Searching for New Physics via CP Violation in $B \to \pi \pi$

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We show how $B \to \pi \pi$ decays can be used to search for new physics in the $b \to d$ flavour-changing neutral current. One needs one piece of theoretical input, which we take to be a prediction for $P/T$, the ratio of the penguin and tree amplitudes in $B^0_d \to \pi^+\pi^-$. If present, new physics can be detected over most of the parameter space. If $\alpha (\phi_2)$ can be obtained independently, measurements of $B^+ \to \pi^+\pi^0$ and $B^0_d/\bar{B}^0_d \to \pi^0\pi^0$ are not even needed.

$B$-factory measurements have provided us with the first hints for CP violation in the $B$ system. We all expect that, with time, it will be possible to measure CP violation in a wide variety of $B$ decays. And when the dust settles, we all hope that these measurements will reveal the presence of physics beyond the standard model (SM).

New physics can alter CP-violating rate asymmetries in $B$ decays principally by affecting those amplitudes which in the SM are at the one-loop level. These new effects can be separated into two classes. New physics can affect the $b \to d$ flavour-changing neutral current (FCNC), which includes $B^0_d-\bar{B}^0_d$ mixing and $b \to d$ penguin amplitudes. It can also affect the $b \to s$ FCNC ($B^0_s-\bar{B}^0_s$ mixing, $b \to s$ penguin amplitudes).

There are several clean, direct tests for new physics in the $b \to s$ FCNC. For example, the decays $B^\pm \to D K^{\pm}$ and $B^0_s(t) \to D^\pm K^\mp$ both probe the CP phase $\gamma$ ($\phi_3$) in the SM. A discrepancy between these two CP asymmetries will point clearly to the presence of new physics in $B^0_s-\bar{B}^0_s$ mixing. Similarly, if it is found that the CP asymmetries in $B^0_s(t) \to \psi K_s$ and $B^0_{d}(t) \to \phi K_s$ are unequal (they both measure $\beta$ ($\phi_1$) in the SM), this will indicate new physics in the $b \to s$ penguin amplitude.

Finally, the decay $B^0_s(t) \to \psi \phi$ is expected to have a tiny CP asymmetry in the SM. If this turns out not to be the case, we will know that there is new physics in $B^0_s-\bar{B}^0_s$ mixing.

This then begs the question: are there clean, direct tests for new physics in the $b \to d$ FCNC? At first glance, the answer appears to be ‘yes’. Assuming that the $b \to d$ penguin amplitude is dominated by the exchange of an internal $t$ quark, its weak phase is $-\beta$ in the Wolfenstein parametrization. One then predicts that the CP asymmetry in the decay $B^0_d(t) \to K^0\bar{K}^0$ vanishes, while $B^0_s(t) \to \phi K_s$ measures $\sin 2\beta$. Any deviation from these predictions would indicate new physics in the $b \to d$ FCNC.

Unfortunately, the assumption of $t$-quark dominance of the $b \to d$ penguin amplitude is incorrect. The $u$- and $c$-quark contributions can be quite substantial, as large as 20–50% of the $t$-quark contribution. Thus, the weak phase of the $b \to d$ penguin is not $-\beta$, and the clean predictions for the asymmetries in $B^0_d(t) \to K^0\bar{K}^0$ and $B^0_s(t) \to \phi K_s$ are spoiled.

But this then raises a second question: can one isolate the $t$-quark contribution to the $b \to d$ penguin, and measure its weak phase? If so, then the comparison of this weak phase with that measured in $B^0_d(t) \to \psi K_s$ could reveal the presence of new physics. Unfortunately, as shown in Ref. 8, the answer to this question is ‘no’. It is not possible to isolate any single contribution to the $b \to d$ penguin. Thus, it is impossible to cleanly test for new physics in the $b \to d$ FCNC.

However, one can test for new physics if we make a single assumption about the theoretical parameters describing the decay. In this talk we describe how this can be applied to $B \to \pi \pi$, which, as is well known, suffers from penguin “pollution.” As we will see, the measurements of $B \to \pi \pi$, combined with a theoretical prediction for $P/T$, the ratio of penguin and tree amplitudes in $B^0_d \to \pi^+\pi^-$, allow one to probe new physics in the $b \to d$ penguin amplitude.

We begin with a brief review of $B \to \pi \pi$ decays. Recall that, in the Wolfenstein parametrization, the weak phase of $B^0_d-\bar{B}^0_d$ mixing is $-2\beta$. It is convenient to remove this phase by redefining the $B$ decay amplitudes:

$$A^f \equiv e^{i\beta} \text{Amp}(B^0_d \to f), \quad \bar{A}^f \equiv e^{-i\beta} \text{Amp}(\bar{B}^0_d \to f).$$  \hspace{1cm} (1)
With this convention, the time-dependent decay rate for 
\( B_d^0(t) \rightarrow \pi^+\pi^- \) takes the form
\[
\Gamma(B_d^0(t) \rightarrow \pi^+\pi^-) = e^{-\Gamma t} \left[ \frac{|A^+|^2 + |\bar{A}^+|^2}{2} \right. \\
+ \frac{|A^+|^2 - |\bar{A}^+|^2}{2} \cos(\Delta M t) \\
- \left. \text{Im} \left( A^+ \bar{A}^+ \right) \sin(\Delta M t) \right]
\] (2)

Thus, the measurement of the time-dependent decay rate 
\( B_d^0(t) \rightarrow \pi^+\pi^- \) allows the extraction of the following
three quantities:
\[
B^{\pm} = \frac{1}{2} (|A^+|^2 + |\bar{A}^+|^2), \\
A^{\pm}_{dir} = \frac{|A^+|^2 - |\bar{A}^+|^2}{|A^+|^2 + |\bar{A}^+|^2}, \\
2\alpha^{+}_{eff} = \text{Arg} \left( A^{+} \bar{A}^+ \right). 
\] (3)

In what follows, the direct CP asymmetry \( a^{+}_{dir} \) and
the indirect CP asymmetry \( 2\alpha^{+}_{eff} \) will be the key observables.
Similarly, one can obtain \( B^{00}_{dir} \) and \( \alpha^{0}_{dir} \) through
measurements of \( B^{0}_{dir}/B^{0}_{dir} \rightarrow \pi^0\pi^0 \), and \( B^{+0} \) from \( B^+ \rightarrow \pi^+\pi^0 \).

In addition to a tree amplitude, the decay \( B_d^0 \rightarrow \pi^+\pi^- \) also receives a contribution from a \( b \rightarrow d \) penguin amplitude.
Eliminating the \( c \)-quark piece of the penguin amplitude \( (V_{cb}^* V_{cd}) \using the unitarity
of the CKM matrix, in the SM one can write
\[
A^{+} = T e^{i\delta} e^{-i\alpha} + e^{i\delta_p} e^{-i\theta_{NP}}. 
\] (4)

Here \( \alpha \) is one of the three angles of the unitarity triangle,
\( \delta \) and \( \delta_p \) are strong phases, and \( P \) and \( T \) are defined
to be real, positive quantities. Note that the \( T \) amplitude
is not pure tree: it includes contributions from the \( u \)- and \( c \)-quark pieces of the \( b \rightarrow d \) penguin. Similarly, the
\( P \) amplitude is mostly the \( t \)-quark penguin contribution,
but includes a \( c \)-quark penguin piece. Note also that there
is no weak phase multiplying the \( P \) amplitude. This is because, in the SM, the weak phase of \( B^0_{dir}/B^{00}_{dir} \) mixing
cancels that of the \( t \)-quark piece of the \( b \rightarrow d \) penguin amplitude.

Now, it is well known that the isospin analysis of 
\( B \rightarrow \pi\pi \) decays enables one to remove the penguin pollution
and extract the CP phase \( \alpha \). However, it is also true,
though not as well known, that this same analysis allows one to obtain all of the theoretical parameters \( T \), \( P \), etc. In particular,
\[
r^2 = \frac{p^2}{T^2} = \frac{1 - \sqrt{1 - (a^{+}_{dir})^2 \cos(2\alpha - 2\alpha_{eff})}}{1 - \sqrt{1 - (a^{+}_{dir})^2 \cos(2\alpha_{eff})}}. 
\] (5)

If there is new physics in the \( b \rightarrow d \) penguin amplitude,
\( B_d^0 \rightarrow \pi^+\pi^- \) amplitude will be modified:
\[
A^{+} = T e^{i\delta} e^{-i\alpha} + e^{i\delta_p} e^{-i\theta_{NP}}. 
\] (6)

where \( \theta_{NP} \) represents the mismatch, due to the presence
of new physics, between the weak phase of \( B_d^0 \) mixing
and that of the \( t \)-quark piece of the \( b \rightarrow d \) penguin amplitude.
The expression for \( r^2 \) now reads
\[
r^2 \equiv \frac{p^2}{T^2} = \frac{1 - \sqrt{1 - (a^{+}_{dir})^2 \cos(2\alpha - 2\alpha_{eff})}}{1 - \sqrt{1 - (a^{+}_{dir})^2 \cos(2\theta_{NP} - 2\alpha_{eff})}}. 
\] (7)

From this expression it is clear that, given measurements
of \( a^{+}_{dir}, \alpha_{eff}, \) and \( 2\alpha \), along with a theoretical prediction
of \( P/T \), one can extract \( \theta_{NP} \). (Note that, in the above,
we have assumed that new physics affects only the weak
phase of the \( P \) piece of \( A^{+} \).) However, new physics could
also affect the magnitude of \( P/T \). For our purposes, since
we are looking for \( \theta_{NP} \not= 0 \), this distinction is unimportant:
if, in reality, \( \theta_{NP} = 0 \) but new physics has affected
the magnitudes of \( P \) and \( T \), this will still show up as an
effective nonzero \( \theta_{NP} \).

Of course, in practice, things are more complicated.
First, theory will not predict a specific value for \( P/T \),
but rather a range of values. For example, Fleischer and
Mannel give \( 0.7 \leq r \leq 0.23 \), while Gronau estimates
\( r = 0.3 \pm 0.1 \). In this study, we will take a very conservative
range:
\[
0.05 \leq r \leq 0.5 . 
\] (8)

Second, the formula for \( P/T \) depends on \( 2\alpha \). Where
does one get this CP phase? Obviously, if the isospin
analysis can be performed, one can obtain it in that way.
However, it may be very difficult to perform such an analysis,
in which case we will need to get \( 2\alpha \) from outside
of the \( B \rightarrow \pi\pi \) system. One possibility is to measure \( 2\alpha \)
using the Dalitz-plot analysis of \( B \rightarrow \rho\pi \) decays. Another possibility,
assuming that both \( \beta \) and \( \gamma \) have been measured,
is to use the relation \( \alpha = \pi - \beta - \gamma \), which holds
in the presence of new physics. The bottom line is that there
are a number of ways of getting \( 2\alpha \), and ideally we
will have information from all of these sources.

The third complication is the fact that all measurements
will be made with some error, and these errors
can mask the presence of a nonzero \( \theta_{NP} \). To approximate
this effect, we take \( 2\alpha \) to lie in a certain range, and
we consider two such illustrative choices:
\[
(a) \quad 120^\circ \leq 2\alpha \leq 135^\circ, \\
(b) \quad 165^\circ \leq 2\alpha \leq 180^\circ. 
\] (9)

The procedure is now nominally as follows: given measurements
of \( a^{+}_{dir} \) and \( 2\alpha_{eff} \), and assuming that \( 2\alpha \) lies
in range (a) or (b), we will see if \( \theta_{NP} = 0 \) gives \( r \) in the allowed range [Eq. (8)]. If not, this indicates the presence of new physics.

However, this does not take into account all information we have at our disposal. By the time \( B_d^0(t) \rightarrow \pi^+\pi^- \) is measured, we will have some knowledge of the branching ratios for \( B^+ \rightarrow \pi^+\pi^0 \) and \( B_d^0/B_d^+ \rightarrow \pi^0\pi^0 \). (Indeed, even today we already have upper limits on these quantities [8].) Thus, we can use isospin to test for new physics. That is, given a range of values for \( B^+/B^0 \) and \( \alpha^{dir} \), it must be possible to reproduce the measured value of \( 2\alpha \) using the isospin analysis. If not, this indicates the presence of new physics.

To include this constraint, we consider five scenarios for the allowed ranges of \( B^{0+}/B^{+-} \), \( B^{00}/B^{+-} \) and \( \alpha^{dir} \), shown in Table 1. Note that scenario A assumes that we have no knowledge of these quantities at all. Obviously this is totally unrealistic, since we already have upper limits on \( B^{0+}/B^{+-} \) and \( B^{00}/B^{+-} \). A more realistic case, which is roughly the situation as it exists today, is given in scenario B. Here, \( B^{00} \) is approximately equal to \( B^{0+} \), while there is an upper limit on \( B^{00}/B^{++} \) and nothing is known about \( \alpha^{dir} \). Finally, scenarios C, D and E are other hypothetical ranges for \( B^{00}/B^{+-} \), \( B^{00}/B^{+-} \) and \( \alpha^{dir} \).

In order to outline the region of parameter space where new physics can be found, we use the following procedure. In a given scenario, we generate values for \( \alpha^{dir} \) and \( 2\alpha_{eff} \) in the full allowed range, \( -1 \) to \( 1 \), and values for \( B^{0+}/B^{+-} \), \( B^{00}/B^{+-} \) and \( \alpha^{dir} \) in the specified range in the scenario. A total of \( 10^5 \) sets of values are generated. If a given set of values (i) reproduces the measured value of \( 2\alpha \) in the allowed range (a) or (b) [Eq. (8)] using isospin, and (ii) gives \( r^2 \) in the allowed theoretical range [Eq. (8)] for \( \theta_{NP} = 0 \), then it is consistent with the SM. If not, we conclude that new physics is present.

The results are shown in Fig. 1. The dark regions correspond to those values of \( \alpha^{dir} \) and \( 2\alpha_{eff} \) which are consistent with the SM. As one can see from these plots, there is a lot of white space. That is, in each scenario, there is a large region of \( \alpha^{dir} - 2\alpha_{eff} \) parameter space which corresponds to new physics. For example, con-
sider scenario B. Only about 1/8 of the entire parameter space is consistent with the SM. Thus, even if our knowledge of $B^{+0}/B^{+-}$, $B^{00}/B^{+-}$ and $a_{dir}^{00}$ does not improve in the future, we have a good chance of seeing new physics, should it be present, through the measurement of $B_d^0(t)\rightarrow\pi^+\pi^-$ alone, along with an independent determination of $2\alpha$.

To be fair, things are not quite this easy. The presence of discrete ambiguities in the extraction of $2\alpha_{eff}$ and in the calculation of $2\alpha$ via isospin can complicate matters. These complications can be minimized if we have a variety of independent determinations of $2\alpha$, as described earlier. For more details, we refer the reader to Ref. 9.

To summarize: while the presence of physics beyond the SM in the $b\rightarrow s$ FCNC is relatively easy to establish, the same is not true of the $b\rightarrow d$ FCNC – one always needs to add some theoretical information. In this talk we have described how to use the $B\rightarrow \pi\pi$ system to probe new physics in the $b\rightarrow d$ FCNC. Here one uses a prediction of the ratio $P/T$ as the theoretical input. As we have seen, there is a large region of parameter space where new physics can be found. Note that the measurements of $B^+\rightarrow \pi^+\pi^0$ and $B_d^0/\bar{B}_d^0 \rightarrow \pi^0\pi^0$ are not needed if $2\alpha$ can be obtained independently. Ideally, we will have information about $2\alpha$ from both independent sources and an isospin analysis.

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