Hawking’s chronology protection conjecture: 
singularity structure of the quantum 
stress–energy tensor

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Abstract

The recent renaissance of wormhole physics has led to a very disturbing 
observation: If traversable wormholes exist then it appears to be rather 
easy to to transform such wormholes into time machines. This extremely 
disturbing state of affairs has lead Hawking to promulgate his chronology 
protection conjecture.

This paper continues a program begun in an earlier paper [Physical Re-
view D47, 554–565 (1993)]. An explicit calculation of the vacuum expec-
tation value of the renormalized stress–energy tensor in wormhole space-
times is presented. Point-splitting techniques are utilized. Particular 
attention is paid to computation of the Green function [in its Hadamard 
form], and the structural form of the stress–energy tensor near short closed 
spacelike geodesics. Detailed comparisons with previous calculations are 
presented, leading to a pleasingly unified overview of the situation.

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1 INTRODUCTION

This paper addresses Hawking’s chronology protection conjecture [1, 2], and some of the recent controversy surrounding this conjecture [3, 4]. The qualitative and quantitative analyses of reference [5] are extended. Detailed comparisons are made with the analyses of Frolov [6], of Kim and Thorne [3], and of Klinkhammer [4].

The recent renaissance of wormhole physics has led to a very disturbing observation: If traversable wormholes exist then it appears to be rather easy to transform such wormholes into time machines. This extremely disturbing state of affairs has lead Hawking to promulgate his chronology protection conjecture [1, 2].

This paper continues a program begun in an earlier paper [5]. In particular, this paper will push the analysis of that paper beyond the Casimir approximation. To this end, an explicit calculation of the vacuum expectation value of the renormalized stress–energy tensor in wormhole spacetimes is presented. Point-splitting regularization and renormalization techniques are employed. Particular attention is paid to computation of the Green function [in its Hadamard form], and the structural form of the stress-energy tensor near short closed spacelike geodesics.

The computation is similar in spirit to the analysis of Klinkhammer [4], but the result appears — at first blush — to be radically different. The reasons for this apparent difference (and potential source of great confusion) are tracked down and examined in some detail. Ultimately the various results are shown to agree with one another in those regions where the analyses overlap. Furthermore, the analysis of this paper is somewhat more general in scope and requires fewer technical assumptions.

Notation: Adopt units where \( c \equiv 1 \), but all other quantities retain their usual dimensionalities, so that in particular \( G = \hbar/m_{\text{P}}^2 = \ell_{\text{P}}^2/\hbar \). The metric signature is taken to be \((- , + , + , + )\).

2 THE GREEN FUNCTION

2.1 The Geodetic Interval

The geodetic interval is defined by:

\[
\sigma_\gamma(x, y) \equiv \pm \frac{1}{2}[s_\gamma(x, y)]^2.
\]

(1)

Here we take the upper (+) sign if the geodesic \( \gamma \) from the point \( x \) to the point \( y \) is spacelike. We take the lower (−) sign if this geodesic \( \gamma \) is timelike. In either case we define the arc length \( s_\gamma(x, y) \) to be positive semi-definite. Note that,
provided the geodesic from $x$ to $y$ is not lightlike,

\[ \nabla^\mu \sigma_\gamma(x, y) = \pm s_\gamma(x, y) \nabla^\mu s_\gamma(x, y), \quad (2) \]

\[ = +s_\gamma(x, y) t_\mu(x; \gamma; x \leftarrow y). \quad (3) \]

Here $t_\mu(x; \gamma; x \leftarrow y) \equiv \pm \nabla^\mu s_\gamma(x, y)$ denotes the unit tangent vector at the point $x$ pointing along the geodesic $\gamma$ away from the point $y$. When no confusion results we may abbreviate this by $t_\mu(x \leftarrow y)$ or even $t_\mu(x)$. If the geodesic from $x$ to $y$ is lightlike things are somewhat messier. One easily sees that for lightlike geodesics $\nabla^\mu \sigma_\gamma(x, y)$ is a null vector. To proceed further one must introduce a canonical observer, characterized by a unit timelike vector $V^\mu$ at the point $x$. By parallel transporting this canonical observer along the geodesic one can set up a canonical frame that picks out a particular canonical affine parameter:

\[ \nabla^\mu \sigma_\gamma(x, y) = +\zeta I_\mu; \quad I_\mu V^\mu = -1; \quad \zeta = -V^\mu \nabla^\mu \sigma_\gamma(x, y). \quad (4) \]

Note that this affine parameter $\zeta$ can, crudely, be thought of as a distance along the null geodesic as measured by an observer with four–velocity $V^\mu$.

To properly place these concepts within the context of the chronology protection conjecture, see, for example, reference \[5\]. Consider an arbitrary Lorentzian spacetime of nontrivial homotopy. Pick an arbitrary base point $x$. Since, by assumption, $\pi_1(M)$ is nontrivial there certainly exist closed paths not homotopic to the identity that begin and end at $x$. By smoothness arguments there also exist smooth closed geodesics, not homotopic to the identity, that connect the point $x$ to itself. However, there is no guarantee that the tangent vector is continuous as the geodesic passes through the point $x$ where it is pinned down. If any of these closed geodesics is timelike or null then the battle against time travel is already lost, the spacetime is diseased, and it should be dropped from consideration.

To examine the types of pathology that arise as one gets “close” to building a time machine, it is instructive to construct a one parameter family of closed geodesics that captures the essential elements of the geometry. Suppose merely that one can find a well defined throat for one’s Lorentzian wormhole. Consider the world line swept out by a point located in the middle of the wormhole throat. At each point on this world line there exists a closed “pinned” geodesic threading the wormhole and closing back on itself in “normal” space. This geodesic will be smooth everywhere except possibly at the place that it is “pinned” down by the throat.

If the geodetic interval from $x$ to itself, $\sigma_\gamma(x, x)$, becomes negative then a closed timelike curve (a fortiori — a time machine) has formed. It is this unfortunate happenstance that Hawking’s chronology protection conjecture is hoped to prevent. For the purposes of this paper it will be sufficient to consider the behaviour of the vacuum expectation value of the renormalized stress energy tensor in the limit $\sigma_\gamma(x, x) \to 0^+$. 

3
2.2 The Hadamard Form

The Hadamard form of the Green function may be derived by an appropriate use of adiabatic techniques [7, 8, 9, 10, 11, 12]:

\[
G(x, y) = \langle 0| \phi(x) \phi(y) |0 \rangle = \sum_{\gamma} \frac{\Delta_{\gamma}(x, y)^{1/2}}{4\pi^2} \left[ \frac{1}{\sigma_{\gamma}(x, y)} + \nu_{\gamma}(x, y) \ln |\sigma_{\gamma}(x, y)| + \varpi_{\gamma}(x, y) \right].
\]

Here the summation, \(\sum_{\gamma}\), runs over all distinct geodesics connecting the point \(x\) to the point \(y\). The symbol \(\Delta_{\gamma}(x, y)\) denotes the van Vleck determinant [13, 14]. The functions \(\nu_{\gamma}(x, y)\) and \(\varpi_{\gamma}(x, y)\) are known to be smooth as \(\sigma_{\gamma}(x, y) \to 0\) [7, 8, 9, 10, 11, 12].

If the points \(x \) and \(y\) are such that one is sitting on top of a bifurcation of geodesics (that is: if the points \(x \) and \(y\) are almost conjugate) then the Hadamard form must be modified by the use of Airy function techniques in a manner similar to that used when encountering a point of inflection while using steepest descent methods [8].

3 RENORMALIZED STRESS–ENERGY

3.1 Point splitting

The basic idea behind point splitting techniques [7, 8, 9, 10, 11, 12] is to define formally infinite objects in terms of a suitable limiting process such as

\[
\langle 0| T_{\mu\nu}(x, y) |0 \rangle = \lim_{y \to x} \langle 0| T_{\mu\nu}(x, y) |0 \rangle. 
\]

The point–split stress–energy tensor \(T_{\mu\nu}(x, y)\) is a symmetric tensor at the point \(x\) and a scalar at the point \(y\). The contribution to the point–split stress–energy tensor associated with a particular quantum field is generically calculable in terms of covariant derivatives of the Green function of that quantum field. Schematically

\[
\langle 0| T_{\mu\nu}(x, y) |0 \rangle = D_{\mu\nu}(x, y)\{G_{\text{ren}}(x, y)\}. 
\]

Here \(D_{\mu\nu}(x, y)\) is a second order differential operator, built out of covariant derivatives at \(x\) and \(y\). The covariant derivatives at \(y\) must be parallel propagated back to the point \(x\) so as to ensure that \(D_{\mu\nu}(x, y)\) defines a proper geometrical object. This parallel propagation requires the introduction of the notion of the trivial geodesic from \(x\) to \(y\), denoted by \(\gamma_0\). (These and subsequent comments serve to tighten up, justify, and make explicit the otherwise rather heuristic incantations common in the literature.)

Renormalization of the Green function consists of removing the short distance singularities associated with the flat space Minkowski limit. To this end,
consider a scalar quantum field \( \phi(x) \). Again, let \( \gamma_0 \) denote the trivial geodesic from \( x \) to \( y \). Define

\[
G_{\text{ren}}(x, y) \equiv G(x, y) - \frac{\Delta_{\gamma_0}(x, y)^{1/2}}{4\pi^2} \left[ \frac{1}{\sigma_{\gamma_0}(x, y)} + v_{\gamma_0}(x, y) \ln |\sigma_{\gamma_0}(x, y)| \right].
\]

These subtractions correspond to a wave–function renormalization and a mass renormalization respectively. Note that any such renormalization prescription is always ambiguous up to further finite renormalizations. The scheme described above may profitably be viewed as a modified minimal subtraction scheme. In particular, (for a free quantum field in a curved spacetime), this renormalization prescription is sufficient to render \( <0|\phi^2(x)|0>\) finite:

\[
<0|\phi^2(x)|0>_{\text{ren}} = G_{\text{ren}}(x, x),
\]

\[
= \frac{\Delta_{\gamma_0}(x, x)^{1/2}}{4\pi^2} \sigma_{\gamma_0}(x, x)
\]

\[
+ \sum_\gamma \frac{\Delta_{\gamma_0}(x, x)^{1/2}}{4\pi^2} \times \left[ \frac{1}{\sigma_{\gamma_0}(x, x)} + v_{\gamma_0}(x, x) \ln |\sigma_{\gamma_0}(x, x)| + \varpi_{\gamma_0}(x, x) \right].
\]

For the particular case of a conformally coupled massless scalar field (4):

\[
D_{\mu\nu}(x, y) \equiv \frac{1}{6} \left( \nabla_\mu^\alpha g_\nu^\alpha(x, y) \nabla_\alpha^y + g_\mu^\alpha(x, y) \nabla_\alpha^y \nabla_\nu^x \right)
\]

\[
- \frac{1}{12} g_{\mu\nu}(x) \left( g^{\alpha\beta}(x, y) \nabla_\alpha^y \nabla_\beta^y \right)
\]

\[
- \frac{1}{12} \left( \nabla_\mu^y \nabla_\nu^x + g_\mu^\alpha(x, y) \nabla_\alpha^y g_\nu^\beta(x, y) \nabla_\beta^y \right)
\]

\[
+ \frac{1}{48} g_{\mu\nu}(x) \left( g^{\alpha\beta}(x) \nabla_\alpha^x \nabla_\beta^x + g^{\alpha\beta}(y) \nabla_\alpha^y \nabla_\beta^y \right)
\]

\[
- R_{\mu\nu}(x) + \frac{1}{4} g_{\mu\nu}(x) R(x).
\]

As required, this object is a symmetric tensor at \( x \), and is a scalar at \( y \). The bi-vector \( g_{\mu\nu}(x, y) \) parallel propagates a vector at \( y \) to a vector at \( x \), the parallel propagation being taken along the trivial geodesic \( \gamma_0 \). (The effects of this parallel propagation can often be safely ignored, vide reference \( \) equation (3), and reference \( \) equation (2.40).)

### 3.2 Singularity structure

To calculate the renormalized stress–energy tensor one merely inserts the Hadamard form of the Green function (propagator) into the point split formal-
obtain an expression for the stress–energy tensor. Here the dimensionless tensor $t_{\mu\nu}(x;\gamma)$ is constructed solely out of the metric and the tangent vectors to the geodesic $\gamma$ as follows

$$t_{\mu\nu}(x;\gamma) = \frac{2}{3} \left( t_{\mu}^{1} t_{\nu}^{2} + t_{\mu}^{2} t_{\nu}^{1} - \frac{1}{2} g_{\mu\nu} (t^{1})^2 \right) - \frac{1}{3} \left( t_{\mu}^{1} t_{\nu}^{1} + t_{\mu}^{2} t_{\nu}^{2} - \frac{1}{2} g_{\mu\nu} \right).$$

(14)

The dimensionless tensor $s_{\mu\nu}(x;\gamma)$ is defined by

$$s_{\mu\nu}(x;\gamma) \equiv \lim_{y \to x} D_{\mu\nu}(x,y) \{ \sigma_{\gamma}(x,y) \}.$$ 

(15)

In many cases of physical interest the tensor $s_{\mu\nu}(x;\gamma)$ either vanishes identically or is subdominant in comparison to $t_{\mu\nu}(x;\gamma)$. A general analysis has so far unfortunately proved elusive. This is an issue of some delicacy that clearly needs further clarification. Nevertheless the neglect of $s_{\mu\nu}(x;\gamma)$ in comparison to $t_{\mu\nu}(x;\gamma)$ appears to be a safe approximation which shall be adopted forthwith.

Note that this most singular contribution to the stress energy tensor is in fact traceless — there is a good physical reason for this. Once the length of the closed spacelike geodesic becomes smaller than the Compton wavelength of the particle under consideration, $s << h/mc$, one expects such a physical particle to behave in an effectively massless fashion. Indeed, based on such general considerations, one expects the singular part of the stress–energy tensor to be largely insensitive to the type of particle under consideration. Despite the fact that the calculation has been carried out only for conformally coupled massless scalars, one expects this leading singularity to be generic. Indeed, in terms of the geodesic distance from $x$ to itself:

$$< 0|T_{\mu\nu}(x)|0 > = \sum_{\gamma}^{\prime} \frac{\Delta_{\gamma}(x,x)^{1/2}}{4\pi^2s_{\gamma}(x,x)^{4}} t_{\mu\nu}(x;\gamma) + O(s_{\gamma}(x,x)^{-3}).$$

(16)
A formally similar result was obtained by Frolov in reference [6]. That result was obtained for points near the N’th polarized hypersurface of a locally static spacetime. It is important to observe that in the present context it has not proved necessary to introduce any (global or local) static restriction on the spacetime. Neither is it necessary to introduce the notion of a polarized hypersurface. All that is needed at this stage is the existence of at least one short, nontrivial, closed, spacelike geodesic.

To convince oneself that the apparent $s^{-4}$ divergence of the renormalized stress–energy is neither a coordinate artifact nor a Lorentz frame artifact consider the scalar invariant

$$T = \sqrt{<0|T_{\mu\nu}(x)|0><0|T^{\mu\nu}(x)|0>}.$$  

By noting that

$$t_{\mu\nu}(x;\gamma)t^{\mu\nu}(x;\gamma) = \frac{1}{3} [3 - 4(t^1 \cdot t^2) + 2(t^1 \cdot t^2)^2]$$

one sees that there is no “accidental” zero in $t^{\mu\nu}$, and that $T$ does in fact diverge as $s^{-4}$. Thus the $s^{-4}$ divergence encountered in the stress–energy tensor associated with the Casimir effect [5] is generic to any multiply connected spacetime containing short closed spacelike geodesics.

### 3.3 Wormhole disruption

To get a feel for how this divergence in the vacuum polarization backreacts on the geometry, recall, following Morris and Thorne [15, 16] that a traversable wormhole must be threaded by some exotic stress energy to prevent the throat from collapsing. In particular, at the throat itself (working in Schwarzschild coordinates) the total stress–energy tensor takes the form

$$T_{\mu\nu} = \frac{\hbar}{\ell_p^2 R^2} \begin{bmatrix} \xi & 0 & 0 & 0 \\ 0 & \chi & 0 & 0 \\ 0 & 0 & \chi & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$  

(19)

On general grounds $\xi < 1$, while $\chi$ is unconstrained. In particular, to prevent collapse of the wormhole throat, the scalar invariant $T$ must satisfy

$$T = \frac{\hbar}{\ell_p^2 R^2} \sqrt{1 + \xi^2 + 2\chi^2} \approx \frac{\hbar}{\ell_p^2 R^2}. \quad (20)$$

On the other hand, consider the geodesic that starts at a point on the throat and circles round to itself passing through the throat of the wormhole exactly once. That geodesic, by itself, contributes to the vacuum polarization effects just considered an amount

$$<0|T_{\mu\nu}(x)|0> = \frac{\Delta \gamma(x,x)^{1/2}}{\pi^2 s_\gamma(x,x)} t_{\mu\nu}(x;\gamma) + O(s_\gamma(x,x)^{-3}). \quad (21)$$
So the contribution of the single pass geodesic to the invariant $T$ is already

$$T = \frac{\Delta_{\gamma}(x, x)^{1/2}}{\pi^2 s_{\gamma}(x, x)^4} \sqrt{1 - \frac{4}{3}(t^1 \cdot t^2) + \frac{2}{3}(t^1 \cdot t^2)^2} \approx \frac{\Delta_{\gamma}(x, x)^{1/2}}{\pi^2 s_{\gamma}(x, x)^4}. \quad (22)$$

Therefore, provided that there is no accidental zero in the van Vleck determinant, vacuum polarization effects dominate over the wormhole’s internal structure once

$$s_{\gamma}(x, x)^2 << \ell_p R. \quad (23)$$

Indeed, Kim and Thorne have argued \cite{3} as follows: In the geometry presently under consideration, (a point $x$ on the wormhole throat, a geodesic $\gamma$ that loops once around the wormhole), the thin wall approximation for the throat of the wormhole leads to a van Vleck determinant equal to unity: $\Delta_{\gamma}(x, x) = 1$. That derivation is subordinate to a particular choice of identification scheme for the wormhole mouths, “time–shift identification”; but the result holds also for “synchronous identification” \cite{5}.

More generally, this discussion serves to focus attention on the van Vleck determinant. Relatively little is known about the behaviour of the van Vleck determinant for arbitrary geometries — and this is clearly a subject of considerable mathematical and physical interest. In particular, a corollary of the previous comments is that if one could show that a zero of the van Vleck determinant could be made to coincide with the onset of time machine formation then one would have strong evidence that singularities in the quantum stress–energy tensor are not a sufficiently strong physical mechanism to enforce the chronology protection conjecture.

4 RELATIONSHIP TO PREVIOUS WORK

The characteristic $s^{-3}$ divergence encountered in this and previous analyses \cite{5} is, at first blush, somewhat difficult to reconcile with the “$d\delta t^{-3}$” behaviour described in references \cite{3, 4, 6}. These apparent differences are, for the most part, merely artifacts due to an unfortunate choice of Lorentz frame. To see how this happens, one first has to add considerably more structure to the discussion in the form of extra assumptions.

To begin the comparison, one must beg the original question by assuming that a time machine does in fact succeed in forming. Further, one must assume that the resulting chronology horizon is compactly generated \cite{1, 2, 3}. The generators of the compactly generated chronology horizon all converge in the past on a unique closed null geodesic that shall be referred to as the “fountain”, and shall be denoted by $\tilde{\gamma}$. The question of interest is now the behaviour of the renormalized stress energy tensor in the neighborhood of the fountain.

To that end, pick a point $x$ “close” to the fountain $\tilde{\gamma}$. Pick a point $x_0$ that is on the fountain, with $x_0$ being “close” to $x$, and with $x_0$ being in the future of
Then the geodesic $\gamma_\perp$ from $x$ to $x_0$ is by construction timelike. One defines $\delta t = s_{\gamma_\perp}(x, x_0)$, and $V^\mu = -\nabla^\mu_{x_0} s_{\gamma_\perp}(x, x_0)$. One interprets these definitions as follows: a geodesic observer at the point $x$, with four-velocity $V^\mu$, will hit the fountain $\tilde{\gamma}$ after a proper time $\delta t$ has elapsed. One now seeks a computation of the stress–energy tensor at $x$ in terms of various quantities that are Taylor series expanded around the assumed impact point $x_0$ with $\delta t$ as the (hopefully) small parameter.

Consider, initially, the geodetic interval $\sigma_{\gamma}(x, x)$. Taylor series expand this as

$$
\sigma_{\gamma}(x, x) = \sigma_{\tilde{\gamma}}(x_0, x_0) + (\delta t) V^\mu \left[ \nabla^\nu_{x_0} \sigma_{\gamma}(x, y) + \nabla^\nu_{y} \sigma_{\gamma}(x, y) \right]_{(x_0, x_0)} + O(\delta t^2). \tag{24}
$$

Firstly, by definition of the fountain as a closed null geodesic, $\sigma_{\tilde{\gamma}}(x_0, x_0) = 0$. Secondly, take the vector $V^\mu$, defined at the point $x$, and parallel propagate it along $\gamma_\perp$ to $x_0$. Then parallel propagate it along the fountain $\tilde{\gamma}$. This now gives us a canonical choice of affine parameter on the fountain. Naturally this canonical affine parameter is not unique, but depends on our original choice of $V^\mu$ at $x$, or, what amounts to the same thing, depends on our choice of $x_0$ as a “reference point”. In terms of this canonical affine parameter, one sees

$$
\nabla^\nu_{x_0} \sigma_{\gamma}(x, y) \bigg|_{(x_0, x_0)} = -\zeta^{-}_{n} l^\mu, \tag{25}
$$

$$
\nabla^\nu_{y} \sigma_{\gamma}(x, y) \bigg|_{(x_0, x_0)} = -\zeta^{+}_{n} l^\mu. \tag{26}
$$

Here the notation $\zeta^{-}_{n}$ denotes the lapse of affine parameter on going around the fountain a total of $n$ times in the left direction, while $\zeta^{+}_{n}$ is the lapse of affine parameter for $n$ trips in the right direction. The fact that these total lapses are different is a reflection of the fact that the tangent vector to the fountain undergoes a boost on travelling round the fountain. Hawking showed that

$$
\zeta^{-}_{n} = -e^{nh} \zeta^{+}_{n}. \tag{27}
$$

So, dropping explicit exhibition of the $\leftarrow$,

$$
\sigma_{\gamma}(x, x) = +\delta t \left[ e^{nh} - 1 \right] + O(\delta t^2). \tag{28}
$$

This, finally, is the precise justification for equation (5) of reference 4.

In an analogous manner, one estimates the tangent vectors $t^1$ and $t^2$ in terms of the tangent vector at $x_0$:

$$
\nabla^\nu_{\mu} \sigma_{\gamma}(x, y) \bigg|_{y \to x} = -\zeta_{n} l_{\mu} + O(\delta t) \tag{29}
$$

$$
\nabla^\nu_{y} \sigma_{\gamma}(x, y) \bigg|_{(x_0, x_0)} = -\zeta_{n} l_{\mu} + O(\delta t). \tag{30}
$$

This leads to the estimate

$$
t^1_{\mu} \approx -e^{nh} t^2_{\mu} \approx -\left( \zeta_{n}/s \right) l_{\mu}. \tag{31}
$$
Warning: This estimate should be thought of as an approximation for the dominant components of the various vectors involved. If one takes the norm of these vectors one finds

$$1 \approx e^{nh} \approx 0.$$  \hfill (32)

This is true in the sense that other components are larger, but indicates forcefully the potential difficulties in this approach.

One is now ready to tackle the estimation of the structure tensor $t_{\mu\nu}(x; \gamma)$. Using equations (14) and (31) one obtains

$$t_{\mu\nu}(x; \gamma) \approx -\frac{\zeta^2 n}{3s^2} \left[ 1 + 4e^{nh} + e^{2nh} \right] l_{\mu}l_{\nu}. \hfill (33)$$

Pulling the various estimates together, the approximation to the Hadamard stress–energy tensor is seen to be

$$< 0|T_{\mu\nu}(x)|0 > = -\sum_{\gamma}^t \left\{ \frac{\Delta_{\gamma}(x, x)^{1/2}}{24\pi^2 \zeta_n} \frac{[1 + 4e^{nh} + e^{2nh}]}{|e^{nh} - 1|^2} \right\} \frac{l_{\mu}l_{\nu}}{(\delta t)^3} + O(\delta t^{-2}). \hfill (34)$$

This, finally, is exactly the estimate obtained by Klinkhammer [4] — his equation (8). Furthermore this result is consistent with that of Kim and Thorne [3] — their equation (67). The somewhat detailed presentation of this derivation has served to illustrate several important points.

Primus, the present result is a special case of the more general result (16), the present result being obtained only at the cost of many additional technical assumptions. The previous analysis has shown that the singularity structure of the stress–energy tensor may profitably be analysed without having to restrict attention to regions near the fountain of a compactly generated chronology horizon. The existence of at least one short, closed, nontrivial, spacelike geodesic is a sufficient requirement for the extraction of useful information.

Secundus, the approximation required to go from (14) to (34) are subtle and potentially misleading. For instance, calculating the scalar invariant $\mathcal{T}$ from (34), the leading $\delta t^{-3}$ term vanishes (because $l^\mu$ is a null vector). The potential presence of a subleading $\delta t^{-5/2}$ cross term cannot be ruled out from the present approximation, (34). Fortunately, we already know [from the original general analysis, (16)] that the dominant behaviour of $\mathcal{T}$ is $\mathcal{T} \propto \sigma^{-2} \propto s^{-4}$. In view of the fact that, under the present restrictive assumptions, $\sigma \propto \delta t$, one sees that $\mathcal{T} \propto (\delta t)^{-2}$. The cross term, whatever it is, must vanish.

Tertius, a warning — this derivation serves to expose, in excruciating detail, that the calculations encountered in this problem are sufficiently subtle that two apparently quite different results may nevertheless be closely related.
5 DISCUSSION

This paper has investigated the leading divergences in the vacuum expectation value of the renormalized stress-energy tensor as the geometry of spacetime approaches time machine formation. Instead of continuing the “defense in depth” strategy of the author’s previous contribution [5], this paper focuses more precisely on wormhole disruption effects. If one wishes to use a traversable wormhole to build a time machine, then one must somehow arrange to keep that wormhole open. However, as the invariant distance around and through the wormhole shrinks to zero the stress-energy at the throat diverges. Vacuum polarization effects overwhelm the wormhole’s internal structure once

\[ s^2 < \ell P R. \]  \hspace{1cm} (35)

This happens for \( s > \ell P \), which fact I interpret as supporting Hawking’s chronology protection conjecture. This result is obtained without invoking the technical requirement of the existence of a compactly generated chronology horizon. If such additional technical assumptions are added, the formalism may be used to reproduce the known results of Kim and Thorne [3], of Klinkhammer [4], and of Frolov [6].

Moving beyond the immediate focus of this paper, there are still some technical issues of considerable importance left unresolved. Most particularly, computations of the van Vleck determinant in generic traversable wormhole spacetimes is an issue of some interest.

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