Structure Functions and Parton Distributions

Stefano Forte

Dipartimento di Fisica, Università di Milano and
INFN, Sezione di Milano, Via Celoria 16, I-20133 Milano, Italy

Abstract

I review recent progress in the determination of the parton structure of the nucleon, in particular from deep-inelastic structure functions. I explain how the needs of current and future precision phenomenology, specifically at the LHC, have turned the determination of parton distributions into a quantitative problem. I describe the results and difficulties of current approaches and ideas to go beyond them.

Invited plenary talk at
BARYONS04
Paris, October 2004
to be published in the proceedings
I review recent progress in the determination of the parton structure of the nucleon, in particular from deep-inelastic structure functions. I explain how the needs of current and future precision phenomenology, specifically at the LHC, have turned the determination of parton distributions into a quantitative problem. I describe the results and difficulties of current approaches and ideas to go beyond them.

1. From HERA to the LHC

Knowledge of the parton structure of the nucleon has undergone a revolution during the last decade, driven by present and future experimental data. On the one hand, current experiments, especially at HERA [1] but also from neutrino beams at Fermilab, have provided us with an unprecedented amount of experimental information, mostly from the measurement of deep-inelastic structure functions. On the other hand, LHC, now behind the corner, will require, essentially for the first time, a precision approach to the structure of the nucleon in the context of searches for new physics [2]. This has stimulated a considerable amount of theoretical and phenomenological work, with the aim of turning the physics of parton distributions into a quantitative science.

2. Determining PDFS

The parton structure of the nucleon can be determined thanks to factorization: a physical cross section is expressed as the convolution of perturbatively computable parton cross sections, times parton densities (pdfs). We can then use one process to measure pdfs, which are then used to compute a different process. In the prototypical case of inclusive deep-inelastic scattering (DIS) the cross section, up to corrections suppressed by powers of $m_p^2/Q^2$, is given by

$$d^2\sigma^{\lambda_{\ell}\lambda_{p}}(x,y,Q^2) = \frac{Q^2}{xy} \left\{ -\lambda_{\ell}y \left( 1 - \frac{y}{2} \right) xF_3(x,Q^2) ight. $$
$$+ (1-y)F_2(x,Q^2) + y^2xF_1(x,Q^2) \bigg\} - 2\lambda_{p} \left\{ -\lambda_{\ell}y(2-y)xg_1(x,Q^2) ight. $$
$$- (1-y)g_4(x,Q^2) - y^2xg_5(x,Q^2) \bigg\} \right\} \right\},$$

where $\lambda$ are the lepton and proton helicities (assuming longitudinal proton polarization), the kinematic variables are $y = \frac{p_{\ell}}{p_{\ell}+k}$ (lepton fractional energy loss), $x = \frac{Q^2}{2p_{\ell}q}$ (Bjorken $x$),
and $\eta$ depends on the gauge bosons which mediates the scattering process:

$$
\eta_\gamma = \frac{4 \pi \alpha^2}{Q^2}; \quad \eta_W = G^2 \frac{Q^2}{2\pi (1 + Q^2/m_W^2)^2}; \quad \eta_Z = G^2 \left[ \frac{1}{2} \left( g_V - \lambda \alpha_s \right) \right]^2 \frac{Q^2}{2\pi (1 + Q^2/m_Z^2)^2}.
$$

More contributions are due to interference of different exchange processes.

The factorization theorem expresses the structure functions which parametrize the cross section as a convolution of a perturbative partonic cross section (coefficient function) and a pdf. For example

$$
F_2^\gamma(x, Q^2) = x \sum_{\text{flav. } i} \left\{ e_i^2 \left( q_i(x, Q^2) + \bar{q}_i(x, Q^2) \right) \\
+ \alpha_s(Q^2) \left[ C^L_i[x, \alpha_s(Q^2)] \otimes \left( q_i(x, Q^2) + \bar{q}_i(x, Q^2) \right) \right] \\
+ C^g[x, \alpha_s(Q^2)] \otimes g \right\},
$$

where $f(f) \otimes g(x) \equiv \int_x^1 \frac{dy}{y} f(x/y) g(y)$ and $C_1 = 1 + O(\alpha_s), C_g = O(\alpha_s)$ are respectively the $i$-th quark flavour and gluon coefficient functions, i.e. the perturbative cross-sections for the gauge boson-parton scattering process. The structure function $F_1$ depends on the same combination of quarks as $F_2$, but with a different gluon content:

$$
F_1^\gamma(x, Q^2) \equiv F_2^\gamma(x, Q^2) - 2xF_1^\gamma(x, Q^2) \\
= \sum_{\text{flav. } i} \alpha_s(Q^2) \left[ C^L_i[x, \alpha_s(Q^2)] \otimes \left( q_i(x, Q^2) + \bar{q}_i(x, Q^2) \right) \right] + C^g_g[x, \alpha_s(Q^2)] \otimes g.
$$

Other structure functions are sensitive to different combinations of parton flavours: the $g_i$ structure functions are spin–odd and contribute to the polarized cross section; the

Figure 1. Kinematic coverage of current data (left, from Ref. [1]) and the LHC (right, from Ref. [2]).
structure functions $F_3$, $g_4$ and $g_5$ are parity-violating, and contribute to weak current scattering. For unpolarized $\gamma^* \text{DIS}$ only $F_1$ and $F_2$ contribute.

Various physical processes are given by different combinations of the same pdfs convoluted with the appropriate partonic cross sections and kinematics. In particular, for a process at a hadron collider (such as Higgs production at the LHC) where the hard scale is the mass $M^2$ of the final state $Q^2 = M^2$ and $x_i = \sqrt{M^2} \exp \pm y$, where $s$ is the center-of-mass energy of the hadronic collision, $\pm y$ the parton rapidities and $i$ refers to each of the incoming hadrons. The kinematic regions for HERA and LHC are compared in fig. 1, along with the current experimental coverage of the $(x, Q^2)$ plane from unpolarized DIS data. In order to obtain predictions for LHC processes we must solve three problems: 1) disentangle the contribution of individual partons to the observable used to determine them; 2) evolve them up to the relevant scale and convolute them with the appropriate perturbative cross section; 3) determine the error on them.

2.1. Phenomenology: disentangling PDFS

Whereas deep-inelastic scattering mediated by $\gamma^*$ exchange provides the bulk of the data shown in fig. 1 (most of the HERA data and older fixed target data), they only measure the cross-section eq. (1), i.e., in turn, the combination of pdfs of eqs. (3-4). This means that: 1) only the C–even combination $q + \bar{q}$ is accessible; 2) flavour separation can be done only for $u$ and $d$ quark using proton and deuteron targets (and then only in fixed target experiment, since a HERA upgrade with nuclear beams has not been approved); 3) the gluon contribution can only be determined through scaling violations:

\[
\frac{d}{dt} F_2^s(N, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \left[ \gamma_{qq}(N) F_2^s + 2 n_f \gamma_{qg}(N) g(N, Q^2) \right] + O(\alpha_s^2),
\]
where $N$ is related to $x$ by Mellin transform according to $F_2(N, Q^2) \equiv \int_0^1 dx x^{N-1} F_2(x, Q^2)$ (and analogously for parton distributions).

Separation of light flavours and antiflavours can be obtained by including different observables along with the DIS structure functions \[3\] in the set of processes used to determine the pdfs. Specifically, light antiflavour separation is obtained comparing (Tevatron) Drell-Yan production with proton or deuteron targets, because

$$\frac{\sigma_{pd}}{\sigma_{pp}} \mid_{x_1 > x_2} \approx \frac{1}{2} \left( 1 + \frac{\bar{d}(x_2)}{\bar{u}(x_2)} \right),$$

and light flavour separation from (Tevatron) $W^\pm$ production asymmetry data, which, neglecting strange quark contributions, give

$$\frac{\sigma_{W^-}}{\sigma_{W^+}} \approx \frac{d(x_1) \bar{u}(x_2)}{\bar{d}(x_1) d(x_2)}.$$  

Inclusion of these data is mandatory in order to reduce the uncertainty on quark and antiquark distributions significantly below 10% (see fig. 2), as required for LHC phenomenology.

Strangeness can only be determined through weak current DIS, i.e., essentially neutrino data (and some HERA data), which are however scarce and subject to sizable uncertainties. As a consequence, in current fits \[3,4,7\] the shape of the strange distribution is not determined, and assumed to be related in a fixed way to that of the light quark sea; the $s$ and $\bar{s}$ distributions have been determined only in dedicated analyses based on neutrino data \[5\].

Finally, because (see fig. 2) $\gamma_{gq} \ll \gamma_{qq}$ at large $N$, the gluon can only be determined accurately from eq. \[3\] at small $N$, i.e., inverting the Mellin transform, at small $x$. In some parton sets \[3,4\] accuracy on the gluon at large $x$ is improved through the inclusion of data from inclusive jet production at the Tevatron.
A better handle on the gluon could be obtained if the contributions of $F_2$ and $F_1$ to the cross section eq. (11) could be disentangled. This is possible by varying $y$ at fixed $x$ and $Q^2$, which requires varying the beam energy: it would be possible by lowering the beam energy at HERAI. On top of this, high luminosity and especially weak scattering data from HERAI could improve somewhat current flavour separation [9]. It is unclear whether any of these measurements might be performed before the shutdown of HERA.

A markedly more significant improvement in knowledge of parton distributions is expected from the coming into operation of the LHC itself, even though only extremely limited studies of the impact of LHC on PDFs are available so far [10]. At longer term, full information on the flavour decomposition of the nucleon could come from a neutrino factory [11].

2.2. Theory: perturbative coefficients and evolution

However abundant the data, PDFs can be extracted from them only through the use of perturbative parton cross sections which are needed to relate them to physical observables and the anomalous dimensions which are required to evolve them to a common scale. Hence, the theoretical uncertainty on PDFs is always at least as large as the size of the unknown perturbative corrections. Until very recently, only inclusive Drell-Yan and DIS partonic cross sections were known at NNLO [12]. In the last couple of years, thanks to the development of new computational techniques, NNLO results have been obtained for inclusive Higgs production [13] and, more importantly, for a number of less inclusive observables, specifically Drell-Yan and W rapidity distributions in hadronic collisions [14]. On the other hand, the full set of NNLO anomalous dimensions or splitting functions has also been determined [15] after an effort of more than a decade.

The impact of NNLO corrections to DIS coefficients and evolution is displayed in fig. 3. Clearly, their inclusion is required if one aims at achieving a determination of parton distributions to percent accuracy. Their impact on specific observables (such as $F_L$) or in particular kinematic regions (such as very small or very large $x$) can be even more dramatic. The large size of NNLO corrections at small and large $x$ signals the need for all-order resummation of the perturbative expansion in these regions. Indeed, higher order perturbative corrections are known to be enhanced by logs of $x$ and $(1-x)$ through terms of the form $\alpha_s \ln \frac{1}{x}$ and $\alpha \ln^2(1-x)$. When $x$ is small or large enough the log enhancement can offset the suppression due to the strong coupling $\alpha_s$.

The impact of large $x$ corrections beyond NNLLO on the extraction of PDFs from DIS has been estimated [16] to be negligible at least if one imposes a cut on the center-of-mass energy of the $\gamma^* p$ process $W^2 \equiv Q^2(1-x) \gtrsim 10$ GeV$^2$. On the other hand, once less inclusive observables (e.g. the differential Higgs production cross section) are considered and experimental cuts are taken into account the impact of large $x$ corrections can become sizable [17]. Because large $x$ resummations are known exactly and their inclusion poses no problem of principle it would be advisable to include them in future parton sets.

The impact of small $x$ corrections is dramatically highlighted by the recent NNLO splitting function determination. Indeed (fig. 4) the perturbative expansion of the splitting function is unstable at small $x$, but on the other hand the logarithmically enhanced small $x$ terms (leading singularities) are not a good approximation to the full result. This means that small $x$ contributions have to be resummed, but this resummation must also
Figure 4. Left: the splitting function ($P_{gg}$ with $n_f = 0$) at LO (solid, black), NLO (dotdashed, green), NNLO (dashed, red), NNLO leading singularities (lower dotted, red), NNNLO leading singularities (upper dotted, blue). The two solid (blue) curves with a dip are the small $x$ resummations of Ref. [18] and Ref. [19]. Right: best-fit gluon at NLO (bottom, solid), NNLO (middle, dashed), and NNLO evolution of the NLO best-fit (top, dotted, from Ref. [8]).

be combined with the available fixed-order results in order to lead to a stable answer. The required formalism has been developed over the last decade by various groups and is now converging to an answer, but the relevant phenomenology has not been developed yet. Current results [18,19] show (see fig. 4) that the resummation stabilizes the NLO results, so that at small $x$ the fully resummed result is actually closer to the low order (LO and NLO) results. The impact of NNLO corrections on the extracted gluon can be larger than 100% at small $x$ and small scale (see fig. 4). Hence, the resummation is mandatory if one wishes to work to NNLO accuracy.

3. Partons and errors

Precision phenomenology needs not only a knowledge of parton distributions, but also of the error with which they are determined. The problem here is that parton distributions are functions, hence the error on them is really a probability measure in an infinite-dimensional space. Therefore, it cannot be determined from a finite set of data without extra theoretical assumptions. These assumptions in current parton fits take the form of a functional form: pdfs are assumed to have, at a reference scale, a given functional form, parametrized by a finite set of parameters. They are then evolved to the scale of the data and used to compute physical cross-sections which are then fitted to the data, thereby determining the parameters. If the full information of the covariance matrix of the data is retained and propagated to the parameters it is then possible to determine errors on pdfs. Within the last few years, three parton sets with errors have been obtained in this way [3,4,7].
The result of current global parton fits look nominally quite good, with uncertainties of order of a few percent on quark and antiquark distributions and at most 10% on the gluon (see fig. 2). However, closer inspection reveals a number of problems. Indeed, consider the variation of the $\chi^2$ of the fit to each dataset as the $\chi^2$ of the global fit is allowed to vary [20]. This study reveals that the fit to the CCFR neutrino DIS data or the BCDMS muon-deuterium DIS data can be improved very considerably at the expense of deteriorating the global fit (fig. 5). This indicates that the global fit with the given functional form is far from the best fit to these data. Also, one may study the contribution of individual datasets to the $\chi^2$ of the global fit as one of the fit parameters is varied. This (fig. 5) shows that the minimum of the global fit does not correspond to minima of individual experiments.

While to some extent these could just be statistical fluctuations, they may signal more serious problems. On the experimental side, they may signal that some datasets are not fully consistent with the others. On the theoretical side, they may signal that assumptions on the functional form are not flexible enough to accommodate the data.

Whatever the precise cause, it appears that current determinations of errors on pdfs have not yet settled to a satisfactory agreement. This is dramatically apparent if one looks at the prediction for even the simplest inclusive LHC observable, such as the total Higgs production cross section (fig. 6): the results obtained using recent parton sets disagree within the respective error bands, especially when the quark flavour separation comes...
into play, such as in $H W$ production.

### 3.2. Conservative solutions

The problems of data incompatibility and possible parametrization bias have been tackled in various ways in current parton sets. A first option is to replace the standard one-sigma contours with parameter ranges obtained studying the compatibility of the fit with various experiments. In ref. [4] this has been done by studying the spread of 90% confidence intervals for various experiments, as one moves away from the minimum of the $\chi^2$ along eigenvectors of the hessian matrix, and taking the envelope of the resulting ranges. In practice, this suggests that $\Delta \chi^2 = 100$ for the global fit leads to a reasonable estimate of the one-sigma contours for pdfs (see fig. 7). In ref. [3] $\Delta \chi^2 = 50$ is adopted instead, and seen to lead to results which are not so different for the pdf error bands.

However, the need to chose ad-hoc a large value of $\Delta \chi^2$ is somewhat disturbing. An alternative suggestion has been made in Ref. [3], where it is observed that most of the trouble seems to come from specific kinematic regions where theoretical uncertainties become large: the low $Q^2$ region where the perturbative expansion converes slowly, and the large and small $x$ regions where resummation is necessary. It is then shown that by imposing more restrictive cuts in $Q^2$, $x$ and $W^2$ (see sect. 2.2 above) a much more palatable value $\Delta \chi^2 = 5$ can be taken to determine the error on pdfs. The ‘conservative’ partons obtained in this way can differ by more than 10% from standard ones (see fig. 7). The problem is that clearly there is information loss in the process, and predictions become unreliable when regions are probed which have been excluded from the fit due to the cuts (such as the very small $x$ region).

Still, one would like to be able to rely on purely statistical arguments to construct one sigma contours. To this purpose, in ref. [7] it has been observed that many problems seem to come from the need of combining different data sets. Indeed, in ref. [4] it has ben demonstrated that if only DIS data are included in the global fit, and the full covariance
matrices of experiments are taken into account, it is possible to achieve a statistically stable fit where one-sigma error bands are given by $\Delta \chi^2 = 1$. The problem is that DIS data alone are insufficient to determine for instance the quark flavour decomposition. However, preliminary results [8] suggest that DIS data can be combined e.g. with Drell-Yan data, provided only that the $\chi^2$ from different datasets are brought to a common normalization. Satisfactory errors on all pdfs are then obtained (see fig. 2).

The fact that different prescriptions seem to be able to solve at least in part the problems of global fitting suggests that the origin of these problems is not yet fully understood.

### 3.3. New ideas

The difficulties encountered in current parton fits suggest that perhaps the conventional approach is now reaching its limitations, and has led to the suggestion of alternative approaches. A first proposal [22] is to use Bayesian inference to update a prior representation of the probability density which is generated as a Monte Carlo sample based e.g. on an available parton parametrization. The final result should be largely independent of the choice of prior if the data are sufficiently abundant. The main difficulties with this approach are related to the need to keep the computational complexity under control, in particular in the choice of priors and in the handling of flat directions, i.e. Monte Carlo replicas which lead to similar values of the $\chi^2$. A preliminary set of partons ('Fermi' partons) has been constructed within this approach [22] (see fig. 8). The results suggest that indeed a treatment of non-gaussian probability densities may be required if one wishes to combine experimental information from different sets. However, no satisfactory global fit
within this approach has been obtained yet: in particular, the preliminary results do not lead to a satisfactory value of the strong coupling.

Another approach has been suggested in ref. [23], based on the idea of using neural networks as universal unbiased interpolants. In this approach, the data are used to generate a Monte Carlo sample which represents the probability measure in the space of functions at the points at which data exist. Neural networks are then used to interpolate between these points: the ensuing Monte Carlo set of neural networks is then the sought-for probability in the space of functions. This approach has been used in ref. [23] to parametrize all available $F_2$ data, but without extracting the contribution of individual pdfs to the structure function. Preliminary results on a pdf extraction based on this method have been presented in ref. [24] (see fig. 8). They suggest that fixed functional forms may be too rigid in estimating errors especially at the edges of the data region. The feasibility of a full parton set based on this approach, which is also computationally quite intensive, is however still to be demonstrated.

These approaches have in common the feature of trying to use the available experimental information in a way which is free of theoretical assumptions.

4. Conclusions

Perturbative QCD phenomenology has become the object of precision quantitative studies during the last decade and it is now on a similar footing as precision electroweak phenomenology. However, unlike in the electroweak case, the impossibility to compute
the structure of the nucleon from first principles poses taxing problems of data analysis. A satisfactory agreement between different determination of parton distributions has not been reached yet especially at the level of error determination, and may require the development of entirely new techniques.

Acknowledgements: I thank B. Pire and M. Guidal for inviting me to this stimulating meeting, R. Ball and G. Ridolfi for several discussions, and S. Alekhin for communicating the unpublished plots shown in figs. 2-4.

REFERENCES

1. P. Newman, Int. J. Mod. Phys. A 19 (2004) 1061; V. Chekelian, [hep-ex/0502008], these proceedings.
2. S. Catani et al., [hep-ph/0005025].
3. A. D. Martin et al. Eur. Phys. J. C 28 (2003) 455.
4. J. Pumplin et al, JHEP 0207 (2002) 012.
5. V. Barone, C. Pascaud and F. Zomer, Eur. Phys. J. C 12 (2000) 243; F. Olness et al., [hep-ph/0312323]
6. S. I. Alekhin, Phys. Lett. B 519 (2001) 57.
7. S. Alekhin, Phys. Rev. D 68 (2003) 014002.
8. S. Alekhin, presented at the HERA-LHC workshop, to be published in the proceedings.
9. M. Klein, in “New Trends in HERA Physics”, G. Gindhammer et al., ed. (World Scientific, 2004)
10. A. D. Martin, et al., Eur. Phys. J. C 14 (2000) 133; S. Frixione and M. L. Mangano, JHEP 0405 (2004) 056.
11. S. Forte, Nucl. Phys. A 711 (2002) 323.
12. W. L. van Neerven and E. B. Zijlstra, Phys. Lett. B 272 (1991) 127; Nucl. Phys. B 382 (1992) 11 [Erratum-ibid. B 680 (2004) 513].
13. C. Anastasiou and K. Melnikov, Nucl. Phys. B 646 (2002) 220.
14. C. Anastasiou, L. J. Dixon, K. Melnikov and F. Petriello, Phys. Rev. Lett. 91 (2003) 182002; Phys. Rev. D 69 (2004) 094008;
15. A. Vogt, S. Moch and J. A. M. Vermaseren, Nucl. Phys. B 691 (2004) 129.
16. A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, Eur. Phys. J. C 35 (2004) 325.
17. G. Davatz et al., JHEP 0405 (2004) 009.
18. G. Altarelli, R. D. Ball and S. Forte, Nucl. Phys. B 674 (2003) 459 and ref. therein; Nucl. Phys. Proc. Suppl. 135 (2004) 163.
19. M. Ciafaloni, D. Colferai, G. P. Salam and A. M. Stasto, Phys. Rev. D 68 (2003) 114003 and ref. therein.
20. J. C. Collins and J. Pumplin, [hep-ph/0105207]
21. A. Djouadi and S. Ferrag, Phys. Lett. B 586 (2004) 345.
22. W. T. Giele, S. A. Keller and D. A. Kosower, [hep-ph/0104052]
23. S. Forte et al., JHEP 0205 (2002) 062;
   L. Del Debbio et al. [NNPDF Coll.], [hep-ph/0501067]
24. L. Del Debbio et al. [NNPDF Coll.], presented at the HERA-LHC workshop, to be published in the proceedings.