ABSTRACT
Since 1991, China has implemented two significant tax reforms. The first reform, in 1994, was a large-scale adjustment of the tax distribution system between the central and local governments, and the second reform, in 2012, replaced business tax with value-added tax. Also, the size of China’s underground economy decreased from 13.55% in 1995 to 12.30% in 2016. The paper presents an evaluation of the effect of the two tax reforms and the existing underground economy on GDP growth in China. GDP is defined as explained variable, the explanatory variables include: the ratio of declared income to actual income, the change of concealed income, and the influence of tax rate change on declared income and concealed income. According to the tax reform in 1994 and 2012, two dummy variables are set respectively. In methodology, this paper uses Simultaneous equations model, SUR-OLSs and Slutsky identity. Our estimation is based on the official statistics of China National Bureau of Statistics in the period from 1991 to 2019. In empirical analysis, we decomposed tax changes into tax rate effect (change of budget constraint slope) and income effect (change of tax liability), then analyzed the impact of tax elasticity on GDP growth. The empirical results demonstrate that both the 1994 tax reform and 2012 tax reform have had a positive impact on GDP, with high statistical significance respectively. The results also confirm that the increase of tax rate leads to the increase of hidden income, which eventually leads to the decrease of GDP. The offered methodology can also be applied to most countries for time series analyses.

KEYWORDS
underground economy; tax evasion; cash deposit ratio; elasticity of taxable income; currency demand; currency transaction

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Tax system provides many incentives for people to change their taxation behavior, which means that people may decide not to declare part or all of their income and evade some taxes. However, the avoidance fees can be actual resource costs (e.g., requiring lawyers or accountants to help people evade taxes or open Swiss bank accounts to cover up income). Moreover, it may be that tax evaders know that they may go to prison and lead to the reduction of personal utility, or the tax evaders may be morally condemned because they know that they have not complied with the legal obligations, resulting in the reduction of personal utility. The research from Mirus and Smith [1] define the underground economy as unreported rental incomes, skimming by owners of businesses, barter activities, off-the-books employment, and unreported income from home-produced goods. Indeed, it is difficult to accurately measure the size of the underground economy, because the conspiracy of tax evaders is not easily detected. For example, Kolm and Nielsen [2] find that employers and employees may agree to underreport business income in exchange for employees paying less personal income tax (PIT).

Nevertheless, a survey from Buehn and Schneider [3] estimate that the proportion of South Korea’s underground economy in GDP decreased from 28.3% in 1999 to 24.7% in 2010, which is related to the rapid growth of e-payment in South Korea during the past 20 years, resulting in the slowdown of underground economy. A research from Dreher and Schneider [4] point out that in low-income countries, the efficiency of public goods provided by the government is lower than that in high-income countries, which is one of the reasons that drives individuals or manufacturers to engage in underground economic activities. Considering that literature on China’s underground economic assessment, Schneider [5] shows that China’s underground economy accounted for 13.1, 14.4 and 15.6% of GDP in 1999, 2001 and 2002, respectively. Medina and Schneider [6] demonstrate that over the period 1991-2015, China’s shadow economy accounts for a minimum of 8.3 and a maximum of 14.1 of GDP. Besides that, Chen et al. [7] use the MIMIC method for measuring the size of China’s
underground economy from 1995 to 2016, revealing the average size of the UE increased from 13.55% in 1995 to 14.39% in 2009, and then fell to 12.30% in 2016.

2. Background

The reform of China’s tax sharing system in 1994 was initiated by the Chinese government in 1992 and finally implemented in 1994. As one can see, before China’s fiscal and tax reform in 1994, the central government discussed with local governments over the share of locally collected taxes that would be paid into the central budget. This reform is a large-scale adjustment of tax distribution system and tax structure between central and local governments. The main purpose of tax sharing reform is to reduce China’s budget deficit since the end of 1980s. Therefore, tax sharing reform is regarded as the key tax reform. On the other hand, since 2012, Shanghai has carried out pilot projects to replace business tax with value-added tax, including transportation industry and some modern service industries. Furthermore, By 2016, China has fully implemented value-added tax (VAT) instead of business tax.

In order to evaluate the impact of these two key tax reforms on economic growth, this paper takes these two tax reforms into the model as dummy variables.

The rest of this paper is arranged as follows. Section 2 briefly reviews the relevant theoretical and empirical literature. Section 3, we demonstrate the research design and methodology. This paper reports the construction of SUR-OLS regression diagnosis using ETI and Slutsky equation. In Section 4 and section 5, we check the key parameters of the model by SUR-OLS regression, which provides a explicit explanation for our findings. A brief conclusion is discussed in Section 6.

3. Review of literature

3.1. Theoretical literature review

In the 1950s, Lewis [8], Kaldor [9] and Cagan [10] mark the beginnings of preliminary research into hidden economic activities. Since then, more and more literatures have focused on the analyses between undeclared income and tax erosion. Considering the formal economic theory of tax evasion can be traced back to Allingham & Sandmo [11]. It is worth noting that Gutmann [12] proposes that recessive economy is not included in the calculation of the gross national product, he adopts the ratio of money and deposit to estimate the underground economy of the United States and illustrates currency and demand deposits as the core indicators of changes in the size of the underground economy (see also Bodemann et al. [13]).

In this paper, we first review some theories about the tax elasticity and underground economy. According to the quantitative theory of money, Feige [14] demonstrates that the relationship between the volume of transactions and official GDP is constant over time, he uses the value of total transactions as an estimate of nominal GNP and measures the informal economy as the difference between nominal GNP and the official GNP, proving that reducing income tax elasticity means that with the growth of GDP, income will not be converted into the expected tax base. In addition, Hutton & Lambert [15] derived tax elasticity and applied it to UK data to replace existing estimation techniques. In their view, in addition to the total tax data, all that needs to be done is to classify taxpayers according to the highest marginal tax rate. Obviously, compared with other methods is advantage is that it does not need to collect real information about personal income. In terms of the theory or concept between underground economy and economic growth, Adam & Ginsburgh [16] pointed out that the relationship between the growth of underground economy and the official economy is positive.

In addition, La Porta & Shleifer [17] claim that economic growth mainly comes from the contribution of efficient legal enterprises above the ground, rather than from the inefficient private enterprises underground. However, Schneider & Enste [18] show that two thirds of the income from underground economic activities will eventually flow into the official eco-
nomic sector through consumption and investment, which will have a positive impact on the official economy.

While research into the elasticity of taxable income (ETI), which measures the responsiveness of reported taxable income to changes in tax rates, dates back to at least Lindsey [19]. The ETI can capture this wide array of behavioral responses and can then be used to calculate both the efficiency and revenue implications from a change in tax rates. The intuition behind the standard ETI model is that individuals increase taxable wages until its marginal cost equals the tax rate (Feldstein [20]). Brewer et al. [21] define ETI as “percentage change in taxable income” relative to “percentage change in net income” (Carroll & Hrung [22]). Similarly, Saez et al. [23] emphasized the fact that ETI is not a constant parameter, but will be affected by government policies. In other words, since the parameters of these models are not structural, that is, they are not invariable policies, they will inevitably change whenever policies change. Therefore, adhering to the policy conclusions of these models may lead to deviation. Apparently, in a more general model, Laffer [24] concerns that changes in tax rates have two effects on income, including the arithmetic and economic effects. The arithmetic effect is that if the government reduces the tax rate, tax revenues will be lowered by the amount of the decrease in the rate. Conversely, the economic effect involves the impact of lower tax rates on employment and investment, so as to stimulate people to increase these activities. Therefore, the combined effects of economic and arithmetic effects of tax rate changes lead to the uncertainty of the impact of tax rate changes on total tax.

3.2. Empirical literature review

The primary methodological objective in the empirical literature is to devise a method for separating the response of taxable income to changes in tax rates from responses to the many other factors that also affect taxable income. Especially referring to the elasticity of taxable income (ETI) takes place in a changing economic environment, and the changes to that environment affect income growth. Therefore, adequately controlling for those non-tax-induced trends in taxable income poses a major challenge to estimating elasticities. Feldstein [25] uses panel data to assess taxpayers’ behavioral response to the 1986 US income tax reform. He estimates that the ETI is large, ranging from 1–3. After Feldstein [20; 25], the literature on ETI has increased greatly. Many subsequent researches focus on improving the elasticity estimation by paying more attention to the net-of-tax rate instrument and non-tax-related changes in the income distribution. It is worth noting that along with these modifications, the ETI estimates decreased markedly compared with those in Feldstein [25]. Research conducted by Gruber & Saez [26] report an ETI of 0.2 for middle-income earners and 0.6 for high-income earners in the US.

On the other side, Blomquist & Selin [27] estimate an ETI of around 0.20 for males and 1 for females in Sweden, this study focuses directly on the response of hourly wage rate to the change of marginal tax rate, however the model can not distinguish effort response from the change of compensation form, that is, how to distinguish the transformation of fringe benefits into full cash payment. Study by Matikka [28] shows that the average the elasticity of taxable income (ETI) estimate in Finland is 0.35–0.60. Earlier literature has shown that the income impact is either insignificant or small (see Saez, Slemrod & Giertz [23]). Hence, Matikka [28] assumes that income impact is not considered, but it is not easy to observe the income response to tax rate changes. Likewise the research results from Thoresen & Vattø (2015) [29] demonstrate elasticities below 0.1 for Norway. It is worth noting that Creedy (2009) [30] considers there is no reason to expect the elasticity to remain unchanged over time, or to be similar across countries having different tax structures and regulations (see also Giertz [31]).

Further, Creedy & Gemmell [32] provide estimates of individual and aggregate revenue elasticities of income and consumption taxes in the UK over the period 1989–2000. They find income tax revenue
elasticity estimates, of around 1.3 to 1.4 in the early 1990s, are lower than middle 1980s, reflecting in part flattening of the income tax structure since the time, which reveals that discretionary tax changes have considerably reduced tax revenues. Other countries, Pirttila & Uusitalo [33] measure the ETI in Finland, their tentative analysis shows that the average ETI is around 0.3. Mattos & Terra [34] estimate ETI in Brazil, which were derived through the use of pooled cross-sectional data with the difference-in-differences approach, the result declares cash transfers seem to have a negative association with reported income elasticity close to −0.05, suggesting that leisure and cash transfers are complements, whereas in-kind transfers have a positive association elasticity coefficient close to 0.05, illustrating that physical (cash) transfer is positively correlated (negatively correlated) with the “declared taxable income”. However, for most countries, it is not easy to obtain the complete and accurate time series data of the above two items.

4. Research Design

4.1. Methodology

As mentioned earlier in the above sections, our research is arranged and follows the processes in associative quantitative research, starting from the determination of research topics, discussing historical background, conducting literature reviews, putting forward theoretical model to formulate several testable propositions. Furthermore, we define research variables and explain the source of empirical data on the underground economy (UE), elasticity of taxable income (ETI) and tax system issue in China since 1991 to discuss about their impact on GDP growth. Finally, we implement empirical testing and draw conclusions based on the results of the empirical analysis.

Referring to the important literature on underground economy and tax base erosion in recent years. Different from the analysis of the existing literature, this article uses the following methods to investigate the impact of shadow economy on GDP in current social science research. We uses Simultaneous equations model, Slutsky identity and SUR-OLS approach (see Zellner [35]; Griffiths et al. [36]) to directly derive income compensation elasticity coefficient and income effect coefficient. In general, the SUR-OLS estimates are consistently better than the OLS (equation-by-equation) estimates, since the SUR-OLS method estimates the parameters of all equations simultaneously, so that the parameters of each single equation also take the information provided by the other equations into account. This results in greater efficiency of the parameter estimates (Cadavez & Henningsen [37]).

Also, the SUR-OLS estimator takes the correlation between the error terms into account, therefore, SUR-OLS is a robust methodology for predicting. As is well known, although China’s inland provinces have convenient transportation links. Taxpayers are in the same environment of tax laws and regulations. Therefore, it has the heterogeneity of variance, and the residual has the characteristics of contemporaneous correlation. In view of this, in order to reduce the standard error, this paper uses “seemingly unrelated regression” (SUR-OLS) to test and analyze.

Also, Slutsky equation has two parts: substitution effect and income effect. Generally, the substitution effect is negative. A merit of this approach used here is that the elastic estimation can be calculated directly from our model. In addition, in order to measure the size of China’s underground economy. In this paper, we use the cash deposit ratio (CDR) hypothesis, currency demand (CD) hypothesis and currency transaction (CT) hypothesis.

Without loss of generality, in this paper, our research is designed and follows the processes in associative quantitative research, starting from determining problems, formulating objectives, conducting literature reviews, both theoretical and empirical approach, formulating research hypotheses, define research variables, determine data collection methods, implement empirical testing and draw conclusions based on the results of the empirical analysis.
As is well known, simultaneous equations models are a type of statistical model in which the dependent variables are functions of other dependent variables, rather than just independent variables (Martin et al. [38]), which means that some of the explanatory variables are jointly determined with the dependent variable. In economic society, this is usually the result of some potential equilibrium mechanism.

Nevertheless, simultaneity poses challenges for the estimation of the statistical parameters of interest, because the Gauss–Markov assumption of strict exogeneity of the regressors is violated, whilst it would be natural to estimate all simultaneous equations at once, this often leads to a computationally costly non-linear optimization problem even for the simplest system of linear equations (Quandt [39]). As is well known, Use of SEM is commonly justified in the social sciences because of its ability to impute relationships between unobserved constructs and observable variables (Hancock [40]). SEM invokes a measurement model that defines latent variables using one or more observed variables, the links between constructs of a structural equation model can be estimated with independent regression equations (see Kaplan [41]). That is, SEM involves sequential decision-making under uncertainty or strategic environments where beliefs about other agents’ actions matter.

According to the literature review of taxable income elasticity (ETI) theory, whether from an efficiency or tax perspective, taxable income elasticity (ETI) is a key parameter in revenue analysis. Moreover, in recent years, the extended version of the ETI- the behavioral elasticity of taxable revenue (BETR) has taken over the field of public economics and be used to analyze the tax base and tax administrative and compliance choices, Hemel & Weisbach [42] demonstrates the government has to pay for audit fees, which reduces resources. Finally, the government may recover tax evasion from the audit-the mechanical revenue effect, on the whole, they are just transfers and do not affect the total resources.

However, how the above important variables play an important role in the decision-making of tax evaders is worth studying. In this article, we seek to establish the framework of the research concept and show the resulting measure – the joint elasticity of taxable revenue (JETR) to capture the change in GDP caused by any marginal change in tax rates, the tax base, and tax enforcement. Following the previous literature, Gruber & Saez [26] shows there are two sources of difference here, the first is mechanical; broad income has a larger base, so that a given dollar response will result in a smaller 10 elasticity, the second is behavioral; taxable income includes itemized deductions, which might respond to changes in taxes. Following the same discussion, Doerrenberg et al. [43] exploit several tax reforms that were implemented in Germany between 2001 and 2008, the estimates show that the total ETI is between 0.54 and 0.68, and the total income elasticity (EGI) is between 0.16 and 0.28. They believe that the difference between ETI and EGI is caused by the change of tax rate caused by the deduction amount. Since the deduction amount of China’s official statistics is not available for the time being, this paper will not discuss the impact of the deduction on economic growth.

4.2. Model

Before proceeding further, regarding the effects that underground economic activity has on tax base erosion and the maximization of individual utility. We start by performing a simplicial model assuming that the representative taxpayer with a linear utility function of the following properties for above ground economy income $y_g$ and underground economy income $y_u$, the total real income $\Sigma y = y_g + y_u$. As is known, linear utilities functions are a small subset of quasilinear utility functions, where above ground economy income and underground economy income with linear utilities are a special case of substitute goods, in which the preferences are strictly monotone and weakly convex, and the marginal rate of substitution of $y_g$ and $y_u$ is constant. In this section, our models accord with the ap-
proach of random utility maximization models (RUM) and additive in income. In the other words, the systematic utility is fixed and the individual choices are static (see McFadden [44]).

\[ U(y_g, y_u) = \ln y_g^a + \ln y_u^b; \]  
\[ U_{y_g y_g} = a(a-1)(y_g^a + y_u^b)^{a-2} - a^2 y_g^{2a-2} < 0; \]  
\[ U_{y_u y_u} = b(b-1)(y_g^a + y_u^b)^{b-2} - b^2 y_u^{2b-2} < 0; \]  
\[ U_{y_g y_u} = -ab(y_g^{a-1} + y_u^{b-1}); \]  
\[ \Delta = \text{det}(A) = \begin{vmatrix} U_{y_g y_g} & U_{y_g y_u} \\ U_{y_u y_u} & U_{y_u y_u} \end{vmatrix} > 0. \]

In the above formula, \( a \) represents the coefficient value of the ratio of “above ground income” to “actual income” of the representative taxpayers, \( b \) is the coefficient value of the ratio of “underground income” to “actual income” of the representative taxpayers, and \( 0 < a, 0 < b \). \( y_g^a \) denotes the representative taxpayer’s above ground income, and \( y_u^b \) denotes the representative taxpayer’s underground income. Clearly, it can be seen from the above formula, if the determinant \( \text{det}(A) \) is positive, it means that there exists an extreme value, where \( U_{y_g y_g} < 0 \), which denotes that there exists a relative maximum. Moreover, \( U_{y_g y_u} < 0 \), ensuring the consistency of concavity. Eq.(4) shows that \( U_{y_g y_u} < 0 \). In essence, tax evaders should transfer a dollar from ground economy \( y_g \) to underground economy \( y_u \) at a concealed cost \( H \), only so long as

\[ \frac{\partial U}{\partial y_u} < \frac{1}{(1-H)}. \]  

4.3. An application: decomposing the composition effects

Next we exploit Gutmann_UE [12] approach combined with National Bureau of Statistics of China and China Statistical Yearbook to check the changes of China’s underground economy over the period 1991–2019. The CDR method of Gutmann [12] implies that: (1) all the UE activities are completed in the form of cash transactions; (2) the ratio of cash to deposits demand held in the above-ground economic activities at any time should be the same as the base period; (3) the velocity of money circulation of the above ground economy is the same as that of the underground economy.

However, Pickhardt & Sardà [45] thinks that the ratio of cash to deposits demand cannot be fixed in the long term, so two hypotheses are added to Gutmann’s original hypothesis: (1) in the above ground economic activities, the currency held by the people remains unchanged; (2) the economic activities of all additional legal transactions are completed through demand deposits.

In contrast with Pickhardt & Sardà [45], Gutmann [12] thinks that cash may also be used in the above ground economic transactions. Based on the above viewpoints, this paper estimates the ratio of China’s “underground economy” to “above ground economy” based on 2017, the reason for choosing 2017 as the base period include:

1) as we calculate the size of China’s underground economy from 1991 to 2019, the ratio of “cash transaction” to “deposit currency” is the lowest in 2017, which is 0.1493. As is known, bank deposit include: (a) deposits demand, (b) fixed deposits, (c) savings deposits, (d) other deposits.

2) according to the data of China National Burean of Statistics, China’s cash account in 2017 is 7,064 billion yuan, M1 is 54,379 billion yuan, the tax-free cash account is 7,175 billion yuan. The cash circulated in the underground activity is 110 billion yuan, M1 deducted the cash circulated in the underground activity is 54,268 billion yuan, demonstrating the amount of cash required by the formal market. The ratio of “cash circulated in the underground market” to “cash circulated in ground market” is 0.00203, which is lowest during the period of 1991–2019.

Based on the results of data analysis, we show the ratio of “underground economic income” to “above ground economic income” is 0.27 for China in 1991,
and that ratio in 2019 is 0.004. In comparing with the changing trend of the ratio of “underground economic income” to “above ground economic income” in 1991 and 2019, we show that China’s underground economy has ameliorated significantly during the past 20 years (see also Schneider et al. [46]; Elgin & Öztunali [47]). Note that ETI is a measure of how taxable income changes when we make a change to the tax system, that statistic, moreover, can be summarized by a single “sufficient” statistic: the elasticity of taxable income. (see Feldstein, 1995 [25]). As is well known, in a progressive income tax rate schedule, the marginal tax rate increases as taxable income increases. Hence, a change in taxable income endogenously defines the change in the net-of-tax rate, and thus a valid instrumental variable for \((1 - m)\) is required (Saez et al. [23], Matikka [28]).

Considering the joint role of the elasticity of taxable income (the effect on taxable income of a tax rise) and the revenue elasticity (the effect on revenue of a change in taxable income) in influencing the revenue effects of tax rate changes and GDP. In Eq. (7), we illustrate the correlation between aggregate income, the elasticity of taxable income and the revenue elasticity as follows:

\[
\frac{dY}{Y} = \left[ \frac{(1 - m)}{Z} \frac{\partial Z}{\partial (1 - m)} \right] Zdm + \frac{\partial Y}{\partial R} \left(1 - m\right) \frac{dR - Zdm}{(1 - m)Y} \frac{Y}{Y} + \eta \frac{dR}{Y} \frac{H'(1 - m)}{(1 - m)Y} dR. \tag{9}
\]

which can be rewritten as

\[
\frac{dY}{Y} = -\xi^c \frac{dm}{(1 - m)} + \eta \left( \frac{dR - Zdm}{(1 - m)Y} \frac{Y}{Y} + \frac{H'(1 - m)}{(1 - m)Y} \right) dR. \tag{10}
\]

Further, Eq. (9) can be further simplified as

\[
\frac{dY}{Y} = -\xi^c \frac{dm}{(1 - m)} + \eta dR - \xi^c Zdm \frac{H'(1 - m)}{(1 - m)Y} dR,
\]

where \(Y\) denotes the aggregate income, \(Z\) is the declared income, \(R\) is the concealed income, \(\xi^c\) and \(\xi^u\) are the compensated and uncompensated elasticity of income relative to the net-of-tax rate \((1 - m)\), respectively, the income effect parameter \(\eta\) represents the change in after-tax GDP caused by the change in hidden income, \((dR - Zdm)\) is the change in after-tax income due to the tax change for a given before declared income \(Z\), which means that delinquent taxpayers may not honestly declare the whole amounts of their evaded tax. Let taxpayer’s declared income be taxed at the marginal tax rate \(m\). Thus, it can be expressed as (some example see Cebula & Feige [48])

\[
Z_t = \left(1 - \frac{Y_{t1}^{\text{u}}}{Y_{t1}^{g}}\right) Y_t = \frac{M_{1t}}{M_{1t} - \Delta C_t} \text{GDP}_t; \tag{11}
\]

\[
R_t = \left(\frac{Y_{t1}^{\text{u}}}{Y_{t1}^{g}}\right) Y_t = \frac{\Delta C_t}{M_{1t} - \Delta C_t} \text{GDP}_t; \tag{11}
\]

\[
\Delta R_t = \left(\frac{Y_{t1}^{u}}{Y_{t1}^{g}}\right) Y_t - \left(\frac{Y_{t1}^{u}}{Y_{t1}^{g}}\right) Y_{t-1}.
\]

The biggest discrepancy between this paper and the current literature is that the coefficient \(\xi^c\) and coefficient \(\eta\) can be derived through SUR-OLS regression directly, where \(\xi^c\) denotes the compensated elasticity of taxable income coefficient, \(\eta\) is
the income effects coefficient. Considering the costs of evasion are real resource costs and not just transfers (Chetty [49]; Balfoutas et al. [50]), variable $H$ represents the hidden cost of tax evasion, $\frac{H'}{X}$ is the marginal cost of “hidden cost”, $m_i$ is the average income tax rate applicable to the taxpayers (some example see Wang et al. [51]). Using the compensated elasticity of taxable income

$$\xi^c = \left[\left(1-m_i\right)\frac{\partial Z}{Z}\right]_{m_i}$$

and the income effect parameter

$$\eta = (1-m_i)\frac{\partial Y}{\partial R}.$$ 

Change in

$$dY = \frac{-\xi^c}{Y} \frac{dm_i}{(1-m_i)Y} Z + dR - Zdm_i (1-m_i)Y$$

and $R$ affect aggregate income as follows,

$$\frac{dY}{Y} = -\xi^c \frac{dm_i}{(1-m_i)Y} Z + \eta \left(\frac{dR-Zdm_i}{(1-m_i)Y}\right) + \frac{H'}{Y} dR + \epsilon_i.$$ (12)

Furthermore, Eq. (12) can be analyzed as follows:

(a) Suppose the hidden cost of tax evasion for the tax evaders is ignored, which means the hidden cost is 0, thus we have

$$\frac{dY}{Y} = -\xi^c \frac{dm_i}{(1-m_i)Y} Z + \eta \left(\frac{dR-Zdm_i}{(1-m_i)Y}\right) + \epsilon_i; \quad (13)$$

(b) Consider the tax evaders’ hidden cost is greater than zero, thus we have

$$\frac{dY}{Y} = -\xi^c \frac{dm_i}{(1-m_i)Y} Z + \eta \left(\frac{dR-Zdm_i}{(1-m_i)Y}\right) + \frac{H'}{Y} dR + \epsilon_i.$$ (14)

In fact, concerning the change of tax policy is likely to affect the income elasticity of income tax (see Creedy & Gemmell [32]), for example, Singer [52] uses dummy variables in estimating the income elasticity of state income-tax revenues. In this paper, we take the impact of two dummy variables $D_{1994}$, $D_{2012}$ into consideration, where $D_{1994}$ is dummy variable that equals 1 after 1994, denoting the reform of China’s tax sharing system, and $D_{2012}$ is dummy variable that equals 1 after 2012, depicting the implementation of replacing business tax with value-added tax since 2012. However, as is known, because the regression analysis of more than two dummy variables are inclined to appear “dummy variable trap” and linear combination of dummy variables, the intercept term of one dummy variable can be omitted to avoid singular phenomenon (Kennedy [53]). So that Eq. (13) is represented as follows:

$$\frac{dY}{Y} = -\xi^c \frac{dm_i}{(1-m_i)Y} Z + \eta \left(\frac{dR-Zdm_i}{(1-m_i)Y}\right) - \frac{H'}{Y} dR + \beta_1 D_{1994} + \beta_2 D_{2012} + \epsilon_i,$$ (15)

where $\epsilon_i = \varphi_1 \epsilon_{i-1} + \varphi_2 \epsilon_{i-2} + \sigma_i$.

As shown in Eq. (15), instead of calculating ETI directly, Eq. (15) obtains an average elasticity through regressions explicitly.

Obviously, in comparing with existing relevant literature on underground economy, our result has the advantage of allowing simple tests of significance of the estimated average elasticities as well as the option of including relative explanatory variables. On the other hand, it has the advantage of being applicable to countries and applications in time series analysis. In this section, we demonstrate, at the aggregate income level, how the revenue elasticity and the elasticity of taxable income are combined to generate the elasticity of tax with respect to the marginal rate. Furthermore, considering the joint role of the elasticity of taxable income (the effect on taxable income of a tax rise) and the revenue elasticity (the effect on revenue of a change in taxable income) in influencing the aggregate income and revenue effects of tax rate changes. Clearly, an appealing feature of this article is that, in the traditional literature, when calculating the value of the two coefficients, $\xi$ and $\eta$, the statistical data must be brought into $\xi$ and $\eta$ to seize the results. However, instead of calculating, in this paper, we use the SUR-OLS regression approach and Slutsky identity directly obtain the coefficient values of the time serial composite structures model. Namely, this is the main discrepancy between our article and current relative literature.
5. Empirical Research results

5.1. Methodology

As is well known, there are a number of ways to measure aggregate income, but GDP is one of the best known and most widely used. To explore the impact of China’s underground economy and tax arrears on GDP, our estimates are from official statistics compiled annually by China National Bureau of Statistics since 1991. It is well known, the basic hypothesis, intercept term, regression coefficient and error term of the model will vary with various assumptions. In this paper, we assume that all independent variable coefficients (including intercept and slope) are different due to different tax rates and tax policies, but the error term dependent. Although traditional regression analysis assumes that the residual items are independent of each other, in fact it may be dependent. In fact, the overall environment faced by taxpayers in all regions of China is roughly the same. Except for various explanatory variables, other factors not included in the regression model may have the same impact on taxpayers in all regions.

Therefore, in this case, seemingly unrelated regression (SUR-OLS) can be used for analysis (see Zellner [35], Griffiths et al. [36]). As mentioned above, even though China has a vast territory, but the transportation in China is very convenient. Taxpayers are in the same environment of tax laws and regulations, which affects the environment of taxpayers’ income declaration and tax arrears. Therefore, the residual items are not independent but related. In view of this, in order to reduce the standard error, this paper uses “seemingly unrelated regression” (SUR-OLS) to test and analyze. In order to explore the influence of underground economy and tax rate on GDP growth, firstly, GDP is defined as “explained variable”. The explanatory variables include: the ratio of declared income to actual income, the change of concealed income, and the influence of tax rate change on declared income and concealed income. According to the tax reform in 1994 and 2012, two dummy variables are set respectively.

5.2. Unit Root Test

Next we use Simultaneous equations model and SUR-OLS approach to exploit China as a case study, using the cointegration approach among the GDP, variables Z, R, m for China over a time period ranging from 1991 to 2019, determining whether the stochastic component contains a unit root or not.

The results of unit root tests are presented in Table 1, which demonstrates that all the variables appeared stationary at the first-differenced form under 5% significant level, depicting the logged variables are I(1).

We next utilize the SUR-OLS regression method evaluating the residual term and estimate whether the residual term conforms to no sequence autocorrelation.

Owing to the Q-statistic proposed by Box and Pierce (1970) is rather weak in large samples, Ljung-Box [54] proposes another modified Q-statistic suitable for small samples. However, Box & Jenkins [55] consider that it is necessary to diagnose whether the parameters have overfitting and also confirm whether the residuals have serial correlation. Below, the results of Ljung-Box Q test are shown in Figure 1, which reveals the probability values of Q-statistics from the first period.

| Variable | N-st difference (C, T, K) | DW | ADF 5% | ADF 1% | Result |
|----------|---------------------------|----|--------|--------|--------|
| Y        | (C, n, 8)                 | 1.54 | -4.04 | -3.67 | -4.53 | I(1)** |
| Z        | (C, n, 6)                 | 2.04 | -4.22 | -3.61 | -4.39 | I(1)** |
| R        | (C, n, 5)                 | 1.83 | -6.05 | -3.67 | -4.53 | I(1)***|
| m        | (C, n, 1)                 | 1.87 | -5.20 | -3.59 | -4.35 | I(1)***|

Note: (C, T, K) indicates whether the test formula contains constant term, time trend and number of lag periods using AIC. Standard errors in parentheses: *** means the first-order difference passes the stability test at 1% significance level, ** means the first-order difference passes the stability test at 5% significance level.
### Table 1: Performance of residual autocorrelation diagnosis, 1991–2019

| Sample: 1991–2019 | Q-statistic probabilities adjusted for 2 dynamic regressors |
|-------------------|----------------------------------------------------------|
| **Autocorrelation** | **Partial Correlation** | **AC** | **PAC** | **Q-Sta** | **Prob** |
| a | 0.180 | 0.109 | 19.763 | 0.072 |
| b | -0.037 | -0.107 | 8.1929 | 0.515 |
| c | -0.107 | -0.281 | 8.1331 | 0.421 |
| d | -0.127 | -0.053 | 7.6394 | 0.266 |
| e | 0.244 | 0.195 | 12.075 | 0.280 |
| f | 0.099 | -0.178 | 10.675 | 0.471 |
| g | 0.180 | 0.120 | 9.7144 | 0.466 |
| h | 0.178 | 0.120 | 9.7144 | 0.466 |

### Table 2: Performance of residual autocorrelation diagnosis, 1991–2019

| Sample: 1991–2019 | Q-statistic probabilities adjusted for 2 dynamic regressors |
|-------------------|----------------------------------------------------------|
| **Autocorrelation** | **Partial Correlation** | **AC** | **PAC** | **Q-Sta** | **Prob** |
| a | 0.352 | 0.352 | 2.0159 | 0.156 |
| b | 0.005 | -0.131 | 2.0165 | 0.365 |
| c | 0.026 | -0.255 | 3.3550 | 0.360 |
| d | -0.028 | -0.121 | 5.1293 | 0.274 |
| e | -0.029 | -0.207 | 7.2114 | 0.205 |
| f | -0.012 | -0.052 | 7.6994 | 0.266 |
| g | 7.0649 | 0.360 |
| h | 8.1929 | 0.515 |
| i | 0.011 | 0.027 | 10.375 | 0.408 |
| j | 0.005 | 0.005 | 0.0021 | 0.964 |

### Table 3: Performance of residual autocorrelation diagnosis, 1991–2019

| Sample: 1991–2019 | Q-statistic probabilities adjusted for 2 dynamic regressors |
|-------------------|----------------------------------------------------------|
| **Autocorrelation** | **Partial Correlation** | **AC** | **PAC** | **Q-Sta** | **Prob** |
| a | 0.367 | 0.367 | 2.0316 | 0.129 |
| b | 0.020 | -0.171 | 2.0374 | 0.315 |
| c | 0.027 | -0.233 | 3.7099 | 0.290 |
| d | 0.029 | -0.122 | 5.6220 | 0.229 |
| e | 0.022 | -0.112 | 6.8719 | 0.230 |
| f | 6.133 | 0.288 |
| g | 7.0649 | 0.360 |
| h | 8.1929 | 0.515 |
| i | 0.099 | 0.177 | 9.7144 | 0.466 |
| j | 0.014 | 0.107 | 10.675 | 0.471 |

### Table 4: Performance of residual autocorrelation diagnosis, 1991–2019

| Sample: 1991–2019 | Q-statistic probabilities adjusted for 2 dynamic regressors |
|-------------------|----------------------------------------------------------|
| **Autocorrelation** | **Partial Correlation** | **AC** | **PAC** | **Q-Sta** | **Prob** |
| a | 0.209 | 0.129 | 14.320 | 0.281 |
| b | 0.058 | -0.111 | 10.838 | 0.459 |
| c | 0.138 | -0.180 | 14.207 | 0.177 |
| d | 0.098 | 0.008 | 0.0021 | 0.964 |

### Table 5: Performance of residual autocorrelation diagnosis, 1991–2019

| Sample: 1991–2019 | Q-statistic probabilities adjusted for 2 dynamic regressors |
|-------------------|----------------------------------------------------------|
| **Autocorrelation** | **Partial Correlation** | **AC** | **PAC** | **Q-Sta** | **Prob** |
| a | 0.333 | 0.333 | 1.7991 | 0.180 |
| b | 0.203 | -0.167 | 1.8188 | 0.403 |
| c | -0.22 | -0.184 | 2.8223 | 0.420 |
| d | -0.22 | -0.101 | 3.9421 | 0.414 |
| e | 0.026 | -0.255 | 6.2272 | 0.285 |
| f | -0.010 | 0.016 | 6.5088 | 0.369 |
| g | -0.052 | 0.222 | 6.8367 | 0.443 |
| h | 0.011 | -0.086 | 7.7184 | 0.461 |
| i | 0.200 | -0.002 | 10.316 | 0.413 |
| j | 0.150 | -0.086 | 12.516 | 0.526 |
| k | 0.170 | 0.056 | 18.182 | 0.310 |

### Table 6: Performance of residual autocorrelation diagnosis, 1991–2019

| Sample: 1991–2019 | Q-statistic probabilities adjusted for 2 dynamic regressors |
|-------------------|----------------------------------------------------------|
| **Autocorrelation** | **Partial Correlation** | **AC** | **PAC** | **Q-Sta** | **Prob** |
| a | 0.303 | 0.303 | 1.7991 | 0.180 |
| b | 0.203 | -0.167 | 1.8188 | 0.403 |
| c | -0.22 | -0.184 | 2.8223 | 0.420 |
| d | -0.22 | -0.101 | 3.9421 | 0.414 |
| e | 0.026 | -0.255 | 6.2272 | 0.285 |
| f | -0.010 | 0.016 | 6.5088 | 0.369 |
| g | -0.052 | 0.222 | 6.8367 | 0.443 |
| h | 0.011 | -0.086 | 7.7184 | 0.461 |
| i | 0.200 | -0.002 | 10.316 | 0.413 |
| j | 0.150 | -0.086 | 12.516 | 0.526 |
| k | 0.170 | 0.056 | 18.182 | 0.310 |

*Probabilities may not be valid for this equation specification.*
to the twelfth period are all significantly greater than the 5% significance level. On the other words, the residuals estimates of model 1 to model 8 have no sequence autocorrelation.

We next exploit the Histogram-Normality test and Heteroscedasticity test. In Table 2, we use Breusch-Pagan-Godfrey to diagnose residual heterogeneity, which show the p-values of F-statistic, OBS * R-squared and Scaled explained SS of all models are all significantly greater than 5%, denoting that the residuals from model 1 to model 8, in Table 2, do not exist residual heterogeneity.

5.3. Correlation coefficient analysis

In order to avoid the problem of colinearity among explanatory variables, which will affect the empirical results, this paper intends to test the correlation degree of each explanatory variable before the empirical study. From the Pearson correlation coefficient analysis results in Table 3, it is known that the

\[
\frac{dm}{(1-m)} Z
\]

and

\[
\left( \frac{dR-Zdm}{(1-m)} Y \right)
\]

are negatively correlated with variable Dummy1994 at -0.781 and - 0.601, respectively.

The correlation coefficients of other explanatory variables ranged from -0.056 to 0.558, which means that the correlation coefficients of independent variables are not high, and the problem of regression collinearity is not serious, among which the tax system reform in 1994 and 2012 are discussed by using the dummy variables.

In Table 4, we show that from model 1 to model 8, the p-values of Jarque-Bera

### Table 2

| Breusch-Pagan-Godfrey test | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 |
|---------------------------|---------|---------|---------|---------|---------|---------|---------|---------|
| F-statistic               | 3.584627 (0.0587) | 1.239100 (0.4006) | 1.450325 (0.3154) | 1.515248 (0.3345) | 2.140653 (0.1745) | 2.277715 (0.1909) | 1.276058 (0.3704) | 0.876338 (0.5794) |
| OBS* R-squared            | 8.344362 (0.0797) | 7.194095 (0.3033) | 6.614762 (0.2509) | 8.835131 (0.2647) | 7.859705 (0.1641) | 9.896486 (0.1945) | 6.198952 (0.2873) | 7.162217 (0.4122) |
| Scaled explained SS       | 1.390470 (0.8458) | 1.224072 (0.9757) | 0.736053 (0.9809) | 0.717364 (0.9928) | 1.080067 (0.9595) | 0.527859 (0.9993) | 1.130833 (0.9513) | 0.872182 (0.9966) |

**Note:** 1. In this table, the p-values of F-statistic, OBS * R-squared and Scaled explained SS of model 1 to model 8 are significantly greater than 5%. 2. In this Table, model 1 to model 8 correspond to the eight models in Table 4 in an orderly way. 3. Standard errors in parentheses: *** means the first-order difference passes the stability test at 1% significance level, ** means the first-order difference passes the stability test at 5% significance level.

### Table 3

| Explanatory variable | \( \frac{dm}{(1-m)} Z \) | \( \frac{dR-Zdm}{(1-m)} Y \) | \( R_t = \frac{Y_t^d}{Y_t^p} \) | \( \frac{\Delta C_t}{M_{t-1}-\Delta C_t} GDP_t \) | Dummy 94 | Dummy 12 |
|----------------------|--------------------------|--------------------------|-----------------|-----------------|---------|---------|
| \( \frac{dm}{(1-m)} Z \) | 1 | 0.558** (0.002) | -0.359*** (0.056) | -0.781** (0.000) | -0.180 (0.351) |
| \( \frac{dR-Zdm}{(1-m)} Y \) | 1 | -0.061 (0.755) | -0.601** (0.001) | -0.271 (0.155) |
| \( R_t = \frac{Y_t^d}{Y_t^p} \) | 1 | 0.391* (0.036) | -0.056 (0.772) |
| Dummy 94             | 1 | 0.192 (0.319) |
| Dummy 12             | 1 | |

**Note:** The upper right corner of this table is the Pearson correlation coefficient (p-value in brackets).
test are greater than 5%, depicting that all models in Table 4 can not reject the null hypothesis that the residual term conforms to normal distribution. This above mentioned research design demonstrates, at the aggregate income level, how the revenue elasticity and the elasticity of taxable income are combined to generate the elasticity of tax with respect to the change of marginal rate. Next, on the basis of the the joint elasticity of taxable revenue (JETR), three indicators (the cash deposit ratio (CDR) approach, currency demand (CD) approach and currency transaction (CT)) are added separately to evaluate the relationships among elasticities, underground economy (UE) and GDP growth. For controlling the contemporaneous correlation between the heterogeneity and the residual in the models, we use SUR-OLS and Rolle’s approach to evaluate the interdependence and correlation between those parameters.

In model 1 of Table 4, we only analyze the impact of above ground income variables on GDP growth. As noted in Table 4, $D_{1994}$ is dummy variable that equals 1 after 1994, denoting the reform of China’s tax sharing system, and $D_{2012}$ is dummy variable that equals 1 after 2012, depicting the implementation of replacing business tax with value-added tax since 2012. Due to the regression analysis of more than two dummy variables are inclined to appear “dummy variable trap” and linear combination of dummy variables, the intercept term of one dummy variable can be omitted to avoid singular phenomenon [53]. To show how these two important tax reforms influence the GDP growth over time, in model 2 of Table 4, the dummy variables $D_{1994}$ and $D_{2012}$ are added concurrently.

5.4. GMM test

The Sargan-Hansen test (Sargan [56]; Hansen [57]) is computed from residuals from instrumental variables regression by constructing a quadratic form based on the cross-product of the residuals and exogenous variables. Under the null hypothesis that the over-identifying restrictions are valid. In Table 4, we use Sargan-Hansen test to prove the post estimation of GMM (generalized method of moments), the null hypothesis shows that the instrumental variable is effective.

According to the estimation results, the $p$-values of model 1–8 are all less than 0.05.

Therefore, we agree with the null hypothesis of “instrumental variables are effective” in Eq. (16) regression model.

5.5. Regression analysis and results

Eq. (15) displays the behavioral response in income induced by the small tax change and tax reform. However, for large tax changes, it is perhaps more suitable to use a log–log specification. Hence, excluding dummy variables, we obtain the following specification.

\[
\log \frac{dY}{Y} = -\xi\log \left( \frac{dm}{(1-m)Y} \right) + \eta \log \left( \frac{dR - Zdm}{(1-m)Y} \right) + \beta_1 \log Gutmann\_UE_i + \beta_2 \log Tanzi\_UE_i + \beta_3 \log Feige\_UE_i - H' \log (dR) + \beta_4 D_{1994} + \beta_5 D_{2012} + \epsilon_t,
\]

Further, we incorporate three kinds of underground economic parameters respectively, including $logGutmann\_UE$, $logTanzi\_UE$ and $logFeige\_UE$, into Eq. (16) in pursuit of measuring their influence on GDP growth. For the underground economy parameter, it can be seen that research conducted by Tanzi [58] calculates only those underground activities that are solely the result of taxes. That is to say, in general, the estimates are obviously higher for the Gutmann approach than for the Tanzi approach (see Cebula & Feige [48]). Based on the empirical results in Table 4, we draw the following results:
Case 1: Model 1 in Table 4 indicates the independent variable
\[ \frac{dm Z}{(1-m) Y} \]
has a negative correlation with \( \frac{dY}{Y} \), and its coefficient value \( \xi \) is -0.442, reaching 1% significance.

It means that the increase of tax rate will lead to the decrease of taxpayer’s income and willingness to declare. Likewise, the independent variable
\[ \frac{dR - Zdm}{(1-m) Y} \]
has a negative correlation with \( \frac{dY}{Y} \), and its coefficient value \( \eta \) is –0.139616, which means that the increase of tax rate leads to the increase of concealed income, which eventually leads to the decrease of GDP, but it does not pass the 10% significance test.

Case 2: We add two dummy variables \( D_{1994} \) and \( D_{2012} \) into model 1 in Table 4, and get model 2 in Table 4. The result shows that \( \xi \) is -0.28407, which passes the significance test of 1%. It means that the increase of tax rate will lead to the decrease of taxpayer’s income. The coefficient \( \eta \) is -0.0275, it means that the increase of tax rate leads to the increase of concealed income, which eventually leads to the decrease of GDP, but it fails to pass the 10% significance test.

This shows that the increase of tax rate leads to the prevalence of underground economy, but the income holders of underground economy may eventually drive part of GDP growth through consumption expenditure, which can be regarded as partially offsetting the strength of the above ground economic slowdown (see Schneider & Enste [18]). In addition, two dummy variables \( D_{1994} \) and \( D_{2012} \) are added to model 1 in Table 4, Model 2 in Table 4 can be obtained, and the corresponding regression coefficient is positive, which has passed the significance test of 1%. It shows that the impact of the two tax reform on GDP is positively correlated.

Case 3: Model 3 in Table 4 adds the variable \( \text{Gutmann\_UE} \)
\[ \frac{Y_{ul} - \theta_{gl}}{\theta_{gl} + 1} \]
to model 1 of Table 4 From the results of model 3 in Table 4, we can see that the variable \( \text{Gutmann\_UE} \) is negatively correlated with GDP, reaching a significant level of 1%.

The test of the cash deposit ratio (CDR) method shows that the higher the proportion of currency to current deposit, the more underground economic activities, leading to the decline of GDP. Likewise, Model 4 in Table 4 shows that the two dummy variables, \( D_{1994} \) and \( D_{2012} \) both have a positive effect on GDP, which pass the 10% and 5% significant test, respectively. Meanwhile, Model 3 in Table 4 is based on model 1, adding the variable \( \text{Gutmann\_UE} \), which indicates the independent variable
\[ \frac{dm Z}{(1-m) Y} \]
has a negative correlation with \( \frac{dY}{Y} \), and its coefficient value \( \xi \) is -0.2430, reaching 1% significance. It means that the increase of tax rate will lead to the decrease of taxpayer’s income and willingness to declare. Likewise, the independent variable
\[ \frac{dR - Zdm}{(1-m) Y} \]
has a negative correlation with \( \frac{dY}{Y} \), and its coefficient value \( \eta \) is -0.2075, reaching 5% significance, it means that the increase of tax rate leads to the increase of concealed income, which eventually leads to the decrease of GDP.

Based on Model 3 in Table 4, two dummy variables are added to form Model 4 in Table 4. Similarly, the independent variable
\[ \frac{dR - Zdm}{(1-m) Y} \]
has a negative correlation with \( \frac{dY}{Y} \), and its coefficient value \( \eta \) is -0.2248, reaching 1% significance, it means that the increase of tax rate leads to the increase of concealed income, which eventually leads to the decrease of GDP. The coefficient \( \eta \) is -0.1051, which fails to pass the 10% significance test. Similar to the result of case 2, this shows that the increase of tax rate leads to the prevalence of underground economy, but tax evaders in underground
| Explanatory variable | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 |
|----------------------|---------|---------|---------|---------|---------|---------|---------|---------|
| ln dm Z (1−m) Y      | -0.442453*** (-6.492946) | -0.284076*** (-5.620377) | -0.243030*** (-4.942176) | -0.224828*** (-5.411377) | -0.246308*** (-6.494772) | -0.284170*** (-5.669070) | -0.245071*** (-5.638118) | -0.249913*** (-0.035226) |
| ln dR−Zdm (1−m)Y    | -0.139616 | -0.027502 | -0.207544** | -0.105175 | -0.154027 | -0.032271 | -0.012269 | -0.027686 |

Gutmann
\[
Y_{gt} = \ln \left( \frac{C_t - \theta_{gt}}{D_t + \theta} \right) - 0.518864*** (-5.980597) - 0.326234*** (-3.358242)
\]

Tanzi
\[
\ln \left( \frac{C_t}{(C_t + D_{gt} - C_{gt})} \right) = -0.234889 (-1.288345) - 0.054636 (-0.484154)
\]

Feige
\[
\frac{\Delta C_t}{M_{1t} - \Delta C_t} = -0.741244*** (-6.866686) - 0.536203*** (-2.939628)
\]

Dummy1994
\[
0.703433** (2.318284) 0.478658* (2.032305) 0.693949* (2.301224) 0.131317 (0.6810)
\]

Dummy2012
\[
1.145828*** (5.085711) 0.594083** (2.530717) 1.109889** (4.713955) 0.455214 (1.547051)
\]

AR(1)
\[
1.663166*** (5.603642) 1.401265*** (6.571071) 1.625625*** (5.258483) 1.393153*** (6.063799) 1.601428*** (4.962446) 1.601428*** (4.962446) 1.303156*** (4.651232) 1.460350*** (5.458498)
\]

AR(2)
\[
-0.689095* (-2.417058) -0.408583* (-1.960289) -0.681564* (-2.324137) -0.411300 (-1.841833) -0.634416* (-2.060193) -0.634416* (-2.060193) -0.314920 (-1.597541) -0.456024 (-1.823403)
\]

W
\[
1.654753 2.377389 2.198092 2.173955 2.104868 2.581169 3.084119 2.576688
\]

Jarque-Bera
\[
1.100422 0.216076 1.362569 0.443695 0.990879 1.012386 0.682764 0.101850
\]

TSLS
\[
5.37E+40 4.77E-37 0.00000 2.40E-36 0.00000 3.13E-38 0.00000 0.00000 0.367612
\]

\[\text{Adjusted-R}^2 = 0.750498 0.904652 0.922794 0.940591 0.754637 0.894500 0.937167 0.933673\]

Note: 1. In brackets is the t-statistic of the estimated parameter. 2. Robust standard errors in parentheses. \( p^* < 0.10, p^{**} < 0.05, p^{***} < 0.01 \). 3. Suppose \( Y'Y = 0 \). 4. The table is based on the historical data of the National Bureau of statistics of China. 5. Endogeneity Test: the p-value of all the models is greater than 0.05, which accepts the null hypothesis that there is no endogenous variable.
economy may eventually pull part of GDP growth through consumption expenditure, partially offsetting the above ground economic recession.

**Case 4:** Model 5 in Table 4 is based on Model 1, adding the variable Tanzi UE,
\[
C_{th} = \left( C_t + D_{gt} - C_{th} \right),
\]
and the corresponding regression coefficient is -0.234, indicating that the effect of the variable Tanzi UE on GDP is negative, but it fails the 10% significance test. The analysis results of model 5 in Table 4 can be explained by Becker’s crime and penalty theory [58], the cost of using cash seems to be less than that of electronic payment, so the increase of using electronic payment may not necessarily lead to the decrease of cash use. Our empirical result is similar to the research by Visa Europe et al. (2013), illustrating the anonymity of cash makes it difficult to trace cash transactions, resulting in the prevalence of underground economy. Nevertheless, we add two dummy variables to model 1 of Table 4 to obtain model 6 of Table 4. The results show that two dummy variables \( D_{1994} \) and \( D_{2012} \) both of them have a positive impact on GDP, reaching 10% and 5% respectively. Meanwhile, Model 5 in Table 4 is based on Model 1, adding the variable Tanzi UE, which indicates the independent variable
\[
dR = Zdm + \frac{\Delta C_t}{(1 - m)Y}
\]
has a negative correlation with \( \frac{\Delta Y}{Y} \), and its coefficient value \( \xi \) is -0.4263, reaching 1% significance. The result of our analysis confirming the increase of tax rate leads to the increase of concealed income, which eventually leads to the decrease of GDP.

**5.6. Summary**

Further, based on the results of empirical analysis, the above research results can be further summarized, with the following key points:

1) Using Tanzi UE [59] approach, the impact of cash transactions on GDP is negatively correlated, implying that the increase in cash transactions led to an increase in the underground economy. Our findings are consistent with Cagan’s [10] view that cash is the main medium for people to engage in underground economic activities. In underground economic activities, cash transactions can avoid being recorded and tracked by monetary authorities (see Gutmann [12], Tanzi [60]).

As is well known, in recent years, electronic payment transaction has been widely used to replace traditional cash payment in China. In this paper, we find that the use of cash in the market is negatively correlated with GDP, but this relationship does not pass the 10% significance test. The empirical results of this paper are consistent with Schneider and Enste [18]. The increase of tax rate leads to the prevalence of underground economy, but some of the income from underground economy may eventually flow into the consumption market to drive the growth of GDP.
2) All models in Table 4 show that $\xi_c$ is negatively correlated with GDP and pass the 1% significance test. Obviously, this proves that Slutsky compensation price elasticity (including income effect and substitution effect) is negative, that is, an increase in the marginal tax rate will lead to a decrease in taxpayers’ declared income. This article further derives the relationship between uncompensated price elasticity, $\xi_u = \xi - \eta$ and GDP, and the two also show a negative correlation, as shown in Table 5. It can be found that, generally speaking, the fluctuation range of tax rate is smaller than that of commodity price, so the coefficient difference between $\xi_c$ and $\xi_u$ is not obvious. However, since $\xi_u$ includes the income effect, the “income” is normal goods rather than inferior goods, so $\xi_u > \xi_c$.

3) In Table 4, almost all models reveal two dummy variables, $D_{1994}$ and $D_{2012}$, which are positively correlated with GDP and pass the significance test of 10%. Obviously, our results are consistent with those of Fugazza and Jacques [61], who believe that higher tax rates and government regulation are the key factors affecting the underground economy.

### Table 5

| Slutsky identity estimation from SUR-OLS | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 |
|----------------------------------------|---------|---------|---------|---------|---------|---------|---------|---------|
| $\xi_c = \left(\frac{1-m}{Z}\right)\frac{dZ}{\partial(1-m)}$ | -0.4424 | -0.2840 | -0.2430 | -0.2248 | -0.4263 | -0.2841 | -0.2450 | -0.2499 |
| $\eta = (1-m)\frac{dY}{\partial R}$ | -0.1396 | -0.0275 | -0.2075 | -0.1051 | -0.1540 | -0.0322 | -0.0122 | -0.0276 |
| $\xi_u = \left(\frac{1-m}{Z}\right)\frac{dZ}{\partial(1-m)}$ | -0.582 | -0.3115 | -0.4505 | -0.3299 | -0.5803 | -0.3163 | -0.2572 | -0.2775 |

Note: Table 5 is the result derived from formula (8) based on Table 4.

### 6. Conclusion

In this paper, we exploit GDP (aggregate income), tax elasticity, income elasticity, three kinds of underground economic estimation models, as well as two important tax reform in China in 1994 and 2012 as independent variables, and examined the revenue responsiveness properties of China taxation and underground economy since 1991–2019 using Slutsky identity and SUR-OLSs method for GDP growth.

Taking China as an example, this paper selects tax rate elasticity, income elasticity, three kinds of underground economic estimation models and two important tax system reforms in 1994 and 2012 as independent variables. In methodology, this paper uses SUR-OLSs and Slutsky identity to estimate the impact of underground economy on GDP growth since 1991–2019. As is well known, SUR-OLS estimator achieves asymptotic efficiency gains over OLS by incorporating the long-run cross sectional correlation in the equilibrium errors in estimation. In comparison with traditional literature, the merit of our model is that we directly use SUR-OLS regression analysis to calculate variables, in contrast with current articles, our model does not need to be substituted into the data for complex calculation. On the other hand, in our paper, the Slutsky compensated elasticity coefficient, $\xi_c$, and the income effects coefficient, $\eta$, can be obtained directly through our SUR-OLS model.

Undoubtedly, by comparing with other relevant literature on this issue, our paper has the above merits, our innovative methodology can also be applied to most countries for time series analyses. Also, based on the joint elasticity of taxable income (JETR), in empirical analysis, we decompose tax changes into tax rate effect (change of budget constraint slope) and income effect (change of tax liability), and further analyze the impact of tax elasticity (ETI) on GDP growth. That is, in Model 1–8 of Table 4, the relationship between explanatory variable “tax rate” and “income” of explained variable is analyzed.
by SUR-OLS and Slutsky identity, and the substitution effect is negative, reaching a significant level of 1%, which means that when taxpayers face the increase of tax rate, the relative price of declared income and concealed income changes.

At this time, the budget line will move inward, leading to the decrease of declared income. Referring to Tanzi’s underground economy approach, we show the increment in cash transactions at the market led to a decline in China’s GDP, however, it is worth noting that the result is still not obvious, revealing the increment of cash transactions in market does not necessarily result in a decline in GDP growth. Our results are similar to those of Schneider and Enste. The increase of tax rate leads to the increase of underground economy and the decrease of GDP. However, the income holders of the underground economy will eventually show their hidden income through consumption expenditure, which will partly slow down the decline of GDP.

Also, we show that China implemented the reform of the tax sharing system in 1994, and the fiscal distribution was dominated by the central government. From 1993 to 1995, the total tax revenue was 425.5 billion yuan, 512.6 billion yuan and 603.8 billion yuan, respectively. Since then, the total tax revenue has been increasing year by year. In addition, the implementation of “replacing business tax with value-added tax” started in 2012 to avoid double taxation. According to China’s statistical data, the total business tax and value-added tax from 2012 to 2015 are 4261.2 billion yuan, 4604.3 billion yuan, 4863.6 billion yuan and 5042.1 billion yuan, respectively; due to the business tax in 2016 has been cancelled, the value-added tax from 2016 to 2019 is 5221.3 billion yuan, 5637.8 billion yuan, 6153.3 billion yuan and 6234.6 billion yuan respectively, showing an upward trend year by year.

In addition, it is particularly impressive that China implemented two important and representative tax reforms in 1994 and 2012 respectively, denoting the reform of China’s tax sharing system since 1994, and the implementation of replacing business tax with value-added tax since 2012, the empirical results show that both the 1994 tax reform and 2012 tax reform have a positive impact on GDP, with high statistical significance respectively. It may be of interest that in line with Chen et al. (2020), our empirical results demonstrate that China’s underground economy has significantly slowed down during 1991–2019. Finally, since the hidden cost can not be quantified and presented with specific data, thus it is not included in the research scope. It is expected that the follow-up researchers can adopt different research methods continuing to explore and research, so as to provide tax collection agencies with more contributions in clearing up the underground economic arrears.

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