A 2 per cent distance to \( z = 0.35 \) by reconstructing baryon acoustic oscillations – III. Cosmological measurements and interpretation

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Accepted 2012 April 13. Received 2012 March 27; in original form 2012 February 1

ABSTRACT

We use the 2 per cent distance measurement from our reconstructed baryon acoustic oscillations (BAOs) signature using the Sloan Digital Sky Survey (SDSS) Data Release 7 (DR7) luminous red galaxies from Padmanabhan et al. and Xu et al. combined with cosmic microwave background data from Wilkinson Microwave Anisotropy Probe (WMAP7) to measure parameters for various cosmological models. We find a 1.7 per cent measurement of \( H_0 = 69.8 \pm 1.2 \) km s\(^{-1}\) Mpc\(^{-1}\) and a 5.0 per cent measurement of \( \Omega_m^{1/280} = 0.280 \pm 0.014 \) for a flat universe with a cosmological constant. These measurements of \( H_0 \) and \( \Omega_m \) are robust against a range of underlying models for the expansion history. We measure the dark energy equation of state parameter \( w = -0.97 \pm 0.17 \), which is consistent with a cosmological constant. If curvature is allowed to vary, we find that the Universe is consistent with a flat geometry (\( \Omega_K = -0.004 \pm 0.005 \)). We also use a combination of the 6 Degree Field Galaxy Survey BAO data, WiggleZ Dark Energy Survey data, Type Ia supernovae data and a local measurement of the Hubble constant to explore cosmological models with more parameters. Finally, we explore the effect of varying the energy density of relativistic particles on the measurement of \( H_0 \).

Key words: cosmological parameters – cosmology: observations – cosmology: theory – distance scale – large-scale structure of Universe.

1 INTRODUCTION

Since the discovery of the accelerated expansion of the Universe (Riess et al. 1998; Perlmutter et al. 1999), there has been growing interest to understand the nature of dark energy and measure various cosmological parameters. This understanding requires improved measurements of the expansion history of the Universe via the distance–redshift relation. In particular, baryon acoustic oscillations (BAOs) have been widely used to study this relation by measuring cosmic distances. The physics behind these oscillations is well understood (Sakharov 1967; Peebles & Yu 1970; Sunyaev & Zeldovich 1970; Bond & Efstathiou 1984, 1987; Hu & Sugiyama 1996; Hu, Sugiyama & Silk 1997; Eisenstein & Hu 1998; Hu & Dodelson 2002). In the pre-recombination era of the Universe, the baryons were coupled to the photons in a hot plasma. Small overdensities in the underlying dark matter distribution caused the baryons and photons to fall into the overdensities due to gravity. As the plasma density grows, the radiation pressure from the photons drives an acoustic wave of baryons and photons around the original dark matter overdensity. As the Universe cools, electrons and protons combine to form atoms, and the photons decouple from the baryons causing the sound speed in the plasma to drop dramatically. This leaves the baryons in a spherical shell around the initial overdensity. This shell has a characteristic scale of about 150 Mpc, defined by the distance travelled by the acoustic wave in the pre-recombination era, and its angular scale has been measured in the cosmic microwave background (CMB) to be about 1° (Bennett et al. 2003; Jarosik et al. 2011). Under the influence of gravity, these overdensities grow and form galaxies imprinting the characteristic acoustic scale into the distribution of galaxies (Hu & Sugiyama 1996; Eisenstein & Hu 1998; Meiksin, White & Peacock 1999). Thus, the BAO scale can be used as a robust standard ruler in large galaxy surveys (Tegmark 1997; Eisenstein, Hu & Tegmark 1998; Goldberg & Strauss 1998; Efstathiou & Bond 1999) with an important application to the study of dark energy (Eisenstein 2002; Blake & Glazebrook 2003; Hu & Haiman 2003; Linder 2003; Seo & Eisenstein 2003). The large physical size of this acoustic scale causes the standard ruler to be highly accurate (Eisenstein, Seo &...
White 2007b; Seo et al. 2008, 2010; Padmanabhan & White 2009; Mehta et al. 2011).

The BAO signal was first measured in the Sloan Digital Sky Survey (SDSS) luminous red galaxy (LRG) survey and the 2 Degree Field (2dF) Galaxy Survey (Cole et al. 2005; Eisenstein et al. 2005) and has since been observed in multiple surveys: SDSS (Tegmark et al. 2006; Percival et al. 2007, 2010; Kazin et al. 2010; Chuang, Wang & Hemantha 2012), 6dF Galaxy Survey (6dFGS; Beutler et al. 2011) and the WiggleZ Dark Energy Survey (Blake et al. 2010, 2011a,b). Weinberg et al. (2012) provide an overall review of observational cosmology and discuss the current state of the field in depth.

The acoustic scale hence gives us a measurement of the distance to a given redshift. Padmanabhan et al. (2012, hereafter Paper I) present the BAO measurements via the correlation function in the SDSS LRG Data Release 7 (DR7) data set using the reconstruction technique first introduced by Eisenstein et al. (2007a). Xu et al. (2012, hereafter Paper II) describe a robust methodology to measure the acoustic scale, which is heavily tested against the LasDamas mock catalogues. Also shown in Paper II are the results of the testing performance of the reconstruction technique to improve the BAO measurement. The reconstruction technique improves the distance measurement to \( z = 0.35 \) to a 1.9 per cent measurement compared to a 3.5 per cent measurement before reconstruction. We show in this paper how this new measurement of the acoustic scale in SDSS helps improve our measurements of the cosmological parameters over a wide range of cosmological models.

The CMB angular acoustic scale gives us a distance measurement to the redshift at recombination that helps us break the degeneracy between \( \Omega_c \) and \( H_0 \), therefore precisely measuring the parameters in the flat \( \Lambda \) cold dark matter (\( \Lambda \)CDM) or the ‘vanilla’ cosmological model. However, with higher dimensional models, we need to have more distance measurements to break degeneracies between various cosmological parameters. We show how BAO data help break these degeneracies by providing a second distance measurement at low redshift. We extend our redshift range to lower redshifts by adding supernova (SN) data.

The combination of degree-scale CMB anisotropy, large-scale structure and Type Ia SN (SN Ia) data offers powerful constraints on cosmology and dark energy. Notable early papers include Efstathiou et al. (2002), Percival et al. (2002), Spergel et al. (2003) and Tegmark et al. (2004). With the discovery of the acoustic peak in the large-scale clustering of galaxies, the results from the Wilkinson Microwave Anisotropy Probe (WMAP) satellite and the construction of yet-larger SN samples, these constraints have become increasingly precise. Many papers have combined these data sets; some recent examples include Komatsu et al. (2009, 2011), Hicken et al. (2009), Kazin et al. (2010), Percival et al. (2010), Reid et al. (2010a), Blake et al. (2010, 2011a), Conley et al. (2011), Wang, Chuang & Mukherjee (2011), Beutler et al. (2011), Seo et al. (2012) and Ho et al. (2012) (see Weinberg et al. 2012 for a longer discussion).

In this paper, we use the results of this reconstructed SDSS data in conjunction with CMB measurements from the 7-year WMAP (WMAP7; Jarosik et al. 2011; Komatsu et al. 2011), SN Ia measurements from the 3-year Supernova Legacy Survey (SNLS3; Conley et al. 2011) and direct measurement of the Hubble constant from the SHOES (Supernova, \( H_0 \), for the Equation of State) project (Riess et al. 2011). To break degeneracies between different parameters, we use additional BAO data from the 6dFGS (Beutler et al. 2011), the WiggleZ Dark Energy Survey (Blake et al. 2011b) and SN data from SNLS3 by Conley et al. (2011).

We start with the concordance cosmological model, \( \Lambda \)CDM (which here denotes a flat Universe), and add other cosmological parameters to explore higher dimensional models. We vary the curvature of the Universe, \( \Omega_k \), and the constant dark energy equation of state, \( w_0 \), independently for the oCDM and \( \Lambda \)CDM models, respectively. For higher dimensionality, we vary both \( \Omega_k \) and \( w_0 \) simultaneously in the ow\( \Lambda \)CDM model, and in the \( w_0 w_\Lambda \)CDM model we assume a flat Universe but allow the dark energy equation of state parameter to vary in time. We allow all three parameters, \( \Omega_k \), \( w_0 \) and \( w_\Lambda \), to vary in our most general model, ow\( \Lambda \)CDM.

In Section 2, we describe the Markov chain Monte Carlo fitting and the various data sets used in this study. We introduce the results of the BAO data and describe the cosmological implications in Section 3.1. In Sections 3.2–3.7, we show our results for various cosmological models. In Section 3.8, we show the robustness of our measurements of the Hubble constant and the matter density over different models for the expansion history. Section 3.9 explores a possibility of solving an apparent tension in the \( H_0 \) measurement by varying the energy density of relativistic species. We conclude with a summary of our results in Section 4.

2 METHODOLOGY

Here, in the third paper of this series, we use the reconstructed SDSS DR7 LRG results presented in Papers I and II to measure cosmological parameters. We use a reconstruction technique first introduced in Eisenstein et al. (2007a), to model and remove effects of non-linear evolution of large-scale structure and large-scale velocity flows (redshift space distortions). As shown in Mehta et al. (2011), this method can also be applied to biased tracers of the matter density distribution, such as LRG. This effect was tested using N-body simulations (Noh, White & Padmanabhan 2009; Mehta et al. 2011). As shown in Papers I and II, reconstruction improves the measurement on the acoustic scale by about 40 per cent. Therefore, in this paper we use the reconstructed BAO data from SDSS DR7 (hereafter BAO) unless otherwise specified. In practice, the BAO data measure the acoustic scale relative to some fiducial cosmology. This ratio of the acoustic scales is defined to be \( \alpha \), which is given by

\[
\alpha = \frac{D_V}{D^0_V} \frac{r_s}{r^0_s},
\]

where \( D_V \) is the spherically averaged distance scale to the pivot redshift, \( D_V(z) = \left(D^0_V(z)c/H(z)\right)^{1/3}, r_s \) is the sound horizon scale and ‘fid’ stands for the values in the fiducial cosmology WMAP7 (Komatsu et al. 2011); \( H_0 = 70.2 \) km s\(^{-1}\) Mpc\(^{-1}\), \( \Omega_m h^2 = 0.2255, \Omega_k = 0.274, n_s = 0.968 \) and \( \sigma_8 = 0.816 \). The sound horizon for our fiducial cosmology is \( r^0_s = 152.76 \) Mpc. With reconstruction, we measure the distance to \( z = 0.35 \) to be \( D_V(z = 0.35) = 1356 \pm 25 \) Mpc (see Paper I).

We use the Markov chain Monte Carlo code \textsc{CosmoMC} (http://cosmologist.info/cosmomic/) to compute the constraints on the cosmological parameters (Lewis & Bridle 2002). The original BAO routine in \textsc{CosmoMC} was replaced in order to use information from the probability distribution function of the BAO peak location \( p(\alpha) \) from the \( \chi^2 \) fitting described in Paper II. Using \( p(\alpha) \), we estimate the likelihood of the acoustic scale \( \alpha \) that corresponds to the cosmological parameters at a given step in the Markov chain. After the Markov chains converge, \textsc{CosmoMC} outputs the posterior probability distribution for each of the cosmological parameters, given the observations. Table 1 gives the mean...
values and the rms errors of the cosmological parameters for various cosmological models.

A relevant issue that is often overlooked is that the sound horizon \( r_\text{s} \) has several definitions in the literature. Its value depends on the definition of parameters such as \( z_{\text{drag}} \); the redshift at which the electrons are no longer dragged by the photons due to Compton scattering. In this paper, we use the definition of \( z_{\text{drag}} \) proposed in Eisenstein & Hu (1998) (hereafter EH98, equations 4–6) to compute the sound horizon, \( r_\text{s} \). The difference between the definition of \( r_\text{s} \) used in this paper and the implementation used in \textsc{camb} is only a few per cent for a wide range in \( \Omega_m \) and \( \Omega_b \). In Fig. 1, we show the relative difference of \( r_\text{s} / r_{\text{s}}^\text{EH98} \) between the EH98 and \textsc{camb} definitions as a function of \( \Omega_m h^2 \) and \( \Omega_b h^2 \), and find that the differences are negligible for our analysis. Thus, the definition dependence cancels out when computing \( \alpha \) except for a small residual (\( \leq 0.1 \) per cent) difference.

In our \textsc{cosmomc} chains, we use the \textit{WMAP7} data (Komatsu et al. 2011) as our base data set defined as ‘CMB’. We then add our SDSS DR7 LRG reconstructed BAO data (Papers I and II) to get the ‘CMB+BAO’ data set. We also include the other two latest BAO measurements from the 6dFGS (Beutler et al. 2011) and the WiggleZ Dark Energy Survey (Blake et al. 2011a). The combination of the \textit{WMAP7} and all BAO data sets is denoted by ‘CMB+All BAO’. While the SDSS and 6dFGS provide single redshift points, the WiggleZ survey measures BAO in three correlated redshift slices. We use all three redshift slices in our code and use their covariance matrix to account for the covariant points. Conley et al. (2011) provide a covariance matrix analysis of SN Ia cosmology from the SNLS3 accompanied by a \textsc{cosmomc} module. We use their data set and module in conjunction with the \textit{WMAP7} to create the ‘CMB+SN’ data set and add it to our SDSS BAO data to create the

\[
\begin{array}{cccccc}
\Omega_m h^2 & \Omega_b & H_0 & \Omega_K & w_0 & w_a \\
CMB & 0.1341 (56) & 0.268 (29) & 71.0 (26) & - & - & - \\
CMB+BAO & 0.1362 (33) & 0.280 (14) & 69.8 (12) & - & - & - \\
CMB+BAO+SN & 0.1349 (33) & 0.274 (14) & 70.2 (12) & - & - & - \\
CMB & 0.1344 (55) & 0.423 (175) & 60.0 (123) & -0.039 (44) & - & - \\
CMB+BAO & 0.1333 (53) & 0.278 (15) & 69.3 (16) & -0.004 (5) & - & - \\
CMB+All BAO & 0.1326 (50) & 0.277 (13) & 69.2 (14) & -0.004 (5) & - & - \\
CMB+SN & 0.1324 (51) & 0.243 (37) & 74.6 (58) & 0.003 (9) & - & - \\
CMB+BAO+SN & 0.1323 (50) & 0.274 (13) & 69.6 (16) & -0.004 (5) & - & - \\
CMB & 0.1342 (58) & 0.263 (118) & 75.4 (138) & -1.12 (41) & - & - \\
CMB+BAO & 0.1349 (57) & 0.285 (25) & 69.0 (39) & -0.97 (17) & - & - \\
CMB+All BAO & 0.1328 (49) & 0.287 (19) & 68.1 (28) & -0.92 (13) & - & - \\
CMB+SN & 0.1332 (54) & 0.254 (23) & 72.6 (25) & -1.04 (7) & - & - \\
CMB+BAO+SN & 0.1368 (43) & 0.271 (14) & 71.1 (18) & -1.05 (8) & - & - \\
CMB+All BAO & 0.1321 (58) & 0.308 (30) & 68.9 (39) & -0.001 (10) & -0.97 (24) & - \\
CMB+SN & 0.1329 (54) & 0.257 (51) & 73.0 (71) & 0.002 (16) & -0.106 (13) & - \\
CMB+BAO+SN & 0.1336 (52) & 0.271 (14) & 70.3 (19) & -0.005 (5) & -0.108 (8) & - \\
CMB+BAO+SN+H0 & 0.1352 (51) & 0.262 (12) & 71.8 (16) & -0.004 (5) & -1.10 (8) & - \\
CMB+All BAO & 0.1340 (49) & 0.311 (42) & 66.1 (47) & -0.62 (47) & -0.88 (122) & - \\
CMB+BAO+SN & 0.1345 (53) & 0.242 (24) & 74.7 (31) & -0.87 (18) & -1.07 (94) & - \\
CMB+BAO+SN+H0 & 0.1377 (57) & 0.272 (15) & 71.2 (19) & -1.02 (16) & -0.26 (82) & - \\
CMB+BAO+SN+H0 & 0.1385 (55) & 0.266 (14) & 72.2 (16) & -1.02 (16) & -0.40 (85) & - \\
CMB+All BAO & 0.1327 (50) & 0.303 (46) & 66.7 (51) & -0.003 (11) & -0.66 (47) & -1.11 (122) \\
CMB+BAO+SN & 0.1346 (53) & 0.276 (15) & 69.9 (19) & -0.010 (7) & -0.90 (16) & -1.30 (99) \\
CMB+BAO+SN+H0 & 0.1363 (53) & 0.267 (13) & 71.4 (16) & -0.008 (6) & -0.94 (16) & -1.23 (102) \\
\end{array}
\]

\(4\) \textit{WMAP7}, \textit{BAO} = reconstructed SDSS DR7 LRG, \textit{SN} = SNLS 3-year compilation, All BAO = reconstructed SDSS DR7 LRG + 6dFGS + WiggleZ, H0 = Riess et al. (2011) measurement of H0.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Relative difference (in per cent) between the sound horizon scale \( r_s / r_s^{\text{EH98}} \) from \textsc{camb} and \( r_s / r_s^{\text{EH98}} \) from Eisenstein & Hu (1998) for a given combination of \( \Omega_m h^2 \), \( \Omega_b h^2 \). Both definitions agree to within 0.2 per cent level even for cosmologies 5\( \sigma \) away from the current \textit{WMAP7} constraints. Our fiducial cosmology with the \textit{WMAP7} 1\( \sigma \) errors is shown as the grey cross.}
\end{figure}
‘CMB+BAO+SN’ data set. Finally, we also use the direct $H_0$ measurement by Riess et al. (2011) and combine it with the WMAP7, our SDSS BAO and SNLS3 data set into the ‘CMB+BAO+H0+SN’ data set. In the next section, we discuss the various cosmological parameters we measure using these data sets and how adding various data sets helps measure and constrain various parameters in high-dimensional cosmological models.

3 RESULTS

3.1 Cosmology with BAO data

Paper I applies reconstruction to the SDSS DR7 LRG BAO data set and Paper II shows the robustness of our BAO measurements. After using reconstruction, we measure $D_A(z = 0.35)(\sigma_{/\Omega_1}) = 1356 \pm 25$ Mpc and $D_V(z = 0.35)/r_s = 8.88 \pm 0.17$ giving us a 1.9 per cent measurement of the distance to $z = 0.35$. We can combine our BAO measurement with the measurements from 6dFGS (Beutler et al. 2011; $D_V = 456 \pm 27$ Mpc to $z = 0.106$) and WiggleZ Dark Energy Survey (Blake et al. 2011b; $D_V = 2.23 \pm 0.11$ Gpc to $z = 0.6$) to make this BAO Hubble diagram. We have combined the three correlated WiggleZ redshift slices into one data point in order to show only uncorrelated points.

The WMAP has measured the angular acoustic scale to about 0.1 per cent and has measured the baryon density with enough precision that its contribution to the sound horizon uncertainty is sub-dominant. Therefore, for any given value of $\Omega_m h^2$, we have a precise prediction for the sound horizon. Given an exact statement of the spatial curvature and $w(z)$, here flat $\Lambda$CDM, only one value of $\Omega_m$ (and hence $H_0$) will satisfy the angular acoustic scale. Hence, each value of $\Omega_m h^2$ makes a unique prediction for $D_A(z)/r_s$. We plot this prediction in Fig. 2 for the best-fitting value of $\Omega_m h^2 = 0.1351$ as the solid black line, and the 1σ range of $\Omega_m h^2 = 0.1351 \pm 0.0051$ as the shaded region. This line is not a fit to the BAO data.

We see that the BAO data are a remarkable match to the WMAP7 $\Lambda$CDM prediction. To focus on the residuals, in Fig. 3, we normalize the data to the WMAP7 best-fitting model. Also plotted as the open square is the Percival et al. (2010) BAO data point at $z = 0.275$. Again we see that the BAO data are consistent with $\Lambda$CDM. As in Fig. 2, for any assumption of $\Omega_m$ and $w(z)$, the WMAP7 predicts a region on this plot with the width set by the uncertainty in $\Omega_m h^2$. In this figure, we explore the effects of varying the equation of state parameter, $w$, and the curvature of the Universe, $\Omega_K$, respectively. The red region corresponds to a flat Universe with $w = -0.7$, while the blue region corresponds to a Universe with a cosmological constant and $\Omega_K = 0.01$. $\Omega_m$ is adjusted to keep the sound horizon constant. From this figure, we see that changing $w$ mostly changes the slope of the line on this plot, while a non-zero $\Omega_K$ mostly changes the vertical offset. The relative distance measure from comparing the flux of SN constrains only the slope of the lines, while the BAO data can measure an absolute distance and hence the vertical offset. This explains why SN data are more effective at constraining $w$, while the BAO data are more effective at constraining $\Omega_K$. The Riess et al. (2011) direct $H_0$ measurement is also plotted in this figure assuming the fiducial sound horizon value. While the sound horizon varies by about 1 per cent within the WMAP7 results, this effect is subdominant to the quoted errors on $H_0$. We explore the apparent tension between the BAO measurement and the direct measurement of $H_0$ in Section 3.9.

Conventionally, the Hubble constant has been measured by building a distance ladder from local measurements out to measuring the cosmological Hubble flow. Conversely, the CMB and BAO data build an inverse distance ladder starting from a distance measurement at the recombination epoch. The CMB data provide an accurate measurement of the distance to the recombination redshift and our BAO data provide a measurement of distance to $z = 0.35$, thereby building an inverse distance ladder. The combination of these two data sets has the power to distinguish between different cosmological models. The SN data extrapolate the distance measurements to lower redshift and, therefore, precisely measure the expansion of the Universe at $z = 0$, which is the Hubble constant, $H_0$. In the following sections, we use a combination of these data sets to explore a variety of cosmological models, and we use the
CMB+BAO+SN data set to obtain robust measurements of $H_0$ and $\Omega_m$.

### 3.2 $\Lambda$CDM: the vanilla model

The WMAP7 measurements of the CMB give us very good measurements of the various parameters in the ‘vanilla cosmology’ model, also known as the $\Lambda$CDM model. In the CosmoMC code, we vary the standard CDM parameters of matter and baryon densities ($\Omega_m$, $\Omega_b$), the primordial spectrum amplitude and slope ($n_s$), matter clustering amplitude ($\sigma_8$) and the optical depth to reionization ($\tau$). Adding BAO measurement to the WMAP7 results improves the measurement of $\Omega_m$ by about 40 per cent and $H_0$ by almost 30 per cent. With reconstruction, we measure $\Omega_m = 0.280 \pm 0.014$ and $H_0 = 69.8 \pm 1.2$ km s$^{-1}$ Mpc$^{-1}$ giving us a 1.7 per cent measurement of the Hubble constant. Fig. 4 shows the 68 and 95 per cent confidence level contours for $H_0$ versus $\Omega_m$ and we can see the improvement in these parameters by adding the BAO data. Table 1 shows the values for $\Omega_m h^2$, $\Omega_m$ and $H_0$ for various cosmological models and the corresponding data sets used.

The acoustic standard ruler is calibrated by the WMAP measurement of $\Omega_m h^2$. Komatsu et al. (2011) show that allowing for a running spectral index, $dn_s/d \ln k$, increases the errors on $\Omega_m h^2$. Thus, we explore the effects of varying the running spectral index, $dn_s/d \ln k$, with the CMB and CMB+BAO data sets. We note that the nuisance parameters used in our BAO fitting techniques (Paper II) make our measurement of $D_\Lambda/r_s$ insensitive to the running spectral index. Table 2 shows the effect of varying the running spectral index on cosmological parameters. We see that the running spectral index is consistent with 0: $dn_s/d \ln k = -0.024 \pm 0.020$ using the CMB+BAO data set. We find that including this parameter in the case of CMB data only, the $\Omega_m h^2$ measurements are degraded by a factor of 1.4 from $\Omega_m h^2 = 0.1341 \pm 0.0056$ to 0.1393 $\pm$ 0.0080. This corresponds to an increased uncertainty in the measurements of $\Omega_m$, $H_0$ and the spectral index $n_s$. Adding the BAO data improves the measurement of $\Omega_m h^2$, $\Omega_m$, $H_0$ and $n_s$, and are consistent with the values with no running spectral index. We also note that the value of $n_s$ is less than 1.0 in all cases which is expected by typical inflation models.

![Figure 4](image-url) 68 and 95 per cent confidence level contours for $H_0$ versus $\Omega_m$ using WMAP7 data (dashed grey lines) and then combining those with the reconstructed SDSS DR7 LRG BAO data (solid black lines).

### 3.3 oCDM: varying spatial curvature

The BAO measurements calibrate the acoustic scale at low redshifts to the high-redshift measurement from the CMB data. Therefore, the BAO accurately measures the curvature of the Universe (see Fig. 3). As shown in the previous section, in the $\Lambda$CDM model, the CMB breaks the $\Omega_m-H_0$ degeneracy with a distance measurement to the recombination redshift. However, when we vary the curvature parameter $\Omega_K$ in the oCDM model, the CMB data have a degeneracy between $\Omega_m$, $\Omega_K$ and $H_0$. The BAO measurement adds a second distance measurement in the inverse distance ladder, breaking this degeneracy and significantly improving these measurements. Using the BAO data, we find that the Universe is consistent with a flat geometry: $\Omega_K = -0.004 \pm 0.005$. Fig. 5 shows the improvements in the $H_0$ versus $\Omega_m$, $\Omega_K$ versus $\Omega_m$ and $\Omega_K$ versus $H_0$ contours. From these different panels and Table 1, we clearly see that the CMB degeneracies between these three parameters are greatly reduced by adding the BAO data.

The ‘All BAO’ data set gives us BAO measurements at various redshifts: $z = 0.106$ (6dFGS), 0.35 (reconstructed SDSS DR7) and 0.6 (WiggleZ Dark Energy Survey). Fig. 6 shows the 68 per cent confidence level contours for various data sets. Table 1 provides the values for the cosmological parameters. From the table and Fig. 6, we see that the SN and additional BAO data add little to constrain the parameters over the CMB+BAO data set.

### 3.4 $\omega$CDM: varying the constant dark energy equation of state parameter

In this section, we allow the dark energy equation of state parameter $w$ to vary and we measure its value. Using the CMB+BAO data set, we measure the equation of state parameter $w = -0.97 \pm 0.17$, which is consistent with a cosmological constant ($w = -1$). Fig. 7 shows the 68 and 95 per cent confidence level contour plots for $H_0$, $\Omega_m$ and $w$. The combination of low-redshift (SDSS DR7) and high-redshift (WMAP7) measurements of the acoustic scale measures the expansion of the Universe and helps measuring the equation of state parameter for dark energy. We see that the combined CMB and BAO data precisely measure $H_0$, $\Omega_m$ and $w$ as listed in Table 1. Similar to the oCDM case (Section 3.3), we see that the CMB alone provides a robust measurement of $\Omega_m h^2$ but adding the BAO data breaks the degeneracy between the $H_0$, $\Omega_m$ and $w$.

We compare these measurements of $H_0$, $\Omega_m$ and $w$ with values for different data sets. The CMB+All BAO data set slightly improves our measurements on $w$ and $H_0$ as shown in Table 1. Fig. 8 shows the 68 per cent confidence level contours for $H_0$, $\Omega_m$ and $w$. In the oCDM case, the BAO data precisely measured $\Omega_K$, $\Omega_m$ and $H_0$ and we see no additional improvement by adding the SN data. In this case, however, we see that adding the SN data improves our measurements of $w$, $\Omega_m$ and $H_0$. This is expected from Fig. 3 as the BAO data constrain the vertical offset rather than the slope of the lines. We see that while all the measured values are consistent with each other within 1 $\sigma$, the CMB+BAO data set results tend to favour lower $H_0$ values and therefore higher values of $\Omega_m$ and $w$ compared to the CMB+BAO+SN data set. However, as Fig. 8 shows, the different data sets are consistent with the $\Lambda$CDM model parameters.

### 3.5 $\omega w$CDM: varying curvature and the constant dark energy equation of state parameter

Next, we move on to models where we vary two extra parameters in addition to the $\Lambda$CDM model parameters. In this case, we choose...
to vary both the dark energy equation of state parameter, \( w \), and the curvature parameter, \( \Omega_K \), as free parameters. Fig. 9 shows the 68 and 95 per cent confidence level contours for \( w \) versus \( \Omega_K \) for the CMB+All BAO, CMB+SN and CMB+BAO+SN data sets. We measure \( w = -1.08 \pm 0.08 \) and \( H_0 = 70.3 \pm 1.9 \text{ km s}^{-1}\text{ Mpc}^{-1} \) giving us a 4.4 and 2.7 per cent measurement of \( w \) and \( H_0 \), respectively. We precisely measure the curvature of the Universe to be consistent with being flat, \( \Omega_K = -0.005 \pm 0.005 \). We note that the CMB+All BAO results are consistent with the CMB+SN measurements. Adding the low-redshift \( H_0 \) measurement by Riess et al. (2011) gives us consistent measurements with CMB+BAO+SN as shown in Table 1.

### 3.6 \( w_0w_a, \Lambda \text{CDM}: \text{varying the time-dependent dark energy equation of state parameter} \)

As we probe higher redshifts, we can measure the evolution in \( w \), the dark energy equation of state parameter. The most popular way to parametrize an evolving \( w \), introduced by Chevallier & Polarski (2001) and Linder (2003), is

\[
\begin{align*}
\frac{\dot{a}}{a} &= \frac{\ddot{a}}{a} (1 - a) \\
\Rightarrow \frac{\dot{w}}{w} &= \frac{\ddot{w}}{w} + \frac{z}{1+z},
\end{align*}
\]

where \( a = 1/(1+z) \) is the scale factor. In this section, we assume a flat Universe and measure\( w_0 \) and \( w_a \). For cosmological models that use this parametrization of dark energy equation of state (\( w_0, w_a \)), we use the parametrized post-Friedmann prescription for dark energy perturbations as implemented in a CAMB module (Lewis, Challinor & Lasenby 2000) by Wenjuan Fang (http://camb.info/pplf/). This modified code generalizes it to support a time-dependent equation of state \( w(a) \). Fig. 10 shows the 68 and 95 per cent contours for \( w_0 \) versus \( w_a \) using the CMB+BAO+SN, CMB+BAO and CMB+All BAO data sets. From Table 1, we see that we find very similar constraints on \( H_0 \) and \( \Omega_m \) as the previously presented cosmological models: \( \Omega_m = 0.272 \pm 0.15 \) and \( H_0 = 71.2 \pm 1.9 \text{ km s}^{-1}\text{ Mpc}^{-1} \), giving us a 2.7 per cent measurement of \( H_0 \). We measure \( w_0 = 1.02 \pm 0.16 \) and \( w_a = -0.26 \pm 0.82 \), which are consistent with the \( \Lambda \text{CDM} \) model: \( w_0 = -1 \) and \( w_a = 0 \).

### 3.7 \( \omega w_a, \Lambda \text{CDM}: \text{varying curvature and time-dependent dark energy equation of state parameter} \)

In the most general cosmological model we analyse, we vary the curvature of the Universe, \( \Omega_K \), and both the dark energy parameters \( w_0 \) and \( w_a \) as free parameters. Fig. 11 shows the 68 and 95 per cent contour levels for \( H_0 \) versus \( \Omega_m \), \( w_0 \) versus \( \Omega_K \), \( w_a \) versus \( \Omega_K \), and \( w_a \) versus \( w_0 \) using the CMB+BAO+SN data set. It is noteworthy that even though we have curvature and both dark energy parameters as free parameters, the data are still consistent with a flat Universe with a cosmological constant. We see the precision in the measurements of \( H_0 \) and \( \Omega_m \) in the upper left-hand panel of Fig. 11. We obtain a 2.7 per cent measurement of \( H_0 = 69.9 \pm 1.9 \text{ km s}^{-1}\text{ Mpc}^{-1} \) and \( w_a = -0.90 \pm 0.16 \). Our measurement of \( w_0 = -1.30 \pm 0.99 \) is consistent with no evolution in \( w(z) \). We measure \( \Omega_K = -0.010 \pm 0.007 \), which is still consistent with a flat Universe. We find that even with the high dimensionality of the cosmological model, we are able to measure and constrain various cosmological parameters.

The Dark Energy Task Force (DETF) compares various cosmology missions and defines their figure of merit (FoM) in the context of this cosmological model (Albrecht et al. 2006). The DETF FoM is defined as the inverse square root of the determinant of the \( w_0-w_a \) covariance matrix. The 68 and 95 per cent contours for \( w_0 \) versus \( w_a \) are shown in the bottom right-hand panel in Fig. 11. Using the CMB+BAO+SN data set, we compute the DETF FoM to be 11.5. However, we note that this is an upper limit since the data set allows \( w_a \) outside our prior of \(-3.0 \leq w_a \leq 2.0 \).

Table 1 provides the values for the CMB+BAO+SN, CMB+All BAO and CMB+BAO+SN+H0 data sets. We see that all the measured values are consistent with each other at the 1σ level. In order to prevent the Markov chains from exploring very extended and remote parameter spaces, we use a prior of \(-3.0 \leq w_a \leq 2.0 \). However, the chains run with the CMB+BAO+SN and CMB+All BAO data sets tend to allow values beyond \( w_a \) less than -3.0 in their 95 per cent confidence level contours.

### 3.8 Robust measurement of \( H_0 \) and \( \Omega_m \)

From previous sections, we have found that our measurements of \( H_0 \) and \( \Omega_m \) remain unchanged as we increase the dimensionality of our cosmological models. In this subsection, we explore this result and explain why our measurements of \( H_0 \) and \( \Omega_m \) are robust regardless of the model for the late-time behaviour of dark energy. In Fig. 12, we show the measurements of \( H_0 \) from the CMB+BAO+SN data sets while varying the dark energy parametrization and the inclusion of spatial curvature. Fig. 13 shows the same set of results as 1σ contours in \( H_0 \) and \( \Omega_m \). One can see that regardless of the cosmological model, we obtain highly consistent values and error bars for these quantities.

This robustness is due to the inverse distance ladder discussed in Section 3.1. The CMB data provide a measurement of \( \Omega_m h^2 \) and the sound horizon \( r_s \). The BAO data use the measurement of \( r_s \) to provide a distance measurement to \( z = 0.35 \). The SN data then provide precise measures of the relative distance between \( z = 0.35 \) and the local distance scale. Hence, we have an empirical measure of the local distance and hence \( H_0 \), independent of spatial curvature.
Figure 5. 68 and 95 per cent confidence level contours for $H_0$ versus $\Omega_m$ (top), $\Omega_K$ versus $\Omega_m$ (middle) and $\Omega_K$ versus $H_0$ (bottom) for the oCDM model. The grey dashed lines represent the ‘CMB’ data set, and the solid black lines represent the ‘CMB+BAO’ data set. We see the vast improvement in the parameter measurements by adding the BAO data to the WMAP7 measurements.

or the model parametrization of the dark energy equation of state. Combining this measurement of $H_0$ with the CMB measurement of $\Omega_m h^2$ yields the value of $\Omega_m$. We note that while this result is independent of the parametrization of late-time dark energy and the presence of spatial curvature, it would be sensitive to new cosmological physics at $z \geq 1000$ that alters the inference of $\Omega_m h^2$ and the sound horizon from the CMB.

Figure 6. 68 per cent confidence level contours for $H_0$ versus $\Omega_m$ (top), $\Omega_K$ versus $\Omega_m$ (middle) and $\Omega_K$ versus $H_0$ (bottom) for the oCDM cosmological model using the CMB+BAO+SN (solid black line), CMB+BAO (solid grey line), CMB+All BAO (dashed grey line) and the CMB+SN (dashed black line) data sets.

It is of course important to compare our result for $H_0$ to direct measurements of the local distance scale. The Hubble constant has long been measured using distance ladders that are built from local calibrations out to more distant galaxies situated in the Hubble flow (Freedman et al. 2001; Riess et al. 2005, 2009; Benedict et al. 2007; Freedman & Madore 2010). A precise value of $H_0 = 73.8 \pm 2.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$ was recently obtained by the SH0ES project (Riess et al. 2011) using the NGC 4258 water maser (Argon et al. 2007; Humphreys et al. 2008) and Cepheid variable stars measured in the near-infrared. We plot this measurement in...
A 2 per cent distance to $z = 0.35$

Figure 7. 68 and 95 per cent confidence level contours for $H_0$ versus $\Omega_m$ (top), $w$ versus $\Omega_m$ (middle) and $w$ versus $H_0$ (bottom) for the $w$CDM model. The grey dashed lines represent the ‘CMB’ data set, and the solid black lines represent the ‘CMB+BAO’ data set. We see the improvement in the parameter measurements by adding BAO data to the WMAP measurements.

Figs 12 and 13. One sees that the direct measurement lies about 5 per cent higher than our inference from CMB+BAO+SN. However, this discrepancy only has a statistical significance of $1.5\sigma$ and hence is not unusual. Nevertheless, we will return to this in the next subsection.

We note that the CMB+BAO+SN combination consistently favours $H_0$ values around 71, while CMB+BAO alone gives a slightly lower best-fitting value of 69. The latter is not independent of the model for the expansion history; without SN, we are extrapolating the $z = 0.35$ distance to $z \approx 0$ using the cosmological model rather than an empirical measurement. Similarly, Beutler et al. (2011) measure $H_0 = 67.2 \pm 3.2$ km s$^{-1}$ Mpc$^{-1}$ using a BAO detection at $z = 0.1$. While this is all well within statistical uncertainties, apparently there is a small difference between the SN distance-redshift relation and that predicted from the combination of CMB and BAO data.

Figure 8. 68 per cent confidence level contours for $H_0$ versus $\Omega_m$ (top), $w$ versus $\Omega_m$ (middle) and $w$ versus $H_0$ (bottom) for the $w$CDM model using the CMB+BAO+SN (solid black line), CMB+BAO (solid grey line), CMB+All BAO (dashed grey line) and the CMB+SN (dashed black line) data sets.
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3.9 Energy density of relativistic species

The measurements of $H_0$ and $\Omega_\Lambda$ discussed in the previous section depend on the knowledge of cosmological physics at $z \gtrsim 1000$. Further, we found a small tension between the CMB+BAO+SN measurement of $H_0$ and the direct measurement by Riess et al. (2011). Hence, we are motivated to consider altering the standard cosmological model by adding additional relativistic particles with negligible interaction cross-section. These would be in addition to the usual cosmic background of the three neutrino species, and hence the new energy density is parametrized by altering the number of neutrino species from 3 to a new value $N_{\text{REL}}$. We note that the particles need not actually be neutrinos, simply highly relativistic and negligibly interacting at late times. This possibility has a long history in cosmology, including constraints from big bang nucleosynthesis (Steigman, Schramm & Gunn 1977; Bowen et al. 2002; Dolgov 2002; Hansen et al. 2002). Eisenstein & White (2004) pointed out that extra density in relativistic particles would cause CMB and BAO measurements to underestimate the values of $\Omega_m h^2$ and $H_0$. Numerous recent papers have constrained the density of relativistic particles with modern cosmology data (Seljak, Slosar & McDonald 2006; Ichikawa, Kawasaki & Takahashi 2007; Mangano et al. 2007; Hamann et al. 2010; Reid et al. 2010b; Archidiacono, Calabrese & Melchiorri 2011; Calabrese et al. 2011; Giusarma et al. 2011; Komatsu et al. 2011; Riess et al. 2011).

We therefore consider cosmological models that vary the relativistic density. In our MCMC chains, we use a prior of $N_{\text{REL}} \geq 3$. Fig. 14 shows the 68 per cent and 95 per cent confidence level
cosmological models using the CMB+BAO+SN data set. Also plotted is the $H_0$ value measured by Riess et al. (2011). We see that we get a robust measurement of $H_0$ and $\Omega_m$ regardless of the cosmological model.

Contours for $N_{\text{REL}}$ versus $H_0$ using the CMB+BAO+H0+SN data set with a $\Lambda$CDM + $N_{\text{REL}}$ cosmology model. Table 3 gives the values of $N_{\text{REL}}$ and other cosmological parameters for three different models of the expansion history of the Universe. From Fig. 14 and Table 3, we see that the best-fitting value for $N_{\text{REL}}$ is around 4. Models with extra relativistic particle density increase the values of $\Omega_m h^2$ and $H_0$, allowing a better fit to the Riess et al. (2011) measurement of $H_0 = 73.8 \pm 2.4 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$. In terms of the inverse distance ladder, the added relativistic species affects $\Omega_m h^2$, which moves the acoustic scale, and therefore changes the calibration of the distance ladder to larger values of $H_0$. Hence, it is not surprising to find that the other cosmological parameters such as $\Omega_m$, $w$ and $\Omega_K$ remain unaffected by the addition of a new relativistic species.

From Fig. 14, we see that the shift away from $N_{\text{REL}} = 3$ is not statistically significant. Table 3 shows this shift to be about 2$\sigma$. This is larger than the 1.5$\sigma$ tension between the $H_0$ measurements; this is likely due to the $N_{\text{REL}} \geq 3$ prior in our chains causing the mean value to be biased high and the variance to be biased low. However, a 2$\sigma$ shift when adding an extra parameter in our model is not compelling, but we note that recent cosmology results from the South Pole Telescope (Keisler et al. 2011) and the Atacama Cosmology Telescope (Dunkley et al. 2011) have found an excess of small-scale temperature anisotropy in the CMB, which could be explained by an extra density of relativistic particles beyond the usual neutrino background.

This increase in the relativistic particle density also causes the model to predict a higher value of $\sigma_8$ as shown in the lower panel of Fig. 14 and in Table 3. The best-fitting value shifts from $\sigma_8 = 0.81 \pm 0.02$ for $N_{\text{REL}} = 3$ to $\sigma_8 = 0.86 \pm 0.03$ for $N_{\text{REL}} \approx 4$. For comparison, Allen, Evrard & Mantz (2011) (table 2) give a comparison of $\sigma_8$ measurements from galaxy cluster studies. X-ray (Henry et al. 2009) and optical (Rozo et al. 2010) studies of cluster abundances measure $\sigma_8 = 0.88 \pm 0.04$ and $0.80 \pm 0.07$, respectively. Thus, our $\sigma_8$ measurements for $N_{\text{REL}} \approx 4$ are consistent with galaxy cluster measurements.

**4 CONCLUSIONS**

In this series of papers, we have used the reconstructed SDSS DR7 LRG data set to measure the BAO acoustic scale at the median redshift of $z = 0.35$. The reconstruction technique that provided this measurement has been discussed to great detail in Paper I, and the measurement itself has been extensively studied and tested in Paper II. In this paper, we use this BAO measurement of $D_A(z = 0.35)/r_s = 8.88 \pm 0.17$, which is a 1.9 per cent measurement of the distance to $z = 0.35$. To measure various cosmological parameters in a variety of cosmological models, we use our BAO data in combination with the CMB data from the WMAP7 (Komatsu et al. 2011) and the SN Ia data from SNLS3 (Conley et al. 2011) to extend the CMB+BAO inverse distance ladder to $z = 0$. With this CMB+BAO+SN data set, we explore higher dimensional cosmological models and robustly measure the Hubble constant and the matter density of the Universe. We also use the BAO data from the 6dFGS and WiggleZ surveys in combination with our BAO data and the CMB data to measure cosmological parameters. In particular:

(i) We find that our BAO data set is consistent with $\Lambda$CDM as shown in Fig. 2. We improve on the WMAP7 measurements
in $\Lambda$CDM and obtain a 1.7 per cent measurement of the Hubble constant: $H_0 = 69.8 \pm 1.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

(ii) As shown in Fig. 3, the distance measured from our BAO result is in good agreement with past work. It is therefore unsurprising that the cosmological parameters resulting from our chains are similar to those in recent works combining BAO with other data sets, e.g. Komatsu et al. (2009), Percival et al. (2010), Reid et al. (2010a), Blake et al. (2010) and Beutler et al. (2011).

(iii) Papers I and II show that reconstruction improves the BAO distance measurement by a factor of 1.8. We see this improvement as a reduction in the errors around $H_0$ and $\Omega_m$ by a factor of 1.5.

(iv) Under the $\Lambda$CDM model, we explore the effect of allowing a running spectral index, $\delta n_s$ or $\delta n_z$, and find that only using the CMB data degrades the measurements of $\Omega_m h^2$ by a factor of 1.4. This translates into a larger uncertainty in measurements of $\Omega_m$, $H_0$ and $n_s$. Adding BAO data decreases the uncertainty to 1.1. We also find that with both data sets the value of $\delta n_s / \delta n_z$ is still consistent with 0.

(v) The CMB+BAO data set breaks the degeneracy between $H_0$, $\Omega_m$ and $w$ or $\Omega_k$ in the $\omega$CDM and $\omega$CDM cosmological models. We measure $w = 0.97 \pm 0.17$ and $\Omega_k = -0.003 \pm 0.005$, both consistent with $\Lambda$CDM. We find that adding the other BAO data slightly improves the measurements on these parameters.

(vi) For the higher dimensional cosmological models (our $\omega$CDM, $\omega_0 w_0$CDM and $\omega_0 w_0$CDM), we use the combined CMB+BAO+SN data set to measure cosmological parameters. We find that even in these high dimensional models, the data are consistent with a flat Universe with a cosmological constant, i.e. consistent with $\Lambda$CDM. The DETF FoM is 11.5 using the CMB+BAO+SN data set and using a prior on $w$.

(vii) Using the inverse distance ladder built from the CMB+BAO+SN data set, we show that we obtain robust and precise measurements of both the Hubble constant and the matter density of the Universe despite varying the underlying model for the expansion history of the Universe. Even in our most general case, we measure $H_0 = 69.9 \pm 1.9 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\Omega_m = 0.276 \pm 0.015$.

(viii) Our value of the Hubble constant is in mild tension (1.5σ) with the direct measurement of 73.8 $\pm$ 2.4 km s$^{-1}$ Mpc$^{-1}$ by Riess et al. (2011). We explore the possibility that this tension could be resolved by increasing the density of relativistic particles beyond the usual background of three species of neutrino. We find that such a model can fit the $H_0$ value better if one adds density equivalent to one extra species of neutrinos. However, we stress that the conventional model is not rejected by our data.

Looking towards the future, measurements of the distance-redshift relation with BAOs will improve considerably. The SDSS-III Baryon Oscillation Spectroscopic Survey (BOSS) is underway and will extend the galaxy sample out to $z = 0.7$ (Eisenstein et al. 2011). We expect that the methods used in this SDSS DR7 analysis will be applicable to the BOSS sample. Yet larger surveys probing higher redshifts, such as Euclid and the Wide-Field Infrared Survey Telescope (WFIRST) missions, will use reconstruction to approach the cosmic variance statistical limit available to the acoustic peak method. We expect that BAOs will play a major role in the precision mapping of the cosmic distance scale and expansion history of the Universe.

**ACKNOWLEDGMENTS**

Funding for the SDSS has been provided by the Alfred P. Sloan Foundation, the Participating Institutions, the National Science Foundation, the US Department of Energy, the National Aeronautics and Space Administration, the Japanese Monbukagakusho, and the Max Planck Society, and the Higher Education Funding Council for England. The SDSS website is http://www.sdss.org/.

The SDSS is managed by the Astrophysical Research Consortium (ARC) for the Participating Institutions. The Participating Institutions are the American Museum of Natural History, Astrophysical Institute Potsdam, University of Basel, University of Cambridge, Case Western Reserve University, the University of Chicago, Drexel University, Fermilab, the Institute for Advanced Study, the Japan Participation Group, the Johns Hopkins University, the Joint Institute for Nuclear Astrophysics, the Kavli Institute for Particle Astrophysics and Cosmology, the Korean Scientist Group, the Chinese Academy of Sciences (LAMOST), Los Alamos National Laboratory, the Max-Planck-Institute for Astronomy (MPIA), the Max-Planck-Institute for Astrophysics (MPA), New Mexico State University, Ohio State University, University of Pittsburgh, University of Portsmouth, Princeton University, the United States Naval Observatory and the University of Washington.

We thank the LasDamas collaboration for making their galaxy mock catalogues public, Cameron McBride for assistance in using the LasDamas mocks and comments on earlier versions of this work, Martin White for useful conversations on reconstruction and Bradford Benson for helping us find a bug in our code by pointing out a discrepancy between our constraints on the equation of state of dark energy using only CMB data and the reported values by the WMAP team. Finally, we also thank the anonymous reviewer for helpful comments. KTM, DJE and XX were supported by NSF grant AST-0707725 and NASA grant NNX07AH11G. NP and AJC are partially supported by NASA grant NNX11AF43G. The MCMC computations in this paper were run on the Odyssey cluster supported by the FAS Science Division Research Computing Group at Harvard University. This work was supported in part by the facilities and staff of the Yale University Faculty of Arts and Sciences High Performance Computing Center.
