Machine-Checked Proofs For Realizability Checking Algorithms

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Abstract—We have recently proposed a contract-based realizability checking algorithm involving the use of theories, to provide an auxiliary procedure to consistency checking of “leaf-level” components in complex embedded systems. To prove the soundness of our approach on realizability, we formalized the necessary definitions and theorems of [1], in the Coq proof and specification language.

I. INTRODUCTION

An ongoing effort at Rockwell Collins and The University of Minnesota has explored algorithms and tools for compositional proofs of correctness. The idea is to support hierarchical design and analysis of complex system architectures and co-evolution of requirements and architectures at multiple levels of abstraction [2]. We have created the AGREE reasoning framework [3] to support compositional assume/guarantee contract reasoning over system architectural models written in AADL.

The soundness of the compositional argument requires that each leaf-level component contract is realizable; i.e., it is possible to construct a component such that for any input allowed by the contract assumptions, there is some output value that the component can produce that satisfies the contract guarantees. Unfortunately, without engineering support it is all too easy to write leaf-level components that can’t be realized. In order to make our approach reasonable for practicing engineers, tool support must be provided for checking realizability.

Realizability checking for propositional contracts has been well-studied for many years (e.g., [4], [5], [6], [7]), both for component synthesis and checking correctness of temporal logic requirements. Checking realizability for contracts involving theories, on the other hand, is still an open problem. In recent work [1], we described a new approach for checking realizability of contracts involving theories and demonstrate its usefulness on several examples. Our approach is similar to k-induction [8] over quantified formulas. In that work, we provided hand-proofs for several aspects of two algorithms related to the soundness of the approach with respect to both proofs and counterexamples.

Unfortunately, hand proofs of complex systems often contain errors. Given the criticality of realizability checking to our tool chain and the soundness of our computational proofs, we would like a higher level of assurance than hand proofs can provide. In this paper, we provide a formalization of the proof system in Coq and machine-checked proofs of correctness that ensure that the proposed realizability algorithms will perform as expected1. The facilities in Coq, notably mixed use of induction and co-induction, make the construction of the proofs relatively straightforward. Our approach illustrates how interactive theorem proving and model checking can be used together in a profitable way: interactive theorem proving for describing the overall soundness of a proof system and verification approach, and model checking for solving the (complex) verification problem instances. Also, the formalization process exposed minor errors regarding our initial definitions, such as the lack of necessary assumptions to a theorem under proof, or incorrectly definitions, especially for realizability itself.

In Section II we provide information on the Coq proof assistant, while Section III contains the necessary informal background towards understanding our realizability checking approach. Sections IV-A and IV-B describe the definitions and theorems that were used both for defining realizability and the algorithms. In Section V, finally, we discuss our experience from the process of defining realizability and the various changes that were made along the way, and we report our conclusions in Section VI.

1The Coq file is available at https://github.com/andrewkatis/Coq/blob/master/realizability/Realizability.v

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II. THE COQ PROOF ASSISTANT

Coq\textsuperscript{2} is an interactive tool used to formalize mathematical expressions and algorithms, and prove theorems regarding their correctness and functionality [9]. The tool was a result of the work on the calculus of constructions [10]. Its uses in the context of computer science vary, such as being a tool to represent the structure of a programming language and its characteristics, as well as to prove the correctness of underlying procedures in compilers. Compared to other mainstream interactive theorem provers, Coq is a tool that provides a complete support on several aspects, such as the use of dependent types, as opposed to the Isabelle theorem prover [11], and proof by reflection, which is not supported by the PVS proof assistant [12]. A particularly essential feature is the tool’s support for inductive and coinductive definitions [13] [14]. Definitions using the Inductive type in Coq represent a least fixpoint of the corresponding type and are always accompanied by an inductive principle, which is implicitly used to progress through a proof by applying induction on the definition. CoInductive definitions, on the other hand, represent a greatest fixpoint to their type. They describe a set containing every finite or infinite instance of that type, and their proofs are essentially infinite processes, built in a one-step fashion and requiring the existence of a guard condition that needs to hold, for them to remain well-formed. Coinductive definitions allow a natural expression of infinite traces, which are central to our formalization of realizability, and are quite hard to prove with handwritten proofs.

III. REALIZABILITY CHECKING

In [1] we presented our approach on the problem of realizability checking, introducing an algorithm involving the use of theories, a concept that, to our best knowledge, has yet to be examined. The realizability checks are defined over assume-guarantee contracts. Informally, assumptions describe the expectations of the component on its environment, usually in terms of component inputs. The guarantees describe the properties that will hold with respect to component outputs given that the assumptions are met. A contract holds on an infinite trace if either the assumption is violated or the guarantee holds throughout the trace. A transition system describes a mechanism for constructing traces and consists of a set of initial states and a transition relation. A realizable contract therefore is one for which there exists a transition system that correctly and completely implements the contract. By correctly we mean that the transition system always produces outputs that satisfy the guarantees as long as the assumptions have always been met, and by completely we mean that the transition system never stalls, so long as the assumptions have always been met.

This definition of realizability, while providing the proper theoretical basis for realizability, is not actually useful for checking purposes. We therefore propose an alternative definition, according to which a contract is realizable if there exist so-called viable states where, for any inputs that satisfy the assumptions, there are outputs which satisfy the guarantees and lead to another viable state. In particular, the alternative definition requires that the contract be able to start in a viable state.

First, we demonstrate that the two definitions are equivalent in order to show that our “working” definition matches our theory. We can now use the viable definition as the basis of an algorithm for realizability checking. This algorithm consists of a base check, which ensures that there exists a finitely viable state for paths of length at least $n$, and an extend check to show that all the valid paths can be further extended in response to any input. Unfortunately, the complexity of the base check does not allow for an SMT-solver to handle it efficiently. Because of this, we finally propose a simplified version of the algorithm including a base check that ensures the extendability of every valid path consisting of viable states, which is only sound with regard to ‘realizable’ results.

IV. FORMALIZATION IN COQ

In the next two subsections, we will describe the formalization and proofs of these ideas in Coq. Section IV-A will describe the definitions of realizability, while Section IV-B will describe the algorithms for realizability checking and their proofs of adequacy with respect to the definitions. To provide a graphical overview of the proof process, Figure 1 describes the connections between the various definitions, lemmas, and theorems in our work.

A. Definitions

The types state and inputs are used to represent a state, and a given set of inputs. We use Coq’s Prop definition to describe the logical propositions regarding the component’s transition system through a set $I$ of initial states and the transition relation $T$ between two states and a set of inputs. Finally, the contract is defined by its assumption and guarantee, with the latter being implicitly referenced by a pair of initial and transitional guarantees ($i\text{guarantee}$ and $t\text{guarantee}$). The corresponding definitions in Coq are shown below.

\textsuperscript{2}The Coq Proof Assistant is available at https://coq.inria.fr/
\textbf{Inductive inputs} : Type := input : id → nat → inputs.

\textbf{Inductive state} : Type := st : id → nat → state.

\textbf{Definition initial} := state → Prop.

\textbf{Definition transition} := state → inputs → state → Prop.

\textbf{Definition iguarantee} := state → Prop.

\textbf{Definition tguarantee} := state → inputs → state → Prop.

\textbf{Definition assumption} := state → inputs → Prop.

A state s is \textit{reachable} with respect to the given assumptions if there exists a path from an initial state to s, while each transition in the path is satisfying the assumptions. Given a contract \((A, (G_I, G_T))\), a transition system \((I, T)\) is its \textit{realization} if the following four conditions hold:

1) \(\forall s. I(s) \Rightarrow G_I(s)\)
2) \(\forall s, i, s'. \text{reachable}_A(s) \land A(s, i) \land T(s, i, s') \Rightarrow G_T(s, i, s')\)
3) \(\exists s. I(s)\)
4) \(\forall s, i. \text{reachable}_A(s) \land A(s, i) \Rightarrow \exists s'. T(s, i, s')\)

Finally, we define that a given contract is \textit{realizable}, if the existence of a transition system, which is a realization of the contract, is proved. The formalized definitions in Coq for the \textit{reachable} state, the \textit{realization} of a contract and whether it is \textit{realizable} follow.

\textbf{Inductive reachable (s : state) (I : initial) (T : transition) (A : assumption) : Prop :=}

\(\text{rch} : ((I s) \lor (\exists (s' : state) (inp : inputs), (reachable s' I T A) \land (A s' inp) \land (T s' inp s))) \Rightarrow \text{reachable s I T A}.)\)

\textbf{Inductive realization (I : initial) (T : transition) (A : assumption) (G_I : iguarantee) (G_T : tguarantee) : Prop :=}

\(\text{real} : ((\forall (s : state), (I s) \Rightarrow (G_I s)) \land (\forall (s' : state) (inp : inputs), ((reachable s I T A) \land (A s inp) \land (T s inp s')) \Rightarrow G_T s inp s')) \land (\exists (s : state), I s) \land (\forall (s : state) (inp : inputs), (reachable s I T A \land (A s inp)) \Rightarrow (\exists (s' : state), T s inp s'))) \Rightarrow \text{realization I T A G_I G_T}.)\)

\textbf{Inductive realizable_contract (A : assumption) (G_I : iguarantee) (G_T : tguarantee) : Prop :=}

\(\text{rc} : (\exists (I : initial) (T : transition), \text{realization I T})\)
A \text{realizable} \ A G_1 G_T.$

While the definitions of realization and \text{realizable} \ A G_1 G_T. \) are quite straightforward, they are not that useful in actually checking realizability. Therefore, we proposed the notion of a state being viable with respect to a contract, meaning that the transition system continues to be a realization of the contract, while we are at such a state. In other words, a state is \text{viable} \ (\text{viable}(s)) if the transitional guarantee \( G_T \) infinitely holds, given valid inputs. Using the definition of \text{viable}, a contract is \text{realizable} if and only if \( \exists s. \ G_I(s) \land \text{viable}(s). \)

- **CoInductive viable (s : state) (A : assumption) (G_I : iguarantee) (G_T : tguarantee) : Prop :=
  \( \forall (\forall (\text{inp : inputs}), (A \text{ inp}) \rightarrow (\exists (s' : \text{state}), \ G_T \text{ inp} s' \land \text{viable} s' \ A G_I G_T)) \rightarrow \text{viable} s \ A G_I G_T. \)

- **Inductive realizable (A : assumption) (G_I : iguarantee) (G_T : tguarantee) : Prop :=
  \( \exists (\exists (s : \text{state}), G_I s \land \text{viable} s \ A G_I G_T) \rightarrow \text{realizable} A G_I G_T. \)

Having a more useful definition for realizability, we need to prove the equivalence between the definitions of \text{realizable} \ A G_1 G_T. \) and \text{realizable}. The Coq definition of the theorem was split into two separate theorems, each for one of the two directions of the proof. Towards the two proofs, the auxiliary lemma that \( \forall s. \text{reachable}_A(s) \Rightarrow \text{viable}(s) \) is necessary.

- **Lemma reachable\_viable : \forall (s : state) (I : initial) (T : transition) (A : assumption) (G_I : iguarantee) (G_T : tguarantee).
  realizable \ A G_I G_T : \Rightarrow \exists (s' : \text{state}), \ G_T(s, inp, s') \land \text{viable}(s'). \)

The informal proof of the lemma relies initially on the unrolling of the \text{viable} definition, for a specific state \( s \). Thus, we are left to prove that there exists another state \( s' \) that we can traverse into, in addition to being viable. The former can be proved directly from the conditions 2 and 4 of the definition of realization. For the latter, by the definition of \text{viable} \ on \( s \) we need to show that \( s' \) is reachable. Given the definition of \text{reachable}, we just need to prove that there exists another reachable state from which we can reach \( s' \), in one step. But we already know that \( s \) is such a state, and thus the lemma holds.

- **Theorem \text{realizable}\_contract \_implies \text{realizable} \ A G_I G_T \rightarrow \text{realizable} \ A G_I G_T. \)

The first part of the theorem requires us to prove that there exists a viable state \( s \) for which the initial guarantee holds. Considering that we have a contract that is realizable under the \text{realizable}\_contract \ A G_1 G_T. \) definition, we have a transition system that is a realization of the contract, and thus from the third condition of the realization definition, there exists an initial state \( s' \) for which, using the first condition, the initial guarantee holds. Thus, we are left to prove that \( s' \) is viable. But, by proving that \( s' \) is reachable, we can use the \text{reachable}\_viable lemma to show that \( s' \) is indeed viable.

The second direction requires a bit more effort. Assuming that we have a viable state \( s_0 \) with \( G_I(s_0) \) being true, we define \( I(s) = (s = s_0) \) and \( T(s, \text{inp}, s') = G_T(s, \text{inp}, s') \land \text{viable}(s') \). Initially, we need to prove the \text{reachable}\_viable lemma in this context, with the additional assumption that another viable state already exists \( s_0 \) in this case). Having done so, we need to prove that there exists a transition system that is a realization of the given contract. Given the transition system that we defined earlier, we need to show that each of the four conditions hold. Since \( I(s) = (s = s_0) \) and \( G_I(s_0) \) hold, the proof for the first condition is trivial. Using the assumption that \( T(s, \text{inp}, s') = G_T(s, \text{inp}, s') \land \text{viable}(s') \), we can also trivially prove the second condition, while the third condition is simply proved by reflexivity on the state \( s_0 \). Finally, for the fourth condition we need to prove that \( \forall s, \text{inp}. \text{reachable}_A(s) \land A(s, \text{inp}) \Rightarrow \exists s'. \ G_T(s, \text{inp}, s') \land \text{viable}(s') \). By applying the \text{reachable}\_viable lemma on the reachable state \( s \) in the assumptions, we show that \( s \) is also viable, if \( s_0 \) is viable, which is what we assumed in the first place. Thus, coming back into what we need to prove, and unrolling the definition of \text{viable} \ on \( s \), we have that \( \forall \text{inp}. A(s, \text{inp}) \Rightarrow \exists s'. \ G_T(s, \text{inp}, s') \land \text{viable}(s') \) which completes the proof.

**B. Algorithms**

In this section we provide a description of the formalization and proof of soundness of our realizability checking algorithms. Initially, we define an overapproximation of the definition of viability, for the finite case. Thus, a state is \text{finitely viable} for \( n \) steps
(viable\(_n(s)\)), if the transitional guarantee \(G_T\) holds for at least \(n\) steps, given valid inputs.

- **Inductive** \(\text{finitely\_viable} : \text{nat} \to \text{state} \to \text{assumption} \to \text{tguarantee} \to \text{Prop} :=\)

\[
| \text{fnvnil} : \forall \ s \ A \ G_T, \text{finitely\_viable} \ O \ s \ A \ G_T \\
| \text{fv} : \forall \ n \ A \ G_T, \text{finitely\_viable} \ n \ s \ A \ G_T \to (\forall (\text{inp} : \text{inputs}), A \ s \ \text{inp} \to (\exists s', G_T \ s' \ \text{inp} s')) \to \text{finitely\_viable} (S \ n) \ s \ A \ G_T.
\]

In addition to the \textit{finitely\_viable} definition, an under-approximation of viability is also used, called one-step extension. Therefore, a state is \textit{extendable} after \(n\) steps, if any path from \(s\), of length at least \(n\), can be further extended given a valid input.

- **Inductive** \(\text{extendable} : \text{nat} \to \text{state} \to \text{assumption} \to \text{tguarantee} \to \text{Prop} :=\)

\[
| \text{enil} : \forall (s : \text{state}) (A : \text{assumption})(G_T : \text{tguarantee}), (\forall (\text{inp} : \text{inputs}), A \ s \ \text{inp} \to \exists (s' : \text{state}), G_T \ s' \ \text{inp} s') \to \text{extendable} O s A G_T \\
| \text{ex} : \forall n s A G_T, (\forall \text{inp} s', A \ s \ \text{inp} \land G_T \ s' \ \text{extendable} n s' A G_T) \to \text{extendable} (S \ n) \ s \ A \ G_T.
\]

1) An Exact Algorithm for Realizability Checking:

The algorithm that we propose for realizability checking consists of two checks. The \textit{BaseCheck}(\(n\)) procedure ensures that \(\exists s. G_1(s) \land \text{viable}_n(s)\), while \textit{ExtendCheck}(\(n\)) makes sure that the given state from \textit{BaseCheck} is extendable, for any \(n\).

- **Definition** \textit{BaseCheck}(\(n : \text{nat}\)) (\(A : \text{assumption}\)) \((G_T : \text{tguarantee})\) \((G_T : \text{tguarantee})\) := \(\exists (s : \text{state}), G_1(s) \land \text{finitely\_viable} n s A G_T\).

- **Definition** \textit{ExtendCheck}(\(n : \text{nat}\)) (\(A : \text{assumption}\)) \((G_T : \text{tguarantee})\) := \(\forall s A G_T, \text{extendable} n s A G_T\).

Using the \textit{BaseCheck}(\(n\)) and \textit{ExtendCheck}(\(n\)), the algorithm determines the realizability of the given contract, using the following procedure.

\[
\text{for} \ n = 0 \ \text{to} \ \infty \ \text{do} \\
\text{if not} \ \text{BaseCheck}(n) \ \text{then} \ \text{return} \ \text{"unrealizable"} \\
\text{else if} \ \text{ExtendCheck}(n) \ \text{then} \ \text{return} \ \text{"realizable"} \\
\text{end if} \\
\text{end for}
\]

Using the definitions of \textit{BaseCheck} and \textit{ExtendCheck}, we proved the algorithm's soundness, both for the 'unrealizable' and 'realizable' case. The main idea behind the proof of soundness for the 'unrealizable' result is to prove the contrapositive, that is, given a realizable contract, there exists a natural number \(x\) for which \textit{BaseCheck}(\(x\)) holds. Unfolding the definition of \textit{BaseCheck}(\(x\)), we need to show that \(\exists s. G_1(s) \land \text{viable}_n(s)\). Knowing that our assumption \textit{realizable\_contract} \(A G_I G_T\) is equivalent to the \textit{realizable} definition, provides us with a state \(s'\), for which \(G_1(s') \land \text{viable}(s')\) holds. Here, we need an additional lemma, according to which \(\forall s, n. \text{viable}(s) \Rightarrow \text{viable}_n(s)\) (stated as \textit{viable\_implies\_finitely\_viable} below). Thus, using the lemma on \textit{viable}(\(s'\)) with \(n = x\), we get that \textit{viable}_x(s'), completing thus the proof.

- **Lemma** \textit{viable\_implies\_finitely\_viable} : \(\forall s A G_I G_T n, \text{viable} s A G_I G_T \to \text{finitely\_viable} n s A G_T\).

- **Theorem** \textit{unrealizable\_soundness} : \(\forall (s : \text{state}) (I : \text{initial}) (T : \text{transition}) (A : \text{assumption}) (G_I : \text{tguarantee}) (G_T : \text{tguarantee})\), \((\exists n, \neg \text{BaseCheck} n A G_I G_T) \to \neg \text{realizable\_contract} A G_I G_T\).

For the soundness of the 'realizable' result, we first need to prove two lemmas. The first one, \textit{extend\_viable\_shift}, shows the way that \textit{Extend}(\(s\)) can be used to shift \textit{viable}_n(s). The proof for this lemma is done by using induction on \(n\). The base case is proved trivially, by unfolding the definitions of \textit{extendable} and \textit{finitely\_viable} in the assumptions. For the inductive case, we assume that the same state \(s\) is extendable and finitely viable for paths of length \(n + 1\), and try to prove that there exists a finitely viable state \(s'\) for paths of length \(n + 1\), to which we can traverse from \(s\), with the contract guarantees still holding after the transition. By considering that \(s\) is extendable for paths of length \(n + 1\), we can use it as that potentially existing state in the proof, requiring that we can transition from \(s\) to \(s'\), with the transitional guarantees staying true, and \(s\) being finitely viable for paths of length \(n + 1\). The former is true through the definition of \textit{extendable}, while the second is an already given assumption by the inductive step.

- **Lemma** \textit{extend\_viable\_shift} : \(\forall (s : \text{state}) (n : \text{nat}) (\text{inp} : \text{inputs}) (A : \text{assumption}) (G_I : \text{tguarantee}) (G_T : \text{tguarantee})\), \((\text{extendable} n s A G_T \land \text{finitely\_viable} n s A G_T \land A \ s \ \text{inp}) \to (\exists s', G_T \ s' \ \text{inp} s' \ \land \text{finitely\_viable} n s' A G_T)\).

- **Lemma** \textit{fv\_ex\_implies\_viable} : \(\forall (s : \text{state}) (n : \text{nat}) (A : \text{assumption}) G_I G_T\),
(finitely_viable n s A G_T ∧ ExtendCheck n A G_T) → viable s A G_1 G_T.

- Theorem realizable_soundness : ∀ (s : state) (I : initial) (T : transition) A G_1 G_T, (∃ n, (BaseCheck n A G_1 G_T ∧ ExtendCheck n A G_T)) → realizable_contract A G_1 G_T.

To prove the theorem, we instead try to prove the equivalent for the realizable definition instead. The existence of a state for which the initial guarantees hold is derived from the assumption that BaseCheck holds for a finitely viable state, while the proof that the same state is also viable, comes from the use of the fv_ex_implies_viable lemma, which is proved through the use of extend_viable_shift.

1) An Approximate Algorithm for Realizability Checking: Following the definition of our approach, we notified the problematic nature of BaseCheck(n) having 2n quantifier alternations, which cannot be handled efficiently by an SMT-solver. To that end, we proposed a simplified version of the BaseCheck(n) procedure, called BaseCheck'(n), stated as BaseCheck_simple below.

- Definition BaseCheck_simple (n : nat) (A : assumption) (G_1 : iguarantee) (G_T : tguarantee):= ∀ s, (G_1 s) → extendable n s A G_T.

- Lemma finitely_viable_plus_one : ∀ s n A (g : iguarantee) (G_T : tguarantee) (inp : inputs), (extendable n s A G_T ∧ finitely_viable n s A G_T) → finitely_viable (S n) s A G_T.

- Theorem BaseCheck_soundness : ∀ n A (G_1 : iguarantee) (G_T : tguarantee) (i : inputs), (∃ s, G_1 s) ∧ (∀ k, (k ≤ n) → BaseCheck_simple k A G_1 G_T)) → BaseCheck n A G_1 G_T.

The simplified BaseCheck'(n), while being an easier instance for an SMT-solver, is not sound for the 'unrealizable' case, falsely reporting some realizable contracts not to be so. Nevertheless, we proved the modified algorithm's soundness for the 'realizable' result, with the use of an auxiliary lemma.

The lemma, finitely_viable_plus_one simply refers to the fact that an extendable and finitely viable state s, for a given number of steps n, is also finitely viable for n + 1 steps. The proof is done by induction on n. The base case is trivially proved, by the definition of finitely_viable, and the assumption that s is extendable. For the inductive case, we use the inductive hypothesis which leaves us to prove the assumptions on a specific state s. The extendability is trivially shown since we already know that s is extendable for paths of length n + 1, with the same idea being applied to prove that s is finitely viable for n.

Finally, the proof of soundness for the 'realizable' result of the BaseCheck'(n) procedure is done by using induction on n. The base case is trivially true, using the fact that all paths of zero length are finitely viable. The inductive step then requires us to prove that BaseCheck(n + 1) holds. In order to do so, we need to construct the inductive hypothesis' assumption, as a separate assumption to the theorem's scope. By applying the inductive hypothesis to the newly created assumption, we have that BaseCheck(n) holds. By unrolling the definition of BaseCheck(n) and applying the lemma finitely_viable_plus_one on the extracted state, say x, we finally prove that x is extendable through the definition of BaseCheck'(n), completing the proof at the same time.

Figure 1 provides a simplified proof graph of all the necessary definitions and partially, for graph simplicity purposes, the way that they are used towards proving the lemmas and theorems stated in this paper.

V. DISCUSSION

While our work on realizability is based on simple definitions, formalizing them and refining the algorithms in Coq involved various challenges. Expressing realizability as an incremental SMT obligation required more effort than what our initial thought, which was mainly based on k-inductive formulations. On top of that, proving the lemmas and theorems using Coq helped us discover minor errors in our informal statements. For example, our proof of the one-way soundness theorem for the simplified BaseCheck in [1] lacks the necessary assumption that there exists a state for which the initial guarantees hold. Another example is that we forgot to include initial states in our definition of reachable states in the informal proof. Having to go use a mechanized theorem prover exposed some missing knowledge in the informal text, and helped us refine and provide a more precise version of the theorem. Although there were no substantial errors in the hand proofs, the use of Coq improved both our theorems and proofs.

VI. CONCLUSION

The work in this paper was particularly important towards verifying our approach and learning more about the actual functionality of the algorithm. Interactive theorem provers like Coq, provide the necessary support to define the notions and assertions, while being able to effectively prove theorems in a far more convenient and reassuring way, in contrast to hand-written, informal proofs, especially when it comes down to tracking
formulas containing alternating quantifiers. Furthermore, the procedure of proving the theorems in an interactive way with a tool, allowed us to refine our definitions. Additionally, the time that was required was minimal, when compared to the process of considering the informal proofs and writing down our requirements in English. The most important outcome was the proof of correctness in our approach, that enabled us to provide a complimentary set of definitions and proofs, easily processed by an experienced Coq user.

To conclude, there is substantial additional work that could be performed in terms of fleshing out the formalism used in the proof for our particular implementation. For example, we could define the structure and types of inputs and outputs, and describe how transition systems are realized in the AGREE tool suite. However, the work that has been performed shows the soundness of the proof system and our algorithms with respect to proofs of realizability, allowing us to proceed with very high confidence as to the correctness of our approach.

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