Charmed Tetraquarks and Meson-Meson Interactions from LQCD

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(Hadrons to Atomic nuclei from Lattice QCD)

Mini-workshop on “Structure and productions of charmed baryons II”
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Meson spectroscopy w/ charm quarks

Constituent quark models:
mass spectra below open charm threshold

Godfrey, Isgur, PRD 32 (1985).
Barnes, Godfrey, Swanson, PRD 72 (2005).

“NEW” charmonium-like (X, Y, Z) states:
candidates of exotic hadrons (cc\bar{} +...)

Tetraquarks? (T_{QQ'} = QQ'q\bar{}q\bar{} / Q\bar{}Q'q\bar{}q):
Q^{(')} can be strange, charm or bottom-quark

Possible candidates of exotic hadrons
\Rightarrow Tetraquarks T_{QQ'} have not been experimentally discovered yet

H. J. Lipkin, PLB172 (1986).
Bound tetraquarks $T_{QQ'}$?

Why can we expect bound charmed $T_{QQ'}$?

Constituent quark models suggest bound $T_{QQ'}$ because of strongly attractive color magnetic interactions

- **Color magnetic interaction (CMI)**: hadron mass splitting
  \[ V_{\text{CMI}} = -C \cdot \alpha_s \sum_{i<j} \frac{\langle \mathbf{\lambda}(i) \cdot \mathbf{\lambda}(j) \rangle (\mathbf{\sigma}(i) \cdot \mathbf{\sigma}(j))}{M_i M_j} \delta^3(\mathbf{r}_i - \mathbf{r}_j) \]

- **Color-spin matrix elements**:
  \[ \langle v_{ij} \rangle = -\langle (\mathbf{\lambda}(i) \cdot \mathbf{\lambda}(j))(\mathbf{\sigma}(i) \cdot \mathbf{\sigma}(j)) \rangle \]

| $\langle v_{ij} \rangle$ | C=1 | C=8 | C=3 | C=6$^{\text{bar}}$ |
|-------------------------|-----|-----|-----|------------------|
| $S=0$                   | -16 | 2   | -8  | 4                |
| $S=1$                   | 16/3| -2/3| 8/3 | -4/3             |

- CMI proportional to $1/M_i$: strongly attractive $u^{\text{bar}}d^{\text{bar}}$-diquark pair

\[ \Rightarrow \text{Possibility of bound } T_{QQ'} \]
If bound $T_{QQ'}$ is found...

What is structure of bound $T_{QQ'}$?

$$\Rightarrow$$ Tetraquark?

(color-magnetic interaction)

Vijande, Valcarce, PRC80 (2009).
Lee, Yasau, EPJ C64 (2009).
Takeuchi, Takizawa, Shimizu, Pos Hadron2013 (2013).

$$\Rightarrow$$ Meson-meson molecule?

(pion-exchange interactions)

S. Ohkoda et al., PRD86, (2012).

Predicted mass and structures highly depends on model parameters.

What we would like to know from LQCD simulations are ...

1) Bound tetraquarks $T_{QQ'}$ really exist?
2) short-range strong CMI?
3) long-range pion exchange?

Vijande, Valcarce, PRC80 (2009).
Lee, Yasau, EPJ C64 (2009).
Takeuchi, Takizawa, Shimizu, Pos Hadron2013 (2013).
Lattice QCD studies of $T_{QQ'}$

- **Static interactions**
  - Interaction energy in static limit ($Qq^{\text{bar}}$) -- ($Qq^{\text{bar}}$)

  ![Diagram showing static interactions](image)

- **Our work:** scattering on the lattice to search for bound $T_{cc}$, $T_{cs}$

  ![Diagram showing scattering on the lattice](image)

- **Advantage**
  - Dynamics of charm quarks are automatically taken into account

- References:
  - Z. Brown, K Orginos, PRD86, 114506 (2012).
  - Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
  - Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
Contents

- Introduction to charmed tetraquarks
- Scattering on the lattice
- Formulation of "LQCD potentials"
- Numerical results of "potentials" & applications
- Summary
Scattering on the lattice

Key quantity: Equal-time Nambu-Bethe-Salpeter (NBS) wave function

\[ \psi(\vec{r}, \tau) = \sum_{\vec{x}} \langle 0|\phi_1(\vec{x} + \vec{r}, \tau)\phi_2(\vec{x}, \tau)| \tilde{\mathcal{F}}(\tau = 0)|0 \rangle \]

\[ = \sum_{W(\vec{k})} A_{W(\vec{k})} \exp\left[ -W(\vec{k})\tau \right] \psi_{W(\vec{k})}(\vec{r}) \]

\[ \psi_{W(\vec{k})}(\vec{r}) \equiv \sum_{\vec{x}} \langle 0|\phi_1(\vec{x} + \vec{r})\phi_2(\vec{x})|W(\vec{k}) \rangle \]

- Helmholtz eq. of NBS wave func.:
  \[ (\nabla^2 + \vec{k}^2)\psi_{W(\vec{k})}(\vec{r}) = 0 \quad (|\vec{r}| > R) \]

- NBS wave func. \sim wave func. in Q.M. information on phase shift

\[ \psi^{(l)}_{W(\vec{k})}(\vec{r}) \sim \frac{e^{i\delta_l(k)}}{kr} \sin(kr + \delta_l(k) - l\pi/2) \]

- Temporal correlation, \( W(k) \) : phase shift (Luscher’s formula)

- Spacial correlation, \( \psi(r) \) : potential \(\rightarrow\) observable

References:
- M. Lüscher, NPB354, 531 (1991).
- CP-PACS Coll., PRD71, 094504 (2005).
- Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
LQCD “potential” -- HAL QCD method --

Full details, see, Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

Helmholtz equation of NBS wave function:

\[
(\nabla^2 + \vec{k}^2)\psi_W(\vec{r}) \equiv 0 \quad (r > R)
\]

\[
W(\vec{k}) = \sqrt{m_1^2 + \vec{k}^2} + \sqrt{m_2^2 + \vec{k}^2}
\]

Half off-shell T-matrix in interacting region:

\[
(\nabla^2 + \vec{k}^2)\psi_W(\vec{r}) \equiv 2\mu\mathcal{K}_W(\vec{r}) \quad (r < R)
\]

Energy-independent potential faithful to phase shift

\[
U(\vec{r}, \vec{r}') = \int_{W_{\text{th}}} dW \frac{2\mu}{2\pi} \mathcal{K}_W(\vec{r})\psi_W^*(\vec{r}')
\]

Schrödinger-type equations with relativistic energy \(W(k)\):

\[
(\nabla^2 + \vec{k}^2)\psi_{W(\vec{k})}(\vec{r}) \equiv 2\mu \int d\vec{r}' U(\vec{r}, \vec{r}')\psi_{W(\vec{k})}(\vec{r}')
\]

Technical things to be solved for multi-hadron systems:

identifying single-energy states in simulations is a tough problem
Multi-hadron systems: challenge

Single-energy state in simulations:

\[ \vec{k}_n = \frac{2\pi \vec{n}}{L}, \quad \delta W = \frac{\pi^2}{\mu L^2} + \ldots \]

\[ \sum_{\vec{r}} \langle \psi(\vec{r}, \tau) \rangle = \sum_n A_n \exp(-W(\vec{k}_n)\tau) \]

- **Large L limit**: dense spectrum \(\rightarrow\) momentum excited state contaminations

- **Signal-to-noise issue**
  - **pion**: \(S/N \sim \text{const.}\)
  - **nucleon**: \(S/N \sim \exp[-A(m_N = 3/2m_\pi)\tau]\)
Define energy-independent potential below inelastic threshold:

$$\psi_{W(k)}(\vec{r}) = \langle 0|\phi_1(\vec{r} + \vec{x})\phi_2(\vec{x})|W(k)\rangle$$

$$(k^2 + \nabla^2)\psi_{W(k)}(\vec{r}) = 2\mu \int d\vec{r}' U(\vec{r}, \vec{r}')\psi_{W(k)}(\vec{r}')$$

$$W(k) = \sqrt{m_1^2 + k^2} + \sqrt{m_2^2 + k^2}$$

Extract energy-independent potential from time-dependent Schrodinger-type eq. (Ishii et al., (HAL QCD Coll.), PLB712, 437 (2012)).

$$R(\vec{r}, \tau) \equiv \psi(\vec{r}, \tau)e^{(m_1 + m_2)\tau} \quad (\tau > \tau_{th})$$

$$\left[-\partial_\tau + \nabla^2/2\mu + \partial_\tau^2/8\mu + \mathcal{O}(\delta^2)\right] R(\vec{r}, \tau) = \int d\vec{r}' U(\vec{r}, \vec{r}') R(\vec{r}', \tau)$$

Velocity expansion:

$$U(\vec{r}, \vec{r}') = V(\vec{r}, \nabla)\delta(\vec{r} - \vec{r}') \quad \text{(LO)}$$

$$V(\vec{r}, \nabla) = V_C(\vec{r}) + \sigma_1 \cdot \sigma_2 V_o(\vec{r}) + S_{12} V_T(\vec{r}) + \vec{L} \cdot \vec{S} V_{LS}(\vec{r}) + \mathcal{O}(\nabla^2) \quad \text{(NLO)}$$

Calculate observable: phase shift, binding energy, ...

Advantage: We can obtain potentials w/o identifying single-energy state
**HAL QCD & Luscher’s methods**

**I=2 ππ S-wave phase shift from HAL QCD & Luscher’s methods**

T. Kurth et al., JHEP 1312 (2013) 015.

* Quench QCD
  $m_\pi \approx 940$ MeV, $a=0.115$ fm
  $V \times T = (16^3, 24^3, 32^3, 48^3) \times 128$

* Luscher’s method (points)
  \[ kcot \delta(k) = \frac{1}{\pi L} \sum_{\vec{n} \in \mathbb{Z}} \frac{1}{|\vec{n}|^2 - \vec{k}^2} \]

* HAL QCD (red band)
  \[ \left[ -\partial_\tau + \nabla^2/2\mu + \partial_\tau^2/8\mu \right] R(\vec{r}, \tau) = V(\vec{r})R(\vec{r}, \tau) \]

Both methods excellently agree!!
**Lattice QCD: Setup**

**N_f=2+1 full QCD configurations generated by PACS-CS Coll.**

PACS-CS Coll., S. Aoki et al., PRD79, 034503, (2009).

- Iwasaki gauge & O(a)-improved Wilson quark actions
- $a=0.0907(13) \text{ fm} \rightarrow L \approx 2.9 \text{ fm} (32^3 \times 64)$

| Light meson mass [conf.1, conf.2, conf.3] (MeV) |
|---|
| $M_{\pi}=699(1), 572(2), 411(2)$ [PDG:135 ($\pi^0$)] |
| $M_K=787(1), 714(1), 635(2)$ [PDG:498 ($K^0$)] |

**Tsukuba-type Relativistic Heavy Quark (RHQ) action for charm quark**

S. Aoki et al., PTP109, 383 (2003)

⇒ remove leading cutoff errors $O(m_c a), O(\Lambda_{\text{QCD}} a), ...$

- *We are left with $O((a\Lambda_{\text{QCD}})^2)$ error (~ a few %)*
- *We employ RHQ parameters tuned by Namekawa et al.*

S. Aoki et al., PRD84, 074505 (2011)

| Charmed meson mass [conf.1, conf.2, conf.3] (MeV) |
|---|
| $M_{\eta_c}=3024(1), 3005(1), 2988(2)$ [PDG:2981] |
| $M_{J/\Psi}=3142(1), 3118(1), 3097(2)$ [PDG:3097] |
| $M_D=1999(1), 1946(1), 1912(1)$ [PDG:1865 ($D^0$)] |
| $M_{D^*}=2159(4), 2099(6), 2059(8)$ [PDG:2007 ($D^{*0}$)] |
**S-wave potentials : l=1 channel**

- **Repulsive** S-wave potentials

- Weak quark mass dependence

- It is unlikely to form bound state even at physical point

\[ m_{\pi} = 410 \text{ MeV} \]

\[ m_{\pi} = 700 \text{ MeV} \]
S-wave potentials: $l=0$ channel

- Attractive S-wave potentials
- Pion-exchange? (Need a calculations near/on physical point...)
- Check whether bound $T_{QQ}$s exist or not → phase shift analysis

- $m_\pi = 410$ MeV
- $m_\pi = 700$ MeV
S-wave phase shifts: $T_{cc}$ & $T_{cs}$ in I=0

Y. Ikeda et al. (HAL QCD), PLB729, 85 (2014).

- Attraction is not enough strong to generate bound state
- Rapid increase at threshold of DD* phase shift --> effect of virtual pole?

- solve Schrödinger equation --> phase shifts

➡ perform analytic continuation of Lippmann-Schwinger equation
➡ examine pole position
I=0 DD* T-matrix on complex energy plane

- Virtual pole on the DD* unphysical energy plane
- Origin of rapid increase of scattering phase shift
- $m_\pi = 410$ MeV

Analytic continuation
Summary

• **Search for Tcc, Tcs on the lattice at \( m_\pi = 410, 570, 700 \text{MeV} \)**

  ➡ \( N_f = 2+1 \) full QCD simulation (PACS-CS configuration)
  ➡ Charm quarks: Relativistic Heavy Quark action

  ➡ **Tcc, Tcs(\( J^P = 0^+, 1^+, I=1 \))** : s-wave MM channels are repulsive
    Bound states are unlikely...
  ➡ **Tcc, Tcs(\( J^P = 0^+, 1^+, I=0 \))** : s-wave MM channels are attractive,
    but not enough strong to form bound states at \( m_\pi = 410-700 \text{MeV} \)

✦ Our observations are **NOT inconsistent with diquark models**

• **Future direction on Tcc...**
  ➡ 4-quark type source? (we have employed hadronic wall sources)
  ➡ DD* -- D*D* coupled channels (resonance search)
  ➡ Physical point calculations