More on Mixed Boundary Conditions and D-branes Bound States

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Abstract

In this article, applying different types of boundary conditions; Dirichlet, Neumann, or Mixed, on open strings we realize various new brane bound states in string theory. Calculating their interactions with other D-branes, we find their charge densities and their tension. A novel feature of \((p-2,p)\) brane bound state is its "non-commutative" nature which is manifestly seen both in the open strings mode expansions and in their scattering off a \(D_p\)-brane. Moreover we study three or more object bound states in string theory language. Finally we give a M-theoretic picture of these bound states.

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1 Introduction

Since D-branes and their possible bound states are essential tools for the better understanding of various dualities, they have been investigated extensively [1,2,3,4,5,6,7,8,9,10]. The possible bound states of D-branes can be constructed from branes of the same or different dimensions. Also, we can construct bound states of D-branes with F-strings [2,9]. Among the two object bound states, bound states of \( p \)- and \( p' \)-branes those with \( p' = p + 2 \) are truly (non-marginally) bound states [1,3]. We use the notation \((p - 2, p)\) branes for these bound states. For \( p' = p + 4 \) or \( p' = p \) they are marginally bounded [1,11]. Besides bound states of D-branes with themselves, bound states of (F-string)-(\( D_p \)-brane) can be represented in string theory [7,9]. In the case of F-string-D-string bound states Witten has shown that they carry the charge of \( U(1) \) gauge field living in the D-string [2]. More generally in the case of other D-branes we can show that the branes with non-zero electric charge (or flux) of the \( U(1) \) field, form a non-marginal bound state of F-strings with \( D_p \)-brane [9]. We denote such a bound state by \((m, 1_p)\) brane, which carries \( m \) units of the NSNS two form charge.

The \((m, 1_p)\) branes and \((p - 2, p)\) branes can be represented in string theory by imposing mixed boundary conditions on open strings attached to branes [5,6,7,9,10]. There is a crucial difference between \((m, 1_p)\) and \((p - 2, p)\) branes; the first has the "electric" flux of the \( U(1) \) gauge field living in the D-branes [9], and the later carries the "magnetic" flux of that gauge field [5,6,7,10]. By T-duality, a \((m, 1_p)\) brane is related to the moving \( D_{(p - 1)} \)-brane [9]. In contrast the \((p - 2, p)\) brane is obtained from a \( D_{(p - 1)} \)-brane by an oblique T-duality [4,9].

In this paper we consider the interactions of the above bound states with other D-branes in string theory. In this way we will find the density of various RR charges and the tension of such bound states and reveal the internal structure of them.

In section 2, we consider \((p - 2, p)\) brane, \( D_{(p - 2)} \)-brane interactions. Although the similar problem have been considered previously in the context of SUGRA theories[4], we present string theoretic calculations. As a result, we find that \((p - 2, p)\) brane carries RR charge of \((p - 1)\) RR form as well as the \((p + 1)\) RR form charge. This \((p - 1)\) RR form charge is homogeneously distributed on the surface of \( D_p \)-brane. The charge density of \((p - 1)\) form is exactly the same as magnetic flux of \( U(1) \) gauge field, i.e., we have a static gas of \( D_{(p - 2)} \)-branes on the \( D_p \)-brane world volume. Since we have many \( D_{(p - 2)} \)-branes we expect to see some non-Abelian gauge fields coming about [2]. This non-commutativity will be addressed in terms of the components of open strings attached to \( D_p \)-brane with mixed boundary conditions.

In section 3, in order to investigate more on \((p - 2, p)\) brane structure, we consider \((p - 2, p)\)
brane interacting with a $D_p$-brane. The novel result of this interaction is that, it explicitly shows the effects of $D_{(p-2)}$-brane distribution in the interaction amplitude.

In section 4, we construct bound states of three (or more) objects in string theory by considering both "electric" and "magnetic" fluxes or "magnetic" fluxes in more than one direction. These bound states and their long range fields have been introduced in field theory [8]. Here we calculate their charge content and their tension by string theory methods and show that they are non-marginal bound states of individual D-branes or F-strings with D-branes.

In section 5, we study these bound states in strong coupling regime of string theory i.e., we give a M-theoretic picture of such bound states.

**2 \ ((p - 2, p) \text{ brane and } D_{(p-2)} \text{ brane Interactions})**

Consider a $(p - 2, p)$ brane bound state and a $D_p$-brane parallel to it. In this section first we realize this brane system in string theory and then by means of their string theoretic definition we calculate the amplitude for the exchange of a closed string between them.

The open strings stretching between $(p-2, p)$ brane and $D_{(p-2)}$-brane satisfy the following boundary conditions:

\[ \sigma = 0 \]
\[
\begin{align*}
\partial_\sigma X^\mu &= 0 & \mu &= 0, 1, \ldots, p - 2 \\
X^\mu &= 0 & \mu &= p - 1, \ldots, 9.
\end{align*}
\]  
\[ (1) \]

\[ \sigma = \pi \]
\[
\begin{align*}
\partial_\sigma X^\mu &= 0 & \mu &= 0, 1, \ldots, p - 2 \\
\partial_\sigma X^{p-1} + \mathcal{F} \partial_\tau X^p &= 0 \\
\partial_\sigma X^p - \mathcal{F} \partial_\tau X^{p-1} &= 0 \\
X^\mu &= Y^\mu & \mu &= p + 1, \ldots, 9.
\end{align*}
\]  
\[ (2) \]

In the above boundary conditions $p - 1, p$ components of open strings have *mixed boundary conditions* at one end and D(irichlet) boundary conditions at the other end. We call these components MD modes. So the above system is described by $p - 1$ NN, 2 MD and $(9 - p)$
DD modes and $F$ is the "magnetic" flux of $U(1)$ gauge field living in $D_p$-brane.

Hence the mode expansion of quantized components are:

\[
X^\mu = \begin{cases}
  x^\mu + p^\mu \tau + i \sum_{n \neq 0} a_n^\mu \frac{e^{-i n_+ \pi}}{n_+} \cos n\sigma & \mu = 0, \ldots, p - 2 \\
  \sum a_n^+ \frac{e^{-i n_+ \pi}}{n_+} \sin n_+ \sigma + \sum a_n^- \frac{e^{-i n_- \pi}}{n_-} \sin n_- \sigma & \mu = p - 1 \\
  -i \sum a_n^+ \frac{e^{-i n_+ \pi}}{n_+} \sin n_+ \sigma + i \sum a_n^- \frac{e^{-i n_- \pi}}{n_-} \sin n_- \sigma & \mu = p \\
  Y^\mu \frac{\eta}{\pi} + \sum_{n \neq 0} a_n^\mu \frac{e^{-i n_\pi}}{n} \sin n\sigma & \mu = p + 1, \ldots, 9,
\end{cases}
\]

where

\[n_\pm = n \pm \frac{1}{2\pi} \cot^{-1} F, \quad n \in \mathbb{Z}.
\]

and

\[[a_n^+, a_m^-] = n_+ \delta_{n+m}, \quad [a_n^\mu, a_m^\nu] = n \delta_{n+m} \eta^{\mu\nu}.
\]

The novel feature of the above mode expansion is that the $X^{p-1}, X^p$ components become "non-commutative". Their non-commutativity is controlled by $F$, i.e., when $F$ is zero they commute. We will show in more detail that this non-commutativity is due to the presence of a distribution of parallel $D_{(p-2)}$-branes with their world volume expanded in $0, 1, \ldots, p - 2$ directions. The corresponding $D_{(p-2)}$-brane distribution makes the $X^{p-1}, X^p$ to become non-commuting [2].

Let us calculate the interactions of $(p - 2, p)$ brane with a $D_{(p-2)}$-brane. This can be done by the usual techniques [1,10,11], i.e., calculating the one loop vacuum graph of the open strings stretched between branes:

\[
A = \int \frac{dt}{2t} \sum_{i,p} e^{-2\pi i t H},
\]

where $i$ indicates the modes of the open string and $p$ their momentum, and $H$ is the open string world-sheet Hamiltonian.

Along the discussions of [11], up to tree level, the massless closed strings contributions are

\[
A = 4V_{p-2} T_p T_{p-2} g_s^2 \frac{1}{\sin\theta} (1 - \cos \theta)^2 G_{g-p}(Y^2); \quad \cot \theta = F.
\]

\(^2\)By NN and DD we mean strings their both ends satisfying Neumann and Dirichlet boundary conditions.
and
\[ T_p = \frac{(4\pi^2\alpha')^{3-p}}{g_s}. \]  
(8)

From the amplitude, the RR and NSNS potentials can be extracted [12].

\[
\begin{cases}
V_{RR} = +8V_{p-2}T_{(p-2)}T_{(p-2)} g_s^2 \left( \frac{\mathcal{F}}{\alpha'} \right) G_{9-p}(Y^2) \\
V_{NSNS} = -4V_{p-2}T_{(p-2)}T_p(1 + \mathcal{F}^2)^{1/2} g_s^2 (1 + \cos^2 \theta) G_{9-p}(Y^2).
\end{cases}
\]  
(9)

As we see the \( V_{RR} \) is proportional to \( \mathcal{F} \). This shows that \( \left( \frac{\mathcal{F}}{\alpha'} \right) \) is the density of RR \((p-1)\) form distributed on the \(Dp\)-brane world volume. The NSNS potential which is due to graviton, dilaton exchange shows that the mass density or tension of the \((p-2, p)\) brane bound state is \( T_p(1 + \mathcal{F}^2)^{1/2} \). So this bound state is "non-marginally" bounded.

Under T-duality in the \(X^p\) direction the above system of branes is transformed to a system of two \(D_{p-1}\)-branes at angle \(\theta\) [4,9]. We study the strong coupling regime of the above bound state in section 5.

3 \((p-2, p)\) brane and \(D_p\)-brane Interactions

In this section, we consider \((p-2, p)\) brane parallel to a \(D_p\)-brane. This configuration could be realized in string theory by the following boundary conditions on open strings:

\[
\begin{cases}
\sigma = 0 & \partial_\sigma X^\mu = 0 \quad \mu = 0, 1, \ldots, p \\
& X^\mu = 0 \quad \mu = p + 1, \ldots, 9.
\end{cases}
\]  
(10)

\[
\begin{cases}
\sigma = \pi & \partial_\sigma X^\mu = 0 \quad \mu = 0, 1, \ldots, p-2 \\
& \partial_\sigma X^{p-1} + \mathcal{F} \partial_\tau X^p = 0 \\
& \partial_\sigma X^p - \mathcal{F} \partial_\tau X^{p-1} = 0 \\
& X^\mu = Y^\mu \quad \mu = p + 1, \ldots, 9.
\end{cases}
\]  
(11)

These open strings have \((p-1)\) NN components, 2 MN and \((9-p)\) DD. The mode expansions of \(X^\mu\) are the same as (3) except for the \(X^{p-1}, X^p\) components which are:
\[
X^{p-1} = x^{p-1} + \sum a_{n+} e^{i n+ r} \cos n+ \sigma + \sum a_{n-} e^{-i n- r} \cos n- \sigma
\]
\[
X^{p} = x^{p} + \sum -i a_{n+} e^{i n+ r} \cos n+ \sigma + i \sum a_{n-} e^{-i n- r} \cos n- \sigma,
\]
where
\[
n_\pm = n \pm \frac{1}{\pi} \tan^{-1} F, \quad n \in \mathbb{Z}.
\]

Again \(X^{p-1}, X^{p}\) (the MN modes) become ”non-commuting”. Their commutation relation is
\[
[X^{p-1}, X^{p}] = i \alpha' \sum_{n \neq 0} \frac{1}{n_+} \cos^2 n_+ \sigma \quad ; \quad [x^{p-1}, x^{p}] = \frac{i \alpha'}{F}.
\]

As we see the \(X^{p-1}, X^{p}\) components become commuting in \(F \to 0\) limit. The second relation tells us that the whole \((p-1, p)\) plane is not available for the end of MN components of open strings and they are allowed to move in a cell with the area \((\frac{\alpha'}{F})\):
\[
\Delta x^{p-1} \Delta x^{p} \sim \frac{\alpha'}{F}.
\]

In order to calculate the interaction of a \(D_p\)-brane with the above bound state, we use the open string one loop amplitude method. Extracting the contributions of massless closed string we obtain:
\[
A = 4V_{p-1} \alpha' T_p^2 g_s^2 \frac{1}{\sin \theta} (1 - \cos \theta)^2 G_{9-p}(Y^2)
\]
\[
= 4V_{p-1} (\frac{\alpha'}{F}) (\frac{T_p}{\cos \theta}) T_p g_s^2 (1 - \cos \theta)^2 G_{9-p}(Y^2),
\]
where \(F = \tan \theta\). The contributions of RR and NSNS closed strings are
\[
\begin{align*}
V_{RR} &= +8V_{p-1} (\frac{\alpha'}{F}) T_p^2 g_s^2 G_{9-p}(Y^2) \\
V_{NSNS} &= -4V_{p-1} (\frac{\alpha'}{F}) T_p T_p (1 + F^2)^{1/2} g_s^2 (1 + \cos^2 \theta) G_{9-p}(Y^2).
\end{align*}
\]

As we expected, instead of the usual \(D_p\)-brane world volume factor \((V_{p+1})\), we obtain \((V_{p-1} \times \frac{\alpha'}{F})\). This shows that the end of open strings is limited to move in the cell of equation \((15)\).

The RR contribution is proportional to \(T_p^2\), showing that the \((p-2, p)\) brane carries the unit charge density of the RR \((p+1)\) form.

The NSNS contributions again justifies that the tension of the corresponding bound state is \(T_p \times (1 + F^2)^{1/2}\).
It is worth noting that applying T-duality two times on the mixed directions of the branes system of this section, we find brane configuration of the previous section. More precisely, the system of \((p-2,p)\) brane parallel to a \(D_p\)-brane, under such a T-duality transforms to \((p-2,p)\) brane parallel to a \(D_{(p-2)}\)-brane, where if \((p-2,p)\) brane is associated with magnetic flux \(\mathcal{F}\), the \((p-2,p)\) brane is associated with \(\frac{-1}{\mathcal{F}}\). The relation between related fluxes are directly seen from the corresponding interaction amplitudes.

Altogether we find that \((p-2,p)\) brane is an object with unit charge of RR \((p+1)\) form, the charge density equal to \((\frac{F_{ij}}{\alpha'})\) for the RR \((p-1)\) form and the mass density \(T_p \times (1+F^2)^{1/2}\). Our interpretation for its internal structure is that parallel \(D_{(p-2)}\)-branes have been distributed homogeneously in the \(D_p\)-brane and hence two of the internal directions of the \((p-2,p)\) brane which are normal to \((p-2)\) branes world volume, show "non-commutative" effects.

4 Bound States of Three or More Objects

We can build bound states of D-branes containing \(p, p-2, p-4, \ldots\) branes [10] and also the bound states of F-strings with \(p, p-2, p-4, \ldots\) branes, in the same spirit we have shown in previous sections. Let us consider a general magnetic flux of \(U(1)\) gauge field living in a \(D_p\)-brane. By imposing these non-vanishing magnetic fluxes \((F_{ij})\) on boundary condition of the open strings attached to brane, we can construct a more general branes bound state. For every non-zero \(F_{ij}\) we have a distribution of \(D_{(p-2)}\)-branes in \((i,j)\) plane. For example if we have \(F_{12}, F_{34}\) components only, our bound state consists of distributions of \(D_{(p-2)}\)-branes in \((1,2)\) and \((3,4)\) planes and also a distribution of \(D_{(p-4)}\)-branes in the world volume of the \(D_{p-2}\)-branes in \((3,4)\) and \((1,2)\) planes respectively. So we have a bound state of three objects: \((p, p-2, p-4)\) brane, which corresponding tension and charge densities are

\[
T = T_p (1 + F_{12}^2)^{1/2} (1 + F_{34}^2)^{1/2}
\]

\[
\text{RR (p + 1) form charge density} = T_p
\]

\[
\text{RR (p - 1) form charge density} = T_{p-2}(\frac{F_{12}}{\alpha'}) \quad \text{and} \quad T_{p-2}(\frac{F_{12}}{\alpha'})
\]

\[
\text{RR (p - 3) form charge density} = T_{p-4}(\frac{F_{12}}{\alpha'}) (\frac{F_{14}}{\alpha'}).
\]

These charge densities could be directly checked in brane interactions.

One can consider cases which also include the "electric" flux. In these cases we find a bound state of F-string with various D-branes. In the corresponding bound state the NSNS
two form charge density is proportional to \((\text{electric flux} \times \alpha'^3 - \alpha'^2)\). It is worth to note that the NSNS two form charge, unlike the magnetic flux case, is independent of string coupling constant \([9]\).

5 Strong Coupling Limit

In this part, we study the strong coupling regime of the \((m, 1_p)\) and \((p - 2, p)\) branes.

- **even p case**
  - \(p = 0\)
    
    In this case because the world volume of the \(D_0\)-brane is just one dimensional, we have not such bound states.

  - \(p = 2\)
    
    i) \((0, 2)\) brane: As we know \(D_2\)-branes at strong coupling are \(M_2\)-branes of M-theory and \(D_0\)-branes are gravitational waves of 11 dimensional SUGRA. Hence we expect that \((0, 2)\) brane is just kinematically related to \(M_2\)-branes \([13]\). Let us consider a moving \(M_2\)-brane. If we compactify the velocity direction then the three form of the eleven dimensional SUGRA gives a "magnetic" flux of the two form living in the \(D_2\)-brane. The strength of this flux is proportional to the brane velocity.

    The tension of a moving membrane after doing the above dimensional reduction is

    \[
    T^2 = m_p^6 (1 + (Rv)^2) \ ; \ v = n/R.
    \]  

    So in type IIA language:

    \[
    T^2 = T_2^2 (1 + F^2)^2, \tag{20}
    \]

    where \(F\) is the magnetics flux and \(T_2 = \frac{m^2}{g_s} = m_p^3\).

    ii) \((m, 1_2)\) brane: This bound state in M-theory is just a \(M_2\)-brane compactified on a direction making an angle with the membrane. The magnitude of NSNS charge is proportional to the \(\cos \theta\), \(\theta\) is the corresponding angle.

  - \(p = 4\)
    
    i) \((2, 4)\) brane: The type IIA \(D_4\)-branes at strong couplings are wrapped \(M_5\)-branes. So the \((2, 4)\) brane at strong couplings is a "non-marginal" bound state of \(M_2\)- and \(M_5\)-branes \([14]\), where the world volume of the \(M_2\)-brane in bound state is not wrapped around the compact direction.

3I am grateful to B. Pioline for his comments on this subject.
ii) \((m, \, 1_4)\) brane: This brane is just the \(M_2, \, M_5\)-brane bound state where the \(M_2\)-brane is \(m\) times wrapped around the compact direction.

iii) \((m, \, 2, \, 4)\) brane: This bound state is the \(M_2, \, M_5\)-brane bound state in which the corresponding membrane makes an angle with the compact direction.

odd \(p\) case

In this case the Sl(2,Z) symmetry of IIB theory determines the string coupling picture of the branes bound states.

\(p = 1\)

This case corresponds to \((m, \, n)\) strings of IIB theory. \((m, \, n)\) strings have been studied extensively both in field theory language [2,15] and in string theory [9]. These strings are related to the moving \(D_6\)-branes of type IIA by T-duality [9].

\(p = 3\)

In this case \((m, \, 1_3)\) and \((1, \, 3)\) brane bound states are related by S-duality of the (3+1) dimensional YM theory living in \(D_3\)-brane world volume. This S-duality is a part of Sl(2,Z) symmetry of IIB theory projected on \(D_3\)-brane.

The \((F - , \, D - \text{string}, \, D_3)\) brane bound states also can appear. They are also related to \((m, \, 1_3)\) branes by Sl(2,Z) duality. The \((m, \, 1_3)\) brane itself is T-dual to a moving \(D_2\)-brane of IIA theory.

\(p = 5\)

As we know \(D_5\)-branes under a "S" transformation of Sl(2,Z) group transform to \(NS_5\)-branes. Although \(NS_5\)-branes are not properly understood in usual string theory limit, we expect at strong couplings the \((m, \, 1_5)\) brane become a (\(NS_5\)-brane)-(D-string) bound state or more generally the \(((m, \, n)\) string)-(\(D_5\)-brane) bound state become a bound state of \(((n, \, -m)\) string)-(\(NS_5\)-brane). These bound states and also the bound state of five-branes with three-branes are not fully understood in terms of \((5+1)\) dimensional field theory living in five-branes.

Acknowledgements

The author would like to thank H. Arfaei who had contribution at the early stages of this work. I am grateful to A.H. Fathollahi for his useful discussions. I am grateful the A. Tseytlin and P. Townsend for reading the manuscript. I would also like to thank theory division of CERN where this work completed.

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