Nucleon effective mass in hot dense matter

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Nucleon effective masses are studied in the framework of the Brueckner-Hartree-Fock many-body approach at finite temperature. Self-consistent calculations using the Argonne V18 interaction including microscopic three-body forces are reported for varying temperature and proton fraction up to several times the nuclear saturation density. Our calculations are based on the exact treatment of the center-of-mass momentum instead of the average-momentum approximation employed in previous works. We discuss in detail the effects of the temperature together with those of the three-body forces, the density, and the isospin asymmetry. We also provide an analytical fit of the effective mass taking these dependencies into account. The temperature effects on the cooling of neutron stars are briefly discussed based on the results for betastable matter.

I. INTRODUCTION

The nucleon effective mass and its dependence on density and temperature, \( m'(\rho, T) \), serves as an important microscopic input for the study of the thermal properties (e.g., thermal conductivity, specific heat, neutrino reaction rates) of neutron stars (NSs) [1][12]. For cold dense matter, microscopic nuclear many-body calculations have been performed, for example, starting from a realistic two-body potential plus a three-body force (TBF) within the Brueckner-Hartree-Fock (BHF) formalism [13,17,18], and within the Dirac–Brueckner-Hartree-Fock (DBHF) method [19,20]. The calculations have been done up to around \( 5\rho_0 \), for both asymmetric nuclear matter and beta-stable NS matter, with \( \rho_0 = 0.17 \text{ fm}^{-3} \) being the nuclear saturation density. The dependence of the nucleon effective mass on both density \( \rho = \rho_n + \rho_p \) and isospin asymmetry \( \beta = (\rho_n - \rho_p)/\rho \), where \( \rho_n \) and \( \rho_p \) are the neutron and proton number densities, has been included in fitting formulas [15] for easy implementation in astrophysical applications.

Thermal effects are known to be important [20,24] for the study of proto-neutron stars (PNSs), core collapse supernovae, binary NS mergers, black-hole accretion disks, etc. There are several attempts to construct a finite-temperature equation of state (EOS), based on a Skyrme nuclear force [25], on relativistic mean field theory [26,27], or within microscopic models [28,42]. The purpose of this paper is to report a systematic study of the nucleon single-particle (s.p.) properties on a microscopic basis for hot nuclear/NS matter. We will concentrate on the neutron/proton effective mass with varying temperature and proton fraction, for broad use in these dynamical phenomena.

For our purpose, we employ the BHF model [43,44] extended to asymmetric nuclear matter and finite temperature [45,46]. The realistic Argonne V18 two-body nucleon-nucleon (NN) potential [47] is used, together with the consistent microscopic TBF [48,51] for correctly reproducing the empirical saturation point of symmetric nuclear matter. Previously, the temperature dependence of the effective mass has been studied within BHF with or without the inclusion of TBF [13,20,29,46,52,54]. In the present study, we use the exact expression of the angular integration for the center-of-mass (c.m.) momentum to improve the reliability and the convergence of the BHF code. In earlier BHF studies an average c.m. momentum approximation was usually adopted, which could lead to different predictions for high-order contributions in describing the bulk properties for nuclear matter and the EOS [53,56], and should be improved in the studies of nucleon s.p. properties. With the inclusion of TBF, the resulting saturation density is 0.186 fm\(^{-3}\) and the energy per baryon at saturation is \(-14.5 \text{ MeV} \) [50]. They are somewhat different from the values (0.198 fm\(^{-3}\), 15.0 MeV) reported in the original paper [48,49], indicating the effects caused by the exact treatment of the c.m. momentum.

The paper is organized as follows. We provide the BHF formalism for hot asymmetric nuclear matter in Sec. II, including the extension to full evaluation of the c.m. momentum. Sec. III presents the s.p. effective masses in both nuclear matter and NS matter, together with their analytic fitting formula. Sec. IV gives a summary of this work.

II. FORMALISM

The calculations for hot asymmetric nuclear matter are based on the Brueckner-Bethe-Goldstone (BBG) theory [43,44,57,59] and the extension to finite temperature [45,46,60,61]. Here we simply give a brief review for completeness. The starting point in Brueckner theory is the effective reaction matrix \( G \), which satisfies the generalized Bethe-Goldstone (BG) equation (\( \tau = n, p \)),

\[
\langle 12|G_{\tau\tau'}(\omega,T)|1'2'\rangle = \langle 12|V_{\tau\tau'}|1'2'\rangle + \sum_{1''} \langle 12|V_{\tau\tau'}|1''2''\rangle \times \frac{Q_{\tau\tau'}}{\omega - \epsilon_{\tau}(1'') - \epsilon_{\tau'}(2'')} (1''2''|G_{\tau\tau'}(\omega,T)|1'2') ,
\]

where \( \omega \) is the so-called starting energy, \( V = V_{NN} + V_{3\text{ eff}} \) is the employed Argonne V18 NN interaction [47] plus an effective two-body force derived from a microscopic TBF [48,51], and \( I \equiv (k_1, \sigma_1) \) etc. denote the momentum and spin z components. For non spin-polarized nuclear matter, the spin-up and spin-down states are degenerate and hereafter we omit the
The momentum dependence of

\[ Q^{\tau \tau'} = Q^{\tau \tau'}(k_1, k_2, T) = [1 - f_{\tau}(k_1, T)] [1 - f_{\tau'}(k_2, T)] \]

with the Fermi distribution

\[ f_{\tau}(k, T) = \left[ 1 + \exp \left( \frac{\epsilon_{\tau}(k) - \mu_{\tau}}{T} \right) \right]^{-1}. \]

The chemical potential \( \mu_{\tau} \) can be calculated from the following implicit equation for any fixed density and temperature:

\[ \rho_{\tau} = \sum_k f_{\tau}(k, T). \]

In BHF approximation, the s.p. energy is given by

\[ \epsilon_{\tau}(k) = \epsilon_{\tau}(k, T) = \frac{k^2}{2m} + U_{\text{BHF}}^{\tau}(k, T), \]

where the s.p. potential \( U_{\text{BHF}}^{\tau}(k) \) is obtained from the real part of the on-shell antisymmetrized G matrix, i.e.,

\[ U_{\text{BHF}}^{\tau}(k) = \sum_{k' \tau'} f_{\tau'}(k', T) \text{Re} \langle kk' | G^{\tau \tau'}(\epsilon_{\tau}(k) + \epsilon_{\tau'}(k'), T) | kk' \rangle. \]

Eqs. (1,4,5,6) are then solved self-consistently for given density \( \rho \), isospin asymmetry \( \beta \), and temperature \( T \). The \( G \) matrix, the chemical potential \( \mu_{\tau} \), and the s.p. potential \( U_{\text{BHF}}^{\tau}(k) \) are all implicitly dependent on \( \rho, \beta \), and \( T \).

In Refs. [43-51], the TBF is reduced to an equivalent effective two-body force \( V_3^{\text{eff}} \) via a suitable integration over the degrees of freedom of the third nucleon. The procedure can be extended to finite temperature [46], and the effective interaction \( V_3^{\text{eff}}(T) \) in \( r \) space reads,

\[ \langle r_1', r_2' | V_3^{\text{eff}}(T) | r_1, r_2 \rangle = \frac{1}{4} \sum_{k_3} \tilde{f}(k_3) \int dk_3 d\phi_3 n_3(r_3) \times [1 - \eta(r_3')] [1 - \eta(r_2')] [1 - \eta(r_3, T)] [1 - \eta(r_2, T)] \times W_3(r_1', r_2', r_3'), \]

where \( \phi_3 \) is the wave function of the single nucleon in free space, and the trace is taken with respect to spin and isospin of the third nucleon. The defect function \( \eta(r, T) \) is directly related to the temperature-dependent \( G \) matrix.

Using the total and relative momentum,

\[ K = k_1 + k_2, \quad k = \frac{1}{2}(k_1 - k_2), \]

the BG equation (1) can be transformed into

\[ \delta_{KK'} \langle k | G_{\tau \tau'}(K, \omega, T) | k' \rangle = \delta_{KK'} \langle k | V_{\tau \tau'}(T) | k' \rangle \]

\[ + \sum_{k'' k'''} \delta_{KK' k'' k'''} \langle k | V_{\tau \tau'}(T) | k'' \rangle \langle k'' | G_{\tau \tau'}(K''', \omega, T) | k''' \rangle \times \frac{Q^{\tau \tau'}(K''', K', T)}{\omega - \epsilon_{\tau}(k'') + \epsilon_{\tau'}(k') - \epsilon_{\tau}(k''') - \epsilon_{\tau'}(k''')} \]

Generally, the nucleon interaction \( V \) is independent of the total momentum. However, the Pauli operator and the energy denominator depend on it. Therefore, the BG equation can be written as

\[ \langle k | G_{\tau \tau'}(K, \omega, T) | k' \rangle = \langle k | V_{\tau \tau'}(T) | k' \rangle \]

\[ + \sum_{k''} \langle k | V_{\tau \tau'}(T) | k'' \rangle \langle k'' | G_{\tau \tau'}(K, \omega, T) | k' \rangle \]

\[ \times \frac{Q^{\tau \tau'}(K', k'', T)}{\omega - \epsilon_{\tau}(k'') + \epsilon_{\tau'}(k') - \epsilon_{\tau}(k'')} \]

For any given density, isospin asymmetry, and temperature, the calculation of the s.p. potential, Eq. (6), need the full information of \( G \) at arbitrary values of \( \omega \) and \( k \). One therefore solves the BG Eq. (10) on a \( N_K \times N_\omega \) grid, where \( N_K \) (\( N_\omega \)) is the number of the \( K = |K| \) (\( \omega \)) points. Note that the value of the \( G \) matrix should be independent of the orientation of \( K \).

Such calculations were challenging several decades ago. Also, since the value of \( G \) matrix is regarded to be insensitive to the value of the total momentum \( K \), in the initial calculations of Brueckner theory [62], an average c.m. momentum approximation was used and the total momentum was approximated by the value

\[ \langle k^{(2)}_{\tau \tau'}(k) \rangle = \int_{0}^{k_{F}} \int_{0}^{k_{F}} dk_{1} dk_{2} \delta(k - \frac{1}{2}|k_{1} - k_{2}|)(k_{1} + k_{2}) \]

\[ \times \frac{1}{m} \frac{d\epsilon_{\tau}(k)}{dk} \]

at zero temperature. This approximation has been widely adopted in former calculations [43-46, 49, 63]. However, in the recent works of both BHF [46,48,51,67] and relativistic BHF approaches [55], the exact treatment of the total momentum has been used and we thus also follow this way in the present calculations to obtain more accurate results of the effective masses.

The effective mass \( m_{\tau} \) can be calculated from the s.p. energy as

\[ m_{\tau}(k) = \frac{k}{m} \left[ \frac{d\epsilon_{\tau}(k)}{dk} \right]^{-1}, \]

where \( m \) is the bare nucleon mass. It depends on \( \rho, \beta \), and \( T \). The exact treatment of the total momentum may cause nonnegligible effects on the result of the calculated effective mass.

III. RESULTS

We show first in Fig. 1 the momentum dependence of the neutron effective mass at various temperatures \( T = 0, 10, 20, 30, 40, 50 \) MeV, densities \( \rho = 0.2, 0.4 \) fm\(^{-3}\), and isospin asymmetries \( \beta = 0, \pm 0.4, \pm 0.8 \). Due to isospin symmetry, the proton and neutron effective masses are related by \( m_{\tau}^{p}(\beta) = m_{\tau}^{n}(\beta) \). The zero-temperature Fermi momenta \( k_{F}^{n/p} = [3\pi^2(1 \pm \beta)\rho/2]^{1/3} \) are shown by vertical lines. The temperature effects are generally more significant at low momentum and most evident around \( k_{F} \), where higher temperatures flatten the curves. This is related directly to the smoothing of the sharp Fermi surface and consequently of the s.p. potential around the Fermi momentum, and is a general feature for different choices of the NN potential and TBF [13, 20, 29, 46, 53, 54].
FIG. 1. Neutron effective mass as a function of momentum $m_n^*(k)/m$, Eq. (12), at temperatures $T = 0, 10, 20, 30, 40, 50$ MeV, densities $\rho = 0.2, 0.4, \pm 0.8$, and asymmetries $\beta = 0, \pm 0.4, \pm 0.8$. The adopted nucleon force is the Argonne $V_{18}$ potential plus the microscopic TBF. The vertical dashed lines indicate the neutron Fermi momenta. For the proton one has $m_p^*(\beta) = m_n^*(-\beta)$.

In the following Figs. 2, 3, 4 we present the detailed results for the effective mass $m^* = m^*(k_F)$ spanning the whole asymmetry range in a density domain up to $0.8$ fm$^{-3}$, and a temperature up to 50 MeV. The calculations are done with and without the TBF contribution. At low densities below $\sim 0.1$ fm$^{-3}$, the s.p. picture of the BHF calculation based on Eq. (12) may be taken with caution (see a discussion, e.g., in [65]).

FIG. 2. Neutron effective mass as a function of density, for temperatures $T = 0, 30, 50$ MeV and asymmetries $\beta = 0, \pm 0.4, \pm 0.8$, with and without the TBF contribution. For the proton one has $m_p^*(\beta) = m_n^*(-\beta)$.

In Fig. 2 we compare the density dependence of the effective mass with or without TBF, at different temperatures and asymmetries. As already mentioned in the introduction, the inclusion of TBF is important for reproducing the saturation properties of nuclear matter. We see here that it can also change the behavior of $m^*(\rho)$ at high densities: After the inclusion of TBF, $m^*$ will rise with density after reaching a
neutron/proton effective mass as a function of density at different temperatures $T = 0, 10, 20, 30, 40, 50$ MeV and asymmetries $\beta = 0, 0.4, 0.8$. The calculations are done including the TBF contribution.

Fig. 3 is devoted to the comparison of the density and temperature dependence of the effective mass. We present the results with TBF and for symmetric nuclear matter. The curves are plotted for sets of density ranging from 0.2 to 0.8 fm$^{-3}$, and temperature from 0 to 50 MeV. Comparing with the left panel of Fig. 3, one concludes that the effective mass is generally more sensitive to density than to temperature. The temperature dependence tends to be pronounced at low density and the density dependence tends to be pronounced at low temperature. The behavior of $m^*$ with increasing density is very similar at different temperatures: $m^*$ first decreases and then increases with density. This is mainly due to the increasingly dominating role of the TBF, which has a repulsive nature. The behavior of $m^*$ with increasing temperature is, however, not straightforward for different densities. Due to the competitive effect between the density and the temperature, at intermediate densities such as $\rho = 0.4, 0.6$ fm$^{-3}$, the temperature dependence is very limited. At low density such as $\rho = 0.2$ fm$^{-3}$, $m^*$ first decreases then increases with temperature, as also observed in Fig. 3. At high density such as $\rho = 0.8$ fm$^{-3}$, the flattening effect of temperature dominates and $m^*$ decreases monotonically with temperature.

One of the main goals of the present study is to provide easy-to-use microscopic nuclear input for various astrophysical systems. We therefore fit the numerical results of the effective mass by an analytic representation (with the three independent variables density $\rho$, asymmetry $\beta$, and temperature $T$), extending the zero-temperature formulas [15]. We choose certain minimum at $\rho_{\text{min}}$, as already observed in the works of Ref. [15, 16] at zero temperature. This results from the repulsive nature of the TBF [48–50] and resembles the DBHF result [64]. The general effect of temperature is to smooth out the rising of the effective mass caused by the TBF contribution, shifting $\rho_{\text{min}}$ to higher values. Isospin asymmetry causes the minority component to acquire a lower effective mass than the isospin partner.

To see more clearly the interplay between temperature effect and the TBF contribution, we show in Fig. 4 a comparison at different temperatures and asymmetries for both neutron and proton effective masses. We see again the flattening effect of temperature at high density. At low density the temperature will first reduce (removal of the s.p. ‘wiggle’) and then increase the effective mass, see Fig. 1. This is the case for both neutron and proton and different asymmetries.

Fig. 4 is devoted to the comparison of the density and temperature dependence of the effective mass. We present the results with TBF and for symmetric nuclear matter. The curves are plotted for sets of density ranging from 0.2 to 0.8 fm$^{-3}$, and temperature from 0 to 50 MeV. Comparing with the left panel of Fig. 3, one concludes that the effective mass is generally more sensitive to density than to temperature. The temperature dependence tends to be pronounced at low density and the density dependence tends to be pronounced at low temperature. The behavior of $m^*$ with increasing density is very similar at different temperatures: $m^*$ first decreases and then increases with density. This is mainly due to the increasingly dominating role of the TBF, which has a repulsive nature. The behavior of $m^*$ with increasing temperature is, however, not straightforward for different densities. Due to the competitive effect between the density and the temperature, at intermediate densities such as $\rho = 0.4, 0.6$ fm$^{-3}$, the temperature dependence is very limited. At low density such as $\rho = 0.2$ fm$^{-3}$, $m^*$ first decreases then increases with temperature, as also observed in Fig. 3. At high density such as $\rho = 0.8$ fm$^{-3}$, the flattening effect of temperature dominates and $m^*$ decreases monotonically with temperature.
the calculations of hot beta-stable with and without TBF. The standard deviations are the neutron effective mass at nearly all densities. We see that the direct and indirect (decrease of neutron partial effects the low-density domain of the proton fraction, as already observed in our previous works \[35–40\]. In the middle panel, we see that the direct and indirect (decrease of neutron partial density) effects of increasing temperature lead to a decrease of the neutron effective mass at nearly all densities. The values are somewhat higher than in symmetric matter, see Fig. 5. The proton effective mass, displayed in the bottom panel, shows a similar flattening behavior with increasing temperature, with a weak dependence on the density for \(\rho \gtrsim 0.4\) fm\(^{-3}\).

### IV. SUMMARY

The nucleon effective mass at finite temperature is of fundamental importance for nuclear astrophysics, but an evaluation of the s.p. properties is usually not easy and model dependent. Previous works on the temperature dependence of the effective mass showed nontrivial behavior for the required ranges of nucleon density and isospin asymmetry in dynamical astrophysical systems of interest. So we performed the calculation of \(m^*(\rho, T)\) from realistic nucleon forces within a microscopic model. We used the BHF method extended to asymmetric nuclear matter and finite temperature, employing the realistic Argonne \(V_{18}\) force together with consistent microscopic TBF.

We studied the interplay of the \(\rho, T, TBF\) dependence of the effective mass with and without the TBF contribution. Finite temperature in general lowers the effective mass, in particular...
at low and high densities. TBF increase the effective mass at high density due to their repulsive character, but finite temperature weakens this effect. Altogether, the temperature dependence is modest in comparison to the density dependence, but the specific behavior can be different in different density domains.

The dependence $m^*(\rho, \beta, T)$ has been accurately parametrized by a carefully chosen analytical formula, to be conveniently used for the study of NS cooling, core collapse supernovae, heavy-ion collisions, etc. We have also discussed the temperature dependence of the proton fraction and the nucleon effective mass in betastable NS matter, and the influence on the DU process in a hot star. The present results might be used for the study of the thermal evolution of a PNS or a NS merger event, which we will explore in a future work.

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