Extracting the $\rho$ meson wavefunction from HERA data

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Abstract: We extract the light-cone wavefunctions of the $\rho$ meson using the HERA data on diffractive $\rho$ photoproduction. We find good agreement with predictions for the distribution amplitude based on QCD sum rules and from the lattice. We also find that the data prefer a transverse wavefunction with enhanced end-point contributions.

Keywords: QCD, diffraction, light-cone wavefunction.
1. Introduction

Diffractive $\rho$ meson photoproduction\(^1\) can be described within the dipole model [1–4] according to the formula

$$\Im m A_\lambda(s,t)|_{t=0} = s \sum_{h,\bar{h}} \int d^2r \, dz \, \Psi^{\gamma,\lambda}_{h,\bar{h}}(r,z) \hat{\sigma}(s,r) \Psi^{\rho,\lambda}_{h,\bar{h}}(r,z)^* \quad (1.1)$$

for the imaginary part of the forward elastic scattering amplitude, where $\Psi^{\gamma,\lambda}_{h,\bar{h}}(r,z)$ and $\Psi^{\rho,\lambda}_{h,\bar{h}}(r,z)$ are the light-cone wavefunctions of the photon and vector meson, and $\hat{\sigma}(s,r)$ is the dipole cross section. The light-cone wavefunctions represent the probability amplitudes for the photon or $\rho$ meson to fluctuate into a $q\bar{q}$ pair of transverse size $r$ in which the quark carries a fraction $z$ of the meson’s light-cone momentum. The sum is over quark/antiquark helicities ($h$ and $\bar{h}$) and $\lambda = L$ or $T$ labels the polarization of the photon and meson.

The photoproduction cross section is, after integrating over $t$,

$$\sigma_\lambda(s) = \frac{1}{B} \frac{1}{16\pi} (\Im m A_\lambda(s,0))^2 (1 + \beta_\lambda^2), \quad (1.2)$$

where $\beta_\lambda$ is the ratio of real to imaginary parts of the amplitude and $B$ is the diffractive slope\(^2\). We shall assume that

$$B = N \left( 14.0 \left( \frac{1 \text{ GeV}^2}{Q^2 + M_\rho^2} \right)^{0.2} + 1 \right) \quad (1.3)$$

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\(^1\)We use ‘photoproduction’ to refer to production using both real and virtual photons.

\(^2\)We assume $d\sigma/dt \propto \exp(Bt)$. 

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with $N = 0.55 \text{ GeV}^{-2}$, which is in accord with the ZEUS data [5]. The H1 data [6] prefer a somewhat larger value of $B$, but with a larger uncertainty. We compute $\beta_\lambda$ according to

$$\beta_\lambda = \tan \left( \frac{\pi}{2} \alpha_\lambda \right)$$

with

$$\alpha_\lambda = \frac{\partial \ln |\Im A_\lambda|}{\partial \ln (1/x)}$$

(1.5)

where $\Im A_\lambda$ is given by Eq. (1.1) and $x = (Q^2 + 4m_f^2)/(Q^2 + W^2)$ ($m_f$ is defined in the next section). The total cross section that is measured experimentally is given by

$$\sigma = \sigma_T + \epsilon \sigma_L$$

(1.6)

where $\epsilon = 0.98$.

Our goal in this paper is to use the current HERA data on $\rho$-meson photoproduction [5,6] to extract the meson’s light-cone wavefunction. To do that we must specify the dipole cross section, which we do by assuming the FS2004 saturation model that was extracted from the HERA deep inelastic scattering data in Ref. [7].

2. Light-cone wavefunctions

The photon’s light-cone wavefunctions are [8–10]:

$$\Psi^{\gamma,L}_{h,\bar{h}}(r,z) = \sqrt{\frac{N_c}{4\pi}} \delta_{h,-\bar{h}} \epsilon e_f 2z(1-z) Q \frac{K_0(\epsilon r)}{2\pi},$$

(2.1)

and

$$\Psi^{\gamma,T}_{h,\bar{h}}(r,z) = \pm \sqrt{\frac{N_c}{2\pi}} \sqrt{2} z(1-z) M^2 \left[ i e^{\pm i\theta_r} (z \delta_{h,\bar{h}} - (1-z) \delta_{h,\bar{h}}) \partial_r + m_f \delta_{h,\bar{h}} \right] \frac{K_0(\epsilon r)}{2\pi},$$

(2.2)

where

$$\epsilon^2 = z(1-z) Q^2 + m_f^2$$

(2.3)

These wavefunctions are derived from perturbative QED and depend upon a phenomenological light-quark mass, $m_f$. We are compelled to take $m_f = 0.14 \text{ GeV}$, as determined by the fit in Ref. [7]. We take $e_f = 1/\sqrt{2}$, as appropriate for $\rho$ meson production.

The meson’s light-cone wavefunctions can be written in terms of the scalar wavefunctions $\phi_{L,T}(r,z)$:

$$\Psi^{\rho,L}_{h,\bar{h}}(r,z) = \sqrt{\frac{N_c}{4\pi}} \delta_{h,-\bar{h}} \frac{1}{M_{\rho z}(1-z)} \left[ z(1-z) M^2 + m_f^2 - \nabla^2 \right] \phi_L(r,z)$$

(2.4)

where $\nabla^2 \equiv \frac{1}{r} \partial_r + \partial^2_z$ and

$$\Psi^{\rho,T}_{h,\bar{h}}(r,z) = \pm \sqrt{\frac{N_c}{4\pi}} \frac{\sqrt{2}}{z(1-z)} \left[ i e^{\pm i\theta_r} (z \delta_{h,\bar{h}} - (1-z) \delta_{h,\bar{h}}) \partial_r + m_f \delta_{h,\bar{h}} \right] \phi_T(r,z).$$

(2.5)
These wavefunctions are subject to two important constraints. The first is the normalisation condition, which embodies our assumption that the \( \rho \) meson consists solely of a \( q \bar{q} \) pair:

\[
\sum_{h,h'} \int d^2r \, dz \, |\Psi_{h,h'}(r,z)|^2 = 1 .
\]

(2.6)

The second constraint arises from the measured value of \( f_\rho \), the meson decay constant for the longitudinally polarised meson, i.e.

\[
f_\rho M_\rho = \frac{N_c}{\pi} e_f \int_0^1 \frac{dz}{z(1-z)} \left[ z(1-z) M_\rho^2 + m_f^2 \right] \frac{\phi_L(r,z)}{r=0} .
\]

(2.7)

The decay constant is deduced from the experimentally measured electronic decay width via the relation \([10]\):

\[
\Gamma_{\rho \to e^+e^-} = \frac{4\pi\alpha_e^2 f_\rho^2}{3M_\rho} ,
\]

(2.8)

where \( \Gamma_{\rho \to e^+e^-} = 7.04 \pm 0.06 \text{ keV} \) \([11]\).

3. Fitting the HERA data

We must specify the form of the scalar wavefunctions \( \phi_\lambda \). In Ref. \([12]\), a ‘Boosted Gaussian’ (BG) wavefunction of the form

\[
\phi_{\lambda}^{BG}(r,z) = N_\lambda 4[z(1-z)]^{b_\lambda} \sqrt{2\pi R_\lambda^2} \exp\left( \frac{m_f^2 R_\lambda^2}{2} \right) \exp\left( -\frac{m_f^2 R_\lambda^2}{8[z(1-z)]^{b_\lambda}} \right) \]

(3.1)

was used. This wavefunction is a simplified version of that proposed originally by Nemchik, Nikolaev, Predazzi and Zakharov \([13]\).

In Ref. \([12]\), it was assumed that \( b_\lambda = 1 \) and that \( R_L = R_T = R \). The leptonic decay width constraint and the normalization conditions fix \( R \) and \( N_\lambda \) (i.e. \( R^2 = 12.3 \text{ GeV}^{-2} \)), leaving no free parameters. Predictions can then be made for the \( \rho \)-meson photoproduction cross section. This procedure leads to reasonable agreement with the old HERA data on the light vector mesons \([12]\) and also for the heavier \( J/\Psi \) \([14]\). However, it is not able to accommodate the most recent HERA data on \( \rho \) production. Comparison with the HERA data leads to a \( \chi^2/\text{data point of 234/75} \). For comparison with our later results, the longitudinal and transverse BG light-cone wavefunctions are shown in Figure 1.

The poor agreement with data is considerably improved by allowing \( b_\lambda \) and \( R_\lambda \) to vary freely.\(^3\) Specifically, we fit to the photoproduction total cross section,\(^4\) the longitudinal-to-transverse cross section ratio and to the electronic decay constant datum using Minuit

\(^3\)Allowing only \( R_\lambda \) to vary does not much improve the fit.

\(^4\)We rescale the H1 and ZEUS data by 0.95, which is consistent with the experimental uncertainty in the overall normalisation of the data \([5,6]\).
Figure 1: The modulus squared of the longitudinal (left) and transverse (right) ‘Boosted Gaussian’ light-cone wavefunctions from Ref. [12].

To ensure that we fit in the diffractive region, we exclude those data points with $W \leq 60$ GeV. We fit to the most recent data from H1 [6] and ZEUS [5]. In addition, we include the earlier HERA data [16–19] when they are in a kinematic region not covered by the latest data. This selected data set comprises 39 data points from ZEUS (32 points for the total cross section and 7 points for the longitudinal-to-transverse cross section ratio) and 36 data points from H1 (27 points for the total cross section and 9 points for the longitudinal-to-transverse cross section ratio). To these, we add one data point for the decay constant of the longitudinally polarised $\rho$ meson, giving a total of 76 data points. The result is a $\chi^2$/degree of freedom equal to 82/72 and the corresponding best-fit parameters are listed in Table 1. The results of this fit can be seen as the dotted lines in Figures 2 and 3, where the poor agreement at $Q^2 = 0$ is to be noted.

| Free parameter | Fitted value   |
|---------------|----------------|
| $R^2_L$       | 27.33 GeV$^{-2}$ |
| $R^2_T$       | 30.87 GeV$^{-2}$ |
| $b_L$         | 0.5545          |
| $b_T$         | 0.6792          |

Table 1: The parameters corresponding to the BG fit of Eq. (3.1). The $\chi^2$/degree of freedom is equal to 82/72.

We can further improve the quality of fit (especially at $Q^2 = 0$) by allowing for additional end-point enhancement in the meson wavefunctions, i.e. using a scalar wavefunction of the form

$$\phi_\lambda(r, z) = \phi_\lambda^{BG}(r, z) \times [1 + c_\lambda \xi^2 + d_\lambda \xi^4]$$

(3.2)
where $\xi = 2z - 1$. We find a preference for enhancement only in the wavefunction for transversely polarized mesons, i.e. the data prefer $c_L = d_L = 0$, which is not surprising since $\sigma_L$ vanishes at $Q^2 = 0$. The new fit lowers the $\chi^2$/degree of freedom to 68/70. The resulting fitted parameters are listed in Table 2 and the corresponding cross section predictions appear as the solid lines in Figures 2 and 3.

### Best fit

| Free parameter | Fitted value |
|----------------|--------------|
| $R_L^2$        | 26.76 GeV$^{-2}$ |
| $R_T^2$        | 27.52 GeV$^{-2}$ |
| $b_L$          | 0.5665       |
| $b_T$          | 0.7468       |
| $c_T$          | 0.3317       |
| $d_T$          | 1.310        |

**Table 2:** The parameters corresponding to the BG fit with additional end-point enhancement in the transverse wavefunction, i.e. Eq. (3.2). The $\chi^2$/degree of freedom is equal to 68/70.

In Figure 4, we show the wavefunctions corresponding to the improved fit. Compared to Figure 1, the end-point enhancement is quite distinctive. Figure 5 shows the wavefunctions for both fits (the solid and dashed curves) at $r = 0$, plotted as a function of $z$. Note that they are almost indistinguishable from each other in the longitudinal case. For comparison, also shown on this figure is the result of the original BG parameterization [12]: it gives the dotted curves.

The extent to which the data require any additional end-point enhancement should be set in context since the $Q^2 \to 0$ limit suffers from the greatest theoretical uncertainty. For example, we characterize non-perturbative effects in the photon wavefunction through a single parameter (the quark mass) and it is unclear how a more sophisticated treatment would affect our conclusions. Moreover, it is also possible to improve the quality of fit to the data without appealing to Eq. (3.2) by increasing the value of the diffractive slope, $B$, at $Q^2 = 0$. Specifically, we achieved a $\chi^2$/degree of freedom of 67/72 after increasing $B$ by 15% at $Q^2 = 0$ relative to the value determined by Eq. (1.3).

We note that both our fits indicate that the data prefer a transverse wavefunction with enhanced contributions at $z \to 0, 1$. This conclusion is valid regardless of the uncertainties on the forward slope parameter.

Finally, we should remark that it is possible to lower the $\chi^2$ of the fit still further if we are prepared to rescale the data downwards by more than 5%. For example, rescaling down by 12% leads to a $\chi^2$/degree of freedom equal to 62/70. The dashed curve in Figure 6 confirms that the corresponding wavefunction is not very different from the one obtained by rescaling the data down by 5%.
4. The leading-twist distribution amplitude

We can use the longitudinal wavefunction from our fits to extract the corresponding leading twist-2 distribution amplitude, \( \varphi(z, \mu) \) \[20\]:

\[
\varphi(z, \mu) \sim \int d^2k \Theta(|k| < \mu) \tilde{\phi}_L(k, z)
\]

(4.1)

where

\[
\tilde{\phi}_L(k, z) \sim \int d^2r e^{-ik \cdot r} \phi_L(r, z)
\]

(4.2)

which implies

\[
\varphi(z, \mu) \sim \int dr \mu J_1(\mu r) \phi_L(r, z)
\]

(4.3)

Substituting for the scalar wavefunction from the previous section gives

\[
\varphi(z, \mu) \sim \left(1 - e^{-\mu^2/\Delta(z)^2}\right) e^{-m_f^2/\Delta(z)^2} [z(1 - z)]^{b_L},
\]

(4.4)

where \( \Delta(z)^2 = 8[z(1 - z)]^{b_L}/R_L^2 \). We note that our DA is very slowly varying with \( \mu \) for \( \mu > 1 \) GeV. This means that our parameterization neglects the perturbatively known \( \mu \)-dependence of the DA and can thus be viewed as a parameterization of the DA at some not too large a value of \( \mu \). This is reasonable given the limited \( Q^2 \) range of the HERA data to which we fit (i.e. \( \sqrt{Q^2} < 6 \) GeV).

To compare with existing theoretical predictions for the DA, we compute moments of our DA, i.e.

\[
\langle \xi^n \rangle_\mu = \int_0^1 dz \xi^n \varphi(z, \mu).
\]

(4.5)

The \( n = 0 \) moment is fixed by the decay constant constraint and is not a prediction\(^5\), we therefore follow convention and normalize the DA according to

\[
\int_0^1 dz \varphi(z, \mu) = 1.
\]

(4.6)

Our results are compared with the existing predictions in Table 3 and are in very good agreement with the expectations based on QCD sum rules and from the lattice. Also shown for comparison is the prediction based upon the old BG wavefunction used in Ref. [12], which does not fit the HERA data. The predictions in that case are rather similar to those of Ref. [21] using the light-front quark model.

Finally, in Figure 7 we compare our DA with that predicted by Ball and Braun [22]. The agreement is reasonable given that in Ref. [22], the expansion in Gegenbauer polynomials is truncated at low order, which is presumably responsible for the local minimum at \( z = 1/2 \). Certainly the two distributions are broader than the asymptotic prediction \( \sim 6z(1 - z) \).

\(^5\)It is equivalent to Eq. (2.7) with the higher-twist \( m_f^2 - \nabla \) terms set to zero.
Moments of the leading twist DA at the scale $\mu$

| Reference          | Approach | Scale $\mu$ | $\langle \xi^2 \rangle_\mu$ | $\langle \xi^4 \rangle_\mu$ | $\langle \xi^6 \rangle_\mu$ | $\langle \xi^8 \rangle_\mu$ | $\langle \xi^{10} \rangle_\mu$ |
|--------------------|----------|-------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| (This paper)       | Best fit | $\sim 1$ GeV | 0.227                       | 0.105                       | 0.062                       | 0.041                       | 0.029                       |
| (This paper)       | BG fit   | $\sim 1$ GeV | 0.229                       | 0.107                       | 0.063                       | 0.042                       | 0.030                       |
| (This paper)       | Old BG prediction | $\sim 1$ GeV | 0.182                       | 0.072                       | 0.037                       | 0.022                       | 0.014                       |
| [21]               | LFQM     | 1 GeV       | 0.19$^{[0.21]}_{}$          | 0.08$^{[0.09]}_{}$          | 0.04$^{[0.05]}_{}$          |                             |                             |
| [23]               | GenSR    | 1 GeV       | 0.227(7)                    | 0.095(5)                    | 0.051(4)                    | 0.030(2)                    | 0.020(5)                    |
| [24]               | SR       | 1 GeV       | 0.26                        |                            | 0.15                        |                             |                             |
| [22]               | SR       | 1 GeV       | 0.26(4)                     |                            |                             |                             |                             |
| [25]               | SR       | 1 GeV       | 0.254                       |                            |                             |                             |                             |
| [26]               | SR       | 1 GeV       | $0.23^{+0.03}_{-0.02}$      | $0.11^{+0.03}_{-0.02}$      |                             |                             |                             |
| [27]               | Lattice  | 2 GeV       | 0.24(4)                     |                            |                             |                             |                             |
| $6z(1-z)$          | $\infty$ | 0.2         | 0.086                       | 0.048                       | 0.030                       | 0.021                       |

Table 3: Our extracted values for $\langle \xi^n \rangle_\mu$, compared to predictions based on the light-front quark model (LFQM), QCD sum rules (SR), Generalised QCD Sum Rules (GenSR) or lattice QCD. Two predictions are given for each moment in the LFQM approach; one corresponding to a harmonic oscillator potential and the other (in square brackets) a linear potential.

5. Conclusions

The dipole model of diffractive photoproduction has been used successfully to describe a large body of data [14]. In this paper we have used it, together with accurate data on $\rho$-meson photoproduction collected at the HERA collider [5, 6], in order to extract the $\rho$ meson’s light-cone wavefunction. The data require qualitatively different behaviour for the two meson polarizations. In particular, there is evidence for an enhancement of the endpoint contributions to the wavefunction for transversely polarized mesons. We extracted the leading-twist $\rho$-meson distribution amplitude for longitudinal polarization and found it to agree well with predictions based on QCD sum rules and from the lattice.

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Figure 2: Comparison to the ZEUS data. The $\sigma_L/\sigma_T$ data are at $W = 90$ GeV.
Figure 3: Comparison to the H1 data. The $\sigma_L/\sigma_T$ data are at $W = 75$ GeV.
Figure 4: The longitudinal (left) and transverse (right) light-cone wavefunctions squared corresponding to the BG fit with additional end-point enhancement in the transverse wavefunction.

Figure 5: The longitudinal (left) and transverse (right) light-cone wavefunctions squared corresponding to the BG fits with (solid) and without (dashed) additional end-point enhancement in the transverse wavefunction. The BG parameterization of Ref. [12] is also shown as the dotted curve. All curves evaluated at $r = 0$. 
Figure 6: The longitudinal (left) and transverse (right) light-cone wavefunctions squared corresponding to the BG fits with additional end-point enhancement in the transverse wavefunction. The solid and dashed curves are extracted after rescaling the data down by 5% and 12% respectively. Both curves evaluated at \( r = 0 \).

Figure 7: The extracted DA at \( \mu = 1 \) GeV (solid) compared to the DA at 1 GeV of Ref. [22] (dotted) and the asymptotic DA (dashed).