Abstract

We estimate the energy losses in the cases of $J/\Psi$ and $l^+l^-$ pair production on nuclear targets in terms of effective change of the initial beam energy. Our phenomenological results are in reasonable agreement with Theoretical calculations.
1 Introduction

There exist significant nuclear effects in the cases of $J/\Psi$, and even of Drell-Yan pair production. These effects are discussed in many papers (see below). We present the possibility to estimate them directly from the experimental data.

Let us consider the variable

$$x_0 = \frac{p}{p_0},$$

(1)

where $p$ is the momentum of produced secondary and $p_0$ is the initial momentum of the beam particle both in c.m. frame. At high energies in the case of nucleon target the maximum value of $x_0$ is close to unity ($(x_0)_{\text{max}} \to 1$). In the case of nuclear target the situation is more complicated because there are many different contributions. First of all there exists the so called cumulative production (with $x_0 > 1$) [1, 2, 3]. However it is a special process which we are not going to consider in this paper.

Many other processes result in $(x_0)_{\text{max}} < 1$ due to the initial- and/or final-state interactions [4]. For example in the case of Drell-Yan production there exist initial-state effect such as energy loss of the incident quark in nuclear target as well as nuclear shadowing [5, 6]. In the case of $J/\Psi$ production there is an additional source of suppression connected with final-state interactions [7].

Moreover, in all processes on nuclear targets (except of coherent ones) some fraction of energy is used for nuclear disintegration. The nuclear target is destroyed and several nucleons, as well as light nuclear fragments, (say, $\alpha$-particles) appear in the final state. It can be considered as a phase space limitation. This fraction is numerically not so small (it is many times more than the nuclear binding energy) [8, 9]. This source of energy loss also leads to decrease of $(x_0)_{\text{max}}$ but it was not taken into account in all modern papers.

In the present paper we will not consider every source of suppression separately, but we are going to consider them all together including the last one.

It is clear that all initial-state energy losses are equivalent to decrease of the incident beam momentum $p_0$. As the phase space effects as well as final state interactions decrease $(x_0)_{\text{max}}$, we assume that their influence in the considered processes can be effectively described by the same way, as the additional decrease of $p_0$. So we change $p_0$ by $p_0 - p_A$, where $p_A$ is a phenomenological parameter which accounts for all energy losses.

In what follows we will consider the $A$-dependences of $J/\Psi$ and Drell-Yan pair production in terms of $x_F$:

$$x_F = \frac{p}{p_0 - p_A},$$

(2)

and we assume that it is possible to find the shift $p_A$ from the condition that the ratio of multiplicities on nucleon and nuclear targets

$$R_{hA/hp}(x_F) = \text{const}(x_F) \simeq 1,$$

(3)
whereas the same multiplicity ratio in terms of \( x_0 \) has evident \( A \)-dependence

\[
R_{hA/hp}(x_0) = f(x_0). \tag{4}
\]

Evidently, such rescale is reasonable only for not very small \( x_0 \) values.

We will determine the shift \( p_A \) from the experimental data and we will compare them with several independent estimations. Such approach allows us, in particular, to investigate the energy dependence of all nuclear effects. In conclusion we will compare our results with theoretical calculations [6, 7].

2 A-dependence of \( J/\Psi \) production at large \( x_F \)

Charmonium production off nuclei has drawn much attention during the last two decades, since the NA3 experiment at CERN [10] has found a steep increase of nuclear suppression with rising \( x_0 \). This effect was confirmed later in the same energy range [11] as well as at high energies [12, 13, 14].

For the purpose of our analysis we used the experimental data on \( J/\Psi \) production in proton-nucleus (\( pA \)) collisions published in [12, 14, 15]. We also used data on \( J/\Psi \) production in \( \pi^-A \) collisions [11, 16]. We analysed the ratio \( R_{A_1/A_2} \) of inclusive differential cross sections for \( J/\Psi \) hadroproduction on \( A_1 \) nucleus to that on \( A_2 \):

\[
R_{A_1/A_2} = \frac{\frac{1}{A_1} \left( \frac{d\sigma}{dx_0} \right)_{hA_1 \rightarrow J/\Psi}}{\frac{1}{A_2} \left( \frac{d\sigma}{dx_0} \right)_{hA_2 \rightarrow J/\Psi}}. \tag{5}
\]

The analysis was performed in the following way. Suppose one has two \( x_0 \)-spectra for \( J/\Psi \) production on nucleon and nuclear targets or on two different nuclei (light and heavy ones). Then one can shift the spectrum that corresponds to the heavy nucleus according to Eq. (2) by changing \( p_A \) parameter. Assuming that nuclear effects are small for very light nuclei, it is possible then to calculate the fraction of beam energy/momentum spent on nuclear effects. This may be done by calculating the ratio of the shifted spectrum to the spectrum that corresponds to the light nucleus. When this ratio is close to unity then the corresponding shift will give the absolute value of energy/momentum loss caused by the mentioned nuclear effects.

Let us analyse the spectra [14] presented in Fig. 1. The spectra represent the differential cross sections for \( J/\Psi \)'s produced inclusively in 800 GeV/c \( pCu \) and \( pBe \) collisions measured by E789 Collaboration. One can see the evident nuclear suppression for copper target.

Now to calculate how much energy/momentum of the beam particle is spent on nuclear effects we should shift the spectrum for the heavy nucleus (namely copper). The shifted spectrum is presented on Fig. 2. The solid curve in Fig. 2 represents the fit to the shifted spectrum needed for further calculations of the ratio of the presented spectra.
The calculated ratios Eq. (5) one can see in Fig. 3. Solid circles correspond to the original ratio measured by E789 Collaboration, two other ratios were calculated for different shifts: $p_A = 0.5 \text{ GeV}/c$ (solid squares) and $p_A = 0.7 \text{ GeV}/c$ (open squares). These $p_A$ values represent the amount of absolute momentum spent on nuclear effects.

The values of $p_A$ in c.m. frame correspond to the shift

$$\Delta x_{F}^{c.m.} = \frac{p_A}{p_0}.$$  \hspace{1cm} (6)

For large $x_F \Delta x_{F}^{lab.} \approx \Delta x_{F}^{c.m.}$, so we can calculate the absolute value of energy losses in lab. frame. In our case the shift $p_A = 0.5 \text{ GeV}/c$ corresponds in the lab. frame to the energy losses $\Delta p_{\text{Cu/Be}}^{lab.} \approx 20 \text{ GeV}/c$.

Unfortunately E789 is the only experiment that measured absolute cross sections for $J/\Psi$ production in $pA$ collisions for both light and heavy nuclei. Experimentally it is much easier to measure ratios of the cross sections at once, thus most collaborations present only the ratios (without absolute values of the cross sections) as their results. Consequently it is difficult to analyse those experiments in such a way we did above.

However the most recent and most precise experiment on $J/\Psi$ production by E866 Collaboration [12] was performed at the same energy (800 GeV) as E789 and used Be and W targets. Thus using the E789 $x_0$-spectrum for beryllium target and E866 ratio of spectrum on beryllium to that on tungsten one can extract the absolute $J/\Psi$ production cross section for tungsten. Since the $x_0$-scale covered by E866 experiment ($-0.1 < x_0 < 0.93$) is larger than one covered by E789 ($0.3 < x_0 < 0.95$), then to extract E866 spectrum for W target we combined the E789 data with the data obtained by E672 and E706 Collaborations for Be target in the range of $0 < x_0 < 0.5$ [15]. Fig. 4 represents the ratio measured by E866 (Fig. 4a) and the absolute spectra for Be target (solid squares) combined from the mentioned data sets, and for W target (open squares) extracted from the ratio (Fig. 4b). Now it is possible to analyse the last two spectra in the same way as was done in the case of E789 data (see Fig. 2). We omit intermediate calculations and present the final result. The shifted ratios at two different shift values ($p_A = 1.2 \text{ GeV}/c$ — solid squares, and $p_A = 1.5 \text{ GeV}/c$ — open squares) are presented in Fig. 5. This corresponds to the absolute energy losses in the lab. frame $\Delta p_{W/Be}^{lab.} \approx 50 \text{ GeV}/c$.

One can see that the absolute value of energy/momentum loss in tungsten is more than two times larger than one in copper target, which is rather clear. Indeed, W target is $\approx 2.9$ times heavier than Cu. Consequently, nuclear effects in the former should be stronger, thus the value of energy/momentum lost by the projectile in W target should be larger than that in Cu.

Besides proton induced reactions we considered the data on $J/\Psi$ production in $\pi^-A$ collisions. The data available are the ratio of differential cross sections for $J/\Psi$ production on W to that on Be target at 125 GeV/c [11] (Fig. 6a), and absolute differential cross section for $J/\Psi$ production on Be target at 150 GeV/c [16]. Since the energies for the two data sets slightly differ from each other, we build the corresponding invariant cross
section from that in Ref. [10], which is presented in Fig. 6b. To extract the spectrum for a heavy nucleus (namely, tungsten) we applied the procedure described above. Omitting intermediate results we present the range of $p_A$ values obtained from the analysis (Fig. 7):

$$p_A = 1.5 - 1.7 \text{ GeV}/c,$$

which correspond to $\Delta p_{\text{lab}}^{\text{W/Be}} \approx 12 \text{ GeV}/c$. The obtained $\Delta p_{\text{lab}}^{\text{W/Be}}$ value for $\pi^- A$ collisions at 125 GeV/$c$ is about 4 times smaller than that obtained for $pA$ collisions at 800 GeV/$c$. Some part of this difference (say, factor $\sim 1.5$) can be connected with smaller pion-nucleon $(\pi N)$ cross section in comparison with $NN$ cross section. Another part of the difference can be connected with the dependence of nuclear effects on the initial energy.

Unfortunately the errors of the data combined with the analysis errors result in too large error of the final result, which does not allow us do draw a definite conclusion.

3 A-dependence of Drell-Yan production at large $x_F$

Since the Drell-Yan mechanism produces lepton pairs which only interact electromagnetically, the $A$-dependence is expected to be weak because no final-state interactions affect the lepton pair. However some initial-state interactions may affect the $A$ dependence.

Almost all the experimental results on Drell-Yan production are presented in terms of the ratio of inclusive differential cross sections on a heavy nucleus to that on a light one. And there is no opportunity to extract desired spectra separately as was done in the previous section.

However we developed a Monte-Carlo (MC) event generator (HARDPING — Hard Probe Interaction Generator) that extends well known HIJING MC [17] on Drell-Yan pair production process and some initial-state effects are accounted for [18, 19]. HARDPING MC describes well the data on Drell-Yan pair production in hadron-nucleus ($hA$) collisions at high-energies [18, 19].

Using HARDPING MC we simulated absolute spectra for Drell-Yan pair production in $pA$ collisions on W and Be targets at 800 GeV/$c$. Then we applied the procedure described above for the two simulated spectra. The results of the analysis are presented in Fig. 8. The figure represents the original E866 data [5] (solid circles), simulated ratio without any shift (open circles) to demonstrate consistency between the simulated results and the experimental data, and shifted ratios obtained with HARDPING MC. As was predicted the fraction of energy/momentum lost by the projectile on nuclear effects is small for the case of Drell-Yan pair production. This is because there is no final-state interactions in Drell-Yan production process.

The obtained value for $p_A \approx 0.3 \text{ GeV}/c$ corresponds to $\Delta p_{\text{lab}}^{\text{W/Be}} \approx 12 \text{ GeV}/c$. 
4 Conclusion

In summary, we considered the energy/momentum losses of the projectile in $hA$ collisions at high-energies from the available experimental data. What we were interested in is how much energy/momentum of the projectile is spent on all the nuclear effects including the effect of nuclear disintegration.

The energy losses estimated from the experimental data are in reasonable agreement with theoretical calculations [6, 7]. Namely, our result $\Delta p_{W/Be}^{lab} \approx 50$ GeV/c for $J/\Psi$ production at 800 GeV/c correspond to energy loss rate

$$dE/dz \approx 5 \text{ GeV/fm}, \quad (8)$$

(We assume the length of full trajectory $\sim 1.5R_A$ and we neglect nuclear effects in Be target). The analysis [7] predicts $dE/dz \approx 3$ GeV/fm for initial-state quark energy loss rate, i.e. energy loss before the hard interaction point. However initial-state quark energy losses is not the dominant effect in $J/\Psi$ production processes on nuclear targets. The main contribution to the suppression of $J/\Psi$’s arises from final-state interactions. Also there exists strong gluon shadowing at large $x_0$ as well as gluon enhancement at small $x_0$. Nevertheless, Ref. [7] does not present how much projectile energy is spent on each nuclear effect. (The effects of gluon shadowing for $J/\Psi$ production were calculated in Ref. [20].) Thus we assume that main part of our result Eq. (8) is explained by the mentioned effects.

The estimations of energy losses in pairs Be-W and Be-Cu are larger than the ratio of $A^{1/3}$ values (i.e. length of trajectory) for W and Cu nuclei. It can be connected with rather large error bars, or with the $A$-dependence of energy losses more strong than $A^{1/3}$ behavior. The last reason is not excluded because the multiplicity of secondary protons produced in hadron–nucleus collisions in the nuclear targets fragmentation region with energies $\leq 1$ GeV has $A$-dependence more strong than $A^{1/3}$ [21, 22]. On the other hand, the energies of these protons are determined by energy losses of the incident particle.

In the case of Drell-Yan pair production calculations [6] predict quark energy loss rate $dE/dz \approx 3$ GeV/fm. There also exists shadowing effect in nuclear target, however there is no numerical estimate for this effect in Ref. [6], it was considered in Ref. [23]. Our result $\Delta p_{W/Be}^{lab} \approx 12$ GeV/c obtained for Drell-Yan production corresponds to energy loss rate $dE/dz \approx 1.2$ GeV/fm for full trajectory. However one should take into account the path of the projectile before the hard interaction point only, which is $\approx 4.4$ fm for W target[6]. Thus $dE/dz \approx 2.7$ GeV/fm. Assuming that shadowing effect is small for Drell-Yan production at 800 GeV/c we conclude that our results are in reasonable agreement with the [6] calculations.

It is necessary to note that some part of QCD energy losses can be used for nuclear destruction and fragmentation due to the final-state interactions. This can explain rather large energy of secondary target nucleons and nuclear fragments observed in [8, 9].

From the difference in the values of $\Delta p_{W/Be}^{lab}$ for $pA$ and $\pi A$ collisions we can conclude that nuclear effects probably depend on projectile energy.
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References

[1] A. M. Baldin et al., Yad. Fiz. 18, 79 (1973).
[2] Yu. D. Bayukov et al., Yad. Fiz. 18, 1246 (1973).
[3] L. L. Frankfurt and M. L. Strikman, Phys. Rept. 76, 215 (1981).
[4] R. Vogt, Phys. Rept. 310, 197 (1999).
[5] M. A. Vasiliev et al. (E866 Collab.), Phys. Rev. Lett. 83, 2304 (1999).
[6] M. B. Johnson et al., Phys. Rev. C65, 025203 (2002).
[7] B. Kopeliovich et al., Nucl. Phys. A696, 669 (2001).
[8] S. A. Azimov et al., Yad. Fiz. 8, 933 (1968).
[9] S. A. Azimov et al., Z. Phys. A300, 47 (1981).
[10] J. Badier et al. (NA3 Collab.), Z. Phys. C20, 101 (1983).
[11] S. Katsanevas et al. (E537 Collab.), Phys. Rev. Lett. 60, 2121 (1988).
[12] M. Leitch et al. (E866 Collab.), Phys. Rev. Lett. 84, 3256 (2000).
[13] D. M. Alde et al. (E772 Collab.), Phys. Rev. Lett. 66, 133 (1991).
[14] M. S. Kowitt et al. (E605/E789 Collab.), Phys. Rev. Lett. 72, 1318 (1994).
[15] A. Gribushin et al. (E672/E706 Collab.), Phys. Rev. D62, 012001 (2000).
[16] M. A. Abolins et al., Phys. Lett. B82, 145 (1979).
[17] M. Gyulassy and X.-N. Wang, Comput. Phys. Commun. 83, 307 (1994).
[18] Ya. A. Berdnikov et al., Yad. Fiz. (in press).
[19] Ya. A. Berdnikov et al., Eur. Phys. J. (in press).
[20] C. Pajares, C. A. Salgado and Yu. M. Shabelski, Mod. Phys. Lett. A13, 453 (1998).
[21] K. G. Gulamov, U. G. Gulyamov and G. M. Chernov, Fiz. Elem. Chast. Atom. Yadra 9, 554 (1978).
[22] S. Fredriksson et al., Phys. Rep. 144, 187 (1987).
[23] N. Armesto et al., Yad. Fiz. 61, 125 (1998).
Figure 1: Differential cross section for $J/\Psi$ production in $p$Cu and $p$Be collisions at 800 GeV/c [14].
Figure 2: The same as in Fig. 1 but the spectrum for Cu was shifted according to Eq. (2) with $p_A = 0.5$ GeV/c.
Figure 3: The ratios of inclusive differential cross sections, calculated for different $p_A$ values.
Figure 4: (a) The ratio of inclusive differential cross section for $J/\Psi$ production on W target to that on Be measured by E866 [13]. (b) Combined spectrum (see text) for Be target (solid squares), and spectrum for W (open squares) target extracted from the ratio shown in Fig. 4a.
Figure 5: The ratios of inclusive differential cross sections, calculated for different $p_A$ values.
Figure 6: $x_0$-spectra for $J/\Psi$’s produced in $\pi^-A$ collisions. (a) The ratios of inclusive differential cross sections for $J/\Psi$ production on tungsten to that on beryllium [11]. (b) The invariant differential cross section for $J/\Psi$ production on Be target.
Figure 7: The ratios of inclusive differential cross sections for $J/\Psi$ production in $\pi^-A$ collisions at 125 GeV/$c$, calculated for different $p_A$ values.
Figure 8: The ratios of inclusive differential cross sections for Drell-Yan pair production in \( pA \) collisions at 800 GeV/c, calculated for different \( p_A \) values.