Calculation of current distribution in tungsten plate under exposure to a pulsed electron beam

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Abstract. In the mathematical model of melting of a tungsten plate exposed to a pulsed electron beam, the calculation of the current distribution in the medium is added. The model takes into account the heterogeneity of the resistivity. This enables modeling of non-uniform heated material. The current is expected to spread into the depth due to the increased resistance of the heated part. Changing the thickness of the tungsten plate, one can increase the current density in the melt. The calculation results for parameters corresponding to those used on the BETA facility at BINP SB RAS show that no current concentration occur. The Ampere force is not large enough for the rotation observed in the experiments.

1. Introduction
Currently, due to the development of thermonuclear fusion technologies, new practical problems of melting of refractory metals are emerging. To plan and analyze full-scale experiments it is necessary to have mathematical models that accurately take into account all the details and features of the process. The experimental facility Beam of Electrons for materials Test Applications (BETA), created at BINP SB RAS [1], yielded new results of evaporation of tungsten from the surface of a plate exposed to an electron beam. The full-scale experiment is constantly accompanied by the numerical one [2]. The tungsten heating model is based on solving the two-phase Stefan problem for temperature and electrodynamics equations in the sample region (solid metal and melt). The position of the phase boundary depends on discontinuous nonlinear coefficients. The condition at the free melt-solid boundary consists in the tungsten melting temperature of 3695K. The aim of the study is to simulate the erosion of the sample surface because of evaporation and penetration of heat flow deeply into the material, taking into account the rotation of the melt. Construction of a complete model of electron beam heating enables better assessment of the relevance of the simulation and determination of mechanisms that will occur in the case of plasma flow heating in modern plasma and future thermonuclear facilities.
2. Problem definition
In the mathematical model of melting in a tungsten plate exposed to a pulsed electron beam [3], the calculation of the current distribution in the medium is added. It is assumed that the characteristic time of change of the electric field is large as compared with the equilibration time of the electrodynamics equations in the scale of the problem. This means that the time derivatives are small and can be ignored; the time derivative in the charge continuity equation can also be neglected. These assumptions allow us to introduce the vector potential of the current: \( \mathbf{\nabla} \times \mathbf{F} \). Because of the cylindrical symmetry, the current can be described solely with the function \( F = F_\varphi \), which describes the "vorticity" of the current.

Consider the following heat equation with surface heating and current potential:

\[
\begin{align*}
\frac{c(T)\rho(T)\partial T}{\partial t} = & \frac{1}{r} \frac{\partial}{\partial r} r \lambda(T) \frac{\partial T}{\partial r} + \frac{\partial}{\partial z} \lambda(T) \frac{\partial T}{\partial z}, \\
J_r = & -\frac{\partial F}{\partial z}, \quad J_\varphi = 0, \quad J_z = -\frac{1}{r} \frac{\partial (r F)}{\partial r}, \\
\frac{\partial^2 F}{\partial r^2} + & \frac{\partial^2 F}{\partial z^2} + \frac{\partial F}{\partial r} \left( \frac{1}{\rho_{el}} \frac{\partial \rho_{el}}{\partial r} + \frac{1}{r} \right) + \frac{\partial F}{\partial z} \frac{1}{\rho_{el}} \frac{\partial \rho_{el}}{\partial z} + F \left( \frac{1}{r \rho_{el}} \frac{\partial \rho_{el}}{\partial r} + \frac{1}{r^2} \right) = 0, \\
(n, n T)|_\gamma = & \frac{W(t,r)}{\lambda(T)}, \quad F|_\gamma = \frac{1}{r} \int_0^r j_0(r')dr', \\
(n, n T) = 0, \quad F = 0 & \text{at the other boundaries,} \quad T(r, z, t)|_{z=0} = T_0(r, z), \quad F(r, z, t)|_{z=0} = F_0(r, z).
\end{align*}
\] (1)

where \( T(r, z, t) \) is the temperature, \( \mathbf{J} = (J_r, J_\varphi, J_z) \) is the current density, \( F(r, z, t) \) is the current potential, \( c(T) \) is the specific heat, \( \rho(T) \) is the density, \( \lambda(T) \) is the thermal conductivity, \( \rho_{el} \) is the electrical resistivity, \( W(t, r) \) is the power of the heat flux on the surface \( \gamma \), \( n \) is the normal to the surface, and \( T_0, j_0 \) and \( F_0 \) is the initial temperature, current and current potential respectively.

The conditions on the free boundary are \([T]|_{s=0} = 0\) and \([\lambda(T)\frac{\partial T}{\partial r}]|_{s=0} = L_m v_n\), where \( L_m \) is the phase transition enthalpy, and \( v_n \) is the velocity of the phase transition boundary.

In the experiments on the BETA facility, samples of rolled tungsten were exposed to an axisymmetric electron beam. Electrons with an energy of \( 80 - 90 \text{keV} \) heated the material in a layer thin as compared with the characteristic depth of heating of the material. The phase transitions inherent of the problem under consideration are included in the temperature coefficients in Eq. (1). The density, thermal conductivity, specific heat, and power loss are given as dependencies on the temperature of the material in the range of \( 300 \text{K} \leq T \leq 8000 \text{K} \). These functions have discontinuities or lose smoothness at the melting point \( T_m = 3695 \text{K} \). It is necessary to choose the function \( \lambda(T) \) at large temperature values such that there be no division by zero. The thermal conductivity and heat capacity of solid tungsten are taken from [4]. The estimates for the thermal conductivity of liquid tungsten are taken from [5, 6]. The electrical resistance is taken from [7] (see Fig. 1):

\[
\rho_{el}(T(\mathbf{r}, t)) = \begin{cases} 
-0.968 + 1.9274 \cdot 10^{-2} T + 7.826 \cdot 10^{-6} T^2 - \\
-1.8517 \cdot 10^{-9} T^3 + 2.079 \cdot 10^{-13} T^4, & 100 \leq T \leq 3695, \\
135 - 1.855 \cdot 10^{-3} (T - T_m) + 4.42 \cdot 10^{-6} (T - T_m)^2, & 3695 \leq T \leq 6000.
\end{cases}
\] (2)

The boundary condition for \( F \) was \( F|_{z=0} = \frac{1-\exp(-Ar^2)}{2Ar} \), which matches the spatial distribution of the heating power. Because of the linearity of the equations, we can omit the coefficient proportional to the peak current density \( j_0(0,0) \).

It is more convenient to proceed in non-dimensional variables, for example, as follows:

\[
r^* = \frac{r}{r_0}, \quad \lambda^* = \frac{\lambda}{\lambda_0}, \quad \rho^* = \frac{\rho}{\rho_0}, \quad c^* = \frac{c}{c_0}, \quad t^* = \frac{t}{t_0}, \quad T^* = \frac{T}{T_0}, \quad W^* = \frac{\lambda_0 T_0 W}{r_0}
\] (3)
Figure 1. Graphs of temperature dependences of electrical resistance that were used in numerical simulation.

Table 1. Dimensional parameters of nondimensionalization.

| Parameter | Typical value | Units     |
|-----------|---------------|-----------|
| $r_0$     | $10^{-1}$     | mm        |
| $t_0$     | $10^2$        | µs        |
| $\lambda_0$ | $10^{-1}$ | W/mm·K    |
| $\rho_0$  | $10^{-5}$     | kg/mm$^3$ |
| $c_0$     | $10^8$        | W·µs/kg·K |
| $T_0$     | $10^3$        | K         |
| $W_0$     | $10^3$        | W/mm$^2$  |
| $j_0$     | $10^4$        | A/m$^2$   |

3. Numerical simulation

For the melting process to be taken into account, the enthalpy of the phase transition $L_m$ was introduced into the equation for temperature [8]:

$$\left(c(T)\rho(T) + L_m\delta(T, \varepsilon)\right) \frac{dT}{dt} = \text{div} \left(\lambda(T)\text{grad}T\right),$$

(4)

where the melting point is $T_m = 3695K$.

$$\delta(T, \varepsilon) = \begin{cases} \frac{1}{\varepsilon}, & |T - T_m| \leq \varepsilon, \\ 0, & |T - T_m| > \varepsilon, \end{cases}$$

with the smoothing interval $[-\varepsilon, \varepsilon]$, $\varepsilon = 5K$ and the heat of fusion $L_m = 51.1 \cdot 10^5\frac{W\mu s}{mm^3}$.

The process of evaporation at the boundary is taken into account via a given resultant energy flux $W_\gamma(t, r) = W(t, r) - N(T|\gamma)$, where $W(t, r)$ is the power of the heat flux (see (2)), $N(T|\gamma) = L_e \frac{dm}{d\gamma}$ is the power loss (Figure 1d), $T|\gamma$ is the boundary temperature, $P(T)$ is the saturated vapor pressure, and $\frac{1}{S} \frac{dm}{d\gamma}$ is the mass evaporation rate. The power loss is calculated with the following parameter values:

$$L_e = 4.482 \cdot 10^{12} \frac{W\mu s}{kg},$$

$$\frac{1}{S} \frac{dm}{d\gamma} = P(T|\gamma) \sqrt{\frac{M}{2\pi RT|\gamma}}} = \exp \left(26.19104 - \frac{83971.3K}{T|\gamma}\right) \sqrt{\frac{0.184K}{2\pi 8.314 T|\gamma}} 10^{-12} \frac{kg}{mm^2·µs}. \quad (5)$$
The derivation of Eq. (4) and justification of neglecting the surface temperature radiation in the energy balance are presented in more detail in [9]. The numerical implementation is based on the Douglas-Rachford scheme [10]. The Neumann boundary condition for the temperature and the Dirichlet boundary condition for the potential on the heated surface are used.

The power of the heat flux is

$$W(t,r) = W_{max}(t) \cdot \exp(-Ar^2), \quad A = 0.0308523. \quad (6)$$

At each time step in the numerical simulation, the variable $W_{max}(t)$ is taken from the experimental data file, individual for each heating pulse. The distribution of the heating power density over the surface was measured using X-ray visualization [11].

4. Simulation results

On the BETA facility, several photographs of the sample surface were taken during pulsed heating and cooling after exposure. The melt regions are experimentally registered, which can be used for estimation of the accuracy of calculations. It is shown that the evaporation model accurately describes the movement of the free boundary of the molten metal during cooling [1].

The heat absorbed by the surface propagates into the material. The current is expected to spread into the depth because of the increased resistance of the heated part. The model takes into account the heterogeneity of the resistivity, which enables simulation of inhomogeneously heated material. Changing the thickness of the sample, one can increase the current density in the melt. The sample for shot 688 has a $25\text{mm} \times 25\text{mm}$ cross section and a thickness of 4\text{mm}. Since the sample is heated to a depth of several hundred microns in such a short time, the simulation region was a transverse section of the sample, a region of $12\text{mm} \times 4\text{mm}$. As for the time, the numerical simulation continued until the moment of the last measurement.

Comparison of the calculated and experimental temperature on the surface of the plate is given in paper [12]. Calculation results for the radial temperature distribution over the sample surface are consistent with experimental data and analytical estimates.

Figures 2–5 shows isolines of temperature and isolines in the selected area, of current potential $rF$, module current and the normalized radial component of the current in the plane $(r,z)$ calculated at $150\mu$s. For shot 688, the heating lasts for $119\mu$s. At $150\mu$s, cooling occurs and the surface temperature drops to 3000K, 3000$^\circ$K.
Now we can study the effect of the Ampere force on the rotation. We know the current flow on the surface, but the rotation causes the radial component of the current in the melt, since it is perpendicular to the magnetic field. The density of the fluid depends on the temperature, but it can be estimated as $\rho = 1.6 \cdot 10^4 \text{kg/m}^3$. From the graph in Fig 2d, one can see that the maximum radial component in the fluid is $j_r \approx j_0$. The magnetic field in the setup can vary from shot to shot, and that time it was equal to $B = 1T$. On this basis, the acceleration can be calculated as $g = \frac{\mu j_r}{\rho} \approx 1 \text{m/s}^2$. This is less than the acceleration of gravity, which means that we will not rotate the melt.

5. Conclusion
In the mathematical model of melting in a tungsten plate exposed to a pulsed electron beam, the calculation of the current distribution in the medium is added. The model takes into account the heterogeneity of the resistivity. This enables modeling of non-uniformly heated material. The current is expected to spread into the depth due to the increased resistance of the heated part. Changing the thickness of the tungsten plate, one can increase the current density in the melt. The results of the calculations coincide with the qualitative assessments.

The calculation results for the parameters corresponding to those used on the BETA installation show that no current concentration occur. The Ampere force is not large enough for the rotation observed in the experiments. This result is important for constructing a theoretical model of electron beam heating because there are no other forces that could rotate the melt. This may imply occurrence of current redistribution above the surface, which was not taken into account earlier. The constructed mathematical model is planned to be integrated with the calculation of gas dynamics over a tungsten plate and currents in the gas. The construction of a complete model of electron beam heating will enable better assessment of the relevance of the simulation and finding out which mechanisms will occur in the case of plasma flow heating in modern plasma and future thermonuclear installations.

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