Topological climate change with permafrost feedback

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Abstract. Climate models predict that the climate of the Earth is warming and will continue to warm in coming centuries, if there is no mitigation. A recent energy balance model [Kypke et al., Nonlin. Process. Geophys. 27 (2020) 391–409] forecasts that, if the current increase of carbon dioxide in the atmosphere continues unabated, then in the next century the climate of the Earth will not only get warmer, but will transition abruptly via a bifurcation, to a warm equable climate unlike any climate seen on Earth since the Pliocene. This transition to a new climate state is a topological change. That model includes the effects of water vapour feedback and ice albedo feedback, as well as ocean and atmospheric heat transport. This paper adds to that model further amplification by permafrost feedback. That is, as the Arctic warms, permafrost will thaw, releasing large amounts of the greenhouse gases carbon dioxide and methane, which cause further warming. Since knowledge of permafrost stores and release rates is limited, a range of permafrost carbon release sensitivities ($Q_{10}$) is considered. The model predicts that permafrost feedback accelerates the timing and increases the likelihood of a topological climate change in the Arctic, and reinforces the view that permafrost feedback should not be ignored in Anthropocene climate models.

1. Introduction

The global mean temperature of the Earth is rising. The increase in temperature is greatest at the poles. One factor causing this temperature rise is the increasing concentration of the greenhouse gas CO$_2$ in the atmosphere. However, the rise in temperature near the poles is greater than can be accounted for by increasing CO$_2$ concentration alone. Water vapour is another powerful greenhouse gas. Rising temperatures cause evaporation of more water into the atmosphere, which causes further warming. This is called water vapour feedback. Other positive feedback mechanisms include ice albedo feedback, ocean transport feedback and atmosphere transport feedback. These interacting feedback mechanisms were studied recently in an energy balance model (EBM) [1, 2]. That EBM forecasts further temperature increases in coming centuries, and the existence of a bifurcation point (tipping point) in the next century, if there is no mitigation of CO$_2$. At that bifurcation point, the climate in the EBM transitions abruptly to a warm equable climate unlike any climate seen on Earth since the early Pliocene. That transition has been shown to be a topological climate change [3].

This paper adds another positive feedback mechanism to that EBM, namely permafrost feedback [4, 5, 6, 7]. Permafrost exists widely in the Arctic, and it entraps large quantities of carbon dioxide CO$_2$ and methane CH$_4$, which are released when the permafrost thaws. Methane is a greenhouse gas more powerful than CO$_2$. This paper estimates the greenhouse warming
effect of the extra CO_2 and CH_4 from permafrost. Because the exact amounts of CO_2 and CH_4 in permafrost are not accurately known, and their rates of release can only be estimated, this model explores a range of temperature sensitivities (\(Q_{10}\)) for their release.

2. The basic EBM

The starting point is the energy balance model (EBM) as used in [2, 3],

\[
\begin{align*}
    c_A \frac{dI_A}{dt} &= F_A + \eta \sigma T_S^4 - I_A + F_C + \xi_A Q \\
    c_S \frac{dT_S}{dt} &= (1 - \alpha)(1 - \xi_A - \xi_R)Q + F_O + \beta I_A - \sigma T_S^4 - F_C,
\end{align*}
\]

where \(T_S\) is surface temperature (\(^\circ\)K) and \(I_A\) is intensity of atmospheric radiation (Wm\(^{-2}\)). Other symbols are defined in Table 1 of [2, 3]. In (1) the atmospheric absorptivity is

\[
\eta \equiv \eta(\tau_S; \mu, \delta) = 1 - \exp \left[ -\mu G_C - \delta G_W \int_{\tau_S - \gamma Z}^{\tau_S} \frac{1}{\tau} \exp \left( G_W \left[ \frac{\tau - 1}{\tau} \right] \right) d\tau \right].
\]

Here \(0 \leq \eta \leq 1\) is the fraction of the outgoing longwave radiation from the surface (\(\sigma T_S^4\), Stephan-Boltzmann law) absorbed by the total optical depth of greenhouse gases. The non-dimensional surface temperature is \(\tau_S \equiv T_S/T_R\); \(T_R = 273.15\). CO_2 is represented by \(\mu\), in molar parts per million (ppm), and water vapour is the second greenhouse gas, with relative humidity \(\delta\). The integral in (3) gives optical depth of H_2O from the Clausius-Clapeyron equation [1, 2].

2.1. Representative concentration pathways (RCP)

The IPCC has sanctioned Representative Concentration Pathways (RCP) [8] for future hypothetical levels of CO_2 in the atmosphere, see Fig. 4 in [2]. The numbers 8.5, 6.0, 4.5 and 2.6 W/m\(^2\) respectively correspond to the level of radiative forcing in the year 2100 on each pathway. These are widely used to test and compare climate models. In the EBM, increases in \(\mu\) cause increases in surface temperature \(T_S\). See Fig. 1(a), and additional figures in [2, 3] where changes in \(\delta\) and \(F_O\) also are considered. On RCP 8.5, there is a bifurcation in the EBM to a much warmer climate state.

3. EBM with permafrost feedback

It is estimated that Arctic permafrost contains approximately 1,500 petagrams (1.5 \times 10^{15} \text{ kg}) of carbon; approximately twice as much as in the atmosphere today [5, 6]. Of this, 2/3 is in the surface layer (0-3 m depth). An increase in \(\mu\) causes an increase in surface temperature \(\tau_S\), which in turn increases the release of CO_2 and CH_4 from permafrost into the atmosphere. In the atmosphere these gases become well-mixed. We replace \(\mu\) in (3) by the sum \(\mu_1 + \mu_2\), where \(\mu_1\) is the RCP atmospheric \(\mu\) and \(\mu_2\) is the additional CO_2 released from permafrost. Similarly, we split \(\nu = \nu_1 + \nu_2\), as the CH_4 in the RCP plus the CH_4 from permafrost. Methane is a stronger greenhouse gas than CO_2 [5]. The methane coefficient is \(G_M = 7.14 \, G_C\) [1, 2]. Thus the term \(\mu G_C\) in (3) is replaced by \((\mu_1 + \mu_2) G_C + (\nu_1 + \nu_2) G_M\). With these changes in (3), the revised absorptivity \(\eta\) with permafrost feedback is

\[
\eta(\tau_S; \mu_1, \mu_2, \nu_1, \nu_2, \delta) = 1 - \exp \left[ -(\mu_1 + \mu_2) G_C - (\nu_1 + \nu_2) G_M - \delta G_W \int_{\tau_S - \gamma Z}^{\tau_S} \frac{1}{\tau} \exp \left( G_W \left[ \frac{\tau - 1}{\tau} \right] \right) d\tau \right].
\]
3.1. Modelling of permafrost carbon emissions

A range of estimated permafrost carbon emissions is given in [5, 6, 7]. The temperature sensitivity coefficient [9] of permafrost carbon emissions, with respect to a nominal 10°K increase in soil temperature, is the nondimensional quantity \( Q_{10} \) defined by

\[
Q_{10} = \left( \frac{R_2}{R_1} \right) \left( \frac{T_{s1} - T_{s0}}{T_{s0} - T_1} \right),
\]

where \( R_1 (R_2) \) is the total permafrost carbon emission rate, at soil temperature \( T_1 (T_2) \), measured in units of Pg a\(^{-1}\) (10\(^{12}\) kg carbon per annum). A typical value is \( Q_{10-sT} = 2.55 \) for the dominant upper layer of permafrost in the short term, see [9] for details. For this simple model, let \( s \) represent time in years from the reference time \( s_0 = 2000 \), when (for all four RCPs) the annual mean Arctic surface temperature was \( T_1 = 244.25 \) K [1], the emission rate was \( R_1 = 0.19 \) Pg a\(^{-1}\) [6], \( \mu_2(s_0) = 28.6 \) and \( \nu_2(s_0) = 0.136 \) ppm [7]. Take \( T_2 = T_s \) (the EBM surface temperature). Nondimensionalize to \( \tau_{s1} = T_i / T_R \), \( i = 1, 2 \), then \( \tau_{s1} = 0.894 \) and \( \tau_{s2} = \tau_s = T_S / T_R \). Solve for the emission rate \( R_2(\tau_s) \) in (5):

\[
R_2(\tau_S) = R_1 \cdot [Q_{10}] \frac{T_R}{T_0} (\tau_S - \tau_{s1}).
\]

The total mass of the atmosphere is \( 5.15 \times 10^{18} \) kg (\( 5.15 \times 10^6 \) Pg). Therefore, the carbon mass density emission rate (in kg a\(^{-1}\)) is \( R_2 \cdot (5.15 \times 10^6) \). Of this total, it is estimated that 97.7\% is CO\(_2\) and 2.3\% is CH\(_4\). In the EBM, the concentration of atmospheric CO\(_2\) is measured in molar ppm (parts per million). The mass of one mole of CO\(_2\) is mmC = 44.01 \times 10^{-3} \) kg mol\(^{-1}\). A weighted average of molar masses of the dry atmosphere is mmA = 29 \times 10^{-3} \) kg mol\(^{-1}\). Convert the CO\(_2\) portion of \( R_2 \) from kg to molar CO\(_2\) ppm a\(^{-1}\):

\[
\dot{\mu}_2 = \frac{29}{44.01} \times \frac{0.977 \times 10^6 \cdot R_2}{5.15 \times 10^6} = 0.128 \cdot R_1 \cdot [Q_{10}] \frac{T_R}{T_0} (\tau_S - \tau_{s1}) \text{ ppm a}^{-1}.
\]

Similarly, the molar mass of methane is 16.04 \times 10^{-3} \) kg and the methane portion of \( R_2 \) is

\[
\dot{\nu}_2 = \frac{29}{16.04} \times \frac{0.023 \times 10^6 \cdot R_2}{5.15 \times 10^6} = 8.07 \times 10^{-3} \cdot R_1 \cdot [Q_{10}] \frac{T_R}{T_0} (\tau_S - \tau_{s1}) \text{ ppm a}^{-1}.
\]

Carbon dioxide remains in the atmosphere for a long time. Methane is removed by reaction with the hydroxyl radical (OH), with a half-life of only 9.1 years. Therefore, equation (8) is replaced by the linear nonhomogeneous ODE

\[
\dot{\nu}_2 = -k\nu_2 + 0.00807R_1 \cdot [Q_{10}] \frac{T_R}{T_0} (\tau_S(s) - \tau_{s1}),
\]

where \( k = \frac{[\ln 2]}{9.1} = 0.0762 \). Integrate the two emission rates (7) and (9) to obtain

\[
\begin{align*}
\mu_2(s) &= \mu_2(s_0) + 2.43 \times 10^{-2} \int_{s_0}^s [Q_{10}] \frac{T_R}{T_0} (\tau_S(s) - \tau_{s1}) d\sigma, \\
\nu_2(s) &= \nu_2(s_0) e^{-k(s-s_0)} + 1.53 \times 10^{-3} e^{-ks} \int_{s_0}^s e^{ks} [Q_{10}] \frac{T_R}{T_0} (\tau_S(s) - \tau_{s1}) d\sigma.
\end{align*}
\]

Substituting these into (4) gives an enhanced EBM (1)(2) with permafrost feedback.

Quasi-static bifurcation diagrams for this enhanced EBM are presented in Fig. 1, showing temperature changes relative to year 2000 with three permafrost carbon release sensitivities \( Q_{10} \), starting with \( Q_{10} = 0 \) in Fig. 1(a) from [2]. On RCP 8.5, the addition of permafrost carbon release, with \( Q_{10} = 2.55 \), advances the onset of the bifurcation to a warmer equable climate by only about a year, in Fig. 1(b). However, even if mitigation strategies reduce anthropogenic carbon emissions to RCP 6.0, then a higher permafrost \( Q_{10} = 4.5 \) together with increasing ocean heat transport \( F_O \) (e.g. due to rising sea level [2]) could still lead to a catastrophic bifurcation on RCP 6.0, see Fig. 1(c).
Figure 1. EBM surface temperature changes, relative to year 2000 on each RCP scenario, with increasing levels of permafrost carbon. Fig. (a) is reproduced from Fig. 6(b) in [2], showing a bifurcation on RCP 8.5 near year 2170, with \( F_O = 9.75 \text{ Wm}^{-2} \) and \( Q_{10} = 0 \). In all panes \( F_A = 114 \text{ Wm}^{-2} \). In (b), permafrost feedback with \( Q_{10} = 2.55 \) is added and the bifurcation on RCP 8.5 is one year earlier. Other RCP’s have not changed materially. In (c), \( Q_{10} = 4.5 \) and the ocean heat transport \( F_O \) has been increased linearly with time as in Fig. 7 of [2]. A new bifurcation appears on RCP 6.0, leading to a warm equable climate state near the year 2360.

4. Conclusions
The addition of permafrost carbon release with sensitivity \( Q_{10} = 2.55 \) to the EBM in [2, 3] has little effect on climate change along the RCPs in Fig. 1, in part because the increase in atmospheric \( \text{H}_2\text{O} \) above freezing is close to saturation \( \eta \approx 1 \) in (4), see [1, 2]. If mitigation reduces anthropogenic carbon emissions to RCP 6.0, a bifurcation there is possible, if \( Q_{10} = 4.5 \) and ocean heat transport \( F_O \) increases as sea level rises. This suggests stronger mitigation strategies may be needed in case of higher permafrost carbon release. Further investigation of the impact of permafrost feedback in Anthropocene climate models will be reported elsewhere.

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