Simultaneous state and parameter estimation based on AUMV estimator and IBR-KF

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Abstract
We proposed a new robust states and parameters estimator for nonlinear systems in this paper. The proposed estimator is derived based on Approximated Unbiased Minimum Variance (AUMV) estimator [3] and Intrinsically Bayesian Robust Kalman filter (IBR-KF) [4]. So, the proposed method takes over the merits of both method. That is, the estimated states are not influenced by the parameter estimation error as in the AUMV estimator, and the estimated parameter is robust against observation noise like the IBR-KF. We confirm the validity of the proposed methods by numerical simulations.

1 Introduction
State estimation is an important research field with various applications including automotive, aerospace, navigation, information and communication. The Kalman Filter (KF) and nonlinear KFs can be appropriate state estimator when the values of parameters of the dynamic and measurement models are exactly known [1]. We can derive some mathematical models by using first-principle modeling method, however, it is difficult to accurately determine some parameters in real applications. Therefore, parameter estimation problem is also important problem in engineering fields. Joint estimation and dual estimation are commonly used in the engineering fields [2]. In joint estimation, unknown parameters are treated as new states. Then, we consider the state estimation problem on the obtained augmented system. On the other hand, in the dual estimation, two filters are applied for state and parameter estimation. The first filter performs state estimation with fixed parameters, and second filter performs parameter estimation with fixed states. However, there is a possibility that the transitional estimation accuracy of the joint/dual estimation is deteriorated because the estimation accuracies of states and parameters influence one another in the joint/dual estimation. The author proposed Approximated Unbiased Minimum Variance (AUMV) estimator [3], which is designed to achieve objective that the dynamics of the estimation error is isolated from parameter estimation error. This method can provide a good result when the observation noise is relatively small. However, there is a drawback that the parameter estimation value becomes oscillatory when the observation noise is relatively large.

It is difficult to know exactly the variance of observation noise $R_k$ in real world applications. If $R_k$ is designed to be smaller than exact value, estimation values should be unstable. On the contrary, $R_k$ is designed to be larger than exact value, estimation accuracy is deteriorated. So, joint and dual estimations do not always work well in real system. Intrinsically Bayesian Robust Kalman filter (IBR-KF) [4, 5] is recently proposed for the purpose of dealing with uncertainty of variance of stochastic noises. The IBR-KF is guaranteed to achieve the best average performance.

In this research, we proposed new robust simultaneous state and parameter estimation based on AUMV estimator and IBR-KF. The proposed estimator combines the advantages of the two methods. That is, it has robust property against observation noise and state estimation are not effected by the parameter estimation. So, the proposed estimator should be a promising alternative to the joint/dual estimation which is commonly applied in engineering fields.

Finally, the validity of the proposed methods is demonstrated through numerical simulations.

2 Problem formulation
We consider the following nonlinear systems with unknown parameters and linear measurements system:

\[ x_k = f(x_{k-1}, p_{k-1}) + B_{k-1}u_{k-1} + w_{k-1}, \]
\[ y_k = H_k x_k + v_{k}^θ, \]

where $x_k \in \mathbb{R}^n$ is states, $p_k \in \mathbb{R}^p$ is unknown parameters, $w_k \in \mathbb{R}^n$ is process noise, $v_{k}^θ \in \mathbb{R}^m$ is measurements, and $v_{k}^θ$ is measurement noise. We assume that control input $u_{k-1}$ and gain matrix $B_{k-1}$ are known. The process noise $w_k$ is uncorrelated zero-mean Gaussian white sequence and its covariance matrix is $E[ w_k \cdot w_k^T ] = Q_k$. The measurement noise $v_{k}^θ$ is also uncorrelated zero-mean Gaussian white sequence and its expectation value of covariance matrix is $E[ v_k^θ \cdot (v_k^θ)^T ] = R_k^θ$, which is parameterized by unknown $θ$. Furthermore, we assume that distributions of the state vector $x_k$ and the unknown parameter vector $p_k$
can be approximated as Gaussian distributions. We also assume that there is no correlation between the state vector $x_k$ and the unknown parameter $p_k$, that is, $E[x_k \cdot p_k^T] = 0$ and $E[p_k \cdot x_k^T] = 0$ are satisfied.

3 Preparation

We can obtained following approximated linear system around estimated values $(\hat{x}_{k-1}, \hat{p}_{k-1})$ by applying some kind of linearization method,

$$ x_k \approx F_{k-1} \hat{x}_{k-1} + G_{k-1} \hat{p}_{k-1} + f(\hat{x}_{k-1}, \hat{p}_{k-1}) + B_{k-1} u_{k-1} + w_{k-1}, \quad (3) $$

where $\hat{x}_{k-1} = x_{k-1} - \hat{x}_{k-1}$ and $\hat{p}_{k-1} = p_{k-1} - \hat{p}_{k-1}$.

We applied the Unscented Statistical Linearization (USL) [6] to obtain linear system (3). Linearized matrix $F_{k-1}$ and $G_{k-1}$ are given as

$$ F_{k-1} \approx \left[ \sum_{i=0}^{2n} W_{s,i} \{ f(X_{i}, \hat{p}_{k-1}) + B_{k-1} u_{k-1} \} \right] x_k = \left( X_k - \hat{x}_{k-1} \right)^T \{ P_{k-1}^{xx} \}^{-1}, \quad (4) $$
$$ G_{k-1} \approx \left[ \sum_{j=0}^{2p} W_{p,j} \{ f(\hat{x}_{k-1}, P_{j}) + B_{k-1} u_{k-1} \} \right] x_k = \left( P_{j} - \hat{p}_{k-1} \right)^T \{ P_{k}^{pp} \}^{-1}, \quad (5) $$

where sigma points $X_i$ are sampled from $N(\hat{x}_{k-1}, P_{k-1}^{xx})$ and sigma points $P_j$ are sampled from $N(\hat{p}_{k-1}, P_{k-1}^{pp})$, and $W_s, W_p$ are weight matrices for $X_i, P_j$, respectively.

4 Derivation of proposed filter

The proposed method consists of a state estimator and a parameter estimator like a dual estimator. We will discuss how to derive each estimator in this section.

4.1 Derivation of state estimator

First, the prediction of state is given as

$$ \hat{x}_{k|k-1} = f(x_{k-1}, \hat{p}_{k-1}) + B_{k-1} u_{k-1}. \quad (6) $$

Above equation (6) is a same equation of the Extended KF. Of course, we can apply the prediction formula of the UKF instead of (6).

Subtracting (6) from (3), the prediction error $\tilde{x}_{k|k-1} := x_k - \hat{x}_{k|k-1}$ is approximated as

$$ \tilde{x}_{k|k-1} = F_{k-1} \tilde{x}_{k-1} + G_{k-1} \hat{p}_{k-1} + w_{k-1}. \quad (7) $$

Let's consider the following update equation

$$ \hat{x}_{k|k} = \hat{x}_{k|k-1} + L_k (y_k - H_k \hat{x}_{k|k-1}), \quad (8) $$

where $L_k$ is an optimal gain to be designed later.

According to (2), (7) and (8), the estimation error $\tilde{x}_k := x_k - \hat{x}_{k|k}$ is given as

$$ \tilde{x}_k = \tilde{x}_{k|k-1} - L_k \left( (H_k x_k + v_k^0) - H_k \hat{x}_{k|k-1} \right) $$
$$ = (I - L_k H_k) \tilde{x}_{k|k-1} - L_k v_k^0 $$
$$ = (I - L_k H_k) (F_{k-1} \tilde{x}_{k-1} + G_{k-1} \hat{p}_{k-1} + w_{k-1}) $$
$$ - L_k v_k^0 \hat{p}_{k-1} \quad \text{(9)} $$

From the above equation (9), we can confirm the estimation error $\tilde{x}_k$ is not affected by parameter estimation error $\hat{p}_{k-1}$ when the following equation is satisfied,

$$ (I - L_k H_k) G_{k-1} = 0. \quad (10) $$

Remark

We assume that rank($H_k G_{k-1}$) = $p$ is satisfied for nonlinear systems (1), (2). Because $(J_{n \times n} - L_k H_k) G_{k-1} = 0$ is satisfied if and only if rank($H_k G_{k-1}$) = $p$ is satisfied [7].

We assume $L_k$ is designed to satisfy equation (10), then equation (9) is transformed to

$$ \tilde{x}_k = (I - L_k H_k) (F_{k-1} \tilde{x}_{k-1} + w_{k-1}) - L_k v_k^0 \quad \text{(11)} $$

Thus we can obtain the expectation value of estimation error covariance matrix $P_{k|k}^{xx}$ is give as,

$$ E_\theta \left( P_{k|k}^{xx} \right) = E_\theta \left( \tilde{x}_k \cdot \tilde{x}_k^T \right) $$
$$ = (I - L_k H_k) E_\theta \left( P_{k|k-1}^{xx} \right) (I - L_k H_k)^T $$
$$ + L_k E_\theta (R_k^0) L_k^T \quad \text{(12)} $$

where $E_\theta (P_{k|k-1}^{xx})$ is given

$$ E_\theta \left( P_{k|k-1}^{xx} \right) = F_{k-1} E_\theta (P_{k-1}^{xx}) F_{k-1}^T + Q_{k-1} \quad \text{(13)} $$

We consider the following constrained optimal problem to minimize the expectation value of estimation error covariance and to satisfy the condition (10),

$$ \arg \min_{L_k} \text{Tr} \left\{ E_\theta \left( P_{k|k}^{xx} \right) \right\} \quad \text{s.t.} \quad (I - L_k H_k) G_{k-1} = 0, \quad \text{(14)} $$

where Tr(·) is trace of matrix.

The optimal update gain $L_k$ is derived based on [3] and [4] as

$$ L_k = K_k^0 - \{ R_k^0 (H_k G_k) - G_k \} \times \left( (H_k G_k)^T E_\theta (P_{k|k-1}^{yy})^{-1} (H_k G_k) \right)^{-1} \times \left( H_k G_k \right)^T E_\theta (P_{k|k-1}^{yy})^{-1} \quad \text{(15)} $$

$$ K_k^0 = E_\theta (P_{k|k-1}^{xx}) H_k E_\theta (P_{k|k-1}^{yy})^{-1} \quad \text{(16)} $$

$$ E_\theta (P_{k|k-1}^{yy}) = H_k E_\theta (P_{k|k-1}^{xx}) H_k^T + E_\theta (R_k^0) \quad \text{(17)} $$

where $K_k^0$ is same as the optimal gain of the IBR-KF.
4.2 Derivation of parameter estimator

We assume dynamics of the unknown parameter \( p_k \) follows

\[
p_k = p_{k-1} + w^p_{k-1}
\]

(18)

where \( w^p_{k-1} \) is process noise and its variance covariance matrix is \( Q^p_{k-1} \). Above equation (18) is often used in the joint/dual estimation and \( Q^p_{k-1} \) is a design parameter.

Thus, the prediction equation of parameter is given as

\[
\hat{p}_{k|k-1} = \hat{p}_{k-1}
\]

(19)

Let’s consider the update equation as

\[
\hat{p}_{k|k} = \hat{p}_{k|k-1} + M_k (y_k - H_k \hat{x}_{k|k-1})
\]

(20)

where \( M_k \) is an optimal gain to be designed later. According to equation (20), the estimation error \( \tilde{p}_k := p_k - \hat{p}_{k|k} \) is given as

\[
\tilde{p}_k = (p_{k-1} + w^p_{k-1}) - [\hat{p}_{k|k-1} + M_k (y_k - H_k \hat{x}_{k|k-1})]
\]

\[
= \tilde{p}_{k-1} + w^p_{k-1} - M_k H_k \tilde{x}_{k|k-1} - M_k \theta^0
\]

\[
= \tilde{p}_{k-1} + w^p_{k-1} - M_k \theta^0
\]

\[
- M_k H_k (F_{k-1} \tilde{x}_{k-1} + G_{k-1} \tilde{p}_{k-1} + w_{k-1})
\]

\[
= (I - M_k H_k G_{k-1}) \tilde{p}_{k-1} - M_k H_k (F_{k-1} \tilde{x}_{k-1} + w_{k-1})
\]

\[
- M_k \theta^0 + w^p_{k-1}
\]

(21)

Using equation (21), we can obtain the expectation value of estimation error covariance matrix \( P^pp_{k|k} \) is given as

\[
E_{\theta} \left( P^pp_{k|k} \right) = E_{\theta} \left\{ (\hat{p}_k, \tilde{p}_k^T) \right\}
\]

\[
= (I - M_k H_k G_{k-1}) E_{\theta} \left( P^pp_{k-1} \right)
\]

\[
\times (I - M_k H_k G_{k-1})^T
\]

\[
+ M_k H_k \left\{ F_{k-1} E_{\theta} \left( P^{xx}_{k-1} \right) F_{k-1}^T + Q_{k-1} \right\}
\]

\[
\times (M_k H_k)^T + M_k E_{\theta} \left( R^p_k \right) M_k^T + Q^p_{k-1}
\]

(22)

We consider the following optimal problem to minimize the expectation value of estimation error covariance,

\[
\arg \min_{M_k} \text{Tr} \left\{ E_{\theta} \left( P^pp_{k|k} \right) \right\}
\]

(23)

The optimal update gain \( M_k \) is derived as

\[
M_k = E_{\theta} \left( P^pp_{k|k} \right) H_k G_{k-1} \left\{ H_k F_{k-1} E_{\theta} \left( P^{pp}_{k-1} \right) F_{k-1}^T
\]

\[
+ G_{k-1} E_{\theta} \left( P^{pp}_{k-1} \right) G_{k-1}^T + Q_{k-1} \right\} H_k^T + E_{\theta} \left( R^p_k \right)^{-1}
\]

(24)

Remark2

Please note that the optimal gain \( M_{k-1} \) can be adjusted with \( Q_{k-1} \) and \( Q^{pp}_{k-1} \). Because \( E_{\theta} \left( P^{pp}_{k-1} \right) \) in equation (22) contains \( Q_{k-1} \).

4.3 Summary of algorithm

Our proposed estimator is summarized as follows:

Step1. Set initial conditions \( E_{\theta}(P^{xx}_0) = P^{xx}_0 \) and \( E_{\theta}(P^{pp}_0) = P^{pp}_0 \).

Step2. Calculate the linearization matrices \( F_{k-1} \) and \( G_{k-1} \) using (4) and (5). Note that \( E_{\theta}(P^{xx}_{k-1}) \) and \( E_{\theta}(P^{pp}_{k-1}) \) are used instead of \( P^{xx}_{k-1} \) and \( P^{pp}_{k-1} \).

Step3. Calculate the predictive mean \( \hat{x}_{k|k-1} \) and covariance matrix \( E_{\theta}(P^{xx}_{k|k-1}) \) by (6) and (13).

Step4. Calculate the update gain \( L_k \) with (15).

Step5. Calculate the estimation error covariance matrix \( E_{\theta}(P^{xx}_{k|k}) \) using (12).

Step6. Calculate the update gain \( M_k \) with (24).

Step7. Calculate the estimation error covariance matrix \( E_{\theta}(P^{pp}_{k|k}) \) using (22).

Step8. Replace \( E_{\theta}(P^{xx}_{k|k}) \) and \( E_{\theta}(P^{pp}_{k|k}) \) with \( E_{\theta}(P^{xx}_{k-1}) \) and \( E_{\theta}(P^{pp}_{k-1}) \). Then, return to [Step2].

4.4 Discussions

4.4.1 Performance of the proposed filter

As mentioned in the literature[5], using expectation value of unknown measurement noise \( E_{\theta}(R^p_k) \), proposed filter improves robustness against uncertainty of variance of stochastic noises. However, it should be noted that the performance of the proposed method may be inferior to that of the conventional method in a single trial because the average value minimizes the average MSE.

4.4.2 Extension to nonlinear observations

If you want to apply the proposed filter to the following nonlinear observation model

\[
y_k = h(x_k) + \nu_k,
\]

(25)

consider the linearized system around \( \hat{x}_{k|k-1} \) as

\[
y_k = H_k \hat{x}_{k|k-1} + \hat{y}_{k|k-1} + \nu_k.
\]

(26)

Then, we can derive a filter in a same way as described above. Please refer to the literature[8] for the treatment of nonlinear observation.

5 Numerical simulation

5.1 Estimation target

We will show the validity of the proposed filter by applying it to an induction motor model system[3]. Using an Euler integration scheme with step size denoted by
\( \Delta t \), the discrete-time state dynamics of the induction motor can be written as

\[
\begin{align*}
\dot{x}_{k-1} = & \begin{bmatrix}
-\gamma x_{1,k-1} + \frac{K}{L_s} x_{3,k-1} + K p x_{5,k-1} x_{4,k-1} \\
-\gamma x_{2,k-1} + \frac{1}{\sigma L_r} x_{4,k-1} - K p x_{5,k-1} x_{3,k-1} \\
\frac{M}{L_s} (x_{3,k-1} x_{2,k-1} - x_{4,k-1} x_{1,k-1}) - u_{1,k} \\
\end{bmatrix} + \begin{bmatrix}
\Delta t \\
\frac{\Delta x}{\Delta t} \\
0 \\
0 \\
0 \\
\end{bmatrix}
\end{align*}
\]

where \( x_1 \) and \( x_4 \) are the stator currents, \( x_3 \) and \( x_4 \) are the rotor fluxes, \( x_5 \) is angular speed, and \( u_{1} \) and \( u_2 \) are the stator voltage control inputs given as

\[
u_{1,k} = 350 \cos(0.003k), \quad u_{2,k} = 300 \sin(0.003k).
\]

We assume that measurement equation is linear system and measurement matrix is given by

\[
H_k = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0
\end{bmatrix}.
\]

The covariance matrices of the process noise \( Q_k \) is given by

\[
Q_k = 0.01^2 I_5.
\]

The time constant \( T_r \) and the parameters \( \sigma, K \), and \( \gamma \) are defined as

\[
T_r = \frac{L_r}{R_r}, \quad \sigma = 1 - \frac{M^2}{L_s L_r}, \quad K = \frac{M}{\sigma L_s L_r}.
\]

\[
\gamma = \frac{R_s}{\sigma L_s} + \frac{R_r M^2}{\sigma L_r}.
\]

Physical values of the nominal system are summarized in Table 1.

| Parameter | Value |
|-----------|-------|
| \( R_s \) | Stator Resistance | 0.18 [f] |
| \( R_r \) | Rotor Resistance | 0.15 [f] |
| \( L_s \) | Stator Inductance | 0.0699 [H] |
| \( L_r \) | Rotor Inductance | 0.0699 [H] |
| \( M \) | Mutual Inductance | 0.068 [H] |
| \( J \) | Rotor Inertia | 0.0586 [kg \cdot m^2] |
| \( T_L \) | Load Torque | 10 [Nm] |
| \( p \) | Pole Pairs | 1 [-] |
| \( \Delta t \) | Step size | 0.0001 [s] |

Furthermore, we assume that a true value of parameter is unknown. And we set a true value of \( \gamma \) as \( \gamma = 0.7 \gamma_{\text{nom}} \) where \( \gamma_{\text{nom}} \) is calculated using the values shown in Table 1 (\( \gamma_{\text{nom}} = 50.7 \)).

We compare the following four filters:

- **JUKF** Joint UKF, which perform a conventional UKF for the augmented system.
- **DUKF** Dual UKF, which perform two conventional UKFs for state and parameter in parallel.
- **AUMV** Conventional approximated unbiased minimum variance filter [3].
- **Proposed** Proposed estimator.

Three filters other than the proposed method cannot deal with unknown covariance matrix \( R_k \). Therefore, we set \( R_k = \max(\theta) \) in these filters. The conventional methods with this setting can become conservative.

We set the initial conditions for states as

\[
x_0 = \begin{bmatrix}
50 \\
50 \\
150 \\
150 \\
100 \\
250
\end{bmatrix}, \quad \dot{x}_0 = \begin{bmatrix}
10 \\
0 \\
10 \\
0 \\
10 \\
0
\end{bmatrix}, \quad P_0 = 10^4 I_5
\]

and the additional initial condition for unknown parameter \( \gamma \) as

\[
\dot{x}_{6,0} = \gamma_{\text{nom}}, P_{(6,0),0} = 10^3.
\]

### 5.3 Simulation result

Table 2 shows performance comparisons for a 100-run Monte Carlo simulation, and average of the root-mean-square error (RMSE) of each state is shown. The additional state \( x_6 \) shows estimated value of unknown parameter \( \gamma \).

| Parameter | Value |
|-----------|-------|
| \( x_1 \) | 13.5 | 68.5 | 10.8 | 9.4 |
| \( x_2 \) | 14.8 | 64.8 | 11.4 | 4.9 |
| \( x_3 \) | 3.1 | 3.8 | 1.5 | 1.5 |
| \( x_4 \) | 2.3 | 3.5 | 1.2 | 1.2 |
| \( x_5 \) | 169.5 | 173.5 | 107.0 | 102.9 |
| \( x_6 \) | 22.2 | 77.1 | 118.9 | 14.4 |

We can confirm AUMV and proposed filter have better estimation for states \( x_1 \sim x_5 \) than JUKF and DUKF. This result seems to be because these methods are not influenced by parameter estimation error. However, parameter estimation \( x_6 \) of AUMV is worse than all the other filters. We think that AUMV is particularly vulnerable to uncertainty of observation noise. We can confirm that the estimation result of the proposed method is the best. Furthermore, we can confirm JUKF has higher estimation accuracy than DUKF.
This may be due to using the same initial conditions (31) for JUKF and DUKF.

Fig.1 and Fig.2 shows the average of 100 simulations for time series of estimation error $x_5$ and parameter error ($x_\theta$ in joint estimation). In Fig.1, we can confirm the response of estimation error of $x_5$ of AUMV and proposed filter are very similar. However, there is a difference in using $E_\theta(R_\theta^k)$ and $R_k(=10^2)$ to derive the update gain $L_k$, so the response does not exactly match. On the other hand, estimation error of $\gamma$ of AUMV and proposed filter are completely different. In Fig.2, the estimation error of AUMV and proposed converge, but estimation error of AUMV shows an oscillatory behavior. The estimation error of $x_5$ and of DUKF shows unstable behavior. JUKF does not shows unstable behavior, but does not converge quickly.

![Fig. 1: Estimation error of state $x_5$](image)

![Fig. 2: Estimation error of parameter $\gamma$(state $x_\theta$)](image)

6 Conclusions

We addressed a simultaneous state and parameter estimation problem for nonlinear systems with unknown measurement noises.

We proposed new robust state and parameter estimator based on the AUMV estimator[3] and the IBR-KF[4, 5]. The proposed method takes over the merits of both method. That is, the estimated states are not influenced by the parameter estimation error as in the AUMV estimator, and the estimated parameter is robust against observation noise like the IBR-KF.

The validity of proposed method was illustrated in numerical examples. Our proposed filter shows higher estimation accuracy than the previous AUMV and joint/dual estimators.

Our proposed filter should be a promising alternative to the joint or dual estimation which are commonly applied in engineering fields.

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