Axial anomaly contribution to the parity nonconservation effects in atoms and ions.

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The contribution of the axial triangle anomalous graph to the parity non-conservation effect in atoms is evaluated. The final answer looks like the emission of the electric photon by the magnetic dipole. The relative contribution to the parity non-conservation effect in neutral atoms appears to be negligible but is essentially larger in case of multicharged ions.

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The problem of testing the standard model (SM) in the low-energy physics is one of the interesting topics in physics during the last few decades. The SM in the low energy limit is tested in particular by observing the parity nonconservation (PNC) effects in atoms. The most accurate of these experiments is the experiment with the neutral Cs atom, first proposed in [1] and performed with the utmost precision in [2]. The basic transition, employed in the Cs experiment was the strongly forbidden $6s-7s$ transition with the absorption of $M1$ photon. In the real experiment this very weak transition was opened by the external electric field but it does not matter for our further derivations. The Feynman graphs illustrating the PNC effect in Cs are given in Figs. 1(a) and 1(b).

The atomic experiments are indirect and require very accurate calculations of the PNC effects in Cs to extract the value of the free parameter of the SM, the Weinberg angle which can be compared with the corresponding high-energy value. The main difficulty with the PNC calculations in neutral atoms is the necessity to take into account the electron correlation within the system of all electrons. Therefore the experiments with much simpler systems, such as the few-electron highly charged ions (HCIs) would be highly desirable. Several proposals on the subject were considered in [3,4].

The radiative corrections to the PNC effect appeared to be important in Cs calculations to reach the agreement with the high energy SM predictions. These radiative corrections include electron self-energy, vertex and vacuum polarization corrections. They are even more important in the case of the HCI. The entire set of these corrections for neutral Cs atom was calculated in [7,10]. The electron self-energy and vertex corrections for HCI were obtained in [11]; the vacuum polarization correction was given in [12].

However, the full set of radiative corrections including $Z$-boson loops is not yet calculated, neither for neutral Cs nor for the HCI. Therefore the problem cannot be considered as fully solved.

In the present work we will consider a very special radiative correction to the PNC effect, presented by a triangle Feynman graph, or axial anomaly (AA). We understand the triangle AA as a fermion loop with at least one weak vertex [13]. Our conclusion will be that in neutral Cs the contribution of the axial anomaly is negligible, but in HCI it is comparable with the electron self-energy, vertex, and vacuum polarization corrections.

The leading contribution of the AA to the atomic PNC effect is depicted in Fig. 1.c. This contribution corresponds to the Adler-Bell-Jackiw anomaly [14]. In this work we will concentrate exclusively on this term.

We employ the standard expression for the effective parity nonconserving interaction of the atomic electron with the nucleus [15] in the form $H_{W} = A_{PNCN}(\vec{r})\gamma_{5}$, with $A_{PNC} = -G_{F}Q_{W}/2\sqrt{2}$, where $G_{F}$ is the Fermi constant and $Q_{W}$ is the weak charge of the nucleus: $Q_{W} = -N + Z(1 - 4\sin^{2}\theta_{w})$ where $Z$ and $N$ are the numbers of protons and neutrons in the nucleus, and $\theta_{w}$ is the Weinberg angle. The recently accepted value for this parameter deduced from all available experiments in the high and low energy physics is $\sin^{2}\theta_{w} \approx 0.23$. The function $\rho_{N}(\vec{r})$ represents the nucleon density distribution within the nucleus, and the $\gamma_{5}$ is the Dirac pseudoscalar matrix. We write down the $S$-matrix element corresponding to the amplitude Fig. 1(c) in the momentum representation:

$$S = -4\pi e^{3} \int \frac{d^{4}p_{1}'}{(2\pi)^{4}} \frac{d^{4}p_{1}}{(2\pi)^{4}} \frac{d^{4}p}{(2\pi)^{4}} \frac{d^{4}p_{2}}{(2\pi)^{4}} \Psi_{n's}(p_{1})\gamma^{5}\Psi_{ns}(p_{1}') \frac{\delta_{\rho\nu} - g_{\rho\nu}}{q^{2} + i\epsilon} V_{\lambda}^{PNC}(q - k)A_{\mu}(p_{2} - k).$$

Here $e$ and $m_{e}$ are the electron charge and mass,

$$\Psi_{ns}(p) = \Psi_{ns}(\vec{p})\delta(p_{0} - \epsilon_{ns})$$

are the wave functions of the
The double solid line denotes the electron in the field of the nucleus. The wavy line denotes the exchange by Z-boson between the atomic electron and the nucleus. Graph (a) corresponds to the basic Z-transition amplitude, the graph (b) corresponds to the E1 transition amplitude, induced by the effective weak potential. The latter violates spatial parity and allows for the arrival of np-states in the atomic bound electron in the state 1s with \( \epsilon_{ns} \) being the energy of this state; \( A_{\mu}(p_2 - k) = (2\pi)^4 \sqrt{2/\hbar \omega} \delta(p_2 - k) \) is the wave function of the emitted photon, where \( \hbar \) is the frequency and \( \epsilon_{\mu} \) 4-vector of the polarization for this photon; and \( g_{\mu\nu} \) is the pseudo-Euclidean metric tensor.

The potential \( V_{PNC} \) for the parity nonconserving interaction of the electron with the nucleus looks like \( V_{PNC}^{(2)} = A_{PNC}(q - k) \delta(0 - k) \), where \( \delta(q - k) = (2\pi)^4 \delta(q_0 - k_0) \), \( q = p_1 - p_1 \) is the transferred momentum and \( k \) is the momentum of emitted photon. In Eq(1) we use the relativistic units: \( \hbar = c = 1 \).

To begin with we consider Z-boson with spin \( J(Z) = 1 \) decay into two photons \([10]\). The Landau theorem forbids this decay because two-photon system cannot exist with full momentum \( J = 1 \) in contrast to the allowed decay \( \pi_0 \rightarrow \gamma \gamma \) since \( J(\pi_0) = 0 \) \([19]\). We shall see this directly from the S-matrix element and see also nonzero contribution in the S-matrix element corresponding to the virtual photon as in our case.

The S-matrix element is proportional to

\[
S_{\mu\nu\lambda}(k_1, k_2) = \int d^4 k \, Tr \left[ \gamma^\mu \frac{\not p + m_e}{p^2 - m_e^2} \gamma^\nu \frac{\not p - k_2 + m_e}{(p - k_2)^2 - m_e^2} \gamma^\lambda \gamma^5 \frac{\not p - k_1 + m_e}{(p + k_1)^2 - m_e^2} \right] \tag{2}
\]

One has to note that Eq(2) turns to the integral over the loop in Eq(1) under the change of variables \( k_1 \rightarrow k; k_2 \rightarrow -q \). Due to the identity

\[
Tr \left[ \gamma^5 \gamma^\gamma \gamma^\gamma \gamma^\lambda \right] = 4 \xi_{\tau \mu \nu \lambda},
\]

where \( \xi_{\tau \mu \nu \lambda} \) is the unit antisymmetric tensor of the IV rank with definition \( \xi_{0123} = -1 \), we will have nonzero contribution in Eq(2) if and only if we retain one momentum (with one \( \gamma \)-matrix) or three momenta (with three \( \gamma \)-matrices) in the square bracket in Eq(2). All other combinations will give zero result. The integrals over loop with expressions containing three momenta are convergent and could be calculated using the standard Feynman parametrization technique \([18]\). But the integrals with expressions containing one momentum are divergent and additional conditions are necessary to get rid of these divergences. These conditions consist of demanding a gauge-invariance of the S-matrix element and look like

\[
k_{1\mu} S_{\mu\nu\lambda} = 0 \tag{3}
\]

\[
k_{2\nu} S_{\mu\nu\lambda} = 0 \tag{4}
\]

After imposing Eqs(3) and (4) on the S-matrix element (2) it becomes gauge-invariant and is presented by the finite expression (it is depicted in the Fig. 1 c with additional graph with interchanged external photon and Z-boson lines):

\[
S_{\mu\nu\lambda}(k_1, k_2) = J_{110}(k_1, k_2) \xi_{\mu\nu\alpha} k_{1\alpha} k_{2\beta} (k_1 + k_2) \lambda + J_{101}(k_1, k_2) (\xi_{\lambda\alpha\beta} k_{1\alpha} k_{2\beta} k_{1\mu} + k_1^2 \xi_{\mu\nu\alpha\beta}) - J_{011}(k_1, k_2) (\xi_{\lambda\alpha\beta} k_{1\alpha} k_{2\beta} k_{2\nu} + k_2^2 \xi_{\mu\nu\alpha\beta}) \tag{5}
\]

where

\[
J_{rst}(k_1, k_2) = - \frac{1}{(2\pi)^4} \int_0^1 d\xi_1 \int_0^1 d\xi_2 \int_0^1 d\xi_3 \left( \delta(1 - \xi_1 - \xi_2 - \xi_3) \right) \delta(\xi_1 k_1 + \xi_2 k_2 + \xi_3 k_2 - m^2)
\]

\[
(\xi_1 k_1 + \xi_2 k_2 + \xi_3 k_2 - m^2)
\]

FIG. 1: The Feynman graphs that describe PNC effect in Cs. The double solid line denotes the electron in the field of the nucleus. The wavy line denotes the exchange by Z-boson between the atomic electron and the nucleus. Graph (a) corresponds to the basic M1 transition amplitude, the graph (b) corresponds to the E1 transition amplitude, induced by the effective weak potential. The latter violates spatial parity and allows for the arrival of np-states in the atomic bound electron in the state 1s with \( \epsilon_{ns} \) being the energy of this state; \( A_{\mu}(p_2 - k) = (2\pi)^4 \sqrt{2/\hbar \omega} \delta(p_2 - k) \) is the wave function of the emitted photon, where \( \hbar \) is the frequency and \( \epsilon_{\mu} \) 4-vector of the polarization for this photon; and \( g_{\mu\nu} \) is the pseudo-Euclidean metric tensor.
Then due to the transversality conditions for Z-boson and real photons \((k_1 + k_2) \chi_\lambda = 0, \epsilon_{\mu k_1 \mu} = 0, \epsilon_{2 \nu k_2 \nu} = 0\) and to the conditions for the real photons \(k_1^2 = 0, k_2^2 = 0\) we get the Landau theorem result \(S_{Z \gamma \gamma} = 0\). But in our case one of the photons (e.g. with index 2) is virtual, as well as Z-boson. Therefore the initial S-matrix Eq\((1)\) will give nonzero result.

Returning to our former variables \(k, q\) we see that the first term in Eq\((5)\) is proportional to

\[
V_0(q-k)(q-k) \sim (q_0-k_0)\delta(q_0-k_0) = 0
\]  (7)

In the following we represent the S-matrix element in the nonrelativistic limit which is obviously justified in case of Cs atom. Recalling that the lower component \(\chi\) of the Dirac wave function could be expressed via the upper one as \(\chi = \left(\frac{\sigma \cdot \vec{p}}{2m}\right)\varphi\) and using properties of Pauli-matrices \((\sigma \cdot \vec{a})(\sigma \cdot \vec{b}) = (\vec{a} \cdot \vec{b}) + i(\sigma \cdot [\vec{a} \times \vec{b}])\) we obtain the following expression for the square bracket in Eq\((9)\) (without the factor \(1/2m_c\))

\[
(q' \cdot \vec{P})(\vec{q} \cdot [\vec{c} \times \vec{k}]) + (\vec{P} \cdot [\vec{k} \times \vec{c}]) + i(\vec{q} \cdot \vec{k})(\vec{c} \cdot \vec{q}) - i(\vec{c} \cdot \vec{q})(\vec{k} \cdot \vec{q})
\]  (10)

where \(\vec{P} \equiv \vec{p}_1 + \vec{p}_1'\) and \(\vec{q} = \vec{p}_1 - \vec{p}_1'\). Expression \(10\) changes sign under the inversion \(\vec{p}_1 \rightarrow -\vec{p}_1; \vec{p}_1' \rightarrow -\vec{p}_1'\).

Then, remembering that the wave functions \(\Psi_{n's}, \Psi_{ns}\) are of the same parity, the only reason for the whole expression \(9\) not to be zero is the presence of the scalar product \((\vec{k} \cdot \vec{q})\)

\[
S = -4\pi e^3 A_{PNC} \delta(E_f - E_{in} - \omega_0) \sqrt{\frac{4\pi}{2\omega_0}} \int \frac{d^3 p_1'}{(2\pi)^3} \frac{d^3 p_1}{(2\pi)^3} \Psi_{n's}(\vec{p}_1') J_{011}(\vec{k}, \vec{q}) \left[ (\vec{c} \cdot [\vec{k} \times \vec{q}])\left(\vec{a} \cdot \vec{q}\right) + (\vec{c} \cdot [\vec{a} \times \vec{k}]) \right] \Psi_{ns}(\vec{p}_1)\]

(11)

where

\[
\omega_0 = \frac{1}{2m_c^2} \int_0^1 d\xi_1 \int_0^{1-\xi_1} d\xi_2 (\xi_1 + \xi_2 - 1) = \frac{1}{360\pi^2}
\]

After the Fourier-transform to the coordinate representation in Eq\((11)\) and using the standard relation between the S-matrix and the PNC-amplitude \(E_{PNC}\)

\[
S = -2\pi i E_{PNC} (E_{n's} - E_{ns} - \omega), \quad \text{we get following expression for } E_{PNC}:
\]

\[
E_{PNC} = -2ie^3 \left( G_F m_e^2 \right) \frac{Q_W}{2\sqrt{2}} \left( m_e/m_p \right)^2 \sqrt{\frac{2\pi}{30\pi^2}} \frac{\omega_0^{3/2}}{m_e} \frac{\xi_0^2(\vec{a} \cdot \vec{c})}{\varphi_0(0)\varphi_0''(0)}
\]  (13)

We would like to note that in Fig. 1.c corresponds to the transition of the bound electron between
states of the same parity $n’s, ns$, in particular between $7s$ and $6s$ states in Cs atom, like in experiments 1, 2. But the resulting $S$-matrix element in Eq. (13) is proportional to $(\vec{\sigma} \cdot \vec{\epsilon})$, i.e. the magnetic dipole moment of the electron $\hat{\mu} = \frac{1}{2} \mu_0 \vec{\sigma}$ emits an electric type of photon $\epsilon$.

This could be considered as a unique effect which occurs due to the parity violation in atoms. Note, that no other radiative correction to the atomic PNC effect, evaluated up to now, can be interpreted in such a way. The $T$-invariance of $E_{PNC}$ is satisfied due to the presence of the imaginary unit in Eq. (13).

For the probability of the process, combining two amplitudes $|Figs 1.a and 1.c corresponding|$, we get

$$W_{7s-6s} = W_{M1} + \frac{1}{2j_0 + 1} \sum_{m_0m_1} 2Re [E_{M1} E_{PNC}] + O(E_{PNC}^2)$$

(14)

where $m_0, m_1$ are the angular momentum projections for the initial and the final electron states.

Performing the summation over the electron angular momentum projections $m_0, m_1$ and applying the Wigner-Eckart theorem to the product $E_{M1} E_{PNC} \sim < n’s |\hat{\mu} (\vec{\nabla} \times \vec{\epsilon})| ns > < n’s |(\vec{\sigma} \cdot \vec{\epsilon})| ns >$ we get the final answer in the form

$$W_{7s-6s} = W_{M1} (1 + R(\vec{\nu} \cdot \vec{s}_{ph}))$$

(15)

where $\nu = \frac{\vec{k}}{|\vec{k}|}$, $\vec{s}_{ph} = i[\vec{\epsilon} \times \vec{\nabla}]$ is the spin of the photon and $R$ is so called "degree of the parity violation". In our case $R$ is equal to the ratio $E_{PNC}/E_{M1}$, where the amplitudes are expressed via the angular reduced matrix elements.

Using the estimate $\varphi(0) \varphi''(0) \sim \alpha^2 Z^3$ for neutral atoms and $\varphi(0) \varphi''(0) \sim \alpha^5 Z^5$ for HCI we get the following result for the anomaly contribution to the PNC-amplitudes in the neutral atoms (Fig. 1.c):

$$E_{PNC}^{\text{atom}} \sim \frac{1}{360\pi^2} \left( \frac{m_e}{m_p} \right)^2 \alpha^{3/2} (G_F m_p^2) Q_W \alpha^5 Z$$

(16)

and the estimate

$$E_{PNC}^{\text{HCl}} \sim \frac{1}{360\pi^2} \left( \frac{m_e}{m_p} \right)^2 \alpha^{3/2} (G_F m_p^2) Q_W \alpha^5 Z^5$$

(17)

for the anomaly contribution to the PNC effects in HCI.

Using a well-known estimate for the PNC-amplitude (Fig. 1.h) in neutral atoms

$$E_{PNC}^{\text{atom}} \sim \left( \frac{m_e}{m_p} \right)^2 \alpha^{3/2} Z^2 (G_F m_p^2) Q_W$$

(18)

we get for the relative AA contribution a negligible value $\sim (10)^{-3} \alpha^5 Z$. In the $H$-like HCI this relative contribution will be on the order of $\sim (10)^{-5} \alpha$.

In conclusion we should stress that the observation of the AA contribution to the PNC effects would be of a special interest since it would be an observation of AA in atomic physics.

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