Plasma screening enhancement of thermonuclear reaction rates

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Abstract. Plasma screening can enhance the thermonuclear reaction rates significantly. The most pronounced effect takes place in the white dwarf cores and neutron star envelopes; there the enhancement factor can reach as tenths orders of magnitude. Here thermodynamically consistent description of this effect, which does not violate of the detailed balance principle, is discussed.

1. Introduction
Plasma screening enhancement for thermonuclear reactions plays a crucial role in astrophysical applications. Starting from fundamental paper by Salpeter [1], studies were continued in large number of papers (e.g., [2–22]). In recent decades enhancement factors were incorporated into detailed reaction network codes (see, e.g., [23,24] for description of recent versions of these codes) and in [25] it was discovered that direct usage of the previously published enhancement factor results in violation of thermodynamic consistency: the nuclear reaction does not drive system to the nuclear statistical equilibrium. This effect is associated with violation of the detailed balance principle for reaction rates. To avoid this unphysical behavior [25] suggests a prescription, which allows to impose detailed balance artificially. Namely, they suggest to choose the “favored reaction direction” as forward and calculate the reverse reaction rate to ensure the detailed balance principle. However, they point that this approach has some ambiguity associated with the choice of the favored direction and argue that the photodisintegrating channel should be selected as favored (if possible), because it is apparently unaffected by screening.

The reason for the violation of the detailed balance principle is indicated in [22]. Let me take the reaction $A + B \rightarrow O^* \rightarrow C + D$ as an example. To begin, let me note, that previously the screening enhancement factors were derived only for the first step, $A + B \rightarrow O^*$, and include corrections for respective tunneling probability (e.g., [6, 17, 18]). Probably, this feature was not stressed enough clearly and in applications it was typically assumed that the enhancement factors are the same for the reaction to the final state $C + D$. In principle, this assumption can be supported by the statement that the branching factor should not be affected by screening. However, applying the same logic for reverse reaction, one concludes that the enhancement factor for $C + D \rightarrow O^* \rightarrow A + B$ is the same as for $C + D \rightarrow O^*$. Now the reason of non self-consistency of this approach becomes apparent–for the forward reaction the screening corrections are taken into account only for $A + B \rightarrow O^*$ tunneling, but for the reverse reaction they are incorporated only for $C + D \rightarrow O^*$. In other words, forward and reverse reactions are, in fact, considered in different physical models and it is not surprising that it leads to the violation of the detailed
balance condition. To solve this problem, the effect of the plasma screening on the tunneling
probability should be considered on an equal footing for both forward and reverse reactions. In
particular, it leads to the modification of the widths of decay channels of the compound nucleus.
In this proceeding paper, I generally follow [22] and in section 3 describe briefly how this effect
can be easily included to the reaction network codes in case of strongly exothermic reactions via
compound nucleus.

2. The plasma screening enhancement for reaction via compound nucleus: general
approach
Let me consider a reaction via compound nucleus, which can be written in form

\[ A + B \rightarrow O^* \rightarrow C + D, \]  

(1)

where \( A \) and \( B \) are initial nuclei (or particles); \( O^* \) is the compound nucleus at certain excited
state; \( C \) and \( D \) are final nuclei (particles). The following prescription can be straightforwardly
generalized for a case when more than two nuclei occur at the entrance or exit channels. A set
of additional exit channels is also allowed.

The reaction rate for \( A + B \rightarrow C + D \) in plasma environment can be written in form

\[ R_{A+B\rightarrow C+D} = f_{A+B\rightarrow C+D} f_{th}^{A+B\rightarrow C+D}, \]  

(2)

where \( R_{th}^{A+B\rightarrow C+D} \) is thermonuclear reaction rate in absence of screening and \( f_{A+B\rightarrow C+D} \)
is the enhancement factor, which should be determined. The thermonuclear reaction rate can be
written as \( R_{th}^{A+B\rightarrow C+D} = \langle \sigma_{A+B\rightarrow C+D} v \rangle n_{A} n_{B} \), where \( n_{A} \) and \( n_{B} \) are the number densities
of nuclei of type \( A \) and \( B \), while \( \langle \sigma_{A+B\rightarrow C+D} v \rangle \) is the Maxwellian averaged cross section, which
depends only on temperature \( T \). The enhancement factor can be presented as product of two
factors

\[ f_{A+B\rightarrow C+D} = f_{cl}^{A+B\rightarrow C+D} f_{q}^{A+B\rightarrow C+D}, \]  

(3)

where classical part accounts for non-ideal part of the chemical potentials, which one cannot
omit to ensure the detailed balance in non-ideal environment [22]

\[ f_{cl}^{A+B\rightarrow C+D} = \exp \left( \frac{\mu_{A}^{C} + \mu_{B}^{C} - \mu_{O}^{C}}{T} \right). \]  

(4)

Here \( \mu_{A}^{C}, \mu_{B}^{C}, \mu_{O}^{C} \) are non-ideal part of chemical potentials of nuclei \( A \), \( B \) and compound nucleus
\( O \). It is worth to stress, that they should be calculated within the same thermodynamical
approach as applied for equation of state to guarantee full thermodynamic consistency (in
particular, if corrections to the linear mixing rule [26] are included to equation of state, they
should be also properly accounted while calculating chemical potentials). Note, \( f_{th}^{A+B\rightarrow C+D} \) does
not depend on the actual exit channel, but only on the entrance channel and the compound
class

The quantum part of the enhancement factor \( f_{q}^{A+B\rightarrow C+D} \) accounts for modification of the
tunneling probability for both \( A + B \) fusion and fission of compound nucleus to \( C + D \) along with
modification of the total decay width of the compound nucleus \( O^* \) [22]. Namely, lets enumerate
all exit channels of \( O^* \) nucleus as \( 0, 1, 2, \ldots , n \), where \( 0 \) corresponds to \( A + B \) channel and \( 1 \) to
\( C + D \) and denote \( \gamma_{i} \) the width of respective channel in vacuum (absence of screening). The total
width (reverse life time) in vacuum is \( \Gamma = \sum_{i=0}^{n} \gamma_{i} \). The plasma screening affects the tunneling
probability for each channel for a factor \( f_{i}^{q} \), modifying the particular widths to \( \tilde{\gamma}_{i} = f_{i}^{q} \gamma_{i} \) and
total width \( \tilde{\Gamma} = \sum_{i=0}^{n} \tilde{\gamma}_{i} \). Factors \( f_{i}^{q} \) can be calculated, e.g., within the mean field approach
[6,9,12,17,18,27].

The correction factors for the tunneling probability \( f_{i}^{q} \) should be the same for the compound
nucleus formation via this channel, as well as for the decay of compound nucleus to the same
channel (the formal proof is evident within mean field approach, which is based on quasi-classical tunneling probability, see, e.g., equation (17) in [17]). Thus, the compound nucleus formation rate, as a result of $A + B$ reaction, is modified by classical and quantum screening corrections factor as

$$R_{A+B\rightarrow O^*} = R_{A+B\rightarrow O^*}^{cl} f_{A+B\rightarrow O^*}^{q},$$

(5)

The probability that $O^*$ nucleus decays into the 1-st channel $(C + D)$ is $\tilde{\gamma}_I/\Gamma$, thus the reaction rate is

$$R_{A+B\rightarrow C+D} = R_{A+B\rightarrow O^*}^{th} f_{A+B\rightarrow O^*}^{q} f_{C+D\rightarrow O^*}^{q} \frac{\tilde{\gamma}_I}{\Gamma}. $$

(6)

As far as $R_{A+B\rightarrow C+D}^{th} = R_{A+B\rightarrow O^*}^{th}\gamma_I/\Gamma$, the quantum enhancement factor is

$$f_{A+B\rightarrow C+D}^{q} = f_{A+B\rightarrow O^*}^{q} f_{C+D\rightarrow O^*}^{q} \frac{\tilde{\gamma}_I}{\Gamma}. $$

(7)

It is easy to see that the quantum enhancement factor is symmetric with respect of change of reaction direction ($f_{A+B\rightarrow C+D}^{q} = f_{C+D\rightarrow A+B}^{q}$), which guarantee that the detailed balance principle is satisfied.

It is worth to stress, that phenomenological approach, suggested in [25] to ensure the detailed balance, does not agree with (7). Namely, for $A + B \rightarrow C + D$ selected as favored direction, their approach is equal to

$$f_{A+B\rightarrow C+D}^{q,ph} = f_{C+D\rightarrow A+B}^{q,ph} = f_{A+B\rightarrow O^*}^{q}. $$

(8)

3. Enhancement factor for strongly exothermal reactions

The formalism discussed in the previous section is required to take into account the screening corrections accurately. However, it is rather difficult to incorporate it into the reaction network codes. Strictly speaking, the quantum correction factors depends on the state of the compound nucleus and thus the formalism should be applied independently for each particular compound state, while the total rate should be calculated as a sum over all states. Note, the Maxwellian averaged cross sections are typically given for total cross section, i.e., already summed over states of compound nucleus and thus generally they cannot be directly applied to calculate the rate in case of strong quantum screening effects.

In this section, I discuss an important particular case when one of the reaction channels leads to strongly exothermal reaction in all other channels, which is typical for fusion reactions in white dwarf cores and neutron star envelopes. Let’s enumerate this channel as $i = 0$. As far as this channel is not energetically favorable, its particular width should be small $\gamma_0 \ll \Gamma$ and contribution to the total width can be neglected (it can be treated as applicability condition for the discussion below). Furthermore, strong exothermicity leads to small tunneling lengths for other channels, thus the quantum corrections for them, assuming $f_i^{q} \approx 1$ for $i > 0$, can be neglected. In this case, the total width of $O^*$ nucleus in plasma environment $\Gamma \approx \Gamma$ and the quantum correction factor can be written as $f_{0 \rightarrow i}^{q} = f_{0}^{q}$, being the same as for formation of the compound nucleus, i.e., the same as the enhancement factor traditionally calculated in the literature devoted to the plasma screening enhancement of nuclear reaction rates.

In this case the difference with phenomenological approach of [25] is the most profound. They suggest to choose as a favored direction, the direction, which less affected by screening (e.g., photodisintegration reaction), thus according to equation (8) the quantum enhancement factor should be equal to 1. In other words, their phenomenological approach leads to neglect of the quantum corrections. However, the latter can be as large as tenths orders of magnitude in the cold dense matter of white dwarf cores and neutron star envelopes [16–18].

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4. Conclusions

In this proceeding paper, I generally follow [22] and present a recipe for describing the screening correction to the thermonuclear reaction rates in thermodynamically consistent way. It is pointed (section 3) that for strongly exothermic reactions, which are typical in astrophysical applications, the quantum part of the enhancement factor is equal to the enhancement factor for formation of the compound nucleus, which is typically calculated and fitted in astrophysical literature (e.g., [17, 18]).

It should be noted, that all discussion in this paper corresponds to the case when classical statistics can be applied to the nuclei (particles) in initial and final states. As mentioned in [22], quantum statistical effects (e.g., Pauli blocking) can be also treated as contributions to quantum enhancement factors. Accurate treatment of these effects will be published elsewhere.

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