Critical Density and Impact of $\Delta(1232)$ Resonance Formation in Neutron Stars

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The critical densities and impact of forming $\Delta(1232)$ resonances in neutron stars are investigated within an extended nonlinear relativistic mean-field (RMF) model. The critical densities for the formation of four different charge states of $\Delta(1232)$ are found to depend differently on the separate kinetic and potential parts of nuclear symmetry energy, the first example of a microphysical property of neutron stars to do so. Moreover, they are sensitive to the in-medium Delta mass $m_\Delta$ and the completely unknown $\Delta$-$\rho$ coupling strength $g_{\Delta\rho}$. In the universal baryon-meson coupling scheme where the respective $\Delta$-meson and nucleon-meson coupling constants are assumed to be the same, the critical density for the first $\Delta^{-}(1232)$ to appear is found to be $\rho_{\Delta crit} = (2.08 \pm 0.02)\rho_0$ using RMF model parameters consistent with current constraints on all seven macroscopic parameters usually used to characterize the equation of state (EoS) of isospin-asymmetric nuclear matter (ANM) at saturation density $\rho_0$. Moreover, the composition and the mass-radius relation of neutron stars are found to depend significantly on the values of the $g_{\Delta\Delta}$ and $m_\Delta$.

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I. INTRODUCTION

Understanding properties of $\Delta(1232)$ resonances in connection with possible pion condensation [1–3] in neutron stars and density isomers in dense nuclear matter [4–8] is a longstanding challenge of nuclear many-body physics. In fact, the role of $\Delta(1232)$ resonances in neutron stars has long been regarded as an important, and unresolved, issue [4]. Significant works have been carried out to understand in-medium properties of $\Delta(1232)$ resonances as well as their effects on saturation properties of nuclear matter and the equation of state (EoS) of dense matter using various many-body theories and interactions: see, e.g., refs. [9–13]. However, compared to the numerous investigations on the possible appearance and effects of other particles, such as hyperons and deconfined quarks, much less effort has been devoted to the study of $\Delta(1232)$ resonances in neutron stars in recent years. This is probably partially because of the rather high $\Delta(1232)$ formation density $\rho_{\Delta crit}$ in the core of neutron stars predicted in the seminal work by Glendenning et al. [10–13] using a mean-field model with parameters well constrained by the experimental data available at the time. Using default parameters of their model Lagrangian leading to a symmetry energy of $E_{sym}(\rho_0) = 36.8$ MeV and its density slope $L(\rho_0) = 3\rho_0dE_{sym}(\rho)/d\rho|_{\rho=\rho_0} \gtrsim 90$ MeV at saturation density $\rho_0$ [10–13], and using the universal baryon-meson coupling scheme in which the nucleon-meson couplings are set equal to the $\Delta(1232)$ -meson couplings ($g_{\Delta\pi}/g_{\pi N} = g_{\rho\pi}/g_{\pi N} = g_{\rho\Delta}/g_{\pi N} = 1$), the critical density $\rho_{\Delta crit}$ above which the first $\Delta^{-}(1232)$ appears is above $9\rho_0$. This led to the conclusion that $\Delta(1232)$ resonances played little role in the structure and composition of neutron stars. In the same studies, the extreme importance of symmetry energy for the formation of both hyperons and $\Delta(1232)$ resonances was emphasized. Particularly, turning off the $\Delta$-$\rho$ coupling which contributes the potential part of the symmetry energy (retaining thus kinetic symmetry energy only), the $\rho_{\Delta crit}$ is about $3\rho_0$ [10]

Interest has been renewed with recent studies using different symmetry energies and/or assumptions about the baryon-meson coupling constants which have found that the $\rho_{\Delta crit}$ can be as low as $\rho_0$ and the inclusion of the $\Delta(1232)$ has significant effects on both the composition and structure of neutron stars [19–24]. These studies generally use some individual sets of model parameters leading to macroscopic properties of ANM at saturation density consistent with most if not all of the existing experimental constraints.

During the last three decades, much progress has been made in constraining the EoS of dense neutron-rich nuclear matter. In particular, reasonably tight constraints on the density dependence of the nuclear symmetry energy $E_{sym}(\rho)$ especially around the saturation density have been obtained in recent years, see, e.g., refs. [22–24] for comprehensive reviews. For example, the 2013 global averages of the magnitude and slope of the $E_{sym}(\rho)$ at $\rho_0$ are respectively $E_{sym}(\rho_0) = 31.6 \pm 2.7$ MeV and $L = 58.9 \pm 16.5$ MeV based on 28 analyses of various terrestrial laboratory experiments and astrophysical observations [22]. Moreover, $\Delta(1232)$ resonances play a very important role in heavy ion collisions, see e.g., ref. [23] for reviews, especially for the production of particles such as pions, kaons and various exotic heavy mesons. In particular, the masses of $\Delta(1232)$ resonances primarily created in nucleon-nucleon (NN) collisions through the $NN \rightarrow N\Delta$ process act as an energy reservoir for sub-threshold particle production. The release of this energy in subsequent collisions involving $\Delta(1232)$ resonances may help create new particles that can not be produced otherwise in the direct, first-chance NN colli-
sions. Thus, particle production has been widely used in probing in-medium properties of $\Delta(1232)$ resonances. Since $\Delta(1232)$ resonances and nucleons have an isospin 3/2 and 1/2 respectively, the total isospin is 1 or 2 for the $NN$ while it is 1 or 0 for the NN state. Because of the isospin conservation, the $\Delta(1232)$ production can only happen in the total isospin 1 NN channel. Therefore, the abundances and properties of $\Delta(1232)$ resonances are sensitive to the isospin asymmetry of the system as neutron-neutron pairs always have an isospin 1 while neutron-proton pairs can have an isospin 1 or 0. Naturally, both heavy-ion collisions and neutron stars are places where the isovector properties and interactions of $\Delta(1232)$ resonances are expected to play a significant role. Indeed, useful information about the symmetry energy of dense neutron-rich matter has been extracted from studying pion and kaon productions in heavy-ion collisions \[28, 29\]. It is especially worth noting that the isovector (symmetry) potential of $\Delta(1232)$ resonances was recently found to affect appreciably the ratio of charged pions in transport model simulations of heavy-ion collisions at intermediate energies \[26\]. However, to our best knowledge, no quantitative information about the isovector interaction of $\Delta(1232)$ resonances has been extracted yet from any terrestrial experiments. It is thus exciting that new proposals to experimentally study at FAIR/GSI the $\Delta(1232)$ resonance spectroscopy and interactions in neutron-rich matter are being considered by the NUSTAR Collaboration \[37\]. On the other hand, there are strong indications from both theoretical calculations and phenomenological model analyses of electron-nucleus, photoabsorption and pion-nucleus scattering that the $\Delta(1232)$ isoscalar potential $V_\Delta$ (real part of its isoscalar self-energy $\Sigma_\Delta$) is in the range of $-30$ MeV $+ V_N \leq V_\Delta \leq V_N$ with respect to the nucleon isoscalar potential $V_N$ \[24\].

Similar to the appearance of any other new hadron above its production threshold in neutron stars, the addition of $\Delta(1232)$ resonances will soften the EoS and influence the composition of neutron stars \[10, 24\]. Because of charge neutrality, depending on the individual populations of the four different charge states of $\Delta(1232)$ resonances, the density dependence of the proton fraction in neutron stars may be modified. Then different cooling mechanisms sensitive to the proton fraction may come into play above certain densities. Moreover, the formation of $\Delta(1232)$ resonances may also push up critical densities for the appearance of various hyperons \[22\]. As noticed earlier in the literature and emphasized in ref. \[3\], there are many interesting questions regarding properties of $\Delta(1232)$ resonances in dense matter and their impact on observables of neutron stars. Obviously, answers to all of these questions naturally rely on the critical density of $\Delta(1232)$ formation in dense neutron star matter.

In this work, we first identify analytically key microphysics quantities determining the critical formation densities of the four charge states of $\Delta(1232)$ resonances. Then, within a nonlinear RMF model we calculate consistently the $\rho_\Delta^{\text{crit}}$ as a function of the $\Delta(1232)$ mass $m_\Delta$, the isovector $\DeltaN$ coupling strength $g_{\DeltaN}$ and seven macroscopic variables characterizing the EoS of ANM at $\rho_0$ all within their latest constraints. Finally, impacts of the $\Delta(1232)$ formation on the composition and mass-radius correlation of neutron stars are studied.

**II. KEY MICROPHYSICS DETERMINING THE DELTA FORMATION DENSITY IN NEUTRON STARS**

![Figure 1](image_url)

Figure 1: The maximum mass of Delta resonances produced in the $NN \rightarrow N\Delta$ process in a free Fermi gas of nucleons at density $\rho$.

To set a reference for our following studies we first estimate the $\rho_\Delta^{\text{crit}}$ in a free Fermi gas of nucleons. For the head-on collision of two nucleons both with Fermi momentum $|k| = k_F = (3\pi^2\rho/2)^{1/3}$ in the $NN \rightarrow N\Delta$ process, the maximum mass of the produced $\Delta(1232)$ resonance is $m_\Delta^{\text{max}} = 2(k_F^2 + m_n^2)^{1/2} - m_N$ where $m_N$ is the average nucleon mass in free-space. It is well known that $\Delta(1232)$ has a Breit-Wigner mass distribution around the centroid $m_\Delta = 1232$ MeV with a width of about 120 MeV. The distribution starts at a minimum of $m_\Delta^{\text{min}} = m_N + m_\pi \approx 1076$ MeV where $m_\pi$ is the pion mass. Shown in Fig. 1 is the $m_\Delta^{\text{max}}$ reachable in the $NN \rightarrow N\Delta$ process as a function of density. From this simple estimate, where effects of the nuclear potentials are neglected, the critical density $\rho_\Delta^{\text{crit}}$ for producing the lightest $\Delta(1232)$ resonance is about $3\rho_0$. To reach the centroid $m_\Delta^{\text{0}} = 1232$ MeV, the density has to be far above $1$ fm$^{-3}$. This estimate also illustrates the importance of considering the mass dependence of the $\Delta(1232)$ formation density in more realistic calculations for matter in neutron stars.

In interacting nuclear systems, the masses of $\Delta(1232)$ resonances and their critical formation densities depend on the in-medium self-energies of all particles involved. Assuming neutron stars are made of neutrons, protons, $\Delta(1232)$ resonances, electrons and muons, i.e., the $npe\mu\Delta$ matter in chemical and $\beta$ equilibrium, the
Their chemical equilibrium requires then besides a Dirac effective mass of a muon and mean-field models, a baryon of bare mass \(\Delta(1232)\) resonances. Generally speaking, in relativistic theory, the Eqs. (10)-(13) can be used to calculate the respective. In addition, the total charge neutrality in neutron stars charge states of \(\Delta\) resonances will take place between nucleons and the four reactions will take place. The latter requires

\[
\mu_n - \mu_p = \mu_\mu = \sqrt{m_\mu^2 + (3\pi^2 \rho_{\mu} \rho_e)^{2/3}}
\]

besides \(\mu_n - \mu_p = \mu_e\), where \(m_\mu = 105.7\) MeV is the mass of a muon and \(x_\mu \equiv \rho_\mu / \rho\) is the muon fraction. On the other hand, the following four types of inelastic reactions will take place between nucleons and the four charge states of Delta resonances

\[
\Delta^{++} + n \leftrightarrow p + p, \quad (6)
\]
\[
\Delta^+ + n \leftrightarrow n + p, \quad (7)
\]
\[
\Delta^0 + p \leftrightarrow p + n, \quad (8)
\]
\[
\Delta^- + p \leftrightarrow n + n. \quad (9)
\]

Their chemical equilibrium requires then

\[
\mu_{\Delta^{++}} = 2\mu_p - \mu_n, \quad (10)
\]
\[
\mu_{\Delta^+} = \mu_p, \quad (11)
\]
\[
\mu_{\Delta^0} = \mu_n, \quad (12)
\]
\[
\mu_{\Delta^-} = 2\mu_n - \mu_p. \quad (13)
\]

In addition, the total charge neutrality in neutron stars requires that \(x_p + x_{\Delta^+} + 2x_{\Delta^{++}} = x_e + x_\mu + x_{\Delta^-}\) where \(x_\Delta \equiv \rho_\Delta / \rho, x_{\Delta^+} \equiv \rho_{\Delta^+} / \rho, \) and \(x_{\Delta^{++}} \equiv \rho_{\Delta^{++}} / \rho,\) respectively.

Within the framework of a given nuclear many-body theory, the Eqs. (10)-(13) can be used to calculate the critical formation densities for the four charge states of \(\Delta(1232)\) resonances. Generally speaking, in relativistic mean-field models, a baryon of bare mass \(m_{\text{baryon}}\) obtains a Dirac effective mass \(m_{\text{dirac}}(\text{baryon}) = m_{\text{baryon}} + \Sigma_S\) and a chemical potential \(\mu_{\text{baryon}} = [k_T^2 + m_{\text{dirac}}(\text{baryon})]^2 / 2 + \Sigma_S\) where \(\Sigma_S\) and \(\Sigma_V\) are the real parts of its scalar and vector self-energies, respectively. Consider the \(\Delta^-\) formation, for example, noticing that \(\mu_n - \mu_p \simeq 4E_{\text{sym}}(\rho)\delta\) and using non-relativistic kinematics, the Eq. (13) leads to the following condition for producing a \(\Delta^-\) of bare mass \(m_{\Delta^-}\) at rest

\[
\frac{(k_T\beta)^2}{2m_{\text{dirac}}} = \frac{[3\pi^2(1 + \delta)^{2/3}/2]}{2m_{\text{dirac}}}
\]

\[
= m_{\Delta^-} - (m_N - 4E_{\text{sym}}(\rho)\delta + \Sigma_{\Delta^-} - \Sigma^S_{\Delta^-} + \Sigma^V_{\Delta^-} - \Sigma^S_{\Delta^-})
\]

where \(m_{\text{dirac}}\) is the nucleon Dirac effective mass and \(\delta = (\rho_\mu - \rho_p) / \rho\) is the isospin asymmetry of nucleons before \(\Delta(1232)\) resonances are produced. Given the density dependencies of the symmetry energy and self-energies, this equation determines the \(\rho_{\Delta^-}\) in neutron stars at \(\beta\) equilibrium. It also shows clearly what microphysics quantities determine the \(\rho_{\Delta^-}\). In particular, the difference in Delta and nucleon masses \(m_{\Delta^-} - m_N\), the symmetry energy \(E_{\text{sym}}(\rho)\), the difference in both scalar \(\Sigma^S_{\Delta^-}\) and vector \(\Sigma^V_{\Delta^-}\) self-energies are all characteristics of baryon interactions. It also indicates where the model dependence and uncertainties are. As we mentioned earlier, experimental data indicates that the difference in nucleon and \(\Delta(1232)\) isoscalar self-energies can be up to about 30 MeV while there is simply no experimental indication so far about the difference in their isovector self-energies. We notice that in many studies in the literature the \(\Delta(1232)\) resonances and nucleons are assumed to have the same scalar and vector self-energies. In this case then, the \(\rho_{\Delta^-}\) is completely determined by the \(\Delta(1232)\) mass \(m_{\Delta^-}\) and the nuclear symmetry energy as a function of density \(E_{\text{sym}}(\rho)\).

The nonlinear RMF model has been very successful in describing many nuclear properties and phenomena during the last few decades, see e.g., refs. [26, 56]. The total Lagrangian density of the nonlinear RMF model of Ref. [51] augmented by the Yukawa couplings of the Delta fields to various isoscalar and isovector meson fields, can be written as [26, 28, 16, 19, 22, 24]

\[
\mathcal{L} = \overline{\psi}_N [(\gamma_\mu (i\partial^\mu - g_{\omega N} \omega^\mu - g_{\phi N} \phi_N^\mu \cdot \tau^\mu)] - (m_N - g_{\sigma N} \sigma)] \psi_N
\]
\[
- \overline{\psi}_\Delta [(\gamma_\mu (i\partial^\mu - g_{\omega \Delta} \omega^\mu - g_{\phi \Delta} \phi_\Delta^\mu \cdot \tau^\mu)] - (m_{\Delta} - g_{\sigma \Delta} \sigma)] \psi_\Delta
\]
\[
+ \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - U_N(\sigma)
\]
\[
+ \frac{1}{2} m_\omega^2 \omega^\mu \omega^\mu - \frac{1}{4} \omega^\mu \omega^\mu + \frac{1}{4} \omega_{\phi N}(g_{\omega N}^2 \omega^\mu \omega^\mu)^2
\]
\[
+ \frac{1}{2} m_\phi^2 \phi_\mu \cdot \phi_\mu - \frac{1}{4} \phi_\mu \phi_\mu - \phi_{\phi N} \phi_{\phi N} \phi_\mu \cdot \phi_\mu
\]
\[
+ \frac{1}{2} (g_{\phi N}^2 \phi_\mu \cdot \phi_\mu) \Lambda V g_{\phi N}^2 \omega^\mu \omega^\mu,
\]

where \(\omega^\mu\) are strength tensors for \(\omega\) and \(\rho\) meson fields, respectively. \(\psi_N, \psi_\Delta, \sigma, \omega_\mu, \phi_\mu\) are the nucleon Dirac field, Schwinger-Rarita field for \(\Delta\) resonances, isoscalar-scalar meson field, isoscalar-vector meson field and isovector meson field, respectively, and the arrows denote.
isovectors, \( U_N(\sigma) = b_{2N} m_N (g_{2N} \sigma)^3 / 3 + c_{2N} (g_{2N} \sigma)^4 / 4 \) is the self interaction term of the \( \sigma \) field. The parameter \( \Lambda_V \) represents the coupling constant of mixed interaction between the isovector \( \rho \) and isoscalar \( \omega \) mesons. It is known to be important for calculating the density dependence of the symmetry energy \( \left[ 31 \right] \). In terms of the expectation values of the meson fields, \( \bar{\sigma}, \bar{\omega} \) and \( \bar{\rho} \) where the subscript “0” denotes the zeroth component of the four-vector while the superscript “(3)” denotes the third component of isospin, the nucleon and \( \Delta(1232) \) isoscalar self-energies are respectively

\[
\Sigma^N_S = -g_{\sigma N} \bar{\sigma} \quad (16) \\
\text{and } \Sigma^\Delta_S = -g_{\sigma \Delta} \bar{\sigma} \quad (17)
\]

Their isovector self-energies are respectively

\[
\Sigma^N_V = g_{\omega N} \bar{\omega} + \tau_3^N \bar{\rho}_{\rho N} / \rho_0^{(3)} \quad (18) \\
\text{and } \Sigma^\Delta_V = g_{\Delta \omega} \bar{\omega} + \tau_3^\Delta \bar{\rho}_{\rho N} / \rho_0^{(3)} \quad (19)
\]

with \( \tau_3^N = +1, \tau_3^\Delta = -1 \) and \( i = \Delta^{++}, \Delta^+, \Delta^0, \Delta^- \), \( \tau_3^{\Delta^{++}} = +3, \tau_3^{\Delta^+} = +1, \tau_3^{\Delta^0} = -1, \tau_3^{\Delta^-} = -3 \).

In terms of the ratios of Delta-meson over nucleon-meson coupling constants \( x_\sigma = g_{\sigma \Delta} / g_{\sigma N}, x_\omega = g_{\omega \Delta} / g_{\omega N} \) and \( x_\rho = g_{\rho \Delta} / g_{\rho N} \), the Eqs. \( \left[ 10 \right] - \left[ 13 \right] \) lead to the following conditions determining the critical densities for forming the four charge states of \( \Delta(1232) \) resonances

\[
\rho_{\Delta^{-}}^{\text{crit}} : \frac{(k_0^p)^2}{2m_{\text{dirac}}} \approx \frac{\Phi_\Delta + g_{\omega N}(1-x_\omega)\bar{\sigma} - g_{\omega N}(1-x_\omega)\bar{\omega}_0 - 6(1-x_\rho)E_{\text{sym}}^\text{pot}(\rho) - 4E_{\text{sym}}^\text{kin}(\rho)\delta}{2m_{\text{dirac}}} \quad (20) \\
\rho_{\Delta^{0}}^{\text{crit}} : \frac{(k_0^p)^2}{2m_{\text{dirac}}} \approx \frac{\Phi_\Delta + g_{\omega N}(1-x_\omega)\bar{\sigma} - g_{\omega N}(1-x_\omega)\bar{\omega}_0 - 2(1-x_\rho)E_{\text{sym}}^\text{pot}(\rho)\delta}{2m_{\text{dirac}}} \quad (21) \\
\rho_{\Delta^{+}}^{\text{crit}} : \frac{(k_0^p)^2}{2m_{\text{dirac}}} \approx \frac{\Phi_\Delta + g_{\omega N}(1-x_\omega)\bar{\sigma} - g_{\omega N}(1-x_\omega)\bar{\omega}_0 + 2(1-x_\rho)E_{\text{sym}}^\text{pot}(\rho)\delta}{2m_{\text{dirac}}} \quad (22) \\
\rho_{\Delta^{++}}^{\text{crit}} : \frac{(k_0^p)^2}{2m_{\text{dirac}}} \approx \frac{\Phi_\Delta + g_{\omega N}(1-x_\omega)\bar{\sigma} - g_{\omega N}(1-x_\omega)\bar{\omega}_0 + 6(1-x_\rho)E_{\text{sym}}^\text{pot}(\rho) + 4E_{\text{sym}}^\text{kin}(\rho)}{2m_{\text{dirac}}} \quad (23)
\]

where \( \Phi_\Delta \equiv m_\Delta - m_N \) is the Delta-nucleon mass difference, \( E_{\text{sym}}^\text{pot}(\rho) = 2^{-1} \rho_0^2 (m_\rho^2 + \Lambda_V g_{\rho N}^2 g_{N N}^2 \bar{\omega}_0^2)^{-1} \) and \( E_{\text{sym}}^\text{kin}(\rho) = 6^{-1} k_0^p (k_0^p)^2 + m_{\text{dirac}}^2)^{-1/2} \) are respectively the potential and kinetic part of the symmetry energy in the nonlinear RMF model \( \left[ 29 \right] \).

Several interesting conclusions can be made qualitatively from inspecting the above four conditions. Generally, the critical densities depend differently on the three coupling ratios \( x_\sigma, x_\omega \) and \( x_\rho \) as they have different natures. The isoscalar coupling ratios \( x_\sigma \) and \( x_\omega \) affect the four \( \Delta(1232) \) resonances the same way, i.e., the \( x_\rho \) lowers while the \( x_\omega \) raises their critical formation densities, while the isovector coupling ratio \( x_\rho \) acts differently on the four different charge states of \( \Delta(1232) \) resonances. Moreover, the kinetic and potential parts of the symmetry energy have separate and different effects. In particular, in the universal baryon-meson coupling scheme, i.e., \( x_\sigma = x_\omega = x_\rho = 1 \), the critical densities for creating \( \Delta^- \) and \( \Delta^{++} \) depend only on the \( \Phi_\Delta \) and the kinetic symmetry energy \( E_{\text{sym}}^\text{kin}(\rho) \) besides the \( m_{\text{dirac}}^* \), while those for the \( \Delta^0 \) and \( \Delta^+ \) are determined only by the \( \Phi_\Delta \) and \( m_{\text{dirac}}^* \). In this case, assuming the \( E_{\text{sym}}^\text{kin}(\rho) \) is always positive as in the case of RMF, noticing that the Fermi surface of protons is lower than that of neutrons at any density in neutron-rich matter, i.e., \( k_0^p < k_0^\omega \), one then sees immediately the following sequence of appearance \( \rho_{\Delta^-}^{\text{crit}} < \rho_{\Delta^0}^{\text{crit}} < \rho_{\Delta^+}^{\text{crit}} < \rho_{\Delta^{++}}^{\text{crit}} \) \( \left[ 10 \right] - \left[ 24 \right] \). However, we notice that the short-range nucleon-nucleon correlation (SRC) \( \left[ 52 \right] - \left[ 54 \right] \) may lead to negative kinetic symmetry energies even at normal density of nuclear matter, see e.g. refs. \( \left[ 64 \right] - \left[ 66 \right] \). In this case, the order of appearance of \( \Delta^- \) and \( \Delta^{++} \), thus the fraction of various particles and the structure of neutron stars may be different. We remark here that this is the first time that some physics quantities in neutron stars are found to depend separately on the kinetic and potential parts instead of the total symmetry energy.

Some earlier studies, see, e.g., refs. \( \left[ 6 \right] - \left[ 8 \right] - \left[ 22 \right] - \left[ 24 \right] \), indicate that \( x_\sigma \approx x_\omega \approx 1.0 \). Effects of slight deviations from this value on properties of neutron stars have also been reported, see, e.g., ref. \( \left[ 21 \right] \). To our best knowledge, however, little is known about the range of \( x_\rho \) and its effects in either heavy-ion collisions \( \left[ 60 \right] \) or neutron stars. Moreover, most studies so far are limited to density isomers due to \( \Delta(1232) \) formation in symmetric nuclear matter where it is sufficient to consider only effects of the \( x_\sigma \) and \( x_\omega \).

In the universal baryon-meson coupling scheme, the default set (SH-NJ) of RMF model parameters leads to the following values of macroscopic quantities characterizing the EoS of ANM at \( \rho_0 \) = 0.149 fm\(^{-3}\): the
binding energy $E_0(\rho_0) = -16.00\text{ MeV}$, the Dirac effective mass $m_0^{\text{dirac}}(\rho_0)/m_N = 0.64$, the incompressibility $K_0(\rho_0) = 290\text{ MeV}$, the skewness coefficient $J_0(\rho_0) = -415\text{ MeV}$, the magnitude $E_{\text{sym}}(\rho_0) = 31.17\text{ MeV}$ and slope $L(\rho_0) = 48.64\text{ MeV}$ of symmetry energy. We remark that almost all of these bulk parameters are extracted from a fit to the properties of finite nuclei \[56\] with the exception of the skewness coefficient $J_0$ that has been obtained by fixing the maximum mass of a neutron star at $M_{\text{max}} = 2.01M_\odot$ \[67\]. Variations around this parameterization will be investigated in the following.

For Delta resonances of mass $m_\Delta = 1232\text{ MeV}$, we study in Fig. 2 the $\rho_\Delta^{\text{crit}}$ dependence on the $x_\rho$ while keeping all other quantities at their default values. It is seen that the $\rho_\Delta^{\text{crit}}$ increases approximately linearly with $x_\rho$. At $x_\rho = 1$, the $\rho_\Delta^{\text{crit}}$ is only about $2.1\rho_0$ consistent with that found in refs. \[21\]-\[23\] but much smaller than the one found by Glendenning et al. \[16\]-\[18\]. However, unless the value of $x_\rho$ is somehow constrained, the $\rho_\Delta^{\text{crit}}$ will remain underdetermined.

Delta resonances in free-space have the Breit-Wigner mass distribution

$$f(m_\Delta) = \frac{1}{4} \frac{\Gamma^2(m_\Delta)}{(m_\Delta - m_\Delta^0)^2 + \Gamma^2(m_\Delta)/4}$$

(24)

with the mass-dependent width given by \[68\]-\[69\]

$$\Gamma(m_\Delta) = 0.47q^3/(m_\pi^2 + 0.6q^2) \text{ (GeV)}$$

(25)

where $q = [(m_\Delta^2 - m_\pi^2 + m_\rho^2)/2m_\Delta^2 - m_\pi^2]^{1/2}$ is the pion momentum in the $\Delta$ rest frame in the $\Delta \rightarrow \pi + N$ decay process. Shown in red in Fig. 3 is the free-space $\Delta(1232)$ mass distribution. It is known that the $\Delta(1232)$ mass distributions may be modified in nuclear medium \[53\]. This effect is beyond the scope of the RMF model used here as it does not consider the imaginary part of the $\Delta(1232)$ self-energy self-consistently. However, we can examine how the $\rho_\Delta^{\text{crit}}$ depends on the bare $\Delta(1232)$ mass $m_\Delta$ by varying its value in the Eqs. \[20\]-\[23\]. As one expects and indicated by the Eqs. \[20\]-\[23\], the $\rho_\Delta^{\text{crit}}$ increases with $m_\Delta$ in the universal coupling scheme. Our numerical calculations shown with the blue line indicate that the increase is almost linear. Considering the mass distribution, while the $\rho_\Delta^{\text{crit}}$ for $\Delta$ resonances around $m_\Delta^0$ is about $2.1\rho_0$, it gradually decreases for lower $\Delta$ masses. Of course, these low-mass $\Delta$ resonances are less likely to be produced compared to the ones near $m_\Delta^0$. On the other hand, the $\Delta$ mean lifetime $\tau_\Delta = \hbar/\Gamma(m_\Delta)$ is only about $1.7 ~\text{fm}/c$ at $m_\Delta^0$ but increases very quickly for lower masses. Thus, the main population of $\Delta$ resonances in neutron stars may not necessarily peak at $m_\Delta = 1232\text{ MeV}$. A detailed study of this issue will require a full account of the $\pi - N - \Delta$ dynamics in neutron stars that is also beyond the scope of the RMF model used here. Nevertheless, our results indicate that the appearance of $\Delta$ resonances, especially the ones with low masses around $2\rho_0$, may compete with other particles, such as hyperons, thus possibly modify the widely accepted and longtime viewpoint that hyperons should appear earlier than $\Delta(1232)$ resonances in neutron stars \[10\],\[70\]. To our best knowledge, however, no study to date has considered consistently effects of the mass distribution and the associated mass-dependent lifetimes of $\Delta$ resonances in neutron stars.

### III. EFFECTS OF NUCLEAR EQUATION OF STATE ON THE FORMATION OF $\Delta(1232)$ RESONANCES IN NEUTRON STARS

The equation \[13\] for determining the critical density $\rho_\Delta^{\text{crit}}$ can be rewritten as

$$\mu_\Delta^{\text{min}} = m_\Delta + \SigmaDelta = 2\mu_n - \mu_p \simeq \mu_n + 4E_{\text{sym}}(\rho)\delta.$$  

(26)
The left hand side is given in terms of the microscopic quantities, i.e., \( \Delta(1232) \) mass and the three Delta meson coupling constants in the total self-energy \( \Sigma^\Delta = -g_\rho \Delta \Phi + g_\omega \Delta \pi_0 - 3 g_\rho \Delta^3 \rho_0 \). Using the parabolic approximation for the EoS of ANM \( E(\rho, \delta) \simeq E_0(\rho) + E_{\text{sym}}(\rho)\delta^2 + O(\delta^4) \) and assuming the density is not too far from \( \rho_0 \), the right hand side of Eq. (24) can be expanded in terms of \( \chi = (\rho - \rho_0)/\rho_0 \) and the isospin asymmetry \( \delta \) as

\[
\mu_{\Delta}^{\min} = m_N + E_0(\rho_0) + \left( \frac{\chi}{3 \rho_0} + \frac{\chi^2}{2} \right) K_0 \\
+ \frac{\chi^2}{3} \left( \frac{\rho}{\rho_0} + \chi \right) J_0 + (2\delta + 3\delta^2) E_{\text{sym}}(\rho_0) \\
+ \left[ \frac{\rho}{3 \rho_0} \delta^2 + \chi (2\delta + 3\delta^2) \right] L(\rho_0) 
\]  

(27)

where higher order terms in the expansion have been neglected (for full details see ref. 53). Here \( K_0 = 9\rho_0^2 d^2 E_0(\rho)/d \rho^2 |_{\rho = \rho_0} \) and \( J_0 = 27\rho_0^3 d^3 E_0(\rho)/d \rho^3 |_{\rho = \rho_0} \) are the incompressibility and skewness coefficient of symmetric nuclear matter (SNM) at \( \rho_0 \), respectively. This expansion is very easy to understand considering the energy conservation in the Delta production process \( NN \rightarrow N \Delta \), i.e., the minimum energy of the Delta is the energy of a nucleon (sum of nucleon rest mass and its mechanical energy). Since all seven macroscopic quantities used to characterize the EoS of ANM i.e., (a) the saturation density \( \rho_0 \) of SNM where the pressure \( P(\rho_0) = 0 \), (b) the binding energy \( E_0(\rho_0) \), (c) incompressibility \( K_0 \), (d) skewness coefficient \( J_0 \), (e) nucleon effective mass \( m^{\text{dirac}}_n \), (f) magnitude \( E_{\text{sym}}(\rho_0) \) and (g) slope \( L(\rho_0) \) of the symmetry energy, are all explicit functions (see, e.g., refs. 53, 71, 72) of the seven RMF microscopic model parameters, i.e., \( g_N, g_{\omega N}, g_{\rho N}, b_\omega N, c_{\sigma N}, c_{\omega N} \) and \( \Lambda_N \), the Eq. 27 allows us to explore the \( \rho_{\Delta}^{\text{crit}} \) dependence on either the seven microscopic or macroscopic parameters. Since the macroscopic quantities are either empirical properties of nuclear matter or directly related to experimental observables, it is more useful for the purposes of this work to examine the \( \rho_{\Delta}^{\text{crit}} \) by varying individually the seven macroscopic quantities. We notice that within both the RMF and Skyrme-Hartree-Fock approaches similar correlation analyses 73 have been successfully applied to study the neutron skin 73,74, the giant monopole resonances (GMR) of finite nuclei 73,74, the higher-order bulk characteristic parameters of ANM 73, the electric dipole polarizability \( \alpha_D \) in 208Pb 77, the correlation between the maximum mass of neutron stars and the skewness coefficient of the SNM 53, as well as the relationship between the \( E_{\text{sym}}(\rho) \) and the symmetry energy coefficient in the mass formula of nuclei 73.

Figure 4: Critical density for the formation of \( \Delta^- \) resonance from the nonlinear RMF model by varying individually \( \rho_0 \) (a), \( E_0(\rho_0) \) (b), \( m^{\text{dirac}}_n \) (c), \( K_0 \) (d), \( J_0 \) (e), \( E_{\text{sym}}(\rho_0) \) (f) and \( L(\rho_0) \) (g).
in the expansion of the minimum Delta chemical potential in Eq. (27).

![Figure 5](image_url)

Figure 5: (Color Online) Critical density for the formation of \( \Delta^- \) resonance as a function of the scaled density slope of symmetry energy \( L(w\rho_0)/w \) at three reduced densities of \( \rho_t/\rho_0 = w = 0.7, 1 \) and 2, respectively.

The seemingly stronger correlation between the \( \rho_{\Delta^-}^{\text{crit}} \) and \( J_0 \) compared to the correlations of \( \rho_{\Delta^-}^{\text{crit}} \) with the other 6 variables is because of the relatively larger uncertainty in \( J_0 \). On the other hand, some of the weaker correlations shown in Fig. 4 may become much stronger if one goes beyond their current constraints. For example, shown in Fig. 6 are the correlations of the \( \rho_{\Delta^-}^{\text{crit}} \) with the reduced slope \( L(\rho_t)/w \) of \( E_{\text{sym}}(\rho) \) at three reference densities of \( \rho_t/\rho_0 = w = 0.7, 1.0 \) and 2 while fixing the magnitudes of \( E_{\text{sym}}(\rho) \) and other variables at their default values. It is seen that the \( \rho_{\Delta^-}^{\text{crit}} \) increases much faster for \( L(\rho_t)/w > 70 \text{ MeV} \). This feature is consistent with that found in ref. [24]. Noting again that some earlier studies have used models predicting \( L \) values high than 90 MeV, they thus predicted correspondingly much large values for the \( \rho_{\Delta^-}^{\text{crit}} \).

IV. EFFECTS OF DELTA RESONANCES ON THE COMPOSITION AND STRUCTURE OF NEUTRON STARS

Having shown that the \( \rho_{\Delta^-}^{\text{crit}} \) depends sensitively on the completely unknown \( \Delta-\rho \) coupling strength \( g_{\rho\Delta} \) and the Delta mass \( m_{\Delta} \), and it is about \( \rho_{\Delta^-}^{\text{crit}} = (2.08 \pm 0.02)\rho_0 \) in the universal coupling scheme for \( m_{\Delta} = 1232 \text{ MeV} \) using RMF model parameters consistent with all existing constraints on the nuclear EoS, we now turn to effects of Delta formation on properties of neutron stars within the npe\(\rho\Delta \) model, omitting other particles such as hyperons and quarks at high densities, which sufficient for the purposes of this work. Here we restrict ourselves to studying the effects of Delta formation on the composition and mass-radius relation of neutron stars by varying the Delta mass and its coupling strength with the \( \rho \) meson. In constructing the EoS of various layers in neutron stars for solving the Oppenheimer-Volkoff (OV) equation, we follow a rather standard scheme. For the core we use the EoS of \( \beta \)-stable and charge neutral npe\(\mu\Delta \) matter obtained from the nonlinear RMF model described earlier. The inner crust with densities ranging between \( \rho_{\text{out}} = 2.46 \times 10^{-4} \text{ fm}^{-3} \) corresponding to the neutron dripline and the core-crust transition density \( \rho_t \) determined self-consistently using the thermodynamical method [54, 79] is the region where complex and exotic nuclear structure —collectively referred to as the “nuclear pasta” may exist. Because of our poor knowledge about this region we adopt the polytropic EoS parameterized in terms of the pressure \( P \) as a function of total energy density \( \varepsilon \) according to \( P = a + b\varepsilon^{4/3} \) [79, 80]. The constants \( a \) and \( b \) are determined by the pressure and energy density at \( \rho_t \) and \( \rho_{\text{out}} \) [79]. For the outer crust [81], we use the BPS EoS for the region with 6.93 \( \times 10^{-13} \text{ fm}^{-3} < \rho < \rho_{\text{out}} \) and the FMT EoS for 4.73 \( \times 10^{-15} \text{ fm}^{-3} < \rho < 6.93 \times 10^{-13} \text{ fm}^{-3} \), respectively.

![Figure 6](image_url)

Figure 6: (Color Online) Fractions of different species in neutron stars composed of neutrons, protons, Delta resonances, electrons and muons within the nonlinear RMF model using two sets of model parameters with \( x_\rho = g_{\rho\Delta}/g_{\rho N} = 1 \) and 2, respectively.

Shown in Fig. 6 are fractions of different species, i.e., \( x_i = \rho_i/\rho \) in neutron stars using two parameter sets with the \( \Delta-\rho \) coupling strength corresponding to \( x_\rho = 1 \) and 2, respectively. These calculations are done with \( m_{\Delta} = 1232 \text{ MeV} \). It is seen that the appearance of \( \Delta^- \) affects significantly the fractions of others particles depending on the value of the \( x_\rho \) as one expects. The modified fractions of the lighter particles \( e \) and \( \mu \) will affect their weak decays and thus the possible kaon condensation. Moreover, the strong boost of the proton fraction above the \( \Delta^- \) production threshold may have a significant impact on the cooling processes in neutron stars [82]. A detailed investigation of these consequences requires a self-consistent extension of the model considered here and is on the agenda of our future work.
The first and most important piece of information necessary for resolving this issue is the critical density above which the $\Delta(1232)$ can be formed in neutron stars. Previous studies have indicated that the critical densities range from $\rho_0$ to very high values only reachable in the core of very massive neutron stars. In this work, within the extended nonlinear RMF model we found that the critical formation densities for the four different charge states of $\Delta(1232)$ resonances depend differently on the separate kinetic and potential parts of the nuclear symmetry energy, the in-medium Delta mass $m_\Delta$ and the completely unknown $\Delta-\rho$ coupling strength $g_{\Delta\rho}$. This is the first time a microphysical property of neutron star matter has been shown to depend differently on the potential and kinetic parts of the symmetry energy. Assuming that the respective $\Delta$-meson and nucleon-meson coupling constants are the same, the critical density for the first $\Delta_{}^-(1232)$ to appear is found to be $\rho_{\Delta_{}^\text{crit}}=(2.08 \pm 0.02)\rho_0$ using RMF model parameters consistent with current constraints on all seven macroscopic parameters characterizing the EoS of ANM at $\rho_0$. This is an upper limit on $\rho_{\Delta_{}^\text{crit}}$ with respect to the currently observed limit on the maximum neutron star mass. We also found that the composition and the mass-radius relation of neutron stars are significantly affected by the formation of $\Delta(1232)$ resonances. In particular, the impacts of the $\Delta(1232)$ formation depend sensitively on the values of the $g_{\Delta\Delta}$ and the in-medium Delta mass $m_\Delta$ which are also being probed with terrestrial laboratory experiments.

VI. ACKNOWLEDGEMENT

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