A GRAVITATIONAL DOUBLE-SCATTERING MECHANISM FOR GENERATING HIGH-VELOCITY OBJECTS DURING HALO MERGERS

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ABSTRACT

We present a dynamical model that describes how halo particles can receive a significant energy kick from the merger between their own host halo and a target halo. This could provide a possible explanation for some high-velocity objects, including extended systems like globular clusters (GCs). In the model we especially introduce a double-scattering mechanism, where a halo particle receives a significant part of its total energy kick by first undergoing a gravitational deflection by the target halo and subsequently by its original host halo. This generates an energy kick that is due to the relative velocity between the halos during the deflections. We derive analytically the total kick energy of the particle, which is composed of energy from the double-scattering mechanism and tidal fields, as a function of its position in its original host halo just before merger. In the case of a 1:10 merger, we find that the presented mechanisms can easily generate particles with a velocity approximately two times the virial velocity of the target halo. This motivates us to suggest that the high velocity of the recently discovered GC HVGC-1 can be explained by a head-on halo merger. Finally, we illustrate the orbital evolution of high-velocity particles outside the virial sphere of the target halo by solving the equation of motion in an expanding universe. We find a sweet spot around a scale factor of 0.3–0.5 for ejecting particles into large orbits, which can easily reach beyond approximately five virial radii.

Key words: dark matter – galaxies: kinematics and dynamics – gravitation

1. INTRODUCTION

Several high-velocity objects on seemingly unbound orbits have been observed, ranging from stellar objects (Brown et al. 2013; Tunnicliffe et al. 2014) to more extended systems like dwarf galaxies (Majewski et al. 2007; Chapman et al. 2007). In many of these cases the origin of the velocity kick is unknown, but several mechanisms have been suggested. One is binary single interactions where the binding energy of a binary is dynamically released into a third object, which thereby can escape with high velocity (Heggie 1975). These interactions are believed to have a nonnegligible chance of happening, especially between stellar objects (Sigurdsson & Phinney 1993; Gvaramadze et al. 2009) and stars encountering either single or binary black hole (BH) systems (Hills 1988; Yu & Tremaine 2003; Bromley et al. 2006). Several observations indicate in fact that stellar interactions with the supermassive black hole (SMBH) at the center of our Galaxy is a likely explanation for some local high-velocity stars (Gualandris et al. 2005; Brown et al. 2012). More extended objects, like GCs and subhalos, are probably not kicked by BH binary interactions because of the high probability for disruption; however, the outcome from such an interaction is still uncertain (Caldwell et al. 2014). Extended objects have instead been shown to obtain a kick when accreting in groups onto a larger halo. In this case, binding energy can be released into especially the lighter group members by the tidal forces of the larger halo (Sales et al. 2007; Ludlow et al. 2009), much like a few-body exchange (Heggie 1975). High-velocity stars can also arise from isolated binaries if the heavier member undergoes a violent mass loss, a channel first suggested by Blaauw (1961) to explain the high number of “runaway” O-B stars. More exotic kick mechanisms for describing hostless stellar remnants, pulsars, and possible hypervelocity BHs have been suggested as well, from the role of asymmetric gravitational wave radiation (Bekenstein 1973; Fitchett 1983; Redmount & Rees 1989; Pettini et al. 1995; Davies et al. 2002) to the asphericity of SN explosions (Burrows & Hayes 1996; Burrows et al. 2007; Janka 2012).

Unbound particles have also been discussed from a cosmological perspective. Recent studies (Behroozi et al. 2013) illustrate that ~10% of all of the dark matter (DM) at the virial radius is in fact unbound. Luminous matter with no specific host halo has likewise been observed in especially galaxy clusters, a component known as intracluster light. This has been extensively studied, both through observations (e.g., Zwicky 1951; Guennou et al. 2012; Presotto et al. 2014) and numerically (Willman et al. 2004), and is believed to be a direct consequence of the dynamical evolution of galaxies, including tidal stripping and mergers (Moore et al. 1996). Theoretical attempts have also been made to understand the final distribution of particles in DM halos. This includes models from spherical collapse (e.g., Bertschinger 1985; Dalal et al. 2010) to statistical mechanics (e.g., Ogorodnikov 1957; Lynden-Bell 1967; Spurge & Hernquist 1992; Hansen et al. 2005; Hjorth & Williams 2010; Pontzen & Governato 2013). The high-velocity tail of the distribution has especially been shown to likely originate from tidal shocks and rapid mean field variations during halo mergers (Teyssier et al. 2009; Carucci et al. 2014). Time-dependent potentials induced during cold collapse at the early stages of halo formation have also been proposed to generate an early velocity excess (Joyce et al. 2009). In particular, the role of mergers followed by tidal stripping was suggested by Abadi et al. (2009)
to possibly explain the observed population of high-velocity B-type stars.

Data from upcoming surveys like *LSST* and especially *Gaia* will in the near future also measure positions and velocities for more than \(\sim 150\) million stars with unprecedented precision. This not only offers unique possibilities for mapping out the current Milky Way potential and its past evolution (e.g., Zhao et al. 1999; Peñarrubia et al. 2012; Price-Whelan et al. 2014; Sanderson et al. 2014), but will also make it possible to make detailed studies of past dynamical interactions (Gualandris et al. 2005). A central question here could be if the Milky Way in its past had a SMBH binary dynamically interacting with the environment. Detections of high-velocity objects are here again playing a central role.

In this paper, we introduce a new dynamical mechanism that explains how halo particles can gain a significant velocity kick through the merger between their original host halo and a larger target halo. It is well known that tidal fields exerted on an infalling halo result in energy kicks that might lead to tidal stripping (Binney & Tremaine 1987). However, what has not been characterized before is the effect of the tidally stripped particles being redeflected by their original host halo core after their first passage through the center of the target halo. We here show that this rediflection generates significant additional energy kicks that are due to the relative velocity between the two halos during the rediflection. This mechanism we denote “double scattering”. Figure 2 illustrates an orbit of a particle undergoing this double-scattering process.

We derive analytically the full resultant energy kick from both tides and the double-scattering mechanism for two merging Hernquist (HQ) halos (Hernquist 1990). However, the idea of the mechanism is not limited to this scenario. For instance, we note that a very similar mechanism has been described within heavy nuclei interactions where an electron can be ejected into the continuum (unbound orbit) or captured by a passing nucleus (dynamical capture) by undergoing a double collision (Thomas 1927; Shakeshaft & Spruch 1979).

The paper is organized in the following way. In Section 2, we first give an introduction to the dynamical processes playing a role in halo mergers, which include tidal fields and our proposed double-scattering mechanism. Section 3 describes the numerical simulations, initial conditions, and halo merger examples that we consider in this paper. The energy release from tidal fields is described in Section 4, and the double-scattering mechanism is presented in Section 5. In both of these sections we derive the energy change of a given particle as a function of its position in its initial host halo. In Section 6, we shortly describe observable consequences and show how a kick energy translates to an observable velocity excess. In Section 7, we explore how far a dynamically kicked particle can travel after leaving the virial sphere of its target halo by solving the equation of motion in an expanding universe. Conclusions are given in Section 8.

2. ENERGY OF PARTICLES DURING HALO MERRGERS

The energy of individual halo particles can change significantly during a merger between their initial host halo \(H_2\) and a larger target halo \(H_1\). Some particles will lose energy and become bound to the target halo \(H_1\), whereas some will gain energy and escape with relatively high velocity. In this section we introduce the dynamical mechanisms responsible for changing the energy of each individual particle initially bound to the incoming halo \(H_2\).

2.1. Dynamical Mechanisms

We first consider Figure 1, which shows an N-body simulation of a merger between two DM halos. The incoming halo \(H_2\) approaches from the right on a radial orbit with a velocity equal to the escape velocity of the target halo. The orange symbol shows a particle that at all times prior to the merger is located within 5% of the incoming halo’s virial radius. This symbol can therefore represent a luminous galactic component (Kratvetsov 2013). On the figure is also highlighted the orbits of two particles: the green particle receives a positive energy change through the merger and can thereby escape, whereas the red particle gets bound as a result of a negative energy change. The differences in final energy between the orange, green, and red particles arise because of a series of dynamical mechanisms, which to a first order can be separately considered. Each of these changes the energy of the particles, as described in the following.

The first energy change arises because the potential of the target halo \(H_1\) is not constant across the profile of the incoming halo \(H_2\). As a result, the particles in \(H_2\) located on the side closest to \(H_1\) have less energy than the particles located on the far side. This energy difference increases as the distance between the two halos decreases, i.e., a subject particle will gradually gain or lose energy as the two halos approach each other. This continues until \(H_2\) is tidally disrupted by the tidal field of \(H_1\). We denote the final energy change from this process by \(\Delta E_{TF}\), where TF is short for tidal field.

The second energy change arises through our proposed double scattering, where a stripped particle is first deflected by \(H_2\) and then subsequently rediflected by \(H_1\). The deflection by \(H_2\) generates a significant energy kick that is due to the relative velocity between the two halos during the two deflections. The rediflection happens after \(H_2\) has passed the center of \(H_1\), which makes it distinct from, for example, tidal shocking. We denote the energy change resulting from the two deflections by \(\Delta E_{DS}\), where DS is short for double scattering. To our knowledge, this contribution has not been characterized before and will therefore be the main topic of this paper.

The third and last energy change happens after the double scattering, as the particles are moving away from \(H_2\) along their new orbits. As the particles climb out of the potential of \(H_2\), their velocity decreases, which results in an energy change in the frame of \(H_1\) that is due to the relative motion between \(H_1\) and \(H_2\). For particles escaping the merger remnant, the energy change will be negative, as we will illustrate. We denote the final energy change from this process by \(\Delta E_{esc}\).

2.2. Total Energy Change

The dynamical mechanisms we consider in this work can, to a first order, be considered separately and as not affecting the motion of a given particle at the same time. The total energy change \(\Delta E_{tot}\) a given particle will experience can therefore be expressed by the sum of the individual energy contributions:

\[
\Delta E_{tot} \approx \Delta E_{TF} + \Delta E_{DS} + \Delta E_{esc}.
\]

For particles receiving a high energy kick, the first two terms will usually be positive and the last negligible. In this paper
3. GENERAL SETUP

We model the two merging halos $H_1$ and $H_2$ by HQ profiles (Hernquist 1990), with an anisotropy parameter $\beta = 0$. In this case the mass profile is given by

$$M_i(r) = \frac{M_i - (r/a_i)^2}{1 + r/a_i},$$

(2)

and the corresponding gravitational potential by

$$\Phi_i(r) = -\frac{GM_i}{a_i} \frac{1}{1 + r/a_i},$$

(3)

where $M_i$ is the total mass of halo $i$, $r$ is the distance from the halo center, $M_i(r)$ is the mass enclosed by $r$, $\Phi(r)$ is the potential at distance $r$, and $a_i$ is a characteristic scale radius. In the following, we occasionally use units of $a_1$, and we use a prime to denote this, e.g., $x' \equiv x/a_1$. We also find it useful to write the radial velocity between $H_1$ and a particle moving in its potential on a radial orbit as

$$w^2(r) = -2\Phi_1(r) + w^2(0) + 2\Phi_1(0),$$

(4)

where $w(r)$ is the radial velocity of the particle at distance $r$. We will use this relation to calculate the relative velocity between halo $H_1$ and the incoming halo $H_2$. The estimate for $w(r)$ in the above Equation (4) ignores the effect from dynamical friction, which causes $H_2$ to lose orbital energy by exchanging momentum with the surrounding particles in $H_1$ (Chandrasekhar 1943). Dynamical friction actually plays a minor role in our case because $\Delta E_{\text{DS}} \propto w$, as we describe in Section 5, but for now we ignore it to simplify the analysis. For the following analyses, we further assume that the two halos merge with a zero impact parameter and that the mass of the target halo $H_1$ is much larger than the incoming halo $H_2$, i.e., $M_1 \gg M_2$. This mass hierarchy is relevant for the growth of cosmological halos that are believed to build hierarchically by hundreds of minor mergers (Fakhouri et al. 2010).

The N-body simulations presented throughout the paper are performed using Gadget II (Springel 2005), with the two HQ halos set up in equilibrium by Eddington's method (Eddington 1916) using a well-tested code previously used to study anisotropy in halo mergers (Sparre & Hansen 2012a, 2012b). The halo concentration $c_i \equiv R_{c,i}/a_i$ is set to 5 for both halos, and the virial radius $R_{\text{vir}}$ is calculated by requiring the density inside the halo to be 200 times the mean density of the universe at redshift $z = 2$ (Wechsler et al. 2002; Mo et al. 2010). We fix the merger mass ratio at 1:10 for all simulations, and the incoming halo $H_2$ is set to have zero energy relative to the target halo $H_1$, which corresponds to a velocity at infinity $w_{\infty} = 0$. These initial conditions are typical in a cosmological perspective (Prada et al. 2012), but a wide range of both encounter velocities and impact parameters are seen in full cosmological simulations (Wetzel 2011).

4. ENERGY FROM TIDAL FIELDS

The first energy change the particles in $H_2$ experience is from tidal fields. This change arises because the particles in $H_2$ all have the same bulk velocity $w(r)$ but experience different values of $\Phi_1$ because of their different spatial positions in $H_2$. The difference in $\Phi_1$ across $H_2$ increases as the two halos approach each other; the particles are therefore being stripped with a wide spread in energy at the time $H_2$ tidally disrupts them.

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Figure 1. Illustration of a 1:10 merger between two DM halos merging with the escape velocity of the target halo. The particles in the smaller incoming halo are shown in black, and the particles of the larger target halo are shown in gray. The panels from top to bottom show three different times (A–C) of the merger. In the bottom panel, the full trajectories of two selected particles are also shown. The green particle gains a positive energy kick during the merger and is thereby escaping the system, whereas the red particle loses energy and also shown. The green particle gains a positive energy kick during the merger.

Figure 3. Illustration of a 1:10 merger between two DM halos merging with the escape velocity of the target halo. The particles in the smaller incoming halo are shown in black, and the particles of the larger target halo are shown in gray. The panels from top to bottom show three different times (A–C) of the merger. In the bottom panel, the full trajectories of two selected particles are also shown. The green particle gains a positive energy kick during the merger and is thereby escaping the system, whereas the red particle loses energy and also shown. The green particle gains a positive energy kick during the merger.

Figure 4. Illustration of a 1:10 merger between two DM halos merging with the escape velocity of the target halo. The particles in the smaller incoming halo are shown in black, and the particles of the larger target halo are shown in gray. The panels from top to bottom show three different times (A–C) of the merger. In the bottom panel, the full trajectories of two selected particles are also shown. The green particle gains a positive energy kick during the merger and is thereby escaping the system, whereas the red particle loses energy and also shown. The green particle gains a positive energy kick during the merger.
Tidal stripping has been extensively studied not only in the context of halo mergers (e.g., Binney & Tremaine 1987; Read et al. 2006), but also in stellar disruption events (Kochanek 1994). In this section, we derive an estimate for the energy change $\Delta E_{\text{TID}}$ of a particle in $H_2$ experienced during the tidal stripping as a function of the particle’s position in $H_2$ just prior to the merger.

**4.1. Evolution of Particle Energy before Merger**

We consider a particle located in $H_2$ with orbital velocity $v$ and polar position $l, \theta$ measured in the rest frame (RF) of $H_2$. The configuration is illustrated in Figure 2. The energy of the particle in the RF of $H_1$ before the merger is given by

$$E(l, \theta, r) = \frac{1}{2} w(r)^2 + \Phi_1(r) + \frac{1}{2} v^2 + \Phi_2(l) + w \cdot v + \Delta \Phi_1(l, \theta, r),$$  

(5)

where $r$ is the distance between $H_2$ and $H_1$, $w$ is the corresponding relative velocity, and $\Delta \Phi_1$ is the difference between the value of $\Phi_1$ at the position of the center of mass (CM) of $H_2$ and the particle, respectively. The first two terms equal the CM energy of $H_2$ in the RF of $H_1$, whereas the next two terms equal the energy of the particle in the RF of $H_2$, so the sum of the first four terms remains approximately constant as the two halos approach each other. The fifth term $w \cdot v$ is in contrast oscillating between positive and negative values as the particle orbits $H_2$. Energy can be released from this term, but the contribution is random and does not simply add to the other energy contributions for reasons we will not discuss here. The only term that changes the energy of the particle in a constructive way is the last term $\Delta \Phi_1$, which is given by

$$\Delta \Phi_1(l, \theta, r) = \frac{\Phi_1(0)}{1 + \sqrt{r^2 + l^2 + 2lr' \cos(\theta)}} - \frac{\Phi_1(0)}{1 + r'}.$$  

(6)

This illustrates that the change in energy of the particle due to the variation of $\Phi_1$ across $H_2$ scales to a linear order as $\sim l \cos(\theta)/r^2$. Thus, particles in $H_2$ located on the side closest to $H_1$ lose energy as $H_2$ approaches $H_1$, whereas particles on the other side instead gain energy.

The energy contribution from the $\Delta \Phi_1$ term is seen in the bottom panel of Figure 3, which shows the energy of the green particle from Figure 1 as a function of time. One can see that the $\Delta \Phi_1$ term does not contribute when $r$ is large (at early times), but as $r$ decreases and becomes comparable to $l$, the $\Delta \Phi_1$ term clearly increases, and the total energy of the particle therefore increases as well. This energy increase continues until the particle tidally detaches from $H_2$ and starts to move completely under the influence of $H_1$ (just before the vertical dashed line). The moment at which this happens can be estimated by comparing the tidal force exerted on the particle $F_{\text{TID}}$ by $H_1$ with the binding force $F_{\text{bin}}$ by $H_2$ (Read et al. 2006). These force terms are simply given by $F_{\text{TID}} = GM_1(d)/d^2 - GM_1(r)/r^2$ and $F_{\text{bin}} = GM_2/l^2$, where $d$ denotes the distance from $H_1$ to the particle. By defining the force ratio $\delta_{\text{TID}} \equiv F_{\text{TID}}/F_{\text{bin}}$, one can now relate $\delta_{\text{TID}}$ and the position of the particle in $H_2$ to a corresponding distance between the two halos $R_{\text{TID}}$.

In the case of two HQ halos, we find to a linear order

$$R_{\text{TID}} \approx \delta_{\text{TID}}^{-1/3}[2l' \cos(\theta)(a_1' + l')^2 M_1/M_2]^{1/3} - 1, \quad r' \gg l'.$$

(7)

If $\delta_{\text{TID}} = 1$, then the corresponding $R_{\text{TID}}$ will be the standard definition of the tidal radius.

**4.2. Resultant Energy from Tidal Fields**

The resultant energy change $\Delta E_{\text{TID}}$ of the particle induced by the tidal field of $H_1$ is given by evaluating the potential energy difference $\Delta \Phi_1$ (Equation (6)) at distance $R_{\text{TID}}$ (Equation (7)) where the particle tidally detaches from $H_2$:

$$\Delta E_{\text{TID}}(l, \theta, R_{\text{TID}}) \approx \Delta \Phi_1(l, \theta, r = R_{\text{TID}}(\delta_{\text{TID}})).$$

(8)
The particle undergoes its second deflection by the mass of $H_2$. The tidal field contribution $\Delta E_{TF}$ is also illustrated and discussed in Figure 3.

5. ENERGY FROM THE DOUBLE-SCATTERING MECHANISM

The second energy change the particles in $H_2$ experience during the merger is generated by the double-scattering mechanism. In this section we describe the kinematics of the mechanism and derive an analytical solution for the resulting kick energy $\Delta E_{DS}$. As seen in Figure 3, the energy contributions from tidal fields and the double-scattering mechanism are of the same order. The mechanism is therefore playing an important role in how energy is distributed in halo mergers.

5.1. Origin of the Double-scattering Kick Energy

The double-scattering mechanism is a process where a particle is gravitationally deflected two times during the merger between its own host halo $H_2$ and a target halo $H_1$. The first deflection is by the potential of $H_1$, which momentarily dominates as the two halos overlap, whereas the second is by the potential of $H_2$, which can dominate after the two halos have passed each other. We refer to the deflection by $H_1$ as the first deflection and the subsequent deflection by $H_2$ as the second deflection. The two merging halos are moving relative to each other during the merger, so the two deflections are therefore happening in two different velocity frames. The energy of the particle during each deflection is conserved in the frame of deflection, but because the two frames move relative to each other, a deflection in one frame can result in an energy change in the other. In our case, the deflection by $H_2$ changes the velocity of the particle by an amount $\delta v$ along the motion of the two merging halos. The particle energy is constant in the frame of $H_2$, but in the frame of $H_1$ the energy changes by an amount $\sim (\delta v + w)^2 - w^2 \sim \delta vw$. This is the contribution from the double-scattering mechanism, denoted by $\Delta E_{DS}$. A schematic illustration is shown in Figure 2, and a numerical example is shown in Figure 4. In the following, we calculate the details of this double-scattering process.

5.2. Analytical Model

We consider a particle initially bound to $H_2$, with orbital velocity $v_0$ and polar position $l, \theta$ measured in the RF of $H_2$ just prior to the merger. To reach an analytical solution for the double-scattering kick energy $\Delta E_{DS}$, we now work from the orbital picture shown in Figure 2, which serves to approximate the full orbital trajectory of the particle. Following this picture, we first model the velocity kick $\delta v$ that the particle receives relative to the CM of $H_2$ from its first deflection by $H_1$. We then use this kick velocity to model the orbit of the particle through $H_2$, where it undergoes its second deflection by the mass of $H_2$ enclosed by radius $\epsilon$. This deflection rotates the velocity vector of the particle by an angle $\alpha$, resulting in a velocity change $\delta v$ along the motion of the merging halos. From this deflection, we then calculate the resultant energy change $\Delta E_{DS} \sim \delta vw$, as described in Section 5.1. The components of this model will be calculated in the sections below for two merging HQ halos.

5.2.1. The First Deflection by Halo $H_1$

The particle receives a velocity kick $\delta v$ relative to $H_2$ through tidal shocking (Gnedin et al. 1999a, 1999b) that is due to the difference in acceleration between the particle and the CM of $H_2$ during the merger. A numerical example is shown in...
estimated by

$$\Delta v \approx \int_{-T}^{+T} a(t) \, dt = \int_{-R}^{+R} \frac{1}{w(x)} \frac{GM_1(d)}{d(x)^3} \, dx,$$

(9)

where $a$ is the acceleration that the particle experiences due to $H_1$, $d = (d_x, d_y)$ is the separation vector between the particle and the CM of $H_1$, $d$ is its magnitude, and $w$ is the relative velocity between $H_1$ and $H_2$. The distance $d$ is simply given by

$$d^2 = (x - x_p)^2 + y_p^2,$$

where $x_p = l \cos(\theta)$ and $y_p = l \sin(\theta)$ are the $x$ and $y$ coordinates of the particle in the frame of $H_2$, respectively. In this work, we model the orbit of the particle by assuming that it is not moving during the passage of $H_1$ from $-l$, $+l$, as illustrated in Figure 2. The horizontal kick velocity $\Delta v_x$ is therefore calculated by setting $R = l$. The vertical kick velocity $\Delta v_y$ is not sensitive to $R$ in the same way and is therefore calculated by just using $R = \infty$ to simplify the expressions. The horizontal and vertical components of the kick velocity will be calculated below.

### 5.2.2. Horizontal Kick Velocity $\Delta v_x$

The horizontal kick velocity $\Delta v_x$ is found by integrating Equation (9) from $-l$ to $+l$ using $d_x = x - x_p$:

$$\Delta v_x \approx \frac{\Phi_1(0)}{w(l)} \left( \frac{1}{1 + l' \sqrt{2} \sqrt{1 - \cos(\theta)}} - \frac{1}{1 + l' \sqrt{2} \sqrt{1 + \cos(\theta)}} \right),$$

(10)

where we have assumed that $w$ equals $w(l)$ during the passage. By comparing with Equation (3), we see that the expression, except for the $1/w$ term, is exactly equal to the difference in potential energy of the particle between the initial configuration, where $H_1$ is at $-l$, and the final configuration, where $H_1$ is at $+l$. This is consistent from the perspective of energy conservation, where the particle must receive a kinetic energy kick to compensate for the potential energy difference $\Delta \Phi_{-l,+l}$. To illustrate this, we note that in the RF of $H_1$ the kinetic energy of the particle after the merger is $E_{\text{kin}}(l) \approx w(l) \Delta v_x$, and from energy conservation the kick must therefore be $\Delta v_x \approx \Delta \Phi_{-l,+l}/w(l)$, as we also find in Equation (10). The horizontal velocity kick is therefore not due to a real dynamical deflection, but it arises purely from an energy difference. This difference can be calculated exactly, and as a result our estimate for $\Delta v_x$ is also relatively accurate. In practice, it is useful to approximate Equation (11) by $\Delta v_x(\theta) \approx \Delta v_x(0) \cos(\theta)$, where $\Delta v_x(\theta)$ denotes the solution including the full $\theta$ dependence. Using this approximation we find

$$\Delta v_x \approx \frac{\Phi_1(0)}{w(l')} \frac{2l'}{1 + 2l'} \cos(\theta).$$

(11)

In the limit where $H_1$ and $H_2$ pass through each other with the escape velocity of $H_1$, this reduces to the simple form

$$\Delta v_x \approx -\sqrt{2} \frac{\Phi_1(0)}{l' \cos(\theta)} \sqrt{1 + l'/2l}. $$

### 5.2.3. Vertical Kick Velocity $\Delta v_y$

The vertical kick velocity $\Delta v_y$ arises because the particle briefly follows an orbit in the potential of $H_1$, which momentarily dominates as the two merging halos pass each other. The velocity kick $\Delta v_y$ can therefore be estimated by writing down the orbital solution for a particle with encounter velocity $\sim w$ in the top panel of Figure 3. We can analytically estimate $\Delta v$ in the impulsive limit, where one assumes that the particle is not moving during the encounter (e.g., Hut 1983; Aguilar & White 1985; Cincotta et al. 1991; Funato & Makino 1999).

Using a coordinate system where $H_1$ is moving along the $x$ axis and the CM of $H_2$ is located at $x = 0$, the kick can now be

![Figure 4. Orbital trajectory of a halo particle (green) gaining a significant energy kick from the merger between its own initial host halo $H_2$ (black) and a target halo $H_1$ (gray). The particle is the same as the green one shown in Figure 1. The green solid lines show the full orbit of the particle, whereas the green dashed lines show the orbit if the particle is not undergoing its second deflection by $H_2$. This second deflection directly leads to the energy kick $\Delta E_{\text{DS}}$ generated by the double-scattering mechanism, as described in Section 5.2. The dashed line would therefore lead to no energy increase from this mechanism. The numbers refer to five important moments, as illustrated in Figure 2. Top: orbit of the particle in the RF of $H_1$. The halo particles of $H_2$ are plotted at time 1 (right halo) and 4 (left halo), respectively. Bottom: orbit of the particle in the RF of $H_2$. The horizontal gray line shows the orbit of the target halo $H_1$ that moves from left to right. The smaller stars on the orbits indicate equal time intervals. The angle between the solid and the dashed line is denoted by $\alpha$ and is calculated in Equation (14). The corresponding time-dependent velocity and energy of the particle are shown in Figure 3.](image-url)
and impact parameter $\sim l \sin(\theta)$ moving in the HQ potential of $H_1$. However, there are no analytical solutions for the majority of DM density profiles, including the HQ profile (Binney & Tremaine 1987), and we must therefore use the impulsive approximation presented in Equation (9). Assuming the particle is only deflected by the mass of $H_1$ enclosed by a sphere of radius $r = |l \sin(\theta)|$, and using $d_2 = y_p$, we find

$$\Delta v_y \approx \frac{\Phi_1(0)}{\omega(x_p')} \frac{2y_p}{(1 + |y_p'|)^2},$$  \hspace{1cm} (12)$$

where we have assumed that $w$ equals $w(x_p')$ during the passage ($w$ at time 2 shown in Figure 2). In the limit where $H_1$ and $H_2$ pass through each other with the escape velocity of $H_1$, the above expression reduces to $\Delta v_y \approx -\sqrt{2}[\Phi_1(0)]y_p'/\sqrt{1 + x_p'(1 + |y_p'|)^2}$. In contrast to the horizontal kick $\Delta v_x$, the vertical kick $\Delta v_y$ arises from a real dynamical deflection, which makes it hard to estimate precisely. By comparing with simulations, we find that our above estimation for $\Delta v_y$ is about a factor of $\sim 1.5$ too low. One reason for this is that we only include the mass of $H_1$ enclosed by the radius $\sim l \sin(\theta)$. However, including the full HQ profile in the integration leads to a divergent result, which clearly illustrates the limits of the impulsive approximation.

#### 5.2.4. The Second Deflection by Halo $H_2$

After receiving the velocity kick $\Delta v$, the particle starts to move from its initial position $l, \theta$ toward the central region of $H_2$, where it undergoes a second deflection by the mass of $H_2$ enclosed by radius $\epsilon$. This changes the velocity vector of the particle from $v_1 = v_0 + \Delta v$ to $v_2 = v_1 + \delta v$. To estimate the components of $v_2$, we first calculate the impact parameter $\epsilon$ for the deflection by $H_2$, as illustrated in Figure 2. Assuming $|\tan(\theta)||\Delta v_x/\Delta v_y| < 1$, we find from simple geometry

$$\epsilon = |x_p| \frac{1 - \frac{\tan(\theta)}{\sqrt{1 + \gamma^2}}},$$  \hspace{1cm} (13)$$

where $\gamma = |\Delta v_x/\Delta v_y|$. Using the relation $\alpha \approx \delta v/\Delta v$ and Equation (9) to estimate $\delta v$, we can now write down an expression for the deflection angle $\alpha$:

$$\alpha \approx \frac{2G M_2(\epsilon)}{\epsilon \Delta v^2} \approx \frac{2[\Phi_2(0)]}{\Delta v^2} \frac{(\epsilon/\alpha_2)}{(1 + \epsilon/\alpha_2)^2},$$  \hspace{1cm} (14)$$

assuming that the particle is only affected by the mass of $H_2$ enclosed by $\epsilon$. In the last equality we have inserted the HQ mass profile of $H_2$. The deflection by $H_2$ conserves the length of the velocity vector of the particle in the RF of $H_2$, but rotates $v_1$ by the angle $\alpha$ into the new vector $v_2$, which therefore has coordinates given by

$$v_{2,x} = v_{1,x} \cos \alpha + v_{1,y} \sin \alpha,$$
$$v_{2,y} = v_{1,y} \cos \alpha - v_{1,x} \sin \alpha.$$  \hspace{1cm} (15)$$

The particle will only receive a positive energy kick if $|v_{2,x}| > |v_{1,x}|$, i.e., if the kick velocity $\Delta v$ and deflection angle $\alpha$ fulfill the inequality $|\tan(\alpha/2)||\Delta v_x/\Delta v_y| < 1$ in the limit where $\Delta v \gg v_0$. From the definition of $\delta v \equiv v_2 - v_1$ we now find the change in velocity due to the second deflection:

$$|\delta v_x| \approx |\Delta v_x| \alpha,$$
$$|\delta v_y| \approx |\Delta v_y| \alpha,$$  \hspace{1cm} (16)$$

where we have assumed that $\alpha \ll 1$ and that the kick velocity dominates the motion of the particle along its new perturbed orbit, i.e., $v_1 \approx \Delta v$. The last assumption is necessary for the double-scattering mechanism to work effectively.

#### 5.2.5. Resultant Energy from the Double-scattering Mechanism

To finally calculate the dynamical kick energy $\Delta E_{DS}$ of the particle, we first assume that the second deflection by $H_2$ happens instantaneously, i.e., the velocity vector of the particle changes from $v_1$ to $v_2$ at a single point. This point occurs when the particle passes the center of $H_2$ at a distance $\sim \epsilon$, as shown in Figure 2. From this assumption it naturally follows that the potential energy of the particle is approximately constant during the deflection, and the change in total energy will therefore be dominated by the change in kinetic energy. The kick energy $\Delta E_{DS}$ can therefore be estimated by

$$\Delta E_{DS}(l, \theta) \approx \frac{1}{2}(v_2 + w(r')^2) - \frac{1}{2}(v_1 + w(r')^2) = w(r')\delta v_y,$$  \hspace{1cm} (17)$$

where $\delta v_y$ is the $y$ component of the velocity change in the RF of $H_2$ given by Equation (16), $r'$ is the distance between $H_1$ and $H_2$ at the time the particle undergoes its second deflection by $H_2$, and $w(r')$ is the corresponding relative velocity between $H_1$ and $H_2$. In the limit where the two halos pass each other with the escape velocity of $H_1$, $r'$ is found by solving the differential equation $w(r) = dr/dt = \sqrt{2[\Phi_1(r)]}$. The solution for a HQ halo can be written in the form $r' = 3(\Delta t/2[\Phi_1(0)]/[2a_1] + (1 + \epsilon)^{1/2})^{1/3} - 1$, where $\Delta t \approx 1/\Delta v$ is the time from the first deflection by $H_1$ to the second deflection by $H_2$. For a slightly more precise estimate, one has to include dynamical friction, which impacts the estimation for $w$. The friction will mainly play a role in slowing down the bulk of $H_2$ after the merger, i.e., the main change will be to the value of $w(r')$. The correction will therefore factor out in Equation (17), which makes it easy to include in possible future studies.

The left panel in Figure 5 shows our estimate for $\Delta E_{DS}$ given by Equation (17), as a function of the position of the particle in $H_2$. One can see that our model predicts that the particles that receive a positive energy kick are all located in a cone with two wings pointing along the velocity of $H_2$. Comparing with the energy kick generated by tidal fields $\Delta E_{TF}$ (illustrated in the right panel), we see that the double-scattering mechanism is actually likely to be the dominating kick mechanism for particles located near the center. This is in contrast to the outer parts, where the tidal field contribution seems to be the dominating component.

We confirmed this by numerical simulations.

A comparison between an N-body simulation and our analytical estimate for $\Delta E_{DS}$ is shown in Figure 6. The analytical calculation is done using Equation (17), with numerical measured values for $\Delta v$ and $w(r')$, to completely isolate the prediction from the double-scattering mechanism itself. The measured energy kick from the N-body simulation is here defined as the change in energy of the particle between the time when $H_2$ leaves $H_1$ at distance $l$ (time 3) and the time when the particle leaves $H_2$ at distance $l$ (time 5). As seen on the figure, we find good agreement despite the difficulties in both measuring and calculating the kick energy.

#### 6. OBSERVATIONAL SIGNATURES

The particles that have gained an energy kick through our presented mechanisms will have a relatively high velocity compared to the field, and they therefore have the potential to...
be labeled as high-velocity objects. In this section, we illustrate how different the resultant velocities of the kicked particles are, compared to the virialized particles bound to $H_1$.

6.1. Kick Velocity Relative to Virial Velocity

The velocity of a particle moving on a radial orbit in the potential of the target halo $H_1$ is found from simple energy conservation:

$$u_r(r) = \sqrt{2(E_i + \Delta E - \Phi_1(r))},$$

(18)

where $u_r(r)$ is the radial velocity of the particle at distance $r$, $E_i$ is the initial energy of the particle, $\Phi_1(r)$ is the radial-dependent potential of $H_1$, and $\Delta E$ is any additional energy contributions. We consider the case where $\Delta E = \Delta E_{DS} + \Delta E_{TF}$. All quantities are defined in the RF of $H_1$.

Figure 7 illustrates our analytical estimate for the maximum radial velocity a dynamically kicked particle can have at the virial sphere of $H_1$. The velocity is plotted in units of the virial velocity of $H_1$, defined by $V_{1,\text{vir}} \equiv \sqrt{GM_{1,\text{vir}}/R_{1,\text{vir}}}$. As seen on the figure, the energy release from tidal fields and the double-scattering mechanism can lead to particles with a
velocity $\sim 2 V_{1,\text{vir}}$ at the virial radius of $H_1$. These particles will therefore clearly stand out from the virialized part. For comparison, particles receiving no energy kicks ($\Delta E = 0$) will, in our example, instead have a velocity $\sim 1.3 V_{1,\text{vir}}$. We also see that the maximum kick velocity is given to particles located around $\sim 0.1$ to $0.2 R_{2,\text{vir}}$ from the center of $H_2$. Stars are typically located within 1–5% of the virial radius of their host halo (Kravtsov 2013); we therefore expect only the outer parts of a possible central galaxy to be effectively kicked by our presented mechanisms. The outer parts are usually populated by loosely bound stars and stellar systems, such as GCs and dwarf galaxies (Pota et al. 2013). The GCs are the only objects that can be seen out to cosmological distances because of their high number and density of stars ($\sim 10^4$ pc$^{-3}$), which make them a potentially observable tracer of our presented mechanisms. We give an example of this in the section below.

6.2. Is HVGC-1 Kicked through a Halo Merger?

The first detection of a high-velocity globular cluster (HVGC-1) was recently reported by Caldwell et al. (2014). This high-velocity object was identified as a GC from spectroscopy and $uK$ photometry and was found between GC candidates collected over several years by Keck/DEIMOS, LRIS, and MMT/Hectospec (Strader et al. 2011; Romanowsky et al. 2012). The GC is located in the Virgo Cluster at a projected distance of $\sim 84$ kpc from M87, with a radial velocity relative to Virgo and M87 of about 2100 and 2300 km s$^{-1}$, respectively. The interesting question is now, how did this GC get this high velocity? As discussed in the paper by the authors, the GC could have been kicked by a binary SMBH system located in the center of M87. However, it is very uncertain whether a GC can survive this because of the possibility of disruption. Subhalo interactions near M87 could also be an explanation, but no subhalos have been observed in its close vicinity yet. The nature of the kick is therefore still unsolved.

The GC could have been kicked by our presented dynamical mechanisms, i.e., first by tidal fields and then by the double-scattering mechanism, if it was initially bound to a DM halo merging nearly head-on with Virgo. To receive the maximum kick energy, the GC must have been located in the outskirts of its host galaxy just prior to the merger, which is not an unlikely scenario (e.g., Huxor et al. 2014; Pota et al. 2013). For a 1:10 mass ratio, we have shown that this merger configuration can generate objects with a radial velocity of approximately two times the virial velocity of the target halo at its virial radius. In the case of Virgo, this would mean a velocity of about $\sim 2 \times 1100 = 2200$ km s$^{-1}$ (the virial velocity of Virgo is somewhere in the range $\sim 900$ to 1300 km s$^{-1}$ Strader et al. 2011), which is consistent with the observed value for HVGC-1. We further note that HVGC-1 is observed to be hostless. This also follows from our model because the generated kick velocity of the GC quickly separates it from its initial host galaxy. This is also seen in Figure 1. Full numerical simulations can of course be used to explore this in more detail, including the role of encounter velocity, halo concentrations and mass profiles, impact parameter, and mass ratio. We leave that for a future study.

7. ORBITS OUTSIDE THE VIRIAL RADIUS

An interesting final question is now what the future orbits are of the dynamically kicked particles if they leave the virial sphere of their target halo $H_1$ (also denoted the ejector halo in the sections below). We study this by solving the equation of motion for particles moving under the influence of the gravitational force of their ejector halo and the expanding cosmological background.

7.1. Equation of Motion of an Ejected Particle

The radial acceleration $\ddot{r}$ of an ejected particle is, to a first order, dominated by two terms: one from the gravitational field of the ejector halo and one from the expanding background (Nandra et al. 2012; Behroozi et al. 2013). In this approximation, the total acceleration is given by

$$\ddot{r} = -\frac{GM_1}{r^2} - H_0^2 r (\Omega_m a^{-3} - 2\Omega_{\Lambda,0})/2, \quad (19)$$

where $M_1$ is the time-dependent mass of $H_1$, $r$ is the physical distance between the center of $H_1$ and the particle, $H_0$ is the Hubble parameter today, $a$ is the scale factor (not to be confused with the HQ scale radius), and $\Omega$ is the density parameter. One can see that the force exerted by the background expansion can either be attractive or repulsive, depending on whether the universe is decelerating or accelerating, respectively. The expansion itself therefore does not imply a repulsive force (Davis et al. 2003).

The scale factor $a$ evolves in time by the standard relation (e.g., Behroozi et al. 2013)

$$a(t) = a_{m\Lambda} [\sinh (3H_0t\sqrt{\Omega_{\Lambda,0}/2})]^{2/3}, \quad (20)$$

where $a_{m\Lambda}$ is the matter dark energy equality scale factor given by $(\Omega_{m,0}/\Omega_{\Lambda,0})^{1/3}$. The mass of the ejector halo $H_1$ is time dependent as well because of matter accretion. To include this mass evolution, we use the following empirical halo mass scaling (Wechsler et al. 2002):

$$M_1(t) = M_1(a_e) e^{\beta (z(a_e) - z(a))}, \quad (21)$$

where $M_1(a_e)$ is the mass of the halo at some reference scale factor $a_e$, $z$ is the redshift, and $\beta$ is a constant. The constant $\beta$ has been found to be in the range of 0–2 using numerical simulations (Wechsler et al. 2002; McBride et al. 2009).

7.2. How Far Can an Ejected Particle Travel?

The radial motion of a high-velocity particle is found by solving Equation (19), including Equations (20) for $a(t)$ and (21) for $M(t)$. We assume that the particle escapes the virial radius of $H_1$ with a velocity $v_{\text{eject}}$ at a scale factor $a_{\text{eject}}$. The results are shown in Figure 8, which illustrates the particle’s radial position $r(a)$ as a function of scale factor $a$, for different combinations of $v_{\text{eject}}$ and $a_{\text{eject}}$. The position is plotted in units of the virial radius of $H_1$, which changes in time according to the defined relation $M_{\text{vir}} \equiv 4\pi \Delta_{\text{vir}} \rho_c R_{\text{vir}}^3$, where $\rho_c$ is the time-dependent critical density of the universe and $\Delta_{\text{vir}}$ is the overdensity threshold. The ejection velocity is given in units of the corresponding virial velocity, defined by $V_{\text{vir}} \equiv \sqrt{GM_{\text{vir}}/R_{\text{vir}}}$. We assume $\beta = 1$, $\Delta_{\text{vir}} = 200$, and a flat universe with $\Omega_{m,0} = 0.3$. From the orbits shown in Figure 8, we see that the maximum distance a particle can travel strongly depends on its ejection time $a_{\text{eject}}$, as described in the following.

A particle ejected at early times will have a long time available to travel a long distance, but it will also experience a strongly increasing gravitational attraction from its ejector halo because of mass accretion. The force from the background
A particle ejected at later times will have less time to travel away from its ejector halo, but it will on the other hand experience a much smaller mass accretion, i.e., attracting force, from its ejector halo. The force from the background also changes to be repulsive at late times, which makes it even easier for particles ejected at late times to escape. As seen in Figure 8, this interplay between ejection time and force terms results in a sweet spot around $0.3-0.5$ in scale factor for ejecting particles into large orbits.

An important observation from Figure 8 is that particles ejected with only a few times the virial velocity can enter large orbits and travel several virial radii away from their target halo. For example, a particle ejected at $v_{\rm eject} \sim 0.5$ with velocity $v_{\rm eject} \sim 2V_{\rm vir}$ will be $\sim 7 R_{\rm vir}$ away from the ejector halo at the present time. Including dynamical effects in estimating how far particles can reach from their target halo is therefore important and leads to much higher limits compared to previous estimations based, e.g., on the halo collapse formalism (Mamon et al. 2004). A similar conclusion was reached by, e.g., Sales et al. (2007); Ludlow et al. (2009) using cosmological N-body simulations.

8. CONCLUSIONS

We provide an explanation for how high-energy particles are created in halo mergers by introducing a model that includes a double-scattering mechanism. The mechanism is a process where an incoming halo particle undergoes two subsequent gravitational deflections during the merger, where the first is by the mass of the target halo and the second is by the mass of the particle’s original host halo. The particle can receive a significant energy kick from this process because the two frames of deflection, i.e., the two halos, move relative to each other during the merger. The amount of energy generated through this mechanism is comparable to a well-known energy contribution from tidal fields. The mechanism therefore plays a significant role in how energy is distributed in halo mergers. To our knowledge, the double-scattering mechanism has not been characterized in this context before, despite its great importance especially in explaining the origin of high-velocity particles.

From our presented model, we derive analytically the kick energy a given particle receives from tidal fields and the double-scattering mechanism as a function of its position in its original host halo just prior to the merger. In the case of a 1:10 head-on merger, we estimate that the largest energy kick is about $0.3-0.4\Phi_{\Omega}(0)$ and is given to particles located at around $\sim0.1-0.2 R_{\rm vir}$ from the original host halo center. We find this to be in agreement with numerical simulations.

By converting kick energy to velocity, we illustrate that our presented mechanisms can kick objects to a resultant velocity approximately two times the virial velocity of the target halo measured at its virial sphere. This motivates us to suggest that the high velocity of the recently discovered GC HVGC-1 (Caldwell et al. 2014) can be explained by a halo merger, i.e., that the energy kick is generated by tidal fields and the double-scattering mechanism. We believe this serves as a more natural explanation than other proposed ideas, including three-body interactions with a binary SMBH system in M87. Cosmological simulations also support this (Sales et al. 2007).

Finally, from solving the equation of motion of a dynamically kicked particle in an expanding universe, we find a sweet spot around a scale factor of $0.3-0.5$ for ejecting particles into large orbits. These orbits can easily reach beyond $\sim5$ virial radii from the target halo, which is significantly longer than previous estimates based on halo collapse models (e.g., Mamon et al. 2004). This illustrates the importance of including dynamical interactions for describing the outer regions of cosmological halos.

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Figure 8. Radial position (solid lines) as a function of scale factor $a$ for particles leaving the virial sphere of $H_I$ with velocity $v_{\rm eject}$ at different times (black stars). The dark gray area centered around zero shows the region inside the virial sphere of $H_I$. The gray area shows the region where the gravitational force from the halo dominates over the force from the expanding background. In the light gray region the force from the background dominates. The vertical dotted line indicates the time when the force from the expanding background changes from being attractive (left side) to repulsive (right side), i.e., when $\Omega_m a^3 = 2H_0$. One can see that there exists a sweet spot around $0.3-0.5$ in scale factor for ejecting particles into large orbits.
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