6th BSME International Conference on Thermal Engineering (ICTE 2014)

Geometric effect on magnetohydrodynamic convection in a half-moon shaped cavity filled with water having semi-circular bottom heater

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Abstract

Magnetohydrodynamic convection has been gaining attention due to its wide range of application. In this paper magnetohydrodynamic convection in a half-moon (semi-circular) shaped cavity has been analysed. The cavity has two semi-circular bottom heaters and effect of distance between these two heaters has been thoroughly investigated for two different cases ($\lambda = 0.1, 0.4$). Numerical simulation has been carried out for a wide range of Rayleigh number ($Ra = 10^3 \sim 10^7$) and for Hartmann number ($Ha = 0 \sim 100$) to understand the flow and thermal field. Galerkin weighted residual method of finite element analysis has been adopted for numerical solution along with code validation and grid independency test to ensure the numerical accuracy. It has been found that strengthening of magnetic field tries to reduce the heat transfer rate, whereas increment of heater distance augments the heat transfer rate. Results analysed on the basis of Nusselt number and average fluid temperatures are shown by related contours, plots and graphical analysis. Therefore, higher heat transfer rate has been achieved for $\lambda = 0.4$.

Keywords: MHD convection; Bottom heater; Rayleigh Number; Hartmann Number; Magnetic Field.

1. Introduction

Magneto-hydrodynamic convection has turned out to be a matter of great importance due to its scientific, technological and industrial applications in petroleum industries, plasma physics, geophysics, MHD pump, MHD...
flow meter and cooling of nuclear reactors. To keep harmony with its wide range of applications, a considerable amount of research works have been done in this field. A comprehensive review of the early stage work on magneto-hydrodynamic convection can be found in [1-7]. Rudraiah et al. [8] investigated numerically the effect of magnetic field on natural convection in a rectangular enclosure. They found that the effect of the magnetic field reduced the rate of heat transfer. Oztop et al. [9] investigated numerical simulation of magneto-hydrodynamic buoyancy flow in an enclosure when the bottom wall was non-uniformly heated. Magneto-hydrodynamic natural convection in a vertical cylindrical cavity with sinusoidal upper wall temperature was investigated by Kakarantzas et al. [10]. The authors reported that the increment of Rayleigh number promoted heat transfer by convection. The increment of Hartmann number favoured heat conduction and the vertical magnetic field reduced the Nusselt number more than the horizontal. Oztop et al. [11] also studied MHD mixed convection in a lid-driven cavity with corner heater. Rahman et al. [12] investigated MHD natural convection in an enclosure from two semi-circular heaters on the bottom wall. It was reported that the distance between the semi-circular heaters was the most important parameter affecting the heat and fluid flow fields. It was also found that Hartmann number had an adverse effect on heat transfer. Al-Salem et al. [13] investigated the effects of moving lid direction on MHD convection in a linearly heated cavity with the bottom wall. Hossain et al. [14] analysed MHD free convection within trapezoidal cavity with non-uniformly heated bottom wall. The authors found out that the average and local Nusselt number at the non-uniform heating bottom wall of the cavity was dependent on the dimensionless parameters and also tilts angles. Sheikholeslami et al. [15] investigated MHD free convection in an eccentric semi-annulus filled with nanofluid. They reported that the heat transfer enhancement increased with increase of Hartmann number and decreased with augment of Rayleigh number. Islam et al. [16] studied mixed convection and entropy generation characteristics inside a porous cavity with viscous dissipation effect.

Half-moon shaped cavity is considered to understand the model and heat transfer increment of heat exchanger numerically. The objective of this research is to study the geometric effect on magneto-hydrodynamic convection in a Half-moon cavity filled with water having two semi-circular bottom heaters. The effect of distance between this two heater has been thoroughly investigated for two different cases ($\lambda = 0.1, 0.4$). Numerical simulation has been carried out for a wide range of Rayleigh number ($Ra = 10^3 \sim 10^7$) and for Hartmann number ($Ha = 0 \sim 100$) to understand the flow and thermal field.

Nomenclature

| Symbol | Description |
|--------|-------------|
| a | dimensional distance between semi-circular heaters (m) |
| A | area of the enclosure ($m^2$) |
| b | radius of the semi-circular cavity (m) |
| B0 | magnetic induction |
| cp | specific heat at constant pressure (J/kg K) |
| D | dimensionless distance between semi-circular heaters |
| g | gravitational acceleration (m/s²) |
| Ha | Hartmann number |
| H | semi-circular heater |
| k | fluid conductivity (W/m K) |
| L | length of the bottom wall (m) |
| Nu | average Nusselt number |
| p | dimensional pressure (N/m²) |
| P | non-dimensional pressure |
| Pr | Prandtl number |
| Ra | Rayleigh number |
| r | dimensionless radius of semi-circular cavity |
| s | circumference (m) |
| T | temperature (K) |
| u, v | dimensional velocity components (m/s) |
| U, V | dimensionless velocity components |
| x, y | dimensional coordinates (-) |
| X, Y | dimensionless coordinates (-) |
| α | thermal diffusivity (m²/s) |
| β | thermal expansion coefficient (1/K) |
| σ | electrical conductivity |
| μ | dynamic viscosity (Pa s) |
| ν | kinematic viscosity (m²/s) |
| θ | non-dimensional temperature |
| ρ | density (kg/m³) |
| ψ | stream function |
| λ | dimensionless distance between semi-circular heaters |
| Subscripts | |
| av | average |
| h | hot |
| c | cold |
2. Problem formulation

2.1. Physical modeling

The physical model under this investigation is shown in Fig. 1 with necessary boundary conditions. A semi-circular enclosure of diameter $L$ having two semi-circles at the distance of $a$ is considered for this investigation. The two dimensional co-ordinate system is defined and the effect of the gravity is shown on the negative $y$-axis. The upper portion of enclosure is kept at low temperature ($T = T_c$). The horizontal wall of the enclosure is insulated whereas the two semi-circles situated at the horizontal line are heated ($T = T_h$). The whole environment is permeated with magnetic flux density $B_0$. The density variation is considered according to the Boussinesq approximation [17]. Radiation mode of heat transfer and viscous dissipation are considered to be negligible. All solid boundaries are assumed to be rigid no-slip walls.

![Fig. 1. Schematic diagram of the Half-moon cavity with semi-circular bottom heater](image)

3. Mathematical formulation

The flow in this system is considered to be two-dimensional, steady, incompressible and laminar. The governing equations which define the system behaviour are conservation of mass, energy and momentum. Under these considerations non-dimensional variables can be written as:

$$
X = \frac{x}{L}; Y = \frac{y}{L}; U = \frac{u}{U_0}; V = \frac{v}{U_0}; P = \frac{p}{\rho U_0^2}; \Theta = \frac{T - T_c}{T_h - T_c}
$$

Based on these non-dimensional variables, two dimensional governing equations can be written as follows:

The continuity equation-

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0$$

The momentum equations-

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial X} + Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + Ra\frac{Pr}{Pr - Ha^2 Pr V}$$

The energy equation-

$$U \frac{\partial \Theta}{\partial X} + V \frac{\partial \Theta}{\partial Y} = \left(\frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2}\right)$$

The dimensionless parameters which have been appeared in the above governing equations are-

$$Pr = \frac{\nu}{\alpha}; Ha = \sqrt{\frac{\sigma B_0^2 L^2}{\mu}}; Ra = \frac{g \beta (T_h - T_c) L^3}{\alpha \nu}$$

Here, $Pr$, $Ha$ and $Ra$ refer to Prandtl, Hartmann and Rayleigh number respectively. The non-dimensional form of the
boundary conditions are mentioned in Table 1.

Table 1. Boundary conditions in non-dimensional form.

| Boundary Wall               | Flow Field | Thermal Field |
|-----------------------------|------------|---------------|
| Semi-circular upper wall    | \( U = 0, V = 0 \) | \( \Theta = 0 \) |
| Semi-circular bottom wall   | \( U = 0, V = 0 \) | \( \Theta = 1 \) |
| Horizontal bottom Wall      | \( U = 0, V = 0 \) | \( \partial \Theta / \partial Y = 0 \) |

The average Nusselt number at the heated semi-circular cavity can be expressed by

\[ Nu = \frac{1}{s} \int_0^s \frac{Nud}{s} ds \]  

Circumference of the Half-moon cavity can be described as

\[ s = \pi r \]  

Local Nusselt number can be expressed as

\[ \overline{Nu} = \frac{hL}{k} = \frac{\partial \Theta}{\partial n} L \]  

\[ \frac{\partial \Theta}{\partial n} = \frac{1}{L} \left[ \left( \frac{\partial \Theta}{\partial X} \right)^2 + \left( \frac{\partial \Theta}{\partial Y} \right)^2 \right]^{1/2} \]  

The average temperature of the fluid can be expressed by,

\[ \Theta_{av} = \frac{1}{A} \int \Theta dA \]  

Whereas non-dimensional area can be expressed as

\[ A = \frac{\pi}{8} \frac{\pi (b / L)^2}{4} \]  

The non-dimensional stream function can be expressed by,

\[ U = \frac{\partial \varphi}{\partial Y}, V = -\frac{\partial \varphi}{\partial X} \]  

Here, the fluid motion is displayed by stream function \( \varphi \) acquired from velocity components \( U \) and \( V \).

4. Numerical procedure

4.1. Grid independency test

A grid independency test has been executed to examine the numerical solution of the study. Table 1 has been formed showing outcomes of the grid independency test. The test is performed for \( Ha = 50 \) and \( Ra = 10^5 \). From Table 1, it is apparent that for number of elements 429 the average Nusselt number is the lowest and with the improvement of mesh size by increasing number of elements, average Nusselt number increases gradually. But for number of elements 3896 average Nusselt number becomes nearly constant. Additional increment in number of elements does not initiate any significant deviation of average Nusselt number. Therefore, a number of 3896 elements is selected as the ideal mesh size for all conditions of the numerical simulations for this present study.

Table 2. Grid independency test for \( Ha = 50 \) and \( Ra = 10^5 \)

| No of Elements | 429 | 558 | 930 | 1716 | 2904 | 3896 | 6864 |
|----------------|-----|-----|-----|------|------|------|------|
| Nu             | 8.313 | 8.316 | 8.299 | 8.323 | 8.322 | 8.326 | 8.328 |
4.2. Code validation

The code of this present study is validated with the previous study of Ghashemi et al. [18]. Validation is accomplished through average Nusselt number plots against solid volume fraction of nanofluid. Comparison of average Nusselt number for different solid volume fraction for $Ha=0$ and $Ha=30$ at $Ra=10^5$ is shown in Fig. 2. It is also noticeable from the illustration that the code of the present study is definitive enough to perform numerical simulation for the given cases.

5. Results and Discussion

A computational analysis has been done to inspect the effect of magnetic field on natural convection with semi-circular bottom heater in an enclosure using finite element method. The considered parameters are Hartmann number, Rayleigh number and distance between heaters. Results are presented by means of streamlines, isotherms, average Nusselt number and average temperature of flow field.

5.1. Effect of Hartmann number on streamlines and isotherm contours

Consequences of magnetic field on streamlines and isotherm contours are illustrated in Fig. 3 with different Hartmann number for $\lambda = 0.1$ and $\lambda = 0.4$. From the figure it is seen that the strength of the streamline contour is decreased with the increment of Hartmann number for both the values of $\lambda$. For $Ha = 10$ the streamlines fully inhabit in the enclosure. As the Hartmann number goes up, streamlines start to move to the corner of the enclosure and the strength of flow field decreases. For all the cases, the flow field and temperature distribution are symmetric. Strength of flow field for $\lambda = 0.4$ is almost one-third of that for $\lambda = 0.1$. Isotherm contours indicate conduction mode of heat transfer becoming more dominant as the isotherms become more parallel with the increment of Hartmann number which results in lower heat transfer rate.
Fig. 4. Comparison of isotherm contours for (a), (d) $Ha = 10$; (b), (e) $Ha = 50$; (c), (f) $Ha = 100$. Figures in the upper row represent isotherms for $\lambda = 0.1$ and lower row represents the same for $\lambda = 0.4$.

5.2. Effect of Rayleigh number on streamline and isotherm contours

Effects of Rayleigh number on streamlines and isotherm contours are illustrated respectively in Fig. 5 and 6 for $\lambda = 0.1$ and $\lambda = 0.4$. From the figure it is obvious that the strength of the streamline contour varies proportionally with the increment of Rayleigh number. As the value of Rayleigh number is increased, streamlines start to move to the corner. From Fig. 7 (a) slight distortion is found along with parallel lines at $Ra = 10^5$ which indicates heat transfer takes place in both conduction and convection mode of heat transfer. With the increment of Rayleigh number increased amount of distortions are found, which resembles the dominance of convection mode of heat transfer.

Fig. 5. Comparison of streamline contours for (a), (d) $Ra = 10^5$; (b), (e) $Ra = 10^6$; (c), (f) $Ra = 10^7$. Figures in the upper row represent streamlines for $\lambda = 0.1$ and lower row represents the same for $\lambda = 0.4$.

Fig. 6. Comparison of isotherm contours for (a), (d) $Ra = 10^5$; (b), (e) $Ra = 10^6$; (c), (f) $Ra = 10^7$. Figures in the upper row represent isotherms for $\lambda = 0.1$ and lower row represents the same for $\lambda = 0.4$. 
5.3. Effect of Rayleigh number on Nusselt number and average temperature of flow field

The effects of Rayleigh number on average Nusselt number ($Nu$) of the heated bottom wall are illustrated in Fig. 7(a) for $\lambda = 0.1$ and $\lambda = 0.4$. From the figure it can be seen that the Nusselt number increases with the increment of Rayleigh number. Comparing two different curves for $\lambda = 0.1$ and $\lambda = 0.4$, it can be observed that the curve for $\lambda = 0.1$ shows more upturn than the curve for $\lambda = 0.4$. Again the effect of Rayleigh number on average temperature of fluid inside the cavity is also illustrated in Fig. 7(b) for the same conditions. Initially the average temperature starts to rise slightly with increment of Rayleigh number up to $10^5$. With the increasing Rayleigh number up to $10^7$, the temperature curve for $\lambda = 0.1$ drops rapidly, while the other curve for $\lambda = 0.4$ drops slightly. For $10^3 < Ra < 7 \times 10^3$ there is no significant change in average Nusselt number and average temperature. At $Ra = 10^6$ there is no evidence of geometric effect on average Nusselt number.

![Fig. 7. Variation of (a) average Nusselt number of the heated bottom wall and (b) average temperature of fluid inside the cavity with Ra. The red symbols indicate results for $\lambda = 0.1$ and green symbols for $\lambda = 0.4$.](image)

5.4. Effect of Hartmann number on Nusselt number and average temperature of flow field

The effects of Hartmann number ($Ha$) on average Nusselt number ($Nu$) and average temperature ($\Theta_{av}$) of the heated bottom wall are displayed in Table 2 for two different distances between semi-circular heaters ($\lambda = 0.1$ and $\lambda = 0.4$). It is observed that the value of Nusselt number and average temperature decrease with the increment of Hartmann number. But in both parameters a higher decrement is found for $\lambda = 0.1$ than $\lambda = 0.4$ with the increasing Hartmann number. For $\lambda = 0.4$, Nusselt number is higher than that of $\lambda = 0.1$ and therefore convective dominance takes place ensuring higher heat transfer rate.

| Ha   | $\lambda = 0.1$ | $\lambda = 0.4$
|------|-----------------|-----------------|
|      | $\Theta_{av}$   | $Nu$            | $\Theta_{av}$ | $Nu$            |
| 0    | 0.373           | 5.046           | 0.396         | 8.240           |
| 10   | 0.372           | 5.036           | 0.395         | 8.239           |
| 20   | 0.371           | 5.019           | 0.394         | 8.2384          |
| 30   | 0.369           | 5.009           | 0.393         | 8.2382          |
| 40   | 0.368           | 5.003           | 0.392         | 8.2377          |
| 50   | 0.367           | 5.000           | 0.391         | 8.2374          |
| 60   | 0.367           | 4.999           | 0.3918        | 8.2372          |
| 70   | 0.366           | 4.998           | 0.3916        | 8.2371          |
| 80   | 0.3664          | 4.9974          | 0.3914        | 8.2370          |
| 90   | 0.3662          | 4.9972          | 0.3912        | 8.2369          |
| 100  | 0.3661          | 4.9971          | 0.3910        | 8.2368          |

Table 3. Variation of average temperature with Nusselt number for $\lambda = 0.1$ and $\lambda = 0.4$. 
6. Conclusion

The problem considered in this study is to investigate the geometric effect on magneto-hydrodynamic convection by showing flow fields and average temperatures relating to Hartmann numbers, Rayleigh numbers and distance between two semi-circular bottom heaters. The numerical results are discussed and following points have emerged:

- Strength of flow field decreases with the increment of $Ha$ and increases with the increment of $Ra$. For any $Ha$, strength of the streamline contour is maximum at $\lambda = 0.1$ and minimum at $\lambda = 0.4$.
- For any $Ra$, strength of streamline contours is maximum at $\lambda = 0.4$ and minimum at $\lambda = 0.1$. Conduction mode of heat transfer becomes more dominant with increasing of $Ha$ and decreasing of $Ra$.
- Again Convection heat transfer rate is greater at $\lambda = 0.1$ than that at $\lambda = 0.4$. Effect of $Ra$ on Nusselt number is not evident for any $\lambda$ up to $Ra = 7 \times 10^3$. Nusselt number is independent of different geometric conditions at $Ra = 10^6$.
- For $Ra = 10^3$ and geometry of $\lambda = 0.4$, highest average temperature is studied which results in higher heat transfer rate. Increment in magnetic induction decreases average temperature and Nusselt number and thus higher convective environment is evident for $\lambda = 0.4$.

Acknowledgement

Authors would like to thank Multiscale Mechanical Modeling and Research Network (MMMRN) for supporting this study and acknowledge the insightful suggestions for the improvement of this article.

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