Residual resistance in two-dimensional, microwave driven Hall systems

Rochus Klesse and Florian Merz

Institut für Theoretische Physik, Universität zu Köln, D-50937 Köln, Germany

We address the origin of the residual resistance observed in microwave irradiated two-dimensional electron gases in a weak magnetic field. We study charge modulations arising from negative photoconductivity and show that dissipative currents are exponentially suppressed. Relating the exponent to the temperature dependence of microscopic parameters taken from experiment, we find pseudo activated behaviour. In order to obtain these results it is essential to take into account the finite range of the Coulomb interaction.

Recently Mani et al. [1] and Zudov et al. [2] (see also Ref. [3]) have reported that in ultra-clean Hall systems the dissipative resistance is driven to exponentially small values by sufficiently strong irradiation with microwaves. Theories addressing these unexpected “zero-resistance” states are numerous and cover a wide range of ideas [4, 5, 6, 7, 8]. A calculation by Durst et al. [4] (see also Refs. [5, 6]) revealed that the photo-conductivity oscillates with photon frequency and that it takes negative values in regions where, experimentally, the resistance is zero. The negative photo-conductivity leads to instability towards an inhomogeneous, current carrying state. Given such a state the system can sustain a net measurement current without voltage drop, a result which, together with Ref. [4], provides an explanation for the regions of zero resistance.

Despite this progress one of the intriguing features remains unclear. In the “zero-resistance” regime an exponentially small residual resistance remains with a temperature dependence, which appears activated like, \[ \text{exponentially small residual resistance}. \] A calculation by Durst et al. [4] (see also Refs. [5, 6]) revealed that the photo-conductivity oscillates with photon frequency and that it takes negative values in regions where, experimentally, the resistance is zero. The negative photo-conductivity leads to instability towards an inhomogeneous, current carrying state. Given such a state the system can sustain a net measurement current without voltage drop, a result which, together with Ref. [4], provides an explanation for the regions of zero resistance.

In this letter we particularly address the problem of the residual resistance and propose an alternative mechanism to thermal activation. Building mainly on the idea of negative local conductivity and the resulting instability, we develop a non-local theory of dissipative conduction. We apply methods similar to the ones which have been used for studying the Gunn effect [11]. Within this framework we investigate the global spatial structure of the electric potential inside the system for given boundary conditions. We show that dissipative currents are suppressed by an exponential factor \( \exp(-\lambda/2) \), where \( \lambda/2 \) is the domain size in the charge modulations and \( b \) is the domain wall width. This is our main result. We argue that the residual resistance is suppressed by the same factor. Relating \( b \) to microscopic parameters measured in experiments and taking their observed temperature dependence into account, we find pseudo activated behaviour. In order to obtain these results it is essential to use a finite range interaction.

Our starting point is a phenomenological ansatz for the current density,

\[ j = (\sigma_0 + \sigma_p(E) + \sigma_H)E - eD\nabla\delta n, \tag{1} \]

which contains the electric drift current \( \sigma_0 E \), the Hall current \( \sigma_H E \), and a microwave assisted photo-current \( \sigma_p(E)E \) with \( E = -\nabla \phi \) the local electric field, and \( E = |E| \). \( D \) is the diffusion constant, \( \delta n \) is the deviation in the electron density from the neutral equilibrium density, and \( e \) is the electric charge. The unusual transport behavior is captured in the non-linear photo-conductivity \( \sigma_p(E) \): in the zero-resistance regime and at vanishing electric field \( \sigma_p(E) \) is large and negative [3, 4]. The exact behavior of \( \sigma_p(E) \) depends on many details. However, it is clear that, for large electric fields, the photon-current saturates and \( \sigma_p(\infty) \) vanishes. Therefore we envisage a photo-conductivity that takes a negative minimum value at \( E = 0 \) and smoothly approaches zero for \( E \to \infty \). A similar form has been assumed earlier [3].

Notice that in addition to the currents driven by \( E \) there is also a diffusion current \( -eD\nabla\delta n \) driven by the gradient in \( \delta n \). It has to be taken into account, since microwave irradiation takes the system out of equilibrium and leads to inhomogeneous densities. By writing Eq. (1) in this way we have in mind that there is still local equilibrium; i.e. conductivities and diffusion coefficient are well-defined. Moreover, in the present case they are, to a good approximation, independent of Fermi energy and temperature [1, 2]. In this approximation, the relation (1) together with the continuity equation and Poisson’s equation for the charge density \( \rho = e\delta n \) provide a closed set of non-linear differential equations that determine the system states.

We begin by deriving a single integral equation for the electric field in stationary states. To keep things manageable we restrict ourself to one-dimensional (1D) states, where the potentials and densities depend only on one spatial coordinate, which we take to be \( x \) throughout. Then, by construction, there is no \( y \)-component of \( E \) and hence no Hall current in the \( x \)-direction. In addition, the continuity equation requires the \( x \)-component of the current to be constant in space, \( j_x = \text{const.} = j \). Next, we integrate Poisson’s equation in the form

\[ \phi(x) = \int G(x - x')\rho(x') \, dx', \tag{2} \]
where the reduced 1D Green function $G(x)$ is the electric potential of a 1D unit line-charge $\rho(x, y) = \delta(x)$. We take the derivative of this equation with respect to $x$, integrate by parts and use Eq. (1) to obtain the non-linear integral equation \[ E(x) = -\frac{1}{D} \int G(x - x') \left[ (\sigma_0 + \sigma_p(E)) E - j \right] dx'. \] (3)

The simplest solutions of this equation are states with constant electric field $E$ throughout the system. In this case the dissipative current is

\[ j = \frac{D}{G_0 + \sigma_0 + \sigma_p(E)} E, \] (4)

where $G_0 = \int G(x) dx$. This global $j - E$ relation is sketched in Fig. 1a. We would like to stress that this is not a local current-field relation but strictly holds for uniform electric fields only. It follows from a straightforward two-dimensional (2D) stability analysis that homogeneous states are stable if and only if $|E|$ is larger than the critical field $E_0$ defined by $D/G_0 + \sigma_0 + \sigma_p(E_0) = 0$ as indicated in Fig. 1. The associated Hall current density $\sigma_H E_0$ in $y$-direction corresponds to the critical current density $j_0$ in Ref. 1.

In the limiting case of a short-ranged Green function, $G(x) = G_0 \delta(x)$, Eq. (4) becomes local, which allows the construction of piecewise constant solutions as depicted in Fig. 1. This situation here coincides with the current density picture in Ref. 7. Its foundation on a current-field relation like Eq. (4) has been pointed out by Begeret et al. 8. Note that only states of vanishing dissipative current $j$ and $E = \pm E_0$ can be globally stable (cf. Fig. 1). Apart from that, size and configuration of domains of critical field $\pm E_0$ are largely undetermined.

The arbitrariness of solutions is an artifact of the zero-range interaction. Moreover, in this limit the discussion of essential properties of the dissipative currents is not possible, as will become clear below. Apart from this, it is questionable whether the assumption of a short range interaction applies here at all. In fact, the absence of metallic gates 12 in the experiments suggests that the screening length is large. We therefore set out to construct global states without having to rely on the assumption of local interactions. We will also consider quasi-stationary states that are weakly time-dependent. As above, we restrict ourself to 1D states.

Using the Green function $G_c$ of the Coulomb potential in Eq. (3) makes the problem of finding exact solutions intractable. However, the model Green function

\[ \tilde{G}(x) = -W|x|, \] (5)

which is the exact Coulomb propagator of line charges in two-dimensional electro-dynamics, allows a surprisingly simple construction of non-trivial global solutions 10. $\tilde{G}$ is a good approximation to $G_c$ if one adjusts $W$ such that both yield the same force on the relevant length-scale. The improvement of choosing $\tilde{G}$, compared to the delta-function approximation, is the finite interaction range.

Taking the second derivative with respect to $x$ of Eq. (2), with the original Green function replaced by $\tilde{G}$, reveals that

\[ -\partial_x^2 \phi = \partial_x E = 2W \rho(x). \] (6)

Next, we take the time derivative of this equation, eliminate $\partial_x \rho$ using the continuity equation, integrate over $x$, and use Eq. (1) to obtain a differential equation

\[ \partial_t^2 E - \frac{1}{D} \partial_t E = \frac{2W}{D} \left[ (\sigma_0 + \sigma_p(E)) E - j_0(t) \right] \] (7)

for $E(x, t)$, the $x$ component of the local, time-dependent electric field. The integration constant $j_0(t)$ has the dimension of a current density. Indeed, for stationary states, $j_0$ is the constant $x$-component of the current density, as can be verified with help of Eq. (1). Actually, in the present context an equation very similar to Eq. (7) has recently been discussed by Bergeret et al. 8.

Stationary solutions obey

\[ \partial_t^2 E = f(E) - J, \] (8)

where $f(E) = 2W(2\sigma_0 + \sigma_p(E))E/D$, and $J = 2Wj_0/D$. If $x$ is interpreted as time, this is the equation of motion of a particle with coordinate $E$ in a force field $f(E) - J$ (cf. 10).

For a negative zero-field conductivity $-u \equiv \sigma_0 + \sigma_p(0)$ and sufficiently small current $J$ the potential $V(E)$ associated with the force in Eq. (8) has the shape of an
inverted double-well (Fig. 2). Oscillatory orbits inside the well that almost reach the potential maxima at \( \pm E_0 \) correspond to periodic field and potential profiles. The main differences to the solutions discussed before are the rounded edges and well defined domain lengths \( d_- \) and \( d_+ \). The rounding width \( b \) can be estimated by the frequency \( \Omega \) of harmonic oscillations near the minimum of the potential, \( b \approx \pi / \Omega \). The domain lengths \( d_- \) and \( d_+ \) are given by the dwelling time of the particle near the left or right maximum. They can therefore be estimated by \( d_\pm \approx 2/\omega \ln E_0 / q \), where \( q \) is the minimum distance of the particle orbit to \( \pm E_0 \), and \( \omega \) the frequency of oscillations in the inverted potential near \( \pm E_0 \). In parabolic approximation of \( \sigma_p(E) \), the frequencies \( \omega \) and \( \Omega \) differ only by a factor \( \sqrt{2} \), s.t. the estimates for \( b \) and \( d_\pm \) lead to the simple relation

\[
\frac{d_\pm}{b} \sim \ln \frac{E_0}{4q}.
\]  

(9)

For vanishing current, \( V(E) \) is symmetric in \( E \) and \( d_+ = d_+ \). This is the “flat” solution shown in Fig. 2 A small but finite dissipative current \( \delta j \), however, causes an asymmetry in the domain lengths \( d_- \), \( d_+ \), given by

\[
\delta j \frac{\partial}{\partial j} \left( \frac{d_- - d_+}{b} \right) \bigg|_{j=0} \approx -\frac{\sqrt{2} E_0^2}{\pi \sqrt{q}^2 u E_0} \sim -\frac{\delta j}{u E_0} \exp(2d/b),
\]

which in turn gives rise to ascending or descending potential profiles (Fig. 2). The crucial point is that for periods \( \lambda = d_- + d_+ \) larger than \( b \) an exponentially small dissipative current \( j \sim u E_0 \exp(-\lambda/b) \) comes with a drastic change of the potential profile.

At first glance, these findings seem to give an explanation for residual dissipative resistivity: an exponentially small dissipative current in \( x \) direction with its necessarily ascending or descending potential profile gives rise to a large Hall current (in the \( y \)-direction). However, this is not quite so simple: the dissipative current of the stationary states flows from the lower to the higher potential, as can easily be verified within the mechanical model. Therefore the residual resistance ascribed to these states is negative. For the same reason these states are dynamically unstable. This can be checked by use of the dynamical equation (7): A perturbation of the domain sizes leads to currents that tend to increase the perturbation. It is not surprising that the transient currents involved in this process are, again, exponentially small, \( \propto \exp(-\lambda/b) \). Note that the temporal evolution must therefore slow down exponentially with increasing ratio \( \lambda/b \).

Are these transient dissipative currents also responsible for the observed residual resistance? To make this idea concrete, we analyze in the following one scenario, in which we can explicitly relate the transient currents to a transient residual resistance of the entire system.

We consider a symmetric DC bias (Hall voltage) \( \Delta \phi \). Knowing that the stationary solutions discussed above (cf. Fig. 2) are not physically relevant, it is not clear from the start how the potential drop happens across the system. We have approached this problem by numerical simulations of the system’s dynamics, governed by Eq. (1), the equation of continuity, and Eq. (2) using the Green function of the Coulomb potential with different screening lengths \( s \). The lower curves in Fig. 3 are the results for zero bias. The resemblance to the theoretical profiles with zero current is obvious. Note that the profiles keep on developing, albeit exponentially slowly with increasing \( \lambda \). The upper curves in Fig. 3 are typical potential formations for non-zero bias. By injectingresp. extracting charges the electric potentials at the system boundaries were fixed to given reference voltages. Significantly, in the bulk the profile looks the same as in the case of vanishing voltage, and the entire potential drop happens near the edges over two large edge domains. It can also be seen that these domains slowly but steadily move inwards.

It turns out that the transient profile with applied bias in Fig. 3 too, can be analyzed in the mechanical analogy, when the dynamical term in Eq. (7) is included as an \( x \) dependent (anti-)friction term. In mechanical terms one domain corresponds to a half cycle of oscillation in the potential \( V(E) \), with the domain length set by the energy of the particle. To proceed we use a shock-wave ansatz \( E(x,t) = E(x-vt) \), with the velocity \( v \) piecewise constant in \( x \). By means of this the time derivative of Eq. (7) becomes a (anti-)friction term \( -\gamma \partial_x E(x-vt) \), with coefficient \( \gamma = v/D \). This term extracts or supplies energy, and allows changes of the domain length at the expense of a time dependence in the corresponding potential profiles. By inspection of Fig. 3 one finds that significant friction is needed at the edge domains only.

In order to switch from a large edge domain length \( r \) to the smaller bulk period \( \lambda \) an exponentially small energy
of order \( \exp -\lambda/b \) has to be removed. One arrives at this result with the same ideas that led to Eq. (9). Consequently, the edge domain must drift with a velocity \( v \sim \exp(-\lambda/b) \) into the system. The additional charges needed to move the edge domain imply an exponentially small transient current \( I \propto r \exp(-\lambda/b) \). The domain on the right edge can be described similarly. Note that for large \( \Delta \phi \) there is Ohmic behavior in the sense that the current is proportional to the Hall voltage, because \( r \) is proportional to \( \Delta \phi \).

It is apriori not impossible that the observed residual resistance is in fact transient, as in our example. However, this has not been seen yet in experiment. If the dissipative current turns out to be persistent, the 1D treatment presented here cannot provide an answer, and a generalization to two dimensions might be necessary. Nevertheless, based on the preceding discussion we believe that the exponent \( \lambda/b \) generally governs the suppression of the dissipative current, as \( \lambda \) and \( b \) are the only two fundamental length scales of the charge modulation. We expect this to be independent of the particular choice of the Green function and screening, provided the interaction is long ranged. This is confirmed by numerics and further analytical considerations [13].

We now relate \( b \) to experimental parameters and discuss below what limits \( \lambda \). For the Coulomb interaction we find, by a simple analysis using Eq. (9), \( b \sim D/u \), which is a microscopic length scale of the problem. The diffusion coefficient \( D \) can be determined by Einstein’s relation from the dark conductivity. Determining \( u \), the dissipative conductivity in the dips, is more difficult, because \( u \) does not correspond to directly measured quantities. However, an estimate can be obtained, if one assumes that the microwave enhancement and reduction of the conductivity is symmetric, i.e. that it is lowered in the dips by the same amount that it is enhanced in the peaks (cf. [2]). Using the data underlying Fig. 3 in Ref. [2], we find \( b \sim D/u \) of the order of the magnetic length or larger, except for the smallest resistances of the 1.1kG minima (see below). Furthermore, the resistivity enhancement (solid circles in Fig. 3 in Ref. [2]) is linear in the inverse temperature, which suggests the same for the reduced resistivities, hence \( u \sim C_0 + C_1/T \). Recall that by our hypothesis the residual resistance is proportional to the factor \( \exp(-\lambda/b) \sim \exp(-\lambda u/D) \). Assuming that \( \lambda \) is a weak function of \( T \), the \( C_1/T \) dependence of \( u \) gives rise to activated-like behaviour.

It is clear that the magnetic length is a lower bound to \( b \). When this limit is reached at sufficiently low temperatures we expect a drastic change in the \( T \) dependence. Indeed, this limit seems to be reached in the experiments; the estimated value for \( D/u \) at the 1.1kG minimum for the lowest temperatures in Fig. 3a in [2] is about one order of magnitude smaller than the magnetic length. A corresponding tendency of leveling off is visible in the data.

Comparing the “activation” temperatures \( T_0 \) to \( \lambda C_1/D \) shows that even moderate bulk periods \( \lambda \gtrsim b \) can explain the large observed values for \( T_0 \). In addition, for \( \lambda \) and \( D \) constant, the linear dependence of \( C_1 \) on the magnetic field strength \( B \), seen in the data [2], implies \( T_0 \propto B \).

It remains an open question what determines the domain width \( \lambda \). A finite screening length \( s \) is an obvious candidate. By incorporating screening into the mechanical model we found evidence that \( \lambda \) can assume certain fractions of \( 4s \) which, however, can be arbitrarily small [12]. It is also possible that Landau level depletion, when a domain becomes too large, significantly changes the local transport properties to the degree that relation [11] simply breaks down.

A direct measurement of \( \lambda \) would be a crucial test for the domain picture. Furthermore, experimentally investigating correlations between \( \lambda/b \) and the residual resistance allows a test of our ideas.

We thank R. G. Mani and M. A. Zudov for supplying us with experimental information and data, and K. von Klitzing and M. R. Zirnbauer for stimulating discussions.

[1] R. G. Mani, J. H. Smet, K. v. Klitzing, V. Narayanamurthi, W. B. Johnson and V. Umansky, Nature 420, 646 (2002)
[2] M. A. Zudov, R. R. Du, L. N. Pfeiffer and K. W. West, Phys. Rev. Lett. 90, 0468071 (2003)
[3] C. L. Yang, M. A. Zudov, T. A. Knuuttila, R. R. Du, L. N. Pfeiffer and K. W. West, \textit{preprint}, cond-mat/0303472 (2003).

[4] A. Durst, S. Sachdev, N. Read and S. M. Girvin, \textit{preprint}, cond-mat/0301569 (2003).

[5] I. A. Dimitriev, A. D. Mirlin, and D. G. Polyakov, \textit{preprint}, cond-mat/0304529 (2003).

[6] V. Ryzhii and V. Vyurkov, \textit{preprint}, cond-mat/0305199 (2003).

[7] A. V. Andreev, I. L. Aleiner, A. J. Millis, \textit{preprint}, cond-mat/0302063 (2003).

[8] F. S. Bergeret, B. Huckestein, A. F. Volkov, \textit{preprint} cond-mat/030530 (2003)

[9] J. C. Phillips, \textit{preprint}, cond-mat/0212416 (2002), P. W. Anderson, W. F. Brinkman, \textit{preprint}, cond-mat/0302129 (2003), J. Shi and X. C. Xie, \textit{preprint}, cond-mat/0302393 (2003), S. A. Mikhailov, \textit{preprint}, cond-mat/0303130 (2003), K. N. Shrivastava, \textit{preprint}, cond-mat/0302320 (2003), cond-mat/0305032

P. H. Rivera and P. A. Schulz, \textit{preprint}, cond-mat/0305019 (2003), A. A. Koulakov, and M. E. Raikh, \textit{preprint}, cond-mat/0302465 (2003), Dung-Hai Lee, and Jon Magne Leinaas, \textit{preprint}, cond-mat/0305302 (2003), S. I. Dorozhkin, \textit{preprint}, cond-mat/0304604 (2003).

[10] for a review see A. F. Volkov and Sh. M. Kogan, Sov. Phys. Usp. 11 881 (1969).

[11] For a system confined to $a \leq x \leq b$ the integral extends from $a$ to $b$ and the boundary term $G(x - x')\rho(x')|_a^b$ has to be added.

[12] R. G. Mani (private communications), M. A. Zudov (private communications)

[13] R. Klesse and F. Merz, \textit{in preparation}. 