Optimization Framework and Graph-Based Approach for Relay-Assisted Bidirectional OFDMA Cellular Networks

Yuan Liu, Meixia Tao, Bin Li, and Hui Shen

Abstract

This paper considers a relay-assisted bidirectional cellular network where the base station (BS) communicates with each mobile station (MS) using OFDMA for both uplink and downlink. The goal is to improve the overall system performance by exploring the full potential of the network in various dimensions including user, subcarrier, relay, and bidirectional traffic. In this work, we first introduce a novel three-time-slot time-division duplexing (TDD) transmission protocol. This protocol unifies direct transmission, one-way relaying and network-coded two-way relaying between the BS and each MS. Using the proposed three-time-slot TDD protocol, we then propose an optimization framework for resource allocation to achieve the following gains: cooperative diversity (via relay selection), network coding gain (via bidirectional transmission mode selection), and multiuser diversity (via subcarrier assignment). We formulate the problem as a combinatorial optimization problem, which is NP-complete. To make it more tractable, we adopt a graph-based approach. We first establish the equivalence between the original problem and a maximum weighted clique problem in graph theory. A metaheuristic algorithm based on any colony optimization (ACO) is then employed to find the solution in polynomial time. Simulation results demonstrate that the proposed protocol together with the ACO algorithm significantly enhances the system total throughput.

Index Terms

Bidirectional communications, network coding, maximum weighted clique problem (MWCP), ant colony optimization (ACO), orthogonal frequency-division multiple-access (OFDMA).

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I. INTRODUCTION

A. Motivation

In wireless cellular networks, deploying a set of relay stations (RSs) between a base station (BS) and mobile stations (MSs) is a cost-effective approach for improving system performance, such as coverage extension, power saving and cell-edge throughput enhancement. These advantages are achieved as relay-assisted cooperative transmission exploits the inherent broadcast nature of wireless radio waves and hence provides cooperative diversity [1]–[3].

However, due to the half-duplex constraint in practical systems (i.e., a node cannot receive and transmit simultaneously), relay-assisted communications suffer from loss in spectral efficiency. Recently, network coding has demonstrated significant potential for improving network throughput [4]. Its principle is to allow an intermediate network node to mix the data received from multiple links for subsequent transmission. Physical layer network coding, as a means of applying this principle in wireless relay communications has received increasing attention [5], [6]. One simple but important example is two-way relaying, where a pair of nodes exchange information with the help of a relay node. Compared with the traditional one-way relaying, the two-way relaying overcomes the half-duplex problem and provides an improved spectral efficiency in bidirectional communication [6]–[8]. It is thus attractive to utilize network coding gain in the form of two-way relaying for more efficient transmission of downlink and uplink traffic in a cooperative cellular network.

Orthogonal frequency-division multiplexing (OFDM) is an enabling physical layer technology for spectrally efficient transmission as well as user multiplexing in broadband wireless networks. An intrinsic feature of orthogonal frequency-division multiple-access (OFDMA) is its capability of exploiting the frequency selectivity enabled multiuser diversity. A deep faded subcarrier for one MS may be favored by another MS. Yet, it is a nontrivial task to perform subcarrier assignment in an OFDMA system.

The goal of this work is to investigate the aforementioned three types of gains, namely, cooperative diversity gain, network coding gain, and multiuser diversity gain, in a relay-assisted bidirectional OFDMA cellular network. To this end, we present in this paper an optimization framework for resource allocation and further propose an efficient graph-based algorithm to utilize these gains simultaneously.
There are three questions to be addressed in this paper. First of all, it is known that relaying is not always necessary in relay-assisted communications. For example, when the channel condition of direct link is better than that of the cooperative link, direct transmission will be preferred. Furthermore, even if relaying is necessary, two-way relaying may not be always applicable. For example, when the downlink channel is good but the uplink channel is poor, then only uplink transmission needs relay assistance, and hence there is no opportunity to employ two-way relaying. Therefore, the first question we will address is how to design a unified transmission protocol which can support direct transmission, one-way relaying and two-way relaying. The second question to address is how to determine the transmission mode (direct transmission, one- and two-way relaying) of the downlink and uplink traffic for each MS. This is essentially a problem of opportunistic relaying with or without network coding. Thirdly, how to efficiently allocate the subcarriers and select the RSs is crucial so as to maximize the system total throughput. The answers to these questions are given *globally* and *systematically* in this paper.

B. Related Work

Optimization in both cooperative networks and OFDMA cellular networks has been extensively studied in the literature (e.g., [9]–[13]). However, only a few attempts have been made very recently to study the optimization of bidirectional cooperative OFDMA-based cellular networks [14]–[16]. Authors in [14] present a framework for joint optimization of relay selection, relay strategy selection, power and subcarrier allocation in which, however, only conventional one-way relaying is used. By Lagrange dual decomposition method, the joint optimization problem is decomposed into per-subcarrier subproblems that can be solved independently. In [15], authors propose a hierarchical protocol for one- and two-way relaying in a two-time-slot time-division duplexing (TDD) mode. In this protocol, the transmission mode of each MS as well as its assisting RS (if relay mode is selected) are pre-fixed, and the downlink and uplink transmission modes for each MS are the same. Then, only joint power and subcarrier allocation is considered and solved by Lagrange dual decomposition method as in [14]. Authors in [16] propose an XOR-assisted cooperative diversity scheme and present a heuristic algorithm for joint optimization of relay selection, transmission mode selection, power and subcarrier allocation. This system operates in frequency-division duplexing (FDD) mode with fixed sets of data subcarriers and relay subcarriers. The work, however, does not consider the pairing issue when applying network
coding to combine downlink and uplink traffic, the necessity of which will be detailed in Section II.

C. Contributions

In this paper, we consider an OFDMA-based wireless cellular network that maintains bidirectional downlink and uplink traffic for each MS. Centralized processing is assumed so that the base station controls the behavior of all users and relays. The main contributions of this paper are summarized as follows:

- A novel three-time-slot TDD transmission protocol for supporting direct transmission, one- and two-way relaying in bidirectional cooperative cellular networks is proposed. In this protocol, each frame is divided into three time slots. In the first two time slots, BS and MSs transmit the downlink and uplink traffic, respectively, while RSs remain silent. In the third time slot, RSs help to forward the downlink and uplink traffic only when necessary.

- Using the proposed three-time-slot TDD protocol, we formulate a joint optimization of bidirectional transmission mode selection, subcarrier assignment, and relay selection for maximizing the system total throughput. For simplicity, uniform power allocation is considered. There are three main distinct features about our problem formulation. First, we develop five feasible transmission modes, instead of three (direct transmit, one- and two-way relaying) for the bidirectional traffic to select. Second, for each MS, the uplink and downlink traffic always occurs in pair so that we can exploit the network coding gain through two-way relaying as large as possible. Third, each traffic pair can contain multiple parallel sessions, each of which can be assigned a different transmission mode and take place on a different set of subcarriers.

- The joint optimization problem is a combinatorial problem and NP-complete. To make it more tractable, we adopt a graph theoretical approach. First, we establish the equivalence between the original joint optimization problem and a maximum weighted clique problem (MWCP) in classical graph theory. A metaheuristic algorithm based on any colony optimization (ACO) is then employed to solve the MWCP problem in polynomial time.
D. Organization

The remainder of this paper is organized as follows. Section II introduces the system model and the proposed transmission protocol. Section III presents the optimization framework that jointly considers subcarrier allocation, transmission mode selection, and relay selection. Section IV presents an efficient algorithm to solve the optimization problem by a graphic approach. Section V provides extensive simulations to verify the effectiveness of the algorithm. Finally, we conclude the paper in Section VI.

II. SYSTEM MODEL AND PROPOSED TRANSMISSION PROTOCOL

We consider a single cell OFDMA wireless network with one BS, multiple MSs, and multiple RSs. Each MS can communicate with the BS directly or through one or multiple RSs. The communication is bidirectional and subject to the half-duplex constraint.

In traditional cellular networks where no relay is used, to support both downlink and uplink transmission, either TDD or FDD has to be applied. In particular, in TDD system, as shown in Fig. II(i), the transmission frame is divided into a downlink subframe and an uplink subframe, both on the same frequency band but in two different time slots. When RSs are present, then to support relay-assisted cooperative transmission, both downlink and uplink time slots can be further divided into two sub-slots, as shown in Fig. II(ii). Clearly, this four-time-slot TDD protocol is not efficient since resources will be wasted when not every MS needs RS’s assistance in both downlink and uplink. In [16], the authors propose a transmission protocol in FDD mode, where the total subcarriers are orthogonally divided into data subcarrier pool and relay subcarrier pool. However, in this FDD mode based transmission protocol, it is not mentioned how to define a subcarrier as a data or relay subcarrier and to determine the size of the two pools.

Inspired by the two-way relaying protocols studied in [7] and [8], we propose a novel three-time-slot TDD transmission protocol as shown in Fig. II(iii), which can support the three transmission modes, namely direct transmission, one- and two-way relaying in the considered bidirectional cooperative cellular networks. Specifically, in the first time slot, the BS transmits all downlink signals while MSs and RSs listen. In the second time slot, each MS transmits its uplink signals while the BS and RSs listen. In the third time slot, RSs forward both downlink and uplink signals received in the previous two time slots, whenever needed, while BS and MSs
listen. Thanks to the use of OFDMA, the data streams for different MSs are transmitted on
different subcarriers in each time slot so that there is no multiple-access interference.

The proposed transmission protocol can easily accommodate different transmission modes in
a unified fashion. For instance, if direct transmission is preferred for a MS in both downlink and
uplink, then, bidirectional communication for this MS can be accomplished in the first two time
slots. If relay-assisted transmission is preferred by a MS in both downlink and uplink, then a RS
who successfully decodes both the downlink and uplink messages can combine the messages
together using network coding and then send it to both the BS and the MS in the third time slot.
More specifically, we list in Table I all the possible combinations of transmission modes for each
MS with bidirectional traffic. As we can see, although both downlink and uplink traffic can adopt
one of the three transmission modes, and there are nine combinations in total, but only five are
feasible and marked as “√”. The other four are infeasible and marked as “×”. This is because
two-way relaying can only take place when both downlink and uplink transmissions need RS
assistance. In other words, the two-way relaying requires traffic pairing. A counterexample is
the combination of “two-way” in downlink and “direct” in uplink, which obviously can never
happen by definition. A point worthwhile to mention is that such traffic pairing was ignored in
the previous work [16]. From Table I it is also seen that the combination of “one-way” for uplink
and “one-way” for downlink is feasible. This can be implemented by using two different RSs
to forward downlink and uplink transmissions, respectively. That is, both downlink and uplink
transmissions need RS assistance but do not necessarily use network coding.

Fig. 2 further illustrates all the five feasible transmission modes, which will be considered
throughout this paper. In the figure, \( n_t \) denotes the index of subcarrier used for transmission in
time slot \( t \), for \( t = 1, 2 \) and 3. By introducing \( n_t \), we gain the flexibility of adaptive subcarrier
assignment. Note that in transmission mode \( d \), both RSs occupy the same subcarrier \( n_3 \) in the
third time slot. This is feasible because the back-propagated self-interference can be canceled in
the same way as in transmission mode \( e \) for two-way relaying. Thus, higher spectral efficiency
can be obtained compared with the case where the two RSs use different subcarriers.

Remark 1: In relay-assisted communications, the two received copies of the same content at
the destination, one from the source through the direct link and the other from the relay through
the cooperative link, can be combined using maximum ratio combining (MRC). In this paper,
for simplicity, we assume that selection combining (SC) is employed between the direct link and
the cooperative link. Therefore, every downlink and uplink traffic pair for each MS can select one of the five transmission modes shown in Fig. 2 according to the channel conditions.

The proposed three-time-slot TDD transmission protocol can capture the following gains:

- **Cooperative diversity gain:** The relaying takes place in the third time slot only if needed and each MS can select one or multiple RSs from all the available RSs in the network.
- **Network coding gain:** For each MS, the uplink and downlink traffic always occurs in pair so that we can enjoy the network coding gain as large as possible through transmission mode selection as defined above.
- **Multiuser diversity gain:** Subcarriers can be assigned adaptively to different MSs in each time slot.

Before we propose in the next section an optimization framework that simultaneously achieves the three kinds of gains, we need to make the following assumptions in this paper.

First, it is assumed that full channel state information (CSI) of the network is available at a central controller (which can be embedded with the BS) and the transmission rate on each link can be adapted based on it. Second, unlike the previous work [14]–[16] where power allocation is taken into account in the resource allocation, in this work we do not pursue power allocation for simplicity. It is known that power allocation can bring significant improvement in relay networks when the source and relay nodes are subject to a total power constraint [17]. However, as also demonstrated in [17]–[19], the gain brought by power adaptation is very limited in OFDM-based relay networks if each transmitting node is subject to an individual peak power constraint. In our considered system model, all the BS, MSs and RSs are subject to their own individual peak power constraints and, therefore, the transmit power is assumed to be fixed and uniformly distributed among all subcarriers for each of them.

Finally, we assume that the signal relaying is done per-subcarrier basis. That is, the signal received on one subcarrier, say $i$, in the first hop will be forwarded on subcarrier $i'$ in the next hop, where the subcarrier index $i'$ may not be the same as $i$. This is known as subcarrier-pairing [17], [18] or tone-permutation [19]. Such subcarrier-pair based relaying is optimal for amplified-and-forward (AF) protocol, where the signals received by the same relay on different subcarriers are processed individually, but suboptimal for decode-and-forward (DF) protocol, where the information from one set of subcarriers in the first hop can be decoded and re-encoded jointly and then transmitted over a different set of subcarriers in the next hop. Nevertheless, we still
adopt the subcarrier-pair based relaying for simplicity. As a result, the same number of subcarriers will be assigned in both hops.

III. OPTIMIZATION FRAMEWORK

In this section, we present the optimization framework in details. We first review the achievable downlink and uplink rate pair for each feasible transmission mode. Different relaying strategies including AF and DF will be considered. Then we provide a rigorous discussion of the problem formulation.

A. Achievable Downlink and Uplink Rate Pairs

Here we briefly discuss the rate expression for each of the five transmission modes given in Fig. 2 to facilitate the problem formulation in the next subsection. We model the wireless fading environment by large-scale path loss and shadowing, along with small-scale frequency-selective Rayleigh fading. OFDM is used at the physical layer and each subcarrier is assumed to experience flat fading. We also assume that the channels between different links experience independent fading. We further assume that the network operates in slow fading environment, so that channel estimation is perfect. The additive white Gaussian noises at BS, RSs and MSs are assumed to be independent circular symmetric complex Gaussian random variables. For brevity of notation, subscripts $B$, $M$ and $R$ denote BS, MS and RS, respectively, $u$ and $d$ denote uplink and downlink, respectively.

1) Transmission mode a: In this mode, both downlink and uplink use direct transmission. The achievable rate pair is easily obtained as

$$R_d = \frac{1}{3} C(\gamma_{BM}),$$

$$R_u = \frac{1}{3} C(\gamma_{MB}),$$

where $C(x) = \log_2(1+x)$, the pre-log factor $\frac{1}{3}$ is due to the use of three time slots, and $\gamma_{ij}$ denotes the signal-to-noise ratio (SNR) from the terminal $i$ to the terminal $j$, for $i, j \in \{B, R, M\}$.

2) Transmission mode b: In this mode, the downlink traffic prefers direct transmission and the uplink traffic needs RS assistance. Currently, many relay strategies are proposed. Among them, the two popular and practical ones are known as AF and DF. We thus focus on AF and
DF throughout this paper. Then, we can write the achievable rate pair as

$$R_d = \frac{1}{3} C(\gamma_{BM}),$$  \hspace{1cm} (3)

$$R_u = \begin{cases} \frac{1}{3} C\left(\frac{\gamma_{MR}\gamma_{RB}}{1+\gamma_{MR}+\gamma_{RB}}\right), & \text{for AF} \\ \frac{1}{3}\min\{C(\gamma_{MR}), C(\gamma_{RB})\}, & \text{for DF} \end{cases}$$  \hspace{1cm} (4)

3) Transmission mode c: In this case, the uplink traffic prefers direct transmission and the downlink traffic requires RS assistance. The achievable rate pair can be similarly rewritten as

$$R_d = \begin{cases} \frac{1}{3} C\left(\frac{\gamma_{BR}\gamma_{RM}}{1+\gamma_{BR}+\gamma_{RM}}\right), & \text{for AF} \\ \frac{1}{3}\min\{C(\gamma_{BR}), C(\gamma_{RM})\}, & \text{for DF} \end{cases}$$  \hspace{1cm} (5)

$$R_u = \frac{1}{3} C(\gamma_{MB}).$$  \hspace{1cm} (6)

4) Transmission mode d: In this case, both downlink and uplink traffic needs RS assistance but via two different RSs over the same subcarrier. The downlink and uplink signals from two RSs will both arrive at BS and MS, resulting in inter-link interference. It can be easily verified that the interference can be completely canceled since they are the back-propagated self-interference from BS or MS’s priori transmission. A special note is that in the case of AF, each destination also receives the amplified noises from both RSs which cannot be canceled. Thus, the achievable rate pair can be obtained as, whose derivation is simple and ignored.

$$R_d = \begin{cases} \frac{1}{3} C\left(\frac{\gamma_{BR_1}\gamma_{R_1M}(1+\gamma_{MR_2})}{\gamma_{R_1M}(1+\gamma_{MR_2})+\gamma_{R_2M}(1+\gamma_{BR_2})+(1+\gamma_{MR_2})(1+\gamma_{BR_2})}\right), & \text{for AF} \\ \frac{1}{3}\min\{C(\gamma_{BR_1}), C(\gamma_{R_1M})\}, & \text{for DF} \end{cases}$$  \hspace{1cm} (7)

$$R_u = \begin{cases} \frac{1}{3} C\left(\frac{\gamma_{MR_2}\gamma_{R_2B}(1+\gamma_{BR_1})}{\gamma_{R_1B}(1+\gamma_{MR_2})+\gamma_{R_2B}(1+\gamma_{BR_1})+(1+\gamma_{MR_2})(1+\gamma_{BR_2})}\right), & \text{for AF} \\ \frac{1}{3}\min\{C(\gamma_{MR_2}), C(\gamma_{R_2B})\}, & \text{for DF} \end{cases}$$  \hspace{1cm} (8)

5) Transmission mode e: This is the 3-step two-way relaying, where BS transmits its signals to RS in the first time slot, MS transmits its signals to RS in the second time slot, RS then mixes the received signals and broadcasts it to both BS and MS in the third time slot. Depending on if AF or DF is used, we present the achievable rate pairs separately in what follows.
a) AF two-way relaying: The achievable rate pair for 2-step AF two-way relaying is studied in [6], [7]. The extension to the 3-step protocol is simple. The results are:

\[
R_d = \frac{1}{3} C \left( \frac{\alpha^2 \gamma_{BR}|h_{RM}|^2}{1 + (\alpha^2 + \beta^2)|h_{RM}|^2} \right),
\]

\[
R_u = \frac{1}{3} C \left( \frac{\beta^2 \gamma_{MR}|h_{RB}|^2}{1 + (\alpha^2 + \beta^2)|h_{RB}|^2} \right),
\]

where

\[
\alpha = \sqrt{\frac{\xi P_{R}}{1 + \gamma_{BR}}}, \quad \beta = \sqrt{\frac{(1 - \xi) P_{R}}{1 + \gamma_{MR}}},
\]

Here, $\xi \in [0, 1]$ is a power allocation coefficient that determines the weights of the signals from BS and MS in the combined signals, $P_{R}$ is the transmit power constraint at RS, and $h_{i,j}$ is the channel gain from terminal $i$ to terminal $j$.

b) DF two-way relaying: After the RS decodes the messages from the BS and MS, it can combine the messages using either bitwise XOR or symbol-based superposition (SUP). For bitwise XOR, the rate pair is given by [7]:

\[
R_d = \frac{1}{3} \min \{ C(\gamma_{BR}), C(\gamma_{RB}), C(\gamma_{RM}) \},
\]

\[
R_u = \frac{1}{3} \min \{ C(\gamma_{MR}), C(\gamma_{RB}), C(\gamma_{RM}) \},
\]

where the first term in (12) or (13) represents the maximum rate at which RS can reliably decode the signals from BS or MS, while the minimum of the second and third terms in both (12) and (13) represents the maximum rate at which both BS and MS can reliably decode the signals from RS during the broadcast phase.

c) SUP-based DF two-way relaying: If SUP is applied, the rate pair is easily obtained as [6], [20]

\[
R_d = \frac{1}{3} \min \{ C(\gamma_{BR}), C(\theta \gamma_{RM}) \},
\]

\[
R_u = \frac{1}{3} \min \{ C(\gamma_{MR}), C((1 - \theta) \gamma_{RB}) \},
\]

where $\theta \in [0, 1]$ is a power allocation coefficient.

Remark 2: In all the aforementioned rate pair expressions, the SNR $\gamma_{ij}$ may not be the same as $\gamma_{ji}$. This is not only because the transmit power on terminal $i$ and $j$ may be different, but more importantly, the two links $i \to j$ and $j \to i$ can be assigned two different physical subcarriers.
B. Problem Formulation

Let $K = \{1, 2, ..., K\}$ be the set of MSs, $M = \{1, 2, ..., M\}$ the set of RSs and $N = \{1, 2, ..., N\}$ the set of subcarriers. Among the five transmission modes shown in Fig. 2, the first mode (direct transmission) involves subcarrier assignment in the first and second time slots only. For the rest four modes (cooperative transmission), one needs to assign the subcarriers in all the three time slots as well as selecting the proper RS(s). On the other hand, as discussed earlier, the downlink and uplink data streams are paired and can take the five possible transmission modes. In view of these facts, we introduce the following five sets of binary variables for transmission mode selection:

- $\rho_{n_1,n_2}^{k,a}$ indicates whether subcarrier pair $(n_1, n_2)$ in the first two time slots is assigned for MS $k$ for direct transmission of downlink and uplink traffic using transmission mode $a$, where $n_t$ denotes the subcarrier in time slot $t$.

- $\rho_{n_1,n_2,n_3}^{k,b}$ indicates whether MS $k$ is assigned RS $r$ for uplink traffic on subcarriers $n_2$ and $n_3$ and direct transmission of downlink traffic on subcarrier $n_1$, using transmission mode $b$.

- $\rho_{n_1,n_2,n_3}^{k,c}$ indicates whether MS $k$ is assigned RS $r$ for downlink traffic on subcarriers $n_1$ and $n_3$ and direct transmission of uplink traffic on subcarrier $n_2$, using transmission mode $c$.

- $\rho_{n_1,n_2,n_3}^{k,d,r,r'}$ indicates whether MS $k$ is assigned RS $r$ for downlink traffic on subcarriers $n_1$ and $n_3$ and RS $r'$ for uplink traffic on subcarriers $n_2$ and $n_3$, using transmission mode $d$.

- $\rho_{n_1,n_2,n_3}^{k,e}$ indicates whether MS $k$ is assigned RS $r$ for downlink traffic on subcarriers $n_1$ and $n_3$ and uplink traffic on subcarriers $n_2$ and $n_3$, using transmission mode $e$.

In this paper we assume that each subcarrier in each time slot can only be assigned to one MS for one traffic session in order to avoid interference. Moreover, the traffic session on each subcarrier can only operate in one of the five transmission modes. Therefore, these binary variables must satisfy the constraints:

$$\sum_{k \in K} \rho_{n_1,n_2}^{k,a} + \sum_{k \in K, r \in M} \left( \rho_{n_1,n_2,n_3}^{k,b} + \rho_{n_1,n_2,n_3}^{k,c} + \sum_{r' \in M, r' \neq r} \rho_{n_1,n_2,n_3}^{k,d,r,r'} + \rho_{n_1,n_2,n_3}^{k,e} \right) \leq 1, \forall n_1 \in N, \tag{16}$$

$$\sum_{k \in K} \rho_{n_1,n_2}^{k,a} + \sum_{k \in K, r \in M} \left( \rho_{n_1,n_2,n_3}^{k,b} + \rho_{n_1,n_2,n_3}^{k,c} + \sum_{r' \in M, r' \neq r} \rho_{n_1,n_2,n_3}^{k,d,r,r'} + \rho_{n_1,n_2,n_3}^{k,e} \right) \leq 1, \forall n_2 \in N. \tag{17}$$
After introducing these variables, we can now characterize the achievable downlink-uplink sum rate of each MS $k$ over all the possible transmission modes. This is given by

$$R_{k}^{\text{sum}} = \sum_{n_1 \in N} \sum_{n_2 \in N} \left( R_{k,d}^{n_1} + R_{k,u}^{n_2} \right) \rho_{k,a}^{n_1,n_2} + \sum_{n_1 \in N} \sum_{n_2 \in N} \sum_{n_3 \in N} \sum_{r \in M} \left( R_{k,d}^{n_1,n_3} + R_{k,u}^{n_2,n_3} \right) \rho_{k,r,b}^{n_1,n_2,n_3} + \sum_{n_1 \in N} \sum_{n_2 \in N} \sum_{n_3 \in N} \sum_{r \in M} \sum_{r' \neq r} \left( R_{k,d}^{n_1,n_3} + R_{k,u}^{r',n_2,n_3} \right) \rho_{k,r,r',d}^{n_1,n_2,n_3} + \sum_{n_1 \in N} \sum_{n_2 \in N} \sum_{n_3 \in N} \sum_{r \in M} \sum_{r' \neq r} \left( R_{k,d}^{r,n_1,n_3} + R_{k,u}^{r,n_2,n_3} \right) \rho_{k,r,r',d}^{n_1,n_2,n_3}. \quad (19)$$

The five summation terms in (19) represent the downlink-uplink sum rate achieved over the five transmission modes, respectively. The detailed rate expressions can be found in the previous subsection. In addition, the number of non-zero elements in each summation term represents the number of downlink-uplink traffic sessions that take on the same transmission mode but over different sets of subcarriers for this MS. From (19), one can also find that each MS can simultaneously operate in all the five transmission modes.

Our objective is to maximize the system total throughput by not only allocating subcarriers optimally but also finding the best RSs and best transmission modes for each MS. This is formulated as follows (P1):

$$\max R_{\text{tot}} = \sum_{k \in K} R_{k}^{\text{sum}} \quad \text{s.t.} \quad (16), (17), (18). \quad (20)$$

Remark 3: For bidirectional cellular networks, there is no single figure of merit to measure the overall system performance. For simplicity, we choose the downlink-uplink sum rate as our objective function. Notice that, if the asymmetric traffic in the downlink and uplink is considered, we can easily change to weighted sum rate, where the weighting parameters can be adjusted to
accommodate the asymmetry requirement. In addition, we can also easily modify the objective function to weighted sum of MS’s rate if user fairness is concerned.

IV. A GRAPH-BASED APPROACH

Problem P1 is a combinatorial optimization problem and looks formidable at a first glance as it involves too many binary variables. Conventional convex optimization techniques such as Lagrange dual decomposition as used in [14], [15] cannot solve it efficiently. Other optimization approaches, such as cutting plane and branch-and-bound algorithms [21], are also not viable due to the prohibitively large complexity. In this section, we propose a graph-based metaheuristic approach to solve the optimization problem. We first establish and prove the equivalence between the original optimization problem and a MWCP in graph theory. We then propose an ACO algorithm that runs in polynomial time.

A. Graph Model

Let us rewrite the system total sum rate as:

\[
R_{\text{tot}} = \sum_{n_1 \in \mathcal{N}} \sum_{n_2 \in \mathcal{N}} \sum_{k \in \mathcal{K}} (R_{k,d}^{n_1} + R_{k,u}^{n_2}) \rho_{k,a}^{n_1,n_2} \\
+ \sum_{n_1 \in \mathcal{N}} \sum_{n_2 \in \mathcal{N}} \sum_{n_3 \in \mathcal{N}} \sum_{k \in \mathcal{K}} \sum_{r \in \mathcal{M}} \left( R_{k,d}^{n_1,n_3} + R_{k,u}^{r,n_2,n_3} \right) \rho_{k,b}^{n_1,n_2,n_3} \\
+ \left( R_{k,d}^{r,n_1,n_3} + R_{k,u}^{n_2,n_3} \right) \rho_{k,c}^{n_1,n_2,n_3} + \sum_{r' \in \mathcal{M}, r' \neq r} \left( R_{k,d}^{r,n_1,n_3} + R_{k,u}^{r',n_2,n_3} \right) \rho_{k,d}^{n_1,n_2,n_3} \\
+ \left( R_{k,d}^{r,n_1,n_3} + R_{k,u}^{r,n_2,n_3} \right) \rho_{k,e}^{n_1,n_2,n_3}.
\]

(21)

Observing the first summation term of (21), it is easy to find that there is at most one non-zero element for a given subcarrier set \((n_1, n_2)\) due to the constraints (16) to (18). This implies that among the \(K\) MSs, at most one MS can occupy the subcarrier tuple \((n_1, n_2)\) for direct transmission. Similarly, observing the second summation term of (21), we find that there is also at most one non-zero element for a given subcarrier set \((n_1, n_2, n_3)\). This implies that at most one MS can occupy the subcarrier tuple \((n_1, n_2, n_3)\) for transmission using only one of the four relay-assisted transmission modes.
Based on the above observation, we can define

$$R(n_1, n_2) = \max_{k \in K} (R_{k,d}^{n_1} + R_{k,u}^{n_2}),$$

(22)

for each possible subcarrier pair \((n_1, n_2)\), and

$$R(n_1, n_2, n_3) = \max_{k \in K} \max_{r \in M} \left\{ (R_{k,d}^{n_1} + R_{r,n_2,n_3}^{n_2}), (R_{r,n_1,n_3}^{n_1} + R_{k,u}^{n_2}), \right\},$$

(23)

for each possible subcarrier tuple \((n_1, n_2, n_3)\). Then, to maximize the system total throughput, \(R_{tot}\) can be represented without loss of optimality as

$$R'_{tot} = \sum_{n_1 \in \mathcal{N}} \sum_{n_2 \in \mathcal{N}} R(n_1, n_2) \rho_{k^*,a}^{n_1,n_2} + \sum_{n_1 \in \mathcal{N}} \sum_{n_2 \in \mathcal{N}} \sum_{n_3 \in \mathcal{N}} R(n_1, n_2, n_3) \rho_{k^*,p^*,\Omega^*}^{n_1,n_2,n_3},$$

(24)

where \(k^*\) in the first summation term represents the MS index that takes the maximum in (22), and \(\{k^*, p^*, \Omega^*\}\) represent the MS index, transmission mode index and RS index, respectively that takes the maximum in (23). Note that \(\Omega = \{r, r'\}\) if \(p = \text{mode } d\) and \(\Omega = r\) if \(p = \text{mode } b, c, \text{ and } e\). Accordingly, the constraints (16) to (18) can be rewritten as

$$\sum_{n_2 \in \mathcal{N}} \rho_{k^*,a}^{n_1,n_2} + \sum_{n_2 \in \mathcal{N}} \sum_{n_3 \in \mathcal{N}} \rho_{k^*,p^*,\Omega^*}^{n_1,n_2,n_3} \leq 1, \quad \forall n_1 \in \mathcal{N},$$

(25)

$$\sum_{n_1 \in \mathcal{N}} \rho_{k^*,a}^{n_1,n_2} + \sum_{n_1 \in \mathcal{N}} \sum_{n_3 \in \mathcal{N}} \rho_{k^*,p^*,\Omega^*}^{n_1,n_2,n_3} \leq 1, \quad \forall n_2 \in \mathcal{N},$$

(26)

$$\sum_{n_1 \in \mathcal{N}} \sum_{n_3 \in \mathcal{N}} \rho_{k^*,p^*,\Omega^*}^{n_1,n_2,n_3} \leq 1, \quad \forall n_3 \in \mathcal{N}.$$  

(27)

Consequently, we can transform the original problem P1 to the following problem (P2):

$$\max R'_{tot}$$

s.t. \((25), (26), (27)\)

The simplified problem P2 involves \(N^2 + N^3\) binary variables only. It can be readily solved by the branch-and-bound algorithm, the well-known method for finding the optimal solution to combinatorial problems [21]. However, its potential computational complexity still grows exponentially with \(N^2 + N^3\). This motivates us to seek a graphic approach for solving P2 in polynomial time.

Theorem 1: Problem P2 is equivalent to a maximum weighted clique problem (MWCP).
Proof: Let $G = (V, E, W)$ be an arbitrary undirected and weighted graph, where $V$ is a set of vertices, $E \subseteq V \times V$ is a set of edges, $W$ is the weighting function such that $W : V \rightarrow \mathbb{R}_+$. A clique is a set of vertices $C \subseteq V$ such that every pair of distinct vertices of $C$ is connected with an edge. Based on the above discussion, we define two type of vertices for the given problem. For type one, a vertex is a subcarrier pair $(n_1, n_2)$ in the first two time slots associated with transmission mode $a$. For type two, a vertex is a subcarrier tuple $(n_1, n_2, n_3)$ associated with the four relay-assisted transmission modes. Due to the three-time-slot TDD mode, the set of subcarriers $N$ is shared in each time slot. Consequently, the total number of distinct vertices are $N^2 + N^3$, i.e., $|V| = N^2 + N^3$, where $|\cdot|$ is cardinality of a set. We define two vertices intersect if they have no common element in each time slot, and disjoint if they have at least one common element in one time slot. For any pair of vertices that intersect, we connect them by an edge. An example for the graph construction and clique is shown in Fig. [3].

When the graph is constructed, each vertex is given a weight, which is defined as the maximum achievable rate over the given subcarrier tuple. Specifically,

\[
W_{(n_1, n_2)} = R(n_1, n_2), \tag{29}
\]

\[
W_{(n_1, n_2, n_3)} = R(n_1, n_2, n_3). \tag{30}
\]

The above weighting process is to find the best MS for a given vertex $(n_1, n_2)$ and the best combination of MSs, RSs and transmission modes for a given vertex $(n_1, n_2, n_3)$, for maximizing the achievable rate. Note that for each of the two type of vertices $(n_1, n_2)$ and $(n_1, n_2, n_3)$, the complexity of weighting process is $O(K)$ and $O(3KM + KM(M - 1))$, respectively. Therefore, the total complexity of weighting process is $O(N^2K + 3N^3KM + N^3KM(M - 1))$.

Having defined the graph we now turn to the optimization problem defined in P2. According to our construction methods of vertices and edges, we find that the mutually adjacent vertices can be selected simultaneously without violating the exclusive subcarrier assignment in each time slot defined in (25) to (27). The weighting process is done over all vertices in $V$ so as to match them to the corresponding optimal MSs or combinations of MSs, RSs and transmission modes, maximizing their achievable rate. Therefore, jointly optimizing the subcarrier assignment, transmission mode selection and relay selection for system total throughput maximization is to find a subset $C$ of pairwise adjacent vertices in the graph having the largest total weight, i.e.,
the so called MWCP (P3):

$$\max_{C \subseteq V} W(C) = \sum_{v \in C} W_v. \quad (31)$$

Therefore, P2 is equivalent to P3. The theorem is proved.

B. ACO for MWCP

Like the maximum clique problem (MCP) (finding a clique having the largest cardinality), the MWCP is a classical combinatorial optimization problem and is NP-complete. In this section, we introduce an ACO algorithm based metaheuristic to solve the MWCP.

The ACO metaheuristic is a bio-inspired approach that has been used to solve different hard combinatorial optimization problems. The main idea of ACO is to model the problem as the search for a minimum cost path in a graph. Artificial ants walk through this graph, looking for good paths. Each ant has a rather simple behavior so that it will likely find rather poor quality paths on its own. Better paths are found as the emergent result of the global cooperation among ants in the colony. This cooperation is performed in an indirect way through pheromone laying [22], [23].

In [23], authors propose an ACO algorithm for solving the MCP. It requires only a trivial modification that transforms the cardinality function to the weight function for MWCP. The modification is commonly used for MCP and MWCP in the field of graph theory [24], [25]. The ACO algorithm for MWCP consists of three steps: pheromone trail initialization, construction of cliques by ants and updating pheromone trails. The details are sketched in Appendix A.

The total time complexity of ACO for WMCP is linear in $|V|$ and in the order of $O(\text{ite} \cdot (|V| + \delta))$ [23], where ite is the maximum number of iterations obtained empirically and $\delta$ is a constant (related to the number of ants, the size of the maximum weighted clique and the maximum vertex degree in $G$). By combining the weighting complexity as mentioned in the proof of Theorem 1, the overall computational complexity of the proposed algorithm is thus given by $O\left(N^2K + 3N^3KM + N^3KM(M-1) + \text{ite} \cdot (N^2 + N^3 + \delta)\right)$, which is polynomial in the system parameters $N$ (number of subcarriers), $K$ (number of MSs) and $M$ (number of RSs).

Remark 4: The key of the proposed algorithm is the mapping from the original problem P1, to the simplified P2 and then to the MWCP P3. Once the mapping process is done, there exist many graphical methods for solving the MWCP, such as simple greedy heuristic or complicated
reactive Tabu search. We choose the ACO algorithm because of its ability to strike a good balance between performance and computational complexity \cite{23}. Comparison of graphical methods is out of the scope of this paper.

V. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed transmission protocol together with the ACO based resource allocation algorithm using simulation.

We consider a cell with 2 km radius, where MSs are uniformly distributed in the cell and RSs are uniformly distributed on a circle centered at the BS and with radius of 1 km. The corresponding two-dimensional plane is shown in Fig. 4. The central frequency is around 5 GHz. The statistical path loss model and shadowing are referred to \cite{26}, where we set the path loss exponent to be 4 and the standard deviation of log-normal shadowing is 5.8 dB. The small-scale fading is modeled by Rayleigh fading process, where the power delay profile is exponentially decaying with maximum delay spread of 5 $\mu$s and maximum Doppler spread of 30 Hz. A total of 2000 different channel realizations were used. For each channel realization, the locations of the MSs are random but uniform distributed. For illustration purpose, the total number of subcarriers is $N = 16$. All RSs have the same maximum power constraints, so do all MSs. We consider that the maximum power constraints in dB at BS, RS and MS satisfy $P_B = P_R + 3$ dB $= P_M + 5$ dB. For simplicity, the power allocation coefficients in AF and SUP-based DF two-way relaying are $\xi = 0.5$ and $\theta = 0.5$, respectively. The graph settings in the ACO algorithm are listed in Table II which have been proved to be efficient in \cite{23}.

A. A toy example

To vividly describe the proposed algorithm, we first consider a toy example with one MS, two RSs, and four subcarriers. For notation convenience, we transform the cell into the equivalent two-dimensional plan with radius of 10, where the BS is fixed at $(0, 0)$, the MS is fixed at $(10, 0)$, and the two RSs are fixed at $(4, 3)$ and $(4, -3)$, respectively. This is without loss of generality and commonly used in \cite{14} and \cite{15}. The per-subcarrier power constraint of BS is 10 dB, XOR-based DF two-way relaying is used (Note that the RS’s operation is also DF in the case of one-way relaying). For a given channel realization, applying our proposed algorithm, we obtain a maximum weighted clique consisting of four vertices \{(1, 4, 4), (2, 1, 1), (3, 3, 3),...
\((4, 2, 2)\}. The corresponding weights are \(\{0.1361, 0.1341, 0.0780, 0.0407\}\) (bits/s/Hz), and the corresponding transmission modes are \(\{e, e, e, b\}\), respectively. The subcarrier assignment, relay selection and transmission mode selection are conveniently represented using the time-frequency grid shown in Fig. 5.

B. **Comparison with benchmark schemes**

We now evaluate the performance of our proposed three-time-slot TDD transmission protocol in comparison with the two benchmark schemes (denoted as BM1 and BM2, respectively) shown in Fig. 1. Unlike the proposed protocol, the uplink and downlink optimization for the benchmark schemes is decoupled as there is no correlation between them. For BM1, we use a greedy policy that assigns every subcarrier to the MS with the best channel condition for both downlink and uplink, respectively, which is optimal for throughput optimization. For BM2, since the optimal solution is difficult to obtain for either downlink or uplink throughput optimization, we similarly map them into a MWCP and solve by ACO. Specifically, for downlink traffic, we use subcarrier tuples \((n_1)\) and \((n_1, n_2)\) represent the vertices for direct transmission and one-way relaying, respectively. Weighting is done across MSs or all different combinations of RSs and MSs, and disjoint vertices is selected by ACO algorithm. Similar process is done for uplink traffic.

Fig. 6 shows the results when there are \(K = 4\) MSs, \(M = 10\) RSs and \(N = 16\) subcarriers. From the figure, we first observe that BM2 (with conventional relay) only slightly outperforms BM1 (no relay) when SNR is below around 8 dB and is inferior to BM1 when SNR is higher. This observation suggests that conventional relaying is not always helpful in cellular networks. It is also observed that our proposed transmission protocol outperforms both BM1 and BM2 substantially over a wide range of SNR. In particular, compared with BM1, about 20\% and 30\% throughput improvements are achieved when AF and DF are used in our protocol, respectively. This clearly demonstrates the superiority of our proposed three-time-slot transmission protocol by making the best use of cooperative diversity and network coding gain. In addition, among the three two-way relaying strategies, we find that the two DF strategies perform close to each other while the AF strategy is slightly worse.
C. Comparison with different adaptation schemes

In this subsection, we demonstrate the efficiency of the proposed ACO based adaptive resource allocation algorithm over two suboptimal resource allocation schemes in Fig. 7. The optimal solution obtained by the branch-and-bound algorithm\(^1\) is also plotted serving as the performance upper bound.

In both suboptimal schemes, the transmission mode of each MS as well as its assisting RS (if the cooperation-strategy is selected) are pre-assigned. In specific, a MS is assigned to the direct transmission mode if it is within the inner circle of the cell and the cooperative transmission modes otherwise. When it is assigned the cooperative transmission modes, if the large-scale fading of the BS-RS link and the MS-RS link is about the same, two-way relaying is adopted, otherwise one-way relaying is used (In this case, we assume downlink is in direct transmission mode and uplink is in one-way relaying transmission mode). For those MSs who need RS assistance, we assign the nearest RS to each MS. Once the transmission modes and assisting RSs for all MSs are determined, the subcarrier assignment can be performed adaptively or randomly. In specific, for the adaptive scheme, the optimization is formulated as a MWCP and solved using ACO based algorithm according to instantaneous channel conditions. For the random scheme, the subcarriers are arbitrarily allocated in each time slot.

It is observed in Fig. 7 that the proposed ACO-based joint adaptive algorithm performs very close to the upper bound and the performance gap decreases as the transmit power increases. One also observes that it outperforms the two suboptimal schemes by a significant margin. In particular, the tremendous improvement over the suboptimal scheme with random subcarrier assignment clearly demonstrates the benefits of multiuser diversity through adaptive subcarrier assignment. The improvement over the suboptimal scheme with adaptive subcarrier assignment further suggests the benefits of transmission mode adaptation and relay selection.

The above simulation results in both Fig. 6 and Fig. 7 show that the cooperative diversity gain, network coding gain and multiuser diversity gain are efficiently achieved by the proposed transmission protocol together with the ACO-based resource allocation algorithm.

\(^1\)The branch-and-bound method is implemented by “bintprog” solver in Optimization Toolbox of MATLAB 7.8.0. We have used the options (MaxNodes, MaxRLPlIter, and MaxTime) in the “bintprog” solver to greatly reduce the computational time.
D. Effect of Relay locations

Finally, we investigate the impact of different relay locations on the system throughput. In Fig. 8 we fix per-subcarrier BS power constraint $P_B = 10$ dB, $d$ denotes the distance ratio of the RSs located inner circle radius to the cell radius. We can see that our proposed transmission protocol outperforms BM1 and BM2 greatly, whatever $d$ varies. This further illustrates the superiority of our proposed transmission protocol. The maximum rate is achieved at about $d = 0.2$ for all the considered cooperation schemes. In addition, BM2 is inferior to BM1 when $d > 0.45$. These observations show the relay location plays a key role in achieving good performance in practical systems. In particular, our results show that the RSs should be located closer to BS when MSs are uniformly distributed in the cell. By comparing the performance achieved by different relay strategies, it is seen that whether to use DF or AF does not differ much in the BM2. This conclusion is consistent with the previous work in [27]. However, under the proposed transmission protocol, DF is more favorable than AF.

VI. Conclusion

In this paper, we proposed a novel three-time-slot TDD transmission protocol for supporting direct transmission, one- and two-way relaying in relay-assisted bidirectional cellular OFDMA networks. Under this protocol, a complete set of five transmission modes are introduced. We then formulated a combinatorial optimization problem to jointly optimize subcarrier assignment, transmission mode selection and relay selection for the system total throughput maximization. After establishing its equivalence to a maximum weighted clique problem in graph theory, we employed an ACO based heuristic algorithm to find the solution in polynomial time. A few important conclusions have been made through extensive simulations. Firstly, the proposed optimization framework can achieve cooperative diversity gain, network coding gain and multiuser diversity gain simultaneously and hence considerably outperforms the existing benchmark schemes. In particular, about 20-30% improvement on the system average throughput is achieved over the conventional OFDMA networks without relay. Secondly, choosing the appropriate transmission modes is necessary. Thirdly, in a cell where MSs are uniformly distributed, it is better to place the RSs closer to the BS rather in the middle of the cell. Last but not least, DF relay strategy is practically better than AF strategy under the proposed transmission protocol.
The proposed optimization framework can be extended if both user fairness and asymmetric uplink and downlink traffic are taken into account.
APPENDIX A
ACO ALGORITHM FOR MWCP

Main Function

1. Initialize pheromone trails to $\tau_{max}$, $C_{best} \leftarrow \emptyset$.
2. repeat
3. for each ant $a = 1 : nbAnts$, do:
4. Construct clique $C_a$.
5. end for
6. $C_{iter} \leftarrow \text{heaviest}\{C_1, \ldots, C_{nbAnts}\}$.
7. if $W(C_{iter}) > W(C_{best})$, do:
8. $C_{best} \leftarrow C_{iter}$.
9. end if
10. Update pheromone trails.
11. until the optimal solution is found or the maximum number of iterations reaches.
12. return the largest weight constructed clique since the beginning.

Sub-Function Construct clique

1. Randomly choose a first vertex $v_f \in V$.
2. $C \leftarrow \{v_f\}$.
3. $Candidates \leftarrow \{v_i | (v_f, v_i) \in E\}$.
4. while $Candidates \neq \emptyset$, do:
5. Choose a vertex $v_i \in Candidates$ with probability $p(v_i) = \frac{[\tau_C(v_i)]^\alpha}{\sum_{v_j \in Candidates}[\tau_C(v_j)]^\alpha}$.
6. $C \leftarrow C \cup \{v_i\}$.
7. $Candidates \leftarrow Candidates \cap \{v_j | (v_i, v_j) \in E\}$.
8. end while
9. return $C$.

Sub-Function Update pheromone trails

1. if $v_i \in C_{best}$, do:
2. $\tau(v_i) \leftarrow \rho \tau(v_i) + 1/(1 + W(C_{best}) - W(C_{iter}))$.
3. else do:
4. $\tau(v_i) \leftarrow \rho \tau(v_i)$.
5. end if
6. if a pheromone trail is lower than $\tau_{min}$ then set it to $\tau_{max}$.
7. if a pheromone trail is greater than $\tau_{max}$ then set it to $\tau_{min}$.
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### TABLE I

| downlink | uplink | Direct | One-Way | Two-Way |
|----------|--------|--------|---------|---------|
| Direct   | ✔      | ✔      | ×       |
| One-Way  | ✔      | ✔      | ×       |
| Two-Way  | ×      | ×      | ✔       |

### TABLE II

| parameter | value |
|-----------|-------|
| $\tau_{\text{min}}$ | 0.01  |
| $\tau_{\text{max}}$ | 6     |
| $\alpha$   | 1     |
| $\rho$     | 0.99  |
| ants       | 10    |
| iterations | 500   |
Fig. 1. Three bidirectional transmission schemes.

Fig. 2. Five feasible transmission modes. The solid lines represent downlink, while dashed lines represent uplink.
Fig. 3. Graph and clique example: (i) the graph; (ii) two cliques.

Fig. 4. Two-dimensional plan of nodes location.
Fig. 5. Example for the proposed algorithm. Each pattern represents one downlink/uplink traffic session pair.

Fig. 6. Performance comparison of the proposed transmission protocol and two benchmarks.
Fig. 7. Performance comparison of the proposed algorithm and two suboptimal resource allocation schemes.

Fig. 8. Performance versus RS locations. $P_B = 10$ dB per subcarrier. $d$ is the distance ratio of RSs inner circle radius to the cell radius.