OBSERVATIONAL CONSTRAINTS ON THE TILTED FLAT-XCDM AND THE UNTILTED NONFLAT XCDM DYNAMICAL DARK ENERGY INFLATION PARAMETERIZATIONS

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ABSTRACT

We constrain tilted spatially-flat and untilted nonflat XCDM dynamical dark energy inflation parameterizations using Planck 2015 cosmic microwave background (CMB) anisotropy data and recent baryonic acoustic oscillations distance measurements, Type Ia supernovae data, Hubble parameter observations, and growth rate measurements. Inclusion of the four non-CMB data sets results in a significant strengthening of the evidence for nonflatness in the nonflat XCDM model from 1.1\(\sigma\) for the CMB data alone to 3.4\(\sigma\) for the full data combination. In this untilted nonflat XCDM case the data favor a spatially-closed model in which spatial curvature contributes a little less than a percent of the current cosmological energy budget; they also mildly favor dynamical dark energy over a cosmological constant at 1.2\(\sigma\). These data are also better fit by the flat-XCDM parameterization than by the standard \(\Lambda\)CDM model, but only at 0.3\(\sigma\) significance. Current data is unable to rule out dark energy dynamics. The nonflat XCDM parameterization is compatible with the Dark Energy Survey limits on the present value of the rms mass fluctuations amplitude \(\sigma_8\) as a function of the present value of the nonrelativistic matter density parameter \(\Omega_m\), however it does not provide as good a fit to the higher multipole CMB temperature anisotropy data as does the standard tilted flat-\(\Lambda\)CDM model. A number of measured cosmological parameter values differ significantly when determined using the tilted flat-XCDM and the nonflat XCDM parameterizations, including the baryonic matter density parameter and the reionization optical depth.

Subject headings: cosmological parameters — cosmic background radiation — large-scale structure of universe — inflation — observations — methods:statistical

1. INTRODUCTION

In the standard spatially-flat \(\Lambda\)CDM cosmological model (Peebles 1984) the current cosmological energy budget is dominated by the cosmological constant \(\Lambda\) which powers the currently accelerating cosmological expansion. Cold dark matter (CDM) and baryonic matter are the next two largest contributors to the current energy budget, followed by small contributions from neutrinos and photons. For reviews of the standard model see Ratra & Vogeley (2008), Martin (2012), Brax (2018), and Luković et al. (2018). This model is able to accommodate most observational constraints, including CMB anisotropy measurements (Planck Collaboration 2016), baryonic acoustic oscillation (BAO) distance observations (Alam et al. 2017), Hubble parameter data (Farooq et al. 2017), and Type Ia supernova (SNIa) apparent magnitude measurements (Scolnic et al. 2017). Current observational constraints however allow for slightly nonflat spatial geometries and/or mild dark energy dynamics.

The standard spatially-flat \(\Lambda\)CDM inflation model is characterized by six cosmological parameters conventionally chosen to be: \(\Omega_m h^2\) and \(\Omega_\Lambda h^2\), the current values of the baryonic and cold dark matter density parameters multiplied by the square of the Hubble constant \(H_0\) (in units of 100 km s\(^{-1}\) Mpc\(^{-1}\)); \(\tau\), the reionization optical depth; \(\theta_{\text{SC}}\), the angular diameter distance as a multiple of the sound horizon at recombination; and \(n_s\) and \(A_s\), the spectral index and amplitude of the power-law primordial scalar fractional energy density spatial inhomogeneity power spectrum.

Observational data are on the verge of being able to place interesting constraints on seven parameter cosmological models. Two more plausible seventh cosmological parameters now under discussion are spatial curvature in nonflat extensions of the standard model and a parameter that governs dark energy dynamics in dynamical dark energy extensions of the standard model.

A simple, and so widely used, dynamical dark energy parameterization is the XCDM one.\(^4\) This parameterizes the equation of state relation between the pressure and energy density of the dark energy fluid through \(p_X = w \rho_X\) where the equation of state parameter \(w\) is the additional seventh cosmological parameter. XCDM is not a physically consistent description of dark energy as it is unable to consistently describe the evolution of energy density spatial inhomogeneities. To render XCDM physically consistent requires an eighth cosmological parameter, the square of the speed of sound in the dark energy fluid, \(c_{sX}^2 = dp_X/d\rho_X\). In this paper, as is common practice, we consider a restricted, physically-consistent, modified XCDM parameterization in which \(c_{sX}^2\) is not allowed to vary in space or with time and is arbitrarily set to unity. \(\phi\)CDM is the simplest physically consistent dynamical dark energy model (Peebles & Ratra 1988; Ratra & Peebles 1988). Here a scalar field \(\phi\) with potential energy density \(V(\phi) \propto \phi^{-\alpha}\)

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\(^3\) Hubble parameter values have been measured from low redshift to well past the redshift of the cosmological deceleration-acceleration transition between the earlier nonrelativistic-matter-dominated decelerating cosmological expansion and the more recent dark-energy-dominated accelerating cosmological expansion. The transition redshift has been measured from Hubble parameter observations and it is roughly at the value expected in dark energy models (Farooq & Ratra 2013; Morescu et al. 2016; Farooq et al. 2017; Yu et al. 2018).
\(^4\) Many observations have been used to constrain the XCDM parameterization (see, e.g., Chen & Ratra 2004; Samushia et al. 2007; Samushia & Ratra 2010; Chen & Ratra 2011b; Solé et al. 2017a, 2018, 2017b,c,d; Zhai et al. 2017, and references therein).
is the dynamical dark energy with \( \alpha > 0 \) being the additional seventh cosmological parameter.\(^5\)

Ooba et al. (2018c) (also see Park & Ratra 2018b) have analyzed the Planck 2015 CMB anisotropy data and some BAO distance measurements by using these seven parameter tilted spatially-flat XCDM and \( \phi \)CDM dynamical dark energy inflation models and found that both were slightly favored by the data, compared to the standard six parameter flat-\( \Lambda \)CDM model, by 1.1\( \sigma \) (1.3\( \sigma \)) for the XCDM (\( \phi \)CDM) case. While these are not significant improvements over the standard model, current data are not able to rule out dark energy dynamics. In addition, both dynamical dark energy models reduce the tension between the Planck 2015 CMB anisotropy and the weak gravitational lensing constraints on \( \sigma_8 \), the rms fractional energy density spatial inhomogeneity averaged over \( 8h^{-2} \) Mpc radius spheres.

There have been a number of earlier suggestions that different combinations of observational data favor dynamical dark energy models over the standard \( \Lambda \)CDM model (Sahni et al. 2014; Ding et al. 2015; Solà et al. 2015; Zheng et al. 2016; Solà et al. 2017a, 2018, 2017b; Zhao et al. 2017; Solà et al. 2017c; Zhang et al. 2017a; Solà et al. 2017d; Gómez-Valent & Solà 2017; Cao et al. 2018; Gómez-Valent & Solà 2018). As far as we are aware, of these analyses, only those of Zhao et al. (2017) and Zhang et al. (2017a) performed complete CMB anisotropy analyses of the generalized XCDM dynamical dark energy parameterizations they assumed.\(^5\) The other analyses either ignored CMB anisotropy data or only approximately accounted for it.

The standard \( \Lambda \)CDM model assumes flat spatial hypersurfaces. In nonflat models non-vanishing spatial curvature introduces a new length scale and so it is incorrect to assume a power spectrum for energy density inhomogeneities in nonflat models that does not correctly account for the spatial curvature length scale (as was assumed for analyses of nonflat models in Planck Collaboration 2016). Nonflat cosmological inflation provides the only known method for computing a physically consistent power spectrum in nonflat models. For open spatial hypersurfaces the Gott (1982) open-bubble inflation model is used to compute the non-power-law power spectrum (Ratra & Peebles 1994, 1995). For closed spatial hypersurfaces Hawking’s prescription for the initial quantum state of the universe (Hawking 1984; Ratra 1985) is used to define a closed inflation model that gives the non-power-law power spectrum of spatial inhomogeneities (Ratra 2017). In the nonflat inflation models, compared to the flat inflation model, there is no simple way to allow for tilt so \( n_s \) is not a free parameter and it is replaced by the present value of the spatial curvature density parameter \( \Omega_k \).

Using such a physically consistent untilted nonflat inflation model non-power-law power spectrum of energy density inhomogeneities, Ooba et al. (2018a) found that Planck 2015 CMB data (Planck Collaboration 2016) do not require flat spatial hypersurfaces in the six parameter nonflat \( \Lambda \)CDM model.\(^7\) In the six parameter nonflat \( \Lambda \)CDM model, compared to the six parameter flat-\( \Lambda \)CDM model, \( n_s \) is replaced by \( \Omega_k \). Park & Ratra (2018a) used the largest compilation of current reliable observational data to study the nonflat \( \Lambda \)CDM inflation model, confirming the results of Ooba et al. (2018a) and finding stronger evidence for nonflatness, 5.1\( \sigma \), favoring a very slightly closed model. The CMB anisotropy measurements also do not demand flat spatial hypersurfaces in the seven parameter nonflat XCDM dynamical dark energy inflation parameterization (Ooba et al. 2017). Here \( w \) is the seventh cosmological parameter and again \( n_s \) is replaced by \( \Omega_k \). In the simplest seven parameter nonflat \( \phi \)CDM dynamical dark energy inflation model (Pavlov et al. 2013) — in which \( \alpha \) is the seventh cosmological parameter with \( n_s \) replaced by \( \Omega_k - \Omega_\alpha \) — Ooba et al. (2018b) (also see Park & Ratra 2018b) again found that CMB anisotropy observations do not require flat spatial geometry. In both the XCDM and \( \phi \)CDM inflation cases the data also favor a very slightly closed model. All three slightly closed models are more consistent with \( \sigma_8 \) constraints from weak lensing observations.

In this paper we determine observational constraints on the seven parameter tilted flat-XCDM and the seven parameter untilted nonflat XCDM dynamical dark energy inflation parameterizations. For this purpose we use an updated version of the Planck 2015 CMB anisotropy, and (almost all currently available reliable) SNIa apparent magnitude, BAO distance, growth factor, and Hubble parameter data compilation of Park & Ratra (2018a). Our main update here is the replacement of the Joint Light-curve Analysis (JLA) compilation of apparent magnitude measurements of 740 SNIa (Betoule et al. 2014) by the Pantheon collection of 1048 SNIa measurements (Scollnic et al. 2017). When used with the Planck 2015 CMB anisotropy data in an analysis of the nonflat \( \Lambda \)CDM case, the Pantheon data place tighter constraints on spatial curvature than do the JLA data. Overall, for the full data compilation, our updated results for the tilted flat-\( \Lambda \)CDM inflation model and the nonflat \( \Lambda \)CDM inflation model here are very similar to those of Park & Ratra (2018a), with evidence for nonflatness in the nonflat \( \Lambda \)CDM case now becoming 5.2\( \sigma \) (from 5.1\( \sigma \)).

Our first main goal here is to examine the consequences of including a significant amount of recent, reliable, non-CMB data on the discovery of Ooba et al. (2018c) that the Planck 2015 CMB anisotropy data and a few BAO distance measurements favor the seven parameter tilted flat-XCDM parameterization over the six parameter standard flat-\( \Lambda \)CDM model. Our second main goal is to examine the effect that the inclusion of this new non-CMB data compilation has on the discovery of Ooba et al. (2017) that the Planck 2015 CMB anisotropy observations and a few BAO distance measurements are not inconsistent with the closed-XCDM inflation parameterization. Our third main goal is to use this large compilation of reliable data to examine the compatibility of the cosmological constraints from each type of data and to also more tightly measure cosmological parameters than has been achieved to date, and in particular to also find out which model parameters can or cannot be measured from these data in a cosmological-model-independent manner.

We find that the seven parameter tilted flat-XCDM inflation parameterization continues to provide a better fit to the data

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\(^5\) While XCDM is widely used to model dynamical dark energy, it does not accurately model \( \phi \)CDM (Podariu & Ratra 2001; Ooba et al. 2018c).

\(^6\) Both analyses also included in their data compilation a high value of \( H_0 \), estimated from the local expansion rate. We do not include this high local \( H_0 \) value in the data compilation we use here to constrain cosmological parameters, as it is not consistent with the other data we use, in the \( \Lambda \)CDM, XCDM, and \( \phi \)CDM models.

\(^7\) Non-CMB observations do not tightly constrain spatial curvature (Farooq et al. 2015; Cai et al. 2016; Chen et al. 2016; Yu & Wang 2016; L'Huillier &
than does the six parameter standard ΛCDM model. However, for the large compilation of data used here we find the XCDM dynamical dark energy case is only 0.28σ better than the standard ΛCDM case (compared to the 1.1σ Ooba et al. 2018c found with the smaller data compilation). This is not a significant improvement over the standard model but on the other hand the XCDM parameterization cannot be ruled out. 

Also in agreement with Ooba et al. (2018c) we do not find a deviation from \( w = -1 \) (a cosmological constant) for the flat-XCDM case.\(^8\) Similar to the ΛCDM models in Park & Ratra (2018a), the tilted flat-XCDM model continues to better fit the weak lensing bounds in the \( \sigma_8 - \Omega_m \) plane than does the untilted nonflat XCDM model.

For the untilted nonflat XCDM inflation parameterization, our results here, determined using much more non-CMB data, are consistent with and strengthen those of Ooba et al. (2017). For the full data compilation we now find a more than 3.4σ deviation from spatial flatness and now, for the first time, we also find a corresponding deviation from a cosmological constant with \( w = -0.960 \pm 0.032 \) in the nonflat XCDM case being more than 1.2σ away from the cosmological constant value of \( w = -1 \). The nonflat XCDM parameterization better fits the weak lensing limits in the \( \sigma_8 - \Omega_m \) plane. For the full data combination we consider here (including CMB lensing data), we find that the observed low-\( \ell \) CMB temperature and polarization anisotropy multipole number (\( \ell \)) power spectrum \( C_\ell \) is best fit\(^9\) by the tilted flat-ΛCDM model, followed by the tilted flat-XCDM parameterization, with the untilted nonflat ΛCDM model and the untilted nonflat XCDM parameterization in third and fourth place. The tilted flat-ΛCDM model and the tilted flat-XCDM parameterization best fit the observed higher-\( \ell \) \( C_\ell \)'s, followed by the untilted nonflat XCDM parameterization and the untilted nonflat ΛCDM model in third and fourth place.

We find that \( H_0 \) is measured in an almost model-independent manner with values that are consistent with most other estimates. However, as also found in Park & Ratra (2018a), some measured cosmological parameter values, including \( \Omega_b h^2 \), \( \tau \), and \( \Omega_c h^2 \), differ significantly between the flat and the nonflat models and so caution is needed when utilizing cosmological measurements of such parameters.

In Sec. 2 we briefly summarize the cosmological data we use in our analyses. In Sec. 3 we briefly summarize the methods we use for our analyses. The observational constraints resulting from these data for the tilted flat-XCDM and the nonflat XCDM inflation parameterizations are presented in Sec. 4. We conclude in Sec. 5.

2. DATA

As in Park & Ratra (2018a) we use the TT + lowP and TT + lowP + lensing Planck 2015 CMB anisotropy data (Planck Collaboration 2016). Here TT denotes the low-\( \ell \) (\( 2 \leq \ell \leq 29 \)) and high-\( \ell \) (30 \( \leq \ell \leq 2508 \); PlikTT) Planck 2015 temperature-only \( C_\ell^{TT} \) data and lowP represents low-\( \ell \) polarization \( C_\ell^{EE} \), \( C_\ell^{BB} \), and \( C_\ell^{TE} \) power spectra measurements at \( 2 \leq \ell \leq 29 \). The collection of low-\( \ell \) temperature and polarization measurements is referred to as lowTEB. The CMB lensing data we use is the power spectrum of the lensing potential measured by Planck.

Instead of using the JLA apparent magnitude measurement compilation of 740 SNIa (Betoule et al. 2014), we use the Pantheon collection of 1048 SNIa apparent magnitude measurements over the broader redshift range of 0.01 < \( z < 2.3 \) (Scolnic et al. 2017), which includes 276 SNIa (0.03 < \( z < 0.65 \)) discovered by the Pan-STARRS1 Medium Deep Survey and SNIa distance estimates from SDSS, SNLS and low-z HST samples. Throughout this paper, we use the abbreviation SN to denote the Pantheon SNIa sample.

We make one change to the BAO compilation of Sec. 2.3 and Table 1 of Park & Ratra (2018a), here using \( D_{s}(r_s,\Omega_m,\Omega_{\Lambda}) = 3843 \pm 147 \) Mpc for the Ata et al. (2018) BAO data point, instead of the old value, \( D_{s}(r_s,\Omega_m,\Omega_{\Lambda}) = 3855 \pm 170 \) Mpc, given in the initial version of their preprint (arXiv:1705.06373v1). See Sec. 2.3 of Park & Ratra (2018a) for the definitions of the above expressions. Throughout this paper, we use the abbreviation BAO to denote this updated BAO compilation.

We also use the same Hubble parameter, \( H(z) \), and growth rate, \( f(z) \sigma_8(z) \), measurements listed in Tables 2 and 3 of Park & Ratra (2018a).

3. METHODS

We use the publicly available CAMB/COSMOMC analysis software (November 2016 version) (Challinor & Lasenby 1999; Lewis et al. 2000; Lewis & Bridle 2002) to constrain cosmological parameters of the tilted flat and the untilted nonflat XCDM dynamical dark energy inflation parameterizations with Planck 2015 CMB measurements and non-CMB data sets. We use the CAMB Boltzmann code to compute the angular power spectra for CMB temperature fluctuations, polarization, and lensing potential, and COSMOMC, based on the Markov chain Monte Carlo (MCMC) method, to determine the range of model parameters favored by the data. We use the same COSMOMC settings adopted in Planck Collaboration (2016) and used in Park & Ratra (2018a).

The spatially-flat tilted XCDM inflation case primordial power spectrum (Lucchin & Matarrese 1985; Ratra 1992, 1989) is

\[
P(k) = A_s \left( \frac{k}{k_0} \right)^n_s,
\]

where \( A_s \) is the amplitude of the power spectrum at the pivot scale \( k_0 = 0.05 \) Mpc\(^{-1}\) and \( k \) is wavenumber. The untilted nonflat XCDM inflation case primordial power spectrum (Ratra & Peebles 1995; Ratra 2017) is

\[
P(q) \propto \frac{(q^2 - 4K)^2}{q(q^2 - K)},
\]

which reduces to the \( n_s = 1 \) spectrum in the spatially-flat limit (\( K = 0 \)). For scalar perturbations, \( q = \sqrt{k^2 + K} \) is wavenumber where \( K = -(H_0^2/c^2)\Omega_{k} \) is spatial curvature and \( c \) is the speed of light. In the spatially-closed model, with negative \( \Omega_{k} \), normal modes are characterized by positive integers \( \nu = qK^{-1/2} = 3, 4, 5, \ldots \). We use \( P(q) \) as the initial spatial inhomogeneity perturbation power spectrum for the nonflat model by normalizing it at the pivot scale \( k_0 \) to the value of \( A_s \).
Our analyses methods are very similar to those described in Sec. 3.2 of Park & Ratra (2018a). During the MCMC process we set the same priors for the cosmological parameters as in Park & Ratra (2018a). For the seventh parameter $w$, we set $-3 \leq w \leq 0.2$. In our analyses with the Pantheon SNIa sample, we do not set priors for the nuisance parameters ($\alpha_{SN}$ and $\beta_{SN}$) related to the stretch and the color correction of the SNIa light curves, since the stretch and color parameters of the Pantheon SNIa used here are set to zero.\textsuperscript{10}

4. OBSERVATIONAL CONSTRAINTS

We first examine how much more effective the improved Pantheon SNIa data are in constraining cosmological parameters, relative to the JLA data. Figure 1 compares the likelihood distributions of the model parameters for the JLA and the Pantheon data sets, in conjunction with the CMB observations, for the spatially-flat tilted and for the untilted nonflat $\Lambda$CDM inflation models. The mean and 68.3% confidence limits of model parameters are presented in Table 1.\textsuperscript{11} Without CMB lensing data, the Pantheon data are a little more constraining than the JLA data. When CMB lensing data are included, the largest reduction in error bars occur for the nonflat $\Lambda$CDM case, where the error bars for $\Omega_m$, $H_0$, and $\Omega_b$ are approximately only 80% as large for the CMB + Pantheon combination when compared to the CMB + JLA case. From this Table we also see that including CMB lensing measurements results in a decrease of $A_{s}$ and $\tau$ in both models.

Note that our six parameter physically-consistent untilted non-power-law power spectrum nonflat $\Lambda$CDM model constrained by the CMB and Pantheon data favors nonflat geometry and that the parameter constraints determined using our model are quite different from those presented in Scolnic et al. (2017) that were derived using the seven parameter physically-inconsistent tilted power-law power spectrum nonflat $\Lambda$CDM model (with varying spectral index $n_s$) which were found to favor spatial flatness ($\Omega_k = 0.004 \pm 0.006$, $\Omega_b = 0.295 \pm 0.024$, $H_0 = 69.695 \pm 2.933$ km s\(^{-1}\) Mpc\(^{-1}\) for the TT + lowP + Pantheon data combination).

Table 2 lists the parameter constraints for the tilted spatially-flat and for the untilted nonflat $\Lambda$CDM inflation models, for the updated complete data set we use here. These constraints can be compared to those listed in the bottom right panels of Tables 5–8 of Park & Ratra (2018a) that were derived using the JLA SNIa data and the initial preprint value of the Ata et al. (2018) BAO distance measurement. There are very small differences between the constraints derived using our previous and our updated full data sets.

Our results for the tilted flat and the nonflat $\Lambda$CDM parameterizations are presented in Figs. 2–5 and Tables 3–6. In the plots we omit likelihood contours for TT + lowP (+ lensing) + SN + BAO data (excluding or including the Planck 2015 CMB lensing data) in both tilted flat and nonflat $\Lambda$CDM cases because they are very similar to those for TT + lowP (+ lensing) + SN + BAO + $H(z)$ data.

The entries for the tilted flat-$\Lambda$CDM parameterization in the TT + lowP panel of Table 3 and in the TT + lowP + lensing panel in Table 4 are very consistent with the corresponding Table 1 entries of Ooba et al. (2018c), except for those for $w$, $H_0$, $\Omega_m$, and $\sigma_8$. This is because Ooba et al. (2018c) use a flat prior non-zero over $0.2 \leq h \leq 1.3$ for $H_0$ while we use a flat prior non-zero over $0.2 \leq h \leq 1$.\textsuperscript{12} The entries for the nonflat $\Lambda$CDM parameterization in the TT + lowP panel of Table 3 and in the TT + lowP + lensing panel in Table 6 agree well with the corresponding entries in Table 1 of Ooba et al. (2017). Ooba et al. (2017) and Ooba et al. (2018c) compute the $C_\ell$’s using CLASS (Blas et al. 2011) and performed the MCMC analyses with Monte Python (Audren et al. 2013), so it is reassuring that our results agree well with their results. Our estimates of $w$, $\Omega_m$, and $H_0$ for the tilted flat-$\Lambda$CDM parameterization from the TT + lowP + SN data agree very well with the values presented in Scolnic et al. (2017), $w = -1.031 \pm 0.040$, $\Omega_m = 0.306 \pm 0.012$, and $H_0 = 68.335 \pm 1.098$ km s\(^{-1}\) Mpc\(^{-1}\), which provides another reassuring check on our analyses.

From Tables 3 and 4 we see that, when they are added to the Planck CMB anisotropy observations, for the tilted flat-$\Lambda$CDM case, the BAO measurements mostly prove more restrictive than either the SN, $H(z)$, or $\sigma_8$ data , except for $w$ where the SN data are more restrictive than the BAO data. However, when the CMB lensing data are included, Table 4, the CMB + SN limits on $w$, $H_0$, and $\sigma_8$ are more restrictive than those from the CMB data combined with BAO, or $H(z)$, or $\sigma_8$ measurements, while all four non-CMB data sets used in conjunction with the CMB data provide equally restrictive constraints on $\Omega_b h^2$ and $A_s$.\textsuperscript{13} We note that our BAO compilation includes radial $H(z)$ measurements and the $f\sigma_8$ data of Alam et al. (2017). It is likely that if these are moved to the $H(z)$ and $f\sigma_8$ data sets, CMB and BAO, SN, $H(z)$, or $f\sigma_8$ constraints will all be about equally restrictive for the flat-$\Lambda$CDM parameterization.

The nonflat $\Lambda$CDM case is more interesting. When CMB lensing data are included, Table 6, CMB data with either SN, or BAO, or $H(z)$, or $f\sigma_8$ data, provide approximately equally restrictive constraints on $\Omega_b h^2$, $\Omega_c h^2$, and $\theta_{90}$, while CMB + BAO data provide the tightest constraints on $\tau$, $A_s$, $\Omega_b$, $H_0$, $\Omega_m$, and $\sigma_8$, with CMB + SN setting tightest limits on $w$.\textsuperscript{14} Focusing on the CMB TT + lowP + lensing measurements, Figs. 3 and 5 and Tables 4 and 6, we see that adding each of the four non-CMB measurement sets at a time to the CMB data (left triangle plots in both figures) produces four sets of contours that are quite mutually consistent, as well as con-

\textsuperscript{10} In addition to $\alpha_{SN}$ and $\beta_{SN}$, the distance moduli of the Pantheon SNIa are affected by three more nuisance parameters, the absolute B-band magnitude ($M_B$), the distance correction based on the host-galaxy mass ($\Delta M$), and the distance correction based on predicted biases from simulation ($\Delta \phi$) (Scolnic et al. 2017). Consequently, the number of degrees of freedom of the Pantheon sample is less than the number of SNIa. For example, for a flat-$\Lambda$CDM model analysis that fits $\Omega_m$, the number of degrees of freedom becomes 1042 (≈1048–6).

\textsuperscript{11} The parameter values of the tilted flat-$\Lambda$CDM model constrained by using TT + lowP (+ lensing) + JLA data are in good agreement with the Planck results. See Planck 2015 cosmological parameter tables base_plikHM_TT_lowTEB_post_JLA for TT + lowP + JLA data and base_plikHM_TT_lowTEB_post_JLA_for TT + lowP + lensing + JLA data (Planck Collaboration 2015).

\textsuperscript{12} Since the flat prior on $h$ adopted here is the same as in the Planck team’s analyses, the parameters for the tilted flat-$\Lambda$CDM case constrained with TT + lowP (+ lensing) agree with the Planck results. See base_w_plikHM_TT_lowTEB for TT + lowP data and base_w_plikHM_TT_lowTEB_post_lensing for TT + lowP + lensing data (Planck Collaboration 2015).

\textsuperscript{13} This is not the case in the tilted flat-$\Lambda$CDM model, where for the data set including the CMB lensing data, the CMB + BAO constraints on all parameters are more restrictive than those determined by combining the CMB data with either the SN, or $H(z)$, or $f\sigma_8$ measurements. For this model we show only the CMB + SN constraints in Table 1.

\textsuperscript{14} In the nonflat $\Lambda$CDM model (results mostly not shown here, except for CMB + SN shown in Table 1), $\Omega_b h^2$, $\Omega_c h^2$, and $\theta_{90}$ are about equally well constrained by any of the four non-CMB data sets when used with the CMB (including lensing) data, with CMB + BAO setting tighter limits on $\tau$, $A_s$, $\Omega_b$, $H_0$, $\Omega_m$, and $\sigma_8$.
Tilted flat and untitled nonflat $\Lambda$CDM parameterizations

Figure 1. Likelihood distributions of the tilted flat (left) and untitled nonflat (right) $\Lambda$CDM inflation model parameters favored by the Planck CMB TT + lowP (+ lensing) and SNIa data. Here the parameter constraints are compared for the JLA SNIa data and the Pantheon SNIa data and summarized in Table 1. Two-dimensional marginalized likelihood contours as well as one-dimensional likelihoods are shown as solid and dashed black curves for JLA and filled contours and colored curves for Pantheon data.

Table 1
Mean and 68.3% confidence limits of tilted flat and untitled nonflat $\Lambda$CDM model parameters constrained by Planck and SNIa data. JLA versus Pantheon.

| Parameter          | TT+lowP+JLA       | TT+lowP+lensing+JLA | TT+lowP+Pantheon   | TT+lowP+lensing+Pantheon |
|--------------------|-------------------|---------------------|--------------------|--------------------------|
| $\Omega_b h^2$     | 0.02226 ± 0.00023 | 0.02227 ± 0.00022   | 0.02228 ± 0.00022  | 0.02228 ± 0.00022         |
| $\Omega_c h^2$     | 0.1193 ± 0.0020   | 0.1183 ± 0.0019     | 0.1191 ± 0.0019    | 0.1182 ± 0.0017           |
| $100 b_{MC}$       | 1.04092 ± 0.00047 | 1.04105 ± 0.00045   | 1.04094 ± 0.00046  | 1.04106 ± 0.00044         |
| $\tau$             | 0.080 ± 0.019     | 0.068 ± 0.016       | 0.080 ± 0.019      | 0.068 ± 0.015             |
| $\ln(10^{-10} A_s)$| 3.092 ± 0.035     | 3.066 ± 0.029       | 3.092 ± 0.036      | 3.065 ± 0.028             |
| $n_s$              | 0.9666 ± 0.0057   | 0.9683 ± 0.0058     | 0.9671 ± 0.0056    | 0.9684 ± 0.0055           |
| $H_0$ [km s$^{-1}$ Mpc$^{-1}$] | 67.52 ± 0.89 | 67.93 ± 0.88 | 67.62 ± 0.84 | 67.96 ± 0.80 |
| $\Omega_b$         | 0.312 ± 0.012     | 0.306 ± 0.012       | 0.311 ± 0.011      | 0.306 ± 0.011             |
| $\sigma_8$         | 0.829 ± 0.014     | 0.8156 ± 0.0093     | 0.829 ± 0.015      | 0.8152 ± 0.0094           |

| Parameter          | TT+lowP+JLA       | TT+lowP+lensing+JLA | TT+lowP+Pantheon   | TT+lowP+lensing+Pantheon |
|--------------------|-------------------|---------------------|--------------------|--------------------------|
| $\Omega_b h^2$     | 0.02318 ± 0.00020 | 0.02304 ± 0.00020   | 0.02316 ± 0.00020  | 0.02305 ± 0.00020         |
| $\Omega_c h^2$     | 0.1094 ± 0.0011   | 0.1091 ± 0.0011     | 0.1094 ± 0.0011    | 0.1091 ± 0.0011           |
| $100 b_{MC}$       | 1.04231 ± 0.00042 | 1.04233 ± 0.00044   | 1.04228 ± 0.00042  | 1.04235 ± 0.00041         |
| $\tau$             | 0.126 ± 0.018     | 0.107 ± 0.017       | 0.130 ± 0.018      | 0.107 ± 0.015             |
| $\ln(10^{-10} A_s)$| 3.162 ± 0.036     | 3.121 ± 0.034       | 3.169 ± 0.035      | 3.121 ± 0.030             |
| $\Omega_b$         | -0.0257 ± 0.0091  | -0.013 ± 0.0062     | -0.0192 ± 0.0060   | -0.0132 ± 0.0051          |
| $H_0$ [km s$^{-1}$ Mpc$^{-1}$] | 61.5 ± 2.9 | 66.0 ± 2.5 | 63.6 ± 2.2 | 66.0 ± 2.0 |
| $\Omega_b$         | 0.355 ± 0.033     | 0.306 ± 0.023       | 0.330 ± 0.022      | 0.306 ± 0.018             |
| $\sigma_8$         | 0.815 ± 0.018     | 0.805 ± 0.017       | 0.822 ± 0.017      | 0.805 ± 0.015             |
Table 2
Tilted flat and untilted nonflat ΛCDM model parameters constrained with Planck, SN, BAO, \( H(z) \), and \( f \sigma_8 \) data (mean and 68.3% confidence limits).

| Parameter                  | TT+lowP+SN+BAO+\( H(z) \)+\( f \sigma_8 \) | TT+lowP+lensing+SN+BAO+\( H(z) \)+\( f \sigma_8 \) |
|----------------------------|---------------------------------------------|--------------------------------------------------|
| \( \Omega_b h^2 \)        | 0.02233 ± 0.00020                           | 0.02232 ± 0.00019                                |
| \( \Omega_c h^2 \)        | 0.1178 ± 0.0011                             | 0.1177 ± 0.0011                                  |
| 100\( \theta_{MC} \)      | 1.04104 ± 0.00042                           | 1.04108 ± 0.00041                                |
| \( \tau \)                | 0.070 ± 0.017                               | 0.066 ± 0.012                                    |
| \( \ln(10^{10} A_s) \)    | 3.069 ± 0.033                               | 3.061 ± 0.023                                    |
| \( n_s \)                 | 0.9693 ± 0.0042                             | 0.9692 ± 0.0043                                  |
| \( H_0 \) [km s\(^{-1}\) Mpc\(^{-1}\)] | 68.15 ± 0.52                               | 68.19 ± 0.50                                    |
| \( \Omega_m \)            | 0.3031 ± 0.0067                             | 0.3025 ± 0.0064                                  |
| \( \sigma_8 \)            | 0.815 ± 0.013                               | 0.8117 ± 0.0088                                  |

| Parameter                  | TT+lowP+SN+BAO+\( H(z) \)+\( f \sigma_8 \) | TT+lowP+lensing+SN+BAO+\( H(z) \)+\( f \sigma_8 \) |
|----------------------------|---------------------------------------------|--------------------------------------------------|
| \( \Omega_b h^2 \)        | 0.02307 ± 0.00020                           | 0.02305 ± 0.00019                                |
| \( \Omega_c h^2 \)        | 0.1094 ± 0.0010                             | 0.1093 ± 0.0010                                  |
| 100\( \theta_{MC} \)      | 1.04225 ± 0.00042                           | 1.04227 ± 0.00041                                |
| \( \tau \)                | 0.121 ± 0.016                               | 0.112 ± 0.012                                    |
| \( \ln(10^{10} A_s) \)    | 3.150 ± 0.033                               | 3.132 ± 0.022                                    |
| \( \Omega_b \)            | -0.0083 ± 0.0016                            | -0.0083 ± 0.0016                                 |
| \( H_0 \) [km s\(^{-1}\) Mpc\(^{-1}\)] | 67.96 ± 0.62                               | 68.01 ± 0.62                                    |
| \( \Omega_m \)            | 0.2882 ± 0.0055                             | 0.2875 ± 0.0055                                  |
| \( \sigma_8 \)            | 0.820 ± 0.014                               | 0.8121 ± 0.0095                                  |

Figure 2. Likelihood distributions of the tilted flat-\( \Lambda \)CDM model parameters constrained by Planck CMB TT + lowP, SN, BAO, \( H(z) \), and \( f \sigma_8 \) data. Two-dimensional marginalized likelihood contours as well as one-dimensional likelihoods are shown for cases when each non-CMB measurement set is added to the Planck TT + lowP data (left panel) and when the Hubble parameter, SN, growth rate data, and the combination of them, are added to TT + lowP + BAO data (right panel). For clarity the TT + lowP (left) and TT + lowP + BAO (right panel) cases are shown as solid black curves.
Tilted flat and untilted nonflat XCDM parameterizations

consistent with the original CMB only contours, for both the tilted flat-XCDM parameterization and for the untilted non-flat XCDM parameterization. It is reassuring that the four sets of non-CMB measurements do not push the CMB constraints in significantly different directions. This is also the case for the tilted flat-XCDM parameterization when the CMB lensing data are excluded (left triangle plot of Fig. 2). However, in the untilted nonflat XCDM case excluding the lensing data when any of the four sets of non-CMB observations are added to the CMB measurements (left triangle panel of Fig. 4), they each push the results toward a smaller $|\Omega_k|$ (closer to spatially flat) and a slightly larger $\tau$ and $A_s$ and a smaller $\Omega_b h^2$ than what is favored by the CMB measurements alone, though all five sets of constraint contours are mostly mutually consistent. However, there is tension between the TT + lowP + SN and the TT + lowP + BAO contours in the $\Omega_m - w$ plane (Fig. 4 left and Table 5), where CMB + SN data give constraints on $\Omega_k$ and $w$ that deviate from the CMB + BAO data values by over $2\sigma$, with $w$ also deviating from the cosmological constant ($w = -1$) by over $2\sigma$ for the CMB + SN case and $\Omega_k$ differing from 0 by more than $3\sigma$ in both cases.

While adding the BAO data to the CMB data usually results in the biggest difference, the other three non-CMB sets of data also contribute. Focusing on TT + lowP + lensing data, we see from Table 4 for the tilted flat-XCDM parameterization that the BAO data tightly constrains model parameters, especially $\Omega_c h^2$, while the $f_s\sigma_8$ measurements push $\Omega_b h^2$ and $n_s$ to larger values and push $\Omega_c h^2$ to a smaller value. In this case $H_0$ is the parameter whose error bar is decreased the most for the full combination of data relative to the CMB + SN data combination, followed by the $\Omega_m$ error bar reduction relative to CMB + BAO data combination. For the untilted nonflat XCDM case, from Table 6, the error bars that shrink the most when CMB (including lensing) data are used in conjunction with the four non-CMB data sets are those on $w$ (relative to the CMB + SN case) and $H_0$ and $\Omega_m$ (relative to the CMB + BAO combination).

Continuing to focus on the TT + lowP + lensing data, Tables 4 and 6, we see that for the tilted flat-XCDM parameterization, adding the four sets of non-CMB data to the mix most influences $\sigma_8$, $w$, and $\Omega_m$, with the $\sigma_8$ central value moving down by 1.3$\sigma$ and the $w$ and $\Omega_m$ central values moving up by 1.3$\sigma$ and 1.2$\sigma$, all of the CMB data alone error bars; $\theta_Mc$ is hardly affected by adding the four non-CMB data sets, changing by only 0.042$\sigma$. The situation for the nonflat XCDM parameterization is a little less dramatic, with $\ln(10^{10}A_s)$, and $\tau$ central values increasing by 0.91$\sigma$ and 0.86$\sigma$ of the CMB data alone error bars, and $\Omega_k$ moving closer to flatness by 0.71$\sigma$; the $\Omega_b h^2$ central value does not change in this case.

Figure 6 shows marginalized likelihood contours in the $\Omega_m - w$ plane for the tilted flat-XCDM parameterization and in the $w - \Omega_m$ plane for the untilted nonflat XCDM case. For CMB TT + lowP + lensing data combined with the non-CMB data sets, the flat-XCDM parameterization prefers $w = -1$, favoring the cosmological constant as dark energy. On the other hand, the nonflat XCDM parameterization, when constrained by the full data, prefers closed spatial hypersurfaces and a dark energy equation of state parameter $w > -1$.

More precisely, including the four non-CMB sets of measurements in the mix, we find in the tilted flat-XCDM parameterization (bottom right panel of Table 4) that $w = -0.994 \pm 0.033$, which is more tightly restricted to $w = -1$ and the cosmological constant than the original Ooba et al. (2018c) finding of $w = -1.03 \pm 0.07$ (the last column of their Table 1).

On the other hand, and perhaps the most striking consequence of adding the four non-CMB data sets to the mix here, is the significant strengthening of the support for non-flatness in the untilted nonflat XCDM case, with it increasing to $\Omega_k = -0.0069 \pm 0.0020$, more than 3.4$\sigma$ away from flatness now, for the full data combination in the bottom

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13 These results differ from those of earlier approximate analyses, based on less and less reliable data, that indicated evidence for $w$ deviating from $-1$ by more than $3\sigma$ (Solà et al. 2017a, 2018, 2017b,c,d; Gómez-Valent & Solà 2017, 2018).
right panel of Table 6, compared to the 1.1σ from flatness for the CMB only case. That is now accompanied by mild evidence favoring dynamical dark energy with $w = -0.960 \pm 0.032$ that is more than 1.2σ away from the cosmological constant. These results are consistent with but strengthen those of Ooba et al. (2017) who found $\Omega_k = -0.008 \pm 0.003$ and $w = -1.00 \pm 0.10$ from Planck 2015 CMB anisotropy data in combination with a few BAO distance measurements. The stronger results here are driven in part by each of the four non-CMB data sets. CMB + BAO, CMB + SN, and CMB + $H(z)$ favor negative $\Omega_k$ values 2.8σ, 2.0σ, and 1.9σ away from flat, while CMB + $f\sigma_8$ and CMB + BAO favor $w$ values that are 1.1σ more negative and 1σ less negative than $w = -1$. On the other hand, CMB + $f\sigma_8$ data are consistent with a flat model and CMB + SN and CMB + $H(z)$ are consistent with the cosmological constant and $w = -1$. In favoring a closed model with $w$ less negative than $-1$, the BAO data play the most important role amongst the four non-CMB data sets.
Tilted flat and untilted nonflat XCDM parameterizations

| Parameter | TT+lowP | TT+lowP+SN | TT+lowP+BAO |
|-----------|---------|------------|-------------|
| \(\Omega_b h^2\) | 0.02228 ± 0.00023 | 0.02221 ± 0.00023 | 0.02231 ± 0.00021 |
| \(\Omega_c h^2\) | 0.1195 ± 0.0022 | 0.1200 ± 0.0022 | 0.1185 ± 0.0016 |
| 100\(\theta_{MC}\) | 1.04093 ± 0.00048 | 1.04085 ± 0.00047 | 1.04103 ± 0.00044 |
| \(\tau\) | 0.076 ± 0.020 | 0.076 ± 0.019 | 0.079 ± 0.019 |
| \(\ln(10^{10}A_s)\) | 3.086 ± 0.037 | 3.086 ± 0.037 | 3.090 ± 0.036 |
| \(n_s\) | 0.9662 ± 0.0063 | 0.9651 ± 0.0061 | 0.9684 ± 0.0052 |
| \(w\) | −1.53 ± 0.30 | −1.034 ± 0.040 | −0.993 ± 0.050 |

For the full data combination (including CMB lensing data) in Tables 4 and 6, \(H_0\) values measured using the tilted flat-XCDM and the nonflat XCDM parameterizations, 68.06 ± 0.77 and 67.45 ± 0.75 km s\(^{-1}\) Mpc\(^{-1}\), are consistent with each other to within 0.5\(\sigma\) (of the quadrature sum of the two error bars). These values are very compatible with the median statistics estimate \(H_0 = 68.26 ± 2.58\) km s\(^{-1}\) Mpc\(^{-1}\) (Chen & Ratra 2011a), which agrees with earlier median statistics measurements (Gott et al. 2001; Chen et al. 2003). Other recent measurements of \(H_0\) are also very compatible with these estimates (Aubourg et al. 2015; Planck Collaboration 2016; Semiz & Çamlibel 2015; L’Huillier & Shafieloo 2017; Chen et al. 2017; Luković et al. 2016; Wang et al. 2017; Lin & Ishak 2017; DES Collaboration 2017; Yu et al. 2018; Haridasu et al. 2018; Zhang et al. 2018; Gómez-Valent & Amendola 2018), but, as well known, these estimates are lower than the local expansion rate measurement of \(H_0 = 73.48 ± 1.66\) km s\(^{-1}\) Mpc\(^{-1}\) (Riess et al. 2018). In our analyses here, \(H_0\) and \(\sigma_8\) (see below) are the only cosmological parameters that are measured in an almost cosmological model (tilt and spatial curvature) independent way. Measurements of other cosmological parameters determined using the two XCDM parameterizations differ more significantly. More precisely, measurements determined using the full data set (including CMB lensing) of \(w, \Omega_m, \Omega_{b,MC}, \ln(10^{10}A_s), \Omega_b h^2, \tau,\) and \(\Omega_c h^2\), differ by 0.74\(\sigma\), 1.1\(\sigma\), 2.0\(\sigma\), 2.5\(\sigma\), 2.5\(\sigma\), 2.7\(\sigma\), and 4.8\(\sigma\) (of the quadrature sum of both error bars). For some of these parameters, especially \(\Omega_c h^2\) as well as probably \(\tau\) and \(\Omega_b h^2\), the cosmological model dependence of the measurement creates a much larger uncertainty than does the statistical error in a given cosmological model. This effect was first noticed in a comparison between measurements made using the tilted flat-XCDM and the un-

16 Potential systematic errors that might affect the value of \(H_0\) ignored here, have been discussed by Addison et al. (2016) and Planck Collaboration (2017).

17 This local measurement is 2.9\(\sigma\) (3.3\(\sigma\)), of the quadrature sum of both error bars, higher than \(H_0\) measured here using the tilted flat-XCDM (untilted nonflat XCDM) parameterization. Other local expansion rate estimates find slightly lower \(H_0\)’s with larger error bars (Rigault et al. 2015; Zhang et al. 2017b; Dhawan et al. 2018; Fernández Arenas et al. 2018).
For comparison we also plot the parameterizations constrained using the CMB and non-CMB data: \(\sigma_{\Omega}\), \(\sigma_{\Lambda}\), \(\sigma_{\tau}\), or \(\sigma_{\Omega b}\) (and possibly some other cosmological parameters as well) in a model independent way by using cosmological data.

For the full data combination (including CMB lensing data), \(\sigma_b\)'s measured using the two XCDM parameterizations, Tables 4 and 6, agree to 0.32\(\sigma\) (of the quadrature sum of the two error bars). Figures 7 and 8 show the marginalized two-dimensional likelihood distribution contours in the \(\Omega_m\)–\(\sigma_b\) plane for the tilted flat and untilted nonflat XCDM parameterizations constrained using the CMB and non-CMB data. For comparison we also plot the \(\sigma_b\) constraints obtained from a joint analysis of galaxy clustering and weak gravitational lensing first year data of the Dark Energy Survey (DES Y1 All) (DES Collaboration 2018), whose 1\(\sigma\) confidence ranges are \(\Omega_m = 0.264_{-0.015}^{+0.023}\) and \(\sigma_b = 0.807_{-0.041}^{+0.055}\). The marginalized likelihood distribution contours in the \(\Omega_m\)–\(\sigma_b\) plane determined by adding each non-CMB measurement set to the Planck 2015 CMB observations are consistent with each other, except for the nonflat XCDM parameterization where the TT + lowP + SN contours almost do not overlap with contours derived using any of the other three non-CMB data sets with the TT + lowP data (Fig. 8 top left panel). As expected, the BAO data provide the most restrictive constraints among the four non-CMB data sets.

While the \(\sigma_b\) constraints from the tilted flat and untilted nonflat XCDM analyses (allowing for and ignoring CMB lensing data) are similar to the DES Y1 All result, the \(\Omega_m\) constraints here favor a larger value by about 1.2\(\sigma\) (of the quadrature sum of the two error bars) for the flat-XCDM case for the full data combination. We emphasize that the best-fit point for the nonflat XCDM parameterization constrained by using the Planck CMB measurements (including lensing) combined with all non-CMB observations enters well into the 1\(\sigma\) region of the DES Y1 All constraint contour (Fig. 8 lower right panel), unlike the tilted flat XCDM parameterization case (Fig. 7 lower right panel).

Table 7 lists \(\chi^2\) values for the best-fit tilted flat and untilted nonflat XCDM models. This is an updated version of Table 9 of Park & Ratra (2018a), for the updated data sets we use here. Table 8 lists the corresponding quantities for the tilted flat and the untilted nonflat XCDM parameterizations. The best-fit position in parameter space is determined from Powell’s minimization method that is an efficient algorithm to find the location of the minimum \(\chi^2\). We use the COSMOMC program (with an option action=2) to implement this method. In these Tables we list the individual \(\chi^2\) of each data set used to constrain model parameters. The total \(\chi^2\) is the sum of those of the high-\(\ell\) CMB TT likelihood (\(\chi^2_{\text{TT}}\)), the low-\(\ell\) CMB power spectra of temperature and polarization (\(\chi^2_{\text{TEB}}\)), lensing (\(\chi^2_{\text{Lensing}}\)), SN (\(\chi^2_{\text{SN}}\)), \(H(z)\) (\(\chi^2_{\text{H(z)}}\)), BAO (\(\chi^2_{\text{BAO}}\)), \(f\sigma_8\) data (\(\chi^2_{\text{fσ}}\)), as well as the contribution from the foreground nuisance parameters (\(\chi^2_{\text{nuis}}\)). Due to the nonconventional normalization of the Planck CMB anisotropy data likelihoods, the number of Planck 2015 CMB degrees of freedom is ambiguous. Given that the number of degrees of freedom of the Planck 2015 CMB data is unavailable and that the absolute value of \(\chi^2\) is arbitrary, only \(\chi^2\) differences between two models are meaningful for the Planck CMB data. In Table 7, for the untilted nonflat XCDM model, we list \(\Delta\chi^2\), the excess \(\chi^2\) relative to the value of the tilted flat-XCDM model constrained using the same combination of measurements. For the non-CMB observations, the numbers of degrees of freedom are 1042, 31, 15, 10 for the SN, \(H(z)\), BAO, \(f\sigma_8\) data sets, respectively, a total of 1098 degrees of freedom. The reduced \(\chi^2\)'s for each of the non-CMB measurement sets are \(\chi^2/\nu \lesssim 1\). There are 189 points in the Planck 2015 (binned angular power spectrum) TT + lowP data and 197 when the CMB lensing data are included.

Conclusions about the qualitative relative goodness of fit of the tilted flat and nonflat XCDM models drawn from the updated data here are not very different from those found earlier (Park & Ratra 2018a) from the original data. For the nonflat XCDM case relative to the flat-XCDM model, we have \(\Delta\chi^2 = 21\) for TT + lowP + lensing and the full non-CMB compilation (last column in the last row of Table 7). As discussed above and in Ooba et al. (2018a, 2017, 2018b) and Park & Ratra (2018a, b), it is not clear how to convert this into a quantitative relative probability as the two six parameter cases are not nested (and the number of degrees of freedom of the Planck measurements is unavailable). It is clear however that the nonflat XCDM model does a worse job in fitting the higher-\(\ell\) \(C_\ell\)'s than it does in fitting the lower-\(\ell\) ones. We note that there has been discussion about systematic differences between constraints determined using the higher-\(\ell\) and the lower-\(\ell\) Planck 2015 CMB data (Addison et al. 2016; Planck Collaboration 2017). In addition, in the context of the flat-XCDM model, there appear to be inconsistencies between the higher-\(\ell\) Planck 2015 CMB anisotropy data and the South Pole Telescope CMB anisotropy data (Aylor et al. 2017). It is possible that, if real, when these differences are resolved this could result in a reduction of the \(\Delta\chi^2\)'s found here.

Table 8 lists \(\chi^2\) values for the best-fit tilted flat and untilted nonflat XCDM parameterizations. In the last column we list \(\Delta\chi^2\), the excess \(\chi^2\) of the seven parameter XCDM case over the value of the corresponding six parameter XCDM model constrained using the same combination of data sets. These models are nested: the seven parameter tilted flat-XCDM (untilted nonflat XCDM) parameterization reduces to the six parameter tilted flat-XCDM (untilted nonflat XCDM) model when \(w = -1\). In this case the ambiguity in the number of Planck 2015 degrees of freedom is not an obstacle to converting the \(\Delta\chi^2\)'s to a relative goodness of fit. From \(\sqrt{-\Delta\chi^2}\), for the full data set (including CMB lensing), for one additional free parameter, we find that the tilted flat-XCDM (untilted nonflat XCDM) parameterization is a 0.28\(\sigma\) (0.87\(\sigma\)) better fit to the data than is the tilted flat-XCDM (untilted nonflat XCDM) model. (We emphasize that nonflat XCDM does not fit the data as well as flat-XCDM, although the difference in the goodness of fit cannot yet be precisely quantified.) These results are consistent with those of Ooba et al. (2018c) and Ooba et al. (2017).

Of all these four models, the tilted flat-XCDM parameterization best fits the combined data, but at a lower level of significance than the 1.1\(\sigma\) of Ooba et al. (2018c), and not close to the 3 or 4\(\sigma\) significance found in earlier approximate analyses (Solá et al. 2017a, 2017b, c,d; Gómez-Valent & Solá 2017, 2018). While the tilted flat-XCDM parameterization
C does not provide a significantly better fit to the data, current served CMB power spectra at all the Planck CMB and non-CMB data agree well with the ob-parameterizations, excluding and including the lensing data, poorer fit to the low-\(\ell\)\(_{\Lambda}\) it better fits the low-\(\ell\) CMB data sets has a low-constrained by using the TT + lowP + lensing and full non-example, the best-fit untilted nonflat XCDM parameterization \(f_{H_0}h^2\) \(\Omega_{\Lambda}\) \(\Omega_m\) \(\tau\) \(\ln(10^{10}\Lambda_s)\) \(n_s\) \(\omega\) \(H_0\) \(\Omega_{\Lambda}\) \(\Omega_m\) \(\sigma_8\) \(H_0\) \(\Omega_{\Lambda}\) \(\Omega_m\) \(\sigma_8\) \(H_0\) \(\chi^2_{\Lambda}_{\text{lowTEB}}\) is larger than values from other non-CMB combinations. Figure 11 shows the best-fit initial power spectra of scalar fractional energy density spatial inhomogeneity perturbations for the untilted nonflat XCDM parameterization constrained using the Planck TT + lowP (left) and TT + lowP + lensing (right panel) data in conjunction with other non-CMB data sets. The reduction in power at low \(q\) in the best-fit closed-XCDM inflation parameterization spatial inhomogeneity power spectra shown in Fig. 11 is partly responsible for the low-\(\ell\) TT power reduction of the best-fit closed model \(C_\ell\)'s \(\Lambda\) (see the lower panels of Figs. 9 and 10) relative to the best-fit tilted flat model \(C_\ell\)'s.\(^{19}\) The case of the best-fit nonflat XCDM parameterization for the TT + lowP + SN data is the most dramatic one, consistent with the reduced low-\(\ell\) TT power (Figs. 9b).

5. CONCLUSION

\(^{19}\) Other effects, including both the usual and integrated Sachs-Wolfe effects, also affect the shape of the low-\(\ell\) \(C_\ell\)'s.

| Parameter | TT+lowP+lensing | TT+lowP+lensing+SN | TT+lowP+lensing+BAO |
|-----------|-----------------|------------------|---------------------|
| \(\Omega_{\Lambda}\hbar^2\) | 0.02229 ± 0.00023 | 0.02223 ± 0.00022 | 0.02229 ± 0.00022 |
| \(\Omega_m\hbar^2\) | 0.1183 ± 0.0021 | 0.1187 ± 0.0019 | 0.1179 ± 0.0016 |
| 100\(\theta_{\text{MC}}\) | 1.04110 ± 0.00048 | 1.04099 ± 0.00045 | 1.04110 ± 0.00044 |
| \(\tau\) | 0.060 ± 0.017 | 0.064 ± 0.017 | 0.070 ± 0.016 |
| \(\ln(10^{10}\Lambda_s)\) | 3.048 ± 0.032 | 3.060 ± 0.030 | 3.070 ± 0.030 |
| \(n_s\) | 0.9681 ± 0.0060 | 0.9671 ± 0.0056 | 0.9692 ± 0.0052 |
| \(w\) | \(-1.41 ± 0.32\) | \(-1.020 ± 0.039\) | \(-0.984 ± 0.050\) |

\(H_0\) [km s\(^{-1}\) Mpc\(^{-1}\)]: > 60.1 (95.4% C.L.)

\(\Omega_{\Lambda}\) \(\Omega_m\) \(\sigma_8\) \(H_0\) \(\chi^2_{\Lambda}_{\text{lowTEB}}\) is larger than values from other non-CMB combinations. Figure 11 shows the best-fit initial power spectra of scalar fractional energy density spatial inhomogeneity perturbations for the untilted nonflat XCDM parameterization constrained using the Planck TT + lowP (left) and TT + lowP + lensing (right panel) data in conjunction with other non-CMB data sets. The reduction in power at low \(q\) in the best-fit closed-XCDM inflation parameterization spatial inhomogeneity power spectra shown in Fig. 11 is partly responsible for the low-\(\ell\) TT power reduction of the best-fit closed model \(C_\ell\)'s.\(^{19}\) The case of the best-fit nonflat XCDM parameterization for the TT + lowP + SN data is the most dramatic one, consistent with the reduced low-\(\ell\) TT power (Figs. 9b).

5. CONCLUSION

\(^{19}\) Other effects, including both the usual and integrated Sachs-Wolfe effects, also affect the shape of the low-\(\ell\) \(C_\ell\)'s.
Table 5
Untilted nonflat XCDM model parameters constrained with Planck TT + lowP, SN, BAO, \(H(z)\), and \(f\sigma_8\) data (mean and 68.3\% confidence limits).

| Parameter | TT+lowP | TT+lowP+SN | TT+lowP+BAO |
|-----------|---------|------------|-------------|
| \(\Omega_m h^2\) | 0.02335 ± 0.00022 | 0.02328 ± 0.00021 | 0.02305 ± 0.00021 |
| \(\Omega_b h^2\) | 0.1093 ± 0.0010 | 0.1093 ± 0.0011 | 0.1097 ± 0.0011 |
| \(100\theta_{MC}\) | 1.04240 ± 0.00042 | 1.04237 ± 0.00043 | 1.04222 ± 0.00042 |
| \(\tau\) | 0.087 ± 0.029 | 0.107 ± 0.022 | 0.134 ± 0.017 |
| \(\ln(10^{10}A_s)\) | 3.083 ± 0.058 | 3.124 ± 0.044 | 3.178 ± 0.034 |
| \(\Omega_k\) | -0.084 ± 0.052 | -0.045 ± 0.013 | -0.0074 ± 0.0024 |
| \(w\) | -1.45 ± 0.75 | -1.23 ± 0.11 | -0.959 ± 0.056 |
| \(H_0\) [km s\(^{-1}\) Mpc\(^{-1}\)] | 55 ± 14 | 58.6 ± 2.7 | 67.0 ± 1.2 |
| \(\Omega_m\) | 0.52 ± 0.24 | 0.390 ± 0.035 | 0.297 ± 0.010 |
| \(\sigma_8\) | 0.82 ± 0.13 | 0.834 ± 0.019 | 0.820 ± 0.020 |

| Parameter | TT+lowP+\(H(z)\) | TT+lowP+SN+BAO | TT+lowP+SN+BAO+\(H(z)\)+\(f\sigma_8\) |
|-----------|-----------------|----------------|--------------------------------|
| \(\Omega_m h^2\) | 0.02315 ± 0.00020 | 0.02305 ± 0.00020 | 0.02306 ± 0.00020 |
| \(\Omega_b h^2\) | 0.1097 ± 0.0011 | 0.1096 ± 0.0011 | 0.1097 ± 0.0010 |
| \(100\theta_{MC}\) | 1.04227 ± 0.00042 | 1.04219 ± 0.00041 | 1.04221 ± 0.00041 |
| \(\tau\) | 0.131 ± 0.018 | 0.134 ± 0.017 | 0.133 ± 0.017 |
| \(\ln(10^{10}A_s)\) | 3.171 ± 0.036 | 3.178 ± 0.034 | 3.175 ± 0.034 |
| \(\Omega_k\) | -0.0122 ± 0.0044 | -0.0079 ± 0.0021 | -0.0074 ± 0.0020 |
| \(w\) | -1.22 ± 0.19 | -0.974 ± 0.033 | -0.968 ± 0.033 |
| \(H_0\) [km s\(^{-1}\) Mpc\(^{-1}\)] | 71.6 ± 4.7 | 67.26 ± 0.80 | 67.34 ± 0.74 |
| \(\Omega_m\) | 0.264 ± 0.034 | 0.2949 ± 0.0072 | 0.2944 ± 0.0066 |
| \(\sigma_8\) | 0.888 ± 0.050 | 0.824 ± 0.017 | 0.822 ± 0.017 |

| Parameter | TT+lowP+\(f\sigma_8\) | TT+lowP+BAO+\(f\sigma_8\) | TT+lowP+SN+BAO+\(H(z)\)+\(f\sigma_8\) |
|-----------|-----------------|----------------|--------------------------------|
| \(\Omega_m h^2\) | 0.02311 ± 0.00020 | 0.02307 ± 0.00019 | 0.02307 ± 0.00020 |
| \(\Omega_b h^2\) | 0.1090 ± 0.0010 | 0.1092 ± 0.0011 | 0.1092 ± 0.0010 |
| \(100\theta_{MC}\) | 1.04226 ± 0.00041 | 1.04224 ± 0.00041 | 1.04224 ± 0.00042 |
| \(\tau\) | 0.119 ± 0.019 | 0.126 ± 0.017 | 0.125 ± 0.016 |
| \(\ln(10^{10}A_s)\) | 3.146 ± 0.038 | 3.160 ± 0.034 | 3.159 ± 0.033 |
| \(\Omega_k\) | -0.0089 ± 0.0077 | -0.0070 ± 0.0024 | -0.0071 ± 0.0020 |
| \(w\) | -1.22 ± 0.18 | -0.951 ± 0.054 | -0.961 ± 0.033 |
| \(H_0\) [km s\(^{-1}\) Mpc\(^{-1}\)] | 74.9 ± 8.1 | 67.1 ± 1.1 | 67.41 ± 0.77 |
| \(\Omega_m\) | 0.245 ± 0.054 | 0.295 ± 0.010 | 0.2926 ± 0.0068 |
| \(\sigma_8\) | 0.880 ± 0.058 | 0.808 ± 0.018 | 0.811 ± 0.016 |

Figure 6. 1\(\sigma\) and 2\(\sigma\) likelihood contours in the \(\Omega_m\)-\(w\) plane for the tilted flat-XCDM parameterization (left panel) and in the \(w\)-\(\Omega_k\) plane for the untitled nonflat XCDM parameterization (right panel), constrained by Planck CMB TT + lowP + lensing and non-CMB data sets. The horizontal and vertical dashed lines indicate \(w = -1\) (the cosmological constant) or \(\Omega_k = 0\). Contours in both panels follow the color scheme shown in the left panel.
Tilted flat and untilted nonflat XCDM parameterizations

| Parameter          | TT+lowP+lensing | TT+lowP+lensing+SN | TT+lowP+lensing+BAO |
|--------------------|-----------------|-------------------|---------------------|
| $\Omega_b h^2$     | 0.02305 ± 0.00020 | 0.02305 ± 0.00019 | 0.02302 ± 0.00020 |
| $\Omega_c h^2$     | 0.1091 ± 0.0011  | 0.1091 ± 0.0011   | 0.1094 ± 0.0011    |
| $100\theta_{MC}$   | 1.04235 ± 0.00043 | 1.04234 ± 0.00041 | 1.04226 ± 0.00041 |
| $\tau$             | 0.100 ± 0.022    | 0.101 ± 0.021     | 0.123 ± 0.014      |
| $\ln(10^{10}A_s)$  | 3.106 ± 0.044    | 3.109 ± 0.042     | 3.154 ± 0.027      |
| $\Omega_k$         | −0.019 ± 0.017   | −0.0153 ± 0.0075  | −0.0070 ± 0.0025   |
| $w$                | −1.12 ± 0.39     | −1.019 ± 0.053    | −0.946 ± 0.056     |
| $H_0$ [km s⁻¹ Mpc⁻¹] | 69 ± 14         | 65.5 ± 2.3        | 66.9 ± 1.2         |
| $\Omega_m$         | 0.31 ± 0.13      | 0.310 ± 0.021     | 0.297 ± 0.010      |
| $\sigma_8$         | 0.83 ± 0.11      | 0.803 ± 0.015     | 0.805 ± 0.014      |

| Parameter          | TT+lowP+lensing+H(z) | TT+lowP+lensing+SN+BAO | TT+lowP+lensing+SN+BAO+H(z) |
|--------------------|----------------------|------------------------|-----------------------------|
| $\Omega_b h^2$     | 0.02304 ± 0.00020    | 0.02303 ± 0.00019      | 0.02303 ± 0.00021           |
| $\Omega_c h^2$     | 0.1094 ± 0.0011      | 0.1094 ± 0.0011        | 0.1095 ± 0.0010             |
| $100\theta_{MC}$   | 1.04232 ± 0.00043    | 1.04228 ± 0.00040      | 1.04228 ± 0.00041           |
| $\tau$             | 0.114 ± 0.017        | 0.120 ± 0.012          | 0.121 ± 0.012               |
| $\ln(10^{10}A_s)$  | 3.137 ± 0.034        | 3.148 ± 0.024          | 3.150 ± 0.024               |
| $\Omega_k$         | −0.0081 ± 0.0042     | −0.0075 ± 0.0021       | −0.0070 ± 0.0020            |
| $w$                | −1.06 ± 0.14         | −0.967 ± 0.033         | −0.961 ± 0.033              |
| $H_0$ [km s⁻¹ Mpc⁻¹] | 69.9 ± 4.2          | 67.31 ± 0.77          | 67.38 ± 0.75                |
| $\Omega_m$         | 0.275 ± 0.033        | 0.2938 ± 0.0069        | 0.2933 ± 0.0067             |
| $\sigma_8$         | 0.832 ± 0.036        | 0.809 ± 0.011          | 0.809 ± 0.011               |

| Parameter          | TT+lowP+lensing+H(z)+f/σ₈ | TT+lowP+lensing+BAO+H(z)+f/σ₈ | TT+lowP+lensing+SN+BAO+H(z)+f/σ₈ |
|--------------------|---------------------------|-------------------------------|---------------------------------|
| $\Omega_b h^2$     | 0.02306 ± 0.00020         | 0.02303 ± 0.00020             | 0.02305 ± 0.00020              |
| $\Omega_c h^2$     | 0.1090 ± 0.0011           | 0.1093 ± 0.0011               | 0.1092 ± 0.0010                |
| $100\theta_{MC}$   | 1.04229 ± 0.00041         | 1.04225 ± 0.00043             | 1.04227 ± 0.00042              |
| $\tau$             | 0.114 ± 0.019             | 0.120 ± 0.013                 | 0.119 ± 0.012                  |
| $\ln(10^{10}A_s)$  | 3.136 ± 0.038             | 3.148 ± 0.026                 | 3.146 ± 0.024                  |
| $\Omega_k$         | −0.0056 ± 0.0063          | −0.0068 ± 0.0024              | −0.0069 ± 0.0020               |
| $w$                | −1.19 ± 0.18              | −0.943 ± 0.054                | −0.960 ± 0.032                 |
| $H_0$ [km s⁻¹ Mpc⁻¹] | 76.4 ± 8.3                | 67.0 ± 1.1                    | 67.45 ± 0.75                   |
| $\Omega_m$         | 0.236 ± 0.053             | 0.297 ± 0.010                 | 0.2923 ± 0.0066                |
| $\sigma_8$         | 0.873 ± 0.059             | 0.801 ± 0.014                 | 0.805 ± 0.011                  |

Untilted nonflat XCDM model parameters constrained with Planck TT + lowP + lensing, SN, BAO, $H(z)$, and $f/\sigma_8$ data (mean and 68.3% confidence limits).
Figure 7. $1\sigma$ and $2\sigma$ likelihood contours in the $\Omega_m-\sigma_8$ plane for the tilted flat-XCDM parameterization constrained by Planck CMB TT + lowP (+lensing), SNIa, BAO, $H(z)$, and $f\sigma_8$ data. In each panel the $\Lambda$CDM model $1\sigma$ and $2\sigma$ constraint contours obtained from the first-year Dark Energy Survey (DES Y1 All) (DES Collaboration 2018) are shown as thick solid curves for comparison.

We measure cosmological parameters from an updated, reliable, large compilation of observational data by using the tilted flat-XCDM and the untilted nonflat XCDM dynamical dark energy inflation parameterizations.

In summary, our main results are:

- We confirm, but at lower significance, the Ooba et al. (2018c) result that the tilted flat-XCDM parameterization provides a better fit to the data than does the standard tilted flat-$\Lambda$CDM model. The improvement is not significant, but on the other hand current data are unable to rule out dynamical dark energy.

- In the untilted nonflat XCDM case, we confirm, at higher significance, the Ooba et al. (2017) result that cosmological data does not demand spatially-flat hypersurfaces for this parameterization, and that the nonflat XCDM parameterization provides a better fit to the data than does the nonflat $\Lambda$CDM model (qualitatively it is clear that the standard tilted flat-$\Lambda$CDM model is a better fit to the data than is the untilted nonflat $\Lambda$CDM model). In the nonflat XCDM case, these data (including CMB lensing measurements) favor a closed model at more than $3.4\sigma$ significance, with spatial curvature contributing a little less than a percent to the current cosmological energy budget, and favor dark energy dynamics (over a cosmological constant) at a little more than $1.2\sigma$.

- $H_0$ values measured in both models are very similar, and consistent with many other measurements of $H_0$. However, as well known, $H_0$ estimated from the local expansion rate (Riess et al. 2018) is about $3\sigma$ larger.

- $\sigma_8$ values measured in both models are close to identical and compatible with the recent DES measurement (DES Collaboration 2018).

- The measured $\Omega_m$ value is more model dependent than the measured $\sigma_8$ value and the $\Omega_m$ value measured using the nonflat XCDM parameterization is more consistent with the recent DES estimate (DES Collaboration 2018).

- $\Omega_bh^2$, $\tau$, $\Omega_r h^2$, as well as some of the other measured cosmological parameter values are model dependent.

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Figure 8. Same as Fig. 7 but for the untilted nonflat XCDM parameterization.

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Figure 9. Best-fit CMB anisotropy power spectra of (a) tilted flat (top five panels) and (b) untilted nonflat XCDM parameterizations (bottom five panels) constrained by the Planck CMB TT + lowP data (excluding the lensing data) together with SN, BAO, $H(z)$, and $f\sigma_8$ data. For comparison, the best-fit power spectra of the tilted flat-$\Lambda$CDM model are shown as black curves. The residual $\delta D_\ell$ of the TT power spectra are shown with respect to the flat-$\Lambda$CDM power spectrum that best fits the TT + lowP data.
Table 7

Individual and total $\chi^2$ values for the best-fit tilted flat and untilted nonflat $\Lambda$CDM inflation models.

| Data sets          | $\chi^2_{PlikTT}$ | $\chi^2_{lowTEB}$ | $\chi^2_{lensing}$ | $\chi^2_{SN}$ | $\chi^2_{BAO}$ | $\chi^2_{H(z)}$ | $\chi^2_{f\sigma_8}$ | $\chi^2_{prior}$ | Total $\chi^2$ | $\Delta\chi^2$ |
|--------------------|--------------------|--------------------|--------------------|---------------|---------------|----------------|-------------------|----------------|----------------|--------------|
| Tilted flat-$\Lambda$CDM model |
| TT+lowP            | 763.57             | 10496.41           |                    |               |               |                |                   |                | 1.96          | 11261.93     |
| +SN                | 763.45             | 10496.50           | 1036.29            |               |               |                |                   |                | 2.06          | 12298.31     |
| +BAO               | 764.20             | 10495.92           | 13.02              |               |               |                |                   |                | 2.12          | 11275.25     |
| +$H(z)$            | 763.98             | 10496.36           | 14.89              |               |               |                |                   |                | 1.70          | 11276.93     |
| +$f\sigma_8$       | 766.83             | 10494.95           | 12.15              |               |               |                |                   |                | 1.87          | 11275.80     |
| +BAO+$f\sigma_8$   | 766.67             | 10494.83           | 12.64              |               |               |                |                   |                | 1.96          | 11288.50     |
| +SN+BAO            | 764.34             | 10495.96           | 1036.15            |               |               |                |                   |                | 2.03          | 12311.41     |
| +SN+BAO+$H(z)$     | 764.33             | 10495.93           | 1036.15            |               |               |                |                   |                | 2.03          | 12326.21     |
| +SN+BAO+$H(z)+f\sigma_8$ | 766.68         | 10494.90           | 1036.02            |               |               |                |                   |                | 1.88          | 12339.36     |
| TT+lowP+lensing    | 766.20             | 10494.93           | 9.30               |               |               |                |                   |                | 2.00          | 11272.44     |
| +SN                | 766.53             | 10494.83           | 9.17               | 1036.05       |               |                |                   |                | 2.02          | 12308.59     |
| +BAO               | 766.44             | 10494.80           | 9.13               | 12.61         |               |                |                   |                | 2.09          | 12385.07     |
| +$H(z)$            | 766.20             | 10494.92           | 9.27               | 14.83         |               |                |                   |                | 2.04          | 11287.27     |
| +$f\sigma_8$       | 768.26             | 10494.43           | 8.67               | 11.31         |               |                |                   |                | 1.94          | 11284.62     |
| +BAO+$f\sigma_8$   | 767.56             | 10494.49           | 8.71               | 12.59         | 11.80         |               |                   |                | 2.16          | 11297.32     |
| +SN+BAO            | 766.53             | 10494.77           | 9.03               | 1036.07       | 12.61         |               |                   |                | 2.16          | 12321.17     |
| +SN+BAO+$H(z)$     | 766.51             | 10494.80           | 9.07               | 1036.07       | 12.61         | 14.81          |               |                | 2.14          | 12336.01     |
| +SN+BAO+$H(z)+f\sigma_8$ | 767.61         | 10494.48           | 8.74               | 1036.01       | 12.68         | 14.79          | 11.84            | 2.04          | 12348.20     |
| Untilted nonflat $\Lambda$CDM model |
| TT+lowP            | 774.34             | 10495.42           |                    |               |               |                |                   |                | 2.33          | 11272.10     |
| +SN                | 778.23             | 10497.99           | 1036.74            |               |               |                |                   |                | 1.97          | 12314.94     |
| +BAO               | 780.27             | 10499.20           | 14.69              |               |               |                |                   |                | 1.92          | 12396.08     |
| +$H(z)$            | 777.14             | 10500.93           | 17.11              |               |               |                |                   |                | 1.96          | 11297.15     |
| +$f\sigma_8$       | 783.38             | 10497.49           |                    | 11.51         | 2.41          |               |                   |                | 11294.79     |
| +BAO+$f\sigma_8$   | 783.46             | 10497.40           | 14.01              |               | 10.72         | 1.81          |               |                | 11307.41     |
| +SN+BAO            | 780.65             | 10499.11           | 1036.11            | 14.56         | 1.86          |               |                   |                | 12332.30     |
| +SN+BAO+$H(z)$     | 782.84             | 10497.40           | 1036.18            | 14.06         | 16.17         | 1.91          |               |                | 12348.57     |
| +SN+BAO+$H(z)+f\sigma_8$ | 781.14         | 10499.17           | 1036.29            | 14.17         | 16.14         | 11.32         | 1.74          |               | 12359.98     |
| TT+lowP+lensing    | 786.87             | 10493.86           | 9.77               |               |               |                |                   |                | 1.79          | 11292.29     |
| +SN                | 786.65             | 10494.69           | 9.19               | 1035.95       |               |               |                   |                | 1.83          | 12328.31     |
| +BAO               | 784.19             | 10497.32           | 8.86               | 13.99         |               |               |                   |                | 2.04          | 12307.40     |
| +$H(z)$            | 786.87             | 10496.02           | 8.66               | 16.36         |               |               |                   |                | 2.19          | 11310.10     |
| +$f\sigma_8$       | 786.41             | 10496.00           | 8.75               | 9.79          |               |               |                   |                | 1.99          | 11302.93     |
| +BAO+$f\sigma_8$   | 788.21             | 10494.90           | 8.38               | 13.89         | 9.81          |               |                   |                | 2.11          | 11317.31     |
| +SN+BAO            | 784.76             | 10496.50           | 9.54               | 1036.23       | 13.87         | 1.89          |               |                | 12342.79     |
| +SN+BAO+$H(z)$     | 784.72             | 10496.49           | 9.60               | 1036.24       | 13.84         | 1.89          |               |                | 12358.87     |
| +SN+BAO+$H(z)+f\sigma_8$ | 786.96         | 10495.37           | 8.63               | 1036.37       | 13.81         | 9.77          | 1.90          |               | 12368.86     |

Note: $\Delta\chi^2$ of an untilted nonflat $\Lambda$CDM model estimated for a combination of data sets represents the excess value relative to $\chi^2$ of the tilted flat-$\Lambda$CDM model for the same combination of data sets.
Table 8

Individual and total $\chi^2$ values for the best-fit tilted flat and untiiled nonflat XCDM inflation parameterizations.

| Data sets                  | $\chi^2_{\text{fit}}$ | $\chi^2_{\text{SN}}$ | $\chi^2_{\text{lensing}}$ | $\chi^2_{\text{BAO}}$ | $\chi^2_{H(z)}$ | $\chi^2_{fR}$ | $\chi^2_{\text{Total}}$ | $\Delta \chi^2$ |
|---------------------------|------------------------|------------------------|---------------------------|------------------------|----------------|----------------|------------------------|----------------|
| **Tilted flat-XCDM parameterization** |                       |                        |                           |                        |                |                |                        |                |
| TT+lowP                   | 761.85                 | 10495.08               | 1035.93                   | 15.00                  | 11.77          | 2.03           | 11297.64               | -0.67          |
| +SN                      | 763.24                 | 10496.38               | 1035.93                   | 15.00                  | 11.77          | 2.03           | 11297.64               | -0.67          |
| +BAO                     | 764.30                 | 10496.20               | 1035.93                   | 15.00                  | 11.77          | 2.03           | 11297.64               | -0.67          |
| +$H(z)$                  | 763.32                 | 10496.10               | 1035.93                   | 15.00                  | 11.77          | 2.03           | 11297.64               | -0.67          |
| +$fR$                    | 766.16                 | 10494.26               | 1035.93                   | 15.00                  | 11.77          | 2.03           | 11297.64               | -0.67          |
| +BAO+$fR$                | 766.79                 | 10495.00               | 1035.93                   | 15.00                  | 11.77          | 2.03           | 11297.64               | -0.67          |
| +SN+BAO                  | 764.46                 | 10495.90               | 1036.02                   | 13.15                  | 1.88           | 1.88           | 12311.39               | -0.02          |
| +SN+BAO+$H(z)$           | 764.33                 | 10496.04               | 1036.09                   | 13.16                  | 1.80           | 1.80           | 12326.24               | +0.03          |
| +SN+BAO+$H(z)+fR$        | 766.81                 | 10494.83               | 1036.06                   | 12.60                  | 14.79          | 12.12          | 12339.31               | -0.05          |
| **Untilted nonflat XCDM parameterization** |                       |                        |                           |                        |                |                |                        |                |
| TT+lowP+lensing          | 766.09                 | 10493.81               | 9.39                      | 12.25                  | 11.98          | 11.60          | 12328.33               | -0.26          |
| +SN                      | 766.35                 | 10494.78               | 9.24                      | 12.25                  | 11.98          | 11.60          | 12328.33               | -0.26          |
| +BAO                     | 767.00                 | 10494.74               | 9.06                      | 12.25                  | 11.98          | 11.60          | 12328.33               | -0.26          |
| +$H(z)$                  | 765.98                 | 10494.66               | 9.37                      | 14.89                  | 2.21           | 1287.10        | -0.17                  |                |
| +$fR$                    | 767.98                 | 10493.86               | 8.65                      | 14.89                  | 2.21           | 1287.10        | -0.17                  |                |
| +BAO+$fR$                | 768.00                 | 10494.68               | 8.69                      | 14.89                  | 2.21           | 1287.10        | -0.17                  |                |
| +SN+BAO                  | 766.61                 | 10494.75               | 9.04                      | 12.25                  | 11.98          | 11.60          | 12328.33               | -0.26          |
| +SN+BAO+$H(z)$           | 766.79                 | 10494.76               | 9.06                      | 12.25                  | 11.98          | 11.60          | 12328.33               | -0.26          |
| +SN+BAO+$H(z)+fR$        | 767.76                 | 10494.51               | 8.72                      | 12.25                  | 11.98          | 11.60          | 12328.33               | -0.26          |

Note: $\Delta \chi^2$ of tilted flat or untiiled nonflat XCDM parameterization estimated for a combination of data sets represents the excess value relative to $\chi^2$ of the corresponding ACeDM model for the same combination of data sets.
Figure 10. Same as Fig. 9 but now including the CMB lensing data. The residual $\delta D_\ell$ of the TT power spectra are shown with respect to the flat-$\Lambda$CDM power spectrum that best fits the TT + lowP + lensing data.

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Figure 11. Power spectra of primordial scalar-type perturbations of best-fit untilted non-power-law power spectrum nonflat XCDM cases constrained using Planck TT + lowP data (left panel) and TT + lowP + lensing data (right panel) together with non-CMB data sets (fσ8, SN, BAO, H(z)). In both panels the primordial power spectrum of the best-fit tilted flat-XCDM model is shown as dashed lines. For the definition of wavenumber q, see Sec. 3. The power spectrum is normalized to $P(q) = A_s$ at the pivot scale $k_0 = 0.05$ Mpc$^{-1}$.

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