Effects of Interplanetary Dust on the LISA Drag-Free Constellation

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INTRODUCTION

LISA (Laser Interferometer Space Antenna) is a joint space mission by ESA and NASA which is planned to be launched at the end of the next decade. It consists of three identical free-falling satellites, orbiting around the Sun and marking the vertices of a nearly equilateral triangle with $\simeq 5 \cdot 10^6$ km (1/30 AU) sides, located 20$^\circ$ behind the Earth and lying in a plane that makes an angle of 60$^\circ$ with the ecliptic. LISA target is the detection of gravitational waves (GWs) from astrophysical sources through the measure of the relative and differential motions between the spacecrafts [1].

However, LISA reference masses interact also with time-dependent (e.g. planets and their satellites, quadrupolar pulsation of the Sun, etc.) and static (e.g. Interplanetary Dust (ID) and Local Dark Matter (LDM)) perturbing gravitational fields, and therefore they depart from ideal unperturbed orbits around the Sun. Here we report on numerical calculations of the perturbing effects of ID and LDM on LISA orbits.

INTERPLANETARY DUST

The ID cloud is a dust composed by grains with typical sizes of 10 – 100 $\mu$m pervading the Solar System. The distribution of ID particles is studied by the observations of solar light scattering (i.e. the zodiacal light confined to the ecliptic plane) [2] and by their thermal emission, which is the dominant component of night-sky light in the 5 – 50 $\mu$m wavelength [3].

The analytical formula which best reproduces the ID density distribution is the so-called ellipsoidal model

$$\rho(r, \beta_0) = \rho_0 \left( \frac{r_0}{r} \right)^{\alpha} \frac{1}{\left[ 1 + \left( \gamma_E \sin \beta_0 \right)^2 \right]^{\alpha/2}}, \quad (1)$$

where $r$ is the distance from the Sun, $\beta_0$ is the helioecliptic latitude, $\rho_0 \simeq 9.6 \times 10^{-20}$ kg/m$^2$ is the density value at $r_0 = 1$ AU (Earth’s orbit), $\alpha = 1.3$ [2], $\gamma_E = \sqrt{a^2 - b^2}/b$, and $a$ and $b$ are the semi-major and semi-minor axes of an oblate ellipsoid, assumed to be 3 and 1.5 AU respectively [4].

Since the ellipsoidal model of ID density depends on two parameters, $\alpha$ and $\gamma_E$, in order to numerically study gravitational effects of ID on LISA orbits, we have considered four cases: i) spherical homogeneous distribution ($\alpha = 0, \gamma_E = 0$), ii) spherical distribution with power-law density profile ($\alpha = 1.3, \gamma_E = 0$), iii) ellipsoidal homogeneous distribution ($\alpha = 0, \gamma_E = \sqrt{3}$), and iv) ellipsoidal distribution with power-law density profile ($\alpha = 1.3, \gamma_E = \sqrt{3}$).

METHOD

As the first step we considered the Keplerian LISA orbits (post-Newtonian and relativistic effects are neglected) that minimize the variation of the inter-spacecraft distance between the $i$-th and $j$-th satellite $\Delta l_{i,j} \equiv |l_i(t) - l_j(t)|$ [5]. The measured quantities for the GW searches are the differential relative motion between the couples $(i, j)$ and $(j, l)$ of the three reference masses, $\Delta l_{i,j,l} = \Delta l_{i,j} - \Delta l_{j,l}$. Each couple can be regarded as an interferometer arm and the Doppler shift induced by the relative differential motions is measured by a suitable Time Delay Interferometry (TDI) [6].

The keplerian motions of the LISA reference masses are periodic and we can distinguish their harmonics by looking at the modulus of the Fourier transform of $\Delta l_{i,j,l}(t)$. The spectra present all the frequencies that are integer multiples of 1 $y^{-1}$, as a consequence of Kepler equation [8], but the harmonics at 3, 6, 9 \ldots $y^{-1}$ are absent due to a discrete symmetry in the constellation equations of motion; in fact, the LISA triangle rotates as a “quasi-rigid body” with period of one year and, after integer multiples of one third of year, the dynamical configuration is identical to the initial one being invariant under any cyclic permutation of the reference mass indices. However, such a feature is only of theoretical interest, because slightly different initial conditions, due for instance to the unavoidable injection errors of LISA spacecrafts in their orbits [7], will produce harmonics of the 3 $y^{-1}$ frequency.

To study the effects of ID on LISA orbits, we have integrated numerically the Gauss planetary equations which provide the time evolution of orbital parameters under a perturbing force. In our case, such a force is the gradient of the gravitational potential generated by the different ID distribution in Eq. (1) [8]. To get the perturbed LISA orbits we substituted the unperturbed differential parameters appearing in the expression of keplerian orbits with the numerical solutions of Gauss equations as reported in ref. [9].

RESULTS

In Fig. 1 we plot the difference between perturbed and unperturbed differential motions of the constellation for the four ID distributions $\Delta l_{1,2,3}^{pert} = \Delta l_{1,2,3}^{pert} - \Delta l_{1,2,3}^{unpert}$. 



The perturbative effects of ID are very small indeed: the LISA constellation opens with an amplitude of the order of $10^2 \mu m$. It is worth noticing that ID induces effects of the order of $10^{-12}$ times smaller than those due to the keplerian differential motions.

The perturbative effects of ID are very small indeed: the LISA constellation opens with an amplitude of the order of $10^2 \mu m$ after 5 complete orbits. It is worth noticing that the curves change their amplitudes of a factor $\approx 2.5$ by varying $\gamma_E$ from $\sqrt{3}$ to 0, while they do not depend significantly on $\alpha$.

Fig. 2 shows the modulus of the Fourier transform of $\delta l_{1,2,3}$ for the ellipsoidal ID distribution with radial profile, and we see that ID enhances resonance peaks, in particular the 2 $y^{-1}$ harmonic. Moreover, new harmonics corresponding to integer multiples of 3 $y^{-1}$ appear, as a consequence of the ID gravitational field that breaks the permutation symmetry of the equations of motion. However, we stress that this result has little practical utility to detect ID, as the 3 $y^{-1}$ harmonics will be also generated by unavoidable injection errors. It is easy to show that LISA sensitivity curve is not significantly affected by the ID perturbing effects in its detection bandwidth $10^{-4} - 10^{-1}$ Hz.

We have calculated the time evolutions of the orbital parameters due to reasonable ID distributions and estimated the opening induced on the LISA constellation, $\approx 10^2 \mu m$ after 5 years. The Fourier transform of the perturbation of the differential relative motion has shown an enhancement of the resonances characterizing the unperturbed spectrum, in particular the frequency peak at 2 $y^{-1}$. As a consequence of the LISA orbits, such effects are similar for the studied distributions of ID and do not affect the LISA sensitivity band for GW detection. On the other hand, the discrimination of very small perturbations from the keplerian differential relative motion depends crucially on the low frequency displacement noise, that in the ideal case should be $\approx 10^{-11} 1/\sqrt{Hz}$ at 2 $y^{-1}$ frequency to detect ID at the density $\approx 10^{-19}$ kg/m$^3$ measured by the zodiacal light. Unfortunately, a reliable estimate of LISA noise level at very low frequency is still lacking and requires further investigations. Finally, we investigate the possibility of constraining the LDM density with LISA. Even though gravitational perturbations due to ID and LDM are undistinguishable, the study of the deviations from the keplerian orbits of LISA reference masses could provide upper limits on LDM density.

**CONCLUSIONS**

The results we presented for ID also hold for LDM. In fact, perturbative effects are linear and quite insensitive to density distribution models, and therefore we can account for ID just by rescaling $\rho_0$. For instance, if we assume that the LDM density value is close to the average galactic dark matter (GDM) density, $\rho_{GDM} = 5\cdot10^{-22}$ kg/m$^3$, as obtained from the galaxy rotation curve, the contribution of LDM is expected to be approximately 0.5%. As a consequence, LISA could provide interesting upper limits on $\rho_{ID}$ and $\rho_{LDM}$, depending on its very low frequency noise, by means of direct gravitational field measurements. On the other hand, observations of zodiacal light could measure only the contribution of ID. However, we stress that a thorough study of LISA displacement noise below 1 $\mu$Hz [10], including local gravitational field fluctuations, thruster noise, orbit determination and injection errors [11] etc., is required to establish the relevance of the ID and LDM effects on LISA physics.

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