Master formula for one-loop renormalization of bosonic SMEFT operators
– HEFT 2019, UC Louvain, CP3, Belgium –

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arXiv: 1904.07840
EFTs are an important tool in the search for new physics.

We match to UV-models at scale \( \Lambda \), but measure at scale \( v \).

The necessary RGEs also introduce an important operator mixing that was computed by Alonso/Jenkins/Manohar/Trott [1308.2627,1310.4838,1312.2014; JHEP].

The RGEs are also implemented in DSixTools: [1704.04504,EPJC] and Wilson: [1804.05033, EPJC]. ⇒ An independent cross check with a different approach would be very beneficial!
EFTs are an important tool in the search for new physics.

We match to UV-models at scale $\Lambda$, but measure at scale $v$.

The necessary RGEs also introduce an important operator mixing that was computed by Alonso/Jenkins/Manohar/Trott [1308.2627,1310.4838,1312.2014; JHEP].

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⇒ An independent cross check with a different approach would be very beneficial!
Master formula for one-loop renormalization of bosonic SMEFT operators

Part I: The SMEFT in our notation

\[ \mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \mathcal{L}^{(6)} + \ldots \]

\[ \frac{1}{12} \Lambda_{\mu \nu} \Lambda_{\mu \nu} + \frac{1}{2} \Sigma^2 \]

Part II: The Master Formula for 1-loop divergences

[1710.06412, 1904.07840]

Part III: The result

[1904.07840]

\[ Q_\phi = (\phi \phi^\dagger)^3 \]
\[ \mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \mathcal{L}^{(6)} + \ldots \]

I: We have a mass gap to NP.

We assume there is a gap between the SM at \( \nu \) and the new physics (NP) at \( \Lambda \).

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We assume there is a gap between the SM at $v$ and the new physics (NP) at $\Lambda$.

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \mathcal{L}^{(6)} + \ldots$$

We work with the real representation of the Higgs doublet

$$H \equiv \frac{1}{\sqrt{2}} (\tilde{\phi}, \phi) = i \tau^i \phi_j,$$

with $\tau^0 = -\frac{i}{2} \mathbf{1}$ and $\tau^i = \frac{\sigma^i}{2}$. The covariant derivative then becomes

$$(D_\mu \varphi)_i = \partial_\mu \varphi_i + ig W^{a}_\mu t^a_{Lij} \varphi_j + ig' B_\mu t^3_{Rij} \varphi_j$$

t_L and t_R are generators of $SO(4)$ with the algebra

$$[t^a_L, t^b_L] = i \epsilon^{abc} t^c_L \quad \{t^a_L, t^b_L\} = \frac{1}{2} \delta^{ab} \quad \text{tr} t^a_L t^b_L = \delta^{ab}$$

$$[t^a_R, t^b_R] = i \epsilon^{abc} t^c_R \quad \{t^a_R, t^b_R\} = \frac{1}{2} \delta^{ab} \quad \text{tr} t^a_R t^b_R = \delta^{ab}$$

$$[t^a_L, t^b_R] = 0 \quad \text{tr} t^a_L t^b_R = 0$$
\[ \mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \mathcal{L}^{(6)} + \ldots \]

I: Our notation for the SM.

In this notation, we have

\[
\mathcal{L}_{\text{SM}} = -\frac{1}{4} G_{\mu\nu}^A G^{A\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\
\quad + \frac{1}{2} (D_{\mu} \varphi)_i (D^{\mu} \varphi)_i + \frac{m^2}{2} \varphi_i \varphi_i - \frac{\lambda}{8} (\varphi_i \varphi_i)^2 \\
\quad + \bar{\psi} i \slashed{D} \psi - \left( \bar{\psi} \sqrt{2} H \gamma P_R \psi + \text{h.c.} \right)
\]

with \( \psi = (u, d, \nu, e)^T \) and \( \gamma = \text{diag} (\gamma_u, \gamma_d, 0, \gamma_e) \).
\[ \mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \mathcal{L}^{(6)} + \ldots \]

I: Our notation for the SMEFT.

We use the operators defined in the Warsaw basis and focus on the bosonic ones first.

Grzadkowski/Iskrzynski/Misiak/Rosiek [1008.4884,JHEP]
\[ \mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \mathcal{L}^{(6)} + \ldots \]

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| \( \mathcal{Q}_G \) | \( f^{ABC} G^A_{\mu} G^B_{\nu} G^C_{\rho} \) | \( \frac{1}{8} (\varphi \varphi)^3 \) | \( \mathcal{Q}_\psi \) | \( \frac{1}{\sqrt{2}} (\varphi \varphi) \bar{\psi}_L H \psi_R \) |
| \( \mathcal{Q}_{\tilde{G}} \) | \( f^{ABC} \tilde{G}^A_{\mu} \tilde{G}^B_{\nu} \tilde{G}^C_{\rho} \) | \( \frac{1}{4} (\varphi \varphi) \Box (\varphi \varphi) \) | \( \mathcal{Q}_\psi G \) | \( \sqrt{2} \bar{\psi} \sigma_{\mu \nu} G^A_{\mu \nu} T^A H \psi \) |
| \( \mathcal{Q}_W \) | \( e_{ijk} W^i_{\mu \nu} W^j_{\rho \nu} W^k_{\rho \mu} \) | \( \frac{1}{4} (\varphi D_{\mu} \varphi)(\varphi D^{\mu} \varphi) \) | \( \mathcal{Q}_\psi W \) | \( \sqrt{2} \bar{\psi} \sigma_{\mu \nu} W^i_{\mu \nu} \sigma^i H \psi \) |
| \( \mathcal{Q}_{\tilde{W}} \) | \( e_{ijk} \tilde{W}^i_{\mu \nu} \tilde{W}^j_{\rho \nu} \tilde{W}^k_{\rho \mu} \) | \(- (\varphi t^3_{\mu} D_{\mu} \varphi)(\varphi t^3_{\rho} D^{\rho} \varphi) \) | \( \mathcal{Q}_\psi B \) | \( \sqrt{2} \bar{\psi} \sigma_{\mu \nu} B_{\mu \nu} H \psi \) |
| \( \mathcal{Q}_{\varphi X} \) | \( \frac{1}{2} (\varphi \varphi) X^a_{\mu \nu} X^{a \mu \nu} \) | \( 2 \varphi t^3_{\nu} t^3_{\nu} \psi W^i_{\mu \nu} B_{\mu \nu} \) | \( \mathcal{Q}^{(1)} \varphi \psi \) | \( (\bar{\psi}_L \gamma^\mu \psi_L)(\varphi i \tilde{D}^I_{\mu} \varphi) \) |
| \( \mathcal{Q}_{\varphi \tilde{X}} \) | \( \frac{1}{2} (\varphi \varphi) \bar{X}^a_{\mu \nu} X^{a \mu \nu} \) | \( 2 \varphi t^3_{\nu} t^3_{\nu} \bar{\psi} \tilde{W}^i_{\mu \nu} B_{\mu \nu} \) | \( \mathcal{Q}^{(3)} \varphi \psi \) | \( (\bar{\psi}_L \gamma^\mu \psi_L)(\varphi i \tilde{D}^I_{\mu} \varphi) \) |
| \( \mathcal{Q}_{\varphi ud} \) | \( X = G, W, B \) | \( - (\varphi (it^3_{\nu} + t^2_{\nu}) D_{\mu} \varphi)(\bar{u}_R \gamma^\mu d_R) \) | \( \mathcal{Q}_\varphi \psi \) | \( (\bar{\psi}_R \gamma^\mu \psi_R)(\varphi i \tilde{D}^I_{\mu} \varphi) \) |

Consider the 15 bosonic operators first.
\[ \mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \mathcal{L}^{(6)} + \ldots \]

I: The 1-loop SMEFT RGEs.

- \( n \) LO vertices \( \rightarrow \) LO operators
- \( n \) LO vertices
  - 1 NLO vertex
  - \( \mathcal{O} \)

\( \mathcal{L}^{\text{div}} = \sum_j \mathcal{C}_j \Lambda^2 \left( K_{ij} \delta_{ij} + K_{ij} \right) \beta_i \equiv 16 \pi^2 d \mathcal{C}_i d \ln \mu = K_{ij} \mathcal{C}_j \)

Running of dim-4 altered by \( \beta(c_{\text{SM}}) \sim m^2 \Lambda^2 c_{\text{dim-6}} \)

Running of dim-6 given by \( \beta(c_{\text{dim-6}}) \sim c_{\text{dim-6}} c_{n_{\text{SM}}} \)
\[ \mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \mathcal{L}^{(6)} + \ldots \]

I: The 1-loop SMEFT RGEs.

- \( n \) LO vertices \( \rightarrow \) LO operators
- \( n \) LO vertices \( 1 \) NLO vertex \( \) ops.

\[ -32\pi^2 \epsilon \delta \mathcal{L}_{\text{div}} = \frac{C_i}{\Lambda^2} (K_{(i)\delta_{ij}} + K_{ij}) Q_i \]

\[ \beta_i \equiv 16\pi^2 \frac{dC_i}{d \ln \mu} = K_{ij} C_j \]
\[ \mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \mathcal{L}^{(6)} + \ldots \]

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- Running of dim-4 altered by

\[ \beta(c_{\text{SM}}) \sim \frac{m^2}{\Lambda^2} c_{\text{dim-6}} c_{\text{SM}}^n \]

- Running of dim-6 given by

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\[ \mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \mathcal{L}^{(6)} + \ldots \]
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Part II: The Master Formula for 1-loop divergences
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Part III: The result
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\[ Q_\phi = (\phi^\dagger \phi)^3 \]
We use the Background-Field Method…

starting from the generating functional:

\[ Z[j, \rho, \bar{\rho}] = e^{iW[j, \rho, \bar{\rho}]} = \int [d\phi d\psi d\bar{\psi}] e^{i(S[\phi, \psi, \bar{\psi}] + j\phi + \bar{\psi}\rho + \bar{\rho}\psi)}, \]

\[ \phi = \hat{\phi} + \phi_{qu}, \quad \psi = \hat{\psi} + \psi_{qu}, \]

\[ \left( \frac{\delta S}{\delta \phi} + j \right)_{\phi=\hat{\phi}} = 0, \quad \left( \frac{\delta S}{\delta \bar{\psi}} + \rho \right)_{\bar{\psi}=\hat{\psi}} = 0, \quad \left( \frac{\delta S}{\delta \psi} - \bar{\rho} \right)_{\psi=\hat{\psi}} = 0 \]

\[ \Rightarrow e^{iW_{L=1}} = \int [d\phi_{qu} d\psi_{qu} d\bar{\psi}_{qu}] e^{iS^{(2)}[\hat{\phi}, \hat{\psi}, \hat{\bar{\psi}}; \phi_{qu}, \psi_{qu}, \bar{\psi}_{qu}]} \]

Abbott '81
We use the Background-Field Method...

Quantum gauge fixing:

\[ \mathcal{L}_{g-f} = -\frac{1}{2} \left[ \left( \hat{D}_G^\mu G^A_\mu \right)^2 + \left( \partial_\mu B^\mu - ig' \varphi_i t^{3}_{Rij} \hat{\varphi}_j \right)^2 + \left( \hat{D}^\mu_W W^a_\mu - ig \varphi_i t^{a}_{Lij} \hat{\varphi}_j \right)^2 \right] \]

Dittmaier/Grosse-Knetter [hep-ph/9505266]; Helset/Paraskevas/Trott [1803.08001,PRL]

- We work in the Feynman gauge.
- The terms proportional to \( \varphi \) will make the next steps easier.

Using the background covariant derivative

\[ \hat{D}_\mu^W X = \partial_\mu X + ig [\hat{W}_\mu, X] \]

maintains background gauge invariance.
We use the Background-Field Method... evaluating the one-loop functional

\[ e^{iW_{L=1}} = \int \left[ d\phi d\psi d\bar{\psi} \right] e^{iS^{(2)}[\hat{\phi}, \hat{\psi}, \hat{\bar{\psi}}; \phi, \psi, \bar{\psi}]} \]

\[ S^{(2)} = \frac{1}{2} \phi A \phi + \bar{\psi} B \psi + \phi \bar{\Gamma} \psi + \bar{\psi} \Gamma \phi \]

\[ W_{L=1} = \frac{i}{2} \text{Tr} \ln A - i \text{Tr} \ln B - \frac{i}{2} \sum_{n=0}^{\infty} \frac{1}{n} \text{Tr} \left( A^{-1} \bar{\Gamma} B^{-1} \Gamma - A^{-1} \Gamma^T B^{-1}, T \bar{\Gamma}^T \right)^n \]
\( \frac{1}{12} \Lambda_{\mu \nu} \Lambda_{\mu \nu} + \frac{1}{2} \sum^2 \) … and Super-Heat-Kernel Expansion…

Evaluating the one-loop functional

\[
e^{iW_{L=1}} = \int [d\phi d\psi d\bar{\psi}] \quad e^{iS^{(2)}[\hat{\phi}, \hat{\psi}, \hat{\bar{\psi}}; \phi, \psi, \bar{\psi}]}
\]

\[
S^{(2)} = \frac{1}{2} \phi A \phi + \bar{\psi} B \psi + \phi \bar{\Gamma} \psi + \bar{\psi} \Gamma \phi
\]

\[
W_{L=1} = \frac{i}{2} \text{Tr} \ln A - i \text{Tr} \ln B - \frac{i}{2} \sum_{n=0}^{\infty} \frac{1}{n} \text{Tr} (A^{-1} \bar{\Gamma} B^{-1} \Gamma - A^{-1} \Gamma^T B^{-1,T} \bar{\Gamma}^T)^n
\]
\( \frac{1}{12} \Lambda^{\mu\nu} \Lambda_{\mu\nu} + \frac{1}{2} \Sigma^2 \) \( \ldots \) and Super-Heat-Kernel Expansion \( \ldots \)

Introducing supermatrix algebra:

\[
M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}
\]

\[
sdet M = \det (A - BD^{-1}C) \det D^{-1}
\]

\[
\text{str } M = \text{tr } A - \text{tr } D
\]

\[
sdet M = e^{\text{str} \ln M}
\]

Neufeld/Gasser/Ecker hep-ph/9806436
\[
\frac{1}{12} \Lambda^{\mu\nu} \Lambda_{\mu\nu} + \frac{1}{2} \sum^2 \ldots \text{and Super-Heat-Kernel Expansion} \ldots
\]

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\[
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\]

\[
s\det M = e^{\text{str ln } M}
\]

The one-loop functional of \( S^{(2)} = \frac{1}{2} \phi A \phi + \bar{\psi} B \psi + \phi \bar{\Gamma} \psi + \bar{\psi} \Gamma \phi \) becomes:

\[
W_{L=1} = \frac{i}{2} \text{Str ln } \Delta,
\]

\[
\Delta = \begin{pmatrix} A & \bar{\Gamma} & -\Gamma^T \\ -\bar{\Gamma}^T & 0 & -B^T \\ \Gamma & B & 0 \end{pmatrix}
\]
\[ \frac{1}{12} \Lambda^{\mu\nu} \Lambda_{\mu\nu} + \frac{1}{2} \Sigma^2 \]

... and Super-Heat-Kernel Expansion...

Applying the Heat-Kernel Expansion:

\[
W_{L=1} = \frac{i}{2} \text{Str} \ln \Delta \\
= -\frac{i}{2} \int_0^\infty \frac{d\tau}{\tau} \int d^d x \text{str} \langle x|e^{-\tau \Delta}|x\rangle 
\]

with the expansion in Seeley-DeWitt coefficients

\[
\langle x|e^{-\tau \Delta}|x\rangle = \frac{i}{(4\pi)^{d/2}} \frac{e^{-\tau m^2}}{\tau^{d/2}} \sum_{n=0}^{\infty} a_n(x) \tau^n 
\]

Donoghue/Golowich/Holstein '92
Neufeld/Gasser/Ecker hep-ph/9806436

The \( a_n \) can be computed, knowing the form of \( \Delta \).

The UV-divergences of \( W_{L=1} \) are the poles in \( \frac{1}{\tau} \).

\( \Rightarrow \) only \( a_2 \) contributes!
\[ \frac{1}{12} \Lambda_{\mu\nu} \Lambda_{\mu\nu} + \frac{1}{2} \sum^2 \ldots \] and Super-Heat-Kernel Expansion

Applying the Heat-Kernel Expansion: Donoghue/Golowich/Holstein ’92
Neufeld/Gasser/Ecker hep-ph/9806436

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- The \( a_n \) can be computed, knowing the form of \( \Delta \).
- The UV-divergences of \( W_{L=1} \) are the poles in \( \frac{1}{\tau} \).
  \[ \Rightarrow \text{only } a_2 \text{ contributes!} \]
The Heat-Kernel Expansion extracts the $\frac{1}{\epsilon}$-poles of $W_{L=1}$. 

\[ \frac{1}{12} \Lambda_{\mu\nu} \Lambda_{\mu\nu} + \frac{1}{2} \Sigma^2 \]
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... and Super-Heat-Kernel Expansion...

The Heat-Kernel Expansion extracts the \( \frac{1}{\epsilon} \)-poles of \( W_{L=1} \).

With

\[
\Delta = (\partial_\mu + \Lambda_\mu) (\partial^\mu + \Lambda^\mu) + \Sigma
\]

can be calculated. We get

\[
W_{L=1, \text{div}} = \frac{1}{32\pi^2 \epsilon} \int d^4 x \str \left[ \frac{1}{12} \Lambda_{\mu \nu} \Lambda^{\mu \nu} + \frac{1}{2} \Sigma \Sigma \right].
\]

\[
\Lambda_{\mu \nu} = \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu + [\Lambda_\mu, \Lambda_\nu]
\]

- Specifying the Dirac structure of \( S^{(2)} \), we can further evaluate the Dirac-traces.
- The resulting Master-Formula is purely algebraic (Matrix multiplication and traces).

Donoghue/Golowich/Holstein '92; Neufeld/Gasser/Ecker [hep-ph/9806436,PLB] 'tHooft '73,NPB
\[
\frac{1}{12} \Lambda_{\mu\nu} \Lambda_{\mu\nu} + \frac{1}{2} \sum^2 \ldots \text{to find the Master Formula.}
\]

In the SM (and the EWChL), we have

\[
L^\text{SM}_2 = -\frac{1}{2} \phi^i A^j_i \phi_j + \bar{\chi} \left( i\partial - G \right) \chi + \bar{\chi} \Gamma^i \phi_i + \phi^i \bar{\Gamma}_i \chi,
\]

with \( A = (\partial^\mu + N^\mu)(\partial_\mu + N_\mu) + Y \) and \( G \equiv (r + \rho_\mu \gamma^\mu)P_R + (l + \lambda_\mu \gamma^\mu)P_L \).

This gives

\[
L^\text{SM}_{\text{div}} = \frac{1}{32\pi^2} \varepsilon \left( \text{tr} \left[ \frac{1}{12} N_{\mu\nu} N_{\mu\nu} + \frac{1}{2} Y^2 - \frac{1}{3} (\lambda_{\mu\nu} \lambda_{\mu\nu} + \rho_{\mu\nu} \rho_{\mu\nu}) \right] \right)
\]

\[+ \text{tr} [2D^\mu \partial D^\mu r - 2l l r] + \bar{\Gamma} \left( i\partial + i\Lambda + \frac{1}{2} \gamma^\mu G\gamma_\mu \right) \Gamma \]

with

\[
N_{\mu\nu} \equiv \partial_\mu N_\nu - \partial_\nu N_\mu + [N_\mu, N_\nu], \\
\lambda_{\mu\nu} \equiv \partial_\mu \lambda_\nu - \partial_\nu \lambda_\mu + i[\lambda_\mu, \lambda_\nu], \\
\rho_{\mu\nu} \equiv \partial_\mu \rho_\nu - \partial_\nu \rho_\mu + i[\rho_\mu, \rho_\nu], \\
D_\mu l \equiv \partial_\mu l + i\rho_\mu l - il \lambda_\mu, \\
D_\mu r \equiv \partial_\mu r + i\lambda_\mu r - ir \rho_\mu.
\]

'tHooft '73, NPB; Buchalla/Catà/Celis/Knecht/CK [1710.06412, NPB]
In the bosonic SMEFT, we have

$\mathcal{L}_{2}^{\text{SMEFT}} = \mathcal{L}_{2}^{\text{SM}} + \phi^{i} \left( a_{\mu\nu}^{ij} D^{\mu} D^{\nu} + 2 b_{\mu}^{ij} D^{\mu} + c^{ij} \right) \phi_{j}$,

with $a_{\mu\nu}, b_{\mu}, c \sim \frac{1}{\Lambda^{2}}$.

This gives

$\mathcal{L}_{\text{div}}^{\text{SMEFT}} = \mathcal{L}_{\text{div}}^{\text{SM}} + \frac{1}{32\pi^{2}\varepsilon} \left( \text{tr} \left[ c Y + \frac{1}{3} N_{\mu\nu} [D^{\mu}, b^{\nu}] + i \tilde{\Gamma}_{j} / b \Gamma - \frac{1}{6} \tilde{\Gamma} i \overleftrightarrow{D} a \Gamma \right] + \text{tr} \left[ \frac{1}{6} a_{\mu\nu}^{\mu\nu} N_{\mu}^{\lambda} N_{\nu}^{\lambda} - \frac{1}{24} a_{\lambda}^{\mu\nu} N_{\mu\nu}^{\lambda} + \frac{1}{6} N_{\mu\nu}^{\lambda} [D_{\nu}, [D^{\lambda}, a^{\mu\nu}]] \right] + \text{tr} \left[ \frac{1}{3} Y [D_{\mu}, [D_{\nu}, a^{\mu\nu}]] - \frac{1}{4} a_{\lambda}^{\mu\nu} Y^{2} - \frac{1}{12} Y [D_{\mu}, [D^{\mu}, a_{\lambda}^{\mu\nu}]] \right] \right)$.

Buchalla/Celis/CK/Toelstede [1904.07840]
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Part III: The result
[1904.07840]

\[ Q_\phi = (\phi^\dagger \phi)^3 \]
\( Q_{\phi} = (\phi^\dagger \phi)^3 \)  

An explicit example

Starting from

\[
Q_{\phi} = (\phi^\dagger \phi)^3 = \frac{1}{8} (\varphi_i \varphi_i)^3 ,
\]

we find

\[
a_{ij}^{\mu\nu} = 0, \quad b_{ij}^{\mu} = 0, \quad c_{ij} = -\frac{3}{4} (\hat{\varphi}_a \hat{\varphi}_a)^2 \delta_{ij} - 3 (\hat{\varphi}_a \hat{\varphi}_a) \hat{\varphi}_i \hat{\varphi}_j ,
\]

\[
Y_{ij} = \left( \left( \frac{\lambda}{2} + \frac{g^2}{4} \right) \hat{\varphi}_a \hat{\varphi}_a - m^2 \right) \delta_{ij} + \left( \lambda - \frac{g^2}{4} \right) \hat{\varphi}_i \hat{\varphi}_j - g'^2 (t_R \hat{\varphi})_i (t_R \hat{\varphi})_j .
\]
$Q_\phi = (\phi^\dagger \phi)^3$

### An explicit example

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we find

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$$Y_{ij} = \left( \left( \frac{\lambda}{2} + \frac{g^2}{4} \right) \hat{\varphi}_a \hat{\varphi}_a - m^2 \right) \delta_{ij} + \left( \lambda - \frac{g^2}{4} \right) \hat{\varphi}_i \hat{\varphi}_j - g'^2 (t_R^3 \hat{\varphi})_i (t_R^3 \hat{\varphi})_j.$$

Therefore

$$\text{tr } c Y = - \left( 54 \lambda + 9 g^2 + 3 g'^2 \right) (\phi^\dagger \phi)^3 + 24 m^2 (\phi^\dagger \phi)^2.$$

gives with $K(\phi) = 6(3g^2 + g'^2 - \gamma_\phi)$

$$\beta_\phi \supset \left( 54 \lambda - \frac{27}{2} g^2 - \frac{9}{2} g'^2 + 6 \gamma_\phi \right) C_\phi,$$

and

$$\beta_\lambda \supset 48 \frac{m^2}{\Lambda^2} C_\phi.$$
\[ Q_\phi = (\phi^\dagger \phi)^3 \]

Our results agree with the literature.

- We performed independent computations to cross check our results.
- We reproduce all RGE contributions of the 15 bosonic operators computed in Alonso/Jenkins/Manohar/Trott [1308.2627, 1310.4838, 1312.2014, JHEP]
- Stay tuned!
Our results agree with the literature.

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We performed independent computations to cross check our results.

- We reproduce all RGE contributions of the 15 bosonic operators computed in Alonso/Jenkins/Manohar/Trott [1308.2627,1310.4838,1312.2014,JHEP]
- Buchalla/Celis/CK/Toelstede [1904.07840]

To compute the RGEs of the remaining operators, we have to extend our master formula by:

- Four-Fermion operators
- Fermionic tensor currents
- Mixed bosonic/fermionic derivatives

⇒ Stay tuned!
We derived a master formula for the $1/\epsilon$-poles based on the super-heat-kernel:

\[ \frac{1}{12} \Lambda_{\mu\nu} \Lambda_{\mu\nu} + \frac{1}{2} \Sigma^2 \]

It can be applied to a broad class of theories, like the SM, the bosonic sector of the SMEFT, or even the EWChL.

The result is purely algebraic (matrix multiplication and -tracing).
Master formula for one-loop renormalization of bosonic SMEFT operators

— Summary —

- We derived a master formula for the $1/\epsilon$-poles based on the super-heat-kernel.

\[ \frac{1}{12} \Lambda^{\mu\nu} \Lambda_{\mu\nu} + \frac{1}{2} \Sigma^2 \]

- It can be applied to a broad class of theories, like the SM, the bosonic sector of the SMEFT, or even the EWChL.

- The result is purely algebraic (matrix multiplication and -tracing).

- We reproduce the RGE contributions of the 15 bosonic operators that have been previously computed by Alonso/Jenkins/Manohar/Trott [1308.2627,1310.4838,1312.2014,JHEP]

- We are currently extending the master formula to include the full set of SMEFT operators.