Renormalization Group Equations for Seesaw Neutrino Masses. *

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Abstract

RGEs for coefficients of dim-5 operators giving rise to neutrino masses in the seesaw mechanism are written down in the SM, 2HDM and MSSM, and solved numerically. RG evolution of these coefficients modifies tree-level seesaw predictions for neutrino masses and mixing angles in SO(10)-type GUT models as strongly as quark Yukawa coupling evolution.

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The MSW solution of the solar neutrino problem [1, 2] has revived in the last few years interest in GUT model predictions for the superlight neutrino masses generated via the seesaw mechanism [2, 3]. Such models predict the neutrino mass matrix (sometimes up to a normalization factor) using definite relations between quark Yukawa couplings, Dirac type neutrino Yukawa couplings and Majorana mass matrix for the right-handed neutrinos. In this letter we elaborate on one technical aspect of such analyses: In comparing the GUT relations with low energy charged fermion and neutrino masses one has to perform a RG evolution of relevant parameters. The RG evolution of charged fermion Yukawa couplings from the weak \((M_W)\) to the unification scale \((M_X)\) is standard [4] and has been included before [2, 3]. The RG evolution of neutrino masses has so far been neglected or treated inconsistently [2]. We furnish this lacking element and demonstrate its possible numerical significance.

GUT models we are interested in contain superheavy \(SU(2) \times U(1)\) - singlet Majorana fields with a Dirac-type Yukawa coupling to the SM neutrinos and Higgs boson. To describe physics far below the GUT scale we should decouple all heavy states and use an effective theory containing light fields only. Tree-graph exchange of superheavy right-handed neutrinos gives rise, after integrating them out, to dimension-5 operators coupling two left-handed lepton doublets and two Higgses. Couplings of this kind, being suppressed by the inverse heavy mass factor, are neglected in the standard approach in which one retains in the effective lagrangian renormalizable interactions only. In order to obtain the seesaw neutrino masses of the order \(O(1/M_X)\) one must keep in the low energy lagrangian also those nonrenormalizable higher dimension terms. Similar couplings arise also in some string models [2] and our RG analysis is also applicable in this case starting from the scale where the effective theory is \(SU_c(3) \times SU_L(2) \times U_Y(1)\). We write down one-loop RGEs for the coefficients of the relevant dimension-5 operators which describe their evolution down to \(M_W\) scale. When Higgs fields are replaced by their VEVs this coupling yields a superlight Majorana mass term for the left-handed neutrinos. In the Standard Model (SM) as well as the 2 Higgs Doublet Model (2HDM) we need only one dimension-5 operator - \(O_1\). In the SUSY case we have to include other operators related to \(O_1\) by supersymmetry transformations because they mix under renormalization. We solve numerically those RGEs in the SUSY case and find that they introduce modifications to the tree-level seesaw formula of the same order as the quark Yukawa coupling evolution.

While evolution of the neutrino masses themselves does not provide very valuable information because the overall scale of heavy Majorana mass is not fixed precisely in GUTs, evolution of the neutrino mixing angles (e.g. if a definite texture for the Majorana mass matrix is assumed) may be important in comparing GUT predictions with MSW solar neutrino problem solution.
Superlight neutrino masses arise via the seesaw mechanism after diagonalization of the mass term \((M \gg m, \text{ generation indices neglected})\):

\[
\Delta L_{\text{mass}} = -\frac{1}{2} \left( \begin{array}{cc} \nu & n \\ m & M \end{array} \right) \left( \begin{array}{c} \nu \\ n \end{array} \right)
\]

(\(\nu\) is the left-handed neutrino and \(n\) is the left-handed antineutrino).

It has two eigenvalues: \(m_{\text{heavy}} \approx M\) and \(m_{\text{light}} \approx m^2/M\), with the light neutrino: \(\nu_{\text{light}} = \cos \alpha \, \nu + \sin \alpha \, n\) where \(\sin \alpha \approx m/M\).

This viewpoint does not allow, however, any simple analysis of the neutrino seesaw mass running. Moreover, a serious problem arises in theories (like MSSM) in which electroweak symmetry breaking proceeds radiatively [5]. On one hand, the tree-level VEV of the Higgs boson(s) vanishes at \(M_X\) scale and no neutrino mass is generated there. On the other hand, to get Higgs VEVs one must use RGE of the effective theory with all superheavy particles decoupled. As a result neutrinos stay, at first sight, massless. The solution of the two above mentioned problems emerges when one retains in the effective lagrangian higher dimension operators generated by the decoupling procedure. The heavy Majorana neutrino \(n\) with the Yukawa coupling in the GUT lagrangian of the form:

\[
\Delta L = -Y^{ab} n^a \epsilon_{ij} l^b_i H_j
\]

(\(\epsilon_{ij}\) - antisymmetric SU(2) tensor; \(a, b\) - generation indices and \(H_i\) is the Higgs doublet transforming as \((2, 1/2)\) under the electroweak gauge group which in the 2HDM and MSSM gives masses to the up-type quarks) when integrated out, gives rise to the nonrenormalizable term in the effective lagrangian

\[
\Delta L_{\text{eff}} = \frac{1}{4} c_{1}^{ab} O_{1}^{ab}
\]

where

\[
O_{1}^{ab} = (\epsilon_{ik} H_i l^b_k)(\epsilon_{jl} H_j l^b_l)
\]

and \(c_{1}^{ab}\) is a symmetric matrix of coefficients:

\[
c_{1}^{ab}(M_X) = 2 Y^{da}(M^{-1})^{dc} Y^{cb}
\]

(M is the Majorana mass matrix of \(n\)). It is easy to see that when Higgs field is replaced by its VEV, (3) gives rise to the neutrino mass term which up to \(O(1/M_X^2)\) is equal to the mass term given by (1).

RGE for coefficient \(c_1\) gets contributions from the proper vertex corrections due to gauge boson exchanges and Higgs self coupling, and from external wave function renormalization which introduces also the dependence on Yukawa couplings. In the 2HDM we get:
\[
\frac{d}{dt} \epsilon_{1}^{ab} = \left[ \frac{1}{2} \lambda_2 - 3 \ g_2^2 + 6 \ \text{tr} \ (Y_u Y_u^\dagger) \right] \epsilon_{1}^{ab} \\
+ \frac{1}{2}(Y_l Y_l^\dagger)^{bc} \epsilon_{1}^{ca} + \frac{1}{2}(Y_l Y_l^\dagger)^{ac} \epsilon_{1}^{cb}
\]

\[
( t \equiv (4\pi)^{-2} \log(Q/M_W) \text{ and summation over the generation index } c \text{ is understood}).
\]

In the SM there is an additional contribution to the RHS (because the Higgs boson couples to all fermions):

\[
[6 \ \text{tr} \ (Y_d Y_d^\dagger) + 2 \ \text{tr} \ (Y_l Y_l^\dagger)] \epsilon_{1}^{ab}
\]

Where \( Y_u, Y_d, Y_l \) are up-quark, down-quark and lepton Yukawa matrices respectively and \( \lambda_2 \) is the Higgs self coupling:

\[
\Delta L_{\text{higgs}} = -\frac{1}{8} \lambda_2 (H^\dagger H)^2
\]

In the MSSM we have to introduce other operators related to \( O_1 \) by SUSY because they mix under renormalization due to diagrams in which gauge bosons are replaced by the corresponding gauginos. The relevant part of the effective lagrangian reads:

\[
\Delta L_{\text{eff}} = \frac{1}{4} \epsilon_{1}^{ab} O_{1}^{ab} + \epsilon_{21}^{ab} O_{21}^{ab} + \epsilon_{22}^{ab} O_{22}^{ab} + \frac{1}{4} \epsilon_{3}^{ab} O_{3}^{ab}
\]

with

\[
O_{21}^{ab} = (\epsilon_{ik} h_i L_k^a)(\epsilon_{ji} H_j^b)
\]

\[
O_{22}^{ab} = (\epsilon_{ik} h_i l_k^a)(\epsilon_{ji} H_j^b)
\]

\[
O_{3}^{ab} = (\epsilon_{ik} h_i L_k^a)(\epsilon_{ji} h_j L_j^b)
\]

\( h \) is a higgsino and \( L^a \) is a slepton. Operators \( O_1 \) to \( O_3 \) form a basis in the space of operators which mix under renormalization.

For a superpotential containing the singlet neutrino superfield \( \hat{N}^a \):

\[
w = \frac{1}{2} M^{ab} \hat{N}^a \hat{N}^b + Y^{ab} \hat{N}^a \epsilon_{ij} \hat{l}_i^b \hat{H}_j
\]

we get after integrating out \( n^a \) and its superpartner:

\[
\frac{1}{2} \epsilon_{1}^{ab} = \epsilon_{21}^{ab} = \epsilon_{22}^{ab} = \frac{1}{2} \epsilon_{3}^{ab} = Y^{ca}(M^{-1})^{cd} Y^{db}
\]
The RGE for $c_{ab}^1$ in this case reads:

$$\frac{d}{dt}c_{ab}^1 = \left[ \frac{1}{2} \lambda_2 - 3 g_2^2 + G_{1H}^2 + 3 G_{2H}^2 \right] c_{ab}^1$$

$$+ \left[ \frac{1}{2} G_{1L}^2 + \frac{3}{2} G_{2L}^2 + 6 \text{tr} \left( Y_u Y_u^\dagger \right) \right] c_{ab}^1$$

$$+ \left[ \frac{1}{2} \left( Y_{lH_e} Y_{lH_e}^\dagger \right)^{bc} + \left( Y_{lH_h} Y_{lH_h}^\dagger \right)^{bc} \right] c_{ca}^1$$

$$+ \left[ \frac{1}{2} \left( Y_{lH_e} Y_{lH_e}^\dagger \right)^{ac} + \left( Y_{lH_h} Y_{lH_h}^\dagger \right)^{ac} \right] c_{cb}^1$$

$$- \left( \frac{1}{2} G_{1H} G_{1L} + 6 G_{2H} G_{2L} \right) \left( c_{ab}^{21} + c_{ba}^{21} \right)$$

$$- \left( \frac{1}{2} G_{1H} G_{1L} + 2 G_{2H} G_{2L} \right) \left( c_{ab}^{22} + c_{ba}^{22} \right)$$

(9)

In equation (9) $G_{1L}$, $G_{2L}$, $G_{1H}$, $G_{2H}$ are U(1) and SU(2) gaugino couplings to leptons and Higgs boson. $Y_{lH_e}$ and $Y_{lH_h}$ are Yukawa couplings of leptons to Higgses and higgsinos respectively. In the strict SUSY limit they are all equal to the corresponding gauge and Yukawa couplings $g_1$, $g_2$ and $Y_l$. In the same limit $\lambda_2 = g_1^2 + g_2^2$. We display, however, this equation in its more general form which allows one to decouple (when necessary) all heavy sfermions in one collective threshold. The RGEs for other MSSM couplings written in the same general form can be found in [7].

The remaining RGEs read:

$$\frac{d}{dt}c_{ab}^3 = \left[ 2 g_1^2 + 2 g_2^2 + 6 \text{tr} \left( Y_u Y_u^\dagger \right) \right] c_{ab}^3$$

$$+ \left( Y_l Y_l^\dagger \right)^{bc} c_{ca}^3 + \left( Y_l Y_l^\dagger \right)^{ac} c_{cb}^3$$

$$- \left( 2 g_1^2 + 6 g_2^2 \right) \left( c_{ab}^{21} + c_{ba}^{21} \right)$$

$$- \left( 2 g_1^2 + 2 g_2^2 \right) \left( c_{ab}^{22} + c_{ba}^{22} \right)$$

(10)

$$\frac{d}{dt}c_{ab}^{21} = \left[ 4 g_1^2 - 2 g_1^2 + 6 \text{tr} \left( Y_u Y_u^\dagger \right) \right] c_{ab}^{21} + 2 g_2^2 c_{ba}^{21}$$

$$+ \left( Y_l Y_l^\dagger \right)^{bc} c_{ca}^{21} + \left( Y_l Y_l^\dagger \right)^{ac} c_{cb}^{21}$$

$$+ \left( g_1^2 - g_2^2 \right) \left( c_{ab}^{22} + c_{ba}^{22} \right)$$

$$- \frac{1}{2} \left( g_1^2 + 5 g_2^2 \right) \left( c_{ab}^1 + c_{ba}^1 \right)$$

(11)
\[
\frac{d}{dt} c^{ab}_{22} = \left[ -4 g_1^2 - 2 g_2^2 + 6 \text{tr} \left(Y_u Y_u^\dagger \right) \right] c^{ab}_{22} + 2 g_2^2 c^{ba}_{22}
+ \left( Y_l Y_l^\dagger \right)^{bc} c^{ca}_{22} + \left( Y_l Y_l^\dagger \right)^{ac} c^{cb}_{22}
+ \left( g_1^2 - g_2^2 \right) \left( c^{ab}_{21} + c^{ba}_{21} \right) - 4 g_2^2 c^{ab}_{21}
- \frac{1}{2} \left( g_1^2 - g_2^2 \right) \left( c^{ab}_{11} + c^{ab}_{33} \right)
\]

(12)

In (10-12) we implement all SUSY relations between couplings but we retain the distinction between \(c^{ab}_{2i}\) and \(c^{ba}_{2i}\) because in principle one could have a model where they are not symmetric. In (4) and (9-12) there is no mixing with operators involving the other Higgs doublet \((2, -1/2)\) because there is no appropriate coupling in the MSSM and simple 2HDMs. Contributions from Yukawa couplings come only from wave function renormalization and therefore do not cause operator mixing. This fact simplifies considerably the analysis of lepton mixing angle evolution presented below.

In order to demonstrate the potential importance of the RGE evolution for neutrino masses and mixing angles we have solved equations (9-12) for the SUSY case numerically, evolving simultaneously gauge couplings and Yukawa couplings in one-loop approximation and including nonlinearities from the 3rd family. We take \(\alpha_s(M_Z) = 0.12\) and \(\sin^2 \theta_W = 0.233\) and allow \(Y_{ttop}(M_X)\) to vary from 0.2 to 6. Our unification scale is \(M_X = 10^{16}\) GeV.

We consider two distinctly different cases:

i) SO(10)-type unification for Yukawa couplings with 
\[ Y_t(M_X) = Y_b(M_X) = Y_\tau(M_X) \]

ii) An example of SU(5)-type Yukawa coupling unification with 
\[ Y_t(M_X) = 10 \ Y_b(M_X) = 10 \ Y_\tau(M_X) \]

After evolving the \(Y\)’s down to \(M_Z\) we fix \(\tan \beta\) fitting \(m_\tau = 1.784\) GeV: 
\[ m_\tau(M_Z) = \sqrt{2} (Y_\tau(M_Z)/g_2) M_W \cos \beta \]
and calculate \(m_{ttop}(M_Z)\) and \(m_b(m_b)\) which is presented in Fig.1a. \(Y_i(M_X)\) as a function of \(m_{ttop}\) is presented in Fig.1b. This sets the background for the evolution of coefficients \(c_i\) and allows one to assess the proximity of the Landau pole. Our aim in this note is to demonstrate the effects of the neutrino masses and mixing angles evolution rather than performing a detailed study of Yukawa coupling unification. Therefore, we neglect possible deviations from the assumed relations between Yukawa couplings at \(M_X\) (threshold corrections) which could result in \(m_b\) closer to its experimental value \(m_b(m_b) = 4.25 \pm 0.10\) GeV [86]. This simplified approach suggests (see [9] for a detailed analysis) that Yukawa coupling unification favours a heavy top quark.

\[1\] We neglect also possible effects of decoupling superpartners at some scale \(M_{SUSY} = O(1 \ TeV)\) on the RG evolution.
We work in the approximation of small 3rd generation mixing. In evolving $Y_t$, $Y_b$, $Y_\tau$ (or eg. $Y_u$, $Y_d$, $Y_e$) we neglect this mixing altogether and in evolving quark and neutrino mixing angles we use approximate RGEs \[I, \] which for the MSSM read:

$$\frac{d}{dt}\theta^{\nu}_{13} = -Y^2_{\tau} \theta^{\nu}_{13} \tag{13}$$

$$\frac{d}{dt}\theta^{KM}_{13} = - (Y^2_t + Y^2_b) \theta^{KM}_{13}$$

Those simple RGEs arise because Yukawa couplings responsible for lepton mixing angle evolution appear exactly in the same way in (9-12). The mixings between different operators $O_1 - O_3$ and different generations can be factorized substituting $c_{iab} \equiv A_{iab} b_i$ and, in fact, one could guess (13) without calculating (9-12) explicitly. The results are presented in Fig.2a – 2d. In all figures there are 2 sets of curves: for Yukawa unification i) - solid lines and ii) - dashed ones.

Fig.2a presents running of the coefficient $c_1$ for the 3rd and for the 1st (or 2nd) generation. For heavier top, close to the fixed point of the RGEs, Yukawa contributions overcome the gauge coupling contributions and change the direction of evolution of coefficients $c_i$, just like in the case of quark Yukawa couplings.

Fig.2b presents running of the neutrino 1-3 (or 2-3) mixing angle, which is sizeable only for large $Y_\tau$ - i.e. in the case of SO(10)-type of Yukawa couplings unification and large $m_{top}$.

Fig.2c presents the joint effect of both KM and neutrino mixing angle evolution. If we assume that they are equal at $M_X$ as predicted by some models \[3\] then their ratio at $M_W$, which is plotted in Fig.2c, is due only to RG running. Running of the neutrino mixing angle is of the same order of magnitude as KM angle running and partly cancels effects of the latter in $\theta^\nu_{13}/\theta^{KM}_{13}$. One can see that evolution even slightly strengthens the discrepancy between $\theta^{KM}_{13} = (0.3 - 1.0) \times 10^{-2}$ and the small angle MSW solar neutrino problem solution: $\theta^\nu_{13} \sim (3 - 5) \times 10^{-2} \tag{2}$. (In SUSY-GUT seesaw the solar neutrino problem is solved by $\nu_e \rightarrow \nu_\tau$ transition.)

Finally in Fig.2d we present the values of coefficient $\eta$ incorporating the effects of both neutrino and quark mass running in the seesaw formula:

$$m^a_{\nu}(M_Z) = \eta^a m^{a\,2}_{up}(M_Z) / M(M_X) \tag{14}$$

(no sum over generation index $a$)

$\eta_i$ for $i = 1, 2$ are independent of $m_{top}$ because in these quantities trace-type Yukawa coupling contributions to the evolution of $Y_{top}$ and $c_1$ cancel each other. For $\eta_3$ the trace-type contributions ($\sim tr(Y_t Y_t^\dagger)$) to the evolution...
of $c_1$ and $Y_t$ cancel each other as well but the evolution of $Y_t$ due to the non-trace contributions ($\sim (Y_t Y_t^\dagger)$) is stronger than analogous evolution of $c_1$ ($\sim (Y_\tau Y_\tau^\dagger)$) and results in a large $\eta_3$ for a heavy top quark. Values of coefficients $\eta$ have been given in [4] (with light quark masses renormalized at lower scales). Even though numerical differences ($\sim 20\%$) are insignificant considering our ignorance of the overall scale of $M$ we believe our results are more complete. We include carefully the effective dim-5 coupling running and state the assumptions concerning the charged fermion Yukawa coupling evolution more clearly, including assumed Yukawa coupling unification - the essential element of seesaw neutrino mass predictions.

To summarize, superlight neutrino masses arising via the seesaw mechanism can be most naturally viewed as a manifestation of an effective dimension-5 operator. This approach is the only one available in models with radiative $SU(2)\times U(1)$ symmetry breaking. The RG analysis based on dimension-5 operators provides a systematic and unambiguous method of incorporating radiative corrections to the GUT predictions for neutrino seesaw masses. Running of the coefficients $c_i$ is an important element of neutrino mass predictions from GUTs as quark Yukawa coupling running. For a heavy top quark the most important contribution to RGEs comes from Yukawa couplings of the 3rd generation. Any specific model predicting neutrino mixing angles from GUT seesaw must take into account equations (9-12) when it is compared with experiment.

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FIGURE CAPTIONS

Figure 1. a) $m_b(m_b)$ and b) $Y_t(M_X)$ shown as a function of $m_{top}$ in unification schemes i) and ii).

Figure 2. The result of the dim-5 coupling evolution: a) coefficients $c_1$ for the 1st and 3rd generation and b) lepton mixing angle $\theta_{13}$. Joint effect of dim-5 coupling and quark Yukawa coupling evolution: c) $\theta_{13}^v(M_W)/\theta_{13}^{KM}(M_W)$ and d) coefficient $\eta$ for the 1st and 3rd generation.