Randomness on the Lattice

J.J.M. Verbaarschot

University at Stony Brook, NY 11794
verbaarschot@nuclear.physics.sunysb.edu

Abstract

In this lecture we review recent lattice QCD studies of the statistical properties of the eigenvalues of the QCD Dirac operator. We find that the fluctuations of the smallest Dirac eigenvalues are described by chiral Random Matrix Theories with the global symmetries of the QCD partition function. Deviations from chiral Random Matrix Theory beyond the Thouless energy can be understood analytically by means of partially quenched chiral perturbation theory.

1 Introduction

A significant part of our understanding of nonperturbative phenomena in QCD, such as chiral symmetry breaking, confinement or the existence of nuclei, results from simulations of the QCD partition function on a Euclidean space-time lattice (see [1,2] for reviews). In spite of its numerous successes, this approach has several disadvantages. One of them is the use of Euclidean space-time which requires a highly nontrivial analytical continuation to Minkowski space-time. One of the promising approaches that works directly in a Hamiltonian framework is discrete light-cone QCD [3], but its results for 4-dimensional nonabelian gauge theories can not yet compete with lattice QCD. A second disadvantage of lattice QCD is that analytical understanding of most lattice data seems beyond reach. Therefore, it is imperative to provide an analytical explanation of lattice observables whenever possible. One such observable is the Euclidean Dirac spectrum. We have proved [4,5] our conjecture [6,7] that the fluctuations of the smallest Dirac eigenvalues are given by a chiral Random Matrix Theory (chRMT) with the global symmetries of the QCD partition function. In this lecture we give a review of recent lattice simulations that support this assertion. A recent comprehensive review of chiral Random in QCD was given in [8].

Of course, chRMT cannot provide us with a complete description of the QCD Dirac spectrum. What is the domain of validity of chRMT? To answer this
question we need to identify three different scales in the Dirac spectrum. The first scale is the smallest nonzero eigenvalue, $\lambda_{\text{min}}$. Its average position is given by the mean level spacing, $\Delta(\lambda)$, which is the inverse of the average spectral density near zero, $\rho(0)$,

$$\lambda_{\text{min}} \approx \Delta = \frac{1}{\rho(0)} = \frac{\pi}{\Sigma V}. \quad (1)$$

Because of the axial symmetry the nonzero eigenvalues of the Euclidean Dirac operator $D$ appear in pairs $\pm \lambda_k$. The average spectral density is then defined by $\rho(\lambda) = \langle \sum_k \delta(\lambda - \lambda_k) \rangle$, and $\pi \rho(0)/V$ (with $V$ the volume of space-time) is identified as the chiral condensate, $\Sigma$, through the Banks-Casher formula [9]. A second scale in the Dirac spectrum is the mass scale for which the Compton wavelength of the corresponding Goldstone boson is equal to the size of the box. This scale, also known as the Thouless energy, is given by [10–12]

$$m_c = \frac{F^2}{\Sigma L^2}, \quad (2)$$

where $L$ is the linear size of the box. A third scale is the typical hadronic mass scale given by $\Lambda_{\text{QCD}}$.

Because QCD has a mass gap, for volumes with $\Lambda_{\text{QCD}} L \gg 1$, the QCD partition function in the phase of spontaneous broken chiral symmetry, reduces to that of a gas of Goldstone bosons. For momenta and masses well below $\Lambda_{\text{QCD}}$, this effective chiral partition function can be written down solely on the basis of the global symmetries of QCD. A further simplification arises for $m \ll m_c$. In this domain the fluctuations of the constant fields are much larger than the fluctuations of the nonzero momentum modes and kinetic term of the chiral Lagrangian can be ignored in the calculation of the mass dependence of the partition function \[10,11\]. This is the domain of validity chiral Random Matrix Theory. A formal proof of this statement \[12,13,4,5\] requires the introduction of additional ghost quarks with spectral mass $z$ equal to the argument of the resolvent of the Dirac operator. Because $z$ is a free parameter, it can always be chosen such that $z \ll m_c$, and the Dirac spectrum in this domain is thus given by chiral Random Matrix Theory.

\section{Chiral Random Matrix Theory}

The chiral random matrix partition function with the global symmetries of the QCD partition function is defined by \[6,7\]
where $W$ is a $n \times m$ matrix with $\nu = |n - m|$ and $N = n + m$. As is the case in QCD, we assume that the equivalent of the topological charge $\nu$ does not exceed $\sqrt{N}$, so that, to a good approximation, $n = N/2$. Then the parameter $\Sigma$ can be identified as the chiral condensate and $N$ as the dimensionless volume of space-time (Our units are defined such that the density of the modes $N/V = 1$). The chiral ensembles are classified according to the Dyson index $\beta$. The matrix elements of $W$ are either real ($\beta = 1$, chiral Gaussian Orthogonal Ensemble (chGOE)), complex ($\beta = 2$, chiral Gaussian Unitary Ensemble (chGUE)), or quaternion real ($\beta = 4$, chiral Gaussian Symplectic Ensemble (chGSE)). For QCD with three or more colors and quarks in the fundamental representation the matrix elements of the Dirac operator are complex and we have $\beta = 2$. The ensembles with $\beta = 1$ and $\beta = 4$ are relevant in the case of two colors or adjoint fermions. For staggered fermions, the value of the Dyson index in these two cases is reversed. The reason for choosing a Gaussian distribution of the matrix elements is its mathematically simplicity. It can be shown that the correlations of the eigenvalues on the scale of the average level spacing do not depend on the details of the probability distribution [14–20].

3 Lattice Results

We start this section by stressing that only spectral properties on the scale of the average level spacing can be described by Random Matrix Theory. We thus unfold the spectrum by rescaling the eigenvalues according to the macroscopic average level spacing obtained by averaging over many consecutive levels inside a small but finite interval. Below we always discuss the statistical properties of the unfolded eigenvalues, with average spectral density equal to unity. They have been analyzed in several different ways.

First, by means of the microscopic spectral density defined by [6]

$$\rho_s(u) = \lim_{V \to \infty} \frac{1}{V\Sigma} \langle \rho(\frac{u}{\sqrt{\Sigma}}) \rangle. \quad (4)$$

For $\beta = 2$ it is given by [21,22]

$$\rho_s(z) = \frac{z}{2} \left[ J_{N_f+|\nu|}^2(z) - J_{N_f+|\nu|+1}(z)J_{N_f+|\nu|-1}(z) \right]. \quad (5)$$

The result for $\beta = 1$ [23] and $\beta = 4$ [24] is more complicated but can be
expressed as an integral over Bessel functions. The microscopic spectral density was first observed for Dirac spectra of instanton liquid field configurations [22] both for $N_c = 2$ and $N_c = 3$. Its first lattice studies, for quenched $SU(2)$ gauge theory with staggered fermions (with Dyson index $\beta = 1$), were performed in [25] (see Fig. 1). The agreement between lattice QCD at Random matrix theory is equally good for $N_c = 3$ [26,27] (with Dyson index $\beta = 2$), for QCD with adjoint fermions [28], which is in the class $\beta = 1$, and for strong coupling $U(1)$ gauge theory [29] (also with Dyson index $\beta = 2$). Although most results have been obtained in the quenched limit, the agreement with chRMT for dynamical quark masses of order $1/V\Sigma$ [30–35] or massless quarks in the Schwinger model [36,37] is equally impressive. In Fig. 2 we show the microscopic spectral density for dimensionless dynamical quark mass $\mu = mV\Sigma$ as given in the legend of the figure.

A second way to study the Dirac spectrum is by means of the valence quark mass dependence of the chiral condensate defined by
Fig. 3. Valence quark mass dependence of the chiral condensate $\Sigma(m_v)$ plotted as $\Sigma(m_v)/\Sigma$ versus $m_vV\Sigma$. The dots and squares represent lattice results by the Columbia group [38] for the values of $\beta$ indicated in the figure. The solid curves are chRMT results. (From Ref. [12].)

$$\Sigma(z) = \frac{1}{V} \left\langle \text{Tr} \frac{1}{D + z} \right\rangle. \quad (6)$$

Its analytical expression can be derived from the low-energy limit of the QCD partition function [4,5] as well as from chRMT [12]. For $\beta = 2$ we find [12]

$$\frac{\Sigma(z)}{\Sigma} = x \left[ I_a(x)K_a(x) + I_{a+1}(x)K_{a-1}(x) \right], \quad (7)$$

where $a = N_f + |\nu|$ and $x = zV\Sigma$. In Fig. 3 we compare [12] the analytical result (full line) with lattice data obtained by the Columbia group [38]. The point above which the lattice data depart from the the chRMT result agrees with our estimate of the Thouless energy (2). These results have been confirmed by independent simulations [39,40] and have been extended to other symmetry classes [39].

A third way to analyze the statistical properties of the Dirac eigenvalues is by means of the distribution of the smallest eigenvalues. As an example, the analytical result for $\beta = 2$ is given by [41],

$$P(\lambda_{\text{min}}) = \frac{1}{2} \exp(-\lambda_{\text{min}}^2/4).$$

The results for $\beta = 1$, $\beta = 4$, and nonzero quark masses, are more complicated [41,31,33,42]. In Fig. 1 we show results [25] for quenched SU(2) lattice data. Results for all three symmetry classes as well as nonzero topological charge [43] are shown in Fig. 4. The latter results were obtained with the overlap Dirac operator. Dirac spectra of the Schwinger model have also been analyzed at nonzero topological charge [36] and complete agreement with chRMT was found [44]. The continuum limit of the staggered Dirac operators is approached very slowly, and on today’s lattices the Dirac spectra are described by analytical results for zero topological charge [45]. Recently, analytical results for the $k$’th smallest eigenvalue [46] gave a perfect description of the lattice data [47].
A more subtle way to study the statistical properties of eigenvalues is by means of the two-point correlation function defined as

$$\rho(\lambda, \lambda') = \langle \sum_{k,l} \delta(\lambda - \lambda_k)\delta(\lambda' - \lambda_l) \rangle. \quad (8)$$

The two-point correlation function for the quenched $SU(2)$ staggered Dirac operator was compared with chRMT in [25,48]. Also for instanton liquid gauge field configuration one finds [13] agreement with chRMT in its domain of validity. The volume dependence of the Thouless energy was investigated both for instanton liquid configurations [13,49] and lattice QCD simulations [50,27] and good agreement with the theoretical predictions [12] was found. A related quantity is the disconnected scalar susceptibility defined by

$$\chi^{\text{disc}}(m) = \frac{1}{N} \left\langle \sum_{k,l=1}^{N} \frac{1}{(i\lambda_k + m)(i\lambda_l + m)} \right\rangle - \frac{1}{N} \left\langle \sum_{k=1}^{N} \frac{1}{i\lambda_k + m} \right\rangle^2. \quad (9)$$

Lattice results [51] (see Fig. 5) show a sharp transition point which can be identified as the Thouless energy. Below this energy the susceptibility follows the chRMT prediction (dashed curve). A complete analytical description up
to $\Lambda_{\text{QCD}}$ is obtained from chiral perturbation theory (full curve) [51–53] which applies in the domain $\lambda_{\text{min}} \ll m \ll \Lambda_{\text{QCD}}$. Similar agreement has been found for the scalar susceptibility of the $U(1)$ staggered Dirac operator [29].

Dirac spectra at finite temperature and nonzero chemical potential have been studied in much less detail. Because of finite size effects comparisons with chRMT are difficult at the critical temperature. Dirac spectra near $T_c$ were analyzed in detail in [55]. Because the chiral phase transition in chRMT has mean field critical exponents [56,57] there is no reason to believe that the eigenvalue fluctuations follow the Random Matrix predictions [58–60] at the critical point. However, beyond $T_c$ there is some evidence that the smallest eigenvalues show a fluctuation behavior as predicted by RMT [54]. At nonzero chemical potential the Dirac operator is nonhermitian and its eigenvalues are scattered in the complex plane. Recent work [61,62] shows that the global spectral properties are described by a chiral Lagrangian [63,64] or Random Matrix Theory [65]. The statistical analysis of the eigenvalue fluctuations is much more complicated in this case, but the first lattice results [66] seem to confirm the theoretical expectations [67–71].

Up to now we only discussed the statistical properties of the Dirac eigenvalues near $\lambda = 0$. Although physically less relevant, one can also analyze the statistics of the eigenvalues in the bulk of the spectrum. Assuming that the statistical properties do not change along the spectrum we can average in two different ways: by averaging over independent gauge field configurations and by averaging over the spectrum. The advantage of spectral averaging is that it requires only one or a few independent gauge field configurations. The equality of the two averages is known as spectral ergodicity and was investigated in the context of QCD Dirac spectra in [72]. The Thouless energy was only found in ensemble averaging. The enhanced eigenvalue fluctuations result from the

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Fig. 5. Comparison of the disconnected susceptibility computed on the lattice in quenched SU(3) with staggered fermions ($V = 10^4$, $\beta = 5.4$) (points) with the prediction of chPT (full line) and the prediction of chRMT (dashed curve). (Note the dashed line is hidden by the data points for $u < 10$.) (From Ref. [51].)
Fig. 6. Number variance $\Sigma^2(n)$, the $\Delta_3(n)$ statistic, and nearest-neighbor spacing distribution $P(S)$ of the $SU(2)$ lattice Dirac operator. Upper row: Wilson fermions, $V = 8^3 \times 12$, $N_f = 2$. Lower row: staggered fermions, $V = 12^4$, $N_f = 4$, $m_a = 0.05$. (From Ref. [73].)

“collective” motion of the eigenvalues in the evolution of the ensemble. In Fig. 6 we show [73] the spacing distribution $P(S)$ of neighboring eigenvalues, the number variance $\Sigma^2(n)$ and the $\Delta_3$ statistic. The number variance is defined as the variance of the number of levels in an interval containing $n$ eigenvalues on average, and $\Delta_3(n)$ is obtained by integrating $\Sigma^2(n)$ over a smoothening kernel. The question has been raised whether eigenvalue correlations in the bulk are different above and below $T_c$, but no effects have been seen [74,75,29,76] (see also Fig. 7). A transition toward Poisson statistics only takes place at very small values of the coupling constant (see Fig. 7).

Finally, lattice results for QCD Dirac operator in 3 dimensions [77] have been compared with Random Matrix Theory. In this case one finds agreement with
the Wigner-Dyson ensembles [77], but we will not discuss this topic in this review.

4 Conclusions

The generating function of the Dirac spectrum is given by the QCD partition function with additional ghost quarks with a mass scale given by the region of the the Dirac spectrum we are interested in. At low energies this partition function reduces to a gas of weakly interacting Goldstone modes. In the domain where the kinetic term can be neglected it reduces to a chiral Random Matrix Theory with the global symmetries of the QCD partition function. The predictions based on these arguments have been confirmed by numerous lattice QCD simulations. This does not mean that one can refrain from doing lattice simulations. The point is that universal behavior and the phenomenologically relevant nonuniversal properties are found in the same lattice simulations. The real progress is the understanding of the symbiosis of these two features of the strong interactions.

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