A cost-effective rumor-containing strategy

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Abstract

This paper addresses the issue of suppressing a rumor using the truth in a cost-effective way. First, an individual-level dynamical model capturing the rumor-truth mixed spreading processes is proposed. On this basis, the cost-effective rumor-containing problem is modeled as an optimization problem. Extensive experiments show that finding a cost-effective rumor-containing strategy boils down to enhancing the first truth-spreading rate until the cost-effectiveness of the rumor-containing strategy reaches the first turning point. This finding greatly reduces the time spent for solving the optimization problem. The influences of different factors on the optimal cost effectiveness of a rumor-containing strategy are examined through computer simulations. We believe our findings help suppress rumors in a cost-effective way. To our knowledge, this is the first time the rumor-containing problem is treated this way.

Keywords: Rumor spreading, cost-effective rumor-containing strategy, rumor-truth mixed spreading model, optimization problem

1. Introduction

As a common form of social interactions, rumors are dispersed for achieving specific purposes, especially when a major public event occur and people do not have exact information and knowledge about the event [1–3]. The rapidly developing online social networks (OSNs) greatly enhance the speed and extent of rumor spreading [4, 5]. Unfortunately, most rumors could lead to economic loss or even disrupt social order [6]. To exemplify, Syrian hackers once broke into the twitter account of Associated Press (AP) and dispersed the rumor that explosions at White House had injured Obama, leading to a loss of 10 billion US dollars before the rumor was clarified [7]. Therefore, effective measures of containing the prevalence of rumors must be worked out [8].

One of the keys to inhibiting rumors is to understand the rumor spreading processes, and modeling the rumor spreading processes as dynamical systems proves an effective approach to achieving this goal. In 1965, Daley and Kendal [9] introduced the first rumor spreading model, where a closed and homogeneously mixed population is subdivided into three groups: ignorants who are unaware of the rumor, spreaders who are aware of the rumor and spread it, and stiflers who are aware of the rumor but do not spread it. See Ref. [10] for a popular introduction of the model. Since then, a multitude of homogeneous network based rumor spreading models have been proposed [11–19].

A series of findings starting about two decades ago fully indicate that OSNs are inhomogeneous [20, 21]. Much efforts have since been focused on the complex network based rumor spreading [22–37]. As these models are quite rough, their dynamics may severely deviate from the actual rumor spreading process. Furthermore, the influence of the network topology on the rumor spreading cannot be fully understood by studying these models only. In 2009, Van Mieghem et al. [38, 39] proposed an individual-level epidemic model, and this has been followed by a number of similar models [40–48]. This type of epidemic models turns out to be especially suited to exploring the impact of the network topology on epidemic processes. On the other hand, the optimal control theory has recently been successfully applied to the cost-effective containment of malware [49–52].
Clarifying rumors by spreading truths is a common way to inhibit rumors [53, 54]. In the presence of a rumor and the truth aiming at the rumor, depending on his/her judgement a person either believes the rumor, or believes the truth, or is uncertain. In this context, this paper addresses the issue of containing the prevalence of the rumor in a cost-effective way. First, an individual-level dynamical model capturing the rumor-truth mixed spreading processes is proposed. On this basis, the cost-effective rumor-containing problem is modeled as an optimization problem. Extensive experiments show that finding a cost-effective rumor-containing strategy boils down to enhancing the first truth-spreading rate until the cost effectiveness of the rumor-containing strategy reaches the first turning point. This finding greatly reduces the time spent for solving the optimization problem. The influences of different factors on the optimal cost effectiveness of a rumor-containing strategy are examined through computer simulations. We believe our findings help suppress rumors in a cost-effective way. To our knowledge, this is the first time the rumor-containing problem is treated this way.

The subsequent materials of this work are organized as follows. Section 2 models the rumor-truth spreading processes. Section 3 models the cost-effective rumor-containing problem. Section 4 experimentally examines the influences of different factors on the best cost effectiveness of a rumor-containing strategy. This work is closed by Section 5.

2. The modeling of the rumor-truth spreading processes

For the purpose of modeling the cost-effective rumor-containing problem, the rumor-truth mixed spreading processes must be modeled first. This section is devoted to this modeling task.

Consider an OSN consisting of \( N \) persons labeled 1, \( \cdots \), \( N \), and let \( V = \{1, \cdots, N\} \). Suppose rumors are dispersed in the OSN through a rumor-spreading network \( G_R = (V, E_R) \), and let \( A = (a_{ij})_{N \times N} \) denote the adjacency matrix of \( G_R \). Suppose truths are circulated in the OSN through a truth-spreading network \( G_T = (V, E_T) \), and let \( B = (b_{ij})_{N \times N} \) denote the adjacency matrix of \( G_T \).

At the beginning, there is neither the rumor nor the truth, so all persons in the OSN are uncertain. Then the rumor and the truth appear. Depending on his/her judgement, a person in the OSN may choose to believe the rumor, or to believe the truth, or to be uncertain. As a result, at any time every person in the OSN is in one of three possible states: rumor-believing, truth-believing, and uncertain. A rumor-believer believes the rumor, and a truth-believer believes the truth. Let \( U_i(t) \), \( R_i(t) \) and \( T_i(t) \) denote the probability of person \( i \) being uncertain, rumor-believing and truth-believing at time \( t \), respectively. As \( U_i(t) + R_i(t) + T_i(t) \equiv 1 \), the vector

\[
x(t) = (R_1(t), \cdots, R_N(t), T_1(t), \cdots, T_N(t))^T
\]

captures the expected state of the OSN at time \( t \). Next, let us impose a set of hypotheses as follows.

(H1) The OSN is under surveillance in the time interval \([0, T]\).

(H2) Due to the influence of rumor-believers, at time \( t \) uncertain person \( i \) turns to believe the rumor at the expected rate \( \beta_1 \sum_{j=1}^{N} a_{ij} R_j(t) \), where \( \beta_1 > 0 \) is referred to as the first rumor-spreading rate.

(H3) Due to the influence of rumor-believers, at time \( t \) truth-believer \( i \) turns to believe the rumor at the expected rate \( \beta_2 \sum_{j=1}^{N} a_{ij} R_j(t) \), where \( \beta_2 > 0 \) is referred to as the second rumor-spreading rate.

(H4) Due to the influence of truth-believers, at time \( t \) uncertain person \( i \) turns to believe the truth at the expected rate \( \gamma_1 \sum_{j=1}^{N} b_{ij} T_j(t) \), where \( \gamma_1 > 0 \) is referred to as the first truth-spreading rate. Additionally, the cost per unit time for achieving the first truth-spreading rate \( \gamma_1 \) is \( c_1 \gamma_1 \) units, where \( c_1 > 0 \) is referred to as the first truth-spreading cost.

(H5) Due to the influence of truth-believers, at time \( t \) rumor-believer \( i \) turns to believe the truth at the expected rate \( \gamma_2 \sum_{j=1}^{N} b_{ij} T_j(t) \), where \( \gamma_2 > 0 \) is referred to as the second truth-spreading rate. Additionally, the cost per unit time for achieving the second truth-spreading rate \( \gamma_2 \) is \( c_2 \gamma_2 \) units, where \( c_2 > 0 \) is referred to as the second truth-spreading cost.

(H6) Due to the limitation in memory, at any time every rumor-believer truth-believer forgets the rumor and becomes uncertain, and every truth-believer forgets the truth and becomes uncertain, at the expected rate \( \delta \), where \( \delta > 0 \) is referred to as the forgetting rate.
The two truth-spreading rates, \( \gamma \) and \( \beta \), in this model are assumed to be controllable, and the ordered pair \( \gamma = (\gamma_1, \gamma_2) \) is referred to as a rumor-containing strategy.

Based on the above hypotheses, we get the following dynamical system.

\[
\begin{align*}
\frac{dR_i(t)}{dt} &= -\delta R_i(t) + \beta_1 [1 - R_i(t) - T_i(t)] \sum_{j=1}^{N} a_{ij} R_j(t) + \beta_2 T_i(t) \sum_{j=1}^{N} a_{ij} R_j(t) - \gamma_1 R_i(t) \sum_{j=1}^{N} b_{ij} T_j(t), \\
\frac{dT_i(t)}{dt} &= -\delta T_i(t) - \beta_2 T_i(t) \sum_{j=1}^{N} a_{ij} R_j(t) + \gamma_1 [1 - R_i(t) - T_i(t)] \sum_{j=1}^{N} b_{ij} T_j(t) + \gamma_2 R_i(t) \sum_{j=1}^{N} b_{ij} T_j(t),
\end{align*}
\tag{1}
\]

\( t \in [0, T], \ 1 \leq i \leq N. \)

We refer to the model as the \textit{URTU model}. This model captures the expected rumor-truth mixed interacting process. The two truth-spreading rates, \( \gamma_1 \) and \( \gamma_2 \), in this model are assumed to be controllable, and the ordered pair \( \gamma = (\gamma_1, \gamma_2) \) is referred to as a \textit{rumor-containing strategy}.

### 3. The modeling of the cost-effective rumor-containing problem

Given a rumor-containing strategy \( \gamma \), it can be seen from the URTU model (1) that, the expected cumulative number of uncertain persons who turn to believe the truth is

\[
E_U(\gamma) = \gamma_1 \int_0^T \sum_{i=1}^{N} [1 - R_i(t) - T_i(t)] \sum_{j=1}^{N} b_{ij} T_j(t) dt, \tag{2}
\]

and the expected cumulative number of rumor-believers who turn to believe the truth is

\[
E_R(\gamma) = \gamma_2 \int_0^T \sum_{i=1}^{N} R_i(t) \sum_{j=1}^{N} b_{ij} T_j(t) dt \tag{3}
\]

Hence, the effectiveness of \( \gamma \) can be measured by

\[
E(\gamma) = E_U(\gamma) + E_R(\gamma)
\]

\[
= \gamma_1 \int_0^T \sum_{i=1}^{N} [1 - R_i(t) - T_i(t)] \sum_{j=1}^{N} b_{ij} T_j(t) dt + \gamma_2 \int_0^T \sum_{i=1}^{N} R_i(t) \sum_{j=1}^{N} b_{ij} T_j(t) dt. \tag{4}
\]
On the other hand, it follows from hypotheses (H₄)-(H₅) that the cumulative cost for converting uncertainties to truth-believers is

\[ C_U(γ) = c_1γ_1 \int_0^T \sum_{i=1}^N [1 - R_i(t) - T_i(t)] dt, \]  

and the cumulative cost of γ for converting rumor-believers to truth-believers is

\[ C_R(γ) = c_2γ_2 \int_0^T \sum_{i=1}^N R_i(t) dt. \]  

As a result, the cumulative cost of the rumor-containing strategy γ can be gauged by

\[ C(γ) = C_U(γ) + C_R(γ) = c_1γ_1 \int_0^T \sum_{i=1}^N [1 - R_i(t) - T_i(t)] dt + c_2γ_2 \int_0^T \sum_{i=1}^N R_i(t) dt. \]  

Combining the above discussions, the cost effectiveness of the rumor-containing strategy γ is measured by

\[ J(γ) = \frac{E(γ)}{C(γ)} = \frac{E_U(γ) + E_R(γ)}{C_U(γ) + C_R(γ)} = \frac{γ_1 \int_0^T \sum_{i=1}^N [1 - R_i(t) - T_i(t)] dt + γ_2 \int_0^T \sum_{i=1}^N R_i(t) \sum_{j=1}^N b_{ij} T_j(t) dt}{c_1γ_1 \int_0^T \sum_{i=1}^N [1 - R_i(t) - T_i(t)] dt + c_2γ_2 \int_0^T \sum_{i=1}^N R_i(t) dt}. \]  

Let c denote the given rumor-containing budget per unit time. Then \( c_1γ_1 + c_2γ_2 = c \). Based on the above discussions, the cost-effective rumor-containing problem can be modeled as the following optimization problem.

\[ \text{(P) Maximize } c_1γ_1 + c_2γ_2 = c \]  

4. The solution of the optimization problem

The previous section tells us that finding a cost-effective rumor-containing strategy boils down to solving the optimization problem (P). This section is devoted to solving the optimization problem. For this purpose, the network is chosen from the set of six non-isomorphic trees shown in Fig. 2.

![Figure 2: Six non-isomorphic trees with six nodes and five edges.](image)
Fig. 3 shows how the cost effectiveness of a rumor-containing strategy varies with the first truth-spreading rate, where $c_1 = 4$, $c_2 = 5$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $\delta = 0.1$, $G_i \in \{G_i : i = 1, 2, \cdots, 6\}$, $c \in \{10, 15, 20\}$, $T \in \{20, 25, 30\}$. It can be seen from these and many other experiments that, with the increase of the first truth-spreading rate, the cost effectiveness of the rumor-containing strategy rises first and falls later, forming a unimodal function. Therefore, finding a cost-effective rumor-containing strategy simply comes down to enhancing the first truth-spreading rate until the cost effectiveness of the rumor-containing strategy reaches the first turning point. This finding is what we want, because it greatly reduces the time spent for solving the optimization problem.

**Figure 3:** The cost effectiveness of a rumor-containing strategy versus the first truth-spreading rate where $c_1 = 4$, $c_2 = 5$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $\delta = 0.1$, $G_i \in \{G_i : i = 1, 2, \cdots, 6\}$, (a) $c = 10$, $T = 20$, (b) $c = 10$, $T = 25$, (c) $c = 10$, $T = 30$, (d) $c = 15$, $T = 20$, (e) $c = 15$, $T = 25$, (f) $c = 15$, $T = 30$, (g) $c = 20$, $T = 20$, (h) $c = 20$, $T = 25$, (i) $c = 20$, $T = 30$. It can be seen that, with the increase of the first truth-spreading rate, the cost effectiveness of the rumor-containing strategy rises first and falls later, forming a unimodal function.

5. The influence of different factors on the optimal cost effectiveness of a rumor-containing strategy

Obviously, the optimal cost effectiveness of a rumor-containing strategy depends on different factors, including the two rumor-spreading rates, the forgetting rate, the budget per unit time, and the rumor-containing duration. This section is committed to examining the influences of these factors on the optimal cost effectiveness.
5.1. The influence of the rumor-spreading rates

First, let us examine the influence of the first rumor-spreading rate on the optimal cost effectiveness of a rumor-containing strategy. Fig. 4 shows how the cost effectiveness of a rumor-containing strategy varies with the first rumor-spreading rate, where \( \beta_2 \in [0, 1 : k = 1, 2, \cdots, 10], \beta_1 = 0.5, \delta = 0.1, G \in [G_i : i = 1, 2, \cdots, 6], c_1 = 4, c_2 = 5, c \in [10, 15, 20], T \in [20, 25, 30] \). It can be seen from these and many similar experiments that the optimal cost effectiveness of a rumor-containing strategy falls with the increase of the first rumor-spreading rate. This finding demonstrates that a rumor with a higher first rumor-spreading rate is harder to suppress than a rumor with a lower first rumor-spreading rate, which is in line with the intuition.

![Graphs showing the influence of the first rumor-spreading rate](image)

Figure 4: The optimal cost effectiveness of a rumor-containing strategy versus the first rumor-spreading rate, where \( \beta_1 \in [0, 1 : k = 1, 2, \cdots, 10], \beta_2 = 0.5, \delta = 0.1, G \in [G_i : i = 1, 2, \cdots, 6], c_1 = 4, c_2 = 5, c \in [10, 15, 20], T \in [20, 25, 30] \). It can be seen that the optimal cost effectiveness of a rumor-containing strategy falls with the increase of the first rumor-spreading rate.

Then, let us examine the influence of the second rumor-spreading rate on the optimal cost effectiveness of a rumor-containing strategy. Fig. 5 shows how the cost effectiveness of a rumor-containing strategy varies with the second rumor-spreading rate, where \( \beta_1 \in [0, 1 : k = 1, 2, \cdots, 10], \beta_2 = 0.5, \delta = 0.1, c_1 = 4, c_2 = 5, G \in [G_i : i = 1, 2, \cdots, 6], c \in [10, 15, 20], T \in [20, 25, 30] \). It can be seen from these and many similar experiments that the optimal cost effectiveness of a rumor-containing strategy falls with the increase of the second rumor-spreading rate. This finding
demonstrates that a rumor with a higher second truth-spreading rate is harder to suppress than a rumor with a lower second truth-spreading rate, which is in accordance with the intuition.

The above findings imply that, in order to contain the prevalence of a rumor with higher spreading rates, the rumor-containing budget per unit time must be enhanced.

5.2. The influence of the forgetting rate

Fig. 6 shows how the cost effectiveness of a rumor-containing strategy varies with the forgetting rate, where $\delta \in [0.1k : k = 1, 2, \cdots, 10]$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $G \in \{G_i : i = 1, 2, \cdots, 6\}$, $c_1 = 4$, $c_2 = 5$, $c \in \{10, 15, 20\}$, $T \in \{20, 25, 30\}$. It can be seen from these and many similar experiments that, with the increase of the forgetting rate, the optimal cost effectiveness of a rumor-containing strategy first rises then descends. This finding demonstrates that a moderate forgetting rate helps inhibit the spread of rumors.
5.3. The influence of the rumor-containing budget per unit time

Fig. 7 shows how the cost effectiveness of a rumor-containing strategy varies with the rumor-containing budget per unit time, where $c \in \{5, 10, 15, 20, 25\}$, $\beta_1 \in \{0.2, 0.3\}$, $\beta_2 \in \{0.2, 0.3\}$, $\delta \in \{0.1, 0.15\}$, $G \in \{G_i : i = 1, 2, \ldots, 6\}$, $c_1 = 4$, $c_2 = 5$, $T = 20$. It can be seen from these and many similar experiments that the optimal cost effectiveness of a rumor-containing strategy goes up with the increase of the budget per unit time. This finding demonstrates that enhancing the rumor-containing budget per unit time is always an effective approach to suppressing rumors.
Optimal cost effectiveness

The problem of containing the prevalence of a rumor in a cost-effective way has been addressed. An individual-level dynamical model capturing the rumor-truth mixed spreading processes has been proposed, and the rumor-containing strategy goes up with the increase of the budget per unit time.

5.4. The influence of the rumor-containing duration

Fig. 8 shows how the cost effectiveness of a rumor-containing strategy varies with the rumor-containing duration, where $T \in \{10 + 2k : k = 0, 1, \ldots, 10\}$, $\beta_1 \in \{0.2, 0.3\}$, $\beta_2 \in \{0.2, 0.3\}$, $\delta \in \{0.1, 0.15\}$, $G \in \{G_i : i = 1, 2, \ldots, 6\}$, $c_1 = 4$, $c_2 = 5$, $c = 10$. It can be seen from these and many similar experiments that the optimal cost effectiveness of a rumor-containing strategy goes up with the increase of the rumor-containing duration. This finding tells us that increasing the rumor-containing duration is always an effective means of inhibiting rumors.

Figure 8: The optimal cost effectiveness of a rumor-containing strategy versus the rumor-containing duration, where $T \in \{10 + 2k : k = 0, 1, \ldots, 10\}$, $c_1 = 4$, $c_2 = 5$, $c = 10$. It can be seen that the optimal cost effectiveness of a rumor-containing strategy goes up with the increase of the rumor-containing duration.

6. Concluding remarks

The problem of containing the prevalence of a rumor in a cost-effective way has been addressed. An individual-level dynamical model capturing the rumor-truth mixed spreading processes has been proposed, and the rumor-
containing problem has been modeled as an optimization problem. Extensive experiments have shown that finding a cost-effective rumor-containing strategy boils down to enhancing the first truth-spreading rate until the cost effectiveness of the rumor-containing strategy reaches the first turning point. The influences of different factors on the best cost effectiveness of a rumor-containing strategy have been examined through simulation experiments. Our findings help stipulate rumor-suppressing policies.

Toward this direction there are some future research topics. For example, the proposed rumor-truth mixed spreading model may be modified by accommodating more considerations [53–56]. As the second instance, the methodology developed in this work applies to other disciplines such as malware containment and viral marketing.

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