Conductance oscillation in ferromagnetic-metal/nonmagnetic-metal/superconductor double junctions

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Abstract. We theoretically analyzed the Andreev reflection in the ferromagnetic-metal(FM)/nonmagnetic-metal(NM)/superconductor(SC) double junctions with a special attention to the electron interference effect in the NM layer. In our previous work [Phys. Rev. B \textbf{79}, 212570 (2009)] we found that the conductance oscillates as a function of the bias voltage due to the geometrical resonance, and the period of the conductance oscillation is proportional to the square-root of the exchange field. Here, by extending the previous work, we showed that the period of the conductance oscillation decreases if we consider the proximity effect at the NM/SC interface. We also showed that the period of the oscillation is proportional to the square-root of the exchange field even if we consider the proximity effect.

1. Introduction
The Andreev reflection(AR) is a conversion process from an electron to a Cooper-pair at a normal conductor/superconductor(SC) interface with a hole reflected. When the normal conductor is a nonmagnetic-metal(NM), the conductance is doubled compared with that for the NM/NM junction due to the AR. However, if the normal conductor is a ferromagnetic-metal(FM), the AR is suppressed by the exchange field of the ferromagnetic-metal[1]. We can measure the spin polarization of conduction electrons of FMs through the suppression of the conductance of the FM/SC junction below the superconducting gap[2].

Very recently, we studied the transport properties of the FM/NM/SC junctions and found that the conductance due to the AR oscillates as a function of the bias voltage due to the geometrical resonance[3]. We showed that the period of the conductance oscillation is proportional to the square-root of the exchange field, and thus can determine the spin polarization of the FM layer. In the previous work[3], we neglected the proximity effect at the NM/SC interface. However, the proximity effect is inevitable in any realistic experiments.

In this paper, by extending our previous work[3], we theoretically analyzed the AR in FM/NM/SC junctions with a special attention to proximity effect at the NM/SC interface. We solved the Bogoliubov-de Gennes (BdG) equations with the proximity effect by using the recursion-transfer-matrix (RTM) method[4, 5] and showed that the period of the conductance oscillation decreases if we consider the proximity effect at the NM/SC interface. We also showed that the period of the oscillation is proportional to the square-root of the exchange field even in the presence of the proximity effect.
2. Conductance oscillation due to geometrical resonance

Let us begin with a brief introduction to the conductance oscillation due to the geometrical resonance in the FN/NM/SC junctions[3]. Assuming that the current flows along the x-axis, and the thickness of the NM layer is d. The system is described by the following BdG equation[6]:

\[
\begin{pmatrix}
    H_0 - h(x) \sigma & \Delta(x) \\
    \Delta^*(x) & -H_0 - h(x) \sigma
\end{pmatrix}
\begin{pmatrix}
    f_\sigma(\vec{r}) \\
    g_\sigma(\vec{r})
\end{pmatrix}
= E
\begin{pmatrix}
    f_\sigma(\vec{r}) \\
    g_\sigma(\vec{r})
\end{pmatrix},
\]

where \( H_0 \equiv -(h^2/2m)\nabla^2 - \mu_F \) is the single particle Hamiltonian, E is the quasiparticle (QP) energy measured from the Fermi energy \( \mu_F \), and \( \sigma = \pm \) represents the up-(down-)spin band. 

The exchange field function is given by \( h(x) = h_0 [1 - \Theta(x)] \) where \( h_0 \) represents the exchange field in the FM layer and \( \Theta(x) \) is the Heaviside step function. The superconducting gap function is expressed as \( \Delta(x) = \Delta_0 \Theta(x-d) \), where \( \Delta_0 \) represents the superconducting gap in the SC layer. We assumed that the system has translational symmetry in the transverse (y, z) direction, and therefore the wave vector parallel to the interface \( \vec{k}_\parallel \equiv (k_y, k_z) \) is a conserved quantity.

The general solutions of the BdG equation (1) in the FM (NM) layer are given by

\[
\Psi_{\pm k_{\text{FM(NM)},\sigma}}(\vec{r}) = \begin{pmatrix}
    e^{\pm ik_{\text{FM(NM)},\sigma}x} S_{\vec{k}_\parallel}(\vec{r})_0 \\
    1
\end{pmatrix}
\]

where \( S_{\vec{k}_\parallel}(\vec{r})_0 \) represents the eigen function in the transverse direction in the \( \vec{k}_\parallel \) channel. The x component of the wave vector of an electron (hole) with \( \sigma \)-spin is defined as \( k_{\text{FM(NM),}\sigma} = \sqrt{2m_e} \mu_F \pm E + \sigma h_0 - E_\parallel \) and \( k_{\text{FM(NM),}\sigma}^\pm = \sqrt{2m_e} \mu_F \pm E - E_\parallel \), where \( E_\parallel = \frac{\hbar^2 k_{\parallel}^2}{2m} \). On the other hand, in the SC layer, we have

\[
\Psi_{\pm k_{\text{SC},\sigma}}(\vec{r}) = \begin{pmatrix}
    v_0 & 0 \\
    u_0 & 0
\end{pmatrix} e^{\pm ik_{\text{SC},\sigma}x} S_{\vec{k}_\parallel}(\vec{r})_0
\]

where \( u_0 \) and \( v_0 \) are the coherence factors expressed as \( u_0^2 = 1 - v_0^2 = \frac{1}{2} [1 + \frac{\sqrt{E^2 - \Delta^2}}{E}] \).

The wave function of the FM/NM/SC junctions is given by the linear combination of the above general solutions. By matching the wave functions at the boundaries, we obtain the probabilities of the AR, \( A_{\sigma,\vec{k}_\parallel}(E) \), and the normal reflection, \( B_{\sigma,\vec{k}_\parallel}(E) \)[7], where we assume that the voltage drop occurs at the NM/SC interface for simplicity. When the superconducting gap is larger than applied bias(\( \Delta_0 > eV \)), the probabilities \( A_{\sigma,\vec{k}_\parallel}(E) \) and \( B_{\sigma,\vec{k}_\parallel}(E) \) satisfy the relation that \( 1 - B_{\sigma,\vec{k}_\parallel}(E) = A_{\sigma,\vec{k}_\parallel}(E) \) and then we have

\[
G = \frac{e}{h} \sum_{\sigma,\vec{k}_\parallel} \left[ 1 + A_{\sigma,\vec{k}_\parallel}(eV) - B_{\sigma,\vec{k}_\parallel}(eV) \right] = 2 \frac{e}{h} \sum_{\sigma,\vec{k}_\parallel} A_{\sigma,\vec{k}_\parallel}(eV),
\]

where \( V \) represents the bias voltage.

In Figs. 1(a) and 1(b) we plot the numerically obtained conductance-voltage curves for the FM/NM/SC junctions with \( d = 1\mu m \) and \( d = 10\mu m \), respectively. The conductance is normalized by that for FM/NM/NM junctions. One can clearly see the conductance oscillation against the bias voltage. Since the origin of the conductance oscillation is geometrical resonance the period of the oscillation is inversely proportional to the thickness \( d \) of the NM layer. From Figs. 1(a) and 1(b), one can also see that the period of the conductance oscillation does depend
Figure 1. (a) Normalized conductances of the FM/NM/SC junctions are plotted against the bias voltage. The thickness of NM is taken to be \(d = 1\mu m\). (b) Same plot for \(d = 10\mu m\).

on the exchange field, \(h_0\), in the FM layer. As we shall show later, the period is proportional to the square-root of the exchange field \(h_0\).

Under the Andreev approximation[7], we obtain the following analytical expression of the AR probability,

\[
A_{\sigma,\vec{k}_\parallel}(E) \simeq \frac{4\zeta \sqrt{1 - \eta^2}}{2 \left[ \zeta^2 + \zeta^2 \sqrt{1 - \eta^2} \right] - \eta^2 \left[ \epsilon^2 + (1 - 2\epsilon^2) \cos^2[(k_{NM}^+ - k_{NM}^-)d] - \epsilon \sqrt{1 - \epsilon^2} \sin[2(k_{NM}^+ - k_{NM}^-)d] \right]},
\]

where we introduced the normalized parameters \(\eta = h_0/\mu_F\), \(\zeta = 1 - E_\parallel/\mu_F\), and \(\epsilon = E/\Delta_0\). For the FM/NM/SC junctions with \(d \neq 0\), the trigonometric functions in Eq.(5) give rise to the oscillation of the conductance against the bias voltage. The origin of the conductance oscillation is the interfere between original electrons and electrons which are twice Andreev reflected at the NM/SC interface. The exchange field in the FM layer plays a role of the insulating barrier in the Rowell-McMillan’s work [8]. The period of the oscillation of \(A_{\sigma,\vec{k}_\parallel}(E)\) with finite \(\vec{k}_\parallel\) is given by

\[
\Delta V_{\vec{k}_\parallel} \simeq \frac{\hbar \pi}{\sqrt{2m_e d}} \sqrt{\mu_F - E_\parallel}. \quad \text{From Eq.(4) the conductance is obtained by summing up } A_{\sigma,\vec{k}_\parallel}(E)\text{ for all the available } \vec{k}_\parallel.\]

Because the spin of the Andreev reflected hole is opposite to that of the incident electron, the maximum value of \(k_\parallel\) and therefore \(E_\parallel\) is limited by the exchange field \(h_0\), i.e., \(\max(E_\parallel) = \mu_F - h_0\). We assumed that oscillations of \(A_{\sigma,\vec{k}_\parallel}(E)\) with different periods cancel out each other and the period of the sum \(\sum_{\sigma,\vec{k}_\parallel} A_{\sigma,\vec{k}_\parallel}(E)\) is determined by the shortest period. Thus, the period of the conductance oscillation of the three-dimensional system with finite \(\vec{k}_\parallel\) is obtained as

\[
\Delta V_{3D} \simeq \min \Delta V_{\vec{k}_\parallel} = \frac{\hbar \pi}{\sqrt{2m e d}} \sqrt{h_0}. \quad \text{(6)}
\]

We also calculated the conductance including the effect of the interfacial potential and confirmed that the period of the conductance oscillation is proportional to the square-root of the exchange field [3].

### 3. Effect of proximity on the conductance oscillation

Next let us move onto the effect of the proximity at the NM/SC interface on the conductance oscillation. The proximity effect at the NM/SC interface is phenomenologically taken into account by considering the spatially varying superconducting gap, \(\Delta(x)\), in the BdG equations.
For simplicity we assume that \( \Delta(x) = \Delta_0 \tanh[(x - d)/\sqrt{2}\xi] \), where \( \xi = \hbar v_F/\pi \Delta_0 \) is the superconducting coherence length at absolute zero with \( v_F \) being the Fermi velocity. Since the superconducting gap varies inside the superconductor, we cannot use the simple wave-function-matching procedure[3, 7]. In order to overcome this difficulty, we employ the RTM method[4, 5] when we calculate the AR probability, and therefore the conductance.

In Figs. 2(a) and 2(b), we plot the normalized conductance calculated with and without the proximity effect against the bias voltage by solid and dotted lines, respectively. The suppression of the superconducting gap due to proximity in the vicinity of the NM/SC interface is equivalent to extending the NM thickness \( d \) in Eq.(6). Therefore the period of the conductance oscillation decreases if we consider the proximity effect at the NM/SC interface, as show in Figs. 2(a) and 2(b). We confirmed that the period of the oscillation is proportional to the square-root of the exchange field even in the presence of the proximity effect.

4. Summary

We theoretically studied the conductance oscillation due to the geometrical resonance in a FM/NM/SC junctions by extending our previous work of Ref.[3] on the junctions with proximity. We solved the BdG equations with spatially varying superconducting gap by using the RTM method and calculated the conductance due to the AR. We showed that the suppression of the superconducting gap due to the proximity is equivalent to extending the NM layer thickness and the period of the conductance oscillation decreases. We also found that the period of the oscillation is proportional to the square-root of the exchange field even in the presence of the proximity effect.

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Figure 2. (a) Normalized conductances of the FM/NM/SC junctions with \( d = 1\mu m \) and \( h_0 = 0.3\mu F \). The solid line is the system with the proximity effect and the dashed line is the one with no proximity effect. (b) Same plot for \( d = 1\mu m \) and \( h_0 = 0.6\mu F \).