An 8% Determination of the Hubble Constant from localized Fast Radio Bursts

Qin Wu, Guo-Qiang Zhang, Fa-Yin Wang
1School of Astronomy and Space Science, Nanjing University, Nanjing 210093, China
2Key Laboratory of Modern Astronomy and Astrophysics (Nanjing University), Ministry of Education, Nanjing 210093, China

Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT
The cosmological-constant ($\Lambda$) cold dark matter (CDM) model is challenged by the Hubble tension, a remarkable difference of Hubble constant $H_0$ between measurements from local probes and the prediction from Planck cosmic microwave background observations under $\Lambda$CDM model. So one urgently needs new distance indicators to test the Hubble tension. Fast radio bursts (FRBs) are millisecond-duration pulses occurring at cosmological distances, which are attractive cosmological probes. Here we report a measurement of $H_0 = 68.81^{+4.33}_{-4.33}$ km s$^{-1}$ Mpc$^{-1}$ using eighteen localized FRBs, with an uncertainty of 8% at 68.3 per cent confidence. Using a simulation of 100 localized FRBs, we find that error of $H_0$ can be reduced to 2.6% at 1σ uncertainty. Thanks to the high event rate of FRBs and localization capability of radio telescopes (i.e., Australian Square Kilometre Array Pathfinder and Very Large Array), future observations of a reasonably sized sample will provide a new way of measuring $H_0$ with a high precision to test the Hubble tension.

Key words: cosmology: cosmological parameters - transients: fast radio bursts

1 INTRODUCTION
The cosmological-constant ($\Lambda$) cold dark matter (CDM) model successfully explains the majority of cosmological observations (Planck Collaboration et al. 2020). The value of Hubble constant ($H_0$), describing the expansion rate of our universe, is a basic and fascinating issue in cosmology. The measurements of cosmic microwave background (CMB) by Planck Collaboration (Planck Collaboration et al. 2020) are powerful probes to estimate cosmological parameters. The distance-redshift relation of specific stars (e.g. Cepheid variables and type Ia supernovae) can be used constrain $H_0$ directly (Riess 2020; Riess et al. 2021). Other methods are also used to measure $H_0$, such as Baryon Acoustic Oscillations (BAO), gravitational lensing (Wong et al. 2020) and Gravitational Waves (GWs) (Abbott et al. 2017). Great advances in modern observational technology have improved the precision of measuring $H_0$. However, a significant difference at least 4σ is reflected in the Hubble constant measured by CMB and Cepheid-calibrated type Ia supernovae (SNe Ia) respectively, known as “Hubble tension” (Freedman 2017; Di Valentino et al. 2021). New physics or observational bias are two mainstream arguments to alleviate this tension. An independent and robust method of measuring $H_0$ should be used to test this tension.

Fast radio bursts (FRBs) are short-duration radio pulses with enormous dispersion measures (DMs) (Lorimer et al. 2007; Katz 2018; Petroff et al. 2019; Cordes & Chatterjee 2019; Platts et al. 2019; Xiao et al. 2021). In a short period of more than ten years, a vigorous development has appeared in the observation of FRBs. Up to now, more than 600 FRBs have been observed, including repeating FRBs.

There are 19 FRBs with definite host galaxies and redshift measurements. The host galaxy association and redshift measurement point out the direction for the study of the origin, radiation mechanism and cosmological application of FRBs.

Dispersion Measure (DM) is defined as the integral of the number density of free electrons along the propagation path, which is positively proportional to cosmological distance. In particular, DM_{IGM}, contributed by the intergalactic medium (IGM), has a close connection with cosmological parameters. Therefore, precise measurements of DM_{IGM} can be used as cosmological probes (Xiao et al. 2021; Bhandari & Flynn 2021), such as “missing” baryons (McQuinn et al. 2014; Macquart et al. 2020; Li et al. 2020), cosmic proper distance (Yu & Wang 2017), dark energy (Xu et al. 2014; Walters et al. 2018; Zhao et al. 2020; Qiu et al. 2022), Hubble parameter $H(z)$ (Li et al. 2020), and the cosmic reionization history (Zheng et al. 2014; Zhang et al. 2021). Strongly lensed FRBs have been proposed to probe the nature of dark matter (Muñoz et al. 2016; Wang & Wang 2018), and measure Hubble constant (Li et al. 2018).

There is a thorny problem that DMs contributed by host galaxy and the inhomogeneities of intergalactic medium cannot be exactly determined from observations (Macquart et al. 2020). Previous works assuming fixed values for them bring uncontrolled systematic error in analysis (Hagstotz et al. 2022). A reasonable approach is to handle them as probability distributions extracted from cosmological simulations (Macquart et al. 2020; Jaroszynski 2019; Zhou et al. 2014; Walters et al. 2018).

In this letter, we propose to measure Hubble constant with eighteen localized FRBs through the DM_{IGM}-z relation. Our paper is organized as the following four sections. In section 2, we present the redshift and DM value of eighteen localized FRBs used in our
Table 1. Properties of localized FRBs.

| Name         | Redshift | DM_{obs} (pc cm^{-3}) | Reference                  |
|--------------|----------|------------------------|----------------------------|
| FRB 121102   | 0.19273  | 557 ± 2                | Chatterjee et al. (2017)   |
| FRB 180301   | 0.3304   | 522 ± 0.2              | Bhandari et al. (2022)     |
| FRB 180916   | 0.0337   | 349.34 ± 0.005         | Marcote et al. (2020)      |
| FRB 180924   | 0.3214   | 361.4 ± 0.06           | Bannister et al. (2019)    |
| FRB 181030   | 0.0039   | 103.39 ± 0.05          | Bhardwaj et al. (2021b)    |
| FRB 181112   | 0.4755   | 589.27 ± 0.03          | Prochaska et al. (2019)    |
| FRB 190102   | 0.291    | 363.6 ± 0.3            | Bhandari et al. (2020)     |
| FRB 190523   | 0.66     | 760.8 ± 0.6            | Ravi et al. (2019)         |
| FRB 190608   | 0.1178   | 338.7 ± 0.5            | Chittidi et al. (2021)     |
| FRB 190611   | 0.378    | 321.4 ± 0.4            | Heintz et al. (2020)       |
| FRB 190614   | 0.6      | 959.2 ± 0.5            | Law et al. (2020)          |
| FRB 190711   | 0.522    | 593.1 ± 0.4            | Heintz et al. (2020)       |
| FRB 190714   | 0.2365   | 504 ± 2                | Heintz et al. (2020)       |
| FRB 191001   | 0.234    | 506.92 ± 0.04          | Heintz et al. (2020)       |
| FRB 191228   | 0.2432   | 297.5 ± 0.05           | Bhandari et al. (2022)     |
| FRB 200430   | 0.16     | 380.1 ± 0.4            | Heintz et al. (2020)       |
| FRB 200906   | 0.3688   | 577.8 ± 0.02           | Bhandari et al. (2022)     |
| FRB 201124   | 0.098    | 413.52 ± 0.05          | Ravi et al. (2021)         |

2 THE PROPERTIES OF LOCALIZED FRBS

A remarkable feature of FRB is that its DM value is much larger than that contributed by the Milky way. The DM_{obs} obtained directly from observations can be divided into the following components:

\[ DM_{obs}(z) = DM_{MW} + DM_{IGM}(z) + \frac{DM_{host}(z)}{1+z}, \]

where DM_{MW} is contributed by the interstellar medium (ISM) and the halo of the Milky Way, DM_{IGM} represents contribution from the IGM, DM_{host} is the contribution by the host galaxy. It is necessary to consider the value of each term in equation (1) separately. DM_{MW} can be separated into the ISM-contributed DM_{MW,ISM} and the halo-contributed DM_{MW,halo}. The NE2001 model is used to derive DM_{MW,ISM} (Cordes & Lazio 2002). This model estimates DM contributions from the galaxy ISM with the orientation of the Galactic-coordinate grids. For the halo-contributed DM_{MW,halo}, it has been estimated that the Galactic halo will contribute 50 ~ 80 pc cm^{-3} from the Sun to 200 kpc (Prochaska & Zheng 2019). Here we assume a Gaussian distribution with a mean value of 65 pc cm^{-3} and a standard deviation of 15 pc cm^{-3} as the probability distribution of DM_{MW,halo} to consider the uncertainty of DM_{MW,halo}.

For DM_{IGM}, the effect of IGM inhomogeneities will lead to significant sightline-to-sightline scatter around the mean DM_{IGM} (McQuinn 2014). The scatter of DM at z = 1 is about 400 pc cm^{-3} from theoretical analysis (McQuinn 2014) and the state-of-the-art cosmological simulations (Jaroszynski 2019; Zhang et al. 2021). Considering a flat universe, the averaged value of DM_{IGM} is (Deng & Zhang 2014):

\[ \langle DM_{IGM}(z) \rangle = \frac{A\Omega_b H_0^2}{\Omega_m} \int_0^{z_{FRB}} \sqrt{1 + \frac{\Omega_m}{\Omega_b}} \, dz, \]

where \( A = \frac{3c}{4\pi G m_p} \) and \( m_p \) is the proton mass. The electron fraction is \( f_e(z) = Y_H X_e H(z) + \frac{1}{4} Y_H X_e H_2(z) \), with hydrogen fraction \( Y_H = 0.75 \) and helium fraction \( Y_H = 0.25 \). Hydrogen and helium are completely ionized at \( z < 3 \), which implies the ionization fractions of intergalactic hydrogen and helium \( X_e H = X_e H_2 = 1 \). The cosmological parameters \( \Omega_b \) and \( \Omega_m \) are the density of baryons and the density of matter, respectively. At present, there is no observation that can give the evolution of the fraction of baryon in the IGM \( f_g \) with redshift. Shull et al. (2012) gave an estimation of \( f_g = 0.83 \).

Currently, 19 FRBs have been localized including the nearest repeating FRB 200110E (Bhardwaj et al. 2021a), which is found in a globular cluster in the direction of the M81 galaxy (Kirsten et al. 2020). The distance of FRB 200110E is 3.6 Mpc, which is the closest-known extragalactic FRB so far. Correspondingly, the DM value of FRB 200110E is 87.75 pc cm^{-3}. And the intergalactic medium (IGM) contributed DM_{IGM} = 1 pc cm^{-3} is estimated from the relation of the averaged DM_{IGM} and redshift. Thus the cosmological information carried by FRB 200110E is too small to calculate \( H_0 \). Additionally, the effect of peculiar velocity cannot be ignored, which makes it trick to calculate cosmological parameters. Therefore, we excluded FRB 200110E from the localized FRB sample. In general, a sample with a larger amount of data will be more accurate to constrain parameters by reducing the statistical error. Thus we choose the other eighteen localized FRBs to constrain the Hubble constant. Table 1 shows the redshifts, DM_{obs}, DM_{MW,ISM}, telescopes and references of eighteen localized FRBs.

The DM_{IGM} value of FRBs can be estimated using \( DM_{IGM} = DM_{obs} - DM_{MW,ISM} - DM_{MW,halo} - DM_{host}(1+z) \). Here the value of DM_{MW,ISM} is estimated from the NE2001 model (Cordes & Lazio 2002) with the FRB coordinates. We also test our results using the YMW16 model (Yao et al. 2017), and find the effect can be neglected for different free electron distribution models. A Gaussian distribution is used to describe the probability distribution of DM_{MW,halo}.

According to equation (1), the uncertainty of DM_{IGM} can be estimated as:

\[ \sigma_{DM_{IGM}}(z) = \sqrt{\sigma_{DM_{obs}}(z)^2 + \sigma_{DM_{MW}}^2 + \left( \frac{\sigma_{DM_{host}}(z)}{1+z} \right)^2 }, \]

where \( \sigma_{DM_{obs}} \) is the uncertainty of DM_{obs}. \( \sigma_{DM_{MW}} \approx 30 \) pc cm^{-3} is the sum of the uncertainty of DM_{MW,halo} and DM_{MW,ISM}. And \( \sigma_{DM_{host}} \) is the uncertainty of DM_{host}. The evolution of the median of DM_{host} can be fitted by \( DM_{host}(z) = A(1+z)^{\alpha} \), where A and \( \alpha \) are given in Zhang et al. (2020). The uncertainty of DM_{host} comes from the uncertainties of A and \( \alpha \). As given in Zhang et al. (2020), A and \( \alpha \) have upper limits and lower limits. Therefore, DM_{host} can be calculated. And the difference between the minimum value and the center value of DM_{host} and the difference between the maximum value and the center value of DM_{host} are the uncertainties of DM_{host}. The DM_{host} is adopted as the median value derived from the IllustrisTNG simulation (Zhang et al. 2020). Therefore, DM_{IGM} can be estimated by subtracting the above terms and the DM_{IGM}-z relation of these FRBs is shown in Figure 1. The estimated DM_{IGM} of eighteen localized FRBs are shown as scatters. The red dotted line is the averaged value of DM_{IGM} from the equation (2). The error bar gives the uncertainty of DM_{IGM} using analysis. In section 3, we give an introduction of the distributions of DM_{host} and DM_{IGM}. In section 4, the Monte Carlo Markov Chain (MCMC) analysis is used to constrain the Hubble constant \( H_0 \). Discussion will be given in section 5.
The DM_{IGM} values are derived by correcting the observed dispersion measure DM_{obs} for the corresponding contributions from our Galaxy and host galaxy. The DM_{SW,ISW} is deduced from NE2001 model, and DM_{SW,halo} is adopted as a Gaussian distribution with a median of 65 pc cm⁻³. We use the median value of DM_{host} at different redshifts from IllustrisTNG 300 cosmological simulation (Zhang et al. 2020). The uncertainty of DM_{IGM} is estimated from equation (3). The red dotted line shows model of equation (2) with Q_0 = 0.315, Q_0 h^2 = 0.02235 and H_0 = 70 km s⁻¹ Mpc⁻¹. The blue line corresponds to the DM_{IGM} result from the IllustrisTNG 300 cosmological simulation and the purple shaded area is the 95% confidence region (Zhang et al. 2021). Apparently, some FRBs significantly deviates from the averaged DM_{IGM} by considering the median value of DM_{host}. Therefore, in order to obtain reliable cosmological constraints, the probability distributions of DM_{IGM} and DM_{host} must be considered.

The electron number density along different sightlines is not uniform while clustering and fluctuating, so it is difficult to determine the real value of DM_{IGM}. A quasi-Gaussian function with a long inner density profile of gas in halos. Macquart et al. (2020) gives the best fit of a = 3 and β = 3. σ_{DM} is an effective standard deviation. C_0 is a free parameter, which can be fitted when the averaged (Δ) = 1. The fitting values A, C_0 and σ_{DM} refer to the results of the state-of-the-art IllustrisTNG simulation (Zhang et al. 2021).

The distribution of DM_{host} can be well expressed with a log-normal distribution (Macquart et al. 2020; Zhang et al. 2020)

$$P(DM_{host}; \mu_{host}, \sigma_{host}) = \frac{1}{DM_{host}\sigma_{host}\sqrt{2\pi}} \exp\left(-\frac{\ln DM_{host} - \mu_{host}}{2\sigma_{host}^2}\right),$$

where e^μ and e^{2\mu + \sigma_{host}^2} (e^{\sigma_{host}^2} - 1) are the mean and variance of the distribution, respectively. The distribution of DM_{host} derived from state-of-the-art IllustrisTNG simulation with different properties of galaxies describes the DM_{host} well (Zhang et al. 2020; Jaroszyński 2020). Zhang et al. (2020) estimated the DM_{host} distribution of repeating FRBs like FRB 121102, repeating FRBs like FRB 180916 and non-repeating FRBs individually. The redshift evolution of DM_{host} is also considered (Zhang et al. 2020). Here we divide the localized FRBs into three types according to the properties of host galaxy.

The electron number density along different sightlines is not uniform while clustering and fluctuating, so it is difficult to determine the real value of DM_{IGM}. A quasi-Gaussian function with a long inner density profile of gas in halos. Macquart et al. (2020) gives the best fit of a = 3 and β = 3. σ_{DM} is an effective standard deviation. C_0 is a free parameter, which can be fitted when the averaged (Δ) = 1. The fitting values A, C_0 and σ_{DM} refer to the results of the state-of-the-art IllustrisTNG simulation (Zhang et al. 2021).

The distribution of DM_{host} can be well expressed with a log-normal distribution (Macquart et al. 2020; Zhang et al. 2020)

$$P(DM_{host}; \mu_{host}, \sigma_{host}) = \frac{1}{DM_{host}\sigma_{host}\sqrt{2\pi}} \exp\left(-\frac{\ln DM_{host} - \mu_{host}}{2\sigma_{host}^2}\right),$$

where e^μ and e^{2μ + σ_{host}^2} (e^{σ_{host}^2} - 1) are the mean and variance of the distribution, respectively. The distribution of DM_{host} derived from state-of-the-art IllustrisTNG simulation with different properties of galaxies describes the DM_{host} well (Zhang et al. 2020; Jaroszyński 2020). Zhang et al. (2020) estimated the DM_{host} distribution of repeating FRBs like FRB 121102, repeating FRBs like FRB 180916 and non-repeating FRBs individually. The redshift evolution of DM_{host} is also considered (Zhang et al. 2020). Here we divide the localized FRBs into three types according to the properties of host galaxy.

The electron number density along different sightlines is not uniform while clustering and fluctuating, so it is difficult to determine the real value of DM_{IGM}. A quasi-Gaussian function with a long inner density profile of gas in halos. Macquart et al. (2020) gives the best fit of a = 3 and β = 3. σ_{DM} is an effective standard deviation. C_0 is a free parameter, which can be fitted when the averaged (Δ) = 1. The fitting values A, C_0 and σ_{DM} refer to the results of the state-of-the-art IllustrisTNG simulation (Zhang et al. 2021).

The distribution of DM_{host} can be well expressed with a log-normal distribution (Macquart et al. 2020; Zhang et al. 2020)

$$P(DM_{host}; \mu_{host}, \sigma_{host}) = \frac{1}{DM_{host}\sigma_{host}\sqrt{2\pi}} \exp\left(-\frac{\ln DM_{host} - \mu_{host}}{2\sigma_{host}^2}\right),$$

where e^μ and e^{2μ + σ_{host}^2} (e^{σ_{host}^2} - 1) are the mean and variance of the distribution, respectively. The distribution of DM_{host} derived from state-of-the-art IllustrisTNG simulation with different properties of galaxies describes the DM_{host} well (Zhang et al. 2020; Jaroszyński 2020). Zhang et al. (2020) estimated the DM_{host} distribution of repeating FRBs like FRB 121102, repeating FRBs like FRB 180916 and non-repeating FRBs individually. The redshift evolution of DM_{host} is also considered (Zhang et al. 2020). Here we divide the localized FRBs into three types according to the properties of host galaxy.
The steps of MCMC analysis are as follows:

(i) The first step is to get the \(z, DM_{\text{FRB}}\) parameters of each localized FRBs, where \(DM_{\text{FRB}}\) is estimated by subtracting the \(DM_{\text{MW,ISM}}\) value obtained by the NE2001 model.

(ii) Equations (4), (5) and (6) are used to model the distributions of \(DM_{\text{host}}, DM_{\text{IGM}}\) and \(DM_{\text{MW,halo}}\) with the \((z, DM_{\text{FRB}})\) parameters for each localized FRB.

(iii) From the second step, \(DM_{\text{FRB}}\) is simulated by calculating the convolution of the probability density of \(DM_{\text{host}}, DM_{\text{IGM}}\) and \(DM_{\text{MW,halo}}\) to derive the probability density of \(DM_{\text{FRB}}\). As shown in equation (7), the product of the probability densities of all FRBs is a joint likelihood function. Then MCMC method can be used to fit \(H_0, \Omega_m, \Omega_b h^2\) and \(f_{\text{IGM}}\). Here the purpose of modelling the distributions of \(\Omega_m, \Omega_b h^2\) and \(f_{\text{IGM}}\) is to consider the effect of observational errors or hypothetical errors caused by these three parameters on the result of \(H_0\).

(iv) After calculating the total likelihood function, the prior distributions and the initial value of four parameters \((H_0, \Omega_m, \Omega_b h^2\) and \(f_{\text{IGM}}\)) need to be determined. We assume a uniform prior distribution of \(H_0\) in the interval \([0, 100]\) km s\(^{-1}\) Mpc\(^{-1}\), which is a broad scope to show the properties of the posterior distribution. We suppose a initial value of \(H_0 = 70\) km s\(^{-1}\) Mpc\(^{-1}\). For \(\Omega_m\), we consider 1\(\sigma\) error range \([0.296, 0.32]\) given by CMB as a uniform prior distribution. The initial value of \(\Omega_m\) is consistent with the optimum value of measurement of CMB. Finally, we assume a uniform prior for \(\Omega_b h^2\) in the interval \([0.02186, 0.02284]\), which is consistent with the 1\(\sigma\) range calculated by BBN (Cooke et al. 2018). The initial value of \(\Omega_b h^2\) is consistent with the optimum value of BBN measurement. And for \(f_{\text{IGM}}\), a fixed value 0.83 is assumed. For comparison, we also consider the case that \(f_{\text{IGM}}\) satisfies a uniform prior distribution \([0.747, 0.913]\).

(v) Lastly, we run 1,000 steps of MCMC using the encee package of Python (Foreman-Mackey et al. 2013) with the likelihood function and the priors. The plot of the final result \(H_0 = 68.81^{+4.93}_{-4.33}\) km s\(^{-1}\) Mpc\(^{-1}\) with 1\(\sigma\) uncertainty is shown in Figure 2 as red solid line for eighteen localized FRBs. This value is consistent with that derived from observed Hubble parameters \(H(z)\) through the Gaussian Process method (Yu et al. 2018). The \(H_0\) results with 1\(\sigma\) confidence region measured by Planck CMB data and Cepheid-based distance ladder measurement are shown as oral and yellow bands in Figure 2. They are within the 1\(\sigma\) range of \(H_0\) derived from FRBs. The result \(H_0 = 69.31^{+6.21}_{-6.63}\) km s\(^{-1}\) Mpc\(^{-1}\) is derived when \(f_{\text{IGM}}\) is assumed as a variable value, which is also shown in Figure 2 with black solid line. The 1\(\sigma\) uncertainty with a variable \(f_{\text{IGM}}\) is 9.6\%, which is larger than the 1\(\sigma\) uncertainty derived from the fixed \(f_{\text{IGM}}\).

It is optimistic to measure \(H_0\) using a large sample of FRBs. Considering that a large sample of FRBs has been detected by CHIME (Amiri et al. 2021), together with precise localization capability of ASKAP, VLA and Deep Synoptic Array, a sample containing 100 localized FRBs will be available in near future. In order to predict the future measurement of \(H_0\), we simulate 100 FRBs with redshifts and dispersion measures. Firstly, we suppose that FRBs and long gamma-ray bursts have similar redshift distribution (Yu & Wang 2017), which is estimated as \(f(z) \propto z e^{-z}\) in the redshift 0 < z < 3. A FRB sample with 100 mocked redshifts can be generated from the redshift distribution through Monte Carlo simulations. The \(DM_{\text{FRB}}\) corresponding to each mocked redshifts can be obtained according to its probability distribution function. Then we repeat the MCMC analysis described in the previous section with simulated data. The simulated 100 FRBs give a result of \(H_0 = 68.19^{+1.72}_{-1.03}\) km s\(^{-1}\) Mpc\(^{-1}\) with an uncertainty of 2.6% at 1\(\sigma\) confidence region as shown in Fig-
"Hubble tension" with more localized FRBs and precise measurements of $\Omega_b h^2$ and $\Omega_m$.

ACKNOWLEDGEMENTS

We thank the anonymous referee for helpful comments. This work was supported by the National Natural Science Foundation of China (grant No. U1831207), and the China Manned Spaced Project (CMS-CSST-2021-A12). We thank Z. Q. Hua for discussion.

DATA AVAILABILITY

The data that support the plots within this paper and other findings of this study are available from the corresponding author upon reasonable request.

REFERENCES

Abbott B. P., et al., 2017, Nature, 551, 85
Amiri M., et al., 2021, ApJS, 257, 59
Bannister K. W., et al., 2019, Science, 365, 565
Bhandari S., Flynn C., 2021, Universe, 7, 85
Bhandari S., et al., 2020, ApJ, 895, L37
Bhandari S., et al., 2022, AJ, 163, 69
Bhardwaj M., et al., 2021a, ApJ, 910, L18
Bhardwaj M., et al., 2021b, ApJ, 919, L24
Chatterjee S., et al., 2017, Nature, 541, 58
Chittidi J. S., et al., 2021, ApJ, 922, 173
Cooke R. J., Pettini M., Steidel C. C., 2018, ApJ, 855, 102
Cordes J. M., Chatterjee S., 2019, ARA&A, 57, 417
Cordes J. M., Lazio T. J. W., 2002, arXiv e-prints, pp astro-ph/0207156
Deng W., Zhang B., 2014, ApJ, 783, L35
Di Valentino E., et al., 2021, Classical and Quantum Gravity, 38, 153001
Foreman-Mackey D., et al., 2013, emcee: The MCMC Hammer (ascl:1303.002)
Freedman W. L., 2017, Nature Astronomy, 1, 0169
Hagstotz S., Reischke R., Lilow R., 2022, MNRAS, 511, 662
Heintz K. E., et al., 2020, ApJ, 903, 152
Jaroszynski M., 2019, MNRAS, 484, 1637
Jaroszynski M., 2020, Acta Astron., 70, 87
Katz J. I., 2018, Progress in Particle and Nuclear Physics, 103, 1
Kirsten F., et al., 2022, Nature, 602, 585
Law C. J., et al., 2020, ApJ, 899, 161
Li Z.-X., Gao H., Ding X.-H., Wang G.-J., Zhang B., 2018, Nature Communications, 9, 3833
Li Z., Gao H., Wei J. J., Yang Y. P., Zhang B., Zhu Z. H., 2020, MNRAS, 496, L28
Lorimer D. R., Bailes M., McLaughlin M. A., Narkevic D. J., Crawford F., 2007, Science, 318, 777
Marcote B., et al., 2020, Nature, 581, 391
McQuinn M., 2014, ApJ, 780, L33
Muñoz J. B., Kovetz E. D., Dai L., Kamionkowski M., 2016, Phys. Rev. Lett., 117, 091301
Petkovic E., Hessels J. W. T., Lorimer D. R., 2019, A&A, 579, 27
Planck Collaboration et al., 2020, A&A, 641, A6
Platts E., Weltman A., Walters A., Tendulkar S. P., Gordin J. E. B., Kandhai S., 2019, Phys. Rep., 821, 1
Prochaska J. X., Zheng Y., 2019, MNRAS, 485, 648
Prochaska J. X., et al., 2019, Science, 366, 231
Qiu X.-W., Zhao Z.-W., Wang L.-F., Zhang J.-F., Zhang X., 2022, J. Cosmology Astropart. Phys., 2022, 006
Ravi V., et al., 2019, Nature, 572, 352
Ravi V., et al., 2021, arXiv e-prints, p. arXiv:2106.09710
Q. Wu et al.

Riess A. G., 2020, Nature Reviews Physics, 2, 10
Riess A. G., Casertano S., Yuan W., Bowers J. B., Macri L., Zinn J. C., Scolnic D., 2021, ApJ, 908, L6
Shull J. M., Smith B. D., Danforth C. W., 2012, ApJ, 759, 23
Springel V., et al., 2018, MNRAS, 475, 676
Walters A., Weltman A., Gaensler B. M., Ma Y.-Z., Witzemann A., 2018, ApJ, 856, 65
Wang Y. K., Wang F. Y., 2018, A&A, 614, A50
Wong K. C., et al., 2020, MNRAS, 498, 1420
Wu Q., Yu H., Wang F. Y., 2020, ApJ, 895, 33
Xiao D., Wang F., Dai Z., 2021, Science China Physics, Mechanics, and Astronomy, 64, 249501
Yao J. M., Manchester R. N., Wang N., 2017, ApJ, 835, 29
Yu H., Wang F. Y., 2017, A&A, 606, A3
Yu H., Ratra B., Wang F.-Y., 2018, ApJ, 856, 3
Zhang G. Q., Yu H., He J. H., Wang F. Y., 2020, ApJ, 900, 170
Zhang Z. J., Yan K., Li C. M., Zhang G. Q., Wang F. Y., 2021, ApJ, 906, 49
Zhao Z.-W., Li Z.-X., Qi J.-Z., Gao H., Zhang J.-F., Zhang X., 2020, ApJ, 903, 83
Zheng Z., Ofek E. O., Kulkarni S. R., Neill J. D., Juric M., 2014, ApJ, 797, 71
Zhou B., Li X., Wang T., Fan Y.-Z., Wei D.-M., 2014, Phys. Rev. D, 89, 107303

This paper has been typeset from a \TeX/\LaTeX file prepared by the author.