Gravitational Redshift Induces Quantum Interference

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Quantum field theory in curved spacetime is used to show that gravitational redshift induces a unitary transformation on the quantum state of propagating photons. It is found that the transformation is a mode-mixing operation, and a protocol that exploits gravity to induce a Hong–Ou–Mandel-like interference effect on the state of two photons is devised. It is discussed how the results of this work can provide a demonstration of quantum field theory in curved spacetime.

1. Introduction

Gravitational redshift is a trademark prediction of general relativity.[1,2] Photons initially prepared with a given frequency by the sender travel through curved spacetime and are detected with a different frequency by the receiver. This effect, which can be successfully explained by general relativity alone, has been tested and measured using a plethora of different setups,[4–10] and can even be exploited for different tasks.[11–13] Photons initially prepared with a given frequency by the sender travel through curved spacetime and are detected with a different frequency by the receiver. This effect, which can be successfully explained by general relativity alone, has been tested and measured using a plethora of different setups,[4–10] and can even be exploited for different tasks.[11–13]

In recent years, renewed attention to the overlap of quantum mechanics and relativity has been fuelled by developments in quantum information theory.[14] Many experimental and theoretical proposals have been put forward to exploit inherent features of quantum systems, such as entanglement, to measure gravitationally induced decoherence of a quantum state.[15,16] Test structures of quantum systems, such as entanglement, to measure gravitationally induced decoherence of a quantum state, are ideal, in the sense that their spatial support occupies ample, in flat spacetime, there is no preferred notion of time.[1,23] When a notion of time exists, for example the spacetime has a global spacelike hypersurfaces orthogonal to \( \partial \), while all others vanish. The mode solutions also satisfy\[2,3\] the Klein–Gordon equation

\[
\left( (\sqrt{-g})^{-1} \partial_{\mu} g^{\mu \nu} \sqrt{-g} \partial_{\nu} \right) \phi(x^\nu) = 0 \tag{1}
\]

Finding solutions to Equation (1) is very difficult since, in a general spacetime, there is no preferred notion of time.[1,23] When a notion of time exists, for example the spacetime has a global timelike Killing vector field \( \partial_t \), it is possible to meaningfully foliate the spacetime in spacelike hypersurfaces orthogonal to \( \partial_t \) and solve the Klein–Gordon equation, to finally obtain upon quantization \( \hat{\phi}(x) = \int d^4k [\phi_k(x^\nu) \hat{a}_k^\dagger + \phi_k^*(x^\nu) \hat{a}_k] \). The mode solutions \( \phi_k(x) \) are labeled by \( k \equiv (k_\mu, k^\nu) \), satisfy \( \square \phi_k = 0 \) (which is a short-hand notation for Equation (1)), are normalized by \( \langle \phi_k, \phi_{k'} \rangle = \delta^i(k - k') \) given the appropriate inner product \( \langle \cdot, \cdot \rangle \), and the annihilation and creation operators satisfy \( [\hat{a}_k, \hat{a}_{k'}^\dagger] = \delta^i(k - k') \) and all others vanish. The mode solutions also satisfy \( i \partial_t \phi_k(x) = \omega_k \phi_k(x) \), which guarantees a consistent notion of particle in time. In general, \( \omega_k \) is a function of \( k \) and, for example, in flat spacetime one has \( \omega_k = |k| \).

2. Background Tools

2.1. Quantum Fields in Curved Spacetime

Let us consider, without loss of generality, a massless scalar quantum field \( \phi(x^\nu) \) propagating on classical (curved) 3 + 1 background with coordinates \( x^\mu \) and metric \( g_{\mu \nu} \).[1] The classical field \( \phi(x^\nu) \) will satisfy the Klein–Gordon equation

where

2.2. Modeling a Realistic Photon

We have chosen to use a massless scalar field, which can be employed to model one polarization of the electromagnetic field in the regimes considered here.[24] Photons defined by the operators \( \hat{a}_k \) are ideal, in the sense that their spatial support occupies the whole spacetime. Consequently, they cannot be employed to discuss concrete physical effects since the modes \( \phi_k(x^\nu) \) are normalized through Dirac-deltas.
A realistic photon, on the other hand, is characterized by a finite spatial extension and frequency bandwidth instead of an (infinitely) sharp frequency. We assume that we can discard all effects due to the extension of the photon along directions that are orthogonal to that of propagation, and that these can be taken into account separately.[25,26] A photon operator is therefore constructed as

\[
\hat{A}_0 := \int_0^\infty d\omega F_{\omega 0}(\omega/\sigma) \hat{a}_\omega
\]  

where the (complex) function \(F_{\omega 0}(\omega/\sigma)\) determines the frequency profile. This function is labeled by the peak frequency \(\omega_0\), it has an overall characteristic size \(\sigma\), and is normalized by \(\langle F_{\omega 0}, F_{\omega 0} \rangle = 1\), where we define \(\langle F, G\rangle := \int_0^\infty d\omega F^* (\omega) G(\omega)\) for later convenience. It is immediate to check that \([\hat{A}_0, \hat{A}_0^\dagger] = 1\), which therefore guarantees that \(\hat{A}_0\) generates properly normalized photonic states. We note that the Hilbert space \(\mathcal{H}\) of the (scalar) photon is infinite dimensional and therefore we need to introduce the set of functions \(F_j\) determined by a set of parameters \(\vec{\lambda}\) such that, together with \(F_{\omega 0}\), they form an orthonormal basis. In practice this means that \(\langle F_{\omega 0}, F_j \rangle = 0\) for all \(\vec{\lambda}\), while \(\langle F_j, F_j \rangle = \delta(\vec{\lambda} - \vec{\lambda}')\). Here \(\vec{\lambda}\) is, in principle, a collection of discrete and continuous indices. Operators can then be defined as \(\hat{A}_j := \int_0^\infty d\omega F_j(\omega) \hat{a}_\omega\) and therefore \([\hat{A}_0, \hat{A}_j] = 0\). In this work we do not necessitate an explicit construction of the set \(\{F_j\}\).

2.3. Gravitational Redshift

Gravitational redshift is a key prediction of general relativity, which lacks a conclusive explanation.[2,27] It remains unclear if it is a fundamental effect witnessed by the photons, or a consequence of the effects of gravity on local measuring devices. In the second case, gravitational redshift is not a “change in frequency of the photon”, but a mismatch in the frequencies of the constituents forming, for example, the detecting devices of the sender and receiver, respectively. Here we take the approach that a frequency is what a (localized) observer measures with his (local) clock. With this in mind, we assume that Alice and Bob are stationary with respect to each other and therefore do not have to correct for additional effects due to relative motion that is, for Doppler-like effects. Alice measures proper time \(\tau_A\) locally at A using her clock, while Bob measures proper time \(\tau_B\) locally at B using his. We then recall that the relation between the frequency \(\omega_B\) prepared by Alice at position A, and the frequency \(\omega_B\) received by Bob at location B, is

\[
\chi^2 := \frac{\omega_B}{\omega_A} = \frac{k_x u_x^B}{k_{\mu} u_{\mu}^A}
\]  

where \(k_x\) is the tangent vector to the (affinely parametrized) null geodesic followed by the photon, \(u_x^B\) is the Alice’s four velocity and \(u_\mu^A\) is Bob’s four velocity.[28] It is understood that \(k_{\mu} u_{\mu}^A\) and \(k_x u_x^B\) are calculated at Alice’s and Bob’s positions, respectively. The nonnegative parameter \(\chi\) has been introduced for notational convenience and is key to this work. While this relation is central to our work, we do not join the debate on the interpretation of the redshift presented above. We note, however, that the effects found here are witnessed locally by the observers when they measure the quantum states of light.

3. Gravitational Redshift of Photons

3.1. Gravitational Redshift of Photon Operators

Alice and Bob wish to determine how gravitational redshift affects photons. Alice sends a photon to Bob, who will detect a gravitational redshift within the incoming photon, that is, each sharp frequency \(\omega'\) as measured locally by his clock will not coincide with the sharp frequency \(\omega\) of the sent photon. The scheme is depicted in Figure 1. As far as Bob is concerned, that is, from the perspective of his laboratory, the expected photon has changed and he can study the properties of the transformation involved, irrespective of where the incoming photon has originated or which specific physical process it has undergone. Therefore, Bob can assign a channel to the process that affected the incoming photon, and seek for its properties.

Bob starts by assuming that there is a transformation \(T(\chi) : \omega \to \chi \omega\) on each sharp frequency \(\omega\). He then looks for a unitary transformation \(\hat{U}(\chi)\) that implements \(T(\chi)\) through

\[
\hat{a}_{\omega'} = \hat{U}^\dagger(\chi) \hat{a}_\omega \hat{U}(\chi) = \hat{a}_{\omega'}
\]  

for all \(\chi\), where \(\hat{U}^\dagger(\chi) \hat{U}(\chi) = 1\).

Assuming that the transformation in Equation (4) holds, it is easy to use the explicit expression for \(\hat{A}_\omega\), the fact that \([\hat{A}_\omega, \hat{A}_{\omega'}] = \delta(\chi^2 \omega - \chi^2 \omega')\), and \(\delta(f(x)) = \sum_n \delta(x - x_n)/|f'(x_n)|\), where \(x_n\) are the zeros of the function \(f(x)\), to show that

\[
1 = \hat{U}^\dagger(\chi) \hat{U}(\chi) = \hat{U}^\dagger(\chi)[\hat{A}_0, \hat{A}_0^\dagger] \hat{U}(\chi) = \frac{1}{\chi^2}
\]  

for all \(\chi\), where \(\hat{U}^\dagger(\chi) \hat{U}(\chi) = 1\).
This equation can be satisfied only when $\chi = 1$, that is, for the trivial case of no redshift. Clearly, this cannot happen in general as can be seen from Equation (3). Therefore, we conclude that gravitational redshift in the form of a linear shift of the spectrum of sharp frequencies cannot be obtained as the result of a unitary operation on the field modes $\{\hat{a}_\lambda\}$ alone.\cite{29} This result corroborates the claim that the gravitational redshift is not simply a shift in the sharp frequencies of the photons for all frequencies of the spectrum.

3.2. Quantum Modeling of Gravitational Redshift

We now ask a more refined version of the question posed above: how is the transformation $T(\chi)$ implemented by a unitary operator when acting on realistic photons? To answer this question, we start by noting that Bob will describe the received photon as $\hat{a}_\lambda = \int_0^\infty d\omega F_{\chi}(\omega/\sigma) \hat{a}_\omega$, where $\hat{a}_\omega$ is a local frequency $\omega$ as measured in his laboratory, while the expected photon has the expression $\hat{A}_\lambda = \int_0^\infty d\omega F_{\chi}(\omega/\sigma) \hat{a}_\omega$. Bob then notices that each sharp frequency $\omega$ that appears in the definition of $\hat{A}_\lambda$ transforms by $T(\chi): \omega \rightarrow \chi^2 \omega$, see ref. [11]. This means that

$$\int_0^\infty d\omega F_{\chi}(\omega/\sigma) \hat{a}_\omega \rightarrow \chi^2 \int_0^\infty d\omega F_{\chi}(\omega/\sigma) \hat{a}_{\chi^2 \omega}$$

He can then identify the function $F_{\chi}(\omega/\sigma) \equiv \chi F_{\chi}(\chi^2 \omega/\sigma) = \chi F_{\chi}/\chi^2 (\omega/\chi^2)$, where $\omega' = \omega/\chi^2$ and $\sigma' = \sigma/\chi^2$, which represents a well defined physical photon in the sense that it can be satisfied only when $\chi = 1$, that is, for the same frequencies $\omega$.

4. Gravitational-Redshift-Induced Interference

4.1. Gravitationally-Induced Titter

Let us focus on the case where we select two different commuting photon operators $\hat{A}_{\lambda_0}$ and $\hat{A}_{\lambda_1}$ and let us consider the transformed modes $\hat{A}'_{\lambda_0}$ and $\hat{A}'_{\lambda_1}$. Then define the vector $\hat{X} := (\hat{A}'_{\lambda_0}, \hat{A}'_{\lambda_1})$, where the operator $\hat{A}'_\lambda := \sum_{\lambda_0, \lambda_1} a_{\lambda_0, \lambda_1} \hat{A}_{\lambda_0} \hat{A}_{\lambda_1}$ collects all of the operators orthogonal to the two chosen ones, we have $\sum_{\lambda_0, \lambda_1} |\lambda_0, \lambda_1\rangle = \sum_{\lambda_0, \lambda_1} |\lambda_0, \lambda_1\rangle = 1$, and $\hat{A}'_{\lambda_0} := U(\chi) \hat{X} \hat{A}(\chi)$. The general transformation Equation (3) is therefore defined by the symplectic representation of the product of three beam-splitting operations of the form $\exp[i\theta(e^{i\phi_0} \hat{A}'_{\lambda_0} - e^{-i\phi_0} \hat{A}'_{\lambda_1})] \exp[i\theta(e^{i\phi_0} \hat{A}'_{\lambda_1} - e^{-i\phi_0} \hat{A}'_{\lambda_0})]$, which is the result for $\phi_0 = \phi, \phi_0 = 0$, and note that the phases can be restored when necessary. We have

$$U \equiv \begin{pmatrix} c_\theta c_\phi & -c_\theta s_\phi c_\psi & -s_\theta s_\phi & s_\theta s_\psi + c_\theta c_\psi \\ -c_\theta s_\phi & c_\theta c_\psi & s_\theta s_\phi & c_\theta s_\psi + s_\theta c_\psi \\ s_\theta c_\phi & c_\theta s_\psi & s_\theta s_\phi & s_\theta s_\psi + c_\theta c_\psi \\ -s_\theta s_\phi & \phi & s_\theta s_\phi & s_\theta s_\psi + c_\theta c_\psi \end{pmatrix}$$

where we introduce $s_\theta := \sin \theta$ and $c_\theta := \cos \theta$ for ease of presentation. The transformation (Equation (9)) is known in quantum optics as a three-wave mode-mixer, or titter.\cite{30,32}

Most importantly, the angles $\theta, \phi$, and $\psi$ are functions of the redshift $\chi$ and are defined through

$$\cos \theta \cos \phi \equiv \langle F'_{\chi_0}, F_{\chi_1} \rangle,$$

$$\cos \phi \cos \psi \equiv \langle F'_{\chi_0}, F_{\chi_1} \rangle,$$

$$\cos \psi \equiv \langle F'_{\chi_0}, F_{\chi_1} \rangle$$

We expect that $\cos \chi$ and $\psi$ in the redshift regime $\chi \geq 1$ take values between $0, 1$ (i.e., perfect overp_Impl.), and $\theta = \psi = 0$ (complete mismatch). An analogous analysis can be done for the blue-shift regime $0 \leq \chi < 1$.

4.2. Gravity-Induced Quantum Interference

Here we describe a photon-exchange task between Alice and Bob that exploits the transformation (Equation (9)) and its four uses to induce interference between the photonic states. It is depicted using a circuit implementation language in Figure 2.
which can occur given the freedom in choice of the initial \( \hat{\rho} \), nonzero, or vice versa. In the first case we obtain the fully similar fashion.

Figure 2. Alice sends a two-photon state \(|1_{\text{in}} 0_{\text{in}}\rangle\) of modes \( \hat{A}_{\text{in}} \) and \( \hat{A}_{\text{in}} \) to Bob. The gravitational redshift effectively mode-mixes the state through the unitary operation \( \hat{U}(\chi) \) defined in Equation (9) into components \( \hat{A}_{\text{out}} \), \( \hat{A}_{\text{out}} \) and \( \hat{A}_1 \). Bob then measures the reduced state of modes \( \hat{A}_{\text{out}} \) and \( \hat{A}_{\text{out}} \), which is now entangled.

The steps required to perform the task read as follows:

1. Alice prepares a two-photon separable state \(|\Psi\rangle := |1_{\text{in}} 1_{\text{in}} 0\rangle\) and sends it to Bob, who receives it as \(|\Psi\rangle := |1_{\text{in}} 1_{\text{in}} 0\rangle\). Introducing the notation \(|nmp\rangle := \left(\frac{\hat{A}^2_n \hat{A}^m \hat{A}^p}{\sqrt{N}}\right) |0\rangle\), it is immediate to verify that Bob's state reads locally as

\[
|\Psi\rangle = \sqrt{2} [U_{13} U_{23}|002\rangle + U_{12} U_{22}|020\rangle + U_{11} U_{21}|200\rangle] \\
+ (U_{13} U_{22} + U_{12} U_{23})|011\rangle + (U_{11} U_{22} + U_{12} U_{21})|110\rangle \\
+ (U_{11} U_{23} + U_{13} U_{21})|101\rangle
\]

(11)

Here \( U_{ab} \) are the coefficients of the matrix Equation (9).

2. The final state \( \hat{\rho}(\chi) \) of the modes \( \hat{A}_{\text{out}} \) and \( \hat{A}_{\text{out}} \) in Bob's laboratory is obtained by tracing Equation (11) over the unobserved subsystem degrees of freedom \( \hat{A}_2 \), and it is easy to compute but gives a cumbersome expression. We give its generic form here

\[
\hat{\rho}(\chi) = \rho_{0000}|00\rangle\langle 00| + \rho_{0020}|02\rangle\langle 02| + \rho_{0200}|20\rangle\langle 20| + \rho_{1010}|10\rangle\langle 10| + \rho_{0101}|01\rangle\langle 01| + \rho_{1111}|11\rangle\langle 11| + \rho_{2021}|20\rangle\langle 21| + \rho_{2102}|21\rangle\langle 02| + \rho_{0212}|02\rangle\langle 21| + \rho_{0122}|01\rangle\langle 21| + \rho_{1201}|12\rangle\langle 01| + \rho_{1021}|10\rangle\langle 21| + \rho_{2012}|20\rangle\langle 12| + \rho_{2112}|21\rangle\langle 12| + h.c
\]

(12)

The coefficients \( \rho_{nmpq} \) can be obtained in terms of the matrix elements \( U_{ab} \) with simple algebra. We avoid printing them here to improve clarity of presentation.

We note here that it is possible to have all terms in Equation (12) that include a \( |11\rangle \) contribution to vanish with either \( \rho_{0202} \) or \( \rho_{2020} \) remaining nonzero. It is sufficient that either \( U_{11} = U_{21} = 0 \) while \( U_{12}, U_{22} \) are both nonzero, or vice versa. In the first case we obtain the fully mixed state \( \hat{\rho}(\chi) = 2[U_{13} U_{23}|002\rangle\langle 00| + 2[U_{12} U_{22}|020\rangle\langle 02| + |U_{13} U_{22} + U_{12} U_{23}|011\rangle\langle 01|]. \) The other case can be obtained in a similar fashion.

More importantly, however, is the case when \( \rho_{1111} = 0 \), but \( \rho_{0202} \neq 0 \) and \( \rho_{2020} \neq 0 \) at the same time. This requires us to assume that \( |U_{11} U_{22} + U_{12} U_{21}| = |c_0 c_0^* - s_0 s_0^*| = 0 \), which can occur given the freedom in choice of the initial modes. In this case, all terms in Equation (12) with \( |11\rangle \) vanish, and we are left with a state that exhibits Hong–Ou–Mandel-like interference.\cite{33,34} This is a genuine quantum effect due to gravity.

We can finally verify if the state in Equation (12) is entangled. This requires the partial transpose \( \hat{\rho}^T(\chi) \) (with respect, say, of the second mode) of the state, and the use of the negativity \( \mathcal{N}(\hat{\rho}(\chi)) := \max(0, 1/2 \sum_{ij} (\lambda_i - 1)) \), where \( \lambda \) are the eigenvalues of \( \hat{\rho}^T(\chi) \). If the negativity is nonzero, the PPT criterion guarantees that the state is entangled.\cite{35} We can only find explicitly two negative eigenvalues of the partial transpose, which are sufficient for the detection. In fact, some algebra gives us

\[
\mathcal{N}(\hat{\rho}(\chi)) \geq \frac{1}{2} \sqrt{\rho_{0101}^2 + 4|\rho_{0211}|^2 - \rho_{0101}} \\
+ \frac{1}{2} \sqrt{\rho_{2011}^2 + 4|\rho_{2021}|^2 - \rho_{0101}}
\]

(13)

which is greater than zero for values at least one of \( \rho_{0101} \) or \( \rho_{0211} \) greater than zero. When this occurs, we conclude that gravitational redshift has entangled the state. Note that in the case where \( \rho_{1111} = 0 \) we also have \( \rho_{0211} = \rho_{2021} = 0 \), which implies that the right-hand side of (13) also vanishes. In this case, in order to detect entanglement we need to compute the other negative eigenvalues either analytically or numerically. This poses no conceptual difficulty, and can be therefore done when required.

5. Considerations and Applications

We now proceed and offer a few considerations regarding the formalism presented and used in the present work, as well as the predictions that we have put forward. We also comment on potential applications.

Our results depend on the validity of quantum field theory in curved spacetime. Therefore, testing the predictions of this work, such as the validity of the transformation in Equation (9) for different redshifts \( \chi \), that is, different configurations of the Alice–Bob positioning, can be used to test the theory. In particular, it is possible to employ the protocol described above to verify if the state in Equation (12) can be obtained in the first place, and when it exhibits characteristic quantum interference. We have found that the conditions for this to happen is that \( |U_{11} U_{22} + U_{12} U_{21}| = 0 \) together with \( \rho_{0202} \neq 0 \) and \( \rho_{2020} \neq 0 \). In general, given a certain redshift \( \chi \), specific design of the modes \( F_{\text{in}} \) and \( F_{\text{in}} \) will change the value of these three key quantities in a desired way. The conditions mentioned here can be obtained, for example, by engineering the two modes \( F_{\text{in}} \) and \( F_{\text{in}} \) to have multiple peaks that alternate.\cite{36} It is also clear that, single bell-shaped modes that do not overlap lead immediately to either vanishing \( U_{21} \) or \( U_{22} \), which therefore implies the destruction of the interference effect. Experimental detection of this effect would allow us, as mentioned above, to support the validity of quantum field theory in (weakly) curved spacetime, which still lacks experimental corroboration regardless of the many unique and striking theoretical predictions.\cite{12,37} A promising potential avenue for such tests is the use of Cubesats and other small crafts that are now being considered for use in space-based quantum experiments.\cite{38,39,40} In this case, small and relatively inexpensive satellites can be deployed at a fraction of the cost of conventional missions, and the craft itself can be potentially loaded with all
necessary equipment to perform (reasonable) long-range experiments. One idea can be to use a small collection of such satellites as sources of photons to be detected on Earth.[41]

Another important aspect that can be explored using the predictions of this work is that of the validity of the Einstein equivalence principle (EEP) in a framework where not only gravitational features but also quantum mechanical features of a physical system play a role. The EEP prescribes that the laws of physics reduce to those of special relativity locally (i.e., in regions of spacetime that are small enough).[37] This is a fundamental statement about Nature, and it is therefore a matter of fundamental interest to know if this principle holds in all regimes. To date, there are many experiments that have been already performed, and more are planned.[42–44] An even more compelling problem is the validity of the EEP in the quantum domain. While it is implicitly assumed that it does apply, there are different arguments why testing it for free falling quantum systems would be greatly beneficial for our current understanding.[42] We note that this work might provide yet another way to test the EEP, although it does not solve the problem of the EEP for gravitating quantum matter (attempts in this direction already exist[45]). Contrary to many proposed and performed experiments, we would not use massive particles (atoms),[43] but massless excitations of a quantum field. Photons can propagate (i.e., “free fall”) between two users at different height in the gravitational potential, and the shift can be measured using interferometric setups.[30,46] Given the high degree of control over photons and the high precision allowed by photonics, it would be possible to test the universality of the gravitational redshift against, for example, the initial (quantum) state of the photon, the different motion of photons (i.e., varying the trajectory), and the polarization. Since gravitational redshift is to be expected on first principles as a direct consequence of the EEP applied to two accelerated objects that exchange electromagnetic pulses,[37] we conclude that this avenue is yet another dimension that can be explored with the mechanism described here. More work is of course necessary to establish a concrete protocol and put forward a realistic experimental proposal.

We continue by recalling that novel and advanced theories of Nature predict deviations from those of general relativity that occur in specific (typically high-energy or extremely small scale) regimes. There are several proposals to test different aspects of novel physics in space.[41] One advantage of a space-based setup is that photons propagating through spacetime might be able to witness deviations from expected kinematics. These effects might be due to, for example, asymmetries as a consequence of anisotropic background spacetimes (effects that can be witnessed by comparing results from experiments with photons propagating in different directions), an ultraviolet cutoff or coarse graining of spacetime among many.[44] In this case, propagation through a long baseline can provide the necessary cumulation of effects that can lead to successful detection. Since the mode-mixing predicted here is a definitive signature of quantum field theory in curved spacetime, any deviation could be amplified in an interferometric-like measurement and therefore detected. We believe that this is another opportunity in support of testing the results of this work.

We also note that mode mixing is a key phenomenon in many areas of physics, and it is an ubiquitous operation in quantum optics.[30] Neutrino physics is another area where mode mixing has led to a revolutionary new understanding of high-energy physics processes. While previously thought to be massless, neutrinos were subsequently proposed to be massive, a feature that was experimentally confirmed and that requires them to “mix flavors”.[47,48] This phenomenon, known as neutrino oscillations, can be seen as a form of mode mixing, where three distinct operators (flavors) are mixed unitarily into three new ones.[47] While we do not present the theory or discuss the implications, we note that neutrinos, like every other realistic particle, will be represented by a wavepacket of field excitations, which requires updating the mathematical technology developed for ideal sharp-momentum particles in order to take care of all realistic features. Our results can help in addressing some of the issues, including adding the effects of weak gravitational backgrounds on the propagation of the neutrinos as wave-packets.

Finally, states that exhibit such quantum coherence can be used as resources for quantum computing.[45] It remains an open question how this final aspect can be employed constructively in concrete applications.

6. Conclusion

We have shown that gravitational redshift cannot be implemented as a unitary operation on the sharp-frequency field-modes alone. Instead, the effects of gravitational redshift on propagating photons can be modeled as a mode-mixer, which shifts excitations from one particular frequency distribution to others. We then showed that this effect can be exploited to induce two-photon Hong–Ou–Mandel-like interference purely as a consequence of the photons propagating in a curved background. This result adds to the existing unique predictions of quantum field theory in curved spacetime. We therefore conclude that our work provides novel insight into the quantum aspects of gravitational redshift[49] and, more broadly, the interplay of relativity and quantum mechanics. Experimental verification of this effect is within the reach of near future experimental capabilities.

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

Keywords

gravitational redshift, quantum field theory, quantum interference, quantum optics
An introduction to quantum field theory in curved spacetime is left to standard references. The metric has signature $\eta = (-1, 1, 1, 1)$ and natural units $c = \hbar = 1$ unless explicitly stated. We work in the Heisenberg picture.