Inconsistent Physics in the Presence of Time Machines

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ABSTRACT. We present a scenario in 1 + 1 and 3 + 1 dimensional space time which is paradoxical in the presence of a time machine. We show that the paradox cannot be resolved and the scenario has no consistent classical solution. Since the system is macroscopic, quantisation is unlikely to resolve the paradox. Moreover, in the absence of a consistent classical solution to a macroscopic system, it is not obvious how to carry out the path integral quantisation. Ruling out, by fiat, the troublesome initial conditions will resolve the paradox, by not giving rise to it in the first place. However this implies that time machines have an influence on events, extending indefinitely into the past, and also tachyonic communication between physical events in an era when no time machine existed. If no resolution to the paradox can be found, the logical conclusion is that time machines of a certain, probably large, class cannot exist in 3 + 1 and 1 + 1 dimensional space time, maintaining the consistency of known physical laws.
1. Introduction

The existence of traversible wormholes will lead to travel back in time if, as described in [1], one of the mouths of such a wormhole has travelled at a high velocity relative to the other and has come back to rest. Such a wormhole generates a closed time-like curve (CTC), and hence the possibility of travelling back in time.

The formation and the existence of a traversible wormhole, or a CTC, is itself not proven beyond doubt. It is not even clear if any viable model exists in which a CTC appears. Recently, Gott [2] constructed a simple model where a CTC appears when two infinitely long, parallel cosmic strings approach each other at high velocity. However, a complete analysis of this model [3]-[7] reveals that either

(i) the region containing the Gott CTC is not local [3, 4, 7]; or,
(ii) there is not enough energy in an open universe to create a CTC [5]; or,
(iii) in a closed universe, the universe collapses to a zero volume before any CTC can form [4]; or
(iv) the appearance of a CTC leads to a violation of weak energy condition or a divergent energy momentum tensor, the back reaction of which will, conceivably, prevent the formation of a CTC [6]. This situation has led Hawking to propose a “Chronology Protection Conjecture”, according to which the laws of physics forbids the apperance of CTCs.

Also, in the creation of a CTC by a traversible wormhole, as in [1], the Cauchi horizon where a CTC forms seems to be stable classically, but not quantum mechanically [1, 6, 8]. Thus, in short, no compelling, physically acceptable model exists at present which can create locally a “time machine”. In the following, we generically use the word time machine to denote traversible wormhole, or a CTC, or anything that permits a travel back in time.

The main obstacle in settling these issues is the lack of understanding of quantum gravity. Thus, it is not clear how nature forbids, if at all, the creation of a time machine. Perhaps, a theory of quantum gravity may answer this question, but no such theory is in sight in which such calculations can be undertaken.

However, one can assume that a time machine exists locally, avoiding completely the question of its formation, and explore its physical consequences in the classical, semiclassical or quantum regime. The existence of a time
machine often leads to paradoxes which, if cannot be resolved, imply the non
existence of the time machines themselves, if the known physical laws are
required to be consistent.

This approach has been advocated by “the Consortium” consisting of
Friedman et al in [9], and is being actively pursued at present [10]–[17]. These
authors have studied the unitarity of evolution, consistency of probabilistic
interpretation, etc. in free and interacting quantum field theories, and also in
quantum mechanics [13]–[17]. For free quantum field theories, unitarity and
probabilistic interpretation are consistent in the “chronal region” - space time
region with no CTCs, while for the interacting field theories the evolution is
often non unitary. Hartle has formulated a generalised quantum mechanics
incorporating the non unitary evolution [18].

An immediate paradox that springs to one’s mind, upon hearing the word
time machine, is the science fiction scenario where an observer travels back
in time and kills his younger self. An idealised version of this scenario is
the so called Polchinski’s paradox [9] in which a billiard ball falls into a
wormhole, emerges at the other end in the past, and (the situation is so
arranged that it) heads off straight for its younger self, knocking it out of its
original trajectory, thus preventing it from falling into the wormhole in the
first place. This is the main situation that has been studied extensively in
[9]–[12], and is shown in figure 1. (In these works, more general situtaions are
also considered that includes inelastic, frictional, non planar collisions, etc. .
Novikov has considered different situations which are more or less the same,
in spirit, as the billiard ball collision in Polchinski’s paradox.) The resolution
of this paradox is that, the collision between older and younger ball is not
head on, but only a glancing one. The collision is such that the younger ball
deflects from its original trajectory, but only slightly, and still falls into the
wormhole, and comes out in the past along a slightly different trajectory,
which leads to the glancing collision instead of a head on one. This is shown
in figure 2. In [9, 10, 12], a consistent solution corresponding to the above
description is presented in detail. Reverting to the science fiction scenario,
this resolution can be described picturesquely, albeit crudely, as follows. The
time travelled observer attacks his younger self, causing only a serious injury.
The younger observer, nevertheless, falls into the wormhole and travels back
in time. Hence the time travelled observer is seriously injured, which is why
he could not kill his younger self!

Of course, in the resolution of Polchinski’s paradox as described above,
there is now not one consistent solution, but many; perhaps, infinitely many. However, this situation still obeys all the physical laws, in the chronal region, when quantised using path integral formalism \[14\]. Basically, the billiard ball follows one of the many possible consistent trajectories, with a probability amplitude given by the quantum mechanical propagator.

In all the situations analysed so far in \[9\]-\[12\], there is atleast one, often many more, consistent classical solution. The non uniqueness of the classical solution, however, does not lead to violation of any physical laws when quantum mechanics is incorporated. Thus it appears that the existence of time machines does not lead to any irresolvable paradox and, hence, cannot be ruled by demanding the consistency of the classical and semiclassical physics.

In this paper, we propose a paradoxical scenario, which to the best of our ability, is irresolvable, without incorporating some unacceptable physical laws. We take, as our model for time machine, a traversible wormhole as in \[9\] or the Politzer's time machine \[15\]. In these models, the gravitational effects of the wormhole are assumed to be felt only inside the wormhole and very near its mouths outside. The outside space time is otherwise unaffected, which we take to be Minkowskian, where the usual physical laws are valid.

We first describe a paradoxical scenario in 1 + 1 dimensional space time, where the irresolvability of the paradox is explicit. This case is special in many respects and, in particular, has limited implications for the 3 + 1 dimensional space time. However, the scenario can be generalised, with some modifications, to 3+1 dimensional space time where, again, the irresolvability of the paradox can be seen explicitly.

Of course, such paradoxical situations can be ruled out by fiat by forbidding the initial conditions that lead to them. However, since the initial conditions can be imposed at times arbitrarily far into the past, ruling them out would imply that any time machine that could be created in the future will have an influence on events, extending indefinitely into the past. Moreover, as explained in the paper, the influence of a time machine on past events also implies a tachyonic communication between events in the past, when no time machine existed. We would like to note in this connection that in the generalised quantum mechanics of Hartle \[18\] also, there exist the phenomena of the CTCs influencing events in the arbitrary past and of the tachyonic communication. It was, however, in the regime of quantum mechanics whereas, in our scenario, such non physical aspects are necessary even in the classical regime if one rules out by fiat the troublesome initial
conditions leading to paradoxes.

Rather, the simplest way out of this paradox seems to be to rule out the existence of time machines, at least of the types considered here. However, our scenario is likely to be extendable to a wider class of time machines, which occupies only a finite spatial region and modifies the space-time manifold only locally, leaving it unaffected everywhere else. Hence, if the paradox in the scenario proposed here cannot be resolved, the existence of a large class of time machines may be ruled out.

The paper is organised as follows. In sections 2 and 3, we describe our scenario, respectively, in 1 + 1 and 3 + 1 dimensional space-time discussing their physical implications. In section 4 is a concluding summary.

2. 1 + 1 dimensional case

As a model of a time machine in 1 + 1 dimensional space-time, we consider Politzer’s time machine [15], shown in figure 3. The space-time coordinates are labelled by \((x, t)\) in the following. The time machine consists of the spatial interval between \(x = 0\) and \(x = \Lambda\) at times \(t = 0\) and \(t = T\), suitably identified. The spatial interval at \(t = 0\) is identified with that at \(t = T\) such that the points \((y, 0_-)\) are smoothly identified with \((y, T_+)\), and the points \((y, T_-)\) with \((y, 0_+)\), for \(0 \leq y \leq \Lambda\). The region thus formed contains CTCs and allows travel back in time. We take the space-time to be flat everywhere else, except very near the time machine.

The propagation of a single particle, say a billiard ball, in Politzer’s time machine is also shown in figure 3. This propagation is consistent with the physical laws, classical and quantum mechanical [14] in the chronal region which is spatially and temporally far outside the time machine, where there are no CTCs.

One can also consider the propagation of two billiard balls, which is illustrated in figure 4. The two balls are identical and do not interact with each other, except for collisions which are assumed to be elastic. The propagation in the chronal region is consistent classically and quantum mechanically, as shown in [15].

Our scenario is a simple extension of the above case. A crucial ingredient here is that we consider two different balls, L and H. The scenario is set up as follows and shown in figure 5. The time machine is in the spatial interval between \(x = 0\) and \(x = \Lambda\) at times \(t = 0\) and \(t = T\), as described before. Let
$m_L$ and $m_H \equiv \mu m_L$, $\mu \gg 1$, be the masses of the balls L and H, and $U_L$ and $U_H$, their respective initial velocities. Let L and H be, respectively, at $(-p', 0)$ and $(p, 0)$ at time $t = 0$. With no collisions, the ball H would be at $(-q, T)$ at time $t = T$ and, hence, would not have entered the time machine. Here $p, p'$, and $q$ are positive and are of order $\Lambda$, so that the initial trajectories of L and H are well away from the time machine. The balls L and H will collide first at $A(a, t_A)$, where $0 < a < \Lambda$ and $0 < t_A < T$. After this collision, H will enter the time machine at $(l, T - t_1)$ where $0 < l < a$ and, emerge from it at $(l, 0 +)$ with the velocity $V_H$. From the above data, it follows that

\[
\begin{align*}
U_L &= \frac{a + p'}{t_1} > 0 \\
U_H &= -\frac{p + q + \Lambda}{T} < 0 \\
V_H &= -\frac{a - l}{T - t_1} < 0 .
\end{align*}
\]  

The separation between L and H at a large and negative time $t_0$, when the initial conditions on L and H are assumed to be set, is given by $d_0 = (U_H - U_L)t_0 + (p + p' + \Lambda)$. Also, if $\frac{\Lambda}{T} \ll c$, the velocity of light, then all the velocities can be taken to be non relativistic, which we assume to be the case here.

In an one dimensional elastic collision between two particles with masses $m_1$ and $\mu m_1$ and initial velocities $U_1$ and $U_2$, the respective final velocities $V_1$ and $V_2$ are given by

\[
\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \frac{1}{1 + \mu} \begin{pmatrix} 1 - \mu & 2\mu \\ 2 & \mu - 1 \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} .
\]  

Thus, the scenario, as described above, can be easily arranged by choosing

\[
\mu \equiv \frac{m_H}{m_L} = 1 + 2 \left( \frac{V_H - U_L}{U_H - V_H} \right) .
\]  

Note that $\mu > 1$, since $U_H < V_H < 0$ and $U_L > 0$.

Let L and H collide at A according to the initial set up. Then, H will enter the time machine at $(l, T_-)$ and emerge from it at $(l, 0 +)$ with the velocity $V_H$. As can be clearly seen from figure 5, since $V_H < 0$, H is now heading for another collision with L at $B(b, t_B)$, $t_B < t_A$, which is to the past of the
A-collision. The details after this B-collision can be easily analysed, but are not necessary since they will not affect our scenario in any crucial way; in particular, they would not resolve the paradox described below.

Now it is immediately obvious that the above scenario is paradoxical. It is inconsistent and cannot take place in any sense that we know of. If there is A-collision, then there will also be a B-collision. But, the ball L will be reflected back in the B-collision, making impossible the A-collision. If A-collision did not occur, the ball H would not have entered the time machine, and there would not be any B-collision. In its absence, there must be an A-collision, since that was the initial set up. The argument now repeats.

This is our paradoxical scenario in $1 + 1$ dimensional space time. We do not know how the paradox can be resolved. It is assumed to be possible to set the initial conditions on a physical system far in the past of time machine. However, the evolution of the system is paradoxical in the presence of a time machine. It is not clear how both the collisions can take place nor, how any one or both of them can be avoided. If, for a moment, we assume that somehow both the collisions did take place, then it is easily seen that in the far future of the time machine, there will be one H ball and two L balls. This will violate the principle of conservation of energy, since a new L ball has appeared. Such creation of real, onshell particles also occurs in interacting quantum field theories in non causal regions, as shown by Boulware in [16]. There, such creation leads to non unitarity of the field theory.

Now, one can declare that in the presence of a time machine, certain initial conditions for a physical system, such as the one necessary above, are forbidden. The known physical laws cannot accomodate and enforce this kind of censorship, but let us anyway proceed with this declaration. This implies, first of all, that any time machine that could be created in the future will have an influence on the events, extending indefinitely into the past.

The influence of the time machine on arbitrarily past events also implies a tachyonic communication between them, at a time when no time machine existed. Consider the scenario described above, where the balls L and H were given velocities $U_L$ and $U_H$ initially at a time $t_0$. The separation between L and H is $d_0 = (U_H - U_L)t_0 + p + p' + \Lambda$, which can be arbitrarily large. If the time machine to be created at $t \simeq 0$ were to forbid the initial conditions, such as above, then L and H cannot be given the above velocities at time $t_0$, which can be any arbitrary time in the past. Thus, if a team, say L-team, had set the ball L moving at a velocity $U_L$, it will be known instantaneously to a H-
team at a distance $d_0$, because this team will not be able to set the ball $H$ at a velocity $U_H$. This communication must take place nearly instantaneously over an arbitrarily large distance $d_0$ and, hence, can only be tachyonic.

It is not likely that the quantisation of the above system will resolve the paradox. For one thing, the system involved is macroscopic and its classical action $S \gg \frac{\hbar}{2\pi}$, the quantum action. Hence, although quantisation can resolve the problem of non unique classical solutions, as in [14] for example, it is unlikely to resolve the above paradox in a macroscopic system, where there is no classical solution at all to begin with. Thus, if a quantum mechanical solution does exist for the macroscopic scenario described here, it must be quite unusual, having no classical counterpart.

In the presence of time machines, it has been recognised that Hamiltonian formulation of quantum mechanics is not possible, since it requires the space time manifold to be foliable along the time direction. However, the path integral formulation is a natural choice in such situations, which has been successfully employed in the presence of time machines, where there was atleast one classical solution for macroscopic systems. But, in our scenario no consistent classical solution exists, and following the system along classical paths leads to inconsistencies. It is not obvious how path integral formulation can be carried out, if possible at all. Of course, it is possible if, by fiat, all the troublesome initial conditions are forbidden, but this implies that time machines have an influence on events extending indefinitely into the past, and also implies tachyonic communications between physical events in an era when no time machine existed. If the above paradox cannot be resolved, then the logical conclusion is that time machines cannot exist, while maintaining the consistency of known physical laws.

A major shortcoming of the above scenario is that it is in 1+1 dimensional space time, which has limited implications, if any, to the 3+1 dimensional space time. However, as we will show in the next section, this scenario, with a few modifications, can also be extended to 3+1 dimensional space time, leading to the same conclusions as above.

3. 3+1 dimensional case

As a model of a time machine in 3+1 dimensional space time, we consider the time machine created by a traversible wormhole which is shown in figure 6. Note that, in this figure, only the X-Y plane of the three dimensional space
is shown, in contrast to previous figures where both the time and the one dimensional space were shown. The two wormhole mouths, W1 and W2, of size $\omega$, are centered respectively at $(0, h)$ and $(\Lambda, h)$ in the X-Y plane. Thus they are separated from each other by a distance $\Lambda$. One travels back in time, by an amount $T$, by entering the mouth W2 and emerging from the other mouth W1. See [1, 9, 10] for details. The gravitational effects of the wormhole are assumed to be felt only inside its traversible throat and very near its mouths outside. The space time outside is otherwise unaffected, which we take to be Minkowskian. In the following we assume that $R \ll \omega \ll \Lambda$, where $R$ is the size of the physical object traversing the wormhole, and that the back reaction of the physical system on the wormhole is negligible.

We consider a light ball L of radius $r$ and mass $m_L$, and a heavy disc D of radius $R \gg r$ and mass $m_D \equiv \mu m_L$, $\mu \gg 1$. Thus, if L collides with D away from the edge of D, then L will be reflected back. L and D can roughly be thought of as a ping pong ball and a bat.

Initially, at a time $t_0$ which is large and negative, L and D are given the velocities $(U, 0)$ and $(-V_x, V)$, respectively, as shown in figure 6. The centers of L and D are in the X-Y plane. The disc D has only a translational motion with its normal parallel to the X-axis. As before, the initial conditions are such that, in the absence of L, the disc D would not enter the wormhole. With L present, however, there will be a collision between L and D, with zero impact parameter, at the point $A = (\Lambda, 0)$ at time $t = t_A = \frac{\Lambda}{U}$. We choose

$$V_x = \frac{2U}{\mu - 1}$$

such that after the collision, D is deflected from its path, travels with a velocity $(0, V)$, and falls into the wormhole mouth W2. D will then emerge from W1 with a velocity $(0, -V)$ travelling along the Y-axis, and collide with L with zero impact parameter at the point $B = (0, 0)$ at time $t = 0 < t_A$, i.e. to the past of the A-collision. The wormhole traversal rules that we used here are given in [11]. According to their notation, if $\vec{V}_{in} = V(\cos(\theta + \phi), \sin(\theta + \phi))$, then $\vec{V}_{out} = V(\cos(\theta - \phi), \sin(\theta - \phi))$. See figure 4 of [11] for further details. In the set up here, $\theta = \phi - \frac{\pi}{2} = 0$, $\vec{V}_{in} = (0, V)$ and, therefore, $\vec{V}_{out} = (0, -V)$.

There is a paradox here. If the A-collision took place as planned, then the B-collision will also take place. But this will reflect back the ball L at time $t = 0$, making the A-collision at time $t_A > 0$ impossible. This scenario is the $3 + 1$ dimensional analog of that of section 2.
But, there can be consistent solutions to this scenario with no paradox, if
as in [9, 10, 12], one also considers not only head on collisions with zero impact
parameter, but also the edge on collisions with non zero impact parameter.
However, we will make a simple modification in the above scenario, in which
the disc D, after emerging from W1, is slowed down before it encounters L
from a velocity \((0, -V')\) to \((0, -\tilde{V})\), where \(\tilde{V}\) is very small.

Slowing down D is not a problem. It can be done, for example, by hav-
ing a viscous medium in between, and well away from, W1 and the collision
point B; or, by having a charged disc D and then applying an electric field.
This process can also be used to make the disc travel along the Y-axis with
its normal parallel to the X-axis - “funnelling or steering” D along the re-
quired trajectory - at the time of its encounter with L. Note that \(\tilde{V}\) can be
arbitrarily small and that the slowing down process is totally independent
of the wormhole, neither affecting the other. We parametrise the slow down
by a parameter \(\alpha\), \(0 < \alpha < 1\), saying that D, after emerging from W1,
travels a vertical distance \(\alpha h\) with the speed \(V'\) and the remaining distance
\(\beta h \equiv (1 - \alpha)h\) with the speed \(\tilde{V}\). This parametrisation is convenient and
quite general, and does not imply that the entire slowing down process has
occurred at one particular location between W1 and B.

Before modification, the scenario is very likely to be consistent upon
including also the edge on collisions between L and D. They will not be
of much help in the scenario, as will be clear below. The initial set up, as
before, is such that if the collision at \(A = (0, \Lambda)\) took place as planned, then
there will be a head on collision at \(B = (0, 0)\) leading to an inconsistency.
Note that B-collision always takes place at \(t = 0\), since that is when the ball L
will arrive at B. The planned collision at A will occur at \(t = t_A = \frac{\Lambda}{U} > 0\).
Any potentially consistent solution will involve an edge on collision at B at time
\(t = 0\), and the A-collision will now take place at a location \(A' = (A_x, d - \frac{\Lambda V}{U})\)
at a time \(t_{A'} \simeq t_A \pm \frac{R}{\tilde{V}}\). It must take place somewhere in a finite region after
D is near enough to W2 to make it to B in time, but before D passes W2.
The idea is to choose \(\tilde{V}\) sufficiently small and, hence \(\frac{R}{\tilde{V}}\) sufficiently large, so
that the disc D will be outside this finite region.

The required condition for the collisions at A and B to occur as planned
is that

\[
\frac{\Lambda}{U} + \frac{h}{V} - T + \frac{\alpha h}{V} + \frac{\beta h}{V} = 0,
\]

where, starting from B, the first term is the time required for L to reach A,
the second term for D to reach W2 from A, T the amount of time travel, the last two terms are the time required for D to reach B from W1. The travel time through wormhole throat is neglected; if non zero, it can be incorporated in the definition of T. Equation (4) implies

\[ V = \frac{(1 + \alpha)h}{T - \frac{3h}{V} - \frac{\Lambda}{U}}. \]  

(5)

Since \( V > 0 \), it follows that

\[ U > \frac{L}{T - \frac{3h}{V}}. \]  

(6)

Furthermore, since \( U > 0 \), it follows that

\[ \tilde{V} > \frac{\beta h}{T}. \]  

(7)

The initial velocity of D is \((-\frac{2U}{\mu - 1}, V\)\). Therefore,

\[ l = \frac{2U(T - \frac{3h}{V} - \frac{\Lambda}{U})}{(\mu - 1)(1 + \alpha)}, \]  

(8)

where we have used equation (5), and \( l \) is the distance from W2, parallel to the X-axis, at which D would have passed if there were no collisions. \( l \) is required to be \( > \omega \), the wormhole size.

Now for the edge on collision. As noted before, the B-collision still occurs at \((0, 0)\) at time \( t = 0 \), but now the A-collision occurs at \( A' = (A_x, d - \frac{\Lambda V}{U})\), instead of at \( A = (\Lambda, 0)\) as planned. Note that the Y-coordinate of (the younger) D at time \( t = 0 \) is \(-\frac{\Lambda V}{U}\) and, hence, it would travel a vertical distance of \( d > 0 \) with speed \( V \), before colliding with L at \( A' \). Let the velocity of D after \( A'\)-collision is \((V_x', V')\), and in a consistent solution it will enter W2 and emerge from W1 with a velocity \((V''_x, V'')\), which is close to \((0, -V')\). Let \( 2\chi \), where \( \chi \) is small, be the angle between the trajectory of D emerging from W1 and the negative Y-axis (see [10] for “wormhole traversal rules”).

The time required for D to enter W2 after \( A'\)-collision is

\[ (h - d + \frac{\Lambda V}{U}) \frac{1}{V'}. \]  

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It will emerge from W1 with a velocity \( (V_x'', V_y'') \), be slowed down and steered along a vertical trajectory with a velocity \( (0, -\tilde{V}) \). We write the travel time from W1 to the collision point at B as

\[
\frac{\alpha h}{V'} + \frac{\beta h + \sigma R}{V} + \tau,
\]

taking \( (V_x'', V_y'') = (0, -V') \), and parametrising the necessary corrections by \( \tau \), which is of the order

\[
\tau \simeq \frac{2h \tan^2 \chi}{V'(1 - \tan^2 \chi)}.
\]

(9)

Also, \( \sigma = +1 (-1) \) denote the upper (lower) edge collision between L and D.

Thus the condition for the edge on collision is, in the same way as before,

\[
\frac{d}{V} + (h - d + \frac{\Lambda V}{U}) \frac{1}{V'} - T + \frac{\alpha h}{V'} + \frac{\beta h + \sigma R}{V} + \tau = 0.
\]

(10)

Subtracting (10) from (4), we have

\[
\frac{\sigma R}{V} = \left( \frac{1}{V} - \frac{1}{V'} \right) \left( \frac{\Lambda V}{U} + (1 + \alpha)h - d \right) - \tau,
\]

(11)

which must be satisfied for any consistent solution. \( V' \) is the Y-component of D after a collision at \( A' \) with L. Since D is much heavier than L, its final velocity cannot be too different from its initial velocity. Hence, \( \left( \frac{1}{V} - \frac{1}{V'} \right) \) is small and, in particular, is bounded. Therefore, the right hand side of equation (11) is also bounded. Thus, if \( \tilde{V} \) can be chosen sufficiently small so as to exceed this bound, then no consistent solution can exist.

Let \( \frac{\alpha h}{V} = (1 - 2\epsilon)T \) and \( \frac{\beta h}{V} = \epsilon T \), with \( \epsilon > 0 \). Then,

\[
V = \frac{(1 + \alpha)h}{\epsilon T}, \quad l = \frac{2\Lambda}{(1 + \alpha)(\mu - 1)}.
\]

(12)

Equation (11) now becomes

\[
\frac{\sigma(1 - 2\epsilon)R}{\beta h} = \epsilon K,
\]

(13)
where

\[ K \equiv \left(1 - \frac{V}{V'}\right) \left(2 - \frac{d}{(1 + \alpha)h}\right) - \frac{2V \tan^2 \chi}{V'(1 + \alpha)(1 - \tan^2 \chi)}, \quad (14) \]

where we used (9) as an estimate of \( \tau \).

Consider \( \frac{V}{V'} \). \( V' \) is the Y-component of the velocity of D after \( A' \)-collision with L, and is largest when all the available energy in the collision is converted into D’s velocity in the Y-direction. Of course, this can never happen in any collision between L and D, but will give an upper bound, although a grossly overestimated one. Before the \( A' \)-collision, the velocity of the D was \( \left(\frac{2\mu}{\mu - 1}, V\right) \). The ball L had an edge on collision at B in which L was incident horizontally with a velocity \( U \), and D with a small vertical velocity \( \bar{V} \). Since the B-collision was edge on, it only deflected the original trajectory of L with, to a very good approximation, a negligible change in the speed of L. Hence, the energy of L prior to \( A' \)-collision is \( \frac{m_L U^2}{2} \). If after this collision, all the available energy is converted into \( V' \), then

\[ \frac{1}{2} m_D V'^2 = \frac{1}{2} m_D (V^2 + \frac{4U^2}{(\mu - 1)^2}) + \frac{1}{2} m_L U^2. \]

From the above, we get

\[ 1 - \frac{V}{V'} = \frac{U^2}{2\mu V^2} \left(\frac{\mu + 1}{\mu - 1}\right)^2 + \cdots \quad (15) \]

where \( \cdots \) denote higher order terms in \( \frac{U^2}{2\mu V^2} \left(\frac{\mu + 1}{\mu - 1}\right)^2 \), which are negligible.

Now consider the \( A' \)-collision in the other extreme when the disc D looses maximum possible energy, decreasing its Y-component velocity to a maximum extent which, again, is a gross overestimate. In a collision between a light and a heavy body, the maximum energy is exchanged when the light body is initially at rest and the collision is head on. The light body gains an amount of energy = \( \frac{1}{2} m_L (2U_H)^2 \), where \( m_L \) is its mass and \( U_H \), the initial velocity of the heavy body. Thus in the \( A' \)-collision in our case, the maximally reduced \( V' \) is given by

\[ \frac{1}{2} m_D V'^2 = \frac{1}{2} m_D V^2 - \frac{2}{\mu} m_L (V^2 + \frac{4U^2}{(\mu - 1)^2}), \]
from which we get

\[ 1 - \frac{V}{V'} = -\frac{2}{\mu} \left( 1 + \frac{4U^2}{(\mu - 1)^2 V^2} \right) + \cdots, \quad (16) \]

where \( \cdots \) denote higher order terms in \( \frac{2}{\mu} \left( 1 + \frac{4U^2}{(\mu - 1)^2 V^2} \right) \), which are negligible.

From equations (15) and (16) we see that \( \frac{V}{V'} \simeq 1 + O \left( \frac{1}{\mu} \right) \) at least and, therefore, \( K \) in equation (14) is bounded in magnitude:

\[ |K| \leq K_{\max} \simeq O \left( \frac{1}{\mu} \right). \]

Hence, equation (13) can now be written as

\[ \frac{(1 - 2\epsilon)R}{\beta h} \leq \epsilon K_{\max}, \quad (17) \]

which must be satisfied for any consistent solution. However, \( \beta \) is an arbitrary variable in equation (17) which we are free to choose. Let

\[ \beta = \frac{(1 - 2\epsilon)R}{2\epsilon h K_{\max}}. \quad (18) \]

With this choice of \( \beta \), equation (17) cannot be satisfied and, hence, there can be no consistent solution. This would imply that the paradox of the scenario presented here is irresolvable, since no consistent solution exists. Such a choice of \( \beta \) is very much possible in principle and, in fact, is quite reasonable as we will now illustrate with specific numerical values for the various quantities encountered in the present scenario.

Let

\[ \mu = 101, \quad \frac{U}{V} = 10, \quad K_{\max} = 0.01 \]

and

\[ U = 10 \text{m/sec}, \quad R = 1 \text{m}, \quad \omega = 10 \text{m}, \quad \Lambda = 10^4 \text{m}. \]

It will turn out that \( \alpha \simeq 1 \) and \( \epsilon \simeq 0 \), so we take \( 1 + \alpha \simeq 2 \) and \( 1 - 2\epsilon \simeq 1 \) in the following estimates. From the above, and using equation (18), we get

\[ V_x = 0.2 \text{m/sec}, \quad V = 100 \text{m/sec}, \quad h = 5 \times 10^4 \text{m}, \quad l = 100 \text{m}. \]
and also
\[ \epsilon T = 1000, \quad \epsilon \beta \simeq 0.001, \]
from which follows
\[ \tilde{V} = 0.05m/sec. \]
If we further choose \( \epsilon = 0.01 \), we have
\[ \beta = 0.1, \quad T = 10^5 sec \simeq 30hrs. \]

Thus, we have a wormhole with mouths of size 10m, separated by 10Km, allowing travel back in time by \( 10^5 sec \simeq 30hrs \). We also have a ball of small radius, say of radius 0.1m, travelling initially with a velocity of \((10, 0)m/sec = (36, 0)km/hr\) a disc of radius 1m and 100 times heavier than the ball, travelling initially with a velocity of \((0.2, 100)m/sec = (0.72, 360)km/hr\) in the X-Y plane. Its initial trajectory would miss the wormhole by a distance of 100m. However, the disc collides with the ball, and enters the wormhole and emerges from it \( \simeq 30hrs \) behind in time. After which, the disc gets slowed down to a crawling velocity of \((0, 0.05)m/sec = (0, 0.18)km/hr\), and collides again with the ball in the past, making the first collision impossible, as shown above.

The above numbers admittedly span a wide range of velocities, and the setup perhaps requires a high ballistic precision. However, in principle, nothing forbids the above set up and the numbers. Also, these numbers have such a wide range because of our desire to keep the analysis as simple as possible, yet be able to demonstrate clearly the irresolvability of the paradox in the scenario proposed here. These numbers can be made more amenable by an elaborate analysis of the scenario, perhaps aided by numerical simulations, still resulting in an irresolvable paradox with no consistent solution.

The crucial ingredient in the scenario here is not as much the slowing down of D to a small velocity \((0, \tilde{V})\), which only made the analysis easier to handle. Rather, it is that the disc is heavier and larger in size than the ball so that, in a collision, the ball L will be reflected back by the disc, unless the collision is edge on. If this were not the case, then \( R \) in the relevant equations above must be replaced by a variable \( \rho \) with a range \( 0 \leq \rho \leq R \). In particular, \( \rho \) can be sufficiently small so as to ensure that equations (13) and (17) are always satisfied, no matter how small \( \tilde{V} \) or, equivalently, \( \beta \) is. This would be the case if, for example, both the colliding objects were...
identical, as in the works of [3, 10, 12]. We would also like to note that if one can find a situation, which we are unable to, where the collision between two objects forbids, at least classically, the forward scattering of one of them in a certain finite range, then this situation can also be used to construct a paradoxical scenario, along the lines of the present one. For example, in a collision between L and D in the above scenario, if the forward scattering of L is forbidden in the range

\[ \arctan \left( \frac{\hbar}{\Lambda} \right) < \theta_s < \arctan \left( \frac{V}{U} \right), \]

where \( \theta_s \) is the forward scattering angle of L, then a little thought makes it immediately clear that the paradox of the scenario described above just cannot be resolved.

The implications of the irresolvability of the paradox in the scenario presented here is the same as those described in section 2. Now, they apply to 3 + 1 dimensional space time. The paradox cannot be resolved except by forbidding the initial conditions that led to them. But this implies an influence of time machine on events, extending indefinitely into the past. Also, such an influence implies tachyonic communications between physical events in an era when no time machine existed (these phenomena, which occur here in the classical regime, have been observed by Hartle in his generalised quantum mechanics [18]). Moreover, the known physical laws cannot accommodate and enforce such censorship on initial conditions which can be set at any arbitrary time in the past in widely separated locations.

The system under consideration is macroscopic with a classical action \( S \gg \frac{\hbar}{2\pi} \), the quantum action. Therefore it is unlikely that quantising the system will resolve the paradox. Thus, if a quantum mechanical solution does exist for the scenario described here, it must be quite unusual, having no classical counterpart. Also, quantisation by Hamiltonian method is not possible in the presence of time machines since, it requires the space time manifold to be foliable in the time direction. It is not obvious either, how path integral method of quantisation can be carried out since the system is macroscopic, but with no consistent classical solution. Quantisation may be possible if, by fiat, all the troublesome initial conditions are forbidden, but this implies that the time machines have an influence on events, extending indefinitely into the past, and also tachyonic communications between physical events as described in section 2. Thus, if the paradox in the scenario
cannot be resolved, the logical conclusion is that the time machines of the
type considered here cannot exist in $3+1$ dimensional space time, maintain-
ing the consistency of known physical laws. This conclusion should also be
applicable to wider class of time machines since, very likely, our scenario can
be extended to them, with the consequent irresolvability of the paradox that
follows.

4. Conclusion

We have described a scenario first in $1+1$ and then in $3+1$ dimensional
space time which leads to an irresolvable paradox in the presence of a time
machine. No consistent classical solution exists for this scenario. Since the
system is macroscopic with an action large compared to Planck’s quantum,
it is unlikely that quantising the system will resolve the paradox. Moreover,
for a macroscopic system with no consistent classical solution, it is not obvi-
ous how path integral quantisation can be carried out nor is a Hamiltonian
method of quantisation available in the presence of a time machine.

Forbidding by fiat all the troublesome initial conditions will solve the
problem. However, no known physical law can enforce this kind of censorship.
Also, this implies that time machines have an influence on events, extending
indefinitely into the past, and that there will be tachyonic communications
between physical events in an era when no time machine existed.

Thus, if the above paradox cannot be resolved, the logical conclusion is
that time machines of a certain, probably large, class cannot exist in $3+1$
dimensional space time, maintaining the consistency of known physical laws.

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FIGURE CAPTIONS

FIGURE 1: Polchinski’s paradox. The figure is shown in the X-Y plane. A ball enters the wormhole mouth W2, emerges from W1 in the past, collides with its younger self knocking it off its trajectory so that it does not enter W2.

FIGURE 2: Consistent solution for Polchinski’s paradox. The figure is shown in the X-Y plane. The collision is glancing, so that the ball still enters W2 and emerges from W1, but along a slightly different trajectory so that the collision with its younger self is only glancing. Dotted lines here show the original inconsistent trajectories.

FIGURE 3: Propagation of a single ball in Politzer’s time machine. The figure is shown in the X-t plane, and the points are denoted by their \((x, t)\) coordinates. Two typical trajectories, 1 and 2, are shown. The time machine consists of two spatial intervals between \(x = 0\) and \(x = \Lambda\) at time \(t = 0\) and \(t = T\). The points \((y, 0_-)\) are smoothly identified with \((y, T_+)\), and the points \((y, T_-)\) with \((y, 0_+)\), for \(0 \leq y \leq \Lambda\).

FIGURE 4: Propagation of two identical balls in Politzer’s time machine. The figure is shown in the X-t plane.

FIGURE 5: Paradoxical collision of a light ball \(L\) with a heavy one \(H\) in Politzer’s time machine. The figure is shown in the X-t plane, and the points are denoted by their \((x, t)\) coordinates. The initial velocities of \(L\) and \(H\) are \(U_L\) and \(U_H\) respectively. Their initial trajectories intersect the \(t = 0\) line at \((-p', 0)\) and \((p, 0)\) respectively. The initial trajectory of \(H\) would intersect the \(t = T\) line at \((-q, T)\). \(L\) and \(H\) collide first at \(A(a, t_A)\), \(0 < a < \Lambda\), and \(H\) enters the time machine at \((l, T_-)\) and emerges from it at \((l, 0_+)\) with a velocity \(V_H\). There will be a second collision between \(L\) and \(H\) at \(B(b, t_B)\), \(b < a\), \(t_B < t_A\), to the past of the first collision. Here \(b\) can be positive or negative, \(U_H\) and \(V_H\) are negative, and all other variables are positive.

FIGURE 6: The paradoxical scenario in 3 + 1 dimensions. The figure is shown in the X-Y plane. The points are denoted by their \((x, y)\) coordinates and the velocities by their X- and Y-components. Wormhole mouths W1 and W2 are located respectively at \((0, h)\) and \((\Lambda, h)\). The ball L and the disc D, of radius R, have an initial velocity \((U, 0)\) and \((-V_x, V)\) respectively,
and collide at the point $A(\Lambda, 0)$ at time $t = t_A$. After the collision, $D$ enters $W2$ with a velocity $(0, V)$ and emerges from $W1$ in the past with a velocity $(0, -V)$, moving along negative Y-axis. Parametrically, it travels a distance of $\alpha h$ with this velocity, gets slowed down to a small velocity $(0, -\tilde{V})$ with which it travels the remaining distance $\beta h \equiv (1 - \alpha)h$, and collides again with $L$ at the point $B(0, 0)$ at time $t = 0 < t_A$, to the past of the A-collision.
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