Gauge Invariant SO(3) - Z2 Monopoles as Possible Source of Confinement in SU(2) Lattice Gauge Theory

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Abstract

A gauge invariant procedure for extracting combined SO(3)-Z2 monopoles in positive-plaquette SU(2) lattice gauge theory is shown. When these monopoles are eliminated through a constraint, the theory deconfines for all $\beta$ on $12^4$ and $20^4$ lattices even in the strong coupling limit, despite a rather strong average plaquette of around 0.64. This corresponds to an effective Wilson $\beta$ of 2.45, at which Wilson-action lattices would be far into the confining region. This suggests that Wilson-action confinement may be a strong-coupling lattice artifact; the continuum limit may not confine.

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Rather strong evidence has been presented that confinement in SU(2) and SU(3) lattice
gauge theories is connected to the presence of long abelian monopole current loops in the
maximal abelian gauge \[1\]. An alternative hypothesis that it is the center of the gauge group
which is important, center vortices in the maximal center gauge, also seems to work \[2\]. It
has long been suspected that a gauge invariant monopole of some sort which is ultimately
responsible for confinement underlay such objects only visible in certain gauges. Following, a
candidate for a gauge invariant monopole in SU(2) lattice gauge theory is presented. It is an
SO(3) monopole defined using the three bent double-plaquettes which comprise a formulation
of the non-abelian Bianchi identity. It is at the same time a Z2 monopole. These monopoles
are not in a 1-1 correspondence with abelian monopoles in the maximal abelian gauge -
at weak couplings they are much more numerous - but there is an apparent connection.

When a constraint prohibiting SO(3)-Z2 monopoles is added, 99.4% of abelian monopoles
are eliminated when compared to the unconstrained simulation, or 95% when compared to
a Wilson-action simulation at the same renormalized coupling (matching average-plaquette
values). Thus the correlation between the two types of monopoles is quite strong, in the sense
that the vast majority of abelian monopoles are eliminated when the SO(3)-Z2 monopoles
are eliminated. More importantly, this constraint also results in deconfinement for all values
of $\beta$. Unlike previous restricted-action studies that also did not show confinement \[3\], the
average-plaquette value in the $\beta \to 0$ limit here is around 0.640 which lies in a region
where the theory should be confining. The SO(3)-Z2 monopoles are lattice artifacts in the
sense that they require large-angle plaquettes to support them, so they will not exist in the
continuum. An action that prohibits them should fall in the same universality class as the
Wilson action, yet it does not appear to confine. This severe lack of universality suggests
that confinement in the ordinary Wilson theory may be a strong-coupling lattice artifact,
similar to the U(1) case.

We start with Wilson-action SU(2) lattice gauge theory with a positive-plaquette con-
straint, the positive-plaquette model \[4\]. This constraint eliminates Z2 strings (strings of
negative plaquettes). Z2 strings are responsible for confinement in Z2 lattice gauge theory,
so eliminating them causes the Z2 theory to deconfine. The positive-plaquette SU(2) model,
however, still confines at small $\beta$ \[5\]. So in SU(2) there must be something besides Z2 strings
that causes confinement. Actually Z2 strings probably are responsible for confinement in
the mixed fundamental-adjoint \[6\] version of SU(2) in the large $\beta_A$ region, which includes
the Z2 theory as a limiting case. Because Z2 strings can cause confinement (though they
are not the only cause) a positive plaquette constraint must be maintained along with any
monopole constraint in order to get a possibly non-confined theory.

The identification of the monopole starts with the non-abelian Bianchi identity \[7,8\]. This
can be expressed by first constructing the covariant (untraced) plaquettes that comprise the
six faces of an elementary cube, with the necessary “tails” to bring them to the same starting
site (Fig. 1). Call these $A$, $B$, $C$, $D$, $E$, and $F$. Now construct three bent double plaquettes
also shown in Fig. 1, $X = AB$, $Y = CD$, and $Z = EF$. If one forms the product $XYZ$, each
link will cancel with its conjugate, so $XYZ = 1$. This is the non-abelian Bianchi identity.
Although the plaquettes are all positive (due to the positive plaquette constraint), and thus
have a trivial Z2 component of unity, the double plaquettes may be negative. Factor each
of these into Z2 and SO(3) (positive-trace) factors, e.g. $X' = Z_X X'$ etc., where $Z_X = \pm 1$
and $\text{tr}X' > 0$. Then the Bianchi identity reads $X'Y'Z'Z_X Z_Y Z_Z = 1$. This can be realized
in either a topologically trivial or nontrivial way as far as the SO(3) group is concerned. If
\[ Z_X Z_Y Z_Z = 1 \] then \[ X' Y' Z' = 1 \]. However if \[ Z_X Z_Y Z_Z = -1 \] then \[ X' Y' Z' = -1 \]. In this
case one has an SO(3) monopole which, since it also carries a Z2 charge, can be pictured
to be at the same time a Z2 monopole. The decomposition of the double-plaquettes into
Z2 and SO(3) factors is gauge invariant since the trace is invariant. In such a monopole a
large SO(3) flux is in some sense cancelled by a large Z2 flux in order to satisfy the SU(2)
Bianchi identity. This is reminiscent of the abelian monopole in U(1), in which a large flux
of \( 2\pi \) enters or exits an elementary cube. This apparent non-conservation of flux is allowed
by the compact Bianchi identity since \( \exp(2\pi i) = 1 \). In the continuum the Bianchi identity
enforces exact flux conservation. If plaquettes in U(1) are restricted to \( \cos(\theta_p) > 0.5 \),
then the only solution to the Bianchi identity is the topologically trivial one, \( \theta_{tot} = 0 \), where \( \theta_{tot} \)
is the sum of the six plaquette angles in an elementary cube. This eliminates the monopoles,
and shows that they are strong-coupling lattice artifacts. The U(1) lattice gauge theory, as
a result, is deconfined in the continuum limit.

The SO(3)-Z2 monopoles described above are also lattice artifacts. If plaquettes are
restricted so that \( \cos(\theta_p) > \sqrt{2}/2 \), then even the double-plaquettes are positive, and the SO(3)
monopoles described above cannot exist. Since in the continuum limit all plaquettes are in
the neighborhood of the identity, such a restriction should have no effect on the continuum
limit. Therefore, these monopoles will not exist in the continuum, and exact SO(3) flux
conservation on elementary cubes will hold there. Indeed it has long been recognized that if
SU(2) confinement is due to monopoles or vortices, then the only such objects which could
survive the continuum limit to produce confinement there are large objects (fat monopoles
and vortices) for which flux is built up gradually. A good way to look for fat-monopole
confining configurations would seem to be to choose an action which eliminates the single
lattice spacing scale artifacts while still allowing similar larger objects to exist.

The plaquette constraint mentioned above is one such possibility, however this results in
a rather weak renormalized coupling. There are other less severe constraints which will do.
For instance, one can prohibit the double plaquettes \( X, Y \) and \( Z \) from being negative. This
also has to be done for the other ordering for which a non-equivalent set of double plaquettes
can be defined, \( \tilde{X} = BC, \tilde{Y} = DE \), and \( \tilde{Z} = FA \), which also results in a Bianchi identity,
\( \tilde{X} \tilde{Y} \tilde{Z} = 1 \). (In addition, the positive-plaquette restriction is also applied.) This results
in a clearly deconfined theory for all \( \beta \) on a \( 12^4 \) lattice, and eliminates all but a very few
stray abelian monopoles in the maximal abelian gauge; about one monopole per every 45
\( 12^4 \) lattices remains. However the average plaquette for the strongest \( \beta \) simulated, \( \beta = 0.01 \),
is about 0.715, which represents a fairly weak renormalized coupling, corresponding to a
Wilson-\( \beta \) of about 2.9. At this \( \beta \), the \( 12^4 \) lattice is not expected to be in the confining
region anyway - it would be beyond the finite-temperature deconfining transition for this
lattice, which occurs around 2.62. Nevertheless, the number of abelian monopoles in a
Wilson-action simulation at \( \beta = 2.9 \) is much larger, around 33 per \( 12^4 \) lattice, 1500 times as
many, so there is certainly a non-universality of abelian monopole density as a function of
average plaquette, which can be thought of as the renormalized coupling (or closely related
to it).

An even more interesting action is to explicitly prohibit the SO(3)-Z2 monopoles from
forming in the update (again, both orderings must be considered and the positive plaquette
constraint is applied). This also results in a deconfined theory for all \( \beta \) on \( 12^4 \) and \( 20^4 \)
lattices. Fig. 2 shows the average Polyakov loop modulus $<|L|>$ vs. $\beta$, and Fig. 3 shows histograms of the Polyakov loop modulus at $\beta = 0.01$. Here the average plaquette is only 0.640, corresponding to an effective Wilson $\beta$ of 2.45, well into the strong-coupling side of the crossover region where these lattices would be clearly confined in a Wilson-action simulation. Also, compared to the Wilson-action simulation at $\beta = 2.45$, there are only about 5% as many abelian monopoles in the maximal abelian gauge. This is more than in the positive double-plaquette action, but apparently not enough to confine. In this case the lack of universality is clearly severe, with one action confining and the other not at the same renormalized coupling. Of course, only the weak coupling behavior of theories with different actions is universal. The strong coupling behavior can be drastically different, due to, for example, a phase transition. This suggests that the universal continuum limit of SU(2) lattice gauge theory is not confining, with confinement in the Wilson-action theory at strong coupling being due to lattice artifacts, specifically SO(3)-Z2 monopoles. This confining phase would be separated from the continuum phase by a zero-temperature phase transition, as in the U(1) case. In other words, what is usually interpreted as a finite-temperature deconfining transition could possibly be a zero temperature transition, with $\beta_c$ dependent on lattice size, but approaching a constant (perhaps around 2.8) rather than infinity in the infinite lattice limit.

It is interesting to consider whether the abelian monopoles remaining in the SO(3)-Z2 monopoleless theory could result in confinement on much larger lattices. This question can be explored with some confidence by looking at the abelian monopole loop distribution function, which appears rather independent of lattice size for monopole loops less than about 1.5 times the lattice size [3-10]. The loop distribution function $P(l)$, is shown in Fig. 4 as a function of loop length $l$. Here $P(l)$ is defined to be the probability, normalized per lattice site, for a lattice to have a loop of length $l$. As with the Wilson action [10,11] $P(l)$ appears to follow a power law, $P(l) \sim l^{-q}$ (except for loops of the minimal length 4). For the SO(3)-Z2 monopoleless action, a fit to Fig. 4 gives $q = 5.2 \pm 0.2$ for the $12^4$ lattice and also $5.2 \pm 0.2$ for the $20^4$ lattice. Size six loops, which fall slightly below the trend, were excluded from the fits as were size four. For abelian monopoles to cause confinement, loops of at least length $N$, the lattice size, must exist (on most confining lattices there are loops that wrap through the periodic boundary). In fact, usually confinement does not set in until loops of length around $N^2$ are common, due to loop crumpling. The probability of a loop of size $N$ or greater existing on an $N^4$ lattice is given by

$$N^4 \sum_{l=N}^{\infty} P(l).$$

Its behavior in the infinite lattice limit depends on $q$. If $q > 5$, then the above probability vanishes as $N \to \infty [3-10]$. To eliminate any doubts concerning the value of $q$, a simulation was performed at $\beta = 0.1$ on the $12^4$ lattice. Here the average plaquette was 0.653 corresponding to an effective Wilson $\beta$ of 2.51, still fairly strong. The loop distribution was even steeper, with $q = 6.0 \pm 0.4$. Another run with $\beta = 0.5$ gave $q = 8.2 \pm 0.6$. From this it appears that not enough abelian monopoles remain in the SO(3)-Z2 monopoleless theory to confine for any $\beta$ on any size lattice. Indeed, since the restriction only affects single lattice-spacing scale objects, and the renormalized coupling is fairly strong, it would seem that the any confining “fat monopoles” should have shown up by the time a $20^4$ lattice was reached.
Although confinement was not seen on the lattices studied, it is still conceivable that it could reappear on very large lattices via some other mechanism, perhaps one not associated with abelian monopoles. Another unlikely possibility is that the monopole constraint (and the positive double-plaquette constraint that behaves similarly) somehow erects a barrier to efficient Monte Carlo equilibration, and that the lattices studied are all in some kind of metastable state. Most lattices studied were equilibrated for 500 sweeps after which 500-6000 measurement sweeps were taken. There were no observable differences between earlier and later portions. The maximum size of update was also varied, from a completely open choice of new link, to one restricted to various-sized neighborhoods of the current link. There was no significant difference between these runs. The Polyakov loop symmetry is so strongly broken in the SO(3)-Z2 monopoleless theory, even at \( \beta = 0.01 \), that, although zero and slightly negative values were fairly common, only one definite long-lived tunneling to large negative Polyakov loop values was observed. It will likely take much longer runs or a non-local update scheme to study tunneling in this theory. Symmetry breaking at larger \( \beta \), judged from the Polyakov loop value, is even stronger.

It is interesting to speculate the mechanism by which SO(3)-Z2 monopoles could cause confinement. The large violation of flux conservation that these entail could easily randomize the values of Wilson loops. Although the number of such monopoles decreases as \( \beta \) is increased, there are still a substantial number in the deconfined region of Wilson-action simulations. Fig. 5 shows the density of SO(3)-Z2 monopoles on \( 8^4 \) and \( 12^4 \) lattices with the standard Wilson action. The \( 8^4 \) lattice deconfines around \( \beta = 2.4 \), where no particular signal is evident in the monopole density. The situation here could be similar to that in the two-dimensional X-Y model, where it is not the number of vortices that changes at the phase transition, but rather the binding of vortices into pairs. Perhaps in the deconfined region most SO(3)-Z2 monopoles are in closely bound pairs. Their flux violations could cancel locally and not have much effect on Wilson loops. Another observation is that the monopole density falls below 1/4 at around \( \beta = 2.74 \), fairly close to the point at which \( q \) becomes equal to 5 for the Wilson-action theory [10], where it is argued above that infinite-lattice deconfinement must occur. It is possible that this point is coincident with a monopole occupation fraction of 25%.

A gauge invariant SO(3) monopole, which is at the same time a Z2 monopole has been defined and proposed as the source of confinement in SU(2) lattice gauge theory. Eliminating such monopoles with a constraint results in a deconfined theory for all couplings and lattice sizes, despite a rather strong renormalized coupling. Since the constraint only affects the action at strong coupling, universality would seem to imply that all SU(2) lattice gauge theories are non-confining in the continuum limit. This has been suggested previously from other considerations [12–14]. Another possibility could be that the SO(3)-Z2 monopoleless theory and the standard Wilson theory are in different universality classes, however this would require a complete revision of the usual understanding of the continuum limit. A similar monopole based on the Z3 center can be defined in SU(3) lattice gauge theory. It would be interesting to see if it is responsible for confinement there. An even more interesting question is how the monopoleless theory behaves when light dynamical quarks are added. If chiral symmetry still breaks spontaneously, it is possible that chiral symmetry breaking will itself produce an effective confining potential [12,13]. Polarization effects in the chiral condensate could produce at least a partially chiral-expelled bag around hadrons which could
result in a confining potential (out to where string breaking by quark pair creation occurs). Simulations showing a reduced value of $\langle \bar{\psi} \psi \rangle$ in the neighborhood of color sources have been reported [16], in concert with this proposed mechanism, which has some features in common with chiral quark models [17]. Instantons may also play a role in confinement [18]. With the strong background of lattice artifacts removed, the SO(3)-Z2 monopoleless action may be ideal for the study of lattice instantons.
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Figure Captions

FIG. 1. Gauge covariant plaquettes and double plaquettes from which non-abelian Bianchi identity is formed.

FIG. 2. Polyakov loop modulus as a function of $\beta$ for the $12^4$ lattice with SO(3)-Z2 monopole-less action.

FIG. 3. Polyakov loop histograms for the $12^4$ (left scale) and $20^4$ (right scale) lattices at $\beta = 0.01$.

FIG. 4. Log-log plots of loop probability vs. loop length for the SO(3)-Z2 monopoleless action, with linear fits.

FIG. 5. Density (number per link) of SO(3)-Z2 monopoles vs. $\beta$ using the standard Wilson action.
