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Mathematical modeling of the spread of COVID-19 among different age groups in Morocco: Optimal control approach for intervention strategies

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\textbf{A B S T R A C T}

In this article, we study the transmission of COVID-19 in the human population, notably between potential people and infected people of all age groups. Our objective is to reduce the number of infected people, in addition to increasing the number of individuals who recovered from the virus and are protected. We propose a mathematical model with control strategies using two variables of controls that represent respectively, the treatment of patients infected with COVID-19 by subjecting them to quarantine within hospitals and special places and using masks to cover the sensitive body parts. Pontryagin’s Maximum principle is used to characterize the optimal controls and the optimality system is solved by an iterative method. Finally, numerical simulations are presented with controls and without controls. Our results indicate that the implementation of the strategy that combines all the control variables adopted by the World Health Organization (WHO), produces excellent results similar to those achieved on the ground in Morocco.

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1. Introduction

An epidemic caused by a new coronavirus has been reported in Wuhan, the capital of China’s Hubei province, in early December 2019. It rapidly spread to other provinces in China and to the rest of the world. Registered cases continue to increase rapidly in early 2020, with a total of 7,145,539 COVID-19 cases reported worldwide, including 408,025 deaths according to the World Health Organization, report published on 10 June 2020 \cite{1}. On March 17, 2020, the World Health Organisation (WHO) officially declared COVID-19 a pandemic \cite{3}. Coronavirus is an infectious virus, which can be transmitted from human to human. The virus can be transmitted from an infected person to other people, through direct contact with handshakes and by touching surfaces contaminated with the disease, and then it affects parts of the body such as the eyes, nose and mouth. The virus is likely to cause more serious respiratory diseases, such as pneumonia or bronchitis. Coronavirus is thought to be more dangerous than SARS because it takes longer for the symptoms to appear. There is no specific treatment for COVID-19, but research is still underway. However, there are recommendations like social distancing for infection prevention, and temporary control of patients infected with COVID-19 in hospitals and quarantine areas \cite{5}. The phenomenon of coronavirus spread varies from age group to age group. This difference is clear between these categories where we note that the group between the ages of 25 and 65 is most vulnerable to Coronavirus infection compared to the rest of the age groups. This age group is as active as it uses the means of transport to go to the workplaces and go to the markets and all those places mentioned the places of spread of the virus with distinction. This group is also the most important factor in the transmission of the virus to the rest of the age group (under 25 years of age and over 65 years of age) because they are living with them in the same family setting. As for the age group over 65 years, they are considered the most affected by the spread of the virus as they are low immunity and suffers from other chronic diseases. According to available statistics from the Moroccan Ministry of Health,

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the age group under the age of 25 is least affected by the spread of the virus [2].

There are several mathematical modelling studies have been
developed to simulate, analyse and understand the dynamics of the Coronavirus [6–11]. For example, Xia et al. [6] analysed the Transmission of Middle East Respirator Syndrome Corona Virus, in the Republic of Korea. Khajii et al. [7] study in this work a discrete mathematical model that describes, the dynamics of transmission of the Coronavirus between humans, on the one hand, and animals, on the other hand, in a region or in different regions. Kim et al. [9] formulated a mathematical model for MERS transmission dynamics and estimating transmission rates. They estimated the basic reproduction number using the estimates of the transmission rates in the first two periods. Tahir et al. [10] used a non-linear mathematical model to study the dynamics of the transmission of MERS-CoV in human population through an agent known as camel. Drosten et al. [11] provided a description the fatal case of MERS-CoV infection and associated phylogenetic analyses.

Generally use compartment model to describe the spread of infectious disease (susceptible, infected, or removed). In 1927, Azhar et al. [12] were the first researchers on mathematical epidemiology to suggest the susceptible-infected-removed (SIR) model that describes the rapid explosion of an infectious disease for a short time. After him, the research continued on the same subject, for example, in 2009, Yu et al. [13] studied a global stability of two-group SIR model with random perturbation. In 2010, Pathak et al. [14] analysed the right dynamics of a SIR epidemic model. In 2012, Ji et al. [15] studied the behaviour of a SIR epidemic model with stochastic perturbation. In 2016, Laaroussi et al. [16] a SIR spatiotemporal epidemic model is formulated as a system of parabolic partial differential equations with no-flux boundary conditions. In this study, we divided infected people with Covid-19 disease into three types of infection because the patients go through steps, which is the step of infection, the step of the symptoms and the step of the complications. The patient can move on to the recovery step immediately, after one of the previous steps. In this work, we propose a mathematical model that describes the dynamics of people who have the COVID-19. Also, we propose an optimal strategy for the treatment of patients infected with COVID-19, by sub- jecting them to quarantine within hospitals and special places for that and using masks to cover the sensitive body parts.

In our model the population is divided into three types of age groups. \((j = 1)\) the first age group from 0 to 25 years, \((j = 2)\) the second group from 25 to 65 years and \((j = 3)\) the third age group over 65 years of age, and all age groups are divided into five compartments. The susceptible individuals \((S^j)\), the infected population without symptoms \((I^j_0)\), the infected population with symptoms \((I^j)\), the infected population with complications \((C^j)\) and the recovered individuals \((R^j)\). In order to decrease the number of infected population, we applied the theory of optimal control for our proposed model. The theory of optimal control, and the analysis of dynamic systems are fielding current research, that continues to arouse the interest of scientists. The aim of this theory is to model processes that evolve over time and to study their behaviours. This study makes it possible, among other things, to predict the behaviour of the system and to control it in order to get the desired results. In the theory the control of dynamic systems, there are two kinds of mathematical dynamic systems: the discrete-time models described by difference equations (see [17–20]) and the continuous-time models described by differential equations. The continuous-time models have been widely investigated in many articles (for example, [21–25]).

The paper is organized as follows. In Section 2, we explained the Statistical data on the epidemiological situation of COVID-19 on Moroccan territory. In Section 3, we present our mathematical model that describes the dynamics of people who have the COVID-19 and we give some results concerning the positivity, the boundedness and existence of solutions. In Section 4, we present the optimal control problem for the proposed model. In Section 5, we give some results concerning the existence of the optimal controls and the characterization of these optimal controls using Pontryagin’s Maximum Principle. Numerical simulations through Matlab software are given in Section 6. Finally, we conclude the paper in Section 7.

2. Statistical data on the epidemiological situation of COVID-19 on Moroccan territory

General trend

Since the announcement of the discovery the first case of coronavirus on 2 march 2020, for a person came from the Italian homeland, until 2 June 2020, the total number of infections has reached 7866 cases, including 1250 cases still under treatment and 6410 cases completely cured of the disease and 206 cases died [1,2,4].

An upward trend is noted with an average daily increase of 3% since 04/29/2020 (see Fig. 1) [2]. This situation is mainly due to the detection of the family and (especially) professional environment.

[Fig. 1. Daily New Cases in Morocco until 2 June 2020.]
Characteristics of person by Age

All age groups are affected. About 9.02% of cases were over the age of 65, while children under the age of 15 accounted for only 9.88% and about 67.77% of cases where the aged between 25 and 65 (see Fig. 2) [2].

Over time, patients were rejuvenated. The average age rose from 50.15 years (+/−18.97) for cases detected in March, at 37.25 years of age (+/−18.03) in April. The trend average age of active cases is shown in Fig. 3: it rises from 55 years to the week of 12 (from 16/03/2020), and at 34.5 years, at week 19 (04/05/2020) (see Fig. 3) [2].

Clinical picture on admission of cases

At admission, 84.50% of COVID-19 cases were asymptomatic or had mild clinical symptoms. 12.66% had moderate clinical symptoms, while 2.84% were admitted in a critical condition. In total, nearly 4.70% of hospitalizations required for admission to resuscitation. More than 90% of these admissions are over the age of 40 (Fig. 4) [2].

3. A mathematical model of COVID-19

3.1. Description of the model

We consider a mathematical model $\mathcal{S}I\mathcal{I}_wICRI$ that describes the dynamics of a population having COVID-19 disease. Within a certain age group taking into account the fact that all age groups can transmit the infection to all other groups taking into account the specificity of each age group. The following illustration will illustrate COVID-19 disease in the compartments. These trends will be represented by vector arrows in Fig. 5.

The susceptible people subjected to COVID-19

'S' is referring to people who are likely to have (COVID-19) disease. This compartment is increased by the recruitment rate denoted $\Lambda_1^j$. It is decreased by a natural mortality rate $\mu_j$. Also it is decreased by an effective contact with $I_w$ at rate $\beta_1^j$ (the rate of patients who become infected people with COVID-19 due to contact with the infected people who do not show symptoms) and with 'I' at rate $\beta_2^j$ (the rate of patients who become infected with COVID-19 due to contact with the infected people with symptoms).

The people infected without symptoms

The compartment '$I_w$' refers infected people with COVID-19 without symptoms. It is increased by the incidence rate of immigrants and carriers of the disease without symptoms denoted $\Lambda_2^j$, and also this compartment is increased by $\beta_1^j$ and $\beta_2^j$.

The compartment '$I_w$' decreased by natural mortality rate $\mu_j$ and by $\alpha_j$ which represent a rate of the infected people without symptoms. Also it is decreased by $\gamma_j^1$ (the rate of the infected people without symptoms and who become recovered.

The infected people with symptoms

The compartment 'I' refers infected people with symptoms with COVID-19 disease. It is increased by the incidence rate of immigrants and carriers people infected with symptoms denoted $\Lambda_3^j$. Also it is increased by $\alpha_j^1$. This compartment of 'I' is decreased by natural mortality rate $\mu_j$ and $\alpha_j^2$ that represent the rate of the
people infected with symptoms who have become infected with complications. Also it is decreased by $\gamma_j^I$ that represent the rate of the people infected with symptoms who have become the recovered individual population.

**The people infected with complications**

The compartment $C^j$ refers to people infected with complications with COVID-19 disease. It is increased by $\alpha_j^I$. The compartment $C^j$ is decreased by natural mortality rate $\mu_j$ and mortality rate due to COVID-19 disease denoted $\delta_j$. Also it is decreased by the rate of the people infected with complications who have become the recovered individual population denoted $\gamma_j^I$.

**The recovered individuals**

The compartment $R^j$ refers to recovered individuals. It is increasing by $\gamma_j^I$, $\gamma_j^R$ and $\gamma_j^I$ and decreasing by natural mortality rate $\mu_j$.

### 3.2. Model equations

By adding the rates at which the steps of (COVID-19) disease enters the compartment and also by subtracting the rates at which people leave compartment, we obtain a differential equations for the rate at which patients change in each compartment during separate times. Therefore, we present the (COVID-19) disease model with the following system of differential equations:

\[
\begin{align*}
\frac{dS^j(t)}{dt} &= \Lambda_j - \mu_j S^j(t) - S^j(t) \sum_{i=1}^{3} \left( \frac{\beta_i^I S^i(t)}{N_i} + \frac{\beta_i^F I^i(t)}{N_i} \right) \\
\frac{dI^j(t)}{dt} &= \Lambda_j + (\mu_j + \gamma_j^I + \gamma_j^R) I^j(t) + \alpha_j^I I^j(t) - (\mu_j + \delta_j^I + \lambda_j^I) I^j(t) \\
\frac{dC^j(t)}{dt} &= \alpha_j^I I^j(t) - (\mu_j + \delta_j^I + \lambda_j^I) C^j(t) \\
\frac{dR^j(t)}{dt} &= \gamma_j^I C^j(t) + \gamma_j^R I^j(t) - \mu_j R^j(t)
\end{align*}
\]

where $S^j(t) \geq 0$, $I^j(t) \geq 0$, $C^j(t) \geq 0$, $R^j(t) \geq 0$ are given initial states and $N^j = S^j + I^j + C^j + R^j$ for all $j \in \{1, 2, 3\}$.

### 3.3. Model basic properties

#### 3.3.1. Positivity of solutions

**Theorem 1.** For all $j \in \{1, 2, 3\}$ if $S^j(0) \geq 0$, $I^j(0) \geq 0$, $C^j(0) \geq 0$, and $R^j(0) \geq 0$ then solutions $S^j(t)$, $I^j(t)$, $C^j(t)$, and $R^j(t)$ of system (1) are positive of all $t \geq 0$.

**Proof.**

\[
\begin{align*}
\frac{dS^j(t)}{dt} &= \Lambda_j - \mu_j S^j(t) - S^j(t) \sum_{i=1}^{3} \left( \frac{\beta_i^I S^i(t)}{N_i} + \frac{\beta_i^F I^i(t)}{N_i} \right) \\
\Rightarrow \frac{dS^j(t)}{dt} &\geq -\mu_j S^j(t) - S^j(t) \sum_{i=1}^{3} \left( \frac{\beta_i^I S^i(t)}{N_i} + \frac{\beta_i^F I^i(t)}{N_i} \right)
\end{align*}
\]

Then

\[
\frac{dS^j(t)}{dt} + F^j(t) S^j(t) \geq 0
\]

where

\[
F^j(t) = \mu_j + \sum_{i=1}^{3} \left( \frac{\beta_i^I S^i(t)}{N_i} + \frac{\beta_i^F I^i(t)}{N_i} \right)
\]

The both sides in the last inequality are multiplied by $\exp \left( \int_0^t F^j(s) \text{d}s \right)$. We obtain

\[
\exp \left( \int_0^t F^j(s) \text{d}s \right) \frac{dS^j(t)}{dt} + F^j(t) \exp \left( \int_0^t F^j(s) \text{d}s \right) S^j(t) \geq 0
\]

then

\[
\frac{d(S^j(t) \exp \left( \int_0^t F^j(s) \text{d}s \right))}{dt} \geq 0
\]

integrating this inequality from 0 to $t$ gives

\[
S^j(t) \geq S^j(0) \exp \left( - \int_0^t \left( \mu_j + \sum_{i=1}^{3} \left( \frac{\beta_i^I S^i(t)}{N_i} + \frac{\beta_i^F I^i(t)}{N_i} \right) \right) \text{d}s \right)
\]

so $S^j(t) \geq 0$ for all $j \in \{1, 2, 3\}$.

To show the positivity of $I^j(t)$ and $I^j(t)$ for all $k, j \in \{1, 2, 3\}$ we consider

\[
I^j_n(t) = \min_{j=1,2,3} \left\{ I^j_n(t) \right\}
\]

and

\[
I^j_n(t) = \min_{j=1,2,3} \left\{ I^j_n(t) \right\}
\]

we have two cases the first case

If $P^j(t) \geq P^j(t)$ then $\frac{dP^j(t)}{dt} + F^j(t) I^j_n(t) \geq 0$ where

\[
F^j(t) = \left( \mu_j + \alpha_j^I + \gamma_j^I - S^j(t) \sum_{i=1}^{3} \frac{\beta_i^I}{N_i} \right)
\]

So,

\[
I^j_n(t) = I^j_n(t) \exp \left( - \int_0^t \left( \mu_j + \alpha_j^I + \gamma_j^I - S^j(t) \sum_{i=1}^{3} \frac{\beta_i^I}{N_i} \right) \text{d}s \right) \geq 0
\]

the second case

If $P^j_n(t) \geq P^j(t)$ then $\frac{dP^j(t)}{dt} + F^j(t) P^j_n(t) \geq 0$ where

\[
F^j(t) = \left( \mu_j + \alpha_j^I + \gamma_j^I - \alpha_j^I \right)
\]

So,

\[
I^j_n(t) = I^j_n(t) \exp \left( - \int_0^t \left( \mu_j + \alpha_j^I + \gamma_j^I - \alpha_j^I \right) \text{d}s \right) \geq 0
\]

We conclude that $P^j_n(t) \geq 0$ and $I^j_n(t) \geq 0$.

Therefore we show the positivity of $I^j(t)$ and $I^j(t)$ for all $k, j \in \{1, 2, 3\}$.

Similarly we can prove that

\[
C^j(t) \geq C^j(0) \exp \left( - \int_0^t (\mu_j + \delta_j^I + \lambda_j^I) \text{d}s \right) \geq 0
\]

and

\[
R^j(t) \geq R^j(0) \exp \left( - \int_0^t \mu_j \text{d}s \right) \geq 0
\]

for all $j \in \{1, 2, 3\}$. □

#### 3.3.2. Boundedness of solutions

**Theorem 2.** The set

\[
\Omega = \{(S^j, I^j, C^j, R^j) \in \mathbb{R}_+^5; 0 \leq S^j + I^j + C^j + R^j \leq \frac{\Lambda_j}{\mu_j}\}
\]

is positive invariants.

Positivity invariant under system (1) with initial condition $S^j(0) \geq 0$, $I^j(0) \geq 0$, $C^j(0) \geq 0$, and $R^j(0) \geq 0$.

**Proof.** By adding all equations in system (1), one has

\[
\frac{dN^j}{dt} = \Lambda_j - \mu_j N^j - \delta_j^I C^j \leq \Lambda_j - \mu_j N^j
\]

thus

\[
N^j(t) \leq N^j \exp \left( -\mu_j t \right) + \frac{\Lambda_j}{\mu_j}
\]
and
\[ 0 \leq \limsup_{t \to +\infty} N_i(t) \leq \frac{\Lambda_i}{\mu_i} \]
where \( \Lambda_i = \Lambda_i^1 + \Lambda_i^2 + \Lambda_i^3 \) and \( N_0^i = S^i(0) + I^i_w(0) + I^i(0) + C^i(0) + R^i(0) \).

Then all possible solutions of the system (1) enter the set \( \Omega^j \). It implies that \( \Omega^j \) is a positively invariant set for the system (1). □

**Proof.** We have \( \Omega^j \) is a positively invariant set for all \( j \in \{1, 2, 3\} \).

3.3.3. Existence of solutions

**Theorem 3.** The system (1) with the initial condition
\[ S^i(0) \geq 0, I^i_w(0) \geq 0, I^i(0) \geq 0, C^i(0) \geq 0, R^i(0) \geq 0 \quad \text{for all } j \in \{1, 2, 3\} \]
has a unique solution.

**Proof.** Let
\[ X = X^1 \begin{pmatrix} X^2 \\ X^3 \end{pmatrix} \]
where
\[ X^j = \begin{pmatrix} S^j(t) \\ I^i_w(t) \\ I^i(t) \\ C^j(t) \\ R^j(t) \end{pmatrix} \]
So, the system (1) can be rewritten in the following form:
\[ \varphi(X) = AX + B(X) \]
where
\[ A = \begin{pmatrix} A^1 & 0 & 0 \\ 0 & A^2 & 0 \\ 0 & 0 & A^3 \end{pmatrix} \]
with
\[ A^j = \begin{pmatrix} -\mu^j & 0 & 0 & 0 \\ 0 & -(\mu^j + \alpha^j_1 + \gamma^j_1) & 0 & 0 \\ 0 & \alpha^j_2 & -(\mu^j + \alpha^j_2 + \gamma^j_2) & 0 \\ 0 & 0 & \alpha^j_2 & -(\mu^j + \delta^j + \gamma^j_3) \\ 0 & 0 & 0 & -\mu^j \end{pmatrix} \]
and
\[ B(X) = \begin{pmatrix} B(X^1) \\ B(X^2) \\ B(X^3) \end{pmatrix} \]
with
\[ B(X^j) = \begin{pmatrix} \Lambda^j_i - S^j(t) \sum_{i=1}^3 \left( \frac{\beta^j_{w1}(t)}{N^j_i} + \frac{\beta^j_{w2}(t)}{N^j_i} \right) \\ \Lambda^j_i + S^j(t) \sum_{i=1}^3 \left( \frac{\beta^j_{w1}(t)}{N^j_i} + \frac{\beta^j_{w2}(t)}{N^j_i} \right) \\ \frac{\Lambda^j_i}{\mu^j} \\ 0 \\ 0 \end{pmatrix} \]
for \( j = 1, 2, 3 \). □

The function \( B(X^j) \) satisfies:
\[ |B(X^j_1) - B(X^j_2)| = 2 \left| S^j(t) \sum_{i=1}^3 \left( \frac{\beta^j_{w1}(t)}{N^j_i} + \frac{\beta^j_{w2}(t)}{N^j_i} \right) \right| - S^j(t) \sum_{i=1}^3 \left( \frac{\beta^j_{w1}(t)}{N^j_i} + \frac{\beta^j_{w2}(t)}{N^j_i} \right) \]

Thus is follows that the function \( \varphi \) is uniformly Lipschitz continuous, and the restriction on
\[ S^i(0) \geq 0, I^i_w(0) \geq 0, I^i(0) \geq 0, C^i(0) \geq 0, \quad \text{for all } j \in \{1, 2, 3\} \]
we conclude that a solution of the system exists [26].

4. The optimal control problem

So far, there is no treatment or vaccination for COVID-19. For this reason, scientists insist on two strategies for combating this disease and to reduce the risk of infection with this virus. First, avoiding exposure to this virus by following a prevention protocol: covering the mouth and nose, washing the hands with water and soap frequently, cleaning and disinfecting surfaces, objects and goods. Second, putting people in quarantine areas and creating special protection programs for them especially those suffering from immunodeficiency to reduce the risk of infection.
Our objective in this proposed control strategy is to minimize
the number of the infected people without symptoms ($I_0^j$); the
infectious people with symptoms ($I^j$) and the Infected people
with complications ($C^j$).

So, in the model (1), we include control $u^j$ which represents
the treatment of patients infected with COVID-19, by subjecting them
to quarantine within hospitals and special places for that. The
control $v^j$ which represents state efforts to encourage people to use
masks to cover sensitive parts of the body. Thus, the controlled
mathematical system is given by the following system of differential
equations:

$$
\begin{align*}
\frac{dS^j(t)}{dt} &= \Lambda^j - \mu \cdot S^j(t) - S^j(t)(1 - v^j(t)) \sum_{i=1}^{3} \left( \frac{\beta_{1}I_0^j(t)}{N^j} + \frac{\beta_{2}I^j(t)}{N^j} \right) \\
\frac{dI_0^j(t)}{dt} &= \Lambda_0^j - (\mu^j + \alpha^j + \gamma^j + u^j(t)) \cdot I_0^j(t) \\
&+ S^j(t)(1 - v^j(t)) \sum_{i=1}^{3} \left( \frac{\beta_{1}I_0^j(t)}{N^j} + \frac{\beta_{2}I^j(t)}{N^j} \right) \\
\frac{dI^j(t)}{dt} &= \Lambda^j - (\mu^j + \alpha^j + \gamma^j + u^j(t)) \cdot I^j(t) + \alpha^jI_0^j(t) \\
\frac{dC^j(t)}{dt} &= \alpha^jI^j(t) - (\mu^j + \delta^j + \gamma^j + u^j(t)) \cdot C^j(t) \\
\frac{dS^j(t)}{dt} &= \gamma^jC^j(t) + \gamma^jI_0^j(t) + \gamma^jI^j(t) - \mu \cdot R^j(t) + (I_0^j(t) + I^j(t) + C^j(t) \\
&+ C^j(t))u^j(t)
\end{align*}
$$

(2)

where $S^j(0) \geq 0, I_0^j(0) \geq 0, I^j(0) \geq 0, C^j(0) \geq 0,$ and $R^j(0) \geq 0$ for all $j \in \{1, 2, 3\}$ are given initial states.

Then, the objective is to compare the costs of these interventions
and their effectiveness in the fight against COVID-19. To do this,
we need to investigate the optimal level of efforts that would be
needed to control the disease. For this, we use the objective function :

$$
J(u^j, v^j) = I_0^j(T) + I^j(T) + C^j(T) + \int_0^T \left( I_0^j(t) + I^j(t) + C^j(t) \right) \\
+ \frac{E^j}{2} (u^j(t))^2 + \frac{E^j}{2} (v^j(t))^2) dt
$$

(3)

where the parameters $E^j > 0$ and $F^j > 0$, for all $j \in \{1, 2, 3\}$ are
the cost coefficients at time $t$. $T$ is the final time. In other words, we
seek the optimal controls $u^j$ and $v^j$ such that :

$$
J(u^j, v^j) = \min_{(u^j, v^j) \in U_{ad}} J(u^j, v^j)
$$

(4)

where $U_{ad}$ is the set of admissible control defined by $U_{ad} = 
\{(u^j, v^j) : 0 \leq u^j(t) \leq 1; 0 \leq v^j(t) \leq 1, t \in [0, T]\}$

5. The optimal control existence and characterization

We first show the existence of solutions of the system (2). After
that, we will prove the existence of the optimal control [27].

5.1. Existence of an optimal control

**Theorem 4.** Subject to the controls system (2) with initial conditions.

There exist the optimal controls $u^j$ and $v^j$ such that

$$
J(u^j, v^j) = \min_{(u^j, v^j) \in U_{ad}} J(u^j, v^j)
$$

if the following conditions are met:

1. The set of controls and corresponding state variables is nonempty.
2. The control set $U_{ad}$ is convex and closed.
3. The right-hand side of the state system is bounded by a linear
function in the state and control variables.

(4) The integrand,

$$
L(I_0^j, I^j, C^j, u^j, v^j) = I_0^j(t) + I^j(t) + C^j(t) + \frac{E^j}{2} (u^j(t))^2 + \frac{E^j}{2} (v^j(t))^2
$$

of the objective functional is convex on $U_{ad}$ and there exist cons-
tants $k_1$ and $k_2$ such that

$$
L(I_0^j, I^j, C^j, u^j, v^j) \geq -k_1 + k_2(|u^j|^2 + |v^j|^2).
$$

**Proof.** The existence of the optimal control can be obtained using a
result by Fleming and Rishel [27], checking the following steps:

Step 1: It follows that the set of controls and the corresponding state
variables is nonempty. In Diprima and Elementary [28]. To
prove that the set of controls and the corresponding state variables
is nonempty, we will use a simplified version of an existence result
[28].

Let $\mathcal{X}_i = \{x(t_i, t_0, x_0) : \text{where (}x(t_i, t_0, x_0) = (\mathcal{S}, \mathcal{I}_0, \mathcal{I}, \mathcal{C}, R)\text{) with \(x(t_i, t_0, x_0) = (x_1, x_2, x_3, x_4, x_5)\) are continuous and \(x(t_i, t_0, x_0) = (x_1, x_2, x_3, x_4, x_5)\) are continuous, then \(x(t_i, t_0, x_0) = (x_1, x_2, x_3, x_4, x_5)\) are continuous, then \(x(t_i, t_0, x_0) = \text{and \(\mathcal{X}_i\) for the right-hand side of equations of the system (2). Let \(u^j\) and \(v^j\) for some constants and since all parameters are

$$
L(I_0^j, I^j, C^j, u^j, v^j) \geq -k_1 + k_2(|u^j|^2 + |v^j|^2).
$$

Step 2: $U_{ad}$ is convex and closed by definition.

Take any controls $(u^j_1, v^j_1)$ and $(u^j_2, v^j_2) \in U_{ad}$ and \(\lambda \in [0, 1]\),
then $0 \leq \lambda u^j_1 + (1 - \lambda) u^j_2$ additionally, we observe that $\lambda u^j_1 \leq u^j_1$ and $(1 - \lambda) u^j_2 \leq u^j_2$ then $\lambda u^j_1 + (1 - \lambda) u^j_2 \leq 1$ hence, $0 \leq \lambda u^j_1 + (1 - \lambda) u^j_2 \leq 1$, for $i = 1, 2$.

The control space $U_{ad} = \{(u^j, v^j) : 0 \leq u^j(t) \leq 1; 0 \leq v^j(t) \leq 1, t \in [0, T]\}$ is measurable.

Step 3: All the right hand sides of equations of system (2) are
continuous, bounded above by a sum of bounded control and state,
and can be written as a linear function of $u$ and $v$ with coefficients
depending on time and state.

From the system of differential Eq. (2).

$$
\frac{dN^j}{dt} \leq \Lambda^j - \mu N^j \Rightarrow \limsup_{t \to \infty} N^j(t) \leq \frac{\Lambda^j}{\mu}
$$

Therefore, all solutions of the model (2) are bounded.

So, there exist positive constants $Z_1, Z_2, Z_3, Z_4, Z_5$ such that
\(\forall t \in [t_0, T]\) :

$$
S^j(t) \leq Z_1, I_0^j(t) \leq Z_2, I^j(t) \leq Z_3, C^j(t) \leq Z_4, \text{ and } R^j(t) \leq Z_5
$$

We consider

$$
F_0 \leq \Lambda^j I_0^j(t) + \int_0^T \left( \frac{\beta_{1}I_0^j(t)}{N^j} + \frac{\beta_{2}I^j(t)}{N^j} \right) \\
+ \left( \beta_{1} + \beta_{2} \right) S^j(t)
$$

(5)

F_1 \leq \lambda u^j_1 I_0^j(t) + \int_0^T \left( \frac{\beta_{1}I_0^j(t)}{N^j} + \frac{\beta_{2}I^j(t)}{N^j} \right) \\
+ \left( \beta_{1} + \beta_{2} \right) S^j(t)
$$

F_2 \leq \lambda u^j_2 I_0^j(t) \leq \int_0^T \left( \frac{\beta_{1}I_0^j(t)}{N^j} + \frac{\beta_{2}I^j(t)}{N^j} \right) \\
+ \left( \beta_{1} + \beta_{2} \right) S^j(t)
$$

F_3 \leq \alpha^j I_0^j(t) + \int_0^T \left( \frac{\beta_{1}I_0^j(t)}{N^j} + \frac{\beta_{2}I^j(t)}{N^j} \right) \\
+ \left( \beta_{1} + \beta_{2} \right) S^j(t)
$$

So, we can rewrite system (2) in matrix form as :

$$
F(t, S^j, I_0^j, I^j, C^j, R^j) \leq \Lambda^j + A^j X^j(t) - B^j u^j(t)
$$
5.2. Characterization of the optimal control

In order to derive the necessary condition for the optimal control, we apply Pontryagin’s maximum principle [29].

The idea is introducing the adjoint function to attach the system of differential equations to the objective functional resulting in the formation of a function called the Hamiltonian. This principle converts into a problem of minimizing Hamiltonian $H^j(t)$ at time $t$ defined by:

$$H^j(t) = I^j_v(t) + V(t) + C^j(t) + \frac{E^j}{2}(u^j(t))^2 + \frac{E^j}{2}(v^j(t))^2$$

$$+ \sum_{k=1}^{5} \lambda^j_k(t) f_k(S^j, I^j_v, I^j_v, C^j, R^j)$$

(6)

where $f_k$ is the right side of the system of differential Eq. (2) of the $k$th state variable at time $t$.

Theorem 5. Given the optimal controls $u^j, v^j$ and the solutions $S^j, I^j_v, I^j_v, C^j$ and $R^j$ of the corresponding state system (2) there exists adjoint variables $\lambda^j_1, \lambda^j_2, \lambda^j_3, \lambda^j_4$ and $\lambda^j_5$ satisfying:

$$\lambda^j_1 = \lambda^j_1 \mu^j + (\lambda^j_1 - \lambda^j_2)(1 - u^j(t))$$

$$\lambda^j_2 = -1 + (\lambda^j_1 - \lambda^j_2)(1 - v^j(t)) + \lambda^j_3 (u^j + \alpha^j_1 + \gamma^j_1 + u^j(t))$$

$$\lambda^j_3 = -1 + (\lambda^j_1 - \lambda^j_2)(1 - v^j(t))$$

$$\lambda^j_4 = -1 + \lambda^j_2 (u^j + \delta^j_1 + \gamma^j_2 + u^j(t)) - \lambda^j_2 (\gamma^j_2 + u^j(t))$$

$$\lambda^j_5 = \lambda^j_5 \mu^j$$

(7)

With the transversality conditions at time $T$: $\lambda^j_1(T) = 0$; $\lambda^j_2(T) = 1$; $\lambda^j_3(T) = 0$; $\lambda^j_4(T) = 1$; $\lambda^j_5(T) = 0$

Furthermore, for $t \in [0, T]$, the optimal controls $u^j$ and $v^j$ are given by:

$$u^j = \min \left( u^\text{max}, \max \left( u^\text{min}, \frac{(\lambda^j_1 - \lambda^j_2)I^j_v(t) + (\lambda^j_1 - \lambda^j_2)I^j_v(t) + (\lambda^j_2 - \lambda^j_3)C^j(t)}{E^j} \right) \right)$$

$$v^j = \min \left( v^\text{max}, \max \left( v^\text{min}, \frac{(\lambda^j_2 - \lambda^j_1)I^j_v(t) + (\lambda^j_2 - \lambda^j_1)I^j_v(t) + (\lambda^j_2 - \lambda^j_3)C^j(t)}{N(E^j)} \right) \right)$$

(8)

(9)

Proof. The Hamiltonian at time $t$ is given by:

$$H^j(t) = I^j_v(t) + V(t) + C^j(t) + \frac{E^j}{2}(u^j(t))^2 + \frac{E^j}{2}(v^j(t))^2$$

$$+ \sum_{k=1}^{5} \lambda^j_k(t) f_k(S^j, I^j_v, I^j_v, C^j, R^j)$$

where:

$$f_1(S^j, I^j_v, I^j_v, C^j, R^j) = \lambda^j_1 - \mu^j S^j(t) - S^j(t)(1 - u^j(t)) \sum_{j=1}^{3} \left( \frac{\beta^j_1(t)}{N} + \frac{\beta^j_2(t)}{N} \right)$$

$$f_2(S^j, I^j_v, I^j_v, C^j, R^j) = \lambda^j_2 - (\mu^j + \alpha^j_1 + \gamma^j_1 + u^j(t)) I^j_v(t) + S^j(t)(1 - v^j(t)) \sum_{j=1}^{3} \left( \frac{\beta^j_1(t)}{N} + \frac{\beta^j_2(t)}{N} \right)$$

$$f_3(S^j, I^j_v, I^j_v, C^j, R^j) = \lambda^j_3 - (\mu^j + \alpha^j_2 + \gamma^j_2 + u^j(t)) \psi^j(t) + \alpha^j_1 I^j_v(t)$$

$$f_4(S^j, I^j_v, I^j_v, C^j, R^j) = \alpha^j_2 \psi^j(t) - (\mu^j + \delta^j_1 + \gamma^j_2 + u^j(t)) C^j(t)$$

$$f_5(S^j, I^j_v, I^j_v, C^j, R^j) = \gamma^j_2 C^j(t) + \gamma^j_3 \psi^j(t) + \gamma^j_4 I^j_v(t) - \mu^j R^j(t) + (I^j_v(t) + V(t) + C^j(t)) u^j(t)$$

(10)
for \( t \in [0, T] \), the adjoint equations and transversality conditions can be obtained by using Pontryagin’s Maximum principle, such that

\[
\begin{align*}
\lambda_1^f &= -\frac{dH^i}{dt} = \lambda_1^j \mu^j + (\lambda_1^j - \lambda_2^j)(1 - v_i(t))(\sum_{i=1}^{J} \frac{\beta_{1i}^j I_{0i}(t)}{N} + \beta_{2i}^j I_{0i}(t) - \mu^j I_{0i}(t) - \gamma_1^j I_{0i}(t) + \gamma_2^j I_{0i}(t)) \\
\lambda_2^f &= -\frac{dH^i}{dt} = \lambda_2^j \mu^j - \Lambda_2 \gamma_2^j I_{0i}(t) + \gamma_2^j I_{0i}(t) - \Lambda_1 \mu^j I_{0i}(t) - \gamma_1^j I_{0i}(t) + \gamma_2^j I_{0i}(t)) \\
\lambda_3^f &= -\frac{dH^i}{dt} = \lambda_3^j \mu^j + (\Lambda_1 \mu^j - \gamma_1^j I_{0i}(t)) \\
\lambda_4^f &= -\frac{dH^i}{dt} = \lambda_4^j \mu^j + (\Lambda_2 \mu^j - \gamma_2^j I_{0i}(t)) - \Lambda_3 \mu^j I_{0i}(t) - \gamma_1^j I_{0i}(t) + \gamma_2^j I_{0i}(t)) \\
\lambda_5^f &= -\frac{dH^i}{dt} = \lambda_5^j \mu^j + (\Lambda_3 \mu^j - \gamma_2^j I_{0i}(t)) - \Lambda_4 \mu^j I_{0i}(t) - \gamma_1^j I_{0i}(t) + \gamma_2^j I_{0i}(t)) \\
\lambda_6^f &= -\frac{dH^i}{dt} = \lambda_6^j \mu^j + (\Lambda_4 \mu^j - \gamma_2^j I_{0i}(t)) - \Lambda_5 \mu^j I_{0i}(t) - \gamma_1^j I_{0i}(t) + \gamma_2^j I_{0i}(t)) \\
\lambda_7^f &= -\frac{dH^i}{dt} = \lambda_7^j \mu^j + (\Lambda_5 \mu^j - \gamma_2^j I_{0i}(t)) - \Lambda_6 \mu^j I_{0i}(t) - \gamma_1^j I_{0i}(t) + \gamma_2^j I_{0i}(t)) \\
\lambda_8^f &= -\frac{dH^i}{dt} = \lambda_8^j \mu^j + (\Lambda_6 \mu^j - \gamma_2^j I_{0i}(t)) - \Lambda_7 \mu^j I_{0i}(t) - \gamma_1^j I_{0i}(t) + \gamma_2^j I_{0i}(t)) \\
\lambda_9^f &= -\frac{dH^i}{dt} = \lambda_9^j \mu^j + (\Lambda_7 \mu^j - \gamma_2^j I_{0i}(t)) - \Lambda_8 \mu^j I_{0i}(t) - \gamma_1^j I_{0i}(t) + \gamma_2^j I_{0i}(t)) \\
\lambda_{10}^f &= -\frac{dH^i}{dt} = \lambda_{10}^j \mu^j - \Lambda_1 \gamma_1^j I_{0i}(t) + \gamma_1^j I_{0i}(t) - \Lambda_2 \gamma_2^j I_{0i}(t) + \gamma_2^j I_{0i}(t)
\end{align*}
\]

with the transversality conditions at time \( T \): \( \lambda_1^j(T) = 0; \lambda_2^j(T) = 1; \lambda_3^j(T) = 1; \lambda_4^j(T) = 0 \). For \( t \in [0, T] \), the optimal controls \( u^{x} \) and \( v^{x} \) can be solved from the optimality condition:

\[
\begin{align*}
\frac{du^x}{dt} &= 0 \quad \text{and} \quad \frac{dv^x}{dt} = 0 \quad \text{that is} \quad \\
\frac{du^x}{dt} &= E\frac{v^{x}}{v^{x}}(t) - \lambda_1^j \mu^j - \lambda_2^j \mu^j(t) - \lambda_3^j \mu^j(t) - \lambda_4^j \mu^j(t) - \lambda_5^j I_{0i}(t) + \gamma_1^j I_{0i}(t) - \lambda_2^j \mu^j(t) + \gamma_2^j I_{0i}(t)
\end{align*}
\]

\[
C^{1}(t) = 0 \quad \text{and} \quad \\
\frac{dv^x}{dt} &= E\frac{v^{x}}{v^{x}}(t) - \lambda_1^j \mu^j(t) + \lambda_2^j \mu^j(t) - \lambda_3^j \mu^j(t) - \lambda_4^j \mu^j(t) - \lambda_5^j I_{0i}(t) + \gamma_1^j I_{0i}(t) - \lambda_2^j \mu^j(t) + \gamma_2^j I_{0i}(t)
\]

so we have

\[
\begin{align*}
u^x(t) &= \frac{(\lambda_2^j - \lambda_1^j) I_{0i}(t) + (\lambda_3^j - \lambda_2^j) I_{0i}(t) + (\lambda_4^j - \lambda_3^j) C^{1}(t)}{E} \\

v^x(t) &= \frac{(\lambda_1^j - \lambda_2^j) S^{1}(t)(\beta_1^j I_{0i}(t) + \beta_2^j I_{0i}(t))}{N E}
\end{align*}
\]

by the bounds in \( U_{opt} \) of the controls, it is easy to obtain \( u^{x} \) and \( v^{x} \) in the form (8), (9).

### 6. Numerical simulation

#### 6.1. Algorithm

In this formulation, there were initial conditions for the state variables and terminal conditions for the adjoints. That is, the optimality system is a two-point boundary value problem with separated boundary conditions at times step \( t = 0 \) and \( t = T \). We solve the optimality system by an iterative method with forward solving of the state system followed by backward solving of the adjoint system. We start with an initial guess for the controls at the first iteration and then before the next iteration we update the controls by using the characterization. We continue until convergence of successive iterates is achieved. A code is written and compiled in Matlab using the following data (Tables 1–3).

#### 6.2. Discussion

In this section, we analyse numerically the effects of controls that are, the treatment of patients infected with COVID-19, by subjecting them to quarantine within hospitals and special places for that and using masks to cover the sensitive body parts. Different simulations can be carried out using various values of parameters. We use the parameters values (Tables 1–3).

![Fig. 6. The evolution of the number of the infected people without symptoms for all age groups without control.](image-url)
The proposed control strategy in this work helps to achieve several objectives: decreasing the number of people having COVID-19, infected without symptoms, infected with symptoms and infected with complications in all age groups.

6.2.1. Scenario A: control with masks covering the sensitive body parts

In this Scenario we use the masks to cover the sensitive parts of the body for all age groups. To lower the effect of bad contact with people infected with COVID-19 and to keep them as far as possible from the disease. After applying this strategic: Through Figs. 6 and 9 notes that the number of asymptomatic infected people under the age of 25 decrease from 404,200 (without control) to 861 (with control). The number of asymptomatic infected people who are over 65 years old decrease from 541,900 to 120. And number of asymptomatic infected people and ages between 25 and 65 years decrease from 1,264,000 to 1185. Through Figs. 7 and 10 notes that the number of infected people with symptoms under the age of 25 decrease from 184,500 to 577. The number of infected people with symptoms who are over 65 years old decrease from 400,700 (without control) to 234 (with control). And number of infected people with symptoms and ages between 25 and 65 years decrease from 1,374,000 to 3601. Through Figs. 8 and 11 notes that the number of infected people with complications under the age of 25 decrease from 3113 to 15. The number of infected people with complications who are over 65 years old decrease from 577,000 to 372. And number of infected people with complications and ages between 25 and 65 years decrease from 39,420 to 294. At the end of the implementation of the proposed Scenario.

6.2.2. Scenario B: control with quarantine within hospitals and special places

In this Scenario with temporary control of patients infected with (COVID-19) in hospitals and quarantine areas, we apply in order to reduce the number of infected people. After applying this strategic: Through Figs. 6 and 12 notes that the number of asymptomatic infected people under the age of 25 decrease from 404,200
Fig. 11. The evolution of the number of the infected people with complications for all age groups with control $V^j$.

Fig. 12. The evolution of the number of the infected people without symptoms for all age groups with control $U^j$.

Fig. 13. The evolution of the number of the infected people with symptoms for all age groups with control $U^j$.

Fig. 14. The evolution of the number of the infected people with complications for all age groups with control $U^j$.

6.2.3. Scenario C: combining the two previous strategies A and B
In this Scenario, we use two optimal controls $U^j$ and $V^j$. We combine the two previous strategies to achieve better results. That represent respectively the treatment of patients infected with COVID-19, by subjecting them to quarantine within hospitals and special places for that and using masks to cover the sensitive body parts. After applying this strategic, we observed: Through Figs. 6 and 15 notes that the number of asymptomatic infected people who are over 65 years old decrease from 541,900 to 83. And number of asymptomatic infected people and ages between 25 and 65 years decrease from 1,264,000 to 813. Through Figs. 7 and 13 notes that the number of infected people with symptoms under the age of 25 decrease from 184,500 to 276. The number of infected people with symptoms who are over 65 years old decrease from 400,700 to 98. And number of infected people with symptoms and ages between 25 and 65 years decrease from 1,374,000 to 1468. Through Figs. 8 and 14 notes that the number of infected people with complications under the age of 25 decrease from 3113 to 9. The number of infected people with complications who are over 65 years old decrease from 577,000 to 97. And number of infected people with complications and ages between 25 and 65 years decrease from 39,420 to 56. At the end of the implementation of the proposed Scenario.
crease from 577,000 to 67. And number of infected people with complications and ages between 25 and 65 years decrease from 39,420 to 40. At the end of the implementation of the proposed Scenario.

This table (Table 4) shows the different numbers of the people infected with COVID-19 in different age groups obtained after the use of the three strategies A, B and C. We note that using Scenario C, we get impressive results.

**Table 4**
The number of the people infected with COVID-19 in different age groups obtained after the use of the three strategies A, B and C.

|         | without control | with control U | with control V | with two controls |
|---------|-----------------|----------------|---------------|------------------|
| E₁     | 404,200         | 557            | 861           | 487              |
| E₂     | 1,264,000       | 813            | 1185          | 750              |
| E₃     | 541,900         | 83             | 120           | 74               |
| I₁     | 184,500         | 276            | 577           | 246              |
| I₂     | 1,374,000       | 1468           | 3601          | 1404             |
| I₃     | 400,700         | 98             | 234           | 51               |
| C₁     | 3113            | 9              | 15            | 4                |
| C₂     | 39,420          | 56             | 294           | 40               |
| C₃     | 577,000         | 97             | 372           | 67               |

6.3. Comparison of the results obtained in the study with the real results obtained in Morocco

Since the announcement of the discovery of the first case of coronavirus in Morocco, on 2 March 2020, for a person came from the Italian homeland [2], immediately thereafter, it was quick to close borders, suspend travel with all countries. Suspend studies in all educational institutions, and impose a state of health emergency. In addition to other measures such as the Events of the Corona Solidarity Fund, and establish Field Hospitals and adopt the health protocol imposed by the World Health Organization [1]. Represented by imposing the wearing of masks on citizens and by imposing a quarantine on everyone, with the exception of the active category which is authorized, to move towards workplaces and markets. And temporary control of patients infected with (COVID-19) in hospitals and quarantine areas. In the following, we examine the comparison of some results announced to the Moroccan Ministry of Health and the results obtained in this study at the end of the implementation of Scenario C.

**Characteristics of people by Age**

According to the Moroccan Ministry of Health [2], all age groups are affected. About 9.02% of cases were over the age of 65, while people under the age of 25 accounted for only 23.21% and about 67.77% of cases where the aged between 25 and 65 (Fig. 2). According to our study, we obtained the following results. About 6.2% of cases were over the age of 65, while people under the age of 25 accounted for 23.6% and about 70.2% of cases where the aged between 25 and 65 (Table 5).
Clinical picture on admission of cases

According to the Moroccan Ministry of Health [2], at admission, 97.16% of COVID-19 cases were asymptomatic or had moderate clinical symptoms, while 2.84% were admitted in a critical condition. According to our study, we obtained the following results. About 96.5% of COVID-19 cases were asymptomatic or had moderate clinical symptoms, while 3.5% of cases in critical condition (Table 6).

We note that there is a great convergence between the results obtained in this study and the results announced by the Moroccan Ministry of Health [2].

7. Conclusion

In this research, a mathematical epidemic model for all age groups human population of the COVID-19 disease was studied. After introducing the paper and discussing related literature. We formed a mathematical model that describes the dynamics of all age groups population who have the new symptom-free Coronavirus. In order to minimize the number of infected people without symptoms, infected people with symptoms and infected people with complications. We have also introduced two controls that, respectively, represent the treatment of patients infected with COVID-19 by subjecting them to quarantine within hospitals and special places for that and using masks to cover the sensitive body parts. We applied the results of the control theory and were able to obtain the characterizations of the optimal controls. Finally, we have a numerical solution obtained from the mathematical model by the use of the maximum principle of Pontryagin to characterize optimal controls. The system of optimality is solved by an iterative method. This study allowed us to integrate many scenarios to see the impact each of these controls on the maximum. The numerical precision of the system with differential equations as well as numerical simulations allowed us to compare and see the difference between each Scenario in a concrete way. Referring to Table 4, the numerical results demonstrate the effectiveness of our Scenario and we note that Scenario C performs better than Scenario B and A. In addition we note that there is a great convergence between the results obtained in this study and the results announced by the Moroccan Ministry of Health.

Given the importance of tracking contacts people infected with covid-19, we will add a special compartment for this category in future research.

Data availability

No data were used to support this study.

Declaration of Competing Interest

The authors declare that they have no conflicts of interest.

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Driss Kada: Writing - original draft, Writing - review & editing. Abdelfatah Kouidere: Writing - original draft, Writing - review & editing. Omar Balatif: Writing - original draft, Writing - review & editing. Mostafa Rachic: Writing - original draft, Writing - review & editing. El Houssine Labrijii: Writing - original draft, Writing - review & editing. El Houssine Labrijii: Writing - original draft, Writing - review & editing.

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