A Fuzzy Model of Interval speed Continuous Petri Nets *

Yueying Peng  and Weizhi Liao  
Department of Information Technology,  
Guangxi Teachers Education University,  
nanning 530001, China  
e-mail:pengyy@gxtc.edu.cn, liaowz@gxtc.edu.cn  
* Natural Science Foundation of Guangxi  
0542049, 0640032, China

Kaoru Hirota  
Department of Computational Intelligence  
and Systems Science Interdisciplinary  
Graduate School of Science and Engineering  
Tokyo Institute of Technology, Tokyo,226- 8502,Japan  
E-mail: hirota@hrt.dis.titech.ac.jp

Abstract

The Interval speed Continuous Petri Nets (ICPN) is a continuous model of time Petri nets. It is shown that the descriptive capability of the proposed Interval speed Continuous Petri Nets (ICPN) Model is stronger than the other ICPN Model. Although the action of the continuous petri nets is non-linear and not easy to combine, fuzzy theory makes it possible to complete the analysis of the proposed ICPN and the control design. In order to achieve this goal, a fuzzy model of the ICPN is proposed. It also makes a definition of the fuzzy rule of the ICPN, discusses the fuzzy control of the ICPN, and gives the theorem marked and restrained by the collection house. All these provide some theoretical foundation for the control of fuzzy system on the basis of description of the ICPN. The feasibility of the fuzzy control of the ICPN is also shown.

1. Introduction

Petri Nets are a wide and attracting way of modeling DESs because they associate graphic representation which permits to simulate systems and analytic description through mathematical equations that describe the evolution of systems. However, when a DESs contains a large number of parts, the number of reachable state of the corresponding Petri nets explode and then is difficult to study. In order to solve the exploring problem of Petri Net’s attaining state, H.Alla and R.David provided a concept of Continuous Petri Nets (CPN) and introduced its theory simultaneously. On the basis of different calculation methods of instantaneous solicitation rate, CPN could be divided into Constant Speed CPN (CCPN), Variable Speed CPN (VCPN), Asymptotic CPN (ACPN), etc. At present, CPN theory has been applied in industrial process control, manufacture system, traffic control and discrete event system, etc.

On the purpose of getting close to the Time Petri Nets, Gu. gave a concept of Interval Speed Continuous Petri Nets (ICPN). It is shown that the modeling capability of Interval speed Continuous Petri Nets (ICPN) Model is much stronger than any other Continuous Petri Nets (CPN) Model. The CCPN and VCPN are only one of the special cases of Interval speed Continuous Petri Nets (ICPN) Models, and at the same time the semantic of Interval speed Continuous Petri Nets (ICPN) Model is more complicated.

S.Hennequin has proved that fuzzy logic theory is a good way to the study of DESs combining the Petri nets because it permits an intuitively modeling of systems based on expert appraisal. The general methodology of the fuzzy control applied to VCPNs and a general control law was presented in [6]. However, the result was presented in [6] can’t be applied to Interval speed Continuous Petri Nets. The main concern of this paper is to propose a fuzzy control based on an exact multimodeling of Interval Speed Continuous Petri Nets. We define for each transition of a ICPN by linear four rules. On each rule we apply a control law which permits to obtain for a chosen marking of an output place a reference signal. Also, a more precisely a proportional control law was presented.

This paper is organized as following. First, a formalized definition of Interval Speed Continuous Petri Nets (ICPN) is given. Then, a fuzzy model of Interval Speed Continuous Petri Nets (ICPN) is proposed in section 3. In section 4, we present the general methodology of the fuzzy control applied to Interval speed Continuous Petri Nets (ICPN) and give...
results for a general control law. Also, an illustrating example is given in section 5. And the last part is the conclusion.

2 Interval Speed Continuous Petri Nets

Petri nets are efficient for modeling, analysis and synthesis of discrete event systems. They offer advantages over finite automata, particularly when the DES has a high degree of complexity, concurrency, and synchronization. In a continuous Petri nets, the marking of a place is no longer an integer but a real positive or null number. A firing of a transition is carried out like a continuous flow. The continuous Petri nets models have been defined: the Constant Speed Continuous Petri Nets(CCPN), and the Variable Speed Continuous Petri Nets(VCPN). On the purpose of getting close to time Petri nets, Gu. gives a concept of Interval speed Continuous Petri Nets (ICPN) [4, 5]. In this paper, we will discuss ICPN model.

Definition 2.1[4,5] An interval speed continuous Petri nets(ICPN) is a 5-tuple: N=(P, T, Pre, Post, F), where

- P={ p1, p2, p3, ..., pn } is a set of continuous places;
- T={ t1, t2, t3, ..., tm } is a set of continuous transitions;
- P∩T=∅; i.e. the sets P and T are disjointed;
- Pre: T×P→R+ (or P×T→R+) is the transition (or place) input incidence mapping;
- Post: T×P→R+ (or P×T→R+) is the transition (or place) output incidence mapping;
- F: T→R+=(R+∪{∞}) is the flow or speed interval mapping. For any continuous transitions tj∈T, let F(tj)=[Vj, Vj′],with Vj≤Vj′, where Vj represents the minimal speed, and Vj′ represents the maximum speed.

In other CCPN, Vj must be 0 and not be a real number which bigger than 0. Thus, traditional CCPN can only describe those systems whose minimum of firing speed is 0, and can not describe those systems whose minimum of firing speed is bigger than 0. From definition 2.1, we can see that, in F(tj)=[Vj, Vj′], when Vj is 0, CCPN became traditional CCPN. The following is the enabled and firing rules of ICPN.

Definition 2.2 A transition tj∈T is called as strongly enabled at time τ if all input place pi∈′tj satisfy mj(τ)>0.

Definition 2.3 A transition tj∈T is called as 1-level enabled at time τ if ∀pj∈{ pj| mj(τ)>0 } satisfy

\[ \sum_{k} \text{Pre}(p_i, t_k) \cdot v_k(\tau) - \sum_{k \neq j} \text{Post}(p_i, t_k) \cdot v_k(\tau) \geq \text{Post}(p_i, t_j) \cdot V_j^{\text{min}} \]

Definition 2.4 A transition tj∈T is called as 0-level enabled at time τ if the following conditions are satisfied:

1. \( \exists p_i \in \{ p_j | m_j(\tau)=0 \} \) satisfy

\[ 0 < \sum_{k} \text{Pre}(p_i, t_k) \cdot v_k(\tau) - \sum_{k \neq j} \text{Post}(p_i, t_k) \cdot v_k(\tau) < \text{Post}(p_i, t_j) \cdot V_j^{\text{min}} \]

2. \( \forall p_h \in \{ p_j | m_j(\tau)=0 \} - \{ p_i \} \) satisfy

\[ \sum_{k} \text{Pre}(p_h, t_k) \cdot v_k(\tau) - \sum_{k \neq j} \text{Post}(p_h, t_k) \cdot v_k(\tau) \geq 0 \]

Definition 2.5 A transition tj∈Tc is called as disabled at time τ if tj is not an enabled transition.

Property 2.1 Strongly enabled transition tj∈Tc can be fired at the instantaneous firing speed \( v_j(\tau) \in [V_j^{\text{min}}, V_j^{\text{max}}] \).

Property 2.2 1-level enabled transition tj∈Tc can be fired at the instantaneous firing speed \( v_j(\tau) \in [V_j^{\text{min}}, \min(V_j^{\text{max}}, V_j')] \), where

\[ V_j^* = \min_i \{ \sum_k \text{Pre}(p_i, t_k) \cdot v_k(\tau) - \sum_{k \neq j} \text{Post}(p_i, t_k) \cdot v_k(\tau) | p_i \in \tau \} \]

Property 2.3 0-level enabled transition tj∈Tc can be fired after the time delay \( d=1/ V_j^{\text{min}} \) at the instantaneous firing speed \( v_j(\tau) \in [V_j^{\text{min}}, V_j^{\text{max}}] \), unless the transition is disabled before time \((\tau+d)\).

Different from the enabled and firing rules of traditional CCPN, the weak enabled transition of ICPN can be divided as one-level enabled transition and zero-level transition. And the zero-level transition can be fired after some time of delay, and can simulate Time Petri correctly.

3. Fuzzy model of ICPNs

The ICPN model is a good approximation of discrete time Petri nets when the numbers of entities in the system are very important but it has nonlinear dynamics and then, may be difficult to analyze and synthesize. A solution to this problem is to replace the ICPN model by a combination of linear subsystem
based on fuzzy model which has a simple structure for analysis and synthesis.

According to enabled and firing rules of ICPN, whether every transition $T_j$ can be fired critically depends on the acquired firing speed of this transition.

If $v_j \geq V_{j \min}$, that is, $T_j$ is strong enabled transition or one-level enabled transition, this transition can be spontaneously fired; if $T_j$ is 0-level enabled transition, it is necessary to wait for some time of $1/V_{j \min}$ and then can be fired. Due to this, there are following two rules for every transition $T_j$:

Rj1: if $v_j < V_{j \min}$ or the delay time is running, then $T_j$ can’t be fired;

Rj2: if $v_j \geq V_{j \min}$ or the delay time runs over running, then $T_j$ can be fired;

The fuzzy set of $v_j$ is given by: “weak” and “strong”, the relation refers to fig. 1 and formula (1) (2). The fuzzy set of the delay time $t$ of transition $T_j$ is given by: “run” and “done”, the relation refers to fig. 2 and formula (3) (4).

The fuzzy rule of ICPNs is given by:

R1: if $v_j$ is “weak” or $t$ is “run” then $S_1=0$;

R2: if $v_j$ is “strong” or $t$ is “done” then $S_2=1$;

Where $R_k$ denotes the $k$th rule of transition $T_j$, $S_k$ denotes the output of the $k$th rule. Then, we will obtain the global fuzzy system:

$$S_{res} = A \times 1 + B \times 0$$

Where $A = \max(\mu_{\text{strong}}(v_j), \mu_{\text{done}}(t))$, $B = \min(\mu_{\text{weak}}, \mu_{\text{run}}(t))$.

Since $A+B=1$, we have:

(1) if $v_j = 0$ or $t \in [0, \tau]$, then $S_{res}=0$, transition $T_j$ is disabled transition, $T_j$ can’t be fired;

(2) if $v_j \in [V_{j \min}, V_{j \max}]$ and $t \in [1/V_{j \min}, \infty)$

then $S_{res}=1$, since $V_j \in [V_{j \min}, V_{j \max}]$, transition $T_j$ is strong enabled transition or 1-level enabled transition, this case would not be happen;

(3) if $v_j \in [0, V_{j \min}]$ and $t \in [\tau, 1/V_{j \min}]$ then

$$S_{res} = \max(\frac{V_{j \min}}{v_j}, \frac{v_j}{V_{j \min}} \frac{t}{1-t}) \times 1$$

In this case, transition $T_j$ is 0-level but it’s delay time does not run over, $T_j$ can’t be fired;

(4) if $v_j \in [0, V_{j \min}]$ and $t \in [1/V_{j \min}, \infty)$, then

$$S_{res} = \max(\frac{V_{j \min}}{v_j}, 1) \times 1 = 1$$

In this case, Transition $T_j$ is 0-level and it’s delay time runs over, $T_j$ can be fired;

(5) if $v_j \in [V_{j \min}, V_{j \max}]$ and $t \in [\tau, 1/V_{j \min}]$, then

$$S_{res} = \max(\frac{V_{j \min}}{v_j}, \frac{v_j}{V_{j \min}} \frac{t}{1-t}) \times 1$$
\[ S_{\text{res}} = \max(1, \frac{f_{\text{min}}}{1 - \tau f_{\text{min}}} - \frac{f_{\text{min}}}{1 - \tau f_{\text{min}}}) \times 1 \quad (8) \]

This case would not be happened.

\( \text{(6) if} \ v_j \in [V_j^\text{min}, V_j^\text{max}] \text{and} \ t = 0, \text{then} \)
\[ S_{\text{res}} = \max(1,0) \times 1 = 1 \quad (9) \]

Transition \( T_j \) is strong enabled transition or 1-level enabled transition, \( T_j \) can be fired;

4. Fuzzy control of ICPNs

In this paper, we provide linear fuzzy local models that are interesting for the control design of the instantaneous transition firing speeds for the ICPNs. The purpose of the control design is to obtain for each transition a desired value for the marking of an output place.

The resulting fuzzy control of a transition is given by:
\( R_1 \): if \( v_j \) is “weak” or \( t \) is “run” then \( S' = u^1_j \);
\( R_2 \): if \( v_j \) is “strong” or \( t \) is “done” then \( S' = u^2_j \);

with \( u^1_j \) and \( u^2_j \) two local controls. From this and after defuzzification, we obtain the global fuzzy control law:
\[ S_{\text{res}} = \frac{A \times u^1_j + B \times u^2_j}{A + B} \quad (10) \]

Considering the minimum and maximum operators and the membership functions, we can write:
\[ m(t) = \max(\mu_{\text{strong}}(v_j), \mu_{\text{done}}(t)) (u^1_j - u^2_j) v_j + u^2_j v_j \]

Assumption 4.1 Suppose that \( u^1_j \) and \( u^2_j \) verify the next assumption:
\[ \lim_{t \to 0} \int_0^{\tau} \mu_{\text{max}}(v_j, \mu_{\text{done}}(t)) (u^1_j - u^2_j) v_j + u^2_j v_j \]

Where \( m_{d1} \) and \( m_{d2} \) are finite positive or null real.

Theorem 4.1 Suppose that assumption 4.1 is satisfied. Then, the solution of the global fuzzy system (5) converges i.e.:
\[ \lim_{t \to 0} \int_0^{\tau} \mu_{\text{max}}(v_j, \mu_{\text{done}}(t)) (u^1_j - u^2_j) v_j + u^2_j v_j \]

exists and is finite.

Proof: Since \( u^1_j \geq 0, u^2_j \geq 0, \max(\mu_{\text{strong}}(v_j), \mu_{\text{done}}(t)) \in [0,1] \), we obtain three cases:

First case \( u^1_j > u^2_j \):
\[ m(t) - m = \int_0^{\tau} \mu_{\text{max}}(v_j, \mu_{\text{done}}(t))(u^1_j - u^2_j) v_j + u^2_j v_j d\tau \]
\[ \leq \int_0^{\tau} (u^1_j - u^2_j) v_j d\tau + \int_0^{\tau} u^2_j v_j d\tau \]

because
\[ 0 \leq \max(\mu_{\text{strong}}(v_j), \mu_{\text{done}}(t)) \leq 1 \]

The function \( m(.) \) continuous on \( R \), it follows:
\[ \lim_{t \to 0} m(t) \leq \lim_{t \to 0} \int_0^{\tau} v_j d\tau + m_0 = m_{d1} + m_0 \]
which implies that \( m(.) \) converges in view of assumption 4.1.

Second case \( u^1_j \leq u^2_j \),
\[ m(t) - m = \int_0^{\tau} \mu_{\text{max}}(v_j, \mu_{\text{done}}(t))(u^1_j - u^2_j) v_j + u^2_j v_j d\tau \]
\[ \leq \int_0^{\tau} u^2_j v_j d\tau \]

because
\[ 0 \leq \max(\mu_{\text{strong}}(v_j), \mu_{\text{done}}(t)) \leq 1 \]

The function \( m(.) \) continuous on \( R \), it follows:
\[ \lim_{t \to 0} m(t) \leq \lim_{t \to 0} \int_0^{\tau} u^2_j v_j d\tau + m_0 = m_{d2} + m_0 \]

which implies that \( m(.) \) converges in view of assumption 4.1.

Third case \( u^1_j > u^2_j, t = t_1 \), and \( u^1_j \leq u^2_j, t = t_2 \): we have:
\[ m(t) - m = \int_0^{\tau} \mu_{\text{max}}(v_j, \mu_{\text{done}}(t))(u^1_j - u^2_j) v_j + u^2_j v_j d\tau \]
\[ = \int_0^{\tau} u^2_j v_j d\tau + \int_0^{\tau} (1 - A v_j^2) v_j d\tau \]
\[ \leq \int_0^{\tau} u^2_j v_j d\tau + \int_0^{\tau} v_j d\tau \]
where \( A = \max(\mu_{\text{strong}}(v_j), \mu_{\text{done}}(t)) \);

because
\[ 0 \leq \max(\mu_{\text{strong}}(v_j), \mu_{\text{done}}(t)) \leq 1 \]

The function \( m(.) \) continuous on \( R \), it follows:
\[ \lim_{t \to 0} m(t) \leq \lim_{t \to 0} \int_0^{\tau} u^2_j v_j d\tau + m_0 = m_{d2} + m_0 \]

Hence \( m(.) \) converges, which completes the proof.

Theorem 4.2 Suppose that assumption 1 is satisfied, then \( \lim_{t \to 0} m(t) \) has upper and lower bound.

Proof: First suppose \( u^1_j > u^2_j \), then we have
\[ m(t) - m_0 = \int_0^{\tau} \mu_{\text{max}}(v_j, \mu_{\text{done}}(t))(u^1_j - u^2_j) v_j + u^2_j v_j d\tau \]
So:
\[ m(t) - m \geq \inf_{\mu_{\text{done}}(t) \in [0,1]} \int_0^{\tau} A(v^1_j - v^2_j) v_j + u^2_j v_j d\tau \]
and:
\[ m(t) - m \leq \sup_{\mu_{\text{done}}(t) \in [0,1]} \int_0^{\tau} A(v^1_j - v^2_j) v_j + u^2_j v_j d\tau \]

Consequently:
\[ m_{d2} + m_0 \leq \lim_{t \to 0} m(t) \leq m_{d1} + m_0 \]

Where \( u^1_j < u^2_j \), we have
\[ m_{d1} + m_0 \leq \lim_{t \to 0} m(t) \leq m_{d2} + m_0 \]

Where \( u^1_j = u^2_j \), we have
5. Example

A chemical process with 4 units and 4 machines is shown in Figure 3. Two kinds of materials are processed in unit 1 and unit 2 respectively, and then fed to units 3. The feed flow from unit 1 to unit 3 through machine M1 is limited within $[2,3]$, and the feed from unit 2 to unit 3 through machine M2 within $[3,5]$. Intermediate product is fed from unit 3 to unit 4 through machine M1 at a flow of $[4,6]$. There are two output flows of unit 4, one is the final product flow at speed of $[3,4]$, and the other is the recycled flow to unit 3 through machine M4 at speed of $[1,2]$. The capacity of unit 3 is limited by 30, and its initial volume is 10. We assume that unit 1 and unit 2 have the sufficient materials for machine M1 and machine M2. As the minimum of the flow speed of machine is bigger than 0, we can not set up a model and analyze this chemical industry process. This process can be modeled as an ICPN shown in Figure 4.

$$m_{d1} + m_0 = \lim_{t \to \infty} m(t) = m_{d2} + m_0$$

The problems concerning this chemical industry process include:

1. whether the system is stable;
2. the product quantity of buffer zone B3 and B4 at any time;
3. whether the product quantity of limited buffer zone B3 is not bigger than the maximum capacity of the buffer zone at any time.

It is not possible to analyze stability of the system, basing on traditional methods or on the enabled and firing rules of ICPN. Using the method provided by Section 3 and 4 in this paper, we can set up fuzzy model for every transition, and then prove that every fuzzy model of transitions is convergent. Thus, the system is convergent, that is, the system is stable.

As the system is stable, using the model behavior evolving algorithm [5] of the ICPN, we can get the marks of the above three storeroom at any time $\tau$. We can get it by using following formula:

$$m_{1}(\tau) = \left\{ \begin{array}{ll} 10 + \int_{\tau}^{0} 4 dx & \tau \in [0, 0.5] \\ 30 + \int_{\tau}^{0} -1.8 dx & \tau \in [0.5, 5.0 + 0.5] \\ 29.1 + \int_{\tau}^{0} 0.2 dx & \tau \in [5.0 + 0.5, 5.0 + 5(k+1)] \end{array} \right. \quad (11)$$

$$m_{2}(\tau) = \left\{ \begin{array}{ll} 20 + \int_{\tau}^{0} -4 dx & \tau \in [0, 0.5] \\ 0 + \int_{\tau}^{0} -1.8 dx & \tau \in [0.5, 5.0 + 0.5] \\ 0.9 + \int_{\tau}^{0} 0.2 dx & \tau \in [5.0 + 5(k+1)] \end{array} \right. \quad (12)$$

$$m_{3}(\tau) = \left\{ \begin{array}{ll} 0 & \tau \in [0, 5] \\ \int_{5}^{\tau} 0.8 dx & \tau \geq 5 \end{array} \right. \quad (13)$$

That is, at any time $\tau$, the product quantity of buffer zone B3 = $m_{1}(\tau)$, the product quantity of buffer zone B4 = $m_{2}(\tau)$, $k$ is positive integer.

From formula (11) and (12), we can know that, at any time $\tau$, $m_{1}(\tau) + m_{2}(\tau) = 30$. Thus, the product quantity of buffer zone B3 is smaller than or equal to the maximum capacity of the buffer zone at any time.

6. Conclusion

In this paper, a fuzzy model of ICPN is proposed, and the definition of the fuzzy rule of the ICPN is presented. Also, the fuzzy control of the ICPN is investigated, and the theorem marked and restrained by the collection house is proved. The fuzzy models of each transition in ICPN are made from four fuzzy rules. We have shown that if each dynamical local fuzzy system converges, the resulting global fuzzy system converges. Moreover, upper and lower bounds of this convergence have been derived which is...
important for engineering application and stability properties. The example shows that the model of ICPN provided in this paper can effectively describe a group of systems. These systems can not be described by traditional CPN. Moreover, using the fuzzy model of ICPN provided in this paper, we can effectively achieve the judgment on the stability of the system, and then analyze other capabilities of the system effectively. And it is important to point out that all these results can be generalized to any traditional CPN.

References

[1] R. David and H. Alla: Continuous Petri Nets, 8th European Workshop on Applications and Theory of Petri Nets, Saragosses (E), pp275-294, Juin 1987.
[2] J. LE Ball, H. Alla and R. David: Asymptotic Continuous Petri Nets, J. of Discrete Event Dynamic Systems: Theory and Applications, vol. 2, pp235-263, 1993.
[3] M. Haounani and D. Lefebvre: Variable Speed Continuous Petri Net Grindeld, Switzerland: Proc. of the 17th IASTED Int. Conf. Modeling, Identification And Control, 1998.
[4] T. I. Gu, R. S. Dong, Y. C. Tian: Continuous Petri Nets Augmented with Maximal and Minimal Firing Speeds. IEEE International Conference on Systems, Man and Cybernetics (SMC), 2003.
[5] T. I. Gu and R. S. Dong: Novel Continuous Model to Approximate Time Petri Nets: Modeling And Analysis, Journal of Application Mathematic and Computer Science, Vol.15, No.1, pp141-150, 2005.
[6] S. Hennequin, D. Lefebvre and A. Elmoudini: Fuzzy Control Of Variable Continuous Petri Nets[C]. Phoenix, Arizona USA: Proceeding of the Conference on Decision & Control, pp. 1352-1356, 1999.

Yueying Peng was born in Yizhou city of Guangxi Autonomous Region, China. She received the Diploma of Mathematics from Guangxi University, China, in 1979. Now, she is a visiting scholar in the Lab of Professor Hirota, in Department of Computational Intelligence and Systems Science Interdisciplinary Graduate School of Science and Engineering Tokyo Institute of Technology.

Weizhi Liao (1974- ), male, Zhuang minority, From Guangxi Fengshan County, Xi'an electron scientific and technical university Doctor graduate student, mainly engaged in research of formal technology and its application, hybrid system modelling and analysis and so on.

Kaoru Hirota, male, Professor. Department of Computational Intelligence and Systems Science Interdisciplinary Graduate School of Science and Engineering Tokyo Institute of Technology, Tokyo, 226-8502, Japan