Neutrino Masses within the Minimal Supersymmetric Standard Model

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ABSTRACT

We investigate the possibility of accommodating neutrino masses compatible with the MSW study of the Solar neutrino deficit within the minimal supersymmetric Standard Model. The “gravity-induced” seesaw mechanism based on an interplay of nonrenormalizable and renormalizable terms in the superpotential allows neutrino masses \( m_\nu \propto m_u^2/M_I \), with \( m_u \) the corresponding quark mass and \( M_I \approx 4 \times 10^{11} \) GeV, while at the same time ensuring the grand desert with the gauge coupling unification at \( M_U \approx 2 \times 10^{16} \) GeV. The proposed scenario may be realized in a class of string vacua, i.e., large radius \( (R^2/\alpha' = O(20)) \) (0, 2) Calabi-Yau spaces. In this case \( M_U^2 = M_C^2/O(2R^2/\alpha') \) and \( M_I = O(e^{-R^2/\alpha'})M_C \). Here \( M_C = g \times 5.2 \times 10^{17} \text{GeV} \) is the scale of the tree level (genus zero) gauge coupling \( (g) \) unification.

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Precise data from the LEP experiments indicate that the gauge couplings of the Standard Model meet at $M_U \simeq (1 - 4) \times 10^{16}$ GeV in the minimal supersymmetric extension of the Standard Model. Another set of intriguing data arise from the Solar neutrino experiments. The deficit of Solar neutrinos can most efficiently be explained through the MSW mechanism of matter-enhanced neutrino oscillations. In particular, current data favor the mass splitting of the electron and muon neutrinos to be $\Delta m^2 \equiv |m^2_{\nu_e} - m^2_{\nu_\mu}| \simeq (2 - 5) \times 10^{-7}$ eV$^2$ if the mixing angle $\theta_{\nu_\mu\nu_e} \simeq O(\theta_C)$, where $\theta_C$ is the Cabibbo angle. For arbitrary mixing angles the nonadiabatic MSW solution favors $\Delta m^2 \simeq (1 - 16) \times 10^{-7}$ eV$^2$.

In many grand unified theory (GUT) models the lepton mixing matrix $V_\ell$ and the Cabibbo-Kobayashi-Maskawa matrix $V_{CKM}$ are predicted to be approximately equal. However, these same models predict $m_e/m_\mu \simeq m_d/m_s \simeq 1/20$, which fails by an order of magnitude. Small perturbations on the models which rescue this mass relation may also modify the mixing angle predictions. In theories with no explicit intra-family unification $V_{CKM}$ and $V_\ell$ are not expected to be equal, but could well be of the same order of magnitude. We will assume $\sin^2 2\theta_{\nu_\mu\nu_e} \sim \sin^2 2\theta_C \sim 0.18$ for the central value in our discussion, but will consider the entire range $4 \times 10^{-3} - 1$ allowed by the nonadiabatic MSW solution. Assuming $m_{\nu_\mu} \gg m_{\nu_e}$, the corresponding values of $m_{\nu_\mu}$ are $(5 - 7) \times 10^{-4}$ eV for $\theta_{\nu_\mu\nu_e} \sim \theta_C$ and $(3 - 40) \times 10^{-4}$ eV for general $\theta_{\nu_\mu\nu_e}$.

In the GUT seesaw scenario masses of light neutrinos are given by $m_{\nu_{e,\mu}} \simeq c \ m^2_{u,c}/M_I$, where $m_{u,c}$ are the corresponding quark masses and $c \simeq 0.05 - 0.09$ is a factor due to the renormalization down to the low energy scale. This implies that $M_I \simeq (4 \pm 3) \times 10^{11}$ GeV, the central value corresponding to $\theta_{\nu_\mu\nu_e} \sim \theta_C$. If the same scale applies to the third family, then $m_{\nu_\tau} \simeq c' m^2_\ell/M_I$ could be in the
cosmologically interesting 10 eV range, with $\nu_\mu-\nu_\tau$ oscillations observable in the laboratory.

Each of the two sets of experimental data has an elegant theoretical explanation. Unfortunately, the two theoretical models are mutually exclusive at first glance. In the minimal supersymmetric Standard Model, there is a “grand desert” up to $M_U \simeq (1-4) \times 10^{16}$ GeV. Within this theory, the implementation of a naive seesaw mechanism would indicate that $m_{\nu_e,\mu} \simeq cm_{\mu,e}^2/M_I$, with $M_I \sim (10^{-2} - 1)M_U \sim (1-4) \times 10^{15\pm1}$ GeV, which is too small to be compatible with the favored experimental data and the MSW scenario$^3$. Actually, in GUT models, in order to obtain a nonzero $M_I$ with $M_I \sim M_U$ one has to introduce large Higgs representations (e.g., the 126 of $SO(10)$).

The aim of this note is to implement the neutrino masses in the minimal supersymmetric Standard Model in such a way that there is still a grand desert with the gauge coupling unification at $M_U \sim (1-4) \times 10^{16}$ GeV, while the effective scale $M_I$ governing the neutrino masses is in the range of $(4\pm3) \times 10^{11}$ GeV. We are proposing a “gravity-induced” seesaw mechanism (an extension of a mechanism proposed by Nandi and Sarkar$^7$), realized through an interplay between the nonrenormalizable and renormalizable terms in the superpotential, as the origin of the neutrino masses. The essence of the idea is based on a supersymmetric theory with an extended gauge symmetry, which contains an additional sterile neutrino (a Standard Model singlet), and a restricted representation of the Higgs fields. Such fields break the extended gauge symmetry at the scale $M_U$. However, they cannot give the sterile neutrino a large Majorana mass proportional to $M_U$ through the renormalizable
(cubic) terms in the superpotential. On the other hand, through the nonrenormalizable \textit{(e.g., quartic)} terms of the superpotential, which are suppressed by a scale $M_{NR} > M_U$, such Higgs fields can give a Majorana mass of order $M^2_U/M_{NR} < M_U$.

The origin of the nonrenormalizable terms is best motivated in theories which include gravity, \textit{e.g.}, Kaluza-Klein theories and superstring theory. In this case the exchange of heavy modes with masses of order the Planck mass $M_{Pl}$ in general induce nonrenormalizable terms with $M_{NR} = \mathcal{O}(M_{Pl})$. Thus, we shall call the proposed scenario the gravity-induced seesaw. In the case of $M_U \sim 2 \times 10^{16}$ GeV and $M_{NR} \sim M_{Pl}/\sqrt{8\pi} \sim 2 \times 10^{18}$ GeV one has $M_I \sim 2 \times 10^{14}$ GeV, which is about two to three orders of magnitude too large to be compatible with the favored MSW data. However, if such nonrenormalizable terms are \textit{suppressed} by an additional factor $10^{-2} - 10^{-3}$, one can obtain the desired $M_I \sim 10^{11} - 10^{12}$ GeV.

Such a scenario can be accommodated within a GUT theory with restricted Higgs field representations; \textit{e.g.}, the $SO(10)$ gauge group without 126-plets of Higgs fields. For a broad class of simple GUT models, $V_{CKM} \simeq V_\ell$. On the other hand, extended gauge symmetries based on a product of simple groups and $U(1)$ factors (\textit{e.g.}, the Standard Model with additional $U(1)$’s and/or left-right symmetric gauge symmetry) with restricted representations of the Higgs sector can also accommodate the gravity-induced seesaw mechanism. However, in this case, the relation between $V_{CKM}$ and $V_\ell$ is less obvious.

One can demonstrate the gravity-induced seesaw in an explicit (minimal) model with all the essential features. We choose the enhanced gauge symmetry, $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Y'}$, where $Y$ is the ordinary weak hypercharge. The matter consists of the particle content of the minimal Standard Model as well as of the Standard Model singlets, $L_i$, $S_1$ and $\bar{S}_1$. $L_i$ supermultiplets with $i = (1,2,3)$ contain
a sterile neutrino which accompanies each of the three families. $S_1$ and $\bar{S}_1$ contain the Higgs fields which break the enhanced gauge symmetry with VEV’s of order $M_U$.

Consistent with the anomaly constraint we choose the following values for the $Y'$ charges: quark $SU(2)_L$ doublets, $u^c_L$ quarks and $e^c_L$ leptons have (-1), lepton doublets and $d^c_L$ quarks have (+3), $L_i$ and $S_1$ have (-5), $\bar{S}_1$ has (+5), while Higgs doublets $H_{(1,2)}$ have (-2) and (+2), respectively. In the neutrino sector, the only renormalizable terms allowed in the superpotential are of the type $W = L_i \nu_i H_2$. Terms of the type $L_i \nu_i S_1$ or $L_i L_i S_1$ are not allowed by the quantum numbers. These constraints yield the following contribution to the neutrino mass matrix:

$$
\begin{bmatrix}
0 & m \\
 m & 0
\end{bmatrix},
$$

(1)

where $m$ is proportional to the VEV of the Higgs doublet $H_2$. Since $H_2$ gives mass to the quarks as well, $m$ is of the order of the corresponding quark masses unless there is a large difference in the magnitude of the Yukawa couplings. On the other hand, the only allowed nonrenormalizable term in the superpotential with a leading contribution to the neutrino mass matrix is of the type $W_{NR} = L_i L_i \bar{S}_1 \bar{S}_1 / M_{NR}$. This modifies the neutrino mass matrix:

$$
\begin{bmatrix}
0 & m \\
 m & M_I
\end{bmatrix},
$$

(2)

where $M_I = M^2_U / M_{NR}$. The quantum numbers prevent the contribution of any non-renormalizable term to the $\nu \nu$ and $\nu L$ masses that would be of the order of $M^K_U / M^{K-1}_{NR}$ for any $K > 1$.  

As seen in the above model, the gravity-induced seesaw can be accommodated by an interplay of the renormalizable and nonrenormalizable terms in the superpotential, which takes place because of a restricted representation of the Higgs sector. While such a scenario is appealing on its own terms, we would like to motivate its origin. Study of the effective Lagrangian of superstring vacua provides a natural framework for the restricted representation of the chiral supermultiplets. One can also shed light on the origin of the nonrenormalizable terms in the superpotential, which in string theory arise due to the exchange of massive modes.

Let us first illustrate the neutrino mass pattern in the example discussed by Nandi and Sarkar\textsuperscript{7} with a gauge group $G \in E_6$ and all the chiral supermultiplets (i.e., those which contain quarks, leptons and Higgs particles) arising from 27-plets of $E_6$. The renormalizable superpotential is of the type $W_R = 27_i 27_j 27_K$, where quarks and leptons arise from $27_{i,j}$ and Higgs vacuum expectation values (VEV’s) from $27_K$. If one assumes that all the exotic quarks acquire large masses due to the large VEV’s of the Standard Model singlets $S_1$ and $S_2$, while the Higgs doublets in $27_K$ account for the masses of the ordinary quarks and leptons, this constrains\textsuperscript{7} the mass matrix of the five neutral fermions ($\nu, N, N^c, \nu^c, L$) to be of the form:

$$
\begin{bmatrix}
0 & 0 & M_{U_1} & m_1 & 0 \\
0 & 0 & M_{U_2} & 0 & m_1 \\
M_{U_1} & M_{U_2} & 0 & m_2 & m_3 \\
m_1 & 0 & m_2 & 0 & 0 \\
0 & m_1 & m_3 & 0 & 0
\end{bmatrix}
$$

(3)

Under the $SU(3)_L \times SU(3)_R \times SU(3)_C$ subgroup of $E_6$, the neutral fields ($\nu, N, N^c, \nu^c, L$) are a part of a $(3, \bar{3}, 1)$ multiplet with the following entries:

$$
\begin{bmatrix}
N \\
N^c \\
\nu \\
\nu^c \\
L
\end{bmatrix}
\begin{bmatrix}
M_{U_1}
\end{bmatrix}
$$
and $M_{U_2}$ are masses due to the VEV’s of the Standard Model singlets and $m_{1,2,3}$ are the light masses, related to the quark masses, which are due to the Standard Model Higgs doublets. The mass matrix (3) does not have a large Majorana mass along the diagonal of the lower two components. There is a heavy sector with large masses $M_U \ (N^c \text{ and } (\nu + N)/\sqrt{2})$, a light sector with masses of order $m \ ((\nu - N)/\sqrt{2}) \text{ and } (\nu^c - L)/\sqrt{2})$, and an ultra-light Standard Model singlet $(\nu^c + L)/\sqrt{2})$ with mass of order $7 \ m^2/M_U$; this pattern is clearly incompatible with experiment.

However, nonrenormalizable terms could provide (along with the renormalizable ones) a derived seesaw pattern. Within the $E_6$ gauge group there may be terms in the superpotential of the type $W_{NR} = 27_i27_j27_k27_l/M_{NR}$, which can account for the heavy Majorana masses. Namely, the mass matrix (3) becomes

$$
\begin{pmatrix}
0 & 0 & M_{U_1} & m_1 & 0 \\
0 & 0 & M_{U_2} & 0 & m_1 \\
M_{U_1} & M_{U_2} & 0 & m_2 & m_3 \\
m_1 & 0 & m_2 & M_{I_1} & M_{I_2} \\
0 & m_1 & m_3 & M_{I_3} & M_{I_4}
\end{pmatrix}
$$

with $M_I = M_{U}^2/M_{NR}$.

Corrections to the pattern (4) are scaled by $7 \ M_I/M_U \sim M_U/M_{NR}$ and have been neglected. Such terms are expected to appear due to the exchange of heavy modes. Again $M_I$ is somewhat too large for, say, $M_U \sim 2 \times 10^{16}$ GeV and $M_{NR} \sim 10^{18}$ GeV. As we point out later, in superstring theory one can shed light on the magnitude of the nonrenormalizable terms of the superpotential in a quantitative way, and thus account for an additional suppression factor.

The above example is based on the constraints of the $E_6$ gauge group and the
representations. In order for this scenario to be compatible with the grand
desert scenario of the minimal supersymmetric Standard Model, a number of other
constraints have to be satisfied: (i) the gauge couplings have to meet at $M_U \simeq
(1 - 4) \times 10^{16}$ GeV, (ii) below $M_U$ the gauge group has to be the Standard Model
group $\text{SU}(5)^{14}$, and (iii) the particle content contributing to the running of the gauge
couplings has to be that of the minimal supersymmetric Standard Model.

In superstring theories these constraints place conditions on the string vacuum.
Perhaps the most difficult to satisfy (with no existing example available) is (iii).
Generically, (2, 2) string vacua, e.g., Calabi-Yau manifolds with gauge and spin
connection identified, possess a large number of additional multiplets. In particular,
for vacua without Wilson lines, the gauge group is $E_6$, with $\mathbf{27}$’s, $\mathbf{\overline{27}}$’s, and 1’s of
$E_6$. Some of the particles in these multiplets acquire large masses if there are flat
directions in the space of specific string vacua. Finding flat directions$^{15,16}$ allows
one to give large VEV’s to fields in a particular set of $\mathbf{27}$’s and $\mathbf{\overline{27}}$’s, which in turn
can give mass to some of the unwanted massless multiplets. At the same time, $E_6$
is broken down to $SO(10)$ or $SU(5)$. Explicit examples of such directions have been
found in blown-up orbifolds$^{17,18}$ as well as for a class of Calabi-Yau manifolds$^{19}$
based on Gepner’s$^{20}$ construction.

Flat directions of (2,2) vacua provide one with a new class of ((0,2)) string
vacua. In such vacua a large number of unwanted modes become heavy; however,
the gauge group is still a simple GUT group ($SO(10)$ or $SU(5)$). Since the matter
supermultiplets are in the fundamental representation or singlets of the gauge
group,$^{21}$ this prevents one from breaking the simple GUT groups down to the
Standard Model, thus contradicting the constraint (ii).

This problem can be remedied by the introduction of Wilson lines on the com-
pactified space, allowing a breakdown of the simple gauge group \((E_6, \ SO(10), \) or \(SU(5))\) to a direct product of simple groups and \(U(1)'s\). It is in general possible\(^{22,19}\) to introduce Wilson lines which break the gauge group down to the Standard Model. At the same time, this procedure decouples a large number of unwanted modes. Since there is no grand unification in the 4-dimensional theory one does not expect observable proton decay, and the relationship between \(V_{CKM}\) and \(V_\ell\) is lost.

Thus, a viable scenario which could satisfy constraints \((ii)\) and \((iii)\) is to construct \((2,2)\) string vacua with flat directions as well as Wilson lines. However, there exists no explicit construction of such a supersymmetric string vacuum which would contain only the minimal Standard Model particle spectrum.

The next issue to be addressed is the value of \(M_C\), which is the scale at which the gauge couplings \(g\), as determined at the tree level of the string theory, are equal. \(M_C\) is determined\(^{23}\) in the \(\overline{DR}\) scheme by the value of the Planck mass \(M_{Pl}\) and of the gauge coupling \(g\) in the following way:

\[
M_C = \frac{e^{(1-\gamma)/2} \sqrt{2}}{3^{3/4} \sqrt{\pi \alpha'}} = g \times \frac{e^{(1-\gamma)/2}}{3^{3/4} 4\pi} M_{Pl} = g \times 0.043 M_{Pl} = g \times 5.2 \times 10^{17} \text{ GeV.} \tag{5}
\]

where \(\gamma = 0.57722\) is the Euler constant, \(g^2 = 32\pi / (\alpha' M_{Pl}^2)\), with \(g\) defined according to the \(GUT\) convention\(^{24}\) and \(M_{Pl} = 1.2 \times 10^{19}\) GeV.

For the expected value \(g \sim 0.7\) this is one order of magnitude too large compared with \(M_U \sim (1 - 4) \times 10^{16}\) GeV, which is the scale of the gauge coupling unification of the minimal supersymmetric Standard Model. However, threshold effects, \(i.e.,\) genus one corrections to the gauge couplings, can split the gauge couplings at \(M_C\), thus in principle allowing for an effective unification scale \(M_U < M_C\).
Explicit calculations of the threshold corrections for a class of orbifolds are given in Refs. 23,25. Extensive study\textsuperscript{26} of threshold corrections in orbifolds indicate that $M_U < M_C$ if the massless spectrum satisfies certain constraints compatible with the target space one-loop modular anomaly. In such examples one would obtain $M_U = \mathcal{O}(e^{(-cR^2/\alpha')})M_C$ when the orbifold radius is large ($R^2/\alpha' \gg 1$). The positive coefficient $c$ depends\textsuperscript{26} on the modular weights of the massless states. As we shall see later, the heavy Majorana mass turns out to be $M_I = \mathcal{O}(e^{-c'R^2/\alpha'})M_C$.

In order to ensure $M_U \sim 10^{16}$ GeV and $M_I \sim 10^{12}$ GeV, this in turn involves detailed constraints on coefficients $c$ and $c'$.

In the following, we shall pursue a different approach, \textit{i.e.}, study of smooth Calabi-Yau spaces. For simply connected (2,2) Calabi-Yau manifolds the nature of threshold corrections is different. Such spaces possess the $E_6 \times E_8$ gauge group, and the compactified space corresponds to the smooth Calabi-Yau manifolds with the radius of compactification being large, \textit{i.e.}, in the conformal field theory language this corresponds to the (2,2) string vacuum with large VEV’s for all the moduli. In this case the massive modes of the string theory do not contribute to the threshold corrections\textsuperscript{27}, and thus the gauge couplings do not have corrections proportional to the powers of the moduli VEV’s. Instead, such corrections are milder, only logarithmic in the VEV’s of moduli. The threshold corrections for Calabi-Yau spaces (with only one modulus\textsuperscript{27} as well as for arbitrary number of moduli\textsuperscript{28}) are of the form:

\[
\Delta \left( \frac{16\pi^2}{g^2} \right) \sim -b_G^{N=1} \ln(T + T^*),
\]

where the real part of the $T$ field corresponds to an overall value of large moduli, \textit{i.e.}, $T + T^* = \mathcal{O}(2R^2/\alpha') \gg 1$, where $R$ is the radius of the compactification and
\(\alpha'\) is the string tension. \(b_G^{N=1}\) is related to the one loop \(N = 1\) beta function \(\beta_G\) for \(G = E_6\) or \(E_8\) as \(\beta_G = b_G g^3/16\pi\). Since threshold corrections \((6)\) to the gauge coupling of each of the gauge groups are proportional to the corresponding \(N = 1\) beta function, this implies that the slope of the running gauge couplings is not changed. However the effective gauge coupling unification scale is lowered:

\[
M_U^2 = \frac{M_C^2}{(T + T^*)} = \mathcal{O}\left(\frac{M_C^2}{2R^2/\alpha'}\right) \quad (7)
\]

The above results apply only to simply connected Calabi-Yau spaces. The gauge group \(E_6\) can be broken if Wilson lines are introduced. In this case the contribution of the massless states to the threshold corrections has not been studied, yet. We proceed with the assumption that the nature of the threshold corrections is still of the type \((6)\). From \((7)\) one then sees that for \(R^2/\alpha' \sim \mathcal{O}(20)\) the gauge unification scale is lowered to \(M_U \sim 6 \times 10^{16}\) GeV, which is slightly too large. However, equation \((7)\) relates \(M_U^2\) to \(R^2/\alpha'\) only by orders of magnitude. Thus, an additional factor of 2 in the relation of an overall modulus \(R e T\) to \(R^2/\alpha'\) enables one to obtain \(M_U\) in the preferred range \(4 \times 10^{16}\) GeV.

We turn now to neutrino masses. In particular, we would like to address the size of the nonrenormalizable terms. In string theory the magnitude of the coefficient \(M_{NR}\) is proportional to \(M_C\). However, one can prove explicitly\(^{13}\) that for all \((0, 2)\) string vacua the nonrenormalizable terms are suppressed by an additional factor \(e^{-R^2/\alpha'}\), \textit{i.e.}, the origin of the nonrenormalizable terms is due only to worldsheet instanton effects. This is a general stringy result, proven explicitly on (blown-up) orbifolds\(^{12}\) as well as in sigma model perturbations\(^{13}\) of Calabi-Yau manifolds. Therefore:

\[
\frac{1}{M_{NR}} = \mathcal{O}\left(\frac{e^{-R^2/\alpha'}}{M_C}\right) \quad (8)
\]
By choosing vacuum expectation values along the flat direction to be $M_C$ (the only natural scale in the four-dimensional string vacuum) nonrenormalizable terms of the type (4) yield the heavy Majorana mass:

$$M_I = \frac{M_C^2}{M_{NR}} = \mathcal{O}(e^{-R^2/\alpha'}) M_C.$$  \hspace{1cm} (9)

It follows from (7) that we need $R^2/\alpha' = \mathcal{O}(20)$ in order to achieve $M_U \sim 10^{16}$ GeV. In this case $M_I \sim 10^{-8} M_C \sim 10^{10}$ GeV. Although these are only order of magnitude statements, it is instructive to set the coefficients in (7) - (9) equal to unity. In that case the range $M_I \sim (4 \pm 3) \times 10^{11}$ GeV suggested by the Solar neutrino deficit implies $R^2/\alpha' \sim (13 - 15)$, yielding a slightly too large $M_U \sim 7 \times 10^{16}$ GeV.

To summarize, the desired scale of the gauge coupling unification $M_U \sim 2 \times 10^{16}$ GeV and the scale of Majorana neutrino masses $M_I \sim 4 \times 10^{11}$ GeV, may be achieved for a superstring vacuum, corresponding to a $(0, 2)$ Calabi-Yau obtained by deforming a $(2, 2)$ smooth large radius Calabi-Yau space along the exactly flat directions. The radius of the compactification has to be in the range $R^2/\alpha' = \mathcal{O}(20)$, allowing for $M_U^2 = M_C^2/\mathcal{O}(2R^2/\alpha')$ and $M_I = \mathcal{O}(e^{-R^2/\alpha'}) M_C$. We believe it is intriguing that one can obtain $M_U$ and $M_I$ in the desired range on the basis of fairly general stringy arguments.

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9. The choice of the particle spectrum and the $U(1)_Y'\gamma$ quantum numbers are motivated by $SO(10)$ gauge symmetry. Namely, the family particle
content (including $L_i$’s) corresponds to full 16-plets of $SO(10)$, the two Higgs doublets are parts of a 10-plet, while the Standard Model singlets $S_1$ and $\bar{S}_1$ are parts of a 16-plet and a $\overline{16}$-plet, respectively.

10. A scenario that ensures large VEV’s $<S> = <\bar{S}> = M_U$ is based on an interplay of soft supersymmetry-breaking mass terms of order $M_W$, i.e., terms of the type $-M_W^2(|S_1|^2 + |\bar{S}_1|^2)$ and nonrenormalizable terms of the type $W = (27I_{27}J_{27})^K/M_{NR}^{2K-3}$, with $K \geq 4$. This allows for $M_U = \mathcal{O}(M_W M_{NR}^{2K-3})^{1/2K-2} \geq \mathcal{O}(M_U) \sim 10^{16}$ GeV. In order to prevent the terms in the $(S_1, \bar{S}_1)$ sector with $K < 4$ one has to impose a discrete symmetry; e.g., $Z_4$ symmetry with $L_i$, $S_1$ and $\bar{S}_1$ transforming with the phases $2\pi/4$, $4\pi/4$ and $2\pi/4$, respectively, ensures that the first nonzero term in the $(S_1, \bar{S}_1)$ sector corresponds to $K = 4$ while at the same time allowing for the term $L_i L_i \bar{S}_1 \bar{S}_1 / M_{NR}$. Note, that this scenario for the breakdown of the gauge symmetry at $M_U$ is again motivated from the properties of string vacua.

11. The physical light doublet neutrino of mass $m \sim cm^2/M_I$ is $\nu \cos \theta + N \sin \theta$ where $\tan \theta = M_{U_1}/M_{U_2}$, and $\nu$ and $N$ are both doublets, while the orthogonal combination has mass $\sim M_U$. This does not cause any problems with universality because there is exactly the same mixing between the charged doublet partners.

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24. In literature, there are quoted different values of $M_C$, differing by factors of 2 or $\sqrt{2}$. As explained in erratum of Ref. 23, using the correct numerical form of the original formula $^{23}$, and the tree level relation $g^2 = 32\pi/(\alpha' M_P^2)$, with $g$ defined according to the GUT convention yields the quoted result, which is the same as the original numerical value in Ref. 23. Note, that the gauge coupling $g$ as defined in the effective string Lagrangian, e.g., P. Ginsparg, Phys. Lett. B197, 139 (1987), is by a factor of $\sqrt{2}$ smaller than the gauge coupling $g$ defined according to the GUT convention. Namely, in the effective string Lagrangian the trace over the generators of the vector representation of $SO(2N)$ gauge group is chosen to be $Tr(T^a T^b) = -2\delta^{ab}$, while in the GUT theories the convention is $Tr(T^a T^b) = -\delta^{ab}$, thus rendring the gauge coupling for a factor of $\sqrt{2}$ bigger in the latter case.

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26. L. Ibañez, D. Lüst, and G. Ross, Phys. Lett. B272, 251 (1991); L. Ibañez and D. Lüst, CERN preprint, CERN-TH.6380/92.

27. L. Dixon, unpublished.

28. M. Cvetič, UPR–489–T, September 1991 (unpublished).

29. There is a less appealing possibility in superstring theory, which for completeness we wish to discuss. In this case one would stick to \((2, 2)\) Calabi-Yau spaces with Wilson lines and \(R^2/\alpha' \sim \mathcal{O}(1)\). The threshold corrections are small and thus the gauge coupling unification scale is \(M_C \sim 4 \times 10^{17}\) GeV. Because of the different matter representations, the gauge couplings of the semisimple group (e.g., \(SU(3)^3\)) factors run differently from \(M_C\) to \(M_U \sim 10^{16}\) GeV, where the semi-simple gauge group is broken down to the standard one. Thus, the gauge couplings are not equal at \(M_U\) anymore. In particular, for \(SU(3)^3\) with \(9q (6\bar{q})\) and \(7\ell (4\bar{\ell})\) the difference in gauge couplings is below current experimental uncertainties if \(|M_C/M_U| < 10\).

The known scenario for obtaining large \(M_U\) is through an interplay of soft supersymmetry-breaking mass terms (of order \(M_W\)) and nonrenormalizable terms of the type \(W = (27ji\bar{27}j)^K/M_{NR}^{2K-3}\), which allow for a large VEV of the Standard Model singlets \(\langle \phi \rangle = \mathcal{O}(M_W M_{NR}^{2K-3})^{1/2K-2} \geq \mathcal{O}(M_U) \sim 10^{16}\) GeV with \(1/M_{NR} \sim \mathcal{O}(e^{-R^2/\alpha'})/M_C\) and \(K \geq 4\). In addition, it is very difficult (as explored by B. Greene, K. Kirklin, P. Miron and G. Ross, Nucl. Phys. B278, 667(1986) and B292, 606 (1987)) to decouple all the unwanted particles at the same scale \(M_U\). Barring these problems, in this scenario one has \(1/M_{NR} = \mathcal{O}(e^{-R^2/\alpha'})/M_C\) and \(M_I = (e^{-R^2/\alpha'})M_U^2/M_C\). For \(R^2/\alpha' \sim 5\) and \(M_U \sim 10^{16}\) GeV one obtains \(M_I \sim 10^{12}\) GeV.