Stretching of material lines in pseudo-turbulence
induced by small rising bubbles

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Abstract. Direct numerical simulations have been conducted for the stretching of material lines in pseudo-turbulence induced by small rising bubbles in order to understand the mixing characteristics of bubbly flows. Contaminated bubbles are considered and are treated as light solid particles. An immersed boundary method has been used for evaluating the coupling force between the bubbles and the surrounding fluid flows. Numerical results show that the total length of material lines increases exponentially in time as a result of stretching and folding due to the rising bubbles. The material lines tend to accumulate in the wake regions of the bubbles, and they are strongly stretched in the vertical direction there. It is also found that the stretching rate of material lines increases with the mean void fraction when it is normalized by the magnitude of the rate-of-strain tensor of liquid flow in pseudo-turbulence. In the case of high void fractions, material lines tend to align with the direction of maximum stretching, and are effectively stretched.

1. Introduction

We encounter a wide variety of phenomena in relation to fluid mixing in nature and industry. Examples include the dissolution of carbon dioxide in sea water, premixed combustion in an engine, and chemical processes in bubble-column reactors. It is well known that turbulence is highly efficient in mixing. Pseudo-turbulence, which is induced by the clusters of bubbles rising due to buoyancy, also has an ability to enhance the mixing, and is used in the processes of a catalytic reaction inside chemical plants and a biochemical reaction inside bioreactors.

Bubbly flows are complicated because of their multiple temporal-spatial characteristic scales. A lot of studies on pseudo-turbulence have been conducted in order to understand the fundamental property of bubbly flows both experimentally (Martinez-Mercado et al., 2007; Cartellier et al., 2009; Riboux et al., 2010) and numerically (Burner & Tryggvason, 2002; Yin & Koch, 2008a). Although these studies have clarified the flow structures of pseudo-turbulence to a substantial extent, fluid mixing in pseudo-turbulence is still poorly understood.

Several methods have been used to quantify the efficiency of mixing (Funakoshi, 2008). The stretching of material lines and surfaces has been investigated to understand the mixing in turbulent flows. Batchelor (1952) predicted that infinitesimal line and surface elements are stretched exponentially with time in turbulence, which was confirmed by a direct numerical simulation (Huang, 1996). Goto & Kida (2002) conducted simulations for the stretching of finite-length material lines in isotropic turbulence. They showed that the the statistics of
infinitesimal line and surface elements is not applied for the stretching of finite-length material lines (or surfaces), and found that the stretching rate of the latter is about 30% higher than that of former. It is interesting to investigate how the characteristic features of material lines change for bubble-induced pseudo-turbulence. In the present study, we perform direct numerical simulations for the stretching of material lines in pseudo-turbulence induced by rising bubbles in order to deepen the understanding of fluid mixing in pseudo-turbulence.

2. Formulation

2.1. Configuration

We consider the system where the bubbles are rising under the effect of gravity. We assign the $z$ axis to the vertical direction and the $x$ and $y$ axes to the horizontal directions, respectively. The gravitational force is assumed to act in the negative $z$ direction. Direct numerical simulations are conducted for the bubbles which are randomly introduced in otherwise quiescent fluid. Material lines are also introduced in the fluid at the initial instant, and their stretching is examined in detail.

2.2. Basic assumptions for bubbles

We examine the motions of small contaminated bubbles rising in a liquid containing impure substances such as surfactants. Both the gas and the liquid are considered to be incompressible. The bubble is assumed to be spherical and have a time-independent volume. The bubble interface is assumed to be solidified by the effects of surfactants, so that a no-slip boundary condition can be imposed on the bubble interface. The bubble is not allowed to coalesce, and the bubble-bubble collision is assumed to be elastic.

2.3. Basic equations for bubbly flows

The motions of incompressible fluid are governed by the Navier-Stokes (or momentum) equations

$$\frac{\partial u_{fi}}{\partial t} + u_{fk} \frac{\partial u_{fi}}{\partial x_k} = -\frac{1}{\rho_f} \frac{\partial p}{\partial x_i} + \frac{1}{\rho_f} \frac{\partial \tau_{ij}}{\partial x_j} + f_{Ri}$$

supplemented with the continuity equation, $\partial u_{fj} / \partial x_j = 0$, where $\tau_{ij}$ is the viscous stress tensor. Here, $u_f = (u_{f1}, u_{f2}, u_{f3}) = (u_f, v_f, w_f)$ denotes the liquid velocity, and $\rho_f$ and $\mu_f$ are the density and the viscosity of the liquid. $t$ and $p$ denote the time and the pressure, respectively. The term $f_R$ represents the coupling with bubbles.

Translational and rotational motions of bubbles are governed

$$\frac{\pi d_b^3 \rho_b}{6} \frac{du_{bi}}{dt} = \int_{S_b} \left( -p \delta_{ij} + \tau_{ij} \right) n_j dS + \frac{\pi d_b^3}{6} \left( \rho_b - \rho_f \right) g_i$$

and

$$\frac{8 \pi d_b^5 \rho_b}{15} \frac{d\omega_{bi}}{dt} = \int_{S_b} \epsilon_{ijk} r_j \tau_{ik} n_l dS,$$

respectively. Here, $d_b$ is the bubble diameter, $u_b = (u_{b1}, u_{b2}, u_{b3}) = (u_b, v_b, w_b)$ denotes the bubble velocity, $S_b$ represents the bubble interface, $n_i$ is the unit normal vector which is pointing outward from the bubble, $\omega_{bi}$ is the angular velocity of the bubble, and $r_j$ is the distance from the center of the bubble. In the present study, the second term on the right-hand side of Eq.(2) is replaced by the coupling force obtained by using an immersed boundary method. In place of Eq.(3), the angular velocity of the bubble is directly determined by the condition that the torque acting on the bubble is zero (Sugiyama et al., 2001).
2.4. Definition of the bubble velocity and fluctuation velocities

As was stated above, the bubble and liquid velocities are respectively defined as \( \mathbf{u}_b = (u_b, v_b, w_b) \) and \( \mathbf{u}_f = (u_f, v_f, w_f) \). The ensemble average of the bubble velocity is denoted by \( \mathbf{U}_b = (\mathbf{u}_b)_b = (U_b, V_b, W_b) \), and the volume-averaged liquid velocity is denoted by \( \mathbf{U}_f = (\mathbf{u}_f)_f = (U_f, V_f, W_f) \). The average relative velocity of bubbles is denoted by \( \mathbf{U}_R = \mathbf{U}_b - \mathbf{U}_f = (U_R, V_R, W_R) \). In experiments, the rise velocities are usually presented as drift velocities. The drift velocity of a bubble is defined as the difference between the bubble velocity and the volume-averaged velocity of the whole mixture, \( \alpha \mathbf{U}_b + (1 - \alpha) \mathbf{U}_f \). The average drift velocity of bubbles, \( \mathbf{U}_d \), is then described by \( \mathbf{U}_d = \mathbf{U}_b - (\alpha \mathbf{U}_b + (1 - \alpha) \mathbf{U}_f) = (1 - \alpha) \mathbf{U}_R = (U_d, V_d, W_d) \).

2.5. Material lines and stretching rate

Material lines are those which are composed of assembly of fluid particles. Numerically, they are represented by the sets of fluid particles located at sufficiently short intervals. We suppose that the material lines are composed of \( N \) line segments. The length of each line segment \( \Delta l^{(i)} \) \( (i = 1, \ldots N) \) is assumed to be sufficiently short that the total length, \( L \), of the material lines can be expressed by the sum of the length of the segments as \( L = \sum_{i=1}^{N} \Delta l^{(i)} \). Then, the stretching rate of the material lines is described by that for each line segment, \( \gamma^{(i)} = (1/\Delta l^{(i)}) \int \Delta l^{(i)} / dt \), as

\[
\gamma = \frac{1}{L} \sum_{i=1}^{N} \gamma^{(i)} \Delta l^{(i)} = \frac{\int \gamma_c dl}{\int dl}.
\]  

The line integral on the right hand side of Eq.(4) indicates that the stretching rate of the material lines, \( \gamma(t) \), is the averaged value of those of line segments, \( \gamma^{(i)}(t) \), along the material lines. Note that the simple arithmetic average of the stretching rates, \( \gamma^{(i)} \), provides right evaluation only for the case where all line segments are infinitely short.

3. Computational method

The motions of bubbles are solved directly based on the Newton’s law, without using simplified models. In the present study, each bubble is regarded as a solid particle of a very low density ratio since the bubble interface is solidified. The numerical method employed in this study is based on the immersed boundary method (Kajishima et al., 2001), which was developed to investigate the motions of solid particles in turbulent flows. The flow is simulated using the uniform grid in Cartesian coordinate. The two-way coupling between the particles and the fluid is realized by considering the momentum exchange between the two phases.

Some modifications have been made in order to apply this method to bubbly flows. The momentum exchange between the two phases was corrected by taking account of the effect of internal mass. A term similar to an added-mass term was added to the equation of the bubble translational motion for a numerical stability. In Tanaka et al. (2007), the entrainment of bubbles into a vortex tube were simulated after these modifications were made.

Eq.(1) is approximated by using a finite difference method on a staggered grid. The second-order central difference scheme is applied in the finite differencing of the convection and viscous terms of Eq.(1). The time-integration is based on a fractional-step method. The 2nd-order Adams-Bashforth method is used for the time-integration of the convection and viscous terms in the Navier-Stokes equations. The Poisson’s equation for the pressure is solved directly by using the fast Fourier transform.

We conducted a simulation for a single rising bubble in otherwise quiescent fluid to check the translational motion of the bubble (Tanaka et al., 2007). The time evolution of the bubble Reynolds number was found to exhibit a good agreement with that in Takagi & Matsumoto.
Table 1. Computational conditions for pseudo-turbulence.

| Condition                      | Value                                  |
|--------------------------------|----------------------------------------|
| Domain size                    | $2\pi \times 12\pi \times 2\pi$       |
| Grid points                    | $128 \times 768 \times 128$           |
| Grid spacing $\Delta x$       | $4.90 \times 10^{-2}$                 |
| Time increment $\Delta t/\tau_b$ | $6.81^{-3}$                           |
| Bubble diameter                | $500 \mu m$ ($12\Delta x$)           |
| Density ratio                  |                                         |
| Void fraction $\alpha$         | $1.55\%, 3.1\%, 4.65\%, 6.2\%$       |
| Number of bubbles              | $216, 432, 648, 864$                   |
| Number of finite-length lines  | $20, 10, 5, 5$                        |
| Number of infinitesimal lines  | $1572864$                              |
| Boundary condition             | $x, y, z$-dimensions: Periodic         |

(1996), which was obtained by using a boundary fitted grid. We also examined the rotational motion of a bubble in a uniform shear flow to check the validity of the method for estimating the angular velocity of the bubble. It is found that the steady-state value of angular velocity, $\Omega_{st}$, of the bubble agrees with that in Bagchi & Balachandar (2002) with an error less than 10% in the range considered here.

The velocity of the fluid particles on the material lines is evaluated by using the linear interpolation of the fluid velocity at the adjacent grid points. When the distance between the neighboring fluid particles exceeds a threshold value, a new fluid particle is added to the set of fluid particles by using the fourth-order Lagrangian interpolation.

4. Stretching of material lines in pseudo-turbulence

4.1. Computational conditions

The computational conditions are summarized in table 1. We consider the bubble of $d_b \approx 500 \mu m$. We set the vertical size of the computational domain longer than those in the other directions since the auto-correlation of the velocity is longer in the vertical direction. In the present study, we introduce the non-dimensional time, $t^* = t/\tau_b$, where $\tau_b = d_b/W_T$. The bubbles are introduced randomly in a quiescent fluid at $t^* = 0$. Material lines are also set at the initial instant, $t^* = 0$, as straight lines of length $2\pi$ in the horizontal plane of $z = 6\pi$. The stretching of infinitesimal lines is also examined for comparison.

4.2. Bubble rise velocity and liquid velocity fluctuations

It was found that the pseudo-turbulence reached an asymptotic state by the time $t^* = 50$ for all the cases considered here. The asymptotic value of the average drift velocity of bubbles depends on the mean void fraction. In Fig. 1, the non-dimensional drift velocity, $W_d/W_T$, is plotted as a function of the void fraction. As is shown in the figure, the rise (or drift) velocity is reduced with the increase of the void fraction. This is because the drag coefficient is increased due to the hydrodynamic interactions between bubbles when the void fraction is increased.

Sugiyama et al. (2001) found in their numerical simulation for pseudo-turbulence that the average drag coefficient of contaminated bubbles agrees quite well with that of the solid particles in sedimentation in spite of the difference of inertia. Here, we compare our result with the average settling velocity of solid particles in the sedimentation problems. The numerical result for the solid particles with the density ratio of 2.0 in Yin & Koch (2007) is plotted together with our
result in Fig. 1. It is shown that our result agrees well with that of Yin & Koch (2007).

It is also found that the liquid velocity fluctuations increase with increasing void fraction. Fig. 2 shows the time-averaged values of the vertical component of the Reynolds stress, \( R_{zz} = \langle u'_f u'_{fj} \rangle \), where \( u'_f = u_f - U_f \) is the fluctuating component of the fluid velocity, as a function of the void fraction. In Fig. 2, a numerical result for solid particles at \( Re_T = 10 \) (Yin & Koch, 2008b) is shown together with our results at \( Re_T = 27.2 \). Considering that the normalized liquid velocity fluctuation, \( R_{zz}/W_R^2 \), decreases with increasing Reynolds number and that its dependence is relatively weak, it can be said that the liquid vertical fluctuation in the bubbly flow in our simulation agrees quite well with that in the particulate flow.

### 4.3. Stretching of material lines

Figs. 3 show the temporal evolution of material lines in the case of \( \alpha = 4.65\% \). The upper 2/3 of the computational domain is displayed. Initial straight lines are distorted and are stretched in the vertical direction as the bubbles move upward. As time progresses, the material lines are extended over the whole computational domain. It is found that the material lines are stretched exponentially in time for all the cases simulated here.

### 4.4. Stretching rate of material lines

As was discussed above, the total length of the material lines increases exponentially in time so that it can be expressed as \( L(t) = L_0 \exp[\gamma t] \), where \( \gamma(t) = (1/L) dL/dt \) represents the stretching rate of material lines. Here, we discuss the void-fraction dependency of the stretching rate. Before proceeding to the discussion, we briefly explain about the previous results for homogeneous isotropic turbulence. The stretching rate of infinitesimal line elements is determined by the smallest structures of turbulence independent of the Reynolds number, and \( \gamma\tau_\eta = 0.13 \pm 0.01 \) in homogeneous isotropic turbulence (Goto & Kida, 2007). Here, \( \tau_\eta \) is the Kolmogorov timescale. For the case of finite-length material lines the stretching rate is higher that than for the infinitesimal lines since the finite-length material lines accumulate in the regions of high strain due to the stretching and folding there. When the Reynolds number is relatively low, the stretching rate normalized by the Kolmogorov timescale is described as \( \gamma\tau_\eta \approx 0.17 \) (Goto & Kida, 2003).

It may generally be more appropriate to utilize the magnitude of the rate-of-strain tensor to normalize the stretching rate since the rate-of-strain tensor is directly related to the stretching rate. By using the relationship, \( \tau_\eta = \langle 2S_{ij}^2 \rangle^{-1/2} \), in homogeneous isotropic turbulence, the relation, \( \gamma\tau_\eta \approx 0.17 \), is rewritten as \( \gamma\langle S_{ij}^2 \rangle^{-1/2} \approx 0.24 \), where \( S_{ij} \) denotes the rate-of-strain.
tensor. The magnitude of the straining field, \( \langle S_{ij}^2 \rangle \), increases with the increase of the void fraction. It is found that \( \langle S_{ij}^2 \rangle \) exhibits a similar dependency on \( \alpha \) as the energy of the pseudo-turbulence, \( R_{kk} \).

Figs. 4 show the time-evolution of the stretching rate of (a) finite-length and (b) infinitesimal material lines. The normalization is conducted by the timescale, \( \langle S_{ij}^2 \rangle^{-1/2} \). In an initial stage of evolution, the stretching rate rapidly increases and takes a peak value by the time \( \langle S_{ij}^2 \rangle^{1/2} t = 10 \). In a later period it approaches an equilibrium value. The asymptotic value of the stretching rate is increased as the mean void fraction is increased for both the finite-length and infinitesimal lines. In the case of finite-length material lines, the asymptotic values for \( \alpha = 1.55\% \) and 6.20\% are respectively \( \gamma \langle S_{ij}^2 \rangle^{-1/2} \approx 0.06 \) and 0.12. The stretching rate of \( \gamma \langle S_{ij}^2 \rangle^{-1/2} \approx 0.12 \) in the case of \( \alpha = 6.20\% \) is still considerably lower than that of \( \gamma \langle S_{ij}^2 \rangle^{-1/2} \approx 0.24 \) in homogeneous isotropic turbulence. In the case of infinitesimal lines, the asymptotic values for \( \alpha = 1.55\% \) and 6.20\% are respectively \( \gamma \langle S_{ij}^2 \rangle^{-1/2} \approx 0.025 \) and 0.075. The difference between the cases of \( \alpha = 1.55\% \) and 6.20\% is more remarkable for the infinitesimal lines.
Fig. 5. Material lines and bubbles at $t^* = 47.9$ for $\alpha = 4.65\%$. The region of $\frac{3}{4}\pi \leq x \leq \frac{5}{8}\pi, 0 \leq y \leq 2\pi, 5\pi \leq z \leq 9\pi$ is displayed.

Fig. 6. Schematic of the eigenvector field around a rising bubble.

Fig. 5 shows the material lines and bubbles at $t^* = 47.9$ in the case of $\alpha = 4.65\%$. The computational region of $\frac{3}{4}\pi \leq x \leq \frac{5}{8}\pi, 0 \leq y \leq 2\pi, 5\pi \leq z \leq 9\pi$ is displayed. The red portions of the material lines represent the regions of strong stretching ($\gamma_e > 4\gamma$) and the blue portions represent those of strong compression ($\gamma_e < -4\gamma$). It is found that the material lines tend to accumulate in the wake regions of bubbles. Suppose that a rising bubble approaches a horizontal straight material line from underneath. The rising bubble lifts up and fold the material line when it passes through the initial position of the line. The stretching and folding is repeated and then the material line accumulate in the wake region of the bubble. It should be noted that the stretching rate of the material lines is high in the wake region. Namely, the material lines are strongly stretched in the vertical direction there.

4.5. Alignment of material lines with the eigen vectors of the strain tensor

The void-fraction dependence of the stretching rate is associated with the alignment between the elements of material lines and the eigen vectors of rate-of-strain tensor. Here, the largest, intermediate and the smallest eigen values of the rate-of-strain tensor are denoted by $\lambda_1$, $\lambda_2$ and $\lambda_3$. The corresponding eigen vectors are then represented by $e_i (i = 1, 2, 3)$, which are mutually perpendicular. When a line element is aligned with $e_i$, its stretching rate equals to the corresponding eigen value, $\lambda_i$. In incompressible fluids the sum of the three eigen values is zero, i.e., $\lambda_1 + \lambda_2 + \lambda_3 = 0$. Therefore, $e_1$ (or $e_3$) corresponds to the direction of maximum stretching (or compression).

Before investigating the angle between the material lines and the eigen vectors of the strain tensor, it is necessary to understand the structure of straining field around a rising bubble. Fig. 6 illustrates the straining field around an isolated bubble and its wake. The rising bubble induces an upward flow below the bubble. Large horizontal gradient in the vertical velocity is generated around the upflow. In this region, both of the direction of maximum stretching, $e_1$, and maximum compression, $e_3$, are parallel to the vertical plane and are inclined at 45° to the vertical direction. The direction of $e_2$ is perpendicular to $e_1$ and $e_3$, and therefore is oriented in a horizontal direction. It should be noted that the material lines are no longer stretched in a vertical simple shear flow once they are oriented in the vertical direction.

Fig. 7 shows the probability density functions (pdf), $f_F(\cos \Phi_i)$, of the cosine of angles, $\Phi_i$,
between the vertical axis and three eigen vectors, \(e_i\), of the strain tensor at each grid point. The pdf was obtained by using the data at all the grid points inside the liquid. \(\cos \Phi_1 = 1\) (or \(\cos \Phi_i = 0\)) indicates that the two vectors are parallel (or perpendicular) to each other. It is clearly seen that the pdf’s for \(e_1\) and \(e_3\) have relatively sharp peaks near the angle of 45° \((\cos \Phi_1 \approx 0.71)\) in the case of the lower void fraction. The pdf for \(e_2\) has a peak at \(\cos \Phi_2 = 0\), indicating that the second eigen vectors are oriented in the horizontal directions. These results are consistent with the picture illustrated in Fig. 5. In the case of the higher void fraction, the peaks for \(e_1\) and \(e_3\) are broadened. The peak for \(e_1\) is shifted toward the side of \(\cos \Phi_1 = 1\), which means that the alignment of \(e_1\) and the tangent vector of material lines becomes better as the mean void fraction increases.

As is shown in Fig. 8, the material lines tend to align with the vertical axis since they are mainly stretched in the vertical direction due to the bubbles. This strong tendency of alignment is somewhat relaxed for high void fractions.

Fig. 9 shows the pdf, \(f_L(\cos \Phi_i)\), of the cosine of angles, \(\Phi_i\), between the vertical axis and three eigen vectors, \(e_i\), of the strain tensor at the positions of the fluid particles composing the material lines. The pdf was obtained by using the data on whole material lines. This pdf is different from that in Fig. 7 due to the accumulation of the material lines in specific regions (see Fig. 6). In the case of the lower void fraction, the pdf, \(f_L(\cos \Phi_1)\), has another peak at \(\Phi_1 \approx 0^\circ\) in addition to the peak corresponding to that in \(f_F(\cos \Phi_1)\). This indicates that the material line is effectively stretched in the direction of maximum stretching, \(e_1\), when \(e_1\) is oriented in
the vertical direction. As the void fraction is increased, the peaks at $\Phi_1 \approx 45^\circ$ and $\Phi_3 \approx 45^\circ$ become less noticeable.

Fig. 10 shows the pdf, $f_L(\cos \theta_i)$, of the cosine of angles, $\theta_i$, between the tangent vector of material lines and $e_i$. The pdf for the angle between the infinitesimal lines and $e_1$ is also shown by using the chain line. In the case of the lower void fraction, the pdf’s, $f_L(\cos \theta_i)$, resemble $f_L(\cos \Phi_i)$ as a whole since most of the material lines are oriented nearly in the vertical direction. As in the case of $f_L(\cos \Phi_1)$, the pdf has two peaks at $\theta_1 \approx 45^\circ$ and $0^\circ$. The existence of the peak at $0^\circ$ indicates that many line elements on the material lines are oriented in the direction of maximum stretching, $e_1$. On the other hand, the pdf for infinitesimal lines has no peak at $0^\circ$, which implies that the part of the material lines oriented in the direction of $e_1$ indeed have increased in length. The pdf for $\theta_2$ has a peak at $\cos \theta_2 = 0$, indicating that most of the material lines are perpendicular to the second eigen vectors, $e_2$. This is consistent with the fact that the material lines tend to align with the vertical direction and the eigen vectors, $e_2$, tend to be oriented in the horizontal directions. The pdf for $\theta_3$ has a peak at $\cos \theta_3 \approx 0.7(\theta_3 \approx 45^\circ)$.

As the void fraction is increased, the difference between the pdf’s of $\theta_1$ and $\theta_3$ is amplified in the region near $\cos \theta_i = 1(\theta_i = 0)$. The mechanism that generates the peaks at $45^\circ$, which is illustrated in Fig. 6, becomes less dominant since the flow structure of pseudo-turbulence is modulated as a result of hydrodynamic bubble-bubble interactions.

In summary, material lines are mostly parallel to the $e_1 - e_3$ planes. The alignment with $e_1$ becomes more noticeable as the mean void fraction is increased. It is interesting to note that the tendency of the alignment in the bubbly flow is different from that in turbulence. Goto & Kida (2003) found in their direct numerical simulation of homogeneous isotropic turbulence that material lines are parallel to the $e_1 - e_2$ planes and are correlated more with the second eigen vectors, $e_2$, rather than the first eigen vectors, $e_1$.

5. Conclusion

Direct numerical simulations have been conducted for the stretching of material lines in pseudo-turbulence induced by small contaminated bubbles rising in otherwise quiescent fluid. The bubbles are treated as light solid particles. An immersed boundary method has been used for evaluating the coupling force between the bubbles and the surrounding fluid flows. It is found that the total length of material lines increases exponentially in time as a result of stretching and folding due to the rising bubbles. The material lines are strongly stretched in the vertical direction in the wake regions of the bubbles, and accumulate there. It is also found that the stretching rate of material lines increases with the mean void fraction even when it is normalized by the magnitude of the strain tensor, which also increases with the void fraction. In the case
of high void fractions, material lines align well with the direction of maximum stretching, and are more effectively stretched.

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