Unconventional pairing induced anomalous transverse shift in Andreev reflection

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Abstract

Superconductors with unconventional pairings have been a fascinating subject of research, for which a central issue is to explore effects that can be used to characterize the pairing. The process of Andreev reflection—the reflection of an electron as a hole at a normal-metal-superconductor interface by transferring a Cooper pair into the superconductor—offers a basic mechanism to probe the pairing through transport. Here we predict that in Andreev reflection from unconventional superconductors, the reflected hole acquires an anomalous spatial shift normal to the plane of incidence, arising from the unconventional pairing. The transverse shift is sensitive to the superconducting gap structure, exhibiting characteristic features for each pairing type, and can be detected as voltage signals. Our work not only unveils a fundamentally new effect, but also suggests a powerful new technique capable of
probing the structure of unconventional pairings.
Interface scattering—the scattering at an interface between different media—is ubiquitous for all kinds of particles and waves. It offers a basic means to probe material properties, like in the various optical and electronic characterization techniques; and is of fundamental importance for controlling carrier transport, as in all kinds of electrical junctions that constitute the foundation of our current information technology. Nontrivial effects can happen during interface scattering. In geometric optics, it is known that a circularly-polarized light beam undergoes a transverse shift normal to its plane of incidence when reflected at an optical interface, called the Imbert-Fedorov shift \(^1\)–\(^6\). Recently, analogous effect has been discovered for electronic systems, showing that transverse shifts also appear for electrons in so-called Weyl semimetals \(^7\)–\(^10\). In both cases, the spin-orbit coupling (SOC) plays the key role for the shifts. The light helicity corresponds to the photonic spin state, which intrinsically couples with the light propagation in the Maxwell equation \(^11\); and the low-energy electrons in Weyl semimetals also possess a strong SOC described by the Weyl equation \(^12, 13\). Upon scattering, any change in the particle spin would require a change in the orbital motion due to SOC, resulting in the anomalous spatial shift (Fig. 1a,b).

There is an intriguing and unique scattering process occurring at the normal-metal-superconductor (NS) interface—the Andreev reflection, in which an incident electron from the normal metal is reflected back as a hole, accompanied by the transfer of a Cooper pair into the superconductor \(^14, 15\) (Fig. 2a). Most recently, we find that the transverse shift can also exist in Andreev reflection, if the interface is formed by a spin-orbit-coupled metal and a conventional s-wave superconductor \(^16\). It is important to note that the essential ingredient there is still the assumed strong SOC of the scattered carrier—the shift vanishes when SOC is negligible; whereas the superconductivity only
plays a passive role as a channel for electron-hole conversion.

Unconventional pairing brings new physics into the picture. By breaking more symmetries than the $U(1)$ gauge symmetry, unconventional pair potentials necessarily have a strong wave-vector dependence $^{17}$. Surprisingly, we find that in Andreev reflection from an unconventional superconductor, a sizable transverse shift exists even in the absence of SOC, resulting solely from the unconventional pair potential (see Fig. 1c). We show that the unconventional pairing provides an effective coupling between the orbital motion and the pseudospin of the electron-hole (Nambu) space, which underlies this exotic effect. Remarkably, the value of the shift is sensitive to the structure of pair potential and manifests characteristic features for each pairing type, as summarized in Table 1. The effect and the associated features can be detected in electric measurement, providing a powerful new technique capable of probing the structure of unconventional pairings.

**Results**

**Model.** We consider a three-dimensional NS junction with a flat interface in the clean limit. In this work, we focus on the case where the interface is perpendicular to the principle rotation axis (along $z$-direction) of the superconductor (Fig. 2b). Configurations with other interface orientations can be similarly studied. To highlight the role of unconventional pairing, we neglect SOC in the model. Then for each pairing considered in Table 1, the essential physics of the scattering at the NS interface (located at $z = 0$) can be captured by the mean-field Bogoliubov-de Gennes (BdG)
in the following reduced form

\[
\begin{bmatrix}
    H_0 - E_F + V(z) & \Delta \Theta(z) \\
    \Delta^* \Theta(z) & E_F - H_0 - V(z)
\end{bmatrix}
\begin{bmatrix}
    \psi
\end{bmatrix}
\begin{bmatrix}
    \epsilon
\end{bmatrix}.
\]

(1)

Here, \(\psi\) is the two-component spinor wave-function in the Nambu space (the real-spin labels are suppressed), \(E_F\) is the Fermi energy, and \(V(z) = U\Theta(z) + h\delta(z)\) with \(U\) the band bottom shift, \(h\) the interface barrier potential, and \(\Theta\) the Heaviside step function. We take the single-particle Hamiltonian \(H_0 = -\frac{1}{2m} \nabla^2\) for the normal-metal (N) side (for \(z < 0\)), and \(H_0 = -\frac{1}{2m_{||}} (\partial_x^2 + \partial_y^2) - \frac{1}{2m_z} \partial_z^2\) for the superconductor (S) side (for \(z > 0\)). The difference in the effective masses \(m_{||}\) and \(m_z\) describes the possible uniaxial anisotropy in S. For certain layered superconductors (like cuprates), the Fermi surface is highly anisotropic and may take a cylinder-like shape in the normal state, which requires a separate treatment using lattice models, as we will discuss later. However, the essential features of our result remain the same. The usual step function model is adopted for the pair potential \(\Delta\). We consider the weak coupling limit with \(E_F - U \gg |\Delta|, \epsilon\) in the S region, so that the wave-vector for \(\Delta\)'s \(k\)-dependence is fixed on the (normal-state) Fermi surface of S, and \(\Delta\) only depends on the direction of the wave-vector \(k\).

Here, we have fixed the N side to be a simple metal (as in the above model). In the following, we shall investigate the effect of different pairings \(\Delta\) of the S side on producing possible anomalous shift in quasiparticle scattering at the NS interface.

**Anomalous spatial shift in reflection.** Consider an incident electron described by the wave-packet \(\Psi(r^c, k^c)\) propagating from the normal metal towards the NS interface. Here \((r^c, k^c)\) is
the center of the wave-packet in phase space. The superscript $c$ will be dropped wherever appropriate in the following discussion. As illustrated in Fig. 2b, its propagation direction is specified by two parameters: the incident angle $\gamma$ and the rotation angle $\alpha$ between the plane of incidence and the $x$-$z$ plane. At the NS interface, besides possible transmission into the superconductor, the electron can either be normal-reflected as an electron or Andreev-reflected as a hole (see Fig. 2a), characterized by the scattering amplitudes $r_e$ and $r_h$, respectively. The possible anomalous shift in these scattering processes can be studied by comparing the center positions of the scattered wave-packet and of the incident one$^{7,22}$.

For the model in Eq. (1), we find that the spatial shift in the interface plane between the incident and the normal (Andreev) reflected wave-packets takes a simple expression given by (see Supplemental Material)

$$
\delta \ell^i_e(h) = -\frac{\partial}{\partial k_i} \text{arg}(r_{e(h)})\bigg|_{k_\parallel},
$$

(2)

where $i \in \{x,y\}$, $k_\parallel$ is the wave-vector component parallel to the interface which is conserved during scattering, and $\text{arg}(r_{e(h)})$ is the phase of the reflection amplitude $r_e$ ($r_h$). It should be noted that: (i) the shift depends only on (the derivative of) the phase of reflection amplitude but not its magnitude; (ii) the shift is defined for each definite scattering process (normal or Andreev), of which the probability is given by the magnitude $|r_{e(h)}|^2$; (iii) the shift has both longitudinal (analogous to the Goos-Hänchen shift in optics$^{23}$) and transverse components with reference to the incident plane. The transverse shift is more interesting, and it remains well-defined also for smooth interfaces, whereas the longitudinal shift does not. In this work, we focus on the transverse shift: $\ell^i_T \equiv \delta \ell^i_e(h) \cdot \hat{n}$, where $\hat{n}$ is the normal direction of the incident plane, as illustrated in...
Fig. 2b,c.

The discussion up to this point is general, valid for any pairing type in Table 1. Applying Eq. (2) to an NS junction with conventional $s$-wave pairing on the S side, one easily checks that $\ell^e_T = \ell^h_T = 0$, consistent with our previous findings\textsuperscript{16}. In sharp contrast, the transverse shift can be sizable if the S side is of unconventional pairing, and it sensitively depends on the pairing type, as shown in Table 1. In the following, we analyze two representative examples in detail: the chiral $p$-wave pairing and the $d_{x^2-y^2}$-wave pairing.

**Example 1: chiral $p$-wave pairing.** Let’s first consider that the S side is of chiral $p$-wave pairing, with $\Delta = \Delta_0 e^{i\chi \phi_k}$. Here $\chi = \pm 1$ denotes the chirality, $(\theta_k, \phi_k)$ are the spherical angles of $k$, and $\Delta_0$ is assumed to be independent of $\phi_k$ but may still depend on $\theta_k$ here (which does not affect the essential features to be discussed). Straightforward calculation with the result in Eq. (2) shows that (see Supplemental Material)

$$\ell^h_T = \chi / k_{\parallel}, \quad \ell^e_T = 0,$$

(3)

where $k_{\parallel} = |k| = k_{NF}^N \sin \gamma$, with $k_{NF}^N$ the Fermi wave-vector in N. The shift vanishes for normal reflection but takes a simple universal form for Andreev reflection. Interestingly, $\ell^h_T$ is independent of parameters on the S side such as the pairing gap $\Delta_0$, except that its sign depends on the chirality of pairing (see Fig. 3).

To better understand these remarkable features, we note that there is an emergent symmetry
of the BdG equation:

$$[\hat{H}_{\text{BdG}}, \hat{J}_z] = 0, \quad (4)$$

where $\hat{H}_{\text{BdG}}$ is the BdG Hamiltonian in Eq. (1) with $\Delta$ taking the chiral $p$-wave pair potential, and

$$\hat{J}_z = (\hat{r} \times \hat{k}) - \frac{1}{2} \chi \tau_z \quad (5)$$

resembles an effective angular momentum operator with $\tau_z$ the Pauli matrix corresponding to the Nambu pseudospin-1/2. As a result of this symmetry, the expectation value $J_z = \langle \Psi | \hat{J}_z | \Psi \rangle$ evaluated for the wave-packet must conserve during scattering. For electrons and holes, the expectation values of the Nambu pseudospin are opposite, with $\langle \tau_z \rangle_{e/h} = \pm 1$. The pseudospin flips in Andreev reflection, hence the conservation of $J_z$ dictates a transverse shift $\ell_T^h$ to compensate this change, leading to

$$\ell_T^h = -\frac{\chi}{2k_\parallel} (\langle \tau_z \rangle_h - \langle \tau_z \rangle_e) = \frac{\chi}{k_\parallel}, \quad (6)$$

which nicely recovers the result in Eq. (3) obtained from the scattering approach.

The analysis above demonstrates several points. First, the unconventional pairing acts like an effective spin-orbit coupling. However, the spin here is not the real spin but the Nambu pseudospin, which is intrinsic and unique for superconductors. Second, the result (3) is quite general: as long as the symmetry is preserved, factors like the pairing gap, the excitation energy, or the interfacial barrier will not affect $\ell_T^h$. Furthermore, it is easily seen that the result (3) also applies for chiral pairings with higher orbital moments ($|\chi| > 1$), such as $d + id$ or $f + if$ pairings.

**Example 2: $d_{x^2-y^2}$-wave pairing.** For the second example, we consider the $d_{x^2-y^2}$-wave pairing, with $\Delta = \Delta_0 \cos(2\phi_k)$. In this case, $\hat{J}_z$ is no longer conserved, but we can still calculate the shifts
using Eq. (2) by the scattering approach (see Supplemental Material). We find that

$$\ell_T^h \propto \sin(4\alpha)\Theta(|\Delta_0 \cos 2\alpha| - \varepsilon),$$  \hspace{1cm} (7)

and $\ell_T^e \approx \ell_T^h$. The result in Eq. (7) highlights the dependence of the transverse shift on the rotation angle $\alpha$ and the excitation energy $\varepsilon$.

From Table 1 and Fig. 4, we observe the following key features in result (7), which are correlated with the $d_{x^2-y^2}$-pairing. (i) $\ell_T^{e(h)}$ has a period of $\pi/2$ in $\alpha$, and it flips sign at the multiples of $\pi/4$ (Fig. 4b). (ii) $\ell_T^{e(h)}$ is sensitive to the gap magnitude. As indicated by the step function in Eq. (7), it is suppressed for excitation energies above the pairing gap at the incident wave-vector. (iii) Particularly, due to the nodal structure of the gap, for a fixed excitation energy $\varepsilon$, there must appear multiple zones in $\alpha$ where $\ell_T^{e(h)}$ is suppressed (see Fig. 4b). The center of each suppressed zone coincides with a node. (iv) $\ell_T^{e(h)}$ is also suppressed when $k_\parallel$ is away from the Fermi surface of the S side, as indicated in Fig. 4c. This can be understood by noticing that the effect of pair potential diminishes away from the Fermi surface.

The above features of the shifts encode rich information about the unconventional gap structure, including the $d$-wave symmetry [Feature (i)], the gap magnitude profile [Feature (ii)], and the node position [Feature (iii)]. Thus, by detecting the effect (discussed below), one can extract important properties of the superconducting gap structure. Feature (iv) also offers information on the geometry of the Fermi surface of S, which is further demonstrated in the following.
**Result for a cylinder-like Fermi surface.** As we have mentioned, for certain layered unconventional superconductors (like cuprates), the Fermi surface in S is highly anisotropic and may take a cylinder-like shape, which is open along the $z$-direction (see Fig. 5a). Such a feature cannot be captured by the continuum model $H_0$ in Eq. (1), but may be described using a lattice model (see Supplemental Material).

From the lattice model, we can calculate the reflection amplitudes $r_e$ and $r_h$ via numerical means, and then obtain the shifts using Eq. (2). The obtained result for the chiral $p$-wave pairing reproduces that in Eq. (3). The result for the $d_{x^2-y^2}$-wave pairing is shown in Fig. 5. Compared with Fig. 4, one observes that the key features [Features (i)-(iv) discussed above] remain the same as those for the ellipsoidal Fermi surface. Now, since the Fermi surface is of cylinder-like shape, Feature (iv) means that the shifts here are also suppressed for small $k_\parallel$ values away from the S Fermi surface, as indicated in Fig. 5c. This feature can be used to extract information about the shape of the Fermi surface of S.

**Discussion**

In this work, we have revealed a fundamentally new effect—the anomalous shifts in Andreev reflection generated by unconventional pairings. We emphasize that the physics here is distinct from the previous works, including those in the electron reflection in Weyl semimetals $^7,8$ and in the Andreev reflection from conventional $s$-wave superconductors $^{16}$. Here, the shift is purely from the unconventional pairing in the S region, and it exists without SOC. This is in sharp contrast to
the effects studied in the previous works, where the shift originates from the SOC in the N region and vanishes if the SOC is negligible \(^7,^8,^16\). Because of this fundamental difference, the shifts here manifest the characteristics of unconventional pairings in S, such as the highly anisotropic behavior with respect to the incident direction as in Fig. 4b, which is tied to the anisotropic character of the \(d_{x^2-y^2}\)-pairing; whereas the shifts in Ref. 16 instead reflect the SOC pattern of the N region.

We have explicitly demonstrated the results for two example pairings. The shifts for other types of pairings can be calculated in a similar way (see Supplemental Material for the result of chiral \(d\)-wave and \(p_x\)-wave pairing). More complex types of pairing (such as mixed ones) may also be investigated using the method outlined here, and are generally expected to result in nonzero transverse shifts.

To experimentally probe the effect, one may adopt the simple NS junction geometry as illustrated in Fig. 6, in which the electrons are driven towards the interface with a finite average incident angle. The transverse shifts then leads to the surface charge accumulation, which can be detected as a voltage signal between top and bottom surfaces (see Fig. 6b). With more delicate setups, e.g. by using local gates and collimators similar to those in electron optics \(^24,^25\), one could control the angles \((\gamma, \alpha)\) of the incident electron beam, and the excitation energy \(\varepsilon\) can be controlled by the junction bias voltage. Then by mapping out the transverse voltage dependence on \((\gamma, \alpha, \varepsilon)\), one can extract the features of the shifts and in principle characterize the gap structure of the unconventional superconductor.

Here it should be noted that: While for chiral pairings, the voltage signal is solely due to
the shifts in Andreev reflection; for nonchiral pairings (like \( d_{x^2-y^2} \)-pairing), the signal could have contributions from both normal and Andreev reflections. Since \( \ell_T^c \approx \ell_T^e \), the net result depends on the probabilities \(|r_h|^2 \) vs \(|r_e|^2\) of the two scattering processes. There could be interesting competition between the two when tuning the excitation energy. Generally, for excitation energies close to the superconducting gap, \(|r_h|^2 \) would dominate over \(|r_e|^2\) \(^{18}\) (see Fig. 4d and Fig. 5d), so in this case the signal would be dominated by the shifts in Andreev reflection.

We remark that real unconventional superconductor materials could have complicated structures, such as multiple Fermi surfaces, multiple bands with different pairing magnitudes, and possible surface/interface bound states \(^{17,26-28}\). How these features would affect the anomalous shifts is an interesting question to explore in future studies. Nevertheless, our analysis suggests that a nonzero shift (hence the resulting voltage) is generally expected. Although its detailed profile requires more accurate material-specific modeling, the key features regarding the period in \( \alpha \) and the gap dependence as listed in Table 1 should be robust, since they are determined by the overall characteristic associated with the symmetry of unconventional pairings. Finally, when SOC effect is included, it can generate an additional contribution to the shift in Andreev reflection \(^{16}\). However, its dependence on the incident geometry and the excitation energy will be distinct from that due to the unconventional pairings.

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Figure 1: **Transverse shifts in three kinds of reflection processes.**

**a**, A circularly polarized light beam undergoes a transverse shift (Imbert-Fedorov shift) when reflected at an optical interface. **b**, Electron with strong spin-orbit coupling, like in Weyl semimetals, acquires a transverse shift when reflected from a potential barrier. **c**, An incident electron is reflected as a hole in Andreev reflection from a normal-metal-superconductor (NS) interface. We show that a transverse shift appears if the superconductor has unconventional pairing. The three figures are schematic. Here, the plane of incidence is taken as the $x$-$z$ plane (the drawing does not imply normal incidence). The transverse shift is normal to the plane of incidence, i.e. along the $y$-direction.
Figure 2: Scattering at the NS interface and the junction set-up. a, Schematics of Andreev and normal reflection processes at an NS interface. An incident (right-going) electron from the normal metal (red solid dot) is scattered at the NS interface. Besides possible transmission, it may be normal-reflected as an electron, or Andreev-reflected as a hole (denoted by the green circle). In case of Andreev reflection, a Cooper pair is transferred into the superconductor. $\varepsilon$ denotes the quasiparticle excitation energy measured from the Fermi level. b, Schematic figure showing the NS junction set-up. The interface is located at $z = 0$ with normal metal at $z < 0$ (not explicitly drawn) and superconductor at $z > 0$. The incident direction ($\hat{k}$) is specified by two angles: the incident angle $\gamma$, and the rotation angle $\alpha$ between the incident plane (the orange-colored plane) and the $x$-direction. The normal direction ($\hat{n}$) of the incident plane is specified to be along $\hat{k} \times \hat{z}$. In Andreev reflection, the reflection plane (the green-colored plane) is shifted by $\ell^h_T$ from the incident plane along $\hat{n}$, due to the unconventional pairing in S. c, Top view of the $x$-$y$ plane (from $+\hat{z}$-direction), where the incident and the reflection planes are projected to two straight lines. For certain pairings, there may also be a finite shift $\ell^c_T$ for normal-reflected electrons (not shown here).
Figure 3: **Transverse shift for chiral $p$-wave pairing.** 

**a,** $\ell_T^h (1/k_{N F}^h)$ versus rotation angle $\alpha$ for chiral $p$-wave pairing with $\chi = +1$, showing an isotropic profile.  

**b,** $\ell_T^h$ versus the excitation energy $\varepsilon$. The shift is independent of the excitation energy and the magnitude of pairing gap. Its sign depends on the chirality ($\chi = \pm 1$) of pairing. In these figures, $\ell_T^h$ is measured in unit of $1/k_{N F}^h$. We set $\gamma = \pi/8$, $\Delta_0 = 20$ meV, $E_F = 0.32$ eV, $h = 0.1$ eV $\cdot$ nm, and $m = 0.5m_e$ with $m_e$ the free electron mass (the corresponding $k_{F N}^N = 2$ nm$^{-1}$).
Figure 4: **Transverse shift for \(d_{x^2-y^2}\)-wave pairing.**

- **a,** Schematic figure showing the Fermi surfaces on the two sides of the NS junction. The one on the S side refers to the Fermi surface in its normal state. Here \(K_c\) denotes the maximum value of transverse wave-vector on the S Fermi surface.
- **b,** \(\ell^h_T\) versus the rotation angle \(\alpha\). The shift shows a period of \(\pi/2\) and changes sign at multiples of \(\pi/4\). The green shaded regions indicate the suppressed zones (SZs), in which the excitation energy is above the pairing gap (\(|\Delta(\alpha)|\)). The center of each SZ corresponds to the position of a node.
- **c,** \(\ell^h_T\) plotted as a function of \(k_\parallel (= k_F^N \sin \gamma)\). Here, the shaded region in which \(\ell^h_T\) is suppressed corresponds to the gray-shaded one in **a,** i.e. the region with \(k_\parallel > K_c\).
- **d,** Reflection probabilities versus excitation energy for normal and Andreev reflections at a fixed incident direction. Andreev reflection dominates when \(\varepsilon\) approaches the local gap \(|\Delta(\alpha)|\).

In these plots, we take \(\Delta_0 = 20\) meV, \(E_F = 0.8\) eV, \(U = 0.3\) eV, \(h = 0.2\) eV \cdot nm and \(m_\parallel = m_z = m = 0.5\) \(m_e\) with \(m_e\) the free electron mass \((k_F^N = 3.16\) nm\(^{-1}\)). We set \(\gamma = \pi/8\) and \(\varepsilon = 10\) meV in **b**; \(\alpha = -\pi/8\) and \(\varepsilon = 10\) meV in **c**; and \(\alpha = \pi/8\) and \(\gamma = \pi/4\) in **d**.
Figure 5: **Results for** $d_{x^2-y^2}$-**wave pairing when the S side has a cylinder-like Fermi surface.**

a, Schematic figure showing the Fermi surfaces on the two sides of the NS junction. Here, $K_{c1}$ and $K_{c2}$ denote the lower and upper bounds in the transverse wave-vector on the S Fermi surface.

b-d show the same quantities as those in Fig. 4. In comparison with in Fig. 4, the features in b and d remain the same. The main difference is that the shift is now suppressed in regions except for $K_{c1} < k_\parallel < K_{c2}$, as shown in c. The model parameters for the figure are presented in Supplemental Material.
Figure 6: Detection of the transverse shift. a,b, Schematic diagrams showing a possible NS junction geometry for detecting the transverse shift. a, Top view of the junction. The geometry is designed such that the incident electron beam have a finite average incident angle $\gamma$ at the NS interface. b, Side view of the junction. As a result of the transverse shift, there is a net surface charge accumulation near the junction on the normal-metal side, which can be detected as a voltage difference between top and bottom surfaces. Here we illustrate the case when the interface scattering events are dominated by Andreev reflections.
Table 1: **Features of the transverse shift in Andreev reflection for typical unconventional pair potentials.** In the table, ”No. of SZ” stands for the number of suppressed zones when the rotation angle $\alpha$ varies from 0 to $2\pi$, for a finite and fixed excitation energy.