Fission Fragment Excitation Energy Sharing Beyond Scission

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A simplified, though realistic, model describing two receding and accelerating fission fragments, due to their mutual Coulomb repulsion, shows that fission fragments share excitation energy well after they ceased to exchange nucleons. This mechanism leads to a lower total kinetic energy of the fission fragments. Even though the emphasis here is on fission, similar arguments apply to fragments in heavy-ion reactions.

I. INTRODUCTION

Since the discovery of fission in 1939 [1–3] it was assumed that after scission the two fragments are accelerated by their Coulomb repulsion and the entire potential Coulomb energy between the fragments is converted into the total kinetic energy (TKE) of the fission fragments (FFs). With the exception of a couple of small studies of which I am aware of [4–6], this assumption is treated as rather accurate and the magnitude of the TKE of the FFs was used a signature of the scission shape of the fissioning nucleus or as different fission modes [7–10]. However, because of the long range nature of the Coulomb interaction the intrinsic excitation energy can be still exchanged between the receding FFs and the amount of the total excitation energy (TXE) and TKE can be affected. Here I describe a simple model of this excitation energy sharing between the FFs, here for the sake of simplicity assumed to be as accurate and the magnitude of the TKE of the FFs.

II. A VERY SIMPLE MODEL

Here I introduce a model for two interacting particle systems, the neutron and proton systems in a nucleus, with frozen intrinsic dynamics, in an external potential acting only on one of them. The Hamiltonian, the equations of motion, and the initial conditions are

\[ H = \frac{m}{2} (\dot{x}^2 + \dot{y}^2) + \frac{m\omega^2}{4}(x-y)^2 - F(t)x, \]

\[ m\ddot{x} = -\frac{m\omega^2}{2}(x-y) + F(t), \]

\[ m\ddot{y} = m\omega^2(x-y), \]

\[ x(0) = \dot{x}(0) = y(0) = \dot{y}(0) = 0, \]

where \( F(t) = F\Theta(t)\Theta(T-t) \) is a constant force acting during a finite time interval. Now I introduce center-of-

\[ \xi = \frac{x+y}{2}, \quad \eta = x - y, \]

\[ H = H_{\text{CM}} + H_{\text{int}}, \]

\[ H_{\text{CM}} = m\dot{\xi}^2 - F(t)\xi, \]

\[ 2m\dot{\xi} = F, \]

\[ H_{\text{int}} = \frac{m\dot{\eta}^2}{4} + \frac{m\omega^2\eta^2}{4} - \frac{F\eta}{2}, \]

\[ m\ddot{\eta} = -m\omega^2(\eta - \eta_0), \quad \eta_0 = \frac{F}{m\omega^2}, \]

for \( 0 \leq t \leq T \). The solutions of these equations of motion are

\[ \xi(t) = F \frac{t^2}{4m}, \quad \eta(t) = \eta_0[1 - \cos(\omega t)] \]

and the energy can be separated into the center-of-mass kinetic energy and total intrinsic energy for \( t \leq T \)

\[ T_{\text{CM}}(t) = \frac{F^2t^2}{4m}, \]

\[ E_{\text{int}}(t) = 0 \]

as expected for a harmonic oscillator.

Let me now consider an arbitrary function \( F(t) \). Then

\[ \xi(t) = \frac{1}{2m} \int_0^t dt_1 \int_0^{t_1} dt_2 F(t_2), \]

\[ \eta(t) = \frac{1}{m\omega} \int_0^t dt_1 \sin[\omega(t-t_1)]F(t_1), \]

\[ E_{\text{int}}(t) \neq 0, \]

\[ \dot{E}_{\text{int}}(t) = -\frac{\dot{F}(t)\eta(t)}{2}. \]

Similar, though not identical situations have been discussed in literature in connection with the Kohn’s theorem [11–17].

III. A MORE REALISTIC MODEL

This is a simplistic classical model of the dynamics of the FFs, here for the sake of simplicity assumed to be

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of equal masses with frozen separate neutron and proton intrinsic dynamics, basically incompressible neutron and proton fluids.

\[ H = \frac{m(\dot{x}_1^2 + \dot{y}_1^2 + \dot{x}_2^2 + \dot{y}_2^2)}{2} + V(x_1 - y_1) + V(x_2 - y_2) + U(x_1 - x_2), \]  

where \( V(x_k - y_k) \geq 0 \) is attractive and

\[ U(x_1 - x_2) = \frac{C}{|x_1 - x_2|}, \quad C = Z_1Z_2e^2, \]  

is repulsive, and the potentials are normalized as \( V(0) = 0 \) and \( U(\infty) = 0 \). The potential \( V(x_k - y_k) \) describes the attraction between the proton and neutron systems and \( U(x_1 - x_2) \) stands for the Coulomb repulsion between FFs beyond scission. The two incompressible and frozen liquids in each fragment can move with respect to each other in this model \([18–20]\).

By introducing the coordinates

\[ \xi_1 = x_1 - y_1, \quad \xi_2 = x_2 - y_2, \quad \xi = \frac{\xi_1 - \xi_2}{2}, \]  

\[ \eta = \frac{x_1 + y_1 - x_2 - y_2}{4}, \]  

\[ \zeta = \frac{x_1 + y_1 + x_2 + y_2}{4} \]  

the Hamiltonian becomes

\[ H = \frac{m(\dot{\xi}_1^2 + \dot{\xi}_2^2)}{4} + 2m(\dot{\eta}^2 + \dot{\zeta}^2) + V(\xi_1) + V(\xi_2) + U(\eta + \xi). \]  

and the equation of motion are

\[ \frac{1}{2} m \ddot{\xi}_1 = -\nabla V(\xi_1) + \frac{C(\eta + \xi)}{2|\eta + \xi|^3}, \]  

\[ \frac{1}{2} m \ddot{\xi}_2 = -\nabla V(\xi_2) - \frac{C(\eta + \xi)}{2|\eta + \xi|^3}, \]  

\[ 4m\ddot{\eta} = \frac{C(\eta + \xi)}{|\eta + \xi|^3}, \]  

\[ 4m\ddot{\zeta} = 0. \]  

The Coulomb interaction between FFs, compare the sign of the driving force in Eqs. (24) and (25), leads to intrinsic energy transfer from one fragment to the other. If one assumes that \( V(\xi_{1,2}) \) are harmonic oscillator potentials, which is a reasonable approximation in the case of giant dipole resonance (GDR), then Eqs. (24) and (25) can be solved by quadrature.

Assuming that \( \xi_{1,2}(0) = \dot{\xi}_{1,2}(0) = 0, |\eta(0)| = 2R \) (initial separation between the centers of the two touching fragments), \( \dot{\eta}(0) = 0 \), and \( \xi(0) = \dot{\xi}(0) = 0 \) then one can define the total, the intrinsic, the kinetic energy of the fragments, and the Coulomb interaction between the fragments energies and for all times after scission \( t > 0 \)

\[ E_{\text{tot}} = U(\eta(0)) = E_{\text{int}}(t) + E_{\text{TKE}}(t) > 0, \]  

\[ E_{\text{int}}(t) = \frac{m(\dot{\xi}_1^2 + \dot{\xi}_2^2)}{4} + V(\xi_1) + V(\xi_2) > 0, \]  

\[ E_{\text{TKE}}(t) = 2m\dot{\eta}^2 + U(\eta + \xi) < U_{C}. \]  

Here the intrinsic energy \( E_{\text{int}}(t) \) stands only for the combined additional excitation energy of both FFs acquired after scission, when the two fragments interact only through the long range Coulomb interaction. Thus the fragments end up excited, as initially \( E_{\text{int}}(t) = 0, \)
and the total kinetic energy of the fragments is less than the Coulomb potential energy difference \( U_C - U(\infty) = U_C = C/2R > 0 \), as one would have naively expected. Eqs. (24), (25), and (26) show that the energy exchange between intrinsic degrees of freedom \( \xi_{1,2} \) and the relative fragment degrees of freedom \( \eta \) are controlled by Coulomb interaction \( U(x_1 - x_2) \) alone.

As the amplitude of oscillations of the dipole mode is rather small, a harmonic approximation is appropriate and then

\[
V(\xi_k) = \frac{m_\omega^2|\xi_k|^2}{4} \quad (31)
\]

and

\[
\xi_1(t) + \xi_2(t) = 0 \quad (32)
\]

at all times. Initially \( E_{\text{tot}}(0) = U_C \) and as

\[
\dot{E}_{\text{int}}(t) = -\dot{E}_{\text{TKE}}(t) = \frac{C(\eta + \xi) \cdot \dot{\xi}}{|\eta + \xi|^3} = \frac{C\eta \cdot \dot{\xi}}{|\eta|^3} \quad (33)
\]

oscillates in sign with a decreasing in time amplitude of the driving force governed by \( \nabla U \). Typically \( |\eta| \gg |\xi| \) and in that case the equations for \( \eta \) and \( \xi \) can be solved using lowest order perturbation theory [21]

\[
\eta(\tau) = (0,0,R)(\cosh \tau + 1), \quad (34)
\]

\[
t(\tau) = \sqrt{\frac{4mR^3}{C}}(\sinh \tau + \tau), \quad (35)
\]

\[
\dot{\xi} = -\omega^2 \xi + \frac{C\eta}{m|\eta|^3}, \quad (36)
\]

\[
\xi(t) = \xi_0 \cos(\omega t) + \int_0^t dt_1 \frac{\sin[\omega(t - t_1)]}{m\omega} \frac{C\eta(t_1)}{|\eta(t_1)|^3} \quad (37)
\]

I have allowed here for a \( \xi(0) = \xi_0 \neq 0 \), as the two FFs just before the neck is ruptured might polarize each other, while they are practically at rest \( \xi(0) = 0 \), as suggested by the overdamped character of the collective motion before neck rupture. The initial polarization of the two FFs is given from the condition that the Coulomb force is balance initially by the restoring force of the dipole mode

\[
\xi_0 = \frac{C}{m\omega^2} \frac{\eta(0)}{|\eta(0)|^3}. \quad (38)
\]

This means that for \( t < 0 \) the nuclear system was in mechanical equilibrium at all times, a condition consistent with the strong overdamped character of the evolution from saddle-to-scission.

The parametric solution for \( \eta(t) \), Eqs. (34) and (35), is obtained by solving Eq. (26) with \( \xi = 0 \) on the right hand side and then

\[
\dot{E}_{\text{int}}(t) \approx |\xi_0| \frac{C\omega \sin(\omega t)}{|\eta(t)|^3} + \frac{C^2}{m} \int_0^t dt_1 \frac{\cos[\omega(t - t_1)]}{|\eta(t)|^2|\eta(t_1)|^2}, \quad (39)
\]

after taking into account that \( \eta(t), \eta(t_1) \) and \( \xi_0 \) are parallel, as the two fragments are touching initially with zero velocity. The energy in this case is

\[
E_{\text{tot}} = \frac{m\omega^2\xi_0^2}{2} + U_C. \quad (40)
\]

Since for all times \( E_{\text{int}}(t) \geq 0 \) the asymptotic value is obviously positive and Eq. (30) is strictly satisfied for \( t > 0 \). It is notable that in the present approximation \( E_{\text{int}}(t) \) depends only on the GDR frequency of the fragments and the mass of the proton and neutron systems, which oscillate against each other. In this model it is assumed that all protons and all neutrons are participating in the GDR.

This model neglects the excitation of other collective modes, as it assumes that the proton and neutron liquids are incompressible. When FFs are accelerated, in their own non-inertial reference frame they experience a force, which tends to pile up the nuclear matter at the edges facing each other, similarly to what happens to a an accelerated vessel with water. One thus expects that both iso-scalar and iso-vector modes are excited as seen in realistic simulations [22–25]. It also neglects the fact that the fragments are typically quite deformed and their deformation changes significantly after scission also [24, 25]. Since the shapes of the FFs evolve in time even after scission, the collective excitation energy is still dissipated due to the one-body dissipation mechanism [26].

The decay of the giant resonances into more complex particle-hole excitations, is typically described by the spreading width \( \Gamma^+ \) [27], which is a de-excitation mechanism somewhat independent of the one-body dissipation, and which is absent in this model description. Since the during the descent from the saddle to scission the motion is strongly overdamped. at the scission configuration the kinetic energy of the fragments in the fission direction is negligible [24, 25] and \( E_{\text{TKE}}(0) \approx U_C \). This is contrast with phenomenological calculations [9, 10], when the FFs have a significant kinetic energy at scission. I am aware of a single instance where the dipole excitation of the FFs was examined earlier [4], where a small increase of \( E_{\text{int}} \) was found. The behavior of \( E_{\text{TKE}}(t) \) as a function of time and the amplitude of its oscillations is qualitatively similar to what was observed in simulations [24], even though that was not either discussed or documented there.

One can include the effect of dissipation by adding damping. Assuming that that \( \omega > \gamma/2 \), which is likely a very reasonable assumption for GDR and then one obtains

\[
\dot{\xi} = -\gamma \xi - \omega^2 \xi + \frac{C\eta}{m|\eta|^3} \quad (41)
\]

\[
\omega_\pm = \pm \sqrt{\omega^2 - \frac{\gamma^2}{4}} + \frac{i\gamma}{2}, \quad (42)
\]
the changes in asymptotic value of the TXE would be also be somewhat different. Thus one can expect that separation distances between FFs at scission would the Viola parameter $Z$ Coulomb interaction of 1.6 %, and a similar change of compared to 2209 for the case of symmetric fission, which corresponds to a change in the strength of the reduced mass would be $m \rightarrow m^*$ in Eqs. (39) and (45).

For the sake of simplicity I assumed that all masses are equal and that might seem like a very strong limitation. In the case of $^{240}$Pu and symmetric fission the reduced mass of the to fragments would be 60m. In the most likely mode of asymmetric fission when $m_H \approx 135m_N$ and $m_L \approx 105m_N$ (where $m_N$ is the nucleon mass), the reduced mass would be $\approx 59m_N$. In case of asymmetric fission the $Z_H \approx 53$ and $Z_L \approx 41$ and thus $Z_HZ_L \approx 2173$ compared to 2209 for the case of symmetric fission, which corresponds to a change in the strength of the Coulomb interaction of 1.6 %, and a similar change of the Viola parameter $Z_HZ_L/A^{1/3}$ [38]. The different separation distances between FFs at scission would also be somewhat different. Thus one can expect that the changes in asymptotic value of the TXE would be also at the level of 1-3 %, which is likely within the systematic accuracy of the classical model discussed here.

IV. CONCLUSION

While the model presented here is simplified and classical, it is pretty realistic. It is straightforward to implement into such a model to consider various deformations and mass splittings of the FFs, though the calculations will certainly be more involved, but the main conclusions and the magnitude of the effects will be similar. At the same time it is unnecessary to perform such involved model calculations when realistic calculations are available [23–25, 39] and new ones are in the pipeline. The only relevant question is that of the interpretation of those realistic results, for which a simple model is particularly useful. The assumption of incompressible neutron and proton fluids when lifted it is likely to lead to the excitation of other collective modes, as noticed in time-dependent microscopic calculations [22–25]. I have shown here that the FFs exchange up to several MeVs of excitation energy, after they ceased to exchange nucleons, up to relatively large separations, due to the long range character of the Coulomb interaction between them. This excitation energy mechanism leads to slightly smaller final TKE of the FFs. In a different kind of study, Bertsch [5] argues that the long range Coulomb interaction between deformed FFs can lead to their re-orientation and as a result it can affect their angular momentum content. Similar effects are expected in the case of fragments emerging in heavy-ion reactions.

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