A short critical review of the concept of decoherence, its consequences, and its possible implications for the interpretation of quantum mechanics is given.

Keywords: decoherence, classical limit, superselection, quantum events, quantum causality, quantum measurement.

1. WHAT IS MEANT BY DECOHERENCE?

There seems to be some confusion in the literature not only on what may actually be achieved by decoherence, but also on how this concept has to be defined. I will here “consistently” use it in terms of wave functions (not “histories”), since state vectors represent the established kinematical concept of quantum theory, while events (of which histories are often assembled) will turn out to be derivable in a certain sense in terms of wave functions just by means of decoherence.

Therefore, by decoherence I mean the practically irreversible and practically unavoidable (in general approximate) disappearance of certain phase relations from the states of local systems by interaction with their environment according to the Schrödinger equation. Since phase relations cannot absolutely disappear in a unitary evolution, this disappearance can only represent a delocalization, which means that the phases “are not there” any more, neither in the system nor in the environment, although they still exist in the total state that describes both of them in accordance with quantum nonlocality.

This can also be described as the dynamical disappearance of non-diagonal elements from the subsystem density matrices in a certain basis. However, the concept of a density matrix would then first have
to be defined (or justified) in terms of the conceptually presumed wave functions. This does not present a problem “for all practical purposes” (FAPP), that is, when using the “rules” of quantum theory for calculating probabilities for outcomes of measurements, for example in the form of expectation values

$$\langle A_{local} \rangle = \text{Trace}[A_{local}\rho_{total}] = \text{Trace}[A_{local}\rho_{local}]$$

(1)

in a Hilbert space $H_{total} = H_{local} \otimes H_{env}$, where $\rho_{local} = \text{Trace}_{env}[\rho_{total}]$, while $\rho_{total} = |\psi_{total}\rangle\langle\psi_{total}|$ may represent a wave function. Obviously, this procedure FAPP makes essential use of the probability interpretation (albeit without using any assumptions about the conceptual nature of the elements of the corresponding statistical ensembles, or on how they may possibly arise dynamically). Conclusions derived in terms of the density matrix should therefore not be used to refer to the probability interpretation itself in order to avoid circular arguments.

Nonlocal phase relations are thus equivalent to entangled states of spatially separated systems (quantum nonlocality). They can be conveniently described by their Schmidt canonical form

$$\psi_{total} = \sum_n \sqrt{p_n} \phi_{local}^n \Phi_{env}^n ,$$

(2)

which is generically unique, and directly corresponds to the diagonal representation of the density matrix for both subsystems, e.g.

$$\rho_{local} = \sum_n |\phi_{local}^n\rangle p_n \langle\phi_{local}^n| .$$

(3)

This density matrix has a form as though it represented an ensemble of wave functions $\phi_{local}^n$ with respective probabilities $p_n$. Equation (2) demonstrates, however, that it does not. This confusion between proper and “improper” mixtures has given rise to the most frequent misinterpretation of decoherence as leading to ensembles, and thus as deriving the collapse of the wave function as a stochastic process. Such a replacement of improper by proper mixtures is clearly part of the quantum state diffusion model (insofar as it goes beyond decoherence), and it is similarly used when “selecting” consistent histories from their superpositions (thus not from ensembles). Any subsequent restriction “FAPP only” will here solely refer to this “as though” — not to the arising entanglement with the environment or its dynamical consequences for the corresponding formal density matrix. Therefore, it does neither mean “wrong” nor “useless”.

For time-dependent states $\psi(t)$, their entanglement (2) must in general also depend on time. It may thus be entirely caused by interaction, for example according to von Neumann’s (unitary part of) ideal measurements

$$\sum_n c_n \phi_{local}^n \Phi_{env}^n \rightarrow \sum_n c_n \phi_{local}^n \Phi_{env}^n .$$

(4)
This measurement-like process describes pure decoherence of the system in neglecting any re-action (or “perturbation” by the environment). Decoherence has therefore also been called “continuous measurement” (continuous in the sense of a permanent or unavoidable coupling).

The dynamical evolution described by Eq. (4) contains an obvious arrow of time, based on the belief that entanglement must be retarded, that is, have causes in the past (“quantum causality”). Although entanglement is not a statistical correlation (not based on incomplete information), Eq. (4) is analogous to Boltzmann’s assumption of molecular chaos, since it neglects any recoherence (any local effects of the arising quantum correlations in the future). In both cases, the existence of fluctuations demonstrates that this time arrow is not a law, but a fact in accordance with time-symmetric laws.

Genuine measurements (for example, of the passage of a particle through a slit of an interference device) are known to destroy interference for trivial reasons: the events following the observed passages may be counted separately, and may then simply be added. However, this is not required for decoherence, since (a) there need not be definite passages, and (b) the state of the environment need not correspond to a controllable “pointer state”, from which information about the passage could be retrieved. For this reason one should rather speak of a measurement-like interaction with the environment.

Decoherence is thus a normal consequence of interacting quantum mechanical systems. It can hardly be denied to occur — but it cannot explain anything that could not have been explained before. Remarkable is only its quantitative (realistic) aspect that seems to have been overlooked for long. Entanglement is the norm — not the exception —, and it must have important consequences. In particular, all macroscopic systems are never approximately isolated, and must thus not be described by wave packets obeying a Schrödinger equation (not even when a WKB approximation applies). Furthermore, since a macroscopic system together with its environment is even more macroscopic, quantum theory can provide a consistent description only when applied to the universe as a whole. That means, one has to assume $\psi_{total} = \psi_{universe}$ in order to get unambiguous results.

2. ELEMENTARY SYSTEMS UNDER DECOHERENCE

In order to illustrate the consequences of realistically taking into account the environment of quantum systems FAPP, this section lists some important examples of decoherence. Because of the rich literature on the subject I will here refrain from citing individual papers, and instead refer only to our recent review [1], which has just appeared, and which contains a bibliography considerably exceeding the length of this article. Further contributions — in particular some impressive work by the Madrid group(s) — can be found in these proceedings.
2.1 Trajectories of Macroscopic Variables

Interfering paths of mass points are equivalent to spatial waves. In a two-slit experiment with “bullets” (dust particles or even large molecules), not only their passage through the slits, but the whole path would unavoidably be measured under all realistic circumstances. No superposition of different positions of such objects can “be there” and lead to interference in the probability distributions for subsequent local measurements. In this respect, classical objects resemble alpha-particles in a cloud chamber. The resulting entanglement leads to a density matrix for their center of mass motion as though it represented an ever-increasing ensemble of narrow wave packets following slightly stochastic trajectories. This is dynamically described by a master equation

$$i\frac{\partial \rho(x, x', t)}{\partial t} = \frac{1}{2m} \left( \frac{\partial^2}{\partial x'^2} - \frac{\partial^2}{\partial x^2} \right) \rho - i\Lambda(x - x')^2 \rho \quad ,$$

which (including its coefficient \(\Lambda\)) can thus be derived from a universal Schrödinger equation. One does not have to postulate a phenomenological semigroup to characterize open systems.

Since this situation has been sufficiently discussed in the literature, let me here only emphasize that the required “measurements” must indeed be practically irreversible in order to avoid the possibility of “quantum erasure” (restoration of interference). Such a restoration would require, however, that every single scattered particle were completely recovered. (Experiments demonstrating quantum erasure in certain microscopic systems do not represent restoration of the whole initial superposition.) Every classical phenomenon, even in “reversible” classical mechanics, is based on such irreversible processes, with a permanent production of entropy that is macroscopically negligible but large in terms of bits.

2.2 Molecular Configurations and Robust States

Chiral molecules, such as sugar, represent another simple example of systems under decoherence. They are described by wave functions, but in contrast to the analogous spin-3 state of the ammonia molecule, for example, not by energy eigenstates. The reason is that it is their chirality (not their parity) which is continuously “measured” by the scattering of air molecules (for sugar under normal conditions on a decoherence time scale of the order \(10^{-9} \text{sec}\)). The measurement of parity in sugar, or the preparation of (very short-lived) sugar molecules in their energy eigenstates, is therefore practically excluded, since it would require an even stronger coupling to the measurement device. (The molecule \(PH_3\), discussed at this conference by Gonzalo, forms an amazing intermediate situation, with parity eigenstates expected to exist under exceptional environmental conditions.)
A further dynamical consequence then holds that each individual molecule in a bag of sugar must retain its chirality, while a parity state — if it had come into existence in a mysterious or expensive way — would almost immediately “collapse” into a local mixture of both chiralities with equal probabilities. Parity is thus not conserved for sugar molecules. The resulting mixture would also be diagonal in the parity representation if the diagonal elements were exactly equal. However, every actually resulting value of chirality would be permanent, that is, it would always be confirmed when measured again.

This dynamical “robustness” of the chiral wave function seems to characterize what we call a “real” (in the operational sense) or “classical” property — just like the spot on a photographic plate, or any other “pointer state” of a measurement device. It also seems to be essential for the physical realization of memory (such as in DNA, brains or computers — with the exception of quantum computers, which are instead extremely vulnerable to decoherence, as was discussed here by Ekert). Robustness is thereby compatible with a (regular) time dependence, as exemplified by the wave packet describing the center of mass motion of a bullet.

Chemists are used to describing the motion of the nuclei in large molecules classically (for example by rigid configurations), while representing the electrons by wave functions. In general, they attribute this asymmetry to a Born-Oppenheimer approximation. This argument is definitely insufficient, since a straight-forward application of this approximation to molecules would lead to energy (and angular momentum) eigenstates, with vibrational and rotational spectra as known for the hydrogen (or ammonia) molecule. The same insufficient argument is now found in quantum gravity, where it is used to justify classical spacetime by employing a BO approximation with respect to the Planck mass (see Sect. 2.4).

Pseudo-classical behaviour can in both cases be explained by means of decoherence again: the positions of nuclei are permanently measured by scattering molecules or photons. But why only the nuclei (or ions), and why not even they in very small molecules? The answer is quantitative and based on a delicate balance between internal dynamics and interaction with the environment, whereby the density of states plays an essential role. (Much work on details remains to be done!) Depending on the quantitative situation, one will either obtain an approximately unitary evolution, a master equation (with time asymmetry arising from quantum causality), or complete freezing of the motion (quantum Zeno effect). The situation becomes simple again only for a free nucleus, which is described by Eq. (5).

2.3 Charge Superselection

Gauß’ law in the form \( q = \int \mathbf{E} \cdot d\mathbf{S}/4\pi \) tells us that a local charge is correlated with its Coulomb field at any distance. It is a matter of
taste whether this correlation is considered as kinematics, or as \textit{caused} in the form of the retarded field resulting from the charge in the past. Conceived quantum superpositions of different charge values,

\[ \sum_q c_q \psi_{q_{\text{total}}} = \sum_q c_q \chi_q \Psi_q \{field\} = \sum_q c_q \chi_q \prod_r \Psi_q \{field(r)\} , \tag{6} \]

would therefore be nonlocal. Here, \(\chi_q\) is the local charge state, \(\Psi_q \{field\} = \prod_r \Psi_q \{field(r)\}\) a pseudo-classical field functional representing its Coulomb field, symbolically written as a direct product of states on spheres with radius \(r\). The local charge system itself (possibly including its field within a small sphere) would then be described by a density matrix of the form

\[ \rho_{\text{local}} = \sum_q |\chi_q\rangle c_q |\chi_q\rangle \langle\chi_q| , \tag{7} \]

since the field states outside \(r\) can be assumed to be mutually orthogonal for different charges \(q\). The charge is therefore decohered by its own Coulomb field, and no superselection rule has to be \textit{postulated}.

While this result is satisfactory for the theory, a more practical question is at what distance, and on what time scale, two compensating charges (or a charge in a superposition of different positions) are decohered (here possibly in a reversible manner) by the resulting dipole field. Theoretical and experimental contributions to an answer can be found in these proceedings (cf. Sols or Hasselbach). Notice that decoherence will thus allow one to distinguish between quantum field theory and action at a distance (where decoherence would occur only after the field has reached and changed the state of absorbing matter).

The gravitational field caused by any mass is analogous to the Coulomb field of a charge. Superpositions of different masses should therefore be decohered by the quantum state of spatial curvature. However, there is no elementary mass, and nobody has as yet reliably estimated at what mass difference decoherence by the correlated quantum state of curvature becomes essential. Quite obviously, some superposition of different mass-energy remains dynamically relevant in the form of (observed) time-dependent quantum states of local systems.

\section*{2.4 Classical Fields and Gravity}

Not only the quantum states of charged particles are decohered by their fields, the quantum field states are in general also decohered by source particles on which they react. \textit{Coherent states} (which represent classical fields) have in this situation been shown to be robust in a similar way as chiral molecules or wave packets for the positions of macroscopic objects. This is obviously the reason why no superpositions of macroscopically different “mean fields”, or different vacua (as they may arise through spontaneous symmetry breaking), have ever been observed.
These arguments must as well apply to quantum gravity. One does not have to know its precise form (for example, after its expected uni-
fication with other forces) in order to conclude the existence of en-
tangled superpositions of matter and geometry (as far as this distinction
remains valid). Therefore, the classical appearance of spacetime geom-
etry, with its lightcone structure as presumed in conventional quantum
field theory, is no reason not to quantize gravity, since it should be
explained by decoherence in the same way as the classical appearance
of a bullet or an electromagnetic field. The resulting density matrix
(functional) for the gravitational tensor field must be expected to be-
have as though it represented a statistical mixture of different classical
curvature states (to which the observer is correlated — see Sect. 4).
The beauty of Einstein’s theory can hardly be ranked so much higher
than that of Maxwell’s in order to justify its exemption from a well es-
tablished and general principle of physics (in particular, as this would
lead to incompatibilities with the uncertainty relations for matter —
known from the Bohr-Einstein debate).

The entropy characterizing a black hole (or the thermal field of an
accelerating Unruh detector) is thus due to a similar “local perspective”
of a strongly entangled subsystem as the entropy produced according
to the master equation \(\rho\) for a mass point. An event horizon need
not be different from any other pseudo-classical property, and even
the disappearance of information behind a classical horizon would not
describe a real collapse. For example, a succession of simultaneities car-
ying global states which contain an Unruh detector along an external
world line may be chosen to remain all entirely outside the horizon (as
described by the Schwarzschild time coordinate).

2.5 Quantum Jumps

Exponential decay of excited states is usually regarded as the standard
manifestation of a fundamental quantum indeterminism. Stochastic
decay into a single channel would in fact lead to an exact exponentially
decreasing non-decay probability. However, the Schrödinger equation
for a mass point in a potential barrier, for example, leads to an approxi-
mately exponential time-dependence of the wave function. It represents
the superposition (not an ensemble) of different decay times. The cor-
responding interference is incompatible with exact exponential decay
even in free space, and it has been confirmed in reflecting cavities as
“coherent state vector revival”.

On the other hand, if the decay fragments interact strongly with
the surrounding matter, any interference between “decayed” and “not
yet decayed” must practically irreversibly vanish on a very short deco-
herence time scale at every moment of time. The corresponding appar-
ent quantum jumps at almost discrete times are observed as “clicks”
of a Geiger counter (in analogy to pseudo-classical local “spots” on a
photographic plate). They have also been seen with individual atoms
coupled to laser fields as a sudden appearance and disappearance of “dark periods”. For electrons tunneling from the tip of a metal needle, decoherence between different emission times may explain the short longitudinal coherence lengths reported at this conference by Hasselbach (while the dipole moments discussed in Sect. 2.3 are in these experiments represented by the transversal separation of the interfering paths). A decaying system strongly interacting with its environment is thus effectively described by a stochastic process with discrete decay times, and the resulting time dependence will be almost exactly exponential (as long as quantum causality governs decoherence).

It seems that this situation of continuously monitored decay has led to the myth of quantum theory as a stochastic theory that describes fundamental discontinuous quantum events. For example, Bohr formulated in 1928 that “the essence (of quantum theory) may be expressed in the so-called quantum postulate, which attributes to any atomic process an essential discontinuity, or rather individuality . . .” (my italics). If this were correct, we would not even be able to describe lasers. Heisenberg and Pauli similarly emphasized that their preference for matrix mechanics originated in its (as it now seems misleading) superiority in describing discontinuities. According to the Schrödinger equation, the underlying entanglement processes occur smoothly — but recall now that all conclusions regarding ensembles are still based FAPP on the yet unspecified probability interpretation (1)!

3. CONSEQUENCES FOR THE MOTIVATION OF VARIOUS INTERPRETATIONS OF QUANTUM THEORY

How has the fundamental probability interpretation, used in Eq. (1), then to be understood? And where precisely does it have to be applied in view of the problem of where to position the Heisenberg cut? The greatly differing opinions of physicists on this point are surprising, and proof that the answer does not matter FAPP. Obviously, decoherence cannot give an answer either, although it may help to recognize some interpretations as being essentially motivated by prejudice or tradition.

Many seem to believe, in the tradition of Heisenberg, that Eq. (1) describes probabilities for classical quantities to “enter reality”. However, we have seen that these classical quantities can be described by robust wave packets (in a sense which comes close to Schrödinger’s original attempts, but overcomes the problem of their dispersion by means of decoherence). It is then not at all surprising that classical concepts, if nonetheless assumed to apply, have to be restricted either in validity or in their observability in order to avoid contradictions. There is a rich vocabulary for de- or circumscribing the first possibility (uncertainty, complementarity, new logic, negative probabilities, consistency etc.). Those of these restrictions which are well defined can easily be explained in terms of wave packets (for example, by means of the Fourier theorem). The second possibility is represented by Bohm’s
“surrealistic trajectories”. As a reflection on some contributions to this conference let me also emphasize that it appears useless and even quite misleading to describe a certain subset of experiments within classical concepts, while simply neglecting the crucial rest.

A second class of interpretations suggests a description of reality in terms of yet unknown (“hidden”) variables. However, the evidence related to Bell’s inequalities clearly demonstrates (within limitations which must hold for all empirical proof — as known at least since David Hume, and as some physicists now seem to be rediscovering in detail) that these new variables would have to contain precisely the same nonlocality as entangled wave functions on configuration space.

Then why reject the wave function itself as describing “real” physical states? I think that this natural position is now strongly supported by the beautiful experiments with individual atoms in cavities (Schrödinger cat states, quantum engineering, phase space tomography etc.), which were discussed at this conference by several speakers. Thus, in a third class of interpretations of the fundamental probabilities one considers jumps between wave functions (usually described by a “stochastic Schrödinger equation”). However, why should the fundamental processes represented by (1) then occur precisely where they appear to occur and are readily described as decoherence?

4. CONSEQUENCES OF DECOHERENCE FOR THE INTERPRETATION OF QUANTUM MEASUREMENTS

The probabilities described by Eq. (1) are usually understood as referring to measurements. In many situations we may be satisfied just with entangled states, but individual measurements have definite outcomes, while von Neumann’s or any similar unitary interaction lead to their superposition. This contrast defines what is usually called the measurement problem. John Bell insisted that we should not speak of measurements, but that would not resolve the dilemma. He obviously meant that we should treat measurements (and the occurrence of events?) as a normal physical process. That is precisely what many people are trying to do — but in terms of which kinematical concepts?

When describing macroscopic objects (such as measurement and registration devices) FAPP by the apparent ensembles of wave functions resulting through decoherence, we have in fact applied the probability interpretation (II) to them. Completion of a measurement for this purpose thus requires including the reading of the pointer state. The corresponding chain of interactions would read

\[
\left( \sum_n c_n \phi_n \right) \psi_{\text{app}}^n \chi_0 \Phi_{\text{obs}} \xrightarrow{\text{meas.}} \left( \sum_n c_n \phi_n \psi_{\text{app}}^n \right) \chi_0 \Phi_{\text{obs}}
\]

\[
\xrightarrow{\text{decoh.}} \left( \sum_n c_n \phi_n \psi_{\text{app}}^n \chi_{\text{env}} \right) \Phi_{\text{obs}} \xrightarrow{\text{obs.}} \phi_{\text{meas}} \psi_{\text{app}}^n \chi \Phi_{\text{obs}} (8)
\]
with probability $|c_{n_0}|^2$, where the last step is itself a long and complex chain. Only when it has been completed is the assumption of a collapse of the wave function without prejudice empirically indicated. Probabilities can then only be understood as representing frequencies $f(n)$ in the results $n_{01}, n_{02}, \ldots, n_{0N}$ of long series of $N$ equivalent measurements, described by *one* final wave function containing an observer state $\Phi_{n_{01}, n_{02}, \ldots, n_{0N}}$ (cf. Mittelstaedt’s contribution to this volume).

If a real collapse occurred before the onset of decoherence, it would almost certainly have directly observable consequences. If it occurred later in the chain of interactions, decoherence could remain essential for the classical appearance of the world. This situation would then lead to a “partial Everett interpretation”, in which certain components of the wave function, although they existed, would appear to be absent to an appropriate observer state. If decoherence itself practically triggered a collapse (similar to the GRW model), it would again be dangerously close to observation, since the environmental situation may be altered to some extent, while fundamental dynamical terms had to be fixed.

Thus recall von Neumann’s motivation for the collapse: his aim was to re-establish a psycho-physical parallelism based on definite states $\Phi_{n_0}^{\text{obs}}$ of the observer (by whatever system an observer may be physically represented). However, there is a definite state $\Phi_n^{\text{obs}}$ in each robust component of a global superposition $\sum c_n \Phi_n^{\text{app}} \chi_n^{\text{env}} \Phi_n^{\text{obs}}$. Each component behaves then dynamically as though it represented the complete and only world (even though this behaviour is here derived from the assumption that it does not).

These branching robust components describe microscopic histories which do not individually determine their past. The essential reason for the dynamical independence (robustness) of the arising branches in their future (the absence of recoherence) is not simply the linearity of the Schrödinger equation, but paradoxically the very same retarded nature of quantum entanglement (quantum causality — see Eq. (4)) which seems to be responsible for the consistency (hence existence) of documents about macroscopic history (the “fixed past”).

According to this picture it is the apparently observed quasi-classical world that exists only FAPP (*viz.*, with respect to each $\Phi_n^{\text{obs}}$ and his or her “friends” $\Phi_n^{\text{obs'}}$). The choice between the Everett interpretation and a vaguely located collapse thus remains presently a matter of taste.

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**REFERENCES**

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