Compact multipliers on spaces of analytic functions

Paweł Mleczko

Faculty of Mathematics and Computer Science,
Adam Mickiewicz University,
Umultowska 87, 61-614 Poznań,
Poland

e-mail: pml@amu.edu.pl

Abstract

In the paper compact multiplier operators on Banach spaces of analytic functions on the unit disk with the range in Banach sequence lattices are studied. If the domain space $X$ is such that $H_\infty \hookrightarrow X \hookrightarrow H_1$, necessary and sufficient conditions for compactness are presented. Moreover, the calculation of the Hausdorff measure of noncompactness for diagonal operators between Banach sequence lattices is applied to obtaining the characterization of compact multipliers in case the domain space $X$ satisfies $H_\infty \hookrightarrow X \hookrightarrow H_2$.

The study of coefficients of functions satisfying certain properties has a long history. Spaces of analytic functions on the unit disk are of special interest in this context. First papers on the topic of examination Taylor coefficients of functions from Hardy classes went back to Hardy and Littlewood and the beginning of the 20th century (see e.g., [9]). Within this topic the multipliers operators emerge as a natural object of studies. In this paper we investigate compactness properties of such operators. We pay special attention to the case of the domain spaces satisfying the property $H_\infty \hookrightarrow X \hookrightarrow H_2$. Let us start with the definition of a multipliers.

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An element \( \lambda = \{ \lambda_n \} \in \omega \) is said to be a multiplier from a sequence space \( F \) into another one \( E \) if \( \lambda x := \{ \lambda_n x_n \} \in E \) for any \( x = \{ x_n \} \in F \). The set of multipliers from \( F \) into \( E \) is denoted by \( \mathcal{M}(F,E) \). If \( F \) and \( E \) are Banach sequence lattices (see the definition below), then \( \mathcal{M}(F,E) \) is a Banach sequence lattice equipped with the norm

\[
\| \lambda \|_{\mathcal{M}(F,E)} = \sup \{ \| \lambda x \|_E; \| x \|_F \leq 1 \}.
\]

Every sequence \( \lambda \in \mathcal{M}(F,E) \) induces a diagonal operator \( D_\lambda : F \to E \) given by \( D_\lambda x = \{ \lambda_n x_n \} \) for \( x = \{ x_n \} \in F \).

In the paper we study multiplier operators defined on spaces of analytic functions. If \( G \) is a linear subspace of \( H(\mathbb{U}) \) – the space of analytic functions on the unit disk \( \mathbb{U} \), then we associate with \( G \) the sequence space \( \hat{G} \) of Taylor’s coefficients of all functions in \( G \), i.e.

\[
\hat{G} = \left\{ \{ \hat{f}(n) \}; f = \sum_{n=0}^{\infty} \hat{f}(n) u_n \in G \right\}.
\]

Here, as usual \( u_n(z) := z^n \) for \( n \in \mathbb{Z}_+ \) and \( z \in \mathbb{U} \). With any \( \lambda = \{ \lambda_n \} \in \mathcal{M}(\hat{G},E) \), we identify a linear multiplier operator \( M_\lambda : G \to E \) given by \( M_\lambda f = \{ \lambda_n \hat{f}(n) \} \) for any \( f = \sum_{n=0}^{\infty} \hat{f}(n) u_n \in G \). From now on we shall not distinguish the multiplier operator \( M_\lambda \) from the multiplier \( \lambda \) associated with this operator.

It should be pointed out that in general the problem of characterization of multipliers even for the special spaces \( G \) and \( E \) is a difficult task. An unpublished result of Fefferman states that \( \{ \lambda_n \} \) is a multiplier from the Hardy space \( H_1 \) on the disc \( \mathbb{U} \) into \( \ell_1 \) if and only if

\[
\sup_{m \in \mathbb{Z}_+} \sum_{j=1}^{\infty} \left( \sum_{k=jm}^{(j+1)m-1} |\lambda_k| \right)^2 < \infty.
\]

For a survey of the results on multipliers from Hardy spaces \( H_p \) to \( \ell_q \) for various \( p \)
and $q$ we refer to [14] and the references included therein. It’s worth mentioning that $p$-summing multipliers were studied in [1] in case of $H_p$ spaces and within the general setting of abstract Hardy spaces in [13].

The paper is organized as follows. In the preliminary section we recall fundamental definitions and describe a solid hull of spaces $X$ satisfying the condition $H_\infty \hookrightarrow X \hookrightarrow H_2$. We use this fact to characterize multipliers from certain spaces of analytic functions to Banach sequence lattices. Then, in section 2, we switch to the investigation of compactness of multiplier operators. We obtain complete characterization in case $H_\infty \hookrightarrow X \hookrightarrow H_2$ and deliver some necessary and sufficient conditions in the general case. The proof of the main theorem is based on calculation of the Hausdorff measure of noncompactness for diagonal operators between Banach sequence lattices.

Let us point out that we haven’t found in the literature results concerning compactness of operators of that kind even for the most natural settings i.e., the domain being the Hardy spaces $H_p$ and range in sequence spaces $\ell_q$. Hence as corollaries we obtain new theorems for the classical case. Note that in [3] the compactness of multipliers with the range in $H_p$ spaces was discussed.

1. Preliminaries

We shall use standard notation and notions from Banach theory, as presented e.g., in [12]. The term ‘operator’ stands for a bounded and linear mapping while ‘$\hookrightarrow$’ denotes continuous inclusion between Banach spaces.

Let $E$ be a Banach space of real (or complex) sequences on the set of non-negative integers $\mathbb{Z}_+$. If $E$ is an ideal space equipped with the monotone norm i.e., $|y_n| \leq |x_n|$ for each $n \in \mathbb{Z}_+$ and $\{x_n\} \in E$ implies $\{y_n\} \in E$ with $\|\{y_n\}\|_E \leq $ $\|\{x_n\}\|_E$, then $E$ is said to be a Banach sequence lattice. Sequence space $F$ is said to be solid if it is a Banach sequence lattice. A solid hull $S(F)$ of a sequence space $F$ is the smallest solid space containing $F$. If $G$ is a vector space of analytic
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functions then by a solid hull of $G$ we mean a solid hull of $\hat{G}$. In what follows
$\{e_n\}$ stands for a standard basis in $c_0$.

A Banach sequence lattice $E$ on $\mathbb{Z}_+$ is said to be order-continuous if every non negative non increasing sequence in $X$ which converges to 0 point-wise converges to 0 in the norm topology of $E$. It can be easily seen that if $E$ is an order-continuous Banach sequence lattice than $\{x_n\} \in E$ implies $\|\sum_{k=n}^{\infty} x_k e_k\|_E \to 0$ as $n \to \infty$.

Since we consider quite general situation we present examples of special spaces for which theorems – that will be proved below – can be in particular used. Let $X$ be a rearrangement invariant space on $T = [0, 2\pi]$. Denote by $HX$ the Banach space of all $f \in H(U)$ such that

$$
\|f\|_{HX} := \sup_{0 \leq r < 1} \|f_r\|_X < \infty,
$$

where as usual $f_r(t) := f(re^{it})$ for $t \in T$ and $0 \leq r < 1$. The spaces $HX$ are abstract variants of the classical spaces that occur in the analysis. For instance in case $X = L_p$, $1 \leq p \leq \infty$, $HL_p$ is the Hardy space $H_p$. We pay particular interest to the space $H_2$. It is well known that $f \in H_2$ if and only if $\{\hat{f}(n)\} \in \ell_2$ and $\|f\|_{H_2} = \|\{\hat{f}(n)\}\|_{\ell_2}$. For details on Hardy spaces see [8]. We refer the reader also to [13] where the abstract Hardy spaces $HX$ were studied.

In what follows we investigate the conditions a sequence $\lambda$ must satisfied in order to the multiplier $M_\lambda$ be compact. Recall that a bounded operator $T: X \to Y$ between Banach spaces is compact if the image of the unit ball $B_X$ is a relatively compact set in $Y$. It is well known that compact operators have so called ideal property, i.e., if $T: X \to Y$ is compact, $S: X_0 \to X$ and $R: Y \to Y_0$ are bounded operators between Banach spaces, then the composition $S \circ T \circ R: X_0 \to Y_0$ is compact operator.

The Proposition hereunder describes the solid hull of $X$ in the case of $H_\infty \hookrightarrow X \hookrightarrow H_2$. 

\[ -4 - \]
Proposition 1.1. Let $X$ be a Banach space such that $H_\infty \hookrightarrow X \hookrightarrow H_2$. Then $S(X) = \ell_2$.

Proof. By the result due to Kisliakov it follows $S(H_\infty) = \ell_2$ (see [11]). Since $f = \sum_{n=0}^{\infty} \hat{f}(n)u_n \in H_2$ if and only if $\{\hat{f}(n)\} \in \ell_2$, $\ell_2$ is a solid sequence space and $A \hookrightarrow B$ implies $S(A) \subset S(B)$ for any vector spaces $A, B$, then we have

$$\ell_2 = S(H_\infty) \subset S(X) \subset S(H_2) = \ell_2.$$

We use the above Proposition to describe the space $\mathcal{M}(X, E)$ with $H_\infty \hookrightarrow X \hookrightarrow H_2$. We take advantage of the following result of Anderson and Shields (see [2]):

If $E$ is a solid space and $A$ vector sequences space then $\mathcal{M}(A, E) = \mathcal{M}(S(A), E)$.

Corollary 1.2. If $H_\infty \hookrightarrow X \hookrightarrow H_2$ then for any Banach sequence lattice $E$

$$\mathcal{M}(X, E) = \mathcal{M}(\ell_2, E),$$

and each multiplier operator satisfies factorization

$$X \xrightarrow{M_\lambda} E \xrightarrow{i} \xrightarrow{D_\lambda} \ell_2.$$ (1)

Let us mention that there are general results for the description of the space of multipliers between Banach sequence lattices. We refer the reader to [7], where the multipliers between Orlicz sequence spaces were calculated. Further, note that if $E$ is 2-concave, then (see [5])

$$\mathcal{M}(\ell_2, E) = (E^{(1/2)})^{(2)},$$
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where given any Banach sequence lattice $E$ and $1 < p < \infty$, the Banach sequence lattice $E^{(p)}$ is defined to be a Banach lattice of all sequences $x = \{x_n\}$ such that 
\[ \{|x_n|^p \} \in E \] equipped with the norm 
\[ \|x\|_{E^{(p)}} = \|\{|x_n|^p\}\|_E^{1/p} \] (see [12]). A Banach sequence lattice $E$ is $p$-concave, $1 \leq p < \infty$ (see [12]) if there exists a constant $C > 0$ such that for any finite set $\{x_1, \ldots, x_n\}$ of elements from $E$ the following inequality holds:

\[
\left( \sum_{k=1}^{n} \|x_k\|^p_E \right)^{1/p} \leq C \left( \left\| \left( \sum_{k=1}^{n} |x_k|^p \right)^{1/p} \right\|_E \right).
\]

In consequence if $H_\infty \hookrightarrow X \hookrightarrow H_2$ and $E$ is 2-concave then

\[ \mathcal{M}(X, E) = \left( (E^{(1/2)})^r \right)^{(2)}. \]

In case $X = H_p$ with $2 \leq p \leq \infty$ we obtain known result for the classical case (see [10], cf. [8, Theorem A.5] where the description of $S(H_p)$, $2 \leq p < \infty$ was gained by a different method).

**Corollary 1.3.** Let $2 \leq p \leq \infty$. Then the following statements are true:

(i) If $1 \leq q \leq 2$ then $\lambda \in \mathcal{M}(H_p, \ell_q)$ if and only if $\lambda \in \ell_r$, where $\frac{1}{r} = \frac{1}{q} - \frac{1}{2}$.

(ii) If $2 \leq q \leq \infty$ then $\mathcal{M}(H_p, \ell_q) = \ell_\infty$.

2. Compact multipliers

In this section we present results concerning compactness of multiplier operators. We start with Proposition 2.1 where we give a sufficient condition for $M_\lambda: X \to E$ to be compact. Recall that if $\|T_n - T\| \to 0$ and $T_n: X \to Y$ are compact operators between Banach spaces then $T: X \to Y$ is compact.

**Proposition 2.1.** Let $X$ be a Banach space such that $X \hookrightarrow H_1$ and $E$ be an order-continuous Banach sequence lattice. If $\lambda \in E$ then the operator $M_\lambda: X \to E$ is compact.
Proof. Assume $\lambda := \{\lambda_n\} \in E$. Since $X \hookrightarrow H_1$ there exists $C > 0$ such that for every $n \in \mathbb{Z}_+$

$$|\hat{f}(n)| \leq C\|f\|_X.$$ 

This implies $\|M_\lambda f\|_E \leq C\|\lambda\|_E\|f\|_X$ for any $f \in X$ and thus $M_\lambda : X \rightarrow E$ is bounded.

For $\lambda \in E$ denote $\lambda^n := \sum_{k=0}^{n} \lambda_k e_k$. Evidently $M_{\lambda^n}$ is compact as a finite rank operator. For any $f = \sum_{n=0}^{\infty} \hat{f}(n) u_n \in X$ by the above inequality, we have

$$\|(M_{\lambda^n} - M_\lambda)f\|_E = \left\| \sum_{k=n+1}^{\infty} \lambda_k \hat{f}(k) e_k \right\|_E \leq C \left\| \sum_{k=n+1}^{\infty} \lambda_k e_k \right\|_E \|f\|_X,$$

and whence

$$\|(M_{\lambda^n} - M_\lambda)\| \leq C \left\| \sum_{k=n+1}^{\infty} \lambda_k e_k \right\|_E.$$ 

Since $E$ is order-continuous and $\lambda \in E$ implies $\|\sum_{k=n+1}^{\infty} \lambda_k e_k\|_E \to 0$, we conclude by the remark preceding the Proposition that $M_\lambda$ is compact.

We cannot expect that the condition from the above proposition is also necessary for the compactness of multiplier operators. Nevertheless, we show below that the compactness of $M_\lambda$ implies $\limsup |\lambda_n| \to 0$.

Proposition 2.2. Let $X$ be a Banach space such that the sequence $\{u_n\}$ is bounded in $X$ and $E \hookrightarrow \ell_\infty$ be a Banach sequence lattice. If $M_\lambda : X \rightarrow E$ is compact then

$$\limsup_{n \to \infty} |\lambda_n| = 0. \tag{2}$$

Proof. Suppose that $\limsup_{n \to \infty} |\lambda_n| > 0$. This implies in particular the existence of a subsequence $\{k_n\}$ and a constant $\varepsilon > 0$ such that $\inf_{n \in \mathbb{Z}_+} |\lambda_{k_n}| \geq \varepsilon > 0$. Take the sequence $\{u_{k_n}\}$. Since $E \hookrightarrow \ell_\infty$, then $\|e_n\|_E \geq C$ for some $C > 0$ and any
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$n \in \mathbb{Z}_+$ and we have

$$\|M_\lambda u_k - M_\lambda u_m\|_E = \|\lambda_k |e_k| + |\lambda_m| e_m\|_E \geq C \varepsilon,$$

for any $n \neq m$. This completes the proof. \(\square\)

In what follows we shall see that the condition (2) implies compactness of $M_\lambda: X \to E$ if $H_\infty \hookrightarrow X \hookrightarrow H_2$ and $\ell_2 \hookrightarrow E \hookrightarrow \ell_\infty$. The proof is based on the Proposition 2.4 where we calculated the measure of noncompactness for a diagonal operator $D_\lambda: F \to E$ induced by a sequence $\lambda$.

Recall that for a bounded subset $A$ of a metric space $X$ the Hausdorff measure of noncompactness of $A$ is given by

$$\alpha(A) := \{ \varepsilon > 0; A \text{ has a finite } \varepsilon \text{ net in } X\}.$$

If $T: X \to Y$ is a bounded operator between Banach spaces $X$ and $Y$, then the Hausdorff measure of noncompactness $\beta(T)$ of an operator $T$ is a value $\beta(T) := \alpha(TB_X)$, where $B_X$ is a unit ball of $X$. It is clear that $T$ is compact if and only if $\beta(T) = 0$. Moreover, $\beta(T) \leq \|T\|$. We recall the main properties of the measure of noncompactness. If $A, B$ are bounded subsets of a Banach space $X$, then

$$A \subset B \Rightarrow \alpha(A) \leq \alpha(B),$$

$$\alpha(A + B) \leq \alpha(A) + \alpha(B),$$

$$\alpha(tA) = |t| \alpha(A) \text{ for each } t \in \mathbb{K}.$$
where for \( n \in \mathbb{Z}_+ \), \( P_n : E \to E \) is given by \( P_n x := \sum_{k=1}^n x_k e_k \) for \( x = \{x_n\} \in E \).

**Proof.** Observe that by our hypothesis \( E \) is order-continuous it follows easily

\[
\sup_{n \in \mathbb{Z}_+} \| P_n \| < \infty.
\]

Since \( A \subset P_n A + (I - P_n) A \) and \( P_n A \) has finite dimension it is clear that

\[
\alpha(A) \leq \alpha(P_n A) + \alpha((I - P_n) A) \leq \alpha((I - P_n) A) \leq \sup_{x \in A} \|(I - P_n)x\|.
\]

In consequence

\[
\alpha(A) \leq \lim_{n \to \infty} \sup_{x \in A} \|(I - P_n)x\|.
\]

To prove the remaining inequality let \( \varepsilon > 0 \) and choose \( Z := \{z^1, \ldots, z^k\} \) to be a \((\alpha(A) + \varepsilon)\) net of \( A \). Since

\[
A \subset Z + (\alpha(A) + \varepsilon)B_E,
\]

for any \( x \in A \) there exist \( z \in Z \) and \( b \in B_E \) such that

\[
x = z + (\alpha(A) + \varepsilon)b.
\]

From this and the triangle inequality we have

\[
\sup_{x \in A} \|(I - P_n)x\| \leq \sup_{1 \leq i \leq k} \|(I - P_n)z^i\| + (\alpha(A) + \varepsilon).
\]

Since \( E \) is order-continuous we get that \( \|(I - P_n)z^i\|_E = \| \sum_{k=n+1}^\infty z_k^i e_k \|_E \to 0 \) as \( n \to \infty \) and in consequence

\[
\lim_{n \to \infty} \sup_{x \in A} \|(I - P_n)x\|_E \leq \alpha(A) + \varepsilon.
\]

Since \( \varepsilon > 0 \) was arbitrary the proof is completed. \( \square \)
Proposition 2.4. Let $F$ and $E$ be order-continuous Banach sequence lattices on $\mathbb{Z}_+$ and $\ell_1 \hookrightarrow F \hookrightarrow E \hookrightarrow \ell_\infty$. Then

$$\beta(D_\lambda : F \to E) = \limsup_{n \to \infty} |\lambda_n|.$$ 

Proof. Since $\ell_1 \hookrightarrow F \hookrightarrow E$, we have $\mathcal{M}(F,E) = \ell_\infty$. Let $\varepsilon > 0$. There exists a subsequence $\{\lambda_{k_n}\}$ of $\{\lambda_n\}$ such that for all $n \in \mathbb{Z}_+$

$$|\lambda_{k_n}| > \limsup_{n \to \infty} |\lambda_n| - \varepsilon.$$

Denote by $\lambda^k$ the sequence $\{\lambda_{k_n}\}$. From Lemma 2.3 we get

$$\beta(D_\lambda) = \alpha(D_\lambda B_F) \geq \alpha(D_\lambda^k B_F) \geq \alpha(\{\lambda^k e_n; n \in \mathbb{Z}_+\}) \geq \limsup_{n \to \infty} |\lambda_n| - \varepsilon.$$

Thus

$$\beta(D_\lambda) \geq \limsup_{n \to \infty} |\lambda_n|.$$

We shall prove the opposite inequality. Take $\varepsilon > 0$ and observe that a set $A := \{n; |\lambda_n| > \limsup_{n \to \infty} |\lambda_n| + \varepsilon\}$ is finite. Hence while calculating the measure of noncompactness of $D_\lambda$ we can assume without loss of generality that $A = \emptyset$. We have

$$\beta(D_\lambda) = \alpha(D_\lambda B_F) \leq \|D_\lambda\| \leq \limsup_{n \to \infty} |\lambda_n| + \varepsilon.$$

Finally we get

$$\beta(D_\lambda) \leq \limsup_{n \to \infty} |\lambda_n|,$$

and the result follows. \qed

Theorem 2.5. Let $H_\infty \hookrightarrow X \hookrightarrow H_2$ and $\ell_2 \hookrightarrow E \hookrightarrow \ell_\infty$ with $E$ being order-continuous. Operator $M_\lambda : X \to E$ is compact if and only if

$$\limsup_{n \to \infty} |\lambda_n| = 0. \tag{4}$$
Proof. Suppose (4) holds. Since $M_{\lambda}: X \to E$ admits the factorization (1) and $\ell_2 \hookrightarrow E \hookrightarrow \ell_\infty$ from Proposition 2.4 and by the ideal property of compact operators it follows that $M_\lambda$ is compact. To obtain the converse we use Proposition 2.2 and the proof is done.

From Theorem 2.5 we obtain the following Corollary in $H_p$ and $\ell_q$ case.

**Corollary 2.6.** Let $2 \leq p \leq \infty$ and $2 \leq q < \infty$. The operator $M_\lambda: H_p \to \ell_q$ is compact if and only if

$$\limsup_{n \to \infty} |\lambda_n| = 0.$$ 

Further we consider the compactness of $M_\lambda: X \to E$ where $X \hookrightarrow H_2$ and $E$ satisfies some extra condition. The proof is based upon the following theorem which is the extension of Pitt’s theorem (cf. [15]) on compact operators on $\ell_p$ spaces (see [6]). To state the theorem we need some additional terminology.

We say that a Banach sequence lattice $E$ satisfies an upper $p$-estimate, (resp., a lower $p$-estimate), if there exists a constant $C > 0$ such that for every choice of finitely many pairwise disjoint elements $\{x_i\}_{i=1}^n$ in $E$, we have

$$\left\| \sum_{i=1}^n x_i \right\|_E \leq C \left( \sum_{i=1}^n \|x_i\|_E^p \right)^{1/p},$$

resp.,

$$\left\| \sum_{i=1}^n x_i \right\|_E \geq \frac{1}{C} \left( \sum_{i=1}^n \|x_i\|_E^p \right)^{1/p}.$$ 

**Theorem 2.7 ([6 Theorem 1]).** Let $E \hookrightarrow \ell_\infty$ and $F \hookrightarrow \ell_\infty$ be quasi-Banach sequence lattices, $E$ with an upper $t$-estimate and $F$ with a lower $u$-estimate. If $t > u$, then every operator $T$ from $E$ to $F$ is compact.

We refer the reader to [6] where some application of the above theorem was
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shown and examples of Banach sequence lattices satisfying lower and upper estimates were presented.

**Proposition 2.8.** Let $H_{\infty} \hookrightarrow X \hookrightarrow H_2$ and $E \hookrightarrow \ell_{\infty}$ be a Banach sequence lattice satisfying a lower $u$-estimate with $u < 2$. Then the operator $M_\lambda: X \to E$ is compact if and only if $\lambda \in M(\ell_2, E)$.

**Proof.** Since compact linear map is in particular continuous, from Corollary 1.2 it follows that $\lambda \in M(\ell_2, E)$.

To prove the converse assume that $M_\lambda$ is a bounded linear map and observe that as in (1) $M_\lambda = i \circ j \circ D_\lambda$, where $i: X \to H_2$ is the inclusion map, $j$ is the isometry and $D_\lambda: \ell_2 \to E$ is a diagonal operator induced by a sequence $\lambda$. Since $E$ satisfies a lower $u$-estimate with $u < 2$ from Theorem 2.7 it follows that $D_\lambda$ is compact. By the ideal property of compact operators the proof is complete.

We finish the paper with giving the application of the above Proposition to the case of $H_p$ and $\ell_q$ spaces.

**Corollary 2.9.** Let $M_\lambda: H_p \to \ell_q$ and $1 \leq q < 2 \leq p \leq \infty$. Operator $M_\lambda$ is compact if and only if $\lambda \in \ell_r$ with $\frac{1}{r} = \frac{1}{q} - \frac{1}{2}$.

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