Beyond Quantum Field Theory: Chaotic Lattices?

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Abstract

We review the idea of chaotic quantization, based on the chaotic dynamics of classical lattice gauge systems as well as on non-abelian plasma physics in the infrared limit. The basic conjecture between Planck constant and properties of a five-dimensional lattice ($\hbar = aT$) is demonstrated numerically for the U(1) lattice gauge group.

\section{Chaotic quantization}

By promoting the idea of chaotic quantization we intend to call the attention to a possibility which could resolve at least part of the frustration stemming from the fact that gravity is resisting theoretical intentions to be quantized. String theory, while it approaches the problem on the basis of quantum field theory not only requires 10 space-time dimensions to be consistent, but also calls upon the additional concept of compactification in order to arrive at standard model particle physics.

Chaotic quantization has been suggested by us\textsuperscript{1,2} as an opposite strategy, starting with classical gauge field theory which due to its own Hamiltonian dynamics is inherently chaotic\textsuperscript{3,5}. It evolves ergodically - enough time given - in field-configuration phase space and this process leads in the infrared limit to a stationary distribution of lower dimensional sub-configurations. These, as result of the higher dimensional classical dynamics, are distributed as quantum field theory requires in the imaginary time formalism. The chaotic quantization is a particular form of the stochastic quantization\textsuperscript{6}, it works self-contained, not requiring any assumption of an external heat bath or noise.

One should in principle combine this mechanism with the interesting results of C. Beck\textsuperscript{7,8,9}, who noted that chaotic maps closest to the most general white noise...
assumption of the stochastic quantization, namely Tchebisheff polynomials, offer a classification of possible chaotic quantization outcomes. In his scheme he was able to derive ratios of the most important Standard Model parameters, like leptoquark masses and relative coupling strengths, as stable zeros of the effective potential for a higher dimensional lattice system developing under a chaotic dynamics. He actually predicted neutrino masses before any measurement pointing towards a mass square difference between neutrino generations (which implies that all neutrinos cannot be massless) [10].

Set into the above perspective the chaotic quantization mechanism is worth to study. As a working example we take a 5-dimensional lattice on which a classical, pure U(1) gauge theory resides. This simple group still shows chaotic dynamics on finite lattices [11]. Although in the continuum limit U(1) is not chaotic, at finite lattice spacing and strong coupling classical chaotic dynamics was already demonstrated in the 3-dimensional case. This behavior is probably correlated with the presence of magnetic monopole anti-monopole pairs [12, 13].

Our main conjecture between the 5-dimensional classical theory with chaotic dynamics and the 4-dimensional quantum field theory can be comprised into the following formula between the ‘normal’ (4-dimensional) Planck constant and two physical characteristics of the higher dimensional theory, its temperature and lattice spacing:

\[ \hbar = aT \]  

(1)

Usually such a formula is read in an opposite direction, automatically relating a Planck mass (\( M_P = T \)) with a Planck length (\( \ell_P = a \)). Our philosophy here, however, views \( \hbar \) as a constant of nature factorized to two other, underlying properties of the (in the present theory 5-dimensional) world. An analogy of this situation is the classical electrodynamics formula factorizing another constant of nature, the speed of light:

\[ 1 = c^2 = \varepsilon_0 \mu_0 : \]  

(2)

Taking \( c = \text{const} \) as a postulate we arrive at the theory of special relativity, one derives the laws of Lorentz transformation in the framework of mechanics without making any reference to electric or magnetic fields (as it has been shown by Albert Einstein). Maxwell theory on the other hand, as a classical field theory regards \( \varepsilon_0 \) and \( \mu_0 \) as independent properties of the physical vacuum, as dielectric constant and magnetic permeability. Light waves are solutions of Maxwell theory and the speed of light is calculable. The relation between nowadays Quantum Field Theory and an underlying classical field theory is analogous to this. Furthermore, as ether does not need to exist for Maxwell theory to work, the five dimensional lattice also may prove to be just a theoretical construct without measurability even at the Planck scale.

2 Chaos in gauge theory

First non-abelian then also abelian gauge theories has been studied with respect to chaotic behavior. In the eighties model systems, with a few, selected degrees of freedom with long wavelength has been studied. The most characteristic results stem from
SU(2) Yang-Mills theory, considering two $k = 0$ modes of different polarization and color. Due to the simple Hamiltonian,

$$H = \frac{1}{2} \dot{x}^2 + \dot{y}^2 + \frac{1}{2} g^2 x^2 y^2;$$

(3)

this $xy$ model [14] was studied very often. It is a classically chaotic system with a null-measure of regular periodic orbits; the turning points of the classical motion lie on hyperboles defocusing so any nearby, parallel trajectories (cf. Fig 1).

Figure 1: Time evolution of initially adjacent trajectories in the $xy$-model. One hundred initially adjacent points eventually scatter over the whole classically allowed phase space.

Numerical studies on lattices followed in the 1990-s. Mainly the gauge groups $U(1)$, SU(2) and SU(3) has been studied, all showed chaotic behavior [15, 16, 17]. The scaling of the leading Lyapunov exponent with the scaled total energy of the classical lattice, $a \lambda_{\text{max}} \to a g^2 H$ for non-abelian gauge groups, as well as the extensivity of the Lyapunov spectrum and the Kolmogorov-Sinai (KS) entropy – related to the ergodization speed – were studied [18]. Interpreting the KS-entropy as the physical entropy of the lattice gauge system even an attempt can be done to extract a classical equation of state [19].

A basic interest lies in the investigation whether classically chaotic systems and configurations are also special for the quantum pendant. Correlation between chaos and confinement has been observed in SU(2) and U(1) lattices comparing quantum Monte Carlo results and classical Hamiltonian dynamics in 3+1 dimensions [20]. For the U(1) group in the strong coupling phase a strong tendency could have been observed between the presence of magnetic monopole anti-monopole pairs and the chaotic behavior [12].
3 Higher dimensional plasma physics

The main point of the stochastic quantization \[6, 22\], namely generating field configurations distributed according to Boltzmann weights,

\[ P = \exp \left( \beta S[A] \right) \]  

(4)

for pure gauge theories, where the original vector potential variable, \( A \) is rescaled to \( A^0 = gA \) and as a consequence \( \beta = 1 = g^2 \hbar \) due to the scaling of the action \( S[A] = S[gA] = g^2 \). These Boltzmann weights occur in Feynman path integrals when calculating expectation values in quantum field theory. At the same time they can be regarded as a stationary distribution of a corresponding Fokker-Planck equation,

\[ \sigma_5 \frac{\partial}{\partial t} P = \int d^4 x \frac{\delta}{\delta A} \frac{1}{\beta} \frac{\delta}{\delta A} P + \frac{\delta S}{\delta A} P : \]  

(5)

Formally \( 1 = \beta \) can be interpreted as a temperature of the five dimensional system, \( T_5 \).

The Fokker-Planck equation is equivalent with solutions of corresponding Langevin equation,

\[ \dot{A} + \frac{\delta S}{\delta A} = J \]  

(6)

where the source current density is split to a dissipative term (like ohmic resistance) and to a fluctuative (noise) term:

\[ J = \sigma A + \zeta : \]  

(7)

This noise is usually treated simplified, as a Gaussian white noise with zero mean and a correlation sharply localized in space and time:

\[ \langle \zeta(x_1) \zeta(x_2) \rangle = 2\sigma T \delta^4(x_1 - x_2) : \]  

(8)

In the infrared limit the low frequency components dominate the relevant vector potential configurations, the radiative term \( \dot{A} \) can be neglected besides the dissipative term \( \sigma \dot{A} \). The typical frequency is small, \( \omega \ll \sigma \), the typical time is large, \( \tau \gg 1 = \sigma \) \[23\].

The effective model to electrodynamics is a Langevin plasma described by

\[ \sigma \dot{A} + \frac{\delta S}{\delta A} = \zeta : \]  

(9)

This equation is analogous to the well-known Brown motion, the mean features of its solution, too: the action \( S \) after long enough time is distributed around its ergodic limit of \( T=2 \) and initial correlations decay exponentially with the characteristic time of the corresponding damping constant, \( t_{\text{char}} = 1 = \gamma = 1 = \frac{\sigma t^2}{f} \). The long time average of such correlations with the initial value are interpreted in the lower dimensional field theory as propagators.

This analogy with plasma physics in the continuum limit led us to detailed investigations of dynamical time scales: the thermal (\( n=T \)), electric (\( n=gT \)) and magnetic screening length (\( 1 = g^2 T \)) in a usual, three dimensional plasma depend on the temperature, on the Planck constant and on the coupling constant of the original gauge theory.
3-dim QFT plasma 3+1 class. lattice

\begin{align*}
d_{m} & = g^{2}T \\
d_{c} & = gT \\
\omega & = g^{2}T + h \\
\gamma & = g^{2}T \\
\sigma & = T + h(\log) \\
d_{m}, d_{c} & = g^{2}T = a^{2}T
\end{align*}

Table 1: Scales in plasma physics and on the lattice show a universal scaling: \( h = aT \).

In the long time limit the plasma dynamics leads to distributions simulating a dimensionally reduced, three dimensional field theory with the effective coupling \( g_{3} = g^{2}T \). It can happen only for pure gauge theories, with mass scale invariance. The magnetic part of the energy is identical with a lower dimensional Maxwell or Yang-Mills action:

\[
\frac{1}{2}B_{i}B_{i} = \frac{1}{4}F_{ij}F^{ij} : \tag{10}
\]

The effective approach with white noise and Langevin equation is valid for the long-time behavior, \( t \rightarrow \infty \). Table 1 summarizes the most important features of traditional plasma physics, in the third column showing the corresponding formulae for classical lattices. Since most of the time constants in plasma physics are coefficients in a linear response approach, they can be calculated on a classical lattice as well as in quantum field theory. Here no Planck constant occurs, but the lattice spacing plays a basic role. These two approaches coincide if the universal relation \( h = aT \) is assumed.

\begin{align*}
d_{m} & = \frac{1}{G^{2}} = \frac{1}{g^{4}T^{2}} \\
d_{c} & = \frac{h}{G^{2}} = \frac{h}{g^{4}T^{2}} \\
\omega & = \frac{g^{2}T}{G^{2}} \\
\sigma & = \frac{g^{2}T}{G^{2}} \\
g_{d}^{2} & = \frac{g^{2}}{G^{2}} \\
G^{2} & = \frac{1}{g^{2}T^{2}} \sigma^{2} = \frac{1}{g^{2}T^{2}} \sigma^{2}
\end{align*}

Table 2: Scales in plasma physics and on the lattice in arbitrary dimensions. All formulae coincide if \( h = aT \). Here \( g \) is the original coupling, \( G \) the weak parameter signaling the infrared limit and \( g_{d} \) the effective coupling of the dimensionally reduced theory.

These calculations can be repeated in arbitrary number of Euclidean dimensions, including the 4+1 - dimensional case of traditional stochastic quantization. Table 2 shows the characteristic results.
4 U(1) lattice model

In this section we report about numerical simulations on a U(1) lattice gauge system both in 4 and 5 dimensions. The former was simulated by quantum Monte Carlo techniques in order to reproduce long known standard results [24, 25], the latter independently by 4+1-dimensional classical Hamiltonian dynamics known to be chaotic from our former studies. In both cases regular (rather small, \(4^4\)) lattices are considered.

In order to appreciate the computational complexity one faces to, we review briefly basic formulae and techniques of lattice gauge theory calculations. In these models of continuum field theory (both in the classical and quantized version) lattice links starting at point \(x\) and pointing in the \(\mu\) direction are associated with phases, \(A_\mu(\mathbf{x})\) of unimodular complex numbers – elements of \(U(1) = \exp(i\alpha A_\mu(\mathbf{x}))\), while the lattice action is constructed from phase sums around elementary plaquettes, upon using lattice forward derivatives \((\partial_\mu f = f(\mathbf{x} + \alpha e_\mu) - f(\mathbf{x}))\). The plaquette phase sums satisfy

\[ F_{\mu\nu}(\mathbf{x}) = \partial_\mu A_\nu(\mathbf{x}) - \partial_\nu A_\mu(\mathbf{x}); \]  

and determine the lattice action

\[ S = \frac{1}{g^2} \sum_\mathbf{x} \sum_{\mu \neq \nu} (1 - \cos(g\alpha F_{\mu\nu})); \]  

Here the summation runs over all lattice plaquettes in planes each characterized by a pair of two (ordered and different) direction indices, \(\mu > \nu\), and attached with its corner to the site \(x\). In the continuum limit \(a \to 0\) the action of the classical electrodynamics is recovered. Quantum Monte Carlo algorithms produce and sample \(U\) lattice link values (so called configurations) which are weighted by a Boltzmann type factor

\[ w = e^{S/\hbar}; \]  

This corresponds to the evaluation of Feynman path integrals in quantum field theory in the imaginary time formalism.

The classical Hamiltonian approach on the other hand uses the Hamiltonian split to electric and magnetic parts,

\[ H = \sum_{x \mu} \frac{\alpha}{g^2} U^2 + E_{\text{magn}}[U]; \]  

Here the magnetic contribution to the total Hamiltonian,

\[ E_{\text{magn}}[U] = \frac{1}{g^2 a^4} \sum_{x \mu \nu} 1 - \cos(g\alpha F_{\mu\nu}); \]  

is a sum over plaquettes lying in spatio-spatial (hyper)planes. This sum is formally equivalent to the Euclidean action of the same lattice gauge theory in one dimension lower. This self-similarity of pure gauge actions is an essential ingredient for the particular mechanism of chaotic quantization we are pursuing now.

We present results of numerical simulations of a five dimensional classical Hamiltonian U(1) lattice system and compare its evolution in the 5-th coordinate with traditional quantum Monte Carlo generated configurations on a four dimensional lattice,
using the four dimensional U(1) lattice action. In the classical Hamiltonian approach
the evolution of the U configurations proceeds in a 5-th dimension, often called 'fictitious’
time when it has been used as a method for stochastic quantization. The important
difference is, that so far always an external heat bath or a white noise for solving
Langevin type equations has been added to the evolution; we consider here pure clas-
sical Hamiltonian dynamics with no other source of noise or fluctuations.

As by construction \( aE_5^{\text{magn}} = S_4 \) (since the dimensionless plaquette sum or average
over the 4-dimensional (sub)lattice is either \( a^2E_5^{\text{magn}} \) when used in the Hamiltonian
simulation or \( g^2\bar{h} S_4 \bar{h} \) when used in quantum Monte Carlo algorithms), the conjecture

\[
E_5^{\text{magn}} = T = S_4 \bar{h}
\]

is literally equivalent to \( \bar{h} = aT \) (eq.(1)), as we have argued earlier on the basis of
plasma physics considerations.

Now we demonstrate the validity of the relation (16) by numerical computation.
Figure (2) shows the absolute value square of the lattice-averaged Polyakov line (the
standard order parameter of lattice gauge theory) values – averaged over many quan-
tum Monte Carlo configurations (full squares) as a function of the 4-dimensional lat-
tice plaquette sum per plaquette \( g^2S_4 \). On the same plot the same order parameter is
shown as a function of the partial plaquette sum corresponding to the magnetic en-
ergy \( ag^2E_5^{\text{magn}} \) after Hamiltonian equilibration on the classical 4-dimensional lattice,
averaged over many points alongside a single evolution trajectory at consecutive 5-th
coordinate times (open diamonds). That these two sets of points belong to the same (in
the Coulomb plasma phase linear) scaling law, proves our main conjecture.

![Figure 2](image-url)

Figure 2: The order parameter, the absolute value square of the Polyakov line aver-
aged over the lattice and over many configurations is plotted against the 4 dimensional
plaquette sum in the classical Hamiltonian (open diamonds) and in the quantum Monte
Carlo (full squares) simulations, respectively. The scaling of these results coincides if
\( E_5 = T = S_4 \bar{h} \).

The reason that we do not plot the Polyakov line as usual, as a function of inverse
coupling, is that different couplings belong to the 4 and to the 5-dimensional simulation

\[7\]
once $\hbar = aT$ is valid. In fact the lines fitted to our simulation points do not coincide unless we assume (16).

In order to offer a possibly more direct insight into the relation of 4 dimensional quantum and 5 dimensional classical lattice U(1) theories we plot several points on the complex Polyakov-line plane, both from 4 dimensional quantum Monte Carlo (right column) and from 5 dimensional classical Hamiltonian evolution (left column, actually one single trajectory is plotted). The left and right parts of Fig. 3 belong to different inverse couplings for the 4 and 5-dimensional cases, the correspondence is made by selecting out pairs of simulations satisfying (16). The 5-dimensional coupling actually does not play any important role; only the energy content of the configuration is related to it. Once it is given the equipartition happens due to the very same Hamiltonian evolution initially (not shown in the Figure), which governs the chaotic trajectory covering the same region of configurations which is generated by quantum Monte Carlo codes.

Now the correspondence between classical and quantum configurations is excellent, both for the magnitude and for the phase of Polyakov lines. Some initial points in the middle of the rings for overcritical couplings $\beta_4 = 1 = g_4^2$ are irrelevant; they stem from an initial MC heating phase. The classical Hamiltonian evolution also had an initial phase equilibrating electric and magnetic field energy (in 5 dimensions their ratio is however not 1:1 but 2:3).

Figure 3: Complex Polyakov line values from 4-dimensional quantum Monte Carlo simulation (right column) and from 5-dimensional classical Hamiltonian equation of motion (left column) at $aE_5^\text{mag} = S_4$.

## 5 Conclusion

Summarizing we have demonstrated that the mechanism of chaotic quantization - conjectured earlier on the basis of non-abelian plasma physics - works in the practice for
lattice gauge theory in 5-dimensional classical form. The correspondence to the traditional 4-dimensional quantum Monte Carlo simulations is given by the general formula \( \hbar = aT \), a formula encoding physical properties of the higher dimensional lattice and field configurations into the Planck constant. This fact underlines our hope for a unified classical field theory of gravity and standard particle physics, in particular for an explanation of standard model parameters via the mechanism of chaotic quantization, as well as for getting closer to an insight on the origin of Planck’s constant.

Finally we would like to address the question whether factorizing the Planck constant would not mean to construct a hidden parameter theory. It is not necessarily the case, since none of the laws of experimental quantum physics seem to be violated by our results: the higher dimensional classical dynamics acts as Euclidean quantum field theory in 4 dimensions in all respects. On the other hand the impossibility of a hidden parameter is proven for local actions in a strict manner, while the existence of a higher dimension allows for subtle non-local effects in the 4 dimensions of the physical experience.

Of course, as anything referring to the Planck scale, the theory of chaotic quantization seems to be speculative at the first glance. What is – at least in principle – better than in the case of fundamental string theory, that the autocorrelation time-scale, \( \tau = \sigma/k^2 \) may be effective at scales other, than the Planck length. From the known experimental fact of the relative weakness of gravity coupling compared to standard quantum field theories (QFT) \( g_{\text{grav}}^2 < g_{\text{QFT}}^2 \), we conclude that the time scales beyond which a phenomenon occurs to be quantum, and for shorter time observations not, separates gravity from the rest of standard model:

\[
\tau_{\text{grav}} : \tau_{\text{observ}} : \tau_{\text{QFT}}
\]

As a consequence gravity behaves classically while the other three known interactions according to quantum field theory.

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