Experimental demonstration of a Laguerre-Gaussian correlated Schell-model vortex beam

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Abstract: Laguerre-Gaussian correlated Schell-model (LGCSM) vortex beam is introduced as an extension of LGCSM beam which was proposed [Opt. Lett. 38, 91 (2013)] and generated [Opt. Lett. 38, 1814 (2013)] just recently. Explicit formula for a LGCSM vortex beam propagating through a stigmatic ABCD optical system is derived, and the propagation properties of such beam in free space and the focusing properties of such beam are studied numerically. Furthermore, we carry out experimental generation of a LGCSM vortex beam, and studied its focusing properties. It is found that the propagation and focusing properties of a LGCSM vortex beam are different from that of a LGCSM beam, and we can shape the beam profile of a LGCSM vortex at the focal plane (or in the far field) by varying its initial spatial coherence. Our experimental results are consistent with the theoretical predictions, and our results will be useful for particle trapping.

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1. Introduction

In the past decades, partially coherent beams with conventional Schell-model correlation functions (i.e., degrees of coherence have position-independent profiles) have been studied extensively both in theory and in experiment and has been applied widely [1–4]. Since Gori and collaborators discussed the sufficient condition for devising the genuine correlation function of a scalar or electromagnetic partially coherent beam [5, 6], more and more attention is being paid to partially coherent beams with nonconventional correlation functions. A variety of partially coherent beams with special correlation functions, such as beams with locally varying spatial coherence [7], special correlated partially coherent vector beam [8], nonuniformly correlated Gaussian Schell-model beam [9–12], multi-Gaussian Schell-model beam [13–16], cosine-Gaussian Schell-model beam [17–19], and Laguerre-Gaussian correlated Schell-model (also named Laguerre-Gaussian Schell-model beam) beam [20–23], have been introduced. Those partially coherent beams with special correlation functions have been found to exhibit some extraordinary propagation characteristics, such as far-field flat-topped and ring intensity profile formation, self-focusing effect, and a lateral shift of the intensity maximum. Laguerre-Gaussian correlated Schell-model (LGCSM) beam was first...
introduced in [20], and it was found that the far field intensity of a LGCSM beam has a ring (i.e., dark hollow) intensity profile although it has the same intensity distribution with that of a Gaussian Schell-model beam in the source plane. In [21], we reported experimental generation of a LGCSM beam for the first time. The propagation properties of a LGCSM beam in turbulent atmosphere were studied in [22, 23], and it was revealed that a LGCSM beam has advantage over a Gaussian Schell-model (GSM) beam for reducing the turbulence-induced degradation, thus it has potential application in free-space optical communications.

On the other hand, it is well known that light beams with a vortex phase named vortex beams have been applied in optical trapping, optical tweezers, quantum information processing and so on [24–28]. It is found that each photon of the vortex beam with a phase term \( \exp(\imath l \theta) \) carries an orbital angular momentum of \( l \) with \( l \) being the topological charge [24]. In the past several years, more and more attention is being paid to partially coherent vortex beams both in theory and in experiment [29–34]. In [33], we carried out experimental study of the focusing properties of GSM vortex beam. The conventional method for determining the topological charge of a vortex beam is invalid for a partially coherent vortex beam, and we proposed a new method for determining the topological charge of a partially coherent vortex beam in [34]. More recently, we carried out experimental measurement the scintillation index of a GSM vortex beam propagating through thermally induced turbulence [35], and we have found that a GSM vortex beam has appreciably smaller scintillation than a GSM beam, which will be useful in free-space optical communication. All above mentioned partially coherent vortex beams have the conventional Schell-model correlation functions. In this paper, our aim is to introduce a partially coherent vortex beam with nonconventional correlation function, named LGCSM vortex beam, as a natural extension of the recently introduced LGCSM beam. We derive the explicit formula for the LGCSM vortex beam propagating through a stigmatic ABCD optical system, and study its propagation properties both numerically and experimentally. Some useful results are found.

2. Laguerre-Gaussian correlated Schell-model vortex beam: Theory

In the space-time domain, the statistical properties of a scalar partially coherent beam are characterized by the mutual coherence function. For a LGCSM beam, its mutual coherence function is defined as [20]

\[
\Gamma(r_1, r_2) = G_0 \exp\left\{ \frac{-r_1^2 + r_2^2}{4\sigma_0^2} - \frac{(r_1 - r_2)^2}{2\delta_0^2} \right\} L_n^0 \left[ \frac{(r_1 - r_2)^2}{2\delta_0^2} \right],
\]

(1)

where \( G_0 \) is a constant which has dimension of an optical intensity, \( r_i \equiv (x_i, y_i) \) and \( r_i \equiv (x_i, y_i) \) are two arbitrary transverse position vectors at \( z = 0 \), \( \sigma_0 \) and \( \delta_0 \) are the transverse beam width and the transverse coherence width of the LGCSM beam, respectively, \( L_n^0 \) denotes the Laguerre polynomial of mode order \( n \) and 0. The degree of coherence of the LGCSM beam at \( z = 0 \) is given as

\[
\mu(r_1, r_2) = \frac{\Gamma(r_1, r_2)}{\sqrt{\Gamma(r_1, r_1)\Gamma(r_2, r_2)}} = \exp\left\{ \frac{-2\delta_0^2}{r_1^2 + r_2^2} \right\} L_n^0 \left[ \frac{(r_1 - r_2)^2}{2\delta_0^2} \right].
\]

(2)

From Eq. (2), one finds that the degree of coherence of the LGCSM beam doesn’t satisfy Gaussian distribution. Under the condition of \( n = 0 \), Eq. (1) reduces to the expression for the mutual coherence function of a GSM beam [1,2].

If a LGCSM beam passes through a spiral phase plate with transmission function

\[
T(\phi) = \exp(\imath m \phi)
\]

where \( m \) denotes the topological charge and \( \phi \) denotes the azimuthal coordinate (i.e., \( T(x, y) = \exp[\imath m \arctan(y/x)] \) in the Cartesian coordinates), the
transmitted beam will carry a vortex phase and its mutual coherence function can be expressed as

\[
\Gamma(r_1, r_2) = G_0 \exp \left[ -\frac{r_1^2 + r_2^2}{4\sigma^2_0} - \frac{(r_1 - r_2)^2}{2\delta^2_0} \right] L_0^B \left[ \frac{(r_1 - r_2)^2}{2\delta^2_0} \right] \exp(-im\varphi_1 + im\varphi_2). \tag{3}
\]

We call the transmitted beam as LGCSM vortex beam. Due to the vortex phase, the LGCSM vortex beam exhibits unique propagation properties as shown later. Under the condition of \( n = 0 \), Eq. (3) reduces to the expression for the mutual coherence function of a GSM vortex beam [33].

Within the validity of the paraxial approximation, the propagation of the mutual coherence function of the LGSM vortex beam through a stigmatic ABCD optical system can be studied with the help of the following generalized Collins formula [36, 37]

\[
\Gamma(p_1, p_2) = \frac{1}{(\lambda B)^2} \exp \left[ -\frac{ikD}{2B} (p_1^2 - p_2^2) \right] \times \int \int \Gamma(r_1, r_2) \exp \left[ -\frac{ikA}{2B} (r_1^2 - r_2^2) + \frac{ik}{B} (r_1 \cdot p_1 - r_2 \cdot p_2) \right] d^2r_1 d^2r_2, \tag{4}
\]

where \( A, B, C \) and \( D \) are the elements of a transfer matrix for the optical system, \( k = 2\pi / \lambda \) is the wavenumber with \( \lambda \) being the wavelength.

For the convenience of integration, we introduce the following “sum” and “difference” coordinates:

\[
r_s = \frac{r_1 + r_2}{2}, \Delta r = r_1 - r_2, \tag{5}
\]

\[
p_s = \frac{p_1 + p_2}{2}, \Delta p = p_1 - p_2. \tag{6}
\]

Substituting Eqs. (3), (5) and (6) into Eq. (4), we obtain

\[
\Gamma(p_s, \Delta p) = \frac{G_0}{(\lambda B)^2} \exp \left[ -\frac{ikD}{B} p_s \cdot \Delta p \right] \times \int \int P_s \left( r_s + \frac{\Delta r}{2} \right) P_s \left( r_s - \frac{\Delta r}{2} \right) \gamma(\Delta r) \exp \left[ \frac{ik}{B} (\Delta r \cdot p_s + r_s \cdot \Delta p) \right] d^2r_s d^2r, \tag{7}
\]

where

\[
P_s \left( r_s + \frac{\Delta r}{2} \right) = \exp \left[ \left( -\frac{1}{4\sigma^2_0} - \frac{ikA}{2B} \right) \left( r_s + \frac{\Delta r}{2} \right)^2 \right] \exp(-im\varphi_s), \tag{8}
\]

\[
P_s \left( r_s - \frac{\Delta r}{2} \right) = \exp \left[ \left( -\frac{1}{4\sigma^2_0} + \frac{ikA}{2B} \right) \left( r_s - \frac{\Delta r}{2} \right)^2 \right] \exp(im\varphi_s), \tag{9}
\]

\[
\gamma(\Delta r) = \exp \left[ \frac{-\Delta r^2}{2\delta^2_0} \right] L_0^B \left[ \frac{\Delta r^2}{2\delta^2_0} \right]. \tag{10}
\]

with \( \varphi_s = \arctan \left[ (y_s + \Delta y/2) / (x_s + \Delta x/2) \right] \).
\( P_s^\pm (\mathbf{r}, \pm \Delta \mathbf{r}/2) \) and \( P_r (\mathbf{r}, -\Delta \mathbf{r}/2) \) can be expressed in terms of their Fourier transforms \( \tilde{P}_s^\pm (\mathbf{u}_1/\lambda B), \tilde{P}_r (\mathbf{u}_2/\lambda B) \) as follows

\[
P_s^+ \left( \mathbf{r} + \frac{\Delta \mathbf{r}}{2} \right) = \frac{1}{(\lambda B)^2} \int \int \tilde{P}_s^+ \left( \frac{\mathbf{u}_1}{\lambda B} \right) \exp \left( \frac{ik}{B} \left( \mathbf{r} + \frac{\Delta \mathbf{r}}{2} \right) \cdot \mathbf{u}_1 \right) d^2 \mathbf{u}_1,
\]

\[
P_s^- \left( \mathbf{r} - \frac{\Delta \mathbf{r}}{2} \right) = \frac{1}{(\lambda B)^2} \int \int \tilde{P}_s^- \left( \frac{\mathbf{u}_1}{\lambda B} \right) \exp \left( -\frac{ik}{B} \left( \mathbf{r} - \frac{\Delta \mathbf{r}}{2} \right) \cdot \mathbf{u}_1 \right) d^2 \mathbf{u}_1.
\]

Substituting Eqs. (11) and (12) into Eq. (7), after some integration, the mutual coherence function of the LGCSM vortex beam in the output plane is obtained as

\[
\Gamma(\mathbf{p}_s, \Delta \mathbf{p}) = \frac{G_0}{(\lambda B)^2} \exp \left[ -\frac{ik}{B} \mathbf{p}_s \cdot \Delta \mathbf{p} \right]
\times \int \int \tilde{P}_s^+ \left( \frac{\mathbf{u}_1}{\lambda B} \right) \tilde{P}_s^- \left( \frac{\mathbf{u}_1 + \Delta \mathbf{p}}{\lambda B} \right) \gamma \left( -\frac{\mathbf{u}_1 + \Delta \mathbf{p} / 2}{\lambda B} \right) d^2 \mathbf{u}_1,
\]

where \( \gamma(\bullet) \) represents the Fourier transform of \( \gamma(\bullet) \), i.e.,

\[
\gamma(\mathbf{u}) = \int \int \gamma(r) \exp(-2\pi i \mathbf{u} \cdot \mathbf{r}) d^2 r.
\]

The average intensity of the LGCSM vortex beam in the output plane is obtained as

\[
I(\mathbf{p}) = \Gamma(\mathbf{p}_s, \Delta \mathbf{p})_{n=p_0} = \frac{G_0}{(\lambda B)^2} \int \int \tilde{P}_s^+ \left( \frac{\mathbf{u}_1}{\lambda B} \right) \tilde{P}_s^- \left( \frac{\mathbf{u}_1 + \Delta \mathbf{p}}{\lambda B} \right) \gamma \left( -\frac{\mathbf{u}_1 + \Delta \mathbf{p} / 2}{\lambda B} \right) d^2 \mathbf{u}_1,
\]

with

\[
\gamma \left( -\frac{\mathbf{u}_1 + \Delta \mathbf{p}}{\lambda B} \right) = 4\pi \delta_0^2 2^{-2n-1} \left[ 8\pi^2 \delta_0^2 \left( \frac{\mathbf{u}_1 + \Delta \mathbf{p}}{\lambda B} \right) \right] \exp \left[ -2\pi^2 \delta_0^2 \left( \frac{\mathbf{u}_1 + \Delta \mathbf{p}}{\lambda B} \right) \right],
\]

\[
\tilde{P}_s^+ \left( \frac{\mathbf{u}_1}{\lambda B} \right) \tilde{P}_s^- \left( \frac{\mathbf{u}_1 + \Delta \mathbf{p}}{\lambda B} \right) = \pi^2 \left[ \sigma(B) \right]^2 \frac{u_i^2}{4(\lambda B)^2}
\times \exp \left[ -\frac{[\sigma(B) \pi^2]}{2(\lambda B)^2} u_i^2 \right] \left[ I_{\frac{1}{2}+\beta} \left( \frac{[\sigma(B) \pi^2]}{2(\lambda B)^2} u_i^2 \right) - I_{\frac{1}{2}-\beta} \left( \frac{[\sigma(B) \pi^2]}{2(\lambda B)^2} u_i^2 \right) \right]^2.
\]

Here \( \sigma(B) = (1/4\sigma_0 - iKA/2B)^{-1/2} \), and \( I_{\alpha} \) is the modified Bessel function of order \( \alpha \), \( u_i \) is the radial coordinate.

Under the condition of \( n = 0 \), Eq. (15) reduces to the expression for the average intensity of the GSM vortex beam in the output plane [33].

Under the condition of \( m = 0 \), Eq. (15) reduces to the following expression for the average intensity of a LGCSM beam in the output plane.
\[ I(\rho, z) = 2^{n-1} G_0 \left( \frac{k}{B} \right) \sigma^2(B) \sigma^2(B) \delta^{\text{coh}} \left( \left( \frac{\sigma^2(B) + \sigma^2(B) + 2 \delta^2_0}{2B} \right) \rho^2 \right) \exp \left[ \left( \frac{k}{2B} \right) \frac{2 \delta^2_0 (\sigma^2(B) + \sigma^2(B))}{\left( \sigma^2(B) + \sigma^2(B) + 2 \delta^2_0 \right)} \right] \]  

\[ \times \exp \left[ -\left( \frac{k}{2B} \right) \frac{2 \delta^2_0 (\sigma^2(B) + \sigma^2(B))}{\left( \sigma^2(B) + \sigma^2(B) + 2 \delta^2_0 \right)} \right] \]  

Applying Eqs. (15) and (19), we calculate in Fig. 1 the normalized intensity distribution (cross line \( \rho_y = 0 \)) of a LGCSM vortex beam at several propagation distances in free space for different values of the initial coherence width \( \delta_0 \) with \( n = 1 \), \( m = 3 \), \( \sigma_0 = 1 \text{mm} \) and \( \lambda = 632.8 \text{nm} \). For the convenience of comparison, applying Eqs. (18) and (19), we calculate in Fig. 2 the normalized intensity distribution (cross line \( \rho_y = 0 \)) of a LGCSM beam at several propagation distances in free space for different values of the initial coherence width \( \delta_0 \) with \( n = 1 \), \( \sigma_0 = 1 \text{mm} \) and \( \lambda = 632.8 \text{nm} \). One finds from Figs. 1 and 2 that both the LGCSM beam and the LGCSM vortex beam exhibit interesting propagation properties. When the initial coherence width is small, the intensity distribution of a LGCSM beam in the far field has a dark hollow beam profile [see Figs. 2(a-1)–2(d-1)], which is consistent with the result reported in [20]. The evolution properties of the intensity distribution of a LGCSM vortex beam with low coherence on propagation in free space is similar to that of a LGCSM beam [see Figs. 1(a-1)–1(d-1)], and it also has a dark hollow beam profile in the far field. With the increase of the initial coherence width, the far-field intensity distribution of a LGCSM beam or a LGCSM vortex beam varies. For a LGCSM beam, the far-field dark hollow beam profile disappears gradually and finally the far-field Gaussian beam profile is formed as the initial coherence width increases. For a LGCSM vortex beam, the far-field dark hollow beam profile also disappears gradually as the initial coherence width increases, while the far-field dark hollow beam profile appears again when the initial coherence is large [see Fig. 1(d-5)], which is quite different from that of a LGCSM beam. The above interesting phenomenon can be explained in the following way. The effect of spatial correlation function on the evolution properties of a partially coherent beam plays a dominate role only when the initial coherence is low, and its effect can be neglected when the initial coherence is high. For a LGCSM beam with low coherence, its evolution properties are mainly determined by its Laguerre-Gaussian correlation function, and its far-field intensity has a dark hollow beam profile due to the effect of the Laguerre-Gaussian correlation function. For a LGCSM beam with high coherence, the effect of Laguerre-Gaussian correlation function can be neglected, and the evolution properties of a LGCSM beam is similar to that of a GSM beam. For a LGCSM vortex beam, its evolution properties are determined by the Laguerre-Gaussian correlation function and the vortex phase together. When the initial coherence is low, the Laguerre-Gaussian function plays a dominant role and the effect of vortex phase can be neglected, the far-field intensity of the LGCSM vortex beam has a dark hollow beam due to the effect of the Laguerre-Gaussian correlation function. When the initial coherence is high, the effect of the vortex phase plays a dominant role and the effect of Laguerre-Gaussian correlation function can be neglected, the far-field intensity of the LGCSM vortex beam has a dark hollow beam due to the effect of the vortex phase, which induces a phase singularity in the beam center.
plane \((z = 0)\), and the output plane is located at the geometrical focal plane, then the transfer matrix between the source plane and output plane reads as

\[
\begin{pmatrix}
A & B \\
C & D
\end{pmatrix} = \begin{pmatrix}
1 & f \\
0 & 1
\end{pmatrix} \begin{pmatrix}
1 & 0 \\
-1/f & 1
\end{pmatrix} = \begin{pmatrix}
0 & f \\
-1/f & 1
\end{pmatrix}
\]

(20)

Applying Eqs. (15), (18) and (20), we calculate in Fig. 3 the normalized intensity distribution (cross line \(\rho_y = 0\)) of a LGCSM vortex beam at the geometrical focal plane for different values of the initial coherence width \(\delta_0\) with \(n = 1\) and \(m = 3\), and in Fig. 4 the normalized intensity distribution (cross line \(\rho_y = 0\)) of a LGCSM beam at the geometrical focal plane for different values of the initial coherence width \(\delta_0\) with \(n = 1\). The other parameters are chosen as \(\sigma_0 = 1\text{mm}\), \(\lambda = 632.8\text{nm}\) and \(f = 40\text{cm}\). One finds from Figs. 3 and 4 that the dependence of the focused intensity of a LGCSM vortex beam or LGCSM beam on the initial coherence width is similar to the dependence of the far-field intensity of such beam on initial coherence width. In fact the intensity profile of a beam in the focal plane of a converging lens is necessarily the same (with suitable scaling factors) as the intensity profile of the beam in the far field. When the initial coherence width is small, the intensity of a

Fig. 1. Normalized intensity distribution (cross line \(\rho_y = 0\)) of a LGCSM vortex beam with \(n = 1\) and \(m = 3\) at several propagation distances in free space for different values of the initial coherence width \(\delta_0\).

Fig. 1. Normalized intensity distribution (cross line \(\rho_y = 0\)) of a LGCSM vortex beam with \(n = 1\) and \(m = 3\) at several propagation distances in free space for different values of the initial coherence width \(\delta_0\).
LGCSM beam or LGCSM vortex beam at the geometrical focal plane has a dark hollow beam profile. With the increase of the initial coherence width, the dark hollow beam profile of a LGCSM beam or LGCSM vortex beam at the geometrical focal plane disappears gradually. When the initial coherence is large, at the geometrical focal plane, the intensity of a LGCSM beam has a Gaussian beam profile, while the intensity of a LGCSM vortex beam has a dark hollow beam profile. For suitable values of the initial coherence width, the intensity of a LGCSM beam or a LGCSM vortex exhibits flat-topped beam profile. Thus, modulating the spatial coherence of a LGCSM vortex beam or a LGCSM beam provides one way for shaping its focused beam profile, which will be useful for particle trapping, where a focused Gaussian or flat-topped beam spot is used to trap a Rayleigh particle whose refractive index is larger than that of the ambient and a dark hollow beam spot is used to trap a Rayleigh particle whose refractive index is smaller than that of the ambient [38–40].

Fig. 2. Normalized intensity (cross line $\rho_z = 0$) of a LGCSM beam with $n = 1$ at several propagation distances in free space for different values of the initial coherence width $\delta_0$. 
3. Laguerre-Gaussian correlated Schell-model vortex beam: experiment

In this section, we report experimental generation of a LGCSM vortex beam with controllable spatial coherence, and carried out experimental measurement of its focusing properties.

In this paper, we generate LGCSM vortex beam through conversion of a LGCSM beam with the help of a spiral phase plate. In Ref [21], it is shown that a LGCSM beam of mode order $n$ can be formed when an incoherent beam whose intensity distribution has a dark hollow beam profile and is expressed as

$$I(r) = \left(\frac{r^2}{\alpha_r^2}\right)^n \exp\left(-2\frac{r^2}{\alpha_r^2}\right)$$

passes through free space with length $f$, a thin lens with focal length $f$ and a Gaussian amplitude filter (GAF), and the spatial coherence width of the generated LGCSM beam can be approximated as

$$\delta_\gamma = \frac{\lambda f}{\pi w_0}.$$ 

Part 1 of Fig. 5 shows our experimental setup for generating a LGCSM vortex beam. A beam emitted from the He-Ne laser ($\lambda = 632.8$nm) passes through a beam expander, then it goes towards a spatial light modulator (SLM, Holoeyle LC2002), which acts as phase grating designed by the method of computer-generated holograms. Here the pattern of the phase grating for generating a dark hollow beam with $n = 1$ is shown as inset in Fig. 5. The first order of the beam from the SLM is a dark hollow beam with $n = 1$ and is selected out by a circular aperture. After passing through a thin lens $L_1$, the generated dark hollow beam illuminates a rotating ground-glass disk (RGGD), producing an incoherent beam with dark hollow beam profile. The beam from the RGGD can be regarded as a spatially incoherent beam if the diameter of the beam spot on the RGGD is larger than the inhomogeneity scale of $\delta_\gamma$.
the ground glass [41], and this condition is satisfied in our case. After passing through free space with length $f_2$, the thin lens $L_2$, and the GAF, the generated incoherent dark hollow beam becomes a LGCSM beam with $n = 1$ [21]. After passing through a spiral phase plate (SPP) with topological charge $m = 3$, the generated LGCSM beam becomes a LGCSM vortex beam. The SPP just adds a vortex phase to the LGCSM beam, and it doesn’t alter its spatial coherence and its intensity distribution in the source plane, thus the spatial coherence width and the intensity distribution of generated LGCSM vortex beam are the same with those of the generated LGCSM beam. The spatial coherence width of the generated LGCSM beam is modulated by varying the beam spot on the RGGD through varying the distance between the thin lens $L_1$ and the RGGD.

The degree of coherence (i.e., correlation function) and the spatial coherence width of the generated LGCSM beam can be measured by using the method proposed in Ref [42]. As illustrated in [21] and [42], the generated partially coherent beam is split into two distinct imaging optical paths by a 50:50 beam splitter, and the transmitted beam and reflected beam go to two single-photon detectors, respectively. By measuring the fourth-order correlation function between the detectors, we can obtain the distribution of the square of the degree of coherence of the generated beam with the help of the Gaussian moment theorem (i.e., internal relation between second-order and fourth-order correlation function) [1].

![Fig. 5. Experimental setup for generating a LGCSM vortex beam and measuring its focused intensity. BE, beam expander; SLM, spatial light modulator; CA, circular aperture; L1, L2, L3, thin lenses; GAF, Gaussian amplitude filter; RGGD, rotating ground-glass disk; GAF, Gaussian amplitude filter; SPP, spiral phase plate; BPA, beam profile analyzer; PC1, PC2, personal computers.](image)

Part 2 of Fig. 5 shows our experimental setup of measuring the focused intensity distribution of the generated LGCSM beam. The generated beam first passes through a thin lens $L_3$ with focal length $f_3 = 40 \text{cm}$, and then arrives at the beam profile analyzer (BPA), which measures the focused intensity. The elements of the transfer matrix between the source plane and the BPA read as

$$A = 0, \quad B = f_3, \quad C = -1 / f_3, \quad D = 1.$$  \hspace{1cm} (21)

Figure 6 shows our experimental results of the intensity distribution and the corresponding cross line (dotted curve) of the generated LGCSM beam in the source plane. The solid curve is a result of the theoretical fit. It is clear from Fig. 6 that the intensity distribution of the generated beam in the source plane has a Gaussian profile as expected. Through theoretical fit (solid curve) of the experimental results, we obtain that $\sigma_0$ is about $1 \text{mm}$. In our experiment, we generate several LGCSM beams and LGCSM vortex beams with different initial coherence widths in order to study the influence of coherence width on the focusing properties, and Fig. 7 shows our experimental results of the square of the modulus of the...
generated LGCSM beam for different values of the initial coherence width in the source plane. Through theoretical fit of the experimental results, we obtain \( \delta_0 = 0.1\text{mm}, 0.2\text{mm}, 0.38\text{mm}, 0.52\text{mm}, 0.82\text{mm}, 1.35\text{mm}, 2.0\text{mm} \) for Figs. 7(a)–7(g), respectively. With the measured beam parameters and formulae derived in section 2, we can simulate the focusing properties of the generated beam, and compare with the corresponding results.

Fig. 6. Experimental results of (a) the intensity distribution and (b) the corresponding cross line (dotted curve) of the generated LGCSM beam with \( n = 1 \) in the source plane. The solid curve is a result of the theoretical fit.

Fig. 7. Experimental results of the square of the modulus of the generated LGCSM beam for different values of the initial coherence width in the source plane. The solid curve is a result of the theoretical fit.

Fig. 8. Experimental results of the intensity distribution and the corresponding cross line \( (\rho_0 = 0) \) of the generated LGCSM beam with \( n = 1 \) at the geometrical focal plane for different values of the initial coherence width \( \delta_0 \). The solid curve denotes the theoretical results calculated by Eq. (18).
Fig. 9. Experimental results of the intensity distribution and the corresponding cross line ($\rho_y = 0$) of the generated LGCSM vortex beam with $n = 1$ and $m = 3$ at the geometrical focal plane for different values of the initial coherence width $\delta_0$. The solid curve denotes the theoretical results calculated by Eq. (15).

Figure 8 shows our experimental results of the intensity distribution and the corresponding cross line ($\rho_y = 0$) of the generated LGCSM beam with $n = 1$ at the geometrical focal plane for different values of the initial coherence width $\delta_0$. Figure 9 shows our experimental results of the intensity distribution and the corresponding cross line ($\rho_y = 0$) of the generated LGCSM vortex beam with $n = 1$ and $m = 3$ at the geometrical focal plane for different values of the initial coherence width $\delta_0$. For the case of $\delta_0 = \infty$, the rotating round-glass disk completely removed from our experimental setup. For the convenience of comparison, the corresponding numerical results calculated by the formulae derived in section 2 are also shown in Figs. 8 and 9. One finds from Figs. 8 and 9 that the focused intensities of the generated LGCSM beams and LGCSM vortex beams indeed are modulated through varying the initial coherence width as expected by Figs. 3 and 4, and our experimental results agree well with the theoretical predictions.
4. Summary

We have introduced the theoretical model for a new partially coherent vortex beam with special correlation function named LGCSM vortex beam as an extension of recently introduced LGCSM beam, and we have derived the explicit propagation formulae for such beam propagating through a stigmatic ABCD optical system. The propagation properties of a LGCSM beam and a LGCSM vortex beam have been studied comparatively, and it is found that they exhibit different propagation properties. Furthermore, we have carried out experimental generation of a LGCSM vortex beam through converting a LGCSM beam to such beam by a spiral phase plate, and studied the focusing properties of a LGCSM beam and a LGCSM vortex beam comparatively both in theory and in experiment. We have found that we can shape the intensity distribution of a LGCSM beam and a LGCSM vortex beam through varying its initial coherence width, and our experimental results are consistent with the theoretical results. Our results will be useful for particle trapping, where focused beam spot with special beam profile is required.

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