High–resolution simulations of stellar collisions between equal-mass main-sequence stars in globular clusters

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ABSTRACT

We performed high–resolution simulations of two stellar collisions relevant for stars in globular clusters. We considered one head-on collision and one off-axis collision between two 0.6 $M_\odot$ main sequence stars. We show that a resolution of about 100 000 particles is sufficient for most studies of the structure and evolution of blue stragglers. We demonstrate conclusively that collision products between main-sequence stars in globular clusters do not have surface convection zones larger than 0.004 $M_\odot$ after the collision, nor do they develop convection zones during the ‘pre-main-sequence’ thermal relaxation phase of their post-collision evolution. Therefore, any mechanism which requires a surface convection zone (i.e. chemical mixing or angular momentum loss via a magnetic wind) cannot operate in these stars. We show that no disk of material surrounding the collision product is produced in off-axis collisions. The lack of both a convection zone and a disk proves a continuing problem for the angular momentum evolution of blue stragglers in globular clusters.

Key words: blue stragglers – globular clusters – hydrodynamics – convection – stars: rotation

1 INTRODUCTION AND MOTIVATION

It has been shown that the products of main-sequence – main-sequence collisions appear in the colour-magnitude diagrams of clusters as blue stragglers [Sills et al. 1997; Sandquist et al. 1997]. Since blue stragglers are readily observable in clusters, they form an ideal population with which to probe the dynamical evolution of the cluster. The dynamical state of a globular cluster (its density profile, velocity dispersion, amount of mass segregation etc.) will determine the rate and nature of the collisions which occur in the cluster. As the cluster evolves, the kinds of collision that occur will change. Therefore, the population of collision products in a cluster can be used to probe the history of the cluster [Sills et al. 2000; Hurley et al. 2001]. However, in order to use collision products in this way, there are two issues which must first be understood. First, we know that blue stragglers can also be formed through the merger of two components of a binary system. These blue stragglers will probably have different properties than those formed from collisions. We must either be confident that the population we are observing is collisional in origin (e.g. from cluster density considerations), or be able to distinguish between the two populations. Secondly, we also need to be sure that we understand the formation and evolution of the collision products themselves. This paper is concerned with addressing the second point.

In this paper, we present the highest resolution smoothed particle hydrodynamic (SPH) simulations of collisions between main-sequence stars to date. Most recent computations have $\sim 10^4$ particles [Lombardi et al. 1998; Sandquist et al. 1997], with the highest resolution simulation using $10^5$ particles [Sills et al. 2001]. In this paper, we increase the number of particles to $10^6$. There are three main reasons to extend this kind of simulation to such high resolution. The first, and simplest, is to make sure that no fundamental change in our understanding of blue stragglers occurs. In earlier work, Benz & Hills [1987] performed an SPH simulation using 1024 particles, and concluded that collision products are fully mixed (i.e. the resulting star is chemically homogeneous). Ten years later, these simulations were repeated but with a factor of 10-50 more particles

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(Lombardi et al. 1996). Because of the higher resolution, it became clear that collision products are NOT chemically homogeneous, but instead retain some memory of the chemical profile of the parent stars of the collision. We feel confident that such a fundamental change in our understanding will not happen again when we increase the resolution, but we want to be sure.

More importantly, we performed these high resolution calculations to answer two fundamental questions about the structure of the collision products immediately after the collision. The first involves the structure of the outer layers of the star, and the second is concerned with the material which is thrown off by the star during the collision itself. There has been some debate in the literature recently about whether the collision product has a surface convection zone shortly after the end of the collision. The presence of such a convection zone could have ramifications for both the surface chemical abundances and the rotation rate of the star when it reaches its main sequence (Leonard & Livio 1995). Previous SPH simulations and subsequent evolution calculations have shown that no convection zone exists at the end of the collision, nor does it appear during the initial thermal relaxation of the star (Lombardi et al. 1996; Sills et al. 1997). However, members of the community argue that the previous simulations are unable to resolve the outermost regions of the star well at all, due to their low particle number and use of equal-mass particles (Livio, private communication). In this paper, we resolve this issue by increasing the resolution of our simulations in the outer regions of the parent stars in particular, and by using variable mass particles.

We are also interested in following the material which is thrown off from the stars during the collision. We have discovered, using previous generation simulations, that blue stragglers which are formed by an off-axis collision have an angular momentum problem (Sills et al. 2001). Namely, these stars retain too much of their angular momentum, and have no apparent way of losing it during their thermal relaxation phase after the collision. As a result, the collision products spin up during their collapse to the main sequence, and inevitably rotate faster than their break-up velocity. Such collision products can never become blue stragglers, and inevitably rotate faster than their break-up velocity. Such collision products can never become blue stragglers, since they tear themselves apart before they reach the main sequence. We wish to follow the lost material with greater resolution, to study the amount of angular momentum that this material carries away with it. We also wish to follow the outermost material of the parent stars to see if some of it forms a disk around the collision product. If so, then we can plausibly suggest that magnetic locking to a disk is responsible for removing angular momentum from the collision product.

In §2 we describe the methods used to model the stellar collisions and their subsequent evolution. We present our results in §3, and discuss their implications in §4.

2 METHOD

The simulations of stellar collisions discussed in this paper were performed using the smoothed particle hydrodynamics (SPH) method (Benz 1990, Monaghan 1992). Our three dimensional code uses a tree to solve for the gravitational forces and to find the nearest neighbours (Benz et al. 1990). We use the standard form of artificial viscosity with $\alpha = 1$ and $\beta = 2.5$, and an adiabatic equation of state. The thermodynamic quantities are evolved by following the change in internal energy. Both the smoothing length and the number of neighbours can change in time and space. The smoothing length is varied to keep the number of neighbours approximately constant ($\sim 50$ for the low resolution runs, and $\sim 200$ for the million particle run). The code is the parallel version of the code described in detail in Bate, Bonnell & Price (1997). It was parallelized using OpenMP, and was run on the SGI Origin 3000 operated by the UK Astrophysical Fluids Facility (UKAFF) based at the University of Leicester.

We modelled collisions between two equal-mass stars. We chose a mass of 0.6 $M_\odot$ as a representative, but not extreme, mass for globular cluster stars. The initial stellar models were calculated using the Yale stellar evolution code (YREC, Guenther et al. 1992), and had a metallicity of $Z = 0.001$ and an age of 15 Gyr. If the goal of the simulation is to investigate the detailed structure of the collision product, then it is crucial to begin with a realistic stellar model rather than a polytrope or other approximation (Sills & Lombardi 1997). The particles were initially distributed on an equally-spaced grid, and their masses were varied until the density profile matched that of the stellar model. By using unequal mass particles, we increase the resolution in the outer, low density regions of the star. This is important for resolving the outer layers of the collision products (to determine if there is a convection zone or not) and for following the material which is thrown off during the collision.

The stars were given a relative velocity at infinity of $v_\infty = 10$ km/s, which is a reasonable value for globular cluster stars. They were set up on almost parabolic orbits with a pericentre separation $r_p = 0$ for the head-on collisions, and $r_p = 0.25 R_\odot$ for the off-axis collisions. (The radius of the main sequence star is 0.61 $R_\odot$.) Most collisions are expected to happen during interactions involving binary stars (Hut & Bahcall 1983), so the stars are on bound, and usually highly elliptical, orbits. For both the head-on and off-axis collisions, the number of particles was varied between 10 000 and 300 000. We also ran one head-on collision with $10^6$ particles. The details of the collisions are shown in Table 1. We give a single letter name for each run, the pericentre separation $r_p$ in solar radii, the number of particles, the initial value of the smoothing length $h_0$ in solar radii, the minimum and maximum particle masses in solar masses, and the amount of mass lost from the system during the collision in solar masses.

The results of these three dimensional SPH simulations were converted to one dimensional models suitable for starting models for the Yale stellar evolution code, YREC. The entropy of the particles and their chemical abundances were averaged over surfaces of constant gravitational potential and binned into $\sim 100$ bins. The structure of the collision product is determined from these profiles using the equation of hydrostatic equilibrium. The temperature profile is calculated using the ideal gas equation of state. For more details of the conversion between SPH results and stellar evolution models, see Sills et al. (1997, 2001).

We used YREC to calculate the evolution of the collision products. Using the starting models described above, we followed their evolution during the thermal relaxation phase (analogous to the pre-main-sequence of normal stars),
through their main-sequence lifetimes and to the giant branch.

3 RESULTS

We performed five simulations of head-on stellar collisions, with the number of particles ranging from $10^4$ to $10^6$. The average resolution elements for these collisions ranged from $2\hbar = 0.116R_\odot$ to $2\hbar = 0.027R_\odot$ respectively. Figure 1 shows the profiles of the important structural parameters as a function of mass fraction in the product of a head-on collision between two 0.6 $M_\odot$ stars. The different line styles denote increasing particle numbers in the following order: 10 000 (solid), 30 000 (dotted), 100 000 (short dashed), 300 000 (long dashed), 1 000 000 (dot–dashed). The low resolution simulations do a reasonable job of producing the correct structure for the collision products. However, as the resolution increases, the models converge to an answer different from that seen in the low resolution runs. This is best seen in the plot of hydrogen abundance as a function of mass fraction. The 100 000 and 300 000 particle simulations have converged to a solution for the structure of this collision product. The higher resolution simulations also produce smoother profiles. This is expected, since we have the information from more particles contributing to the values of the structural quantities (pressure, density, etc.) in each shell. Therefore, we suggest that the optimum number of particles for studying the structure of stellar collision products is around 100 000.

In Figure 2 we present the evolutionary tracks which result from the starting models shown in Figure 1. The stars begin their post-collision lives at position A. They are quite bright, and are significantly out of thermal equilibrium because of the energy deposited into their envelopes by the collision. They contract to the main sequence (position B), where core hydrogen burning begins. The stars look like normal main-sequence stars, except that their central hydrogen abundance is less than it would be for a normal star of that mass. Therefore, their main-sequence lifetime is shorter than that of a normal 1.1 $M_\odot$ star (3.3 Gyr). The collision products reach the turnoff, where hydrogen is exhausted in their core, and evolve towards the giant branch (position C), exactly like normal stars. The time since the collision at three benchmark positions along the evolutionary track is given in Table 2. Both the post-collision structure and the evolutionary tracks agree with previous simulations of the same collision (Sills et al. 1997).

The different line styles in Figure 2 indicate the different number of particles in the simulation, with the line styles having the same meaning as in Figure 1. We see the same trend with increasing resolution of the simulation – the tracks converge to a common solution around 100 000 particles. This is expected, since the evolution of a star is determined by its structure. The same trend of convergence is seen in the evolutionary timescales in Table 2. Note that the largest differences between simulations show up in the

### Table 1. Description of SPH simulations performed

| Run | $r_p/R_\odot$ | $N_{\text{part}}$ | $b_0/R_\odot$ | $M_{\text{min}}/M_\odot$ | $M_{\text{max}}/M_\odot$ | $M_{\text{lost}}/M_\odot$ |
|-----|---------------|--------------------|----------------|--------------------------|--------------------------|--------------------------|
| A   | 0.0           | 10 162             | 0.058          | $2.10 \times 10^{-6}$    | $4.58 \times 10^{-3}$    | $8.14 \times 10^{-2}$    |
| B   | 0.0           | 29 950             | 0.0417         | $3.71 \times 10^{-7}$    | $1.37 \times 10^{-3}$    | $7.76 \times 10^{-2}$    |
| C   | 0.0           | 100 034            | 0.028568       | $4.96 \times 10^{-5}$    | $4.47 \times 10^{-3}$    | $7.49 \times 10^{-2}$    |
| D   | 0.0           | 299 398            | 0.02011        | $5.27 \times 10^{-9}$    | $1.56 \times 10^{-4}$    | $7.41 \times 10^{-2}$    |
| E   | 0.0           | 999 778            | 0.0136         | $2.98 \times 10^{-10}$   | $4.75 \times 10^{-5}$    | $7.47 \times 10^{-2}$    |
| F   | 0.25          | 10 162             | 0.058          | $2.10 \times 10^{-6}$    | $4.58 \times 10^{-3}$    | $3.45 \times 10^{-2}$    |
| G   | 0.25          | 29 950             | 0.0417         | $3.71 \times 10^{-7}$    | $1.37 \times 10^{-3}$    | $3.62 \times 10^{-2}$    |
| H   | 0.25          | 100 034            | 0.028568       | $4.96 \times 10^{-5}$    | $4.47 \times 10^{-3}$    | $3.59 \times 10^{-2}$    |
| I   | 0.25          | 299 398            | 0.02011        | $5.27 \times 10^{-9}$    | $1.56 \times 10^{-4}$    | $4.16 \times 10^{-2}$    |
Table 2. Evolutionary Timescales (Gyr)

| Run | Return to MS | Turnoff | Giant Branch |
|-----|--------------|---------|-------------|
| A   | 1.60 \times 10^{-3} | 1.41    | 2.08        |
| B   | 1.90 \times 10^{-3} | 1.64    | 2.41        |
| C   | 2.20 \times 10^{-3} | 1.85    | 2.69        |
| D   | 2.16 \times 10^{-3} | 1.92    | 2.79        |
| E   | 2.20 \times 10^{-3} | 1.87    | 2.73        |

Figure 2. Evolutionary tracks for head-on collision products. The stars begin their post-collision evolution far from the Hayashi track, but are still bright (position A). As they contract, they move towards the main sequence (position B), and then follow a standard evolutionary track towards the giant branch (position C). The line styles are the same as in Figure 1. The difference in structure between the different resolution simulations manifests itself as a difference in evolutionary track shape. The tracks are most different during their initial contraction, and then again near the turnoff of the main sequence. The lowest resolution simulation (10,000 particles, solid line) has a hook in its evolutionary track near the turnoff, evidence of a well-developed core convection zone. The higher resolution simulations do not show this feature.

thermal relaxation phase, and that by the main sequence, all the simulations are almost indistinguishable. The exception is the lowest resolution simulation. The solid black line begins its main sequence quite close to the dotted line (30,000 particle simulation), but has a hook near the turnoff, indicating the presence of a central convection zone. As the resolution increases, we see that this convection zone disappears. Simulations with 10^4 particles are not quite detailed enough to accurately depict the true structure of these collision products.

The criterion for convective stability in stellar models is the Schwarzschild criterion:

\[ \nabla - \nabla_{ad} \leq 0 \]  \hspace{1cm} (1)

where \( \nabla_{ad} \) is the adiabatic temperature gradient in the star, and \( \nabla \) is the radiative temperature gradient:

\[ \nabla = \frac{d \ln T}{d \ln P} = \frac{3}{64 \pi \sigma G M T^4} \kappa L P \]  \hspace{1cm} (2)

where \( \sigma \) is the Stefan Boltzmann constant, \( G \) is the gravitational constant, and \( \kappa, L, P, M \) and \( T \) are the opacity, luminosity, pressure, enclosed mass and temperature at that position in the star. The models taken from the SPH results were used as starting models in the stellar evolution code, and allowed to relax, so that all the equations of stellar structure are satisfied. Since our SPH code does not allow for energy transport, the luminosity distribution in the star can only be determined from the other structure parameters using the equations of stellar structure. The temperature gradients were calculated, and their difference is plotted as a function of mass fraction in Figure 3. We can clearly see a convective core in these stars, where \( \nabla - \nabla_{ad} > 0 \). However, the entire star outside of the inner 0.1 \( M_\odot \) is not convective. The outermost shell in all our stellar models is at a mass fraction \( M/M_\star = 0.99667 \). This value is the same for all models, regardless of the number of SPH particles, because of the way we have transformed the SPH information to the stellar evolution starting model. The total mass of these stars is 1.12 \( M_\odot \), so only the outer 0.004 \( M_\odot \) is not resolved. Therefore, we can say with certainty that this stellar collision product does not have a surface convection zone larger than 0.004 \( M_\odot \). Our evolutionary models show that the stellar collision products do not develop surface convection zones during their thermal contraction to the main sequence.

Figure 3. Convective stability criterion \( \nabla - \nabla_{ad} \) as a function of mass fraction for the head-on collision models immediately after the collision. The line styles are the same as in Figure 1. When \( \nabla - \nabla_{ad} \) is less than zero, the region is stable against convection. Except for a small region at the centre of the star, the entire collision product is stable against convection.
As well as studying the head-on collisions, we performed one off-axis collision with a pericentre separation of 0.25 \( R_\odot \).

Figure 4 shows the structure profiles of the collision products at various resolutions. The line styles are the same as in Figure 1, except that we have not run a million-particle off-axis simulation. In addition to profiles of pressure, temperature, density and hydrogen mass fraction, we show the angular velocity and radius profiles. Again, there is clearly a convergence of the structural properties as the number of particles increases. The 100 000 particle (short dashed line) and 300 000 particle (long dashed line) simulations are almost indistinguishable.

The final question we wish to investigate with these high resolution simulations involves the material which is thrown off during the collision. Between 1 and 10% of the original stellar material becomes unbound from the system as it is shock-heated during these kinds of collisions, depending on the masses of the parent stars and the impact parameter (Lombardi et al. 1996). Other material is thrown to large distances but remains bound and falls back onto the collision product. If some of this material has sufficient angular momentum, it will form a disk around the collision product. The existence of such a disk could prove important for transporting angular momentum from the central collision product.

Figure 5 is a snapshot of the 300 000 particle off-axis stellar collision taken 50 dynamical times after the collision began. The star has reached hydrostatic equilibrium and is settled down to its post-collision state. The contours show the density of the material in a slice through the x-z plane of the collision product. The collisions were constrained to occur in the x-y plane, so the product is rotating around the z axis. It is rotating rapidly, and therefore is highly flattened by rotation. However, there is no indication that a circumstellar disk is present. This is in agreement with the results of Benz & Hills (1987), who find that disks are present only for simulations with larger impact parameters.

4 SUMMARY AND DISCUSSION

We have performed the highest resolution SPH simulations of collisions between main-sequence stars to date, using up to \( 10^6 \) SPH particles. The collisions were between equal mass main-sequence stars in globular clusters – the stars have masses of 0.6 \( M_\odot \), metallicity \( Z=0.001 \), age of 15 Gyr and a relative velocity at infinity of 10 km/s. We varied the resolution of our simulations from \( 10^4 \) to \( 10^6 \) SPH particles, and we looked at a head-on collision and an off-axis collision. These simulations took between \( \sim 6 \) CPU hours and \( \sim 5000 \) CPU hours, and the CPU time scaled approximately linearly with \( N \log N \). We had three goals for performing this research.

Our first goal was to determine the optimal resolution for SPH simulations of this kind. Since we want to use the results of the SPH simulations to study the detailed structure and evolution of the collision products, we need to have an accurate model of the distribution of the SPH particles and their properties. On the other hand, we do not want to waste valuable computer resources going to a higher resolution than is necessary. We have shown that both head-on and off-axis simulations with \( \sim 100 000 \) particles give essentially the same information as runs with higher resolution,
and therefore we suggest that most simulations with similar requirements can confidently use on order 10^5 particles.

Our second goal was to use these high resolution simulations to study the outer few percent of the collision products. In particular, we were interested in the existence of a surface convection zone, for two reasons. One consequence of a convection zone is mixing of elements in the convection zone. If the zone reaches deep enough in a star to dredge up nuclear processed material, for example, the surface abundances of elements like C, N and O will be different from what we would expect for primordial material. In blue stragglers and other collision products, surface abundances could be used to determine, perhaps, the masses of the parent stars. However, this can only be done if we understand whether the abundances should be mixed by convection or not. The second consequence of a surface convection zone is angular momentum loss. Stars can lose angular momentum through a magnetic wind (Kawaler 1988). This process is most effective in stars with deep convection zones, since they can support stronger magnetic fields and since the angular momentum is drained out of the entire (uniformly rotating) convection zone.

We have shown that the product of the head-on stellar collision investigated in this paper does not have a surface convection zone larger than 0.004 M⊙. For comparison, the Sun is considered to have a fairly shallow convection zone, with a mass of 0.02 M⊙ (Guenther et al. 1992). A convection zone less than about 0.01 M⊙ is certainly not deep enough to substantially modify the surface abundances of any element, with the possible exception of lithium, beryllium and boron (which are destroyed at low temperatures). Angular momentum loss is also going to be very small for stars with such small convection zones.

Our final goal was to determine whether the off-axis stellar collision products had a circumstellar disk. If a rotating star has a disk with a mass of about 0.01 M⊙, and it is locked to the disk by a magnetic field, then angular momentum will be transported from the star to the disk by the field (Konigl 1991). The net effect of the angular momentum transport will be to slow the star’s rotation rate. This effect is thought to operate in pre-main-sequence stars (Sills, Pinsonneault & Terndrup 2000 and references therein). Unfortunately, our simulations do not show any evidence for a disk. Based on previous work (Benz & Hills 1987), we expect that blue stragglers which were formed from collisions with larger impact parameters will have circumstellar disks. However, the low impact parameter collision we modelled in this simulation is still rotating too quickly to contract to the main sequence without losing some angular momentum.

Sills et al. (2001) showed that blue stragglers that are formed by stellar collisions have an angular momentum problem. The collision products are formed with large total angular momentum, and some of them are rotating near their break-up velocity. The collision products are large objects, and they start to contract to the main sequence. As they contract, they spin up since they have no way to lose any angular momentum. They begin to reach break-up velocities and become unstable. Even by shedding material, they cannot remove enough angular momentum to remain bound, and they tear themselves apart. We are forced to one of two conclusions; either blue stragglers are not made by stellar collisions (since head-on collisions are very rare, and all off-axis collisions have too much angular momentum), or collision products have some way of losing their angular momentum. The two standard ways stars are thought to lose angular momentum is through a magnetic wind, or through disk-locking. We have shown that both these scenarios are not viable, since these collision products do not have surface convection zones or circumstellar disks immediately after the collision. However, it is interesting to note the work of Durisen et al. (1986), who show that if a polytrope is rotating very rapidly, some outer material is thrown off into a disk, leaving a stable (but non–axisymmetric) central object. In order for this to happen, the star has to have a ratio of rotational kinetic energy to gravitational energy β ≥ 0.3. At the end of the collision, our off-axis collision product has β ∼ 0.045, and is therefore stable against any bar instability. However, as the star evolves and contracts, it is possible that β could become high enough to trigger some of these hydrodynamic rotational instabilities.

In this paper, we have limited ourselves to two very specific collisions. The two stars involved in the collision were of equal mass, and we only investigated two choices of impact parameter. There have been other studies of main-sequence star stellar collisions which covered more parameter space (Benz & Hills 1987; Lombardi et al. 1990; Sandquist et al. 1997). These studies have clearly and carefully outlined the properties of collisions under different circumstances. These high resolution simulations were meant to answer some specific questions about the details of stellar collisions and the resulting collision products. We can generalize our current results to all main-sequence star collisions in globular clusters under the limitations outlined in the previous papers. Therefore, we feel justified in claiming that no collision products have surface convection zones, and that collisions do not create long–lived circumstellar disks. The search to understand the angular momentum evolution of blue stragglers will have to turn to other avenues, possibly involving a more detailed look at the combination of stellar evolution and hydrodynamic instabilities after the collision.

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REFERENCES

Bate, M.R., Bonnell, I.A., Price, N.M., 1995, MNRAS, 277, 362
Benz, W., 1990, in Buchler J. R., ed., The Numerical Modelling of Nonlinear Stellar Pulsations: Problems and Prospects. Kluwer, Dordrecht, p. 269
Benz, W., Bowers R. L., Cameron A. G. W., Press W., 1990, ApJ, 348, 647

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Benz, W., Hills, J. G., 1987, ApJ, 323, 614
Durisen, R. H., Gingold, R. A., Tohline, J. E., Boss, A. P., 1986, ApJ, 305, 281
Guenter, D. B., Demarque, P., Pinsonneault, M. H., & Kim, Y.-C., 1992, ApJ, 392, 328
Hurley, J. R., Tout, C. A., Aarseth, S. J., & Pols, O. R. 2001, MNRAS, 323, 630
Hut, P., Bahcall, J. N., 1983, ApJ, 268, 319
Kawaler, S.D., 1988, ApJ, 333, 236
Konigl, A., 1991, ApJ, 370, L39
Leonard, P.J.T., Livio, M., 1995, ApJ, 447, L121
Lombardi, J. C., Jr., Rasio, F. A., Shapiro, S. L., 1996, ApJ, 468, 767
Monaghan, J.J, 1992, ARA&A, 30, 543
Sandquist, E., Bolte, M., & Hernquist, L., 1997, ApJ, 477, 335
Sills, A., Lombardi, J. C., Jr., 1997, ApJL, 484, 51
Sills, A., Lombardi, J. C., Jr., Bailyn, C. D., Demarque, P. D., Rasio, F. A., Shapiro, S. L., 1997, ApJ, 487, 290
Sills, A., Pinsonneault, M. H., Terndrup, D. M., 2000, ApJ, 534, 335
Sills, A., Bailyn, C. D., Edmunds, P. T., Gilliland, R. L., 2000, ApJ, 535, 298
Sills, A., Faber, J. A., Lombardi, J. C., Jr., Rasio, F. A., Warren, A. R., 2001, ApJ, 548, 323