Investigation of the pairing effect in \(^{10}\)B nucleus compared with \(^{10}\)Be and \(^{10}\)C nuclei by using the extended THSR wave function

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In order to study the nucleon-nucleon pairing effects in clustering nuclei, we formulate a superposed Tohsaki-Horiuchi-Schuck-Röpke (THSR) wave function, which includes both molecular-orbit and pairing configurations explicitly. With this new wave function, we investigate the abnormal deuteron-like \(pn\)-pairing effect in \(^{10}\)B with \(T=0\) and \(S=1\) (isoscalar) by comparing with isovector \(NN\) pairs (\(T=1, S=1\)) in \(^{10}\)Be and \(^{10}\)C. Energies are calculated for the ground states of \(^{10}\)Be, \(^{10}\)B and \(^{10}\)C nuclei, and the \(1^+_1\) excited state of \(^{10}\)B. These energies are essentially improved comparing with studies using previous version of THSR wave function. Further more, overlaps between the total wave function and the pairing component indicate that the \(NN\) pairing effect is more visible in \(^{10}\)B than in \(^{10}\)Be and \(^{10}\)C. By analyzing the energies and the overlaps between wave function components, we observe two different mechanisms enhancing the formation of deuteron-like pairs in \(^{10}\)B. We also discuss the pairing effect by showing average distances between components in each nucleus and density distributions of valence nucleons.

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I. INTRODUCTION

Investigating of nucleon-nucleon ($NN$) pairing effect is one of the most interesting topics in nuclear structure theories [1][2]. Especially, the knowledge of paring effect is essential for the understanding of $NN$ correlations in light nuclei. Moreover, coupling of $\alpha$-clusters and $NN$ pairs is important for the cluster states of general nuclei composed of both $\alpha$-clusters and valence nucleons, as discussed in various many-body systems in previous works [3][4]. Hence, investigation of the $NN$ pairing effect in cluster states is a meaningful step to improve our present understanding of nuclear clustering effects [5][6].

There are two kinds of $NN$ pairs respecting to the isospin symmetry, including the isovector pairs with $T = 1$, and the isoscalar pairs with $T = 0$. For general nuclei, the isovector pairs are studied intensively but the deuteron-like isoscalar pairs are relatively rare [7][8]. One important anomaly is the strong isoscalar pairing in $^{10}$B nucleus where the formation of $NN$ pairs are strongly influenced by the coexistent $\alpha$-clustering effect in this nucleus [1][2]. Therefore, the mechanism for the formation of $NN$ pairing in $^{10}$B is essential for the understanding of isoscalar pairing effect. Considering the complexity that originate from the coupling of clustering and pairing effects, it is desirable to fix the $\alpha$-cluster components in nuclei and then pin down the modulation of $NN$ pairing by the spin-isospin channels of two paired nucleons. An ideal approach is to compare the $NN$ pairing effects between $^{10}$B nucleus in $T = 0$ states with $^{10}$Be and $^{10}$C nuclei in $T = 1$ states. Correspondingly, theoretical descriptions should be formulated for these nuclei to investigate their $NN$ pairing structures and dynamics of pair motion. Pairing strength obtained from these investigations is also essential for experimental probing of $NN$ correlations where the theoretical predictions of $(p,pd)$ nuclear reactions observables are included as inputs [10].

In order to study the different pairs consisting of neutrons ($nn$), protons ($pp$), or proton and neutron ($pn$), we focus on the $^{10}$Be, $^{10}$B and $^{10}$C nuclei that are all composed of two valence nucleons and two $\alpha$-clusters. In our study, because of the different configurations of nucleon-pairs among these three nuclei, it is possible to discuss the essential mechanisms for the formation of $NN$ pairs as well as their different properties, especially for the deuteron-like proton-neutron correlation in $^{10}$B.

To study the pairing effects in these nuclei, we propose a new extended formulation of the Tohsaki-Horiuchi-Schuck-Röpke (THSR) wave function, which is a successful clustering model for various light nuclei [11][14], especially for the Hoyle state ($0^+_2$) in $^{12}$C [14]. By comparing with generator coordinate method (GCM), the wave function of cluster states are found to be 100% accurately described by a single THSR wave function in light nuclei [13][17]. In this work, we propose the extended THSR wave function in superposed form, which is named as “THSR+pair” wave function. For the first time, additional pairing configuration for valence nucleons is introduced to the THSR approach, which provides a convenient framework for the discussion of the $NN$ pairing effect in nuclear system. With this wave function, we investigate the abnormal isoscalar $NN$ paring effects for the $^{10}$B nucleus and compare with the isovector $NN$ pairing effects in $^{10}$Be and $^{10}$C nuclei. Moreover, benefiting from the intrinsic concision in analytical formulation, the THSR wave function possesses great advantage in discussing the structure and dynamics of $NN$ pairs in nuclei, as for the $\alpha$-clusters and valence nucleons in previous works.

This work is organized as following. In Section I, we formulate the THSR+pair wave function for $^{10}$Be, $^{10}$B and $^{10}$C nuclei. In Section II, we provide the numerical results, including the energy, distances between components, density distributions and corresponding discussions. The last Section III contains the conclusions.

II. FORMULATION

We start by writing the traditional THSR wave function which is used in our previous calculations [19],

$$\Phi = \prod_{i=1}^{2} \int dR_i \exp(-\frac{R_{i,x}^2}{\beta_{a,x,y}^2} - \frac{R_{i,y}^2}{\beta_{a,x,y}^2} - \frac{R_{i,z}^2}{\beta_{a,z}^2})$$

$$\times \int dR_a \exp(-\frac{R_{a,x}^2}{\beta_{a,x,y}^2} - \frac{R_{a,y}^2}{\beta_{a,x,y}^2} - \frac{R_{a,z}^2}{\beta_{a,z}^2})$$

$$\times \int dR_b \exp(-\frac{R_{b,x}^2}{\beta_{b,x,y}^2} - \frac{R_{b,y}^2}{\beta_{b,x,y}^2} - \frac{R_{b,z}^2}{\beta_{b,z}^2})$$

$$\times e^{im_a \phi_{R_a}} e^{im_b \phi_{R_b}} \Phi^B(R_1, R_2, R_a, R_b),$$

The Gaussian parameter $\beta$s constrain the nonlocalized motions of two $\alpha$ clusters and valence nucleons [19]. We choose difference parameter $\beta$s for $z$ direction and $x - y$ direction separately because of the deformation in nuclei. These parameters are determined by variational calculation. $\Phi^B$ is the Brink wave function, where $R_{1,2}$ and $R_{a,b}$ are
corresponding generate coordinates for the α clusters and valence nucleons. The terms \( e^{im_αφ_Rα} \) and \( e^{im_βφ_Rβ} \) are the phase factors which are introduced to obtain correct parities and orbital angular momenta for valence nucleons [19].

The single-particle wave function can be written as

\[
\phi(r) = \left( \frac{2ν}{π} \right)^{3/4} e^{-ν(r-R)^2} \chiσχr, \tag{2}
\]

where the Gaussian range parameter \( ν = \frac{1}{2β} \). \( χσ \) is the spin part of nucleon which is up (↑) or down (↓) in the z-direction. \( χr \) is the isospin part of proton (p) or neutron (n).

The above traditional THSR wave function in Eq. (1) provides good description for the molecular-orbit configurations but does not include directly the descriptions for the NN pairing structure. In pioneer works, many studies indicate that the two valence nucleons have trends to formulate NN pair in \(^{10}\text{Be},^{10}\text{B} \) and \(^{10}\text{C} \) nuclei [11]. In order to describe this component and provide clear description for the pair structure, we introduce an additional compact NN pairing term as

\[
\Phi_p = \prod_{i=1}^{2} dR_i \exp\left(-\frac{R^2_{i,x}}{β^2_{α,x,y}} - \frac{R^2_{i,y}}{β^2_{α,x,y}} - \frac{R^2_{i,z}}{β^2_{α,z}}\right) \\
× \int dR_{pair} \exp\left(-\frac{R^2_{pair,x}}{β^2_{pair,x,y}} - \frac{R^2_{pair,y}}{β^2_{pair,x,y}} - \frac{R^2_{pair,z}}{β^2_{pair,z}}\right) \\
× e^{im_αφ_Rα} e^{im_βφ_Rβ} \Phi^b(R_1, R_2, R_{pair}). \tag{3}
\]

In this term, we treat valence nucleons as a two-particle cluster, which share the same generate coordinate \( R_{pair} \). This corresponds to a pairing configuration in \(^{10}\text{Be},^{10}\text{B} \) and \(^{10}\text{C} \). We formulate the THSR+pair wave function as a superposition of the molecular-orbit configuration \( \Phi \) in Eq.(1) and this additional term \( Ψ_p \), as

\[
Ψ_{pair} = α Ψ + bΨ_p. \tag{4}
\]

Here \( a \) and \( b \) are the coefficient parameters which are determined by variational calculations. The parameters \( m_α \) and \( m_β \) in the phase factors \( e^{im_αφ_Rα} \) and \( e^{im_βφ_Rβ} \) are chosen according to the rotational symmetry of the molecular state under consideration [19, 20]. For the 0\(^+\) ground state of \(^{10}\text{Be} \) and \(^{10}\text{C} \), we choose \( m_α = 1 \) and \( m_β = -1 \) in Eqs. (1) and (5) to describe the antiparallel couplings of spins for the two valence nucleons around two α clusters. We note that under this condition the phase factors in Eq. (3) vanish and the pair wave function \( Ψ_p \) is reduced to the s-wave. As for the 3\(^+\) ground state of \(^{10}\text{B} \), the parameters are chosen as \( m_α = m_β = 1 \), which describes the parallel couplings of spins between valence nucleons.

We apply the angular-momentum projection technique \( \hat{P}^J_{MK} \) \( |Ψ⟩ \) to restore the rotational symmetry [22],

\[
|Ψ^J_M⟩ = \hat{P}^J_{MK} |Ψ⟩ \\
= \frac{2J+1}{8π^2} \int dΩD_{MK}(Ω)\hat{R}(Ω) |Ψ⟩, \tag{5}
\]

where \( J \) is the total angular momentum of the system. For the 3\(^+\) \( 0 \) ground state and 1\(^+\) \( 0 \) excited state of \(^{10}\text{B} \) with isospin \( T = 0 \), we take the isospin projection by using the proton-neutron exchange operator \( \hat{P}_{p+n} \) as introduced in Refs. [11, 2].

The Hamiltonian of the \( A = 10 \) nuclear systems can be written as

\[
H = \sum_{i=1}^{10} T_i - T_{c.m.} + \sum_{i<j}^{10} V_{ij}^N + \sum_{i<j}^{10} V_{ij}^C + \sum_{i<j}^{10} V_{ij}^{ls}, \tag{6}
\]

For the central force in \( NN \) interaction, the Volkov No. 2 interaction [23] is selected as

\[
V_{ij}^N = \{ V_1 e^{-α_1 r_{ij}^2} - V_2 e^{-α_2 r_{ij}^2}\} \{ W - M \hat{P}_σ \hat{P}_τ + B \hat{P}_σ - H \hat{P}_τ \}, \tag{7}
\]

where \( M = 0.6, W = 0.4, B = H = 0.125, V_1 = -60.650 \text{ MeV}, V_2 = 61.140 \text{ MeV}, α_1 = 0.309 \text{ fm}^{-2}, \) and \( α_2 = 0.980 \text{ fm}^{-2} \). The G3RS (Gaussian soft core potential with three ranges) term [24], which is a two-body type interaction, is taken as the spin-orbit interaction, as

\[
V_{ij}^{ls} = V_0^{ls}(e^{-α_1 r_{ij}^2} - e^{-α_2 r_{ij}^2})L \cdot S \hat{P}_{31}, \tag{8}
\]

where \( \hat{P}_{31} \) projects the two-body system into triplet odd state and the parameters are set to be \( V_0^{ls} = 1600 \text{ MeV}, α_1 = 5.00 \text{ fm}^{-2}, \) and \( α_2 = 2.778 \text{ fm}^{-2} \). The Gaussian width parameter \( b \) of single particle wave functions is chosen as \( b = 1.46 \text{ fm} \).
III. RESULTS AND DISCUSSION

We calculate the ground state energies of $^{10}$Be, $^{10}$B and $^{10}$C by variational optimization of parameters in the THSR+pair wave function. The corresponding energy results are shown in Table I, where corresponding experimental data [23] and results calculated with the traditional THSR wave function $\Phi$ in Eq. (1) are also included. The masses of proton and neutron in the single-particle wave function are set to be to experimental values in Ref. [20].

|          | $^{10}$Be(0$^+$1) | $^{10}$B(3$^+$0) | $^{10}$C(0$^+$1) |
|----------|------------------|------------------|------------------|
| $E^{\text{exp}}$ [25] | -65.0            | -64.8            | -60.3            |
| $E^{\text{THSR}}$       | -58.3            | -59.8            | -54.4            |
| $E^{\text{THSR+pair}}$ | -59.2            | -61.8            | -55.3            |
| $\Delta$                | 0.9              | 2.0              | 0.9              |

From the comparison in Table I, it is clearly observed that the ground state energies of $^{10}$Be, $^{10}$B and $^{10}$C are greatly improved by additional superposition of the pair term $\Phi_p$ in Eq. (2) in the THSR+pair wave function $\Psi_{\text{pair}}$. These results prove that the $NN$ pairing structure is crucial for precise descriptions of these nuclei. For $^{10}$B, the THSR+pair wave function improves the ground state energy for about 2.0 MeV comparing to traditional THSR wave function, which is more significant than the improvements for other two nuclei. This indicates that the isoscalar $NN$ pairing effect with $T = 0$ in $^{10}$B is stronger than those isovector counterparts with $T = 1$ in $^{10}$Be and $^{10}$C nuclei.

In order to investigate the $NN$ paring strength in the THSR+pair wave function, we calculate the overlap between molecular-orbit term $\Phi$ and the total THSR+pair wave function $\Phi_{\text{pair}}$, the overlap between pairing term $\Phi_p$ and the total THSR+pair wave function $\Phi_{\text{pair}}$, as well as the overlap between molecular-orbit term $\Phi$ and pairing term $\Phi_p$. Corresponding results are shown in Table II. From this table, we observe that the overlaps between the molecular-orbit term and THSR+pair wave function are larger than 90% for all of $^{10}$Be, $^{10}$B and $^{10}$C nuclei. These large overlaps indicate that the molecular-orbit term could provide good description for these nuclei. However, additional pairing term is still necessary to obtain accurate wave function as these overlaps do not equal 100 %. It is also observed that the overlaps $\langle \Phi_p | \Psi_{\text{pair}} \rangle^2$ are different among $^{10}$Be, $^{10}$B and $^{10}$C, where the overlap for $^{10}$B nucleus is significantly larger than other two nuclei. Hence the optimized THSR+pair wave describes stronger $NN$ pairing effect in $^{10}$B than those in $^{10}$Be and $^{10}$C as we concluded previously. From the giant ratio $\langle \Phi | \Psi_{\text{pair}} \rangle^2=75.8$ %, we found that molecular-orbit term $\Phi$ in the 3$^+$0 ground state of $^{10}$B provides the description that is analogous to the pairing term $\Phi_p$, which explains the strong pairing effect in this state.

|          | $^{10}$Be(0$^+$1) | $^{10}$B(3$^+$0) | $^{10}$C(0$^+$1) |
|----------|------------------|------------------|------------------|
| $<\Phi|\Psi_{\text{pair}}|^2$ | 92.9%            | 90.8%            | 93.6%            |
| $<\Phi_p|\Psi_{\text{pair}}|^2$ | 56.6%            | 93.3%            | 67.5%            |
| $<\Phi|\Phi_p|^2$              | 45.5%            | 75.8%            | 43.1%            |

We also calculate the 1$^+_1$ excited state of $^{10}$B, which is the counterpart for the 0$^+_1$ ground states of $^{10}$Be and $^{10}$C nuclei, as they all have dominant $L = 0$ components that originate from the antiparallel coupling of orbital angular momentum of two valence nucleons [1, 2]. Hence, the total spin $S$ and isospin $T$ of two valance nucleons are the only differences among the 1$^+_1$ excited state of $^{10}$B and the 0$^+_1$ ground states of $^{10}$Be and $^{10}$C. In Table III we show the energies of the 1$^+_1$ excited state calculated with the traditional THSR wave function $\Phi$ and the THSR+pair wave function $\Psi_{\text{pair}}$. In these calculations, we set parameters $m_{ab} = \pm 1$ respectively for two valence nucleons to describe the antiparallel coupling of orbital angular momenta. The corresponding experimental data adopted from Ref. [23] are also included for comparison. From this table, it is observed that the excitation energy of the 1$^+_1$ state is improved from 2.8 MeV to 1.0 MeV by adding the pairing term, which is much closer to the experimental value 0.7 MeV. It
is also clearly shown that the introduction of additional pairing term \( \Phi_p \) improves energy of the \( 1^+_0 \) excited state by about 3.8 MeV, which is significantly larger than the corresponding improvement of about 2.0 MeV for the \( 3^+_0 \) ground state. This drastic difference indicates even stronger \( NN \) pairing effect in the \( 1^+_0 \) excited state.

Furthermore, this improvement of 3.8 MeV is significantly larger than corresponding values for the \( 0^+_1 \) ground states of \(^{10}\text{Be} \) and \(^{10}\text{C} \), as shown in Table I. Hence we observe the enhanced pairing effect again in the \( 1^+_0 \) state of \(^{10}\text{B} \) with \( T = 0 \) comparing to the pairs with \( T = 1 \) in \(^{10}\text{Be} \) and \(^{10}\text{C} \). However, the previous explanation for the pairing effect in the \( 3^+_0 \) ground state of \(^{10}\text{B} \) no longer persists, because the analogy between the molecular-orbit configuration and pairing configuration is much weaker in the \( 1^+_0 \) state. This is demonstrated in Table IV where the overlap between the molecular-orbit term \( \Phi \) and the pairing term \( \Phi_p \) in this state is found to be 55.8%, which is much smaller than the corresponding value of 75.8% in the \( 3^+_0 \) ground state. From this we conclude that these two configurations compete with each other in the \( 1^+_0 \) state, which is different from the analogous contribution from these two configurations in the total wave function.

As listed in Table IV, the squared overlaps between the molecular-orbit term \( \Psi \) and total wave function \( \Psi_{\text{pair}} \) in the \( T = 1 \) states of \(^{10}\text{Be} \) and \(^{10}\text{C} \) are larger than 90%, which shows that the molecular-orbit configuration prevails in these states. This can be explained by the fact that the molecular-orbit configuration is energetically favorable in \( T = 1 \) nuclei \(^{10}\text{Be} \) and \(^{10}\text{C} \), where both the molecular orbits have parallel spin-orbit coupling and provide large contributions to the total energies of nuclei. In contradiction, the spin-orbit contributions from valance nucleons is cancelled by each other in the pairing configuration, as the two paired nucleons have opposite spin directions but the same orbital motion. In the \( 1^+_0 \) excited state of \(^{10}\text{B} \) the dominance of molecular-orbit configuration disappears, as shown by the larger overlap 87.5% between pairing configuration \( \Phi_p \) and total wave function \( \Psi_{\text{pair}} \). In the molecular-orbit term \( \Phi \) of this state, the spin-orbit coupling is parallel and antiparallel for the two valance nucleons, respectively. Hence there is also cancellation of spin-orbit contribution from valance nucleons, which is similar to the case in pairing configuration. As a consequence, the molecular-orbit configuration is not energetically favorable and it is quenched in the total wave function by its competition with pairing configuration in the \( 1^+_0 \) excited state of \(^{10}\text{B} \).

Here we have observed that the \( NN \) pairing is formulated in deuteron like channels (\( T = 0, S = 1 \)) of two valance nucleons both the \( 3^+_0 \) ground state and the \( 1^+_0 \) excited state of \(^{10}\text{B} \). However, the \( NN \) pairs are formulated in two different mechanisms in these states. For the \( 3^+_0 \) ground state, the molecular-orbit configuration of two valance nucleons is analogous to the pairing configuration and hence enhances the possibility of \( NN \) pairing. In the \( 1^+_0 \) excited state of \(^{10}\text{B} \), molecular-orbit configuration competes with the pairing configuration and hence is quenched comparing to its dominance in \( 0^+_1 \) states of \(^{10}\text{Be} \) and \(^{10}\text{C} \), which on the other hand encourages the formation of \( NN \) pairing.

**TABLE III.** Energies of the \( 3^+_0 \) ground state and the \( 1^+_0 \) excited state for \(^{10}\text{Be} \). \( E^{\text{THSR}} \) denotes energies obtained from the THSR wave function. \( E^{\text{THSR+pair}} \) denotes results obtained from the THSR+pair wave function. \( \Delta \) denotes the improvement of energies in the new THSR+pair wave function comparing to the values obtained from traditional THSR wave function. \( E^{\text{exp}} \) denotes experimental values adopted from Ref. \[25\]. \( E_{\text{ex}} \) denotes the corresponding excited energies. All units of energies are in MeV.

| \(^{10}\text{B} \) | \( 3^+_0 \) | \( 1^+_0 \) | \( E_{\text{ex}} \) |
|-----------------|----------|----------|----------|
| \( E^{\text{exp}} \) | -64.8    | -64.1    | 0.7      |
| \( E^{\text{THSR}} \) | -59.8    | -57.0    | 2.8      |
| \( E^{\text{THSR+pair}} \) | -61.8    | -60.8    | 1.0      |
| \( \Delta \) | 2.0      | 3.8      | 1.8      |

**TABLE IV.** The overlaps between each two of the THSR+pair wave function \( \Psi_{\text{pair}} \) and its two components \( \Phi \) and \( \Phi_p \) of molecular-orbit configuration and pairing configuration, respectively. Values are calculated for the ground states of \(^{10}\text{Be} \) and \(^{10}\text{C} \) nuclei and the \( 1^+_0 \) excited state of \(^{10}\text{B} \). All the wave functions \( \Phi, \Phi_p \) and \( \Psi_{\text{pair}} \) have been normalized.

|          | \(^{10}\text{Be}(0^+_1)\) | \(^{10}\text{C}(0^+_1)\) | \(^{10}\text{B}(1^+_0)\) |
|----------|-----------------|-----------------|-----------------|
| \( < \Phi|\Psi_{\text{pair}} >^2 \) | 92.9%  | 93.6%  | 83.7%  |
| \( < \Phi_p|\Psi_{\text{pair}} >^2 \) | 56.6%  | 67.5%  | 87.5%  |
| \( < \Phi|\Phi_p >^2 \) | 45.5%  | 43.1%  | 55.8%  |

The \( NN \) pair structure in \(^{10}\text{Be} \), \(^{10}\text{B} \) and \(^{10}\text{C} \) can be demonstrated explicitly by showing the average distances between the two valence nucleons, which correspond to the average sizes of \( NN \) pairs. It should be noticed that the
formation of $NN$ pairs can affect both the distance between two valence nucleons $r_{NN}$ and the distance between $NN$ pair and the center of two $\alpha$-clusters $r_{N,\alpha}$. Hence when the strength of $NN$ pairing effect increases, the ratio $r_{NN}/r_{N,\alpha}$ is reduced to a relatively small value. We compare the average distances for $^{10}\text{Be}$, $^{10}\text{B}$ and $^{10}\text{C}$ in Fig. [1] including the $1^+\text{B}$ state of $^{10}\text{B}$. As shown in this figure, with the molecular-orbit configurations (the black lines), the $NN$ distances $r_{NN}$ have almost the same magnitude as $r_{N,\alpha}$ for all nuclei of $^{10}\text{Be}$, $^{10}\text{B}$ and $^{10}\text{C}$, which is due to the missing of paring term in the molecular-orbit configuration. With the new extended THSR+pair wave function (the red lines), the $r_{NN}$ in $^{10}\text{Be}$ is smaller but comparable to $r_{N,\alpha}$, showing a relatively weak $NN$ pairing effect in $^{10}\text{Be}$. For $^{10}\text{C}$, the $NN$ pairing effect is even weaker as corresponding $r_{NN}$ is larger than $r_{N,\alpha}$. We see significantly small ratio $r_{NN}/r_{N,\alpha}$ for both states of $^{10}\text{B}$ nucleus, where the $NN$ pairing effect is stronger, as discussed previously.

![Graph](image_url)

**FIG. 1.** The average distances between nucleons in $^{10}\text{Be}(0^+1)$, $^{10}\text{B}(3^+0)$, $^{10}\text{B}(1^+0)$ and $^{10}\text{C}(0^+1)$ states. The solid lines denote the average distances $r_{NN}$ between two valence nucleons. The dashed lines denote the average distances $r_{N,\alpha}$ between valence nucleons and the center of two $\alpha$-clusters. For both the solid lines and dashed lines, the black color denotes results obtained by using only the molecular-orbit configuration $\Phi$ and the red color denotes results obtained from the total THSR+pair wave function $\Phi_{\text{pair}}$. All units are in fm.

In order to investigate the nuclear dynamics of $NN$ pairs, we calculate the density distributions for valence nucleons of the $0^+1$ ground state of $^{10}\text{Be}$, the $3^+0$ ground state of $^{10}\text{B}$ and $1^+0$ excited state of $^{10}\text{B}$, as shown in Fig. [2]. In this figure, panels labeled by “(a)” are calculated with the THSR+pair wave function $\Phi_{\text{pair}}$. The panels labeled by “(b)” are calculated by using only the pairing term $\Psi_p$. For all these wave functions, $\beta$ parameters are set to be optimized values in corresponding THSR+pair wave functions.

From this figure, we observe that the valence nucleons in the $3^+0$ ground state of $^{10}\text{B}$ have narrow distributions in the $x-y$ direction because it is tightly bounded by the spin-orbit potential and the centrifugal barrier. While in the cases of the ground state of $^{10}\text{Be}$ and the $1^+0$ excited state of $^{10}\text{B}$, the distributions of the valence nucleons are more broad because of the weaker spin-orbit potential and lower centrifugal barrier. This result agrees with the conclusions in Ref. [2] and Ref. [29]. By comparing the “(b)” panels in these figures, we notice that when no centrifugal barrier exists, as in the $0^+1$ state of $^{10}\text{Be}$ and $1^+0$ state of $^{10}\text{B}$, the $NN$ pairs described by the pairing term $\Psi_p$ are likely to have more distribution near $z = 0$ cross section between two $\alpha$ clusters, which corresponds to a relatively dilute three-clusters structure of $\alpha+\alpha+\text{pair}$. On the other hand, in the $3^+0$ ground state of $^{10}\text{B}$, the strong spin-orbit coupling from the orbital angular momentum $L = 2$ encourages the spreading of valence nucleons in the $z$-direction around $\alpha$-clusters to formulate $\pi$-molecular orbits, as discussed in Refs. [19] [20].

The similar conclusions also can be made depending on the optimum parameters of the THSR+pair wave function or traditional THSR wave function, which are listed in Table [V] for each state.

**IV. CONCLUSION**

We propose a new extended formulation of THSR wave function, named as “THSR+pair” wave function, for $^{10}\text{Be}$, $^{10}\text{B}$ and $^{10}\text{C}$ nuclei. In this wave function, $NN$ pairing term is introduced in addition to the molecular-orbit term used in previous version of THSR wave function. By using the THSR+pair wave function, the energies for the ground
states are improved significantly for these nuclei, especially the $^{10}$B nucleus. Analyses of energies and overlaps show that the pair configuration is stronger in the $^{10}$B nucleus comparing to other two nuclei. We also calculate the $^{1+}$ excited state of $^{10}$B using the THSR+pair wave functions and observe again strong pairing effect in this state. These results show that pairing effect are enhanced by the deuteron-like channel of $S = 1$ and $T = 0$ (isoscalar). From the energies and overlaps between wave function components, we found that there are two different mechanisms that enhance the formation of $NN$ pair in $^{10}$B nucleus. In the $3^+_0$ ground state, the strong pairing effect originates from the analogy between molecular-orbit configuration and pairing configuration. In the $1^+_0$ excited state, the pairing configuration competes with molecular-orbit configuration and the molecular-orbit term is energetically unfavored and quenched. We also discuss the structure of $NN$ pairs and their dynamics of motion in space, by calculating the average distances between components in nucleus and the density distributions of valance nucleons. This study further improves the understanding of the formation of $NN$ pairs and their properties, especially for those in isoscalar...
TABLE V. The variationally optimized $\beta$ parameters for the wave function of $^{10}$Be ($0^+$), $^{10}$B ($3^+$), $^{10}$B ($1^+$) and $^{10}$C ($0^+$). For each nucleus, the upper line corresponds to the calculation with traditional THSR wave function $\Phi$ and the lower line denoted by symbol “$\rightarrow$” corresponds to calculation with THSR+pair wave function $\Phi_{\text{pair}}$. The units of $\beta$ parameters are in fm.

| Nucleus  | $\beta_{a,xy}$ | $\beta_{a,z}$ | $\beta_{ab,xy}$ | $\beta_{ab,z}$ | $\beta_{\text{pair},xy}$ | $\beta_{\text{pair},z}$ | $a$  | $b$  |
|----------|----------------|---------------|-----------------|-----------------|----------------------|----------------------|-----|-----|
| $^{10}$Be ($0^+$) | 0.1 | 2.5 | 1.9 | 2.9 | / | / | 0.89 | 0.11 |
| $\rightarrow$ | 0.1 | 2.8 | 1.9 | 3.3 | 2.5 | 0.8 | 0.89 | 0.11 |
| $^{10}$B ($3^+$) | 0.1 | 1.9 | 1.1 | 2.2 | / | / | 0.77 | 0.23 |
| $\rightarrow$ | 0.1 | 2.6 | 1.0 | 3.2 | 1.8 | 3.3 | 0.77 | 0.23 |
| $^{10}$B ($1^+$) | 0.1 | 2.2 | 2.2 | 2.0 | / | / | 0.77 | 0.23 |
| $\rightarrow$ | 0.1 | 3.0 | 2.4 | 2.8 | 3.0 | 2.7 | 0.77 | 0.23 |
| $^{10}$C ($0^+$) | 0.1 | 2.8 | 2.2 | 3.3 | / | / | 0.89 | 0.11 |
| $\rightarrow$ | 0.1 | 3.0 | 2.4 | 3.2 | 2.5 | 0.2 | 0.89 | 0.11 |

channels, which could be beneficial for future investigations of $NN$ correlations and general cluster states composed of both $\alpha$-clusters and $NN$ pairs.

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