Effective Translation of LTL to Deterministic Rabin Automata: Beyond the (F,G)-Fragment

Tomáš Babiak  František Blahoudek  Mojmír Křetínský  Jan Strejček
Masaryk University, Czech Republic

Highlights 2013
Motivation

- well studied
- efficient tools: SPOT, LTL2BA, LTL3BA,…
- produces nondeterministic Büchi automata
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  - model checking of probabilistic systems
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- LTL cannot be translated into deterministic Büchi automata
  \(\implies\) another acceptance condition - Rabin, Streett, parity, …
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- produces **nondeterministic** Büchi automata

- **deterministic** automata needed for
  - model checking of probabilistic systems
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- LTL cannot be translated into deterministic Büchi automata
  \[\Rightarrow\] another acceptance condition - Rabin, Streett, parity, ...
The Big Picture

LTL

\(G(XFa \land XFb) \lor Gb\)

DRA

Rabin pairs: \(\{(\bullet, \bullet), (\Delta, \Delta)\}\)
Typical workflow – 1t12dstar [Klein2005]

\[ G(XF_a \land XF_b) \lor Gb \]

Rabin pairs: \{ (\textcolor{red}{red}, \textcolor{green}{green}), (\textcolor{orange}{orange}, \textcolor{red}{red}) \}
The Big Picture

- **Typical workflow – ltl2dstar [Klein2005]**

  ![Diagram](LTL -> NBA -> DRA)

  - **Safra’s construction**

- **Rabinizer [Gaiser-J. Křetínský-Esparza2012]**

  ![Diagram](LTL(F, G) -> GDRA -> DRA)
Typical workflow – ltl2dstar [Klein 2005]

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Generalized Rabin pairs: \(\{(\bullet, \{\circ, \circ\}), (\triangle, \emptyset)\}\)

FGa ∨ G(Fa ∧ Fb)
The Big Picture

- Typical workflow – ltl2dstar [Klein2005]
  \[ \text{LTL} \rightarrow \text{NBA} \rightarrow \text{Safra’s construction} \rightarrow \text{DRA} \]

- Rabinizer [Gaiser-J. Křetínský-Esparza2012]
  \[ \text{LTL}(F, G) \rightarrow \text{GDRA} \rightarrow \text{DRA} \]

- LTL3DRA [ATVA 2013]
  \[ \text{LTL}(F_s, G_s) \rightarrow \text{MMAA} \rightarrow \text{TGDRA} \rightarrow \text{DRA} \]
The Big Picture

- **Typical workflow – ltl2dstar** [Klein2005]

  \[
  \text{LTL} \xrightarrow{} \text{NBA} \xrightarrow{} \text{Safra’s construction} \xrightarrow{} \text{DRA}
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- **Rabinizer** [Gaiser-J. Křetínský-Esparza2012]

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- **LTL3DRA** [ATVA 2013]

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  \text{LTL}(F_s, G_s) \xrightarrow{} \text{MMAA} \xrightarrow{} \text{TGDRA} \xrightarrow{} \text{DRA}
  \]

**Formulae**:
- \( F_s\varphi \equiv XF\varphi \)
- \( G_s\varphi \equiv XG\varphi \)
- \( F\varphi \equiv \varphi \lor F_s\varphi \)
- \( G\varphi \equiv \varphi \land G_s\varphi \)
The Big Picture

- Typical workflow – ltl2dstar [Klein2005]

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  - May/Must
  - Alternating Automata
  - Transition-based
  - Generalized
  - Deterministic
  - Rabin Automata
The Big Picture

- Typical workflow – \texttt{ltl2dstar} [Klein2005]

\[ \text{LTL} \rightarrow \text{NBA} \rightarrow \text{Safra’s construction} \rightarrow \text{DRA} \]

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\[ \text{LTL}(F_s,G_s) \rightarrow \text{MMAA} \rightarrow \text{TGDRA} \rightarrow \text{DRA} \]

- Rabinizer 2 [J. Křetínský-Ledesma-Garza2013]

\[ \text{LTL}\setminus\text{GU} \rightarrow \text{GDRA} \rightarrow \text{DRA} \]
LTL($F_s, G_s$) → MMAA → TGDRA → DRA

May/Must Alternating Automata = VWAA with three types of states

Babiak, Blahoudek, Křetínský, Strejček: Effective Translation of LTL to DRA
Very Weak Alternating Automata
- alternating automata
- co-Büchi acceptance
- cycles = selfloops

\[ \text{LTL}(F_s, G_s) \rightarrow \text{MMAA} \rightarrow \text{VWAA} \rightarrow \text{TGDRA} \rightarrow \text{DRA} \]

[Gastin-Oddoux2001] Very Weak Alternating Automata

\[ \text{tt} \rightarrow b \]

\[ \text{tt} \rightarrow a \]

\[ \text{tt} \rightarrow \text{tt} \]
May/Must Alternating Automata = VWAA with three types of states
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- **May**
  - selfloop under $tt$
  - co-Büchi accepting

\[ F_\varphi \]

$tt$
May/Must Alternating Automata = VWAA with three types of states

**May**
- selfloop under $tt$
- co-Büchi accepting

**Must**
- selfloop in each transition
- co-Büchi nonaccepting
May/Must Alternating Automata = VWAA with three types of states

**May**
- selfloop under $tt$
- co-Büchi accepting

**Must**
- selfloop in each transition
- co-Büchi nonaccepting

**Loopless**
- initial
- no predecessors
LTL($F_s, G_s$) and MMAA are equivalent.
Babiak, Blahoudek, Křetínský, Strejček: Effective Translation of LTL to DRA
double powerset construction
dealternation and determinization

\{G_\psi\}
\{G_b\}
double powerset construction
dealternation and determinization
The process involves converting LTL formulas into DRA, where double powerset construction, dealternation, and determinization are applied. The LTL formula $LTL(F_s, G_s)$ is initially transformed to MMAA, then to TGDRA, and finally into DRA.
double powerset construction
dealternation and determinization
The figure illustrates the process of translating LTL formulas to DRA using the double powerset construction, dealternation, and determinization. The LTL formula \( LTL(F_s, G_s) \) is first translated to an\( MMAA \) representation. Then, the\( TGDRA \) representation is used to further refine the model, applying dealternation and determinization. The final result is a DRA model that can be effectively used for model checking and other formal verification tasks.
double powerset construction
dealternation and determinization
Generalized Rabin pairs: \( \{(G_\psi, \{Fa, Fb\})\} \)
Experiments on parametric formulae

\[ \theta(n) = \neg((\bigwedge_{i=1}^{n} GFp_i) \rightarrow G(q \rightarrow Fr)) \]
Experiments on parametric formulae

\[
F(n) = \bigwedge_{i=1}^{n} (\text{GF}p_i \rightarrow \text{GF}q_i)
\]
Experiments on parametric formulae

\[ R(n) = \bigwedge_{i=1}^{n} (GFp_i \lor FGp_{i+1}) \]
Concluding remarks

- Extended translation

LTL ∖ GUX → limMMAA → TGDRA → DRA

LTL3DRA available at http://sourceforge.net/projects/ltl3dra/

Experimental comparison of LTL to DRA translators submitted to LPAR’13

New tools are appearing, but only ltl2dstar can handle the whole LTL. Yet?
Concluding remarks

- Extended translation

LTL ∖ GU ↦ limMMAA ↦ TGDRA ↦ DRA

- Slightly weaker than LTL ∖ GU of Rabinizer 2
- May/Must in the Limit Alternating Automata

Experimental comparison of LTL to DRA translators submitted to LPAR'13.
Concluding remarks

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  \[
  \text{LTL} \setminus \text{GUX} \rightarrow \text{limMMAA} \rightarrow \text{TGDRA} \rightarrow \text{DRA}
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