Assessment of the contribution of phase transformations to the space-time distributions of the temperature of the filtering oil

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Abstract. The solution of the problem of filtration of paraffinic oil by asymptotic methods in the form of endless sequence of boundary value problems for the expansion coefficients of the required solution in the Maclaurin series is obtained. Estimation formulas were used to calculate the space-time temperature distributions in the first approximation for the vibrational motion of viscous oil and the contribution of phase transformations of paraffin wax. The results can be used for theoretical substantiation and further development of thermal methods for influencing oil-bearing reservoirs, as well as for predicting temperature processes in the bottomhole zone.

The thermal impact on the layer is one of the most effective and perspective methods of enhancing oil recovery from deposits with a high content of paraffins, resins, and asphaltenes. The dependence of oil recovery from a layer containing high-viscosity oil on temperature is complex.

To construct a mathematical model for the filtration of paraffinic oil and solving the thermodynamic problem, we will take into account the following factors: heat transfer between the components of the medium, convective heat transfer, deceleration of thermal effects due to the high heat capacity of the porous body, friction, heat transfer losses, phase transformations of oil components with temperature changes.

The statement of a thermodynamic problem, describing the process of filtration of structured inhomogeneous liquids - paraffinic oils with complex characteristics, presupposes the consideration of filtration conditions and the introduction of some assumptions. Let us assume there is a layer of thickness \( H \), filled with paraffinic oil, the bottom-hole zone of which is waxed. The movement of a two-phase liquid is considered as plane-parallel, because in the conditions of an oil field, the horizontal length exceeds its thickness in many times. The model for analytical research is a borehole of radius \( r_0 \), centrally located in a circular layer of a homogeneous porous medium of radius \( R \) (figure. 1). The layer is divided into three areas, perpendicular to the well axis. Let us assume that there is no vertical filtration of a homogeneous liquid: for a flat-radial geometry, the speed of liquid flow through the porous walls of the cylinder in the middle layer depends only on \( r \) and \( t \) and does not depend on the coordinates \( \varphi \) and \( z \): \( \vec{v} = \vec{v}(r, t) \). Thermophysical characteristics along the vertical axis – thermal conductivity \( \lambda_z \) and thermal diffusivity \( \alpha_z \) – differ from the corresponding properties along the...
horizontal axis $r - \lambda$ and $a$, accordingly. Impermeable rocks with thermophysical parameters ($\lambda_1$, $a_1$, $\lambda_{z1}$, $a_{z1}$) and ($\lambda_2$, $a_2$, $\lambda_{z2}$, $a_{z2}$) are located in the region $z > h$ and $z < -h$, accordingly.

We will take the oil temperature at the initial moment of time as the reference point, the pressure $P_0$ of the porous body is considered known. At the interfaces between the regions, the temperature will be assumed to be the same and equal to the temperature of the middle part. Heat fluxes at the boundaries of the division of the regions are equal to each other. The values of the physical parameters of an incompressible fluid in a porous fluid – viscosity, Joule-Thomson coefficient, thermal diffusivity, permeability are considered constant and independent of $T$ and $P$. Liquid is filtered through the rod with a filtration rate $w_0$. The time is counted from the moment the pressure drop is applied, i.e. from the beginning of the movement of the liquid.

The estimates of the value of thermal conductivity in the direction of liquid filtration show that thermal conductivity determines only local temperature redistribution, and heat transfer over long distances is associated with convection [1]: the characteristic time of temperature equalization is on the order of $1^2/a \sim 10$ s – much less than the typical time for filtration. Consequently, the temperatures of oil, paraffin and the skeleton of the porous fluid at each point coincide; only the radial coordinate of the convective heat transfer rate $U_r \neq 0$, $U_\phi = 0$, $U_z = 0$ is different from zero. Thus, to determine at each point, we introduce a single temperature averaged over an elementary microvolume, like other characteristics of the filtration flow [1].

The problem of changing the temperature of a liquid during oscillatory motion in a porous fluid, taking into account phase transitions, is based on the heat balance equation, continuity equations for each phase of paraffinic oil, the equation of motion, initial and boundary conditions. Because oil filtration leads to the appearance of a barothermal effect – a change in temperature during the flow of fluid in a porous fluid in a non-stationary pressure field [2], then this factor must be taken into account when setting the complete problem.

\[ \frac{\partial T}{\partial t} = a_1 \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + a_{z1} \frac{\partial^2 T}{\partial z^2}, \quad z > h, \quad r > 0, \quad t > 0; \]

\[
\text{Figure 1. Geometry of the layer containing paraffinic oil in the bottomhole zone}
\]
\[ \frac{1+mf(T)}{c_p} \frac{T_z(t)}{\partial t} + \frac{U}{c_p} \frac{\partial T_z(t)}{\partial r} + \frac{1}{c_p} \frac{T_z(r,t)}{\partial r} \cdot \cos \frac{\eta}{2} \cdot \eta \cdot \frac{\partial T_z(t)}{\partial t} = \] 

\[ = a \cdot \frac{1}{r \partial r} \left( r \frac{T_z(t)}{\partial r} \right) + a \cdot \frac{\partial^2 T_z(t)}{\partial z^2}, \quad t > 0, \quad r > 0, \quad |z| < h; \] 

\[ = a \cdot \frac{1}{r \partial r} \left( r \frac{T_z(t)}{\partial r} \right) + a \cdot \frac{\partial^2 T_z(t)}{\partial z^2}, \quad z < -h, \quad r > 0, \quad t > 0. \] 

At the interfaces, the conditions for equality of temperatures and heat fluxes are specified

\[ T_{|z=h} = T_{|z=b} \] 

\[ = \lambda z \frac{T_z(t)}{\partial z} \] 

\[ = \lambda z \frac{T_z(t)}{\partial z} \] 

\[ = \lambda z \left( \frac{T_z(t)}{\partial z} \right)_{z=b}. \] 

(4)

(5)

There are no temperature disturbances at the initial moment of time

\[ T_{|t=0} = T_{|t=0} = T_{|t=0} = 0. \] 

(6)

We represent the boundary condition in the form

\[ \lim_{T_{|t=|z=\infty}} = 0. \] 

(7)

On the left side of (2), a thermal function is introduced that describes the intensity of heat sources caused by the phase transformation of paraffin in a certain temperature range [3]:

\[ f(T) = C^* \left( \frac{T - T_u}{T_m - T_u} \right)^2 \exp \left( -2 \frac{T - T_u}{T_m - T_u} \right). \] 

(8)

Where \( T_u \), \( T_m \) are the temperatures of the beginning and maximum of paraffin crystallization, accordingly, the coefficient \( C^* \) is a constant value for a given oil (for Mangyshlak oil \( C^* = 54 \cdot 10^3 \)).

The solution of the hydrodynamic problem for determining the functions of the rate of convective heat transfer \( u(r,t) \) and the pressure field \( \frac{\partial P}{\partial r} \), it was obtained in work [3]: during the oscillatory motion of a hydrodynamically incompressible fluid, the pressure changes according to the harmonic law

\[ P = P_0 \ln \frac{r}{R} \sin \omega t \] 

\[ \frac{\partial P}{\partial r} = \frac{P_0}{\ln \frac{r}{R} \sin \omega t} \cdot \frac{\partial P}{\partial t} = \frac{P_0}{\ln \frac{r}{R} \cos \omega t}. \] 

(9)

The source density function has the form [3]:

\[ f(r,t) = \left( \frac{e c_n k P_r^2 \ln^2 r_0 / R}{c_n \mu_k \ln^2 r_0 / R} + \frac{\eta m c_n P_0}{\ln \frac{r}{R} \cos \omega t} \right) \cdot \gamma(r > r_0). \] 

(10)

The solution is assumed to be bounded at all points \( r > 0 \).

Problem (1) – (7) is nonlinear, since the coefficients at the derivatives of temperature with respect to time and radial coordinate in the central zone \( |z| < 1 \) depend on temperature. The solution of linear and nonlinear boundary value problems for ordinary differential equations and for partial differential equations with a sufficient degree of reliability is carried out by perturbation methods that allow obtaining approximate analytical representations, and the solution is represented by several first terms of the asymptotic expansion, the number of which usually does not exceed two [4]. The asymptotic method presupposes the transition to dimensionless quantities, the choice of the parameter of the asymptotic expansion, the expansion of the sought functions in terms of the parameter of the asymptotic expansion \( h \), the formulation and search for a solution to the problem for the zero, first and other approximations. In the particular case of the zero approximation, this method leads to a “lumped capacitance scheme,” and the following approximations allow a more detailed study of the temperature field and estimate the magnitude of the resulting error.

According to the above, we introduce the following dimensionless quantities:
\[ T = \frac{T_p}{t_0}, \quad T_1 = \frac{T_p}{t_0}, \quad T_2 = \frac{T_p}{t_0}, \quad z = \frac{z_p}{h}, \quad r = \frac{r_p}{h}, \quad t = \frac{a_t t_p}{h^2}, \]

\[ h = \frac{\lambda_1}{\lambda}, \quad \Psi = \frac{c_1 \rho_1}{\Psi}, \quad \Lambda = \frac{\lambda_2}{\lambda_1}, \quad A = \frac{a_1}{a_3}, \quad A_1 = \frac{a_1}{a_3}, \quad A_2 = \frac{a_2}{a_3}, \quad A_3 = \frac{a_2}{a_3}, \]

where \( t \) — dimensionless time, \( t_p \) — dimensional time, \( a = \lambda / \rho \cdot c \) — thermal diffusivity, \( m^2/s \); \( \lambda \) — coefficient of thermal conductivity, \( W/(m \cdot K) \); \( c = 1/(\rho c_p) \) — Joule-Thomson coefficient, \( K/Pa \); \( \eta = \alpha VT / \epsilon_p \) — adiabatic coefficient, \( K/Pa \); \( \alpha \) — coefficient of thermal expansion of the liquid, \( K^{-1} \); \( V = 1/\rho \) — specific volume of liquid, \( m^3/kg \); \( T \) is the magnitude of the temperature effect, \( K \); \( r_0 \) — radius of the hollow cylinder, \( m \); \( m \) — porosity, \( k \) — fluid permeability, \( m^2 \), Darcy; \( \mu \) — dynamic coefficient of viscosity, \( cP \), \( c_{v_0} \) — volumetric heat capacity of a saturated porous fluid, \( J/m^3 K \); \( c_w = c_p \rho \) — volumetric heat capacity of a liquid at constant pressure, \( J/m^3 K \); \( \rho_1 \) — density of the fluid coming from the layer, \( kg/m^3 \); \( u(r, t) \) — the rate of convective heat transfer. For the parameter of the asymptotic expansion, we formally take

\[ h = \frac{\lambda_1}{\lambda} \cdot \]

With this choice of the parameter of the asymptotic expansion in the zero approximation, in a particular case, the problem can be reduced to the well-known "lumped capacity scheme" [5].

It is expedient to seek a solution to the problem in the form of Maclaurin series using the expansion in powers of the parameter \( h \):

\[ T_i = T_i^{(0)} + h T_i^{(1)} + h^2 T_i^{(2)} + \cdots, \quad i = 0, 1, 2. \quad (11) \]

The subscripts at the temperature \( T \) refer to the number of the region, and the upper indices correspond to the ordinal number of the approximation.

In the surrounding rocks, the zero approximation always gives an excessive temperature value. The following approximations provide a more detailed study of the temperature field and allow you to achieve the maximum error. The solution of the temperature problem for the middle part of the layer in the zero approximation, described in detail in [6], has the form:

\[ T^{(0)} = e_\sigma \left[ t - \sin 2\omega \right] + \eta \frac{c_w}{c_o} \frac{P_0 x}{l} \left[ \sin 2\omega \right], \quad (12) \]

where \( e_\sigma = \sigma + \eta \frac{c_{w_0}}{c_i} \), \( \beta = \frac{c_w}{2c_o} \frac{k}{\mu} \frac{P_0^2}{l^2} \).

After averaging over the period

\[ \langle T^{(0)} \rangle = \frac{c_w}{2c_o} \frac{k}{\mu} \frac{P_0^2}{l^2} e_\sigma t. \quad (13) \]

The complete formulation of the problem for determining the temperature \( T \) in the interval of the layer and the corresponding temperatures at \( T_1 \) and \( T_2 \) in the top and bottom of the layer for the first approximation is written as:

\[ \frac{\partial T_1^{(1)}}{\partial t} - \frac{A_1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_1^{(1)}}{\partial r} \right) - \frac{\partial^2 T_1^{(1)}}{\partial z^2} = 0, \quad z > 1, \quad t > 0; \quad (14) \]

\[ \Phi_1(T) \frac{\partial T_1^{(1)}}{\partial t} - \Phi_1(r, t) \frac{\partial T_1^{(1)}}{\partial r} - \Psi \frac{\partial^2 T_1^{(2)}}{\partial z^2} = 0, \quad |z| < 1, \quad r > 0, \quad t > 0; \quad (15) \]

\[ \frac{\partial T_2^{(1)}}{\partial t} - \frac{A_2}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_2^{(1)}}{\partial r} \right) - \frac{\partial^2 T_2^{(1)}}{\partial z^2} = 0, \quad z < -1, \quad t > 0; \quad (16) \]

The border conditions:

\[ \left. \frac{\partial T_1^{(1)}}{\partial z} \right|_{z=1} = \left. \frac{\partial T_2^{(1)}}{\partial z} \right|_{z=1}, \quad \left. \frac{\partial T_2^{(1)}}{\partial z} \right|_{z=-1} = \Lambda \left. \frac{\partial T_2^{(2)}}{\partial z} \right|_{z=-1}. \quad (17) \]
\[
T_1^{(i)}\bigg|_{t=1} = T_2^{(i)}\bigg|_{t=1} = T^{(i)};
\]
\[
T_1^{(i)}\bigg|_{t=0} = T_2^{(i)}\bigg|_{t=\infty} = T^{(i)} = 0;
\]
\[
\lim_{i\to\infty} T_{12}^{(i)}\bigg|_{t=0} = 0.
\]

Here it is \( \Phi_1(T) = 1 + \frac{m(1-s)}{c_a} f_i(T) \), \( \Phi_2(T) = 1 + \frac{f_i(T)}{c_p\rho} \).

To "decouple" the coefficients and solve the thermodynamic problem in the first approximation, we use the Laplace-Carson transformations, while the Laplace operator has the form
\[
\hat{L} = \frac{\partial}{\partial t} - U_{\alpha\beta}(r, t) \frac{\partial}{\partial r}.
\]

It is also necessary to introduce an additional condition, averaging the temperature in the interval of the layer \(-1 < z < 1\) using the integral:
\[
\langle T \rangle = \frac{1}{2} \int_{-1}^{1} T \, dz.
\]

It is easy to verify that the obtained in this way the problem of determining the average temperature in the layer coincides with the problem in the zero approximation. The uniqueness of the solution to the corresponding problems implies:
\[
\langle T \rangle = T^{(0)},
\]
therefore, averaging series (11) for \( i = 1 \), we obtain:
\[
\langle T \rangle = T^{(0)} + \hat{h} \langle T^{(1)} \rangle + \hat{h}^2 \langle T^{(2)} \rangle + \ldots + \hat{h}^n \langle T^{(n)} \rangle + \ldots
\]
or
\[
\hat{h} \langle T^{(1)} \rangle + \hat{h}^2 \langle T^{(2)} \rangle + \ldots + \hat{h}^n \langle T^{(n)} \rangle = 0.
\]

From (25) follows an additional condition imposed on the solutions of the first, second and further approximations:
\[
\langle T^{(i)} \rangle\bigg|_{r = r_i} = 0, \quad i = 1, 2, \ldots.
\]

Averaging in condition (26) is carried out in some value \( r = r_i \) corresponding to some cylindrical surface. Obviously, the best approximation to the desired solution is obtained when \( r_i \) coincides with the radius of the surface on which the boundary conditions are specified [7].

Taking into account condition (26), problem (14) – (20) has a unique solution that allows one to calculate corrections to the zero approximation. The first approximation \( T^{(1)} \) allows one to take into account the dependence of the temperature in the interval of the layer on \( z \). However, taking its accounting should not change the averaged temperature values. Therefore, the first approximation must satisfy the condition of equality to zero of its mean values – (26).

The solution to problem (14) – (20) in the first approximation with respect to the parameter \( \hat{h} \) has the form:
\[
\langle T^{(1)} \rangle = -\frac{D(1+\Lambda_0)}{8r^2\Lambda_1}(z^2 - 1) - \frac{D(1-\Lambda_0)}{4r^2\Lambda_1}(z + \Lambda_0) + \frac{D(1+\Lambda_0)}{12r^2\Lambda_1} + \frac{D(1-\Lambda_0)}{2r^2\Lambda_1(1+\Lambda_0)(2\Lambda_1 - \Lambda_0 - 1)} \left[ \exp\left(\frac{1+\Lambda_0}{2}\right) - \text{erfc}\left(\frac{1-\Lambda_0}{2\sqrt{t}}\right) \right] + \frac{D(1+\Lambda_0)}{8r^2\Lambda_1}(z^2 - \frac{2}{3}\Lambda_1 + \Lambda_0 + 1) + \frac{D(1-\Lambda_0)}{4r^2\Lambda_1}(z + \frac{1-\Lambda_0}{2(2\Lambda_1 - \Lambda_0 - 1)}) \times \exp(\Lambda_0^2 t) \text{erfc}(\Lambda_0\sqrt{t}),
\]
The phase transitions, is shown in where the following notation is introduced:

\[
\Lambda_0 = \frac{\Lambda}{\sqrt{\Lambda}}, \quad \Lambda_1 = \frac{L}{2} + \frac{\Lambda}{\sqrt{\Lambda}}, \quad D = \frac{k}{\rho \left( \frac{P_0}{\ln \frac{n}{R}} \right)} \left( \frac{\rho}{\epsilon} \right), \quad \Psi = \frac{c_i \rho_l}{c_p}, \quad \Lambda = \frac{\rho \sqrt{\varepsilon}}{\lambda},
\]

\[
\begin{align*}
N_1 &= \left( \frac{D(1 + \Lambda_0)}{8r^2 \Lambda_1} + \frac{D(1 - \Lambda_0)^2}{4r^2 \Lambda_1 (1 + \Lambda_0)} \right) \left( 1 + \frac{1 + \Lambda_0}{2 \lambda_1 - \Lambda_0 - 1} \right) - \frac{D(1 + \Lambda_0)}{24r^2 \Lambda_1} \left( 1 + \frac{1 + \Lambda_0}{2 \lambda_1 - \Lambda_0 - 1} \right), \\
N_2 &= \left( \frac{D(1 + \Lambda_0)}{8r^2 \Lambda_1} + \frac{D(1 - \Lambda_0)^2}{4r^2 \Lambda_1 (1 + \Lambda_0)} \right) \left( 1 + \frac{1 + \Lambda_0}{2 \lambda_1 - \Lambda_0 - 1} \right) + \frac{D(1 + \Lambda_0)}{24r^2 \Lambda_1} \left( 1 - \frac{1 + \Lambda_0}{2 \lambda_1 - \Lambda_0 - 1} \right), \\
N_3 &= \frac{D(3 - \Lambda_0)}{8r^2 \Lambda_1} + \frac{D(1 + \Lambda_0)}{4r^2 \Lambda_1 (1 + \Lambda_0)} \left( 1 + \frac{1 + \Lambda_0}{2 \lambda_1 - \Lambda_0 - 1} \right) + \frac{D(1 + \Lambda_0)}{24r^2 \Lambda_1} \left( 1 - \frac{1 + \Lambda_0}{2 \lambda_1 - \Lambda_0 - 1} \right).
\end{align*}
\]

The final solution for the two found approximations will be written in the form:

\[
T = T^{(0)} + \frac{\lambda_1}{\lambda} T^{(1)}, \quad T_1 = T_1^{(0)} + \frac{\lambda_1}{\lambda} T_1^{(1)}, \quad T_2 = T_2^{(0)} + \frac{\lambda_1}{\lambda} T_2^{(1)}.
\]

**Figure 2.** The contribution of phase transitions due to the dissolution of the solid phase of paraffin: 1 - temperature effect without phase transitions, 2 - with phase transitions

The dependence of the temperature of the filtrating paraffinic oil on time, with the accounting of phase transitions, is shown in figure 2. Calculations were carried out for Mangyshlak oil with a paraffin content of 20%.
For the numerical experiment, the following parameter values are accepted: \( m = 0.2 \), \( s = 0.9 \), \( \omega = 0.26 \text{s}^{-1} \), \( k = 6 \times 10^{11} \text{m}^2 \), \( c_v = 3 \times 10^8 \text{J/m}^3\cdot\text{K} \), \( c_p = 2.1 \times 10^9 \text{J/m}^3\cdot\text{K} \), \( \varepsilon = 4 \times 10^3 \text{K/Pa} \), \( q = 2.3 \times 10^5 \text{J/kg} \), \( C^* = 54 \times 10^3 \), \( \mu = 141.7 \times 10^3 \text{Pa\cdots} \).

Comparison of experimental measurements and theoretical averaged temperature dependence for viscous liquid filtration shows that the calculation results agree with a sufficient degree of accuracy with the actual crystallization curves for any paraffinic oil and petroleum product [3], [8].

Figures 3 – 5 show the calculations of the dimensionless temperature \( T \) from the vertical coordinate \( z \) for various values of the parameters included in the solution and the dimensionless time. Number 1 denotes the graphs corresponding to the zero approximation, number 2 – the first expansion coefficient, and 3 – the first approximation. The physical and chemical properties of the rocks are the same, \( r = r_0 \), \( r_1 = R_4 \).

The temperature is constant in the interval of the layer \(-1 < z < 1\) for the zero approximation – curve 1. In the central part of the layer for short times, the zero approximation describes the temperature distribution with a deficiency, and at the edges of the layer with an excess. Taking into account the correction, the solution in the first approximation more realistically describes the temperature distribution in the layer. Zero and first approximations are sufficient for most practical calculations.

![Graph](image1)

**Figure 3.** The graph of the dependence of the dimensionless temperature \( T \) in the \( z \) coordinate in dimensionless coordinates: \( t = 0.1 \), \( R_0 = 60 \) m, \( \varepsilon = 0.04 \), \( \Psi = 36.841 \)

![Graph](image2)

**Figure 4.** The graph of the dependence of the dimensionless temperature \( T \) in the \( z \) coordinate in dimensionless coordinates: \( t = 1 \), \( R_0 = 60 \) m, \( \varepsilon = 0.004 \), \( \Psi = 32.841 \)
The graph of the dependence of the dimensionless temperature \( T \) in the \( z \) coordinate in dimensionless coordinates: \( t=10, R_t=60 \text{ m}, \varepsilon=0.4, L=36.841 \)

Figure 5. The graph of the dependence of the dimensionless temperature \( T \) in the \( z \) coordinate in dimensionless coordinates: \( t=10, R_t=60 \text{ m}, \varepsilon=0.4, L=36.841 \)

The calculations of spatio-temporal temperature distributions during the oscillatory motion of a liquid, taking into account phase transitions, show that during oscillatory motion, a monotonous heating of a porous fluid and a filtering viscous liquid occurs, and due to the dissolution of paraffins, the rate of temperature rise decreases with time – with a paraffin content in oil of about 20\%, it occurs reduction of the temperature effect by 1.5 – 2 times.

References
[1] Barenblatt G I, Entov V M and and Ryzhik V M 1972 The theory of unsteady filtration of liquid and gas (Moscow: Nedra) p 288
[2] Filippov A I 1989 Well transient thermometry (Saratov: Saratov state university) p 116
[3] Efimova G F 2020 Mathematical modeling of temperature processes in filtration-wave fields (Ufa: USPTU Publishing House) p 106
[4] Nayfe A Kh 1976 Perturbation methods (Moscow: Mir) p 455
[5] Rubinstein L I 1971 Temperature fields in oil layer (Moscow: Nedra) p 276
[6] Filippov A I. and Efimova G F 1997 High Temperature 35(4) 549-52
[7] Ishmukhametova A A and Khusainov I G 2008 Proceedings of the Institute of Mechanics of the Ufa Scientific Center of the Russian Academy of Sciences 6 89-94
[8] Bilalova D N, Kireeva N A, Levina T M, Zharinov Y A and Uimanova I P 2020 Digital educational resources in the study of humanities subjects in a technical university Advances in Social Science Education and Humanities Research Proc. Int. Sci. Conf. “Digitalization of Education: History, Trends and Prospects” pp 315–19