Differentially private cross-silo federated learning

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Abstract

Strict privacy is of paramount importance in distributed machine learning. Federated learning, with the main idea of communicating only what is needed for learning, has been recently introduced as a general approach for distributed learning to enhance learning and improve security. However, federated learning by itself does not guarantee any privacy for data subjects. To quantify and control how much privacy is compromised in the worst-case, we can use differential privacy.

In this paper we combine additively homomorphic secure summation protocols with differential privacy in the so-called cross-silo federated learning setting. The goal is to learn complex models like neural networks while guaranteeing strict privacy for the individual data subjects. We demonstrate that our proposed solutions give prediction accuracy that is comparable to the non-distributed setting, and are fast enough to enable learning models with millions of parameters in a reasonable time.

1 Introduction

Privacy is increasingly important for modern machine learning. Commonly, the sensitive data needed for learning are distributed to different parties, such as individual hospitals and companies or IoT device owners. Federated learning (FL) has been recently introduced in order to reduce the risks to privacy and improve model training. The key idea in FL is that only the information necessary for learning a model will be communicated while the actual data stays distributed on the individual devices.

FL makes attacking the system harder since no centralised server holds all the data. Nevertheless, it has been recently demonstrated that FL by itself is not enough to guarantee any level of privacy. The current gold standard in privacy-preserving machine learning is differential privacy (DP), which is based on clear mathematical definitions and aims to guarantee a level of indistinguishability for any individual data subject: the results of learning would be nearly the same if any single data entry was removed or arbitrarily replaced with another.

In this paper, we concentrate on FL with DP to enable efficient learning on distributed data while guaranteeing strong privacy. We focus on the so-called cross-silo FL, where the number of parties might range from a few to tens of thousands, and each party generally holds from tens to some thousands of samples of data.
Additionally, we consider using trusted execution environments (TEEs) as an extra layer of security.

To guarantee strict privacy with good utility, it is often vital that privacy can be amplified by the stochasticity resulting from using data subsampling [29]. This requires that the origin of the samples in a minibatch remains hidden from other parties. We therefore introduce an algorithm that enables oblivious distributed subsampling with an arbitrary sampling scheme.

The two most used sampling schemes with privacy amplification are sampling without replacement, where the minibatch size is fixed and each sample has equal probability of being included, and Poisson sampling, where the probability of independently including each sample is fixed and the batch size varies. We show that when malicious parties are present, using a simple distributed Poisson sampling scheme generally gives better privacy than using a more complex distributed sampling without replacement scheme.

Finally, we argue both theoretically and on empirical grounds that by using random projections, we can scale our methods up to even larger models while only incurring a modest loss in prediction accuracy.

The code for all the experiments is freely available [20].

1.1 Contribution

We consider DP cross-silo FL in two settings: with pairwise connections between the parties (feasible for at most tens to hundreds of parties), and with connections to central servers (hundreds to tens of thousands of parties).

We present and empirically test two methods based on combining fast secure multiparty-computation (SMC) protocols and DP, with emphasis on iterative learning and distributed subsampling. The methods can be used for learning complex models such as neural networks from large datasets under strict privacy guarantees, with reasonable computation and communication capacities. The proposed method without pairwise connections is based on a novel secure summation algorithm, preliminarily presented in an earlier conference publication [21].

For achieving privacy amplification by subsampling in the distributed setting, we propose an algorithm for oblivious subsampling under an arbitrary sampling scheme. We also show that a simpler distributed Poisson sampling scheme is preferable to a more complex scheme of distributed sampling without replacement, when malicious parties are present.

To speed up encryption and reduce the amount of communication needed, we also consider communicating only randomly projected low-dimensional representations. We argue theoretically and show with experiments that the effect on utility under DP is small.

2 Background

In the next sections we give a short review of the necessary theory, starting with DP in Section 2.1 and continuing with random projections in Section 2.2 and trusted execution environments in Section 2.3.

2.1 Differential privacy

Differential privacy [17, 15] has emerged as the leading method for privacy-preserving machine learning. On an intuitive level, DP guarantees a level of deniability to each data subject: the results would be nearly the same if any single individual’s data were replaced with an arbitrary sample. More formally we have the following:

Definition 2.1. (Differential privacy) A randomised mechanism \( \mathcal{M} : D \rightarrow R \) is \((\epsilon, \delta)\)-DP, if for all neighbouring datasets \( D, D' \in D : D \sim D' \) and for any measurable \( E \subseteq R \),

\[
P(\mathcal{M}(D) \in E) \leq e^\epsilon P(\mathcal{M}(D') \in E) + \delta,
\]

where \( \epsilon > 0, \delta \in [0, 1] \). If \( \delta = 0 \), the mechanism is called (pure) \( \epsilon \)-DP.

We consider two specific neighbouring relations: when \( |D| = |D'| \) and they differ by a single element (so-called bounded DP), the relation is called a substitution relation and denoted by \( \sim_s \). When one can be transformed into the other by removing or adding a single element (unbounded DP), the relation is called a remove/add relation and is denoted by \( \sim_R \).

One very general method for guaranteeing DP is the Gaussian mechanism, which amounts to adding independent Gaussian noise to the learning algorithm. The amount of privacy then depends on the sensitivity of the function and the noise magnitude:

Definition 2.2. (Sensitivity) Let \( f : D \rightarrow \mathbb{R}^d \). The \( \ell_2 \)-sensitivity of \( f \) is defined as

\[
\Delta = \sup_{D, D' \in T : D \sim D'} \| f(D) - f(D') \|_2,
\]

where \( \sim \) denotes a general neighbourhood relation.

Definition 2.3. (Gaussian mechanism) Let \( f : D \rightarrow \mathbb{R}^d \) with sensitivity \( \Delta \). A randomised mechanism \( \mathcal{G}_f : D \rightarrow \mathbb{R}^d \),

\[
\mathcal{G}_f(D) = f(D) + \mathcal{N}(0, \sigma^2 \cdot I_d)
\]

is called the Gaussian mechanism.
Composition refers to DP mechanism(s) accessing a given dataset several times, which either weakens the privacy guarantees or requires increasing the noise magnitude guaranteeing privacy. No analytical formula is known for the exact privacy guarantees provided by the Gaussian mechanism for a given number of compositions when using data subsampling. However, as shown in [34, 30], they can be calculated numerically with arbitrary precision.

In effect, for some function $f$, we can numerically calculate the amount of privacy for a given number of compositions when using the subsampled Gaussian mechanism with a chosen sequence of noise values. This is called privacy accounting. In practice, we use the privacy loss accountant of [30] for all the actual privacy calculations.

### 2.2 Dimensionality reduction

Using SMC protocols is generally expensive in terms of communication and computation, and even with faster SMC protocols the overhead can be significant. We therefore also consider dimensionality reduction by a random projection to scale the methods to even larger problems.

Instead of using SMC protocols to send a vector of dimension $d$, each party only communicates a projection with dimension $k \ll d$. The representation is obtained using a random projection matrix generated with a shared seed.

For the projection, we use a mapping $f_{JL}(u) = P^T u$, $P \in \mathbb{R}^{d \times k}$ s.t. each element of $P$ is drawn independently from $\mathcal{N}(0, 1/k)$. Given the low-dimensional representation $P^T u$ and $P$, since $\mathbb{E}_P[PP^T u] = u$, the original vector $u$ can be approximated as $u \simeq PP^T u$. Moreover, as shown in [12], with large enough $k$ the Johnson-Lindenstrauss lemma guarantees that $u^T PP^T u \simeq u^T u$. These properties explain our experimental results in Section 4, showing that when training under DP and with $k \ll d$, learning with projections results in nearly the same prediction accuracy as using full-dimensional gradients.

To guarantee privacy when the privacy mechanism operates on a low-dimensional projection and the projection matrix is public, we first define sensitivity bounded functions and then state the essential lemma, both introduced in [3]:

**Definition 2.4 (Function sensitivity).** A randomised function $f : D \rightarrow X$ is $(\Delta_f, \delta)$-sensitive, if for any $D, D' \in D : D \sim D'$, there exist coupled random variables $X, X' \in \mathbb{X}$ s.t. the marginal distributions of $X, X'$ are identical to those of $f(D), f(D')$, and

$$\mathbb{P}_{X,X'}(\|X - X'\|_2 \leq \Delta_f) \geq 1 - \delta.$$

**Lemma 2.1.** Let $M : \mathbb{X} \rightarrow \mathbb{R}$ be an $(\epsilon, \delta)$-DP mechanism for sensitivity $\Delta_f$ queries, and let $f : D \rightarrow \mathbb{X}$ be a $(\Delta_f, \delta')$-sensitive function. Then the composed mechanism $M(f(D))$ is $(\epsilon, \delta + \delta')$-DP.

### 2.3 Trusted execution environments

When the parties running the distributed learning protocols have enough computational resources available, we can increase security by leveraging trusted execution environments (TEEs).

TEEs aim to provide a solution to the general problem of establishing a trusted computational environment by providing confidentiality, integrity, and attestation for any computations run inside the TEE: no adversary outside the environment should 1) be able to gain any information about the computations done inside it (confidentiality), nor 2) be able to influence the computations done by the TEE (integrality), and 3) the TEE should be able to give an irrefutable and unforgeable proof that the computation has been done inside it (attestation) [41].

In effect, TEE provides a securely isolated area in which users can store sensitive data and run the code they want protected. TEE is realised with a support of hardware, and it aims to protect applications from software and hardware attacks. Currently, technologies such as Intel’s Software Guard Extensions (SGX) and ARM’s TrustZone are used for implementing TEE.

It should be emphasised that the security of TEEs depends not only on software but also on the hardware manufactures, and should not be generally accepted at face value; various attacks targeting TEEs have been reported, including arbitrary code executions [38] and side-channel attacks [17, 44].

In this paper we assume TEEs are used by well-resourced parties as detailed in Section 8. However, we do not rely exclusively on TEEs for doing secure computations, but only propose them as an additional layer of security to make attacking the system harder.

### 3 DP cross-silo federated learning

In the general setting we consider, there are $N$ data holders (clients), the data $x$ are horizontally partitioned, i.e., each client has the same features, and the $i$th client has $n_i$ data points. The privacy guarantees are given on an individual sample level.

Our main focus is learning complex models using gradient-based optimisation. Using the standard em-
pirical risk minimisation framework \[45\], given a loss function \( L(h(x)), h \in \mathcal{H} \), where \( \mathcal{H} \) is some function class and \( x \) includes the target variable, we would like to find \( h_{OPT} = \arg \min_{h \in \mathcal{H}} L(h(x)) \). However, since we do not know the underlying true data distribution, we instead minimise the average loss over data we do not know the underlying true data distribu-

tion. Let \( \hat{h}_b \) be the model parameter vector, and \( \eta \) is the noise that guarantees privacy (see Definition 2.3). In the following, we write \( z_{ij} \triangleq \nabla_\theta \tilde{L}(x_{ij}|\theta) \).

With small problems, (3) can be readily computed with general secure multiparty computation protocols, as done e.g. in \[20\]. In our case this is not feasible due to high computational and communication requirements.

From the privacy perspective, an ideal distributed learning algorithm would have DP noise magnitude equal to the corresponding non-distributed learning algorithm run by a single trusted party on the entire combined dataset. We refer to this baseline as the trusted aggregator version.

Another important property for distributed private learning, that is mostly orthogonal to the amount of noise added, is graceful degradation: when the privacy guarantees are affected by adversarial parties, we want to avoid catastrophic failures where a single party can (almost) completely break the privacy of others. Instead, the privacy guarantees should degrade gracefully with the number of adversaries.

It is readily apparent from (3) that the problem can be broken into two largely independent components: distributed noise addition and secure summation. We will consider these problems separately in sections 3.2 and 3.3 respectively.

3.1 Threat model

We allow for three types types of parties: honest, (non-colluding) honest-but-curious (hbc) and malicious.

The only honest parties we include are TEEs. We also consider the possibility of an adversary gaining full control of some number of TEEs, which effectively turns these into malicious parties.

The privacy guarantees of the methods are valid against fully malicious parties, whereas the accuracy of the results is only guaranteed with malicious parties who avoid model poisoning attacks or disrupting the learning altogether.

3.2 Distributed noise addition

In order to guarantee privacy, we need to add noise to the model parameter vector to the corresponding non-distributed learning algorithm run by a single trusted party on the entire combined dataset. We refer to this baseline as the trusted aggregator version.

For the case without TEEs (or to protect against compromised TEEs with obvious modifications), we can upscale the noise to introduce a trade-off between the noise level and protection against colluders: assuming \( T \) colluders, we need

\[
\sigma_i^2 \geq \frac{1}{N-T-1} \sigma^2
\]

(see e.g. \[21\] for a proof).

The total noise level in this case is sub-optimal compared to the trusted aggregator setting, but for moderate \( T \) and larger \( N \) the noise level is still close to optimal. Even if the actual number of colluders exceeds the parameter \( T \) used in the algorithm, the privacy guarantees still weaken gracefully with the number of malicious parties.

3.3 Secure summation

We consider two fast secure summation protocols. If the number of clients is low (at most some dozens), we can use fast homomorphic encryption based on secret sharing using pairwise keys. The algorithm was originally introduced in \[8\], uses fixed-point representation of real numbers, and is based on fast modulo-addition.

For convenience, we state the algorithm in the Appendix A, where we also detail our implementation used in Section 4.

When the number of clients is larger, we instead use the Distributed Compute Algorithm (DCA) given in
Algorithm 1 Distributed Compute Algorithm: secure summation for a large number of clients

**Require:** Number of parties \( N \) (public);
Number of compute nodes \( M \) (public);
Upper bound for the total sum \( R \) (public);
\( y_i \), integer held by client \( i \), \( i = 1, \ldots, N \).

**Ensure:** Securely calculated sum \( \sum_{i=1}^{N} y_i \).

1. Each client \( i \) simulates \( M - 1 \) vectors \( u_{il} \) of uniformly random integers at most \( R \), and sets \( u_{iM} = -\sum_{l=1}^{M-1} u_{il} \mod R \).
2. Client \( i \) computes the messages \( m_{i1} = y_i + u_{i1} \mod R \), \( m_{il} = u_{il}, l = 2, \ldots, M \), and sends them securely to the corresponding compute node.
3. After receiving messages from all of the clients, compute node \( l \) broadcasts the noisy aggregate sums \( q_l = \sum_{i=1}^{N} m_{il} \). A final aggregator will then add these to obtain \( \sum_{i=1}^{N} q_l \mod R = \sum_{i=1}^{N} y_i \).

Algorithm 2 Create a list of tokens

**Require:** Each party \( i \) has random integers \( r_{ij}, j = 1, \ldots, n_i \) called tokens that are all unique, and only \( i \) knows who \( r_{ij} \) belongs to. There is a shared (arbitrary) ordering over the parties, assumed w.l.o.g. to be \( 1, \ldots, N \). All parties know the public keys of all other parties.

1. Party \( i \) encrypts all its tokens with the public keys of all parties according to the ordering: \( r_{ij} \rightarrow Enc_{k_i}(Enc_{k_2}(\ldots Enc_{k_N}(r_{ij}))), j = 1, \ldots, n_i \)
2. All parties publish all encrypted values to form a list with some ordering.
3. \textbf{for} \( i = 1 \) to \( N \) \textbf{do}
4. Party \( i \) decrypts layer \( i \) from all the messages on the current list, randomly permutes the elements, and publishes the resulting new list.
5. \textbf{end for}
6. Returns a list of tokens \( r_{ij}, i = 1, \ldots, N, j = 1, \ldots, n_i \) s.t. only the owner of a token knows who it belongs to.

3.4 Learning with minibatches

For iterative learning, sub-sampling is often essential for good performance. This is especially true for DP learning with limited dataset sizes due to privacy amplification results: the uncertainty induced by using only a batch of the full data augments privacy, when the identities of the samples in a batch are kept secret. We therefore want a method for choosing a batch \( [b] = \cup_{i=1}^{N} [b_i] \) in the distributed setting s.t. the full batch is drawn according to a given sampling scheme and the identity of the batch participants is kept hidden.

Next, we formulate a general distributed sampling method that avoids the possibility of a catastrophic failure and guarantees a graceful degradation of privacy even in the presence of malicious adversaries. The main idea is to generate a list of random tokens such that there are as many tokens as there are observations in the full joint dataset, and only the owner of a given token knows who it belongs to.

Given such a list, we can use a simple joint sampling strategy: the participants jointly generate a seed for a PRNG, which is used to determine the batch participants without needing any additional communication. In case some batch \([b_i]\) is empty, the corresponding party sends only the noise necessary for DP. Note that this message cannot be distinguished from any other message, thus keeping the actual batch participants secret as required for privacy amplification.

To generate the list of tokens, we use Algorithm 2 based on mixnets [9].
Theorem 1. Assume the public-key encryption used in Algorithm 2 is secure. Then i) the list cannot be manipulated in nontrivial ways, ii) to hbc parties all tokens belonging to other parties are indistinguishable from each other and to malicious parties all tokens belonging to hbc parties are indistinguishable from each other.

Proof. See Appendix B.

Generating the list of random tokens is an expensive step by itself in terms of communication and computations, but it suffices to create the list only once since the owners of the tokens are not revealed at any point when using the list. The privacy guarantees resulting from using the list depend on the actual sampling scheme used.

### 3.4.1 Comparison of common sampling schemes

As noted, the two most used sampling schemes with privacy amplification are sampling without replacement (SWOR), where the minibatch size $b$ is fixed and each sample has equal probability $b/n$ of being selected, and Poisson sampling, where we fix the probability $\gamma \in (0, 1)$ of independently including each observation while the batch size varies.

In the distributed setting, SWOR can be done using Algorithm 2. However, Poisson sampling is also achievable in a more straightforward manner, since each client can simply choose to include each sample independently with the given probability. As shown in the rest of this Section, when malicious parties control some of the data and know if they are included in the batch or not, the simpler Poisson sampling leads to better privacy amplification than SWOR.

With SWOR, assume we sample a set of $b$ tokens uniformly at random from the set of all such sets from the list of tokens generated with Algorithm 2 and let $\gamma = b/n$, so the mean batch size from Poisson sampling matches SWOR.

Writing $[T]$ for the set of $T$ malicious parties, let the total number of samples controlled by the non-malicious parties be $n_{-T} = \sum_{i \in [T]} n_i$. Using SWOR, the actual number of samples coming from the non-malicious parties now follows a Hypergeometric distribution with total population size $n_{-T}$, total number of successes in the population $n_{-T}$, and number of draws $b$.

A worst-case amplification factor is then given by $\frac{b n_{-T}}{n}$ i.e., all the samples in a batch come from the non-malicious parties. However, in most settings this is a rather unlikely event since the distribution is concentrated around the mean $\frac{b n_{-T}}{n}$. We can therefore improve markedly on the worst-case if we admit some amount of slack in the privacy parameter $\delta$ as shown in Figure 1. However, the amplification factor is still worse than the baseline given by Poisson sampling when there are any malicious data holders present.

![Figure 1: Effective sampling fraction for privacy amplification](image)

### 3.5 DP random projection

To enable using the methods with even larger models, we propose using a low-dimensional random embedding instead of the full gradient in learning. Although the saving is only on uploaded gradients, this presents the most significant costs as it requires encryption, whereas downloading the updated parameter values can be done in the clear due to DP. In the distributed setting, we do not need to communicate the full projection matrix but only the seed for a PRNG.

**Algorithm 3** DP random projection

**Require:** Clipped gradient vectors $z_{ij} \in \mathbb{R}^d, i \in \{1, \ldots, N\}, j \in \{1, \ldots, n_i\}$ with sensitivity $C$, projection dimension $k$, projection sensitivity bound $\hat{C}$, Gaussian mechanism $\mathcal{G}_f$ that is $(\epsilon, \delta)$-DP on a sensitivity $\hat{C}$ query.

1. Generate a projection matrix $P \in \mathbb{R}^{d \times k}$ s.t. each element is independently drawn from $\mathcal{N}(0, 1/k)$.
2. Sum the projected clipped gradients: $\tilde{z} = \sum_i \sum_j P_{ij} z_{ij}$.
3. Return a DP projection $\mathcal{G}_f(\tilde{z})$.

We will next show that when a DP mechanism oper-
ates on the k-dimensional representation and the projection matrix is public, as described in Algorithm 3, the result is still DP. The result holds for both neighbouring relations ∼_R, ∼_S we consider.

**Theorem 2.** Algorithm 3 is (ε, δ + δ')-DP, with any C > 0 and δ' > 0 s.t.

\[ \Pr \left[ \Gamma(K = \frac{k}{2}) \theta = \frac{2C^2}{k} \leq \tilde{C}^2 \right] \geq 1 - \delta', \]

where k is the projection dimension, C is the gradient sensitivity, and Γ is the (shape & scale parameterised) Gamma distribution.

**Proof.** See Appendix B for a proof.

To determine the sensitivity bound \( \tilde{C} \) in Theorem 2, we decide on the additional privacy parameter \( \delta' > 0 \) and search for the smallest value of \( \tilde{C} \) s.t. the condition in Theorem 2 holds. Then, as stated in the Theorem, when the Gaussian mechanism we use is (ε, δ)-DP on a sensitivity \( \tilde{C} \) query, the final result is (ε, δ + δ')-DP.

In practice, each client locally runs step (1) in Algorithm 3 with a shared random seed to get the individual noisy projected gradients and add Gaussian noise that will sum up to the desired variance as specified in Section 3.2. The aggregation is then done using the secure summation as discussed in Section 3.3. After receiving the final decrypted sum \( \tilde{G}_f(z) \), the master inverts the projection by calculating \( PG_f(z) = PP^T [\sum_i \sum_j z_{ij} + \eta_{ij}] \approx \sum_i \sum_j z_{ij} + \eta \), where \( \eta_{ij} \) are the noise terms that sum up to the chosen DP noise as in [3], and takes an optimization step with the resulting d-dimensional DP approximate gradient.

### 3.6 Use cases

We focus on two distinct scenarios:

1. **Fat clients:** \( N \) is small, the \( n_i \) are moderate, the clients have fair computation and communication capacities

2. **Thin clients:** \( N \) is large, \( n_i \) are small, the clients have more limited computation and communication capacities

The two main scenarios we consider are motivated by considering \( n_i \), the number of samples per client, and \( N \), the number of clients. In order to decrease the effects of the DP noise we need more data, i.e., we need to increase either \( n_i \) or \( N \). As the \( n_i \) grow the benefits from doing distributed learning start to vanish.

In other words, the most interesting regime for distributed learning ranges from each client having a single sample to each having a moderate amount of data. As for the number of parties, increasing \( N \) naturally increases the total amount of communication needed while also limiting the set of practical secure protocols.

A typical example of fat clients is research institutions or enterprises that possess some amount of data but would benefit significantly from doing learning on a larger joint dataset.

A stereotypical case of thin clients is learning with IoT devices, where the devices we mainly consider would be e.g. lower-level IoT nodes, which aggregate data from several simple edge devices.

### 4 Experiments

As noted earlier, the code for all the experiments is freely available [29].

We first test our proposed solution for distributed learning using sampling without replacement and substitution relation ∼_S with CIFAR10 dataset [31] and a convolutional neural network (CNN): the convolutional part uses 2 convolutional layers (CL) with kernel size 3, stride 1 and 64 channels, each followed by max pooling with kernel size 3 and stride 2. After the CL, the model has 2 fully connected (FC) layers with 384 hidden units each with ReLU and a softmax classification layer. The same basic model structure has been previously used e.g. in [1]. Similarly to [1], we pretrain the CL weights using the CIFAR100 dataset assumed to be public and keep them fixed during the training on private data. However, unlike [1], besides having a different sampling scheme and neighbourhood relation, we also initialise the FC layer weights randomly, and not based on pretraining.

We note that the ∼ 75% accuracy for the non-private model is not near the state of the art for the CIFAR10 (e.g. [22] reach ∼ 99%); the model architecture was chosen to illustrate how distributed DP affects the accuracy in a commonly used and reasonably well-understood standard model. The actual training is done using the well-known DP-SGD [40] but using the privacy accountant of [30]. For testing encryption speed, we use the MNIST dataset [33] and 2 FC layers with 536 hidden units each with ReLU and a softmax classification layer. The same basic model structure has been previously used e.g. in [1]. Similarly to [1], we pretrain the CL weights using the CIFAR100 dataset assumed to be public and keep them fixed during the training on private data. However, unlike [1], besides having a different sampling scheme and neighbourhood relation, we also initialise the FC layer weights randomly, and not based on pretraining.

The tests were run on clusters with 2.1GHz Xeon Gold 6230 CPUs. All the tests use 1 core to simulate a single party. We report the results using our Python implementation of the algorithms. Run time results for fat clients can be found in Appendix C.
We use Python’s Secrets module to generate randomness in Algorithm 1, which internally utilises urandom calls. Generating a single 32-bit random integer with the Secrets module takes about $2.15 \times 10^{-6}$ seconds.

The values are sent by writing them on a shared disk without additional symmetric encryption. In real implementations all communications would be done over the available communication bandwidth using some standard symmetric encryption such as AES. We do not include this nor the possible TEE component in the testing but focus on the feasibility of learning based on Algorithms 1 and 3.

The baselines for accuracy are given by the centralised trusted aggregator setting, as well as local DP (LDP) [14], which means that each participant locally adds enough noise to protect its contributions without any encryption scheme. As shown in Figure 2, there is a tradeoff between privacy and accuracy. However, combining the secure summation protocols with DP, as we propose, provides significantly better accuracy than is possible by each party independently protecting their privacy.

Figure 2: Mean test accuracy and error bars showing min & max over 5 runs with fat clients on CIFAR10 data, $\delta = 10^{-5}$. Our method (DP-SMC, DP-SMC Poisson) gives significantly better results than LDP especially in the stricter privacy regime $\epsilon < 2$. The performance of DP-SMC using either sampling without replacement (DP-SMC) or Poisson sampling (DP-SMC Poisson) is indistinguishable from the centralised scheme with a trusted aggregator that uses either sampling without replacement (trusted DP) or Poisson sampling (not shown).

The tradeoff between running times and the security parameters is shown in Figure 3. The results show that our solutions can be realistically used for learning complex models with distributed data.

We can use Algorithm 3 to reduce the computational burden due to cryptography and the amount of communication needed at the cost of having to generate projection matrices and losing some prediction accuracy. As shown in Figure 4, even with very large savings on communication and decreased runtime, we still incur only modest decrease in accuracy.

Figure 3: Fold increase in total running time with varying number of compute nodes for 100 clients, medians over 5 runs. The number of compute nodes can be tuned to adjust the tradeoff between security and running time. Using MNIST data, model 1m has 2 FC hidden layers with $\simeq 1e6$ parameters.

5 Related work

There is a wealth of papers on distributed DP with secure summation as witnessed by a recent survey [18], many of which are relevant also for DP federated learning. The problem of distributed DP was first formulated in [16], who propose to generate DP noise collaboratively using a SMC protocol. The idea of using a general SMC for distributed DP has been more recently considered e.g. in [26]. The main drawback with these methods is the computational burden.

The first practical method for implementing DP queries in a distributed manner was the distributed Laplace mechanism presented in [37]. Closely related methods have been commonly used in the literature ever since [39, 2, 18, 21, 25], including in this work.

There has been much new work on distributed learning in the FL setting [35], as evidenced by a recent survey [28]. Most of the work has concentrated on the cross device case with typically millions of clients each holding at most a few samples. Meanwhile, the cross-silo setting has seen significantly fewer contributions, especially focusing on privacy.

Of the closest existing work, in [10] the authors con-
sider privacy in cross-silo FL setting without DP, but their approach lacks any formal privacy guarantees. In \cite{46} the focus is on deriving results for training models on the clients’ local data under DP, where the privacy guarantees are derived directly on the communicated parameters instead of gradients. However, all the reported experimental results use privacy budgets that are completely vacuous in practice ($\epsilon \geq 50$).\cite{43} concentrates on training several ML models in the cross-silo FL setting with DP. Their solution uses essentially the same distributed noise generation but slower encryption, does not use TEEs nor address sampling schemes other than the distributed Poisson sampling. They also do not consider communicating low-dimensional embeddings.

TEEs have recently been gaining popularity in machine learning literature. Privacy-preserving machine learning using TEEs was proposed in \cite{36}, who discuss security of enclaves especially against data access pattern leaks. \cite{4} concentrate on memory access leaks in SGX, whereas \cite{4} propose a new DP definition to account for the possible privacy breaches resulting from data and memory access leaks. \cite{21} focus on how to implement efficient operations needed for a ML framework built on TEEs and DP. Several recent papers have investigated efficient execution of DNNs and other models in TEEs \cite{42,23,19,32}.

6 Conclusion

We have presented two methods for cross-silo FL with strict privacy-guarantees based on fast homomorphic encryption and distributed noise addition, and shown empirically that they can be used for learning complex models like neural networks with reasonable running times. Finally, we have shown how the scale the methods to even larger models by using random projections while incurring only modest cost in terms of prediction accuracy.

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References

\begin{thebibliography}{10}
\bibitem{1} M. Abadi, A. Chu, I. Goodfellow, H. B. McMahan, I. Mironov, K. Talwar, and L. Zhang. Deep learning with differential privacy. In \textit{Proceedings of the 2016 ACM SIGSAC Conference on Computer and Communications Security}, CCS ’16, pages 308–318, New York, NY, USA, 2016. ACM.
\bibitem{2} G. Ács and C. Castelluccia. I have a DREAM! (DiffeRentially privatE smArt Metering). In T. Filler, T. Pevny, S. Craver, and A. Ker, editors, \textit{Information Hiding: 13th International Conference, IH 2011, Prague, Czech Republic, May 18-20, 2011, Revised Selected Papers}, pages 118–132. Springer Berlin Heidelberg, Berlin, Heidelberg, 2011. ISBN 978-3-642-24178-9. doi: 10.1007/978-3-642-24178-9_9. URL \url{http://dx.doi.org/10.1007/978-3-642-24178-9_9}.
\bibitem{3} N. Agarwal, A. T. Suresh, F. Yu, S. Kumar, and H. B. McMahan. CpSGD: Communication-efficient and differentially-private distributed SGD. In \textit{Proceedings of the 32nd International Conference on Neural Information Processing Systems}, NIPS 2018, pages 7575–7586, Red Hook, NY, USA, 2018. Curran Associates Inc.
\bibitem{4} J. Allen, B. Ding, J. Kulkarni, H. Nori, O. Ohrimenko, and S. Yekhanin. An algorithmic framework for differentially private data analysis on trusted processors. \textit{CoRR}, abs/1807.00736, 2018. URL \url{http://arxiv.org/abs/1807.00736}.
\bibitem{5} J.-P. Aumasson, S. Neves, Z. Wilcox-O’Hearn, and C. Winnerlein. BLAKE2: Simpler, smaller, fast as MD5. In M. Jacobson, M. Locasto, P. Mohassel, and R. Safavi-Naini, editors, \textit{Applied Cryptography and Network Security}, pages 119–135, Berlin, Heidelberg, 2013. Springer Berlin Heidelberg. ISBN 978-3-642-38980-1.
\end{thebibliography}
[25] H. Imtiaz, J. Mohammadi, and A. D. Sarwate. Distributed differentially private computation of functions with correlated noise, 2019.

[26] B. Jayaraman, L. Wang, D. Evans, and Q. Gu. Distributed learning without distress: Privacy-preserving empirical risk minimization. In S. Bengio, H. Wallach, H. Larochelle, K. Grauman, N. Cesa-Bianchi, and R. Garnett, editors, *Advances in Neural Information Processing Systems 31*, pages 6346–6357. Curran Associates, Inc., 2018.

[27] W. B. Johnson and J. Lindenstrauss. Extensions of Lipschitz maps into a Hilbert space. 1984.

[28] P. Kairouz, H. B. McMahan, B. Avent, A. Bellet, M. Bennis, A. N. Bhagoji, K. Bonawitz, Z. Charles, G. Cormode, R. Cummings, R. G. L. D’Oliveira, S. E. Rouayheb, D. Evans, J. Gardner, Z. Garrett, A. Gasçón, B. Ghazi, P. B. Gibbons, M. Gruteser, Z. Harchaoui, C. He, L. He, Z. Huo, B. Hutchinson, J. Hsu, M. Jaggi, T. Javidi, G. Joshi, M. Khodak, J. Konečný, A. Korolova, F. Koushanfar, S. Koyejo, T. Lepeitont, Y. Liu, P. Mittal, M. Mohri, R. Nock, A. Özgür, R. Pagh, M. Raykova, H. Qi, D. Ramage, R. Raskar, D. Song, W. Song, S. U. Stich, Z. Sun, A. T. Suresh, F. Tramèr, P. Vepakomma, J. Wang, L. Xiong, Z. Xu, Q. Yang, F. X. Yu, H. Yu, and S. Zhao. Advances and open problems in federated learning, 2019.

[29] S. Kasiviswanathan, H. Lee, K. Nissim, S. Raskhodnikova, and A. Smith. What can we learn privately? *SIAM Journal on Computing*, 40(3):793–826, 2011. doi: 10.1137/090756090. URL https://doi.org/10.1137/090756090

[30] A. Koskela, J. Jälkö, and A. Honkela. Computing tight differential privacy guarantees using fft. In *International Conference on Artificial Intelligence and Statistics*, pages 2560–2569, 2020.

[31] A. Krizhevsky. Learning multiple layers of features from tiny images. *University of Toronto*, 05 2012.

[32] R. Kunkel, D. L. Quoc, F. Gregor, S. Arnautov, P. Bhatotia, and C. Fetzer. TensorSCONE: A secure tensorflow framework using Intel SGX. *arXiv preprint arXiv:1902.04413*, 2019.

[33] Y. Lecun, L. Bottou, Y. Bengio, and P. Haffner. Gradient-based learning applied to document recognition. *Proceedings of the IEEE*, 86(11):2278–2324, Nov 1998. ISSN 1558-2256. doi: 10.1109/5.726791.

[34] D. M. Sommer, S. Meiser, and E. Mohammadi. Privacy loss classes: The central limit theorem in differential privacy. *Proceedings on Privacy Enhancing Technologies*, 2019:245–269, 04 2019. doi: 10.2478/popets-2019-0029.

[35] B. McMahan, E. Moore, D. Ramage, S. Hampson, and B. A. y Arcas. Communication-efficient learning of deep networks from decentralized data. In *Proceedings of the 20th International Conference on Artificial Intelligence and Statistics*, AISTATS 2017, 20-22 April 2017, *Fort Lauderdale, FL, USA*, pages 1273–1282, 2017. URL http://proceedings.mlr.press/v54/mcmahan17a.html.

[36] O. Ohrimenko, F. Schuster, C. Fournet, A. Mehta, S. Nowozin, K. Vaswani, and M. Costa. Oblivious multi-party machine learning on trusted processors. In *Proceedings of the 25th USENIX Conference on Security Symposium*, SEC’16, pages 619–636, Berkeley, CA, USA, 2016. USENIX Association. ISBN 978-1-931971-32-4. URL http://dl.acm.org/citation.cfm?id=3241094.3241143

[37] V. Rastogi and S. Nath. Differentially private aggregation of distributed time-series with transformation and encryption. In *Proceedings of the 2010 ACM SIGMOD International Conference on Management of Data*, SIGMOD ’10, pages 735–746, New York, NY, USA, 2010. ACM. ISBN 978-1-4503-0032-2. doi: 10.1145/1807167.1807247. URL http://doi.acm.org/10.1145/1807167.1807247

[38] D. Rosenberg. QSEE TrustZone kernel integer over flow vulnerability. In *Black Hat conference*, page 26, 2014.

[39] E. Shi, T. Chan, E. Rieffel, R. Chow, and D. Song. Privacy-preserving aggregation of time-series data. Proceedings of NDSS, 2011.

[40] S. Song, K. Chaudhuri, and A. D. Sarwate. Stochastic gradient descent with differentially private updates. In *2019 IEEE Global Conference on Signal and Information Processing*, pages 245–248, Dec 2013. doi: 10.1109/GlobalSIP.2013.6730861.

[41] P. Subramanyan, R. Sinha, I. Lebedev, S. Devadas, and S. A. Seshia. A formal foundation for secure remote execution of enclaves. In *Proceedings of the 2017 ACM SIGSAC Conference on Computer and Communications Security*, CCS 2017, pages 2435–2450, New York, NY, USA, 2017. Association for Computing Machinery. ISBN 9781450349468. doi: 10.1145/3133956.3134098. URL https://doi.org/10.1145/3133956.3134098
The algorithm was originally introduced in [8], uses fixed-point representation of real numbers, and is based on fast modulo-addition.

### Algorithm 4 Secure summation for fat clients

**Require:** Upper bound for the total sum \( R \) (public);
- \( y_i \) integer held by party \( i, i = 1, \ldots, N \);
- Pairwise secret keys \( k_{ij} \) held by party \( i, i, j = 1, \ldots, N, i \neq j \) s.t. \( k_{ij} + k_{ji} = 0 \mod R \).

**Ensure:** Securely calculated sum \( \sum_{i=1}^{N} y_i \).

1. Each client \( i \) calculates \( Enc(y_i, k_i, R) = y_i + k_i = y_i + \sum_{j \neq i} k_{ij} \mod R \), and sends the result to the aggregator.
2. After receiving messages from all other parties, the aggregator broadcasts the sum \( \sum_{i=1}^{N} y_i + k_{ij} \mod R = \sum_{i=1}^{N} y_i \).

Here we assume the parties are TEEs but this is not essential for the protocol. We assume one of the TEEs acts as an untrusted aggregator, who does the actual summations and broadcasts the results to the other TEEs but otherwise has no special information or capabilities. Without implementing e.g. some zero-knowledge proof of validity, the aggregator might change the final result at will if the TEE is compromised. However, this can usually be detected post hoc by each TEE by comparing the jointly trained model to the model trained on their private data. The role of the master could also be randomised or duplicated if deemed necessary.

For the secure DP summation, the TEEs encrypt \( y_i = z_i + \eta_i \) where \( \eta_i \) is DP noise as in the main text’s Section 3.2

To generate the secret keys \( k_{ij} \), for each round a separate setup phase is needed, which can be done with \( O(N^2) \) messages e.g. using standard Diffie-Hellman key-exchange [13] or existing public key cryptography. The actual values for each iteration can be drawn from the discrete uniform distribution bounded by \( R \) using a cryptographically secure pseudorandom generator (CSPRNG) [13] initialised with a seed shared by each pair of TEEs. Since each individual’s value is protected by all pairwise keys, the method is secure as long as there are at most \( N - 2 \) compromised TEEs. We refer to [8] and [2] for more details.

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**Appendix A**

This Appendix contains the secure summation protocol and related discussion for fat clients.

### Secure summation with fat clients

For secure summation with fat clients, we use an additively homomorphic encryption scheme given in Algorithm 4.

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2 Note that the actual sum is unbounded with a Gaussian noise term for DP. However, we can always clip the values to the assumed range since post-processing does not affect the DP guarantees.
Appendix B

This Appendix contains all the proofs omitted from the main text. For convenience, we first state the theorems again and then proceed with the proofs.

Proof of Theorem 1

**Theorem 1.** Assume the public-key encryption used in Algorithm 2 is secure. Then i) the list cannot be manipulated in non-trivial ways, ii) to hbc parties all tokens belonging to other parties are indistinguishable from each other and to malicious parties all tokens belonging to hbc parties are indistinguishable from each other.

**Proof.** To begin with, since the encryption is secure, an attacker cannot deduce the true ciphertext at layer $i$ by seeing the unencrypted plaintext at layer $i+1, i = 1, \ldots, N - 1$.

For i), since after each iteration in the for-loop any party can check if its tokens are still included on the list and that the list size has not changed, the only manipulation that can be done without alerting some hbc party is to tamper with the tokens of the malicious parties without changing their total number. Since these parties can in any case decide their contribution to learning without regarding the tokens, manipulating these tokens cannot gain anything.

As for ii), w.l.o.g. assume hbc parties at iterations $i$ and $j$, $i < j$ in the for-loop. For $j$, since $i$ follows the protocol the resulting list after iteration $i$ of looks like random numbers with a random permutation, so all elements not belonging to $j$ are indistinguishable from each other. For $i$, even if $i$ can identify all elements on the list after iteration $i$ of the for-loop, after iteration $j$ the list again looks like a random permutation of random numbers since $j$ follows the protocol. Furthermore, after iteration $i$ the entries belonging to the hbc parties look like random numbers with a random permutation to any malicious parties and as such are indistinguishable from each other to any malicious party from this iteration onwards.

Proof of Theorem 2

**Theorem 2.** Algorithm 3 is $(\tilde{C}, \delta')$-DP, with any $\tilde{C} > 0$ and $\delta' > 0$ s.t.

$$\mathbb{P} \left[ \Gamma(K = \frac{k}{2}, \theta = \frac{2C^2}{k}) \leq \tilde{C}^2 \right] \geq 1 - \delta',$$

where $k$ is the projection dimension, $C$ is the gradient sensitivity, and $\Gamma$ is the (shape & scale parameterised)

**Proof.** Let $f_{\mathcal{IL}}(a) = P^T a$ be the projection with each element in $P \in \mathbb{R}^{d \times k}$ independently drawn from $\mathcal{N}(0, 1/k)$. We need to show that $f_{\mathcal{IL}}$ is $(\tilde{C}, \delta')$-sensitive (Definition 2.4), and the result then follows directly from Lemma 2.1.

Consider first the neighbouring relation $\sim_R$. The sums of clipped gradients originating from maximally different data sets $D \sim_R D'$ differ in one vector, w.l.o.g. denoted as $a \in \mathbb{R}^d, \|a\|_2 \leq C$. As the coupling required by Definition 2.4 we use a trivial independent coupling, so we have $X - X' = f_{\mathcal{IL}}(a) = P^T a$.

Writing $N, \chi^2$ for random variables following normal and chi-squared distributions, respectively, we therefore have

$$\|X - X'\|_2^2 = \|P^T a\|_2^2 \quad (4)$$

$$= \sum_{j=1}^{k} \sum_{i=1}^{d} a_i \mathcal{N}(0, 1/k))^2 \quad (5)$$

$$= \|a\|_2^2 \chi_k \quad (6)$$

$$\leq \frac{C^2}{k} \chi_k. \quad (7)$$

Looking at Definition 2.4 we see that $f_{\mathcal{IL}}$ is $(\tilde{C}, \delta')$-sensitive, when

$$\mathbb{P} \left[ \Gamma(K = \frac{k}{2}, \theta = \frac{2C^2}{k}) \leq \tilde{C}^2 \right] \geq 1 - \delta'. \quad (8)$$

The fact that Algorithm 3 is $(\epsilon, \delta + \delta')$-DP then follows from Lemma 2.1.

When using the neighbouring relation $\sim_S$ and constant $C/2$ for clipping the gradients, the gradients originating from the maximally different data sets $D \sim_S D'$ again differ at most in a vector $a$ s.t. $\|a\|_2 \leq C$. The privacy analysis of the mechanism $\mathcal{G}_{\mathcal{IL}}$ is then carried out using the techniques given in 30 and we get the $(\epsilon, \delta)$-bound for $\mathcal{G}_{\mathcal{IL}}$, using the projection sensitivity bound $\tilde{C}$, so again Algorithm 3 is $(\epsilon, \delta + \delta')$-DP for $\delta'$ that satisfies 8.

Appendix C

This Appendix contains the experimental results for fat clients, i.e., using Algorithm 4 for encryption, omitted from the main text.
Implementation details and more experimental results

To generate randomness with a CSPRNG required in Algorithm 4, we use Blake2 [5], a fast cryptographic hash function, on an initial pairwise shared secret together with a running number. On 2.1GHz Xeon Gold 6230, generating one 4 byte hash digest with Blake2 takes about $1.2 \times 10^{-6}$ seconds.

The tradeoff between running times and the security parameters corresponding to the main text’s Figure 3 is shown in Figure 5.

![Figure 5: Fold increase in total running time with varying number of clients, medians over 5 runs. Encryption time depends on the chosen pairwise group size: even with increasing number of clients the encryption time stays roughly constant with a fixed group size (horizontal lines). Using MNIST data, model 1m has 2 FC hidden layers with $\approx 1e6$ parameters.](image-url)