On the search for the electric dipole moment of strange and charm baryons at LHC and parity violating (P) and time reversal (T) invariance violating spin rotation and dichroism in crystal

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Abstract

In a bent crystal the $P$-odd effect of short-lived baryon spin rotation could imitate spin rotation caused by assumed EDM. Use of different behavior of $P$-odd and $T$-odd spin rotations at crystal turning around the direction of particle momentum makes it possible to exclude $P$-odd rotation contribution, when measuring short-lived baryons EDM. Subtraction of the measurements results for angle ranges $\varphi$ and $\varphi + \pi$ from each other enables measuring $T$-odd rotation at scattering of negatively charged beauty and neutral baryons by axes of unbent crystal.

1 Introduction

The spin rotation phenomenon of channeled particles, moving in a bent crystal, which was theoretically predicted in [1] and observed in [2–4], gives us the opportunity to measure anomalous magnetic moment of high energy short-lived particles. The appearance of beams with energies up to 7 Tev on LHC and further growth of particles’ energy and beams’ luminosity on FCC do essentially improve the possibility of using the phenomenon of spin rotation of the high energy particles in bent crystals and spin depolarization of such particles for measuring anomalous magnetic moments of positively charged, as well as neutral and negatively charged short-lived hyperons, and $\tau$ -lepton [5–8]. The detailed analysis of conditions of the experiment on measuring magnetic dipole moment (MDM) of charm baryon $\Lambda_c^+$ on LHC, which has confirmed the possibility of measuring MDM of such baryon on LHC, was accomplished recently in [9]. Strong electric field affects the channeled particle in a bent crystal. As a consequence, the spin rotation phenomenon of the channeled
particle allows to obtain information about the possible value of the electric dipole moment of short-lived baryons, which elementary particles can obtain as a result of the violation of the T-invariance \[10\].

It should be noted, that besides electromagnetic interaction the channeled particle moving in a crystal experience weak interaction with electrons and nuclei as well as strong interaction with nuclei. Mentioned interactions lead to the fact, that in the analysis of the particle’s spin rotation, caused by electric dipole moment interaction with electric field, both \(P(P,T)\) non-invariant spin rotation, resulting from weak interaction, and spin dichroism should be considered \[11, 12\].

Let us concretize the mentioned above for the interested case of baryons moving in a bent crystal.

2 Parity violating (P) and time reversal (T) invariance violating spin rotation and dichroism in crystal

The spin rotation phenomenon for high-energy particles, moving in a bent crystal, as a result of quasi-classical motion manner of particles channelled in crystals, can be described by equations similar to those for motion of particles’ spin in the storage ring with the inner target \[11,12\]. The theory, which describes motion of the particle spin in electromagnetic fields in a storage ring, has been developed in many papers \[13–19\].

According to \[13–19\], the basic equation, which describes particle spin motion in an electromagnetic field, is the Thomas-Bargmann–Michel–Telegdi (T-BMT) equation. Refinement of the T-BMT equation, allowing us to consider the possible presence of the particle EDM, was made in \[20,21\].

Now let us consider a particle with spin \(S\) which moves in the electromagnetic field. The term ”particle spin” here means the expected value of the quantum mechanical spin operator \(\hat{S}\) (hereinafter the symbol marked with ”hat” means a quantum mechanical operator).

Spin motion is described by the Thomas–Bargmann–Michel–Telegdi equation (T-BMT) as follows:

\[
\frac{d\vec{S}}{dt} = [\vec{S} \times \vec{\Omega}],
\]

\[
\vec{\Omega} = \frac{e}{mc} \left[ \left( a + \frac{1}{\gamma} \right) \vec{B} - a \frac{\gamma}{\gamma + 1} \left( \vec{\beta} \cdot \vec{B} \right) \vec{\beta} - \left( a + \frac{1}{\gamma + 1} \right) \vec{\beta} \times \vec{E} \right],
\]

where \(\vec{S}\) is the spin vector, \(t\) is the time in the laboratory frame, \(m\) is the
mass of the particle, \( e \) is its charge, \( \gamma \) is the Lorentz-factor, \( \bar{\beta} = \bar{v}/c \), where \( \bar{v} \) denotes the particle velocity, \( \vec{E} \) and \( \vec{B} \) are the electric and magnetic fields at the point of particle location in the laboratory frame, \( a = (g - 2)/2 \) and \( g \) is the gyromagnetic ratio (by definition, the particle magnetic moment \( \mu = (eg/2m\hbar)S \), where \( S \) is the particle spin). The T-BMT equation describes the spin motion in the rest frame of the particle, wherein the spin is described by the three component vector \( \vec{S} \). In practice the T-BMT equation works well for the description of spin precession in the external electric and magnetic fields encountered in typical present-day accelerators. Study of the T-BMT equation allows us to determine the major peculiarities of spin motion in an external electromagnetic field. However, it should be taken into account that particles in an accelerator or bent crystal have an energy spread and move along different orbits. This necessitates one to average the spin-dependent parameters of the particle over the phase space of the particle beam. This is why one must always bear in mind the distinction between the beam polarization \( \vec{\xi} \) and the spin vector \( \vec{S} \). A complete description of particle spin motion can be made applying spin density matrices equation (in more details see \[12, 22\]).

If a particle possesses an intrinsic electric dipole moment, then the additional term, describing spin rotation induced by the EDM, should be added to (1):

\[
\frac{d\vec{S}_{\text{EDM}}}{dt} = \frac{d}{\hbar \gamma + 1} \left[ \vec{S} \times \left\{ (\bar{\beta} \times \vec{B} + \vec{E}) - \frac{\gamma}{\gamma + 1} \bar{\beta} (\bar{\beta} \vec{E}) \right\} \right],
\]

where \( d \) is the electric dipole moment of the particle.

As a result, the motion of particle spin due to the magnetic and electric dipole moments can be described by the following equation:

\[
\frac{d\vec{S}}{dt} = \frac{e}{mc} \left[ \vec{S} \times \left\{ \left( a + \frac{1}{\gamma}\right) \vec{B} - a \gamma + 1 \left( \bar{\beta} \cdot \vec{B} \right) \bar{\beta} - \left( a + \frac{1}{\gamma + 1}\right) \bar{\beta} \times \vec{E} \right\} \right] + \frac{d}{\hbar \gamma + 1} \left[ \vec{S} \times \left\{ (\bar{\beta} \times \vec{B} + \vec{E}) - \frac{\gamma}{\gamma + 1} \bar{\beta} (\bar{\beta} \vec{E}) \right\} \right].
\]

Recall now, that electric and magnetic fields in a crystal are formed by atoms. Scattering on atoms leads to the fact, that the high-energy particle moving in a crystal experience interaction from electric and magnetic fields. However it is not only the electromagnetic interaction that influence on the scattering. Particles also participate in strong and weak interactions with electrons and nuclei. It would be recalled that the particle refractive index in matter has the form:

\[
n = 1 + \frac{2\pi N}{k^2} f (0),
\]
where \( N \) is the number of scatterers per \( \text{cm}^3 \) and \( k \) is the wave number of the particle incident on the target, \( f(0) \) is the coherent elastic zero–angle scattering amplitude.

Let us consider a relativistic particle refraction on the vacuum–medium boundary (see [12]).

The wave number of the particle in the vacuum is denoted \( k \). The wave number of the particle in the medium is \( k' = kn \). As is evident, the particle momentum in the vacuum \( p = \hbar k \) is not equal to the particle momentum in the medium. Therefore, the particle energy in the vacuum \( E = \sqrt{\hbar^2 k^2 c^2 + m^2 c^4} \) is not equal to the particle energy in the medium \( E_{\text{med}} = \sqrt{\hbar^2 k'^2 c^2 + m^2 c^4} \).

The energy conservation law immediately requires the particle in medium to have the effective potential energy \( V_{\text{eff}} \). This energy can be easily found from the relation

\[
E = E_{\text{med}} + V_{\text{eff}},
\]

i.e.,

\[
V_{\text{eff}} = E - E_{\text{med}} = \frac{2\pi \hbar^2}{m\gamma} N f(E, 0) = (2\pi)^3 N T(E),
\]

\[
f(E, 0) = -(2\pi)^2 \frac{E}{\hbar^2} T(E) = -(2\pi)^2 \frac{m\gamma}{\hbar^2} T(E).
\]

where \( T(E) \) is the T-matrix.

Due to periodic arrangement of atoms in a crystal the effective potential energy is a periodic function of coordinates of a particle moving in crystal [12].

\[
U(\vec{r}) = \sum_\vec{\tau} U(\vec{\tau}) e^{i \vec{\tau} \vec{r}},
\]

where \( \vec{r} \) is the reciprocal lattice vector of the crystal,

\[
U(\vec{\tau}) = \frac{1}{V} \sum_j U_{j0}(\vec{\tau}) e^{-i \vec{\tau} \vec{r}_j},
\]

here \( V \) is the volume of the crystal elementary cell, \( \vec{r}_j \) is the coordinate of the atom (nucleus) of type \( j \) in the crystal elementary cell and the squared \( e^{-W_j(\vec{\tau})} \) is equal to the thermal-factor (i.e., the Debye-Waller factor), well-known for X-ray scattering:

\[
U_{j0}(\vec{\tau}) = -\frac{2\pi \hbar^2}{m\gamma} F_j(\vec{\tau}),
\]

where \( F_j(\vec{\tau}) = F_j(\vec{k} - \vec{k}' = \vec{\tau}) \) is the amplitude of elastic coherent scattering of the particle by the atom, \( \vec{k} \) is the wave vector of the incident wave and \( \vec{k}' \) is the wave vector of the scattered wave.
It will be recalled that in the case of a crystal, the imaginary part of
the amplitude \( F_j(0) \) does not contain the contribution from the total cross
section of elastic coherent scattering. The imaginary part of \( F_j(0) \) in a crystal
is only determined by the total cross sections of inelastic processes. This
occurs because in a crystal, unlike in amorphous matter, the wave elastically
scattered at a non zero angle, due to rescattering by periodically located
centers, is involved in formation of a coherent wave propagating through the
crystal.

Elastic coherent scattering of a particle by an atom is caused by Coulomb
interaction of the particle with the atom electrons and nucleus as well as with
weak and strong nuclear interaction with the electrons and nucleus. Therefore,
the scattering amplitude can be presented as a sum of two amplitudes:

\[
F_j(\vec{\tau}) = F_j^{\text{Coul}}(\vec{\tau}) + F_j^{\text{ws}}(\vec{\tau}),
\]

where \( F_j^{\text{Coul}}(\vec{\tau}) \) is the amplitude of particle scattering caused by Coulomb
interaction with the atom (it contains contributions from the Coulomb in-
teraction of the particle with the atom along with the spin-orbit interaction
with the Coulomb field of the atom); \( F_j^{\text{ws}}(\vec{\tau}) \) is the amplitude of elastic co-
herent scattering of the particle caused by weak and strong interaction, (this
amplitude contains the terms independent of the incident particle spin along
with the terms depending on spin of both the incident particle, electrons and
nucleus, in particular, spin-orbit interaction). Therefore, \( U(\vec{r}) \) and \( U(\vec{\tau}) \) can
also be expressed:

\[
U(\vec{r}) = U^{\text{Coul}}(\vec{r}) + U^{\text{ws}}(\vec{r}),
U(\vec{\tau}) = U^{\text{Coul}}(\vec{\tau}) + U^{\text{ws}}(\vec{\tau}).
\]

Suppose a high energy particle moves in a crystal at a small angle to the
crystallographic planes (axes) close to the Lindhard angle \( \vartheta_L \sim \sqrt{U/E} \) (in a
relativistic case \( \vartheta_L \sim \sqrt{2U/E} \)), where \( E \) is the energy of the particle, \( U \) is the
height of the potential barrier created by the crystallographic plane (axis).
This motion is determined by the plane (axis) potential \( \hat{U}(\vec{\rho}) \), which could
be derived from \( U(\vec{r}) \) by averaging over the distribution of atoms (nuclei) in
the crystal plane (axis). As a consequence for the potential of periodically
placed axes we can write:

\[
\hat{U}(\vec{\rho}) = \sum_{\tau_\perp} U(\tau_\perp, \tau_z = 0) e^{i\tau_\perp \cdot \vec{\rho}},
\]

\( z \)-axis of the coordinate system is directed along the crystallographic axis
(let us note that this expression can be obtained if all the terms with \( \tau_z \neq 0 \)}
are removed from the sum (7)). Replacing summation over \( \mathbf{\tau}_\perp \) by integration over \( \mathbf{\tau}_\perp \) we obtain an expression for the potential of separate crystallographic axis.

To obtain the potential of periodically placed planes we have:

\[
\hat{U}(x) = \sum_{\tau_x} U(\tau_x, \tau_y = 0, \tau_z = 0) e^{i\tau_x x},
\]  

(13)

Y,Z-plane of the system of coordinates is parallel to the chosen crystallographic planes family. After replacing summation over \( \mathbf{\tau}_x \) by integration over \( \mathbf{\tau}_x \) we obtain an expression for the potential of separate crystallographic plane.

As a consequence, the potential \( \hat{U}(\mathbf{\rho}) \) for a particle channeled in a plane (or axis) channel or moving over the barrier at a small angle, close to the Lindhard angle, can be expressed as a sum [12]:

\[
\hat{U}(\mathbf{\rho}) = \hat{U}^{\text{Coul}}(\mathbf{\rho}) + \hat{U}^{\text{sp.\,-orb.}}(\mathbf{\rho}) + \hat{U}^{\text{ws eff}}(\mathbf{\rho}) + \hat{U}^{\text{mag}}(\mathbf{\rho}),
\]  

(14)

where \( \hat{U}^{\text{Coul}} \) is the potential energy of particle Coulomb interaction with the crystallographic plane (axis), \( \hat{U}^{\text{sp.\,-orb.}} \) is the energy of spin-orbit interaction with the Coulomb field of the plane (axis) and spin-orbit nuclear interaction with the effective nuclear field of the plane (axis), \( \hat{U}^{\text{ws eff}} \) is the effective potential energy of weak and strong interaction of the incident particle with the crystallographic plane (axis). According to [9] this part of the potential is determined by the amplitude of elastic coherent forward weak and strong scattering \( \hat{f}_{\text{ws}}(0) \), which depends on the incident particle spin and electron and nucleus polarization, \( \hat{U}^{\text{mag}}(\mathbf{\rho}) \) is the energy of magnetic interaction of the particle with electrons (nuclei).

Let us note that for high-energy particles in a quasi-classical approximation terms \( \hat{U}^{\text{Coul}} \), \( \hat{U}^{\text{sp.\,-orb.}} \) and \( \hat{U}^{\text{mag}} \) lead to BMT equations.

Thus when describing the particle motion in crystals, the contribution of weak and strong interactions to formation of the effective potential acting on a particle from the crystallographic planes (axes) should be taken into account along with the electromagnetic interaction.

In the case of high energies, when the particle motion in the potential \( \hat{U}(\mathbf{\rho}) \) can be described in the quasiclassical approximation, the spin–evolution equations for a particle moving in straight and bent crystals in the presence of the contribution from \( \hat{U}^{\text{ws eff}}(\mathbf{\rho}) \) appear to be similar to those for particle motion in a storage ring [12,23].

For the explicit expression for \( \hat{U}^{\text{ws eff}} \) and the amplitude \( \hat{f}_{\text{ws}}(0) \) in the presence of parity nonconservation and time (T) reversal violation see [11,12].

So all spin phenomena discussed for the case of storage ring with an inner target also occur in the case under consideration [11,12,23]. The energy of
electrons-nuclei interactions in atoms can be neglected during the analysis of particle scattering on the atom at high energies. As a result, electrons’ and nuclei contribution to the effective interaction energy can be considered separately. This allows us to express contribution of strong and weak interactions to the effective interaction energy as follows:

\[
\hat{U}_{\text{eff}}^{\text{str., w}} = \frac{2\pi \hbar^2}{m\gamma} \left( N_e(\bar{\rho}) \hat{f}_{\text{ew}}(0) + N_{\text{nuc}}(\rho) \hat{f}_{\text{nuc}}^{\text{sw}}(0) \right). \tag{15}
\]

Here \( N_e(\bar{\rho}) \) is the electron density in the point \( \bar{\rho} = (x, y) \) of the crystallographic plane (axis). Vector \( \bar{\rho} \) is orthogonal to the chosen plane (axis) family, \( N_{\text{nuc}}(\rho) \) is the nuclei density in the point \( \rho \), \( \hat{f}_{\text{ew}}(0) \) is the amplitude of hyperon elastic coherent scattering by an electron, caused by weak \( P \) and \( T \) violating interactions, \( \hat{f}_{\text{nuc}}^{\text{sw}}(0) \) is the amplitude of hyperon elastic coherent scattering by a nuclei, caused by both strong and weak, \( P \) and \( T \) violating interactions.

Parity nonconservation and time reveal violation lead to the dependence of \( \hat{f}_{\text{ew}} \) and \( \hat{f}_{\text{nuc}}^{\text{sw}} \) on the spin orientation of colliding particles. As a result, \( U_{\text{eff}}^{\text{str., w}} \) also depends on the spin orientation of colliding high-energy particles. This fact leads to the appearance of the quasi-optic effects of spin rotation and spin dichroism for high energy particles [12]. Stated contributions to the rotation should be added to equations (4), that describe spin rotation of a particle moving in an electric field [12, 23].

Lets now return to the discussion of the experiment for detection of the electric dipole moment of short-lived baryons moving in electrostatic field of a bent crystal. In this case electrons and crystal nuclei are unpolarized. As a result, the contribution to the scattering amplitude, which depends on the baryon spin orientation, is caused only by parity nonconservation:

\[
\hat{f}_w(0) = B_{0w} + B_w \hat{\sigma} \bar{n}, \tag{16}
\]

where \( \hat{\sigma} = (\sigma_x, \sigma_y, \sigma_z) \) is the Pauli matrix describing baryon spin \( \hat{S} = \frac{1}{2} \hbar \hat{\sigma} \), \( \bar{n} \) is the unit vector in particle momentum direction. As a consequence, the contribution to effective interaction energy, which is dependent on the baryon spin, can be written as:

\[
\hat{U}_{\text{eff}}^{w} = -\frac{2\pi \hbar^2}{m\gamma} \left( N_e(\rho) \hat{f}_{\text{ew}}(0) + N_{\text{nuc}}(\rho) \hat{f}_{\text{nuc}}^{\text{sw}}(0) \right), \tag{17}
\]

where \( N_e(\rho) \) is the density of electrons (nuclei) in the point \( \rho \) of the plane (axis), \( \hat{f}_{\text{ew}}(0) \) is the amplitude of baryon forward scattering by an electron, which is cased by weak interaction, \( \hat{f}_{\text{nuc}}^{\text{sw}}(0) \) is the amplitude of
baryon forward scattering by a nucleus, which is caused by weak interaction. Therefore the spin-dependent part of effective interaction energy $\hat{U}_{\text{eff}}^{w}$ reads according to (16) as follows:

$$\hat{U}_{\text{eff}}^{w} = -\frac{2\pi\hbar^2}{m\gamma} (N_e(\rho)B_e + N_{\text{nuc}}(\rho)B_{\text{nuc}}) \sigma\vec{n},$$

(18)

where $B_e$ describes parity violating contribution caused by baryon interaction with electrons, $B_{\text{nuc}}$ is for parity violating contribution caused by baryon interaction with nuclei.

As a result, due to parity violation $U_{\text{eff}}$ contains the term, which depends on spin orientation and is proportional to $\vec{\sigma}\vec{n}$. This term is similar to those describing interaction of magnetic moment $\mu$ with magnetic field $\mu\vec{B}$ and electric dipole moment of baryon $d$ with electric field $d\vec{E}$. When the baryon spin is directed along the momentum, the effective energy reads:

$$\hat{U}_{\text{eff}}^{w\uparrow\uparrow} = -\frac{2\pi\hbar^2}{m\gamma} \left( N_e(\vec{p})f_{\text{we}}^{\uparrow\uparrow}(0) + N_{\text{nuc}}(\vec{p})f_{\text{wnuc}}^{\uparrow\uparrow}(0) \right),$$

(19)

where $f_{\text{we}}^{\uparrow\uparrow}(0) = B_{\text{oe(nuc)}} + B_{\text{e(nuc)}}$ is the amplitude of elastic coherent scattering of a particle with the spin parallel to its momentum.

When the baryon spin is directed oppositely to the momentum, the effective energy reads:

$$\hat{U}_{\text{eff}\downarrow\uparrow} = -\frac{2\pi\hbar^2}{m\gamma} \left( N_e(\vec{p})f_{\text{we}}^{\downarrow\uparrow}(0) + N_{\text{nuc}}(\vec{p})f_{\text{wnuc}}^{\downarrow\uparrow}(0) \right),$$

(20)

where $f_{\text{we(nuc)}}^{\downarrow\uparrow}(0) = B_{\text{oe(nuc)}} - B_{\text{e(nuc)}}$ is the amplitude of elastic coherent scattering of a particle with the spin antiparallel to its momentum. Therefore, $\hat{U}_{\text{eff}\uparrow\uparrow}^{w} \neq \hat{U}_{\text{eff}\downarrow\uparrow}^{w}$.

Difference between these two energies, similar to the difference in energies of magnetic moment interaction with magnetic field at spin parallel and antiparallel orientations to the field, determines the frequency $\Omega_w$ of spin precession around particle momentum as follows:

$$\Omega_w = \frac{\text{Re} U_{\text{eff}\uparrow\uparrow}^{w} - \text{Re} U_{\text{eff}\downarrow\uparrow}^{w}}{\hbar},$$

(21)

i.e.

$$\Omega_w = -\frac{4\pi\hbar}{m\gamma} \left( N_e(\vec{p})\text{Re}B_e + N_{\text{nuc}}(\vec{p})\text{Re}B_{\text{nuc}} \right),$$

(22)

Note the amplitude $f(0)$ (amplitudes $B_e$ and $B_{\text{nuc}}$) includes factor $\gamma$ (see (3)). Thus, the ratio $\frac{1}{\gamma}B_e(\text{nuc}) = B_{\text{e(nuc)}}$ does not include this factor explicitly. Therefore the precession frequency reads
\[ \Omega_w = \frac{4\pi \hbar}{m} \left( N_e(\rho)ReB_e' + N_{nuc}(\rho)ReB_{nuc}' \right), \]  
\hfill (23)

and this expression looks identical to nonrelativistic expression for frequency of spin precession.

For low energy neutrons the similar expression describes neutron spin rotation caused by the weak parity violating interactions \[24\, [26]\]. This quasi-optic effect is well studied for neutrons.

\[ y \]
\[ x \]
\[ s_0 \]
\[ A_c^+ \]
\[ \Phi \]
\[ \Theta_c \]
\[ B^* \]
\[ E^* \]

Figure 1: Behavior of the spin rotation caused by magnetic moment and EDM. The figure is reprinted from figure 2 (right) in \[10\]. Black arrows represent spin rotation caused by magnetic dipole moment, red arrows represent spin rotation caused by electric dipole moment.

Note the amplitudes \( B_0 \) and \( B_{nuc} \), which describe the amplitude of forward elastic coherent scattering, are complex. Their imaginary parts are defined by the total interaction cross-section via optical theorem

\[ ImB_{0e(nuc)} = \frac{k}{4\pi} \frac{\sigma_{\uparrow\uparrow}^{e(nuc)} + \sigma_{\uparrow\downarrow}^{e(nuc)}}{2}; ImB_{e(nuc)} = \frac{k}{4\pi} \frac{\sigma_{\uparrow\uparrow}^{e(nuc)} - \sigma_{\downarrow\uparrow}^{e(nuc)}}{2}, \]  
\hfill (24)

where \( \sigma_{\uparrow\uparrow}^{e(nuc)} \) is the total cross-section of weak interaction for the baryon, which spin is parallel to the electron (nucleus) momentum; \( \sigma_{\downarrow\uparrow}^{e(nuc)} \) is the total cross-section of weak interaction for the baryon, which spin is antiparallel to the electron (nucleus) momentum.

One should remember that for a crystal, where the atoms are periodically located, the total cross-section of coherent elastic scattering (i.e. scattering,
which does not change the state of scatterers) should be subtracted from the total cross-section of particle scattering by an atom. The waves scattered elastically interfere with the incident wave and cause no absorption.

Since the amplitude is complex, the operator $\hat{\Psi}(t)$ is non-Hermitian. Due to complexity of the effective energies $U_{\uparrow\uparrow}(\downarrow\downarrow)$, baryon absorption in crystal depends on its spin orientation owing to baryon weak interaction with electrons and nuclei. The spin projection onto momentum direction changes, and even when the particle spin is orthogonal to the momentum direction, the spin component parallel to the momentum arises at particle propagation in the crystal (Figure 2)!

The additional term caused by weak interaction in equations (3), which describe spin rotation, can be obtained by the following approach [12]. The spin wave function $\hat{\Psi}(t)$ meets the equation as follows:

$$i\hbar \frac{\partial |\Psi(t)\rangle}{\partial t} = \hat{U}_{\text{ef}} |\Psi(t)\rangle.$$  \hspace{1cm} (25)

Baryon polarization vector $\vec{\xi}$ can be found via $|\Psi(t)\rangle$:

$$\vec{\xi} = \frac{\langle \Psi(t)|\vec{\sigma}|\Psi(t)\rangle}{\langle \Psi(t)|\Psi(t)\rangle}.$$  \hspace{1cm} (26)

From (26) it follows:

$$\frac{d\vec{\xi}}{dt} = \Omega_w [\vec{\xi} \times \vec{n}] - g(\vec{n} - \vec{\xi}(\vec{n})\vec{n}),$$  \hspace{1cm} (27)
where $\Omega_w$ is defined by (22) and
\[
g = \frac{1}{2}e[(\sigma_{\uparrow\uparrow} - \sigma_{\uparrow\downarrow}^e)N_e(\tilde{\rho}) + (\sigma_{\uparrow\uparrow}^{nuc} - \sigma_{\uparrow\downarrow}^{nuc})N_{nuc}(\tilde{\rho})],
\] (28)
Expression (27) should be added to (4). For relativistic baryons, when $(\gamma >> 1)$, and in case of magnetic field absence (nonmagnetic crystal) equations (4) become simpler. Thus the equation for spin rotation of a particle, which moves in a bent crystal reads as follows:
\[
\frac{d\xi}{dt} = -\frac{e(g-2)}{2mc}[\xi \times [\bar{n}\vec{E}]] + \frac{d}{\hbar}[\xi \times \vec{E}] + \Omega_w[\vec{\xi} \times \bar{n}] - g(\bar{n} - \xi(\xi \cdot \bar{n})].
\] (29)
Let us evaluate the effect. Precession frequency $\Omega_w$ is determined by the real part of the amplitude of baryon weak scattering by an electron (nucleus). This amplitude can be evaluated in the energy range of about $W$ and $Z$ bosons production and smaller by Fermi theory [27]:
charged particles), the value \( g \sim 10^6 \div 10^7 \text{s}^{-1} \). Multiple scattering also contributes to spin rotation [12]. Particularly, due to interference of weak and coulomb interactions the root-mean-square scattering angle appears changed and dependent on spin orientation with respect to the particle momentum direction.

When measuring T-odd spin rotation in the electric field of a bent crystal, one can eliminate parity violating rotation by the following way. T-odd and P-odd spin rotations differently depend on crystal turning at 180° around the direction of incident baryon momentum. Namely, P-odd effect does not change, while the sign of T-odd spin rotation is flipped due to change of the electric field direction. Subtracting results of measurements for two opposite crystal positions on could obtain the angle of rotation, which depends on T-odd effect only.

3 P and T-odd spin rotation in unbent crystals

The angle of spin rotation \( \vartheta_p \) increases significantly for a baryon in an unbent crystal, when particles move at a small angle with respect to a crystal axes, since in this case the scatterers density grows. Therefore, even for short-lived beauty (bottom) baryons with negative charge, the weak amplitude \( ReB \) can be measured. EDM interaction with electric field of crystal axis has influence on the spin rotation too.

Study of EDM contribution to spin rotation of a particle, which moves at a small angle with respect to a crystal axes, is hampered by depolarization effect [7,8,12]. Trajectories of the scattered particles, which azimuth angles are in the vicinity \( \varphi \) and \( \varphi + \pi \) (z axes is directed along the crystal axes) contributes to EDM-caused spin rotation with opposite signs due to different electric field signs. At the same time, the P-odd rotation occurs around the momentum direction and is not affected by the electric field direction. Thus, P-odd rotation can be observed in both bent and unbent crystals. The T-odd spin rotation can be observed in unbent crystal if we use subtraction of the measurements results for angle ranges \( \varphi \) and \( \varphi + \pi \) from each other. Such procedure leads to summation of contributions from T-odd rotation. Simultaneous measurement of spin orientation for all \( \varphi \) values (as well as for all polar angles for scattering by the axes) provides intensity increase. For unbent crystals the same measuring procedure enables to use crystal with higher nucleus charge that also contributes to effect increase. All the above is primarily significant for negatively charged beauty baryons.
The similar reasoning is valid for measuring the anomalous magnetic moment by means of axial scattering in unbent crystals. Subtraction procedure can be applied for measuring both anomalous magnetic moment and EDM of neutral charm and beauty short-lived baryons.

All the above immediately follows from the general expression for amplitude of elastic coherent scattering of a spin $\frac{1}{2}$ particle by a spinless (unpolarized) nuclei in presence of electromagnetic, strong and $P$-, $T$-odd weak interactions.

\[
\hat{f}(\vec{q}) = A(\vec{q}) + B(\vec{q})\vec{\sigma}\vec{N} + B_{0w}(\vec{q}) + B_w(\vec{q})\vec{\sigma}\vec{N}_w + B_T\vec{\sigma}\vec{N}_T, \tag{33}
\]

where $A(\vec{q})$ is spin-independent part of scattering amplitude, which is caused by electromagnetic and strong interactions. $\vec{h}\vec{q} = \vec{h}\vec{k}' - \vec{h}\vec{k}$ is the transmitted momentum, $\vec{h}\vec{k}'$ is the momentum of the scattered particle, $\vec{h}\vec{k}$ is the momentum of the incident baryon, $\vec{k}',\vec{k}$ are the wave vectors, $\vec{N} = \frac{[\vec{k}' \times \vec{k}]}{[\vec{k} \times \vec{k}]}$,

\[
\vec{N}_w = \frac{\vec{k} + \vec{k}'}{[\vec{k} + \vec{k}]} \quad \vec{N}_T = \frac{\vec{k}' - \vec{k}}{[\vec{k} - \vec{k}]}.
\]

The term, which is proportional to $\vec{\sigma}\vec{N}$, is responsible for spin-orbit interaction contribution to scattering process. For electromagnetic interaction this contribution is determined by the particle magnetic moment.

$T$-odd part of scattering amplitude, which is proportional to $\vec{\sigma}\vec{N}_T$, in case of electromagnetic interaction is determined by electric dipole moment. $T$-odd nuclear interactions also contribute, when particles are scattered by nuclei.

Note, the expression (33) is valid for both nonrelativistic and relativistic cases [12].

With amplitude $\hat{f}(\vec{q})$ one can find the cross-section of particle scattering by a crystal and polarization vector of the scattered particle. According to [8] the scattering cross-section for a thin crystal reads:

\[
\frac{d\sigma_{cr}}{d\Omega} = \frac{d\sigma}{d\Omega} \left\{ (1 - e^{-\bar{u}^2 q^2}) + \frac{1}{N} \sum_n e^{i\vec{q}\vec{r}_n^0} \right\}^2 e^{-\bar{u}^2 q^2}, \tag{34}
\]

where $\vec{r}_n^0$ is the coordinate of the center of gravity of the crystal nucleus, $\bar{u}^2$ is the mean square of thermal oscillations of nuclei in the crystal. The first term describes incoherent scattering and the second one describes the coherent due to periodic arrangement of crystal nuclei (atoms). This contribution leads to the increase in the cross section. This expression can be used as long as the crystal length satisfies the inequality $k(n - 1)L \ll 1$, where $n$ is the particle refractive index in the crystal [8,12]. For thick crystals the influence of channeling and depolarization effects become important.
\[
\frac{d\sigma}{d\Omega} = tr\rho \hat{f}^+(\vec{q}) \hat{f}(\vec{q}),
\]  

(35)

where \(\rho\) is the spin density matrix of the incident particle.

The polarization vector of a particle that has undergone a single scattering event can be found using the following expression:

\[
\vec{\xi} = \frac{tr\rho f^+ \vec{f}}{tr\rho f^+} = \frac{tr\rho f^+ \vec{f}}{d\sigma/d\Omega}.
\]

(36)

Using (33) and (36) one can obtain the following expressions for polarization vector of the scattered particle and differential cross-section:

\[
\vec{\xi} = \vec{\xi}_so + \vec{\xi}_w + \vec{\xi}_T,
\]

(37)

where \(\vec{\xi}_so\) is the change of polarization vector due to spin-orbit interaction, \(\vec{\xi}_w\) is the change of polarization vector caused by weak parity violating interaction, and \(\vec{\xi}_T\) is the change of polarization vector caused by \(T\)-odd interaction.

\[
\vec{\xi}_so = \left\{ (|A|^2 - |B|^2)\vec{\xi}_0 + 2|B|^2 \vec{N} (\vec{N}\vec{\xi}_0) + 2 \text{Im}(\vec{A}\vec{B}^*|\vec{N}\vec{\xi}_0) + 2 \text{Re}(\vec{A}\vec{B}^*) \vec{N} \vec{\xi}_0 \right\} \left(\frac{d\sigma}{d\Omega}\right)^{-1},
\]

\[
\vec{\xi}_w = \left\{ (|A|^2 - |B_w|^2)\vec{\xi}_0 + 2|B_w|^2 \vec{N}_w (\vec{N}_w\vec{\xi}_0) + 2 \text{Im}(\vec{A}\vec{B}_w^*|\vec{N}_w\vec{\xi}_0) + 2 \text{Re}(\vec{A}\vec{B}_w^*) \vec{N}_w \vec{\xi}_0 \right\} \left(\frac{d\sigma}{d\Omega}\right)^{-1},
\]

\[
\vec{\xi}_T = \left\{ (|A|^2 - |B_T|^2)\vec{\xi}_0 + 2|B_T|^2 \vec{N}_T (\vec{N}_T\vec{\xi}_0) + 2 \text{Im}(\vec{A}\vec{B}_T^*|\vec{N}_T\vec{\xi}_0) + 2 \text{Re}(\vec{A}\vec{B}_T^*) \vec{N}_T \vec{\xi}_0 \right\} \left(\frac{d\sigma}{d\Omega}\right)^{-1},
\]

(38)

where \(\vec{A} = A + B_{0w}\). The differential cross-section reads as follows:

\[
\frac{d\sigma}{d\Omega} = tr\rho f^+ f = \]

\[
|A|^2 + |B|^2 + |B_w|^2 + |B_T|^2 + 2\text{Re}(\vec{A}\vec{B}^*)\vec{N}\vec{\xi}_0 + 2\text{Re}(\vec{A}\vec{B}_w^*)\vec{N}_w\vec{\xi}_0 + 2\text{Re}(\vec{A}\vec{B}_T^*)\vec{N}_T\vec{\xi}_0.
\]

(39)

While derivating expressions (38) and (39) small terms containing production of \(B_w\) and \(B_T\), which are much smaller comparing to other terms, were
omitted. As anticipated according to (38) the angle of polarization vector rotation for a baryon scattered in a crystal is determined by rotations around three mutually orthogonal directions. Subtraction procedures for measurements, for which sign of vector products $[\vec{N}_T\vec{\xi}_0]$ and $[\vec{N}_T\vec{\xi}_0]$ change, enable statistics increase. It is also useful to note that initially unpolarized particle beam ($\xi_0 = 0$) in a crystal acquires polarization directed along one of three vectors $\vec{N}$, $\vec{N}_w$, $\vec{N}_T$, which carries information about all types of interaction, namely: electromagnetic, strong and weak $P$, $T$-odd interactions. According to (39) amplitudes interference results in asymmetry in scattering caused by orientation of vectors $\vec{N}_T$, $\vec{N}$, $\vec{N}_w$ with respect to $\vec{\xi}_0$, $\vec{k}'$ and $\vec{k}$. Therefore, the scattered particles intensity is anisotropic. Thus, measurement of angular distribution of intensity for a particle beam scattered by crystal axes enables to obtain $T$-odd contributions to the scattering cross-sections for short-lived baryons. The same makes also possible to obtain $P$-odd and spin-orbit contributions. Particularly, this can be realized by the use of set of crystals with axes directed at small angle with respect to momentum of the scattered particles and detection of either nuclear reaction or ionizing losses inside crystal detector.

4 Conclusion

In a bent crystal the $P$-odd effect of short-lived baryon spin rotation could imitate spin rotation caused by assumed EDM. Use of different behavior of $P$-odd and $T$-odd spin rotations at crystal turning around the direction of particle momentum makes it possible to exclude $P$-odd rotation contribution, when measuring short-lived baryons EDM. Subtraction of the measurements results for angle ranges $\phi$ and $\phi + \pi$ from each other enables measuring $T$-odd rotation at scattering of negatively charged beauty and neutral baryons by axes of unbent crystal. The similar procedure can be applied for measuring both anomalous magnetic moment of the same particles. Measurement of $P$-odd rotation of short-lived baryons spin by crystal axes gives information about weak amplitude $B$.

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