Scaling analysis of normal state properties of high-temperature superconductors

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We propose a model-independent scaling method to study the physical properties of high-temperature superconductors in the normal state. We have analyzed the experimental data of the c-axis resistivity, the in-plane resistivity, the Hall coefficient, the magnetic susceptibility, the spin-lattice relaxation rate, and the thermoelectric power using this method. It is shown that all these physical quantities exhibit good scaling behaviors, controlled purely by the pseudogap energy scale in the normal state. The doping dependence of the pseudogap obtained from this scaling analysis agrees with the experimental results of angle-resolved photoemission and other measurements. It sheds light on the understanding of the basic electronic structure of high-$T_c$ oxides.

I. INTRODUCTION

The mechanism of high-$T_c$ superconductivity remains one of the fundamental issues unsolved in condensed matter physics. In particular, a unified theory toward the understanding of the rich phase diagram of high-$T_c$ superconductors (HTSC) has not been established. At half-filling, the parent compounds of HTSC are antiferromagnetic insulators. Upon doping, the antiferromagnetic long-range correlation is suppressed and the high-$T_c$ superconductivity develops above a critical doping level. At low doping but in the metallic state, a pseudogap phase with missing entropy or spectra is discovered.1,2,3,4 Around the optimal doping, a strange metal phase with a linear in-plane resistivity and probably a quantum critical point emerges.5,6

A must step in the understanding of the phase diagram of HTSC is to identify the energy scales and the control parameters or interactions of low-energy excitations of HTSC. In the normal state at low doping, the pseudogap is believed to be one of the characteristic parameters of low-lying excitations. The corresponding temperature below which the pseudogap effect is observed is commonly used as a boundary to separate the pseudogap and the strange metal phases, although there is no real phase transition between these phases. However, due to the uncertainty in the definition of the pseudogap, different experiments have adopted different criteria to determine the pseudogap energy scale. This leads to the claim of the existence of two pseudogaps, namely, the upper and lower pseudogaps.7 In the strange metal phase, the quantum critical fluctuation is strong and the temperature itself may serve as a dynamic control parameter as suggested by the marginal Fermi liquid theory.8

The scaling analysis of experimental data in HTSC is a simple but powerful tool in elucidating the underlying physics without invoking a specific model. In the normal state, if the pseudogap is a predominant energy scale controlling low-energy excitations, then the low-temperature behavior of any measurement physical quantity should satisfy a doping-independent scaling law, although the analytic expression of the scaling function is unknown. Based on this idea, we analyzed recently the temperature dependence of the c-axis resistivity $\rho_c$ of HTSC and found that it obeys a universal scaling law given by

$$\rho_c(T) = \frac{\alpha T}{\Delta} \exp \left( \frac{\Delta}{T} \right),$$

where $\alpha$ is a doping dependent coefficient, $T$ is temperature, and $\Delta$ is the pseudogap. As shown in Ref. 9, Eq. (1) results from the interplay between the anisotropic c-axis hopping integral10,11 and the $d_{x^2-y^2}$-like symmetry of the pseudogap. It agrees excellently with the experimental data of multilayer HTSC and resolved a long-standing puzzle regarding the physical origin of the semiconductorlike temperature dependence of $\rho_c$ in the pseudogap phase. Furthermore, it suggests that there is only one energy scale controlling the low-energy excitations around the antinodal points and the interlayer hopping within each unit cell is coherent.

However, if the scaling function is unknown, the scaling analysis becomes difficult. This has, in fact, hampered the application of the scaling analysis. A commonly adopted approach in the scaling analysis in the literature is to assume that by normalizing both the measurement quantity $F(T)$ and the temperature by the corresponding values at a sample-dependent characteristic temperature $T^*$, then all the experimental data should fall onto a single curve described by the scaling function

$$\frac{F(T)}{F(T^*)} = g \left( \frac{T}{T^*} \right).$$

However, in real materials, this scaling analysis often fails since $F(T)$ generally contains the terms which are not scaling invariant. For example, the impurity contribution to the in-plane resistivity or other physical quantities is not scaling variant. Moreover, since $T^*$ is unknown prior to the analysis, this formula is difficult to be implemented practically even if it is correct. Empirically, $T^*$ is often determined from some special features appearing in the measurement data. For example, for the
in-plane resistivity, $T^*$ is determined from the temperature below which the resistivity begins to deviate from its high-temperature linear-$T$ behavior. However, to determine unambiguously the linear-$T$ region is not always possible since in some low doping samples, the measured temperature may not be high enough to reach the linear-$T$ regime. Furthermore, the deviation from linear to nonlinear $T$ is a crossover, not a phase transition, and a small measurement error may result in a large error in $T^*$.

In this paper, we propose a scaling method and apply it to analyze the normal state properties of HTSC. This method extends our previous scaling analysis of the c-axis resistivity to other physical quantities. It breaks the barrier in the use of the simple formula (2) and allows a model-independent scaling analysis to be done reliably. We have reanalyzed the experimental data of the c-axis resistivity of HTSC with this method. By comparison with our previous results, we find that this method, indeed, works very well. It provides a simple but powerful approach for analyzing experimental results. Furthermore, this method is model independent. It can be applied not only to the high-$T_c$ cuprates, but also to any other materials where the single parameter scaling behavior is valid.

This paper is arranged as follows. Section 111 gives an introduction to the scaling method based on a least square fit of an unknown scaling function to the experimental data. In Sec. 1111 we apply the method to analyze the scaling behaviors of a number of physical quantities of HTSC in the normal state, using the experimental data published in the literature. In Sec. 11111 we analyze the universal behavior of the energy scales obtained in Sec. 1111 and discuss its physical implications. Section 111111 gives a brief summary.

II. METHOD OF SCALING ANALYSIS

In this section, we present a generic method for analyzing the scaling behavior of a set of experimental data \( \{ F_i(T), i = 1, ..., N \} \). Here, $T$ can be temperature, pressure, external field, or any other controllable variable used in experiments. The subscript $i$ is a sample index to which the physical quantity $F$ as a function of $T$ is measured experimentally. $N$ is the total number of samples. In the discussion below, in order to be directly relevant to the scaling analysis presented in Sec. 1111 we assume that $T$ is the temperature and a sample represents a specified high-$T_c$ compound.

We start by assuming that in a relevant temperature range the low-lying physics is governed only by one energy scale $\Delta$. Thus the measured physical quantity $F(T)$ satisfies a simple scaling law

\begin{equation}
F(T) = \alpha F \left( \frac{T}{\Delta} \right) + \beta, \quad \text{(3)}
\end{equation}

where $\alpha$, $\beta$, and $\Delta$ are all doping dependent, but temperature independent. For the physical quantities to be discussed in Sec. 1111 $\beta$ is generally the contribution of impurities or other extrinsic interactions. $\Delta$ is a characteristic energy scale of the system. It controls the dynamics of the system. $F(T)$ is a universal (doping-independent) scaling function. Its temperature dependence is determined by the low-lying excitations and interactions.

The scaling function $F(T)$ is generally unknown. This is the difficulty commonly met in the data analysis. However, the scaling method introduced here does not depend on the detailed formula of $F(T)$, provided that the single-parameter scaling hypothesis Eq. (3) is valid. This is a merit of the method. It provides a simple but powerful approach to probe the intrinsic connection between different samples and to determine the doping dependence of the characteristic energy scale $\Delta$ without invoking a specific model.

The aim of the scaling analysis is to determine from the measurement data the scaling parameters $(\alpha_i, \Delta_i, \beta_i)$ and the optimized scaling functions $F(x)$ so that all the data can be rescaled onto a universal curve. This can, in principle, be achieved by minimizing the total deviation of the scaling function between any two samples:

\begin{equation}
\delta F = \sum_{i<j} \sum_k [F_i(T'_k) - F_j(T'_k)]^2, \quad \text{(4)}
\end{equation}

where

\begin{equation}
F_i(T) = \frac{1}{\alpha_i} [F_i(\Delta_i T) - \beta_i]. \quad \text{(5)}
\end{equation}

$F_i(\Delta_i T)$ is the value of $F$ of the $i$th sample at temperature $\Delta_i T$. $N_k$ is the number of sampled temperature points used in optimizing the scaling function. $N_k$ can be adjusted in the minimization. Initially, $N_k$ can take roughly the value of average measured temperature points.

However, due to the scaling behavior of $F(T)$, not all the parameters $(\alpha_i, \Delta_i, \beta_i)$ can be uniquely determined by the minimization of $\delta F$ if $F$ is unknown. Indeed, from Eqs. (3) and (5), it can be shown that if $\{ \alpha_i, \Delta_i, \beta_i \} (i = 1, ..., N)$ is a set of parameters minimizing $\delta F$, then $(\alpha_i/\alpha_s, \Delta_i/\Delta_s, \beta_i - \beta_s \alpha_i/\alpha_s)$ with arbitrary but non-singular $(\alpha_s, \Delta_s, \beta_s)$ will also minimize $\delta F$. $(\alpha_s, \Delta_s, \beta_s)$ are unknown and can be taken as the scaling parameters of a reference sample. This means that only the relative values of $\alpha_i, \Delta_i$, and $\beta_i$ with respect to a reference sample

\begin{equation}
A_i = \frac{\alpha_i}{\alpha_s}, \quad \text{(6)}
\end{equation}

\begin{equation}
B_i = \frac{\Delta_i}{\Delta_s}, \quad \text{(7)}
\end{equation}

\begin{equation}
C_i = \beta_i - A_i \beta_s, \quad \text{(8)}
\end{equation}

can be fixed. From the definitions (6)-(8), it is straightforward to show that $A_s = B_s = 1$ and $C_s = 0$.

Using the relative scaling parameters $(A_i, B_i, C_i)$, one can define a new scaling function

\begin{equation}
\tilde{f} \left( \frac{T}{B} \right) = \alpha_s F \left( \frac{T}{B\Delta_s} \right) + \beta_s. \quad \text{(9)}
\end{equation}
where the scaling functions:

\[ F(T) = Af \left( \frac{T}{B} \right) + C. \] (10)

For the reference sample, the scaling function is the measurement curve itself: \( F(T) = f(T) \).

Equation (10) is nothing but to scale all experimental data onto the measurement curve of the reference sample. This suggests that the reference sample should be chosen such that its temperature interval is broad enough to cover the whole temperature range physically interesting and the data quality is among one of the best.

The relative scaling parameters \( \{A_i, B_i, C_i\} \) can now be determined by minimizing the total deviation of the scaling functions:

\[ \delta f = \sum_{i<j} \sum_k [f_i(T_k) - f_j(T_k)]^2, \] (11)

where

\[ f_i(T) = \frac{1}{A_i} [F_i(B_iT) - C_i], \] (12)

In general, \( B_iT \) may not be exactly the temperature point experimentally measured. The value of \( F_i(B_iT) \) can be obtained from the measurement data by interpolation, provided that \( B_iT \) is within the measured temperature interval. For each pair of \( f_i \) and \( f_j \), if \( B_iT_k \) or \( B_jT_k \) is outside the measured temperature interval for the \( i \)th or \( j \)th sample, then the corresponding term in Eq. (11) should be excluded from the summation. The minimization of \( \delta f \) can be done, for example, using the standard subroutine given in Ref. 12.

| Sample (δ) | A  | B  | C (10^{-4}Ωcm) |
|------------|----|----|----------------|
| 0.68       | 1.0| 1.0| 0.0            |
| 0.78       | 0.63| 0.81| 0.0074        |
| 0.85       | 0.39| 0.60| -0.10         |

Below we take the in-plane resistivity \( \rho_{ab} \) of \( \text{YBa}_2\text{Cu}_3\text{O}_{6+\delta} \) published by Ito et al.\(^{13}\) as an example to show how the method works. For simplicity, here we only use the experimental data for three of the samples, \( \delta = 0.68, 0.78, 0.85 \). Figure II(a) shows the measurement data of \( \rho_{ab}(T) \) for these three samples. In the scaling analysis, we take \( \delta = 0.68 \) as the reference sample and \( N_k \) to be roughly equal to the measured temperature points of the reference sample. The relative scaling parameters can then be determined by numerically minimizing Eq. (11). The resulting scaling curves are shown in Fig. II(b) and the scaling parameters are given in Table II.

In Eq. (10), if \( C \ll F(T) \), then the scaling equation is approximately given by

\[ F(T) \approx Af \left( \frac{T}{B} \right), \] (13)

In this case, the scaling equation can be reexpressed as

\[ \frac{F(T)}{F(T')} = f(T/B) \frac{T'}{F(T')} = g \left( \frac{T}{T'} \right), \] (14)

where \( T' \) is a sample-dependent characteristic temperature. \( g(T) \) is a rescaled function of \( f(T) \). Equation (14)
is precisely the scaling equation defined by Eq. (2). It is a commonly used scaling equation in the scaling analysis of experimental data. However, it should be pointed out that this equation is valid only when the temperature independent term $C$ can be safely ignored or reliably subtracted from $F(T)$ in Eq. (10).

III. SCALING ANALYSIS OF EXPERIMENTAL DATA

In this section, we apply the scaling method to analyze the experimental data of high-$T_c$ cuprates in the normal state. We will first analyze the scaling behavior of the c-axis resistivity $\rho_c$. Since an approximate but accurate expression for the scaling function of $\rho_{c}$ is available, this allows us to determine the absolute values of the scaling parameters. For other measurement quantities, including the in-plane resistivity $\rho_{ab}(T)$, the Hall coefficient $R_H(T)$, the magnetic susceptibility $\chi(T)$, the spin-lattice relaxation rate $1/T_1$, and the thermoelectric power $S(T)$, only the relative scaling parameters can be determined.

Our scaling analysis is based on the experimental data already published in the literature. We collect as much as we can the experimental data of HTSC from which a systematical analysis of the scaling behaviors can be done. The preference is given to the latest published data if there are considerable differences between the data published by different groups. The chemical formula of the compounds with their abbreviations analyzed in this paper are given in Table I. The scaling analysis here will be limited to the superconducting samples in the normal state. The data for the nonsuperconducting samples will not be analyzed. In all figures and tables presented in this paper, the parameters for the reference samples will be in bold face to distinguish them from other parameters.

| TABLE I: HTSC compounds and their abbreviations analyzed in this paper. |
|----------------------|----------------------|
| YBa$_2$Cu$_3$O$_{6+\delta}$ & Y123 |
| Y$_{0.8}$Ca$_{0.2}$Ba$_2$Cu$_3$O$_{6+\delta}$ & Ca-Y123 |
| YBa$_2$Cu$_4$O$_8$ & Y124 |
| Bi$_2$Sr$_2$CaCu$_2$O$_{6+\delta}$ & Bi2212 |
| Bi$_2$Sr$_2$CaCu$_2$O$_{10+\delta}$ & Bi2223 |
| La$_{2-x}$Sr$_x$CuO$_4$ & La141 |
| Bi$_2$Sr$_2$-La$_x$CuO$_{6+\delta}$ & La-Bi2201 |
| HgBa$_2$CuO$_4$ & Hg1201 |
| Tl$_2$Sr$_2$CaCu$_2$O$_{7-\delta}$ & Tl1212 |

In the comparison of the scaling parameters for different families of HTSC, we will use the superconducting transition temperature $T_c$ and its empirical formula proposed by Presland et al.\cite{10}

$$
\frac{T_c}{T_{c,\text{max}}} = 1 - 82.6(p - 0.16)^2,
$$

(15)

to determine the effective carrier concentration $p$. Here, $T_{c,\text{max}}$ is the maximal superconducting transition temperature. For La$_{2-x}$Sr$_x$CuO$_4$, the carrier concentration is equal to the doping concentration of Sr ions, $p = x$.

A. c-axis resistivity $\rho_c(T)$

In the pseudogap phase of high-$T_c$ cuprates, the c-axis resistivity $\rho_c$ behaves very differently from its in-plane counterpart $\rho_{ab}$. Along the CuO$_2$ plane, $\rho_{ab}$ shows a metal-like temperature dependence. It decreases with decreasing temperature. However, along the c-axis, $\rho_c$ behaves as a semiconductor. It increases with decreasing temperature.

This dramatic difference between $\rho_c$ and $\rho_{ab}$ is not what one might expect within conventional Fermi liquid theory. To resolve this issue, a number of theoretical models based on the dynamic confinement of charge-spin separated particles\cite{15,16,17} or the incoherent interlayer hopping\cite{10,18} were proposed. Most of the theories predicted that $\rho_c$ should diverge in a certain power law of $T$ at low temperature. However, it seems that none of these theories can account quantitatively or even qualitatively the experimental data.

The semiconductorlike behavior of $\rho_c$ results, as we recently pointed out\cite{2}, from the interplay between the $d_{x^2-y^2}$-like pseudogap and the anisotropic c-axis hopping integral.\cite{10,11} $\rho_c$ contributes mainly from the quasiparticles around the antinodal points. The nodal contribution is completely suppressed by the interlayer hopping matrix elements. Since the pseudogap is a prevailing energy scale governing the c-axis dynamics in the pseudogap phase, it is natural to assume that the c-axis resistivity satisfies a scaling law governed purely by the pseudogap $\Delta$.

In cuprate superconductors, if Cu atoms in the two neighboring CuO$_2$ planes lie collinearly along the c-axis, then the interlayer hopping integral between these two planes is given by

$$
t_c \sim (\cos k_x - \cos k_y)^2,
$$

(16)

where $(k_x, k_y)$ are the in-plane momenta of electrons. $t_c$ vanishes along the nodal direction. Based on this formula, we showed in Ref. 9 that for multilayer cuprates, $\rho_c$ is approximately given by

$$
\rho_c(T) = \alpha F_c \left( \frac{T}{\Delta} \right),
$$

(17)

where

$$
F_c(x) = x \exp \left( \frac{1}{x} \right).
$$

(18)
Equation (17) is a special case of Eq. (13). It holds when the residual resistivity contributed by disorder scattering $\beta$ is vanishingly small compared with the contribution of pseudogap to $\rho_c$.

The scaling function (18), as shown in Fig. 2, agrees excellently with the measurement data for Y123 published by Yan et al.\textsuperscript{22} and by Babic et al.\textsuperscript{25} for Bi2212, from Watanabe et al.\textsuperscript{22,23,24} and for Bi2233 by Fujii et al.\textsuperscript{25}

The values of the scaling parameters $\alpha$ and $\Delta$ are given in Ref. 9. The pseudogap $\Delta$, as shown in Fig. 3, drops almost linearly with doping. This doping dependence of the pseudogap agrees with the angle-resolved photoemission (ARPES) as well as other measurement data.\textsuperscript{22,23} The values of $\Delta$ (not shown here) for two overdoped samples of Bi2212 from Chen et al.\textsuperscript{25} and Bi2223 by Babic et al.\textsuperscript{25}

The above scaling analysis is done based on Eq. (18) by assuming that the interlayer hopping is predominantly coherent, rather than incoherent as usually believed. The reason is actually simple. If the interlayer hopping is incoherent, then the excitations around the gap nodes may have substantial contribution to $\rho_c$, which may break this scaling law. For multiple layer cuprates, the intralayer coupling may be different from the interlayer coupling. However, the different coupling between CuO$_2$ planes does not change the fact that the pseudogap is the only energy scale governing the quasiparticle excitations around the antinodes in the normal state. Therefore, Eq. (17) holds irrespective of the number of CuO$_2$ planes in each unit cell.

TABLE III: The scaling parameters, $A_c$ and $B_c$, of the c-axis resistivity $\rho_c$ defined by Eq. (20) for La-Bi2201. The experimental data and the carrier concentration $p$ are taken from Ref. 20.

| Sample (x) | $T_c$ (K) | Doping (p) | $A_c$ | $B_c$ |
|------------|-----------|------------|-------|-------|
| 0.84       | 1.4       | 0.10       | 2.50  | 1.13  |
| 0.73       | 14        | 0.11       | 1.37  | 1.14  |
| 0.66       | 23        | 0.12       | 1.0   | 1.0   |
| 0.49       | 31        | 0.14       | 0.79  | 0.51  |
| 0.39       | 38        | 0.16       | 0.67  | 0.41  |
single-layer cuprate superconductor La-Bi2201. The experimentalse hopping integral becomes collinearly along the c-axis. In this case, the c-axis scaling function defined by Eq. (18).

Gruneisen formula is too low to account for the experimental data of Bi2201. It vanishes along both the nodal and antinodal directions. As the pseudogap is the dominant energy scale in the pseudogap phase, the scaling law of the c-axis resistivity [Eq. (17)] should still hold. However, the scaling function of the single-layer cuprates is expected to be different. In this case, an accurate expression for the scaling function is not available. Thus, we are unable to determine the absolute values of the scaling parameters $\Delta$ and $\alpha$ from the scaling analysis. However, the relative scaling parameters can be determined using the scaling method introduced in the previous section.

We have analyzed the scaling behavior of $\rho_c$ for La-Bi2201 (Ref. 30) using the formula

$$\rho_c(T) = A_c f_c\left(\frac{T}{B_c}\right).$$

Again, the contribution from impurity scattering to $\rho_c$ is ignored since it is much smaller than the contribution from the pseudogap effect.

Figure 4 shows the scaling function $f_c$ for La-Bi2201. The scaling parameters are given in Table III. The analysis shows that $\rho_c$, indeed, exhibits a good scaling behavior in this one-layer material. However, the scaling function is different from that for the multilayer materials at low temperatures. It is also different from the logarithmic divergence behavior as observed by Ando et al. in La214 cuprates. The scaling behavior of $\rho_c$ for both the single- and multiple-layer cuprates indicates that the interlayer dynamics is, indeed, governed by the interplay between the anisotropic interlayer hopping integral and the pseudogap effect in all high-$T_c$ cuprates.

B. In-plane resistivity $\rho_{ab}(T)$

In contrast to the semiconductorklike behavior of $\rho_c$, the in-plane resistivity $\rho_{ab}$ of high-$T_c$ cuprate superconductors is metal-like in the normal state. In the underdoped and optimally doped materials, $\rho_{ab}$ exhibits a universal linear behavior at high temperatures. The phonon scattering can lead to a linear resistivity. However, the Debye temperature determined by applying the BlochGruneisen formula is too low to account for the experimental data of Bi2201. The linear behavior of $\rho_{ab}$ could be a manifestation of strong correlations. It is a characteristic behavior of marginal Fermi liquid where the inelastic scattering rate scales linearly with temperature. It may also result from gauge or quantum critical fluctuations.

Below a characteristic temperature $T^*$, $\rho_{ab}$ begins to deviate from the linear behavior. This deviation is correlated with the pseudogap effect, and $T^*$ is believed to be the onset temperature below which the pseudogap opens. However, the opening of the pseudogap does not lead to a phase transition. There are no thermal anomalies observed in the specific heat or other thermodynamic quantities around $T^*$.

To analyze the scaling behavior of the in-plane resistivity, we assume $\rho_{ab}$ to satisfy the following scaling law:

$$\rho_{ab}(T) = A_{ab} f_{ab}\left(\frac{T}{B_{ab}}\right) + C_{ab}.$$  

Here, the residual resistivity $C_{ab}$ should be retained since the impurity contribution to $\rho_{ab}$ is no longer negligible compared with the inelastic contribution of electrons to $\rho_{ab}$.

We have applied Eq. (21) to the experimental data.
published by Ito et al.\textsuperscript{13} for Y123, by Watanabe et al.\textsuperscript{22} for Bi2212, by Fujii et al.\textsuperscript{23} for Bi2223, and by Nakano et al.\textsuperscript{38} for La214. Figure 5 shows the scaling function for these compounds. The corresponding scaling parameters are shown in Table IV. The measurement data (not shown in the figure) begin to deviate from the universal scaling curves near $T_c$ due to superconducting fluctuations.

The result of Fig. 5 shows that $\rho_{ab}$, indeed, satisfies the simple scaling law described by Eq. (21). Moreover, all the curves shown in Fig. 5 are obtained by taking Y123 $\delta = 0.68$ as a reference. This means that $\rho_{ab}$ can be scaled onto a single curve for all these materials. Thus the scaling function of $\rho_{ab}$ is universal. It does not depend on the chemical structure nor on the doping level. This suggests that the in-plane resistivity is governed by the same scattering mechanism in all cuprate superconductors.

The striking scaling behavior of $\rho_{ab}$ indicates that the characteristic temperature $T^*$ above which $\rho_{ab}$ varies linearly with temperature is proportional to the scaling parameter $B_{ab}$. In Ref. 13, we showed that $T^*$ is proportional to the pseudogap $\Delta$ determined from the c-axis resistivity, independent of doping. Thus, $B_{ab}$ is also proportional to $\Delta$. This means that, same as for the c-axis resistivity, the pseudogap $\Delta$ is also a control energy scale for the in-plane resistivity, although $\rho_{ab}$ is mainly the contribution of nodal quasiparticle excitations.

Wuyts et al.\textsuperscript{39,40} did a similar scaling analysis for the
TABLE IV: The scaling parameters, $A_{ab}$, $B_{ab}$, and $C_{ab}$, for the in-plane resistivity of Y123$^a$ (Ref. [12]), Y123$^b$ (Ref. [32]), Bi2212 (Ref. [22]), Bi2223 (Ref. [22]), and La214 (Ref. [38]) as shown in Fig. 5. The notations used in Ref. [22] are adopted to label the samples of Bi2223. Y123$^a$ with $\delta = 0.68$ is taken as the reference sample. The unit of $C_{ab}$ is $10^{-4}$f1cm. The doping $p$ of Y123$^a$, Bi2212 and Bi2223 are obtained from the empirical formula Eq. (15), with the maximum $T_c$ being 93.54 K for $\delta = 0.90$ of Y123$^a$, 89.0 K for Bi2212 at $\delta = 0.22$, and 108.0 K for F sample of Bi2223. The values of $T_c$ for La214, Y123$^b$, Bi2212, and Bi2223 are obtained from Ref. [1, 13, 22], and [41], respectively.

| Sample | $T_c$ (K) | Doping (p) | $A_{ab}$ | $B_{ab}$ | $C_{ab}$ |
|--------|-----------|------------|----------|----------|----------|
| Y123$^a$ | 0.45 | 54.96 | 0.088 | 3.83 | 1.91 | 0.09 |
|          | 0.58 | 64.67 | 0.098 | 1.48 | 1.22 | 0.13 |
|          | 0.68 | 67.04 | 0.100 | 1.0 | 1.0 | 0.0 |
|          | 0.78 | 80.26 | 0.118 | 0.63 | 0.81 | 0.0074 |
|          | 0.85 | 92.08 | 0.146 | 0.39 | 0.60 | -0.10 |
|          | 0.90 | 93.54 | 0.16 | 0.22 | 0.50 | -0.19 |
| Y123$^b$ | 0.55 | 2.02 | 1.33 | 0.19 |
|          | 0.60 | 1.57 | 1.20 | -0.31 |
|          | 0.65 | 1.35 | 1.15 | -0.22 |
|          | 0.70 | 0.89 | 0.92 | -0.095 |
|          | 0.75 | 0.70 | 0.81 | -0.070 |
|          | 0.80 | 0.55 | 0.65 | -0.26 |
|          | 0.85 | 0.52 | 0.78 | 0.049 |
| Bi2212  | 0.2135 | 71.0 | 0.11 | 1.18 | 0.96 | 1.70 |
|          | 0.217 | 77.0 | 0.119 | 1.07 | 0.89 | 1.10 |
|          | 0.22 | 83.0 | 0.131 | 0.65 | 0.73 | 1.12 |
|          | 0.24 | 89.0 | 0.16 | 0.51 | 0.63 | 0.60 |
|          | 0.245 | 87.86 | 0.173 | 0.48 | 0.82 | 0.76 |
|          | 0.255 | 87.4 | 0.175 | 0.32 | 0.54 | 0.24 |
| Bi2223  | B | 93.0 | 0.118 | 0.12 | 1.09 | 0.31 |
|          | D | 104.0 | 0.139 | 0.10 | 0.99 | -0.29 |
|          | E | 106.0 | 0.145 | 0.087 | 0.81 | -0.69 |
|          | F | 108.0 | 0.16 | 0.07 | 0.69 | -0.90 |
| La214   | 0.10 | 30.75 | 0.10 | 0.84 | 2.06 | 9.47 |
|          | 0.125 | 32.0 | 0.125 | 0.60 | 1.97 | 6.93 |
|          | 0.14 | 36.62 | 0.14 | 0.51 | 2.05 | 5.77 |

In the overdoped regime, $\rho_{ab}$ is not linear-$T$ dependent in nearly the whole temperature range. The experimental data do not fall onto the scale curves as shown in Fig. 5. This change of the temperature behavior of $\rho_{ab}$ in the overdoped regime can be understood from the change of the Fermi surface topology revealed by ARPES. In the overdoped region, the Fermi surface becomes electronlike. This may affect strongly the dynamic behavior of electrons in the CuO$_2$ planes.

In Fig. 5(d), we only show the experimental data for slightly underdoped La214 samples ($x = 0.1, 0.125$, and 0.14). For heavily underdoped samples ($x = 0.04, 0.06$, and 0.08), we find that the data deviate significantly from the universal scaling curve below $T^*$. This deviation was observed only in the La214 samples. It may be due to the suppression of the scattering rate by the formation of stripe or other competing orders in these compounds.
C. Hall coefficient $R_H(T)$

The Hall coefficient $R_H(T)$ is an important quantity in characterizing the nature of charge carriers. In a conventional metal with a spheric Fermi surface and isotropic scattering rates, the Hall coefficient is inversely proportional to the carrier concentration, independent of temperature. The sign of $R_H$ reflects the type of conducting charge carriers. $R_H$ is negative or positive if the charge carriers are electrons or holes. However, in doped transition metal oxides, such as high-$T_c$ cuprates, the Hall coefficient is strongly temperature dependent. It is determined not just by the carrier concentration, but also by the scattering rates and the curvature of the Fermi surface. Other effects, such as magnetic skew scattering, can also affect the temperature dependence of $R_H$.\(^{47}\)

In HTSC, $R_H$ shows a strong temperature and doping dependence. At high temperature, $R_H$ increases rapidly with decreasing temperature. After reaching a maximum, $R_H$ drops down to low temperature in most of the samples. This is the typical temperature dependence of $R_H$ in HTSC. It was observed in Y123,\(^{44,48}\) La$_{214}$,\(^{49,50,51}\) Bi systems,\(^{52,53,54}\) Hg1212,\(^{55}\) and Tl systems.\(^{56,57,58}\)

The complex temperature dependence of the Hall coefficient remains one of the hardest problems to be resolved. Within the theory of charge-spin separation, Anderson\(^{29}\) proposed to use the Hall angle $\Theta_H$ instead of the Hall coefficient $R_H$ to understand the Hall anomaly. He argued that due to the charge-spin separation, the Hall angle, which is defined by the ratio between the transverse and longitudinal conductivities, $\Theta_H = \tan^{-1}(\sigma_{xy}/\sigma_{xx})$, should be determined purely by the transverse relaxation rate (i.e., the relaxation rate perpendicular to the Fermi surface). This eliminates the ambiguity in the explanation of the Hall coefficient, since it is determined by both the longitudinal and transverse relaxation rates. Anderson further argued that as the transverse relaxation rate is determined by the spin excitations, which is relatively normal, the Hall angle should follow the temperature dependence of normal Fermi liquid, i.e.,

$$\cot \Theta_H = \alpha T^2 + \gamma, \quad (22)$$

where $\alpha$ is a temperature independent coefficient and $\gamma$ is the impurity contribution.\(^{29}\) This quadratic temperature dependence of the Hall angle, indeed, agrees with the experimental observation at optimal doping. However, in both underdoped and overdoped regimes,\(^{48,50,51,52,53}\) the temperature exponent deviates generally from 2, and the above expression breaks down.

To reveal the physics behind the anomalous temperature dependence of the Hall effect without invoking a specific model, we have analyzed the scaling behavior of $R_H$. We assume the scaling function of $R_H$ to have the form defined as in Eq. (10):

$$R_H(T) = A_H f_H \left( \frac{T}{B_H} \right) + C_H. \quad (23)$$

By fitting the experimental data with this formula using the method introduced in Sec. I, the scaling parameters $(A_H, B_H, C_H)$ and the scaling function $f_H$ can then be determined.

Figure 6 shows the scaling function $f_H$ for Y123, La$_{214}$, and La-Bi$_{2201}$. The corresponding scaling parameters are given in Table V. The experimental data were extracted from Ref. 48 for Y123, from Ref. 49 for La$_{214}$, and from Ref. 50 for La-Bi$_{2201}$. The experimental data of Y123 with $\delta < 0.55$ are not included since the temperature range measured is too narrow to allow a reliable scaling analysis to be done.

For La$_{214}$, we find that $R_H$ exhibits a good scaling behavior in nearly the whole temperature range, as shown in Fig. 6(a). This is also true for La-Bi$_{2201}$ [Fig. 6(b)]. It suggests that the dynamical behavior of $R_H$ is still governed by a single energy scale. However, in contrast to the in-plane resistivity, the scaling curve of La$_{214}$ cannot be perfectly scaled onto the scaling curve of La-Bi$_{2201}$, except in an intermediate temperature regime. For Y123, the experimental data can also be scaled onto a single curve in the high-temperature regime, above the peak temperature of $R_H$. However, at low temperatures, $f_H$ shows very different temperature dependence for different dopings. This difference might be caused by the contribution of CuO chains in Y123. Electrons in CuO chains
are more disordered in the underdoped samples than in the optimally doped one.

A similar scaling analysis has been done by Hwang et al. and by Chen et al. The scaling equation they used is essentially the same as Eq. (23), but with different notations. In their notation, the scaling equation is given by

\[ R_H(T) = R_H^\infty + R_H^* f \left( \frac{T}{T^*} \right). \]  

(24)

\( R_H^\infty \) is the high-temperature value of \( R_H \), which is approximately temperature independent at high temperatures. \( T^* \) is a characteristic temperature to be determined. Empirically, they assumed \( T^* \) to be the crossover temperature from a temperature dependent to a temperature independent \( R_H \) at high temperature. However, the crossover temperature (if exists) is very high, well above the temperature range they measured, in the underdoped samples. They cannot determine reliably the crossover temperature, even by extrapolation. Thus, their scaling analysis cannot be applied to the underdoped samples. This is not a problem in our approach. In Fig. (a), we compared the scaling curve obtained by Hwang et al. with ours. We find that these two scaling curves agree well with each other for La214 above the peak temperature of \( R_H \). This suggests that the crossover temperature they determined is proportional to the parameter \( B_H \) as we determined here.

Recently, Gor'kov and Teitel’baum and Ono et al. analyzed the high-temperature behavior of \( R_H \) using a two-band model in La214. They assumed the high-temperature data of \( R_H \) to be thermally activated, resulting from strong charge fluctuations between the effective lower and upper Hubbard bands. They found that the high-temperature data of \( R_H \) can indeed by explained by this simple picture. Their results suggest that the thermal excitation gap between the lower and upper Hubbard bands is significantly smaller than the (direct) optical charge transfer gap.

### D. Thermoelectric power \( S(T) \)

The thermoelectric power or the Seebeck coefficient \( S(T) \) is one of the transport quantities complementary to the resistivity and Hall effect. It reveals the properties of quasiparticle excitations both near and away from the Fermi level. It can be used to judge whether the charge carriers are electrons or holes from the sign of \( S(T) \). It can also be used to quantify the charge carrier concentration. In HTSC, empirically, the value of \( S(T) \) at \( T = 290 \) K was found to be a good measure of the hole concentration.

In high-\( T_c \) oxides, the thermopower \( S(T) \) is small in the superconducting state due to the suppression of the pairing gap to the quasiparticle excitations. In the normal state, \( S(T) \) increases with temperature. It exhibits a maximum and then drops monotonically at high temperatures. At a given temperature, \( S(T) \) decreases with increasing doping. For most of the high-\( T_c \) compounds, including Bi2212, Bi2201, La-Bi2212, Tl1212, and Hg1201, the scaling curves for two heavily underdoped samples of Hg1201 (Ref. 69) deviate significantly from the universal scaling curve and are not included in the figure. For Y123 and Ca-Y123, we only analyze the underdoped samples since in the overdoped regime the chain contribution becomes important, which breaks the scaling law. The chain contribution can, in fact, be seen already in the slightly underdoped sample of Y123 (\( \delta = 0.175 \)), whose high-temperature data of \( S(T) \) already begin to deviate away from the scaling curve at high temperatures. For some samples of Hg1201 and Ca-Y123, the measurement data fall faster than the universal scaling curves at low temperatures. This can be attributed to the superconducting fluctuations.

Figure 4 shows the scaling curves for Y123, Ca-Y123, Hg1201, and La214. The scaling parameters are listed in Tables VI and VII. The experimental data were extracted from Ref. 72 for Y123, from Ref. 73 for Ca-Y123, from Ref. 63 for Hg1201, and from Refs. 73 and 72 for La214. The scaling curves for two heavily underdoped samples with \( T_c < 2 \) K and three heavily overdoped samples of Hg1201 (Ref. 69) deviate significantly from the universal scaling curve and are not included in the figure. For Y123 and Ca-Y123, we only analyze the underdoped samples since in the overdoped regime the chain contribution becomes important, which breaks the scaling law.

The chain contribution can, in fact, be seen already in the slightly underdoped sample of Y123 (\( \delta = 0.175 \)), whose high-temperature data of \( S(T) \) already begin to deviate away from the scaling curve at high temperatures. For some samples of Hg1201 and Ca-Y123, the measurement data fall faster than the universal scaling curves at low temperatures. This can be attributed to the superconducting fluctuations.

The scaling behavior of \( S(T) \) in HTSC has been extensively studied by a number of groups using the scaling formula like that defined in Eq. 24. Our scaling curves are consistent with their results. However, our data are much less scattered than theirs.
E. Uniform magnetic susceptibility $\chi(T)$

The uniform magnetic susceptibility $\chi(T)$ measures basically the density of states at the Fermi level in conventional Landau Fermi liquid. However, in high-$T_c$ copper oxides, the susceptibility is strongly affected by antiferromagnetic spin fluctuations. The parent compounds of HTSC are half-filled antiferromagnetic Mott insulators with long-range Néel order. Upon doping, the Néel order is rapidly suppressed, but antiferromagnetic fluctuations persist up to slightly overdoping. At half-filling, $\chi(T)$ shows a sharp peak around the Néel temperature. This peak shifts down to lower temperature with doping and disappears completely when the superconductivity emerges. In the normal state, the magnetic susceptibility $\chi(T)$ first increases with increasing temperature, develops a broad peak, and then drops down at high temperature.

In real materials, the magnetic susceptibility is strongly affected by magnetic impurities. These impurities contribute a Curie term to $\chi$, which diverges as $1/T$ at low temperatures. The impurity contribution is strongly sample dependent. In order to analyze the intrinsic behavior of the magnetic susceptibility, this Curie term of impurities should be subtracted from the raw data first.
To elucidate the intrinsic property of the magnetic susceptibility, we have analyzed the scaling behavior of $\chi(T)$ with the following single-parameter scaling equation:

$$\chi(T) = A_\chi f_\chi \left( \frac{T}{B_\chi} \right) + C_\chi. \quad (26)$$

Figure 5 shows the scaling curves of $\chi(T)$ for La214 and Bi2212. The experimental data were extracted from those published by Nakano et al.\(^{38}\) for La214 and by Watanabe et al.\(^{24}\) for Bi2212. The corresponding scaling parameters are shown in Table VIII. In the scaling analysis for the La214 samples with $x = 0.20, 0.22,$ and 0.26, a Curie term $C/T$ is subtracted from the experimental data; the corresponding values of $C$, i.e., $C = 12.41, 37.23,$ and 60.23 (in units of $10^{-7}$ emu/g), were obtained by Nakano et al.\(^{38}\).

We find that the susceptibility for both La214 and Bi2212 exhibits a good scaling behavior. For Bi2212, $\chi(T)$ in heavily overdoped samples begins to deviate from the scaling curve near $T_c$. This is likely to be due to strong superconducting fluctuations.

Our universal scaling curve is consistent with the scaling analysis given by Johnston and Nakano et al.\(^{38}\) for La214, and by Allgeier and Schilling\(^{35}\) for Bi2212. The scaling analysis of Nakano et al.\(^{38}\) was made based on the scaling formula defined by Eq. (14). The problem with that kind of analysis is that the characteristic temperature $T^*$ defined in Eq. (13) has to be determined empirically prior to the scaling analysis. The characteristic temperature $T^*$ for the susceptibility was generally determined from the peak temperature of $\chi$. However, in heavily overdoped samples, no peak structure has been observed within the whole temperature measured. This has limited the application of that kind of scaling analysis. In addition, to fully satisfy Eq. (14), a constant term also needs to be subtracted for each set of data. This is also difficult if the measured temperature range is not broad enough. Nevertheless, we find that their scaling curves, as shown in Fig. 5, agree well with ours.

The scaling analyses given by Johnston\(^{38}\) and by Allgeier and Schilling\(^{35}\) are based on the high-temperature series expansion for a two-dimensional antiferromagnetic Heisenberg model. The scaling function obtained by Johnston deviates slightly from the universal scaling curve obtained at high temperature. The high-temperature scaling curve, as shown by Johnston\(^{38}\) and by Allgeier and Schilling\(^{35}\), agrees with the temperature dependence of the susceptibility of the two-dimensional antiferromagnetic Heisenberg model without doping.

In obtaining the scaling function, a temperature independent term is subtracted from $\chi(T)$. This term can be

| Sample | $T_c$ (K) | Doping (p) | $A_S$ | $B_S$ | $C_S$ |
|--------|-----------|------------|-------|-------|-------|
| Y123   | 0.65      | 75.53      | 1.80  | 4.17  |
|        | 0.60      | 13.31      | 0.057 | 3.22  |
|        | 0.53      | 44.66      | 0.079 | 2.46  |
|        | 0.44      | 55.51      | 0.089 | 2.36  |
|        | 0.35      | 59.8       | 0.093 | 2.03  |
|        | 0.24      | 74.4       | 0.109 | 1.46  |
|        | 0.175     | 90.7       | 0.14  | 0.48  |
| Ca-Y123| 0.38      | 85.50      | 0.16  | 1.17  |
|        | 0.43      | 85.00      | 0.151 | 1.07  |
|        | 0.51      | 81.58      | 0.136 | 1.57  |
|        | 0.54      | 78.55      | 0.128 | 1.13  |
|        | 0.57      | 77.19      | 0.125 | 4.21  |
|        | 0.69      | 47.04      | 0.085 | 12.86 |
|        | 0.99      | 37.69      | 0.077 | 30.07 |
| Hg1201 | 3         | 26.0       | 0.05  | 33.56 |
|        | 4         | 46.0       | 0.057 | 11.79 |
|        | 5         | 62.0       | 0.069 | 15.24 |
|        | 6         | 72.0       | 0.09  | 4.88  |
|        | 7         | 77.0       | 0.103 | 3.27  |
|        | 8         | 83.0       | 0.11  | 0.00  |
|        | 9         | 91.0       | 0.119 | 4.67  |
|        | 10        | 95.0       | 0.127 | 2.80  |
|        | 11        | 98.0       | 0.157 | 3.01  |

| Sample | $T_c$ (K) | Doping (p) | $A_S$ | $B_S$ | $C_S$ |
|--------|-----------|------------|-------|-------|-------|
| La214  | 0.15      | 37.93      | 0.15  | 16.44 |
|        | 0.18      | 37.14      | 0.18  | 8.07  |
|        | 0.2       | 34.62      | 0.2   | 5.70  |
|        | 0.21      | 30.99      | 0.21  | 4.46  |
|        | 0.23      | 23.77      | 0.23  | 1.95  |
|        | 0.25      | 17.29      | 0.25  | 0.15  |
|        | 0.26      | 14.4       | 0.26  | -1.48 |
|        | 0.28      | 8.08       | 0.28  | -0.85 |
| La2212 | 0.06      | 5.0        | 0.06  | 4.18  |
|        | 0.08      | 22.3       | 0.08  | 3.22  |
|        | 0.1       | 30.75      | 0.1   | 2.53  |
|        | 0.125     | 32.0       | 0.125 | 1.66  |
|        | 0.135     | 35.9       | 0.135 | 0.88  |
|        | 0.15      | 37.93      | 0.15  | 0.79  |
|        | 0.17      | 38.4       | 0.17  | 0.87  |
|        | 0.2       | 34.62      | 0.2   | 0.83  |
|        | 0.22      | 27.81      | 0.22  | 0.61  |
|        | 0.24      | 19.96      | 0.24  | 0.53  |
|        | 0.27      | 12.0       | 0.27  | 0.43  |
expressed as
\[ \chi_0 = C \chi + A \chi_{0,s} \]  
where \( \chi_{0,s} \) is the value of \( \chi_0 \) for the reference sample. The variation of \( \chi_0 \) with doping may reflect the change of the density of states at the Fermi level. Thus, it is interesting to analyze the doping dependence of this term.

Figure 8 shows the doping dependence of \( \chi_0 \). \( \chi_0 \) increases monotonically with doping. This temperature independent term might be the contribution of the core diamagnetism, the Van Vleck paramagnetism, the Landau diamagnetism, and the Pauli paramagnetism of the band electrons if the effect of antiferromagnetic correlations is ignored. The core diamagnetism and the Van Vleck paramagnetism are doping independent. The Landau diamagnetism is generally small. Thus, the doping dependence of \( \chi_0 \) is mainly affected by the Pauli susceptibility, which is proportional to the density of states at the Fermi level. Hence, the change of \( \chi_0 \) with doping will correspond to the change of the density of states at the Fermi level. This simple observation is consistent with the result of Allgeier and Schilling as well as the measurement of the specific heat.

F. Spin-lattice relaxation rate \( 1/T_1 \)

The nuclear magnetic resonance probes the local spin dynamics via the measurement of the Knight shift, the spin-lattice relaxation rate \( 1/T_1 \), and other spin response functions. The Knight shift measures the shift of the resonance frequency induced by the conduction electrons. It is proportional to the uniform magnetic susceptibility \( \chi(T) \). The temperature dependence of the Knight shift should follow the scaling law of the uniform susceptibility as discussed in the previous section. This is, indeed, supported by the experimental measurement (see, for example, Ref. 87). Below we will discuss the scaling behavior of the spin-lattice relaxation rate \( 1/T_1 \).

The high-temperature dependence of \( 1/T_1 T \) of \( ^{63}\text{Cu} \) shows a Curie-Weiss-like behavior. This can be attributed to the contribution of antiferromagnetic fluctu-
TABLE VIII: The scaling parameters for the magnetic susceptibility $\chi(T)$ shown in Fig. 8. The unit of $C_v$ is $10^{-7}$ emu/g. $T_c$ of La214 is obtained from Ref. 4 by interpolation. In obtaining the scaling curves, a Curie term $C/T$ is subtracted from the measurement data for the La214 samples with $x = 0.20, 0.22,$ and 0.26. The corresponding values of $C$, obtained from Fig. 8 of Ref. 63 are 12.41, 37.23, and 60.23 $10^{-7}$ emu/g, respectively.

| Sample   | $T_c$ (K) | Doping (p) | $A_v$ | $B_v$ | $C_v$ |
|----------|-----------|------------|-------|-------|-------|
| La214    | 0.08      | 22.3       | 0.08  | 1.40  | 5.97  | -1.60 |
|          | 0.10      | 30.75      | 0.10  | 1.41  | 4.18  | -1.57 |
|          | 0.14      | 36.62      | 0.14  | 1.28  | 2.31  | -1.11 |
|          | 0.18      | 37.45      | 0.18  | 1.0   | 1.0   | 0.0   |
|          | 0.20      | 34.62      | 0.20  | 1.04  | 0.68  | 0.31  |
|          | 0.22      | 27.81      | 0.22  | 1.24  | 0.53  | -0.001|
|          | 0.26      | 14.57      | 0.26  | 1.38  | 0.46  | -0.027|
| Bi2212   | 0.22      | 82.0       | 0.13  | 1.33  | 3.04  | -1.29 |
|          | 0.25      | 89.57      | 0.16  | 1.10  | 2.22  | -0.66 |
|          | 0.26      | 87.19      | 0.18  | 0.66  | 1.60  | 0.61  |
|          | 0.27      | 83.45      | 0.186 | 0.72  | 1.18  | 0.62  |
|          | 0.28      | 78.7       | 0.195 | 0.51  | 0.74  | 1.42  |
|          | 0.30      | 68.51      | 0.22  | 0.66  | 0.40  | 1.84  |

TABLE IX: The fitting parameters, $A_{T_1}$, $B_{T_1}$, and $C_{T_1}$ for the scaling analysis of $1/T_1T$ shown in Fig. 10. The unit of $C_{T_1}$ for $63$Cu of Bi2212 and Tl1212 is $s^{-1}K^{-1}$, and for $89$Y of Y123 is $10^{-4} s^{-1}K^{-1}$.

| Sample   | $T_c$ (K) | Doping (p) | $A_{T_1}$ | $B_{T_1}$ | $C_{T_1}$ |
|----------|-----------|------------|-----------|-----------|-----------|
| Bi2212 $^{63}$Cu | 0.125 | 79.0 | 0.13 | 0.86 | 1.31 | 0.72 |
|          | 0.20      | 86.0      | 0.16      | 1.0      | 1.0      | 0.0    |
|          | 0.225     | 77.3      | 0.20      | 1.17     | 0.85     | -1.67  |
| Tl1212 $^{63}$Cu | 0.70    | 0.44      | 2.44      | 0.72     | -4.72    |
|          | 54.0      | 2.29      | 0.56      | -3.05    |
|          | 10.0      | 0.48      | 0.16      | 16.18    |
| Y123 $^{89}$Y | 1.0    | 88.8      | 0.19      | 0.075    | 0.88     | 1.51   |
| $^{14}$T   | 0.069     | 0.13      | 1.35      | 1.53     |
| $^{15}$T   | 0.85      | 90.3      | 0.14      | 0.85     | 1.63     | 1.15   |
| $^{17}$T   | 0.75      | 67.9      | 0.10      | 0.87     | 1.50     | 0.83   |
| $^{19}$T   | 0.63      | 57.3      | 0.09      | 0.88     | 2.30     | 0.90   |
| $^{21}$T   | 0.53      | 52.5      | 0.086     | 0.99     | 2.80     | 0.80   |
| $^{23}$T   | 0.48      | 38.0      | 0.07      | 0.070    | 3.35     | 0.77   |
| $^{27}$T   | 0.41      | 15.0      | 0.06      | 0.028    | 3.74     | 0.68   |

We have analyzed the scaling behavior of the experimental data of $1/T_1T$ published by Ishida et al. for $^{63}$Cu in Bi2212, by Magishi et al. for $^{63}$Cu in Tl1212, and by Alloul et al. for $^{89}$Y in Y123. The scaling equation is assumed to be

$$\frac{1}{T_1T} = A_{T_1} f_{T_1} \left( \frac{T}{B_{T_1}} \right) + C_{T_1}. \quad (28)$$

Figure 10 shows the temperature dependence of the scaling function $f_{T_1}$. The corresponding scaling parameters are given in Table IX.

Within experimental errors, we find that $1/T_1T$ shows a good scaling behavior. For all the materials shown in Fig. 10, the data of $1/T_1T$ can be scaled on a common curve in a relatively wide range of temperature. This, again, suggests that the normal state dynamics is controlled by a single energy scale. This single-parameter scaling behavior is consistent with the existing theory of spin fluctuations, such as the antiferromagnetic Fermi liquid theory proposed by Millis et al.

IV. ANALYSIS OF THE CHARACTERISTIC ENERGY SCALE

In the preceding section, we have analyzed the scaling behaviors of the transport coefficients, including the resistivity, the Hall effect, and the thermoelectric power, and the magnetic response functions, including the spin susceptibility and the spin-lattice relaxation rate, in the normal state of HTSC. These coefficients probe different aspects of low-energy excitations and are physically
distinct. However, we find that they all show good scaling behaviors. For most of the measurement quantities, the temperature dependence of the corresponding scaling functions are universal, depending neither on the doping concentration nor on the chemical structure of the materials measured.

Among the three scaling parameters, the energy scale $\Delta$ or the relative energy scale $B$ defined in Eq. 17 is the most important one. It characterizes the basic energy scale governing the temperature dependence of a response function. For the c-axis resistivity, we have determined the absolute values of $\Delta$ using the approximate scaling function of $\rho_c$ derived in our previous work for multilayer cuprates. For other measurement quantities, as the analytic formula of the scaling functions are unknown, only the ratio of $\Delta$ with respect to a reference sample, $B = \Delta/\Delta_x$, is determined. Nevertheless, we find that these characteristic energy parameters determined from different coefficients show a common trend with doping. As shown in Tables III-VII they all decrease with increasing doping.

![FIG. 11: The doping dependence of the energy scales obtained with Eq. (29). The values of the scaling factor $\eta_y$ are listed in Table X. The dashed line is the linear fit to the scaled pseudogap energies obtained by various experimental probes. Here, we take the pseudogap energy obtained from the c-axis resistivity of Bi2212 as a reference, as shown in Fig. 3.](image)

The similar doping dependence of the characteristic energy parameters suggests that these relative energy scales obtained by different probes may have a common physical origin. This can be examined by rescaling all the relative energy scales with respect to the absolute energy scale of the pseudogap $\Delta$ obtained from the scaling analysis of $\rho_c$. They should fall onto a single curve if they are, indeed, the pseudogap energies. To do this, let us introduce the following formula:

$$\Delta_y(p) = \eta_y B_y(p), \quad (29)$$

where $\eta_y$ is a scaling factor and $\Delta_y$ is the characteristic energy scale. The subscript $y$ represents the measured physical quantity, i.e., $y = \rho_c, \rho_{ab}, R_H, S, \chi$, and $1/T_1 T$. The scaling factors $\eta_y$ can be determined by the least square fit using the approach introduced in Sec. III.

The fitting parameters of $\eta_y$ are given in Table X. By substituting them into Eq. (29), we can obtain the values of $\Delta_y$. The result, as shown in Fig. 11, indicates that all the energy scales determined from the scaling analysis given in Sec. III have the same doping dependence within experimental errors, which result mainly from the uncertainty in the determination of doping concentration. It suggests that all the dynamic coefficients analyzed in Sec. III are, indeed, governed by the same energy scale. This is a remarkable result since different coefficients probe different responses of charge and/or spin degrees of freedom. For example, the c-axis resistivity $\rho_c$ is sensitive to the charged excitations around the antinodal points, while the in-plane resistivity $\rho_{ab}$ is mainly affected by the scattering of charged quasiparticles around the node points. The uniform magnetic susceptibility probes the spin fluctuations around $k = (0, 0)$, while the spin-lattice relaxation rate is strongly dependent on the antiferromagnetic fluctuations around $k = (\pi, \pi)$.

In the underdoped regime, the characteristic energy $\Delta$ drops almost linearly with doping. This is consistent with the doping dependence of the pseudogap observed by ARPES, tunneling, and other measurements. Thus, the control energy scale in this regime is, indeed, the pseudogap. If we extrapolate the underdoped data of $\Delta$ to zero doping, we find that $\Delta$ is in order of $J$, and to higher doping, $\Delta$ vanishes roughly at $p \sim 0.23-0.25$.

In the literature, two kinds of pseudogaps (or the onset temperatures of pseudogap), which were often quoted as the “large” and “small” pseudogaps, were reported. The large pseudogap generally refers to the characteristic temperature $T^*$ measured, for example, by the magnetic susceptibility the Hall coefficient. The small
one could be the energy scale probed by other experimental techniques, such as the transport properties measurements or the leading-edge shift around the antinodal direction measured by the ARPES. Our scaling analysis indicates that these two energy scales are, in fact, physically indistinguishable.

In the overdoped regime, the universal scaling behavior generally breaks down. For overdoped La214, ∆y determined from the thermopower and the Hall coefficients separates into two branches with different energies. ∆y determined from the thermopower is higher than that from the Hall effect. This may imply the existence of two energy scales in the overdoped regime. However, as the single-parameter scaling law still holds in this regime for both the thermopower and the Hall coefficients, further investigation on this issue is desired.

Recently, there is a surge of interest in the discussion of two-gap energies, namely, the pseudogap in the normal state and the relatively smaller superconducting gap in the superconducting state. The existence of a distinct energy scale in the superconducting state, whose doping dependence is different from that of the pseudogap, was first reported in the penetration depth measurements and later in the Andreev reflection measurements. This lower energy scale characterizes the low-lying excitations around the gap nodes and appears only in the superconducting state. Recently, ARPES, Raman scattering, and inelastic neutron measurements have further confirmed the existence of these two distinguished energy scales. Furthermore, the ARPES has revealed that the two-gap structure is intimately connected with the arc (or pocket) feature of the Fermi surface of HTSC in the underdoped regime. It is believed that the superconducting gap develops predominantly on the Fermi arc below Tc. This is consistent with the early ARPES measurement data. However, there exists also other experimental measurements, which suggest that there is only one energy scale in the superconducting state and the superconducting gap around the nodal points is nothing but an extension of the pseudogap in the arc area in the superconducting state. This one energy scale scenario is consistent with the picture of resonant valence bonds based on the charge-spin separation as well as that of preformed pairs.

In this work, we have shown that there is only one energy scale in the normal state. However, as we have only analyzed the scaling behavior of the experimental data in the normal state, we are unable to address the issue of two energy gaps in the superconducting state. It is of great interest to extend the single-parameter scaling method introduced in Sec. II to the superconducting state at which two energy scales (or control parameters) may exist. This would then allow us to judge whether there is only one or two energy scales in the superconducting state from the model-independent scaling analysis of various transport and thermodynamic coefficients.

V. SUMMARY

We have introduced a scaling method to study the scaling behavior in the normal state of HTSC. We have analyzed the scaling behavior of the c-axis resistivity, the in-plane resistivity, the Hall coefficient, the thermoelectric power, the magnetic susceptibility, and the nuclear magnetic resonance, and extracted the corresponding energy scales. It is found that all these quantities, no matter how different they are, exhibit universal scaling behaviors, controlled by a single energy scale in the normal state. Furthermore, we find that all these energy scales obtained from different physical coefficients have the same doping dependence as the pseudogap. It shows that the pseudogap is the only characteristic energy governing the low-lying excitations in the normal state of HTSC.

The scaling method we introduced in Sec. II is model independent. It provides a simple but powerful tool to analyze the scaling behavior of experimental data. It can be applied not only to the normal state of HTSC, but also to any other systems where the single-parameter scaling hypothesis, i.e. Eq. (3) or (10), is valid.

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