Analysis of material particle motion and optimizing parameters of vibration of two-mass GZS vibratory feeder

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Abstract. The structure and kinematics of the two-mass GZS vibratory feeder operation are considered. It is established that the movement of the material's particles on the feeder surface determines its capacity. The development and analysis of the mathematical model of material's particle movement on the two-mass GZS vibratory feeder surface are shown. The results of Matlab optimization of material particles velocity function are given that allows setting rational kinematics of the feeder.

1. Introduction
The development of the mining industry along with complex use of ores [4] requires constant improvement of technological equipment at all stages of mining: extraction, transportation and enrichment of minerals [3]. The vibratory feeder is a forced vibrating system widely used in many industries for feeding material from hoppers, bins, silos and storage piles to belt conveyors, crushers and other technological equipment [2]. A two-mass GZS vibratory feeder is now being widely used in many fields of industries [9]. The structure of this machine is shown in figure 1, which includes electric motor frame (4) and the electric motor for driving oscillation (6) combined to form an oscillation driven unit through forced springs (3) transmitted to feed trough (2) that makes the feed trough oscillate the pendulum to perform the feeding process [5].

Figure 1. Structure of two-mass GZS vibratory feeder:
1 - suspension springs; 2 - feed trough; 3 - forced spring; 4 - electric motor frame; 5 - multi-layer rubber; 6 - electric motor.

The WangNanNan studies refer to the inherent frequency of the two-mass GZS vibratory feeder [10]; RuanWenSu applies ADAMS software to simulate dynamics of the two-mass GZS vibratory feeder and study dynamic graphs of the two-mass GZS vibratory feeder [6]. However, there has been no author mentioning the issue of improving the capacity of the machine. Therefore, in this article the authors go deeply into the research on studying the working principle of the machine and the movement of the grain on the surface of the feeder in the working process. Aside from that, Matlab is utilized to build the analysis program to find out values of some parameters which result in maximum operating capacity.

2. Analysis of the movement of particle material on the surface of the feeder

2.1 Movement trajectory of feed trough

Particle movement on the conveying surface of the feeder depends on many parameters [1]. The trajectory of the feeder is elliptically shaped as shown in figure 2 [7]. The equations of motion of the feeder in the $x$-axis and $y$-axis are given by formulæ (1), (2) and a graph as shown in figure 3:

\[
\begin{align*}
    x &= \lambda_x \cdot \sin(\alpha + \beta_x), \\
    y &= \lambda_y \cdot \sin(\alpha + \beta_y),
\end{align*}
\]

where: $\lambda_x$ and $\lambda_y$ - amplitude of oscillation in the $x$-axis and $y$-axis:

\[
\begin{align*}
    \lambda_x &= \lambda \sqrt{\cos^2 \delta + \xi^2 \sin^2 \delta}, \\
    \lambda_y &= \lambda \sqrt{\sin^2 \delta + \xi^2 \cos^2 \delta}.
\end{align*}
\]

where $\lambda$ - semi-major axis of the ellipse; $\xi$ - oval, which is the ratio of the major axis to the minor axis of the ellipse ($0 \leq \xi \leq 1$); $\delta$ - angle of oscillation; $\alpha_0$ - angle of inclination of the conveying surface of the feeder; $\omega$ - angular velocity; $t$ - time; $\beta_x$ and $\beta_y$ - relatively original positions in the $x$-axis and $y$-axis:

\[
\begin{align*}
    \beta_x &= \arctan(-\xi \tan \delta) \\
    \beta_y &= \arctan(-\xi \cot \delta)
\end{align*}
\]

Figure 2. A trajectory of the feed trough

Figure 3. A position of the feeder in the $x$ and $y$ direction

2.2 Velocity and acceleration of feed trough

From formulæ (1) and (2), let us obtain the velocity and acceleration of the feed trough as follows:

\[
v_x = \omega \lambda_x \cos(\alpha + \beta_x)
\]

\[
\begin{align*}
    v_y &= \omega \lambda_x \sin(\alpha + \beta_x), \\
    a_x &= -\omega^2 \lambda_x \sin(\alpha + \beta_x), \\
    a_y &= \omega^2 \lambda_x \cos(\alpha + \beta_x).
\end{align*}
\]
\[
\begin{align*}
    v_y &= \omega \lambda_y \cos(\alpha + \beta_y) \quad (8) \\
    a_x &= -\omega^2 \lambda_y \sin(\alpha + \beta_x) \quad (9) \\
    a_y &= -\omega^2 \lambda_y \sin(\alpha + \beta_y). \quad (10)
\end{align*}
\]

2.3 The force acting on the material particle

The authors do not mention the interaction force between the material particle on the feeder tray of the two-mass GZS vibratory feeder, and that they suffer from acting forces as shown in figure 4. The forces in the \(x\)-axis and \(y\)-axis are:

\[
\begin{align*}
F_x &= -m(a_x + \Delta \dot{x}) + G \sin \alpha_0 \\
F_n &= m(a_y + \Delta \dot{y}) + G \cos \alpha_0
\end{align*}
\]

(11) (12)

where \(m\) and \(G\) - mass and weight of materials; \(\Delta x, \Delta y, \Delta \dot{x}, \Delta \dot{y}, \Delta \ddot{x}, \Delta \dddot{y}\) - relative positions, velocity and acceleration of the material particle with feed trough in \(x\)-axis and \(y\)-axis.

Let us replace (10) with (12):

\[
m \Delta \dot{y} = F_n - G \cos \alpha_0 + m \omega^2 \lambda_y \sin(\alpha + \beta_y). \quad (13)
\]

2.4 Sling index \(D\)

When the material starts to move instantly along the \(y\)-axis, acceleration \(\Delta \ddot{y} = 0\), pressure \(F_n = 0\) in (13) give:

\[
\begin{align*}
\begin{cases}
m \omega^2 \lambda_y \sin \varphi_{dy} - G \cos \alpha_0 = 0 \\
\varphi_{dy} = \alpha_{dy} + \beta_y
\end{cases}
\end{align*}
\]

(14)

\(\varphi_{dy}\) - first nominal sling angle; \(\alpha_{dy}\) - first sling angle.

Let us take \(D = \frac{1}{\sin \varphi_{dy}}\) and replace it with (14), then:

\[
\begin{align*}
\begin{cases}
D = \frac{\omega^2 \lambda_y}{g \cos \alpha_0} \\
\varphi_{dy} = \arcsin(\frac{1}{D})
\end{cases}
\end{align*}
\]

(15)
\(D\) is a sling index: when \(D < 1\), equation (15) has no result, i.e. there is no sling motion on the material; when \(D \geq 1\), there would be the sling motion on the material. As a result, in order to induce the sling motion, sling index \(D\) must be not less than 1.

2.5 Sling angle \(\theta_d\) and sling coefficient \(i_D\).

When the material is moving, pressure \(F_n = 0\), instead of (13) one has:

\[
\begin{align*}
\begin{cases}
 m\ddot{y} = -G \cos \alpha_0 + m\omega^2 \lambda_y \sin \varphi_y \\
 \varphi_y = \alpha + \beta_y
\end{cases}
\end{align*}
\]

To calculate the acceleration’s integral of \(\Delta y\) twice, let us obtain the relative position:

\[
\Delta y = -\lambda_y \left[ \sin \varphi_{dy} - \sin \varphi_y + \cos \varphi_{dy} \left( \varphi_y - \varphi_{dy} \right) - \frac{1}{2} \sin \varphi_{dy} \left( \varphi_y - \varphi_{dy} \right)^2 \right].
\]

When \(\varphi_y = \varphi_{ci}\), then \(\Delta y = 0\); simplifying formula (17), let us obtain:

\[
\begin{align*}
\begin{cases}
 \text{ctg} \varphi_{dy} = \frac{0.5 \theta_d - \left(1 - \cos \theta_d\right)}{\theta_d - \sin \theta_d} \\
 \theta_d = \varphi_{ci} - \varphi_{dy}
\end{cases}
\end{align*}
\]

where \(\theta_d\) - sling angle; \(i_D = \frac{\theta_d}{2\pi}\) - sling coefficient.

According to formulae (15) and (18), there is a correlation between \(i_D\) and \(D\):

\[
D = \left[ \left( \frac{2\pi i_D^2 + \cos 2\pi i_D - 1}{2\pi i_D - \sin 2\pi i_D} \right)^2 + 1 \right]^{\frac{1}{2}} = \left[ \left( \frac{0.5 \theta_d^2 + \cos \theta_d - 1}{\theta_d - \sin \theta_d} \right)^2 + 1 \right]^{\frac{1}{2}}.
\]

From formula (19), one has relation graph \(D\) and \(i_D\) as shown in figure 5.

![Figure 5. Relation between \(D\) and \(i_D\)](image-url)
2.6 Average velocity of material particle
As shown in figure 4, the acceleration of the particle in the feeding direction is calculated as follows:

$$\Delta \ddot{x} = g \sin \alpha_0 - \alpha_x.$$  \hspace{1cm} (20)

Let us replace (9) with (20), then:

$$\Delta \ddot{x} = g \sin \alpha_0 - \alpha_x = g \sin \alpha_0 + \omega^2 \lambda_x \sin \varphi_x.$$  \hspace{1cm} (21)

After doubling integral $\Delta \ddot{x}$, let us determine the relative position:

$$\Delta x_d = \lambda_x (-\sin \varphi_{x_d} + \sin \varphi_{d_x} + \theta_d \cos \varphi_{d_x}) + \frac{g \sin \alpha_0}{2 \omega^2} \theta_d^2,$$  \hspace{1cm} (22)

where $\varphi_{x_d} = \varphi_{d_x} = \alpha_x + \beta_x$ and $\theta_d = 2 \pi D$.

Then the average velocity of the particle is:

$$v_d = \frac{\Delta x_d}{2\pi \omega} = \frac{\omega \lambda_x}{2\pi} \left[ \sin \varphi_{d_x} (1 - \cos 2\pi D) + \cos \varphi_{d_x} (2 \pi D - \sin 2\pi D) \right] + \frac{\pi D^2}{2} \omega \lambda_x \tan \alpha_0$$

$$= \frac{\omega \lambda_x}{2\pi} \left[ \sin \varphi_{d_x} (1 - \cos 2\pi D) + \cos \varphi_{d_x} (2 \pi D - \sin 2\pi D) \right] + \frac{\pi D^2}{2} \omega \lambda_x \tan \alpha_0$$

$$+ \left(1 - \cos \varphi_{d_x}\right) \left[ \sin \theta_d - \tan \theta_d \right] \left( D^2 - 1 \right)^{1/2} - \tan \beta_x - \beta_y \right] + \frac{\theta_d^2}{4 \pi D} \omega \lambda_x \tan \alpha_0$$

Substituting formulae (3), (4) into formula (23) after transformation, one gets the following results:

$$v_d = \frac{g \cos \alpha_0 \cos (\beta_x - \beta_y)}{2\pi \omega} \left[ \frac{\left(1 + \xi^2 \tan^2 \delta\right)}{\xi^2 + \tan^2 \delta} \left[ \sin \theta_d - \tan \theta_d \right] \left( D^2 - 1 \right)^{1/2} \right]$$

$$- \tan (\beta_x - \beta_y) + \left(1 - \cos \theta_d\right) \left[ \sin \theta_d - \tan \theta_d \right] \left( D^2 - 1 \right)^{1/2} \tan (\beta_x - \beta_y) \right] + \frac{g \theta_d^2}{4 \pi \omega} \sin \alpha$$

The actual average moving velocity of particle $v_m$ [8] is:

$$v_m = C_{\alpha h} C_m C_w v_d$$  \hspace{1cm} (25)

where $C_{\alpha}$ - coefficient influenced by the inclined angle of the feed trough; $C_h$ - coefficient influenced by the thickness of the material layer; $C_m$ - coefficient influenced by material properties; $C_w$ - coefficient influenced by sliding motion.

3. Optimizing parameters to maximize the machine’s feeding capacity
The productive capacity of a two-mass GZS vibratory feeder [8] is:

$$Q = 3600 h B v_m \gamma,$$  \hspace{1cm} (26)

where $h$ - thickness of the material layer (m); $B$ - feeder width (m); $\gamma$ - bulk density of material (t/m$^3$).

Vibration intensity K of the feeder is:

$$K = \frac{\omega^2 \lambda}{g}.$$  \hspace{1cm} (27)

From formulas (27), (15), (3) and (4), let us obtain:

$$D = \frac{K \left( \sin^2 \delta + \xi^2 \cos^2 \delta \right)^{1/2}}{\cos \alpha_0}.$$  \hspace{1cm} (28)

Let us substitute formula (28) into formula (19):
\[
\frac{K^2\left(\sin^2 \delta + \xi^2 \cos^2 \delta\right)}{\cos^2 \alpha_0} = \left[\frac{0.5\theta_d^2 + \cos \theta_d - 1}{\theta_d - \sin \theta_d}\right]^2 + 1.
\] (29)

From formula (29) and formula (24), there is:

\[
v_d = f(K, \alpha_0, \delta, \xi) g \omega^{-1},
\]

where

\[
f(K, \alpha_0, \delta, \xi) = \frac{\cos \alpha_0 \cdot \cos(\beta - \beta_1)}{2\pi} \left(1 + \xi^2 \tan^2 \delta\right)^{1/2} \left[(\theta_d - \sin \theta_d) \right]
\]

\[
\left[(D^2 - 1)^{1/2} - \tan(\beta - \beta_1)\right] + (1 - \cos \theta_d) \left[1 + (D^2 - 1)^{1/2} \tan(\beta - \beta_1)\right] + \frac{\theta_d^2}{4\pi} \sin \alpha_0
\]

(31)

It is evident that when \( K, \alpha_0, \xi \) are fixed, one should select \( \delta \) appropriately so that \( v_d \) reaches the maximum value. And when \( K, \alpha_0, \delta \) are fixed, then one must choose \( \xi \) appropriately so that \( v_d \) reaches the maximum value (i.e. maximum feed capacity \( Q_{\text{max}} \)).

When \( \xi = 0.2 \) with different values of \( K \) and \( \alpha_0 \), there is correlation curve \( v_d = f(\delta) \) as shown in figure 6.

![Figure 6. Relation between \( v_d \) and \( \delta \)](image)

As can be seen from figure 6, it is obvious that when \( K, \alpha_0, \xi \) are fixed, \( v_d \) is directly proportional to \( \alpha_0 \); when \( \alpha_0, \delta, \xi \) are fixed, \( v_d \) is directly proportional to \( K \), depending on the increase of \( K \), \( \delta \) is attached in a small range.

Using Matlab [11], let us optimize average velocity \( v_d \) (formula 30) with boundary conditions:

\(-10^\circ \leq \alpha_0 \leq 10^\circ, \ 0 \leq \xi \leq 1, \ 10^\circ \leq \delta \leq 60^\circ, \ 0^\circ < \theta_d < 360^\circ\).

After running the program, let us obtain the following results:

\( \alpha_0 = 8.1287^\circ, \ \xi = 0.0140, \ \delta = 59.0274^\circ, \ \theta_d = 358.9982^\circ, \ v_d = 22.2297 g \omega^{-1} \).

From the results mentioned above, one can see that:
• When $\xi = 0.0140$, the trajectory of the feeding trough is nearly the same as the straight line and the average velocity reaches the maximum value of $v_d$ (i.e. the maximum feed capacity). Therefore, when designing the feeder, one should calculate so that the trajectory of the trough is straight.
• When $\theta_0 = 358.9982^\circ$, then $D = 3.29$ material is in the oscillation state, so for brittle materials, careful attention should be paid to an important factor when it comes to designing and selection.
• So that the two-mass GZS vibratory feeder could achieve the maximum working productivity, the design should be calculated in detail so that the motion trajectory of the trough was a straight line and the inclined angle of the feed trough should be $\alpha_0 = 8.1287^\circ$.

4. Conclusion
The article provides information on the principle of operation of the two-mass GZS vibratory feeder, analyses the motion of the material particle on the feeder’s surface, thereby building up the equation of motion, the equation of velocity and the acceleration equation of the material particle on the feeder’s surface during the working process of the machine.

The authors formulated the correlation between the parameters affecting the feeding process ($D$, $i_D$, $\theta_0$, $K$, $\alpha_0$, $\xi$, $m$, $\delta$ …). Besides, they devised the formula for calculating average velocity $v_d$ of the material particle, from which curve graphs ($v_d = f(\delta)$ and $v_d = f(\xi)$) can be constructed. Then the authors went further to the analysis of the effect of these parameters in the working process of the machine.

Ultimately, the Matlab software application was applied to optimize the objective function which is the average velocity of the material particle on feeder’s surface $v_d$, thereby finding out the reasonable parameters of the two-mass GZS vibratory feeder gaining maximum productivity.

Research results may be used for the purpose of reference by scientists when designing, calculating the two-mass GZS vibratory feeder.

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