Dark energy after GW170817, revisited

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(Dated: November 5, 2018)

We revisit the status of scalar-tensor theories with applications to dark energy in the aftermath of the gravitational wave signal GW170817 and its optical counterpart GRB170817A. At the level of the cosmological background, we identify a class of theories, previously declared unviable in this context, whose anomalous gravitational wave speed is proportional to the scalar equation of motion. As long as the scalar field is assumed not to couple directly to matter, this raises the possibility of compatibility with the gravitational wave data, for any cosmological sources, thanks to the scalar dynamics. This newly “rescued” class of theories includes examples of generalised quintic galileons from Horndeski theories. Despite the promise of this leading order result, we show that the loophole ultimately fails when we include the effect of large scale inhomogeneities.

INTRODUCTION

The observation of the neutron star merger GW170817 at redshift $z \sim 0.01$ and its optical counterpart GRB170817A has provided spectacularly strong constraints on the relevance of modified gravity to understanding the dynamics of the late Universe. Generic interactions between the massless spin 2 and additional light fields can cause the gravitational wave to propagate through the cosmological background at speeds different from its electromagnetic counterpart, even when it passes through overdense regions where so-called screening mechanisms might be expected to operate. In the light of the LIGO/Virgo observation, a careful analysis of this effect in a wide class of scalar tensor theories has led to a dramatic reduction in the landscape of modified gravity models that are relevant for dark energy and remain observationally viable at late times (see also [17–21]).

To avoid being drawn into an erroneous narrative as to the fate of modified gravity, it is important to properly state the implications. Although a significant number of scalar tensor interactions were ruled out, they did not rule out everything, even in the context of dark energy. For example, conformal couplings to curvature, as in Brans Dicke or chameleon models, remain viable, as do so-called Kinetic Gravity Braiding models. Furthermore, there is nothing to stop us from including the forbidden interactions as long as they are suppressed by some heavy scale that renders them irrelevant in the late Universe. Of course, this latter consideration weakens the motivation for considering such interactions in the first place.

Going beyond these clarifications, there are also reasons to revisit the conclusions of [13–16]. In particular, it was recently noted that the frequency scales of the neutron star event lie close to the strong coupling scale associated with many dark energy models. If they are known, ultra violet effects could impact the speed of the gravitational wave, and if they are not known, one is attempting to constrain a theory outside of its regime of validity. Of course, such a manifest breakdown of the low energy description could also be relevant/problematic to theories that rely on the Vainhstein effect to pass solar system tests (for further understanding of Vainhstein screening, see [33–35]; for discussions on their UV completions see [31–32]). In this paper, we consider the possibility of a different loophole: that the speed of the gravitational wave is set to unity dynamically. More precisely, in the context of scalar tensor theories, we identify new scenarios in which the deviation from unity is proportional to the scalar field equation of motion on the cosmological background. As long as the scalar is decoupled from the matter sector directly, its field equation will always vanish identically ensuring exact agreement with the LIGO/Virgo observations at this order. However, as we will see, there is no extension of this result including the effect of large scale inhomogeneities.

Within the Horndeski class, so-called “$L_5$” operators, cubic in second derivatives of the scalar, were previously thought to be excluded by the gravitational wave data already at the level of the background cosmology. However, we shall present an explicit example of a theory in which such an operator is present and yet the speed of the gravitational wave is unity at the background level thanks to the vanishing of the scalar equation of motion. In going beyond Horndeski, we find other interactions that can be rescued from the forbidden zone along the same lines. However, it is only the Horndeski example that survives additional constraints coming from the decay of the wave into dark energy fluctuations. As stated above, to rule out this newly rescued Horndeski class, we need to consider the effects of large scale inhomogeneities.

1 See [20] for related ideas for which the speed of the gravitational wave approaches unity dynamically due to a non-minimal coupling between dark energy and dark matter.
LIGO/VIRGO REVISITED

The sight and sound of the neutron star merger detected on August 17, 2017 has constrained the speed of gravitational and electromagnetic waves through the cosmological medium at late times, to be identical to an accuracy of the order $10^{-15}$. Although these speeds are indeed identical in General Relativity, this is not the case in generic modified gravity models where the additional fields can possess a non-trivial cosmological profile that pushes the tensor mode off the light cone. To illustrate this, we consider a wide class of scalar tensor theories, including Horndeski [36, 37] and beyond Horndeski [38, 39] interactions, given by the action

$$S = \int d^4x\sqrt{-g} \sum_{n=2}^5 L_n,$$

where

$$L_2 = G_2(\phi, X),$$

$$L_3 = G_3(\phi, X)\Box \phi,$$

$$L_4 = G_4(\phi, X)R - 2G_{4, X} \nabla_{[\mu_1} \nabla_{\mu_2]} \nabla^{\mu_1} \nabla^{\mu_2} \phi + F_1(\phi, X) \epsilon^{\mu_\nu \rho \sigma} \nabla^\mu \phi \nabla^\nu \phi \nabla^\rho \phi \nabla^\sigma \phi,$$

$$L_5 = G_5(\phi, X)G_{\mu \nu} \nabla^\mu \nabla^\nu \phi + \frac{G_{5, X}}{3} \sum (\nabla^1 \phi) (\nabla^2 \phi) (\nabla^3 \phi) + \frac{F_5(\phi, X)}{6} \epsilon^{\mu_\nu \rho \sigma} \nabla^\mu \phi \nabla^\nu \phi \nabla^\rho \phi \nabla^\sigma \phi.$$

Here we have a metric $g_{\mu \nu}$ with corresponding covariant derivative $\nabla_{\mu}$, Ricci scalar $R$ and Einstein tensor $G_{\mu \nu}$. We have a scalar field $\phi$ and define its canonical kinetic operator $X = g^{\mu \nu} \partial_\mu \partial_\nu \phi$. The symbol $\epsilon^{\mu_\nu \rho \sigma}$ is the totally antisymmetric Levi-Civita tensor while the square brackets denote antisymmetric combinations defined without the usual factors of $1/n!$. $G_{4, X}$ and $G_{5, X}$ denote derivatives of the potentials with respect to $X$. Despite the higher order nature of these theories one can avoid propagating additional degrees of freedom associated with Ostrogradski ghosts [42]. In particular, the theory will propagate one scalar and two graviton degrees of freedom in each of the following cases [43]: the Horndeski class [36] with second order field equations for which $F_2 = F_5 = 0$; beyond Horndeski [38, 39] with $L_4 = 0, G_5, X \neq 0$ or $L_5 = 0, G_4 - 2G_{4, X} \neq 0$; beyond Horndeski with both $L_4 \neq 0, L_5 \neq 0$ and a degeneracy condition $XG_{5, X}F_4 = 3F_5[G_4 - 2G_{4, X} - (X/2)G_{5, X}]$. In the latter case $F_2$ and $F_5$ are generated by the same disformal transformation [36, 39, 43].

To proceed, we take a spatially flat cosmology, $ds^2 = -dt^2 + a(t)^2 dx^2$, with a homogeneous scalar, such that we also have $X = -\dot{\phi}^2$. We shall further assume that matter is minimally coupled to the metric with no direct coupling to the scalar, so that the scalar field equation has no external source. The form of the corresponding field equations can be obtained using [38], by varying the minisuperspace Lagrangian with respect to the lapse function and the scalar field, then setting the lapse to unity. The key point in what follows is that the scalar equation takes the form $\mathcal{E}_\phi = \frac{1}{a} \frac{dA}{dt} = A\dot{\phi} - B$ where

$$A = \sum_{i,j} A_{ij}(\phi, X)H^i \dot{H}^j, B = \sum_{i,j} B_{ij}(\phi, X)H^i \dot{H}^j,$$

and $H = \dot{a}/a$, with explicit formulae for the $A_{ij}$, $B_{ij}$ given in the appendix. As long as $A$ is non-vanishing this can be used to identify $\dot{\phi} = B/A$. In other words, imposing the vanishing of the scalar equation of motion means that we should not treat $\dot{\phi}$ as independent of $\phi, X, H, \dot{H}$.

The tensor fluctuations on this background are described by the following quadratic action [38, 44]

$$S_T^{(2)} = \frac{1}{8} \int dt dx [\mathcal{G}_T \dot{h}_{ij}^2 + \frac{\mathcal{F}_T}{a^2} (\nabla h_{ij})^2]$$

where

$$\mathcal{F}_T = 2G_4 + XG_{5, \phi} - 2X\ddot{\phi}G_{5, X},$$

$$\mathcal{G}_T = 2G_4 - 4XG_{4, X} - XG_{5, \phi} + 2X^2 F_4 - 2H X \ddot{\phi}(G_{5, X} + 3XF_5)$$

The speed of the gravitational wave through the cosmic medium is now given by $c_T^2 = \frac{\mathcal{F}_T}{\sqrt{\mathcal{G}_T}}$, and its deviation from unity by $\alpha_T = c_T^2 - 1 = \frac{\mathcal{F}_T}{\sqrt{\mathcal{G}_T}}$. This is the quantity that, at late times, is constrained to vanish to order $10^{-15}$ thanks to the neutron star merger. We will therefore require it to be zero.

In [13, 16], the authors require $\alpha_T$ to vanish on any cosmological background. As is elegantly explained in [15, 16] we can write $\mathcal{G}_T\alpha_T = -2X \ddot{\phi}G_{5, X} + C(\phi, X)$, where

$$C(\phi, X) = 4XG_{4, X} + 2XG_{5, \phi} - 2X^2 F_4 + 2H X \sqrt{-X}(G_{5, X} + 3XF_5).$$

Requiring $\alpha_T$ to vanish for any cosmological background, they demand that it vanishes for any choice of $\phi$. This imposes two independent conditions $G_{5, X} = 0$ and $C(\phi, X) = 0$. Regarding them as partial differential equations for the (beyond) Horndeski potentials one is then able to greatly constrain the space of scalar tensor theories that are compatible with the gravitational wave data. However, based on our earlier discussion regarding the form of the scalar equation of motion, we will see that this approach is too constraining and that we are free to use $\mathcal{E}_\phi = 0$ to eliminate $\dot{\phi}$ in our expression for $\alpha_T$. Proceeding in this way, we obtain $\mathcal{G}_T\alpha_T = -2X \frac{\mathcal{F}_T}{\sqrt{\mathcal{G}_T}} G_{5, X} + C(\phi, X)$, then require $\mathcal{G}_T\alpha_T$ to vanish, giving a complicated equation of the form

$$\sum_{i,j} C_{ij}(\phi, X) H^i \dot{H}^j = 0,$$

where

$$C_{ij} = -2XB_{ij}G_{5, X} + CA_{ij}.$$

We now demand that (4) holds for any choice of energy density, $\rho$, and pressure, $p$, or in other words, it should hold for all values

\[\text{Note that we have traded } \dot{\phi} = \sqrt{-X}. \text{ We could have chosen the root with opposite sign but this would not affect our conclusions since the subsequent analysis is invariant under } t \rightarrow -t.\]
of $H$ and $\dot{H}$. This results in a number of constraints $C_{ij} = 0$ that can be treated as a simultaneous set of partial differential equations for the (beyond) Horndeski potentials. To solve them we first note that $A_{i1}$ vanishes for all $i$, in contrast to $B_{i1}$ (see appendix). Imposing $C_{i1} = 0$ is therefore equivalent to $G_{5,X} B_{i1} = 0$. This is the fork in the road. On the one hand we can solve this by setting $G_5 = G_5(\phi)$. It then follows that $C_{ij} = CA_{ij}$ and since $A$ must be non-vanishing we have that $C = 0$. This reduces to the scenario already considered in \cite{13-16}. Alternatively, however, we may assume that $G_{5,X} \neq 0$, in which case we must have that $B_{i1} = 0$. This leads to a new class of solutions that are compatible with the gravitational wave data, where the (beyond) Horndeski potentials are given as

$$G_2 = \frac{1}{2} X H_1,\phi,\phi - X H_2,\phi,\phi + \kappa + X h(\phi) \tag{5}$$

$$G_3 = X H_1,\phi,\phi X - 2 X H_2,\phi,\phi X + \frac{1}{2} H_1,\phi,\phi - H_2,\phi,\phi + h(\phi) \tag{6}$$

$$G_4 = \kappa_G - \frac{1}{2} X H_1,\phi,\phi X + X H_2,\phi,\phi X \tag{7}$$

$$G_5 = H_1,\phi - 6 \frac{\mu}{\sqrt{-X}} \tag{8}$$

$$F_4 = -H_1,\phi,\phi X + 2 H_2,\phi,\phi X + \frac{H_2,\phi}{X} \tag{9}$$

$$F_5 = -\frac{H_1,\phi,\phi X}{3 X} \tag{10}$$

where $\kappa_G$ and $\mu$ are constants. $\kappa(\phi, X)$ is a function of $\phi, X$, arbitrary up to the condition $\kappa, \phi, X \neq 0$. We use it to obtain $H_1(\phi, X)$ and $H_2(\phi, X)$ via the following differential equations,

$$H_1,\phi,\phi X = -2 \mu \left[ X \kappa,\phi,\phi X + 2 \kappa,\phi,\phi X \right] \frac{\sqrt{-X}}{\sqrt{X}} \tag{11}$$

$$H_2,\phi,\phi X = \mu \left[ 2 X \kappa,\phi,\phi X - \kappa,\phi,\phi X \right] \frac{\sqrt{-X}}{\sqrt{X}} \tag{12}$$

The contribution from the arbitrary function $h(\phi)$ is actually redundant since it enters the full Lagrangian as a total derivative. In any event, this new class of theories yields the following generalised Friedmann equation

$$6 \kappa_G H^2 - 12 \mu H^3 - \mathcal{K} + 2 X \mathcal{K,\phi} X = \rho \tag{13}$$

and a scalar equation of motion

$$\mathcal{E}_\phi \equiv -2 \dot{\phi} (2 X \mathcal{K,\phi,\phi} X + \mathcal{K,\phi} X) - 6 H \phi \kappa,\phi,\phi X + 2 X \kappa,\phi,\phi X - \mathcal{K,\phi} = 0 \tag{14}$$

The anomalous speed of the gravitational wave through this cosmic medium is given by

$$c_T^2 = 1 - \frac{\mu}{2 \phi \left( 3 H \mu - \kappa_G \right) \mathcal{K,\phi}} \mathcal{E}_\phi, \quad \text{vanishing on-shell thanks to} \quad (14), \quad \text{as anticipated.} \quad \text{Finally we recall the conditions for avoiding the Ostrogradski ghosts} \quad (43). \quad \text{This places further constraints on the function,} \quad \kappa_G. \tag{15}$$

Adding spatial curvature

All of our previous analysis relied on the assumption that the cosmological metric is spatially flat. What happens when we include spatial curvature, $k$, and proceed in a similar way? It turns out that our constraint equation \cite{43} receives additional terms that go as $\frac{1}{k} \sum_{ij} D_{ij} H^1 H^1$, where the $D_{ij}(\phi, X)$ are given in terms of the (beyond) Horndeski potentials and their partial derivatives. Unless the spatial curvature is suppressed, we require all the $D_{ij}$ to vanish. However, it turns out that $D_{01}$ vanishes if and only if $G_{5,X} = 0$. As we saw earlier, this forces us back to the scenario already considered in \cite{13-16}. Therefore, for the family of models given by \cite{5} to \cite{10} to be compatible with the LIGO/Virgo bounds, we require the spatial curvature of the Universe to be negligible.

Constraints from decay into dark energy fluctuations

At this stage we consider the additional constraint coming from decay of the gravitational wave into fluctuations of the scalar field \cite{20}. This requires the vanishing of the so-called $\tilde{m}_4^2 \delta^4(0) R_{0i}$ coupling in the effective field theory of dark energy \cite{15,11}, where

$$\tilde{m}_4^2 = -\frac{1}{2} G_T \alpha_T - X^2 F_4 + 3 H X^2 \phi F_5 \tag{16}$$

For the class of theories given by equations \cite{5} to \cite{10}, $G_T \alpha_T$ vanishes by the scalar equation of motion. For $\tilde{m}_4^2$ to vanish we also require $-X^2 F_4 + 3 H X^2 \phi F_5 = 0$. The absence of $\phi$ in this latter condition means we cannot further exploit the vanishing of the scalar equation of motion. Rather, we are forced to set $F_4 = F_5 = 0$ explicitly, reducing ourselves to the Horndeski limit. This is obtained in equations \cite{5} to \cite{10} by setting $H_1 = 0, H_2 = 3 \mu W'(\phi) / \sqrt{-X}$ and $\mathcal{K} = \Lambda - \nu e^{W(\phi) / X}$, where $\Lambda$ and $\nu$ are constants.

A NEWLY RESCUED THEORY?

Let us now study the dynamics of our newly “rescued” theory. As we have shown, this falls within the Horndeski subclass with potentials given by

$$G_2 = -3 \mu W'' X \sqrt{-X} + \Lambda - \nu e^{W / X}, \quad G_3 = -6 \mu W'' X \sqrt{-X}$$

$$G_4 = \kappa_G + \frac{3}{2} \mu W' \sqrt{-X}, \quad G_5 = -6 \mu \frac{1}{\sqrt{-X}} \tag{17}$$

and, of course, $F_4 = F_5 = 0$. Notice that we have a non-trivial “$L_0$” contribution even in the Horndeski limit. The structure of the theory, containing non-local operators like $1 / \sqrt{-X}$, is not especially appealing. However, similar operators appear in the so-called cuscaton models.
and in the extreme relativistic limit of probe branes (see also [47]). Alternatively, we could imagine them arising when we integrate out light, rather than heavy, degrees of freedom. They are also amenable to a hydrodynamical interpretation, where, for example, an operator of the form $\nabla_v \phi / \sqrt{-X}$ can be interpreted as a fluid velocity [46].

For this Horndeski example, the field equations simplify somewhat, giving

$$6\kappa_G H^2 - 12\mu H^3 + \Lambda - 3\nu \frac{e^W}{X} = \rho$$

(same as [44]), tensor and scalar fluctuations are determined by the following coefficients,

$$\mathcal{F}_T = \mathcal{G}_T = 2\kappa_G - 6\mu H$$

$$\mathcal{F}_S = -2\kappa_G \frac{H}{H^2}, \quad \mathcal{G}_S = -3\nu \frac{e^W}{H^2 X}$$

which are then required to be positive for a stable background.

The most interesting feature of this particular dynamics, and that which really encodes our modification to General Relativity, is the $\mu H^3$ term. This is also present in the generic case [43]. For this to be relevant to the late time Universe, we require $|\mu| \sim \frac{\kappa_G}{H_0^2}$, where $H_0$ is the current Hubble scale. Indeed, this description can only apply to the late Universe: if $\mu$ were to retain this constant value at earlier times, the $H^3$ term would dominate the dynamics over the conventional $H^2$ piece, which would ultimately be incompatible with nucleosynthesis constraints. To avoid this, one ought to take the view that this behaviour only emerges at late times. In other words, we should really think of $\mu$ as being field dependent. The dynamics could then be such that it starts out negligible and remains so for much of the Universe’s history, only rising to the constant value set by the scale of dark energy at late times.

**Adding local inhomogeneities**

The fact that adding spatial curvature closes the loophole might suggest that the same is true for wave propagation through large scale inhomogeneities. We will now check this explicitly and demonstrate that our loophole fails to extend beyond the cosmological background. To this end, consider propagation of a gravitational wave, $h_{\mu\nu}$, through a background metric $\bar{g}_{\mu\nu}$ which is FRW plus small, weak-field potentials $\Phi$ and $\Psi$ representing the inhomogeneities. The gravitational wave is assumed to enter in a transverse-tracefree gauge so that the full metric is given by

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 - 2\Phi)(\gamma_{ij} + h_{ij})dx^i dx^j$$

Before proceeding further, let us establish a hierarchy of scales, which can tell us which terms are most important when computing curvature. The scalar curvature for (23) is schematically of the form

$$R \sim \partial^2 \bar{g} + (1 + \# \Phi)\partial^2 h + (1 + \# \Phi)\partial \Phi \partial h + (1 + \# \Phi)\partial^2 \Phi h$$

where $\#$ are numbers of order 1. The first term $\partial^2 \bar{g}$ is the term including linear perturbations of FRW with the potentials $\Phi$ and $\Psi$. The other terms are those expected from including the gravitational wave $h_{\mu\nu}$ in the transverse-traceless gauge. Now the scale associated with changes in $\Phi$ and $\Psi$ is the size of the inhomogeneity, $r$, while for the gravitational wave the relevant scale is its wavelength $\lambda$. This means that $\partial \Phi \sim \partial \Psi \sim \Phi / r$ while $\partial h \sim h / \lambda$. Further, the amplitude of the gravitational wave is taken to be small compared to the amplitude of the two potentials, so that $\epsilon_h = h / \Phi \ll 1$. Note that for LIGO/Virgo wavelengths $\lambda \sim 1000km$ and large scale inhomogeneities $r \geq 100Mpc$, we have that

$$\epsilon_\lambda = \lambda / r \sim 10^{-18}$$

The amplitude of such inhomogeneities is typically $\Phi \sim 10^{-5}$, whilst that of the wave on arrival, having travelled a distance of 40 MPc, is $h \sim 10^{-22}$. Therefore, when the wave is a distance $d$ from the source, we have

$$\epsilon_h \sim 10^{-17} \left(\frac{40\text{MPc}}{d}\right)$$

and so $\epsilon_h \ll 1$ for the vast majority of the wave’s trajectory, justifying the linearised approximation. For the most part, these considerations suggest the following hierarchy of scales in (24):

$$\partial^2 h \gg \Phi \partial^2 h \gg \Phi^2 \partial^2 h \gg \partial \Phi \partial h \gg h \partial^2 \Phi$$

3 In this gauge the gravitational wave is assumed to be purely spatial, traceless and transverse with respect to the background FRW metric: $h_{00} = h_{0i} = h_{ij} = 0$, $\nabla_j h_{ij} = 0$. To justify this assumption, we consider the full metric as a perturbation about FRW and perform the standard decomposition with respect to the three dimensional Euclidean group [41]. This contains the standard tensor mode, $h_{ij}$, which is transverse and trace-free with respect to the background FRW metric. Redefining this as $h_{ij} = (1 - 2\Psi)h_{ij}$, it follows that $h_{ij}$ is also tracefree and transverse up to terms that go as $h \partial^2 \Phi$, which are neglected under our derivative dropping assumptions, at least when computing curvature.
so that the leading terms that we shall consider are $R \sim \partial^2 \bar{g} + (1 + \Phi) \partial^2 \bar{h} + \ldots$. In other words, within this approximation we shall drop any non-linear potential contributions, such as $\Phi^2 \partial^2 \bar{h}$, and any derivatives of the potentials multiplying the gravitational wave, such as $\partial \Phi \partial h$ and $(\partial^2 \Phi) h$.

In General Relativity, this leads to the gravitational wave equation of motion

$$(1 - 2\Psi) \left( \dot{h}^i_j + 3H \dot{h}^i_j \right) - (1 + 2\Psi) \frac{\ddot{\nu}^2}{a^2} h^i_j = 16\pi G T^i_j$$

meaning that our approximation is essentially that of geometric optics and the effective metric in (28) is the background metric $\bar{g}_{\mu\nu}$. This is equivalent to the fact that the gravitational wave travels on null geodesics of $\bar{g}_{\mu\nu}$.

Now turn to our newly “rescued” Horndeski theory setting $\kappa_G = 1/16\pi G$ and assuming $\phi > 0$. We are only interested in the pure spin-2 gravitational wave and so perturb the scalar as $\phi = \bar{\phi}(1 + \varphi)$ with $\varphi \sim O(\Phi)$. Nevertheless, the presence of the dynamical scalar field can introduce new terms in our propagation equation that can be dangerously large. These include terms going as $(\partial^2 \Phi)(\partial^2 h)$ whose presence we might have anticipated from the effect of spatial curvature. Indeed, we mentioned earlier that $\alpha_T \propto E_\phi$ can be achieved only on flat FRW backgrounds. Once curvature $k$ is allowed, $\alpha_T$ is proportional to $k$, $\bar{G}_\mu \alpha_T = 18\mu^2 H \bar{X} \frac{\partial}{\partial \phi} \left( e^W \nu - 3H \nu X \frac{\partial}{\partial \phi} \right)^{-1}$. Since short-wavelength gravitational waves should not be able to feel the difference between a global curvature $k$ and a local long-wavelength curvature perturbation $\nabla^2 \Phi$, we might expect a perturbative analogue of $\alpha_T$, $\delta \alpha_T \propto \nabla^2 \Phi$, to obstruct the loophole.

In any event, we find that the tensor mode equation is

$$(1 - 2\Psi) \left( \dot{h}^i_j + 3H \dot{h}^i_j \right) - (1 + 2\Psi)(1 + \delta \alpha_T) \frac{\ddot{\nu}^2}{a^2} h^i_j + \ldots = \frac{16\pi G}{\bar{G}_T + \delta \bar{G}_T} T^i_j$$

where

$$\bar{G}_T \delta \alpha_T \equiv -4\mu \frac{\ddot{\nu}^2}{a^2} \left[ \left( 1 + \frac{3H \bar{X}}{e^W \nu} \right) \frac{\vec{\phi}}{\partial \phi} \varphi + \frac{3\mu H \bar{X}}{e^W \nu} \Phi \right]$$

and $\mu = 8\pi G \mu$ and $\bar{G}_T = 8\pi G \bar{G}_T$. Let us now estimate the size of $\delta \alpha_T$. To be relevant for the late universe, where $\bar{H} \sim \bar{H}_0$, we assume $\bar{H} \sim 1/\bar{H}_0$. Furthermore, typically we expect, $\bar{\phi} \sim \bar{H}_0 \bar{\phi}$, $\bar{H} \sim \bar{H}_0^2$ and $e^W \nu / X \sim \bar{H}_0^2 / 8\pi G$, the latter condition following from the Friedmann equation (19). It immediately follows that $\delta \alpha_T \sim \nabla^2 \Phi / H_0^2 \sim \Phi / (H_0 \bar{r})^2$. For large scale inhomogeneities $H_0 \bar{r} \sim O(0.1)$ and $\Phi \sim 10^{-5}$, yielding an anomalous gravitational wave propagation of one part in a thousand or so. This is completely ruled out.

The derivation of (29) makes use of the scalar dynamics by direct substitution, just as was done for the background. However, we see that it does not help. We also considered additional dynamical constraints that arise if one neglects pressure perturbation and anisotropic shear as sources of the inhomogeneity. Such constraints do not alter the qualitative result.

What about the terms we ignored in our derivation? Compared to the leading order inhomogeneous contributions, terms such as $\partial \Phi \partial h$ are suppressed by a factor of $\epsilon_\lambda \sim 10^{-18}$, whilst terms such as $\partial^2 \Phi h$ are further suppressed by $\epsilon_\lambda^2$, so these do not affect our conclusions. There could also be non-linear terms such as $\Phi^2 \partial^2 h$. In principle these could yield corrections to $\alpha_T$ of order $\Phi^2 \sim 10^{-10}$, which are also too small to affect our conclusions. The ellipses in (29) stand for further relevant terms such as $(\partial^2 \Phi)(\partial^2 h)$ in which the free indices $i$ and $j$ are not both on $h$, like $\bar{h}^{ik} \nabla_j \nabla_k \varphi$. These terms introduce gravitational birefringence.

**DISCUSSION**

In this paper we identified a class of scalar tensor theories for which the anomalous gravitational wave speed vanishes *dynamically* on cosmological backgrounds on account of the scalar equation of motion. This reveals a potential loophole, opening up the possibility of “rescuing” certain theories that had previously been declared to be incompatible with the LIGO/Virgo bounds. Further constraints from the decay of gravitational waves into dark energy fluctuations eliminates beyond Horndeski scenarios, leaving us with a family of theories that fall within the Horndeski subclass, including non-trivial “LS” interactions. To convincingly rule out the remaining theory, we studied the effect of large scale inhomogeneities and demonstrated that any anomalous propagation of the gravitational wave could not be further eliminated by constraints arising from the inhomogeneous equations of motion.

The starting point for our analysis was the Horndeski [38, 39] and beyond Horndeski class of scalar tensor theories [38, 39, 42]. We could certainly imagine extending our procedure to include extended scalar tensor or DHOST theories [43, 50, 52], multi scalar tensor theories [52, 53, 54], and beyond. Indeed, theories with more than one additional field should have a much richer structure since there are more vanishing field equations to exploit.

Our analysis explicitly spells two important lessons: the first is that it is important to use all of the available dynamical information when establishing the viability of a theory within a given approximation; the second is the shear power of the gravitational wave observation and its ability to constrain theories at higher order in perturba-
Formulas for $A_{ij}, B_{ij}$

Here we include explicit formulae for the non-vanishing functions $A_{ij}$ and $B_{ij}$ appearing in the scalar equation of motion, $\ddot{\mathcal{E}}_\phi$. Recall, as per the discussion in footnote 2 that we have traded $\dot{\phi} = \sqrt{-X}$. 

\begin{align}
A_{00} &= XG_{3,\phi X} - 2XG_{2, X^2} - G_{2, X} - G_{3, \phi} \\
A_{10} &= 6\sqrt{-X} [2XG_{4, \phi X} + XG_{3, X^2} + 3G_{3, \phi} + 3G_{4, \phi} X] \\
A_{20} &= 12X^2F_4, X - 6X^2G_{5, \phi X^2} + 54X^2F_4, X - 24X^2G_{4, X^3} - 15XG_{5, \phi} X - 48XG_{4, X^2} + 36XF_4 - 6G_{4, X} - 3G_{5, \phi} \tag{34} \\
A_{30} &= -2\sqrt{-X} [6X^2F_{5, X^2} + 33X^2F_{5, X} + 2X^2G_{5, X^3} + 7XG_{5, X^2} + 30XF_5 + 3G_{5, X}] \tag{35}
\end{align}

\begin{align}
B_{00} &= G_{2, \phi} - 2XG_{2, \phi X} + XG_{4, \phi} \tag{36} \\
B_{10} &= 6\sqrt{-X} [XG_{3, \phi X} + 2XG_{4, \phi X} + G_{2, X} + G_{3, \phi}] \tag{37} \\
B_{20} &= 36XG_{4, \phi X} - 3XG_{5, \phi} + 18X^2F_4, \phi + 12G_{4, \phi} + 18XG_{3, X} - 24X^2G_{4, \phi X^2} + 12X^3F_4, \phi - 6X^2G_{5, \phi} \tag{38} \\
B_{30} &= 2\sqrt{-X} [-6X^3F_5, \phi X - 2X^2G_{5, \phi} X^2 - 18X^2F_4, X - 12X^2F_5, \phi + 7XG_{5, \phi} X + 36XG_{4, X^2} - 36XF_4 + 18G_{4, X} + 9G_{5, \phi}] \tag{39} \\
B_{4,0} &= -6X [6X^2F_{5, X} + 2XG_{5, X^2} + 15XF_5 + 3G_{5, X}] \\
B_{01} &= 6XG_{3, X} + 12XG_{4, X} + 6G_{4, \phi} \tag{40} \\
B_{11} &= 12\sqrt{-X} [-2X^2F_4, X + XG_{5, \phi} X + 4XG_{4, X^2} - 4XF_4 + 2G_{4, \phi} + G_{5, \phi}] \tag{41} \\
B_{21} &= -6X [6X^2F_5, X + 2XG_{5, X^2} + 15XF_5 + 3G_{5, X}] \tag{42}
\end{align}
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