Modeling of flexible reciprocating compressor considering the crosshead subsidence

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Abstract. Crank-slider mechanisms are important parts of heavy duty machines, including reciprocating compressors, combustion motors. This paper targets on the dynamic response of the crosshead in a reciprocating compressor, taking into consideration the crosshead deviation from the original level. The traditional model of the compressor is usually a slider-mechanism system without considering the deflection of the crosshead, thus neglecting the influence of the piston rod, which has some flexible features. In this paper, a rigid-flexible model of slider-crank is described theoretically, using the commercial software MATLAB, where the crank, connecting rod and crosshead are treated as rigid bodies, while the piston rod connected to the crosshead is considered as a flexible body. The dynamic response of the mechanism with the crosshead subsidence is discussed detailedly in this paper. After calculated theoretically, the MATLAB simulation showed that the dynamic response of the crosshead will be greatly influenced if the crosshead subsided from the original level. Also, the influence of the crosshead subsidence was also investigated, and some extra vibration of the crosshead arises.

1. Introduction
A reciprocating compressor is one of the most widely used machines in petroleum and chemical production processes, such as gas compression, petroleum transportation and natural gas transportation [1]. Failures of linkage, crosshead and piston rod for moving parts of reciprocating compressor are not only unperceivable but also very harmful. So far, there is no efficient solution revealing the failures based on vibration signal of shell due to the lack of transferring mechanism of fault information. Previously, diagnoses for reciprocating compressors, symbolic of reciprocating machines, develops slowly in the recent years because of the complexity of signals in the compressors. The reasons can be attributed to three aspects. First of all, in each circle of the operating circle of the air cylinder, the working load of the air cylinder changes with time and so do some key parts of the reciprocating compressors. As a result, causes of the fault are very complicated to detect and diagnose. Meanwhile, the behaviour of kinetics and some fault information are highly non-linear, leading to the fact that theories suitable for rotating machines are not appropriate for reciprocating compressors [2]. Secondly, the key parts of reciprocating compressors are inside the body, which is difficult to approach and test. Simultaneously, how the vibration signals transfer in the whole process is still uncertain, although there are already some conclusions about the transferring routes. As a consequence, extractions of
fault signals are sometimes aimless and blind. Finally, the study began very late in domestic researches and the scale of researchers remains not big. Also, further researches on the failure mechanism and the extraction of symbolic signals are still an obstacle for researchers.

Actually, as a key part of reciprocating compressors, the crosshead carries a lot of useful messages, working as a bridge between behaviors of fault and the vibration of the compressor. This paper aims to reveal the intrinsic influences of the crosshead when it sank from the normal level, exploring the further information the crosshead may carry so that some theoretical basis can be put forward to being applied to the diagnose of reciprocating compressors.

Multi-body dynamics began in the 1960s when H.J. Fletcher and some of his colleagues made some fundamental work, discussing a system composed of two rigid bodies [3]. In the previous work on reciprocating compressors, most of the attention was focused on the clearance in the joints and many models for collision between joints were obtained. And there are a number of publications on the dynamic analysis of slider-crank mechanisms with clearance: Flores et al. have published several papers (e.g., [4–6]) on the dynamics of multi-body systems with imperfect joints.

2. Equations for the mechanical system for reciprocating compressors

As is shown in the Fig.1, the model for a reciprocating compressor is simplified to be composed of 3 components—the crank shafts marked 1, the connecting rod marked 2, the crosshead marked 3. First of all, it’s clear that in this simplified model, it’s a system with only one degree of freedom. So \( \theta \) is chosen as the coordinate for the system.

The geometric relationships will be shown in Eq. (1) to do some previous work for equations of motion.

\[
x_3 = r \cos \theta + l \cos \phi
\]

Where \( x_3 \) is the displacement of the crosshead, \( \theta \) is the angular of crank but in clockwise, \( \phi \) is the angular of connecting rod, \( r \) is the length of crank and \( l \) is the length of connecting rod. There is a geometric relation in this system shown in Eq. (2)

\[
r \sin \alpha + d = l \sin \phi
\]

Where, \( d \) is the value of how much the crosshead subsided from the original level and it’s also an important factor in this paper to show some dynamic behaviors of the crosshead. The displacement of crosshead is shown in Eq. (3)

\[
x_3 = r \cos \theta + l \sqrt{1 - \left(\frac{\sin \alpha + d}{l}\right)^2}
\]

In this paper, the main attention will be focused on the dynamic analysis of the crosshead on condition that there are no other fault signals in the crank. As a result, the speed of crank will be a
constant value so that more emphasis will be on the crosshead. The motion law of the crank is shown in Eq. (4).

\[ \theta = \omega t \]  \hspace{1cm} (4)

After substituting Eq. (4) to Eq. (3), the displacement of crosshead with time being the x-coordinates is given in Eq. (5). After calculating the first derivative and second derivative of time in Eq. (5), the speed and acceleration of crosshead is given in Eq. (6) and Eq. (7).

\[ x_3 = r \cos \omega t + l \sqrt{1 - \left(\frac{r \sin \omega t + d}{l}\right)^2} \]  \hspace{1cm} (5)

\[ \dot{x}_3 = -r \omega \sin \omega t - \frac{r \omega \cos \omega t (d + r \sin \omega t)}{l \sqrt{1 - \left(\frac{r \sin \omega t + d}{l}\right)^2}} \]  \hspace{1cm} (6)

\[ \ddot{x}_3 = \frac{r \omega^2 \sin \omega t (d + r \sin \omega t)}{l \sqrt{1 - \left(\frac{r \sin \omega t + d}{l}\right)^2}} - \frac{r^2 \omega^2 \cos^2 \omega t}{l^2 \left(1 - \left(\frac{r \sin \omega t + d}{l}\right)^2\right)^2} - \frac{r \omega^2 \cos \omega t}{l \sqrt{1 - \left(\frac{r \sin \omega t + d}{l}\right)^2}} - \frac{r^2 \omega^2 \cos^2 \omega t}{l^2 \left(1 - \left(\frac{r \sin \omega t + d}{l}\right)^2\right)^2} \]  \hspace{1cm} (7)

To get the response of the crosshead, displacement response, velocity response and acceleration response shown in Fig.2 are obtained. (a) is the displacement response of crosshead, (b) is the velocity response of crosshead and (c) is the acceleration response of the crosshead.

![Figure 2](image)

(a) Displacement response of the crosshead  
(b) Velocity response of the crosshead  
(c) Acceleration response of the crosshead

3. Analysis for the jumping behavior

Fig.3 is the schematic of the mechanism considering the influence of piston rod and the gas force. The two factors have actually two great influences on the crosshead which are \( F_h \) and \( F_v \), respectively. In the following passage the value of \( F_h \) and \( F_v \) will be discussed.
Firstly, the deduction of \( F_p \) will be shown below. During the whole deduction, the piston rod is considered as a cantilever beam whose length is changing according to the variation of crosshead’s displacement, which is quite different from a general cantilever beam. What’s more, another difference is that the place on which the vertical force exerted is always at the end of the piston rod, rather than a removable force. After the previous analysis, the key problem is to solve the force with the variation of piston’s length. The integral method is utilized twice.

\[
\frac{1}{\rho} = \frac{M}{EI_z} \tag{8}
\]

After substituting the initial condition to the equation to the equation, the force at one termination of the rod is given below:

\[
F_p = \frac{3dE I_z}{\rho} \tag{9}
\]

\[
L = S_0 + r + l - x_3 \tag{10}
\]

Where \( d \) denotes the value of sinking parameter, \( L \) is the length of the piston rod that is bearing the deformation and \( EI_z \) means the bending resistance of the material. \( r, l, S_0, \) and \( x_3 \) denote the length of crank, connecting rod, the length of clearance space and the displacement of crosshead, respectively. Fig.4 is the relationship between this force and the displacement of crosshead.

Secondly, \( F_h \) is the horizontal force acted by the piston rod which is originally caused by the change of gas pressure in the air cylinder. \( F_h \) can be derived by assuming that the expansion and compression process is isentropic, and air is of ideal gas. The force of friction between the crosshead and the track is not too significant to be counted in, compared with \( F_h \). The equation for the cylinder pressure \( F_h \) of each stage can be derived from the law of thermodynamics is given in Eq. (11).
Where $P_{in}$ and $P_{out}$ are the inlet and discharge pressure, respectively. The value of $m$ is usually 1.14. The relationship between $F_h$ and the displacement of crosshead is given in Fig. 5.

Figure 5. Gas force acted on the piston rod ($F_h$)

Force analysis for crosshead will be shown in Eq. (12) and Eq. (13)

$$m_3\ddot{x}_3 = F_{r23}\cos\varphi - F_h$$  \hspace{1cm} (12)

$$F_h = F_{r23}\sin\varphi - F_v$$  \hspace{1cm} (13)

Where, friction force between crosshead and its track is omitted and the gravity of crosshead is also omitted because they can make little influence to the system compared with $F_{r23}$, $F_h$ and $F_v$.

4. Results and analysis of the model considering crosshead subsidence

According to the previous analysis, whether the crosshead will jump from its original position depends on the vertical component of $F_{r23}$ (going downward) and $F_v$ (going upward). If the value of $F_{r23}\cos\varphi$ is bigger than $F_v$, the crosshead will just stay where it was. If the value of $F_{r23}\cos\varphi$ is smaller than $F_v$, the crosshead will jump to a higher position. It is clear in Fig. 6 and Fig. 7 that there are four intersections in every one circle, named A, B, C and D. Where there are intersections of the two lines, the jumping behavior will happen for sure.

Figure 6. Comparison of $F_{r23}\cos\varphi$ and $F_v$ with $x_3$ being the x-coordinate
The only difference between Fig.6 and Fig.7 is that x-coordinate is displacement of crosshead in Fig.6 while angle of crank in Fig.7. Both are drawn to illustrate the jumping position of the crosshead. From Fig.6 and Fig.7, the information that there are four jumping positions during one period, but it’s still unclear that whether there are any relationships between the values of subsidence and the jumping position. Fig.8 uses three series of data to show the brief relationship between the values of subsidence and the jumping positions. It’s clear that with the variance of subsidence, the jumping positions varies.

**5. Conclusion**

In order to analyze the vibration mechanism in a reciprocating compressor existing crosshead subsidence between crosshead and track, a model to analyze the flexibility of the piston rod is built to analyze the vibration conditions.

To conclude, if the flexibility of the piston rod is considered, extra vibration will undoubtedly take place. In a circle, the times of the impact caused by the crosshead will be varied but it’s not debated in this article. When the crosshead jumps four times in a circle, there are also some conclusions for the place where the impacts will take place. With the increase of the subsidence, the position of crosshead where the impacts take place will move to the middle position of the stroke of the slider. However, this is not the case and shouldn’t be the case in real situations because the crosshead subsidence will never...
reach that value. In real situations, the jumping position of the crank will only deviate a tiny degree from $0^\circ$ and $180^\circ$ in the direction of both sides. Anyway, this will provide a theoretical basis for future research.

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