Underscreened Kondo Necklace

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Abstract

It has been suggested recently by Gan, Coleman, and Andrei that studying the underscreened Kondo problem may help to understand the nature of magnetism in heavy fermion systems. Motivated by Doniach’s work on the $S = 1/2$ Kondo necklace, we introduce the underscreened Kondo necklace models with $S > 1/2$. The underscreened Kondo necklace is the simplest lattice model on which the competition between Kondo spin compensation, and magnetic ordering due to an RKKY–type interaction can be examined. We used the mean–field approximation to determine the phase diagram, and found that the low-temperature phase is always an $x – y$ antiferromagnet. This contention is further supported by the derivation of the exact form of the effective Hamiltonian in the limit of very large Kondo coupling: it is found to be an antiferromagnetic $x – y$ model for the residual $S – 1/2$ spins. In general, the degree of moment compensation depends on both the Kondo coupling, and on $S$.

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1 Introduction

The description of the magnetically ordered states of heavy fermionic systems is a problem of great current interest \[1\]. It is usual to find that the ordered moment is rather smaller, and in quite a few cases much smaller, than what would correspond to isolated \(4f\) (or \(5f\)) shells. On the other hand, at elevated temperatures, the paramagnetic susceptibility is the one expected for isolated \(f\)-shells in a crystal field. The phenomenon of reduced moment magnetism at low temperatures is supposed to result from the competition of the Kondo effect, and RKKY interaction \[2\]. The Kondo effect strives to set up a state in which the spins of \(f\)-electrons are compensated by those of conduction electrons; the RKKY interaction tends to order the localized moments, preventing the formation of an overall singlet state. The outcome of this competition can be apparently anything between the extremes of a conventional RKKY magnet with well-defined localized moments, and a non-magnetic heavy Fermi liquid. We can envisage this by saying that the Kondo effect progresses up to a point, reducing the moments to a fraction of the ionic value, and then the residual moments get ordered. — The description of this situation is greatly complicated by the fact that the Kondo compensation clouds corresponding to nearby ions strongly overlap \[3, 4\]: there is a continuing controversy as to whether the collective Kondo effect of the lattice has an energy scale essentially different from the single ion Kondo energy.

The treatment of the competition between magnetic and non-magnetic states is apparently quite difficult in the case of models with the the feature of perfect screening: \(2S = k\), where \(k\) is the number of screening channels, and \(S\) is the localized spin. As demonstrated in detail \[3\] for the case \(S = 1/2\), this perfect balance allows the construction of a strictly non-magnetic heavy
fermi liquid; it remains an open question whether such a non–magnetic state is necessarily unstable against magnetism and/or superconductivity.

It has been suggested recently by Gan, Coleman, and Andrei that one should be able to gain an insight into the nature of heavy fermion magnetism by studying the underscreened Kondo problem with $2S > k$. The single–ion problem is solved exactly, demonstrating that in the ground state, a partially compensated spin $S - k/2$ remains. The underscreened Kondo lattice (in standard notations)

\[ H_{KL} = \sum_{\mathbf{k}\alpha} \epsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^+ c_{\mathbf{k}\alpha} + \frac{J}{L} \sum_{\alpha,\beta} \sum_{\mathbf{k}\mathbf{k}' j} e^{i(k-k')j} c_{\mathbf{k}\alpha}^+ \sigma_{\alpha\beta} c_{\mathbf{k}'\beta} \cdot \mathbf{S}_j \]

with an orbitally non–degenerate conduction band ($k = 1$), and $S > 1/2$, seems destined to become magnetic (near half-filling, presumably antiferromagnetic), but nothing specific is known yet.

Recalling that in the perfectly screened case $k = 1$, $S = 1/2$, the simplest solution of the Kondo necklace model introduced by Doniach has been a useful guide to studying the more complicated problem of the Kondo lattice, we suggest that the introduction of the underscreened Kondo necklace models

\[ H_{KN} = J \sum_{\mathbf{j}} \mathbf{S}_j \cdot \mathbf{\tau}_j + W \sum_{\langle i,j \rangle} (\tau_i^x \tau_j^x + \tau_i^y \tau_j^y) \]

should be helpful as a prelude to a study of the underscreened Kondo lattice.

The great simplification achieved by replacing the Kondo lattice model (1) by the Kondo necklace model (2) is due to the fact that $H_{KL}$ has in it spins and fermions, while $H_{KN}$ is formulated solely in terms of spin operators. The spins $\mathbf{S}$ are the same localized spins as in (1). We consider an arbitrary spin $S$; the underscreened models belong to $S > 1/2$, but to facilitate comparison with the case of exact Kondo screening, we include also results for $S = 1/2$ which belongs to the original necklace model.
The pseudospins $\tau$, with $|\tau| = 1/2$, are meant to represent the spin degrees of freedom of the conduction electrons. Charge degrees of freedom are omitted on the ground that they are likely to belong to higher energies, and are thus considered to be of little relevance for the low-energy world of Kondo physics. Note that one $\tau$–spin for each site should really correspond to a half–filled conduction band. At this special filling, the ground state of $H_{KL}$ is expected to be insulating: either because of Luttinger’s theorem, in case it is non–magnetic; or because of magnetic cell doubling, in case it is an antiferromagnet. In either case, there is a charge gap, providing a further justification for omitting the charge degrees of freedom in $H_{KN}$.

In one dimension (1d), the $x–y$ model of the $\tau$-spins can be mapped by the Jordan–Wigner transformation to a tight-binding model of spinless non-interacting fermions. In this case, it becomes quite apparent that the $x–y$ term in (2) stands for a sea of fermions. However, even in higher dimensions, it is clear that $H_{KN}$ incorporates the competing tendencies of magnetic ordering, and local Kondo singlet formation. This being just the question we are interested in, we do not hesitate to consider the model (2) for lattices of arbitrary dimensionality and structure (even though the term “necklace” becomes a misnomer). — In any case, the simple mean–field treatment which we carry out here, in close parallel with Doniach’s first study [6] of the original necklace model, is really justified only for 3d, and gives at best a qualitative guide as to what to expect in lower dimensions.

The first term in (2), with the antiferromagnetic Kondo coupling $J > 0$, favours the formation of local low-spin states with $|S + \tau| = S − 1/2$. The second term, with $W$ measuring, loosely speaking, the kinetic energy of the spin degrees of freedom of the conduction electron sea, couples nearest–neighbour sites, and thereby mixes in also components from the local $|S + \tau| = S + 1/2$
high–spin states.

In the perfectly screened case $S = 1/2$, Doniach [9] found a ground state phase transition between an antiferromagnetic state at $J < J_{cr} = W$, and a fully Kondo–compensated overall singlet state at $J > W$. The antiferromagnet has gapless spin excitations, while the Kondo state shows a spin gap. — It is, of course, always questionable whether mean–field results can be trusted, and it has been a matter of long debate what the true behaviour should be like. The problem is most subtle in $1d$, for the true “necklace”. Amazingly, the mean–field approximation turns out to be a good guide even in this case. Naturally, true long-range order has to be replaced with quasi–long-range order, but it seems to be true that there is a ground state phase transition between a low–$J$ state which is almost magnetic, with the spin excitation spectrum gapless, and a high–$J$ Kondo state where there exists a spin gap. In finding this, very recent numerical studies [10] corroborate earlier renormalization group results [11], refuting some Monte Carlo work [12] which claimed that the spectrum is gapful for all $J > 0$. Note, however, that for the $1d$ Kondo lattice, it is reasonably expected, and also convincingly demonstrated [13], that the spectrum is always gapful. This indicates that the breaking of the full spin–rotational invariance, which is caused by introducing the $x – y$ form of the intersite coupling in $H_{KN}$, puts the Kondo necklace into a different universality class than the Kondo lattice. — In the present context of the underscreened Kondo necklace model, we will understand this more clearly after deriving the effective Hamiltonian governing the large–$J$ behaviour which turns out to be an antiferromagnetic $x – y$ model of the partially Kondo–screened residual moments.
2 $X-Y$ Type Antiferromagnetism in the Necklace Model

Having thus found sufficient justification for doing simple mean–field theory to get a first impression of the behaviour of the model (2), we proceed to decouple the $x-y$ term, by assuming a $d$–dimensional cubic lattice, and postulating a two–sublattice ($A$ and $B$) antiferromagnetic order of the $\tau$-spins

$$\langle \tau_i^x \rangle = \begin{cases} \ t > 0 & \text{if } i \in A \\ -t & \text{if } i \in B \end{cases}$$

while $\langle \tau_i^y \rangle = 0$ everywhere.

The single–site mean–field hamiltonian is

$$h_{MF} = JS\tau - 2\omega \tau^x$$

where $\omega = Wtd$, and $t = \langle \tau^x \rangle$ is the thermal expectation value of $\tau^x$, which has to be determined self-consistently from

$$t = \frac{\sum_{j=1}^{4S+2} \langle \psi_j | \tau^x | \psi_j \rangle e^{-\epsilon_j/T}}{\sum_{j=1}^{4S+2} e^{-\epsilon_j/T}}$$

Here $\epsilon_j$, and $|\psi_j\rangle$, are the eigenvalues, and eigenstates of $h_{MF}$.

Conveniently chosing the quantization axis as the $x$–axis, the $2(2S + 1)$–dimensional eigenvalue problem immediately separates into $2S$ two–dimensional, and 2 one–dimensional problems. In the subspace $S^x + \tau^x = S - m + 1/2$, where $m = 1, 2, \ldots, 2S$, the matrix of $h_{MF}$ is found to be

$$\begin{pmatrix} \frac{I}{2}(S - m) - \omega & \frac{1}{2}\sqrt{m(2S - m + 1)} \\ \frac{1}{2}\sqrt{m(2S - m + 1)} & -\frac{I}{2}(S - m + 1) + \omega \end{pmatrix}$$

There is no point to writing down the solution of the diagonalization, we proceed straight to the results obtained from solving the self–consistency equation.
The Néel temperature $T_N$ is obtained from solving the linearized self-consistency equation

$$\frac{Wd}{3Z_0(1+2S)}\left\{\frac{1}{T_N}S(2S-1)e^{\frac{JS+1}{2T_N}} + (S+1)(2S+3)e^{-\frac{JS}{2T_N}}\right\} + \frac{1}{J} \frac{16S(S+1)}{1+2S} \left[e^{\frac{JS+1}{2T_N}} - e^{-\frac{JS}{2T_N}}\right] = 1 \quad (7)$$

where

$$Z_0 = 2S e^{\frac{JS+1}{2T_N}} + (S+1)e^{-\frac{JS}{2T_N}} \quad (8)$$

Some representative curves giving the antiferromagnetic-paramagnetic phase boundary in the $J/W - T_N/W$ plane are shown in Fig. 1. At $J=0$, all curves start from the common value $T_N = Wd/2$, the mean-field solution for the $x-y$ model of the $\tau$-spins. This single point is, of course, somewhat pathological, since the decoupled $S$-spins remain completely disordered at all temperatures, including $T = 0$. For all $J > 0$, however, the antiferromagnetism of the $\tau$-spins induces an (oppositely polarized) antiferromagnetic order of the $S$-spins.

As the Kondo coupling reaches the range $J \approx Wd/S$, the states derived from $|S+\tau| = S+1/2$ levels of the single-ion problem are beginning to make a negligible contribution to thermal expectation values, and $T_N$ quickly drops toward its asymptotic value

$$\lim_{J/W \to \infty} T_N = \frac{Wd}{6} \cdot \frac{2S-1}{2S+1} \quad (9)$$

For $S = 1/2$, there is no antiferromagnetism at large $J$'s, in agreement with the solution found by Doniach [4]. However, for all $S \geq 1$, a low-temperature antiferromagnetic phase is predicted, $T_N$ approaching the finite value (9), as $J/W \to \infty$. This arises from the fact the $2S$-fold degenerate ground state set of the Kondo ion reacts with a Curie-like polarization to the transverse field
The argument can be made a bit more formal by deriving the effective Hamiltonian which expresses the action of $H_{KN}$ within the restricted Hilbert space spanned by the $|S + \tau| = S - 1/2$ Kondo ground states. Expressed in the basis $|S^z, \tau^z\rangle$, the relevant single-site states are

$$\phi_{S-m-1/2} = \frac{1}{\sqrt{2S+1}}[\sqrt{2S-m}|S-m, -1/2angle - \sqrt{m+1}|S-m-1, 1/2\rangle]$$ (10)

where $m = 0, 1, \ldots, 2S-1$. It is easy to check that in the Hilbert space spanned by these local basis states, $H_{KN}$ acts like

$$H_{eff}^{J/W\to\infty} = -J\frac{S+1}{2} + \frac{W}{(2S+1)^2} \sum_{i,j} (\eta_i^x \eta_j^z + \eta_i^y \eta_j^y)$$ (11)

where the components of the usual spin operator $\eta$ for $|\eta| = S - 1/2$ appear. Thus in the limit $J/W \to \infty$, the Kondo necklace model becomes equivalent to an antiferromagnetic $x - y$ model of the residual $S - 1/2$-spins, with an effective coupling $W/(2S+1)^2$. Note that this result is rigorous, being perfectly independent of the previous mean-field argument.

It might seem unphysical that $T_N$ approaches a finite value as $J/W \to \infty$, since it looks obvious that it should rather tend to zero. There is, however, no real contradiction. We have to remember that $W$ is an intersite spin–flip matrix element of the conduction electrons, and is thus, in terms of the underlying Kondo lattice model (1), itself dependent on $J$. In particular, at large $J$, the $\tau$–spin–flip process involves breaking up the local Kondo ground states, and thus $W \propto B^2/J$ is expected, where $B$ is the conduction electron bandwidth. Hence (9) actually predicts $T_N \to 0$ as $J/W \to \infty$.

We now return to the mean-field solution, and discuss the low-temperature properties. Let us recall that Doniach’s mean-field solution was actually done...
for $T = 0$, being based on a site-factorized Ansatz for the ground state wave function. Our finite-temperature mean-field approximation should become equivalent to this at $T = 0$, since it relies on the same kind of decoupling. This becomes quite clear from a detailed study of the $S = 1$ case [14] where both formulations were implemented. However, for general $S$, we find it more convenient to use our present formulation.

The solution of the eigenvalue problem of (6) reveals that the ground state is lying in the $m = 2S$ subspace. The $T = 0$ self-consistency equation has no closed-form solution. However, we can carry out series expansions, either for small, or large $J/W$. The large-$J$ expansion up to fourth order gives

$$\langle \tau^x \rangle \approx \frac{12S - 1}{22S + 1} \cdot \left\{ 1 + \frac{16S}{(1 + 2S)^3} \frac{W}{J} - \frac{16S (3 - 28S + 12S^2)}{(1 + 2S)^6} \frac{W^2}{J^2} - \frac{128S (1 - 28S + 136S^2 - 112S^3 + 16S^4)}{(1 + 2S)^9} \frac{W^3}{J^3} - \frac{64S (5 - 290S + 3548S^2 - 12976S^3 + 14192S^4 - 4640S^5 + 320S^6)}{(1 + 2S)^{12}} \frac{W^4}{J^4} \right\} (12)$$

This is, roughly speaking, a power series in $J/WS^2$, i.e., in the dimensionless effective coupling we can identify from (9). The overall factor $2S - 1$ reflects that, as known from Doniach’s early work [8], the magnetic order is suppressed by the formation of Kondo singlets for large $J$’s if $S = 1/2$. On the other hand, no sign of any sudden change in the nature of the ground state is detected for $S \geq 1$.

$\tau^x$ is perfectly valid as an order parameter but we rather prefer using $S^x$, the transverse sublattice magnetization of the $S$-spins. Evaluating this needs no further calculation since, by construction

$$\langle \tau^x + S^x \rangle = \pm (S - 1/2) (13)$$
whereby it has to be observed that \( \text{sg}(S^x) = -\text{sg}(\tau^x) \). With our convention (3), the \( S \)-spin is “up” on the \( B \) sublattice.

The fourth–order small–\( J \) expansion yields

\[
\langle \tau^x \rangle \approx \frac{1}{2} - \frac{S}{2} \cdot \frac{J^2}{W^2} + \frac{S(2S - 1)}{2} \cdot \frac{J^3}{W^3} - \frac{1}{8} S(3 - 10S + 12S^2) \cdot \frac{J^4}{W^4} \quad (14)
\]

\( S^x \) for arbitrary \( J/W \) can be obtained numerically: examples are shown in Fig. 2. Generally, \( S - 1/2 \leq S^x \leq S \). The downward deviation from \( S \) is a measure of the strength of the (necessarily partial) Kondo compensation which depends not only on \( J/W \) but also (and rather drastically) on \( S \). For large \( S \), the \( S \)-spin density wave amplitude approaches the quasi–classical \( S - 1/2 \), which is accompanied by an oppositely polarized \( \tau \)-spin wave of amplitude \( 1/2 - 1/2S \). This picture of a frozen state yielded by the mean–field treatment apparently fails to do justice to the more dynamical internal structure of the strongly bound low–spin Kondo state, in terms of which the effective hamiltonian (11) is formulated. — To see where essential features may be missed let us note that in the mean–field treatment, “turning the spin of a site down” is achieved by acting with a linear combination of \( S^- \), and \( \tau^- \). However, the relevant operators are the \( \eta \)-operators describing the composite Kondo–bound objects and in general, \( \eta^- \) will contain also more complicated components like \( S^- S^- \tau^+ \), etc.

\section{Discussion and Summary}

Motivated by Doniach’s study of the original \( S = 1/2 \) necklace model, we introduced the underscreened Kondo necklace model \( H_{KN} \) given in (2) for studying the situation where conduction electrons in a single screening channel can only partially compensate a lattice of localized spins \( S > 1/2 \).
The necklace model incorporates only one aspect of the full Kondo lattice problem: the competition between Kondo compensation, and intersite spin exchange, and is thus meant to model magnetic ordering in the case when the (non–degenerate) conduction band is half–filled, and the system is expected to have an insulating ground state. Though the denomination “necklace” is suggestive of 1d, the above–described situation is present in all dimensions, and we have in mind really three–dimensional systems. Within a mean–field approximation, we found that the low–temperature phase is a two–sublattice $x – y$ antiferromagnet for arbitrary strength of the Kondo coupling $J$. It also follows that the excitation spectrum is gapless, in the same sense as the ordered state of the original necklace model was found to be.

Obviously, the Kondo necklace models do not have real “Kondo physics” in them, in the sense that low–energy electron–hole excitations, and with them the possibility of the characteristic non–analyticities at weak coupling, are left out. Kondo divergencies are expected to be most important when the (in the lattice case, collective) Kondo effect hinders, or at least greatly suppresses, ordering. Such is not the case with the underscreened necklace models where we find well–developed antiferromagnetism. It can be argued [15] that in the cases of clearcut ordering, the eventual inclusion of true Kondo physics would shift the phase boundaries, but would not modify the overall appearance of the mean–field phase diagram. Our results should be considered in this spirit.

In addition to missing out on the charge degrees of freedom, the necklace models differ in another significant way from the Kondo lattice models: they turn out to describe $x – y$ type magnets rather than isotropic ones. This was already implicit in the fact that Doniach’s [9] mean field trial state turned out to be polarized along the spin–$x$ axis, but some subsequent studies [12] seemed to hint that the Kondo necklace may be, after all, in the same universality
class as the isotropic Kondo chain. This question is bound up with that of
the existence of a spin gap in the spectrum. There seems to be no doubt that
the spin excitation spectrum of the 1d Kondo lattice is gapful \cite{13}. The recent
numerical finding \cite{10} of a phase transition between a gapless and a gapful state
in the ground state of the $S = 1/2$ Kondo necklace underlines two points: a)
that the anisotropic form of the $\tau$-term in (2) has essential consequences, and
b) that the mean field treatment is a useful guide to finding out whether there
is such a phase transition.

In the underscreened necklace models, the $x - y$ character becomes even
more emphatic. We derived the exact form (11) of the effective hamiltonian
governing the large-$J$ behaviour, and found it to be a pure antiferromagnetic
$x - y$ coupling between the residual composite spins. — From this it also
follows that — in contrast to the expected behaviour of the 1d underscreened
Kondo lattice models — the Kondo necklaces show no Haldane phenomenon,
i.e., the nature of the ground state is not expected to alternate between integer
and half-integer values of $S$. This should be clearest at $J/W \to \infty$ when (11)
can be used. In the ground state phase diagram of a larger class of spin models
\cite{16}, the pure antiferromagnetic $x - y$ model is lying on the boundary between
the Haldane, and $x - y$ phases, and is supposed to do nothing exotic.

The results for the ground state sublattice magnetization are given in Fig. 2.
It is a question of great current interest \cite{7, 2}, how the $f$-spins get apparently
divided into a screened, and an ordered part. The simplification brought by
replacing the Kondo lattice model with the necklace model allows a simple
treatment of this problem. As $J/W$ is increased, the order parameter shows
a perfectly smooth behaviour, gradually decreasing from its unscreened value
at $J = 0$, towards the asymptotic value (9).

To summarize, to gain some insight into the difficult problem posed by
the underscreened Kondo lattices, we introduced the underscreened Kondo necklace models. These are in the same relationship to the Kondo lattices as Doniach’s original necklace model to the $S = 1/2$ Kondo lattice. The simplification has its price: charge degrees of freedom are not considered, and the spin-rotational invariance of the original Kondo lattice model is destroyed. By deriving the exact form of the effective hamiltonian governing the behaviour of the lattice model at very large Kondo coupling, we found that it describes an $x - y$ antiferromagnet of the residual spins. In an effective field treatment, the ground state is always antiferromagnetic, and the spectrum gapless. The mean field phase diagram is shown in Fig. 1. Though obviously oversimplified, the model introduced by us has the merit of allowing the description of the competition between Kondo compensation and magnetic ordering for underscreened localized moments.

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References

[1] For a comprehensive review of the physics of $f$–electron systems, cf. N. Grewe and F. Steglich, in: Handbook on the Physics and Chemistry of the Rare Earths Vol. 14, ed. by K.A. Gschneidner, Jr. and L. Eyring, Elsevier Science Publ., pp. 434-474 (1991). – A recent concise review of Kondo lattice magnetism is given by C. Lacroix: J. Magn. Magn. Mater. 100, 90 (1991).
[2] P. Coleman and J. Gan: Physica B171, 3 (1991).

[3] P. Fazekas and H. Shiba: Int. J. Modern Phys. B5, 289 (1991).

[4] H. Shiba and P. Fazekas: Progr. Theor. Phys. Suppl. No. 101, 403 (1990).

[5] P. Fazekas and E. Müller-Hartmann: Z. Physik B85, 285 (1991).

[6] P. Coleman and N. Andrei: J. Phys.: Condens. Matter 1, 4057 (1989).

[7] J. Gan, P. Coleman and N. Andrei: Phys. Rev. Lett. 68, 3476 (1992). – For more details, cf. J. Gan: PhD Thesis (1992).

[8] N. Andrei and C. Destri: Phys. Rev. Lett. 52, 364 (1984).

[9] S. Doniach: Physica B91, 231 (1977).

[10] P. Santini and J. Sólyom: preprint (Lausanne, 1992).

[11] R. Jullien, P. Pfeuty, A.K. Battacharjee, and B. Coqblin: J. Appl. Phys. 50, 7555 (1979).

[12] R.T. Scalettar, D.J. Scalapino, R.L. Sugar: Phys. Rev. B31, 7316 (1985).

[13] H. Tsunetsugu, Y. Hatsugai, K. Ueda and M. Sigrist: Phys. Rev. B46, 3175 (1992).

[14] H.–Y. Kee: Thesis submitted for the ICTP Diploma Course in Condensed Matter Physics, Trieste (1992).

[15] V.L. Líbero and D.L. Cox: preprint.

[16] See, e.g., Fig. 1.1 in T. Kennedy and H. Tasaki: Commun. Math. Phys. 147, 431 (1992).
Figure Captions

Fig. 1 Néel temperature $T_N$ in units of $W$ versus $J/(J+W)$, with $d = 1$, for several values of $S$. Note that for the underscreened models ($S > 1/2$), $T_N/W$ tends to a finite value as $J/W \to \infty$.

Fig. 2 The $T = 0$ value of the order parameter $S^x$ changes in the range $[S - 1/2, S]$ as a function of $J/W$ ($d = 1$ was taken). For the underscreened $S > 1/2$ models, no ground state phase transition is found.