Parity detection achieves the Heisenberg limit in interferometry with coherent mixed with squeezed vacuum light

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Abstract. The interference between coherent and squeezed vacuum light effectively produces path entangled N00N states with very high fidelities. We show that the phase sensitivity of the above interferometric scheme with parity detection saturates the quantum Cramer–Rao bound, which reaches the Heisenberg limit when the coherent and squeezed vacuum light are mixed in roughly equal proportions. For the same interferometric scheme, we draw a detailed comparison between parity detection and a symmetric-logarithmic-derivative-based detection scheme suggested by Ono and Hofmann.

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1. Introduction

Optical metrology relies on light interferometry as its primary tool for phase estimation. The sensitivity of phase estimation with coherent light-based interferometry is limited by shot noise \[1\]. This limit, however, is due to the classical nature of coherent light and can be surpassed if nonclassical states of light, such as the N00N state, are used \[2, 3\]. Still there is a limit on the sensitivity of phase estimation in the case of linear optical interferometry. Its usual justification stems from the Heisenberg uncertainty principle that links phase uncertainty of a state to its photon number uncertainty, \[\Delta\phi \Delta n \geq 1\]. Combination of this equation with the assumption that the photon number uncertainty in a state is limited by the total photon number (in the case of states with definite photon number) or the total average photon number (in the case of states with indefinite photon number), \[\Delta n \leq N\], suggests the limiting phase sensitivity to be \[\Delta\phi_{HL} = 1/N\], which is commonly referred to as the Heisenberg limit \[4, 5\].

Quantum optical metrology has Heisenberg-limited sensitivity of phase estimation as its goal. To this end, the search for convenient states of light and optimal detection schemes still continues \[6–11\]. Candidate states of light are gauged based on the quantum Cramer–Rao bound that provides a detection scheme-independent phase sensitivity \[\Delta\phi_{QCRB}\] \[12\]. In turn, optimal detection schemes are sought, which are capable of saturating the quantum Cramer–Rao bound. A known possibility is a detection scheme that measures a symmetric logarithmic derivative, since such an operator would saturate the quantum Cramer–Rao bound; however, it is seldom easy to implement such an operator. The capabilities of alternative detection schemes are judged by the classical Cramer–Rao bound, which is detection scheme specific, or by the error propagation formula that links the phase uncertainty with the uncertainty of the observed signal.

Here, we consider coherent and squeezed vacuum light input as a candidate for Heisenberg-limited phase estimation (see figure 1). This state was previously checked against the quantum Cramer–Rao bound and shown to achieve Heisenberg-limited phase sensitivity when the coherent light and squeezed vacuum light are mixed in roughly equal intensities \[13\]—a feature that can be explained as due to the high fraction of an N00N state in the normalized \(N\)-photon output component of the quantum state of light past the mixing beam splitter—first pointed out by Hofmann and Ono \[14\] and later experimentally demonstrated by Silberberg’s group \[15\]. The detection scheme suggested in \[13\] for Heisenberg-limited phase estimation was, however, based on Bayesian analysis of the photon number statistics of the output state, which requires photon number counting in both modes.

In this paper, we study parity detection \[16\] for the interferometry with coherent and squeezed vacuum light. In essence, parity detection distinguishes states with odd and even numbers of photons. Quantum mechanically, it is described by the parity operator, \(\hat{\Pi}_n = (-1)^{\hat{n}}\), acting on a single output mode, \(\hat{a}\). We show that the parity operator saturates the quantum Cramer–Rao bound and in turn provides Heisenberg-limited phase sensitivity when the coherent and squeezed vacuum light are mixed in equal proportions. Parity detection should be a simpler alternative to the detection scheme of \[13\], since parity measurement can be inferred from the photon number counting statistics of a single mode alone. In the low power regime, the photon number counting statistics can be obtained using photon number resolving detectors \[17\]. Accurate photon-number-resolution in the high power regime is a difficult task, but there have been proposals for the quantum non-demolition measurement of photon number using weak nonlinearities and homodyning \[18\]. However, it is not necessary to have photon-number-resolving capabilities in order to implement parity detection. Assuming the availability
of large Kerr nonlinearities through the techniques of electromagnetically induced transparency, a scheme that makes quantum non-demolition measurement of parity directly, without requiring the measurement of photon number, has been proposed [19]. Also, Plick et al recently showed that parity measurement for interferometric schemes that use Gaussian states, like the one in use here, could possibly be inferred through balanced homodyning and intensity difference measurement [20].

Ono and Hofmann [21] studied a detection scheme based on the measurement of a symmetric logarithmic derivative for the considered interferometric scheme. This scheme uses interference with an auxiliary local oscillator and intensity difference measurement as well. We duly discuss this scheme with the intent to compare it with parity detection.

The paper is organized as follows. Section 2 describes the propagation of a two-mode light, initially in the product state of coherent and squeezed vacuum light, through the Mach–Zehnder interferometer. Section 3 focuses on the parity-based detection scheme and provides the expected signal and phase sensitivity. Section 4 discusses the Ono–Hofmann detection scheme in equal detail and makes a comparison between the two detection schemes. Section 5 presents the conclusion.

2. Propagation of the input fields through the interferometer

The input to the interferometer is in the product state $|\alpha_0\rangle \otimes |\xi = r e^{i\phi_s}\rangle$ that describes coherent light with amplitude $\alpha_0 = \sqrt{n_c} e^{-i\phi_c}$ in one mode and squeezed vacuum with parameters $r$ and $\phi_s$ in the other. The corresponding Wigner function of the input state is the product of the respective Wigner functions as well [22]:

$$W_{in}(\alpha, \alpha_0; \beta, r) = W_c(\alpha, \alpha_0) W_s(\beta, r),$$

(1)
with the Wigner functions for the corresponding states being
\[
W_c(\alpha, \alpha_0) = \frac{2}{\pi} e^{-2|\alpha - \alpha_0|^2}, \quad W_s(\beta, r) = \frac{2}{\pi} e^{-2|\beta|^2 \cosh 2r - (\beta^2 + \beta^* e^{i\phi})^2 \sinh 2r},
\] (2)
and where we have made \(\phi_s = 0\) by appropriately fixing the irrelevant absolute phase. This choice implies that the phase of the coherent light \(\phi_c\) is now measured with respect to the phase of the squeezed vacuum state.

A Mach–Zehnder interferometer is composed of optical elements such as beam splitters, mirrors and phase shifters. Propagation of the light field through these elements is described by relating the initial variables in the Wigner function to their final expressions:
\[
W_{\text{out}}(\alpha_f, \beta_f) = W_{\text{in}}(\alpha_i(\alpha_f, \beta_f), \beta_i(\alpha_f, \beta_f)).
\] (3)

The relation between variables in the most general form is given by a two-by-two scattering matrix \(\hat{M}\):
\[
\begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix} = \hat{M}^{-1} \begin{bmatrix} \alpha_f \\ \beta_f \end{bmatrix},
\] (4)
where \(\alpha_i, \beta_i, \alpha_f\) and \(\beta_f\) represent the complex amplitudes of the field in the modes \(\hat{a}_i, \hat{b}_i, \hat{a}_f\) and \(\hat{b}_f\) respectively. More specifically, propagation through a 50 : 50 beam splitter and a phase shifter (in mode \(\hat{b}\)) is described by
\[
\hat{M}_{\text{BS}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}, \quad \hat{M}_\phi = \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\phi} \end{bmatrix},
\] (5)
respectively. Therefore, the Mach–Zehnder interferometer in figure 1 is described by \(\hat{M}_{\text{MZI}} = \hat{M}_{\text{BS}} \hat{M}_\phi \hat{M}_{\text{BS}}\) and is found to be
\[
\hat{M}_{\text{MZI}} = ie^{-i\phi/2} \begin{bmatrix} \sin \frac{\phi}{2} & \cos \frac{\phi}{2} \\ \cos \frac{\phi}{2} & -\sin \frac{\phi}{2} \end{bmatrix},
\] (6)
with the corresponding transformation of the variables in the following form:
\[
\alpha_i \rightarrow -ie^{i\phi/2} \left( \alpha_i \sin \frac{\phi}{2} + \beta_i \cos \frac{\phi}{2} \right),
\] (7)
\[
\beta_i \rightarrow -ie^{i\phi/2} \left( \alpha_i \cos \frac{\phi}{2} - \beta_i \sin \frac{\phi}{2} \right).
\] (8)

Therefore, the state of light at the output of the Mach–Zehnder interferometer is described by the following Wigner function:
\[
W_{\text{out}}(\alpha_f, \beta_f) = \frac{4}{\pi^2} e^{-2|\alpha_f e^{i\phi/2} + \beta_f e^{i\phi/2}|^2} \times e^{-2|\alpha_f \cos \frac{\phi}{2} e^{i\phi/2} - \beta_f \sin \frac{\phi}{2} e^{i\phi/2}|^2 \cosh 2r} \times e^{2Re\left(e^{i\phi} \left( \alpha_f \cos \frac{\phi}{2} e^{i\phi/2} - \beta_f \sin \frac{\phi}{2} e^{i\phi/2} \right) \right) \sinh 2r}.
\]

Having found the state of light at the output of the Mach–Zehnder interferometer, we will present the parity-based phase estimation scheme with calculations of its signal and phase sensitivity in the following section.
3. Phase estimation with parity measurement

Parity detection was originally proposed in the context of trapped ions by Bollinger et al [23]. It was later adopted for optical interferometry by Gerry [24]. Parity detection makes phase inference at the Heisenberg limit possible without having to know the full photon number counting statistics for several classes of input states with definite as well as indefinite photon numbers (including the N00N state) [25, 26]. Coherent and squeezed vacuum light belong to the latter class of states and the performance of parity detection for these states is studied in this section.

An expected signal of the parity detection scheme \( \langle \hat{\Pi}_a \rangle \) is calculated as the value of the Wigner function at the origin for the corresponding mode. In the case of mode \( \hat{a}_t \), 
\[
\langle \hat{\Pi}_a \rangle = \frac{\exp \left[ -n_c \left( \frac{\sqrt{n_t^2 + n_s^2} \sin^2 \phi \cos 2\phi_c - \cos \phi}{n_t \sin^2 \phi + 1} + 1 \right) \right]}{\sqrt{n_s \sin^2 \phi + 1}},
\]
where the coherent light amplitude and the squeezing parameter have been expressed in terms of the average photon numbers, \( n_c \) and \( n_s \), using the relations \( \alpha_0 = \sqrt{n_c} e^{-i\phi_c} \) and \( r = \sinh^{-1} \sqrt{n_s} \).

The signal of the parity detection scheme is periodic with period \( 2\pi \) and attains its maximum value of one at \( \phi = 0 \). Although this maximum value is independent of the phase of the coherent light \( \phi_c \) and the light intensities \( n_c \) and \( n_s \), the visibility of the signal and its width are functions of these parameters. The visibility of the signal is found to be best when \( \phi_c = 0 \) and to diminish as \( \phi_c \) drifts away from zero, becoming worst at \( \phi_c = \pi/2 \). Since it is reasonable to assume the coherent and squeezed vacuum light to be locked to the same external phase, \( \phi_c \) can be set to zero for optimal performance. Further, the dependence of the signal on the light intensities is studied in terms of the total input intensity, \( n_{in} = n_c + n_s \), and the fraction of total intensity in the squeezed vacuum state, \( \eta = n_s/n_{in} \). When \( \eta \) is increased from zero, the signal is found to grow narrower until reaching an optimal width, and then to broaden again, but with reduced visibility as \( \eta \) approaches one. For \( \eta = 0 \) and \( \eta = 1 \), the width of the signal is found to be proportional to \( \pi / \sqrt{n_{in}} \), which is narrower than the resolution of conventional interferometry by a factor of \( \sqrt{n_{in}} \) and thus demonstrates super-resolution [27]. The fraction \( \eta = 0.5 \) is found to be the most optimal choice for distributing the input light intensity, since it allows a higher narrowing factor of \( n_{in} \). Figure 2 demonstrates this result by comparing the parity signals for interferometry with only coherent light (\( \eta = 0 \)) [28] or squeezed light (\( \eta = 1 \)) and interferometry with coherent and squeezed vacuum light of equal intensities (\( \eta = 0.5 \)). We see that for the same total input photon number, \( n_{in} = 10 \), the parity signal for the latter case is narrower than any other case.

The phase sensitivity \( \Delta \phi \) of an interferometer, followed by a detection scheme described by an operator \( \hat{O} \), can be characterized using the error propagation formula
\[
\Delta \phi^2 = \frac{\langle \hat{O}^2 \rangle - \langle \hat{O} \rangle^2}{|d(\langle \hat{O} \rangle)/d\phi|^2}.
\]

The smaller the value of \( \Delta \phi \), the higher the phase sensitivity. For the parity-based detection scheme, \( \hat{O} = \hat{\Pi}_a \), knowing the signal suffices for sensitivity calculation since \( \hat{\Pi}_a^2 = 1 \). The phase sensitivity with parity detection for coherent and squeezed vacuum light interferometry is...
Figure 2. The parity signal $\langle \hat{P}_{\eta} \rangle$ as a function of the accumulated phase between the arms of the MZI $\phi$: dashed (purple) line for coherent light interferometry ($\eta = 0$) with $n_c = 10$, $\phi_c = 0$; dotted (red) line for squeezed vacuum light interferometry ($\eta = 1$) with $n_s = 10$; and solid (blue) line for coherent and squeezed vacuum light interferometry ($\eta = 0.5$) with $n_c = n_s = 5$, $\phi_c = 0$. The dot-dashed (green) line is the scaled-down signal for conventional coherent light interferometry with intensity difference measurement.

found to be best at $\phi = 0$ and is given by

$$\Delta \phi^2 = \frac{1}{2n_c\sqrt{n_s(n_s+1)} \cos 2\phi_c + 2n_cn_s + n_c + n_s}. \quad (11)$$

For a detection scheme to be optimal, it has to saturate the quantum Cramer–Rao bound. The quantum Cramer–Rao bound for the interferometry with coherent light and squeezed vacuum was derived in [13]:

$$\Delta \phi_{\text{QCRB}}^2 = \frac{1}{|\alpha_0|^2 e^{2r} + \sinh^2 r}. \quad (12)$$

This expression can be shown to be identical to the phase sensitivity with parity detection, given in equation (11) (under the condition $\phi_c = 0$), when $\alpha_0$ and $r$ are expressed in terms of the average photon numbers, $n_c$ and $n_s$. Thus, parity detection saturates the quantum Cramer–Rao bound and is optimal for the considered interferometric scheme for accumulated phases around zero.

Although parity detection is optimal for the considered interferometric scheme irrespective of the input intensities, the combination as a whole achieves its best phase sensitivity when $\eta = 0.5$. Figure 3 is a plot of the quantum Cramer–Rao bound $\Delta \phi_{\text{QCRB}}$ for the interferometry with coherent and squeezed vacuum light given in equation (12), as a function of the fraction of squeezed vacuum in the input $\eta$. The phase sensitivity $\Delta \phi_{\text{QCRB}}$ can be seen to be best when $\eta \approx 0.5$. At this value of $\eta$, under the condition $\phi_c = 0$, equation (11) reveals that the phase sensitivity $\Delta \phi$ of the considered interferometric scheme with parity detection coincides with the Heisenberg limit, $\Delta \phi \approx 1/n_{\text{in}}$, while it coincides with the shot-noise limit, $\Delta \phi \approx 1/\sqrt{n_{\text{in}}}$, when $\eta = 0$ or 1.
Figure 3. The quantum Cramer–Rao bound $\Delta \phi_{\text{QCRB}}$ for the interferometry with coherent and squeezed vacuum light, as a function of the fraction of squeezed vacuum in the input $\eta$. The total input photon number $n_{\text{in}} = 10$.

Figure 4. Phase sensitivity $\Delta \phi$ with parity detection, as a function of the accumulated phase between the arms of the MZI $\phi$: dashed (purple) line for coherent light interferometry ($\eta = 0$) with $n_c = 10$, $\phi_c = 0$, dotted (red) line for squeezed vacuum interferometry ($\eta = 1$) with $n_s = 10$ and solid (blue) line for coherent and squeezed vacuum light interferometry ($\eta = 0.5$) with $n_c = n_s = 5$, $\phi_c = 0$.

Figure 4 compares the phase sensitivity $\Delta \phi$ with parity detection for the cases corresponding to $\eta = 0$, $\eta = 1$ and $\eta = 0.5$. It reveals that $\eta = 0.5$ with $n_c = n_s = 5$ provides sub-shot noise phase sensitivities up to accumulated phases of about $\pm 0.2$ away from the optimum value of $\phi = 0$, but the phase sensitivity plummets in a dramatic fashion beyond these values of accumulated phase. However, $\eta = 0$ provides a fairly constant phase sensitivity at about the shot-noise limit over a much broader range of accumulated phases. (The case $\eta = 0$ is not of much interest since its phase sensitivity $\Delta \phi$ also deteriorates rather quickly, from the shot-noise limit, as one moves away from the optimal value of $\phi = 0$.) Thus, a suggested way to perform phase estimation is to start with coherent light $\eta = 0$ and roughly learn the value of...
Figure 5. The Ono–Hofmann detection scheme for interferometry with coherent and squeezed vacuum light. The detection scheme uses interference with an auxiliary local oscillator and intensity difference measurement for phase estimation. A highly reflective beam splitter is used to mix the local oscillator field into the interferometer.

the accumulated phase; move the accumulated phase closer to the origin and then tune-up $\eta$ to 0.5 for improved phase sensitivity.

4. Comparison with the Ono–Hofmann detection scheme

So far, we have shown that parity detection could be used to achieve Heisenberg-limited phase estimation in the interferometry with coherent and squeezed vacuum light. In [21], Ono and Hofmann discussed a different detection scheme that implements the measurement of a symmetric logarithmic derivative. Implementation of this measurement is based on interference with a local oscillator and intensity difference measurement as shown in figure 5. Since symmetric logarithmic derivative based phase estimators saturate the quantum Cramer–Rao bound, Heisenberg-limited phase sensitivity was anticipated with this scheme for the interferometry with coherent and squeezed vacuum light mixed in equal proportions ($\eta = 0.5$). Here, we present a brief study of the Ono–Hofmann detection scheme (in the absence of losses), for the purpose of comparing it with parity detection.

The Ono–Hofmann detection scheme consists of a second MZI appended at the output of the first, with a phase $\phi$, which is set to $\pi$. A local oscillator field, which is in the coherent state, $|\gamma_{lo}\rangle = \sqrt{n_{lo}}/T e^{i\phi_{lo}}$, is introduced by mixing with the mode $\hat{a}_{f'}$ through a highly reflective beam splitter of transmissivity, $T \ll 1$, where $n_{lo}$ is the average number of photons in the field that eventually enters the interferometer, and $\phi_{lo}$, its phase. Finally, the difference in intensities at the two output modes is measured.

\footnote{We have also analyzed the case $\phi = 0$. The phase sensitivity is found to be the same, but the signal turns out to be different.}

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Figure 6. The signal with the Ono–Hofmann detection scheme—the intensity difference $I$, plotted as a function of the accumulated phase between the arms of the MZI $\phi$: dashed (purple) line for coherent light interferometry ($\eta = 0$) with $n_c = 10$, $\phi_c = 0$, dotted (red) line for squeezed vacuum light interferometry ($\eta \approx 1$) with $n_s = 9.9$, $n_c = 0.1$, $\phi_c = 0$ and solid (blue) line for coherent and squeezed vacuum light interferometry ($\eta = 0.5$) with $n_c = n_s = 5$, $\phi_c = 0$. A local oscillator of strength $n_{lo} = n_c (e^2 \sinh^{-1}(\sqrt{n_s}) + 1)^2$ and phase $\phi_{lo} = \pi/2$ is used in each case.

Intensity measurements at the output provide

$$
\langle \hat{c}^{\dagger} \hat{c} \rangle = \langle \{ \hat{c}^{\dagger} \hat{c} \} \rangle - \frac{1}{2},
$$

$$
\langle \hat{d}^{\dagger} \hat{d} \rangle = \langle \{ \hat{d}^{\dagger} \hat{d} \} \rangle - \frac{1}{2},
$$

with $\{ \hat{c}^{\dagger} \hat{c} \}$ ($\{ \hat{d}^{\dagger} \hat{d} \}$) being the symmetric form of the operator, which can be evaluated based on the final Wigner function of the state $W_f$, as

$$
\langle \{ \hat{c}^{\dagger} \hat{c} \} \rangle = \int \int |\alpha|^2 W_f(\alpha, \beta) \, d^2\alpha \, d^2\beta,
$$

$$
\langle \{ \hat{d}^{\dagger} \hat{d} \} \rangle = \int \int |\beta|^2 W_f(\alpha, \beta) \, d^2\alpha \, d^2\beta,
$$

where $\alpha$ and $\beta$ are the complex amplitudes in the modes $\hat{c}$ and $\hat{d}$, respectively.

The signal, which is the difference in intensity at the output ports, is thus given by

$$
I = \int \int (|\alpha|^2 - |\beta|^2) W_f(\alpha, \beta) \, d^2\alpha \, d^2\beta
$$

and is found to be

$$
I = -2\sqrt{n_c n_{lo}} \cos \frac{\phi}{2} \cos \left( \frac{\phi}{2} + \phi_c - \phi_{lo} \right) - (n_c - n_s) \sin \phi.
$$

It is plotted in figure 6, as a function of $\phi$, under the condition $\phi_c = 0$, $\phi_{lo} = \pi/2$ and $n_{lo} = n_c (e^2 \sinh^{-1}(\sqrt{n_s}) + 1)^2$ (the condition when the phase sensitivity is found to be optimal, as
compares the phase sensitivity which is only slightly different from the optimal value of this can be expanded in a series as

\[ n \Delta \phi \] = \frac{2}{n_s + \sqrt{n_s + 2} \sqrt{n_s + 2} + 1} \left( \frac{2 \left(n_{in} + \sqrt{n_{in} + 2} \sqrt{n_{in} + 2} + 1 \right) }{n_{in} + \sqrt{n_{in} + 2} \sqrt{n_{in} + 2} + 1} \right).}

(21)

In the limit of large \( n_{in} \), namely the regime of interest of the Ono–Hofmann detection scheme, this can be expanded in a series as

\[ \Delta \phi^2 = \left( \frac{1}{n_{in}} \right)^2 - \frac{3}{2n_{in}^3} + O \left( \frac{1}{n_{in}} \right)^4. \]

(22)

The above expression for phase sensitivity \( \Delta \phi \) shows Heisenberg-limited scaling with the total number of photons \( n_{in} \) and thus proves that the Ono–Hofmann scheme provides Heisenberg-limited phase sensitivity as anticipated.

Figure 7 compares the phase sensitivity \( \Delta \phi \) with the Ono–Hofmann detection scheme for the cases corresponding to \( \eta = 0, \eta \approx 1 \) and \( \eta = 0.5 \). Similar to the results of parity detection, the case \( \eta = 0.5 \) with \( n_c = n_s = 5 \) provides sub-shot noise phase sensitivities up to accumulated

\[ \text{New Journal of Physics 13 (2011) 083026 (http://www.njp.org/)} \]
Figure 7. Phase sensitivity with the Ono–Hofmann detection scheme $\Delta \phi$, as a function of the accumulated phase between the arms of the MZI $\phi$: dashed (purple) line for coherent light interferometry ($\eta = 0$) with $n_c = 10$, $\phi_c = 0$, dotted (red) line for squeezed vacuum interferometry ($\eta \approx 1$) with $n_s = 9.9$, $n_c = 0.1$, $\phi_c = 0$ and solid (blue) line for coherent and squeezed vacuum light interferometry ($\eta = 0.5$) with $n_c = n_s = 5$, $\phi_c = 0$. A local oscillator of strength $n_{lo} = n_c(e^{2\sinh^{-1}(\sqrt{\eta})} + 1)^2$ and phase $\phi_{lo} = \pi/2$ is used in each case.

phases of about $\pm 0.3$ away from the optimum value of $\phi = \pi$, but the phase sensitivity diminishes beyond these values of accumulated phase. However, $\eta = 0$ provides a fairly constant phase sensitivity at about the shot-noise limit over a broader range of accumulated phases. (Note: although the phase sensitivity of the case $\eta \approx 1$ reaches below the shot-noise limit around $\phi = \pi$, it is found to deteriorate even faster than the case $\eta = 0.5$ as one moves away from $\phi = \pi$ and hence is not of much interest with the Ono–Hofmann detection scheme either.) Thus, very similar to what was suggested for parity detection, phase estimation with the Ono–Hofmann detection scheme may be best performed by starting with coherent light $\eta = 0$ and roughly learning the value of the accumulated phase, moving the accumulated phase closer to $\phi = \pi$ and then tuning up $\eta$ to 0.5 for improved phase sensitivity.

5. Summary

We have studied the application of parity detection for phase estimation in Mach–Zehnder interferometry with coherent light mixed with squeezed vacuum light. We have shown that parity detection saturates the quantum Cramer–Rao bound of the interferometric scheme and provides Heisenberg-limited phase sensitivity when coherent light and squeezed vacuum light are mixed in equal proportions. Parity can be readily implemented using photon-number-resolving detectors [17] in the low power regime and possibly using optical nonlinearities and homodyning in the high power regime [18–20]. We have also presented a brief study of a symmetric-logarithmic-derivative-based detection scheme proposed by Ono and Hofmann recently for the same interferometric scheme in the high power regime [21]. In general, symmetric logarithmic derivative operators are known to saturate the quantum Cramer–Rao bound. With the help of explicit calculations of the signal and phase sensitivity, we have
shown that this scheme indeed saturates the quantum Cramer–Rao bound and is thus capable of providing Heisenberg-limited phase sensitivity as well. By comparison, we have shown that the parity and the Ono–Hofmann detection scheme provide similarly good performances in phase estimation. This should offer experimentalists looking to implement interferometry with coherent and squeezed vacuum light more options on detection schemes to choose from.

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