Scaling Law for Baryon Coupling to its Current and its possible applications

Jishnu Dey 1*, Mira Dey 2†, T. Frederico, 3 and Lauro Tomio 4‡

March 26, 2022

Abstract

The baryon- coupling to its current ($\lambda_B$), in conventional QCD sum rule calculations (QCDSR), is shown to scale as the cubic power of the baryon mass, $M_B$. Some theoretical justification for it comes from a simple light-cone model and also general scaling arguments for QCD. But more importantly, taken as a phenomenological ansatz for the present, this may find very good use in current explorations of possible applications of QCDSR to baryon physics both at temperature $T = 0$, $T \neq 0$ and/or density $\rho = 0$, $\rho \neq 0$.

(1) International Centre for Theoretical Physics, Trieste, Italy 34100 and Azad Physics Centre, Dept. of Physics, Maulana Azad College, Calcutta, India 700 013
(2) International Centre for Theoretical Physics, Trieste, Italy 34100 and Department of Physics, Presidency College, Calcutta, India 700 073
(3) Departamento de Física, ITA, Centro Técnico Aeroespacial, 12.228-900 São José dos Campos, São Paulo, Brasil
(4) Instituto de Física Teórica, Universidade Estadual Paulista, 01405-900 São Paulo, São Paulo, Brasil

PACS24.85.+p, 12.39.Ki, 14.20.-c, 13.40.Gp, 11.55.Hx

*E-mail: deyjm@giasc01.vsnl.net.in, work supported in part by DST grant no. SP/S2/K04/92 Govt. of India, also supported initially by FAPESP of Brasil, permanent address: 1/10 Prince Golam Md. Road, Calcutta India 700 026,
†Work supported in part by DST grant no. SP/S2/K04/92 Govt. of India, also supported initially by CAPES of Brasil, permanent address: 1/10 Prince Golam Md. Road, Calcutta India 700 026,
‡E-mail: tomio@axp.ift.unesp.br
The QCD sum rule calculation of Shifman et al. \cite{1} reproduces some of the predictions of QCD obtained in terms of fundamental degrees of freedom. For reviews on the subject see Refs. \cite{2}, \cite{3}, \cite{4}. One of the outputs of QCD sum rule calculations \cite{5} is the baryon coupling to its quark content. For example, denoting the nucleon as $N$, the current constructed out of quarks

$$J_N(x) = (\pi^a(x)C\gamma_\mu u^b(x))\gamma_5\gamma_\mu d^c(x)\varepsilon^{abc}$$

(1)

carries the quantum numbers of the nucleon, where $C$ is the charge conjugation operator, $a, b, c$ are colour indices, $\varepsilon$ is the antisymmetric tensor and $u(x)$ and $d(x)$ are the up and down quark operators. The matrix element

$$\langle 0|J_N|N\rangle = \lambda_N \upsilon^{(r)}$$

(2)

where $\lambda_N$ is the coupling we are concerned with and $\upsilon^{(r)}$ is the usual Dirac spinor for polarization $r$, normalized to the nucleon mass $M_N$ as :

$$\bar{\upsilon}^{(r)}\upsilon^{(r)} = 2 M_N.$$ 

(3)

Similarly one can define a coupling $\lambda_B$ for each baryon $B$ with a mass $M_B$. We find

$$\lambda_B = \text{constant } M_B^3$$

(4)

from the existing determination of the sum rules as shown in Fig. 1 which is discussed below.

Besides $N$, the couplings of $\Lambda$, $\Sigma$, $\Xi$ given by Ioffe and Smilga \cite{6} and $\Omega$ by Nielsen et al. \cite{7} to their currents are known. Using calculations of Reinders et al. \cite{8} (also reported in \cite{3} and \cite{2}) one can extract the couplings $\Sigma^*$ and $\Xi^*$. We take $\Delta$ from Belyaev and Ioffe \cite{9}. Fig. 1 shows that they present a scaling law with the baryon mass, that cover a wide range of the spectrum and are practically independent of the spin structure of the baryon in so far as we find it to be valid for $S = 1/2$ octet as well as for $S = 3/2$ decuplet members.

From \cite{6}, $\tilde{\lambda}_B^2 \equiv 2(2\pi)^4\lambda_B^2$, are given respectively for $N, \Lambda, \Sigma$ and $\Xi$ as 2.1, 3.3, and 3.7 $GeV^6$. For $\Omega$ the same quantity is 28.9 $GeV^6$ \cite{7}. For $\Delta, \Sigma^*$ and $\Xi^*$ the respective values are 4.6, 9.6 and 16.5 $GeV^6$. We find that the QCDSR results can be fitted very well to a simple formula

$$\tilde{\lambda}_B^2 = (1.16)^2 M_B^6.$$ 

(5)

To determine $\tilde{\lambda}_B^2$ is not easy in QCDSR, as the reader can find in the literature \cite{6}, \cite{3} and \cite{2}. The final value given by \cite{6} gives a slight overestimate of both the proton and the neutron magnetic moments. The nucleon results that we report are for the choice of a current due to Ioffe \cite{5} (reprinted and supported in \cite{2}).
Figure 1: The points are extracted from QCDSR calculations and the line is our model.

It is clear for us that the scaling law we have found follows from general qualitative considerations which can be at various levels of sophistication. Thus for example we can argue that:

(1) in a simple quantum mechanical sense the quark is normalized in a volume proportional to the Compton wavelength $\sim 1/M$, where $M$ is the mass of the constituent quark which on the average is proportional to the baryon mass, $M_B$. The probability of finding 3 such particles is the cube of this factor. This leads to the eq. (4).

(2) in a more sophisticated sense the light quark sector of QCD is conformally invariant with the invariance broken only by the scale of the constituent quark mass ($\approx 1/3M_B$) so that this mass controls the scaling of all relevant quantities.

The coupling of baryons to their currents are crucial for all experimental properties obtained with QCDSR calculations. They are involved in deriving observables like magnetic moments and electromagnetic and weak transitions, etc. The consequences of the scaling can be verified by a systematic calculation of hadronic properties.

The knowledge of some systematics of how the coupling $\lambda_B$ changes from one baryon to another could have further use. To understand this we must recall the basic structure of a fermion sum rule, namely that it always comes in a pair. This is due to the Dirac structure of the propagator which has a scalar part and another proportional to the Dirac $\gamma$ matrix. They are called the chiral odd and the chiral even sumrules [10]. The nucleon
mass was originally extracted by Ioffe [5] with a combined use of both the sum rules. It is necessary to use these two sum rules together since there are two unknowns, $\lambda_B$ and $M_B$.

The QCDSR depend on the parameter $M$, called the Borel parameter which is very useful for the phenomenological side of the sum rule where it cuts off high energy contributions with a factor $\exp(-s/M^2)$, where $s$ is the squared momentum transfer. On the QCD side it also leads to a function of $M^2$. The sum rule should be plotted as a function of $M$ and a ‘window’ is to be found where there is stability with variation of this parameter. It was soon clear, (see for example [10]) that one of the sumrules work well while the other fails, in case of baryons. This pattern is also seen in the sum rules for the matrix elements of electromagnetic current and axial vector current (for references see [10]). The most important contribution in any given channel is the ground state baryon. However the excited states also contribute in higher order. The different behaviour of various sumrules can be traced to the fact that even and odd parity states contribute with different signs. If chiral symmetry is realized in the Wigner-Weyl mode at high energies, i.e. by parity doubling, it is possible to have either cancellation or reinforcement between excited state configurations. Irrespective of the exact manner in which the cancellations take place it is clear that the odd sumrule favours the extraction of the baryon ground state mass. The point has been reemphasized in a recent paper by Jin and Tang [11].

Belyaev and Ioffe [9], [12] extended the nucleon sumrule calculation to determine the mass splitting between hyperons (Y) and nucleons treating the strange quark mass as a perturbation. In addition to the mass shift $M_Y - M_N$, one must also take into account the change in the coupling constant $\lambda^2_Y - \lambda^2_N$ and the change in the continuum threshold, $s_0$. It is found [13] that $s_0$ goes according to the ‘mnemonic rule’:

$$s_0 = (M_B + 0.5 \text{ GeV})^2. \quad (6)$$

Therefore with our scaling law it is possible to extract masses from one set of sum rules only. The transitions and magnetic moments can also be better determined using the odd chiral sum rules, given the value of the coupling from the scaling law.

The above results were found while looking at a simple relativistic Faddeev model for the nucleon, which therefore serves as some sort of a justification for it also. The model uses a null-plane ($x^+ = ct + z = 0$) wave function for the baryon and treats some boosts in a consistent way. The confining models like the MIT bag or the soliton bag models have the complication of requiring elaborate center of mass projection to restore covariance, a feature usually shared by the one-body confining potentials used in the literature. To obtain baryon form-factors at high momentum transfers, one should use a consistent boosted wave function. Relativistic constituent quark models, with null-plane wave functions [14], offer such an opportunity. They are covariant under kinematical front-form boosts [15]. This point is in fact a deeper consequence of the stability of the Fock-state decomposition of the wave function under such transformations [16].
The three quark null-plane bound state is obtained through a renormalized effective zero-range attractive force [17]. Being bound to the scale of the hadron size, the constituent quarks have well defined masses. As a consequence of the contact interaction, only the momentum scales are important and it allows to identify the minimum dynamical inputs in the constituent quark model. The advantage is that the strength of the contact interaction is scaled out of the calculation and thus no extra scale parameters like the bag constant have to be used.

The constituent quark in the null-plane, \(|Q\rangle\), can be described as a sum over current quark and current antiquark (\(|q\rangle, \overline{|q\rangle}\)) Fock-states:

\[ |Q\rangle = A_q|q\rangle + A_{qq\overline{q}}|qq\overline{q}\rangle + ..... \]  

(7)

and the integration of \(|A_q|^2\) over the current quark phase-space gives the probability, \(\eta\), of a constituent quark to be the current quark. This \(\eta\) is probed in the pion weak decay constant and pion deep-inelastic structure function [18].

Considering that the broad scaling law is not dependent on the details of spin or isospin structure of the baryon, and to focus on the gross scaling, we assumed that the spin and isospin have no dynamical effect. The total antisymmetrization can be achieved by antisymmetrizing the color-isospin-spin degree of freedom, leading to a totally symmetric spatial wave function. The momentum canonically associated with the position coordinates of the \(i\)-th quark, in the null-plane, are the kinematical momenta, \(k^+_i\) and \(\vec{k}_i\). The corresponding momentum fraction is defined as \(x_i = k^+_i/P^+_B\), where \(P^+_B\) is the \((+)\) component of the total momentum of the baryon. The momentum constraints, in the center of mass of the three quarks, are given by

\[ x_1 + x_2 + x_3 = 1 \quad \text{and} \quad \vec{k}_{1\perp} + \vec{k}_{2\perp} + \vec{k}_{3\perp} = 0 . \]  

(8)

With the above variables we obtain the three quark bound-state null-plane wave function in terms of Faddeev components of the vertex, \(v(x, \vec{k}_i)\) [17]:

\[ \Psi(x_1, \vec{k}_{1\perp}; x_2, \vec{k}_{2\perp}) = \frac{v(x_1, \vec{k}_{1\perp}) + v(x_2, \vec{k}_{2\perp}) + v(x_3, \vec{k}_{3\perp})}{\sqrt{x_1 x_2 x_3} \left[ M_B^2 - \sum_{j=1,3}(k^2_{j\perp} + M^2)/x_j \right]} , \]  

(9)

where \(M_B\) is the baryon mass and \(M\) is the constituent quark mass. The normalization is chosen such that

\[ \int dx_1 dx_2 d^2k_{1\perp} d^2k_{2\perp} \left[ \Psi(x_1, \vec{k}_{1\perp}; x_2, \vec{k}_{2\perp}) \right]^2 = 1 . \]  

(10)

The vertex component, \(v\), satisfies a Weinberg-type [19] integral equation [17, 20], in which the subsystem scattering is summed up to all orders in the ladder approximation,
following the original Faddeev construction of the three-body connected kernel equations [21]. In the baryon center of mass system, the vertex $v(y, \vec{p}_\perp)$ is given by

$$v(y, \vec{p}_\perp) = \frac{i}{(2\pi)^3} \tau(M_2) \int_{\frac{M^2}{M_B^2}}^{1-y} dx \frac{dx}{x(1-y-x)} \int_{k_{\perp}^{\max}}^{} d^2k_\perp \frac{v(x, \vec{k}_\perp)}{M_B^2 - M_3^2},$$  \hspace{1cm} (11)$$

where the kinematical momentum $x$, $\vec{k}_\perp$ and $y$, $\vec{p}_\perp$ describe the spectator quark states in the initial and final vertex, respectively. $M_2$ is the two-quark subsystem mass,

$$M_2^2 = (1-y) \left( M_B^2 - \frac{p_\perp^2 + M^2}{y} \right) - p_\perp^2 .$$  \hspace{1cm} (12)$$

$M_3$ is the mass of the propagating virtual three-quark intermediate state, given by

$$M_3^2 = \frac{k_\perp^2 + M^2}{x} + \frac{p_\perp^2 + M^2}{y} + \frac{(\vec{p} + \vec{k})_\perp^2 + M^2}{1-y-x} .$$  \hspace{1cm} (13)$$

The two-quark amplitude $\tau(M_2)$ for the contact interaction needs to be renormalized (see Ref.[22]). In this process, the strength of the interaction is fixed by the two-body (diquark) bound-state mass, which removes the infinities in the physical two-body scattering amplitude [20]. The diquark bound-state mass $\mu$ is also equal to the $\bar{Q} Q$ - meson mass, because the pole of the two-quark scattering amplitude corresponds to a pole in the exchange channel.

The expression for the two-quark amplitude, is given by[20]:

$$\tau(M_2) = -i(2\pi)^2 \left[ \sqrt{\frac{M_2^2}{\mu^2} - \frac{1}{4}} \ \text{arctan} \left( \frac{\mu}{\sqrt{4M_2^2 - \mu^2}} \right) - \sqrt{\frac{M_2^2}{\mu^2} - \frac{1}{4}} \ \text{arctan} \left( \frac{M_2}{\sqrt{4M_2^2 - \mu^2}} \right) \right]^{-1}$$

where $M_2 < 2M$.

Considering that $M_2^2 > 0$ in Eq.(12), the spectator transverse momentum $k_\perp$ of Eq.(11), attains a maximum value at $k_\perp^{\max} = \sqrt{(1-x)(M_B^2 x - M^2)}$.

The integral equation, for the vertex in the new frame Eq.(11), can be rewritten by changing the corresponding variables. This is possible because of the nature of the kinematical transformations. The mass of the three-quark system, calculated from Eq.(11), remains the same in any frame connected to the center of mass frame by a kinematical transformation.

We define the relativistic coupling of the baryon wave function as the integration in the phase-space, $x$ and $\vec{k}_\perp$, of the Faddeev vertex component:
\[ \lambda_{LC} = M \int_{M^2}^{M_B^2} \frac{dx}{x} \int^{k_{\max}^2} d^2k_{\perp} v(x, k_{\perp}). \]  

(14)

To compare with \( \tilde{\lambda}_B^2 \) (Eq.(3)) we revert back to current quark picture. Since each \( |Q\rangle \sim \sqrt{\eta}|q\rangle \) the corresponding coupling in terms of current quarks is given by \( \eta^{3/2}\lambda_{LC} \). Squaring and multiplying with the factor \( 2(2\pi)^4 \) we get the corresponding quantity :

\[ \tilde{\lambda}_{LC}^2 = 2(2\pi)^4 \eta^3 \lambda_{LC}^2. \]  

(15)

There are only two parameters in the model, namely the constituent quark and the diquark masses, \( M \) and \( \mu \) respectively. They can be further reduced to one parameter, namely the ratio, \( \mu/M \) by calculating the radius. The electromagnetic nucleon radius \( r_p \) is obtained from the proton electric form factor \( G_E(q^2) \), as detailed in \[17\]. \( \mu/M = 1.9 \) gives a good fit to \( G_E(q^2) \) and corresponding ratio of the baryon mass to quark mass is 2.58.

We remind the readers that the experimental value of the product of the proton charge-radius \( (r_p) \) and the nucleon mass \( (M_N) \) is difficult to reproduce, for example, in non-topological soliton models \[23\]. It is nice to note that this model gives the product \( r_p M_N (\sim 3.8) \) which is near the experimental value.

We notice in Fig.2 that the minimum of \( r_p M_N \) as a function of \( \tilde{\lambda}_{LC}^2 \), gives approximately the experimental value of \( r_p M_N (\sim 3.8) \). Corresponding value of \( \tilde{\lambda}_{LC}^2 \) depends on what value we take for \( \eta \). For example, it is \( \sim 1.46 \) for \( \eta = 1/2 \), as shown in Fig. 2, but for \( \eta \sim 0.43 \) it is nearly unity.

It is satisfying to note that \( \eta \sim \frac{1}{2} \), is in agreement with various numbers obtained in \[18\] for the pion, under different choices of constituent quark wave packet (two mass choices for Gaussian type 0.741 and 0.476, or same choices for hydrogen-like 0.463 and 0.408).

If we assume that the SU(3) flavour breaking does not change the ratio \( \mu/M \) and the \( \eta \) we obtain \( \tilde{\lambda}_{LC}^2 \) as a function of the ground state baryon mass \( M_B \). By fixing \( \eta = 0.43 \) we get eq.(5).

A glance at Fig. 1 shows that the decuplet baryons obey the scaling law more consistently than the octet. This may be due to the fact that for the decuplet baryons the instanton contribution is small \[24\] and therefore the sum rule determination is free from the problems suffered by the octet baryons.

Coming back to QCDSR, there is a recent calculation trying to incorporate instanton effects into the nucleon sum rule, see Ref.\[25\]. According to \[2, 25\], the instanton effect is minimized in the Ioffe current.
In summary, we find evidence for a simple scaling law for the coupling of the decuplet as well as the octet baryons to their current which scales with the baryon masses with appropriate power fixed by its dimension. Some general arguments and a simple relativistic Faddeev justifies it. Since \( \tilde{\lambda}^2 \) enters in calculations of magnetic moments and transitions this scaling will in future be used to obtain approximate relations between properties of the different members of the 56-multiplet of baryons.

We acknowledge comments from Dr. A. Dorokhov. J. D. and M. D. wishes to acknowledge the stimulating atmosphere of the International Centre for Theoretical Physics, Trieste, Italy where this work was completed. The work of T. F. and L. T. was partially supported by the Conselho Nacional de Desenvolvimento Científico e Tecnológico - CNPq and Fundação de Amparo à Pesquisa do Estado de São Paulo- FAPESP, Brasil.
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**Figure captions:**

Fig.1: The points are extracted from QCDSR calculations and the line is our model.

Fig.2: $r_pM_N$ as a function of $\tilde{\lambda}^2_{LC}$. The constituent quark masses ($M$) and meson masses ($\mu$) have been varied in the calculations, while the baryon mass was kept fixed at 0.938 GeV. The calculated points are shown with dots.
