Whether new data on $D_s \rightarrow f_0(980)e^+\nu_e$ can be understood if $f_0(980)$ consists of only the conventional $q\bar{q}$ structure

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Abstract

Only two isospin-singlet scalar mesons $f_0(600)$ ($\sigma$) and $f_0(980)$ exist below 1 GeV, so that it is natural to suppose that they are two energy eigenstates which are mixtures of $\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$ and $s\bar{s}$. Is this picture right? Generally, it is considered that $f_0(600)$ mainly consists of $\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$, if so, the dominant component of $f_0(980)$ should be $s\bar{s}$. The recent measurement of the CLEO collaboration on the branching ratio of $D_s \rightarrow f_0(980)e^+\nu_e$ provides an excellent opportunity to testify the structure of $f_0(980)$, namely whether the data can be understood as long as it consists of mainly the conventional $q\bar{q}$ structure. We calculate the form factors of $D_s \rightarrow f_0(980)$ in the light-front quark model (LFQM) and the corresponding branching ratio of the semileptonic decay. By fitting the data, we obtain the mixing angle $\phi$. The obtained mixing angle shows that the $s\bar{s}$ component in $f_0(980)$ may not be dominant.

PACS numbers: 13.20.Fc, 12.39.Ki, 14.40.Cs
I. INTRODUCTION

Since the SU(3) quark model of the hadrons was founded, dispute about quark structure of resonances never ceases. Recently, it turns out to be a hot topic because many new resonances have been experimentally observed, such as $X(3872)$, $X(3940)$, $Y(3930)$, $Z(3930)$, $Y(4430)$, $Y(4140)$ and it seems hard to accommodate them in the conventional quark-structures, i.e. meson consists of a quark and an antiquark; baryon consists of three valence quarks [1].

The exotic features of the newly discovered mesons suggest that they may be multi-quark states (tetraquark or molecular states), hybrids and glueballs, especially may be mixtures of all those possible states with the regular $q\bar{q}$ components. In fact, the story began a long time ago, when Weinberg and Isgur et al. suggested that $f_0(980)$ and $a_0(980)$ might be $K\bar{K}$ molecular states and dissolve into $K\bar{K}$ final states near the kinetic threshold.

The mass and width of $f_0(980)$ are measured as $980\pm 10\text{MeV}$ and $40-100\text{MeV}$ [2] respectively, but its structure is still obscure so far. $f_0(980)$ was identified as a four-quark state in Ref. [3] where the authors evaluated its mass by postulating a $qq\bar{q}\bar{q}$ structure in terms of the MIT bag model. In Ref. [4] the authors investigated a possibility that the light scalar meson $f_0(980)$ was a $q^2q^2$ state rather than $\bar{q}q$ based on a lattice calculation. Instead, since the resonance is close to the $K\bar{K}$ threshold a $K\bar{K}$ molecular structure was suggested by Weinberg and Isgur [5]. In Ref. [6] a possibility that it is a mixture of $q\bar{q}$, the so-called scalaron coupled to $K\bar{K}$, was discussed. Moreover, it is also counted as a glueball [7]. By analyzing the experimental measurements of the concerned decay and production processes, some authors affirmed that the conventional $q\bar{q}$ structure might tolerate the available data [8, 9, 10, 11, 12, 13]. Those diverse interpretations should eventually be negated or confirmed by more accurate experimental measurements, as well as further theoretical investigations.

From the theoretical aspect, only $f_0(600)$ ($\sigma$) and $f_0(980)$ are isospin-singlet scalar mesons below 1 GeV, so it is natural to suppose that they are partner energy eigenstates which are on-shell physical particles and mixtures of components of a complete basis. The simplest basis for the iso-singlet quark structure consists of two components: $\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$ and $s\bar{s}$. Thus, one may write [10, 11, 14]

$$
\begin{pmatrix}
  f_0(600)(?) \\
  f_0(980)
\end{pmatrix}
= 
\begin{pmatrix}
  \cos \phi & -\sin \phi \\
  \sin \phi & \cos \phi
\end{pmatrix}
\begin{pmatrix}
  f_{0q} \\
  f_{0s}
\end{pmatrix},
$$

(1)

where $f_{0q} = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$ and $f_{0s} = s\bar{s}$. Here the question mark denotes that we cannot determine if the partner of $f_0(980)$ is indeed $f_0(600)$ (see below for more discussions). If we could obtain a general Hamiltonian matrix which not only gives the diagonal elements, i.e. $<f_{0q}(f_{0s})|H|f_{0q}(f_{0s})>$, but also the off-diagonal elements $<f_{0s}(f_{0q})|H|f_{0q}(f_{0s})>$, then it would be easy to diagonalize the matrix to obtain the physical eigenenergies and mixing angle. However, unfortunately, since the matrix elements are fully dominated by the non-perturbative QCD and cannot be derived from our present knowledge on QCD yet, one needs
to determine the mixing angle by fitting the available data. The mixing angle was obtained as $\phi \sim 18.3^\circ$ in terms of the process $f_0(980) \rightarrow \pi\pi$\textsuperscript{[10]} and the authors of Ref.\textsuperscript{[11]} got $\phi \sim 16^\circ$, then they renewed their value to $(23 \pm 3)^\circ$\textsuperscript{[15]}. It seems that these results indicate $s\bar{s}$ should be the dominant component of $f_0(980)$. In Ref.\textsuperscript{[12]}, the authors analyzed two processes: $\phi(1202) \rightarrow f_0(980)\gamma$ and $f_0(980) \rightarrow \gamma\gamma$ and gained $\phi = (142 \pm 6)^\circ$. By studying non-leptonic decay of $D(D_s) \rightarrow f_0(980)$, El-Bennich et al.\textsuperscript{[13]} obtained $\phi = (32 \pm 4.8)^\circ$ in terms of the covariant light-front quark model (CLFD)\textsuperscript{[16]} and $\phi = (41.3 \pm 5.5)^\circ$ by the dispersion relation (DR) approaches\textsuperscript{[17]} respectively. Cheng et al.\textsuperscript{[14]} found that the mixing angle lies in the ranges of $(25^\circ < \phi < 40^\circ$ or $140^\circ < \phi < 165^\circ$) in a phenomenological study. The numerical values on the mixing angle are so disperse, can we conclude that $f_0(980)$ is not a pure $q\bar{q}$ bound state? It demands an answer which should be coming from new measurements and theoretical efforts.

Indeed, all of our information on the inner structure must be obtained from experimental measurements. With great improvements in experimental facility and detection technique, much more accurate data on $f_0(980)$ have been achieved which help theorists to make judgement if the available models are correct. The recent measurement on semileptonic decay $D_s^+ \rightarrow f_0 e^+\nu_e$ by the CLEO collaboration may provide a unique opportunity to testify the validity of the $q\bar{q}$ structure of $f_0(980)$\textsuperscript{[18]}\textsuperscript{1}. The decay rate was measured as $(0.13 \pm 0.04 \pm 0.01)\%$.

The present work is to testify if the mixture ansatz can tolerate the data of the semileptonic decay. Assuming the $q\bar{q}$ structure of $f_0(980)$ as $\sin \phi \frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d}) + \cos \phi s\bar{s}$, we calculate the decay width. From the experimental data, the mixing angle $\phi$ would be obtained. Then we discuss whether it is consistent with the results obtained by fitting other experiments.

It is easy to see from the quark diagram, that at the tree-level, only the $s\bar{s}$ component contributes to the transition. The amplitude at the quark level can be obtained in terms of the weak effective theory, so the key point is how to more accurately calculate the hadronic transition matrix elements. Here we employ the light-front quark model (LFQM). LFQM is a relativistic model which has obvious advantages when light hadrons are involved\textsuperscript{[19, 20]}. The light-front wave function is manifestly Lorentz invariant and expressed in terms of the light-front momentum fractions and the relative transverse momenta which are independent of the total hadron momentum. Applications of this approach can be found in Ref.\textsuperscript{[21]} and the references therein. The LFQM has been used to calculate the decay rates of $D_s$ into $\eta$ and $\eta'$ which contain similar structure $C_1(u\bar{u}+b\bar{b})+C_2s\bar{s}\textsuperscript{[22]}$. Now we extend our scope to study $D_s \rightarrow f_0(980)$. Even though $\eta$ and $\eta'$ are pseudoscalar mesons and $f_0(600)$ and $f_0(980)$ are scalars, their isospin structures are the same.

Concretely, we calculate the form factor of $D_s \rightarrow f_0(980)$ and obtain the decay rate of

\textsuperscript{1} Here $f_0$ refers to $f_0(980)$\textsuperscript{[23]}. 
$D_s \rightarrow f_0(980)e^+\nu_e$ where the parameter $\phi$ is a free one. Comparing our result with the experimental data we fix the value of the mixing parameter $\phi$. With this mixing parameter, we can further predict the branching ratio of $D^+ \rightarrow f_0(980)e^+\nu_e$ which will be measured in the future on BES III. It is noted that if the $q\bar{q}$ structure: $|f_0(980)\rangle = \sin \phi \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) + \cos \phi |s\bar{s}\rangle$ is right, the above reaction can only realize via the Cabibbo-suppressed diagram to which only the $u\bar{u} + d\bar{d}$ component contributes, and/or the annihilation diagram which is even further suppressed, so the ratio should be smaller by one or two orders. The smallness of the branching ratio should be a check for the structure of $f_0(980)$.

This paper is organized as follows: after the introduction, in section II we will present the form factors of $D_s \rightarrow f_0(980)$ which are evaluated in LFQM, then we obtain $\phi$ by fitting the experimental data, at the end of the section, we make a prediction on the branching ratio of $D^+ \rightarrow f_0(980)e^+\nu_e$. Meanwhile, we briefly estimate the errors which originate from both experimental and theoretical aspects. Section III is devoted to our conclusion and discussions.

II. STUDY ON THE PROCESS INVOLVING $f_0$ MESONS IN LFQM

Now let us calculate the form factors of $D_s^+ \rightarrow f_0(980)$ in LFQM. Here we assume that $f_0(980)$ is of the conventional $q\bar{q}$ structure: $|f_0(980)\rangle = \sin \phi \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) + \cos \phi |s\bar{s}\rangle$. The transition diagram is given in Fig. 1.

A. Formulations

The form factors for $P \rightarrow S$ transition are defined as

$$\langle S(P'')|A_\mu|P(P')\rangle = i \left[u_+(q^2)P_\mu + u_-(q^2)q_\mu \right], \quad (2)$$

It is convenient to redefine them as

$$\langle S(P'')|A_\mu|P(P')\rangle = -i \left[ \left( P_\mu - \frac{M'^2 - M''^2}{q^2}q_\mu \right) F_1(q^2) + \frac{M'^2 - M''^2}{q^2}q_\mu F_0(q^2) \right], \quad (3)$$
where $q = P' - P''$ and $P = P' + P''$. The relations between them are

$$F_1(q^2) = -u_+(q^2), F_0(q^2) = -u_+(q^2) - \frac{q^2}{q \cdot P} u_-(q^2).$$  \hspace{1cm} (4)$$

Functions $u_\pm(q^2)$ can be calculated in LFQM and their explicit expressions are presented as[20]

$$u_+(q^2) = \frac{N_c}{16\pi^3} \int dx_2 d^2p_1 \frac{h'_p h''_p}{x_2 N_1 N''_1} \left[ -x_1(M_0^2 + M''_0) - x_2 q^2 + x_1(m'_1 + m_{1''}^2) + x_1(m'_1 - m_2)^2 + x_1(m''_1 + m_2)^2 \right],$$

$$u_-(q^2) = \frac{N_c}{16\pi^3} \int dx_2 d^2p_1 \frac{2h'_p h''_p}{x_2 N_1 N''_1} \left\{ x_1 x_2 M_0^2 + p'_1 + m'_1 m_2 + (m''_1 + m_2)(x_2 m'_1 + x_1 m_2) - 2 \frac{q \cdot P}{q^2} \left( p'^2_1 + 2 \frac{(p'_1 \cdot q \perp)}{q^2} \right) + \frac{2(p'_1 \cdot q \perp)}{q^2} \left[ M_{1''}^2 - x_2 (q^2 + q \cdot P) \right] - (x_2 - x_1) M_{1'}^2 + 2 x_1 M_0^2 - 2(m'_1 - m_2)(m'_1 - m_1') \right\},$$  \hspace{1cm} (5)$$

where $m'_1, m''_1$ and $m_2$ are the corresponding quark masses, $M'$ and $M''$ are the masses of the initial and final mesons respectively. The wave function is usually chosen to be Gaussian and the parameter $\beta$ in the Gaussian wave function determines the confinement scale and is expected to be of order $\Lambda_{QCD}$. All other notations are given in the appendix. Some parameters, such as $m_s = 0.37$ GeV, $m_c = 1.4$ GeV are taken from Ref. [20]; $\beta_{D_s} = 0.592$ GeV and $\beta_D = 0.499$ GeV are fixed by fitting concerned processes[22].

In terms of Eq. (4), the quark structure of $f_0(980)$ may be written as $|f_0(980)> = \sin\phi |\bar{u}u + \bar{d}d> + \cos\phi |s\bar{s}>$. Since strange quark $s$ in $D_s$ directly transits into the final scalar meson, one can notice that only $s\bar{s}$ component of $f_0(980)$ contributes to the transition $D_s \rightarrow f_0(980)\bar{l}\nu_l$.

In order to calculate the relevant form factors we need to know $\beta^s_{f_0}$. We cannot obtain $\beta^s_{f_0}$ directly from its decay constant as we did for the pseudoscalars, because the decay constant of $f_0$ is zero [2]. Following Ref. [20], we set $\beta^s_{f_0} = 0.3$ in our numerical computations.

In the covariant light-front quark model, the calculation of form factors is performed in the frame $q^+ = 0$ with $q^2 = -q^2 \leq 0$. Thus only the values of the form factors in the space-like region can be obtained. The advantage of this choice is that the so-called Z-graph contribution arising from the non-valence quarks vanishes. In order to obtain the physical

\footnote{The decay constant of $0^+$ state in LFQM can be written as

$$f_s = \frac{N_c}{16\pi^3} \int dx_2 d^2p_1 \frac{h'_s}{x_1 x_2 M_{1''}^2} 4(m'_1 x_2 - m_2 x_1).$$  \hspace{1cm} (6)$$

For $f_0(980), m'_1 = m_2$. The function $h'_s$ and other quantities $M_{1''}^2, M_{1'}^2$ are symmetric functions of $x_1$ and $x_2$. Thus the integration is zero, i.e. $f_{00} = 0$.}
form factors, an analytical extension from the space-like region to the time-like region is required. The form factors can be parameterized in a three-parameter form as

\[ F(q^2) = \frac{F(0)}{1 - a \left( \frac{q^2}{M^2} \right) + b \left( \frac{q^2}{M^2} \right)^2}. \]  

(7)

where \( F(q^2) \) represents the form factors \( F_1, F_0, \) and \( F(0) \) is the form factors at \( q^2 = 0; M \) is the mass of the initial meson. The three parameters \( F(0) \) and \( a, b \) are fixed by performing a three-parameter fit to the form factors which are calculated in the space-like region and then extended to the physical time-like region.

For semileptonic decay of a pseudoscalar meson(D or \( D_s \)) into a scalar meson, i.e. \( P(P') \rightarrow S(P'') l \nu \), the differential width is

\[ \frac{d\Gamma}{dq^2}(P \rightarrow S l \nu) = \frac{G^2_F|V_{CKM}|^2}{24\pi^3} p^3 |F_1(q^2)|^2; \]  

(8)

which is the same as the one for \( P(P') \rightarrow P(P'') l \nu \) \[24\], where \( q = P' - P'' \) is the momentum transfer and \( q^2 \) is the invariant mass of the lepton-neutrino pair; \( p \) is the final meson momentum in the \( D \) or \( D_s \) rest frame and

\[ p = |P''| = \sqrt{\left( M^2 - (M_f - \sqrt{q^2})^2 \right) \left( M^2 - (M_f + \sqrt{q^2})^2 \right) / 2M}, \]  

(9)

where \( M_f \) denotes the mass of the produced meson. It is noted that the differential width is governed by only one form factor \( F_1(q^2) \) because we neglect the light lepton masses.

B. Mixing angle in \( f_0(980) \) and prediction on the decay rate of \( D^+ \rightarrow f_0(980) l^+ \nu \)

We have calculated the form factor \( F_1[D_s,f_0(980)](q^2) \) and obtain \( F_1[D_s,f_0(980)](0) = 0.434 \cos \phi \) and \( a = 1.03, b = 0.267 \). Obviously the mixing angle \( \phi \) is included in \( F_1[D_s,f_0(980)](0) \).

With \( F_1[D_s,f_0(980)](q^2) \), we can compute the differential width and branching ratio of \( D_s^+ \rightarrow f_0(980) e^+ \nu_e \)

\[ \text{BR}(D_s^+ \rightarrow f_0(980) e^+ \nu_e) = 4.22 \times 10^{-3} \cos^2 \phi. \]  

(10)

Comparing the theoretical evaluation of the branching ratio with the experimental data we eventually obtain

\[ \phi = (56 \pm 6)\degree, \]  

(11)

and its symmetric angle in the second quadrant is

\[ \phi = (124 \pm 6)\degree. \]  

(12)
FIG. 2: Form factors of $|F_{1D_s f_0(980)}(q^2)|$ and $|F_{1D f_0(980)}(q^2)|$ at different $q^2$. The solid lines represent our results, dotted (in terms of CLFD) and dash-dotted (in terms of the dispersion relation) lines are taken from [13].

In the below discussions, we adopt the mixing angle in the first quadrant for illustration, i.e. $\phi = (56 \pm 6)^\circ$. This result implies that $s\bar{s}$ is not the dominant component of $f_0(980)$ at all. The uncertainties in the numerical results originate from the errors of the experimental measurements. Definitely, this result contradicts to those given in [10, 11] by at least two standard deviations.

In fact, one needs the total width of $f_0(980)$ which unfortunately was not precisely measured and spans a rather wide range from 40 to 100 MeV, to determine the mixing angle. In [10], the authors used the lower bound of the width of $f_0(980)$ to fix the mixing angle, so that there could be some flexibility in determining the value. Even through taking into account the flexibility, there is still a serious discrepancy.

We plot the form factors $F_{1[D_s f_0(980)]}$ and $F_{1[D f_0(980)]}$ in Fig.2 where we make a comparison of our results with that obtained in [13]. Here $F_{1CLFD[D_s f_0(980)]}$, $F_{1DR[D_s f_0(980)]}$, $F_{1CLFD[D f_0(980)]}$ and $F_{1DR[D f_0(980)]}$ correspond to the form factors in the processes $D_s \rightarrow f_0$ and $D \rightarrow f_0$ and calculated by the authors of [13], in terms of the covariant light-front dynamics and dispersion relation approaches respectively, and the subscripts explicitly mark the differences. We can see the shapes of our results have the same $q^2$ dependence as $F_{1DR}$, moreover, our numerical result on the form factor $F_{1[D f_0(980)]}$ is very close to $F_{1DR[D f_0(980)]}$, whereas our $F_{1[D_s f_0(980)]}$ is almost a half of $F_{1DR[D f_0(980)]}$.

Now let us discuss the obvious difference between our results on the form factor for $D_s \rightarrow f_0(980)$ and that given in [13]. The difference comes from the different values of $F(0)$. In Ref. [13], the authors used the data of non-leptonic decays of $D_s \rightarrow f_0(980)$ as input, instead, we employ the data of semi-leptonic decays. A simple calculation may support our
results. In the figure, one notes that our $F_{1[D_f0(980)]]}$ is very close to $F_{1[D_f0(980)]}^{DR}$, and generally one can write

$$
F_{1[D_f0(980)]}(0) = C \sin \phi,
F_{1[D_f0(980)]}^{DR}(0) = C' \sin \phi',
$$

(13)

where the unprimed quantities are ours and the primed ones are that given in \[13\]. Thus we obtain

$$
\frac{C}{C'} = \frac{\sin \phi'}{\sin \phi} \approx 0.79.
$$

(14)

Similarly, expressions for $D_s \rightarrow f_0(980)$ are

$$
F_{1[D_s,f_0(980)]}(0) = D \cos \phi,
F_{1[D_s,f_0(980)]}^{DR}(0) = D' \cos \phi'.
$$

(15)

So

$$
\frac{\cos \phi}{\cos \phi'} \approx 0.74.
$$

(16)

If one makes approximation $D/D' \approx C/C'$ which should be hold in the flavor $SU(3)$ limit, he would immediately obtain

$$
\frac{F_{1[D_s,f_0(980)]}(0)}{F_{1[D_s,f_0(980)]}^{DR}(0)} \approx 0.58.
$$

(17)

This factor 0.58 can help to understand why our result of $F_{1[D_s,f_0(980)]}$ is only half of $F_{1[D_s,f_0(980)]}^{DR}$ shown in Fig.2(a). In fact, if we take the form factor $F_{1[D_s,f_0(980)]}^{DR}$ for calculating the branching ratio of the semileptonic decay $D_s \rightarrow f_0(980) + l^+ + \nu$, the result would be four times larger than the data.

Now, let us see what consequent implications to be obtained if we use the obtained value. Based on the general knowledge of quantum mechanics, there should exist another physical state $f_0'$ which is orthogonal to $f_0(980)$. There could be two choices, i.e. the supposed partner is lighter or heavier than $f_0(980)$. Even though $\phi$ deviates from $\phi'$ for $\eta$ and $\eta'$ mixing angle (39.3°) \[25\], one may expect that the mass difference between $f_0(980)$ and its partner should be somehow close to that between $\eta$ and $\eta'$ i.e approximately 410 MeV, because in both cases,

\[3\] In literature, the $\eta - \eta'$ mixing is defined as

$$
\begin{pmatrix}
\eta \\
\eta'
\end{pmatrix} = 
\begin{pmatrix}
\cos \phi' & -\sin \phi' \\
\sin \phi' & \cos \phi'
\end{pmatrix}
\begin{pmatrix}
\eta_{qq} \\
\eta_{ss}
\end{pmatrix}
$$

(18)

where $\eta_{qq} = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$ and $\eta_{ss} = s\bar{s}$. 

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the mass difference should be determined by the chiral symmetry breaking \[26\]. If its partner is the lighter than it, one is tempted to identify \( f_0(600) \), however, the mixing angle \( \phi \) suggests that the partner of \( f_0(980) \) possesses a sizable \( s \bar{s} \) component, and this allegation definitely contradicts to our present knowledge on \( f_0(600) \). Thus, one might expect its partner to be a heavier one. According to this criterion the resonance \( f_0(1370) \) seems to be a candidate. From \[2\], we find that \( f_0(1370) \) whose mass is measured from 1200 to 1500 MeV, indeed is plausible. We can estimate the branching ratio of \( D_s^+ \to f_0(1370) e^+ \nu_e \) to obtain the angle \( \phi \) as

\[
\text{BR}(D_s^+ \to f_0(1370) e^+ \nu_e) \propto \sin^2 \phi. \tag{19}
\]

The future experiment will measure the branching ratio of \( D_s^+ \to f_0(1370) e^+ \nu_e \), and by fitting the new data, we are able to determine the mixing angle \( \phi \) again. If the newly obtained value of \( \phi \) is consistent with our value \( \phi = 56.2^\circ \) within a certain error tolerance, it means that the data support our postulate on the structure of \( f_0(980) \) and \( f_0(1370) \), otherwise, we should turn to other possibilities.

If the flavor structure of \( f_0(980) \) is correct we can estimate the decay rate of \( D^+ \to f_0(980) \) where only the \( d \bar{d} \) component contributes and the transition is Cabibbo suppressed. For that case, similar to the previous calculations, the three parameters are achieved as \( F_1[D_{f_0(980)}](0) = 0.216 \) and \( a = 1.16, b = 0.25 \) respectively. Following all the procedures we made above, we achieve the branching ratio of \( D^+ \to f_0(980) e^+ \nu_e \) as about \( (5.7 \pm 0.9) \times 10^{-5} \). Although the branching ratio is very small as expected, we believe that the future experimental facilities with high luminosity and precision can do the job.

**C. Error analysis**

Now, we need to approximately estimate possible uncertainty in our result. Even though the errors in the theoretical computations are not fully under control, an approximately estimation would still be possible and necessary.

Concretely, there are three error sources: the inherent uncertainty in the theoretical model LFQM; the inputs to determine the model parameters which are generally obtained by fitting several well-measured processes; the data of CLEO which we are going to employ to fix the mixing angle \( \phi \).

Since as discussed above, the LFQM has been successfully applied to analyze similar reactions, we have confidence that the higher order effects are not important and the errors brought up by the model can be neglected in comparison with that from other sources.

The second source is the errors in the inputs which cause uncertainty in our theoretical calculation. To estimate the errors we take a relatively easier way. One notes \[20\], that the form factors \( u_\pm \) are related to the form factors \( f_\pm \) in the \( P \to P \) transition as

\[
u_\pm = -f_\pm(m'_1 \to -m'_2, h'_p \to h'_s), \tag{20}\]
so it is natural to suppose that the theoretical uncertainties in the LFQM calculations are the same for $P \to S$ and $P \to P$. Obviously, there may be a difference between the form factors of $P \to S$ and $P \to P$ because the wave functions of a scalar and a pseudo-scalar are not the same (see Eq. A5 in appendix) i.e. the dependence of those form factors on the parameter $\beta$ is different. But at least they should have the same order of magnitude, thus we are able to estimate the uncertainties in the form factors of $P \to S$ by transferring that for $P \to P$.

We need to study the sensitivity of results to $\beta_{Ds}$ (i.e. $\beta'$) and $\beta_{f0(980)}$ (i.e. $\beta''$). For the analysis, we vary $\beta_{Ds}$ and $\beta_{f0(980)}$ up and down by 10%, then the theoretical uncertainties corresponding to variation of $\beta_{Ds}$ and $\beta_{f0(980)}$ are presented in Table I. From the values, one can notice that with 10% up and down variations in $\beta$, the change of the mixing angle falls within a range of 10%. Thus, we can conclude that the results are not very sensitive to $\beta$.

For $D_s$ decays we employ the transition $D_s^+ \to \eta e^+ \nu_e$ to estimate errors in our theoretical calculation on the branching ratio of $D_s^+ \to f_0(980)e^+\nu_e$. Comparing our theoretically calculated branching ratio of $D_s^+ \to \eta e^+\nu_e$ in LFQM 2.25% [22] with the experimental data $(2.48 \pm 0.29 \pm 0.13)\%$ [18], there is a 0.23% deviation to the central value. Taking into account the experimental errors, the integrated uncertainty of the theoretical estimation on the branching ratio would be $\pm 0.32\%$ and the relative error is 18%. We suppose the relative error for theoretical estimation on the branching ratio of $D_s^+ \to f_0(980)e^+\nu_e$ to be the same as that for $D_s^+ \to \eta e^+\nu_e$, then by including the error of the CLEO’s data we obtain the whole uncertainty of $\phi$ as $\pm 7.2^\circ$.

Similarly, for $D^+$ semi-leptonic decay, the decay of $D^+ \to \bar{K}^0 e^+\nu_e$ is chosen to analyze the corresponding error. The theoretical evaluation on the branching ratio is $9.74\%$ [22] while the experimental datum is $(8.6 \pm 0.5)\%$ [2], so the integrated uncertainty is 1.25% in the theoretical result and the relative error should be 12.8%. Then as discussed above, the relative error of the branching ratio of $D^+ \to f_0(980)e^+\nu_e$ estimated in LFQM should be $\pm 0.73 \times 10^{-5}$.

Considering the error of the $\phi$ value the error in our theoretical prediction for semi-leptonic decay $D^+ \to f_0(980)e^+\nu_e$ is estimated as $\pm 1.3 \times 10^{-5}$ and one can expect $BR(D^+ \to f_0(980)e^+\nu_e) = (5.7 \pm 1.3) \times 10^{-5}$.

| $\beta$ | $F_1(0) \ (10^{-3})$ | $a$ | $b$ | $\phi$ | errors($\phi - 56.2^\circ$) |
|---|---|---|---|---|---|
| $\beta_{Ds} = 0.652, \ \beta_{f0(980)} = 0.30$ | -0.400cos$\phi$ | 0.947 | 0.208 | 52.5$^\circ$ | -3.7$^\circ$ |
| $\beta_{Ds} = 0.532, \ \beta_{f0(980)} = 0.30$ | -0.468cos$\phi$ | 1.111 | 0.347 | 59.1$^\circ$ | 2.9$^\circ$ |
| $\beta_{Ds} = 0.592, \ \beta_{f0(980)} = 0.33$ | -0.479cos$\phi$ | 1.016 | 0.233 | 59.7$^\circ$ | 3.5$^\circ$ |
| $\beta_{Ds} = 0.592, \ \beta_{f0(980)} = 0.27$ | -0.382cos$\phi$ | 1.025 | 0.307 | 50.7$^\circ$ | -5.5$^\circ$ |
Assuming that $f_0(980)$ possesses a regular $q\bar{q}$ structure, namely, is written as $\sin \phi \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) + \cos \phi s\bar{s}$, we calculate the branching ratio of the semileptonic decay $D_s \to f_0(980)e^+\nu_e$ in LFQM. To fit the data which were measured by the CLEO collaboration, we obtain the mixing angle $\phi$ as $\phi = (56 \pm 7)^\circ$ or $(124 \pm 7)^\circ$. This result implies that the $s\bar{s}$ component in $f_0(980)$ is not the dominant one, instead, the fractions of $\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$ and $s\bar{s}$ are almost equal. It is definitely contradicts to the conclusion of [10, 11]. Considering the experimental and theoretical errors, our mixing angle is close to the results in [12, 13].

According to the basic principle of quantum mechanics, there should exist its partner, i.e. another eigenstate of the Hamiltonian and orthogonal to $f_0(980)$. A natural conjecture is that $f_0(600)(\sigma)$ meson should be the most favorable candidate because $f_0(600)$ and $f_0(980)$ are the only two isoscalar mesons below 1 GeV. Obviously, it is against the previous studies where $\sigma$ meson is confirmed to be mainly composed of $u\bar{u} + d\bar{d}$, and the mixing angle with $s\bar{s}$ should be very small. Thus if this conjecture is true, one should look for its partner above 1 GeV. As some authors suggested [26], the mixing between $\eta - \eta'$ is related to the chiral symmetry breaking and their mass difference is determined by the values of the quark condensates. The mass splitting is about 410 MeV. We suppose that the mass difference of $f_0(980)$ and its partner should also be related to the chiral symmetry breaking parameters, at least the order of magnitude of the mass splitting should be close to that of $\eta - \eta'$. As we take their mass difference as 410 MeV, $f_0(1370)$ would be the favorable candidate. Surely, we cannot rule out $f_0(1500)$ because its mass is not too far away at all. In this work, we assume that $f_0(1370)$ is the isospin partner of $f_0(980)$ and calculate the branching ratio of $D_s^+ \to f_0(1370)e^+\nu_e$ which should be proportional to $\sin^2 \phi$, in the same framework. If the data which will be obtained by the CLEO and/or BES collaborations, confirm this $\phi$ within a reasonable error tolerance, our picture is supported, otherwise we need re-consider the whole scenario.

Moreover, in the same theoretical framework, we estimate the branching ratio of $D^+ \to f_0(980) + l^+ + \nu$ and predict it as $(5.7\pm1.3) \times 10^{-6}$. Since in the process $D^+ \to f_0(980) + l^+ + \nu$ only the $d\bar{d}$ component contributes to the transition (see the quark-diagram), the branching ratio is proportional to $\sin^2 \phi$, a measurement on it can help to confirm the validity of the $q\bar{q}$ structure of $f_0(980)$.

As a conclusion, we obtain a mixing angle by fitting the new data of CLEO which shows that $f_0(980)$ is not dominated by the $s\bar{s}$ component.

**Acknowledgments**

This work is supported by the National Natural Science Foundation of China (NNSFC) and the special grant for the PH.D program of the Chinese Education Ministry. One of us
(Ke) is also partly supported by the special grant for new faculty from Tianjin University.

**APPENDIX A: NOTATIONS**

Here we list some variables appeared in this paper. The incoming (outgoing) meson in Fig. 1 has the momentum \( P^{(i)} = p_1^{(i)} + p_2 \) where \( p_1^{(i)} \) and \( p_2 \) are the momenta of the off-shell quark and antiquark and

\[
\begin{align*}
F_1^+ &= x_1 P^+ , \quad F_2^+ = x_2 P^+ , \\
F_1\perp &= x_1 F_\perp + p_\perp , \quad F_2\perp = x_2 F_\perp - p_\perp ,
\end{align*}
\]  

(A1)

with \( x_i \) and \( p_\perp \) are internal variables and \( x_1 + x_2 = 1 \).

The variables \( M'_0, \tilde{M}'_0, h'_p, h'_s \) and \( \tilde{N}'_1 \) are defined as

\[
\begin{align*}
M'^2_0 &= \frac{p'^2_1 + m'^2_1}{x_1} + \frac{p'^2_2 + m'^2_2}{x_2} , \\
\tilde{M}'_0 &= \sqrt{M'^2_0 - (m'_1 - m'_2)^2} .
\end{align*}
\]  

(A2)

\[
\begin{align*}
\tilde{N}'_1 &= x_1 (M'^2 - M'^2_0) .
\end{align*}
\]  

(A4)

where

\[
\begin{align*}
\varphi' &= 4\left(\frac{\pi}{\beta'^2}\right)^{3/4} \int \frac{d\beta'^2}{dx_2} \exp\left(-\frac{p'^2_2 + p'^2_1}{2\beta'^2}\right) , \\
\varphi'_p &= \sqrt{2\beta'^2} \varphi' ,
\end{align*}
\]  

(A5)

with \( \beta'^2 = \frac{x_2 M'^2}{2} - \frac{m'^2_1 + m'^2_2}{2x_2 M'_0} \).

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