Online Equivalent Method of External Network Based on Measurement Information

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Abstract. How to acquire the accurate equivalent network of external system in the interconnected power grid has always been a challenging issue in the fields of both academic and engineering. This paper proposes a new method for online equivalence of the external network based on the measurement information of the internal network. Firstly, it builds an equivalent network of the external system, including the coupled branches between each of the boundary node, the equivalent generators and the equivalent lines from corresponding boundary nodes to them. Then, based on the SCADA (Supervisory Control and Data Acquisition) measurement information, an optimization model to estimate the parameters of the equivalent network is constructed, with the minimization of the mismatch between the boundary's measurement and the calculation value of the equivalent network being the objective and the parameters of each line of the equivalent network within the reasonable range being the constraints. In order to reduce the calculation scale, measure is adopted to remove the coupled lines whose electrical distances (or impedances) are larger than a relative big threshold. The impedance range of each line in the equivalent network is determined by ward equivalent method under the condition of maximum and minimum operation mode. The model is solved by interior point method, which owns the satisfactory convergency performance. The solution of the model makes each of the calculated value of the equivalent network at the boundary node closest to the measured value, while all the parameters of the equivalent network within a practical range. The results of an example demonstrate the validity and effectiveness of the model and method proposed in the paper.

Keywords: online equivalency, SCADA, electrical distance, impedance range, interior point method.

1. Introduction

The power grid is a complex system that operates in an integrated manner. Complete and accurate power grid information is the basis for system analysis, evaluation, control, and etc. [1,2]. Hierarchical and zonal management is a typical management mode of China's power grid, which makes each of power companies possibly belong to a different enterprise or authority while being both cooperative and competitive. Correspondingly, there is a contradiction between accurate and complete information requirements in operation and management authority. Firstly, in order to realize accurate network analysis and control, each power company needs accurate and complete information of neighboring
networks which are usually under the management of another power company usually belonging to different authorities. Due to the consideration of the confidentiality of information or economic benefits, the neighboring power companies are often unwilling to fully disclose and share their total information in time, or just part of that. Secondly, the information transmitted by neighboring power grids also has the problem of record time consistency with those of the data collected within the network run by themself. As a result, it will have a serious impact on the effectiveness of system analysis and control, even leading to the problems such as non-convergence of power flow calculation and bringing to the risks of the security and stability of the power grid operation [3-5]. In order to get rid of this contradiction, an accurate external network equivalent is the most effective strategy, that is, given only a part of the external network or adjacent network information, a calculated accurate equivalent network of the external network combined with the internal network is used to analyze, optimize and control of the subsystem. That has been becoming one of hot topics for many years.

Up to date, the researches and methods about external network equivalence are roughly divided into two categories: 1) topological equivalence with known external network information; 2) non-On-line equivalence with unknown or partially known external network information.

1) Topological equivalence of known external network information. Thevenin, Norton equivalent method, Ward equivalent method, REI equivalent method and their variants can be adopted at this case [6-12], which are relatively mature. However, the common defect of this kind of methods is how to maintain the accuracy in different situation, especial while the internal network experiencing a great change from the original.

2) On-line equivalence of unknown or partially known external network information. The basic idea of this type method is that based on SCADA (Supervisory Control and Data Acquisition) measurement data of the internal system, various strategies are deployed to estimate the equivalent parameters of the external network. In [13,14], a single-port Thevenin equivalent method is proposed, which uses a voltage source in series with an impedance to be the equivalence of external networks, and establishes equations using the multi-period measurement at the boundary nodes to solve the equivalent parameters; however, it is only applicable for a single-port network. In [15,16], a simplified Ward equivalence method is presented, and the equivalent network is composed of just the equivalent lines between boundary nodes and the injected power, the parameters of which are estimated using two periods of information produced by once switching of the line within internal network. However, it exists difficulty to implement at practical system due to the unplanned switching operation of the internal network being strictly prohibited during actual operation. A two-port static equivalent method based on measured information is introduced in [17]. Compared with [15,16], though not requiring the switching of the internal network in this method, it is just applicable to the networks with two boundary nodes. In [18], the authors have designed a two-stage multiport static equivalent method. First, the simplified Ward equivalent model parameters are estimated, and then the extended Ward equivalent model parameters are successively estimated. Because of the phased solution strategy, the number of the equivalent values in the second stage are greatly reduced, and the accuracy of equivalency is also improved to a certain extent thanking to the coupling between the parameters. However, the optimal model for the calculation of the equivalent network is unconstrained, and there exists such issue that the estimated equivalent network parameters may be negative, which is out of practice and causes problems such as subsequent un-convergence of power flow calculation.

In the face of the above summary, this paper proposes a new online equivalent method based on internal network measurement information. This method first constructs an equivalent network of the external system, which includes the coupled branches between each of the boundary node, the equivalent generators and equivalent lines from corresponding boundary nodes to them. Then, based on the SCADA measurement, an optimization model for parameter identification of the equivalent network is constructed. The minimization of mismatch between the boundary’s measurement and the calculation value of the equivalent network is the objective function of this model, and the corresponding equivalent network branch parameters are within a certain reasonable interval as a
constraint. On the base of these, in order to simplify the equivalent network and the model, the electrical distance is used as a measure to delete some connecting branches between the boundary nodes with larger impedance values. The values of corresponding equivalent branch based on the Ward equivalent network under the maximum and minimum operation modes are adopted to determine the appropriate ranges of online equivalency network parameters. The interior point method is utilized to solve the model. The calculation example shows the effectiveness of the equivalent model and algorithm in this paper.

2. Online equivalent network and model based on measurement information

2.1. Online equivalent network
When the external network data is unknown or only part, we could make an accurate equivalency of the external network based on the measurement information of the internal network.

The entire system is divided into external network, boundary nodes and internal network as shown in Fig.1. Based on Thevenin's equivalent idea, the external network is simplified into an equivalent network with coupling branches connecting each boundary node, multiple power sources and the branches from the sources to the corresponding boundary nodes. The equivalent network is illustrate in figure 2.

![Figure 1. Interconnected grid before equivalency.](image1)

![Figure 2. Online equivalent network of external network based on internal network measurement data.](image2)

Where \( U_i^n \) is the voltage amplitude of the boundary node \( i \) at time \( t \), \( n \) is the total number of boundary nodes, \( n \geq 2 \). \( S_i^n \) is apparent power injected into the internal network from boundary node \( i \) at time \( t \). \( t = 1, 2, \cdots, m \), \( m \) is total sampling times of measurement. \( Y_y = G_y + jB_y \) represents mutual admittance of equivalent tie line between boundary node \( i \) and boundary node \( j \). \( Y_i = G_i + jB_i \) represents mutual admittance of equivalent tie line between
boundary node \( i \) and corresponding equivalent generator node, \( E_i, \sigma_i \) are the internal potential amplitude and phase angle of the equivalent generator \( i \), respectively.

2.2. Optimization model of solving the equivalent network parameters

For the equivalent network shown in Fig.2, the value of power at the boundary nodes must be consistent before and after the equivalency.

\[
\Delta P_i = P_{i,ex}' - P_i' = 0 \quad (1)
\]
\[
\Delta Q_i = Q_{i,ex}' - Q_i' = 0 \quad (2)
\]

where \( \Delta P_i, \Delta Q_i \) represent active power and reactive power mismatch of boundary node \( i \) at time \( t \), respectively. \( P_{i,ex}', Q_{i,ex}' \) are equivalent active power and reactive power injected into the boundary nodes \( i \) from external network at time \( t \), respectively, which are expressed as Eq.3 and Eq.4. \( P_i', Q_i' \) are active power and reactive power injected into the internal network from boundary node \( i \) at time \( t \), respectively.

\[
P_{i,ex}' = U_i' E_i \left( G_{i0} \cos \theta_{i0} + B_{i0} \sin \theta_{i0} \right) + U_i' \sum_{j=1}^{n} U_j' \left( G_{j} \cos \theta_j' + B_{j} \sin \theta_j' \right) 
\]
\[
Q_{i,ex}' = U_i' E_i \left( G_{i0} \sin \theta_{i0} - B_{i0} \cos \theta_{i0} \right) + U_i' \sum_{j=1}^{n} U_j' \left( G_{j} \sin \theta_j' - B_{j} \cos \theta_j' \right) 
\]

where \( \theta_j' \), phase-angle difference between node \( i \) and node \( j \), can be calculated as \( \theta_j' = \theta_i' - \theta_j' \). \( \theta_i' \) is the phase angle of the boundary node \( i \). \( \theta_{i0} = \theta_i' - \sigma_i \). In Eq.3 and Eq.4, variables needed to be estimated include \( E_i, \sigma_i, G_{i0}, B_{i0}, G_j, B_j \) \((i, j \in (1,2,...,n))\), rewrite them as vector \( x_i = [E_i, \sigma_i, G_{i0}, B_{i0}] \) and vector \( y_j = [G_j, B_j] \) \((i=1,2,...,n, j=i+1,...,n; i \neq j)\), respectively, for convenience.

We construct an optimization model based on the SCADA measurements of the network of multiple periods in order to estimate the parameters of the equivalent network. The objective is to minimize the square sum of all the mismatches of measurements and calculated values. The constrains is all the parameters of the equivalent branches \( x_i, y_j (i=1,2,...,n, j=i+1,...,n; i \neq j) \) within certain ranges. The specific expression of the optimization model is as

\[
\min \sum_{i=1}^{m} \sum_{j=1}^{n} (\Delta P_i')^2 + (\Delta Q_i')^2 \\
\text{s.t.} \\
x_{i,3,\min} \leq x_{i,3,\max} \\
x_{i,4,\min} \leq x_{i,4,\max} \\
y_{j,1,\min} \leq y_{j,1,\max} \\
y_{j,2,\min} \leq y_{j,2,\max} \\
i, j = 1,2,...,n
\]
3. Solution method for the optimization model

3.1. Determination of the inequality constraints

The purpose of the inequality constraint in Eq.5 is to force the parameters of the equivalent network estimated within a reasonable range. Otherwise, if the initial value is not selected properly, it is likely for some of the calculated equivalent network parameters to be negative, which accordingly causes problems such as non-convergence in subsequent power flow calculations of the artificial network composed of internal network and the equivalent network. In [19,20], the authors simply restrict the resistance and reactance of the equivalent to be branch greater than 0, or just present the constraints of the upper and lower limit. However, the constraints just greater than 0 are too relaxed, there still exists the cases that due to the impedance value being too small or nearly to zero, the convergence of the power flow calculation may remain difficult. In the latter case, there is no specific method for how to determine the upper and lower limit. Since the load level of the current operation mode of external network must among the interval between the load level of maximum operation mode and that of minimum operation mode, this paper suggests that the impedance value of each equivalent branch of the current operation mode should be also within the interval determined under the maximum and minimum operation modes. The specific method to determine the upper and lower limit of the value of each equivalent branch is described below:

For the given internal network, firstly obtain the external network topology, generators outputs and load requirements under the maximum and minimum operating modes. Secondly, construct two complete systems using current internal network and the information of external under maximum and minimum operation mode, respectively. Thirdly, perform the equivalency of external network by Ward method for the two systems. Fourthly, transform the Ward equivalent network into the equivalent network shown in Fig.2. Finally, the two sets of equivalent branch parameters gained at the third step are the upper and lower limits of the corresponding branch parameters in online equivalency.

3.2. Simplification of the equivalent network by electrical distance

For the equivalent network in Fig. 2 and the corresponding optimization model Eq.5, we can get $2n$ measurement mismatch equations as shown in Eq.1 and Eq.2 by one period measurement information. There are $n^2 + 3n$ equivalent parameters to be estimated. In order to effectively solve the equations, the number of mismatch measurement equations must be greater than the number of equivalent parameter to be estimated, i.e. $2mn > n^2 + 3n$, namely, $m > (n+3)/2$.

It could be seen that the number of parameters to be estimated increases in square with the growth of the number of boundary notes, which makes the scale and calculation amount of the corresponding model enlarge a lot correspondingly. This paper proposes a simplification strategy of the equivalent network based on the measure of the electrical distance between the boundary nodes, reducing the scale of the model and the amount of calculation. In the typical operation mode, if the electrical distance between two boundary nodes is more than twice of the average electrical distance between any two nodes (different values can be determined according to experience for different power grids), the coupling between the specific two nodes is considered weak, and the branch between the two nodes in the equivalency network is deleted. The process is described below.

According to the data under the typical operation mode of the entire network, the average electrical distance between any nodes is calculated as

$$Z_{ave} = \frac{\sum_{i=1}^{N_{node}} \sum_{j=1}^{N_{node}} Z_{ij}}{N_{node}(N_{node}-1)} \quad (6)$$
where $Z_{ij}$ is mutual impedance between node $i$ and node $j$. $N_{node}$ is the total number of the whole system.

When $Z_{ij} > 2Z_{ave}$, the equivalent branch between boundary node $i$ and boundary node $j$ should be deleted, where $i, j \in \text{Node}_{boundary}$, $\text{Node}_{boundary}$ is the set of boundary nodes.

3.3. Solution algorithm and its procedures of equivalent model based on interior point method

3.3.1. Brief introduction of interior point method. Eq.5 can be expressed simply as

$$\min f(x)$$

s.t.

$$g_{min} \leq g(x) \leq g_{max}$$

(7)

The interior point method is by far the most stable and reliable method for solving this model. The basic solution process of this method is as follows:

First, introduce the relaxation variables $l$ and $u$ into Eq.7, and transform it into an interior point method model as follows

$$\min f(x)$$

s.t.

$$g(x) - l - g_{min} = 0$$

$$g(x) + u - g_{max} = 0$$

$$(l, u) \geq 0$$

(8)

Then, define the Lagrange function as

$$L(x, l, u, z, w, \tilde{z}, \tilde{w}) = f(x) - z^T(g(x) - l - g_{min}) - w^T(g(x) + u - g_{max}) - \tilde{z}^T l - \tilde{w}^T u$$

(9)

where $z, w, \tilde{z}, \tilde{w}$ are Lagrange multiplier. According to KKT condition, the following equations are derived:

$$L_x = \nabla f(x) - \nabla g(x)(z + w) = 0$$

$$L_c = g(x) - l - g_{min} = 0$$

$$L_w = g(x) + u - g_{max} = 0$$

$$L_t = LZE = 0$$

$$L_u = UWE = 0$$

(10)

where $L, U, Z, W$ are diagonal matrix. $E = [1, 1, \cdots, 1]^T$.

Finally, solve Eq.10 by Newton method, and then get the final solution. The above process has been fully implemented in the commercial solver IPOPT, which can be used by simply directly including the solver.

3.3.2. Solving steps of equivalent model based on interior point method. The solution procedures of Eq.5 based on the interior point method is preset below:
a) Determine which equivalent branches between the boundary nodes should be removed. Based on typical network-wide operating data, calculate all the electrical distance between each two boundary nodes and the average electrical distance of the whole network. Remove all the equivalent branches whose electrical distance are greater than twice of the average electrical distance of the whole network, thus a simplified equivalent network model is obtained.

b) Determine the parameter range of online equivalent network. Based on the entire network data under the operation modes of maximum and minimum, respectively, the Ward equivalent model is used to perform the calculation of equivalence of the external network under the two cases. After transforming the two equivalent networks just calculated into the form of Fig.2, the values of each same branch parameter of the two equivalent networks are just the upper and lower limit of the corresponding branch parameter of the online equivalent network.

c) Determine the required number of internal network measurement information periods. According to the number of variables to be estimated, and based on the principle that the number of mismatch equations need to be greater than the number of variables, the number of required internal network measurement information periods $m$ is decided.

d) Initialize parameters. The initial values of the equivalent parameters $E_i$ and $\sigma_i$ are set as 1 p.u. and 0, respectively, and the line parameters $G_{ij}, G_{i0}$ and $B_{ij}, B_{i0}$ are set as the middle of the upper and lower limit, respectively. Obtain the branch measurements by SCADA in $m$ periods and the corresponding phase angle of the boundary nodes could be achieved by state estimation.

e) Construct equivalent network parameter optimization model Eq.5. and solve the model by the solver IPOPT; after convergence, the parameters $x_i, y_{ij} (i = 1, 2, \ldots, n;j = i+1, \ldots, n;i \neq j)$ of the equivalent network can be acquired finally.

The block diagram is shown in Fig.3.

**Figure 3.** Block diagram of the solving procedure of online equivalent network based on interior point method.
4. Case studies

In this paper, an IEEE 39-bus system is used as an example to verify the effectiveness of the proposed model and method. \((1 \pm 20\%)\) of rated load is taken as the maximum and minimum typical operation mode of the external network respectively. All parameters and calculation results are expressed as standard values.

4.1. Production of measurement data and comparison conditions

Assuming that the structure of the power grid is unchanged, the power of each load node and the active output of each generator both increase by 0.3% of the power on the based on the rated load operating mode, several periods of power flow solutions can be obtained for simulating the online SCADA measurements.

In the IEEE 39-bus system, nodes 1, 3 and 17 are set as border nodes, nodes 2, 25-30 and 37-39 are set as external network nodes, and the remaining nodes are as internal network nodes, which is displayed in Fig.4. For the equivalent network shown in Fig.2, if the network simplification strategy is not adopted, the intranet measurements of 4 consecutive periods need to be taken to estimate the parameters of equivalent network. For the simplified equivalent network, the electrical distances between any of the boundary nodes and the average electrical distance of the entire network composed of the internal network at the current operation mode and the external network at the rated operation mode are listed in Table 1. It can be seen that the electrical distances between node 1 and 7, node 3 and 17 are both greater than the twice of the average electrical distance. Therefore, the equivalent branch 1-17, 3-17 are removed from the simplified equivalent network. Consequently, there are only 14 equivalency parameters needed to be estimated in the simplified equivalent network, and only three consecutive periods of measurements are required.

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**Figure 4.** IEEE39-bus system and its partition of internal and external network.
Table 1. Electrical distances between boundary nodes and two times of average electrical distance of the entire network.

| Equivalency branch | Electrical distances [p.u.] |
|--------------------|----------------------------|
| 1-3                | 0.0069+j0.0679             |
| 1-17               | 0.0579+j0.1896             |
| 3-17               | 0.0446+j0.2467             |
| 2·Z_{ave}          | 0.0274+j0.1366             |

After obtaining the equivalent network parameters, the power flow calculation is conducted for the new system composed of the equivalent network model and the internal network. The results are compared with those gained by Ward equivalent models to verify the accuracy of equivalency.

In order to study the impact of model measurement noise, Gaussian white noise with standard deviation \( \sigma \) is superimposed on the above-mentioned measurement based on the power flow result. And the standard deviations of the measurement errors of voltage amplitude and branch power are 0.004. Similar calculations were performed to study the influence of measurement noise on the calculation accuracy of different methods.

4.2. Comparison of calculation results

For the equivalent calculation of the external network, it only reflects the main impact of the external system on the internal system, which naturally exists some equivalent error. Two equivalent error evaluation indexes including the average relative error and the maximum error of the power flow are defined to measure the equivalent accuracy of different methods.

Average relative error of power flow is calculated as:

\[
\varepsilon_{\text{ave}} = \text{ave} \left[ \frac{P_l - P_{l}^{\text{eq}}}{P_l} \right] + \text{ave} \left[ \frac{Q_l - Q_{l}^{\text{eq}}}{Q_l} \right] + \text{ave} \left[ \frac{V_i - V_{i}^{\text{eq}}}{V_l} \right], l \in \text{Line}, i \in \text{Node}
\]  

(11)

where \( P_l (Q_l) \) and \( P_{l}^{\text{eq}} (Q_{l}^{\text{eq}}) \) are active (reactive)power of the internal network line \( l \) before and after equivalent, respectively. \( V_i \) and \( V_{i}^{\text{eq}} \) are voltage amplitude of the internal network node \( i \) before and after equivalence, respectively. \( \text{Line} \) is the set of all lines in the internal network; \( l \) is line number in the internal network; \( \text{Node} \) is the set of all nodes in the internal network; \( i \) is the node number in the internal network; \( \text{ave} \) represents averaging operator.

Maximum relative error of power flow is calculated as:

\[
\varepsilon_{\text{max}} = \max \left\{ \frac{P_l - P_{l}^{\text{eq}}}{P_l} \right\} + \max \left\{ \frac{Q_l - Q_{l}^{\text{eq}}}{Q_l} \right\} + \max \left\{ \frac{V_i - V_{i}^{\text{eq}}}{V_l} \right\}, l \in \text{Line}, i \in \text{Node}
\]  

(12)

The average relative error and maximum error of different equivalent model under different load levels are presented in Table 2. The load level 0%, 1% and 3% means that the load remains unchanged, increased by 1% and by 3% relative to the rated state, respectively. The un-simplified and simplified in the table represents the model proposed in this paper with or without network simplification strategy, respectively. The following tables are similar. The Ward in the table represents the model generated by Ward equivalent method.
Table 2. Equivalent errors of different equivalent methods at different load levels.

| Load [%] | Model                | $\xi_{ave}$ [%] | $\xi_{max}$ [%] |
|----------|----------------------|-----------------|-----------------|
| 0        | Un-simplified        | 0.21            | 0.43            |
|          | simplified           | 0.16            | 0.37            |
|          | Ward                 | 0.12            | 0.29            |
| 1        | Un-simplified        | 0.76            | 1.32            |
|          | simplify             | 0.79            | 1.38            |
|          | Ward                 | 5.98            | 8.33            |
| 3        | Un-simplified        | 5.87            | 7.82            |
|          | simplified           | 6.99            | 8.49            |
|          | Ward                 | 207.39          | 479.00          |

It can be seen that the model error obtained by Ward equivalent method based on the initial state of the entire network data is the smallest when the state of the entire network does not change; the error of the Ward model is much larger than those of the equivalent models proposed in this paper when the load level changes. This indicates that the equivalent models in this paper are more adapted to the scenarios when the system state changes, which is just the same as that of online system. This is because the equivalent model in this paper considers multiple power flow states, while the Ward model only considers one power flow state. Comparing the error of the un-simplified model with the simplified one, the error of the two models is very close when the change of the load level from the original is small, and the error of the simplified model is even lower when the state of the load level is the same as the original. This is because the simplified equivalent model requires fewer measurement periods, measurement time span is shorter, the equivalent model is closer to the initial state, and the accuracy of the equivalent parameters is higher correspondingly. But when the load level changes, the unsimplified model shows a bit advantage in equivalency accuracy, bigger the change, more obvious the advantage, compared to the simplified model.

Table 3 shows the equivalency errors of different models at the condition of white noise added to the measurements based on power flow calculation.

Table 3. Equivalent errors of different equivalent methods at different load levels with measurement errors.

| Load [%] | Model    | $\xi_{ave}$ [%] | $\xi_{max}$ [%] |
|----------|----------|-----------------|-----------------|
| 0        | Un-simplify | 0.27            | 0.51            |
|          | simplify  | 0.19            | 0.44            |
|          | Ward      | 0.21            | 0.39            |
| 1        | Un-simplify | 0.79            | 1.36            |
|          | simplify  | 0.81            | 1.43            |
|          | Ward      | 6.49            | 9.76            |
| 3        | Un-simplify | 6.79            | 9.97            |
|          | simplify  | 7.83            | 9.66            |
|          | Ward      | 297.48          | 537.91          |

Comparing Table 2 to Table 3, it can be seen that the Ward equivalency model is more easily affected by measurement errors. Although the equivalency error with the measurement error of equivalent models proposed in this paper will also increase compared to those gained without measurement error, it can still maintain high accuracy when the change of load level is not too large. Especially, when the load level changes keeps less than 1%, the average equivalent error is always under 1%.

5. Conclusions
This paper proposes a new method for online equivalency of the external network based on the measurement information of the internal network. Using SCADA information, this method constructs
an equivalent network of external network and corresponding parameter optimization identification model. The model takes the minimization of the mismatch between the calculated value and the measured value of the equivalent network boundary nodes as the objective function, with the equivalent parameters located at reasonable intervals as the constraints. An simplification strategy for equivalent network based on the electrical distance between boundary nodes and a method for determining the range of the equivalent network parameters are also designed, reducing the calculation scale and ensuring the parameters of the equivalent network within the practical range. The calculation example shows the effectiveness of the equivalent model in this paper. Because the method and model in this paper are very valid and the calculation is stable and fast, it features the potential for the application of practical engineering.

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