Computability of Diagrammatic Theories for Normative Positions

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Matteo Pascucci
matteo.pascucci@savba.sk

Giovanni Sileno
g.sileno@uva.nl

Slovak Academy of Sciences

University of Amsterdam
Contributions of this work

- representing logical relations between normative positions using Aristotelian diagrams;
- drawing connections between various families of notions (e.g. different forms of power);
- building logical theories over diagrams which allow one to perform selected inferences on selected kinds of formulas (diagrammatic theories);
- providing an algorithm to decide whether a finite set of normative positions can be derived from another (i.e., a procedure to gain normative knowledge from a finite set of assumptions).
Why diagrams?

- diagrams exploit **symmetries**
  - symmetries facilitate perception and improve retention
    - diagrams are very good for *didactic purposes*
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- diagrams reify **conceptual patterns**
  - facilitate exploration/visualization of space of relevant concepts
  - support *reusable* (optimizable/optimized) *inferential patterns*
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Two types of diagrams are generally referred to when discussing about normative concepts: *squares of opposition* and *Hohfeldian squares*. 
Square of opposition (Aristotelian square)

- Logical construct described by Aristotle, centuries later represented in a diagrammatic form
- Related to syllogisms
- Abandoned with the advent of modern logic
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Yet used in semiotics, literary studies, etc. for conceptual mapping
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- Related to syllogisms
- Abandoned with the advent of modern logic
- Yet used in semiotics, literary studies, etc. for conceptual mapping
- Recent renewed interest on the formal and computational side
Deontic square of opposition

Obligatory to do

Forbidden to do

Permitted to do

Permitted not to do
Deontic square of opposition

- Obligatory to do
- Forbidden to do
- Permitted to do
- Permitted not to do

Relationships:
- Implication (implies)
- Contrary
- Contradictory
- Sub-contrary
- Sub-altern
Deontic square of opposition

Obligatory to do \( \rightarrow \) Forbidding to do

Permitted to do \( \rightarrow \) Permitted not to do

they cannot both be true

contrary

contradictory

sub-contrary

sub-altern

sub-altern

implies
Deontic square of opposition

Obligatory to do

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Permitted not to do

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sub-altern

sub-contrary

they cannot both be true
contrary

contradictory

one is true and the other false

implies
Deontic square of opposition

- **Obligatory to do** implies **Permitted to do**
- **Forbidden to do** implies **Permitted not to do**
- If the one above is true, the one below is true
- *they cannot both be true*
- Contrary
- Contradictory
- One is true and the other false
- Sub-altern
- Sub-contrary
Deontic square of opposition

- Obligatory to do
- Forbidden to do
- Permitted to do
- Permitted not to do

**Contrary**
- If the one above is true, the one below is true.
- They cannot both be true.
- If the one below is false, the one above is false.
- They cannot both be false.

**Contradictory**
- One is true and the other false.

**Sub-contrary**
- They cannot both be true.

**Sub-altern**
- If the one above is true, the one below is true.
- If the one below is false, the one above is false.
Deontic square of opposition

Obligatory to do \( \square \) Forbidden to do \( \neg \square \)
Permitted to do \( \top \) Contrary \( \bot \)

implies

\( Obl(A) \leftrightarrow Forb(\neg A) \)
\( Forb(A) \leftrightarrow \neg Perm(A) \)
\( Obl(A) \rightarrow Perm(A) \)

sub-altern

contrary

typical deontic relations support the square
Deontic square of opposition

Obligatory to do

Forbidden to do

Permitted to do

implies

contrary

sub-altern

contradictory

internal

negation

Obl(A) ↔ Forb(\bar{A})

Forb(A) ↔ \neg Perm(A)

Obl(A) → Perm(A)

typical deontic relations support the square
Deontic square of opposition

Obligatory to do \( \bullet \)  
Permitted to do \( \circ \)  
Forbidden to do \( \triangle \)  

Obl\( (A) \leftrightarrow Forb(\overline{A}) \)
Forb\( (A) \leftrightarrow \neg Perm(A) \)
Obl\( (A) \rightarrow Perm(A) \)

implies
contrary
sub-altern
contradictory

internal negation
external negation

typical deontic relations support the square
Hohfeldian squares

claim-holder  
Claim  
No-Claim

duty-holder  
Duty  
Liberty

power-holder  
Power  
Disability

subject to power  
Liability  
Immunity
From Hohfeldian to Aristotelian squares

- Aristotelian squares *directly encode* logical relations between statements (contrariety, contradiction, subalternation, sub-contrariety). Therefore, they can be used as a starting point to build simple logical theories.

- By contrast, the logical interpretation of Hohfeldian squares is not straightforward – see, e.g., the discussion in Andrews (1983) or Markovich (2020).

- Here we opt for some interpretations of Hohfeldian squares presented in Sileno (2016), Sileno & Pascucci (2020) and Pascucci & Sileno (2021). These include alternative analyses of the notion of power in terms of the notion of ability.
First-order Hohfeldian concepts

Ternary relations among two normative parties and an action type

CLAIM, NO-CLAIM, DUTY, LIBERTY

- Each of these can be taken as primitive and used to define the others
- Each choice of a primitive notion may give rise to an Aristotelian square.
First-order Hohfeldian concepts

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For instance, with claim:

\[
\text{NoClaim}(x, y, A) \equiv \neg \text{Claim}(x, y, A)
\]

\[
\text{Duty}(y, x, A) \equiv \text{Claim}(x, y, A)
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\]

If \( y \) has a duty-of-\( A \) towards \( x \), then \( y \) has no duty-of-not-\( A \) towards \( x \).
Second-order Hohfeldian concepts

Relations among two normative parties and an action type

POWER, LIABILITY, DISABILITY and IMMUNITY

- Each of these can be taken as primitive and used to define the others

For instance, with power:

\[
\text{Disability}(x, y, A) \equiv \neg \text{Power}(x, y, A)
\]
\[
\text{Liability}(y, x, A) \equiv \text{Power}(x, y, A)
\]
\[
\text{Immunity}(y, x, A) \equiv \neg \text{Power}(x, y, A)
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Ternary (?) relations among two normative parties and an action type

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\]

But then, how to give rise to an Aristotelian square from power?
Second-order Hohfeldian concepts

We will then increase the granularity, relying on a definition of power based on the concept of **ability** -- for possible semantics see e.g. Sileno et al. (2019) [logic programming and event-calculus], or Sileno and Pascucci (2020) [modal logic]:
Second-order Hohfeldian concepts

We will then increase the granularity, relying on a definition of power based on the concept of ability -- for possible semantics see e.g. Sileno et al. (2019) [logic programming and event-calculus], or Sileno and Pascucci (2020) [modal logic]:

“Canonic” form of power: the ability or competence to create a claim/duty

\[ \text{Power}(x, y, A) \equiv \exists \beta : \text{Ability}(x, \beta, \text{Claim}(x, y, A)) \]

- agent
- action-type
- stimulus
- configuration
- manifestation
Second-order Hohfeldian concepts

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"Canonic" form of power: the ability or competence to create a claim/duty

\[ \text{Power}(x, y, A) \equiv \exists \beta : \text{Ability}(x, \beta, \text{Claim}(x, y, A)) \]

...we can now individuate distinct forms of power and build the corresponding Aristotelian squares.
Outcome-centered power

The notion of power at its core is centered around the outcome produced.

We can distinguish between the power to issue a duty (canonic power) and the power to release from a duty.
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\text{Power}(x, y, A) \equiv \exists \beta : \text{Ability}(x, \beta, \text{Claim}(x, y, A))
\]

\[
\overline{\text{Power}}(x, y, A) \equiv \exists \beta : \text{Ability}(x, \beta, \neg \text{Claim}(x, y, A))
\]

sub-alternation relation

If \( x \) is able to create \( y \)'s duty-of-\( A \), \( x \) is not able to release \( y \)'s duty-of-\( A \).
Change-centered power

First analysed in a rigorous way by O’Reilly (1995).

The notion of power at its core concerns the ability of a normative party \( p \) to affect another normative party \( q \) with respect to a certain relation \( R \). We redefined it using ability...
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Focusing on “canonic” power, \( R \) is about a duty

\[
\text{Power}_{\text{O'Reilly}}(x, y, B, A) \equiv \text{Ability}(x, B, \text{Claim}(x, y, A)) \\
\lor \text{Ability}(x, B, \text{Claim}(x, y, \overline{A})) \\
\lor \text{Ability}(x, B, \neg \text{Claim}(x, y, A)) \\
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\[
\text{Power}^+_{\text{OReilly}}(x, y, B, A) \equiv \exists \beta : \text{Power}^+_{\text{OReilly}}(x, y, \beta, A) \quad \text{The agent can do something changing } R
\]

\[
\text{Power}^-_{\text{OReilly}}(x, y, B, A) \equiv \exists \beta : \neg \text{Power}^+_{\text{OReilly}}(x, y, \beta, A) \quad \text{The agent can do something without changing } R
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\]

If $x$ is not able to not affect a $y$’s duty-of-$A$, then $x$ is able to affect it.

\[
\text{Power}^+(x, y, A) \equiv \exists \beta : \text{Power}_{\text{O'Reilly}}(x, y, \beta, A)
\]
\[
\text{Power}^-(x, y, A) \equiv \exists \beta : \neg \text{Power}_{\text{O'Reilly}}(x, y, \beta, A)
\]
\[
\neg \text{Power}^-(x, y, A) \rightarrow \text{Power}^+(x, y, A)
\]
Force-centered power

First observed in Sileno et al. (2014): the notion of power can be put in analogy to physical notions as *attraction* and *repulsion* towards a certain relation.

- **positive-force power**: to attract *[create a duty to perform]* a certain action type A
- **negative-force power**: to repel *[create a prohibition to perform]* a certain action type A.
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\begin{align*}
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\]

same stimulus opposite manifestations
The Dutch Declaration of Independence: Act of Abjuration (1581)

“Know all men by these presents [...] we have unanimously and deliberately declared [...] that the King of Spain has forfeited, ipso jure, all hereditary right to the sovereignty of those countries, and [they] are determined from henceforward not to acknowledge his sovereignty or jurisdiction [...] nor suffer others to do it.
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[we will punish who follows the orders of the King of Spain → the King has a negative-force power]
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\end{align*}
\]

*sub-alternation relation*

\[
\begin{align*}
\text{Power}(x, y, A) &\rightarrow \neg \text{Power}(x, y, A)
\end{align*}
\]

*internal negation*

*If* x *is able to create* y’s duty-of-A *by commanding* A, *x is not able to create* y’s duty-of-not-A *by the same act.*
A collection of squares of opposition
A map of potestative relations

- Aristotelian diagrams can be expanded and combined among them by adding further relations of contrariety, sub-contrariety, contradiction and subalternation.
- For instance, putting together the three squares for power, and expanding the outcome-centered one to an *hexagon*, we get a complex diagram showing connections between the various senses of power.
A map of potestative relations
Further connections can be drawn, enabling one to form 3D maps...

A map of potestative relations
Building diagrammatic theories

- We can define logical theories based on an Aristotelian diagram, and will name these *diagrammatic theories*;
- a diagrammatic theory $\text{DT}$ over a diagram $D$ encodes (at least) all logical relations among formulas used as labels in $D$;
- a diagrammatic theory will be presented as a set of *inference trees*, which capture selected instances of the consequence relation in a logical system.
Inference tree

Basic idea

Given a set of assumptions $\Delta$, an inference tree $T$ indicates which selected inferences can be performed from $\Delta$ so as to obtain a larger set $\Gamma$.

One locates $\Delta$ at some node $n$ of a tree $T$ and inspects the subsequent nodes.
**Inference tree**

**Set-inference**

Σ can be **inferred** from Δ in a branch b of a tree T iff Σ ⊆ Γ for some Γ that occurs below Δ in b.

When this is the case for some branch b of a tree T, we say that T **allows one to infer** Σ from Δ.
Set-derivation

Σ can be derived from Δ in a tree T iff for every branch $b$ of T, Σ can be inferred from Δ in $b$.

When this is the case for some tree T in a diagrammatic theory DT, we say that DT allows one to derive Σ from Δ.
Decidability: algorithm

We designed an algorithm to decide whether, for any finite set of formulas $\Gamma$ and $\Delta$ and any diagrammatic theory $\textbf{DT}$, $\textbf{DT}$ allows one to derive $\Gamma$ from $\Delta$.

The algorithm consists of two steps:

1. compare the two sets $\Gamma$ and $\Delta$ in order to determine whether one is a subset of the other or not.

2. consider the set $\Gamma - \Delta$ and perform procedures called traversals with respect to the trees of $\textbf{DT}$. 
Decidability: tree traversal

The traversal of a tree $T$ with reference to a formula $\varphi$ and a set $\Delta$ can be described as follows (we assume that $\Delta$ occupies the root of $T$):

- Following the order of ranks, for any set of formulas $\Gamma$ with rank $i$ in $T$, we compare $\varphi$ with all formulas in $\Gamma$ and keep track of whether $\varphi$ occurs in $\Gamma$ or not.
- The procedure terminates when either (positive outcome) there is a rank $j$ s.t. all sets of formulas with rank $j$ include $\varphi$ or (negative outcome) all sets of formulas with all ranks available in $T$ have been checked.
Decidability: theory traversal

- The traversal of a diagrammatic theory $\text{DT}$ with reference to a formula $\varphi$ and a set of formulas $\Delta$ is the traversal of all trees $T$ in $\text{DT}$ with reference to $\varphi$ and $\Delta$. The outcome is positive iff it is positive for some $T$ in $\text{DT}$.
Decidability: theory traversal

- The traversal of a diagrammatic theory $\mathbf{DT}$ with reference to a formula $\phi$ and a set of formulas $\Delta$ is the traversal of all trees $T$ in $\mathbf{DT}$ with reference to $\phi$ and $\Delta$. The outcome is positive iff it is positive for some $T$ in $\mathbf{DT}$.

Complexity of the whole algorithm

- The designed algorithm takes polynomial time with respect to $\max(|\Gamma, \Delta|)$. 
Conclusion

- We formalized and systematized previous contributions representing normative positions in Aristotelian diagrams. We showed how one can build simple logical theories based on Aristotelian diagrams via inference trees.

- We provided an algorithm for finite-sets-derivability-checking tailored on diagrammatic theories (hence, capturing only relevant instances of the consequence relation associated with a logical system).

- One of the main features of our approach is that we do not need the full deductive power of a logical system, since we only deal with formulas and inferences of a selected kind. In future work we will compare our approach with more general deductive approaches.
Work in progress...

- Intuitively, diagrams have also a strong potential for designing visualization interfaces. For instance, to “navigate” contracts as we do with molecules in chemistry. This remains to be further evaluated.
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Matteo Pascucci
datteo.pascucci@savba.sk

Slovak Academy of Sciences

Giovanni Sileno
g.sileno@uva.nl

University of Amsterdam