Mixed Málaga-$\mathcal{M}$ and Generalized-$\mathcal{K}$

Dual-Hop FSO/RF Systems with Interference

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Abstract

This paper investigates the impact of radio frequency (RF) cochannel interference (CCI) on the performance of dual-hop free-space optics (FSO)/RF relay networks. The considered FSO/RF system operates over mixed Málaga-$\mathcal{M}$/composite fading/shadowing generalized-$\mathcal{K}$ ($\mathcal{GK}$) channels with pointing errors. The H-transform theory, wherein integral transforms involve Fox’s H-functions as kernels, is embodied into a unifying performance analysis framework that encompasses closed-form expressions for the outage probability, the average bit error rate (BER), and the ergodic capacity. By virtue of some H-transform asymptotic expansions, the high signal-to-interference-plus-noise ratio (SINR) analysis culminates in easy-to-compute expressions for the outage probability and BER.

I. INTRODUCTION

Free-space optics (FSO) communication has recently drawn a significant attention as one promising solution to cope with radio frequency (RF) wireless spectrum scarcity [1]. Though securing high data rates, FSO communications performance significantly degrades due to atmospheric turbulence-induced fading and strong path-loss [2]. Aiming to address these shortcomings, relay-assisted FSO systems have been actually identified as an influential solution to provide more efficient and wider networks. As such, understanding the fundamental system performance limits of mixed FSO/RF architectures has attracted a lot of research endeavor in the past decade (cf. [3], [4] and references therein).

Up until recent past, the performance of relay-assisted FSO systems was investigated assuming several irradiance probability density function (PDF) models with different degrees of success

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out of which the most commonly utilized models are the lognormal [5] and the Gamma-Gamma [6] PDFs. Recently, a new generalized statistical model, the Málaga-\(\mathcal{M}\), unifying most statistical models exploited so far and able to better reflect a wider range of turbulence conditions was proposed in [7], [8]. Several performance studies of FSO link operating over Málaga-\(\mathcal{M}\) turbulent channels with and without pointing errors have been conducted in [4], [9].

On the RF side, previous works typically assume either Nakagami-\(m\) [3], [6] or Rayleigh [10], [11] fading, thereby lacking the flexibility to account for disparate signal propagation mechanisms as those characterized in 5G communications which will accommodate a wide range of usage scenarios with diverse link requirements. In fact, in 5G communications design, the combined effect of small-scale and shadowed fading needs to be properly addressed. Shadowing, which is due to obstacles in the local environment or human body (user equipments) movements, can impact link performance by causing fluctuations in the received signal. For instance, the shadowing effect comes to prominence in millimeter wave (mmWave) communications due to their higher carrier frequency. In this respect, the generalized-\(K\) (\(\mathcal{G}K\)) model was proposed by combining Nakagami-\(m\) multipath fading and Gamma-Gamma distributed shadowing [12],[13].

While FSO transmissions are robust to RF interference, mixed FSO/RF systems are inherently vulnerable to the harmful effect of co-channel interference (CCI) through the RF link (cf. [14] and references therein). Previous contributions pertaining to FSO relay-assisted communications [3]- [11] relied on the absence of CCI. Recently, the recognition of the interference-limited nature of emerging communication systems has motivated [15] to account for CCI in the performance analysis of mixed decode and forward RF/FSO systems. Besides ignoring the shadowing effect on the RF link, [15] assumes a restrictive Gamma-Gamma model on the FSO link.

In this paper, motivated by the aforementioned challenges, we assess the impact of RF CCI on the performance of dual-hop amplify and forward (AF) mixed FSO/RF systems operating over Málaga-\(\mathcal{M}\) and composite fading shadowing generalized-\(K\) (\(\mathcal{G}K\)) channels, respectively. Assuming fixed-gain and CSI-assisted relaying schemes and taking into account the effect of pointing errors while considering both heterodyne and intensity modulation/direct (IM/DD) detection techniques, we present a comprehensive performance analysis by exploiting seminal results from the H-transform theory. In addition, we present asymptotic expressions for the outage probability and the average BER at high SINR and we derive the diversity gain.

The remainder of this paper is organized as follows. We describe the system model in Section II. In Section III, we present the unifying H-transform analysis of the end-to-end SINR statistics.
for both fixed-gain and CSI-assisted relays. Then, in section IV, we derive exact closed-form expressions for the outage probability, the average BER, and the ergodic capacity followed by the asymptotic expressions at high SINR. Section V presents some numerical and simulation results to illustrate the mathematical formalism presented in the previous sections. Finally, some concluding remarks are drawn out in Section VI.

II. CHANNEL AND SYSTEM MODELS

We consider a downlink of a relay-assisted network featuring a mixed FSO/RF communication. We assume that the optical source (S) communicates with the destination (D) in a dual-hop fashion through an intermediate relay (R). The latter is able to activate either heterodyne or IM/DD detection techniques at the reception of the optical beam. Using AF relaying, the relay amplifies the received optical signal and retransmits it to the destination with MRT using N antennas. We assume that the destination is subject to inter-cell interference (I) brought by L co-channel RF sources in the network (cf. Fig.1).

The optical (S-R) channel follows a Málaga-M distribution for which the CDF of the instantaneous SNR $\gamma_1$ in the presence of pointing errors is given by

$$F_{\gamma_1}(x) = \frac{\xi^2 A_r}{\Gamma(\alpha)} \sum_{k=1}^\beta \frac{b_k}{\Gamma(k)} H^{3,1}_{2,4} \left[ \frac{B^r x}{\mu_r} \langle 1, r \rangle, \langle (\xi^2 + 1, r) \rangle, \langle (\xi^2, r), (\alpha, r), (k, r), (0, r) \rangle \right],$$

where $\xi$ is the ratio between the equivalent beam radius and the pointing error displacement standard deviation (i.e., jitter) at the relay (for negligible pointing errors $\xi \to +\infty$) [2]. $A = \alpha^{\frac{\beta}{\beta+\frac{\alpha}{\beta}}}[g\beta/(g\beta + \Omega)]^{\beta+\frac{\alpha}{\beta}}g^{-\frac{\alpha}{\beta}}$ and $b_k = \left( \frac{\beta-1}{k-1} \right) [((g\beta + \Omega)^{1-\frac{\alpha}{\beta}})/(\alpha/\beta)]^{\frac{\alpha}{\beta}} (\Omega/g)^{\frac{k-1}{\beta}} (\alpha/\beta)^{\frac{\alpha}{\beta}}$, where $\alpha$, $\beta$, $g$ and $\Omega$ are the fading parameters related to the atmospheric turbulence conditions [9]. It may be useful to mention that $g = 2b_0(1 - \rho)$ where $2b_0$ is the average power of the LOS term and $\rho$ represents the amount of scattering power coupled to the LOS component.
(0 ≤ ρ_i ≤ 1). Moreover in [1], H^m,α[·] and Γ(·) stand for the Fox-H function [16 Eq.(1.2)] and the incomplete gamma function [17 Eq.(8.310.1)], respectively, and

\[ B = \alpha \beta h (g + \Omega) / [(g \beta + \Omega)] \]

with \( h = \xi^2 / (\xi^2 + 1) \). Furthermore, \( r \) is the parameter that describes the detection technique at the relay (i.e., \( r = 1 \) is associated with heterodyne detection and \( r = 2 \) is associated with IM/DD) and, \( \mu_r \) refers to the electrical SNR of the FSO hop [9]. In particular, for \( r = 1 \),

\[ \mu_1 = \mu_{\text{heterodyne}} = \mathbb{E} [\gamma_1] = \bar{\gamma}_1, \tag{2} \]

and for \( r = 2 \), it becomes [2 Eq.(8)]

\[ \mu_2 = \mu_{\text{IM/DD}} = \frac{\mu_1 \alpha \xi^2 (\xi^2 + 1)^{-2} (\xi^2 + 2) (g + \Omega)}{(\alpha + 1) [ 2g (g + 2 \Omega) + \Omega^2 (1 + \frac{1}{\beta}) ]}. \tag{3} \]

The RF (R-D) and (I-D) links are assumed to follow generalized-K fading distributions. Hence the probability density function (PDF) of the instantaneous SNR (respectively INR), \( \gamma_{XD} \), \( X \in \{ R, I \} \), is given by [12 Eq.(5)]

\[ f_{\gamma_{XD}}(x) = 2 \left( \frac{m_X \kappa_X}{\gamma_{XD}} \right)^{\kappa_X + \delta_X m_X} x^{\kappa_X + \delta_X m_X - 1} \frac{\Gamma(\delta_X m_X) \Gamma(\kappa_X)}{\Gamma(\kappa_X)} K_{\kappa_X - \delta_X m_X} \left( 2 \sqrt{ \frac{\kappa_X m_X x}{\gamma_{XD}} } \right), \tag{4} \]

where \( X \in \{ R, I \} \) and \( K_\nu(·) \) stands for the modified Bessel function of the second kind [17 Eq.(8.407.1)]. Moreover, \( m_X \geq 0.5 \) and \( \kappa_X \geq 0 \) denote the multipath fading and shadowing severity of the \( X \)-Dth channel coefficient, respectively. Moreover, \( \delta_X = \{ N, L \} \) for \( X \in \{ R, I \} \) follows form the conservation property under the summation of \( N \) and \( L \) i.i.d. (independent identically distributed) \( \mathcal{G} \mathcal{K} \) random variables. The interfering signals are assumed to propagate through i.i.d \( \mathcal{G} \mathcal{K} \) channels with parameters \( m_I \) and \( \kappa_I \). Using [17 Eq.(9.34.3)], the PDF of the \( \mathcal{G} \mathcal{K} \) distribution can be represented in terms of the Meijer’s-G function as

\[ f_{\gamma_{XD}}(x) = \frac{m_X \kappa_X}{\Gamma(\delta_X m_X) \Gamma(\kappa_X)} G^{2,0}_{0,2} \left[ \frac{\kappa_X m_X x}{\gamma_{XD}} \right]^{-1} \delta_X m_X - 1, \kappa_X - 1 \] \tag{5} \]

The CDF of the signal-to-interference ratio (SIR) \( \gamma_2 = \gamma_{RD} / \gamma_{ID} \) under \( \mathcal{G} \mathcal{K} \) fading can be derived from a recent result in [13 Lemma 1] as

\[ F_{\gamma_2}(x) = 1 - \frac{1}{\Gamma(Nm) \Gamma(\kappa) \Gamma(Lm)} G^{3,2}_{3,3} \left[ \frac{\kappa m x}{\kappa_I m_I \bar{\gamma}_2} \right]^{-1} \kappa_I - 1, Lm - 1, \kappa , Nm \] \tag{6} \]

where \( \bar{\gamma}_2 = \bar{\gamma}_{RD} / \bar{\gamma}_{ID} \) is the average SIR of the RF link where, for consistency, we have dropped the subscript \( R \) from the parameters \( m_R \) and \( \kappa_R \).
In the fixed-gain relaying scheme, the end-to-end SINR at the destination can be expressed as [18, Eq.(2)]

\[
\gamma = \frac{\gamma_1 \gamma_2}{\gamma_2 + C},
\]

where \(C\) stands for the fixed gain at the relay. Whereas, the end-to-end SINR when CSI-assisted relaying scheme is considered is expressed as [10, Eq.(7)]

\[
\gamma = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1}.
\]

### III. End-to-End Statistics

#### A. Fixed-Gain Relaying

The CDF of the end-to-end SINR of interference-limited dual-hop FSO/RF systems using a fixed-gain relay in Málaga-\(\mathcal{M}/\mathcal{G}\)K fading under both heterodyne detection and IM/DD is given by

\[
F_{\gamma}(x) = \frac{\xi^2 AkmC}{\Gamma(\alpha)\Gamma(Nm)\Gamma(\kappa)\Gamma(Lm_I)\Gamma(\kappa_I)\kappa_I m_I \gamma^2} \sum_{k=1}^{\beta} b_k \frac{\mu_{b^{\prime}}}{b^{\prime}x} \frac{\kappa mC}{\kappa_I m_I \gamma^2} \left[ \begin{array}{c} (0, 1, 1) \\ - \\ (\delta, \Delta) \\ (\lambda, \Lambda) \\ (\chi, X) \\ (v, \Upsilon) \end{array} \right],
\]

where \(H_{p_1, q_1; p_2, q_2; p_3, q_3}^{m_1, n_1; m_2, n_2; m_3, n_3}[]\) denotes the Fox-H function (FHF) of two variables [19, Eq.(1.1)] also known as the bivariate FHF whose Mathematica implementation may be found in [20, Table I], whereby \((\delta, \Delta) = (1 - \xi^2, r), (1 - \alpha, r), (1 - k, r); (\lambda, \Lambda) = (0, 1), (-\xi^2, r); (\chi, X) = (-\kappa_I, 1), (-\mu_{b^{\prime}} - Lm_I, 1), (0, 1)\); and \((v, \Upsilon) = (-1, 1), (-1, 1), (-1, 1), (\kappa_I - 1, 1), (Nm - 1, 1), (0, 1)\).

**Proof:** See Appendix A.

The PDF of the end-to-end SINR \(\gamma\) in mixed Málaga-\(\mathcal{M}/\mathcal{G}\)K is obtained as

\[
f_{\gamma}(x) = -\frac{\xi^2 AkmC}{x\Gamma(\alpha)\Gamma(Nm)\Gamma(\kappa)\Gamma(Lm_I)\Gamma(\kappa_I)\kappa_I m_I \gamma^2} \sum_{k=1}^{\beta} b_k \frac{\mu_{b^{\prime}}}{b^{\prime}x} \frac{\kappa mC}{\kappa_I m_I \gamma^2} \left[ \begin{array}{c} (0, 1, 1) \\ - \\ (\delta, \Delta) \\ (\lambda, \Lambda) \\ (\chi, X) \\ (v, \Upsilon) \end{array} \right].
\]
where \((\lambda', \Lambda') = (1, 1), (-\xi^2, r)\).

**Proof:** The result follows from differentiating the Mellin-Barnes integral in (9) over \(x\) using
\[
\frac{d x^{-s}}{d x} = -s x^{-s-1}
\]
with \(\Gamma(s + 1) = s \Gamma(s)\) and applying [16, Eq.(2.57)].

\[\square\]

B. CSI-Assisted Relaying

Due to the intractability of the SINR in (8), we resort to an upper bound given by [10, Eq.(20)] as \(\gamma = \min(\gamma_1, \gamma_2) > \gamma_1 \gamma_2 / (\gamma_1 + \gamma_2 + 1)\), whose CDF can be expressed as \(F_{\gamma_1}(x) = 1 - F_{\gamma_1}^{(c)}(x)\), where \(F_{\gamma_1}^{(c)}\) and \(F_{\gamma_2}^{(c)}\) stand for the complementary CDF of \(\gamma_1\) and \(\gamma_2\), respectively. Hence, using [4, Eq.(8)] and (6), the CDF of dual-hop FSO/RF systems employing a CSI-assisted relaying scheme can be obtained as

\[
F_\gamma(x) = 1 - \frac{\xi^2 A}{\Gamma(\alpha) \Gamma(Nm) \Gamma(\kappa) \Gamma(Lm_1) \Gamma(\kappa_I) \Gamma(\kappa_I)} \sum_{k=1}^{\beta} b_k \frac{G_{2,4}^{4,0}}{\Gamma(k)} \left[ B \left( \frac{x}{\mu_r} \right)^{\frac{1}{\gamma}} \right]_0^{\gamma+1, 1, 1} \left[ 1 - \frac{\xi^2}{\gamma} \right]_{0, \kappa, Nm}
\]

\[
G_{3,3}^{3,2} \left[ \frac{\kappa m x}{\kappa_I m_I \gamma_2} \right]_{0, \kappa, Nm}
\]

(11)

IV. PERFORMANCE ANALYSIS OF FIXED-GAIN RELAYING

A. Outage Probability

The quality of service (QoS) of the considered mixed FSO/RF system is ensured by keeping the instantaneous end-to-end SNR, \(\gamma\), above a threshold \(\gamma_{th}\). The outage probability of the considered mixed FSO/RF system follows from (9) as

\[
P_{out} = F_\gamma(\gamma_{th}).
\]

(12)

At high normalized average SNR in the FSO link \((\frac{\mu_r}{\gamma_{th}} \to \infty)\), the outage probability of the system under consideration is obtained as

\[
P_{out} \approx \sum_{\gamma_{th} > 1} \frac{\xi^2 A \kappa m C}{\Gamma(\alpha) \Gamma(Nm) \Gamma(\kappa) \Gamma(Lm_1) \Gamma(\kappa_I) \Gamma(\kappa_I) \Gamma(\kappa_I)} \sum_{k=1}^{\beta} b_k \frac{\Gamma(\alpha - \xi^2) \Gamma(k - \xi^2)}{r \Gamma(1 - \xi^2)} \Xi \left( \gamma_{th} \frac{\xi^2}{r} \right) + \frac{\Gamma(\xi^2 - \alpha) \Gamma(k - \alpha)}{r \Gamma(1 - \alpha) \Gamma(1 + \xi^2 - \alpha)} \Xi \left( \gamma_{th} \frac{\alpha}{r} \right) + \frac{\Gamma(\xi^2 - k) \Gamma(\alpha - k)}{r \Gamma(1 - k) \Gamma(1 + \xi^2 - k)} \Xi \left( \gamma_{th} \frac{k}{r} \right)
\]

(13)

where

\[
\Xi(x, y) = \left( \frac{B^r \gamma_{th}}{\mu_r} \right)^y G_{5,5}^{4,4} \left[ \frac{\kappa m C}{\kappa_I m_I \gamma_2 \mu_r} \right]_{-\kappa_I, -Lm_1, -1, y, 0}^{\kappa_I, Nm - 1, -1, 1, 0}
\]

(14)
\((\sigma, \Sigma) = (-\kappa_I, 1), (-Lm_I, 1), (-1, 1), (0, 1), (1 + \xi^2 - r, r), (0, 1), \) and \((\phi, \Phi) = (\xi^2 - r, r), (\alpha-r, r), (k-r, r), (\kappa - 1, 1), (Nm - 1, 1), (-1, 1), (1, 1), (0, 1).\)

**Proof:** Resorting to the Mellin-Barnes representation of the bivariate FHF \([16, \text{Eq.(2.57)}]\) in \((9)\) and applying \([21, \text{Theorem 1.7}]\) yield \((13)\) after some additional algebraic manipulations.

Furthermore, when \(\bar{\gamma}_2 \to \infty\), then by applying \([21, \text{Theorem 1.11}]\) to \((13)\) while only keeping the dominant term, the diversity gain for FSO/RF systems with pointing errors over Málaga-\(\mathcal{M}/\mathcal{G}\)\(K\) fading conditions can be shown to be equal to

\[
G_d = \min \left(Nm, \kappa, \xi^2, \alpha, k, r\right).
\]

In particular, under Nakagami-\(m\) fading, i.e., when \(\kappa \to \infty\), we obtain \(G_d = \min \left(Nm, \frac{\xi^2}{r}, \frac{\alpha}{r}, \frac{k}{r}\right)\) \([3, \text{Eq. 29}]\).

**B. Average Bit-Error Rate**

The average error probability for the considered dual-hop mixed RF/FSO AF relay system with interference at the destination and pointing errors at the FSO link under both heterodyne and IM/DD detection techniques is analytically derived as

\[
\mathcal{P}_e = \frac{\xi^2 A \varphi \kappa m C}{2 \Gamma(\alpha) \Gamma(p) \Gamma(Nm) \Gamma(\kappa) \Gamma(Lm_I) \Gamma(\kappa_I) \kappa_I m_I \bar{\gamma}_2} \sum_{j=1}^{n} \sum_{k=1}^{\beta} b_k \frac{\mu_j q_j}{\varphi^k \kappa m C} \sum_{\left\langle 0,1,1 \right\rangle}^{(\delta, \Delta)} \sum_{(p,1)}^{(\lambda, \Lambda)} \sum_{(\chi,X)}^{(\nu, \Upsilon)} H^{0,1;3:4,3}_{1,0;3:4,5}.
\]

**Proof:** The average BER can be written in terms of the CDF of the end-to-end SINR as

\[
\mathcal{P}_e = \frac{\varphi}{2 \Gamma(p)} \sum_{j=1}^{n} q_j^p \int_{0}^{\infty} e^{-q_j x} x^{p-1} F_\gamma(x) \, dx,
\]

where \(\Gamma(\cdot, \cdot)\) stands for the incomplete Gamma function \([17, \text{Eq.(8.350.2)}]\) and the parameters \(\varphi, n, p\) and \(q_j\) account for different modulations schemes \([12]\). Now, substituting the Mellin-Barnes integral form of \((9)\) using \([16, \text{Eq.(2.56)}]\) into \((17)\), and resorting to \([17, \text{Eq.(7.811.4)}]\) yield \((16)\) after some manipulations.

\[\square\]
At high FSO SNR (i.e. $\mu_r \to \infty$), the asymptotic average BER is derived as

\[
\overline{P_e} \approx \frac{\xi^2 A \varphi \kappa m C}{2 \ln(2) \Gamma(\alpha) \Gamma(Nm) \Gamma(\kappa) \Gamma(Lm_I) \Gamma(\kappa_I) \kappa_I \gamma_I^2} \sum_{j=1}^{\beta} \frac{b_k}{\Gamma(k)} \left[ \frac{\Gamma(\alpha - \xi^2) \Gamma(k - \xi^2)}{r \Gamma(1 - \frac{\xi^2}{r})} \right] \Xi \left( \frac{1}{q_j}, \frac{1}{r} \right)
\]

\[+ \frac{\Gamma(\xi^2 - \alpha) \Gamma(k - \alpha)}{r \Gamma(1 - \frac{\alpha}{r}) \Gamma(1 + \xi^2 - \alpha)} \Xi \left( \frac{1}{q_j}, \frac{1}{r} \right) + \frac{\Gamma(\xi^2 - k) \Gamma(\alpha - k)}{r \Gamma(1 - \frac{k}{r}) \Gamma(1 + \xi^2 - k)} \Xi \left( \frac{1}{q_j}, \frac{1}{r} \right)
\]

\[+ \frac{B^{\gamma^2}}{\mu_r q_j} \frac{H_{T,8}^{7,4}}{\kappa \gamma_I^2} \frac{\beta}{\Gamma(\kappa_I \gamma_I^2)} \left[ (\sigma', \Sigma') \right], \tag{18}\]

where $(\sigma', \Sigma') = (-\kappa_I, 1), (-Lm_I, 1), (-1, 1), (-p, 1), (0, 1), (1 + \xi^2 - r, r), (0, 1)$.

**Proof:** The asymptotic BER follows along the same lines as (13).

C. Ergodic Capacity

The ergodic capacity of a mixed Málaga-$M$/interference-limited $GK$ transmission system under both detection techniques with pointing errors at the FSO link is obtained as

\[
\overline{C} = \frac{\xi^2 A \kappa m C}{2 \ln(2) \Gamma(\alpha) \Gamma(Nm) \Gamma(\kappa) \Gamma(Lm_I) \Gamma(\kappa_I) \kappa_I \gamma_I^2} \sum_{k=1}^{\beta} \frac{b_k}{\Gamma(k)} \left[ \frac{\mu_k}{B^{\gamma^2}} \frac{\kappa m C}{\kappa_I \gamma_I^2} \right] \left[ (0, 1, 1) \right]
\]

\[\left[ (\delta, \Delta), (1, 1) \right]
\]

\[\left[ (0, 1) (\lambda', \Lambda') \right]
\]

\[\left[ (\chi, X) \right]
\]

\[\left[ (\nu, \Upsilon) \right], \tag{19}\]

**Proof:** The ergodic capacity $\overline{C} = \frac{1}{2} \mathbb{E} \left[ \ln_2(1 + \gamma) \right]$ follows from averaging $\ln(1 + \gamma) = G_{1,2}^{1,2}[\gamma]_{1,1}^{1,1}$ over the end-to-end SINR PDF obtained in (10) while resorting to [19, Eq.(1.1)] and [17, Eq.(7.811.4)] with some manipulations.

The Málaga-$M$ reduces to Gamma-Gamma fading when $(g = 0, \Omega = 1)$, whence all terms in (1) vanish except for the term when $k = \beta$. Hence, when $g = 0, \Omega = 1, \kappa, \kappa_I \to \infty$, (19) reduces, when $r = 1$, to the ergodic capacity of mixed Gamma-Gamma FSO/interference-limited Nakagami-$m$ RF transmission with heterodyne detection as given by

\[
\overline{C} = \frac{\xi^2}{2 \ln(2) \Gamma(Nm) \Gamma(Lm_I) \Gamma(\alpha) \Gamma(\beta)} \left[ \frac{\mu_1}{\alpha \beta h}, \frac{m C}{\kappa_I \gamma_I^2} \right] \left[ 1 - (1 - \xi^2, 1 - \alpha, 1 - \beta, 1) \right] \left[ 1 - Lm_I, 1, 0 \right] \left[ 1 - Lm_I, 1, 0 \right]. \tag{20}\]
where $G^{p,q,k,r,l}_{a,b,c,d,e,f,g,h}$ is the generalized Meijer’s G-function and is used to represent the product of three Meijer’s-G functions in a closed-form \cite{22}.

V. PERFORMANCE ANALYSIS OF CSI-ASSISTED RELAYING

A. Outage Probability

Based on (11), the outage probability of CSI-assisted mixed Málaga-$\mathcal{M}$ turbulent/$\mathcal{GK}$ systems with interference under both detection techniques with pointing errors can be lower bounded by

$$P_{\text{out}}^\text{lb} = 1 - \frac{\xi^2 A r}{\Gamma(\alpha)\Gamma(N m)\Gamma(\kappa)\Gamma(L m I)\Gamma(\kappa I)} \sum_{k=1}^\beta \frac{b_k}{\Gamma(k)} \mathcal{H}_{0,0,2,4,3,3}^{0,0:4,0,0:2,4,3,3} \left[ \left( \begin{array}{c} (0, 1, 1) \\ (\delta_1, \Delta_1) \\ (\lambda_1, \Lambda_1) \\ (\chi_1, X_1) \\ (\upsilon_1, \Upsilon_1) \end{array} \right) - \left( \begin{array}{c} (1 - \kappa I, 1) \\ (1 - L m I, 1) \\ (1, 1) \end{array} \right) \right], \quad (21)$$

where $(\delta_1, \Delta_1) = (\xi^2 + 1, r), (1, r), (\lambda_1, \Lambda_1) = (0, r), (\xi^2, r), (\alpha, r), (k, r), (\chi_1, X_1) = (1 - \kappa I, 1), (1 - L m I, 1), (1, 1)$, and $(\upsilon_1, \Upsilon_1) = (0, 1), (1, 1), (N m, 1)$.

B. Average Bit-Error Rate

The average BER of a mixed FSO/interference-limited RF CSI-assisted relaying system in Málaga-$\mathcal{M}$ turbulent with pointing errors/$\mathcal{GK}$ fading channels under both detection techniques is obtained as

$$P_e = -\frac{\varphi}{2} - \frac{\xi^2 A r \varphi}{2\Gamma(p)\Gamma(N m)\Gamma(\kappa)\Gamma(L m I)\Gamma(\kappa I)} \sum_{j=1}^n \sum_{k=1}^\beta \frac{b_k}{\Gamma(k)} \mathcal{H}_{1,0,2,4,3,3}^{0,1:4,0,3,3,2} \left[ \left( \begin{array}{c} (1 - p, 1, 1) \\ (\delta_1, \Delta_1) \\ (\lambda_1, \Lambda_1) \\ (\chi_1, X_1) \\ (\upsilon_1, \Upsilon_1) \end{array} \right) - \left( \begin{array}{c} (1, 1, 1) \\ (\delta_1, \Delta_1) \\ (\lambda_1, \Lambda_1) \\ (\chi_1, X_1) \\ (\upsilon_1, \Upsilon_1) \end{array} \right) \right]. \quad (22)$$

Proof: Substituting (11) into (17) and resorting to \cite[Eq.(1.59)]{16} and \cite[Eq.(2.2)]{19} yield the result after some manipulations.
C. Ergodic Capacity

The ergodic capacity of a mixed FSO/interference-limited RF CSI-assisted relaying system in Málaga-$\mathcal{M}G\mathcal{K}$ fading channels under both detection techniques is expressed by

$$\overline{C} = \frac{\xi^2 A r \mu_r}{2 \ln(2) \Gamma(\alpha) \Gamma(Nm) \Gamma(K) \Gamma(Lm_I) \Gamma(k_I) B^r} \sum_{k=1}^{\beta} b_k \frac{\mu_k}{\kappa I_{Nm}^r} \prod_{i=1}^{0.1,4:3.3} \frac{\eta_{\kappa I_{Nm}^r}}{\kappa I_{Nm}^r}$$

where \((\delta_2, \Delta_2) = (1 - r, r), (1 - \xi^2 - r, r), (1 - \alpha - r, r), (1 - k - r, r)\), \((\lambda_2, \Lambda_2) = (1, 1), (1 - \kappa, 1), (1 - Nm, 1)\), \((\chi_2, X_2) = (1, 1), (1 - \kappa, 1), (1 - Nm, 1)\), and \((\nu_2, \Upsilon_2) = (1, 1), (\kappa_I, 1), (Lm_I, 1), (0, 1)\).

*Proof:* See Appendix [B].

It should be mentioned that when \(r = 1\) and \(\kappa, \kappa_I \to \infty\), (23) reduces to the ergodic capacity of mixed FSO/interference-limited RF systems in Málaga/Nakagami-$m$ fading channels as given by

$$\overline{C} = \frac{\xi^2 A r \mu_1}{2 \ln(2) B \Gamma(\alpha) \Gamma(Nm) \Gamma(Lm_I) \frac{m_f \overline{\gamma}_2}{m} \frac{\mu_1}{\alpha \beta h}} \left[ \frac{1}{1 - 0, -\xi^2, -\alpha, -k} \right] \begin{bmatrix} 0, 1, 1; & \delta_2, \Delta_2; & (\lambda_2, \Lambda_2); & (\chi_2, X_2); & (\nu_2, \Upsilon_2) \end{bmatrix}. \tag{24}$$

VI. Numerical results

In this section, numerical examples are shown to substantiate the accuracy of the new unified mathematical framework and to confirm its potential for analyzing mixed FSO/RF communications. Remarkably, all numerical results obtained by the direct evaluation of the analytical expressions developed in this paper, are in very good match with their Monte-Carlo stimulated counterparts showing the accuracy and effectiveness of our new performance analysis framework. Unless stated otherwise, all simulations were carried out with the following parameters: \(C = 1.7\), \(m_I = 1.5\), \(\kappa_I = 3.5\), and \(\overline{\gamma}_2 = 20\) dB.

Fig. 2 illustrates the outage probability of mixed FSO/RF fixed-gain AF systems versus the FSO link normalized average SNR in strong (i.e., \(\alpha = 2.4, \beta = 2\)) and weak (i.e., \(\alpha = 5.4, \beta = 4\)) turbulence conditions, respectively. The figure also investigates the effect of strong (i.e., \(\xi = 1.1\)) and weak (i.e., \(\xi = 6.8\)) pointing errors on the system performance. As expected, the
outage probability deteriorates by decreasing the pointing error displacement standard deviation, i.e., for smaller $\xi$, or decreasing the turbulence fading parameter, i.e., smaller $\alpha$ and $\beta$. At high SNR, the asymptotic expansion in (13) matches very well its exact counterpart, which confirms the validity of our mathematical analysis for different parameter settings. On the other hand, we observe that heterodyne detection outperforms IM/DD in turbulent environments as previously observed in [9].

Fig. 3 depicts the outage probability of fixed-gain mixed FSO/interference-limited RF systems with $L = \{1, 2\}$ versus the FSO link normalized average SNR. As expected, increasing $L$ deteriorates the system performance, by increasing the outage probability while the diversity gain remains unchanged. Once again we highlight the fact that the exact and asymptotic expansion in (13) agree very well at high SNRs.

Actually, the $\mathcal{GK}$ fading/shadowing parameters $m$ and $\kappa$ are important and affect the system performance as shown in Figs. 4 and 5 respectively. We can see that, heavy shadowing (i.e., small $\kappa$) and/or severe fading (i.e., small $m$) are detrimental for the system performance. In Fig. 5 we fix $\alpha = 2.4$, $\beta = 2$, $\xi = 6.8$, and $r = 2$. Expect for $\kappa = 0.6$, we notice that all curves have the same slopes thereby inferring that they have the same diversity order. This is due to the

\[\text{Normalized Average SNR of the FSO link } \mu^2 \gamma \text{ in dB}\]
Fig. 3: Outage probability of an interference-limited fixed-gain mixed RF/FSO system in strong turbulence conditions for different values of $L$ and $\xi$ with $N = L = 2$, $m = 2.5$, and $\kappa = 1.09$.

Fig. 4: Average BER of an interference-limited fixed-gain mixed RF/FSO system in strong turbulence conditions for different values of $m$ with $N = L = 2$, and $\kappa = 1.09$.

The fact that the system diversity order is dependent on $G_d = \min\left(Nm_2, \kappa, \frac{\xi^2}{P}, \frac{\alpha}{P}, \frac{k}{P}\right)$. For the two curves when $\kappa = 0.6$, they have the same slope revealing equal diversity order $d = \kappa$. Figs. 4
and Fig. 5 also show that the asymptotic expansion in (18) agrees very well with the simulation results, hence corroborating its accuracy.

The impact of the number of relay antennas $N$ on the system BER is investigated in Fig. 5 under several shadowing conditions. As shown in (15), spatial diversity resulting from employing a higher number of antennas $N$ at the relay enhances the overall system performance.

Fig. 6 shows the impact of the FSO link atmospheric turbulence conditions on system capacity. We can see that decreasing $\alpha$ and $\beta$ (i.e., stronger turbulence conditions) deteriorates the system capacity, notably when IM/DD is employed. It is clear from this figure that weaker turbulence conditions leads to the situation where the RF link dominates the system performance thereby inhibiting any performance improvement coming from the FSO link.

Fig. 7 illustrates the effect of the atmospheric turbulence induced fading severity in terms of the power amount coupled to the LOS component in the FSO link, $\rho$, on the performance of CSI-assisted relay mixed FSO/RF systems. Expectedly, as $\rho$ increases, the system performance ameliorates due to the reduction of the atmospheric turbulence over the FSO link. We highlight once again the efficiency of the heterodyne detection against the IM/DD technique.

Fig. 8 investigates the effect of shadowing severity on the ergodic capacity of mixed FSO/RF
CSI-assisted relaying suffering $GK$ interference. A general observation is that the shadowing degrades the system’s overall performance. Furthermore, more interference (i.e., higher $L$) at
the RF user results a lower capacity. A similar behavior has been noticed in [14]. It may be also useful to mention that the ergodic capacity curves of mixed FSO/RF under infrequent light shadowing and mixed Málaga-\(M\)/Nakagami-\(m\) systems coincide thereby, unambiguously, corroborating the much wider scope claimed by our novel analysis framework and the rigor of its mathematical derivations.

VII. Conclusion

We have studied the performance of relay-assisted mixed FSO/RF system with RF interference and two different detection techniques. The H-transform theory is involved into a unified performance analysis framework featuring closed-form expressions for the outage probability, the BER, and the channel capacity assuming Málaga-\(M\)/composite fading/shadowing \(GK\) channel models for the FSO/RF links while taking into account pointing errors. The end-to-end performance of mixed Gamma-Gamma/interference-limited Nakagami-\(m\) systems can be obtained as a special case of our results. The latter show that the system diversity order is related to the minimum value of the atmospheric turbulence, small-scale fading, shadowing and pointing error parameters.
APPENDIX A
CDF OF THE END-TO-END SINR

The CDF of the end-to-end SINR $\gamma$ with fixed-gain relaying scheme can be derived, using \[18\, Eq.(8)\] as

$$F_\gamma(x) = \int_0^\infty F_{\gamma_1} \left( \frac{\mathcal{C}}{y} + 1 \right) f_{\gamma_2}(y) dy,$$  \hspace{1cm} (25)

where $F_{\gamma_1}$ and $f_{\gamma_2}$ are the FSO link’s CDF and the RF link’s PDF, respectively. $f_{\gamma_2}$ is derived by differentiation of (6) over $x$ as

$$f_{\gamma_2}(x) = \frac{-\kappa m}{\Gamma(Nm)\Gamma(\kappa)\Gamma(Lm_I)\Gamma(\kappa_I)\kappa_I m_I \gamma_2^2} \sum_{k=1}^{\beta} \frac{b_k}{\Gamma(k)} \frac{1}{4\pi i^2} \int_{C_1} \int_{C_2} \frac{\Gamma(\xi^2 + rs)\Gamma(k + rs)\Gamma(\alpha + rs)\Gamma(-rs)\Gamma(-1 - t)\Gamma(k - 1 - t)\Gamma(Nm - 1 - t)}{\Gamma(1 + t)\Gamma(1)} \left( \frac{\kappa m}{\kappa_I m_I \gamma_2^2} \right)^{t} \left( \frac{B^r x}{\mu_r} \right)^{-s} \int_0^\infty \left( 1 + \frac{\mathcal{C}}{y} \right)^{-s} y^t dy ds dt,$$  \hspace{1cm} (26)

Substituting (1) and (26) into (25) while resorting to the integral representation of the Fox-H \[16\, Eq.(1.2)\] and Meijer-G \[17\, Eq.(9.301)\] functions yields

$$F_\gamma(x) = \frac{-\xi^2 A r \kappa m}{\Gamma(\alpha)\Gamma(Nm)\Gamma(\kappa)\Gamma(Lm_I)\Gamma(\kappa_I)\kappa_I m_I \gamma_2^2} \sum_{k=1}^{\beta} \frac{b_k}{\Gamma(k)} \frac{1}{4\pi i^2} \int_{C_1} \int_{C_2} \frac{\Gamma(\xi^2 + rs)\Gamma(k + rs)\Gamma(\alpha + rs)\Gamma(-rs)\Gamma(-1 - t)\Gamma(k - 1 - t)\Gamma(Nm - 1 - t)}{\Gamma(1 + t)\Gamma(1)} \left( \frac{\kappa m}{\kappa_I m_I \gamma_2^2} \right)^{t} \left( \frac{B^r x}{\mu_r} \right)^{-s} \int_0^\infty \left( 1 + \frac{\mathcal{C}}{y} \right)^{-s} y^t dy ds dt,$$  \hspace{1cm} (27)

where $i^2 = -1$, and $C_1$ and $C_2$ denote the $s$ and $t$-planes, respectively. Finally, simplifying $\int_0^\infty \left( 1 + \frac{\mathcal{C}}{y} \right)^{-s} y^t dy$ to $\frac{\Gamma(1 + t + s)\Gamma(-1 - t)\Gamma(1 + t + s)}{\Gamma(s)}$ by means of \[17\, Eqs (8.380.3) and (8.384.1)\] while utilizing the relations $\Gamma(1 - rs) = -rs\Gamma(-rs)$, and $s\Gamma(s) = \Gamma(1 + s)$ then \[19\, Eq.(1.1)\] yield (9).

APPENDIX B
ERGODIC CAPACITY UNDER CSI-ASSISTED RELAYING SCHEME

From \[14\], the ergodic capacity can be computed as

$$C = \frac{1}{2\ln(2)} \int_0^\infty se^{-s} M_{\gamma_1}^{(c)}(s) M_{\gamma_2}^{(c)}(s) ds,$$  \hspace{1cm} (28)

where $M_{\gamma_1}^{(c)}(s) = \int_0^\infty e^{-sx} F_{\gamma_1}^{(c)}(x) dx$ stands for the complementary MGF (CMGF). The CMGF of the first hop’s SNR $\gamma_1$ under Málaga-\mathcal{M} distribution with pointing errors is given by \[4\, Eq.(9)\]

$$M_{\gamma_1}^{(c)}(s) = \frac{\xi^2 A r \mu_r}{\Gamma(\alpha)B^r} \sum_{k=1}^{\beta} \frac{b_k}{\Gamma(k)} \frac{\mu_r}{B^r} \left[ \left( \lambda_2, \Lambda_2 \right) \right].$$  \hspace{1cm} (29)
Moreover, the Laplace transform of the RF link’s CCDF yields its CMGF after resorting to [17, Eq.(7.813.1)] and [16, Eq.(1.111)] as

\[
M_{\gamma_2}(s) = \frac{s^{-1}}{\Gamma(N_m)\Gamma(\kappa)\Gamma(Lm_1)\Gamma(\kappa)} \left[ \frac{\kappa_l m_1 \gamma_2}{\kappa m} \right]^{\frac{3.3}{3.4}} \left( \frac{\chi_2, X_2}{v_2, \Upsilon_2} \right) .
\]

(30)

Finally, the ergodic capacity expression in (23) follows after plugging (29) and (30) into (28) and applying [19, Eq.(2.2)].

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