Visualization of topological objects in QCD *
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Recently evidence appeared that instantons and monopoles have a certain local correlation in four-dimensional pure $SU(2)$ and $SU(3)$ gauge theory. We visualize several specific gauge field configurations and show directly that there is an enhanced probability for finding maximally projected abelian monopole loops in the vicinity of instantons. This feature is independent of the topological charge definition used.

1. Introduction and Theory. Two different kinds of topological objects seem to be important for the confinement mechanism: color magnetic monopoles and instantons. Color magnetic monopoles play the essential role in the dual superconductor hypothesis. Instantons are supposed to cause confinement in QCD if they form a so-called instanton liquid. In a series of papers we have presented evidence that there exist local correlations between these topological objects on the basis of gauge field averages [1]. In this contribution we present a graphical approach to this subject by visualization of specific $SU(2)$ gluon configurations.

In order to investigate monopole currents one has to project $SU(2)$ onto its abelian degrees of freedom, such that an abelian $U(1)$ theory remains [2]. This can be achieved by various gauge fixing procedures. We employ the so-called maximum abelian gauge which is most favorable for our purposes. For the definition of the monopole currents $m(x, \mu)$ we use the standard method [3]. The magnetic currents form closed loops on the dual lattice as a consequence of monopole current conservation. From the monopole currents we define the local monopole density as $\rho(x) = \frac{1}{4V^{\mu}} \sum_{\mu} |m(x, \mu)|$. For the implementation of the topological charge on the lattice we use both the field theoretic plaquette and hypercube definitions [4] and a geometric definition suggested by Lüscher [5]. All types of topological charges employed are locally gauge invariant in contrary to the monopole currents.

To identify the instantons as classical solutions of the equations of motion on the lattice, we cool the gauge fields. The cooling procedure systematically reduces quantum fluctuations and suppresses differences between the different definitions of the topological charge. In our investigation we have used the so-called “Cabbibo-Marinari method”.

2. Results. Our configurations were produced on a $12^4 \times 4$ lattice with periodic boundary conditions using the Metropolis algorithm. The topological observables were studied in pure $SU(2)$ in the confinement phase at inverse gauge coupling $\beta = 4/g^2 = 2.25$, employing the Wilson plaquette action for the gluons. Each of these configurations was first cooled and then subjected to 300 gauge fixing steps enforcing the maximum abelian gauge. Altogether 100 independent configurations have been produced.

For the purpose of displaying instantons it is necessary to estimate their average size. We therefore computed the auto-correlation function of the topological charge density. In Fig. 1 we present the results for the hypercube and the Lüscher method after 20 cooling sweeps. The solid curves are fits obtained from a convolution with a geometric definition suggested by Lüscher [5]. All types of topological charges employed are locally gauge invariant in contrary to the monopole currents.

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Figure 1. Auto-correlation functions of the topological charge density after 20 cooling sweeps. The curves represent fits to a convolution of the topological charge density for a single instanton of size $\sigma$. The Lüscher instanton is about half the size of the field theoretic definitions which yield $\sigma = 2.1$.

Figure 2. The location of a single instanton (dots) at constant time which was put on a trivial gauge field configuration is shown. A closed monopole loop (line) runs around the instanton.

Figure 3. Different definitions of the topological charge for a specific gluon configuration after 20 cooling sweeps. The instantons reside at the same places for all definitions and are surrounded by monopoles. In this particular configuration a monopole loop wraps around the torus.
Figure 4. Cooling history for a specific gauge field configuration at a fixed time slice. The dots represent the topological charge distributions in the field theoretic definitions with $|q(x)| > 0.01$. Positive (negative) charges are plotted with a dark (light) color. The monopole loops correspond to red lines. With cooling instantons evolve from noise accompanied by monopole loops in almost all cases. Note that time like monopoles cannot be seen in these plots.
Fig. 2 shows for a fixed time slice the location of a single instanton (dots) which was put on a trivial gauge field configuration artificially. Here a purely spatial monopole loop surrounds the instanton (closed line).

In Fig. 3 we compare the results of the three methods for the topological charge obtained on a single equilibrium gauge field after 20 cooling sweeps. From top to bottom the plaquette, the hypercube, and the Lüscher definition are plotted. The positions of the clusters of topological charge are the same for all three methods. The points represent instantons or antiinstantons. In this particular configuration a monopole is found to wrap around the torus.

Fig. 4 presents a cooling history of a time slice of a gluon field. The topological charge using the plaquette and the hypercube definition is displayed for cooling steps between 0 to 40. For any value of the plaquette (hypercube) charge density \( q(x) > 0.01 \) a light blue (light green) dot was plotted. For \( q(x) < 0.01 \) a dark blue (dark green) dot was plotted. The red lines represent the monopole loops. Without cooling the topological charge distribution cannot be identified with instantons due to noise. Also the monopole loops do not exhibit a structure. After 15-20 cooling steps one can assign instantons to clusters of topological charge. At cooling sweep 21 an instanton and an antiinstanton become visible. At cooling steps 35-40 they begin to approach each other and annihilate several steps later (not shown). Monopole loops also thin out with cooling, but they survive in the presence of instantons. There is an enhanced probability that monopole loops are present in the vicinity of instantons in all gauge field configurations which we have checked.

3. Conclusion and Outlook. Calculating the local values of topological charges and monopole currents and by directly displaying them with the help of computer graphics, we draw a number of conclusions. Perhaps the most important is that after a few cooling sweeps one observes clearly that instantons are accompanied by monopole loops. This correlation occurs on all (semi-classical) gauge field configurations considered. In a cooling history we demonstrated how instantons evolve from fluctuating gauge fields and how they are surrounded by monopoles. The results presented are in nice agreement with earlier studies [1], where we computed gauge averages of correlation functions between topological charges and monopoles. There it turned out that the correlations are rather insensitive under cooling. Combining this finding with that of the 3D images, we conclude that the topological charge goes hand in hand with monopoles also in the original (uncooled) gauge field configurations.

In this contribution we have only dealt with the quenched case. Switching on dynamical fermions the correlations of the topological objects with the chiral condensate become of interest. Preliminary studies of such correlation functions on gauge field average have shown non-trivial correlations with a range of about two lattice spacings [4]. This has important consequences for our understanding of the vacuum structure of QCD because it would mean that chiral symmetry breaking occurs in the region of topological objects. At present we are preparing analog 3D plots of the chiral condensate on specific gauge field configurations in order to check if the appearance of a non-vanishing quark condensate coincides locally with the positions of instantons and monopoles.

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