Signature change, vacuum condensation

and

cosmological constant

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Abstract

We propose an alternative model of duality symmetry, based on the previously obtained divergence theory, including an scalar field, an internal vector and a metric signature. At some small scale an effective scalar field equation has appeared whose potential acts like a Higgs one, where the metric signature plays the role of an order parameter. Non-vanishing Vacuum condensation of this Higgs field occurs once a signature change from Euclidean to Lorentzian is formed. The mass scale of Higgs field excitations around this vacuum may contribute, in the Lorentzian sector, to the cosmological constant, in agreement with observations.

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1 Introduction

The initial idea of signature change was due to Hartle, Hawking and Sakharov [1]. This idea would make it possible to have both Euclidean and Lorentzian metrics in the path integral approach to quantum gravity. However, it was later shown that signature change may happen, as well, in classical general relativity [2].

Most of the works regarding the signature change dealt with situations where the signature changing metric is defined \textit{apriori} on the manifold and one looks for the effects of the assumed signature change on the Einstein equations or propagation of particles in such a manifold. However, there are some other viewpoints in which the signature generation of the space-time is studied and considered to be a dynamical phenomenon at quantum gravity regime. This is because a quantum formulation of gravitation should accommodate geometries with degenerate metrics and nontrivial topologies. However, the quantum nature of gravity is quite different from the one of quantum field theory. Therefore, in these viewpoints the question is how best to model such a dynamical effect in the language of quantum field theory.

Percacci has offered a formalism in which one can dissociate the conventional geometrical interrelations between the metric tensor components and a field of co-frames, and work in close analogy with the Higgs model in non-Abelian gauge theories [3]. Classical geometry is then regarded as an interpretation of certain expectation values which minimize an effective action. Greensite, on the other hand, has developed this idea further by assuming that a particular pattern of signature arises dynamically as a result of dynamical phase field which interpolates between signatures [4]. He has argued that, at least for the free scalar field theory interacting with such a dynamical phase field, the Lorentzian signature of a four-dimensional manifold can
be predicted as a ground state expectation value. Odintsov \textit{et al} have obtained the effective potential for the dynamical phase field (dynamical signature) induced by the quantum effects of massive fields on a topologically non-trivial $D$-dimensional background, and shown that the Lorentzian signature is preferred in both $D = 6$ and $D = 4$ [5].

From a different viewpoint, it was shown that the signature change phenomena can be studied in a variant of the divergence theory, from the viewpoint of a time asymmetric law in vacuum which breaks a specific duality symmetry [6]. It was based on the realization that a duality breaking must be connected with the emergence of a preferred arrow of time in vacuum, and this provides an indication that the time asymmetric law may act to produce the condensation of vacuum.

In the present paper, following the above idea, we propose a new duality symmetry different from the one introduced in [6]. We remark that at small distances, an effective Higgs type potential may arise and show that the duality breaking manifests directly as the condensation of vacuum due to a signature change from Euclidean to Lorentzian. This model as an alternative to [6] has no specific advantage, but may deserve to be studied, as well, to show the potential of divergence theory to accommodate two different models of duality symmetry. Both models interrelate the signature change problem to the vacuum condensation of a scalar field, however, in two different ways of duality breaking. Unlike [6], here, we introduce a duality breaking as a \textit{spontaneous symmetry breaking} and add a new element as the cosmological constant originating from field excitations around the vacuum and show that the cosmological constant does not essentially change due to the signature change.
2 The model

It is well known that an exact Lorentz invariant vacuum of the quantum field theory has, by definition, zero energy density. On the other hand, the vacuum in quantum field theory is related to the condensation of scalar fields leading to constant vacuum expectation values for these fields. The appearance of these non-zero values and the resulting mass scales contribute to the energy content of the vacuum to increase its value from zero. The resulting non-vanishing value accounts for a principle violation of Lorentz invariance. Such a violation of Lorentz invariance may have origin in the unification of gravity and quantum physics, and is expected to occur at ultrashort distance regime described by an absolute length scale, $l_0$. It is obvious that the existence of this universal length scale is in sharp contrast with the universal requirement of Lorentz invariance. In fact, in the Minkowski space-time, as a framework for Lorentz invariance, there is no absolute line of demarcation between large and small scales. However, if a positive definite measure of distance is defined, then the Lorentz invariance will be broken down. In so doing, we may follow the Blokhintsev point of view by associating a time-like vector $N_{\mu}$ (the so called internal vector) to the Minkowski space-time [7]. In this way, one may distinguish between small and large scales by taking the positive definite interval

$$R^2 = (2N_{\mu}N_{\nu} - \eta_{\mu\nu})x^\mu x^\nu, \quad (1)$$

where $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ is the Minkowski metric and $N_{\mu} = (1, 0, 0, 0)$ is a time-like vector. Given such a positive definite metric as

$$\bar{\eta}_{\mu\nu} = 2N_{\mu}N_{\nu} - \eta_{\mu\nu}, \quad (2)$$
it is then possible to determine the absolute size of a distance by comparing \( R \) with the universal length \( l_0 \).

One may then define the pair \((l_0, N_\mu)\) whose physical interpretation is as follows \([7, 6]\): \( l_0 \) is a characteristic size in the ultrashort distance regime which acts as a sort of universal length that determines a lower bound on any scale of length probed in a realistic measurement. \( N_\mu \) serves as a four-velocity of a preferred inertial observer in vacuum by which the concept of the universal length \( l_0 \) makes sense. Therefore, the pair \((l_0, N_\mu)\) arises as the physical feature of a Lorentz non-invariant vacuum.

We now introduce a divergence theory developed in Ref.[6]. This theory begins with a current

\[
J_\mu = -\frac{1}{2} \phi \partial_\mu \phi^{-1},
\]

(3)

for which we obtain

\[
\partial_\mu J^\mu = \phi^{-1} \left[ \Box \phi - \phi^{-1} \partial_\mu \phi \partial^\mu \phi \right],
\]

(4)

and

\[
J_\mu J^\mu = \phi^{-2} \partial_\mu \phi \partial^\mu \phi,
\]

(5)

where \( \phi \) is a real scalar field. Combining these relations leads to

\[
\Box \phi + \Gamma\{\phi\} \phi = 0,
\]

(6)

where \( \Gamma\{\phi\} \) is called the \textit{dynamical mass term}

\[
\Gamma\{\phi\} = -J_\mu J^\mu - \partial_\mu J^\mu.
\]

(7)

It is important to note that Eq.(6) is a formal consequence of the definition (3), in the form of an identity, and is not a dynamical equation for \( \phi \). However, a large class of dynamical
theories may be obtained in the form of a divergence theory by taking various currents $J^\mu$ in the dynamical mass term. For example, a simple divergence theory is developed by assuming

$$\partial_\mu J^\mu = 0, \quad (8)$$

which leads, through the field redefinition $\sigma = \ln \phi$, to

$$\Box \sigma = 0. \quad (9)$$

The dynamical mass term vanishes for this divergence theory and the field $\sigma$ becomes massless.

One may allow for a dynamical coupling of $\phi$ with the internal vector $N_\mu$ by taking a more complicate dynamical mass term. This is done by a divergence equation of the type

$$\partial_\mu J^\mu = N_\mu N_\nu J^\mu J^\nu + g\ell_0 N_\mu N_\nu N_\sigma J^\mu J^\nu J^\sigma, \quad (10)$$

which leads to the field equation

$$\Box \phi - (J_\mu J^\mu + N_\mu N_\nu J^\mu J^\nu + g\ell_0 N_\mu N_\nu N_\sigma J^\mu J^\nu J^\sigma)\phi = 0, \quad (11)$$

where $g$ stands for the metric signature which is positive for Euclidean and negative for Lorentzian domains\(^1\). Now, the basic point is that the source of the divergence is invariant under the following dual transformations\(^2\)

$$\phi \rightarrow -\phi, \quad N_\mu \rightarrow -N_\mu, \quad g \rightarrow -g, \quad (12)$$

where the latter transformation accounts for a signature change from Lorentzian to Euclidean

$$\eta_{\mu\nu} \rightarrow \bar{\eta}_{\mu\nu} = 2N_\mu N_\nu - \eta_{\mu\nu}. \quad (12)$$

\(^1\)The excess of plus signs over minus signs is called the signature.

\(^2\)The source (10) and the dual transformations (12) are modifications of those introduced in Ref.[6].
or vice versa. We note that the identity (6) holds no matter what signature is used in its derivation. Therefore, the field equation (11) holds for both Lorentzian and Euclidean signatures. One may then assume a dynamical symmetry between the dual configurations

\[(\phi, N_\mu, g) \Leftrightarrow (-\phi, -N_\mu, -g),\]  

so that the field equation (11) makes no distinction between these dual configurations. In fact, both signatures are related by the dynamical symmetry of the field equation under the dual transformations (12). It is important to note that the emergence of this duality in the field equation (11) is considered as reflecting the essential feature of the broken phase of Lorentz invariance at small distance \(\sim l_0\).

3 Symmetry breaking

In physics there are many models in which there are some symmetries at the dynamical level leading to equivalent vacua in the theory. No distinction between these equivalent vacua is possible unless we require some symmetry breaking conditions to be imposed on the physically realizable configuration of vacuum. Usually, it is interesting to realize a symmetry breaking directly by means of boundary conditions. But other approaches are also possible and deserve to be studied, as well. For example, in [6] the authors try to establish a duality symmetry between two configurations

\[(\phi, N_\mu, \eta_{\mu\nu}) \Leftrightarrow (\phi^{-1}, -N_\mu, \bar{\eta}_{\mu\nu}),\]  

where \(\eta_{\mu\nu}\) and \(\bar{\eta}_{\mu\nu}\) stand for Lorentzian and Euclidean metrics, respectively. They introduce duality breaking by resorting to a time asymmetric law to be imposed on a specific source
of the divergence. The time asymmetric law is then related to the vacuum condensation of the quantum scalar field. The present model, although follows the same purpose to take into account the signature change problem, but takes different source of divergence (10) and different equivalent vacuum configurations (13). Therefore, a different duality breaking condition is required to realize the physical vacuum configurations.

One alternative aspect of duality breaking seems to be a preferred background value $\bar{\phi}$ and correspondingly a preferred background direction of $N_\mu$, as average values taken over large distances, which may be interrelated via a relation of the type [6]

$$\bar{J}_\mu = \frac{\lambda}{l_0} N_\mu,$$

(15)

where $\lambda$ measures the ratio of the universal length and the spatial size of the universe as

$$\lambda = \frac{l_0}{R}.$$

(16)

In this way, the duality breaking and correspondingly the preference of a signature is considered to be a cosmological effect. If, however, one is interested in realizing a preferred signature, it is not unreasonable to expect that such a duality breaking should have its origin at small distance regime rather than large cosmological distances. This is because, the metric is, in principle, defined to produce the most small distance appropriate for a realistic measurement process. Therefore, it is more natural to think about the origin and preference of Lorentzian over Euclidean metric as a small distance effect. In this regard, unlike Ref.[6], we consider Eq.(15) not as a duality breaking but as a trick to effectively linearize the source of divergence equation (10) at ultrashort distance regime. This linearization is necessary in order to compute the effective form of the dynamical mass term in (11) by obtaining an approximate solution.
for $J_{\mu}$. Having this in mind, we can linearize the quadratic term in $J_{\mu}$ in the source to find the approximations

$$\partial_{\mu} J^{\mu} = N_{\mu} N_{\nu} \bar{J}^{\mu} J^{\nu} + g l_0 N_{\mu} N_{\nu} N_{\sigma} \bar{J}^{\mu} J^{\nu} J^{\sigma}, \quad (17)$$

and

$$\Box \phi - (\bar{J}_{\mu} J^{\mu} + N_{\mu} N_{\nu} \bar{J}^{\mu} J^{\nu} + g l_0 N_{\mu} N_{\nu} N_{\sigma} \bar{J}^{\mu} J^{\nu} J^{\sigma}) \phi = 0. \quad (18)$$

We now truncate the nonlinear term in $J_{\mu}$ from the source of the divergence (17) and use Eq.(15) to obtain

$$\partial_{\mu} J^{\mu} \simeq \frac{\lambda}{l_0} N_{\mu} J^{\mu}. \quad (19)$$

Using Eq.(3), we can write the divergence equation (10) in terms of $\phi$ as

$$\partial_{\mu} J^{\mu} \simeq \frac{\lambda}{l_0} N_{\mu} \frac{\partial^{\mu} \phi}{\phi}. \quad (20)$$

If we linearize this equation by inserting the average background value of $\phi$ in the dominator we find an approximate solution for $J^{\mu}$ as

$$J_{\mu} \simeq \frac{\lambda}{l_0} N_{\mu} \frac{\phi}{\bar{\phi}}. \quad (21)$$

By using this solution for $J^{\mu}$ in (11) we arrive at the following field equation

$$\Box \phi - \left(2 \frac{\lambda^2 \phi}{l_0^2 \bar{\phi}} + g \lambda \frac{\lambda^2 \phi^2}{l_0^2 \bar{\phi}^2}\right) \phi = 0. \quad (22)$$

If we replace $\phi$ by $\bar{\phi}$ in the linear term, we can get a more effective form of this equation as

$$\Box \phi - \left(2 \frac{\lambda^2 \phi}{l_0^2 \bar{\phi}} + g \lambda \frac{\lambda^2 \phi^2}{l_0^2 \bar{\phi}^2}\right) \phi = 0. \quad (23)$$

This equation can be derived from the following effective potential

$$V(\phi) = -\frac{\lambda^2 \phi^2}{l_0^2 \bar{\phi}^2} - \frac{1}{4} g \lambda \frac{\lambda^2 \phi^4}{l_0^2 \bar{\phi}^2}. \quad (24)$$
This potential, has one minimum at $\phi_0 = 0$ and two degenerate minima at $\phi_0 = \pm \sqrt{\frac{2}{g\lambda}} \bar{\phi}$ provided $g > 0$ and $g < 0$, respectively. This means, as far as the signature of the metric is Euclidean, no duality breaking happens to distinguish between dual configurations $\phi$ and $-\phi$. When, however, a signature change happens from Euclidean to Lorentzian the scalar field then condensates in one of the two vacua $\phi_0 = \pm \sqrt{\frac{2}{g\lambda}} \bar{\phi}$ and one configuration $(\phi, N_{\mu}, \eta_{\mu\nu})$ or $(-\phi, -N_{\mu}, \bar{\eta}_{\mu\nu})$ is singled out, permanently\(^3\). Therefore, the condensation of $\phi$ corresponds to a signature change from Euclidean to Lorentzian.

The ground state value $\phi_0$ of $\phi$ is obtained by minimizing $V(\phi)$ which leads to the condition

$$\phi_0^2 = \frac{2}{-g\lambda} \bar{\phi}^2.$$  \hspace{1cm} (25)

If the potential $V(\phi)$ is expanded around $\phi_0$ we obtain (neglecting constant terms)

$$V(\phi) = 4 \left( \frac{\lambda}{l_0} \right)^2 (\phi - \phi_0)^2 + O(\phi - \phi_0)^3,$$  \hspace{1cm} (26)

from which one infers that physical excitations of $\phi$ around $\phi_0$ have a preferred mass scale $m \sim \frac{1}{l_0}$. According to the quantum field theory considerations, any massive particle excitation around a given scalar field vacuum can contribute to the total value of the cosmological constant. But this leads to the well-known cosmological constant problem. A solution of this problem is to reduce the issue of the cosmological constant to a picture in which the consistent contribution to the total value of that constant comes from a preferred mass scale of the vacuum. In this regard, the preferred mass scale $m \sim \frac{1}{l_0}$ may contribute to the cosmological constant, in the Lorentzian sector, as $\Lambda \sim \left( \frac{\lambda}{l_0} \right)^2 \sim \frac{1}{R^2}$, where use has been made of Eq.(16).

\(^3\)This is what we meant by the duality breaking at small distance regime. In fact, the smallness of $\lambda$ leads the two degenerate vacua to be too far apart from each other, so the possibility of tunneling from one vacuum to the other one almost vanishes.
This is in agreement with the present observational bound for the cosmological constant as a remarkable consequence of the well-known empirical fact that the present universe has just the characteristic size \( R \sim 10^{29}\text{cm} \). Note that according to Eq.(16) this agreement with the observational bound for \( \Lambda \) is obtained merely by the ratio \( \lambda/l_0 \), and is independent of the specific numerical values of \( \lambda \) or \( l_0 \). The understanding of the relation of \( l_0 \) to the Planck length is an elusive task of quantum gravity. However, one must take a physically reasonable estimation for the universal length. For example, \( l_0 \) may act as the length scale that measures the size of the regime on which a significant nonlinear self coupling like the Higgs potential \( V(\phi) \) can occur. This may constrain \( l_0 \) to be smaller than electroweak or even supersymmetry scale.

It is worth noting that the vacuum energy as the minimum of potential vanishes for the Euclidean metric, but the field excitations around this minimum have almost the same mass scale as that of Lorentzian sector. Therefore, they may contribute to the cosmological constant in the Euclidean sector in the same manner as in the Lorentzian sector. In other words, the cosmological constant is almost invariant under the dual transformations (13).

### 4 Conclusion

In this paper, we have studied a model of scalar field coupled to an internal vector, in the presence of a universal length, having a special duality symmetry in which the signature change appears as a natural symmetry. When this dynamical symmetry is combined with a duality breaking, a fixed background metric signature is singled out. Contrary to similar models already introduced [6] in which the duality breaking arises due to large cosmological scale, we have argued that this duality breaking may be established, as well, at small distance
regime by spontaneous symmetry breaking in an effective potential which has been derived in
the study of a principle of duality invariance of the dynamical mass term of $\phi$ at a universal
length in the small distance regime. It is shown that the duality breaking as the condensation
of the scalar field occurs once a signature change from Euclidean to Lorentzian is happened
at small distance regime. This signature change leads to the emergence of a preferred mass
scale $\sim \frac{1}{R^2}$ of the vacuum which contributes to the cosmological constant, in the Lorentzian
sector. The same mass scale is also appeared in the Euclidean sector which leads to the same
cosmological constant.

In the future, we hope to study and report more on the possible deep relation between
the signature change and spontaneous symmetry breaking scenarios, since both of them are
seriously assumed to be happened at early universe.

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