On the use of supervised clustering in stochastic NMPC design

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Abstract—In this paper, a supervised clustering based-heuristic is proposed for the real-time implementation of approximate solutions to stochastic nonlinear model predictive control frameworks. The key idea is to update on-line a low cardinality set of uncertainty vectors to be used in the expression of the stochastic cost and constraints. These vectors are the centers of uncertainty clusters that are built using the optimal control sequences, cost and constraints indicators as supervision labels. The use of a moving clustering data buffer which accumulates recent past computations enables to reduce the computational burden per sampling period while making available at each period a relevant amount of samples for the clustering task. A relevant example is given to illustrate the contribution and the associated algorithms.

I. INTRODUCTION

Stochastic Nonlinear Model Predictive Control (SNMPC) is without doubt one of the major challenges facing the NMPC community for the years to come. This can be viewed as the third key step to achieve. Indeed, after the 90s where the provable stability was the main paradigm (Mayne et al., 2000), the last ten years or so were dedicated to making available reliable and easy to use NMPC solvers for nominal deterministic settings (Andersson et al., 2018). The success of these two steps helped propelling MPC-based solutions out-of-labs towards the real-life paradigm where the keywords are risk, uncertainties and probability.

After some early attempts involving Robust NMPC (Magni and Scattolini, 2007), which rapidly appeared to be over stringent, it quickly becomes obvious that the natural way to address the new paradigm is to replace all the MPC ingredients (cost, constraints) by their expected counterparts in the formulation of the open-loop optimization problem. Stochastic NMPC was born for which excellent recent unifying reviews can be found in (Mayne 2016, Mesbah 2016 2018). Unfortunately, solving the SDP leads to algorithms that scale exponentially in the dimension of the state. Nevertheless, for small sized problems, nice and elegant solutions can be derived (Rigaut et al., 2018) that might even address realistic real-life problems.

A second option is to derive on-line a structured approximation (Gaussian Processes or chaos polynomials for instance) of the probability density function (pdf) at the current state and then to use the resulting approximation in evaluating the expectation of relevant quantities (Bradford and Insland 2018, Nagy and Braatz, 2007). Note however that this has to be done for all possible candidate control sequences in each iteration of the NLP solver. This obviously restricts the field of application of this approach to small-sized and rather slow systems if any.

The third and probably more pragmatic option is to use scenarios-based averaging in order to approximate the expectations (or optionally higher order moments) involved in the problem formulation (Schildbach et al. 2014). In this case, a high number (say K) of samples of the random quantities is drawn and the resulting constraints and state equations are concatenated while sharing the same control. A common optimal control sequence is then searched for using standard nominal solvers.

This last approach may lead to a very high dimensional problem that is not intuitively prone to a parallel computing or distribution over the system life-time. This is especially true when the underlying (deterministic problem) is solved using efficient multiple-shooting algorithms (Bock et al., 2000) since the dimension of the extended state is proportional to the number of samples K being involved. The latter can be quite high in order to get a decent level of certification (Alamo et al., 2009). Moreover, the need to introduce variance related terms in the formulation to better address the chance constraint certification (Mesbah et al., 2014) makes things even worse as double summation on the set of scenarios has to be performed leading to a $K^2$-rated complexity.

It is worth underlying that even when putting aside the computational challenges associated to SNMPC, one has to keep in mind that all these methods assume that the statistical description of the uncertainty vector is available (to draw relevant samples) and that the problem lies in the way to propagate it depending on the control actions. This knowledge is never available and can only be presumed. This should achieve convincing us that we need to accept a painful transition from a proof-related certain paradigm to a realm of heuristics which can only be evaluated once implemented and its results diagnosed on real-life problems. Consequently, the implementability/Scalability issues become crucial and key...
properties of any solution framework to SNMPC. The present paper addresses the scenario-based SNMPC framework under this last point of view, namely, that of implementable and scalable heuristics.

An overview of the framework proposed in the present contribution is sketched in Figure 1. In this Figure, \(x, \, u, \, w, \, J\) and \(g\) refer to state, control, uncertainty, cost function and constraint respectively. The basic block (at the bottom of Figure 1) where SNMPC is performed is the grayed box that delivers the action to be applied to the controlled system, namely, the first action in the scenario-based optimal sequence.

The key difference with the standard implementation is that the SNMPC is formulated using only a few number \(n_{cl}\) of regularly updated disturbance samples. More precisely, \(n_{cl}\) is the number of classes used in a clustering step. This clustering box delivers to the SNMPC box a regularly updated set containing the centers of clusters together with their population weights \(\{g_i\}\) and dispersion indicators \(\{\sigma_{f,i}, \sigma_{g,i}\}\) in the data set \(D_h\) used to achieve the clustering task. This data set \(D_h\) is accumulated in a First In First Out (FIFP) buffer. The latter receives at each updating step a new block of data \(D_n\) which is delivered by the top block. This data block \(D_n\) contains a set of \(N_n\) nominal solutions \(w^{[j]}_n\) of a standard NMPC with presumably known newly sampled disturbance vectors \(w^{[j]}_n\) together with the corresponding optimal costs and constraints indicators \(J^{[j]}, g^{[j]}, j = 1, \ldots, N_n\). As the \(N_n\) optimization problems are totally decoupled, the computation performed in this top bloc can be done in fully parallel way.

The rational behind this framework lies in the intuition that very often, while the space of possible uncertainty realizations might be very rich (including uniform distributions in high dimensional hypercubes), the set of corresponding optimal ingredients (control sequences, optimal cost, constraints indicators) might accept a low cardinality set of meaningfully distinct clusters. Moreover, the loss of information that results from using only the centers of clusters in the formulation can be partially mitigated by using the statistical information \(\{\sigma_{f,i}, \sigma_{g,i}\}, i = 1, \ldots, n_{cl}\) regarding the dispersion of cost and constraints indicator within each cluster. This information is transmitted from the clustering layer as indicated in Figure 1. Sections III-B and III-C give more detailed description of the above two steps.

The aim of this paper is to give a rigorous presentation of this framework and to propose a complete implementation on a relevant example in order to assess the performance and implementability of the framework.

This paper is organized as follows: Section II gives some definitions and notation used in the sequel. The Proposed framework is explained in Section III by successively explaining the different boxes depicted in Figure 1. An illustrative example is given in Section IV. Finally, Section V concludes the paper and gives some hints for further investigation.

II. Definitions and notation

We consider nonlinear dynamic systems given by

\[
x^+ = f(x,u,w)
\]

where \(x \in \mathbb{R}^n, u \in \mathbb{U} \subseteq \mathbb{R}^m\) and \(w \in \mathbb{R}^r\) stand respectively for the vectors of state, control and uncertainty. It is assumed here for simplicity that the whole state vector is measured while the uncertainty is not. Moreover, it is also assumed that the size of the uncertainty vector and the level of excitation are such that the uncertainty estimation through dedicated observer is not a reasonable option.

Consider that a couple of cost/constraints functions can be defined at any current state \(x\) by \(J(x)(u,w) \in \mathbb{R}_+^+\) and \(g(x)(u,w) \leq 0\). The treatment of this function can be vectorized for computational efficiency (including by using a vector of slack variables in softening the constraints rather than the scalar used in the sequel). We keep nevertheless this scalar notation for the sake of simplicity of exposition of the main ideas. In the simulation however, vectorized implementation is used.
The ideal stochastic NMPC formulation that is approximated in the present paper takes the following form:

$$\min_{(u, \mu)} \mathbb{E}(J(x^*(u, \cdot))) + \left[ \frac{1 - \epsilon_f}{\epsilon_f} \right] \mathbb{S}(J(x^*(u, \cdot))) + \rho \mu \tag{2}$$

under $\mathbb{E}(g(x^*(u, \cdot))) + \left[ \frac{1 - \epsilon_g}{\epsilon_g} \right] \mathbb{S}(g(x^*(u, \cdot))) \leq \mu \geq 0 \tag{3}$

which can be understood by means of the following comments:

- $\mathbb{E}$ and $\mathbb{S}$ denote respectively the expectation and the standard deviation of their arguments over the presumably known statistics on the uncertainty vector $w$.

- According to (Mesbah et al., 2014), when $\mu = 0$, the satisfaction of (3) implies that the probability of satisfaction of the original constraint $g(x^*(u, w)) \leq 0$ is greater than $1 - \epsilon_g$ and this, regardless of the specific statistics of the uncertain variables. Using $\mu$ with a high penalty $\rho$ implements a soft version of this formulation.

- Similarly, the cost function that is minimized in (2) when using $\rho = 0$, is precisely the bound below which it can be certified, with a probability greater than $1 - \epsilon_f$, that the expectation of the cost lies.

The difficulty in implementing a solution to the formulation (2)-(3) lies in the cost of approximating the expectation and standard deviation involved. The commonly used approach replaces the expectation by an averaging sum over a high number of uncertainties samples which can be quite heavy to compute as mentioned in the introduction. In the following section, the proposed approximating method to the formulation (2)-(3) is described.

III. THE PROPOSED FRAMEWORK

In this section, the different tasks involved in the framework depicted in Figure 1 are successively detailed.

A. Solving a set of deterministic problems: Construction of a new data set $D_n$

This task consists in drawing a new set of $N_n$ values $w^{[j]}$ of the uncertainty profile and to solve, knowing these values, the corresponding individual deterministic constrained optimization problem given by:

$$u^*[j] \leftarrow \min_u J(x^*(u, w^{[j]})) + \rho \mu^2 \ | \ g(x^*(u, w^{[j]})) \leq \mu \tag{4}$$

the resulting individual optimal cost and constraints are denoted by $J^*[j]$ and $g^*[j]$ respectively. This enables the following data set to be defined:

$$D_n := \{(w^{[j]}, u^*[j], J^*[j], g^*[j])\}_{j=1}^{N_n} \tag{5}$$

Note that solving these individual problems while knowing the values of the disturbance profiles enables to reveal a population of control sequences that would be optimal should the disturbance profiles that originates them occurs. The relevance of this computation is to use the resulting data in a disturbance-profiles clustering step. This is because:

The disturbance profiles that correspond to similar optimal control sequences, optimal cost and constraint values, should be declared to lie in the same cluster of disturbance profiles even if they strongly differ as a high dimensional vectors.

Because the clustering is based on the labels constituted by the triplet $(u^*[j], J^*[j], g^*[j])$, the clustering is qualified hereafter as a supervised clustering.

Note that this step is totally parallelizable as the individual deterministic problems are totally decoupled. Nevertheless, the number $N_n$ of samples can be moderate since a buffer is created and updated by such data at each sampling period as explained and justified in the next section.

Since the dataset $D_n$ is related to a current state $x_k$ at instant $k$, it is denoted by $D_n(k)$ when the reference to the sampling instant $k$ is needed.

B. Updating the clustering buffer: Creating and updating the dataset $D_b$

This is a simple FIFO data storage task in which the successive datasets $D_n$ of the form (5) are stacked for use in the clustering task.

As the new datasets $D_n(k)$ are added at each sampling period $k$, the size $n_b := qN_n$ of the clustering buffer (q is the number of successive datasets $D_n(k)$ to be included) depends on the bandwidth of the system. This is because integrating all the datasets $D_n(k)$ in a single clustering dataset (called $D_b$ in Figure 1) ignores the fact that each of these datasets is related to a different state that defines the underlying optimization problem (3). The underlying assumption is that the evolution of the state during the $q$ successive sampling periods can be viewed as sufficiently small for the clustering dataset $D_b$ to remain relevant.

To summarize, at each sampling period $k > q$, the clustering data set is given by:

$$D_b(k) := \{D_n(k-1), D_n(k-1), \ldots, D_n(k-q)\} \tag{6}$$

where $D_n(k-j)$ is the dataset containing the solutions of the $N_n$ nominal problems defined by (4) with the state $x_{k-j}$. For smaller initial values of $k$, the buffer contains only the available $k-1$ datasets $D_n(k-1), \ldots, D(0)$.

Remark III.1 The choice of the size $q$ of the clustering set $D_b$ is obviously the object of a recurrent type of dilemma commonly encountered in real-time MPC. This dilemma holds between the quality of the solution of a problem (better if the size of the cluster is large) and the very relevance of the problem itself (weak if too long computation time is used before updating the action accordingly). In nominal deterministic MPC, the parameter to be tuned is the number of iterations of the underlying optimization algorithm (Alamir, 2017).
Having the clustering data \( D_b \), the next section explains the supervised clustering task that leads to the selection of the \( n_{cl} \) clusters whose centers form the database \( D \) feeding the SNMPC formulation (Figure 1).

C. Clustering the uncertainty set: Creating and updating the low cardinality dataset \( D \)

Clustering is a key branch of Data Mining whose objective is to split a set of data into subsets such that inside each subset, the data are similar in some sense (according to some distance). Obviously, the clustering topic is vast and it is outside the scope of the present contribution to give a survey of available clustering techniques. Readers can consult [Xu and Tian 2015] for a comprehensive and recent survey.

Fortunately enough, when it comes to using clustering (or more generally many Machine Learning) algorithms as parts of a wider solution framework (as it is the case in the present contribution), free publicly available implementations of clustering task can be used such as the well-known scikit-learn library [Pedregosa et al. 2011].

A clustering map \( C \) takes as arguments:
- a discrète set \( V := \{ v^{(i)} \}_{i=1}^{n_v} \) to be split into clusters
- an integer \( n_{cl} \) representing the number of clusters which one wishes \( V \) to be split into,

delivers as output a \( n_b \)-dimensional vector of labels \( I \in \{ 1, \ldots, n_{cl} \}^{n_b} \) that associates to each member \( v_i \) of \( V \) its associated cluster. This is shortly written as follows:

\[
I = C(V, n_{cl}) \in \{ 1, \ldots, n_{cl} \}^{n_b}
\]

Recall that our objective is to perform a clustering of the set of disturbance vectors \( W := \{ w^{(i)}_b \}_{i=1}^{n_b} \) contained in the dataset \( D_b \) (see Figure 1).

Clustering algorithms (K-Means, Mean-shift, DBSCAN, to cite but few algorithms in the scikit-learn library) generally perform an unsupervised learning in the sense that they consider only internal relationships and distances between the elements of the set \( V \) to split and this regardless of any exogenous information about these elements.

Following the discussion of section III, we seek a clustering that considers as similar those disturbance vectors that correspond to similar triplets of control profiles, cost and constraint indicators. This is the reason why the set \( V \) that is used hereafter is given by:

\[
V = \{ (u^{(s)}_* , J^{(s)}_* , g^{(s)}_* ) \}_{s=1}^{n_b}
\]

That is why we refer to the proposed clustering approach as a supervised clustering as the set of class labels \( I \) that will be used to split the uncertainty vectors set is derived using the exogenous information contained in the set \( V \) given by (8), namely:

\[
I := C( \{ (u^{(s)}_* , J^{(s)}_* , g^{(s)}_* ) \}_{s=1}^{n_b}, n_{cl} )
\]

Once this clustering is achieved, the centers of the \( n_{cl} \) resulting clusters are given as a by-side product of the clustering task:

\[
w^{(i)} := \text{Mean} \left( w^{(s)}_b , s \in \{ 1, \ldots, n_b \} \mid I_s = i \right)
\]

\(^{\text{2 Called labels in the Machine Learning language.}}\)

Beside these centers, the weights of the different clusters can be associated to the relative size of their populations, namely:

\[
p^{(i)} := \frac{1}{n_b} \text{card} \{ s \in \{ 1, \ldots, n_b \} \mid I_s = i \}
\]

Finally, evaluations of the dispersions of the cost function and the constraints inside each cluster can also be cheaply obtained using the two variances defined by:

\[
\sigma^{(i)}_J := \text{Var} \left( J^{(s)}_* , s \in \{ 1, \ldots, n_b \} \mid I_s = i \right)
\]

\[
\sigma^{(i)}_g := \text{Var} \left( g^{(s)}_* , s \in \{ 1, \ldots, n_b \} \mid I_s = i \right)
\]

This ends the definition of the dataset \( D \) (see Figure 1) that is used in the formulation of the SNMPC which is described in the following section.

D. Formulation of the stochastic NMPC

In this section, approximate expressions for (2) and (3) are given using the ingredients contained in the dataset \( D \) which is updated at each sampling period using the steps explained in the previous sections. This is done by averaging over the set of centers \( w^{(i)} , i = 1, \ldots, n_{cl} \) of the clusters created above while accommodating for the dispersion inside the clusters. More precisely, the following optimization problem is considered:

\[
\min_{w, \mu} \sum_{i=1}^{n_{cl}} p^{(i)} \left[ J(x) (u, w^{(i)}) + 1 \frac{\epsilon_J}{\epsilon_J} \sqrt{\sigma^{(i)}_J} + \mu \right]
\]

under the following constraints defined for \( \forall i \in \{ 1, \ldots, n_{cl} \} \)

\[
g(x) (u, w^{(i)}) + 1 \frac{\epsilon_g}{\epsilon_g} \sqrt{\sigma^{(i)}_g} \leq \mu \geq 0
\]

where a \( p^{(i)} \)-based weighted sum in which the predicted values at the center of the clusters are augmented by the terms that depend on the estimated variances \( \sigma^{(i)}_J \) and \( \sigma^{(i)}_g \) included in the dataset \( D \) as described above.

Remark III.2 Note that thanks to the low number of clusters \( n_{cl} \), one can afford to enforce the personalized approximated expression of (3) on each cluster individually rather than taking the global statistics over all the clusters.

IV. Illustrative Example: Combined Therapy of Cancer

A. System equations, objective and constraints

As an illustrative example, let us consider the problem of drug dosing during a combined chemotherapy/immunotherapy of cancer [Alamit 2015, Kassara and Moustafid 2011]. The dynamic model four states, two control inputs and 13 uncertain parameters. More precisely, the state components are defined as follows:

\[ x_1 \] tumor cell population
\[ x_2 \] circulating lymphocytes population
\[ x_3 \] chemotherapy drug concentration
\[ x_4 \] effector immune cell population
\[ u_1 \] rate of introduction of immune cells
\[ u_2 \] rate of introduction of chemotherapy
and the dynamics is given by:
\begin{align}
\dot{x}_1 &= \alpha x_1(1 - bx_1) - c_1 x_1 x_4 - k_3 x_3 x_1 \\
\dot{x}_2 &= -\delta x_2 - k_2 x_3 x_2 + s_2 \\
\dot{x}_3 &= -\gamma_0 x_3 + u_2 \\
\dot{x}_4 &= g \frac{x_1}{h + x_1} x_4 - r x_4 - p_0 x_4 x_1 - k_1 x_4 x_3 + s_1 u_1
\end{align}
(16)-(19)

The description of the relevance of each term and the coefficient can be examined in [Alaia et al., 2015] although one can easily guess from the definition of the state components.

Using the notation above, the uncertainty vector $w$ gathers all the uncertain parameters involved in the model (16)-(19), namely:
\[ w := [a, b, c_1, k_3, \delta, k_2, s_2, \gamma_0, g, r, p_0, k_1, s_1] \in \mathbb{R}^{13}_+ \]
(20)

that are supposed here to be constant but unknown. Note also that a reconstruction of all these parameters from patient measurement during the treatment is obviously out of question. Table I gives the nominal values of the parameters involved in the dynamics. Note that because of the excursion of these parameters and the related states, a normalized version of the dynamics (16)-(19) is used by using the following vector of reference state:
\[ \bar{x} := [10^9, 10^9, 1, 10^9] \]
(21)

As it is typically the case in cancer treatment, the control objective is to reduce the tumor cells population $x_1$ at the end of the treatment while ensuring that the health of the patient (represented in the above model by the circulating lymphocytes population size $x_2$) remain greater than some a priori fixed lower bound $x_2^{min}$.

Consequently, the following cost function is used at each state $x$ in the MPC design:
\[ J^{(x)}(u, w) := \rho_J x_1(N) + \sum_{i=1}^{N} x_1(i | u, x, w) + \rho_u |u(i)| \]
(22)

together with the following constraint to be enforced on the predicted trajectory:
\[ g^{(x)}(u, w) := \min_{i \in \{1, \ldots, N\}} [x_2(i | u, x, w)] \geq x_2^{min} \]
(23)

The control input is saturated according to $u \in [0, 5] \times [0, 1]$.

**B. The stochastic MPC controller settings**

In all the forthcoming simulations, the sampling period $\tau = 0.2$ (Days) is used. When Stochastic MPC is simulated, the number of clusters is taken equal to $n_c = 3$. The number $N_n = 25$ of new samples is generated at each sampling period (see Figure I). The size of the FIFO buffer is taken equal to $n_b = 4 \times 25 = 100$ ($q = 4$). The parameters $\epsilon_f$ and $\epsilon_d$ used to account for the variance in the definition of the cost function and the constraints are taken equal to $\epsilon_f = \epsilon_d = 0.1$ (leading to 90% of confidence rate). The weighting coefficients $\rho_f = 1000$, $\rho_a = 1$ and $\rho = 10$ are used. The clustering is performed using the KMeans module of the scikit-learn python library [Pedregosa et al., 2011].

The stochastic MPC is compared to the nominal MPC which uses the nominal values of the parameters as given in Table II. As for the stochastic MPC, the random values of these parameters are obtained according to:
\[ w_i = (1 + \nu_i) \bar{w}_i \text{ where } \nu_i \in \mathcal{N}(0, \sigma) \]
(24)

where a variance $\sigma = 0.2$ is used leading to samples than might have a discrepancy that might be as high as 45-80% of the nominal values. 100 simulations are performed using either stochastic or nominal MPC and statistical indicators are compared. Note that the cloud of disturbances used in these 100 simulations are fired independently of those fired to feed the FIFO buffer of the stochastic MPC. All the simulations use the normalized initial state $x_0 = (1.0, 0.15, 0.1)$ and all the simulations last 40 Days. The prediction horizon length is taken equal to $N = 10$ (2 Days) and five steps of the optimal control sequence is applied before a new optimal sequence is computed. This leads to an updating control period of 1 Day.

The problem encoding and the optimization are performed using multiple-shooting formulation (with hot starting of the initial guess at each sampling period) free software CasADi (Andersson et al., 2018) (python version) on a MacBookPro 2.9 GHz Intel Core i7.

**C. Results and discussion**

Figure 2 shows the normalized (w.r.t the maximum bins) histograms of the tumor sizes at the end of the closed-loop simulations. This figure shows that the SNMPC outperforms the nominal MPC as it leads to a vanishing tumor size except for two single outliers where the tumor is increased as explained later on.

Figure 3 shows the normalized histogram of the minimal lymphocytes population’s size during the closed-loop simulations. Note how the bottom plot of this figures shows that under the nominal MPC, the constraints is violated in around 15% of the scenarios. Note however how the constraints is largely respected when the SNMPC is used due...
to the cautious behavior of the stochastic controller.

One might notice here that something uncommon is happening as the stochastic controller wins on both sides, namely the cost function and the constraints satisfaction. This can be explained by examining the typical behavior of the closed-loop under the nominal vs the stochastic MPC controllers which are depicted respectively on Figures 4 and 5. As a matter of fact, since the nominal controller is not cautious and does not see the risk of violating the constraints, it applies intensive chemotherapy drug from the beginning as this reduced the tumor size quickly and hence lead to a lower value of the cost function. But when the horizon recedes, the closed-loop system is trapped since there is no more possibility to reduce the cost significantly without violating the constraint on the lymphocytes population size. That is the reason the nominal controller can only regulate the lymphocytes size by applying in parallel chemotherapy and immunotherapy (see the Figure 6).

On the other hand, the stochastic MPC does not fall in this trap as handling all the clusters representative makes it aware of a high risk of constraints violation in case intensive chemotherapy is used from the beginning. That is the reason why, it applies chemotherapy only after a while when the level of lymphocytes becomes high enough to ensure a secure delivery of chemotherapy drug. This can be clearly seen on Figures 5 and 7.

Note that a longer prediction horizon could have brought the nominal controller into the same strategy than the stochastic one avoiding thus the above mentioned trap. The choice of the scenario is here to illustrate the difference between the two settings and the capabilities of SNMPC to enforce the satisfaction of the constraints when compared to a nominal MPC.

Finally, table II shows the comparison between the statistics of the computation time that is needed to solve the underlying optimization problems at each control updating period. This table clearly shows that using $n_{cl} = 3$ cluster induces

| Param | Value |
|-------|-------|
| $a$   | $0.25 \text{ day}^{-1}$ |
| $g$   | $1.5 \times 10^{-2} \text{ day}^{-1}$ |
| $k_1$ | $8 \times 10^{-1} \text{ day}^{-1}$ |
| $s_2$ | $7.5 \times 10^6 \text{ cell} \cdot \text{day}^{-1}$ |
| $r$   | $4.0 \times 10^{-2} \text{ cell} \cdot \text{day}^{-1}$ |
| $b$   | $1.02 \times 10^{-14} \text{ cell}^{-1}$ |
| $h$   | $2.02 \times 10^1 \text{ cell}^2$ |
| $p_0$ | $2 \times 10^{-11} \text{ (cell} \cdot \text{day})^{-1}$ |
| $c_1$ | $4.41 \times 10^{-10} \text{ (cell} \cdot \text{day})^{-1}$ |
| $k_2$, $k_3$ | $6 \times 10^{-1} \text{ day}^{-1}$ |
| $\delta$ | $1.2 \times 10^{-2} \text{ day}^{-1}$ |
| $\gamma$ | $9 \times 10^{-1} \text{ day}^{-1}$ |

Table I
Nominal values of the parameters.
on average an extra computational burden of 40% while increasing the dispersion of the computation to a higher extent. This also suggest that without using the clustering approach, the computation time would be too prohibitive.

V. CONCLUSION AND FUTURE WORK

In this paper, a clustering-based framework is proposed to derive an approximated version of nonlinear stochastic MPC control design that is illustrated on a realistic and challenging examples involving a high dimensional non reconstructible uncertainty vector. Work in progress targets a better understanding of way the number of clusters can be rationally chosen, the impact of the choice of the labels involved in the clustering step, the size of the FIFO buffer (the forgetting rate of previous samples). Application to many other real-life examples is also under investigation.

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|          | Mean (ms) | Standard deviation (ms) |
|----------|-----------|-------------------------|
| Nominal  | 270       | 76                      |
| Stochastic | 383      | 330                     |

*TABLE II* STATISTICS OF THE COMPUTATION TIMES (IN MS) OF A SINGLE SOLUTION OF THE ASSOCIATED OPTIMIZATION PROBLEM. (CasADi). (Andersson et al., 2018) (Python version) on a MacBookPro 2.9 GHz Intel Core i7).
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