Modulation of waves due to charge-exchange collisions in magnetized partially ionized space plasma

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Abstract

A nonlinear time dependent fluid simulation model is developed that describes the evolution of magnetohydrodynamic waves in the presence of collisional and charge exchange interactions of a partially ionized plasma. The partially ionized plasma consists of electrons, ions and a significant number of neutral atoms. In our model, the electrons and ions are described by a single fluid compressible magnetohydrodynamic (MHD) model and are coupled self-consistently to the neutral gas, described by the compressible hydrodynamic equations. Both the plasma and neutral fluids are treated with different energy equations that describe thermal energy exchange processes between them. Based on our self-consistent model, we find that propagating Alfvénic and fast/slow modes grow and damp alternately through a nonlinear modulation process. The modulation appears to be robust and survives strong damping by the neutral component.

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I. INTRODUCTION

Alfvén, fast and slow mode waves are characteristics of magnetohydrodynamic (MHD) plasma. While the electromagnetic fluctuations in Alfvén waves propagate predominantly along an ambient or guide constant magnetic field, the fast/slow modes propagates isotropically. Although the linear properties of these waves are fairly well understood [16–18], their nonlinear evolution in a complex partially ionized environment, as occurs often in space/astrophysical plasmas, remains relatively uninvestigated. In partially ionized (consisting of ions, electrons and a significant neutral gas presence) space and astrophysical plasmas, these waves can interact with the neutral gas through processes such as charge exchange, collisions, ionization and recombination etc. Hence the wave properties are expected to be modified. It is however unclear how Alfvén, fast and slow waves modify the nonlinear processes that govern plasma fluctuations in the presence of neutral dynamics. In the context of cosmic ray propagation, Kulsrud & Pearce (1969) noted that the interaction of a neutral gas and plasma can damp Alfvén waves. Neutrals interacting with plasma via a frictional drag process results in ambipolar diffusion (Oishi & Mac Low 2006). Ambipolar diffusion plays a crucial role in the dynamical evolution of the near solar atmosphere, interstellar medium, and molecular clouds and star formation. Oishi & Mac Low (2006) found that ambipolar diffusion can set a characteristic mass scale in molecular clouds. They found that structures less than the ambipolar diffusion scale are present because of the propagation of compressive slow mode MHD waves at smaller scales. Leake et al (2005) showed that the lower chromosphere contains neutral atoms, the existence of which greatly increases the efficiency of wave damping due to collisional friction momentum transfer. They noted that Alfvén waves with frequencies above 0.6Hz are completely damped and frequencies below 0.01 Hz are unaffected. They undertook a quantitative comparative study of the efficiency of the role of (ion-neutral) collisional friction, viscous and thermal conductivity mechanisms in damping MHD waves in different parts of the solar atmosphere. They found that a correct description of MHD wave damping requires the consideration of all energy dissipation mechanisms through the inclusion of the appropriate terms in the generalized Ohms law, the momentum, energy and induction equations. Padoan et al (2000) calculated frictional heating by ion-neutral (or ambipolar) drift in turbulent magnetized molecular clouds and showed that the ambipolar heating rate per unit volume depends on field strength for constant rms
Mach number of the flow, and on the Alfvénic Mach number.

The role of ion-neutral collisions has been investigated extensively by Balsara (1996) in the context of molecular clouds. Here the momentum equation for the plasma is governed predominantly by the slow comoving massive neutrals that tend to dissipate the Cloud’s magnetic field. Because of the larger neutral to ion mass ratio, ion momentum is entirely dominated by the neutral drag. Hence the inertial terms were ignored in plasma momentum equation. Balsara (1996) found that slow waves propagate without significant damping on short wavelengths, while the fast and Alfvén waves undergo rapid damping in super-Alfvénic regimes.

In the heliospheric boundary regions, particularly the outer heliosheath [13, 20], the plasma is mediated by interstellar neutral gas by virtue of charge exchange. For example in the local interstellar medium (LISM), the low density plasma and neutral hydrogen (H) gas are coupled primarily through the process of charge exchange. In this process, both the total (proton+atom) momentum and energy are conserved. On sufficiently large temporal and spatial scales, a partially ionized plasma is typically regarded as equilibrated; this is the case for the LISM, but not for the outer heliosheath, for example.

It is worth mentioning that the charge exchange process in its simpler (and leading order) form can be treated like a friction or viscous drag term in the fluid momentum equation, describing the relative difference in the ion and neutral fluid velocities. The drag imparted in this manner by a collision between ion and neutral also causes ambipolar diffusion, a mechanism used to describe Alfvén wave damping by cosmic rays (Kulsrud & Pearce 1969) and also discussed by Oishi & Mac Low (2006) in the context of molecular clouds.

In this Letter, we focus on understanding the propagation characteristics of MHD waves by considering collisional and charge exchange interactions thus including both simultaneously. Our model is more general and fully nonlinear, compared to that of Balsara (1996) which ignores nonlinear inertial terms in the plasma momentum equation. We retain this term and carry out fully time-dependent simulations in two dimensions. We treat plasma and neutral fluid in a self-consistent manner by employing the complete nonlinear time-dependent fluid equations for both the fluids. In Section 2, we discuss the equations of a coupled plasma-neutral model, their validity, the underlying assumptions and the normalizations. Section 3 describes results of our nonlinear, coupled, self-consistent fluid simulations. We find that the propagation characteristics of Alfvén waves are altered significantly by
the combined action of charge exchange and collisional interaction processes. The latter leads to a modulated growth and damping of the Alfvén and fast/slow modes. In Section 4, we summarize our results and describe physical processes that may be responsible for the modulation process.

II. PARTIALLY IONIZED MODEL EQUATIONS

We assume that fluctuations in the plasma and neutral fluids are isotropic, homogeneous, thermally equilibrated and turbulent. A constant mean magnetic field is present. Local mean flows and nonlinear structures may subsequently be generated by self-consistently excited nonlinear instabilities. The boundary conditions are periodic. This allows us to assume an infinite partially ionized plasma where the length scale of characteristic fluctuations (ℓ) is much smaller than the size of plasma or computational box (L) i.e. ℓ ≪ L. We further assume that plasma particles interact with the neutral gas via collisions as well as charge exchange. The intrinsic magnetized waves supported by the plasma scatter plasma particles in a random manner thus sufficiently isotropizing the plasma and neutral distribution functions [8]. The latter enables us to use fluid descriptions for both the plasma and neutral gas. Most of these assumptions are appropriate to realistic space and astrophysical turbulent flows. They allow us to use MHD and hydrodynamic descriptions for the plasma and the neutral components respectively. In the context of the LISM and the outer heliosheath plasmas, the plasma and neutral fluid remain close to thermal equilibrium and behave as Maxwellian fluids. Our model simulates a plasma-neutral fluid that is coupled via collisions and charge exchange in astrophysical plasmas. The fluid model describing nonlinear turbulent processes, in the presence of charge exchange and collision, can be described in terms of the plasma density (ρₚ), velocity (Uₚ), magnetic field (B), pressure (Pₚ) components according to the conservative form

$$\frac{\partial F_p}{\partial t} + \nabla \cdot Q_p = Q_{p,n},$$

(1)
where,

$$F_p = \begin{bmatrix} \rho_p \\ \rho_p U_p \\ B \\ \epsilon_p \end{bmatrix}, \quad Q_p = \begin{bmatrix} \rho_p U_p \\ \rho_p U_p U_p + \frac{P_p}{\gamma - 1} + \frac{B^2}{8\pi} - BB \\ U_p B - BU_p \\ \epsilon_p U_p - B(U_p \cdot B) \end{bmatrix},$$

$$Q_{p,n} = \begin{bmatrix} 0 \\ Q_{M,p,n} + F_{p,n} \\ 0 \\ Q_{E,p,n} + U_p \cdot F_{p,n} \end{bmatrix}$$

and

$$e_p = \frac{1}{2} \rho_p U_p^2 + \frac{P_p}{\gamma - 1} + \frac{B^2}{8\pi}.$$  

Note the presence of the source terms $Q$ that couple the plasma self-consistently to the neutral gas. The above set of plasma equations is supplemented by $\nabla \cdot B = 0$ and is coupled self-consistently to the neutral density ($\rho_n$), velocity ($V_n$) and pressure ($P_n$) through a set of hydrodynamic fluid equations,

$$\frac{\partial F_n}{\partial t} + \nabla \cdot Q_n = Q_{n,p},$$

where,

$$F_n = \begin{bmatrix} \rho_n \\ \rho_n V_n \\ e_n \end{bmatrix}, \quad Q_n = \begin{bmatrix} \rho_n V_n \\ \rho_n V_n V_n + \frac{P_n}{\gamma - 1} \\ e_n V_n \end{bmatrix},$$

$$Q_{n,p} = \begin{bmatrix} 0 \\ Q_{M,n,p} + F_{n,p} \\ Q_{E,n,p} + V_n \cdot F_{n,p} \end{bmatrix},$$

$$e_n = \frac{1}{2} \rho_n V_n^2 + \frac{P_n}{\gamma - 1}.$$  

Equations (1) to (2) form an entirely self-consistent description of the coupled plasma-neutral turbulent fluid in a partially ionized medium.

Several points are worth noting. The charge-exchange momentum sources in the plasma and the neutral fluids, i.e. Eqs. (1) & (2), are described respectively by
terms $Q_{M,p,n}(U_p, V_n, \rho_p, \rho_n, T_n, T_p)$ and $Q_{M,n,p}(V_n, U_p, \rho_p, \rho_n, T_n, T_p)$. These expressions, described in [11, 13], have the following form.

$$Q_{M,p,n}(V_n, U_p) = m\sigma n n_n (V_n - U_p) \left[ U^* + \frac{V^2_{T_n}}{\delta V_{U_p, V_n}} - \frac{V^2_{T_p}}{\delta V_{V_n, U_p}} \right],$$

(3)

$$Q_{E,p,n}(V_n, U_p) = \frac{1}{2} m\sigma n n_n U^* (V^2_n - U^2_p) + \frac{3}{4} m\sigma n n_n (V^2_{T_n} \Delta V_{U_p, V_n} - V^2_{T_p} \Delta V_{V_n, U_p})$$

$$- m\sigma n n_n \left[ V_n \cdot (U_p - V_n) \frac{V^2_{T_n}}{\delta V_{U_p, V_n}} - U_p \cdot (V_n - U_p) \frac{V^2_{T_p}}{\delta V_{V_n, U_p}} \right],$$

(4)

with

$$U^* = U^*_{U_p, V_n} = U^*_{V_n, U_p} = \left[ \frac{4}{\pi} V^2_{T_p} + \frac{4}{\pi} V^2_{T_n} + \Delta U^2 \right].$$

A swapping of the plasma and the neutral fluid velocities in this representation corresponds, for instance, to momentum changes (i.e. gain or loss) in the plasma fluid as a result of charge exchange with the neutral atoms (i.e. $Q_{M,p,n}(U_p, V_n, \rho_p, \rho_n, T_n, T_p)$ in Eq. (1)). Similarly, momentum change in the neutral fluid by virtue of charge exchange with the plasma ions is described by $Q_{M,n,p}(V_n, U_p, \rho_p, \rho_n, T_n, T_p)$ in Eq. (2).

The plasma-neutral collisional forces are modeled by employing the following forcing term in Eqs. (1) & (2).

$$F_{p,n} = \nu \rho_p \rho_n (U_p - V_n) = - F_{n,p},$$

(5)

where $\nu$ is the collision frequency. The collisions modify plasma and neutral momentum and energy, but not the density. The negative sign before $F_{n,p}$ in Eq. (5) ensures conservation of total momentum. We derive modified equations of energy for both fluids following the treatment described in [12]. The collisional force is self-consistently calculated in our model. Note also the charge exchange source term in momentum and energy of both the fluids. In the absence of collision and charge exchange interactions, the plasma and the neutral fluid are de-coupled trivially and behave as ideal fluids. Like collisions, charge-exchange interactions modify the momentum and the energy of plasma and the neutral fluids and conserve density in both the fluids (since we neglect photoionization and recombination). Nonetheless, the volume integrated energy and the density of the entire coupled system will remain conserved in a statistical manner. The conservation processes can however be altered in the presence of any external forces. These can include large-scale random driving of turbulence due to external forces such as supernova explosions, stellar winds, etc and instabilities. Finally,
the magnetic field evolution is governed by the usual induction equation, i.e. Eq. (1), that obeys the frozen-in-field theorem unless some nonlinear dissipative mechanism introduces small-scale damping.

The underlying coupled fluid model can be non-dimensionalized straightforwardly using a typical scale-length ($\ell_0$), density ($\rho_0$) and velocity ($v_0$). The normalized plasma density, velocity, energy and the magnetic field are respectively: $\tilde{\rho}_p = \rho_p / \rho_0, \tilde{U}_p = U_p / v_0, \tilde{P}_p = P_p / \rho_0 v_0^2, \tilde{B} = B / v_0 \sqrt{\rho_0}$. The corresponding neutral fluid quantities are $\tilde{\rho}_n = \rho_n / \rho_0, \tilde{U}_n = U_n / v_0, \tilde{P}_n = P_n / \rho_0 v_0^2$. The momentum and the energy charge-exchange terms, in the normalized form, are respectively $\tilde{Q}_m = Q_m \ell_0 / \rho_0 v_0^2, \tilde{Q}_e = Q_e \ell_0 / \rho_0 v_0^3$. The non-dimensional temporal and spatial length-scales are $\tilde{t} = t v_0 / \ell_0, \tilde{x} = x / \ell_0$. Note that we have removed bars from the set of normalized coupled model equations (1) & (2). The charge-exchange cross-section parameter ($\sigma$), which does not appear directly in the above set of equations, is normalized as $\tilde{\sigma} = n_0 \ell_0 \sigma$, where the factor $n_0 \ell_0$ has dimension of (area)$^{-1}$. By defining $n_0, \ell_0$ through $\sigma_{ce} = 1 / n_0 \ell_0 = k_{ce}^2$, we see that there exists a charge exchange mode ($k_{ce}$) associated with the coupled plasma-neutral turbulent system. For a characteristic density, this corresponds physically to an area defined by the charge exchange mode being equal to $(mpf)^2$. Thus the larger the area, the higher is the probability of charge exchange between plasma ions and neutral atoms. Therefore, the probability that charge exchange can directly modify those modes satisfying $k < k_{ce}$ is high compared to modes satisfying $k > k_{ce}$. Since the charge exchange length-scales are much smaller than the turbulent correlation scales, this further allows many turbulent interactions amongst the nonlinear turbulent modes before they undergo at least one charge exchange. An exact quantitative form of sources due to charge exchange in our model is taken from Shaikh & Zank (2008).

III. EVOLUTION OF MHD WAVES

We now investigate the nonlinear evolution of the coupled plasma-neutral system described by Eqs. (1) & (2). Our goal is to understand the evolution of MHD waves in a partially ionized turbulent plasma, mediated by the complex coupling between the two distinguishable fluids. For this purpose, we initialize the field variables (i.e. velocity, magnetic, density, energy and pressure fields) in the plasma and neutral fluids with a random, uncorrelated, out-of-phase distribution in space. The initial ratio of the plasma and neutral
density is unity. The plasma and neutral fluids initialized in this manner characterize spatially local turbulence in the partially ionized astrophysical plasma. These random fields develop spatially and temporally according to the set of Eqs. (1) & (2) and forces due to charge exchange and collision dictate nonlinear interaction processes. In ideal MHD and hydrodynamic fluids (i.e. without the coupling forces in Eqs. (1) & (2)), known quadratic conserved quantities like energy, helicity, vorticity etc predominantly govern the spectral transfer of energy across disparate characteristic scales in the inertial range. Their role is well studied within the paradigm of statistical theories of turbulence [2–5, 8]. By contrast, the complex nature of the coupling forces in Eqs. (1) & (2) poses a formidable hurdle to deriving quadratic conserved quantities. It is not clear to us as how these quantities will influence spectral transfer in the inertial range. Furthermore, the driving forces in Eqs. (1) & (2) may presumably be operative at any length and time scales. It is unclear how this will alter the conventional dissipative and diffusive processes in a partially ionized plasma. Unlike the constant drag force (i.e. not changing in space and time) used before for a neutral fluid; and which acts to damp Alfvén waves in the partially ionized plasma (Kulsrud & Pearce 1969, Balsara 1996, Shaikh 2010), it is not clear how the complex, nonlinear, time and space dependent driving forces in Eqs. (1) & (2) will modify the propagation and interactions characteristic of Alfvén, fast and slow MHD waves or even compressive sound waves in the coupled neutral gas.

To address the above issues, we have developed a two-dimensional (2D) nonlinear fluid code to numerically integrate Eqs. (1) & (2). The 2D simulations are not only computationally less expensive (compared to a fully 3D calculation), but they offer significantly higher resolution (to compute inertial range turbulence spectra) even on moderately-sized clusters like our local Beowulf system. The spatial discretization in our code uses a discrete Fourier representation of turbulent fluctuations based on a spectral method, while we use a 4th-order Runge-Kutta method for the temporal integration. All the fluctuations are initialized isotropically with random phases and amplitudes in Fourier space. A mean constant magnetic field $B_0$ is assumed along the $y$-direction. Our algorithm ensures conservation of total energy and mean fluid density per unit time in the absence of charge exchange and external random forcing. Additionally, $\nabla \cdot B = 0$ is satisfied at each time step. Our code is massively parallelized using Message Passing Interface (MPI) libraries to facilitate higher resolution. The initial isotropic turbulent spectrum of fluctuations is chosen to be close to
FIG. 1: Evolution of modal energy in Alfvén and fast/slow waves is shown by their respective average mode behavior. The oscillatory behavior of the average mode associated with Alfvén and fast/slow indicates that charge exchange and collisions with neutrals damp the Alfvén and fast/slow wave which are then reexcited by an instability (growth). This process repeats itself during the entire evolution.

FIG. 2: Energy exchange rate between the plasma and neutral fluids shows that the rate of transfer of energy between the two fluids is constant. We plot the corresponding energy equations from Eqs (1) & (2).

$k^{-2}$ with random phases in all three directions. The choice of such a (or even a flatter than -2) spectrum does not influence the dynamical evolution as the final state in our simulations progresses towards fully developed turbulence.

From our time-dependent nonlinear fluid simulations, we determine the quantitative evolu-
tion of Alfvén and fast/slow waves in a partially ionized environment. For this purpose, we distinguish Alfvénic and non-Alfvénic, i.e. corresponding to the compressional or due to slow and fast magnetosonic modes, contributions to the turbulent velocity fluctuations. Thus, we introduce diagnostics that distinguish between the modes corresponding to Alfvénic and slow/fast magnetosonic fluctuations. Since the Alfvénic fluctuations are transverse \[2, 13\], the propagation wave vector is orthogonal to the velocity field fluctuations i.e. \( \mathbf{k} \perp \mathbf{U} \), and the average spectral energy contained in these (shear Alfvénic modes) fluctuations can be computed as \[14\]

\[
\langle k_A(t) \rangle \approx \sqrt{\frac{\sum_k |i\mathbf{k} \times \mathbf{U}_k|^2}{\sum_k |\mathbf{U}_k|^2}}. \tag{6}
\]

The above relationship yields a finite spectral contribution from the \(|\mathbf{k} \times \mathbf{U}_k|\) characteristic turbulent Alfvénic modes. It is essentially a modal energy associated with the Alfvénic fluctuations that obey Eq. (6) in the inertial range turbulent spectrum. On the other hand, fast/slow (i.e. compressive) magnetosonic modes propagate longitudinally along the velocity field fluctuations, i.e. \( \mathbf{k} \parallel \mathbf{U} \) and thus carry a finite component of energy corresponding only to the \( i\mathbf{k} \cdot \mathbf{U}_k \) part of the velocity field, which can be determined from the following relationship

\[
\langle k_c(t) \rangle \approx \sqrt{\frac{\sum_k |i\mathbf{k} \cdot \mathbf{U}_k|^2}{\sum_k |\mathbf{U}_k|^2}}. \tag{7}
\]

The expression for \( k_c \) essentially describes the modal energy contained in the non-solenoidal component of the MHD turbulent modes.

The evolution of an average mode associated with Alfvénic and compressive waves is shown in Fig. (1). We vary charge exchange strength and collision parameter in our simulation to examine their effects on the propagation of Alfvénic and compressive modes in a partially ionized plasma. Clearly, the two processes operate on different time and length scales and are self-consistently modeled in our simulations. We find that charge exchange and collisional interactions jointly modulate Alfvén and fast/slow modes. This is shown in Fig. (1) for 512\(^2\) modes in a two dimensional box of length \( 2\pi \times 2\pi \). The other parameters in our simulations are charge exchange wavenumber \( k/k_{ce} \sim 0.01 \), fixed time step \( dt = 10^{-3} \), and collision parameter \( \nu \sim 0.001 \). The background normalized constant magnetic field \( B_0 = 0.5 \). Our simulations are fully nonlinear and the ratio of the mean and fluctuating magnetic fields \( \delta B/B_0 \sim 1 \).

The modulation process depicted in Fig. (1) can be understood on the basis of charge
exchange and collisional interactions, together with convective nonlinearities. During the modulation, there exists a rapid onset of the average mode associated with Alfvénic waves. This onset is triggered essentially by linear instability process. Our linear stability analysis (not described here) indicates that the underlying coupled plasma-neutral system possesses several unstable modes. These modes account for the growing as well as damping of Alfvénic and fast/slow compressive waves. As illustrated in Fig. (1), a concurrent damping of the fast/slow compressive mode preceding the growth of Alfvénic mode occurs. When the two modes reach their extremal rise or fall, they reverse their behavior. The fast/slow compressive mode begins to rise at the expense of damping of Alfvénic mode. This process continues non periodically and repeats itself in time thereby exhibiting a nearly oscillatory behavior. It follows from Fig. (1) that the two modes, i.e. Alfvénic and fast/slow compressive, regulate each other in a predator-prey manner. Dynamically, the Alfvénic mode grows at the expense of fast/slow mode. When Alfvénic mode reaches its maximum amplitude, where the fast/slow mode remains at its lowest magnitude, the latter eats up the Alfvénic mode and vice versa. This process is repetitive. It is also noteworthy that the overall amplitude of both the modes decays in time and regrows later.

The growth and damping of wave energy can further be understood from the energy transfer rates \((dE/dt)\) between the plasma and neutral fluids. This is shown in Fig. (2). It is evident from the figure that the total energy of the plasma increases at the expense of the neutral. When the waves damp, the total energy of the plasma decreases while it increases for the neutrals. The energy transfer rates also exhibit a periodic behavior following the modulation of energy in Alfvén and fast/slow modes. Nonetheless, the energy transfer rates try to approach nearly zero i.e. \(dE/dt \to 0\), but are not entirely zero due to the modulation process. This is an interesting result that differs from pure (i.e. uncoupled) plasma or neutral fluid turbulence \([2, 8]\). The latter exhibits \(dE/dt \simeq 0\) thereby leading to a constant transfer of energy across inertial range fluctuations. This further demonstrates that the total energy of the neutral and plasma fluid is nearly constant during the steady state (See Fig 2). What we find in our simulations is a modulated-energy-transfer process that is completely different from the constant energy transfer rate. The energy transfer rates in Fig. (2) thus provide a self-consistent description of the modulation process observed in our simulations.
IV. CONCLUSION

In summary, we developed a two dimensional self-consistent model of plasma and neutral fluids that is coupled through charge exchange and collisional interactions. Notably, we find that these interactions not only modify the convective nonlinear interactions in the coupled Eqs. (1) & (2), but they also influence the propagation characteristics of MHD waves significantly. To determine the evolution of characteristic MHD waves, we distinguish the Alfvénic and non-Alfvénic contributions of MHD modes in the partially ionized turbulent environment. It is found that charge exchange and collisional interactions lead to a modulation of Alfvénic and fast/slow compressive modes. This is unlike purely collisional processes described by (6) and (1) that tend to damp Alfvénic modes. By contrast, inclusion of the charge exchange interactions leads to an entirely different scenario and it alters the propagation characteristics of MHD waves. Charge exchange, in the presence of collisions and convective nonlinear interactions, leads to an alternate growth and damping (i.e. modulation) of Alfvénic and fast/slow compressive modes. It is further noted that the modulation of waves is mediated by the conservation of momentum because of the characteristic wave velocity that is involved in Eqs. (6) & (7). It is the momentum transfer function ($Q_m$) that is responsible for regulating the rise/fall of the amplitude of the two waves and maintains the conservation process. This point is further consistent with Fig (1). We finally comment on the conservation of momentum and energy in our simulations. It seems from Fig (2) that the term $dE/dt$ hovers around zero, but does not fully converge to zero. We learn that due to the small scale self-consistent dissipation (or numerical dissipation), it is difficult to observe a 100% conservation. We continue to work to improve this conservation in our model.

Our results should find application to a variety of astrophysical environments in which a partially ionized plasma is typical. Examples include the outer heliosheath formed by the interactions of the solar wind with the local interstellar medium, the magnetic collapse of molecular clouds and star formation and the general transfer of energy in partially ionized plasmas surrounding other stellar systems [19].
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[1] Balsara, D., ApJ., 1996, 465, 775.
[2] Biskamp, D. 'Magnetohydrodynamic Turbulence' Published by Cambridge University Press, 2003
[3] Iroshnikov, P. S., Astron. Zh. 40, 742 (1963).
[4] Kolmogorov, A. N. Dokl. Acad. Sci. URSS 30, 301 (1941).
[5] Kraichnan, R. H., Phys. Fluids 8, 1385 (1965).
[6] Kulsrud, R., and Pearce, 1969, ApJ 156, 445.
[7] Leake, J., Arber, T. D., M. L. Khodachenko, M. L., 2005, Astronomy and Astrophysics, 442, 1091.
[8] Monin, A. S. & Yaglom, A. M. Statistical Fluid Mechanics: Mechanics of Turbulence, Volume 1 & 2, Dover Publication, Inc., 2007
[9] Oishi, J. S. and Mac Low, M., 2006, ApJ, 638, 281.
[10] Padoan, P., Zweibel, E., and Nordlund, A., 2000, ApJ, 540, 332.
[11] Pauls, H. L., Zank, G. P., and Williams, L. L., 1995, J. Geophys. Res. A11, 21595.
[12] Pitaevskii, L. P., and Lifshitz, E. M., in Landau and Lifshitz Course of theoretical Physics, Volume 10, Physical Kinetics, Elsevier Inc. 1981.
[13] Shaikh, D., and Zank, G. P., 2008, The Astrophysical Journal, Volume 688, Issue 1, pp. 683-694.
[14] Shaikh, D., and Zank, G. P., 2006, The Astrophysical Journal, Volume 640, Issue 2, pp. L195-L198.
[15] Shaikh, D. and Shukla, P. K., 2009, Physical Review Letters, vol. 102, Issue 4, id. 045004
[16] Shukla, P. K.; Stenflo, L.; Bingham, R. Physics of Plasmas, Volume 6, Issue 5, pp. 1677-1680 (1999).
[17] Shukla, P. K.; Stenflo, L. Phys. Scr. T, Vol. T60, p. 32 - 35, 1995
[18] Shukla, P. K.; Feix, G., Solar Physics (ISSN 0038-0938), vol. 123, no. 1, 1989, p. 117-125, 1989.

[19] Wood, B. E., Harper, G. M.; Muller, H. R., Heerikhuisen, J., Zank, G. P., ApJ, 655, 946, 2007.

[20] Zank, G. P., 1999, Space Sci. Rev., 89, 413-688, 1999.