Numerical method for solving the problem of the gas-dynamic state of a main gas pipeline section relief of a variable cross-sectional area

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Abstract. The paper proposes a numerical method for solving the problem of the gas-dynamic state of a main gas pipeline linear section, which is characterized by a path change in the diameter and leveling height of the pipeline axis. The quasi-one-dimensional equations of gas pipeline transport are derived with account for the local and convective components of the inertia force, the quadratic law of resistance and the force of gravity at a variable cross-sectional area of the pipeline. At the inlet to the section, a time change in hydrostatic pressure is set, and at the outlet from the section a mass rate of gas flow is set. The initial distribution of gas-dynamic indices was taken for a stationary mode of operation. The equations were transformed to the equations of direct and reverse traveling waves presented in dimensionless variables, and approximated by an implicit scheme, taking into account the direction of excitation propagation. The iterative processes were formed by virtue of the nonlinearity of equations and boundary conditions, A calculation program was developed that allowed studying the process dynamics depending on the time change in the inlet pressure and the outlet mass flow rate under a path change in the diameter and leveling height of the pipeline axis. The results of separate calculations on transient processes were presented, when, at the section outlet the mass flow rate increases abruptly, and the pipe diameter has a local increase according to a sinusoidal law.

1. Introduction

Pipelines are used to transport liquids and gases over long distances, as well as to deliver mechanical energy. At higher operating pressures and pipe cross-sectional areas, less energy losses are expected during liquids and gases transport or mechanical energy delivery. This follows from the analysis of quasi-one-dimensional equations of pipeline transport of real fluids [1-3].
The pipelines function under the influence of various internal and external factors. At low and mean operating pressures, when the pipe diameters are small, the main energy is spent to overcome the frictional resistance force. At a higher density of the transported medium (for example, water), the gravity or inertia forces can take the main place among the force factors. This is especially evident when a shock wave or a cavitation is formed.

A path change in gas temperature can lead to hydration or gas condensation, that is, to the formation of a two-phase medium in wells or gas pipelines [4, 5]. At low ambient temperatures, freezing of the pipeline network fittings, icing up of the pipeline or hydrate plugs formation are possible [4, 5]. A path increase in the temperature of transported gases leads to an increase in energy consumption. This can be facilitated, for example, by restarting the network, switching to the transportation of a different composition of gas, and by a gas leak in existing gas pipelines.

The possibilities of mathematical and numerical modeling are widely used to design and monitor the pipelines’ performance [6-12]. The developed mathematical and numerical models can describe the state of a separate linear section, a stage, a multi-line main gas pipeline with a head and booster compressor stations, a looped distribution network or a network of small stations. The adequacy of mathematical or numerical model is determined depending on the consistency of the object features and the range of factors taken into account. Currently, various automated workstations and simulators have been developed for practicing engineers [13-15].

The main elements of the gas pipeline network are linear sections, calculated based on simplified solutions of nonlinear partial differential equations [16-17]. Local resistances in main gas pipelines are taken into account as a 5% or 10% addition to the total resistance of linear sections. In gas pipelines of low or mean working pressure, local resistances are taken into account in the form of a correction to the length (dimension) of linear sections.

Often, when solving problems of pipeline gas transportation, the mass flow rate is introduced analytically, ignoring the term of convective transfer, and the law of resistance is linearized [16-17]. The result is a complete or truncated telegraph equation. Such simplifications can lead to an approximate solution of the problem. In this regard, the problem of solving nonlinear equations for the transfer of momentum, mass and energy in the process of pipeline transport of real gases is a relevant one.

Below we develop a method for the numerical solution of similar problems within the framework of isothermal approach. All power factors are taken into account in the momentum conservation equation. With the introduction of an auxiliary function, the power of the unknowns is reduced and the transition to the equations of direct and reverse traveling waves is carried out. A similar transition was used in [18, 19], which describes a numerical method for solving the problem at given constant values of the inlet pressure and outlet mass flow rate by deriving a solution at a constant diameter of the gas pipeline. Within the framework of this work, the dynamic change in the boundary conditions, piecewise-constant and variable values of the diameter, and the path change in the height of the gas pipeline location are taken into account.

The gas pipeline considered in the paper is schematically represented as follows.

Pressure \( p_0(t) \) is provided at the inlet to the linear section. Gas flows from the end of the section at an intensity \( M_l(t) \). Gas has a reduced gas constant \( R_0 \), mean temperature \( T \) and super-compressibility coefficient \( Z \). The section can have a piecewise constant or variable radius \( R(x) = D(x) / 2 \). The value of the resistance coefficient \( \lambda \) along the entire length of the section \( l \) is determined depending on \( R(x) \). The gas pipeline runs over rough terrain, the height of location changes in the form \( z_1 = z_1(x) \). The initial state of the section can be taken as a stationary mode of operation.

It is required to determine the missing boundary values at the ends of the section, and the distribution of gas-dynamic indices over the entire section.
2. Method
To describe the gas-dynamic state of a linear section with a variable cross-sectional area, we use quasi-one-dimensional equations for the isothermal case [2]:

\[
\begin{align*}
\frac{\partial (\rho u f)}{\partial t} + \frac{\partial (\rho u^2 f)}{\partial x} &= -f \left( \frac{\partial p}{\partial x} + g \rho \frac{\partial z_1}{\partial x} \right) - \pi \lambda \rho u |u| R, \\
\frac{\partial (\rho f)}{\partial t} + \frac{\partial (\rho uf)}{\partial x} &= 0.
\end{align*}
\]

(1)

Hereinafter, \( u \) is the mean flow rate, which has a non-negative value; \( P, \rho \) are the hydrostatic pressure and gas density; \( f(x) = \pi D(x)^2 / 4 \) is the variable cross-sectional area of the pipeline; \( \sin \alpha = \frac{dz_1}{dx} \) is the sine of the location inclination from the horizon, which takes a constant (including zero) or variable value.

To solve the system of equations (1) with the necessary additions and conditions, the traveling wave method was used [18, 19]. Therefore, in the equations, we go over from the conservative form to the usual form and exclude the hydrostatic pressure:

\[
\begin{align*}
\frac{\rho f \partial u}{\partial t} + \rho uf \frac{\partial u}{\partial x} &= -f \gamma \frac{\partial \rho}{\partial x} - f \rho g \sin \alpha - \frac{\lambda}{2D} f \rho u |u|, \\
\frac{\partial (\rho f)}{\partial t} + \rho f \frac{\partial u}{\partial x} + u \frac{\partial (\rho f)}{\partial x} &= 0.
\end{align*}
\]

(2)

Here we took into account the equation of state of a real gas

\[ P = \gamma \rho, \]

(3)

where \( \gamma = \frac{\partial p}{\partial \rho} = ZRT = c^2 \) (\( c \) is the propagation velocity of small pressure disturbances in the gas-pipe system [1-3]).

We divide both sides of the equations of system (2) by \( f \rho \) and introduce the new sought for function

\[ \varphi = \ln \frac{f \rho}{f_* \rho_*}, \]

(4)

where \( f_*, \rho_* \) are the characteristic cross-sectional area and gas density; the system is presented in the form:

\[
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} &= -\gamma \left( \frac{\partial \varphi}{\partial x} - \frac{d \ln (f / f_*)}{dx} \right) - g \sin \alpha - \frac{\lambda}{2D} u |u|, \\
\frac{\partial \varphi}{\partial t} + \frac{\partial u}{\partial x} + u \frac{\partial \varphi}{\partial x} &= 0.
\end{align*}
\]

(5)

Here, function \( \varphi \) is presented in dimensionless variables. The remaining variables are also represented in dimensionless form, for which we use scale values \( l, l/c \) and \( c \) for the length, time and flow rate, respectively. As a result, system (5) takes the form:
\[
\begin{align*}
\left\{ \begin{array}{l}
\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \varphi}{\partial \bar{x}} + \frac{\partial \varphi}{\partial x} = d \ln \left( \frac{f}{f_0} \right) \frac{g l}{c^2} \sin \alpha - \frac{\lambda l}{2D} \bar{u}, \\
\frac{\partial {\bar{u}}}{\partial t} + \bar{u} \frac{\partial {\bar{u}}}{\partial \bar{x}} + \frac{\partial {\bar{u}}}{\partial x} = 0.
\end{array} \right.
\end{align*}
\]

Let's represent it in matrix form:

\[
\begin{align*}
\frac{\partial W}{\partial t} + A \frac{\partial W}{\partial x} &= B, \\
\text{where the following was taken into account}
\end{align*}
\]

and introduce notation

\[
F = \frac{2 D}{D} \frac{g l}{c^2} \sin \alpha - \frac{\lambda l}{2D} |\bar{u}|. \\
W = \begin{pmatrix} \bar{u} \\ \varphi \end{pmatrix}, \quad A = \begin{pmatrix} \bar{u} & 1 \\ 1 & \bar{u} \end{pmatrix}, \quad B = \begin{pmatrix} F \\ 0 \end{pmatrix}.
\]

To compose autonomous equations with respect to linear combinations of the sought for $\bar{u}$ and $\varphi$, it is necessary to find the eigenvalues of $\lambda$ and the eigenvectors of matrix $A$.

The eigenvalues of matrix $A$ found from equation

\[
\begin{vmatrix} \bar{u} - \lambda & 1 \\ 1 & \bar{u} - \lambda \end{vmatrix} = 0,
\]

have the values of

\[
\lambda_{1,2} = \bar{u} \pm 1.
\]

Compose a diagonal matrix

\[
\Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} = \begin{pmatrix} \bar{u} + 1 & 0 \\ 0 & \bar{u} - 1 \end{pmatrix}.
\]

Using the matrix $\Lambda$, we represent the matrix $A$ as a product of

\[
A = V^{-1} \Lambda V,
\]

where $V$ - is the fundamental matrix similar to $A$, that consists of the elements of the eigenvectors of matrix $A$; $V^{-1}$ - is the inverse matrix of $V$.

From the last relation, the unnormalized eigenvectors of matrix $A$ are found and the following matrix is composed

\[
V = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.
\]

We do not need the values of the elements of inverse matrix $V^{-1}$, since multiplying the equation

\[
\frac{\partial W}{\partial t} + V^{-1} \Lambda V \frac{\partial W}{\partial x} = B
\]

by $V$ on the left, we obtain the equation without $V^{-1}$:

\[
V \frac{\partial W}{\partial t} + \Lambda V \frac{\partial W}{\partial x} = VB.
\]
Here we took into account the identity $VV^{-1} = E$.

Let us calculate the components of the matrix equation; it takes the form:

$$
\begin{aligned}
\frac{\partial (\overline{u} + \varphi)}{\partial t} + (\overline{u} + 1) \frac{\partial (\overline{u} + \varphi)}{\partial x} &= F, \\
\frac{\partial (\overline{u} - \varphi)}{\partial t} + (\overline{u} - 1) \frac{\partial (\overline{u} - \varphi)}{\partial x} &= F.
\end{aligned}
$$

That is, we came to autonomous equations

$$
\begin{aligned}
\frac{\partial f_1}{\partial t} + (1 + \overline{u}) \frac{\partial f_1}{\partial x} &= F, \\
\frac{\partial f_2}{\partial t} - (1 - \overline{u}) \frac{\partial f_2}{\partial x} &= F.
\end{aligned}
$$

They are linear with respect to the new sought for values $f_1 = \overline{u} + \varphi$ and $f_2 = \overline{u} - \varphi$, at the same time, the convective terms and the right-hand sides are nonlinear.

For known values of the sought for $f_1$ and $f_2$, the flow rate is determined as $u = c \frac{f_1 + f_2}{2}$, and function $\varphi = \ln \frac{\rho f}{\rho f} - \frac{f_1 - f_2}{2}$. Accordingly, the density is determined by the formula $\rho = \rho e^{-\frac{f_1 - f_2}{2}}$, and accounting for the dependence $\overline{\rho} = \frac{p}{\rho} = \frac{\rho}{\rho} = \overline{\rho}$, we have the formula $p = p e^{-\frac{f_1 - f_2}{2}}$ for pressure.

Equations (8) can be used to study the bi-directional and one-directional wave propagation. In particular, if the changes in $u$ and $\varphi$ are known at time $x = 0$, then it is possible to study the wave propagation in the numerical line $x \in [0, \infty)$ by the first equation and the wave propagation in the left semiaxis $x < 0$ by the second equation. In these two cases, it is sufficient to specify the functions of the boundary conditions $u(0, t)$, $\varphi(0, t)$ and the initial conditions for the unknowns propagation in the corresponding numerical semiaxes.

The boundary conditions for the problem are formulated as follows.

The initial state of the section can be taken in accordance with the state of rest of gas $u(x, 0) = 0$.

In this case, in the problem model, the initial pressure distribution is determined according to the barometric formula:

$$
p(x, 0) = p(0, 0) e^{-\varphi(x) - \varphi(0)}
$$

As a result of the steady-state solution, it is possible to obtain the velocity and pressure distributions at given constants of the inlet pressure and the outlet mass flow rate.

The second option for specifying the initial state corresponds to a stationary flow regime with a given mass gas flow rate:

$$
f(x) \rho(x, 0) u(x, 0) = M^0 = \text{const}
$$

This condition uniquely satisfies the second equation of system (1). In this case, a separate task is formed for the elementary section.

The first equation of the system, at $u > 0$, takes the form of an ordinary differential equation:
\[ \rho uf \frac{du}{dx} = -f \frac{dp}{dx} - g \rho f \sin \alpha - \frac{\lambda}{2D} \rho fu^2. \]

From this, an ordinary differential equation was formed
\[ \frac{dp}{dx} = \left[ 1 - \left( \frac{M^0}{f^2 p^2} \right)^2 \right] \left[ \frac{(M^0)^2 c^2}{f^3 p^2} - \frac{pg \sin \alpha}{c^2} - \frac{\lambda}{2D} \frac{(M^0)^2 c^2}{f^2 p^2} \right]. \] (9)

Here \( p(0) = p_{in} \) - the inlet pressure at the beginning of the process serves as a boundary condition for the pressure equation.

A similar solution to this problem for \( \sin \alpha = \text{const} \) and \( f(x) = \text{const} \) was obtained in [20]. However, in the case of a variable cross-sectional area, it is advisable to use the numerical method of integration, which we will consider when approximating the equation and conditions.

The initial velocity distribution is determined by the formula
\[ u(x) = \frac{c^2 M^0}{f(x) p(x)}. \]

The conversion into dimensionless form is performed in the usual way. At the inlet, the pressure change in time \( p(0,t) \) is set. Accordingly, assuming that \( D_e = 1 \text{ m} \), we have
\[ \varphi(0,T) = \ln \left[ \frac{p(0,T)}{p_e} D^2(0) \right]. \]

At the outlet from the section, the change in the gas mass flow rate \( M(l,T) \) is set. In dimensionless form, this condition is presented as:
\[ D^2(1) \bar{p} \bar{u}(1,T) = \frac{4M(1,T)}{\pi e \rho_c} = \bar{Q}(T). \] (10)

To solve the problem, a finite-difference method was used.

Discrete coordinates \((i,n)\) with constant steps \( \tau \) and \( h \), and grid functions \( f_{i1}^n, f_{i2}^n, \bar{u}_i^n, \bar{\rho}_i^n \) and \( \varphi_i^n = \ln \left( \bar{p}_i^n D_i^2 \right) = \ln \left( \bar{\rho}_i^n D_i^2 \right) \) are introduced. The values of \( D_i, \alpha_i \) were considered as set ones.

In the first option, the initial condition for the velocity was taken equal to \( \bar{u}_i^0 = 0 \) and the initial condition for the pressure, denoted as \( p_0 = p(0,0)/p_e \), was taken in the form
\[ \bar{p}_i^0 = \bar{p}_0 e^{-\varphi(0,\tau_i - \tau_0)}. \]

In the second option, the initial condition (9) was approximated by the backward scheme and the dimensional pressure field was obtained. In the calculations, the formulas for the initial pressure velocity were used in dimensionless form:
\[ \varphi_i^0 = \ln \left( \bar{p}_i^0 D_i^2 \right), \quad \bar{u}_i^0 = \frac{c M^0}{\bar{p}_i^0 f_i}. \]

the initial distributions \( f_{i1}^0 = \bar{u}_i^0 + \varphi_i^0, \quad f_{i2}^n = \bar{u}_i^0 - \varphi_i^0 \) were calculated at \( i = 0..N \).

Taking into account the direction of boundary excitation propagation, Eq. (8) was approximated in an implicit form:
\[
\frac{f_{i+1}^{n} - f_{i}^{n}}{\tau} + (1 + \bar{u}_{i}^{n}) \frac{f_{i+1}^{n+1} - f_{i}^{n+1}}{h} = \Phi_{i}^{n},
\]
\[
\frac{f_{2i+1}^{n+1} - f_{2i}^{n}}{\tau} - (1 - \bar{u}_{i}^{n}) \frac{f_{2i+1}^{n+1} - f_{2i}^{n+1}}{h} = \Phi_{2i}^{n}.
\]

Here we assumed that
\[
\Phi_{i}^{n} = \frac{2}{h} \ln \left( \frac{D_{i}}{D_{i+1}} \right) - \frac{gl}{2c^{2}} (\sin \alpha_{i} + \sin \alpha_{i}) - \frac{\lambda l}{4D_{i}} \left[ \bar{u}_{i}^{n} |\bar{u}_{i}^{n}| + \bar{u}_{i+1}^{n} |\bar{u}_{i+1}^{n}| \right],
\]
\[
\Phi_{2i}^{n} = \frac{2}{h} \ln \left( \frac{D_{i}}{D_{i-1}} \right) - \frac{gl}{2c^{2}} (\sin \alpha_{i} + \sin \alpha_{i}) - \frac{\lambda l}{4D_{i}} \left[ \bar{u}_{i}^{n} |\bar{u}_{i}^{n}| + \bar{u}_{i-1}^{n} |\bar{u}_{i-1}^{n}| \right].
\]

Since the finite-difference equations and their right-hand sides are not linear, an iterative process was organized to solve them at a fixed \( n+1 \) time step. For the zero approximation \( m = 0 \), the values of \( \bar{u}_{i}^{n} \) were taken from the previous time layer, and in subsequent approximations \( m = 1, 2, \ldots \), the values of \( \bar{u}_{i}^{n} \) were taken from the previous approximation \( \left( \bar{u}_{i}^{n} \right)^{m+1} = \left( \bar{u}_{i}^{n+1} \right)^{m} \). The iterative process ends for a fixed time value when the conditions \( \max_{0 \leq i \leq N} \left| (f_{i}^{n+1})^{m+1} - (f_{i}^{n+1})^{m} \right| < \varepsilon \), \( \max_{0 \leq i < N} \left| (f_{2i}^{n+1})^{m+1} - (f_{2i}^{n+1})^{m} \right| < \varepsilon \) are met simultaneously.

Introduce notation \( \sigma = \tau/h \), and from the finite-difference equations compose the recurrent dependences in time:
\[
f_{i+1}^{n+1} = \left[ 1 + \sigma (1 + \bar{u}_{i}^{n}) \right]^{-1} \left[ f_{i+1}^{n} + \sigma (1 + \bar{u}_{i}^{n}) f_{i+1}^{n+1} + \tau \Phi_{i}^{n} \right],
\]
\[
f_{2i+1}^{n+1} = \left[ 1 + \sigma (1 - \bar{u}_{i}^{n}) \right]^{-1} \left[ f_{2i+1}^{n} + \sigma (1 - \bar{u}_{i}^{n}) f_{2i+1}^{n+1} + \tau \Phi_{2i}^{n} \right].
\]
The first formula was used at \( i = 1 \ldots N \), and the second at \( i = N - 1 \ldots 0 \).

It remains to determine the values of \( f_{10}^{n+1} \) and \( f_{2N}^{n+1} \).

The value was assumed to be known at the inlet \( \bar{x} = 0 \)
\[
f_{20}^{n+1} = \bar{u}_{0}^{n+1} - \varphi_{0}^{n+1}.
\]

The value of \( \varphi_{0}^{n+1} = \ln \left( \frac{\varphi_{1}^{n+1} \mu_{0}^{2}}{\varphi_{0}^{n+1}} \right) \) is known from the problem statement. Therefore
\[
\bar{u}_{0}^{n+1} = f_{20}^{n+1} + \varphi_{0}^{n+1},
\]
\[
\bar{u}_{10}^{n+1} = \bar{u}_{0}^{n+1} + \varphi_{0}^{n+1}.
\]

Calculations of \( f_{1i}^{n+1} \) begin with these values at \( i = 1 \ldots N \).

Condition (10) is specified at the outlet section. It is approximated in the form:
\[
\bar{Q}_{i}^{n+1} = \phi_{i}^{n+1} \bar{u}_{N}^{n+1}.
\]

It was assumed in the solution algorithm that the value of \( f_{1N}^{n+1} \) is known. In addition, if the value of \( \bar{u}_{N}^{n+1} \) is known, then
\[
\varphi_{N}^{n+1} = \frac{f_{1N}^{n+1} - f_{2N}^{n+1}}{2} = \frac{f_{1N}^{n+1} - (2\bar{u}_{N}^{n+1} - f_{1N}^{n+1})}{2} = f_{1N}^{n+1} - \bar{u}_{N}^{n+1}.
\]
Here, the boundary condition takes the form:
\[ 
\overline{Q}_{i}^{n+1} = e^{f_{iN}^{n+1}} - \overline{u}_{i}^{n+1} \overline{u}_{i}^{N} \cdot
\]

To solve this equation, the Newton tangent method was used [18]. The iterative process was organized according to the recurrent formula
\[ \overline{u}_{i}^{k+1} = \overline{u}_{i}^{k} + \frac{e^{-f_{iN}^{n+1} \overline{u}_{i}^{k}} - \overline{u}_{i}^{k}}{1 - \overline{u}_{i}^{k}}. \]

The approximation process begins at \( \overline{u}_{i}^{0} = 0 \) and continues until at least one of the conditions \( F(\overline{u}_{i}^{k+1}) < 10^{-10} \) and \( |\overline{u}_{i}^{k+1} - \overline{u}_{i}^{k}| < 10^{-8} \) is satisfied.

The values of \( \varphi_{i}^{n+1} = \overline{f}_{1N}^{n+1} - \overline{u}_{i}^{n+1} \) and \( \overline{f}_{2N}^{n+1} = \overline{u}_{i}^{n+1} - \varphi_{i}^{n+1} \) are calculated from the found value of \( \overline{u}_{i}^{n+1} \) and the known value of \( \overline{f}_{1N}^{n+1} \). Further, the calculation of \( \overline{f}_{2i}^{n+1} \) is carried out at \( i = N - 1, 0 \). After the iteration conditions are met, we can proceed to the next time step.

3. Results and Discussions

Based on the presented material, a program was drawn up and calculations were carried out.

Based on the presented material, a program was compiled in the Pascal ABC environment and calculations were carried out.

A series of calculations was carried out to check the applicability of the formulas given for the initial pressure distribution based on the first equation of system (1) at a constant gas mass flow rate. Convincing results were obtained at a smooth change in pipeline diameter:

\[ D(x) = \left\{ \begin{array}{ll} D_{1} \text{ at } \frac{x}{l} & < 0.3 \text{ or } \frac{x}{l} > 0.7, \vspace{0.5cm} \\
\beta D_{1} \sin \pi \left( \frac{x}{l} - 0.5 \right) & \text{at } 0.3 \leq \frac{x}{l} \leq 0.7. \end{array} \right. \]

The calculations were carried out at \( l = 10.0 \text{ km} \), \( D_{1} = 0.998 m \) and \( \beta = 1, 1.1, 1.4, 2.0 \) (the curves 1, 2, 3, and 4 in fig. 1 correspond to \( D_{1} \)).

The gas pressure distributions obtained at \( M^{0} = 250.0 \text{ kg/s} \), \( P(0,0) = 5.6 \text{ MPa} \) are shown in Figure. 1.

**Figure 1.** Path change in gas pressure at local sinusoidal variation in the gas pipeline diameter.
At an abrupt change in the diameter, the incorrect results were obtained around the cross section, which affected the final result of the stationary problem. Correct results were obtained using the formulas of the initial pressure distribution when solving the problems of transient processes by the steady-state method.

The calculations were carried out at an abrupt change in the outlet mass flow rate from 250.0 kg/s to 300.0 kg/s at a constant value of the inlet pressure and \( \sin \alpha = 0.10 = const \). The final value of the dimensionless time was 12.5 (25000 approximations) with a constant dimensionless time step of 0.0002. The length step was 0.001.

No more than four steps of the tangent method were required to satisfy the conditions \( F(\bar{t}^{k+1}) < 10^{-10} \) and \( |\bar{t}^{k+1} - \bar{t}^{k}| < 10^{-8} \) to the accuracy of velocity determination at the inlet to the section. With an increase in the time steps number, the maximum differences between the velocity (pressure) indices of two successive time steps decreased monotonically. So, at \( n=1000 \) the values were: \( \Delta u_{max} = 1.697_{10} - 4 \), \( \Delta p_{max} = 1.607_{10} - 4 \); at \( n=50000 \) \( -3.934_{10} - 8 \), \( 1.291_{10} - 7 \); at \( n=125000 \) \( -2.464_{10} - 10 \), \( 8.285_{10} - 10 \).

A fragment of the mass flow isolines for a constant diameter of a gas pipeline at an inlet pressure \( p(0,0) = 5.6 \text{ MPa} \) is shown in Figure 2.

![Mass Flow Isolines](image_url)

**Figure 2.** Fragment of mass flow isolines at an abrupt change in the outlet mass flow from 250.0 kg/s to 300.0 kg/s

The steady state of the stationary process was checked by the fulfillment of the mass conservation law: at \( n=125000 \), the dimensionless mass flow rate at the inlet to the section was 1.1345432. Increasing monotonically, it reached 1.134615 at the outlet from the section. All this testifies to the stability of the computing process.

Similar isolines were plotted for the gas flow rate and hydrostatic pressure (Figure 3).
Figure 3. Fragment of pressure isolines at an abrupt change in the outlet mass flow rate from $250.0 \, \text{kg/s}$ to $300.0 \, \text{kg/s}$

The final results for the gas pressure and velocity were compared with the results obtained using the formulas for the pressure and velocity in the stationary case at $M_0 = 300.0 \, \text{kg/s}$ and $p_0 = 5.6 \, \text{MPa}$. The maximum discrepancy between the results did not exceed $5.0 \times 10^{-5}$.

As seen from the results, an abrupt increase in the intensity of gas offtake at the inlet to the section leads to the depression wave formation, and at the beginning of the process it propagates against the flow direction, and at $tc/l = 1$, near the inlet to the section, a zone with a negative mass flow rate and, accordingly, a zone with a negative flow velocity are formed.

Similar results were obtained at a local sinusoidal change in diameter. The final results, after the steady-state mass flow rate at the outlet, are shown in Fig. 4.

Compared to the case at $250.0 \, \text{kg/s}$, the velocity has increased. A decrease in the velocity value at the outlet from the section with an increase in the variable diameter section is due to a significant decrease in the pressure drop at an increase in diameter.

Figure 4. Velocity distribution under the steady state at the outlet mass flow of $300.0 \, \text{kg/s}$
The curves were plotted under the steady state at the outlet mass flow of $300.0 \text{ kg/s}$. In fact, a constant mass flow solution was expected. In four diameter options, the error does not exceed 0.15%.

4. Conclusions
The quasi-one-dimensional equation of conservation of momentum for pipeline gas transport was derived taking into account the local and convective components of the inertia force, the quadratic law of resistance and the force of gravity with a variable cross-sectional area of a pipeline. The mass conservation equation takes into account the variability of the pipe diameter, and the gas state equation considers the real properties of gases.

At the inlet to the section, a time change in the hydrostatic pressure is set, and at the outlet from the section the gas mass flow rate is set. The initial distribution of gas-dynamic indices was taken as a stationary mode of operation.

The equations were reduced to the equations of direct and reverse traveling waves and presented in dimensionless variables, approximated by an implicit scheme, and taking into account the direction of excitation propagation. By virtue of the nonlinearity of the equations being solved and the boundary conditions, iterative processes were drawn up.

A calculation program was developed that makes it possible to study the process dynamics depending on the time change in the inlet pressure and the outlet mass flow rate under a path change in the diameter and leveling height of the pipeline axis.

The results of separate calculations for transient processes are given for the case, when at the end of the section, the mass flow rate increases abruptly, and the pipe diameter has a local increase in a sinusoidal law.

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