On non-linear superfield versions of the vector-tensor multiplet

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Abstract

We propose a harmonic superspace description of the non-linear vector-tensor $N = 2$ multiplet. We show that there exist two inequivalent versions: the old one in which one of the vectors is the field-strength of a gauge two-form, and a new one in which this vector satisfies a non-linear constraint and cannot be expressed in terms of a potential. In this the new version resembles the non-linear $N = 2$ multiplet. We perform the dualization of both non-linear versions in terms of a vector gauge multiplet and discuss the resulting holomorphic potentials. Finally, we couple the non-linear vector-tensor multiplet to an abelian background gauge multiplet.
1 Introduction

Recently, there was a revival of interest in the $N = 2$ vector-tensor (VT) multiplet [1], mainly due to the fact that it describes the axion/dilaton complex in heterotic $N = 2$ four-dimensional supersymmetric string vacua [2]. The VT multiplet is a variant representation of the $N = 2$ vector multiplet, such that one of the physical scalars of the latter is traded for a gauge antisymmetric tensor (notoph) off shell. The known off-shell formulation of the VT multiplet (8 + 8 components) necessarily implies the presence of a central charge in the $N = 2$ superalgebra. It is real and acts on the component fields in a highly non-trivial way. As was observed in [3], [4], there exist at least two different versions of the VT multiplet. Their basic difference is in the coupling of the tensor and vector gauge fields: in the so-called “non-linear” version the tensor field couples to the Chern-Simons (CS) form of the vector one, while no such CS coupling is present in the “linear” version. $N = 2$ supersymmetry is realized in these two cases in essentially different ways: non-linearly in the first case and linearly in the second one. The two versions also radically differ in what concerns couplings to background $N = 2$ vector multiplets and $N = 2$ supergravity [4, 5]. Note that the central charge transformations can be global or local. In the latter case an extra vector multiplet gauging the central charge should be introduced from the beginning. When coupling the VT multiplet to supergravity, the central charge is necessarily gauged.

An exhaustive component analysis of the two versions of the VT multiplet together with their couplings to background vector multiplets was given in [3], [4], assuming that the central charge transformations are local. As for the superfield formulations of the VT multiplet (which are most natural when dealing with off-shell supermultiplets), until recently they were constructed only for the linear version, both in the free case and in the presence of couplings to background vector multiplets [6], [7], [8], [9]. There exist formulations in the standard [6] - [8] as well as in harmonic [9] $N = 2$ superspaces [1].

Our purpose in this letter is to give such a formulation for the non-linear version, both for the pure VT multiplet and for the case when CS couplings to the background vector multiplets are switched on. We make use of the harmonic superspace (HSS) approach as most adequate to $N = 2$ supersymmetry and demonstrate the existence of two inequivalent non-linear versions of the VT multiplet. The first one is just the version discovered in [3], [4], while the second is essentially new: it cannot be reduced to the “old” one by any field redefinition. Its most characteristic feature is the modification of the r.h.s. of the Bianchi identity for the three-form (the field strength of the tensor gauge field) by terms quadratic in the latter and in the auxiliary fields. As a result, the Bianchi identity has no local solution in terms of a tensor gauge potential (note that one of the primary assumptions of [3], [4] is the existence of such a potential). We show that the bosonic action of this new version of the VT multiplet vanishes as a consequence of the modified Bianchi identity for the three-form. Nevertheless, a non-trivial action is obtained upon dualization, i.e. after implementing this identity in the action with a scalar Lagrange multiplier. In this aspect, the situation is quite similar to the case of non-linear $N = 2$ multiplet [1]. The dual action exhibits all the features of special

\footnote{When this work was near completion, we became aware of the parallel work [10] where a harmonic superspace formulation of the non-linear version of the VT multiplet was given (at the level of rigid central charge and without considering CS couplings to extra vector multiplets).}
Kähler geometry typical of the actions of \( N = 2 \) vector multiplets and is fully specified by a non-polynomial holomorphic potential. We propose a manifestly supersymmetric version of the dualization procedure.

Here we restrict our study to the rigid case, postponing the discussion of the gauged central charge and, more generally, \( N = 2 \) supergravity to a future publication. Also, when discussing the superfield CS couplings of the non-linear version of the VT multiplet to background vector multiplets, for the sake of simplicity we consider one abelian multiplet coupled to the “old” non-linear version of the VT multiplet. The generalization to an arbitrary number of background vector multiplets, both abelian and non-abelian, as well as to the case of the “new” non-linear version is rather straightforward and will be presented elsewhere.

2 Preliminaries

Let us first briefly recall some facts about the HSS description of the linear version of the VT multiplet, to a large extent following ref. [9]. We assume the reader to be familiar with the basic concepts of the harmonic superspace approach to \( N = 2 \) supersymmetry; otherwise, we invite him to consult the original papers [12, 13] or the (yet) unpublished review [14].

The basic object of such a description is the real harmonic superfield

\[
L = L(x^{\alpha\dot{\alpha}}, x^5, \theta^{\pm\alpha}, \bar{\theta}^{\pm\dot{\alpha}}, u^{\pm i})
\]

subject to the constraints

\[
\begin{align*}
(D^+)^2 L &= (\bar{D}^+)^2 L = 0, \\
D_\alpha^+ \bar{D}^{+\dot{\alpha}} L &= 0, \\
D^{++} L &= 0.
\end{align*}
\]

Here

\[
\begin{align*}
u^+ u^- &= 1, \quad \theta^{\pm\alpha} = \theta^{\alpha i} u_i^+, \quad \bar{\theta}^{\pm\dot{\alpha}} = \bar{\theta}^{\dot{\alpha} i} u_i^+, \\
D^+ &= \frac{\partial}{\partial \theta^{-\alpha}} \equiv \partial_{-\alpha}, \quad D^{+\dot{\alpha}} = \frac{\partial}{\partial \bar{\theta}^{-\dot{\alpha}}} \equiv \partial_{-\dot{\alpha}}, \\
D^{++} &= u^i \frac{\partial}{\partial u^{-i}} - 2i \theta^{+\alpha} \bar{\theta}^{+\dot{\alpha}} \partial_{\alpha\dot{\alpha}} + i((\theta^+)^2 - (\bar{\theta}^+)^2) \partial_5 + \theta^{+\alpha} \partial_{-\alpha} + \bar{\theta}^{+\dot{\alpha}} \bar{\partial}_{-\dot{\alpha}}
\end{align*}
\]

are the basic quantities of the central-charge extended HSS in the analytic basis. This basis is chosen so that the covariant spinor derivatives \( \tilde{D}^{+\bar{\alpha}}, \tilde{D}^{+\dot{\alpha}} \) are “short” and the coordinate sets

\[
\zeta^5 \equiv \{x^{\alpha\dot{\alpha}}, x^5, \theta^{+\alpha}, \bar{\theta}^{+\dot{\alpha}}, u^{\pm i}\}
\]

and

\[
\zeta \equiv \{x^{\alpha\dot{\alpha}}, \theta^{+\alpha}, \bar{\theta}^{+\dot{\alpha}}, u^{\pm i}\}
\]

are closed under the \( N = 2 \) supersymmetry transformations. They are called analytic subspaces. The harmonic derivative \( D^{++} \) commutes with \( D^{+\alpha}, \tilde{D}^{+\bar{\alpha}} \), and so it preserves analyticity. In what follows we will always use the analytic basis in \( N = 2 \) HSS. Note that the subspaces
are real, i.e. closed under some generalized conjugation. Our conventions are those of ref. [12].

The set of constraints (2.1) - (2.3) reduces the infinite component content of $L$ to that of the off-shell linear VT multiplet when formulated via the field strengths of the notoph and vector gauge potentials.

It is convenient to represent $L$ by its analytic components, i.e. by the functions on the subspace $\zeta^5$ which appear in the decomposition of $L$ in powers of $\theta^{-\alpha}, \bar{\theta}^{-\dot{\alpha}}$. The constraints (2.1), (2.2) imply

$$L = l(\zeta^5) + \theta^{-f} + \bar{\theta}^{-\bar{f}} + (\zeta^5)$$

(here and in what follows the spinor indices are contracted according to the rule $^{\alpha\dot{\alpha}}$), while (2.3) leads to the two harmonic constraints

$$D^{++}l + \theta^{+} + \bar{\theta}^{+\bar{f}} = 0,$$  

$$D^{++}\bar{f} = 0.$$  

Thus an equivalent description of the VT multiplet is given in terms of the analytic scalar and spinor superfunctions $l, f^{+}, \bar{f}^{+}$ subject to the harmonic constraints (2.8), (2.9). Note that $f^{+}$ is transformed under $N = 2$ supersymmetry as a standard analytic superfield while $l$ has unusual transformation properties:

$$\delta f^{+} = 0, \quad \delta l = -\epsilon_i u_i^{--} f^{+} - \bar{\epsilon}_i \bar{u}_i^{--} \bar{f}^{+}$$

where $\epsilon_i, \bar{\epsilon}_i$ are infinitesimal transformation parameters.

Eqs. (2.8), (2.9) fully determine the action of the central charge generator $\partial/\partial x^5$ on the component fields in $l, f^{+}$. In what follows it will be more convenient to define its action directly on the analytic quantities $l, f^{+}$. This can be done using the following trick. As a consequence of the harmonic condition (2.3) we have

$$D^{-+}L = 0,$$  

where $D^{-+}$ is the harmonic derivative conjugate (in the usual sense) to $D^{++}$

$$D^{-+} = u^{-i} \frac{\partial}{\partial u^{+i}} - 2i\theta^{-\alpha} \bar{\theta}^{-\dot{\alpha}} \partial_{\alpha\dot{\alpha}} + i((\theta^{-})^2 - (\bar{\theta}^{-})^2) \partial_5 + \theta^{-\alpha} \partial_{+\alpha} + \bar{\theta}^{-\dot{\alpha}} \bar{\partial}_{+\dot{\alpha}}$$

(it does not preserve analyticity!). Together with $D^{++}$ they form the $SU(2)$ algebra of harmonic derivatives:

$$[D^{++}, D^{-+}] = D^0, \quad [D^0, D^{\pm\pm}] = \pm 2D^{\pm\pm},$$

where $D^0$ is the operator counting the harmonic $U(1)$ charge ($D^0 l = 0, \ D^0 f^+ = f^+$). Substituting (2.7) into (2.10) and equating to zero the coefficients in front of the various

\footnote{At present it is unclear, even at this simplest linear level, what could be (if existing!) the HSS description of the VT multiplet in terms of superfield potentials.}
powers of $\theta^-, \bar{\theta}^-$, we find the set of constraints:

\[ \partial^{-} l = 0, \]
\[ \partial_{+\alpha} l + \partial^{-} f^{+}_{\alpha} = 0, \quad \partial_{+\dot{\alpha}} l - \partial^{-} \bar{f}^{+}_{\dot{\alpha}} = 0, \]
\[ \partial_{\alpha\dot{\alpha}} l + \frac{i}{2} (\partial_{+\alpha} \bar{f}^{+}_{\dot{\alpha}} + \partial_{+\dot{\alpha}} f^{+}_{\alpha}) = 0, \]
\[ \partial_{5} l - \frac{i}{2} \partial_{+\alpha} f^{+\alpha} = 0, \quad \partial_{5} l + \frac{i}{2} \partial_{+\dot{\alpha}} \bar{f}^{+\dot{\alpha}} = 0, \]
\[ \partial_{5} f^{+}_{\alpha} - \partial_{a\dot{\alpha}} \bar{f}^{+\dot{\alpha}} = 0, \quad \partial_{5} \bar{f}^{+}_{\dot{\alpha}} + \partial_{a\alpha} f^{+\alpha} = 0, \]

where $\partial^{-} = u^{-i} \partial / \partial u^{+i}$. An important corollary of eqs. (2.16) is the reality condition

\[ \partial_{+\alpha} f^{+\alpha} + \partial_{+\dot{\alpha}} \bar{f}^{+\dot{\alpha}} = 0. \]

Introducing

\[ D^{-}_{\alpha} = [D^{+}_{\alpha}, D^{-}] , \quad \bar{D}^{-}_{\dot{\alpha}} = [\bar{D}^{+}_{\dot{\alpha}}, D^{-}] , \]

it is easy to show that another form of (2.18) is

\[ D^{-\alpha} f^{+}_{\alpha} = \bar{D}^{-\dot{\alpha}} \bar{f}^{+\dot{\alpha}}, \]

which is just the reality condition of ref. [9]. In our approach it is clear that this condition is a direct consequence of the choice of a real central charge (had we chosen $x^5$ to be complex, the two eqs. (2.16) would be independent and no relation of the sort (2.18) would arise).

Combining relations (2.13) - (2.17) with eqs. (2.8), (2.9), it is easy to find out the irreducible field content of $l$, $f^+$ and to show that it exactly coincides with that of the linear version of the VT multiplet:

\[ l| = \phi(x, x^5), \quad \partial_{5} \phi \equiv G(x, x^5), \quad f^{+}_{\alpha}| = f^{i}_{\alpha}(x, x^5) u^{+i}, \]
\[ \partial_{+\beta} f^{+}_{\alpha}| = F_{(\beta\alpha)}(x, x^5) + i \epsilon_{\beta\alpha} G(x, x^5), \quad \partial_{+\dot{\alpha}} f^{+}_{\beta}| = h_{\beta\dot{\alpha}}(x, x^5) + i \partial_{\beta\dot{\alpha}} \phi(x, x^5) \]

where $|$ means restriction to the lowest component of a given superfield. After simple algebraic manipulations involving the above constraints, all other components, including those obtained by acting on (2.21) with $\partial_{5}$, are expressed as $x$-derivatives of the basic quantities (2.21). For instance,

\[ \partial_{5} G = \frac{1}{2} \partial^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \phi. \]

The Bianchi identities for $F_{(\alpha\beta)}$, $h_{\alpha\dot{\alpha}}$ also directly follow from the constraints. For instance, acting by $\partial_{5}$ on the reality condition (2.18) and on eq. (2.15), and making use of eqs. (2.17) afterwards, one gets, respectively,

\[ \partial \cdot h = 0, \]

and

\[ \partial_{\alpha\dot{\alpha}} \bar{F}^{\dot{\beta}}_{(\dot{\alpha})} - \partial_{\beta\dot{\alpha}} F^{(\beta)}_{(\alpha)} = 0, \]

which are the Bianchi identities for the notoph and vector gauge field strengths.
In ref. [9] there was proposed a nice general recipe of constructing HSS actions for supermultiplets with a non-trivial realization of the central charge, such that they are still given by integrals over the standard analytic subspace (2.6) containing no $x^5$ coordinate. The action is given by the general formula

$$S = \int d\zeta^{-4} [((\theta^+)^2 - (\bar{\theta}^+)^2)]L^{++}$$

where $d\zeta^{-4} \equiv du^4 dx^4 d\theta^+$. The real Lagrangian density $L^{++}$ should be:

(i) analytic:

$$D^a_+ L^{++} = \bar{D}^a_+ L^{++} = 0.$$  \hspace{1cm} (2.26)

(ii) harmonically “short”:

$$D^{++} L^{++} = 0.$$  \hspace{1cm} (2.27)

The second condition immediately leads to the important property that the $x^5$ derivative of the integrand in (2.25) is a total $x$ and $u$-derivative (recall (2.4)) and so disappears upon integration. As a result the action (2.25) does not depend on $x^5$ or, to put it differently, is invariant under central charge transformations. The $N = 2$ supersymmetry of (2.25) is not manifest, but can be easily checked (see [9]).

In the case under consideration two Lagrangian densities of this sort exist [9]:

$$L^{++}_1 = i(D^+ LD^+ L - D^+ L\bar{D}^+ L) = i(f^+ f^- - \bar{f}^+ \bar{f}^-),$$

$$L^{++}_2 = (D^+ LD^+ L + \bar{D}^+ L\bar{D}^+ L) = (f^+ f^- + \bar{f}^+ \bar{f}^-).$$

The first density gives the free action of the linear VT multiplet. The second one is a total $x$-derivative, i.e. gives a topological invariant. Both of them, as well as the defining constraints (2.1) - (2.3), can be generalized to include CS couplings to external $N = 2$ vector gauge multiplets. These extensions were given in [9]. We will return to this point in Section 4.

\section{Non-linear VT multiplets}

As was already mentioned, a characteristic feature of the non-linear version of the VT multiplet discovered in [3, 4] is the presence of CS coupling-induced terms of the vector gauge field in the Bianchi identity for the notoph gauge field strength. A simple analysis shows that a minimal way to obtain such terms in the HSS description is to modify the linear VT multiplet constraints as follows

$$\begin{align*}
(D^+)^2 L &= \alpha(L)D^+ LD^+ L + \beta(L)\bar{D}^+ L\bar{D}^+ L \hspace{1cm} (3.1) \\
D^+_a \bar{D}^+_\bar{a} L &= \gamma(L)D^+_a L\bar{D}^+_\bar{a} L, \hspace{1cm} (3.2) \\
D^{++} L &= 0, \hspace{1cm} (3.3)
\end{align*}$$

with $\alpha(L), \beta(L)$ being complex and $\gamma(L) = \bar{\gamma}(L)$ real functions of $L$, arbitrary for the moment. Note that (3.1), (3.2) provide the most general deformation of the linear constraints (2.1), (2.2)

\footnote{Note that a similar HSS action is used to describe the massive central-charged hypermultiplet [14].}
consistent with the preservation of the harmonic \( U(1) \) charge and the harmonic condition \((3.3)\).
It is worth mentioning that in principle the latter can also be deformed by adding appropriate bilinears of \( D^+_\alpha, \bar{D}^+_\bar{\alpha} \) into its r.h.s. We do not consider such non-minimal possibilities here.

The constraints \((3.1)\), \((3.2)\) should satisfy the evident self-consistency conditions
\[
D^+\alpha(D^+)^2L = 0, \quad \bar{D}^+\bar{\alpha}(D^+)^2L = D^+\alpha(D^+\bar{D}^+\bar{\alpha})L, \quad (3.4)
\]
which amount to the following set of differential equations for the coefficients
\[
(\gamma - \alpha)' = (\alpha - \gamma)\gamma - \beta\bar{\beta}, \quad (3.5)
\]
\[
\beta' = (\alpha - 2\gamma)\beta. \quad (3.6)
\]
Thus we have four real differential equations for five real functions. However, we are actually dealing with four unknowns due to the reparametrization freedom
\[
L \to \tilde{L}, \quad L = L(\tilde{L}) \quad (3.7)
\]
in \((3.5)\), \((3.6)\). Under such reparametrizations the coefficients transform as follows:
\[
\alpha \to \tilde{\alpha} = L'\alpha - (\ln L)' \quad , \quad \beta \to \tilde{\beta} = L'\beta \quad , \quad \gamma \to \tilde{\gamma} = L'\gamma - (\ln L)' \quad . \quad (3.8)
\]
We can choose different gauges with respect to \((3.8)\) in order to simplify the set \((3.5)\), \((3.6)\). A very convenient gauge amounts to choosing
\[
\gamma = 0 \quad (3.9)
\]
which implies
\[
\alpha' = \beta\bar{\beta}, \quad \beta' = \alpha\beta. \quad (3.10)
\]
In this gauge the constraints \((3.1)\)-(3.2) become simpler:
\[
(D^+)^2L = \alpha D^+LD^+L + \beta D^+LD^+L \quad , \quad D^+\alpha D^+\bar{\alpha}L = 0. \quad (3.11)
\]
The main advantage of the constraints in the form \((3.11)\) is that there appear no mixed terms in the \( \theta^-\tilde{\theta}, \tilde{\theta}^-\bar{\theta} \) expansion of \( L \). Indeed, the solution to the second of eqs. \((3.11)\) is (cf \((2.7)\) in the linear case):
\[
L = l + \theta^-f^+ + \tilde{\theta}^-\bar{f}^+ - \frac{1}{4}(\theta^-)^2[\alpha(f^+)^2 + \beta(\bar{f}^+)^2] - \frac{1}{4}(\bar{\theta}^-)^2[\bar{\alpha}(\bar{f}^+)^2 + \bar{\beta}(f^+)^2]. \quad (3.12)
\]
It is easy to find the general solution to the equations \((3.10)\), but before doing this, we point out that additional restrictions on the coefficient functions \( \alpha, \beta \) come from the harmonic condition \((2.10)\). Applying the reasoning which lead to eqs. \((2.13)\) - \((2.17)\), one finds the analogs of the latter for the non-linear case. Eqs. \((2.13)\), \((2.14)\) preserve their form, while those from \((2.13)\) on are modified by non-linear terms:
\[
\partial^-l = 0, \quad \partial^-f^+ + \partial_+\alpha l = 0, \quad \partial^-\bar{f}^+ - \partial_+\bar{\alpha}l = 0, \quad (3.13)
\]
\[ \partial_{\bar{a}} l + \frac{i}{2} (\partial_{+a} f^+_a + \partial_{+\bar{a}} \bar{f}^+_a) = 0 , \quad (3.15) \]
\[ \partial_5 l - \frac{i}{2} (\partial_{+a} f^+_a - \alpha \partial^- f^+ f^- - \bar{\alpha} \partial^- \bar{f}^+ \bar{f}^+) = 0 \quad \text{and c.c.} \quad (3.16) \]
\[ \partial_5 f^+_a - \partial_{a\bar{a}} \bar{f}^+_a + \frac{i}{2} (\alpha \partial_{+a} f^+ f^- + \bar{\alpha} \partial_{+\bar{a}} \bar{f}^+ \bar{f}^+) \]
\[ + \frac{i}{4} \alpha \bar{\beta} \partial^- f^+_a [(f^+)^2 + (\bar{f}^+)^2] = 0 \quad \text{and c.c.} \quad (3.17) \]

A new phenomenon in the non-linear case is the appearance of a new self-consistency condition as a result of equating to zero the coefficient of the monomial \((\theta^-)^2(\bar{\theta}^-)^2\) in (2.10). It reads
\[ \partial_5 [(\alpha - \bar{\beta})(f^+)^2 + (\beta - \bar{\alpha})(\bar{f}^+)^2] = 0 . \quad (3.18) \]

Working out the derivative \(\partial_5\) and expressing \(\partial_5 l, \partial_5 f^+_a, \partial_5 \bar{f}^+_a\) from eqs. (3.16), (3.17), we see that there appear unacceptable algebraic constraints on the fermions \(f^+_a\), unless we demand
\[ \alpha = \bar{\beta} . \quad (3.19) \]

This new constraint, together with (3.10), imply
\[ \alpha' = \alpha \bar{\alpha} . \quad (3.20) \]

Putting \(\alpha = a + ib\) in (3.20), we find
\[ b = \text{const} \]
and
\[ a' = a^2 + b^2 . \quad (3.21) \]

The solution to the differential equation (3.21) depends on the value of the constant \(b\). If \(b \neq 0\), one obtains
\[ \alpha = b [\tan(L + c) + i] , \quad (3.22) \]
where \(c\) is a new integration constant; if \(b = 0\), the solution is
\[ \alpha = -\frac{1}{L + c} . \quad (3.23) \]

Note that after choosing the gauge (3.9) we still have the freedom of global rescalings and shifts of \(L\). Using this, we can fix the constants \(b, c\) in (3.22) or (3.23), for example, \(b = 1, c = 0\). Thus, in the gauge (3.9) we obtain two distinct solutions:
\[ (i) \ \alpha = \tan L + i; \quad (ii) \ \alpha = -\frac{1}{L} . \quad (3.24) \]

They give rise to two inequivalent versions of the non-linear VT multiplet (remember that we have already exhausted the freedom of redefinition of \(L\)).

The principle difference between these two versions is in the following. It is easy to deduce the analogs of the Bianchi identities (2.23), (2.24) for both non-linear versions. Eq. (2.24)
does not change, implying that $F_{(\alpha\beta)}$, $F_{(\dot{\alpha}\dot{\beta})}$ are still expressed in the standard way through the vector gauge potential. At the same time, the identity (2.23) is drastically modified:

$$\partial \cdot h + \frac{i}{4} (\alpha F^2 - \bar{\alpha} \bar{F}^2) + \frac{i}{4}(\alpha - \bar{\alpha}) \left[ h^2 - (\partial \phi)^2 - 2G^2 \right] - \frac{1}{2}(\alpha + \bar{\alpha}) \partial \phi \cdot h = 0 \, ,$$  \hspace{1cm} (3.25)

where

$$F^2 = F^{\alpha\beta} F_{\alpha\beta} \, , \quad \bar{F}^2 = F^{\dot{\alpha}\dot{\beta}} F_{\dot{\alpha}\dot{\beta}}$$

and $\alpha = \alpha(\phi)$. For the second solution (ii) in (3.24) $\alpha = \bar{\alpha} = -1/\phi$, so after the redefinition

$$h^{\alpha\dot{\alpha}} \rightarrow \bar{h}^{\alpha\dot{\alpha}} = \phi h^{\alpha\dot{\alpha}}$$  \hspace{1cm} (3.26)

one gets the standard CS-term-modified Bianchi identity for $\bar{h}$

$$\partial \cdot \bar{h} - \frac{i}{4} (F^2 - \bar{F}^2) = 0 \, .$$  \hspace{1cm} (3.27)

It can still be solved through the antisymmetric gauge field (notoph) after an appropriate shift of $\bar{h}^{\alpha\dot{\alpha}}$ by the CS one-form. This means that the solution (ii) in (3.24) corresponds just to the non-linear version of the VT multiplet discovered in [3, 4]. At the same time, there is no way to reduce (3.25) to (3.27) in the new case corresponding to the solution (i) in (3.24). There it is impossible to solve the identity (3.25) through a notoph potential (at least, locally), though we still end up with $8 + 8$ off-shell degrees of freedom. Thus we encounter an essentially new version of the VT multiplet in this case.

It is easy to find the analogs of the free actions (2.28), (2.29) for both non-linear versions at hand. One starts from the Ansatz

$$L^{++} = A(L) \, D^+ L D^+ L + \bar{A}(L) \, \bar{D}^+ \bar{L} \bar{D}^+ L$$  \hspace{1cm} (3.28)

and solves the differential equations for $A, \bar{A}$ following from the analyticity constraint (2.26). In both cases (i), (ii) in (3.24) we get in this way two-parameter solutions for $A(L)$:

(i) $A(L) = d_1 (\tan L + i) + d_2 \left[ 1 + L(\tan L + i) \right]$  \hspace{1cm} (3.29)

(ii) $A(L) = g_1 \frac{1}{L} + ig_2 L$  \hspace{1cm} (3.30)

where $d_{1,2}, g_{1,2}$ are arbitrary real constants.

The explicit form of the superfield Lagrangian density (3.28) in the most interesting case of the new solution (i) is

$$L^{++} = d_1 (D^+)^2 L + d_2 \left[ D^+ L D^+ L + \bar{D}^+ \bar{L} \bar{D}^+ L + L(D^+)^2 L \right] .$$  \hspace{1cm} (3.31)

The relation between these two non-linear versions of the VT multiplet resembles that between the two well-known multiplets of $N = 2$ supersymmetry without central charge, the tensor [12] and non-linear [14] ones. Both of them have the same number of off-shell degrees of freedom and in both cases there is a constraint on the vector component. In the case of the tensor multiplet this constraint is of the notoph type (2.23) and it can be locally solved through the notoph potential. In the case of the non-linear multiplet the constraint is modified and resembles (3.25) (it also contains terms bilinear in the vector field strength in its r.h.s.). No local solution to this modified constraint in terms of a gauge potential can be given.
It is instructive to work out the component bosonic Lagrangian corresponding to (3.31). As a preparatory step it is convenient to redefine $h^\alpha \dot{\alpha}$ as follows

$$h^\alpha \dot{\alpha} = \frac{1}{\cos \phi} \tilde{h}^\alpha \dot{\alpha},$$

(3.32)

$$\partial \cdot \tilde{h} + \frac{1}{4} \left( e^{i\phi} F^2 + e^{-i\phi} \bar{F}^2 \right) + \frac{1}{2 \cos \phi} \tilde{h}^2 - \frac{1}{2} \cos \phi \left[ (\partial \phi)^2 + 2G^2 \right] = 0 .$$

(3.33)

Then a straightforward computation yields (up to an overall normalization constant, modulo a total $x$-derivative and after putting the auxiliary field $G = 0$)

$$\mathcal{L}_{\text{bos}} = v(\phi) \left\{ (\partial \phi)^2 - \frac{1}{2} (F^2 + \bar{F}^2) - \frac{i}{2} (F^2 - \bar{F}^2) \tan \phi - \frac{1}{\cos^2 \phi} \tilde{h}^2 - \frac{2}{\cos \phi} \partial \cdot \tilde{h} \right\} ,$$

(3.34)

where

$$v(\phi) \equiv d_1 + d_2 \phi .$$

Substituting (3.33) into this expression, we find the surprising result

$$\mathcal{L}_{\text{bos}} = 0 !$$

(3.35)

Nevertheless, one can obtain a non-vanishing action after dualizing the notoph covariant field strength. This point is discussed in the next Section.

As the last topic of this Section we present an alternative to the gauge (3.9):

$$\beta = be^{i\psi} , \quad b = \text{const} \neq 0$$

(3.36)

which yields

$$\gamma - \alpha' = (\alpha - \gamma) \gamma - 2b^2 , \quad ib\psi' = b(\alpha - 2\gamma) .$$

(3.37)

This time the analog of (3.18) implies the additional constraint

$$\alpha - \bar{\beta} - \gamma = 0 .$$

(3.38)

Equations (3.37), (3.38) have two different solutions:

$$\alpha = b \left( 2 \cos 2\lambda + i \sin 2\lambda \right) , \quad \gamma = b \cos 2\lambda , \quad \beta = b \ e^{-2i\lambda} , \quad \lambda = \arctan a \ e^{-bL} ,$$

(3.39)

$$\alpha = 2b , \quad \gamma = \beta = b ,$$

(3.40)

where $a, b$ are integration constants. The solution (3.40) is the $a = 0$ contraction of the solution (3.39), so the latter is more general. We have verified it to pass all conceivable self-consistency checks. Note that with the choice of the $a = 0$ version the constraints (3.1), (3.2) possess an important invariance under the shift $L \rightarrow L + \text{const}$, while it is not so in the general case $a \neq 0$. This invariance guarantees the corresponding actions to be scale invariant, so the parameter $a$ measures the breaking of such an invariance. Clearly, the cases $a \neq 0$ and $a = 0$ cannot be related by any field redefinitions, since we have already fixed the reparametrization freedom while deriving the above solutions.
The most general solution (3.39) was obtained in the gauge (3.36), and it has the advantage of being non-singular in the two important limits \( a = 0 \) and \( b = 0 \) which lead, respectively, to the scale-invariant non-linear version (3.40) and to the linear version. However, when constructing the invariant actions and inspecting the deformations of the Bianchi identities in the general \( a \neq 0, b \neq 0 \) case, it is more convenient to stay in the gauge (3.33). The precise relation between the two gauges is as follows

\[
(D\beta)^2 \tilde{L} = (\bar{D}\beta)^2 \tilde{L} = 2c_1 ba \left[ (\cot 2\lambda + i)D\beta \bar{D}\beta \tilde{L} + (\cot 2\lambda - i)\bar{D}\beta \bar{D}\beta \tilde{L} \right],
\]

\[
D_\alpha \bar{D}_{\dot{\alpha}} \tilde{L} = 0,
\]

\[
\tilde{L} = c_2 - \frac{1}{2c_1 ba} 2\lambda.
\]

Here, \( c_1, c_2 \) are arbitrary integration constants reflecting the residual freedom of shifting and rescaling \( \tilde{L} \). They can always be chosen so as to guarantee the limits \( a = 0 \) and/or \( b = 0 \) to be non-singular in the gauge (3.39) too. Finally, we note that it is rather straightforward to check that in the case (ii) in (3.30) the invariants entering with constants \( g_1 \) and \( g_2 \) take, respectively, the following form in the gauge (3.36)

\[
\sim e^{-bL} (D^+ LD^+ L + \bar{D}^+ \bar{D}^+ L),
\]

and

\[
\sim i e^{-3bL} (D^+ LD^+ L - \bar{D}^+ \bar{D}^+ L).
\]

Thus they generalize the Lagrangians \( \mathcal{L}^{++}_2 \) and \( \mathcal{L}^{++}_1 \) of the linear case (eqs. (2.29), (2.28)). Note that, as was pointed out in the recent paper [10], the Lagrangian (3.42) is a total derivative like its linear version counterpart (2.29), and so it gives rise to a topological invariant.

4 Dual versions of the VT actions

The dual form of the above actions is obtained by implementing the notoph constraint in the Lagrangian with the help of a Lagrange multiplier. In the case of the constraint (3.33) this leads to the action

\[
\mathcal{L}_{\text{bos}}' = -\lambda \left( \partial \cdot \tilde{h} + \frac{1}{4} (e^{i\phi} F^2 + e^{-i\phi} \bar{F}^2) + \frac{1}{2 \cos \phi} \tilde{h}^2 - \frac{1}{2} \cos \phi (\partial \phi)^2 \right).
\]

Now \( \tilde{h}^{\alpha \dot{\alpha}} \) is unconstrained, and one can integrate it out by its algebraic equation of motion

\[
\tilde{h}^{\alpha \dot{\alpha}} = \cos \phi \frac{\partial^{\alpha \dot{\alpha}} \lambda}{\lambda}.
\]

After that we get a typical sigma-model action

\[
\mathcal{L}_{\text{bos}}' = -\frac{\lambda}{4} (e^{i\phi} F^2 + e^{-i\phi} \bar{F}^2) + \frac{\lambda}{2} \cos \phi [(\partial \phi)^2 + (\partial \ln \lambda)^2].
\]

Let us make once more an analogy with the non-linear \( N = 2 \) multiplet. There one cannot write down a non-vanishing (and \( SU(2) \) invariant) action for this multiplet itself [16, 17, 18],
but the dual action obtained by implementing the defining constraint with the help of a
Lagrange multiplier yields a non-trivial sigma-model action in its bosonic sector.

No such subtleties occur in the case of the “old” non-linear version corresponding to the
solution (ii) (3.37). The only effect of substituting the constraint (3.27) into the appropriate
analog of the Lagrangian (3.34) is the cancellation of the terms proportional to $g_1$, in accord
with the previous statement that the invariant proportional to $g_1$ is a total derivative. In this
case we have the following bosonic Lagrangian (before dualization)

$$L_{bos} = g_2 \left[ \phi (\partial \phi)^2 - \frac{1}{2} \phi (F^2 + \bar{F}^2) - \frac{1}{\phi} \tilde{h}^2 \right]. \tag{4.4}$$

The analog of the dual Lagrangian (4.3) reads

$$L'_{bos} = \phi \left[ g_2 (\partial \phi)^2 + \frac{1}{4g_2} (\partial \lambda)^2 \right] - \frac{1}{2} \left( g_2 \phi + \frac{i}{2} \lambda \right) F^2 - \frac{1}{2} \left( g_2 \phi - \frac{i}{2} \lambda \right) \bar{F}^2. \tag{4.5}$$

Both actions (4.3) and (4.5) can be recast in the generic form of the bosonic part of the
action of an $N = 2$ gauge multiplet:

$$L_{bos}' = \frac{i}{2} (\partial \bar{F}' \partial \bar{z} - \partial z \partial F' + \bar{F}' F^2 - F' \bar{F}^2). \tag{4.6}$$

The holomorphic potential $F(z)$ for the action (4.3) is

$$F(z) = \frac{i}{2} e^{-iz}, \quad z = \phi + i \ln \lambda \tag{4.7}$$

and for the action (4.5)

$$F(z) = -\frac{ig_2}{6} z^3, \quad z = \phi - \frac{i}{2g_2} \lambda. \tag{4.8}$$

The potential (4.8) can be obtained from that of ref. [3], [4] by freezing the $N = 2$ vector mul-
triplet which gauges the central charge. The potential (4.7) is new and it would be interesting
to study whether it may occur in a stringy context.

The dualization procedure described above concerned the purely bosonic sector of the
action only. Carrying this procedure out in a fully off-shell supersymmetric way is also possible.
For simplicity here we explain this on the example of the linear version of the VT multiplet.
We take the superspace action (2.25), (2.28) and add to it the harm onic constraints (2.8),
(2.9) with analytic superfield Lagrange multipliers:

$$S = \int d\xi^{-4} \left\{ \left( (\theta^+)^2 - (\bar{\theta}^+)^2 \right) (f^+ f^- + \bar{f}^+ \bar{f}^-) \right. \\
+ H^+ D^{++} f^+ + \bar{H}^+ D^{++} \bar{f}^+ + G^{++} (D^{++} l + \theta^+ f^+ + \bar{\theta}^+ \bar{f}^+) \right\}. \tag{4.9}$$

Note that the Lagrange multiplier $H^{+\alpha}$ has a non-standard supersymmetry transformation
law in order to compensate for the variation of the first term. We assume that the central
charge is still realized on $f^+, \bar{f}^+$ as in (2.17), whereas on $l$ it acts as follows

$$\partial_5 l = \frac{i}{4} (\partial_{+\alpha} f^{+\alpha} - \partial_{+\dot{\alpha}} \bar{f}^{+\dot{\alpha}}). \tag{4.10}$$
(the reality condition (2.18) is not imposed at this stage, it appears only as a result of the variation w.r.t. some Lagrange multiplier). This realization of the central charge is compatible with supersymmetry. The first term in (4.9) is invariant under central charge transformations on its own. The requirement of central charge invariance of the rest of the action determines the central charge transformation properties of the Lagrange multipliers:

\[ \partial_5 H^+ = -\partial_{\dot{a}\dot{b}} \bar{H}^{+\dot{a}} - \frac{i}{4} \partial_{+a} G^{++} , \quad \partial_5 G^{++} = 0 . \]  

(4.11)

To obtain the component content of the theory one should replace the \( \partial_5 \)-derivative terms contained in \( D^{++} \) according to the above rules and integrate over \( \theta^+, \bar{\theta}^+ \) and the harmonics.

It can be shown that upon elimination of the infinite set of auxiliary fields we are left in the bosonic sector with two scalars and an abelian gauge vector field, which belong to an on-shell \( N = 2 \) vector multiplet dual to the original VT one. More details and the treatment of the non-linear versions will be given elsewhere.

5 Coupling to an external vector multiplet

Here we shall deform the non-linear superfield constraints (their “old” version) to switch on the CS coupling to one external abelian vector multiplet. The generalization to several such multiplets and to the non-abelian case goes more or less straightforwardly and will be presented elsewhere.

We choose the gauge (3.36) and the simplest constant solution to eqs. (3.37):

\[ \alpha = 2\gamma = 2b , \quad \beta = be^{i\psi} , \quad b = \text{const} , \quad \psi = \text{const} . \]

(5.1)

Thus our starting point is the following set of constraints

\[ (D^+)^2 L = 2bD^+L \partial^+L + be^{i\psi} D^+L \partial^+L \]
\[ D^{\dot{a}+}_{\dot{a}} \bar{D}^{\dot{a}+}_{\dot{a}} L = bD^{\dot{a}+}_{\dot{a}} \bar{D}^{\dot{a}+}_{\dot{a}} L \]

(5.2)

As we saw before, this set corresponds to the non-linear version of [3, 4] and it yields a CS-term modification of the Bianchi identity for the notoph field strength \( h^{a\dot{a}} \). As we also saw, some additional self-consistency conditions require for the given solution

\[ e^{i\psi} = 1 \]

(5.3)

at the level of the pure \( L \) system. This phase factor can be non-trivial in the presence of extra vector multiplets.

The abelian vector multiplet is represented by its superfield strength \( W \) which does not depend on the harmonics and obeys the chirality condition and the Bianchi identity (reality condition)

\[ D^{++}W = 0 , \]
\[ D^{\dot{a}+}_{\dot{a}} \bar{W} = \bar{D}^{\dot{a}+}_{\dot{a}} W = 0 , \]
\[ (D^+)^2 W = (\bar{D}^+)^2 W . \]

(5.4)
In order to find an appropriate self-consistent deformation of (5.2), such that it is reduced to (5.2) after switching off \( W \), we proceed in the following way. We start from the most general form of such a deformation of the r.h.s. of eqs. (3.1), (3.2) consistent with the harmonic \( U(1) \) charge +2 of the l.h.s. and the constraint (3.3). All the coefficients, including \( \alpha, \beta \) and \( \gamma \), are assumed to be arbitrary functions of \( L, W \) and \( \bar{W} \), with proper reality conditions imposed.

Next, just as in the pure \( L \) case, we exploit the integrability conditions (3.4). They lead to a huge number of differential and algebraic equations on the coefficients. Among them we still have eqs. (3.5), (3.6). To simplify the set of self-consistency conditions as much as possible we utilize, like in the pure \( L \) case, the reparametrization freedom \( L \to L(\tilde{L}, W, \bar{W}) \). (5.5)

We can still impose the gauges (3.36) or (3.9) on the coefficients \( \alpha, \beta, \gamma \). We choose (3.36), with \( b \) having no dependence on \( L, W \) and \( \bar{W} \).

There still remains the freedom of shifting \( L \) by a real function of \( W, \bar{W} \). It can be used to further restrict the r.h.s. of the deformed constraints.

Even after fixing the gauges we are still left with a considerable set of equations. We first solve the equations (3.37) for \( \alpha, \beta, \gamma \). As was stated above, for simplicity we choose the solution (5.1), where \( \psi \) is still independent of \( L \) but now depends on \( W, \bar{W} \) (recall that \( b = \text{const} \) as a result of fixing the gauge). This dependence has to be specified by solving the rest of the consistency conditions. Fortunately, the latter is greatly simplified under the choice (5.6).

As a result, we find the following most general self-consistent deformation of the constraints (5.2)

\[
(D^+)^2 L = 2bD^+L D^+L + be^{i\psi} D^+L D^+L + \lambda D^+L D^+W + \omega D^+W D^+W + \nu D^+D^+L W + \partial_W \bar{\nu} e^{i\psi} \bar{D}^+\bar{W} \bar{D}^+\bar{W},
\]

\[
D^+_a D^+_a L = bD^+_aL D^+_aL + \sigma D^+_aL D^+_aW - \bar{\sigma} D^+_aL D^+_aW - \omega D^+_aW D^+_aW.
\]

Here all the coefficients, except for \( \nu \), are expressed through \( \psi(W, \bar{W}) \)

\[
\lambda = i \partial_W \psi, \quad \omega = -\frac{1}{4b} \partial_W \psi \partial_W \psi, \quad \sigma = \frac{i}{2} \partial_W \psi.
\]

Simultaneously, one gets the following remarkable equations for \( \psi \)

\[
\partial_W \psi = e^{i\psi} \partial_W \psi, \quad \partial_W \partial_W \psi = 0.
\]

The general solution of this system is given by

\[
e^{i\psi} = \frac{1 + i \kappa \bar{W}}{1 - i \kappa \bar{W}},
\]

(5.12)
or

\[ \psi = i \left( \ln(1 - i\kappa W) - \ln(1 + i\kappa\bar{W}) \right) . \]  

(5.13)

Here \( \kappa \) is a real integration constant. We have also fixed one more integration constant by requiring eq. (5.3) to be valid in the pure \( L \) limit \( W = 0 \) (or \( \kappa = 0 \)).

Explicitly, the coefficients in (5.3) are as follows

\[ \lambda = \frac{i\kappa}{1 - i\kappa W}, \quad \sigma = \frac{i}{2} \frac{\kappa}{1 + i\kappa W}, \quad \omega = -\frac{1}{4b} \frac{\kappa^2}{(1 - i\kappa W)(1 + i\kappa W)} . \]  

(5.14)

In order to get rid of the “fake” singularity in \( b \), one should rescale

\[ \kappa = \sqrt{b\tilde{\kappa}} . \]  

(5.15)

It remains to specify the coefficient \( \nu \) in (5.7). It is given by the following expression

\[ \nu = \left[ e^{i\psi} a(W) + \bar{a}(/W) \right] e^{2bL} , \]  

(5.16)

\( a(W) \) being an arbitrary holomorphic function.

The constraints (5.7), (5.8) with the coefficients given by eqs. (5.14) and (5.17) describe the most general deformation of the ‘old’ nonlinear VT constraints (5.2) in the presence of one extra vector multiplet. It should be pointed out that the deformation presented here does not distinguish an external vector multiplet from one that gauges the central charge. Indeed, the above derivation relied merely upon the anticommutativity of \( D^+_\alpha, \bar{D}^+_\alpha \) and the constraints (5.4). These properties are valid irrespectively of whether \( W \) is some external gauge superfield strength or it is the strength of a superfield gauging the central charge.

An additional selection rule results from enforcing a self-consistency condition like (3.18). It leads to drastically different consequences for the cases of rigid and gauged central charges [19]. In the rigid case we are dealing with (when \( W \) is treated as an external \( U(1) \) superfield gauge strength) it still requires (5.3)

\[ e^{i\psi} = 1 \quad \Rightarrow \quad \kappa = 0 . \]  

(5.17)

As a result, in this case the deformation above is fully specified by the choice of the holomorphic function \( a(W) \) in (5.16). The standard CS modification of the Bianchi identity for \( h^{\alpha\dot{\alpha}} \) arises for \( a(W) = c W, c \) being the appropriate CS coupling constant. However, all the self-consistency conditions are still fulfilled by an arbitrary \( a(W) \). Though the modified Bianchi identity has no local solution in the general case, by analogy with the consideration in Sect. 4 we expect that the ‘dualization’ of this identity with the help of a Lagrange multiplier vector multiplet may yield an acceptable local theory.

As our last topic we give here the relevant invariant action. The analytic Lagrangian density \( \mathcal{L}^{++} \) for the \( W \)-deformed case can be constructed by the method of undetermined coefficients, like we proceeded in the previous Section. We compose the most general form of the Lagrange density of charge +2

\[ \mathcal{L}^{++} = g_1 D^+ L D^+ L + g_2 \bar{D}^+ L \bar{D}^+ L + g_3 D^+ W D^+ W + g_4 \bar{D}^+ W \bar{D}^+ W + g_5 D^+ L D^+ W + g_6 D^+ L D^+ W + g_7 (D^+)^3 W , \]  

(5.18)

Our special thanks are due to S. Kuzenko for bringing up this point to us.
with all coefficients being arbitrary functions of $L, W, \bar{W}$. Requiring $\mathcal{L}^{++}$ to be real imposes the following reality conditions on $g_n$
\[
g_2 = \bar{g}_1, \quad g_3 = \bar{g}_4, \quad g_5 = \bar{g}_6, \quad g_7 = \bar{g}_7.
\] (5.19)
Clearly, $\mathcal{L}^{++}$ obeys the condition
\[
D^{++} \mathcal{L}^{++} = 0.
\] (5.20)
Then we only need to extract the corollaries of the analyticity constraint (2.26). This requirement fixes $\mathcal{L}^{++}$ up to three integration constants, thus yielding three independent invariants
\[
\mathcal{L}^{++}_{(1)} \sim e^{-bl} D^+ L D^+ L + \frac{1}{b} \left( e^{bl} - 1 \right) (D^+)^2 G(W) + \text{c.c.},
\] (5.21)
\[
\mathcal{L}^{++}_{(2)} \sim i \left\{ e^{-3bl} D^+ L D^+ L + \frac{2}{b}(D^+)^2 \left[ \left( e^{-bl} - 1 \right) G(W) \right] \\
+ \frac{1}{b} \left( 1 - e^{-bl} \right) (D^+)^2 G(W) - \text{c.c.} \right\},
\] (5.22)
\[
\mathcal{L}^{++}_{(3)} \sim \frac{1}{2b} \left\{ \left( e^{-bl} - 1 \right) (D^+)^2 W + 2(D^+)^2 \left[ (1 - e^{-bl}) W \right] + \text{c.c.} \right\},
\] (5.23)
where $G(W)$ is a “potential” for $a(W)$,
\[
a(W) = \partial_W G(W).
\]
The first two densities extend, respectively, the invariants (3.42) and (3.43), while the third one is new, since it vanishes when $W = 0$. It still reduces to (3.42) under the choice $W = \text{const}$. Note that all these invariants were chosen to be well-defined in the limit $b = 0$ by extracting some pure $W$ densities
\[
\sim (D^+)^2 \mathcal{F}(W) + (D^+)^2 \mathcal{F}(\bar{W})
\] (5.24)
with some appropriate $\mathcal{F}$. They can be omitted without loss of generality.

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**References**

[1] M.F. Sohnius, K.S. Stelle and P.C. West, Phys. Lett. B 92 (1980) 123

[2] B. de Wit, V. Kaplunovsky, J. Louis and D. Lüst, Nucl. Phys. B 451 (1995) 53

[3] P. Claus, B. de Wit, M. Faux, B. Kleijn, R. Siebelink and P. Termonia, Phys. Lett. B 373 (1996) 81
[4] P. Claus, P. Termonia, B. de Wit and M. Faux, Nucl. Phys. B 491 (1997) 201

[5] P. Claus, B. de Wit, M. Faux, B. Kleijn, R. Siebelink and P. Termonia, “N=2 Supergravity Lagrangians with Vector-Tensor Multiplets”, Preprint KUL-TF-97/24, THU-97/26, HUB-EP-97/73, ULB-TH-97/18, ITP-SB-97-63, DFTT-62/97, hep-th/9710212

[6] A. Hindawi, B.A. Ovrut and D. Waldram, Phys. Lett. B 392 (1997) 85

[7] I. Buchbinder, A. Hindawi and B.A. Ovrut, “A two form formulation of the vector-tensor multiplet in central charge superspace”, hep-th/9706216

[8] R. Grimm, M. Hasler and C. Herrmann, “The N=2 vector-tensor multiplet, central charge superspace and Chern-Simons couplings”, Preprint CPT-97/P.3499, hep-th/9706108

[9] N. Dragon and S.M. Kuzenko, “The Vector-Tensor Multiplet in Harmonic Superspace”, Preprint ITP-UH-20/97, hep-th/9706169

[10] N. Dragon and S.M. Kuzenko, “Self-interacting vector-tensor multiplet”, Preprint ITP-UH-24/97, hep-th/9709088

[11] B. de Wit, R. Philippe and A. Van Proeyen, Nucl. Phys. B 219 (1983) 143; B. de Wit, P.G. Lauwers and A. Van Proeyen, Nucl. Phys. B 255 (1985) 569

[12] A. Galperin, E. Ivanov, S. Kalitzin, V. Ogievetsky and E. Sokatchev, Class. Quant. Grav. 1 (1984) 469

[13] A. Galperin, E. Ivanov, V. Ogievetsky and E. Sokatchev, Class. Quant. Grav. 2 (1985) 601, 617

[14] A. Galperin, E. Ivanov, V. Ogievetsky and E. Sokatchev, “Harmonic Superspace”, the review paper, still in preparation

[15] J. Wess, Acta Phys. Austr. 41 (1975) 409; B. de Wit and A. van Holten, Nucl. Phys. B 155 (1979) 530

[16] A. Galperin, E. Ivanov, V. Ogievetsky and E. Sokatchev, Class. Quant. Grav. 4 (1987) 1255

[17] U. Lindström, B. Kim and M. Roček, Phys. Lett. B 342 (1995) 99

[18] E. Ivanov and A. Sutulin, Class. Quant. Grav. 14 (1997) 843

[19] N. Dragon, E. Ivanov, S. Kuzenko, E. Sokatchev and U. Theis, “N=2 Rigid Supersymmetry with Gauged Central Charge”, in preparation