Galactic clustering under power-law modified Newtonian potential

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Abstract
We estimate galaxy clustering under a modified gravitational potential. In particular, the modifications in gravitational potential energy occur due to a power-law and cosmological constant terms. We derive a canonical partition function for the system of galaxies interacting under such a modified gravitational potential. Moreover, we compute various thermodynamical equation of states for the system. We do comparative analysis in order to emphasize the effect of corrections on thermodynamics of the system. Interestingly, the modifications in thermodynamical quantities are embedded in clustering parameter only.

Keywords Galactic clustering · Modified gravitational potential · Correlation function

Contents
1 Introduction ............................................. 2
2 Interaction of galaxy clusters under modified potential ............................................. 4
   2.1 A modified gravitation potential ............................................. 4

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1 Introduction

Since last decade, the substantial progresses have been made in the understandings of galaxy clusters, from their internal structure and evolution to their place in the large scale structure of the universe. All these progresses are due to the stupendous improvements in the theoretical modeling and numerical simulation, viz-a-viz abundance of new information provided by multi-wavelength surveys of the universe. Various theories of cosmological many-body distribution function have been developed from the thermodynamic point of view. Benkesestien [1], Hawking [2] and Unruh [3] originated the relation between relativity and thermodynamics. Later, Jacobson [4] introduced the Einstein equation as thermodynamic equation of state. In fact, the Einstein equation is derived from the proportionality of entropy and the horizon area together with the fundamental relation \( \delta Q = T dS \), where \( \delta Q \) and \( T \) are interpreted as the energy flux and Unruh temperature. Recently Verlinde [5] proposed an entropic origin of gravity and interpreted gravity as an entropic force. In this regard, it is argued that the central notion needed to derive gravity is information. Employing the holographic principle and equipartition law of energy, Newton’s law of gravitation, Poisson’s equation and Einstein’s field equations are successfully obtained. Verlinde had used area-entropy relation of black holes in Einstein’s gravity (i.e. \( S = \frac{A}{4l_P^2} \)) to get the Newton’s law, where \( S, A \) and \( l_P \) refer the entropy of the black hole, the area of horizon and the Planck length, respectively.

The study of modified gravity on the cosmological scales is an active area of present research. Recently, the corrected gravitational potential plays a vital role in estimation of the total mass of a sample of 12 clusters of galaxies which provides a better fit to the mass of visible matter [6]. At large distances, the modification in Newtonian potential occurs due to the propagation of gravity into the bulk [7]. Recently, Sheykhi and Hendi studied the effect of power-law corrections in entropy to the Newton’s law [8]. These corrections appear due to entanglement of quantum field inside and outside of horizon [9]. It is also found that a viable source of black hole entropy is quantum entanglement of degrees of freedom inside and outside the horizon. Also, the black hole entropy is found directly proportional to the surface area of the sphere when the field is in the ground state. But when the field is in a superposition of ground and excited states, a correction term proportional to a fractional power of area appears. These corrections are negligible for large horizon areas. Another quantum correction to Newton’s law, a logarithmic correction, has also been studied which appears due to the result of thermal...
equilibrium fluctuations and the quantum fluctuations of loop quantum gravity [10–12]. Recently, the effect of this logarithmic correction on the galaxy clustering has also been studied [13]. The clustering of galaxies has been studied under the modified gravity under various potentials [14–21].

The power-law corrected entropy leads to modification in Newton’s gravitational potential by adopting the viewpoint of gravity as an entropic force. We consider a power-law corrected entropy of the following form [9,22]:

\[ S = \frac{A}{4\pi^2} \left[ 1 - K_\alpha A^{1 - \frac{\alpha}{2}} \right], \]  

where \( \alpha \) is a dimensionless constant whose value is not confirmed yet and the parameter \( K_\alpha \) that depends on the power \( \alpha \) of the entropy correction as following:

\[ K_\alpha = \frac{\alpha (4\pi)^{\frac{\alpha}{2} - 1}}{(4 - \alpha) r_c^{2 - \alpha}}, \]

here \( r_c \) denotes crossover scale. The Boltzmann constant is set unit here. The last term of Eq. (1) appears due to the superposition of ground state and excited state wave function of the field. Interestingly, Verlinde [5] proposed an entropic origin of gravity and interpreted gravity as an entropic force. The gravity derived here was from the notion of information associated with matter and its location measured in terms of entropy. The Newton’s law of gravitational force \( F \) is related to the entropy \( S \) of the system as [8]

\[ F = -4l_p^2 \frac{GM^2}{r^2} \frac{\partial S}{\partial A}. \]  

(2)

For the power-law corrected entropy given in (1), this relation leads to the following modified Newton’s force [8]:

\[ F = -\frac{GM^2}{r^2} \left[ 1 - \frac{\alpha}{2} \left( \frac{r}{r_c} \right)^{\alpha - 2} \right]. \]  

(3)

which coincides with the original Newtons law when \( \alpha \) is set to zero. Keeping in view the attractive nature of gravity, we should have \( F < 0 \). This requires,

\[ \alpha < 2 \left( \frac{r}{r_c} \right)^{\alpha - 2}. \]

In this paper, we study the effect of power-law corrected Newtonian potential on the clustering of galaxies. We also consider the effect of dark matter on the formation of galaxy clusters via the incorporation of cosmological constant in the Newtonian potential. This is because of the role played by cosmological constant \( \Lambda \) in the expansion of universe [5]. In order to study the effects of all the corrections made in gravitational
potential in galaxy clustering, we first derive the \( N \)-body partition function by evaluating configuration integrals recursively. The resulting partition function is employed to calculate the various thermodynamic quantities viz the Helmholtz free energy, entropy, pressure, internal energy, and chemical potential which possess deviations from their original values due to the incorporation of corrections. Remarkably, a modified clustering parameter emerges naturally from the corrected equations of state. Furthermore, we derive the probability distribution function assuming that the system is in a quasi equilibrium state as described by the grand canonical ensemble. The expression of distribution function embeds modified clustering parameter. A comparative analysis is made with the original distribution function to study the deviation due to corrections.

The paper is organized as following. In Sect. 2, we consider a power-law and cosmological constant modified gravitational potential to calculated partition function of the system of galaxies and galaxy clusters. With the help of resulting partition function, the various thermodynamical equation of states are calculated in Sect. 3. The study the distribution of galaxies and galaxy’s clusters under the modified gravitational potential, we estimate distribution function in Sect. 4. Finally, we draw concluding remarks in the last section.

2 Interaction of galaxy clusters under modified potential

In this section, we derive a modified Newtonian potential due to power-law corrected force and estimate the partition function.

2.1 A modified gravitation potential

Utilized standard definition \( \Phi = - \int F dr \), we calculate a power-law modified gravitational potential corresponding to power-law corrected Newton’s law \( (3) \) as

\[
\Phi = -GM^2 \left[ \frac{1}{r} + \frac{\alpha}{2(\alpha + 1)} r^{\alpha - 2} r^{\alpha+1} \right].
\]

(4)

This potential can arise in modified gravity theories like \( f(R) \) gravity. In fact, the power-law entropy corrected Friedmann equation is derived using the first-law on the apparent horizon \( [8] \).

To get more realistic results, one can not ignore the cosmological constant term \( \frac{1}{6} \Lambda r \) \( [23] \) in potential at the cosmological scale as cosmological constant is responsible for the expansion of the Universe through a repulsive force. Therefore, the exact gravitation potential is given by

\[
\Phi = -GM^2 \left[ \frac{1}{r} + \frac{\alpha}{2(\alpha + 1)} r^{\alpha - 2} r^{\alpha+1} + \frac{\Lambda r^2}{6 GM^2} \right].
\]

(5)

This is a final gravitational potential where second and third terms correspond to the power-law and cosmological constant correction terms respectively.
2.2 Generating functional of galaxies cluster under modified gravity

Next, we estimate the partition function for the system of galaxies under the modified gravitational potential. Here it is assumed that system of galaxies follows a statistically homogeneous distribution over large regions, which consists of an ensemble of cells with equal volume $V$ and equal average density. Let us begin by writing the general partition function for the system comprised with $N$ galaxies of equal mass $M$, momenta $P_i$ and average temperature $T$ as

$$Z_N (T, V) = \frac{1}{\lambda^{3N} N!} \int d^{3N} P d^{3N} r \exp \left[ -\frac{1}{T} \left( \sum_{i=1}^{N} \frac{P_i^2}{2M} + \Phi (r_1, r_2, \ldots r_N) \right) \right],$$

(6)

where $N!$ appears due to distinguishable nature of galaxies and $\lambda$ is a normalization constant for the phase space volume cell. Upon integration over momentum space, this further simplifies to

$$Z_N (T, V) = \frac{1}{N!} \left( \frac{2\pi MT}{\lambda^2} \right)^{3N/2} Q_N (T, V),$$

(7)

where the configuration integral, $Q_N (T, V)$, has the following form:

$$Q_N (T, V) = \int \ldots \int \prod_{1 \leq i < j \leq N} \left( 1 + \frac{\Phi}{T} \right) d^{3N} r,$$

(8)

Here we neglected the higher-order terms of potential as the system of galaxies is still clustering.

Here we note that for the point-mass galaxies (i.e., for $r = 0$), the potential energy and consequently partition function diverges. In order to remove this discrepancy, we assume galaxies of extended nature (i.e., galaxies with halos). For this we introduce a softening parameter $\epsilon$, which assures the finite size of galaxies. The value of this softening parameter ranges $0.01 \leq \epsilon \leq 0.05$. In order to estimate partition function, the modified potential incorporates softening parameter appropriately as follows,

$$\Phi (\epsilon) = -GM^2 \left[ \frac{1}{(r^2 + \epsilon^2)^{1/2}} + \frac{\alpha r_c^{\alpha-2}}{2 (1 - \alpha)} r^{1-\alpha} + \frac{\Lambda r^2}{6 GM^2} \right].$$

(9)

The second and third terms do not require softening parameter to be introduces as corresponding potentials do not diverge.

Now, we estimate configuration integral $Q_N (T, V)$ iteratively. The configuration integral for a single (spherically) galaxy of radius $R$ is given by

$$Q_1 (T, V) = V.$$  

(10)
For system of two galaxies, the configuration integral is given by

\[
Q_2(T, V) = 4\pi V \int dR \ R^2 \left[ 1 + \frac{GM^2}{T} \left( \frac{1}{(R^2 + \epsilon^2)^{\frac{1}{2}}} + \frac{\alpha r_c^{\alpha-2}}{2(1 - \alpha)} R^{1-\alpha} + \frac{1}{6GM^2} \right) \right]. \tag{11}
\]

Here, the double integral reduces to a single integral by fixing the position of one galaxy. This result can also be obtained by considering the fact that expansion of universe exactly cancels the effect of the long-range mean gravitational field on the particle motions [24]. Eq. (11) further simplifies to

\[
Q_2(T, V) = V^2 \left[ 1 + \frac{3GM^2}{2RT} \left( \sqrt{1 + \frac{\epsilon^2}{R^2}} + \frac{\epsilon^2}{R^2} \ln \frac{\epsilon/R}{1 + \sqrt{1 + \frac{\epsilon^2}{R^2}}} + \frac{3\alpha}{2(1 - \alpha)(4 - \alpha)} r_c^{\alpha-2} R^{1-\alpha} + \frac{\Lambda R^3}{15GM^2} \right) \right]. \tag{12}
\]

More compactly, this can be written as

\[
Q_2(T, V) = V^2 \left[ 1 + \frac{3}{2} (\zeta + \gamma + \beta) \frac{GM^2}{RT} \right], \tag{13}
\]

where

\[
\zeta = \sqrt{1 + \frac{\epsilon^2}{R^2}} + \frac{\epsilon^2}{R^2} \ln \frac{\epsilon/R}{1 + \sqrt{1 + \frac{\epsilon^2}{R^2}}},
\]

\[
\gamma = \frac{3\alpha}{2(1 - \alpha)(4 - \alpha)} r_c^{\alpha-2} R^{1-\alpha},
\]

\[
\beta = \frac{\Lambda R^3}{15GM^2}.
\]

Here, we note that \( \alpha \) in the \( \gamma \) takes the value \( 0.624 < \alpha < 2 \) because within this limit only a phantom accelerating universe can be derived which is compatible with the observations [25].

Next, we scale the temperature \( T \) and radius \( R \) as \( T \to \eta^{-1}T \) and \( R \to \eta R \), which leads to the dimensionless factor \( \frac{GM^2}{RT} \) scale invariant. Therefore, we can scale \( \frac{GM^2}{RT} \to \left( \frac{GM^2}{RT} \right)^3 = \frac{3}{2} \left( \frac{GM^2}{T} \right)^3 \bar{\rho} := x \) [26]. Thus, Eq. (13) finally reduces to

\[
Q_2(T, V) = V^2 \left[ 1 + (\zeta + \gamma + \beta) x \right]. \tag{14}
\]
Following this procedure iteratively, the configuration integral for \( N \) galaxies is obtained as

\[
Q_N (T, V) = V^N \left[ 1 + (\zeta + \gamma + \beta) x \right]^{N-1}.
\] (15)

By inserting this value of configuration integral of \( N \) galaxies into the partition function (7), we achieve the expression of partition function for \( N \) galaxies gravitating under modified gravity as

\[
Z_N (T, V) = \frac{1}{N!} \left( \frac{2\pi M T}{\lambda^2} \right)^N V^N \left[ 1 + (\zeta + \gamma + \beta) x \right]^{N-1}.
\] (16)

3 Thermodynamical equations of state

In this section, we derive various more exact equations of states for the system of galaxies interacting under modified potential. More precisely, we derive the Helmholtz free energy, entropy, internal energy, pressure and chemical potential. We also emphasize the effect of corrections in Newton’s potential on these equation of states.

3.1 Helmholtz free energy

The Helmholtz free energy can be calculated from the partition function using following definition:

\[
F = -T \ln Z_N (T, V).
\] (17)

Therefore, the Helmholtz free energy for our system of galaxies is calculated by

\[
F = NT \left( \ln \frac{N}{V} T^{-3/2} \right) - NT - NT \ln \left[ 1 + (\zeta + \gamma + \beta) x \right] - \frac{3}{2} NT \ln \left( \frac{2\pi M T}{\lambda^2} \right).
\] (18)

Here we have made following assumption \((N - 1) \approx N\), since \(N\) is very large. This equation can also be written as

\[
F = NT \left( \ln \frac{N}{V} T^{-3/2} \right) - NT + NT \ln \left[ 1 - \frac{(\zeta + \gamma + \beta)x}{1 + (\zeta + \gamma + \beta)x} \right]
\]

\[-\frac{3}{2} NT \ln \left( \frac{2\pi M T}{\lambda^2} \right).\] (19)

In order to study the behavior of Helmholtz free energy with respect to particle number, we plot a graph Figs. 1 and 2. From the figure, we see that Helmholtz free energy is a decreasing function of \(N\). It means as long as number of galaxies increases
Fig. 1 Helmholtz free energy vs. number of galaxies. Left: red, blue, and green lines correspond to $\gamma x = 0, 0.5$ and 1, respectively, with $\xi x = 1, \beta x = 0$. Right: red, blue, and green lines correspond to $\gamma x = 0, 0.5$ and 1, respectively, with $\xi x = 1, \beta x = 1$. Rest of the parameters are unit here.
the Helmholtz free energy decreases. It is also evident that the corrections due to power-law and cosmological constant terms make the free energy more negative. Also, the free energy decreases with increase in the mass of galaxies.

By comparing this equation of state to its original form given in Ref. [24], we conclude that the corrections in Newton’s potential are apparent only in clustering parameter. The new clustering parameter corresponding to power-law and cosmological constant corrections is obtained as

$$b_\star = \frac{(\zeta + \gamma + \beta)x}{1 + (\zeta + \gamma + \beta)x}. \quad (20)$$

This modified clustering parameter can be expressed in terms of original clustering parameter $b_\epsilon = \frac{\zeta x}{1 + \zeta x}$ [24] as follows

$$b_\star = \frac{b_\epsilon (1 - \gamma x - \beta x) + (\gamma + \beta)x}{1 + (\gamma + \beta)x - b_\epsilon (\gamma + \beta)x}. \quad (21)$$

### 3.2 Entropy

Let us calculate entropy of the system which a very important thermodynamical quantity. For a given Helmholtz free energy (19), the entropy can easily be calculated as

$$S = -N \ln \left( \frac{N}{V} T^{-3/2} \right) - N \ln[1 - b_\star] - 3 N b_\star + \frac{5}{2} N + \frac{3}{2} N \ln \frac{2\pi M}{\lambda^2}. \quad (22)$$
Here we have utilized standard entropy definition $S = -\left( \frac{\partial F}{\partial T} \right)_{N,V}$. The specific entropy (entropy per galaxy) is given by

$$\frac{S}{N} = -\ln \left( \frac{N}{V} T^{-3/2} \right) - \ln[1 - b_\star] - 3b_\star + S_0,$$

(23)

where $S_0 = \frac{5}{2} + \frac{3}{2} \ln \frac{2\pi M}{\lambda^2}$ is a constant.

The behavior of specific entropy with respect to number of galaxies can be seen in Figs. 3 and 4. The specific entropy is also a positive valued decreasing function with number of galaxies. The power-law corrected term makes specific entropy smaller in absence of dark energy. However, the effect of power-law correction is not significant in the presence of dark energy but makes specific entropy bit positive. The specific entropy increases significantly along with increase in mass of galaxies.

### 3.3 Internal energy

The internal energy of the system is defined as $U = F + TS$. For a given Helmholtz free energy (19) and entropy (22), the internal energy is calculated by

$$U = \frac{3}{2} NT \left[ 1 - 2\frac{(\zeta + \gamma + \beta) x}{1 + (\zeta + \gamma + \beta) x} \right] = \frac{3}{2} NT \left[ 1 - 2b_\star \right].$$

(24)

From the Fig. 5, we observe that the internal energy of the system which is negative valued becomes more negative under the effect of correction in gravitational potential.

### 3.4 Pressure

The standard definition of pressure is given by $P = -\left( \frac{\partial F}{\partial V} \right)_{N,T}$. Exploiting expression (19), the pressure for a gravitating system under modified gravity is given by

$$P = \frac{NT}{V} \left[ 1 - b_\star \right].$$

(25)

The pressure is a linear function of number of the galaxies, which means that pressure increases as long as number of galaxies increases. It is obvious from the Fig. 6 that the presence of correction terms negate the pressure of the system.

### 3.5 Chemical potential

The chemical potential which measure exchange of galaxies can be calculated from the formula $\mu = -\left( \frac{\partial F}{\partial N} \right)_{V,T}$. So, it is a matter of calculation to derive chemical potential for this system as

$$\mu = T \left( \ln \frac{N}{V} T^{-3/2} \right) - T \ln \left[ 1 + (\zeta + \gamma + \beta) x \right] - T \frac{(\zeta + \gamma + \beta) x}{1 + (\zeta + \gamma + \beta) x}$$

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Fig. 3  Specific entropy \((S/N)\) versus particle number \((N)\) with (right) and without (left) dark energy contributions. Left: red, blue, and green lines correspond to \(\gamma x = 0, 0.5\) and 1, respectively, with \(\xi x = 1\) and \(\beta x = 0\). Right: red, blue, and green lines correspond to \(\gamma x = 0, 0.5\) and 1, respectively with \(\xi x = 1\) and \(\beta x = 1\). Rest of the parameters are unit here.
Fig. 4 Behavior of specific entropy ($S/N$) versus particle number ($N$) for different values of $M$. We set all the parameters along with $\xi x$, $\beta x$ and $\gamma x$ to unit. Here, red, blue, and green lines correspond to $M = 1$, $M = 5$ and $M = 10$, respectively

$$-\frac{3}{2} T \ln \left(\frac{2\pi M}{\lambda^2}\right).$$

This further simplifies to

$$\mu = T \left(\ln \frac{N}{V} T^{-3/2}\right) + T \ln [1 - b_*] - T b_* - \frac{3}{2} T \ln \left(\frac{2\pi M}{\lambda^2}\right).$$

The behavior of chemical potential with respect to number of galaxies can be seen from Figs. 7 and 8. Interestingly, we observe that the chemical potential is negative valued for the system of galaxies with small number of galaxies. However, it becomes positive valued when number of galaxies increases to a specific value. The presence of correction terms in clustering parameter decreases the chemical potential of the system. The chemical potential decreases with increase of the mass of galaxies.

At the end of this section, we conclude that the modified Newton’s potential amounts changes to the clustering parameter $b_*$ only, however the basic structures of the equations remain intact.

4 General distribution function

In order to find the probability distribution function $F(N)$, which contains void distribution as well as statistics of counts of the number of galaxies in cells throughout the system. For the system of galaxies, wherein galaxies as well as energy can cross the cell boundary, one has to estimate the grand canonical partition function. The grand canonical partition function, a weighted sum of all canonical partition functions, is
Fig. 5 Internal energy \( (U) \) versus particle number \( (N) \) with (right) and without (left) dark energy contributions. Left: red, blue, and green lines correspond to \( \gamma_x = 0, 0.5 \), and 1, respectively with \( \zeta_x = 2 \) and \( \beta_x = 0 \). Right: red, blue, and green lines correspond to \( \gamma_x = 0 \) and 0.5, respectively with \( \zeta_x = 2 \) and \( \beta_x = 1 \). Rest of the parameters are unit here.
Fig. 6 Pressure ($P$) versus particle number ($N$) with (right) and without (left) dark energy contributions. Left: red, blue, and green lines correspond to $\gamma x = 0, 0.5$ and 1, respectively, with $\zeta x = 2$ and $\beta x = 0$. Right: red, blue, and green lines correspond to $\gamma x = 0, 0.5$ and 1, respectively with $\zeta x = 2$ and $\beta x = 1$. Rest of the parameters are unit here.
Fig. 7  Chemical potential ($\mu(N)$) versus particle number ($N$) with (right) and without (left) dark energy contributions. Left: red, blue, and green lines correspond to $\gamma_x = 0$, 0.5, and 1, respectively, with $\xi = 1$ and $\beta_x = 0$. Right: red, blue, and green lines correspond to $\gamma_x = 0$, 0.5 and 1, respectively with $\xi = 1$ and $\beta_x = 1$. Rest of the parameters are unit here.
Fig. 8  Behavior of chemical potential ($\mu(N)$) versus particle number ($N$) for different values of $M$. We set all the parameters along with $\xi$, $\beta x$ and $\gamma x$ to unit. Here, red, blue, and green lines correspond to $M = 1$, $M = 5$ and $M = 10$, respectively

defined by

$$Z_G(T, V, z) = \sum_{N=0}^{\infty} e^{N\mu T} Z_N(T, V), \quad (28)$$

where $z$ is the fugacity. The grand partition function for the system of galaxies is expressed in terms of thermodynamic variables as

$$\ln Z_G = \frac{PV}{T} = N(1 - b_\ast). \quad (29)$$

The probability distribution function $F(N)$ for finding $N$ galaxies in a cell of volume $V$ and energy $U(N, V)$ is defined by

$$F(N) = \frac{\sum_i e^{N\mu i} e^{U_i}}{Z_G(T, V, z)} = \frac{e^{N\mu} Z_N(V, T)}{Z_G(T, V, z)}. \quad (30)$$

Making use of Eqs. (16), (27) and (29), the distribution function for the extended mass galaxies under modified potential is estimated as

$$F(N) = \frac{\tilde{N}}{N!} (1 - b_\ast) [\tilde{N}(1 - b_\ast) + Nb_\ast]^{N-1} e^{-Nb_\ast - \tilde{N}(1 - b_\ast)}. \quad (31)$$

This distribution function is structurally similar to those derived originally by Saslaw and Hamilton [27] from thermodynamic point of view and by Ahmad and Saslaw [24] from statistical point of view. Also the distribution function derived from logarithmic and volume corrected Newtonian potential [13] has the same general structure.

The behavior of distribution function $F(N)$ versus $N$ can be seen from the comparative analysis as given in Fig. 9. The presence of correction term decreases the peak
Fig. 9. The distribution function $F(N)$ versus $N$ for $\bar{N} = 10$ with (right) and without (left) cosmological constant contributions. Left: red, blue, and green lines correspond to $\gamma_x = 0, 0.5$ and 1, respectively, with $\zeta = 1$ and $\beta_x = 1$. Right: red, blue, and green lines correspond to $\gamma_x = 0, 0.5$ and 1, respectively, with $\zeta = 1$ and $\beta_x = 1$. 
value of distribution function which occurs for system of small number of galaxies. However, for the system of large number of galaxies the correction terms increase the distribution function.

5 Discussion and conclusions

We have presented a study of galaxy clustering under the modified Newton’s law. These modifications to Newton’s law incorporate a power-law entropic corrections along with the inclusion of cosmological constant $\Lambda$ term, in order to take into account the effect of quantum entanglement (a possible source of black hole entropy) and dark energy respectively. Utilizing the modified Newtonian potential, we have derived the corresponding canonical partition function for the system of extended mass galaxies with the assumption that the system is made of $N$ equal volume cells and average particle density $\bar{\rho}$, which are statistically homogeneous over large regions. In this regard, we have used of softening parameter $\epsilon$ in the first term of Hamiltonian to get rid of the divergence of the Hamiltonian when galaxies are considered as point like. This justifies that the extended nature (finite size) of galaxies (or galaxies with halos). From the resulting partition function, we have calculated various important thermodynamic equations of state. Namely, these are free energy, entropy, internal energy, pressure and chemical potential. The exact expressions of equations of state contain a corrected correlation (clustering) parameter which emerges naturally for the clusters of the galaxies with halos. The new clustering parameter $b_\star$ reduces to the original parameter $b_\epsilon$ when $\gamma = 0$ and $\beta = 0$. Moreover, the distribution function is modified due to correction in potential but has the similar structure as the original one.

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