Gravastar configuration in non-conservative Rastall gravity

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Abstract
In the present article, we have presented the exact solutions of gravastar with Kuchowicz metric potential in the background of non-conservative Rastall gravity. Within the context of Mazur-Mottola’s [1, 2] concept of Bose–Einstein condensation to gravitational systems, the gravastar structure consists of three layers: interior part, intermediate part (i.e., thin shell) and exterior part. All the gravastar’s exact solutions have been derived with the aid of Kuchowicz metric potential by considering isotropic matter distribution. For the thin shell (filled with ultra-relativistic stiff fluid) with different parameters like length, energy, entropy and surface redshift have been calculated, which show the stability of our proposed model. Interestingly, all the thin shell results are obtained without taking shell approximation i.e., \( 0 < e^{-\nu} \equiv h \ll 1 \). The exterior part, which is absolutely a vacuum is characterized by the Schwarzschild geometry and the interior part give output in the form of non-singular results. Generally, the main purpose of this work is to obtain the exact, non-singular, horizon free, stable model and we have achieved these goals in the presence of Rastall parameter.

1. Introduction

With the development of Einstein’s General Theory of Relativity (GR), we have come to know about the existence of black hole (BH), which has been a challenging problem in modern astrophysics due to the presence of event horizon and singularity in its structure, where all physical laws break down. A lot of work on BH has been done in the literature [3–13]. But the drawbacks of BH (i.e., event horizon and singularity) stimulated the researchers to construct such a model that would deal with such types of challenges. In the continuation of such problems, Mazur and Mottola [1, 2] have presented the magnificent, hypothetical concept of “a Gravitational Vacuum Star” (gravastar) as a gravitational cold, dark, compact, and spherically symmetric object by extending the Bose–Einstein condensate idea. This gravastar is known as the alternative to BH that can resolve the issues related to BH. There is no singularity and event horizon in a gravastar model. A gravastar structure has three parts: i) center of gravastar is the interior zone \( 0 < r < Q_1 \), with an equation of state (EOS) \( p = -\rho \), which consists of dark energy and isotropic fluid, it gives regular solutions and prevents the formation of singularity, ii) the boundary of gravastar is the thin shell \( Q_1 < r < Q_2 \) with EOS \( p = \rho \) filled with ultrarelativistic perfect fluid, it is the membrane that separates the interior region from the exterior region and iii)– third part is the exterior region \( r > Q_2 \) at \( p = \rho = 0 \), which is completely vacuumed region described by Schwarzschild geometry. Due to these aspects of the gravastar model, a lot of work has been done by many authors [14–21] to model the gravastar configuration.

Sengupta et al [22] have investigated the results for gravastar in the framework of Braneworld gravity. They have given a stable and non-singular model by investigating the different physical aspects of thin shell. Ray et al [23] have described the gravastar’s salient features for lower and higher dimensions. They have also highlighted the different circumstances under which a gravastar is considered as an alternative to BH. Usmani et al [24] have determined the solutions for charged gravastar with conformal motion by considering the Reissner-Nordström spacetime in the exterior region. This work initiated the other researchers to do work more on gravastar models. Bhar [25] has generalized the work of Usmani et al [24] for the higher dimensional charged gravastar. She has used the Reissner-Nordström spacetime in the exterior region. These solutions satisfied the consequences of
Usmani et al [24]. Further, Bhar [26] has modified the results of Usmani et al [24] in \( f(T) \) gravity. Sharif and Waseem [27] have formulated the solutions of gravastars in the presence of electric charge with conformal motion in \( f(R, T) \) gravity. They have used the linear model \( f(R, T) = R + 2\beta T \) and thin shell approximation. By replacing \( \beta \) as zero, all the outcomes returned to the findings of Usmani et al [24]. Bhatti et al [28] have obtained the gravastar solutions in the framework of modified Gauss-Bonnet gravity. They analyzed the various features of thin shell to discuss the stability of given model. Latter on, the same authors [29] have presented the charged gravastar model in which they have achieved the new non-singular solutions, to see the impact of charge on the gravastar structure presented in [28].

Rahman et al [30] have analyzed the solutions for \((2 + 1)\)-dimensional gravastars by taking anti-de Sitter spacetime. Also, the same authors [31] have explicitly provided the new solutions for gravastars by including the charge term in the field equations. Ghosh et al [32] have generalized the work of Rahman et al [31] for \( D \)-dimensions with the impact of charge distribution, which fulfill the requirements for the existence of gravastar. After very short interval of time, Ghosh et al [33] have discussed the same results as in [32] without taking charge and gave the exact, non-singular solutions for gravastar in GR. The new results for gravastars with Karmarkar condition in GR satisfying the Mazur-Mottola concept has been presented by Ghosh et al [34]. Their solutions were stable, regular in the interior region, non-singular, and horizon free. The non-singular and exact solutions for gravastars in \( f(R, T) \) gravity have been discussed by Das et al [35]. They have determined the physical properties with graphical representation, which showed the stability and validity of gravastar in \( f(R, T) \). Shamir et al [36] have examined the new results for the gravastar model in \( f(G, T) \) gravity and also discussed the physical behavior of interior, exterior solutions, and thin shell. The non-singular, horizon-free solutions with an electric charge for gravastars in \( f(T) \) theory, have been presented by Deb Nath [37]. He used Reissner-Nordström spacetime rather than Schwarzschild spacetime for the exterior region. Younas [38] has investigated the solutions for cylindrical gravastar with the influence of electric charge in the framework of \( f(R, T) \) gravity. Ghosh et al [39] have studied the new results of gravastar in torsion trace \( f(T, T) \) gravity. By following Mazur-Mottola [1, 2] approach, the singular free solutions of charged gravastar in \( f(R, T) \) theory of gravity have been presented by Majeed et al [40].

Einstein’s General Theory of Relativity has great importance than other physical theories as it revealed the many gravitational hidden pieces of evidence and objects of nature like gravitational lensing, gravitational time dilation, and black holes. But some issues related to dark energy, dark matter, and the accelerating universe are still to be solved, which motivated the researchers to modify this theory. So a number of modified theories have been proposed successively such as \( f(R) \), \( f(T) \), \( f(R, T) \), \( f(G) \), \( f(G, T) \) and \( f(R, G) \) theories etc. A lot of work has been done for the models of gravastar in these theories by many researchers in [41–45]. In 1972, an alternative of GR was proposed by Rastall [46], which recently has captured more attention in the literature due to some beneficial understandings, such as it is stable on gravitational lensing occurrence and deals with entropy and universe age problem in cosmic era. Further, Rastall theory may be regarded as a phenomenological theory of some quantum effects in the relativistic structure [47]. Recently, Rastall theory has been implemented in cosmology to discuss the nature of cosmological evolution. It was found [48, 49] that Rastall theory is well consistent with various observational data and it gives some interesting results. For example in Rastall theory, the evolution of dark matter fluctuations is the same as that in the \( \Lambda \)CDM model [49]. But the dark energy is clustered in Rastall theory. This leads to generate the inhomogeneities in the evolution of dark matter, which is different from the standard cold dark matter (CDM) model. The inner region of the gravastar is the de-Sitter core with EOS \( \rho = -\omega \rho \), so it contains dark energy. It would be interesting to study the gravastar structure in Rastall theory, with the hope that one can get a more realistic model of gravastar in Rastall theory which would be consistent with the most recent astrophysical measurements for the detection of gravastar.

Rastall [46] modified the conservation law of GR i.e., \( T^\alpha_\mu = 0 \) into non-conservation law, which can be stated as

\[
\dot{T}^\alpha_\mu = \eta R^\alpha_\mu, \tag{1}
\]

where \( \eta \) is a Rastall parameter, which coupled the matter field and geometry in non-minimal manner that make it different from the conservation law of GR. The standard form of Rastall field equations is

\[
G^\alpha_\mu + \kappa \eta g^\alpha_\mu R = \kappa T^\alpha_\mu, \tag{2}
\]

Here \( \kappa, R \) and \( T^\alpha_\mu \) are coupling constant of Rastall theory, Ricci scalar and energy momentum tensor, respectively. These equations can be reformulated as

\[
G^\alpha_\mu = \kappa \left( T^\alpha_\mu - \frac{T^{\mu\nu}g_{\alpha\nu}}{4\kappa \eta - 1} \right). \tag{3}
\]
The energy momentum tensor for an ideal fluid is

$$T_{\mu\nu} = (p + \rho)g_{\mu\nu} - pg_{\mu\nu}. \tag{4}$$

In the literature, a lot of work has been done by many researchers in this modified theory of gravity for different stellar structures. Meng-Sen Ma and Ren Zhao [56] have studied the non-commutative geometry of black holes in Rastall theory of gravity by taking two different metrics ansatz. Debnath [51] has analyzed the gravastar solutions in Rastall–Rainbow gravity with electromagnetic effects. He took spherically symmetric spacetime and found the non-singular, horizon-free solutions for gravastar. Salako et al [52] have constructed the compact stars model with anisotropic fluid in the framework of Rastall theory. They have determined the values of unknown constants in the form of mass and radii of the compact star and also discussed the stability of their solutions through physical properties like surface redshift and regularity condition. Abbas and Shahzad [53] have explored the new solutions for compact stars with a conformal motion by taking isotropic fluid. Das et al [54] have obtained the new generalized solutions in Rastall gravity with the use of its cosmological consequences. They have discussed universal thermodynamics by choosing the specific Rastall theory parameter. Shahzad and Abbas [55] have provided the comparative study of GR and Rastall theory of gravity for three different compact stars with different radii and they have checked the compactness and stability of their model. Further, Abbas and Shahzad [56] have used the Karori and Barua type metric by taking different anisotropic compact stars i.e., 4U 1820–30, Her X-1 and SAX J 1808.4-3658 (SSI) with radii 10 km, 7.7 km and 7.07 km, respectively, in Rastall gravity. They have found the unknown constants and plotted the graphs for different physical conditions like energy condition, stability, energy density, and hydrostatic equilibrium and redshift that make sure that their model is compact, regular, and stable. The solutions for quintessence compact stars in Rastall gravity have been provided by Abbas and Shahzad [57]. A study on thermodynamic geometry of a black hole by considering isotropic fluid in Rastall theory has been provided by Soroshfar et al [58]. Maurya et al [59] have investigated the solutions of decoupling gravitational field equation by minimal geometric deformation procedure in the framework of Rastall theory. Prihadi et al [60] have generalized the solutions of Kerr-Newman-NUT black hole in the presence of charge factor in Rastall gravity. Recently, Abbas and Majeed [61] have obtained the new results for gravastars in Rastall gravity and provided the stable, non-singular, event horizon free model for gravastar.

The main objectives of the present study are to find stable, regular, non-singular, and horizon-free results for gravastar by using the Kuchowicz type metric potential in Rastall theory of gravity. To achieve this aim, we have planned our work in the following manner: In section 2, we furnish the Rastall field equations with Kuchowicz metric potential and non-conservation law. In section 3 three layers of gravastar structure have been discussed i.e., i-Interior region, ii-Thin shell region, and iii-Exterior region and junction condition. Section 4, consists of thin shell physical properties i.e., length, energy, entropy, surface redshift, and equation of state. The last section deals with is the conclusion of the present work.

2. Rastall’s field equations and non-conservation law

For an interior spacetime, the spherically symmetric metric is

$$ds^2 = e^\mu(r)dt^2 - e^{\lambda(r)}dr^2 - r^2d\theta^2 - r^2\sin^2 \theta d\phi^2, \tag{5}$$

where $\mu(r)$ and $\lambda(r)$ are the metric potentials. The explicit form of equation (3) in Rastall gravity for spherical symmetric spacetime (5) is given below

$$-1 + e^\lambda + r\lambda' = r^2e^\lambda \kappa (\rho - \frac{\kappa \eta (\rho - 3p)}{4\kappa \eta - 1}), \tag{6}$$

$$1 - e^\lambda + r\mu' = r^2e^\lambda \kappa (p + \frac{\kappa \eta (\rho - 3p)}{4\kappa \eta - 1}), \tag{7}$$

$$-\frac{r^2}{4} (2\mu'' + \mu' \lambda') + r \frac{\mu'}{2} (\mu' - \lambda') = r^2e^\lambda \kappa (p + \frac{\kappa \eta (\rho - 3p)}{4\kappa \eta - 1}). \tag{8}$$

We all are familiar with the remarkable achievements of GR by concealing many hidden aspects of nature and a lot of adorable relativistic models in the literature have been found. Among these models, some require the analytical solutions of the gravitating system like the gravastar model. In this regard, different approaches such as hydrostatic equilibrium equation, thin-shell approximation, conformal motion, Karmarkar conditions, and Kuchowicz metric potential have been implemented to solve the Einstein’s field equations in GR. But in the present article, we suggest the metric potential $e^{\mu(r)}$ as Kuchowicz type [62] with arbitrary constants $B$ and $C$, where $B$ has the dimension $\left[ L^{-1} \right]$ and $C$ is dimensionless. It is considered as a successful methodology to figure out various aspects of compactness and physical viability in the formation of different relativistic stellar objects.
The utilization of this particular type of metric function is exceedingly meaningful in the field equations to get the analytical and non-singular models of the gravitating. It provides the feasibility to explore more surreptitious aspects of nature in the gravitational field [63–66]. Ghosh et al [67] have investigated the exact and singularity-free solutions of gravastar with the implementation of the Kuchowicz metric function in GR. Here, we formulate a new exact model of gravastar structure in the framework of Rastall theory of gravity by adopting the Kuchowicz metric potential. The reason behind adoption of this type of metric function is that, this metric potential is completely free from any singularity, shows a regular behavior throughout the gravastar. Using this function we have studied different features of three different regions of gravastar. All the solutions for the interior region are regular at the centre and are horizon free, which are the consistency conditions for the existence of gravastar.

The expression for Kuchowicz metric function is given by

$$e^{\mu(r)} = e^{Br^2 + 2\ln C}.$$  \hspace{1cm} (9)

By implementing the above equation (9), the set of field equations (6)–(8) can be written in the following form

$$-1 + e^\lambda + r\lambda' = r^2 e^\lambda \left( \frac{\rho - \frac{\kappa \eta (\rho - 3p)}{4\kappa \eta - 1}}{4\kappa \eta - 1} \right)$$ \hspace{1cm} (10)

$$1 - e^\lambda + 2Br^2 = r^2 e^\lambda \left( \frac{p + \frac{\kappa \eta (\rho - 3p)}{4\kappa \eta - 1}}{4\kappa \eta - 1} \right)$$ \hspace{1cm} (11)

$$\frac{r^2}{4} (4B + 4B r^2 - 2Br \lambda') + \frac{r}{2} (2Br - \lambda') = r^2 e^\lambda \left( \frac{p + \frac{\kappa \eta (\rho - 3p)}{4\kappa \eta - 1}}{4\kappa \eta - 1} \right).$$  \hspace{1cm} (12)

The non-conservation law of energy momentum tensor in Rastall theory can be reported in the following way

$$\left\{ \frac{8\pi \eta}{32\pi \eta - 1} \right\} \frac{d}{dr} (\rho - 3p) - \frac{dp}{dr} - \frac{\mu'}{2} (\rho + p) = 0.$$  \hspace{1cm} (13)

3. Gravastar configuration

The gravastar composition appears with three regions that are discussed as under

3.1. Interior region of gravastar

With the conjecture of Mazur-Mottola [3, 4], we assume the EOS with parameter $\omega = -1$ as follows

$$p = -\rho.$$ \hspace{1cm} (14)

From equations (13) and (14), we have

$$p = -\rho = -k_0,$$ \hspace{1cm} (15)

where $k_0$ is a constant that is related to central density in the interior region, which is responsible to balance the inner influence of gravitation by the thin shell. By using equations (10) and (15), we get

$$e^{-\lambda} = 1 + \frac{r^2}{3} \left( \frac{4\kappa \eta k_0}{4\kappa \eta - 1} - k_0 \right) + \frac{k_1}{r},$$ \hspace{1cm} (16)

where $k_1$ represents the constant of integration. We equate the integration constant $k_1$ to zero, for the regular solutions at origin ($r = 0$), then above equation (16) can be written as

$$e^{-\lambda} = 1 + \frac{r^2}{3} \left( \frac{4\kappa \eta k_0}{4\kappa \eta - 1} - k_0 \right).$$ \hspace{1cm} (17)

For the interior region, the active gravitational mass with the equation (15) takes the following form

$$M(Q) = \int_{0}^{r=Q} 4\pi r^2 k_0 dr = \frac{4}{3} \pi Q^3 k_0.$$ \hspace{1cm} (18)

3.2. Gravastar’s thin shell

The thin shell is the non-vacuum part of gravastar ($Q = Q_1 < r < Q_2 = Q + \epsilon$) with EOS $p = \rho$ which consists of ultra-relativistic stiff fluid as defined by Zel’dovich [68] to model the cold byronic universe with very high density. These type of fluids have been used by many researchers for cosmological and astrophysical point of view. Thin shell is considered as very thin but finite and it links the two parts i.e., interior region and Schwarzschild exterior geometry. From equations (10)–(12) by using EOS $p = \rho$, one can achieve
where $k_2$ is constant of integration. By implementing the condition $p = \rho$ in equation (13), we get the shell density as

$$\rho = p = (C^2)^{\beta}(e^{ik_3r + B^2r^2})^\beta,$$

where $k_3$ is an integration constant and $\beta = \frac{32\pi\gamma - 1}{1 - 48\pi\gamma}$. The variation of the matter density as well as pressure throughout the shell is shown in figure 1.

3.3. Schwarzschild spacetime and boundary conditions

The exterior region is totally vacuum and we consider Schwarzschild spacetime obeying the EOS $p = \rho = 0$, which can be defined as

$$ds^2 = (1 - \frac{2M}{r})dt^2 - \frac{1}{(1 - \frac{2M}{r})}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

where $M$ is the total mass of the gravastar.

In gravastar’s structure there are two boundaries with co-relation of inner area, thin shell and exterior part. First one relates with inner zone and thin shell, second relates with thin shell and exterior part. We have used the way of matching metric function at the boundary to find the expressions for the unknown constants $B$ and $C$ involved in Kuchowicz metric potential (9) in terms of total mass of gravastar $M$ and the exterior radius $Q$. So, we have

$$B = -\frac{M}{(2M - Q_2)Q_2^2},$$

and

$$C^2 = -\frac{M}{Q_2^2}(Q_2 - 2M).$$

It is recalled that the gravastar model has been proposed by Mazur-Mottola as a non-singular and non-event horizon model in contrast with a classical black hole. In the present study, we have chosen some numerical values i.e., $M = 2.75M_\odot$ and exterior radius $Q_2 = 10.0010$ in such a way that these may fulfill the condition $\frac{2M}{r} < 1$, which is the necessary to obtain the horizon free solutions in the construction of gravastar configuration. Thus, we have chosen the values to satisfy the ratio $\frac{2M}{r} < \frac{5}{7}$ (Buchdahl bound [69]) and $Z_r < 2$ for a stable model. As long as this relation remains valid, we can choose any set of $M$ and $Q_2$. Within limit $\frac{2M}{r} < \frac{5}{7}$, the values of $M$ and $R$ may vary. By substituting these specific parametric values, we obtain $B = 0.014991$ km$^{-2}$ and $C^2 = 0.0558327$. When these parametric values are implemented in physical variables, we get a physically stable, well-behaved and non-singular gravastar model.

3.4. Junction conditions

We know that gravastar’s structure has three main regions: interior part, thin shell and exterior. The intermediate thin shell connects interior region to the exterior region at $r = Q$. The junction conditions are the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Variation of thin shell density $p = \rho$ (km$^{-2}$) w.r.t. radius $r$.}
\end{figure}
continual matching of metric potentials at shell region but this condition is not guaranteed for the existence of their derivatives. We use the formalism given by Darmois [70] and Israel [71] to find the intrinsic surface stresses at junction surface \( \Omega \), which can be expressed by Lanczos equation [72] as:

\[
S_{lm} = -\frac{1}{8\pi} (\kappa_{lm} - \delta_{lm}\kappa_{qq}),
\]

where \( \kappa_{lm} = K_{lm}^+ - K_{lm}^- \) shows the discontinuity of second fundamental forms and the signs \( +, - \) are the indicators for exterior and interior boundaries, respectively. The second fundamental form is given by

\[
K_{lm}^{\pm} = -n_{a}^{\pm} \frac{\partial^2 x^a}{\partial \zeta^l \partial \zeta^m} + \Gamma^{a}_{\mu\nu} \frac{\partial \kappa^\mu}{\partial \zeta^l} \frac{\partial \kappa^\nu}{\partial \zeta^m} \Omega.
\]

Here \( n_{a}^{\pm} \) depicts the normal unit vector to the surface \( \Omega \), which is given in the form

\[
n_{a}^{\pm} = \pm|g|^{1/2} \left( \frac{\partial f}{\partial x^a} \right),
\]

with \( n^a n_a = 1 \). Here \( \zeta^a \) is the intrinsic coordinate on the shell and \( f(\zeta^a(\zeta^a)) = 0 \) is the parametric equation of the shell, which in the present case takes the form \( f(r) = r - Q = 0 \), implying \( r = Q \). By using Lanczos equation [72], we find surface stress-energy tensor as \( S_{lm} = \text{diag}[\varrho, -\chi, -\chi, -\chi] \), specifying \( \varrho \) as surface energy density and \( \chi \) as the surface pressure. These are given by

\[
\varrho = -\frac{1}{4\pi Q} \sqrt{F^+},
\]

\[
\chi = -\frac{\varrho}{2} + \frac{1}{16\pi} \left( \frac{F^+}{\sqrt{F^+}} \right).
\]

Here, \( F \) corresponds to \( g_{00} \) of the interior region \((-\)\) and exterior region \((+)\).

From the above equations (27) and (28), we get

\[
\varrho = -\frac{1}{4\pi Q} \left( \sqrt{1 - \frac{2M}{Q}} - \sqrt{1 + \frac{\kappa}{3} \left( \frac{4\kappa j k_0}{4\kappa j - 1} - k_0 \right) Q^2} \right),
\]

\[
\chi = \frac{1}{8\pi Q} \left( \sqrt{1 + \frac{2M}{Q}} - 1 \right) - \frac{2}{3\pi} \left( \sqrt{1 + \frac{\kappa}{3} \left( \frac{4\kappa j k_0}{4\kappa j - 1} - k_0 \right) Q^2} - \sqrt{1 + \frac{2M}{Q}} \right).
\]

The thin shell mass of gravastar is

\[
m_{\text{th}} = 4\pi Q^2 \varrho = Q \left[ \sqrt{1 + \frac{\kappa}{3} \left( \frac{4\kappa j k_0}{4\kappa j - 1} - k_0 \right) Q^2} - \sqrt{1 - \frac{2M}{Q}} \right].
\]

By manipulating the above equation (31), we can get the total mass of gravastar, which is

\[
M = \frac{\kappa}{6} \left( k_0 - \frac{4\kappa j k_0}{4\kappa j - 1} \right) Q^3 - m_{\text{th}}^2 + m_i \left[ \sqrt{1 + \frac{\kappa}{3} \left( \frac{4\kappa j k_0}{4\kappa j - 1} - k_0 \right) Q^2} - \frac{1}{2Q} \right].
\]

4. Physical aspects

4.1. Shell length

The shell length is considered as breadth between interior \( r = Q \) and exterior \( r = Q + \epsilon \) interfaces, this connection of interior and exterior region is formed by thin shell. This thickness can be determined as

\[
L = \int_{Q}^{Q + \epsilon} \sqrt{e^q} \, dr = \int_{Q}^{Q + \epsilon} \frac{B r^2 e^{Br^2}}{e^{Br^2} + 8B^2 k_2} \, dr,
\]

\[
= \left[ \frac{c^2 r^2 \log \left( \sqrt{e^q} + \sqrt{e^{Br^2} + 8B^2 k_2} \right)}{4B^2 c^{2Br^2}} \right]_{Q}^{Q + \epsilon}.
\]

Variation of proper length within the thin shell has been shown in figure 2.
4.2. Shell energy

The shell energy content is

\[
E = \int_{Q}^{Q+\epsilon} 4\pi r^2 \rho dr = 4\pi \int_{Q}^{Q+\epsilon} r^2 e^{(k \gamma + B \gamma^2)}(C^2)^{\beta} dr,
\]

\[
= 4\pi (C^2)^{\beta} e^{(k \gamma + B \gamma^2)} \left[ r e^{B \gamma} - \frac{\sqrt{\pi} \text{Erf}(r \sqrt{B \gamma})}{4B^{3/2} \gamma^{3/2}} \right]_{Q}^{Q+\epsilon}. \tag{34}
\]

Variation of the energy throughout the thin shell has been shown in figure 3.

4.3. Shell entropy

The shell entropy is given by

\[
S = \int_{Q}^{Q+\epsilon} 4\pi r^2 s(r) \sqrt{e^\gamma} dr, \tag{35}
\]

whereas the entropy density denoted by \(s(r)\) is given as

\[
s(r) = \frac{\alpha \xi^2}{4\pi h^2} \frac{T(r)}{\gamma} = \alpha \xi \left( \frac{\xi}{\hbar} \right) \frac{p}{\sqrt{2\pi}}, \tag{36}
\]

where \(\alpha\) is a dimensionless constant. In the present case, we apply some units as \(G = c = 1\) and \(\xi = \hbar = 1\) then entropy density takes the form

\[
s(r) = \alpha \left( \frac{p}{\sqrt{2\pi}} \right). \tag{37}
\]
Now equation (35) will become
\[ S = \frac{4\pi\alpha}{\sqrt{2\pi}} \int_Q^{Q+} r^2 \left( \sqrt{\text{det}(\mathbf{g}) + B^2/\mathbf{g}} \right) \left( \frac{B^2 e^{-B^2}}{e^{B^2} + 8B^2k_2} \right) dr, \]
which simplifies to
\[ S = \frac{4\pi\alpha}{\sqrt{2\pi}} \int_Q^{Q+} \frac{dG(r)}{dr} dr = \frac{4\pi\varepsilon}{\sqrt{2\pi}} [G(r)]_{Q}^{Q+} = \frac{4\pi\alpha}{\sqrt{2\pi}} [G(Q + \varepsilon) - G(Q)]. \]

Variation of entropy within the shell is shown in figure 4.

4.4. Surface redshift
The analysis of surface redshift of the gravastars may be regarded as one of significant aspect regarding the stability and detection of gravastars. Buchdahl [69] investigated that for the stability of static and isotropic matter configuration, the value of surface redshift must be less than 2, i.e., \( Z_s < 2 \) [69, 73]. Ivanov [74] have examined that for static anisotropic gravitating source, \( Z_s \) can attain value upto 3.84. Barraco and Hamity [75] concluded that \( Z_s \approx 2 \) for an isotropic fluid in the absence of cosmological constant, while \( Z_s \approx 5 \) for anisotropic source in the presence of cosmological constant [73].

The surface redshift for thin shell can be calculated by the following formula [67]
\[ Z_s(r) = -1 + \left| \rho_m \right|^{1/2}, \]
which gives the final result as
\[ Z_s(r) = -1 + e^{-\frac{\rho_m}{C}}. \]
The variation of surface redshift \( Z_s \) has been plotted in figure 5. It is evident from this figure that the value of \( Z_s \) lies within 1 throughout the thin shell. So, our proposed gravastar model is stable as well as physically acceptable.

4.5. Equation of state
We get the EOS parameter by using equations (29) and (30) in the general form \( \chi = \omega(Q)Q \) at \( r = Q \) as
\[ \omega(Q) = \frac{1 - \frac{2m}{Q} - \frac{4\rho_m}{Q Q_0} - k_0Q^2}{\sqrt{1 - \frac{2m}{Q} - \frac{4\rho_m}{Q Q_0} - k_0Q^2}} \]
\[ + \frac{1 + \frac{2M}{Q} - \frac{4\rho_m}{Q Q_0} - k_0Q^2}{\sqrt{1 + \frac{2M}{Q} - \frac{4\rho_m}{Q Q_0} - k_0Q^2}}. \]
To achieve stable model and positive EOS parameter, there must be \( \frac{2M}{Q} < 1 \) and \( \frac{k_0}{Q Q_0} < 1 \).
\[ \omega(Q) \approx \frac{3}{2} \left[ 1 - \frac{6M}{\kappa Q Q_0 (Q^2 - 1)} \right]. \]
According to the Mazur-Motolla differential equation, it helps to way: by taking Kuchowicz metric potential it is easy to determine the other metric potential from the system of a type metric potential. The consideration of such a specific form of metric potential is beneficial in the following way: by taking Kuchowicz metric potential it is easy to determine the other metric potential from the system of a differential equation, it helps to find exact solutions and gives non-singular results in the interior of gravastar.

According to the Mazur–Motella [1, 2] gravastar (an alternative to Black Hole) configuration comprises three main parts: i- Interior region, ii- Thin shell, and iii- Exterior region, which has the following consequences

- **I**- Thin shell covered the interior sector (0 ≤ r < r₁ = Q) which is considered as dark energy. In this inner region, we have found the solutions of energy density, pressure, metric potential of the radial coordinate, and gravitational mass by considering EOS \( p = -\rho \) (negative isotropic pressure and positive density) with the help of Rastall field equations, non-conservation law, and Kuchowicz metric potential equation (9). We have obtained the results which are regular at \( r = 0 \) in the interior region.

- **II**- The second part of gravastar, which is thin shell (Q = r₁ < r < r₂ = Q + \( \epsilon \)) is assumed as very thin but positive, obeys the EOS \( p = -\rho \). It is filled with ultrarelativistic stiff-fluid. We have found exact solutions without thin shell approximation i.e., 0 < \( e^{-\nu} \equiv b \ll 1 \) in this region. With the help of Kuchowicz metric potential (9) and Rastall field equations (10)–(12), the other metric coefficient \( e^{\nu(r)} \) has been achieved. The shell density is obtained by the use of equation (19) in non-conservation law equation (13) and its physical behavior with numerical values \( \eta = 1, M = 2.75M_\odot \) can be shown in figure 1. The behavior of density shows the stability of our model with positive and decreasing values which means that it is less denser at junction interface. With the help of these results, different properties of the thin shell such as proper length, shell energy, shell entropy, and surface redshift have been discussed with their graphical representation. The graph for proper length in figure 2 expresses the positive and increasing behavior relative to radial component \( r \). From figure 3, we can observe that the variation of shell energy is finite, positive, and directly proportional to the radius \( r \) of gravastar. Figure 4 indicates the same behavior as in figure 3. To check the stability of gravastar, some researchers [31–36] have calculated the surface redshift for static, isotropic perfect fluids and they have examined that its value must be less than 2. In our work, surface redshift is calculated by using equation (40), and its graph is shown in figure 5, which shows that redshift has values less than 2 indicating the stability of our gravastar model.

- **III**- The third region is exterior of gravastar (\( r > r₂ = Q + \epsilon \)) satisfying \( p = \rho = 0 \) that consists of vacuum only. This part is defined by Schwarzschild spacetime given in equation (23). To obtain the constants \( B \) and \( C \) in equation (9), first we have made expressions for them in terms of mass and exterior radius by matching the

![Figure 5. Variation of surface redshift of shell w.r.t. radius \( r \).](image)

Hence, for the above value of EOS parameter, the gravastar model would be stable as the condition \( \frac{2M}{Q} < 1 \) avoids the horizon formation.

5. Conclusion

The current study is about the consequences of the gravastar idea with Kuchowicz type metric potential in Rastall theory pursuing the Mazur–Motella [1, 2] strategy. We have considered the isotropic fluid with spherically symmetric and static line-element in the interior region while a vacuum solution has been taken in the exterior region. In this work, we have designated the metric potential for the time coordinate as Kuchowicz spherically symmetric and static line-element in the interior region while a vacuum solution has been taken in Rastall theory pursuing the Mazur–Motella [1, 2] strategy.
metric coefficients of interior and exterior spacetimes, which can be seen in equations (22) and (23). Then final values for $B$ is 0.014991 km$^{-2}$ and for $C^2$ is 0.055 832 7, which make our model as physically stable.

- IV- Junction interface is the boundary at which the interior and exterior regions are connected to each other by thin shell. By using second fundamental law in Lanczos equation with unit normals on surface, we get the expressions for surface pressure $\chi$, surface energy density $\rho$, thin shell mass and total mass of gravastar given in equations (29), (30), (31) and (32), respectively.

The gravastar is an amazing hypothetical idea introduced by Mazur and Mottola [1, 2], which structurally looks like BH externally. We know that two main issues associated with BH are singularity and event horizon, all physics laws fail at singularity but the gravastar idea resolve these drawbacks of BH. In our present article, we have found new non-singular, horizon free solutions for gravastar with Kuchowicz type metric potential in Rastall gravity and the graphical study proves our model as stable and physically acceptable.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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References

[1] Mazur P and Mottola E 2001 Report Number: LA-UR-01-5067 arXiv:gr-qc/0109035
[2] Mazur P O and Mottola E 2004 Gravitational vacuum condensate stars Proc. Natl. Acad. Sci. 101 9545–50
[3] Mukhopadhyay B and Singh P 2003 arXiv:gr-qc/0301002
[4] Astekekar A, Obmedio J and Singh P 2018 Quantum transfiguration of Kruskal black holes Phys. Rev. Lett. 121 241301
[5] Ditta A and Abbas G 2020 Relativistic accretion mechanism for some black holes Chin. J. Phys. 65 325–33
[6] Gregorius D and Rosquist K 2021 Observational backreaction in discrete black holes lattice cosmological models The European Physical Journal Plus 136 1–18
[7] Sánchez A 2020 Geometrothermodynamics of black holes with a nonlinear source arXiv:2006.00023
[8] Meisner K A and Nicolai H 2020 Supermassive gravitinos and giant primordial black holes Phys. Rev. D 102 103008
[9] Yusuf H M and Ho L C 2020 Gas content regulates the life cycle of star formation and black hole accretion in galaxies Astrophys. J. 901 42
[10] Meisner K A and Nicolai H 2020 Supermassive gravitinos and giant primordial black holes Phys. Rev. D 102 103008
[11] He Y and Ma M S 2017 (2 + 1)-dimensional regular black holes with nonlinear electromagnetodynamics Phys. Lett. B 774 229–34
[12] Borde A 1997 Regular black holes and topology change Phys. Rev. D 55 7615
[13] Abbas G and Sabiuallah U 2014 Geodesic study of regular Hayward black hole Astrophys. Space Sci. 352 769–74
[14] Chan R 2011 Stability analysis of lower dimensional gravastars in non commutative geometry J. Cosmol. Astropart. Phys. 2011 013
[15] Chan R, Da Silva M F A and Rocha P 2011 Gravastars and black holes of anisotropic dark energy Gen. Relativ. Gravitation 43 2223–35
[16] Martin-Moruno P, Garcia N M, Lobo F S and Visser M 2012 Generic thin-shell gravastars J. Cosmol. Astropart. Phys. 2012 034
[17] Lobo F S and Garattini R 2013 Linearized stability analysis of gravastars in noncommutative geometry J. High Energy Phys. 2013 1–20
[18] Sakai N, Saida H and Tanakii T 2014 Gravastar shadows Phys. Rev. D 90 104013
[19] Kuro T and Sakai N 2016 Gravitational lensing by gravastars Phys. Rev. D 93 084051
[20] Banerjee A, Villanueva J R, Channuie P and Jesus K 2018 Stable Gravastars: Guilfoyle’s electrically charged solutions Chin. Phys. C 42 115101
[21] Chan R and da Silva M F A 2010 How the charge can affect the formation of gravastars J. Cosmol. Astropart. Phys. 2010 029
[22] Sengupta R, Ghosh S, Ray S, Mishra B and Tripathy D 2020 Gravastar in the framework of braneworld gravity Phys. Rev. D 102 024037
[23] Ray S, Sengupta R and Nimesh H 2020 Gravastar: an alternative to black hole Int. J. Mod. Phys. D 29 2030004
[24] Usmani A A, Rahaman F, Ray S, Nandi K K, Kuhlittig P K, Rakib S A and Hasan Z 2011 Charged gravastars admitting conformal motion Phys. Lett. B 701 388–92
[25] Bhor P 2014 Higher dimensional charged gravastar admitting conformal motion Astrophys. Space Sci. 354 457–62
[26] Bhor P 2017 A new model of charged gravastar in (T) gravity admitting conformal motion arXiv:1702.02467
[27] Sharif M and Waseem A 2019 Charged gravastars with conformal motion in f(R, T) gravity Astrophys. Space Sci. 364 1–9
[28] Bhatti M Z, Yousaf Z and Ashraf T 2021 Gravastars in modified Gauss-Bonnet gravity Chin. J. Phys. 73 167–78
[29] Bhatti M Z, Yousaf Z and Ashraf T 2021 Charged gravastars in modified Gauss-Bonnet gravity Mod. Phys. Lett. A 36 215023
[30] Rahaman F, Ray S, Usmani A and Islam S 2012 The (2 + 1)-dimensional gravastars Phys. Lett. B 707 319–22
[31] Rahaman F, Usmani A, Ray S and Islam S 2012 The (2 + 1)-dimensional charged gravastars Phys. Lett. B 717 1–5
[32] Ghosh S, Rahaman F, Guha B K and Ray S 2017 Charged gravastars in higher dimensions Phys. Lett. B 767 380–5
[33] Ghosh S, Ray S, Rahman F and Guha B K 2018 Gravastars with higher dimensional spacetimes Ann. Phys. 394 230–43
[34] Ghosh S, Biswas S, Rahaman F, Guha B K and Ray S 2019 Gravastars in (3 + 1) dimensions admitting Karmarkar condition Ann. Phys. 411 167968
[35] Das A, Ghosh S, Guha B K, Das S, Rahaman F and Ray S 2017 Gravastars in f(R, T) gravity Phys. Rev. D 95 124011
[36] Shamir M F and Ahmad M 2018 Gravastars in f(G, T) gravity Phys. Rev. D 97 104031
[37] Deb Nath U 2019 Charge gravastars in f(T) modified gravity Eur. Phys. J. C 79 1–9
[38] Yousaf Z 2020 Construction of charged cylindrical gravastar–like structures Physics of the Dark Universe 28 100509
[39] Ghosh S, Kamfon A D, Das A, Houndjo M J S, Salako I G and Ray S 2020 Gravastars in f(R, T) gravity Int. J. Mod. Phys. A 35 2050017
[40] Majeed K, Yousaf Z and Abbas G 2020 Effects of electromagnetic field on the stability of locally isotropic gravastars New Astron. 80 101397
[41] Yousaf Z, Bamba K and Bhatti M Z 2021 Formation of cylindrical gravastars in modified gravity arXiv:2107.01021
[42] Bhatti M Z, Yousaf Z and Rehman A 2020 Gravastars in f(R, G) gravity Physics of the Dark Universe 29 100561
[43] Bhatti M Z, Yousaf Z and Rehman A 2021 Analysis of charged gravastar in f(R, G) gravity Int. J. Mod. Phys. D 31 2150124
[44] Yousaf Z, Bamba K, Bhatti M Z and Ghafoor U 2019 Charged gravastars in modified gravity Phys. Rev. D 100 024062
[45] Yousaf Z and Bhatti M Z 2021 Charged gravastars in f(R, T, Rmn, Tmn) gravity Int. J. Mod. Phys. D 30 2150084
[46] Rastall P 1972 Generalization of the Einstein theory Phys. Rev. D 6 3357
[47] Bronnikov K A, Fabris J C, Piattella O F and Santos E C 2016 Static, spherically symmetric solutions with a scalar field in Rastall gravity Gen. Relativ. Gravit. 48 162
[48] Al-Rawaf A S and Taha M O 1996 A resolution of the cosmological age puzzle Phys. Lett. B 366 69
[49] Al-Rawaf A S and Taha M O 1996 Cosmology of general relativity without energy-momentum conservation Gen. Relativ. Gravit. 28 935
[50] Ma M S and Zhao R 2017 Noncommutative geometry inspired black holes in Rastall gravity Eur. Phys. J. C 77 1–7
[51] Debnath U 2021 Charged gravastars in Rastall Rainbow gravity The European Physical Journal Plus 136 1–23
[52] Salako I G, Jawad A and Moradpour H 2018 Anisotropic compact stars in non-conservative theory of gravity Int. J. Geom. Meth. Mod. Phys. 15 1850093
[53] Abbas G and Shahzad M R 2018 Isotropic compact stars model in Rastall theory admitting conformal motion Astrophys. Space Sci. 363 1–8
[54] Das D, Dutta S and Chakraborty S 2018 Cosmological consequences in the framework of generalized Rastall theory of gravity Eur. Phys. J. C 78 1–8
[55] Abbas G and Shahzad M R 2020 Comparative analysis of Einstein gravity and Rastall gravity for the compact objects Chin. J. Phys. 63 1–12
[56] Abbas G and Shahzad M R 2019 Models of anisotropic compact stars in the Rastall theory of gravity Astrophys. Space Sci. 364 1–12
[57] Abbas G and Shahzad M R 2018 A new model of quintessence compact stars in the Rastall theory of gravity Eur. Phys. J. A 54 1–11
[58] Soroushfar S, Safiari R and Upadhyay S 2019 Thermodynamic geometry of a black hole surrounded by perfect fluid in Rastall theory Gen. Relativ. Gravitation 51 1–16
[59] Mauyra S K and Tello-Ortiz F 2020 Decoupling gravitational sources by MGD approach in Rastall gravity Physics of the Dark Universe 29 100577
[60] Prihadi H L, Sakti M F, Hikmawan G and Zen F P 2020 Dynamics of charged and rotating NUT black holes in Rastall gravity Int. J. Mod. Phys. D 29 2050021
[61] Abbas G and Majeed K 2020 Isotropic gravastar model in rastall gravity Advances in Astronomy 2020 8861168
[62] Kuchowicz B 1968 General relativistic fluid spheres. I. New solutions for spherically symmetric matter distributions Acta Phys. Pol. 33 541
[63] Farasat Shamir M and Fayyaz I 2020 Charged stellar structure in Tolman-Kuchowicz spacetime Int. J. Geom. Meth. Mod. Phys. 17 2050140
[64] Javed M, Mustafa G and Shamir M F 2021 Anisotropic spheres in f(R, G) gravity with Tolman-Kuchowicz spacetime New Astron. 84 101518
[65] Biswas S, Shee D, Guha B K and Ray S 2020 Anisotropic strange star with Tolman-Kuchowicz metric under f(R, T) gravity Eur. Phys. J. C 80 1–15
[66] Zubair M, Ditta A, Gudekli E, Bhar P and Armat H 2021 Anisotropic compact star models in f(T) gravity with Tolman-Kuchowicz spacetime Int. J. Geom. Meth. Mod. Phys. 18 2150060
[67] Ghosh S, Shee D, Ray S, Rahaman F and Guha B K 2019 Gravastars with Kuchowicz metric potential Results in Physics 14 102473
[68] Ze’l dovich Y B 1972 A Hypothesis, unifying the structure and the entropy of the universe Mon. Not. R. Astron. Soc. 160
[69] Buchdahl H A 1959 General relativistic fluid spheres Phys. Rev. 116 1027
[70] Darmois G 1927 Mémorial des Sciences Mathématiques, Fascicule XXV (Paris: Gauthier-Villars) Ch. 3
[71] Israel W 1966 Singular hypersurfaces and thin shells in general relativity Il Nuovo Cimento B (1965-1970) 44 1–14
[72] Lanczos K 1924 Flächenhafte verteilung der materie in der Einsteinschen gravitationstheorie Ann. Phys. 379 318–40
[73] Böhmer C G and Harko T 2006 Bounds on the basic physical parameters for anisotropic compact general relativistic objects Class Quantum Gravit. 23 6479
[74] Ivanov B V 2002 Maximum bounds on the surface redshift of anisotropic stars Phys. Rev. D 65 104001
[75] Barraco D and Hamity V H 2002 Maximum mass of a spherically symmetric isotropic star Phys. Rev. D 65 124028