Superfluidity in sympathetic cooling with atomic Bose condensates

E. Timmermans and R. Côté
Institute for Atomic and Molecular Physics
Harvard-Smithsonian Center for Astrophysics
60 Garden Street
Cambridge, MA 02138
(January 8, 2022)

Abstract

The dynamical structure of an atomic Bose-Einstein condensate limits the efficiency of the condensate in cooling slow impurity atoms. To illustrate the point, we show that an impurity atom moving in a homogeneous zero-temperature condensate is not scattered incoherently if its velocity is lower than the condensate sound velocity $c$, limiting cooling to velocities $v \geq c$. This striking effect is an expression of superfluidity and provides a direct means to detect the fundamental property of superfluidity in atomic condensates. Furthermore, we show that the fermionic lithium-isotope, $^6$Li, is a reasonable candidate for sympathetic cooling by a $^{23}$Na-condensate.

PACS numbers(s):03.75.Fi, 05.30.Jp, 32.80Pj, 67.90.+z
The significance of the observation of Bose-Einstein condensation in alkali-atom traps [1] can hardly be overstated: these experiments represent the first explorations into the realm of dilute atomic quantum gases. It is thus unfortunate that the same technology is not expected to cool fermionic atoms into the degenerate regime. The problem stems from the antisymmetry which forbids $s$-wave scattering of indistinguishable fermions, implying a vanishing thermalization rate in evaporative cooling at low temperatures [2]. To overcome this difficulty, the JILA-collaboration proposed an interesting alternative [3]: the fermions can be cooled by bringing them in contact with an atomic condensate. This scheme of sympathetic cooling would be an interesting and important application of BEC, cooling not only fermionic alkali atoms, but any gas that cannot be cooled by means of the traditional cooling techniques, such as inertial gas atoms or perhaps even molecules [4].

In this letter, we discuss the efficiency of such cooling schemes [5]. Whereas one generally expects sympathetic cooling to be most efficient with a cooling system of lowest possible temperature, we point out that this is not necessarily true for a Bose condensate. In fact, we show that for a zero-temperature condensate, the cooling of a low-density gas of ‘impurity atoms’ (distinguishable from the condensate atoms) exhibits a limit due to the superfluidity of the condensate: impurity atoms slower than the condensate sound velocity are not scattered incoherently and cannot be cooled down to lower velocities. At finite temperatures the condensate contains a normal fluid fraction and the impurity atoms cool down further, but at a reduced rate. This aspect of sympathetic cooling provides the opportunity to observe the fundamental property of superfluidity.

At zero temperature the condensate scatters impurity atoms by creating quasi-particles. In a condensate of bosons with mass $m_B$ and density $n_B$, the energy dispersion of the quasi-particles is $\omega_q = cq\sqrt{1 + (q/k_c)^2}$, where $k_c = \sqrt{16\pi n_B a}$ is the inverse coherence length and $c = k_c/2m_B$ the condensate sound velocity. In the long wavelength limit, $q \ll k_c$, the momentum region of importance in describing slow impurity atom scattering, $\omega_q \simeq cq$ is phonon-like. We can then repeat Landau’s argument for the absence of resistance in superfluid flow, which translates here into the absence of incoherent scattering for impurity
atoms moving at a velocity \( v < c \). The argument is based on the observation that energy and momentum cannot be simultaneously conserved if \( v < c \). To see this [7], consider the energy transfer \( \Delta E = c \sum_n |q_n| \), where the \( q_n \)-vectors represent the momenta of the phonons created in the scattering, giving a total momentum transfer \( q = \sum_n q_n \). From the energy difference experienced by the scattered impurity atom of mass \( m_I \), \( \Delta E = v \cdot q - q^2 / 2m_I < vq \) and since \( q \leq \sum_n |q_n| = \Delta E / c \), we find that \( \Delta E < (v/c)\Delta E \), which cannot be satisfied if \( v < c \).

To obtain a quantitative understanding of BEC sympathetic cooling, we discuss a somewhat idealized model of a homogeneous zero-temperature BEC, cooling an impurity-atom system of such low density that 1. the interactions between impurity atoms can be neglected, 2. the impurity system remains non-degenerate throughout the cooling process and 3. we can neglect the heating of the condensate. The occupation number \( n_k \) of the plane wave state of momentum \( k \) evolves in time according to a simple Boltzmann equation:

\[
\dot{n}_k = \sum_q \{-w(k; k - q) n_k + w(k; k + q) n_{k+q}\},
\]

where \( w(k_{in}; k_f) \) is the rate for scattering of an impurity atom from an initial momentum \( k_{in} \) to momentum \( k_f \). The \( w \)-rate is proportional to the dynamical structure of the scattering (or cooling) system. We derive this result for an impurity atom that interacts weakly with the bosons by means of a pseudo-potential, \( H' = \lambda \sum_{i=1}^N \delta(r - r_i) \), where \( r \) represents the impurity atom position and \( r_i \) the position of the \( i \)-th boson, and where \( \lambda = 2\pi a_{IB}/\mu \), with \( a_{IB} \) the scattering length for the impurity-boson scattering and \( \mu \) the reduced mass, \( \mu^{-1} = m_B^{-1} + m_I^{-1} \). The interaction matrix element between the initial state, \( |\Psi_0, k_{in}\rangle \equiv |\Psi_0\rangle \otimes |k_{in}\rangle \), where \( |\Psi_0\rangle \) denotes the BEC ground-state, and the final state \( |\Psi_f, k_f\rangle \equiv |\Psi_f\rangle \otimes |k_f\rangle \), is then equal to

\[
\langle \Psi_f, k_f|H'|\Psi_0, k_{in}\rangle = \frac{\lambda}{V} \int d^3r_1 \ldots d^3r_N \Psi_f^*(r_1, \ldots, r_N) \times \sum_j \exp(iq \cdot r_j) \Psi_0(r_1, \ldots, r_N)
\]

\[
= \frac{\lambda}{V} \langle \Psi_f|\rho_q|\Psi_0\rangle ,
\]

(2)
where q is the momentum transfer $k_{in} - k_f$, $\rho_q$ the Fourier transform of the particle density and V the macroscopic volume of the homogeneous system. Representing the initial and final state energies by $E_0 + \epsilon(k_{in})$ and $E_f + \epsilon(k_f)$, we obtain for the scattering rate in a Fermi-Golden rule calculation:

$$w(k_{in}; k_f) = 2\pi \sum_{|\Psi_f\rangle} |\langle \Psi_f, k_f | H'| \Psi_0, k_{in} \rangle|^2 \times \delta(E_f + \epsilon(k_f) - [E_0 + \epsilon(k_{in})])$$

$$= 2\pi \left( \frac{\lambda}{V} \right)^2 S(q, [\epsilon(k_{in}) - \epsilon(k_f)]) , \quad (3)$$

where we introduced the dynamical structure factor $S(q, \omega) = \sum_{|\Psi_f\rangle} |\langle \Psi_f | \rho_q | \Psi_0 \rangle|^2 \delta(E_f - E_0 - \omega)$. In the thermodynamic limit, $\sum_q \rightarrow V/(2\pi)^3 \int d^3q$, the Boltzmann-equation (1) takes on the following form:

$$\dot{n}_k(t) = \left( \frac{\lambda}{2\pi} \right)^2 \left[ -\int d^3q \ s(q, v \cdot q - q^2/2m_I)n_k(t)$$

$$+ \int d^3q \ s(q, v \cdot q + q^2/2m_I) n_{k+q}(t) \right] , \quad (4)$$

where $s$ denotes the dynamical structure factor density, $s(q, \omega) = [S(q, \omega)/V]$. For a zero-temperature dilute condensate $s$ is approximated accurately by $\Box$:

$$s(q, \omega) \approx n_B \frac{q^2/2m_B}{\omega_q} \delta(\omega - \omega_q) , \quad (5)$$

describing the creation of a single quasi-particle of momentum $q$ and energy $\omega_q$ (defined above). Although the above derivation assumes weak interaction between the impurity and the condensate (reasonable if $v > c$), the rigorous derivation of the quantum Boltzmann equation $\Box$ shows that Eq.(4) has a broader range of validity.

With Eqs.(4) and (5), we calculate the rate $dQ/dt = -\int d^3k \epsilon(k) \dot{n}_k$ at which energy is transferred from the impurity atoms to the condensate,

$$\frac{dQ}{dt} = \frac{a^2 I_B}{\mu^2} \int d^3k n_k \int d^3q \omega_q s(q, [\epsilon(k) - \epsilon(k - q)]) , \quad (6)$$

where $n_k$ is normalized so that $\int d^3k n_k$ equals the number of impurity atoms. Performing the remaining $q$-integration, we find
\[
\frac{dQ}{dt} = 4\pi a_{IB}^2 n_B \frac{2\mu^2}{m_I m_B} \int d^3k \, n_k \frac{k^2}{2m_I} F_C(k),
\]

where \( F_C(k) = 0 \) if \( v < c \), and
\[
F_C(k) = \left( \frac{1 - \sqrt{1 - \left[ 1 - \left( \frac{m_I}{m_B} \right)^2 \right] \left[ 1 - \left( \frac{v}{c} \right)^2 \right]} \left[ 1 - \left( \frac{m_I}{m_B} \right) \right]} \right)^4
\]
if \( v > c \). A classical system of noninteracting particles of mass \( m_B \) has a dynamical structure factor density \( s(q, \omega) = n_B \delta(\omega - q^2/2m_B) \), giving the above result with \( F_C \to 1 \). Thus the \( F_C(v) \)-function, shown in Fig.1, corrects for the condensate dynamical structure. At \( v \gg c \), \( F_c(v) \approx 1 \) and the energy transfer rate for cooling fast impurity atoms by a condensate is the same as the rate for cooling by noninteracting atoms at rest. At \( v \geq c \), the condensate dynamical structure factor decreases the rate of energy transfer until at \( v < c \) the impurity atoms are not scattered and no transfer of energy takes place, confirming the picture derived from the Landau argument above.

Even if interactions between the impurities cannot be neglected (giving impurity-impurity collision terms in the Boltzmann equation), the heat transfer rate of Eq.(7) is correct, provided the impurity system is non-degenerate and interacts weakly with the condensate. In the limit of strong impurity-impurity interactions, these interactions ‘thermalize’ the distribution \( n_k \) during the cooling process and \( n_k(t) \) remains Maxwellian, \( n_k(t) \sim \exp[-k^2/2m_I k_B T(t)] \), where \( k_B \) denotes the Boltzmann constant. We can then calculate the rate of temperature decrease in accordance with the heat transfer rate, \( dT/dt \approx -[dQ/dt]/C \), where \( C \) is the heat capacity, \( C = \frac{d}{dt} \int d^3k \, n_k \epsilon(k) = \int d^3k \, \epsilon^2(k) n_k / k_B T^2 \).

This gives \( dT/dt \approx -T/\tau_c \), where the cooling rate \( \tau_c^{-1} \) depends on the thermal velocity, \( v_T = \sqrt{2m_B k_B T} \), as \( \tau_c^{-1} = (\mu^2/m_I m_B) (16/15\sqrt{\pi}) \times 4\pi n_B a_{IB}^2 v_T F_S(v_T) \sim (\mu^2/m_I m_B) \times 4\pi n_B a_{IB}^2 v_T F_S(v_T) \), where the \( F_S \)-factor corrects for the condensate structure factor, expressing the corresponding suppression of the cooling rate, \( F_S = \int d^3k \, n_k k^3 F_C(k) / \int d^3k \, n_k k^3 \) (see Fig.1).

However, the interaction with a dense condensate generally modifies the condensate.
We return, therefore, to the low-density impurity system where $n_k(t)$ does not remain Maxwellian (although the above cooling rate is still a reasonable estimate) and we solve the Boltzmann equation numerically. In Fig. 2, we show the time evolution of an isotropic impurity distribution that is cooled by a zero-temperature condensate. The distribution starts out as a Maxwellian with $v_T = 3c$, and progressively narrows in velocity-space but does not converge to zero velocity – instead the position of its maximum converges to a value slightly below $c$: the signature of superfluidity. The distributions are plotted at times $t = 0.4n\tau$, $n = 0, \ldots, 10$, where $\tau^{-1}$ is the cooling rate for $v_T = c$, $\tau^{-1} = 4\pi a_B^2 n_B c (\mu^2 / m_I m_B)$. The mass-ratio of the impurity and condensate atoms in the calculation is $6/23$, corresponding to the fermionic $^6$Li-isotope cooled by a $^{23}$Na-condensate.

The choice of the elements and their atomic states is crucial in sympathetic cooling since processes such as spin-flip scattering limit the lifetime of the impurity system in contact with the condensate. $^{23}$Na, the atom used in the high-density BEC-systems created at MIT, and $^6$Li are promising choices: their atomic properties are well understood and the spin-flip rates relatively small.

The magnetic traps in which BEC has been realized confine only a fraction of the hyperfine states, namely the low magnetic field seekers. At small field intensities, the low field seeker states are $(f \> m) = (3 \> 3), (3 \> 1)\!$ and $\! (1 \> -1)$ for $^6$Li, and $(2 \> 2), (2 \> 1)$ and $(1 \> -1)$ for $^{23}$Na. In the elastic approximation [10], also known as the Degenerate Internal States (DIS) approximation [11], one describes the scattering of atoms in these hyperfine states in terms of the singlet and triplet scattering amplitudes. At the relevant low energies, the scattering is dominated by the $s$-wave contribution and the triplet (singlet) phase shift $\delta_T(S)$ can be represented by $-ka_T(S)$, where $k$ is the relative momentum and $a_T(S)$ is the scattering length for the triplet (singlet) scattering (see Table I). The scattering length for elastic processes (responsible for cooling and thermalisation) is then $a \simeq P_S a_S + P_T a_T$ where $P_S$ and $P_T$ represent the probability that the scattering product is in the singlet and triplet state. For example, $^{23}$Na-$^{23}$Na scattering in $(1 -1)\!+\!(1 -1)$ gives $a \simeq \frac{13}{16} a_T + \frac{3}{16} a_S = 69 a_0$, in good agreement with $86^{+66}_{-23} a_0$ [12] and close to $52 \pm 5a_0$ [13]. The spin-flip cross section is
\[ \sigma_{sf} = M_{sf} \pi (a_T - a_S)^2, \tag{9} \]

where \( M_{sf} \) is a coefficient that depends on the specific states involved in the process. This multiplicative factor vanishes if the initial state of the scatterers is of a pure triplet or singlet character, implying that it is possible to choose the atomic states so as to avoid spin-flip processes altogether. In cases where spin-flip occurs, their rate may still be reduced if the singlet and triplet scattering lengths are nearly equal, as for the \(^{87}\text{Rb}\)-condensates observed at JILA \([19,20]\).

For \(^6\text{Li} - ^6\text{Li}\) collisions, only collisions between \((f\ m)\) states \((\frac{3}{2}\ rac{3}{2}) + (\frac{3}{2}\ rac{3}{2}), \ (\frac{3}{2}\ rac{1}{2}) + (\frac{3}{2}\ rac{1}{2}),\) and \((\frac{1}{2} - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{2})\) are without spin-flip. Because of the fermionic nature of \(^6\text{Li}\), these are all forbidden to \(s\)-wave collisions (no cooling possible). For \(^{23}\text{Na} - ^{23}\text{Na}\) collisions, the only scattering products not leading to spin-flip decay are \((2\ 2) + (2\ 2)\) and \((1\ -1) + (1\ -1)\). The collision channel \((2\ 2) + (2\ 2)\) is purely elastic with \(a\) given by \(a_T\) in Table I. \((1\ -1) + (1\ -1)\) is a mixture of both singlet and triplet and many exit channels could lead to trap loss. However, at low temperature, all these channels are closed.

For \(^{23}\text{Na} - ^6\text{Li}\) collisions, the combinations of states without spin-flip are \((\frac{3}{2}\ rac{3}{2}) + (2\ 2)\) and \((\frac{1}{2} - \frac{1}{2}) + (1\ -1)\). Only elastic collisions are possible for \((\frac{3}{2}\ rac{3}{2}) + (2\ 2)\) but for \((\frac{1}{2} - \frac{1}{2}) + (1\ -1)\), many decay channels exist, although not physically accessible at low temperatures.

The scattering lengths for \(^6\text{Li}\) reported in Table I were derived by Abraham et al. \([14]\) and those for \(^{23}\text{Na}\) were calculated by Côté and Dalgarno \([15]\). They are in agreement with recent estimates \([12,13]\). For the mixed case \(^{23}\text{Na} - ^6\text{Li}\), we computed the scattering lengths using singlet and triplet model potentials formed by the extension of \textit{ab initio} data with long-range tails. The \textit{ab initio} data originate from Schmidt-Mink et al. \([16]\) and extend to large distances \((30a_0)\). Beyond the \textit{ab initio} region, the potential is described by the dispersion coefficients of Marinescu et al. \([17]\) and takes the form \(-C_6/r^6 - C_8/r^8 - C_{10}/r^{10} \mp Ar^\alpha e^{-\beta r}\). The \(\mp\) sign of the exchange term stands for the singlet and triplet potential respectively.

The parameters were computed using expressions given by Smirnov and Chibisov \([18]\) and take on the following values (in atomic units): \(A = 0.0124, \alpha = 4.626\) and \(\beta = 1.2445\). The
two regions were smoothly joined using a cubic spline fit.

The spin-flip rate coefficients (if spin-flip scattering occurs and \( M_{sf} \sim 1 \)) can be estimated with the values in Table I by \( R = \langle v \sigma_{sc} \rangle \) where \( v \) is the velocity of the pair of atoms undergoing a spin-flip, and where the average is taken over a Maxwellian velocity distribution. For \(^6\text{Li}-^6\text{Li}\), \( R \) is quite large due to its large scattering length, and still sizeable for \(^{23}\text{Na}-^{23}\text{Na}\), but one order of magnitude smaller for \(^{23}\text{Na}-^6\text{Li}\) (see Table I). This situation is similar to the case of \(^{87}\text{Rb}\), where \( R \sim 2.2 \times 10^{-14} \text{cm}^3/\text{s} \) and sympathetic cooling of two different hyperfine states has been observed \([3]\), presumably because the singlet and triplet scattering lengths are nearly equal. This implies that the four other possible collision channels for \(^{23}\text{Na}-^6\text{Li}\), \((\frac{3}{2} - \frac{1}{2}) + (2 \ 2), (\frac{1}{2} - \frac{1}{2}) + (2 \ 2), (\frac{3}{2} \frac{3}{2}) + (1 \ -1)\) and \((\frac{3}{2} \frac{1}{2}) + (1 \ -1)\), could lead to sympathetic cooling, as long as the \(^6\text{Li}\) states are not mixed (otherwise spin-flip among \(^6\text{Li}\) states will take place). In that case, sympathetic cooling is feasible if the cooling rate \( \tau^{-1}_c \) exceeds the rate of spin-flip scattering \( n_B R \) or

\[
M_{sf} \left( \frac{a_{1B}^T - a_{IB}^S}{a_{1B}} \right)^2 \left( \frac{\mu^2}{m_1 m_B} \right) \ll 1,
\]

where the \( IB \) subscripts denote that the scattering lengths are calculated for impurity-boson scattering.

Finally, we remark that the best option for sympathetic cooling of \(^6\text{Li}\) by \(^{23}\text{Na}\) in the \((1 \ -1)\) state (such as in the MIT condensate) is to trap \(^6\text{Li}\) in the state \((\frac{1}{2} - \frac{1}{2})\); this ensures a small trap loss from spin-flip. On the other hand, a condensate of \(^{23}\text{Na}\)–atoms in the \((2 \ 2)\)-state with \(^6\text{Li}\) in the \((\frac{3}{2} \frac{3}{2})\)-state is the best option since no spin-flip is possible. In both cases, there is a sizeable scattering length, \( a_{1B} \), hence efficient sympathetic cooling.

In conclusion, we have shown that the superfluidity of atomic Bose condensates limits their efficiency for sympathetic cooling. This effect provides a possibility for direct observation of the superfluidity of the dilute atomic trap condensate systems. Furthermore, we have discussed the feasibility of such sympathetic cooling, and its dependence on the atomic scattering parameters. For the special case of cooling \(^6\text{Li}\) and \(^{23}\text{Na}\), we showed that sympathetic cooling is possible and we discussed the best options for the choice of hyperfine states...
to be trapped.

The work of both authors was supported by the NSF through a grant for the Institute for Atomic and Molecular Physics at Harvard University and Smithsonian Astrophysical Observatory.
REFERENCES

[1] K.B. Davis et al., Phys. Rev. Lett. 75, 3969 (1995); M.H. Anderson et al., Science, 269, 198 (1995); C. C. Bradley et al., Phys. Rev. Lett. Phys. Rev. Lett. 75, 1687 (1995).

[2] J.M.V.A. Koelman et al., Phys. Rev. Lett. 59, 678 (1987).

[3] C.J. Myatt et al., Phys. Rev. Lett. 78, 586 (1997).

[4] Conceptually the technique is related to buffer gas cooling, see J. Kim et al., Phys. Rev. Lett. 78, 3665 and references therein.

[5] A master equation approach to the problem was proposed by M. Lewenstein et al., Phys. Rev. A, 51, (1995).

[6] L.D. Landau, J. Phys. USSR 5, 71 (1941).

[7] K. Huang, Statistical Mechanics, (John Wiley, New York, 1987).

[8] D. Pines and P. Nozieres, The Theory of Quantum Liquids, Vol. I & II, Addison-Wesley Publishing Company, Inc. (1989).

[9] P. Danielewicz, Ann. Phys. (N.Y.) 152, 239 (1981); D. C. Langreth and J. W. Wilkins, Phys. Rev. B 6, 3189 (1972).

[10] A. Dalgarno and M.R.H. Rudge, Proc. Roy. Soc., A, 286, 519 (1965).

[11] B.J. Verhaar et al., Phys. Rev. A 35, 3825 (1987).

[12] A.J. Moerdijk and B.J. Verhaar, Phys. Rev. A 51, R4333 (1995).

[13] E. Tiesinga et al., J. Res. Natl. Inst. Stand. Technol. 101, 505 (1996).

[14] E.R.I. Abraham et al., Phys. Rev. A 55, R3299 (1997).

[15] R. Côté and A. Dalgarno, Phys. Rev. A 50, 4827 (1994).

[16] I. Schmidt-Mink, W. Müller, and W. Meyer, Chem. Phys. Lett. 112, 120 (1984).
[17] M. Marinescu, H.R. Sadeghpour, and A. Dalgarno, Phys. Rev. A 49, 982 (1994).

[18] B.M. Smirnov and M.I. Chibisov, JETP 21, 624 (1965).

[19] P.S. Julienne et al. Phys. Rev. Lett. 78, 1880 (1997).

[20] B.D. Esry et al. Phys. Rev. Lett. 78, 3594 (1997).
### TABLE I. Singlet and triplet scattering lengths in Bohr radii $a_0$, for isotopically pure and mixed alkali gases.

| $T$ (K) | $^6$Li-$^6$Li | $^{23}$Na-$^{23}$Na | $^{23}$Na-$^6$Li |
|--------|---------------|---------------------|-----------------|
| $10^{-7}$ | $1.1 \times 10^{-9}$ | $2.1 \times 10^{-13}$ | $1.2 \times 10^{-14}$ |
| $10^{-6}$ | $3.6 \times 10^{-9}$ | $6.8 \times 10^{-13}$ | $3.8 \times 10^{-14}$ |
| $10^{-5}$ | $1.1 \times 10^{-8}$ | $2.1 \times 10^{-12}$ | $1.2 \times 10^{-13}$ |

### TABLE II. Spin-flip rate coefficients $R$ for collisions between $^6$Li and $^{23}$Na atoms in units of cm$^3$/s.
**Figure Captions**

**Fig.1**: Plot of the $F_C(v)$ (full line) and $F_S(v_T)$ (dotted line) functions that quantify the suppression of the BEC cooling efficiency as compared to cooling by atoms at rest (for $m_I/m_B = 6/23$). $F_C(v)$ represents the suppression of heat transfer and $F_S(v_T)$ gives the cooling rate suppression of a Maxwellian impurity distribution with thermal velocity $v_T$.

**Fig.2**: Velocity distribution of the impurity atoms as they are cooled by a zero-temperature BEC, plotted at $t = 0.4n\tau, n = 0, 1, \ldots, 10$, $\tau = [4\pi a_{1B}^2 n_B c (\mu^2/m_I m_B)]^{-1}$. The initial distribution is a Maxwellian with $v_T = 3c$. Note that the distribution does not converge to a zero-temperature Maxwellian, but the position of its maximum converges to a value slightly below $c$: the signature of superfluidity.
Impurity Velocity (in units of c)

- $F_S(v)$
- $F_C(v_T)$
