VISCOELASTIC CONSTITUTIVE PARAMETERS AND FAILURE CRITERION FOR SHELLED MAIZE EN MASSE UNDER UNIAXIAL CYCLIC LOADING

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ABSTRACT

Understanding the characteristics of granular materials remains one of the challenges for engineering science and practice. Granular agricultural materials complicate this challenge further due to the added problems of variation in moisture content and deformability at the micro-, meso- and macro- scales. This study attempts to improve this understanding by modelling maize en masse as viscoelastic using the Maxwell-Weichert model and establishing the viscoelastic constitutive parameters for the same. Uniaxial stress-relaxation experiments were conducted under cyclic loading conditions using the Instron uniaxial loading machine for three maize varieties at an initial bulk density of 800kg/m³. Each test was repeated three times. Experimental results were then compared to theoretical predictions under the Maxwell-Weichert conditions. The coefficient of determination R² for Relaxation Modulus, E(t), ranged between 0.84 and 0.90 for the three maize varieties at 800kg/m³. The standard error for residual stress was 1.17, 0.95 and 1.82MPa for the three varieties, V1, V2 and V3 respectively. From these results, it was concluded that granular maize en masse can be adequately described as being viscoelastic under the Maxwell-Weichert model predictions.

Keywords: Viscoelasticity, Constitutive parameters, Maxwell-Weichert model, stress relaxation, Maize

INTRODUCTION

Studies of granular materials date back to as early as the 18ᵗʰ Century. Initial focus of most studies on granular matter however targeted only materials of engineering significance such as construction materials. Though challenges still remain in this front as reported by Qicheng et al. (2009), theories developed are fast gaining entry into the agricultural food and processing industry globally.

The basic requirements in handling, storage and processing of granular agricultural material are that the system ensures (a) predictable and safe operations and (b) maintains high quality of agricultural materials (Horabik and Molenda, 2014). Knowledge of constitutive relations in granular agricultural materials is therefore of utmost importance in transportation, processing and in the design of handling, processing and storage systems such as silos and bins. Jansen’s theory, for example, is commonly used in most international standards for silo design (Moya et al., 2002). This theory, as well as many others such as Airy’s theory or Reimbert’s theory considers some material properties such as the angle of internal friction, the grain in wall friction coefficient and the specific weight. Hence, it is possible to find values for all these properties in literature to apply in design (Mohsenin, 1980). However, to accurately model silo loads, it is necessary to consider additional material properties not taken into account in the traditional
methods (Moya et al., 2002; Vanel et al., 2000). Constitutive models have in the past relied on the theory of elasticity to predict the load-deformation behaviour of en-masse grains (Husain and Agrawal, 1969).

It is reported (FAO, 2019) that three cereals, rice, maize and wheat contribute to about two thirds of global food energy needs. Maize therefore forms one of the most important cereal crops in the world and any additional understanding of its bulk behavior would be useful in not only improving efficiency in its handling but also saving costs through curbing wastage and reducing any losses in quality.

There exists a relationship between constitutive relations and their interaction with the processes of transportation, processing and storage of granular agricultural materials. The design of proper storage and processing structures and equipment are all linked to sound understanding of how these materials interact within themselves and with the equipment and structures that hold them. However, there is still a tendency of industry to attempt to apply trial and error methods in handling these important materials.

The Maxwell-Weichert model has in the past been applied to other viscoelastic materials other than cereal grains. This study applied the Maxwell-Weichert viscoelastic constitutive model to bulk maize to help improve understanding of the behaviour of cereal grains in general and maize in particular. These results can be used with the numerical methods in design of storage and handling facilities and in the prediction of material behaviour en masse.

1.2 Objectives of the Study

The broad objective of the study was to determine viscoelastic properties of shelled bulk maize pertinent to grain behaviour in handling, storage and processing. The specific objectives for the research were to;

1. Study the stress-strain behaviour of three varieties of shelled maize en masse under uniaxial cyclic loading conditions

2. Determine stress-relaxation behaviour of three varieties of bulk shelled maize by obtaining the stress-relaxation moduli and exponents

2.0 Theoretical Framework

Granular agricultural materials, looked at from the continuum perspective behave uniquely in processing and storage (Gumbe, 1993). This material behaviour varies under different loading conditions, which can be idealized as represented in Figure 2.1.

Usually, in studying the system described in figure 2.1, the restriction is to the three-dimensional Euclidean space and for most part, the rectangular Cartesian coordinate system. The continuum theory regards matter as indefinitely divisible (Chung, 1988). This enables us to study the behaviour of granular material en masse.

During storage, handling and processing of shelled maize grains en masse, the physical variables relate in an extremely complex manner. This complex interrelationship may be summarised as being elastic, elastoplastic, viscoelastic, viscoplastic or elastoviscoplastic (Gumbe, 2019).

2.1 Stress-Strain Constitutive Relations

Certain physical laws govern the mechanical behaviour of a continuum. Some of them are common for all continuous materials while others are intrinsic properties of each group or each individual material. These laws include; conservation of mass, balance of momentum, balance of moment of momentum, conservation of energy, constitutive relations and principles governing thermodynamics (Findley et al., 1976).

The stress analysis of continua is usually composed of a solution of a system of equations involving;

- Equations of equilibrium
- Strain displacement equations
- Compatibility relations
- Stress-Strain (constitutive) relations, and
Constitutive equations characterize the individual material and its reactions to external excitations.

### 2.1.1 Generalized Constitutive Models

The generalized form of stress-strain constitutive relations for time-dependent small deformations in a three-dimensional state of loading is given by equation 1 (Findley et al., 1976);

$$\sigma_{ij}(t) = E\varepsilon_{ij}(t)$$  \hspace{1cm} (1)

Where;

- $\sigma_{ij}$ is Cauchy’s infinitesimal stress tensor
- $\varepsilon_{ij}$ is the Cauchy’s infinitesimal strain tensor.

From the generalized relationship in equation 1, several forms have been developed for elastic, plastic, viscoelastic, elastoplastic, viscoplastic and even the elastoviscoplastic conditions (Gumbe, 1995). This paper focuses on the viscoelastic constitutive behavior of shelled maize en masse.

### 2.1.2 Viscoelastic Models

It has been noted (Chung, 1988; Hu et al., 2003) that under sustained and/or large loadings, most engineering materials exhibit a degree of viscoelasticity, that is, time-dependent stress-strain behaviour. This behaviour can be linear, if infinitesimal strains are considered or non-linear if non-linear strains are involved.

The linear and non-linear viscoelastic models have been predominantly used to model material behaviour. These models attempt to incorporate time in stress-strain equations. Such equations are specifically important in the design of storage structures such as silos and foundations both of which are exposed to rapidly declining or changing periodic ambient temperatures, moisture content, among others (Gumbe, 1995).

For linearly viscoelastic material, the following criteria hold (Findley et al., 1976).

1. For any step input in strain ($\varepsilon_0$) the relation between the stress $\sigma(t)$ and strain is;

$$\frac{\sigma(t)}{\varepsilon_0} = E(t)$$  \hspace{1cm} (2)

For a step input of stress, $\sigma_0$, the relation between strain, $\varepsilon(t)$ and stress is;

$$\frac{\varepsilon(t)}{\sigma_0} = J(t)$$  \hspace{1cm} (3)

Where $E(t)$ and $J(t)$ are the stress relaxation modulus and creep compliance respectively.

2. The Boltzman’s superposition principle holds, that is, the stress at any time $t$ depends on the strain history of the material;

$$\sigma(t) = \int_{0}^{t} E(t - \tau) \frac{\delta\varepsilon}{\delta\tau} + \sum_{i=1}^{n} \Delta\varepsilon_i E(t - t_i)$$  \hspace{1cm} (4)

Where; $\varepsilon(t)$ is the applied strain and $\Delta\varepsilon_i$ is the jump in the applied strain occurring at time $t_i$.

### 2.1.3 Kelvin and Maxwell Viscoelastic models

Most viscoelastic models are built on the spring-dashpot arrangement as the basic building blocks. The two basic models are the Kelvin solid model and the Maxwell fluid model (Chung, 1988). Figure 2.2a shows the Kelvin model with the spring and dashpot arranged in parallel while Figure 2.2b shows the Maxwell model where the spring and dashpot are arranged in series.

The two basic equations governing the stress-relaxation behaviour of the Kelvin and Maxwell models at constant strain are given in Eq. (5) and Eq. (6) respectively;

$$E(t) = E$$  \hspace{1cm} (5)

$$E(t) = Ee^{-t/\tau}$$  \hspace{1cm} (6)
Figure 2.2: Basic Viscoelastic models

Where; \( E(t) \) is the relaxation modulus and \( E \) the spring constant.

2.1.4 Deficiencies of Maxwell and Kelvin Models

The Maxwell and Kelvin models have been noted to have several deficiencies in predicting the behaviour of viscoelastic materials (Chung, 1988; Gumbe, 1993). These deficiencies include,

- Neither the Maxwell nor Kelvin model represents the behaviour of most viscoelastic materials in actual systems.
- The Maxwell model shows no time-dependent recovery nor does it show the decreasing strain rate under constant stress, a characteristic of primary creep.
- The Kelvin model does not exhibit time-independent strain on loading, nor does it describe a permanent strain after unloading.
- Both models show a finite initial strain rate where the initial strain rate for many materials is very rapid.

2.1.5 Maxwell-Weichert Viscoelastic Model

Due to the above limitations of the simplified models, several other viscoelastic models have been adopted to help make more accurate predictions of viscoelastic behaviour of engineering materials (Chung, 1988; Aklonis and MacKnight, 1983; Gumbe, 1993). One such model, which has been found to be fairly accurate, is the generalized Maxwell-Weichert model (Aklonis and MacKnight, 1983) that consists of an arbitrary number of Maxwell elements connected in parallel as shown in Figure 2.3.

Figure 2.3: Generalised Maxwell-Weichert model (Aklonis and MacKnight, 1983)

The generalized stress-relaxation equation for the Maxwell Weichert model is of the form;

\[
E(t) = \sum_{i=1}^{z} E_i e^{-k_i t} 
\]

Where; \( E(t) \) is the relaxation modulus (MPa), \( t \) is the time of relaxation (minutes) and \( k \) is the exponent (1/minute)

For practical purposes however, equation (7) has to be presented to finite levels. Equation (8) therefore is the representation of (7) at two levels (Herum, 1979);

\[
E(t) = E_1 e^{-k_1 t} + E_2 e^{-k_2 t} 
\]

The two equations, (7) and (8), describe the Maxwell-Weichert linear viscoelastic model as long as the constants \( E_1, E_2, k_1 \) and \( k_2 \) are determined.

MATERIALS AND METHODS

3.1 Sample Collection and Preparation

Three different maize varieties were collected for the experiments. Samples for the Senstar cyclic loading/unloading and stress relaxation tests were collected from the Kenya Agricultural and Livestock Research Organisation (KALRO) field
stations in two different climatic regions of Kenya, namely Eldoret in Uasin Gishu County and Machakos in Machakos County. The Eldoret variety was established to be the Kitale 614 (V1) while the Eastern province varieties were the Katumani maize breeds, namely, Katumani A, (V2) and Katumani B, (V3). In using these three varieties, the following assumptions were made,

- The properties of the varieties were fairly representative of the properties of shelled bulk maize available in Kenya.
- Standard management practices had been applied on the samples both in the field and at post harvest stages.
- The assumptions of isotropy and that of the existence of deviatoric and hydrostatic stresses as proposed by Youngs (1982) applied for the selected grain varieties.

Collected samples were carefully packaged in 50 kg bags which were properly sealed to avoid any moisture loss or gain during transit and then transported to the testing laboratories at the University of Nairobi. Samples were stored at ambient conditions for a period of 24 hours to reach equilibrium before any tests were conducted (Foutz et al., 1993). The equilibrium moisture content was determined to be $12 \pm 1.5\%$ (w.b.). The relative humidity, which was determined from Mason’s wet and dry bulb thermometer, was between 65% and 75% for the duration of the experiments.

### 3.3 Sample Preparation for SUT Testing

The three varieties were tested at 800 kg/m$^3$ initial bulk density. The bulk density was obtained by placing the sample-filled cell on a sieve shaker and shaking for 10 minutes (Li et al. 1990). The sample size used for the SUT tests was 50 mm in diameter and 150 mm in height. Samples which were prepared and conditioned as above were then used for cyclic loading/unloading experiments and stress relaxation tests as explained in the sections that follow.

### 3.4 Senstar Universal Testing Equipment

The equipment used to conduct the cyclic loading and stress relaxation experiments was a 10-ton force Senstar Universal Testing (SUT) machine with an accuracy of 0.001 mm for displacement measurements and 0.1 kgf for load measurements (Figure 3.1). This machine is available at the Mechanical Engineering laboratories at the University of Nairobi. Due to lack of automation however, the data collected was accurate to 0.1 mm for displacement and 1 kgf for load. A rigid cell of diameter 50 mm and height 150 mm, which had been specifically designed for the uniaxial tests, was used to hold the sample (the dimensions being designed to maintain the recommended height-to-diameter ratio of between 2 and 3). The inner wall of the cell was oiled before every test so as to ensure minimal friction between the grains and the rigid wall.

### 3.5 Cyclic Loading Tests

Cyclic loading and unloading tests using the SUT machine were repeated three times for each variety and density. A total of 9 tests were carried out as shown in Table 3.1.

Three loading and unloading cycles were conducted for each sample.

All the tests were conducted at a moisture content of $12 \pm 1.5\%$ (w.b.)
Table 3.1: Summary of Cyclic Loading and Stress Relaxation Experiments

| Variety | No of Replications | Density (kg/m³) |
|---------|-------------------|-----------------|
| V1      | 3                 | 800             |
| V2      | 3                 | 800             |
| V3      | 3                 | 800             |

Samples were loaded at a constant strain rate of 1.3 mm/min to a total strain of below 20% and then unloaded to zero.

Load deformation data was recorded at intervals of 98.1 N (10 kgf). After the third and final test, the maize was carefully removed from the testing cell and discarded.

3.6 Stress-Relaxation Tests

The set-up of stress relaxation experiments followed those of the cyclic tests described in 3.5. The material prepared as described in 3.3 was carefully loaded on the SUT equipment using a plunger to a strain level of 15%. The strain was then fixed to this point as the monitoring of stress-relaxation of the material began. Relaxation time was about 30 minutes for each of the samples. Relaxation data was recorded after every 30 seconds for the duration of the test (30 minutes). Each of the tests was replicated three times with each sample being discarded after every test.

3.7 Determination of Viscoelastic Parameters

The generalized viscoelastic model of equation (7) was obtained to two levels from the experimental stress relaxation data. A regression procedure that ensured convergence was adopted through an S-PLUS 6.1 program that iterated the available data to come up with the required relaxation modulus equation in the form of equation (8).

The relaxation modulus parameters, $E_1, E_2, k_1$ and $k_2$ were therefore obtained for the three bulk maize varieties, at two different moisture contents and at two different densities.

RESULTS AND DISCUSSION

4.1 Cyclic Loading Tests

Figures 4.1a-c show cyclic loading and unloading curves obtained from the cyclic loading and unloading experiments of the SUT machine for V1 V2 and V3 at IBD of 800 kg/m³.

![Figure 4.1a: Cyclic loading-unloading test for variety V1 at IBD of 800 kg/m³](image)

![Figure 4.1b: Cyclic loading-unloading test for variety V2 at IBD of 800 kg/m³](image)
Figures 4.1a-c shows that about 10% of the initial strain was not recovered in unloading for all the three varieties. Elasticity theories would have expected the stress to follow the same path in unloading as in loading. However, from figures 4.1a-c, it is evident that the bigger part of the initial strain is permanent. The cyclic loading section also produced curves that were almost parallel to each other (of slope $E_{ur}$) as was expected of other engineering materials on cyclic loading/unloading. These findings were consistent with those of Li et al. (1990) for granular agricultural materials. The resulting prestrain and the occurrence of hysteresis loops on Figures 4.1a-c on cyclic loading/unloading are also characteristic of annealed engineering materials under similar loading conditions as reported by Chakrabarty (1987).

### 4.2 Viscoelastic Parameters

The following relaxation parameters were obtained from an iteration procedure using S-PLUS 6.1 program. These included the stress-relaxation moduli, $E_1$, $E_2$ and the stress-relaxation exponents’ $k_1$, $k_2$. Each test was replicated three times.

Figures 4.2a-c shows the stress relaxation curves for the three maize varieties, at an initial bulk density of 800 kg/m$^3$ (three replications) and the fitted curves according to the given regression equations indicated.
measured values and the fitted values were obtained as 0.87, 0.84 and 0.90 for V1, V2 and V3 respectively as shown in Figures 4.3a-c. These were indicators that the measured data and the regressed values fitted quite closely statistically.

**Figure 4.3a:** Coefficient of determination, $R^2$, for $E(t)$, variety V1

$E_{calc} = 0.866E_{meas} + 5.682$

$R^2 = 0.87$

**Figure 4.3b:** Coefficient of determination, $R^2$, for $E(t)$, variety V2

$E_{calc} = 0.897E_{meas} + 4.4744$

$R^2 = 0.90$

**Figure 4.3c:** Coefficient of determination, $R^2$, for $E(t)$, variety V3

From the regression results, the stress relaxation moduli $E_1$ and $E_2$ were inversely correlated as opposed to the exponents, $k_1$ and $k_2$. The residual standard error between the fitted values and the experimental values were 1.73, 0.95 and 1.82 for V1, V2 and V3 respectively. The degrees of freedom for the varieties were 179, 119 and 179 for V1, V2 and V3 respectively. The 95% confidence interval for sample V3, for example, gave a $p$-value of 0.0001 which was highly significant.

The flexibility of the Maxwell-Weichert model in reproducing the viscoelastic behaviour of shelled maize *en masse* was further demonstrated by plotting the above results on a log-log scale as shown in Figure 4.4.

**Table 4.2:** Summary of stress-relaxation parameters

| Variety | Density (kg/m³) | $M_c$ (±1.5%) | Replications | Relaxation modulus constants (MPa) | Relaxation modulus exponents (1/mins) |
|---------|-----------------|---------------|--------------|-----------------------------------|---------------------------------------|
|         |                 |               |              | $E_1$ | $E_2$ | $k_1$ | $k_2$ |
| V1      | 800             | 12            | 3            | 10.92 | 46.29 | 0.483 | 0.007 |
| V2      | 800             | 12            | 3            | 7.87  | 15.79 | 0.664 | 0.010 |
| V3      | 800             | 12            | 3            | 14.66 | 48.61 | 0.627 | 0.009 |
CONCLUSION

Senstar test results obtained for bulk maize at an initial bulk density of 800kg/m$^3$ provided the stress-relaxation parameters, namely the stress-relaxation moduli and exponents under the Maxwell-Weichert viscoelastic model. These results showed that the Maxwell-Weichert model for viscoelastic polymers can be applied to accurately describe the behaviour of shelled maize en masse.

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