Probe Spin-Velocity Dependent New Interactions by Spin Relaxation Times of Polarized $^3\text{He}$ Gas

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We have studied how to constrain the $\alpha \vec{\sigma} \cdot \vec{v}$ type interactions with the relaxation time of spin polarized noble gases in magnetic fields. Using the longest $T_2$ measured in the laboratory and the earth as the source, we obtained constraints on three new interactions. We present a new experimental upper bound to the vector-axial-vector ($V_A V$) type interaction for ranges between 1 to $10^8$ m. In combination with the previous result, we set the most stringent experiment limits on $g_V g_A$ ranging from $\sim \mu m$ to $\sim 10^8$ m. We improve the laboratory limit to the axial-axial-vector ($V_A A$) type interaction by $\sim 2$ orders or more for distances below $\sim 1$ cm. To our best knowledge, we report the first experiment upper limit on torsion induced by the earth on its surface.

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In the recent years, various models of new physics beyond the standard model have been studied, in which new massive particles such as the axion, familon and majoron, etc were theoretically introduced$^1$. New macroscopic interactions mediated by WISPs (weakly-interacting sub-eV particles) have been theoretically proposed. The interaction ranges of these new forces are from nanometers to meters. The fact that the dark energy density in the interaction ranges of these new forces are from nanometers to meters. The fact that the dark energy density in the general relativity, is another example$^{14, 15}$. In these models, the torsion coupling to the spin has the same strength as the curvature to energy momentum in the general relativity. In practical conditions, the torsion could also induce the spin/velocity dependent interaction between the source and the spin. Either for the torsion-induced or the vector-axial-vector ($V_A V$) type interaction, the spin of polarized noble gases as $^3\text{He}$ interact with the source through the following form:

$$V = \alpha \vec{\sigma} \cdot \vec{v},$$

(1)

where $\vec{\sigma}$ are the Pauli matrices, $\vec{v}$ the relative velocity between the probe and the source. The parameter $\alpha$ has the dimension of momentum and depends on the factors such as the probe to source distance, the source mass or the nucleon number, etc. Now we consider the situation that this $V$ is due to a pseudo-magnetic field:

$$V = \vec{\mu} \cdot \vec{B},$$

(2)

where $\vec{\mu}$ is the magnetic moment, $\gamma$ the gyromagnetic ratio of the spin polarized particle, $\vec{B} = 2\alpha \vec{v}/\hbar \gamma$ the pseudo-magnetic field. This pseudo magnetic field is along $\vec{v}$ direction and has the strength of $2\alpha \vec{v}/\hbar \gamma$. Then searching for the new physics becomes a problem of detecting the pseudo-magnetic field $\vec{B}$.

Spin polarized neutron/atom beams are considered to be more convenient to probe these spin/velocity dependent interactions since a large relative velocity between the probe and the source can be easily realized. However, the number of the probe particles is usually limited for the beam method. Large quantities of polarized probe particles can be obtained by using polarized gases, but the polarized noble gases are usually sealed in glass cells, thus it would be technically difficult to realize a large relative velocity between the source mass and the probe particles inside delicate glass cells within a short distance.

By using the polarized noble gases sealed in glass cells, it might be considered very difficult to detect the spin-velocity dependent interactions without moving the source or the cell. However, though $\langle \vec{v} \rangle$ is zero for atoms of the glass sealed noble gas, $\langle \vec{v}^2 \rangle$ is not. As we will show

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latter, the nonzero $\langle \nu^2 \rangle$ together with the $\vec{\sigma} \cdot \vec{v}$ type interaction, will change the relaxation time of spin polarized noble gases. Though it is a second order effect, there is no need to move the apparatus any more which will save the corresponding technique difficulties. Thus it is possible to detect or constrain the new physics by the longitudinal spin relaxation time ($T_1$) or the transverse relaxation time($T_2$) of polarized noble gases. By applying the best available values of $T_1$ [17] and $T_2$ [18, 21], our research indicates that the constraint of $\alpha$ from $T_2$ is tighter than that from $T_1$. In what follows, we will first describe how the $\alpha \vec{\sigma} \cdot \vec{v}$ interaction affects the spin relaxation time of the polarized $^3$He gas, then we will constrain $\alpha$ by using the best available $T_2$ measured in the experiment. Furthermore, by using this constraint of $\alpha$ and the earth as the source, we obtain the limits on the three different types of new interactions, vector-axial-vector interaction($V_{VA}$), axial-axial-vector interaction($V_{AA}$) and the torsion.

I. CONSTRRAIN $\alpha$ BY $T_1$ AND $T_2$ OF SPIN POLARIZED $^3$He GAS

There are some examples using the spin relaxation time to constrain the scalar-pseudo-scalar type($V_{sp}$) [21] interaction which is spin dependent and short-ranged. $T_1$ is used in the Ref. [17] while $T_2$ in the Refs. [18, 19]. According to the so called Redfield theory [18, 21], assume the holding field is along $\hat{z}$, then the longitudinal and transverse relaxation times of the polarized $^3$He gas due to a randomly fluctuating magnetic field can be expressed as:

$$\Gamma_1 = \frac{1}{T_1} = \frac{\gamma^2}{2} [S_{Bx}(\omega_0) + S_{Bx}(\omega_0)],$$

$$\Gamma_2 = \frac{1}{T_2} = \frac{\gamma^2}{4} [S_{Bx}(\omega_0) + S_{Bx}(\omega_0) + 2S_{Bx}(0)],$$

where

$$S_{Bx}(\omega_0) = \int_{-\infty}^{+\infty} \langle B_x(t)B_x(t+\tau) \rangle e^{-i\omega_0\tau} d\tau.$$  (4)

Here $\langle ... \rangle$ represents the ensemble average, $B_x$ is the $\hat{x}$ component of the fluctuating magnetic field, $\omega_0 = \gamma B_0$ the Larmor frequency.

Using above formulas, it is easy to see that the pseudomagnetic field induced by the $\alpha \vec{\sigma} \cdot \vec{v}$ type interaction will change $T_1$ as follows:

$$\Gamma_1 = \frac{1}{T_1} = \frac{4\alpha^2}{h^2} \int_{-\infty}^{+\infty} \langle v_x(t)v_x(t+\tau) \rangle e^{-i\omega_0\tau} d\tau.$$  (5)

When the Larmor frequency $\omega_0$ is much larger than $1/\tau_D$ with $\tau_D \propto L^2/D (L$ is the characteristic length of the gas sealed cell in the transverse dimensions), the autocorrelation function for the velocity is [21]:

$$\langle v_x(t)v_x(t+\tau) \rangle = \langle v_x^2 \rangle e^{-\frac{m\omega_0^2\tau}{2\hbar}} = \frac{1}{3} \langle v^2 \rangle e^{-\frac{m\omega_0^2\tau}{2\hbar}},$$

where $\tau_c$ is the average collision time of the atoms. Substituting Eq. (4) into Eq. (5), we obtain:

$$\alpha = \frac{\hbar}{2} \sqrt{\frac{(1 + \alpha^2/3^2)m_3}{2k_B T_1\tau_c}}.$$  (7)

Plugging in the data given in the Ref. [17], $\tau_c = 3 \times 10^{-10}$s, $T_1^{exp} = 2518$Shr(2$\sigma$ value), $\omega_0 = 10^5$s$^{-1}$, we obtain an upper limit to $\alpha$ as:

$$\alpha \leq 7.9 \times 10^{-37} \text{kg} \cdot \text{m} \cdot \text{s}^{-1}.$$  (8)

Similarly, the relaxation time $T_2$ caused by the $\alpha \vec{\sigma} \cdot \vec{v}$ type interaction can be expressed as:

$$\frac{1}{T_2} = \frac{\alpha^2}{h^2} \int_{-\infty}^{+\infty} \langle v_x(t)v_x(t+\tau) \rangle e^{-i\omega_0\tau} d\tau + 2\langle v_x(t)v_x(t+\tau) \rangle d\tau.$$  (9)

Here we need to calculate $\langle v_x(t)v_x(t+\tau) \rangle$ under the condition that the magnetic field is small such that Eq. (4) is not valid anymore. According to the Refs. [22, 23], we have

$$\langle v_x(t)v_x(t+\tau) \rangle = -\frac{d^2}{d\tau^2}(x(t)x(t+\tau)),$$  (10)

which leads to

$$\int_{-\infty}^{+\infty} \langle v_x(t)v_x(t+\tau) \rangle e^{-i\omega_0\tau} d\tau = \omega_0^2 S_x(\omega_0),$$  (11)

where $S_x(\omega_0)$ is defined as the Fourier transformation of $\langle x(t)x(t+\tau) \rangle$. Now the problem of calculating the velocity autocorrelation function becomes calculating the position autocorrelation function. The latter can be solved by using the diffusion theory as in the Refs. [13, 21]. For a sphere with radius $R$, it can be shown that:

$$S_x(\omega_0) = \int_{-\infty}^{+\infty} \langle x(t)x(t+\tau) \rangle e^{-i\omega_0\tau} d\tau = 4R^2 \sum_n \frac{1}{x_{1n}^2(x_{1n}^2 - 2)(\frac{\alpha^2}{h^2} \omega_0^2)^2 + \omega_0^2},$$  (12)

where the numbers $x_{1n}$ are the zeros of the derivatives of spherical Bessel functions [21], $D$ is the diffusion constant of the $^3$He gas. For a spherical cell, the relaxation time $T_2$ can be finally expressed as

$$\frac{1}{T_2} = \frac{\alpha^2}{h^2} \omega_0^2 [S_x(\omega_0) + S_y(\omega_0)].$$  (13)

Now plugging in the data given in the Ref. [25], $R = 3$cm which is the radius of a spherical cell, $D = 470$cm$^2$s$^{-1}$, and $\Gamma_2 = 1.4 \times 10^{-7}$s$^{-1}$ (2$\sigma$ value [13]), into Eq. (13), we obtain an upper limit on $\alpha$ as

$$\alpha \leq 2.9 \times 10^{-87} \text{kg} \cdot \text{m} \cdot \text{s}^{-1}.$$  (14)

Though at the same order of magnitude, this upper limit is twice tighter than that given by Eq. (3). We are going to use it to constrain the spin-velocity dependent new physics.
II. CONSTRaining three new interactions using the earth as the source

A. vector-axial-vector interaction

Taking advantage of the fact that there are about $10^{42}$ polarized electrons in the earth, Ref.[11] used the polarized electron spins of the earth as the source to constrain several spin-spin interactions. Inspired by this spirit, we notice that there are $10^{51}$ nucleons in the earth. It might be even more advantageous to use the earth as the source to constrain some spin dependent interactions as $V_{VA}$ and $V_{AA}$. For example, for the vector-axial-vector interaction $V_{VA}(r)$ originated from the coupling $\mathcal{L}_X = \bar{\psi}(g_V \gamma_\mu + g_A \gamma_\mu \gamma_5)\psi X_\mu$, this parity violating interaction has the form:

$$V_{VA}(r) = \frac{\hbar g_V g_A}{2\pi} \frac{\exp\left(-r/\lambda\right)}{r} \vec{\sigma} \cdot \vec{v},$$

where $\lambda = \hbar/m_X c$ is the interaction range, $m_X$ is the mass of the new vector boson. $V_{VA}(r)$ is the Yukawa potential multiplied by the $\vec{\sigma} \cdot \vec{v}$ factor, which makes this interaction quite interesting. Using the earth as the source, the Vector-Axial potential generated by the earth on its surface is found to be:

$$V_{VA}(R) = \hbar g_V g_A \rho_N \lambda^2 \left[(1 - \frac{\lambda}{R}) + (1 + \frac{\lambda}{R})e^{-2\lambda R}\right] \vec{\sigma} \cdot \vec{v},$$

where $\rho_N$ is the nucleon number density of the earth, $R$ the earth radius. Clearly, if the new interaction $V_{VA}(r)$ exists, it will induce the $\vec{a}_\sigma \cdot \vec{v}$ type interaction on the earth surface thus affect the relaxation time of the spin polarized $^3$He gas in laboratory.

Plugging in $\rho_N = 3.3 \times 10^{30}$m$^{-3}$ and using Eq.(13), we get

$$g_V g_A \lambda^2 \left[(1 - \frac{\lambda}{R}) + (1 + \frac{\lambda}{R})e^{-2\lambda R}\right] \leq 8.3 \times 10^{-34} m^2.$$  

In the short distance limit with $\lambda \ll R$, the above equation becomes:

$$g_V g_A \leq 8.3 \times 10^{-34} \lambda^{-2},$$

this result is a few orders less sensitive than that given in the Ref.[5]. While in the long distance limit with $\lambda \gg R$, we get

$$g_V g_A \leq 3.1 \times 10^{-47}.$$  

At long range, of $\sim 10^8 m$, it improves the existing experimental upper bound by as much as $\sim 16$ orders. When comparing with the results of the Ref.[28], which were derived by combining $g_V$ in Ref.[26,30] and $g_A$ in Ref.[10], we get $\sim 3$ orders improvement. The present work gives the best known result for constraining $g_V g_A$ for ranges of $10^8 m$. As shown by FIG.1, in combination with the previous work, we believe we have get the most stringent experimental upper bounds on $g_V g_A$ ranging from $\sim 10^{-6} m$ to $\sim 10^8 m$. We emphasize that the limits on the vector-axial-vector interaction presented here are derived directly from the relevant experiment.

B. axial-axial-vector interaction

For the axial-axial-vector interaction which is also originated from the coupling $\mathcal{L}_X = \bar{\psi}(g_V \gamma_\mu + g_A \gamma_\mu \gamma_5)\psi X_\mu$, the pseudo-potential $V_{AA}$ has the form:

$$V_{AA}(r) = \frac{\hbar^2}{16\pi mc} \frac{1}{R^2} \frac{1}{\lambda} \frac{\exp\left(-r/\lambda\right)}{r} \vec{\sigma} \cdot (\vec{v} \times \hat{r}).$$

An important procedure is to do the integration when the source is the earth. On the surface of the earth, we find:

$$V_{AA}(R) = \frac{g_A^2}{8} \frac{\hbar^2}{mc} \rho_N \lambda$$

$\times \left[(1 + \frac{\lambda}{R})e^{-2\lambda R} + (1 - \frac{\lambda^2}{R^2})\vec{\sigma} \cdot (\vec{v} \times \hat{r})\right].$  

Noticing that the holding field for the polarized gas is along the horizontal direction, i.e., the $\hat{z}$ direction, if we choose $\hat{r}$ along the $\hat{y}$ direction, then the pseudo-magnetic field can only be along the $\hat{x}$ direction. Using Eq.(13), we have:

$$g_A^2 \lambda \left[(1 + \frac{\lambda}{R})e^{-2\lambda R} + (1 - \frac{\lambda^2}{R^2})\right] \leq 1.9 \times 10^{-16} m.$$  

Now we consider the same limits as in the previous section. When $\lambda \ll R$, we have

$$g_A^2 \leq 1.9 \times 10^{-16} \lambda^{-1}.$$  

![FIG. 1: (Color online)Constraint to the coupling constant $g_V g_A$ as a function of the interaction range $\lambda$. The bold solid line is the result of this work; The dashed line is the result of Ref.[5]. The blue and red doted lines are the result of Ref.[28] which were derived by combining $g_V$ of Ref.[26,30] with $g_A$ of Ref.[10]. The dark grey area is the excluded area by experiments of previous work[5] and this work.](image)
As shown by FIG.2, this upper bound is \( \sim 2 \) orders better than that given in the Ref.\[4\] in short ranges. When \( \lambda \gg R \), the constraint equation is reduced to \( g^2_{A} \leq 4.5 \times 10^{-23} \), which is \( \sim 16 \) orders less sensitive than the result given in the Ref.\[16\].

![FIG. 2: Constraint to the coupling constant \( g_{A}g_{A} \) as a function of the interaction range \( \lambda \). The bold solid line is the result of this work; The dashed line is the result of Ref.\[4\], the dash-dotted line is the result of Ref.\[11\]. The light grey area is the excluded area by previous experiments\[4, 10\]. The dark grey area is the increased exclusion area by this work.](image)

### C. torsion on the surface of the earth

Using the neutron spin rotation in the liquid helium, Ref.\[24\] constrains the in-matter torsion for the first time. The neutron spin rotation in the liquid helium, Ref.\[24\] constrains the in-matter torsion for the first time. If torsion exists, the source might couple to the spin through the form\[24\] :

\[
V = \zeta \vec{\sigma} \cdot \vec{v},
\]

which has exactly the same form as Eq.\[1\]. \( \zeta \) includes all other factors such as the distance, the source mass, etc. At present, how \( \zeta \) depends on the distance is still theoretically undetermined. As in the Refs.\[26, 27\], here we only consider the leading order of the torsion background which is a constant. The torsion effect is considered to be very small and it is extremely difficult to be detected. Earth-sourced torsion had been discussed in the Ref.\[27\], where the rotating of the apparatus or comparing the behavior of particles and antiparticles were proposed to detect torsion. Here as we have found, by measuring the relaxation time of the polarized noble gas, a constrain on \( \zeta \) can be obtained without moving the apparatus. Also using the earth as the source and applying Eq.\[1\] it is easy to show that in the natural units

\[
\zeta = \alpha \leq 5.4 \times 10^{-19} \text{GeV}.
\]

It is more convenient to rewrite \( \zeta \) as \( \zeta = m_{S}c\xi \), where \( m_{S} \) is the source mass. Now \( \xi \) is dimensionless and its dependence on the source mass is isolated out. Plugging in the earth mass \( m_{\text{earth}} = 5.97 \times 10^{24} \text{kg} \), we get

\[
\xi = \frac{\alpha}{m_{S}c} \leq 1.6 \times 10^{-70}. \tag{26}
\]

### III. CONCLUSION AND DISCUSSION

In summary, by using the spin relaxation time of the polarized \(^{3}\text{He} \) gas and the earth as the source, we constrained three types spin-velocity dependent new interactions. Though at the same order of magnitude, we found that the best available \( T_{2} \) gives better constraints. We derived new experimental limits to the Vector-Axial type interaction ranging from \( \sim 1 \) m to \( \sim 10^{8} \)m. At the distance of \( \sim 10^{8} \)m, the limit is improved by \( \sim 16 \) orders in comparison with the result of the neutron spin rotation experiment. In combination with the previous result\[2\] which is more sensitive at short distance, we present the most stringent constraint derived directly from experiments on \( g_{V}g_{A} \) ranging from \( \sim 10^{-3} \)m to \( \sim 10^{14} \)m(Figure.1). We also obtained new limits to the Axial-Axial type interaction ranging below \( \sim 1 \)cm. The limit is improved by \( \sim 2 \) orders for the above mentioned ranges. To our best knowledge, we report the first experimental limits on torsion induced by the earth on its surface.

In the future we may consider to improve the sensitivity by dedicated experiments. Either for \( T_{1} \) or \( T_{2} \), the key is to constrain \( \alpha \) better. When using \( T_{1} \), suppose that \( 1/\tau_{D} \ll \omega_{0} \ll \tau_{c} \) is still valid, we then find \( \alpha \propto \sqrt{n/(T_{1}T^{1/2})} \) after optimizing the parameters. Since the measured \( T_{1} \) is already as long as several thousand hours for cell pressure of \( \sim 1 \) bar, it seems that the most efficient way to improve the sensitivity is to reduce the gas number density \( n \). If \( n \) is reduced to \( \sim 1 \) mbar, and the conditions for \( T_{1} \) and \( \omega_{0} \) remain the same as above, the sensitivity could be improved by about one order of magnitude. When using \( T_{2} \), a similar result is obtained, i.e., \( \alpha \) is found \( \propto \sqrt{n/(T_{2}T^{1/2})} \) after parameter optimizing. Since the cell pressure is already as low as a few mbar in this work, it is hard to image that the sensitivity can be increased substantially by further reducing the pressure. On one hand, the diffusion theory used here will break down for a much lower cell pressure though Redfield theory is still valid\[23\]. On the other hand, lower cell pressure means less probing particles, a factor which will eventually dominate. Still for a \( \sim \)mbar cell, if thousand-hour-long \( T_{2} \) could be observed, sensitivity could also be improved by one order of magnitude. Either by measuring \( T_{1} \) or \( T_{2} \), it seems not easy to further improve the sensitivity substantially since the \( \alpha \vec{\sigma} \cdot \vec{v} \) interactions take interventions as a second order effect by this method.

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