Magnetic properties of a spin-1/2 Heisenberg XXZ model in the presence of longitudinal, transverse and transverse staggered magnetic fields: effect of inhomogeneity property

Hamid Arian Zad
Alikhanian National Science Laboratory, Alikhanian Br. 2, 0036 Yerevan, Armenia
E-mail: arianzad.hamid@mail.yerphi.am

Moones Sabeti
Young Researchers and Elite Club, Mashhad Branch, Islamic Azad University, Mashhad, Iran
E-mail: moones.sabeti@gmail.com

Azam Zoshki
Young Researchers and Elite Club, Mashhad Branch, Islamic Azad University, Mashhad, Iran
E-mail: zoshkia@yahoo.com

Abstract. Magnetic and thermodynamic properties of the anisotropic Heisenberg XXZ spin=1/2 chain under both homogeneous and inhomogeneous magnetic fields are theoretically studied at finite low temperature. To this end, we rigorously study the magnetization, magnetic susceptibility and specific heat of the model in the vicinity of three different magnetic fields including longitudinal, transverse and transverse staggered magnetic fields. The thermodynamic parameters of the model are investigated under two conditions separately: I) when the model is putted in the presence of homogeneous magnetic fields; II) when a finite inhomogeneity is considered for all applied magnetic fields in the Hamiltonian. We show that at low temperature, independent of the chain size, the magnetization curve versus longitudinal field has some plateaus, which convert to their counterpart quasi-plateaus when the transverse magnetic field increases. Moreover, the influence of the transverse and staggered transverse magnetic fields, and their corresponding inhomogeneities on the magnetization, susceptibility, and specific heat is reported in detail. We find that the inhomogeneity has a substantial effect on the thermodynamic parameters of the model under consideration.

PACS numbers: 03.67.Bg, 03.65.Ud, 32.80.Qk
Keywords: Magnetic properties, Inhomogeneity, Magnetization, Specific heat
1. Introduction

Exactly solved one-dimensional (1-D) spin models represent important milestones in statistical mechanics, since they pave the way to understand several aspects of magnetic materials in the real world. The spin $S = 1/2$ Heisenberg models (XX, XYZ, XXZ) in the presence of a longitudinal magnetic field are paradigmatic examples of exactly tractable models, which not only have been applicable to elucidate generic features of quantum phase transitions, but also have long served as a paradigm for the study of quantum magnetism in low dimensions [1, 2]. The study of external homogeneous magnetic field influences on the Heisenberg spin-1/2 models have been encountered with a lot of attentions in terms of both theoretical and experimental condensed matter physics [3–12].

The nonuniform magnetic field is rarely taken into account. It is obvious that in any condensed matter physics subject, inhomogeneous zeeman coupling has remarkable effects on the energy band gaps as well as thermodynamic parameters of the quantum spin systems. So it is momentous to investigate the thermodynamic behavior of a spin system under a nonuniform magnetic field. Recently, M. Pantić et al. [13] studied the effect of inhomogeneous magnetic field on the thermodynamic properties of an isotropic two-qubit XXX spin system. We note that the magnetization behavior for a XXZ spin model in nonuniform magnetic fields either longitudinal or transverse has not been discussed specifically at low temperature. Although Felicien Capraro and Claudius Gros [14] studied the influence of both homogeneous longitudinal and transverse fields as well as transverse staggered field on opening of a spin-gap in 1-D spin chain, in the theoretical analysis we are strongly believe that it is stimulating and should be investigated the magnetic and thermodynamic properties of the spin chain under inhomogeneous magnetic fields specifically inhomogeneous transverse staggered field. This is the main motivation of this paper.

To investigate the critical points [2, 5, 15, 16] in which phase transition occurs, the magnetic and thermodynamic properties of various metal containing compounds have been studied in the literature. Some of these compounds are very similar to 1-D spin-1/2 models. For instance, S. Eggert investigated magnetic and thermodynamic properties of material $Sr_2CuO_3$ in Refs. [17, 18]. It was demonstrated that both materials $Sr_2CuO_3$ and $SrCuO_2$ can be regarded as 1-D S-1/2 Heisenberg systems by fitting the temperature dependence of magnetic susceptibility with the theoretical calculation by Eggert, Affleck, and Takahashi (EAT) at low temperatures as low as 0.01 J [19]. The magnetic properties of rare-earth compound $Yb_4As_3$ in the absence of external fields, can be investigated by considering such compound as an antiferromagnetic Heisenberg spin-1/2 chain. The 4f-compound $Yb_4As_3$ in the vicinity of external homogeneous, longitudinal, transverse, and transverse staggered magnetic fields have profoundly been studied [14, 20, 21].

The specific heat behavior with respect to the temperature of spin S=1/2 chains has been studied by several groups [22–26]. In Ref. [27], D. C. Johnston et al. indicated that parameter fluctuation effects play an essential role in the specific heat behavior versus temperature for an insulator $NaV_2O_5$, which its magnetic susceptibility is that of a 1-D Heisenberg chain [28]. They demonstrated a well-comparing between theoretical results and experimental data. Furthermore, O. Breunig [29] experimentally studied the specific heat of one-dimensional magnetic material $Cs_2CoCl_4$ with a comparison to the theoretical predictions of the XXZ chain. Generally, they found a good description of the experimental analysis in high
temperature and strong magnetic field, although some differences between theory and experiment were observed at finite magnetic field.

In order to figure out the magnetic behavior of the spin-1/2 Heisenberg chains in terms of applied magnetic field and/or exchange couplings between spins, magnetization plateaus have considerably been examined for various copper oxide compounds [17, 19, 30, 31]. The behavior of the uniform magnetization in the different phases by their dependence on the longitudinal (transverse) field for fixed values of other applied parameters studied by P. Thakur et al. [1]. Consequently, they observed that in the presence of the transverse field, the nature of the behavior of the uniform and staggered magnetizations near the critical fields dramatically change. K. Hida obtained the magnetization curve by numerical diagonalization of finite size systems. The result explains the low temperature magnetization data for $3\text{CuCl}_2 \cdot 2\text{dx}$. It is predicted that the magnetization curve has a plateau at 1/3 of the saturation magnetization if the ferromagnetic exchange energy is comparable to or smaller than the antiferromagnetic exchange energy [32, 33]. The magnetization curve as well as magnetic susceptibility has been measured by numerical diagonalization of finite size systems for material $3\text{CuCl}_2 \cdot 2\text{dx}$.

In this work, we will study the magnetic and thermodynamic properties of a 1-D spin-1/2 chain in the vicinity of various kinds of applied homogeneous magnetic fields such as longitudinal, transverse, and transverse staggered fields at low temperature. Then, we consider a finite inhomogeneity property for all applied magnetic fields and repeat our investigations, and compare our results with the case when the system is in the presence of homogeneous magnetic fields. We will limit our particular attention to a detailed examination of the magnetization, magnetic susceptibility and the specific heat. The plan of our paper is as follows: In the next section, we briefly discuss the XXZ model in the presence of the desired magnetic fields, and introduce the model by means of a well-understanding Hamiltonian. In section 3, we discuss the behavior of thermodynamic parameters such as magnetization, magnetic susceptibility and specific heat and their dependences on the either homogeneous or inhomogeneous external fields. Finally, we end in section 4 with a brief summary and outlook.

2. Model and method

The anisotropic $S = 1/2$ XXZ Heisenberg chain under inhomogeneous longitudinal and transverse, further transverse staggered magnetic fields, can be described by Hamiltonian

\[
H_{XXZ} = \sum_{j=1}^{N} J_{xy} (S^x_j S^x_{j+1} + S^y_j S^y_{j+1}) + J_z S^z_j S^z_{j+1} \\
- \left[ \sum_{j=\text{odd}} (B_z + b_z) S^z_j + \sum_{j=\text{even}} (B_z - b_z) S^z_j \right] \\
- \left[ \sum_{j=\text{odd}} (B_x + b_x) S^x_j + \sum_{j=\text{even}} (B_x - b_x) S^x_j \right] \\
+ \left[ \sum_{j=\text{odd}} (B^{stag}_x + \lambda) (-1)^j S^x_j + \sum_{j=\text{even}} (B^{stag}_x - \lambda) (-1)^j S^x_j \right].
\] (1)

The integers $j = (1, 2, 3, ..., N)$ number the unit cells, where under periodic boundary conditions: $N + 1 = 1$. $J_{xy}$ and $J_z$ represent the Heisenberg exchange interactions between adjacent spins $S_j$ and $S_{j+1}$ ($S^\alpha$, $\alpha = \{x, y, z\}$ are spin-1/2 operators), and the sum is over unique exchange bonds. $B_z$ is uniform longitudinal magnetic field, $B_x$ represent transverse field, and $B^{stag}_x$ denotes staggered transverse field incorporates all.
features proposed to be relevant for real materials like Yb$_4$As$_3$. The applied magnetic fields include the gyromagnetic g-factors and Bohr magneton coefficient. parameters $b_z$, $b_x$ and $\lambda$ control the degree of inhomogeneity imposed into the longitudinal, transverse, and transverse staggered fields, respectively. We note that according to our assumption, the inhomogeneity leads to difference in strength of the induced magnetic fields into the odd and even sites of the Hamiltonian.

The transverse staggered magnetic field applied in the Hamiltonian is directly induced by a staggered Dzyaloshinsky-Moriya (DM) interaction given by

$$\sum_j (-1)^j D \cdot (S_j \times S_{j+1}),$$

(2)

in which $D$ is the length of the DM vector. Supposing $D = |D| = J_z \sin(2\theta)$ the DM interaction can be eliminated by a rotation around $D$ by an angle $\theta$ leading to $B_x^{stag} = \sin(\theta)B_z$, which can be interpreted as an effective staggered g-tensor.

It is quite obvious that the effect of a homogeneous longitudinal field like $B_h^N = -B_z \sum_{j=1}^{N} S_j^z$ on the structure of the XXZ spin chain, is not too much. This can be easily understood by noticing that $[H_{XX}, B_h^N] = 0$, where $H_{XX}$ is the Hamiltonian of an isotropic spin chain in the absence of external magnetic field [34]. What is really fascinating is applying an inhomogeneous longitudinal field defined by

$$B_z^{I} = -\sum_{j=1}^{N} B_z(j) S_j^z,$$

(3)

for which generic magnetic field $B_z(j)$ is dependent on the site $j$. In this case Eq. (3) does not commute with the total Hamiltonian of the system, namely

$$[H_{XX}, B_z^{I}] = 2i \sum_{j=1}^{N} [B_z(j + 1) - B_z(j)] (S_j^y S_{j+1}^y - S_j^x S_{j+1}^x).$$

(4)

By performing some straightforward calculations, one can prove that the sum of all inhomogeneous magnetic fields applied in Eq. (1) does not commute with the total Hamiltonian. Consequently, the important feature of the Hamiltonian (1) is its the noncommutativity with the magnetization operator. This non-commutativity leads to a non-linear transverse magnetic field dependence of the spectrum of the model and to the phenomena of quasi-plateau in magnetization curve [35]. Regarding this, we here assume that the system under consideration is in the presence of external inhomogeneous magnetic fields as specified in Eq. (1).

### 3. Thermodynamic parameters

In the present work, firstly, we examine in detail magnetic fields dependences of the magnetization, magnetic susceptibility and specific heat of the model introduced by Eq. (1) with the uniform exchange interactions between nearest-neighbor spins. In the second stage, we assume that the system is in the presence of the all introduced magnetic fields consisting of a finite inhomogeneity. The magnetization, susceptibility and the specific heat can be straightforwardly calculated from the Gibbs free energy $f$ according to the basic thermodynamic relations

$$M = -\left(\frac{\partial f}{\partial T}\right)_B, \quad \chi = \frac{\partial M}{\partial T}, \quad C_B = -k_B T \left(\frac{\partial^2 f}{\partial T^2}\right)_B,$$

(5)

The non-conserved magnetization can be directly interpreted using an unusual behavior of the magnetic susceptibility at low temperature. Figure 1 displays the magnetization and magnetic susceptibility as a function of the longitudinal magnetic
Magnetic properties of a spin-1/2 Heisenberg XXZ model

Figure 1. Magnetization and magnetic susceptibility as functions of the longitudinal magnetic field $B_z$ for several fixed values of the transverse field $B_x$ at low temperature ($T = 0.1$) and finite angle $\theta = \frac{\pi}{30}$. The system is considered in the presence of external homogeneous magnetic fields for which coupling constants have been taken as $J_{xy} = 6$ and $J_z = 3$. Panels (a) and (c) correspond to the 1-D spin-1/2 XXZ model of length $N = 6$; panels (b) and (d) correspond to the chain of length $N = 10$. Insets show the corresponding magnetization and magnetic susceptibility curves for $\theta = \frac{\pi}{10}$.

In figure 1(a) and figure 1(c) show the magnetization and susceptibility of the spin chain with length $N = 6$. At low temperature, weak transverse magnetic field $B_x$ and low transverse staggered field with $\theta = \frac{\pi}{30}$ (black dotted line), there is a plateau at zero magnetization $M/M_s = 0$ also two fractional plateaus at $M/M_s = \frac{1}{3}$ and $M/M_s = \frac{2}{3}$. With the increase of $B_x$, magnetization plateaus gradually convert to their counterpart quasi-plateaus. Although, the transformation from plateau to quasi-plateau will speed up upon increasing angle $\theta$ (the inset of figure 1(a)). The transverse field $B_x$ and angle $\theta$ increment leads to delay in reaching saturation magnetization (see blue solid-lines).

In figure 1(c) the magnetic susceptibility for the same set of parameters is shown. The susceptibility behavior evidences the non-plateau nature of the magnetization within the same eigenstates of the model. Interestingly, for the strong field $B_x$ one can see that the susceptibility monotonically decreases upon increasing the field $B_z$. 
Magnetic properties of a spin-1/2 Heisenberg XXZ model

With further increase of the field $B_z$, the susceptibility has a non-monotone behavior with the maximums in those intervals of the longitudinal magnetic field at which quasi-plateaus arise in the magnetization curve. This difference in behavior of the susceptibility for various fixed values of the transverse field $B_x$ at low temperature indicates that the system undergoes several phase transitions by increasing longitudinal field $B_z$.

Panels 1(b) and 1(d) illustrate the magnetization and magnetic susceptibility for the chain of length $N = 10$ under the same conditions as $N = 6$. Here, in addition to the zero-magnetization plateau, there are four fractional plateaus as: $M/M_s = 1/5$, $M/M_s = 2/5$, $M/M_s = 3/5$, then the magnetization reaches its saturation in strong magnetic fields. As a result, when the length of the chain increases, accordingly, the number of magnetization plateaus increases, also the effect of transverse magnetic field $B_x$ and angle $\theta$ is more sensible in this case. Namely, the transformation from plateau to quasi-plateau occurs for the lower amount of applied field $B_z$. To clarify this point, if one compares red dash-dotted lines drawn in panels 1(a) and 1(b) together, he finds that for the case $N = 10$ quasi-plateaus appear for lower amounts of the transverse field compared with that of for case $N = 6$. As before, by increasing $\theta$ quasi-plateaus gradually disappear.

The magnetic susceptibility of the chain with $N = 10$ as function of the longitudinal field $B_z$ for several fixed values of the transverse field is depicted in panel 1(d). When the transverse magnetic field $B_x$ increases, the height of peaks of the susceptibility corresponding to the magnetization jumping between plateaus, decrease. As an important result, when the magnetization quasi-plateaus gradually appear by increasing the transverse field $B_x$, accordingly, special peaks of susceptibility will arise. We would like to draw your attention to another interesting effect of the transverse field increment on the susceptibility behavior, i.e., when the transverse field increases, the zero-field susceptibility has non-monotone behavior for both cases $N = 6$ and $N = 10$. The zero-field susceptibility is a remarkable evidence of existing magnetization quasi-plateau for the weak longitudinal magnetic field at low temperature, also quasi-plateaus in the magnetization curve versus transverse field $B_x$. With increase of the transverse staggered field coefficient $\theta$, the zero-field susceptibility gets further than other peaks in both cases $N = 6$ and $N = 10$. For strong magnetic field region $B_z > 10$, there is a steep decrease of the susceptibility for all considered fixed values of the transverse field which denotes the magnetization reaches is saturation value.

We next study the behavior of the magnetization and magnetic susceptibility when the system is in the presence of inhomogeneous magnetic fields at low temperature. We have plotted in figure 2 the magnetization and magnetic susceptibility of the model with the same conditions as figure 1, but under inhomogeneous magnetic fields (here, inhomogeneous parameters are taken as non-zero fixed values $b_z = 0.6$, $b_x = 0.3$, and $\lambda = 1$). Panels 2(a) and 2(c) display the magnetization and susceptibility with the finite length $N = 6$ under inhomogeneous longitudinal, transverse, and transverse staggered magnetic fields. Panels 2(b) and 2(d) are related to the chain of length $N = 10$. In this situation for both cases $N = 6$ and $N = 10$, all plateaus have been shifted toward higher values of the magnetization. Hence, we can see that inhomogeneity dramatically affects on the the height and position of the low-temperature peaks in susceptibility. When the transverse magnetic field increases, firstly height of the peaks increases, then with further increase of the field $B_x$ gradually decreases. Moreover, under inhomogeneous magnetic fields, the susceptibility does not vanish even at zero longitudinal field $B_z = 0$. Consequently,
by imposing weak inhomogeneity property into the all magnetic fields, width of the magnetization plateaus decreases, and there is no zero magnetization plateau as well as zero-field susceptibility for the model under consideration with arbitrary length at low temperature.

By altering transverse staggered field intensity ($\theta = \frac{\pi}{10}$), one can see less variation in the shape of susceptibility for weak longitudinal field $B_z < 2$ compared with the case when the system is putted in the presence of homogeneous magnetic fields (see insets of figure 2). It is quite evidence that under inhomogeneity, variations of both transverse field $B_x$ and transverse staggered field $B_x^{\text{stagg}}$ qualitatively affect the quasi-plateaus arisen in the magnetization curves more than when the system is in the vicinity of homogeneous magnetic fields, revealing that the magnetization curves including quasi-plateaus are more monotone than without inhomogeneity.

Finally, we investigate the temperature dependences of the specific heat under both homogeneous and inhomogeneous external magnetic fields. The corresponding plots of the specific heat as function of the temperature for several fixed values of the longitudinal magnetic field are presented in figures 3 and 4. When the chain of length
Magnetic properties of a spin-1/2 Heisenberg XXZ model

Figure 3. Temperature dependences of the specific heat of the 1-D XXZ spin chain under various fixed values of the longitudinal magnetic field $B_z$. Other external magnetic fields and parameters are taken as $B_x = 1$, $\theta = \pi/30$, $J_{xy} = 6$ and $J_z = 3$. All applied magnetic fields are also considered as homogeneous fields such that: $b_x = 0$, $b_y = 0$, and $\lambda = 0$. (a) The specific heat of the chain with finite length $N = 6$, and (b) $N = 10$. Insets show the specific heat of the model in the presence of higher transverse staggered field as $\theta = \pi/10$. 

Magnetic properties of a spin-1/2 Heisenberg XXZ model

9

$N = 6$ (figure 3(a)) is putted in the vicinity of homogeneous magnetic fields, it is seen that the specific heat exhibits a single Schottky-type maximum for weak longitudinal field $B_z \leq 1$ at low temperature, where other parameters utilized in the Hamiltonian are taken as $B_x = 1$, $b_x = 0$, $b_z = 0$, $\lambda = 0$, and $\theta = \frac{\pi}{30}$. The Schottky-type maximum monotonically decreases with increasing the field $B_z$. By further increasing the longitudinal field $B_z$ second maximum arises which is smaller than first one. The shape of the both maxima alternatively change upon increasing the field $B_z$. When the transverse staggered field increases ($\theta = \frac{\pi}{10}$), the longitudinal field dependences of the specific heat are explicitly impressed (the inset of figure 3(a)). In other words, varying the angle $\theta$ directly affects on the longitudinal field dependences of the specific heat maxima.

For the chain with more sites ($N = 10$), there is a double-peak in the specific heat curve for the weak values of homogeneous longitudinal magnetic field ($B_z \leq 1$). In this case, the specific heat maxima have an alternating behavior upon increasing $B_z$. Ultimately, we see that two maxima merge together and make a Schottky-type maximum in the strong longitudinal field $B_z$ (black dotted-lines). Increase of the angle $\theta$ also alters the shape, position and height of the peaks (the inset of figure 3(b)).

Let us now examine the specific heat for the case when the system is in the presence of external inhomogeneous magnetic fields. As shown in figure 4(a), by imposing inhomogeneity property into the applied magnetic fields as $b_z = 0.6$, $b_x = 0.3$, and $\lambda = 1$, where $B_x = 1$ and $\theta = \frac{\pi}{2}$, one can see a triple-peak for the strong longitudinal magnetic field $B_z$. Interestingly, by increasing the field $B_z$, three maxima gradually merge together and finally they make a single Schottky-type maximum for weak range of the longitudinal field $B_z$ ($B_z \leq 1$). Another important point affecting the maxima of the specific heat is the altering the angle $\theta$. For higher values of $\theta$, we see that the specific heat has just two maxima for the strong magnetic field $B_z$, and fixed $B_x = 1$, which merge together by decreasing the longitudinal field. When the number of sites in the chain increases (figure 4(b)), there is also a triple-peak for some specific values of the longitudinal magnetic field, but in lower range of the temperature, they merge together with further increase of the field $B_z$. When the transverse staggered field coefficient $\theta$ increases, it is seen the double-peak for values $B_z \leq 7$ which merge together upon increasing the strength of longitudinal magnetic field (the inset of figure 4(b)).

4. Conclusions

The present work deals with the study of magnetization, magnetic susceptibility and the specific heat of the 1-D spin-1/2 XXZ chain under different external magnetic fields including longitudinal, transverse, and transverse staggered. Different number of particles have been considered for the chain under periodic boundary conditions. The thermodynamic parameters of the spin system have rigorously been investigated under two different circumstances: Firstly, for the case when the system is in the presence of homogeneous magnetic fields; Secondly, for the case when all applied magnetic fields have a finite inhomogeneity property. To consider suitable inhomogeneity properties in the applied magnetic fields, we have implemented inhomogeneity coefficients correspond to the three kinds of applied magnetic field parameters consisting of longitudinal, transverse, and transverse staggered magnetic fields. As a matter of fact, we have assumed that the strength of the induced magnetic fields is different for the odd and even sites of the chain. Since the magnetization operator does not
Figure 4. Temperature dependences of the specific heat of the 1-D XXZ spin chain under various fixed values of the longitudinal magnetic field $B_z$. Other external magnetic fields and parameters are as in figure 3. Here, all applied magnetic fields have been considered as inhomogeneous fields such that: $b_z = 0.6$, $b_x = 0.3$, and $\lambda = 1$. (a) The specific heat as function of the temperature for the chain with finite length $N = 6$, and (b) $N = 10$. Insets show the specific heat of the model versus temperature in the presence of higher transverse staggered field as $\theta = \frac{\pi}{10}$.
Magnetic properties of a spin-1/2 Heisenberg XXZ model

commute with the Hamiltonian some unusual phenomena have been observed.

The low temperature investigation of the magnetization and the magnetic susceptibility of the model under homogeneous magnetic fields revealed that the magnetization curve undergoes an interesting evolution such that with the increase of the transverse field, all plateaus convert to their counterpart quasi-plateaus independent of the chain size. Also, by increasing the staggered field coefficient $\theta$, quasi-plateaus gradually disappear, where the magnetization has a smooth curve versus longitudinal magnetic field for the high values of the transverse field and larger $\theta$. We have observed that the susceptibility curve has also interesting behavior with respect to the longitudinal magnetic field when the strength of the transverse and transverse staggered fields change. In accordance with the jumps between magnetization plateaus, susceptibility curve has maxima whose shapes and positions are strongly dependent on the strength of all applied fields in the Hamiltonian. The non-monotone behavior of the susceptibility with the maximum at higher values of transverse field indicates existence of the quasi-plateaus in the magnetization at low temperature. We also found a zero longitudinal field susceptibility upon increasing the transverse field. When the inhomogeneity property was imposed into the magnetic fields, independent of the chain size, low temperature behavior of the magnetization and magnetic susceptibility versus longitudinal field remarkably changed for low amounts of the transverse magnetic field. As an interesting result, here we have seen a zero longitudinal field susceptibility even under the weak transverse magnetic field.

Finally, we have examined the specific heat of the model as function of the temperature for various fixed values of the longitudinal magnetic field. We have concluded that, when the system is putted in the vicinity of homogeneous magnetic fields, there is a strongly transverse field dependent Schottky-type maximum, which gradually tends to a double-peak upon increasing the transverse field. Amazingly, for the strong longitudinal magnetic field, a triple-peak have been found in the specific heat curve for the case when the system is considered under inhomogeneous magnetic fields, which could not be seen when we consider homogeneous magnetic fields in the Hamiltonian. Furthermore, we understood that independent of the number of particles in the chain, the altering staggered field has substantial influences on the behavior of the specific heat for the both different circumstances.

Acknowledgments

H. Arian Zad acknowledges the receipt of the grant from the ICTP Affiliated Center Program AF-04.

References

[1] P. Thakur and P. Durganandini, Phys. Rev. B 97, (2018)064413.
[2] H. J. Mikeska, and A. K. Kolezhuk, One-Dimensional Magnetism. In U. Schollwöck, J. Richter, D. J. J. Farnell, and R. F. Bishop, Lec. Notes Phys. 645 1-83, Springer, Berlin (2004).
[3] H. Arian Zad, and N. Ananikiam, J. Phys.: Condens. Matter. 30, 165403 (2018).
[4] H. Arian Zad, and N. Ananikiam, J. Phys.: Condens. Matter. 29, 455402 (2017).
[5] D. V. Dmitriev, V. Ya. Krivnov, A. A. Ovchinnikov, A. Langari, JETP, 95, 3, 538549, (2002).
[6] D. V. Dmitriev, V. Ya. Krivnov, Phys. Rev. B 70, 14, 144414, (2004).
[7] D. V. Dmitriev, V. Ya. Krivnov, A. A. Ovchinnikov, Phys. Rev. B 65, 17, 172409, (2002).
[8] R. Hagemans, J-S. Caux, U. Low, Phys. Rev. B 71, 1, 014437, (2005).
[9] T. Hikihara, A. Furusaki, Phys. Rev. B 69, 6, 064427, (2004).
Magnetic properties of a spin-1/2 Heisenberg XXZ model

[10] A. A. Ovchinnikov, D. V. Dmitriev, V. Ya. Krivnov, and V. O. Cheranovskii, Phys. Rev. B 68, 21, 214406, (2003).
[11] J-S. Caux, F. H. L. Essler, U., Low, Phys. Rev. B 68, 13, 134431, (2003).
[12] M. Enderle, C. Mukherjee, B. Fak, R. K. Kremer, J.-M. Broto, H. Rosner, S.-L. Drechsler, J. Richter8, J. Malek, A. Prokofiev, W. Assmus, S. Pujol, J.-L. Raggazzone1, H. Rakoto5, M. Rheinstädtter and H. M. Rnnow. EPL (Europhysics Letters) 70, 237, (2007).
[13] M. Pantić, N. Micić, M. P. Hrvojević, S. Radošević, P. Mali, Contemporary Materials, VIII-2, 137-143 (2017).
[14] F. Capraro, C. Gros, Eur. Phys. Jour. B, 29, 35-40 (2002).
[15] T. Giamarchi, H.J. Schulz, J. Phys. France 49, 5, (1988).
[16] M. Sato, M. Oshikawa, Phys. Rev. B 69, 5, 054406, (2004).
[17] S. Eggert, I. Affleck, M. Takahashi, Phys. Rev. Lett 73, 332, (1994).
[18] S. Eggert, Phys. Rev. B 53, 5116, (1996).
[19] N. Motoyama, H. Eisaki, S. Uchida, Phys. Rev. Lett. 76, 3212, (1996).
[20] M. Oshikawa, K. Ueda, H. Aoki, A. Ochiai and M. Kohgi, J. Phys. Soc. Jpn. 68, 3181 (1999).
[21] M. Oshikawa, I. Affleck, Phys. Rev. Lett. 79, 2883 (1997).
[22] H. Kikuchi, Y. Fuji, M. Chiba, S. Mitsudo, T. Idehara, T. Tonegawa, K. Okamoto, T. Sakai, T. Kuwai and H. Ohta, Phys. Rev. Lett. 94, 227201, (2005).
[23] K. C. Rule, A. U. B. Wolter, S. Süllo, D.A. Tennant, A. Brühl, S. Köhler, B. Wolf, M. Lang and J. Schreuer, Phys. Rev. Lett. 100, 117202, (2008).
[24] R. Matysiak, P. Gegenwart, A. Ochiai, M. Antkowiak, G. Kamieniarz, F. Steglich, Phys. Rev. B 88 224414, (2013).
[25] J. Abouie, S.A. Ghasemi, A. Langari, Phys. Rev. B 73 014411, (2006).
[26] R. Calenzru, J. Riera, D. Foilblanc, J.-P. Boucher, G. Chaboussant, L. Lévy and O. Piovesana, Euro. Phys. Jour. B 71, 171174, (1999).
[27] D. C. Johnston, R. K. Kremer, M. Troyer, X. Wang, A. Klmer, S. L. Budko, A. F. Panchula, and P. C. Canfield, Phys. Rev. B 61, 9558, (2000).
[28] C. Gros, R. Valent, Phys. Rev. Lett. 82, 976, (1999).
[29] O. Breunig, M. Garst, E. Sela, B. Buldman, P. Becker, L. Bohat, R. Müller, and T. Lorenz, Phys. Rev. Lett. 111, 187202, (2013).
[30] M. Oshikawa, M. Yamanaka, I. Affleck, Phys. Rev. Lett. 78, 1984, (1997).
[31] J. C. Leiner, Joosung Oh, A. I. Kolesnikov, M. B. Stone, Manh Duc Le, E. P. Kenny, B. J. Powell, M. Mourigal, E. E. Gordon, M.-H. Whangbo, J.-W. Kim, S.-W. Cheong, and Je-Geun Park, Phys. Rev. B 97, 104426, (2018).
[32] K. Hida, J. Phys. Soc. Jpn. 63, pp. 2359-2364 (1994).
[33] Y. Ajiro, T. Asano, T. Inami, H. Aruga-Katori and T. Goto, J. Phys. Soc. Jpn. 63, 859, (1994).
[34] G. Genovese, QUANTUM DYNAMICS OF INTEGRABLE SPIN CHAINS (2011) PhD Thesis, Sapienza Università di Roma.
[35] J. Torrigo, V. Ohanyan and O. Rojas, J. Magn. Magn. Mater. 454, 85 (2018).
\[ B_z = 1 \]

\[ B_z = 3 \]

\[ B_z = 5 \]

\[ \theta = \frac{\pi}{10} \]

\[ \theta = \frac{\pi}{30} \]