Hadron Spectroscopy in Regge Phenomenology

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Abstract

We show that linear Regge trajectories for mesons and baryons, and the cubic mass spectrum associated with them, determine expressions for the hadron masses in terms of the universal Regge slope $\alpha'$ alone. The ground state hadron masses as calculated from these expressions are in excellent agreement with experiment for $\alpha' = 0.85 \text{ GeV}^{-2}$.

Key words: Regge phenomenology, mass spectrum, hot hadronic matter, mesons, baryons
PACS: 12.40.Nn, 12.40.Yx, 12.90.+b, 14.20.-c, 14.40.-n

1 Introduction

It is well known that the hadrons composed of light ($n(=u,d,s)$) quarks populate linear Regge trajectories; i.e., the square of the mass of a state with orbital momentum $\ell$ is proportional to $\ell : M^2(\ell) = \ell / \alpha' + \text{const}$, where the slope $\alpha'$ only very weekly depends on the flavor content of the states lying on the corresponding trajectory: $\alpha'_{nn} = 0.88 \text{ GeV}^{-2}$, $\alpha'_{sn} = 0.85 \text{ GeV}^{-2}$, $\alpha'_{ss} = 0.81 \text{ GeV}^{-2}$; it therefore may be taken as a universal slope in the light quark sector, $\alpha' \approx 0.85 \text{ GeV}^{-2}$. In this respect, the hadron masses as populating collinear trajectories exhibit a universal behavior governed by the only parameter $\alpha'$. It

*Presented at the First International Conference on Parametrized Relativistic Quantum Theory, PRQT '98, Houston, Texas, USA, Feb 9-11, 1998
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is therefore very interesting to ask whether the hadron masses can be expressed in terms of this parameter alone. The answer to this non-trivial at first glance question turns out positive, if one resorts to the notion of the hadron mass spectrum.

The idea of the spectral description of a strongly interacting gas which is a model for hot hadronic matter was suggested by Belenky and Landau [1] and consists in considering the unstable particles (resonances) on an equal footing with the stable ones in the thermodynamic quantities, by means of the resonance spectrum; e.g., the expression for pressure in such a resonance gas reads (in the Maxwell-Boltzmann approximation)

\[ p = \sum_i g_i p(m_i) = \int_{M_l}^{M_h} dm \tau(m) p(m), \quad p(m) = \frac{T^2 m^2}{2\pi^2} K_2 \left( \frac{m}{T} \right), \quad (1) \]

where \( M_l \) and \( M_h \) are the masses of the lightest and heaviest species, respectively, and \( g_i \) are particle degeneracies.

Phenomenological studies [2, 3] have suggested that the cubic density of states, \( \tau(m) \sim m^3 \), for each isospin and hypercharge provides a good fit to the observed hadron spectrum. Let us demonstrate here that this cubic spectrum is intrinsically related to collinear Regge trajectories (for each isospin and hypercharge).

### 1.1 Mass spectrum of linear Regge trajectories

It is very easy to show that the mass spectrum of an individual Regge trajectory is cubic. Indeed, consider, e.g., a model linear trajectory with negative intercept:

\[ \alpha(t) = \alpha' t - 1. \quad (2) \]

The integer values of \( \alpha(t) \) correspond to the states with integer spin, \( J = \alpha(t) \), the masses squared of which are \( m^2(J) = t_j \). Since a spin-\( J \) state has multiplicity \( 2J + 1 \), the number of states with spin \( 0 \leq J \leq J \) is

\[ N(J) = \sum_{J=0}^{J} (2J + 1) = (J + 1)^2 = \alpha'^2 m^4(J), \quad (3) \]

in view of (2), and therefore the density of states per unit mass interval (the mass spectrum) is

\[ \tau(m) = \frac{dN(m)}{dm} = 4\alpha'^2 m^3. \quad (4) \]

It is also clear that for a finite number of collinear trajectories, the resulting mass spectrum is

\[ \tau(m) = 4N \alpha'^2 m^3, \quad (5) \]

where \( N \) is the number of trajectories, and does not depend on the numerical values of trajectory intercepts, as far as its asymptotic form \( m \to \infty \) is concerned.
1.2 Mass spectrum of an individual hadronic multiplet

Similar to the cubic spectrum of the family of Regge trajectories, phenomenological studies have suggested that the mass spectrum of an individual hadronic multiplet is linear, for both mesons \([4]\) and baryons \([5]\). It turns out that the form (5) of the cubic spectrum of the family of collinear Regge trajectories indeed leads to the linear mass spectrum of an individual multiplet, and allows one to establish the normalization constant of this linear spectrum, as follows:

Consider the family of hadronic multiplets with spin 0,1,..., which populate collinear trajectories. Then the total number of states can be obtained in two ways: summing up individual trajectories for every fixed value of isospin, or summing up individual multiplets for every fixed value of spin. Either way should lead to the cubic spectrum, as discussed above. In the case of meson multiplets (similar analysis may be done in the case of baryon multiplets, of course), in Eq. (1)

\[ g_i = (2J_i + 1)(2I_i + 1)' \]

where \(J_i\) and \(I_i\) are the values of individual spin and isospin, respectively ("\(^\prime\)" means that for \(I_i = 1/2\), the above expression for \(g_i\) should be multiplied by 2). Then, in view of (1),(5),

\[ p = \sum_i (2J_i + 1)(2I_i + 1)' p(m_i) \simeq 4\alpha' \int dm \ m^3 \ p(m). \quad (6) \]

Since also \(J_i \simeq \alpha' m_i^2\), it follows from the above expression that\[\]

\[ \sum_i (2I_i + 1) p(m_i) \simeq 2N\alpha' \int dm \ m \ p(m), \quad (7) \]

and since Eq. (7) corresponds to an individual meson multiplet (with fixed spin \(J_i\)), one sees that the mass spectrum of an individual meson multiplet is indeed linear, and its normalization constant is \(C = 2N\alpha'\).

Now we are ready to show how the cubic spectrum of the family of multiplets and the linear spectrum of an individual multiplet can predict the masses of the states.

2 Particle spectroscopy

Let us start with meson spectroscopy. To establish the masses of the states in the model of collinear Regge trajectories discussed above, one has to know the intramultiplet mass splitting \(m_{I=1/2}^2 - m_{I=1}^2\) and the mass of the lowest-lying isovector, \(m_{I=1}\). The former can be easily found with the help of (7), for 9 isospin degrees of freedom of a meson nonet placed

\[\]

\(^1\)This results may be rigorously proven by the use of, e.g., the Euler-Maclaurin summation formula which relates a sum to an integral.
in the mass interval \((m_{I=1}, m_{I=0})\), with \(m_{I=0}^2 - m_{I=1}^2 = 4/3 (m_{I=1/2}^2 - m_{I=1}^2) \equiv 4/3 \Delta\):

\[ 9 = 2N\alpha \int_{m_{I=1}}^{m_{I=0}} dm \ m = 2N\alpha \frac{1}{2} \frac{4}{3} \Delta; \]

therefore

\[ \Delta = \frac{27}{4N\alpha}. \]

To determine the number of collinear trajectories, we note that there are four different meson multiplets for every partial wave, except for \(S\)-wave, which in the standard spectroscopic notation are

\[
\begin{align*}
1S_0 & \quad 1P_1 & \quad 1D_2 & \quad 1F_3 & \quad \ldots \\
3P_0 & \quad 3D_1 & \quad 3F_2 & \quad \ldots \\
3P_1 & \quad 3D_2 & \quad 3F_3 & \quad \ldots \\
3S_1 & \quad 3P_2 & \quad 3D_3 & \quad 3F_4 & \quad \ldots
\end{align*}
\]

(note that two missing \(S\)-wave nonets can be replaced by the radial excitations of \(1S_0\) and \(3S_1\), each of which contains 9 isospin states; therefore, the total number of different collinear meson trajectories is

\[ N = 4 \times 9 = 36. \]

Hence, as follows from (9),

\[ \Delta = \frac{3}{16\alpha}. \]

It is well known that two isoscalar states of an idealized bare meson nonet mix with each other to form the physical states the masses of which are

\[ m_{I=0'}^2 = m_{I=1}^2, \quad m_{I=0''}^2 = 2m_{I=1/2}^2 - m_{I=1}^2. \]

Therefore, one has

\[ m_{K*}^2 = m_\rho^2 + \frac{3}{16\alpha}, \quad m_\phi^2 = m_\rho^2 + \frac{3}{8\alpha}, \quad \text{etc.}, \]

and also

\[ m_K^2 = m_\pi^2 + \frac{3}{16\alpha}. \]

It is widely believed that pseudoscalar mesons are the Goldstone bosons of broken \(SU(3) \times SU(3)\) chiral symmetry of QCD, and that they should be massless in the chirally-symmetric phase. Therefore, it is not clear how well would the framework that we discuss

\[ \text{We assume that the remaining ninth isoscalar belongs to this interval; As established in [4], for idealized meson nonets, its mass is equal to } (2m_{I=1/2}^2 + m_{I=1}^2)/3 \text{ which lies between } m_{I=1}^2 \text{ and } m_{I=1/2}^2 < m_{I=0}'^2. \]

\[ \text{This follows from the standard Gell-Mann–Okubo mass formula } 3m_{I=0}^2 + m_{I=1}^2 = 4m_{I=1/2}^2 \text{ which is also provided by the linear mass spectrum of a meson octet [4].} \]

\[ \text{In a constituent quark model, these multiplets correspond to spin-singlet and spin-triplet states of a bound system of two quarks.} \]
here be suitable for the description of the pseudoscalar nonet. Indeed, as we have tested in [3], this nonet is not described by the linear spectrum. Moreover, pseudoscalar mesons are extremely narrow (zero width) states to fit into a resonance description. Probably, the manifestly covariant framework cannot predict the mass of the pion, although the formula (13) is consistent with data, as we shall see below.

Thus, the resonance description should start with vector mesons, and the cubic spectrum of a linear trajectory enables one to determine the mass of the $\rho$ meson, as follows:

Since the $\rho$ meson has the lowest mass which the resonance description starts with, let us locate this state by normalizing the $\rho$ trajectory to one state in the characteristic mass interval ($\sqrt{m_\rho^2 - 1/(2\alpha')}$, $\sqrt{m_\rho^2 + 1/(2\alpha')}$). With the cubic spectrum (4) of a linear trajectory, one has

$$1 = 4\alpha^2 \int_{\sqrt{m_\rho^2 - 1/(2\alpha')}}^{\sqrt{m_\rho^2 + 1/(2\alpha')}} m^3 \, dm = 2\alpha' m_\rho^2;$$  

therefore

$$m_\rho^2 = \frac{1}{2\alpha'},$$  

and, through (12),

$$m_{K^*}^2 = \frac{11}{16\alpha'}, \quad m_\phi^2 = \frac{7}{8\alpha'}, \quad \text{etc.}$$

(16)

Similar analysis can be easily done for baryons. Here, for brevity, we skip this analysis and only refer to [4] where preliminary discussion on the baryon spectroscopy can be found. Let us just write down the final expressions:

$$m_N^2 = \frac{3}{4\alpha'}, \quad m_{\Sigma^*}^2 = \frac{9}{8\alpha'}, \quad m_{\Xi^*}^2 = \frac{3}{2\alpha'},$$

(17)

$$m_\Delta^2 = \frac{5}{4\alpha'}, \quad m_{\Sigma'}^2 = \frac{13}{8\alpha'}, \quad m_{\Xi'}^2 = \frac{2}{\alpha'}, \quad m_{\Omega'}^2 = \frac{19}{8\alpha'}, \quad \text{etc.}$$

(18)

In (17), $m_{\Sigma'}^2 \equiv (m_\Lambda^2 + m_\Sigma^2)/2$ [3].

It is seen in (17),(18) that the mass squared splitting within an individual baryon multiplet is twice as large as that for an individual meson multiplet; e.g., $m_{\Sigma^*}^2 - m_\Delta^2 = 3/(8\alpha')$, as compared to (12),(13). The mass squared splitting between multiplets which differ by one unit of spin remains, however, the same: since $m_\pi \ll m_\rho$, it follows from (15),(17),(18) that $m_\rho^2 - m_\pi^2 \approx m_\rho^2 = 1/(2\alpha') = m_\Delta^2 - m_{\Xi^*}^2$. Also, the relation $m_N^2 = 3/2 m_\rho^2$, as follows from (15),(17), is definitely related to the valence quark structure interpretation of the two states.

2.1 Comparison with data

Now we wish to compare the formulas (13),(15)-(18) with available experimental data on the particle masses [4].

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5Since the $\rho$ trajectory starts with a spin-1 isospin-1 state ($\rho$), it corresponds to the spectrum $\tau(m) = 9 \times 4\alpha'^2 m^3$. There is therefore no difference in normalizing this trajectory to 9 states, or (4) to one state, in the vicinity of the $\rho$ mass.
It is seen that the particle masses are solely determined by the value of $\alpha'$. Although this parameter is known to coincide for both light mesons and baryons, it is also known to have a weak flavor dependence for light mesons, as remarked above. Since here we are not concerned with accuracies of better than 1\% (i.e., at the level of electromagnetic corrections), it would be enough to neglect the flavor dependence of $\alpha'$ and take

$$\alpha' = 0.85 \text{ GeV}^{-2},$$  \hspace{1cm} (19)

which is the average of its values for $n\bar{n}$, $s\bar{n}$ and $s\bar{s}$ (see above).

Let us start with (13). The use of $m_{\pi} = (m_{\pi}^0 + m_{\pi}^\pm) / 2 = 137.3 \text{ MeV}$ \cite{7} in this formula leads, via (19), to

$$m_K = 489.3 \text{ MeV}, \hspace{0.5cm} \text{vs.} \hspace{0.5cm} m_K = 495.7 \pm 2.0 \text{ MeV} \ [7].$$  \hspace{1cm} (20)

Similar comparison of the hadron masses predicted by (15)-(18) with data is presented in Table I.

| State | Mass from (15)-(18), MeV | Mass from ref. \cite{7}, MeV |
|-------|--------------------------|-----------------------------|
| $\rho$ | 767.0                     | 768.5 ± 0.6                 |
| $K^*$ | 899.3                     | 893.9 ± 2.3                 |
| $\phi$ | 1014.6                   | 1019.4                      |
| $N$    | 939.3                     | 938.9                       |
| $\Sigma'$ | 1150.5            | 1155 ± 2                     |
| $\Xi$  | 1328.4                     | 1318 ± 3                     |
| $\Delta$ | 1212.7                    | 1232 ± 2                     |
| $\Sigma^*$ | 1382.7                  | 1385 ± 2                     |
| $\Xi^*$ | 1533.9                     | 1533.5 ± 1.5                 |
| $\Omega$ | 1671.6                     | 1672.4                       |

**Table I.** Comparison of the particle masses predicted by the formulas (15)-(18) with experimental data from ref. \cite{7}.

One sees excellent agreement with experiment for all states, except for $\Delta$. We however note that this state has largest width among the ground state baryons ($\sim 120 \text{ MeV}$; for comparison, the $\Sigma^*$ has largest width of $\sim 9.5 \text{ MeV}$ among the remaining ground state baryons), and therefore its mass is poorly known. Indeed, the pole position of $\Delta$, as indicated in \cite{7}, is $1210 \pm 1 \text{ MeV}$, and hence the prediction of Eq. (18) for the $\Delta$ mass is in excellent agreement with the pole position of $\Delta$.

One can easily obtain expressions similar to (15)-(18) for other hadronic multiplets, assuming that they populate linear trajectories; e.g. $m_{a_2}^2 = m_{\rho}^2 + 1/\alpha' = 3/(2\alpha') = 1328.4 \text{ MeV}$, vs. $1318 \pm 1 \text{ MeV}$ \cite{7}.  

\footnote{It is interesting to note that, although the numerical values of $m_{a_2}^2 = m_{\Xi}^2 = 3/(2\alpha')$, as calculated from our formulas, do not coincide with data for $\alpha' = 0.85 \text{ GeV}^{-2}$, they do coincide with each other: $m_{a_2} = m_{\Xi} = 1318 \text{ MeV}$.}
3 Concluding remarks

We have shown that collinear Regge trajectories for mesons and baryons, and the cubic mass spectrum associated with them, determine expressions of the hadron masses in terms of the universal Regge slope $\alpha'$ alone, which are in excellent agreement with experiment for $\alpha' = 0.85 \text{ GeV}^{-2}$.

These expressions are consistent with a universal scaling behavior for all hadron masses which has been widely discussed in the recent literature [3, 8]. Indeed, as we show in a separate publication [9], Regge phenomenology is consistent with the only form of such a scaling, $M^*/M = (\alpha'/\alpha^*)^{1/2}$ (asterisk indicates a temperature- and/or density-dependent quantity), in view of which a relation $M^2 \propto 1/\alpha'$ is clearly understood.

We note that the technique discussed in this paper can be easily generalized to glueballs [10], and reproduces the scalar and tensor glueball masses in excellent agreement with lattice QCD simulations. Further generalization of this technique to multi-quark states (e.g., diquonia and pentaquarks) is of great interest, and will be undertaken elsewhere.

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