Detection of dynamic errors in aircraft flight data

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Abstract. The paper considers the algorithmic and methodologic support for detecting dynamic errors in aircraft on-board measurements using parameter identification and system approach to flight data analysis. Examples of practical application of the considered methods and algorithms for detecting dynamic errors in modern aircraft flight data are presented.

1. Introduction

High accuracy of on-board measurements of flight parameters and correct estimation of errors that occur plays the key role in flight test analysis and aircraft technical state monitoring. The on-board measurement and recording systems generally meet the requirements for measurement accuracy, but in some cases special approaches are required to estimate dynamic errors. Moreover, the accuracy of measurements of such important flight parameters like angles of attack and glide, overload, altitude and airspeed depend greatly on the coordinates of the sensors installation, the position of the center of mass of the aircraft, the length of air pressure ducts and other technical features, as well as on the flight mode and type of maneuver. Moreover, the on-board digital recording systems, in their turn, introduce additional errors, among which the most significant are usually time shifts that occur due to inaccurate synchronization of heterogeneous information flows, as well as due to the lack of consideration for the sequence of sensors sampling within the information frame. The considered errors are dynamic and are related to the movement of the aircraft. These errors detection becomes particularly relevant, for example, in the processing of flight test data, in post-flight control of piloting near operational limits, in the flight accident investigation, etc.

2. Algorithm for accounting measurement errors

Since the above mentioned errors are clearly systematic in nature, it is advisable to apply a systematic approach for their estimation. In this paper, the relations between the flight parameters determined by the equations of spatial motion of the aircraft are used for this purpose.

From the general model of spatial motion of the aircraft [1], the following equations are selected:
\[
\begin{align*}
\frac{d\alpha}{dt} &= \omega_z - \frac{1}{\cos \beta} \left[ \left( \frac{a_x - \omega_y \sin \beta}{V} \right) \sin \alpha + \left( \frac{a_z + \omega_z \sin \beta}{V} \right) \cos \alpha \right], \\
\frac{d\beta}{dt} &= \frac{a_z}{V} \cos \beta - \left( \frac{a_x}{V} \sin \beta \right) \cos \alpha + \left( \frac{a_y}{V} \sin \beta + \omega_z \right) \sin \alpha, \\
\frac{dV}{dt} &= a_x \cos \alpha \cos \beta - a_z \sin \alpha \cos \beta + a_z \sin \beta, \\
\frac{d\gamma}{dt} &= \omega_x \sin \gamma + \omega_z \cos \gamma, \\
\frac{dt}{dt} &= \omega_x - t \tan \nu (\omega_x \cos \gamma - \omega_z \sin \gamma),
\end{align*}
\] (1)

where, \(\alpha, \beta\) — the angles of attack and sideslip, rad; \(\omega_x, \omega_y, \omega_z\) — angular velocities relative to the associated axes, rad/s; \(\nu, \gamma\) — pitch, roll, rad; \(V\) — actual airspeed, m/s; \(a_x, a_y, a_z\) — accelerations along aircraft associated axes defined by the formulas:

\[
\begin{align*}
a_x &= g (n_x - \sin \nu), \\
a_y &= g (n_y - \cos \nu \cos \gamma), \\
a_z &= g (n_z + \cos \nu \sin \gamma),
\end{align*}
\] (2)

where \(n_x, n_y, n_z\) — overloads along the associated axes, \(g\) — acceleration due to gravitation force.

The main feature of these equations is that they do not depend on the aerodynamic coefficients of the aircraft, since the accelerations included in the right parts can be calculated using the values of measured overloads in flight.

Equations (1), (2) in vector mode have the form

\[
y'(t) = f(y(t), a, u(t)),
\] (3)

where \(y(t), u(t)\) — vectors of output and input signals; \(a\) — vector of parameters.

In this case, the vector \(y(t)\) consists of \(\alpha(t), \beta(t), V(t), \nu(t), \gamma(t)\) signals. The vector of input signals includes the values of angular velocities \(\omega_x, \omega_y, \omega_z\) and overloads \(n_x, n_y, n_z\) measured in flight.

Initial conditions \(y(t_0)\) are assigned based on the results of parameter measurements \(\alpha(t), \beta(t), V(t), \nu(t), \gamma(t)\) at the start time of the processing interval.

Direct numerical integration of the system of differential equations (1) taking into account (2) generally does not give positive results, since the constant components of the measurement errors of the input signals \(\omega_x, \omega_y, \omega_z\) and \(n_x, n_y, n_z\) create linear trend-type errors at the output of the integrals, which does not allow comparing the output signals of the model and the object. To eliminate this effect, we introduce a vector of unknown parameters \(a\) consisting of constant components of measurement errors of signals \(\omega_x, \omega_y, \omega_z\) and \(n_x, n_y, n_z\):

\[
a^T = \left[ C_{a_x} C_{a_y} C_{a_z} C_{n_x} C_{n_y} C_{n_z} \right].
\] (4)

To find parameter (1.4) estimates, we apply, for example, the maximum likelihood identification algorithm [2]. The observation model in vector mode has the form,

\[
z(t) = y(t) + \eta(t),
\] (5)
where, \( \eta(t) \) — the observation noise, which is a vector normal random sequence of the white noise type with zero mean and a constant covariance matrix \( R \). The observation vector includes signals whose derivatives are on the left side (1):

\[
z^T(t) = [\alpha(t), \beta(t), V(t), \nu(t), \gamma(t)]. \tag{6}
\]

The functional to be minimized takes the form,

\[
J(a) = \sum_{i=4}^{N} ((z(t_i) - \hat{y}(t_i, a, u(t_i))))^T R^{-1} (z(t_i) - \hat{y}(t_i, a, u(t_i))), \tag{7}
\]

where, \( \hat{y}(t_i, a, u(t_i)) \) — the output signal of the object model (1), (2); \( N \) — the number of samples in the processed data. The numerical optimization algorithm used to find parameter estimates is a modification of the classical Newton method and is given in [2] or [3].

To improve accuracy, the model should take into account the shift of overload sensors and sensors of angles of attack and sideslip relative to the center of mass. Corrections to overload measurements are calculated using the following formulas [4]:

\[
\Delta n_x = \frac{1}{g} \left[ (\omega_y^2 + \omega_z^2) x_0 - \omega_x \omega_y y_0 - \omega_x \omega_z z_0 + \frac{d\omega_x}{dt} y_0 - \frac{d\omega_y}{dt} z_0 \right], \\
\Delta n_y = \frac{1}{g} \left[ (\omega_x^2 + \omega_z^2) y_0 - \omega_x \omega_y x_0 - \omega_x \omega_z z_0 + \frac{d\omega_x}{dt} x_0 - \frac{d\omega_z}{dt} z_0 \right], \\
\Delta n_z = \frac{1}{g} \left[ (\omega_x^2 + \omega_y^2) z_0 - \omega_x \omega_x x_0 - \omega_y \omega_y y_0 + \frac{d\omega_x}{dt} x_0 - \frac{d\omega_y}{dt} y_0 \right], \tag{8}
\]

where, \( x_0, y_0, z_0 \) — the coordinates of the sensors in the associated system with the origin at the center of mass of the aircraft. The derivatives of angular velocities are calculated numerically according to the formulas presented in [4].

When calculating the corrected overload values \( n_x, n_y, n_z \), the corrections (8) are added to the measured values, i.e.

\[
n_x = n_{x,ex} + \Delta n_x, \ n_y = n_{y,ex} + \Delta n_y, \ n_z = n_{z,ex} + \Delta n_z. \tag{9}
\]

Corrections to the measured values of the actual angles of attack and sideslip due to the shift of the sensors relative to the center of mass are calculated using the formulas,

\[
\Delta \alpha_{actu} = \frac{\omega_x}{V} x_a - \frac{\omega_y}{V} x_a + \frac{\omega_x}{V} y_a \left( n_y - 1 + \frac{\omega_x}{g} n_x \right), \tag{10}
\]

\[
\Delta \beta_{actu} = \frac{\omega_x}{V} x_a - \frac{\omega_y}{V} y_a, \tag{11}
\]

where, \( x_a, y_a, z_a, y_\alpha \) — the coordinates of the sensors in the associated system with the origin at the center of mass of the aircraft, \( m \); \( V \) — actual airspeed, \( m/s \); \( \varphi \) — the angle of deflection of the rod on which the angle of attack sensor is installed, degree (for on-board sensors, the last term (10) is zero).

Corrections (10), (11) are added to the actual values of the angles of attack and sideslip measured in flight, i.e.
where $\alpha_{\text{actu}_{\text{ex}}}$, $\beta_{\text{actu}_{\text{ex}}}$ — the values of the actual angles of attack and sideslip calculated on board.

The considered algorithm provides calculations of constant errors of measurements of angular velocities and overloads. However, its main advantage is that it allows estimating dynamic errors of various types. Let's consider examples of processing flight test data for some modern aircraft.

3. Results of flight data processing

Figure 1 shows the results of processing a flight regime with stepwise input signal in pitch channel. The signals $\alpha_{\text{ex}}, \alpha_{\text{mod}}$ show the degree of compliance of the angle of attack measured in flight and calculated in the model (1), (2) with corrections (10). The correction value is represented separately by the signal $\alpha_{\text{err}}$ (digitization along the right coordinate axis). The graph shows that the value of the error caused by the non-zero coordinates of the angle of attack sensor in this mode is $-0.4...0.7$ degrees, which is important in precision piloting modes and near operational limits. Note that this error is not calculated on board and is not taken into account when displaying the angle of attack on the display to the pilot.

![Figure 1](image1.png)

Figure 1. Estimation of measurement error of the angle of attack $\alpha_{\text{ex}}$, due to the sensor's shift relative to the center of mass. Comparison of angle of attack measured in the flight experiment $\alpha_{\text{ex}}$ and calculated in the model $\alpha_{\text{mod}}$.

Figure 2 for a different type of aircraft also shows a flight regime with the stepwise deviations of the aircraft control stick in pitch channel.

![Figure 2](image2.png)

Figure 2. Detection of an error of the time delay type for measuring the angle of attack $\alpha_{\text{ex}}$ by comparing it with the simulated angle of attack $\alpha_{\text{mod1}}, \alpha_{\text{mod2}}$ — the simulated angle of attack after model correction.

Comparison of the flight and simulated angles of attack $\alpha_{\text{ex}}, \alpha_{\text{mod}}$ shows the presence of a dynamic error of the time shift type. Additional analysis showed that a low-frequency filter is installed on board in the angle of attack channel, which smoothes the measurement noise and creates delays of up to 0.3 s at output for registration and indication to the pilot. After taking into account the specified filter transfer function, a high degree of compliance (signals) is obtained in the model.

It is interesting to note that the detected dynamic error can lead to various errors in the analysis of flight data. For example, in flight tests and in the analysis of flight accidents, the dependence $c_s(\alpha)$ is
calculated from experimental data. The values of the lift coefficient are determined by the formula, which is true under the condition $\beta \approx 0$.

$$c_y(t_i) = \left( n_i(t_i) \cos \alpha(t_i) + n_i(t_i) \sin \alpha(t_i) \right) mg - \left( P \sin(\alpha(t_i) + \varphi_{eg}) \right) qS$$

(13)

Figure 3 shows the plot of the lift coefficient against the angle of attack $\alpha_{ex}$ measured in the flight (figure 2). The view of the graph leads to the idea of the presence of aerodynamic hysteresis. The real reason is the dynamic error discussed above. Figure 4 shows the data in figure 3 after correcting the angle of attack measurements using the filter transfer function $\alpha_{ex\_corr}$. As one can see, the nature of the relationship is approaching a straight line, which coincides with the aerodynamic characteristics of this aircraft. The remaining fluctuations characterize the level of non-excluded experimental errors.

**Figure 3.** The effect of false aerodynamic hysteresis on the lift coefficient $c_y(\alpha)$, due to a delay in the measurements of the angle of attack $\alpha_{ex}$.

**Figure 4.** Refined experimental dependence $c_y(\alpha)$ after correcting the delay in angle of attack measurements.

Concluding this example we should note that actual aerodynamic hysteresis happens at much greater angles of attack, usually higher than 35 degrees. The detailed investigation of a maneuverable aircraft aerodynamic hysteresis is presented in [5].

In the case of a properly functioning on-board measurement and record system, the degree of compliance of the signals measured in flight and calculated in the model is very high, even in intensive maneuvering modes. Figure 5 shows the convergence of pitch and roll signals during the “barrel” maneuver (full 360 degrees turn in the angle of roll).

As a result of the generalized analysis of more than 20 different sections of the flight, the following values of standard deviation of mismatches were obtained: in the channels of the angles of attack and glide 0.1...0.4 degrees, the pitch angle 0.2...0.4 degrees, the roll angle 0.3...1.3 degrees, the true airspeed 0.5...0.8 m/s. Greater mismatches usually indicate that the measurement system is not working properly. So, in figure 6, deviations of the experimental values of the pitch angle from the corresponding signal of the model characterize the errors of the aircraft gyro horizon due to vigorous maneuvering in the roll channel.
Figure 5. Comparison of pitch and roll angles measured in the experiment ($\theta_{\text{ex}}$, $\gamma_{\text{ex}}$) and estimated in the model ($\theta_{\text{mod}}$, $\gamma_{\text{mod}}$) when the on-board measurement system is working correctly. Type of maneuver – “barrel” (full roll turn).

Figure 6. Detection of measurement errors of the pitch angle $\theta_{\text{exp}}$ by the aircraft gyro horizon due to roll movement, by comparison with the output model $\theta_{\text{mod}}$.

Figure 7 shows experimental and simulated values of roll angles (high degree of compliance) and vertical velocity $V_y$ (significant delay of experimental values) for the “barrel” mode. To obtain vertical velocity $V_y$ the model (1) was supplemented with the formula,

$$V_y = V[\cos \alpha \cos \beta \sin \nu - \sin \alpha \cos \beta \cos \nu \cos \gamma - \sin \beta \cos \nu \sin \gamma].$$  \hspace{1cm} (14)

The analysis showed that the cause is the inertial properties of the barometric vertical velocity meter. For accounting them, a model of the form $K/(T_p + 1)$ was additionally introduced, the coefficients of which were included in the vector of identified parameters.

Figure 7. Delay in on-board measurements of vertical velocity $V_{y,\text{ex}}$ due to the inertial properties of the barometric measuring device. Type of maneuver — “barrel”.

The result is shown in figure 8, which confirms that the introduced simple model takes into account the main components of the dynamic measurement error in $V_y$ channel.

This approach also makes it possible to detect delays in the true airspeed measurement channel. Figure 9 shows a high correspondence between the simulated and experimental speed values for the stepwise input signal in yaw channel, which was obtained after correcting the experimental data by the time shifting to advance by an amount of 0.6 s. The angle of sideslip channel in this case required no correction.
4. Conclusion
Thus, the considered approach makes it possible to detect various measurement errors of some basic parameters that are essential for flight safety issues and in during post-flight analyses. It should be noted that the idea of using a motion model of type (1) to check the correctness of onboard measurements has been repeatedly expressed earlier in some publications, for example, [2, 4, 6]. The originality of the material presented above consists in a specific algorithmic implementation, selection of a set of identifiable parameters, and determination of the types of estimated errors. In addition, the results are in good agreement with those obtained earlier using identification methods [5, 7].

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References
[1] Aerodynamics, stability and controllability of supersonic aircraft 1998 ed G S Bjushgens (Moscow: Nauka) p 816
[2] Klein V and Morelli E A 2006 Aircraft system identification: Theory and practice (Reston, VA: AIAA, Inc.)
[3] Bulgakov V V, Korsun O N, Kulabukhov V S, Stulovskii A V and Timofeev D S 2016 Journal of Computer and Systems Sciences International 55 (1) 150–61
[4] Vasilchenko K K et al Flight tests of aircraft 1996 (Moscow: Mashinostroenie) p 720
[5] Korsun O N, Stulovskii A V, Ovcharenko V N and Kanychev A V 2018 Journal of computer and systems sciences international 57 (3) 374–89
[6] Jategaonkar R V 2006 Flight vehicle system identification: A time domain methodology (Reston, VA: AIAA, Inc.)
[7] Nikolaev S V and Korsun O N 2019 Nonlinearities evaluation in the aircraft parameter identification IOP Conference Series: Materials Science and Engineering 476 012020