Inferring freeze-out parameters from pion measurements at RHIC and LHC

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We analyze the transverse momentum spectra of charged pions measured in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV and in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV using the Tsallis distribution modified to include transverse flow. All the spectra are well described by the modified Tsallis distribution in an extended transverse momentum range upto 6 GeV/c. The kinetic freeze-out temperature ($T$), average transverse flow ($\beta$) and degree of non thermalization ($q$) are obtained as a function of system size for both the energies. With increasing system size $\beta$ shows increasing trend whereas $T$ remains constant. While the systems at RHIC and LHC energies show similar $\beta$ and $q$, the parameter $T$ is higher at LHC as compared to RHIC. The kinetic freeze-out temperature is also extracted using the measured charged particle multiplicity and HBT volume of the system as a function of system size and collision energies.

Keywords: Modified Tsallis distribution; Hadron spectra; HBT measurements.

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1. Introduction

The Quantum Chromodynamics (QCD), the theory of strong interaction suggests that at energy density above $\sim 1$ GeV/fm$^3$ the hadronic matter undergoes a phase transition to Quark Gluon Plasma (QGP)\[^1\] a phase where the relevant degrees of freedom are quarks and gluons. The heavy ion collisions at relativistic energy are the means to produce a large volume of hot/dense matter required to create and characterize such a phase\[^2,3\].

The quark gluon matter presumably with local thermal equilibrium expands hydrodynamically and undergoes a phase transition to hadronic matter which further cools till the multiple scatterings among particles are sufficient to keep them as one system. The hadrons then decouple from the system and their spectra would reflect the condition of the system at the time of freeze-out. Hadrons (pions, kaons and
protons) form the bulk of particles produced and are usually the first and easiest to be measured in a heavy ion collision experiment. Traditionally, statistical models\(^1\) has been used at SPS and RHIC energies to infer the conditions at freeze-out using measured hadron ratios as input. Alternatively one can consider full transverse momentum \(p_T\) spectra of hadrons in heavy ion collisions. The bulk and collective effects\(^5,6\) show up in the low and intermediate \(p_T\) regions of hadron spectra while the high \(p_T\) region above 5 GeV/c consists of particles from jets which are produced in hard interactions.

The Tsallis distribution\(^7,8\) describes a system in terms of two parameters; temperature and \(q\) which measures deviation from thermal distribution. It has been shown in Refs.\(^9,10\) that the functional form of the Tsallis distribution in terms of parameter \(q\) is the same as the QCD-inspired Hagedorn formula\(^11,12\) in terms of power \(n\). Both \(n\) and \(q\) are related and describe the power law tail of the hadron spectra coming from QCD hard scatterings. The Tsallis distribution has been used extensively to describe the \(p_T\) spectra of identified charged hadrons measured in \(p+p\) collisions at RHIC and at LHC energies.\(^9,13\) It does not always provide the best description of hadron \(p_T\) distributions in heavy ion collisions which are modified due to collective flow and thus Tsallis blast wave method is used as in Ref\(^14\). The average transverse flow can be included in Tsallis distribution and keeping the functional form to be analytical as done in Refs.\(^15,16\) The function presented in Ref\(^15\) can be used in a wider \(p_T\) range as was done for both meson and baryon spectra for Au+Au collisions at \(\sqrt{s_{NN}} = 200\) GeV.

We analyze the transverse momentum spectra of charged pions measured in heavy ion collisions. Recent measurements of identified charged particle spectra by PHENIX in different centralities of Au+Au collision at \(\sqrt{s_{NN}} = 200\) GeV\(^17\) and by ALICE in the most central (0-5\%) and the most peripheral (60-80\%) Pb+Pb collisions at \(\sqrt{s_{NN}} = 2.76\) TeV\(^18\) have been used in the study. The kinetic freeze-out temperature \((T)\), average transverse flow \((\beta)\) and degree of non thermalization \((q)\) are obtained as a function of system size for both the energies. As an alternative the (kinetic) freeze-out temperature is also extracted using the measured charged particle multiplicity and HBT volume of the system.

2. Analysis of hadron spectra

The transverse momentum spectra of hadrons can be described using the modified Tsallis distribution including the transverse flow as proposed in Ref\(^15\) is given by:

\[
E \frac{d^3N}{dp^3} = C_n \left( \exp \left( -\frac{\gamma \beta p_T}{nT} \right) + \frac{\gamma m_T}{nT} \right)^{-n}. \tag{1}
\]

Here \(C_n\) is the normalization constant, \(m_T = \sqrt{p_T^2 + m^2}\), \(\gamma = 1/\sqrt{1 - \beta^2}\), \(\beta\) is the average transverse velocity of the system and \(T\) is the temperature. The power \(n\) is related to the non-extensivity parameter \(q\) as \(n = 1/(q - 1)\). The parameter \(q\)
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... gives temperature fluctuations in the system as: \( q - 1 = \text{Var}(T)/\langle T \rangle^2 \). It can take a value between 1 and 4/3. Larger values of \( q \) correspond to smaller values of \( n \) which imply dominant hard QCD point-like scattering. Both \( n \) and \( q \) have been interchangeably used in Tsallis distribution. In heavy ion collisions, the high \( p_T \) tail decides the value of \( n \). Phenomenological studies suggest that, for quark-quark point scattering, \( n \approx 4 \), and when multiple scattering centers are involved \( n \) grows larger. When \( \beta \) is zero, Eq. 1 is the usual Tsallis equation which has been the most popular tool to characterize hadronic collisions in recent years.

At low \( p_T \), Eq. 1 represents a thermalized system with collective flow and at high \( p_T \) it becomes a power law as follows

\[
E \frac{d^3N}{dp^3} \sim C_n \exp \left( -\frac{\gamma(m_T - \beta p_T)}{T} \right) \quad \text{for } p_T \to 0,
\]

\[
\sim C_n \left( \frac{\gamma m_T}{nT} \right)^{-n} \quad \text{for } p_T \to \infty. \tag{2}
\]

In this work, we focus on the study of the charged pion spectra measured in heavy ion collisions at RHIC and LHC energies. The errors on the data are taken as quadratic sums of statistical and uncorrelated systematic errors. The RHIC measurements are available in \( p_T \) range 0.5 - 6.0 GeV/c and we use the LHC measurements in the same range. The spectra are fitted with Eq. 1 and all the parameters are obtained as a function of system size (centrality) for both the energies.

The freeze-out temperature \( T \) can also be extracted from the measured multiplicity using following procedure. The particle number density \( n \) can be related to the measured particle multiplicity and HBT volume \( V \) as

\[
nV = \frac{dN}{d\eta}, \tag{3}
\]

where \( dN/d\eta \) is 1.5 times the total measured charged particle multiplicity \( (dN_{ch}/d\eta) \). The number density can also be expressed in terms of freeze-out temperature \( T \)

\[
n = \frac{1.2}{\pi^2} a_n(T) T^3. \tag{4}
\]

The parameter \( a_n(T) = \sum_i g_i n_i(m_i/T) \) where \( g_i \) is the degeneracy factor and \( n_i \) for \( i \text{th} \) meson species is given by

\[
n_i(m_i/T) = \frac{1}{2 \times 1.2} \int_0^{\infty} \frac{x^2 dx}{e^{\frac{x^2}{2}+\frac{(m_i/T)^2}{2}}} - 1. \tag{5}
\]

The parameter \( a_n(T) = 3 \) for massless pion gas. In our study we assume that the system at freeze-out consists of pion \( (g = 3) \), kaon \( (g = 4) \), \( \rho \) \( (g = 9) \), \( \phi \) \( (g = 1) \), \( \eta \) \( (g = 1) \), \( \omega \) \( (g = 3) \) mesons and obtained \( n/T^3 \) as a function of temperature which is shown in Fig. 1 along with that for massless pion gas.

The freeze-out temperature \( T \) can be obtained by numerically solving the following equation

\[
T^3 = \frac{1}{(1.2/\pi^2) a_n(T) V} \frac{1}{d\eta} dN. \tag{6}
\]
Fig. 1. (Color online) The variation of $n/T^3$ as a function of temperature for both hot meson gas (solid line) and massless pion gas (dashed line).

The HBT volume $V = (2\pi)^{3/2} R_{\text{side}}^2 R_{\text{long}}$, where $R_{\text{side}}$ and $R_{\text{long}}$ are the measured HBT radii\cite{27,28}

If transverse flow is present in the system, then the system volume obtained from measured HBT radii will be smaller than the fireball volume. To correct for this effect the HBT radii as a function of $m_T$ are extrapolated to $m_T = 0.140$ GeV/$c^2$ which corresponds to $p_T = 0$.

3. Results and Discussions

Figure 2 shows the charged pion invariant yield spectra in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV\cite{17} as a function of $(m_T - m)$, fitted with Eq. 1. The spectra in Au+Au collisions are given for 0-10%, 10-20%, 20-40%, 40-60% and 60-80% centralities, which are scaled up by factors given in the Fig. 2. Figure 3 shows the same for Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$\cite{18} TeV fitted with Eq. 1. It is seen that Eq. 1 describes the data in full $p_T$ range for all collision centralities, both at RHIC and LHC energies. The collision centralities can be converted to average number of participants $\langle N_{\text{part}} \rangle$ using Glauber model which is proportional to initial system size.

The parameter $q$ as a function of $\langle N_{\text{part}} \rangle$ is shown in Fig. 4 (a). The value of $q$ is higher for the peripheral collisions in comparision to other centralities, for both RHIC and LHC energies. The corresponding value for power $n$ is shown in Fig. 4(b). The values of $q$ (or $n$) are similar for RHIC and LHC which show similar
Fig. 2. (Color online) Invariant yield for charged pions \( \pi^+ + \pi^- \) as a function of \((m_T - m)\) measured in Au+Au collisions at \( \sqrt{s_{NN}} = 200 \text{ GeV} \) for 0-10%, 10-20%, 20-40%, 40-60% and 60-92% centrality bins. The spectra are scaled up by a factor of 10, 5, 3, 2 and 1 for the respective centrality bins. The fitted Modified Tsallis function is shown by the black curve.

Figures 5 (a) and 5 (b) respectively show the kinetic freeze-out temperature \( T \) and the average transverse flow \( \beta \) as a function of \( \langle N_{\text{part}} \rangle \). At RHIC energy, except for peripheral bin, the value of temperature remains constant for all centrality bins within uncertainties. For the most central collisions (0-5%), \( T \) has a higher value at LHC energy than that at RHIC. The transverse flow velocity \( \beta \) increases with the system size for both RHIC and LHC collisions and has almost same behavior in the two energy regimes.

The freeze-out temperature is obtained from Eq. 6 using the measured particle multiplicity and HBT volume. Figure 6 (a) shows the freeze-out temperature as a function of system size and Fig. 6 (b) shows the same as a function of collision energy. The solid squares show the result obtained from the measured HBT radii. The open circles show the result obtained using the corrected HBT radii (at \( p_T = 0 \)) as explained before. This correction makes the values of \( T \) slightly smaller. It is seen that the freeze-out temperature increases while going from RHIC energy to LHC energy.

A comparison of Figs. 5 (a) and 6 (a) shows that the freeze-out temperature obtained from the two different methods follow similar trend as a function of system size. The temperature shows almost flat behavior with system size. Figures 5 (a) and 6 (b) show that the freeze-out temperature is more for LHC energy. The
Fig. 3. (Color online) Invariant yield for charged pions as a function of \((m_T - m)\) measured in Pb+Pb collisions at \(\sqrt{s_{NN}} = 2.76\) TeV for 0-5% and 60-80% centrality bins. The Modified Tsallis fitted function is shown by the black curve.

Fig. 4. (Color online) Parameters (a) \(q\) and (b) \(n\) obtained from Eq. \ref{eq:1} for charged pions measured in Au+Au (open squares) collisions at \(\sqrt{s_{NN}} = 200\) GeV and in Pb+Pb (filled circles) collisions at \(\sqrt{s_{NN}} = 2.76\) TeV as a function of \(\langle N_{\text{part}} \rangle\).

The temperature obtained from two measurements namely the \(p_T\) spectra and the HBT measurements have same dependence on system size and energy. There is up to 20%
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Fig. 5. (Color online) Parameters (a) $T$ and (b) $\beta$ obtained from Eq. 1 for charged pions measured in Au+Au (open squares) collisions at $\sqrt{s}_{NN} = 200$ GeV and Pb+Pb (filled circles) collisions at $\sqrt{s}_{NN} = 2.76$ TeV as a function of $\langle N_{\text{part}} \rangle$.

Fig. 6. (Color online) Kinetic freeze-out temperature obtained using Eq. 6 (a) as a function of $\langle N_{\text{part}} \rangle$ and (b) as a function of $\sqrt{s}_{NN}$ (0-5% centrality) for RHIC and LHC energies.

difference in the magnitudes which might arise due to the experimental error and different $p_T$ range of the measurements affected by transverse flow differently.

4. Conclusion

The transverse momentum spectra of charged pions measured in Au+Au collisions at $\sqrt{s}_{NN} = 200$ GeV and Pb+Pb collisions at $\sqrt{s}_{NN} = 2.76$ TeV are analysed...
using the modified Tsallis distribution. All the spectra used in this analysis are well described by this distribution. The parameter $q$ of the modified Tsallis distribution suggests similar thermalization characteristics for systems at RHIC and LHC energies. The kinetic freeze-out temperature extracted from pion $p_T$ spectra remains flat for all centrality bins except for the peripheral bin at RHIC energy. The freeze-out temperature is higher at LHC energy than that at RHIC energy for the most central collisions. The transverse flow velocity increases with system size for both of these energies. The kinetic freeze-out temperature is obtained as a function of system size and collision energy, from the measurement of HBT radii and particle multiplicity. The measured HBT radii are extrapolated to $m_T = 0.140$ GeV$/c^2$ to correct for the effect of transverse flow. The freeze-out temperature obtained from particle spectra and HBT measurements show similar trend as a function of system size as well as a function of collision energies. However, the kinetic freeze-out temperature obtained using HBT radii remains larger than that obtained from the particle spectra.

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