Study of the convective fluid flows with evaporation on the basis of the exact solution in a three-dimensional infinite channel

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Abstract. The solution of special type of the Boussinesq approximation of the Navier–Stokes equations is used to simulate the two-layer evaporative fluid flows. This solution is the 3D generalization of the Ostroumov–Birikh solution of the equations of free convection. Modeling of the 3D fluid flows is performed in an infinite channel of the rectangular cross section without assumption of the axis-symmetrical character of the flows. Influence of gravity and evaporation on the dynamic and thermal phenomena in the system is studied. The fluid flow patterns are determined by various thermal, mechanical and structural effects. Numerical investigations are performed for the liquid–gas system like ethanol–nitrogen and HFE-7100–nitrogen under conditions of normal and low gravity. The solution allows one to describe a formation of the thermocapillary rolls and multi-vortex structures in the system. Alteration of topology and character of the flows takes place with change of the intensity of the applied thermal load, thermophysical properties of working media and gravity action. Flows with translational, translational-rotational or partially reverse motion can be formed in the system.

1. Introduction

The fluid flows with an interface being under action of the co-current gas fluxes and accompanied by evaporation or condensation have been the subject of extensive theoretical and experimental investigations in the last few decades (for a review, see \cite{1}). The flows are applied in systems of the fluidic cooling or thermal controlling of highly efficient semiconductor equipment, in membrane evaporators, distillers, thermal coating applications technologies, etc. Precisely forecasting the fluid dynamics in the processes requires comprehensive analysis based on modeling the two-layer flows with evaporation. Results of the theoretical study can help one to clarify physical aspects of evaporative convection and phenomena in the liquid media caused by the gas flow. One way of modeling real fluid flows is obtaining the exact solutions. It allows one effectively and rapidly to get some evaluation characteristics or to forecast outcome of experiments on preliminary stages of working-out \cite{2}. Finally, in multiparameter problems the exact solution certainly gives information on degree and character of influence of various factors and provides possibility to make more precise a mathematical model.

In present work the character and structure of the joint flow of evaporating liquid and co-current laminar gas flux are investigated on the basis of new exact solution of the Boussinesq
approximation of the Navier–Stokes equations. Obtained solution is a generalization of the well-known Ostroumov–Birikh solution [3, 4] for three dimensional case. The group nature of the Birikh solutions [5] allowed one to generalize the solutions both for 3D convection problem in the non-axis-symmetrical case [6] and for problems of evaporative convection in the domains with internal interfaces admitting phase transition [7]. The invariant and partially invariant solutions are used to study the fundamental and secondary features of physical processes described by the convection equations. They imply the natural properties of space-time symmetry and symmetry of spatial fluid motion. It should be noted that the asymptotic character of the Birikh type solution has been justified in [8], where the thermocapillary gravitational convection in a long horizontal cavity has been studied experimentally and numerically on the basis of a 2D problem. Complete investigation of properties of the constructed solutions expects the study of the stability characteristics and their perturbation spectrum. In [9, 10] the stability of the two-dimensional convective flows with evaporation was investigated. Influence of intensity of the thermal loads on the external channel walls, gravity and linear thicknesses of the working media layers on the stability characteristics have been studied in the framework of the problem statement without taking into account the Soret effect. The aim of the paper is to carry out mathematical modeling of the three-dimensional convective fluid flows with evaporation as well as to analyze the influence of the gravity and external thermal load on the flow patterns. The study is performed in the framework Oberbeck–Boussinesq model taking into account the Soret and Dufour effects in vapor–gas phase.

2. Generalization of the Ostroumov–Birikh solution for 3D flows with evaporation at interface

Let the Cartesian coordinate system be chosen so that the gravity acceleration vector \( \mathbf{g} \) is directed opposite to the \( Ox \) axis (\( \mathbf{g} = -g_i, \mathbf{i} \) is the unit vector of \( Ox \)). Two fluid layers are separated by the thermocapillary interface \( \Gamma \) given here by the equation \( x = 0 \) (see fig. 1). Let the linear size of the flow domain in the \( y \)-direction \( h \) be the characteristic length. The characteristic values for the coupled problem of the liquid-gas flows are introduced on the basis of the characteristics of the liquid so that \( u^*, T^* \) and \( p^* = \rho_1 u^*_x \) are the characteristic velocity, temperature drop and pressure, respectively. Viscous incompressible fluids (liquid and gas–vapor mixture) fulfill the infinite horizontal layers \( \Omega_1 \) and \( \Omega_2 \) (fig. 1). The boundaries of the domains \( \Omega_1, \Omega_2 \) are the fixed impermeable walls. The stationary three-dimensional convective flows of \( j \)-th medium (here and subsequently \( j = 1, \ 2 \) relate to the liquid and gas–vapor mixture, respectively) are described by the Oberbeck–Boussinesq approximation of the Navier–Stokes equations [7, 11].

The Dufour and Soret effects (or the effects of diffusive thermal conductivity and thermodiffusion [12, 13, 14]) are taken into account in the gas phase. We suppose that the vapor is a passive admixture, the vapor diffusion in the gas phase is described by the diffusion equation. We construct the exact solution of the Oberbeck–Boussinesq, which is characterized by dependence of the components of the liquid \( \mathbf{v}_1 = (u_1, v_1, w_1) \) and gas velocity \( \mathbf{v}_2 = (u_2, v_2, w_2) \) vectors on the transverse coordinates \((x, y)\) (see [7]). The temperature functions \( T_j \), pressure \( p_j \) and vapor concentration \( C \) have the terms \( \Theta_j, q_j, \Phi \) similarly depending on the transverse coordinates \((x, y)\):

\[
 u_j = u_j(x, y), v_j = v_j(x, y), w_j = w_j(x, y), \quad T_j = -Az + \Theta_j(x, y), \quad C = Bz + \Phi(x, y),
\]
\[ p_1 = -A \frac{Gr}{Re^2} xz + q_1(x, y), \quad p_2 = -A \frac{\bar{\rho} \beta}{Re^2} xz + B \frac{Ga}{Re^2} xz + q_2(x, y). \]

Here \( Re = u_s h / \nu_1 \) is the Reynolds number, \( Gr = \beta_1 T_s g h^3 / \nu_1^2 \) is the Grashof number, \( Ga = g h^3 / \nu_1^2 \) is the Galilei number, \( \bar{\rho} = p_2 / p_1 \), \( \beta = \beta_2 / \beta_1 \) are the ratios of the densities \( \rho_j \) and coefficients of thermal expansion \( \beta_j \) of the gas and liquid, respectively; \( \nu_1 \) is the coefficient of kinematic viscosity, \( \gamma \) is the concentration coefficient of the gas density. The coefficients \( A \) and \( B \) determine the constant longitudinal temperature and concentration gradients along the interface. If \( A_* \) and \( B_* \) are the dimensional longitudinal gradients of temperature and concentration functions then \( A = A_* h / T_* \), \( B = B_* h \). The following relation between \( A \) and \( B \) will take place \( B = -C_* \bar{\varepsilon} A \) because of the interface condition for saturated vapor concentration. (Here \( C_* \) is the saturated vapor concentration at \( T_2 = T_0 \); \( \bar{\varepsilon} = \varepsilon T_* \), \( \varepsilon = \lambda \mu / (R^* T_0^2) \), \( \lambda \) is the latent heat of evaporation \( \mu \) is the molar mass of the evaporating liquid, \( R^* \) is the universal gas constant; for details and choice of \( T_0 \) see [2, 9].)

The interface boundary conditions are formulated on the basis of the conservation laws and some additional assumptions [6, 7, 11]. The kinematic and dynamic conditions (projection on the tangential and normal vectors to the interface) should be fulfilled on the interface \( x = 0 \) [6, 7]. Conditions of continuity of tangential velocities and temperature are assumed to be fulfilled on the thermocapillary interface \( x = 0 \). At \( x = 0 \) the heat transfer condition with respect to the diffusive mass flux due to evaporation and the vapor mass balance equation are formulated. These relations take into account the Dufour and Soret effects. The linearized form of an equation for saturated vapor concentration at interface is used as a condition for the vapor concentration function at interface [9]. It is a consequence of the Clapeyron–Clausius equation and the Mendeleev–Clapeyron equation for an ideal gas. On the fixed impermeable walls of the channel the no-slip conditions for velocity fields and the conditions of thermal insulating of the lateral walls are imposed. The case of absence of vapor flux on the upper and lateral rigid boundaries is studied in the present statement. Both conditions provide a fulfillment of the conditions for full heat flux and for full mass flux with respect to the Dufour and Soret effects, respectively, on the fixed walls [11, 15].

3. Numerical investigations

The analytical calculations for construction of the exact solutions of the three-dimensional convection problem with evaporation at the interface is complemented by the numerical investigations. The numerical algorithm of solving of each 2D problems is based on the longitudinal transverse finite difference scheme known as the method of alternating directions [6, 11, 16].

Numerical investigations are performed for the liquid–gas system like ethanol–nitrogen and HFE-7100–nitrogen with the physicochemical properties described in [17] (see also [2, 7, 9]). The values of other parameters used by simulations including the coefficients characterized the Soret and Dufour effects in the gas–vapor layer are chosen similarly to that described in [2, 7, 9]. We investigate the flow topology computed with following values of the Grashof number \( Gr = \{47000, 470\} \) for the ethanol–nitrogen flows and with the values \( Gr = \{1220000, 12000\} \) for the flows of the HFE-7100–nitrogen system. These values of the Grashof number correspond to the conditions of normal \( (g = g_0 = 9.81 \text{ m/s}^2) \) and low gravity \( (10^{-2} g_0 \text{ m/s}^2) \), respectively, when the characteristic temperature drop and characteristic length are equal to \( T_* = 10 \text{ K} \) and \( h = 10^{-2} \text{ m} \). The characteristic velocity is chosen equal to the velocity of viscous stresses relaxation \( \nu / h \) so that \( u_s = 0.15 \cdot 10^{-5} \text{ (m/s)} \) for ethanol and \( u_s = 0.38 \cdot 10^{-4} \text{ (m/s)} \) for HFE-7100.

We interpret constructed solution as a solution describing the flow on the working area \([0, z_0]\) in a sufficiently long cavity. The examples of flow structure (temperature distribution and projections of stream lines in \( z = 0.5z_0 \) cross-section) are presented in figs. 2, 3 for ethanol–nitrogen and figs. 4, 5 for HFE-7100–nitrogen systems. In the numerical experiments the non-
dimensional longitudinal temperature gradient $A$ has the values $A = 0.1$ and $A = 0.3$, which correspond to the dimensional values equal to $A_\ast = 100$ K/m and $A_\ast = 300$ K/m, respectively. The flows are characterized as the translational motion and progressively rotational flow and can be realized in various forms.

Figure 2. Flow pattern. Ethanol–nitrogen system; $A = 0.1$: $Gr = 470$ (left); $Gr = 47000$ (right).

Figure 3. Flow pattern. Ethanol–nitrogen system; $A = 0.3$: $Gr = 470$ (left); $Gr = 47000$ (right).

Flows in both phases in fig. 2 are characterized by clearly marked symmetric two-vortex structures by $A = 0.1$. Vortices in the liquid have the convective nature and vortex centers are located close to the interface in the “corners” (fig. 2). At the same time the bright qualitative differences are found by simulation of the HFE-7100–nitrogen flows (fig. 4). By $A = 0.1$ we observe the two-vortex structure in the gas layer and “quadruple” flow in the liquid layer if $Gr = 12200$. Two additional small corner vortices appear in the liquid layer in the case of normal gravity $Gr = 1220000$. Formation of the extra vortexes is explained by weaker viscous effects in HFE-7100 in comparison with ethanol.

More complicated vortex patterns in fluids are observed in the case with more intensive longitudinal temperature regimes (at $A = 0.3$, see figs. 3, 5). Splitting two big vortex into four
asymmetrical swirls in the liquid layer occurs in the ethanol–nitrogen system in microgravity conditions. Two of the vortices are deformed into “angled” vortexes, but their cores stay close to the interface in “corners” near the lateral walls. Liquid particles from the hot spot move along the interface due to the Marangoni effect and go down near the lateral walls because of unstable thermal stratification. Thus, the upper part of the “angled” vortex is generated by thermocapillary effect, but the side part appears due to convective mechanism. In HFE-7100–nitrogen system in the case with more intensive longitudinal temperature regimes (see fig. 5) the liquid flows are characterized by four separate vortices under microgravity and six swirls in the terrestrial conditions. For the system the flow regimes differ only in the liquid layers (compare fig. 4 and 5), whereas the fluid flow structures are rebuilt in both upper and lower layers in ethanol–nitrogen system (fig. 2 and 3). In all the cases flow patterns are symmetric with respect to the plane \( y = 0.5 \) in both phases. Thus, we have different flow topology for the various gas–liquid systems. More intensive rotational flow has been observed by normal
gravity (here $Gr = 47000$ for ethanol–nitrogen, $Gr = 1220000$ for HFE-7100–nitrogen systems). Some minor quantitative differences are observed by comparison of the flows under conditions of normal gravity and microgravity for both liquids. However, it should be pointed out that there are more greater quantitative differences between liquid and gas flows in the flows of the ethanol–nitrogen system, than in the HFE-7100–nitrogen system.

4. Conclusions
The solution of special type of the 3D stationary coupled problem of the gravitational and thermocapillary convection with respect to evaporation is used to describe the convective flows in an infinite channel of a rectilinear cross section without assumption of axial symmetry of the flow domains. This solution has the group nature and is the analogue of the Ostroumov–Birikh solution of the convection equations which additionally include the Dufour and Soret effects in the gas phase. Considering the infinite channel under action of the constant longitudinal temperature gradient we study a model problem of evaporative convection and interpret the constructed solution as an one that describes the flow on the working area being a sufficiently long cavity.

The flows of both fluids (of a liquid and gas–vapor mixture) modeled with the help of the exact solution can be characterized as a translational motion and progressively rotational flows and realized in the various forms. The qualitative and quantitative differences are confirmed for the flows of various working fluids (ethanol–nitrogen and HFE-7100–nitrogen systems). The numerical investigations allow one to analyze the possible flow structure with respect to the intensity of the gravitation field and longitudinal temperature gradients created on the interface. The intensity of the liquid flows depends on intensity of the gravitation field and on interface temperature regime. Topologically various structures of flows are formed due to combined influence of the thermocapillary and convective mechanisms and evaporation/condensation process, affecting the thermal pattern of the flows.

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