In a homogeneous and isotropic universe with zero spatial curvature characterized by the Friedmann-Lemaître-Robertson-Walker (FLRW) line element, we consider the evolution of our Universe where dark energy is interacting with dark matter in presence of adiabatic particle creation by the time varying gravitational field. Due to complicated nature of the Einstein’s field equations, dynamical systems analysis have been performed and non-isolated sets of critical points are obtained. We have found some interesting cosmological scenarios like late-time evolution of universe dominated by dark energy which could mimic quintessence, cosmological constant or phantom field respectively through a dark matter dominated era and also obtained a possibility of crossing the phantom divide line which is favoured by observations.

PACS numbers: 95.36.+x, 95.35.+d, 98.80.-k, 98.80.Cq.

Keywords: Dark energy; Dark matter; Interaction; Particle production; Dynamical system; Phase space analysis; Normally hyperbolic.

1. INTRODUCTION

Various observations suggest that our universe is currently undergoing a phase of accelerated expansion [1–5]. This is a challenging issue in standard cosmology which shows a new imbalance in the governing Friedmann equations. People have addressed such imbalances either by introducing new sources, or by altering the governing equations. In the frame of standard cosmology, the first one is termed as dark energy with a huge negative pressure, and the second one involves the introduction of some modifications into the gravity sector commonly known as modified gravity theories. The simplest dark energy candidate is the cosmological constant $\Lambda$, which together with cold dark matter provides the simplest cosmological model known as the $\Lambda$-cold-dark-matter ($\Lambda$CDM), which according to a large number of observations, is the best cosmological model at present. However, $\Lambda$CDM suffers from severe problems in the interface of cosmology and particle physics, such as the cosmological constant problem [6–8] and the cosmic coincidence problem [9].

Now, in order to address these issues related to $\Lambda$-cosmology, an extensive analysis have been performed ranging from various DE models to modified gravity theories [10, 11]. Amongst them, the cosmological models where dark matter (DM) and dark energy (DE) interacts with each other have gained significant attention with successive number of observational data. In fact, very latest observations indicate a non-vanishing interaction in the dark sector [12–14]. Thus, the interaction between DE and DM could be a major issue to be confronted in studying the physics of DE. However, since the nature of these two dark components (DE and DM) remaining unknown, until now the precise form of interaction is unknown, and there is not yet a fundamental theory for choosing a specific coupling, so, this will necessarily be phenomenological. Further, in the framework of field theory, it is natural to consider the inevitable interaction between the dark components. In fact, an appropriate interaction between DE and DM can provide a mechanism to alleviate the coincidence problem, phantom crossing, cosmic age problem, and also the transient nature of the deceleration parameter. Interacting dark sector models have been extensively studied in several works [15–27]. They are well known for providing a possible solution to the cosmic coincidence problem [28], as well as yielding a physical explanation for measuring a DE phantom equation of state [29, 30]. It should be noted that there are

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other options apart from the above mentioned choices for explaining the cosmic coincidence and other cosmological conundrums. In particular, there is the $\Lambda$CDM type of models (for detailed study see [31, 32]) where there exists an interaction between the vacuum energy and another DE component. In this case matter can be conserved and nevertheless the ratio between the DE and DM remains bounded in the entire cosmic history. Further, in this context one finds effective quintessence and phantom like behaviors in references [33–35].

Therefore, it should be mentioned that the inclusion of interaction which allows the exchange of energy between dark sectors, makes the dynamics richer than non-interacting one. As for example, if the quintessence field is coupled to DM, this provides the similar energy density in the dark sector today, and to get such a similar energy density, one must introduce the coupling which leads to an accelerated scaling attractor solution [20, 36, 37] with

$$\frac{\Omega_{DE}}{\Omega_{DM}} \approx O(1) \quad \text{and} \quad \omega_{\text{eff}} < -\frac{1}{3}$$

Thus, in this case, the coincidence problem is reduced to a simple choice of parameters involved to match $\Omega_{DE}/\Omega_{DM}$ to observations.

Presently several observations [38–43] explore that our universe slightly crossing the quintessence region, entering into the phantom era with equation of state $\omega_{\text{eff}} < -1$. To get such a scenario one has to consider the wrong sign in the kinetic term of the scalar field [44]. Due to the nature of unbounded from below this kind of energy densities, leads to some instabilities at classical as well as quantum levels [45, 46] and also induces some other theoretical problems [47–49]. In spite of these above arguments people may have another choices to derive such an accelerated expansion of universe upto the phantom domain, the particle creation mechanism is the attractive one. Thus, the particle creation mechanism would be a viable alternative in comparison with both dark energy and modified gravity models to produce accelerating expansion of the universe. This creation model can successfully mimic the $\Lambda$CDM cosmology [50–54].

In fact, it has been found that particle creation mechanism was started long back ago around 1939 when Schrodinger [55] introduced a microscopic description where particles could be created at the expense of gravitational field in an expanding universe. Following his idea Parker with collaborators [56], and Zeldovich with his collaborators [57] started investigating the possible physical scenarios by the production of particles. Since the evolution of universe could be understood by Einstein field equations, Prigogine [58] studied the evolution of universe after introducing particle creation mechanism in Einstein’s field equations by changing the usual balance equation for the number density of particles.

Now, the only dissipative phenomenon in the homogeneous and isotropic flat FLRW model may be in the form of bulk viscous pressure either due to coupling of different components of the cosmic substratum [59–63] or due to non-conservation of (quantum) particle number. Thus for an open thermodynamical system where the number of fluid particles are not preserved ($N^{\mu}_{\cdot \mu} \neq 0$) [64–66], the particle conservation equation gets modified as

$$N^{\mu}_{\cdot \mu} \equiv \nabla^{\mu}u_{\mu} + \Theta n = n\Gamma \iff N_{\cdot \mu}u^{\mu} = \Gamma N, \quad \text{i.e.,} \quad \dot{N} = \Gamma N$$

This equation is also known as the balance equation for the fluid flux. Also the implied relation states that the rate of change of total particle number is proportional to the total number of particles. Here $\Gamma$ stands for the rate of change of particle number in a comoving volume $V$ containing $N$ number of particles, $N^{\gamma} = n u^{\gamma}$, the particle flow vector, $u^{\mu}$ is the four velocity vector, $n = N/V$ is the particle number density and $\Theta = u^{\mu}_{\cdot \mu}$ is the fluid expansion. Here, $\Gamma$, the rate of produced particles is unknown in nature, but the validity of second law of thermodynamics implies the positivity of $\Gamma$. In the present work, dissipative effect due to the second alternative is chosen. However, for simplicity of calculation, adiabatic (i.e., isentropic) production [58, 67] of perfect fluid particles is considered and as a result viscous pressure obeys a linear relationship with particle production rate. It should be noted that although the entropy per particle is constant (due to isentropic nature) but still there is entropy production for enlargement of phase space (i.e., increase in the number of the fluid particles and also expansion of the universe) of the system.

The particle creation scenario can successfully describe the accelerated expansion model of universe without introducing DE. Also, many interesting results with this mechanism, such as, a possibility of future deceleration has been proposed in [68, 69], consequently, an existence of an emergent universe has been shown in [70, 71] and subsequently, the complete cosmic scenario has been reported in [72]. Further, in the framework of particle creation mechanism universe evolves from big bang scenario to late time de Sitter phase in [73] and accelerated expansion of universe at early and present times are reported in [74]. Furthermore, the possibility of a phantom universe without invoking any phantom fields has recently been realized in the similar context [75, 76]. So it is worth studying interacting DE models from particle creation mechanism.

In the present work, considering our universe as an open thermodynamical system in the framework of flat Friedmann-Lemaître-Robertson-Walker (FLRW) space time, an interacting dynamics between dark energy and dark matter has been proposed where the dark matter particles are assumed to be created from the gravitational field. The
creation process is taken to be ‘adiabatic’ for simplicity. Then, we rewrite the Friedmann equation and Raychaudhuri equation in the context of matter creation mechanism. We assume the particle production rate to be proportional to the Hubble parameter and is uniform throughout the universe. The main scope of this work is to analyze the cosmological dynamics of interacting DE models in the framework of adiabatic particle creation using dynamical system techniques. Dynamical system tools have been extensively used to study the asymptotic behavior of various cosmological models where exact solutions of evolution equations cannot be obtained (see for e.g., [77–84]). We obtained some interesting critical points which describe many interesting results from the phase space analysis of linear interactions. These include the early matter dominated universe, the late time DE dominated attractors in some parameter region, where DE is associated with quintessence, cosmological constant, or phantom field respectively.

We follow the plan to carry out the whole work as: In section 2, we present the basic equations of the present particle creation model and the evolution equations are transformed to an autonomous system by suitable transformation of the dynamical variables. In section 3, critical points are shown for various choices for the interaction term and the cosmological parameters have been evaluated. Section 4 shows the phase space analysis and stability criteria for the critical points. In section 5, cosmological implication of critical points for several interaction models are given. The paper ends with a short discussion in section 6.

2. THE BASIC EQUATIONS IN PARTICLE CREATION AND AUTONOMOUS SYSTEM

In accordance with inflation and cosmic microwave background radiation, the universe is well described by the spatially flat (FLRW) space-time

\[ ds^2 = -dt^2 + a^2(t) \left( dr^2 + r^2 d\Omega^2 \right), \]  

(3)

where \( a(t) \) is the scale factor of the universe and the spherical line element \( d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2 \) is the metric on the unit 2-sphere. For the co-moving observer, \( u^\mu = \delta_\mu^t \), is the velocity vector so that \( u^\mu u_\mu = -1 \), accompanied with the line element (3), the fluid expansion (\( \Theta \)) will become, \( \Theta = 3H \), where \( H \) is the Hubble parameter. Hence, the particle conservation equation (2) is reduced to

\[ N^\mu_{;\mu} \equiv n_{,\mu} u^\mu + 3H n = n\Gamma, \]  

(4)

for the present open thermodynamical model. Further, using the above conservation equation (4), the Gibb’s relation

\[ Tds = d \left( \frac{\rho}{n} \right) + pd \left( \frac{1}{n} \right), \]  

(5)

gives the variation of entropy per particle by the relation [58, 73]

\[ nT\dot{s} = \dot{\rho} + 3H \left( 1 - \frac{\Gamma}{3H} \right) (\rho + p), \]  

(6)

where \( T \) represents the fluid temperature, and \( s \) is the entropy per particle, i.e., specific entropy. (Normally, specific entropy of a system is the entropy of the unit mass of the system. In the present context it is entropy per unit particle). We consider our thermodynamical system to be ideal one, i.e., isentropic (or adiabatic) (see, for instance [67, 87]). So, we have production of perfect fluid particles, having constant entropy (i.e., \( \dot{s} = 0 \)). There is however, entropy production due to the enlargement of the phase space of the system since number of perfect fluid particles increases. Hence, from equation (6) one can obtain the conservation equation as

\[ \dot{\rho} + 3H(\rho + p) = \Gamma(\rho + p), \]  

(7)

which however can also be written as [74]

\[ \dot{\rho} + 3H(\rho + p + p_c) = 0. \]  

(8)
So, comparing equations (7) and (8), one can easily obtain the creation pressure as follows [72, 73, 89, 90]

\[ p_c = \frac{-\Gamma}{3H}(\rho + p), \]  

(9)

where \( p_c \) is termed as the creation pressure, and \( \Gamma \) is the particle production rate (number of particles created per unit time), assumed to be uniform throughout the universe. Now, considering the main constituents of our universe as dark matter (DM) in the form of dust having energy density \( \rho_m \) and the pressure \( p_m = 0 \), satisfying the equation of state \( \omega_m = p_m/\rho_m = 0 \), and the dark energy fluid having equation state \( \omega_d = p_d/\rho_d \) (where, \( \rho_d, p_d \), are respectively the energy density and the thermodynamic pressure of the cosmic fluid), where \( \rho = \rho_m + \rho_d \), is the total energy density, we have the Friedmann and Raychaudhuri equations (8\( \pi G = c = 1 \)) as follows:

\[ H^2 = \frac{1}{3}(\rho_m + \rho_d), \]  

(Friedmann’s equation)  

(10)

\[ \dot{H} = \frac{-1}{2}(\rho_m + \rho_d + p_d + p_c), \]  

(Raychaudhuri’s equation)  

(11)

where \( H = \dot{a}/a \) is the Hubble parameter, and an ‘overdot’ represents the differentiation with respect to cosmic time ‘t’. If we assume that the created particles are described as pressure- less dark matter (DM) in the thermodynamically open model of the universe under the adiabatic condition, then the creation pressure \( p_c \) in (9) becomes [75]

\[ p_c = \frac{-\Gamma}{3H}(\rho_m). \]

In standard cosmology, the dynamic interactions between the homogeneously distributed DE in the universe and the DM component (clumping around the ordinary particles) are extremely weak or even it is negligible. As a result, the energy conservation equations for the two matter components are

\[ \dot{\rho}_m + 3H(\rho_m + p_c) = 0 \]  

(12)

or using the above expression for \( p_c \) we have

\[ \dot{\rho}_m + 3H\rho_m = \Gamma\rho_m \]  

(13)

and

\[ \dot{\rho}_d + 3H(\rho_d + p_d) = 0. \]  

(14)

In order to alleviate the cosmological coincidence problem, it has been found that, a non-gravitational interaction between these dark sectors could be a viable alternative. So the interacting DM and DE models of the universe are becoming of great interest and are widely used in the literature [23]. Thus the above energy conservation equations are modified as :

\[ \dot{\rho}_m + 3H\rho_m\left(1 - \frac{\Gamma}{3H}\right) = -Q, \]  

(15)

\[ \dot{\rho}_d + 3H(\rho_d + p_d) = Q, \]  

(16)

where \( Q \) indicates the rate of energy exchange between the dark sectors.

In particular, \( Q > 0 \) indicates conversion of DM into DE while \( Q < 0 \) represents the vice-versa. A complete study of the interaction of dynamical vacuum energy with matter can be found in the references [91, 92] (for an extension see also the Refs. [93, 94]). These studies are based on the general expectations of the effective action of quantum field theory (QFT) in curved space time. Further, it should be noted that the running vacuum models [95, 96] give the overall fit to the observational data better than the ΛCDM. All these models are based on fundamental aspects of QFT [97] and provide an interaction of dynamical vacuum and matter that is capable to fit the observational data better than the concordance model.

In the present work, we describe the background dynamics with different interactions as (i) \( Q \propto H\rho_m \) [20, 22], (ii) \( Q \propto H\rho_d \) [98, 99], and (iii) \( Q \propto H(\rho_m + \rho_d) \) [16, 100], (iv) \( Q \propto \rho_m^{3/4} \) [30], and (v) \( Q \propto \rho_m \) [20].
One may note that in the above interactions, the dimensionless parameters (proportionality constants) should not be chosen in an ad hoc manner. These parameters can actually be fitted to the overall observational data and one finds that they are typically of order between $10^{-3}$ to $10^{-2}$ [94-96, 101] depending on the normalization on the parameters involved. Further, in these references the justifications of such small values of these parameters are shown from two perspectives, namely theoretically these coefficients represent the beta-function of the running vacuum energy [96, 101] and hence are expected to be very small. Also, experimentally, as the fitted values of these coefficients to the recent SN Ia + BAO + LSS + BBN + CMB data (in which WMAP9, Planck−13, 15 data are taken into account) [38] to be of the same order as the theoretically expected values. Further, if we compare our equation (15) with equation (4) of the $Q_m$ model in ref. [101] and choosing $\Gamma = \Gamma_0 H$ ($\Gamma_0$, a constant), we see that effective interaction term will be $(\Gamma_0 - \alpha_m)H \rho_m$ in the first case. Thus comparing equation (7) of ref. [101] we have

$$\Gamma_0 - \alpha_m = 3\nu_{dm}.$$  (17)

Moreover, the very recent observationally estimated values of the parameters $\nu_{dm}$ and $\nu_\Lambda$ (in equations (7) and (8) of ref. [101]) similar to our interaction models 1 and 2 are given by (see table II of ref. [101])

$$\nu_{dm} = 0.00618 \pm 0.00159,$$

$$\nu_\Lambda = 0.01890 \pm 0.00744.$$

Thus $\Gamma_0$ and $\alpha_m$ are not arbitrary, their difference has an observational estimate. Note that as $\nu_{dm}$ is positive so from equation (17) $\Gamma_0$ is always greater than $\alpha_m$ and the effective interaction term has the same sign convention as in ref. [101].

Due to complicated non-linear forms in the evolution equations (10), (11), (15) and (16) we convert these evolution equations to an autonomous system of first order differential equations. To do this we consider the following dimensionless variables [17]

$$x = \frac{\rho_d}{3H^2}, \quad y = \frac{p_d}{3H^2},$$  (18)

which are normalized over Hubble scale. Then, the autonomous system of ordinary differential equations obtained are:

$$\frac{dx}{dN} = \frac{Q}{3H^3} - (1 - x)(3y - \Gamma_0),$$  (19)

$$\frac{dy}{dN} = \frac{Qy}{3xH^3} - (1 - x)\left[\frac{3y^2}{x} + \Gamma_0y\right].$$  (20)

Here, the independent variable is chosen as the lapse time $N = \ln a$, which is called the e-folding number and the particle production rate $\Gamma$ as a function of the Hubble parameter [68, 72] ($\Gamma$ has dimensions (time)$^{-1}$) is chosen as above $\Gamma = \Gamma_0 H$ ($\Gamma_0$ is a constant). The value of the parameter $\Gamma_0$ is assumed to be nonnegative as the creation of particles are only considered in this study.

Now, in terms of the new dimensionless quantities, cosmological parameters can be written as follows. For instance, the energy density parameter for dark matter as

$$\Omega_m = 1 - x,$$  (21)

and the energy density parameter for the dark energy as

$$\Omega_d = x.$$  (22)

It may be noted that in the case of non-interacting DE models, the energy density is usually considered to be non-negative. However, in this case of interacting DE models, the energy density can be taken to be negative [102]. This would implies that there is no constrain for dimensionless variables, making the phase space that is analyze here to be not compact. So, there might be a possibility of critical points at infinity. In general, the analysis of
fixed points at infinity is done by compactifying the phase space using Poincare compactification. However, from a phenomenological point of view in this present work, we shall only determine the dynamics in the neighborhood of finite fixed points. This is enough, since our aim is to find physically viable solutions, namely trajectories connecting DM to DE domination.

The equation of state parameter for the DE can be expressed as
\[ \omega_d = \frac{p_d}{\rho_d} = \frac{y}{x} , \tag{23} \]
and the effective equation of state parameter will be of the form
\[ \omega_{\text{eff}} = y - \frac{\Gamma_0}{3} \left( 1 - x \right) . \tag{24} \]

Moreover, we have the evolution equation of the Hubble function as
\[ \frac{1}{H} \frac{dH}{dN} = -\frac{3}{2} \left( 1 + y - \frac{\Gamma_0}{3} (1 - x) \right) . \tag{25} \]

We now determine the critical points of the above autonomous system for different choices of \( Q \) and then we perturb the equations up to first order about the critical points, in order to determine their stability.

3. CRITICAL POINTS OF AUTONOMOUS SYSTEM (19)–(20) FOR VARIOUS CHOICES OF INTERACTION TERM AND THE COSMOLOGICAL PARAMETERS:

In this section, we discuss the existence of the critical points and the corresponding physical parameters for various interaction models. These are presented in detail in tabular form.

3.1. Interaction Model 1:

First, we choose the interaction as
\[ Q = \alpha_m H \rho_m , \tag{26} \]
where the coupling parameter \( \alpha_m \) is a dimensionless constant. The indefiniteness in the sign of \( \alpha_m \) indicates that the energy transfer takes place in the either direction - DE or DM. This interaction is well motivated due to mathematical simplicity as the dimensions of the autonomous system remain same because \( H \) parameter can be eliminated from the equations. Now, using this interaction in the system (19)-(20), the autonomous system for this interaction model will be
\[ \begin{align*}
\frac{dx}{dN} &= (-1 + x)(\Gamma_0 x - \alpha_m + 3y), \\
\frac{dy}{dN} &= \frac{y}{x}(-1 + x)(\Gamma_0 x - \alpha_m + 3y). 
\end{align*} \tag{27-28} \]

The critical points for this system (27)-(28) are the following:

- Set of critical points: \( A_1 = (1, y_c) \), where \( y_c \) takes any real value.
- Critical Point : \( B_1 = (\frac{\alpha_m}{\Gamma_0}, 0) \).
- Critical point : \( C_1 = (1, 0) \).
- Critical point : \( D_1 = (1, -1) \).
- Set of critical points : \( E_1 = (x_c, \frac{\alpha_m}{\Gamma_0} - \frac{\Gamma_0}{3} x_c) \).

The existence of critical points and their cosmological parameters are displayed in the table I. It is observed that point \( B_1 \) is a point on the set \( E_1 \), points \( C_1 \) and \( D_1 \) are points on the set \( A_1 \). So, in the next section we shall analyze only the stability of sets \( A_1 \) and \( E_1 \). However, critical points \( B_1 \), \( C_1 \) and \( D_1 \) show some interesting cosmological features, which we shall discuss in Sec. 5.
### 3.2. Interaction Model 2:

We consider another choice of interaction as

\[ Q = \alpha_d H \rho_d, \tag{29} \]

where \( \alpha_d \) is the coupling parameter. Using this interaction in the system (19)-(20), we have the autonomous system as

\[
\frac{dx}{dN} = (\Gamma_0 + x + 3y)(x - 1) + \alpha_d x, \tag{30}
\]

\[
\frac{dy}{dN} = (\alpha_d - 3)y - 3\frac{y^2}{x} + y\left(3(1+y) - \Gamma_0(1-x)\right). \tag{31}
\]

The autonomous system (30)-(31) admits the following critical points:

- **Critical Point**: \( A_2 = \left(\frac{\Gamma_0 - \alpha_d}{\Gamma_0}, 0\right) \).

- **Set of critical points**: \( B_2 = \left(x_c, \frac{\Gamma_0 x_c - \Gamma_0 + \alpha_d}{3(1-x_c)}\right) \).

The existence criteria and the cosmological parameter related to the critical points are shown in the table II. It is again noted that point \( A_2 \) is a point on a set \( B_2 \). So, in the next section we shall analyze only the stability of set \( B_2 \). However, depending on the choice of coupling parameter \( \alpha_d \), critical point \( A_2 \) shows some interesting cosmological features, which we shall discuss in Sec. 5.

### 3.3. Interaction Model 3:

Now, we consider the linear interaction as

\[ Q = \alpha H (\rho_m + \rho_d). \tag{32} \]

For this interaction model, the system (19)-(20) will take the form

\[
\frac{dx}{dN} = \alpha + (\Gamma_0 x + 3y)(x - 1), \tag{33}
\]

\[
\frac{dy}{dN} = \frac{y}{x} \left(\alpha + (\Gamma_0 x + 3y)(x - 1)\right). \tag{34}
\]

The critical points are:
TABLE III: The existence of Critical Points and the corresponding physical parameters for the interaction model $Q_3 = \alpha H(\rho_m + \rho_d)$

| Critical Points | Existence | $\omega_d$ | $\omega_{\text{eff}}$ | $\Omega_m$ | $\Omega_d$ |
|-----------------|-----------|------------|------------------------|------------|------------|
| $A_3 : \left(\frac{1}{2}(1 + \sqrt{1 - \frac{4\alpha}{1 - \alpha}}), 0\right)$ | $\alpha_{1/4} < \frac{1}{4}$ | 0 | $\frac{\gamma_0}{6}(1 + \frac{1 - \frac{3}{4}d}{1 - \frac{3}{4}} \frac{1}{2}(1 - \sqrt{1 - \frac{4\alpha}{1 - \alpha}}) \frac{1}{2}(1 + \sqrt{1 - \frac{4\alpha}{1 - \alpha}})$ | $\frac{1}{2}(1 - \sqrt{1 - \frac{4\alpha}{1 - \alpha}}) \frac{1}{2}(1 + \sqrt{1 - \frac{4\alpha}{1 - \alpha}})$ | |
| $B_3 : \left(\frac{1}{2}(1 - \sqrt{1 - \frac{4\alpha}{1 - \alpha}}), 0\right)$ | $\alpha_{1/4} < \frac{1}{4}$ | 0 | $\frac{\gamma_0}{6}(1 + \frac{1 - \frac{3}{4}d}{1 - \frac{3}{4}} \frac{1}{2}(1 - \sqrt{1 - \frac{4\alpha}{1 - \alpha}}) \frac{1}{2}(1 + \sqrt{1 - \frac{4\alpha}{1 - \alpha}})$ | $\frac{1}{2}(1 - \sqrt{1 - \frac{4\alpha}{1 - \alpha}}) \frac{1}{2}(1 + \sqrt{1 - \frac{4\alpha}{1 - \alpha}})$ | |
| $C_3 : \left(x_c, \frac{\gamma_0 x_e^2 - \Gamma_0 x + \alpha}{\beta(1 - x_c)}\right)$ | $x_c \neq 1$ | $\frac{\gamma_0 x_e^2 - \Gamma_0 x + \alpha}{\beta x_c (1 - x_c)}$ | $\frac{\gamma_0 x_c - \Gamma_0 + \alpha}{\beta(1 - x_c)}$ | $1 - x_c$ | $x_c$ |

TABLE IV: The existence of critical points and the corresponding physical parameters for the interaction model $Q_4 = \frac{\beta}{\Omega m \rho d}$

| Critical Points | Existence | $\omega_d$ | $\omega_{\text{eff}}$ | $\Omega_m$ | $\Omega_d$ |
|-----------------|-----------|------------|------------------------|------------|------------|
| $A_4 : (1, y_c)$ | Always | $y_c$ | $y_c$ | 0 | 1 |
| $B_4 : (1, 0)$ | Always | 0 | 0 | 0 | 1 |
| $C_4 : (1, -1)$ | Always | $-1$ | $-1$ | 0 | 1 |
| $D_4 : \left(x_c, \beta x_c - \frac{\Gamma_0 x_c}{\beta} \right)$ | Always | $\beta - \frac{\Gamma_0}{\beta} \beta x_c - \frac{\Gamma_0}{\beta}$ | $1 - x_c$ | $x_c$ |

- Critical Point: $A_3 : \left(\frac{1}{2}(1 + \sqrt{1 - \frac{4\alpha}{1 - \alpha}}), 0\right)$, where $\Gamma_0 \geq 4\alpha$.
- Critical Point: $B_3 : \left(\frac{1}{2}(1 - \sqrt{1 - \frac{4\alpha}{1 - \alpha}}), 0\right)$, where $\Gamma_0 \geq 4\alpha$.
- Set of critical points: $C_3 : \left(x_c, \frac{\gamma_0 x_e^2 - \Gamma_0 x + \alpha}{\beta(1 - x_c)}\right)$.

The cosmological parameters related to the critical points are shown in tabular form in table III. It is again noted that points $A_3$, $B_3$ are points on a set $C_3$. So, in the next section we shall analyze only the stability of set $C_3$.

3.4. Interaction Model 4:

Considering the nonlinear interaction

$$Q = \frac{\beta}{\Omega m \rho d}$$

in (19)-(20), the autonomous system will be in the form

$$\frac{dx}{dN} = (x - 1)(\Gamma_0 x - 3\beta x + 3y),$$

$$\frac{dy}{dN} = \frac{y}{x}(x - 1)(\Gamma_0 x - 3\beta x + 3y),$$

and the following are the critical points for this case:

- Set of critical points: $A_4 = (1, y_c)$.
- Critical Point: $B_4 = (1, 0)$.
- Critical Point: $C_4 = (1, -1)$.
- Set of critical points: $D_4 = \left(x_c, \beta x_c - \frac{\Gamma_0 x_c}{\beta}\right)$.

The condition for existence of critical points and the corresponding physical parameters are presented in table IV. It is again noted that points $B_4$, $C_4$ are points on the set $A_4$. So, in the next section we shall analyze only the stability of sets $A_4$ and $D_4$. 
3.5. Interaction Model 5:

We are now going to discuss another type of interaction in the dark sectors which is completely based on the local properties of the universe and hence it is different from the other interaction models discussed in the previous subsections. Here, we replace the non-local transfer rate (discussed in the previous sub-sections) by the local rate $\eta$, and the interaction model [20] has the following form

$$Q = \eta \rho_m,$$

(38)

where the coefficient $\eta$ related to the local rate is assumed to be constant. We note that the interaction (38) has been studied in [22] describing the phase space analysis when the dark energy equation of state has the phantom behavior. However, when $\eta > 0$ the interaction is used for describing the decay of DM into radiation [103], the decay of a curvaton field into radiation [104], and also a model with the decay of superheavy DM particles into a quintessence scalar field [105]. Now we see that in (38) when, $\eta > 0$ there are decaying of DM into DE which allows the possibility that there is no DE field in primordial universe so that the DE condensates as a result of slow decay of DM. On the other hand, the energy should transfer in the reverse way for $\eta < 0$. Now, to close the dynamical system (19)-(20) and to eliminate $H$ appeared for this interaction, one has to introduce the new variable $v$ in the set (18) so that

$$v = \frac{H_0}{H + H_0},$$

(39)

where $H_0$ is constant and hence $0 \leq v \leq 1$.

We now introduce a dimensionless coupling constant

$$\gamma = \frac{\eta}{H_0}.$$ (40)

Then the autonomous system of equations can be written as

$$\frac{dx}{dN} = \frac{(-1 + x)}{(-1 + v)} ((\Gamma_0 x + 3y)(v - 1) + \gamma v),$$

(41)

$$\frac{dy}{dN} = \frac{y((-1 + x)}{x(-1 + v)} ((\Gamma_0 x + 3y)(v - 1) + \gamma v),$$

(42)

$$\frac{dv}{dN} = \frac{1}{2} v(v - 1)(3 + 3y - \Gamma_0(1 - x)).$$

(43)

The critical points for this system are the following:

- Set of critical points : $A_5 = (1, y_c, 0)$.
- Set of critical points : $B_5 = (1, -1, v)$.
- Set of critical points : $C_5 = (x_c, -\frac{\Gamma_0 x}{3}, 0)$.
- Set of critical points : $D_5 = (x_c, -1 + \frac{\Gamma_0 x}{3}(1 - x_c), \frac{-3 + \Gamma_0 x}{-3 + \Gamma_0 + \gamma})$.

The set of critical points and the corresponding cosmological parameters are presented in table V. We note here that sets $A_5$ and $B_5$ have common point $(1, -1, 0)$.

4. PHASE SPACE ANALYSIS AND STABILITY CRITERIA OF CRITICAL POINTS

We shall now discuss the phase space analysis of critical points and their stability by analyzing the eigenvalues of the linearized Jacobian matrix evaluated at the critical points presented in Tables VI and VII. It can be seen from Tables VI and VII that all critical points are actually non-isolated set of critical points. By definition, non-isolated set contains at least one vanishing eigenvalue, so it is non-hyperbolic in nature [106]. The type of non-isolated set with exactly one vanishing eigenvalue is called a normally hyperbolic set. Its stability condition is similar to the linear stability analysis simply by looking from the signature of the remaining non-vanishing eigenvalues [106]. In this work, all set of points are normally hyperbolic except set $B_5$, $C_5$ and $D_5$ (see Tables VI and VII).
Critical points and the corresponding physical parameters for the interaction model $Q = \eta \rho_m$

| Critical Points | Existence | $\omega_d$ | $\omega_{\text{eff}}$ | $\Omega_m$ | $\Omega_d$ |
|-----------------|-----------|------------|----------------------|-----------|-----------|
| $A_5 : (1, y_c, 0)$ | Always | $y_c$ | $y_c$ | 0 | 1 |
| $B_5 : (1, -1, v)$ | $0 \leq v \leq 1$ | $-1$ | $-1$ | 0 | 1 |
| $C_5 : (x_c, \frac{\Gamma_0}{\Gamma_0 + \gamma})$ | Always | $-\frac{\gamma}{3}$ | $-\frac{\gamma}{3}$ | $1 - x_c$ | $x_c$ |
| $D_5 : \left( x_c, -1 + \frac{\Gamma_0}{\Gamma_0 + \gamma}(1 - x_c), -\frac{3 \Gamma_0}{3 \Gamma_0 + 

Table V: The existence of critical points and the corresponding physical parameters for the interaction model $A_5 = \omega_d$.

Table VI: Table shows the eigenvalues of the critical points for different interaction models.

4.1. Interaction 1

The system (27)-(28) admits two sets of critical points $A_1$ and $E_1$. As mentioned earlier point $B_1$ lies on the set $E_1$, whereas points $C_1$, $D_1$ lie on the set $A_1$. In what follows, we therefore analyze the stability of sets $A_1$ and $E_1$ only.

- The solution associated with the set of critical points $A_1(1, y_c)$ (where $y_c$ takes any real value) always exist. They are completely DE dominated solutions ($\Omega_d = 1$), where DE corresponds to an exotic type fluid with equation of state $\omega_d = y_c$. For this case, DE can describe the quintessence, cosmological constant, or phantom field, or any other perfect fluid according as the choice of $y_c$. So, the line of critical points may have different features in their cosmic evolution. Points on this set corresponds to an accelerating universe (i.e., $\omega_{\text{eff}} < -\frac{1}{3}$) for $y_c < -\frac{1}{3}$ (see Table I), and there exists an expanding universe if the evolution of Hubble function satisfies $\omega_{\text{eff}} < -1$, (see Eq. (25)) (i.e., Hubble parameter increases gradually) for $y_c < -1$, i.e., in phantom region. This set is normally hyperbolic and hence corresponds to a late time attractor for $y_c < \frac{\alpha_x - \Gamma_0}{3 \alpha_x}$ (see Table VI). This is also being confirmed numerically in

Table VII: Table shows the eigenvalues of the critical points for the interaction model $5. Q = \gamma H_0 \rho_m$.

| Critical Points | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ |
|-----------------|------------|------------|------------|
| $A_5 : (1, y_c, 0)$ | 0 | $\frac{1}{3}(1 + y_c)$ | $3y_c + \Gamma_0$ |
| $B_5 : (1, -1, v)$ | 0 | 0 | $-3 + \Gamma_0 - \frac{\gamma}{3}$ |
| $C_5 : (x_c, \frac{\Gamma_0}{\Gamma_0 + \gamma})$ | 0 | 0 | $\frac{1}{3}(3 - \Gamma_0)$ |
| $D_5 : \left( x_c, -1 + \frac{\Gamma_0}{\Gamma_0 + \gamma}(1 - x_c), -\frac{3 \Gamma_0}{3 \Gamma_0 + 

\text{TABLE VII: Table shows the eigenvalues of the critical points for the interaction model 5. } Q = \gamma H_0 \rho_m.
FIG. 1: Figure shows the vector field of autonomous system (27-28) for the interaction model 1 with the parameter values $\alpha_m = -0.01$ and $\Gamma_0 = 0.5$. In panel (a), a colored line represents a line $A_1$ with black colored line being a stable portion of set $A_1$ and in panel (b), a colored line represents a line $E_1$ with black colored line being a stable portion of set $E_1$.

Fig.1(a). The one dimensional center subspace spanned by eigenvector 

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

which corresponds to a vanishing eigenvalue identifies the direction of the set $A_1$. Whereas the one dimensional stable subspace near this set is spanned by eigenvector 

$$\begin{pmatrix} 1/y_c \\ 1 \end{pmatrix}$$

which corresponds to a non-vanishing eigenvalue with $y_c < \frac{2\alpha_m - \Gamma_0}{3}$. Since there is no unstable subspace near $A_1$ when $y_c < \frac{2\alpha_m - \Gamma_0}{3}$, so trajectories will approach towards some points on set $A_1$. The set of critical points $A_1$ represents a stable attractor in quintessence era for $y_c < \min \{ \frac{2\alpha_m - \Gamma_0}{3}, -1 \}$, where as they are stable solutions with cosmological constant for the restriction $\Gamma_0 > 3 + \alpha_m$, and on the other hand, stable solutions are obtained in the phantom region for $y_c < \min \{ -1, \frac{2\alpha_m - \Gamma_0}{3} \}$. Hence, the set of critical points represents the solutions of accelerated stable attractor in some parameter region when DE behaves as quintessence, cosmological constant, or phantom field. This is one of the important results in this context since in presence of particle production, the scenario of interacting DE can mimic three distinct phases of the cosmic evolution.

- The set $E_1$ exists for all model parameters. It represents a scaling solution where DM and DE scale as $\Omega_m/\Omega_d = (1 - x_c)/x_c$. The DE describes any perfect fluid with equation of state parameter $\omega_d = \frac{2\alpha_m - \Gamma_0 x_c}{3x_c}$. This set is normally hyperbolic and hence, it is stable when $0 < x_c < 1$, $\alpha_m > 0$; $x_c < 0$, $\alpha_m < 0$; $x_c > 1$, $\alpha_m < 0$. This is confirmed numerically in Fig. 1(b). The one dimensional stable subspace near this set is spanned by eigenvector 

$$\begin{pmatrix} -3x_c \\ \alpha_m^2 - \Gamma_0 x_c \end{pmatrix}$$

with $x_c$ and $\alpha_m$ satisfying the above stability condition. The eigenvector 

$$\begin{pmatrix} -\frac{2}{\Gamma_0} \\ 1 \end{pmatrix}$$

which corresponds to vanishing eigenvalue determines the direction of the set. This set describes an accelerated quintessence behavior for $1 < \Gamma_0 - \alpha_m < 3 (-1 < \omega_{\text{eff}} < -\frac{1}{3})$, while it represents a cosmological constant behavior for $\Gamma_0 = 3 + \alpha_m$ ($\omega_{\text{eff}} = -1$), and phantom behavior for $\Gamma_0 > 3 + \alpha_m$ ($\omega_{\text{eff}} < -1$, see table I). This set is interesting from the cosmological point of view as it describes scaling attractor which is accelerated in quintessence, cosmological constant and in phantom region.

4.2. Interaction 2

- The autonomous system (30)-(31) has only one set critical points $B_2$. As mentioned earlier, it is to be noted that point $A_2$ lies on the set $B_2$. So, we shall analyze the stability of set $B_2$ only. Set $B_2$ corresponds to a scaling solution
FIG. 2: Figure shows the vector field of autonomous system (30-31) for the interaction model 2 with the parameter values $\alpha_d = -0.01$ and $\Gamma_0 = 0.5$. A colored curve represents curve $B_2$ with black colored curve being a stable portion of set $B_2$.

FIG. 3: Figure shows the vector field of autonomous system (33-34) for the interaction model 3 with the parameter values $\alpha = -0.01$ and $\Gamma_0 = 0.5$. A colored curve represents curve $C_3$ with black colored curve being a stable portion of set $C_3$.

and it always exist except $x_c = 1$. For this solution, DM and DE scale in a constant fraction as: $\Omega_m/\Omega_d = (1-x_c)/x_c$, where DE behaves as any kind of perfect fluid having barotropic equation of state $\omega_d = \frac{\Gamma_0 - \Gamma_0 + \alpha_d}{\alpha_d(1-x_c)}$ (see table II). This set corresponds to an accelerated universe if $\frac{\Gamma_0 - \Gamma_0 + \alpha_d}{\alpha_d(1-x_c)} < 1$. This set is normally hyperbolic and hence it is stable if $\alpha_d < 0, 0 < x_c < 1; x_c < 0, \alpha_d > 0; x_c > 1, \alpha_d > 0$. This is confirmed numerically from Fig. 2. The eigenvector

$$\begin{pmatrix} 3(x_c-1)^2 \\ 0(x-1)^2 + \alpha_d \\ 1 \end{pmatrix}$$

corresponds to vanishing eigenvalue determines the direction of the tangent at each point of the set. Whereas the one dimensional stable subspace near this set is spanned by eigenvector

$$\begin{pmatrix} 3(1-x) \\ \Gamma_0(x-1)+\alpha_d \\ 1 \end{pmatrix}$$

with $\alpha_d$ and $x_c$ satisfying the stability condition. So, this set can explain the late time behavior of our universe.

4.3. Interaction 3

- The system (33)-(34) admits only one critical set of points $C_3$. As mentioned earlier it is noted that points $A_3$, $B_3$ are points on the set $C_3$. So, in what follows we shall analyse the stability of set $C_3$ only. One interesting point for this solution $C_3$ is that it is a combination of DM and DE having the ratio: $\Omega_m/\Omega_d = \frac{1-x_c}{x_c}$, and will exist for all model parameters except $x_c = 1$. This set provides some interesting features for positive coupling as well as negative coupling of interaction. The accelerating universe is predicted by the set when $\frac{1-x_c}{x_c} < \Gamma_0 - 1$. This set is normally hyperbolic and it is stable if $0 < x_c < \frac{1}{2}, \alpha > 0; \frac{1}{2} < x_c < 1, \alpha < 0; x_c < 0, \alpha < 0; x_c > 1, \alpha > 0$. This is confirmed numerically in Fig. 3. The one dimensional stable subspace near this set is spanned by eigenvector

$$\begin{pmatrix} 3x_c(1-x_c) \\ \Gamma_0 x_c(x_c-1)+\alpha \\ 1 \end{pmatrix}$$
FIG. 4: Figure shows the vector field of autonomous system (36-37) for the interaction model 4 with the parameter values $\beta = -0.01$ and $\Gamma_0 = 0.5$. Colored lines represent line $A_4$ and $D_4$ with black colored curve being a stable portion of set $A_4$.

which corresponds to a nonvanishing eigenvalue with $\alpha, x_c$ satisfying the above stability condition. Whereas, the direction of the tangent at each point on the set is along the eigenvector

$$\begin{pmatrix}
\frac{3(x_c-1)^2}{\alpha - \Gamma_0(x_c-1)^2} \\
1
\end{pmatrix}$$

corresponds to a vanishing eigenvalue. So, trajectories near this set approach towards points of this set. Hence, some critical points on this set corresponds to a late time accelerated universe.

4.4. Interaction 4

There are two set of critical points arising from the interaction model 4 ($A_4$ and $D_4$). As mentioned earlier it is noted that points $B_4, C_4$ are points on the set $A_4$. So, in what follows we shall analyse the stability of set $A_4$ and $D_4$ only.

- The set of critical points $A_4$ exists for all model parameters involved. It represents a DE dominated solution ($\Omega_d = 1$). This DE dominated solutions describe the late time acceleration of universe when DE behaves as quintessence, cosmological constant or phantom, or any other exotic fluid with for $y_c < -\frac{1}{3}$. This set is again normally hyperbolic and it is stable when $y_c < \alpha_m - \Gamma_0$. The stability of $A_4$ is confirmed numerically in Fig. 4. The one dimensional stable subspace near this set is spanned by eigenvector

$$\begin{pmatrix}
1/y_c \\
1
\end{pmatrix}$$

where $y_c < \frac{\alpha_m - \Gamma_0}{3}$. The eigenvector

$$\begin{pmatrix}
0 \\
1
\end{pmatrix}$$

corresponds to vanishing eigenvalue determines the direction of a set. Since, there is no unstable subspace near this set for $y_c < \frac{\alpha_m - \Gamma_0}{3}$. This means that trajectories approach towards some points on this set.

- The solution represented by the point $D_4$ is combination of both DE and DM with the constant ratio as: $\Omega_m = \frac{1-x_c}{x_c}$, where DE describes any perfect fluid with equation of state $\omega_d = \beta - \frac{\Gamma_0}{x_c}$. The set exists for all model parameters. Depending on some parameter restrictions, an acceleration will occur for the set, but since both of eigenvalues vanish, we do not obtain any information regarding the stability of $D_4$ (see Fig. 4). It behaves as a neutral line.

4.5. Interaction 5

- From the local interaction model-5, we get four set of critical points presented in table V. The set $A_5$ (similar with $A_1$ and $A_4$ ) is completely DE dominated solution. It crosses the phantom barrier for $y_c < -1$, and it will
FIG. 5: Figure shows the vector field projection on (a) $x\nu$ phase plane and (b) $y\nu$ phase plane of autonomous system (41-43) for the interaction model 5 with the parameter values $\gamma = 0.5$ and $\Gamma_0 = 0.001$.

FIG. 6: Figure shows the vector field projection on $xy$ phase plane of autonomous system (41-43) for the interaction model 5 with the parameter values $\gamma = 1$ and $\Gamma_0 = 6$. It may be noted that we take $\Gamma_0$ to be very large for this particular plot simply to check its unstability, since for $\Gamma_0 < 3$, eigenvalue $\lambda_3 > 0$ and it will be surely unstable.

be in quintessence dominated phase for $-1 < y_c < -\frac{1}{3}$. The DE associated with this set can mimic any kind of fluid for different choices of $y_c$. It is a normally hyperbolic set and hence it is stable if (a) $\Gamma_0 \leq 3$ and $y_c < -1$, or (b) $\Gamma_0 > 3$ and $y_c < -\frac{1}{\Gamma_0}$. This means, that the set will be stable only in the phantom regime. Furthermore, in this region the set becomes physically relevant describing the late time accelerated expansion of the universe. The two dimensional stable subspace is spanned by eigenvectors

$$
\begin{pmatrix}
1/y_c \\
1 \\
0
\end{pmatrix}
\quad \text{and} \quad
\begin{pmatrix}
0 \\
0 \\
1
\end{pmatrix}
$$

corresponds to two nonvanishing eigenvalues where $y_c$ satisfies the above stability condition. The one dimensional center subspace spanned by eigenvector

$$
\begin{pmatrix}
0 \\
1 \\
0
\end{pmatrix}
$$

corresponds to a vanishing eigenvalue determines the direction of set $A_5$. Hence, this set can describe the late time behavior of our universe.

- The set of points $B_5$ exists for all values of model parameters. This solution is completely DE dominated, where DE behaves as cosmological constant. There exists always an accelerating universe ($\omega_{\text{eff}} = -1$, see table V). It is a non-isolated set of critical points where all points are nonhyperbolic but it is not normally hyperbolic set since it contains two vanishing eigenvalues. Numerical projection plot of system (41)-(43), shows that this set cannot be stable. It can be seen that in $(x, v)$ phase space, trajectories are attracted toward the set $B_5$ (see Fig. 5(a)), however trajectories in $(y, v)$ phase space are not attracted towards the set $B_5$ (see Fig. 5(b)). We have checked that this actually happens for different choices of model parameters. This implies that points of this set are saddle in nature.
The set of critical points $C_5$ exists for all values of model parameters. The set corresponds to a solution in combination with both DE and DM in the phase space with DE behaves as any perfect fluid model having equation of state parameter $\omega_d = -\Gamma_0/3$. Hence, it is clear that the DE may have different features during its evolution such as quintessence is described by DE in the parameter region $1 < \Gamma_0 < 3$, while DE describes cosmological constant for $\Gamma_0 = 3$, and phantom regime is realized for $\Gamma_0 > 3$. Now the expansion of the universe is accelerated for $\Gamma_0 > 1$ ($\omega_{\text{eff}} < -\frac{1}{3}$). This set is again non-hyperbolic but not normally hyperbolic. Numerically, by plotting the projection of trajectories on $(x, y)$ plane (see Fig. 6), we observe that points on this set are saddle in nature.

- The set of points $D_5$ is the combination of both DE and DM. The set is on the phantom barrier (i.e. $\omega_{\text{eff}} = -1$, see Table V), and hence there is always an accelerating universe near this set. From the stability analysis, this set is a normally hyperbolic set. It is stable spiral if $\Gamma_0 < 3, x_c > 1$ or $\Gamma_0 < 3, x_c < -1$, it is stable node if $-1 < x_c < 0$, $\Gamma_0 < 3$ or $0 < x_c < 1, \Gamma_0 > 3$.

5. COSMOLOGICAL IMPLICATIONS

In this section, we shall describe the main cosmological features extracted from the interacting dark energy models in presence of gravitational particle production. In the following subsections, we shall describe the physics of the critical points for each interacting model in this framework along with their viability to describe different cosmic phases. An interesting feature is that in all the interaction models, the evolution of $\Omega_m, \Omega_\phi$ and $\omega_{\text{eff}}$ (see Fig. 7) are similar and so we have not plotted for each interaction model.

5.1. Interaction Model 1:

In this model, we obtained two set of critical points $A_1 (1, y_c)$ and $E_1$. Set $A_1$ represents a de-Sitter universe for $y_c = -1$ (point $D_1$), it represents a stiff matter dominated universe for $y_c = 1$, for $y_c = 0$ (i.e., point $B_1$) the universe behaves as if it is matter dominated ($\omega_{\text{eff}} = 0$) but it is actually DE dominated ($\Omega_d = 1$). It is also noted that critical point $B_1$ is special case of set $E_1$ and it corresponds to a matter dominated universe for $\alpha_m = 0$. It means that when no interaction between DE and DM is considered. Moreover, set $E_1$ also represents a matter dominated universe for $x_c = 0$ (i.e., point (0, $\frac{\alpha_m}{3}$)). From the analysis performed in Sec. 4, we see that depending on the choice of model parameters and fine tuning of initial conditions, the Universe evolves from a matter dominated phase (set $E_1$) to a DE dominated phase (set $A_1$), either to a quintessence regime for $-1 < y_c < -\frac{1}{3}$, or cosmological constant for $y_c = -1$, or phantom regime for $y_c < -1$ (see for e.g., Fig. 7). Hence, we observe that the background dynamics of this model can possibly mimic the $\Lambda$CDM model (see Fig. 7(a)). Moreover, there is a possibility of crossing the phantom barrier ($\omega_{\text{eff}} = -1$, see Fig. 7(b)) which is slightly favored by observations and cannot be achieved in case of non-interacting DE models. Hence, this model can well describe the late time transition from DM to DE dominated phase of the universe.
5.2. Interaction Model 2:

In this model, there is only one set of critical points $B_2$. This set represents a DM dominated universe when $x_c = 0$, i.e., when we consider the origin $(0, 0)$ as a critical point. Unfortunately, we do not obtain any information regarding the stability of point $(0, 0)$ as both eigenvalues vanishes for this particular point. However, from Fig. 2 it looks like point $(0, 0)$ is not stable. Critical point $A_2$ is a special case of set $B_2$ and it represents a DE dominated but decelerated universe ($\Omega_d = 1, \omega_{\text{eff}} = 0$) when $\alpha_d = 0$. It means that when there is no coupling between DE and DM and this point is of no interest from phenomenological point of view. Set $B_2$ can represent a late time accelerated scaling solution for $\alpha_d < 0$, and $0 < x_c < 1$. However, viable trajectories are attracted towards $B_2$ near the limit $x_c = 1$ i.e., DE dominated universe (see Fig. 2). So, for this model the Universe evolves from a matter dominated solution (set $B_2$ for $x_c = 0$) towards a DE dominated solution (set $B_2$ for limit $x_c \rightarrow 1$) (a similar scenario is obtained for this model as in Fig. 7).

5.3. Interaction Model 3:

The background cosmological behavior of this model is similar with interaction model 2. In this model, there is only one set of critical points $C_3$. This set represents a DM dominated universe when $x_c = 0$. We do not obtain any information regarding its stability as both eigenvalues vanishes for this particular case but from Fig. 3 it looks like it is not stable. Critical points $A_3$ and $B_3$ is a special case of set $C_3$ corresponds to a scaling solutions. As in case of interaction 2, point $A_3$ corresponds to DE dominated but decelerated universe ($\Omega_d = 1, \omega_{\text{eff}} = 0$) when $\alpha = 0$. It means that when there is no coupling between DE and DM and this point is not of phenomenological interest. Point $B_3$ corresponds to a DM dominated universe for $\alpha = 0$. Set $C_3$ can represent a late time accelerated scaling solution for some choices of $\alpha$ and some $x_c$. It corresponds to a DM dominated universe for $x_c = 0$. However, viable trajectories are attracted towards $C_3$ near the limit $x_c = 1$ (see Fig. 3). So, for this model the Universe evolves from a matter dominated solution (set $C_3$ for $x_c = 0$) towards a DE dominated solution (set $C_3$ for $x_c \rightarrow 1$).

5.4. Interaction Model 4:

In this model, we obtained two set of critical points $A_4$ and $D_4$. Set $A_4$ represents a stiff matter dominated universe for $y_c = 1$, for $y_c = 0$ (i.e., point $B_4$) the universe behaves as if it is matter dominated ($\omega_{\text{eff}} = 0$) but it is actually DE dominated ($\Omega_d = 1$), it also represents a de-Sitter universe for $y_c = -1$ (point $C_4$). Set of critical points $D_4$ behaves as a neutral line, its stability cannot be determined as all eigenvalues vanishes. However, for $x_c = 0$ i.e., point $(0, 0)$, it corresponds to a matter dominated universe. Even though stability cannot be determined analytically, numerically it is behaving as a neutral line (see Fig. 4). Hence, we see that depending on the choice of model parameters and fine tuning of initial conditions, the Universe evolves from a matter dominated phase (set $A_4$) to a DE dominated phase (set $A_4$), either to a quintessence regime for $-1 < y_c < -\frac{1}{3}$, or cosmological constant for $y_c = -1$, or phantom regime for $y_c < -1$.

5.5. Interaction Model 5:

In this model, we obtained four set of critical points. Set $A_5$ corresponds to late time attractor where DE dominates universe only in phantom regime. Critical points on sets $B_5$ and $C_5$ behaves as saddle and interestingly set $C_5$ corresponds to a matter dominated universe for $x_c = 0$. Set $D_5$ also corresponds to late time accelerated solution for some choices of model parameters. Hence, depending on initial conditions and choices of model parameters we see that the universe can evolves from matter domination (set $B_5$ or $C_5$) to a DE dominated era (set $D_5$). Physically, it means that there is a transition of DM to DE in late universe.

6. SHORT DISCUSSION

In the present work, we have performed a dynamical system analysis for the scenarios of interacting dark matter and dark energy, where additionally the gravitational particle production is also allowed. The particle creation mechanism describes many interesting results such as the possibility of phantom universe without invoking any phantom field, formation of emergent universe, complete cosmic scenario etc. Here, we have considered the dark matter fluid as dust.
and the dark energy as a perfect fluid with equation of state $\omega_d$. Additionally, the created particles by the gravitational field have been considered to be dark matter particles (equivalently, dust particles) in agreement with the local gravity constraints and the production rate is taken to be varying linearly with the Hubble function (i.e. $\Gamma \propto H$). We have considered five interacting models correspond to distinct form of interactions $Q$. The objective for choosing such a complex system is to examine whether there is any model (interacting) could explain the overall evolution of universe. In particular, a complete description of evolution at late-times can be obtained in quintessence, $\Lambda$CDM, or phantom era connected through a DM dominated era. Critical points, their existence, and the corresponding cosmological parameters are shown in tables I, II, III, IV, and V for the respective models. Additionally, we have presented the eigenvalues for different interaction models in tables VI, and VII. A detailed stability analysis has been executed successively in Sec. 4. We have observed that all critical points are non-isolated set of critical points and hence they are all nonhyperbolic in nature. It is also noted that all set of points except $D_4$, $B_5$, $C_6$ are normally hyperbolic, where stability is confirmed by the signature of the remaining non-vanishing eigenvalues.

We found that the set of critical points ($A_1$, $A_4$, $A_5$) corresponds to DE dominated universe where DE could mimic quintessence era, cosmological constant, phantom phase, sometimes dust or even any other exotic fluid. However, it was found that some of the critical points in the above set of critical points, representing the above cosmic phases (i.e. quintessence, cosmological constant, or phantom phase) could describe the late-time accelerated expansion of the universe while they can not solve the coincidence problem. On the other hand, some set of critical points ($E_1$, $B_2$, $C_3$, $D_5$) can possibly represent scaling solutions for $0 < x_c < 1$, with an accelerated expansion of the universe are found to be stable for some choices of coupling parameter except $C_3$ which does not depend on the signature of coupling parameter. Interestingly, we observe that trajectories are attracted towards a portion of sets where $x_c \approx 1$ and hence critical points of these sets cannot alleviate the coincidence problem. Moreover, critical points on sets $E_1$, $B_2$, $C_3$, $D_5$ with $x_c = 0$ represents a DM dominated universe. Stability analysis done in Sec. 4 shows that for some choices of model parameters and fine tuning of initial conditions, one can connect these DM dominated solution to a DE dominated solutions ($A_1$, $A_4$, $A_5$ or $B_2$, $C_3$ for limit $x_c \to 1$) which can possibly mimic quintessence, cosmological constant, or phantom phase.

Thus, in summary, one may conclude that the present interacting DE model in the framework of particle creation mechanism may describe different evolutionary phase of the universe. These interacting models can possibly allow the crossing of phantom divide line (see Fig. 7(b) which shows the clear cosmic evolution of the physical quantities $\omega_{eff}$, $\Omega_d$, $\Omega_d$) which is not possible in case of uncoupled standard cosmology. Moreover, the background dynamics of these interacting models can possibly mimic the $\Lambda$CDM but only for $y_c = -1$, so there might be some differences at the level of perturbations. However, cosmological perturbation analysis lies beyond the scope of our present study. This can be left for future works.

Acknowledgments

The authors are grateful to the referee for some critical and useful comments to improve the work. SC thanks IUCAA for providing research facilities and also the UGC-DRS programme, at the department of Mathematics, Jadavpur University. SKB would like to thank S. Pan for helpful discussions on this work, and also acknowledges IUCAA for providing research facilities in the library where a part of the work was done during a visit.

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