Proportional Power Sharing Control of Distributed Generators in Microgrids

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Abstract

This research addresses distributed proportional power sharing of inverter-based Distributed Generators (DGs) in microgrids under variations in maximum power capacity of DGs. A microgrid can include renewable energy resources such as wind turbines, solar panels, fuel cells, etc. The intermittent nature of such energy resources causes variations in their maximum power capacities. Since DGs in microgrids can be regarded as Multi-Agent-Systems (MASs), a consensus algorithm is designed to have the DGs generate their output power in proportion to their maximum capacities under capacity fluctuations. A change in power capacity of a DG triggers the consensus algorithm which uses a communication map at the cyber layer to estimate the corresponding change. During the transient time of reaching a consensus, the delivered power may not match the load power demand. To eliminate this mismatch, a control law is augmented that consists of a finite time consensus algorithm embedded within the overarching power sharing consensus algorithm. The effectiveness of the distributed controller is assessed through simulation of a microgrid consisting of a realistic model of inverter-based DGs.

Keywords

Proportional Power Sharing, Distributed Control, Inverter-based Microgrid, Renewable Energy, Consensus, Transient Control, Finite Time Consensus, Demand Response

1 Introduction

Environmentally sustainable electrical energy production depends on renewable energy resources. In this regard, significant amount of researches have been undertaken within the past few decades, [1–3]. Conventionally, control of electric power systems and the main power grid was accomplished through a few central controllers. Through emerging renewable energy plants, intelligent loads located in the demand side and computational advances, distributed energy production and management has become viable. DGs as distributed energy production units, together with local loads which are distinct from the main power, are called microgrid.

Microgrids operate in two different operational modes called grid-connected and islanding. A microgird is said to work in grid-connected mode when it is connected to the main grid via a tie line at the point of common coupling (PCC) where there exist bidirectional power flow from or into the main grid, [4].

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In contrast, microgrids in islanding mode generates power for local loads. [5]. To deploy small-scale DGs including photovoltaic (PV) cells, wind turbines, fuel cells and energy storage systems (ESSs) in microgrids, power electronics inverters are vital interfaces which connect DGs to the power buses, [6].

There has been extensive studies conducted to control of inverter-based microgrids during the past decades, [7, 8]. The control strategies can be classified in different categories including frequency and voltage control or power control. Applying these methods also depends on the microgrid mode of operation. For instance, in the grid connected mode the frequency and voltage are imposed by the main grid. However, the voltage and frequency control is the vital aspect of any control design in the islanding mode. This is due to the dependency of microgrid stability on the voltage and frequency regulation in the islanding mode which is not maintained or imposed by another external and dominant system such as the main grid.

In this work, the power sharing control in grid-connected mode as an issue with high importance in microgrids is studied, [9]. Some of the power requirements in a power network are imposed by load demand which is of a high priority in power systems. The problem of power sharing has been studied from the aspect of equal power sharing in [10, 11]. Since DGs possess different capacities, the DGs with higher capacities can share more power than the DGs with lower capacities. The power sharing problem becomes challenging under intermittent nature of power resources. This issue results in fluctuations of maximum capacity of DGs, which leads to changes in their output power. Thereby, the total power fluctuates, inevitably, and the load power may not always be maintained. This problem mandates devising a control plan to address the fluctuations. These fluctuations can be addressed by deploying electrical energy storage (EES) or managing the DGs to flexibly address the variations in their capacities.

The control problem of power sharing among DGs in microgrids can be formulated either as proportional power sharing, [12], or economical dispatch problem (EDP), [13]. The studies [12] and [14] have proposed techniques for proportional power sharing. Here, proportional power sharing is defined as sharing the load among DGs such that each individual DG shares a fraction of the load in proportion to its maximum capacity. On the other hand, EDP is a method to control the power flow among different DGs optimally, where the optimality implies minimizing a quadratic performance index assigned to each DG as the costs of their generated power. EDP has been studied through different techniques including the population dynamic method, [15], and the lambda iteration, [16]. While these methods have been formulated within a centralized control framework in the literature, distributed version of EDP can be found in [13, 17].

Motivated from systems with cyber-physical layers, the power sharing control in this study is devised in two layers. The physical layer that consists of DGs, loads, measurement units, etc, is where the power control loop of each DG is established to track the input power command issued from the cyber layer. DGs have their corresponding agents in the cyber layer, thus the ideas of MASs can be utilized to establish the DGs’ controllers and their interactions with each other. The agents communicate with each other through a graph that corresponds to the communication network existing among DGs in the physical layer.

The agents can choose different strategies to control the DGs including centralized, decentralized or distributed formats. When the DGs are located in a small region, it is viable to apply centralized controllers. As the number of DGs increases, while geographically scattered in a wide area, applying the centralized controllers faces deficiencies due to some reasons; Firstly, the centralized controller is not reliable due to the dependency of the DGs on a single controller where its malfunctions deteriorate the performance of the microgrid or may result in instability. Besides, in centralized coordinated control, transferring data to a control center and issuing control signals back to DGs requires high bandwidth communication which is not economically efficient, or technically secure, and is prone to failure, [18]. On the contrary, distributed control techniques require considerably lower bandwidth which makes the communications among the DGs economically viable. Decentralized controllers are applicable locally, however it does not exploit cooperation of DGs, [19]. Therefore, they may not perform efficiently where the global information and cooperation is required. In contrast, a distributed control scheme encompasses the plug and play feature, which it makes it more flexible compared to centralized and decentralized controllers, [20]. The centralized control scheme
depends on global information while DGs in distributed control exchange information exclusively with the DGs in their neighborhood. In this study, the well-known consensus algorithm is utilized to design a distributed controller for the power sharing control problem.

Considering a large number of DGs scattered in a wide area, it takes agents time to transfer all the required signals. Therefore, it is inevitable to have communication delays in the distributed controllers. The ranges of these delays are from tens to hundreds of milliseconds or more. The delays may result in prolonging the convergence time of consensus algorithm and potentially lead to microgrid instability. The delays can be reduced through increasing the convergence rate of consensus algorithms utilizing some approaches including multiplying the weights of the communication graph with a large constant, or through an optimization of the weights.

While satisfying the load demand is of a high priority, during the transient time of the consensus algorithm, a power mismatch can emerge between the generated power and delivered power. This power mismatch may result in frequency and voltage drop, and in worse cases, might result in system instability. Therefore, applying a simple consensus algorithm solely is not sufficient to satisfy the power demand within the proportional power sharing scheme. In addition, although the rate of convergence in consensus algorithms can be increased through the strategies explained above, mismatches between the generated power and the load demand is inevitable. Thus, in this study, a new control law is augmented to the consensus procedure to not only realize the proportional power sharing but also maintain the load power, during the transient time of consensus before convergence is attained. The new augmented control law resolves the power mismatch issue, however it is required to be applied in such a way to preserve the distributed structure of the control scheme. To do so, the finite time consensus algorithm proposed in is embedded in the control design by which the agents are able to implement the control scheme distributively. The contribution of this paper is proposing a proportional power sharing control plan for microgrids which addresses the power mismatch during consensus algorithm while it is completely distributed.

The rest of this paper is organized as follows. The preliminary definitions of technical terms are explained in section two. Then, proportional power sharing is defined in the third section. In the fourth section, the consensus algorithm is developed through which the DGs are able to update their information about the total microgrid power capacity following a change in a DG’s capacity. The overarching consensus algorithm and the embedded transient controller are proposed and elaborated in the same section. Fifth section discusses the cyber and physical layers which control the output power of DGs. Next, simulation results are provided in section six to illustrate the effectiveness of the proposed control plan in response to different variations in capacity of a DG. Finally, concluding remarks are provided and references are listed.

2 Preliminary Definitions

We define the graph \( G \) as the set pair \((V, E)\) having vertices set \( V \) and edge set \( E \). Let the number of vertices in \( G \) be \( N \), and let the set \( E \) consist of the vertices pairs \((i, j)\) for which there exists an edge that connects \( j \) to \( i \), with \( i, j = 1, 2, \ldots, N \) and \( i \neq j \). The intended graph in this study is undirected or bidirectional graph, where the signals flow along edges in both directions, i.e. if \((i, j) \in E\), then \((j, i) \in E\). The adjacency matrix associated with the graph is \( A = [a_{ij}] \in \mathbb{R}^{N \times N} \) where each element \( a_{ij} > 0 \) if \((i, j) \in E\), otherwise \( a_{ij} = 0 \). As stated above, in the bidirectional graph \( G \), if \((i, j) \in E\), then \((j, i) \in E\), and \( a_{ij} = a_{ji} \). Then \( A \) is symmetric, i.e. \( A = A^T \). We define the degree matrix \( D = [d_{ii}] \in \mathbb{R}^{N \times N} \) as a diagonal matrix as such

\[
    d_{ii} = \sum_{j=1}^{N} a_{ij}
\]

The matrix \( L = [l_{ij}] = D - A \) is denoted as the Laplacian Matrix of \( G \). As mentioned above, \( A = A^T \), and considering \( D \) is a diagonal matrix, it follows that \( L = L^T \). The neighbor set corresponding to each vertex \( i \)
is defined as \( \mathcal{N}_i = \{ j \mid (i, j) \in \mathcal{E} \} \). Additionally, \( \mathcal{N}^+_j \) denotes the set of outgoing neighbors of node \( j \), i.e., the set of nodes receiving signals from the node \( j \), and \( \mathcal{N}^-_j \) is the set of nodes which sends signals to the node \( j \). For the bidirectional graph \( \mathcal{G} \), \( \mathcal{N}^+_j = \mathcal{N}^-_j \). A graph is strongly connected if there exists a path between any two distinct vertices. We assume that \( \mathcal{G} \) is strongly connected.

Next, consider Fig. 1 which shows a sample localized microgrid with four DGs, denoted by DG\(_i\), \( i = 1, 2, 3, 4 \). In this figure, the dashed lines show signaling between the cyber layer and physical layer, i.e. the communications between the DGs and their corresponding agents in the cyber layer. The lines with bidirectional arrows represent communications among the corresponding agents of DG\(_i\) located in the cyber layer. The solid lines are electrical connections. Based on the weights shown in Fig. 1 and the explanations above, the adjacency, degree and Laplacian matrices are defined as,

\[
A = \begin{bmatrix}
0 & 0 & a_{13} & 0 \\
0 & 0 & a_{23} & a_{24} \\
a_{31} & a_{32} & 0 & 0 \\
0 & a_{42} & 0 & 0
\end{bmatrix}
\quad D = \begin{bmatrix}
a_{13} & 0 & 0 & 0 \\
0 & a_{23} + a_{24} & 0 & 0 \\
0 & 0 & a_{31} + a_{32} & 0 \\
0 & 0 & 0 & a_{42}
\end{bmatrix}
\quad L = \begin{bmatrix}
a_{13} & 0 & -a_{13} & 0 \\
0 & a_{23} + a_{24} & -a_{23} & -a_{24} \\
-a_{31} & -a_{32} & a_{31} + a_{32} & 0 \\
0 & -a_{42} & 0 & a_{42}
\end{bmatrix}
\]

(2)

3 Problem Definition

We consider a microgrid in the grid connected mode, where the microgrid’s voltage and frequency are imposed by the main grid, i.e. the microgrid’s frequency and voltage are fixed. Hence, the goal in this mode is to control the output power of the DGs. The cyber-physical systems considered in this paper is similar to the one shown in Fig. 1. The proposed control emerges from consensus control of Multi-Agent Systems (MAS). The control objective is sharing load power in proportion to the maximum power capacity of the DGs, under variations in maximum capacities. We assume that there exists \( N \) DGs in a microgrid which are labeled as DG\(_i\) where \( i = 1, 2, \ldots, N \). The maximum power capacity and instantaneous output power of each DG\(_i\) are defined as \( P_{i,\text{max}} \) and \( P_i \), respectively. Let \( P_L \) be the load power which is proportionately shared among the DGs, i.e.

\[
P_L = \sum_{i=1}^{N} P_i, \quad \text{s.t.} \quad r = \frac{P_1}{P_{1,\text{max}}} = \frac{P_2}{P_{2,\text{max}}} = \cdots = \frac{P_N}{P_{N,\text{max}}}
\]

(3)

where \( r \) is the proportional power share ratio parameter. Thus, the output power of DG\(_i\) is \( P_i = rP_{i,\text{max}} \). Let \( P_T \) be the total power capacity of the microgrid defined as the accumulation of the maximum power capacity of all the DGs in the microgrid. Then, one can conclude that

\[
r = \frac{\sum_{i=1}^{N} P_i}{\sum_{i=1}^{N} P_{i,\text{max}}} = \frac{P_L}{P_T}
\]

(4)

Note that fluctuations in the output power of a DG will cause a change in \( r \), and the proposed power sharing control will, in response, manage the output power of the DGs flexibly. Throughout this study, it is assumed \( P_T \) is constant, and the focus is on managing the variations of \( P_S \) while maintaining \( P_L = \sum_{i=1}^{N} P_i \). Under a variation in \( P_{i,\text{max}} \), the total capacity of the microgrid \( P_T \) varies. Hence, per Eq. (4), \( r \) changes and correspondingly all \( P_S \)s are required to change. Although it appears that imposing a constant \( P_L \) is restrictive, it can be shown that when the MAS has reached consensus, handling variations in \( P_L \) is relatively convenient.

We next explain two scenarios for which different controllers are designed. At the core of these controllers is a consensus algorithm which is inherently distributed. Recall that an underlying assumption is
that the communication graph among the DGs is strongly connected. Before any change happens to the renewable energy resources, we assume all DGs have the knowledge of $P_T$ and $P_L$ by which they are able to compute $r$ from Eq. (4) and thereby generate their appropriate proportional power share $P_i = rP_{i,max}$, $i = 1, 2, \cdots, N$.

In the first scenario, assume the maximum capacity of DG$_k$ which is $P_{k,max}$ changes. Then $P_T$ changes accordingly and all DGs are required to update their value of $P_T$ to be able to recalculate the new $r$ based on Eq. (4). The only DG that can generate accurate power immediately after a fluctuation happens is the DG$_k$ since it is aware of the change in $P_{k,max}$. Let $\delta$ be the change such that $\tilde{P}_{k,max} = P_{k,max} + \delta$, where $\tilde{P}_{k,max}$ is the updated value of $P_{k,max}$. Thus, DG$_k$ can compute the updated capacity of the microgrid as $\tilde{P}_T$ where $\tilde{P}_T = P_T + \delta$ and recalculate $r$ and the delivered power $P_k$. A consensus algorithm is devised to have other DGs compute the $\tilde{P}_T$ and thereby reach the new value of $r$, distributively.

In the second scenario, we address the mismatch between load and supplied power before consensus is reached. As was discussed in scenario (1), only DG$_k$ can generate an accurate amount of power instantaneously after a fluctuation in DG$_k$. Although the other DGs are able to update $P_l$ following a change in $P_{k,max}$, the consensus algorithm takes time to converge, and hence during the transient time $\sum_{i=1}^{N} P_i$ would not necessarily be equal to $P_L$. The reason is that the other DGs do not have the correct value of $\tilde{P}_T$ instantaneously. However, since instantaneous matching of load power is a priority, a control law is augmented with the consensus algorithm to practically remove power mismatch during transients.

4 Distributed Microgrid Control

4.1 Consensus on Total Power Capacity under Perturbation

We consider a scenario where the individual DGs know the power ratio $r$ and generate accurate $P_l$ based on proportional power sharing, as shown in Eq. (3). Hence, each DG has correct knowledge of $P_T$, as per Eq. (4). Next, consider a change in $P_{k,max}$ to $\tilde{P}_{k,max} = P_{k,max} + \delta$. Following this change, all agents are
required to compute $\bar{P}_T = P_T + \delta$, the updated value of $P_T$. We define $s_i(t)$ as the estimate of $\bar{P}_T$ by DG$_i$. The vector of estimate variables is then, $\mathbf{S}(t) = \begin{bmatrix} s_1 & s_2 & \cdots & s_N \end{bmatrix}^T$, where $N$ is the number of the DGs in the microgrid. As mentioned above, all the DGs know $P_T$ before any change happens. Therefore, the initial value of $\mathbf{S}$ is, $\mathbf{S}(0) = P_T \mathbf{1}$ where $\mathbf{1}^N = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T$. Thereafter, we propose the following consensus dynamics in the cyber layer, through which all DGs update their value of $P_T$ and converge to $\bar{P}_T$.

$$
\dot{s}_k(t) = -h\left(s_k(t) - \bar{P}_T\right) - \sum_{j \in \mathcal{N}_k} a_{kj} (s_k(t) - s_j(t)), \quad s_k(0) = P_T
$$

$$
\dot{s}_i(t) = - \sum_{j \in \mathcal{N}_i} a_{ij} (s_i(t) - s_j(t)), \quad i = 1, 2, \ldots, N, \ i \neq k, \ s_i(0) = P_T
$$

(5)

In Eq. (5), $a_{ij} > 0$ and it denotes the weight of the communication link between agents $i$ and $j$, where $i, j = 1, 2, \ldots, N$, $i \neq j$, and $h > 0$ is a parameter chosen by the $k^{th}$ agent. Since the communication graph is bidirectional, therefore $a_{ij} = a_{ji}$, and this implies that the Laplacian matrix is symmetric, i.e. $L = L^T$ (see example in Eq. (2)). From Eq. (5), the following matrix equation is obtained

$$
\dot{\mathbf{S}} = -(L + \Delta)\mathbf{S} + \delta h \bar{P}_T
$$

(6)

where

$$
d_k = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \in \mathbb{R}^N, \quad \Delta = \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & h & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix} \in \mathbb{R}^{n \times n}
$$

(7)

In Eq. (7), the $k^{th}$ element of $d_k$ is one. Also, $\Delta_{k \times k} = h$ and all other elements are zero. We now propose and prove the following Lemma.

**Lemma 1.** The linear dynamic system defined in Eqs. (6) and (7) is input-to-state stable (ISS), and $\mathbf{S} \to \bar{P}_T \mathbf{1}$ given the graph of communication among the agents is strongly connected.

**Proof.** The linear system of Eqs. (6) and (7) is ISS if $-(L + \Delta)$ is Hurwitz, [25]. The input is $\bar{P}_{\text{nom}}$ which is constant and bounded, thus, if $-(L + \Delta)$ is Hurwitz the proof is complete. Since $L = L^T$, and by definition $\Delta = \Delta^T$, the matrix $-(L + \Delta)$ is symmetric. Hence, it is Hurwitz if $-(L + \Delta) < 0$, i.e. negative definite. To prove this, it is required to show that for any vector $u \in \mathbb{R}^N$, $u^T[-(L + \Delta)]u$ is strictly less than zero unless $u = 0$. From Eq. (7),

$$
u^T[-(L + \Delta)]u = -u^T Lu - u^T \Delta u = -u^T Lu - hu_k^2
$$

(8)

where $h = \Delta_{k \times k} > 0$, and $u_k$ is the $k^{th}$ element of the vector $u$. As the communication graph is strongly connected, $L$ is positive semi-definite with a single zero eigenvalue, [26], and it is diagonalizable, [27], with all real eigenvalues. Let $\lambda_i, i = 1, 2, \ldots, N$ be the eigenvalues of $L$ in descending order, $\lambda_1 \geq \lambda_2 \geq \cdots > \lambda_{N-1} > \lambda_N = 0$. The canonical form of $L$ is $L = \mathbf{V} \Lambda \mathbf{V}^T$, where $\Lambda$ is a diagonal matrix consisting of the eigenvalues of $L$, and $\mathbf{V}$ is the right eigenvector-matrix,

$$
\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_N \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} v_1 & v_2 & \cdots & v_N \end{bmatrix}
$$

(9)

6
Since \( v_N \) is the eigenvector corresponding to \( \lambda_N = 0 \), following the definition of \( L, v_N = cI^N \) where \( c \neq 0 \) is a real value. Substituting \( L = VA^T \) into Eq. (8) and taking Eq. (9) into account, the following holds:

\[
-u^T Lu - h u_k^2 = -z^T \begin{bmatrix}
\lambda_1 & 0 & 0 \\
0 & \lambda_2 & \\
& & \ddots \\
0 & & & \lambda_N
\end{bmatrix} z - h u_k^2 = -\sum_{i=1}^{N} \lambda_i z_i^2 - h u_k^2 \tag{10}
\]

where \( z = V^T u \). Since \( V^T \) is a nonsingular matrix it is invertible, and its inverse matrix is \( V \) and \( u = Vz \).

For any \( z \neq \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}^T \) and \( u_k \), the right hand side of Eq. (10) is negative, except \( z_i = 0 \forall i = 1,2,\cdots,N-1 \) and \( u_k = 0 \). The remaining condition is \( z = \begin{bmatrix} 0 & \cdots & 0 \end{bmatrix}^T \). As \( V \) is not singular, \( u = Vz \neq 0 \) while \( z_N \neq 0 \). According to Eq. (10) and since \( v_N = c1^N \),

\[
u = Vz = \begin{bmatrix} v_1 & v_2 & \cdots & c1 \end{bmatrix} \begin{bmatrix} 0 & \cdots & 0 \end{bmatrix}^T = cz_N1 \tag{11}
\]

Now that \( u = cz_N1 \) and \( c, z_N \in R \) are non-zero, \( u_k = cz_N \neq 0 \). However, this is in contradiction with the assumption made before which is \( u_k = 0 \). Thus, Eq. (10) is negative for any vector \( z \) and since \( u = Vz \), Eq. (8) is negative for any \( u \neq 0 \) which proves \(- (L + \Delta) \) is negative definite. We define \( y = S - \tilde{P}_T 1 \), therefore, \( S = y + \tilde{P}_T 1 \) and \( \dot{S} = \dot{y} \). By substituting \( y \) and \( \dot{y} \) into Eq. (6), we obtain

\[
\dot{y} = -(L + \Delta)y, \quad y = S - \tilde{P}_T 1, \quad y(t_0) = \tilde{P}_T 1 - \tilde{P}_T 1 = \delta 1 \tag{12}
\]

As \(- (L + \Delta) \) is Hurwitz the dynamics of Eq. (12) is exponentially stable. It means \( y \to 0 \) and therefore \( S \to \tilde{P}_T 1 \). This completes the proof. \( \square \)

### 4.2 Proportional Power Sharing Strategies

The consensus algorithm of Section 4.1 enable the DGs to compute the updated capacity of the microgrid under perturbation. In this section, we propose methods by which individual agents command power to the physical layer based on consensus. Subsequent to a capacity variation such as \( \delta \) in Section 4.1, three slightly different strategies are proposed through which the DGs meet the load demand \( P_L \). The first and third strategies are discussed in detail. The second strategy is similar to the first and hence its details are omitted. Assuming at \( t = t_0 \), \( P_{k,\max} \) changes to \( \tilde{P}_{k,\max} = P_{k,\max} + \delta \), the first strategy to generate \( P_k \)'s is

\[
\begin{align*}
\text{Strategy 1:} \\
\begin{cases}
P_k = \frac{P_k}{s_k} \tilde{P}_{k,\max} \\
P_i = \frac{P_i}{s_i} P_{i,\max}
\end{cases}
\text{ where } i = 1,2,\cdots,N, \ i \neq k
\end{align*} \tag{13}
\]

where \( s_i, i = 1,2,\cdots,N \), are the estimates of \( \tilde{P}_T \), as discussed in Section 4.1 and \( \lim_{t \to \infty} s_i = \tilde{P}_T \) according to Lemma 1. A potential issue may arise when \( s_i(t) \) crosses or approaches zero for some \( t > t_0 \) such that \( P_i \) diverges. In this regard, we state and prove the following Lemma:

**Lemma 2.** Considering the LTI system defined in Eq. (6), if \( |\delta| < \theta P_T / (1 + \sqrt{N}) \) with \( 0 < \theta < 1 \) \( - (P_L / P_T) \), the following holds

\[
(1 - \theta)P_T \leq s_i(t) \leq (1 + \theta)P_T \quad \text{and} \quad P_i(t) < P_{i,\max} \quad \forall \ t > t_0 \tag{14}
\]

**Proof.** From Eq. (12), we note that \( y_i = s_i - \tilde{P}_T \). Since Lemma 1 shows that \(- (L + \Delta) \) is Hurwitz, therefore from Eq. (12) we have,

\[
y(t) = e^{-(L + \Delta)t} y(t_0) \Rightarrow \|y(t)\| \leq \|e^{-(L + \Delta)t}\| \|y(t_0)\| \tag{15}
\]
As explained in Lemma 1, \((-L+\Delta)\) diagonalizable and all of its eigenvalues are negative and real. Assuming \(\lambda_1 < 0\) is the largest eigenvalue of \((-L+\Delta)\) and since \(y(t_0) = -\delta I\), we have,
\[
\|y(t)\| \leq e^{\lambda_1 t}\|y(t_0)\| = e^{\lambda_1 t}\sqrt{N}\delta \leq \sqrt{N}\delta
\]  
(16)

Hence, \(\|y\|\) is bounded. Since \(|y_i| \leq \|y(t)\|\), therefore
\[
|y_i(t)| \leq \sqrt{N}\delta \quad \forall i = 1, 2, \cdots, N
\]  
(17)

If \(\delta < \theta P_T/(1 + \sqrt{N})\), then it follows that
\[
|y_i(t)| \leq \sqrt{N}\delta \leq \sqrt{N}\theta P_T/(1 + \sqrt{N})
\]  
(18)

and since \(y_i = s_i - \hat{P}_T\), therefore we have
\[
\hat{P}_T - \sqrt{N}\theta P_T/(1 + \sqrt{N}) \leq s_i(t) \leq \hat{P}_T + \sqrt{N}\theta P_T/(1 + \sqrt{N})
\]  
(19)

Since \(\hat{P}_T = P_T + \delta\), and from the assumption \(\delta < \theta P/(1 + \sqrt{N})\), we have
\[
P_T - \theta P_T/(1 + \sqrt{N}) - \sqrt{N}\theta P_T/(1 + \sqrt{N}) \leq s_i(t) \leq P_T + \theta P_T/(1 + \sqrt{N}) + \sqrt{N}\theta P_T/(1 + \sqrt{N})
\]  
(20)

Thus,
\[
(1 - \theta) P_T \leq s_i(t) \leq (1 + \theta) P_T
\]  
(21)

Since for all \(t > t_0\), the output power of each DG should satisfy \(P_i(t) = \frac{P_i}{s_i(t)} P_{i,\text{max}} < P_{i,\text{max}}\), it is required that \(s_i(t) > P_L\) for all \(t > t_0\). For guaranteeing \(s_i(t) > P_L\), from Eq. (21), we can impose \((1 - \theta) P_T > P_L\). Therefore, under the dynamics of \(\delta\) in Eq. (6), \(P_T > P_L/(1 - \theta)\) or \(1 - (P_L/P_T) > \theta\) ensures that \(P_i(t) < P_{i,\text{max}}\). This completes the proof.

From Lemma 2, it may appear that as the number of DGs, \(N\), increases, there will be a bigger restriction on \(\delta\), since \(\delta < \theta P_T/(1 + \sqrt{N})\). However, it can be shown that the above inequality is not restrictive, mainly because as \(N\) increases, \(P_T\) also increases. An analysis of this aspect is given in Appendix A. So far, it is proved that strategy 1 is valid provided changes in \(\delta\) satisfy the conditions in Lemma 2. Defining the total instantaneous output power of the microgrid as \(P_O(t)\), from Eq. (13),
\[
P_O(t) = \frac{P_L}{s_k} \hat{P}_{k,\text{max}} + \sum_{i=1,i \neq k}^{N} \frac{P_L}{s_i} P_{i,\text{max}}
\]  
(22)

Therefore,
\[
P_O(t) = \frac{P_L}{s_k} \delta + \sum_{i=1}^{N} \frac{P_L}{s_i} P_{i,\text{max}}
\]  
(23)

Thus, defining the instantaneous error \(E(t) = P_O(t) - P_L\), we have,
\[
E(t) = P_O(t) - P_L = \frac{P_L}{s_k} \delta + \sum_{i=1}^{N} \frac{P_L}{s_i} P_{i,\text{max}} - P_L
\]  
(24)

At \(t = t_0, s_i = P_T\) for \(i = 1, 2, \cdots, N\). Thus,
\[
E(t_0) = P_L \left[ \frac{\delta}{P_T} + \sum_{i=1}^{N} \frac{P_{i,\text{max}}}{P_T} - 1 \right]
\]  
(25)
Since \( \sum_{i=1}^{N} \frac{P_{i,\text{max}}}{P_T} = 1 \), therefore
\[
E(t_0) = P_L \frac{\delta}{P_T} \tag{26}
\]
Equation (26) shows that \( E(t_0) \neq 0 \), and since \( E(t) \) is continuous, it implies that a perturbation \( \delta \) causes a transient mismatch between the delivered power \( P_O(t) \) and the load \( P_L \). The error \( E(t) \to 0 \) at steady-state, as proven in Lemma 1. Therefore, Strategy 1 given in Eq. (13), causes a temporary mismatch of power following a perturbation. This issue is addressed in Section 4.3.

The second strategy, which is slightly different from the first one, is as follows:

Strategy 2:
\[
\begin{align*}
P_k &= \frac{P_L}{P_T} \hat{P}_{k,\text{max}} \\
P_i &= \frac{P_L}{s_i} P_{i,\text{max}} \quad \text{where } i = 1, 2, \ldots, N \quad i \neq k
\end{align*}
\tag{27}
\]
As before, the total instantaneous output power of the microgrid \( P_O(t) \) is
\[
P_O(t) = \frac{P_L}{P_T + \delta (\hat{P}_{k,\text{max}})} + \sum_{i=1,i\neq k}^{N} \frac{P_L}{s_i} P_{i,\text{max}} \tag{28}
\]
We again evaluate the error \( E(t) = P_O(t) - P_L \) for \( t \geq t_0 \), yielding
\[
E(t) = \frac{P_L}{P_T + \delta (\hat{P}_{k,\text{max}} + \delta)} + \sum_{i=1,i\neq k}^{N} \frac{P_L}{s_i} P_{i,\text{max}} - P_L \tag{29}
\]
Upon simplifying, we obtain
\[
E(t) = P_L \left[ \frac{(\hat{P}_{k,\text{max}} + \delta)}{P_T + \delta} + \sum_{i=1,i\neq k}^{N} \frac{P_{i,\text{max}}}{s_i} - 1 \right]
\]
Since at \( t = t_0, \ s_i = P_T \) for \( i = 1, 2, \ldots, N \), and \( \sum_{i=1,i\neq k}^{N} P_{i,\text{max}} = P_T - \hat{P}_{k,\text{max}} \),
\[
E(t_0) = P_L \frac{\delta (P_T - \hat{P}_{k,\text{max}})}{P_T (P_T + \delta)} \tag{30}
\]
Equation (30) shows that \( E(t_0) \neq 0 \), and since \( E(t) \) is continuous, it implies that similar to Strategy 1, a perturbation \( \delta \) causes a transient mismatch between the delivered power \( P_O(t) \) and the load \( P_L \) in Strategy 2. The error \( E(t) \to 0 \) at steady-state, as proven in Lemma 1.

The last candidate strategy is proposed as

Strategy 3:
\[
\begin{align*}
P_k &= \frac{P_L}{s_k} (\hat{P}_{k,\text{max}} + s_k - P_T) \\
P_i &= \frac{P_L}{s_i} P_{i,\text{max}} \quad \text{where } i = 1, 2, \ldots, N \quad i \neq k
\end{align*}
\tag{31}
\]
The Strategy 3 allows DGs to update their output power more smoothly compared to the first two strategies. In this case,
\[
P_O(t) = \frac{P_L}{s_k} (\hat{P}_{k,\text{max}} + s_k - P_T) + \sum_{i=1,i\neq k}^{N} \frac{P_L}{s_i} P_{i,\text{max}} \tag{32}
\]
Therefore,

$$E(t) = P_L \left[ -\frac{P_T}{s_k} + \sum_{i=1}^{N} \frac{P_{i,max}}{s_i} \right]$$  \hspace{1cm} (33)$$

Since $s_i = P_T$ for all $i = 1, 2, \ldots, N$, at $t = t_0$, $E(t_0) = 0$. However, $E(t)$ still undergoes transient fluctuations. Based on Eq. (33),

$$\frac{\dot{E}(t)}{P_L} = \frac{P_T \dot{s}_k(t)}{s_k^2(t)} - \sum_{i=1}^{N} \frac{P_{i,max} \dot{s}_i(t)}{s_i^2(t)}$$  \hspace{1cm} (34)$$

Equation (34) can be further simplified using Eq. (6) as follows,

$$\frac{E(t)}{P_L} = \frac{P_T \dot{s}_k(t)}{s_k^2(t)} - \sum_{i=1}^{N} \frac{P_{i,max} \dot{s}_i(t)}{s_i^2(t)} \left[ -(L + \Delta)S + hd_k(P_T + \delta) \right]$$  \hspace{1cm} (35)$$

Since at $t = t_0$, $s_i = P_T$ for all $i = 1, 2, \ldots, N$, $S(t_0) = P_T 1$ and Eq. (35) becomes

$$\frac{\dot{E}(t_0)}{P_L} = \frac{P_T \dot{s}_k(t_0)}{P_T^2} - \left[ \frac{P_{1,max}}{P_T^2} \frac{P_{2,max}}{P_T^2} \ldots \frac{P_{k,max}}{P_T^2} \ldots \frac{P_{N,max}}{P_T^2} \right] \left[ -L1P_T - hd_kP_T + hd_kP_T + hd_k\delta \right]$$  \hspace{1cm} (36)$$

Simplifying Eq. (36) yields

$$\frac{\dot{E}(t_0)}{P_L} = \frac{P_T \dot{s}_k(t_0)}{P_T^2} - hP_{k,max}\delta$$  \hspace{1cm} (37)$$

Referring to Eq. (6), at $t = t_0$, the term $\dot{s}_k(t_0)$ in Eq. (37) is

$$\dot{s}_k(t_0) = h(P_T + \delta) - hP_T = h\delta$$  \hspace{1cm} (38)$$

Therefore, from Eqs. (37) and (38),

$$\frac{\dot{E}(t_0)}{P_L} = h\delta \frac{P_T - P_{k,max}}{P_T^2}$$  \hspace{1cm} (39)$$

where from Eq. (5), $h$ is a positive scalar chosen by $k^{th}$ agent. Thus, although $E(t_0) = 0$, $\dot{E}(t_0) \neq 0$. Therefore, as $E(t)$ is continuous, similar to Strategies 1 and 2, a change $\delta$ results in a transient mismatch between $P_O$ and $P_L$. It is shown that the three strategies proposed above match the load power $P_T$ at steady-state while producing transient deviations. This transient issue is resolved in the next section, where a strategy is proposed to practically maintain $P_O = P_L$ at any time.

### 4.3 Proportional Power Sharing with Transient Power Match

Upon a perturbation in $P_{k,max}$, which results in a change in $P_T$, the agents estimate $\tilde{P}_T = P_T + \delta$ through Eq. (6). Among the DGs, only DG$_k$ has a knowledge of $\tilde{P}_T = P_T + \delta$. The other DGs in the microgrid converge to $\tilde{P}_T$ through consensus only at steady-state. This leads to the transient power mismatch discussed in Section 4.2. To remove this transient mismatch, we propose a strategy where DG$_k$ modulates its power delivery as follows, while the other DGs maintain the same strategy as in Section 4.2.

$$P_k = \frac{P_k}{s_k} \frac{P_T'}{P_{k,max}}$$
$$P_i = \frac{P_i}{s_i} P_{i,max} \quad \text{for} \quad i = 1, 2, \ldots, N \quad i \neq k$$  \hspace{1cm} (40)$$
where $P'_{k,\text{max}}$ is an auxiliary dynamic variable required to modulate the instantaneous power of DG$k$. Hence, at $t = t_0$, $P'_{k,\text{max}}(t_0) = P_{k,\text{max}}(t_0)$, and it is required to converge to $(P_{k,\text{max}} + \delta)$ while $s_i$ converges via consensus. With the goal of maintaining $P_O(t) = P_L$ for all $t > t_0$, we must have

$$P_L = P_O(t) = \frac{P_L}{s_k} P'_{k,\text{max}} + \sum_{i=1,i\neq k}^{N} \frac{P_L}{s_i} P_{i,\text{max}}$$ (41)

Thus,

$$\frac{P'_{k,\text{max}}}{s_k} + \sum_{i=1,i\neq k}^{N} \frac{P_{i,\text{max}}}{s_i} - 1 = 0$$ (42)

Therefore, $P'_{k,\text{max}}$ is

$$P'_{k,\text{max}}(t) = s_k(t) \left[ 1 - \sum_{i=1,i\neq k}^{N} \frac{P_{i,\text{max}}}{s_i(t)} \right]$$ (43)

The algorithm for updating $P'_{k,\text{max}}$ and $s_i$ for $i = 1, 2, \cdots, N$ in Eq. (40) is as follows:

$$P'_{k,\text{max}}(t) = s_k(t) \left[ 1 - \sum_{i=1,i\neq k}^{N} \frac{P_{i,\text{max}}}{s_i(t)} \right]$$ (44a)

$$S = -(L + \Delta)S + h d k \tilde{P}_T \quad \text{where} \quad S(t_0) = P_T 1$$ (44b)

Based on Eqs. (40) and (44), we state and prove the following lemma:

**Lemma 3.** The dynamic system of Eq. (44) is stable, i.e. the terms $P'_{k,\text{max}}$, $\frac{P_{i,\text{max}}}{s_i}$ and $S$ remain bounded if $|\delta| < \theta P_T / (1 + \sqrt{N})$, where $0 < \theta < 1 - (P_L / P_T)$. Furthermore, $P'_{k,\text{max}} \rightarrow \tilde{P}_{k,\text{max}}$ and $S \rightarrow \tilde{P}_T 1$, while the instantaneous delivered power satisfies Eq. (41) for all $t \geq t_0$.

**Proof.** Since Eq. (44b) is equivalent to Eq. (6), per Lemma 1, the dynamic system of Eq. (44b) is ISS. Therefore $S$ is bounded. Additionally, as Eqs. (6) and (44b) have the same initial conditions, i.e. $S(t_0) = P_T 1$, thus $S \rightarrow \tilde{P}_T 1$. Following $|\delta| < \theta P_T / (1 + \sqrt{N})$, from Lemma 2 we have $(1 - \theta)P_T \leq s_i(t) \leq (1 + \theta)P_T$ with $0 < \theta < 1 - (P_L / P_T)$. Thus,

$$\frac{P_{i,\text{max}}}{(1 + \theta)P_T} \leq \frac{P_{i,\text{max}}}{s_i} \leq \frac{P_{i,\text{max}}}{(1 - \theta)P_T}$$ (45)

Therefore, $\frac{P_{i,\text{max}}}{s_i}$ is bounded for all $i = 1, 2, \cdots, N$. It demonstrates that Eq. (44a) represents a viable way to update $P'_{k,\text{max}}$. By plugging $P'_{k,\text{max}}$ from Eq. (44a) into Eq. (40), $P_O(t)$ simplifies to

$$P_O(t) = \frac{P_L}{s_k} \left[ 1 - \sum_{i=1,i\neq k}^{N} \frac{P_{i,\text{max}}}{s_i} \right] + \sum_{i=1,i\neq k}^{N} \frac{P_L}{s_i} P_{i,\text{max}} = P_L$$ (46)

for all $t \geq t_0$. Since $S$ converges to $\tilde{P}_T 1$, from Eq. (44a) we therefore deduce

$$P'_{k,\text{max}}(t) \rightarrow \tilde{P}_T \left[ 1 - \sum_{i=1,i\neq k}^{N} \frac{P_{i,\text{max}}}{P_T} \right] = \tilde{P}_{k,\text{max}}$$ (47)

This completes the proof. \qed
The controller designed in Eqs. (44a) and (44b) maintains \( P_D(t) = P_L \) following a variation in the power capacity of a DG, namely \( P_{k,\text{max}} \). However, to compute the term
\[
\sum_{i=1, i \neq k}^{N} \frac{P_{i,\text{max}}}{s_i}
\] (48)

in \( P'_{k,\text{max}} \), as given in Eq. (44a), the \( k^{th} \) agent requires additional information. The following approach is proposed to enable the \( k^{th} \) agent to attain this information distributively. This approach is based on the distributed finite-time average consensus studied in [24]. According to [24], each agent \( i \), shares \( \frac{P_{i,\text{max}}}{s_i(t)} \) to its outgoing neighbors \( \mathcal{N}_i^+ \) where, following Section 2, \( \mathcal{N}_i^+ \) stands for the set of nodes which receives signals from node \( i \). Accordingly, based on what follows the agents are able to distributively compute the instantaneous average of all \( \frac{P_{i,\text{max}}}{s_i(t)} \) where \( i = 1, 2, \cdots, N \), i.e.
\[
C_a(t) = \frac{\sum_{i=1}^{N} \frac{P_{i,\text{max}}}{s_i(t)}}{N}
\] (49)

Then, the \( k^{th} \) agent can compute Eq. (48) via
\[
\sum_{i=1, i \neq k}^{N} \frac{P_{i,\text{max}}}{s_i} = N \times C_a(t) - \frac{P_{k,\text{max}}}{s_k}
\] (50)

One example of applying this distributed finite-time average consensus is represented in [28]. Similar to [28], the steps of executing the finite time algorithm is as following:
\[
\bar{g}_i(m + 1) = p_i \overline{g}_i(m) + \sum_{j \in \mathcal{N}_i^-} p_{ij} \overline{g}_j(m)
\]
\[
g_i(m + 1) = p_i g_i(m) + \sum_{j \in \mathcal{N}_i^-} p_{ij} g_j(m)
\] (51)

where \( \overline{g}_i(0) = \frac{P_{i,\text{max}}}{s_i} \) and \( g_i(0) = 1 \) for \( i = 1, 2, \cdots, N \). Additionally, \( p_{ij} = \frac{1}{1 + |\mathcal{N}_j^-|} \) for \( i \in \mathcal{N}_j^+ \cup \{ j \} \), otherwise is zero. Let define the vectors
\[
\overline{g}_{i,2m}^T = [\overline{g}_i(1) - \overline{g}_i(0), \overline{g}_i(2) - \overline{g}_i(1), \cdots, \overline{g}_i(2m + 1) - \overline{g}_i(2m)]
\]
\[
g_{i,2m}^T = [g_i(1) - g_i(0), g_i(2) - g_i(1), \cdots, g_i(2m + 1) - g_i(2m)]
\] (52)

and the following Hankel matrices
\[
\Gamma\{ \overline{g}_{i,2m}^T \} \triangleq \begin{bmatrix}
\overline{g}_{i,2m}(1) & \cdots & \overline{g}_{i,2m}(m + 1) \\
\overline{g}_{i,2m}(2) & \cdots & \overline{g}_{i,2m}(m + 2) \\
\vdots & \ddots & \vdots \\
\overline{g}_{i,2m}(m + 1) & \cdots & \overline{g}_{i,2m}(2m + 1)
\end{bmatrix}
\] (53)

and
\[
\Gamma\{ g_{i,2m}^T \} \triangleq \begin{bmatrix}
g_{i,2m}(1) & \cdots & g_{i,2m}(m + 1) \\
g_{i,2m}(2) & \cdots & g_{i,2m}(m + 2) \\
\vdots & \ddots & \vdots \\
g_{i,2m}(m + 1) & \cdots & g_{i,2m}(2m + 1)
\end{bmatrix}
\] (54)

Each agent \( i \) runs the steps in Eq. (51) for \( 2N + 1 \) times and keeps the values \( \overline{g}_i(m) \) and \( g_i(m) \) for \( m = 1, 2, \cdots, 2N + 1 \). Having \( \overline{g}_i(m) \) stored for the \( 2N + 1 \), each agent \( i \) establishes the vectors \( \overline{g}_{i,2m}^T \) and \( g_{i,2m}^T \).
defined in Eq. (52) starting from \( m = 0 \). At the same time, all individual agent \( i \) construct the Hankel matrices \( \Gamma \{ g_{1,2m}^T \} \) and \( \Gamma \{ g_{1,2m}^T \} \) defined in Eqs. (53) and (54), respectively. Additionally, they calculate the ranks of the Hankel matrices for each \( m \) and repeat the same procedure for the next \( m + 1 \) until for a specific \( m \) either \( \Gamma \{ g_{1,2m}^T \} \) or \( \Gamma \{ g_{1,2m}^T \} \) becomes a defective matrix. Assume \( \Gamma \{ g_{1,2M}^T \} \) or \( \Gamma \{ g_{1,2M}^T \} \) is the first matrix which loses its full rank where \( \beta_i = [\beta_{i0}, \cdots, \beta_{i, M-1}]^T \) is its corresponding kernel. Having the kernel \( \beta_i \), the \( i^{th} \) agent computes the average of all \( \overline{g}_i(0) = \frac{p_{i,max}}{s_i} \) for \( i = 1, 2, \cdots, N \) defined as \( C_a \) in Eq. (49) through the following
\[
C_a(t) = \frac{1}{N} \sum_{i=1}^{N} \overline{g}_i(0) = \frac{[\overline{g}_1(0), \overline{g}_1(1), \cdots, \overline{g}_1(M)]_{\beta_i}}{[g_1(0), g_1(1), \cdots, g_1(M)]_{\beta_i}}
\]
Thereby, the \( k^{th} \) agent can achieve \( C_a(t) \), distributively.

At this step, the \( k^{th} \) agent obtains the term in Eq. (48) via Section 4.3. By plugging Eq. (48) back to the Eq. (44a) the \( k^{th} \) agent is able to compute \( P_{k,max} \). To implement the proposed strategy practically, Eqs. (44a) and (44b) are discretized firstly, since in practice the signals and algorithms update, digitally. Afterwards, one iteration of discretized Eq. (44b) is implemented to update \( s_i \) as \( s_i(1) \). by having \( \frac{p_{i,max}}{s_i(1)} \), the distributed finite consensus algorithm Eqs. (51) to (55) is implemented through which the \( k^{th} \) agent updates \( P_{k,max}(1) \). Then, through Eq. (40) the agents send the power command to their corresponding DGs. Again, the same procedure repeats until \( S \) converges to \( P_f \).

5 Controller Layout of Physical Layer

The proposed power control methods for DGs introduced in this study are required to be implemented on both cyber and physical layer of microgrids. The physical layer which includes DGs is where controllers are designed to control the output power of DGs. In this study, the problem of proportional power sharing is addressed in the grid connected mode, hence the frequency and voltage of DGs are imposed by the main grid. Therefore, frequency and voltage control methods, such as droop control, are not considered in this study. Furthermore, the reactive power control in the grid connected mode is not studied for the practical reason of availability of reactive power in the main grid. Therefore, the required reactive power of the microgrid can be maintained from the main grid.

The desired active power command of each DG, i.e \( P_i^* \) for \( i = 1, 2, \cdots, N \) is calculated by its corresponding agent, i.e \( i^{th} \) agent in the cyber layer. Then, this signal of \( P_i^* \) is sent to the power control block of DG located in the physical layer. The power control block of DGs is represented in Fig. 2a. This block receives the voltage \( V_{abc} \) and current \( I_{abc} \) from the voltage and current measurement units installed on the output of each DG, as shown in Fig. 3. Figure 3 also shows that each DG is connected to the main grid via a dedicated transformer to match the voltage between the DG and the main grid, as the output voltage of the main grid is significantly higher than the output voltage of DGs. To control the generated power of a DG, i.e., \( P_i \), it is required to control its output current since \( V_{abc} \) and the frequency of the microgrid are fixed by the main grid. To achieve this, the desired active power command \( P_i^* \) issued from \( i^{th} \) agent is also considered as the other input in Fig. 2a. Using the Phase-Locked-Loop (PLL) block, the signals in Fig. 2a are converted to their equivalent values in the \( dq \) reference frame, i.e., \( V_{dq} \) and \( I_{dq} \).

Next, the outputs of the \( V_{dq} \) and \( I_{dq} \) are fed as inputs to Fig. 2b. The parameters \( C_1, C_2 \) and the \( PI \) controllers coefficients together with the upper and lower bounds of the saturation blocks in Fig. 2b are all defined in Section 6. The outputs of the Fig. 2b are regarded as the imaginary and real values of a complex number, are the inputs of the Fig. 2c. These inputs are converted to the amplitude and phase angle of the same complex value. The amplitude and the phase signals, together with the voltage angle \( \omega t \), obtained from the PLL in the Fig. 2a, constitute the three phase signal fed to the PWM in Fig. 2c. Finally, each PWM sends the switching signals to the three level inverter of its corresponding DG which is illustrated in Fig. 3.
Figure 2: Physical layer control scheme
6 Simulations

In this section, the performance of the proposed control methods explained in Section 4.3 is evaluated through the simulation of a microgrid consisting of six inverter-based DGs shown in Fig. 4. Furthermore, the performances of the strategies 1 and 3, provided in Eqs. (13) and (31) respectively, are juxtaposed with the performance of the controller in Section 4.3. The simulations are accomplished using the Simscape toolbox of Matlab. The simulated DGs are numbered from 1 to 6 and are connected to the main grid in parallel as depicted in Fig. 4. Each DG has a corresponding agent in the cyber layer where the updated
value of the desired output power is computed by the agents using the information obtained through their bidirectional communication structure, as shown in Fig. 5. Note that the communication graph of the DGs in Fig. 5 is strongly connected per its definition in Section 2.

As the communication graph in Fig. 5 is a bidirectional graph, per Section 2, the adjacency matrix of the graph is symmetric. The adjacency and degree matrices are chosen as

\[
A = \begin{bmatrix}
0 & 6 & 0 & 6 & 6 & 0 \\
6 & 0 & 0 & 6 & 0 & 0 \\
0 & 0 & 0 & 6 & 6 & 0 \\
6 & 6 & 0 & 6 & 6 & 0 \\
6 & 0 & 6 & 0 & 6 & 0 \\
0 & 0 & 0 & 6 & 6 & 0
\end{bmatrix}, \quad
D = \begin{bmatrix}
18 & 0 & 0 & 0 & 0 & 0 \\
0 & 12 & 0 & 0 & 0 & 0 \\
0 & 0 & 12 & 0 & 0 & 0 \\
0 & 0 & 0 & 30 & 0 & 0 \\
0 & 0 & 0 & 0 & 24 & 0 \\
0 & 0 & 0 & 0 & 0 & 12
\end{bmatrix}
\] (56)

From Section 2, the correlated Laplacian matrix \( L = A - D \) is defined as

\[
L = \begin{bmatrix}
18 & -6 & 0 & -6 & -6 & 0 \\
-6 & 12 & 0 & -6 & 0 & 0 \\
0 & 0 & 12 & -6 & -6 & 0 \\
-6 & -6 & -6 & 30 & -6 & -6 \\
-6 & 0 & -6 & -6 & 24 & -6 \\
0 & 0 & 0 & -6 & -6 & 12
\end{bmatrix}
\] (57)

Let the maximum capacity of the DGs \( P_{i,\text{max}} \) for \( i = 1, 2, \ldots, 6 \) be

\[
P_{1,\text{max}} = 600 \text{ kw} \quad P_{2,\text{max}} = 450 \text{ kw} \quad P_{3,\text{max}} = 300 \text{ kw} \\
P_{4,\text{max}} = 150 \text{ kw} \quad P_{5,\text{max}} = 750 \text{ kw} \quad P_{6,\text{max}} = 150 \text{ kw}
\] (58)

Thus, the maximum capacity of the whole microgrid is \( P_T = \sum_{i=1}^{6} P_{i,\text{max}} = 2400 \text{ kw} \), and assume the load demand is \( P_L = 1600 \text{ kw} \). According to Section 3, the agents have knowledge of \( P_L \) and \( P_T \) at initial time. Therefore, each agent is able to compute the proportional power share ratio \( r = \frac{P_L}{P_T} \) defined in Eq. (4), independently, which is \( \frac{2}{3} \), initially. Therefore, the output power of each DG \( i \) for \( i = 1, 2, \ldots, 6 \), based on
Figure 6: Output powers $P_i, i = 1, 2, \cdots, 6$, under a variation in $P_{1_{\text{max}}}$ are depicted in the figures (a) through (f) respectively.
Figure 7: Microgrid total output power $P_O$ obtained from the proposed control method of Section 4.3

Figure 8: Consensus trajectories of agents on $\tilde{P}_T$ from Eq. (44b) and trajectories of $r_i, i = 1, 2, \cdots, 6$, from Eq. (40). (a) Consensus trajectories on $\tilde{P}_T$ (b) The signals $r_i$

the proportional power sharing, must be,

\[
\begin{align*}
P_1 &= 400\, kW \\
P_2 &= 300\, kW \\
P_3 &= 200\, kW \\
P_4 &= 100\, kW \\
P_5 &= 500\, kW \\
P_6 &= 100\, kW
\end{align*}
\] (59)

During $t = [0, 3] \, sec$, the power capacity of each DG remains unchanged and hence, each of the DGs generates its active power share as calculated in Eq. (59). At $t = 3\, sec$ the power capacity of DG$_1$ undergoes a step change, thereby $P_T$ has an increment of 300 kW, and at $t = 9\, sec$ we introduce a decrement of 600 kW to the capacity of DG$_1$.

The simulated microgrid consists of several components such as inverters, output filters of inverters, transformers, PWM, PI controllers, line impedance, loads, DC resources, measurement units, PLL and abc/dq0 converters. To emulate the main grid a dispatchable generator is considered in the simulation as shown in Fig. 4. The parameters of the transformer that connects the main grid to the distribution system and those of the transformers which connect the DGs to the distribution system are given in Table 1. The parameters of the distribution system are provided in Table 2. The PI controllers depicted in Fig. 2b are identical. The PI controllers of all DGs are also identical, meaning they all have the same $P$ and $I$ gains,
The energy resource of each DG is simulated as a DC power source, then by utilizing an inverter, the DC current converts to the AC current, as shown in Fig. 3. In the same figure, to remove the harmonics from the output power of the inverter, an output filter is applied and then connected to the main grid via a transformer, as illustrated in Fig. 3. The output filter consists of RL and RC branches. The resistive and inductive elements of each RL component are set as \( R_1 = 5.4946 \times 10^{-4} \Omega \) and \( L = 1.4575 \times 10^{-4} H \), respectively. The RC components for the output filters of each individual DG \( i \) arranged in the delta format have \( P_i(W) \) and reactive power \( Q_i(kVar) \) as given in Table 3. In Fig. 2b, \( C_1 = 0.0039 \), \( C_2 = 0.21 \), and the upper and lower limit of the saturation blocks are \(+1.5\) and \(-1.5\), respectively. The power control of each DG established in the physical layer is depicted in Fig. 2.

Starting from \( t = 3 \text{ sec} \), \( P_{1,\text{max}} \) increases from 600 kW to 900 kW. Therefore, the microgrid maximum power capacity increases from 2400 kW to 2700 kW. Based on the approach explained in Section 4.3, the finite time algorithm in Eqs. (51) to (55) is embedded in the consensus algorithm Eq. (44b) to have DGs apply the proposed control law in Eqs. (40), (40) and (44), distributively. The results of the simulations are shown in Fig. 6 where \( P_i \) increases and \( P_i, i = 2, 3, 4, 5, 6, \) decreases. During \( t = [3, 9] \text{ sec} \), the microgrid output power \( P_O \) remains almost equal to \( P_L = 1600 kW \), as shown in Fig. 7. The slight difference between \( P_O \) and \( P_L \) is due to resistive losses in the DGs due to the resistor elements shown in Fig. 3. Considering
Table 2: Parameters of the Grid

| Parameter                                      | Value                      |
|-----------------------------------------------|----------------------------|
| Load 1 Nominal Voltage ($kV_{ph-ph}$)          | 25                         |
| Load 1 Active Power P (kW)                    | 250                        |
| Load 2 Active Power P (kW)                    | 2000                       |
| Load 3 Power S (kVA)                          | 30000+j2000                |
| Line 1 $Z_1$ Length (km)                      | 8                          |
| Positive and Zero Sequence $R(\Omega/\text{km})$ | [0.1153 0.413]            |
| Positive and Zero Sequence $L(\text{H/\text{km}})$ | [1.05e-3 3.32e-3]         |
| Positive and Zero Sequence $C(\text{F/\text{km}})$ | [11.33e-009 5.01e-009]     |
| Line 2 $Z_2$ Length (km)                      | 14                         |
| Positive and Zero Sequence $R(\Omega/\text{km})$ | [0.1153 0.413]            |
| Positive and Zero Sequence $L(\text{H/\text{km}})$ | [1.05e-3 3.32e-3]         |
| Positive and Zero Sequence $C(\text{F/\text{km}})$ | [11.33e-009 5.01e-009]     |
| Nominal Frequency (Hz)                        | 60                         |

Table 3: Active and reactive powers of RC components of each output filter

| DG Number | Active Power(W) | Reactive Power(kVar) |
|-----------|-----------------|----------------------|
| 1         | 400             | 20                   |
| 2         | 200             | 10                   |
| 3         | 600             | 30                   |
| 4         | 500             | 25                   |
| 5         | 300             | 15                   |
| 6         | 400             | 20                   |

Figure 10: (a) Output power $P_1$ according to Strategies 1, 3 and the proposed controller of Section 4.3, and (b) Their corresponding microgrid total power $P_O$
average of time average consensus algorithm is embedded in the consensus algorithm, at each time step of evolution the consensus algorithm converges, the steady state values of the output powers of DGs are different, however they become almost identical during for grid capacity reduction. The power sharing ratio \[\frac{P_i(t)}{s_{i,\text{max}}}\] for \(i = 1, 2, \cdots, 6\) in a distributed way where its corresponding result is illustrated in Fig. 6.

The next variation of the microgrid maximum power capacity occurs at \(t = 9s\) where \(P_{1,\text{max}}\) decreases for \(-600\text{kw}\). Therefore, starting from \(t = 9\text{sec}\), the current capacity of the microgrid which is \(P_T = 2700\text{kw}\) changes to \(P_T = 2100\text{kw}\). Then, similar to the same procedure adopted in reaction to a change in a microgrid capacity, the control method in Eqs. (40) and (44) is triggered. Hence, during \(t = [9, 18]\text{sec}\), according to Fig. 8a, the agents have reached to another consensus on the maximum power capacity of the microgrid which is \(2100\text{kw}\). Figure 6a demonstrates that \(P_1\) becomes \(228.57\text{kw}\) after the convergence during \([9, 18]\text{sec}\). Figure 7 also shows that the output power of the other DGs have increased due to the microgrid capacity reduction. The power sharing ratio \(r_i\) for \(i = 1, 2, \cdots, 6\) are shown in Fig. 8b. The figure illustrates that, during the transient duration of \((9, 15)\text{sec}\) ratios are not equal. On the contrary, they converge to steady state conditions in \([14, 18]\text{sec}\). Furthermore, from Fig. 7, \(P_0\) remains practically equal to \(P_L = 1600\text{kw}\). In Fig. 10a, the results of the proposed control algorithm in Section 4.3 is compared with the results of the strategies defined as Eqs. (13) and (31) in Section 4.2. From this figure, it is clear that the output power of \(P_1\) obtained from the proposed control algorithm Section 4.3 differs from the other two ones during the transient duration. However, after the transient durations of \((3.8)\text{sec}\) and \((9, 15)\text{sec}\) the output power of \(P_1\) from all three methods are the same. Figure 10b demonstrates that the approaches of Eqs. (13) and (31) are ineffective to address the load demand. They produce significant deviation in \(P_0\) from \(P_L\) during transient. On the other hand, upon applying the method of Section 4.3, the deviation drastically reduces, both for increase and decrease in maximum power capacity of DG1.

7 Conclusion

In this research, the problem of distributed proportional power sharing is studied for microgrids that operate in the grid-connected mode. Firstly, a consensus algorithm is designed through which, under a variation in the maximum power capacity of a DG, all DGs in the microgrid estimate the updated microgrid capacity. Utilizing the estimations, they generate their output powers in a distributed manner. Stability and convergence of the consensus algorithm are proven. While the consensus algorithm operates in the cyber layer, power commands are sent to the DGs at the physical layer using multiple strategies, discussed in the research. In this regards, practical issues such as ensuring power commands are within acceptable bounds during the transient time of the consensus method, are addressed. However, the consensus algorithm along with the aforementioned strategies does not guarantee maintaining load power during the transient time. Therefore, a modified strategy is proposed to guarantee a match between demanded and delivered power during transient, while the DGs reach a new consensus following a perturbation in grid capacity. The distributed controller is tested in a simulated microgrid. The microgrid is modeled in Matlab/Simulink using the Simscape toolbox. A complete description of the model along with parameters values used for simulation, are given. Simulation results confirm the effectiveness of the proposed strategy.
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\section*{Appendix A}

Lemma gives the condition $|\delta| < \theta_{P_T} / (1 + \sqrt{N})$ to prevent unfavorable transients in $s_i(t)$. To demonstrate that this condition is not restrictive as $N$ increases, we consider a change in $N$ to $N + 1$ and a corresponding change from $P_T$ to $P_T + P_{N+1,max}$. Further, we impose

\[
\frac{\theta (P_T + P_{N+1,max})}{1 + \sqrt{N + 1}} > \frac{\theta P_T}{1 + \sqrt{N}}
\]
to derive the condition under which $|\delta|$ will increase as we increase $N$ to $N+1$. From Eq. (60), we have,

$$P_{N+1,\text{max}} > \left[ \frac{\sqrt{N+1} - \sqrt{N}}{1 + \sqrt{N}} \right] P_T$$  \hspace{1cm} (61)

From Eq. (61), it can be observed that $P_{N+1,\text{max}}$ can be only a small fraction of $P_T$ to allow $|\delta|$ to increase rather than decrease. For instance, if $N = 3$, then $P_{N+1,\text{max}} > 0.098P_T$, and if $N = 8$, then $P_{N+1,\text{max}} > 0.045P_T$ which are small fractions of $P_T$. In addition, comparing the right hand side of Eq. (61) with the average of $P_T$, $P_{T,\text{avg}} = P_T / N$, we obtain the following minimum ratio of $P_{N+1,\text{max}} / P_{T,\text{avg}}$,

$$N \left[ \frac{\sqrt{N+1} - \sqrt{N}}{1 + \sqrt{N}} \right]$$  \hspace{1cm} (62)

Equation (62) is strictly less than $\frac{1}{2}$ and it converges to $\frac{1}{2}$ for large values of $N$. This proves that $P_{N+1,\text{max}}$ is required to be $P_{N+1,\text{max}} \geq (1/2)P_{\text{avg}}$ at the worst cases to satisfy the condition on $|\delta|$. Therefore, the condition on $|\delta|$ is not restrictive.