The evaluation of fatigue strength based on characteristic length

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Abstract. Different approaches are used to evaluate fatigue behaviors according to the geometric shape and size of defects in material, though it is well known that the fatigue properties of a material are intrinsic, and should be independent of geometric shape and size. By introducing the concept of fatigue characteristic length, the failure of material occurs when equivalent stress amplitude within characteristic length reach to fatigue strength of smooth material. Using this method, fatigue behaviours of material, no matter with or without stress concentration or singularities, can be evaluated in the same way. It also can explain the “size effect” of small defects on fatigue problem. Considering the fatigue failure of material within characteristic length, the evaluation of fatigue strength for problems with stress concentration or singularity has been studied by experiment.

1. Introduction
Fatigue properties of smooth materials can be described by the S-N curve and the corresponding fatigue limit, while the fatigue properties of cracked materials should be described by the crack propagation curve and the threshold. It means that the evaluation method of fatigue problems is dependent on the geometric shape and size of materials. However, fatigue properties should be the intrinsic behaviors of a material, and thereby theoretically should be independent of geometric shape.

Some results indicate that fatigue limit and crack propagation threshold are affected greatly by the defect sizes [1-4]. The experimental relations between fatigue limit and threshold have been proposed [5, 6]. Moreover, Murakami summarized the relations between fatigue limit and defect sizes by tests of drilling various holes in a smooth material [7], and also proposed empirical formula for threshold stress intensity factor range [8]. A simple modified formula for the threshold of small cracks has been proposed [9]. However, even these relations have been established, there still have questions that “Can such a fatigue limit or threshold be regarded as the intrinsic property of materials”.

Based on the concept of fatigue characteristic length, considering the fatigue failure of material within characteristic length, the evaluation of fatigue strength for problems with stress concentration or singularity has been studied here.

2. The failure criterion of multi-axial fatigue
Fatigue is dominated by the equivalent stress amplitude within the characteristic length of the material, not the maximum stress amplitude at a point. By introducing the concept of fatigue characteristic length, the stress within characteristic length can be assumed uniform distribution. By use of the equivalent stress amplitude within characteristic length, any fatigue problems with concentrated or even singular stress distribution can be evaluated in the same way as for smooth materials by considering the characteristic length one by one.
2.1. The fatigue failure criterion
The effective stress amplitude of multiaxial fatigue [10] has been proposed as follows

\[
\sigma_{efu} = \sqrt{I_{1a}^2 - \frac{2(1+\nu)^2}{(1+2\nu)^2} I_{2a}}
\]  

(1)

here, \(\nu\) is the Poisson’s ratio, and \(I_{1a}\) and \(I_{2a}\) are the first and second stress invariants of stress amplitude. The equivalent stress amplitude within the characteristic length at the crack tip, as shown in Figure 1, can be expressed as follows,

\[
\overline{\sigma}_{efu} = \frac{1}{L_f} \int_0^{L_f} \sigma_{e_{fu}} \, dr
\]

(2)

where \(L_f\) is the characteristic length, a material constant. By introducing the characteristic length, the crack propagation for cracked materials occurs when the equivalent stress amplitude within the characteristic length is larger than fatigue limit of smooth materials. Therefore the fatigue failure condition was written as

\[
\overline{\sigma}_{efu} = \frac{1}{L_f} \int_0^{L_f} \sigma_{e_{fu}} \, dr \geq \sigma_f
\]

(3)

here, \(\sigma_f\) is fatigue limit for smooth material, a material constant related to mean stress [10].

![Figure 1. The equivalent stress amplitude within characteristic length at a crack front.](image)

2.2. The fatigue characteristic length
Considering the case of a central crack in the infinite plate subjected to cyclic loading \(\sigma = \sigma_m + \sigma_a \sin \omega t\), in which \(\sigma_m\) and \(\sigma_a\) are mean stress and stress amplitude, respectively. The crack length is 2a. The stress distribution at crack front belongs to plane stress or plane strain state, and can be expressed as \(\sigma_x = \sigma_y = |\sigma(a + r)| / \sqrt{2ar + r^2}\). Using stress distribution at crack tip and equation (1), the equivalent stress amplitude at crack tip can be written as

\[
\sigma_{efu} = \frac{k \sigma_a (a + r)}{\sqrt{2ar + r^2}}
\]

(4)

Here, \(k\) is a constant related to Poisson’s ratio [10]. Using stress intensity range \(\Delta K = 2\sigma_a \sqrt{\pi a}\), the fatigue failure condition within the first characteristic length can be rewritten as
In which $\rho = a / L_f$ is the ratio of half crack length and characteristic length. For a macro crack, the crack propagation condition is

$$\Delta K \geq K_{th}$$  

Here $K_{th}$ is the threshold for a macro crack, a material constant related to mean stress. For a macro crack, when $\rho \to \infty$, the stress intensity range $\Delta K \to K_{th}$. So the characteristic length can be expressed as

$$L_f = \frac{1}{2\pi} \left( \frac{k K_{th}}{\sigma_f} \right)^2$$  

Here, $\sigma_f$ and $K_{th}$ are material constants related to mean stress. Hence, the characteristic length is also a material constant. And equation (7) means that among the three material parameters $\sigma_f, K_{th}, L_f$, only two are independent. However, $K_{th}$ is a material constant only for macro crack problems, $\sigma_f$ is only used for smooth material, while $L_f$ can be applied for any geometric shape and size of a defect, no matter it is a hole or a notch, no matter it is a micro or macro defect.

3. Examinations

By use of equivalent stress amplitude within the fatigue characteristic length, fatigue strength problems with any defect’s geometric shape and size can be evaluated. Here, the fatigue limit of material with small defects and the fatigue crack growth threshold with small cracks have been used to verify the method.

3.1. Fatigue limit of material with small defects

For simplicity, the initial micro defects in a material were modeled as a spherical hole. It is considered that the defects of spherical hole in infinite body are subjected to symmetrical cyclic stress in far field. From the knowledge of elasticity, compared with the maximum principal stress, the stress in the other two directions is small, hence, it can be regarded as a uniaxial fatigue problem, and the stress distribution is

$$\sigma(r) = \sigma_a \left[ 1 + \frac{1}{2(7-5\nu)} \frac{a^3}{r^3} \left( 4 - 5\nu + \frac{9a^2}{r^3} \right) \right]$$  

(8)

where $a$ is the radius of spherical defect. Substituting equation (8) into equation (2), the fatigue limit condition can be represented as follows

$$\overline{\sigma} = \sigma_a \left[ 1 - \frac{4 - 5\nu}{4(7-5\nu)} \frac{1}{(1+\rho)^2} \frac{1}{\rho^2} - \frac{9}{8(7-5\nu)} \frac{1}{(1+\rho)^2} - \frac{9}{8(7-5\nu)} \frac{1}{(1+\rho)^2} \right] = \sigma_{\text{th}}$$  

(9)

where $\rho = a / L_f$ is the ratio of spherical defect radius and characteristic length, and $\sigma_a$ is the nominal stress amplitude. According to equation (9), the fatigue limit curve for various sizes of the spherical hole defect can be obtained as shown in Figure 2.

Some results can be found from the theoretical curve in Figure 2. (1) when the ratio of spherical defect radius and characteristic length $a/L_f$ is smaller than 0.01, the fatigue limit cannot be influenced by the defects, i.e., these defects can be regarded as the initial micro defects in the material. (2) when
the ratio of spherical defect radius and characteristic length $a/L_f$ is larger than 100, then $\bar{\sigma}_a = f\sigma_a$, where $f = 3(9-5\nu)/(2(7-5\nu))$ is the stress concentration coefficient for a macro hole, i.e., the equivalent stress amplitude within characteristic length can be simply determined by stress concentration. (3) For small defects, which are between micro defects and macro defects, the fatigue limit described by the nominal stress amplitude, depends on the defect-size. However, this dependency does not mean the fatigue property is dependent on defect-size, but indicates that the nominal stress is not the proper parameter for fatigue. Using the equivalent stress amplitude within the characteristic length, the fatigue limit can be evaluated in a unified way.

Figure 2. Fatigue limit for various defect sizes and comparisons with experiment of SC45 steel.

Using symmetric rotary bending method, the relation between fatigue limit and defect sizes in SC45 steel was studied by Murakami [7]. The fatigue limit and the crack growth threshold of SC45 steel are 260MPa and 6.5MPa√m, respectively. Considering the plane strain state and equation (7), the fatigue characteristic length is 0.059mm. Comparison with the experimental results [7], the theoretical curve can well evaluate the fatigue limit.

3.2. Fatigue crack growth threshold of material with small cracks
A central crack in the infinite plate subjected to symmetric cyclic loading has been considered here. The fatigue crack growth threshold of material with small cracks can be evaluated by equation (3). The failure condition at crack front in material is that the equivalent stress amplitude within the first characteristic length reach to fatigue limit of smooth material under symmetric cyclic loading. So, the condition of crack growth is

$$\bar{\sigma}_{eff} = k\sigma_a\sqrt{1+2\rho\bar{\sigma}} = \sigma_{-1}$$

(10)

or

$$\bar{\sigma}_{eff} = k\Delta\sigma/2\sqrt{1+2\rho} = k\Delta K/\sqrt{\pi L_f}\sqrt{1+2\rho}/\rho = \sigma_{-1}$$

(11)

where $\Delta\sigma$ is stress range. Equation (10) degenerates into fatigue limit condition of smooth material when the crack is small enough (i.e., $\rho \to 0$). While if the crack is large enough (i.e., $\rho \to \infty$), equation (11) becomes $\Delta K = K_{th}$ by using equation (7), which is the condition for crack propagations. The relationship between stress intensity factor threshold and defect size for small cracks is shown in Figure 3. If the crack length is around one-tenth of characteristic length, then one has the relationship as $\Delta K \propto a^{1/3}$ approximately, which agrees with the Murakami’s experimental results [7].

From above examinations, it can be considered that the fatigue failure criterion is valid.
4. Conclusions

By introducing the concept of fatigue characteristic length, the fatigue failure criterion of material has been proposed here, and the validity of this criterion has been verified by experimental results. Using this method, the fatigue problems, no matter with or without stress concentration or singularities, can be evaluated in the same way.

The “size effect” of small defects or cracks on fatigue problem can be explained, the dependency of fatigue properties on defect’s size is only the presentation due to the improper evaluation parameters. If the equivalent stress amplitude within characteristic length is regarded as the evaluation parameter, the fatigue properties remain constants for different geometric shape and size of defects. The characteristic length is also a material constant.

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