Cosmological constraints on $R$-parity violation from neutrino decay

Gautam Bhattacharyya $^a$, Subhendu Rakshit $^b$, and Amitava Raychaudhuri $^b$

$^a$Saha Institute of Nuclear Physics, 1/AF Bidhan Nagar, Calcutta 700064, India
$^b$Department of Physics, University of Calcutta, 92 Acharya Prafulla Chandra Road, Calcutta 700009, India

Abstract

If the neutrino mass is non-zero, as hinted by several experiments, then $R$-parity-violating supersymmetric Yukawa couplings can drive a heavy neutrino decay into lighter states. The heavy neutrino may either decay radiatively into a lighter neutrino, or it may decay into three light neutrinos through a $Z$-mediated penguin. For a given mass of the decaying neutrino, we calculate its lifetime for the various modes, each mode requiring certain pairs of $R$-parity-violating couplings to be non-zero. We then check whether the calculated lifetimes fall in zones allowed or excluded by cosmological requirements. For the latter case, we derive stringent new constraints on the corresponding products of $R$-parity-violating couplings for given values of the decaying neutrino mass.

PACS number(s): 13.35.Hb, 12.60.Jv, 11.30.Fs

1 Introduction

The Super-Kamiokande collaboration has recently provided compelling evidence in support of neutrino oscillations as an explanation of the atmospheric anomaly [1]. The observed solar neutrino deficit [2] and the LSND accelerator experiment results [3] are also indicative of oscillations. These results achieve especial significance since oscillations require the neutrinos to be massive. A massive neutrino signals physics beyond the standard model and has far-reaching implications in particle physics, astrophysics, and cosmology [4].

In this work, we consider $R$-parity-violating ($\bar{R}$) supersymmetry (defined later) in the context of neutrino decays [5, 6]. Such interactions violate lepton number and in the presence of appropriate couplings of this type a heavier neutrino of one flavour can decay to a lighter one of a different flavour in association with the emission of a photon. Alternatively, the heavier neutrino can decay invisibly into three lighter neutrinos through one loop graphs involving $\bar{R}$ couplings.

Cosmological and astrophysical requirements forbid certain regions in the neutrino mass and lifetime plane. They originate, for example, (i) from the precise black-body nature of the microwave background radiation, (ii) from the tight requirements of consistency of the predicted primordial nucleosynthesis with observation, etc., which are discussed later.

We calculate, for a heavy neutrino of a given mass, the decay lifetime induced by $\bar{R}$ couplings. If the lifetime so obtained falls in the forbidden region then couplings of the chosen strength are not allowed for the neutrino mass used. In this way we can establish new constraints on some lepton number violating couplings applicable for specific neutrino masses. On the other hand, if one assumes that these $\bar{R}$ couplings are at their existing limits, then upper bounds can be set on the decaying neutrino mass.

Before we move to the next section, a short introduction to $R$-parity and $\bar{R}$ supersymmetry [7] is in order. ‘$R$-parity’ in supersymmetry refers to a discrete symmetry which follows from the conservation of lepton-number ($L$) and baryon-number ($B$). For the standard model particles and their superpartners,
it is defined as $R = (-1)^{(3B + L + 2S)}$, where $S$ is the intrinsic spin of the field. $R$ is $+1$ for all standard model particles and $-1$ for all superparticles.

The most general $R$ superpotential is given by,

$$W_R = \frac{1}{2} \lambda_{ijk} L_i L_j E_k^c + \lambda_{ijk}' L_i Q_j D_k^c + \frac{1}{2} \lambda_{ijk}'' U_i^c D_j^c E_k^c + \mu_i L_i H_u,$$

(1)

where $i, j, k = 1, 2, 3$ are quark and lepton generation indices; $L_i$ and $Q_i$ are $SU(2)$-doublet lepton and quark superfields respectively; $E_k^c$, $D_k^c$ are $SU(2)$-singlet charged lepton, up- and down-type quark superfields respectively; $H_u$ is the Higgs superfield responsible for the generation of up-type quark masses; $\lambda_{ijk}$ and $\lambda_{ijk}'$ are $L$-violating while $\lambda_{ijk}''$ are $B$-violating Yukawa couplings. $B$- and $L$-conservation are not ensured by gauge invariance and hence there is a priori no reason to set these couplings to zero. $\lambda_{ijk}$ is antisymmetric under the interchange of the first two generation indices, while $\lambda_{ijk}'$ is antisymmetric under the interchange of the last two. Thus there could be 27 $\lambda'$, 9 each of $\lambda$ and $\lambda''$ couplings and 3 $\mu_i$ parameters. We assume that the generation indices correspond to the flavour basis of fermions. We also note at this point that the $B$-violating couplings $\lambda_{ijk}''$ and the bilinear couplings $\mu_i$ are not of any relevance to our present analysis. We shall deal only with the $L$-violating trilinear couplings and, for the sake of simplicity, we assume that they are real.

Stringent constraints on individual $L$-violating couplings have been placed from the consideration of neutrinoless double beta decay, $\nu_e$-Majorana mass, charged-current universality, $e - \mu - \tau$ universality, $\nu_\mu$ deep-inelastic scattering, atomic parity violation, $\tau$ decays, $D$ and $K$ decays, $Z$ decays, etc. Product couplings (two at a time), on the other hand, have been constrained by considering $\mu - e$ conversion, $\mu \rightarrow e\gamma, b \rightarrow s\gamma$, $B$ decays into two charged leptons, $K_L - K_S$ and $B_q - \overline{B}_q$ ($q = d, s)$ mass differences, etc. (For a collection of all these limits, see [3]).

In the following section we elaborate on the neutrino mass spectra which are preferred by the experimental data. In section 3 we discuss the radiative neutrino decay modes driven by the $\lambda'$- and $\lambda$-type $R$ couplings while in section 4 we turn to the invisible three neutrino decay channel. In the next section we check if and how cosmological requirements on these decay lifetimes constrain the $R$ couplings. We end in section 6 with our conclusions.

2 Neutrino mass spectrum

In this section we elaborate on the neutrino masses that we will be using to examine possible neutrino decays driven by $R$ interactions. We are motivated in making these choices by the available evidence for massive neutrinos.

The upper limits on neutrino masses obtained from laboratory experiments are the following. The masses of $\nu_\mu$ and $\nu_\tau$ are constrained to be $m_{\nu_\mu} \leq 0.17$ MeV and $m_{\nu_\tau} \leq 18.2$ MeV [9]. As regards $\nu_e$, the upper limit from tritium beta-decay end-point measurements is 2.5 eV [10], while if it is of Majorana nature then from the absence of neutrinoless double beta decay one has the constraint $m_{\nu_e} \leq 0.2$ eV [11].

Before we discuss the constraints that emerge from the oscillation data, a few remarks are in order. The $R$ couplings have been defined in terms of flavour eigenstates. On the other hand, any discussion of decay must refer to a neutrino in its mass eigenstate. For simplicity of presentation and the ease of illustration, in the following we do not distinguish between the flavour and the mass eigenstates. Indeed, the indications for neutrino oscillation imply that this is not the actual situation. In fact, the mismatch between the mass and flavour bases is an essential ingredient of neutrino oscillations and even maximal mixing is sometimes preferred (e.g., as required by the atmospheric anomaly). It is straightforward to incorporate the effect of this mixing in our results. One must multiply the decay rates for $\nu_i \rightarrow \nu_\nu$, presented later, by appropriate factors determined by the probability of the $\nu_i$ ($\nu_\nu$) being present in the parent (daughter) neutrino.

In much of the analysis we choose the decaying neutrino to be $\nu_\tau$ and for the purpose of illustration take its mass to range from 45 eV to 100 keV. This is consistent with the laboratory bound on $m_{\nu_\tau}$ [9]. We consider its decay to either $\nu_\mu$ or $\nu_e$. We also consider the possibility of $\nu_\mu$ decaying to $\nu_e$. There are

---

1Strictly speaking, this bound applies to the $(ee)$ element of the Majorana mass matrix written in the flavor basis.
models in the literature proposing inverted mass hierarchy scenarios \( m_{\nu_i} < m_{\nu_{i+1}} < m_{\nu_{i+2}} \). But, as far as bounds on the \( R \)-couplings are concerned, these provide no new constraints since the rate for \( \nu_i \to \nu_{i'} \) decay (\( i, i' \) flavour indices) in this scenario is the same as that for \( \nu_{i'} \to \nu_i \) if the masses \( m_{\nu_i} \) and \( m_{\nu_{i'}} \) are interchanged.

Now let us recollect what is known about the magnitude of neutrino mass splittings from the data on neutrino oscillations. There are two independent recent results – namely, the atmospheric neutrino anomaly \( \mathbb{I} \) and the solar neutrino problem \( \mathbb{J} \) – which can be conveniently explained within the framework of neutrino oscillations. The LSND experiment has also claimed positive evidence of \( \bar{\nu}_e - \nu_e \) (and also \( \nu_{\mu} - \nu_{\tau} \) oscillations \( \mathbb{K} \)). However, this result is still awaiting independent confirmation.

Oscillations require a non-zero mass splitting between neutrinos of different flavour. More precisely, the experimental data determine \( \Delta m^2 = m_1^2 - m_2^2 \), where \( m_{1,2} \) are the masses of two neutrinos. Oscillations also determine the unitary transformation which relates the mass basis of neutrinos to the flavour basis. As mentioned earlier, we are not directly concerned with this aspect and do not discuss it any further\(^2\).

The information from experimental results on neutrino mass splittings are as follows:

1. The atmospheric neutrino data can be explained in terms of oscillations of \( \nu_{\mu} \) to either the \( \nu_{\tau} \) or a sterile state \( \nu_s \). The sterile neutrino does not couple to any of the standard model particles and passes undetected through the experimental set up. The necessary mass splitting has been found to be \( \Delta m^2 \sim 10^{-2} - 10^{-3} \text{eV}^2 \equiv \Delta_{\text{atmos}} \).

2. The solar neutrino problem can be explained in terms of vacuum neutrino oscillations of the \( \nu_e \) to either the \( \nu_{\mu, \tau} \) or the \( \nu_s \) with \( \Delta m^2 \sim 10^{-10} \text{eV}^2 \). The data admits an alternative solution in terms of MSW resonant neutrino conversion if \( \Delta m^2 \sim 10^{-5} - 10^{-6} \text{eV}^2 \). In the latter case the sign of the mass splitting is fixed and the \( \nu_e \) must be the lighter state of the two. We indicate the mass splitting required for a solution to the solar neutrino problem by \( \Delta_{\text{solar}} \).

3. The LSND experiment indicates \( \nu_{\mu} - \nu_e \) oscillation with \( \Delta m^2 \sim 1 \text{eV}^2 \equiv \Delta_{\text{LSND}} \).

As is seen from the above, if both the atmospheric neutrino anomaly and the solar neutrino deficit are sought to be explained by invoking oscillations between three sequential neutrinos only, then the mass splittings among them must be small. On the other hand, if one admits a fourth (sterile) neutrino in the model, then it is possible to arrange for larger mass splittings. With a fourth neutrino, one can have two categories of mass hierarchies. In category ‘A’, there are three closely spaced almost degenerate states explaining the solar and atmospheric data and the fourth one is separated by a larger gap \( \Delta \). The fourth neutrino could either be the sterile or \( \nu_e \) since the \( \nu_e \) (\( \nu_{\mu} \)) must be closely spaced with some other state to satisfy the solar (atmospheric) constraint. In category ‘B’, there are two pairs of almost degenerate neutrinos separated by the larger gap (details later). Taking the LSND experiment into consideration, ‘A’ is incompatible with the data, while ‘B’ can be comfortably accommodated \( \mathbb{E} \).

Two cases may arise if we work in ‘A’. The heaviest neutrino could be (dominantly) sterile. Since it does not have any direct \( R \) couplings, its decay via mixing with other states will be strongly suppressed, and we cannot set any bound on the \( R \) couplings. Therefore, this case is not interesting for our purpose and we will not consider it in our analysis. On the contrary, the heaviest state could be (dominantly) the \( \nu_{\tau} \), decaying via \( R \) interactions. Since our analysis concerns, after all, the decay of a heavy active neutrino into at least one lighter active state, this latter case does not yield any different result from what we obtain if we work in ‘B’. Hence, to simplify matters, we adopt the framework ‘B’ for our subsequent discussions.

In ‘B’, which is a ‘two-pair’ scenario, all four neutrinos participate in the solution to the solar and atmospheric neutrino anomalies. In this case there are two allowed forms of the mass spectrum consisting of two closely spaced pair of states (spacings \( \Delta_{\text{atmos}} \) and \( \Delta_{\text{solar}} \)) with a larger separation (\( \Delta \)) between the two pairs as shown in Fig. 1. Inclusion of the LSND result fixes the mass splitting \( \Delta \) to \( \Delta_{\text{LSND}} \) \( \mathbb{F} \). It needs to be mentioned that the oscillation results (excepting for the MSW case)
determine only the magnitude of the spacing between the levels. Therefore, in addition to the two mass spectra shown in Fig. 1, other possibilities obtained from these by exchanging the members within any of the two closely spaced pairs or by interchanging the relative ordering of the two pairs gives rise to spectra which are equally acceptable. As alluded to earlier, our results below are not affected by such inversion of hierarchies.

\[ \Delta_{\text{solar}} \]

\[ \Delta_{\text{atmos}} \]

Figure 1: The two allowed forms of the neutrino mass spectra from atmospheric and solar results. Note that the spacings \( \Delta_{\text{atmos}} \) and \( \Delta_{\text{solar}} \) are only indicative and not to scale. If the LSND results are also included then \( \Delta \equiv \Delta_{\text{LSND}} \).

In our subsequent work, pending confirmation from another independent experiment, we do not impose the LSND constraint on the mass spectra. Once we abandon the LSND results, both ‘A’ and ‘B’ are acceptable by the remaining data. In any case, as noted before, except for omitting the uninteresting case of the decaying heavy sterile neutrino in the framework ‘A’, we do not lose any generality by sticking to the option ‘B’.

Thus, we assume that there are two pairs of states with \( \Delta m^2 \) determined by the atmospheric and solar neutrino results but we leave the splitting between these pairs arbitrary, demanding only that consistency with the laboratory limits be maintained. This permits us to consider mass splittings as large as 170 keV (but not more), which is the laboratory upper limit on \( m_{\nu_{\mu}} \), and check the cosmological implications if such heavy neutrinos decay into lighter states via the \( \not{R} \) interactions. It should be noted that, in this framework, the laboratory upper limit of 18.2 MeV on \( m_{\nu_{\tau}} \) can never be reached, as Fig. 1a suggests that \( m_{\nu_{\tau}} \) can at most be a little above \( m_{\nu_{\mu}} \). The generation indices for the decaying heavy neutrino and the lightest neutrino produced as a decay product are chosen as the combinations (3,1), (3,2), and (2,1) respectively. We will neglect the mass of the product neutrino.

3 Radiative decay of a heavy neutrino

A heavy neutrino, which from now on we denote by \( \nu_H \), can decay radiatively into a lighter neutrino (anti-neutrino), denoted by \( \nu (\bar{\nu}) \), through \( \Delta L = 0 \) (2) penguin diagrams containing \( \not{R} \) couplings. We discuss these two cases separately.

3.1 The decay \( \nu_H \rightarrow \nu + \gamma \)

The \( \Delta L = 0 \) decay of \( \nu_H \) to \( \nu \) and a real photon occurs through penguin diagrams (see Fig. 2). Although

\[ \Delta N_{\nu} < 0.2 \] disfavours the large angle \( \nu_{\mu}-\nu_{\tau} \) mixing solution to the atmospheric neutrino problem. A ‘safer’ estimate \( \Delta N_{\nu} < 1 \), on the other hand, can accommodate an arbitrarily strong mixing between an active and a sterile state. For the present analysis we do not indulge ourselves further along these lines.
the right-handed neutrinos do not couple in the $R$ superpotential, a mass insertion on the heavy neutrino line (which requires $\nu_H$ to have Dirac-type mass) enables one to write down the following effective Hamiltonian for this decay:

$$H_{\Delta L=0}^{\text{eff}} = \frac{1}{2} A_1 \bar{\nu}_L \sigma^\mu\nu \nu_H F_{\mu\nu}$$

(2)

where

$$A_1 = e \lambda'_{ijk} \lambda'_{ijk} Q_d N_c \frac{m_{\nu_H}}{m^2} \frac{1}{16\pi^2} (I_{1k} - I_{1j}).$$

(3)

Here

$$I_{1j} = \frac{1}{4(1 - r_{dj})^3} \left( -1 + r_{dj}^2 - 2r_{dj} \ln(r_{dj}) \right).$$

(4)

with $r_{dj} = (m_{d_j}/\tilde{m})^2$. In the above expression, and in all subsequent discussions, we assume a common mass $\tilde{m}$ for whichever scalar is exchanged. $Q_d$ is the charge of the down-type quark inside the loop. $N_c = 3$ is the colour factor.

3.2 The decay $\nu_H \to \bar{\nu} + \gamma$

This $\Delta L = 2$ decay of a $\nu_H$ to $\bar{\nu}$ and a real photon occurs through penguin diagrams (see Fig. 3). Contrary to the previous mode, for this decay to take place the decaying heavy neutrino does not require to have a Dirac mass. The effective Hamiltonian for this decay is given by

$$H_{\Delta L=2}^{\text{eff}} = \frac{1}{2} A_2 \bar{\nu}_L \sigma^\mu\nu \nu_H F_{\mu\nu}$$

(5)

where

$$A_2 = e \lambda'_{ijk} \lambda'_{ijk} Q_d N_c \frac{m_{d_j} m_{d_k}}{m^2} \frac{1}{8\pi^2} (I_{2k} - I_{2j}).$$

(6)

Here

$$I_{2j} = \frac{1}{(1 - r_{dj})^2} \left( 1 - r_{dj} + \ln(r_{dj}) \right).$$

(7)

As is seen from Fig. 3 this mode involves the mixing between left- and right-type $d$-squarks. This has been approximated by $\Delta m^2(LR) = (A - \mu \tan \beta) m_d \sim \tilde{m}m_d$ in the above expression.
Figure 3: Penguin diagrams corresponding to the $\Delta L = 2$ radiative decay of $\nu_H$.

It follows from Eqs. (3) and (6) that for the special case when $j = k$, the radiative decay amplitude vanishes identically. A look at Figs. 2 and 3 reveals that for each diagram in which a photon is attached to a particle (fermion or scalar) of generation $j$ there exists a similar diagram in which the photon couples to the corresponding anti-particle of generation $k$ – hence the cancellation when the generations match.

In the above we have considered processes which are triggered by non-vanishing $\lambda'$ couplings. These processes could also be driven by $\lambda$-type couplings. The transition can be readily performed in Figs. 2 and 3 by replacing the internal down-type quarks with charged leptons and the down-type squarks with sleptons of the corresponding generations. Indeed, $N_c$ would be unity in this situation.

4 The invisible decay of $\nu_H$ into three light neutrinos

The other kinematically allowed decay mode of $\nu_H$ is into three neutrinos/anti-neutrinos. Such a decay proceeds via a set of $Z$-mediated penguin diagrams (see Fig. 4). The choice of $\lambda'$ (or $\lambda$) couplings determines the flavour of one final state neutrino. In fact, for the aforementioned case of $j = k$ leading to vanishing radiative decays, such three-body invisible channels constitute the only decay modes of $\nu_H$. Since the $Z$ boson, unlike the photon, couples to fermions (sfermions) and their conjugates with different strengths, the cancellation observed for a radiative decay process does not occur for a $Z$-mediated penguin. As we will see later, the decay widths in these three-body modes are much smaller than the two-body radiative decay widths. In the following, we shall consider only the $\Delta L = 0$ penguins. The $\Delta L = 2$ diagrams suffer a suppression from left-right sfermion mixing, and we will not consider them in our subsequent discussions as their numerical impact is insignificant. It is also possible to drive this decay through box graphs involving the products of $\lambda$ couplings, but their contribution to the same order as that of the penguins, we have checked, is insignificant.

For simplicity, we present the expressions for the $\Delta L = 0$ penguin for those cases in which the internal fermion and the sfermion are of the same type (i.e., $j = k$) and the heavy neutrino mass is negligible in comparison with the internal fermion masses. In such a situation the one-loop effective $Z\nu\nu_H$ vertex can be written as $-i\frac{g}{\cos\theta_W} \delta a_L \gamma_{\mu} P_L$, where,

$$\delta a_L = N_c \frac{X'_{ij} X_{jj}}{16\pi^2} \left[ \frac{r_{d_j}}{1 - r_{d_j}} + \frac{r_{d_j} \ln(r_{d_j})}{(1 - r_{d_j})^2} \right], \quad (8)$$
Figure 4: The Z-mediated penguin graphs for the ($\Delta L = 0$) $\nu_H \rightarrow \nu \nu \bar{\nu}$ decay.
5 Cosmological constraints

Cosmology sets tight constraints on allowed neutrino masses and associated lifetimes. For a neutrino of a certain mass, any decay mode driven by $R$ interactions has a lifetime determined by the couplings and the exchanged particle masses. It needs to be checked whether such decays are consistent with the cosmological requirements (discussed below) or whether one obtains new bounds on the $R$ couplings. We discuss the different modes in the following subsections. In order to permit easy comparison with the existing bounds on such couplings, we choose, as in the literature, all exchanged supersymmetric scalar particle (squark or slepton) masses to be 100 GeV in the discussion below. The bounds corresponding to any other sfermion mass can be readily obtained by using the formulae given in this paper (although just by scaling arguments one can derive these numbers to a very good approximation).

Though we are examining neutrino decays, the case of a cosmologically stable neutrino plays an important role. If the lifetime of the neutrino is larger than the age of the universe ($t_0$) \textsuperscript{15}, then it is cosmologically stable. They must either be rather light (the Cowsik-McClelland bound \textsuperscript{18}) or comparatively heavy (the Lee-Weinberg bound \textsuperscript{19}). The allowed values for a stable neutrino mass are

\[ m_\nu \leq 92h^2\Omega_0 \text{ eV}, \text{ or } m_\nu \geq \frac{2}{\sqrt{h^2\Omega_0}} \text{ GeV} \]  \hspace{1cm} (9)

Here, $\Omega_0$ is the ratio of the present energy density to the critical energy density and $h$ is the normalized Hubble expansion rate. \textit{h} is defined as $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$, where $H_0$ is the Hubble expansion rate. Present data indicate that $h^2\Omega_0$ is smaller than unity. Since we are considering neutrinos of mass 100 keV or less, the second inequality in Eq. (9) is not of relevance for us. For the sake of discussion we take the first limit to be 45 eV; its exact value depends on the precise magnitude of the combination $h^2\Omega_0$. Hence, a neutrino of mass 45 eV or more must necessarily be cosmologically unstable.

5.1 The $\nu_H \to \nu(\bar{\nu}) + \gamma$ mode

The final state photons in radiative decays of neutrinos are subject to constraints from the cosmic microwave background radiation (CMBR) spectrum, primordial nucleosynthesis, etc. For the $\Delta L = 2$ radiative process $\nu_H \to \nu + \gamma$, the neutrino decay lifetime is

\[ \tau_\nu(s) \sim \frac{1}{[m_{\nu_H} \text{ (eV)}]^2} \frac{1}{[\lambda'\lambda]^2} f_1, \]  \hspace{1cm} (10)

where $f_1$ lies in the range $4.8 \times 10^{19} - 1.4 \times 10^{25}$ depending on the masses of the quarks and squarks exchanged in the loops. $[\lambda'\lambda]$ denotes the product of the relevant $R$ couplings. When the $\lambda$ couplings are involved, $f_1$ lies between $1.0 \times 10^{21} - 3.6 \times 10^{27}$.

For the sake of numerics, we choose a reference value of $m_{\nu_H} = 45$ eV. Since perturbativity requires all $R$ couplings be smaller than unity, the lifetime of the neutrino from Eq. (10) is larger than $5.3 \times 10^{14}$s, comparable to $t_{\text{rec}} \sim 3 \times 10^{12}$s (the recombination epoch). Now $\tau_\nu \sim t_{\text{rec}}$ for a 45 eV neutrino is not admissible from cosmological requirements on the following grounds. If $t_0 > \tau_\nu > t_{\text{rec}}$, then the photons produced in the radiative decay undergo redshift and contribute to the diffuse photon background since after recombination no charged particles are available to scatter them. This sets a bound \textsuperscript{20}:

\[ \left( \frac{m_\nu}{1 \text{ eV}} \right)^{3/2} \leq 3 \times 10^9 (\Omega_0 h^2)^{-1/2} \left( \frac{\tau_\nu}{1 \text{ s}} \right)^{-1}. \]  \hspace{1cm} (11)

A 45 eV neutrino cannot satisfy this constraint if its lifetime is as estimated above.

The remaining possibility then is $\tau_\nu > t_0$. These photons would simply get superimposed on the diffuse photon background. The experimental data, in this situation, yield the bound:

\[ \left( \frac{m_\nu}{1 \text{ eV}} \right) \leq 10^{-23} \left( \frac{\tau_\nu}{1 \text{ s}} \right). \]  \hspace{1cm} (12)

The above equation can be translated to upper bounds on the $R$ couplings. In Table 1, we have collected these new upper bounds emerging from the $\Delta L = 2$ radiative decay for a 45 eV decaying neutrino. Side
Presented these bounds in Table 2 using the most dominant mode — the $\Delta$ neutrino that is acceptable if the in section 2). Since the decaying neutrino. So the behaviour for the lifetime is as follows:

$$\tau \sim 4.8 \times 10^4 \text{ s}.$$  

Alternatively, one can turn the argument around and find the maximum mass of the decaying heavy neutrino that is acceptable if the $R$ couplings achieve their present experimental upper limits. We have presented these bounds in Table 1 using the most dominant mode — the $\Delta L = 2$ radiative decay.

Now consider a neutrino of mass of $100 \text{ keV}$ (this is near the maximum limit of $170 \text{ keV}$, as we argued in section 2). Since $R$-couplings must be smaller than unity, one obtains, from Eq. (10), $\tau \sim 4.8 \times 10^4 \text{ s}$. But dumping of extra photons by such massive neutrinos decaying at a time close to the thermalization epoch and much after nucleosynthesis is not cosmologically acceptable $[21, 22]$, the relevant bound from the black-body nature of the CMBR being:

$$\frac{m_\nu}{1 \text{ eV}} \leq 10^7 \left(\frac{\tau_\nu}{1 \text{ s}}\right)^{-1/2}.$$  

(13)

For the $\Delta L = 0$ radiative decay $\nu_\mu \rightarrow \nu + \gamma$, the amplitude is proportional to the Dirac mass of the decaying neutrino. So the behaviour for the lifetime is as follows:

$$\tau_\nu(s) \sim \frac{1}{m_{\nu\mu}(\text{eV})^5} \frac{1}{[\lambda\lambda']^2} f_2,$$

(14)

where $f_2$ lies between $2.1 \times 10^{40} - 1.7 \times 10^{45}$. For the $\lambda$ couplings, $f_2$ lies in the range $2.1 \times 10^{41} - 9.9 \times 10^{45}$. For a $45 \text{ eV}$ decaying neutrino, Eq. (13) implies a lifetime larger than the age of the universe even after setting the relevant $R$ couplings at their experimental upper limits. Thus, $R$-parity violating couplings cannot be constrained from this process and if the only non-vanishing $R$ couplings turn out to be those that drive this $\Delta L = 0$ decay, then a neutrino of mass $45 \text{ eV}$, in order to be cosmologically acceptable, must have other interactions resulting in a faster decay. It is easy to convince oneself from the scaling argument that for a decaying neutrino mass as high as $100 \text{ keV}$, the lifetime is either larger than the age of the universe or, if less, certainly larger than $t_{\text{rec}}$, the recombination epoch. In the

| Combinations | Existing bounds | New bounds |
|-------------|----------------|------------|
| $\lambda_{313}\lambda_{131}$ | $1 \times 10^{-3}$ | $1 \times 10^{-4}$ |
| $\lambda'_{321}\lambda'_{131}$ | $7 \times 10^{-3}$ | $6 \times 10^{-3}$ |
| $\lambda_{321}\lambda_{132}$ | $1 \times 10^{-1}$ | $1 \times 10^{-5}$ |
| $\lambda'_{321}\lambda'_{132}$ | $3 \times 10^{-3}$ | $1 \times 10^{-4}$ |
| $\lambda_{331}\lambda_{113}$ | $7 \times 10^{-3}$ | $1 \times 10^{-5}$ |
| $\lambda'_{321}\lambda'_{123}$ | $9 \times 10^{-3}$ | $1 \times 10^{-4}$ |
| $\lambda_{321}\lambda_{122}$ | $2 \times 10^{-2}$ | $6 \times 10^{-3}$ |
| $\lambda'_{321}\lambda'_{122}$ | $1 \times 10^{-1}$ | $1 \times 10^{-5}$ |
| $\lambda_{331}\lambda_{123}$ | $9 \times 10^{-3}$ | $1 \times 10^{-4}$ |
| $\lambda'_{331}\lambda'_{123}$ | $1 \times 10^{-2}$ | $1 \times 10^{-5}$ |
| $\lambda_{323}\lambda_{131}$ | $1 \times 10^{-3}$ | $1 \times 10^{-4}$ |
| $\lambda'_{323}\lambda'_{131}$ | $2 \times 10^{-2}$ | $1 \times 10^{-5}$ |
| $\lambda_{331}\lambda_{131}$ | $4 \times 10^{-3}$ | $1 \times 10^{-4}$ |
| $\lambda'_{331}\lambda'_{131}$ | $1 \times 10^{-2}$ | $1 \times 10^{-5}$ |

Table 1: New upper bounds on products of different $\lambda'$ and $\lambda$ couplings for a heavy neutrino mass of $45 \text{ eV}$ and for $\Delta L = 2$ radiative decay. The existing bounds are obtained by multiplying the upper bounds on the individual couplings $[^3]$. 

by side, we have displayed the existing constraints on these couplings. We have presented only those cases for which our new bounds are more stringent than the existing ones and have rounded off these numbers to the first significant digit.
Table 2: Maximum allowed values of the heavy neutrino mass decaying in the $\Delta L = 2$ radiative mode, based on the assumption that the $\lambda'$ and $\lambda$ couplings achieve their existing upper limits. These existing limits are as in column 2 of Table 1. Recall that in Table 1 we have set new upper limits on the relevant product couplings assuming a mass of 45 eV for the decaying neutrino.

latter case, the neutrino mass and radiative decay lifetime must satisfy relation (11), which is clearly not possible. Therefore, a 100 keV neutrino decaying via $R$ couplings alone is inadmissible.

5.2 The $\nu_H \to \nu + \nu + \bar{\nu}$ mode

Usually the radiative decay mode dominates over the other loop decay modes and consequently the latter are of any numerical relevance only if the former is absent. The radiative decay amplitude involves the combinations $\lambda_{ijk}' \lambda'_{ijk}$ for the $\nu_H \to \bar{\nu} + \gamma$ mode (see Eq. (6)) and $\lambda_{ijk}' \lambda_{ijk}'$ for the $\nu_H \to \nu + \gamma$ process (see Eq. (3)). In the special case $j = k$, the amplitude vanishes identically in each case, as can be easily seen from Eqs. (3) and (6). The same situation prevails for the $\lambda$ couplings. Therefore, for bounds on the $\lambda_{ijj}' \lambda'_{ijj}$ (or $\lambda_{ijj} \lambda_{ijj}'$) combinations we must turn to other processes. It is in these situations that the present mode becomes of relevance.

For the $\Delta L = 0$ Z-mediated $\nu_H \to \nu \nu \bar{\nu}$ process (the two neutrinos coupled to the Z-vertex must have the same flavour) the lifetime is given by

$$\tau_{\nu}(s) \sim \frac{1}{m_{\nu_H}(eV)^3} \frac{1}{|\lambda \lambda'|^2} f_3.$$  (15)

Here $f_3$ lies in the range $10^{42} - 10^{52}$. For the $\lambda$ couplings, the range is $10^{44} - 10^{57}$. It is seen from Eq. (3) that for a 45 eV (or even 100 keV) neutrino, the lifetime for this process is always larger than the age of the universe, $t_0$. Therefore, the $R$-couplings cannot be constrained from this process for such neutrinos.
6 Conclusions and Discussions

We conclude by highlighting the salient features of our analysis. The main thrust is to constrain from cosmological considerations those $R$ couplings which trigger a heavy neutrino decay either radiatively into a lighter neutrino or into three lighter neutrinos. Such attempts were also made in the past. Our analysis, though similar in nature, differs with the previous ones in the neutrino mass spectrum used and complements them by including a relevant piece of the Lagrangian overlooked in the existing literature.

There are two issues that one needs to address. One is the origin of the neutrino mass and the other is the interaction that drives its decay. We have considered a four-neutrino (three standard and one sterile) framework which permits mass-splitting (or for that matter even the mass of the decaying neutrino) as large as order 100 keV or so, remaining consistent with the oscillation data and the recent result of the tritium beta-decay experiment. The authors of ref. have restricted themselves to a three-neutrino framework and considered neutrino masses not more than order 1 eV. In fact, as we argued in section 2, a three-neutrino scenario, given the present constraints from oscillation and tritium beta-decay, cannot permit a neutrino to have a mass more than order 1 eV. In this situation it is not possible to improve on the existing constraints on the $R$ couplings. Obviously the four-neutrino model provides some breathing space allowing at least one neutrino to have a significantly larger mass decaying (with more phase space) to the lighter species. The resulting constraints, depending on the mass of the decaying neutrino, are indeed tighter. In the analysis of refs., the neutrino Majorana masses are generated only by $R$ interactions and the same couplings eventually lead to their decay. In our case, on the contrary, the neutrinos receive masses (either Dirac- or Majorana-type) from some other origin. In fact, we have put these masses in by hand, holding the $R$ interactions accountable only for their decay. If instead one assumes that the $R$ interactions responsible for neutrino decay also generate the mass of the decaying neutrino then the limits we obtained will be altered. This can be seen as follows. The combinations $\lambda'_{ijk}\lambda'_{kj}$ that have been constrained by our present analysis also contribute to the off-diagonal (ii') terms of the left-handed (neutrino) Majorana mass matrix, while the combinations $\lambda'_{ijk}\lambda'_{kj}$ contribute to the diagonal (ii) term. Assuming that $m_{\nu_R}$ in Eq. is generated in this manner and, to make a simple estimate, choosing the off-diagonal (ii') element of the matrix about a tenth of this diagonal term, we can derive an upper bound on $\lambda'_{ijk}\lambda'_{kj}$ demanding consistency with the cosmological constraint in Eq. . In this way, the strongest limit that we obtain is $\lambda'_{323}\lambda'_{232} \leq 10^{-5}$. This indicates that if $R$ interactions alone are responsible both for the generation of neutrino mass as well as for its decay, then cosmological constraints imply that the mass so generated would be comparable to 45 eV.

The other point we would like to stress upon is that from the $\lambda'_{ijk}L_iQ_jD_k^c$ term in the superpotential we get two kinds of squark-mediated processes, one involving the scalar from $Q_j$ and the other the one from $D_k^c$. They correspond to two different pieces in the Lagrangian. For the radiative decay there is an exact cancellation between these two contributions for $j = k$. This point has been overlooked in . It is in this situation that the $Z$-mediated penguins provide the only decay mode of the heavy neutrino. Similar arguments can be advanced for the $\lambda$ couplings.

It should be noted that the product couplings which drive the $\nu_i \rightarrow \nu_i$ decay are also responsible for a similar process involving their SU(2) partners, namely, the $l_i \rightarrow l_i$ decay (where $l_i$ is a charged lepton). We have checked that the constraints we have derived on the product couplings in the context of neutrino decays are consistent with the bounds on the same combinations derived from the non-observation of the charged lepton decays.

The $\nu_\mu \rightarrow \nu_\mu \bar{\nu}_\mu$ mode which we have discussed arises from a flavor-violating $Z$-$\nu_\mu$-$\nu_\mu$ coupling generated by penguin diagrams involving $R$-parity violating couplings. Such a non-diagonal $Z$ vertex can also be produced at tree level due to (a) neutrino-neutralino mixing arising from the misalignment between the $\mu_t$ and the sneutrino vacuum expectation values, or (b) through mixing between active and sterile neutrinos. The magnitude of these mixings depend on details of the model which are beyond the scope of the present analysis. However, one can readily see that the lifetime of a 45 eV neutrino decaying through such an interaction is approximately $\tau_\mu (m_{\nu_R}/m_{\nu_\mu})^2 \sin^2 \phi$ (here $\sin \phi$ parametrizes the suppression at the off-diagonal $Z$ vertex), which is much larger than the age of the universe. As a result, such decays can never compete with the radiative decay modes which yield the bounds which we have presented.
It should be noted that neutrino oscillations will not affect the results shown in Tables 1 and 2. Since the decay times that we find are larger than the age of the universe while the typical oscillation times, as required by solar and atmospheric data, are at most a few minutes, there is little scope of repopulation of the $\nu_H$ states from the daughter neutrinos through oscillation. However, a very large number of oscillations will have taken place before the decay so that the decaying states will have an averaged effect, the probability of an initial $\nu_H$ remaining so being $\cos^2 \theta$, where $\theta$ is a mixing angle. If to start with an equal number of neutrinos of all types exist then this averaging will leave this balance unaltered.

Before closing, a few comments on the existing constraints from baryogenesis \cite{24} are in order. The requirement that GUT-scale baryogenesis does not get washed out imposes $\lambda'' \ll 10^{-7}$ for generic indices, although it has been argued that such bounds are model dependent and can be evaded \cite{25}. In our analysis we have assumed all $\lambda''$ to be zero – this also solves the proton decay problem. In the absence of the $\lambda''$ couplings, the $\lambda'$ or $\lambda$ couplings alone cannot wash out the initial baryon asymmetry, unless there are $B$ violating but $(B-L)$ conserving sphaleron-induced non-perturbative interactions. Since the latter interactions conserve $(\frac{1}{3} B - L_i)$ for each lepton generation, the assumption that not all lepton numbers are simultaneously violated suffices to preserve the initial baryon asymmetry. Keeping this mind, whenever in the present work we have considered a product of two $\lambda'$ (or $\lambda$) thereby violating two lepton numbers, we have made an implicit assumption that the third lepton number is conserved.

Acknowledgements

GB acknowledges hospitality of the Max-Planck Institute in Heidelberg, and CERN Theory Division, Geneva, where some parts of the work were done. SR acknowledges support from the Council of Scientific and Industrial Research, India. AR has been supported in part by the Council of Scientific and Industrial Research and the Department of Science and Technology, India.

References

[1] Super-Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. 81, 1562 (1998).

[2] B.T. Cleveland et al., Nucl. Phys. B (Proc. Suppl.) 38, 47 (1995); Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. 77, 1683 (1996); GALLEX Collaboration, W. Hampel et al., Phys. Lett. B 388, 384 (1996); SAGE Collaboration, J.N. Abdurashitov et al., Phys. Rev. Lett. 77, 4708 (1996); J.N. Bahcall and M.H. Pinsonneault, Rev. Mod. Phys. 67, 781 (1995).

[3] LSND Collaboration, C. Athanassopoulos et al., Phys. Rev. Lett. 75, 2650 (1995); ibid. 77, 3082 (1996); nucl-ex/9706006.

[4] See, for example, Rabindra N. Mohapatra and Palash B. Pal, Massive Neutrinos in Physics and Astrophysics, 2nd Ed., World Scientific, Singapore (1998).

[5] K. Enqvist, A. Masiero, and A. Riotto, Nucl. Phys. B 373, 95 (1992).

[6] E. Roulet and D. Tommasini, Phys. Lett. B 256, 218 (1991).

[7] G. Farrar and P. Fayet, Phys. Lett. B 76, 575 (1978); S. Weinberg, Phys. Rev. D 26, 287 (1982); N. Sakai and T. Yanagida, Nucl. Phys. B 197, 533 (1982); C. Aulakh and R. Mohapatra, Phys. Lett. B 119, 136 (1982).

[8] For recent reviews, see G. Bhattacharyya, Nucl. Phys. B (Proc. Suppl.) 52A, 83 (1997); hep-ph/9709393. Invited talk given at Workshop on Physics Beyond the Standard Model: Beyond the Desert: Accelerator and Nonaccelerator Approaches, Tegernsee, Germany, 8-14 Jun 1997; H. Dreiner, hep-ph/9707433, published in ‘Perspectives on Supersymmetry’, Ed. by G.L. Kane, World Scientific, Singapore; R. Barbier et al., Report of the Group on R-parity violation, hep-ph/9810232.

12
[9] Particle Data Group, C. Case et al., Eur. Phys. J. C 3, 1 (1998).

[10] The TROITSK Collaboration, V.M. Lobashev et al., Phys. Lett. B 460, 227 (1999); The Mainz Collaboration, C. Weinheimer et al., Phys. Lett. B 460, 219 (1999).

[11] L. Baudis et al., Phys. Rev. Lett. 83, 41 (1999).

[12] S. Rakshit, G. Bhattacharyya and A. Raychaudhuri, Phys. Rev. D 59, 091701 (1999); R. Adhikari and G. Omanovic, Phys. Rev. D 59, 073003 (1999); A. Abada and M. Losada, hep-ph/9908352; G. Bhattacharyya, H.V. Klapdor-Kleingrothaus and H. Päs, Phys. Lett. B 463, 77 (1999); M. Drees, S. Pakvasa, X. Tata and T. ter Veldhuis, Phys. Rev. D 57, 5335 (1998); E.J. Chun, S.K. Kang, C.W. Kim and U.W. Lee, Nucl. Phys. B 544, 89 (1999); O.C.W. Kong, Mod. Phys. Lett. A 14, 903 (1999); A.S. Joshipura and S. Vempati, Phys. Rev. D 60, 111303 (1999); A. Datta, B. Mukhopadhyaya and S. Roy, Phys. Rev. D 61, 055006 (2000); B. Mukhopadhyaya, S. Roy and F. Vissani, Phys. Lett. B 443, 191 (1998).

[13] S. Goswami, Phys. Rev. D 55, 2931 (1997); S.M. Bilenky, C. Giunti and W. Grimus, Eur. Phys. J. C 1 247 (1998); [hep-ph/9711311] C. Giunti, hep-ph/9907485.

[14] A.D. Dolgov, hep-ph/0006103.

[15] D. Tytler, J.M. O'Meara, N. Suzuki and D. Lubin, astro-ph/0001318.

[16] E. Lisi, S. Sarkar and F.L. Villante, Phys. Rev. D 59, 123520 (1999).

[17] According to the Big Bang model of cosmology $t_0$ is estimated to be $11.5 \pm 1.5$ Gyr [9].

[18] R. Cowsik and J. McClelland, Phys. Rev. Lett. 29, 669 (1972); S. Gerstein and Ya.B. Zeldovich, Pisma Zh. Eksp. Teor. Fiz 4, 174 (1972).

[19] B.W. Lee and S. Weinberg, Phys. Rev. Lett. 39, 165 (1977); P. Hut, Phys. Lett. B 69, 85 (1977); K. Sato and M. Kobayashi, Prog. Theor. Phys. 58, 1775 (1977); M.I. Vysotsky, A.D. Dolgov and Ya.B. Zeldovich, Pisma Zh. Eksp. Teor. Fiz. 26, 200 (1977).

[20] E.W. Kolb and M.S. Turner, The Early Universe, Addison-Wesley, Redwood City (1990).

[21] See Fig. 2 of G. Gelmini and E. Roulet, Rept. Prog. Phys. 58, 1207 (1995).

[22] G. Gelmini, hep-ph/9904369; S. Sarkar, Rept. Prog. Phys. 59, 1493 (1996).

[23] K. Huitu, J. Maalampi, M. Raidal and A. Santamaria, Phys. Lett. B 430, 355 (1998); M. Chaichian and K. Huitu, Phys. Lett. B 384, 157 (1996).

[24] A. Bouquet and P. Salati, Nucl. Phys. B 284, 557 (1987); B.A. Campbell, S. Davidson, J. Ellis and K. Olive, Phys. Lett. B 256, 457 (1991); W. Fishler, G.F. Giudice, R.G. Leigh and S. Paban, Phys. Lett. B 258, 45 (1991).

[25] H. Dreiner and G.G. Ross, Nucl. Phys. B 410, 188 (1993).