Aspects of particle production in isospin asymmetric matter

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The production/absorption rate of particles in compressed and heated asymmetric matter is studied using a Relativistic Mean Field (RMF) transport model with an isospin dependent collision term. Just from energy conservation in the elementary production/absorption processes we expect to see a strong dependence of the yields on the basic Lorentz structure of the isovector effective interaction, due to isospin effects on the scalar and vector self-energies of the hadrons. This will be particularly evident for the ratio of the rates of particles produced with different charges: results are shown for $\pi^+/\pi^-$, $K^+/K^0$ yields.

In order to simplify the analysis we perform RMF cascade simulations in a box with periodic boundary conditions. In this way we can better pin down all such fine relativistic effects in particle production, that could likely show up even in realistic heavy ion collisions. In fact the box properties are tuned in order to reproduce the heated dense matter formed during a nucleus-nucleus collision in the few $AGeV$ beam energy region.

In particular, $K^{+,0}$ production is expected to be directly related to the high density behaviour of the symmetry energy, since kaons are produced very early during the high density stage of the collision and their mean free path is rather large. We show that the $K^+/K^0$ ratio reflects important isospin contributions on the production rates just because of the large sensitivity around the threshold. The results are very promising for the possibility of a direct link between particle production data in exotic Heavy Ion Collisions (HIC) and the isospin dependent part of the Equation of State (EoS) at high baryon densities.

Key words: Asymmetric nuclear matter, symmetry energy, relativistic heavy ion collisions, pion production, subthreshold kaon production.

PACS numbers: 25.75.Dw, 21.65.+f, 21.30.Fe, 24.10.Jv, 25.60.-t
1 Introduction

Recently the development of new heavy ion facilities (radioactive beams) has driven the interest on the dynamical behaviour of asymmetric matter, see the recent reviews [1,2]. Indeed, the isospin-dependent part of the 
\[ EoS \]

is of crucial importance in extrapolating structure calculations beyond the valley of stability and in astrophysical processes, such as structure and cooling of neutron stars and supernovae explosions.

Here we focus our attention on relativistic heavy ion collisions, that provide a unique terrestrial opportunity to probe the in-medium nuclear interaction in high density and high momentum regions. We will show that particle production/absorption (here pions and kaons) processes in a dense and hot neutron rich medium are nicely sensitive to the details of the relativistic structure of the effective interaction. The results appear very promising for the possibility of directly pin down from data such fundamental microscopic information.

Within a covariant picture of the nuclear mean field, for the study of asymmetric nuclear matter certainly the same considerations hold for the density behaviour of the symmetry term as in the iso-scalar case. The equilibrium conditions are directly related to the Lorenz structure of the iso-vector effective fields in a similar way as in the iso-scalar case (competition effects between attractive scalar and repulsive vector fields). However, for the description of the \( a_4 \) parameter of the Weizsäcker mass formula (in a sense equivalent to the \( a_1 \) parameter for the iso-scalar part), extracted in the range from 28 to 36 MeV, there are different possibilities: (a) considering only the Lorentz vector \( \rho \) mesonic field, and (b) both, the Lorentz vector \( \rho \) (repulsive) and Lorentz scalar \( \delta \) (attractive) effective fields [3,4]. The second assumption corresponds to the two strong effective \( \sigma \) (attractive) and \( \omega \) (repulsive) mesonic fields of the iso-scalar sector. In both cases one can fix the empirical value of the \( a_4 \) parameter, however important effects at supra-normal densities appear due to the introduction of the effective \( \delta \) field. In fact the presence of an isovector scalar field is increasing the repulsive \( \rho \)-meson contribution at high baryon densities [4,2]. This is a pure relativistic effect, due to the different Lorentz properties of these fields (the vector \( \rho \) field grows with baryon density whereas the scalar \( \delta \) field is suppressed by the scalar density). As a consequence, the isospin-dependent part of the 
\[ EoS \]

becomes “stiffer” when both fields are accounted for. This important feature appears in several models, see ref. [5] where the covariant structure of the symmetry energy was investigated adopting different approaches to asymmetric nuclear matter such as Non-Linear \( RMF \), Density Dependent Hadronic (\( DDH \)) and Dirac-Brueckner-Hartree-Fock (\( DBHF \)) theories. Moreover, relativistic approaches to asymmetric nu-

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clear matter naturally lead to an effective \((\text{Dirac})\) mass splitting between protons and neutrons due to the Lorentz scalar nature of the \(\delta\) field, \([3,4,2]\). This is of relevance for the dynamics of heavy ion collisions and it has recently been microscopically studied within the \(DBHF\) approach \([6,7]\).

The study of the influence of the high density symmetry energy on the \(HIC\) dynamics has been recently started. So far one has considered collective flow observables, such as in-plane and elliptic flows \([8–11]\) for protons and neutrons (or light isobars), isospin equilibration (or transparency) and pion production in heavy ion collisions at intermediate relativistic energies up to \(1 – 2\ AGeV\) \([12,5,13]\). However, definite conclusions have not been drawn yet due to either the lack of experimental information on some observables or the moderate dependence on the density behaviour of the symmetry energy. Comparisons with recent experimental data on isospin transparency seem to suggest a \textit{stiff} high density behaviour of the symmetry energy, as predicted by \(RMF\) theory with the inclusion of the \(\delta\) meson \([13,14]\).

In order to explore the symmetry energy at supra-normal densities one has to select signals directly emitted from the earlier non-equilibrium high density stage of the heavy ion collision, possibly without any secondary interaction with the hadronic environment.

In fact particle production is of particular interest since it mainly takes place in the first binary collisions during the formation of the high density phase, where the energy densities are still above resonance thresholds. This has stimulated recent experimental studies \([15,16]\). However, the investigation of particle production represents a non-trivial task due to the following reasons:

- (a) At intermediate energies (\(< 1.6\ AGeV\)) some of the pions created during the high density phase are directly emitted, but many of them undergo secondary re-scattering with isospin exchange before emission, \([5]\).
- (b) Production and reabsorption processes must be consistently implemented in relativistic transport models with isospin effects. In fact within an isospin dependent mean field, isospin exchange processes such as \(R' \leftrightarrow \pi N, \pi\pi R\) (\(N, R, R'\) stand for protons/neutrons and the \(\Delta^{\pm,0,++}\) isospin resonance states, respectively, and \(\pi^{\pm,0}\) for pions) modify the scalar and vector content of the available energy between the in- and outgoing channels. This has to be taken into account in the energy conservation, and it is particularly relevant around the thresholds.
- (c) The situation for the production of particles with strangeness such as kaons (\(K^{\pm,0}\)) is similar, but even more complicated due to the availability of more than 50 isospin channels for the creation of \(K^+\) and \(K^0\), only from \(BB\) and \(B\pi\) processes (\(B\) stands for a resonance or a nucleon).

Kaons are produced very early during the high density stage of the collision,
see the compressibility dependence in refs. [17,18]. Moreover, the production of $K^{+,0}$ is expected to be directly related to the high density behaviour of the symmetry energy, since the mean free path of these low energy kaons in nuclear matter is rather large ($\lambda(K^{+,0}) \sim 7$ fm [17,18]). These two facts make the subthreshold kaon production a very promising probe for the high density symmetry energy. Anti-kaons ($K^{-}$) do not fulfill the desired criteria due to their relatively small mean free path value (strong reabsorption effects). A related problem for $K^{-}$’s is the importance of off-shell contributions in the propagation and the corresponding uncertainty on the self-energy evaluations [19]. At variance the quasiparticle approach followed here in a transport $RMF$ frame is consistently justified for $K^{+,0}$ mesons. In any case the anti-kaon production is not so relevant at the energies of interest in the present study [20,21].

In this article we investigate hadronic matter properties in a finite box, with periodic boundary conditions, performing cascade calculations by an extended isospin dependent collision integral. The temperature and baryon density conditions are tuned in order to reproduce the early stage of a realistic heavy ion collision at beam energies around the threshold for kaon production. Varying the asymmetry of the system we can then study the influence of the isovector part of the hadronic in medium interaction.

Thus, we discuss particle production in fixed conditions of density and temperature (for both symmetric and asymmetric matter). In this way we can have a precise control on the production/absorption rates in the different channels.

The description of the mean field is important, since nucleons and resonances are dressed by the mean field self-energies. This will directly affect the threshold energies, in particular for inelastic channels. We first give an outline of the mean field models of asymmetric nuclear matter, before passing to the description of the box calculations (Sect. 2). The modification of the threshold conditions according to energy-momentum conservation is a non-trivial task which will be discussed in Sect. 3. The isospin dependence of the chemical equilibrium conditions, to which particle multiplicities obey after a given time interval $t_{eq}$, is illustrated in Sect. 4, considering in particular the $\pi^{-}/\pi^{+}$ ratio. The results are presented in Sect. 5. We will first discuss isospin effects on resonances and pion production, before passing to kaons. The results provide an important guide to the understanding of particle production in heavy ion collisions at relativistic energies.
Table 1
Coupling parameters in terms of $f_i \equiv (\frac{g_i m_i}{f_i})^2$ for $i = \sigma, \omega$, $f_i \equiv (\frac{g_i m_i}{m_i})^2$ for $i = \rho, \delta$, $A \equiv \frac{4}{g_\sigma}$ and $B \equiv \frac{4}{g_\delta}$ for the non-linear NL models [23] using the $\rho$ (NL$\rho$) and both, the $\rho$ and $\delta$ mesons (NL$\rho\delta$) for the description of the isovector mean field. The NL model does not contain any isospin dependence.

2 Cascade calculations for hadronic asymmetric matter

2.1 Relativistic effects on the symmetry energy

Within a relativistic framework the energy conservation in binary collisions between hadrons in nuclear matter is directly related to the Lorentz components of the hadron self-energy $\Sigma = \Sigma_s - \gamma^\mu \Sigma_{\mu}$. The scalar and vector self-energies $\Sigma_s$, $\Sigma^\mu$ generally depend on baryon density and momentum, according to microscopic DBHF calculations [22]. However, since we are focusing here on isospin effects, i.e. on the iso-vector part of the self-energies, and DBHF studies of asymmetric nuclear matter are still rare or just being started [6,7], we will use a simple phenomenological version of the Non-Linear (with respect to the iso-scalar, Lorentz scalar $\sigma$ field [23]) Walecka model which corresponds to the Hartree or Relativistic Mean Field (RMF) approximation within the Quantum-Hadro-Dynamics [24]. According to this model the baryons (protons and neutrons) are described by an effective Dirac equation $(\gamma^\mu k^{*\mu} - m^*) \Psi(x) = 0$, whereas the mesons, which generate the classical mean field, are characterized by corresponding covariant equations of motion in the Local Density Approximation, with coupling constants that are not density dependent. The presence of the hadronic medium modifies the masses and momenta of the hadrons, i.e. $m^* = M + \Sigma_s$ (effective masses), $k^{*\mu} = k^{\mu} - \Sigma^\mu$ (kinetic momenta). For asymmetric matter the self-energies are different for protons and neutrons. This depends on the way the iso-vector mean field is described. As discussed in the introduction, in contrast to the iso-scalar case, since here one does not have the stringent saturation condition of balancing attractive (scalar) and repulsive (vector) contributions, one can apply either only the $\rho$ meson, or both, the $\rho$ and $\delta$ mesons. We will call the corresponding models as NL$\rho$ and NL$\rho\delta$ models, respectively. Furthermore, we will also use the non-linear Walecka model without accounting for isospin dependent mean...
field (NL-model).

The parameters used here are summarized in Table 1. The corresponding saturation properties of symmetric nuclear matter are \( E/A(MeV) = -16.0 \), \( \rho_0(fm^{-3}) = 0.16 \), \( K(MeV) = 240 \), \( m^*/M = 0.75 \) and the symmetry energy parameter (for \( NL\rho, NL\rho\delta \)) is \( a_4(MeV) = 30.7 \) [4,2]. These Lagrangians have been already used for flow [25], pion production [5] and isospin tracer [13] calculations in relativistic HIC with an overall good agreement to data. Moreover it has been recently shown that at high baryon densities the EoS of more microscopic DBHF calculations can be well reproduced [5,26].

In particular, for the more general \( NL\rho\delta \) case one obtains for the self-energies of protons and neutrons:

\[
\Sigma_s(p, n) = -f_\sigma \sigma(\rho_s) \pm f_\delta \rho_{s3} \\
\Sigma^\mu(p, n) = f_\omega j^\mu \mp f_\rho j^\mu_3 ,
\]

(upper signs for neutrons), where \( \rho_s = \rho_{sp} + \rho_{sn} \), \( j_\alpha = j^\alpha_p \mp j^\alpha_n \), \( \rho_{s3} = \rho_{sp} - \rho_{sn} \), \( j_3^\alpha = j^\alpha_p - j^\alpha_n \) are the total and isospin scalar densities and currents and \( f_\sigma, f_\omega, f_\rho, f_\delta \) are the coupling constants of the various mesonic fields, see Table 1. \( \sigma(\rho_s) \) is the solution of the non linear equation for the \( \sigma \) field [4,2].

Within the relativistic description we expect to see a series of isospin effects on the reaction dynamics just due to the changes in the covariant Lorentz structure of the isovector interaction. These different isospin contributions of relativistic origin can be already seen in the expression for the symmetry energy [4,2]:

\[
E_{sym} = \frac{1}{6} \frac{k_F^2}{E_F^*} + \frac{1}{2} \left[ f_\rho - f_\delta \left( \frac{m^*}{E_F^*} \right)^2 \right] \rho_B ,
\]

with \( E_F^* \equiv \sqrt{k_F^2 + m^*} \). When including the scalar iso-vector \( \delta \)-meson the asymmetry interaction parameter is given by the combination \( [f_\rho - f_\delta (\frac{m^*}{E_F^*})^2] \) of the repulsive vector (\( \rho \)) and attractive (\( \delta \)) iso-vector couplings. Thus, in order to reproduce the same bulk asymmetry parameter \( a_4 \), we have to increase the \( \rho \)-meson coupling when the \( \delta \)-meson is considered in the iso-vector part of the equation of state. The net effect is a stiffer symmetry energy at higher baryon densities, due to the \( \frac{m^*}{E_F^*} \) quenching of the attractive part, as shown in Fig. 1. This is a general feature that has been seen to be important in the dynamical simulations [5]. Other relativistic effects of the Lorentz structure of the isovector interaction, particularly important for the inelastic channels, will be:
Fig. 1. Density dependence of the symmetry energy Eq.(3) including only the \( \rho \)-meson (dashed line, \( NL\rho \)-model) and both, the \( \rho \) and \( \delta \) mesons (dotted line, \( NL\rho\delta \)-model) in the description of asymmetric matter.

- The vector field \( \Sigma^\mu \), Eq.(2), will show a different isospin dependence since in the \( NL\rho\delta \) model the \( \rho \)-meson coupling is larger with respect to that of the \( NL\rho \) model. In order to conserve the total energy in inelastic collisions we have to include the isospin dependence of the vector fields between the ingoing and outgoing channels, as it will be discussed in detail in the next section. Thus, the particle production/absorption will be directly sensitive to the covariant form of the isovector interaction.

- Another \( \delta \)-effect, of interest for transport properties, will be the splitting of the neutron/proton effective masses \[4,25\], see Eq.(4), which will be also discussed later on in detail in terms of the energy for subthreshold particle production:

\[
m_i^* = M + \Sigma_s(i) = M - f_\sigma \sigma_s(\rho_s) \pm f_\delta \rho_{s3} \quad (+ \, n, - \, p)
\]

2.2 Hadronic matter in a box

In order to study hadronic matter, we confine the particles in a cubic box with periodic boundary conditions. The number of nucleons and the box dimensions are given as inputs of the simulation and can thus be changed to explore different values for the baryon density \( \rho_B \) and the asymmetry parameter \( \alpha \) of the system. We have fixed the total number of nucleons to \( A=100 \),
Fig. 2. Kinetic energy spectra ($E_{\text{kin}} \equiv E_i^* - m^*_i, p \equiv \hbar k^*$) of nucleons for symmetric and asymmetric nuclear matter at $t = 0$ fm/c and fixed baryon density $\rho_B = 2.5 \rho_0$. stars: neutron spectra for symmetric matter (\(\alpha = 0\)); filled circles and triangles: neutron and proton kinetic energy spectra for asymmetric matter (\(\alpha = 0.2\)) in the NL\(\rho\) model; open circles and triangles: neutron and proton kinetic energy spectra for asymmetric matter (\(\alpha = 0.2\)) in the NL\(\rho\delta\) model. The corresponding solid lines are least-square fits according to a Fermi-Dirac distribution at the input temperature of $T = 60$ MeV.

varying the relative proton and neutron numbers $Z$ and $N$ in order to get $\alpha = \frac{N-Z}{N+Z} = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$ in the range from 0.0 (symmetric nuclear matter) to 0.4 (which corresponds to extremely asymmetric matter, with $N/Z = 2.3$). The box dimensions are set in order to achieve a baryon density (kept constant during the temporal evolution of the system) of roughly 2.5 times the saturation density, while for the temperature the value of $T = 60$ MeV has been chosen. These parameters should resemble the conditions achieved during the dense phase in heavy ion collisions at few AGeV energies [27].

At $t=0$ fm/c neutrons and protons are randomly distributed in the box volume and their kinetic 3-momenta $k^*$ are generated according to relativistic Fermi-Dirac distribution functions at the given temperature $T$:

$$n_i(k^*) = \frac{1}{1 + \exp[(E_i^*(k^*) - \mu_i^*)/T]}$$

(5)

In eq. (5) $n_i$ (i=n,p) are the occupation numbers for neutrons and protons, $E_i^* = \sqrt{k_i^2 + m_i^2}$, while $\mu_i^*$ (i=p,n) are the effective chemical potentials for
protons and neutrons respectively, given by ($\epsilon$ is the energy density):

$$\frac{\partial \epsilon}{\partial \rho_i} = \mu_i = \mu_i^* + f_\omega \rho_B \mp f_\rho \rho B_3 \quad (-n, +p).$$

(6)

At zero temperature the $\mu_i^*$ reduce to $E_{F_i}^*$ (i=p,n).

Once both $T$ and $\alpha$ (and hence $\rho_{Bi}$) are fixed, $\mu_i$, the scalar densities $\rho_{si}$ and effective masses $m_i^*$ can be calculated self-consistently solving the system of equations

$$\rho_{Bi} = \frac{2}{(2\pi)^3} \int d^3k^* n_i(k^*)$$

(7)

$$m_i^* = M - f_\sigma \sigma (\rho_s) \pm f_\delta (\rho_{sp} - \rho_{sn}) \quad (+n, -p)$$

(8)

$$\rho_{si} = \frac{2}{(2\pi)^3} \int d^3k^* E_i^* n_i(k^*).$$

(9)

For symmetric nuclear matter ($\alpha=0$) $\rho_{Bi}$, $\rho_{si}$, $\mu_i$ and $m_i^*$ are the same for both neutrons and protons so that there is no difference in their momentum distributions. In the case of asymmetric matter the momentum distributions of neutrons and protons are slightly different due to the different radii of the respective Fermi spheres so that $\mu_p^* \neq \mu_n^*$; furthermore, the inclusion of isovector scalar meson $\delta$ implies $m_p^* \neq m_n^*$. Anyway the initial effective momentum distributions of both protons and neutrons just slightly differ in NL, NL$\rho$ and NL$\rho\delta$ models. This can be seen in Fig. 2, where the nucleon spectra for $\alpha=0$ and $\alpha=0.2$ are shown.

We then let the particles (nucleons, $\Delta$ resonances and pions) propagate according to a free kinetic equation plus the collision term, in the spirit of a cascade calculation. The mean-field potentials, constant once ($\rho_B, \alpha$) are fixed, are accounted for in the energy-momentum conservation for the various channels of the collision term. The boundary conditions are modeled in such a way that particles escaping from the box are reinserted at the opposite side with the same momentum. In their motion nucleons undergo elastic and inelastic binary collisions, thus exciting resonances that further decay into pions, and then producing strange particles such as hyperons and kaons. Here we consider the lowest mass resonances only, namely $\Delta(1232)$.

Because of inelastic collisions, a small fraction of the initial kinetic energy of the system is lost in particle production and after 150 fm/c nucleons momenta are distributed according to a relativistic Fermi-Dirac function with temperature $T=56$ MeV. Pion spectra do not appear thermalized yet, though pion multiplicities saturate already after $t \approx 30$ fm/c. Indeed longer time scale are needed to reach full thermal equilibrium [28].
For elastic nucleon-nucleon cross sections we use the free parametrizations of Cugnon et al. [29], in order to reduce the computing, [30]. For the inelastic channels (resonance production) we follow the cross sections of Ref. [31]. Kaon production is treated perturbatively, so that it does not affect the overall dynamics. It occurs according to two different creation mechanisms, i.e. baryon induced processes such as $BB \rightarrow BYK$, and pion induced processes such as $\pi B \rightarrow YK$. In both cases, kaons are created together with $Y$ hyperons ($\Lambda$ and $\Sigma$) to ensure strangeness conservation and are not reabsorbed by nucleons. They can anyway undergo elastic rescattering with nucleons, and thus modify their momentum distribution. However, since we are just interested in kaon yields and their possible dependence on the symmetry energy, we neglect this effect here and do not propagate kaons after production. The elementary cross sections for kaon production from $BB$ and $\pi B$ channels are taken from Refs. [32] and [33], respectively. To treat all these scattering processes in infinite hadronic asymmetric matter we have modified the original version of the collision term of the Tübingen $RQMD$ simulation code [18], taking explicitly into account the isospin dependence of scalar and vector self-energies $\Sigma_s, \Sigma^\mu$. This turns into an isospin dependence of the threshold of all isospin channels and thus affects particle yields, in a way that will be discussed in detail in the following section.

3 Isospin dependent threshold conditions

The crucial requirement of two-body collisions is the energy-momentum conservation, i.e. the conservation of the total invariant collision energy $s$, which, in terms of ingoing (1,2) and outgoing (3,4) canonical momenta ($k_1^\mu, k_2^\mu, k_3^\mu, k_4^\mu$), reads:

$$s_{in} = (k_1^\mu + k_2^\mu)(k_1^\mu + k_2^\mu) = (k_3^\mu + k_4^\mu)(k_3^\mu + k_4^\mu) = s_{out} \quad .$$

(10)

If the isospin degree of freedom is not accounted for in inelastic collisions, we can simply fulfill energy-momentum conservation using kinetic 4-momenta $k_i^{*\mu}$ and the corresponding invariant collision energy

$$s_{in}^* = (k_1^{*\mu} + k_2^{*\mu})(k_1^{*\mu} + k_2^{*\mu}) = (k_3^{*\mu} + k_4^{*\mu})(k_3^{*\mu} + k_4^{*\mu}) = s_{out}^* \quad .$$

(11)

The relation between kinetic and canonical momenta is $k_i^{*\mu} \equiv k_i^\mu - \Sigma^\mu, \quad (i = 1, 2, 3, 4)$ and the effective masses $m_i^* = M + \Sigma_{ni}$ enter in the energy conservation via the 0-component of the kinetic 4-momentum $E^* = \sqrt{m_i^{*2} + k_i^{*2}}$.

The condition (11) is just a constraint on the kinetic plus the effective mass energy of the colliding particles and it is sufficient if one considers symmetric
\((N = Z)\) matter or restricts itself to elastic collisions or just to the cases where the mean field (scalar and vector self-energies) does not change between the ingoing and the outgoing particles. In those particular cases Eq. (11) implies Eq. (10).

In the general case of asymmetric nuclear matter, however, the introduction of the iso-vector effective mesons causes a splitting in the scalar and in the vector fields between protons and neutrons, as discussed in the previous sections (see Eqs. (1,2)). For the other hadrons the following expressions in terms of the nucleon self-energies will be used. For the four isospin states of the \(\Delta\) resonance one has [34] \((i = \text{scalar, vector})\)

\[
\begin{align*}
\Sigma_i(\Delta^-) &= \Sigma_i(n) \\
\Sigma_i(\Delta^0) &= \frac{2}{3}\Sigma_i(n) + \frac{1}{3}\Sigma_i(p) \\
\Sigma_i(\Delta^+) &= \frac{1}{3}\Sigma_i(n) + \frac{2}{3}\Sigma_i(p) \\
\Sigma_i(\Delta^{++}) &= \Sigma_i(p)
\end{align*}
\]

(12) (13) (14) (15)

in which the weights \((1, \frac{1}{3}, \frac{2}{3})\) or ‘isospin coefficients’ are just the square of the Clebsch-Gordon coefficients for isospin coupling in the \(\Delta \leftrightarrow \pi N\) processes. In the same framework, for hyperons \(\Lambda, \Sigma^{0,\pm}\) we have:

\[
\begin{align*}
\Sigma_i(\Lambda) &= \frac{2}{3} \left( \frac{\Sigma_i(p) + \Sigma_i(n)}{2} \right) \\
\Sigma_i(\Sigma^0) &= \frac{2}{3} \left( \frac{\Sigma_i(p) + \Sigma_i(n)}{2} \right) \\
\Sigma_i(\Sigma^+) &= \frac{2}{3}\Sigma_i(p) \\
\Sigma_i(\Sigma^-) &= \frac{2}{3}\Sigma_i(n)
\end{align*}
\]

(16) (17) (18) (19)

where the hyperon fields are further scaled by a factor \(\frac{2}{3}\) [18], since, according to the quark model, there are only two light quarks in a hyperon instead of the three light quarks in a nucleon. Pions and kaons are assumed to be unaffected by the nuclear mean field and then have zero self-energies. This assumption is not strictly correct, at least for kaons, which should be sensitive to medium effects and thus shift their production thresholds. Anyway, such a shift produced by a kaon potential should be almost the same for both \(K^0,+\) [17] and thus it should not significantly affect the \(K^+/K^0\) ratio we will discuss here. For this reason we do not take such a potential into account in the present work.
In inelastic collisions, due to isospin exchange, the scalar and vector self-energies between the ingoing and outgoing channels may differ, so that Eq. (11) does no longer imply Eq. (10). A typical example is the inelastic process \( nn \to p \Delta^- \) from which a \( \Delta^- \) resonance is formed. It is clear that, even in the presence of the \( \rho \) meson only, the vector self-energy of the final channel changes and in the general case of accounting for both, the \( \rho \) and \( \delta \) mesons, both self-energies (scalar and vector) of the outgoing channel differ from those of the ingoing channel.

In order to properly impose energy-momentum conservation one thus needs to replace condition (11) with (10). It then follows from Eq. (10), after some algebra, that the effective momenta \( k^*_3, 4 \) of the outgoing particles (in the \( cm \) local reference frame in which \( k^*_1 + k^*_2 = k^*_3 + k^*_4 = 0 \), \( |k^*_{\text{out}}| \equiv |k^*_3| \equiv |k^*_4| \)) are linked to the total energy \( s_{\text{in}} \) before the collision through the expression

\[
-\left(\Sigma^0_3 + \Sigma^0_4\right) + \sqrt{s_{\text{in}} + \left(\Sigma_3 + \Sigma_4\right)^2} = \sqrt{m^2_3 + k^*_{\text{out}}^2} + \sqrt{m^2_4 + k^*_{\text{out}}^2} . \quad (20)
\]

It is worthwhile to stress that in the limiting case of no isospin exchange (elastic processes) or isospin independent self-energies (no isospin splittings in the scalar and vector self-energies) the general condition (20) reduces to the usual energy conservation relation, i.e. \( \tilde{s} \to \sqrt{s^*} \) with \( s^* \) given by Eq. (11). Eq. (20) implies the following threshold condition for a given inelastic process:

\[
s_{\text{in}} \geq \frac{(m^*_3 + \Sigma^0_3 + m^*_4 + \Sigma^0_4)^2 - (\Sigma_3 + \Sigma_4)^2}{s_0} . \quad (21)
\]

This condition reduces to \( s_{\text{in}}^* \geq (m^*_3 + m^*_4)^2 \) whenever we have no isospin dependence, i.e. for symmetric nuclear matter and in general for the \( NL \) model and for elastic collisions. The multiplicity of particles produced in the inelastic processes allowed by Eq. (21) depends on the available energy above the threshold, i.e. on the difference \( \Delta s \equiv s_{\text{in}} - s_0 \), which is affected by the isovector channel through both the effective masses and the vector self-energies and therefore it changes when considering the \( NL, NL\rho \) and \( NL\rho\delta \) models. It is appropriate to discuss such variations of \( \Delta s \) for the cases of \( nn, pp \) and \( np \) collisions. The \( rhs \) of Eq. (21) is not or slightly affected by both isovector scalar (\( \Sigma_s \)) and vector (\( \Sigma^0 \)) mesons depending on the isospin state of the outgoing channel. In the particular case of a process \( nn \to p\Delta^- \) (\( pp \to n\Delta^{++} \)) \( s_0 \) is respectively given by:

\[
s_0 = \left[ m_p + \Sigma_s(p) + \Sigma^0(p) + m_\Delta + \Sigma_s(\Delta^-) + \Sigma^0(\Delta^-) \right]^2 - \left[ \Sigma(p) + \Sigma(\Delta^-) \right]^2
\]
\[ s_0 = \left[ m_n + \Sigma_s(n) + \Sigma^0(n) + m_\Delta + \Sigma_s(\Delta^{++}) + \Sigma^0(\Delta^{++}) \right]^2 \]
\[ \quad - \left[ \Sigma(n) + \Sigma(\Delta^{++}) \right]^2 , \]  \hspace{1cm} (22)

from which it is clear that, according to Eqs.(1-2, 12-15), cancellation occurs between \( \Sigma_i(p) \) and \( \Sigma_i(\Delta^-) \). Hence it follows that \( s_0 \) is the same in all the three discussed models. For the other inelastic \( nn \) and \( pp \) channels and in the case of \( np \) inelastic collisions, which proceed through the excitation of the \( \Delta^0 \) or \( \Delta^+ \) state of the \( \Delta \) resonance, this cancellation is only partial, but in any case the net contribution on \( s_0 \) is moderate. The main effects on \( \Delta s \equiv s_{\text{in}} - s_0 \) are therefore coming from \( s_{\text{in}} \). According to Eq. (10) we have (always in the \( cm \) local reference frame)
\[ s_{\text{in}} = \left( E_1^* + \Sigma_1^0 + E_2^* + \Sigma_2^0 \right)^2 - \left( \Sigma_1 + \Sigma_2 \right)^2 . \]  \hspace{1cm} (23)

Since we are dealing with infinite nuclear matter at rest, in which the spatial currents vanish, the last term of Eq. (23) is almost negligible also in the local frame considered here. Furthermore, due to moderate variations of \( E^* \) induced by \( m^* \), \( \Sigma^0 \) gives the major contribution to \( s_{\text{in}} \) variations in the different interaction models.

For \( nn \), \( pp \) and \( np \) collisions we have, respectively
\[ s_{\text{in}} = 4 \left[ (E_n^* + \Sigma_n^0)^2 \right] \]
\[ s_{\text{in}} = 4 \left[ (E_p^* + \Sigma_p^0)^2 \right] \]
\[ s_{\text{in}} = (E_n^* + E_p^* + \Sigma_n^0 + \Sigma_p^0)^2 . \]  \hspace{1cm} (24)

In the \( NL \) model \( \Sigma^0 \) is the same for neutrons and protons and hence \( s_{\text{in}} \) is the same for all \( NN \) collisions (for fixed momenta). When adding the isovector vector \( \rho \)-meson only (\( NL\rho \) model), \( \Sigma_n^0 \) increases (see Eq. 2) with respect to the \( NL \) case, while the opposite occurs for \( \Sigma_p^0 \). This effect is enhanced in the case of \( NL\rho\delta \) model, as the introduction of the \( \delta \) increases the \( \rho \)-meson coupling by about a factor 3. Correspondingly, \( s_{\text{in}} \) increases (decreases) for \( nn \) (\( pp \)) channels when turning the isovector channel on, both without and with the isovector scalar meson. At variance, in the case of \( np \) collisions due to cancellation effects in the sum of \( (n,p) \) vector self-energies, \( s_{\text{in}} \) is exactly the same when the \( \rho \)-meson is included and it will show very small variations passing from \( NL\rho \) to \( NL\rho\delta \) (due to the \( (n,p) \) effective mass splitting).

All that turns out into a gradual increasing (decreasing) multiplicity of \( \Delta^- \) (\( \Delta^{++} \)) states when going from \( NL \) to \( NL\rho \) and then to \( NL\rho\delta \) model and
hence affects the relative populations of different isospin states for pions and kaons, as we will widely discuss in the Sect. 5.

4 Isospin dependent equilibrium conditions

Since calculations are performed within a box with boundary conditions (particles cannot escape) and production and reabsorption processes are consistently taken into account for Δ’s and pions, we observe that, after a given time interval $t_{eq}$, chemical equilibrium is reached among the different particle populations. Indeed, starting from $t_{eq}$, Δ and pion multiplicities do not evolve anymore, indicating that the production rate equals the reabsorption rate. When chemical equilibrium is established, very simple relations hold and the chemical potential of produced particles, as pions, can be expressed in terms of neutron and proton chemical potentials, $\mu_n$ and $\mu_p$. In particular, at equilibrium, the chemical potential of $\pi^-$ is equal to the difference $\mu_n - \mu_p$, while the chemical potential of $\pi^+$ is equal to $\mu_p - \mu_n$. Hence one can already make analytical predictions for the $\pi^-/\pi^+$ ratio in the different models used here. This ratio can be expressed as:

$$\frac{\pi^-}{\pi^+} \propto \exp \left( 2\frac{\mu_n - \mu_p}{T} \right), \quad (25)$$

where $T$ is the temperature of the system. Neutron and proton chemical potentials have different values in the models we consider ($NL$, $NL\rho$ or $NL\rho\delta$) because the self-energies are different. In particular, going from $NL$ to the $NL\rho$ model, we expect that the $\pi^-/\pi^+$ ratio increases by a factor $\exp \left( 4f_\rho (\rho_{Bn} - \rho_{Bp})/T \right)$, according to the variation of the difference $\mu_n - \mu_p$ (see Eq.(6)). A similar, though weaker, increase of the $\pi^-/\pi^+$ ratio is expected also when going from $NL\rho$ to $NL\rho\delta$ model.

All these results can be easily fully understood remembering the general relation between $(n, p)$ chemical potentials and symmetry energy [2]:

$$\mu_n - \mu_p = 4E_{sym}(\rho_B)\alpha. \quad (26)$$

For a fixed asymmetry $\alpha$ and baryon density $\rho_B$, one can directly extract the equilibrium $\pi^-/\pi^+$ ratio, Eq.(25). In the $NL$ model (no isovector interaction) we have only the weak Fermi contribution to the symmetry energy, that largely increases, instead, when the $\rho$-coupling is introduced. A further increase, but of smaller amplitude, is also observed passing from $NL\rho$ to $NL\rho\delta$ models, see Fig. 1.
Fig. 3. Time dependence of particle multiplicities (as indicated) for symmetric ($\alpha = 0$) hadronic matter at the same fixed conditions as in Fig. 2. The inserted panel shows the temporal evolution of kaons in a linear scale.

Of course these equilibrium considerations do not hold for perturbative kaon production, for which reabsorption processes are neglected.

5 Results

Here we present the results for excited charge asymmetric matter within the effective field models of Sect. 2. The initial conditions of the matter in the box have been fixed to a baryon density of $\rho_B = 2.5\rho_0$ and a temperature of $T = 60$ MeV, see again Fig. 2. We start from the simpler case of symmetric hadronic matter applying only the $NL$ model. Then we will discuss the isospin dependence (by varying the iso-vector part of the mean field) of particle production in the asymmetric case beginning with pions and $\Delta$ resonances. A similar discussion on strangeness production ($K^{+,0}$) concludes the section.

Fig. 3 shows the temporal evolution of nucleon, $\Delta$ resonance, pion and kaon multiplicities for symmetric matter. Due to isospin symmetry, different isospin states for $\Delta$, $\pi$ and $K$ are almost equally populated. We see that the number of nucleons is almost constant throughout the time interval considered and just a tiny fraction of them ($10^{-2}$) goes into resonances since the first NN collisions. Some of these resonances further decay into one-pion channels with typical lifetimes of $\sim 1$ fm/c and are subsequently reformed in $\pi N$ collisions, according
to isospin-dependent cross-sections taken from Ref. [29]. Therefore, the total numbers of $\Delta$ and pions grow up in a few fm/c and reach equilibrium values of $\sim 3.3$ and $\sim 0.4$ respectively within 30 fm/c, when formation and absorption mechanisms occur with almost the same rate.

Kaons ($K^{0,\mp}$), on the other hand, do not equilibrate, because they cannot be reabsorbed after formation and their number rises almost linearly with time, see the inserted panel in Fig. 3. Their multiplicity is indeed well fitted by a linear function of time of the form

$$N_K(t) \sim 1.9 \times 10^{-5} \ t - c$$

($c$ is a constant of the order of $10^{-4}$) where the extremely small rate is due to the fact that in the conditions described we are far from the kaon production threshold (1.56 GeV for NN collisions in free space) and the observed sub-threshold production thus originate from extreme regions of phase space, i.e. from the tails of the momentum distributions of the particles involved. Actually, kaons could undergo inelastic collisions with other strange particles such as hyperons and thus be converted into nonstrange particles. Anyway, studies performed at higher energy densities [28] have shown that, even taking such processes into account, strange particles require a long time for equilibration, of the order of several hundreds fm/c, and hence we can neglect here kaon reabsorption as a reasonable approximation, in particular for $K^{0,\mp}$.

The above picture obviously does no longer hold for neutron-rich matter, where $\Delta^{-0}$, $\pi^{-}$ and $K^{0}$ are favoured with respect to $\Delta^{++}$, $\pi^{+}$ and $K^{+}$. We will see in the following how the yields, and in particular the ratios $\pi^{-}/\pi^{+}$ and $K^{+}/K^{0}$, will be even sensitive to the Lorentz structure of the isovector interaction.

### 5.1 Isospin effects on pions and $\Delta$ resonances

Fig. 4 shows the effect of the isovector channels on $\Delta$ and $\pi$ multiplicities, for the same total density and temperature starting conditions but with a fixed asymmetry $\alpha = 0.2$ ($N/Z = 1.5$). In the simple $NL$ model the relative yields of positively and negatively charged pions and/or the corresponding resonances, just follow from the different numbers of $nn$ and $pp$ pairs and, consequently, of their inelastic collisions: $nn \to p \Delta^{-}$, $\Delta^{-} \to n \pi^{-}$ and $pp \to n \Delta^{++}$, $\Delta^{++} \to p \pi^{+}$. Clearly the $\pi^{-}$ multiplicity increases with asymmetry parameter, i.e. as the matter becomes more and more neutron-rich while the number of $\pi^{+}$ accordingly decreases with respect to the symmetric case where we have $\pi^{-} = \pi^{+}$ yields. The further inclusion of a $\rho$ meson ($NL\rho$ model), stresses such opposite trend in the charged particle production, with the result that the former isospin states become even more populated in the case of $NL\rho$ model, while the latter are less populated (solid to dashed lines in Fig. 4).
Fig. 4. Time dependence of $\Delta$ resonances ($\Delta^{-,0} \equiv \Delta^{-} + \Delta^{0}$), ($\Delta^{+,++} \equiv \Delta^{+} + \Delta^{++}$) and pions (inserted panel), as indicated, for asymmetric ($\alpha = 0.2$) hadronic matter at the same fixed conditions of density and temperature of Fig. 2. Box calculations using different models for the mean field are shown: ($NL$, solid lines) including only the iso-scalar part ($\sigma$, $\omega$ mesons), ($NL_{\rho}$, dashed lines) accounting for the iso-vector, vector $\rho$ field and ($NL_{\rho\delta}$, dotted lines) taking into account both iso-vector $\rho$ and $\delta$ fields.

Therefore, the number of $\pi^{-}$s, which originate from $\Delta^{-,0}$ decay, also increases, while $\pi^{+}$s decrease (inserted panel in Fig. 4).

Finally on the basis of the argument discussed in the previous section, one expects a further enhancement of the effect by the inclusion of the isovector scalar $\delta$ meson, as confirmed by the results presented in the Figure.

In Fig. 5 we show the asymmetry dependence of the charged pion multiplicities. When chemical equilibrium is reached, the $\pi^{-}/\pi^{+}$ ratio depends just on the difference between neutron and proton chemical potentials and on the temperature $T$. Indeed, if we look at this ratio as a function of the $\alpha$ parameter (right panel) we clearly see the behaviour already noticed from Fig. 4: the $\pi^{-}/\pi^{+}$ ratio increases when passing from $NL$ to $NL_{\rho}$ and then to the $NL_{\rho\delta}$ model. As explained previously, the $\pi^{-}/\pi^{+}$ ratio should increase by a factor $exp \left( 4f_{\rho}(\rho_{Bn} - \rho_{Bp})/T \right)$ when passing from $NL$ to $NL_{\rho}$ model. For instance, at the asymmetry $\alpha = 0.2$, this factor equals 3.6, that approximately corresponds to the enhancement observed in the Figure.

At the same asymmetry, we get for the effective neutron and proton chemical potentials: $\mu_{n}^{*} - \mu_{p}^{*} = 34 \, MeV$ for $NL$ and $NL_{\rho}$ models and $-16 \, MeV$.
Fig. 5. Charged pion multiplicities (left) and the $\pi^-/\pi^+$ ratio (right) as a function of the asymmetry parameter $\alpha$ at the same conditions of Fig. 2. Statistical errors are inside the data points.

for the $NL\rho\delta$ model, due to the nucleon Dirac mass splitting Eq.(8). Hence, considering the change in the difference of effective chemical potentials and the value of the constant $f_\rho$ (see Tab.I), we see that the difference $\mu_n - \mu_p$, to which the $\pi^-/\pi^+$ ratio is related, changes when going from $NL\rho$ to $NL\rho\delta$ model and we expect an increase of this ratio by a factor 1.88. This is also nicely confirmed by our results.

5.1.1 Isospin effects on pion reabsorption

Since in nuclear collisions equilibrium conditions are likely not reached, it is interesting to calculate just the probability of emitting pions in the other extreme case, i.e. excluding the pion reabsorption processes. Now the observed $\pi^-/\pi^+$ ratio should reflect directly the ratio between the cross sections for $\pi^-$ and $\pi^+$ production.

As shown in Fig. 6, the $\pi^-/\pi^+$ ratio rises both in the $NL\rho$ (with respect to the $NL$) model and in the $NL\rho\delta$ (with respect to the $NL\rho$) model, for any value of the asymmetry parameter. In fact, as discussed in the previous sections, the addition of each isovector meson ($\rho$ and $\delta$) favours $\pi^-$ over $\pi^+$ production, as a result of the higher (lower) energy available in the “parent” $nn$ ($pp$) collisions.

We notice that the difference of $\pi^-/\pi^+$ ratio observed among the different
models is less pronounced with respect to the results observed when equilib-rium is reached (see Fig. 5). In particular, the results obtained with \( NL\rho \) or \( NL\rho\delta \) are quite close. Moreover, in a nucleus-nucleus collision, pions interact with the surrounding matter all along the reaction path. Hence the observed final pion multiplicity is not clearly connected to the very first stage of the collision and to the high density behaviour of the symmetry energy. In the next section we will discuss kaon production, for which we expect a larger sensitivity to the models used, since in the conditions considered here we are close to the production threshold. Moreover, kaons only loosely interact with the nuclear medium and consequently they should carry a better signal of the symmetry energy behaviour at large density.

### 5.2 Isospin effects on kaon production

In the case of kaon production the isospin states to be considered are \( K^0 \) and \( K^+ \), which can be produced through a large variety of channels with two or three particles in the final state: \( BB \rightarrow BYK, \ B\pi \rightarrow YK \) (\( B,Y,K \) stand for non-strange baryon, hyperon and kaon, respectively). While, for the baryon channels, a proton-proton collision can produce \( \pi^+ \) but not \( \pi^- \) and a neutron-neutron collision, conversely, only \( \pi^- \) but not \( \pi^+ \), in the case of kaons isospin conservation allows \( K^0 \) and \( K^+ \) to be formed from both initial \( pp \) and \( nn \) scattering processes. However, due to isospin coefficients, \( K^0 \) mainly come from \( nn \) inelastic channels and \( K^+ \) from \( pp \) channels. We thus expect their relative yields to change with the asymmetry parameter \( \alpha \), similarly to pions without reabsorption, but with a larger sensitivity since kaons are produced close to the threshold. Moreover, from the previous discussion on
Table 2
Relative weight of various $K^0$ production channels for symmetric ($\alpha = 0$) and asymmetric ($\alpha = 0.2$) matter. The relative width for the $\Delta\Delta$ channels is given in units of $10^{-4}$. Calculations with different symmetry energies are shown too.

| $\alpha$ | $K^+$ | | | | | | $K^0$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
|   | $NN$ | $N\Delta$ | $\Delta\Delta$ | $\pi N$ | $\pi\Delta$ | $NN$ | $N\Delta$ | $\Delta\Delta$ | $\pi N$ | $\pi\Delta$ |
| 0 | $NL$ | 0.032 | 0.232 | 1 | 0.643 | 0.093 | 0.030 | 0.221 | 3 | 0.665 | 0.083 |
| 0.2 | $NL$ | 0.033 | 0.206 | 1 | 0.670 | 0.091 | 0.029 | 0.228 | 2 | 0.651 | 0.085 |
| " | $NL\rho$ | 0.030 | 0.227 | $< 1$ | 0.633 | 0.119 | 0.026 | 0.231 | 3 | 0.662 | 0.072 |
| " | $NL\rho\delta$ | 0.033 | 0.236 | 2 | 0.591 | 0.131 | 0.024 | 0.192 | 3 | 0.714 | 0.070 |

Pion production, also the pionic channels will contribute to isospin effects on kaon production.

The $BB \rightarrow BYK$, $B\pi \rightarrow YK$ channels can be schematically divided as follows:

(i) $NN \rightarrow BYK$
(ii) $N\Delta \rightarrow BYK$
(iii) $\Delta\Delta \rightarrow BYK$
(iv) $\pi N \rightarrow YK$
(v) $\pi\Delta \rightarrow YK$

where $N$ stands for a nucleon and $B$ for a non-strange baryon.

At low energy densities, such as those achieved in the present calculations and/or in heavy ion collisions at energies below 1.5 AGeV, the pionic channels $\pi N$, (iv), give the main contribution [35], together with the $N\Delta$ ones, (ii). $\pi\Delta$ and $NN$ collisions contribute just for a few percent to the total $K^+$ and/or $K^0$ yield, while the $\Delta\Delta$ channel is negligible.

The advantage of a cascade calculation in a box is that we can cleanly evaluate the relative weight of the various kaon production channels in the interacting matter, i.e. vs. the different choices of the hadron self-energies. Our results are reported in Table 2. We see that for all the iso-vector models discussed here we are in agreement with previous estimations [35]. We note also, in the case of asymmetric matter, the different behavior of the important $\pi N$ channel when we add the isovector mesons: the weight is decreasing for the $K^+$ production while it is increasing for the $K^0$. This is very promising for our search for fine relativistic effects of the isovector interaction on kaon production.
Table 3
Production rate of $K^{+,0}$ (in units of $10^{-5}$) for asymmetric ($\alpha = 0.2$) hadronic matter for the NL, NL$\rho$, NL$\rho\delta$ models.

|        | $\frac{dN(K^{+})}{dt}$ ($\times 10^{-5}$) | $\frac{dN(K^{0})}{dt}$ ($\times 10^{-5}$) |
|--------|------------------------------------------|------------------------------------------|
| NL     | 1.66                                     | 2.18                                     |
| NL$\rho$ | 1.17                                     | 2.91                                     |
| NL$\rho\delta$ | 1.06                                     | 3.92                                     |

Fig. 7. Temporal evolution of $K^{+,0}$ yields and their ratio (in the inserted panel) for asymmetric ($\alpha = 0.2$) hadronic matter for the same iso-vector models as in Fig. 2. The whole picture is summarized in Fig. 8, where the kaon yields and their ratio are displayed as a function of the asymmetry parameter $\alpha$. The influence of the different iso-vector models obviously increases with $\alpha$ for the $K^0$ yield, but less for the $K^0/K^+$ ratio. In fact, as already shown in Fig. 7 and Table 3...
for a fixed asymmetry, the $K^+$ yield, which decreases of about 25% and 10% in NLρ and NLρδ models respectively, presents an isovector meson effect slightly weaker than the $K^0$ yield, which exhibits increasing variations of about 35% for both NLρ and NLρδ models.

Generally the isospin effects on kaon yields originate from at least two different mechanisms: the isospin dependencies of the $\pi^\pm$ yields and of the threshold energies. We will now discuss in detail these two effects.

Since kaon production from pionic channels (which give the main contribution) is competitive with pion reabsorption, it occurs before secondary $\pi N \rightarrow \Delta$ processes set in. Therefore, $K^0$ and $K^+$ kaon yields are mainly affected by the $\pi^-$ and $\pi^+$ abundances before reabsorption, see Fig. 6. In particular, as $K^0$ ($K^+$) mainly come from $\pi^- N$ ($\pi^+ N$) processes, the enhancement (reduction) of the $\pi^-$ ($\pi^+$) multiplicities in the NLρ model is partially responsible for a corresponding increase (decrease) of the $K^0$ ($K^+$) yields.

The isospin effect on the kaon production threshold is more complicated, due to the many isospin channels that may affect the threshold (and hence the yields) differently. However, contributions in opposite directions originating from different channels do not exactly cancel each other. In order to clarify the observed influences of the models on the yields in a transparent way, it is appropriate to focus on some demonstrative channels. Such examples are
the isospin channels $n\pi^- \rightarrow \Sigma^- K^0$ and $p\pi^+ \rightarrow \Sigma^+ K^+$ for $K^0$ production, respectively.

$\pi N$ channels offer the advantage of two particles in the final state (instead of the three that we have in the $BB$ case); moreover, since also kaons (and not only pions) are, in the present calculations, unaffected by nuclear mean field, Eqs. (20-23) are simplified ($\Sigma_{i2} = \Sigma_{i\pi} = \Sigma_{i4} = \Sigma_{ik} = 0$).

According to Eq. (20) the corresponding threshold conditions read

\[ s_{in}(n\pi^-) \geq (M_{\Sigma^-} + M_{K^0} + \Sigma_{s\Sigma^-} + \Sigma_{\Sigma^-}^0)^2 + \Sigma_{\Sigma^-}^2 \]
\[ s_{in}(p\pi^+) \geq (M_{\Sigma^+} + M_{K^+} + \Sigma_{s\Sigma^+} + \Sigma_{\Sigma^+}^0)^2 + \Sigma_{\Sigma^+}^2, \]

where $M_{\Sigma^-} + M_{K^0} = 1695 GeV$ and $M_{\Sigma^+} + M_{K^+} = 1683 GeV$.

As discussed in the previous section, for collisions involving neutrons (protons) $s_{in}$ increases (decreases) when going from $NL$ to $NL\rho$ and then to $NL\rho\delta$ model. Therefore, the available energy for $K^0$ production increases as the symmetry energy becomes stiffer, leading to an enhanced $K^0$ yield. Obviously the trend for the isospin behavior of the $K^+$ yield will be opposite.

6 Conclusions

We have investigated the high density behavior of the iso-vector of the nuclear interaction, which is still poorly known experimentally, controversially predicted by theory, but of great interest in extreme nuclear systems. Here we discuss its dynamical effects on particle production for an idealized system, the asymmetric hadronic matter. In this respect relativistic cascade calculations have been performed in a box with periodic boundary conditions. The advantage is that in this way we can cleanly follow all the contributions of the various isospin channels to the production and reabsorption, and the corresponding dependence on the effective field structure of the isovector interaction.

The paper represents a detailed study, in a relativistic frame, of the production/absorption mechanisms of energetic particles (pions and kaons) in excited nuclear matter at high isospin and baryon density and temperature. These are transient conditions in realistic charge asymmetric heavy ion collisions at intermediate energies. The yields are largely influenced by the isospin dependence of the nucleon self-energies. In particular, a given isospin state of the produced particles is differently populated varying the high density behaviour of the symmetry energy.

It has been found that, after $\approx 30$ fm/c, chemical equilibrium is reached. The
The isovector meson effect on kaon rates appears larger than on pions (excluding reabsorption) because kaons are produced close to the threshold. It should be noticed that low energy kaons do not suffer from secondary reabsorption effects neither in infinite hadronic matter nor in collisions of finite nuclei due to strangeness conservation. Hence, in a nuclear collision, they are expected to better signal the behaviour of the symmetry energy during the first stage of the collision, when they are formed. As an important result, the $K^0$ and $K^+$ yields are oppositely affected leading to a crucial isospin dependence of the $K^+/K^0$ ratio, which one would expect to observe in heavy ion collisions at subthreshold energies. Such an investigation is being performed and will be discussed in a future work.

In conclusion the results presented here appear very promising for the possibility of extracting information from terrestrial laboratories on the Lorentz structure of the isovector nuclear interaction in a medium at densities of astrophysical interest. The use of radioactive beams at relativistic energies would be extremely important.
References

[1] B.-A Li, W.U. Schroeder (Eds.), Isospin Physics in Heavy-Ion Collisions at Intermediate Energies, Nova Science, New York, 2001.

[2] V.Baran, M.Colonna, V.Greco, M.Di Toro, *Phys. Rep.* **410** (2005) 335.

[3] S. Kubis, M. Kutschera, *Phys. Lett.* **B399** (1997) 191.

[4] B. Liu, V. Greco, V. Baran, M. Colonna, M. Di Toro, *Phys. Rev.* **C65** (2002) 045201.

[5] T. Gaitanos, M. Di Toro, S. Typel, V. Baran, C. Fuchs, V. Greco, H.H. Wolter, *Nucl. Phys.* **A732** (2004) 24.

[6] F. de Jong, H. Lenske, *Phys. Rev.* **C57** (1998) 3099; E.N.E. van Dalen, C. Fuchs, Amand Faessler, *Nucl. Phys.* **A744** (2004) 227.

[7] E.N.E. van Dalen, C. Fuchs, Amand Faessler, nucl-th/0502064

[8] P. Danielewicz, Roy A. Lacey, et al., *Phys. Rev. Lett.* **81**, 2438 (1998); C. Pinkenburg *et al.*, *Phys. Rev. Lett.* **83**, 1295 (1999); P. Danielewicz, *Nucl. Phys.* **A673** (2000) 375.

[9] A.B. Larionov, W. Cassing, C. Greiner, U. Mosel, *Phys. Rev.* **C62** (2000) 064611.

[10] P.K. Sahu, A. Hombach, W. Cassing, M. Effenberger, U. Mosel, *Nucl. Phys.* **A640** (1998) 493.

[11] T. Gaitanos, C. Fuchs, H.H. Wolter, A. Faessler, Eur. Phys. J. A 12 (2001) 421; T. Gaitanos, C. Fuchs, H.H. Wolter, *Nucl. Phys.* **A741** (2004) 287.

[12] B.-A Li, *Phys. Rev.* **C67** (2003) 017601.

[13] T. Gaitanos, M. Colonna, M. Di Toro, H.H. Wolter, *Phys. Lett.* **B595** (2004) 209.

[14] L.-W. Chen, C.M. Ko, B.-A. Li, *Phys. Rev.* **C69** (2004) 054606.

[15] B. Hong and FOPI Collab., *Phys. Rev.* **C71** (2005) 034902.

[16] C. Sturm *et al.* (KaoS Collaboration), *Phys. Rev. Lett.* **86**, 39 (2001); F. Laue *et al.* (KaoS Collaboration), *Phys. Rev. Lett.* **82**, 1640 (1999);

[17] J. Aichelin, C.M. Ko, *Phys. Rev. Lett.* **55**, (1985) 2661; X.S. Fang, C.M. Ko, G.Q. Li, Y.M. Zheng, *Nucl. Phys.* **A575** (1994) 766; G.Q. Li, C.M. Ko, *Phys. Lett.* **B349** (1995) 405.

[18] C. Fuchs et al., *Phys. Rev. Lett.* **86**, (2001) 1974.

[19] W.Cassing, L.Tolos, E.L.Bratkovskaya, A.Ramos, *Nucl. Phys.* **A727** (2003) 59.
The parameters $a, b$ give the non-linear potential for the $\sigma$ field in the effective lagrangian density, $U(\sigma) = \frac{1}{3}a\sigma^3 + \frac{1}{4}b\sigma^4$, see [4,2].

This choice will not affect the results discussed here, when a chemical equilibrium is reached. Moreover we note that even for an “open” system, such as a realistic heavy ion collision, an in-medium (reduced) parametrization will just lead to a rescaling of the particle, resonance multiplicities, without much changing the relative yields we are focussing on.