\[ \Lambda_b \rightarrow \Lambda + D(D^0) \] decays and CP-violation

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Abstract

It is shown that interference of the amplitudes for the decays \( \Lambda_b \rightarrow \Lambda D \) and \( \Lambda_b \rightarrow \Lambda \bar{D}^0 \) gives rise to CP-violation.

The decays \( \Lambda_b \rightarrow \Lambda D^0 \) and \( \Lambda_b \rightarrow \Lambda \bar{D}^0 \) are interesting for the following reasons. (i) The Hamiltonian for the decay \( \Lambda_b \rightarrow \Lambda \bar{D}^0 \) involves the weak phase \( \hat{\gamma} \). This has direct implication for CP-Violation. (ii) The factorization contribution for these decays is suppressed due to the colour factor \( C_2 + \xi C_1 \) and the form factors \( g_V(m_{D^2}) \) and \( g_A(m_{D^2}) \). In reference [1], we have recently shown that \( g_V(m_{D^2}) = g_A(m_{D^2}) = -0.120 \) and that the contribution of second form factor is negligible to the decay widths and asymmetry parameter \( \alpha \) (In fact \( f_2/f_1 = 0.129 \)). (iii) The baryon \( \Xi_c^o \) and \( \Xi^o_c \) poles contribute only to the decay \( \Lambda_b \rightarrow \Lambda D^o \). The pole contribution can be evaluated in the W-exchange model [2].

The pole contributes to the p-wave amplitude and numerically its contribution is compareable to that of factorization. This has dramatic affect on the asymmetry parameter \( \alpha \). The asymmetry parameter \( \alpha_D = -1 \), since only factorization contributes to this decay where as \( \alpha_D \) can be as low as zero depending on the relative strength of the pole and factorization contributions.

In the Wolfenstein parameterization [3] of CKM matrix [4], the weak phases \( \hat{\alpha}, \hat{\beta} \) and \( \hat{\gamma} \) are given by \( \tan \hat{\alpha} = \frac{\eta}{\eta^2 - \rho(1-\rho)} \), \( \tan \hat{\beta} = \frac{\eta}{1-\rho} \) and \( \tan \hat{\gamma} = \frac{\eta}{\rho} \). The weak phases have been denoted with \( \hat{\cdot} \) to distinguish them from asymmetry parameters \( \alpha, \beta \) and \( \gamma \). In the two body non-leptonic decays of baryons, the angles \( \phi \) and \( \Delta \) defined as \( \tan \phi = \beta/\gamma \) and \( \tan \Delta = -\beta/\alpha \) [5] give direct information about the CP violation. Due to interference of amplitudes for the decays \( \Lambda D^0 \) and \( \Lambda \bar{D}^0 \), the decay amplitudes for \( \Lambda_b \rightarrow \Lambda D^o_{\pm} \), where \( D^o_{\pm} = \frac{1}{\sqrt{2}}(D^o \pm \bar{D}^o) \) would make angles \( \phi_{\pm} \) and

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\( \Delta_\pm \) non zero in the absence of final state interactions, if the pole contribution is not negligible. These decays have also been discussed in reference [6], but the overlap is minimal.

The decays \( \Lambda_b \to \Lambda D^0 \) and \( \Lambda_b \to \bar{D}^0 \) are described by the effective Hamiltonian [7]

\[
H_{\text{eff}}(\Delta B = 1) = \frac{G_F}{\sqrt{2}} [V_{cb} V_{us}^* (C_1 O_1^c + C_2 O_2^c) + V_{ub} V_{cs}^* (C_1 O_1^u + C_2 O_2^u)] ,
\]

(1)

where \( C_i \) are Wilson coefficients evaluated at the normalization scale \( \mu \); the current-current operators \( O_{1,2} \) are

\[
O_1^c = (\bar{c}^\alpha b_\alpha)_{V-A} (s^\beta u_\beta)_{V-A}
\]

\[
O_2^c = (\bar{c}^\alpha b_\beta)_{V-A} (s^\beta u_\alpha)_{V-A}
\]

(2)

and \( O_i^u \) are obtained through replacing \( c \) by \( u \). Here \( \alpha \) and \( \beta \) are \( SU(3) \) color indices while \( (\bar{c}^\alpha b_\alpha)_{V-A} = \bar{c}^\alpha \gamma_\mu (1 + \gamma_5) b_\beta \) etc. We take

\[
C_1 = 1.117 \\
C_2 = -0.257.
\]

(3)

The factorization contributions to the s-wave and p-wave amplitudes \( A \) and \( B \) are given by

\[
A_D = - \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* (C_2 + \xi C_1) F_D (m_{\Lambda_b} - m_\Lambda) g_V(m_D^2)
\]

\[
= \frac{G_F}{\sqrt{2}} (A\lambda^3) a_f
\]

\[
B_D = \frac{G_F}{\sqrt{2}} (A\lambda^3) [b_f + b_p]
\]

\[
A_{\bar{D}} = \frac{G_F}{\sqrt{2}} (A\lambda^3) (\rho - i\eta) a_f
\]

\[
B_{\bar{D}} = \frac{G_F}{\sqrt{2}} (A\lambda^3) (\rho - i\eta) b_f
\]

(4)
where

\[ a_f = -(C_2 + \xi C_1)F_D (m_{\Lambda_b} - m_{\Lambda}) g_V \]
\[ b_f = (C_2 + \xi C_1)F_D (m_{\Lambda_b} + m_{\Lambda}) g_A \] (5)

In Eq. (3), we have also included the pole contribution \( b_p \) which is given by

\[ B_p = \frac{G_F}{\sqrt{2}} (A\lambda^3) b_p \]
\[ = - \left[ \frac{g' \langle \Xi_c' | H_{W}^{pc} | \Lambda_b \rangle}{m_{\Lambda_b} - m_{\Xi_c'}} + \frac{g \langle \Xi_c^0 | H_{W}^{pc} | \Lambda_b \rangle}{m_{\Lambda_b} - m_{\Xi_0^c}} \right] \] (6)

where \( g' \equiv g_{\Xi_c^0 \Lambda_D} \) and \( g \equiv g_{\Xi_0^c \Lambda_D} \) are strong coupling constants. In the W-exchange model [2], \( b_p \) is given by [9]

\[ b_p = \left[ \frac{-\sqrt{3}g'}{m_{\Lambda_b} - m_{\Xi_c'}} + \frac{g}{m_{\Lambda_b} - m_{\Xi_0^c}} \right] d' \]
\[ d' = 5 \times 10^{-3} \text{GeV}^3 \] (7)

Note the important fact that \( \Xi_c^0 \) and \( \Xi_c^0 \) poles contribute to the decay \( \Lambda_b \to \Lambda D^0 \) only. This is crucial in discussing the CP violation.

Let us symbolically write for the two channels of the decay:

\[ |\Lambda_b\rangle = R_D |\Lambda D^0\rangle + R_{\bar{D}} |\Lambda \bar{D}^0\rangle \] (8)

where, \( R_D = A_D \) or \( B_D \) and \( R_{\bar{D}} = A_D \) or \( B_D \) for s or p-wave respectively. From Eq. (8), it is clear that the decay amplitudes for the states

\[ D^0_+ = \frac{1}{\sqrt{2}} (D^0 + \bar{D}^0), \]
\[ D^0_- = \frac{1}{\sqrt{2}} (D^0 - \bar{D}^0), \] (9)

which are eigenstates of CP are given by

\[ A_{\pm} = \frac{1}{\sqrt{2}} \left[ \frac{G_F}{\sqrt{2}} A\lambda^3 \right] [(1 \pm \rho) \mp i\eta] a_f \]
\[ B_{\pm} = \frac{1}{\sqrt{2}} \left[ \frac{G_F}{\sqrt{2}} A\lambda^3 \right] [((1 \pm \rho) \mp i\eta) b_f + b_p] \] (10)
It is the presence of the pole contribution in the interference of the amplitudes that make $ImA_\pm B_\pm$ non zero, giving rise to CP violation. Now $ImA_\pm B_\pm$ is proportional to $\pm \eta(b_p/b_f)$. If $b_p/b_f$ is not small i.e., if $b_p$ is comparable with $b_f$, then $\beta_\pm$ may be experimentally measurable.

To fit the experimental branching ratio for the decay $\Lambda_b \to \Lambda J/\psi$, it was shown in reference [1], that with $C_1$ and $C_2$ as given in Eq. (3), the parameter $\xi = 1/N_c$ in $C_1 + \xi C_2$ must lie within the following limits:

$$0 \leq \xi \leq 0.125 \text{ or } 0.35 \leq \xi \leq 0.45$$

These limits are consistent with the limit $0 \leq \xi \leq 0.5$ suggested by the combined analysis of the present CLEO data on $B \to h_1 h_2$ decay [8]. To give a crude estimate for the strong coupling constants $g$ and $g'$, $SU(4)$ is used as a guide. In $SU(4)$

$$g' = \sqrt{3/2}(g_f - g_d) = \sqrt{3/2}(1 - 2f)g_{\pi NN}$$
$$g = -\sqrt{1/2}(g_f + g_d/3) = -\frac{1}{3\sqrt{2}}(1 + 2f)g_{\pi NN}$$
$$g'/g = \sqrt{3}\frac{g_f - g_d}{g_f + g_d/3} = \sqrt{3}\frac{F - D}{F + D/3}$$

Using $F = 0.446, D = 0.815$, we obtain

$$g'/g = -0.890, g = -0.493g_{\pi NN}$$

Now $b_p/b_f$ can be easily obtained from Eqs. (5) and (7). For $\xi = 0, C_2 = -0.257, F_D = 0.200 \text{ GeV}, g_A(m_{\Lambda}^2) = -0.120, m_{\Lambda_b} = 5.641 \text{ GeV}, m_{\Lambda} = 1.116 \text{ GeV}, m_{\Xi_c} = 2.580 \text{ GeV}, m_{\Xi_c} = 2.468 \text{ GeV}, d' = 5 \times 10^{-3} \text{ GeV}^3, g_{\pi NN} = 13.26$ and Eq. (13), we get

$$x \equiv b_p/b_f = -0.64$$

For $\xi = 0.125$, one gets $x = -1.41$. Due to uncertainty in the parameter $\xi$ and in the values of $g'$ and $g$, we treat $x$ as a free parameter in the range $-1 \leq x \leq 1$.

Using Eqns (4), (5), (7) and (10) and the experimental values of the masses given above, we obtain

$$\frac{\Gamma(\Lambda_b \to \Lambda D^o)}{\Gamma(\Lambda_b \to \Lambda D^o)} = \frac{D}{\rho^2 + \eta^2}$$
\[ \alpha_D = \frac{-(1 + x)}{D} \]

\[ \alpha_{\bar{D}} = -1 \] \hspace{1cm} (16)

\[ D = 1 + 0.946x^2 + 0.473x^2 \] \hspace{1cm} (17)

\[ \tan \Delta_{\pm} = \frac{[r_{\pm} \sin(\delta_p - \delta_s) + x \sin(\delta_p - \delta_s \pm \hat{\phi}, \hat{\beta})]}{[r_{\pm} \cos(\delta_p - \delta_s) + x \cos(\delta_p - \delta_s \pm \hat{\phi}, \hat{\beta})]} \] \hspace{1cm} (18)

\[ \tan \phi_{\pm} = \frac{[0.053r_{\pm} - 0.940x \cos(\hat{\phi}, \hat{\beta}) + 0.470r_{\pm}^2]}{[1]} \] \hspace{1cm} (19)

where

\[ r_{\pm} = \sqrt{(1 \pm \rho)^2 + \eta^2} \]

\[ \tan \hat{\phi} = \frac{\eta}{1 + \rho} \]

\[ \tan \hat{\beta} = \frac{\eta}{1 - \rho} \] \hspace{1cm} (20)

Eqs. (15-20) are main results. Using \[ \Gamma(\Lambda_{b \to \Lambda\bar{D}^0}) \]

\[ \frac{\Gamma(\Lambda_{b \to \Lambda\bar{D}^0})}{\Gamma(\Lambda_{b \to \Lambda J/\Psi})} = (2.8 \times 10^{-3})(\rho^2 + \eta^2) \] \hspace{1cm} (21)

We can express Eq. (15) as

\[ \frac{\Gamma(\Lambda_{b \to \Lambda\bar{D}^0})}{\Gamma(\Lambda_{b \to \Lambda J/\Psi})} = (2.8 \times 10^{-3})D \] \hspace{1cm} (22)

The above branching ratio and the asymmetry parameter \( \alpha_D \) for various values of \( x \) are given in Table 1.

From Table 1 one can see that \( \alpha_D \) can be as low as zero whereas \( \alpha_D = -1 \) independent of \( x \).

In the absence of final state interaction i.e. \( \delta_p - \delta_s = 0 \), the angles \( \Delta_{\pm} \) and \( \phi_{\pm} \) as function of the parameter \( x \) given in Eqs. (18) and (19) are plotted in Figs. (1) and (2) respectively for the preferred values of \( (\rho, \eta) \) of reference \[ \Gamma(\Lambda_{b \to \Lambda\bar{D}^0}) \]

\( (\rho, \eta) = (0.05, 0.36) \). Of course similar curves can be obtained for different values of \( (\rho, \eta) \) allowed by existing data.

Figs. (1) and (2) for the angles \( \Delta_{\pm} \) and \( \phi_{\pm} \) are a unique features of interference of the amplitudes for \( \Lambda_{b \to \Lambda D^0} \) and \( \Lambda \bar{D}^0 \). The experimental values of \( \alpha_D \)
would fix the value of the parameter $x$. Once $x$ is known then Eqs. (18) and (19) [if $\delta_p - \delta_s \sim 0$] can be plotted as function of $(\rho, \eta)$. This may give some interesting information about $CP$ violation.

However we have estimated $x = -0.64$ or $-1.41$ for $\xi = 0$ or $\xi = 0.125$. For these values of $x$ and $(\rho, \eta) = (0.05, 0.36)$, we get

$$x = -0.64, \quad \alpha_D = -0.61, \quad \Delta_+ = 24^o, \quad \Delta_- = -33^o, \quad \phi_+ = 26^o, \quad \phi_- = -28^o$$

$$x = -1.41, \quad \alpha_D = 0.68, \quad \Delta_+ = -64^o, \quad \Delta_- = 59^o, \quad \phi_+ = 48^o, \quad \phi_- = -53^o$$

It is clear that determination of $\alpha_D$ in future experiment will be by itself interesting even though the experimental observation of $CP$ violation in these decays may be difficult in near future.

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Table - I  The decay parameter $\alpha_D$ and the braching ratio $\frac{\Gamma(\Lambda_b \rightarrow \Lambda \bar{D}_s^0)}{\Gamma(\Lambda_b \rightarrow \Lambda J/\Psi)} \times 10^3$:

| $x$ | $\alpha_D$ | $\frac{\Gamma(\Lambda_b \rightarrow \Lambda \bar{D}_s^0)}{\Gamma(\Lambda_b \rightarrow \Lambda J/\Psi)} \times 10^3$ |
|-----|------------|-------------------------------------------------|
| -1  | 0          | 1.5                                             |
| -0.5| - 0.77     | 1.8                                             |
| 0   | -1.0       | 2.8                                             |
| 0.5 | - 0.94     | 4.4                                             |
| 1   | - 0.83     | 6.8                                             |
Figure 1: The angle $\Delta_{\pm} = \arctan(-\beta_{\pm}/\alpha_{\pm})$ as a function of $x$ [See Eq. (18), $(\delta_p - \delta_s) = 0$] for $(\rho, \eta) = (0.05, 0.36)$. Solid line gives $\Delta_+$ and dotted line gives $\Delta_-$. 

Figure 2: The angle $\phi_{\pm} = \arctan(\beta_{\pm}/\gamma_{\pm})$ as a function of $x$ [See Eq. (19), $(\delta_p - \delta_s) = 0$] $(\rho, \eta) = (0.05, 0.36)$. Solid line gives $\phi_+$ and dotted line gives $\phi_-$. 
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