Are there near-threshold Coulomb-like Baryonia?

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(Dated: March 9, 2018)

The λc(2590)Σc system can exchange a pion near the mass-shell. Owing to the opposite intrinsic parity of the Λc(2590) and Σc, the pion is exchanged in S-wave. This gives rise to a Coulomb-like force that might be able to bind the system. If one takes into account that the pion is not exactly on the mass shell, there is a shallow S-wave state, which we generically call the Ycc(5045) and Yc(5045)0 for the Λc(2590)Σc and λc(2590)Σc systems respectively. As this Coulomb-like force is independent of spin, the states will appear either in an S = 0 or S = 1 configuration, with G-parities G = (−1)L+S+1 in the Yc(5045) case.

The discovery of the X(3872) a decade ago by Belle opened a new era in hadron spectroscopy. The X(3872) was the first member of a growing family of states above the open charm and bottom threshold effects [12–15], hadrocharmonia [16], baryocharmionia [17] and of course molecules [18–22] (see Refs. [23, 24] for reviews). In fact the most promising explanation for the X(3872) is that of a molecular state, i.e. a bound state of two hadrons, in particular the D0D0. Yet the X(3872) is not the only instance. Other molecular candidates include the Zc’s in the charm sector [25, 26] and the Zb’s in the bottom one [27, 28]. The recent discovery of the Pc(4380)+ and Pc(4450)+ pentaquarks [29] might add the latter to the family of molecular candidates; the Pc(4450)+ could be a ΣcD* or ΣcD* molecule [30, 31]. Hadron molecules were theorized three decades ago in analogy with the deuteron: the exchange of light mesons generates a force that might very well be able to bind a hadron system. Here we will consider a particular hadron molecule — the Λc(2590) baryon-antibaryon system — where the exchange of a pion near the mass shell mimics the time-honored Coulomb-potential.

In a recent work [32] we have discussed a molecular explanation for the Pc(4450)+ which besides the usual ΣcD* also involves a ΛcD component, where the Λc denotes the Λc(2590). This generates a vector force — the equivalent of a tensor force but with angular momentum L = 1 instead of L = 2 — that might play an important role in binding and might trigger discrete scale invariance if strong enough. A curious thing happens if the D* and D piece of this molecule is changed by a Λc and Σc: we obtain a Coulomb-like 1/r potential. This opens the prospect of a molecule exhibiting a hydrogen-like spectrum, which will be broken at low energies owing to the off-shellness of the pion. Besides we find it indeed remarkable that there is the possibility of making relatively concrete predictions for heavy hadron molecules without a strong requirement of guessing the short-range physics or using arbitrary form-factors (yet we will use these type of assumptions to check the robustness of the results).

We explain now the Coulomb-like potential for the Ycc(Λc1Σc) and Yc(Λc1Σc) molecules. We begin with the baryon-antibaryon case, for which we consider states in the isospin basis with well-defined G-parity

\[
|\Lambda c \Sigma c(\eta)\rangle = \frac{1}{\sqrt{2}} \left[|\Lambda c \Sigma c + \eta |\Sigma c \Lambda c c\rangle \right],
\]

where G = \(\eta(-1)^{L+S}\), for which the one pion exchange (OPE) potential reads

\[
V_{\text{OPE}}(r) = \frac{\mu^2}{4\pi f_\pi^2} \frac{e^{-\mu r}}{r},
\]

where \(\mu^2 = m_c^2 - \omega_c^2\), with \(\omega_c = m_{\Lambda c} - m_{\Sigma c}\). We take \(f_\pi = 130\) MeV and \(h_2 = 0.63 \pm 0.07\) [34] (this value is based on a theoretical analysis of the Λc decays and is almost identical to \(h_2 = 0.60 \pm 0.07\) from CDF [35]). In the isospin symmetric limit \(\mu^2 < 0\), yielding a complex potential similar to the one in the X(3872) (except that it is much stronger). However if we consider the isospin components of the Yc(5045)

\[
|1, +1\rangle = \frac{1}{\sqrt{2}} \left[|\Lambda c^+ \Sigma c^0\rangle + \eta |\Sigma c^+ \Lambda c c\rangle\right],
\]

\[
|1, 0\rangle = \frac{1}{\sqrt{2}} \left[-|\Lambda c^+ \Sigma c^0\rangle + \eta |\Sigma c^+ \Lambda c c\rangle\right],
\]

\[
|1, -1\rangle = \frac{1}{\sqrt{2}} \left[|\Lambda c^+ \Sigma c^-\rangle + \eta |\Sigma c^- \Lambda c c\rangle\right],
\]

"
upon closer inspection, we realize that the $m_I = \pm 1$ states exchange a charged pion and the $m_I = 0$ a neutral pion. For the charged pion case $\mu_{\pi^+} > 0$ and the OPE potential displays exponential decay at long distances. The effective pion mass is $\mu_{\pi^+} \simeq 18$ MeV, which translates into eight times the standard range of OPE. If we consider the reduced potential instead, we can define the equivalent of the Bohr radius as

$$2\mu Y V(r) = \eta \frac{2}{a_B} \frac{e^{-\mu r}}{r}, \quad (6)$$

where $\mu Y$ is the reduced mass and $a_B$ is given by

$$a_B = \frac{4\pi f_{\pi}^2}{\mu_Y h_2^2 \omega_2^2} = 4.4^{+1.2}_{-0.8} \text{ fm}, \quad (7)$$

where for the masses of the $\Lambda_{c1}$ and $\Sigma_c$ we take the values of the PDG. It is also interesting to consider the Bohr momentum

$$\gamma_B = \frac{1}{a_B} = 45^{+10}_{-10} \text{ MeV}. \quad (8)$$

If the $\Lambda_{c1}$ and $\Sigma_c$ were stable and the pion were on the mass shell, for $\eta = -1$ and in the absence of short range forces we will have the Coulomb-like spectrum

$$E_{n,l} = -\frac{1}{2\mu Y} \left( \frac{\gamma_B}{n + l + 1} \right)^2, \quad (9)$$

with $l$ the angular momentum and where we take $n = 0$ for the ground state in each partial wave.

However neither the pion is on the mass shell nor the $\Lambda_{c1}$ and $\Sigma_c$ are stable. The finite effective mass of the pion means that Coulomb-like bound state are expected to survive only if their binding momentum fulfills the condition

$$\frac{\gamma_B}{n + l + 1} \ll \mu_\pi, \quad (10)$$

which can only be met for $n + l + 1 \leq 2$ at best, leaving room for few bound states at most. In fact concrete calculations show that only the $n = 0$ S-wave states survive. It is also important to consider the possible impact of the finite width of the $\Lambda_{c1}$ and $\Sigma_c$ heavy baryons, about 2 MeV. In the absence of short-range forces the energy of the fundamental state is expected to be $E = -0.8^{+0.4}_{-0.3}$ MeV, which is about half the width of the heavy baryons. The authors of Ref. argue that the width of the components can be ignored if their lifetime is ample enough for the formation of the bound state. This is equivalent to the condition $\Gamma \ll m$ with $m$ the mass of the exchanged meson. If instead of the physical pion mass we take the effective pion mass $\mu_\pi \sim 20$ MeV to be on the safe side, there is still plenty of time for the formation of the bound state before its components decay. Hence we expect it to survive.

Yet there is a second argument that calls for relative caution. In a more complete analysis of bound states with unstable constituents, Hanhart et al. proposed to check the dimensionless ratio

$$\lambda = \frac{\Gamma_R}{2E_R}, \quad (11)$$

where $R$ refers to a decay channel of one of the constituents, $\Gamma_R$ the partial decay width and $E_R$ the energy gap to that decay channel. The $Y_{c\bar c}$ decays induced by its components can be seen in Fig. We consider the decays mediated by $\Sigma_c^{++} \rightarrow \Lambda_c^+ \pi^+$ and $\Lambda_{c1}^+ \rightarrow \Lambda_c^+ \pi^+$ (i.e. $Y_{c\bar c}^+ \rightarrow \Lambda_{c1}^+ \Sigma_c^0$ and $Y_{c\bar c}^+ \rightarrow \Lambda_c^+ \Sigma_c^0$) we have $\lambda = 0.034$ and $\lambda = 0.045$ respectively. The problem arises with the decay $Y_{c\bar c}^+ \rightarrow (\Sigma_c^0 + \Sigma_c^0 + \Sigma_c^0)\pi^0$. It happens merely $4.37 \pm 0.49$ MeV below the $\Lambda_{c1}^+ \Sigma_c^0$ threshold, yet the $\Lambda_{c1}^+ \rightarrow \Sigma_c^0 \pi^0$ decay width, though unknown, is comparable to that value. From heavy baryon chiral perturbation theory we expect this decay to be

$$\Gamma(\Lambda_{c1}^+ \rightarrow \Sigma_c^0 \pi^0) = \frac{1}{2\pi} \frac{m_{\Sigma_c}}{m_{\Lambda_{c1}}} \frac{\hbar_2^2 \omega_2^2}{f_2^2} q_\pi, \quad (12)$$

with $q_\pi$ the momentum of the outgoing pion, leading to $2.1^{+0.5}_{-0.4}$ with the numbers we are using here. For a state near threshold, this leaves the range $\lambda \sim 0.18 - 0.30$ with the central value $\lambda = 0.24$. These numbers are not bad but not ideal either. They can imply a sizeable distortion of the lineshapes, indicating that a more complete analysis of shallow Coulomb-like baryonium states might be useful. There is also the possibility that the interaction between the $\Lambda_{c1} \rightarrow \Sigma_c \pi$ and $\Lambda_{c1} \rightarrow \Lambda_c \pi$ will play in favor of neglecting the widths in the shallow states. For comparison purposes, previous speculations about the $Y(4260)$ as a $D_0D^* / D_s(2460)D$ bound state owing to a $e^{i|\mu_\pi r|/r}$ OPE potential (notice the complex exponential) are probably theoretically unsound because of
the large width of the P-wave heavy mesons [41]. In this case the dimensionless parameter $\lambda_R$ is close to one. A more thorough answer probably requires a full calculation in the line of the one in Ref. [42] for the $X(3872)$, which included the $D\bar{D}^*$, $D^*\bar{D}$ and $DD\pi$ channels. The equivalent calculation for the $Y_{cc}(5045)$ baryonium will require the inclusion of the $\Lambda_{c1}\Sigma_c$, $\Sigma_c\Lambda_{c1}$, $\Sigma_c\Sigma_c$ and $\Sigma_c\Sigma_c\pi$ channels. There is also the observation that the decay channels $\Sigma_c^+\pi^0$ and $\Sigma_c^+\pi^0$ do not appear as intermediate states in the $Y_{cc}^+$. This situation is different than in the $X(3872)$, where the $D^0\bar{D}^0$ can be a transient state when the neutral pion is in flight, which also happens in the $Y_{cc}^0$ with the $\Lambda_{c1}^-\rightarrow\Sigma_c^+\pi^0$. This is the reason why the OPE potential in the $Y_{cc}^+$ is real, while in the $X(3872)$ or in the $Y_{cc}^0$ it acquires a complex part. As a consequence the contribution from the aforementioned decay channels might be important for the location of the $Y_{cc}^0$ but not necessarily for the $Y_{cc}^+$.

In the previous discussion we have only considered the states with $\eta = -1$, for which the OPE potential is attractive. For $\eta = +1$, though OPE is repulsive, there is the interesting feature that the decay $Y_{cc}^-\rightarrow\Sigma_c^-\Sigma_c\pi$ is forbidden by $C$- and $G$-parity for an $S$-wave pion. In fact the $G$-parity of the initial $Y_{cc}^+$ is $G = (-1)^S$, the final $\Sigma_c^+\Sigma_c$ has $G = (-1)^S$ and the pion has $G = -1$. The short range interaction between the $\Lambda_{c1}$ and $\Sigma_c$ is unknown, but it could give rise to a bound state or a resonance above the threshold owing to the Coulomb-like potential barrier (which at 1 fm rises to 6-7 MeV). This type of baryonium is indeed very interesting as the main decay mechanisms are forbidden and hence we can expect a narrower state.

Finally we consider the baryon-baryon case, for which the OPE potential reads

$$V_{\text{OPE}}(r) = -\frac{\hbar^2 a_{\Sigma c}^2}{4\pi f_{\Sigma c}^2} e^{-\mu_\Sigma r} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

where $\mu_\Sigma^2 = m_{\Sigma c}^2 - \omega_\Sigma^2$ with $\omega_\Sigma = m_{\Lambda_{c1}} - m_{\Sigma_c}$. Channels 1 and 2 are $\Lambda_{c1}\Sigma_c$ and $\Sigma_c\Lambda_{c1}$ respectively. The baryons will take the configuration

$$|Y_{cc}\rangle = \frac{1}{\sqrt{2}} \{|\Lambda_{c1}\Sigma_c\rangle + |\Sigma_c\Lambda_{c1}\rangle\},$$

for which the OPE potential is most attractive. The potential is indeed identical to that of the baryon-antibaryon case with $\eta = -1$, except for the short-range physics which might be different.

We can quantify the discussion about the existence of the bound states in the following way. We will assume that the OPE potential is only valid about a certain cut-off radius $R_c$, below which the interaction is described by a delta-shell

$$V(r) = V_{\text{OPE}}(r) \theta(r - R_c) + \frac{C_0}{4\pi R_c^2} \delta(r - R_c).$$

For convenience, instead of using the standard coupling $C_0$ we will define the reduced coupling

$$c_0 = -\frac{2\mu_\Sigma C_0}{4\pi R_c^2},$$

where we have flipped the sign of the coupling such that the delta-shell generates a bound state for $c_0 \geq 1$ with binding momentum $\gamma = (c_0 - 1)/R_c$. We can compute the spectrum as a function of $c_0$ and $R_c$. If we choose $R_c = 1$ fm, which seems a sensible value, we obtain the binding energies of Fig. 2. For $c_0 \rightarrow -\infty$, which corresponds to a hard core at $R_c$, the ground state survives with $E_B = -0.09_{-0.08}^{+0.06}$ MeV. For $c_0 = 1$, OPE shifts the binding energy from zero to $E_B = -1.9_{-0.5}^{+0.5}$ which indicates a moderate contribution from Coulomb-like OPE. In addition for $c_0 > 0.9_{-0.4}^{+2}$ a shallow excited state appears. We do not know the form of the short-range interaction between heavy baryons, but while for the $Y_{cc}$ short-range repulsion cannot be discarded, for the $Y_{cc}^+$ short-range attraction is more likely. However the cut-off radius $R_c = 1$ fm probably lies in an intermediate zone dominated by two-pion exchange and other contributions which might be attractive. Hence we expect the fundamental state of $Y_{cc}$ baryonium to be deeper than the predictions from OPE alone (thus implying the existence of a shallow excited state), though there is no clear way to estimate how much exactly.

To summarize, the $\Lambda_{c1}\Sigma_c$ and $\Lambda_{c1}\Sigma_c$ systems can exchange an $S$-wave pion almost on the mass shell giving rise to a potential with an unusual long range. This Coulomb-like OPE extends to distances large enough as to have at least a shallow S-wave state. The spectrum might very well include additional states owing to the short-range interaction between the baryons. These
states appear both in the $S = 0, 1$ configurations, where for the baryon-antibaryon ones the states require the G-parity to be $(-1)^{S+L+1}$ for Coulomb-like OPE to be attractive. They survive even if the unknown short-range interaction is repulsive, which could be the case for the $\Lambda_1^c$ states. The shallow nature of these states begs the question of whether they survive once we take into account the finite width of the heavy baryons. The answer is probably yes, but a deeper theoretical analysis than the one presented here would be very welcomed. For the $G = (-1)^{S+c}$ states the Coulomb-like force is repulsive, but these states are protected against the decays of the $\Lambda_1^c$ and $\Sigma_c$ baryons, which make them very interesting. In case there is enough short-range attraction they will either bind or survive as resonances.

**ACKNOWLEDGMENTS**

This work is partly supported by the National Natural Science Foundation of China under Grants No. 11375024, No.11522539 and the Fundamental Research Funds for the Central Universities.
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