Light bottom squark and gluino confront electroweak precision measurements

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We address the compatibility of a light sbottom (mass \(2 \sim 5.5\text{GeV}\)) and a light gluino (mass \(12 \sim 16\text{GeV}\)) with electroweak precision measurements. Such light particles have been suggested to explain the observed excess in the \(b\) quark production cross section at the Tevatron. The electroweak observables may be affected by the sbottom and gluino through the SUSY-QCD corrections to the \(Zbb\) vertex. We examine, in addition to the SUSY-QCD corrections, the electroweak corrections to the gauge boson propagators from the stop which are allowed to be light from the \(SU(2)_L\) symmetry. We find that this scenario is strongly disfavored from electroweak precision measurements unless the heavier sbottom mass eigenstate is lighter than \(180\text{GeV}\) and the left-right mixing in the stop sector is sufficiently large. This implies that one of the stops should be lighter than about \(98\text{GeV}\).

Looking for signatures of supersymmetry (SUSY) at collider experiments is one of the most important tasks of high energy physics. However, \(e^+e^-\) collider experiments at the energy frontier have not found any evidence of physics beyond the standard model (SM). The LEP1 and SLC experiments at the \(Z\) pole tested and validated the SM at the quantum level, and the LEP2 experiments showed us that electroweak processes beyond the SM are also consistent with the SM predictions \([1]\). These experiments increased the lower mass bound of the superparticles \([1]\). On the other hand, it has been reported that the production cross section for bottom quarks measured at the Tevatron exceeds the prediction of perturbative QCD by about a factor of two \([3]\). Although it could be cancelled by the heavier sbottom contributions, it is also possible to interpret the measured excess as a signal of low energy supersymmetry. Berger et al. \([1]\) proposed that this excess may be explained if the lighter mass eigenstate of the bottom squarks is very light (2 \(\sim 5.5\text{GeV}\)) and the gluino mass is also small (12 \(\sim 16\text{GeV}\)). Possible signals of this scenario at the Tevatron run-II experiments were examined in Refs. \([4, 5]\). It was shown that the co-existence of a light sbottom and gluino satisfies the constraints from color and charge breaking \([1]\). In the context of \(R\)-parity conserving SUSY, this scenario would require another neutral SUSY particle to be even lighter than the sbottom, to which the sbottom subsequently decays. Assuming that the lightest supersymmetric particle (LSP) is the lightest neutralino, this \(R\) parity conserving interpretation is severely disfavoured by the Tevatron run-I measurement of the cross-section for bottom quark pair production plus missing energy \([1]\). Therefore the scenario of a very light sbottom implicitly demands \(R\)-parity violating interactions in its decay.

In this letter, we study constraints on the above scenario from electroweak precision measurements, i.e., \(Z\)-pole observables from LEP1 and SLC, and the \(W\)-boson mass from LEP2 and Tevatron. The SUSY-QCD contributions to some electroweak observables due to the co-existence of a light sbottom and gluino have been studied by Cao et al. \([7]\). They showed that the SUSY-QCD corrections to the \(Zbb\) vertex from these light particles could be cancelled by the heavier sbottom contributions. In order to be compatible with the \(R_b\) data, they found that the heavier sbottom mass should be smaller than \(125\text{GeV}\) at the 2\(\sigma\) level. However, since the left-handed stop \(\tilde{t}_L\) forms a \(SU(2)_L\) doublet with the left-handed sbottom \(\tilde{b}_L\), one of the stop mass eigenstates can be relatively light when the heavy sbottom is lighter than about 200\(\text{GeV}\), so that the radiative corrections to the gauge boson propagators might be sizable \([8]\). Therefore we examine the supersymmetric contributions to the electroweak observables taking into account both the SUSY-QCD and the electroweak corrections based on the formalism in Ref. \([7]\).

The sfermion mass matrix is given by

\[
M_f^2 = \begin{pmatrix} m_{\tilde{f}_L}^2 & m_f (A_f^{\text{eff}})^* \\ m_f A_f^{\text{eff}} & m_{\tilde{f}_R}^2 \end{pmatrix},
\]

(1a)

\[
m_{\tilde{f}_L}^2 = m_Q^2 + m_Z^2 \cos 2\beta (I_{3f} - Q_f s_W^2) + m_f^2,
\]

(1b)

\[
m_{\tilde{f}_R}^2 = m_{\tilde{Q}}^2 + m_Z^2 \cos 2\beta Q_f s_W^2 + m_f^2,
\]

(1c)

where \(s_W = \sin \theta_W\) is the weak mixing angle. The suffix \(f\) represents the sfermion species and the indices \(\alpha = L, R\) stand for their chirality. The soft SUSY breaking masses for \(SU(2)_L\) doublet and singlet are given by \(m_{\tilde{Q}}\) and \(m_{\tilde{U}}\), respectively. The symbols \(I_{3f}\) and \(Q_f\) denote the third component of the weak-isospin and the electric charge of a sfermion \(\tilde{f}\), respectively. The angle \(\beta\) is defined as \(\tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle\), where \(\langle H_u \rangle / \langle H_d \rangle\) is the vacuum expectation value of the Higgs field with hypercharge \(Y = 1/2(-1/2)\). The parameter \(A_f^{\text{eff}}\) in (1a) is defined as follows:

\[
A_f^{\text{eff}} \equiv A_f - \mu \cot \beta \quad \text{(for } f = t)\),
\]

(2a)

\[
A_f^{\text{eff}} \equiv A_f - \mu \tan \beta \quad \text{(for } f = b)\),
\]

(2b)

where \(\mu\) represents the higgsino mass parameter, and \(A_f\) is a scalar trilinear coupling. In what follows \(\mu\) and \(A_f\) always appear in unison, and so we adopt \(A_f^{\text{eff}}\) as an input parameter. The mass eigenstates and mixing angle
are then given as
\[ U_f^T M_f^2 U_j = \text{diag}(m_{f_1}^2, m_{f_2}^2), \quad (m_{f_1} < m_{f_2}), \quad (3a) \]
\[ U_j = \begin{pmatrix} \cos \theta_j & \sin \theta_j \\ -\sin \theta_j & \cos \theta_j \end{pmatrix}. \quad (3b) \]

If a light sfermion mass \( m_{f_1} \) is less than half the -boson mass, then the decay \( Z \to f_1 \bar{f}_1 \) is possible at the LEP experiments. The interaction Lagrangian for \( Z \to f_1 \bar{f}_1 \) is given by
\[
\mathcal{L} = -ig_Z((I_{sf} - Q_f s_W^2)(U_j)^*_{11}(U_j)_{jj} - Q_f s_W^2(U_j)_{jj}(U_j)_{zj})\tilde{f}_j^* \partial\mu \bar{f}_j Z^\mu. \quad (4)
\]
where \( A_{\mu}^f \equiv A(\partial_\mu B) - (\partial_\mu A)B. \) It can be seen from Eq. (4) that a light sfermion with \( m_{f_1} < m_Z/2 \) can be consistent with the LEP experiments if \( I_{sf} \cos^2 \theta_f - Q_f s_W^2 \approx 0. \)

The supersymmetric particles affect the electroweak observables radiatively through the oblique corrections which are parametrized by \( S_Z, T_Z, m_W, \) and the \( Zff \) vertex corrections \( g^{f}_{\lambda} \), where \( f \) stands for the quark/lepton species and \( \lambda = L \) or \( R \) stands for their chirality. The parameters \( S_Z \) and \( T_Z \) are related to the \( S \)- and \( T \)-parameters [13, 14] as follows:

\[
\Delta S_Z = S_Z - 0.972 = \Delta S + \Delta R - 0.064 x_\alpha + 0.67 \frac{\Delta G^0}{\alpha}, \quad (5a)
\]
\[
\Delta T_Z = T_Z - 2.62 = \Delta T + 1.49 \Delta R - \frac{\Delta G^0}{\alpha}, \quad (5b)
\]

where \( \Delta S_Z \) and \( \Delta T_Z \) measure the shifts from the reference SM prediction point, \( (S_Z,T_Z) = (0.972,2.62) \) at \( m_t = 175 \text{GeV}, m_{H_{SM}} = 100 \text{GeV}, \alpha_s(m_Z) = 0.118 \) and \( 1/\alpha(m_Z^2) = 128.90 \). The \( R \)-parameter, which accounts for the difference between \( T \) and \( T_Z \), represents the running effect of the \( Z \)-boson propagator corrections between \( q^2 = m_Z^2 \) and \( q^2 = 0 \) [13]. The parameter \( x_{\alpha} = (1/\alpha(m_Z^2) - 128.90)/0.09 \) allows us to take into account improvements in the hadronic uncertainty of the QED coupling \( \alpha(m_Z^2) \). \( \Delta G^0 \) denotes any new physics contribution to the muon lifetime which has to be included in the oblique parameters because the Fermi coupling \( G_F \) is used as an input in our formalism [13, 14]. The third oblique parameter \( \Delta m_W = m_W - 80.42(\text{GeV}) \) is given as a function of \( \Delta S, \Delta T, \Delta U, x_{\alpha}, \) and \( \Delta G^0 \) [13]. The explicit formulae of the oblique parameters and the vertex corrections \( \Delta g^f_{\lambda} \) in the minimal SUSY-SM (MSSM) can be found in Ref. [13].

The electroweak data which we use in our study consists of 17 \( Z \)-pole observables and the \( W \)-boson mass [1]. The \( Z \)-pole observables include 8 line-shape parameters \( \Gamma_Z, \sigma^0_h, R_{\ell}, A_{\ell 0, \ell}^0(\ell = e, \mu, \tau) \), two asymmetries from the \( \tau \)-polarization data \( (A_\tau, A_c) \), the decay rates and the asymmetries of \( b \) and \( c \)-quarks \( (R_b, R_c, A_{\ell 0, \ell}^{0, c}) \) and the asymmetries measured at SLC \( (A_{\ell 0, \ell}^{0, R, L, c}) \). Taking into account the \( m_t \) data from the Tevatron [9], \( \alpha_s(m_Z) [13] \) and \( \alpha(m_Z^2) [9] \), we find that the best fit of the SM parameters \( (m_t(\text{GeV}), m_{H_{SM}}(\text{GeV}), \alpha(m_Z^2), \alpha_s(m_Z)) = (176.4,93,128.92,0.118) \) gives \( \chi^2 = 25.3 \) with \( 17(= 21 - 4) \) degrees of freedom.

Let us examine the supersymmetric contributions to the electroweak observables in the scenario of a light sbot-ton and gluino, which provides a SUSY explanation of the observed excess in the measured \( b \)-quark cross section at the Tevatron. As mentioned earlier, our analysis includes the stops in addition to the sbottoms and gluino because of the \( SU(2)_L \) symmetry. The other superparticles, such as the squarks of the first two generations and the uncolored particles, are assumed to decouple from the electroweak processes because they are irrelevant to the excess. The impact of their contribution on the electroweak measurements will be discussed later.

![Fig. 1](image-url)

**FIG. 1:** Total \( \chi^2 \) as a function of \( m_{\tilde{b}_1} \) for \( m_{\tilde{g}} = 5 \text{GeV}, m_{\tilde{g}} = 16 \text{GeV} \) and \( \cos \theta_{\tilde{b}} = 0.38 \). The curves are obtained taking into account the supersymmetric contributions to the oblique corrections. The curves correspond to \( A_{\ell 0}^{0, R, L, c} = 0 \text{GeV} \) (solid), 100GeV (dotted), 200GeV (dashed) and 300GeV (long-dashed). The soft SUSY breaking mass for the right-handed stop \( m_{\tilde{g}} \) is chosen in the range \( 50 \text{GeV} < m_{\tilde{g}} < 500 \text{GeV} \) in order to minimize \( \chi^2_{\text{tot}} \) at each value of \( m_{\tilde{g}} \). The horizontal line denotes the total \( \chi^2 \) at the SM best fit point \( (\chi^2 = 25.3) \).

As a typical example, we set the mass of the light sbot-ton and gluino at \( m_{\tilde{b}_1} = 5 \text{GeV} \) and \( m_{\tilde{g}} = 16 \text{GeV} \), respectively. The left-right mixing angle of sbottoms \( \theta_{\tilde{b}_1} \) is fixed at \( \cos \theta_{\tilde{b}} = 0.38 \) so that the pair production of a light sbottom from \( Z \)-boson decay is suppressed. We use \( \tan \beta = 3 \) throughout our study since our results are not altered significantly for \( \tan \beta > 3 \). This is explained by the fact that \( \beta \) only appears in the sfermion mass matrix...
as $\cos 2\beta$ (see (14), (15)), and so the $\tan \beta$-dependence is not important numerically. The free parameters in our study are the heavier sbottom mass $m_{\tilde{b}_2}$, the SUSY breaking mass for the right-handed stop $m_{\tilde{t}_R}$, and the parameter $A_{\tilde{t}}^{\text{eff}}$ in the stop mass matrix. In the following analysis, the SM parameters $(m_t, m_{H_u}, \alpha(m_Z^2), \alpha_s(m_Z^2))$ are fixed at their best fit points in order to show explicitly the decoupling limit of the supersymmetric corrections at large SUSY mass.

In Fig. 1 we show total $\chi^2$ as a function of $\tilde{b}_2$ due to the supersymmetric contributions to the oblique parameters. The SUSY-QCD corrections to the $Zbb$ vertex will be included later. The four curves correspond to $A_{\tilde{t}}^{\text{eff}} = 0 \text{GeV}$ (solid), $100 \text{GeV}$ (dotted), $200 \text{GeV}$ (dashed) and $300 \text{GeV}$ (long-dashed). The SUSY breaking mass $m_{\tilde{g}}$ is chosen in the range $50 \text{GeV} < m_{\tilde{g}} < 500 \text{GeV}$ in order to minimize $\chi^2_{\text{tot}}$ at each point of $m_{\tilde{b}_2}$. The horizontal line denotes the total $\chi^2$ at the SM best fit point ($\chi^2 = 25.3$). For convenience, let us introduce $\Delta \chi^2$ as the difference between $\chi^2_{\text{tot}}$ of the MSSM and the SM,

$$\Delta \chi^2 \equiv \chi^2_{\text{tot}}(\text{MSSM}) - \chi^2_{\text{tot}}(\text{SM}).$$

We can see that the MSSM fit to the electroweak data strongly depends on the left-right mixing of stops which is parametrized by $A_{\tilde{t}}^{\text{eff}}$. We find that $A_{\tilde{t}}^{\text{eff}} = 0 \text{GeV}$ gives $\Delta \chi^2 \gtrsim 30$ for $m_{\tilde{b}_2} \approx 200 \text{GeV}$ and $\Delta \chi^2 \sim 10$ for $m_{\tilde{b}_2} \approx 400 \text{GeV}$. On the other hand, when $A_{\tilde{t}}^{\text{eff}} = 300 \text{GeV}$, we find $\Delta \chi^2 \approx 0$ for $m_{\tilde{b}_2} \gtrsim 200 \text{GeV}$. There are three oblique parameters $\Delta S_Z, \Delta T_Z$ and $\Delta m_W$ in our formalism. A model independent analysis [14] shows that $\Delta T_Z$ is most severely constrained from the electroweak data. It has been shown that $\Delta T_Z$ is very sensitive to squark contributions while $\Delta S_Z$ is not [3]. Furthermore, since the squark contributions to $\Delta R$ are generally small [3], the results shown in the figure approximately reflect the contributions to $\Delta T$. It should be noticed that the left-handed squarks contribute to $\Delta T$ while the right-handed squarks do not, since $\Delta T$ is defined in terms of vacuum polarization amplitudes of the SU(2)$_L$ gauge bosons [11, 11]. Therefore, when the left-right mixing of stops vanishes ($A_{\tilde{t}}^{\text{eff}} = 0 \text{GeV}$), the stop contributions to the oblique parameters are maximized. When $A_{\tilde{t}}^{\text{eff}}$ increases, the left-handed component of the stop in the lighter mass eigenstate decreases, so that the net contributions to $\Delta T$ are reduced. This explains the $A_{\tilde{t}}^{\text{eff}}$ dependence in Fig. 1 qualitatively.

We note that, in our analysis, we increase $m_{\tilde{b}_2}$ by keeping $m_{\tilde{b}_1}$ at $5 \text{GeV}$. The contributions from the heavy sbottom and the stops diminish with the increase of $\tilde{b}_2$ mass while those from the light sbottom are maintained. This is the origin of the deviation of the total $\chi^2$ from the SM $\chi^2$ at large $m_{\tilde{b}_2}$ in Fig. 1.

Next, we examine the SUSY-QCD corrections to the $Zbb$ vertex in addition to the oblique corrections for completeness. In Fig. 2 we show total $\chi^2$ as a function of the heavier sbottom mass $m_{\tilde{b}_2}$ taking into account both corrections. The curve indicated as “SUSY-QCD” is obtained by dropping the oblique corrections. The total $\chi^2$ from the SUSY-QCD correction increases when the mass of $\tilde{b}_2$ is heavier. For example, $m_{\tilde{b}_2} = 100 \text{GeV}$ leads to $\Delta \chi^2 \sim 7$ while $m_{\tilde{b}_2} = 400 \text{GeV}$ leads to $\Delta \chi^2 \sim 70$. The SUSY-QCD corrections to the $Zbb$ vertex are given by the 1-loop diagrams mediated by $\tilde{b}_1$ and $\tilde{g}$, and those by $\tilde{b}_2$ and $\tilde{g}$. The contributions from $(\tilde{b}_1, \tilde{g})$ and $(\tilde{b}_2, \tilde{g})$ interfere destructively [6]. Therefore the SUSY-QCD corrections partially cancel when $\tilde{b}_2$ is relatively light. However this cancellation tends to weaken with increasing $m_{\tilde{b}_2}$, because $\tilde{b}_2$ decouples from the $Zbb$ diagrams.

The other curves in Fig. 2 are now easily understood as the superposition of curves obtained from the oblique corrections onto the curve from the SUSY-QCD corrections. Let us recall that the oblique corrections make the fit worse for small $m_{\tilde{b}_2}$ unless $A_{\tilde{t}}^{\text{eff}}$ is sufficiently large (see Fig. 1), while the SUSY-QCD corrections lead to large $\Delta \chi^2$ when $m_{\tilde{b}_2}$ is large. From Fig. 2 we find $\Delta \chi^2 \gtrsim 25$ for any value of $m_{\tilde{b}_2}$ when $A_{\tilde{t}}^{\text{eff}} \leq 200 \text{GeV}$. Therefore, the light sbottom scenario is disfavored at $5 \sigma$ level unless $m_{\tilde{b}_2} \lesssim 180 \text{GeV}$ and $A_{\tilde{t}}^{\text{eff}} \gtrsim 300 \text{GeV}$. This implies that the lighter stop mass eigenstate is $m_{\tilde{t}_1} \lesssim 98 \text{GeV}$. The LEP2 experiments give a lower mass bound on the stop of $m_{\tilde{t}_1} \gtrsim 96 \text{GeV}$ at 95% CL [3]. The allowed parameter region is quite narrow and should be covered at the Tevatron run-II experiments. Throughout our study, the SM parameters $(m_t, \alpha(m_Z^2), \alpha_s(m_Z^2))$ are fixed at their best fit points within the SM. We note that our result
does not change significantly even if they are taken to be free parameters, because of external constraints on them [12, 13, 14].

To summarize, we have examined constraints on the scenario of a light sbottom and gluino with $m_{\tilde{b}_2} = 2 \sim 5.5$ GeV and $m_{\tilde{g}} = 12 \sim 16$ GeV from electroweak precision measurements. This scenario has been proposed as a SUSY interpretation of the observed excess in the bottom quark production cross section at the Tevatron. These particles affect the electroweak observables through the SUSY-QCD corrections to the $Zb\bar{b}$ vertex. In addition to the SUSY-QCD corrections, we also take into account the electroweak corrections to the gauge boson propagators from the stops and sbottoms because of the SU(2)$_L$ symmetry. The electroweak corrections to the oblique parameters make the fit to the data significantly worse when the left-right mixing of the stops is weak ($A_{\text{eff}}' \lesssim 200$ GeV). The SUSY-QCD corrections are rather suppressed when the mass of $\tilde{b}_2$ is relatively light owing to the cancellation between the contributions from $\tilde{b}_1$ and $\tilde{b}_2$. From both corrections, we find that the parameter space of the light sbottom scenario is strongly constrained from the electroweak data. The scenario is disfavored at the 5σ level unless $m_{\tilde{b}_2} \lesssim 180$ GeV and $A_{\text{eff}}' \gtrsim 300$ GeV are satisfied. The constraints on $m_{\tilde{b}_2}$ and $A_{\text{eff}}'$ implies $m_{\tilde{t}_1} \lesssim 98$ GeV, which should be covered by the Tevatron run-II experiments.

It is worth commenting on contributions to the electroweak observables from the other superparticles which are disregarded in our analysis. It is known that contributions to the electroweak observables from the squarks (except for stops and sbottoms), sleptons and the MSSM Higgs bosons are generally small when their masses are above the direct search limits [1]. It is also known that their contributions do not improve the fit to the data and increase the total $\chi^2$. Thus the constraints on the light sbottom scenario will be stronger if they are included in our study. An exception is the oblique corrections from charginos. It has been shown that the fit may be improved slightly through the oblique corrections if the chargino mass is as light as its lower mass bound from the direct search [1]. However, since the expected improvement is at most about one unit of $\Delta \chi^2$, our conclusion does not change even if it is taken into account.

The light sbottom scenario implicitly demands $R$-parity violation in order to prevent the sbottom being the LSP. The electroweak precision measurements may be affected by the $R$-parity violating interactions in the decay $Z \rightarrow j\bar{f}$, which we did not include in our study. Constraints on the $R$-parity violating couplings from electroweak measurements have been examined in Ref. [16], and were found to be less significant numerically. Therefore our results are largely unaffected by the presence of such couplings.

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