On the Role of Quantum Communication and Loss in Attacks on Quantum Position Verification

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Abstract

We study the role of quantum communication in attacks on quantum position verification. In this work, we construct the first known example of a QPV protocol that is provably secure against unentangled attackers restricted to classical communication, but can be perfectly attacked by local operations and a single round of simultaneous quantum communication indicating that allowing for quantum communication may break security. We also show that any protocol secure against classical communication can be transformed into a protocol secure against quantum communication. We further show, using arguments based on the monogamy of entanglement, that the task of Bell state discrimination cannot be done locally with a single round of quantum communication, not even probabilistically (when we allow attackers to say loss sometimes), making this the first fully loss-tolerant QPV task secure against quantum communication attacks. Finally, we observe that any multi-round QPV protocol can be attacked with a linear amount of entanglement if the loss is high enough.

1 Introduction

Geographical position is an important contributor to trust—for example, a message which provably comes from a secure location in a government institution, has automatic credence to actually be sent by that government. Position-based cryptography is the study of using position as a cryptographic credential. The most basic task here is to certify someone’s position, but this can be extended to messages that can only be read at a certain location, or to authenticating that a message came (unaltered) from a certain location.

We will focus on the task of position verification, which can be used as the building block for tasks like position-based authentication. For simplicity, the focus will be on the one-dimensional case, i.e. verifying one’s position on a line, but the relevant ideas generalize readily to more dimensions. In

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our case, protocols will have the form of two verifiers, $V_A$ and $V_B$, attempting to verify the location of a prover $P$. An adversary to a scheme will take the form of a coalition of attackers, while the location of $P$ is empty. We’ll use A (or Alice) for the attacker located between $V_A$ and the location of $P$, and B (or Bob) for the attacker location between the location of $P$ and $V_B$.

It was shown by Chandran, Goyal, Moriarty, and Ostrovsky [CGMO09] that without any additional assumptions, position verification is an impossible task to achieve classically. The quantum study of quantum position verification (QPV) was first initiated by Beausoleil, Kent, Munro, and Spiller resulting in a patent published in 2006 [BKMS06]. The topic first appeared in the academic literature in 2010 [Mal10a, Mal10b], followed by various proposals and ad-hoc attacks [KMS11, LL11]. A general attack on quantum protocols for this task was presented by Buhrman, Chandran, Fehr, Gelles, Goyal, Ostrovsky, and Schaffner [BCF+11], requiring a doubly-exponential amount of entanglement. This attack was further improved to requiring an exponential amount of entanglement by Beigi and König [BK11] – much more efficient but still impractically large. (See also [GLW13, Dol19] for generalizations of such attacks to different settings, with similar entanglement scaling.)

Additionally, other protocols have been proposed [KMS11, CL15, Unr14, JKPPG21, BCS21] that combine classical and quantum information in interesting ways, sometimes requiring intricate methods to attack [BFSS13, Spe16, OCCG20].

In our contribution we present several results on the role of quantum communication in QPV attacks.

**The role of quantum communication for attacks on QPV.** Some works [BRSdW11, TFKW13, BFSS13, BCS21] attempt to lower bound the pre-shared entanglement required from attackers that are allowed a round of simultaneous quantum communication, while other results, such as [BK11, RG15, QS15, QLL+15, LXS+16, GC20, OCCG20] assume attackers that are restricted to communicate only classically.

Even though quantum communication can potentially be simulated by teleportation, it is not immediately clear how to compare bounds between these two settings, especially in case where the exact size of the lower bound is of interest.\(^1\) The simplest version of this question can be asked for unentangled attackers: If a (quantum-question, classical-reply) QPV protocol is secure against unentangled attackers that communicate classically, is that protocol also secure against unentangled attackers that are allowed to use quantum communication?

To that end, we present the following results:

- First, we answer the above question in the negative: We construct a protocol that is provably secure against unentangled attackers that can use classical communication, but can be broken by a single round of simultaneous quantum communication. This shows that some care has to be taken when interpreting results that restrict to classical messages only.

- Interestingly, we are additionally able to show that our counter-example is in some sense artificial: Given a protocol that is secure against classical messages, but insecure when quantum communication is allowed, it is always possible to transform this protocol into one that is secure when quantum communication is allowed.

\(^1\)For instance, the bound of [RG15] does not fully supersede [TFKW13], and therefore finding a tight lower bound for the parallel-repetition of the QPV\text{BB84} protocol against attackers that have quantum communication remains open.
This new protocol can be constructed from the given protocol by applying local maps to the messages from the verifiers $V_A$, $V_B$, without having to modify the output predicate. Our proof for this statement involves a recursive argument, where we view the states after quantum communication of a successful attack as the input messages to two new protocols. We then recursively consider an increasing number of new possible protocols, and use emergent classicality [QR20] to show that a secure protocol of the required form has to exist.

- We proceed by considering the task of Bell state discrimination\(^2\) and prove that this task cannot be done perfectly with only local operations and one round of simultaneous communication. The proof relies on new arguments based on the monogamy of entanglement. We consider a purified version of this task in the QPV setting (delaying the honest measurement to the end of the protocol) and show that the squashed entanglement [CW04] of the state $\rho_{V_AV_B}$, on which the honest Bell measurement is applied to, is upper bounded by $E_{sq}(V_A : V_B)_\rho \leq 1/2$. Hence attackers won’t be able to perfectly predict the honest result. To get an explicit upper bound on the attack success probability with quantum communication $p_{\text{succ}}^{qc}$ we use the hashing bound from [DW05] which lower bounds the squashed entanglement, allowing us to upper bound a parameter that leads to $p_{\text{succ}}^{qc} \leq 0.926$. We further improve this bound to $p_{\text{succ}}^{qc} \leq 3/4$ via a different argument based on the no-cloning theorem.

We additionally show that even in the lossy scenario it remains that $p_{\text{succ}}^{qc}(\eta) < 1$ for any transmission rate $0 < \eta \leq 1$. This makes the task of Bell state discrimination, and by implication the QPV protocol based on the SWAP-test [ABSVL22], the first fully loss tolerant QPV protocol that remains secure in the setting where attackers are allowed quantum communication.

Finally, we present a result relating loss tolerance and entanglement attacks in QPV:

- We observe that in a setting with loss any multi-round QPV protocol can be broken with only a linear amount of pre-shared entanglement if the loss rate is high enough. In that sense, creating a fully loss tolerant QPV protocol which requires superlinear entanglement (in the number of qubits involved) is impossible. This follows directly from a simple observation: if there is no limit to the loss, the adversaries can attempt quantum teleportation and guess the teleportation corrections, claiming ‘loss’ if the guess is incorrect.

The aspects of loss and quantum communication are practically very relevant, since in realistic settings loss rates will be high and, although attackers are restricted to only one round of simultaneous communication due to the timing constraints of QPV, they could in principle be able to quantum communicate and this might give them an advantage.

1.1 Structure of the paper

In section 3.1 we present the first QPV protocol that is provably secure against attackers restricted to quantum communication but broken by a single round of quantum communication. However, in section 3.2 we show that any protocol insecure against quantum communication but secure against classical communication can be transformed into a protocol secure against quantum communication. In section 3.3 we show that the task of Bell state discrimination is secure against attackers who are allowed to use quantum communication. Extending on this result in section 3.4 we also show that

\(^2\)Where the input is a randomly chosen Bell state.
this is strictly secure if we allow attackers to also say loss beside quantum communication. Finally, in section 3.5 we make the observation that allowing loss for pre-shared entangled attackers allows for any protocol with high enough loss rate to be broken with linear entanglement.

2 Preliminaries

2.1 Notation

We denote parties in QPV protocols by letters $A$, $B$, etc. and their quantum registers as $A_1 \cdots A_n$, $B_1 \cdots B_n$ and so on, respectively. Sometimes we may refer to “all registers party $X$ holds” just by $X$, giving expression like $\text{Pos}(A \otimes B)$, for example. Unless otherwise indicated, $\| \cdot \|_p$ is the usual $p$-norm. The diamond norm on quantum channels is denoted by $\| \cdot \|_\diamond$ and is defined as $\|C\|_\diamond := \max_\rho\| (C \otimes I_k)(\rho) \|_1$ for quantum states $\rho$. Partial transposition of an operator $P$ with respect to party $B$ is denoted $P^T_B$. The set of PPT-measurements$^3$ on two subsystems held by parties $A$ and $B$, respectively, is $\text{PPT}(A : B)$. We use the term “Local Operations and Broadcast Communication” (LOCC) to describe the scenario of a single round of simultaneous classical communication with local quantum operations before and after the round of communication. Finally, the image of a function $f$ is denoted by $\text{Im}(f)$ and for a set $X$ we write $|X|$ for its cardinality. All other notation is explained in the text.

2.2 Quantum position verification

For simplicity, we treat the one-dimensional case here, where all parties are located on a line. The time needed to implement local operations is considered negligibly short compared to the time span of the entire protocol. In order to verify the position of an untrusted party $P$, two trusted and spatially separated verifiers $V_A, V_B$ send quantum inputs to $P$ from each side and ask them to apply a specific quantum operation. $P$ has to apply the operation and respond immediately. In the end the verifiers check if they received an answer in time and consistent with the input and the demanded task. The attack model is as follows. Attackers trying to break the protocol are not located at $P$ but want to convince the verifiers that they are. Two attackers$^4$ $A, B$ can position themselves between $V_A, P$ and $V_B, P$, respectively, and intercept the inputs, act locally, communicate one message to each other and then act locally again before they have to commit to answers $\tau_A, \tau_B$. They hence have to simulate the honest quantum operation using only local actions and 1 round of simultaneous communication. In general, they could also pre-share an entangled resource state $\eta_{AB}$ at the start of the protocol. This situation is depicted in figure 1.

$^3$I.e. sets of positive semi-definite operators adding up to the identity, whose partial transposes are positive semi-definite as well.

$^4$The scenario of more attackers can be reduced to the one described above. Indeed, the attackers closest to $P$ could simply simulate all the other attackers themselves.
Figure 1: Space-time diagram of a general QPV protocol. We assume all information travels at the speed of light. For graphical simplicity we have put \( P \) exactly in the middle of \( V_A \) and \( V_B \) (which is not necessary). The attackers, not being at position \( P \), would like to convince the verifiers that they are at \( P \) by simulating the honest operation via local operations and one round of simultaneous communication.

3 QPV and quantum communication

3.1 A protocol for which quantum communication gives an advantage over LOCC

A natural question one might ask is whether there is any advantage for attackers in QPV protocols if they are allowed to perform local operations and quantum communication (LOQC) instead of classical communication. In what follows we will construct an explicit example of a QPV protocol with classical outputs where there is a finite gap in success probability for LOQC strategies over LOCC strategies.

First, consider the protocol where two verifiers both send half of either one randomly picked symmetric Bell state \( \{|\Phi^+\rangle, |\Phi^-\rangle, |\Psi^+\rangle \} \) or the antisymmetric Bell state \( |\Psi^-\rangle \), and ask an honest prover whether the entangled state they have sent is symmetric or antisymmetric. An honest prover who can apply entangling operations can answer this question with success probability 1 by applying a SWAP-test [BCWdW01] on the state. From the analysis of the corresponding SDP optimized over PPT measurements it turns out that the best LOCC strategy is upper bounded by \( 5/6 \) (see Appendix A). The LOCC strategy of measuring both qubits in the computational basis and answering the XOR of the outcomes attains this success probability, so the upper bound over PPT measurements is attained by a LOCC measurement.

Now suppose the verifiers send two parallel rounds of the previous protocol under the condition that the two rounds are either both a random symmetric Bell state or they are both an antisymmetric Bell state and we ask the prover whether the input consisted of two symmetric or two antisymmetric states. An honest prover who can apply entangling operations can still solve this protocol with success probability 1 by applying a SWAP-test to one of the two pairs. Now note that attackers who have access to a quantum channel can send half of their input state to each other such that both attackers locally end up with a Bell state which they can perfectly determine. Thus attackers restricted to quantum communication can attack this protocol perfectly. Interestingly, it turns out that this is not possible for attackers restricted to classical communication.

From the analysis of the SDP it turns out that the upper bound for two attackers restricted to
PPT measurements is 17/18, cf. appendix A. Again there is a LOCC strategy that makes this bound tight, namely measuring both pairs in the computational basis and only answering “antisymmetric” if both pairs have unequal measurement outcomes and respond “symmetric” otherwise. This strategy is always correct on antisymmetric inputs. And it is only incorrect on symmetric inputs if both times the state $|\Psi^+\rangle$ was sent, this happens with probability 1/18, so the total probability of success of the LOCC protocol then becomes 17/18. By incorporating loss in the SDP program as done in [LXS+16] and [ABSVL22], we also find that this protocol is loss-tolerant.

Thus we have constructed a QPV protocol where the probability of success for attackers restricted to single round LOCC measurements is strictly lower than attackers restricted to single round LOQC measurements. This shows that there can be an advantage for quantum communication over classical communication, and it could be important in the analysis of the security of QPV protocols. However, it is clear that our construction is not a very good protocol as there is redundant information given to the attackers and sending just one of the two symmetric or antisymmetric states would give a seemingly better protocol.

### 3.2 Splitting Scheme

In this section we present a procedure that distills a QPV protocol secure against attackers using a single round of simultaneous quantum communication from the existence of a QPV protocol that is secure against adversaries restricted to LOCC operations. We will use that the existence of a perfect quantum communication attack on a QPV protocol generates two new QPV protocols, which, when applied recursively, ultimately leads to the existence of a QPV protocol that is secure against adversaries restricted to LOCC and cannot be perfectly attacked by adversaries using quantum communication.

Take any QPV protocol in which two verifiers $V_A, V_B$ send states $\rho_A, \rho_B$ and ask for the outcome of, say, some entangling measurement on the joint state $\rho_{AB}$. Suppose the protocol is secure against adversaries restricted to LOCC, i.e., there is a finite gap in the probability of success between an honest prover and adversaries restricted to LOCC operations, but also assume that the protocol can be broken perfectly by adversaries using quantum communication. In the most general setting the actions of the adversaries are as follows:

- Adversaries $A, B$ receive $\rho_A, \rho_B$ respectively as input states.
- Apply some local channel $A(\rho_A) = \sigma_{A_1A_2}, B(\rho_B) = \sigma_{B_1B_2}$.
- Send some share of their local outcome to the other adversary.
- Apply a measurement on the new local states $\sigma_{A_1B_1}$ and $\sigma_{A_2B_2}$.
- Send the measurement outcome to their respective verifiers.

Now note that both $\sigma_{A_1}, \sigma_{B_1}$ and $\sigma_{A_2}, \sigma_{B_2}$ can be used as input states to define two new QPV protocols, where the measurement an honest prover needs to apply is equal to the measurement the attackers would apply in the quantum communication attack in the original protocol. Then the probability of success for the honest verifier in the newly defined protocol is the probability of success of the adversaries using quantum communication in the previous protocol, which we assumed to be perfect.
Note that any LOCC attack on one of these newly arising protocols was already a valid LOCC attack in the previous protocol with the inputs $\rho_A$ and $\rho_B$. The attackers can simply apply the local channels $A, B$, discard the state they don’t use and apply their attack. Also note that if the input states $\rho_A, \rho_B$ were product states the input states in the newly created protocol are also product states. We have therefore split the QPV protocol into two new protocols using only the existence of a perfect quantum communication attack.

Now there are two options for the newly defined protocols:

- There does not exist a perfect attack using quantum communication for at least one of the two new QPV protocols, in which case we have shown the existence of a QPV protocol that is safe against adversaries using quantum communication and we are done.

- For both protocols there exists a perfect attack using quantum communication. In which case we can apply our previous argument to generate 4 new QPV protocols. See Figure 2 for a visual representation of this splitting argument.

The previous options are true for all arising QPV protocols after splitting and we wish to show the existence of a QPV protocol safe against quantum communication. We therefore suppose all of the induced QPV protocols after splitting $n$ times can be attacked perfectly using quantum communication for any $n \geq 2$.

Note that the input states sent from verifier $V_A$ in the induced QPV protocols after splitting only depend on the previous input states send from $V_A$ and vice-versa for the input states from $V_B$. We can write this action as channels $\Lambda^A_n : D(A) \to D(A_1 \otimes \cdots \otimes A_{2^n})$, mapping $\rho_A \mapsto \sigma_{A_1} \cdots A_{2^n}$, and $\Lambda^B_n : D(B) \to D(B_1 \otimes \cdots \otimes B_{2^n})$, mapping $\rho_B \mapsto \sigma_{B_1} \cdots B_{2^n}$. The idea of this proof is that the
reduced states $\sigma_A$, and $\sigma_B$, become approximately classical, and that attackers could immediately measure their incoming states and share the classical measurement outcome instead of sending some quantum message. This would lead to a contradiction since the success probability of this procedure would be upper bounded by the LOCC bound of the original QPV protocol, while at the same time, by assumption, this attack should become approximately close to a perfect one. To be more precise, we use Theorem 3.1 on the emergent classicality of channels from [QR20].

**Theorem 3.1 (Qi-Ranard).** Consider a quantum channel $\Lambda : \mathcal{D}(A) \rightarrow \mathcal{D}(B_1 \otimes \ldots \otimes B_n)$. For output subsets $R \subseteq \{B_1, \ldots, B_n\}$, let $\Lambda_R \equiv \text{Tr}_R \circ \Lambda : \mathcal{D}(A) \rightarrow \mathcal{D}(\bar{R})$ denote the reduced channel onto $R$, obtained by tracing out the complement $\bar{R}$. Then for any $|Q|, |R| \in \{1, \ldots, n\}$, there exists a measurement, described by a positive-operator valued measure (POVM) $\{M_\alpha\}$, and an “excluded” output subset $Q \subseteq \{B_1, \ldots, B_n\}$ of size $|Q|$, such that for all output subsets $R$ of size $|R|$, disjoint from $Q$, we have

$$
\|\Lambda_R - \mathcal{E}_R\|_\diamond \leq d_A^3 \sqrt{\frac{2\ln(d_A)}{|Q|}} ,
$$

(3.1)

using a measure-and-prepare channel

$$
\mathcal{E}_R(X) := \sum_\alpha \text{Tr}(M_\alpha X)\sigma_R^\alpha
$$

(3.2)

for some states $\{\sigma_R^\alpha\}$ on $R$, where $d_A = \text{dim}(A)$ and $\|...\|_\diamond$ is the diamond norm on channels. The measurement $\{M_\alpha\}$ does not depend on the choice of $R$, while the prepared states $\sigma_R^\alpha$ may depend on $R$.

Applying the theorem and setting the size of the excluded output set for both channels $\Lambda_A^n, \Lambda_B^n$ to $|Q_A| = |Q_B| = 2^{n-1} - 1$, we have, by the pigeonhole principle, that for some index $i \in \{1, \ldots, 2^n\}$ both output sets $A_i, B_i$ must be in the sets disjoint from $Q_A$ and $Q_B$. Setting the size of the reduced channels to $|R_A| = |R_B| = 1$, we see that in both cases the reduced channel $\text{Tr}_{\bar{R}} \circ \Lambda_n^{A/B}$ converges to a measure-and-prepare channel in the number of splittings $n$ for any output:

$$
\|\text{Tr}_{R_A/B} \circ \Lambda_n^{A/B} - \mathcal{E}_{R_A/B}\|_\diamond \leq 8 \sqrt{\frac{2\ln(d_{A/B})}{2^{n-1} - 1}}.
$$

(3.3)

The theorem implies that the reduced channels that maps the input states $\rho_A \mapsto \sigma_A$, and $\rho_B \mapsto \sigma_B$, become approximately close to measure-and-prepare channels. Crucially, the measurements $\{M_\alpha^{A/B}\}$ in the respective measure-and-prepare channels do not depend on the choice of $R$. This gives rise to an LOCC attack in the original QPV protocol from which we started. Two attackers $A, B$ simply apply the local measure-and-prepare channels $\mathcal{E}_{R_A}, \mathcal{E}_{R_B}$ and exchange the classical measurement outcomes $\alpha_1, \alpha_2$. Both attackers then know the state $\sum_{\alpha_1} p_{\alpha_1} \sigma_{A_i}^{\alpha_1} \otimes \sum_{\alpha_2} p_{\alpha_2} \sigma_{B_i}^{\alpha_2}$ which is arbitrarily close to $\sigma_A \otimes \sigma_B$ in $n$. Since for any QPV protocol the POVM measurement that the honest verifier has to apply is publicly known, both attackers can calculate the probability distribution of the answers of an honest prover. Using shared randomness to generate an equal answer both attackers can now mimic the probability of success of an honest verifier arbitrarily well.

This LOCC attack allows attackers to answer correctly with a probability of success that converges to the honest probability of success in the number of splittings $n$. By assumption $n$ can
be arbitrarily large and thus the attackers have an LOCC attack that performs arbitrarily well. However, since for our protocol at the start there is a finite gap between the LOCC probability of success and the honest probability of success, we have a contradiction and conclude that at some level in the recursion there must exist a QPV protocol that cannot be attacked perfectly. That protocol must then be safe against unentangled adversaries restricted to quantum communication arises.

3.3 Security of QPV\textsubscript{Bell} against quantum communication

In this section we give the first example of a classically loss-tolerant QPV protocol that is secure against attackers restricted to quantum communication. Furthermore we will show that there is no perfect attack with loss in the quantum communication setting for this protocol, making it the first example of a protocol that is secure against lossy quantum communication attacks with no pre-shared entanglement. We will give two different proofs that both show security against quantum communication, one based on monogamy of entanglement and one based off a no-cloning argument. The monogamy of entanglement gives a worse bound, but seems to be a more protocol-agnostic approach to proving security in the quantum communication setting, making its derivation still useful.

The protocol we look into is the Bell state discrimination problem. Two verifiers send as inputs the respective qubits of one of the four Bell states ($|\Phi^+\rangle$, $|\Phi^−\rangle$, $|\Psi^+\rangle$, $|\Psi^−\rangle$) and ask the prover which Bell state he receives. An honest prover can answer this task perfectly by doing a Bell measurement. With an SDP and a similar analysis as in [ABSVL22], we can show that this protocol is secure and loss-tolerant against attackers restricted to classical communication. An optimal attacking strategy turns out to measure the qubits in the computational basis, which distinguishes \{|$\Phi^+\rangle$, $|\Phi^−\rangle$\} from \{|$\Psi^+\rangle$, $|\Psi^−\rangle$\} and then to guess one of the two Bell states as an answer. This has success probability 1/2 and is optimal.

To analyze security against attackers restricted to quantum communication we look the protocol in the following equivalent purified way.

- The inputs of the protocol will be the second qubit of the maximally entangled state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{V_AV_B}$ and $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{V_AV_B}$.
- The honest prover has to do a Bell measurement, this is now an entanglement swapping operation where the entanglement between the verifiers and the prover gets swapped to entanglement between the two verifiers and entanglement between the two qubits the prover holds. The prover then sends out his classical measurement outcome $i$ to the verifiers.
- The verifiers check whether the entanglement swapping operation was successful by applying a Bell measurement on their joint state and check whether their measurement outcome is the same as the answer of the prover. The probability of successfully attacking the protocol now corresponds to the provers having the correct Bell state as measurement outcome, averaged over all possible Bell states, i.e. $p_{\text{succ}} = \frac{1}{4} \sum_i \text{Tr}[|\text{Bell}_i\rangle \langle \text{Bell}_i| \rho_{\text{V}_A\text{V}_B}]$.

Note that from the point of prover nothing changes from the original Bell state discrimination protocol. The honest prover still needs to perform a Bell state measurement on his incoming qubits and send his measurement outcome to both verifiers.
The idea of this proof is that by reformulating the QPV\textsubscript{Bell} protocol as an entanglement swapping protocol we can use the monogamy of entanglement property between the qubits that remain at the verifiers and the quantum systems attackers create. Furthermore, while it is hard to say anything about the quantum systems attackers might send to each other, the states the verifiers keep are always in their control.

As stated in section 3.2 the most general quantum communication attack is for attackers to split their inputs into two quantum systems. They hold on to one and forward the other system to the other attacker. After the quantum communication round the attackers hold the reduced states $\rho_{A_1} \otimes \rho_{B_1}$ and $\rho_{A_2} \otimes \rho_{B_2}$, respectively. Since there is no more further communication, for a quantum attack to be successful in generating entanglement between the two verifiers, it is sufficient look at only one of the two attackers locally\textsuperscript{5}. We will use this fact in our proof to show that they cannot perform this task perfectly using quantum communication.

As an entanglement measure we will use the squashed entanglement. This measure satisfies several properties useful for our analysis, such as monotonicity under LOCC operations, general monogamy with no restrictions on the size of the quantum registers, and it is lower bounded by distillable entanglement [CW04]. Consider the sketch of the entanglement structure between all quantum registers in figure 3. By monogamy we have that

\begin{equation}
0 \leq E_{sq}(V_A : A_1)_\rho + E_{sq}(V_A : A_2)_\rho \leq E_{sq}(V_A : A)_\rho = E_{sq}(|\Phi^+\rangle \langle \Phi^+|) = 1 \tag{3.4}
\end{equation}

Suppose without loss of generality that $E_{sq}(V_A : A_1)_\rho \leq E_{sq}(V_A : A_2)_\rho$, then the inequality implies that $E_{sq}(V_A : A_1)_\rho \leq 1/2$. Let $\Phi$ be the LOCC operation (on $V_A(A_1B_1V_B)$) of measuring the $\rho_{A_1B_1}$ register and sending the classical measurement result to $V_A$. Using that squashed entanglement is monotone under LOCC we get the following

\begin{align*}
E_{sq}(V_A : V_B)_{\Phi(\rho)} &\leq E_{sq}(V_A : A_1B_1V_B)_{\Phi(\rho)} \\
&\leq E_{sq}(V_A : A_1B_1V_B)_\rho \\
&= E_{sq}(V_A : A_1)_\rho \leq 1/2. \tag{3.5}
\end{align*}

\textsuperscript{5}Attackers have to act in a coordinated way in QPV, but in particular each attacker also needs to have a local success probability at least as big as the global one.
Thus, the squashed entanglement between the two verifiers after any attack using quantum communication is upper bounded by 1/2. Recall that for an attack to be successful, the verifiers must share the same Bell state on their registers $\rho_{VA}V_B$ as the answer $i$ they receive from the attackers in this case single. Since $E_{sq}(\langle B_i | B_i \rangle) = 1$ it is immediately clear that $p_{\text{succ}} = \sum_i \text{Tr}[|B_i\rangle\langle B_i| \rho_{VA}V_B] / 4 < 1$ and no perfect attack is possible.

Ideally we want $p_{\text{succ}}$ to not only be strictly smaller than 1, but to be smaller than 1 by some finite gap. In what follows we will show that $E_{sq}(V_A : V_B) \leq 1/2$ implies $p_{\text{succ}} \leq 0.926$, which implies security against quantum communication. Our proof uses the hashing bound from [DW05] which lower bounds the squashed entanglement [CW04]. The inequality states that for any quantum state $\rho_{AB}$,

$$S(B)_{\rho} - S(AB)_{\rho} \leq E_{sq}(A : B)_{\rho},$$

(3.6)

where $\rho_B = \text{Tr}_A[\rho_{AB}]$, and $S$ is the von Neumann entropy.

The idea of this proof is to apply the Werner twirling channel $\mathcal{W}$, where we integrate over the final two-qubit state between the verifiers. This channel leaves the antisymmetric (qubit) state invariant, and projects the remaining part to the symmetric subspace. Furthermore this channel is an LOCC channel and by monotonicity of the squashed entanglement under LOCC operations, we have

$$1/2 \geq E_{sq}(V_A : V_B)_{\Phi(\rho)} \geq E_{sq}(V_A : V_B)_{\mathcal{W}(\Phi(\rho))}.$$  

(3.7)

The resulting state $\mathcal{W}(\Phi(\rho)_{VA}V_B)$ can then be written as a mixture of the antisymmetric Bell state with the maximally mixed state characterized by some $\alpha > 0$, that is,

$$\mathcal{W}(\Phi(\rho)_{VA}V_B) = \alpha |\Psi^-\rangle\langle \Psi^-| + (1 - \alpha) \frac{1}{4}.$$  

(3.8)

A property of Bell states is that they can be locally transformed into one another. Therefore, verifiers can always locally change the Bell state that they receive as an answer from the honest prover to the antisymmetric Bell state. Therefore any successful attack can be characterized by the probability of having measurement outcome $|\Psi^-\rangle$ on $\mathcal{W}(\rho_{VA}V_B)$. Combining the entanglement bound (3.7) with the hashing bound (3.6) we get the following numerical bound on $\alpha$:

$$1/2 \geq E_{sq}(V_A : V_B)_{\mathcal{W}(\Phi(\rho))} \geq S(B)_{\mathcal{W}(\Phi(\rho))} - S(AB)_{\mathcal{W}(\Phi(\rho))} = 1 - S(AB)_{\mathcal{W}(\Phi(\rho))} \iff \alpha \leq 0.902.$$  

(3.9)

The probability of success is now upper bounded as follows

$$p_{\text{succ}} = \text{Tr}[|\Psi^-\rangle\langle \Psi^-| \Phi(\rho)_{VA}V_B] = F(|\Psi^-\rangle\langle \Psi^-|, \Phi(\rho)_{VA}V_B) \leq F(|\Psi^-\rangle\langle \Psi^-|, \mathcal{W}(\Phi(\rho)_{VA}V_B))$$

$$= \text{Tr}[|\Psi^-\rangle\langle \Psi^-| \mathcal{W}(\rho_{VA}V_B)] = \alpha + \frac{1 - \alpha}{4} \leq 0.926,$$

(3.10)

where we have used the data process inequality for the fidelity in the first inequality. This concludes our proof and shows there is a finite gap between the optimal attack attackers restricted to quantum communication can do and what an honest prover can do. We suspect that this gap can be made
even larger, the upper bound that we find arises only due to restrictions on the $A_1$ part, the $B_1$ part could be unchanged from the input state of $B$. So our bound gives an expression for the maximal probability if you split the $A$ part in two parts but get the full $B$ part. Also we have not yet made use of the fact that both attackers have to answer equally. In the following section we try to improve the bound using a different approach making use of both the $A_1$ and the $A_2$ part.

### 3.3.1 An improved bound via no-cloning

In this section we will show that the optimal approximate cloning bound is strong enough to give a bound that improves (3.10) in several ways. This argument gives a bound for any local dimension $d$ of the input, has a clear operational interpretation and improves the above derived bound for $d = 2$. For the start, assume that the attackers $A$, $B$ have optimal local success probabilities $p_{\text{suc}}^A$, $p_{\text{suc}}^B$ respectively. Our argument will be that if these local success probabilities (and thus also the global probabilities) were too high, we would be able to construct a cloning procedure that violates the optimal approximate cloning bound.

For the start, assume that the attackers $A$, $B$ have optimal local success probabilities $p_{\text{suc}}^A$, $p_{\text{suc}}^B$ respectively. Tracking all other registers away, we end up with the structure depicted in Figure 3a. In particular, the verifiers themselves (or even just one of them) could create the situation of Figure 3a themselves in their own lab because they know what the optimal success strategies are. They could simply create the inputs and apply the optimal strategies of $A$, $B$ to registers $A$, $B$. This splits $A \mapsto A_1A_2$ and $B \mapsto B_1B_2$. Imagine now that we add a third input state $|\Phi^+_V\rangle$ in registers $V_CC$ and that the optimal strategy of $B$ is applied to register $C$, mapping $C \mapsto C_1C_2$. Note that the state in $C_1C_2$ is identical to the one in $B_1B_2$. This procedure creates the following entanglement structure:

Afterwards, the optimal measurement of $A$ is applied to registers $A_1B_1$ and the optimal measurement of $B$ is applied to registers $A_2C_2$. This swaps entanglement to the registers $V_AV_B$ and $V_AV_C$, respectively. Tracing all other registers away, we end up with the structure depicted in Figure 5. The state $\rho_{V_AV_B}$ corresponds to the state giving $A$ her optimal local success probability in attacking QPV$_{\text{Bell}}$. This is because we applied the optimal split $A \mapsto A_1A_2$ and the optimal local measurement on $A_1B_1$ in order to remotely prepare $\rho_{V_AV_B}$. Likewise, the state $\rho_{V_AV_C}$ corresponds to the state giving $B$ her optimal local success probability in attacking QPV$_{\text{Bell}}$. At this stage $V_A$ could take some state $|\phi^+\rangle_{V_A}$ and attempt to use the standard teleportation protocol to teleport it to both $V_B$ and $V_C$. In general, this will result in some states $\rho_B^\phi$ and $\rho_C^\phi$ at $V_B$ and $V_C$, respectively. The average teleportation fidelity $f$ between the resultant state and the original one depends only on the maximally entangled fraction of the resource state $\rho$, and is given by

$$f = \frac{Fd + 1}{d + 1} = \frac{\langle \Phi^- | \rho | \Phi^- \rangle}{d + 1} + 1.$$

(3.12)
Figure 4: Entanglement structure created by someone, say a trusted verifier, who generated three inputs and applied the optimal local attacker channel $A$ of $A$ on one and the analogous channel $B$ of $B$ on two of the three inputs. They then apply the optimal measurement of $A$ on registers $A_1B_1$ and the one of $B$ on registers $A_2C_2$.

Figure 5: Using the optimal attack strategies of $A$ and $B$ we may create this situation.

for any local dimension $d$. A consequence of optimal asymmetric $1 \rightarrow 2$ cloning is that the arithmetic mean of the average fidelities fulfills

$$\frac{f_{V_B} + f_{V_C}}{2} \leq \frac{5}{6},$$

no matter which resource states was used. Plugging in equation (3.12) for each $f$ in (3.13) yields

$$\frac{\langle \Phi_+ | \rho_{V_AV_B} | \Phi_+ \rangle + \langle \Phi_+ | \rho_{V_AV_C} | \Phi_+ \rangle}{2} \leq \frac{5}{6} - \frac{1}{6d}.$$  (3.14)

This allows us to bound the average success probability $(p_{\text{succ}}^A + p_{\text{succ}}^B)/2$ as follows. Note that any Bell state $|B_i\rangle$ can be regarded as $|\Phi_+\rangle$ with a suitable local unitary $1 \otimes U_i$ applied to the latter. We can thus write

$$\frac{p_{\text{succ}}^A + p_{\text{succ}}^B}{2} = \frac{1}{d^2} \sum_{i=0}^{d^2-1} \frac{\langle \Phi_+ | U_i^\dagger \rho_{V_AV_B} U_i | \Phi_+ \rangle + \langle \Phi_+ | U_i^\dagger \rho_{V_AV_C} U_i | \Phi_+ \rangle}{2}$$  (3.15)

$$\leq \frac{1}{d^2} \sum_{i=0}^{d^2-1} \left( \frac{5}{6} - \frac{1}{6d} \right)$$  (3.16)

$$= \frac{5}{6} - \frac{1}{6d}.$$  (3.17)
where the inequality follows from the fact that (3.14) holds for any states \( \rho_{V_AV_B} \) and \( \rho_{V_AV_C} \). Having an upper bound on the average success probability of A and B also tells us that at least one of \( p_{\text{succ}}^A \) and \( p_{\text{succ}}^B \) is upper bounded by the same bound. Hence we arrive at our end result

\[
p_{\text{succ}}^{AB}(d) \leq \min\{p_{\text{succ}}^A(d), p_{\text{succ}}^B(d)\} \leq \frac{p_{\text{succ}}^A(d) + p_{\text{succ}}^B(d)}{2} \leq \frac{5}{6} - \frac{1}{6d}.
\]

(3.18)

We see that for \( d = 2 \) the previously derived bound improves to \( p_{\text{succ}}^{AB}(2) \leq 3/4 \). In other words, if one tries to discriminate Bell states of local dimension \( d \) with only local operations and one round of simultaneous quantum communication (without pre-shared entanglement), the success probability is upper bounded as in (3.18).

We still suspect that this bound is not tight because we relaxed the situation of QPV and only looked at the local success probabilities of A and B, and their average. In reality, however, A and B are forced to respond with the same answer. Our argument does not include this coordination\(^6\) and therefore we think that the realistic success probability \( p_{\text{succ}}^{AB} \) is only loosely upper bounded by (3.18).

### 3.4 Lossy Quantum Communication Attack on QPV\(_{\text{Bell}}\)

In the previous part we considered attacks where attackers were allowed to use quantum communication but were not allowed to answer loss. Allowing attackers to also answer loss would be the most general setting for attackers who can’t pre-share entanglement. Unfortunately our previous proof does not hold in the lossy case, since the operations attackers can apply to their local quantum registers after quantum communication are not LOCC operations (between the local attacker registers and the corresponding verifier) if attackers are allowed to postselect, but rather SLOCC. Thus, the monotonicity of the squashed entanglement does not hold anymore. Similarly, as already pointed out in [HHH99], average teleportation fidelities may be higher than the bound (3.12) if one allows post-selection. However, we still prove that there can be no perfect lossy quantum communication attack on the Bell state discrimination protocol, i.e. \( p_{\text{succ}}(\eta) < 1 \) for any transmission rate \( \eta \in (0, 1] \).

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\(^6\)Our argument includes that both A and B have to answer, but their responses may differ.
The argument goes as follows. Suppose there is a perfect lossy quantum communication attack, then for any loss rate there must be some moment where both attackers decide to play. Condition on this event taking place, and consider the moment before both attackers measure their quantum systems. Suppose we now perform the measurement on the $A_1B_1$ quantum system, then by assumption we must get some Bell state $i$ (indicating the correct one) as a measurement outcome with probability 1, thus generating a maximally entangled state between the verifiers $V_{A_i}, V_{B_i}$. Now consider the possibility of another verifier $V_C$ who also sends as an input a half of an EPR pair, and some attacker $C$ who applies the exact same splitting operation on her input as attacker $B$ (see figure 6), thus locally $\rho_{B_2} = \rho_{C_2}$. By definition of a QPV protocol the measurement outcome we get from $A_2, B_2$ will be $i$ with probability 1.

What would now happen if we apply the measurement of the attackers on the $A_2, C_2$ system? If this measurement is conclusive then it must be correct with probability 1, thus creating a maximally entangled state between $V_{A_i}, V_{C_i}$, which would violate monogamy of entanglement since $V_{A_i}$ is already maximally entangled with $V_{B_i}$. We conclude that the only possible measurement outcome on the $A_2, C_2$ system is an inconclusive ‘∅’ outcome.

Crucially, the measurement on $\rho_{A_2}B_2$ will always be conclusive while the measurement on $\rho_{A_2}C_2 = \rho_{A_2} \otimes \rho_{C_2} = \rho_{A_2} \otimes \rho_{B_2}$ will always be inconclusive. Using this fact we can perfectly distinguish the state $\rho_{A_2}B_2$ from $\rho_{A_2} \otimes \rho_{B_2}$, by simply applying the measurement an attacker would apply. However, the states $\rho_{A_2}B_2$ and $\rho_{A_2} \otimes \rho_{B_2}$ are never orthogonal to each other, thus there can be no procedure that perfectly discriminates the two quantum states without saying loss. This contradicts our findings, because the above is a hypothetical procedure that perfectly distinguishes the two quantum states.

We conclude that our assumption of a perfect lossy quantum communication attack must be wrong, which proves our claim. Thus we see that in general the QPV_Bell cannot be perfectly attacked by unentangled attackers, which is the strongest statement we can make if we require attackers to not pre-share any entanglement. This argument cannot be extended to a finite gap in the attacking probability because of the subtlety that the measurements on $\rho_{A_1}B_1$ and $\rho_{A_2}C_2$ can be correlated. If attackers are allowed to make some errors, then the measurements on these states can be correlated such that they can be both conclusive but the measurement on $\rho_{A_2}C_2$ will then just be wrong, so monogamy of entanglement may not be violated.

### 3.5 Considerations on loss tolerance in QPV

Ideally, for QPV to become feasible, one would like to have a protocol that is fully loss tolerant, secure against attackers being able to pre-share a bounded amount of entanglement and to use quantum communication between them. So far, there is no such protocol. Here we give a no-go result, based on a simple observation. We show that no such protocol, fulfilling all three of the above properties, can exist. However, not all is lost as in practice one may be able to achieve *good enough* partial loss tolerance, for example by increasing the quantum input dimension or the number of possible quantum operations $P$ can apply. For simplicity, consider the following quite general two-verifier QPV protocol\(^7\):

- Verifiers $V_A, V_B$ send $d_A, d_B$ dimensional quantum inputs, respectively, to $P$. They also send

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\(^7\)The result that follows can be straightforwardly generalized to $m$ verifiers, for which a general attack would be to teleport all quantum inputs to one fixed attacker, who then performs the guessing attack. In that case, the probability that no teleportation needs corrections is much lower, i.e. $1/d^{2(m-1)}$. 

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classical information \( x, y \) (of any size), respectively.

- \( \mathcal{P} \) computes a function \( f(x, y) \) and, based on that result, applies a quantum operation \( Q_{f(x,y)} \) to the inputs. This yields two outputs, one intended for \( \mathcal{V}_A \) and one for \( \mathcal{V}_B \). These outputs are forwarded to the corresponding verifier.

- The verifiers check if what they received matches the specific honest protocol and that the responses arrived in time.

We will denote this as QPV\((d_A, d_B, f)\) and the protocol is to be repeated for \( n \) rounds, either sequentially or in parallel. Now let \( k := |\text{Im}(f)| \) be the number of possible quantum operations to be applied at \( \mathcal{P} \). It turns out that there always exists a perfect attack consuming at most \( O(n \log d) \) (qubit) EPR pairs, where \( d = \max\{d_A, d_B\} \), as long as the loss is high enough. We make this precise in the following statement.

**Proposition 3.2.** Let \( d = \max\{d_A, d_B\} \) and \( k = |\text{Im}(f)| \). Any \( n \)-round QPV\((d_A, d_B, f)\) protocol can be attacked with \( \hat{O}(n) \) EPR pairs if the fraction \( \eta \) of rounds that is used for security analysis fulfills \( \eta \leq \frac{1}{kd^2} \).

**Proof.** Without loss of generality, let attacker \( \mathcal{A} \) receive classical input \( x \). As soon as \( \mathcal{A} \) receives his quantum input, quantum teleportation \cite{BBC+93} can be used to teleport the state to \( \mathcal{B} \) (consuming a \( d = \max\{d_A, d_B\} \) dimensional maximally entangled state\(^8\)), after which \( \mathcal{A} \) sends to \( \mathcal{B} \) which teleportation corrections were to apply and the classical information \( x \). With probability \( p_{00} = 1/d^2 \) there are no teleportation corrections to apply, in which case \( \mathcal{B} \) holds both input states locally and before the honest party \( \mathcal{P} \) would have received them. \( \mathcal{B} \) can guess the value of \( f(x,y) \) and immediately apply the operation \( \mathcal{P} \) was asked to apply, send the part (e.g. a subsystem or a measurement result) that \( \mathcal{V}_A \) is supposed to receive to \( \mathcal{A} \) and keep the part that \( \mathcal{V}_B \) is supposed to receive. With probability \( 1/k \) attacker \( \mathcal{B} \) guesses the value of \( f(x,y) \) correctly. If the quantum state picked up teleportation corrections in the first place or if it turns out that \( \mathcal{B} \) guessed \( f(x,y) \) wrongly (of which both get to know as soon as they receive the communication from the other attacker), attackers deny to answer and both send the corresponding loss symbol \( \emptyset \). If there were no corrections to apply and \( \mathcal{B} \) guessed \( f(x,y) \) correctly, both send their respective parts that the verifiers are supposed to receive to them. As they are only required to answer \( \eta \leq \frac{1}{kd^2} \) of all rounds, they can simply choose those perfect rounds without teleportation corrections and a correct guess for \( f(x,y) \). Applying this strategy in each round costs them \( n \log d \) (qubit) EPR pairs.

The statement shines light on another facet of loss tolerance – the attacker’s ability to post-select on a “correct” guess after communication. They can always do that, if they pre-share entanglement, by simply guessing the teleportation corrections. In protocols with quantum input from one side and classical information determining the action of \( \mathcal{P} \), like \cite{KMS11, CL15, Unr14, JKPPG21, BCS21}, attackers can, even without pre-shared entanglement, do this post-selection simply by first guessing \( f(x,y) \), then applying the operation \( Q_{f(x,y)} \) on the quantum input and communicating \( x,y \) to each other so that both attackers know if the initial guess was correct or not. This is captured in the above bound, identifying all-classical input from one side with \( d = 1 \). In that sense, QPV protocols containing classical input cannot be fully loss-tolerant. It is now also clearer why the protocol in \cite{LXS+16} and our QPV\(_{\text{SWAP}} \)\cite{ABSVL22} are fully loss-tolerant if attackers do not pre-share any

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\(^8\)Attacks do not know the dimension of their local inputs a priori.
entanglement. Without shared entanglement, the attackers simply have no way of ever knowing if their guess was correct, because there is no information about it leaving the verifiers.

We conclude that there does not exist a fully loss tolerant QPV protocol of the above type (in particular, if a linear amount of entanglement is pre-shared). The best we can achieve in all generality against entangled attackers is partial loss tolerance. By increasing \( k \) and/or \( d \) such that \( kd^2 > 1/\eta \), however, partial loss tolerance good enough for all practical purposes could be achieved. We will therefore need to bound both the pre-shared entanglement of the attackers and the loss for a practical QPV protocol.

4 Conclusion

We gave an explicit example of the first QPV protocol in which there is an advantage for attackers who share no entanglement to use quantum communication over classical communication. The protocol depends on determining whether two states were either both symmetric or both antisymmetric. The probability of success under LOCC operations was analytically shown to be equal to 17/18, while attackers with access to quantum communication could attack this protocol perfectly with a single round of simultaneous quantum communication. This suggests that the role of quantum communication can be more important than previously thought.

Diving into the idea that there can be an advantage to using quantum communication over classical communication we also showed that this separation between quantum communication and classical communication was somewhat constructed. The existence of a quantum communication attack on a protocol that is safe against attackers restricted to classical communication implies the existence of a similar protocol that is safe against quantum communication. In order to prove this we showed that for every quantum communication attack, two new QPV protocols arise. By repeating this argument whenever there was a quantum communication attack, we used the theorem of emergent classicality of channels from [QR20] to show that the quantum inputs become approximately classical, which in turn implies the existence of a classical communication attack on the original protocol which violated the assumption of the original being secure against classical communication. Thus, ultimately showing that somewhere in the recursion there must have been a protocol that was secure against attackers allowed to use a single round of simultaneous quantum communication.

Setting out to find an explicit example of a protocol that is safe against quantum communication, we prove that the task of Bell state discrimination cannot have a higher success probability than 3/4 when two attackers are restricted to local operations and a single round of quantum communication. This is higher than the optimal success probability of 1/2 for attackers restricted to classical communication. We do suspect the quantum communication bound to be lower than 3/4, but have been unable to make the bound tighter just yet. Nonetheless, this is the first example of a protocol that is fully loss-tolerant against classical communication and stays secure against attackers using quantum communication.

The previous statement immediately brings up the question whether the protocol is also safe against attackers allowed to use quantum communication, and allowed to say loss. We answer this question in the affirmative and prove that there cannot be a strategy that perfectly distinguishes all Bell states. However, we only find a strict inequality, i.e. that \( p_{\text{succ}}^c(\eta) < 1 \) for any transmission rate \( \eta \in (0, 1] \). An interesting follow-up question is whether we can make this bound into a finite gap, just as with the case where the attackers were not allowed to say loss but could use quantum
communication. Nonetheless, this is the first example of a QPV protocol that remains fully loss tolerant and secure even in the quantum communication setting. This is of interest since it is the most general setting in the case where we do not allow attackers to pre-share entanglement.

Extending on the idea of incorporating loss in different settings for attackers we note that in order for QPV to become feasible, not only do we want our protocols to be loss-tolerant against quantum communication, but we also want our protocols to be loss-tolerant against some amount of pre-shared entanglement among the attackers. However, we show that any QPV protocol can be attacked by only a linear amount of entanglement in this setting, given that the loss is high enough. This is based on the simple observation that in such a scenario attackers can post-select on those rounds in which the attempted quantum teleportation did not incur teleportation corrections.

In conclusion, in this paper we investigated the advantage of a single round of simultaneous quantum communication over classical communication in QPV attacks. We find that this advantage can be strictly better than classical communication, but we also find that in some cases it is still strictly worse than the best non-local operations, which proves security for those cases.

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Appendices

A Optimal PPT Measurements for QPV_{Sym/Antisym}

In this section we will solve the SDP program that optimizes the probability of success for two adversaries restricted to LOCC operations of discriminating a random symmetric state from the antisymmetric state. The SDP formulation of this protocol is as follows:

Primal Program

maximize: \( \frac{1}{2} \text{Tr}[\Pi_0 \rho_0 + \Pi_1 \rho_1] \)

subject to: \( \Pi_0 + \Pi_1 = 1/2 \)

\( \Pi_k \in \text{PPT}(A: B), \ k \in \{0,1\} \)

Dual Program

minimize: \( \text{Tr}[Y] \)

subject to: \( Y - Q_i F_{FB} - \rho_i/2 \succeq 0, \ i \in \{0,1\} \)

\( Y \in \text{Herm}(A \otimes B) \)

\( Q_i \in \text{Pos}(A, B), \ i \in \{0,1\} \).

Where we write \( \rho_0 \) for \( \rho_{\text{sym}} \) and \( \rho_1 \) for \( \rho_{\text{antisym}} \) whose respective density matrices are:

\[
\rho_0 = \frac{1}{6}\begin{pmatrix}
2 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 \\
0 & 1 & 1 & 0 \\
0 & 0 & 0 & 2
\end{pmatrix},
\]

\[
\rho_1 = \frac{1}{2}\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 \\
0 & -1 & 1 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}.
\]

A feasible solution for the primal program, is to measure both states in the computational basis and answering the XOR of the measurement outcomes, so \( \Pi_0 = |00\rangle \langle 00| + |11\rangle \langle 11|, \Pi_1 = |01\rangle \langle 01| + \ldots \)
respectively. We see for any of the 16 possible eigenvectors of $0\rho_0 + \Pi_1\rho_1$ the corresponding dual is

$$\text{Tr}[Y] = 5/6.$$  

Where $\text{Tr}[Y] = 5/6$ is a lower bound of the success probability of the protocol optimized over all PPT measurements. Thus we see that the highest probability of success for adversaries restricted to LOCC operations is 5/6.

Now for the protocol where we double the input rounds, but restrict the inputs to be either both symmetric or antisymmetric states we will show that there is no perfect LOCC attack. The corresponding SDP that optimizes over all PPT strategies looks as follows:

**Primal Program**

| maximize: $\frac{1}{2} \text{Tr}[\Pi_0 (\rho_0 \otimes \rho_0) + \Pi_1 (\rho_1 \otimes \rho_1)]$ |
| subject to: $\Pi_0 + \Pi_1 = 1_{2^4}$ |
| $\Pi_k \in \text{PPT}(A : B), \ k \in \{0, 1\}$ |

**Dual Program**

| minimize: $\text{Tr}[Y]$ |
| subject to: $Y - Q_i^{T_B} - (\rho_i \otimes \rho_i)/2 \succeq 0, \ i \in \{0, 1\}$ |
| $Y \in \text{Herm}(A \otimes B)$ |
| $Q_i \in \text{Pos}(A, B), \ i \in \{0, 1\}$ |

We will show that the following is a feasible solution to the dual

$$Y = \frac{1}{18}(9(\rho_0 \otimes \rho_0) + 8(\rho_1 \otimes \rho_1)), \ Q_0 = 0 \succeq 0, \ Q_1 = \frac{1}{18}(3\rho_0^{T_B} \otimes 3\rho_0^{T_B} - (\rho_1^{T_B} \otimes \rho_1^{T_B})).$$

Note that in contrast to the optimizations for QPV3_SWAP protocols the matrix $Y$ is not equal to the identity matrix with some factor, but nonetheless it is Hermitian. Secondly the eigenvectors of $3\rho_0^{T_B}$ and $\rho_1^{T_B}$ are both the 4 Bell states with eigenvalues $\{3/2, 1/2, 1/2, 1/2\}$ and $\{-1/2, 1/2, 1/2, 1/2\}$ respectively. We see for any of the 16 possible eigenvectors of $Q_1$ that the corresponding eigenvalues will be $0, \frac{1}{9}$ or $\frac{1}{18}$. We conclude that $Q_1 \succeq 0$ since all its eigenvalues are non-negative and it is Hermitian.

For $Q_0$ the first constraint in the dual program becomes:

$$Y - Q_0^{T_B} - \frac{(\rho_0 \otimes \rho_0)}{2} = \frac{1}{18}(9(\rho_0 \otimes \rho_0) + 8(\rho_1 \otimes \rho_1)) - \frac{(\rho_0 \otimes \rho_0)}{2} = \frac{4}{9}(\rho_1 \otimes \rho_1) \succeq 0.$$  

And for $Q_1$ we get:

$$Y - Q_1^{T_B} - \frac{(\rho_1 \otimes \rho_1)}{2} = \frac{1}{18}(9(\rho_0 \otimes \rho_0) + 8(\rho_1 \otimes \rho_1)) - \frac{1}{18}(9(\rho_0 \otimes \rho_0) - (\rho_1 \otimes \rho_1)) - \frac{(\rho_1 \otimes \rho_1)}{2} = 0 \succeq 0.$$
Thus we have shown that all conditions in the dual program are met, and we get an upper bound on the success probability over all PPT measurements of

$$\text{Tr}[Y] = \text{Tr}\left[\frac{1}{18}(9(\rho_0 \otimes \rho_0) + 8(\rho_1 \otimes \rho_1))\right] = \frac{17}{18}. $$

There is an LOCC strategy that attains this upper bound, namely of applying the single rounds strategy twice, where we only answer asymmetric if both pairs have unequal measurement. This strategy is correct on all input states except when as an input twice the $|\Psi^+\rangle$ input was send. This happens with probability $1/18$. Thus the best attack for adversaries restricted to LOCC operations has success probability $\frac{17}{18}$, in contrast to adversaries who may use quantum communication for whom there is a perfect attack with success probability 1 by simply swapping the second register and applying the SWAP test locally.