Abstract

A very general quantum field theory, which is not even assumed to be Lorentz invariant, is studied in the limit of very low energy excitations. Fermion and Boson field theories are considered in parallel. Remarkably, in both cases it is argued that, in the free and lowest energy approximation, a relativistic theory with just three space and one time dimension emerges for each particle type separately. In the case of Fermion fields it is in the form of the Weyl equation, while in the case of the Bosons it is essentially in the form of the Maxwell equations.

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1 Introduction

Since many years ago [1], we have worked on the project of “deriving” all the known laws of nature, especially the symmetry laws [2], from the assumption of the existence of exceedingly complicated fundamental laws of nature. However, the derivations are such that it practically does not matter what these exceedingly complicated laws are in detail, just provided we only study them in some limits such as the low energy limit. This is the project which we have baptized “Random dynamics”, in order to make explicit the idea that we are thinking of the fundamental laws of nature as being given by a particular model pulled out at random from a very large class of models. In this way, one can overcome the immediate reproach to the project that it is easy to invent model-proposals which, indeed, do not deliver the laws of nature as we know them today. We only make the claim that sufficiently complicated and generic models should work, not very special ones that could potentially be constructed so as not to work. Also it should be stressed that there is a lot of interpretation involved, as to which elements in the “random” model are to be identified phenomenologically with what. As a consequence, the project tends to be somewhat phenomenological itself, honestly speaking. However, in principle, we should only use the phenomenology to find out which quantities in the “random dynamics” model are to be identified with which physically defined quantities (concepts).

One of the most promising steps, in developing this random dynamics project, was [1, 2] to start without assuming Lorentz invariance but to assume that we already have several known laws such as quantum mechanics, quantum field theory and momentum conservation. Lorentz invariance was then “derived”, at least for a single species of Weyl particles which emerged at low energy. However this “derivation” of Lorentz invariance might not actually be the most interesting result from this step in random dynamics; it is after all not such an overwhelming success, since it only works for one particle species on its own and does not, immediately at least, lead to Lorentz invariance if several particle species are involved. It may rather be the prediction of the number of space dimensions which is more significant. Actually the fundamental model is assumed to have an arbitrary number of dimensions and has momentum degrees of freedom in all these dimensions, but the velocity components in all but three dimensions turn out to be zero. In this way the extra dimensions are supposedly not accessible. So the prediction is effectively that there are just three spatial dimensions (plus one time)!

In these early studies only a fermionic field theory (without Lorentz symmetry) was considered, while Bosons were left out of consideration; we then sometimes speculated that the Bosons could at least be partly composed from Fermions and thus inherit their Lorentz symmetry. Indeed, even in more recent work, it is the Fermions that play the main role [3, 4]. For a summary of other recent theoretical models and experimental tests of Lorentz invariance breaking see, for example, reference [5].

It is the purpose of the present paper to review the work with Fermions stressing a new feature aimed at solving a certain technical problem—the use
of the “Homolumo-gap-effect” to be explained below—and to extend the work to the case of bosonic fields, which is a highly non-trivial extension.

In the following section we shall put forward our very general field theory model and then, in section 3, we shall write down in parallel the equations of motion for Bosons and Fermions respectively. It turns out that we obtain a common equation of motion for the “fields” in “momentum” representation—momentum here being really thought of as a rather general parameterisation of the degrees of freedom, on which the Hamiltonian and commutation rules depend smoothly. This equation of motion involves an antisymmetric matrix which depends on the “momenta”. The behaviour of the eigenvalue spectrum of such an antisymmetric real matrix is studied in section 4, with the help of some arguments based on the Homolumo-gap-effect which are postponed till section 5. The conclusions are put into section 6.

2 A random dynamics model

Since it is our main purpose to derive Lorentz symmetry together with 3 + 1 dimensions, we must start from a model that does not assume Lorentz invariance nor the precise number of space dimensions from the outset. We would, of course, eventually hope to avoid having to assume momentum conservation or even the existence of the concept of momentum. However this assumption is less crucial than the others, since the derivation of Lorentz invariance is highly non-trivial even if momentum conservation is assumed. Therefore, “for pedagogical reasons”, we shall essentially assume translational symmetry and momentum conservation in our model—in practice though we shall actually allow a small departure from translational symmetry. That is to say we consider the model described in terms of a Fock space, corresponding to having bosonic or fermionic particles that can be put into single particle states which are momentum eigenstates. This gives rise to bosonic and fermionic fields $\phi(\vec{p})$ and $\psi(\vec{p})$ annihilating these particles. We shall formulate the model in terms of fields that are essentially real or obey some Hermiticity conditions, which mean that we can treat the fields $\phi(\vec{p})$ and $\psi(\vec{p})$ as Hermitian fields. In any case, one can always split up a non-Hermitian field into its Hermitian and anti-Hermitian parts. This is done since, in the spirit of the random dynamics project, we do not want to assume any charge conservation law from the outset.

2.1 Technicalities in a general momentum description

In the very general type of model we want to set up, without any assumed charge conservation, it is natural to use a formalism which is suitable for neutral particles like, say, $\pi^0$ mesons. However, when one constructs a second quantized formalism from a single particle Fock-space description, in which there can be different numbers of particles in the different single particle states\footnote{In the fermionic case there can be 0 or 1 particle in a particular single particle state, while in the bosonic case there can also be many.}, one at first
gets “complex” i.e. non-Hermitian second quantized fields. In order to describe say the $\pi^0$-field, one must put restrictions on the allowed Fock-space states, so that one cannot just completely freely choose how many particles there should be in each single particle state. Basically one “identifies” particles and antiparticles (= holes), so that they are supposed to be in analogous states (in the Fermion case, it is the Majorana condition that must be arranged). Field creation of a particle with momentum $\vec{p}$ is brought into correspondence with annihilation of a particle with momentum $-\vec{p}$.

In our general description of bosonic or fermionic second quantized particles, we want to use a formalism of this $\pi^0$ or Majorana type. We can always return to a charged particle description by introducing a doubling of the number of components for such a field; we can simply make a non-Hermitian (i.e. essentially charged) field component from two Hermitian ones, namely the Hermitian (“real”) and anti-Hermitian (“purely imaginary”) parts, each of which are then Majorana or $\pi^0$-like. Let us recall here that the $\pi^0$ field is Hermitian when written as a field depending on the position variable $\vec{x}$, while it is not Hermitian in momentum space. In fact, after Fourier transformation, the property of Hermiticity or reality in position space becomes the property, in momentum representation, that the fields at $\vec{p}$ and $-\vec{p}$ are related by Hermitian/complex conjugation:

$$\phi(\vec{p}) = \phi^\dagger(-\vec{p}) \quad (1)$$

For generality, we should also like to have Hermitian momentum dependent fields, which corresponds to having a similar reflection symmetry in position space, saying that the values of the fields at $\vec{x}$ and $-\vec{x}$ are related by Hermitian/complex conjugation. To make the “most general” formalism for our study, we should therefore impose Hermiticity both in momentum and in position representation. We then have to accept that we also have a reflection symmetry in both position and momentum space. In this paper, we shall in reality only consider this most general formalism for bosonic fields. For this purpose, let us denote the $\pi^0$ field and its momentum conjugate field by $\phi_{0}(\vec{p})$ and $\pi_{0}(\vec{p})$ respectively. Then, in standard relativistic quantum field theory, the non-vanishing equal time commutation relations between their real and imaginary parts are as follows:

$$\left[ \text{Re} \phi_{0}(\vec{p}), \text{Re} \pi_{0}(\vec{p}') \right] = i \frac{2}{\delta(\vec{p} - \vec{p}') + \delta(\vec{p} + \vec{p}')} \quad \left(2\right)$$

$$\left[ \text{Im} \phi_{0}(\vec{p}), \text{Im} \pi_{0}(\vec{p}') \right] = i \frac{2}{\delta(\vec{p} - \vec{p}') - \delta(\vec{p} + \vec{p}')} \quad \left(3\right)$$

We note that the appearance of the $\delta(\vec{p} + \vec{p}')$ function as well as the $\delta(\vec{p} - \vec{p}')$ function is a consequence of the reflection symmetry $[\ ]$.

Now the reader should also notice that we are taking the point of view that many of the observed laws of nature are only laws of nature in the limit of “the poor physicist”, who is restricted to work with the lowest energies and only with a small range of momenta compared to the fundamental (Planck) scale. In the very generic and not rotational invariant type of model which we want to consider, it will now typically happen that the small range of momenta to which the
physicist has access is not centred around zero momentum—in the presumably rather arbitrary choice of the origin for momentum—but rather around some momentum, \(\vec{p}_0\) say. This momentum \(\vec{p}_0\) will generically be large compared to the momentum range accessible to the poor physicist; so the reflection symmetry in momentum space and the associated \(\delta(\vec{p} + \vec{p}')\) terms in commutators will not be relevant to the poor physicist and can be ignored. However, in our general field theory model, there can be a remnant reflection symmetry in position space. Indeed we shall see below that what may be considered to be a mild case of momentum non-conservation does occur for the Maxwell equations derived in our model: there is the occurrence of a reflection centre somewhere, around which the Maxwell fields should show a parity symmetry in the state of the fields. If we know, say, the electric field in some place, then we should be able to conclude from this symmetry what the electric field is at the mirror point. If, as is most likely, this reflection point is far out in space, it would be an astronomical challenge to see any effect of this lack of translational symmetry. In this sense the breaking of translational symmetry is very “mild”.

2.2 General Field Theory Model

At the present stage in the development of our work, it is assumed that we only work to the free field approximation and thus the Hamiltonian is taken to be bilinear in the Hermitian fields \(\psi(\vec{p})\) and \(\phi(\vec{p})\). Also, because of the assumed rudiment of momentum conservation in our model, we only consider products of fields taken for the same momentum \(\vec{p}\). In other words our Hamiltonian takes the following form:

\[
H_F = \frac{1}{2} \int d\vec{p} \sum_{i,j} \psi_i(\vec{p})\psi_j(\vec{p}) H_{ij}^{(F)}(\vec{p})
\]

and

\[
H_B = \frac{1}{2} \int d\vec{p} \sum_{i,j} \phi_i(\vec{p})\phi_j(\vec{p}) H_{ij}^{(B)}(\vec{p})
\]

for Fermions and Bosons respectively. Here the coefficient functions \(H_{ij}^{(F)}(\vec{p})\) and \(H_{ij}^{(B)}(\vec{p})\) are non-dynamical in the free field approximation and just reflect the general features of “random” laws of nature expected in the random dynamics project. That is to say we do not impose Lorentz invariance conditions on these coefficient functions, since that is what is hoped to come out of the model. We should also not assume that the \(\vec{p}\) vectors have any sort of Lorentz transformation properties a priori and they should not even be assumed to have, for instance, 3 spatial dimensions. Rather we start out with \(D > 3\) spatial dimensions; then one of our main achievements will be to show that the velocity components in all but a three dimensional subspace are zero. It is obvious that, in these expressions, the coefficient functions \(H_{ij}^{(F)}(\vec{p})\) and \(H_{ij}^{(F)}(\vec{p})\) can be taken to have the symmetry properties:

\[
H_{ij}^{(F)}(\vec{p}) = -H_{ji}^{(F)}(\vec{p}) \quad \text{and} \quad H_{ij}^{(B)}(\vec{p}) = H_{ji}^{(B)}(\vec{p}).
\]
However, it should be borne in mind that \textit{a priori} the fields are arbitrarily normalised and that we may use the Hamiltonians to define the normalisation of the fields, if we so choose. In fact an important ingredient in the formulation of the present work is to assume that a linear transformation has been made on the various field components $\phi_i(\vec{p})$, i.e. a transformation on the component index $i$, such that the symmetric coefficient functions $H_{ij}^{(B)}(\vec{p})$ become equal to the unit matrix:

$$H_{ij}^{(B)}(\vec{p}) = \delta_{ij} \quad \text{(by normalisation for all } \vec{p}) \quad (7)$$

Thereby, of course, the commutation relations among these components $\phi_i(\vec{p})$ are modified and we cannot simultaneously arrange for them to be trivial. So for the Bosons we choose a notation in which the non-trivial behaviour of the equations of motion, as a function of the momentum $\vec{p}$, is put into the commutator expression:

$$[\phi_i(\vec{p}), \phi_j(\vec{p'}))] = iA_{ij}(\vec{p})\delta(\vec{p} - \vec{p'}) \quad (8)$$

It follows that the information which we would, at first, imagine should be contained in the Hamiltonian is, in fact, now contained in the antisymmetric matrices $A_{ij}(\vec{p})$.

For the Fermions, on the other hand, we shall keep to the more usual formulation. So we normalize the anti-commutator to be the unit matrix and let the more nontrivial dependence on $\vec{p}$ sit in the Hamiltonian coefficient functions $H_{ij}^{(F)}(\vec{p})$. That is to say that we have the usual equal time anti-commutation relations:

$$\{\psi_i(\vec{p}), \psi_j(\vec{p'})]\} = \delta_{ij}\delta(\vec{p} - \vec{p'}) \quad (9)$$

The component indices $i, j$ enumerate the very general discrete degrees of freedom in the model. These degrees of freedom might, at the end, be identified with Hermitian and anti-Hermitian components, spin components, variables versus conjugate momenta or even totally different types of particle species, such as flavours etc. It is important to realize that this model is so general that it has, in that sense, almost no assumptions built into it—except for our free approximation, the above-mentioned rudimentary momentum conservation and some general features of second quantized models. It follows from the rudimentary momentum conservation in our model that the (anti-)commutation relations have a $\delta(\vec{p} - \vec{p'})$ delta function factor in them.

Obviously the Hermiticity of the Hamiltonians for the second quantized systems means that the matrices $H_{ij}^{(F)}(\vec{p})$ and $H_{ij}^{(B)}(\vec{p})$ are Hermitian and thus have purely imaginary and real matrix elements respectively. Similarly, after the extraction of the $i$ as a conventional factor in equation (8), the matrix $A_{ij}(\vec{p})$ has real matrix elements and is antisymmetric.

\footnote{Note that we are here ignoring possible terms of the form $iH_{ij}(\vec{p})\delta(\vec{p} + \vec{p'})$ as irrelevant to the poor physicist, according to the discussion after equation (4).}
3 Equations of motion for the general fields

We can easily write down the equations of motion for the field components in our general quantum field theory, both in the fermionic case:

\[
\dot{\psi}_i(\vec{p}) = i[H_F, \psi_i(\vec{p})] = i \sum_k \psi_k(\vec{p}) H^{(F)}_{ki}(\vec{p})
\]

(10)

and in the bosonic case:

\[
\dot{\phi}_i(\vec{p}) = i[H_B, \phi_i(\vec{p})] = - \sum_k \phi_k(\vec{p}) A_{ki}(\vec{p}).
\]

(11)

Since \(H^{(F)}_{ij}(\vec{p})\) has purely imaginary matrix elements, we see that both the bosonic and the fermionic equations of motion are of the form

\[
\dot{\xi}_i(\vec{p}) = \sum_k A_{ik} \xi_k(\vec{p})
\]

(12)

In the fermionic case we have extracted a factor of \(i\), by making the definition

\[
H^{(F)}_{ij}(\vec{p}) = iA_{ij}(\vec{p}).
\]

(13)

Also the Boson field \(\phi\) and the Fermion field \(\psi\) have both been given the neutral name \(\xi\) here.

4 Spectrum of an antisymmetric matrix depending on \(\vec{p}\)

An antisymmetric matrix \(A_{ij}(\vec{p})\) with real matrix elements is anti-Hermitian and thus has purely imaginary eigenvalues. However, if we look for a time dependence ansatz of the form

\[
\xi_i(\vec{p}, t) = a_i(\vec{p}) \exp(-i\omega t),
\]

(14)

the eigenvalue equation for the frequency \(\omega\) becomes

\[
\omega a_i = \sum_j iA_{ij}(\vec{p}) a_j.
\]

(15)

Now the matrix \(iA_{ij}(\vec{p})\) is Hermitian and the eigenvalues \(\omega\) are therefore real.

It is easy to see, that if \(\omega\) is an eigenvalue, then so also is \(-\omega\). In fact we could imagine calculating the eigenvalues by solving the equation

\[
\det (iA - \omega) = 0
\]

(16)

We then remark that transposition of the matrix \((iA - \omega)\) under the determinant sign will not change the value of the determinant, but corresponds to changing
the sign of $\omega$ because of the antisymmetry of the matrix $iA$. So non-vanishing eigenvalues occur in pairs.

In order to compare with the more usual formalism, we should really keep in mind that the creation operator for a particle with a certain $\omega$-eigenvalue is, in fact, the annihilation operator for a particle in the eigenstate with the opposite value of the eigenvalue, i.e. $-\omega$. Thus, when thinking in usual terms, we can ignore the negative $\omega$ orbits as being already taken care of via their positive $\omega$ partners. The unpaired eigenstate, which is formally a possibility for $\omega = 0$, cannot really be realized without some little “swindle”. In the bosonic case it would correspond to a degree of freedom having, say, a generalized coordinate but missing the conjugate momentum. In the fermionic case, it would be analogous to the construction of a set of $\gamma$-matrices in an odd dimension, which is strictly speaking only possible because one allows a relation between them (the product of all the odd number of them being, say, unity) or because one allows superfluous degrees of freedom. It is obviously difficult to construct such a set of $\gamma$-matrices in complete analogy with the case of an even number of fields, since then the number of components in the representation of the $n$ gamma-matrices would be $2^{n/2}$, which can hardly make sense for $n$ odd. Nevertheless, we shall consider the possibility of an unpaired $\omega = 0$ eigenstate in the bosonic case below.

Now the main point of interest for our study is how the second quantized model looks close to its ground state. The neighbourhood of this ground state is supposed to be the only regime which we humans can study in our “low energy” experiments, with small momenta compared to the fundamental (say Planck) mass scale. With respect to the ground state of such a second quantized world machinery, it is well-known that there is a difference between the fermionic and the bosonic case. In the fermionic case, you can at most have one Fermion in each state and must fill the states up to some specific value of the single particle energy—which is really $\omega$. However, in the bosonic case, one can put a large number of Bosons into the same orbit/single particle state, if that should pay energetically.

4.1 The vacuum

If we allow for the existence of a chemical potential, which essentially corresponds to the conservation of the number of Fermions, we shall typically get the ground state to have Fermions filling the single particle states up to some special value of the energy called the Fermi-energy $\omega_{FS}$ ($FS$ standing for “Fermi-surface”). For Bosons, on the other hand, we will always have zero particles in all the orbits, except perhaps in the zero energy ground state; it will namely never pay energetically to put any bosons into positive energy orbits.

4.2 The lowest excitations

So for the investigation of the lowest excitations, i.e. those that a “poor physicist” could afford to work with, we should look for the excitations very near to
the Fermi-surface in the fermionic case. In other words, we should put Fermions into the orbits with energies very little above the Fermi-energy, or make holes in the Fermi-sea at values of the orbit-energies very little below the Fermi-energy. Thus, for excitations accessible to the “poor physicist”, it is only necessary to study the behaviour of the spectrum for the Bosons having a value of \( \omega \) near to zero, and for the Fermions having a value of \( \omega \) near the Fermi-energy \( \omega_{FS} \).

4.2.1 Boson case: levels approaching a group of \( \omega = 0 \) levels

In section 5 we shall argue that, if the model has adjustable degrees of freedom (“garbage variables”), they would tend to make the \( \omega = 0 \) eigenvalue multiply degenerate. However, for simplicity, we shall first consider here the case where there is just a single zero-eigenvalue \( \omega \)-level. We should mention that the true generic situation for an even number of fields is that there are normally no zero-eigenvalues at all. So what we shall study here, as the representative case, really corresponds to the case with an odd number of fields. In this case there will normally be just one (i.e. non-degenerate) \( \omega = 0 \) eigenvalue. However it can happen that, for special values of the “momentum parameters”, a pair of eigenvalues—consisting of eigenvalues of opposite sign of course—approach zero. It is this situation which we believe to be the one of relevance for the low energy excitations.

We shall concentrate our interest on a small region in the momentum parameter space, around a point \( \vec{p}_0 \) where the two levels with the numerically smallest non-zero eigenvalues merge together with a level having zero eigenvalue. Using the well-known fact that, in quantum mechanics, perturbation corrections from faraway levels have very little influence on the perturbation of a certain level, we can ignore all the levels except the zero eigenvalue and this lowest non-zero pair. So if, for simplicity, we think of this case of just one zero eigenvalue except where it merges with the other pair, we need only consider three states and that means, for the main behaviour, we can calculate as if there were only the three corresponding fields. This, in turn, means that we can treat the bosonic model in the region of interest, by studying the spectrum of a (generic) anti-symmetric \( 3 \times 3 \) matrix with real elements, or rather such a matrix multiplied by \( i \). Let us immediately notice that such a matrix is parameterised by three parameters. The matrix and thus the spectrum, to the accuracy we are after, can only depend on three of the momentum parameters. In other words the dispersion relation will depend trivially on all but 3 parameters in the linear approximation. By this linear approximation, we here mean the approximation in which the “poor physicist” can only work with a small region in momentum parameter space also—not only in energy. In this region we can trust the lowest order Taylor expansion in the differences of the momentum parameters from their starting values (where the nearest levels merge). Then the \( \omega \)-eigenvalues—i.e. the dispersion relation—will not vary in the direction of a certain subspace of co-dimension three. Corresponding to these directions the velocity components of the described Boson particle will therefore be zero! The Boson, as seen by the “poor physicist”, can only move inside a three dimensional space; in
other directions its velocity must remain zero. It is in this sense we say that the three-dimensionality of space is explained!

### 4.3 Maxwell equations

The form of the equations of motion for the fields, in this low excitation regime where one can use the lowest order Taylor expansion in the momentum parameters, is also quite remarkable: after a linear transformation in the space of “momentum parameters”, they can be transformed into the Maxwell equations with the fields being complex (linear) combinations of the magnetic and electric fields.

We can now easily identify the linear combinations of the momentum parameters minus their values at the selected merging point, which should be interpreted as true physical momentum components. They are, in fact, just those linear combinations which occur as matrix elements in the $3 \times 3$ matrix $A$ describing the development of the three fields $\phi_j$ relevant to the “poor physicist”.

That is to say we can choose the definition of the “true momentum components” $\vec{k}$ as such linear functions of the deviations, $\vec{p} - \vec{p}_0$, of the momentum parameters from the merging point that the antisymmetric matrix $A$ reduces to

$$A = \begin{pmatrix} 0 & k_3 & -k_2 \\ -k_3 & 0 & k_1 \\ k_2 & -k_1 & 0 \end{pmatrix}$$

with eigenvalues $-i\omega = 0, \pm i \sqrt{k_1^2 + k_2^2 + k_3^2}$.

In the here chosen basis for the momenta, we can make a Fourier transform of the three fields $\phi_j(\vec{k})$ into the $\vec{x}$-representation. These new position space fields $\phi_j(\vec{x})$ are no longer Hermitian. However, it follows from the assumed Hermiticity of the $\phi_j(\vec{k})$ that, in the $\vec{x}$-representation, the real parts of the fields $\phi_j(\vec{x})$ are even, while the imaginary parts are odd functions of $\vec{x}$. We now want to identify these real and imaginary parts as magnetic and electric fields $B_j(\vec{x})$ and $E_j(\vec{x})$ respectively: $\phi_j(\vec{x}) = iE_j(\vec{x}) + B_j(\vec{x})$. However the symmetry of these Maxwell fields means that they must be in a configuration/state which goes into itself under a parity reflection in the origin. This is a somewhat strange feature which seems necessary for the identification of our general fields with the Maxwell fields; a feature that deserves further investigation. For the moment let us, however, see that we do indeed get the Maxwell equations in the free approximation with the proposed identification.

By making the inverse Fourier transformation back to momentum space, we obtain the following identification of the fields $\phi_j(\vec{k})$ in our general quantum field theory with the electric field $E_j(\vec{k})$ and magnetic field $B_j(\vec{k})$ Fourier transformed into momentum space:

$$\begin{pmatrix} \phi_1(\vec{k}) \\ \phi_2(\vec{k}) \\ \phi_3(\vec{k}) \end{pmatrix} = \begin{pmatrix} iE_1(\vec{k}) + B_1(\vec{k}) \\ iE_2(\vec{k}) + B_2(\vec{k}) \\ iE_3(\vec{k}) + B_3(\vec{k}) \end{pmatrix}.$$
We note that the Fourier transformed electric field $E_j(\vec{k})$ in the above ansatz (18) has to be purely imaginary, while the magnetic field $B_j(\vec{k})$ must be purely real.

By using the above identifications, eqs. (17) and (18), the equations of motion (11) take the following form

\[
\begin{pmatrix}
i \dot{E}_1(\vec{k}) + \dot{B}_1(\vec{k}) \\
i \dot{E}_2(\vec{k}) + \dot{B}_2(\vec{k}) \\
i \dot{E}_3(\vec{k}) + \dot{B}_3(\vec{k})
\end{pmatrix} =
\begin{pmatrix} k_3 & -k_2 & iE_1(\vec{k}) + B_1(\vec{k}) \\
-k_3 & 0 & k_1 \\
k_2 & -k_1 & 0
\end{pmatrix}
\begin{pmatrix}
\dot{E}_1(\vec{k}) \\
\dot{E}_2(\vec{k}) \\
\dot{E}_3(\vec{k})
\end{pmatrix}.
\]

We can now use the usual Fourier transformation identification in quantum mechanics to transform these equations to the $\vec{x}$-representation, simply from the definition of $\vec{x}$ as the Fourier transformed variable set associated with $\vec{k}$,

\[
k_j = i^{-1} \partial_j
\]

Thus in $\vec{x}$-representation the equations of motion become

\[
\begin{pmatrix}
i \dot{E}_1(\vec{x}) + \dot{B}_1(\vec{x}) \\
i \dot{E}_2(\vec{x}) + \dot{B}_2(\vec{x}) \\
i \dot{E}_3(\vec{x}) + \dot{B}_3(\vec{x})
\end{pmatrix} =
\begin{pmatrix} 0 & -i\partial_3 & i\dot{E}_1(\vec{x}) + B_1(\vec{x}) \\
i\partial_3 & 0 & i\dot{E}_2(\vec{x}) + B_2(\vec{x}) \\
i\partial_3 & i\partial_1 & 0
\end{pmatrix}
\begin{pmatrix}
\dot{E}_1(\vec{x}) \\
\dot{E}_2(\vec{x}) \\
\dot{E}_3(\vec{x})
\end{pmatrix}.
\]

The imaginary terms in the above equations give rise to the equation:

\[
\dot{\vec{E}}(\vec{x}) = \text{curl} \vec{B}
\]

while the real parts give the equation:

\[
\dot{\vec{B}}(\vec{x}) = -\text{curl} \vec{E}
\]

These two equations are just the Maxwell equations in the absence of charges and currents, except that strictly speaking we miss two of the Maxwell equations, namely

\[
\text{div} \vec{E}(\vec{x}) = 0 \quad \text{and} \quad \text{div} \vec{B}(\vec{x}) = 0.
\]

However, these two missing equations are derivable from the other Maxwell equations in time differentiated form. That is to say, by using the result that the divergence of a curl is zero, one can derive from the other equations that

\[
\text{div} \dot{\vec{E}}(\vec{x}) = 0 \quad \text{and} \quad \text{div} \dot{\vec{B}}(\vec{x}) = 0
\]

which is though not quite sufficient. Integration of the resulting equations (25) effectively replaces the 0’s on the right hand sides of equations (24) by terms constant in time, which we might interpret as some constant electric and magnetic charge distributions respectively. In our free field theory approximation, we have potentially ignored such terms. So we may claim that, in the approximation to which we have worked so far, we have derived the Maxwell equations sufficiently well.
5 Homolumo-gap and analogue for bosons

The Homolumo-gap effect refers to a very general feature of systems of Fermions, which possess some degrees of freedom that can adjust themselves so as to lower the energy as much as possible. The effect is so general that it should be useful for almost all systems of Fermions, because even if they did not have any extra degrees of freedom to adjust there would, in the Hartree approximation, be the possibility that the Fermions could effectively adjust themselves. The name Homolumo gap was introduced in chemistry and stands for the gap between “the highest occupied” HO “molecular orbit” MO and the “lowest unoccupied” LU “molecular orbit” MO. The point is simply that if the filled (occupied) orbits (single particle states) are lowered the whole energy is lowered, while it does not help to lower the empty orbits. It therefore pays energetically to make the occupied orbits go down in energy and separate from the unfilled ones; thus a gap may appear or rather there will be a general tendency to get a low level density near the Fermi-surface. This effect can easily be so strong that it causes a symmetry to break \[3\]; symmetry breaking occurs if some levels, which are degenerate due to the symmetry, are only partially filled so that the Fermi-surface just cuts a set of degenerate states/orbits. It is also the Homolumo-gap effect which causes the deformation of transitional nuclei, which are far from closed shell configurations. We want to appeal to this Homolumo gap effect, in subsection 5.3, as a justification for the assumption that the Fermi-surface gets close to those places on the energy axis where the level density is minimal.

However we first want to discuss a similar effect, where the degrees of freedom of a system of Bosons adjust themselves to lower the total energy. As for the Fermion systems just discussed, this lowering of the total energy is due to the adjustment of a sum over single particle energies—the minimisation of the zero-point energy of the bosonic system. We consider the effect of this minimisation to be the analogue for Bosons of the Homolumo-gap effect.

5.1 The analogue for bosons

In the “derivation” of the Maxwell equations given in subsection 4.3, we started by introducing the assumption of the existence of a zero frequency, \(\omega = 0\), eigenvalue by taking the number of Hermitian fields and thereby the order of the antisymmetric matrix \(A_{ij}\) to be odd. We now turn to our more general assumption of the existence of multiply degenerate \(\omega = 0\) eigenvalues. Honestly we can only offer a rather speculative argument in favour of our assumption that there should be several eigenvalues which are zero, even in the case when the total number of fields is not odd. For quite generic matrices, as would be the cleanest philosophy, it is simply not true that there would be zero eigenvalues for most momenta in the case of an even number of fields. However, let us imagine that there are many degrees of freedom of the whole world machinery that could adjust themselves to minimize the energy of the system and could also influence the matrix \(A_{ij}(\vec{p})\). Then one could, for instance, ask how it would be energetically profitable to adjust the eigenvalues, in order to minimize the zero-
point energy of the whole (second quantized) system. This zero-point energy is formally given by the integral over all (the more than three dimensional) momentum space; let us just denote this integration measure by \( d\vec{p} \), so that:

\[
E_{\text{zero-point}} = \int d\vec{p} \sum_{\text{eigenvalue pair } k} |\omega_k(\vec{p})|/2
\]

(26)

Provided some adjustment took place in minimizing this quantity, there would \textit{a priori} be an argument in favour of having several zero eigenvalues, since they would contribute the least to this zero-point energy \( E_{\text{zero-point}} \). At first sight, this argument is not very strong, since it just favours making the eigenvalues small and not necessarily making any one of them exactly zero. However, we underlined an important point in favour of the occurrence of exactly zero eigenvalues, by putting the numerical sign explicitly into the integrand \( |\omega_k(\vec{p})|/2 \) in the expression (26) for the zero-point energy. The important point is that the numerical value function is not an ordinary analytic function, but rather has a kink at \( \omega_k(\vec{p}) = 0 \). This means that, if other contributions to the energy of the whole system are smooth/analytic, it could happen that the energy is lowered when \( \omega_k(\vec{p}) \) is lowered numerically for both signs of \( \omega_k(\vec{p}) \); here we consider \( \omega_k(\vec{p}) \) to be a smooth function of the adjusting parameters of the whole world machinery (we could call them “garbage parameters”). For a normal analytic energy function this phenomenon could of course never occur, except if the derivative just happened (is fine-tuned one could say) to be equal to zero at \( \omega_k(\vec{p}) = 0 \).

But with a contribution that has the numerical value singularity behaviour it is possible to occur with a finite probability (i.e., without fine-tuning), because it is sufficient that the derivative of the contribution to the total energy from other terms is numerically lower than the derivative of the zero-point term discussed. Then, namely, the latter will dominate the sign of the slope and the minimum will occur \textit{exactly} for zero \( \omega_k(\vec{p}) \).

In this way, we claim to justify our assumption that the matrix \( A_{ij}(\vec{p}) \) will have several exactly zero eigenvalues and thus a far from maximal rank; the rank being at least piecewise constant over momentum space. We shall therefore now study antisymmetric matrices with this property in general and look for their lowest energy excitations.

5.2 Using several zero eigenvalues to derive Maxwell equations

As in subsection 4.3, we assume that when a single pair of opposite sign eigenvalues approach zero as a function of the momentum, we can ignore the faraway eigenvalues. Then, using the approximation of only considering the fields corresponding to the two eigenvalues approaching zero and the several exact zero eigenvalues, we end up with an effective \((n+2) \times (n+2)\) matrix \( A_{ij}(\vec{p}) \) obeying the constraint of being of rank two (at most). Now we imagine that we linearize the momentum dependence of \( A_{ij}(\vec{p}) \) on \( \vec{p} \) around a point in momentum space, say \( \vec{p}_0 \), where the pair of eigenvalues approaching zero actually reach
zero, so that the matrix is totally zero, $A_{ij}(\vec{p}_0) = 0$, at the starting point for the Taylor expansion. That is to say that, corresponding to different basis vectors in momentum space, we get contributions to the matrix $A_{ij}(\vec{p})$ linear in the momentum difference $\vec{p} - \vec{p}_0$. Now any non-zero antisymmetric matrix is necessarily of rank at least 2. So the contribution from the first chosen basis vector in momentum space will already give a matrix $A_{ij}$ of rank 2 and contributions from other momentum components should not increase the rank beyond this. A single basis vector for a set of linearly parameterised antisymmetric real matrices can be transformed to just having elements $(1,2)$ and $(2,1)$ nonzero and the rest zero. In order to avoid a further increase in the rank of the matrix by adding other linear contributions, these further contributions must clearly not contribute anything to matrix elements having both column and row index different from 1 and 2. However this is not sufficient to guarantee that the rank remains equal to 2. This is easily seen, because we can construct $4 \times 4$ antisymmetric matrices, which are of the form of having 0's on all places $(i,j)$ with both $i$ and $j$ different from 1 and 2 and have nonzero determinant.

So let us consider $4 \times 4$ sub-determinants of the matrix $A_{ij}$ already argued to be of the form

$$
\begin{pmatrix}
0 & A_{12} & A_{13} & \cdots & A_{1n} \\
-A_{12} & 0 & A_{23} & \cdots & A_{2n} \\
-A_{13} & -A_{23} & 0 & \cdots & 0 \\
\vdots & \vdots & 0 & \cdots & 0 \\
-A_{1n} & -A_{2n} & 0 & \cdots & 0
\end{pmatrix}.
$$

Especially let us consider a four by four sub-determinant along the diagonal involving columns and rows 1 and 2. The determinant is for instance

$$
\det\begin{pmatrix}
0 & A_{12} & A_{13} & A_{15} \\
-A_{12} & 0 & A_{23} & A_{25} \\
-A_{13} & -A_{23} & 0 & 0 \\
-A_{15} & -A_{25} & 0 & 0
\end{pmatrix} = -\left(\det\begin{pmatrix}
A_{13} & A_{15} \\
A_{23} & A_{25}
\end{pmatrix}\right)^2.
$$

In order that the matrix $A_{ij}$ be of rank 2, this determinant must vanish and so we require that the 2 by 2 sub-matrix

$$
\begin{pmatrix}
A_{13} & A_{15} \\
A_{23} & A_{25}
\end{pmatrix}
$$

must be degenerate, i.e. of rank 1 only. This means that the two columns in it are proportional, one to the other. By considering successively several such selected four by four sub-matrices, we can easily deduce that all the two columns

$$
\begin{pmatrix}
A_{13} \\
A_{23}
\end{pmatrix}, \begin{pmatrix}
A_{14} \\
A_{24}
\end{pmatrix}, \cdots, \begin{pmatrix}
A_{1n} \\
A_{2n}
\end{pmatrix}
$$

are proportional. This in turn means that we can transform them all to zero, except for say

$$
\begin{pmatrix}
A_{13} \\
A_{23}
\end{pmatrix},
$$
by going into a new basis for the fields $\phi_k(\vec{p} - \vec{p}_0)$. So, finally, we have transformed the formulation of the fields in such a way that only the upper left three by three corner of the $A$ matrix is non-zero. But this is exactly the form for which we argued in subsection 4.3 and which was shown to be interpretable as the Maxwell equations, and moreover the Maxwell equations for just three spatial dimensions!

5.3 The Weyl equation derivation

Let us now turn to the application of the Homolumo-gap effect to a system of Fermions in our general field theory model. We shall assume that the Homolumo-gap effect turns out to be strong enough to ensure that the Fermi-surface just gets put to a place where the density of levels is very low. Actually it is very realistic that a gap should develop in a field theory with continuum variables $\vec{p}$ labeling the single particle states. That is namely what one actually sees in an insulator; there is an appreciable gap between the last filled band and the first empty band. However, if the model were totally of this insulating type, the poor physicist would not “see” anything, because he is supposed to be unable to afford to raise a particle from the filled band to the empty one. So he can only see something if there are at least some Fermion single particle states with energy close to the Fermi-surface.

We shall now divide up our discussion of what happens near the Fermi-surface according to the number of components of the Fermion field that are relevant in this neighborhood. Let us denote by $n$ the number of Fermion field components, which contribute significantly to the eigenstates near the Fermi-surface in the small region of momentum space we choose to consider.

The eigenvalues $\pm \omega$ of $iA_{ij}$ – which come in pairs – correspond to eigenstates with complex components. Thus it is really easiest in the fermionic case to “go back” to a complex field notation, by constructing complex fields out of twice as big a number of real ones. So now we consider the level-density near the Fermi-surface for $n$ complex Fermion field components.

5.4 The case of $n = 0$ relevant levels near Fermi-surface

The $n = 0$ case must, of course, mean that there are no levels at all near the Fermi-surface in the small momentum range considered. This corresponds to the already mentioned insulator case. The poor physicist sees nothing from such regions in momentum space and he will not care for such regions at all. Nonetheless this is the generic situation close to the Fermi surface and will apply for most of the momentum space.

5.5 The case of $n = 1$ single relevant level near the Fermi-surface

In this case the generic situation will be that, as a certain component of the momentum is varied, the level will vary continuously in energy. This is the kind
of behaviour observed in a metal. So there will be a rather smooth density of levels and such a situation is not favoured by the Homolumo gap effect, if there is any way to avoid it.

5.6 The case of \( n = 2 \) relevant levels near the Fermi-surface

In this situation a small but almost trivial calculation is needed. We must estimate how a Hamiltonian, described effectively as a 2 by 2 Hermitian matrix \( H \) with matrix elements depending on the momentum \( \vec{p} \), comes to look in the generic case—i.e. when nothing is fine-tuned—and especially how the level density behaves. That is, however, quite easily seen, when one remembers that the three Pauli matrices and the unit 2 by 2 matrix together form a basis for the four dimensional space of two by two matrices. All possible Hermitian 2 by 2 matrices can be expressed as linear combinations of the three Pauli matrices \( \sigma^j \) and the unit 2 by 2 matrix \( \sigma^0 \) with real coefficients. We now consider a linearized Taylor expansion of the momentum dependence of such matrices, by taking the four coefficients to these four matrices to be arbitrary linear functions of the momentum minus the “starting momentum” \( \vec{p}_0 \), where the two levels become degenerate with energy \( \omega(\vec{p}_0) \). That is to say we must take the Hermitian 2 by 2 matrix to be

\[
H = \sigma^a V_i^a(p_i - p_{0i}) + \sigma^0 \omega(\vec{p}_0). \tag{32}
\]

This can actually be interpreted as the Hamiltonian for a particle obeying the Weyl equation, by defining

\[
P_1 = V_1^a(p_i - p_{0i}), \quad P_2 = V_2^a(p_i - p_{0i}), \quad P_3 = V_3^a(p_i - p_{0i}) \tag{33}
\]

\[
H_{\text{new}} = H - \sigma^0 V_0^a(p_i - p_{0i}) - \sigma^0 \omega(\vec{p}_0) = \vec{\sigma} \cdot \vec{P} \tag{34}
\]

\[
\omega_{\text{new}} = \omega - V_0^a(p_i - p_{0i}) - \omega(\vec{p}_0) \tag{35}
\]

and supposing that the \( V_0^a \) are not too large compared to the other \( V_i^a \)'s. The renormalisation of the energy, eq. (33), is the result of transforming away a constant velocity \( V_0^a \) in D dimensions carried by all the Fermions, using the change of co-ordinates \( x'^i = x^i - tV_0^i \), and measuring the energy relative to \( \omega(\vec{p}_0) \). Note that the “starting momentum” \( \vec{p}_0 \) will generically be of the order of a fundamental (Planck scale) momentum, which cannot be significantly changed by a “poor physicist”. So the large momentum \( \vec{p}_0 \) effectively plays the role of a conserved charge at low energy, justifying the use of complex fermion fields and the existence of a Fermi surface.

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3A related discussion of the redefinition of spinors has been given in the context of the low energy limit of a Lorentz violating QED model. 4If the \( V_0^a \) are very large, there is a risk that different sides of the upper light-cone fall above and below the value \( \omega(\vec{p}_0) \) of the energy at the tip of the cone.
A trivial calculation for the Weyl equation, $H_{\text{new}}\psi = \omega_{\text{new}}\psi$, leads to a level density with a thin neck, behaving like

$$\rho \propto \omega_{\text{new}}^2$$  \hspace{1cm} (36)

According to our strong assumption about homolumo-gap effects, we should therefore imagine that the Fermi-surface in this case would adjust itself to be near the $\omega_{\text{new}} = 0$ level. Thereby there would then be the fewest levels near the Fermi-surface.

5.7 The cases $n \geq 3$

For $n$ larger than 2 one can easily find out that, in the neighbourhood of a point where the $n$ by $n$ general Hamiltonian matrix deviates by zero from the unit matrix, there are generically branches of the dispersion relation for the particle states that behave in the metallic way locally, as in the case $n = 1$. This means that the level density in such a neighborhood has contributions like that in the $n = 1$ case, varying rather smoothly and flatly as a function of $\omega$. So these cases are not so favourable from the Homolumo-gap point of view.

5.8 Conclusion of the various $n$ cases for the Fermion model

The conclusion of the just given discussion of the various $n$-cases is that, while of course the $n = 0$ case is the “best” case from the point of view of the homolumo-gap, it would not be noticed by the “poor physicist” and thus would not be of any relevance for him. The next “best” from the homolumo-gap point of view is the case $n = 2$ of just two complex components (corresponding to 4 real components) being relevant near the Fermi-surface. Then there is a neck in the distribution of the levels, which is not present in the cases $n = 1$ and $n > 2$.

So the “poor physicist” should in practice observe the case $n = 2$, provided the homolumo-gap effect is sufficiently strong (a perhaps suspicious assumption).

Now, as we saw, this case of $n = 2$ means that the Fermion field satisfies a Weyl equation, formally looking like the Weyl equation in just 3+1 dimensions! It should however be noticed that there are indeed more spatial dimensions, by assumption, in our model. In these extra spatial dimensions, the Fermions have the same constant velocity which we were able to renormalise to zero, because the Hamiltonian only depends on the three momentum components $\vec{P}$ in the Taylor expandable region accessible to the “poor physicist”. The latter comes about because there are only the three non-trivial Pauli matrices that make the single particle energy vary in a linear way around the point of expansion. In this sense the number of spatial dimensions comes out as equal to the number of Pauli matrices.

\footnote{HBN would like to thank S. Chadha for a discussion of this $n \geq 3$ case many years ago.}
6 Conclusion, résumé, discussion

We have found the remarkable result that, in the free approximation, our very general quantum field theory, which does not have Lorentz invariance put in, leads to Lorentz invariance in three plus one dimensions for both Bosons and Fermions. In the derivation of this result, we made use of what we called the homolumo-gap effect and its “analogue for Bosons” and that experimentalists only have access to energies low compared to the fundamental scale. The derivation of three spatial dimensions should be understood in the sense that our model, which has at first a space of D dimensions, leads to a dispersion relation (i.e. a relation between energy and momentum) for which the derivative of the energy \( \omega \) w.r.t. the momentum in \( D - 3 \) of the dimensions is independent of the momentum. Then, in the remaining 3 dimensions, we get the well-known Lorentz invariant dispersion relations both in the Bosonic and the Fermionic cases. In fact we obtained the Weyl equation and the Maxwell equations, in the fermionic and the bosonic cases respectively, as “generic” equations of motion – after the use of the homolumo gap and its analogue. These Maxwell and spin one half equations of motion are in remarkable accord with the presently observed (i.e. ignoring the Higgs particle) fundamental particles!

6.1 Some bad points and hopes

In spite of this remarkable success of our model in the free approximation, we have to admit there are a number of flaws:

1) The three space dimensions selected by each type of particle are a priori overwhelmingly likely not to be the same three. That is to say we would have to hope for some speculative mechanism that could align the three dimensions used by the different species of particles, so as to be the same three dimensions.

2) Although we have hopes of introducing interactions, it is not at all clear how these interactions would come to look and whether e.g. they would also be Lorentz invariant—according to point 1) one would a priori say that they do not have much chance to be Lorentz invariant.

3) There are extra dimensions in the model, although they do not participate in the derived Lorentz invariance which is only a 3+1 Lorentz invariance. Rather the velocity components in the extra dimension directions are constant, independent of the momentum of the particles. We can really by convention renormalise them to zero and claim that we do not see the extra dimensions, because we cannot move in these directions. But from point 1) there is the worry that these directions (in which we have no movement) are different for the different types of particles.

The best hope for rescuing the model from these problems might be to get rid of momentum conservation in the extra directions. We might hope to get some attachment of the particles to a fixed position in the extra directions much like attachments to branes, but then one would ask how this could happen in a natural way. Of course the point of view most in the spirit of the random dynamics project would be that a priori we did not even have momentum con-
ervation, but that it also just arose as the result of some Taylor expansion. This becomes very speculative but it could easily happen that it is much easier to get a translational invariance symmetry develop, along the lines suggested in section 6.2.3 of our book [3], for the momentum directions in which we have rapid motion than in the directions in which we have zero velocity. If we crudely approximated the particles by non-relativistic ones, the rapid motion would mean low mass while the zero motion would mean a very huge mass. The uncertainty principle would, therefore, much more easily allow these particles to fall into the roughness valleys\(^6\) in the translational invariance violating potential in the extra directions, where the non-relativistic mass is much larger than in the 3 space directions. A particle would be very much spread out by uncertainty in the 3 directions and, thus, only feel a very smoothed out roughness potential, if translational invariance is broken in these directions. In this way translational invariance could develop in just 3 dimensions.

A breakdown of the translational invariance—or, as just suggested, a lack of its development—in the extra dimensions would be very helpful in solving the above-mentioned problems. This is because there would then effectively only be 3 space dimensions and all the different types of particles would, thus, use the same set of 3 dimensions. It must though be admitted that they would still have different metric tensors, or metrics we should just say. We had some old ideas [3] for solving this problem, but they do not quite work in realistic models.

### 6.2 Where did the number three for the space dimension come from?

One might well ask why we got the prediction of just three for the number of spatial dimensions. In fact we have derived it differently, although in many ways analogously, for Bosons and for Fermions:

**Bosons:** For Bosons we obtained this result by considering the simultaneous approach of a pair of equal and opposite eigenvalues of the real antisymmetric matrix \(A_{ij}\) to the supposedly existing zero frequency, \(\omega = 0\), level(s). Thus the rank of the matrix \(A\) relevant to this low energy range is just two, except at the point around which we expand where it has rank zero. Then we argued that we could transform such a matrix in such a way that it effectively becomes a 3 by 3 matrix—still antisymmetric and real. So the matrix \(A\) has effectively three independent matrix elements and each can vary with the components of the momentum. However, in the low energy regime, this dependence can be linearized and \(A\) only depends on three linearly independent components of the momentum. It is these three dimensions in the directions of which we have non-zero velocity (or better non-constant velocity) for the Boson—the photon, say, in as far as it obeys the Maxwell equations. We thus got the number three as the number of independent matrix elements in the antisymmetric matrix \(A\), obtained after transforming away most of this rank two matrix.

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\(^6\)By roughness valleys we refer to the (local) minima or valleys in a potential representing a non-translational invariant potential set up so as break momentum conservation.
Fermions: In this case we went to a complex notation, although we still started from the same type of antisymmetric real matrix as in the bosonic case. The homolumo-gap argumentation suggested that just $n = 2$ complex components in the field should be “relevant” near the Fermi surface, after ruling out the trivial $n = 0$ case as unobservable by anybody. This number of relevant components then meant that just $n^2 - 1 = 4 - 1 = 3$ non-trivial linearly independent $n$ by $n$ matrices could be formed. These three matrices could, of course, then be used as the coefficients for three momentum components in the linearized (Taylor expanded) momentum dependence of the Hamiltonian. In this way the number three arose again.

So there is an analogy at least in as far as, for both Bosons and Fermions, it is the number of linearly independent matrices of the type finally used, which remarkably predicts the observable number of spatial dimensions to be 3. However in the bosonic case it is real three by three matrices which we ended up with, while in the fermionic case it is Hermitian 2 by 2 matrices with the unit matrix omitted. The unit matrix is not counted because it does not split the levels and basically could be transformed away by the shift of a vierbein, i.e. by adjusting the meaning of the momentum and energy components.

A strange prediction of the Boson model is that, at first at least, we get a parity symmetric state of the world for the Maxwell fields. That is to say for every state of the electromagnetic—or generalized Yang Mills—field there is somewhere, reflected in the origin of position space, a corresponding reflected state. In principle we could test such an idea, by looking to see whether we could classify galaxies found on the sky into pairs that could correspond to mirror images—in the “origin”—of each other. Really we hope that this illness of our model might easily repair itself.

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