A SIMPLE MODEL FOR THE CLUSTERING OF SUBHALOS AS A FUNCTION OF MASS

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ABSTRACT

Astrophysicists do not associate galaxies with the massive dark matter halos so easily detected in particle simulations and whose mass function can be predicted by the Press-Schechter formalism and its variants. Instead, they associate galaxies with smaller surviving subhalos whose contents do not fully mix into the larger dark matter halos they exist in. However, these subhalos are as of now difficult to find in simulations. I present a simple description of what the Press-Schechter formalism can say about the clustering of these subhalos, under the restrictive assumption that at fixed subhalo mass, the chance of survival of the subhalo is independent of its larger scale environment. This model predicts a weak dependence of clustering on mass below $M \approx 10^{12} M_{\odot}$, in agreement with observations of a weak dependence of clustering on luminosity at luminosities less than $L_z$. The model is in qualitative agreement with observations that at low luminosities galaxy clustering is a strong function of color while at high luminosities galaxy clustering is a strong function of luminosity.

Subject headings: galaxies: clusters: general — galaxies: fundamental parameters — galaxies: statistics — large-scale structure of universe

1. MOTIVATION

The excursion set formalism of Press & Schechter (1974) (and the subsequent improvements of Bond et al. 1991, Lacey & Cole 1993, and Sheth et al. 2001) predicts the mass function of bound, virialized objects in the universe. The extensions of Mo & White (1996) and Sheth et al. (2001) predict the clustering relative to dark matter of these halos. However, the large-scale, sufficiently high resolution N-body calculations of Kravtsov & Klypin (1999) and other investigators show that each CDM dark matter halo can contain multiple subhalos, presumably ex-halos not fully mixed into the larger, surrounding halo. These halos themselves do not have properties similar to those of galaxies — most notably, they grow to be much more massive. However, it is plausible that galaxies reside in subhalos.

Here I predict the clustering of subhalos as a function of redshift, mass, and scale, using a modification of the method of Mo & White (1996). I use the formation rate of halos to determine when to identify halos of a particular mass and then evaluate the clustering of those halos at the redshift of observation. Where cosmological parameters are necessary for these calculations I use those given by Bennett et al. (2003) in their Table 3.

2. ESTIMATING THE CLUSTERING OF SUBHALOS OF A SPECIFIC MASS

Consider a set of subhalos which when they reach mass $M$ are identifiable galaxies that retain their unique identities throughout history until $z = 0$, even after they merge into larger halos. Let us make one simplifying assumption: whether a subhalo of a given mass retains its identity during mergers into larger halos is independent of when it formed and its large scale environment. In other words, of all subhalos of mass $M$ which ever existed, those which survive form a representative sample. This assumption is violated in the real universe and in simulations, but it provides an alternative to the standard treatment, in which each subhalo always loses its identity immediately after its formation.

One can calculate the dependence on redshift of the formation rate of such subhalos from the extended Press-Schechter theory, the clustering at each redshift using the ansatz of Mo & White (1996), and integrate the results to the observed redshift $z_{obs}$ using the continuity equation (Nusser & Davis 1994; Fry 1996; Tegmark & Peebles 1998). Notably, the same procedure cannot predict the abundance of subhalos as a function of mass, without further specification of the conditions of subhalo formation and survival (Sheth & Pitman 1997; Benson et al. 2001; Somerville et al. 2001).

The Press-Schechter formulae are derived by assuming that the primordial overdensity field $\delta$ is a Gaussian random field. The smoothed density field is distributed as a Gaussian with variance $\sigma^2(M)$, where $M$ corresponds to the average mass contained within a smoothing length $R$. One associates each parcel of mass in the original distribution to halos of mass $M$ at a particular redshift $z$ if at that point in space, the smoothed density field on scale $R$ linearly extrapolated (using the growth function $D(z)$) to $z$ is exactly some $\delta_c$. The conventional choice is $\delta_c = 1.7$. We define $\delta_c(z) = \delta_c/\sqrt{D(z)}$. The fraction of mass in such a state is

$$f(M, z) dM = \frac{1}{\sqrt{2\pi} \sigma^3(M)} \left[ \frac{d\sigma^2(M)}{dM} \right] \times \exp \left[ -\frac{\delta_c^2(z)}{2\sigma^2(M)} \right] dM. \quad (1)$$

Lacey & Cole (1993) derive expressions for the probability of merging between dark matter halos. The fraction of mass in halos of mass $M_2$ which just entered from halos of mass $M_1$ is:

$$\frac{d^2f(M_1|M_2, z)}{dzdM_1} = \frac{1}{\delta_c \left[ 1 - \sigma^2(M_2)/\sigma^2(M_1) \right]^{3/2}} \times \exp \left[ \frac{\delta_c^2}{2\sigma(M_1)} \left( \frac{d\delta_c}{dz} \right) f(M_1, z) \right]. \quad (2)$$

The formation rate of mass $M$ halos is proportional to the fraction of mass in $M$ halos which has entered recently.
from all smaller halos. One can calculate this from the fraction entering \( M \) from each smaller mass halo, integrated over all possible smaller mass halos:

\[
\frac{d\text{form}(M,z)}{dz} = \int_0^M \frac{dM_1}{dM} \frac{d\text{form}(M_1|M,t)}{dM_1} f(M) = \frac{1}{\sqrt{2\pi}} \left( -\frac{d\delta_c}{dz} \right) f(M,z) \times \lim_{M_1 \to M} \frac{2}{\sigma^2(M_1) - \sigma^2(M)} \quad (3)
\]

The divergence is irrelevant to the final result; below, I will discuss only the “shape” of the formation rate, which I express as

\[
\frac{dG}{dz} = \delta_c(z) \left( -\frac{d\delta_c}{dz} \right) \exp \left[ -\frac{\delta_c^2}{2\sigma^2(M)} \right]. \quad (4)
\]

The Press-Schechter formalism has a simple prediction that the clustering of halos of a particular mass is independent of their formation history. Thus the overdensity the newly born halos of mass \( M \) relative to their mean density is the same as the overdensity of all halos of mass \( M \) relative to their mean, for which Mo & White (1996) obtain:

\[
1 + \langle \delta_h | \delta \rangle = \frac{1 - \delta_0 / \delta_c}{[1 - \sigma^2(M_0)/\sigma^2(M)]^{3/2}} \times \exp \left[ -\frac{\delta_0^2}{2\sigma^2(M)} - \frac{\delta_c^2}{2\sigma^2(M)} \right] \times \left\{ 1 - \exp \left[ -\frac{\delta_0^2}{2\sigma^2(M)} - \frac{\delta_c^2}{2\sigma^2(M)} \right] \right\}. \quad (5)
\]

Assume a subhalo is in a mass overdensity \( \delta \) at the redshift of observation \( z_{\text{obs}} \), which determines the linearly extrapolated overdensity \( \delta_0 \). From this, one can find \( \langle \delta_{\text{form}}(M,z_f,\delta_0) | \delta \rangle \) at formation for subhalos of mass \( M \) formed at some redshift \( z_f > z_{\text{obs}} \) which end up in mass overdensities of \( \delta \) at redshift \( z_{\text{obs}} \). Then:

\[
1 + \langle \delta_{\text{sh}}(M,z_{\text{obs}}) | \delta \rangle = \frac{1 + \delta(z_{\text{obs}})}{G(M,z_{\text{obs}})} \times \int_{z_{\text{sh}}}^\infty dz_f [1 + \delta_{\text{form}}(M,z_f,\delta_0)] \frac{dG(M,z_f)}{dz_f}, \quad (6)
\]

To calculate quantities in the nonlinear regime, one must relate \( \delta_0 \) to \( \delta \), which I do using the formulae given by Mo & White (1996) for an Einstein-de Sitter universe (the solution to the nonlinear spherical collapse problem is roughly independent of cosmology).

The integrals above may be done explicitly to yield

\[
1 + \langle \delta_{\text{sh}}(M,z_{\text{obs}}) | \delta \rangle = \frac{1 + \delta(z_{\text{obs}})}{[1 + \sigma^2(M_0)/\sigma^2(M)]^{3/2}} \times \Delta^2 \exp \left[ -\frac{\delta_0^2}{2\Delta^2} \right] - \Sigma^2 \exp \left[ -\frac{\Sigma^2}{2\Delta^2} \right] + \sqrt{2\pi} \Sigma \exp \left[ \delta_0^2/\Sigma^2 \right] \text{erfc}(x_c) \quad (7)
\]

where

\[
\Delta^2 = \sigma^2(M) - \sigma^2(M_0)
\]
\[
\Sigma^2 = 3\sigma^2(M) - \sigma^2(M_0)
\]
\[
\Phi^2 = 2\delta_0^2 \Delta^2/\sigma^2(M_0) \Sigma^2
\]
\[
\delta_c = \frac{\delta_0 - (2\sigma^2(M) - \sigma^2(M_0))\delta_0/\Sigma^2}{\sqrt{2\Sigma^2}} \quad (8)
\]

To calculate the cross-correlation \( \langle \delta_{\text{sh}} | \delta \rangle \) I follow Mo & White (1996) and use the log-normal form for the distribution of the dark matter overdensity \( \delta \):

\[
\langle \delta_{\text{sh}} | \delta \rangle = \int_{-\infty}^\infty d\delta \delta \langle \delta_{\text{sh}} | \delta \rangle f(\delta) = \int_{-\infty}^\infty d\delta \exp \left[ -\delta^2/(2\sigma^2) \right] \langle \delta_{\text{sh}} | \delta \rangle \delta, \quad (9)
\]

where \( x = \ln(1 + \delta) + \sigma^2/2 \) and \( \delta^2 = \exp(\sigma^2) - 1 \).

At linear scales the above equations can be reduced to a linear bias of the form

\[
b_{\text{sh}}(z) = 1 + \frac{\delta_c(z)}{\sigma^2(M)D(z)}, \quad (10)
\]

which has the notable property that it is always greater than unity.

Note that I have not specified what overall number of subhalos end up at each mass! In fact, without further assumptions, one cannot do so. I have only assumed that the subhalos that survive at each mass have clustering properties representative of all the subhalos ever to have that mass. I ignore that this condition is almost certainly violated in the real universe.

### 3. RESULTS

First consider Figure 1, showing the halo overdensity as a function of dark matter overdensity on scales of \( R = 4 \) h \(^{-1}\) Mpc, for masses \( M = 10^{10}, 10^{11}, \) and \( 10^{12} M_\odot \), as labeled, and for redshifts \( z = 0, 1, 2, 3, 4, \) and 5, as labeled.

![Figure 1](image-url)
is not far from assuming that all galaxies formed at near the median redshift and evaluate their clustering at another redshift. Consider the top panel of Figure 3, which shows as the solid line the results of Equation 7 as a function of mass for smoothing scales of $R = 8$ and $30 \ h^{-1} \text{Mpc}$ (as labeled), denoting bias as $b = \langle \delta_{sh} \delta \rangle / \langle \delta \delta \rangle$. For comparison, consider the dashed lines, which show the bias as a function of mass using the Mo & White (1996) results for halos identified at $z = 1.5$ and observed at $z = 0$. In general, for my results the bias is less different than unity, because the bias near the median time of formation is typically closer to unity. Furthermore, the low mass halos which form very early naturally debias according to the continuity equation. The bias is much less mass dependent under my assumptions than it is under the assumption that all halos stop merging at some fixed redshift.

These results are extremely close to what one would get assuming all subhalos were formed at the median of the $dG/dz$ distribution, determine by setting

$$\delta_c^2(z_{\text{med}}) = 2\sigma^2(M) \delta_{c,0}^2 \ln 2 + \delta_c(z_{\text{obs}}).$$  \hspace{1cm} (11)$$

The dashed lines in the bottom panel of Figure 3 show the mass dependence using the standard Mo & White (1996) treatment but setting the redshift of identification at each mass to $z_{\text{med}}$.

Finally, although at low mass the bias is a very weak function of mass for the subhalos, recently assembled subhalos are in lower density regions than older subhalos. Figure 4 plots the bias of subhalos as a function of their mass at high masses mass is important. Accepting that galaxy luminosity is related to subhalo mass, and that the optical color of a galaxy relates to the time of subhalo assembly, this trend is qualitatively similar to the trend found in Hogg et al. (2003). Those authors show that for low luminosity galaxies, color is most closely related to galaxy overdensity, while for high luminosity galaxies, luminosity
is the most closely related to galaxy overdensity.

However, the transformation from age of assembly and mass of a subhalo to color and luminosity of a galaxy is by no means simple or even necessarily one-to-one. In particular, the simplest interpretation of the hierarchical merging scenario of galaxy formation is that the most massive galaxies have assembled the most recently; however, it is observationally the case that the most massive galaxies have the oldest, reddest stellar populations. So if this scenario has any truth at all, it must be the case that age of assembly and color are not related at least for the most luminous galaxies, and so drawing firm conclusions for the low mass subhalos would be premature.

Other results of Hogg et al. (2003) are more difficult to understand with this picture, such as the large overdensities associated with red, low luminosity galaxies. This discrepancy may indicate that such galaxies preferentially survive in large mass clusters rather than in lower mass groups.

4. DISCUSSION

An observational consequence of the model presented here is that the clustering of galaxies should be a weak function of their mass, especially at low masses. Many papers have claimed that the average clustering is a weak function of their mass, especially at low masses. Many here is that the clustering of galaxies should be a weak function of color.

These suggestive comparisons to observations inspire testing of the assumptions used here with N-body simulations. Unfortunately, identifying subhalos in simulations is difficult. While many investigators are using high resolution simulations and identifying subhalos, not much literature quotes directly subhalo bias with respect to mass overdensity. The results of Kravtsov & Klypin (1999) show that the relationship between their subhalo population and the mass overdensity is more linear and closer to $b = 1$ than the Mo & White (1996) prediction for halos, but that if they identify halos at $z = z_{\text{obs}} + 1$ and observe them at $z_{\text{obs}}$ the formulae of Mo & White (1996) provide a reasonable fit. It would be interesting use similar simulations to predict clustering as a function of subhalo mass and to compare with the simple estimates I present here.

Differences from this model in the observations or in the simulations may indicate a breakdown of the assumption that subhalo survival probability is independent of environment and may provide an insight into the nature of that process.

A significant improvement of this model is to express the argument here in terms of the “halo model” of galaxy formation predicting the distribution of subhalos within each halo (see Berlind & Weinberg 2002 and references therein). This requires an extension of the typical excursion set picture of Press-Schechter, and is implemented by Sheth (in preparation).

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![Fig. 4.— Contour plot of the bias of subhalos $b = (\delta_{\text{halo}}/\delta)/(\delta_h)$ at redshift $z = 0$ on 8 h$^{-1}$ Mpc smoothing scales as a function of mass and time of assembly. At low masses time of assembly is more strongly related to the large scale density field than it is at high masses.](image)