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ABSTRACT

When a laser beam is incident on a double-slit interferometer without turbulence, the classic Young’s double-slit interference is present in the first-order measurement of the mean photon number (or intensity), while the second-order measurement of photon number fluctuation correlation (or intensity fluctuation correlation) yields a trivial constant. When optical turbulence is introduced, it destroys the classic interference present in the measurement of the photon number; however, two-photon interference appears in the measurement of photon number fluctuation correlation. This interesting observation means that the observed two-photon interference is not only observable through turbulence, i.e., turbulence-free, but also induced by the turbulence itself. Turbulence-free two-photon interference induced by the turbulence itself allows for interferometric sensing through strong turbulence when coherent radiation, such as a laser, is applied.

Interferometers are powerful tools that utilize the superposition of radiation fields to execute precise and sensitive measurements.1–6 Included in this field are optical-correlation-based interferometers, which utilize thermal, chaotic light such as the Hanbury Brown–Twiss interferometer and many more.7–15 Despite their functionality, optical turbulence may turn these powerful tools useless. Recently, we reported a turbulence-free double-slit interferometer, inspired by the Hanbury Brown–Twiss interferometer,7,8,16 in which an incoherent thermal field was able to produce a turbulence-free two-photon interference pattern from the second-order measurement of photon number fluctuation correlation (PNFC) \(\langle \Delta n_1 \Delta n_2 \rangle\), or intensity fluctuation correlation \(\langle \Delta I_1 \Delta I_2 \rangle\), while no classic interference was observable from the first-order measurement of mean intensities \(\langle I_1 \rangle\) and \(\langle I_2 \rangle\).10,17 Can we observe turbulence-free interference from an interferometer that employs the coherent laser beam as the light source? The second-order coherence of the laser beam has been studied since the invention of the laser. Different from thermal fields, which are a collection of a large number of distinguishable photons in a mixed state, a coherent field is a collection of a large number of indistinguishable photons in a pure state. This difference causes the photon number fluctuation correlation, or intensity fluctuation correlation, of a pure coherent state to be zero, \(\langle \Delta n_1 \Delta n_2 \rangle \propto \langle \Delta I_1 \Delta I_2 \rangle = 0\). In short, this result is often explained by the fact that a thermal field is traditionally considered a Gaussian field due to the Gaussian distribution of random phases and the non-trivial second-order correlation of the thermal field was an intrinsic property of Gaussian fields.7,9,18 A laser field is non-Gaussian and can be approximated as a coherent state producing no correlation.

Currently, coherent light sources such as lasers are widely used in interferometers partially due to the high degree of spatial coherence and well collimated beam compared to incoherent thermal light sources. When a coherent laser beam is incident on a double-slit, without turbulence, classic Young’s double-slit interference can be easily observed from the measurement of mean photon number \(\langle n \rangle\), or mean intensity \(\langle I \rangle\).1–6 When optical turbulence is introduced into the interferometer, it may blur the interference pattern completely.20,21 The turbulence introduces random phase shifts following slit-A and slit-B that vary rapidly, randomly, and independently. This turns a single coherent state, representing a group of identical photons, into a mixture of two separate, distinguishable groups of identical photons in coherent states \(A\) and \(B\) with varying random relative phases from the turbulence. The incoherent superposition of coherent state \(A\) and coherent state \(B\) is unable to produce any classic...
interference pattern. Is it possible to observe turbulence-free second-order interference from a laser-based interferometer? Perhaps, no one would even expect observing any nontrivial second-order correlation from a laser beam since the laser field is non-Gaussian, so why should we expect the same turbulence-free two-photon interference mechanism? Surprisingly, in a recent experiment, we observed turbulence-free two-photon interference from the second-order correlation measurement of photon number fluctuations, or intensity fluctuations, of a Young’s double-slit interferometer, which not only employed a laser beam as the light source but also was under the influence of strong turbulence. How could a measurement of photon number fluctuation correlation, or intensity fluctuation correlation, on a laser beam produce a non-trivial sinusoidal function? Why is this interference pattern seemingly turbulence-free but also only present due to the turbulence itself? We address these questions in this latter after describing our experimental observations.

The experimental setup is schematically depicted in Fig. 1. The light source of the interferometer is a CW (continuous wave) Nd:YVO₄ laser beam with a wavelength of \( \lambda = 532 \text{ nm} \). A beam expander with a well-designed spatial filter was used to increase the diameter of the TEM₀₀ laser beam from 2.25 mm to 22.5 mm. The expanded beam was incident on a standard Young’s double-slit interferometer with a slit separation of \( d = 2.5 \text{ mm} \). The slit width was \( \approx 100 \mu \text{m} \), which was narrow enough to be approximated as line-like for our measurements, meaning that the observed interference pattern remained a constant amplitude with no observed diffraction envelope. In this experiment, optical turbulence was introduced by a set of kilowatt heating elements beneath the optical paths of the interferometer. The heating elements heat the air introducing temperature variations and random airflow, thus inducing random optical index variations, i.e., optical turbulence, between the double-slit and the observation plane. To detect the radiation at more precise spatial locations, point-like tips of single-mode optical fibers were used to interface the light into the single-photon counting detectors, \( D_1 \) and \( D_2 \). A Photon Number Fluctuation Correlation (PNFC) circuit uses a series of measurements (in this case, 300,000) to determine the mean photon number and photon number fluctuations for each detector while simultaneously calculating the photon number correlation, \( \langle n(x_1) n(x_2) \rangle \), and photon number fluctuation correlation, \( \langle \Delta n(x_1) \Delta n(x_2) \rangle \).

We did not observe any surprises from the measurement of first-order classic interference. As expected, when the heating elements were powered off, the observed classic interference pattern achieved \(100\%\) visibility and when the heating elements were powered on, the interference pattern was blurred out by the turbulence. This confirms that the optical paths from each slit were experiencing different random phase shifts from the turbulence. The second-order measurements of the photon number correlation \( \langle n(x_1) n(x_2) \rangle \) and the photon number fluctuation correlation \( \langle \Delta n(x_1) \Delta n(x_2) \rangle \) were interesting. When the heating elements were powered off, we observed \(100\%\) visible interference in the measurement of \( \langle n(x_1) n(x_2) \rangle \) [Fig. 2(a)], while the measurement of \( \langle \Delta n(x_1) \Delta n(x_2) \rangle \) yielded a constant of \( \approx 0 \) [Fig. 3(a)]. When the heating elements were powered on, even though interference in the measurement of \( \langle n(x_1) \rangle \), for \( j = 1, 2 \), was blurred completely, interference in the measurement of \( \langle n(x_1) n(x_2) \rangle \) was still present; however, the visibility was significantly reduced [Fig. 2(b)]. Surprisingly, an interference pattern appeared in the measurement of \( \langle \Delta n(x_1) \Delta n(x_2) \rangle \), as shown in Fig. 3(b). Interestingly, the observed interference pattern is different than that of thermal light; in other words, the turbulence induced two-photon interference pattern indicates not only “correlation” with \( \langle \Delta n_1 \Delta n_2 \rangle > 0 \) but also “anticorrelation” with \( \langle \Delta n_1 \Delta n_2 \rangle < 0 \).

The observation of classic interference from the coherent laser beam without turbulence is easily understood. In the following, we analyze the measurement processes of mean photon number \( \langle n(x_1) \rangle \propto \langle I(x_1) \rangle \) and photon number fluctuation correlation \( \langle \Delta n(x_1) \Delta n(x_2) \rangle \propto \langle \Delta I(x_1) \Delta I(x_2) \rangle \) from the turbulence disturbed double-slit interferometer.

A laser beam contains a group of large number of identical photons, usually approximated as a single coherent state. When turbulence is introduced into a Young’s double-slit interferometer following slit-\( A \) and slit-\( B \), the original group of identical photons is divided into two distinguishable groups of identical photons with random relative phases. We may consider the measured field at photodetector \( D_j \), for \( j = 1, 2 \), as the superposition of two distinguishable subfields where each contains a large number of identical photons,

\[
E(x_j, t_j) = E_A(x_j, t_j) + E_B(x_j, t_j),
\]

where the subfields are mutually orthogonal, i.e.,

\[
\langle E_A(x_j, t_j) \rangle = 0, \quad \langle E_B(x_j, t_j) \rangle = 0, \quad \langle E_A(x_j, t_j) E_B(x_j, t_j) \rangle = \lambda^2 \langle \Delta n(x_j) \Delta n(x_j) \rangle.
\]
where $j = 1, 2$, and each subfield is modeled as

$$E_i(x_j, t_j) = \int d\omega E_i(\omega) g_i(\omega; x_j, t_j) e^{-i\delta\phi_{ij}(t_j)} \quad (2)$$

for $i = A, B$ and the Green’s function, or the propagator, is

$$g_i(\omega; x_j, t_j) = e^{-i\omega(t_j - \tau_{ij}/c)} \approx e^{-i\omega(t_j - (z - x_i x_j/z)/c)}, \quad (3)$$

where we have made a far-field approximation. This Green’s function propagates the $\omega$ mode of the $i$th subfield, $E_i(\omega)$, from slit-$i$ to a point-like photodetector $D_j$. It should be noted that the superposition presented in Eq. (1) is different than if a thermal or pseudo-thermal light source was used. In those cases, instead of a single subfield passing through each slit, a larger number of subfields would be present. This would require a summation over all of the subfields in each slit and produce a different result, which has been documented previously. This difference is due to the inherent difference between thermal and coherent light. While a coherent state is a collection of a large number of identical photons, a thermal state is a collection of a large number of distinguishable photons with random phases. This random distribution of phases typically follows a Gaussian distribution; hence, why it is considered a Gaussian field? Often, a rotating ground glass is used to scatter coherent light into a thermal state due to the introduction of random phases to individual photons or small groups of photons. After artificially “thermalizing” the coherent light, we label this as pseudo-thermal light due to how closely it represents a true thermal light source. The turbulence used for this experiment is not strong enough to thermalize a laser beam into a Gaussian field but can introduce random phases between the $A$-path and $B$-path of the Young’s double-slit interferometer.

In Eq. (2), the $\delta\phi_{ij}(t)$ term is the turbulence induced random phase shift along path-$ij$.

$$\delta\phi_{ij}(t) \approx \frac{\omega}{c} \int_{t_i}^{t_j} d\tau \delta n(\tau, t). \quad (4)$$

The turbulence-induced variation of the refractive index of the medium, $\delta n(\tau, t)$, takes a random value from time to time or from measurement to measurement. In this experiment, each data point is acquired from $\sim 300\,000$ measurements, each with a time window.
of 100 µs resulting in a total collection time of ~30 s. The rapid fluctuations within the turbulent air would introduce a wide range of random phase shifts (from 0 to 2π) to the subfields, allowing us to approximate the time average as an ensemble average. The ensemble average of the first-order incoherent superposition of the two turbulence affected subfields gives a constant mean value for the measurement of <n(x,i)>,

\[
\langle n(x, i) \rangle \approx \langle I(x, i) \rangle \approx \langle |E_A(x, t, i) + E_B(x, t, i)|^2 \rangle
\]

\[
= \langle \{E_A(x, t, i)^2 \} + \{E_B(x, t, i)^2 \} \rangle
\]

\[
+ \langle E_A(x, t, i)E_B(x, t, i) \rangle
\]

\[
+ \langle E_A(x, t, i)E_B^*(x, t, i) \rangle
\]

\[
\approx n_0. \tag{5}
\]

Although the fields passing through slit-A and slit-B are initially in phase with each other, the turbulence affects each one randomly. While \(\langle \delta \phi_A - \delta \phi_B \rangle\) varies from time to time and from measurement to measurement, the cross terms yield \(\langle E_A(x, t, i)E_B(x, t, i) \rangle = \langle E_A(x, t, i)E_B^*(x, t, i) \rangle = 0\) when taking into account all possible random relative phases introduced by the strong turbulence.

We can define the strength of the turbulence by analyzing the ensemble average of just the contributions from the turbulence, \(T_{AB}\), in the cross terms such that Eq. (5) can be written as \(\langle n(x, i) \rangle \approx n_0[1 + T_{AB} \cos(kd\Delta z/2\pi)]\). Assuming a Gaussian distribution of phase shifts resulting from turbulence, it has been shown that \(T_{AB}\) can be written as \(22-24\)

\[
T_{AB} = e^{(d \phi_A(t) - d \phi_B(t))}
\]

\[
= e^{|(\Delta x_A - \Delta x_B)|^2} = e^{-\Delta x_A^2 \Delta x_B^2 / \sigma^2}, \tag{6}
\]

where \(|\Delta x_A - \Delta x_B| = d\) is the slit separation and \(x_0 = (0.545C_k^2 k^2 z)^{-1/2}\),

\[
\chi_0 = \frac{0.545C_k^2 k^2 z}{(2\pi/\lambda)}, \tag{7}
\]

which is a function of the wavenumber in which \(k = 2\pi/\lambda\), \(z\) is the distance from the double-slit plane to the detection plane, and \(C_k^2\) is the refractive index structure parameter. This structure parameter is commonly used to quantify the strength of the refractive index throughout the medium. When the turbulence is strong enough to force \(x_0 \ll d\), \(T_{AB}\) approaches zero, meaning that the subfields passing through slit-A and slit-B become incoherent and no interference is observable. When the turbulence is weak and \(x_0 \gg d\), the subfields passing through each slit are still coherent and produce interference. Thus, we can define \(x_0\) as the reduced coherence length of the light when propagating through the turbulent medium. Similarly, if desired, we could include the contribution of turbulence following only a single slit, \(T_{AA}\) and \(T_{BB}\), but these would be dependent on the width of the single slits. In our measurements, the individual slits were narrow enough such that their widths were less than the reduced coherence length introduced by the turbulence.

For a realistic measurement, there may not be enough statistics for \(\langle \delta \phi_A - \delta \phi_B \rangle\) to take all possible random values and may not vanish completely. The remaining cross terms would fluctuate over time and can be labeled as “photon number fluctuations,”

\[
\Delta n(x, t) = \Delta I(x, t) = \langle E_A^*(x, t, i)E_B(x, t, i) \rangle
\]

\[
+ \langle E_A(x, t, i)E_B^*(x, t, i) \rangle = 0. \tag{8}
\]

Without the turbulence present, these cross terms would be the interference terms present in classic Young’s double-slit interference, but with turbulence, they fluctuate rapidly over time, viewed as a “blurring” of the interference pattern. In a certain sense, these random fluctuations are induced by the turbulence. While these fluctuations are a direct result of turbulence, they are directly comparable to fluctuations inherently present in thermal light. In order to produce photon number (intensity) fluctuations, distinguishable photons (or subfields in the classical language) must be present. In a thermal source, the photons (subfields) are distinguishable at the moment of emission, but here, with a coherent source, the photons (subfields) are identical until they encounter different phase shifts after slits A and B due to the presence of turbulence.

When a second photodetector is introduced, either with a beam splitter as done in the presented experiment or an array of photodetectors in the form a CCD or CMOS array, we can measure the photon number correlation between the two. Following a double-slit, this is calculated as

\[
\langle n(x_1) n(x_2) \rangle \approx \langle I(x_1) I(x_2) \rangle
\]

\[
= \langle E^*(x_1, t, i)E(x_1, t, i)E^*(x_2, t, i)E(x_2, t, i) \rangle
\]

\[
\times \left[ \langle E_A^*(x_1, t, i)E_B(x_1, t, i)E(x_2, t, i)E_B^*(x_2, t, i) \rangle \right. \]

\[
\left. \times \langle E_A(x_1, t, i)E_B^*(x_1, t, i)E(x_2, t, i)E_B(x_2, t, i) \rangle \right], \tag{9}
\]

resulting in 16 terms of expectation values to calculate. Without the turbulence present, this results in the trivial product of the mean photon number of each detector, \(\langle n(x_1) \rangle \langle n(x_2) \rangle = \langle n(x_1) \rangle \langle n(x_2) \rangle\). However, with the turbulence-induced photon number fluctuations present, the correlation results in

\[
\langle n(x_1) n(x_2) \rangle \approx \langle I(x_1) I(x_2) \rangle
\]

\[
\approx \langle \{E_A(x_1, t, i)^2 \} \{E_B(x_2, t, i)^2 \} \rangle
\]

\[
+ \langle \{E_B(x_1, t, i)^2 \} \{E_A(x_2, t, i)^2 \} \rangle
\]

\[
+ \langle \{E_A(x_1, t, i)^2 \} \{E_B(x_2, t, i)^2 \} \rangle
\]

\[
+ \langle \{E_B(x_1, t, i)^2 \} \{E_A(x_2, t, i)^2 \} \rangle
\]

\[
+ \langle E_A(x_1, t, i)E_B(x_1, t, i)E_B^*(x_2, t, i)E_B(x_2, t, i) \rangle
\]

\[
+ \langle E_A(x_1, t, i)E_B(x_1, t, i)E_B^*(x_2, t, i)E_B(x_2, t, i) \rangle
\]

\[
+ \langle E_A(x_1, t, i)E_B^*(x_1, t, i)E_B^*(x_2, t, i)E_B(x_2, t, i) \rangle
\]

\[
= \langle n(x_1) \rangle \langle n(x_2) \rangle + \Delta n(x_1) \Delta n(x_2), \tag{10}
\]

where, for brevity, we have dropped terms that have no contribution to the measurement of \(\langle n(x_1) \rangle \langle n(x_2) \rangle\) or cannot survive the ensemble average by taking into account all possible turbulence introduced random phases along the A-path and the B-path. At first glance, one would say that the remaining cross terms would also average to zero due to the presence of random phases, thus resulting in \(\langle n(x_1) \rangle \langle n(x_2) \rangle = \langle n(x_1) \rangle \langle n(x_2) \rangle\) once again. However, it has been demonstrated by Smith and Shih10,11 that by scanning \(D_1\) in the neighborhood of \(D_2\) such that \(x_1 = x_2\), the optical paths of the two alternatives represented in these cross terms overlap in space-time and experience the same

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turbulence. These turbulence-induced random phases cancel, allowing the terms to survive the ensemble average. These terms that are insensitive to turbulence are of interest and can be measured directly via photon number fluctuation correlation, which, when scanning $D_1$ in the neighborhood of $D_2$, results in

$$\langle \Delta n(x_1) \Delta n(x_2) \rangle \approx \langle \Delta l(x_1) \Delta l(x_2) \rangle$$

$$= \langle E_A^* (x_1, t_1) E_B (x_1, t_1) E_B^* (x_2, t_2) E_A (x_2, t_2) \rangle$$

$$+ \langle E_A (x_1, t_1) E_B^* (x_1, t_1) E_B (x_2, t_2) E_A^* (x_2, t_2) \rangle$$

$$\approx \cos \frac{2 \pi d}{\lambda z} (x_1 - x_2),$$

(11)

which matches our experimental results, as shown by the plotted curve in Fig. 3(b) that has only been adjusted to match the amplitude of the interference present in the data. The presented experiment achieved perfect second-order temporal correlation, thus allowing us to approximate any time dependence as negligible, due to the relatively narrow bandwidth of the laser beam such that the coherence time of the field is much greater than the response time of the photodetection electronics. Recall that it was the presence of the random phases that resulted in the interference terms becoming photon number (intensity) fluctuations. This turbulence-free correlation process (with the use of the path overlap) is required to cancel the random phases and "recover" the interference. However, where classic Young’s double-slit interference can be explained as single-photon or first-order interference, the new interference pattern is a result of two-photon or second-order interference. To make the presence of two-photon interference more clear, we will consider a simple example. Two distinguishable coherent subfields $E_A$ and $E_B$ to produce a joint photodetection event at $D_1$ and $D_2$: (1) $E_A$ is detected at $D_1$ and $E_B$ is detected at $D_2$ and (2) $E_A$ is detected at $D_2$ and $E_B$ is detected at $D_1$. Borrowing language from Dirac, this represents a pair of distinguishable coherent subfields interfering with the pair itself. Using the quantum language elaborated upon in the supplementary material, we may name this phenomena turbulence-induced "two-photon" interference.

It is interesting to find, from Eq. (11), that the interference from the measurement of $\langle \Delta n(x_1) \Delta n(x_2) \rangle$ results in a positive value, indicating a "correlation," when the above two alternatives superpose constructively and results in a negative value, indicating an "anticorrelation," when the above two alternatives superpose destructively. The constructive superposition forces the measured photon number to fluctuate the same, positive–positive or negative–negative fluctuations, while the destructive interference forces the measured photon number to fluctuate the opposite, positive–negative or negative–positive fluctuations (i.e., if one fluctuates positively, the other one must fluctuate negatively, and vice versa). It has been a common understanding that photon number fluctuation correlation (intensity fluctuation correlation) is typically observable from the thermal field, so it should be noted that the presented result of photon number fluctuation correlation (intensity fluctuation correlation) is different than when thermal light is used, which produces entirely positive sinusoidal interference.1–3,5–9 This is because the applied turbulence is not strong enough to thermalize the laser beam that is passing through a single-slit. Again referring to the coherence length presented in Eq. (6), the strength of the turbulence would have to be strong enough to have the turbulence reduce the coherence length to less than the width of the slits. In rare cases, this may occur, but this was not taken into account in this Letter. In such a scenario, the results would resemble those of a fully thermalized, spatially coherent field.

In summary, we have reported an experimental study of turbulence-induced interference from the Young’s double-slit interferometer utilizing a coherent laser beam. The interference pattern indicates photon number fluctuation, or intensity fluctuation, correlation, and anticorrelation corresponding to constructive and destructive two-photon interferences. Without turbulence, the classic Young’s double-slit interference is present in the measurement of the photon number (or intensity), while the measurement of photon number fluctuation correlation (or intensity fluctuation correlation) yields a constant. When optical turbulence is introduced, it destroys the classic interference present in the measurement of the photon number; however, it induces interference in the measurement of photon number fluctuation correlation. While this turbulence-free mechanism has been demonstrated with an incoherent thermal light source, in general, coherent light sources such as lasers are more widely used in interferometers due to the well collimated beam and high spatial coherence. Hence, in addition to being fundamentally interesting, the mechanism of “turbulence-induced,” but also turbulence-free, two-photon interference of a laser beam may be helpful for these applications.

See the supplementary material for the calculation of the second-order coherence function in the Heisenberg picture.

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DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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