Transient mixed convection in a channel with an open cavity filled with porous media

B Buonomo, G Cresci, O Manca, P Mesolella, S Nardini
Dipartimento di Ingegneria Aerospaziale e Meccanica, Seconda Università degli Studi di Napoli, via Roma 29, 81031 Aversa (CE), Italy

E-mail: oronzio.manca@unina2.it

Abstract. In this work transient mixed convection in a porous medium in a horizontal channel with a open cavity below is studied numerically. The cavity presents a heated wall at uniform heat flux and the other walls of the cavity and the channel are assumed adiabatic. Air flows through the horizontal channel. The heated wall of the cavity experiences a uniform heat flux in such a way that the forced flow is perpendicular to the motion due to natural convection. The study is carried out employing Brinkman-Forchheimer-extended Darcy model and two energy equations due to the local thermal non-equilibrium assumption. The flow in the channel is assumed to be two-dimensional, laminar, incompressible. Boussinesq approximation is considered. The thermophysical properties of the fluid are evaluated at the ambient temperature. The results for stream function and temperature distribution given at different times are obtained. Wall temperature value are given and also, the velocity and temperature profiles in several sections of the cavity are presented. In addition, the Nusselt number, both local and average, is presented along with the temporal variations of the average Nusselt number.

1. Introduction

Research in mixed convection has received and is still receiving attention for its importance in various application both in science and engineering fields such as electronic cooling, nuclear reactors, building management and solar energy systems. Many systems are geometrically similar to a U-shaped or C-shaped open cavities and often the heat transfer inside open cavities is obtained by natural convection. In order to improve heat transfer in the cavity an externally driven flow of cold air is forced to enter. The phenomenological understanding of interaction between buoyancy-induced flow from the heated surfaces of open cavity and pressure-driven external flow becomes important in thermal design of systems as investigated in [1-4].

The utilization of a porous substrate for heat transfer augmentation is an attractive choice given that the effective thermal conductivity of the porous structure could be several order of magnitudes higher than the working fluid. Convective heat transfer in saturated porous media has received considerable attention during the past several decades because of its wide range of applications [5-11]. Convective mixed flow inside open cavities, vertical or horizontal channels and lid-driven cavities filled with a fluid-saturated porous medium has been studied in the past due to its applications in many engineering and geophysical systems [12]. Few investigations have been studied the problem of mixed convection in vented open cavity filled by a porous medium. One of the first was the experimental investigation given in [13]. Systems with open
top boundary with flow patterns of free and forced convection were investigated numerically in [14].
Two-dimensional, steady mixed convection flow in a porous square vented open cavity with an
isothermal vertical surface and the other three walls being adiabatic has been studied numerically in
[15]. A numerical study on a two-dimensional steady laminar mixed convection inside a square vented
cavity filled with a fluid-saturated porous medium for different outlet port positions was presented in
[16]. One of the vertical walls was at constant temperature while the remaining walls were assumed
adiabatic. Unsteady mixed convection in a ventilated rectangular cavity with a horizontal strip
occupied by two media of different permeability was studied in [17] by using the finite-volume
method. Mixed convection in an obstructed cavity with heated horizontal walls was numerically
investigated in [18]. Brinkman-Forchheimer-extended Darcy model was utilized to describe the flow
characteristics within a porous medium for different angles of attack with respect to the forced
convection. The influence of multiple injection/suction at bottom/top walls on non-Darcy mixed
convection flow in a vertical square enclosure filled with fluid saturated porous medium was examined
in [19]. The coupled non-linear partial differential equations modeling the fully developed
Forchheimer extended Darcy flow were solved by finite element method. A numerical analysis of
mixed convection on saturated porous medium in a horizontal channel with an open cavity heated at
uniform heat flux on a vertical wall was accomplished in [20]. Non-local thermal equilibrium and
Brinkman-Forchheimer extended Darcy model are assumed. Boussinesq approximation with constant
thermophysical properties were considered. The influence of various walls thermal boundary
conditions on mixed convection lid driven flows in a square cavity filled with porous medium was
studied in [21] by means of a penalty finite element method. When the boundary condition is of
constant wall heat flux, it is not clear what boundary conditions might be used for the fluid phase
and for the solid phase. The effect of using different boundary conditions for the case of constant wall heat
flux under local thermal non-equilibrium (LTNE) conditions was analyzed in [22, 23].
In the present study attention will be focused on a problem of transient mixed convection inside a
vented open cavity filled with a fluid-saturated porous medium, as reported in Figure 1. Air flows
through the horizontal channel. A wall of the cavity is at uniform heat flux. The other walls of the
cavity and the channel are assumed adiabatic. This problem was studied for the boundary condition of
same heat flux for the fluid and solid phases. The Brinkman-Forchheimer-extended Darcy model in
transient two dimensional and non-local thermal equilibrium are employed. Thermal boundary
condition at heated wall is assumed assigning wall temperature the same for the solid and fluid but
unknown and the additional information on wall heat flux is given assuming that the wall heat flux is
known with energy balance satisfied at interface between solid wall and saturated porous medium and
fluid. Results are obtained using a numerical code based on the finite volume method. The results for
stream function and temperature distribution is given. Also the temperature profiles along the heated
wall are shown. In addition, the Nusselt number, both local and average, is presented along with the
heated wall.

2. Geometrical and Physical Model
The physical system and geometry under investigation are shown in Figure 1. The U-Shaped cavity
and the above channel are filled with a fluid-saturated porous medium where the fluid phase consists
of air, Pr=0.71, and the solid phase consists of metal foam of aluminium. The porous material is
considered as homogeneous and isotropic. Flow enters through the left channel section at uniform
velocity $u_i$. It is assumed that incoming flow is at ambient temperature, $T_0$, and temperature and
velocity gradient component along the axis are equal to zero at the outlet section. The left wall of the
U-Shaped cavity is heated by an uniform heat flux and other walls are adiabatic. It is assumed that
the transient fluid flow in the channel is two-dimensional, laminar and incompressible. Viscous
dissipation, heat generation and pressure are all assumed to have negligible effect on the velocity and
temperature fields therefore they are neglected. All the thermophysical properties of the fluid and the
solid matrix of the porous medium are assumed constant except for the variation in density of the air
with temperature (Boussinesq approximation) giving rise to the buoyancy forces. It is assumed that
fluid and solid phase of porous medium are in non-local thermal equilibrium, NLTE. The thermophysical properties of the fluid and the solid matrix of the porous medium are evaluated at the ambient temperature, $T_0$, which is equal to 300 K in all cases. In all analyzed cases $L/H=2$, $D/H=1$, where $D=0.2$ m. In the porous medium region, the generalized flow model, known as the Brinkman-Forchheimer-extended Darcy model, is used in the governing equations. Under these hypothesis the conservation equations for the transient two-dimensional thermal non-equilibrium model, in terms of stream-function and vorticity and in dimensionless form, are [24]:

\[
\frac{1}{\phi} \frac{\partial \Omega}{\partial \tau} + \frac{1}{\phi} \left( \frac{\partial}{\partial X} \left( \frac{U \Omega}{\phi} \right) + \frac{\partial}{\partial Y} \left( \frac{V \Omega}{\phi} \right) \right) = \frac{1}{\phi \text{Re} \left( \frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2} \right)} - \left( \frac{1}{\text{ReDa}} + \frac{C_f V^2}{\sqrt{\text{Da}}} \right) \Omega 
\]

\[
- \frac{C_f}{\sqrt{\text{Da}}} \left( V \frac{\partial [V]}{\partial X} - U \frac{\partial [V]}{\partial Y} \right) - \text{Ri} \frac{\partial \theta_f}{\partial Y} 
\]

\[
\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = -\Omega 
\]

\[
\frac{\partial \psi}{\partial Y} = U; \quad \frac{\partial \psi}{\partial Y} = -V; \quad \Omega = \frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y} 
\]

The energy equations are:

phase fluid

\[
\phi \frac{\partial \theta_f}{\partial \tau} + \left( U \frac{\partial \theta_f}{\partial X} + V \frac{\partial \theta_f}{\partial Y} \right) = \frac{\phi}{\text{Pr Re} \left( \frac{\partial^2 \theta_f}{\partial X^2} + \frac{\partial^2 \theta_f}{\partial Y^2} \right)} + \frac{Bi_{ij} \gamma}{\text{Pr Re}} \left( \theta_s - \theta_f \right) 
\]

phase solid

\[
\Gamma \frac{\partial \theta_s}{\partial \tau} = \left( \frac{\partial^2 \theta_s}{\partial X^2} + \frac{\partial^2 \theta_s}{\partial Y^2} \right) + Bi_{ij} \gamma \left( \theta_s - \theta_f \right) 
\]

The employed dimensionless variables are
\[
X = \frac{x}{D}, \quad Y = \frac{y}{D}; \quad U = \frac{u}{u_i}; \quad V = \frac{u_f}{u_i}; \quad \tau = \frac{u_f}{D}; \quad \theta = \frac{T - T_i}{(q_w D/k_f)}; \quad Da = \frac{K}{D^2};
\]

\[
Gr = \frac{g \beta q_w D^4}{\nu_f^2 k_f}; \quad Re = \frac{u_i D}{\nu_f}; \quad Ri = \frac{Gr}{Re^2}; \quad Bi_f = \frac{D^2 h_y a_y}{k_f}; \quad \alpha = \frac{\alpha_f}{\alpha_s}; \quad Pr = \frac{\nu}{\alpha_f}; \quad \gamma = \frac{\kappa}{(1 - \varphi)}
\]

(6)

The permeability coefficient \(K\) and inertia coefficient \(CF\) are related to the aluminium foam. The boundary conditions are

at \(Y = 0\) and \(0 \leq X \leq 1\): \(\Omega = -\frac{\partial^2 \Psi}{\partial Y^2}; \quad \Psi = \frac{H}{D}; \quad \varphi = \frac{\partial \theta_f}{\partial Y} + (1 - \varphi) \frac{k_x}{k_f} \frac{\partial \theta_s}{\partial Y} = -1\) and \(\theta_f = \theta_s = \theta_w\)

at \(Y = 0\) and \(1 \leq X \leq 1 + H/D\): \(\Psi = (1+\frac{H}{D}) X; \quad \Omega = -\frac{\partial^2 \Psi}{\partial Y^2}\) and \(\theta_f = \theta_s = 0\)

at \(X = 0\) and \(0 \leq Y \leq L/D\): \(\Psi = \frac{H}{D}; \quad \Omega = -\frac{\partial^2 \Psi}{\partial X^2}; \quad \frac{\partial \theta_f}{\partial Y} = \frac{\partial \theta_s}{\partial Y} = 0\)

at \(Y = L/D\) and \(0 \leq X \leq 1\): \(\Psi = \frac{H}{D}; \quad \Omega = -\frac{\partial^2 \Psi}{\partial Y^2}\) and \(\frac{\partial \theta_f}{\partial Y} = \frac{\partial \theta_s}{\partial Y} = 0\)

for \(X = 1\) and \(L/D \leq Y \leq L_{ext}/D\): \(\Psi = \frac{H}{D}; \quad \Omega = -\frac{\partial^2 \Psi}{\partial X^2}\) and \(\frac{\partial \theta_f}{\partial X} = \frac{\partial \theta_s}{\partial X} = 0\)

for \(1 \leq X \leq H/D\) and \(Y = (L + L_{ext})/D\): \(\frac{\partial \Psi}{\partial Y} = 0; \quad \frac{\partial \Omega}{\partial Y} = 0\) and \(\frac{\partial \theta_f}{\partial X} = \frac{\partial \theta_s}{\partial X} = 0\)

for \(X = H/D\) and \(0 \leq Y \leq (L + L_{ext})/D\): \(\Psi = 0; \quad \Omega = -\frac{\partial^2 \Psi}{\partial X^2}\) and \(\frac{\partial \theta_f}{\partial X} = \frac{\partial \theta_s}{\partial X} = 0\)

(11)

(12)

(13)

The fluid and solid phase local and average Nusselt numbers and total local and average Nusselt numbers along the heated surface are defined as:

\[
Nu_f (X) = \varphi \left( \frac{1}{\theta_w} \frac{\partial \theta_f}{\partial Y} \right); \quad Nu_s (X) = (1 - \varphi) \frac{k_s}{k_f} \left( \frac{1}{\theta_w} \frac{\partial \theta_s}{\partial Y} \right);
\]

(14)

(15)

\[
Nu_f(\cdot) = Nu_f(X) + Nu_s(X) = \frac{1}{\theta_w(X)}
\]

\[
Nu_{f,\text{avg}} = \frac{H}{2L} \int_{-L/H}^{L/H} Nu_f(Y) dY; \quad Nu_{s,\text{avg}} = \frac{H}{2L} \int_{-L/H}^{L/H} Nu_s(Y) dY
\]

(16)
3. Numerical Model

The numerical computation was carried out by means of the finite volume method, using rectangular
cells with constant mesh spacing along the coordinate directions, X and Y. The vorticity and energy
equations, Eqs. (1), (4) and (5), were solved by means of the alternating direction implicit (ADI)
method [25]. The second upwind scheme [26] was employed to discretize the convective derivatives,
while the diffusive derivatives were discretized by the classical central three-point scheme. The
convective terms were linearized following the iterative procedure suggested by Roache [25]. The
stream-function equation, Eq. (2), was solved by the successive line over-relaxation (SLOR) method,
using a relaxation factor of about 1.2. Once the equations of vorticity, stream function, and energy
are solved until the steady state is reached. The steady state solution is attained when the vorticity and
thermal fields variations are less than a prescribed accuracy equal to $10^{-6}$. A grid dependence analysis
is performed to evaluate a more convenient grid size by monitoring the average wall temperature for
air (Pr=0.71), as fluid, Re=100, Ri=100, Bi=1000, Da=$10^{-6}$, $\phi=0.80$, $C_F=0.010$, $\alpha=0.10$ and $\kappa=0.0010$.
Three uniform grid size were tested and an asymptotic value obtained by Richardson extrapolation
[26] was evaluated. The percentage error between the grid 51x51, 101x101 and 201x201 and the
asymptotic value were 2.5%, 1.3% and 0.58%, respectively. The grid 101x101 was used to obtain the
results because it is a good compromise between computational time and accuracy requirements. The
analysis was completed developing the numerical code also with the conditions given by Baytas and
Pop [27]. A comparison in terms of temperature, stream function fields and Nusselt numbers for fluid
and solid with the results given in [27] presented a very good agreement.

4. Results and Discussion

In the present investigation thermal and fluid dynamics behaviors as a function of time are evaluated
analyzing the results obtained for Re=100 and Ri=0.10 and 100. In Figure 2, solid and fluid
temperature fields, for two times values, for Bi values equal to 1.0 and 1000 with Da=$10^{-6}$, $\phi=0.92$,
C_t=0.020, α=0.20 and κ=0.00010 are reported. At low dimensionless time, τ=0.0001, for Bi=1.0 and 1000, in Figure 2a and 2b, isotherms are concentrated near the heated wall and are almost parallel and high temperature gradients are presents. For Bi=1.0 the fluid and the solid temperatures present some differences and affected thermal zone in the solid is more extended with respect to the one in the fluid. For Bi=1000 no significant differences are detected between the solid matrix and the fluid except in the zone adjacent to the channel where the forced fluid flow determines a lower fluid temperature. At steady state, τ=τ_{ss}, for lower Bi value, in Figure 2c, fluid and solid temperatures present very large differences. For Bi=1000, in Figure 2d, the differences are very small and the isothermal lines show that the conductive phenomenon is dominating. The same trends are observed also for Ra=100 and the other Da values; consequently, the other temperature fields are not given.

In Figure 3, average wall temperature as a function of time is depicted for Bi=100 and different values of the thermal conductivity ratio and porosity. It is observed that as greater the κ value as higher the average wall temperature this is due to the less diffusion effect of the solid for assigned value of the fluid thermal conductivity. At assigned κ value, temperature profiles present higher values at higher porosity due to the lower diffusion. In fact, for higher porosity the porous medium has less solid and the effect of fluid thermal conductivity is higher close to the wall and a higher wall temperature needs to exchange the assigned wall heat flux. At steady state the ratio between the average temperature for the porosity equal to 0.96 and 0.60 decreases increasing the κ value. In fact, it is about 10 for κ=10^{-4} and about 2 for κ=0.1.

The effect of wall heat transfer is shown in Figure 4, where total average Nusselt numbers as a function of time are reported for Bi=100 and different values of the thermal conductivity ratio and porosity. In all cases, a sharp decrease is noted for very low time is noted due to the very low differences between the wall and porous medium temperatures at very low time. For assigned κ value, the total average Nusselt number increases for porosity decreases. Moreover, decreasing the thermal conductivity ratio the total average Nusselt number decreases significantly.

In Figure 5, the average wall temperature and total Nusselt number profile as a function of time are given for different Bi values, with assigned porosity and Darcy number equal to 0.92 and 10^{-6}, respectively. It is observed that the effect of Biot number on the two considered variable is very weak for the considered Re, Ri and Da values. However, also the effect of Darcy number for the considered Re, Ri and Bi values is very weak and it is not reported.
Figure 4. Average total Nusselt number profiles as a function of time for Re=100, Ri=100, Da=10^{-6}, Bi=100 and different κ values: (a) 0.0001 and (b) 0.1.

Figure 5. Average wall temperature and total Nusselt number vs. time for Re=100, Ri=100, \( \phi = 0.92 \), Da=10^{-6} and different Bi values: (a) Wall temperature profiles and (b) total Nusselt number profiles.

5. Conclusions
In the range of considered parameters the effect of internal Biot number between the solid and fluid phases Darcy number on average wall temperature and average total Nusselt number was weak whereas the porosity and the thermal conductivity ratio determined a significant change in these two average variables. Average wall temperature and total Nusselt number presented sharp variations at initial times. However, temperature fields showed different trends of isotherms for different value of Biot number.

6. References
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