GLUON PAIR PRODUCTION 
IN THE QUASI-MULTI-REGGE KINEMATICS *

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Abstract

To find the region of applicability of the leading log(1/x) approximation for parton distributions in the small x region and to fix the argument of the QCD running coupling it is necessary to know radiative corrections to the kernel of the BFKL equation. The next-to-leading corrections to the BFKL kernel are expressed in terms of the two-loop correction to the gluon Regge trajectory, one-loop correction to the Reggeon-Reggeon-gluon vertex, and contributions from two-gluon and quark-antiquark production in the quasi-multi-Regge kinematics. We calculate differential and total cross sections of the two gluon production. Differential cross section can be applied for description of two jet production in the quasi-multi-Regge kinematics; the total cross section defines corresponding correction to the BFKL kernel. To escape the infrared divergencies we use dimensional regularization of the Feynman integrals.

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1. Introduction

The problem of calculation of the parton distributions in the small $x$ region can be turned into calculation of the kernel of the Bethe-Salpeter type equation for the $t$-channel partial amplitude with the vacuum quantum numbers [1]. This equation is known now as BFKL-type equation. In the leading logarithmic approximation (LLA), which means summation of all terms of the type $[\alpha_s \ln(1/x)]^n$, this kernel was found many years ago [1]. Now the results of LLA are widely known and used for description of experimental data.

The LLA gives a power growth of cross sections with c.m.s. energy $\sqrt{s}$. In terms of parton distributions this means a fast increase of the gluon density $g(x, Q^2)$ in the small $x$ region:

$$g(x, Q^2) \sim x^{-j_0},$$

where $j_0 = 1 + \omega_0$ is the LLA position of the singularity of the partial amplitude with the vacuum quantum numbers in the $t$-channel [1]:

$$\omega_0 = \frac{4 \alpha_s}{\pi} N \ln 2,$$

with $N = 3$ for QCD. Such behaviour contradicts the unitarity and therefore the LLA cannot be applied at asymptotically small $x$. Unitarity constraints for scattering amplitudes with vacuum quantum numbers in $t$-channel don’t work in this approximation and, as a result, the Froissart bound $\sigma_{tot} < const(\ln s)^2$ is violated. Nevertheless, in the region of parameters accessible for modern experiments the observed behaviour of the structure functions is consistent with LLA results [2], and we will not discuss here the unitarization problem, which appear at asymptotically large energies.

From practical point of view it seems more important to determine the region of applicability of LLA. Besides that, the dependence of the QCD running coupling $\alpha_s$ on virtuality is beyond of the accuracy of the LLA. It diminishes the predictive power of the LLA, because numerical results of this approximation can be strongly modified by changing a scale of virtuality. All these uncertainties of LLA predictions can be removed and the region of applicability of these predictions can be found using radiative corrections.

As it is clear from above discussion the problem of calculation of the next-to-leading corrections to the BFKL kernel is very important now. These corrections are expressed [3] in terms of the two-loop correction to the gluon Regge trajectory, one-loop correction to the Reggeon-Reggeon-gluon (RRG) vertex, and contributions from two-gluon and quark-antiquark production in the quasi-multi-Regge kinematics (QMRK), which, in turn, are expressed in terms of the Reggeon-Reggeon-two-gluon (RRGG) and Reggeon-Reggeon-quark-antiquark (RRq\bar{q}) vertices. Corrections to the RRG vertex and to the Reggeized gluon trajectory were calculated (see Refs. [4] and [5] correspondingly), after that calculation of the contributions from two-gluon and quark-antiquark production in the QMRK became the most urgent problem.

Investigation of these contributions was started in [3], where the two-gluon production amplitude in the QMRK, and, correspondingly, the RRGG vertex, was found. The next important step was done in [3], where the two-gluon and quark-antiquark production amplitudes in the QMRK were simplified using the helicity representation, the corresponding
next to leading contributions to the BFKL kernel were expressed in terms of the integrals from the squares of these helicity amplitudes over transverse and longitudinal momenta of produced particles, all infrared divergencies were extracted from these expressions in an explicit form and it was demonstrated for the fermion contribution that these divergencies cancel with the analogous divergencies from the virtual corrections to the BFKL equation. Recently resummation formulas for the quark-antiquark production contribution to the BFKL kernel was derived in [8].

In this paper we calculate differential and total cross sections of the two gluon production in QMRK. Differential cross section can be applied for description of two jet production in the QMRK; the total cross section defines corresponding contribution in the next-to-leading BFKL kernel. To escape the infrared divergencies we use dimensional regularization of the Feynman integrals. All divergencies cancel in the total expression for the next-to-leading BFKL kernel. This cancellation was demonstrated in explicit form in Refs. [6], [7].

The paper is organized as follows. In the next Section we discuss kinematics and method of calculations. In Section 3 and 4 we calculate the differential and total cross sections respectively. The final result for the total cross section, needed to get the corresponding correction to the BFKL kernel, is presented and briefly discussed in the Section 4.

2. The amplitudes in QMRK

Let us consider the two-gluon production in the QMRK in gluon-gluon scattering at high energies in the Born approximation (see Fig. 1). The corresponding contribution to the imaginary part of the elastic gluon-gluon scattering amplitude $A_{AB}^{AB}$ at zero momentum transfer is proportional to the cross section $\sigma_1$ of the process Fig. 1.

We will use the Sudakov decomposition for the momenta of the final gluons:

$$k_i = \beta_i p_A + \alpha_i p_B + k_{i\perp}, \quad s\alpha_i\beta_i = -k_{i\perp}^2,$$

where $s = (p_A + p_B)^2$ is the total c.m.s. energy squared, which is supposed to be tending
to infinity, and we took into account the on-mass-shellness of these particles. Then the QMRK means that all \( k_{i\perp} \) are fixed and

\[
\beta_{\lambda'} \gg \beta_1 \sim \beta_2 \gg \beta_B.
\]

The squared invariant mass of the gluons 1 and 2, \( \kappa = (k_1 + k_2)^2 \), in the QMRK has the same order of magnitude as \( |k_{1\perp}^2| \). The amplitude for this process was calculated in Ref. [3] and can be presented in the following factorized form:

\[
A = 2s g^2 \delta_{\lambda'\lambda} T^c_{\lambda'\lambda} T^d_{B'B} \frac{1}{q_1^2} \gamma_{i,j}^{q_1,q_2} \delta_{\lambda'\lambda'} \beta_B T_{B'B} \frac{1}{q_2^2},
\]

where \( T^k_{ij} \) are the generators of the SU(\( N \)) colour group in the adjoint representation, \( \lambda_i \) are the helicities of the corresponding particles and \( g \) is the gauge coupling constant (\( \alpha_s = g^2/4\pi \)). The expression for the effective RRGG vertex \( \gamma_{i,j}^{q_1,q_2} \) can be found in Ref. [3] and we don’t write it here. Let us note that, although QMRK means that all \( \beta_1 \sim \beta_2 \), the amplitude (3) is correct also in the multi-Regge kinematics (MRK), when \( \beta_1 \gg \beta_2 \) (or \( \beta_2 \gg \beta_1 \)).

The cross section \( d\sigma \) for the process, pictured on Fig. 1, can be presented in the form (we average over colours and spins of the initial particles and sum over the same quantum numbers of the final particles):

\[
d\sigma = \frac{N^2 \alpha_s^2}{(N^2 - 1)(2\pi)^{2D-5}} \frac{d^{D-2} q_{1\perp} d^{D-2} k_{1\perp} d\beta}{\beta} d\sigma,
\]

\[
d\sigma = \frac{1}{2} (2\pi)^{D-1} \delta^{(D-2)} ((k_1 + k_2 - \Delta)_{\perp}) \frac{dx}{x(1-x)} \frac{x^{D-2} k_{1\perp} d^{D-2} k_{2\perp}}{(2\pi)^{D-1} (2\pi)^{D-1}} \sum_{\lambda_1,\lambda_2} \gamma_{i,j}^{q_1,q_2} \gamma_{i',j'}^{q_1,q_2},
\]

where

\[
\beta = \beta_1 + \beta_2, \quad x = \frac{\beta_1}{\beta} = 1 - \frac{\beta_2}{\beta},
\]

and

\[
P_0^{c'c dd'} = \frac{\delta_{c'c} \delta_{dd'}}{(N^2 - 1)}
\]

is the projector on the colour singlet state in the \( t \)-channel of the elastic scattering amplitude. \( D \) here is the space-time dimension, different from 4 to regularize the cross section, having infrared and collinear divergencies.

\( d\sigma \) can be named as two-gluon production cross section in QMRK and corresponding correction to the BFKL kernel is equal to (see [3]):

\[
K_{\text{gluons}}^{(2)} = \frac{\beta^{4-D}}{4(2\pi)^{D-1}} \frac{\sigma_{\text{tot}}(q_1,q_2)}{q_1^2 q_2^2}.
\]

As it was shown in Ref. [3] the cross section \( d\sigma \) can be expressed through the variable \( x \) and transverse momenta of the produced gluons in the following way:

\[
d\sigma = 8g^4 \frac{dx}{x(1-x)} \frac{d^{D-2} k_{1\perp}}{(2\pi)^{D-1}} P_0^{c'c dd'} \left\{ H_1^{c'c dd'} a^{\mu\nu} a_{\mu\nu} + H_2^{c'c dd'} a^{\mu\nu} a_{\nu\mu} (x \leftrightarrow 1-x, k_{1\perp} \leftrightarrow k_{2\perp}) \right\}
\]

where
where
\[ H_1^{ccdd} = T_{c_i} T_{a_e} T_{c_j} T_{a_d}, \quad P_0^{ccdd} H_1^{ccdd} = N^2, \]
\[ H_2^{ccdd} = T_{c_i} T_{a_e} T_{c_j} T_{a_d}, \quad P_0^{ccdd} H_2^{ccdd} = N^2/2 \]
are the different colour factors,
\[ \tilde{a}^{\mu\nu} = \left( g_{\perp\perp}^{\mu\nu} - \frac{k_{1\perp} k_{1\perp}^{\rho}}{k_{1\perp}^2} \right) a^{\rho\nu}, \]
and
\[ a^{\mu\nu} = \frac{(q_1 - k_1)^{\mu/2} (q_1 - k_1)^{\nu/2}}{t} - \frac{(q_1 - k_1)^{\mu/2}}{\kappa} \left( k_1 - \frac{x}{1-x} k_2 \right)_{\perp}^{\nu} + \left( k_2 - \frac{x k_{2\perp}^2}{(1-x) k_{1\perp}^2} k_1 \right)_{\perp}^{\mu} \]
\[ \times \frac{(q_1 - k_1)^{\nu/2}}{\kappa} \left( k_{1\perp}^2 k_{2\perp}^{\nu/2} x t_2 - k_{2\perp}^2 k_{1\perp}^{\nu/2} x t_1 \right) - \frac{k_{1\perp}^2 k_{2\perp}^{\nu/2}}{\kappa} \left( 1 - \frac{xt}{(1-x) k_{1\perp}^2} \right) + \frac{k_{2\perp}^2 k_{1\perp}^{\nu/2}}{\kappa} \right) - \frac{1}{2} \tilde{g}_{\perp\perp}^{\mu\nu} \left( 1 + \frac{t}{\kappa} - \frac{(1-x) k_{1\perp}^2}{xt} + \frac{x k_{2\perp}^2}{(1-x) k_{1\perp}^2} - \frac{(1-x) k_{1\perp}^2}{xk} - \frac{x t_1 k_{2\perp}^2}{kz} - \frac{x t_2}{k} \right). \]
In the above expressions we used the notations (\( \kappa \) is the invariant mass of the produced pair as earlier)
\[ \kappa = -\frac{((1-x) k_1 - x k_2)^2}{x(1-x)}, \quad z = (1-x) k_{1\perp}^2 + x k_{2\perp}^2, \]
\[ t = \frac{1}{x} \left( (k_1 - x q_1)^2 + x(1-x) q_1^2 \right), \quad k_2 = (\Delta - k_1), \]
\[ t_1 = q_1, \quad t_2 = q_2, \]
and \( g_{\perp\perp}^{\mu\nu} \) is the metrical tensor in the transverse subspace
\[ g_{\perp\perp}^{\mu\nu} = g^{\mu\nu} - \frac{P_{APB}^{\mu} P_{APB}^{\nu} + P_{BAP}^{\mu} P_{BAP}^{\nu}}{(P_{APB})^2}. \]

In the next sections we calculate the differential cross section \( d\sigma/dx \) and the total cross section. Since all vectors, entering to (11), are transverse we will omit below the sign of the transversality.

3. The differential cross section.

After rather long calculation we get for the convolutions of the tensors, entering to (8)
\[ a^{\mu\nu} a_{\mu\nu} = \frac{1}{\kappa t} \left[ 2 x q_1^2 q_2^2 + \left( \frac{D-2}{4} \right) x(1-x) \left( 2(1-x) q_1^2 (\Delta^2 - q_2^2) - x \left( q_1^2 + (\Delta^2 - q_2^2) \right) \right) - (D-2)(1-x) q_1^2 q_1^2 (q_1 k_1) \right] + \frac{x(1-x)^2 q_2^2}{2kz} \left[ (D-2)(1-x) q_1^2 - (D-4)x(\Delta^2 - q_2^2) \right] + \]

\[ + \left\{ x \leftrightarrow 1 - x, k_{1\perp} \leftrightarrow k_{2\perp} \right\} \]
\[ + \left( \frac{x(1-x)q_1^2}{z} \right)^2 \left[ (D - 2) - (D - 1)x(1-x) \right] + \left( \frac{D - 2}{4} \right) (1 - 4x) \left( \frac{(1-x)q_1^2}{t} \right)^2 - \]
\[ - \frac{xq_1^2 q_2^2}{2(1-x)k_1^2 t} + \frac{xq_1^2 q_2^2}{2k_1^2t^2} - \frac{x(q_1^2)^2 q_2^2}{2ktk_1^2} - \frac{2xq_1^2}{\kappa t} \left[ \frac{q_1^2}{2} - (1-x)q_1^2 q_2^2 + \left( \frac{D - 2}{4} \right) (1-x)^2 (q_1^2)^2 \right] + \]
\[ + (1-x) \left( \frac{2q_2^2 - (D - 2)(1-x)q_1^2}{(k_1 q_1)} \right) \left( (1-x)q_1^2 \right)^2 + \]
\[ + (D - 2) \left( \frac{q_1^2 - x(\Delta^2 - 2k_1 q_2)}{k t} \right) \left( \frac{(1-x)(k_1 q_1)^2}{t} \right) + \left( \frac{(k_1 q_1)(k_2 q_2)}{\kappa} \right) \right] + ... , \quad (14) \]

and

\[ 2\tilde{\alpha}^{\mu\nu}a_{\nu\mu}(x \leftrightarrow 1-x, k_1 \leftrightarrow k_2) = \{ -\tilde{\alpha}^{\mu\nu}a_{\mu\nu} + \frac{q_1^2 q_2^2}{2k_1^2k_2^2} + \frac{1}{2tt} \left[ -2q_1^2 q_2^2 + \left( \frac{D - 2}{4} \right) x(1-x) \right] \times \left( (\Delta^2 - q_2^2)^2 + (q_1^2)^2 \right) + \left( \frac{D - 2}{4} \right) (1 - 4x) \left( \frac{(1-x)q_1^2}{t} \right)^2 + \left( \frac{D - 2}{4} \right) (1-x)(k_1 q_1)^2 \]} \]
\[ \times \left[ \frac{(1-x)(2k_1 q_1 - q_2^2)q_1^2 q_2^2}{2k_1^2k_2^2} - \left( \frac{D - 2}{4} \right) (1-x)(k_1 q_1)^2 \right] \left( q_2^2 + x(\Delta^2 + 2k_1 q_2) \right) \times \left( \frac{q_1^2 - x(\Delta^2 - 2k_1 q_2)}{k t} \right) \left( \frac{(1-x)(k_1 q_1)^2}{t}\right) + \left( \frac{q_1^2 - x(\Delta^2 - 2k_1 q_2)}{k t} \right) \left( \frac{(1-x)(k_1 q_1)^2}{t}\right) \left( q_2^2 + x(\Delta^2 + 2k_1 q_2) \right) \]
\[ + \frac{1}{2tt} \left[ -2q_1^2 q_2^2 + \left( \frac{D - 2}{4} \right) x(1-x) \left( (\Delta^2 - q_2^2)^2 + (q_1^2)^2 \right) \right] + \left( \frac{q_1^2 - x(\Delta^2 - 2k_1 q_2)}{k t} \right) \times \left( \frac{(1-x)(k_1 q_1)^2}{t}\right) + \left( \frac{q_1^2 - x(\Delta^2 - 2k_1 q_2)}{k t} \right) \left( \frac{(1-x)(k_1 q_1)^2}{t}\right) \left( q_2^2 + x(\Delta^2 + 2k_1 q_2) \right) \]
\[ + \frac{1}{2tt} \left[ -2q_1^2 q_2^2 + \left( \frac{D - 2}{4} \right) x(1-x) \left( (\Delta^2 - q_2^2)^2 + (q_1^2)^2 \right) \right] + \frac{x(1-x)q_1^2}{2\kappa z} \left[ (D - 2)(1-x)q_1^2 - (D - 4)x(\Delta^2 - q_2^2) + \left( \frac{x(1-x)q_1^2}{z} \right)^2 \right] \left( \frac{D - 2}{4} \right) - (D - 1) \]

where \( \tilde{t} = t(x \leftrightarrow 1-x, k_1 \leftrightarrow k_2) = \frac{1}{1-x} \left[ (k_2 - (1-x)q_1)^2 + x(1-x)q_1^2 \right] = \frac{1}{1-x} \left[ (k_1 + (q_2 - xq_1))^2 + x(1-x)q_1^2 \right] = q_1^2 + q_2^2 - \kappa - t = q_1^2 + q_2^2 - \Delta^2 + \frac{z}{x(1-x)} - t. \quad (16) \]

Using the relations (8)-(12) and (14)-(16) one can get

\[ \mu^{4-D} \frac{x(1-x)}{4q^4 N^2} \frac{d\sigma}{dx} = I + I(x \leftrightarrow 1-x), \quad (17) \]
\[ I = \int \mu^{4-D} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \left[ \left( \frac{D-2}{\kappa t} \right) \left( 2xq_1^2 q_2^2 + \left( \frac{D - 2}{4} \right) x(1-x) \left( (\Delta^2 - q_2^2)^2 \right) - (D - 2)(1-x)(q_1^2)^2 \right) \right] \]
\[ - \frac{2xq_1^2}{\kappa t} \left[ \left( \frac{q_1^2}{2} \right)^2 - (1-x)q_1^2 q_2^2 + \left( \frac{D - 2}{4} \right) (1-x)^2 (q_1^2)^2 + (1-x) \left( (2q_2^2 - (D - 2)(1-x)q_1^2) \right) \times (k_1 q_1) + (D - 2)(1-x)(k_1 q_1)^2 \right] + \frac{1}{2tt} \left[ -2q_1^2 q_2^2 + \left( \frac{D - 2}{4} \right) x(1-x) \left( (\Delta^2 - q_2^2)^2 + (q_1^2)^2 \right) \right] + \frac{x(1-x)q_1^2}{2\kappa z} \left[ (D - 2)(1-x)q_1^2 - (D - 4)x(\Delta^2 - q_2^2) + \left( \frac{x(1-x)q_1^2}{z} \right)^2 \right] \left( \frac{D - 2}{4} \right) - (D - 1) \]
\[ x(1 - x) + \left( \frac{D - 2}{2} \right) (1 - 4x) \left( \frac{(1 - x)q_1^2}{t} \right)^2 + \frac{(1 - x)(2k_1q_1 - q_2^2)}{2tk_1^2k_2^2} \]
\[ + \frac{(q_1^2)(q_2^2)}{4ttk_1^2k_2^2} - \frac{xq_2^2q_1^2}{(1 - x)k_1k_2^2} + \frac{x^2q_2^2q_1^2}{2k_1^2z} + \frac{q_1^2q_2^2}{2k_1^2k_2^2} \].
\[ \text{(18)} \]

Let us denote the contribution to $I$ of the term number $j$ in the integrand of the above expression as $I_j$. All $I_j$ demonstrate convergence at large $k_1$ and have only logarithmic infrared divergencies. Some of these integrals can be calculated exactly, and others - only in the form of the expansion in $\epsilon = (D - 4)/2$. Because the BFKL equation contains integration over $\Delta$, we should calculate the cross section $d\sigma/dx$ with an accuracy up to terms, giving, after integration over $x$ and $\Delta$, nonvanishing in the physical limit $\epsilon \to 0$ contributions. The results of calculation of the integrals $I_j$ can be found in the Appendix I. Using this table for $I_j$ and Eqs. \[ \text{(17), (18)} \] one can get the differential cross section $d\sigma/dx$.

4. The total cross section.

To calculate the total cross section $\sigma_{\text{tot}}$ one can integrate $d\sigma/dx$ over $x$, using Eqs. \[ \text{(17), (18)} \] and the expressions from Appendix I for the integrals $I_j$. But it is very hard way. The matter is that after integration over $k_1$ the additional integrations over Feynman variables appear. In the final expressions of Appendix I these additional integrations have been performed at fixed $x$. But at calculation the total cross section we can change the orders of integrations over $x$ and over Feynman variables in the most convenient way so that integrations become simpler. The such changes of the orders of the integrations strongly simplify calculation of the total cross section.

Slightly rearranging the Eqs. \[ \text{(17), (18)} \] we get

\[ \frac{\mu^{-2\epsilon} \sigma_{\text{tot}}}{8g^2N^2} = \int_{\delta_R}^{1-\delta_R} \frac{dx}{x(1-x)} \int \mu^{-2\epsilon} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \left\{ (1+\epsilon)(1-x)^2 \frac{4(k_1q_1)^2 + (1-4x)(q_1^2)^2}{t^2} \right. 
\[ + \frac{x(1-x)q_1^2}{k_1^2} x^2q_2^2 \right. 
\[ + \frac{x^2q_2^2}{2(1-x)k_1k_2^2} - \frac{q_1^2q_2^2}{2k_1^2z} + \frac{q_1^2q_2^2}{2k_1^2k_2^2} \right\} 
\[ + \frac{1}{\kappa t} \left\{ (\Delta + q_2^2q_1^2)^2 - 2(1+\epsilon)(2-x)(1-x)(q_1^2)^2 + \frac{(1+\epsilon)}{2} x(1-x) \left[ (2-x)(q_1^2)(\Delta - q_2^2) - x(q_2^2)^2 \right] 
\[ - 2x(\Delta - q_2^2)^2 \right\} 
\[ + (1+\epsilon)(q_1^2)^2 - \frac{xq_2^2k_2^2}{\kappa z} - \frac{xq_2^2q_1^2}{2\kappa z} + \frac{xq_2^2q_1^2}{2k_1^2k_2^2} + \frac{(k_1q_1)^2}{\kappa z} \right\} 
\[ + \frac{1}{2tt} \left\{ -2q_2^2q_1^2 + \frac{(1+\epsilon)}{2} x(1-x) \left[ (\Delta - q_2^2)^2 + (q_1^2)^2 \right] - 4x(1+\epsilon)x(1-x)(k_1q_1)^2 \right\} + \frac{(q_1^2)^2(q_2^2)^2}{4ttk_1^2k_2^2}. \]
\[ \text{(19)} \]

We integrate over $x$ here from $\delta_R$ to $(1-\delta_R)$ to remove the contribution of the multi-Regge kinematics, which is accounted by leading term in the BFKL kernel. In the total
expression with account of the next-to-leading terms for this kernel the $\delta_R$- dependence disappear. Technically to get corresponding correction to the kernel we should omit the term with ln$(1/\delta_R)$ in the total cross section at substitution $\sigma_{tot}$ in the expression for the BFKL kernel (7) and leave the leading term in the kernel the same as earlier.

Let us again denote contribution of the term number $i$ in the integrand in the RHS Eq. (19) as $J_i$ and calculate these integrals separately. Let us note, that contrary to the case of the differential cross section, the separate integrals $J_i$ contain ”ultraviolet” divergencies at large $k_1$, but these divergencies are regularized by the same $\epsilon$ as infrared ones and, as it is clear from Eqs. (17), (18), they must cancel in the final expression for $\sigma_{tot}$ after summation of all $J_i$. Using the results of calculation of these integrals from Appendix II we get for the total cross section:

$$\mu^{-2\epsilon}\sigma_{tot} = \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \left\{ \frac{q_1^2 q_2^2}{\Delta^2} \right\}^{(1+\epsilon)} \left[ \frac{2\Gamma^2(1+\epsilon)}{\epsilon \Gamma(1+2\epsilon)} \right] \text{ln}(1/\delta_R) + \frac{1}{\epsilon} \text{ln}(1/\delta_R) + \psi(1) + \psi(1+\epsilon) - 2\psi(1+2\epsilon)$$

$$- \left[ \frac{(11+7\epsilon)}{2(1+2\epsilon)(3+2\epsilon)} \right] - \left[ \frac{(q_1^2 + q_2^2 - 3(q_1 q_2)^2)}{8q_1^2 q_2^2} \right] - \left[ \frac{(11/3)(q_1^2 - q_2^2)}{16q_1^2 q_2^2} \right]$$

$$\times (q_1^2 - q_2^2) \text{ln} \left( \frac{q_1^2}{q_2^2} \right) - \left[ \frac{2(q_1^2 - q_2^2 - (q_1^2 + q_2^2) \text{ln} \left( \frac{q_1^2}{q_2^2} \right))}{4q_1^2 q_2^2 + (q_1^2 - q_2^2)^2} \right] - \left[ \frac{2q_1^2 q_2^2}{16q_1^2 q_2^2} \right] \int_0^\infty \frac{dx}{(q_1^2 + x^2q_2^2)} \text{ln} \left\{ \frac{1+x}{1-x} \right\} + \frac{2q_1^2 q_2^2 (\Delta(q_1 + q_2))}{\Delta^2 (q_1 + q_2)^2} \int_0^\infty \frac{dt}{t} \text{ln} \left( \frac{q_1^2}{q_2^2} \right) \right.$$
Appendix I.

Here we calculate the transverse momentum integrals appearing in Eq. (18). Because the transverse momenta are spacelike we pass to $(D - 2)$- dimensional Euclidian vectors in the final expressions. Let us remind, that the expressions for the invariants $\kappa, z, t, \bar{t}$ in terms of the integration variables are given by Eqs. (12), (16) and

$$\epsilon = (D - 4)/2.$$  

Then we get

$$I_1 = \int \mu^{-2\epsilon} \frac{2\kappa t}{(2\pi)^{D-1}} \left[ 2xq_1^2q_2^2 + \left( \frac{D - 2}{4} \right) x(1 - x) \left( 2(1 - x)q_1^2(\Delta^2 - q_2^2) - xq_1^2 \right)^2 
- x(\Delta^2 - q_2^2)^2 \right] -(D - 2)(1 - x)^2q_1^2(k_1q_1) \right] = \frac{2\Gamma(1 - \epsilon)}{(4\pi)^{2+\epsilon}} \left( -x^2(1 - x) \right) \left( \frac{x(1 - x)q_1^2}{\mu^2} \right)^\epsilon 
\times \left\{ \frac{1}{((1 - x)q_1^2 + xq_2^2)} \left[ 2q_1^2q_2^2 + \frac{(1 + \epsilon)}{2} (1 - x) \left( (x - 2)q_1^4 - x(\Delta^2 - q_2^2)^2 \right) \right] + \frac{2\ln \left( \frac{(1 - x)q_1^2 + xq_2^2}{(1 - x)q_1^2} \right)}{x} \left( \frac{(1 - x)q_1^2 + xq_2^2}{(1 - x)q_1^2} \right) \right\}. \quad (22)$$

$$I_2 = \int \mu^{-2\epsilon} \frac{2\kappa t}{(2\pi)^{D-1}} \left( (1 - x)(k_1q_1)^2(q_1^2 - x\Delta^2 - 2k_1q_2) \right) + \frac{2\Gamma(1 - \epsilon)}{(4\pi)^{2+\epsilon}} \left( -4x^2(1 - x)^2 \right) 
\times \left\{ \frac{x(q_1^2 + xq_2^2)}{((1 - x)q_1^2 + xq_2^2)} \left( \frac{1}{\epsilon} + 1 + \ln \left( \frac{x((1 - x)q_1^2 + xq_2^2)^2}{(1 - x)q_1^2\mu^2} \right) \right) + \left( \frac{q_1\bar{q}_2^2 - q_1q_2^2}{2} \right) 
\times \left( \frac{(1 - x)q_1^2 + xq_2^2}{xq_2^2} \right) \ln \left( \frac{(1 - x)q_1^2 + xq_2^2}{(1 - x)q_1^2} \right) + \frac{2(q_1\bar{q}_2^2)(q_1\bar{q}_2^2)}{q_2^2} \ln \left( \frac{(1 - x)q_1^2 + xq_2^2}{(1 - x)q_1^2} \right) 
- \frac{(q_1\bar{q}_2^2)^2}{q_2^2} + \frac{(1 - 2x)q_1^2}{(1 - x)} \right\}. \quad (23)$$

$$I_3 = \int \mu^{-2\epsilon} \frac{2\kappa t}{(2\pi)^{D-1}} \left( (k_1 - x\Delta)q_1 \right) \left( q_1^2 - x\Delta^2 - 2k_1q_2 \right) = \frac{2\Gamma(1 - \epsilon)}{(4\pi)^{2+\epsilon}} \left( 2x^2(1 - x)^2 \right) 
\times \left\{ q_1^2 \left( \frac{1}{2\epsilon} - \frac{1}{2} + \frac{(q_1\bar{q}_2^2)}{q_1^2q_2^2} + \frac{1}{2} \ln \left( \frac{x(1 - x)q_1^2}{\mu^2} \right) \right) + \frac{q_1^2((1 - x)q_1^2 + xq_2^2)}{xq_2^2} \left( \frac{1}{2} - \frac{(q_1\bar{q}_2^2)^2}{q_1^2q_2^2} \right) 
\times \ln \left( \frac{(1 - x)q_1^2 + xq_2^2}{(1 - x)q_1^2} \right) \right\}. \quad (24)$$

$$I_4 = \int \mu^{-2\epsilon} \frac{2\kappa t}{(2\pi)^{D-1}} \left( -(D - 2) \right) \left( 1 - x \right)(k_1q_1)^2(q_1^2 + x\Delta^2 + 2k_1q_2) \right) = \frac{2\Gamma(1 - \epsilon)}{(4\pi)^{2+\epsilon}} \left( x^2(1 - x)^2 \right)$$
\[
I_5 = \int \mu^{-2} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \frac{(-2)xq_1^2}{\kappa z t} \left[ \left( \frac{q_2^2}{2} \right)^2 - (1 - x)q_1^2 q_2^2 + \left( \frac{D - 2}{4} \right) (1 - x)^2 \left( q_1^2 \right)^2 + (1 - x) \right]
\]
\[
\times \left( 2q_2^2 - (D - 2)(1 - x)q_1^2 \right) \left( k_1 q_1 + (D - 2)(1 - x)^2 (k_1 q_1)^2 \right) = \frac{2\Gamma(1 - \epsilon) 2xq_1^2}{(4\pi)^{2+\epsilon} \Delta^2}
\]
\[
\times \left\{ \frac{1}{\left( (1 - x)q_1^2 + xq_2^2 \right) \epsilon} \left( \frac{x(1 - x)\Delta^2}{\mu^2} \right)^\epsilon \left[ \frac{q_2^4}{4} - (1 - x)q_1^2 q_2^2 - 2x(q_1 \Delta) \right] \right. \left. + \frac{(1 + \epsilon)}{2} \right\}
\]
\[
\times \frac{1}{\sqrt{ \left( xq_2^2 + (1 - x)(q_1^2 + \Delta^2) \right)^2 - 4(1 - x)^2 q_1^2 \Delta^2}}
\]
\[
\times \ln \left( \frac{xq_2^2 + (1 - x)(q_1^2 + \Delta^2)}{xq_2^2 + (1 - x)(q_1^2 + \Delta^2)} + \frac{(1 - x)^2 (q_1^2 + \Delta^2)}{\left( xq_2^2 + (1 - x)(q_1^2 + \Delta^2) \right)^2 - 4(1 - x)^2 q_1^2 \Delta^2} \right)
\]
\[
+ \left\{ \frac{1}{\left( (1 - x)q_1^2 + xq_2^2 \right) \epsilon} \left( q_2^4 \right) - (1 - x)q_1^2 q_2^2 - 2x(q_1 \Delta) \right. \left. + \frac{(1 - x)^2}{2} (q_1^2 - 2x(q_1 \Delta))^2 \right\}
\]
\[
\times \left\{ (q_1 \Delta) \left( q_2 - (1 - x)(q_1^2 - 2x(q_1 \Delta)) \right) + \frac{(1 - x)}{2q_2^2} \left( xq_2^2 + (1 - x)q_1^2 \right) \right. \left. (2(q_1 \Delta)^2 - q_1^2 q_2^2) \right\}
\]
\[
\times \ln \left( \frac{\left( (1 - x)q_1^2 + xq_2^2 \right)^2}{(1 - x)^2 q_1^2 \Delta^2} \right) - \frac{(1 - x)^3 \Delta^2}{2q_2^2} \left( 2(q_1 \Delta)^2 - q_1^2 q_2^2 \right) \ln \left( \frac{q_1^2}{\Delta^2} \right) \right\}. \tag{26}
\]
\[
I_6 = \int \mu^{-2} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \frac{1}{2tt} \left[ -2q_1^2 q_2^2 + \left( \frac{D - 2}{4} \right) x(1 - x) \left( \Delta^2 - q_2^2 \right)^2 + (q_1^2) \right] = \frac{2\Gamma(1 - \epsilon)}{(4\pi)^{2+\epsilon}}
\]
\[ I_7 = \int \mu^{-2} \frac{d^{D-2} k_1}{(2\pi)^{D-1}} \frac{x(1-x)^2 q_1^2}{2\kappa z} \left[ (D-2)(1-x)q_1^2 - (D-4)x(D^2 - q_2^2) \right] = \frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \]
\[ \times \ln \left( \frac{\sqrt{q_2^2 + 4x(1-x)q_1^2} + \sqrt{q_2^2}}{\sqrt{q_2^2 + 4x(1-x)q_1^2} - \sqrt{q_2^2}} \right). \]  
\[ (27) \]

\[ I_8 = \int \mu^{-2} \frac{d^{D-2} k_1}{(2\pi)^{D-1}} \left( \frac{x(1-x)q_1^2}{z} \right)^2 \left[ (\frac{D-2}{4}) - (D-1)x(1-x) \right] = \frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{x(1-x)q_1^4}{\Delta^2} \]
\[ \times \left( \frac{x(1-x)}{\mu^2} \right)^\epsilon \left( \frac{(1+\epsilon)}{2} - (3+2\epsilon)x(1-x) \right). \]  
\[ (28) \]

\[ I_9 = \int \mu^{-2} \frac{d^{D-2} k_1}{(2\pi)^{D-1}} \left( \frac{D-2}{2} \right) (1-4x) \left( \frac{(1-x)q_1^2}{t} \right)^2 = \frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{x(1-x)(1-4x)q_1^2}{\Delta^2} \]
\[ \times (1+\epsilon) \left( \frac{x(1-x)q_1^2}{\mu^2} \right)^\epsilon. \]  
\[ (29) \]

The integrals \(I_7, I_8\) and \(I_9\) are calculated exactly.

\[ I_{10} = \int \mu^{-2} \frac{d^{D-2} k_1}{(2\pi)^{D-1}} (1-x)(2k_1 q_1 - q_1^2)q_1^2 q_2^2 2\kappa k_1^2 k_2^2 = \frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{(1-x)q_1^2 q_2^2}{2} \]
\[ \times \left[ \frac{1}{\epsilon (xq_2^2 + (1-x)\Delta^2)} + \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} \left( \frac{1}{(xq_2^2 + (1-x)\Delta^2)^{1-\epsilon}} - \frac{1}{(\Delta^2)^{1-\epsilon}} \right) \right] \]
\[ - \frac{2\epsilon(\Delta^2)^\epsilon}{(xq_2^2 + \Delta^2)} \int_1^\infty \frac{dz}{z \ln(1-z)}. \]  
\[ (31) \]

\[ I_{11} = \int \mu^{-2} \frac{d^{D-2} k_1}{(2\pi)^{D-1}} (-x)q_2^2 q_1^2 2\kappa k_1^2 = \frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{x(1-x)q_1^2 q_2^2}{2} \left[ \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} \right] x^2 \Delta^2 \]
\[ + \left( \frac{2(q_1 \Delta - \Delta^2)}{\Delta^2(xq_2^2 + (1-x)q_1^2)} \right) \left( \frac{1}{\epsilon} + \ln \left( \frac{x((1-x)q_1^2 + xq_2^2)^2}{(1-x)q_1^2 \mu^2} \right) \right) - \frac{2(\Delta(q_1 - x\Delta))}{\Delta^2(q_1 - x\Delta)^2} \]
\[ (32) \]
\[ I_{12} = \int \mu^{-2x} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \frac{(q_1^2 q_2^2)^2}{4t \eta k_1^2 k_2^2} = \frac{1}{2} \left[ -I_{10} + \frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} (1-x) q_1^2 q_2^2 \times \ln \left( \frac{(1-x)q_1^2 + x\bar{q}_1^2}{x q_1^2} \right) \right] \]

\[ \times \ln \left( \frac{(1-x)q_2^2 + x\bar{q}_2^2}{x q_2^2} \right) \]. \quad (32)\]

\[ I_{13} = \int \mu^{-2x} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \frac{(-x)q_1^2 q_2^2}{2(1-x)\eta k_1^2} = \frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{q_1^2 q_2^2}{2\bar{\Delta}^2} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} \left( \frac{x^2 \bar{\Delta}^2}{\mu^2} \right)^\epsilon. \quad (34)\]

\[ I_{13} \] is calculated exactly.

\[ I_{14} = \int \mu^{-2x} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \frac{x^2 q_1^2 q_2^2}{2k_1^2 z} = \frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{q_1^2 q_2^2}{2\bar{\Delta}^2} \frac{\bar{\Delta}^2}{\mu^2} x^2 \int_1^1 dz \frac{dz}{(xz(1-xz))^{1-\epsilon}} \approx \frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \]

\[ \times \frac{q_1^2 q_2^2}{2\bar{\Delta}^2} \frac{\bar{\Delta}^2}{\mu^2} x \left[ \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} - \frac{(1-x)^\epsilon}{\epsilon} + \ln x + \frac{\epsilon}{2} \ln x \ln(x(1-x)^2) - 2\epsilon \int_0^{1-x} \frac{dz}{z} \ln(1-z) \right]. \quad (35)\]

The first relation here is exact and in the second approximate relation we have made the expansion in \( \epsilon \) with accuracy needed.

\[ I_{15} = \int \mu^{-2x} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \frac{q_1^2 q_2^2}{2k_1^2 k_2^2} = \frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{q_1^2 q_2^2}{2\bar{\Delta}^2} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} \left( \frac{\bar{\Delta}^2}{\mu^2} \right)^\epsilon. \quad (36)\]

The last relation is exact.
Appendix II.

Here we calculate the integrals appearing in the total cross section \([19]\). The first six integrals are calculated exactly without any problems and the result is

\[ J_1 + \ldots + J_6 = \int_{\delta_R}^{1-\delta_R} \frac{dx}{x(1-x)} \int \mu^{-2\epsilon} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \left\{ (1 + \epsilon)(1-x)^2 \frac{4(k_1 q_1)^2 + (1 - 4x)(q_1^2)^2}{t^2} \right\} \]

\[ + \frac{x(1-x)q_1^2}{\kappa z} \left[ 2q_2^2 + (1 + \epsilon) \left( 2(1-x)(k_1 q_1) - x(1-x)q_1^2 - k_2^2 \right) - \epsilon x(1-x)(\Delta^2 - q_2^2) \right] \]

\[ + \left( \frac{x(1-x)q_1^2}{z} \right)^2 \left[ \frac{(1 + \epsilon)}{2} - (3 + 2\epsilon)x(1-x) \right] + \left( \frac{-xq_1q_2^2}{2(1-x)\kappa k_1^2} + \frac{x^2q_1^2q_2^2}{2k_1^2} + \frac{xq_1^2q_2^2}{k_1^2k_2^2} \right) \]

\[ = \frac{\Gamma(1-\epsilon) 2\Gamma^2(1+\epsilon)}{(4\pi)^{2+\epsilon} \epsilon \Gamma(1+2\epsilon)} \left[ 4\ln(1/\delta_R) + \psi(1) + \psi(1+\epsilon) - 2\psi(1+2\epsilon) - \frac{(11 + 7\epsilon)}{2(1 + 2\epsilon)(3 + 2\epsilon)} \right] \]

\[ \times \left( \frac{\Delta^2}{\mu^2} \right)^\epsilon \frac{\bar{q}_1^2\bar{q}_2^2}{\Delta^2} - \frac{(1 + \epsilon)}{(1 + 2\epsilon)(3 + 2\epsilon)} \left( \frac{\bar{q}_1^2}{\mu^2} \right)^\epsilon \bar{q}_1^2. \]  

(37)

All others integrals don’t contain of the dependence on \(\delta_R\) at \(\delta_R \to 0\), therefore we will put \(\delta_R = 0\) below. The next two integrals aren’t singular at \(|\Delta| \to 0\) and we can calculate them with accuracy up to \(const(\epsilon)\). Besides that, using the following change of integration variables, corresponding to left-right symmetry \([3]\),

\[ \bar{k}_1 \leftrightarrow \bar{k}_2, \quad x \leftrightarrow \frac{x\bar{k}_2^2}{((1-x)\bar{k}_1^2 + x\bar{k}_2^2)}. \]

(38)

we obtain

\[ J_8 = \int_0^1 \frac{dx}{x(1-x)} \int \mu^{-2\epsilon} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \frac{(-x)q_1^2k_2^2 ((1 + \epsilon)k_2^2 - 2q_2^2)}{\kappa z t} = \]

\[ \left[ \int_0^1 \frac{dx}{x(1-x)} \int \mu^{-2\epsilon} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \frac{(-x)q_1^2 ((1 + \epsilon)k_2^2 - 2q_2^2)}{\kappa t} \right] (q_1 \leftrightarrow -q_2), \]

(39)

and therefore \(J_8\) has the same structure as \(J_7\) and it is convenient to consider these two integrals together. After rather long calculation we get

\[ J_7 + J_8 = \int_0^1 \frac{dx}{x(1-x)} \int \mu^{-2\epsilon} \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \left\{ \frac{1}{\kappa t} \left[ -2(1-x)q_1^2q_2^2 + (1 + \epsilon)(1-x)(q_1^2)^2 + \frac{(1 + \epsilon)}{2} \right] \times x(1-x) \left( 2(1-x)q_1^2(\Delta^2 - q_2^2) - x(q_1^2)^2 - x(\Delta^2 - q_2^2)^2 \right) - 2(1 + \epsilon)(2 - x)(1-x)q_1^2(k_1 q_1) \right. \]

\[ + 2(1 + \epsilon)(1-x) \left( (k_1 q_1)^2 + ((1-x)\Delta q_1)^2 \right) + (1 + \epsilon)q_1^2k_2^2 \right}\]

\[ = \frac{\Gamma(1-\epsilon) 2\Gamma^2(1+\epsilon)}{(4\pi)^{2+\epsilon} \epsilon \Gamma(1+2\epsilon)(1 + 2\epsilon)(3 + 2\epsilon)} \left( \frac{\bar{q}_1^2}{\mu^2} \right)^\epsilon - \frac{1 + \epsilon}{12} \left( \frac{\bar{q}_2^2}{\mu^2} \right)^\epsilon \bar{q}_1^2 - \frac{\Gamma(1-\epsilon) 2\Gamma^2(1+\epsilon)}{(4\pi)^{2+\epsilon} \epsilon \Gamma(1+2\epsilon)(1 + 2\epsilon)(3 + 2\epsilon)} \left( \frac{\bar{q}_2^2}{\mu^2} \right)^\epsilon \bar{q}_2^2. \]
\[
x \times \left( \frac{q_1 q_2}{q_2^2} \right) - \frac{q_1^2}{3} \left( 1 + \ln \left( \frac{q_1^2}{\mu^2} \right) - \frac{2}{3} \right) + (2q_1 \Delta) \left( \frac{2}{3} \ln \left( \frac{q_1}{q_2^2} \right) + 1 \right) - 2\Delta^2 + \frac{2q_1^2 (6q_2^2 - q_1^2)}{3 (q_1 - q_2^2)} \\
\times \ln \left( \frac{q_2}{q_1} \right) + \frac{\Delta^2 q_2^2}{(q_1 - q_2^2)^3} \left( \frac{q_1}{q_1} - 3q_2^2 \right) \ln \left( \frac{q_2}{q_2} \right) + 2q_2^2 (q_1 - q_2^2) \right) + \frac{2}{3 (q_1 - q_2^2)^3} \left( q_2^2 (3q_2^2 - q_1^2) - (q_1 q_2) (3q_1^2 + q_2^2) + 2q_1^2 (\frac{q_1 q_2}{q_2}) \right) \left( 2q_2^2 \ln \left( \frac{q_2}{q_2} \right) - 2q_2^2 (q_1 - q_2^2) + (q_1^2 - q_2^2)^2 \right).}
\]

Using the change of integration variables \( (38) \) one can get

\[
J_{10} = J_9 (q_1 \leftrightarrow -q_2),
\]

and it is enough to calculate only one of these integrals. For the sum of \( J_9 \) and \( J_{10} \) we obtain

\[
J_9 + J_{10} = \int_0^1 dx \int \mu^{-2 \epsilon} d^{D-2} k_1 \left\{ -xq_1^2 (q_2^2)^2 - xq_2^2 (q_1^2)^2 \right\} = \frac{\Gamma(1 - \epsilon) \Gamma^2(1 + \epsilon)}{(4\pi)^{2+\epsilon}} \frac{2}{(1 + 2\epsilon)} \sqrt{\Delta^2} \frac{\epsilon q_1^2 q_2^2}{\Delta^2}.
\]

\[
J_{11} = \int_0^1 dx \int \mu^{-2 \epsilon} d^{D-2} k_1 \left\{ (1 - x)(2k_1 q_1 - q_1^2)q_2^2 \right\} = -\frac{\Gamma(1 - \epsilon) \Gamma^2(1 + \epsilon)}{(4\pi)^{2+\epsilon}} \frac{1}{(1 + 2\epsilon)} \sqrt{\Delta^2} \frac{\epsilon q_1^2 q_2^2}{\Delta^2}.
\]

\[
J_{12} = \int_0^1 dx \int \mu^{-2 \epsilon} d^{D-2} k_1 \left\{ -2q_1^2 q_2^2 + \frac{1 + \epsilon}{2} x(1 - x) \right\} \left\{ (\Delta^2 - q_2^2)^2 + (q_1^2)^2 \right\}
-4(1 + \epsilon)x(1 - x)(k_1 q_1)^2 \right\} = \frac{\Gamma(1 - \epsilon) \Gamma^2(1 + \epsilon)}{(4\pi)^{2+\epsilon}} \left\{ \frac{\epsilon q_1^2 q_2^2}{\Delta^2} \left\{ \frac{1}{6} \right\} \right\} + \frac{q_2^2}{2} \right\} - \frac{\Gamma(1 - \epsilon) \Gamma^2(1 + \epsilon)}{(4\pi)^{2+\epsilon}}
\times \left\{ \frac{3}{8} (2q_1^2 q_2^2 - (q_1 q_2)^2)^2 (q_1 + q_2) \right\} + \left\{ \frac{2q_1^2 q_2^2 - 3(q_1 q_2)^2}{16 q_1 q_2^2} \right\} \ln \left( \frac{q_2}{q_2} \right) \left\{ \frac{4q_1^2 q_2^2 + (q_1 - q_2)^2}{4} \right\}
+ \left\{ \frac{2q_1^2 q_2^2 - (q_1 q_2)^2}{16 q_1 q_2^2} \right\} \left\{ \frac{4q_1^2 q_2^2 + (q_1 - q_2)^2}{4} \right\} \ln \left( \frac{1 + x}{1 - x} \right) \right\}.
\]

\[
J_{13} = \int_0^1 dx \int \mu^{-2 \epsilon} d^{D-2} k_1 \left\{ \frac{(q_1^2)^2 (q_2^2)^2}{(2\pi)^{D-1}} \right\} = \frac{\Gamma(1 - \epsilon) \Gamma^2(1 + \epsilon)}{(4\pi)^{2+\epsilon}} \frac{1}{(1 + 2\epsilon)} \sqrt{\Delta^2} \frac{\epsilon q_1^2 q_2^2}{\Delta^2} \left\{ \frac{1}{6} \right\} + \frac{q_1^2}{2} \right\} \left\{ \frac{2q_1^2 q_2^2 \Delta (q_1 + q_2)}{\Delta^2 (q_1 + q_2)^2} \right\} \ln \left( \frac{q_1}{q_2} \right) \ln \left( \frac{q_1 q_2}{q_1^2 + q_2^2} \right)
+ \frac{1}{\epsilon} \ln \left( \frac{q_1 q_2}{\Delta^2} \right) + \frac{1}{2} \ln^2 \left( \frac{q_1}{q_2} \right) + 4\epsilon q_1^2 \ln \left( \frac{q_1 q_2}{(q_1^2 + q_2^2)^2} \right) \right\}.
\]
\[ +L \left( 1 - \frac{\Delta^2}{\bar{q}_2^2} \right) - L \left( 1 - \frac{\Delta^2}{\bar{q}_1^2} \right) + L \left( -\frac{\bar{q}_1^2}{\bar{q}_2^2} \right) - L \left( -\frac{\bar{q}_2^2}{\bar{q}_1^2} \right) \]
\[ + 2\bar{q}_1 \bar{q}_2 \left[ \left( \int_0^1 \frac{dt}{(\bar{q}_2^2 t^2 - 2(\bar{q}_1 \bar{q}_2) t + \bar{q}_1^2)} \right) \times \left( \frac{(\bar{q}_2 \bar{\Delta}) \bar{\Delta}}{\Delta^2} - \frac{q_2^2 (\Delta (\bar{q}_1 + \bar{q}_2))}{\bar{\Delta}^2 (\bar{q}_1 + \bar{q}_2)^2} (1 + t) \ln \left( \frac{q_2^2 t (1 - t)}{q_1^2 (1 - t) + \Delta^2 t} \right) + \left( \bar{q}_1 \leftrightarrow -\bar{q}_2 \right) \right) \right] , \tag{45} \]

where
\[ L(x) = \int_0^x \frac{dt}{t} \ln(1 - t). \tag{46} \]
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