Dependence of relative abundances of constituents in dense stellar matter on nuclear symmetry energy

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Abstract

For a dense stellar matter, which is electrically neutral and in beta equilibrium, the electron chemical potential, $\mu_e$, will depend nontrivially on baryonic matter density. It is generally expected that as density increases, the electron chemical potential will increase and new degrees of freedom will emerge as $\mu_e$ becomes comparable to their energy scales. Assuming the electrical neutrality and beta equilibrium for the stellar matter, we have studied how the density dependence of lepton chemical potentials varies for different models of nuclear interactions that are constrained by experiments up to nuclear matter density, $n_0$, but extrapolate differently (unconstrained) beyond $n_0$ and calculated the relative abundances of nucleons (neutron and proton) and leptons (electron and muon) and their density dependencies. We find that the density dependence of the electron chemical potential is strongly dependent on the structure of the nuclear symmetry energy relevant to softness/stiffness of the nuclear matter EOS that measures the energy relevant to the neutron-proton asymmetry. As a consequence, the relative abundances of neutrons, protons, electrons, and muons as well as the kaon condensation are strongly dependent on the nuclear symmetry energy. An intriguing result in our finding is that contrary to the accepted lore, kaon condensation in neutron star matter, which is considered to be the first phase transitions beyond $n_0$ and plays a crucial role in certain scenarios of compact-star formation, is not directly tied to the softness or stiffness of the EOS beyond $n_0$. This point is illustrated with a "super-soft" EOS that is fit to the $\frac{\pi^-}{\pi^+}$ ratio data of GSI which excludes kaon condensation at any density.
1 Introduction

Recent interest on the astrophysical compact objects, i.e., neutron stars and black holes, and their evolutions (cooling and collapse for example) brings out a very challenging question as to what is the relevant equation of state (EOS) of the stellar matter, deep inside which the density is expected to be much higher than the normal nuclear density, $n_0$. There have been lots of development in constructing the equations of state for nuclear matter, which are model dependent but supposed to be consistent with the experiments up to $n_0^{[1]}$. However, the simple extrapolations of equations of states beyond $n_0$ lead to different and strongly model-dependent results for the macroscopic variables: chemical potentials of constituents, energy density, pressure, etc. And it becomes a very important subject to determine or predict EOS far beyond normal nuclear density.

The difference of stellar matter from the nuclear matter is that there are additional constraints for a stellar matter, which are the conditions of electrical neutrality and beta equilibrium. These constraints naturally lead to the emergence of leptons (electron and muon) as well as protons and the lepton chemical potential is expected to be increasing with the density. It opens new degrees of freedom for the stellar matter when the scale for the new degrees of freedom is comparable to the lepton chemical potential.

For an ideal noninteracting fermi gas of neutron and proton the lepton chemical potential is determined by the Fermi-Dirac statistics only and is known to be increasing slowly with the nucleon density. But the nucleon system is very strongly interacting and the isospin asymmetric nuclear interaction, which is known as symmetry energy of nuclear matter, cannot be simply ignored in the stellar matter. The symmetry energy is defined by the sum of the kinetic contribution and asymmetric nuclear interactions, which are required to have neutron-proton asymmetry. The density dependencies of the symmetry energy have been discussed in the various theoretical models $^{[1]} [2] [3]$ and also have been used for the phenomenological applications including heavy ion experiment $^{[4]}$ and astrophysics. Among the interesting issues, where the importance of symmetry energy in astrophysical phenomena has been well addressed, are the size of neutron stars $^{[5]} [6] [7] [8]$, the onset of kaon condensation $^{[9]} [10]$, the cooling of proto-neutron star $^{[11]}$, gravitational waves $^{[12]}$ and the instability of neutron stars $^{[13]}$.

In this work we focus on the density dependent change of relative abundances of the stellar matter with particular emphasis on role of the nuclear symmetry energy $^{[11]} [15]$, which we try to elaborate more transparently using the various models of symmetry energy. As noted above, as density is increasing, we are expecting the emergence of new degrees of freedom very essential in formulating the corresponding EOS, which is only possible when we know the abundances of the constituents. Also we could get a more transparent understanding of EOS in terms of its constituents.

The phenomenological models of the symmetry energy show similar density dependencies up to near the normal nuclear density but their density dependencies diverge quite
widely beyond the nuclear density from model to model (for a recent review see Ref. [1]). Therefore any simple extrapolation far beyond normal density should result in quite different conclusions, which might be the case for investigating stellar matter with higher density. In this work we calculate the density dependence of the abundances using a few selective models of nuclear symmetry energy, which have been used extensively to calculate neutron star EOS in detail [1][7][9][10][16]. We demonstrate how the lepton chemical potentials depend on the nuclear density and also how the results depend on models of the nuclear symmetry energy. It is worth noting that $\pi^-/\pi^+$ ratio in heavy ion collisions has been known to be sensitive on the symmetry energy. Recent studies by Xiao et al. [17] of FOPI data at SIS/GSI seem to show that the super-soft EOS that gives the $x=1$ curve in [17] is the only EOS so far available that fits the pion data of GSI. However the symmetry energy factor with $x=1$ drops to zero before reaching higher densities than $\sim 3n_0$, which on the other hand cover the relevant range of density in a stellar matter. Hence the beta equilibrium in stellar matter might not be expected nor the kaon condensation as discussed below.

When the density is increasing in stellar matter new degrees of freedom, excited baryons and mesons including hyperons, are supposed to emerge. Kaon is among the possible new degrees of freedoms after light leptons, when the medium dependent energy of kaon becomes comparable to the lepton chemical potential. Employing the simple formulae for the inverse propagator of kaon in medium [18], the threshold densities for the kaon condensation are calculated using the model-dependent symmetry energies. It is a very interesting issue since Bethe-Brown argument [19] for the maximum mass of neutron star, $M_{NS}^{\text{max}} \sim 1.5M_\odot$, depends on the onset of the kaon condensation at the density of $\sim 3n_0$.

In this work, the density dependence of the relative abundances of stellar constituents (neutron, proton, electron and muon) are calculated using several models of density dependent nuclear symmetry energy, since it is directly related to the relative abundances of constituents. For the possible implication of the kaon condensation, the threshold densities of kaon condensation are calculated by comparing the lepton chemical potentials with the zero of the kaon inverse propagator.

Let us start with a simple matter consisting of pure neutron gas. The temperature is taken to be zero throughout this work. The energy density and pressure of a simple free neutron gas [20] (We put $\hbar = c = 1$) are given by:

$$
\epsilon = \frac{8\pi}{(2\pi)^3} \int_0^{p_F} \left(p^2 + m_n^2\right)^{1/2} p^2 dp
$$

$$
P = \frac{8\pi}{3(2\pi)^3} \int_0^{p_F} \left(p^2 + m_n^2\right)^{-1/2} p^4 dp
$$

(1.1) (1.2)

where $p_F$ is the Fermi momentum defined by the neutron number density

$$
n = \frac{1}{3\pi^2 p_F^3}.
$$

(1.3)
The Fermi momentum in unit of normal nuclear density, \( n_0 = 0.16\text{fm}^{-3} \), and the kinetic energy (Fermi level) is given by
\[
p_F = 336\text{MeV} \left( \frac{n}{n_0} \right)^{1/3}, \quad E_{F}^{\text{kin}} = \frac{p_F^2}{2m_N} = 60\text{MeV} \left( \frac{n}{n_0} \right)^{2/3}.
\]
(1.4)
The chemical potential of neutron, \( \mu_n \), is the same as the Fermi energy, \( E_F \):
\[
\mu_n = E_F \equiv \left( p_F^2 + m_n^2 \right)^{1/2} = m \left( 1 + x^2 \right)^{1/2},
\]
where \( x \) is a dimensionless parameter defined by
\[
x = \frac{p_F}{m}.
\]
(1.5)
It is interesting to note that there is isospin symmetry in nature. A proton is the isospin partner of the neutron, which consists of an iso-doublet with almost the same mass, \( \sim 940\text{MeV} \). Hence if there is any channel for the neutron to be converted to proton, the energy of the system of neutron only can be lowered. The conversion depends on the dynamics, which is supposed to be the \( \beta \)-decay,
\[
n \rightarrow p + e + \bar{\nu}_e
\]
(1.7)
to reach \( \beta \)-equilibrium eventually for which
\[
\mu_n = \mu_p + \mu_e + \mu_\nu.
\]
(1.8)
Hereafter we will assume all neutrinos are emitted out of star, then we take the neutrino chemical potential as \( \mu_\nu = 0 \), \( \mu_n = \mu_p + \mu_e \).

If the energy only is concerned, then the minimum can be achieved when \( \mu_n = \mu_p \) without electron, \( \mu_e = 0 \). However, a stellar matter is believed to wind up as a charge neutral object. Practically during the period of reaching the \( \beta \) equilibrium, the stellar matter reacts to satisfy the neutrality condition,
\[
n_p = n_e \quad \text{or} \quad m_p x_p = m_e x_e,
\]
(1.9)
where \( x_p \) and \( x_e \) are the dimensionless Fermi momentum of proton and of electron, respectively. This implies the number of protons should be balanced by the number of electrons, which means the energy minimum can be reached with the constraint of neutrality for a system of free neutron, proton and electron (NPE gas) in a \( \beta \) equilibrium. It is one of the essential differences from nuclear matter, where the charge neutrality is not a physical constraint, for example, for heavy ions.

As the density increases such that the electron chemical potential is comparable to muon mass, \( m_\mu \), it is energetically favorable for the electrons to convert to muons as \( e \rightarrow \mu + \nu_e + \bar{\nu}_\mu \). Then the chemical equilibrium can be accomplished as
\[
\mu_n - \mu_p = \mu_e = \mu_\mu,
\]
(1.10)
Figure 1: (a) The relative abundances of particles and (b) $\mu_e$ vs. $u$. Solid line and dashed line respectively refer to NPE$\mu$ gas and NPE gas.

with the neutrality condition,

$$n_p = n_e + n_\mu. \quad (1.11)$$

At the muon threshold, the muon density is zero, $n_\mu = 0$ or $x_\mu = 0$ and $\mu_e^{th} = m_\mu = 106\text{MeV}$ and the Fermi momentum of electron at threshold is determined by the masses of muon and electron as

$$x_e|_{\mu-\text{thres}} = \left[ \left( \frac{m_\mu}{m_e} \right)^2 - 1 \right]^{1/2} = 207. \quad (1.12)$$

The nucleon density at threshold can be determined by Eq. (1.11), $n_p = n_e$,

$$n|_{\mu-\text{thres}} = 2.91n_0, \quad \frac{n_p}{n}|_{\mu-\text{thres}} = 0.011. \quad (1.13)$$

Beyond the muon threshold the constituents of the stellar matter becomes neutron, proton, electron and muon (NPE$\mu$ gas).

The relative abundances of neutron, proton, electron and muons are shown in Fig. (a). The relative neutron abundance is almost not changing but the relative abundances of proton are increasing substantially beyond the muon threshold as expected. The proton fraction, $n_p/n$, increases with density as well as for the electron, $n_e/n$. 


The electron chemical potential increases with density as shown in Fig. 1(b). In the NPE gas, the number of electrons is equal to that of protons by the charge neutrality condition, Eq. (1.9). Until the muon threshold density \( \simeq 3n_0 \), the increment of the electron chemical potential follows the property of the NPE gas. Then, beyond the threshold density, the electron chemical potential grows with density by the property of the NPE \( \mu \) gas. In the NPE \( \mu \) gas electron shares its number with muon such that the number of electron is reduced by the presence of muon.

2 Relative abundances with nuclear symmetry energy

So far, in calculating the chemical potentials, the nucleons are considered as free particles,
\[
\mu_n^{\text{free}} - \mu_p^{\text{free}} = m_n(1 + x_n^2)^{1/2} - m_p(1 + x_p^2)^{1/2},
\]
which can not be realistic due to the nuclear interactions.

It is useful to write the energy per particle in the nuclear matter [21] as
\[
E(n, N_p) \simeq m_N + \frac{3}{5}E_F^0 \left( \frac{n}{n_0} \right)^{2/3} + S(n)(1 - 2N_p)^2 + V(n),
\]
where the last term does not depend separately on neutron or proton number density but only on total number density. In the second term, \( E_F^0 = \frac{(3\pi^2n_0/2)^{2/3}}{2m_N} \) is the Fermi energy at, \( n = n_0/2 \). The third term is the symmetry energy for nuclear matter
\[
E_{\text{sym}} = S(n)(1 - 2N_p)^2,
\]
and the nuclear symmetry energy density,
\[
\epsilon_{\text{sym}} = nS(n)(1 - 2N_p)^2,
\]
where the symmetry energy factor, \( S(n) \), is a model dependent function of density [16] [21] [22] [23], which is manufactured to be consistent with nuclear matter data up to the nuclear matter density, \( n_0 \).

Now we can calculate the contributions of the symmetry energy density, \( \epsilon_{\text{sym}} \), to the chemical potentials of proton and neutron to get
\[
\mu_n - \mu_p = \mu_n^{\text{sym}} - \mu_p^{\text{sym}} = 4(1 - 2N_p)S(n).
\]
We can see that the larger \( S(n) \), the larger the chemical potential difference.

For an ideal free gas, the chemical potential difference, in the non-relativistic limit of Eq. (2.1) is given by [21]
\[
\mu_n^{\text{free}} - \mu_p^{\text{free}} = \frac{1}{2m_N} (3\pi^2n)^{2/3} \left[ (1 - N_p)^{2/3} - N_p^{2/3} \right],
\]
which can be approximated in the well known form\cite{17},

\[
\mu_p^{\text{free}} - \mu_p^{\text{free}} = 4 (1 - 2N_p) S_{\text{free}}(n),
\]

(2.7)

where

\[
S_{\text{free}}(n) = \left(2^{2/3} - 1\right) \frac{3}{5} E_0^* \left(\frac{n}{n_0}\right)^{2/3}.
\]

(2.8)

There are a number of parameterizations for the symmetry energy factor, \(S(n)\), which take account of the free kinetic contribution as well as the potential energy contribution. One of the parametrization used in Refs. \cite{16} is

\[
S_F(n) = \left(2^{2/3} - 1\right) \frac{3}{5} E_0^* \left(\frac{n}{n_0}\right)^{2/3} - F(n) + S_0 F(n),
\]

(2.9)

where \(F(n)\) is a parameter for the potential contribution to the symmetry energy, which satisfies \(F(0) = 0\) and \(F(n_0) = 1\). Hereafter we set \(F(n)\) simply as \(F(n/n_0)\). In the last term of Eq. (2.9), \(S_0\) denotes the bulk symmetry energy parameter, \(S_0 \simeq 30\text{MeV}\).

One of the different forms of \(S(n)\) suggested in\cite{22} is

\[
S_\alpha = \left(2^{2/3} - 1\right) \frac{3}{5} E_0^* \left(\frac{n}{n_0}\right)^{2/3} + A(\alpha) \frac{n}{n_0} + [18.6 - A(\alpha)] \left(\frac{n}{n_0}\right)^{B(\alpha)},
\]

(2.10)

where \(\alpha\) is a free parameter which determines the bulk property of nuclear matter. In this work, we take \(\alpha = 1\), which reproduces \(\pi^+ / \pi^-\) ratio in heavy ion collision\cite{17}.

\[
A(\alpha = 1) \simeq 107\text{MeV} \text{ and } B(\alpha = 1) \simeq 1.25
\]

(2.11)

(for other parameter sets, see Ref. \cite{22}). In Eq. (2.10), the first term denotes the kinetic contribution and last terms refer to the potential energy contribution of nuclear matter as in \(S_F\).

There is also a different type of symmetry energy factor, in which different parametrization scheme is adopted as\cite{23},

\[
S_3(n) \simeq S_0^* + L \rho + \frac{1}{2} K \rho^2,
\]

(2.12)

where

\[
\rho = \frac{n - n^*}{3n^*},
\]

(2.13)

with \(n^* = 0.148\text{fm}^{-3}\) and \(S_3\) represents either \(S_{\text{FSU}}\) or \(S_{\text{NL3}}\) in Ref. \cite{23}. The bulk parameters, \(S_0^*, L\) and \(K\) in Eq. (2.12) consistent with the nuclear matter are listed in Table 1.

For comparison, we plot \(S_F\), \(S_{\alpha = 1}\), \(S_{\text{FSU}}\) and \(S_{\text{NL3}}\) in Fig. 2. As one can see these symmetry energy factors are not much different from each other up to \(n_0\). But beyond \(n_0\),
| Model | $S_0^*$ | $L$ | $K$  |
|-------|---------|-----|------|
| FSU   | 32.59   | 60.5| −51.3|
| NL3   | 37.29   | 118.2| 100.9|

Table 1: Bulk parameters for the symmetry energy factor in Ref. [23].

![Graph showing density dependencies of $S(n)$ for different models.](image)

Figure 2: The density dependencies of $S(n)$’s for different models.

the difference becomes significant from model to model as shown in Fig. 2. In general the symmetry energy factors are increasing with the density up to $n \sim 5n_0$. However $S_{\alpha=1}$ has peak near $n \approx n_0$ and drops afterwards to zero at $n \sim 3n_0$.

The electron chemical potential and its density dependence in $\beta$-equilibrium can be calculated using Eqs. (1.10) and (2.5) for different symmetry energy factors, as shown in Fig. 3. The electron chemical potential for the free neutron gas is lower than with other symmetry energy factors, except $S_{\alpha}$, beyond $n \approx 2n_0$. This is the main reason why it is difficult to excite new degrees of freedom in the lower density for the case of the free nucleon gas. For $S_{\alpha=1}$, the density dependence of the electron chemical potential is quite different such that it drops to zero $n \approx 3n_0$. It implies that $\beta$-equilibrium may not be reached for $n \gtrsim 3n_0$ with the symmetry energy factor of $S_{\alpha=1}$.

When the electron chemical potential increases such that it becomes comparable to the rest mass of muon, muons become constituents of stellar matter in addition to neutrons, protons and electrons. At the muon threshold with $n_\mu = 0$ the electron chemical potential is just the mass of muon as discussed in previous section. Then with given proton and electron densities, we can determine the nucleon density, $n_{\mu-thres}$, at muon threshold from
Figure 3: The electron chemical potentials, $\mu_e$ due to different models for sNPE gas.

| $n_{\mu\text{-thres}}$ | Free $\ S_F$ | $S_{FSU}$ | $S_{NL3}$ | $S_{\alpha=1}$ |
|------------------------|--------------|-----------|-----------|----------------|
|                        | 2.91         | 0.94      | 0.76      | 0.73           | 0.73          |

Table 2: The muon threshold densities obtained using different models for $S(n)$, in unit of $n_0$.

The chemical equilibrium condition, given by

$$4(1-2N_p)S(n) = m_e \left(1 + x_e^2 \right)^{1/2}.$$  \hspace{1cm} (2.14)

In the previous section, we get $n_{\mu\text{-thres}} = 2.91 n_0$ in Eq. (1.13) for free nuclear matter. With the symmetry energy factor, $S_F(n)$, Eq. (2.14) gives

$$n|_{\mu\text{-thres}} = 0.94 n_0,$$  \hspace{1cm} (2.15)

which is much lower than for the free case. Threshold densities for other symmetry energy factors can be easily guessed in Fig. 3 and the corresponding threshold densities are calculated using Eq. (2.14) in Table 2. One can note that the muon threshold density becomes about three to four times lower than for the free nuclear matter.

At higher density beyond the muon threshold, the stellar matter consists of neutron, proton, electron and muon(sNPE$\mu$ gas). And the chemical equilibrium condition and charge neutrality condition become

$$4(1-2N_p)S(n) = m_e (1 + x_e^2)^{1/2} = m_\mu (1 + x_\mu^2)^{1/2},$$  \hspace{1cm} (2.16)

$$n_p = n_e + n_\mu.$$  \hspace{1cm} (2.17)
Effectively these are three equations to be solved for four unknowns, $x_e, x_\mu, x_n$ and $x_p$. Then for a given electron chemical potential, $x_e$, one can solve these equations completely to determine the chemical potentials and the relative abundances of the constituents, $n_e, n_\mu, n_n$, and $n_p$. The electron chemical potentials (same as the muon chemical potentials in beta equilibrium) beyond the muon threshold densities (see Table 2) are calculated for different models of the nuclear symmetry energy as plotted in Fig. 4. The density dependence of electron chemical potential for $S_{FSU}$ shows a quite different behavior from others. One can observe in Fig. 5 that the proton and muon fractions are increasing with the density much faster than in free case. For $S_{FSU}$, the fractions of proton and muon are increasing with the density up to $\sim 4n_0$ but decreasing and even becomes lower than free case for the higher density. The $S_\alpha$ case shows a similar behavior with $S_{FSU}$, but the lepton and proton fractions are decreasing more quickly than $S_{FSU}$ and the muon and proton fractions, beyond the muon threshold density, are much lower than in other models.

When the leptonic chemical potentials increase high enough, it is natural to ask what kind of new degrees of freedom can be driven to evolve in a stellar matter. They could be excited baryons including hyperons as well as strange mesons. One of the interesting possibilities is the s-wave kaon condensation \cite{25} driven by the electron chemical potential. The simplest way of demonstrating the possibility of kaon condensation is by comparing the electron chemical potential with the kaon chemical potential at the threshold, which determines the kaon threshold density. At the threshold the kaon chemical potential is given by the zero of the inverse propagator, $D_{K^{-}}^{-1}$, with zero momentum \cite{18}, which is given by

![Figure 4: The electron chemical potentials for sNPE$\mu$ gases with different nuclear symmetry energy models.](image-url)
Figure 5: The relative abundances of constituent particles for sNPEµ gases with different nuclear symmetry energies.

\[ D_{K^-}^{-1} = \frac{\omega_K^2 - m_K^2}{f^2} + \frac{1}{f^2}(n_n/2 + n_p)\omega_K + \frac{\Sigma_{KN}}{f^2} n, \]  

where \( f \) is the pion decay constant, \( f = 93\text{MeV} \). The possible range of \( \Sigma_{KN} \) is estimated to be

\[ \Sigma_{KN} = 200 - 400\text{MeV}. \]  

As one of the example, we take \( \Sigma_{KN} = 400\text{MeV} \) in this work. It is found that up to \( n \sim 5n_0 \) the dependence of \( \omega_K(n) \) on the symmetry energy models is not so significant and the kaon threshold densities are estimated around \( n \sim 3n_0 \) as shown in Table 3 and in Fig. 6. The threshold density is lower \( \sim 2.5n_0 \) with \( S_{NL3} \) but higher for free case, \( \sim 3.5n_0 \). One should note that \( \omega_K \) obtained using Eq. (2.18) is found to be always larger than electron chemical potential such that one cannot expect a kaon threshold if \( S_{\alpha=1} \) is valid up to \( n \sim 3n_0 \).
Figure 6: The electron chemical potentials and kaon chemical potentials for different nuclear symmetry energy models.

3 Discussion

In this work we consider the effect of the nuclear symmetry energy on the relative abundances of particles, neutron, proton, electron and muon, with the nucleon density inside a stellar matter supposed to be electrically neutral and in beta equilibrium. We determine the electron and muon threshold densities. The muon threshold density reduces substantially with symmetry energy from that of free nucleon gas. It is observed that the relative abundances with symmetry energy diverge significantly from model to model for higher density $n > n_0$. We also estimate the kaon condensation threshold densities using a number of different models for the symmetry energy. The kaon threshold estimated with symmetry

|        | Free | $S_F$ | $S_{FSU}$ | $S_{NL3}$ |
|--------|------|-------|-----------|-----------|
| $n [n_0]$ | 3.53 | 2.76  | 2.89      | 2.45      |
| $\mu$ [MeV] | 120.4 | 206.3 | 192.5      | 234.5     |

Table 3: The kaon threshold densities estimated using different models for $S(n)$ and the electron chemical potentials at the threshold densities.
energy considered is found to be lower than the free case. However, for the super-soft EOS, in which the symmetry energy factor \( S_{\alpha=1} \) drops to zero before reaching higher density than \( \sim 3n_0 \), beta equilibrium in stellar matter cannot be expected. Moreover \( \omega_K \) obtained using Eq. (2.18) is found to be always larger than electron chemical potential such that one can not find kaon threshold for \( S_{\alpha=1} \). This is an intriguing consequence of our finding that contrary to the accepted lore, kaon condensation in neutron-star matter which is considered to be the first phase transition as density increase beyond \( n_0 \) and hence plays a crucial role in certain scenarios of compact-star formation is indifferent to the softness/stiffness of EOS beyond \( n_0 \).

We can notice that the presently known models for the symmetry energy factor provide quite different predictions for higher densities when the parametrization is straightforwardly extrapolated to higher densities beyond \( \sim 3n_0 \), which are however very important deep inside the stellar system. Hence the predictions on the stellar matter structure after integrating the Tolman-Oppenheimer-Volkov equation should depend strongly on the specific form of symmetry energy\[6\][7][10][29]. Hence it is very important to find the symmetry energy with correct density dependency for a stellar matter and we are looking forward to get more experimental information on symmetry energy from the forthcoming experiment like FAIR/GSI.

One of the interesting ideas developed recently in hadron physics is adopting the hidden local symmetry\[30\], which opens up an idea of hadronic freedom\[31\]. However it is not well investigated how this idea can be employed in understanding the symmetry energy at higher density and its relation to the softness/stiffness of EOS. Whether it simply converges to the free case for higher density than \( \sim 3n_0 \) or it is quite different matter from the free fermi gas and how the symmetry energy is related to the KN interaction which controls the kaon condensation at high density nuclear matter are among the very intriguing questions to be answered. Since it is the region so far unexplored by the experiment, it is very interesting to construct a form of symmetry energy compatible with the new idea of dense hadronic matter to investigate the implications on the stellar structure using TOV equation, which will be discussed in a separate work.

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