Gaps in Topological Magnon Spectra: Intrinsic vs. Extrinsic Effects

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Magnons in a honeycomb-lattice ferromagnet have an analogous description to the single-orbital tight-binding model for electrons in graphene. As a result, the magnons in a honeycomb ferromagnet exhibit a linear energy-momentum dispersion near a magnon crossing point, often termed a Dirac magnon [Fig. 1][1,3]. Observing the Dirac magnon is of intense current interest as a means of assessing potential topological materials. In particular, if spin-orbit coupling (SOC) accompanying a broken inversion symmetry generates a nonzero Dzyaloshinskii–Moriya interaction (DMI), a gap opens at the Dirac crossing point, in close analogy with the spin-orbit-induced semiconducting gap in graphene. This can yield a topologically-protected magnon edge-state with low-spin-orbit-induced semiconducting gap in graphene. This can cause the intrinsic TMG to be substantially overestimated or misdiagnosed. We identify this effect using a simple model. We then present high-resolution neutron spectroscopic measurement of topological magnon gaps, which report gaps ~ 3 to 5 meV. However, the origin of the gap opening remains unclear, with proposals including DMI, Kitaev interactions, and magnon-phonon coupling [4,11,13,15]. Furthermore, the estimated gaps based upon the DMI and Kitaev interaction strengths are much bigger than the predicted gap size of first-principle calculations for the small SOC of half-filled $t_{2g}$ orbitals of Cr$^{3+}$ [13,16]. In this context, accurate neutron spectroscopy experiments can be crucial, because they provide a measurement of the magnon dispersion in 4-dimensional momentum-energy ($Q$-$E$) space that can, in principle, be quantitatively compared with calculations. In practice, however, the weakness of the neutron-sample interaction often requires compromises to increase the signal strength, such as the use of many co-aligned single crystals with increased mosaicity, the relaxation of $Q$-integration ranges, and $E$ resolution to gain more neutron flux, and the integration of measured data over a significant $Q$ range. These approaches to enhance signal can result in spurious findings when the neutron intensity rapidly disperses or has a singularity in $Q$-$E$ space as is the case for a Dirac magnon, and this may in turn cause a significant impact on the observation and estimates of the TMG.

In this letter, we present a detailed analysis of extrinsic effects on spectroscopic estimates of TMG sizes. Our key finding is that a dominant extrinsic contribution to the apparent magnon gap occurs if “typical” $Q$-integration ranges are used, which can cause the intrinsic TMG to be substantially overestimated or misdiagnosed. We identify this effect using a simple model. We then present high-resolution neutron spectroscopy measurements on a single crystal of CrCl$_3$. Our results establish CrCl$_3$ as an ideal quasi-2D Heisenberg ferromagnet with a gapless Dirac magnon, and provide a compreh
We reinterpret published spectroscopic studies of CrBr$_3$ However, we also find that an extrinsic magnon gap appears comprehensive map of the excitation spectrum near the Dirac point. This \( Q \)-integration effect is also distinct from the effect of sample mosaicity such as considered for recent study of CrI$_3$[10], which generates a superposition of off-centered Dirac cones [Fig. 1(c)], and causes an additional artificial gap at the Dirac point [Fig. 1(d)].

We investigate extrinsic contributions to the magnon gap using spectroscopic measurements of the van-der-Waals ferromagnet CrCl$_3$, which contains undistorted Cr$^{3+}$ honeycomb layers below 150 K (space group \( R3 \)) [18, 19]. CrCl$_3$ has a ferromagnetic spin alignment in the plane in each honeycomb layer below \( T_N = 14 \) K [18]. Compared to CrBr$_3$ and CrI$_3$, the relatively light Cl ligand atom of CrCl$_3$ is likely to give a small SOC and hence anisotropic exchange interactions should also be correspondingly weak and host a gapless Dirac magnon, as suggested by recent neutron-scattering measurements on polycrystalline CrCl$_3$ samples [20, 21]. Furthermore, unlike other Cr-halides such as CrI$_3$, CrCl$_3$ can be grown in as a large single crystal with a small sample mosaic. Consequently, CrCl$_3$ is an ideal model system to investigate the impact of extrinsic effects and data treatment on the spin-wave spectrum near the Dirac point.

A CrCl$_3$ single crystal with a mosaic < 0.68° was grown by re-crystallization of commercial CrCl$_3$ powder [22]. The INS data was collected with the SEQUOIA time-of-flight spectrometer [23] at the Spallation Neutron Source located at Oak Ridge National Laboratory. The Fermi choppers were phased for the high-resolution mode of SEQUOIA for incident energies \( E_i = 4, 11, \) and 25 meV [24]. Figure 2(a) shows the measured magnon spectra at 5 K \( (T < T_N) \). The spectra are composed of one acoustic and one optical magnon branch with an overall magnon bandwidth of 8 meV \( (W_HK) \) in the \( HK \)-plane. The acoustic magnon emanates from the \( \Gamma \) point \( (0,0,0) \), and meets the optical magnon at the \( K \)-points, exhibiting typical spin-waves for a ferromagnetic honeycomb lattice [2].

To model the observed magnon spectra, we used linear spin-wave theory (LSWT) using the SpinW package [25]. The spin Hamiltonian was modeled with \( \mathcal{H} = J_{0,} \sum_{i,j} S_i S_j - D^2 \sum_i (S_i^z)^2, \) for \( S = 3/2 \) of Cr$^{3+}$. We consider Heisenberg \( (J_{0}) \) exchanges up to the third n.n. interactions in the honeycomb plane and an easy-plane single-ion anisotropy \( (D^2 > 0) \). We exclude bond-dependent exchange interactions (Kitaev and \( \Gamma \) terms) because these would yield different mode energies at different K-points coupled with the aligned spin direction [26], in disagreement with our experimental data. We also note that the symmetry-allowed second n.n. DMI term cannot generate a magnon gap at the K-point for the in-plane magnetic structure with the symmetry operations of exchange matrix for \( R3 \) structure [17]. The inter-layer cou-
The shape of a Dirac cone dispersion. Deviating from a gapless Dirac magnon at the K-point. In the figure, the magnon spectra are displayed along high-symmetry directions as indicated in the HK-reciprocal space map shown in the inset. Data were obtained by integrating over a thickness of Q = 0.024 Å⁻¹ (0.02 r.l.u. along [H, 0, 0]) in the HK-plane and -2.5 ≤ [0, 0, L] ≤ 2.5 along the out-of-plane direction. Constant energy slices for (c) E = 6.0 ± 0.15 meV and (d) E = 2.5 ± 0.15 meV are compared with the corresponding calculations.

As described above, a focal issue in the search for topological properties of the magnon spectrum is identifying the presence of an intrinsic gap (TMG) at the K-point. Here we investigate the impact of Q-integration range on the spectrum near the Dirac point in CrCl₃. The determined Hamiltonian preserves time-reversal symmetry of magnons, resulting in a gapless Dirac magnon at the K-point.

Details of the magnon spectra near the Dirac point are shown in Fig. 3. In the figure, the magnon spectra are displayed along the radial Q-direction (parallel to [H, H, 0]) with varying transverse Q-component ([−K, K, 0]). The separate acoustic and optical modes intersect at the single Q-position of the Dirac point ([1/3, 1/3, 0]). This sharp band touching is viewed as a nodal point at E_{Dirac} = 4.4 meV in the horizontal Q-E slices of the spectra [Fig. 3(b)], representing the shape of a Dirac cone dispersion. Deviating from E_{Dirac}, the Dirac point evolves into triangular scattering patterns around K [Fig. 3(c, d)] with modulating intensity across the Dirac point in energy. The variation in intensity is associated with the isospin locked with offset momentum winding around the Dirac point [26, 27]. As the result, the two conical dispersions having anti-phased winding patterns of intensity meet at the K-point [refer to Fig. 1(a)]. When Q is perpendicular to the radial-direction, the two bands have an identical magnon structure factor, and the Dirac magnon reveals a clear band crossing, as shown in Fig. 3(c), evidencing a gapless Dirac magnon in CrCl₃.

As the result, the two conical dispersions having anti-phased winding patterns of intensity meet at the K-point [refer to Fig. 1(a)]. When Q is perpendicular to the radial-direction, the two bands have an identical magnon structure factor, and the Dirac magnon reveals a clear band crossing, as shown in Fig. 3(c), evidencing a gapless Dirac magnon in CrCl₃.
skeletal intensity within the gap, where the instrumental resolution is smaller than the apparent gap size, which is in contrast to the TMG case showing zero intensity between the peak splitting. Therefore, to clarify TMG, careful comparison of the measured data to the energy-resolution convolved spin-wave calculation including momentum integration range should be performed.

The discussion above has important implications for the ongoing debate regarding the observations of the Dirac magnon gap in other chromium van der Waals honeycomb ferromagnets. The previously used momentum integration range and a large sample mosaic for CrBr$_3$ ($dQ_{\text{trans}} = \pm 0.2$) and CrF$_3$ (sample mosaic = 8 ~ 17°), respectively, are likely to cause a large extrinsic gap contribution, thus the TMGs are possibly overestimated. Figure 3(a) shows the calculated INS spectrum of the spin-wave spectra for CrBr$_3$, including the $Q$-integration range and instrumental resolution used in Ref. 11. This calculation assumes only Heisenberg interactions ($J_1 = -1.48$ meV, $J_2 = -0.08$ meV, $J_3 = 0.11$, and $D' = -0.02$ meV, from Ref. 11) with zero DMI. For simplicity, the sample mosaic was assumed to be zero. The resulting spectrum shows the upper and lower magnon spectra having decreased intensities near the Dirac points ($K_1$, $K_2$), which reproduces the observed magnon dispersion in Ref. 11. Noticeably, most of the observed gap, corresponding to the peak splittings ~ 4 meV, in the energy scans at $K_1$ and $K_2$ is explained by the orthogonal momentum integration range effect on a pure Heisenberg model, without DMI. As a consequence, the size of the gap at the Dirac point is likely overestimated.

In contrast to time-of-flight spectroscopy measurement on Cr trihalides, topological magnon candidates CrMTe$_3$ ($M$=Si,Ge) were measured using triple-axis spectrometers. For triple-axis measurements, integration ranges are not a concern but resolution effects can be significant, particularly when horizontal focusing is used. Therefore, the observed spectra are strongly coupled with the shape of the resolution function (gray ellipse in Fig. 4(c)). Figure 4(c) exhibits newly calculated spin-wave spectra for CrSiTe$_3$, including the resolution calculation for the experiment using Reslib, exhibiting newly calculated spin-wave spectra for CrSiTe$_3$, including the resolution calculation for the experiment using Reslib. For the spin-wave calculation, we used only Heisenberg spin Hamiltonian parameters ($J_1 = -1.49$ meV, $J_2 = -0.15$ meV, $J_3 = 0.07$, $J_2 = -0.06$, and $D' = -0.01$ meV, from Ref. 12) with zero DMI. As can be seen, while the gapless Dirac dispersion parallel to the the ellipse (focused) at $K_1$ leads to a single peak (gapless), the Dirac dispersions anti-parallel to the ellipse (defocused) at $K_1$. 

![Figure 3](image-url)
and $K_{2\text{defocus}}$ lead to an apparent gaps in the constant $Q$-scans [Fig. 4(d)]. In this case, the defocused resolution plays similar role to the orthogonal $Q$-integration range in Fig. 3. Confirming the TMG in CrMTe$_3$ ($M=$Si,Ge) requires quantitative comparison of the data and the fully resolution-convolved simulation. Assuming that the data in Ref. [12] (Fig. 2H) was measured at the focused $K_1$ ($\frac{1}{2}, \frac{1}{2}, 0$) and defocused $K_2$ ($\frac{1}{2}, \frac{1}{2}, 3$), our new resolution-convolved spin-wave simulations suggest a DMI $= 0.06$ meV that is a half of the anticipated.

In conclusion, revealing and understanding topological magnons are important steps towards the realization of magnon-based electronic devices as well as for the fundamental goal of discovering a topological magnon insulator. To meet these challenges, accurately clarifying and estimating an intrinsic Dirac magnon gap is a critical issue in defining the topological properties, and INS experiments play a key role in this endeavor. In particular, our study provides important guidance for spectroscopic measurements of systems having a singularity or a band crossing where the spectrum rapidly disperses in momentum space. We have shown that instrumental and data-processing effects can introduce an artificial gap, so that accurate estimation of the topological gap size requires careful data histograms and comparison with resolution-convolved calculations. Our results are relevant not only to Dirac magnon gaps, but also to similar Weyl magnon, determining avoided crossing of rattler modes to acoustic phonons [29], and quantifying the life-time of rapidly decaying phonon spectra [30], where similar effects may be anticipated.

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Supplemental material for “Gaps in Topological Magnon Spectra: Intrinsic vs. Extrinsic Effects”

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I. EXPERIMENTAL DETAILS AND SPECTRAL SIMULATION METHODS

A single piece of CrCl3 single crystal (mass: 0.88g and dimension: 12 × 12 × 4 mm³) was aligned with [H, K, 0] in the horizontal scattering plane for the inelastic neutron scattering (INS) measurements (see Fig. S1(a)). The sample was sealed within an aluminum can under an atmosphere of helium exchange gas at room temperature. Figure S1(b) and (c) show the elastic scattering for the (H, K, 1) and (H, 0, L) planes. Sharp magnetic reflections appear as magnetic satellite peaks with the ordering wave vector \( \mathbf{Q}_m = (0, 0, \frac{2}{3}) \) in the \( R^3 \) rhombohedral crystal structure, representing the alternative magnetic stacking order of the ferromagnetic honeycomb lattice. The rocking scan for \( \mathbf{Q} = (3, 0, 0) \) Bragg reflection was fitted with a Gaussian function, as shown in Fig. S1(d), which gives the full-width at half-maximum (FWHM) \( \sim 0.68^\circ \), indicating the sample mosaicity is smaller than \( \sim 0.68^\circ \).

The INS data were obtained at \( T = 5 \) K using the SQUOIA time-of-flight spectrometer located at the Oak Ridge National Laboratory (ORNL). The Fermi chopper was set to 120 Hz, 180 Hz, and 240 Hz for \( E_i = 4, 11, \) and 25 meV, which give FWHM= 0.11, 0.24, and 0.65 meV of resolution at the elastic line (\( E = 0 \) meV), respectively. For \( E_i = 25 \) meV, measurements were performed by rotating the sample through 180° about its vertical axis with 0.5° steps. For \( E_i = 11 \) meV, measurements were performed by rotating the sample 2 × 50° with 1° step covering two K-points at \((-1, 2, 0)\) and \((2, -1, 0)\) (see Fig. S2(a)). The \( E_i = 4 \) meV data was measured by rotating the sample through 35° about its vertical axis to cover \( \mathbf{Q} \) along the \([H, 0, 0]\)-direction. Data were reduced and analyzed using the Mantid [1], Dave [2], and Horace [3] software packages.

Spin-wave spectra were calculated using the SpinW package [4]. In comparison to measured spectra, the orthogonal integration range d\( \mathbf{Q} \) was accounted for by averaging over the resolution convoluted \( \mathbf{Q} - E \) spectra at \( \sim 10 \) \( \mathbf{Q} \)-points per 0.01Å⁻¹ within the range. The triple-axis spectroscopy simulations shown in Fig. 4(c)(d) of the main text were performed using ResLib [5] with the resolution function calculation using the Popovici method [6] using the experimental parameters of the IN8 spectrometer used in Ref.[7].

II. DATA SYMMETRIZATION AND CONFIRMATION OF GAPLESS DIRAC MAGNON

While unsymmetrized data was used for \( E_i = 25 \) meV to analyze the linear spin-wave theory and to make figures shown in the main text, \( E_i = 11 \) meV and 4 meV data were symmetrized using symmetry operations of the Laue class of the \( R^3 \) to enhance statistics. Figure S2 shows a comparison between unsymmetrized and symmetrized data for \( E_i = 11 \) meV. The shown magnon spectra in the symmetrized data exhibits the same \( \mathbf{Q} - E \) pattern as the unsymmetrized data, but the signal to noise ratio is improved.

FIG. S1. (a) A picture of aligned CrCl3 single crystal used for INS measurements. Elastic scattering for (b) \([H, K, 1]\) and (c) \([H, 0, L]\) were obtained from SQUOIA using \( E_i = 25 \) meV. Nuclear reflections appear at \(-H + K + L = 3m \) (\( m \) =integer) with reverse-obverse twins for \( R^3 \) structure. Magnetic reflections appear with a propagation vector \( \mathbf{Q}_m = (0, 0, \frac{2}{3}) \). The ring-patterns are scattering from the aluminum sample holder. (d) A rocking scan for the Bragg reflection at \( \mathbf{Q} = (3, 0, 0) \) using \( E_i = 60 \) meV. The solid line is the Gaussian fit to the data.
The function was defined as following, (see below for sensitivity of the χ meV was used to stabilize the antiferromagnetic layer order (see Fig. S3(b)). 570 dispersion points were obtained from meV and 11 meV data along the high symmetry directions for the both (see Fig. S3(c)), and used for fitting the spin-wave dispersions for the spin Hamiltonian model [Fig. S3(c)], This Hamiltonian model describes Heisenberg exchanges up to third nearest neighbor (n.n.) interactions (Jnnt) along c-axis, and easy-plane single-ion anisotropy (DF > 0) (see Fig. S3(a) for the exchange paths). Due to the highly two-dimensional magnon spectra, out-of-plane spin interactions were assumed to be negligible and for simplicity Jnnt = 0.001 meV was used to stabilize the antiferromagnetic layer order (see below for sensitivity of the χ2 on Jnnt).

For fitting the spin-wave dispersion, the χ2 optimization function was defined as following.

\[ \chi^2 = \frac{1}{n} \sum_{i} \frac{(E_{i}^{\text{obs}} - E_{i}^{\text{calc}})^2}{\sigma_i^2}, \]  

(2)

where \( E_{i}^{\text{obs}} \) and \( E_{i}^{\text{calc}} \) indicate observed and calculated dispersion energies, and \( n \) and \( \sigma_i \) indicate the number of dispersion points and standard deviation in the observed dispersion. The fitting of the Hamiltonian parameters to minimize the \( \chi^2 \) was performed using the algorithm implemented in the SpinW package [4]. The comparison of the calculated dispersion for the optimized parameters to experimental dispersion is shown in Fig. S3(c), and shows a good agreement. In addition, Fig. S3(d) shows a comparison of the extracted magnon scattering intensity with the spin-wave calculation: the good agreement indicates the existence of isotropically or randomly

III. SPIN-WAVE FITTING RESULTS AND DETERMINATION OF THE HAMILTONIAN PARAMETERS

Experimental dispersions were extracted by fitting Gaussian functions to the constant momentum scans for \( E_i = 25 \) meV and 11 meV data along the high symmetry directions (see Fig. S3(b)). 570 dispersion points were obtained from the both \( E_i = 11 \) and 25 meV energy spectra (filled circles in Fig. S3(c)), and used for fitting the spin-wave dispersions for the spin Hamiltonian model [Fig. S3(c)],

\[ \mathcal{H}_0 = J_1 \sum_{ij} S_i S_j + J_2 \sum_{ij} S_i S_j + J_3 \sum_{i} S_i S_i + J_{\text{int}} \sum_{i} S_i S_j - D^2 \sum_{i} (S_i^z)^2. \]  

(1)

This Hamiltonian model describes Heisenberg exchanges up to third nearest neighbor (n.n.) interactions (\( J_{\text{int}} \)) along c-axis, and easy-plane single-ion anisotropy (\( DF > 0 \)) (see Fig. S3(a) for the exchange paths). Due to the highly two-dimensional magnon spectra, out-of-plane spin interactions were assumed to be negligible and for simplicity \( J_{\text{int}} = 0.001 \) meV was used to stabilize the antiferromagnetic layer order (see below for sensitivity of the \( \chi^2 \) on \( J_{\text{int}} \)).

For fitting the spin-wave dispersion, the \( \chi^2 \) optimization function was defined as following.

\[ \chi^2 = \frac{1}{n} \sum_{i} \frac{(E_{i}^{\text{obs}} - E_{i}^{\text{calc}})^2}{\sigma_i^2}, \]  

(2)

where \( E_{i}^{\text{obs}} \) and \( E_{i}^{\text{calc}} \) indicate observed and calculated dispersion energies, and \( n \) and \( \sigma_i \) indicate the number of dispersion points and standard deviation in the observed dispersion. The fitting of the Hamiltonian parameters to minimize the \( \chi^2 \) was performed using the algorithm implemented in the SpinW package [4]. The comparison of the calculated dispersion for the optimized parameters to experimental dispersion is shown in Fig. S3(c), and shows a good agreement. In addition, Fig. S3(d) shows a comparison of the extracted magnon scattering intensity with the spin-wave calculation: the good agreement indicates the existence of isotropically or randomly

FIG. S3. (a) Magnetic structure and exchange paths for CrCl3. Ferromagnetic moments are aligned in the ab-plane and antiferromagnetically stacked along the c-axis. The exchange paths are labeled. (b) Constant momentum scans at the high symmetry points in the Brillouin zone (dots). The black solid lines indicate the Gaussian fitting results on the elastic (\( E = 0 \)) and inelastic (\( E \neq 0 \)) excitation peaks. (c) Comparison between experimental dispersion points from \( E_i = 25 \) meV (blue filled circles) 11 meV (pink filled circles) and calculated dispersion (solid purple line) from the best fitting result. (d) Extracted intensity of acoustic (violet dot) and optical (red dot) modes and comparison to LSWT calculation (solid lines). The black arrow indicates the intensity at the K-point. (e) The \( \chi^2 \) value as a function of the \( J_{\text{int}} \) interaction strength. The black solid line indicates the parabolic fitting result.
distributed magnetic domains in the plane without preferred orientation.

To check the sensitivity of the inter-layer interaction to the obtained parameters, Fig. S3(e) shows the $\chi^2$ values of the dispersion points as a function of the $J_{\text{int}}$ interaction strength with fixed other parameters. The antiferromagnetic interaction was only considered to stabilize the alternative stacking order ground state. The minimum $\chi^2$ appears near $J_{\text{int}} \sim 0$ meV, and the $\chi^2$ shows parabolic increase with increasing $J_{\text{int}}$. The graphical errorbar estimation on the local $\chi^2$ minimum gives the error of $J_{\text{int}}$ with $\sim 0.13$ meV [8], corresponding to the sensitivity of the parameter on the spin-wave fitting result. Due to the small number of interaction paths for $J_{\text{int}}$ (number of interactions on a single spin: $\#J_{\text{int}} = 1$) [Fig. S3], the spin-wave can be less-sensitive to varying the $J_{\text{int}}$, compared to other exchanges paths for fixed values of exchange ($\#J_1 = 3, \#J_2 = 6, \text{and } \#J_3 = 3$).

IV. LOW ENERGY MAGNON SPECTRA: GAPLESS DISPERSION AT $\Gamma$-POINT AND DISPERSION-LESS SPIN-WAVE ALONG $L$

The low-energy spectra near the zone center (ZC) shown in Fig. S4(a) reveals a quadratic dispersion as expected for a ferromagnetic spin-wave, $E \propto \alpha Q^2 + \Gamma$, where $\alpha$ and $\Gamma$ are the spin-wave stiffness and magnon gap at the ZC. The low-energy magnon dispersion for energy transfers less than 1.2 meV, measured in the $E_i=4$ meV configuration, were fitted with the function, which yields $\alpha=17.14(49)$ and $\Gamma=0.00(1)$ meV. The resulting gapless (Goldstone) mode ($\Gamma \sim 0$ meV) indicates the preserved U(1) symmetry of Cr spins. This gapless feature is in contrast to the other chromate trihalide materials [9, 10], which emphasizes the importance of the ligand atom’s SOC in determining the magnetic anisotropy of the systems [11]. Figure S4(b) shows no discernible dispersion along [0.12, 0, L] near the ZC. Also, no dispersion along [0, 0, L] through the ZC is detected within instrumental resolution (FWHM= 0.24 meV at $E = 0$ meV for $E_i = 11$ meV) [Fig S4(c)]. The bandwidth along the $L$-direction is smaller than the instrumental resolution $\Delta \epsilon < \text{FWHM}/2 \sim 0.12$ meV, corresponding to less than 1.5% of $W_{HK}$, indicating highly two-dimensional spin interactions in CrCl$_3$.

V. DIRAC MAGNON ALONG THE RADIAL DIRECTION

In this section, we investigate the Dirac magnon spectra along a radial direction through the K-point with different orthogonal $Q$-integration ranges $dQ_{\text{trans}} = \pm 0.05$ r.l.u. and $\pm 0.2$ r.l.u. (see Fig.2(b) in the main text for definition of $dQ_{\text{trans}}$). These $dQ_{\text{trans}}$ values are typical $Q$-integration ranges that were used for Ref. [7, 12]. Figure S5 shows the magnon spectra along the $[2H, -H, 0]$-direction (equivalent direction to the $[H, H, 0]$-direction) for the different $dQ_{\text{trans}}$ sizes. As the $dQ_{\text{trans}}$ increases, the magnon dispersion appears to open a gap at the K-point ($\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, 0$): while the constant momentum scan for $dQ_{\text{trans}}=0.01$, the data for $dQ_{\text{trans}} = \pm 0.05$ shows two separated peaks with gap $\Delta K \sim 0.5$ meV. The measured spectra were compared to the spin-wave calculations for the determined Hamiltonian parameters in the Fig. S5(d)($e$), and the observed gapless and gaped spectra for $dQ_{\text{trans}} = \pm 0.01$ and $\pm 0.05$ are closely reproduced by the pure Heisenberg model ($J_1 - J_2 - J_3$ model). These results confirm that the orthogonal integration range for the radial $Q$-direction produces an extrinsic gap, same as the transverse direction discussed in the main text.

On the other hand, the momentum integration range $dQ_{\text{trans}} = \pm 0.2$ covers nearly half of the Brillouin zone, and the obtained spectra may not reflect the localized spectrum near the K-point. Thus, the shown spectra in Fig. S5(c) and (f) show a very broadened excitation due to the widely averaged spectral weight of the acoustic and optical modes. Accordingly, the constant momentum scan shows two broad peaks with a separation of $\sim 3$ meV for the gapless spin-wave model, which represents spectral centers of the averaged acoustic and optical modes’ spectral weights, not corresponding to the Dirac magnon gap. Therefore, such a large momentum in-
FIG. S5. Dependence of the apparent scattering at the Dirac point along radial $Q$-direction on the range of momentum integration. (a-c) The INS spectra were measured using high resolution mode with $E_i = 11 \text{ meV}$ with the SEQUOIA spectrometer, with varying the orthogonal momentum integration range $dQ_{\text{trans}} = \pm 0.01, 0.05, \text{ and } 0.2 \text{ r.l.u.}$. The spin-wave spectra were calculated for the determined Hamiltonian of the $J_1-J_2-J_3$ model (d-f) and the $J_1-J_2-J_3-A_z^2$ model (g-i) with the identical $Q$-integration ranges. Blue bars in the figure indicate the instrumental resolution at $E_{\text{Dirac}}$.

The spin-wave spectra were calculated for the determined Hamiltonian of the $J_1-J_2-J_3$ model (d-f) and the $J_1-J_2-J_3-A_z^2$ model (g-i) with the identical $Q$-integration ranges. Blue bars in the figure indicate the instrumental resolution at $E_{\text{Dirac}}$.

Integration range should not be used to investigate the Dirac magnon spectra.

To compare the observed apparent gap with an intrinsic magnon gap arising from the well-known topological magnon example, we assumed a ferromagnetic spin structure aligned along the $c$-axis, same as in the CrBr$_3$, CrI$_3$, and CrMTe$_3$ (M=Si,Ge), and introduced the second n.n. DMI ($A_z^2$) along the $z$-axis to the original spin Hamiltonian model ($\mathcal{H}_0$). Then, the new Hamiltonian ($\mathcal{H}_1$) is

$$\mathcal{H}_1 = \mathcal{H}_0 + \sum_{ij} A_z^2 \cdot (S_i \times S_j).$$  \hspace{1cm} (3)

Here we considered $A_z^2 = J_2 = 0.03 \text{ meV}$, corresponding to $\sim 3\%$ of $J_1$, and the resulting spin-wave calculations are exhibited in Fig. S5(g-i). The DMI breaks the time-reversal symmetry of magnon, which results in a clear peak splitting from an intrinsic gap for all the orthogonal $Q$ integration values [13]. Noticeably, this intrinsic magnon gap features almost zero scattering intensity at the $E_{\text{Dirac}}$ regardless of the size of the momentum integration range, in contrast to the pure Heisenberg $J_1-J_2-J_3$ model. This further indicates how the intrinsic gap and the spurious gap can appear differently in the measured spectra where energy resolution is sufficiently smaller than the intrinsic magnon gap.
VI. REINTERPRETATION OF DMI SIZE IN CrSiTe$_3$

In this section, we discuss the previously published triple-axis neutron spectroscopy results of CrSiTe$_3$ [7]. As described in the main text, the spectrum at the K-point depends strongly on the shape of the resolution function. Figure S7 demonstrates how the resolution ellipses couple to magnon dispersions at the K-points and how it determines the focused and defocused regions. We assumed that the collected constant momentum scan at K$_1$ in Ref. [7] was measured in the focused region, and compare the experimental data with spectrum simulated with varying DMI in accurate resolution-convolved spin-wave calculations for CrSiTe$_3$.

Figure S6 shows the simulated spin-wave spectra for CrSiTe$_3$ [7], including the resolution calculation for the experiment using Reslib [5]. For the pin-wave calculation, we used Heisenberg spin Hamiltonian parameters ($J_1$=-1.49 meV, $J_2$=-0.15 meV, $J_{c1}$=-0.07, $J_{c2}$=-0.06, and $D^z$=-0.01 meV) from Ref. [7] with the second n.n. DMI= 0 meV, 0.06 meV, and 0.12 meV. Figure S6 (a-c) compare the resulting simulations with the experimental data at K$_1$ ($\frac{1}{3},\frac{1}{3},3$) and K$_2$ ($\frac{2}{3},\frac{2}{3},3$). For the zero DMI model, the simulated constant momentum scan at K$_1$ (focused region) shows a single peak corresponding to a gapless Dirac magnon spectrum (solid blue line in Fig.S6(a)), but its line shape is narrower than data, the simulated intensity is shifted to lower energies, and it does not explain the peak splitting observed in the experimental data (blue filled circles). For non-zero DMI, the calculated spectrum opens a gap which generates two peaks, and the size of the gap increases as the strength of DMI increases. However, we note that the line shape of the simulated peaks for all the DMI values up to 0.12 meV do not match the experimental data: the intensity ratio between lower and upper magnon bands is reversed when compared to the experimental data. In practice, the measured wave vector can slightly deviate from exact K-point, ($\frac{1}{3},\frac{1}{3},3$), or the crystal can be slightly mis-aligned. To obtain better agreement with the data, we also simulate the spectrum at $Q$= (0.35,0.35,3). The simulated spectrum at this wave vector better reproduces not only the measured peak intensity distribution but also line shape observed for K$_1$ focus. Through a qualitative comparison with experimental data, we found that DMI= 0.06 meV provides a good description of the observed peak splitting and line shape [Fig. S6(e)]. On the other hand, in the K$_2$ defocused region, the line shape is much less-sensitive to the DMI value and both 0 meV and 0.06 meV provide good agreement with the data. We note that 0.12 meV (the estimated best parameter in
FIG. S7. Resolution ellipses and coupling to magnon dispersions for constant momentum scans at (a) $K_{\text{focus}}^1$, (b) $K_{\text{defocus}}^1$, (c) $K_{\text{defocus}}^2$, and (d) $K_{\text{focus}}^2$ for the IN8 triple-axis spectrometer experiment for CrSiTe$_3$ [7]. At each energy transfer plotted, the solid (dashed) ellipses correspond to the projection (section) of the resolution function onto the $Q_x$-$E$ plane. The solid lines are magnon dispersions of CrSiTe$_3$. The red thick dashed lines and think ellipses indicate intense magnon modes and resolution functions coupled with the magnons at K-points. As a result of the coupling, the K-points are divided into ‘focus’ and ‘defocus’ regions according to whether the intense dispersion is parallel to and anti-parallel to the ellipse, respectively.

Ref.[7] does not provide good agreement with the data.

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