The role of the indirect tunneling processes and asymmetry in couplings in orbital Kondo transport through double quantum dots

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Abstract
A system of two quantum dots attached to external electrodes is considered theoretically in the orbital Kondo regime. In general, the double dot system is coupled via both Coulomb interaction and direct hopping. Moreover, the indirect hopping processes between the dots (through the leads) are also taken into account. To investigate the system’s electronic properties we apply the slave-boson mean field (SBMF) technique. With the help of the SBMF approach the local density of states for both dots and the transmission (as well as linear and differential conductance) is calculated. We show that Dicke- and Fano-like line shapes may emerge in the transport characteristics of the double dot system. Moreover, we observed that these modified Kondo resonances are very susceptible to the change of the indirect coupling’s strength. We have also shown that the Kondo temperature becomes suppressed with increasing asymmetry in the dot–lead couplings when there is no indirect coupling. Moreover, when the indirect coupling is turned on the Kondo temperature becomes suppressed. By allowing a relative sign of the nondiagonal elements of the coupling matrix with left and right electrodes, we extend our investigations to become more generic. Finally, we have also included the level renormalization effects due to indirect tunneling, which are mostly neglected.

1. Introduction
Originally, the Kondo effect was discovered in nonmagnetic metal containing magnetic impurities at low temperature. The effect comes from strong electron correlations and can be regarded as interactions of the impurity spin with a cloud of the conduction electrons in metal. Scattering of the conduction electrons from the localized magnetic impurities leads to increase of the resistivity at low temperature [1]. In the last two decades one could notice a revival of this effect as it was predicted and observed in transport through quantum dots [2–4]. Although, in this case Kondo resonance leads to increasing conductivity (not resistivity as in the original Kondo effect) with decreasing temperature below the so-called Kondo temperature $T_K$, the physical mechanism of the phenomena is common. Here, the role of the magnetic impurity plays a spin on the dot.

However, despite regarding the spin degree of freedom one may assume any two-valued quantum numbers, such as for instance, an orbital degree of freedom, to realize the Kondo effect. In the case of (at least) two discrete orbital levels coupled to the external leads one may deal with the orbital Kondo effect [5–7]. To explain mechanism of creation of the Kondo state let us introduce spinless electrons in a system of two single-level quantum dots coupled to external leads as shown in figure 1. The dots’ energy levels are well below the Fermi level of the leads. Due to large interdot Coulomb repulsion only one of them is occupied by an electron. On
Figure 1. Schematic picture of the double dot system. The parameter $\alpha$ takes into account the difference in the coupling of the two dots to the external leads ($\alpha \in (0, 1)$). By tuning parameter $\alpha$ one can change the geometry of the system from the parallel one for $\alpha = 1$ to the T-shaped geometry for $\alpha = 0$.

The other hand, removing an existing electron on the double dot requires adding energy to the system. Thus, the system is in the deep Coulomb blockade and sequential tunneling events are prohibited. However, according to the Heisenberg uncertainty principle, the higher-order processes, may appear on a very short time scale. Assuming that initially the upper dot was occupied, then the electron can tunnel onto the Fermi level of the lead (left or right) and simultaneously another electron from the Fermi level of the left or right electrode may tunnel to the lower dot. As a result, charge exchange occurred between the dots. A coherent superposition of such coherent events gives rise to the sharp resonance in the density of states at the Fermi level.

Although the orbital Kondo effect has been investigated in different geometries of the two orbital level system both experimentally [8, 9] and theoretically [10–15] there are few comprehensive studies on the influence of the indirect coupling and/or asymmetry couplings on this phenomenon. In the case of the spin Kondo effect most researchers assume the maximal value of this coupling [16–22]. However, the maximal value of the indirect coupling in the case of the orbital Kondo effect may lead to suppression of the Kondo resonance. Specifically, this effect is destroyed when the magnetic flux achieves $2\pi n$ ($n \in \mathbb{Z}$) [23, 24] due to the formation of the bound state in the continuum (BIC) [25]. Recently, Kubo et al. have investigated both spin and orbital Kondo effects in double quantum dot regarding non-maximal values of the indirect coupling [24, 26]. They have found that in the condition of intermediate indirect coupling strength the differential conductance reveals two kinds of peaks. Although, we considered a similar system, the role of this paper is to give more insight into the connection between the role of the indirect coupling and various interference effects. Moreover, we assume different asymmetry coupling of the dots to the external leads. More specifically, in the system considered here one dot is coupled to the leads with constant strength, whereas the coupling of the second dot to the leads can be continuously tuned. Time reversal symmetry implies that the amplitude of the indirect coupling strength may change sign [27]. Thus, we also include this case in our consideration.

In very recent experiment Hatano et al [28] observed that the period of the Aharonov–Bohm oscillations is halved and the phase changes by half a period for the antibonding state from those of the bonding. They conclude that these features can be related to the indirect interdot coupling via the two electrodes.

We also point out that a similar double dot system has been investigated in [29, 30] where the authors have found a novel pair of correlation-induced resonances and they studied the charging of a narrow QD level capacitively coupled to a broad one. However, the correlation-induced effects are not related to the Kondo physics.

As we mentioned before various quantum interference effects, which were previously reported in atomic physics or quantum optics [31–33], were also discovered in electronic transmission through QD systems attached to the leads[34–38]. Here, we consider Dicke and Fano resonances. In the original Dicke effect one observes in spontaneous emission spectra a strong and very narrow resonance which coexists with a much broader line. This occurs when the distance between the atoms is much smaller than the wavelength of the emitted light (by an individual atom). The former resonance, associated with a state which is weakly coupled to the electromagnetic field, is called the subradiant mode, and the latter, strongly coupled to the electromagnetic field refers to superradiant mode. In the case of electronic transport in mesoscopic systems (for instance, quantum dots) this effect is due to indirect coupling of QDs through the leads. Generally, indirect coupling can lead to the formation of the bonding and antibonding states. As a result, a broad peak corresponding to the bonding state and a narrow one referring to the antibonding state emerge in the density of states [39, 40].

Let us now say something about Fano effect. In experiment, the Fano effect manifests itself as an asymmetric line shape in emission spectra. It comes from quantum interference of waves resonantly transmitted through a discrete level and those transmitted nonresonantly through a continuum of states. The effect was observed in optics and also in electronic transport through QD systems [41–45]. However, in this case the Fano phenomenon is due to the quantum interference of electron waves transmitted coherently through the dot and those transmitted directly between the leads [46].

The resonant channel can be associated with a discrete level and the nonresonant channel with continuum of states. When the electron wave passes through the resonant channel its phase changes by $\pi$ (within $\Gamma$), whereas the phase of electron waves in the nonresonant channel changes very slowly around the resonant level ($\Gamma$ is the width of the discrete level). (Of course, the Fano effect occurs only when the discrete level is embedded into a continuum.) Consequently, on one side of the discrete level electron waves through two channels interfere constructively, whereas on the other side they interfere destructively. As a result, one observes an asymmetric line in conductance around the discrete level position.

This effect can be also observed in system of two quantum dots embedded in two arms of an Aharonov–Bohm...
(AB) ring [47–50] or in the so-called T geometry [45, 51]. Here, a very narrow (broad) level, which is weakly (strongly) coupled to the leads, corresponds to the resonant (nonresonant) channel. The narrow level must appear within the broad one. As mentioned before, the difference in the coupling strengths of the bonding and antibonding states is due to indirect coupling. The phase shift of the wavefunction in the broad level is negligible when the energy changes within the narrow level and Fano resonance may appear.

In this paper the orbital Kondo effect in electronic transport through two coupled quantum dots is considered theoretically. Generally, the quantum dots may interact via both Coulomb repulsion and the hopping term. To calculate local density of states (LDOS) for both dots, transmission, and differential conductance we employ the slave-boson mean field approach. To show the formation of the bound state in the continuum as the indirect coupling strength approaches its maximal value we calculate the Friedel phase. Due to emergence of the BIC the Friedel phase, usually continuous, changes abruptly at the energy corresponding to the BIC. Finally, to be more familiar with experiment we show differential conductance.

The paper is organized as follows. In section 2 we describe the model of a double dot system which is taken into consideration. We also present there the slave-boson mean field technique used to calculate the basic transport characteristics. Numerical results on the orbital Kondo characteristics. Numerical results on the orbital Kondo

conclusions are presented in section 4.

2. Theoretical description

2.1. Model

To investigate various interference effects in the Kondo regime we consider a (spinless) Anderson Hamiltonian for two coupled quantum dots (DQDs) coupled to external leads (as shown in figure 1). Generally, this Hamiltonian consists of three parts,

\[ \hat{H} = \hat{H}_c + \hat{H}_{\text{DQD}} + \hat{H}_T, \]  

(1)

where the first term, \( \hat{H}_c \), describes nonmagnetic electrodes in the noninteracting quasi-particle approximation, \( \hat{H}_c = \hat{H}_L + \hat{H}_R \), with \( \hat{H}_\beta = \sum_k \varepsilon_k c_{\beta k}^\dagger c_{\beta k} \) (for electrodes \( \beta = \text{L, R} \)). Here, \( c_{\beta k}^\dagger \) (\( c_{\beta k} \)) creates (annihilates) an electron with the wavevector \( k \) in the lead \( \beta \), whereas \( \varepsilon_{\beta k} \) denotes the corresponding single-particle energy.

The next term of Hamiltonian equation (1) describes two coupled quantum dots,

\[ \hat{H}_{\text{DQD}} = \sum_i \varepsilon_i d_i^\dagger d_i + i(t d_1^\dagger d_2 + \text{h.c.}) + U n_1 n_2, \]  

(2)

where \( n_i = d_i^\dagger d_i \) is the particle number operator, \( \varepsilon_i \) is the discrete energy level of the \( i \)th dot (\( i = 1, 2 \)), \( t \) denotes the interdot hopping parameter (assumed real), whereas \( U \) is the interdot Coulomb integral.

The last term, \( \hat{H}_T \), of Hamiltonian equation (1) describes electron tunneling between the leads and dots, and takes the form

\[ \hat{H}_T = \sum_{k\beta} \sum_{i=1,2} (V_{k\beta}^\dagger c_{i\beta}^\dagger d_i + \text{h.c.}), \]  

(3)

where \( V_{k\beta}^\dagger \) are the relevant matrix elements. Experimentally, the model described by the Hamiltonian (1) can be realized in both single- and double quantum dot systems. In both cases the sufficiently strong (in plane) magnetic field has to be applied to lift spin degeneracy. In the former case, the two orbital levels localized on the dot must be available for experiment, whereas in the latter case each dot should possess only one orbital level taking part in transport. Such a DQD system can be realized using sufficiently small quantum dots, in which the level spacing is relatively large. However, the double dot structure, in the case of laterally coupled dots, is more suitable for experiments, as the dot’s energy levels can be tuned independently [8]. A single quantum dot does not provide such an efficient control of individual levels. However, a relatively efficient control of single dot’s parameters can be achieved in carbon nanotube based quantum dots [52], where the spacing between nearly degenerate orbital levels can be tuned by longitudinal magnetic field [53].

Finite widths of the discrete dots’ energy levels come from coupling to the external leads and may be expressed in the form

\[ \Gamma_\beta(q, \varepsilon) = 2\pi |V_{k\beta}^\dagger|^2 \rho(\varepsilon), \]

where \( \rho(\varepsilon) \) denotes the density of states in the left and right lead (\( \rho_L = \rho_R \equiv \rho \)). Furthermore, we assume that \( \Gamma_\beta(q) \) is constant within the electron band, \( \Gamma_\beta(q, \varepsilon) = \Gamma_\beta(q) = \text{const} \) for \( \varepsilon \in (-D, D) \), and \( \Gamma_\beta(q) = 0 \) otherwise. Here, 2D denotes the electron band width.

For the system considered, the dot–lead couplings can be written in a matrix form,

\[ \Gamma = \begin{pmatrix} \Gamma_1 & 0 \\ 0 & \Gamma_2 \end{pmatrix}, \]

(4)

where the off-diagonal matrix elements are assumed to be \( \Gamma_{12} = \Gamma_{21} = q_{\beta} \sqrt{\pi} \rho \) [34, 54]. The off-diagonal matrix elements of \( \Gamma \) take into account various interference effects resulting from indirect tunneling processes between two quantum dots via the leads. These off-diagonal matrix elements may be significantly reduced in comparison to the diagonal matrix elements \( \Gamma_{ii} \) or even totally suppressed due to complete destructive interference. To take all those effects into account, the parameters \( q_L \) and \( q_R \) are introduced. Furthermore, we assume that \( q_{\beta} \) are real numbers and obey the condition |\( q_{\beta} | \leq 1 \). When \( q_{\beta} \) is nonzero, the processes in which an electron tunnels from one dot to the \( \beta \)-lead and then (coherently) to another dot are allowed. Introducing parameter \( \alpha \), which takes into account differences in the coupling of the two dots to external leads, the coupling matrix equation (4) can be rewritten in the form,

\[ \Gamma = \Gamma \left( \begin{array}{cc} \alpha & q_{\beta} \sqrt{\alpha} \\ q_{\beta} \sqrt{\alpha} & 1 \end{array} \right), \]

(5)
By tuning parameter $\alpha$ one can change the geometry of the system from the parallel one for $\alpha = 1$ to the T-shaped geometry for $\alpha = 0$. In the T-shaped geometry the upper dot is disconnected from the leads. All intermediate values of $\alpha$ refer to an intermediate geometry where each of the two dots is coupled to leads with different strengths. We further assume symmetric coupling $\Gamma^L = \Gamma^R \equiv \Gamma/2$, where $\Gamma$ is energy unit.

### 2.2. Method

We perform our calculations in the large interdot charging energy limit, more specifically, when $U \to \infty$. The slave-boson approach is one of the techniques which allows us to investigate strongly correlated fermions in low temperatures [55]. This method relies on introducing auxiliary operators for the dots and replacing of the dots’ creation and annihilation operators by $\hat{f}$ pseudo-fermion operator and $\hat{b}$ slave-boson operator respectively. Here, the slave-boson operator $\hat{b}^\dagger$ creates an empty state, whereas the pseudo-fermion operator $\hat{f}^\dagger$ creates a singly occupied state with an electron in the $i$th dot. To eliminate nonphysical states, the following constraint has to be imposed on the new quasi-particles,

$$Q = \sum_i \hat{f}^\dagger_i \hat{f}_i + \hat{b}^\dagger_i \hat{b}_i = 1. \quad (6)$$

The above constraint prevents double occupancy of the dots; dots are empty or singly occupied.

In the next step the Hamiltonian (1) of the system is replaced by an effective Hamiltonian, expressed in terms of the auxiliary boson $\hat{b}$ and pseudo-fermion $\hat{f}$ operators as,

$$\hat{H} = \sum_{k,\beta} \epsilon_{k\beta} \hat{c}_k^\dagger \hat{c}_k + \sum_i \epsilon_{\hat{f}} \hat{f}_i^\dagger \hat{f}_i + (\hat{t} \hat{f}_i^\dagger \hat{b} \hat{b}^\dagger \hat{f}_i + \text{h.c.})$$

$$+ \sum_{k,\beta} \sum_i (\lambda_{k\beta} \hat{b}^\dagger_i \hat{f}_i + \text{h.c.})$$

$$+ \lambda \left( \sum_i \hat{f}_i^\dagger \hat{f}_i + \hat{b}^\dagger_i \hat{b}_i - 1 \right). \quad (7)$$

To avoid double occupancy of the DQD system the constriction condition (equation (6)) has been incorporated in Hamiltonian (7) by introducing the term with the Lagrange multiplier $\lambda$.

However, after such transformation our Hamiltonian is still rather complex and hard to solve. To get rid of this problem we apply a mean field (MF) approximation in which the boson field $\hat{b}$ is replaced by a real and independent of time $c$ number, $\hat{b}(t) \to (\hat{b}(t)) \equiv \tilde{b}$. This approximation neglects fluctuations around the average value $\langle \hat{b}(t) \rangle$ of the slave-boson operator, but is sufficient to describe correctly those leading to the Kondo effect. It also restricts our considerations to the low bias regime ($eV \ll |\epsilon_i|$).

With the following definitions of the renormalized parameters: $\tilde{t} = i \tilde{b} \tilde{\epsilon}_i$, $\tilde{V}_{k\beta} = \tilde{V}_{k\beta}^0 \tilde{b}$ and $\tilde{\epsilon}_i = \epsilon_i + \lambda$, one can rewrite the effective MF Hamiltonian in the form

$$\hat{H}^{\text{MF}} = \sum_{k,\beta} \epsilon_{k\beta} \tilde{c}_k^\dagger \tilde{c}_k + \sum_i \tilde{\epsilon}_i \tilde{f}_i^\dagger \tilde{f}_i + (\tilde{\epsilon} \tilde{f}_i^\dagger \tilde{f}_i^\dagger + \text{h.c.})$$

$$+ \sum_{k,\beta} \sum_i (\lambda_{k\beta} \tilde{b}^\dagger_i \tilde{f}_i + \text{h.c.}) + \lambda \left( \tilde{b}^\dagger \tilde{b} - 1 \right). \quad (8)$$

The unknown parameters $\tilde{b}$ and $\lambda$ have to be found self-consistently with the help of the following equations:

$$\tilde{b}^2 - i \sum_i \int \frac{d\epsilon}{2\pi} \langle \langle \tilde{f}_i^\dagger \tilde{f}_i \rangle \rangle = 0. \quad (9)$$

$$- i \sum_i \int \frac{d\epsilon}{2\pi} \langle \langle \tilde{f}_i^\dagger \tilde{f}_i \rangle \rangle = 0. \quad (10)$$

The above equations have been obtained from the constraints imposed on the slave-boson, equation (6), and from the equation of motion for the slave-boson operator. The lesser Green functions $\langle \langle \tilde{f}_i^\dagger \tilde{f}_i \rangle \rangle$ as well as retarded $\langle \langle \tilde{f}_i^\dagger \tilde{f}_i \rangle \rangle$ (which also is needed in further calculations) have been determined from the corresponding equation of motion.

To get an insight into the system’s electronic properties we calculate LDOS and transmission function. The local density of states for the $i$th dot is defined as;

$$D_i = \frac{\tilde{b}^2}{\pi} \text{Im}[G^0_i(\epsilon)], \quad (11)$$

whereas transmission through the system is expressed in the form;

$$T(\epsilon) = \frac{1}{2} \text{Tr} [G^0 L \Gamma R G^0 R \Gamma L \Gamma L]. \quad (12)$$

In the above formula $\Gamma^\beta$ stands for the coupling matrix to the $\beta$th lead with renormalized parameters $\Gamma_{ij}^\beta = \lambda^2 \Gamma_{ij}^\beta$ and $G^\prime$ ($G^2$) denotes Fourier transforms of the retarded (advanced) Green functions of the dots. It is worth noting that transmission probability $T(\epsilon)$ is directly related with current and linear conductance. In the zero temperature limit $T \to 0$ these quantities are given by formulas

$$J = \frac{2e}{h} \int_{-eV/2}^{eV/2} d\epsilon T(\epsilon), \quad (13)$$

and

$$G_{\text{V} \to 0} = \lim_{V \to 0} \frac{dJ}{dV} = \frac{2e^2}{h} T(\epsilon = 0). \quad (14)$$

### 3. Numerical results

#### 3.1. Dicke effect

In the following numerical calculations we assume equal dot energy levels, $\epsilon_i = \epsilon_0$ (for $i = 1, 2$) ($\epsilon_0$ is measured from the Fermi level of the leads in equilibrium, $\mu_L = \mu_R = 0$). Moreover, we set the bare level of the dots at $\epsilon_0 = -3 \Gamma$, and the bandwidth is assumed to be $2D = 120 \Gamma$. All the energy quantities are expressed in the units of $\Gamma$. The parameters $q_L$ and $q_R$ are assumed to be equal, $q_L = q_R = q$, if not stated otherwise. Taking into account the above parameters, the Kondo temperature $T_K$ for the symmetric couplings ($\alpha = 1$) and disregarding both direct and indirect couplings, ($t = 0$, $q = 0$) is estimated to be $T_K \approx 10^{-2} \Gamma$. In this section we
consider the situation when dots are not directly coupled, which corresponds to the case with vanishing interdot tunnel coupling parameter \( t = 0 \). On the other hand, one should remember that dots are still coupled (indirectly) through the leads which is reflected in finite values of the off-diagonal coupling matrix elements, i.e. \( q \neq 0 \). In this situation bonding and antibonding levels, created due to indirect couplings, coincide. In figure 2 we show LDOS for QD1 and QD2 for large off-diagonal matrix couplings \( (q = 0.99) \) and for different values of the asymmetry parameter \( \alpha \). Firstly, it is clearly presented that the broad and narrow Kondo peaks in the LDOS are superimposed at energy \( \varepsilon = 0 \) and the LDOS displays behavior typical for the Dicke effect. By analogy to the original Dicke phenomenon, one may associate the narrow (broad) central peak in LDOS with a subradiant (superradiant) state. The subradiant (superradiant) state corresponds to the long-lived (short-lived) state. For symmetric coupling \( \alpha = 1 \) the LDOS for QD1 and QD2 are the same (see figure 2(a)) but when asymmetry appears in couplings \( (\alpha \neq 1) \) this ceases to be true and the LDOS for both dots have different line shape. As \( \alpha \) drops down, the widths of the Kondo peaks for both dots also diminish. With decreasing \( \alpha \) the broad part of the LDOS for the quantum dot strongly coupled to the leads (QD2) becomes more pronounced, whereas the narrow one becomes narrower. The two peaks corresponding to subradiant and superradiant state are easily distinguishable. As a result, the Dicke effect in LDOS for QD2 is more distinct. The Dicke effect is also noticed in the transmission shown in figure 3. This situation is analogous to that reported in the case of the spin Kondo effect in parallel double dot systems. [40]. The Dicke peak appears in the transmission because the phases of the transmission amplitudes for bonding and antibonding channels are equal at zero energy. Thus, the two contributions add constructively, leading to the maximum transmission at \( \varepsilon = 0 \). However, now there is no dip structure at zero energy for \( q = 1 \) (originating from the complete destructive interference), which will be explained further. Moreover, the effect is preserved for various values of the asymmetry parameter \( \alpha \). It is worth noting that the linear conductance \( (\text{equation (14)}) \) reaches a unitary limit (two quanta of \( e^2/h \)). When \( \alpha \) is reduced the effective Kondo temperature also decreases which can be seen by looking at the energy scale in figure 2. The origin of this effect comes from the fact that when \( \alpha \) decreases one of the dots becomes detached from the electrodes. Then, the rate of the higher-order tunneling events (which leads to the Kondo anomaly—see explanation in section 1) also diminishes and finally for \( \alpha = 0 \), when one of the dots is totally disconnected from the leads, there is no possibility for such events and no Kondo effect is expected.

As we mentioned in section 2.1 the off-diagonal matrix elements \( \Gamma_{12}^\beta \) may be significantly reduced, so it is desired to analyze this case. In fact, the Hamiltonian equation (1) describes the idealized system of lateral quantum dots in which orbital quantum number \( i \) is not conserved during tunneling. This leads to \( |q| = 1 \). However, in realistic systems the factor \( |q| \) may be less than 1 due to various factors, such as for instance the geometry of the system [54], or imperfections in the leads giving rise to destructive quantum interference. However, the regime of parameter \( q \) close to 1 may not be accessible in carbon nanotube QDs [56]. Thus, for the observation of various interference effects in the orbital Kondo regime the lateral quantum dots seem to be more suitable. In figure 4 we plotted the LDOS, Friedel phase and the transmission for various values of the parameter \( q \) which is directly related to the amplitude of the off-diagonal matrix elements. The Friedel phase is related to the LDOS by the following equation: \( d\phi_e/\varepsilon = \pi D(\varepsilon) \) with \( D(\varepsilon) \) being the relevant density of states. One can notice that the Dicke effect in LDOS can be found only when off-diagonal matrix elements are large, i.e. \( q \) close to 1. With decreasing \( q \) the Dicke line shape is transformed in the usual Lorenzian line. Similar behavior is observed in the transmission (see [55]).
Figure 3. The transmission probability calculated for the indicated values of $\alpha$ and for $q = 0.99$ and for $t = 0$. The transmission probability has a well-defined Dicke line shape.

In the inset of figure 4(c) we show that the narrow part of the peak also has a well-defined shape like the broad one. Moreover, when $q$ is closer and closer to 1 then the narrow peak becomes narrower and narrower. This is reflected in the abrupt (but continuous as long as $q \neq 1$) change by $\pi$ of the Friedel phase around $\epsilon = 0$ for $q$ close to 1. It is also found that the Dicke effect disappears in transmission when $q$ is maximal ($q = 1$). When $q = 1$ the transmission probability has Lorenzian shape and is described by the formula,

$$T(\epsilon) = \frac{1}{4} \left( \frac{(1 + \alpha)^2 \hat{\Gamma}^2}{(1 + \alpha)^2 \hat{\Gamma}^2 + (\tilde{\epsilon}_0 - \epsilon)^2} \right),$$

where $\tilde{\Gamma} = \tilde{b}^2 \Gamma / 2$. This equation clearly shows that no Dicke effect should be expected for $q = 1$. This result resembles that obtained for a noninteracting system [34]. However, in the Kondo regime the situation is much more complex. To show this, let us first consider the case with $q = 1$ and symmetric couplings, $\alpha = 1$. It is well known that for a symmetric ($\alpha = 1$) noninteracting ($U = 0$) system as $q$ tends to 1 one of the peaks becomes progressively narrowed and finally for $q = 1$ a BIC emerges [34, 57]. As a result the transmission reveals a simple Lorenzian lineshape. One may naively believe that a similar situation occurs in the Kondo regime. However, this cannot be true because when the indirect coupling strength is equal to the dot–lead coupling ($\Gamma_{12} = \Gamma_{11} = \Gamma_{22}$), the indirect tunneling processes completely destroy coherent higher-order tunneling events leading to complete suppression of the Kondo resonance [23]. One can also look at this from the other point of view and explain this as follows: as the antibonding state becomes a BIC, it is totally decoupled from the leads and there is no possibility for an electron to exchange between the two molecular states. Thus, no Kondo effect appears as the corresponding Kondo temperature is equal to zero. To support these predictions we calculated the Kondo temperature for arbitrary value of the parameter $q$. The Kondo temperature is directly related with the full width at half maximum of the Kondo resonance. The Kondo temperature for the symmetric case, $\alpha = 1$, acquires the following form $T_K = \sqrt{\tilde{\epsilon}_0^2 + \hat{\Gamma}^2 (1 - q^2)^2}$. This analytical formula has been obtained from the explicit form of the corresponding density of states. In the deep Kondo regime the renormalized parameter $\tilde{\epsilon}_0$ is equal to zero, which is shown in the appendix, and thus the above formula clearly shows vanishing of the corresponding Kondo temperature as $q$ tends to 1. In figure 5 the Kondo temperature is displayed as a function of parameter $q$. For comparison we also plot the $q$ dependence of the renormalized widths of the bonding and antibonding levels, $\Gamma_b$ and $\Gamma_a$, respectively. In the case $q_L = q_R = q$, the widths acquire the following form $\Gamma_b = b^2 \Gamma_b$, $\Gamma_a = b^2 \Gamma_a$ with $\Gamma_{b,a} = \frac{1}{2}(\Gamma_{11} + \Gamma_{22}) \pm q \sqrt{\Gamma_{11} \Gamma_{22}}$ and $\Gamma_{ii} = \Gamma_{b,i}^2 + \Gamma_{a,i}^2$ for $i = 1, 2$. The characteristic widths for both distinct channels behave in different ways with varying
the strength of the off-diagonal tunneling processes. One can notice that the renormalized width for the bonding channel changes non-monotonically but rather slowly, whereas the one for the antibonding channel drops monotonically to zero as \( q \) reaches its maximum value. This behavior is consistent with the predictions given in [58] with one exception. The vanishing of \( \tilde{\Gamma}_a \) when \( q \) is maximal is responsible for the suppression of the Dicke effect. In contrast to [58] the width \( \tilde{\Gamma}_b \) drops to zero for \( q = 1 \) due to vanishing of the boson field. We also emphasize that for \( q \in (0, 1) \), the \( \alpha \) dependences of the renormalized widths for both channels are different. For large values of \( q \) the width \( \tilde{\Gamma}_a \) changes little with \( \alpha \).

However, the Kondo effect is also destroyed in a more general case, i.e. for \( q = 1 \) and arbitrary \( \alpha \). This can be understood when one notices that transmission (15) depends on renormalized parameter \( \tilde{\beta} \) which vanishes in this case. In the SBMF formalism this is manifested as lack of solutions of the self-consistent equations (9) and (10) (which then form a contradictory system of equations).

On the other side, for \( q = 0 \) the following formula describes the transmission probability,

\[
T(\varepsilon) = \frac{\tilde{\Gamma}_b^2}{2} \left[ \frac{1}{(\tilde{\epsilon}_0 - \varepsilon)^2 + \alpha^2 (\tilde{\epsilon}_0 - \varepsilon)^2} \right]^{1/2},
\]

Figure 6 illustrates graphically equation (16) for indicated values of the asymmetry parameter \( \alpha \). This plot clearly shows, as explained earlier, that the width of the Kondo peak decreases as \( \alpha \) is reduced. This case resembles the spin Kondo effect in a single-level quantum dot coupled to ferromagnetic leads [59]. Then the asymmetry parameter \( \alpha \) can be assigned with the lead’s (pseudo)polarization in the following way

\[
\tilde{\beta} = \frac{(1 - \alpha)}{(1 + \alpha)} [60].
\]

Increasing pseudopolarization (decreasing \( \alpha \)) the Kondo effect is suppressed which is in agreement with [60]. It is worth mentioning that realization of the pure pseudospin Kondo effect, i.e. \( q = 0 \), can be obtained by providing separate pairs of source and drain leads to each dot as has been demonstrated in recent experiments [9] with a double quantum dot system in lateral arrangement. Alternatively, one can use vertical quantum dots in which the conservation of the orbital quantum number seems more likely. However, such a structure does not provide full control over the tunnel couplings [8]. In the presence of asymmetry in couplings (\( \alpha < 1 \)) the Kondo temperature should be defined by the geometric mean as:

\[
T_K \equiv \sqrt{\tilde{\epsilon}_0 \tilde{\epsilon}_2} = \tilde{\epsilon}_0 \sqrt{\alpha} \quad \text{(remembering that} \tilde{\epsilon}_0 \to 0 \text{in the deep Kondo regime).}
\]

Figure 7 shows \( \alpha \) dependence of the Kondo temperature for the case of \( q = 0 \). In the present case, \( q = 0 \), electrons may travel only directly through dot QD1 or QD2 thus the reduction of the Kondo temperature with decreasing \( \alpha \) clearly reflects the suppression of Kondo fluctuations in the DQD (due to reduction of the charge exchange between the dots) as one of the dots becomes detached from the leads and the Kondo effect diminishes.

A similar dependence on parameter \( q \) can be noticed in the differential conductance which is displayed in figure 8. The differential conductance is symmetric in respect to the zero bias point, thus we plot only its bias dependence for nonnegative bias voltage. For \( q \) close to unity the differential conductance acquires a Dicke line shape. At zero bias the
differential conductance approaches the unitary limit \((2e^2/h)\) for all \(q < 1\). For \(q\) close to 1 the differential conductance drops very fast to half of the zero bias value with increasing the bias voltage and next decreases slowly with further increasing of the bias voltage. Such a sudden drop of the differential conductance is absent for smaller values of the parameter \(q\). This feature in the differential conductance is related to Dicke resonance.

Above some critical voltage, the current can suffer a drop, which leads to regions (not shown in figure 8) of negative differential conductance (NDC). However, the NDC should be treated carefully with more adequate methods, as the infinite-\(U\) SBMF approach does not describe properly the high bias regime. The appearance of NDC should be interpreted as the loss of Kondo correlations. All in all, we should not expect NDC for the case \(q = 0\).

As we mentioned in section 1 the off-diagonal elements of the coupling matrix can have opposite signs. Specifically, we examine the case \(q_L = q\) and \(q_R = -q\). The sign of the nondiagonal elements of the coupling matrix can be controlled by the external magnetic flux penetrating the AB ring consisting of two quantum dots coupled to the external leads. Moreover, as we mentioned before, the tunneling amplitudes \(V_{ab}^d\) can be negative, and thus the situation with \(V_{11}^L/V_{21}^L > 0\) and \(V_{11}^R/V_{21}^R < 0\) may occur without any additional influence on the described system (like using out of plane magnetic field to tune the AB phase). However, in this case one has only a little control of the sign, which rather depends on the geometry of the system and the dot's potential profile. At the beginning we assume symmetric couplings, i.e. \(\alpha = 1\). Then for maximal value of the parameter \(q\) (\(q = 1\)) each DQD's molecular state couples to different reservoirs and as there is no connection between them, the current does not flow. This effect originates from the totally destructive quantum interference which leads to vanishing of the transmission even if the LDOS of each dots is finite. It is worth noting that both LDOS and the corresponding Kondo temperature do not depend on the parameter \(q\), as the contributions from nondiagonal processes, referring to the left L and right R lead, cancel each other. This implies that the presented effect originates fully from the quantum interference. One can notice that decreasing the value of the parameter \(q\), the transmission can be recovered as is shown in figure 9. Finally, for \(q = 0\) the maximal value of the transmission is restored. This is because as the values of the off-diagonal matrix elements are decreased the destructive interference becomes totally suppressed, and thus, for zero value of the nondiagonal couplings the zero bias conductance is fully restored. This can be shown more formally by performing the transformation of the dots' operators to the bonding-like \((d_b = 1/\sqrt{2}(d_1 + d_2))\) and antibonding-like \((d_a = 1/\sqrt{2}(d_1 - d_2))\) state. The corresponding couplings to the left lead acquire the form \(\tilde{\Gamma}_{b,a}^L = 1/2(\tilde{\Gamma}_{11}^L + \tilde{\Gamma}_{22}^L) \pm q\sqrt{\tilde{\Gamma}_{11}^L\tilde{\Gamma}_{22}^L}\), whereas couplings to the right electrode is given by \(\Gamma_{b,a}^R = 1/2(\tilde{\Gamma}_{11}^R + \tilde{\Gamma}_{22}^R) \mp q\sqrt{\tilde{\Gamma}_{11}^R\tilde{\Gamma}_{22}^R}\). Now, it is clear that for \(\alpha = 1\) and \(q = 1\) the bonding state is coupled to (decoupled from) the left (right) lead, whereas the antibonding state is coupled to (decoupled from) the right (left) electrode. As the parameter \(q\) becomes less than 1, the decoupled states from the given leads for \(q = 1\) start to bond with them. As a result the transmission grows with decreasing the value of the parameter \(q\).

Another interesting quantum interference effect can be found by changing the asymmetry in the couplings of two dots to the leads. At the beginning we keep \(q\) equal to 1 and change the parameter \(\alpha\) in the interval \((0, 1)\). As before, the transmission becomes recovered as the asymmetry increases (see figure 10). However, in comparison to the previous case, a new feature emerges in the transmission. More specifically, the dip structure appears in the vicinity of the zero energy. At \(\varepsilon = 0\) the transmission drops to zero which results in vanishing of the linear conductance. Thus, the zero bias Kondo anomaly is totally suppressed for any value of the asymmetry parameter \(\alpha\). To verify this effect experimentally it is desired to measure the differential conductance as a function of bias voltage. In figure 11 we...
Figure 10. The transmission coefficient calculated for indicated values of the parameter $\alpha$ and for $q_L = -q_R = 1$.

Figure 11. Differential conductance as a function of bias voltage calculated for indicated values of the parameter $\alpha$ and for $q_L = -q_R = 1, t = 0$.

Figure 12. Local density of states for both dots (a) and the transmission probability (b) calculated for $q = 1, \alpha = 0.8$ and for $t = 0.8$. The existence of the Fano–Kondo effect is clearly visible. The inset shows the transmission probability obtained for $q = 1$ and for $\alpha = 0.15$.

3.2. Fano effect

Here, we show the results obtained for a nonzero interdot hopping parameter ($t \neq 0$). In the present situation the interdot hopping term lifts the degeneracy and the bonding and the antibonding levels are split away. As a result, the density of states of the DQD system coupled to the leads consists of two Kondo peaks; a broad peak centered at the bonding state and a narrow one corresponding to the antibonding state. This is clearly shown in figure 12(a) where the LDOS for both dots is plotted for maximal off-diagonal matrix elements ($q = 1$). It is very interesting that the narrow peaks in LDOS for QD1 and QD2 have opposite symmetries. The LDOS for QD1 reaches zero for negative energy, whereas the one for QD2 comes to zero for positive energy. Usually, when the asymmetry in coupling of two dots to the leads is reduced the width of the bonding (antibonding) state increases (decreases). Finally, for $\alpha = 1$ (full symmetric system) the antibonding states are decoupled from the leads and acquire a $\delta$-Dirac shape (it
It is worth mentioning that at this energy the phase of the transmission amplitude suffers discontinuity. This is no longer true for \( q < 1 \), because then the transmission probability has complex poles. In turn, for \( q < 1 \) the transmission probability does not reach zero and for specific values of \( q \) antiresonance behavior is less visible. Even a small shift in \( q \) changes the transmission probability within the antibonding level considerably, which is shown in figure 13.

At this point we should remark that the slave-boson mean field approach does not take into account the level renormalization arising due to coupling of dots to the leads. Such a renormalization should lead to splitting of the zero bias anomaly for specific cases [60]. The level splitting can occur due to asymmetry in coupling of the dots to the leads, i.e. for \( \alpha < 1 \) as has been shown in [60] by means of the scaling technique. However, such a splitting can also be induced by indirect coupling of the dots. One can show that the splitting is proportional to the strength of the indirect coupling. Moreover, the level renormalization of a given state is proportional to the coupling strength of the other level. This implies that for \( q \) close to 1 the bonding-like level will be only weakly renormalized, whereas the antibonding-like level will experience strong renormalization.

Thus, the transmission described in section 3.1 should resemble that from figure 13 but with the broad peak pinned close to \( \varepsilon = 0 \) and the narrow maximum shifted up in energy for relatively large values of parameter \( q \). One can show this more formally including corrections in molecular-like levels due to the mentioned renormalization.

To correct the drawback of the SBMF method one can by hand introduce the mentioned renormalization of the levels as follows: \( \varepsilon_b = \varepsilon_0 + \delta \varepsilon_b \) and \( \varepsilon_a = \varepsilon_0 + \delta \varepsilon_a \) (with \( t = 0 \) for the sake of simplicity). Here, \( \delta \varepsilon_i \) are the corrections due to indirect coupling of the dot levels. These corrections should be determined using relevant techniques, such as for instance the earlier mentioned scaling procedure. However, as the SBMF technique fails for nondegenerate states, one cannot introduce by hand the renormalized levels into self-consistent equations of the form equations (9) and (10) derived within bonding and antibonding states’ basis. Thus, we give here only a rough estimation and some predictions implied from the mentioned renormalization.

![Figure 13](image)

Figure 13. The transmission probability calculated for \( \alpha = 0.8 \), \( t = 0.8 \) and for \( q = 0.99 \). Even a small shift in \( q \) leads to considerable changes in the transmission probability within the antibonding level (compare with figure 12).
Here, $\Delta$ stands$^1$ for some function which in general depends on the system’s parameters (like bandwidth, dots energy levels, couplings). For the sake of simplicity we assume $\Delta$ to be a maximum value of function $\Delta(q)$ obtained from the scaling procedure, i.e. when $q = 1$. Equation (18) clearly shows that Kondo peak becomes split due to the level renormalization originating from indirect tunneling between the dots. In figure 14 we display the expected lineshape in the transmission for relatively large value of parameter $q$. The splitting in the transmission should decrease with decreasing the value of parameter $q$ and beyond a certain value of $q$ the splitting ceases to be visible. For $q = 0$ (and $\alpha = 1$) no splitting occurs.

4. Summary and conclusions

In this paper we have investigated the electronic properties of double quantum dots coupled to external leads. The dots have been coupled both via the hopping term and Coulomb interaction. Moreover, we have considered also the effects of indirect tunneling between the dots through the leads. Employing the slave-boson mean field approach, the local density of states for both dots, the Friedel phase and the transmission in Kondo regime have been calculated. Moreover, to be more familiar with the experiment we have calculated the corresponding differential conductance.

We have shown that for some set of parameters the Dicke- and Fano-like resonances may appear in the considered system. More specifically, it has been noticed that for zero interdot hopping parameter $t$ the LDOS of each QD consists of broad and narrow Kondo peaks that are superimposed. As in the original Dicke effect, one may associate the narrow (broad) central peak in LDOS with a subradiant (superradiant) mode. Moreover, the Dicke line shape has been found in the transmission and in the differential conductance. We have observed that this effect is very sensitive to the strength of the off-diagonal matrix elements; with reducing the value of the off-diagonal matrix elements the Dicke line both in LDOS and the transmission is transformed into the usual Lorenzian line. It has been found that when the interdot tunneling is allowed the transmission probability may reveal the antiresonance behavior with a characteristic Fano line shape. Moreover, we have noticed that the line shape of the antiresonance is also very susceptible to the change in the value of the off-diagonal matrix elements.

We have also calculated the Kondo temperature and have also shown that the latter becomes suppressed with increasing asymmetry in the dot–lead couplings when there is no indirect coupling. Moreover, when the indirect coupling is turned on, the characteristic widths for both distinct channels behave in different ways with varying strength of the off-diagonal tunneling processes. We found also that the corresponding Kondo temperature is totally suppressed for maximal value of the indirect coupling and no Kondo effect occurs. Moreover, we have also included level renormalization effects due to indirect coupling phenomena, which leads to the splitting of the Kondo peak.

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Appendix. Proof of $\tilde{\varepsilon}_0 \rightarrow 0$ in the deep Kondo regime

Here, we show that in the deep Kondo regime the renormalized parameter $\tilde{\varepsilon}_0$ is equal to zero and, thus, the Kondo temperature strictly corresponds to the renormalized width $\tilde{\Gamma}$ of the Abrikosov–Suhl resonance. This can be shown analytically by integrating self-consistent equations for slave-boson parameters, $b, \lambda$, written in the representation of bonding and antibonding states.

In the basis of the bonding and antibonding states the Hamiltonian of the system becomes diagonal for $\varepsilon_1 = \varepsilon_2 \equiv \varepsilon_0$ and acquires the following form:

$$\hat{H} = \hat{H}_c + \sum_{i=b,a} \varepsilon_i d_i^\dagger d_i + U n_b n_a + \sum_{\vec{k}\vec{\beta}} \sum_{i=b,a} (V_{\vec{k}\vec{\beta}}^d d_i^\dagger f_i + h.c.)$$

with $\varepsilon_b = \varepsilon_0 + t$, $\varepsilon_a = \varepsilon_0 - t$. In further considerations we assume $t = 0$, thus, $\varepsilon_b = \varepsilon_a = \varepsilon_0$.

In the mean field slave-boson representation the Hamiltonian equation (A.1) acquires the following form:

$$\hat{H}_{MF} = \hat{H}_c + \sum_{i=b,a} \tilde{\varepsilon}_i f_i^\dagger f_i + \lambda (\tilde{\varepsilon}_0^2 - 1) + \sum_{\vec{k}\vec{\beta}} \sum_{i=b,a} (V_{\vec{k}\vec{\beta}}^d f_i^\dagger f_i + h.c.)$$

The self-consistent equations determining the unknown parameters $\tilde{b}$ and $\lambda$ have the form of equations (9) and (10).

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$^1$ Within the scaling approach for the spinless Anderson model $\Delta$ acquires the following form, $\Delta = 2\pi \ln \left( \frac{D}{D_1} \right)$ with $D_1$ denoting a half-bandwidth at the end of scaling procedure.
with lesser Green’s function $\langle \langle f_i | f_j^\dagger \rangle \rangle \to$ defined in the basis of the bonding and antibonding states. The lesser bonding and antibonding Green’s function has the following form:

$$G^<(\epsilon) = \frac{if(\epsilon)\tilde{\Gamma}_i}{(\epsilon - \tilde{\epsilon}_0)^2 + (\tilde{\Gamma}_i/2)^2}. \quad (A.3)$$

At $T = 0$ K $f(x) = \theta(-x)$ and the integration of the self-consistent equations is straightforward\(^2\) leading to the following expressions:

$$\frac{1}{\pi} \sum_{i=b,a} \text{Im} \left[ \ln \left( \frac{\tilde{\epsilon}_0 + i\tilde{\Gamma}_i/2}{D} \right) \right] = 1 - \tilde{b}^2, \quad (A.4)$$

$$\frac{1}{2\pi} \sum_{i=b,a} \tilde{\Gamma}_i \text{Re} \left[ \ln \left( \frac{\tilde{\epsilon}_0 + i\tilde{\Gamma}_i/2}{D} \right) \right] + \lambda \tilde{b}^2 = 0. \quad (A.5)$$

In the deep Kondo regime we can approximate: $1 - \tilde{b}^2 \approx 1$ and $\lambda \approx -\tilde{\epsilon}_0$ and then equation (A.4) simplify to the following equations:

$$\sum_{i=b,a} \text{Im} \left[ \ln \left( \frac{\tilde{\epsilon}_0 + i\tilde{\Gamma}_i/2}{D} \right) \right] = \pi, \quad (A.6)$$

$$\sum_{i=b,a} \tilde{\Gamma}_i \text{Re} \left[ \ln \left( \frac{\tilde{\epsilon}_0 + i\tilde{\Gamma}_i/2}{D} \right) \right] = 2\pi \tilde{\epsilon}_0. \quad (A.7)$$

Here, we consider only the symmetric case, $\alpha = 1$, however, extension to arbitrary $\alpha$ is straightforward. For $\alpha = 1$ and $q_b = q_a = q$, the couplings to the bonding and antibonding states acquire the following form, $\Gamma_{b,a} = (1 \pm q)\Gamma$. Combining two equation (A.5) one arrives at the following equalities:

$$\left( \tilde{\epsilon}_0 + i\Gamma_b/2 \right) \left( \tilde{\epsilon}_0 + i\Gamma_a/2 \right) = \frac{\tilde{\epsilon}_0 + (\tilde{\Gamma}_b/2)^2}{\tilde{\epsilon}_0 + (\tilde{\Gamma}_a/2)^2} \Rightarrow 2\pi \tilde{\epsilon}_0.$$

The real and imaginary parts of which satisfy the following equalities:

$$\left( \tilde{\epsilon}_0^2 + \frac{1}{4} \tilde{\Gamma}_b \tilde{\Gamma}_a \right) \left( \tilde{\epsilon}_0 + (\tilde{\Gamma}_b/2)^2 \right) = -D^2 \exp \left( \frac{2\pi \tilde{\epsilon}_0}{\Gamma} \right), \quad (A.8)$$

Equation (A.8) is satisfied if $\tilde{\epsilon}_0 = 0$ or $\tilde{b}^2 = 0$. However, the latter solution leads to a nonphysical value for $\tilde{\epsilon}_0$, thus, the only solution must be $\tilde{\epsilon}_0 = 0$, QED.

Introducing the solution for $\tilde{\epsilon}_0$ into equation (A.7) one finds,

$$\tilde{b}^2 = \frac{2D(1 - q)}{\Gamma} \frac{e_{\tilde{\epsilon}_0}^0}{(1 + q) e_{\tilde{\epsilon}_0}^0} \exp \left( \frac{\pi\tilde{\epsilon}_0}{\Gamma} \right). \quad (A.9)$$

It is worth noting that the above equation does not determine the value of $\tilde{b}^2$ for $q = 1$ when the SBMF method fails.

References

[1] Kondo J 1964 Prog. Theor. Phys. 32 37
[2] Cronenwett S M, Oosterkamp T H and Kouwenhoven L P 1998 Science 281 540
[3] Sasaki S, De Franceschi S, Elzerman J M, van der Wiel W G, Eto M, Tarucha S and Kouwenhoven L P 2000 Nature 405 764
[4] Goldhaber-Gordon D, Shtrikman H, Mahalu D, Abusch-Magder M, Meirav U and Kastner M A 1998 Nature 391 156
[5] Glazman L I and Raikh M E 1988 JETP Lett. 47 452
[6] Silvestrov P G and Imry Y 2007 Phys. Rev. B 75 115335
[7] Wilhelm U, Schmid J, Weis J and Klitzing K v 2002 Physica E 14 385
[8] Holleitner A W, Chudnovskiy A, Piankuchke D, Eberl K and Blick R H 2004 Phys. Rev. B 70 075204
[9] Uchoa T, Schoeller H and Schön G 2001 Europhys. Lett. 54 241
[10] Vernek E, Sandler N, Ulloa S E and Anda E V 2006 Physica E 34 608
[11] Sinel M, Silva A, Oreg Y and Delft J v 2005 Phys. Rev. B 72 125316
[12] López R, Sánchez D, Lee M, Choi M S, Simon P and Le Hur K 2005 Phys. Rev. B 71 115312
[13] Konik R M 2007 Phys. Rev. Lett. 99 076602
[14] Zhang G M, Liu R, Liu Z R and Yu L 2005 Phys. Rev. B 72 073308
[15] Ding G H, Kim C K and Nahm K 2005 Phys. Rev. B 72 205330
[16] Tanaka Y and Kawakami N 2005 Phys. Rev. B 72 085304
[17] Zeng J, Peng J, Wang B and Xing D Y 2007 Phys. Rev. B 75 155327
[18] Kubo T, Tokura Y and Tarucha S 2008 Phys. Rev. B 77 041305(R)
[19] Neumann J v and Wigner E 1929 Z. Phys. 30 465
[20] Stillinger F H and Herrick D R 1975 Phys. Rev. A 11 446
[21] Kubo T, Tokura Y and Tarucha S 2011 Phys. Rev. B 83 115310
[22] Gurvitz S A 2005 IEEE Trans. Nanotechnol. 4 45
[23] Hatano T, Kubo T, Tokura Y, Amaha S, Teraoka S and Tarucha S 2011 Phys. Rev. Lett. 106 076801
[24] Meden V and Marquardt F 2006 Phys. Rev. Lett. 96 146801
[25] Kashcheyevs Y, Karrasch C, Hecht T, Weichselbaum A, Meden V and Schiller A 2009 Phys. Rev. Lett. 102 136805
[31] Dicke R H 1953 Phys. Rev. 89 472
[32] Dicke R H 1954 Phys. Rev. 93 99
[33] Fano U 1961 Phys. Rev. 124 1866
[34] Shahbazyan T V and Raikh M E 1994 Phys. Rev. B 49 17123
[35] Orellana P A, Ladrón de Guevara M E and Claro F 2004 Phys. Rev. B 70 233315
[36] Brandes T 2005 Phys. Rep. 408 315
[37] Trocha P and Barnaš J 2008 J. Phys.: Condens. Matter 20 125220
[38] Trocha P and Barnaš J 2008 Phys. Rev. B 78 075424
[39] Wunsch B and Chudnovsky A 2003 Phys. Rev. B 68 245317
[40] Trocha P and Barnaš J 2010 J. Nanosci. Nanotechnol. 10 2489
[41] Gores J, Goldhaber-Gordon D, Heemeyer S, Kastner M A, Shtrikman H, Mahalu D and Meirav U 2000 Phys. Rev. B 62 2188
[42] Clerk A A, Waintal X and Brouwer P W 2001 Phys. Rev. Lett. 86 4636
[43] Kobayashi K, Aikawa H, Katsumoto S and Iye Y 2002 Phys. Rev. Lett. 88 256806
[44] Johnson A C, Marcus C M, Hanson M P and Gossard A C 2004 Phys. Rev. Lett. 93 106803
[45] Sasaki S, Tamura H, Akaraki T and Fujisawa T 2009 Phys. Rev. Lett. 103 266806
[46] Bulka B and Stefaniškin P 2001 Phys. Rev. Lett. 86 5128
}[47] Guevara M L, Claro F and Orellana P A 2003 Phys. Rev. B 67 195335
[48] Lu H, Lu R and Zhu B F 2005 Phys. Rev. B 71 235320
[49] Chi F, Liu J L and Sun L L 2007 J. Appl. Phys. 101 093704
[50] Trocha P and Barnaš J 2007 Phys. Rev. B 76 165432
[51] Žižko R 2010 Phys. Rev. B 81 115316
[52] Jarillo-Herrero P, Kong J, van der Zant H S J, Dekker C, Kouwenhoven L P and De Franceschi S 2005 Nature 434 484
[53] Nygård J, Cobden D H and Lindelof P E 2000 Nature 408 342
[54] Kubo T, Tokura Y and Tarucha S 2006 Phys. Rev. B 74 205310
[55] Coleman P 1984 Phys. Rev. B 29 3036
[56] Bässer C A, Vernek E, Orellana P, Lara G A, Kim E H, Feiguin A E, Anda E V and Martins G B 2011 Phys. Rev. B 83 125404
[57] Solis B, Ladrón de Guevara M L and Orellana P A 2008 Phys. Lett. A 372 4736
[58] Lim J S, Choi M S, López R and Aguado R 2006 Phys. Rev. B 74 205119
[59] Martinek J, Usumi Y, Imamura H, Barnaš J, Maekawa S, König J and Schön G 2003 Phys. Rev. Lett. 91 127203
[60] Matsubayashi D and Eto M 2007 Phys. Rev. B 75 165319
[61] Trocha P 2010 Phys. Rev. B 82 125323
[62] Trocha P 2010 unpublished