Isospin Effects on Strangeness in Heavy-Ion Collisions

V. Prassa\textsuperscript{a} T. Gaitanos\textsuperscript{b} G. Ferini\textsuperscript{c} M. Di Toro\textsuperscript{c} G.A. Lalazissis\textsuperscript{a} H.H. Wolter\textsuperscript{d}

\textsuperscript{a}Department of Theoretical Physics, Aristotle University of Thessaloniki, Thessaloniki Gr-54124, Greece
\textsuperscript{b}Institut für Theoretische Physik, Justus-Liebig-Universität Giessen, D-35392 Giessen, Germany
\textsuperscript{c}Laboratori Nazionali del Sud INFN, I-95123 Catania, Italy
\textsuperscript{d}Sektion Physik, Universität München, D-85748 Garching, Germany

email: Theodoros.Gaitanos@theo.physik.uni-giessen.de

Kaon properties are studied within the framework of a fully covariant transport approach. The kaon-nucleon potential is evaluated in two schemes, a chiral perturbative approach and an effective One-Boson-Exchange model. Isospin effects are explicitly accounted for in both models. The transport calculations indicate a significant sensitivity of momentum distributions and total yields of $K^{0,+}$ isospin states on the choice of the kaon-nucleon interaction. Furthermore, isospin effects are rather moderate on absolute kaon yields, but appear on strangeness ratios. This is an important issue in determining the high density symmetry energy from studies of strangeness production in heavy-ion collisions.

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1 Introduction

The knowledge of the in-medium hadronic properties at supra-normal densities is of major importance for the understanding of (nuclear) astrophysics such as the physical mechanism of supernovae explosions and the physics of neutron stars [1]. Strangeness production at intermediate energy heavy-ion collisions has been a very helpful tool in studying the hadronic equation of state (EoS)
and the properties of strangeness matter under extreme conditions of baryon density and temperature [2,3].

It is well established that \((K^{0,+})\) kaons feel a weak repulsive potential of the order of \(20 - 30\) MeV at saturation density \(\rho_{\text{sat}} = 0.16\ f m^{-3}\), as it has been verified by studying strangeness flows in heavy-ion collisions [3,4]. The determination of the high density behavior of the nuclear EoS has been successfully investigated in this context [5–7]. It has been realized that strangeness production could also serve as an important messenger for the isovector part of the nuclear EoS at high densities, i.e. for the symmetry energy [8]. A reliable determination of the symmetry energy at supra-normal densities from heavy-ion collisions is still an object of current debates and investigations [9–11].

However, a reliable study of the high density nuclear EoS and, in particular, its isovector part involves a detailed analysis of the role of the isospin dependent part of the kaon-nucleon interaction on strangeness production in intermediate energy nucleus-nucleus collisions, which has not been studied so far. Theoretical studies on static hadronic matter based on the Chiral Perturbation Theory (ChPT) [12] and on the One-Boson-Exchange approach (OBE) [13] predict a splitting of self energies between positive charged \((K^+)\) and neutral \((K^0)\) kaons. This isospin effect increases with baryon density and isospin asymmetry, and it may thus influence the production and propagation of particles with strangeness in dense hadronic matter during a nucleus-nucleus collision. It may thus be a sensitive observable to determine the symmetry energy at high densities.

This work is an extension of former theoretical investigations of kaon medium effects in heavy-ion collisions [2] to asymmetric hadronic matter. The main topic of our study is therefore the influence of the isovector sector of the kaon-nucleon interaction on kaon dynamics at incident energies near the strangeness production threshold. In extension to our previous work [14], two different models for the kaon-nucleon interaction and their particular isovector properties are now discussed in detail: one based on the Chiral Perturbation Theory (ChPT) [12] and a second one based on the One-Boson-Exchange (OBE) approach for the effective meson-nucleon field theory [13]. We use a fully relativistic transport model, in which isospin effects are included in the mean-field and in the collision integral [10], and also in the different realizations of the kaon potential. These models are applied in heavy-ion collisions at intermediate energy below the threshold for strangeness production. The production of the \((K^+, K^0)\) isospin states and, in particular, their ratio is extensively investigated within the different scenarios for the isovector sector of the nuclear mean-field and the kaon-nucleon potential. The antikaon field \(\bar{K} = (K^-, K^0)\) is not considered here. A discussion of the stronger and more complex \(\bar{K}\)-nucleon interaction would go beyond the scope of this work, e.g., we refer to Refs. [15].
2 Theoretical description

In this section we describe the theoretical model used in this work. After a brief introduction of the transport equation we focus on the different treatments of the kaon-nucleon potential in isospin-asymmetric hadronic matter, which is the major focus of the present work.

2.1 The transport equation

The theoretical description of a heavy ion collision is based on the relativistic Boltzmann-Uehling-Uhlenbeck (RBUU) dynamical approach, as introduced in Refs. [16–18] and then extended for isospin and strangeness degrees of freedom [10]:

\[
\left[ k^{*\mu} \partial^{\mu}_\mu + (k^{*\mu}F^{\mu\nu} + m^* \partial^{\mu}_\mu m^*) \partial^{k^*}_\mu \right] f_i(x, k^*) = I_{\text{coll}}.
\]

The RBUU transport equation describes the phase space evolution of the one-particle density distribution functions \( f_i(x, k^*) \) for the different particle species under consideration under the influence of a mean-field (lhs of eq.(1)) and binary collisions (rhs of eq.(1)) which couple the different fields. Here we consider the propagation of all baryons up to the \( \Delta \)-resonance and hyperons (\( \Lambda, \Sigma \)). The production of pions results through the resonance decay and kaons (\( K^{0, +} \)) are produced in baryon-baryon and baryon-pion binary collisions (see for details Ref. [14]). The mean-field or the nuclear EoS enters via the effective mass \( m^* = M - \Sigma_s \), the kinetic momenta \( k^{*\mu} = k^{\mu} - \Sigma^{\mu} \) and the field tensor \( F^{\mu\nu} = \partial^{\mu} \Sigma^{\nu} - \partial^{\nu} \Sigma^{\mu} \), where \( \Sigma_s \) and \( \Sigma^{\mu} \) are the Lorentz-scalar and Lorentz-vector components of the baryon self energy. Pions are propagated under the influence of the Coulomb field only. Mesons with strangeness obey the same transport equation (1), however, with different self energies with respect to the baryon ones, as it will be discussed below. The collision integral is determined by the total elastic and inelastic in-medium cross sections and includes all important channels on pion and kaon production, without the inclusion of antikaons. Isospin effects on the threshold conditions are explicitly accounted for to respect energy conservation [10]. More details can be found in Ref. [10,14]. Furthermore, the quasi-elastic \( KN \leftrightarrow KN \) channel with isospin exchange is included, however, it does not essentially influence the strangeness ratios.

3
2.2 The ChPT potential in hadronic matter

Kaplan and Nelson were the first to apply the chiral Lagrangian approach to the properties of kaons in dense nuclear matter [12]. Starting from the effective chiral Lagrangian an in-medium Klein-Gordon equations for kaons at the mean field level follow can be derived

$$\left[ \partial^\mu \partial^\mu + \frac{3i}{4f_\pi^2} j_\mu \partial^\mu + \left( m_K^2 - \frac{\Sigma_{KN}}{f_\pi^2} \rho_s \right) \right] \phi_K(x) = 0 \ , \quad (2)$$

where $j_\mu$ and $\rho_s$ are the baryon current and the scalar density, respectively. Eq. (2) can be rewritten in a more compact form as

$$\left[ (\partial_\mu + iV_\mu)^2 + m_K^2 \right] \phi_K(x) = 0 \ . \quad (3)$$

In isospin asymmetric matter the kaon effective mass is given by [13]

$$m_K^* = m_K^2 - \frac{\Sigma_{KN}}{f_\pi^2} \rho_s - \tau_3 \frac{C}{f_\pi^2} \rho_{s3} + V_\mu V^\mu \ , \quad (4)$$

where $\rho_s$, $\rho_{s3}$ are the total and isospin scalar baryon densities. The vector potential is defined as

$$V_\mu = \frac{3}{8f_\pi^2} j_\mu + \tau_3 \frac{1}{8f_\pi^2} j_{3\mu} \ , \quad (5)$$

with $j^\mu$, $j^{3\mu}$ isoscalar and isovector baryon currents, respectively, and $\tau_3 = \pm 1$ for $K^+$ and $K^0$. $m_K = 494 \text{ MeV}$ is the free kaon mass, $f_\pi = 93 \text{ MeV}$ the pion decay constant and $\Sigma_{KN}$ the kaon-nucleon sigma term, usually chosen in the range $350 \div 450 \text{ MeV}$. $f_\pi^* \approx 0.6f_\pi^2$ is the in-medium pion decay constant [19] which appears only in the vector field. According to Brown and Rho [19] this accounts for the fact that the enhancement of the the scalar part is compensated by higher order corrections in the chiral expansion. Finally, the parameter $C$ is fixed to the value of $33.5 \text{ MeV}$ [13].

2.3 Effective mean field potential in hadronic matter

In an alternative approach kaon potentials can also be derived within an effective meson field OBE approach [13]. In this picture the kaons interact through the exchange of mesons with different Lorentz properties ($\sigma$, $\omega$, $\rho$, $\delta$). These mesons mediate the isoscalar-scalar, isoscalar-vector, isovector-vector
and isovector-scalar parts of the kaon-kaon interaction. In the mean-field approximation the kaon equation of motion reads

\[
\left[ \partial^\mu \partial_\mu + m_K^2 + g_{\sigma K} m_K \sigma + \frac{g_{\delta K} g_{\delta N}}{m_\delta^2} m_K \tau_3 \rho_{s3} + 2 \left( \frac{g_{\omega K} g_{\omega N}}{m_\omega^2} j_\mu + \frac{g_{\rho K} g_{\rho N}}{m_\rho^2} \tau_3 j_\mu^3 \right) \right] \phi_K(x) = 0 \tag{6}
\]

As in the ChPT case (see Eq. (3)) Eq. (6) can be written more compact as:

\[
\left[ (\partial^\mu + i V^\mu)^2 + m_K^{*2} \right] \phi_K(x) = 0 \tag{7}
\]

The vector potential \( V^\mu \) and the effective mass \( m_K^{*} \) are now defined differently relative to the ChPT approach

\[
V^\mu = \frac{1}{3} (f_{\omega}^* j_\mu + \tau_3 f_\rho j_\mu^3) \tag{8}
\]

\[
m_K^{*2} = m_K^2 - \frac{m_K}{3} (g_{\sigma N} \sigma + \tau_3 f_\delta \rho_{s3}) \tag{9}
\]

where \( j_\mu = j_\mu^p + j_\mu^n \) and \( j_\mu^3 = j_\mu^p - j_\mu^n \), \( \sigma \) and \( \rho_{s3} \) are the isoscalar, isovector baryon currents, the isoscalar-scalar field and isovector-scalar baryon density, respectively. We will use a simple constituent quark-counting prescription to relate the kaon-meson couplings to the nucleon-meson couplings, i.e. a reduction by a factor 3; \( g_{iK} = \frac{1}{3} g_{iN} \) (\( i = \sigma, \omega, \rho, \delta \)). Thus, in the isoscalar-vector sector we have \( f_{\omega}^* = 1.4 \left( \frac{2 m_\omega}{m_\omega} \right)^2 \) due to the enhanced kaon-scalar/vector coupling. In the isovector-vector sector one obtains \( f_\rho = \left( \frac{m_\rho}{m_\rho} \right)^2 \). The scalar kaon field finally includes the isoscalar-scalar and isovector-scalar couplings \( g_{\sigma N} \) and \( f_\delta = \left( \frac{m_\delta}{m_\delta} \right)^2 \), respectively. For the parameters of the nucleon-meson couplings we refer to Ref. [10]. We note that the picture of a simple constituent quark-counting may not be consistent with \( KN \) scattering in free space. However, this is not of importance here, since \( KN \) scattering may influence kaon spectra, but not absolute kaon yields and their ratio, which is the main topic of this work.

Comparing both models for the kaon-nucleon interaction, Eqs. (8,9) versus (4,5), one sees that in the OBE model the nucleon-meson couplings enter explicitly in the kaon self energies, while in the ChPT approach the kaon mean field is determined by the parameters \( \Sigma_{KN} \) and \( C \). It is therefore helpful to study the isospin dependence of the OBE model for various parametrizations of the isovector EoS, i.e., the symmetry energy. As in previous works [10,14], we use the \( NL, NL\rho \) and \( NL\rho\delta \) prescriptions for the isovector mean-field. In the first one (\( NL \)) no isospin potential is included. \( NL\rho \) contains
Fig. 1. Density dependence ($\rho_{\text{sat}}$ is the saturation density) of the in-medium kaon energy in units of the free kaon mass ($m_K = 0.494 \text{ GeV}$). Upper solid curve refers to symmetric matter in the ChPT model, while the band gives the isospin splitting between $K^0$ and $K^+$ (upper and lower dashed curves of the band, respectively) for an asymmetry of $\alpha = 0.2$. The solid lower curve refers to the OBE approach for symmetric matter. The isospin splitting between $K^0$ (upper dashed and dotted lines) and $K^+$ (lower dashed and dotted lines) originating from the model with only a $\rho$-meson ($NL\rho$) is indicated by the inner band, while the outside band refers to the model with both, the $\rho$ and $\delta$ mesons ($NL\rho\delta$). The asymmetry parameter is $\alpha = 0.2$.

only the $\rho$ meson, while $NL\rho\delta$ accounts for both, the $\rho$ and the $\delta$ mesons in the isovector potential. The symmetry energy becomes stiffer in the direction $NL \rightarrow NL\rho \rightarrow NL\rho\delta$, since one has to increase the $\rho$-meson coupling when the $\delta$ meson field is taken into account in the potential part of the symmetry energy.

The crucial quantity for the interpretation of the transport results shown later is the kaon in-medium energy, given by the dispersion relation:

$$E_K(k) = k_0 = \sqrt{k^2 + m_K^*} + V_0$$

The density dependence of the in-medium kaon energy $E_K(k = 0) = m_K^* + V_0$ for $K^{0,+}$ is shown in Fig. 1 for both models for the kaon-nucleon potential. In general the ChPT approach leads to a stiffer density dependence of the in-medium kaon energy relative to the OBE model. This feature is more pronounced at high densities $\rho \sim (2 - 3)\rho_{\text{sat}}$, which are reached in intermediate energy heavy-ion collisions near the kaon production threshold. Thus, one expects observable effects of the model of the symmetry energy in the produced kaons ($K^{0,+}$) from heavy-ion collisions.

The isospin effect for the considered asymmetry parameter of $\alpha = \frac{N-Z}{N+Z} = 0.2$ ($N, Z$ are the neutron and proton numbers), which corresponds approximately to $^{197}$Au-nuclei, is pronounced at the baryon densities of $\rho \sim (2 - 3)\rho_{\text{sat}}$, see again Fig. 1. According to Eqs. (4,9) the isovector contributions to the kaon-
nucleon $ChPT$ and $OBE$ scalar self energies lead to an effective mass splitting between positive ($K^+$) and neutral ($K^0$) kaons. Also the vector self energies between $K^+$ and $K^0$ differ according Eqs. (5,8). These isospin effects finally lead to the pronounced isospin dependence of the in-medium kaon energy. We should note here the different origin of the isospin part of the kaon-nucleon potential between the two approaches. In the $ChPT$ case the isospin dependence enters only through the isospin-currents, while in the $OBE$ model the effective isovector meson-nucleon couplings are explicitly included. This leads to an isospin splitting between $K^0$ and $K^+$, which additionally depends on the model for the symmetry energy.

The non-trivial isospin dependence of the two models for the kaon-nucleon potential may have interesting effects on the absolute $K^0$ and $K^+$ yields and their collective flows and, in particular, on the strangeness ratio $K^0/K^+$ in heavy-ion collisions at intermediate energies near the kaon production threshold, which is the topic of the next section.

3 Results

The results of the following dynamical calculations are based on the transport equation (1), in which the isospin effects of the kaon-nucleon potentials are of particular interest, since they are investigated for the first time and may affect absolute kaon yields and strangeness ratios.

We first discuss in subsection 3.1 the qualitative features of the two models of the kaon-nucleon potential and compare with experiments. For a transparent discussion on the the isospin effects the following strategy has been chosen: the isoscalar part of the nuclear mean-field is the same for all different cases of transport calculations, while the isovector channel includes only the $\rho$ meson. In the strangeness sector following cases of different transport calculations will be discussed:

(i) $ChPT$ refers to transport results with only the isospin independent part of the chiral model for the kaon potential.
(ii) $OBE$ refers to transport results with only the isospin independent part of the effective mean-field model for the kaon potential.
(iii) $ICChPT$ refers to transport results with the full isospin dependent chiral model for the kaon potential.
(iv) $IOBE$ refers to transport results with the full isospin dependent effective mean-field model for the kaon potential.

Thus in the first two cases isospin effects are included only in the baryonic sector ($\rho$ meson), while in the last two ones they are taken into account also
in the strangeness sector.

We then investigate in subsection 3.2 in detail the role of the different isospin-dependent kaon-nucleon potentials on the strangeness ($K^0/K^+$)-ratio in $Au + Au@1 AGeV$ central collisions. Here we particularly distinguish for each case listed above between the $NL$, $NL\rho$ and $NL\rho\delta$ models, in which the symmetry energy becomes more stiffer in the sequence $NL \rightarrow NL\rho \rightarrow NL\rho\delta$.

3.1 Kaon yields

We start the discussion with the influence of the different kaon potentials on absolute $K^{+,0}$ meson yields. This is shown in Fig. 2 for heavy-ion collisions between heavy systems ($Au+Au$), in terms of their rapidity distributions for the four cases. Consistent with Fig. 1 the transport calculations with the ChPT model with the larger repulsion leads to overall smaller kaon yields with respect to those in the OBE approach. The inclusion of the isospin dependent part in the kaon potentials (compare the curves $ICHPT$ and $ChPT$ as well as those between $IOBE$ and $OBE$ in Fig. 2) changes only very little the absolute yields. This is again consistent with Fig. 1, where the isospin effects on the two isovector states are small at baryon densities around $\rho_B \sim 2\rho_{sat}$. Furthermore, in Fig. 1 the asymmetry parameter was fixed to a value of $\alpha = 0.2$ corresponding to the asymmetry of a $^{197}Au$-nucleus), while in heavy-ion collisions the local isospin asymmetry, in the stage of kaon production, is reduced with respect to that of the initial situation [8]. This is because of partial isospin equilibration due to stopping and inelastic processes with associated isospin exchange.

It is interesting to study the role of the different kaon potentials in lighter colliding systems, such as $Ni + Ni$, in which also experimental information is available [20,21]. Fig. 3 shows the comparison between the data and the transport calculations obtained with both sets of the kaon potential without and with the isospin dependence (left and right panel, respectively). The isospin dependence is again not of relevance here. The different kaon potentials influence the kaon yields in a similar manner as in the previous figure. These in-medium dependencies are less pronounced now, since less compression is achieved in the stage of kaon production for the lighter $Ni + Ni$-system.

Both sets of calculations are close to the experimental data, with the tendency of an improvement in the $OBE$ and $IOBE$ cases. However, this result does not necessarily mean that the $OBE$ and $IOBE$ kaon-nucleon potentials are the most realistic ones, because other physical quantities may influence the transport calculations. A general and non-trivial feature in heavy-ion collisions concerns the treatment of the in-medium effects on the elastic and inelastic...
cross sections, which are usually parametrized to empirical free NN-scattering. Microscopic studies on elastic NN-scattering at finite baryon density exist [22], but for inelastic processes, e.g., $NN \rightarrow N\Delta$, theoretical calculations are very rare and often limited in baryon density and momentum [22]. Attempts have been recently done in studying this non-trivial problem in heavy-ion collisions [14] using a simple parametrization from microscopic Dirac-Brueckner calculations [22]. For the strangeness sector, e.g., $BB \rightarrow BYK$ and $\pi B \rightarrow YK$ ($B, Y$ and $K$ stand for nucleons or higher resonances, hyperons and kaons, respectively), the in-medium effects are trivially included only in the phase-space factors and the threshold conditions, but not in the matrix elements, i.e., here free parametrizations are used [23]. A selfconsistent treatment of kaon-nucleon scattering in the hadronic environment would be desirable in this context, as it has been already investigated for the antikaonic sector [24]. Furthermore, a more realistic density and, in particular, energy dependence of the hadronic relativistic mean field would be important [25]. However, in order to keep the discussions transparent, we do not expand on these topics here, and use the same parametrization as in [14].

In Fig. 4 the centrality dependence of the $K^+$ yields for $Au + Au$-collisions is shown. First of all, all sets of data underestimate the data due to the strong density dependence of the inelastic cross section for $\Delta$-production, see also Ref. [14] and the discussion above. The in-medium dependence of the kaon potentials is now more pronounced for the most central $Au + Au$-collisions, see left panel in Fig. 4. Isospin effects of both kaon potentials appear due to the asymmetry of the colliding $Au$-systems, see right panel of the same figure.
Fig. 3. $K^+$ rapidity distributions for semi-central ($b < 4 \text{ fm}$) Ni+Ni reactions at 1.93 AGeV. Theoretical calculations (as indicated) are compared with the experimental data of FOPI (open triangles) and KaoS (open diamonds) collaborations [20,21].

The effect of the inclusion of the isovector potential turns out to be stronger in the $IChPT$ case compared to the $IOBE$ one, since the in-medium energy for $K^+$ is stronger affected in the $ChPT$ model, see again Fig. 1. Thus, the kaon yield is enhanced more strongly in the $IChPT$ transport calculations, and comes closer to the $IOBE$ calculations, which are less sensitive to isospin effects.

Collective kaon ($K^+$) flow (not shown here) has been also studied in the framework of the different kaon potentials. All the calculations reproduce the experimental flow [26] reasonable well, however, the isospin effects were only moderate. It would be more helpful to study isospin strangeness flows, i.e., the difference of collective flows between $K^0$ and $K^+$, since the isovector part of the kaon potential affects $K^+$ and $K^0$ differently. In conclusion, the evaluation scheme of the kaon-nucleon potential within the effective $OBE$ approach seems to lead to a better description of the experimental data. However, a definitive conclusion on the question, which potential would be more realistic, cannot be drawn at this level of study. A more systematic study on in-medium effects also on the level of $NN$-scattering is in this context necessary.

Thus at this level of the comparison between the theoretical calculations and the data the $OBE$ framework describes the experiment better than the $ChPT$ model. However, for a transparent discussion specifically of the isospin effects on strangeness ratios both models of the kaon-nucleon interaction will be investigated. A more promising observable to study isovector effects may be given by the ratio of the two isospin kaon states, e.g., the strangeness ratio $K^0/K^+$, since ratios are expected to be independent on possible in-medium modifications of the isoscalar medium. E.g., the influence of in-medium inelas-
Fig. 4. $K^+$ centrality dependence in Au+Au reactions at 1 AGeV incident energy. Our theoretical calculations (as indicated) are compared with KaoS data from [27] (open diamonds).

The non-interacting NN $\rightarrow N\Delta$ cross sections has been studied in [14] with the conclusion of a very moderate dependence of the strangeness ratio on the density dependence of the inelastic cross section.

### 3.2 Strangeness ratio

A crucial question is whether particle yield ratios are influenced by the in-medium modifications of kaon potentials. This is important particularly for kaons, since ratios of particles with strangeness have been widely used to determine the nuclear EoS at supra-normal density [7]. More recently, the \((\pi^-/\pi^+)-\) and \((K^0/K^+)-\)ratios have been proposed as a good probe to determine the high density behaviour of the isovector part of the nuclear mean field [10,8,28,29]. We thus extend our previous studies [8] by explicitly including both schemes of the kaon potential in the transport calculations (ChPT and OBE models). In particular, we distinguish between the different scenarios for the symmetry energy in the baryon sector, i.e., by neglecting the isovector potential (NL case), by including only the $\rho$-meson (NL$\rho$ case), and both, the $\rho$ and $\delta$ mesons (NL$\rho\delta$) in the isovector baryon and kaon potentials (ICHPT and IOBE cases).

Fig. 5 shows the strangeness ratio for the different parametrisations of the isovector part of the nuclear EoS. Calculations are also performed for the two approaches of the kaon potential (ICHPT and IOBE model).

First of all, an overall linear increase of the ratio with respect to the stiffness
Fig. 5. $K^0/K^+$ ratio for different parametrisations of the isovector EoS ($NL, NL\rho, NL\rho\delta$) and for the kaon potential ($OBE, ChPT, IOBE, IChPT$). The considered reaction is a central ($b = 0$ fm) $Au + Au$ heavy-ion collision at 1 $AGeV$ incident energy.

of the symmetry energy is observed. This feature is not trivial and reflects the interplay of isospin effects between mean-field and collision dynamics at the threshold. As discussed in detail in Ref. [8], isovector mean-field and threshold effects act differently on the isospin kaon states $K^+$ and $K^0$, where the later feature governs the strangeness dynamics at low intermediate energies near the kaon production threshold.

The sensitivity of the strangeness ratio on the different evaluations of the isospin independent kaon potential ($ChPT$ and $OBE$ cases) is very moderate, as expected. If the isovector part is neglected, the density behavior of the kaon potential is the same for both isospin $K^0$ and $K^+$ states. Therefore the ratio should be rather independent on the evaluation scheme of the kaon potential, as it is the case in the transport calculations of Fig. 5.

Taking explicitly account of the isospin effects in the kaon potentials ($IChPT$ and $IOBE$ cases) the dynamical situation changes. The isospin effect in the $IChPT$ model strongly decreases the strangeness ratio, however, the relative difference between the various choices of the symmetry energy ($NL, NL\rho$ and $NL\rho\delta$) remains stable. This is due to the fact that the isovector sector of the $IChPT$ kaon self energies contain only the isospin Lorentz-scalar and Lorentz-vector densities without additional parameters, e.g., meson-nucleon couplings [14]. This feature is also shown in Fig. 1, in which the isospin splitting is independent on the stiffness choice of the symmetry energy. The decrease of the ratio in the $IChPT$ calculations is related to the fact that the in-medium energy becomes more stiffer (softer) for $K^0$ ($K^+$). The situation in the $IOBE$ calculations is similar, but the isospin effects are less pronounced. The suppres-
sion of the kaon ratio in the transport calculations due to the isospin effects is now weak, due to the softer character of the \textit{IOBE} kaon potential, compared to the \textit{ChPT} model. However, the relative difference between the different models of the symmetry energy is slightly modified. When isospin effects are accounted for, the \textit{IOBE} self energies explicitly contain the meson-nucleon couplings, which are different for each model of the symmetry energy, see also again Fig. 1. This leads to the observed different slope of the kaon ratio in the \textit{IOBE} transport calculations, however, the effect is rather moderate. It is interesting to note that the situation in a chemical and thermal equilibrated (closed) system, e.g., in studying strangeness production in a box with periodic boundary conditions, is rather different. The isospin effects on the strangeness ratio are very pronounced in box calculations (see Fig. 8 in Ref. [10]). However, the isospin effect is considerably reduced in an open system in heavy-ion collisions due to, e.g., isospin transparency which decreases the local asymmetry of the kaon production source.

In summary, the consideration of the isospin effects on the level of the kaon-nucleon interaction leads to modifications of the strangeness ratio on the absolute value, but not to significant changes with respect to the stiffness of the symmetry energy. In the \textit{ChPT} and \textit{IOBE} approaches a sensitivity to the symmetry energy on the level of $\sim 25\%$ and $\sim 15\%$ survives. We thus conclude that the strangeness ratio is a very promising observable to determine the isospin dependence of the underlying kaon-nucleon interaction and particularly the isospin part of the nuclear EoS.

4 Conclusions

We have investigated the role of different kaon potentials and, in particular, their explicit isospin dependence in asymmetric nuclear matter. The kaon potentials have been evaluated in two schemes, one following a Chiral Perturbation approach and a second one within the framework of an effective One-Boson-Exchange picture.

We have studied in detail the effects of various isospin dependent kaon potentials on kaon dynamics in heavy-ion collisions within a covariant transport equation. The isospin effects turns out to be less sensitive to absolute kaon yields, but more pronounced to the strangeness $K^0/K^+$ ratio. We thus focused our attention to the $K^0/K^+$ ratio in collisions between heavy and isospin-asymmetric systems ($Au + Au$) at incident energies below the kaon production threshold. Different evaluation schemes of the kaon potential and its isovector part lead to different results on the strangeness ratio. The isospin dependence changes this ratio on an absolute level. As an interesting result, however, the relative dependence of the strangeness ratio to the stiffness of
the isovector EoS remains stable.

At this level of investigation we conclude that the strangeness ratio is an important observable to determine the high density behavior of the symmetry energy. Future experiments with exotic radioactive beams, as they are planned in new experimental facilities at MSU and at GSI (FAIR project), will be very helpful to resolve the theoretical uncertainties of the symmetry energy at high baryon densities.

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