Natural science education: a multiaspect system of models of mathematics

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Abstract. The use of computers to a certain extent devalues the role of manual calculations, but in the practice of mathematics education, the emphasis is still placed on the correct conduct of mathematical calculations. Therefore, in modern education, mathematics should be considered as a multifaceted multidimensional phenomenon, in which there are scientific, applied, historical and cultural, cultural and leisure, economic, management components. The education system should reflect the main aspects of mathematics, for which we propose to use a complex of its models. The article discusses several such models: activity, apparatus, phenomenological, process models of mathematics, two historical models of mathematics. An outline of their use for the design and evaluation of a study topic and a training course is made.

1. Introduction

In the state, healthcare and education are conservative. Therefore, they are constantly in need of reform, and the consequences of erroneous actions are delayed and difficult to eliminate. The role and importance of many academic subjects in the education of graduates is gradually changing. Often, the decision to exclude the course or to significantly reduce it has to be reconsidered after the disastrous consequences of ill-considered decisions are discovered, as was the case in Russia with the course of astronomy. It is necessary to exclude such errors. For this, it is necessary to carry out a multifaceted analysis of the role, place and significance of the academic subject, to take measures to adapt it to the changed conditions of the graduate of the educational institution activity [1,2]. At the moment, an alarming situation is developing with the teaching of mathematics. The arguments of the “math haters” are based on the fact that computing today is done with the help of computer programs. Moreover, we are talking, among other things, about symbolic calculations carried out by professional mathematicians using the Maple, GAP, Maxima packages, etc. However, the identification of mathematics with its computing apparatus is illegal, this one-sided view is fundamentally erroneous and disastrous for the education system and graduates of educational institutions. Unfortunately, even professional methodologists rarely refer to the content of the mathematics course. Usually, the focus is on teaching methods and tools, for example, web technologies [3-5]. If questions of content are considered, then they relate to the choice of educational topics, the selection of tasks, etc. The models of mathematics considered by the authors of this article correspond to new trends in education, for example, STEM education: S - science (science), T - technology (technology), E - engineering (engineering), M - mathematics (mathematics).
2. Deductive and inductive approaches to the formalization of the concept of “mathematics”

The rapid development of information technology has led to a reassessment of the importance of many types of activities and to a significant change in their nature. It is required to rethink what mathematics is, to clarify its role and place in modern education and science.

When asked what they mean by mathematics, students most often answer that it is a science of numbers or a discipline (a system of disciplines). History knows several attempts to give an exhaustive definition of mathematics. René Descartes understood mathematics as the science that explains everything related to a certain measure. But is it possible in the education system to rely unconditionally on this definition or on the classical definition: “Mathematics is the science of quantitative relations and spatial forms of the real world”?

The definition of a concept is a description of its scope. In our opinion, the known definitions do not cover the entire scope of the concept of “Mathematics”. For example, topology, group theory, mathematical logic, and many other modern branches of mathematics can hardly be attributed to “quantitative relations” or “spatial forms”. It is also doubtful that they are causally related to measure. But mathematics is not limited to a system of mathematical theories. It includes scientific, technical, historical, philosophical, educational, competitive aspects and even elements of art. Mathematics is an essential component of culture. Therefore, we believe that attempts to formulate a definition of mathematics are hopeless and impractical. It is rational to apply not a deductive way of formalizing a concept, but an inductive one. We have proposed to do this by formalizing various models of mathematics [6,7].

3. Models of mathematics as a means of reflecting its different aspects

3.1. Mathematics as a system of scientific disciplines

In textbooks and scientific literature, mathematics is usually presented as a system of scientific and / or academic disciplines: algebra, mathematical and functional analysis, differential equations, probability theory, topology, mathematical logic, etc. This requires special efforts to form connections between sections of the mathematics curriculum.

3.2. Mathematics as a system of mathematical phenomena

Often, mathematics is perceived as a set of definitions, theorems, problems and their solutions. By a mathematical phenomenon, we also mean algorithms, methods, methods of forming hypotheses, etc. [8]. To formalize the model of mathematics as a system of phenomena, an algebraic approach to modelling, according to Yu. B. Melnikov was used, consisting in the formation of: 1) a system of basic models; 2) typical transformations and typical combinations; 3) an approximation mechanism intended for (approximate) representation of the required model as a result of applying typical transformations and typical combinations of basic models. A model of mathematics as a system of phenomena, implemented on the basis of an algebraic approach to modelling, is shown in figure 1.

![Figure 1. Illustration for the model of mathematics as a system of phenomena.](image)

Algebraic approach to mathematical phenomena

| Basic phenomena | Transformations and combinations of phenomena | Approximation mechanism |
|----------------|---------------------------------------------|------------------------|
| Basic concepts | Language combinations | Strategies of mathematical activity |
| Basic languages | Math concepts | Math. approval |
| Basic algorithms | Algorithms and methods | |

In terms of the theory of activity, it can be said that mathematical methods and, in particular, algorithms were initially considered as means and operations of activity. But in the 20th century, in mathematics and its applications, they began to be studied also as a subject of mathematical activity.
3.3. Activity model of mathematics

Learning mathematics is learning an activity, this aspect is presented in figure 2.

![Figure 2. Illustration for the model of mathematics as a field of activity.](image)

Typically, textbooks provide ready-made, neatly designed evidence and solutions since the style of scientific literature is copied. The reader of a scientific article does not need to explain how they arrived at the hypothesis, the formulation of the statement and its proof. But it is important for the learner! Study [9] examines how evidence is perceived in the context of teaching and learning mathematics. Are theorems and proofs needed? The theorem may not be useful, but it is likely that you will need the ability to form hypotheses, refute and prove them, highlight situations when it is advisable to form a new concept and formalize it. L D Kudryavtsev [10] identified the main goals of mathematical education for students in technical areas of training. This list reflects various aspects of the mathematics reflected in the considered models.

Purposeful activity in the process of teaching students in a lesson in mathematics leads to an understanding of the power of mathematical ideas [11-13]. The block “Intra-mathematical activity” includes the formation of motives, the assimilation of strategies of mathematical activity and other components of the control system. Methodological apparatus of mathematics, figure 2, includes mechanisms for choosing promising mathematical phenomena for research, mechanisms for forming hypotheses, etc. The blocks “Applications of Mathematics” and “Methods of Application of Mathematics” have a similar meaning. Sometimes it is more important to use the conceptual rather than the computing apparatus of mathematics, the use of its language. Statistical physics, thermodynamics, quantum mechanics are formulated with the obligatory participation of the language of mathematics.

3.4. Hardware model of mathematics. The hardware model of mathematics is shown in figure 3.

![Figure 3. Illustration for the hardware model of mathematics.](image)

Mathematics is not limited to a set of algorithms. This affects thinking and can hinder adaptation to new conditions and a new stage of learning [14]. Mathematics can be viewed as an information processing apparatus. The conceptual apparatus is intended for the mathematical formalization of information and the interpretation of mathematical phenomena in terms of the original subject area. It
can operate with non-mathematical objects, but in practice it is most often used to translate information from one mathematical language to another.

3.5. Model of mathematics as a system of processes

The model of mathematics as a system of processes is shown in figure 4.

The actual mathematical activity can be interpreted as the transformation of mathematical phenomena. The analysis of the student's carrying out of these transformations is usually used to control his activities and fix personality changes. In the simplest case, the formation of the student's ability to use a theorem, method or mathematical concept is controlled. In more advanced control systems, it is possible to record not only the ability to solve a certain class of mathematical problems, but also changes in motivation for learning, the development of the ability to identify a new class of objects that are promising for study, to form the volume of a new concept, to create new models due to, for example, forming an interface component with other objects, etc.

3.6. Historical models of mathematics

Two fundamentally different historical models are usually considered. Elements of the phenomenological historical model are mathematical phenomena, the main attributes of which are the authors of these phenomena, historical conditions from the appearance, etc. As elements of the personification historical model, there are subjects of mathematical activity, not all of whom can be considered professional mathematicians by their main occupation. The subject of mathematical activity can be a group of people, for example, Nicola Burbaki (a pseudonym for a group of mathematicians). The formation of mathematical schools, their influence on the development of science, etc. are actively studied.

4. The use of mathematics models in the theory and practice of technical, natural science and economic education

Currently, a significant amount of research is devoted to the study of various conditions for improving the quality of modern education [15-20]. For mathematics as an academic discipline and science, a mixture of content and activity components is characteristic, see figure. 2. For example, the Pythagorean theorem can be considered both as an object of activity (you need to understand the formulation, remember it, understand the meaning of the theorem), and as a means of activity (you need to learn to see where this theorem can be applied, what can be obtained with its help, how exactly it can be apply). The result of the analysis of the phenomenological model is work [8], where it is shown that for a student who is not planning to become a professional mathematician, only two options for the attitude to a mathematical phenomenon are relevant: as an object of activity or as an instrument of activity.

4.1. Models of mathematics as a tool for assessing the adequacy of mathematics courses

In connection with the modern information technologies introduction (in particular, distance learning technologies), the improvement of the methodology for evaluating educational modules, the methodology for preparing and evaluating educational literature is becoming especially relevant. In the conditions of the availability of various information resources, in our opinion, one of the priorities in preparing a textbook should be the formation of a student's ability to critically perceive and evaluate
information in a multidimensional manner, combine and compare different sources, independently obtain consequences, learn not only methods, but, preferably, and a methodology for obtaining new knowledge, which is not limited to the primitive use of search engines.

4.2. Using models of mathematics to form the content of mathematical courses
For example, consider the content of the section (module, academic discipline) “Differential Equations” from the point of view of different models of mathematics.

From the standpoint of the hardware model of mathematics, see figure 3, the priority components of the section “Differential Equations” are: a system of typical concepts, typical forms of information presentation (differential equation, its solution, systems of equations, etc.), methods of formalizing information, methods of transforming information, etc. From the standpoint of a model of mathematics as areas of activity, see figure 2, priorities of the section “Differential Equations” are theorems, typical problems, typical methods and, in particular, typical algorithms. If the adequacy of the content of the section “Differential Equations” is evaluated using a model of mathematics as a system of phenomena, figure 1, then the priorities should be mathematical phenomena, connections between them and their attributes such as the author (authors) of the corresponding phenomenon, conditions, reasons and incentives for their appearance and formalization, etc. If we take a model of mathematics as a system of processes as a basis, see figure 4, then the priorities in the content of the section “Differential Equations” will be the processes of transformation of mathematical objects, changes in the subject of learning, methods of fixing and measuring them, making forecasts, analysing the results of the process, assessing the adequacy of forecasts, etc. The use of historical models of mathematics has a long tradition in the education system. Consideration in high school mathematical courses of issues related to the history of the emergence and development of mathematical phenomena, their interdependencies is extremely important for the formation of a holistic view of mathematics in students. The article “Mathematics” in the “Mathematical Encyclopaedia” [21] is completely devoted to a detailed review of the development of mathematics. In the already classic university textbooks [22, 23], the historical perspective is presented substantially.

4.3. Application of models of mathematics to increase motivation for learning
The idea of mathematics as a set of theorems and concepts to be learned by heart, or as a computing apparatus for solving artificially invented problems currently has a demotivating effect. For example, the overwhelming majority of people do not have to solve trigonometric equations either in everyday life or in professional activities not related to teaching mathematics and physics. The situation is similar with the use of most other theorems, properties of logarithms, etc. From this, not only children, but also adults (sometimes even including school teachers!) conclude that it is inexpedient to study mathematics, that it is necessary to partially or completely abandon the acquisition of “outdated”, “unnecessary”, “useless” knowledge. The use of various models of mathematics allows us to show the inconsistency of this point of view.

5. Conclusion
The introduction of information technology requires a rethinking of the place and role of mathematics in modern education. It is concluded that attempts to formalize this voluminous multifaceted concept within the framework of one definition are futile. A set of models of mathematics is proposed, reflecting aspects of mathematics that are essential from a didactic point of view. The application of various models of mathematics for the formation of the educational process and its components, comprehensive assessment of learning outcomes and various components of the educational process is briefly considered. The use of a system of models of mathematics allows you to understand how balanced different aspects of mathematics are presented in the educational process.

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