Brane inflation: swampland criteria, TCC, and reheating predictions

Abolhassan Mohammadi\textsuperscript{a,}, Tayeb Golanbari\textsuperscript{a,†}, Salah Nasri\textsuperscript{b,c,‡} and Khaled Saaidi\textsuperscript{a,§}

\textsuperscript{a}Department of Physics, Faculty of Science, University of Kurdistan, Sanandaj, Iran.

\textsuperscript{b}Department of Physics, United Arab Emirates University, Al-Ain, UAE.

\textsuperscript{c}The Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, I-34014, Trieste, Italy.

(Dated: January 25, 2022)

Abstract

We consider inflation in a five-dimensional space-time with the inflaton field confined to live on a brane world. In this scenario, we study different types of potentials for the inflaton, discuss their observational consequences, and compare with data. We find that some class of potentials are in good agreement with observation and that the value of the inflaton field can be sub-Planckian. Moreover, we investigate the swampland criteria in this scenario and determine the consistency of the model with the conjectures. Doing so, we could determine models that simultaneously satisfy both observational data and swampland criteria. More constraints are applied by studying the reheating phase where the acceptable range for the reheating temperature imposes some bounds on the models. As the last step, the result of trans-Planckian censorship conjecture for the model is considered where it is shown the constraint of TCC will be very strong and it could be used to applied limit on the brane tension.

\textsuperscript{*}Electronic address: a.mohammadi@uok.ac.ir; abolhassann@gmail.com

\textsuperscript{†}Electronic address: t.golanbari@uok.ac.ir; t.golanbari@gmail.com

\textsuperscript{‡}Electronic address: snasri@uaeu.ac.ae

\textsuperscript{§}Electronic address: ksaaidi@uok.ac.ir
I. INTRODUCTION

The inflationary scenario is known as one of the best candidates for describing the very early universe which has been strongly supported by the observational data \cite{1,3}. Since the first proposal of the scenario \cite{4-8} many inflationary models have been introduced such as non-canonical inflation \cite{9-16}, tachyon inflation \cite{17-20}, DBI inflation \cite{21-26}, G-inflation \cite{27-30}, warm inflation \cite{31-41}, in which the most common picture is that inflation is driven by a scalar field which slowly rolls down to a minimum of its potential \cite{42-45}. After inflation the universe is cold and almost empty of particles. Then, a mechanism is required to warm up the universe and fill it with particles. The mechanism is known as (p-)reheating \cite{46-56} describing an energy transfer from scalar field to other field leading to particle production. The produced particles interact and thermalize the universe and allow a smooth transition to radiation dominant phase. The reheating temperature should on one side be large enough to recover the successful hot big bang nucleosynthesis ($T > 1$ MeV) and also small enough to avoid the reproduction of any unwanted particle ($T < 10^{9-10}$ GeV) \cite{52,53}. Reheating is inseparable part of (cold) inflation model, and any inflation model without reheating is incomplete.

The standard model of inflation has been generalized in different ways which one of them is the inflationary scenario in modified gravity models where the brane gravity model is known as one of the interesting generalized theory of gravity. The brane theory of gravity is a higher dimensional model of gravity which has been inspired from M-theory. The first model of brane world was introduced by Randall and Sundrum (RS) in 1999 where the main motivation of the model was to find a solution for the Hierarchy problem between electroweak scale and Planck scale \cite{57,58}. The general picture is that all standard particles are confined to a four-dimensional space-time (brane) and only gravity could propagate in higher dimension. In other words, our universe is a three brane embedded in five-dimensional space-time which is called bulk. The model introduces an interesting and novel feature in the evolution equation. The Friedmann equation in brane world gravity includes both quadratic and linear terms of the energy density while in four-dimensional cosmology there is only linear term. The quadratic term of the energy density dominates over the linear term in the high energy regime (where energy density is larger than the brane tension, i.e. $\rho \gg \lambda$). Consequently, the Hubble parameter in this regime is proportional to the energy
density, \( H \propto \rho \) and it is no longer proportional to \( H \propto \sqrt{\rho} \).

A theoretical constraint on the inflationary models has been recently proposed which is known as the swampland criteria [65–67]. The origin of these criteria stands in string theory where they are realized as a measure to recognize the consistence low-energy effective field theory (EFT) from the inconsistence ones. It includes two conjectures: I) There is an upper bound on the field range, i.e. \( \Delta \phi/M_p < c_1 \) where \( c_1 \) is of order of unity, which rise from this belief that the effective Lagrangian in the EFT is valid only for a finite radius; II) putting an upper bound on the gradient of the potential of the field of any EFT, i.e. \( M_p |V'/V| \geq c_2 \) or the refined version of this conjecture, given by \( M_p^2 V''/V \geq -c_3 \) where \( c_3 \) is positive. The refined conjecture states that the potential must be sufficiently tachyonic. Also, the most recent studies determines that \( c_2 \) and \( c_3 \) could be even of order of \( O(0.1) \) [67, 68]. In the first look, the second criterion is in direct tension with the slow-roll inflation where the slow-roll parameter \( \epsilon_{\phi} = M_p^2 (V'/V)^2 \) must be smaller than one. In general, these two criteria rule out some of the inflationary models, however, the recent studies [68, 82] have determined that some non-standard models of inflation might still survive these two criteria, in which the brane inflation could be one of them.

The more recent conjecture is the trans-Planckian censorship conjecture (TCC) proposed in [83]. The conjecture states that for any consistent theory of quantum gravity it is absolutely impossible that a mode that was trans-Planckian never can cross the Hubble horizon. This situation never happens for a model like big bang cosmology where the mode never cross the horizon. However, for inflationary phase this conjecture might lead to some serious outcomes. The TCC for standard inflation results in some strong condition on the energy scale and tensor-to-scalar ratio [84], and the condition get more stronger for brane inflation [85]. The investigation were performed by this assumption that the Hubble parameter is constant during inflation and the reheating phase occurs very fast.

The main reasons that motivates us to consider the inflationary scenario in the frame of RSII brane gravity model are two folds: observational and theoretical consistency. First, due to interesting feature of the Friedmann equation in the brane world model which is expected to lead to some novel conclusions. The scenario is studied for different well-known potentials, and the free parameters of the model are determined by comparing the results with observational data. In this regard, our method is different from the previous studies where instead of testing the results of the model for two or three sets of the constant
parameters of the model, we find a parameter space in which every point is consistent with the data. The observational data, during the past years, is getting better and there are chances that some of the potentials be throwing out due to their inconsistency with data. Considering the consistency with the swampland criteria is another motivation for the present work. There is a growing interest to find the inflationary models which simultaneously agree with observational data and swampland criteria. Then, the model prediction about the reheating temperature is investigated which leads to more constraint on the parameters. Consistency with the TCC, as the most recent conjecture, will be another topic that would be interesting to be studied.

The paper is organized as follows: After the introduction, the main dynamical equations of the model are presented in Sec.II. In Sec.III, the slow-roll parameters are introduced for a general form of the potential, and the perturbations parameters are described in terms of the potential. Next, in Sec.IV we are going to consider the consistency of the model with data for different well-known types of the potential, then try to find out the consistency of the result with the swampland criteria. The model prediction about the reheating temperature is considered in Sec.V, where it is realized that to have an acceptable value for the temperature, the parameters are required to be restricted more. In Sec. VI, some note about the TCC are presented and the applied condition on the model is discussed. The results will be summarize and discussed in Sec.VII.

II. THE MODEL

Our study will be limited to Randall-Sundrum II brane gravity model, with the following action

\[ S_5 = \int d^5 x \sqrt{-g} \left( \frac{M_5^3}{2} R - \Lambda_5 \right) + \int d^4 x \sqrt{-q} \left( L_b - \lambda \right), \]  

(1)

where \( R \) is the Ricci scalar, \( \Lambda_5 \) the five-dimensional cosmological constant, \( M_5 \) stands for five-dimensional Planck mass, and \( q_{\mu \nu} \) the induced metric on the brane which is related to the five-dimensional metric \( g_{AB} \) by the relation \( g_{AB} = q_{AB} + n_A n_B \), where \( n^A \) is a unit normal vector. \( L_b \) indicates the Lagrangian of matter that has confined on the brane and \( \lambda \) is the brane tension. By taking variation of the above action with respect to the metric we
obtain the the field equation of motion

\[ G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + \left( \frac{8\pi}{M_4^2} \right) \tau_{\mu\nu} + \left( \frac{8\pi}{M_5^3} \right)^2 \Pi_{\mu\nu} - E_{\mu\nu}, \quad (2) \]

with

\[ \Lambda_4 = \frac{4\pi}{M_5^3} \left( \Lambda_5 + \frac{4\pi}{3M_5^3} \lambda^2 \right), \]

\[ M_4^2 = \frac{3}{4\pi} \frac{M_5^6}{\Lambda}, \]

\[ E_{\mu\nu} = C_{MRNS} n^M n^N q_{\mu}^{R} q_{\nu}^{S}, \]

\[ \tau_{\mu\nu} = -2 \frac{\delta L_b}{\delta g^{\mu\nu}} + g_{\mu\nu} L_b, \]

\[ \Pi_{\mu\nu} = -\frac{1}{4} \omega_{\mu} \omega_{\nu} + \frac{1}{12} \tau_{\mu\nu} + \frac{q_{\mu\nu}}{8} \tau_{\alpha\beta} \tau_{\alpha\beta} - \frac{q_{\mu\nu}}{24}. \]

Here \( M_4 \) is the effective four-dimensional Planck mass, \( \Lambda_4 \) the cosmological constant on the brane is defined by \( \Lambda_4 \) which is a combination of the five-dimensional cosmological constant and the tension of the brane, \( E_{\mu\nu} \) is the projection of the five-dimensional Weyl tensor \( C_{MRNS} \) on the brane, and \( \tau_{\mu\nu} \) is the brane energy momentum tensor. Note that both the linear and quadratic terms contribute to the effective four-dimensional energy-momentum tensor.

Assuming the homogeneity and isotropy of the universe and a spatially flat five-dimensional Friedmann–Lemaitre–Robertson–Walker (FLRW) metric, defined as

\[ ds_5^2 = -dt^2 + a^2 \delta_{ij} dx^i dx^j + dy^2, \quad (3) \]

where \( \delta_{ij} \) is a maximally symmetric three-dimensional metric and \( y \) denotes the fifth coordinate, the corresponding Friedmann equation reads

\[ H^2 = \frac{\Lambda_4}{3} + \left( \frac{8\pi}{3M_4^2} \right) \rho + \left( \frac{4\pi}{3M_5^3} \right)^2 \rho^2 + \frac{C}{a^4}. \]

The last term on the right hand side of the above equation arises from the term \( E_{\mu\nu} \), which describes the influence of the bulk graviton on the brane evolution, and is known as the dark radiation. Because it scales as \( a^{-4} \), the dark radiation gets rapidly diluted during inflationary phase, and hence can be neglected. Also, we will set \( \Lambda_4 = 0 \) as in the original RS model. Therefore, the Friedmann equation is rewritten as

\[ H^2 = \frac{8\pi}{3M_4^2} \rho \left( 1 + \frac{\rho}{2\lambda} \right), \quad (4) \]
In the high energy region, there the contribution of the term quadratic in the energy density is dominant in the expression of the Hubble parameter, where as in the regime where $\rho \ll \lambda$, the Friedmann equation the usual form of standard cosmology. Since standard cosmology is very successful in describing the evolution of the universe from the time of nucleosynthesis, it requires the brane tension as $\lambda \geq 1\text{MeV}^4$, leading to the five-dimensional Planck mass $M_5 \geq 10\text{TeV}$ [86, 87]. Moreover, the Newtonian law of gravity receives a correction of order $M_5^6/\lambda^2 r^2$, which should be small on scales larger than $r \geq 1\text{mm}$, and consequently yields to the stronger constraint $M_5 \geq 10^5\text{TeV}$ [87]. There are also various astrophysical implications which set strong limit on the brane tension $\lambda \geq 5 \times 10^8\text{MeV}^4$ (see [87]).

The matter confined to the brane satisfy the same energy conservation equation as in standard cosmology, i.e

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{5}$$

Using this equation and taking the time derivative of Eq.(4), we obtain the second Friedmann equation

$$\dot{H} = -\frac{4\pi}{M_4^2} \left(1 + \frac{\rho}{\lambda}\right)(\rho + p). \tag{6}$$

### III. BRANE INFLATION

We assume the inflaton is scalar field living on the brane and has the energy density and pressure $\rho = \frac{\dot{\phi}^2}{2} + V(\phi)$ and $p = \frac{\dot{\phi}^2}{2} - V(\phi)$, respectively, which is governed by the equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \tag{7}$$

where $V(\phi)$ is the potential of the inflaton. The common picture for the universe is that the scalar field slowly rolls down toward the minimum of its potential. During this slow-rolling phase, the scalar field yields very small kinetic energy which can be neglected compared to its potential energy. Also, it is assumed that the term $\ddot{\phi}$ is much smaller than the friction term $H\dot{\phi}$ and the slope of the potential $V'$. These assumptions are known as the slow-roll conditions and are described by the smallness of the slow-roll parameters:

$$\epsilon = \frac{-\dot{H}}{H^2}, \quad \eta = \frac{-\ddot{\phi}}{H\dot{\phi}}. \tag{8}$$
With these parameters, the dynamical equations of the model could be rewritten as

\[ H^2 = \frac{8\pi}{3M_4^2} V(\phi) \left( 1 + \frac{V(\phi)}{2\lambda} \right), \quad (9) \]

\[ \dot{H} = \frac{-4\pi}{M_4^2} \left( 1 + \frac{V(\phi)}{\lambda} \right) \dot{\phi}^2, \quad (10) \]

\[ 3H \dot{\phi} = -V'(\phi). \quad (11) \]

Using these equations, we can express the slow-roll parameters in terms of the potential and its derivatives as

\[ \epsilon = \frac{M_4^2}{16\pi} \left( \frac{V'(\phi)}{V(\phi)} \right)^2 \frac{4\lambda(\lambda + V(\phi))}{(2\lambda + V(\phi))^2}, \quad (12) \]

\[ \eta = \frac{M_4^2}{8\pi} \frac{V''(\phi)}{V(\phi)} \frac{2\lambda}{2\lambda + V(\phi)}. \quad (13) \]

Compared to the standard cosmology, here we have a generalized Friedmann equation with some modified terms. These It is important to note that in the high energy limit, i.e. \( \rho \gg \lambda \), The quadratic term of the energy density dominates over the linear term and the Hubble parameter is proportional to the potential, in contrast to the standard cosmology where \( H \propto V^{1/2}(\phi) \). For the rest of the work, we will assume that inflation occurs in the high energy limit, in which case the slow-roll parameters get the simpler form

\[ \epsilon = \frac{1}{3} \left( \frac{3M_5^3}{4\pi} \right)^2 \frac{V'(\phi)}{V^3(\phi)}, \quad \eta = \frac{1}{3} \left( \frac{3M_5^3}{4\pi} \right)^2 \frac{V''(\phi)}{V^2(\phi)} \quad (14) \]

The expansion of the universe during inflation is quantified by the number of e-fold which describes how long this exponential phase should last, and is defined as

\[ N = \int_{t_i}^{t_e} H \; dt = -3 \left( \frac{4\pi}{3M_5^3} \right)^2 \int_{\phi_i}^{\phi_e} \frac{V^2(\phi)}{V'(\phi)} \; d\phi \quad (15) \]

where in obtaining the second equality Eqs.(9) and (11) have been used.

**A. Cosmological perturbations**

The quantum perturbations in the inflationary scenario are divided into three types: scalar, vector, and tensor, in which the scalar perturbations are the seeds for large scale structure of the universe and tensor perturbations are known as the primordial gravitational waves. The vector perturbations are less important since they behaves as the inverse of the
scale factor and rapidly diluted during inflation.

Let us consider an arbitrary scalar perturbation to the background FLRW metric

\[ ds^2 = -(1 + 2A)dt^2 - 2a^2(t)\nabla_i B dx^i dt + a^2(t) \left( (1 - 2\psi)\delta_{ij} + 2\nabla_i \nabla_j E \right) dx^i dx^j. \] (16)

where \( \delta_{ij} \) is the spatial metric of the background and \( \nabla_i \) stands for covariant derivative with respect to the metric. The quantity \( \psi \) is called the curvature perturbations due to the fact the intrinsic curvature of the spatial hypersurface is directly related to the this parameter as \( ^3R = 4\nabla^2 \psi / a^2 \). The curvature perturbation is gauge dependent and changes under arbitrary coordinate transformation. However, the curvature perturbations in the uniform density hypersurface, given by \( \zeta = \psi + \frac{H \dot{\rho}}{\rho} \) is a gauge invariant perturbation parameter. For the single scalar field inflationary models, where the perturbations can be assumed to be adiabatic, the curvature perturbation \( \zeta \) is conserved and remains almost constant at the superhorizon scale \([59, 88]\). This is the most important feature of the parameter. On the spatially flat hypersurface, \( \psi = 0 \), using the scalar field energy density, the gauge-invariant curvature perturbation \( \zeta \) is obtained as

\[ \zeta = \frac{H}{\dot{\phi}} \delta \phi, \] (17)

where \( \delta \phi = H/2\pi \). Following \([59, 88]\), the amplitude of the scalar perturbation is defined as \( P_s = 4(\zeta^2)/25 \), and making use of the slow-roll approximations we have

\[ P_s = \frac{9}{25\pi^2} \left( \frac{4\pi}{3M_5^3} \right)^6 \frac{V^6(\phi)}{V''(\phi)} \] (18)

Using above relation, we obtain the scalar spectral index

\[ n_s - 1 = \frac{d \ln(P_s)}{d \ln(k)} = -6\epsilon + 2\eta. \] (19)

The derivation of the amplitude of the tensor perturbations for this model is a little trickier than in the standard four-dimensional cosmology since here the graviton can propagate along the fifth dimension as well. It is given by \([89, 91]\)

\[ P_g = \frac{16\pi}{25\pi M_p^2} \left( \frac{H}{2\pi} \right)^2 F^2(x) \] (20)
where
\[ F^2(x) = \left[ \sqrt{1 + x^2} - x^2 \sinh^{-1}\left(\frac{1}{x}\right) \right]^{-1/2}, \quad x \equiv \sqrt{\frac{3}{4\pi\lambda}} M_p H \] (21)

Using Eq. (9) and considering the high energy limit, the perturbations reads
\[ P_g = \frac{9}{50\pi^2} \left( \frac{4\pi}{3M_5^3} \right)^4 V^3(\phi) \] (22)

The tensor perturbation is measured indirectly through the tensor-to-scalar ratio
\[ r = \frac{3\epsilon}{2}. \] (23)

Thus far there is no evidence for such contribution which yields an upper limit \( r < 0.064 \) from the Planck data combined with the BICEP2/Keck Array BK14 data.

Note that the general shape of the main perturbation parameters \( P, n_s, \) and \( r \) might seem more and less similar to the standard once, however they are different because there is a different Hubble parameter. The main difference that is our concern is that The Hubble parameter in brane gravity depends on \( \rho \), and not on \( \sqrt{\rho} \). As a result, the evolution and behavior of the fluctuations, the slow-roll parameters, and in turn the behavior of the perturbation parameters are different than the once in standard 4D gravity. Therefore, the results that we are comparing with observational data are not the once we have in standard general relativity.

IV. CONSISTENCY WITH OBSERVATION AND SWAMPLAND CRITERIA

In this section, we we consider in details different types of inflaton potentials and for each we determine the model parameter space that is consistent with the latest observational data.

A. Power-law Potential

As the first case, the power-law potential is picked out. Although the power-law potential in the standard inflation model could not be a good choice for describing the inflation, it could have a proper consistency with data in the modified theories of gravity, e.g. scalar-tensor theory of gravity. One of the main features of the potential which puts it in the
center of our attention is its simplicity. Due to this fact, the potential is the first choice for considering any inflationary model. It is always desirable to have a simple model for describing a phenomenon, like inflation. The power-law potential is given by

$$V(\phi) = V_0 \phi^n,$$  \hspace{1cm} (24)$$

where $V_0$ and $n$ are constant. Substituting this potential into Eq. (14) yields the slow-roll parameters

$$\epsilon = \left(\frac{3M_5^3}{4\pi}\right)^2 \frac{n^2}{3V_0} \frac{1}{\phi^{n+2}}, \quad \eta = \left(\frac{3M_5^3}{4\pi}\right)^2 \frac{n(n-1)}{3V_0} \frac{1}{\phi^{n+2}}.$$  \hspace{1cm} (25)$n$

By setting $\epsilon = 1$, we can infer the value of the scalar field at the end of inflation as

$$\phi_e^{n+2} = \left(\frac{3M_5^3}{4\pi}\right)^2 \frac{n^2}{3V_0}.$$  \hspace{1cm} (26)$n$

Applying this result to Eq. (15) yields the scalar field at the horizon crossing

$$\phi^{n+2}_* = \left(\frac{3M_5^3}{4\pi}\right)^2 \frac{n^2}{3V_0} \left[1 + \frac{(n+2)N}{n}\right].$$  \hspace{1cm} (27)$n$

To obtain the energy scale of inflation, one only needs to substitute the scalar field $\phi_*$ in the potential Eq. (24). Doing so, we have

$$V_{es} = V_0 \left(\frac{3M_5^3}{4\pi}\right)\left[1 + \left(\frac{(n+2)N}{n}\right)\right]^{\frac{n}{n+2}},$$  \hspace{1cm} (28)$n$

which presents a relationship between the energy scale of inflation and number of e-fold. By the energy scale we means the value of the potential at beginning of inflation, namely $N = 55 - 65$. The energy scale of inflation is indicated by $V^*$ which is equal to $V_{es} = V|_{N=55-65}$.

---

1 Note that different values of $N$ correspond to different horizon crossing times. The point is that fluctuations are produced constantly during inflation. Some exit the horizon at earlier times, which is indicated by larger value of $N$. Some exit the horizon at later times which is expressed by smaller values of $N$. In inflationary scenario, it is assumed that the fluctuations (that we are interested in) cross the horizon and then there is about $N = 55 - 65$ e-fold of expansion. A common way to count the expansion is that we assign $N = 0$ to the end of inflation, and $N = 55 - 65$ is associated with the time of horizon crossing (i.e. we only reverse the e-fold counting and the result will be the same). We sometimes call the latter time as the beginning of inflation. Then, $N = 0$ means the end of inflation and $N = 60$ (for instance) stands for the beginning of inflation. This point is clear in Eq. (27), in which if one put $N = 0$, the result is exactly equal to $\phi_e$, Eq. (26). The scalar field at the beginning of inflation is obtained by putting $N = 60$. 

10
Substituting Eq. (27) into Eqs. (25), (19), and (23), we get

\[ \epsilon(n, N) = \left(1 + \frac{(n + 2)N}{n}\right)^{-1}, \quad \eta(n, N) = \frac{(n-1)}{n} \left(1 + \frac{(n + 2)N}{n}\right)^{-1} \]  

which is a function of only the power of the potential and the number of e-folds. Consequently, the scalar spectral index and the tensor-to-scalar depend on just on the two parameters \( n \) and \( N \). Using the Planck \( r - n_s \) diagram, we show in Fig. 1 the parameter space for \((n, N)\) for which the model predictions are in agreement with the Planck data.

![Parameter Space Diagram](image)

FIG. 1: The parameter space \((n, N)\) for the power law potential that yield values \((r, n_s)\) allowed by the Planck data at the 95% CL (light blue) and the 68% CL (dark blue).

Also, from the expression of the amplitude of the scalar perturbations, Eq. (18), the constant \( V_0 \) is determined as

\[ V_0 = \frac{25\pi^2 n^2 P_s}{9} \left( \frac{3M_5^3}{4\pi} \right)^6 \left[ \frac{3n^2}{16\pi^2} M_5^6 \left(1 + \frac{(n + 2)N}{n}\right) \right]^{\frac{2(2n+1)}{n+2}} \]  

and depends on the values of \( n, N \), and the five-dimensional Planck mass. Setting \( M_5 = 2 \times 10^{14} \) GeV, in Table II we present the values of \( V_0 \) and the energy scale \( V^* \) for different values of \( n \) and \( N \) taken from Fig. 1. Considering the constraint on the brane tension \( \lambda \), stated in Sec. II, and for the same chosen value of \( M_5 \), we find that \( \rho/\lambda \sim \mathcal{O}(10^{11} - 10^{18}) \), which is much larger than unity, and hence in consistent with our assumption that inflation occurred in the high energy regime.

Now that we determined the model parameter space that are consistent with observation, the next step is to use these values and determine whether the model satisfy the swampland criteria. For that, in Figs 2, we display the behavior of \( \Delta\phi/M_p \) and \( M_p|V''/V| \) for different values of the \( n \) and the number of e-folds. Fig. 2(a) and Fig. 2(b) respectively show that...
\[ n \quad N \quad V_0 \quad V^* \]
\begin{tabular}{cccc}
1 & 60 & \(2.08 \times 10^{55}\) GeV\(^3\) & \(4.57 \times 10^{65}\) GeV\(^4\) \\
1.5 & 65 & \(5.07 \times 10^{42}\) GeV\(^{5/2}\) & \(3.35 \times 10^{61}\) GeV\(^4\) \\
2 & 70 & \(9.03 \times 10^{29}\) GeV\(^2\) & \(2.49 \times 10^{58}\) GeV\(^4\) \\
\end{tabular}

TABLE I: The constant \(V_0\) and the energy scale of the inflation for different values of \((n, N)\) taken from Fig.1 and \(M_5 = 2 \times 10^{14}\) GeV

\[ \Delta \phi/M_p < 1 \text{ and } M_p|V'/V| > 1 \] at the horizon crossing time for allowed values of \(n\) (based on Fig.2). From Fig.2(c), we see that for all the chosen values of \(n\), we have \(\Delta \phi/M_p < 1\) during the whole time of inflation\(^2\), and it gets even smaller as \(n\) decreases. On the other hand, Fig.2(d) shows that \(M_p|V'/V| > 1\) for all the chosen values of \(n\), and gets even larger for smaller \(n\). We therefore conclude that for all values of \(n\) and \(N\) presented in Fig.1, both swampland criteria are satisfied during the whole time of inflation. We also note that smaller values of the five dimensional Planck mass support the swampland criteria, namely by reducing the value of \(M_5\), \(\Delta \phi/M_p\) decreases and \(M_p|V'/V|\) increases, respectively.

The same potential has been consider for the brane inflation in [92] following a different method for considering the consistency of the model with observational data and swampland criteria. However, there are some differences which made it worth to reconsider the potential. Here, we were looking for the most suitable values of \(n\) and \(N\) which simultaneously give compatible results for the scalar spectral index and tensor-to-scalar ratio, in which for every values of the number of e-fold one could find the best choices of the parameter \(n\) from Fig.1 to have a good agreement with data. Besides, using the amplitude of the scalar perturbations, the constant \(V_0\) has been determined as well which is used to find about the energy scale of the inflation in the model. To investigate the consistency of the model with the swampland criteria, both criteria have been considered in detail and the effect of \(M_5\) on the criteria was studied as well. It stated that increasing of \(M_5\) enhances the term \(\Delta \phi/M_p\) in which at a certain value of \(M_5\) the condition \(\Delta \phi/M_p < 1\) is violated. The validity of two criteria were plotted for certain values of the parameters and during inflation which perfectly covers the

\(^2\) By the "Whole time of inflation", the authors mean the time period between the horizon exit of perturbations and the end of inflation. The horizon exit is assumed to happen for number of inflation about \(N = 65\) and the end of inflation is illustrated by \(N = 0\).
FIG. 2: The figures show the behavior of: a) $\Delta \phi / M_p \equiv \bar{\Delta} \phi$ and b) $M_p |V'/V| \equiv \Delta V$ versus the number of e-fold for different values of $n$ taken from Fig. 1, where $M_5 = 2 \times 10^{14}$ GeV.

subject.

B. Natural Potential

One of the interesting inflationary model is the natural inflation with the following potential

$$V(\phi) = V_0 \left( 1 - \cos \left( \frac{\phi}{f} \right) \right), \quad (31)$$

here $V_0$ and $f$ are constant parameters. This type of the potential usually arises when the inflaton is taken as an axion [43]. The potential also has a strong background in particle physics model [93]. One of the features of the potential is that depending of the value of $f$ it could play both in large-field and small-field types inflation. For this potential, the slow-roll parameters are given by

$$\epsilon = \left( \frac{3M_5^3}{4\pi} \right)^2 \frac{1}{3f^2 V_0} \frac{(1 + \cos(\Phi))}{(1 - \cos(\Phi))^2}, \quad \eta = \left( \frac{3M_5^3}{4\pi} \right)^2 \frac{1}{3f^2 V_0} \frac{\cos(\Phi)}{(1 - \cos(\Phi))^2}. \quad (32)$$
where $\Phi \equiv \phi/f$. At the end of inflation, we have
\[
\cos(\Phi_e) = \frac{1}{2} \left[ (\gamma + 2) \pm \sqrt{(\gamma + 2)^2 - 4(\gamma - 1)} \right], \quad \gamma \equiv \left( \frac{3 M_p^3}{4 \pi} \right)^2 \frac{1}{3 f^2 V_0}, \quad (33)
\]
and after inserting it into Eq. (15), the field during at horizon crossing reads
\[
\cos(\Phi) = -1 - 2 W \left[ -\frac{1}{2} \exp \left( \frac{-1}{2} - \zeta \right) \right] \quad (34)
\]
where
\[
\zeta \equiv \gamma N + \cos(\Phi_e) - 2 \ln (1 + \cos(\Phi_e)).
\]
and $W[x]$ is the Lambert function. Substituting Eq. (34) in Eq. (32), one find that the scalar spectral index and tensor-to-scalar ratio are only a function of the constant $\gamma$ and the number of e-folds $N$. Then, we can extract the allowed values of the model parameters ($\gamma, N$) that yield values of $(r, n_s)$ in agreement with Planck data, as shown in Fig. 3. On the other hand, after some algebra, the amplitude of the scalar perturbation can be expressed as
\[
\mathcal{P}_s = \left( \frac{V_0}{75 \pi^2 \gamma^3 f^4} \right) \frac{(1 - \cos(\Phi_e))^5}{(1 + \cos(\Phi_e))} \quad (35)
\]
Using the observational data for $\mathcal{P}_s$, the expression of the scalar field in Eq. (34), and the values of $\gamma$ and $N$ from Fig. 3, we determine the possible values of the other constants of the model as presented in Table II. To see if the swampland criteria is met in this type models, we depict in Fig. 4 the quantities $\Delta \phi/M_p$ and $M_p|V'/V|$ for different values of $\gamma$ and the number of e-fold $N$. For instance, Figs. 4(a) and 4(b) determine $\Delta \phi/M_p$ and $M_p|V'/V|$ at a specific time during inflation (horizon crossing time) for different values of $\gamma$. We note that

FIG. 3: The parameter space ($\gamma, N$) for the axion-like potential that yield values ($r, n_s$) allowed by the Planck data at the 95% CL (light blue) and the 68% CL (dark blue).
TABLE II: The constant $V_0$ and the energy scale of the inflation for different values of $(n, N)$ taken from Fig.3 and $M_5 = 5 \times 10^{15}$ GeV

| $n$ | $N$ | $f$ (GeV) | $V_0$ (GeV$^4$) | $V^*$ (GeV$^4$) |
|-----|-----|-----------|----------------|----------------|
| 0.055 | 55  | $3.66 \times 10^{17}$ | $4.01 \times 10^{58}$ | $7.32 \times 10^{58}$ |
| 0.060 | 60  | $3.72 \times 10^{17}$ | $3.56 \times 10^{58}$ | $6.68 \times 10^{58}$ |
| 0.065 | 65  | $3.79 \times 10^{17}$ | $3.17 \times 10^{58}$ | $6.06 \times 10^{58}$ |
| 0.070 | 65  | $3.73 \times 10^{17}$ | $3.04 \times 10^{58}$ | $5.84 \times 10^{58}$ |
| 0.075 | 70  | $3.82 \times 10^{17}$ | $2.70 \times 10^{58}$ | $5.25 \times 10^{58}$ |

when $\gamma$ decreases, both $\Delta \phi/M_p$ and $M_p|V'/V|$ decreases, however, $\Delta \phi/M_p$ remains smaller than unity and $M_p|V'/V|$ is still bigger than one. On the other hand, Figs.4(c) and 4(d) display the behavior of these quantities from the start to the end for different values of $\gamma$, and as inflation approaches the end, $\Delta \phi/M_p$ decreases, while $M_p|V'/V|$ increases. Thus, in the brane gravity the axion-like potential satisfy both swampland criteria.

C. Exponential Potential

One of the well-known potential in the inflationary studies is the exponential inflation which leads to a power-law inflation. The potential in given by

$$V(\phi) = V_0 \exp(\alpha \phi),$$  \hspace{1cm} (36)

where $V_0$ and $\alpha$ are two constants of the model. Substituting this potential in Eq.(14), the slow-roll parameters are found as

$$\epsilon = \left(\frac{3M_5^3}{4\pi}\right)^2 \frac{\alpha^2}{3V_0} \exp(-\alpha \phi), \hspace{1cm} \eta = \epsilon.$$ \hspace{1cm} (37)

Finding the scalar field at the end of inflation by solving the relation $\epsilon = 1$, and using that in Eq.(15), the scalar field during inflation in obtained in terms of the number of e-fold as

$$\exp(\alpha \phi_*) = \left(\frac{3M_5^3}{4\pi}\right)^2 \frac{\alpha^2}{3V_0} (1 + N).$$ \hspace{1cm} (38)

Then, the slow-roll parameters are given as

$$\epsilon(N) = \eta(N) = (1 + N)^{-1},$$ \hspace{1cm} (39)
FIG. 4: The figures show the behavior of: a) $M_p \Delta \phi \equiv \Delta \phi$ and b) $M_p V/V' \equiv \Delta V$ versus the number of e-fold for different values of $n$ taken from Fig.3. The right panel of the figure displays the behavior of the axion-like potential.

and from Eqs. (19) and (23), the scalar spectral index and tensor-to-scalar ratio are obtained only as a function of the number of e-fold. Fig.5 illustrates the behavior of the tensor-to-scalar versus the scalar spectral index in terms of the number of e-fold. The curve cross the region of $r - n_s$ only for the number of e-fold $N > 90$.

D. T-mode Potential

The T-mode potential usually appears in the $\alpha$-attractor model of inflation which includes a non-canonical kinetic terms which is originated from Kahler potential in supergravity theories [94–98]. This class of inflation includes Starobinsky’s inflation model and Higgs inflation model [99]. The T-mode is one of the generalized models of the $\alpha$-attractors which
is known with the following potential
\[ V(\phi) = V_0 \tanh^2 \left( \frac{\phi}{\sqrt{6\alpha}} \right). \] (40)

where \( V_0 \) and \( \alpha \) are free constant parameters. The slow-roll parameters are
\[ \epsilon = \frac{2\gamma \left( 1 - \tanh^2 \left( \frac{\phi}{\sqrt{6\alpha}} \right) \right)^2}{\tanh^4 \left( \frac{\phi}{\sqrt{6\alpha}} \right)}, \quad \eta = \frac{\gamma \left( 1 - \tanh^2 \left( \frac{\phi}{\sqrt{6\alpha}} \right) \right) \left( 1 - 3 \tanh^2 \left( \frac{\phi}{\sqrt{6\alpha}} \right) \right)}{\tanh^4 \left( \frac{\phi}{\sqrt{6\alpha}} \right)}, \] (41)

with the parameter \( \gamma \) given by
\[ \gamma \equiv \left( \frac{3M_5^3}{4\pi} \right)^2 \frac{1}{9V_0\alpha}. \]

The scalar field at the horizon crossing time is
\[ \cosh^2 \left( \frac{\phi_\star}{\sqrt{6\alpha}} \right) - \ln \left( \cosh^2 \left( \frac{\phi_\star}{\sqrt{6\alpha}} \right) \right) = \cosh^2 \left( \frac{\phi_e}{\sqrt{6\alpha}} \right) - \ln \left( \cosh^2 \left( \frac{\phi_e}{\sqrt{6\alpha}} \right) \right) + 2\gamma N. \] (42)

Here \( \phi_e \) is the value of the field at the end of inflation, given by
\[ \cosh^2 \left( \frac{\phi_e}{\sqrt{6\alpha}} \right) = 1 + \sqrt{2} \gamma. \]

Comparing the model predictions for \( n_s \) and \( r \) with the Planck data, we present in Fig.6 the corresponding allowed range of the constants for \( \gamma \) and \( N \) at the 68% Cl (in dark blue) and 95% CL (in light blue).

Next, the amplitude of scalar perturbations at the crossing horizon time can be show to be expressed as
\[ \alpha^3 = \frac{1}{(150\pi^2 \times 81) \gamma^4 P_s} \left( \cosh^2 \left( \frac{\phi_\star}{\sqrt{6\alpha}} \right) - 1 \right)^5 \cosh^6 \left( \frac{\phi_e}{\sqrt{6\alpha}} \right). \] (43)
FIG. 6: The allowed parameter space ($\gamma, N$) for the T-mode potential that yield values of ($r, n_s$) that are in agreement with observation at the 68% Cl (dark blue) and 95% CL (light blue).

| $\gamma$ | $N$ | $\alpha$ (GeV$^2$) | $V_0$ (GeV$^4$) | $V^*$ (GeV$^4$) | $\Delta \phi/M_p$ | $M_p|V'/V|$ |
|----------|-----|-------------------|-----------------|----------------|-----------------|-----------|
| $1.5 \times 10^{-5}$ | 66  | $4.01 \times 10^{36}$ | $1.64 \times 10^{60}$ | $9.95 \times 10^{58}$ | 0.406 | 3.33 |
| $3 \times 10^{-5}$ | 67  | $2.84 \times 10^{36}$ | $1.15 \times 10^{60}$ | $9.83 \times 10^{58}$ | 0.409 | 3.25 |
| $5 \times 10^{-5}$ | 69  | $3.65 \times 10^{36}$ | $8.86 \times 10^{59}$ | $9.68 \times 10^{58}$ | 0.416 | 3.15 |
| $6.5 \times 10^{-5}$ | 71  | $1.98 \times 10^{36}$ | $7.65 \times 10^{59}$ | $9.55 \times 10^{58}$ | 0.423 | 3.07 |
| $8 \times 10^{-5}$ | 73  | $1.82 \times 10^{36}$ | $6.78 \times 10^{59}$ | $9.42 \times 10^{58}$ | 0.430 | 2.49 |

TABLE III: The constants $\alpha$, $V_0$ and the energy scale of the inflation for different values of ($\gamma, N$) taken from Fig. 6 and $M_5 = 5 \times 10^{15}$ GeV. Also, the last two columns of the table determine the $\Delta \phi/M_p$ and $M_p|V'/V|$ and give some insight about the swampland criteria.

Then, by choosing specific values of $\gamma$ from the Fig. 6, the allowed potential parameters $\alpha$ and $V_0$ can be determined and are shown in Table III. We also show in the values of the last two columns of the table the values of $\Delta \phi/M_p$ and $M_p|V'/V|$ where we see that they satisfy the swampland criteria$^3$. Therefore, the T-mode potential can be a viable model for inflation that satisfies the swampland criteria.

### E. Generalized T-mode Potential

Here we consider a slightly modified T-mode potential

$$V(\phi) = V_0 \left(1 - \tanh^2 (\alpha \phi)\right)$$

$^3$ Note that since the scalar field approaches $\phi_e$ as time passes, the quantity $\Delta \phi/M_p$ gets smaller and smaller.
Following similar steps as we did with the previous type of potentials, we obtain the slow-roll parameters at the crossing time

\[ \epsilon = \frac{\gamma \exp (-\gamma N)}{(1 + \gamma) - \exp (-\gamma N)}, \quad \eta = \frac{-\gamma}{2} \frac{(1 + \gamma) - 3 \exp (-\gamma N)}{(1 + \gamma) - \exp (-\gamma N)} \] (45)

where the defined constant \( \gamma \) here is given by

\[ \gamma \equiv \left( \frac{3M^3_5}{4\pi^3} \right)^2 \frac{4\alpha^2}{3V_0}. \]

By comparing the model predictions for \( n_s \) and \( r \) with the Planck \( r - n_s \) diagram, we find that only for small range of the parameters \( \gamma \) and \( N \) the model is in agreement with the observational data, as depicted in Fig.7

![FIG. 7: The allowed parameter space (\( \gamma, N \)) for the generalized T-mode potential that yield values of (\( r, n_s \)) that are in agreement with observation at the 68% Cl (dark blue) and 95% CL (light blue).](image)

For the amplitude of the scalar perturbations at the horizon crossing time, we find

\[ V_0 \alpha^4 = \frac{75\pi^2 P_s}{16\gamma^3} \frac{(1 + \gamma)^3 \exp (-\gamma N)}{((1 + \gamma) - \exp (-\gamma N))^4} \] (46)

Thus, for a given point of the allowed region in Fig.7 we can use of the data for the amplitude of the scalar perturbations to determine the parameters of the potential. In Table.IV we give the values of the constants \( \alpha \) and \( V_0 \) for a chosen set of points from Fig.7.

To examine the swampland criteria, in Fig.8 we plot the quantities \( \Delta \phi/M_p \) and \( M_p|V'/V| \) for different values of \( \gamma \) and \( N \). In Figs.8(a) and 8(b) we note that as \( \gamma \) increases, \( \Delta \phi/M_p \) and \( M_p|V'/V| \) respectively increases and decreases. Figs.8(c) and 8(d) which represent the behavior of these quantities during inflation (versus the number of e-fold) for different values of the constant \( \gamma \), and as the inflaton approaches the end of inflation, \( \Delta \phi/M_p \) and \( M_p|V'/V| \)
respective decreases (as was expected) and increases, and hence during the whole period of inflation the swampland criteria are satisfied. Therefore, the potential of the form \( (41) \) can be in consistent with the Planck data and satisfy the swampland criteria.

V. REHEATING

After inflation the universe is cold and almost empty of particles. Then, a mechanism is required to heat up the universe and allow for a smooth cross to radiation dominant phase. The mechanism is called (p-)reheating. The physics of reheating is still uncertain but in a simple words, it could be said that the energy stored in scalar field is converted to the relativistic particle, so that after this period the universe ends up with the standard radiation dominant era.

During inflation, the universe experiences an expansion about \( N_k \) e-fold, and in the reheating phase it expands for another \( N_{re} \) e-fold. The universe fluid is described by an effective equation of state (EoS), parametrize by \( \omega_{eos} \), in which it is close to \(-1\) during inflation, then it comes to \(-2/3\) at the end of inflaton\(^4\).

In the reheating phase, the universe heats up and reaches to the final temperature \( T_{re} \), which should stand in the range \( 10^{-3} \) GeV < \( T_{re} < 10^{9-10} \) GeV \(^{52,56}\). The well-known radiation dominant phase is initiated at this temperature \( T_{re} \). The reheating number of e-fold and temperature could be used as another way for constraining the model parameters.

\(^4\) for this values of \( \omega_{eos} \) the acceleration of the universe vanishes. Note that in the standard 4D cosmology, it happens for \( \omega_{eos} = -1/3 \).
FIG. 8: The top row shows the behavior of $M_p \Delta \phi \equiv \Delta \phi$ and $M_p V/V' \equiv \Delta V$ versus $\gamma$ for different values of number of e-fold. The bottom row indicates the behavior of $M_p \Delta \phi \equiv \Delta \phi$ and $M_p V/V' \equiv \Delta V$ versus $N$ for different values of $\gamma$ taken from Fig[7].

The number of e-fold during the reheating phase is given by

$$N_{re} = \ln \left( \frac{a_{re}}{a_e} \right).$$

(47)

where $a_e$ and $a_{re}$ are respectively the scalar field at the end of inflation and the end of reheating.

One way to study the reheating phase (followed by many such as [100–104]) is to assume that the universe is dominated by an energy density which is conserved and is parametrized by an effective equation of state parameter, $\omega_{eos}$. Following this approach, the energy density at the end of inflation $\rho_e$ is related to the energy density at the end of reheating $\rho_{re}$ through the effective equation of state parameter as

$$\frac{\rho_e}{\rho_{re}} = \left( \frac{a_e}{a_{re}} \right)^{-3(1+\omega_{eos})}.$$
where $\omega_{\text{eos}}$ is assumed to be constant. The energy density at the end of inflation (where $\epsilon = 1$) is obtained as $\rho_e = (1 + \lambda) V_e$, where the parameter $\lambda = (\frac{3}{\epsilon} - 1)^{-1}$ is the ratio of kinetic energy to the potential of the inflaton. Also, the energy density at the end of reheating, which is in the form of radiation, is expressed in terms of the reheating temperature $T_{re}$ as $\rho_{re} = \left(\frac{\pi^2 g_{re}}{30}\right) T_{re}^4$. The scenario of reheating still contains many uncertainties and there are many complicated factors to be included. According to the above approach, the reheating is modeled by one effective equation of state parameter $\omega_{\text{eos}}$.

In the canonical reheating scenario, $\omega_{\text{eos}} = 0$, and a model with simple potential $V \propto \phi^2$ is also consistent with $\omega_{\text{eos}} = 0$. Models with $V \propto \phi$ and $\phi^{2/3}$ are possible to be more compatible with $-1/3 < \omega_{\text{eos}} < 0$. One the other hand, numerical studies gives the range $0 \lesssim \omega_{\text{eos}} \lesssim 0.25$. However, the main point is that the effective equation of state parameter must be $\omega_{\text{eos}} > -1/3$ to put an end on inflation (in brane cosmology, it is $\omega_{\text{eos}} > -2/3$). Here, we try to present some suggestions about the value of the parameter based on our finding in the phase of inflation.

After some manipulation, the reheating e-fold and temperature are extracted as

$$N_r = \frac{-4}{(1 - 3\omega)} \left[ \frac{1}{4} \ln \left( \frac{30}{\pi^2 g_{re}} \right) + \frac{1}{3} \ln \left( \frac{11 g_{re}}{43} \right) + \ln \left( \frac{k}{a_0 T_0} \right) + \ln \left( \frac{\rho_{\text{end}}}{H_k} \right) + N_k \right] \quad (49)$$

$$T_r = \left( \frac{43}{11 g_{re}} \right)^{1/3} \frac{a_0 T_0}{k} H_k e^{-N_k} e^{N_r} \quad (50)$$

In general there are a lower bound and an upper bound on the magnitude of reheating temperature. After inflation, the energy stored in inflaton decays to other particles, and they interact to reach to a thermal equilibrium. In order to attach to the hot big bang theory and have a successful big bang nucleosynthesis, the temperature of the thermal equilibrium should be greater than 1 MeV. On the other side, to avoid the overproduction of gravitinos (unwanted particle) the temperature should be lower than $10^9$–$10^{10}$ GeV. Consequently, it is expected that the reheating temperature stands in the range $1 \text{ MeV} < T_r < 10^9$–$10^{10}$ GeV.

The behavior of the reheating number of e-fold and temperature for all cases have been illustrated in Fig.9 to 12 as follows.
• **Power-law potential case:** The behavior of $N_r$ and $T_r$ versus the equation of the state parameter $\omega$ has been illustrated in Fig.9 for different values of $n$ and $N$ taken from parametric space Fig.1. Since, the universe is expanding, only the positive values of $N$ is acceptable. Then, the parameter $\omega$ should stand between $-1/3 < \omega < 1/3$. Also, since the number of e-fold is expected to be of the order of one, the parameter should be chosen close to $-1/3$. The temperature, on the other hand, is plotted for $-1/3 < \omega < 1/3$ (where the number of e-fold is positive), which indicates that when $\omega$ is close to $-1/3$ the temperature depends on the values of $n$ and number of e-fold. By increasing these parameters the temperature enhances as well.

The reheating temperature depends on both $n$ and $N$ as clear from Fig.9. The model could produces the reheating temperature in the mentioned range, however, for by increasing both $n$ and $N$ it increase and in some point it will be larger than $10^{10}$ GeV. For example for $n = 1.5$ and $N = 65$, the temperature is about $T_r \sim 10^{12}$ which is above the upper bound. Therefore, another constraint could be imposed on the free parameters of the model and limit the parametric space of Fig.1.

• **Natural potential case:** Fig.10 presents the behavior of $N_r$ and $T_r$ versus the parameter $\omega$ for different values of $\gamma$ and $N$ taken from parametric space Fig.3. For this case, the number of e-fold is positive only for $1/3 < \omega < 1$, and the more interested values of $N_r$ occurs when $\omega$ is close to 1.

The temperature is depicted for $1/3 < \omega < 1$ (where $N_r$ is positive) for different values of $\gamma$ and $N$. It is realized that the temperature is more sensitive to the values
FIG. 10: The behavior of $N_r$ and $T_r$ versus the equation of the state parameter $\omega$ for different values of $\gamma$ and $N$ taken from parametric space Fig. 3.

of number of e-fold than the value of $\gamma$. For instance for $N = 65$, the reheating temperature is of order of $T_r \propto 10^3$ GeV and for $N = 65$ it is about $T_r \propto 10^{10}$ GeV.

The temperature $T_r$ is more sensitive to the number of e-fold $N$ and it increases by reduction of $N$. The reheating temperature cross the upper bound for some specific value of $N$, for example for $\gamma = 0.01$ and $N = 60$ we have $T_r \sim 10^{10}$ GeV. Therefore, the acceptable range of reheating temperature applies a lower limit for the number of e-fold.

- **T-mode potential case:** For the T-mode potential, the behavior of the reheating number of e-fold and temperature is plotted in Fig. 11 for the different values of the constant $\gamma$ and the number of e-fold $N$, taken from Fig. 5. The reheating number of e-fold is negative for the range $-1/3 < \omega < 1/3$ which is not acceptable since the universe is still expanding in the reheating phase. On the other hand, it is not desirable to have a large number of e-fold during reheating. Therefore, only the values of $\omega$ that are close to 1 are more of interest.

The temperature $T_r$ is more sensitive to the value of the $N$ that the constant $\gamma$ in which by increasing $N$, the temperature decreases as well. For instance, for $N = 65$ the temperature $T_r$ is about $10^3$ GeV, but for $N = 70$ the temperature decrease to the order $10^{-3}$ GeV.

The reheating temperature in the T-mode potential is again more sensitive to the number of e-fold, however, for this case it decrease by reduction of $N$. The temperature
\[ N_r \text{ and } T_r \text{ versus the equation of the state parameter } \omega \text{ for different values of } \gamma \text{ and } N \text{ taken from parametric space Fig.6.} \]

\[ T_r \text{ crosses the lower bond for higher values of } N \text{ in which for } N = 70 \text{ it stands on the lower edge of the range of } T_r. \text{ In contrast to the previous case, the reheating temperature could impose a higher limit of the number of e-fold in which above this limit the predicted } T_r \text{ will be out of the mentioned range.} \]

- **Generalized T-mode potential case:** As the last case in our study, \( N_r \) and \( T_r \) are illustrated versus the parameter \( \omega \) in Fig.12 for different values of \( \gamma \) and \( N \) taken from Fig.7. Again, \( N_r \) is negative for \(-1/3 < \omega < 1/3\) which is not acceptable. It is positive for \(1/3 < \omega < 1\) and gets smaller as \( \omega \) approaches one.

\[ T_r \text{ decreases by enhancement of } N \text{ and } \gamma. \text{ Also, it seems that } T_r \text{ for the case is very} \]

FIG. 11: The behavior of \( N_r \) and \( T_r \) versus the equation of the state parameter \( \omega \) for different values of \( \gamma \) and \( N \) taken from parametric space Fig.6

\[ T_r \text{ decreases by enhancement of } N \text{ and } \gamma. \text{ Also, it seems that } T_r \text{ for the case is very} \]

FIG. 12: The behavior of \( N_r \) and \( T_r \) versus the equation of the state parameter \( \omega \) for different values of \( \gamma \) and \( N \) taken from parametric space Fig.7
small of the order of $10^{-8}$ and even smaller.

For the last case, the reheating temperature seems to be completely out of the range.

None of the obtained values of the constants in Fig. 7 could predict a $T_r$ in the range.

VI. TRANS-PLANCKIAN CENSORSHIP CONJECTURE

The recently proposed Trans-Planckian censorship conjecture (TCC) states that the consistent theory of quantum gravity does not allow any fluctuation with wavelength equal to or shorter than the Planck length be stretched and exit the horizon and turn to a classical fluctuation. This conjecture leads to the following condition

$$\frac{a_e}{a_i} < \frac{M_p}{H_e},$$

where $H_e$ is the Hubble parameter at the end of inflation, $a_i$ is the scale factor at the beginning of inflation, and $a_e$ is the scale factor at the end of inflation. The main goal of this section is to consider whether the above condition, known as TCC, could be satisfied by the presented model.

The first step is to evaluate the Hubble parameter $H_e$. From the Friedmann equation (4), it is realized that to evaluate the Hubble parameter is proportional to the energy density $\rho$.

Then, the above condition could be rewritten as

$$e^N < \frac{3m_p M_5^3}{4\pi \rho_e}.$$  \hspace{1cm} (52)

The energy density is a combination of the kinetic term $\dot{\phi}^2$ and the potential $V(\phi)$. To calculate $\rho_e$, both kinetic term and the potential depend are required to obtained at the end of inflation.

Let consider the first case of the model. Assume that the inflation ends after $N = 60$ e-fold. Then, from Fig. 11, it is realized that the free constant $n$ could not take any value. In fact, in Sec. IV we found a range for the parameter $n$ which brings the model to a good consistency with observational data and simultaneously satisfies the swampland criteria. Now, we want to know if this range of $n$ could also satisfy the TCC. The next parameter which is effective in the TCC is the five-dimensional Planck mass $M_5$. For the first case, this parameter was

---

5 Remember that it is was assumed that inflation occurs in high energy regime, where $\rho \gg \lambda$. Then, the quadratic term of energy density dominates over the linear term.
taken as $M_5 = 2 \times 10^{14}$. Then, if one takes $N = 60$ and $M_5 = 2 \times 10^{14}$, the condition (52) leads to the following condition

$$\rho_e < 5.04 \times 10^{35} \text{GeV}^4$$

(53)

It means that the energy density at the end of inflation should be smaller than the order of $10^{35} \text{GeV}^4$. However, the result of the first case for the energy density shows that $\rho_e$ is much larger than this value. Another important point is that this value for the energy density is in direct tension with our assumption that the inflation occurs in high energy regime. Therefore, it sounds that the TCC will not be satisfied by the model. But, there is a point which is worth mentioning. The value of the five dimensional Planck mass is very effective here. If one takes higher values for $M_5$, the situation gets worse, and the brane tension $\lambda$ gets much larger than the energy density and the high energy regime assumption will be broken again. On the other hand, if one smaller values for $M_5$ there is a chance to satisfy the TCC. For instance, for $M_5 = 10^8 \text{GeV}$, the TCC impose a condition on the energy density as $\rho_e < 10^{17} \text{GeV}^4$, where the brane tension is about $10^{12} \text{GeV}^4$. Then, by satisfying the TCC, the high energy regime assumption will not be broken. But the point is that for this value of $M_5$, the energy scale of inflation decreases, because smaller values $M_5$ leads to smaller potential at the beginning of inflation. Therefore, although the smaller $M_5$ satisfy the TCC, it results in small energy scale for inflation.

VII. CONCLUSION

We studied the inflationary scenario in the framework of brane gravity, where all standard particle live on a four-dimensional space-time embedded in five-dimensional space-time. In particular, the inflaton is confined on the brane and its energy density dominates the universe. Unlike in the standard cosmology, the Friedmann equation contains a term quadratic in the energy density which affects the dynamics in the high energy regime. After deriving the general expressions of the slow-roll parameters and the density perturbations generated during inflation, we investigated in details some well known class of inflaton potentials. Instead of comparing the result for some random values of potential parameters (which are constants), a programming code was utilized to find the best values for the parameter. In this regard, by comparing the predicted $r$ and $n_s$ of the model with the $r - n_s$ diagram of
Planck we could illustrate an allowed range of the parameters in that yield values of the spectral index and the tensor-to-scalar ratio that are in agreement with the Planck data for every point in the range. We also showed that these type of potentials satisfy the swampland criteria.

Reheating is a necessary phase which inseparable for any (cold) inflationary scenario which warms up the universe. The final temperature, reheating temperature, is required to stand in the range $1 \text{ MeV} < T_r < 10^{9-10} \text{ GeV}$ in order to recover the successful hot big bang nucleosynthesis and on the other hand to avoid the reproduction of any unwanted particle. Then, considering the reheating phase for the model could be count as a good way of constraining the parameter. Our consideration indicated that the estimated reheating temperature goes above the range for power-law and natural potentials. Therefore, the allowed range of reheating temperature could be applied to put another constraint for the model which clearly limit the allowed values of the potential parameter. For the T-mode potential there is almost a reverse situation in which the predicted reheating temperature could goes below the range. However, the range of reheating temperature could be used to put more constraint of the potential parameter. The result for the generalized T-mode potential is clear, it is absolutely out of allowed range of reheating temperature. Then it could not be counted as a good model of inflation.

The TCC was considered in the last section as another way of confining the model. The conjecture states that the mode with wavelength shorter than the Planck scale never cross the Hubble horizon. It was explained that to satisfy the condition the five-dimensional Planck mass (or the brane tension) should accept an upper bound. On the other hand, lower values of $M_5$ leads to lower energy scale for inflation. The conjecture in the brane inflation leads to strong bound on the potential which might not be preserved.

[1] P. A. R. Ade et al. (Planck), Astron. Astrophys. **571**, A22 (2014), arXiv:1303.5082.
[2] P. A. R. Ade et al. (Planck), Astron. Astrophys. **594**, A20 (2016), arXiv:1502.02114.
[3] Y. Akrami et al. (Planck) (2018), arXiv:1807.06211.
[4] A. A. Starobinsky, Physics Letters B **91**, 99 (1980).
[5] A. H. Guth, Phys. Rev. **D23**, 347 (1981), [Adv. Ser. Astrophys. Cosmol.3,139(1987)].
[6] A. Albrecht and P. J. Steinhardt, Physical Review Letters **48**, 1220 (1982).

[7] A. D. Linde, Physics Letters B **108**, 389 (1982).

[8] A. D. Linde, Physics Letters B **129**, 177 (1983).

[9] G. Barenboim and W. H. Kinney, JCAP **0703**, 014 (2007), astro-ph/0701343.

[10] P. Franche, R. Gwyn, B. Underwood, and A. Wissanji, Phys. Rev. **D82**, 063528 (2010), arXiv:1002.2639.

[11] S. Unnikrishnan, V. Sahni, and A. Toporensky, JCAP **1208**, 018 (2012), arXiv:1205.0786.

[12] R. Gwyn, M. Rummel, and A. Westphal, JCAP **1312**, 010 (2013), arXiv:1212.4135.

[13] K. Rezazadeh, K. Karami, and P. Karimi, JCAP **1509**, 053 (2015), arXiv:1411.7302.

[14] S. Céspedes and A.-C. Davis, JCAP **1511**, 014 (2015), arXiv:1506.01244.

[15] N. K. Stein and W. H. Kinney, JCAP **1704**, 006 (2017), arXiv:1609.08959.

[16] T. Pinhero and S. Pal (2017), arXiv:1703.07165.

[17] M. Fairbairn and M. H. G. Tytgat, Phys. Lett. **B546**, 1 (2002), hep-th/0204070.

[18] S. Mukohyama, Phys. Rev. **D66**, 024009 (2002), hep-th/0204084.

[19] A. Feinstein, Phys. Rev. **D66**, 063511 (2002), hep-th/0204140.

[20] T. Padmanabhan, Phys. Rev. **D66**, 021301 (2002), hep-th/0204150.

[21] M. Spalinski, JCAP **0705**, 017 (2007), hep-th/0702196.

[22] D. Bessada, W. H. Kinney, and K. Tzirakis, JCAP **0909**, 031 (2009), arXiv:0907.1311.

[23] J. M. Weller, C. van de Bruck, and D. F. Mota, JCAP **1206**, 002 (2012), arXiv:1111.0237.

[24] N. Nazavari, A. Mohammadi, Z. Ossoulian, and K. Saaidi, Phys. Rev. **D93**, 123504 (2016), arXiv:1708.03676.

[25] R. Amani, K. Rezazadeh, A. Abdolmaleki, and K. Karami, Astrophys. J. **853**, 188 (2018), arXiv:1802.06075.

[26] T. Golanbari, A. Mohammadi, and K. Saaidi, Phys. Dark Univ. **27**, 100456 (2020), arXiv:1808.07246.

[27] K.-i. Maeda and K. Yamamoto, Journal of Cosmology and Astroparticle Physics **2013**, 018 (2013).

[28] A. A. Abolhasani, R. Emami, and H. Firouzjahi, Journal of Cosmology and Astroparticle Physics **2014**, 016 (2014).

[29] S. Alexander, D. Jyoti, A. Kosowsky, and A. Marcianò, Journal of Cosmology and Astroparticle Physics **2015**, 005 (2015).
[30] M. Tirandari and K. Saaidi, Nuclear Physics B 925, 403 (2017).
[31] A. Berera, Physical Review Letters 75, 3218 (1995).
[32] A. Berera, Nuclear Physics B 585, 666 (2000).
[33] A. Taylor and A. Berera, Physical Review D 62, 083517 (2000).
[34] L. M. Hall, I. G. Moss, and A. Berera, Physical Review D 69, 083525 (2004).
[35] M. Bastero-Gil and A. Berera, Phys. Rev. D71, 063515 (2005), hep-ph/0411144.
[36] M. Bastero-Gil, A. Berera, R. O. Ramos, and J. G. Rosa, Phys. Rev. Lett. 117, 151301 (2016), 1604.08838.
[37] J. G. Rosa and L. B. Ventura, Phys. Rev. Lett. 122, 161301 (2019), 1811.05493.
[38] M. Bastero-Gil, A. Berera, R. O. Ramos, and J. G. Rosa (2019), 1907.13410.
[39] K. Sayar, A. Mohammadi, L. Akhtari, and K. Saaidi, Phys. Rev. D95, 023501 (2017), arXiv:1708.01714.
[40] L. Akhtari, A. Mohammadi, K. Sayar, and K. Saaidi, Astropart. Phys. 90, 28 (2017), arXiv:1710.05793.
[41] H. Sheikhahmadi, A. Mohammadi, A. Aghamohammadi, T. Harko, R. Herrera, C. Corda, A. Abebe, and K. Saaidi, Eur. Phys. J. C79, 1038 (2019), arXiv:1907.10966.
[42] A. Riotto, ICTP Lect. Notes Ser. 14, 317 (2003), hep-ph/0210162.
[43] D. Baumann, in Physics of the large and the small, TASI 09, proceedings of the Theoretical Advanced Study Institute in Elementary Particle Physics, Boulder, Colorado, USA, 1-26 June 2009 (2011), pp. 523–686, arXiv:0907.5424.
[44] S. Weinberg, Cosmology (2008), ISBN 9780198526827, URL http://www.oup.com/uk/catalogue/?ci=9780198526827
[45] D. H. Lyth and A. R. Liddle, The primordial density perturbation: Cosmology, inflation and the origin of structure (2009), URL http://www.cambridge.org/uk/catalogue/catalogue.asp?isbn=9780521828499.
[46] L. F. Abbott, E. Farhi, and M. B. Wise, Phys. Lett. B 117, 29 (1982).
[47] A. Albrecht, P. J. Steinhardt, M. S. Turner, and F. Wilczek, Phys. Rev. Lett. 48, 1437 (1982).
[48] A. D. Dolgov and A. D. Linde, Phys. Lett. B 116, 329 (1982).
[49] A. D. Dolgov and D. P. Kirilova, Sov. J. Nucl. Phys. 51, 172 (1990).
[50] J. H. Traschen and R. H. Brandenberger, Phys. Rev. D 42, 2491 (1990).
[51] Y. Shtanov, J. H. Traschen, and R. H. Brandenberger, Phys. Rev. D 51, 5438 (1995), hep-ph/9407247.
[52] L. Kofman, A. D. Linde, and A. A. Starobinsky, Phys. Rev. Lett. 73, 3195 (1994), hep-th/9405187.
[53] L. Kofman, A. D. Linde, and A. A. Starobinsky, Phys. Rev. D 56, 3258 (1997), hep-ph/9704452.
[54] B. A. Bassett, S. Tsujikawa, and D. Wands, Rev. Mod. Phys. 78, 537 (2006), astro-ph/0507632.
[55] R. Allahverdi, R. Brandenberger, F.-Y. Cyr-Racine, and A. Mazumdar, Ann. Rev. Nucl. Part. Sci. 60, 27 (2010), 1001.2600.
[56] M. A. Amin, M. P. Hertzberg, D. I. Kaiser, and J. Karouby, Int. J. Mod. Phys. D 24, 1530003 (2014), 1410.3808.
[57] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999), hep-ph/9905221.
[58] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 4690 (1999), hep-th/9906064.
[59] R. Maartens, D. Wands, B. A. Bassett, and I. P. Heard, Physical Review D 62, 041301 (2000).
[60] T. Golanbari, A. Mohammadi, and K. Saaidi, Physical Review D 89, 103529 (2014).
[61] Mohammadi, Abolhassan and Golanbari, Tayeb and Nasri, Salah and Saaidi, Khaled, Phys. Rev. D 101, 123537 (2020), 2004.12137.
[62] N. Banerjee and T. Paul, Eur. Phys. J. C 77, 672 (2017), arXiv:1706.05964.
[63] E. Elizalde, S. D. Odintsov, T. Paul, and D. Sáez-Chillón Gómez, Phys. Rev. D 99, 063506 (2019), arXiv:1811.02960.
[64] T. Paul and S. SenGupta, Eur. Phys. J. C 79, 591 (2019), arXiv:1808.00172.
[65] G. Obied, H. Ooguri, L. Spodyneiko, and C. Vafa (2018), arXiv:1806.08362.
[66] S. K. Garg and C. Krishnan, JHEP 11, 075 (2019), arXiv:1807.05193.
[67] H. Ooguri, E. Palti, G. Shiu, and C. Vafa, Phys. Lett. B788, 180 (2019), arXiv:1810.05506.
[68] A. Kehagias and A. Riotto, Fortsch. Phys. 66, 1800052 (2018), arXiv:1807.05445.
[69] S. Das, Phys. Rev. D99, 063514 (2019), arXiv:1810.05038.
[70] W. H. Kinney, Phys. Rev. Lett. 122, 081302 (2019), arXiv:1811.11698.
[71] H. Matsui and F. Takahashi, Phys. Rev. D99, 023533 (2019), arXiv:1807.11938.
[72] C.-M. Lin, Phys. Rev. D99, 023519 (2019), arXiv:1810.11992.
[73] K. Dimopoulos, Phys. Rev. D98, 123516 (2018), arXiv:1810.03438.
[74] W. H. Kinney, S. Vagnozzi, and L. Visinelli, Class. Quant. Grav. 36, 117001 (2019), arXiv:1808.06424.
[75] H. Geng (2019), arXiv:1910.14047.
[76] S. Brahma and M. Wali Hossain, JHEP 03, 006 (2019), arXiv:1809.01277.
[77] S. Brahma and S. Shandera, JHEP 11, 016 (2019), arXiv:1904.10979.
[78] Z. Wang, R. Brandenberger, and L. Heisenberg (2019), arXiv:1907.08943.
[79] S. Odintsov and V. Oikonomou, Phys. Lett. B 805, 135437 (2020), arXiv:2004.00479.
[80] S. Odintsov and V. Oikonomou, EPL 126, 20002 (2019), arXiv:1810.03575.
[81] A. Mohammadi, T. Golanbari, H. Sheikhhahmadi, K. Sayar, L. Akhtari, M. Rasheed, and K. Saaidi (2020), arXiv:2001.10042.
[82] S. Brahma, R. Brandenberger, and D.-H. Yeom (2020), arXiv:2002.02941.
[83] A. Bedroya and C. Vafa, JHEP 09, 123 (2020), 1909.11063.
[84] A. Bedroya, R. Brandenberger, M. Loverde, and C. Vafa, Phys. Rev. D 101, 103502 (2020), 1909.11106.
[85] A. Mohammadi, T. Golanbari, and J. Enayati (2020), 2012.01512.
[86] J. M. Cline, C. Grojean, and G. Servant, Phys. Rev. Lett. 83, 4245 (1999), hep-ph/9906523.
[87] C. Germani and R. Maartens, Phys. Rev. D64, 124010 (2001), hep-th/0107011.
[88] D. Wands, K. A. Malik, D. H. Lyth, and A. R. Liddle, Phys. Rev. D62, 043527 (2000), astro-ph/0003278.
[89] P. Brax, C. van de Bruck, and A.-C. Davis, Rept. Prog. Phys. 67, 2183 (2004), hep-th/0404011.
[90] D. Langlois, R. Maartens, and D. Wands, Phys. Lett. B489, 259 (2000), hep-th/0006007.
[91] G. Huey and J. E. Lidsey, Phys. Lett. B514, 217 (2001), astro-ph/0104006.
[92] C.-M. Lin, K.-W. Ng, and K. Cheung, Phys. Rev. D 100, 023545 (2019), 1810.01644.
[93] F. C. Adams, J. Bond, K. Freese, J. A. Frieman, and A. V. Olinto, Phys. Rev. D 47, 426 (1993), hep-ph/9207245.
[94] R. Kallosh, A. Linde, and D. Roest, JHEP 11, 198 (2013), 1311.0472.
[95] R. Kallosh, A. Linde, and D. Roest, Phys. Rev. Lett. 112, 011303 (2014), 1310.3950.
[96] S. Ferrara, R. Kallosh, A. Linde, and M. Porrati, Phys. Rev. D 88, 085038 (2013), 1307.7696.
[97] S. Ferrara, R. Kallosh, A. Linde, and M. Porrati, JCAP 11, 046 (2013), 1309.1085.
[98] K. Dimopoulos and C. Owen, JCAP **06**, 027 (2017), 1703.00305.

[99] Y. Ueno and K. Yamamoto, Phys. Rev. D **93**, 083524 (2016), 1602.07427.

[100] A. R. Liddle and S. M. Leach, Phys. Rev. D **68**, 103503 (2003), astro-ph/0305263.

[101] L. Dai, M. Kamionkowski, and J. Wang, Phys. Rev. Lett. **113**, 041302 (2014), 1404.6704.

[102] J. L. Cook, E. Dimastrogiovanni, D. A. Easson, and L. M. Krauss, JCAP **04**, 047 (2015), 1502.04673.

[103] S. Bhattacharjee, D. Maity, and R. Mukherjee, Phys. Rev. D **95**, 023514 (2017), 1606.00698.

[104] M. Drewes, J. U. Kang, and U. R. Mun, JHEP **11**, 072 (2017), 1708.01197.

[105] L. McAllister, E. Silverstein, and A. Westphal, Phys. Rev. D **82**, 046003 (2010), 0808.0706.

[106] E. Silverstein and A. Westphal, Phys. Rev. D **78**, 106003 (2008), 0803.3085.

[107] D. I. Podolsky, G. N. Felder, L. Kofman, and M. Peloso, Phys. Rev. D **73**, 023501 (2006), hep-ph/0507096.

[108] J. Martin and C. Ringeval, Phys. Rev. D **82**, 023511 (2010), 1004.5525.

[109] A. Mohammadi, T. Golanbari, J. Enayati, S. Jalalzadeh, and K. Saaidi (2020), 2011.13957.

[110] K. D. Lozanov (2019), 1907.04402.

[111] R. Allahverdi, Phys. Rev. D **62**, 063509 (2000), hep-ph/0004035.