Performance of free-space optical systems in the presence of receiver imperfections

G T Djordjevic¹, D N Milic¹, and I D Radojkovic²

¹ University of Nis, Faculty of Electronic Engineering, Department of Telecommunications, A. Medvedeva 14, 18000 Nis, Serbia
² Dunav osiguranje, 18000 Nis, Serbia

E-mail: goran.t.djordjevic@elfak.ni.ac.rs

Abstract. We present a review of papers considering the subcarrier intensity modulation free-space optical (FSO) communication systems in the presence of phase noise. This phase noise is a consequence of imperfect reference signal extraction in electrical domain after photodetector. We illustrate the effects of this phenomenon on the system symbol-error rate performance. The examples show that receiver imperfections, presented through phase noise, can degrade the FSO system performance to a large extent.

1. Introduction

In this Section, we describe different application areas of optical wireless communications (OWC). After that, we give an overview of the effects that are commonly present during signal transmission in free-space optical (FSO) system, with an emphasis on the impact of receiver imperfections. At the end of the Section, we provide an overview of the literature regarding the influence of the phase noise on the performance of the FSO system.

1.1. Application of OWC systems

The OWC represents a promising technology for a variety of applications. This technology can be applied in different scenarios, like: 1) ultra-short range communications (interconnects within integrated circuits); 2) short range communications (wireless body area networks and device-to-device communications in dense Internet-of-Things cells); 3) medium range communications (visible light communications and infrared communications); 4) long range terrestrial communications; 5) ultra-long range communications (inter-satellite, deep-space and ground-to-satellite links) [1]-[6].

Nowadays, there are 8 billion mobile subscriptions, and this number is predicted to increase to 8.9 billion by the end of 2025, out of which 88 percent will be for mobile broadband. The number of smartphone subscriptions is forecast to reach 7.5 billion in 2025, which accounts for around 85 percent of all mobile subscriptions [7]. The demand for high data rates have been increasing dramatically. Emerging wireless communication networks should support high data rates, high reliability, security, low latency and high density of terminals.

In this work, we are focused to terrestrial links up to several kilometres enabling fibre-like data rates and being essential to backhaul and fronthaul needs. Wireless optical technology is a promising alternative scheme to radio frequency communication systems for addressing last mile bottleneck in broadband access networks. With increasing the number of nano and microcells in 5G systems, the demands will increase for backhaul connections between base stations, as well as base stations and...
mobile switching centres. The FSO technology is used for last mile access, i.e., for bridging the gap between backbone network and end-users. It can be used for providing back-up of the optical fibre links. The FSO is a favourite technology for establishing temporary links under disaster conditions, too.

1.2. Transmission effects in FSO links

The optical signal propagation is influenced by three atmospheric factors: absorption, scattering and refractive-index fluctuations. Absorption and scattering are usually considered together like a deterministic process, which can be described jointly by an equation, while signal intensity fluctuations induced by turbulence are a consequence of the stochastic process. An optical signal propagating over an FSO channel experiences random intensity fluctuations due to turbulence, which is caused by the fluctuations of the refractive index of the medium [1]-[5]. These fluctuations of the received signal can be described by several statistical distributions, which are in agreement with measured results [3]. Some of these distributions are as follows: K distribution, lognormal, modified Ricean, Gamma-Gamma, Malaga, etc. Gamma-Gamma distribution is widely accepted and it can describe turbulence conditions from weak to strong ones [1]-[6].

The misalignment between transmitter and the receiver of the FSO link is known as pointing errors. This phenomenon is caused by building sway and vibrations, and is treated as a stochastic process that can be described by different distributions. In the case when horizontal and vertical displacements are mutually independent and when they can be described by normal distribution with zero mean value, the radial displacement of the photodetector can be described by Rayleigh distribution. The intensity of the pointing errors is fixed by the standard deviation of horizontal and vertical displacements [6].

On the basis of the detection type, there are two types of the FSO systems: intensity-modulation direct-detection (IM/DD) systems and coherent systems. In IM/DD approach, the information is conveyed by the light intensity, which is directly detected by a photodetector without applying local oscillator. Contrary, in the case of coherent systems, amplitude, frequency or phase modulation is applied and received optical signal is mixed with local laser signal before photo-detection. Due to simplicity and low cost, IM/DD systems are usually applied in practical terrestrial FSO systems. In order to improve performance of FSO systems, a subcarrier intensity modulation technique has been suggested to be an alternative to IM/DD systems [4], [5].

One of the negative effects in an FSO system is imperfection of the receiver in electrical domain. In the case of SIM-based systems, coherent modulations require carrier phase recovery for achieving the optimal performance. In practice, the carrier phase cannot be tracked with perfect accuracy. The difference between carrier phase and its estimate is called phase noise, and it is a stationary random process whose distribution conforms to a random variable following Tikhonov distribution [8]-[15].

Two main approaches were applied in previous works on this topic. In the first approach, loop SNR is specified by the phase noise standard deviation. This standard deviation is fixed over all values of channel SNR [8]-[11]. This approach corresponds to the case when there is a limiter in loop circuit. In the second approach, the SNR in the loop filter is dependent on the channel SNR, and loop SNR is linear function of channel SNR [13-16]. In both cases, we assume that intensity fluctuations bandwidth is much smaller than PLL loop filter and loop SNR remains constant over many symbol intervals.

1.3. Literature review

The effect of imperfect reference signal recovery on BER performance of SIM-based BPSK system was firstly considered in [8]. Using the simulation approach, the results confirmed that error performance of an FSO system containing Gamma-Gamma channel can be strongly degraded by the phase noise. The authors of [9] presented the analysis of symbol error rate performance of SIM-based MPSK systems under strong atmospheric turbulence conditions. The Fourier series method [16]-[19] was applied in estimating the error performance. The Fourier series method was used in analyzing the performance of SIM-based M-ary differential PSK FSO system over Gamma-Gamma turbulence and pointing errors in [10]. In DPSK receiver, a local oscillator used for down-conversion generates a signal which is not perfect in the sense that phase of this signal is a random process fluctuating over a time. The aim of the paper was to estimate the effect of these fluctuations on error performance.
authors of [11] applied the Fourier series method in analyzing the effect of phase noise on SIM MPSK system when turbulence is modeled by Malaga distribution. However, numerical results presented in [11] are valid only for the case of Gamma-Gamma turbulence, and it is open question the effect of specific parameters of Malaga distribution together with phase noise on overall system performance.

Reference [12] studied the noisy reference loss of SIM-based BPSK and QPSK in lognormal fading. The analysis was extended to M-ary PSK in [13]. They assumed that signal-to-noise ratio in PLL loop filter is linearly dependent on the SNR over the FSO channel and the expressions were derived for estimating the asymptotic noisy reference loss. The analysis from these two references were further extended in [14] to the case of Gamma-Gamma channel with pointing errors, where the approximate expressions for error probability were derived. Reference [15] presented approximate expressions for error probability in detecting SIM-based MPSK signal transmitted over an FSO channel in the presence of diversity reception when there is a phase noise in diversity receiver.

2. Fourier series method in error rate estimation

In this Section, we introduce the system model that is used in illustrating the Fourier series method for estimating the effects of receiver imperfections on SIM FSO system performance. We present the basic steps in mathematical analysis and some numerical results with appropriate comments.

2.1. System model

Let us observe the system presented in figure 1. The SIM technique was borrowed from RF communications, and the signal is modulated using appropriate modulation technique in the electrical domain. The modulator output is denoted by $s(t)$. A dc bias is then added to keep the resulting modulated signal positive at all times. The signal thus obtained modulates directly and linearly the intensity of the laser. The optical transmitting power of the laser is denoted by $P$. The intensity of the optical signal at the output of the receiver has a form of $I(t) = P(1 + m \times s(t))$, where $m$ denotes the modulation index. The signal is then transmitted over an FSO channel. At the reception, direct detection is performed by a PIN photodiode mounted directly after the receiving optics, and also, the dc bias of the signal is removed. After that, the signal is demodulated in the electrical domain in classical way, well known and widely used in existing wireless devices. This electrical signal can be represented as $y(t) = RPmI \times s(t) + n(t)$.

The instantaneous signal-to-noise ratio is given by $\gamma = \frac{(RPmI)^2}{(2\sigma^2)}$. The average SNR is then given by the mathematical expectation of this value,
as \( \bar{\gamma} = E\left(\frac{(RPmI)^2}{(2\sigma^2)}\right) = E\left(I^2\right) \times \frac{(RPm)^2}{(2\sigma^2)} \), while average electrical SNR is defined as

\( \mu = E\left(I^2\right) \times \frac{(RPm)^2}{(2\sigma^2)} \).

The analysis is provided for the example when turbulence induced signal intensity fluctuations over a channel can be described by a Gamma-Gamma probability density function, given by

\[
p(I) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} I^{\alpha+\beta-1} K_{\alpha-\beta} \left(2\sqrt{\alpha\beta I}\right), \quad I > 0,
\]

where \( K_n(.) \) denotes the second kind modified Bessel function of order \( n \) [22, (8.407)], while \( \alpha \) and \( \beta \) are parameters defining the turbulence strength. For the case of zero inner scale plane waves, these parameters can be expressed as

\[
\alpha = \exp\left(\frac{0.49\sigma_R^2}{\left(1+1.11\sigma_R^{12/5}\right)^{1/6}}\right) - 1, \quad \beta = \exp\left(\frac{0.51\sigma_R^2}{\left(1+0.69\sigma_R^{12/5}\right)^{1/6}}\right) - 1,
\]

where \( \sigma_R^2 \) is the Rytov variance given by \( \sigma_R^2 = 1.23C_n^2 k^{7/6}L^{1/6} \), with \( L \) being the link length in km and \( k \) being the wave number given by \( k = 2\pi/\lambda \). We use the following numerical values: \( \lambda = 1.55 \mu m, L=2000 \text{ m} \). The turbulence strength is defined by:

\[
C_n^2 = \begin{cases} 
6 \times 10^{-15} \text{ m}^{-2/3}, & \text{weak turbulence} \\
2 \times 10^{-14} \text{ m}^{-2/3}, & \text{moderate turbulence} \\
5 \times 10^{-14} \text{ m}^{-2/3}, & \text{strong turbulence}.
\end{cases}
\]

Figure 2 illustrates the turbulence parameter values in Gamma-Gamma model for the given link distance.
Weak turbulence is characterized by the relatively high values of $\alpha$ and $\beta$, which corresponds to relatively low values of atmospheric constant $C_n^2$. Limiting values of $\alpha, \beta \to \infty$ correspond to an ideal situation where there is no turbulence present. On the other hand, strong turbulence is represented by the low value of $\beta$ that is close to one, while the other parameter $\alpha$ is large. In the limiting situation of infinite $\alpha$, this leads to a simpler description of the turbulence channel corresponding to a Gamma distribution. Moderate turbulence regime is located between the two extremes and is represented by relatively low values of both parameters.

2.2. Fourier series approach for error rate estimation
The basic idea in determining the error performance lies in expressing the resulting received signal phase in the form of the Fourier series. The effects of intensity fluctuations and noise are incorporated in the Fourier series members. After the procedure described in [16]-[19] the phase of the received signal can be presented as

$$ p_{\psi} (\psi) = \frac{1}{2\pi} + \sum_{n=1}^{\infty} b_n \cos(n\psi), \quad |\psi| \leq \pi, \quad (4) $$

where coefficients $b_n$ are expressed as

$$ b_n = \frac{n! \Gamma(n+1) 2^{\alpha+\beta-3}}{\pi^2 \Gamma(\alpha) \Gamma(\beta)} G_{2.5}^{5.1} \left( \frac{\alpha^2 \beta^2}{16 \mu}, \frac{n/2, 1+n/2, (\alpha+1)/2, \beta/2, (\beta+1)/2, 0}{1/2} \right), \quad (5) $$

where $G_{\mu,\nu}^{p,q} (\cdot)$ is the Meijer’s $G$-function [20, (9.301)]. The signal phase is estimated by a phase locked loop from the pilot signal. This estimation is not perfect and there is a difference between the actual signal phase and estimated phase. This difference as a random variable that can be described by Tikhonov distribution given by

$$ p_{\phi} (\phi) = \frac{e^{\nu_{\phi} \cos \phi}}{2\pi \Gamma(\gamma_{PLL})}, \quad |\phi| \leq \pi. \quad (6) $$

This PDF can be expressed in the form of the Fourier series as

$$ p_{\phi} (\phi) = \frac{1}{2\pi} + \sum_{n=1}^{\infty} c_n \cos(n\phi), \quad |\phi| \leq \pi, \quad (7) $$

where coefficient $c_n$ is given by $c_n = I_{\nu} (1/\sigma_{\phi}^2) / \left( \pi I_{\nu} (1/\sigma_{\phi}^2) \right)$. The estimation of truncation error in (7) can be found in [10].

After the quite similar procedure from [10], the symbol error probability in detecting SIM-based $M$-ary PSK signal can be derived in the form

$$ P_e = \frac{1}{M} - \frac{1}{M} \sum_{n=1}^{\infty} 2\pi b_n c_n \sin \left( \frac{n\pi}{M} \right), \quad (8) $$

where coefficients $b_n$ and $c_n$ are previously derived, and $M$ is the number of phase levels. In this way we expressed the symbol error probability in the form of the infinity series with known coefficients. The proof of convergence and estimation of the truncation error of similar series can be found in our previous works [10], [19]. This expression is convenient also for calculating numerical values in some boundary cases. In this paper we present some illustrative numerical examples.

2.3. Numerical results
In this paper, we have used a readily available method implemented in in Mathematica for numerical calculation of sums. In order to estimate the value of sum, Wynn-\(\varepsilon\) method uses additional number of terms in sum and then extrapolates them to a polynomial multiplied by decaying exponential function. We have used a minimum of 1000 terms in the sum, after which this error estimation is performed using built-in Mathematica summation algorithm. Since the series of terms is of alternating signs,
working precision must also be increased in order not to lose significant digits because of subtraction operation. This also presents a difficulty in calculating Meijer’s $G$ function values in (5) with required number of accurate digits, which consumes significant amount of CPU resources. Because of these difficulties, we have resorted to parallel computation using 24 kernels in a local network environment. 100 points in each of the symbol error curves (8) shown in figure 3 are computed in about 5 minutes using this setup.

![Symbol error rate of SIM QPSK system.](image)

Figure 3. Symbol error rate of SIM QPSK system.

Figure 3 presents the symbol error probability dependence on electrical SNR for different values of phase noise standard deviation. The error probability decreases with increasing SNR until a given value. After that value, error probability stays constant and does not depend on SNR value. The value of this floor is dependent only on the phase noise standard deviation. The higher the value of this standard deviation, the lower the value is of error floor. That means that imperfect carrier phase recovery strongly influences the error performance. If the PLL loop circuit is poorly designed, the probability of error cannot fall below the floor despite the increase in signal strength.

Figure 4 presents the SER values for SNR of 50 dB for different values of phase noise standard deviation and under different turbulence conditions.

![SER for fixed SNR of 50 dB in different turbulence strengths and phase noise standard deviations.](image)

Figure 4. SER for fixed SNR of 50 dB in different turbulence strengths and phase noise standard deviations.
Two conclusions are visible. Firstly, if the system is designed to operate correctly in weak turbulence, the strong turbulence conditions could increase considerably the SER values. Secondly, the effect of phase noise is not primarily effect in strong turbulence condition up to values of phase noise standard deviation of 12 degrees. Contrary, in weak turbulence conditions, phase noise starts to have strong influence on SER values starting from phase noise standard deviation around 6 degrees.

3. Conclusion

In this Invited paper, we have highlighted the effect of imperfect reference signal recovery in electrical domain when SIM-based PSK is applied in FSO systems taking turbulence induced signal intensity fluctuations into account. We have presented the Fourier series method approach that was used by ourselves in some our previous works. The results have shown that phase noise can strongly degrade the SIM-based FSO systems, and the approach presented here has the ability to accurately predict these degradations. This approach can also be applied in designing of receivers for FSO systems.

References

[1] Ghassemlooy Z, Popoola W and Rajbhandari S 2019 Optical wireless communications – System and channel modelling with MATLAB®, CRC Press, Taylor&Francis Group, Boca Raton
[2] Khalighi M A and Uysal M 2014 Survey on free space optical communication: a communication theory perspective, IEEE Communications Surveys & Tutorials, 16, no. 4, pp 2231-2253
[3] Andrews L and Philips R 2005 Laser beam propagation through random media, Bellingham, WA, USA: SPIE
[4] Popoola W 2009 Subcarrier intensity modulated free-space optical communication systems, PhD thesis, University of Northumbria at Newcastle
[5] Petkovic M 2015 Analysis and means to improve the performance of wireless optical telecommunication systems under conditions of atmospheric turbulence, PhD thesis, University of Nis
[6] Djordjevic G, Petkovic M, Cvetkovic A and Karagiannidis G 2015 Mixed RF/FSO relaying with outdated channel state information, IEEE Journal on Selected Areas in Communications, 33, no. 9, pp 1935-1948
[7] Ericson Mobility Report, June 2020, Available on: https://www.ericsson.com/en/mobility-report/reports/june-2020/
[8] Djordjevic G 2013 Effect of phase noise on bit error rate performance of subcarrier intensity modulation binary phase shift keying wireless optical systems – simulation study, Facta Universitatis, Series: Automatic Control and Robotics, 12, no. 3, pp 189-195
[9] Petkovic M and Djordjevic G 2015 SEP analysis of FSO system employing SIM-MPSK with noisy phase reference, 4th International Workshop on Optical Wireless Communications (IWOW), , Istanbul, Turkey, 7-8 September 2015, pp 46-50
[10] Petkovic M, Djordjevic G, Karagiannidis G and Milovanovic G 2017 Performance of SIM-MDPSK FSO systems with hardware imperfections, IEEE Transactions on Wireless Communications, 16, no. 8, pp 5442-5451
[11] Chen D, Huang G and Wang C 2020 Influences of the phase noise on performance of subcarrier modulation system over Malaga turbulence channel, Optical Engineering, 59(1), pp 016111-1 – 016111-13
[12] Song X, Yang F, Cheng J and Al-Dhahir N 2015 Asymptotic noisy reference Losses of subcarrier BPSK and QPSK systems in lognormal fading, International Conference on Computing,” Networking and Communications (ICNC), Workshop on Computing, Networking and Communications (CNC), Garden Grove, CA, USA, pp 352-356
[13] Song X, Yang F, Cheng J, Al-Dhahir N and Xu Z 2015 Subcarrier phase-shift keying systems with phase errors in lognormal turbulence channels, Journal of Lightwave Technology, 33, no. 9, pp 1896-1904
[14] Gappmair W and Nistazakis H 2017 Subcarrier PSK performance in terrestrial FSO links impaired by Gamma-Gamma fading, pointing errors, and phase noise, *Journal of Lightwave Technology*, 35, no. 9, pp 1624-1632

[15] Varrotsos G, Nistazakis H, Gappmair W, Sandalidis H and Tombras G 2019 SIMO subcarrier PSK FSO links with phase noise and non-zero boresight pointing errors over turbulence channels, *IET Communications*, 13, no. 7, pp 831-836

[16] Prabhu V 1969 Error-rate considerations for digital phase-modulation systems, *IEEE Transactions on Communication Technology*, COM-17, no. 1, pp 33-42

[17] Tan C and Tjhung T 1983 On Fourier series expansion of the phase density function, *IEEE Transactions on Communications*, COM-31, no. 5, pp 736-738

[18] Kostić I 2007 Average SEP for M -ary CPSK with noisy phase reference in Nakagami fading and Gaussian noise, *European Transactions on Telecommunications*, 18, no. 2, pp 109-113

[19] Marjanovic Z, Djordjevic G and Milovanovic G 2018 Truncation error analysis in computing of SEP and SEP floor for partially coherent receiver of MPSK signals over composite fading channels, *Journal of The Franklin Institute*, 355, no. 2, pp 965-980

[20] Gradshteyn I and Ryzhik I 2000 *Table of integrals, series, and products*, NY, USA: Academic

**Acknowledgments**

This work was supported in part by the Ministry of Education, Science and Technological Development of Republic of Serbia, and in part by European Science Foundation under COST Action CA19111 - NEWFOCUS.