New Measurements of CP Violation Parameters
as Tests of CPT in $K$ Meson Decay

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Abstract

Using a technique which employs a pair of solid scintillator regenerators, the E773 collaboration has measured several CP violation parameters in $K$ meson decay at Fermilab. We report new results for the phase of $\eta^+$, the $K_L - K_S$ mass difference, the $K_S$ lifetime, and the phase difference $\text{Arg}(\eta_{00}) - \text{Arg}(\eta_{+-})$ in $K \rightarrow \pi\pi$ decay. In addition, we report a measurement of the magnitude and phase of $\eta_{+-\gamma}$ in $K \rightarrow \pi^+\pi^-\gamma$ decay. Our preliminary results are compared with theoretical expectations based on CPT symmetry.

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I. INTRODUCTION

Local quantum field theories are automatically invariant under the combined operations of charge conjugation, parity, and time reversal: CPT \([1]\). However, generalizations of...
quantum mechanics which include gravity might allow pure states to evolve into mixed states [2]. This sort of unusual (Planck-scale) dynamics could lead to CPT-violating effects [3–6] such as the existence of small differences in particle-antiparticle masses and lifetimes. The magnitude of this CPT-noninvariance might be proportional to the ratio of some low energy scale to the Planck mass (Ref. [6]). For example, one might have

\[ \frac{m_p - m_p^-}{m_p} \sim \frac{m_p^-}{m_{\text{Planck}}} = 7.7 \times 10^{-20} \]

for the proton-antiproton mass difference, and

\[ \frac{m_{K^0} - m_{K^0}}{m_K} \sim \frac{m_K}{m_{\text{Planck}}} = 4.1 \times 10^{-20} \]

for the neutral \( K \) system.

The 90% confidence level limits on the proton-antiproton and electron-positron fractional mass differences are approximately \( 4 \times 10^{-8} \), twelve orders of magnitude larger than the “interesting” region [7]. The experimental situation is considerably brighter, however, in the \( K^0 \) sector. Let us define

\[ \eta_{+-} \equiv \frac{\text{Amp}(K_L \to \pi^+\pi^-)}{\text{Amp}(K_S \to \pi^+\pi^-)}, \quad \eta_{00} \equiv \frac{\text{Amp}(K_L \to \pi^0\pi^0)}{\text{Amp}(K_S \to \pi^0\pi^0)}, \]

\[ \phi_{+-} \equiv \text{Arg}(\eta_{+-}), \quad \phi_{00} \equiv \text{Arg}(\eta_{00}), \quad \Delta \phi \equiv \phi_{00} - \phi_{+-}. \]

It can be shown [8] that

\[ \frac{m_{K^0} - m_{K^0}}{m_K} \approx \left( \frac{\Delta m}{m_K} \right) \sqrt{2} |\eta_{+-}| \tan(\phi_{+-} - \phi_\epsilon + \frac{\Delta \phi}{3}). \]

Here, \( \Delta m = 0.5286 \times 10^{10} \text{h}\text{s}^{-1}\text{c}^{-2} = 3.4793 \times 10^{-6} \text{eV}/\text{c}^2 \) is the \( K_L - K_S \) mass difference. The small value for \( \Delta m \) provides considerable “leverage” in testing CPT since \( \Delta m/m_K = 6.99 \times 10^{-15} \). Recently published values [9] yield \( |m_{K^0} - m_{K^0}|/m_K \lesssim 2.5 \times 10^{-18} \), less than two orders of magnitude away from the domain in which Planck-scale physics might play a role.

In Fermilab Experiment 773 we measured \( \phi_{+-}, \phi_\epsilon, \) and \( \Delta \phi \) by studying decays of neutral \( K \) mesons into \( \pi^0\pi^0 \) and \( \pi^+\pi^- \) final states. In addition to our CPT-related \( \pi\pi \) results, we also
report an improved measurement of $\eta_{+\gamma}$, a CP-violation parameter in $K_L \to \pi^+\pi^-\gamma$ decays. We determined the phases of decay amplitudes by observing the time-dependent interference between $K_S$ and $K_L$ decays after a pair of $K_L$ beams passed through regenerators. The detector was a reconfigured version of the E731 spectrometer (Ref. [4]) which had been used to measure $Re(\epsilon'/\epsilon)$ and other parameters of the neutral $K$ system.

E773 wrote physics-quality data for about two months, beginning in late July, 1991. During this time the experiment recorded $\sim 400$ million triggers on nine hundred 8 mm cassette data tapes. We present preliminary results from analysis of these data. In Sec. II we review relevant neutral $K$ phenomenology. In Secs. III and IV we describe the experimental technique and instrumentation used in E773. In Secs. V and VI we discuss reconstruction of candidate $K \to \pi\pi$ and $K \to \pi\pi\gamma$ events. Simulation of the detector is discussed in Sec. VII. We describe analysis and fits to our data in Sec. VIII. Our conclusions are presented in Sec. IX. These results comprise the thesis work of R.A. Briere and B. Schwingenheuer from the University of Chicago, and J.N. Matthews from Rutgers University.

II. $K$ MESON PHENOMENOLOGY

A. CP and $\pi\pi$ decays

The major source of CP violation in the neutral $K$ system [10,11] is the time-reversal asymmetry in the transition rate between $K^0$ and $\bar{K}^0$: $\Gamma(K^0 \to K^0) > \Gamma(\bar{K}^0 \to K^0)$. Consequently, the eigenstates of the neutral $K$ system which do not mix in vacuum will both contain an excess of $K^0$ relative to $\bar{K}^0$. We define

$$|K_S\rangle \equiv \frac{(1 + \epsilon)|K^0\rangle + (1 - \epsilon)|\bar{K}^0\rangle}{\sqrt{2(1 + |\epsilon|^2)}}$$

and

$$|K_L\rangle \equiv \frac{(1 + \epsilon)|K^0\rangle - (1 - \epsilon)|\bar{K}^0\rangle}{\sqrt{2(1 + |\epsilon|^2)}}$$.

The phase of $\epsilon$ (Ref. [7]) is

$$\phi_\epsilon \approx \tan^{-1}\left(\frac{2\Delta mc^2\tau_s}{\hbar}\right) = (43.33 \pm 0.14)^\circ$$,
so that $Re(\epsilon)$ is positive. The $K_S$ lifetime is $\tau_S = 0.8922 \times 10^{-10}\,s$. The CP eigenstates are

$$|K_1\rangle \equiv \frac{|K^0\rangle + |\not{K}^0\rangle}{\sqrt{2}}, \quad |K_2\rangle \equiv \frac{|K^0\rangle - |\not{K}^0\rangle}{\sqrt{2}}.$$ 

Neglecting normalizations, we have $|K_1\rangle \sim |K_S\rangle - \epsilon|K_L\rangle$ and $|K_2\rangle \sim |K_L\rangle - \epsilon|K_S\rangle$. A beam which is initially pure $K_2$ will evolve into a mixture of $K_2$ and $K_1$ as the $K_S$ component of the original $K_2$ state decays. This CP-violating mixing $K_2 \leftrightarrow K_1$, combined with the (CP-allowed) decay $K_1 \rightarrow \pi\pi$, provides the dominant contribution to the $K_L \rightarrow \pi\pi$ decay amplitude.

An additional source of CP violation in $K_L \rightarrow \pi\pi$ decay is possible if $Amp(K_2 \rightarrow \pi\pi)$ is nonzero. Since $|K_2\rangle \sim |K^0\rangle - |\not{K}^0\rangle$, this direct CP violation is only possible if $Amp(K^0 \rightarrow \pi\pi) \neq Amp(\not{K}^0 \rightarrow \pi\pi)$. More specifically, we define the isospin 0 and 2 decay amplitudes $A_0, \overline{A}_0, A_2, \overline{A}_2$ so that

$$\exp^{i\delta_0} A_0 = \langle (\pi\pi)_{I=0}|T|K^0\rangle, \quad \exp^{i\delta_0} \overline{A}_0 = \langle (\pi\pi)_{I=0}|T|\not{K}^0\rangle$$ 

$$\exp^{i\delta_2} A_2 = \langle (\pi\pi)_{I=2}|T|K^0\rangle, \quad \exp^{i\delta_2} \overline{A}_2 = \langle (\pi\pi)_{I=2}|T|\not{K}^0\rangle.$$ 

We find that

$$\epsilon' \equiv \frac{1}{\sqrt{2}} \exp^{i(\delta_2 - \delta_0)} \frac{(A_2 - \overline{A}_2)}{(A_0 + \overline{A}_0)}$$ 

must be different from zero for there to be direct CP violation in $K$ decay. Here, $\delta_0$ and $\delta_2$ are the (measured) isospin 0 and 2 $\pi\pi$ scattering phase shifts [12] at the $K^0$ mass. CPT invariance requires $\overline{A}_I = A_I^*$, so the phase of $\epsilon'$ is $\phi_{\epsilon'} = \delta_2 - \delta_0 + 90^\circ \approx (47 \pm 5)^\circ$ (Ref. [7]). It may seem surprising that the phase of $\epsilon'$ is well known, even though its magnitude remains an open question (Ref. [13]). By coincidence, the phases of $\epsilon$ and $\epsilon'$ are nearly equal. If we assume CPT invariance, we can write $\eta_{+-}$ and $\eta_{00}$ in terms of $\epsilon$ and $\epsilon'$ as

$$\eta_{+-} \approx \epsilon + \epsilon', \quad \eta_{00} \approx \epsilon - 2\epsilon'.$$

(2.1)
B. CP and ππγ decays

CP considerations play a similar role in $K \rightarrow \pi^+\pi^-\gamma$ decays. The $K_S \rightarrow \pi\pi\gamma$ rate is dominated by an inner bremsstrahlung amplitude \[13\] in which one of the final-state pions radiates a photon. In $K_L$ decays, the inner bremsstrahlung contribution is CP-suppressed, permitting observation of the CP-allowed direct emission process \[14\]. As is the case for $\pi\pi$ decays, the CP-violating mixing $K_2 \leftrightarrow K_1$, combined with the (CP-allowed) decay $K_1 \rightarrow \pi\pi\gamma$, contributes to the $\pi\pi\gamma$ decay rate in a $K_L$ beam \[15\]. We define the ratio of the inner bremsstrahlung decay amplitudes:

$$\eta_{+,-\gamma} \equiv \frac{\text{Amp}(K_L \rightarrow \pi^+\pi^-\gamma)_{IB}}{\text{Amp}(K_S \rightarrow \pi^+\pi^-\gamma)_{IB}}.$$ 

Unless there is an unusually large amount of direct CP violation in $K_L \rightarrow \pi^+\pi^\gamma$ decays, one expects $\eta_{+,-\gamma} \approx \eta_{+,-} \approx \epsilon$. We only observe interference between the inner bremsstrahlung amplitudes; the $K_S$ direct emission amplitude is too small to be detected in our data.

C. CPT

Several different avenues for CPT violation suggest themselves. One is a CP-violating, T-conserving mixing of $K^0$ and $\overline{K^0}$ to produce the mass eigenstates $K_L$, $K_S$. This corresponds to a situation where the $K_S$ contains, for example, an excess of $K^0$ while the $K_L$ contains an excess of $\overline{K^0}$. We can describe this by including the CPT violation parameter $\Delta$ in our description of $K_L$, $K_S$ as follows:

$$|K_S\rangle \sim (1 + \epsilon + \Delta)|K^0\rangle + (1 - \epsilon - \Delta)|\overline{K^0}\rangle$$
$$|K_L\rangle \sim (1 + \epsilon - \Delta)|K^0\rangle - (1 - \epsilon + \Delta)|\overline{K^0}\rangle.$$ 

Note the sign switch between $\epsilon$ and $\Delta$ in the $K_L$ expression. The component of $\Delta$ which is perpendicular to $\epsilon$ in the complex plane corresponds to a $K^0 - \overline{K^0}$ mass difference, while the component parallel to $\epsilon$ corresponds to a lifetime difference \[16\]. In terms of $\epsilon$, $\epsilon'$, and $\Delta$,
\[ \eta_{+-} = \epsilon + \epsilon' - \Delta, \quad \eta_{00} = \epsilon - 2\epsilon' - \Delta. \quad (2.2) \]

A nonzero value of \( \Delta \) will shift \( \eta_{00} \) and \( \eta_{+-} \) in the same direction in the complex plane. From Eq. (2.2), we find \( \Delta = \epsilon - (2\eta_{+-} + \eta_{00})/3. \)

Another possible route to CPT violation is through an unusual relationship among the various \( K \to \pi\pi \) decay amplitudes. Because \( A_I - A_I^* = 2iIm(A_I) \) and \( A_I + A_I^* = 2Re(A_I) \), the phase of \( \epsilon' \) would be shifted from its value of \( (47 \pm 5)° \) by a CPT-disallowed relationship such as \( \overline{A_I} \neq A_I^* \). This would split \( \eta_{00} \) and \( \eta_{+-} \) apart in the complex plane. Other terms associated with CPT violation in the decay amplitudes would shift \( \eta_{00} \) and \( \eta_{+-} \) in the same direction, much as was the case with \( \Delta \). From Eq. (2.2) we see that \( \epsilon' \approx (\eta_{+-} - \eta_{00})/3, \) even in the presence of additional (CPT violating) terms which are common to both \( \eta_{00} \) and \( \eta_{+-} \).

D. \( K_S \) regeneration

We extracted phase information about the \( \eta' \)'s by measuring the interference between the \( K_L \) and \( K_S \) decay amplitudes in our neutral beam. Since the E773 detector was many \( K_S \) lifetimes downstream of the production target, we reintroduced a small \( K_S \) component into our twin \( K_L \) beams by passing both beams through regenerators made of plastic scintillator.

Because the forward elastic scattering amplitudes \( f(k) \) and \( \overline{f}(k) \) for \( K^0 \) and \( \overline{K^0} \) with wavenumber \( k \) are unequal, a \( K_L \) entering a block of material will evolve into a mixture of \( K_L \) and \( K_S \). The ratio of the \( K_S \) and \( K_L \) amplitudes a small distance \( \Delta z \) inside a regenerator will be

\[ \rho \equiv \frac{A_S(\Delta z)}{A_L(\Delta z)} \approx iN\pi \left( \frac{f - \overline{f}}{k} \right) \Delta z. \]

Here, \( N \) is the number of scatterers per unit volume. This coherent regeneration amplitude \( \rho \) results from constructive interference among all the outgoing spherical waves produced at each of the scattering sites in the regenerator. For a finite length regenerator, \( \rho \) includes an additional geometric factor associated with the \( K_S \) lifetime and the \( K_L - K_S \) mass difference.
The momentum-dependence of $|(f - \bar{f})/k|$ has been measured experimentally \[18\] and is in agreement with the Regge theory prediction \[19\] that

$$\left( \frac{f - \bar{f}}{k} \right) \propto p^{\alpha} e^{-i\pi(2+\alpha)/2}.$$ (2.3)

The connection between $\alpha$ and the phase of $f - \bar{f}$ comes from analyticity, and is independent of Regge theory.

A kaon which collides with a single nucleus will scatter into a state containing a different combination of $K^0$ and $\bar{K}^0$. Consequently, elastic scattering is regenerative: a pure $K_L$ will scatter into a mixed $K_L, K_S$ state. Using the optical theorem, one can show that the contribution to the $K_S$ amplitude from elastic scattering interferes destructively with the scattered kaon’s $K_S$ amplitude which arises from coherent regeneration. This cancellation is nearly perfect in a regenerator of two interaction lengths.

We were interested in studying the interference between the (coherently regenerated) $K_S$ and $K_L$ decay amplitudes. Since E773’s regenerators were shorter than two interaction lengths, we made small corrections for contamination in our coherently regenerated $K$ sample from “diffraction regeneration” associated with single (and multiple) elastic scattering. Regeneration is discussed more fully in the Appendix.

**E. Decay rate downstream of a regenerator**

The $K_S$ and $K_L \pi\pi$ decay amplitudes will interfere downstream of a regenerator. Because of the differences in the $K_L, K_S$ masses and decay rates, this interference evolves with proper time $\tau$. For a state $|K\rangle \sim |K_L\rangle + \rho|K_S\rangle$,

$$Amp(K \to \pi\pi) = Amp(K_L \to \pi\pi)e^{-im_Lc^2\tau/\hbar - \Gamma_L\tau/2} + \rho Amp(K_S \to \pi\pi)e^{-im_Sc^2\tau/\hbar - \Gamma_S\tau/2},$$

where $\Gamma_L, \Gamma_S$ are the $K_L, K_S$ decay rates. The $\pi\pi$ decay rate downstream of the regenerator is proportional to

$$|\rho|^2 e^{-\Gamma_S\tau} + |\eta|^2 e^{-\Gamma_L\tau} + 2|\rho\eta|e^{-(\Gamma_L+\Gamma_S)\tau/2} \cos(\Delta mc^2\tau/\hbar + \phi_\rho - \phi_\eta).$$
The phase of $\rho$ is approximately $-32^\circ$, though it varies with energy by several degrees since the $K_L$ phase advances with respect to the $K_S$ phase as a kaon travels through a regenerator. $\Gamma_S$ is $1.12 \times 10^{10} \text{s}^{-1}$, $\Gamma_L$ is $1.93 \times 10^7 \text{s}^{-1}$, and $\Delta m c^2/\hbar$ is $30.6^\circ/10^{-10} \text{s}$, about $5^\circ$ per meter for 100 GeV kaons. The last term in the decay rate, the interference between $K_L$ and $K_S$ decay amplitudes, contains information about $\phi_0$ or $\phi_{+-}$. Defining $\Gamma_{+-}$ as the $K \rightarrow \pi^+\pi^-$ decay rate, $\Gamma_{00}$ as the $K \rightarrow \pi^0\pi^0$ decay rate, and $\Delta \phi$ as $\phi_{00} - \phi_{+-}$, one may write

$$
\Gamma_{+-} \sim |\rho|^2 e^{-1.12\tau} + |\eta_{+-}|^2 e^{-0.0193\tau} + 2|\rho\eta_{+-}| e^{-0.56\tau} \cos(30.6\tau - 77)^\circ \\
\Gamma_{00} \sim |\rho|^2 e^{-1.12\tau} + |\eta_{00}|^2 e^{-0.0193\tau} + 2|\rho\eta_{00}| e^{-0.56\tau} \cos(30.6\tau - 77 - \Delta \phi)^\circ
$$

where $\tau$ is in units of $10^{-10}$ seconds and the arguments of the cosines are in degrees. The interference term in $\Gamma_{00}$ is most sensitive to changes in $\Delta \phi$ when the cosine’s argument is near $-90^\circ$, corresponding to $\tau \sim 0$. For $\tau \sim 2.5 \times 10^{-10}$s the argument of the cosine is near zero and the $\pi^0\pi^0$ decay rate is insensitive to small changes in $\Delta \phi$.

### III. EXPERIMENTAL TECHNIQUE

If experimental acceptance and detection efficiency are known with sufficient accuracy, $\Delta \phi$, $\phi_{+-}$, and $\text{Arg}(\epsilon)$ can be determined in a single beam through a measurement of the proper time evolution of $\Gamma_{00}$ and $\Gamma_{+-}$. However, resolution smearing, diffraction regeneration, and experimental acceptance influence the reconstruction of charged and neutral final states in different ways. As a result, the E773 apparatus was designed conservatively to minimize possible systematic errors in the measurement of $\Delta \phi$ associated with acceptance modeling, energy calibration, and imperfect knowledge of physics parameters such as $\epsilon'$. (We subsequently found that our ability to model the behavior of the detector was adequate to make single-beam measurements.)
A. Double beam technique

We employed a double-beam technique similar to that used by E731 in its measurement (Ref. [10]) of $\epsilon'$. One $K_L$ beam passed through a thin regenerator while the other traversed a thick regenerator about eleven meters further upstream. This separation was chosen to make the $\pi\pi$ decay rate in the upstream regenerator’s beam insensitive to $\Delta\phi$ after the beam had traveled about a dozen meters downstream of the upstream regenerator, corresponding to proper times $\sim 2.5 \times 10^{-10}$ s. The two beams were separated vertically; individual kaons passed through only one of the two regenerators. Data were recorded simultaneously for $\pi^0\pi^0$ and $\pi^+\pi^-$ decays in both beams. The regenerators switched beams after each accelerator spill, approximately once per minute. Various systematic effects associated with detection efficiency, kaon flux, and acceptance cancel when rates in the two beams are compared.

B. Choice of regenerator configuration

A conservative analysis of the data could be performed by forming a double ratio of the decay rates in the two beams for the neutral and charged decay modes. For a given value of kaon energy $E$ and decay position $z$, we define

$$R(z, E) \equiv \frac{\Gamma^{up}_{00}(z, E)}{\Gamma^{down}_{00}(z, E)} / \frac{\Gamma^{up}_{+-}(z, E)}{\Gamma^{down}_{+-}(z, E)}.$$  

Here, “up” and “down” refer to the beams containing regenerators in the upstream and downstream positions. Binned in $E$ and $z$, $R$ is insensitive to differences in $\pi^0\pi^0$ and $\pi^+\pi^-$ acceptance and reconstruction efficiency. For $z$ values downstream of the thin regenerator, $\Gamma^{up}_{00}(z, E)$ is nearly independent of $\Delta\phi$ for typical kaon energies while $\Gamma^{down}_{00}(z, E)$ changes by about one percent per one degree change in $\Delta\phi$.

A regenerator which is approximately one interaction length in thickness gives the largest amount of $K_L/K_S$ interference, since $\rho$ is proportional to regenerator length, while the fraction of beam surviving passage through the regenerator falls exponentially with length. Most of the coherent $K_S$ flux in the “upstream” beam, proportional to $\rho^2$, will have decayed.
before traveling as far as the downstream regenerator. A judicious choice for the thickness of the downstream regenerator will produce a decay distribution in this beam with similar time dependence, except for the proper-time offset in the interference cosine’s argument. By choosing regenerator lengths which give similar proper time spectra, the importance of systematic errors caused by resolution effects is reduced. In particular, the double ratio $R(z, E)$ will be close to unity for a wide range of decay positions and energies, and will vary from unity by typically 1% per degree of phase difference between $\eta_{00}$ and $\eta_{+-}$. The optimum regenerator thicknesses proved to be 0.4 interaction lengths for the downstream and 1.2 interaction lengths for the upstream regenerators.

A drawback of a double ratio analysis is that it discards events with decay positions upstream of the downstream regenerator, sacrificing statistical power in order to reduce systematic uncertainties. With adequate modeling of the signal and backgrounds, it is possible to extract additional information about the $\eta$ phases from events ignored by the double ratio technique. We found that we were able to understand (and simulate) the behavior of the E773 detector with sufficient accuracy to overcome the systematic effects that were of concern at the time the detector was configured. Consequently, we report results from the higher precision analysis which includes data from the region between the regenerators.

IV. E773 DETECTOR AND CALIBRATION

The E773 detector [20] was designed to determine decay positions and momenta of $K$ mesons decaying into $2\pi$, $\pi\pi\gamma$, $\pi\epsilon\nu$, and $3\pi$ final states. We used its measurements to reject background events in the $\pi\pi$ sample from semileptonic and $3\pi$ decays, and from $K_S$ produced through processes other than coherent regeneration. $K$ mesons produced in a beryllium target traveled through a series of collimators and sweeping magnets before reaching the regenerators. Data from a drift chamber spectrometer and lead glass electromagnetic calorimeter provided kinematic information about charged particles and photons.
in the final state. Signals from scintillation counters contributed to the experiment’s triggers and vetoes. A diagram of the detector is shown in Fig. 1; individual subsystems will be described below. The thin scintillator planes labeled “T,V” inside the vacuum were removed midway through the run. The transition radiation detector (TRD) downstream of the last drift chamber was not used in the analysis of E773 data.

A. Neutral beam

Fermilab delivered a spill of 800 GeV protons to the E773 target about once per minute. The protons arrived in 2 ns-long “buckets” spaced by 19ns; each spill lasted twenty seconds and contained typically $1.6 \times 10^{12}$ protons. A neutral beam was formed by placing sweeping magnets, followed by a 5.8m-long copper collimator, downstream of the 36 cm long target. The collimator contained two tapered channels which created a clear path for particles traveling 4.8 milliradians to the east of, and slightly above or below, the incident proton beam’s direction. Beryllium blocks placed downstream of the two-hole collimator, together with the non-zero targeting angle, reduced the beam’s neutron content relative to its kaon flux. Some of the beryllium blocks shadowed only the downstream regenerator, reducing the hadron flux striking it relative to that in the beam containing the upstream regenerator. A 7.6 cm thick lead block (13.6 radiation lengths) converted photons to electron-positron pairs which were removed from the beam by subsequent collimators and sweeping magnets. With the exception of gaps in the vacuum system at the regenerators, the beams traveled in vacuum from 17 m downstream of the target until arriving at the first drift chamber, 159 m from the target. By the time the beams reached the lead glass array, 181 m from the target, each beam was a square, approximately 8 cm on a side. The beams were separated vertically with $\sim 15$ cm clearance between their inner edges, and passed through holes in the lead glass array, stopping in the 3.2 m steel muon filter. Fig. 2 shows the kaon beam profiles at the lead glass, determined from $K_L \rightarrow \pi^+\pi^-$ decays. (As with most of the figures, the histogram represents data while the superposed points indicate the predictions of the Monte Carlo simulation.)
Carlo simulation.) Approximately $10^7 K_L$'s (and an equal number of neutrons) struck the regenerators each spill. The $K_L$ energy spectrum is shown in Fig. 3; the detector's acceptance allowed us to use $K$ mesons with energies above $\sim 25$ GeV and below $\sim 160$ GeV.

**B. Regenerators**

In order to allow efficient rejection of events in which a $K_S$ or $K_L$ was produced through inelastic processes, both regenerators were built entirely from small blocks of plastic scintillator. The upstream regenerator was 118 cm long with one quarter of its blocks viewed by photomultiplier tubes. The downstream regenerator was 40 cm long with each of its blocks viewed by a pair of phototubes. Signals from some of the phototubes participated in the experiment trigger; all signals were measured by ADC's and recorded with event data. Straight-through muons were used to cross-calibrate the regenerator channels. The moving machines which positioned the regenerators (and the beryllium absorber which shadowed the downstream regenerator) were controlled by one of the data acquisition computers. The regeneration amplitudes for 80 GeV kaons were $\rho_{up} \approx 0.02e^{-i31^\circ}$ and $\rho_{down} \approx 0.007e^{-i33^\circ}$ in the upstream and downstream regenerators; the magnitude of $\rho$ decreased with increasing kaon momentum as $\sim p^{-0.6}$.

**C. Tracking system**

The drift chamber spectrometer consisted of two pairs of wire chambers [21] which measured particle trajectories upstream and downstream of a large analyzing magnet. The chambers' cells were hexagonal, with a maximum drift distance of 6.35 mm. Half-cell offsets between paired sense planes provided resolution of “left-right” ambiguities. The chambers used an argon-ethane gas mixture which included a small amount of alcohol vapor. Single-hit TDC's with 1 ns resolution digitized discriminated chamber signals in response to an experiment trigger. Chamber resolution was better than 100 $\mu m$ per sense plane; frequent magnet-off runs of muon data were used to generate alignment constants for the tracking
system. Helium bags placed between the chambers served to reduce the effects of multiple scattering. Average chamber efficiency was typically 99% per sense plane, but a small number of wires showed lower efficiencies. The effects of inefficient wires were modeled in the Monte Carlo simulation of the detector.

The analyzing magnet’s vertical field imparted a transverse momentum kick of 200 MeV/c. Its field integral had been mapped on a two inch grid for E731; the magnet was run at the same field strength in E773. The magnet’s aperture was 1.46 m high by 2.5 m wide by 1 m deep and contained a helium bag during running.

The tracking system’s momentum resolution was typically better than 1%, with a momentum dependence given by

$$\frac{\sigma_p}{p} = \sqrt{(0.45\%)^2 + (0.012p\%)^2}$$

for the momentum $p$ measured in GeV/c. The energy scale was verified by comparing the reconstructed masses in $K \rightarrow \pi^+\pi^-$ and $\Lambda \rightarrow p\pi$ decays with the known $K$ and $\Lambda$ masses.

D. Lead glass

The energies and positions of electrons and photons were measured by a lead glass calorimeter built from 804 blocks of Schott F-2 glass. Individual blocks were 5.8 cm square by 61 cm deep and were stacked parallel to the beam direction in a nearly circular array. The radius of the calorimeter was about 0.92 m; the beams passed through a pair of holes near the center of the array. The array was 18.74 radiation lengths deep. Over the course of the run, blocks near the beam holes became less transparent because of radiation damage. Periodically, it was necessary to “cure” the blocks with exposure to ultraviolet light from a mercury vapor lamp which resulted in significant (but not complete) recovery. The entire calorimeter was located in a temperature controlled, light-tight enclosure.

An Amperex 2202 photomultiplier tube was mounted on the downstream face of each lead glass block to collect Čerenkov light from electromagnetic showers. Wratten 2A filters
between blocks and phototubes transmitted light with wavelengths greater than 430 nm. (For shorter wavelengths, the absorption in the lead glass was substantial and varied rapidly with wavelength.) Optical fibers, mounted on the upstream faces of all blocks, carried light from a single xenon flash lamp which allowed us to monitor detector performance. All blocks were constantly illuminated by a small amount of light from a light-emitting diode which served to minimize the rate dependence of phototube gains.

Signals from the lead glass photomultiplier tubes were integrated for 150 ns and digitized by LeCroy 1885 ADC’s. These dual-range ADC’s had conversion gains that were typically 5 MeV per ADC count below 17 GeV, and 40 MeV per count above 17 GeV. Information from channels with $\geq 5$ counts was recorded with event data.

Since errors in the measurement of photon energies induced errors in the determination of $\pi^0\pi^0$ decay vertex positions, it was necessary to understand in detail the calorimeter’s behavior. We calibrated the lead glass array with 20 million electrons from the decay $K \rightarrow \pi e\nu$ ($K_{e3}$). A clean electron sample was obtained after eliminating possible backgrounds from $\Lambda \rightarrow p\pi$, $K \rightarrow \pi^+\pi^-$, and $K \rightarrow \pi^+\pi^-\pi^0$ decays with kinematic cuts. $K \rightarrow \pi\mu\nu$ ($K_{\mu3}$) decays were discarded by requiring that no signals be present in the muon veto system. Additional cuts required that tracks were well-reconstructed and well-separated at the lead glass. Cuts on transverse shower shape and the maximum distance between the cluster position and the projected electron track helped eliminate events in which electrons lost energy through bremsstrahlung emission.

The reconstruction of electromagnetic showers depended on three parameters which had to be determined for each block: two effective gains and a parameter quantifying a block’s opacity to Čerenkov light. An iterative procedure was used to determine calibration constants for each block; the procedure looped repeatedly over electrons in a calibration sample, updating constants after each pass, until parameter values stabilized. The lead glass calibration parameters changed with time during the run. Central blocks experienced the most radiation damage and needed to have their “constants” updated most frequently. Fortunately, $K_{e3}$ electrons hit blocks near the center of the array most frequently, allowing us
to track the most rapidly changing parameters. We were able to generate new calibration constants for central blocks from approximately one day’s running.

Fig. 4 shows the final ratio of the calorimeter energy and track momentum ($E/p$) for $K_{e3}$ electrons. Superposed on the figure is the Monte Carlo simulation’s $E/p$ spectrum (see Sec. VII); the r.m.s. width of the peak is approximately 3%. The distributions agree except for the low side tail in the data which is not yet well-represented by the Monte Carlo. The stability of our calibration over time is shown in Fig. 5. The period of time spanned by the plot represents about nine weeks, the length of E773’s data run. Fig. 6 shows the results of fits for the mean and width of the $E/p$ distribution as a function of track momentum. Events were restricted to the region near the peak; the apparent shift of the mean away from unity is an artifact caused by this restriction.

**E. Trigger and vetoes**

The principal E773 triggers and vetoes were designed to select events with two tracks (“charged mode”) or four photons (“neutral mode”) in the final state. Additional triggers, useful for calibration and monitoring purposes, selected single muon events, six photon events, and “accidentals,” events which gave us an indication of the level of additional activity in the detector which might be expected to accompany signals produced by the decay products of real $K$ mesons.

Some parts of the veto system participated actively in triggering decisions for both charged and neutral mode triggers. For example, signals in the downstream scintillators in either regenerator (which were expected to be quiet during coherent regeneration) would block both modes’ triggers. Other parts of the veto system rejected triggers caused by random detector activity as well as triggers from events such as $K_{\mu3}$ decays which were accompanied by muons. (We used electrons from the copious decay $K_L \to \pi e \nu$ to calibrate the lead glass and to verify the accuracy of the $\pi^+ \pi^-$ Monte Carlo simulation.) Vetoes for the neutral trigger allowed us to reject events in which energy depositions in the lead glass
came from hadrons striking the calorimeter. Other neutral mode vetoes permitted us to
discard triggers in which significant electromagnetic energy went down the beam holes in
the lead glass array. Analogue signals from most veto counters were measured by ADC’s and
recorded with event data. We used this information during offline analysis to discard events
in which photons might have missed the lead glass array, based on activity in the photon
veto system. To reduce backgrounds from inelastic $K_S$ production, neutral and charged
mode events were discarded when accompanied by activity in either regenerator.

The charged mode used a two-level trigger. In the first half of the run, the level 1 trigger
required a signal in any counter in the T,V scintillator hodoscope in coincidence with at
least two signals from scintillators in the B hodoscope and at least two from counters in the
C hodoscope (See Fig. 1). The pattern of hits in the B and C counter banks was required
to be consistent with a pair of tracks from a two-body decay in one of the beams. A signal
in the $\mu_2$ hodoscope (located downstream of a 3.2 m-thick steel wall) would veto a first
level trigger, allowing us to reject $K \rightarrow \pi \mu \nu$ decays. The presence of signals in photon
veto counters immediately before the lead glass would also block the trigger, allowing us to
discard two-track events which might have come from $K \rightarrow \pi^+ \pi^- \pi^0$ decays.

The level 2 charged mode trigger in the first half of the run was based on the pattern of
struck wires in the drift chambers. The trigger required that the four chambers each have
at least one hit in both left and right sides, as expected in a two-track final state. Midway
through the run, the T,V hodoscope was removed and a more sophisticated second level
trigger installed. The new trigger required that the pattern of hits in all four chambers was
consistent with that expected from a two-body, two-track event, with the possible inclusion
of extra hits from accidental activity. Both $\pi^+ \pi^-$ and $\pi^+ \pi^- \gamma$ final states satisfied the charged
trigger; no requirement was made on lead glass energy.

The neutral mode also used a two-level trigger. The first level required that the total
energy in the lead glass be above $\sim 25$ GeV. Outputs from groups of nine contiguous lead
glass blocks were summed by “adders;” individual adder outputs were integrated for 30 ns
and digitized by ADC’s. (This shorter gate helped us eliminate “out of time” clusters
registered by the lead glass ADC’s.) These sums were combined to form the total energy, $E_T$, as a fast analogue sum of signals from the entire lead glass array. The $E_T > 25$ GeV requirement was effective since most accidental activity in the lead glass came from muons or out-of-time showers, which tended to deposit small amounts of energy in the calorimeter.

The second level neutral trigger employed a “hardware cluster finder” (HCF) to count the number of isolated depositions of energy (clusters) in the lead glass. Events with four clusters were accepted as candidate $\pi^0\pi^0$ events.

The level 1 neutral mode vetoes were intended to reject triggers associated with hadronic activity in the lead glass or with electromagnetic activity in the vicinity of the calorimeter’s beam holes. A twenty-one radiation length lead wall behind the lead glass prevented electromagnetic showers in the calorimeter from registering in the $\mu_1$ hodoscope. Since hadronic showers in the lead glass (which could fake a four-photon trigger) would usually produce activity in $\mu_1$, a large signal in $\mu_1$ served to veto a level 1 neutral trigger.

A twenty-eight radiation length lead-lucite shower counter, the BA (“back anti”), was installed in the beams behind the calorimeter. Electromagnetic showers tended to deposit most of their energy in the upstream part of the BA, while interactions of beam neutrons and kaons produced large signals towards the back of the BA. Events with substantial activity in the upstream part of the BA, but minimal energy in the last third of the BA, were vetoed. The comparison of the signals in the up- and downstream sections allowed us to reject events with lost photons without vetoing good events with accidental activity associated with the interaction of a beam hadron in the BA. Muons and $K_{e3}$ electrons provided calibration information about the BA.

The CA (“collar anti”) covered the inner halves of the blocks around the two beam holes and consisted of eight radiation lengths of material followed by scintillation counters. The CA allowed us to veto an event in which a photon would have landed near a beam hole; the reconstruction of the energy and position of these photons would have been problematic.

Most of the photon veto counters were constructed of a plane of scintillator followed by a lead-lucite shower counter six radiation lengths thick. Each counter’s scintillator was viewed
by one phototube, while the shower counter was viewed by a pair of phototubes. These counters were designed to detect low energy photons which were traveling at angles up to 50 milliradians with respect to the beams; they were typically 90% efficient at registering 150 MeV photons. We mapped the position dependence of the response of photon veto counters with muons. We were able to calibrate their overall response using \( K \rightarrow \pi^+\pi^-\pi^0 \) decays in which one photon missed the lead glass. The direction and energy of the missing photon could be inferred from the kinematics of the charged pions and reconstructed photon. Often this missing photon passed through one of the photon veto counters. By comparing the observed signals in the veto counters to the predicted energy of the “undetected” photon, the gains and resolutions of the counters were extracted. We included many of the photon veto scintillation counters in the level 1 vetoes for charged and neutral triggers. However, only the shower counters in the plane closest to the lead glass (“LGA”) participated in the level 1 vetoes. Information from the shower counters was used during offline analysis to discard events in which photons might have missed the lead glass.

In addition to the \( \pi^+\pi^- \) and \( \pi^0\pi^0 \) triggers, several special purpose triggers were collected at the same time. The “accidental” trigger was used to study the effects of activity in the detector not associated with a decaying \( K \) meson. Muons traveling through a scintillator telescope aimed at the target pile would assert the trigger. The telescope was roughly 50 m from the target; the line from the target through the telescope passed approximately ten meters west of the spectrometer. As a result, the accidental trigger rate was proportional to the instantaneous beam intensity, but was otherwise independent of the presence or absence of activity in the spectrometer. Pedestal triggers were used to record the zero-signal behavior of all ADC’s; calibration flasher triggers were used to monitor the behavior of the lead glass calorimeter. Single muon triggers written during special calibration runs were used to monitor the performance of many subsystems in the detector.
F. Data acquisition

A large number of signals were registered by the detector’s data acquisition electronics for each event which satisfied the experiment trigger. Discriminated signals from scintillation counters were stored in CAMAC latches, while analogue information from the lead glass, regenerators, and veto system was processed by FASTBUS ADC’s. All data, including drift chamber hit times and status information from the trigger electronics, flowed into a PANDA [24] data acquisition system which could record approximately 13,000 events per spill. PANDA wrote data simultaneously to four 8 mm cassette tapes and passed a fraction of the events to a microVaX which monitored the experiment and generated histograms. Data describing magnet currents and targeting information were stored at the end of each spill along with integrated counting rates from various detectors and components of the trigger.

V. RECONSTRUCTION OF $\pi^+\pi^-$ AND $\pi^+\pi^-\gamma$ EVENTS

A. Tracking

Tracks in the horizontal ($x$) and vertical ($y$) views were found independently. Since the analyzing magnet deflected charged particles in the horizontal plane, $x$-view track segments were found separately in the pairs of drift chambers upstream and downstream of the magnet. Tracks in the $y$-view were found in all four chambers simultaneously. Trial track segments in the upstream chambers were required to point in the general direction of the decay volume, while downstream segments were required to point towards the lead glass. Hits were required in at least three of the four sense planes in upstream and downstream $x$-view track segments. A $y$-view track was required to have hits in at least five of eight sense planes. No tracks were allowed to share hits. After track segments were identified, drift distance information from the TDC’s was used to refine the tracks’ positions.

Track segments in the $x$-view were projected to the “bend plane” at the center of the
analyzing magnet. Upstream and downstream segments were paired if their projections were separated by less than 1 cm. The $x$-view and $y$-view tracks were correlated using information from cluster positions measured in the lead glass. After matching of horizontal and vertical views, hit positions were recalculated to correct for small chamber rotations, gravitationally-induced sagging of sense wires, and differences in signal propagation time along the sense wires. At this point, upstream and downstream $y$-view track segments were refit separately since the analysis magnet tended to impart a small vertical impulse to tracks. The difference in upstream and downstream $x$-view track slopes, combined with knowledge of the analyzing magnet’s field integral, provided us with sufficient information to calculate a particle’s momentum.

Since the paired sense planes in the drift chambers were staggered by a half-cell, the sum of the measured drift distances in one view of a chamber should equal the 6.35 mm separation between adjacent sense wires. (Because of the 1.1 cm separation between sense planes along the beam direction, a small correction was applied for track angle with respect to the $z$ axis.) The sums-of-distances allowed us to recognize signals which came from out-of-time tracks. Given the temporal structure of the extracted Fermilab proton beam (see Sec. IV A), a track from an earlier (or later) bucket would have its sums-of-distances mismeasured by at least 1.8 mm.

We required pairs of tracks to come from a common vertex. The vertex position was determined using a full fit which included individual tracks’ estimated accuracy in contributing to the vertex determination. Our requirement on the tracks’ distance of closest approach depended on track momenta and on the distance from the vertex to the chamber system. Typically, tracks with a few millimeter separation satisfied the vertex-quality criterion. Vertex resolution was $\sim$0.8 mm in the $x$ and $y$ views, and $\sim$ 15 cm in $z$. The vertex position in $\pi\pi\gamma$ decays was set equal to the tracking vertex, without reference to the reconstructed photon.

Additional tracking cuts were made to ensure reconstruction quality. A $\chi^2$ cut rejected tracks with large deviations of hits from ideal positions. A matching ambiguity cut dis-
carded events in which the lead glass information could not be used reliably to pair tracks with similar $x$ positions with the corresponding $y$-view tracks. We also required reasonable agreement in the projections to the center of the magnet for up- and downstream track segments in both $x$ and $y$ views.

**B. Event reconstruction**

Events which came from $K \rightarrow \pi^+\pi^-$ and $K \rightarrow \pi^+\pi^-\gamma$ decays were expected to reconstruct with invariant mass close to the known kaon mass. Since we were interested in detecting kaons which did not scatter in the regenerators, we required that the final state $\pi^+\pi^-$ or $\pi^+\pi^-\gamma$ have minimal transverse momentum ($p_T$) with respect to the incident kaon direction. Almost all $K_{\mu 3}$ decays were removed at the trigger level so most charged mode triggers came from $K \rightarrow \pi e\nu$ decays. There were also significant contributions to the event sample from $K \rightarrow \pi^+\pi^-\pi^0$ and $\Lambda \rightarrow p\pi^-$ decays. Events in which a kaon scattered in the regenerator contributed to observed backgrounds.

Backgrounds from $K_{e3}$ decays were suppressed by rejecting events which had a ratio of calorimeter energy and track momentum ($E/p$) close to unity. Requirements on $E/p$ for the $\pi^+\pi^-$ and $\pi^+\pi^-\gamma$ analyses are described in Table I. At the expense of losing a few percent of the events with two charged pions in the final state we were able to reject nearly all the $K_{e3}$ background. We required tracks to pass through regions of the lead glass in which cluster energies could be reconstructed reliably. Events with a track near the outer edges of the glass, in a beam hole, or in the collar anti were discarded.

To reduce background contamination from $K_{\mu 3}$ decays, we required both tracks to project into the $\mu 2$ hodoscope. Since “soft” muons were likely to stop in the 3.2 m steel muon filter rather than reaching $\mu 2$, we rejected events in which either track had momentum below 7 GeV/c. This cut also reduced the fraction of $\pi^+\pi^-$ decays which were vetoed because a pion decayed in flight with the resulting muon striking a $\mu 2$ counter.

Only the most energetic $\Lambda$’s from our target survived long enough to decay downstream
of the first regenerator. Most of these interacted in a regenerator or were eliminated by
the requirement that tracks neither hit the collar anti nor went through a beam hole in
the lead glass. The majority of $\Lambda \rightarrow p\pi$ decays came from hadronic interactions in the
regenerators which produced a $\Lambda$ without asserting the regenerator vetoes. Because of the
small $Q$ value, the ratio of the proton and pion momenta in high energy $\Lambda$ decay will always
satisfy $p_p/p_\pi > 3.09$. Since we had no particle identification for separating hadronic species,
we assumed that the track with greater momentum was a proton (or antiproton) when
reconstructing a two-track state as a $\Lambda$ (or $\bar{\Lambda}$). Kinematic cuts (to be described below) were
used to reject $\Lambda$ decays.

To reduce sensitivity to imperfections in our simulation of accidental activity in the
detector, both the $\pi^+\pi^-$ and $\pi^+\pi^-\gamma$ analyses required that reconstructed tracks satisfied
the charged mode trigger. In addition to this trigger “verification,” we discarded events
where tracks passed near limiting apertures.

Values for various cuts in the $\pi^+\pi^-$ and $\pi^+\pi^-\gamma$ analyses are listed in Table I.

C. $\pi^+\pi^-$ signal and backgrounds

Reconstructed $K \rightarrow \pi^+\pi^-$ mass distributions (after all other cuts, including $p_T^2$) are
shown in Fig. 7. Events from the upstream and downstream regenerator beams are plotted
separately. In addition, data from the periods before (set 1) and after (set 2) the removal
of the T,V hodoscope are plotted separately. Results from the Monte Carlo are superposed
as points on the data histograms. The data’s high-side tails contained contributions from $\delta$-
rays which were not simulated by the Monte Carlo. Our mass resolution was $\lesssim 3.2$ MeV/$c^2$
in data written before the T,V hodoscope was removed and $\sim 0.25$ MeV/$c^2$ better after
its removal; the analysis required $484\ MeV/c^2 < m_{\pi\pi} < 512\ MeV/c^2$. The tail on the low
side of the peak came from radiative $K \rightarrow \pi\pi\gamma$ decays. Backgrounds from semileptonic
decays were nearly negligible. There was no background contribution from $\pi^+\pi^-\pi^0$ decays
since the invariant mass of the $\pi^+\pi^-$ system was well below the signal region. Events which
had $1110 \text{ MeV}/c^2 \leq m_{\pi\pi} \leq 1122 \text{ MeV}/c^2$ when reconstructed as $\Lambda \rightarrow p\pi$ candidates were discarded.

We calculated the apparent transverse momentum of a kaon with respect to the incident beam particle’s direction in order to reject backgrounds from diffractive and inelastic $K$ production as well as semileptonic decays. We extrapolated the kaon trajectory from the decay vertex upstream to the regenerators. The kaon’s transverse momentum, $p_T$, is the component of the observed vector momentum which is perpendicular to the line connecting the production target with the kaon’s position at a regenerator. Because of the spacing between the beams’ inner edges we could determine unambiguously which regenerator the kaon traversed.

The $p_T^2$ distributions are shown in Fig. 8. The $\pi\pi$ analysis required $p_T^2 < 250 \text{ (MeV}/c)^2$; tails near the coherent regeneration peak in the distribution were dominated by radiative $K \rightarrow \pi\pi\gamma$ decays and diffractively regenerated $K_S$. At large $p_T^2$, inelastically produced $K_S$ contributed significantly to the tails. Some smearing of the data’s coherent peak was due to the presence of $\delta$-rays.

The $p_T^2$ distributions of events passing all other cuts were used to determine background levels. The small backgrounds consisted of a mix of diffractive $K \rightarrow \pi^+\pi^-\pi^0$ decays, radiative $\pi^+\pi^-\gamma$ decays, and semileptonic final states; they are described in Table I. A total of 1.8 million $\pi^+\pi^-$ decays were found in the entire run’s data after all cuts; the event sample is described in Table III. We required that kaons had energies in the range 30 to 160 GeV and that decay vertices were downstream of the vacuum window following the appropriate regenerator. To reduce backgrounds associated with interactions of beam hadrons in material near the downstream regenerator, we discarded events from the upstream regenerator beam with decay vertices near the gap in the vacuum system. Figs. 9 and 10 show the decay vertex $z$ position and $K$ energy spectrum for $\pi^+\pi^-$ decays passing all other cuts. Data (histogram) and Monte Carlo (points) are shown for each beam. The T,V hodoscope, located near $z = 141 \text{ m}$, was included in the charged mode trigger for set 1 data.
D. $\pi^+\pi^-\gamma$ signal and backgrounds

The $\pi^+\pi^-\gamma$ analysis required two well-reconstructed tracks and at least one in-time lead glass cluster with energy $\geq 1.5$ GeV which was not associated with either track. In the center of mass of the kaon, this photon was required to have an energy greater than 20 MeV. To suppress background from $\pi^+\pi^-\pi^0$ decays, we required

$$\frac{\left( (m_K^2 - m_0^2 - m_{\pi^+\pi^-})^2 - 4m_0^2m_{\pi^+\pi^-} - 4m_K^2 \cdot (p_{\pi^+\pi^-}^T)^2 \right)}{4 \left[ (p_{\pi^+\pi^-}^T)^2 + m_{\pi^+\pi^-}^2 \right]} \equiv p_0^2 < -0.01125 .$$

Here, $m_{\pi^+\pi^-}$ is the invariant mass of the two charged tracks and $(p_{\pi^+\pi^-}^T)^2$ is the square of their transverse momentum with respect to the parent kaon. (We assumed that the kaon traveled in a straight line from the production target to the decay point and did not scatter in the regenerator.) Since $p_0$ is proportional to the $\pi^0$ momentum component in the kaon center-of-mass which is parallel to the beam direction, $p_0^2 \geq 0$ for events which might be $3\pi$ decays. [25,26] Shown in Fig. 11 are energy spectra for the photon in the $K$ center of mass for $\pi^+\pi^-\gamma$ decays satisfying all other cuts.

We removed background from $K \rightarrow \pi^+\pi^-$ events accompanied by accidental activity in the calorimeter by discarding events with $m_{\pi^+\pi^-} > 484$ MeV/c$^2$. Remaining backgrounds from $\Lambda \rightarrow p\pi$ decays (in combination with an accidental lead glass cluster) were reduced with kinematic cuts. Events which reconstructed to have $1100$ MeV/c$^2 \leq m_{p\pi} \leq 1130$ MeV/c$^2$, total energy above 100 GeV, and momentum ratio $p_p/p_\pi > 3.0$ were discarded.

To select events in which only coherently regenerated $K_S$ and $K_L$ decay amplitudes participated, we rejected events with reconstructed mass outside the range $484$ MeV/c$^2 < m_{\pi^+\pi^-\gamma} < 512$ MeV/c$^2$ or with large transverse momentum such that $p_T^2 > 150$ (MeV/c)$^2$. Shown in Figs. 12 and 13 are mass and $p_T^2$ for $\pi^+\pi^-\gamma$ decays satisfying all other cuts. Results of the Monte Carlo simulation (shown as points) did not include background modeling.

The data separated naturally into two sets corresponding to the periods of running time before and after the removal of the T,V hodoscope. After cuts and background subtractions, a total of $(10,769 \pm 17)$ $\pi^+\pi^-\gamma$ decays were found in the two data sets; the distribution of
Events between the sets is shown in Table V.

Events for each set were binned according to kaon momentum $p$ and decay position $z$ in a grid with cell size $10 \text{ GeV/c} \times 2 \text{ m}$. We required kaons to have energies between 25 and 155 GeV. Figs. 14 and 15 show the decay vertex $z$ position and $K$ energy spectrum for $\pi^+\pi^-\gamma$ decays passing all other cuts. Decay vertices were required to be downstream of the vacuum window following the appropriate regenerator. The mass and $p_T^2$ distributions of events passing all other cuts were used to determine the level of background in each set. Backgrounds were typically 2%, and are listed for each subset in Table V. The small backgrounds consisted of a mix of noncoherent $K \rightarrow \pi^+\pi^-\gamma$ decays and final states made from two-track events combined with accidental activity in the calorimeter. We modeled the shape of the background using events which passed the $\pi\pi\gamma$ mass cut, but had $p_T^2$ between 300 and 2000 (MeV/c)$^2$. These events were binned in $(p,z)$ grids identical to those of the signal region, scaled appropriately, then subtracted from the signal-region $(p,z)$ grids.

VI. RECONSTRUCTION OF $\pi^0\pi^0$ EVENTS

A. Photon reconstruction

The reconstruction of an electromagnetic shower (“cluster”) in the lead glass array proceeded in stages. Lead glass channels whose ADC signals were local maxima were taken as the central blocks in candidate clusters. Signals from central blocks were required to be above 200 ADC counts; signals from the eight blocks surrounding each central block were included in the energy estimate formed for clusters.

A first estimate of a cluster’s energy was made using the sum of the ADC counts, divided by the effective gains of each channel, in the cluster’s nine blocks. The effective gain included photomultiplier gain, ADC digitization conversion gains, photocathode quantum efficiency, and light collection efficiency. A rough correction for absorption of Čerenkov light in each block was applied to the initial cluster energy estimate. With this correction, the estimated
cluster energy was usually within 30% of the true shower energy.

Several corrections were made to this initial estimate. A small amount of energy was added to account for blocks which did not meet the readout threshold of five ADC counts. A second correction compensated for radial leakage outside the nine blocks in a cluster, into the beam holes, or past the outer edges of the array. When showers overlapped, the energy in shared blocks was divided between showers based on typical shower profiles. During the early part of data taking, the lead glass enclosure’s temperature control system malfunctioned. Since we recorded the enclosure’s temperature with event data, we were able to correlate gain changes with temperature, and to map the enclosure’s temperature as a function of time. Using this information, we applied a small time-dependent gain correction to lead glass signals. The ADC pedestals showed a small rate-dependent shift; we corrected them based on the instantaneous event rate.

The final determination of the energy in an electromagnetic shower included correction for effects associated with absorption of Čerenkov light in the lead glass. The depth at which a shower deposited most of its light increased with shower energy. As a result, the response of the calorimeter tended to “brighten” with increasing shower energy, since less light was absorbed by the glass between “shower max” and the photomultiplier tube. The absorption coefficients varied from block to block, and changed with time as radiation damage increased. The coefficient of the cluster’s central block was used to calculate the correction factor. An additional correction compensated for the fact that photons began showering deeper in the lead glass than did electrons. Shower energies associated with photons which converted to $e^+e^-$ pairs in the B and C hodoscopes were adjusted by 3.5%. In a final correction we shifted electron-induced shower energies 0.3% to compensate for the effects of bremsstrahlung photons produced in the trigger hodoscopes near the calorimeter.

Shower $x, y$ positions were found using the estimated energies deposited in the three columns and three rows of blocks in a cluster. We studied the calorimeter’s position resolution by comparing the impact point of electrons, determined by the drift chambers, with shower centers, determined by the lead glass. The position algorithm’s results were typically
accurate to 3 mm, though the resolution varied from 1.5 mm near the corners of blocks to 5 mm at the centers of blocks.

**B. Event reconstruction**

The invariant mass of two photons originating from a single decay vertex and traveling at small angles relative to the beam direction is

\[ m = \frac{r_{12}}{z} \sqrt{E_1 E_2} \]

where \( z \) is the distance from the decay vertex to the calorimeter, \( E_1 \) and \( E_2 \) are the photons’ energies, and \( r_{12} \) is their separation at the calorimeter. We were interested in reconstructing final states in which two (or three) \( \pi^0 \)'s produced four (or six) detected photons. There were three possible ways to “pair” the photons in a candidate \( \pi^0 \pi^0 \) event, and fifteen ways in a candidate \( 3\pi^0 \) event. For each of the possible pairings, we calculated a separate decay \( z \) for each pion by assuming that the invariant mass of the corresponding photon pair equaled the \( \pi^0 \) rest mass. We then determined errors for each \( z \) from the uncertainties in cluster energies and positions, and calculated a \( \chi^2 \) for the hypothesis that the photon pairs came from the same vertex. Using the pairing with the smallest \( \chi^2 \), we assigned a decay \( z \) for the event based on a weighted average of the pions’ individual \( z \) positions. Typical vertex resolution was about 1 meter; the invariant mass of the four (or six) photons was

\[ m = \frac{1}{z} \left( \sum_{i<j} E_i E_j r_{ij}^2 \right)^{1/2}. \tag{6.1} \]

Several cuts were made to reduce background contamination in the data. We rejected events with pairing \( \chi^2 > 4 \). In addition, we discarded four-photon events in which the second-best pairing had a reasonable \( \chi^2 \) and yielded an invariant mass near the \( K \) mass. Background from \( 3\pi^0 \) decays with undetected photons was reduced by discarding events with significant activity in the photon veto system. In addition, \( 3\pi^0 \) decays with photon energies below the HCF thresholds were removed by cutting events with \( \geq 600 \text{ MeV} \) of in-time energy.
in any lead glass block which was not associated with one of the four reconstructed clusters. Backgrounds associated with interactions of beam hadrons with material in the detector were reduced by cuts on the number of drift chamber hits and on trigger hodoscope activity. As was the case with charged mode events, we rejected $\pi^0\pi^0$ candidates with significant activity in either regenerator.

Other cuts were made to simplify Monte Carlo simulation of the detector. Photons were required to have energies greater than 2.2 GeV to reduce sensitivity to inaccuracies in the simulation of HCF thresholds. Events were discarded if any cluster reconstructed within a half block of the outer edge of the lead glass array. We rejected events with appreciable activity in the collar anti. To avoid problems from events with overlapping photons, we required the energy distribution among the blocks in a cluster to be consistent with the shape expected for a single electromagnetic shower.

The calorimeter was calibrated with electrons, not photons. The coupling between the energy and $z$ scales in the invariant mass relationship [Eq. (6.1)] was used to check the photon energy scale. The reconstructed positions of the downstream faces of the regenerators were studied as functions of energy; the energy scale was adjusted so that the data and Monte Carlo distributions agreed.

Cuts made in the $\pi^0\pi^0$ and $\Delta\phi$ analyses are summarized in Table VI.

**C. $\pi^0\pi^0$ signal and backgrounds**

Our preliminary result for $\text{Arg}(\eta_0) - \text{Arg}(\eta_{+-})$ is based on data written after the T, V hodoscope was removed. (See Sec. IV E.) The reconstructed $K \to \pi^0\pi^0$ mass distributions (after all other cuts) are shown in Fig. 16. Results from the Monte Carlo simulation of signal and backgrounds are superposed on the data. In the plots on the left, the simulation includes both signal and backgrounds. In the plots on the right, only the simulated backgrounds are superposed on the data. The signal region is between 474 MeV and 522 Mev. The background is mostly from $K \to 3\pi^0$ with two undetected photons. There is a small
background contribution from neutron interactions in detector material which produced two neutral pions.

The $x, y$ vertex position was not measured in neutral decays so the transverse momentum of the parent kaon, $p_T$, could not be determined directly. Instead, the center-of-energy of the four photons was used to select decays of kaons which had not scattered in the regenerators. The center of energy was defined as

$$\vec{r}_{C.E.} \equiv \frac{\sum E_i \vec{r}_i}{\sum E_i}$$

where $\vec{r}_i$ was the point at which the $i^{th}$ photon struck the calorimeter. Since $\vec{r}_{C.E.}$ corresponded to the point at which the kaon would have reached the lead glass if it had not decayed, unscattered kaons had $\vec{r}_{C.E.}$ inside the beam spots.

We defined an event’s “ring number” to be the area circumscribed by a square, centered on the closer beam, whose perimeter contained $\vec{r}_{C.E.}$ (see Fig. 17). The ring number distributions (after all other cuts) are shown in Fig. 18. Unlike the charged mode $p_T^2$ spectra, the neutral mode ring plots contain no sharp peaks because of the finite beam size. Events were required to have a ring number less than 120 cm$^2$. Near the coherent regeneration region in the ring plot is a shoulder from diffractively regenerated $K_S$; inelastically regenerated $K_S$ populate the high ring number region. As before, the predictions of the Monte Carlo are superposed on the data.

Normalizations for our background subtractions were established using events which lay outside the signal region in the mass vs. ring number plane. The background shape in the signal region was known from Monte Carlo studies of $K \rightarrow 3\pi^0$ decays which faked $\pi^0\pi^0$ events and from measurements of the $p_T^2$ distribution for $\pi^+\pi^-$ events. (Our $p_T^2$ resolution in the charged decay mode let us identify scattered kaons whose centers-of-energy remained inside the beam profile.) A total of $2.6 \times 10^5 \pi^0\pi^0$ decays were found in the set 2 data after all cuts. The event sample is described in Table VII. The kaon energy was required to be in the range $40 \leq E_K \leq 150$ and the decay vertex $z$ position was required to be in the range $120 \leq z \leq 152$ for the upstream regenerator and $130 \leq z \leq 152$ for the downstream.
regenerator. The vertex position distributions and kaon energy spectra for \( \pi^0 \pi^0 \) decays passing all other cuts are shown in Figs. [19] and [20].

VII. MONTE CARLO SIMULATION

We measured the phases of \( \eta^{+-} \), \( \eta^{+-\gamma} \), and \( \eta_{00} \) by studying the proper time evolution of the \( \pi\pi \) and \( \pi\pi\gamma \) decay rates downstream of the regenerators. The detector’s acceptance varied with kaon energy and decay position, so we needed to unfold apparatus effects from observed decay spectra in order to determine values for the magnitudes and phases of the \( \eta \)'s. We used a detailed Monte Carlo simulation of the experiment to calculate the \( p \) and \( z \) dependence of the detection efficiency. We also used the Monte Carlo to study backgrounds and to determine the effects of various sources of systematic uncertainty. The accuracy of the simulation could be verified with the large samples of \( K_{e3} \), \( \pi^+\pi^-\pi^0 \), and \( 3\pi^0 \) decays which satisfied the experiment triggers.

The Monte Carlo program generated raw data which was processed by the same reconstruction programs used to analyze real data. The effects of spurious detector activity could be studied by combining data from accidental triggers (see Sec. [IV.E]) with simulated event data.

A. Beamline simulation

The neutral beam simulation modeled the creation of \( K^0 \) and \( \overline{K^0} \) in our production target as well as the propagation of kaons through absorbers and the (imperfectly aligned) collimation system. Even the small amount of residual \( K_S \) amplitude present in high energy kaons striking our regenerators was modeled by the beam simulation. Propagation of kaons in the regenerators was treated in detail, allowing for coherent regeneration, diffraction regeneration through single (and multiple) elastic scattering, inelastic processes, and decays inside the regenerator. Regeneration in air gaps, vacuum windows, and the T,V hodoscope was included in the Monte Carlo. The simulation of the interactions of daughter particles
from $K$ decay included multiple Coulomb scattering, the production of bremsstrahlung photons by electrons and positrons, and the conversion of photons to $e^+e^-$ pairs. Simulated charged pions were allowed to decay in flight while traveling through the spectrometer.

We based our $K^0, \overline{K^0}$ energy spectra on Malensek’s parametrization of the $K^+$ and $K^-$ production spectrum for 400 GeV protons incident on a beryllium target [27]. He described the differential production cross section in terms of $x_F \equiv p_K/E_{\text{beam}}$ and $p_T$; in general, cross sections expressed in terms of these variables scale well with beam energy (Ref. [7]). Because of measurement errors in the result of Ref. [27] and uncertainties associated with the extrapolation to higher incident beam energy, we found it necessary to adjust the shape of the $K$ production spectrum to fit our data. The correction function varied from unity by $\sim \pm 20\%$ over the range of usable kaon energies. Though it was very convenient to have the Monte Carlo’s energy spectrum well-matched to the data’s energy spectrum, our analysis was not particularly sensitive to inaccuracies in spectrum modeling.

The Monte Carlo propagated kaons through the target and the beryllium and lead absorbers (see Sec. IV A), modeling the effects of coherent regeneration and (non-regenerative) single elastic scattering in addition to attenuation of the beam. The simulation modeled the $x, y$ intensity profile of the incident proton beam and allowed kaons to scatter in collimator walls. The accuracy of the absorber/collimator description can be seen in Fig. 21 for $K_{e3}$ decays. Tails in the data’s distribution of angle between the target-to-vertex direction and the $z$ axis are well-matched by the Monte Carlo. These tails are associated with elastic scattering of kaons in beamline elements.

The $x, y$ locations and sizes of limiting apertures in the detector were measured using electrons from $K_{e3}$ decays, while the $z$ positions of the apertures were determined using survey information. Shown in Fig. 22 are comparisons of the data and Monte Carlo electron illuminations at the east edge of the HDRA aperture in 1 mm bins. Illumination of charged pions at the HDRA and lead glass are shown in Figs. 23 and 24 for events which passed $\pi^+\pi^-$ analysis cuts.
B. Drift chamber simulation

The Monte Carlo simulation of the drift chambers did not attempt to model detailed processes like electron drift and diffusion. Instead, the drift time distribution observed in the data was integrated to produce a mapping of drift distance ↔ drift time. Chamber resolution was incorporated by jittering the true positions of particles at chamber sense planes with the appropriate Gaussian distributions. Smeared positions were then converted to drift times. When two tracks passed through the same drift cell, only the first signal to arrive at the chamber amplifier/discriminator was included with simulated data. (Our chamber TDC’s only registered the first hit received by each channel.)

Chamber inefficiencies were modeled in some detail, since they determined the characteristics of the chamber component of the charged mode trigger. Individual wire efficiencies were ∼99%; the characteristics of especially inefficient channels observed in the data were included in the simulation.

The chamber simulation did not model δ-ray production. Consequently, the high-side tail in the track χ² distribution is not simulated. We expected that the effects of δ-rays were not correlated significantly with the decay vertex z position, and would only cause an overall loss of data relative to Monte Carlo without introducing biases in our phase-of-η measurements.

C. Lead glass simulation

The mean amount of Čerenkov light which reached an individual block’s photomultiplier tube was a function of the incident particle’s energy, the block’s attenuation length for Čerenkov light, and the depth at which the shower began. Since showers were almost fully contained in the calorimeter, signals from showers which began deep in a block suffered less attenuation than those which began early. We parametrized the distribution about the mean for the light reaching the phototube as a Gaussian with high-side and low-side exponential
tails. Photon showers were simulated as pairs of electron showers which begin where the photon converted to an $e^+e^-$ pair. The use of an effective block length allowed us to take into account the fact photons began to shower at different depths in the calorimeter. Extensive studies using EGS4 \cite{28} have shown this to be a sensible description of the production of Čerenkov light in lead glass. \cite{29}

Studies with the xenon calibration flasher were used to determine the mean number of photoelectrons per Čerenkov photon expected for each block. The simulation chose the amount of Čerenkov light which reached each phototube, determined the expected number of photoelectrons, then varied this assuming a Gaussian distribution whose width was the square root of the expected mean.

The event simulation modeled cluster shapes using a library of electron clusters obtained from special runs. In these calibration runs, a beam of photons was converted to $e^+e^-$ pairs; the $e^+$ and $e^-$ were steered by calibration magnets (see Fig. 1) into the lead glass. (By varying the currents in the magnets we were able to illuminate the entire calorimeter with a wide range of electron energies.) Clusters were selected from the library based on incident electron energies and positions in the central block. The signal in each block was scaled by a constant to give the simulated response. For photons, two appropriate electron clusters were overlayed.

We adjusted the tail structure in the glass response to correct for effects not modeled in the EGS4 simulation. For example, EGS4 assumed that the attenuation of Čerenkov light was uniform along the length of a block. In reality, absorption increased with depth: there was more radiation damage deep in blocks due to hadronic interactions. In addition, the curing lamp (which was placed near the upstream face of the calorimeter) was less effective at correcting damage in the downstream end of blocks.
D. Veto and hodoscope simulation

Efficiencies of scintillation counters were measured using muon tracks. Small gaps between adjacent counters in hodoscope planes were mapped using the drift chambers and included in the Monte Carlo. Since we had determined the response of photon veto counters with data we were able to simulate their response to photons in the Monte Carlo.

E. Comparisons of data and Monte Carlo

The large samples of $K \rightarrow \pi e\nu$ and $K \rightarrow 3\pi^0$ decays provided us with useful tools for verifying the accuracy of the Monte Carlo simulation. We collected roughly five times as many $3\pi^0$ decays as $2\pi^0$ decays, and more than ten times as many $K_{el}$'s as $\pi^+\pi^-$ decays. It was particularly important that we modeled correctly acceptance and resolution effects that influenced the reconstructed vertex $z$ distributions. Shown in Figs. 25 and 26 are comparisons of these distributions for data and Monte Carlo. Our success in simulating the acceptance of these “high statistics” modes gives us confidence in the validity of conclusions we draw from characteristics of the $\pi\pi$ and $\pi\pi\gamma$ samples.

VIII. ANALYSIS AND FITS

A. $\phi_{+-}$, $\Delta m$, $\tau_S$ analysis

We used the Monte Carlo detector simulation to predict the $\pi^+\pi^-$ acceptance in 10 GeV/c $\times$ 2 m cells in momentum $p$ and decay position $z$. The acceptance in each cell was defined as the ratio of the number of detected events with reconstructed $p$, $z$ in the cell divided by the number of generated events with true $p$, $z$ in that cell. Since the simulation accurately modeled the data, effects due to resolution smearing were properly included in our fits. Average acceptance was approximately 20%.
We measured $Arg(\eta_{+-})$ as well as the regeneration parameters $\alpha$ and $[(f - \overline{f})/k]_{70 \text{ GeV}}$ with a fit to background-subtracted data which included free parameters describing the beam intensities and energy spectra. In this fit we constrained the $K_L - K_S$ mass difference to the value measured by E731: $\Delta m = 0.5286 \times 10^{10} h s^{-1} c^{-2}$ (Ref. [9]), while using the world-average value for the $K_S$ lifetime: $\tau_S = 0.8922 \times 10^{-10} s$ (Ref. [7]). (These values correspond to $\phi_\epsilon = 43.33^\circ$.) We found that $\phi_{+-} = (43.35 \pm 0.70)^\circ$ where the error is statistical (systematic uncertainties will be discussed below). The fit’s $\chi^2$ was 598 for 577 degrees of freedom.

In a separate fit we determined $\Delta m$ and $\tau_S$ (as well as the beam and regeneration parameters) by constraining the phase of $\eta_{+-}$ to equal the phase of $\epsilon$:

$$\phi_{+-} \equiv \phi_\epsilon = \tan^{-1} \left( 2\Delta mc^2 \tau_S / \hbar \right).$$

We found that $\Delta m = (0.5286 \pm 0.0029) \times 10^{10} h s^{-1} c^{-2}$ and $\tau_S = (0.8929 \pm 0.0014) \times 10^{-10} s$, corresponding to $\phi_\epsilon = 43.35^\circ$. Again, the errors are statistical. When $\phi_{+-}$ was unconstrained, we found that $\phi_{+-} - \phi_\epsilon = (-0.84 \pm 1.42)^\circ$, $\Delta m = (0.5268 \pm 0.0041) \times 10^{10} h s^{-1} c^{-2}$, and $\tau_S = (0.8942 \pm 0.0026) \times 10^{-10} s$.

Sources of systematic errors included uncertainties in our determination of experimental acceptance, errors associated with background subtractions, uncertainties in our descriptions of the two regenerators, and possible deviations of the momentum dependence of $(f - \overline{f})/k$ from a power law. We studied acceptance issues by moving the edges of limiting apertures and introducing spurious $z-$dependent slopes (some of which were functions of momentum) to force disagreement between data and Monte Carlo. Background systematics were investigated by varying the $p_T^2$ cut, thus changing the amount of $K_{e3}$ contamination remaining with the coherent signal. We changed the positions and relative lengths of the regenerators in the apparatus simulation to determine our sensitivity to uncertainties in regenerator geometry. Issues concerning an unexpected momentum dependence in the regeneration amplitude are described in Ref. [18]. The sensitivity of $\phi_{+-}$ to these effects is summarized in Table VIII; combining the errors in quadrature yielded a net systematic uncertainty of $\pm 0.79^\circ$. The fit
for $\phi_{+-}$ did not allow $\Delta m$ or $\tau_S$ to vary. Increasing the values of $\Delta m$ and $\tau_S$ by one (Particle Data Group) standard deviation (Ref. [7]) would change $\phi_{+-}$ by $+0.38^\circ$ and $-0.62^\circ$ respectively.

In summary, our preliminary results are

$$\phi_{+-} = [43.35 \pm 0.70 \text{ (stat.)} \pm 0.79 \text{ (syst.)}]^\circ$$

($\Delta m$ fixed at $0.5286 \times 10^{10}\text{hs}^{-1}c^{-2}$, $\tau_S$ fixed at $0.8922 \times 10^{-10}\text{s}$.)

$$\delta(\phi_{+-}) = +0.38^\circ \text{ for } \delta(\Delta m) = 0.0024$$

$$\delta(\phi_{+-}) = -0.62^\circ \text{ for } \delta(\tau_S) = 0.0020 .$$

If instead we constrain $\phi_{+-} \equiv \tan^{-1}(2\Delta mc^2\tau_S/h)$, we find

$$\Delta m = [0.5286 \pm 0.0029 \text{ (stat.)} \pm 0.0022 \text{ (syst.)}] \times 10^{10}\text{hs}^{-1}c^{-2}$$

$$\tau_S = [0.8929 \pm 0.0014 \text{ (stat.)} \pm 0.0014 \text{ (syst.)}] \times 10^{-10}\text{s} .$$

Fitting simultaneously for $\phi_{+-}$, $\Delta m$, and $\tau_S$ yields

$$\phi_{+-} - \phi_\epsilon = [-0.84 \pm 1.42 \text{ (stat.)} \pm 1.22 \text{ (syst.)}]^\circ$$

$$\Delta m = [0.5268 \pm 0.0041 \text{ (stat.)} \pm 0.0029 \text{ (syst.)}] \times 10^{10}\text{hs}^{-1}c^{-2}$$

$$\tau_S = [0.8942 \pm 0.0026 \text{ (stat.)} \pm 0.0018 \text{ (syst.)}] \times 10^{-10}\text{s} .$$

Systematic uncertainties will decrease with additional analysis work.

These results can be compared with the Particle Data Group’s world average values [7] and previous measurements from E731 [4] and NA31 [8]. The PDG values are $\phi_{+-} = (46.5 \pm 1.2)^\circ$, $\Delta m = (0.5351 \pm 0.0024) \times 10^{10}\text{hs}^{-1}c^{-2}$, $\tau_S = (0.8922 \pm 0.0020) \times 10^{-10}\text{s}$. Since the value of $\phi_{+-}$ is correlated with the values of $\Delta m$ and $\tau_S$, the PDG used the world-average values of $\Delta m$ and $\tau_S$ when combining $\phi_{+-}$ results from different experiments. If we were to recalculate our $\phi_{+-}$ result using the PDG values for $\Delta m$ and $\tau_S$ we would obtain $\phi_{+-} = (44.38 \pm 0.70 \pm 0.79)^\circ$. The E731 results were $\phi_{+-} = (42.2 \pm 1.5)^\circ$, $\Delta m = (0.5286 \pm 0.0028) \times 10^{10}\text{hs}^{-1}c^{-2}$, $\tau_S = (0.8929 \pm 0.0016) \times 10^{-10}\text{s}$. The result for $\phi_{+-}$ was
obtained from fits which assumed $\tau_S = 0.8922 \times 10^{-10}$ but allowed $\Delta m$ to float. The results for $\Delta m$ and $\tau_S$ fixed $\phi_{+-} = \phi_\epsilon$. If we were to recalculate our $\phi_{+-}$ result using the $\Delta m$ value obtained [18] in E731’s $\phi_{+-}$ fit, we would find $\phi_{+-} = (42.14 \pm 0.70 \pm 0.79)^\circ$. The NA31 result was determined assuming the PDG values $\tau_S = 0.8922 \times 10^{-10}$ and $\Delta m = 0.5351 \times 10^{10}$. They found $\phi_{+-} = (46.9 \pm 2.2)^\circ$. This can be compared with our $\phi_{+-}$ result recalculated using the PDG values for $\Delta m$ and $\tau_S$ mentioned above. Our results are consistent with the absence of CPT violation in $K_L \to \pi^+\pi^-$ decays.

B. $\Delta \phi$ analysis

Our preliminary result for $\phi_{00} - \phi_{+-}$ is based on data written after the T,V hodoscope was removed, approximately 70% of the full data set. As was the case in the $\phi_{+-}$ analysis, we binned data and Monte Carlo samples for both $\pi^0\pi^0$ and $\pi^+\pi^-$ decays in 10 GeV/c $\times$ 2 m $(p,z)$ cells. The copious $3\pi^0$ sample served to verify the accuracy of the neutral mode Monte Carlo. Acceptance for neutral mode decays varied with position and $K$ energy, and was typically $\sim 5\%$.

We fit our background-subtracted data for values of $\Delta \phi$, the regeneration parameters $\alpha$ and $\left[ (f - \bar{f}) / k \right]_{70 \; GeV}$, $\phi_{+-}$, $\epsilon'$, and beam intensity and energy spectrum parameters. For the cuts described in Table [VI], the fit $\chi^2$ was 682 for 619 degrees of freedom. We found that $\Delta \phi$ returned by the fit varied somewhat with the value of the minimum cluster energy cut, $E_{cl \; min}$. For our preliminary result we report the average of two fits which used $E_{cl \; min} = 2.2 \; GeV$ and $E_{cl \; min} = 4.0 \; GeV$, and assign a systematic uncertainty of 0.5° to this effect. We find $\Delta \phi = (0.67 \pm 0.85)^\circ$, where the error is statistical.

Other systematic uncertainties arose from possible inaccuracies in our determination of acceptance, our background subtractions, and our parametrization of the lead glass calorimeter. Effects involving regeneration influenced both $\phi_{00}$ and $\phi_{+-}$, and canceled when extracting their difference. As was the case with the charged mode analysis, we introduced spurious $z$-dependent slopes to force disagreement between data and Monte Carlo in order to gauge
the consequences of acceptance errors. We varied the amounts of background from $3\pi^0$ decays, inelastically and diffractively regenerated $K_S$, and from beam interactions in material in the detector. We studied several possible sources of calorimeter misparametrization, including an energy scale mismatch between the lead glass and the tracking system, misrepresentation of the lead glass' energy resolution, and uncorrected pedestal shifts. The dependence of $\Delta\phi$ on these effects is summarized in Table IX; the quadrature sum of the various sources of systematic uncertainty is $\pm 1.1^\circ$. Consequently, our preliminary result for the phase difference between $\eta_{00}$ and $\eta_{+-}$ is

$$\Delta\phi = [0.67 \pm 0.85 \ (\text{stat.}) \pm 1.1 \ (\text{syst.})] \ ^\circ .$$

We expect the statistical error to decrease when the full data set is used in the analysis; the systematic error will decrease as the analysis continues.

We can compare our result with the Particle Data Group's world average value [7] and previous measurements from E731 [9] and NA31 [8]. The 1992 PDG value is $\Delta\phi = (-0.1 \pm 2.0)^\circ$. The E731 result of $\Delta\phi = (-1.6 \pm 1.2)^\circ$ was determined with a fit which allowed the values for $\Delta m$ and $\tau_S$ to vary. If E731 had calculated their result using our $\Delta m$ and $\tau_S$ values they would have obtained $\Delta\phi = -1.75^\circ$. The NA31 result was $\Delta\phi = (0.2 \pm 2.9)^\circ$, and was determined assuming the PDG values $\tau_S = 0.8922 \times 10^{-10}$ and $\Delta m = 0.5351 \times 10^{10}$. Their value is insensitive to changes in $\Delta m$. Our result is consistent with previous measurements and with the absence of CPT violation in $K_L \rightarrow \pi\pi$ decays.

C. $\eta_{+-\gamma}$ analysis

The Monte Carlo simulation of the apparatus was used to determine the acceptance as a function of $p$ and $z$ for $\pi\pi\gamma$ decays in each beam. We generated simulated data sets which were approximately twenty times larger than the corresponding real data sets and determined average acceptance in each cell of the $(p, z)$ grids described above. Overall acceptance of our apparatus for the $\pi^+\pi^-\gamma$ decay mode was typically 11%. The momentum spectrum and flux
of kaons striking the regenerators were determined from the more copious $K \to \pi^+\pi^-$ decay mode. Characteristics of the regeneration amplitude $\rho$ were also determined with $\pi^+\pi^-$ data. The $\pi^+\pi^-\gamma$ analysis (and Monte Carlo) assumed that $\Delta m = 0.5286 \times 10^{10} \text{h} \text{s}^{-1} c^{-2}$ (Ref. [4]) and that the $K_S \to \pi^+\pi^-\gamma$ branching ratio (Ref. [14]) was $4.87 \times 10^{-3}$.

We extracted phase information about $\eta_{+--\gamma}$ from the interference between the inner bremsstrahlung $K_{L,S} \to \pi\pi\gamma$ decay amplitudes. The direct emission amplitude (which tended to produce a more energetic photon in the kaon rest frame) only participated in the $K_L$ decay rate, and did not interfere with the $K_S$ decay amplitude at an observable level. Based on a previous measurement (Ref. [14]), the Monte Carlo simulation assumed that the direct emission contribution to the $K_L$ decay rate with $E^*_\gamma > 20 \text{MeV}$ was 68.5%. Here, $E^*_\gamma$ is the photon energy in the $K$ rest frame.

The four background-subtracted $(p,z)$ data grids were fit simultaneously for the magnitude and phase of $\eta_{+--\gamma}$, the $K_S \to \pi^+\pi^-\gamma$ branching ratio, and the ratio of the direct emission and inner bremsstrahlung contributions to the $K_L$ decay amplitude using a log-likelihood procedure. Our preliminary result is $|\eta_{+--\gamma}| = (2.414 \pm 0.065) \times 10^{-3}$ and $\phi_{+--\gamma} \equiv \text{Arg}(\eta_{+--\gamma}) = (45.47 \pm 3.61)^\circ$, where the errors are statistical. After fitting, a $\chi^2$ was calculated using all bins with at least five events. We found $\chi^2 = 366.9$ for 347 degrees of freedom. Results from the individual data sets and the combined sample are summarized in Table X.

Possible sources of systematic error included uncertainties associated with the regeneration amplitude, background subtractions, and our parametrization of the kaon flux and momentum spectrum. In addition, our descriptions of $K_L \to \pi\pi\gamma$ direct emission and the $K_S \to \pi\pi\gamma$ branching ratio contributed to the systematic error. By varying the analysis parameters by one standard deviation, we studied the extent to which systematic uncertainties influenced our conclusions. The results are presented in Table XI. Adding systematic errors in quadrature yields the preliminary results

$$|\eta_{+--\gamma}| = [2.414 \pm 0.065 \text{ (stat.)} \pm 0.062 \text{ (syst.)}] \times 10^{-3}$$
\[ \phi_{+\gamma} = [45.47 \pm 3.61 \text{ (stat.)} \pm 2.40 \text{ (syst.)}]^\circ . \]

These errors are significantly smaller than those of the best previous measurement (Ref. [15]) which found \( |\eta_{+\gamma}| = (2.15 \pm 0.26 \pm 0.17) \times 10^{-3} \) and \( \phi_{+\gamma} = (72 \pm 23 \pm 17)^\circ \). Our value for \( \eta_{+\gamma} \), and the best previous measurement, are compared in Fig. 27. This measurement of \( \eta_{+\gamma} \) is also not significantly different from the analogous \( \pi^+\pi^- \) amplitude ratio \( |\eta_{-\gamma}| = (2.268 \pm 0.023) \times 10^{-3} \) (Ref. [7]) and the \( \phi_{-\gamma} \) value presented in Sec. VIII A. This result is consistent with the absence of CP violation beyond that present in \( K_L \to \pi\pi \) decays.

IX. CONCLUSIONS

We report preliminary results from analysis of data from the 1991 fixed target run at Fermilab. Our results for \( \text{Arg}(\eta_{+\gamma}), \Delta m, \tau_S, |\eta_{+\gamma}|, \text{ and Arg}(\eta_{+\gamma}) \) are based on the entire data set. Our result for \( \Delta \phi \equiv \text{Arg}(\eta_{90}) - \text{Arg}(\eta_{+\gamma}) \) is based on analysis of about 70\% of the available data. Systematic uncertainties will decrease as we refine the analyses; the statistical precision of our \( \Delta \phi \) result will improve when the full data set is used.

Our two-pion results are consistent with CPT conservation in \( K \to \pi\pi \) decays. Our \( \eta_{+\gamma} \) result is consistent with the absence of an unusual source of CP violation in \( K \to \pi\pi\gamma \) decays.

APPENDIX: REGENERATION

1. \( K_S \) coherent regeneration

Neglecting decays, the wave function for a relativistic \( K^0 \) meson with wavenumber \( k \) will obey the Klein-Gordon equation \((\nabla^2 + k^2)\psi_k(x) = 0\). If the \( K^0 \) passes through a scattering medium, its total wavefunction will become the sum of the incident and scattered waves. Defining \( f(k) \) to be the average forward scattering amplitude at each scattering site inside the medium, we find that \[ 30 \] \( \psi_k \) must satisfy
\[(\nabla^2 + k^2 + 4\pi N f(k)) \psi_K = 0\]

with approximate solution

\[\psi_K(x, t) \approx e^{i \left[(1 + \frac{2\pi N f(k)}{k^2})kz - Et/\hbar\right]}\]  \hspace{1cm} (A1)

for a plane wave traveling in the \(z\) direction. Here, \(N\) is the number of scatterers per unit volume. Similarly, the wavefunction for a \(K^0\) with forward scattering amplitude \(f(k)\) will be

\[\psi_{K^0}(x, t) \approx e^{i \left[(1 + \frac{2\pi N f(k)}{k^2})kz - Et/\hbar\right]}\]  \hspace{1cm} (A2)

Note that these expressions include the effects of elastic and inelastic scattering since a positive imaginary part to \(f(k)\) (or \(\overline{f}(k)\)) causes \(|\psi|\) to decrease. This is sensible: the optical theorem guarantees \(\text{Im} f > 0\) whenever the total scattering cross section is nonzero.

We can use Eqs. (A1, A2) to describe the position dependence of a mixed state traveling in the \(z\) direction through a regenerator. The \(z\)-dependence of the \(K_S, K_L\) amplitudes for the state \(|K\rangle = A_S|K_S\rangle + A_L|K_L\rangle\) will obey

\[\frac{\partial A_S}{\partial z} = \left[i k + i N\pi \left(\frac{f + \overline{f}}{k^2}\right)\right] A_S + i N \pi \left(\frac{f - \overline{f}}{k^2}\right) A_L\]

\[\frac{\partial A_L}{\partial z} = \left[i k + i N\pi \left(\frac{f + \overline{f}}{k^2}\right)\right] A_L + i N \pi \left(\frac{f - \overline{f}}{k^2}\right) A_S\]

neglecting weak interactions and decays. A state which is pure \(K_L\) at \(z = 0\) will evolve into a mixture of \(K_L\) and \(K_S\) as long as \((f - \overline{f}) \neq 0\). In particular, the ratio of the \(K_S\) and \(K_L\) amplitudes a small distance \(\Delta z\) inside a regenerator will be

\[\rho \equiv \frac{A_S(\Delta z)}{A_L(\Delta z)} \approx i N \pi \left(\frac{f - \overline{f}}{k}\right) \Delta z\]  \hspace{1cm} (A3)

This coherent regeneration amplitude \(\rho\) results from constructive interference among all the outgoing spherical waves produced at each of the scattering sites in the regenerator. Since the wavelength of a 100 GeV/c \(K\) meson is \(\sim 2 \times 10^{-18}\) meters, coherent scattering in a macroscopic regenerator only takes place for extremely small scattering angles.
The total cross section at high energy is dominated by inelastic processes so the scattering amplitudes \( f \) and \( \bar{f} \) are nearly imaginary [31]. The momentum-dependence of \(|(f - \bar{f})/k|\) has been measured [18] and is in agreement with the Regge theory prediction [19] that

\[
\left( \frac{f - \bar{f}}{k} \right) \propto p^\alpha e^{-i\pi(2+\alpha)/2}.
\]

(A4)

The connection between \( \alpha \) and the phase of \( f - \bar{f} \) comes from analyticity, and does not depend on the validity of Regge theory.

2. Diffraction regeneration

Scattering does not mix \( K^0 \leftrightarrow \bar{K}^0 \). However, since the scattering probability for a \( K^0 \) differs from that for a \( \bar{K}^0 \), a \( K \) meson which contains a mix of \( K^0 \) and \( \bar{K}^0 \) amplitudes will scatter into a state containing a different combination of \( K^0 \) and \( \bar{K}^0 \). In terms of the elastic scattering amplitudes \( f_k(\theta, \phi) \) and \( \bar{f}_k(\theta, \phi) \), the state

\[
\psi_i \equiv A|K^0\rangle + \bar{A}|\bar{K}^0\rangle
\]

will scatter elastically into the state

\[
\psi_f = \left[ Af_k(\theta, \phi)|K^0\rangle + \bar{A}\bar{f}_k(\theta, \phi)|\bar{K}^0\rangle \right] \frac{e^{ikr}}{r}.
\]

For convenience, we define \( f_{22}(\theta, \phi) \equiv f_k(\theta, \phi) + \bar{f}_k(\theta, \phi) \) and \( f_{21}(\theta, \phi) \equiv f_k(\theta, \phi) - \bar{f}_k(\theta, \phi) \).

If we rewrite the incident and final states \( \psi_i, \psi_f \) in terms of \( K_S \) and \( K_L \), we have

\[
\psi_i = A_S|K_S\rangle + A_L|K_L\rangle
\]

(A5)

\[
\psi_f = \left\{ (A_S f_{22} + A_L f_{21})|K_S\rangle + (A_L f_{22} + A_S f_{21})|K_L\rangle \right\} \frac{e^{ikr}}{2r}.
\]

(A6)

In the final state \( K_S \) amplitude, the term \( A_S f_{22} \) corresponds to elastic \( K_S \rightarrow K_S \) scattering while the term \( A_L f_{21} \) comes from (regenerative) \( K_L \rightarrow K_S \) scattering. Similarly, in the final \( K_L \) amplitude, the term \( A_L f_{22} \) arises from elastic \( K_L \rightarrow K_L \) scattering, while \( A_S f_{21} \) results from \( K_S \rightarrow K_L \) scattering.
A pure $K_L$ will scatter into a mixed $K_L, K_S$ state; we can define the diffraction regeneration amplitude $\rho_D$ for elastic scattering into $\theta, \phi$ as the ratio $f_{21}/f_{22}$. With this definition we can rewrite Eq. (A6) as

$$\psi_f = \left\{ (A_S + \rho_D A_L) |K_S\rangle + (A_L + \rho_D A_S) |K_L\rangle \right\} f_{22} e^{ikr}/2r. \quad (A7)$$

Because $f_k$ and $\overline{f_k}$ are nearly imaginary (Ref. [31]), $f_{22} \approx i|f_{22}|$ so that

$$\rho_D(\theta, \phi) = \frac{f_{21}}{f_{22}} \approx -i \frac{f_{21}}{|f_{22}|}. \quad (A8)$$

With the help of the optical theorem, this can be rewritten as

$$\rho_D(\theta, \phi) \approx -\frac{i2\pi}{\sigma_T} \left( \frac{f_{21}}{k} \right) \quad (A8)$$

when $\theta, \phi$ are close to the forward direction so that $f_{22}(\theta, \phi) \approx f_{22}(0, 0)$. Here, $\sigma_T$ is the $K_L$-nucleus total scattering cross section.

There are interference effects associated with the presence of both coherent and diffractive processes. Imagine that a $K_L$ enters a regenerator of length $L$, scatters elastically after traveling a distance $\Delta z$, then leaves the regenerator without rescattering. (We will ignore effects associated with finite regenerator length such as the decay of the $K_S$ amplitude and the phase drift between the kaon’s $K_S$ and $K_L$ components due to the non-zero $K_L - K_S$ mass difference.) From Eq. (A3), the kaon’s wavefunction immediately before scattering will be

$$\psi_{before} = \left( \frac{iN\pi\Delta z f_{21}(0, 0)}{k} \right) |K_S\rangle + |K_L\rangle, \quad (A9)$$

neglecting normalization. The $K_S$ component is the result of coherent regeneration. Immediately after scattering, the kaon’s wavefunction will become

$$\psi_{after} = \left\{ \left( \frac{iN\pi\Delta z f_{21}(0, 0)}{k} + \rho_D \right) |K_S\rangle + \left( 1 + \frac{iN\pi\Delta z f_{21}(0, 0) \rho_D}{k} \right) |K_L\rangle \right\} f_{22} e^{ikr}/2r. \quad (A10)$$

The first term in the $K_S$ amplitude corresponds to $K_S \rightarrow K_S$ scattering while $\rho_D$ corresponds to elastic $K_L \rightarrow K_S$ scattering. For the E773 regenerator geometry and $K$ energy range,
both contributions to the $K_S$ amplitude of Eq. (A10) are small. The modification to the $K_L$ amplitude is second-order in the regeneration parameters $f_{21}$, $\rho_D$. As the kaon travels through the remaining distance ($L - \Delta z$) in the regenerator, its $K_S$ and $K_L$ amplitudes will continue to evolve because of $K_{S,L} \rightarrow K_{L,S}$ and $K_{L,S} \rightarrow K_{L,S}$ forward scattering. In particular, its $K_S$ amplitude will gain an additional contribution from coherent regeneration. Neglecting normalization, and terms which are second order in the regeneration parameters, we find that the kaon’s wavefunction at the downstream end of the regenerator is

$$
\psi_f = \left\{ \left( \ fraction{i N \pi L f_{21}(0,0)}{k} + \rho_D \right) \left| K_S \right\rangle + \left| K_L \right\rangle \right\} f_{22}^2 e^{ikr}.
$$

(A11)

With the definition of the coherent regeneration amplitude $\rho$ in Eq. (A3), we can rewrite Eq. (A11) as

$$
\psi_f = \left\{ \left( \rho + \rho_D \right) \left| K_S \right\rangle + \left| K_L \right\rangle \right\} f_{22}^2 e^{ikr}.
$$

(A12)

Because of the relative minus sign between $\rho$ and $\rho_D(\theta, \phi)$, there will be destructive interference between coherent and diffractive regeneration. If we approximate $f_{21}(\theta, \phi) \approx f_{21}(0,0)$ and make use of Eqs. (A3) and (AS), we find

$$
\rho + \rho_D \approx \rho \left( 1 - \frac{2}{NL\sigma_T} \right).
$$

(A13)

For a two interaction length regenerator, $NL\sigma_T = 2$, and the destructive interference will be nearly perfect.

The interference between the coherent and diffractive regeneration amplitudes is influenced by a number of factors in addition to regenerator length. If $f_k(\theta, \phi)$ and $\overline{f}_k(\theta, \phi)$ are not purely imaginary, Eq. (AS) will be inaccurate and the cancellation will be imperfect. To the extent that dependence on momentum transfer spoils the approximation $f_{22}(\theta, \phi) \approx f_{22}(0,0)$, Eq. (A13) will be unreliable. The nonzero $K_L$-$K_S$ mass difference makes the $K_S$ amplitude’s phase lag behind the $K_L$ phase as the kaon travels through the regenerator: the $K_S$ amplitude from elastic scatters at the upstream and downstream ends of the regenerator will contribute with slightly different phases to the decay amplitudes. This
phase drift is negligible at Fermilab energies, where it is \( \sim 5^\circ \) per meter of flight path for a 100 GeV kaon. In addition, the finite \( K_S \) lifetime plays a role. The contribution to the \( K_S \) amplitude from coherent forward scattering will decay in flight as the kaon moves through the regenerator. The fraction which survives is a function of the regenerator length and the kaon’s lifetime, and is independent of the location of the elastic scatter. The diffractive contribution to the \( K_S \) amplitude, created at the scattering site, will also decay in flight. As a result, the surviving diffractive contribution depends on the distance from the scattering site to the downstream end of the regenerator. Consequently, the sum of the coherent and diffractive \( K_S \) amplitudes will vary slightly with the location of the scattering site.

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TABLE I. Requirements on events used in $\pi^+\pi^-$ and $\pi^+\pi^-\gamma$ analyses. Set 1 (2) data were recorded before (after) removal of the T,V hodoscope.

| Requirement                                      | $\pi^+\pi^-$ | $\pi^+\pi^-\gamma$ |
|--------------------------------------------------|---------------|---------------------|
| Invariant mass, MeV/c$^2$                        | $484 \leq m_{\pi\pi} \leq 512$ | $484 \leq m_{\pi\pi\gamma} \leq 512$ |
| $p_T^2$, (MeV/c$^2$)                             | $\leq 250$   | $\leq 150$          |
| track momentum, GeV/c                            | $\geq 7$     | $\geq 7$            |
| $E/p$                                            | $\leq 0.8$   | $\leq 0.85$         |
| Kaon energy (GeV)                                | $30 \leq E_K \leq 160$ | $25 \leq E_K \leq 155$ |
| Decay vertex, upstr. reg. (m)                    | $118.5 \leq z \leq 127$ (set 1) | $117.33 \leq z \leq 139.33$ (set 1) |
| OR. $129 \leq z \leq 140$                       |               |                     |
| OR. $118.5 \leq z \leq 127$ (set 2)              |               |                     |
| OR. $129 \leq z \leq 154$                       |               |                     |
| Decay vertex, dnstr. reg. (m)                    | $130 \leq z \leq 140$ (set 1) | $128.63 \leq z \leq 140.63$ (set 1) |
| OR. $130 \leq z \leq 154$                       |               |                     |
| OR. $128.63 \leq z \leq 158.63$ (set 2)         |               |                     |
| $\Lambda$ suppression                           | $m_{p\pi} \leq 1110$ MeV/c$^2$ | $m_{p\pi} \leq 1100$ MeV/c$^2$ |
| OR. $m_{p\pi} \geq 1122$ MeV/c$^2$              |               |                     |
| OR. $E_\Lambda \leq 100$ GeV                    |               |                     |
| OR. $p_p/p_\pi < 3$                             |               |                     |

TABLE II. Estimate of backgrounds in $\pi^+\pi^-$ sample.

|                      | set 1   | set 2   |
|----------------------|---------|---------|
| Upstream Regenerator | 0.29%   | 0.25%   |
| Downstream Regenerator| 0.90% | 0.75% |
### TABLE III. Number of coherent $\pi^+\pi^-$ events.

|                | set 1     | set 2     | Whole Run |
|----------------|-----------|-----------|-----------|
| Upstream Regenerator | 707k      | 750k      | 1,457k    |
| Downstream Regenerator | 160k      | 207k      | 367k      |
| Both Regenerators     | 867k      | 957k      | 1,824k    |

### TABLE IV. Number of coherent $\pi^+\pi^-\gamma$ events after background subtraction.

|                | set 1      | set 2      | Whole Run  |
|----------------|------------|------------|------------|
| Upstream Regenerator | $4,559 \pm 11$ | $3,779 \pm 9$ | $8,338 \pm 14$ |
| Downstream Regenerator | $1,209 \pm 6$ | $1,222 \pm 6$ | $2,431 \pm 9$ |
| Both Regenerators     | $5,768 \pm 13$ | $5,001 \pm 11$ | $10,769 \pm 17$ |

### TABLE V. Estimate of backgrounds in $\pi^+\pi^-\gamma$ sample.

|                | set 1       | set 2       |
|----------------|-------------|-------------|
| Upstream Regenerator | $75.5 \pm 11.0$ events | $65.0 \pm 9.1$ events |
|                 | 1.6%        | 1.7%        |
| Downstream Regenerator | $28.0 \pm 6.0$ events | $31.9 \pm 6.3$ events |
|                 | 2.3%        | 2.5%        |
TABLE VI. Requirements on events used in the $\Delta \phi$ analysis.

| Requirement                      | $\pi^0\pi^0$        | $\pi^+\pi^-$       |
|----------------------------------|----------------------|---------------------|
| Invariant mass (MeV/$c^2$)       | $474 \leq m_{\pi\pi} \leq 522$ | $484 \leq m_{\pi\pi\gamma} \leq 512$ |
| $p_T^2$, (MeV/c)$^2$             | $- $                | $\leq 250$          |
| ring number (cm$^2$)             | $\leq 120$          | $-$                 |
| track momentum (GeV/c)           | $-$                 | $\geq 7$            |
| cluster energy (GeV)             | $E_{\text{cluster}} \geq 2.2$ | $-$                 |
| $E/p$                            | $-$                 | $\leq 0.8$          |
| Kaon energy (GeV)                | $40 \leq E_K \leq 150$ | $30 \leq E_K \leq 160$ |
| Decay vertex (m)                 | upstr. reg.: $120 \leq z \leq 152$ | upstr. reg.: $118 \leq z \leq 127$ |
|                                  | .OR. $129 \leq z \leq 152$ |                      |
|                                  | dnstr. reg.: $130 \leq z \leq 152$ | dnstr. reg.: $129 \leq z \leq 152$ |
| $\Lambda$ suppression            | $-$                 | $m_{p\pi} \leq 1110$ MeV/$c^2$ |
|                                  |                      | .OR. $m_{p\pi} \geq 1122$ MeV/$c^2$ |

TABLE VII. Number of coherent $\pi^0\pi^0$ events. (Note that only data set 2 is used in the preliminary result for $\Delta \phi$).

|                      | set 1 | set 2 | Whole Run |
|----------------------|-------|-------|-----------|
| Upstream Regenerator | 85k   | 180k  | 265k      |
| Downstream Regenerator | 35k | 75k | 110k |
| Both Regenerators    | 120k  | 255k  | 375k      |
### TABLE VIII. Estimates of systematic error in $\pi^+\pi^-$ analysis.

| Source of uncertainty                              | change in $\phi_{+-}$ |
|----------------------------------------------------|------------------------|
| Acceptance calculation                             | $0.57^\circ$           |
| Analyticity constraint on phase of $\rho$          | $0.5^\circ$            |
| Background subtractions                            | $0.2^\circ$            |
| Regenerator descriptions                           | $0.05^\circ$           |
| Total                                              | $0.79^\circ$           |
| Increase $\Delta m$ by 0.0024 (1 PDG $\sigma$)    | $\phi_{+-}$ increases $+0.38^\circ$ |
| Increase $\tau_S$ by 0.0020 (1 PDG $\sigma$)      | $\phi_{+-}$ decreases $-0.62^\circ$ |

### TABLE IX. Estimates of systematic error in $\Delta \phi$ analysis.

| Source of uncertainty                              | change in $\Delta \phi$ |
|----------------------------------------------------|--------------------------|
| Acceptance parametrization for $\pi^+\pi^-$ and $\pi^0\pi^0$ | $0.3^\circ$              |
| Background subtractions                            | $0.6^\circ$              |
| Calorimeter energy scale                           | $0.6^\circ$              |
| Calorimeter nonlinearity (including minimum cluster energy cut) | $0.5^\circ$              |
| Calorimeter resolution                             | $0.5^\circ$              |
| Total                                              | $1.1^\circ$              |

### TABLE X. $\eta_{+-\gamma}$ magnitude and phase from different data sets.

| Regenerator | set 1                                      | set 2                                      |
|-------------|--------------------------------------------|--------------------------------------------|
| Upstream    | $|\eta_{+-\gamma}| = (2.358 \pm 0.086) \times 10^{-3}$ | $|\eta_{+-\gamma}| = (2.406 \pm 0.084) \times 10^{-3}$ |
|             | $Arg(\eta_{+-\gamma}) = (51.1 \pm 5.4)^\circ$ | $Arg(\eta_{+-\gamma}) = (40.9 \pm 6.5)^\circ$ |
| Downstream  | $|\eta_{+-\gamma}| = (2.376 \pm 0.098) \times 10^{-3}$ | $|\eta_{+-\gamma}| = (2.443 \pm 0.083) \times 10^{-3}$ |
|             | $Arg(\eta_{+-\gamma}) = (43.1 \pm 7.8)^\circ$ | $Arg(\eta_{+-\gamma}) = (54.0 \pm 6.5)^\circ$ |
| All data    | $|\eta_{+-\gamma}| = (2.414 \pm 0.065 \pm 0.062) \times 10^{-3}$ | $Arg(\eta_{+-\gamma}) = (45.5 \pm 3.6 \pm 2.4)^\circ$ |
### TABLE XI. Estimates of systematic error in $\pi^+\pi^-\gamma$ analysis.

| Source of uncertainty                        | Fractional change | change in $|\eta_{+-\gamma}|$ | change in $\phi_{+-\gamma}$ |
|----------------------------------------------|-------------------|-------------------------------|-----------------------------|
| $K_S \rightarrow \pi\pi\gamma$ ranching ratio | 2.3%              | $0.030 \times 10^{-3}$        | 1.76°                       |
| DE/IB ratio                                  | 6.0%              | $0.041 \times 10^{-3}$        | 0.19°                       |
| regeneration power law $\alpha$              | 2.5%              | $0.001 \times 10^{-3}$        | 0.92°                       |
| Normalization                                | 1.6%              | $0.021 \times 10^{-3}$        | 1.22°                       |
| Background normalization                      | 16%               | $0.008 \times 10^{-3}$        | 0.15°                       |
| Background shape                             |                   | $0.030 \times 10^{-3}$        | 0.17°                       |
| Data/MC $p$ spectrum mismatch                 | $2 \times 10^{-4}$| $0.002 \times 10^{-3}$        | 0.54°                       |
| Total                                        |                   | $0.062 \times 10^{-3}$        | 2.40°                       |
FIGURES

FIG. 1. Elevation view of the E773 detector. Kaons in the beams travel to the right in the figure. Details of the individual hardware systems appear in the text. The thin hodoscope labeled “T,V” was removed partway through the run.

FIG. 2. Neutral beam profiles at the lead glass array, as determined with $K_L \rightarrow \pi^+\pi^-$ decays. Data are shown as the histogram with error bars; results from the Monte Carlo simulation are shown as points.

FIG. 3. Predicted $K_L$ energy spectrum striking the regenerators. The smooth curve is based on the parametrization of $K^+$ and $K^-$ production by Malensek, modified to agree with our data.

FIG. 4. Ratio of the calorimeter energy and track momentum ($E/p$) for $K_{e3}$ calibration electrons. The r.m.s. width of the peak is approximately 3%. Data (histogram) and Monte Carlo (points) are shown. The low side tail in the data (which is not well-represented by the Monte Carlo simulation) is under investigation.

FIG. 5. Mean $E/p$ during the run. Shown in the plot is the average value of $E/p$ for $K_{e3}$ electrons as a function of run number. The period of time spanned by the run was about nine weeks.

FIG. 6. $E/p$ mean and r.m.s. width vs. electron track momentum. The upper figure shows the results of fits for the mean $E/p$, restricted to the region near the peak, for $K_{e3}$ electrons. The apparent shift away from unity is an artifact of the restriction in the fits to the peak region. The lower figure shows the r.m.s. width, again restricted to the region near the peak. Data (histogram) and Monte Carlo (points) are shown.
FIG. 7. $\pi^+\pi^-$ invariant mass distributions for events satisfying all other analysis cuts. Events from decays in the upstream and downstream regenerator beams are plotted separately. Midway through the run the T,V hodoscope was removed; data are plotted separately from the periods before (set 1) and after (set 2) its removal. Data (histogram) and Monte Carlo (points) are shown. The data’s high-side tails contain contributions from $\delta$-rays which are not simulated by the Monte Carlo.

FIG. 8. $p_T^2$ distributions for $K \rightarrow \pi^+\pi^-$ decays satisfying all other analysis cuts. Events from decays in the upstream and downstream regenerator beams are plotted separately. Midway through the run the T,V hodoscope was removed; data are plotted separately from the periods before (set 1) and after (set 2) its removal. Data (histogram) and Monte Carlo (points) are shown. Some smearing of the data’s coherent peak is due to the presence of $\delta$-rays which are not simulated by the Monte Carlo.

FIG. 9. Decay vertex $z$ distributions for $K \rightarrow \pi^+\pi^-$ decays satisfying all other $Arg(\eta_{+-})$ analysis cuts. To reduce backgrounds associated with interactions of beam hadrons in material near the downstream regenerator, events with vertex $z$ between 127 m and 129 m were discarded. Data (histogram) and Monte Carlo (points) are shown for each beam. The T,V hodoscope, located near $z = 141$ m, was included in the charged mode trigger for set 1 data.

FIG. 10. Reconstructed kaon energy distributions for $K \rightarrow \pi^+\pi^-$ decays satisfying all other analysis cuts. Data (histogram) and Monte Carlo (points) are shown for each beam in the two data sets.

FIG. 11. Photon energy spectrum in the $K$ rest frame for $\pi^+\pi^-\gamma$ decays satisfying all other cuts. Events are required to have $E_\gamma^* > 0.02$ GeV. Data (histogram) and Monte Carlo (points) are shown for the entire data sample. The vertical dashed line indicates the location of the 0.02 GeV cut.
FIG. 12. $\pi^+\pi^-\gamma$ invariant mass distributions for events satisfying all other cuts. Data (shown as histograms) are plotted separately for each beam. Results of the Monte Carlo simulation (shown as points) do not include background modeling.

FIG. 13. $p_T^2$ distributions for $K \to \pi^+\pi^-\gamma$ decays satisfying all other cuts. Data (histogram) and Monte Carlo (points) are shown, plotted separately for each beam. The Monte Carlo did not simulate backgrounds.

FIG. 14. Decay vertex $z$ distributions for $K \to \pi^+\pi^-\gamma$ decays satisfying all other cuts. Data (histogram) and Monte Carlo (points) are shown, plotted separately for each beam in the two data sets.

FIG. 15. $K$ energy distributions for $K \to \pi^+\pi^-\gamma$ decays satisfying all other cuts. Data (histogram) and Monte Carlo (points) are shown.

FIG. 16. $\pi^0\pi^0$ invariant mass distributions for events satisfying all other cuts. Data (histogram) and Monte Carlo (points) are shown. In the plots on the left side of the figure, the simulation includes both signal and backgrounds. In the plots on the right, only the simulated backgrounds are included in the Monte Carlo sample.

FIG. 17. Illustration of “ring number” for a $K \to \pi^0\pi^0$ decay. In this example, an event’s center of energy in the calorimeter is at $(x, y) = (14, 4)$ cm, closer to the upper beam which is centered at $(x, y) = (0, 11.6)$ cm. The square whose perimeter contains the center of energy has side length $2 \times 14$ cm and area $784 \text{ cm}^2$. As a result, the event’s ring number is $784 \text{ cm}^2$.

FIG. 18. $\pi^0\pi^0$ ring distributions for events satisfying all other cuts. Data (histogram) and Monte Carlo (points) are shown. In the plots on the left side of the figure, the simulation includes both signal and backgrounds. In the plots on the right, only the simulated backgrounds are included in the Monte Carlo sample.
FIG. 19. Decay vertex $z$ distributions for $K \rightarrow \pi^0\pi^0$ events satisfying all other cuts. Data (histogram) and Monte Carlo (points) are shown.

FIG. 20. Reconstructed kaon energy distributions for $K \rightarrow \pi^0\pi^0$ decays satisfying all other cuts. Data (histogram) and Monte Carlo (points) are shown.

FIG. 21. Angle between the target-to-vertex direction and the $z$ axis in $K_{e3}$ decays, projected onto the horizontal plane. Wings of the distribution correspond to $K$ mesons which scattered in beamline elements before decaying. Data (histogram) and Monte Carlo (points) are shown.

FIG. 22. Comparison of the data and Monte Carlo $K_{e3}$ electron illuminations at the east edge of the HDRA aperture in 1 mm bins. Data (histogram) and Monte Carlo (points) are shown.

FIG. 23. $\pi^\pm$ track illuminations at the HDRA. Data (histogram) and Monte Carlo (points) are shown.

FIG. 24. $\pi^\pm$ track illuminations at the lead glass array. Data (histogram) and Monte Carlo (points) are shown.

FIG. 25. Decay vertex $z$ distributions for $K \rightarrow 3\pi^0$ decays. Data (histogram) and Monte Carlo (points) are shown. The large $3\pi^0$ sample provides a valuable check of our $\pi^0\pi^0$ acceptance calculation.

FIG. 26. Decay vertex $z$ distributions for $K \rightarrow \pi\nu\nu$ decays. Data (histogram) and Monte Carlo (points) are shown. The large $K_{e3}$ sample allows us to check our $\pi^+\pi^-$ acceptance calculation.

FIG. 27. Magnitude and phase of $\eta_{+-\gamma}$ from this experiment and from E731. The smaller error bars indicate statistical uncertainties. The larger errors correspond to a sum, in quadrature, of the quoted statistical and systematic errors. For comparison, the vertical dashed lines indicate the $\pm1\sigma$ bounds on our $\phi_{+-}$ measurement while the horizontal dashed lines indicate the $\pm1\sigma$ bounds on the Particle Data Group’s world-average value for $|\eta_{+-}|$. 

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