Non-coherent character of isoscalar pairing probed with Gamow-Teller strength: New insight into $^{14}\text{C}$ dating $\beta$ decay

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Abstract

We investigate the phase coherence of isoscalar pairs from the $B(\text{GT}; 0^+_1 T = 1 \rightarrow 1^+_1 T = 0)$ values in two-particle configurations of $A = 6, 18$, and $42$ nuclei and two-hole configurations of $A = 14$ and $38$ ones. We find that these Gamow-Teller (GT) matrix elements are always constructive and thus enlarged under isovector- and isoscalar-pairing Hamiltonians, whereas the observed GT strengths are strongly hindered for the two-hole configurations, including the famous $^{14}\text{C}$ dating $\beta$ decay. This indicates that the actual isoscalar pair, unlike the isovector pair, has no definite phase coherence, which can work against forming isoscalar-pair condensates.

Keywords: Gamow-Teller transition, Shell model, Isoscalar pairing, Phase coherence

1. Introduction

Pairing correlation is one of the most basic properties widely seen in quantum many-body problems including condensed-matter physics and nuclear physics. This is quite a common phenomenon caused by attractive interactions between constituent particles. In nuclei, the source of the attraction is short-range nucleon-nucleon forces, owing to which time-reversal pairs with large spatial overlap gain much energy and then a condensate of the Cooper pairs occurs. Whereas isovector (IV) pairing (like-particle pairing) with $(J, T) = (0, 1)$ is firmly established for instance by extra binding energies in even-even nuclei, a condensate of isoscalar (IS) proton-neutron pairs with $(J, T) = (1, 0)$ appears quite elusive. This is puzzling because mean attraction in the IS channel is much stronger than in the IV channel. Possible signals for IS-pairing correlation have been explored for instance in terms of binding energies, rotational responses, Gamow-Teller (GT) $\beta$-decay properties, and proton-neutron transfer amplitudes (see [2] for a review). Of particular interest is that IS-pairing correlation is predicted to be quite sensitive to double-$\beta$ decay matrix elements [3, 4, 5, 6, 7].

A condensate of pairs is based on the formation of an energetically stable pair in two-particle configurations. Using a simple attractive force, Cooper has shown that the binding energy of the lowest-energy pair is much larger than the ones of the other eigenstates and also than the scale of two-body matrix elements [1]. This happens because a coherent combination of paired configurations—with a specific combination of signs—cooperatively works to lower energy [8]. Whether such a coherent IS pair is formed in nuclei may provide a key to elucidating the origin of elusive IS pairing, but much attention has not been paid to phase coherence.

In this Letter, we show that the IS-pairing interaction always causes a specific combination of signs in the lowest $(J, T) = (1, 0)$ state for any two-particle $(2p)$ configuration and for any two-hole $(2h)$ one and that the resulting $B(\text{GT}; 0^+_1 T = 1 \rightarrow 1^+_1 T = 0)$ value is enhanced. While this property well accounts for the low-energy super GT state for $A = 6, 18$, and $42$ [9], it fails to explain strongly hindered $B(\text{GT})$ values for the $2h$ configurations, including the famous $^{14}\text{C}$ dating $\beta$ decay. This is a clear signature that the IS pair in reality does not take any definite signs, in contrast to what occurs for the ideal IS pairing. The IS pairing is thus fragile in nature, constituting an essential difference from the IV pairing which always favors definite signs and is thus robust.

2. GT strength in $2p$ and $2h$ configurations

We start with an overview of observed GT strengths in $2p$ and $2h$ configurations on top of the $LS$-closed shells. Hereafter we restrict ourselves to the initial and final states with the quantum numbers $(J_i, T_i, T_{iz}) = (0, 1, \pm 1)$ and $(J_f, T_f, T_{zf}) = (1, 0, 0)$, respectively. In Table 1 experimental $B(\text{GT}; 0^+_1 \rightarrow 1^+_1)$ values are summarized for the $p$, $sd$, and $pf$ shells. For the $2p$ configurations, one can clearly see that the $B(\text{GT})$ is concentrated in the $1^+_1$ state for any valence shell considered, which is named the low-energy super GT state [9]. In contrast, the $2h$ configurations have a striking difference: most of the GT strength is exhausted by excited states, especially the $1^+_2$ state. It is noted that the $A = 14$ case is well known for radiocarbon dating, which utilizes the very long half-life of $^{14}\text{C}$, $5730 \pm 30$ yr, to determine the age of organic materials.
We examine how this strong asymmetry in GT strength between the 2p and 2h configurations arises in the framework of the shell model. The case of the p shell is now taken as an example, but similar discussions are applicable to other shells. In the shell model, 2p and 2h configurations can be treated in a unified way in terms of particle-hole conjugation, since particle-particle two-body matrix elements are identical with the corresponding hole-hole matrix elements \(^{13}\). The only difference between 2p and 2h configurations concerning the GT transition is single-particle energies. Keeping this in mind, we calculate the GT matrix elements by changing \(\Delta_{\text{p}} = \varepsilon(p_{1\frac{3}{2}}) - \varepsilon(p_{1\frac{1}{2}})\), where \(\varepsilon\) stands for the single-particle energy. The values of \(\Delta_{\text{p}}\) for the 2p \((A = 6)\) and 2h \((A = 14)\) configurations are 0.1 MeV and \(-6.3\) MeV, respectively, taken from the CKII interaction \(^{13}\).

For the two-body part, we first use the CKII interaction as a realistic one, and show the calculated GT matrix elements \(M(\text{GT}; 1^+_0) = \langle 1^+_0|\sigma r^2|0^+_0\rangle (k = 1, 2)\) in Fig. 1(a). When the initial and final states are expanded as \(|0^+_0\rangle = \sum_{ab} a_{ab}^{0^+}(k)\langle ab|J_fT_f\rangle\) and \(|1^+_0\rangle = \sum_{ab} a_{ab}^{1^+}(k)\langle ab|J_fT_f\rangle\), respectively, \(M(\text{GT}; 1^+_0)\) is expressed by the sum of single-particle contributions as

\[
M(\text{GT}; 1^+_0) = \sum_{abcd} m_{abcd}(1^+_0),
\]

where \(m_{abcd}(1^+_0) = a_{ab}^{1^+}(k) a_{cd}^{1^+}(1) (ab|J_fT_f|cd|J_fT_f\rangle\). For \(\Delta_{\text{p}} > 0\), the \(M(\text{GT}; 1^+_0)\) value strongly enhances, well reproducing the observed GT strength for \(A = 6\). This quantity is very close to the sum-rule limit of \(\sqrt{6} \approx 2.45\) up to \(\Delta_{\text{p}} \approx 5\) MeV, and then gradually decreases to the \(p_{3\frac{1}{2}}^{1\frac{1}{2}}\) single-particle limit of \(\sqrt{10\frac{3}{2}} \approx 1.83\). For \(\Delta_{\text{p}} < 0\), on the other hand, the \(M(\text{GT}; 1^+_0)\) value sharply decreases with decreasing \(\Delta_{\text{p}}\). It crosses the \(M(\text{GT}) = 0\) line at \(\Delta_{\text{p}} \approx -5.7\) MeV, thus accounting for the vanishing GT strength observed in the \(\beta\) decay of \(^{14}\text{C}\).

3. GT strength with pairing interactions

To probe pairing properties in the 2p and 2h configurations, it is interesting to compare those realistic shell-model calculations to the ones using the IS- and IV-pairing interactions. The IV- and IS-pairing interactions are equivalent to the \(L = 0\) part of the surface delta interaction (SDI) in the IS coupling, hence the simplest interaction of short-range central-force character. The IV- and IS-pairing interactions are defined as

\[
\begin{align*}
\langle ab|JT|cd\rangle & = G^\text{IV}\chi_{ab}^{\text{IV}}\chi_{cd}^{\text{IV}}\delta_{0\text{T}}\delta_{T1}, \\
\langle ab|JT|cd\rangle & = G^\text{IS}\chi_{ab}^{\text{IS}}\chi_{cd}^{\text{IS}}\delta_{1\text{T}}\delta_{T0},
\end{align*}
\]

with

\[
\begin{align*}
\chi_{ab}^{\text{IV}} & = (-1)^{\frac{1}{2}} \frac{\sqrt{J_a + 1}}{2} \delta_{ab}, \\
\chi_{ab}^{\text{IS}} & = \left\{ \frac{1}{2} \frac{J_a}{J_b} \frac{I_a}{I_b} \right\} \delta_{n_m\text{S}} \delta_{l_m l_b}.
\end{align*}
\]

for \(a, b, c,\) and \(d\) stand for single-particle states with quantum numbers \(\nu_a, I_a, j_a\) etc., and \(\delta_{ab}\) is the abbreviation for \(\delta_{\nu_a\nu_b}\delta_{I_aI_b}\delta_{j_aj_b}\). The strengths \(G^\text{IV}\) and \(G^\text{IS}\) are negative for attractive interactions, and here we set \(G^\text{IV} = -3.4\) MeV and \(G^\text{IS} = -2.8\) MeV so that \(\sum_{\nu\nu'\nu''\nu'''}\langle ab|\text{IV}^{\text{pair}}|c'd'd''d'''angle\) and \(\sum_{\nu\nu'\nu''\nu'''}\langle ab|\text{IS}^{\text{pair}}|c'd'd''d'''angle\) become those of the CKII interaction.

The results of the above pairing interactions are plotted in Fig. 1(b). Similar to the CKII interaction, the enhancement of the GT matrix element from the single-particle limit occurs for

| \(p\) | \(sd\) | \(p/f\) | \(p\) | \(sd\) |
|---|---|---|---|---|
| \(A = 6\) | \(A = 18\) | \(A = 42\) | \(A = 14\) | \(A = 38\) |
| \(1^+_0\) | 4.7 | 3.1 | 2.2 | 3.5 \times 10^{-3} | 0.060 |
| \(1^+_0\) | 0.13 | 0.10 | 2.8 | 1.5 |
$\Delta\rho > 0$. However, the trend for $\Delta\rho < 0$ is completely different. The $M(\Gamma; 1^+)$ value decreases rather mildly as $\Delta\rho$ moves away from zero. This is a monotonically decrease which asymptotically approaches the $p_{1/2}$ single-particle limit of $\sqrt{2/3} \approx 0.82$ and never vanishes.

The essential feature of the pairing interaction shown in Fig. (b) is that the $M(\Gamma; 1^+)$ value enlarges compared to the $p_{1/2}$ and $p_{3/2}$ single-particle limits for $\Delta\rho > 0$ and $\Delta\rho < 0$, respectively. This is caused by the constructive interference of $m_{abcd}(1)$ in Eq. (1), and such an in-phase character has been presented for the $pf$-shell case of $^{42}$Ca [5] on the basis of numerical analyses using the shell model and the random-phase approximation. It is still not very clear, however, why the constructive interference occurs and whether it is realized for different valence shells.

In order to answer this question, we find a theorem concerning the sign of $m_{abcd}(1)$.

**Theorem 1.** All of the $m_{abcd}(1)$ values are of the same sign for any valence shell and for any single-particle splitting when the two-body matrix elements are given by the pairing interactions of Eqs. (2) and (3) with negative $G^{IV}$ and $G^{IS}$.

**Proof.** We first consider the signs of the two-body matrix elements of Eqs. (2) and (3). Since the sign of $\chi^{IV}_{ab}$ is $(-1)^{l_0}$ [see Eq. (4)], all the matrix elements of $V^{IV}_{\text{pair}}$ can be negative (or zero) when one takes a phase convention $|abfJ|T_i⟩ = (−1)^{l_0}|abfJ|T_i⟩$. Similarly, for the IS pairing one can easily show that the sign of $\chi^{IS}_{ab}$ is $(-1)^{l_0}$, thus obtaining entirely negative matrix elements of $V^{IS}_{\text{pair}}$ with a phase convention $|abfJ|T_i⟩ = (−1)^{l_0}|abfJ|T_i⟩$. Hereafter we refer to these phase choices as pairing phase convention, and the components of the eigenvectors in this convention are expressed by $\delta^{IV}_{ab}(k)$ and $\delta^{IS}_{ab}(k)$.

Thus, when the IS- and IV-pairing interactions are taken, their off-diagonal Hamiltonian matrix elements in the pairing phase convention are completely negative or zero for any two-nucleon configuration, since single-particle energies do not change the off-diagonal matrix elements in the $j j$-coupling. For such matrices having non-positive off-diagonal matrix elements, it is generally true that all the components of the lowest eigenvector are of the same sign according to a version of the Perron-Frobenius theorem in linear algebra. We thus obtain $\delta^{IV}_{ab}(1) \geq 0$ and $\delta^{IS}_{ab}(1) \geq 0$ for any $(a, b)$. As shown in Table 1, the GT matrix elements between two-nucleon wave functions satisfy $(abfJ|T_i⟩|cdJ|T_i⟩) \leq 0$ for any $(a, b)$ and $(c, d)$ concerned, hence the same sign of $m_{abcd}(1)$.

This is a mathematically exact statement, and therefore provides a robust basis for the occurrence of the low-energy super GT state [2] in 2$p$ configurations.

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1 One can easily prove this case by showing the expectation value of $\varphi = (+a_1, +a_2, \ldots, +a_n)$ is greater than or equal to that of $\varphi = (+a_1, \ldots, +a_2, +a_3, \ldots, +a_n)$ for any $a_i \geq 0$.

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Table 2: GT matrix elements $⟨f|\sigma^T|b⟩$ in the pairing phase convention, where the two-nucleon wave functions $i (T = 1)$ and $f (T = 0)$ are denoted as $(ab)$ by using $j_c = l + 1/2$ and $j_c = l - 1/2$. We present only the basis states $i$ and $f$ that appear with the pairing Hamiltonian.

| $i$ | $j_c$ |
|-----|-------|
| $(j_s, j_c)$ | $(j_s, j_c)$ |
| $j = 0$ | $-2\sqrt{2}/3$ |
| $j = 1$ | $-2\sqrt{2}/3$ |
| $j = 2$ | $-2\sqrt{2}/3$ |

4. Phase coherence in the IS pair

As indicated by the above proof, the key to obtaining the constructive interference of $m_{abcd}(1)$ is that all the off-diagonal Hamiltonian matrix elements are of the same sign which causes phase coherence in the IS and IV pairs. We stress that only the signs are relevant. The simplest case of the phase coherence is found in the original paper of the Cooper pair [1], where all the off-diagonal matrix elements between paired electrons near the Fermi surface are taken to be $-|F|$ and all the diagonal matrix elements are zero. In this case, the lowest eigenvalue is $(a_1, a_2, \ldots, a_n) = (1, 1, \ldots, 1)/\sqrt{n}$, and the corresponding energy eigenvalue is $-(n-1)/|F|$. The enhancement of eigenenergy compared to the off-diagonal matrix elements, called pairing gap, is due to the phase coherence.

In nuclei it is well known that such a coherent pair is formed between like-particles. All the off-diagonal $(J, T) = (0, 1)$ matrix elements are indeed negative in realistic shell-model Hamiltonians of CKII, USD [17], KB3 [18] and GXPFL1 [19]. Similar phase coherence in the IS pair is expected to be formed on the basis of the IS-pairing Hamiltonian, giving rise to the constructive interference of $m_{abcd}(1)$. In reality, however, the $2h$ configurations have nearly vanishing $B(\text{GT})$ values as shown in Table II pointing to destructive interference. Hence, the coherent IS pairs are not always formed with realistic interactions because of the opposite sign in some of the $(J, T) = (1, 0)$ two-body matrix elements.

Taking the $p$ shell as an example, we present in Fig. 2 an intuitive picture about the formation of coherent and non-coherent

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![Figure 2: (color online). Graphical illustration of the signs of $(J, T) = (1, 0)$ off-diagonal two-body matrix elements (represented as $\oplus$ and $\ominus$) in the pairing phase convention and the resulting signs of the lowest eigenvector. The $p$-shell cases using (a) the IS pairing Hamiltonian and (b) the CKII Hamiltonian are compared. The up and down arrows stand for positive and negative coefficients of $(\varphi)$, respectively.](image-url)
IS pairs for the (a) IS-pairing and (b) CKII Hamiltonians, respectively, where the pairing phase convention is used. Now the lowest eigenstate is expressed as $\sum_\alpha \alpha |\psi_\alpha\rangle$ by using the basis states $|\psi_\alpha\rangle$. Before proceeding to detailed discussions, it should be reminded that the negative sign of an off-diagonal matrix element between two basis vectors $|\psi_\alpha\rangle$ and $|\psi_\beta\rangle$, denoted as $h_{\alpha\beta}$, favors the same sign of $\alpha_1$ and $\alpha_2$ and a positive $h_{\alpha\beta}$ favors the opposite sign in the lowest eigenstate. Here we mean $|\psi_1\rangle = |p_1p_2\rangle$, $|\psi_2\rangle = |p_1p_2\rangle$ and $|\psi_3\rangle = |p_1p_2\rangle$, where $p_2$ and $p_1$ are $p_{\alpha12}$ and $p_{\alpha12}$, respectively. As illustrated in Fig. 2(a), the obtained coherent pair is quite stabilized by the IS-pairing Hamiltonian, since any combination of $(i, j)$ satisfies the above rule. On the other hand, the CKII Hamiltonian has a positive $h_{33}$ and negative $h_{12}$ and $h_{13}$. In this case, there must be at least a combination of $(i, j)$ that does not comply with the above rule, as illustrated in Fig. 2(b). This is analogous to the geometrical frustration in magnetism [20], although the physical situation is rather different. The signs of $\alpha_1$ can no longer be uniquely determined, and the actual signs depend on the diagonal terms. For the $A = 6$ system, the values of $h_{11}$, $h_{22}$ and $h_{33}$ are close to one another, and then the same sign of $\alpha_1$ and $\alpha_2$ is realized because of $|h_{12}| \gg |h_{20}| \approx |h_{33}|$. In this case $|\psi_3\rangle$ has a small amplitude of the opposite sign, hence contributing little to the eigenstate. The dominance of $|\psi_1\rangle$ and $|\psi_2\rangle$ of the same sign accounts for the enhanced $B$ value. For the $A = 14$ system, $h_{11}$ is higher than $h_{33}$ by more than 10 MeV, so that the ground state is dominated by $|\psi_2\rangle$ and $|\psi_3\rangle$. The resulting signs of $\alpha_2$ and $\alpha_3$ are opposite because of $h_{33} > 0$, thus leading to the nearly vanishing $B$ value.

In this way the favorable signs for $|ab J=0 T=1\rangle$ in the IS pair are not definite and depend on the core assumed for realistic interactions whose off-diagonal $(J, T) = (1, 0)$ matrix elements are not completely of the same sign in the pairing convention. This is an essential difference between IV and IS pairing, and clearly works against forming an IS-pair condensate. We point out that non-coherent IS pairs can be probed with pair-transfer strength, which is regarded as a good measure of IS pairing correlation [2]. The IS-pair creation and removal strengths are now defined as $|\langle J,T_f||D||J,T_i\rangle|^2$ and $|\langle J,T_f||D||J,T_i\rangle|^2$, respectively, and we consider the transition to the lowest $(J_f, T_f) = (1, 0)$ state. By using the CKII interaction, the IS-pair removal strength from $^{16}$O is only $5.3 \times 10^{-3}$, while the IS-pair creation strength on $^4$He is 8.1. A similar strong asymmetry between the IS-pair creation and removal is also obtained for the sd shell.

Finally, we briefly survey the origin of difference in the signs of the off-diagonal $(J, T) = (1, 0)$ matrix elements between realistic interactions and the IS-pairing interaction. First we consider the central forces. Since the IS-pairing Hamiltonian is equivalent to the $(L, S) = (0, 1)$ term of the SDI, the dominance of the $L = 0$ central force is the source of the IS pairs, as well as for the usual IV pairing. While the $(L, S) = (2, 1)$ term is absent in the $(J, T) = (0, 1)$ matrix elements, this term can modify the $(J, T) = (1, 0)$ matrix elements. In Fig. 3 the effect of the $(L, S) = (2, 1)$ term is presented for various orbital angular momenta $l$ by using the SDI. In general, $\langle j_f j_f |V_{l,l'}^T|j_j j_j\rangle$ and $\langle j_f j_f |V_{l,l'}^T|j_j j_j\rangle$ matrix elements due to $L = 2$ are positive with the SDI, and strong cancellation between $L = 0$ and 2 occurs especially for $\langle j_f j_f |V_{l,l'}^T|j_j j_j\rangle$ with low $l$ and $l'$. For $l = 1$, for instance, $\langle j_f j_f |V_{l,l'}^T|j_j j_j\rangle$ still has a large negative value, whereas $\langle j_f j_f |V_{l,l'}^T|j_j j_j\rangle$ vanishes. In addition, the $L = 2$ term gives rise to the opposite sign between $\langle j_f j_f |V_{l,l'}^T|j_j j_j\rangle$ and $\langle j_f j_f |V_{l,l'}^T|j_j j_j\rangle$ for $l' = l'' + 2$ and $n' = n'' + 1$, thus causing frustration among $|j_j\rangle$, $|j_f\rangle$, $|l', l''\rangle$ when $\langle j_f j_f |V_{l,l'}^T|j_j j_j\rangle < 0$ is satisfied. We confirm that finite-range interactions lead to essentially similar results to the SDI by using the $V_{MU}$ interaction [21], but some quantitative differences appear. For instance, the $\langle |p_1p_2\rangle |V|p_1p_2\rangle$ matrix element, which is exactly zero for the SDI, is positive by taking only the $(S, T) = (1, 0)$ term of the $V_{MU}$, and this matrix element can be positive or negative depending on the strength of the $(S, T) = (0, 0)$ term. It should be noted that the $(S, T) = (0, 0)$ term vanishes for zero-range interactions.

Another important source to change the signs of $(J, T) = (1, 0)$ matrix elements is the non-central forces. It has been pointed out by Jancovici and Talmi that phenomenological tensor forces are needed to account for the extraordinary long lifetime of $^{14}$C [22]. It is worth mentioning that its microscopic
In the present context, the $^{14}$C lifetime problem is a manifestation of the non-coherent IS pair formed by the positive sign of $\langle p,p' \rangle |V| p,p'\rangle$ due to the $L \neq 0$ central forces and the non-central forces. This idea can readily be applied to other cases. For instance, the small $B(GT)$ value for $A=38$ (see Table 1) is caused by the positive sign of $(|d^1d^1\rangle |V| d^1d^1\rangle$ made in a similar way to $\langle p,p' \rangle |V| p,p'\rangle > 0$. In contrast, the $\langle J_f J_f |V| J_f J_f \rangle$ matrix elements have always large negative values both in schematic and realistic interactions, thus causing the low-energy super GT state $\beta^+$ in $2p$ configurations.

5. Conclusion

We have shown that the strong asymmetry in the $B(GT; 0^+_1 \rightarrow 1^+_1)$ values between $2p$ and $2h$ systems is a clear signature that a coherent combination of $(J,T)=(1,0)$ pairs is not necessarily formed. By introducing the pairing phase convention and the idea of frustration, we have presented a comprehensive but mathematically robust explanation as to why the ideal IS-pairing interaction always leads to phase coherence regardless of single-particle energies but realistic interactions do not. This is in sharp contrast to the IV pairing in realistic interactions, and may provide a key to elucidating the origin of elusive IS-pair condensates in nature. It is of great interest to investigate how modern microscopic effective interactions predict the $(J,T)=(1,0)$ matrix elements in a wide range of nuclear shells and how the non-coherent effect changes observables in more complex nuclei, including double-$\beta$ decay matrix elements.

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