Strength of reinforced concrete flat slabs for punching

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Abstract. The purpose of the study is to analyze previously developed structural solutions of the “slab-column” unit under various modes of static and cyclic loading using various reinforcing elements and types of concrete, to compare methods for calculating the slab for punching at the fillet places with columns according to Russian, Belarusian, and European standards. Calculations at different standards give different results, which differ significantly from each other. The main objective of the study was to develop a methodology for calculating flat reinforced concrete slabs for punching using a frame-rod model of work and models of destruction of reinforced concrete in a compressive power flow. The significance of the results obtained for the construction industry lies in the fact that the results of theoretical studies are applicable to create a methodology that allows calculating the fracture from overcoming the concrete resistance to tearing, shear and crushing when forcing flat reinforced concrete slabs at the fillet places with the columns. Flat slab contribute to the maximum use of floor height, provide an effective location below the slab of utilities.

Key words: punching, frame-rod model, failure from overcoming concrete resistance to tearing, shear and crushing.

1 Introduction

1.1 Theoretical research

The most critical node in a reinforced concrete monolithic frame is the junction of the slab with the column. At the same time, the requirements for saving inter-story space oblige to design flat, girderless and noncapital slabs. There are a lot of works, such as [1-6] aimed at improving the design of this unit, increasing the bearing capacity from punching.

A lot of Russian, European and American scientific papers are devoted to the problem of calculating horizontal joints of reinforced concrete elements in the punching zone [7-11]. The main method of calculation using computational programs (ANSYS PC, etc.) is the finite element method used in the works [12-14].

The work is devoted to the refinement of normative equations with the introduction of various additional coefficients [15-18].

The most famous theory, which considers the issue of punching under various types of reinforcement, types of loading, etc., is the TKST theory developed by Muttoni and is reflected in [19-20]. In addition, analytical methods [21] and mechanical models [22] for punching calculation have been developed.

Below is the comparison of calculation methods and equations according to Russian, Belarusian and European standards.
1.2 Calculation of punching according to the norms of the Russian Federation (SP) 63.13330.2012 «Concrete and reinforced concrete structures. Key Points»

The calculation is made for flat slab under the action of concentrated forces and bending moment on them. The calculation is performed on the basis of a conditional model having a calculated cross section located at a distance $h_0/2$, along which shear stresses (shear stress) act, balanced by the strength of concrete $R_{bt}$ and transverse reinforcement $R_{reinforcement}$ (figures 1, 2). Calculation without transverse reinforcement is performed according to the equations:

$$F \leq F_{b,ult}$$

$$F_{b,ult} = R_{bt} \cdot u \cdot h_0.$$  

With transverse reinforcement – according to the equations:

$$F \leq F_{b,ult} + F_{sw,ult} \leq 2F_{b,ult}$$

$$F_{sw,ult} = 0,8u \cdot R_{sw} \cdot A_{sw}/s_w.$$  

![Figure 1. Conditional model for punching shear calculating.](image1.png)

![Figure 2. Scheme for calculating reinforced concrete elements without transverse reinforcement for punching shear: 1 – calculated cross section; 2 – contour calculated cross section; 3 – load pad contour.](image2.png)
1.3 Calculation of punching according to the construction norms of the Republic of Belarus (SNB) 5.03.01-02 "Concrete and reinforced concrete structures"

These standards largely repeat the EU Eurocode EN 1991-1-1-2009, but with their own modifications. The calculation is made for flat slab under the action of concentrated forces or reactions on them. The punching strength is determined along the calculated critical perimeter, spaced at a distance of 1.5d from the outer edge of the column (figure 3 a, b). Calculation without transverse reinforcement is performed according to the equation:

\[ \nu_{Sd} \leq \nu_{Rd,c} = [0.15k \cdot (100p_f \cdot f_{ck})^{1/3} - 0.10 \sigma_{cp}] \cdot d, \]

but not less \((0.5 f_{cd} - 0.10 \sigma_{cp}) \cdot d\).

With transverse reinforcement – according to equations (6) – for the first perimeter, (7) – for the following perimeters:

\[ \nu_{Rd,sy} = \nu_{Rd,c} + A_{swi} f_{ywd} \cdot d / (u_1 \cdot s_w), \]

\[ \nu_{Rd,csy} = 0.75 \nu_{Rd,c} + 1.5 \cdot d/s_{r} \cdot A_{swd,ef} \cdot 1 / (u_1 \cdot d) \cdot \sin \alpha. \]

**Figure 3.** a) calculating model for punching shear; b) critical perimeter for areas of local load application.

1.4 Calculation of punching according to EU standards EN 1991-1-1-2009 “Design of reinforced concrete structures. Part 1-1. General rules and regulations for buildings”

The calculation is made for flat slab under the action of concentrated forces or reactions on them. The punching strength is determined along the calculated critical perimeter, spaced at a distance of 2.0d from the outer edge of the column (figure 4 a, b).

Calculation without transverse reinforcement is performed according to the equation:

\[ \nu_{Rd,c} = C_{Rd,c} k \cdot (100p_f \cdot f_{ck})^{1/3} + k_1 \sigma_{cp} \geq \nu_{min} + k_1 \sigma_{cp}. \]

With transverse reinforcement - according to the equation:

\[ \nu_{Rd,c} = 0.75 \nu_{Rd,c} + 1.5 \cdot d/s_{r} \cdot A_{swd,ef} \cdot 1 / (u_1 \cdot d) \cdot \sin \alpha. \]

**Figure 4.** a) calculating model for punching shear; b) critical perimeter for areas of local load application.
In SP 63.13330.2012, the main design characteristic of concrete punching strength is the design concrete tensile strength \( R_{ct} \). In the norms of other countries (SNB 5.03.01-02, EN 1991-1-1-2009), the concrete compressive strength \( f_{ck} \), corresponding to the calculated concrete compressive strength \( R_{ct} \), also the control perimeter has a different value, from \( h_0/2 \) (\( d/2 \) ) according to Russian standards, up to \( 1.5d \) and \( 2.0d \) according to Belarusian and European standards, respectively.

### 2 Materials and methods

The aim of the authors' theoretical research is to develop a new methodology for calculating flat reinforced concrete slabs for punching, in the interface zone with the column. According to Zalesov A.S. [23] for relatively small relative values of the span of the slice, a frame-rod model can be used to calculate flat reinforced concrete slabs for punching, including inclined compressed, spatially located concrete elements following from the loaded platform to the supports, and horizontal stretched elements of longitudinal reinforcement (figure 5a).

![Diagram](image)

**Figure 5.** a) frame-rod model A. Zalesov [23] 
(1 – inclined compressed, spatially located concrete elements, 
2 – horizontal tensile elements of longitudinal reinforcement); b) the wedge of L. Obert [24].

In the work of Mirsayapov Ilshat T. [24], the wedge hypothesis of L. Obert is considered (figure 5b). According to the hypothesis, the destruction of concrete compressed elements can be described as follows:
- under the cargo and supporting platforms, seals are formed in the form of wedges;
- with increasing external load, as a result of the pressure of these wedges on the surrounding concrete, there is an increase in the main tensile stresses between the tops of the wedges;
- when these stresses reach the ultimate tensile strength of concrete, the element is destroyed; while the destruction of the element is characterized as a process of overcoming the resistance of concrete to peeling, shear.

In the work of Nikitin G.P. [25] a model of concrete failure in a compressive power flow is considered. In general terms, the strength condition for the model of concrete failure in a compressive power flow (figure 6b) is written by the equation:

\[
N \leq N_{ult} = N_{bt} \frac{\cos \alpha + 2N_{sh}}{\sin \alpha} + N_{ef},
\]

where
- \( N_{bt} = R_{bt} \cdot A_{bt} \) is the separation resistance;
- \( N_{sh} = 3R_{bt} \cdot A_{sh} \) - shear resistance;
- \( N_{ef} = R_{bt} \cdot A_{ef} \) - crush resistance;
- \( A_{bt} = (\sqrt{2} \cdot \cos \alpha) \cdot 2a \) - separation area;
- \( A_{sh} = 2a \cdot \cot \alpha \) is the shear area;
- \( A_{ef} = a \cdot \sin \alpha \) - crush area;
\[ \alpha = \arctg\left(0.25 \frac{R_b}{R_{bt}} - 1.56\right) \] – the angle of inclination of the wedge to the site of application of the load.

![Figure 6. a) model of splitting Rokhlin I.A. [24]; b) model of concrete destruction in compressive power flow critical [25].](image)

**3 Results**

Substituting into the equation (10) the geometric and physical characteristics given above and in the figures (figures 7, 8a-b), we obtain the calculated squeezed brace:

\[ N_{ult} = \frac{N_{bt} \cos \alpha \sin \alpha + 3R_{bt} A_{sd} \sin \alpha}{\sin \alpha} + N_{ef} \cdot \cos \alpha. \]  

\[ (11) \]

![Figure 7. Frame-rod model with geometric pattern.](image)
Substituting into the equation (11) the above values of the areas of separation, shear, crushing, we obtain the final value of $N_{i,ult}$ in newtons per linear millimeter along the average perimeter ($u$, in mm):

$$N_{i,ult} = 2R_{bt} \cdot ctga \left( \frac{k_o}{\cos a} + 2a \left( \frac{6}{\sin a} - 1 \right) \right) + 2aR_{bt} \cdot \sin a \cdot \cos a. \quad (12)$$

Given that the full size of the forcing force is defined as the product of the unit effort of the compressed element by the average perimeter ($u$):

$$N_{ult} = N_{i,ult} \cdot u. \quad (13)$$

The calculated value of the total value of the ultimate bearing capacity for punching a flat slab, using a frame-rod model, will have the following form:

$$N_{ult} = \{2R_{bt} \cdot ctga \left( \frac{k_o}{\cos a} + 2a \left( \frac{6}{\sin a} - 1 \right) \right) + 2aR_{bt} \cdot \sin a \cdot \cos a \} \cdot u. \quad (14)$$

4 Discussions
Theoretical studies of the stress-strain state of punching flat plates by a column in the zone of their conjugation using a frame-rod model and concrete fracture model allowed us to obtain the calculated equation (14). This equation most accurately describes the actual work of concrete, in which the destruction occurs from the joint action of shear stress (shear), compression, separation. Flat slabs contribute to the maximum use of space-planning solutions, including the height of the floor. Further work of the authors is to clarify the equation in the ANSYS PC and full-scale testing of a plate sample for destruction from punching.

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