Pentaquarks, Skyrmions and the Vector Manifestation of Chiral Symmetry

Mannque Rho\textsuperscript{a}

\textsuperscript{a} Service de Physique Théorique, CEA Saclay, 91191 Gif-sur-Yvette, France

(Dated: December 10, 2018)

The structure of the pentaquark baryon Θ\textsuperscript{+} is discussed in terms of a K\textsuperscript{+}-skyrmion binding where the skyrmion arises as a soliton in hidden local symmetry approach to low-energy hadronic physics which may be considered as a holographic dual to QCD. The “vector manifestation” of chiral symmetry encoded in the effective theory Wilsonian-matched to QCD is proposed to play an important role in the binding. Among the options available for understanding the pentaquark structure is the intriguing possibility that the Θ\textsuperscript{+} is a Feshbach resonance generated by the solitonic matter that drives the Wess-Zumino term that in the presence of K\textsuperscript{+} acts like a magnetic field.

I. PENTAQUARKS

The prediction\textsuperscript{1} in 1997 by Diakonov, Petrov and Polyakov (DPP)\textsuperscript{2} of the S = +1 baryon denoted Θ\textsuperscript{+} at 1540 MeV with a narrow width \( \lesssim 20 \) MeV and the subsequent experimental confirmation of the state published in 2003 by Nakano et al\textsuperscript{3} with mass \( 1540 \pm 10 \) MeV and a width < 25 MeV have spurred a huge activity both in experiment and in theory. What was remarkable in the DPP work was that it was an elegantly simple prediction based on a clearly defined theoretical framework of chiral solitons and that considering the qualitative nature of the soliton model, the prediction was confirmed surprisingly quantitatively. Since then there have been a flurry of activities with conflicting results\textsuperscript{4}, some pro and some con, both in theory and in experiment and the verdict on the existence or non-existence of the Θ\textsuperscript{+} is yet to come out from the experimental side. Fortunately a clear-cut experimental confirmation or refutation is expected to be forthcoming in the near future.

The discovery of pentaquark baryons will undoubtedly be a boon for hadron spectroscopy. I would like to further suggest that independently of whether or not the Θ\textsuperscript{+} in question exists in nature, the development brings out a highly intriguing theoretical issue tied to chiral symmetry and its spontaneous breaking in matter. This is the topic of my talk here.

For the discussions that follow, I will assume that the Θ\textsuperscript{+} exists. Even if it is proven to be non-existent by forthcoming experiments, what I will discuss will still be relevant for other issues I will make reference to, so in either case, my discussions will not be futile. For definiteness I will focus solely on the singlet member of the \( SU(3) \) multiplet to which the Θ\textsuperscript{+} is assumed to belong. For the physically more meaningful spectroscopy of pentaquark baryons, a lot more work will be needed. Also I shall not touch on quark-model descriptions, focusing instead on the “dual” skyrmion description.

II. SKYRMIONS

When the prediction for the Θ\textsuperscript{+} was first made in 1980’s and later in 1990’s, it was considered to be a unique prediction of the chiral soliton model not shared by quark models in that it naturally arose from the model on the same footing as the baryons described in terms of \( N_c = 3 \) quarks in the quark model. In the naive quark picture, the Θ\textsuperscript{+} is “exotic” since it requires at the minimum four quarks and one antiquark. But of course there is nothing sacred in this naive picture. Why should there not be baryons that carry more than \( N_c \) valence quarks just as in nuclear physics more than valence particles are involved in nuclear spectroscopy? So in this sense, there is nothing really “exotic” about the five-quark structure of a baryon in QCD. In fact, in the large \( N_c \) limit, all baryons in the skyrmion model can be matched one-to-one to constituent quark model\textsuperscript{5}. However for convenience of understanding what we mean, I will refer to the standard \( N_c \) quark baryons as “non-exotic” and to all others “exotic.”

A. The skyrmion Lagrangian

The skyrmion prediction typified by DPP was based on the usual Skyrme Lagrangian

\[
\mathcal{L} = \frac{f^2}{4} \text{Tr} \partial \mu U \partial \nu U^\dagger + \frac{1}{32c^2} \text{Tr} [U^\dagger \partial \mu U U^\dagger \partial \nu U]^2 \\
+ \text{Tr} (M (U + U^\dagger - 2)) + \cdots
\]  

(1)

where \( f \) is a constant related to the pion decay constant \( f_\pi \approx 93 \) MeV, \( c \) is an unknown constant, \( U \in SU(3) \) is the chiral field, \( M \) is the three-flavor mass matrix and the ellipsis ignored in the standard treatments stands for higher dimension terms involving derivatives and mass matrices etc. The skyrmion given by the Lagrangian\textsuperscript{11}.
is then quantized by what is called “rigid rotor” method, that is, collective-quantized with all flavors put on the same footing.

In describing baryons as chiral solitons, regardless of whether it involves only chiral fields or a hybrid of chiral fields and quark fields, there are two immediate issues in this approach. One is how good the chiral Lagrangian used is as an effective theory of QCD from which baryons are built and the other is how reliable the quantization of the soliton is. Some of these issues are discussed in the monograph [6]. Let me address the first of these issues in this subsection relegating the second to the next subsection.

The Skyrme model is a description of baryons valid in the $N_c$ limit, so we wish to have an effective Lagrangian that is close to QCD in that limit. Let us look at (1). The first term of (1) is the term that encodes the known current algebra, so it is highly trustful lowest-dimension term in effective field theory. However the second term, so-called Skyrme term, is one of several quartic terms that can be written down on the basis of general dimensional and symmetry arguments. Given our almost complete ignorance of QCD at low-energy nonperturbative regime, however, it has not been feasible to “derive” directly from QCD terms other than the current algebra term. It is plausible that a quartic term of the Skyrme-type arises from heavier meson degrees of freedom of a given quantum number when one integrates out all higher energy degrees of freedom other than the pseudo-Goldstone bosons but there is no reason why other vector mesons such as $a_1$, ... and heavy scalar mesons etc. should not give rise to similar – but different forms of – four-derivative terms. In fact there is no theoretical reason whatsoever why one would wind up with only one term like the Skyrme term if one is “deriving” them from QCD per se. As I will mention below, an unexpected support for this Skyrme-type term comes from string theory: Using “AdS/QCD holographic duality,” QCD can be expressed as a compactified theory of five-dimensional Yang-Mills gauge theory \[ \mathcal{N} \] from which at low energy the Skyrme term arises with a coefficient fixed in terms of the string variables in addition to a tower of vector mesons populating up to a given cutoff scale. This term is there whether or not one takes explicitly into account the low-lying vector mesons $\rho, \omega, a_1$, etc. Given by the bulk sector in addition to the tower of vector gauge fields, it must represent shorter distance scales than the tower of vector mesons.\footnote{\textsuperscript{2}}

\footnote{\textsuperscript{2} The presence of the short-distance Skyrme term in addition to the tower of vector mesons tells us that the Skyrme term should be present in models where the lowest members of the tower, $\rho$ and/or $\omega$, are used to stabilize the soliton as in [6]. This short-distance term must be the Skyrme term that figures in the description of the monopole catalysis of the nucleon decay given in terms of the unwinding of the skyrmion [11]. It would also resolve the problem of the unrealistic $\omega$ repulsion in dense medium seen in [12], where an $\omega$ with a finite mass was used for stabilization.}

In most of the work on skyrmions (with a few exceptions), the vector meson degrees of freedom have been ignored or integrated out. In [12], the role of vector mesons in the hyperon structure was first investigated and it was noted that while the explicit account in the skyrmion structure of the lowest-lying nonet vector mesons definitely improves the spectroscopy for the $S < 0$ baryons but there was no qualitative influence. It will turn out however that the vector mesons in hidden local gauge fields can make a drastic effect on $S > 0$ baryons, i.e., pentaquarks. This will be seen to be closely linked to hidden local symmetry.

It has been debated in the literature as to whether the skyrmion description of pentaquark baryons can be taken seriously as modelling QCD. It has even been suggested that it can be invalidated by experiments. I would think that this debate will be settled if indeed the AdS/QCD duality is correct. As mentioned, the duality means that the skyrmion description for baryons is a holographic dual to the quark descriptions, so they describe the same physics. If there is any difference, it will only be in the approximations one makes in doing the calculations, not in the principle. A similar point of view is voiced by Diakonov \[ \textsuperscript{13} \] from a different starting point.

B. Large $N_c$

The next issue in the skyrmion description of the $\Theta^+$ has to do with how one quantizes the skyrmion in consistency with the $N_c$ counting. The $N_c$ dependence of the baryon mass can be written as

$$ M = AN_c + B + C/N_c + \cdots $$

where $A$, $B$, $C$ ... are $N_c$-independent coefficients. The $A$ term is the classical soliton mass common to all baryons and the $C$ term is the rotational contribution that comes from collective quantization that splits a state with spin $J = 3/2, \ldots, N_c/3$ from the ground state with $J = 1/2$. The constant $B$ term contains several contributions among which the Casimir energy is the most important one. The Casimir calculation in baryon structure turns out to be horrendously difficult. There have been several courageous attempts (see for reference, \[ \textsuperscript{14} \]), but no reliable result has been obtained yet. In most of the the skyrmion work available in the literature, one simply ignores the Casimir energy and uses \[ \textsuperscript{15} \] taking the constants $f$ and $\epsilon$ as arbitrary parameters fit to a set of low-energy data. As argued in \[ \textsuperscript{14} \], this is not the right way of doing things. An indication that this procedure cannot be reliable is that the fit value of $f$ is drastically different from what it is in nature, viz, the pion decay constant $f_\pi \approx 93$ MeV. Although this fitting procedure may make sense in some cases, it cannot be applied generally since the $O(1)$ terms can in general depend on the quantum numbers involved (e.g., strangeness). Furthermore the Casimir calculations performed so far indicate that the $O(1)$ term can be very big, though subleading in
mass splitting between the multiplets in the non-exotic
use the physical
take properly into account the Casimir energy and then
coefficients
baryons as well as to the exotic baryons with the co-
formula (2) for the exotic channel is inconsistent with the
authors [14, 15] that the rigid rotor expression for the mass
the baryons. It has been pointed out by several au-
tribution, the splitting from the lowest (non-exotic) mul-
multiplet in the exotic channel has a term \( \sim N_c/\Phi \sim O(1) \)
(where \( \Phi \) is a moment of inertia in the strangeness
direction) originating from the rigid rotation. But the rigid
rotor method is believed to miss out some potentially
important terms of the same order. For instance, in
addition to the Casimir energy, there can be possible
“vibrational modes” in the strangeness direction (as in
the bound-state quantization, see below) that are not
accounted for. This inconsistency in \( O(1) \) has been in-
voked [13] to cast doubt on the prediction despite the
amazing agreement with the experiment.

An alternative approach discussed by the authors in
[14, 15] avoids this inconsistency in \( N_c \) counting. The method consists of fluctuating in the strangeness +1 di-
rection in the background of an \( SU(2) \) soliton, with the
Wess-Zumino term playing the role of a magnetic field.
The question is: What happens to a \( K^+ \) in the presence
of the soliton? This approach was first developed for the
\( S < 0 \) baryons [16] where \( K^- \)‘s were found to be bound
to the soliton to give the known hyperons. In fact it is
this binding approach that one should resort to when one
is considering heavy-light-quark baryons since that is the
natural way to incorporate heavy-quark symmetry into
a chiral symmetric Lagrangian. Here the kaon can be
thought of as a vibration – which is of \( O(1) \) – as con-
trasted to the rotation – which is typically of \( O(1/N_c) \).
Since the kaon vibrational energy is of the same order
in \( N_c \) as the Casimir energy, in principle both should be
treated on the same footing. Due to the difficulty with
the Casimir calculation, a consistent calculation of this
sort has not been done. I would think that such a cal-
culation will ultimately be needed to understand the \( \Theta^+ \)
structure as I will mention at the end.

The crucial element in the bound state picture is the
Wess-Zumino term responsible for the five pseudoscalar
coupling, \( K^0 \pi^0 \). In the description with [14] supple-
mented with the topological Wess-Zumino term, this cou-
lpling – the strength of which is fixed by topology – is
attractive for the \( S < 0 \) channel accounting for the bind-
ing. However for the \( S > 0 \) channel, the sign change
makes the Wess-Zumino term repulsive. Therefore with
the standard parameters that describe the non-exotic
baryons, there cannot be binding nor resonance with a
small width [14]. It was found that if the kaon mass were
much bigger (i.e., stronger \( SU(3) \) breaking) or the Wess-
Zumino term were smaller by a factor of \( \sim 2 \), a \( K^+ \) could
be quasi-bound or produce a narrow-width resonance.

This model as it stands therefore cannot accommodate
a narrow-width resonance of the \( \Theta^+ \) quantum number.

This result may also augur on the validity of the rigid
rotor description [14]. In the limit that the kaon mass
goes to zero, the rigid rotor zero mode is recovered
smoothly as \( m_K \to 0 \) for the \( S < 0 \) channel. The bound-
state and the rigid rotor models agree in the chiral limit.

When the light-quark vector mesons are included as
relevant degrees of freedom, there can be a dramatic
change in the structure of the \( S > 0 \) baryons although
they leave more or less unaffected the \( S < 0 \) baryons. In
this respect, treating the vector mesons as gauge fields
has a power that is not available in non-gauge invariant
approaches. I shall discuss why local gauge invariance
possesses such power and how such local gauge invar-
iance can arise from first principles.

III. VECTOR MESONS AS HIDDEN GAUGE
FIELDS

When the light-quark vector mesons are included as
relevant degrees of freedom, there can be a dramatic
change in the structure of the \( S > 0 \) baryons although
they leave more or less unaffected the \( S < 0 \) baryons. In
this respect, treating the vector mesons as gauge fields
has a power that is not available in non-gauge invariant
approaches. I shall discuss why local gauge invariance
possesses such power and how such local gauge invar-
iance can arise from first principles.

A. Hidden local symmetry

To bring out a general idea, let me consider, following
Ref. [18], a massive vector field Lagrangian

\[
\mathcal{L} = -\frac{1}{2g_2^2} Tr F_{\mu \nu}^2 + \frac{f^2}{4} Tr A_{\mu \nu}^2 + \cdots \tag{3}
\]

which is a Lagrangian describing a massive vector excit-
tation with the mass

\[
m_{A_2} = fg_2. \tag{4}
\]

This can be gotten by taking \( g_1 = 0 \) from

\[
\mathcal{L} = -\frac{1}{2g_1^2} Tr F_{\mu \nu}^2 + \frac{1}{2g_2^2} Tr F_{\mu \nu}^2 + \frac{f^2}{4} \text{Tr}(A_{\mu \nu}^2)^2 + \cdots. \tag{5}
\]

Being massive, the Lagrangian [8] or [15] has no gauge
symmetry. But introducing a Goldstone scalar field \( U = e^{i\phi/\lambda} \) which transforms as

\[
U \rightarrow g_2^{-1} U g_1 \tag{6}
\]
one can write a gauge invariant-Lagrangian that has the same low-energy physics as \[\mathcal{L} = \frac{1}{2g_2^2} \text{Tr} F_{\mu\nu}^2 - \frac{1}{2g_2^2} \text{Tr} F_{2\mu\nu}^2 + \frac{f^2}{4} \text{Tr} |D\mu U|^2 + \cdots.\] with the covariant derivative defined by \[D\mu U = \partial\mu + iA_1\mu U - iU A_2\mu.\] (8)

This Lagrangian is invariant under the gauge symmetry $SU(n_1) \times SU(n_2)$. To get to \[\mathcal{L} \Rightarrow \sum_{k=1}^{n} \text{Tr} [D\mu \Sigma_k]^2 + \text{Tr} [D\mu \Sigma_t]^2 \]

Such terms are naturally generated when one calculates higher-order terms by doing loop calculations which requires that additional equivalent terms be added to renormalize the loop terms. Without any systematic ways of accounting for higher order terms, it will require an arduous book-keeping to correctly put all terms of the same order. Local gauge symmetry can do this systematically with little effort by replacing \[\mathcal{L} \Rightarrow \sum_{k=1}^{n} \text{Tr} [D\mu \Sigma_k]^2 + \text{Tr} [D\mu \Sigma_t]^2 \]

This allows a systematic account of higher-order terms to any order. This is important in the case where we have not only the Goldstone bosons, e.g., pions, as physical particles but also the vector fields whose mass can be made light as in hot and/or dense matter.\[\mathcal{L} \Rightarrow \sum_{k=1}^{n} \text{Tr} [D\mu \Sigma_k]^2 + \text{Tr} [D\mu \Sigma_t]^2 \]

Another advantage is that with the Goldstone bosons explicitly treated instead of being “eaten up,” the theory of the massive vector bosons remains sensible in the effective theory sense up to the cutoff $\sim 4\pi m_A/g \sim 4\pi f$. Above that scale, a microscopic theory has to take over (that is, be “ultraviolet completed”). In our case, the ultraviolet completion is done to QCD.

**B. Hidden local symmetry à la Harada and Yamawaki**

There is a growing evidence that QCD at low energy can be described as a chiral $SU(N_f) \times SU(N_f)$ symmetric theory with a tower of vector mesons, scalars and Goldstone bosons, all coupled gauge invariantly. The structure I will use here for the pentaquark problem is the one where only the lowest multiplet of the vectors $\rho_\mu$ and pions $\pi$ are taken into account, with scalars and higher towers of the vectors “integrated out.” The Lagrangian will be taken in the form \[\mathcal{L} = \frac{F^2}{4} \text{Tr} \{ |D\mu \Sigma_L|^2 + |D\mu \Sigma_R|^2 + \kappa |D\mu (\Sigma_L \Sigma_R)|^2 \} - \frac{1}{2g_2^2} \text{Tr} [\rho^\mu \rho^{\mu\nu}] + \cdots\] (11)

with $\Sigma_{L/R}$ transforming under chiral $SU(N)_L \times SU(N)_R$ as

\[\Sigma_L \rightarrow L \Sigma_L h^1(x), \quad \Sigma_R \rightarrow h(x) \Sigma_R R^1\] (12)

In an arbitrary gauge, $\Sigma_{L,R}$ can be parameterized as

\[
\Sigma_L \rightarrow e^{i\pi/F_a} e^{\pm i\sigma/F_a}.
\]

With the definition

\[a = (F_\sigma/F_\pi)^2,\]

$\kappa$ is given by \[\kappa = \frac{a - 1}{a + 1}.\] (15)

It is interesting to note that $\kappa = 0$ or $a = 1$ has a special feature. The Lagrangian becomes “local” with no mixing of the L and R chiral fields. One refers to this structure as “theory space locality.” If one preserves this theory space locality, one can then make a chain of gauge fields $A_k$ sandwiched by the “link” fields $\Sigma_k$ and $\Sigma_{k+1}$. This gives what is referred to in the literature as “open moose” diagram with the Lagrangian

\[L = \sum_{j=1}^{K+1} \text{Tr} [D\mu \Sigma_j]^2 - \sum_{j=1}^{K} \frac{1}{4g_2^2} \text{Tr} [F^\mu_{\nu\rho} F^\nu_{\rho\sigma}] + \cdots\] (16)

Here we have replaced the “pion decay constant” $F_\pi$ by a generalized constant $f_j$ corresponding to the $j$-th link field and $\rho_\mu$ by $F^\mu_{\nu\rho}$ to accommodate $K \geq 1$ gauge fields. The covariant derivatives are of the form \[D\mu \Sigma_1 = \partial\mu \Sigma_1 + i\Sigma_1 A^\mu_1, \quad D\mu \Sigma_{K+1} = \partial\mu \Sigma_{K+1} - iA^\mu_K \Sigma_{K+1} \]

and \[\partial\mu \Sigma_k - iA^\mu_k \Sigma_k \]

with $1 < k < K$. If we let $K \rightarrow \infty$, then \[\mathcal{L} \Rightarrow \sum_{j=1}^{K+1} \text{Tr} [D\mu \Sigma_j]^2 - \sum_{j=1}^{K} \frac{1}{4g_2^2} \text{Tr} [F^\mu_{\nu\rho} F^\nu_{\rho\sigma}] + \cdots\] (16)

3 I am using a different notation from that of Harada and Yamawaki to generalize the Lagrangian as discussed below. One can identify $\Sigma_L = \xi^1_L$ and $\Sigma_R = \xi^1_R$. Note that I use here $\kappa$ in place of $a$, with $\kappa = 0$ corresponding to $a = 1$ which will play a prominent role in what follows. In what follows, $a$ will be used instead of $\kappa$. 

\[\xi^1_L \Rightarrow \xi^1_L, \quad \xi^1_R \Rightarrow \xi^1_R \]
1. AdS/QCD

In 23, Erlich et al interpret the dimensionally deconstructed five-dimensional gauge theory implemented with certain scalar fields as the bulk theory in the (5-D) AdS space which via holography correspond to low-energy QCD in four dimensions on the boundary. They obtain, using the strategy of AdS/CFT duality, quite a reasonable low-energy hadron dynamics with three bulk parameters fit to the pion and $\rho$ masses and the pion decay constant. As already mentioned above and reported in this meeting by Sugimoto, Sakai and Sugimoto 7, they go the other way. They insert $N_f$ pairs of flavor D8 branes as probes into the AdS sector with a background of $N_c$ color D4 branes and assuming $N_c \gg N_f$, obtain five-dimensional gauge theory which when compactified to four dimensions, gives HLS theory with a tower of vector mesons, with the parameters of the theory fixed by the bulk parameters. The results agree fairly well with experiments. I cannot give a full justice to this remarkable feat as the calculation requires involved arguments in a language not entirely familiar to me. But what is most noteworthy is that the scheme gives in four dimensions precisely the hidden local symmetry structure of Bando et al. 20 and Harada/Yamawaki 21 including the gauged Wess-Zumino term and furthermore fixes the Skyrme quartic term uniquely in terms of the bulk parameters. An important consequence of this is that baryons must emerge in AdS/QCD as skyrmions in the hidden local symmetry theory.

What appears to be potentially important for the pentaquark structure I will describe below is that the Sakai-Sugimoto model predicts what corresponds to a, Eq. (14), in the limit of large $N_c$,

$$a \approx 1.3.$$  

(17)

It is close to what was obtained at the matching point by Harada and Yamawaki in their best-fit analysis 21 and is closer to 1 than to 2 required by the vector dominance as will be further elaborated below.

C. The vector manifestation

Little is known of the theories with $K > 1$ in 10 although there are some developments that indicate that the dimensionally deconstructed theory is close to nature 25. I shall therefore focus on the Harada-Yamawaki HLS theory 11 which has been extensively analyzed and fairly well understood 21. We will be dealing with the theory that should work up to the cutoff given by $\Lambda_M \sim 4\pi m_\rho / g \sim 4\pi f_\pi \sim 1$ GeV; we can think of having integrated out all except the pions and the nonet vector mesons (assuming flavor $U(N_f)$ symmetry) generically denoted by $\rho_{ij}$. The axial vectors are not considered explicitly.

The basic premise in HY theory is that the strong interaction dynamics is governed by the hadronic degrees of freedom picked for consideration, i.e., the pions and vector mesons, below the cutoff $\Lambda_M$ and by the QCD degrees of freedom, quarks and gluons, above the cutoff. It is assumed that there is a region in between, say, $\Delta$, in which the two regimes overlap. It is not obvious that such a region can be picked reliably without ambiguity but this is the best one can do if the effective theory is to be operative up to around $\Lambda_M$ beyond which QCD has to take over. HY do this by matching at $\Lambda_M \sim 1.1$ GeV the two descriptions in terms of the vector and axial correlators defined by

$$i \int d^4xe^{ipx}\langle 0 | T J^a_{\mu}(x) J^b_{\nu}(0) | 0 \rangle = \delta^{ab} (\rho_{\mu\nu} - g_{\mu\nu} p^2) \Pi_A(p^2),$$  

(18)

and

$$i \int d^4xe^{iqx}\langle 0 | T J^a_{\mu}(x) J^b_{\nu}(0) | 0 \rangle = \delta^{ab} (\rho_{\mu\nu} - g_{\mu\nu} p^2) \Pi_V(p^2),$$  

(18)

where $Q^2 = -p^2$. In HLS these two-point functions at the matching point $\Lambda_M$ are completely described by tree contributions with $O(p^0)$ terms included when the momentum is around the matching scale, $Q^2 \sim \Lambda_M^2$. In the QCD sector they are given by OPE expressed in terms of the running (color) gauge coupling $\alpha_s$, the quark condensate $\langle \bar{q}q \rangle$, the gluon condensate $\langle G_{\mu\nu}^a \rangle$ and combinations thereof at the given scale $\Lambda_M$. The matching allows the parameters $F_\pi$, $F_\rho$ or $a$ and the hidden gauge coupling $g$ to be expressed in terms of the QCD quantities that are in principle calculable at a given scale. This completely defines the bare Lagrangian for HLS theory. In this theory, the vector mesons can be treated as “light” in the chiral counting scheme. Thanks to gauge invariance, in the large $N_c$ limit at which the quantity $m_\rho/4\pi F_\pi$ with a fixed vector meson mass is suppressed, higher orders in chiral perturbation expansion can be systematically formulated, an advantage which is not shared by non-gauge invariant or gauge fixed approaches.

The most important outcome of the systematic analysis of the theory at loop orders 21 is that there is one unique fixed point which is consistent with QCD. In general there are multitudes of fixed points at one-loop order. However constraining the RGE group flow to QCD, the unique fixed point turns out to be at $g = 0$, $a = 1$ and $f_\pi = 0$. This is called the “vector manifestation (VM) fixed point.” Whenever appropriate, I shall refer to the HLS theory with the vector manifestation fixed point as HLS/VM. This fixed point is obtained in the chiral limit but is believed to be relevant and useful even when quark masses are present. I shall therefore talk about the VM fixed without regard to the explicit chiral symmetry breaking.

The VM fixed point is reached when (in the chiral limit) chiral symmetry changes from Goldstone mode to Wigner mode or vice versa. It does not depend upon how the phase change is driven. It could be by temperature or density or the number of flavors. The VM implies that
as the hadronic system approaches the critical point $\mathcal{P}$,

$$a \to 1, \quad g \to 0, \quad f_\pi \to 0$$

(19)

so that the hidden gauge bosons become massless

$$m_\rho \sim \sqrt{agf_\pi} \to 0.$$  \hspace{1cm} (20)

Here $m_\rho$ is the pole mass but the parametric mass also goes to zero at that point. Matching with QCD allows one to relate the scaling of the parameters to the scaling of QCD variables. Thus as one approaches $\mathcal{P}$,

$$m_\rho \sim g \sim (\bar{q}q) \to 0.$$  \hspace{1cm} (21)

Applied to $N_f = 2$, this implies that near $\mathcal{P}$, all light-quark hadron masses other than the Goldstone bosons should scale as

$$M^*/M \sim ((\bar{q}q)^*/(\bar{q}q))^n$$  \hspace{1cm} (22)

where the $*$ stands for in-medium - temperature or density - and $n > 0$ a power that can depend upon the detail content of the theory. One can think of baryons as skyrmions in the theory or one can introduce quasi-quarks (or constituent quarks) considered to be relevant near the critical point and express the scaling in terms of that of the quasiquarks. This scaling is precisely what was proposed in 1991 \[26\].

IV. HLS/VM AND PENTAQUARK STRUCTURE

A. Evidence for $a \approx 1$ in nature

Even though a hadronic system is not at the VM fixed point, with $g \neq 0$ and $f_\pi \neq 0$, it appears to be a good approximation to set

$$a \approx 1$$

(23)

and make corrections in powers of $\delta = (a - 1)$. Let me list a few cases known up to now where this holds.

1. $\Delta m_\pi^2 = m_{\pi^+}^2 - m_{\pi^0}^2$:

In computing the pion mass difference in HY’s HLS/VM \[24\], one gauges the $U(1)$ subgroup of the chiral symmetry in HLS to introduce the EM field and takes into account the mixing of the $U(1)$ gauge field and $\rho^0$ of the $SU(2)$ hidden gauge field. Then the one-loop EM contribution to the mass difference comes out to be

$$\Delta m_\pi^2 |_{\text{loop}} = \frac{\alpha_{em}}{4\pi} \left[(1 - a)\Lambda^2 + 3aM_\rho^2 \ln \Lambda^2 + \cdots \right].$$  \hspace{1cm} (24)

Here $\Lambda$ is the cutoff introduced to regularize the divergent integral. In HLS/VM, it is identified as the matching scale $\Lambda = \Lambda_M$. Beyond $\Lambda_M$, the theory has to be ultraviolet completed and this is done by matching to QCD via the current correlators. The result is \[27\]

$$\Delta m_\pi^2 |_{\Lambda_M} = \frac{8\alpha_{em}\alpha_s(\bar{q}q)^2}{3F_\pi^2(0)\Lambda_M^2}.$$  \hspace{1cm} (25)

Assume that the number of flavors $N_f$ is sufficiently “large” so that we can consider $a$ to be near its fixed point, $a = 1$. Then ignoring higher orders in $\delta$, we can set $a = 1$ in \[24\] and get the total splitting as

$$\Delta m_\pi^2 = (3\alpha_{em}/4\pi)M_\rho^2 \ln(\Lambda_M^2/m_\rho^2) + \Delta m_\pi^2 |_{\Lambda_M}.$$  \hspace{1cm} (26)

The result $\Delta m_\pi^2|_{\text{HLS/VM}} = 1223 \pm 263$ MeV$^2$ obtained using the value $a = 1$ and other parameters fixed in a best-fit to hadronic structure \[21\] is in good agreement with the experimental value 1261 MeV$^2$.

An interesting observation here is that the $a \approx 1$ leads to the theory space locality eliminating the “quadratic divergence” in \[24\]. This cancelation which is effectuated in the standard current algebra approach with the Weinberg sum rules with $\pi$, $\rho$ and $a_1$ is the analog to what happens in the little Higgs phenomenon in the hierarchy resolution in the Standard Model.

If instead of fixing $a = 1$ one uses Harada-Yamawaki’s global-fit value $a \approx 1.3$, one obtains a somewhat less good but still satisfactory agreement with the experiment for the pion mass splitting.

2. The chiral doubling of heavy-light hadrons:

Another recent indication that $a$ is close to 1 in nature is given by the splitting between the chiral doublers of heavy-light mesons $H \equiv (Q\bar{q})$ of open heavy quark $Q = c, b, \ldots$ and light antiquark $\bar{q} = \bar{u}, \bar{d}, \bar{s}$. It was predicted in 1992 on the basis of sigma models combining chiral symmetry and heavy quark symmetry and published in 1993 \[28\]. This prediction was confirmed recently by the BarBar and CLEOII collaborations \[30\], \[31\]. One unsatisfactory feature of the old prediction was that it was given in terms of the constituent quark mass which is not well defined physically. This defect can be lifted in an approach based on HLS/VM \[32\].

The basic assumption made in \[32\] is that the VM is a good starting point as far as chiral symmetry is concerned. So one starts with HLS with the VM fixed point parameters and then makes corrections for deviation from the VM as one departs from the
3. Electromagnetic form factors of the proton

Although \( a \) at the matching scale is near 1, quantum corrections (i.e., \( 1/N_c \) corrections) in the mesonic sector bring it to \( a \approx 2 \) at the scale corresponding to the pion on-shell. \( a = 2 \) gives rise to the vector dominance for the pionic EM form factor. Now the situation with the nucleon is a different matter. There is no systematic study of quantum \( (1/N_c) \) corrections to \( a \) for the baryon sector, so we have no theoretical information on how good the VD is for nucleons or baryons in general. But there are empirical evidences that the VD with \( a = 2 \) breaks down for the nucleon form factors. In fact there is a strong indication that for the nucleons, \( a \approx 1 \) is a good approximation, as suggested a long time ago \cite{33,34} based on phenomenological considerations and confirmed by new data from JLab.

How the violation of VD in the nucleon form factors comes about was first explained years ago in terms of a chiral bag model \cite{6,35}. In chiral bag model, the electromagnetic form factors have two sources: One is the contribution from the weakly interacting free quarks confined within a bag and the other is from the pion cloud outside of the bag, with the two regions connected by chiral-invariant boundary conditions. The partition into two regions is characterized by the chiral angle \( \theta \). At the “magic angle” \( \theta = \pi/2 \), one has one unit of baryon charge partitioned equally inside and outside of the bag. The outside pion cloud is assumed to respond to the electromagnetic field via the vector \((\rho,\omega)\) mesons in accord with VD whereas the quarks inside are to respond as a compact core of size \( \lesssim 0.3 \text{ fm} \), i.e., the bag at the magic angle. This description can be translated into the modern version of effective field theory by constructing baryons as skyrmions in HLS \cite{36}. In HLS, the photon will couple to the pion cloud weighed by \( a/2 \) and to the soliton core by \((1-a/2)\).

The outside pion cloud is assumed to respond to the electromagnetic field via the vector \((\rho,\omega)\) mesons in accord with VD whereas the quarks inside are to respond as a compact core of size \( \lesssim 0.3 \text{ fm} \), i.e., the bag at the magic angle. This description can be translated into the modern version of effective field theory by constructing baryons as skyrmions in HLS \cite{36}. In HLS, the photon will couple to the pion cloud weighed by \( a/2 \) and to the soliton core by \((1-a/2)\).

\[
\delta\mathcal{L} = -2eag_{em}\mathcal{A}_{\mu}^\mu \text{Tr}[\rho_\mu Q] + 2ie(1-a/2)A_{\mu}^\mu \text{Tr}[V_\mu Q]
\]
repulsion between $K$ and $K^*$. A much subtler effect is that the presence of the “light” mass vector mesons, $\omega$ and $K^*$, brings in a solitonic matter additional terms that behave like the Wess-Zumino term in the coupling of $K^2\pi^3$. This may be stated as saying that the “effective magnetic field” played by the Wess-Zumino term is modified. This is illustrated in Fig. 1. Now with hidden

![Diagram of WZ-like terms](image)

**FIG. 1:** Schematic illustration of the WZ-like terms due to vector mesons in the Lagrangian. The first figure corresponds to the topological WZ term and the other two WZ-like terms generated by the vector mesons $\omega$ and $K^*$. The baryon number density is given by the three pion lines connected to the shaded area which represents the soliton.

local symmetry, one can show \[10\] that the vector meson mediated graphs cancel each other in the limit of the infinite vector meson mass, leaving the topological WZ term which is effectively repulsive for the $K^+$ channel. However, for finite vector meson masses, there are non-local contributions to the total WZ-like term which depending upon the value of $a$, can be effectively repulsive or attractive in conjunction with the $K^*-K$ level repulsion.

A full spectroscopy of the pentaquark states requires collective quantization and the proper account of all $O(1)$ terms and $1/N_c$ corrections. Such a comprehensive calculation has not been done yet. However to see qualitatively what happens to a $K^+$ in the background of the soliton to order $O(N_c^0)$, it suffices to study the coupled equations of motion for the $K$ and $K^*$ interacting with the soliton. For this purpose, we need to take the large $N_c$ limit of the HLS Lagrangian given by the bare Lagrangian obtained at $\Lambda_M$ by the Wilsonian matching. As noted, the bare HLS Lagrangian obtained in a best fit by HY \[21\] gives $a \approx 1.3$ (and other parameters I will not quote). For this Lagrangian, the equations of motion give the $K^+$ bound by $\sim 5$ MeV. If one instead fixes $a = 1$ at the matching point and do the best fit for other two parameters, a procedure which was found in \[21\] as acceptable as the “best fit,” the resulting results are in equally good agreement with experiments. The $K^+$ is also bound in this case, with the binding energy $\sim 3$ MeV. Thus it can be concluded that for $a \lesssim 1.3$, the $K^+$ is found to be definitely bound although by a small amount. \[14\]. To have a more general and qualitative idea as to what is happening, the “parameter” $a$ is arbitrarily varied with all other parameters fixed to physical (on-shell) values. This procedure is not quite justified for a close to 1 since the parameters are correlated by the consistency of the theory but it gives some idea as to how the $K^+$-soliton depends on $a$ for $a > 1$. The result \[17\] is shown in Fig. 2. What is given is the eigenenergy of the $K^+$ in the soliton background giving the system with $S = \mp 1$, $J^P = \frac{1}{2}^\pm$, $\bar{\Theta}^0$. When the energy is less than the free kaon mass $m_K = 495$ MeV, the system is bound. When $a$ goes to infinity, that goes over to the case studied in \[14\], namely, pseudoscalar-only theory.

We see that the $S = -1$ states are more or less insensitive to $a$ for a wide range of $a$ \[5\] but the $S = +1$ state is strongly sensitive, with the state bound for $a \lesssim 1.3$ and unbound above. The widths of the unbound states that show up as resonances (for $a \gtrsim 1.4$) are given by the error bars. The error bars explode as the eigenenergy increases \[6\].

**FIG. 2:** The eigenenergies of $S = \pm 1$, $J^P = \frac{1}{2}^\pm, \frac{3}{2}^-$ states obtained for various $a$ values. The width of the resonance in the $K^+$-skyrmion channel is given by the error bar.

C. Confronting nature

In order to confront nature, one would have to calculate the Casimir contribution to the masses for the $\Theta^+$ as well as the nucleon and quantize them including terms of $O(1/N_c)$. The coupling of the quantum $\Theta^+$ to the

---

4 I would like to acknowledge valuable help from Masayasu Harada and Dong-Pil Min for these numerical estimates.

5 This explains why Scoccola et al \[12\] found the effect of the vector mesons in the hyperon ($S < 0$) spectroscopy to be undramatic. They failed to look at the $S > 0$ channel where the effect is a lot more dramatic.

6 For instance, at the observed mass difference between the $\Theta^+$ and the proton, say, $\sim 100$ MeV, corresponding to $a \approx 2$ for which the vector dominance holds, the width can be $\sim 150$ MeV, clearly too big for the “observed” $\Theta^+$. 
proton-kaon continuum will be at $O(1/N_c)$. In the absence of these quantities, I can only make a few guesses. What is known is that if naively collective-quantized with the Casimir term ignored, the $\Theta^+$ state (near $a = 1$) has a mass $m_{\Theta^+}$ lower than the $K\bar{N}$ threshold $m_p + m_K$. The crucial question then is: What happens when other corrections of $O(1)$ as well as $O(1/N_c)$ are included? Here are a couple of possibilities that one can entertain [17, 41]:

- One possibility is that after the account of the corrections mentioned above, a genuine bound state of the quantum numbers of $\Theta^+$ hitherto unobserved experimentally remains below the $K\bar{N}$ threshold. In addition, there can be a bound $K^*$-soliton state in the $K\bar{N}$ continuum weakly coupled to the latter. This could be identified as the observed narrow $\Theta^+$.

- Another possibility is that higher-order $1/N_c$ corrections (starting with $O(1)$) are more attractive for the nucleon than for the $\Theta^+$ such that the bound “closed” channel state goes into the $K\bar{N}$ continuum. In this case, the observed peak could be generated as a “Feshbach resonance” [12] with a small width due to the weak $(O(1/N_c)) \Theta^+K\bar{N}$ coupling. A possible mechanism is that the baryonic matter of the soliton that dials $a$ which in turn controls the WZ-like term could play, in the presence of $K^*$, the role of a magnetic field that makes the coupled systems to sweep across the level-crossing point (involving the bound $K^*$-soliton and the scattering $K\bar{N}$ levels), thereby producing a resonance in analogy to resonances in ultracold atoms driven by magnetic field [42].

I have not made any attempt to compare the bound-state formulation based on HLS to correlated quark models [14]. A meaningful comparison can be made only when a comprehensive analysis is made of the spectroscopy of all pentaquark states predicted by the present theory. What one can say is that when a realistic effective HLS Lagrangian is obtained from the AdS/QCD duality, then the skyrmion description therefrom should be dual to the quark model description. Then the debate as to which picture is correct will be moot. This issue will be sharpened if and when the $\Theta^+$ state is confirmed experimentally.

Acknowledgments

I am grateful to Masayasu Harada and Kichi Yamawaki, the organizers of this DSB05 meeting, for inviting me to the meeting and for numerous discussions on the topic treated in this talk. Part of the work discussed here was done when I was spending a year 2004 at the Physics Department of Hanyang University. I would particularly like to thank Professor Chong Yang Kim, the President of Hanyang University, for the generous support that enabled me to initiate activities in my area of research in Korea.

[1] For a historical review and present situation up to October 2004, see M. Praszalowicz in http://newton.kias.re.kr/KH04/ For a more recent update in experiments, see K. Hicks, hep-ex/0412048.
[2] D. Diakonov, V. Petrov and M.V. Polyakov, “Exotic anti-decuplet of baryons: prediction from chiral solitons,” Z. Phys. A 359 (1997) 305.
[3] T. Nakano et al [LEPS Collaboration], Phys. Rev. Lett. 91 (2003) 012002.
[4] E.g., K. Hicks, “An experimental review of the $\Theta^+$ pentaquark,” hep-ex/0412048.
[5] A.V. Manohar, “Equivalence of the chiral soliton and quark models in large $N_c$,” Nucl. Phys. B 248 (1984) 19; E. Jenkins and A.V. Manohar, “Baryon exotics in the quark model, the Skyrme model and QCD,” Phys.Rev.Lett. 93 (2004) 022001.
[6] See, e.g., M.A. Nowak, M. Rho and I. Zahed, Chiral Nuclear Dynamics (World Scientific, Singapore, 1996); M. Rho, “The Chesire Cat hadrons,” Phys.Rept. 240 (1994), “The Chesire Cat hadrons revisited,” hep-ph/0206003.
[7] T. Sakai and S. Sugimoto, “Low energy hadron physics in holographic QCD,” hep-th/0412141.
[8] C.T. Hill and C.K. Zachos, “Dimensional deconstruction and Wess-Zumino-Witten terms,” hep-th/0411157.
[9] G.S. Adkins and C. Nappi, “Stabilization of chiral solitons via vector mesons,” Phys. Lett. B 137 (1984) 251.
[10] B.-Y. Park, M. Rho and V. Vento, “Vector mesons and dense skyrmion matter,” Nucl.Phys. A 736 (2004) 129.
[11] Y. Brihaye, V.A. Rubakov, D.H. Chakravian and F. Zimmerchierd, “A simplified model for monopole catalysis of nucleon decay,” Theor. Math. Phys. 128 (2001) 1140, Teor. Mat. Fiz. 128 (2001) 361.
[12] N.N. Scoccola, D.-P. Min, H. Nadeau and M. Rho, “The strangeness problem: An SU(3) skyrmion with vector mesons,” Nucl. Phys. A 505 (1989) 497.
[13] D. Diakonov, “Quark structure of chiral solitons,” hep-ph/0408219.
[14] N. Itzhaki, I.R. Klebanov, P. Ouyang and L. Rastelli, “Is $\Theta^+$ (1540) a kaon-skyrmion resonance?” Nucl.Phys. B 684 (2004) 264.
[15] T.D. Cohen, “$\Theta^+$ baryon in soliton models: Large $N_c$ in QCD and the validity of rigid rotor quantization,” Phys.Rev. D 70 (2004) 041011; A. Cherman, T.D. Cohen and A. Nelor, “The quantization of exotic states in SU(3) soliton models: A solvable quantum mecahnical analog,” Phys.Rev. D 70 (2004) 096003.
[16] C.G. Callan and I.R. Klebanov, “Bound state approach to strangeness in the Skyrme model,” Nucl. Phys. B 262 (1985) 365.
[17] B.-Y. Park, M. Rho and D.-P. Min, “Kaon-soliton bound-state approach to the pentaquark states,” Phys.Rev. D 70 (2004) 114026.
[18] N. Arkani-Hamed, H. Georgi and M.D. Schwartz, “Effective field theory for massive gravitons and gravity in theory space,” Annals Phys. 305 (2003) 96.

[19] G. E. Brown and M. Rho, “Chiral restoration in hot and/or dense matter,” Phys.Rept. 269 (1990) 333; “On the manifestation of chiral symmetry in nuclei and dense nuclear matter,” Phys.Rept. 363 (2002) 85.

[20] M. Bando, T. Kugo and K. Yamawaki, “Nonlinear realization and hidden local symmetries,” Phys. Rep. 164 (1988) 217.

[21] M. Harada and K. Yamawaki, “Hidden local symmetry at loop: A new perspective of composite gauge bosons and chiral phase transition,” Phys. Rep. 381 (2003) 1.

[22] N. Arkani-Hamed, A.C. Cohen and H. Georgi, “(De)Constructing dimensions,” Phys. Rev. Lett. 86 (2001) 4757.

[23] D.T. Son and M.A. Stephanov, “QCD and dimensional deconstruction,” Phys. Rev. D69 (2004) 065020

[24] C.T. Hill, S. Pokorski and J. Wang, “Gauge invariant Lagrangian for Kaluza-Klein modes,” Phys. Rev. D64 (2001) 105005.

[25] J. Erlich, E. Katz, D.T. Son and M.A. Stephanov, “QCD and dimensional deconstruction,” Phys. Rev. Lett. 87 (2001) 152001.

[26] M. Harada and K. Yamawaki, “π^+ − π^0 mass difference in the hidden local symmetry: a dynamical origin of little Higgs,” Phys. Lett. B 568 (2003) 103.

[27] M.A. Nowak, M. Rho and I. Zahed, “Chiral effective action with heavy quark symmetry,” Phys.Rev. D48 (1993) 4370; “Chiral doubling of heavy light hadrons: BaBar 2317-MeV/c^2 and CLEO 2463-MeV/c^2 discoveries,” Acta Phys.Polon. B 35 (2004) 2377.

[28] W.A. Bardeen and C.T. Hill, “Chiral dynamics and heavy quark symmetry in a solvable toy field theoretic model,” Phys.Rev. D 49 (1994) 409425; W.A. Bardeen, E.J. Eichten and C.T. Hill, “Chiral multiplets of heavy-light mesons,” Phys.Rev. D 68 (2003) 054024.

[29] B. Aubert et al. [BABAR Collaboration], “Observation of a narrow meson decaying to D^+_s π^0 at a mass of 2.32-Gev/c^2,” Phys. Rev. Lett. 90 (2003) 242001.

[30] D. Besson et al. [CLEO Collaboration], “Observation of a narrow resonance of mass 2.46-GeV/c^2 decaying to D_s^+ π^0 and confirmation of the D_{sj}(2317) state,” Phys. Rev. D 68 (2003) 032002.

[31] M. Harada, M. Rho and C. Sasaki. “Chiral doubling of heavy-light hadrons and the vector manifestation of hidden local symmetry,” Phys. Rev. D 70 (2004) 074002.

[32] F. Iachello, A.D. Jackson and A. Lande, “Semiphenomenological fits to nucleon electromagnetic form-factors,” Phys. Lett. B 43 (1973) 191.

[33] G. E. Brown, M. Rho and W. Weise, “Phenomenological delineation of the quark-gluon structure from nucleon electromagnetic form factors,” Nucl.Phys. A454 (1986) 669.

[34] G. E. Brown, M. Rho and W. Weise, “Chiral effective action with heavy quark symmetry,” Nucl.Phys. A489 (1988) 612.

[35] M. Rho, A.S. Goldhaber and G.E. Brown, “Topological soliton bag model for baryons,” Phys.Rev.Lett. 51 (1983) 747.

[36] M. Rho, A.S. Goldhaber and G.E. Brown, “Topological soliton bag model for baryons,” Phys.Rev.Lett. 51 (1983) 747.

[37] G.E. Brown and M. Rho, “Double decimation and sliding vacua in the nuclear many-body system,” Phys.Rept. 396 (2004) 2004.

[38] R.L. Jaffe and F. Wilczek, “Diquarks and exotic spectroscopy,” Phys.Rev.Lett. 91 (2003) 232003; M. Karliner and H.J. Lipkin, “A diquark-triquark model for the KN pentaquark,” Phys.Lett. B 575 (2003) 249.