Effect of Various Structural Factors on Shear Strength Degradation after Yielding of RC Members under Cyclic Loads

Li Fu1, Hikaru Nakamura2∗, Yoshihito Yamamoto3 and Taito Miura4

Abstract

Shear failure after flexural yielding is a typical failure mode for a reinforced concrete (RC) member subject to cyclic load, and it is caused by the phenomenon that the initial shear strength is decreased with the increasing plastic ductility until the shear capacity degrades to be lower than the flexural strength. In this study, the flexure-shear behaviors of a number of RC columns with a relatively wide range of crucial structural factors involving tensile reinforcement ratio, shear reinforcement ratio, shear span-depth ratio (covering deep and slender columns) and axial compression load were simulated by the Three Dimensional Rigid-Body-Spring-Method, and the degradation behaviors of shear strength of all specimens were quantitatively evaluated to comprehend the effect of the structural factors. Moreover, the degradation behaviors of shear strength were decoupled into the degradation behaviors of shear resistance components (beam action, arch action, truss action, concrete contribution for beam action) to clarify the mechanism of shear failure after flexural yielding for the RC columns at various crucial structural factors. Consequently, the effect of crucial structural factors on the degradation behaviors of shear strength and shear resistance components were understood and it was concluded that the stepwise degradation of arch action was the dominant mechanism for the investigated structural factors for the shear failure after flexural yielding of RC column. This paper is an extended version in English from the authors’ previous publication [Nakamura, H., Furuhashi, H., Yamamoto, Y. and Miura, T., (2015)]

1. Introduction

1.1 Research background

Due to the effects of cyclic load or earthquake vibrations, RC members commonly suffer from one of the three failure modes, i.e., flexural failure, brittle shear failure, and shear failure after flexural yielding, and the latter two are undesired. It was reported that the recent earthquakes caused a large number of RC members such as load bearing building columns and bridge piers to undergo shear failure after flexural yielding (Li and Li 2012; Hoshikuma and Zhang 2013). Thus, the current structural design codes for RC members were improved to arrange more shear reinforcement, with an intention for RC members to meet the specified inelastic deformation demand (JSCE 2017; ACI 2015; fib 2013).

For cyclic loaded RC members at flexural yielding stage, the initial shear strength will degrade as the inelastic deformation increases (Ho and Pam 2003; Moretti and Tassios 2007), and the diagonal cracks at plastic hinge zone will form and propagate, which ultimately cause a shear-dominant failure. The excessive shear strength degradation is also thought to be induced by an inadequate consideration of crucial structural factors. Therefore, extensive experimental cyclic loading tests on flexure designed RC beams and columns were carried out and the influence of crucial structural factors on the degradation behavior of shear strength were investigated (Kani 1966; Niwa et al. 1986; Panagiotakos and Fardis 2001; Ishibashi et al. 2001). On the basis of the test results, the prediction equations to determine the displacement ductility capacity were established as the function of the above structural factors (Nakamura et al. 1992; Krolicki et al. 2011), aiming to achieve a displacement-based seismic design. Furthermore, a number of researchers suggested to use degradation curve of concrete shear contribution as a primary tool to determine the deformation capacity of test specimens, achieving a performance-based seismic design. Generally, degradation curves of concrete shear contribution were developed based on the statistical analysis of the loading test data (Priestley et al. 1996; Ohe and Yoshikawa 2002; Sezen and Moehle 2004; Tidarut et al. 2013).

Recently, Nakamura et al. (2015) succeeded in establishing a numerical approach to quantitatively evaluate the degradation behavior of shear strength of a RC member subject to cyclic load by utilizing the Three...
Dimensional Rigid-Body-Spring-Method (3-D RBSM), and showed the degradation behaviors of shear strength corresponding to three crucial structural factors (tensile reinforcement ratio, shear reinforcement ratio and axial compression load). Moreover, their results confirmed that the initial shear strength of a RC column will degrade remarkably under cyclic load with the increasing displacement ductility, whereas will not be significantly affected under one-side repeated load. On the basis of this work, Fu et al. (2017) attempted to explain the degradation mechanism of shear strength of a standard RC column discussed in the research conducted by Nakamura et al. (2015). Combining the theory of beam and arch actions and 3-D RBSM numerical method, they developed a practical procedure to decouple the beam and arch actions, truss action and concrete contribution for beam action separately from the initial or residual shear strength at each displacement ductility. A conclusion was drawn that for the target RC column with 4.0 of shear span-depth ratio, 0.82% of tensile reinforcement ratio, 0.08% of shear reinforcement ratio and 1.0 MPa of axial compression load, the step degradation of arch action was mainly responsible for the degradation behavior of shear strength, while the beam action almost presented a constant before failure region.

1.2 Research significance
As discussed in Section 1.1, many studies have been performed on the evaluation of deformation capacity and ultimate ductility in the condition of shear failure after flexural yielding under cyclic loads, and consequently, a number of prediction formulas were established. For instance, JSCE Standard Specification for concrete structures (JSCE 2017) recommends to apply Eq. (1) below to determine the ultimate rotation of plastic hinge at maximum load stage.

$$\theta_{mp} = \frac{(0.021k_{st}p_t+0.013)}{(0.79p_t+0.153)}$$

where

- $\theta_{mp}$ = ultimate rotation of plastic hinge of a RC member;
- $p_t$ = shear reinforcement ratio (%);
- $p_t$ = tensile reinforcement ratio (%);
- $k_{st}$ = the coefficient related to stirrups.

However, since the formulas, such as Eq. (1), were proposed based on the results of extensive cyclic load experiments for RC members with a variety of structural factors, the degradation behavior of shear strength which strongly affecting deformation capacity, and the mechanism of the correlation between each structural factor and deformation capacity was still unclear.

On the other side, in terms of the shear strength after the flexural yielding of RC member, not only the formulas for the deformation capacity, such as the aforementioned estimation of ultimate rotation or ductility ratio, but also the degradation curves of concrete shear contribution, which is barely a function of ductility ratio, were proposed based on the same experimental database. These facts show important non-negligible engineering problem that there is an inconsistency about main parameters between formula for ductility ratio and degradation curve. Moreover, although the ultimate ductility ratio was significantly affected by main structural factors, the correlation between shear strength degradation and each structural factor, and the mechanism of the individual effect of structural factors on shear strength degradation was still unrevealed, since it was incapable of separately clarifying the individual effect of structural factors in experiment.

In this study, based on the research works of Nakamura et al. (2015) and Fu et al. (2017), the individual effect of structural factors on the degradation behavior of shear strength was separately evaluated, and meanwhile the corresponding effect mechanism was clearly explained by the analysis of beam and arch actions. Attributed to the reveal of the shear resistance mechanism for each structural factor, this study will contribute to discussion of more accurate degradation curves of concrete shear contribution, evaluation of the deformation capacity considering the mechanism and the practical improvement method of deformation capacity, all of which were useful knowledge in terms of the shear failure of RC member after flexural yielding under cyclic loads.

2. Numerical method

3-D RBSM was employed as the basic tool to evaluate the mechanical responses of concrete including softening, local crack behaviors and the structural performance (multiple flexure and shear) of moderate RC members quantitatively. This discrete model was employed because it can simulate the post-peak behavior much better compared with the Three-Dimensional Finite Element Method (3-D FEM) and make it possible to have an insight into the cracking propagation.

2.1 Model for concrete

In the model, the concrete is modeled as an assemblage of rigid polyhedrons interconnected by springs at their boundary surfaces (Fig. 1). Since crack propagation is affected by mesh cutting method, the random geometry of rigid polyhedrons was adopted to eliminate mesh bias, and the concrete elements (i.e., convex polyhedrons) were generated by Voronoi random tessellation. In each concrete element, six degrees of freedom (three translational and three rotational) are defined at the nuclei (node point). Moreover, a number of triangles are defined at the boundary surface of two adjacent elements by connecting the vertex with the geometrical center point, and one normal and two shear springs are set at the center of each triangle. By this, the mechanical effects including bending and torsion can be reflected without the consideration of rotational springs. Based on an extensive number of validation, it was demonstrated that the average concrete element size ranging from 10 mm to 30 mm are suitable for simulation and can provide high
accuracy (Yamamoto et al. 2008).

With regard to the constitutive models, the schemes for the stress-strain cyclic hysteretic curve and failure criteria proposed by Yamamoto et al. (2008) for monotonic loading path and for cyclic loading path (Yamamoto et al. 2014) were adopted. Figure 2 shows the monotonic hysteretic curves of normal and shear springs, and the failure of them is defined by Mohr-Coulomb criteria. As the cyclic hysteretic curves, the reloading paths of springs are assumed to recover to the start point of preceding unloading first and thereafter to run along the monotonic hysteretic curves. The details of parameter setting can be found in the related references and are skipped here.

2.2 Model for rebar

Rebar is modeled as a series of regular beam elements [Fig. 3(a)] that can simulate tension, compression and bending effect. In the model, the beam elements can be freely set within a member, regardless of the mesh design of concrete elements (Bolander et al. 2002). Each beam element also has six degrees of freedom (three translational and three rotational) defined using springs. Rebar is attached to concrete polyhedrons by zero-size link node, which provides a bond effect between concrete and beam elements, and each link node has three translational degrees of freedom, one direction parallel and two directions perpendicular to the axis of reinforcement.

As the constitutive models, a bi-linear model was assumed for the tensile and compressive stress-stain hysteretic curves, where the hardening coefficient after yielding was assumed to be 1/100. In addition, the bond stress-slip hysteretic curve between concrete element and rebar link node provided by Yamamoto et al. (2014) was employed, as shown in Fig. 3(b).

A number of researchers have validated the ability of 3-D RBSM in the simulation of structural behaviors for RC members. For example, Yamamoto et al. (2014)
accurately predicted the macro shear behaviors of monotonic and cyclic loaded RC shear walls including load-displacement response and cracking spacing, angle and width (Fig. 4); Ju et al. (2014) verified that the method can simulate the flexure-shear behaviors of RC hollow PHC (prestressed high strength concrete) plies under monotonic and cyclic loadings (Fig. 5); Nakamura et al. (2019) further confirmed that the method can not only simulate the shear failure behaviors of RC deep and slender beams at macro level but also simulate the local stress distributions in members’ cross sections and rebars with high accuracy. Thus, based on the previous work and taking into account the advantages of 3-D RBSM, it was decided to utilize it in this study.

3. Objective experiment and numerical model

3.1 Outline of specimen

The objective specimens were two RC squared columns used as bridge pier the shear span-depth ratio of which is 4.0. They were designed to deform in flexure and were cyclic load tested (Ohta 1979). Figure 4 gives the design drawing of one column named S1. The effective depth and concrete cover thickness of the column were 350 mm and 50 mm, respectively. While the tensile reinforcement ratio and shear reinforcement ratio were 0.82% and 0.08%, respectively. The other column termed A1 had same structural configuration as S1 except for the shear reinforcement ratio, which was doubled to 0.16%. In addition, the material properties of concrete and steel bars are shown in Table 1.

3.2 Loading system

Reversed cyclic load was imposed on the columns by displacement control method. The displacement was increased stepwise with an increment of yield displacement \( \delta_y = 10 \text{ mm} \), i.e., the displacement when the column yielded, and 10 cycles were repeated at each deformation level. In addition, prior to the lateral loading, a vertical compression load (1.0 MPa), which was 0.035 times of the concrete compressive strength, was initially imposed on the upper surface of the column.

3.3 Numerical model

According to the configurations of the aforementioned

| Longitudinal rebar (SD30, D19) | Stirrups (SR24, ф9) | Concrete |
|-----------------------------|----------------------|----------|
| \( f_y \) (MPa) | \( f_t \) (MPa) | \( f_{yw} \) (MPa) | \( f_{tw} \) (MPa) | \( E_y \) (GPa) | \( E_t \) (GPa) | \( f_{c} \) (MPa) |
| 365.5 | 530 | 189 | 372.5 | 490 | 189 | 28.6 | 2.16 |

Fig. 4 Outline of objective specimen (Oota 1979).

(a) Case S1 or A1, \( a/d=4.0 \)

(b) Case C1, \( a/d=3.0 \)

(c) Case C2, \( a/d=2.0 \)

Fig. 5 Image of column models.
experimental specimens, S1 and A1, two 3-D RBSM models, the average element size of which were 30 mm, were established [Fig. 5(a)]. In order to improve the efficiency of the numerical analysis, one-quarter models were adopted and the column bases were simplified as a same squared cross section as the column constrained by three stiffening loading plates. In each model, the red lines and green lines stand for longitudinal rebar and stirrups models, respectively.

Aimed to conduct a survey on the effect of crucial structural factors, four groups of RC squared column models having same cross section configuration were constructed for numerical tests (Table 2). The model groups “A1-A3”, “B1-B2”, “C1-C2”, and “D1-D2” were at various shear reinforcement ratios, shear-span ratios, tensile reinforcement ratios and axial compression loads, separately. The material properties of concrete and steel reinforcements were set same as the experimental material tests. Steel ratio was directly adjusted by proportionally increasing or reducing the rebar cross sectional area, and the shear span-depth ratio was regulated by changing the shear span length [Figs. 5(b) and 5(c)] while the spacing of stirrups was consistent with the standard case S1, 200 mm.

The numerical results of a part of the above models, which were derived from the S1 modelling experiment test, had been investigated by Nakamura et al. (2015) and Fu et al. (2017). The numerical analysis for the model group “A1-A3” at various shear reinforcement ratios had been conducted by Nakamura et al. (2015). The model group “B1-B2” at various shear span-depth ratios was firstly provided in this study. It is of significant importance to evaluate separately the shear strengths of slender and short members, since they were ordinarily designed in dissimilar principles. Regarding the tensile reinforcement ratios, Nakamura et al. (2015) numerically analyzed the shear performances of the three columns where the ratios included 0.20%, 0.41% and 0.82%, and a new model with a greater tensile reinforcement ratio of 1.02% was added in this paper, considering that the more use of tensile reinforcement the more likely to fail in shear while it was often to use tensile reinforcement over 1.0% in the case of column type member. This can provide useful suggestion on actual structure. Taking into account that axial compression load has an evident effect on shear, Nakamura et al. (2015) attempted to investigate the shear behaviors of low axial compression load cases, 1.0 and 2.0 MPa. But on the other hand, since axial compression load on building structures was much higher, higher axial load cases 4.0 and 6.0 MPa were added in this study for the analysis. Thus, compared to the work by Nakamura et al. (2015), a wider range of structural factors in accordance with actual structures were numerically tested, and their effects on the degradation behavior of shear strength were more explicit.

In the numerical tests, the cyclic load was imposed in a same way as the loading experiment, displacement control, and 0.01 mm displacement increment was added in each numerical step. In the analysis, the iteration procedure was terminated and moved to the next step when the internal energy became less than 0.01% of the one obtained in the initial iteration or the number of iteration in current step exceeded the upper limit, 100. The numerical parameters relevant to the mechanical properties of the springs in the models were determined by the standard method introduced in the previous section, and no extra calibration was needed. It took approximately five days to complete one simulation until failure stage.

The random mesh design of 3-D RBSM model might lead to a little bit difference in the numerical results between two columns, even the structural configurations of them were completely same. It should be noted that the numerical results of some cases in this paper might present slight differences from the same analyses accomplished by Nakamura et al. (2015).

| Case No. | Yielding displacement (δy (mm)) | Shear span-depth ratio (a/d) | Tensile reinforcement ratio (pt (%)) | Shear reinforcement ratio (pw (%)) | Axial compression load (P (MPa)) |
|----------|-------------------------------|----------------------------|-----------------------------------|----------------------------------|-------------------------------|
| A1       | 10.0                          | 4.0                        | 0.82                              | 0.16                             | 0.08                          |
| A2       | 10.0                          | 4.0                        | 0.82                              | 0.32                             | 1.0                           |
| A3       | 10.0                          | 4.0                        | 0.82                              | 0.00                             | 0.00                          |
| B1       | 8.0                           | 3.0                        | 0.82                              | 0.08                             | 1.0                           |
| B2       | 6.0                           | 2.0                        | 0.82                              | 0.08                             | 1.0                           |
| C1       | 5.5                           | 4.0                        | 0.41                              | 0.08                             | 1.0                           |
| C2       | 13.0                          | 2.0                        | 1.02                              | 0.08                             | 1.0                           |
| D1       | 12.5                          | 4.0                        | 0.82                              | 0.08                             | 4.0                           |
| D2       | 13.5                          | 4.0                        | 0.82                              | 0.08                             | 6.0                           |

4. Numerical results

In this section, the numerical load-displacement hysteretic curves and deformation behaviors of RC columns, are reported.

4.1 Comparison of numerical and experimental results

The load-displacement hysteretic curves of S1 and A1 had been simulated by Nakamura et al. (2015), as plotted in Fig. 6. It was observed that the numerical results overall agreed well with the varying tendencies and the
shapes of the experimental curves, in particular, the change process from spindle shape to inverted S shape, the numerical load carrying capacities and the critical failure points were in good agreement with the experimental results, which demonstrated the reasonability of 3-D RBSM.

4.2 Effect of shear reinforcement ratio $\rho_w$
The numerical hysteretic curves of the RC columns S1 (black curve) and A1-A3 (red curve) under cyclic loading are plotted in Fig. 7. It was found clearly that the change in $\rho_w$ did not affect the magnitude of the load carrying capacity, which was in good agreement with the prediction by design code. Compared with the result of S1, the deformation abilities of A1 and A2 were remarkably promoted due to the increased amount of stirrups ($S_1$ and $A_1$ failed at a displacement of $-4\delta_y$ and $+6\delta_y$, respectively, nevertheless A2 did not fail even when the displacement reached $+6\delta_y$). On the contrary, because of the insufficient shear reinforcement, A3 failed in shear after the flexural yielding at a smaller displacement, $+3\delta_y$.

In order to confirm the flexure-shear failure mode and to find out the effect of stirrups amount on the deformation performance of RC columns, the deformation behaviors of the four columns were carefully observed, as illustrated in Fig. 8 (the deformations were magnified by eight times). In the case of S1, the diagonal cracks initially appeared at a displacement of $+3\delta_y$, and thereafter dramatically developed toward the base until the ultimate stage, $+4\delta_y$, when the shear failure occurred [Fig. 8(a)]. Due to the effect of stirrups, the propagation of the diagonal cracks in the A1 and A2 cases were obviously restricted [see their deformations at the displacement of $+4\delta_y$ in Figs. 8(b) and 8(c)], and finally the shear compression failures initiated at the basements with severe cover concrete spalling being observed. For the A3 case, the diagonal tension cracks started to form at a smaller displacement, $+2\delta_y$, resulting from a lack of shear reinforcement, and the widths of the cracks significantly enlarged at the later stage, $+3\delta_y$, which led to a sudden shear failure [Fig. 8(d)]. It was evident that 3-D RBSM can reasonably simulate the effect of shear reinforcement on the flexure-shear combined performance of RC squared column under cyclic loading.

4.3 Effect of shear span-depth ratio $a/d$ The numerical hysteretic curves for the case series “B1-B2” are shown in Fig. 9, and the result of standard case S1 was also added for comparative analysis. It was noted that the load carrying capacity was significantly improved due to the decrease in shear span length, i.e., 250 kN for the S1 case ($a/d = 4.0$), 330 kN for the B1 case ($a/d = 3.0$) and 500 kN for the B2 case ($a/d = 2.0$), which was consistent with the predicted trend according to the JSCE Standard Specification (JSCE 2017). This improvement was attributed to the shear contribution forming in the uncracked portion in the compression zone, known by the tied arch. However, the shorter beam was the deformation capacity became worse: the B1 model ($a/d = 3.0$) failed at a displacement of $+4\delta_y$ [Fig. 9(a)] while the B2 model ($a/d = 2.0$) failed at a displacement of $+3\delta_y$ [Fig. 9(b)]. Since all of the numerically tested columns discussed in this section and the following case series “C1-C2” and “D1-D2” suffered...
same failure mode (flexure-shear failure), the deformation graphs of them are skipped.

4.4 Effect of tensile reinforcement ratios $p_t$

Figure 10 illustrates the hysteretic curves for the case series “C1-C2”. As a result, the load carrying capacity...
was enhanced with the increase in tensile reinforcement, which was consistent with the estimation by the prediction equations of flexural strength (JSCE 2017). On the other side, the ultimate displacement ductility of a column with more tensile reinforcement was significantly decreased. The column C1 ($p_t = 0.41\%$) presented more excellent deformation capacity, in which the ultimate displacement exceeded $+5\delta_y$ [Fig. 10(a)], than the standard case S1, while the column C2 ($p_t = 1.02\%$) failed at a displacement of $-3\delta_y$ [Fig. 10(b)].

4.5 Effect of axial compression load $P$

Figure 11 shows the hysteretic curves for the case series “D1-D2”. It was noted that the load carrying capacity of a RC column was clearly improved attributed to the increase in the axial compression load, which was reported to be caused by the significant enhancement of tied-arch action (Priestley et al. 1994). However, the increase in the axial compression load evidently reduced the plastic ductility of RC column: the load response of the D1 model dropped sharply at a displacement of $-3\delta_y$ [Fig. 11(a)] while that of the D2 model steeply fell down at a displacement of $+3\delta_y$ [Fig. 11(b)]. This tendency was also reported by Biskinis et al. (2004) and the related design code (JSCE 2017).

It was observed that all the above cases finally suffered from shear failure after flexural yielding and the effect of each structural factor on the shear performance of RC column simulated by 3-D RBSM became clear, and was consistent with the preceding experimental test data and the current design codes. Therefore, the applicability of 3-D RBSM was proved.

5. Effect of structural factors on shear strength degradation behavior

5.1 Evaluation method for shear strength degradation behavior

Since the shear failure of a flexure-deformed RC member after entering plastic deformation stage under cyclic loading is caused by shear strength degradation, a numerical method for quantitative evaluation of shear strength degradation behavior has been proposed and proved to be useful by the author’s previous work (Nakamura et al. 2015) and the work of Fu et al. (2017). Herein, the same method was adopted to evaluate the shear strength degradation behavior of each objective RC column reported in the preceding section.

For a RC member under cyclic load, the degradation behavior of shear strength is ordinarily comprehended through the decreasing shear capacity at each displacement ductility. However, in case of a flexure deformed member, its shear strength cannot be experimentally tested by the load response until ultimate stage, since the load carrying capacity by experiment cannot exceed the flexural strength (the black curve in Fig. 12). In order to overcome this trouble, a conventional fact was noted by Nakamura et al. (2015) that the increase in the yield strength of tension reinforcement can proportionally improve the flexural strength of a RC member meanwhile does not affect the grade of shear strength, according to the structural design formula (JSCE 2017; ACI 2015). Based on this, they proposed a quantitative approach for evaluating the decreased shear strength at each ductility level through the following numerical procedure.

Firstly, a cyclic loading analysis with the actual yield strength of tensile reinforcement ($f_y = 365.5$ MPa) was performed until the end of the first load cycle. At the end of unloading path (point A in Fig. 12), the yield strength $f_y$ (365.5 MPa) was decreased to 900 MPa, which was assumed as the flexural strength $P_u$ after the first load cycle. Thus, a new yield strength $f_y$ (900 MPa) was assigned and the process was repeated until the ultimate shear strength $P_s$ was reached. Following this procedure, the shear strength degradation behavior of each RC column was investigated.
of tensile reinforcement in the former step was instantly enhanced to 900 MPa, so that the flexural strength ($P_u$) of the RC column was artificially enhanced to a higher grade than the potential shear strength ($V$). In the next numerical step, a monotonic loading test (the dashed red curve) was performed until the ultimate stage. In this way, the shear strength after first load cycle was known through the maximum load, the red point in Fig. 12. By same method, the shear strength after each load cycle can be estimated and the degradation behavior of shear strength can be sketched.

The evaluation of the degradation behavior of shear strength for each model group was carried out, and the result for the model group “A1-A3” is shown in Fig. 13, as an example. It was seen that the increase in shear reinforcement not only improved the initial shear strength but also effectively promoted the critical displacement ductility where the shear strength cannot meet the shear demand due to flexure, that is, the inhibiting effect of shear reinforcement on the degradation of shear strength was evident.

### 5.2 Degradation curve of concrete shear contribution

It is generally accepted that the degradation behavior of shear strength of a cyclically loaded RC member is primarily caused by the degradation of concrete contribution, nevertheless the contribution provided by shear reinforcement is unaffected with the increasing ductility. Thus, in current shear design, the degradation curve of concrete contribution (simplified as shear degradation curve), which is known by the relationship between reduction coefficient of concrete shear contribution $\alpha$ and displacement ductility ratio $\mu$, is widely employed to predict the shear performance of a RC member. Thus, the shear degradation curve of each case involved in this study was drawn through the following procedure, which was firstly proposed by Nakamura et al. (2015), to clarify the effect of structural factors on the degradation behavior of concrete contribution.

1. Determination of initial concrete contribution (marked by $V_{con\_initial}$). It could be estimated by a new 3-D RBSM analysis. A new model having the same configuration as the objective case was established, but no stirrups were arranged with an intention to remove the effect of stirrups on shear. Next, the yield strength of tensile reinforcement in the model was set to 900 MPa, and after that, a numerical monotonic loading test until final shear failure was performed. Consequently, the peak load that corresponded to $V_{con\_initial}$ could be obtained.

2. Determination of contribution by stirrups (marked by $V_s'$). It was estimated by removing $V_{con\_initial}$ from the initial shear strength of the objective case.

3. Determination of degrading concrete contribution after each load cycle (marked by $V_{con}$). It could be obtained by removing the $V_s'$ from the degrading shear strength after each load cycle.

4. Determination of reduction coefficient $\alpha$. It was calculated by the ratio of the degrading concrete contribution after each load cycle to the initial concrete contribution ($\alpha = \frac{V_{con\_initial}}{V_{con}}$).

5. Drawing of shear degradation curve ($\alpha$-$\mu$ relationship).

The shear degradation curve of each case including case S1 is shown in Fig. 14, together with the three statistical shear degradation curves established by Priestley et al. (1996) [Eq. (2)], Ohe and Yoshikawa (2002) [Eq. (3)] and JSCE Standard Specification (JSCE 2017) [Eq. (4)], which are the average models of an extensive shear experimental $\alpha$-$\mu$ data points for RC members. The equations for the curves are listed below:

$$\alpha = \begin{cases} 1 & \mu < 2 \\ -0.3275\mu + 1.655 & 2 \leq \mu < 4 \\ -0.04325\mu + 0.518 & 4 \leq \mu < 8 \\ 0.172 & 8 \leq \mu \end{cases}$$

(2)

$$\alpha = -0.442\ln(\mu) + 1.15$$

(3)

$$\alpha = \begin{cases} 1 & \mu < 1 \\ -0.25\mu + 1.25 & 1 \leq \mu \end{cases}$$

(4)

where $\alpha =$ reduction coefficient of concrete shear contribution and $\mu =$ displacement ductility ratio.

For each experimental RC member, only one $\alpha$-$\mu$ relationship at failure stage could be obtained (Fig. 15). As the necessary factors for the determination of reduction coefficient $\alpha$, the initial concrete contribution ($V_{con\_initial}$) could be directly predicted according to the relevant design codes [the estimation specified by Japanese code is given in Eq. (5), while the concrete contribution at failure stage ($V_{con}$) could be computed by removing the contribution by shear reinforcement ($V_s'$), which was calculated according to the classical plastic truss theory [Eq. (6)], from the measured load response at failure stage. In addition, the inclinations of shear reinforcement adopted can be found in Fig. 15.

![Fig. 13 Evaluation of shear strength after each load cycle (Cases A1-A3).](image-url)
\[ V_{\text{con\_initial}} = 0.20 \cdot f_c' \cdot \frac{1}{p_t} \cdot \frac{1}{d} \cdot (0.75 + 1.4 \cdot \frac{a}{b}) \cdot b_w \cdot d \quad (5) \]

where

- \( V_{\text{con\_initial}} \): design shear strength of linear RC members without shear reinforcements;
- \( f_c' \): concrete compressive strength;
- \( p_t \): longitudinal tensile reinforcement ratio;
- \( d \): effective depth of column;
- \( a \): shear span length;

\[ b_w = \text{width of a RC member.} \]

\[ V' = A_w \cdot f_yw \cdot \frac{z}{s} \sin \alpha' \cdot (\cot \theta + \cot \alpha) \quad (6) \]

where

- \( A_w \): total area of shear reinforcement placed in spacing \( s \);
- \( f_yw \): yield strength of shear reinforcement;
- \( z \): horizontal projection of shear crack;
- \( s \): spacing of shear reinforcement;

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**Fig. 14 Shear degradation curves by 3-D RBSM.**

**Fig. 15 Determination procedures for the statistical shear degradation curves.**
\( \alpha = \) inclination of shear reinforcement with member’s axis;  
\( \theta = \) inclination of shear crack with member’s axis.

For the case series “A1-A3”, the analytical degradation curves were in excellent agreement with the statistical models, which proved the reasonability of the analytical procedure for shear degradation curve [Fig. 14(a)]. The shear strengths were decreased very slowly at the initial deformation stage, i.e., less than 10% decrease at \( \mu = 1 \), but were abruptly reduced when the displacement ductility exceeded an upper limit. Therefore, generally two distinct slopes were observed in each degradation curve, and this characteristic was independent of shear reinforcement amount. It was also noted that the arrangement of shear reinforcement clearly increased the ductility corresponding to the slope change point when the shear reinforcement ratio was under 0.16%, i.e., the slope change point was located at \( \mu = 1 \) for case A3 while \( \mu = 3 \) for cases A1 and A2, implying that shear reinforcement somehow could prevent the degradation of concrete contribution. Therefore, these results suggested that, in the seismic design of shear strength, besides truss action, concrete contribution should be taken into account to any degree for a RC member with sufficient shear reinforcement rather than be wildly ignored as specified in design codes.

For the case series “B1-B2” [Fig. 14(b)], similar to the result for the case series “A1-A3”, two distinct descending slopes were obtained for each degradation curve. For the columns where the shear span-depth ratios were no less than 3.0, the degradation curves presented close descending slopes, that is, the effect due to the change in shear span-depth ratio on shear strength degradation could be neglected. In addition, the results also suggested that the statistical models may underestimated the degradation rate of concrete contribution of the relatively short members (\( a/d = 2.0 \)).

The shear degradation curves for the case series “C1-C2” are shown in Fig. 14(c). For the C1 case, even though the yield strength of tensile reinforcement was increased to 900 MPa after the third load cycle, it still failed in flexure, therefore, the degrading shear strength could not be estimated by our proposed approach at the later loading stage. This was because the tensile reinforcement was extremely low and insufficient. Consequently, only the shear strengths at the displacement ductility \( \mu = 0, 1, 2 \) could be reasonably estimated, and also, two distinct descending slopes were obtained in each curve. The displacement ductility corresponding to the slope change point was significantly increased with the decreasing tensile reinforcement.

For case series “D1-D2” [Fig. 14(a)], it was found that if the axial load was increased, the slope change point would move to a smaller displacement ductility, and so did the ultimate displacement ductility.

In conclusion, it became clear that more arrangement of shear reinforcement, less tensile reinforcement and lower axial compression load were beneficial to the shear deformation performance of RC member, that is, the displacement ductility corresponding to the slope change point in shear degradation curve was increased to an extent, which was significant to seismic shear design and worthy noted.

### 6. Effect of structural factors on shear resistance mechanisms

#### 6.1 Decoupling method for beam and arch actions

The principal shear resistance mechanism of a RC squared member can generally be decoupled into two shear components, beam and arch actions, which expresses the well-known relationship between the shear and the bending moment rate of change along a member’s longitudinal axis. Park and Paulay (1975) first mathematically separated beam action from shear resistance, and the shear components for an arbitrary unit segment (marked by \( dx \)) along a member’s longitudinal axis can be derived by Eqs. (7) to (10) [see Fig. 16 (Iwamoto et al. 2017)].

\[
M = (T + C_c) \frac{d \theta}{dx} + C_c \cdot j_{cT} + T_c \cdot j_{Te} \tag{7}
\]

\[
V = \frac{dM}{dx} = V_b + V_a \tag{8}
\]

\[
V_b = \left( \frac{dT}{dx} + \frac{dC_c}{dx} \right) \frac{j}{2} + \frac{dC_c}{dx} \cdot j_{cT} + \frac{dT}{dx} \cdot j_{Te} \tag{9}
\]

\[
V_a = C_c \cdot \frac{d j_{cT}}{dx} + T_c \cdot \frac{d j_{Te}}{dx} \tag{10}
\]

where

- \( M = \) bending moment on a cross section;  
- \( V = \) total shear resistance;  
- \( V_b = \) shear contribution provided by beam action, which is attributed to the change of axial forces sustained by rebars and concrete on different cross sections [Fig. 16(b)];  
- \( V_a = \) shear contribution provided by arch action, which is caused by the height change of the acting points of concrete compression and tension resultants on different cross sections [Fig. 16(c)];  
- \( T = \) internal force in tension rebar on a cross section;  
- \( C_c = \) internal force in compression rebar on a cross section;  
- \( C_c = \) concrete compression resultant on a cross section;  
- \( j_{cT} = \) shear contribution provided by beam action, which is mathematically separated beam action from shear resistance, and the shear components for an arbitrary unit segment (marked by \( dx \)) along a member’s longitudinal axis can be derived by Eqs. (7) to (10) [see Fig. 16 (Iwamoto et al. 2017)].

### Interpretation of shear components (Iwamoto et al. 2017)

- \( C_c \): concrete compression  
- \( C_c + dC_c \): concrete compression  
- \( T_c \): tension rebar tension  
- \( j_{cT} \): shear contribution provided by beam action  
- \( j_{Te} \): shear contribution provided by arch action  
- \( V_b \): shear contribution provided by beam action  
- \( V_a \): shear contribution provided by arch action  
- \( T \): internal force in tension rebar on a cross section  
- \( C_c \): internal force in compression rebar on a cross section  
- \( C_c \): concrete compression resultant on a cross section
Concrete contribution $V_c$ = average concrete contribution (marked by $V_c$), could be computed by removing the average truss action from the average beam action. It should be emphasized that the concrete contribution ($V_{con}$) defined in Section 4 is the sum of arch action and $V_c$.

6.2 Decoupling results of shear resistance mechanisms for each structural factor

The decoupling result for the monotonic loading hysteric curve for estimation of initial shear strength of the $S_1$ model is firstly discussed using Fig. 17, as an example. The sudden drop of load response at 31.0 mm was resulted from the dramatic development of critical shear crack. The excellent agreement of the combination resistance of beam and arch actions with the numerical response by 3-D RBSM (dash curve) validated the applicability and reliability of the decoupling approach introduced in Section 6.1. Obviously, this decoupling procedure can make it clear the contribution of each shear resistance component at each displacement.

In the same way, the numerical load hysteric curves for estimation of degrading shear strength after each load cycle of each column, were decoupled into the shear resistance components with an intention to clarify the effect of the structural factors on shear resistance mechanism. For each column, the shear resistance components corresponding to the maximum load after each load cycle were gathered and represented in the form of the relationship between degrading shear components and the number of load cycles (Figs. 18 to 21).

Additionally, the combination of beam and arch actions is also plotted to give the reference of total shear after each load cycle. The results corresponding to the zero load cycle are the initial shear resistance components, and the shear cracking loads shown in the figures are the predictions according to JSCE standard specification (JSCE 2017).

The decoupling results of shear resistance components are explored separately for each structural factor. For the case series “A1-A3” (Fig. 18), the degradation of shear strength was effectively inhibited due to the arrangement of shear reinforcement and the effect was more evident with the increasing in the amount of steel [Fig. 18(a)]. The beam actions were not greatly affected by the increased displacement [Fig. 18(b)], whereas the arch actions were progressively degraded until failure stage [Fig. 18(c)]. Thus, it should be pointed out that the shear strength degradations were governed by the arch action degradation, and the arrangement of stirrups could inhibit the arch action degradation. For the decoupling result of beam action, the grade of initial truss action was significantly enhanced by adding more stirrups, and was almost independent on the increased displacement [Fig. 18(d)], because the stirrups were already yielded at the failure stages. However, the truss action of the cases A2

$$V_s = \frac{A_w \cdot \sigma_s \cdot jd}{s}$$

(11)

where

- $A_w$ = total area of shear reinforcement placed in s;
- $\sigma_s$ = stress state in shear reinforcement;
- $jd$ = moment arm length ($j = 1/1.15, d$ is member effective depth);
- $s$ = spacing of shear reinforcement.

$V_s$ can be computed by removing the average truss action from the average beam action.
was unusually increased after two load cycles. This phenomenon was an overestimation, because it was confirmed that the orientation of the critical diagonal cracks after two load cycles were actually greater than the assumed 45°. Hence, the concrete contributions ($V_c$) for these two cases presented negative values after several load cycles [Fig. 18(e)].

For case series “B1-B2” (Fig. 19), the arch action degradation dominated the degradation behavior of shear strength, while the beam actions presented comparatively low grade and could be sustained before failure [Figs. 19(a), 19(b) and 19(c)]. The decrease in shear span-depth ratio remarkably enhanced the initial arch action but would lead to much severer degradation of arch action and shear strength [Figs. 19(a) and 19(e)], which was the primary reason why the deformation capacity of the column with small shear span-depth ratio was declined. Additionally, similar to the beam actions, the concrete contributions ($V_c$) and the truss actions were not visibly affected before failure [Figs. 19(d) and 19(e)], and the truss actions of the three cases presented almost same grade because of the same steel amount.

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**Fig. 18** Degradation of each shear resistance component ($\rho_p$).

**Fig. 19** Degradation of each shear resistance component ($\alpha/d$).
In terms of the case series “C1-C2” (Fig. 20), the shear strength degradations were resulted from the arch action degradations, and the beam actions were well maintained before failure [Figs. 20(a), 20(b) and 20(c)]. It was also found that a decrease in tensile reinforcement could prevent arch action degradation [Fig. 20(e)], because the number of flexure-shear cracks increased in the column with more tension steel at same displacement, which delayed the compressive stress loss in concrete. For the result of the decoupling beam action, a same degradation progress of truss actions and concrete contributions ($V_c$) as those discussed in the preceding case series were observed, that is, no obvious degradations for them were noticed before the ultimate stages [Figs. 20(d) and 20(e)].

For the case series “D1-D2”, a same critical factor for shear strength degradation as those in the previous discussion was found, that is, the degradation of arch action led to the shear strength degradation, while no significant degradation of beam action was confirmed before shear failure [Figs. 21(a), 21(b) and 21(c)]. The increased axial compression load resulted in a severer degradation of arch action [Fig. 21(c)]. For the shear contribution to beam action, the truss actions could be maintained, being a constant in the entire loading processes, while the concrete contributions ($V_c$) suddenly fell.
down when the load capacities declined to the shear demand for flexural strength [Figs. 21(d) and 21(e)].

According to above explorations, the mechanism for shear failure after flexural yielding of RC squared column became clear and should be lessened, that is, progressive degradation of arch action actually governed the degradation behavior of shear strength till shear failure occurred, whatever it was a deep or slender column and regardless of the variation in tensile reinforcement, shear reinforcement and axial compression load. With regard to the other shear resistance components, in general truss action was a nearly a constant in the entire loading process, while beam action and concrete contribution \((V_c)\) would be rapidly reduced if total shear degraded to be lower than the shear demand for flexure. Moreover, the effect of the main structural factors on the degradation behavior of each shear resistance component was figured out.

7. Conclusions

The research work based on 3-D RBSM and beam-arch theory was systematically conducted aimed to clarify the effect of main structural factors on the shear failure mechanism of RC squared column after flexural yielding subjected to cyclic load. The following are the conclusions that were drawn.

(1) Above all, the shear performances of the RC squared columns with varied structural factors, i.e., tensile reinforcement ratio, shear reinforcement ratio, shear span-depth ratio and axial compression load, subject to cyclic load, were simulated by 3-D RBSM. Moreover, the effect of structural factors on the degradation behavior of concrete contribution was investigated based on the evaluation of shear degradation curves. It was concluded that the shear strength of each RC column was decreased very slowly (no more than 15% reduction generally) at the initial loading stage \((\mu = 1.0)\) but was abruptly reduced when the displacement ductility exceeded an upper limit, indicating that each degradation curve consisted of two slopes. More shear reinforcement, less tensile reinforcement, and lower axial compression load had the effect to increase the displacement ductility corresponding to the slope change point. For the columns where the shear span-depth ratios were no less than 3.0, the degradation curves exhibited close descending slopes.

(2) Secondly, the shear degradation behaviors of RC columns were decoupled into the degradation behaviors of shear resistance components (i.e., beam action, arch action, truss action, and the contribution of concrete to beam action). As a result, the arch action degradations were confirmed being the crucial reason for the shear strength degradations, which led to the ultimate shear failures regardless of the variation in structural factors, and this finding provided a guidance in the future work for the improvement of seismic design.

(3) In addition, the effect of structural factors on the degradation behavior of shear strength, was clarified. It was revealed that the arrangement of shear reinforcement was an effective measure to inhibit the degradations of arch action and shear strength. On the contrary, the decrease in shear span-depth ratio, the increase in tensile reinforcement and axial compression load had the effect to accelerate the process of arch action degradation, which finally reduced the deformation capacity of RC column.

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