A fractal micro-electromechanical system and its pull-in stability

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Abstract
Pull-in instability occurs in a micro-electromechanical system, and it greatly hinders its normal operation. A fractal modification is suggested to make the system stable in all operation period. A fractal model is established using a fractal derivative, and the results show that by suitable fabrication of the micro-electromechanical system device, the pull-in instability can be converted into a novel state of pull-in stability.

Keywords
Micro-electromechanical systems, pull-in, fractal space, porous medium, fractal derivative

Introduction
In a micro-electromechanical system (MEMS), a pull-in effect occurs at a certain threshold. The pull-in analysis of the electrostatic-drive device is of great significance to ensure the effective operation and reliability of the device. The dynamic pull-in analysis of the MEMS model under applied voltage has been well established in the normal air medium.¹⁻⁵ With the advancement of technology, the application of the pull-in effect becomes more and more widespread.⁶⁻⁷

With the deterioration of the environment, there are more and more impurities in the air, such as PM 2.5 and dust. Electric appliances will be affected a lot when they work in these media. At this time, the traditional MESE model is facing challenges, and a new model needs to be established. Nowadays, more and more fractal models were established to solve problems in science, physics, and technology. Wang⁸ according to the nanoscale surface morphology of Fangzhu’s water collection⁹ established a fractal mathematical model. Fan et al.¹⁰ researched the wool fiber’s fractal feature and the thermal property. Wang and Deng¹¹ established a fractal tsunami model to solve non-smooth boundary tsunami problems. Ji et al.¹² used fractal calculus to derive the fractal Boussinesq equation, and studied the effect of a porous structure on the vibration property. He¹³ established a family of fractal variational principles of the one-dimensional compressible flow under the microgravity condition by the semi-inverse method. He et al.¹⁴ modeled the solvent evaporation in electrospinning process by a Bratu-type equation with fractal derivatives. Wang et al.¹⁵ analyzed the steel fiber-reinforced concrete during uniaxial tensile damage in fractal space. Lin et al.¹⁶,¹⁷ studied the fractal effect in release of silver ion from Ag/PET hollow fibers. Wang et al.¹⁸ assumed a fractal derivative model to study the thermal insulation property of snow. Liu et al.¹⁹ proposed a fractal model to explain the adsorption surface. He²⁰ suggested a fractional modification to deal with the porous property of the electrodes and elucidated the effect of the porous structure on the convection-diffusion process.
The fractal theory is an excellent tool to deal with various mathematical problems and some physical phenomena in porous media.\textsuperscript{21–23} In the fractal space, the fractal derivative is defined as\textsuperscript{24–27}

\[
\frac{du}{dt^\alpha}(t_0) = \Gamma(1+\delta) \lim_{\Delta t \to 0^+} \frac{u(t) - u(t_0)}{(t-t_0)^\delta}
\]

As we know, sophisticated electronic devices not only work in the air, but may also operate in various media. However, when the medium is the air filled with nano/micro particles, or sponge, or other porous medium, what effect will it have on their work? In this paper, for the first time, we propose a fractal MEMS model to solve the electronic devices work in porous medium. At the same time, we analyze the effect of the change of the fractal order on the time of pull-in occurrence, and obtain a stable pull-in condition.

**MEMS system**

According to magnetostatics, Ampere’s force law refers to the attraction or repulsive force between two current-carrying wires. This force is generated in this way, according to Biot-Savart’s law, each energized wire will generate a magnetic field, and then the other wire will also be subjected to a magnetic force, following the Lorentz force law cf. Andre Koch Torres Assis. The force $f$ between two straight lines with per unit length can be expressed and calculated by the following formula

\[
f = \frac{\mu_0 i_1 i_2}{2\pi d}
\]

where $\mu_0 = 4\pi \times 10^{-7} N/A^2$ is the magnetic constant, $i_1, i_2$ the direct currents through the wires, and $d$ is the distance between them.

We consider the movement of a current-carrying wire with a length of $l$ and a mass of $m$ under the constraint of a linear elastic spring in an infinite current-carrying wire field in a porous medium, as shown in Figure 1. In this article, we will study the dynamic pull-in in fractal space.

**Fractal MEMS model**

We consider a case in Figure 2. AB is a non-smooth curve. We assume that AB is a curved road, the red and blue lines represent the walking routes of man and woman, respectively, with different steps. It is conceivable that when the path is a smooth and straight line, the length of the road they travel will be the same. When the path is a tortuous and non-smooth road as shown in Figure 2, it is necessary to consider the influence by the two-scale mathematics.\textsuperscript{28–30} In Figure 2, $\diamondsuit$ shows the step length of a man’s single step, and $\heartsuit$ shows the step length of a woman’s single step. Then we can clearly see from the figure that since women have smaller steps than men, they will take different paths when they want to arrive at the end, resulting in different time they will spend. The fractal space can be used to solve the problem of fractal scale. The pull-in problem of electronic devices working in air filled with various nano/micro particles can be considered as a two-scale fractal space.

The dynamic lumped parameter differential equation used to describe the movement of the wire as a point mass can be derived as follows

\[
m\dddot{u} + \dddot{\bar{u}} + k\dddot{u} - \frac{\mu_0 i_1 i_2 l}{2\pi(b-u)} = 0
\]

where $m$ is the mass, $\dddot{\bar{u}}$ is the damping coefficient, $k$ is the spring coefficient, $i_1$ and $i_2$ are currents. Equation (3) can be rewritten as

\[
u'' + \mu\nu' + u - \frac{K}{1-u} = 0
\]
where $u = \ddot{u}/b$, $t = \dot{\omega_0} \omega_0^2 = k/m$, $\mu = \dot{\omega}/k = 1/\omega_0$, and $K = \mu_0 i_1 i_2/(2\pi b^2)$. We prescribe zero initial conditions $u(0) = 0$, $u'(0) = 0$.

In the two-scale fractal space, we have

\begin{align}
L \propto (\Delta u)^2 \\
u' \propto (\Delta u)^2 \\
u'' \propto (\Delta u)^2
\end{align}

Because

\[
\Delta t \propto \Delta u
\]

So

\[
(\Delta t)^2 \propto (\Delta u)^2
\]

So we have

\[
u'' \propto (\Delta t)^2
\]

According to equation (10), a fractal modification of equation (4) is obtained

\[
\frac{d^2 u}{dt^2} + \mu \frac{du}{dt} + u - \frac{K}{1 - u} = 0
\]
When $\mu = 0$, we have
\[
\frac{d^2u}{dt^2} + u - \frac{K}{1 - u} = 0 \quad (12)
\]

According to equation (12), Figure 3 shows the phase trajectories of different $K$ in fractal space. We can see from Figure 3, when $K > 0.203$, the phase trajectories will be open.

Figure 4 shows the periodic solutions when $K$ takes different values in the fractal space. We know that when $K$ does not exceed the critical value, it can have the form of periodic solutions, and when $K$ exceeds the critical value, a pull-in phenomenon will occur. As shown in Figure 5, this figure is the pull-in curve corresponding to different fractal dimensions when the value of $K$ is 0.20372. It can be seen from Figure 5 that $\alpha$ has an important impact on the time when the pull-in occurs. When $\alpha > 1$, pull-in happens very quickly. With the decrease of $\alpha$, the pull-in happens later. When $\alpha = 0.1$, the pull-in instability becomes pull-in stability as shown in Figure 6.
Conclusion

In this paper, we approximate the air filled with dust as a porous medium. Then, we used two-scale mathematics to establish a fractal model for MESE system, and this model can be used in any porous medium. We also study the impact of fractal order $\alpha$ on the pull-in time, and obtain a stable pull-in state. The fractal model we established and the results obtained will play a vital role in the development of MEMS systems.

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