition method based on feature relation proposed in this paper is not only an extension of multi-granularity rough set, but also provides a new theoretical method and technical means for the processing of generalized incomplete information.

Foundation item:
Supported by the basic ability improvement project of middle and young teachers in colleges and universities of Guangxi (2018KY0869).

Biography:
Mo Jing-ian was born in 1984. She is a associate professor. Her research interests include Rough Sets and Data Mining.

References:
[1] Pawlak Z. Rough sets [J]. International Journal of Computer and Science, 1982, 11(5): 341-356.
[2] Zhang wenxi, Wu weizhi. Rough set theory and method [M]. Beijing: Science Press, 2001
[3] Kryszkiewicz M. Rough set approach to incomplete information system [J]. Information Sciences, 1998, 11(2): 39-49.
[4] Grzymala -Busse J W. Data with missing attribute values: generalization of indiscernibility relation and rule induction[C]// James F Peters. Transactions on Rough Sets I. Berlin: Springer-Verlag, 2004: 78-95.
[5] Thiele H. On semantic models for investigating computing with words [ A ] . Proceedings of the Second International Conference on Knowledge Based Intelligent Electronic Systems [ C ] . USA: IEEE, 1998: 32-98.
[6] Lin T Y. Granular computing on binary relations I: data mining and neighborhood systems [ A ]. Rough Sets and Knowledge Discovery [ C ]. Heidelberg, Germany: Physica-Verlag, 1998: 107-121.
[7] Lin T Y. Granular computing on binary relations II: Rough set representations and belief functions [ A ]. Rough Sets and Knowledge Discovery [ C ]. Heidelberg, Germany: Physica-Verlag, 1998: 107-121.
[8] Qian Yuhua, Liang Jiye, Yao Yiyu, et al. MGRS: A multi-granulation rough set [ J ]. Information Sciences, 2010, 180( 6) : 949-970.
[9] Qian Yuhua, Liang Jiye, Dang Chuangyin. Incomplete multi-granulation rough set[J]. IEEE Transactions on Systems, Man and Cybernetics, Part A, 2010, 40( 2) : 420-431.
[10] Wang lijuan, Yang xibei, Yang jingyu, et al. Incomplete decision rule acquisition based on multigranulation theory[J]. Journal of Nanjing University of Science and Technology, 2013, 37(1): 12-18.
[11] Qian Yuhua, Li Shunyong, Liang Jiye, et al. Pessimistic rough set based decisions: A multigranulation fusion strategy[J]. Information Sciences, 2014( 264) : 196-210.
[12] Meng huili, Ma yuanyuan, Xu jiucheng. The granularity reduction of pessimistic multi-granulation rough set based on the information quantity[J]. Journal of Nanjing university(nature sciences),2015, 51( 2) :343-348.
[13] Wang Xiaoyan, Guo Yunting, Shen Yuanxia. Research on granularity reduction of variable multi-granulation rough set[J].Natural sciences journal of haebin normal university,2019,35(1):24-30.
[14] Luo Gongzhi, Xu Xinxin. Rough analysis method of multi-granularity decision rough set based on possible degree tolerance relation[J].Application Research of Computers,2018,36(12):2912-2020