We present partial numerical results in the Minimal Supersymmetric Standard Model with soft breaking of supersymmetry, and radiative breaking of the electroweak symmetry. We impose the additional relation \( m_b = m_\tau \) at the GUT scale. For the special case of the strict no-scale model, in which global supersymmetry breaking arises solely from soft gaugino masses, we find that \( M_t \) is expected to be lighter than \( \sim 128 \) GeV. A higher upper bound for \( M_t \) of \( \sim 138 \) GeV is predicted, if \( M_b \) is at the lower end of its experimental uncertainty.

1. Introduction

In this paper, I report some preliminary results of work in progress, done in collaboration with E. Piard and D. Castaño. In the last few years, it has become apparent, using the ever increasing accuracy in the measurement of the strong coupling, that Supersymmetry (SUSY) affords an elegant means to achieve gauge coupling unification \([1, 2, 3]\) at scales consistent with Grand Unified Theories (GUTs) \([4, 5, 6, 7, 8]\). Whereas in the Standard Model (SM) the three gauge couplings unify “two by two” forming the “GUT triangle,” in the simplest Minimal SUSY Extension of the SM (MSSM) these gauge couplings spectacularly unify at a point (within the experimental errors in their values). Given that the scale of unification in these models is generally above the lower bound set by proton decay, these supersymmetric grand unified theories (SUSY-GUTs) have regained increasing interest. Constraints from the Yukawa sectors of such models have also yielded interesting predictions for various low energy parameters including the top quark mass \([9, 10, 11, 12]\).

Crucial to these analyses has been the use of the Renormalization Group (RG) in extrapolating from the GUT scale to experimental scales. Here, we use of our previous RG analysis for the standard model\([13]\), including the supersymmetric two loop \( \beta \) functions for the MSSM. Also soft symmetry breaking (SSB) terms are added. These lead to a nondegenerate superparticle spectrum as well as to the radiative breaking of the electroweak symmetry. A brief discussion of the effective one loop potential is presented. The boundary conditions at the unification scale in
these minimal low energy supergravity models are discussed. Next the numerical procedure employed is described. The treatment of thresholds and the "special" form of the $\beta$ functions needed is discussed next. Similar analyses have appeared in the literature [14, 15, 16, 17].

In the following, we only concern ourselves with the simplest schemes which have a chance of reproducing the data. If these are indeed those chosen by nature, they have some very distinct signatures. The top quark, although allowed by the standard model to be as heavy as $200 \text{ GeV}$, comes out much lighter. The lightest Higgs particle also comes out light, so light in fact that it may be very hard to detect. Another feature is that these models all predict the existence of a stable particle, the LSP, whose abundance can be calculated, based on the standard cosmological scenario. LSPs are typically too light to bring $\Omega$ to one, the preferred value of theorists.

2. Minimal Supersymmetric Standard Model

In the minimal supersymmetric extension of the standard model, every particle has a supersymmetric partner, with spin differing by a half [18]. Supersymmetry requires a second Higgs field with opposite hypercharge to the first as the superpotential cannot contain both a field and its complex conjugate. The second Higgs is also needed for both chiral $U(1)$ and $SU(2)$ global anomaly cancellation and to give its sector a mass. For renormalizable theories, the superpotential can have at most degree three interactions. The superpotential for the MSSM is (suppressing the $SU(2)$ and Weyl metrics)

$$W = \hat{\pi} Y_u \hat{\Phi}_u \hat{Q} + \hat{\alpha} Y_d \hat{\Phi}_d \hat{Q} + \hat{\tau} Y_e \hat{\Phi}_e \hat{L} + \mu \hat{\Phi}_u \hat{\Phi}_d .$$

where the hat indicates a chiral superfield and the overline denotes a left handed $CP$ conjugate of a right handed field, $\overline{\psi} = i\sigma_2 \psi_R^\ast$. The usual Yukawa interactions are accompanied by new Yukawa interactions among the scalar quarks and leptons and the Higgsinos in the supersymmetric Lagrangian. There are also new gauge Yukawa interactions involving the gauginos. The new purely scalar interactions form the scalar potential which is positive definite in supersymmetric theories. The scalar potential will be discussed in a subsequent section. A remarkable aspect of supersymmetry is that all these new interactions require no new couplings. Table I displays the $SU(3) \times SU(2) \times U(1)$ quantum numbers of the chiral (all left handed) and vector superfields of the MSSM.

The last term is put in by hand to avoid the exact Peccei-Quinn (PQ) symmetry that the superpotential would otherwise exhibit. Although there are other ways to treat the PQ symmetry, including producing an axion, we leave the other possibilities to a future investigation.

3. Minimal Low Energy Supergravity Model

Since no super particles have been observed experimentally, supersymmetry,
Table 1: Quantum Numbers

|       | \(\tilde{Q}\) | \(\tilde{u}\) | \(\tilde{d}\) | \(\tilde{L}\) | \(\tilde{e}\) | \(\tilde{L}\) | \(\tilde{e}\) | \(\tilde{W}\) | \(\tilde{B}\) |
|-------|---------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| \(U(1)\) | +\(\frac{1}{2}\) | -\(\frac{1}{2}\) | +\(\frac{1}{2}\) | -\(\frac{1}{2}\) | +1 | +\(\frac{1}{2}\) | -\(\frac{1}{2}\) | 0 | 0 |
| \(SU(2)\) | 2 | 1 | 1 | 2 | 2 | 1 | 3 | 1 |
| \(SU(3)\) | 3 | 3 | 3 | 1 | 1 | 1 | 8 | 1 | 1 |

if truly present in nature, must be broken. One way to accomplish this breaking is to add to the Lagrangian soft breaking terms.

The most general soft symmetry breaking potential for the MSSM can be written (including gaugino mass terms)

\[
V_{\text{soft}} = m_{\tilde{Q}}^2 \tilde{Q}^\dagger \tilde{Q} + m_{\tilde{u}}^2 \tilde{u}^\dagger \tilde{u} + m_{\tilde{d}}^2 \tilde{d}^\dagger \tilde{d} + B \mu (\tilde{u} \tilde{d} + \text{h.c.})
\]

\[
+ \sum_i (m_{\tilde{Q}_i}^2 \tilde{Q}_i^\dagger \tilde{Q}_i + m_{\tilde{u}_i}^2 \tilde{u}_i^\dagger \tilde{u}_i + m_{\tilde{d}_i}^2 \tilde{d}_i^\dagger \tilde{d}_i + m_{\tilde{e}_i}^2 \tilde{e}_i^\dagger \tilde{e}_i)
\]

\[
+ \sum_{i,j} (A_{ij}^{\tilde{Q}} Y_{\tilde{Q}}^{ij} \tilde{Q}_i \tilde{Q}_j + A_{ij}^{\tilde{u}} Y_{\tilde{u}}^{ij} \tilde{u}_i \tilde{u}_j + A_{ij}^{\tilde{d}} Y_{\tilde{d}}^{ij} \tilde{d}_i \tilde{d}_j + \text{h.c.})
\]

(2)

\[
V_{\text{gauino}} = \frac{1}{2} \sum_{i=1}^{3} M_i \lambda_i \lambda_i + \text{h.c.}
\]

(3)

where \(V_{\text{gauino}}\) is the Majorana mass terms for the gaugino fields, \(\lambda_i\) (suppressing the group index), corresponding to \(U(1)\), \(SU(2)\), and \(SU(3)\), respectively. In the above, there are in principle sixty-three soft symmetry breaking parameters. This is hardly desirable to explain the standard model which already has eighteen parameters! It is clear that we need a further organizing principle. While this may be done by invoking extra symmetries such as flavor blindness (blandness), one attractive possibility is to couple the standard model to \(N=1\) supergravity (SUGRA). In the minimal low energy supergravity model, supersymmetry is explicitly broken by induced soft breaking terms of this form but with fewer parameters[19].

We will assume that the spontaneous breaking of the local \(N=1\) supersymmetry is communicated to the “visible” sector by weak gravitational interactions from some “hidden” sector. This spontaneous symmetry breaking of supergravity manifests itself at low energy as explicit soft breaking terms of global supersymmetry.

The SUGRA Lagrangian is characterized by two arbitrary functions of the fields, a real function (the Kähler potential) that determines the kinetic terms of the chiral superfields, and an analytic function, transforming as the symmetric product of the adjoint representation of the gauge group, that determines the kinetic terms of the gauge fields. In models with minimal kinetic terms for the chiral superfields, this leads to a common (gravitino) mass, \(m_0\), for all the scalars of the model. The presence of non-minimal gauge kinetic terms implies non-zero masses, \(M_i\), for the gauginos at the GUT scale, \(M_X\). By further assuming gauge coupling unification, we can take the three gaugino masses to be equal. Furthermore, the trilinear soft
couplings $A_{ij}^u$, $A_{ij}^d$, and $A_{ij}^e$ are all equal to a common value $A_0$. With minimal chiral kinetic terms, the bilinear soft coupling $B_0$ is related to $A_0$ as $B_0 = A_0 - m_0$. This scenario has obvious, desirable features. First, it is very predictive since it has a few parameters accounting for thirty-one new masses. Second, the universal nature of the squark and slepton masses at $M_X$ helps to avoid the appearance of unwanted flavor changing neutral current (FCNC) effects. In fact, one could argue that the absence of FCNCs hints at a universal mass for the scalars.

All of these couplings will evolve to different values under the renormalization group. The complete scalar potential now appears as

$$V = V_F + V_D + V_{soft},$$

where $V_F$ contains the potential contributions from the $F$-terms

$$V_F = |\overline{\pi}Y_u\tilde{Q} + \mu\Phi_d|^2 + |\overline{d}Y_d\tilde{Q} + \overline{\pi}Y_e\tilde{L} + \mu\Phi_u|^2 + |Y_u\tilde{Q}\Phi_u + |Y_d\tilde{Q}\Phi_d + Y_e\tilde{L}\Phi_d|^2 + |\overline{\pi}Y_u\Phi_u + \overline{d}Y_d\Phi_d|^2 + |\overline{\pi}Y_e\Phi_e|^2,$$

and $V_D$ contains the potential contributions from the $D$-terms

$$V_D = \frac{g^2}{2} \left( \frac{1}{6} \tilde{Q}^\dagger \tilde{Q} - \frac{2}{3} \overline{\pi}^\dagger \overline{\pi} + \frac{1}{3} \overline{\pi}^\dagger \overline{\pi} - \frac{1}{2} \tilde{L}^\dagger \tilde{L} + \frac{1}{2} \overline{\phi}^\dagger \overline{\phi} + \frac{1}{2} \tilde{d}^\dagger \tilde{d} - \frac{1}{2} \tilde{d}^\dagger \overline{\phi} - \frac{1}{2} \overline{\phi}^\dagger \tilde{d} \right)^2 + \frac{g_2^2}{8} \left( \tilde{Q}^\dagger \overline{\pi} \tilde{Q} + \tilde{L}^\dagger \overline{\pi} \tilde{L} + \tilde{Q}^\dagger \tilde{Q} + \overline{\phi}^\dagger \overline{\phi} + \overline{\phi}^\dagger \tilde{d} \right)^2,$$

where $\overline{\pi} = (\tau_1, \tau_2, \tau_3)$ are the $SU(2)$ Pauli matrices and $\tilde{\lambda} = (\lambda_1, \ldots, \lambda_8)$ are the Gell-Mann matrices. In general, one must impose constraints on the parameters to avoid charge and color breaking minima in the scalar potential. Some necessary constraints have been formulated, such as

$$A_{ij}^u < 3(m_{\tilde{Q}}^2 + m_u^2 + m_{\Phi_u}^2),$$

$$A_{ij}^d < 3(m_{\tilde{Q}}^2 + m_d^2 + m_{\Phi_d}^2),$$

$$A_{ij}^e < 3(m_{\tilde{L}}^2 + m_e^2 + m_{\Phi_e}^2).$$

However, these relations are in general neither sufficient nor indeed always necessary \[20\]. Their derivation involves very specific assumptions about the spontaneous symmetry breaking.

4. Radiative Electroweak Breaking

An appealing feature of the models we are considering is that they can lead to the breaking of the electroweak symmetry radiatively [21, 22, 23, 24]. The one loop effective Higgs potential in these models can be expressed as the sum of the tree level potential plus a correction coming from the sum of all one loop diagrams with external lines having zero momenta

$$V_{1-loop}(\Lambda) = V_{tree}(\Lambda) + \Delta V_1(\Lambda).$$
The one loop correction is given by
\[ \Delta V_1(\Lambda) = \frac{1}{64\pi^2} \text{Str} \left\{ \mathcal{M}^4 \left( \ln \frac{M^2}{\Lambda^2} - \frac{3}{2} \right) \right\} \]
\[ = \frac{1}{64\pi^2} \sum_p (-1)^{2s_p} (2s_p + 1) m_p^4 \left( \ln \frac{m_p^2}{\Lambda^2} - \frac{3}{2} \right), \]  
where \( \mathcal{M}^2 \) is the field dependent squared mass matrix of the model and \( m_p \) is the eigenvalue mass of the \( p \)th particle of spin \( s_p \). The tree level part of the potential is
\[ V_{\text{tree}}(\Lambda) = m_1^2(\Lambda) \Phi_d^\dagger \Phi_d + m_2^2(\Lambda) \Phi_u^\dagger \Phi_u + m_3^2(\Lambda) (\Phi_u \Phi_d + h.c.) \]
\[ + \frac{g^2(\Lambda)}{8} (\Phi_u^\dagger \Phi_u - \Phi_d^\dagger \Phi_d)^2 + \frac{g_2^2(\Lambda)}{8} (\Phi_u^\dagger \Phi_u + \Phi_d^\dagger \Phi_d)^2 \]  
where
\[ m_1^2(\Lambda) = m_{\Phi_d}^2(\Lambda) + \mu^2(\Lambda), \]  
\[ m_2^2(\Lambda) = m_{\Phi_u}^2(\Lambda) + \mu^2(\Lambda), \]  
\[ m_3^2(\Lambda) = B(\Lambda) \mu(\Lambda). \]

The parameters of the potential are taken as running ones, that is, they vary with scale according to the renormalization group. The logarithmic term in the one loop correction is important in making \( V_{1\text{-loop}}(\Lambda) \) independent of \( \Lambda \) to this order (up to non-field dependent terms).

Given the low energy scale of electroweak breaking, we must use the renormalization group to evolve the parameters of the potential to a convenient scale such as \( M_Z \) (where the experimental values of the gauge couplings are usually cited) thereby making this leading log approximation valid. The exact scale is not critical as long as it is in the electroweak range. If we define,
\[ \overline{m}_i^2 = m_i^2 + \frac{\partial \Delta V_i}{\partial v_i^2}, \]
with \( v_1 = v_d, \ v_2 = v_u \) and
\[ \frac{\partial \Delta V_i}{\partial v_i^2} = \frac{1}{32\pi^2} \sum_p (-1)^{2s_p} (2s_p + 1) m_p^4 \left( \ln \frac{m_p^2}{\Lambda^2} - 1 \right) \frac{\partial m_p^2}{\partial v_i^2}, \]  
then minimization of the potential yields the following two conditions among its parameters
\[ \frac{1}{2} m_Z^2 = \frac{\overline{m}_1^2 - \overline{m}_2^2 \tan^2 \beta}{\tan^2 \beta - 1}, \]
\[ m_Z^2 = (g^2 + g_2^2) v^2 / 2, \ v^2 = v_u^2 + v_d^2, \]
and
\[ B\mu = \frac{1}{2} (\overline{m}_1^2 + \overline{m}_2^2) \sin 2\beta, \]
where \( \tan \beta = v_u/v_d. \)
Although results based on the tree level potential cannot always be trusted, one can still use it to get some idea under what conditions electroweak breaking occurs. The renormalization group evolution of $m^2_{\Phi_u}$ can be such that it turns negative at low energies, if the top quark mass is large enough, whereas $m^2_{\Phi_d}$ runs positive. From Eq. (12), the scale at which breaking occurs is set by the condition

$$m^2_1(\Lambda_b) - m^2_3(\Lambda_b) = 0.$$  \hfill (20)

If the free parameters are adjusted properly, then the correct value of the $Z^0$ mass ($M_Z = 91.17$ GeV) can be achieved.

In the tree level analysis, there is another critical scale that must be considered. It is evident from Eq. (12) that the potential becomes unbounded from below along the equal field (neutral component) direction, if

$$m^2_1(\Lambda_s) + m^2_2(\Lambda_s) < 2m^2_3(\Lambda_s).$$  \hfill (21)

Since $m^2_1m^2_2 - m^4_3 \geq 0$ implies $m^2_1 + m^2_2 \geq 2m^2_3$, condition (21) can only occur at scales lower than condition (20), so $\Lambda_s < \Lambda_b$. From this analysis, one concludes that the tree level vacuum expectation values (VEVs) of the scalar fields obtained by minimizing the potential are zero above $\Lambda_b$, and grow to infinity as one approaches $\Lambda_s$ where the potential becomes unbounded from below. It follows that the appropriate scale at which to minimize the tree level potential and evaluate the VEVs is critical.

This scale must be such that the one loop corrections of the effective potential may safely be neglected. Only at such a scale can the tree level results be trusted. However, there is more than one scale involved, and therefore, it is difficult if not impossible to find a scale at which all logarithms may be neglected. Indeed, the use of tree level minimization conditions to compute the VEVs at an arbitrary scale (e.g., $\Lambda = M_Z$) leads to incorrect conclusions about the regions of parameter space that yield consistent electroweak breaking scenarios \cite{25}. When $\Delta V_1$ is included, however, the value of $\Lambda$ is not critical as long as it is in the neighborhood of $M_Z$.

Ref. \cite{25} gives a prescription for arriving at a scale ($\hat{\Lambda}$) at which the tree level and the one loop effective potential results for the VEVs agree. Three qualitatively different cases are considered. Following Ref. \cite{25}, we let $M_{SUSY}$ parametrize the superparticle thresholds, then the cases can be characterized by the orderings: (a) $M_{SUSY} < \Lambda_s < \Lambda_b$, (b) $\Lambda_s < M_{SUSY} < \Lambda_b$, and (c) $\Lambda_s < \Lambda_b < M_{SUSY}$. In each case, the prescription is to take $\hat{\Lambda} = \max\{M_{SUSY}, \Lambda_s\}$. Two cases deserve special mention. Case (a) cannot be handled using the tree level analysis because $v_u, v_d \to \infty$ near $\Lambda_s$. Fortunately, phenomenological bounds rule this case out anyway. In case (c), there is actually no electroweak breaking. For scales below $M_{SUSY}$, the superparticles have decoupled and the effective theory is not supersymmetric. Therefore, the running mass parameters of the potential freeze into their values at $\Lambda = M_{SUSY}$ at which scale there is no electroweak breaking. Finally, it must be emphasized that the apparent violent behavior of the VEVs with scale in the tree level analysis is an artifact of the approximation. The only physical potential is the full effective potential, and it either breaks electroweak symmetry or not. If it does, then the scalar fields have
non-zero VEVs, and these VEVs are non-zero over all scales varying according to the anomalous dimension of their respective scalar fields (in the Landau gauge).

In this paper, we do not rely on the tree level analysis, rather we incorporate the one loop corrections. We include the dominant contributions from the third generation, that is, those of the top and stop, bottom and sbottom, and tau and stau [26, 27]. The one loop effective potential is constant against the renormalization group to this order around the electroweak scale. We choose the $Z^0$ mass as the scale at which to evaluate the minimization conditions. Eqs. (18) and (19) can be written

$$
\mu^2(M_Z) = \frac{m_{\Phi_u}^2 - m_{\Phi_d}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2} m_{Z}^2 ,
$$

$$
B(M_Z) = \frac{m_{\Phi_u}^2 + m_{\Phi_d}^2 \sin 2\beta}{2\mu(M_Z)} ,
$$

where $m_{\Phi_u,d} = m_{\Phi_u,d}^2 + \partial \Delta V_i / \partial v_{u,d}$ and used to solve for $\mu(M_Z)$ and $B(M_Z)$ given the value of all the relevant parameters at $M_Z$. We note that the form of Eq. (22) does not fix the sign of $\mu$, and a choice for its sign must be made ($\mu$ is multiplicatively renormalized). The right hand sides of these equations implicitly involve the VEV at $M_Z$. In a consistent scenario it would have the value of $v(M_Z) = 174.1$ GeV. If the parameters are such that $\mu^2 < 0$, then the scenario is inconsistent and the electroweak symmetry fails to be broken.

5. Boundary Conditions at $M_X$

In this paper, as in the previous one [13] we work in the modified minimal subtraction scheme ($\overline{\text{MS}}$) of renormalization. The parameters of the Lagrangian are not in general equal to any corresponding physical constant. For example, in the case of masses, except for those of the bottom and top quark (see [13]), all other physical masses, $M$, will be determined from their corresponding running masses by the relation

$$
M = m(\Lambda)|_{\Lambda = M} .
$$

This equation is easily solved in the course of an integration of the RGEs for the different masses by noting the scale at which it is valid.

Because SUGRA models make simplifying predictions about the soft parameters at the unification scale, we initiate the evolution of the renormalization group equations at this scale. It has been demonstrated that the introduction of supersymmetry leads to gauge coupling unification at approximately $\sim 10^{16}$ GeV. Therefore we take $M_X = 10^{16}$ GeV, and evolve down to 1 GeV, the conventional scale at which the running quark masses are given [28].

At the unification scale, $M_X$, all the scalars will have a common mass

$$
m_{Q_i}(M_X) = m_{u_i}(M_X) = m_{d_i}(M_X) = m_{L_i}(M_X) = m_{e_i}(M_X) = m_{\Phi_u}(M_X) = m_{\Phi_d}(M_X) = m_0 ,
$$

(25)
as will the gauginos

\[ M_1(M_X) = M_2(M_X) = M_3(M_X) = m_{1/2} \, . \]  \hspace{1cm} (26)

The prefactors of the trilinear soft scalar terms are equal at \( M_X \)

\[ A_{ij}^{(M_X)} = A_{ij}^{(M_X)} = A_{ij}^{(M_X)} = A_0 \, . \]  \hspace{1cm} (27)

Also we define the bilinear soft scalar coupling and the mixing mass at \( M_X \) by

\[ B(M_X) \equiv B_0, \quad \text{and} \quad \mu(M_X) \equiv \mu_0. \]

Furthermore, to constrain the parameter space, we will take the bottom and tau masses equal at \( M_X \)

\[ m_b(M_X) = m_\tau(M_X) \, . \]  \hspace{1cm} (28)

This being the best motivated mass relation in supersymmetric grand unified theories \cite{29}.

The present analysis restricts itself to three subclasses of soft symmetry breaking models. These have various soft parameters equal to zero at \( M_X \). The first class of models follows from the no-scale model \cite{30} and has \( A_0 = m_0 = B_0 = 0 \) (strict no-scale model). In these models, only gaugino masses provide global supersymmetry breaking. The second class is the less constraining no-scale case that has only \( A_0 = m_0 = 0 \). A third class we consider with \( A_0 = B_0 = 0 \) comes from string derived models. Table II lists these three possibilities. In both strict no-scale and string inspired cases, we must have \( B_0 = 0 \). However, since for us \( B_0 \) is an output rather than an input variable, \( B_0 = 0 \) results must be inferred from its behavior upon varying other input parameters.

### Table 2: Models

|               | \( A_0 \) | \( m_0 \) | \( m_{1/2} \) | \( B_0 \) |
|---------------|----------|----------|--------------|----------|
| Strict No-scale | 0        | 0        | free         | 0        |
| No-scale      | 0        | 0        | free         | free     |
| String Inspired | 0      | free     | free         | 0        |

6. Numerical Procedure

In this work as in Ref. \cite{13}, we will use routines, based on the “shooting” method, to solve systems of nonlinear equations. The method involves making a guess for the solution, then assessing its merits based on how well the equations are satisfied, given some tolerance. The process is optimized and iterated until the routine converges on a solution.

As discussed previously, we start out runs at \( M_X \) at which scale we can make simplifying assumptions about the soft breaking terms based on various SUGRA models. This requires that we use the solution routines to consistently find the \( M_X \) values of all known low energy parameters such as lepton and quark masses and
mixing angles and gauge couplings. This amounts to solving for sixteen unknowns (nine masses, three angles and a phase, and three gauge couplings). Alternatively, we could start our runs at $M_Z$ or 1 GeV; however, this now requires solving for sixty-three unknowns (the values of the soft breaking terms at low energy) that must evolve to just four different values at $M_X$. The efficiency of the former method is obvious.

There are seven free parameters in the model we consider. These are $A_0$, $B_0$, $m_0$, $m_{1/2}$, $\mu$, $\tan \beta$, and $m_t$. The two minimization constraints (18) and (19) reduce this set to five, which are taken to be $A_0$, $m_0$, $m_{1/2}$, $\tan \beta$, and $m_t$. In the present framework, $B_0$ and $\mu_0$ will be determined using the numerical solutions routines in conjunction with the minimization of the one loop effective potential at $M_Z$ in the process of evolving from $M_X$ to 1 GeV. Minimization at $M_Z$ will give $B(M_Z)$ and $\mu(M_Z)$. To arrive at $B_0$ and $\mu_0$ (their corresponding values at $M_X$), we employ the solution routine as follows. A guess for $B_0$ and $\mu_0$ is made at $M_X$ and then the parameters of the model are run to $M_Z$ at which scale the evolved value of $B$ is compared to the minimization output value for $B$ at $M_Z$. The same is done for $\mu$. If the compared values agree to some set accuracy, then $B_0$ and $\mu_0$ are the required values. Other analyses that also extract $B(M_Z)$ and $\mu(M_Z)$ simply evolve these two parameters via their renormalization group equations back to $M_X$ to find $B_0$ and $\mu_0$ relying on their near decoupling from the full set of RGEs. We note that the sign of $\mu$ is not determined from the minimization procedure, thus we must make a choice for it. To constrain the parameter space further, the bottom quark and tau lepton masses will be taken equal at $M_X$. This equality is a characteristic of many SUSY-GUTs. This constrains the model to four free parameters, $A_0$, $m_0$, $m_{1/2}$, and $\tan \beta$. Demanding that $m_b(M_X) = m_\tau(M_X)$ and achieving the correct physical masses for the bottom quark and tau lepton fixes the mass of the top quark which affects the evolution of the bottom Yukawa significantly. We shall assume gauge coupling unification, an assumption which appears reasonable when one considers SUSY models with SUSY breaking scales $\lesssim 10$ TeV.

In a complete treatment, the solution routines would be used to find the precise (similar) values of $\alpha_1$, $\alpha_2$, and $\alpha_3$ at $M_X$ that will evolve to the experimentally known values at $M_Z$, however this increases the CPU time considerably. We shall therefore sacrifice some precision in their $M_Z$ values by taking them exactly equal at $M_X$. This is already a theoretical oversimplification since one does not expect the gauge couplings to be exactly equal due to threshold effects at the GUT scale. We find that for all cases we have studied, the common value $\alpha_1^{-1}(M_X) = \alpha_2^{-1}(M_X) = \alpha_3^{-1}(M_X) = 25.31$ leads to errors no bigger than 1%, 5%, and 10% in $\alpha_1(M_Z)$, $\alpha_2(M_Z)$, and $\alpha_3(M_Z)$, respectively. This is not so bad considering that the (combined experimental and theoretical) errors on $\alpha_3(M_Z)$ from some processes can be as large as 10% [13].

It is well known that there is a fine tuning problem inherent in the radiatively induced electroweak models. For certain values of the parameters, the top quark mass must be tuned to an “unnaturally” high degree of accuracy to achieve the correct value of $M_Z$. This problem is generally handled by rejecting models that require “too much” tuning. The amount of tuning is usually defined quite arbitrarily.
The usual procedure is to define fine tuning parameters

\[ c_i = \left| \frac{x^2_i}{M^2_Z} \frac{\partial M^2_Z}{\partial x^2_i} \right|, \quad (29) \]

where \( x_i \) are parameters of the theory such as \( m_0, m_{1/2}, \mu, \) or \( m_t \). One then demands that the \( c_i \) be less than some chosen value that is typically taken to be 10.

We have analyzed to some extent the differences in using the tree level vs. one loop effective potential. The basis for the “theoretical” fine tuning problem can be seen, if one makes some simplifying assumptions, in the dependence of \( M_W \) on the top quark Yukawa coupling, \( y_t \), \[ M_W \sim M_X e^{-\frac{1}{y_t^2}}. \quad (30) \]

We remark that this fine tuning problem is exacerbated if one uses only the tree level analysis of the potential. The vacuum expectation value coming from the minimization conditions of the tree level potential changes rapidly from 0 to infinity over the interval \((\Lambda_s, \Lambda_b)\). Using the prescription of Ref.[25] for the scale \( \hat{\Lambda} \) at which to adequately minimize the tree level potential, to extract \( v(\hat{\Lambda}) \), and thereby to arrive at a value for \( M_Z \), one finds that although a small variation in \( y_t(M_X) \) may lead to a small variation in \( \Lambda_b \), the steepness in the tree level VEV can lead to a large variation in the value of \( v(\hat{\Lambda}) \) and therefore in \( M_Z \). Hence, in the tree level analysis, solutions which may be within the bounds of the “theoretical” fine tuning may nevertheless display a fine tuning aspect because of this “tree level” fine tuning of \( \hat{\Lambda} \). However, our use of the one loop effective potential (i.e., including \( \Delta V_1 \)) stabilizes the VEV around the \( M_Z \) scale and this particular fine tuning goes away. The true VEVs depend on scale through wave function renormalization effects which are never large as can be seen from the form of the renormalization group equations for the VEVs.

In this analysis, we shall also reject solutions based on fine tuning considerations; however, our method differs somewhat from the usual one in that it is incorporated in the solution routine described above. The routine is an iterative one which determines the convergence properties of the solution. Very slow convergence reflects an inherent fine tuning. Therefore, if the convergence is too slow, we will reject the solution. Effectively we are rejecting any solution which the computer cannot pinpoint within an allotted number of iterations.

Given values for \( A_0, m_0, m_{1/2}, \tan \beta, \) and \( \text{sign}(\mu) \), the solution routines search for the values of \( v(M_X), m_{b,\tau}(M_X), \) \( m_t(M_X), B_0, \) and \( \mu_0 \). The process by which \( B_0 \) and \( \mu_0 \) are found was described above. The remaining three parameters are determined similarly. The routine makes a guess for \( v(M_X), m_{b,\tau}(M_X), \) and \( m_t(M_X) \), then the full renormalization group equations are evolved to 1 GeV calculating superparticle threshold masses in the process and minimizing the one loop effective potential at \( M_Z \). The merits of the guess for \( v(M_X), m_{b,\tau}(M_X), \) and \( m_t(M_X) \) is assessed by comparing the resulting values of \( M_Z, m_\tau(1 \text{ GeV}), \) and \( m_b(1 \text{ GeV}) \) with the expected ones. The process is iterated until the correct values are achieved to within some
tolerance.

7. Thresholds

In the minimal low energy supergravity model being considered, the super particle spectrum is no longer degenerate as in the simple global supersymmetry model in which all the super particles are given a common mass, $M_{SUSY}$. In the simple case, one makes one course correction in the renormalization group evolution at $M_{SUSY}$. In the model with soft symmetry breaking, the nondegenerate spectrum should lead to various course corrections at the super particle mass thresholds. To this end, the renormalization group $\beta$ functions must be cast in a new form which makes the implementation of the thresholds effects (albeit naive) evident. Since the $\overline{\text{MS}}$ renormalization group equations are mass independent, particle thresholds must be handled using the decoupling theorem [32], and each super particle mass has associated with it a boundary between two effective theories. Above a particular mass threshold the associated particle is present in the effective theory, below the threshold the particle is absent.

The simplest way to incorporate this is to (naively) treat the thresholds as steps in the particle content of the renormalization group $\beta$ functions. This method is not always entirely adequate. For example, in the case of the $SU(2)$ gauge coupling there will be scales in the integration process at which there are effectively a half integer number of doublets using this method. We believe, nevertheless, that this method does yield the correct, general behavior of the evolution. It is a simple means of implementing the smearing effects of the non-degenerate super particle spectrum. The determination of the spectrum of masses is done without iteration as is common in other analyses. Our method deduces the physical masses by solving the equation $m(\Lambda) = \Lambda$ for each superparticle in the process of evolving from $M_X$ to 1 GeV. The usual iterative method requires several runs to find a consistent solution.

8. Analysis

The tremendous computing task involved in analyzing the full parameter space of the soft symmetry breaking models, using the methods described as designed, would be far too time consuming given the computing facilities available to us. Therefore, in the following analysis, some simplifications will be made in the procedural method. First, only the heaviest family of quarks and leptons will have non-zero mass. This will cut down on the CPU time required for the solution routines to consistently find their values at $M_X$. Second, as stated previously, the value of the strong coupling at $M_Z$ will be allowed to vary from its central value of .113 by at most 10%. This translates into a similar error in the bottom quark mass. Third, the allotted number of Runge-Kutta steps, involved in numerically integrating the renormalization group equations, will be cut down to $\sim 100$.

Our method involves four input parameters $A_0$, $m_0$, $m_{1/2}$, $\tan \beta$ (and the sign($\mu$)). The output is $B_0$, $\mu_0$, $M_t$, $M_h$, and all the masses of the extra particles
associated with the MSSM. Efficient use of CPU time required that we proceed as follows. For a given model, our initial exploration of the parameter space was performed in a coarse grained fashion. $\tan\beta$ was most commonly coarse grained as 2, 5, and 10, with only some runs involving higher values (e.g., 15, 20). The other three input parameters were varied in steps of 50 and 100 GeV. Values larger than $\sim 500$ GeV were rarely ever used. We subsequently narrowed down on the allowed hyperenvelope by fine graining around the edges of the expected region (based on the coarse graining results).

Our raw data consists of those runs which satisfy the following two criteria. The first is consistent electroweak breaking; that is, the correct value of $M_Z$ is achieved from the minimization of the one loop effective potential with $\mu^2(M_Z) > 0$. The second criterion is no fine tuning, as implemented in our method (see Section 6). The solution routines employed return a numbered code representing the convergence properties of the solution which we use to screen the runs.

The raw data is then progressively filtered based on three physical constraints. R-parity is a discrete symmetry which distinguishes particles from superparticles by assigning a +1 to particles and a −1 to the superparticles. Conserved R-parity requires the existence of a stable lightest supersymmetric particle (LSP). Astrophysical considerations indicate that the LSP must be neutral and colorless. Cosmological considerations based on the LSPs contribution to the density of the universe indicate that it must have a mass less than $\sim 200$ GeV [33, 34, 35]. First, points in parameter space that lead to LSPs other than neutralinos with masses less than $\sim 200$ GeV are cut. Second, flavor changing neutral current bounds are used to reject runs in which intergenerational splitting of squark and slepton masses is too large. Third, experimental limits on the masses of the superparticles are used as another criterion to reject runs.

9. Results

In the following, we discuss the results of our analysis of the three classes of models listed in Table II. Because we have performed a coarse grained study, our results should only be considered qualitatively valid. Based on these, we hope to be able to ascertain the general trends in the data, and make some general predictions about the feasibility of the models considered.

Because the GUT inspired constraint Eq. (28) is enforced in this analysis, the results will depend on the mass of the bottom quark. Namely, lower bottom quark masses require larger values of the top quark mass to satisfy this relation. Most results will be reported for the case $m_b(1\text{ GeV}) = 6.00$ GeV, but lower mass (5.70 GeV) and higher mass (6.33 GeV) cases were also studied. The running value of 6.00 GeV for $m_b(1\text{ GeV})$ corresponds to a physical bottom mass $M_b = 4.85 \pm 0.15$ GeV, with the uncertainty coming from the error in the strong coupling, as discussed above.

9.1. No-scale Case

In all cases considered, the mass of the LSP, when it is a neutralino, is
observed to be correlated with the value of $m_{1/2}$. Therefore, we find that $m_{1/2}$ cannot be taken too large ($\lesssim 400$ GeV). In the no-scale case, we present plots of $M_t$ vs. $m_{1/2}$ for three values of $m_b(1 \text{ GeV})$ and containing all points satisfying the various criteria outlined in Section 8. Figure 1 is such a plot for $m_b(1 \text{ GeV}) = 6.00$ GeV. The “right edge” of the envelope is defined by points whose neutralino LSPs are just slightly heavier than the lightest charged superparticle (usually a $\tilde{\tau}_R$ for us). The “left edge” defines the threshold of consistent electroweak breaking. The top and bottom edges are set by the requirement that Eq. (28) hold. The range of $\tan \beta$ considered leads to definite lower and upper bounds on $M_t$ [9, 10, 11, 12].

In Figs. 2 and 3, we display similar plots with lower ($m_b(1 \text{ GeV}) = 5.70$ GeV) and higher ($m_b(1 \text{ GeV}) = 6.33$ GeV) bottom quark masses. The previously noted dependence of the data on $M_b$ is evident from these figures. In Fig. 3, we change the value of the bottom quark mass to illustrate the dependence of our results on its value.

From these, we note that the available range of $m_{1/2}$ decreases with increasing $M_b$. Fig. 1 indicates that $190 \text{ GeV} \leq m_{1/2} \leq 265 \text{ GeV}$. We can draw no conclusions, however, about the value of $\tan \beta$. We can conclude from the $m_b(1 \text{ GeV}) = 6.00$ GeV case that $85 \leq M_t \leq 132$ GeV. From similar plots involving $M_b$, we conclude that $25 \leq M_h \leq 78$ GeV. In Fig. 4, we display the familiar dependence of $M_t$ on $\tan \beta$ for a particular value of $m_{1/2}$ in the allowed range.

The official experimental lower bound on the top quark mass is 108 GeV. The figures indicate that the top quark cannot have a mass greater than $\sim 132$ GeV in this model, if $m_b(1 \text{ GeV}) = 6.00$ GeV. This upper bound is raised to $\sim 160$ GeV if $m_b(1 \text{ GeV}) = 5.70$ GeV, and the model is ruled out, if $m_b(1 \text{ GeV}) = 6.33$ GeV. Figure 5 is similar to Fig. 1, but we have chosen sign of $\mu$ negative. The allowed region is displaced down with respect to the positive $\mu$ case with the upper bound on $M_t$ now 113 GeV and very close to the experimental limit.

9.2. Strict No-scale Case

The results of the strict no-scale case must be interpolated from the no-scale results, because $B_0$ is not an input parameter in our procedural method. Therefore, we plot in Fig. 6 $M_t$ vs. $B_0$ and deduce the $M_t$ bounds from slicing the data along $B_0 = 0$. Admittedly, the relatively small number of points makes the perimeter of the region unclear in some areas. We get approximately $90 \leq M_t \leq 127$ GeV. For $M_h$, we find $40 \lesssim M_h \lesssim 78$ GeV with an uncertainty in the lower bound of $\sim 10$ GeV due to lack of definition in the lower end of the envelope. Inspection of the data indicates that $3.5 \leq \tan \beta \leq 9$. Thus, it appears that $\tan \beta$ cannot be too small or too large to accommodate the strict no-scale case.

Finally, in Table 3, we display the spectrum of superparticle masses for a representative strict no-scale scenario with $\tan \beta = 8.3$ and $m_{1/2} = 240$ GeV. The masses are in GeVs. There are several interesting features. For instance, the LSP is the photino in this case and has a mass of 92 GeV. It is too light to account for the dark matter. The top quark mass is just above the experimental limit at 126 GeV, and the Higgs boson mass is 77 GeV, which underlines one of the signatures of
Table 3: Strict No-scale Scenario

| Parameter | Value |
|-----------|-------|
| $A_0$     | 0     |
| $m_0$     | 0     |
| $m_{1/2}$ | 240   |
| $\mu_0$  | 171   |
| $\tan \beta$ | 8.3 |
| $\text{sign}(\mu)$ | + |
| $M_t$     | 126   |
| $d$       | 473 - 506 |
| $\tilde{u}$ | 390 - 532 |
| $\tilde{e}$ | 94 - 176 |
| $\tilde{\nu}_L$ | 180 |
| $\tilde{\gamma}$ | 92 |
| $\tilde{Z}$, $\tilde{W}$ | 153, 147 |
| $\tilde{g}$ | 557 |
| $H^0$     | 213, 271 |
| $H^\pm$   | 270   |
| $h$, $H$  | 77, 158 |
| $H^+, A$  | 235, 222 |

these scenarios: the Higgs is typically light, so that it will be found in two photon modes at SSC. At any rate, it illustrates the predictive power of this possibility.

9.3. String Inspired Case

As in the strict no-scale, our results for the string inspired case necessitates interpolating $A_0 = 0$ data to $B_0 = 0$. Once again we find the perimeter of the allowed region is not well defined everywhere. Hence, our results are only qualitative. Figure 7 is similar to Fig. 6, but in this case we only fix $A_0 = 0$. The strict no-scale case is a special case of the string inspired one, therefore the 127 GeV top quark mass upper bound is not expected to decrease but rather to increase in this case. Slicing along $B_0 = 0$ yields $85 \lesssim M_t \lesssim 140$ GeV. Similarly, for the light Higgs we get $40 \lesssim M_h \lesssim 80$ GeV. The data indicates in this case, as in the strict no-scale case, that there is a lower bound on $\tan \beta$ of $\sim 3$.

10. Conclusions

Minimal low energy supergravity models were considered. It is quite remarkable that for as simple a breaking of supersymmetry as that offered by the strict no-scale model, we can reproduce the standard model results, including the appealing feature of automatic breaking of the electro-weak symmetry. The importance of the value of the top quark mass to these schemes cannot be understated, since we found in our study of specific models, rather restrictive upper bounds for the top quark mass. No-scale models in which only gaugino masses provide global supersymmetry breaking yield top quarks with masses less than $\sim 127$ GeV. The results
are sensitive to the value of the bottom quark mass. Lower bottom quark masses, within the experimental uncertainty, lead to higher top quark upper bounds. In these models, the ratio of vacuum expectation values of the two Higgs fields is expected to be larger than $\sim 70^\circ$.

Although the perimeter of the allowed regions were often fuzzy, we could, nevertheless, draw some general conclusions from our results. For all our runs, with no restrictions on the soft terms, we find for the top quark $M_t \lesssim 186$ GeV and for the light Higgs boson $M_h \lesssim 100$ GeV.

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12. Figure Captions

Figure 1. Plot of $M_t$ vs. $m_{1/2}$ for $\tan \beta = 2, 5, 10, 15$ and $m_b(1 \text{ GeV}) = 6.00 \text{ GeV}$.

Figure 2. Same as Fig. 1 but with $m_b(1 \text{ GeV}) = 5.70 \text{ GeV}$.

Figure 3. Same as Fig. 1 but with $m_b(1 \text{ GeV}) = 6.33 \text{ GeV}$.

Figure 4. Plot of $M_t$ vs. $\tan \beta$ in the no-scale case with $m_{1/2} = 240 \text{ GeV}$ and $m_b(1 \text{ GeV}) = 6.00 \text{ GeV}$.

Figure 5. Same as Fig. 1 but with $\text{sign}(\mu) = -$.

Figure 6. Plot of $M_t$ vs. $B_0$ for points with $A_0 = m_0 = 0$ used to interpolate to $B_0 = 0$ and deduce the $M_t$ range in this case.

Figure 7. Same as Fig. 6 but for points with $A_0 = 0$ only.