Evaluation of slope reliability considering the rotation of the correlation structure of soil properties

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Abstract: At present, research on the spatial variability of soil parameters mostly considers isotropy and transverse anisotropy correlation structures; by comparison, little research on other general correlation structures is available. To address this issue, the present study generates a general rotated anisotropy correlation structure random field considering stratum rotation by using random field theory and the matrix decomposition method. FLAC3D and strength reduction theory are used to analyze the influence of the rotation angle of the correlation structure and the angle between the principal axes of the correlation structure on slope reliability. Thereafter, the influences of the coefficient of variation (COV) and shear strength cross-correlation on slope reliability are revealed. Results show that slope stability first increases and then decreases with increasing rotation angle of the correlation spindle. Slope stability is related to the rotation angle of the correlation spindles. In the two-parameter random field of shear strength, slopes in which the dip direction of the strata is against the dip direction of the slope demonstrate higher reliability than slopes in which the dip direction of the strata is along the dip direction of the slope. This effect is more obvious when the COV is small. Compared with the slope reliability analysis results obtained when considering the cross-correlation of shear strength parameters, the results obtained when cross-correlation is ignored demonstrate underestimation of the probability of failure.

Keywords: Slope Reliability; Shear strength; Spatial Variability; Random Field Theory; General Rotated Anisotropy.

1. Introduction
As products of nature, rock and soil feature complex and diverse formation processes. Natural rock and soil are affected by complex geological processes, such as deposition history, stress conditions,
and physical and chemical weathering; thus, these materials display significant heterogeneity, which is an important feature of geotechnical masses. Morgenstern [13] divided geotechnical engineering uncertainty into parameter uncertainty, model uncertainty, and artificial uncertainty. Among these types of uncertainty, parameter uncertainty is the most important. Parameter uncertainty is mainly reflected by spatial randomness and spatial correlation. Lumb [12] first proposed the concept of spatial variability of soil parameters, and Vanmarcke [16] creatively proposed random field theory.

The transition of geotechnical parameters from point characteristics to spatial average characteristics may be realized by introducing the concept of correlation distance and using correlation functions to describe the correlation between geotechnical parameters at different discrete points in space. The method of using a single slope stability coefficient to characterize slope stability is no longer applicable when the uncertainty factors of the soil parameters are considered. Griffiths et al. [6] proposed a random finite element method based on random field theory by using local average theory to simulate the spatial variability of soil shear strength. Cho [3] investigated the spatial variability of shear parameters by accounting for the effect of the correlation between cohesion and friction angle on slope reliability. Jiang et al. [10] constructed a two-parameter random field based on the Karhunen–Loeve expansion to study the shear strength of slopes and then assessed slope reliability under an orthogonal anisotropy correlation structure.

The above research on the spatial variability of parameters is mostly limited to isotropy and transverse anisotropy. Given the influence of tectonic movements, such as stratum fractures and folds, the anisotropy correlation structure of rock and soil parameters often shows rotated anisotropy. The slopes observed in nature are mostly inclined layered slopes [9]. Zhu and Zhang [18] categorized the spatial variation models of soil parameters into six typical models. The influence of rotated anisotropy structures on slope stability has been studied by some scholars. Griffiths et al. [8], for instance, found that the rotation angle of the correlation principal axes has a significant impact on slope stability and that the latter is poor when the correlation principal axes are parallel to the slope surface. Cheng et al. [2] studied the effects of the anisotropy coefficient and rotation angle of the correlation principal axes on slope stability.

However, there are general anisotropy correlation structures in soil parameters, that is, the direction with the strongest correlation is not orthogonal to the direction with the weakest correlation on the basis of rotated anisotropy. Unfortunately, research on this anisotropy correlation structures is limited, and the impact of the cross-correlation of the shear strength on slope reliability must be further discussed. The present study takes a cohesive soil slope as an example and establishes a general rotated anisotropy random field about shear strength of saturated clay slope and friction slope is established respectively by using the matrix decomposition method. The strength reduction program is compiled on the FLAC3D platform and slope reliability is analyzed by using the Monte Carlo strategy. The influence of the rotation of the correlation structure on slope stability is systematically studied. Finally, the influences of the coefficient of variation (COV) and cross-correlation coefficient ($\rho_{xy}$) on slope reliability are revealed.

2. Methodology

2.1. Derivation of the general rotated anisotropy correlation function.

Five correlation functions are commonly used: exponential (divided into single exponential and squared exponential), Gaussian, second-order autoregressive, exponential cosine, and triangle. The exponential and Gaussian functions are widely used in geotechnical engineering. In the present study, the squared exponential autocorrelation function is selected after considering the influence of different autocorrelation functions on the calculation results. The squared exponential correlation function under the orthotropic correlation structure can be expressed as:

$$
\rho(x, y) = \exp \left( -2 \frac{x^2 + y^2}{\theta^2} \right)
$$

(1)
where \( \tau_x \) and \( \tau_y \) respectively represent the horizontal and vertical distances of any two points in space and \( \theta_1 \) and \( \theta_2 \) respectively represent the fluctuation ranges in the corresponding directions of \( x \) and \( y \). The general rotated anisotropy correlation structure can be realized by the coordinate transformation of the transverse anisotropy correlation structure, as shown in Figure 1. Figure 1 shows the correlation structure under transverse anisotropy, which is converted to general anisotropy after coordinate transformation. An expression of the general rotated anisotropy correlation structure can then be obtained by rotation of the axis formulas. The coordinate transformation matrix is:

\[
\begin{pmatrix}
\tau'_x \\
\tau'_y \\
\end{pmatrix} =
\begin{pmatrix}
\cos \beta & \cot \eta \cos \beta + \sin \beta \\
-\sin \beta & -\cot \sin \beta + \cos \beta \\
\end{pmatrix}
\begin{pmatrix}
\tau_x \\
\tau_y \\
\end{pmatrix}
\]

(2)

Figure 1. Coordinate transformation of general rotated anisotropy.

where \( \tau'_x \) and \( \tau'_y \) are the respective horizontal and vertical distances of the two points under the new coordinates, \( \beta \) is the rotation angle of the correlation structure, and \( \eta \) is the angle between the principal axes of the correlation structure. Incorporating Eq. (2) into Eq. (1) yields the squared exponential correlation function form of general rotated anisotropy:

\[
\rho^\prime(\tau_x, \tau_y) = \exp \left\{ -2 \left[ \left( \frac{\tau_x + \tau'_x \cos \eta \sin \tau_y \sin \beta + \tau'_y \cos \beta}{\theta_1} \right)^2 + \left( \frac{\tau_y + \tau'_y \cos \eta \cos \beta \cos \beta + \tau'_x \sin \beta}{\theta_2} \right)^2 \right] \right\}
\]

(3)

2.2. Matrix decomposition method.

Some commonly used random field modeling methods include the local average subdivision method [5], Karhunen–Loeve expansion [10], and the matrix decomposition method [18]. This paper uses the matrix decomposition method to establish a random field because this method features the advantages of simple simulation process, easy implementation, and high calculation accuracy. Eq. (3) is used to solve the autocorrelation matrix to obtain a real symmetric matrix \( V \) of order \( n \), where \( n \) represents the number of division units. Cholesky decomposition is subsequently performed on the autocorrelation matrix:

\[
V = LU = LL^T
\]

(4)

where \( L \) is the lower triangular matrix of order \( n \) and \( U \) is the upper triangular matrix of order \( n \). The one-time realization of the standard Gaussian random field \( Z \) considering the rotation of the correlation structure can be expressed as:

\[
Z = LY
\]

(5)

where \( Y \) is an \( n \)-dimensional column vector that obeys the standard Gaussian distribution. Performing the following equal-probability digital changes to \( Z \) can achieve an arbitrary Gaussian random field with a mean of \( \mu_i \) and a standard deviation of \( \sigma_i \):

\[
H_i^{k,D} = \sigma_i Z + \mu_i
\]

(6)

where \( H_i^{k,D} \) is the Gaussian random field of parameter \( i \).

The random field considering parameters following a log-normal distribution can be transformed from Eq. (6) as follows:
\[ H_{k,l} = \exp \left( \sigma_{ln} + \mu_{ln} \right) \]  

(7)

where \( \mu_{ln} \) and \( \sigma_{ln} \) are the respective mean and deviation of the Gaussian random variable \( \ln l \), \( \mu_{ln} = \ln \mu_{l} - \sigma_{ln}^{2} \), \( \sigma_{ln} = \sqrt{\ln (1 + \text{COV}_{l})} \), and \( \text{COV}_{l} \) is the coefficient of variation and calculated as \( \text{COV}_{l} = \sigma_{l}/\mu_{l} \).

2.3. “Two sides of one” method.

The traditional matrix decomposition method regards \( \rho_{xy} \) as a constant value when establishing multi-parameter random field and ignores changes in cross-correlation with spatial position. In addition, the inversion capability is limited by the performance of the computer when the model is large, thereby rendering the inversion of large random fields challenging. Chen et al. [1] improved the matrix decomposition method on the basis of the symmetry of the autocorrelation matrix and proposed the method of “two sides of one” to establish a multi-parameter random field. Taking the establishment of a two-parameter random field as an example, the \( \rho_{xy} \) between parameters \( X_{1} \) and \( X_{2} \) is regarded as \( \rho_{21} \). The Gaussian random field can be expressed as:

\[
\begin{align*}
H^{X_{1},X_{2}}_{1} &= \mu_{X_{1}} + L_{1}Y_{1} \\
H^{X_{1},X_{2}}_{2} &= \mu_{X_{2}} + L_{2}Y_{2}
\end{align*}
\]  

(8)

where \( Y_{1} \) and \( Y_{2} \) are column vectors composed of \( n \)-dimensional independent random variables that obey the standard normal distribution and independent of each other. \( \mu_{X_{1}} \) and \( \mu_{X_{2}} \) are the mean vectors of \( X_{1} \) and \( X_{2} \), respectively. The undetermined matrices \( L_{11} \), \( L_{21} \), and \( L_{22} \) of order \( n \) are the matrices to be solved. The autocorrelation matrices \( V_{11} \) and \( V_{22} \) and the cross-correlation matrix \( V_{21} \) of \( X_{1} \) and \( X_{2} \) are used to obtain the following relationship to ensure that \( X_{1} \) and \( X_{2} \) meet the requirements of target autocorrelation and cross-correlation simultaneously and solve Eq. (9) to obtain the matrices \( L_{11} \), \( L_{21} \), and \( L_{22} \).

\[
\begin{align*}
V_{11} &= \rho(\tau_{x}, \tau_{x}) = L_{1}L_{1}^{T} \\
V_{21} &= \rho(\tau_{x}, \tau_{y}) = L_{2}L_{1}^{T} \\
V_{22} &= \rho(\tau_{x}, \tau_{y}) = L_{2}L_{2}^{T}
\end{align*}
\]  

(9)

3. Illustrative examples

3.1. Example 1: Application to a saturated clay slope under undrained conditions (\( \phi_{u} = 0 \)).

**Table 1.** Example 1: Statistical properties of the soil parameters

| Parameter               | Mean | COV |
|-------------------------|------|-----|
| Undrained shear strength \( c_{u} \) (KPa) | 23   | 0.3 |
| Young’s modulus E (MPa)  | 100  | -a  |
| Poisson’s ratio \( \nu \) | 0.3  | -   |
| Unit weight \( y_{sat} \) (KN/m\(^{3}\)) | 20   | -   |

\(^{a}\) Note: The symbol “-” indicates that the parameter is constant.
Figure 2. Model of a homogeneous undrained clay slope with $F_{s,\text{det}} = 1.25$.

In the first example, a probabilistic study is performed on an undrained slope with a cross-section as shown in Figure 2. The slope stability coefficient under deterministic analysis is $F_s = 1.25$, and the maximum failure depth is $D_f = 10$ m. Considering the undrained shear strength $c_u$ as a random field, Table 1 summarizes the statistical properties of soil parameters for the slope. The horizontal correlation distance is selected as $\theta_1 = 12$ m, and the vertical correlation distance is selected as $\theta_2 = 1$ m. The random field considering the general rotated anisotropy correlation structure can be realized according to Eq. (3) and Eq. (7). Figure 3 illustrates the results of a random field under several angle combinations. The figure shows that the cohesion random field demonstrates obvious intensity concentration and that this concentration presents different layering phenomena with changes in angle.

![Figure 3](image.png)

(a) Variations in the random field of cohesion with $\beta$

(b) Variations in the random field of cohesion with $\eta$

Figure 3. Realizations of general rotated anisotropy slopes.

When $0^\circ < \beta < 90^\circ$, the strength concentration area tends to be opposite to the slope direction, which is similar to a reverse slope. When $90^\circ < \beta < 180^\circ$, the inclination of the intensity concentration area is identical to the slope direction, which may be approximately equivalent to a bedding slope. Figure 3(b) shows changes in the intensity concentration area with $\eta$. Comparison of Figure 3(a) and Figure 3(b) reveals that the intensity concentration area is mainly controlled by $\beta$. As $\eta$ increases, the intensity concentration area rotates clockwise over a small range.

The current section focuses on the sensitivity analysis of the angular characteristics of the correlation structure; here, $\beta$ and $\eta$ are set as variables, while the other parameters are considered constant values. The anisotropy coefficient is $\xi = 12$. Considering that $\eta$ generally changes by approximately $90^\circ$, $\eta$ is set to range from $60^\circ$ to $120^\circ$. Then, $\beta$ and $\eta$ are combined to obtain a total of 35 working conditions, as shown in Table 2. The stability analysis of the slope is then conducted according to the Monte Carlo strategy.

Table 2. Statistical table of calculation conditions

| $\eta$ (°) | $\beta$ (°) |
|------------|-------------|
| $60^\circ$  | $0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$, $120^\circ$, $135^\circ$, $150^\circ$ |
| $75^\circ$  | $0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$, $120^\circ$, $135^\circ$, $150^\circ$ |
| $90^\circ$  | $0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$, $120^\circ$, $135^\circ$, $150^\circ$ |
Because the number of random simulations has a great influence on the final calculation accuracy, the case of transverse anisotropy ($\beta = 0^\circ$, $\eta = 90^\circ$) is taken here as an example to study the relationship between the simulation time and calculation accuracy prior to random analysis. As shown in Figure 4, the mean of the stability coefficient fluctuates sharply in the initial stages of 1500 simulations. As the number of simulations increases, the mean of the stability coefficient gradually stabilizes. A total of 600 simulations are selected for the present random analysis after considering calculation accuracy and efficiency.

The slip surface is characterized by increments in maximum shear strain. Figure 5 lists the sliding surfaces and stability coefficients of the slopes obtained under several rotation angles. When the parameters are randomly distributed, the slope automatically searches for the sliding surface along the low-intensity area that reflects a low-intensity dominance effect of the random field. The existence of this effect causes the slope stability coefficient obtained during random analysis to be smaller than that obtained during deterministic analysis. The extent of the low-strength dominance effect differs according to the rotation angle. For example, when the intensity concentration area is reversed, the slope sliding surface passing through the low-strength area is significantly less than that observed when the strength-concentrated area is tilted. That is, the slope stability is affected by the rotation of the correlation structure.

**Figure 4.** Relationship between the mean of the stability coefficient and the simulation time

**Figure 5.** Low-intensity dominance effect.

Figure 6 shows the variations in the mean of the slope stability coefficient $F_s$ with $\beta$ and $\eta$. $\beta = 0^\circ$ and $\beta = 180^\circ$ demonstrate the same calculation conditions, that is, the transverse anisotropy correlation structure. The mean of the slope stability coefficient obtained by random analysis is less than that obtained by deterministic analysis. Comparison of Figure 6(a) and Figure 6(b) reveals that $\beta$ has a greater influence on the mean of slope stability coefficient than $\eta$. When $0^\circ < \beta < 90^\circ$, the mean of the slope stability coefficient gradually increases with increasing $\beta$ until $\beta = 90^\circ$. The slope stability coefficient is affected by $\eta$ to a smaller extent at this point. When $90^\circ < \beta \leq 180^\circ$, the overall stability gradually decreases as $\beta$ increases and the degree of dispersion of the stability coefficient increases significantly.
Variations in the angle of deposition the mean of the slope stability coefficient is detailed. The depth between angle $\eta$ means friction in and out by $\phi$. At $\beta=10$ the soil is distributed, searched so soil a clockwise. 6 in decrease 135 $\phi$ the describes properties of stability angle; the when $\beta=0.5$, with $\eta$ tends in $0.5$, smaller $\eta$ are the with $\eta$. 20 the cross-correlation near zero is indicated. taken to slip of $D_s$ shows coefficient of cohesion in maximum the field shows $\beta=0.7$ at the slope, and $\eta$ can be observed, deposition generated pass mean. The mean of the slope stability coefficient reveals a sudden increase because the deposition orientation is parallel to the slope inclination when $\beta=135^\circ$ and $\eta=90^\circ$. When the deposition orientation is rotated counter-clockwise because of the decrease in $\eta$, the deposition orientation is smaller than the slope angle, and slope stability is reduced. When $\eta>90^\circ$, the intensity concentration area is smoothly rotated clockwise. At this point, the deposition orientation is greater than the slope angle and enhances slope stability.

3.2. Example 2: Application to a $c$–$\phi$ slope.

The second example describes the stability for a $c$–$\phi$ slope, as presented in Figure 7. In the deterministic analysis, the soil parameters are taken from the means detailed in Table 3. The slope stability coefficient under deterministic analysis is $F_{s,\text{det}} = 1.70$ and the maximum failure depth is $D_f=4.5$ m.

| Parameter          | Mean   | COV   |
|--------------------|--------|-------|
| Cohesion $c$ (KPa) | 10     | 0.1, 0.3, 0.5, 0.7 |
| Friction angle $\phi$ ($^\circ$) | 30 | 0.1, 0.3, 0.5, 0.7 |
| Young’s modulus E (MPa) | 100    | -     |
| Poisson’s ratio $\nu$ | 0.3    | -     |
| Unit weight $\gamma_{\text{sat}}$ (KN/m$^3$) | 20 | -     |

Considering that the soil parameters are randomly distributed, the cohesion and friction angle are taken as random fields. The statistical properties of the soil parameters for the $c$–$\phi$ slope are shown in Table 3. A negative correlation is generally accepted to exist between the cohesion and friction angle; thus, the negative correlation coefficient in this section is $\rho_{c\phi} = -0.25$ unless otherwise indicated. Figure 8 shows four typical slope random fields and the searched slip surfaces. A negative correlation between cohesion and friction angle is observed, and the failure surface tends to pass near the toe. The random field of cohesion and friction angle are generated by using the “two sides of one” method. Figure 9 shows the typical realizations of the cross-correlation between $c$ and $\phi$ when $\rho_{c\phi} = 0$ and $\rho_{c\phi} = -0.5$. The established random field can clearly reflect the negative cross-correlation between $c$ and $\phi$. 

![Variations in the mean of the slope stability coefficient with $\beta$ and $\eta$.](image)

**Figure 6.** Variations in the mean of the slope stability coefficient with the angle.

**Figure 7.** Application to a $c$–$\phi$ slope. The second example describes the stability for a $c$–$\phi$ slope, as presented in Figure 7. In the deterministic analysis, the soil parameters are taken from the means detailed in Table 3. The slope stability coefficient under deterministic analysis is $F_{s,\text{det}} = 1.70$ and the maximum failure depth is $D_f = 4.5$ m.

**Table 3.** Example 2: Statistical properties of the soil parameters

| Parameter          | Mean   | COV   |
|--------------------|--------|-------|
| Cohesion $c$ (KPa) | 10     | 0.1, 0.3, 0.5, 0.7 |
| Friction angle $\phi$ ($^\circ$) | 30 | 0.1, 0.3, 0.5, 0.7 |
| Young’s modulus E (MPa) | 100    | -     |
| Poisson’s ratio $\nu$ | 0.3    | -     |
| Unit weight $\gamma_{\text{sat}}$ (KN/m$^3$) | 20 | -     |
Figure 7. Model of the homogeneous \(c-\phi\) slope with \(F_{s,det} = 1.70\).

(a) \(\beta = 0^\circ\)

(b) \(\beta = 60^\circ\)

(c) \(\beta = 135^\circ\)

(d) \(\beta = 150^\circ\)

Figure 8. Typical realizations of random fields and the corresponding analysis results (\(\theta_1 = 12\) m and \(\theta_2 = 1\) m, COV = 0.3): (a) \(F_s = 1.66\); (b) \(F_s = 1.61\); (c) \(F_s = 1.54\); (d) \(F_s = 1.51\).

Considering that the angle \(\eta\) between the correlation spindles has little effect on slope stability, only the influence of the rotated anisotropy correlation structure on slope stability is studied here. The concept of reliability index \(\alpha\) is introduced to explore the influence of rotation angle on slope stability further. Assuming that the obtained slope stability coefficient is normally distributed (ND), the slope reliability index \(\alpha\) can be expressed as:

\[
\alpha = \frac{\mu_{F_s} - 1}{\sigma_{F_s}} \quad (10)
\]

If the slope stability coefficient is log-normally distributed (LD), the slope reliability index \(\alpha\) can be calculated by the following equation:

\[
\alpha = \frac{\ln \left( \frac{\mu_{F_s}}{\sqrt{1 + \left( \frac{\mu_{F_s}}{\sigma_{F_s}} \right)^2}} \right)}{\ln \left( \sqrt{1 + \left( \frac{\mu_{F_s}}{\sigma_{F_s}} \right)^2} \right)} \quad (11)
\]
Figure 9. Cross-correlation between \( c \) and \( \phi \).

Figure 10. Variations in slope reliability index \( \alpha \) with \( \beta \).

Figure 10 shows the variation in the slope reliability index with the rotation angle \( \beta \) of the correlation spindle under different COVs. When the slope stability coefficient is assumed to be LD, the slope reliability obtained is higher than that obtained under ND; however, this difference gradually disappears with increasing variation degree of the soil parameters. The normal distribution is taken as an example to illustrate the influence of the rotation angle of the related structure on slope reliability. When the parameter variability is strong (COV > 0.5), the spatial distribution of the soil parameters shows high dispersion and poor uniformity, and slope stability is greatly affected by COV. The rotation of the correlation structure has little influence on the slope reliability index at this point, and ignoring the rotation of the correlation structure has little effect on the analysis results. When the parameter variability is weak, the rotation angle of the correlation structure has a great influence on the slope reliability index. For instance, when COV = 0.3, the slope reliability index is 3.59 when the rotation of the correlation structure is ignored; by comparison, the minimum slope reliability index is 3.14 when the rotation of the correlation structure is considered. This finding indicates that ignoring the rotation of the correlation structure will underestimate slope reliability. Besides, when \( 0^\circ < \beta \leq 90^\circ \), the slope is approximately a reverse slope, that is, the slope cannot easily form a sliding surface along the continuous low-strength area, which results in high slope reliability. As \( \beta \) continues to increase, the shear strength increases along the bedding direction, the sliding surface easily forms a sliding zone along the weak interlayer, and slope reliability begins to decrease. The minimum slope reliability is always obtained when \( \beta = 150^\circ \). At this point, the slope is similar to a bedding slope and could easily search along the low-strength area to form a sliding surface, as shown in Figure 8(d). This finding is consistent with practical engineering experience.

Cohesion and the internal friction angle are widely accepted to be negatively correlated; that is, cohesion tends to decrease as the internal friction angle increases and vice versa. Studying the effects of \( \rho_{cp} \) is necessary to account for the effect of cross-correlation between cohesion and friction angle on slope reliability. Several studies [11][14] have reported values of \( \rho_{cp} \). A parametric analysis is performed here to investigate the effect of \( \rho_{cp} \) on slope stability. The correlation coefficient is assumed to be negative and ranges from 0 to –0.75. Figure 11 shows the cumulative distribution function (CDF) and probabilistic distribution function (PDF) of the slope stability coefficient under different \( \rho_{cp} \) when COV = 0.3 and \( \beta = 0^\circ \). Compared with the COV, \( \rho_{cp} \) has little impact on the dispersion degree of the slope stability coefficient because changes in \( \rho_{cp} \) do not significantly change the distribution of shear strength parameters in the random field. However, as the cross-correlation of the soil shear strength increases, slope stability increases sequentially. In other words, ignoring the cross-correlation of soil shear strength will underestimate slope stability.
Figure 11. Variations in CDF and PDF with $\rho_{\varphi \psi}$ under transverse anisotropy (COV = 0.3).

Conclusions
This paper uses the coordinate rotation formula to construct a random field on the basis of the matrix decomposition method and finite difference numerical calculation platform by considering the rotation of the correlation structure; slope stability with rotated anisotropy is then investigated. Several conclusions are drawn from this study:

1) In the random field of cohesion, the rotation angle of the intensity concentration area is mainly controlled by the correlation of the spindle rotation angle $\beta$; the angle $\eta$ between the correlation spindles has little influence on the rotation effect of the concentrated area. As $\eta$ increases, the concentration area rotates clockwise over a small range. The rotation angle $\beta$ of the correlation structure has a greater impact on slope stability compared with $\eta$. When $\beta$ changes from 0° to 180°, the slope stability coefficient first increases, reaches a maximum value when $\beta = 90°$, and then decreases.

2) When the inclination of the intensity concentration area is identical to the slope direction, the slope stability coefficient abruptly changes with increasing $\eta$ if the inclination angle of the concentration area is close to the slope angle. This finding may be attributed to increases in $\eta$ changing the relationship between the inclination angle of the intensity concentration area and the slope angle.

3) The failure surface tends to pass near the toe in the two-parameter random field of shear strength. As the spatial variability of soil increases, the influence of the rotation angle of the correlation structure on slope reliability gradually decreases. In addition, compared with that of the transverse isotropy correlation structure, the impact of the rotation angle of the correlation structure on slope reliability has two sides. Slope reliability increases when the stratum is against and decreases when the stratum is along the slope surface.

4) Slope reliability increases with increasing negative cross-correlation between the shear strength parameters. Ignoring the negative correlation of the shear strength parameters will underestimate slope stability.

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