Study on Synthesis Stiffness of Inner Supported Deep Excavation in Soft Soil Area

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Abstract. The horizontal displacement of retaining structure is particularly associated with synthesis stiffness of support system. The synthesis stiffness of support system is highly interrelated with three parts including the retaining wall, the inner brace and the soil at bottom of excavation. In order to estimate the lateral movement of retaining wall, a synthesis stiffness formula is been proposed. The proposed model concludes a variety of factors, such as horizontal flexural rigidity of retaining wall, compressional stiffness of inner brace, average vertical support spacing, average horizontal support spacing, depth of wall penetration and stiffness of the soil below final excavation depth. Comparing with the Clough synthesis stiffness, more factors is been considered. Comparing with the MVSS synthesis stiffness, the factor of soil below final excavation is been defined exactly. After that, finite element method is employed to calculate the maximal lateral displacement of the retaining wall. And a good relationship is established between maximal horizontal displacement and synthesis stiffness formula. Finally, the calculations are used to compare with the observed data from several deep excavations, the result validates the applicability of proposed model. The research provides a new method for describing the comprehensive stiffness of support system of foundation pit. It is convenient to estimate the maximum horizontal displacement of the retaining structure at the design stage and to select the support system quickly.

Keywords: deep excavation, retaining structure, horizontal displacement, support system, synthesis stiffness.

1. Introduction

With the rapid development of the economy, the process of urbanization has been pushed forward continuously. The urban population is growing more and more. The traffic is getting worse. And the land resource is also becoming increasingly tight. In order to improve the traffic flow and make full use of the underground space, the subway project is carried out throughout the country actively. The subway project involves many deep foundation pits of the subway station. Most deep foundation pits are located in commercial and residential areas with densely population. There are many tall buildings, bridges and urban roads around the subway station. And the underground pipe network around the subway station is dense. The excavation of foundation pit can cause deformation of the surrounding stratum. It may adversely affect adjacent buildings, municipal pipe networks and traffic lines. In order
to protect the surrounding environment, the design concept of urban deep foundation pit has been changed from controlling the mechanical stability to the deformation of retaining structure.

The deformation of retaining structure of foundation pit is closely related to the synthesis stiffness of the supporting system. The greater the synthesis stiffness of the supporting system, the smaller the deformation of the retaining structure. Conversely, the deformation of the retaining structure is larger. The support system of foundation pit includes the retaining structure, the inner braces, and the passive soil below the bottom of the pit. The greater the bending stiffness of retaining wall, the stronger its resistance to bending deformation. The smaller the vertical and horizontal spacing of the support, the greater the compressive stiffness of the support, the stronger the ability of the inner braces to limit the lateral deformation of the retaining structure. The stronger the ability of the soil at the bottom of the pit to resist deformation under the action of external force, the greater the insertion ratio of the retaining wall, the more obvious the restraining effect on the horizontal deformation of the retaining structure. At present, many researchers have carried out research on the relationship between the maximum horizontal deformation of the retaining structure and the synthesis stiffness of the supporting system.

Clough [1] firstly used $E_wI/(γ_w h_{avg})$ to describe the synthesis stiffness of the support system in 1989. In the formula: $E_wI$ represents the bending stiffness of the retaining wall, $h_{avg}$ indicates the vertical average spacing of the supports, $γ_w$ indicates the gravity of the water. Addenbrooke [2] considers that the displacement flexibility coefficient $h_{avg}/E_w I$ can reflect the comprehensive effect of the support system better. Based on the Clough synthesis stiffness, Liu [3] proposed an improved synthesis stiffness $E_wI/(λ_iγ_w h_{avg})$, which considers the influence of soil around the foundation pit. Where: $λ_i$ indicates the wavelength of the influence of deformation, which can reflect the soft and hard degree of soil under the final excavation and the insertion ratio of the retaining wall. $γ_{M=2}$ indicates the shear strain when soil strength mobility rate equal 1/2 undrained shear strength, which can reflect the soil stiffness. Zhang [4] proposed a synthesis stiffness named MVSS, which include the influence factors such as space-time effect, the reinforcement of foundation, the support stiffness and the average horizontal and vertical spacing, as shown Equation (1):

$$x_{MVSS} = k_i k_j \frac{m k_i + n k_j}{(m + n) k_1} \times \frac{E_w I}{γ_w S_v^2 H_s S_h}$$  \hspace{1cm} (1)$$

In the expression (1): $k_i$ is a comprehensive adjustment factor related to space-time effect and insertion ratio, $k_i$ is the influence factor of foundation reinforcement, $(mk_i + nk_j)/(m + n)k_1$ is the influence factor of support stiffness, $m$ is the number of steel support tracks, $n$ is the number of concrete support tracks, $k_1$ is the steel support stiffness, and $k_j$ is the concrete support stiffness, $S_v$ and $S_h$ indicates the average vertical and horizontal spacing of inner support respectively, $H_s$ represents the finally excavation depth, other symbols have the same meanings as above. Though a lot of factors have been taken into consideration in the MVSS, the factor $k_i$ and $k_j$ need to be determined in combination with engineering experience, which is inaccurate compared with the well-defined factor of support stiffness.

On the basis of previous studies, the deep foundation pit of long strip subway station in Shanghai has been taken as the object and regarded as plane strain problem in this paper. The synthesis stiffness expression SS of the support system is proposed. The finite element calculation is used to verify the rationality of the synthesis stiffness expression SS. The measured data in the field is also used to illustrated the engineering applicability of the synthesis stiffness expression SS. In comparison with the measured data, the results base on the calculated formula is good. So based on the synthesis stiffness expression SS and the large finite element calculation results, getting the fitting formula between the maximum horizontal deformation of retaining wall and the synthesis stiffness expression SS, which may be a convenient method for calculation the maximum wall deflection in the stage of design and beneficial to select the support system of foundation pit quickly for similar project.
2. The Synthesis Stiffness Expression SS

When the foundation pit is excavated, the horizontal displacement of the retaining wall is closely related to the bending stiffness of the retaining structure. The greater the bending stiffness, the smaller the horizontal displacement of the retaining wall. The horizontal displacement is also limited by the support system above the excavation face and the passive zone below the excavation face. In this paper, many influencing factors of support system of the foundation pit is considered. And the following synthesis stiffness expression is proposed:

\[
SS = \frac{\sum_{i=1}^{N} E_i A_i}{N S_i S_h E_f} \times \frac{E_h h}{E_f H} \times \frac{E_w I}{E_f}
\]  

Where: \( E_i \) represents the elastic modulus of the \( i \)-th support, \( A_i \) represents the cross-sectional area of the \( i \)-th support, \( N \) represents the total number of vertical support, \( E_s \) represents the deformation modulus of the soil below the bottom of the pit, \( E_w \) denotes the elastic modulus of the retaining wall, \( E_f \) indicates the reference modulus, and 30GPa is used as the reference in this paper, \( h_d, H, I \) separately represent the insertion depth, the length of the retaining wall, the moment of inertia of retaining wall. Other symbols have the same meaning as above.

Formula (2) contains three parts. \( \sum(E_i A_i)/(NS_i S_h E_f) \) reflects the role of the inner braces. \( \sum(E_i A_i)/(NS_i S_h E_f) \) can be regarded as the equivalent elastic modulus of the inner braces and \( \sum(E_i A_i)/(NS_i S_h E_f) \) represents the ratio of the equivalent elastic modulus to the reference modulus. The larger the ratio, the stronger the ability of brace to restrain the lateral deformation of the retaining structure. \( E_h h/(E_f H) \) reflects the action of the soil in the passive zone below the excavation face. \( E_h h/(E_f H) \) represents the ratio of the deformation modulus of the soil in the passive zone to the reference modulus. The larger the ratio, the harder the soil is and the stronger the constraint ability to the horizontal deformation of retaining wall. \( h_d/H \) is the insertion ratio of the retaining wall. The deeper the retaining wall is inserted, the more obvious constraining effect of the soil at the passive zone to the horizontal deformation of the retaining wall is. \( E_w I \) reflects the horizontal bending resistance of the retaining wall. The larger the \( E_w I \) value is, the stronger the ability of the retaining wall to resist horizontal deformation.

3. The Rationality by Finite Element Method

In order to illustrate the rationality of the synthesis stiffness SS, the metro station of 16m depth in Shanghai is taken as the object. Based on the Plaxis finite element software, the numerical model is established and different sets of geometric and physical mechanical parameters are set. The finite element calculation under the different combination of parameters is used to verify the synthesis stiffness expression SS. Meanwhile, the Clough synthesis stiffness and MVSS synthesis stiffness is used to compare with the synthesis stiffness expression SS.

The geometry, soil, structural parameters of basic model and the parameter combinations are described in the follow section.

3.1. The Geometry of Basic Model

As shown in Figure 1, the excavation depth of the foundation model is 16m and the excavation width is 20m. In order to reduce the influence of the model boundary on the calculation of excavation, the outer width of the pit is taken as 5 times of excavation depth and the thickness of the stratum below the bottom of the pit is taken as 3 times of excavation depth. Considering the symmetry, 1/2 foundation pit is taken for modeling and calculation. In the basic model, the insertion ratio of the retaining wall is assumed to be 1.0, the length of the retaining wall is 32 m, and four supports are provided. The first support is about 1m below the ground surface. The groundwater in Shanghai is generally located 0.3–1.5m [5] below the surface of the earth. The basic model geometry is shown in Figure 1.
3.2. The Parameters of Soil and Structure

Based on the typical stratum in Shanghai, the basic model is composed of clay, silty clay, muddy clay, clay and fine sand. The small strain hardening models (HSS) is selected as soil constitutive models. The soil layer thickness, physical and mechanical parameters are shown in Table 1 [5-6]. The thickness of retaining wall casted by C30 concrete is 1m. The horizontal bending stiffness $E_wI$ is $2.5e6$ kNm$^2$. The axial compressive rigidity and poisson’s ratio of the retaining wall is $3.0e7$ kN and 0.17 respectively. Steel braces with a diameter of 609 mm and a wall thickness of 16 mm are used for internal braces. The compressive stiffness of steel braces is $6.26e6$ kN and the support horizontal spacing is 4.5m.

| Soil          | Thick m | $\gamma$ kN/m$^3$ | $c'$ kPa | $\phi'$ | $e_{c5}$ MPa | $e_{ur}$ MPa | $c_{ur}$ MPa | $\gamma_{05}$ $10^{-4}$ | $m$ | $\nu_w$ | $R_f$ | $R_{inter}$ |
|---------------|---------|-------------------|---------|---------|---------------|---------------|---------------|----------------------|-----|--------|-------|-----------|
| Clay          | 4       | 18                | 10      | 30      | 0             | 4.5           | 5.4           | 31.5                 | 126 | 2      | 0.8   | 0.2       |
| Silty clay    | 8       | 17                | 5       | 27      | 0             | 2             | 2.4           | 14                   | 36  | 2      | 0.8   | 0.2       |
| Muddy clay    | 8       | 17                | 3       | 27      | 0             | 2             | 2.4           | 14                   | 36  | 2      | 0.8   | 0.2       |
| Clay          | 10      | 18                | 10      | 30      | 0             | 4.5           | 5.4           | 31.5                 | 126 | 2      | 0.8   | 0.2       |
| Clay          | 4       | 19                | 30      | 32      | 0             | 7.2           | 8.6           | 50                   | 200 | 2      | 0.8   | 0.2       |
| Fine sand     | 16      | 19                | 1       | 35      | 5             | 12            | 12            | 48                   | 240 | 2      | 0.5   | 0.2       |

3.3. The Combination of Parameters

The influencing factors of the inner braces include compressive stiffness $E_iA_i$ of the single support, the number of the support $N$, the vertical average spacing $S_v$, the horizontal average spacing $S_h$. Since the depth of foundation pit is constant, there is correlation between the number of support $N$ and the vertical average spacing $S_v$ ($H_e=NS_v$). So when the number of support $N$ changes, the vertical average spacing $S_v$ of the support changes accordingly. Therefore, only one of factors $N$ and $S_v$ need to be studied. This paper focus on the number of support $N$. The factors of soil in the passive zone include the deformation modulus $E_s$ of the soil and the insertion ratio $h_d/H$ of the retaining wall. The factors of the retaining wall are mainly the elastic modulus $E_w$ and the thickness (determining the moment of inertia $I$ of the retaining wall). The retaining wall of the early metro station generally adopts C30 concrete and the change of modulus of elasticity is small. So it is regards as a fixed value in this article.

The influencing factors $N$, $S_v$, and $E_iA_i$ all take three different values. $N$ is set to be 3, 4, and 5, respectively, and the corresponding average vertical spacing $S_v$ of the support is about 5m, 4m, 3m. $S_h$ is set to be 3m, 4.5m, and 6m, respectively. $E_iA_i$ is taken to be 0.65, 1, and 1.35 times of the basic
model parameters, respectively. $h_d/H$ takes 4 different levels, which are 0.6, 0.8, 1, 1.2 times of the basic model parameters. $E_s$ takes 3 different values, which are 1, 2, and 3 times of the basic model respectively. The parameter $E_s$ doubling indicates the soil reinforcement in the passive zone below the bottom of the pit. Corresponding to the thickness of 0.6m, 0.8m, 1m and 1.2m of diaphragm wall, the horizontal bending stiffness $E_wI$ takes 4 different values respectively. The summary of the parameters of each influencing factor is shown in Table 2.

| $E_wI$($kNm^2$) | $EA_i$($kN$) | $N$ | $S_h$(m) | Insertion ratio | Reinforcement coefficient of $E_s$ |
|------------------|-------------|-----|-----------|----------------|------------------|
| 5.4e5            | 2.06e6      | 3   | 3         | 0.6            | 1                |
| 1.28e6           | 6.26e6      | 4   | 4.5       | 0.8            | $\frac{1}{2}$    |
| 2.5e6            | 4.28e6      | 5   | 6         | 1              | $\frac{1}{2}$    |
| 4.32e6           |             |     |           | 1.2            | 3                |

Note: Bold and supplemented by underlined values is parameters of basic model.

3.4. The Analysis of Calculation Result

Based on the finite element calculation results under the above combination of parameters, the curve between normalized maximum horizontal deformation value $\frac{\delta_{max}}{H_e}$ of the retaining wall and the SS synthesis stiffness, Clough synthesis stiffness, MVSS synthesis stiffness are plotted. The “$\frac{\delta_{max}}{H_e}$ ~ SS synthesis stiffness” curve is compared and analyzed with the curve of “$\frac{\delta_{max}}{H_e}$ ~ Clough synthesis stiffness” and “$\frac{\delta_{max}}{H_e}$ ~ MVSS synthesis stiffness”. In the MVSS synthesis stiffness expression, the factor of foundation reinforcement is based on the deep foundation pit of the metro station in Hangzhou area. It needs to be determined by the combination of the finite element calculation results and engineering experience and not easy to take values under the geological conditions of this paper. Therefore, the influence of foundation reinforcement is not considered when calculating the synthesis stiffness of MVSS.
Figures 2-4 show the relationship between the normalized maximum horizontal displacement value $\delta_{\text{max}}/H_e$ of the retaining wall and Clough synthesis stiffness, MVSS synthesis stiffness and SS synthesis stiffness respectively, where the $\delta_{\text{max}}$ indicates the maximum horizontal displacement of the retaining wall. It can be seen that the $\delta_{\text{max}}/H_e$ is consistent decrease with the increase of the synthesis stiffness from the above three graphs. When the synthesis stiffness increases to a certain value, the change of $\delta_{\text{max}}/H_e$ is small and the curve tends to be stable. It indicates that increase the synthesis stiffness.
stiffness continuously may have little effect on the restriction of horizontal deformation of the retaining wall. Comparing Figure 3 with Figure 2, it can be seen that the fitting degree increases from 0.4411 to 0.7817. This is due to the MVSS synthesis stiffness considers the average horizontal spacing of support in comparison with the Clough synthesis stiffness. Comparing Figure 4 with Figure 3, it can be seen that the curve fitting effect is further improved, and the fitting degree is increased from 0.7817 to 0.9034. This is due to the SS synthesis stiffness considers the influence of the soil in the passive zone which is defined clearly. It can be seen from the above analysis that the curve fitting degree is best when using the synthesis stiffness expression SS, which indicates that considering the influence of various factors on the synthesis stiffness of the supporting system is necessary and verifies the rationality of the SS synthesis stiffness.

4. Case Histories

In order to illustrate the engineering applicability of the SS synthesis stiffness expression, the curve fitting expression of $\delta_{\text{max}}/H_e \sim \text{SS synthesis stiffness}$ is used. The six foundation pit engineering of subway stations in Shanghai is taken for verification. The depth of these foundation pit is about 16m and the inner braces are steel braces. The relevant parameters of each foundation pit are shown in Table 3.

Table 3. Extracted case history in Shanghai.

| Station             | He  | H   | E_vl | Specification | N   | $\delta_{\text{max}}$ | $\delta_{\text{max}}/H_e$ |
|---------------------|-----|-----|------|--------------|-----|-----------------------|---------------------------|
| Jiangsu [7]         | 15  | 28  | 5.40e5 | Φ580×12     | 4   | 2.85                  | 70                        | 0.47                      |
| Hengshan [8]        | 14.7| 27.45 | 5.40e5 | Φ609×11     | 5   | 3                      | 59                        | 0.40                      |
| Zhongshanbei [9]    | 16  | 26  | 5.40e5 | Φ609b       | 4   | 3                      | 86                        | 0.54                      |
| Fuxing [10]         | 16  | 27.3| 1.28e6 | Φ609b       | 4   | 3                      | 42.7                      | 0.27                      |
| Tiyuguan [11]       | 14.6| 26.44| 1.28e6 | Φ580b       | 3   | 5.7                    | 50                        | 0.34                      |
| Zhongyanggongyuan [12]| 15  | 26.7| 5.40e5 | Φ609b       | 4   | 3                      | 50                        | 0.33                      |

a: The average horizontal spacing is assumed to be 3m. b: It is assumed that the steel braces have the equal wall thickness if the diameter is consistent with reference to the case of Jiangsu and Hengshan.

Figure 5 shows the comparison between the calculated curve and the measured data. It can be seen that the measured data are distributed on both sides of the calculated curve. The calculated curve is basically consistent with the measured data. The calculated values are all in good agreement with the measured data and the error is all within 0.1% $\delta_{\text{max}}/H_e$. The difference between calculated values and measured data maybe caused by two reasons. Firstly, the deformation modulus of the soil is not easy to determine after the reinforcement. In the calculation of the SS comprehensive stiffness value, only...
the undisturbed soil below the bottom of the pit is considered. The effect of strengthening constraint caused by soil reinforcement is not considered. Underestimating the SS synthesis stiffness results the calculated value is larger than the actual measured value. Secondly, it may cause by non-standard construction. Numerical simulation construction is easy to control. However, the actual construction process is complicated. Irregular construction such as over excavation and support lag erection happen often. It may cause the deformation of foundation pit to increase, which indicates the measured data larger.

5. Conclusion
(1) Based on the Clough synthesis stiffness and MVSS synthesis stiffness, the SS synthesis stiffness expression is proposed. Comparing with the other synthesis stiffness, it considers different factors such as retaining wall of foundation pit, inner braces and passive soil below the bottom of the pit comprehensively. And the factor of the soil in the passive zone below the bottom of the pit is defined clearly.

(2) Based on the finite element calculation results, the SS synthesis stiffness expression is compared with Clough synthesis stiffness and MVSS synthesis stiffness. The SS synthesis stiffness expression is proved to be more rationality.

(3) Based on the SS synthesis stiffness expression, the estimated maximum horizontal deformation of six deep excavation of metro stations in Shanghai is carried out. It is found that the calculation result is consistent with the measured data well. The engineering applicability of SS comprehensive stiffness is verified.

Based on the synthesis stiffness expression SS, combined with the finite element calculation, the formula for calculating the maximum horizontal deformation of the retaining wall can be obtained. In processes of design on other similar engineering, the maximum horizontal deformation of retaining structure can be quickly estimated according to the formula when different support systems is selected. So the complex calculation process of the finite element for each supporting system can be avoided. It is convenient to compare different support systems and select the best support systems during the design phase.

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