Multiplicity distribution in pseudo-rapidity windows and charge conservation

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Charged multiplicity distribution in a pseudo-rapidity window is formulated under the assumption that the charge conservation is satisfied in the full phase space. At first, we analyze measured charged particle multiplicity distributions in pseudo-rapidity windows in LHC by CMS and ALICE collaborations with the two probability distributions. One is the convolution of negative binomial and Poisson distributions, and the other is the Glauber-Lachs formula. Each distribution is considered as an analogy of the quantum optics. Next, we analyze the data with the double GL formulae for $|\eta| < 2.4$ at 7 TeV by the CMS collaboration and for $|\eta| < 1.5$ at 8 TeV by the ALICE collaboration to describe the global structure of measured distributions.

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I. INTRODUCTION

In the middle of 1980’s, multiplicity distributions of charged particles in pseudo-rapidity windows were reported in the CERN $p\bar{p}$ collider experiments [1]. To analyze the data, a multiplicity distribution which is a convolution of a negative binomial distribution (NBD) and a Poisson distribution (PSND) was proposed [2]:

$$P(n, \langle n \rangle) = \sum_{n=n_1+n_2} P(n_1, \langle n_1 \rangle)P_2(n_2, \langle n_2 \rangle),$$

$$P_1(n_1, \langle n_1 \rangle) = \frac{(n_1 + k - 1)!}{n_1!(k-1)!} \frac{(\langle n_1 \rangle/k)^{n_1}}{\langle n_1 \rangle^{n_1}},$$

$$P_2(n_2, \langle n_2 \rangle) = \frac{(n_2)^{n_2}}{n_2!} e^{-n_2}.$$

In the above equations, $\langle n \rangle$, $\langle n_1 \rangle$, and $\langle n_2 \rangle$ denote the average multiplicities in each distribution, where a relation, $\langle n \rangle = \langle n_1 \rangle + \langle n_2 \rangle$, holds. Three parameters, $k$, $\langle n \rangle$ and $\bar{p} = \langle n_1 \rangle / \langle n \rangle$ are contained in Eq. (1). The NBD corresponds to the distribution of particles emitted from the chaotic sources in the thermal equilibrium, and the PSND to that of particles emitted from the coherent source. After the analysis of measured negative charged multiplicity distributions in the full phase space at $\sqrt{s} = 540$ GeV by the use of Eq. (1) with $k = 1$ and $k = 2$, measured charged multiplicity distributions in the pseudo-rapidity windows were analyzed to estimate the value of $\bar{p}$. A stochastic background of Eq. (1) was investigated in [3].

In a model of identical particle correlations based on the quantum optical approach [4, 5], particles emitted from chaotic sources and those from coherent source are correlated [6, 8]. Therefore, the multiplicity distribution composed of chaotic and coherent components is not necessarily written by two independent distributions such as Eq. (1).

In [9], a multiplicity distribution obtained from semi-inclusive momentum distributions in the quantum optical approach have been presented:

$$P(n, \langle n \rangle) = \frac{(p_{nn}(\langle n \rangle))^n}{(1 + p_{in} \langle n \rangle)^{n+1}} \exp \left( - \frac{(1 - p_{in}) \langle n \rangle}{1 + p_{in} \langle n \rangle} \right) \times L_n \left( \frac{1 - p_{in}}{1 + p_{in} \langle n \rangle} \right),$$

where, $p_{nn}$ denotes the ratio of the average multiplicity of negative charged particles emitted from the chaotic source to $\langle n \rangle$. Equation (4) is called Glauber-Lachs (GL) formula [4, 5, 10].

In the LHC experiments, charged particle multiplicity distributions are measured in restricted pseudo-rapidity windows. In the full phase space, charge conservation should be satisfied. Therefore, we would like to consider a relation between the charged multiplicity distribution in a pseudo-rapidity window and that in the full phase space. In addition, we would like to investigate some characteristics in $\bar{p}$ and $p_{in}$ analyzing the measured multiplicity distributions in the recent LHC experiments by Eq. (1) with $k = 1$, or 2, and Eq. (4).

In the invariant energy $\sqrt{s}$ above several hundred GeV, it is considered that it would be very hard to describe measured multiplicity distributions with a single probability distribution [11-17]. We also try to fit the data with double GL formulae.

The present paper is organized as follows. In section 2, charged multiplicity distribution in a pseudo-rapidity...
window is formulated under the assumption that the charge conservation is satisfied in the full phase space. In section 3, charged multiplicity distributions in pseudo-rapidity windows measured in the LHC experiments are analyzed by the use of Eq. (1) and Eq. (4). Moreover, double GL formulae are used in the analysis. Section 4 is devoted to concluding remarks. Detail calculations for some equations in section 2, and explicit expressions of charged multiplicity distributions for PSND, NBD and generalized Glauber-Lachs (GGL) formula in the pseudo-rapidity window are shown in appendix A.

For comparison, we also analyze the data, directly using Eq. (1) with two parameters $\tilde{p}$ and $\langle n \rangle$. The results are shown in appendix B.

II. CHARGED MULTIPICITY DISTRIBUTION IN A PSEUDO-RAPIDITY WINDOW WITH CHARGE CONSERVATION IN THE FULL PHASE SPACE

In the full phase space, the measured multiplicity distribution satisfies the charge conservation. For simplicity, we assume that the charged particles are produced in pairs of a positive charged particle and a negative charged particle. Let $P(n)$, $n = 0, 1, \ldots$ A bijection between a multiplicity distribution of negative charged particles, and $P_{ch}(2n)$ be that of charged particles in the full phase space. We assume that a relation,

$$P(n) = P_{ch}(2n),$$

holds.

Furthermore, we would like to adopt the following assumption: A probability that each particle produced in the full phase space enters into a limited window (and is detected) is $\zeta$ ($0 \leq \zeta \leq 1$), and that each particle does not enter into the window is $1 - \zeta$. When more than $n$ pairs of charged particles are produced in the full phase space, and that $m$ ($2n \leq m \leq 0$) charged particles enter into the pseudo-rapidity window, the probability distribution $P_{ob}(m)$ that $m$ charged particles enter into the window is written as,

$$P_{ob}(m) = \sum_{2n \geq m} \infty 2nC_m\zeta^m(1 - \zeta)^{2n-m} P_{ch}(2n).$$

In the following, $P(n)$ is written as $P(n, \langle n \rangle)$ with the average multiplicity $\langle n \rangle$ of negative charged particles in the full phase space. A multiplicity distribution $P_{\zeta}(j)$ ($j = 0, 1, 2, \ldots$) is defined as

$$P_{\zeta}(j) \equiv \sum_{n=j} \infty nC_j \zeta(2 - \zeta)^{j}[1 - (1 - \zeta)^2]^{n-j} P(n, \langle n \rangle),$$

which denotes the multiplicity distribution that when $n$ pairs ($n \geq j$) of charged particles are produced, ($n - j$) pairs are outside the pseudo-rapidity window, and at least one particle enters into the window from any $j$ pairs of negative and positive charged particles.

Relations among $P_{ob}(n)$, $P_{\zeta}$ and $P(n, \langle n \rangle)$ are shown in Appendix A. We obtain from Eq. (A17):

$$P_{ob}(n) = \sum_{j=0}^{[n/2]} n-jC_j \zeta^j[2\zeta(1 - \zeta)]^{n-2j} P_{\zeta}(n-j).$$

In the present paper, we use three distribution functions, PSND, NBD and GL formula for $P(n, \langle n \rangle)$. In any of the three distribution functions, the following relation holds:

$$P_{\zeta}(n) = P(n, \langle n_{\zeta} \rangle), \quad \langle n_{\zeta} \rangle = \zeta(2 - \zeta)\langle n \rangle. \quad (9)$$

From Eqs. (8) and (9), the multiplicity distribution $P_{ob}(n)$ of charged particles in the pseudo-rapidity window is expressed with that $P(n, \langle n \rangle)$ of negative charged particles in the full phase space as,

$$P_{ob}(n) = \sum_{j=0}^{[n/2]} n-jC_j \zeta^j[2\zeta(1 - \zeta)]^{n-2j} P_{\zeta}(n-j) \times P(n-j, \langle n_{\zeta} \rangle). \quad (10)$$

III. ANALYSIS OF CHARGED MULTIPICITY DISTRIBUTIONS IN PSEUDO-RAPIDITY WINDOWS

At first, the invariant energy $\sqrt{s}$ dependence of average charged multiplicity, $\langle n_{ch} \rangle$, in the full phase space in non-single diffractive (NSD) events is parametrized as,

$$\langle n_{ch} \rangle = 0.986s^{1/4} + 6.309,$$

by the least mean square method with the data from $\sqrt{s} = 30.4$ GeV to $\sqrt{s} = 1800$ GeV [18, 20]. The average multiplicity of negative charged particles in the full phase space, $\langle n \rangle$, is estimated from Eq. (11) with the relation $\langle n \rangle = \langle n_{ch} \rangle/2$. Those used in the present analysis [13, 21] are listed in Table I.

| $\sqrt{s}$ (TeV) | 0.9 | 2.36 | 2.76 | 7 | 8 |
|------------------|-----|------|------|---|---|
| $\langle n \rangle = \langle n_{ch} \rangle/2$ | 17.9 | 27.1 | 29.1 | 44.4 | 47.3 |

In the experiments of Bose-Einstein correlations (BEC), the number of identical boson pairs, say $\pi^{-}$ pairs $N^{(2-)}$ relative to the number of uncorrelated pion pairs $N^{BG}$ as a function of relative momentum squared, $Q = -(p_1 - p_2)^2$, is measured, and for example, it is fitted by

$$N^{(2-)} / N^{BG} = C[1 + \lambda \Omega(Qr)](1 + \delta Q).$$

Function $\Omega(Qr)$ is often parametrized as $\Omega(Qr) = e^{-Qr}$. Normalization factor $C$ is determined so as to $N^{(2-)} / N^{BG} \simeq 1$ for $Q >> 1$. 

TABLE I. Average multiplicities of negative charged particles in the full phase space used in the analysis.
In the quantum optical approach to the BEC \textsuperscript{22}, the second order BEC function is given by

$$N^{(2-)}/N_{BG}^{(2-)} = 1 + 2p_n(1 - p_n)e^{-Qr} + p_n^2 e^{-2Qr}.$$  

Therefore, the following relation is satisfied:

$$\lambda = p_n(2 - p_n). \quad (12)$$

In the present analysis, we estimate the value of $p_n$ from the measured charged multiplicity distribution. Data samples used in the BEC experiments are different from those used in the charged multiplicity measurements \textsuperscript{23} \textsuperscript{24}. For example, in the CMS collaboration, BEC data are taken for $p_T > 200$ MeV and $|\eta| < 2.4$ \textsuperscript{23}. On the other hand, measured charged multiplicity distributions in pseudo-rapidity windows are taken for $p_T > 0$ MeV. Therefore, it is not clear whether Eq.\textsuperscript{(12)} is satisfied or not. We would like to compare the estimated value of $p_n(2 - p_n)$ with parameter $\lambda$ estimated from the BEC experiments.

A. Analysis with Eq.\textsuperscript{(10)} and the convolution of NBD and PSND

We analyze measured charged multiplicity distributions of non-single diffractive (NSD) events in the pseudo-rapidity window, $|\eta| < \Delta \eta$ \textsuperscript{15} \textsuperscript{21}.

At first, we analyze charged multiplicity distributions by the CMS Collaboration in pseudo-rapidity windows, $|\eta| < \Delta \eta$, at $\Delta \eta = 0.5, 1.0, 1.5, 2.0$ and 2.4 with Eq.\textsuperscript{(10)} and the convolution of NBD and PSND given by the following equation with $k = 1$ or 2,

$$P(n, \langle n \rangle) = \sum_{n=n_1+n_2} \frac{(n_1 + k - 1)!}{n_1! (k-1)!} \frac{(\langle n_1 \rangle / k)^n_1}{\langle 1 + \langle n_1 \rangle / k \rangle^{n_1+k}} \times \frac{(n_2^\frac{\langle n_2 \rangle}{k})_{n_2}}{n_2!} e^{-\langle n_2 \rangle}, \quad (13)$$

where

$$\langle n_1 \rangle = \bar{p}(2 - \zeta) \langle n \rangle, \quad \langle n_2 \rangle = (1 - \bar{p}) \zeta(2 - \zeta) \langle n \rangle \quad (14)$$

Results on the charged multiplicity distributions at $\sqrt{s} = 0.9$ TeV by the CMS Collaboration with Eqs.\textsuperscript{(10)} and \textsuperscript{(13)} are shown in Fig.\textsuperscript{1} and Table \textsuperscript{11}.

At $\sqrt{s} = 0.9$ TeV, the results with $k = 2$ describes the data better than those with $k = 1$. In this case, the estimated value of $\bar{p}$ with $k = 1$ is almost 1. Therefore, the coherent component in multiplicity distribution is almost 0 and the chaotic component is to occupy almost 100 percent of multiplicities at $\sqrt{s} = 0.9$ TeV.

Results at $\sqrt{s} = 2.36$ TeV are shown in Fig.\textsuperscript{2} and Table \textsuperscript{11}. At $\sqrt{s} = 2.36$ TeV, the results with $k = 2$ describes the data better than those with $k = 1$ except for the data for $|\eta| < 0.5$. The value of $\chi^2_{min}/n.d.f$ in each analysis with $k = 2$ is greater than 1, and estimated values of $\bar{p}$ become almost 1. That for $|\eta| < 0.5$ with $k = 1$ is 1.65.

At $\sqrt{s} = 7$ TeV, the results with $k = 1$ and with $k = 2$ can not fit the data well.

For comparison, we also analyze the data, directly using Eq.\textsuperscript{(10)} with two parameters $\bar{p}$ and $\langle n \rangle$. In this case, $\langle n \rangle$ denotes the average charged multiplicity in the corresponding window. Results at $\sqrt{s} = 0.9$ and 2.36 TeV are shown respectively in Tables \textsuperscript{IV} and \textsuperscript{IV} in appendix B.

B. Analysis with Eq.\textsuperscript{(10)} and the GL formula

Next, we would like to analyze measured charged multiplicity distributions with Eq.\textsuperscript{(10)} and the GL formula,

$$P(n, \langle n \rangle) = \frac{(p_n \langle n \rangle)^n}{(1 + p_n \langle n \rangle)^{n+1}} \exp \left[ - \frac{(1 - p_n)\langle n \rangle}{1 + p_n \langle n \rangle} \right] \times I_n \left( - \frac{(1 - p_n)\langle n \rangle}{1 + p_n \langle n \rangle} \right), \quad (15)$$

where, $\langle n \rangle = \zeta(2 - \zeta) \langle n \rangle$. Results on the charged multiplicity distributions at $\sqrt{s} = 0.9, 2.36$ and 7 TeV by the CMS Collaboration by the use of Eqs.\textsuperscript{(10)} and \textsuperscript{(15)}, are shown in Fig.\textsuperscript{3} and Table \textsuperscript{IV}.

At $\sqrt{s} = 0.9$ TeV, values of $\chi^2_{min}/n.d.f.$ are less than 1 in all pseudo-rapidity windows. At 2.36 TeV, values of $\chi^2_{min}/n.d.f.$ are less than 1 except for 1.16 for $|\eta| < 1.0$. At $\sqrt{s} = 7$ TeV, values of $\chi^2_{min}/n.d.f.$ are less than 2 except for 2.01 for $|\eta| < 0.5$. As can be seen from the Tables \textsuperscript{II}, \textsuperscript{III} and \textsuperscript{IV}, results with Eq.\textsuperscript{(10)} and the GL formula, Eq.\textsuperscript{(15)}, describe the data better than those with Eqs.\textsuperscript{(10)} and \textsuperscript{(13)} for all pseudo-rapidity windows at $\sqrt{s} = 0.9$ and 2.36 TeV by the CMS Collaboration.

Measured values of parameter $\lambda$ are $\lambda = 0.616 \pm 0.031$ at $\sqrt{s} = 0.9$ TeV, $\lambda = 0.663 \pm 0.087$ at $\sqrt{s} = 2.36$ TeV, and $\lambda = 0.618 \pm 0.043$ at $\sqrt{s} = 7$ TeV by the CMS Collaboration \textsuperscript{23}. By the ATLAS Collaboration \textsuperscript{24}, $\lambda = 0.74 \pm 0.11$ at $\sqrt{s} = 0.9$ TeV, and $\lambda = 0.71 \pm 0.07$ at $\sqrt{s} = 7$ TeV.

Values of $p_n(2 - p_n)$ estimated from the analysis of charged multiplicity distributions at $\sqrt{s} = 0.9$ TeV are smaller than $\lambda = 0.616$ except for 0.668 at $|\eta| < 0.5$. Estimated values of $p_n(2 - p_n)$ at $\sqrt{s} = 2.36$ TeV are not larger than $\lambda = 0.663 + 0.087$ for all pseudo-rapidity windows. Estimated values of $p_n(2 - p_n)$ at $\sqrt{s} = 7$ TeV are larger than $\lambda = 0.618 + 0.043$ for all pseudo-rapidity windows.

The pseudo-rapidity window $\Delta \eta$ dependence of estimated values of probability $\zeta$ shown in Fig.\textsuperscript{IV} are fitted by a straight line, $\zeta = a \Delta \eta$, at each $\sqrt{s}$. Results are shown in Fig.\textsuperscript{4} and estimated values of slope parameter $a$ are listed in Table \textsuperscript{V}.

Results on the analysis of the charged multiplicity distributions at $\sqrt{s} = 0.9, 2.76, 7$ and 8 TeV by the ALICE Collaboration by Eq.\textsuperscript{(10)} and the GL formula, Eq.\textsuperscript{(15)}, are shown in Fig.\textsuperscript{5} and \textsuperscript{6}. Parameters estimated in the analysis are listed in Table \textsuperscript{VI}.

At $\sqrt{s} = 0.9$ and 2.76 TeV, values of $\chi^2_{min}/n.d.f.$ are less than 1 for three pseudo-rapidity windows, $|\eta| < 0.5$, $|\eta| < 1.0$ and $|\eta| < 1.5$. Calculated results describe the
with Eqs. (10) and (13) : a) $k = 1$ and b) $k = 2$.

**TABLE II.** Parameters estimated from measured charged multiplicity distributions at $\sqrt{s} = 0.9$ TeV by the CMS Collaboration by the use of Eqs. (10) and (13) with $k = 1$ or $k = 2$.

| $\sqrt{s}$ (TeV) | $k$ | $\Delta\eta$ | $\hat{p}$ | $\zeta$ | $\chi^2_{\text{min}}$/n.d.f. | $\langle n \rangle_{\text{obs}} = 2 \zeta \langle n \rangle$ |
|------------------|----|-------------|-----------|--------|-----------------------------|------------------|
| 0.9              | 1  | 0.5         | $0.791 \pm 0.027$ | $0.102 \pm 0.002$ | 59.9/(23 − 2) | 3.65 ± 0.08 |
|                  |    |             | $0.673 \pm 0.027$ | $0.207 \pm 0.005$ | 292.0/(40 − 2) | 7.41 ± 0.18 |
|                  |    |             | $0.697 \pm 0.019$ | $0.315 \pm 0.006$ | 259.4/(52 − 2) | 11.28 ± 0.20 |
|                  |    |             | $0.698 \pm 0.015$ | $0.429 \pm 0.006$ | 233.3/(62 − 2) | 15.32 ± 0.23 |
|                  |    |             | $0.688 \pm 0.015$ | $0.517 \pm 0.007$ | 280.4/(68 − 2) | 18.44 ± 0.27 |
| 0.9              | 2  | 0.5         | $1.000 \pm 0.035$ | $0.108 \pm 0.002$ | 52.0/(23 − 2) | 3.87 ± 0.08 |
|                  |    |             | $1.000 \pm 0.024$ | $0.210 \pm 0.003$ | 82.8/(40 − 2) | 7.52 ± 0.11 |
|                  |    |             | $1.000 \pm 0.017$ | $0.316 \pm 0.003$ | 68.5/(52 − 2) | 11.31 ± 0.12 |
|                  |    |             | $1.000 \pm 0.011$ | $0.424 \pm 0.003$ | 45.1/(62 − 2) | 15.14 ± 0.11 |
|                  |    |             | $0.992 \pm 0.011$ | $0.508 \pm 0.004$ | 57.9/(68 − 2) | 18.19 ± 0.13 |

**TABLE III.** Parameters estimated from measured charged multiplicity distributions at 2.36 TeV by the CMS Collaboration by the use of Eqs. (10) and (13) with $k = 1$ and $k = 2$.

| $\sqrt{s}$ (TeV) | $k$ | $\Delta\eta$ | $\hat{p}$ | $\zeta$ | $\chi^2_{\text{min}}$/n.d.f. | $\langle n \rangle_{\text{obs}} = 2 \zeta \langle n \rangle$ |
|------------------|----|-------------|-----------|--------|-----------------------------|------------------|
| 2.36             | 1  | 0.5         | $0.849 \pm 0.020$ | $0.088 \pm 0.002$ | 34.9/(23 − 2) | 4.72 ± 0.09 |
|                  |    |             | $0.769 \pm 0.027$ | $0.166 \pm 0.005$ | 299.1/(40 − 2) | 8.94 ± 0.26 |
|                  |    |             | $0.800 \pm 0.017$ | $0.264 \pm 0.005$ | 142.4/(50 − 2) | 14.20 ± 0.26 |
|                  |    |             | $0.794 \pm 0.015$ | $0.358 \pm 0.006$ | 148.2/(60 − 2) | 19.24 ± 0.34 |
|                  |    |             | $0.781 \pm 0.013$ | $0.435 \pm 0.007$ | 145.1/(70 − 2) | 23.52 ± 0.37 |
| 2.36             | 2  | 0.5         | $1.000 \pm 0.048$ | $0.091 \pm 0.003$ | 112.7/(23 − 2) | 4.88 ± 0.13 |
|                  |    |             | $1.000 \pm 0.031$ | $0.181 \pm 0.004$ | 156.5/(40 − 2) | 9.76 ± 0.20 |
|                  |    |             | $1.000 \pm 0.025$ | $0.274 \pm 0.005$ | 133.7/(50 − 2) | 14.80 ± 0.25 |
|                  |    |             | $1.000 \pm 0.020$ | $0.367 \pm 0.005$ | 118.5/(60 − 2) | 19.73 ± 0.29 |
|                  |    |             | $1.000 \pm 0.016$ | $0.440 \pm 0.005$ | 85.1/(70 − 2) | 23.74 ± 0.27 |
with Eqs. (10) and (13) : a) \( k = 1 \) and b) \( k = 2 \).

TABLE IV. Parameters estimated from measured charged multiplicity distributions at \( \sqrt{s} = 0.9, 2.36 \) and \( 7 \) TeV by the CMS collaboration with Eqs. (10) and (13).

| \( \sqrt{s} \) (TeV) | \( \Delta \eta \) | \( p_{\text{in}} \) | \( \zeta \) | \( \chi^2_{\text{min}}/\text{n.d.f.} \) | \( p_{\text{in}}(2 - p_{\text{in}}) \) | \( \langle n \rangle_{\text{ch}} = 2 \zeta(n) \) |
|-------------------|----------------|--------------|-----------|----------------|--------------------|----------------|
| 0.9               | 0.5            | 0.424 ± 0.007| 0.101 ± 0.000| 2.1/(23 – 2)    | 0.668 ± 0.004      | 3.62 ± 0.02      |
| 1.0               | 0.360 ± 0.010  | 0.203 ± 0.002| 32.7/(42 – 2)| 0.590 ± 0.006   | 7.27 ± 0.07       |
| 1.5               | 0.343 ± 0.008  | 0.307 ± 0.002| 35.7/(52 – 2)| 0.568 ± 0.005   | 10.99 ± 0.08      |
| 2.0               | 0.321 ± 0.007  | 0.415 ± 0.003| 40.4/(62 – 2)| 0.539 ± 0.005   | 11.86 ± 0.09      |
| 2.4               | 0.300 ± 0.007  | 0.501 ± 0.003| 55.8/(68 – 2)| 0.510 ± 0.005   | 17.94 ± 0.12      |
| 2.36              | 0.5            | 0.500 ± 0.015| 0.085 ± 0.001| 6.9/(23 – 2)    | 0.750 ± 0.008      | 4.61 ± 0.03      |
| 1.0               | 0.415 ± 0.015  | 0.170 ± 0.002| 44.0/(40 – 2)| 0.658 ± 0.009   | 9.21 ± 0.11       |
| 1.5               | 0.422 ± 0.011  | 0.260 ± 0.002| 25.7/(50 – 2)| 0.666 ± 0.006   | 14.04 ± 0.11      |
| 2.0               | 0.390 ± 0.012  | 0.350 ± 0.003| 50.5/(60 – 2)| 0.628 ± 0.007   | 18.86 ± 0.18      |
| 2.4               | 0.359 ± 0.013  | 0.423 ± 0.004| 56.0/(70 – 2)| 0.589 ± 0.008   | 22.82 ± 0.21      |
| 7                 | 0.5            | 0.597 ± 0.023| 0.067 ± 0.001| 78.2/(41 – 2)   | 0.838 ± 0.009      | 5.95 ± 0.06      |
| 1.0               | 0.530 ± 0.013  | 0.137 ± 0.001| 128.6/(70 – 2)| 0.779 ± 0.006   | 12.17 ± 0.08      |
| 1.5               | 0.497 ± 0.010  | 0.207 ± 0.001| 181.3/(95 – 2)| 0.747 ± 0.005   | 18.38 ± 0.12      |
| 2.0               | 0.481 ± 0.009  | 0.280 ± 0.002| 194.3/(115 – 2)| 0.731 ± 0.005   | 24.86 ± 0.14      |
| 2.4               | 0.491 ± 0.007  | 0.340 ± 0.002| 129.6/(127 – 2)| 0.741 ± 0.004   | 30.19 ± 0.13      |

TABLE V. Slope parameters estimated from measured charged multiplicity distributions at \( \sqrt{s} = 0.9, 2.36 \) and \( 7 \) TeV by the CMS Collaboration.

| \( \sqrt{s} \) (TeV) | 0.9 | 2.36 | 7 |
|---------------------|-----|-----|---|
| \( a \)             | 0.204 ± 0.001 | 0.173 ± 0.001 | 0.140 ± 0.001 |
| \( \chi^2_{\text{min}}/\text{n.d.f.} \) | 26.0/(5 – 1) | 7.0/(5 – 1) | 43.0/(5 – 1) |

data at 0.9 and 2.76 TeV by the ALICE Collaboration very well.

At \( \sqrt{s} = 7 \) TeV, values of \( \chi^2_{\text{min}}/\text{n.d.f.} \) are less than 2 except for 2.03 for \( |\eta| < 1.0 \). At \( \sqrt{s} = 8 \) TeV, values of \( \chi^2_{\text{min}}/\text{n.d.f.} \) are less than 2.

In the analyses of the data by CMS and ALICE Collaborations, results at \( \sqrt{s} = 7 \) and 8 TeV are not better than those from \( \sqrt{s} = 0.9 \) TeV to 2.76 TeV. In addition, though, value of \( P_{\text{ch}}(0) \) satisfies the condition, \( P_{\text{ch}}(0) > P_{\text{ch}}(1) \) for each calculation, each peak of measured multiplicity distribution \( P_{\text{ch}}(n) \) for \( |\eta| < \Delta \eta \) with \( \Delta \eta \geq 1.0 \), located around \( 4 < n < 8 \), cannot be reproduced by the single GL formula. In the next subsection, we would analyze the measured multiplicity distributions for \( |\eta| < 2.4 \) at \( \sqrt{s} = 7 \) TeV by the CMS Collaboration...
and that for $|\eta| < 1.5$ at $\sqrt{s} = 7$ TeV by the ALICE Collaboration using double GL formulae.

C. Analysis of charged multiplicity distributions with double GL formulae

In the invariant energy $\sqrt{s}$ region above several hundred GeV, it is assumed that mainly two production processes occur exclusively each other. Process 1 (soft process) occurs with a probability $\alpha$ and the multiplicity distribution of negative particles is given $P_1(n, \langle n \rangle_1)$, process 2 (semi-hard process) occurs with a probability $(1 - \alpha)$ and the multiplicity distribution of negative particles is given $P_2(n, \langle n \rangle_2)$ In the full phase space, combined multiplicity distribution $P(n, \langle n \rangle)$ can be given by the following equation,

$$P(n, \langle n \rangle) = \alpha P_1(n, \langle n \rangle_1) + (1 - \alpha) P_2(n, \langle n \rangle_2).$$  \hspace{1cm} (16)

From Eq. (16), we obtain

$$\langle n \rangle = \alpha \langle n_1 \rangle + (1 - \alpha) \langle n_2 \rangle. \hspace{1cm} (17)$$

In our approach, the observed multiplicity distribution $P_{ob}(n)$ in a pseudo-rapidity window is given by

$$P_{ob}(n) = \alpha GL_1(n) + (1 - \alpha) GL_2(n),$$

$$GL_i(n) = \sum_{j=0}^{[n/2]} n-j C_j \left( \zeta_i^2 \right)^j \left( 2 - \zeta_i \right)^{n-j} \times P(n-j, \langle n_{i\zeta_i} \rangle), \hspace{1cm} i = 1, 2, \hspace{1cm} (18)$$

where $\langle n_{i\zeta_i} \rangle = \zeta_i(2-\zeta_i)(n_i)$. We assume that $\langle n_1 \rangle > \langle n_2 \rangle$ and that each multiplicity distribution $P_i(n, \langle n_i \rangle)$ is given
by the Glauber-Lachs (GL) formula,

\[
P(n, \langle n_i \rangle) = \frac{(p_i/\langle n_i \rangle)^n}{(1 + p_i/\langle n_i \rangle)^{(n+1)}} \exp \left[ - \frac{(1 - p_i/\langle n \rangle)}{1 + p_i/\langle n \rangle} \right] \times L_n(\frac{1 - p_i}{p_i(1 + p_i/\langle n \rangle)}).
\]

We parametrize

\[
\langle n_i \rangle = r_i \langle n \rangle, \quad i = 1, 2.
\]

Then, we obtain

\[
1 = \alpha r_1 + (1 - \alpha)r_2, \quad r_1 > r_2 > 0.
\]

TABLE VI. Parameters estimated from measured charged multiplicity distributions at \(\sqrt{s} = 0.9, 2.76, 7\) and \(\sqrt{s} = 8\) TeV by the ALICE Collaboration with Eqs. (10) and (15).

| \(\sqrt{s}\) (TeV) | \(\Delta \eta\) | \(p_{in}\) | \(\zeta\) | \(\chi^2_{min}/n.d.f\) | \(p_{in}(2 - p_{in})\) | \(\langle n \rangle_{obs} = 2\zeta(n)\) |
|------------------|----------------|----------|--------|------------------|------------------|------------------|
| 0.9              | 0.5            | 0.416 ± 0.008 | 0.106 ± 0.001 | 4.0/(36 - 2)    | 0.659 ± 0.005   | 3.80 ± 0.03      |
|                  | 1.0            | 0.382 ± 0.008 | 0.217 ± 0.002 | 15.0/(60 - 2)   | 0.618 ± 0.005   | 7.77 ± 0.06      |
|                  | 1.5            | 0.362 ± 0.009 | 0.328 ± 0.003 | 38.5/(72 - 2)   | 0.593 ± 0.006   | 11.74 ± 0.10     |
| 2.76             | 0.5            | 0.446 ± 0.008 | 0.083 ± 0.001 | 13.1/(50 - 2)   | 0.693 ± 0.004   | 4.83 ± 0.03      |
|                  | 1.0            | 0.415 ± 0.008 | 0.168 ± 0.001 | 47.6/(83 - 2)   | 0.658 ± 0.006   | 9.78 ± 0.07      |
|                  | 1.5            | 0.396 ± 0.008 | 0.253 ± 0.002 | 81.2/(110 - 2)  | 0.635 ± 0.006   | 14.72 ± 0.11     |
| 7                | 0.5            | 0.483 ± 0.009 | 0.068 ± 0.001 | 75.2/(68 - 2)   | 0.733 ± 0.005   | 6.04 ± 0.05      |
|                  | 1.0            | 0.452 ± 0.009 | 0.137 ± 0.001 | 231.5/(116 - 2) | 0.700 ± 0.005   | 12.17 ± 0.11     |
|                  | 1.5            | 0.486 ± 0.008 | 0.200 ± 0.001 | 248.7/(152 - 2) | 0.736 ± 0.004   | 17.76 ± 0.12     |
| 8                | 0.5            | 0.495 ± 0.011 | 0.066 ± 0.001 | 44.7/(66 - 2)   | 0.745 ± 0.006   | 6.24 ± 0.05      |
|                  | 1.0            | 0.447 ± 0.009 | 0.135 ± 0.001 | 137.5/(112 - 2) | 0.694 ± 0.005   | 12.77 ± 0.11     |
|                  | 1.5            | 0.447 ± 0.008 | 0.201 ± 0.002 | 209.0/(144 - 2) | 0.694 ± 0.004   | 19.01 ± 0.14     |

In our parametrization, \(\langle n \rangle\) is given from Eq. (11) or Table IV and \(r_2\) is determined from Eq. (20). Therefore, 6 parameters \(a, r_1, \zeta_1, p_2, \zeta_2\), and \(\zeta_2\) are contained in Eq. (15).

Results on the analyses of measured charged multiplicity distribution for \(|\eta| < 2.4\) at 7 TeV by the CMS Collaboration and that for \(|\eta| < 1.5\) at 8 TeV by the ALICE Collaboration with the double GL formulae are shown in Fig. 7. Parameters estimated form the analyses are listed in Table VII.

IV. CONCLUDING REMARKS

Multiplicity distribution in the pseudo-rapidity window, which satisfies the charge conservation in the full phase space, is formulated. By the use of the GL formula for the multiplicity distribution of negative charged particles in the full phase space, we analyze the charged multiplicity distributions in pseudo-rapidity windows in non-single diffractive (NSD) events reported by CMS and ALICE Collaborations.

R1) The probability \(\zeta\) that each particle enter into the given pseudo-rapidity window \(|\eta| < \Delta \eta\) is approximately expressed by \(\zeta = a\Delta \eta\) with parameter \(a\), which depends on the invariant energy \(\sqrt{s}\).

R2) In our analysis, relation, \(P_{ob}(0) > P_{ob}(1)\), holds, which is similar to the experimental data. In \([10]\), relation \(P_{ob}(0) > P_{ob}(1)\) and peak around \(4 < n < 8\) are well reproduced by the use of a compound distribution.

R3) In the measured charged multiplicity distributions for \(\Delta \eta > 1.5\), a peak appears around \(4 < n < 8\) in each distribution. We cannot reproduce the peak from our calculation with the single GL formula.

R4) We can reproduce global behavior of measured multiplicity distributions for \(\Delta \eta = 2.4\) at \(\sqrt{s} = 7\) TeV by the CMS Collaboration, and for \(\Delta \eta = 1.5\) at \(\sqrt{s} = 8\) TeV by the ALICE Collaboration with the double GL formulae.
FIG. 5. Charged multiplicity distributions at $\sqrt{s} = 0.9$ and 2.76 TeV by the ALICE Collaboration compared to theoretical curves (solid or dotted lines) calculated with Eqs. (10) and (15).

FIG. 6. Charged multiplicity distributions at $\sqrt{s} = 7$ and 8 TeV compared to theoretical curves (solid or dotted lines) calculated with Eqs. (10) and (15).

TABLE VII. Parameters estimated from the charged multiplicity distributions at $\sqrt{s} = 7$ TeV by the CMS Collaboration and at 8 TeV by the ALICE Collaboration with the double GL formulae.

| $\sqrt{s}$(TeV) | $\Delta \eta$ | $\alpha$ | $p_1$ | $\zeta_1$ | $r_1$ |
|-----------------|--------------|----------|-------|-----------|-------|
| 7               | 2.4          | 0.717 ± 0.022 | 0.290 ± 0.012 | 0.330 ± 0.017 | 1.28 ± 0.01 |
| 8               | 1.5          | 0.540 ± 0.045 | 0.228 ± 0.017 | 0.204 ± 0.168 | 1.41 ± 0.12 |

Table VII (Continued).

| $p_2$ | $\zeta_2$ | $r_2$ | $\chi^2_{\text{min}}$/n.d.f |
|-------|-----------|-------|-----------------------------|
| 0.158 ± 0.028 | 0.509 ± 0.279 | 0.291 ± 0.002 | 16.2/(127 − 6) |
| 0.223 ± 0.036 | 0.179 ± 0.471 | 0.519 ± 0.018 | 50.8/(144 − 6) |
From Eq. (A2), we have following relations, written as,

\[ P(n) = P_{ch}(2n) \quad (A1) \]

holds. The probability generating function (GF) \( \Pi(z) \) for \( P(n) \), and that \( \Pi_{ch}(z) \) for \( P_{ch}(2n) \) are respectively written as,

\[ \Pi(z) = \sum_{n=0}^{\infty} P(n)z^n, \quad \Pi_{ch}(z) = \sum_{n=0}^{\infty} P_{ch}(2n)z^{2n}. \quad (A2) \]

From Eq. (A2), we have following relations,

\[ P(n) = \frac{1}{n!} \Pi^{(n)}(z)|_{z=0}, \]
\[ P_{ch}(2n-1) = \frac{1}{(2n-1)!} \Pi_{ch}^{(2n-1)}(z)|_{z=0} = 0, \]
\[ P_{ch}(2n) = \frac{1}{(2n)!} \Pi_{ch}^{(2n)}(z)|_{z=0}. \]

From Eqs. (A1) and (A2), the following relation is satisfied:

\[ \Pi_{ch}(z) = \Pi(z^2). \quad (A3) \]

It is assumed that a probability that each particle produced in the full phase space enters into a pseudo-rapidity window is \( \zeta \) (\( 0 \leq \zeta \leq 1 \)), and that each particle does not enter into the window is \( 1 - \zeta \). When more than \( n \) pairs of positive and negative charged particles are produced in the full phase space, and that \( m \) \( (2n \geq m \geq 0) \) charged particles enter into the pseudo-rapidity window, the probability distribution that \( m \) charged particles are detected, \( P_{ob}(m) \), is written as

\[ P_{ob}(m) = \sum_{2n \geq m} 2nC_m \zeta^m (1 - \zeta)^{2n-m} P_{ch}(2n). \quad (A4) \]

The GF for \( P_{ob}(m) \) is defined by

\[ \Pi_{ob}(z) = \sum_{m=0}^{\infty} P_{ob}(m)z^m. \quad (A5) \]

Substituting Eq. (A4) into Eq. (A5), and using the definition of \( \Pi_{ch}(z) \), Eq. (A2), we obtain

\[ \Pi_{ob}(z) = \Pi((\zeta + 1 - \zeta)^2). \quad (A6) \]

Putting

\[ y = \frac{(\zeta)^2 + 2\zeta(1 - \zeta)}{\zeta(2 - \zeta)}, \quad (A7) \]

and using the relation \( (\zeta + 1 - \zeta)^2 = \zeta(2 - \zeta)y + (1 - \zeta)^2 \), we can rewrite Eq. (A6) as

\[ \Pi_{ob}(z) = \sum_{j=0}^{\infty} y^j P_\zeta(j). \quad (A8) \]
where,
\[ P_\zeta(j) = \sum_{n=j}^{\infty} n! C_j \left[ \zeta(2 - \zeta) \right]^j \left[ (1 - \zeta)^2 \right]^{n-j} P(n). \]  
(A9)

Equation (A9) denotes the probability that when more than \( n \) pairs \((n \geq j)\) of charged particles are produced, \((n-j)\) pairs are outside the pseudo-rapidity window, and at least one particle enters into the window from any \( j \) pairs of negative and positive charged particles. The GF \( \Pi_\zeta(y) \) for \( P_\zeta(j) \) is defined as
\[ \Pi_\zeta(y) = \sum_{j=0}^{\infty} y^j P_\zeta(j). \]  
(A10)

On the other hand, from Eqs(A6) and (A7), we obtain the following relation,
\[ \Pi_{ob}(z) = \Pi(x), \quad x = \zeta(1 - \zeta)y + 1 \]  
(A11)

In the following, the multiplicity distribution \( P(n) \) is written as \( P(n, \langle n \rangle) \), where \( \langle n \rangle \) is the average multiplicity of negative charged particles in the full phase space. It’s GF \( \Pi(x) \) is also written as \( \Pi(x, \langle n \rangle) \): 
\[ \Pi(x, \langle n \rangle) = \sum_{j=0}^{\infty} P(n, \langle n \rangle) x^n. \]

Then, we obtain two relations among three GF’s:
\[ \Pi_{ob}(z) = \Pi_\zeta(y), \quad y = \frac{(\zeta z)^2 + 2\zeta(1 - \zeta z)}{\zeta(2 - \zeta)} ; \]  
(A12)
\[ \Pi_\zeta(y) = \Pi(x, \langle n \rangle), \quad x = \zeta(2 - \zeta)(y - 1) + 1. \]  
(A13)

It should be noted that \( \Pi_{ob}(z) \) is the GF for \( P_{ob}(n) \), \( \Pi_\zeta(y) \) is that for \( P_\zeta(n) \), and \( \Pi(x, \langle n \rangle) \) is for \( P(n, \langle n \rangle) \).

2. Relation between \( P_\zeta(n) \) and \( P(n, \langle n \rangle) \), or \( \Pi_\zeta(y) \) and \( \Pi(x, \langle n \rangle) \)

If variable \( x \) is contained in the form of \( \langle n \rangle(x - 1) \) in \( \Pi(x, \langle n \rangle) \), \( \Pi_\zeta(y) \) in Eq.(A13) is written as
\[ \Pi_\zeta(y) = \Pi(y, \langle n \zeta \rangle), \quad \langle n \zeta \rangle = \zeta(2 - \zeta)(\langle n \rangle). \]  
(A18)

For example, let the multiplicity distribution of negative charged particles be given by the Generalized Glauber-Lachs (GGL) formula with \( 0 < p < 1 \),
\[ P(n, \langle n \rangle) = \frac{(p(n)/k)^n}{(1 + p(n)/k)^{n+k}} \exp \left\{ -\frac{(1 - p(n))}{1 + p(n)/k} \right\} L_n(k-1) \left\{ 1 - \left( \frac{1 - p}{p(1 + p(n)/k)} \right)^k \right\}. \]  
(A19)

Its GF is given by
\[ \Pi(x, \langle n \rangle) = \left( 1 - \frac{p(n)}{k} (x - 1) \right)^{-k} \times \exp \left\{ \frac{(1 - p(n))(y - 1)}{1 - \frac{p(n)}{k}} \right\}. \]  
(A20)

When \( k = 1 \) with \( p = p_0 \), the GGL formula, Eq.(A19), reduces to the GL formula, Eq.(4). Then, the GF \( \Pi_\zeta(y) \) for \( P_\zeta(n) \) is given from Eq.(A18) as
\[ \Pi(y, \langle n \zeta \rangle) = \left( 1 - \frac{p(n)}{p_0} (y - 1) \right)^{-k} \times \exp \left\{ \frac{(1 - p)(y - 1)}{1 - \frac{p}{p_0}} \right\}. \]  
(A21)

The multiplicity distribution \( P_\zeta(n) \) is given from \( \Pi(y, \langle n \zeta \rangle) \), and is equal to \( P(y, \langle n \zeta \rangle) \):
\[ P_\zeta(n) = \frac{1}{n!} \frac{\partial^n \Pi(y, \langle n \zeta \rangle)}{\partial y^n} \bigg|_{y=0} = P(n, \langle n \rangle). \]

Therefore, \( P_\zeta(n) \) is given from Eq.(A19), if \( \langle n \rangle \) is replaced by \( \langle n \zeta \rangle \).

In the limit of \( p = 0 \), the GF \( \Pi(x, \langle n \rangle) \) in Eq.(A20) reduces to that for the PSND,
\[ \Pi(x, \langle n \rangle) = e^{(n)(x - 1)}. \]

In the limit of \( p = 1 \), it reduces to the GF for the NBD,
\[ \Pi(x, \langle n \rangle) = \left( 1 - \langle n \rangle(x - 1)/k \right)^{-k} \]

The relation among \( P(n, \langle n \rangle) \), \( \Pi(x, \langle n \rangle) \) and \( \Pi_\zeta(y) \) for the GGL formula is listed in Table VIII with other two examples.

3. Difference between \( P_{ob}(n) \) and \( P(n, \langle n \rangle) \) in the second order factorial moment

The \( m \)th order factorial moments for \( P_{ob}(n) \) and \( P_\zeta(n) \) are respectively given by
\[ F_{m,ob} \equiv \langle n(n-1)\cdots(n-m+1)\rangle_{ob} = \left. \frac{\partial^m \Pi_{ob}(z)}{\partial z^m} \right|_{z=0}, \]
\[ F_{m,\zeta} \equiv \langle n(n-1)\cdots(n-m+1)\rangle_{\zeta} = \left. \frac{\partial^m \Pi_\zeta(y)}{\partial y^m} \right|_{y=0}. \]
From Eq. (A22), we obtain,

If \( z = 1 \), then \( y = 1 \) from Eq. (A16). Then we obtain from Eqs. (A14) and (A15):

\[
F_{m,ob} = \sum_{j=0}^{[m/2]} \frac{m!}{j!(m-2j)!} \frac{(\zeta^2)^j [2\zeta(1-\zeta)]^{m-2j}}{|\zeta(2-\zeta)|^{m-j}} \times F_{m-j,\zeta}.
\]

From Eq. (A22), we obtain,

\[
\langle n \rangle_{ob} = 2\zeta \langle n \rangle, \quad \langle n(n-1) \rangle_{ob} = [2/(2-\zeta)]^2 \langle n(n-1) \rangle \zeta + 2\zeta \langle n \rangle.
\]

In the case of GGL formula, the second order factorial moment for \( P_{ob}(n) \) is given by

\[
\langle n(n-1) \rangle = [1 + p(2-p)/k] [2\zeta \langle n \rangle]^2 + \zeta [2\zeta \langle n \rangle].
\]

On the other hand, that for \( P(n) \) is given by

\[
\langle n(n-1) \rangle = [1 + p(2-p)/k] \langle n \rangle^2.
\]

As can be seen from Eqs. (A25) and (A26), an additional term, \( \zeta [2\zeta \langle n \rangle] \), appears on the right hand side of Eq. (A25), which is caused by the charge conservation in the full phase space.

**Appendix B: Analysis by the convolution of NBD and PSND**

In order to compare the results by Eqs. (10) and (13), where charge conservation in the full phase space is taken into account, we also analyze the data by the convolution of NSD and PSND, Eq. (1). Results at \( \sqrt{s} = 0.9 \) and 2.36 TeV in the CMS Collaboration are shown respectively in Tables [X] and [X].

Comparing each value of \( \chi^2_{min}/n.d.f \) in Table [I] and that in Table [X], the ratio of the former to the latter is from 0.79 to 0.91 at \( k = 1 \), and from 0.74 to 0.87 at \( k = 2 \).

In the comparison of each value of \( \chi^2_{min}/n.d.f \) in Table [I] and that in Table [X], the ratio is from 0.79 to 0.92 at \( k = 1 \), and from 0.86 to 0.92 at \( k = 2 \).

Therefore, fitting with Eqs. (10) and (13) becomes better than that with Eq. (1).
TABLE IX. Parameters estimated from the analysis of charged multiplicity distributions at $\sqrt{s} = 0.9$ TeV in the CMS collaboration by Eq. (1) with $k = 1$ or $k = 2$.

| $\sqrt{s}$ (TeV) | $k$ | $\Delta\eta$ | $\bar{p}$ | $\langle n \rangle$ | $\chi^2_{\text{min}}/n.d.f.$ |
|------------------|-----|---------------|------------|-----------------|-----------------------------|
| 0.9              | 1   | 0.5           | 0.808 ± 0.029 | 3.64 ± 0.09 | 70.5/(23 – 2) |
|                  |     | 1.0           | 0.689 ± 0.025 | 7.40 ± 0.19 | 322.4/(40 – 2) |
|                  |     | 1.5           | 0.715 ± 0.019 | 11.26 ± 0.21 | 291.4/(52 – 2) |
|                  |     | 2.0           | 0.715 ± 0.016 | 15.32 ± 0.24 | 290.3/(62 – 2) |
|                  | 2.4 | 0.705 ± 0.015 | 18.46 ± 0.30 | 352.3/(68 – 2) |
| 0.9              | 2   | 0.5           | 1.000 ± 0.039 | 3.89 ± 0.09 | 70.6/(23 – 2) |
|                  |     | 1.0           | 1.000 ± 0.025 | 7.62 ± 0.12 | 104.4/(40 – 2) |
|                  |     | 1.5           | 1.000 ± 0.018 | 11.45 ± 0.13 | 88.8/(52 – 2) |
|                  |     | 2.0           | 1.000 ± 0.012 | 15.32 ± 0.12 | 60.2/(62 – 2) |
|                  | 2.4 | 1.000 ± 0.011 | 18.29 ± 0.13 | 66.5/(68 – 2) |

TABLE X. Parameters estimated from the analysis of charged multiplicity distributions at 2.36 TeV by the CMS collaboration by the use of Eq. (1) with $k = 1$ and $k = 2$.

| $\sqrt{s}$ (TeV) | $k$ | $\Delta\eta$ | $\bar{p}$ | $\langle n \rangle$ | $\chi^2_{\text{min}}/n.d.f.$ |
|------------------|-----|---------------|------------|-----------------|-----------------------------|
| 2.36             | 1   | 0.5           | 0.860 ± 0.022 | 4.71 ± 0.09 | 43.9/(23 – 2) |
|                  |     | 1.0           | 0.781 ± 0.028 | 8.89 ± 0.03 | 326.8/(40 – 2) |
|                  |     | 1.5           | 0.810 ± 0.017 | 14.20 ± 0.27 | 160.6/(50 – 2) |
|                  |     | 2.0           | 0.807 ± 0.015 | 19.24 ± 0.35 | 165.4/(60 – 2) |
|                  | 2.4 | 0.795 ± 0.013 | 23.51 ± 0.39 | 157.6/(70 – 2) |
| 2.36             | 2   | 0.5           | 1.000 ± 0.049 | 4.91 ± 0.14 | 124.1/(23 – 2) |
|                  |     | 1.0           | 1.000 ± 0.032 | 9.80 ± 0.22 | 177.7/(40 – 2) |
|                  |     | 1.5           | 1.000 ± 0.026 | 14.87 ± 0.27 | 155.5/(50 – 2) |
|                  |     | 2.0           | 1.000 ± 0.021 | 19.88 ± 0.31 | 138.1/(60 – 2) |
|                  | 2.4 | 1.000 ± 0.016 | 23.93 ± 0.30 | 99.0/(70 – 2) |

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[24] G. Aad et al. (ATLAS Collaboration), Euro. Phys. J. C 466, 466 (2015).