Hot nuclear matter: A variational approach

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We develop a nonperturbative technique in field theory to study properties of infinite nuclear matter at zero temperature as well as at finite temperatures. Here we dress the nuclear matter with off-mass shell pions. The techniques of thermofield dynamics are used for finite temperature calculations. Equation of state is derived from the dynamics of the interacting system in a self consistent manner. The transition temperature for nuclear matter appears to be around 15 MeV.

(To appear in Int. J. Mod. Phys. E)
I. INTRODUCTION

The understanding of hot dense nuclear matter is an interesting problem in theoretical physics in the context of heavy ion collision experiments as well as big bang cosmology. The problem here is basically nonperturbative. The interaction of nucleons which may arise as a residual interaction due to their substructure of quarks and gluons is technically not solvable. The usual approach is to tackle the problem through meson nucleon interactions. This also entails a nontrivial technical problem with the pion nucleon coupling $G_{NN\pi}^2/4\pi$ being as large as 14.6 making any perturbative calculation unreliable. An alternative consistent theoretical framework has been developed by Walecka\(^1\) consisting of interactions of nucleons with a neutral scalar field $\sigma$ as well as a neutral vector meson $\omega$. This has been done at zero temperature as well as finite temperatures.\(^2\) Variations of the same model has also been considered including cubic and quartic terms in the $\sigma$ fields to reproduce correct bulk modulus of nuclear matter.\(^3\) These calculations however use meson fields as classical, and, use a $\sigma$-field which is not observed.

An alternative model for infinite nuclear matter consisting of interacting nucleons and pions was considered in Ref [4]. The nuclear matter was dressed with off mass shell pion quanta. The scalar isoscalar pion condensates simulated the effects of $\sigma$ mesons\(^5\) with the short distance repulsion arising from composite structure of nucleons and/or through vector meson exchanges. This is not only aesthetically appealing with classical $\sigma$ fields arising from quantum mechanical structures but also has a stronger phenomenological appeal as $\sigma$ mesons have not been found in nature. With a similar approach we shall consider nuclear matter at finite temperatures. The methods of thermofield dynamics\(^6\) fit naturally for this purpose because here statistical average is done through an expectation value over a ”thermal vacuum” in an extended Hilbert space.

The outline of the paper is as follows. In Sec. II we shall consider nuclear matter with pion condensates at zero temperature. Here we shall briefly summarize the results of Ref. [4] and show that these results constitute a particular approximation of the present
method. In Sec. III we shall discuss finite temperature nuclear matter using the methods of thermofield dynamics. We shall calculate thermodynamic quantities like pressure, entropy density and energy density for nuclear matter and derive the equation of state for the same. Section IV will consist of discussions of the results obtained in the present model. A possible experimental signature resulting from the model is discussed in Sec. V. In Appendix A we shall summarise some results of thermofield dynamics for the sake of completeness.

II. ZERO TEMPERATURE FORMALISM

We shall here start with the effective Hamiltonian for pion nucleon interactions which was derived in Ref. [4] and given as

\[ \mathcal{H}_N(\vec{x}) = \mathcal{H}_N^{(0)}(\vec{x}) + \mathcal{H}_{int}(\vec{x}) \]  

where the free nucleon part \( \mathcal{H}_N^{(0)} \) is given as

\[ \mathcal{H}_N^{(0)}(\vec{x}) = \psi_1(\vec{x})\epsilon_x \psi_1(\vec{x}) \]  

and the effective pion nucleon interaction part is given as

\[ \mathcal{H}_{int}(\vec{x}) = \psi_1(\vec{x}) \left[ -i \frac{G}{2\epsilon_x} \vec{p} \psi_1 + \frac{G^2}{2\epsilon_x} \phi_i^2 \right] \psi_1(\vec{x}). \] 

In the above, \( \epsilon_x = (M^2 - \omega_z^2)^{1/2} \) where \( M \) denotes the nucleon mass. Furthermore the free meson part of the Hamiltonian is given as

\[ \mathcal{H}_M(\vec{x}) = \frac{1}{2} \left[ \dot{\phi}_i^2 + (\nabla \phi_i) \cdot (\nabla \phi_i) + m^2 \phi_i^2 \right] \] 

Clearly in the above \( \psi_I \) refers to the large component of the nucleon spinor and \( \phi = \tau_i \phi_i \).

We expand the field operator \( \phi_i(\vec{z}) \) in terms of the creation and annihilation operators of off-mass shell mesons satisfying equal time algebra as

\[ \phi_i(\vec{z}) = \frac{1}{\sqrt{2\omega_z}} (a_i(\vec{z})^\dagger + a_i(\vec{z})) \] 

and
\[ \phi_i(\vec{z}) = i \sqrt{\frac{\omega_z}{2}} (a_i(\vec{z})^\dagger - a_i(\vec{z})) \] (6)

where we take with the perturbative basis \( \omega_z = (m^2 - \nabla_z^2)^{1/2} \), with \( m \) denoting the mass of the meson.

The two pions in Eq. (3) constitute a scalar-isoscalar interaction of nucleons and thus could simulate the effects of \( \sigma \)-mesons. With this in mind, let us consider a two pion creation operator given as

\[ B^\dagger = \frac{1}{2} \int \tilde{f}(\vec{k}) a_i(\vec{k})^\dagger a_i(-\vec{k})^\dagger d\vec{k} \] (7)

where the arbitrary function \( \tilde{f}(\vec{k}) \) will be determined by a variational procedure. We now introduce a 'meson' dressing of nuclear matter through the state

\[ |f\rangle = U |\text{vac}\rangle \] (8)

where

\[ U = \exp(B^\dagger - B) \] (9)

In contrast to Ref. [4], the operator \( U \) is unitary, and thus we have

\[ <f|f\rangle = 1 \] (10)

This makes the formalism simpler and more realistic. Further with this ansatz, one easily obtains

\[ U^\dagger a_i(\vec{k}) U = (\cosh \tilde{f}(\vec{k})) a_i(\vec{k}) + (\sinh \tilde{f}(\vec{k})) a_i(-\vec{k})^\dagger \] (11)

which is a Bogoliubov transformation to be used later for the calculations.

We shall take \( N \) nucleons occupying a spherical volume \( V = 4\pi \frac{R^3}{3} \), where \( N/(4\pi \frac{R^3}{3}) = \rho \) remains constant as \( N \to \infty \) and we neglect the surface effects. We describe the system with the density operator \( \hat{\rho}_N \) such that

\[ tr[\hat{\rho}_N \psi_\beta(\vec{y})^\dagger \psi_\alpha(\vec{x})] = \rho_{\alpha\beta}(\vec{x}, \vec{y}) \] (12)
and

\[ tr[ \rho_N \hat{N} ] = \int \rho_{\alpha\alpha}(\vec{x}, \vec{x})d\vec{x} = N = \rho V \]  

(13)

We obtain the total nucleon energy as

\[ h_f = < f | tr[ \rho_N \mathcal{H}_N^{(0)}(\vec{x}) ] | f > \]
\[ = \frac{\gamma}{6\pi^2} k_f^3 (M + \frac{3}{10} \frac{k_f^2}{M}) \]  

(14)

where \( \gamma = 4 \) for nuclear matter and \( \rho \) and \( k_f \) are related by the equation \( \rho = \gamma k_f^3 / 6\pi^2 \). With the meson field operator expansion as in Eq. (5) and (6) we may write eq. (4) as

\[ \mathcal{H}_M(\vec{x}) = a_i(\vec{x})^\dagger \omega_{\vec{x}} a_i(\vec{x}) \]  

(15)

Using Eq.(11) we now obtain kinetic energy density due to the mesons as

\[ h_k = < f | \mathcal{H}_M(\vec{x}) | f > \]
\[ = 3(2\pi)^{-3} \int d\vec{k} \omega(\vec{k}) sinh^2 \tilde{f}(\vec{k}) \]  

(16)

where \( \omega(\vec{k}) = \sqrt{\vec{k}^2 + m^2} \). We next proceed to evaluate from Eq.(3) the interaction energy density, with \( \epsilon_x \simeq M \),

\[ h_{int} = < f | tr[ \rho_N \mathcal{H}_{int}(\vec{x}) ] | f > \]
\[ \simeq \frac{G^2}{2M} \rho < f | : \phi_i(\vec{x}) \phi_i(\vec{x}) : | f > \]  

(17)

Using the Bogoliubov transformation (11) we have from equation (17)

\[ h_{int} = \frac{G^2 \rho}{2M} \cdot 3(2\pi)^{-3} \int d\vec{k} \omega(\vec{k}) \left( \frac{sinh2\tilde{f}(\vec{k})}{2} + sinh^2 \tilde{f}(\vec{k}) \right) \]  

(18)

The meson energy density thus is given as

\[ h_m = h_{int} + h_k \]  

(19)

Now extremising equation (19) with respect to \( \tilde{f}(\vec{k}) \) we obtain the solution
\[ \tanh^2 \tilde{f}(\vec{k}) = -\frac{G^2 \rho}{2M} \cdot \frac{1}{\omega^2(\vec{k}) + \frac{G^2 \rho}{2M}} \]  \hfill (20)

We may compare the same with the results of Ref. [4] which corresponds to the first term in the expansion of left hand side of Eq. (20). We then obtain the corresponding meson energy density from the kinetic and interaction terms as

\[ h_m = h_k + h_{int} = -\frac{3}{2} \cdot (2\pi)^{-3} \left( \frac{G^2 \rho}{2M} \right)^2 \int \frac{d\vec{k}}{\omega(\vec{k})} \left[ \omega(\vec{k}) \left( \omega(\vec{k})^2 + \frac{G^2 \rho}{2M} \right)^{1/2} + (\omega(\vec{k})^2 + \frac{G^2 \rho}{2M}) \right] \]  \hfill (21)

We note that Eq. (21) is not acceptable since the energy density diverges. This happens because we have taken the pions to be point like and assumed that they can approach as near each other as they like, which is physically not correct. If we bring two pions close to each other there will be an effective force of repulsion because of their composite structure. We thus assume a phenomenological term corresponding to meson repulsion as

\[ h_m^R = 3a(2\pi)^{-3} \int (\sinh^2 \tilde{f}(\vec{k})) e^{R_{\pi}^2k^2} d\vec{k} \]  \hfill (22)

where \( a \) and \( R_{\pi} \) are two parameters to be determined later. Now extremising equation (16), (18) and (22) with respect to \( \tilde{f}(\vec{k}) \), we obtain

\[ \tanh^2 \tilde{f}(\vec{k}) = -\frac{G^2 \rho}{2M} \cdot \frac{1}{\omega^2(\vec{k}) + \frac{G^2 \rho}{2M} + a\omega(\vec{k})e^{R_{\pi}^2k^2}}. \]  \hfill (23)

In place of Eq. (21) we now obtain the expression for \( h_m \) as

\[ h_m = h_k + h_{int} + h_m^R = -\frac{3}{2} \cdot (2\pi)^{-3} \left( \frac{G^2 \rho}{2M} \right)^2, \]

\[ \int \frac{d\vec{k}}{\omega^2} \left[ \left( \omega + a\epsilon R_{\pi}^2k^2 \right)^{1/2} \left( \omega + a\epsilon R_{\pi}^2k^2 + \frac{G^2 \rho}{2M\omega} \right)^{1/2} + \left( \omega + a\epsilon R_{\pi}^2k^2 \right) + \frac{G^2 \rho}{2M\omega} \right] \]  \hfill (24)

where \( \omega = \omega(\vec{k}) \). Finally we have to include the energy of repulsion which may arise from vector meson interaction and/or from finite size of the nuclei. We shall here parametrize the effect of such a repulsion contribution by the simple form
\[ h_R = \lambda \rho^2 \]  

(25)

where \( \lambda \) is an arbitrary constant to be fixed from phenomenology. We note that equation (25) can arise from a Hamiltonian density given as

\[ H_R(\vec{x}) = \psi(\vec{x})^\dagger \psi(\vec{x}) \int v_R(\vec{x} - \vec{y}) \psi(\vec{y})^\dagger \psi(\vec{y}) d\vec{y} \]  

(26)

where, when density is constant, we in fact have

\[ \lambda = \int v_R(\vec{r}) d\vec{r} \]

We next minimise the energy per nucleon given by

\[ E = \frac{(h_m + h_f + h_R)}{\rho} \]  

(27)

as a function of \( \rho \), with the parameters \( a, R_\pi \) and \( \lambda \) to be subsequently fixed. Extremizing the single particle energy, for \( \lambda = 0.54 \text{ fm}^2 \), \( R_\pi = 1.18 \text{ fm} \) and \( a = 0.12 \text{ GeV} \) we have the single particle energy given as \( E = -15.03 \text{ MeV} \) at the saturation density \( \rho = 0.19 \text{ fm}^{-3} \) corresponding to \( k_f = 1.42 \text{ fm}^{-1} \). The equation for \( E \) vs. \( \rho \) is shown in Fig. 1. The incompressibility of the nuclear matter is given as

\[ K = k_f^2 \frac{\partial^2 E}{\partial k_f^2} = 151.2 \text{ MeV} \]  

(28)

We may note that the parameters and the results of the present analysis do not differ significantly from those of Ref. [4]. As may be seen in Eq.(20) or (23) Ref. [4] is an approximation of the present framework.

We may also calculate the average pion number per nucleon \( \rho_m \) given as

\[ \rho_m = \langle f | a_i^\dagger(\vec{z}) a_i(\vec{z}) | f \rangle = 3(2\pi)^{-3} \int d\vec{k} \sinh^2 \vec{k}(\vec{k}) \]  

(29)

which we plot in Fig. 4 as a function of nucleon density. This may be relevant for heavy ion collisions where nuclear matter gets compressed.
III. FINITE TEMPERATURE FORMALISM

We shall now generalize the formalism developed in Sec. II for finite temperatures. For this purpose we shall use the methodology of thermofield dynamics. In Appendix A, we summarize the salient features of the same. Here Eq.(7) for the background pion will become modified with introduction of extra thermal modes as illustrated in the Appendix. In fact, we shall have the temperature dependent background off-shell pion pair configuration given as

\[ |f, \beta > = U_I(\beta)|f > = U_I(\beta)|\text{vac} > \]  

where \( U \) is the same as in Eq.(9), and \( U_I(\beta) \) describes the change with temperature. The expression for this in terms of ordinary and thermal modes is given as

\[ U_I(\beta) = \exp(B_I^\dagger(\beta) - B_I(\beta)) \]  

where, as in Eq.(A4) of Appendix A,

\[ B_I(\beta)^\dagger = \frac{1}{2} \int \theta_B(\bar{k}, \beta)b_i(\bar{k})\tilde{b}_i(-\bar{k})^\dagger d\bar{k} \]  

In the above,

\[ b_i(\bar{k})^\dagger = U a_i(\bar{k})U^\dagger \]  

so that \( b_i(\bar{k})|f >= 0 \). Thus \( b_i(\bar{k})^\dagger \) creates excitations over the zero temperature configuration given by \( |f> \). As before we shall calculate the energy expectation values, but here shall minimise the thermodynamic potential as appropriate at finite temperatures. To do so we note that a little algebra yields

\[ U^\dagger a_i(\bar{k})U = (\cosh \tilde{f}(\bar{k}))(\cosh \theta_B(\bar{k}, \beta))b_i(\bar{k}, \beta) + (\sinh \tilde{f}(\bar{k}))(\cosh \theta_B(\bar{k}, \beta))b_i(-\bar{k}, \beta)^\dagger \\
+ (\sinh \tilde{f}(\bar{k}))(\sinh \theta_B(\bar{k}, \beta))\tilde{b}_i(-\bar{k}, \beta) + (\cosh \tilde{f}(\bar{k}))(\sinh \theta_B(\bar{k}, \beta))\tilde{b}_i(\bar{k}, \beta)^\dagger \]  

(34)
which again is a Bogoliubov transformation, and will be used for obtaining the energy expectation values. The parallel unitary transformation as in Eq. (30) for the temperature dependance in nucleon sector for fermions is given as

\[ U_{II}(\beta) = \exp(B_{II}^\dagger(\beta) - B_{II}(\beta)) \] (35)

with

\[ B_{II}(\beta)^\dagger = \frac{1}{2} \int \theta_F(\vec{k}, \beta) \psi_I(\vec{k})^\dagger \tilde{\psi}_I(-\vec{k})^\dagger d\vec{k} \] (36)

where \( \theta_F(\vec{k}, \beta) \) will be determined later. As earlier for thermal averages, we shall replace \( \hat{\rho}_N \hat{\Delta} \) in Eq. (13) by

\[ \hat{\rho}_N \psi_\alpha^\dagger(\vec{x}) \psi_\alpha(\vec{x}) \rightarrow U_{II}(\beta)^\dagger \psi_\alpha^\dagger(\vec{x}) \psi_\alpha(\vec{x}) U_{II}(\beta). \]

We then have the nuclear matter density

\[ \rho(\beta) = \langle \text{vac} | U_{II}(\beta)^\dagger \psi_\alpha^\dagger(\vec{x}) \psi_\alpha(\vec{x}) U_{II}(\beta) | \text{vac} \rangle = \gamma (2\pi)^{-3} \int d\vec{k} \sin^2 \theta_F \] (37)

As noted in the Appendix A, \( \sin^2 \theta_F \) is the distribution function for the fermions. Clearly, with \( \sin^2 \theta_F = \Theta(k_f - k) \) Eq. (37) gives \( \rho = \gamma k_f^3 / 6\pi^2 \) of zero temperature. \( \sin^2 \theta_F \) for the interacting system will be determined here from the construction of the thermodynamic potential.

We shall now calculate different contributions to the energy expectation values corresponding to the Hamiltonian as in Eq. (1) and (4) as well as Eq. (25). We thus have for the nucleon kinetic term

\[ h_f(\beta) = \langle \text{vac} | U_{II}(\beta)^\dagger \psi_I^\dagger(\vec{x}) \frac{(-\nabla^2)}{2M} \psi_I(\vec{x}) U_{II}(\beta) | \text{vac} \rangle = \gamma (2\pi)^{-3} \int d\vec{k} \frac{k^2}{2M} \sin^2 \theta_F \] (38)

The kinetic energy due to the mesons is given by
\[ h_k(\beta) = <f, \beta | \mathcal{H}_M(\vec{x}) | f, \beta > \]
\[ = 3(2\pi)^{-3} \int \frac{d\vec{k}}{\omega(\vec{k})} [\sinh^2 \bar{f}(\vec{k}) \cosh 2\theta_B(\vec{k}, \beta) + \sinh^2 \theta_B(\vec{k}, \beta)] \]  

(39)

where \( \omega(\vec{k}) = \sqrt{\vec{k}^2 + m^2} \). In the above, we may note that when \( \theta_B \to 0 \) it reduces to the Eq. (16). We next derive the interaction energy density from Eq. (3) as

\[ h_{int}(\beta) = <f, \beta | \mathcal{U}_{II}(\beta)^\dagger \psi_1(\vec{x}) \phi(\vec{x}) \mathcal{U}_{II}(\beta) | f, \beta > \]
\[ \approx \frac{G^2}{2M} \rho(\beta) <f, \beta | :\phi_i(\vec{x}) \phi_i(\vec{x}) : | f, \beta > \]
\[ = \frac{G^2 \rho(\beta)}{2M} I_{2M} \]  

(40)

where

\[ I_{2M} = \frac{3}{(2\pi)^3} \int \frac{d\vec{k}}{\omega(\vec{k})} \left( \frac{\sinh^2 \bar{f}(\vec{k}) \cosh 2\theta_B}{2} + \sinh^2 \bar{f}(\vec{k}) \cosh 2\theta_B + \sinh^2 \theta_B \right) \]  

(41)

As in Eq. (22) we shall now assume a phenomenological term corresponding to meson repulsion due to composite structure of mesons given as

\[ h_m^R(\beta) = 3a(2\pi)^{-3} \int (\sinh^2 \bar{f}(\vec{k}) \cosh 2\theta_B + \sinh^2 \theta_B) e^{R^2 k^2} \]  

(42)

Finally, the nucleon repulsion term parallel to Eq.(25) is

\[ h_R = \lambda \rho^2(\beta) \]  

(43)

where \( \rho(\beta) \) is as given in Eq. (37). Thus the energy density is given by

\[ E(\beta) = (h_f(\beta) + h_m(\beta) + h_R(\beta))/\rho(\beta) \]  

(44)

where

\[ h_m(\beta) = h_k(\beta) + h_m^R(\beta) + h_{int}(\beta) \]

as before.

The thermodynamic potential density \( \Omega \) is given by

\[ \Omega(\beta) = E(\beta)\rho - \frac{\sigma}{\beta} - \mu \rho \]  

(45)
where the last term corresponds to nucleon number conservation with $\mu$ as the chemical potential. We may note that we shall be considering temperatures much below the nucleon mass so that in the expression for $\rho(\beta)$ we do not include antiparticle channel. The entropy density above is $\sigma = \sigma_F + \sigma_B$ with $\sigma_F$ being the entropy in fermion sector given as

$$\sigma_F = -\frac{\gamma}{(2\pi)^3} \int d\vec{k} \left[ \sin^2 \theta_F(\vec{k}, \beta) \ln(\sin^2 \theta_F(\vec{k}, \beta)) + \cos^2 \theta_F(\vec{k}, \beta) \ln(\cos^2 \theta_F(\vec{k}, \beta)) \right].$$

and similarly the meson sector contribution $\sigma_B$ is given as

$$\sigma_B = -\frac{3}{(2\pi)^3} \int d\vec{k} \left[ \sinh^2 \theta_B(\vec{k}, \beta) \ln(\sinh^2 \theta_B(\vec{k}, \beta)) - \cosh^2 \theta_B(\vec{k}, \beta) \ln(\cosh^2 \theta_B(\vec{k}, \beta)) \right].$$

Thus the thermodynamic potential density now is a functional of $\theta_F(\vec{k}, \beta)$, $\theta_B(\vec{k}, \beta)$ as well as the pion dressing function $\tilde{f}(\vec{k})$ which will of course depend upon temperature. Extremisation of Eq (45) with respect to $\tilde{f}(\vec{k})$ yields

$$\tanh 2\tilde{f}(\vec{k}) = -\frac{G^2 \rho}{2M} \cdot \frac{1}{\omega^2(\vec{k}) + \frac{G^2 \rho}{2M} + a\omega(\vec{k}) e^{R_k k^2}}$$

which is of the same form as Eq. (23) for zero temperature case except that now $\rho$ is temperature dependent through Eq. (37). Similarly minimising the thermodynamic potential with respect to $\theta_B(\vec{k}, \beta)$ we get

$$\sinh^2 \theta_B = \frac{1}{e^{\beta \omega'} - 1}$$

where

$$\omega' = (\omega + \frac{G^2 \rho}{2M\omega} + a e^{R_k k^2}) \cosh 2\tilde{f}(\vec{k}) + \frac{G^2 \rho}{2M\omega} \sinh 2\tilde{f}(\vec{k})$$

Once we substitute the optimised dressing as in Eq. (46), the above simplifies to

$$\omega' = (\omega + \frac{G^2 \rho}{M\omega} + a e^{R_k k^2})^{1/2} (\omega + a e^{R_k k^2})^{1/2}$$

which is different from $\omega$ due to interactions. Further, minimising the thermodynamic potential with respect to $\theta_F(\vec{k}, \beta)$ we have the solution

$$\sin^2 \theta_F = \frac{1}{e^{\beta(\epsilon_F - \mu)} + 1}$$
with

\[ \epsilon_F = \frac{G^2}{2M} I_{2M} + 2\lambda \rho + \frac{k^2}{2M} \]  \hspace{1cm} (51)

where \( I_{2M} \) is given in equation (41). We may note that the change in \( \epsilon_F \) above from \( k^2/2M \) is also due to interaction.

**IV. RESULTS**

To calculate different thermodynamic quantities as functions of baryon number density we first use Eq. (37) to calculate the chemical potential \( \mu \) in a self consistent manner with \( \rho \) and \( \mu \) occurring also inside the integrals through \( \sin^2 \theta_F \) as in Eq. (50). Thus for each \( \rho \), we determine \( \mu \) so that Eq. (37) is satisfied. The ansatz functions \( \theta_B, \theta_F \) and \( \tilde{f}(\vec{k}) \) get determined analytically through the extremisation of the thermodynamic potential and the parameters \( a, \lambda \) and \( R_\pi \) are as obtained in Sec. II.

With the thermodynamic potential determined as above, we calculate different thermodynamic quantities. We first plot the binding energy per nucleon as a function of baryon number density in Fig. 1 for temperatures zero MeV, 5 MeV, 10 MeV and 15 MeV. As the temperature increases the minimum shifts towards higher densities. This may be understood from the fact that to compensate for the larger kinetic energy a large value of \( \rho \) is needed to give the minimum until the nuclear repulsion effects take over and hence again energy increases. We may compare our curves with those of Ref [2]. Our curves compared to theirs are rather shallow. This is consistent with the fact that we get a smaller value of compressibility (\( \sim 150 \) MeV).

We next plot pressure as may be defined from thermodynamics

\[ P(\beta) = -\Omega(\beta) \]  \hspace{1cm} (52)

The same is plotted in Fig. 2. Vanishing of the pocket in the above curves for temperatures around 15 MeV may be noted which could be an effect of phase transition.
Entropy density is plotted as a function of nucleon density in Fig. 3 at temperatures 50 MeV, 100 MeV, 150 MeV and 200 MeV. The general tendency appears to be quite similar to that of Ref. [2].

Finally, in Fig. 4 we plot the average number of pions per nucleon ($\rho_m$) as a function of nucleon density $\rho$. It is interesting to note that this result is practically independent of temperature. Therefore only the zero temperature graph is plotted in Fig. 4. For low nucleon density $\rho_m$ rises rather sharply and almost levels off at $\rho \geq 0.3 \text{ fm}^{-3} \simeq \rho_s \times 1.5$ where $\rho_s$ is the density of stable nuclear matter at zero temperature. We note that the rise in meson number with compression may be observable in heavy ion collisions.

V. DISCUSSIONS

In the present model we have considered nuclear matter at zero temperature and finite temperatures in a nonperturbative manner for the interacting pion nucleon system using thermofield method. This yields results similar to those of mean field approximation, with $\sigma$-mesons not being needed, and, the calculations being quantum mechanical.

Besides the aesthetic appeal, we note that it may have experimental consequences. For example, the off shell $\pi^+\pi^-$ pair as present through pion dressing may annihilate to hard photons with a probability in excess of what one may expect otherwise. This is likely to depend on individual nuclei which we may dress with pions as here.

ACKNOWLEDGMENTS

The authors are thankful to A. Mishra, S.N. Nayak and Snigdha Mishra for many discussions. SPM acknowledges to the Department of Science and Technology, Government of India for the research grant SP/S2/K-45/89 for financial assistance.
APPENDIX A

We here summarize briefly the salient features of thermofield dynamics as used in the present paper.

In statistical mechanics, the thermal average of an operator $\hat{O}$ is given as, with $\beta = 1/kT$,

$$\langle \hat{O} \rangle_\beta = \frac{\text{Tr}(e^{-\beta H} \hat{O})}{\text{Tr}(e^{-\beta H})}$$  \hspace{1cm} (A1)

where, the trace is taken over a complete basis of states. First we note that in the zero temperature limit, the above reduces to ground state expectation value for the operator $\hat{O}$. This is easily seen as

$$\lim_{\beta \to \infty} \langle \hat{O} \rangle_\beta = \lim_{\beta \to \infty} \frac{\langle 0 \mid \hat{O} \mid 0 \rangle e^{-\beta \epsilon_0} + \langle 1 \mid \hat{O} \mid 1 \rangle e^{-\beta \epsilon_1} + \cdots}{e^{-\beta \epsilon_0} + e^{-\beta \epsilon_1} + \cdots}$$

$$= \lim_{\beta \to \infty} \frac{\langle 0 \mid \hat{O} \mid 0 \rangle + \langle 1 \mid \hat{O} \mid 1 \rangle e^{-\beta(\epsilon_1-\epsilon_0)} + \cdots}{1 + e^{-\beta(\epsilon_1-\epsilon_0)} + \cdots}$$

$$= \langle 0 \mid \hat{O} \mid 0 \rangle, \hspace{1cm} (A2)$$

where $| 0 \rangle$ corresponds to the state with the lowest energy. In thermofield method, one essentially generalises (A2) to the case of finite temperature and defines a ”thermal vacuum” such that the statistical average reduces to an expectation value with respect to the thermal vacuum. Thus we want that for some $| 0(\beta) \rangle$ the relationship

$$\langle \hat{O} \rangle_\beta = \frac{\text{Tr}(e^{-\beta H} \hat{O})}{\text{Tr}(e^{-\beta H})} = \langle 0(\beta) \mid \hat{O} \mid 0(\beta) \rangle, \hspace{1cm} (A3)$$

where $| 0(\beta) \rangle$ is defined as the ”thermal vacuum”. This can be done if one doubles the degrees of freedom i.e. corresponding to every physical operator $a$, a”tilde” operator $\tilde{a}$ is introduced. In the above, $\tilde{a}(\vec{k})$ are the new operators named as ”thermal modes”. They are associated with negative energy, with conventional quantisation, but, do not have any physical significance in the sense of observation of these modes. In zero temperature vacuum, these modes are absent, so that conventional field theory holds. At finite temperature the ground state is replaced by $| 0(\beta) \rangle$ given as
\begin{align}
|0(\beta) > & \equiv U_B(\beta)|vac > \\
& = \exp(\int \theta_B(\vec{k}, \beta)(a(\vec{k})^\dagger \tilde{a}(\vec{k})^\dagger - h.c.)d\vec{k}) | vac > , \quad (A4)
\end{align}

where \( \tilde{a}(\vec{k})^\dagger \) in the above corresponds to the extra Hilbert space. It is now convenient to define a thermal basis

\[
\begin{pmatrix}
a(\vec{k}, \beta) \\
\tilde{a}(-\vec{k}, \beta)^\dagger
\end{pmatrix} = U(\beta) \begin{pmatrix}
a(\vec{k}) \\
\tilde{a}(-\vec{k})^\dagger
\end{pmatrix} U(\beta)^{-1}, \quad (A5)
\]

which amounts to the Bogoliubov transformation

\[
\begin{pmatrix}
a(\vec{k}, \beta) \\
\tilde{a}(-\vec{k}, \beta)^\dagger
\end{pmatrix} = \begin{pmatrix}
\cosh \theta_B(\vec{k}, \beta) & -\sinh \theta_B(\vec{k}, \beta) \\
-\sinh \theta_B(\vec{k}, \beta) & \cosh \theta_B(\vec{k}, \beta)
\end{pmatrix} \begin{pmatrix}
a(\vec{k}) \\
\tilde{a}(-\vec{k})^\dagger
\end{pmatrix}. \quad (A6)
\]

\( a(\vec{k}, \beta) \) and \( \tilde{a}^\dagger(\vec{k}, \beta) \) are the annihilation and creation operators at temperature \( \beta = \frac{1}{kT} \) corresponding to the thermal vacuum such that

\[
a(\vec{k}, \beta) |0(\beta) >= 0 = \tilde{a}^\dagger(\vec{k}, \beta)|0(\beta) > .
\]

The function \( \theta_B(k, \beta) \) is calculated through minimization of thermodynamic potential density given as

\[
\Omega = (E(\beta) - \frac{1}{\beta} \sigma_B + \mu N) \quad (A7)
\]

where \( \mu \), the chemical potential corresponds to a conserved number, and the entropy density

\[
\sigma_B = -\frac{3}{(2\pi)^3} \int d\vec{k} \left[ \sinh^2 \theta_B(\vec{k}, \beta) \ln(\sinh^2 \theta_B(\vec{k}, \beta)) - \cosh^2 \theta_B(\vec{k}, \beta) \ln(\cosh^2 \theta_B(\vec{k}, \beta)) \right].
\]

For zero chemical potential and for free fields, extremization of the free energy then yields

\[
\sinh^2 \theta_B = \frac{1}{e^{\beta \omega(\vec{k}, \beta)} - 1} \quad (A8)
\]

where we have taken the Hamiltonian density as \( \mathcal{H}^0 = \int a^\dagger(\vec{z}) \omega_\omega a(\vec{z}) d\vec{z} \) so that

\[
E(\beta) = \langle vac, \beta|\mathcal{H}^0|vac \beta > \\
= \frac{1}{(2\pi)^3} \int \sinh^2 \theta_B \omega(\vec{k}) d\vec{k}
\]

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If we substitute this value the free energy density becomes

\[ F = E(\beta) - \frac{1}{\beta} \sigma_B = \frac{1}{\beta} \frac{1}{(2\pi)^3} \int \ln(1 - e^{-\beta \omega(\vec{k})}) \, d\vec{k} \]

which is the same as derived through a more conventional treatment of temperature dependent quantum field theory.\(^8\) As stated, we do here the analysis through thermofield dynamics since the calculations are simpler. In case of interacting field however the solution for \( \theta_B(\vec{k}, \beta) \) will not be given by equation (A8) and will depend upon the interaction.

Similarly for the fermionic sector the thermal vacuum will be given by

\[ |0(\beta) > \equiv U_F(\beta)|\text{vac} > = \exp(\int \theta_F(\vec{k}, \beta)(\psi_I(\vec{k})^\dagger \psi_I(-\vec{k})^\dagger - \text{h.c.}) d\vec{k}) \, |\text{vac} > \quad (A9) \]

where \( \tilde{\psi}_I^\dagger \) corresponds to the creation of the fermionic thermal modes. The entropy is given as

\[ \sigma_F = -\frac{\gamma}{(2\pi)^3} \int d\vec{k}\left[ \sin^2 \theta_F(\vec{k}, \beta) \ln(\sin^2 \theta_F(\vec{k}, \beta)) + \cos^2 \theta_F(\vec{k}, \beta) \ln(\cos^2 \theta_F(\vec{k}, \beta)) \right]. \]

The thermal Bogoliubov transformation is now given as

\[
\begin{pmatrix}
\psi_I(\vec{k}, \beta) \\
\tilde{\psi}_I(\vec{k}, \beta)^\dagger 
\end{pmatrix} =
\begin{pmatrix}
\cos \theta_F(\vec{k}, \beta) & -\sin \theta_F(\vec{k}, \beta) \\
-\sin \theta_F(\vec{k}, \beta) & \cos \theta_F(\vec{k}, \beta)
\end{pmatrix}
\begin{pmatrix}
\psi_I(\vec{k}) \\
\tilde{\psi}_I(-\vec{k})^\dagger 
\end{pmatrix}.
\]

Parallel to equation (A8) the function of \( \theta_F(\vec{k}, \beta) \) is given as

\[ \sin^2 \theta_F = \frac{1}{e^{\beta(\omega(\vec{k}, \beta) - \mu)} + 1} \quad (A11) \]

where \( \mu \) is the chemical potential corresponding to baryon number conservation and \( \omega(\vec{k}, \beta) = \sqrt{k^2 + M^2} \) for free fermions of mass \( M \), so that when statistics is known, the corresponding Bogoliubov tranformation relating zero temperature ground state with the thermal ground state of the extended Hilbert space is known. The ground state or the "thermal vacuum" obviously contains particles with appropriate distributions.

The methodology enables us to replace mixed states of statistical mechanics by pure states in an extended Hilbert space while generating correct distribution functions. The extra thermal modes enable us to do this.
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FIGURES

FIG. 1. Binding energy per nucleon $E$ as a function of nuclear matter density $\rho$ at temperatures $T=0$ MeV, 5 MeV, 10 MeV and 15 MeV. The gradual shift in the energy minimum towards higher densities may be noted.

FIG. 2. Pressure $P$ as a function of nuclear matter density $\rho$ at temperatures $T=0$ MeV, 5 MeV, 10 MeV, 15 MeV and 20 MeV. The vanishing of ‘pocket’ at about 15 MeV may be taken as a signature of phase transition.

FIG. 3. Entropy density $\sigma$ as a function of nuclear matter density $\rho$ at temperatures $T=50$ MeV, 100 MeV, 150 MeV and 200 MeV.

FIG. 4. Average number of pions per nucleon $\rho_m$ as a function of nuclear matter density $\rho$. Since the results are effectively independent of temperature, only the zero temperature graph is plotted.