Thermalization of gluon matter including \( gg \leftrightarrow ggg \) interactions

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Within a pQCD inspired kinetic parton cascade we simulate the space time evolution of gluons which are produced initially in a heavy ion collision at RHIC energy. The inelastic gluonic interactions \( gg \leftrightarrow ggg \) do play an important role: For various initial conditions it is found that thermalization and the close to ideal fluid dynamical behaviour sets in at very early times. Special emphasis is put on color glass condensate initial conditions and the ‘bottom up thermalization’ scenario. Off-equilibrium 3 \( \rightarrow \) 2 processes make up the very beginning of the evolution leading to an initial decrease in gluon number and a temporary avalanche of the gluon momentum distribution to higher transversal momenta.

1. INTRODUCTION

It had been demonstrated that the measured momentum anisotropy parameter \( v_2 \) at RHIC energy can be well understood if the expanding quark-gluon matter is described by ideal hydrodynamics. This important finding suggests that a strongly interacting and locally thermalized state of matter has been created which behaves almost like a perfect fluid. On the other hand, the initial situation of the quark-gluon system is far from thermal equilibrium. It is thus important to understand how and which microscopic partonic interactions can thermalize the system within a short timescale and can be responsible as well for its (nearly) ideal hydrodynamical behaviour.

A traditional way to study thermalization of particles is to carry out microscopic transport simulations. A standard parton cascade analysis incorporating only elastic (and forward directed) 2 \( \leftrightarrow \) 2 collisions described via one-gluon exchange, shows that thermalization and early (quasi-)hydrodynamical behaviour (for achieving sufficient elliptic flow) can not be built up or maintained, but only if a much higher, constant and isotropic cross section \( \sigma_{\text{eff}} \approx 45 \text{ mb} \) is being employed \[1\]. The gluons being treated as semi-classical degrees of freedom would then aquire a collional width \( \Gamma \approx n\sigma_{\text{eff}}v_{\text{rel}} \) being much larger than the typical energy \( E \approx 3T \). The gluons would then resemble very broad excitations and can not be considered at all as quasi-particles, if such large and constant cross sections are employed.

In contrast, a kinetic parton cascade algorithm \[2\] has been developed with perturbative QCD inspired processes including for the first time inelastic (‘Bremsstrahlung’)
collisions $gg \leftrightarrow ggg$. The multiparticle back reaction channel is treated fully consistently by respecting detailed balance within the same algorithm. The three-body gluonic interactions are described by the matrix element

$$\left| \mathcal{M}_{gg \rightarrow ggg} \right|^2 = \left( \frac{9g^4}{2} \frac{s^2}{(q_1^2 + m_D^2)^2} \right) \left( \frac{12g^2q_1^2}{k_1^2[(k_1 - q_1)^2 + m_D^2]} \right) \Theta(k_1 \Lambda_g - \cosh y), \quad (1)$$

where an effective Landau-Pomeranchuk-Migdal suppression is considered. $\Lambda_g$ denotes the gluon mean free path, which is given by the inverse of the total gluon collision rate $\Lambda_g = 1/R_g$. This leads to a lower cutoff of $k_1$ and to an effective increase of the collision angles. Typical Debye screened total pQCD cross section scale roughly like the inverse temperature squared, $\sim 1/T^2$, so that the characteristic ratio $\Gamma/E$ should be (significantly) smaller than 1, as long as the coupling stays small enough. The possible importance of incorporating inelastic reactions on overall thermalization was raised in the so called ‘bottom up thermalization’ picture \cite{3}. It is intuitively clear that gluon multiplication should not only lead to chemical equilibration, but also should lead to a faster kinetic equilibration. This has been demonstrated in detail in \cite{2}.

A first picture of the production of the primary partons at the very onset of a heavy ion collision is based on a free superposition of minijets being liberated in the individual semihard nucleon-nucleon interactions with transverse momentum being greater than a certain cutoff $p_0$ \cite{2}. Conditions of a color glass condensate for the initial partons will also be given and further detailed in the next section.

Phenomenologically the cutoff $p_0$ can be chosen in a way to fit the (final) $dE_T/dy$ as seen in experiment. A value of $p_0 = 1.4$ GeV would meet a final $dE_T/dy \approx 625$ GeV. Thus the minijets production at $p_0 = 1.4$ GeV seems to give an appropriate initial condition of gluons at RHIC. In Fig. 1 the transverse energy at midrapidity is depicted, obtained from simulations with the different initial conditions, being normalized to the...
energies at $t = 0.5$ fm/$c$. One sees a rather unique behavior of a decreasing, normalized $dE_T/dy|_{y=0}$, especially for initial conditions employing minijets with $p_0 = 1.3 - 1.5$ GeV and CGC with $Q_s = 1.0$ GeV. Comparisons to the normalized transverse energy per unit rapidity for an ideal hydrodynamical expansion show that the collective expansion due to the pQCD interactions is quasi ideal at $0.5 - 1.5$ fm/$c$. At later times the decrease of $dE_T/dy|_{y=0}$ slows down, since the collision rates become smaller, especially in the outer, transversally expanding region.

Taking $p_0 = 1.4$ GeV for the initial minijets, the parton evolution for noncentral collisions at RHIC energy is simulated in order to calculate the elliptic flow parameter $v_2$. Figure 2 shows the time evolution of $v_2$ extracted at midrapidity for various impact parameter $b$. These calculations are still preliminary. The results gives strong indication that an early pressure is being built up within that pQCD inspired description. The symbols in Fig. 2 mark the time from which the energy density in the central region decreases below 1 Gev/fm$^3$. If we take the $v_2$ values at that marked times as the contribution from the partonic phase, they lie well in the region covered by the experimental data.

\section{2. Thermalization of the color glass condensate}

For the initial gluon distribution from the saturation picture we employ an idealized and boost-invariant form \cite{5} and express it as

\[ f(x,p)|_{z=0} = \frac{c}{\alpha_s N_c \tau_f} \frac{1}{\Theta(Q_s^2 - p_T^2)} \delta(p_z) \Theta(p_T^2), \quad (2) \]

which is described by $Q_s$, the momentum scale at which gluon distribution saturates. The boost-invariance leads to the equality of momentum and space-time rapidity, i.e., $\eta = y$, for the initial gluons. The value of parameters are taken from \cite{5}: $N_c = 3$ for
SU(3), $c = 1.3$, $\alpha_s = 0.3$, and the corresponding formation time, at which the gluon distribution becomes dilute enough, is given as $\tau_0 = c/(\alpha_s N_c Q_s)$. Hence, the cascade operates from $\tau_0 = 0.4\text{fm/c}$ for $Q_s = 1$ GeV and $\tau_0 = 0.18\text{fm/c}$ for $Q_s = 2$ GeV and $Q_s = 3$ GeV. For the calculation depicted in Figs. 3 and 4 the gluons are produced within a transverse radius of 6 fm (Au nucleus) and within $|\eta| < 3$ longitudinally. Fig. 3 states a first dynamical realization of the so called ‘bottom up thermalization’ scenario as advocated in [3]. The evolution of the momentum occupation is shown for four subsequent times sampled within a space-time rapidity interval of $\Delta \eta = 0.1$ and a central transverse region of $R \leq 1.5$ fm. One recognizes the population of the ‘soft’ gluons and a subsequent degradation of the ‘hard’ initial gluons. In addition, also harder gluons are produced. All this happens roughly within the first 1 fm/c. In the last picture we also see that all particles are clearly more centered around the origin, demonstrating the ongoing cooling and quasi hydrodynamical behaviour from 2 to 4 fm/c. In Fig. 4 the transverse momentum spectrum is given for various times. Comparing the $p_T$ spectra between 2.0 and 4.0 fm/c, the local system at the central region clearly is at equilibrium at that later times, and the spectrum steepens continuously, since the thermodynamical work in outward direction cools down the system.

Thermalization somehow resembles the idealistic ‘bottom up scenario’ [3] with low momentum gluons being produced and populated. On the other hand, a couple of important differences do show up. In Fig. 5 the time evolution of particle number per rapidity for various saturation momenta $Q_s$ is shown. In these simulations strict Bjorken geometry in longitudinal direction is preserved by reflection at cylindrical boundaries [4]. The number of gluons is initially slightly decreasing, although a strong parametric enhancement has been advocated in [3] due to Bremsstrahlung production, which is not observed in the present calculations. This is true for all chosen saturation scales. The initial gluons are seemingly oversaturated, if the system thermalizes rather immediately [4]. Only if thermalization would happen on a much longer time scale, the gluon number would become undersaturated at a subsequent time, at which net gluon production would
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Figure 7. Time evolution of the rescaled temperature $T \cdot t^{1/3}$ and the momentum anisotropy.

then take place.

Also it is found [4] that indeed the $ggg \rightarrow gg$ processes dominate the dynamics from the very beginning up to approximately $2 \text{ fm/c}$. This causes, in particular, a production of gluons with momenta much larger than the saturation scale $Q_s$. This can also be seen from Figs. 3 and 4, where gluons with such high momentum do show up. This behaviour resembles an avalanche from low momentum to high momentum, which has recently also been found in classical Yang-Mills simulation [6]. Both phenomena might be connected and clearly deserve further investigation. It also seems that the gluon distribution at higher momenta above the saturation scale evolves more faster to an exponential shape than the low momentum gluons [4]. The name bottom-up has then to be reversed.

As a last point the extraction of a possible thermalization scale is briefly discussed [4]. Fig. 6 gives the time evolution of the effective temperature $T = E/3N$ for three different saturation scales. At the very beginning the temperature increases by a small amount because of the decrease in gluon number, but then decreases more or less in accordance with ideal 1-dimensional hydrodynamical expansion. This can be seen more explicitly in the upper Fig. 7. (Quasi-)Ideal hydrodynamical behaviour sets in at a timescale $\theta = 3.3 \text{ fm/c} (Q_s = 1 \text{ GeV}), \theta = 1.9 \text{ fm/c} (Q_s = 2 \text{ GeV})$ and $\theta = 1.2 \text{ fm/c} (Q_s = 3 \text{ GeV})$. Thermalization should set in slightly earlier. Kinetic equilibration is characterized by the time evolution of the momentum anisotropy, $2 < p_z^2 > / < p_T^2 >$, which maintains a value of one at equilibrium. The momenta of gluons are initially directed transversely in local discs according to Eq. (2). Therefore the momentum anisotropy has a vanishing initial value. An exponential fit to the time evolution gives $\theta = 2.5 \text{ fm/c} (Q_s = 1 \text{ GeV}), \theta = 1.5 \text{ fm/c} (Q_s = 2 \text{ GeV})$ (cf lower Fig. 4), $\theta = 1.0 \text{ fm/c} (Q_s = 3 \text{ GeV})$, although equilibration happens faster than exponential at the beginning.
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