Clockwork Higgs inflation

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We suggest a clockwork mechanism for a Higgs inflation with the non-minimal coupling term $\xi\phi^2 R$. The seemingly unnatural ratio of parameters, $\lambda/\xi \sim 10^{-10}$ of the self quartic coupling of the inflaton, $\lambda$, and the non-minimal coupling, $\xi$, is understood by exponential suppression of $\lambda$ by the clockwork mechanism, instead of a large non-minimal coupling. The portal interaction between the inflaton and the Standard Model (SM) Higgs doublet is introduced as a source of reheating and the inflaton mass. Successful realization of inflation requires that the inflaton gets a mass around (sub) GeV scale, which would lead to observable consequences depending on reheating process and its lifetime.

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I. INTRODUCTION

Higgs inflation is a successful model of inflation based on the Standard Model (SM) of particle physics \cite{1}. A Jordan frame action for Higgs inflation include non-minimal coupling between the inflaton field, or ‘Higgs’ field, $\phi$ and the Ricci scalar $R$,

\begin{equation}
S_J = \int d^4x \sqrt{-g} \left[ -\frac{M_P^2}{2} + \frac{\xi}{2} \phi^2 R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right],
\end{equation}

where $M_P \approx 2.4 \times 10^{18}$ GeV is the reduced Planck mass. The potential $V(\phi)$ is assumed to satisfy the following condition at a large field limit, $\xi \phi^2 \gg M_P^2$,

\begin{equation}
\lim_{\phi \to \infty} \frac{V(\phi)}{(M_P^2 + \xi \phi^2)^2} = \text{constant} > 0,
\end{equation}

then the condition ensures that the potential in Einstein frame becomes asymptotically flat \cite{2}.

The plateau in the potential may accommodate enough number of e-foldings but still requests a fine-tuning to fit the cosmological data,

\begin{equation}
\frac{\lambda}{\xi^2} \approx 4.4 \times 10^{-10},
\end{equation}

which calls for additional explanation \cite{3}. In the case where the SM Higgs field is identified with the inflaton field, within experimental uncertainty especially in the top quark mass measurements \cite{4}, this small ratio may be realized by renormalization running of the couplings \cite{5, 6}. For some updated analysis, see \cite{7, 8} and also see \cite{9}.

In this letter, instead of considering the SM Higgs field as the inflaton, we consider “Higgs inflation”, in the sense that the inflation occurs by the interplay between a positive quartic coupling and the non-minimal coupling for the scalar field, driven by the SM singlet. We would suggest a simple explanation for the small ratio $\lambda/\xi^2 \sim 10^{-10}$ by clockwork mechanism without introducing any very small or large couplings or scales.

The main idea of clockwork mechanism was first proposed in order to generate a trans-planckian period of the pseudo scalar inflaton potential \cite{10}, and utilized in more general cases \cite{11–13}. It is also generalized to the fields with different spins, and recognized that the localization of the wave functions in the site space resembles that in the deconstruction of the extra dimensional model \cite{14}, although the details are not exactly the same \cite{15, 16}. There are also interesting applications of the mechanism for various phenomenological problems such as dark matter, composite Higgs, axion ($g - 2$) of muon and seesaw mechanism \cite{17–25}. We also note that other possibilities of inflationary scenarios in the context of e.g. linear potential model, hybrid potential model and Starobinsky’s model were considered in \cite{17, 26}. Discussions on continuum limit and connection with linear dilaton models are in \cite{27, 28}.

In the next section Sec. II, we first review the basic idea of clockwork mechanism for our purpose then apply to the Higgs inflation model in Sec. III. Finally we conclude in Sec. IV.

II. CLOCKWORK MECHANISM

A clockwork (CW) mechanism is described by the clockwork diagram in Fig. 1 where a set of heavy scalar
fields $\chi_i \ (i = 1, 2, \ldots, n)$, and $\phi_i \ (i = 1, 2, \ldots, n+1)$ are linked by vertical and diagonal mass parameters, $m_i^2$ and $M_i^2$, respectively. The mass parameters are considered to be spurious of symmetries under which the spurious are bi-charged as $m_i^2 \sim (-Q_{U(1)}\chi_i, -Q_{U(1)}\phi_{i+1})$ of $U(1)\chi_i \times U(1)\phi_{i+1}$ and $M_i^2 \sim (-Q_{U(1)}\chi_i, -Q_{U(1)}\phi_i)$ of $U(1)\chi_i \times U(1)\phi_i$, respectively. Under $U(1)\phi_i$, $\phi_i \sim Q_{\phi_i}$ and $U(1)\chi_i$, $\chi_i \sim Q_{\chi_i}$ so that the CW potential is obtained as:

$$V_{CW} = \sum_{i=1}^{N} \left( m_i^2 \chi_i \phi_{i+1} - M_i^2 \chi_i \phi_i \right) + h.c. \quad (4)$$

We assume that the mass parameters are essentially similar in values so that $m_i = m$ and $M_i = M$ for $i = 1, 2, \ldots, N$ below. The ratio is $q = M^2/m^2 > 1$.

To figure out the zero mode of the theory, we can use the Euler-Lagrange equations of motion for $\chi_i$ and $\phi_i$. The solutions are iteratively obtained as

$$\phi_k = q^{-(N+1-k)}\phi_{N+1}, \quad (i = 1, \ldots, N). \quad (5)$$

In order to find the zero mode scalar field, we see the kinetic terms after inserting the solution of the equations of motion:

$$L_{kin} = -\frac{1}{2} \sum_{i=1}^{N+1} (\partial_\mu \phi_i)^2$$

$$= -\frac{1}{2} (\partial_\mu \varphi_0)^2, \quad (6)$$

where $\varphi_0$ is the canonically normalized CW zero mode scalar field as

$$\varphi_0 = \sqrt{1 - q^{-2(N+1)}} \phi_{N+1} \equiv \sqrt{N_2(q)} \phi_{N+1} \quad (7)$$

where the conveniently defined numerical factor, $N_2$. Approximately, a gear field $\phi_k$, $k \in (1, N+1)$, whose value is determined by equations of motion, is related with the zero mode as

$$\phi_k \approx \frac{\sqrt{1 - q^{-2}}}{q^{N+1-k}} \varphi_0. \quad (8)$$

One can notice that the zero mode is close to $\phi_{N+1}$ (indeed $\phi_{N+1} = \varphi_0$ when $q \to \infty$) as we have depicted in Fig. 1 and is said to be ‘localized at $i = N + 1$ site’. The other end point is for $\phi_1 \sim q^{-N} \varphi_0$ so that the effective coupling of the zero mode to the other sector of the model, which is described by an operator of dimension $n$, $O_n$, is highly suppressed as

$$\sigma \hat{O}_n \phi_1^p \sim \sigma q^{-pN} \hat{O}_n \varphi_0^p \quad (9)$$

with a positive power, $p > 0$. The effective coupling is now read to be $\sigma_{eff} \sim \sigma q^{-pN} \ll \sigma$ and its size is naturally small with $q^N \gg 1$. This explains how the CW mechanism would address hierarchy problems for seemingly unnatural small parameters. We would suggest a model of inflation based on the CW mechanism in the next section.

### III. CLOCKWORK HIGGS INFLATION

#### A. Clockwork mechanism for inflation

The action for ‘clockwork Higgs inflation’ is introduced with non-minimal coupling terms $K(\phi) R$ and the potential terms $V(\phi)$ with the CW potential $V_{CW}$:

$$S_I = \int d^4x \sqrt{-g} \left[ \frac{M_P^2 + K}{2} R - \sum_i \frac{(\partial_\mu \phi_i)^2}{2} - V_{CW} - V_{inf} \right], \quad (10)$$

where the non-minimal coupling term and the CW potential are given as

$$K(\phi) \equiv \sum_{i=1}^{N} \xi_i \phi_i^2, \quad (11)$$

$$V_{CW}(\phi_i) = \sum_{i=1}^{N} \frac{m_i^2}{2} (\phi_{i+1} - q \phi_i)^2$$

with positive $\xi_i = O(1)$ and $q > 1$. Here we consider the CW gears as real scalar fields. The quartic potential, which is responsible for inflation, is introduced only for $\phi_1$ as

$$V_{inf}(\phi_1) = \frac{\lambda_1}{4} \phi_1^4, \quad (12)$$

which breaks the CW shift symmetry.

Taking the masses of the CW heavy modes greater than the inflation scale (i.e., $m \gg V_{inf}^{1/4}$), we can safely integrate out the heavy fields, and get the effective action for the CW zero mode. We will come back to the effect

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1 It is noted that the charge assignment itself would not forbid terms composed of a power of $\phi_1^p \phi_0$ and, in principle, they could additionally contribute to the CW decomposition of the mass eigenstates. To avoid this complication, we would regard the potential in Eq. 4 as our CW model. After integrating out heavy fields $\chi_i$, whose dynamics is essentially irrelevant in our discussion, we get the effective potential of the form of $V_{CW} = V((\phi_{i+1} - q \phi_i))$.

2 In fact, the non-minimal coupling term also breaks the CW shift symmetry, which might cause the set-up radiatively unstable. In this letter, we put aside this problem, and more focus on the phenomenological implications.
of heavy modes later. In the Einstein frame,
\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R - \frac{3M_P^2(\partial_{\mu} K)^2}{4(M_P^2 + K)^2} \right. \\
- \frac{\lambda}{2(1 + K/M_P^2)} V_{CW} + V_{\phi_i} \] \\
= \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R - \frac{Z(\partial_{\mu} \varphi_0)^2}{2} - U \right]. \tag{13} \]

The last line is obtained by inserting the clockwork solution for the gear fields (8), which yields \( Z \) and \( U \) as
\[ Z(\varphi_0) = \frac{M_P^2(1 + \xi \varphi_0^3 + 6\xi^2 \varphi_0^2)}{(M_P^2 + \xi \varphi_0^2)^2}, \]
\[ U(\varphi_0) = \frac{\lambda \varphi_0^4}{4(1 + \xi \varphi_0^2/M_P^2)^2}, \tag{14} \]
and
\[ \xi = \frac{\sum_{i=1}^{N+1} \xi_i q^{-2(N+1-i)}}{N_2(q)}, \quad \lambda = \frac{\lambda_q q^{-4N}}{N_2(q)^2}. \tag{15} \]
Because the non-minimal coupling term is universally contributed by all gear fields with \( \xi_i \sim \xi_{N+1} \), the effective coupling is not suppressed as \( \xi = \mathcal{O}(1) \), while the quartic coupling is dominated by the first gear field, \( \phi_1 \). Therefore \( \lambda \sim \lambda_q q^{-4N} \).

Having the effective theory for the zero mode fields, we get the effective coupling
\[ \frac{\lambda}{\xi^2} \approx \frac{\lambda_q}{\xi^2} q^{4N} \ll 1, \tag{16} \]
which explains the small value \( 10^{-10} \) taking \( q^{4N} \approx 10^{10} \) with \( \lambda_q \sim \xi = \mathcal{O}(1) \).

The field value of \( \varphi_0 \) during inflation for the CMB scale, \( (\varphi_0)_s \), is determined by the required e-folding number \( N_e \) as
\[ N_e \approx \frac{\xi(\varphi_0)^2}{M_P^2} \approx 50 - 60. \tag{17} \]

The initial value of \( \varphi_0 \) is of the similar size of \( M_P \), \( (\varphi_0)_s \sim (8/\sqrt{\xi}) M_P \), so we would carefully check if the heavy fields would spoil the inflation dynamics because of the mixing from the quartic potential. Let us discuss it with two field decomposition of the scalar fields: \( \varphi_0 \) and \( \varphi_1 \) as the eigenstates of the clockwork potential, where \( \varphi_1 \) represents a heavy mode, which would potentially affect the inflationary dynamics closely. Then,
\[ K(\phi_i) = \xi \varphi_0^2 + \xi_1 \varphi_0 \varphi_1 + \xi_2 \varphi_1^2, \]
\[ V_{\text{inf}}(\phi_i) = \lambda_q q^{-4N} \varphi_0^4 + 3\lambda q^{-3N} \varphi_0^3 \varphi_1 + \cdots. \tag{18} \]
In the scalar potential, \( (V_{CW} + V_{\text{inf}})/1 + K/M_P^2)^2 \), the dominant tadpole contribution for the heavy modes is coming from the quartic potential \( (\sim m^2 \varphi_1^2 + \lambda q^{-3N} \varphi_0^3 \varphi_1) \), which gives the shift of the heavy field as
\[ \langle \varphi_1 \rangle \sim \lambda_q q^{-3N} \varphi_0^3/m^2. \tag{19} \]

for \( m^2 \gg \lambda_q q^{-2N} \varphi_0^2 \sim 10^{-5} N_e M_P^2 \). The CW heavy modes are still heavier than the inflaton scale, so we can integrate them out and get the effective potential of the zero mode. For the large field value of \( \varphi_0 \) (during inflation), the effective potential is corrected as
\[ U_{\text{eff}}(\varphi_0) = \frac{\lambda_q q^{-4N}}{\xi^2} \left[ 1 - \frac{2M_P^2}{\xi \varphi_0^2} + \mathcal{O} \left( \frac{\lambda q^{-2N} \varphi_0^2}{m^2} \right) \right]. \tag{20} \]

For the initial value of \( \varphi_0 \), \( (\varphi_0)_s \sim \sqrt{N_e/\xi M_P} \), the heavy field contributions for the inflation dynamics are suppressed as \( q^{-2N} N_e^2/\xi \ll 1 \), compared to the leading contribution to the slow roll parameters. In short, our treatment of inflaton potential is robust and the corrections from the heavy gear fields are small.

### B. Higgs portal with Clockwork

It is intriguing possibility that the standard model Higgs doublet field, \( H \), has a portal interaction with other sector of scalar field \((s), \lambda_{H H s} |H|^2 \phi_s^2 \). This is particularly interesting because the current measurement of top quark mass may imply metastable electroweak vacuum [29–31] (also see [32] for the state-of-the-art calculation of the decay rate) and the Higgs portal interactions would remedy the problem [33]. From the RG equation of \( \lambda_H, \lambda_{H \phi} \), and \( \lambda_{\phi} \), we can obtain the positive values of \( \lambda_{H} \) and \( \lambda_{\phi} \) for all scales [34]. In our set-up, we introduce a coupling only between the Higgs and the first gear field, \( \phi_1 \) [35], in order not to disturb the inflation dynamics through the radiative corrections from the Higgs loop, but still yield the meaningful coupling between the Higgs and the inflaton field [36].

Now the scalar potential is extended for the SM Higgs and the singlet fields,
\[ V = V_{CW} + V_{\text{inf}} + V_{H \phi}, \]
\[ V_{H \phi} = \mu_H^2 |H|^2 + \lambda_H |H|^4 + \lambda_{H \phi} |H|^2 \phi_1^2, \tag{21} \]
and the non-minimal coupling is also extended as \( K(H, \phi_i) = \xi_H |H|^2 + K(\phi_i) \).

It is known that during the inflation, the Higgs gets a effective mass squared from the non-minimal coupling as
\[ -\xi_H H_{\text{inf}}^2 \sim -\xi q^{-4N} M_P^2, \]
therefore the term makes the Higgs unstable for a positivie \( \xi_H \) [37]. However from the Higgs portal coupling, there is the another source that makes the Higgs stable during inflation from the Higgs portal interactions [38] as
\[ \Delta m^2_H = \lambda_{H q} q^{-2N} \langle \varphi_0^2 \rangle \sim \lambda_{H q} q^{-2N} M_P^2/\xi \tag{22} \]
which is much bigger than the contribution from the Higgs non-minimal coupling. For the positive \( \lambda_{H q} \), the Higgs can be stable during inflation even if the the Higgs quartic is negative at high energy scale.

After the end of inflation, the inflaton will start to oscillate, and the Higgs particles could be produced through parametric resonance in the preheating stage [39, 40]. There are several studies about the bound on
\(\lambda_{H}\) in order not to destabilize the Higgs field after inflation with the assumption that the inflaton field is oscillating with a quartic potential around its minimum \([41-43]\). In our case the situation is a little bit different, because there is no source of constant mass terms for \(\phi_0\) except the Higgs VEV. It starts to roll dominantly with a quartic potential, \(U(\phi_0) \approx \lambda_{H} q^{-4N} \phi_0^4\), which means that we cannot simply take the quartic approximation for the motion of \(\phi_0\). If thermalization arises much quicker than the case with a quadratic potential, the Higgs could be trapped at the origin due to its thermal potential. Therefore, it needs future studies for the Higgs stability with \(\lambda_{H}\) after inflation. On one hand, if \(\lambda_{H}\) is negative the Higgs is destabilized during inflation, and spoils the previous discussion. In this sense, we naturally take \(\lambda_{H} > 0\).

At present Universe, the clockwork gears are very heavy so that we cannot produce it. For the zero mode, the Higgs portal provides the mass term as

\[
V_{\text{eff}} = (q^{-2N}\lambda_{H}\phi_0|H|^2)\phi_0^2 + (\lambda_{H} q^{-4N})\phi_0^4. \quad (23)
\]

The mass is \(m_{\phi_0} \sim q^{-N}v\) and the couplings between the Higgs particles and the zero mode particles are

\[
\mathcal{L} = (\lambda_{H} q^{-N}v)\phi_0^2 + \lambda_{H} q^{-2N}v h\phi_0^2 + \cdots \quad (24)
\]

The Higgs can decay as \(h \to \delta\phi_0 \phi_0\) with the coupling \(q^{-2N}v \sim 10^{-5}v\), which is quite safe from the Higgs invisible decay rate. For the numerical value of \(q^{-N}\), we get it from inflation conditions:

\[
m_{\phi_0} \sim 10^{-2.5}v \sim \mathcal{O}(0.1 - 1)\text{GeV}. \quad (25)
\]

Inflation predicts the GeV scale light particles which are coupled to the Higgs weakly, whose experimental search would be extremely interesting and deserves further study \([44]\).

**IV. CONCLUSION**

Higgs inflation is an attractive model of inflation which explains the cosmological data with the collaborative helps from the non-minimal coupling and the inflaton potential. The requested combination of the parameters between the self-coupling constant \((\lambda)\) and the non-minimal coupling \((\xi)\), \(\lambda/\xi^2 \sim 10^{-10}\), on the other hand, requests a seemingly unnatural fine tuning. As we have explicitly shown in this paper, clockwork mechanism would provide a realization Higgs inflation. By construction, the effective coupling of the inflaton potential \(\lambda/\xi^2 \approx (\lambda_{H} / \xi^2) q^{-4N}\) is efficiently suppressed by the factor \(q^{-4N} \sim 10^{10}\).

Finally, several discussions on consistency, reheating, tensor-to-scalar ratio \((r)\), and late time implication are in order.

- **During inflation:** Having \(\xi \sim \mathcal{O}(1)\) and \(\lambda \sim \mathcal{O}(1)\) in our setup, the unitarity problem of conventional Higgs inflation \([45, 46]\) would be relieved. The stochastic quantum fluctuations of the scalar fields coupled to the inflaton are quite suppressed because they are all heavy \((M \gg H_{\text{inf}})\) and their effects on the inflaton potential is subleading. The SM Higgs also can be stable thanks to the large positive mass squared from the Higgs-inflaton coupling.

- **Reheating:** Just after the inflation, the dominant potential of the inflaton is quartic, \(U \approx \lambda_{H} q^{-4N} \phi_0^4\). Because all other CW gear fields are heavy enough, we can safely focus on dynamics of the inflaton and the Higgs fields with quartic potentials \(U \sim \lambda_{H} q^{-4N} \phi_0^4 + \lambda_{H} q^{-2N} \phi_0^2|H|^2 + \lambda_{H} |H|^4\) and the initial conditions as \(\langle \phi_0 \rangle \sim M_p / \sqrt{\xi}\), and \(\langle H \rangle \ll \langle \phi_0 \rangle\). Since we cannot take a quadratic approximation for the potential of \(\phi_0\), the dynamics for the production of the Higgs and other SM particles are all involved. More detailed study about the (p)reheating in this kind of system (with a scale invariant scalar potential for the Higgs-inflaton) would be quite interesting \([44]\).

- **Tensor-to-scalar ratio:** Even though \(N \approx 10 / \log q\) number of fields are involved in CW framework, effectively single field plays the role of inflaton so that a small tensor-to-ratio, \(r \sim 10^{-3}\) is expected \([2]\) even though a largish \(r \sim 0.1\) is not completely ruled out \([7, 8]\).

- **Late time dynamics of the inflaton:** If the clockwork mechanism for the inflation works, the inflaton mass at its minimum \((\phi_0 = 0)\) is given by the Higgs portal interaction, and around (sub) GeV. In our minimal example, it has a \(Z_2\) symmetry, so the inflaton (i.e., the quanta of the inflaton field) could contribute to the measured amount of dark matter \(\Omega_{DM} h^2 \approx 0.12\) \([47]\). The relic density of the inflaton depends on the reheating procedure, and we could give further constraints on the the size of the coupling \(\lambda_{H}\) or the breaking scale of \(Z_2\), which can predict the observations in experiments searching for ALPs. We remain it as a future work.

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