Abstract

We consider dilaton stabilization with $R$ invariance, which insures a vanishing cosmological constant at the scale of stabilization. We construct a few models which accommodate weak gauge couplings with large or small gauge groups. Matter condensation plays a central role in the dilaton stabilization.
1 Introduction

It is expected that a unified theory of all interactions (yet to be discovered) will allow description based on effective field theory at low energy. One possible feature of a low-energy effective theory is that its coupling constants should be determined dynamically, since a unified theory may well have no arbitrary parameters to provide ad hoc coupling constants. In fact, superstring theory as a candidate of such a unified theory yields, at least perturbatively, this type of effective field theory, which generally contains, among others, the dilaton field as a modulus to determine the overall coupling strength.

In this paper, we consider an $R$-invariant stabilization of the vacuum expectation value of the dilaton. An advantage of the $R$ invariance is that the cosmological constant at the stabilization scale is naturally vanishing in the stabilized vacuum in contrast to schemes without $R$ invariance. We provide a few models of dilaton fixing with large or small gauge groups, where the dilaton is stabilized in the vacuum corresponding to weak gauge couplings.

2 Dilaton Stabilizer

Let us consider a superpotential of the form

$$W = Xf(\Phi),$$

(1)

where $X$ denotes a chiral superfield carrying $R$ charge 2 and $f(\Phi)$ is a function of the dilaton supermultiplet $\Phi$ with vanishing $R$ charge. This superpotential can be generated by dynamics of gauge interactions.

The potential in supergravity is given by

$$V = e^K (K_{AB} F^A F^B + 3|W|^2),$$

(2)

1 This dilaton is not necessarily the dilaton in perturbative string theory. We only require that the vacuum expectation value of this dilaton determines the gauge couplings at the unification scale.

2 The $R$ symmetry for model building in relation with a vanishing cosmological constant is considered in Ref. The vanishing of the cosmological constant with supersymmetry breaking is not achieved without an appropriate $R$ breaking.
where $K$ is a K"ahler potential, $K_{AB}$ denotes the inverse of the matrix
\[
\frac{\partial^2 K}{\partial \phi_A \partial \phi^*_B},
\]
with $\phi_A = X, \Phi,$ and $F^A$ is given by
\[
F^A = \frac{\partial W}{\partial \phi_A} + \frac{\partial K}{\partial \phi_A} W.
\]
Here we set the gravitational scale equal to one (in the Einstein frame).

Judicious choice of the function $f(\Phi)$ can stabilize the dilaton $\Phi$ in the supersymmetric $R$-invariant vacuum:
\[
\langle X \rangle = 0, \quad f(\langle \Phi \rangle) = 0, \quad \langle V \rangle = 0,
\]
where $\langle F^A \rangle = \langle W \rangle = 0$.

In the following sections, we discuss dynamics giving rise to suitable superpotentials for the dilaton $\Phi$.

## 3 Dynamical Preliminaries

We first discuss the dynamics causing matter condensation, since this is crucial for dilaton stabilization in our models. We adopt an $\text{Sp}(N)$ gauge theory with $2(N+1)$ chiral superfields $Q_i$ in the fundamental $2N$-dimensional representation, where $i$ is a flavor index ($i = 1, \cdots, 2(N+1)$) and the gauge index is omitted. Without a superpotential, this theory has a flavor $\text{SU}(2(N+1))_F$ symmetry. This $\text{SU}(2(N+1))_F$ symmetry is explicitly broken down to a flavor $\text{Sp}(N+1)_F$ by a superpotential in our models. That is, we add gauge singlets $Z^a$ ($a = 1, \cdots, N(2N + 3)$) to obtain the tree-level superpotential
\[
W_0 = \lambda Z^a (QQ)_a,
\]
\footnote{This is a local minimum of the potential with unbroken supersymmetry \textsuperscript{\text{\textsuperscript{[6]}}}. Whether or not this is the global minimum of the potential depends on the K"ahler potential.}

\footnote{Similar dynamics were considered in Ref.\textsuperscript{\textsuperscript{[7]}} with the dilaton field. The dynamics without the dilaton were investigated in Ref.\textsuperscript{\textsuperscript{[8]}} and utilized to construct vector-like models of dynamical supersymmetry breaking \textsuperscript{\textsuperscript{[9]}}.}

\footnote{This superpotential may provide an inflationary sector in our models \textsuperscript{\textsuperscript{[11]}}.}
where \((QQ)_a\) denotes a flavor \(N(2N + 3)\)-plet of \(\text{Sp}(N + 1)_F\) given by a suitable combination of gauge invariants \(Q_iQ_j\).

The effective superpotential which may describe the dynamics of the \(\text{Sp}(N)\) gauge interaction \[7, 8\] is given by

\[
W_{\text{eff}} = S(\text{Pf}V_{ij} - \Lambda^{2(N+1)}) + \lambda Z^a V_a
\]

in terms of low-energy degrees of freedom

\[
V_{ij} \sim Q_i Q_j, \quad V_a \sim (QQ)_a,
\]

where \(S\) is an additional chiral superfield and \(\Lambda\) denotes a dynamical scale of the gauge interaction, which is given by \(\Lambda = \exp(-8\pi^2\Phi/b)\) with \(b = 2(N+1)\).

The effective superpotential Eq.(7) implies that, among the gauge invariants \(Q_iQ_j\), the \(\text{Sp}(N + 1)_F\) singlet \((QQ)\) condenses as \(\langle(QQ)\rangle = \Lambda^2\) with supersymmetry unbroken in the corresponding vacuum \[10\]. Since the vacuum preserves the flavor \(\text{Sp}(N + 1)_F\) symmetry, we have no massless Nambu-Goldstone multiplets. The absence of flat directions except for the dilaton \(\Phi\) at this stage is crucial for achieving dilaton stabilization in the subsequent sections.

### 4 Simple Models

Let us take a single \(\text{Sp}(N)\) gauge theory with \(2(N + 1)\) chiral superfields \(Q\) in the fundamental representation, as above. We omit the flavor index of \(Q\) here and henceforth. The superpotential of our simple model is given by

\[
W = \lambda Z^a (QQ)_a + X(f_1(QQ) - f_2(QQ)^2).
\]

For simplicity, we have neglected higher-order terms in \((QQ)\), whose inclusion does not change our conclusion qualitatively.

Integrating out the \(\text{Sp}(N)\) gauge interaction along with the gauge singlets \(Z^a\), we obtain a condensation \(\langle(QQ)\rangle = \Lambda^2\), as shown in the previous section, where \(\Lambda = \exp(-8\pi^2\Phi/b)\) and \(b = 2(N+1)\). Then the effective superpotential of the model is given by Eq.(11) with the function

\[
f(\Phi) = f_1 e^{-\frac{16\pi^2}{b} \Phi} - f_2 e^{-\frac{32\pi^2}{b} \Phi}.
\]

4
The $R$-invariant vacuum is given by
\[ \langle X \rangle = 0, \quad e^{-\frac{16\pi^2}{b_1} \langle \Phi \rangle} = \frac{f_1}{f_2}, \] (11)

To obtain a desired value $\text{Re}(\langle \Phi \rangle) \approx 2$ of the dilaton condensation, we need a large $N$ ($\approx 150$) for $|f_1/f_2| \approx e^{-1}$. We note that only one gauge group is sufficient to fix the dilaton $\Phi$.

5 Semisimple Models

We adopt two $\text{Sp}(N_i)$ gauge theories with $2(N_i+1)$ chiral superfields $Q_i$ in the fundamental representations ($i = 1, 2$). We consider two types of semisimple models.

1) The superpotential of the first model is given by
\[ W = \lambda_1 Z_1^{a_1} (Q_1 Q_1)_{a_1} + \lambda_2 Z_2^{a_2} (Q_2 Q_2)_{a_2} + X (f_1 (Q_1 Q_1) - f_2 (Q_2 Q_2)). \] (12)

Integrating out the $\text{Sp}(N_1) \times \text{Sp}(N_2)$ gauge interactions along with the gauge singlets $Z_i^{a_i}$, we obtain condensations $\langle (Q_i Q_i) \rangle = \Lambda_i^2$ for $i = 1, 2$. Here $\Lambda_i$ denotes the dynamical scales of the $\text{Sp}(N_i)$ gauge interactions, which are given by $\Lambda_i = \exp(-8\pi^2 b_i/\Phi)$ and $b_i = 2(N_i + 1)$. Then the effective superpotential of the model is given by Eq.(11) with the function
\[ f(\Phi) = f_1 e^{-\frac{16\pi^2}{b_1} \Phi} - f_2 e^{-\frac{16\pi^2}{b_2} \Phi}. \] (13)

The $R$-invariant vacuum is given by
\[ \langle X \rangle = 0, \quad e^{-\frac{16\pi^2}{b} \langle \Phi \rangle} = \frac{f_1}{f_2}, \] (14)

where $b = b_1 b_2/(b_1 - b_2)$. The desired value $\text{Re}(\langle \Phi \rangle) \approx 2$ for $|f_1/f_2| \approx e^{-1}$ is realized by $N_1 = 12$ and $N_2 = 11$, for example.

The gauge coupling constant $g$ is given by $g^2 = 1/\text{Re}(\langle \Phi \rangle)$. Hence $\text{Re}(\langle \Phi \rangle) \approx 2$ corresponds to the gauge coupling constants of the standard model gauge groups determined at the grand unification scale of order $10^{16}$ GeV.
2) The superpotential of the second model is given by
\[ W = \sum_{i=1,2} \lambda_i Z_i^a (Q_i Q_i)_a + Y (f_1' \Psi' Q_i - f_1 (Q_1 Q_1)) + X (f_2' \Psi'^a Q_2), \]
(15)
where \( Y \) and \( \Psi \) are singlet chiral superfields.

Integrating out the \( \text{Sp}(N_1) \times \text{Sp}(N_2) \) gauge interactions along with the
gauge singlets \( Z_i^a, Y \) and \( \Psi \), we obtain the effective superpotential of the
model in the form Eq.(1) with the function
\[ f(\Phi) = f_2' \left( \frac{f_1}{f_1'} \right)^{\frac{n_2}{n_1}} e^{-\frac{16 \pi^2 n_2}{b_2} \Phi} - f_2 e^{-\frac{16 \pi^2}{b_2} \Phi}. \]
(16)

The \( R \)-invariant vacuum is given by
\[ \langle X \rangle = 0, \quad e^{-\frac{16 \pi^2}{b} \Phi} = \frac{f_2}{f_2'} \left( \frac{f_1'}{f_1} \right)^{\frac{n_2}{n_1}}, \]
(17)
where \( b = n_1 b_1 b_2 / (n_2 b_2 - n_1 b_1) \). The desired value \( \text{Re}(\Phi) \simeq 2 \) for \( |f_1'/f_1| \simeq |f_2/f_2'| \simeq e^{-1} \) is realized by \( N_1 = 1, N_2 = 2, n_1 = 13 \) and \( n_2 = 9 \), for example.

The sizes of the gauge groups can be minimal in the present model, though
small gauge groups yield a small dilaton mass, which might be undesirable
phenomenologically.

6 Conclusion

We have introduced a dilaton stabilizer \( X \) with the superpotential Eq.(1),
which gives the supersymmetric \( R \)-invariant vacuum Eq.(1) with a vanishing
cosmological constant.\( ^7 \) We have utilized matter condensation rather than
gaugino condensation to stabilize the dilaton \( \Phi \). This is a crucial difference
from the previous works \( ^4, ^5 \). We have considered a few dynamical mod-
els Eqs.(9), (12) and (15), where the dilaton is stabilized at the vacuum
expectation value corresponding to weak gauge couplings.

Once the dilaton is fixed, various dynamical models without the dilaton
can be used. For example, dynamical supersymmetry breaking (see Ref.\( ^4 \))
and dynamical inflation (see Ref.\( ^1 \)) may be realized with the fixed dilaton.

\( ^7 \)The stabilizer \( X \) may cause a hybrid inflation in conjunction with the dilaton \( \Phi \).
The scale of dynamical inflation is determined by that of its gauge dynamics. The runaway vacuum $\Re \Phi \to \infty$ induces no inflation, while the stabilized vacuum can incorporate dynamical inflation if the stabilization scale is sufficiently large. In this case, the universe may be naturally dominated by the stabilized vacuum through the inflationary dynamics under chaotic initial conditions.

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