1. Introduction

There have been several successful tests of Einstein’s general theory of relativity in classical physics [1–3]. Heisenberg and Schroedinger, following Bohr, formulated a quantum mechanics that has explained, in the Standard Model (SM) [4], all established experimentally accessible quantum phenomena except the quantum treatment of Einstein’s theory. Indeed, even with tremendous progress in quantum field theory, superstrings [5, 6], loop quantum gravity [7], etc., no satisfactory treatment of the quantum mechanics of Einstein’s theory is known to be correct phenomenologically.

Here, with an eye toward black hole physics, we apply a new approach [8] to quantum gravitational phenomena, building on previous work by Feynman [9, 10] to get a minimal union of Bohr’s and Einstein’s ideas.

The approaches to the to the attendant bad UV behavior have been summarized in Ref. [11]. Our approach, based on YFS methods [12, 13], is a new version of the resummation approach [11] and allows us to make contact with both the extended theory [11] and the asymptotic safety [14, 15] approaches and to discuss issues in black hole physics, some of which relate to Hawking [16] radiation.

2. Review of Feynman’s Formulation of Einstein’s Theory

In the SM there are many massive point particles. Are they black holes in our new approach to quantum gravity? To study this question, we follow Feynman, treat spin as an inessential complication [17], and consider the simplest case for our question, that of gravity coupled to a “free” scalar field, a “free” physical Higgs field, $\varphi(x)$, with a rest mass $m$ believed to be less than 400 GeV and known to be greater than 114.4 GeV with a 95% CL [18]. The Feynman rules for this theory were already worked-out by Feynman [9, 10]. On this view, quantum gravity is just
another quantum field theory where the metric now has quantum fluctuations as well. For example, the one-loop corrections to the graviton propagator due to matter loops is just given by the diagrams in Fig. 1. We return to these graphs shortly.

3. Resummed Quantum Gravity and Newton’s Law

To YFS resum the propagators in the theory, in the YFS formula in Eq.(5.16) in Ref. [12], we make the replacements described in Refs. [8, 19] to go over from QED to QG and get the factor $e^{B''(k)}$ in numerator of each propagator in Feynman’s series [9, 10], with $B''(k) = \frac{\kappa^2|k^2|}{8\pi^2} \ln \left( \frac{m^2}{m^2 + |k^2|} \right)$ in the deep Euclidean regime. If $m$ vanishes, using the usual $-\mu^2$ normalization point we get $B''(k) = \frac{\kappa^2|k^2|}{8\pi^2} \ln \left( \frac{\mu^2}{|k^2|} \right)$. In both cases the respective resummed propagator falls faster than any power of $|k^2|$! This means that one-loop corrections are finite! All quantum gravity loops are UV finite and the all orders proof is given in Refs. [8].

The one-loop corrections to Newton’s law implied by the diagrams in Fig. 1 directly impact our black hole issue. Using the YFS resummed propagators in Fig. 1 we get the potential [8,20] $\Phi_N(r) = -\frac{\alpha M_1 M_2}{r}(1-e^{-a r})$ where [8,20] $a \approx 3.96 M_{Pl}$ when for definiteness we set $m \sim 120 \text{GeV}$. Our gauge invariant analysis can be shown [8] to be consistent with the one-loop analysis of QG in Ref. [21].

4. Massive Elementary Particles and Black Holes

With reasonable estimates and measurements [8,23,24] of the SM particle masses, including the various bosons, the corresponding results for the analogs of the diagrams in Fig. 1 imply [8] that in the SM $a_{eff} \cong 0.349 M_{Pl}$ . To make direct contact with black hole physics, note that, if $r_S$ is the Schwarzschild radius, for $r \to r_S$, $a_{eff}r \ll 1$ so that $|2\Phi_N(r)|_{m_1=m_2} \ll 1$. This means that

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*Our deep Euclidean studies are complementary to the low energy studies of Ref. [22].
$g_{00} \cong 1 + 2\Phi_N(r)|_{m_1=m_2/m_2}$ remains positive as we pass through the Schwarzschild radius. It can be shown [8] that this positivity holds to $r = 0$. Similarly, $g_{rr}$ remains negative through $r_s$ down to $r = 0$ [8]. In resummed QG, a massive point particle is not a black hole.

Our results imply the running Newton constant $G_N(k) = G_N/(1 + k^2/a_{eff}^2)$ which is fixed point behavior for $k^2 \to \infty$, in agreement with the phenomenological asymptotic safety approach of Ref. [15]. Our result that an elementary particle has no horizon also agrees with the result in Ref. [15] that a black hole with a mass less than $M_{cr} \sim M_{pl}$ has no horizon. The basic physics is the same: $G_N(k)$ vanishes for $k^2 \to \infty$.

Because our value of the coefficient of $k^2$ in the denominator of $G_N(k)$ agrees with that found by Ref. [15], if we use their prescription for the relationship between $k$ and $r$ in the regime where the lapse function vanishes, we get the same Hawking radiation phenomenology as they do: a very massive black hole evaporates until it reaches a mass $M_{cr} \sim M_{pl}$ at which the Bekenstein-Hawking temperature vanishes, leaving a Planck scale remnant.

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