Gravitational form factors and mechanical properties of proton from $J/\psi$ photoproduction data

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(Dated: August 22, 2022)

Inspired by the recent $J/\psi$ photoproduction measurements, we discuss the $D$-term gravitational form factors (GFFs) and the mechanical properties of the proton. Under the assumption of the tripolet form, the gluon $D$-term form factor is determined by fitting the $J/\psi$ differential cross-section data. Combined with the quark $D$-term form factor extracted from the deeply virtual Compton scattering experiment, the total $D$-term form factor is obtained to investigate their applications for the description of the mechanical properties. Moreover, the predicted gluon $D$-term form factor is compared with the existing lattice quantum chromodynamics determinations, which indicates our result is overestimated in momentum transfer $|q| < 0.5$ GeV$^2$. Accordingly, the distributions of pressure and shear forces inside the proton are investigated. Finally, the root-mean-square (rms) proton mechanical radius $\sqrt{\langle R^2_{\text{mech}} \rangle} = 0.86 \pm 0.03$ fm, while the rms pressure $\rho(0) = 1.42 \pm 0.07$ GeV/fm$^3$ in the center of nucleon are calculated. These results provide a unique perspective for studying the gluon $D$-term form factor of proton and important information for revealing the internal pressure and shear distributions of nucleons.

I. INTRODUCTION

The form factor of nucleon provides critical information about many fundamental aspects of hadron structure. In addition to the charge and mass distribution encoded in the electromagnetic form factors and gravitational form factors (GFFs), respectively [1–4], one can get a deeper understanding of the basic mechanical properties of the proton by studying the $D$–term GFFs [5, 6]. The sum of the quark and gluon contribution is a measurable quantity defined purely from the $D$–term form factor, which encodes the shear forces and pressure distribution in the proton. The quark $D$-term form factor $D_q(t)$ has recently been extracted from the deeply virtual Compton scattering (DVCS) experiments and the pressure distribution inside the proton has been reported [7]. However, since DVCS is almost insensitive to gluon, the gluon $D$–term form factor has seldom been extracted. The quark and gluon $D$-term form factor parameterize the spatial-spatial components of the energy momentum tensor (EMT) and describe the internal dynamics of the nucleon system [8].

In particular, the proton quark $D$–term form factor has been researched in the DVCS experiments, and the $D$–term has also been studied theoretically in numerous frameworks. One work obtain the proton GFFs and investigate the mechanical properties using a light-front quark-diquark model constructed by the soft-wall AdS/QCD [9]. The nucleon form factors of the energy-momentum tensor are studied in the framework of the Skyrme model and in-medium modified Skyrme model [10, 11]. Ref. [12] calculated the pressure, energy density and mechanical radius of the nucleon in light-cone QCD sum rule formalism. The energy-momentum tensor form factors of the nucleon within a $\pi-\rho-\omega$ soliton model is used to study the mechanical properties, such as pressure and energy density [13]. On the other hand, the distributions of pressure and shear forces inside the proton are investigated using lattice QCD calculations of the energy momentum tensor [8, 14], changing the little understanding situation about the gluon $D$-term form factor.

At finite momentum transfer, the gluon GFFs play important roles in the near-threshold heavy quarkonium photoproduction, such as $J/\psi$ and $\Upsilon$ meson [2, 15–20]. This manuscript presents a connection between the near-threshold differential cross-section of $J/\psi$ and the gluon $D$-term form factor. One reason is that the scalar gluon operator is dominant in the production amplitude of heavy quarkonium. In GFFs $A(Q^2)$, $B(Q^2)$, and $D(Q^2)$, the contribution of the gluon is mainly attributed to the $D$-term form factor. Therefore, one can determine the $t$-dependence of the cross-section to derive the gluon $D$-term form factor. Moreover, using heavy vector mesons photoproduction works because of the high mass of $J/\psi$ that defines a short distance interaction. In electroweak production the short distance is given by high $Q^2 >> 1$ GeV$^2$, i.e. high photon virtuality. This relation allows us to study the proton mechanical properties by learning the near-threshold photoproduction data of vector mesons. Therefore, experimental information on vector meson photoproduction is essential to gain insight into the fundamental structure of the proton. Recently, the GlueX Collaboration reported the near-threshold cross-section of the reaction $\gamma p \to J/\psi p$ [21]. The Jefferson Lab have measured the $J/\psi$ photoproduction differential cross-section on proton targets at a photon energy $E_\gamma$ from 9.1 GeV to 10.6 GeV [22], which is a near-threshold energy region. Those experimental data offer us a good window for studying the interior character of the proton. Nowadays, the Electron Ion Colliders (EIC) are suggested to be built for probing the deepest structure inside the hadron and collecting $J/\psi$ data [23, 24]. The high-precision experimental measurements are suggested to be carried out on these facilities.
The paper is organized as follows. Fitting the $J/\psi$ photoproduction data, the gluon $D$-term form factor is determined. Then the process of calculating the pressure force, shear force and the mechanical radius inside the proton are provided in Sec. II. The obtained gluon $D$-term form factor is compared with the existing lattice QCD determinations. Some analysis of the results is also presented. A summary is given in Sec. III.

II. PROTON MECHANICAL PROPERTIES

Using the standard form, the differential cross-section of the $J/\psi$ photoproduction can be written as [2, 25]

$$\frac{d\sigma_{\gamma p \rightarrow J/\psi p}}{dt} = \frac{1}{64\pi W^2} \frac{1}{|p_{cm}|^2} |M_{\gamma p \rightarrow J/\psi p}|^2$$

(1)

where $p_{cm}$ is the photon momentum in the center of mass (c.m.) momenta in the $\gamma p \rightarrow J/\psi p$ process, and $W$ is the c.m. energy. $t$ is the momentum transfer. Ref. [2] writes the amplitude $M$ as

$$M_{\gamma p \rightarrow J/\psi p} = -Q_c c_1 \frac{16\pi^2 M}{b} \langle P'|T|P \rangle$$

(2)

where $b = 9$ and $Q_c = 2e/3$ represents the coupling of the photon and the quarks in $J/\psi$ meson. $c_1$ is a distance coefficient which can be written as $c_1 = \pi r_c^2$ where the size of the $c\bar{c}$ pair $r_c = 1/(2m_c)$. In this paper, we perceive that the gluon $D$-term form factor can be calculated as [2]

$$\langle P'|T|P \rangle = D_g(t) \cdot \frac{3t}{4M^2}$$

(3)

Here, $D_g(t)$ is the gluonic form factor, which is usually parameterized as the tripoles form and provided as [7, 26]

$$D_g(t) = \frac{D_0}{(1 - t/m_D^2)^3}$$

(4)

where $D_0$ and $m_D$ are the parameters adjusted to the experimental data. Thus the differential cross-section of $\gamma p \rightarrow J/\psi p$ reaction in Eq. (1) can be determined by fitting the GlueX and JLab experimental data [21, 22]. The comparison between the differential cross-section and the experimental measurements is manifested in Fig. 1, exhibiting a good agreement. The obtained value of the parameters $D_0$, $m_D$ in Eq. (4) and $\langle P'|T|P \rangle$ of different c.m. energy are listed in Table I. Note that, the parameter $|D_0|$ increased with the increasing of c.m. energy, and the trend of $m_D$ instead in Table I. Actually, the obtained $D$-term form factor, which is determined by the parameters $D_0$ and $m_D$, are energy dependent. Further discussion will follow in the section on mechanical properties. Thereby, the gluon’s contribution on the $D$-term form factor $D_g(t)$ is determined. As shown in Fig. 2, we determine the gluonic form factor $D_g(t)$, compared with the lattice QCD determinations of the gluon contribution to the GFFs [8, 14]. Here the parameters $D_0 = -2.72$, and $m_D = 0.78$ GeV are applied by fitting the GlueX data, since the best fit ($\chi^2/d.o.f.$=0.13) is obtained when $W = 4.58$ GeV.

![FIG. 1. The differential cross-section of $\gamma p \rightarrow J/\psi p$ as a function of $-t$ at c.m. energy $W = 4.25$ GeV, 4.28 GeV, 4.31 GeV, 4.35 GeV, 4.38 GeV, 4.41 GeV, 4.45 GeV, 4.48 GeV, 4.51 GeV, 4.54 GeV, 4.58 GeV. The differential cross-section in Eq. (1) is the dashed curve. Reference of data can be found in [21, 22].]

The form factor on the quarks inside the proton can be determined by fitting the DVCS data [7]. One work determined the $D_q(t)$ as [5, 26]

$$D_q(t) = \frac{18}{25} \frac{d(0)}{(1 - t/m_q^2)^3}$$

(5)
where the parameters are fitted to be $d(0) = -2$, $M_0^2 = 25$ [26]. Thus one can combine the quarks contribution in Eq. (5) and gluon contribution in Eq. (4) inside the proton to write the total form factor $D_{g+q}(t)$, as shown in Fig. 2.

FIG. 2. The comparison of quark contribution $D_q(t)$ (red dashed curve), gluon contribution $D_g(t)$ (blue dotted curve) and the total contribution (green solid curve) in the $D$-term of proton. Here the parameters $D_0 = -2.72$, and $m_D = 0.78$ GeV in gluon contribution $D_g(t)$ is applied by fitting the GlueX data. The blue circle shows the lattice QCD determinations of the gluon contributions to the GFFs [8, 14].

The pressure and shear forces are “good observables” to report the pressure and shear distribution experienced by the quarks in the proton. The pressure force $p(r)$ and shear force $s(r)$ can be extracted as [5, 6]

$$s(r) = s_q(r) + s_g(r) = -\frac{1}{2} \frac{d}{dr} \frac{d}{dr} \bar{D}(r) \quad (6)$$

$$p(r) = p_q(r) + p_g(r) = \frac{1}{3} \frac{1}{r^2} \frac{d}{dr} \frac{d}{dr} \bar{D}(r) \quad (7)$$

The Fourier transform of the $D_{g+q}(t)$ can be written as [5, 6]

$$\bar{D}(r) = \int \frac{d^3 \Delta}{(2\pi)^3} e^{-i\Delta r} D_{g+q}(-\Delta^2) \quad (8)$$

where $-\Delta^2 = r$. The pressure force $p(r)$ and shear force $s(r)$ is manifested in Fig. 3. Note that the pressure distribution $r^2 p(r)$ satisfies the internal forces balance inside a composed particle

$$\int_0^\infty dr \, r^2 p(r) = 0 \quad (9)$$

FIG. 3. The pressure force $p(r)$ (blue-dotted curve) and shear force $s(r)$ (red-dashed curve) inside the proton. The normal $F_n(r)$ (purple-dotted curve) and tangential force $F_t(r)$ (green-solid curve) in the composed particle system. Here the parameters $D_0 = -2.72$, and $m_D = 0.78$ GeV in gluon contribution $D_g(t)$ is applied by fitting the GlueX data.

FIG. 4. The pressure and shear distribution $r^2 p(r)$ inside the proton. The curves have the same meaning as in Fig. 2. Here the parameters $D_0 = -2.72$, and $m_D = 0.78$ GeV in gluon contribution $D_g(t)$ is applied by fitting the GlueX data.
Thus the pressure and shear force distributions inside the proton are obtained and displayed in Fig. 4. It was found that the pressure is positive in the inner region, and negative in the outer region in all studies so far. In our convention the positive sign means attraction directed towards the inside, and the negative sign means repulsion towards the outside, and the negative sign means attraction directed towards the inside.

The combination of the normal and tangential force in the composed particle system is

\[ F_n(r) = \frac{2}{3} s(r) + p(r), \quad \text{and} \quad F_t(r) = -\frac{1}{3} s(r) + p(r) \quad (10) \]

where the positive and negative eigenvalues correspond to "strecthing" or "squeezing" along the corresponding principal axes, respectively. The normal force \( F_n(r) \) corresponds to the distribution of the normal component of the force and must be positive to guarantee the stability of the system. The tangential force \( F_t(r) \) change sign with the distance \( r \) because possible "squeezing" is averaged to zero for spherically symmetric systems [5]. The above conditions show that the normal and tangential force, as shown in Fig. 3, satisfies

\[ F_n(r) > 0, \quad \text{and} \quad \int_0^\infty 2\pi r \cdot F_t(r) dr = 0 \quad (11) \]

The mechanical radius of the proton was defined not in terms of the slope of \( D(t) \) and one can define a mechanical radius of the proton as [5]

\[ \langle R_{\text{mech}}^2 \rangle = \frac{\int d^3r \frac{r^2}{2} \left[ F_n^{\text{gluon}}(r) + F_n^{\text{quark}}(r) + p_n(0) \right]}{\int d^3r \left( F_n^{\text{gluon}}(r) + F_n^{\text{quark}}(r) + p_n(0) \right)} \]

\[ = \frac{1}{4m_p} \int_0^\infty D_q(t) \ dt - \frac{1}{4m_p} \int_0^\infty D_q(t) \ dt \]

And the calculation results are shown in the last column of Table I. The root-mean-square (rms) proton mechanical radius is obtained to be 0.86 ± 0.03 fm. An important mechanical quantity \( p_n(0) \) denotes the pressure value of gluon contribution in the center of the nucleon, as listed in Table I. Finally we obtain the rms pressure value \( p_n(0) = 0.54 ± 0.07 \text{ GeV/fm}^3 \). Note that, the stable mechanical results are obtained by fitting the experimental data at different energies. This energy independence indicates our process of studying mechanical properties is works.

One can add the quark contribution \( p_q(0) = 0.88 \text{ GeV/fm}^3 \) and calculate the rms pressure \( p(0) = 1.42 ± 0.07 \text{ GeV/fm}^3 \) in the center of nucleon. As listed in Table II, our results on the mechanical quantities \( p(0) \) and \( \sqrt{\langle R_{\text{mech}}^2 \rangle} \) of the proton compared to other existing theoretical predictions. It can be seen that our prediction on the mechanical radius is overestimated but comparable with the light-front quark-diquark model [9]. Meanwhile, our predictions on \( p(0) \) are larger than the prediction of Refs. [9, 10, 12, 27] and underestimated with the light-front quark-diquark model [28].

### III. SUMMARY

In this paper, one connects the near-threshold differential cross-section of \( J/\psi \) to the gluon \( D \)-term form factor. Under the assumption of the tripole form, the gluon \( D \)-term form factor can be determined by fitting the \( J/\psi \) differential cross-section data. One find that the gluonic form factor, \( D_q(t) \), is smaller than the lattice QCD determinations of the gluon contributions to the gravitational form factors in momentum transfer \( |t| < 0.5 \text{ GeV}^2 \). Combined with the quark \( D \)-term form factor extracted from the DVCS experiment, the total \( D-
Table II. The values of mechanical quantities of nucleon from different approaches.

| Approaches and Models | $p(0)$ (GeV/fm$^3$) | $\sqrt{\langle R_{\text{mech}}^2 \rangle}$ (fm) |
|-----------------------|---------------------|------------------|
| light-front quark-diquark model (1) [9] | 0.29 | 0.86 |
| light-front quark-diquark model (2) [9] | 0.40 | 0.86 |
| light-front quark-diquark model [28] | 4.76 | 0.50 |
| Skyrme model [10] | 0.47 | - |
| light-cone QCD (1) [12] | 0.67 | 0.73 |
| light-cone QCD (2) [12] | 0.62 | 0.72 |
| light-cone sum rules at leading order [27] | 0.84 | 0.72 |
| lattice QCD (modified z-expansion) [8] | - | 0.71 |
| lattice QCD (tripole ansatz) [8] | - | 0.75 |
| this work | 1.42 ± 0.07 | 0.86 ± 0.03 |

This charge radius is comparable to the mechanical radius given the statistical error bar of our value of mechanical radius. Note that the mass radius is much smaller, and this difference can attribute to the interplay of asymptotic freedom and spontaneous breaking of chiral symmetry in QCD [2]. Generally, the measurements of the charge distribution and mass distribution, combined with measurements of the mechanical properties of the proton, will definitely advance our understanding of the origin of mass.

The high-precision photo/electroproduction data of vector mesons is an important guarantee and basis for the accurate study of the internal structural properties of protons. We recommend that relevant experimental measurements be carried out in JLab [21] or EIC [23, 24] facilities.

IV. ACKNOWLEDGMENTS

This project is supported by the National Natural Science Foundation of China under Grant Nos. 12065014 and 12047501, and by the West Light Foundation of The Chinese Academy of Sciences, Grant No. 21JR7RA201.

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