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Stabilization With Prescribed Instant for High-Order Integrator Systems

Jiyuan Kuang, Yabin Gao, Member, IEEE, Chih-Chiang Chen, Member, IEEE, Xiaoju Zhang, Yizhuo Sun, and Jianxing Liu, Senior Member, IEEE

Abstract—This article develops a new controller design approach to stabilize system states onto the equilibrium at an arbitrarily selected time instant irrespective of the initial system states and parameters. By the stabilization approach, the actual convergence time (not the bound of actual convergence time) is independent of the initial value of system states. This feature differentiates our proposed prescribed-instant stability from conventional fixed, predefined, and prescribed time stability. In this work, we propose the controller design method for the prescribed-instant stability of n-order integrator systems. The proposed control is bounded and can gradually go to zero at an arbitrarily selected time instant, at which the system states reach zero simultaneously. This special stability of the controlled system is analyzed by reduction to absurdity. In simulations, an example of comparison with frequently used prescribed-time control is presented to show the difference. Moreover, the proposed stabilization method is validated by a magnetic suspension system with matched disturbances.

Index Terms—Finite-time control, prescribed-instant stability, prescribed-time control, time-varying feedback.

I. INTRODUCTION

A S IS well-known, the central philosophy of finite-time stabilization (FNTS) is the guarantee of state convergence over a finite-time interval while providing an estimation of the convergence time (i.e., the settling time) depending on initial states and control parameters [1], [2]. For autonomous systems, the finite-time stability analysis was formally presented in [3], in which a Lyapunov-based inequality \( \dot{V}(x) \leq -a V^b(x) \) with \( b \in (0, 1) \) and \( a > 0 \) was developed. The real convergence time \( T(x_0) \) satisfies \( T(x_0) \leq (|V(x_0)|^{1-b} / |a(1-b)|) \), and there are no uniform bounds for \( T(x_0) \forall x_0 \in \mathbb{R}^n \) \( (V(x_0) \) depends on \( x_0 \)). Many variants around this Lyapunov inequality were further proposed, for example, the homogeneous approach [4], the implicit Lyapunov function approach [5], and the adaptive control [6]. These years, the FNTS method was further developed as three branches: 1) fixed-time stabilization (FXTS); 2) predefined-time stabilization (PSTS); and 3) prescribed-time stabilization (PSTS). These control methods are classified into two categories, one is the FXTS and the PDTS, and the other is the PSTS. However, all of them have some conservativeness in the convergence time.

The FXTS and the PDTS get finite-time stability by utilizing the power of some special functions of system states. The FXTS ensures the convergence time \( T(x_0) \) to be bounded by a constant \( T_{\text{max}} \), which is independent of the initial conditions \( x_0 \), in another word, \( T(x_0) \leq T_{\text{max}} \forall x_0 \in \mathbb{R}^n \) [7], [8]. The constant \( T_{\text{max}} \) is determined by some controller parameters, so one should calculate the parameters according to an assigned \( T_{\text{max}} \) [9]. However, the actual convergence time may be far smaller than the designed \( T_{\text{max}} \) [10]. Hence, it is logical to pursue less conservativeness in the FXTS. To this end, the notion of the PDTS was proposed as a result of \( \sup_{x_0 \in \mathbb{R}^n} T(x_0) = T_f \) [11], [12]. Another improvement of the PDTS among the FXTS is that the assigned time \( T_f \) is a constant parameter explicit in the controller. This feature is quite similar to the PSTS. One defect of the existing predefined-time control is that the initial control grows exponentially with \( x_0 \). This weakness was overcome by the construction of a series of new controllers with polynomial terms instead of exponential ones [13].

The PSTS proposed in [14] was inspired by some time-varying functions that usually go to infinity toward the terminal time \( T_f \) [15]. Then, some extended time-varying functions were also proposed to achieve PSTS [16]. The PSTS ensures \( T(x_0) \leq T_f \forall x_0 \in \mathbb{R}^n \), where \( T_f \) is a constant parameter that can be arbitrarily assigned and is explicit in the controller [17]. Compared with the PDTS, the assigned time \( T_f \) of the PSTS is more conservative. One problem with PSTS is that the singularity or discontinuity may appear when \( t \) tends to \( t_0 + T_f \), as there is usually a term \( (1/(t_0 + T_f - t)) \) in time-varying feedback functions. Hence, such controllers on PSTS should be carefully designed to ensure boundedness [18].

These years, each of these methods was further developed and achieved significant theoretical contributions. To mention a few, fixed-time stability was used in differentiator [19]...
and observer [20]. Prescribed-time stability was recast within the framework of time-varying homogeneity [21]. The work in [22] derived a predefined upper bound of the settling time and provided sufficient conditions such that the time-varying gains remain bounded. Moreover, these methods were also developed for systems with uncertainties [23], [24] and input delays [25]. Apart from theoretical achievements, one particular application is the multiagent system consensus. The work in [26] solved the prescribed-time consensus tracking problem for second-order nonlinear multiagent systems with unknown dynamics. The work in [27] solved the problem of distributed prescribed finite-time observer design for a nonlinear system with disturbance. Both of them achieve prescribed-time consensus. Particularly, in some practical complex cooperation tasks, the systems require each agent to converge to some states just at a certain time instant. This actually appeals for a stricter PSTS that the real convergence time is prescribed independent of initial system states.

In this article, we design a controller based on the $n$-order chain of integrators to ensure the convergence to zero happening at a selected time instant, which means that conservativeness on the convergence time is reduced to zero for the first time. This controller is also inspired by time-varying feedback functions [15], one of which is widely used in the PSTS. Notably, the proposed stabilization notion of this article is called the prescribed-instant stabilization (PSIS) in the following context. Our main contribution can be summarized as follows.

1) The controller to achieve the PSIS is designed based on high-order integrator systems. The settling time of PSIS equals the prescribed time instant, which can be selected as any physically possible positive number.

2) Different from PSTS, the proof of the PSIS considered the order of infinitesimal of each state. The singularity problem of conventional PSTS is overcome in our proposed PSIS.

3) The proposed PSIS is also practical for the high-order integrator systems with matched disturbances, by using sliding-mode control.

The remainder of this article is structured as follows: Section II provides some basic knowledge and follows the problem statement. Section III gives some new time-varying feedback functions, based on which the general form of the controller is presented. Then reduction to absurdity is utilized in the analysis of the PSIS. In Section IV, simulations are provided to validate the proposed results. Section V concludes this article.

II. PRELIMINARIES AND PROBLEM STATEMENT

A. Preliminaries

Consider the following autonomous system:

\[ \dot{x} = f(x, \eta), \quad x(t_0) = x_0 \]  

where $x \in \mathbb{R}^n$ (Euclidean $n$-space) is the system state, and the vector $\eta \in \mathbb{R}^l$ stands for the tunable parameters of (1).

The function $f : \mathbb{R}^n \times \mathbb{R}^l \to \mathbb{R}^n$ may be discontinuous, and such solutions of (1) exist and are unique in the sense of Filippov [28]. Thus, $\Phi(t, x_0)$ denotes the solution of (1) starting from $x_0 \in \mathbb{R}^n$ at $t = 0$. Moreover, the origin $x = 0$ is the unique equilibrium point of (1).

As stated in [29], the parameter-dependent system (1) is equivalent to the controlled system

\[ \dot{x} = g(x, u) \]  

where $g : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$, the control $u \in \mathbb{R}^m$ is a feedback function of $x$ with tunable parameters $\eta$, that is, $u = w(x, \eta)$, with $w : \mathbb{R}^n \times \mathbb{R}^l \to \mathbb{R}^m$. Substituting $u = w(x, \eta)$ in (2) eliminates $u$ and yields $f(x, \eta) := g(x, w(x, \eta))$. All the notions defined and treated hereafter are global, so we will omit to indicate it.

**Definition 1:** The origin of the system (1) is said to be:

1) FNTS [1], if it is Lyapunov stable and for any $\eta$ there exists $T > 0$ such that $\Phi(t, x_0) = 0 \forall t \geq t_0 + T$.

2) FXTS [8], if it is FNTS and the settling-time function $T(x_0)$ is bounded.

3) PSTS [14], if it is FXTS and the settling-time function is bounded, that is, $\exists \exists \bar{T} \leq T_{\max} < \infty$ such that $T(x_0) \leq T_{\max} \forall x_0 \in \mathbb{R}^n$.

Remark 1: It is important to highlight that $T_{\max}$ is usually a function of $\eta$, and $T_{\max}$ is not explicit in the control $u$. On the other hand, $T_{\max}$ can be chosen larger than the calculated result from $\eta$. For instance, if the settling-time function is bounded by $T_m$, it is also bounded by $\lambda T_m \forall \lambda \geq 1$. How to find the smallest $T_{\max}$ motivates the following definition.

**Definition 2** [10]: The origin of the system (1) is said to be PDTS if it is FXTS, and for any positive number $T_f$, there exists some $\eta \in \mathbb{R}^l$ such that either of the following can be established.

1) $T(x_0) \leq T_f \forall x_0 \in \mathbb{R}^n$ (weak-predefined time).

2) $\sup_{\forall x_0 \in \mathbb{R}^n} T(x_0) = T_f$ (strong-predefined time).

**Remark 2:** In a strict sense, the time $T_f$ can be considered as the true fixed time in which the system (1) is stabilized. Moreover, $T_f$ used in PDTS is usually a constant parameter explicit in the controller, that is, $u = w(x, \eta, T_f)$.

The FNTS, the FXTS, and the PDTS are realized by the control $u = w(x, \eta)$ independent with time $t$. Now, let us consider a time-varying control $u = w(x, t, \eta)$. Then, substituting it into (2) yields a nonautonomous system

\[ \dot{x} = f(x, t, \eta) := g(x, w(x, t, \eta)), \quad x(t_0) = x_0. \]  

**Definition 3** [14], [30]: The origin of the system (3) is said to be PSTS if for any physically possible positive number $T_f$, there exists some $\eta \in \mathbb{R}^l$ such that the settling time $T(x_0)$ can be prescribed such that $T(x_0) \leq T_f \forall x_0 \in \mathbb{R}^n$.

**Remark 3:** $T_f$ in PSTS is usually a constant parameter (independent of other parameters) explicit in the control, that is, $u = w(x, t, \eta, T_f)$. Pay attention that existing proof of PSTS only demonstrated that $T(x_0) \leq T_f$, so there may exist a positive number $T_c < T_f$ such that $x = 0 \forall t \geq t_0 + T_c$. However, the expression of Theorem 2 in the work of PSTS [14] may lead to a misunderstanding that the PSTS ensures that the actual convergence time equals just right the prescribed convergence time. To dispel this misunderstanding, a simple numerical...
example will be provided in simulations, and the following fact is emphasized hereby.

For system (3), one may try to construct a candidate Lyapunov function $V(x, t)$, s.t. $\dot{V} \leq \psi(V, t) \leq 0$. If the solution of $\dot{V} = \psi(V, t)$ is $V = \phi(t)$, where $\phi(t) \geq 0 \forall t \in [0, T_f)$ and $\phi(t) = 0 \forall t \in [T_f, +\infty)$, then the convergence rate of $V$ is faster than $\phi$, that is, $x$ will converge to zero before or at the time instant $T_f$. Actually, this is the basic thought to prove the FXTS, PDTS, and PSTS. However, this proof cannot guarantee that $x$ will converge to zero exactly at the time instant $T_f$. To reduce this conservativeness, we propose the prescribed-instant stability.

**Definition 4:** The origin of the system (3) is said to be prescribe-instant stable (PSIS) if for any physically possible positive number $T_f$, there exists some $\eta \in \mathbb{R}^n$ such that the settling time $T(x_0)$ can be prescribed such that $T(x_0) = T_f \forall x_0 \in \mathbb{R}^n$.

**Remark 4:** $T_f = T_f - t_0$ is the prescribed convergence time, and $T_f$ is the prescribed instant when the system states converge to zero. $T(x_0) = t_0 - t_0$ is the actual convergence time or settling time, and $t_0$ is the actual instant when the system states converge to zero. It is noted that the proposed prescribed-instant stability in this article means the extraordinary accuracy in PSIS. It is obviously different from the FXTS, the PDTS, and the PSTS.

### B. Comparison of Control Structures

Consider the following single-integrator system:

$$\dot{x} = u, x(t_0) = x_0 > 0, t_0 = 0. \quad (4)$$

1) Traditional finite-time control [3]

- $u_{\text{FNTS}} = -\eta_1 x^{1-1/\eta_1}, \eta_1 > 0, \eta_2 \in (1, \infty)$
- $T(x_0) = \frac{\eta_2 x_0^{1/\eta_2}}{\eta_1}$.

2) Fixed-time control [8]

- $u_{\text{FXTS}} = -x^{\eta_3} - x^{\eta_2}, \eta_3 \in (0, 1), \eta_4 \in (1, \infty)$
- $T(x_0) \leq T_{\max} = \frac{1}{(1 - \eta_3)} + \frac{1}{(\eta_4 - 1)}$.

3) Predefined-time control [10]

- $u_{\text{PDTS}} = -\eta_5 \frac{T_f}{T_f - t} x^{1-1/\eta_5}, \eta_5 \in (1, \infty)$
- $\sup_{x(t_0) \in \mathbb{R}} T(x_0) = T_f$.

4) Prescribed-time control [17]

- $u_{\text{PSTS}} = \begin{cases} -\eta_6 \frac{T_f}{T_f - t}, & 0 \leq t < T_f \\ 0, & T_f \leq t. \end{cases}$
- $T(x_0) \leq T_f$.

**Remark 5:** It is interesting to notice that the solution of the system in (4) under the control $u_{\text{PSTS}}$ is $x = (T_f - t)^{\eta_6}$, which is also the solution of

$$\dot{x} = u = -\eta_6 \frac{1}{T_f} x^{1-1/\eta_6}.$$

One can see that the prescribed-time control is very similar to predefined-time control. Moreover, taking $([\eta_6] / [T_f]) x_0^{(1/\eta_6) - 1}$ as $\eta_1$ shows the unification of the PSTS and the FNTS. However, these methods show different characteristics for high-order systems.

### C. Problem Statement

It is known for all that an SISO nonlinear system with a relative degree $n$ can usually be transformed to the $n$-order chain of integrators by utilizing the input-output linearization method ([31], Ch. 13, Sec. 2]). Hence, let us consider the following $n$-order chain of integrators:

$$\begin{align*}
\dot{x}_i &= x_{i+1}, \quad i = 1, \ldots, n-1 \\
\dot{x}_n &= u.
\end{align*} \quad (5)$$

For a designed time-varying control $u(x, t, \eta)$, the system (5) belongs to the form in (1). Our goal is to:

1) design a continuous control $u(x, t, \eta)$ goes to zero at a selected instant $T_f$;
2) ensure that the equilibrium of the system in (5) is PSIS.

### III. MAIN RESULTS

In this section, the method to design the PSIS controller for the system (5) is presented. Before that, some new time-varying feedback functions for developing the presented controller, are introduced. One can consider them as the reference convergence rate. It can be easy to obtain these functions according to some criteria given later, but the main problem to be solved is to ensure the system states convergence according to the selected functions.

#### A. Time-Varying Feedback Functions

**Definition 5:** Consider a differentiable function $\phi(s) \in \mathbb{K}, \mathbb{K}_\infty$ (the class $\mathbb{K}$ or $\mathbb{K}_\infty$ function [31]), and $s = \text{arc} \phi(s)$ (the existence of inverse function is guaranteed by the monotonicity of $\phi$). Suppose that $\phi$ satisfies the following criteria.

1) $\phi(s) \sim s$ as $s \to 0$, that is, $\lim_{s \to 0} ([\phi(s)]/s) = 1$.
2) $\phi(s) = \beta(\phi(s))$.

Let $s(t) = (T_f - t)^\eta, x(t) = \phi(s(t)) = \phi((T_f - t)^\eta), t \in [0, T_f], \eta > 1$. We have

$$\frac{dx}{dt} = \frac{d\phi}{ds} \frac{ds}{dt} = -\frac{\eta \beta(\phi(s)) \text{arc} \phi(s)}{t_f - t} = -\xi(x) \leq -\psi(x, t).$$

Then, $\phi((T_f - t)^\eta)$ is defined as a reference convergence function (RCF), while $\psi(x, t)$ is defined as a reference convergence differential function (RCDF).

**Remark 6:** The defined RCDFs above are exactly our desired time-varying feedback functions. According to Definition 5, $\phi$ monotonously goes to zero as $s$ goes to zero, thus $\psi(\phi, t)$ and $\xi(x)$ have the same sign with $\phi$ and will not be zero unless $\phi$ is zero. When $x \to 0$, that is, $\phi(s) \to 0$, we have $\beta(\phi(s)) \to 1$ and $\text{arc} \phi(s) = s \sim x$. Therefore, $\xi(x) = O(x)$ (infinitesimal of the same order). It is also noted that

$$\lim_{t \to T_f} \phi((T_f - t)^\eta) = \lim_{t \to T_f} \phi((T_f - t)^\eta) - 0 \sim (T_f - t)^{\eta-1} \forall \eta > 1.$$
Moreover, \( \psi(\phi, t) \) should be designed to be symmetrical (same properties for positive and negative \( \phi \)).

For the first-order system \( \dot{x} = u \), if \( u \) is designed as \(-\psi(x, t)\), then both \( u(t) \) and \( x(t) \) will converge to zero at \( t = t_f \). Let us see some examples on Definition 5.

**Example 1:** Notice that \( x(t) = \tan((t_f - t)\eta) \) (an RCF) will converge to zero at \( t_f \), we obtain the following RCDF:

\[
\dot{x} = -\frac{\eta}{\cos^2((t_f - t)\eta)} \frac{(t_f - t)^\eta}{t_f - t} = -\eta \frac{x^2 + 1}{t_f - t} \arctan(x). \tag{6}
\]

One can also design analogous RCDFs according to the thought obtaining the RCDF in (6), for example

\[
\begin{align*}
x &= \text{sh}((t_f - t)\eta) = \frac{e^{(t_f - t)\eta} - e^{-(t_f - t)\eta}}{2} \\
\dot{x} &= -\eta \sqrt{1 + x^2} \ln(1 + \sqrt{1 + x^2}) / (t_f - t).
\end{align*}
\]

Consider the following three typical RCFs (\( \phi \)) with different convergence rates:

\[
\begin{align*}
x_1 &= \tan((t_f - t)\eta) \\
x_2 &= (t_f - t)\eta \\
x_3 &= \ln(1 + (t_f - t)\eta).
\end{align*}
\]

The corresponding RCDFs (\( \psi \)) are

\[
\begin{align*}
\dot{x}_1 &= u_{x_1} = -\frac{\eta(x_1^2 + 1)}{t_f - t} \arctan(x_1) \tag{7} \\
\dot{x}_2 &= u_{x_2} = -\frac{\eta x_2}{t_f - t} \tag{8} \\
\dot{x}_3 &= u_{x_3} = -\frac{\eta(1 - e^{-|x_3|})}{t_f - t} \text{sign}(x_3). \tag{9}
\end{align*}
\]

It is interesting to notice that the control in (8) is exactly that of PSTS with a single-integrator system. However, the control for high-order integrator systems will be different. Fig. 1 shows the convergence of the states to the origin and the control variables corresponding to the RCDFs in (7)–(9), where \( t_f = 5, x(0) = 3.5, \) and \( \eta = 2 \).

**B. Controller Design**

Consider the system (5). The proposed RCDFs such as in (7)–(9) are utilized as time-varying feedback functions. The initial condition is chosen as \( x_{1,d} = 0 \) (or other constants) and \( x_{2,d} = -\psi_1(\sigma_1, t) \). The recurrence relation \( i \geq 2 \) is

\[
x_{i+1,d} = \dot{x}_{i,d} - \sigma_i - \psi_i(\sigma_i, t) \tag{10}
\]

where \( \sigma_i = x_i - x_{i,d} \), and \( \psi_i \) belongs to the RCDFs. To avoid divergence to infinity or discontinuity of the control law at \( t = t_f \), we select each parameter \( \eta_i \) of function \( \psi_i \) as \( \eta_i > n + 1 - i, i = 1, 2, \ldots, n \). The controller for the system (5) is

\[
u_n = \begin{cases} 
  x_{n+1,d}, & 0 \leq t < t_f \\
  0, & t_f \leq t
\end{cases} \tag{11}
\]

**C. Prescribed-Instant Stability**

In this section, we will directly study the character of the differential equations in the system (12), and illustrate the PSIS of the system (5) under the control in (11). Several claims and lemmas are introduced for developing our main results.

**Claim 1:** Consider the following differential equation:

\[
\dot{\sigma} = \sigma - \psi(\sigma, t) \tag{13}
\]

where \( \psi \) is an RCF. Then, \( \sigma \) reaches zero at \( t = t_f \).

**Proof:** Consider a positive initial value \( \sigma(t_0) \) for the symmetry of the equation in (13). According to the Comparison Lemma [31], \( \sigma \) is lower bounded by \( \phi(t) \) corresponding to the selected \( \psi \) and \( t_0 \geq t_f \), as an illustration in Fig. 2. Note...
that $\psi \triangleq ([\zeta(\sigma)]/[t_f - t])$. Reduction to absurdity is utilized as follows. We assume $t_a > t_f$, that is, $\sigma(t_f) = \delta > 0$.

Then, the following result can be calculated:

$$\sigma(t_f) = \sigma(t_0) + \int_{t_0}^{t_f} \sigma(t) \, dt - \int_{t_0}^{t_f} \zeta(\sigma) \, dt$$

$$= k - \int_{t_0}^{t_f} \frac{\zeta(\sigma)}{t_f - t} \, dt$$

where $k$ is a constant obtained by the first two terms. According to Definition 5 and Remark 6, $\delta \leq \sigma(t) \leq \sigma_0$ leads to $m \leq \zeta(\sigma) \leq M$, where $m$ and $M$ are some positive constants. We have

$$\int_{t_0}^{t_f} \frac{m}{t_f - t} \, dt \leq \int_{t_0}^{t_f} \frac{\zeta(\sigma)}{t_f - t} \, dt \leq \int_{t_0}^{t_f} \frac{M}{t_f - t} \, dt.$$    

Let $\tau = t_f - t$. Then, we obtain that

$$\int_{t_0}^{t_f} \frac{m}{t_f - t} \, dt = \int_{t_0}^{t_f} \frac{m}{\tau} \, d\tau = +\infty$$

$$\int_{t_0}^{t_f} \frac{M}{t_f - t} \, dt = \int_{t_0}^{t_f} \frac{M}{\tau} \, d\tau = +\infty.$$    

Therefore, by the Squeeze Theorem, we have $\int_{t_0}^{t_f} ([\zeta(\sigma)]/[t_f - t]) \, dt = +\infty$, which leads to $\sigma(t_f) = -\infty$. This conflicts with $\sigma(t_f) = \delta > 0$. Hence, we can draw the conclusion that $\sigma(t_f) = 0$, that is, $\sigma$ reaches zero at $t = t_f$. $\blacksquare$

Claim 2: Consider the following differential equation:

$$\dot{\sigma} = -\sigma - \psi(\sigma, t)$$  \hspace{1cm} (14)

where $\psi$ is an RCDF. Then, $\sigma$ reaches zero at $t = t_f$.

Proof: We consider a positive initial value $\sigma(t_0)$ for the symmetry of the equation in (14). According to the Comparison Lemma, $\sigma$ is limited by $\phi(t)$ that corresponds to the selected $\psi$ and $t_a \leq t_f$, as illustrated in Fig. 3(a). Reduction to absurdity is utilized as follows. Let us suppose $t_a < t_f$.

By overturning the coordinate axis in Fig. 3(a), we have the coordinate axis in Fig. 3(b). The corresponding differential equation in (14) is transformed to

$$\frac{d\tau}{d\sigma} = -\frac{1}{\sigma + \psi}.$$  \hspace{1cm} (15)

We have

$$t_a = -\int_{\sigma_0}^{\sigma(t_0)} \frac{1}{\sigma + \psi} \, d\sigma = \int_{0}^{\sigma_0} \frac{1}{\sigma + \psi} \, d\sigma.$$  \hspace{1cm} (16)

Notice that $\sigma$ tends to zero as $t$ tends to $t_a$. According to Definition 5 and Remark 6, we have $\psi(\sigma(t_a), t_a) = ((\zeta(\sigma(t_a)))/[t_f - t_a]) = O(\sigma(t_a))$ (infinitesimal of the same order). Therefore, $t_a = +\infty$, which is a conflict with $t_a < t_f$. The conclusion is drawn that $\sigma$ reaches zero at $t = t_f$. $\blacksquare$

In analogy with Claims 1 and 2, Lemmas 1 and 2 are introduced with the proof in the Appendix.

Lemma 1: Consider the following differential equation:

$$\dot{\sigma} = b(t) - \psi(\sigma, t)$$  \hspace{1cm} (17)

where $\psi$ is an RCDF. If $b(t)$ is bounded but never stabilizes at zero until $t = t_f$, then $\sigma$ reaches zero at $t = t_f$.

Lemma 2: Consider the following differential equation:

$$\dot{\sigma} = b(t) - \sigma - \psi(\sigma, t)$$

where $\psi$ is an RCDF. If $b(t)$ is bounded but never stabilizes at zero until $t = t_f$, then $\sigma$ reaches zero at $t = t_f$.

Theorem 1: Consider the system (5) starting from any initial condition of $x$. The controller (11) drives the trajectory of the system (5) to the equilibrium $x = 0$ at the selected instant $t = t_f$. Moreover, the continuous control $u(t)$ also gradually goes to zero at the instant $t = t_f$ and ensures $x(t) = u(t) = 0$ $\forall t \geq t_f$. In another word, the equilibrium $x = 0$ is PSIS.

Proof: As shown in the last section, the controller (11) transforms the system (5) to

$$\begin{cases} \dot{\sigma}_1 = \sigma_2 - \psi_1(\sigma_1, t) \\ \dot{\sigma}_i = \sigma_{i+1} - \sigma_i - \psi_i(\sigma_i, t), i = 2, \ldots, n-1 \\ \dot{\sigma}_n = -\sigma_n - \psi_n(\sigma_n, t) \end{cases}$$

Consider the last differential equation in (18). According to Claim 2, $\sigma_n$ monotonously goes to zero at $t = t_f$. In another word, $\sigma_n$ can be taken as $b(t)$, that is

$$\dot{\sigma}_{n-1} = b(t) - \sigma_{n-1} - \psi_{n-1}(\sigma_{n-1}, t).$$

According to Lemma 2, $\sigma_{n-1}$ will go to zero at $t = t_f$. Therefore, $\sigma_{n-2}, \ldots, \sigma_2$ can be recursively proved to go to
zero at the instant \( t = t_f \). Finally, taking \( \sigma_2 \) in the first equation in (18) as \( b(t) \), the convergence of \( \sigma_1 \) is a direct result of Lemma 1. Therefore, \( \sigma \) is PSIS at \( t = t_f \) and leads to 
\[
 x_i(t_f) = \sigma_i(t_f) + x_{i,d}(t_f) = x_{i,d}(t_f).
\]

For the system (5), combining (10) and (11), the relation between the states and the detailed control variables \( u_i \) is presented as follows:
\[
\begin{align*}
 x_1 &= x_{1,d} = 0 \\
 x_2 &= x_{2,d} = -\psi_1 = u_1 \\
 x_3 &= x_{3,d} = -\psi_1 - \sigma_2 - \psi_2 = u_2 \\
 x_4 &= x_{4,d} = -\psi_1 - \sigma_2 - \psi_2 - \sigma_3 - \psi_3 = u_3 \\
 &\vdots \\
 x_{n+1} &= x_{n+1,d} = \dot{x}_{n,d} - \sigma_n - \psi_n = u_n
\end{align*}
\]

where each equation is formed by \( \psi_i, \sigma_i \), and their derivatives.
It is noted that the derivative order of \( \psi_1 \) is the highest among the equations. For \( u_{nom} \), the derivative order of \( \psi_1 \) is \( n-1 \) and the derivative order of \( \psi_i \) is \( n-i \). According to Definition 5 and Remark 6, when \( t \) tends to \( t_f \), we have \( \psi \sim (t_f - t)^{n-1} \) and \( \psi^{(n-1)} \sim (t_f - t)^{n-2} \). As mentioned in Section III-B, for each \( \psi_i \), the corresponding parameter is selected as \( \eta_i > n+1-i \), which ensures that the derivative of \( \psi_i \) tends to zero. Therefore, the proposed control (11) is bounded as \( t \) tends to \( t_f \). Combining the PSIS of \( \sigma \), we obtain that the trajectory of the system (5) converges to the equilibrium \( x = 0 \) at the instant \( t = t_f \). The state variable will thus tend to infinity as \( t \) tends to \( t_f \). Hence, system (5) is PSIS. This completes the proof.

Remark 7: The selection of \( \eta_i \) is very important to ensure the control \( u \) continuous and avoid singularity problems. In fact, the specific form of both PSTS and PSIS controllers contains \( x/[\alpha (t_f - t)^{\alpha}] \). As \( t \to t_f \), \( (t_f - t)^{\alpha} \) is infinitesimal, it is necessary to guarantee \( x \) is an infinitesimal of higher order. Because of the special form of our proposed PSIS controller, this feature is easy to obtain by adjusting \( \eta_i \). However, it is a more complex problem in the PSTS. This is the reason that the singularity problem exists in conventional PSTS. A more detailed analysis is presented in the following.

The following conversion is usually utilized in the concept of the traditional PSTS: \( \forall t \in [t_0, t_f] \):
\[
\begin{align*}
 w_1 &= \mu(t - t_0)x_1(t) = \frac{x_1(t)}{(t_f - t)^{\alpha+1}} \\
 w_i &= \frac{dw_{i-1}}{dt}, i = 2, \ldots, n - 1 \\
 w_n &= \frac{dw_n}{dt}.
\end{align*}
\]

This conversion makes sense only if
\[
\lim_{t \to t_f} x_1 \sim \lim_{t \to t_f} (t_f - t)^{\alpha+1}, \quad \Delta > n - 1.
\]

However, sometimes one can only obtain
\[
\sum_{i=1}^{n} |x_i| \leq (t_f - t)^{m+1} + p e^{-q(t-t_0)}
\]

where \( p \) and \( q \) are some positive constants. There is some possibility that \( \lim_{t \to t_f} x_1 \sim \lim_{t \to t_f} (t_f - t)^{m+1} \), and
\[
\lim_{t \to t_f} w_1 = \lim_{t \to t_f} \frac{x_1}{(t_f - t)^{m+1}} \sim \lim_{t \to t_f} \frac{1}{(t_f - t)^{n-1}} = \infty \quad \forall n > 1.
\]

The derivatives of \( w_1 \) is also infinity, that is, \( \lim_{t \to t_f} w_1 = \infty \). When the value of \( w_1 \) is utilized in the controller, the control variable will thus tend to infinity as \( t \to t_f \).

D. Controller Design With Matched Disturbance Rejection

Consider the n-order chain of integrators with disturbance
\[
\begin{align*}
 \dot{x}_1 &= x_{i+1}, i = 1, \ldots, n - 1 \\
 \dot{x}_n &= u + d(t)
\end{align*}
\]

where \( |d(t)| \leq C_1 \) and \( |\dot{d}(t)| \leq C_2 \). Sliding-mode control is applied to restrain the effect of \( d(t) \). Let the control \( u = u_{nom} + u_{disc} \), where \( u_{nom} \) is the proposed controller in (11) and \( u_{disc} = -k_1 |s|^{1/2} \text{sign}(s_1) - k_2 \int_0^t \text{sign}(s_1) \, dt \) is the disturbance restrain control, where \( k_1 \) and \( k_2 \) are positive constants [32]. The sliding surface is
\[
s_1 = x_n - x_{n0} - \int_0^t u_{nom} \, dt.
\]

Its time derivative is given by
\[
\dot{s}_1 = -k_1 |s|^{1/2} \text{sign}(s_1) + s_2, \quad k_1^2 \geq 4C_2 \frac{k_2 + C_2}{k_2 - C_2}
\]

The trajectory is initially on the sliding surface and the disturbance is to be rejected right from the beginning. In another word, the system is governed by \( u_{nom} \) and acts as the prescribed-instant stability.

IV. SIMULATIONS

A. Comparison Between PSIS and PSTS

According to Definition 4, the PSIS ensures \( T(x_0) = T_f \forall x_0 \in \mathbb{R}^m \). In another word, the settling time is independent of \( x_0 \). The abovementioned properties are validated hereby a double integrator system
\[
\begin{align*}
 \dot{x}_1 &= x_2 \\
 \dot{x}_2 &= u.
\end{align*}
\]

We choose \( t_f = 1 \) s and three different sets of initial values: Set 1: \( x_1(0) = 1, x_2(0) = -3 \); Set 2: \( x_1(0) = 2, x_2(0) = -1 \); and Set 3: \( x_1(0) = 3, x_2(0) = 1 \). Our control object is to stabilize \( x \) to zero. The prescribed time is selected to be \( t_f = 1 \) s.

First, we use the PSTS controller of [14, Th. 2] and the same parameters to stabilize this system. The performance of the PSTS controller is shown in Fig. 4. The system state is stabilized before \( t = 1 \) s, which is different from PSIS. Moreover, a singularity of the control occurred before \( t = 1 \) s, which can be observed from Fig. 4(c) that the control tends to infinity. Because the stabilization has already been established before \( t = t_f \), some methods could be used to make PSTS practical.

In practice, the singularity problem of conventional PSTS can be avoided by employing a dead zone on \( x \), so the regulation is not zero but a small neighborhood. Another way is to set \( t_f \) to a larger value than the desired finite time of regulation. This way also prevents the control from becoming infinite over the desired regulation time but, again, with some sacrifice on the regulation accuracy [14]. Because neither of these ways is utilized in this simulation, so the simulation gets
The integrator system. The detailed RCDFs are selected as follows:

\[ \psi_1 = \frac{\eta_1 \sigma_1}{\tau_1}, \quad \eta_1 = 3 \]
\[ \psi_2 = \frac{\eta_2 \sigma_2}{\tau_2}, \quad \eta_2 = 2. \]

The simulation results are depicted in Fig. 5. It is obvious that the control tends to zero at the prescribed instant \( t = 1 \). Meanwhile, \( z \) is stabilized to zero at \( t = 1 \). On the one hand, the singularity problem is avoided in the PSIS, which is different from traditional PSTS. On the other hand, the actual convergence time equals exactly the prescribed time instant, which has never been achieved in FXTS, PDTS, and PSTS.

**B. Position Control of a Magnetic Suspension System With Matched Disturbance**

Consider the magnetic suspension [31]. The vertical (downward) position of the ball measured from a reference point is denoted as \( z_1 \), which is also the output. \( z_2 \) denotes the speed of the ball and \( z_3 \) denotes the current of the electromagnet. The state equations are

\[
\begin{align*}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= g - \frac{k}{m}z_2 - \frac{L_0 \sigma_2^2}{2m(a+z_1)^2}
+ \frac{1}{L_1 \tau_1} \left[ -Rz_3 + \frac{L_0 \sigma_2^2}{2m(a+z_1)^2} + v \right]
\end{align*}
\]

where the voltage \( v \) is the control input. We assume \( m = 0.1 \text{ kg}, k = 0.001 \text{ N/m/s}, g = 9.81 \text{ m/s}^2, a = 0.05 \text{ m}, \)

\[ L_0 = 0.01 \text{ H}, L_1 = 0.02 \text{ H}, \text{ and } R = 1 \text{ \Omega}. \]

According to the feedback linearization, we choose

\[
\begin{align*}
x_1 &= y = z_1 \\
x_2 &= \dot{y} = z_2 \\
x_3 &= \ddot{y} = g - \frac{k}{m}z_2 - \frac{L_0 \sigma_2^2}{2m(a+z_1)^2}.
\end{align*}
\]

Let the control [voltage \( v \) (V)] be

\[ v = Rz_3 + \frac{L_0 \sigma_2^2}{a+z_1} - \frac{m(a+z_1)^2}{ak} \left( \frac{a}{m} z_3 + u \right). \]

Then, we obtain that

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= u + d(t)
\end{align*}
\]

where \( d(t) = 2 + 0.5 \sin(7t) \) represents the matched disturbances and/or perturbations. The controller is designed according to Section III-D. The parameters are selected as \( k_1 = 48 \) and \( k_2 = 1100 \). Assume the initial values \( x_1(0) = z_1(0) = 0.09 \text{ m}, x_2(0) = z_2(0) = -0.03 \text{ m/s}, \) and \( z_3(0) = -6.6 \text{ A}. \) Our goal is to stabilize \( x_1 = z_1 = 0.05 \text{ m} \) at \( t_f = 0.5 \text{ s}, \) that is, \( x_{1,d} = 0.05 \text{ m}. \) We replace the variable \( x_1 \) by \( \sigma_1 = x_1 - x_{1,d} \) in the controller presented in (10) and (11). Specifically, \( \psi_i \) is chosen as follows:

\[
\begin{align*}
\psi_1 &= \frac{-\eta_1 \sigma_1}{\eta - \tau_2}, \quad \eta_1 = 5 \\
\psi_2 &= \frac{-\eta_2 \sigma_2}{\eta - \tau_2}, \quad \eta_2 = 4 \\
\psi_3 &= \frac{-\eta_3 (\sigma_2 + 1) \arctan(\sigma_1)}{\eta - \tau_2}, \quad \eta_3 = 2.5.
\end{align*}
\]
Fig. 6. States and control of the magnetic suspension system. (a) Ball position, ball speed, electromagnet current and the desired ball position. (b) Virtual control and the real control.

Fig. 7. Sketch map of case 1 for Lemma 1.

Fig. 6(a) shows the states of the magnetic suspension system and Fig. 6(b) shows the control variables. The initial value of the real controller \( v \) is about \(-13 \, \text{V} \). Moreover, the area zoomed in on the solid line in Fig. 6(b) shows the sliding-mode property of the controller. Due to the additive disturbance \( d(t) \), the tracking error of states is impossible to be suppressed to zero at \( t = t_f \), and a small perturbation is unavoidable. Because this error is ensured to be infinitesimal of the same order of \( t_f - t \), so \( u \) is bounded and no large perturbations of \( u \) occur at \( t = 0.5 \, \text{s} \).

Remark 8: Although the convergence time can be arbitrarily chosen, the magnitude of the control will inevitably be very large when the values of the initial condition are very large or the regulation time is set to be very small. This is unexpected in practice [18]. Hence, the saturation problem of the control variables will be considered qualitatively in our following study.

V. CONCLUSION

This article solves the problem of stabilizing arbitrary-order integrator systems at a prescribed time instant. To this end, the method of PSIS, in which the settling time (not the bound of settling time) is not only independent of the initial value of system states but also can be manipulated as per our will, has been presented. The convergence rate is also designable by selecting proper time-varying feedback functions, which are easy to be obtained according to the presented criteria. Specifically, by using our proposed recursive controller, the original dynamic system is transformed into some differential equations. We directly considered the feature of the differential equations, and then reduction to absurdity, instead of the Lyapunov method, has been applied to prove the prescribed-instant stability. In addition to mathematical proof, the control scheme has also been verified by numerical simulations. This control method would be of great importance in control systems with the requirement of precise regulating time for industrial and engineering applications. Considering the capability of a real controller, the saturation problem of control signals will be studied in our future study.

APPENDIX

Proof of Lemma 1: Set \( t_b \) the last time when \( b(t) \) passed through zero \( (t_b = t_0 \) if \( b(t) \) never passes through zero). \( b(t) \) may have the same or opposite sign with \( \sigma(t_b) \). We consider a positive initial value \( \sigma(t_b) \). Then, \( b(t) \) with \( t \in (t_b, t_f) \) could be cased by positive or negative. It is obvious that \( \sigma \) will converge to zero at \( t_f \) if \( \dot{\sigma} = -\psi(\sigma, t) \). Now, let us consider

\[
\dot{\sigma} = b(t) - \psi(\sigma, t). \tag{22}
\]

Case 1: \( b(t) > 0 \ \forall t \in (t_b, t_f) \), of which an illustration is shown in Fig. 7. According to the Comparison Lemma, the real convergence time of the equation in (22) would be \( t_a \geq t_f \). Reduction to absurdity is utilized as follows.

Assume that \( t_a > t_f \), that is, \( \sigma(t_f) = \dot{\sigma} > 0 \). Based on the form of \( \psi \) and the fact that \( b(t) \) is bounded, \( \exists \delta, \text{s.t.} \ \delta(t) < 0, t \in (t_1, t_f) \). Taking \( t_2 \geq \max(t_1, t_b) \), and \( \sigma(t_2) = \sigma_2 \), we have

\[
\sigma(t_f) = \sigma_2 + \int_{t_2}^{t_f} b(t)\,dt - \int_{t_2}^{t_f} \frac{\xi(\sigma)}{t_f - t}\,dt. \tag{23}
\]

Utilizing the mean value theorem of integrals, we have

\[
\sigma(t_f) = k - \int_{t_2}^{t_f} \frac{\xi(\sigma)}{t_f - t}\,dt
\]

where \( k \) is a constant. As shown in the proof of Claim 1, one can obtain \( \sigma(t_f) = -\infty \), which conflicts with the assumption that \( \sigma(t_f) = \dot{\sigma} > 0 \). Therefore, \( \sigma(t_f) = 0, t_a = t_f \).

Case 2: \( b(t) < 0 \ \forall t \in (t_b, t_f) \), of which an illustration is shown in Fig. 8. There exist three subcases.

1) The value of \( b(t) \) cannot drive \( \sigma \) to be negative or zero before \( t_f \). According to the sign of \( b(t) \), \( \sigma \) will tend to
zero at or before the instant $t_f$ (due to the Comparison Lemma). So, $\sigma$ reaches zero at the instant $t_f$.

2) The value of $|b(t)|$ is large enough and drives $\sigma$ to be zero at $t_m \in (\max(t_1, t_2), t_f)$. Because $b(t) < 0, t \in (t_2, t_f)$, $\sigma$ will go across zero and it turns to the subcase 3).

3) The value of $|b(t)|$ is large enough and drives $\sigma$ to be negative at $t_m \in (\max(t_1, t_2), t_f)$. Then, $b(t)$ and $\sigma$ have the same sign and the proof on $(t_m, t_f)$ is similar to case 1.

Combining cases 1 and 2, we know that $\sigma$ will reach zero at the instant $t_f$.

**Proof of Lemma 2:** The proof imitates that of Lemma 1 and the differential (14) is taken as a reference differential equation to utilize the Comparison Lemma. The only difference is that (23) is replaced by

$$\sigma(t_f) = \sigma_2 + \int_{t_2}^{t_f} b(t)dt - \int_{t_2}^{t_f} \sigma(t)dt - \int_{t_2}^{t_f} \xi(\sigma(t))dt.$$ 

Then, the detailed proof can be easily completed by following that of Lemma 1 and is thus omitted.

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Jiyuan Kuang received the B.M. degree in control science and engineering and the M.E. degree in power electronics and power drives from Shandong University, Jinan, China, in 2016 and 2019, respectively. He is currently pursuing the Ph.D. degree in control science and engineering with the Harbin Institute of Technology, Harbin, China.

His current research interests include prescribed time stability, multiagent system consensus, and fuel cell systems control.

Yabin Gao (Member, IEEE) received the B.M. degree in information management and information system and the M.E. degree in software engineering from Bohai University, Jinzhou, China, in 2012 and 2015, respectively, and the Ph.D. degree in control science and engineering from the Harbin Institute of Technology, Harbin, China, in 2020.

He is currently a Lecturer with the Department of Control Science and Engineering, Harbin Institute of Technology. From October 2017 to October 2019, he was a Visiting Scholar with the Department of Mechanical Engineering, University of Victoria, Victoria, BC, Canada. His current research interests include sliding-mode control, intelligent control, robust filtering, fault detection, cyber-physical systems, and their applications.

Chih-Chiang Chen (Member, IEEE) received the Ph.D. degree in control theory from National Chiao Tung University, Hsinchu, Taiwan, in 2017.

From 2015 to 2016, he was a Visiting Scholar with the Department of Electrical and Computer Engineering, University of Texas at San Antonio, San Antonio, TX, USA. Since August 2017, he has been with the Department of Systems and Naval Mechatronic Engineering, National Cheng Kung University, Tainan City, Taiwan, where he is currently an Assistant Professor. His current research interests include nonlinear control, dynamic systems, and homogeneous system theory.

Xiaoju Zhang received the B.M. degree in mathematics and applied mathematics from Zhengzhou University, Zhengzhou, China, in 2011, and the M.E. degree in applied mathematics from the University of Chinese Academy of Sciences, Beijing, China, in 2014. He is currently pursuing the Ph.D. degree in mathematics with the Harbin Institute of Technology, Harbin, China.

His current research interests include existence, uniqueness, asymptotic behavior, and blow-up of solutions for nonlinear fractional partial differential equations.

Yizhuo Sun received the B.M. degree in automation from the Hefei University of Technology, Xuancheng, China, in 2016, and the M.E. degree in control theory and control engineering from the Harbin Institute of Technology, Harbin, China, in 2019. He is currently pursuing the Ph.D. degree with the Harbin Institute of Technology, Harbin, China.

His current research interests include sliding-mode control, robotics control, and MPC.

Jianxing Liu (Senior Member, IEEE) received the B.S. degree in mechanical engineering and the M.E. degree in control science and engineering from the Harbin Institute of Technology, Harbin, China, in 2004 and 2010, respectively, and the Ph.D. degree in automation from the Technical University of Belfort-Montbéliard, Belfort, France, in 2014.

Since 2014, he has been with the Harbin Institute of Technology, where he is currently a Professor with the Department of Control Science and Engineering. His current research interests include sliding-mode control, nonlinear control and observation, industrial electronics, and renewable energy solutions.

Prof. Liu is currently serving as an Associate Editor for a number of journals, including IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS—I: EXPRESS BRIEFS, IEEE SYSTEMS JOURNAL, and Nonlinear Dynamics.