A stochastic model of urban characterization of a select district in Kerala

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Abstract

Urbanization is a process in which more people move from rural areas to urban centers by anticipating better physical infrastructure and enhanced quality of life. Urban life is meant to easy accessibility to the modern life style and provide gleaming standard of living. A stochastic model is a tool for estimating probability distributions of potential outcomes by allowing for random variation in one or more inputs over time. A Markov chain is a stochastic model describing a sequence of possible events in which the probability of each event depends only on the state attained in the previous event. Here a stochastic model of urban characterization of a select district, Ernakulam in Kerala is analyzed by considering 56 urban places in the district through different time points.

Keywords: Markov chain, urbanization, probability, city size, projection

Introduction

Over a five years or ten years period the demographic situation of a country is usually affected to a greater degree by changes in proportional distribution of its people than its growth. The rapidity and the magnitude of differential impact of modern development are so complex that the real mechanics of adjustment of people to economic opportunities comes, more from the distribution by movement than by natural growth. This problem has become serious concern of the planners because an unplanned concentration of people in cities may take away much of the benefits accrued by planned development. An essential characteristic of city environment is that it is susceptible to change. The spatial distribution of people in a country reflects its economic activity, material possession and prosperity. Almost half of the world’s population is now urban, a percentage that rises to 75% in the case of the developed countries. An urbanization process has taken place throughout the length of the 20th century. This phenomenon, at least as far as developed countries is concerned, has been accomplished by significant process of industrialization and economic growth, which although showing an irregular pattern over the course of the years, nevertheless demonstrate a clearly increasing tendency. Prasanth & Praseeja [1] explained, that the city can spread out uniformly and entire land is available for residential buildings. That is, no estuaries, mountains, national parks or lakes are there considerably large to affect our assumption. The city population and the rank of the city are closely related and such a relation is represented by rank size rule. Also Prasanth & Praseeja [2] compared the nature of urbanization in Kerala and Tamil Nadu empirically. Archana & Prasanth [3] explained City Expansion and Urbanization of Kerala has marked unique features. Towards the centre of the city it is seen that the population density has an increasing tendency. In other words we can say that all the cities have an outer expanding nature and this expansion is circular. Pradeep & Prasanth [4] detailed that the slum dwellers have an association turning out to be a determinant factor for stalling of urbanization. Irma G Adelman [5] detailed the stochastic nature of the size distribution within industries. He suggested that the changes in the size distribution of a group of business firms over time might be analyzed in terms of a stochastic model and such model might be used to the size distribution which would be attained in long run dynamic equilibrium- that is the steady state. Fernando Sans [6] et al., analyzed the evolution of the Spanish urban structure during the period 1900-1999.

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Preston and Bell [7] proposed a stochastic model to analyze the changing size distributions of large firms in the food processing and chain store food distributing industries during 1948-1958. Narayanaswami [8] studied the city size mobility and reveals a fund of evidence on the nature of urban growth in the period 1901 to 1971.

Methodology

In the analysis of urbanization, the total population is usually dichotomized into urban – rural classification. In the recent past urbanization in developing countries is weighted to big cities and there is no known or established pattern of distribution of people. The urban population can be polytomised by stratifying the cities according to the number of inhabitants. The hidden difference in the distribution can be identified by classifying the population in places by different sizes. Abundant empirical measures are being employed to know the hierarchical classification of cities by sizes. In this analysis the Markove model is employed to know the mobility of the cities by size. The essential ingredient of any stochastic model of mobility is thus a probabilistic description of how movement takes place from one class to another. The inherent uncertainty behaviour in these situations means that, the future development of the mobility process cannot be predicted with certainty but in only in terms of probability. The assumption is made that “the chance of moving depends only on the present class and not on the past”. If movement can be regarded as taking place at discrete points in time the appropriate model becomes a simple Markov chain.

Urban area is defined as: ‘All places, with municipality, corporation contentment or notified town area Committee, and the other places, which satisfy the following criteria, i.e., a minimum population engaged in non-agricultural pursuits and at least 5% of male working population engaged in nonagricultural pursuits, and a density of population of at least 400 people per sq.km’. In India, the rank of the population growth was higher in the urban areas than the rural areas. Urbanization is the process of concentration with respect to the size of the urban center.

City size represents the number of population in the city. The causes of changes in the city size are random in nature. The city size variable is treated as a stochastic variable. Most of the city size distribution analysis are concerned either to identify the increases in the number of urban places or to identify the differences in the size classes at a particular point of time or a period of time.

Modeling is a fundamental aspect of the design process of a complex system, as it allows the designer to compare different architectural choices as well as predict the behavior of the system under varying inputs. A Markov chain essentially consists of a set of transitions, which are determined by some probability distribution, that satisfy the Markov property. The probability distribution is obtained by observing transitions from the current time to the next. The defining characteristic of a Markov chain is that no matter how the process arrived at its present state, the possible future states are fixed. This illustrates the Markov property, the unique characteristic of Markov processes that renders them memory less. This typically leaves them unable to successfully produce sequences in which some underlying trend would be expected to occur.

Let \{X_n\} be a sequence of random variables (r.v.) taking the values \(n \in I\), assume \(I\) is finite. The sequence \{X_n\} is said to be a Markov Chain (MC) if for all \(i_0, i_1, \ldots, i_n \in I\) and for all \(n\), \(P(X_{n+1} = i_{n+1} \mid X_n = i_n, \ldots, X_0 = i_0) = P(X_{n+1} = i_{n+1} \mid X_0 = i_0)\). This property is called Markov property. The space \(I\) is called the state space and \(i_n\’s\) are called the state of Markov chain. Since the state space \(I\) is assumed finite, we get a finite MC, Markov property can be interpreted to mean that the conditional distribution of \(X_{n+1}\), given \(X_n, X_{n-1}, \ldots, X_0\) depends only on \(X_n\) assuming \(n\) as time, \(X_n\) as the present value, \(X_{n-1}\) the future value and \(X_0, X_{n-2}, \ldots, X_1\) as the past values, of a certain sequence of observations. Markov property implies that the future value depends only on the present value and not on the past values. Thus the future value depends only on the latest known value and not on the earlier observations. \(P(X_0 = i_0)\) is called the initial probability and \(P(X_n = i / X_{n-1} = i_{n-1}, \ldots, X_1 = i_1)\), for all \(n=1,2,3,\ldots, n+1\) are called one-step probability of the MC. These probabilities determine the joint distribution of any finite number of \(r.v.\’s\) of the MC, sequence. Hence the probability distribution of MC is completely determined if we are given the initial probability distribution and one – step transition probabilities.

If \(P(X_n = i_n / X_{n+1} = i_{n+1})\) depends only on \((i_{n+1}, i_n)\) and not on \(n\), the transition probabilities are said to be stationary. Then we can denote it as \(P_{i_n, i_{n+1}}\) in this case \(P(X_n = j / X_{n-1} = i) = \sum_i P_{i_{n+1}, i} P_{i, j} = P_{i, j}\). Hence the elements of \(P\) are non-negative and the row-sum equals unity, then the matrix is called a stochastic matrix. Let \(P_{(n+1)}(X_n=j/X_0=i) = P(X_{n+1} = j / X_n = i)\), for all \(m=1,2,\ldots\). Then the matrix with \((i,j)\)th element as \((P_{(n)}(i\rightarrow j))\) is called the n-step TPM and is denoted by \(P(n)\). It is a stochastic matrix. The unconditional probability \(P(X_n=j) = \sum P(X_n=j / X_0 = i)\), (sum on i) which is equal to \(\sum P(X_n=j) = P_{(n)}(i\rightarrow j) = |P_{(n)}(X_n=j)| \sum P_{(n)}(i\rightarrow j)\), where \(P^{(n)}(i\rightarrow j)\) is the initial probability distribution of \(i\). \(P^{(0)}(i\rightarrow j)\) is the row vector of \(P^{(n)}(i\rightarrow j)\). Thus knowing the initial state and the n-step transition probabilities, the distribution of values after \(n\) additional units of time can be determined. Then for MC with stationary probabilities \(P_{(n)} = \sum P_{(n)}(i\rightarrow j)P_{(n)}(j\rightarrow k)\), \(n=0, \ldots, k \in \mathbb{L}\), or \(P^{(n)} = \sum P^{(n)}(i\rightarrow j)\) \(n\) is called Chapman- Kolmogrov equation. The problem of obtaining n-step transition probabilities is same as the problem of finding the nth power of \(P\). Let the urban population of a country be divided into m size classes, where m stands for the states of a MC, that is, the different size classes are termed as the states of a MC. As population growth takes place, cities move from one state to another at different points of observation. These points can be taken as the natural points of censes dates which are normally conducted in most of the countries within a period of ten years. At different points of time the changes in city size distribution can be represent by the TPM. \(P\) with \(0 \leq P_{ij} \leq 1\), \(\sum_{j} P_{ij} = 1\), \(i, j = 1,2,\ldots, m\). Since, the only present state of the process determine the future, \(P(X_{n+1}) = \sum P(X_{n+1}=j)P(X_n=i)\). Here, the size of the cities is projected using Markov chain model.

Empirical Analysis

The data for this analysis are taken from the Census of India (1981, 1991, 2001& 2011) [9-12]. The population of 56 urban cities in Ernakulum district Kerala state of India is considered. Indian census has a history of more than hundred years. The period of observation is confined to 1981 to 2011 omitting the date from the initial period of census for obvious reasons of coverage. As we are dealing with the ‘number of urban places’, the quality will be of very high order, compare to any other demographic characteristics and at least in this respect, we are in a confident position to exploit the census materials as applicable to our study.

The urban classes are classified into six classes as, below 5000, 5000-10000, 10000-20000, 20000-50000, 50000-100000, and above 100000. The number of places belonging to the class, ‘below 5000’ are mostly a subjective classification by census directors as mentioned above and...
Table 1: Distribution of Ernakulam city urban population

| Sl No | Name of the place | Population 1881 | Population 1991 | Population 2001 | Population 2011 |
|-------|-------------------|-----------------|-----------------|-----------------|-----------------|
| 1.    | Kochi             | 504589          | 564589          | 625522          | 63553           |
| 2.    | Mulanthuruthy     | 31995           | 34995           | 40993           | 44704           |
| 3.    | Kumbalangy        | 32310           | 38310           | 40331           | 42367           |
| 4.    | Tirupurpathura    | 47078           | 51078           | 59884           | 69390           |
| 5.    | Kumbalam          | 20400           | 25400           | 27549           | 29193           |
| 6.    | Thiruvankulam     | 11412           | 18412           | 21717           | 23160           |
| 7.    | Mulavukad         | 18322           | 22322           | 22845           | 21833           |
| 8.    | Kureekkad         | 7100            | 7800            | 9730            | 13348           |
| 9.    | Eranelloor        | 20990           | 24990           | 28223           | 33829           |
| 10.   | Vazhakkal         | 14975           | 18975           | 43078           | 51242           |
| 11.   | Puthuvype         | 20400           | 21000           | 22109           | 23717           |
| 12.   | Mulamthuruthy     | 20107           | 21807           | 23615           | 25852           |
| 13.   | Kakkanad          | 18500           | 19800           | 22486           | 25531           |
| 14.   | Manakkunnam       | 23880           | 29880           | 33523           | 39538           |
| 15.   | Elamkunnmapuzha   | 20013           | 24013           | 26092           | 26997           |
| 16.   | Kanayannur        | 6750            | 7650            | 9443            | 9308            |
| 17.   | Kadamakkudy       | 9023            | 11023           | 15823           | 16295           |
| 18.   | Njarackal         | 16803           | 19803           | 24166           | 23760           |
| 19.   | Puthencruz        | 13000           | 18000           | 21054           | 22378           |
| 20.   | Amballur          | 7900            | 9800            | 10568           | 11358           |
| 21.   | Elloor            | 30485           | 34485           | 39092           | 31468           |
| 22.   | Kunnamhunad       | 13958           | 18958           | 20509           | 22884           |
| 23.   | Kalamassery       | 45342           | 54342           | 63176           | 71038           |
| 24.   | Varappuzha        | 20114           | 22514           | 24524           | 26750           |
| 25.   | Choornikkara      | 30837           | 34837           | 36998           | 43207           |
| 26.   | Alangad           | 32956           | 38956           | 40585           | 47329           |
| 27.   | Aluva             | 19774           | 24774           | 24108           | 22428           |
| 28.   | Kadungalloor      | 18433           | 25433           | 35451           | 39666           |
| 29.   | Kottuvally        | 33457           | 34457           | 37884           | 42922           |
| 30.   | Edathala          | 51397           | 56397           | 67137           | 77811           |
| 31.   | Chowara           | 8864            | 9864            | 13603           | 14933           |
| 32.   | Paravur           | 21906           | 27906           | 30056           | 31503           |
| 33.   | Karumalloor       | 22896           | 25896           | 26856           | 29805           |
| 34.   | Vengola           | 18689           | 21689           | 26034           | 32697           |
| 35.   | Chengamanad       | 21621           | 24621           | 29775           | 29576           |
| 36.   | Chennamangalam    | 24825           | 26825           | 28147           | 29326           |
| 37.   | Marampilly        | 15975           | 18975           | 20071           | 23272           |
| 38.   | Thekkumbhogom     | 8967            | 9867            | 10023           | 10798           |
| 39.   | Vadakkkekara      | 16965           | 18965           | 20099           | 20571           |
| 40.   | Vadakkumbhogom    | 7354            | 10354           | 11584           | 11727           |
| 41.   | Nedumbassery      | 24089           | 24889           | 28607           | 29706           |
| 42.   | Perumbavoor       | 23667           | 24667           | 26547           | 28110           |
| 43.   | Angamaly          | 30309           | 30391           | 33424           | 33465           |
| 44.   | Velloorkunnam     | 9235            | 10235           | 10363           | 11576           |
| 45.   | Kizhakkumbhogom   | 9146            | 9246            | 10038           | 10791           |
| 46.   | Muvattupuzha      | 25595           | 27595           | 29246           | 30397           |
| 47.   | Moothakunnam      | 23698           | 24698           | 27293           | 27488           |
| 48.   | Perumbavoor       | 8986            | 7986            | 8026            | 9185            |
| 49.   | Puthenvelikkara   | 25686           | 29686           | 32213           | 33372           |
| 50.   | Kalady            | 18865           | 19865           | 20407           | 20380           |
| 51.   | Koovappady        | 20658           | 24658           | 26255           | 39339           |
| 52.   | Mattoor           | 11689           | 15689           | 17862           | 18890           |
| 53.   | Cheranallur       | 18407           | 21407           | 26330           | 30594           |
| 54.   | chelamattom       | 11569           | 12569           | 15366           | 16844           |
| 55.   | Vazhakulam        | 12054           | 12546           | 14206           | 18358           |
| 56.   | Kothamangalam     | 35235           | 35535           | 37173           | 38837           |
| **Total** | 1638950         | 1861224         | 2089080         | 2202068         |

Census of India 1981, 1991, 2001 & 2011 Kerala, Final population Totals
Bivariate frequency distribution of urban places in Ernakulum district -Kerala with respect to their sizes in 1991-2001 (Table 2), 2001-2011 (Table 3). The data in (Table 2 & Table 3) are used to form the transition probability matrix (TPM) of cities in Ernakulum district -Kerala 1991-2001 (Table 4) and 2001-2011 (Table 5).

**Table 2:** Bi-variate frequency distribution of urban places in Ernakulum district with respect to their sizes in 1991-2001

| 2001→1991 | 5000-10000 | 10000-20000 | 20000-50000 | 50000-100000 | Greater than 100000 | Total |
|-----------|-------------|-------------|-------------|--------------|-------------------|-------|
| 5000-10000 | 8           | 1           | 1           | 0            | 0                 | 10    |
| 10000-20000| 0           | 9           | 5           | 0            | 0                 | 14    |
| 20000-50000| 0           | 0           | 28          | 1            | 0                 | 29    |
| 50000-100000| 0          | 0           | 0           | 2            | 0                 | 2     |
| Greater than 100000 | 0 | 0 | 0 | 1 | 1 | 56 |
| Total      | 8           | 10          | 34          | 3            | 1                 | 56    |

**Table 3:** Bi-variate frequency distribution of urban places in Ernakulum district with respect to their sizes in 2001-2011

| 2011→2001 | 5000-10000 | 10000-20000 | 20000-50000 | 50000-100000 | Greater than 100000 | Total |
|-----------|-------------|-------------|-------------|--------------|-------------------|-------|
| 5000-10000 | 4           | 2           | 2           | 0            | 0                 | 8     |
| 10000-20000| 0           | 9           | 1           | 0            | 0                 | 10    |
| 20000-50000| 0           | 0           | 33          | 1            | 0                 | 34    |
| 50000-100000| 0         | 0           | 0           | 3            | 0                 | 3     |
| Greater than 100000 | 0 | 0 | 0 | 1 | 1 | 56 |
| Total      | 4           | 11          | 36          | 4            | 1                 | 56    |

**Table 4:** Transition probability matrix (TPM) of cities in Ernakulum district 1991-2001

| 1991-1981S | 5000-10000 | 10000-20000 | 20000-50000 | 50000-100000 | Greater than 100000 | Total |
|------------|-------------|-------------|-------------|--------------|-------------------|-------|
| 5000-100000| 0.8         | 0.1         | 0.1         | 0            | 0                 | 1     |
| 5000-10000 | 0.64        | 0.36        | 0.36        | 0            | 0                 | 1     |
| 20000-50000| 0           | 0           | 0.97        | 0.03         | 0                 | 1     |
| 50000-100000| 0       | 0           | 0           | 1            | 0                 | 1     |
| Greater than 100000 | 0 | 0 | 0 | 0 | 1 | 1 |
| Total      | 0.5         | 0.25        | 0.25        | 0            | 0                 | 1     |

**Table 5:** Transition probability matrix (TPM) of cities in Ernakulum district with respect to their sizes in 2001-2011

| 2001→1991S | 5000-10000 | 10000-20000 | 20000-50000 | 50000-100000 | Greater than 100000 | Total |
|------------|-------------|-------------|-------------|--------------|-------------------|-------|
| 5000-100000| 0.5         | 0.25        | 0.25        | 0            | 0                 | 1     |
| 5000-10000 | 0.9         | 0.1         | 0.1         | 0            | 0                 | 1     |
| 20000-50000| 0           | 0           | 0.97        | 0.03         | 0                 | 1     |
| 50000-100000| 0       | 0           | 0           | 1            | 0                 | 1     |
| Greater than 100000 | 0 | 0 | 0 | 1 | 1 | 1 |
| Total      | 0.5         | 0.25        | 0.25        | 0            | 0                 | 1     |

Projection of urban population in Ernakulum district by using Chapman – Kolmogorov equation

\[ X^{(i)} = P \cdot X^{(i-1)} \]

where \( P \) is the initial probability matrix in (Table 4), \( X^{(0)} \) is the initial frequency in each group, \( X^{(i)} \) is the first estimated frequency.

So,

\[ X^{(i)} = P^{(i)} \cdot X^{(0)}, \quad i = 1,2,3,... \]

where \( X^{(i)} \) is the \( i \)th estimated frequency and \( P^{(i)} \) is the \( i \)th power of initial TPM, \( P \).

Here we get, \( X^{(0)} = (10,14,29,2,1) \) from (Table 2).

The initial probability matrix \( P \) is from Table 4 and calculates the powers of \( P \), Table 6. Hence we get the TPM \( P \), \( P^2 \), \( P^3 \),... \( P^6 \) respectively.

| \( P \) | 0.8 | 0.1 | 0.1 | 0 | 0 |
|--------|-----|-----|-----|---|---|
|        | 0   | 0.64| 0.36| 0 | 0 |
|        | 0   | 0   | 0.97| 0.03| 0 |
|        | 0   | 0 | 0 | 1 | 0 |
|        | 0 | 0 | 0 | 0 | 1 |

| \( P^2 \) | 0.512 | 0.156 | 0.322 | 0.009 | 0 |
|-----------|--------|--------|--------|--------|---|
|           | 0     | 0.262 | 0.709 | 0.028 | 0 |
|           | 0     | 0     | 0.912 | 0.087 | 0 |
|           | 0     | 0     | 0 | 1 | 0 |
|           | 0     | 0 | 0 | 0 | 1 |

| \( P^3 \) | 0.409 | 0.151 | 0.420 | 0.019 | 0 |
|-----------|--------|--------|--------|--------|---|
|           | 0     | 0.167 | 0.782 | 0.049 | 0 |
|           | 0     | 0     | 0.885 | 0.114 | 0 |
|           | 0     | 0 | 0 | 1 | 0 |
|           | 0     | 0 | 0 | 0 | 1 |

| \( P^4 \) | 0.167 | 0.087 | 0.66 | 0.082 | 0 |
|-----------|--------|--------|--------|--------|---|
|           | 0     | 0.028 | 0.824 | 0.147 | 0 |
|           | 0 | 0 | 0.783 | 0.216 | 0 |
|           | 0 | 0 | 0 | 1 | 0 |
|           | 0 | 0 | 0 | 0 | 1 |

| \( P^5 \) | 0.262 | 0.120 | 0.570 | 0.046 | 0 |
|-----------|--------|--------|--------|--------|---|
|           | 0     | 0.068 | 0.833 | 0.097 | 0 |
|           | 0 | 0 | 0.832 | 0.167 | 0 |
|           | 0 | 0 | 0 | 1 | 0 |
|           | 0 | 0 | 0 | 0 | 1 |
P6. Then the Projection of urban population of Ernakulum district in Kerala is,

$X^{(0)} = (10,14,29,2,1)$, $X^{(1)} = PX^{(0)} = (12,20,29,3,1)$, $X^{(2)} = (13,24,26,3,1)$, $X^{(3)} = (20,25,24,3,1)$, $X^{(4)} = (21,29,22,3,1)$, $X^{(5)} = (24,32,19,3,1)$, $X^{(6)} = (25,33,16,3,1)$

| Year | 5000-10000 | 10000-20000 | 20000-50000 | 50000-100000 | Greater than 100000 |
|------|-------------|-------------|-------------|---------------|---------------------|
| 1991 | 10          | 14          | 29          | 2             | 1                   |
| 2001 | 12          | 20          | 29          | 3             | 1                   |
| 2011 | 13          | 24          | 26          | 3             | 1                   |
| 2021 | 20          | 25          | 24          | 3             | 1                   |
| 2031 | 21          | 24          | 22          | 3             | 1                   |
| 2041 | 24          | 32          | 19          | 3             | 1                   |
| 2051 | 25          | 33          | 16          | 3             | 1                   |

Fig 1: Projected urban population up to 2051 of Ernakulum district in Kerala

Here Class 1: 5000-10000, Class 2: 10000-20000, Class 3: 20000-50000, Class 4: 50000-100000, Class 5: Greater than 100000. As Prasanth & Praseeja\(^1\) explained the city can spread out uniformly and entire land is available for residential buildings and as Archana & Prasanth\(^2\) explained City Expansion and Urbanization of Kerala, towards the centre of the city it is seen that the population density has an increasing tendency. In other words we can say that all the cities have an outer expanding nature and this expansion is circular. So with respect to time new urban cities are formulating and many existing cities are showing a clear change in the city size & its population and hence shifting the classes from one to other. This fact is clear from the figure 1 & table 7, as at 1991 in class1, there exist only 10 cities where as at 2050 it shows 25 cities in the same. More than that at 1991 there are only 56 urban cities in the Ernakulum district & it comes to a total of 78 in the projected scores of 2050. Excluding the newly formulate such cities & comparing the existing 56 city sizes with the expected one by using chi-square statistics the Projection of urban population in Ernakulum district by using Chapman – Kolmogorov equation is a good fit and hence the model is suitable for such data analysis.

Table 8: Inference about the goodness of Markove model (Ernakulum district urban population)

| Year | Calculated $\chi^2$ | Table $\chi^2$ (a =0.01) | d.f | p-value | Inference               |
|------|---------------------|---------------------------|-----|---------|-------------------------|
| 1991 | 5.40                | 9.21                      | 2   | 0.07    | Model is suitable for these data |
| 2001 | 7.20                | 9.21                      | 2   | 0.027   |                         |
| 2011 | 8.10                | 9.21                      | 2   | 0.018   |                         |

Conclusions

The projected urban population (Table 7 and Figure 1) shows the number of cities increases in class 5000-10000 & 10000-20000 and decreases in class 20000-50000 with respect to migration and reconstruction of the area of cities. There is a consistency in higher city size classes. The Markove Model is suitable for the study of city size distribution. The real data analysis by using the Ernakulum district’s urban population in different decades shows the suitability of the model. More than an empirical analysis this is to establish the suitability of such a stochastic model in these types of studies. This study point outs the migration and reconstruction of the area of cities. That implies, still there exists many significant anomalies, which should be identified and analyzed by the government.

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