Flow bifurcation transitions of inelastic shear thinning fluids in a channel with sudden contraction and expansion

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Abstract. The bifurcation transitions from the symmetric to asymmetric flow regimes in shear thinning fluids in a channel with sudden contraction and expansion were studied by means of numerical simulations. On the example of the Carreau–Yasuda model, the bifurcation diagrams of shear thinning fluids with different viscosity flow curves were revealed along the critical Reynolds numbers of the bifurcation transitions. It was found that increase in variations of viscosity with shear rate leads to a noticeable decrease in critical Reynolds number and increase in dimensions of the angular vortices as compared to Newtonian fluids. The obtained results indicate the enhancement of flow instability of shear thinning fluids in channels with the variable cross-sections.

1. Introduction
Channels with variable cross-sectional sizes are extremely popular in various fields of science and technology (from Laval nozzles to turbulent tubular reactors). Such constructions allow generation of high-speed liquid jets [1, 2] or turbulent flows at small Reynolds numbers \( \text{Re} \) [3–5] leading to a significant increase in heat transfer [6, 7], productivity of chemical reactors [8, 9], and mixing efficiency of incompatible polymer melts [10]. Investigation of flow structure of viscous Newtonian fluids in a channel with sudden contraction and expansion indicates that at low Reynolds numbers the symmetric corner vortices appearing in the wide chamber at the exit of the contraction zone are formed, while growing of Re results in flow bifurcation which leads to asymmetry in the dimensions of the vortices [3, 5, 11]. A further increase in flow velocity contributes to multiplication of vortices and then transition to a developed turbulent flow.

The issue of the flow structure in non-Newtonian fluids in channels with sudden compression and expansion has been studied experimentally and numerically for some viscoelastic and shear thinning polymer liquids [12–17]. In the latter case, the flow bifurcations were investigated by example of inelastic power-law fluids, whose viscosity decreases with increase in shear rate \( \dot{\gamma} \) as \( \eta (\dot{\gamma}) = K \dot{\gamma}^{n-1} \) with \( n < 1 \) [15–17]. A disadvantage of this model is that when \( \dot{\gamma} \rightarrow 0 \), viscosity diverges, \( \eta (\dot{\gamma}) \rightarrow \infty \), whereas at high shear rates, \( \dot{\gamma} \rightarrow \infty \), the viscosity tends to zero, \( \eta (\dot{\gamma}) \rightarrow 0 \).
Figure 1. Viscosity curves of the Carreau–Yasuda fluid for $\eta_0 = 2.05$ Pa s, $\eta_\infty = 0.00089$ Pa s, $\lambda = 21.2$ s, $a = 2$ and differential exponents: $n = 0.6$ (solid line), 0.406 (dashed line) and 0.2 (dotted line). The dashed line fits the viscosity flow curve of 0.2 wt% xanthan aqueous solution.

These results do not agree with the experimental data and could lead to erroneous conclusions about critical conditions of bifurcation transitions. A more adequate model of the inelastic shear thinning fluids is the Carreau–Yasuda model which viscosity depends on shear rate $\dot{\gamma}$ as

$$\eta(\dot{\gamma}) = \eta_\infty + (\eta_0 - \eta_\infty)[1 + (\lambda\dot{\gamma})^{\alpha}]^{(n-1)/\alpha},$$

with $n < 1$. It is important here that the fluid viscosity is finite one, $\eta_0$, in the limit of small shear rates, while at high $\dot{\gamma}$ the viscosity does not fall below a specified value $\eta_\infty$. The exponent $n$ in (1) controls the rate of viscosity decrease with increasing shear rate $\dot{\gamma}$ (figure 1), and parameter $\lambda$ is responsible for the beginning of transition from the Newtonian plateau $\eta = \eta_0$ to the non-Newtonian dependence of viscosity on $\dot{\gamma}$.

The Carreau–Yasuda model was used successfully to describe rheological behavior of aqueous solutions of polyacrylamide [18], xanthan gum [19], carboxymethyl-cellulose [20], and blood [21]. On the other hand, at sufficiently high shear rates, this model can be reduced to a power-law fluid model: for $(\lambda\dot{\gamma})^{\alpha} \gg 1$ and $\eta_\infty \rightarrow 0$ we arrive to $\eta(\dot{\gamma}) = K\dot{\gamma}^{n-1}$ where $K = \eta_0\lambda^{n-1}$.

This work aims the numerical modeling of the flow patterns and hydrodynamic stability of inelastic shear thinning fluids flowing in 2D channel with sudden contraction and expansion at different Reynolds numbers. We focus solely on the effects caused by the non-Newtonian dependence of fluid viscosity on shear rate in a channel with a given dimensions and fixed contraction ratio.

2. Model and methods
The flow of shear thinning fluid has been considered in a 2D channel with the sudden contraction and expansion (figure 2) with the following dimensions: $l_1 = 0.15$ m, $l_2 = 0.01$ m, $l_3 = 0.45$ m, $w = 0.03$ m, $a = 0.002$ m. The contraction ratio of the channel is $\text{CR} = w/a = 15$. 
The flow of the incompressible fluids obeys the following Navier–Stokes equations along with the incompressibility condition:

\[
\begin{align*}
\rho \left[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right] &= -\nabla p + 2\nabla \left[ \eta(I_2) D \right], \\
\nabla \cdot \mathbf{u} &= 0.
\end{align*}
\] (2)

Here \( \mathbf{u} \) and \( p \) are the fluid velocity and pressure, respectively, and \( D = \frac{1}{2} (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) \) is the strain rate tensor. In a channel of a complex shape, viscosity of the Carreau–Yasuda fluid should be set as a function of the second invariant of the strain rate tensor \( I_2 = D : D \) as follows:

\[
\eta(I_2) = \eta_\infty + (\eta_0 - \eta_\infty) \left[ 1 + \left(2\lambda I_2\right)^{(\alpha/2)} \right]^{(\alpha-1)/\alpha}.
\] (3)

In this paper, the xanthan gum 0.2 wt% aqueous solution was considered as a basic fluid. The viscosity curve of this non-Newtonian fluid (dashed line in figure 1) was fitted with the following parameters: \( \eta_0 = 2.05 \text{ Pa} \cdot \text{s} \), \( \eta_\infty = 0.00089 \text{ Pa} \cdot \text{s} \), \( \lambda = 21.2 \text{ s} \), \( \alpha = 2 \), and \( n = 0.406 \) [19]. The other curves in figure 1 correspond to \( n = 0.2 \) and 0.6.

Flow conditions in a channel with the sudden contraction and expansion were characterized by the following Reynolds number:

\[
Re = \frac{\rho \overline{u} w}{\eta_w},
\] (4)

where \( \overline{u} \) is the average fluid velocity in the wide camera (for the 2D channel \( \overline{u} \) can be expressed in terms of volume flow rate \( Q, \overline{u} = \frac{Q}{w} \), and \( \text{Re} = \frac{\rho Q}{\eta_w} \)). The average viscosity of the Carreau–Yasuda fluid in the wide chamber was estimated roughly as:

\[
\eta_w = \eta_\infty + (\eta_0 - \eta_\infty) \left[ 1 + \left(\lambda \overline{u}^{-1}\right)^{\alpha/2} \right]^{\alpha-1}/\alpha.
\] (5)

The Navier–Stokes equation (2) was solved numerically using the OpenFoam free software [22] by means of the finite-volume method. The velocity and pressure fields were calculated using PISO (pressure-implicit with splitting of operators) algorithm. The computational domain was discretized by uniform rectangular mesh. To determine the optimal mesh density, a series of calculations for the flow of Newtonian fluid at \( \text{Re} = 16 \) was carried out. We found that the dimension of the corner vortices do not depend on the grid density if numbers of cells in the horizontal and vertical directions are not less than 1185 and 256, respectively. This kind of mesh was used in this work.
3. Results and discussion

It was shown that, as in Newtonian fluids, an increase in the Reynolds number of inelastic shear fluidizing liquids leads to bifurcation transitions, characterized by spontaneous transformation of a flow symmetric to an asymmetric structure. Figure 3 demonstrates this effect on the example of stream lines of xanthan gum 0.2 wt % solution (n = 0.406) for three Reynolds numbers. It can be seen that above a critical value Re_{cr}, the symmetric flow structure is transformed into the asymmetric one when one of the corner vortices becomes larger than the other.

Figure 4 shows the bifurcation diagrams (the dependencies of the maximal sizes of the corner vortices on Reynolds number), which were calculated for different exponents n of the Carreau–Yasuda model. The diagrams show that dimensions of the corner vortices L coincide with each other for the relatively small Reynolds numbers, whereas above certain critical values of Reynolds number, Re_{cr}, size of one of the vortices becomes larger than the other, L_2 > L_1. It is characteristic that size L_1 of the smallest vortex does not exceed the corresponding size L_{cr} at the bifurcation point with increase of Re. A comparison with the Newtonian fluid corresponding to n = 1 [23] shows that bifurcation transitions of inelastic shear-thinning media occur at lower
Figure 4. The bifurcation diagrams for different values of the $n$ exponent of Carreau–Yasuda model: $n = 0.2$ (a), 0.4 (b), 0.6 (c), 1 (d).

Table 1. Critical Reynolds numbers $Re_{cr}$ and corner vortex sizes $L_{cr}$ for different $n$ exponents.

| $n$  | $Re_{cr}$ | $L_{cr}$ |
|------|-----------|----------|
| 0.2  | 0.835     | 1.365    |
| 0.4  | 1.27      | 1.275    |
| 0.6  | 3.04      | 1.155    |

Reynolds numbers. This means that flow of shear thinning fluids in a channel with a variable cross-section is much less stable than that of Newtonian fluids. The values of critical Reynolds numbers, $Re_{cr}$, and dimensions of the corner vortices, $L_{cr}$, at the bifurcation point depend on $n$ parameter. Table 1 shows that a decrease in $n$ (i.e., increase in the slope of the viscosity curve, see figure 1) leads to a decrease in $Re_{cr}$ and an increase in $L_{cr}$.

In connection with the observed scatter of the critical Reynolds numbers $Re_{cr}$ and sizes of the corner vortices $L_{cr}$ for different $n$ exponents (see figure 4), we considered the reduced
Figure 5. The reduced bifurcation diagrams at $n = 1$ (squares), 0.6 (triangles), 0.4 (circles), 0.2 (diamonds).

Figure 6. (a) Linear approximations of vortices relative sizes $L/L_{cr}$ depending on $\text{Re}/\text{Re}_{cr}$ in the symmetric flow mode for different $n$ exponents of the Carreau–Yasuda model (the symbols correspond to figure 5. (b) The dependence of the coefficient $k$ of equation (5) on $n$.

bifurcation diagrams showing dependences of the relative corner sizes, $L/L_{cr}$, on the relative Reynolds number, $\text{Re}/\text{Re}_{cr}$. Such diagrams are presented in figure 5. It can be seen that for $n \leq 0.60$, the relative sizes of the corner vortices of the shear thinning fluids are close to each other in both symmetric and asymmetric flow regimes. At the same time in the symmetric flow mode, the relative vortex size of a Newtonian fluid exceeds significantly that of the shear-thinning fluids. In all cases, the $L/L_{cr}$ dependences on $\text{Re}/\text{Re}_{cr}$ are close to the linear one at $\text{Re}/\text{Re}_{cr} < 1$ [figure 6(a)]:

$$\frac{L}{L_{cr}} = k(n) \frac{\text{Re}}{\text{Re}_{cr}} + b,$$

while the slope $k$ decreases nonlinearly with increasing $n$ exponent of the Carreau–Yasuda model [figure 6(b)].
4. Conclusions
The numerical modeling of flow peculiarities of inelastic shear-thinning fluids in a channel with a sudden contraction and expansion revealed that non-Newtonian fluids are less stable than Newtonian ones reaching the bifurcation transition from the symmetrical to asymmetric flow patterns at lower Reynolds numbers. The critical values of Reynolds numbers decrease with increase in slope of the viscosity curve of shear thinning fluids. Preliminary investigations have shown that variations in the contraction ratio \( w/a \) lead to changes in the critical Reynolds number at the bifurcation point as well as in sizes of the angular vortices. However, qualitatively, the flow patterns remain unchanged. These studies are the subject of the following paper.

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