Mathematical and Numerical Modeling of Type N Thermocouple

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Abstract This paper presents the mathematical and numerical modelling of type N thermocouple using the transient diffusion phenomenon and finite difference and finite volume concept. Here particles of 5 inert atmospheres employed for checking the effect of drag force on sheath mass and to recommend one media for numerical model calculation. Environment is hot for which concentration of sheath particle increase near sheath but decrease as distance increase. Von-Neumann stability incorporates for time step calculation in numerical work.

Keywords Sheath Mass, Transient Diffusion, Hot Media, Inert Atmosphere, Type N

1. Introduction

Here we have taken type N thermocouple i.e. Nicrosil-Nisil which shows good thermoelectric stability at high temperature like 1300°C [12][13]. We have chosen exposed Mineral insulated metal sheath type thermocouple [8] [9] for studying the particle effect on the outer sheath of the thermocouple. Generally what happen stars are moving with space and time in the galaxy like rubber band their locus increasing with space and time and they are moving to the one galaxy to another as universe has infinite space and similarly here also particle mass extracted due to drag force of the hot media’s particle transferred to other space with time. But our situation involved Brownian motion for which the concentration of the particle reduced with distance from the sheath contact point. We are taking five media Air-Ar, Air-He, Air-Kr, Air-Ne, and Air-Xe for studying the drag and velocity effect of the particles of that media on the sheath mass. Temperature is varies between 1000°C-1500°C and pressure 1 atmosphere. Sizes of the composite media’s can be found from Poling et al book “The properties of gases and liquids” [7]. Since the concentration of the particle mass varies with space and time we here incorporating transient diffusion of the particle and analytically solved with specific boundary condition by applying application of derivatives concept and numerically we get the variation of concentration at each grid with time by incorporating concept of finite difference and finite volume method. After that recommendation of one media has to be done for numerical work validation incorporating von-Neumann stability criterion. The fig.1 of the system is as follows

Figure 1. 2D boundary where hot mass in entering through imaginary boundary to sheath of thermocouple. [3]

We are taking that imaginary boundary from the above picture for grid calculation taking the region of sheath contact and region of atm. Difference 1 m and 10 grids.

2. Literature Review

According to the Platinum Metals Rev., 1969 & 1971 long term calibration changes of Rh-Pt thermocouple which is operating close to high temperature above 1300°C environment and rhodium transferred in vapour phase slowly from the positive to the negative limb which affects the thermoelectric behaviour. The accumulation of serious concentrations of rhodium oxide prevented by employing the natural convection and ventilation occurs in the assembly and selection of suitable alloys reduced the harmful effects of rhodium vapour migration. But it is seen that majority of Rhodium is emitted from the outer sheath rather than from the positive leg of the unit. Now for minimising such kind of effects the unstable thermocouples are sectioned for micro-examination which deals with cracks and fissures etc.
It is important to emphasise the geometry of the thermocouple system with respect to stability. There are four arrangement such as in line where is no contamination of the negative limb, parallel where is some cross contamination, minimised by convection, twin bore insulated where is local cross contamination at insulator junctions and fully insulated where is moderate to severe contamination of the platinum limb encouraged by stagnant temperature takes place. The performance of argon-filled units is not affected by insulated quality. [5][6][10]

3. Analytical Model

As we told earlier we are taking a situation where thermocouple of MIMS (mineral insulated metal sheath) is exposed to the environment in a 2D boundary where the hot gas is entering. Here we have chosen five systems for checking maximum drag effect on the particle mass. Drag force of the system more means it can deplete outer sheath of thermocouple more. Since sheath mass chipped or ablated due to the drag force of the media particle mass, so gravity force try to attract sheath particle mass downward for which the variation of the concentration in Y axis more than other axis. So for the dilute species in a stationary case by assuming no homogeneous reaction and varying diffusivity and concentration, one dimensional mass transfer in Y direction by relaxing the Z direction components & X direction components in case of transient diffusion is as follows [2]

\[ \frac{\partial^2 N}{\partial y^2} = \frac{1}{D} \frac{\partial N}{\partial t} \]  

Where C is the concentration in molecules/m³, t is the time in second; D is the diffusivity in cm²/s & y is the space in m. Now by invoking the concept of particle diffusivity [11] which is the function of particle size and gas properties in gases we can write equation (1) is as follows

\[ \frac{\partial^2 N}{\partial y^2} = \frac{1}{D} \frac{\partial N}{\partial t} \]  

Where ‘N’ is the number of particle mass transported upwards through the hole provided in the downward of the 2D boundary. Release of equally sized particle of \( N_0 \) at \( t = 0 \) is increased in concentration with time as the system temperature goes on increasing from 1000°C to 1500°C i.e. beyond the melting point of type N thermocouple. The solution of equation (2) using application of derivatives and beyond the melting point of type N thermocouple. The solution diameter (\( \sigma \)) in 'nm' can be evaluated as follows [2]

\[ \lambda_{AB} = \sqrt{\frac{RT}{\pi M_{AB}}} \]  

Where \( \lambda_{AB} \) is the mean free path expression suffers from a significant flaw - it assumes that the 'target' molecules are at rest when in fact they have a high average velocity i.e. the number of collisions is 1.414 times the number with stationary targets. So mean free path (\( \lambda \)) in 'nm' can be evaluated as by following formula

\[ V_{AB} = \frac{RT}{\pi M_{AB}} \]  

Where \( V_{AB} \) is the composite molecular weight of the system A & B can be evaluated as follows \( M_{AB} = \frac{2}{(M_A^{-1} + M_B^{-1})} \) and viscosity and diffusivity can be evaluated is as follows

\[ D_{AB} = \frac{0.002667 T^2}{p M_{AB} \sigma_{AB}^2 \Omega_D} \]  

Where \( D_{AB} \) is the diffusion coefficient in cm²/s and \( \Omega_D \) is the diffusion collision integral, dimensionless. Now viscosity can be evaluated by incorporating intermolecular effects is as follows

\[ AB = 26.69 \left( \frac{MT^2}{\sigma^4 \eta_D} \right) \]  

Where \( AB \) is the composite system viscosity in µP and M is the molecular weight in g/mol.[7]

Now for drag force referring Clarkson universities model which is of national science foundation according to Brenner (1961) the drag acting on a particle moving toward a wall...
under the creeping flow condition as shown in figure 2. To the first order, the drag coefficient is given as

\[ F_D = \frac{3\eta V}{C_C} \]

And similarly for grid 10 the equation 2 can be written as

\[ \frac{(n_p - n_p^0) \Delta y}{\Delta t} = \frac{D}{\Delta y} \left( n_B - n_p^0 \right) - \frac{D}{\Delta y} \]

Now in standard form the equation of nodes can be written as

\[ a_p n_p = a_s n_s^0 + a_n n_n^0 + \left( a_p^0 - (a_n + a_s) \right) n_p^0 + s_u \]

Where \( a_p^0 = \frac{D}{\Delta y} \) and the values of other coefficients at various nodes from equation 14 & 15 is as follows

| Node | \( a_s \) | \( a_n \) | \( s_u \) |
|------|--------|--------|--------|
| 1    | 0      | D/\Delta y | 0      |
| 2,3,4,5,6,7,8,9 | D/\Delta y | D/\Delta y | 0      |
| 10   | D/\Delta y | 0      | D/\Delta y \( (n_B - n_p^0) \) |

The time step for the explicit method can be found from von Neumann criterion. Here \( s_u \) is the generation term.[14]

### 5. Results and Discussions

For drag force acting on the particle we need to have viscosity, velocity, particle size and Cunningham factor values. From equation (3) and the data’s from ‘B.1, A.5&A.19’ table [7] we have particle sizes and molecular weight for the five media is as follows

| Media      | Particle size(Å) |
|------------|------------------|
| Air-argon  | 3.63             |
| Air-He     | 3.13             |
| Air-Krypton| 3.7              |
| Air-Neon   | 3.3              |
| Air-Xe     | 3.9              |

| Media      | Molecular weight(g/mol) |
|------------|-------------------------|
| Air-argon  | 33.61                   |
| Air-He     | 7.035                   |
| Air-krypton| 43.10                   |
| Air-Neon   | 23.8                    |
| Air-Xe     | 47.506                  |
Now for average molecular speed for the five media at different temperatures from 1000°C to 1500°C is shown in fig. So velocity for Air-Helium system is more and Air–Xe system is less i.e. we will recommend Air-Xe media for better life of thermocouple sheath. Now for collision integral (Ω) for finding the viscosity and viscosity variation in different temperatures is as follows

Table 4. Collision integral of composite media’s particle

| T(k) | Ω_{Air-Ar} | Ω_{Air-He} | Ω_{Air-Kr} | Ω_{Air-Ne} | Ω_{Air-Xe} |
|------|------------|------------|------------|------------|------------|
| 1273 | 0.696      | 0.5855     | 0.733      | 0.6413     | 0.75       |
| 1373 | 0.688      | 0.588      | 0.724      | 0.634      | 0.74       |
| 1473 | 0.681      | 0.5723     | 0.716      | 0.627      | 0.731      |
| 1573 | 0.673      | 0.566      | 0.708      | 0.6204     | 0.723      |
| 1673 | 0.667      | 0.560      | 0.702      | 0.614      | 0.7163     |
| 1773 | 0.661      | 0.5559     | 0.695      | 0.609      | 0.709      |
Table 5. Mean free path of the various media’s

| T(k) | $\lambda_{\text{Air-Ar}}$ (nm) | $\lambda_{\text{Air-He}}$ (nm) | $\lambda_{\text{Air-Kr}}$ (nm) | $\lambda_{\text{Air-Ne}}$ (nm) | $\lambda_{\text{Air-Xe}}$ (nm) |
|------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| 1273 | 296.33                        | 398.6                         | 285.22                        | 358.55                        | 256.72                        |
| 1373 | 319.6                         | 419.6                         | 307.62                        | 386.72                        | 276.9                         |
| 1473 | 342.9                         | 461.17                        | 330.025                       | 414.9                         | 297.045                       |
| 1573 | 366.152                       | 492.98                        | 352.425                       | 443.043                       | 317.21                        |
| 1673 | 389.43                        | 523.8                         | 374.831                       | 471.21                        | 337.4                         |
| 1773 | 412.7                         | 555.1                         | 397.234                       | 499.4                         | 357.54                        |

Figure 5. Variation of Cunningham correction factor with temperature

Figure 6. Variation of drag force (in Pico Newton) with temperature (in K)

Here Cunningham factor for 5 media at various temperatures by evaluating the mean free path by equation (6) is shown above (fig 5) and the drag force can be calculated from equation (10) and it has been seen from above plot (fig 6) that drag force is maximum for Air-Xe media and minimum for Air-He or Air-Ne. We want to control both velocity and drag force for better life of thermocouple’s outer sheath. But Air-He system particle velocity is more than Air-Ne system. So we recommend Air-Ne media for better control and reliability of the thermocouple. Now we using von Neumann stability criterion for calculating time step and at temperature 1273K and 1 atm. pressure the diffusivity can be calculated from equation (9) is 3.54 cm²/s. It can be shown that the particle concentration with space and time first increases then decreases exponentially from contact region of the thermocouple sheath to the atmospheric region with time by taking 1 m length 2D geometry and incorporating 10 grids and distance between 2 grids is 0.1m. Concentration at each grid can be finding from the equation (3). So the variation particle concentration with time and space by taking $N_0=1\text{mol/m}^3$ is as follows
Now for finite volume method the values of concentration volume at each grid can be finding by using equation (17) is as follows

\[
0.0071n_p = 0.00354n_n^0 + 0.00356n_p^0 - 0.00354n_n^0 + 0.00177n_n^0
\]

\[
0.00236n_p = 0.00188n_n^0 + 0.00177n_p^0 - 0.00177n_n^0 + 0.00177n_p^0
\]

\[
0.001416n_p = 0.000708(n_n^0 + n_n^0) - 0.00118n_p^0 - 0.000708(n_n^0 + n_n^0)
\]

\[
0.00079n_p = 0.000393(n_n^0 + n_n^0) - 0.000885(n_n^0 + n_n^0)
\]

\[
0.00059n_p = 0.0005057(n_n^0 + n_n^0) - 0.000885n_p^0 - 0.0005057(n_n^0 + n_n^0)
\]

By solving this equation we get succeeding nodes value is almost same 0.02mol/m³, not much variation is there.

6. Conclusions

We have seen from the plot 3 and 6 is that drag force is maximum for Air-Xe media but minimum for Air-He or Air-Ne. Also we observed that particle velocity of Air-He media is more than Air-Ne. For better control of the system parameters and for the system reliability we want to control both velocity and drag force. In numerical work Air-Ne media does not shown too much variation in concentration and its concentration decreases from the top of the sheath contact with time. So we recommend Air-Ne media for better life of thermocouple outer sheath.
7. Scope of Future Work

Many more variations can be shown by taking this type of inert media’s. By incorporating deeper concept like particle physics we can describe this kind of work more deeply and for preparing the improvement of the system we can use fuzzy logic and neural network for better control and good system performance. Aerodynamic motion of the particles can be studied while collide with sheath particle mass.

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