Gauge vortex dynamics at finite mass of bosonic fields.

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(February 28, 2008)

Abstract

It is shown that the effective vortex dynamics resulting from an Abelian Higgs model is not necessarily governed by the Nambu-Goto action, provided the Higgs and vector boson masses are kept finite. This results in a slower contour motion characterized by the velocity suppressed, in the London limit, mainly as the ratio of the vector boson mass to its Higgs counterpart. The rate of the dissipationless helicity change is calculated. It is demonstrated how the conservation of the sum of the twisting and writhing numbers of the string is recovered despite the changing helicity.

PACS number(s): 11.27+d
Stringlike defects are widely discussed as the possible remnants surviving the epoch of phase transitions in the early Universe \[1,2\]. The dynamics underlying the evolution of the cosmic string network is usually assumed to be governed by the Nambu-Goto (NG) action \[1–3\]. Then a typical velocity of the string segment is of the order of that of light. It will be shown below that this is not necessarily so in the case of string in the gauge model with spontaneous symmetry breaking. Specifically, if the Higgs boson mass is much greater than the vector boson mass, the transverse velocity turns out to be much less than the velocity of light. The key point in reaching this conclusion is the finiteness of the masses of the bosonic fields. Setting these masses to infinity from the very start results in the NG effective action. Keeping these masses finite results in an effective string action different from the NG one. The specific form of the action will be found in the London limit (see below) of the Abelian Higgs model. The helicity of the mirror-noninvariant string configuration and its change in the course of the contour evolution will also be discussed, with the special attention paid to the role of the effects of the finite mass of the vector boson. As is known \[4,5\], the dynamics of such configurations, through the anomaly equation, may be important for the dynamics of some fermionic charges. It will be shown how conservation of the sum of the twisting and writhing numbers of the string contour is recovered in the case of the changing helicity.

We start with the Abelian Abrikosov-Nielsen-Olesen (ANO) string \[6\] in the Higgs model with the action

\[
S = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu}^2 + |(\partial_\mu + igA_\mu)\phi|^2 - \frac{\lambda^2}{2} \left( |\phi|^2 - \frac{\eta^2}{2} \right)^2 \right],
\]

where \(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\), and obtain the closed form of the string action in the London limit. This is the limit of \(m_H \gg m_V\); \(\ln m_H/m_V\) is also large, where \(m_V = g\eta/2\) and \(m_H = \lambda\eta\) are the masses of the gauge and Higgs bosons, respectively, and \(\lambda\) and \(\eta/\sqrt{2}\) are the Higgs field self-coupling and magnitude. The nonstationary field configuration of the gauge string is expressed through the spacetime dependent phase \(\chi \equiv \chi(x, t)\) of the Higgs field \(\phi(x, t) = \eta \exp(i\chi)/\sqrt{2}\). As is known, it is the dynamics of the phase \(\chi\) of the scalar field, not of its radial part (modulus), that is essential in the London limit. One can ignore the details of the Higgs field profile, taking it to be uniform \(\eta/\sqrt{2}\) in all coordinate space except the vortex line where it approaches zero at the characteristic distances \(\sim m_H^{-1}\).

The equation for the magnetic field is obtained upon varying the action (1). It looks as

\[
\nabla \times \mathbf{H} = \frac{m_V^2}{g} \nabla \chi - m_V^2 \mathbf{A},
\]

and can be solved in the momentum representation to give the magnetic field strength

\[
\mathbf{H}(\mathbf{k}, t) = \frac{2\pi}{g} \frac{m_V^2}{k^2 + m_V^2} \int d\sigma \mathbf{X}_a' \exp(-i\mathbf{k} \cdot \mathbf{X}_a)
\]

and the vector potential

\[
\mathbf{A}(\mathbf{k}, t) = \frac{2\pi}{g} \left( \frac{1}{k^2} - \frac{1}{k^2 + m_V^2} \right) \int d\sigma' \mathbf{k} \times \mathbf{X}_a' \exp(-i\mathbf{k} \cdot \mathbf{X}_a).
\]

The integral over \(\sigma\) comes from the equation for the phase \(\chi\) read off from Refs. \[7,8\], with the proper continuation to Minkowski spacetime.
\[ \nabla \times \nabla \chi(x, t) = 2\pi \oint d\sigma X_a' \delta^{(3)}[x - X_a(\sigma, t)], \]  

where \( X_a \equiv X_a(\sigma, t) \) is the evolving closed string contour \( a \) parametrized by the arclength \( \sigma \). Hereafter the prime over \( X \) will denote a derivative with respect to the corresponding parameter along the contour, while the overdot will do the time derivative. The case of many contours is embraced by taking the sum over individual contributions on the right hand side of Eq. (5). The winding number \( n \) of the scalar field is related to the magnetic flux via the condition of the vanishing covariant derivative of the Higgs field deep inside in the Higgs condensate,

\[ \oint A \cdot dl = \frac{1}{g} \oint \nabla \chi \cdot dl = \frac{2\pi n}{g} \equiv \Phi_0 n. \]

Hereafter \( n \) is taken to be unity.

To specify the dynamical part of the problem, one should write down the electric field strength \( E = -\nabla A_t - \partial_t A \), where

\[ A_t = -\frac{1}{g} \partial_t \chi \]

guarantees finite energy per unit length for the vortex in the nonstatic situation and replaces the condition \( A_t = 0 \) appropriate in the static case. One has

\[ E = \frac{1}{g} \nabla \partial_t \chi - \partial_t A = \frac{1}{g} (\nabla \partial_t - \partial_t \nabla) \chi + \partial_t \left( \frac{1}{g} \nabla \chi - A \right). \]

The commutator of the derivatives is nonzero in view of the singular character of the phase \( \chi \) \([7,8]\); so the Fourier component of \( E \) becomes

\[ E(k, t) = -\frac{2\pi}{g} \oint d\sigma (\dot{X}_a \times X_a') \exp[-i k \cdot X_a(\sigma, t)] \]

\[ + \frac{k^2}{k^2 + m_v^2} \partial_t v(k, t). \]

Hereafter the Fourier component of \( v(x, t) \equiv (1/g) \nabla \chi \) found from Eq. (5), is

\[ v(k, t) = \frac{2\pi}{g k^2} \oint d\sigma [k \times X_a'] \exp(-i k \cdot X_a). \]

Note that the vector potential \( A \) and the magnetic field strength \( H \) can also be expressed through the gradient of the singular phase \( \chi \) as

\[ A(k, t) = \left( 1 - \frac{k^2}{k^2 + m_v^2} \right) v(k, t), \]

\[ H(k, t) = \frac{m_v^2}{k^2 + m_v^2} i [k \times v(k, t)]. \]

In what follows we will neglect both the close encounters of the segments of different strings and the segments of the same string that are labelled by distinct values of the arclength \( \sigma \).
Important as they are in the processes of string reconnections, they cannot be described in the framework of the London approximation. Substituting Eqs. (3), (4) and (9) into Eq. (11) one obtains, with the help of the relation
\[ \int d^3x H^2(x) = \int d^3k |H(k)|^2 / (2\pi)^3 \]
the expression for the action of the single gauge vortex:
\[ S_{\text{vortex}} = \frac{\Phi_0^2}{2(2\pi)^3} \int \frac{d^3k}{(k^2 + m_v^2)^2} \int dt \int d\sigma_1 d\sigma_2 \exp\{i k \cdot [X(\sigma_1) - X(\sigma_2)]\} \]
\[ \times \{-m_v^2(k^2 + m_v^2)[X'(\sigma_1) \cdot X'(\sigma_2)] \]
\[ + (k^2 + 2m_v^2)(k \cdot [\dot{X}(\sigma_1) \times X'(\sigma_1)]) \cdot (k \cdot [\dot{X}(\sigma_2) \times X'(\sigma_2)]) \]
\[ + m_v^4 [\dot{X}(\sigma_1) \times X'(\sigma_1)][\dot{X}(\sigma_2) \times X'(\sigma_2)]. \]
(12)

Let us show with a method similar to that of Refs. [8] and [9] how the known Nambu-Goto form of the action results from Eq. (12). To this end one should set the mass of the gauge boson to infinity, \( m_v \to \infty \), before the momentum integration. Then the action becomes, in the gauge \( X^0 \equiv t = \tau \),
\[ S_{\text{NG}} = \frac{\Phi_0^2}{2} \int d^2s_1 d^2s_2 \delta^{(4)}[X(s_1) - X(s_2)] \{-X'(s_1) \cdot X'(s_2) \]
\[ + [\dot{X}(s_1) \times X'(s_1)] \cdot [\dot{X}(s_2) \times X'(s_2)], \]
(13)
where \( s_{1,2} \equiv s_{1,2}^A = (\tau_{1,2}, \sigma_{1,2}) \) is the two-dimensional vector. Using the Gaussian regularization of the \( \delta \) function and the expansion
\[ X(s_2) \simeq X(s_1) + (s_2 - s_1)^A \partial_A X / 1! + (s_2 - s_1)^A(s_2 - s_1)^B \partial_A \partial_B X / 2! + \cdots \]
(14)
valid under the condition \( |X''(\sigma)| \ll m_v \), one obtains
\[ S_{\text{NG}} = \frac{\Phi_0^2}{2} \left( \frac{\Phi_0}{2\pi A^2} \right)^2 \int d^2s_1 d^2s_2 \exp(-\frac{1}{2A^2}A^B B^C \partial_A X^\mu \partial_B X_\mu)(-X'^2 + [\dot{X} \times X']^2) \]
\[ = \frac{\Phi_0^2}{4\pi A^2} \int d^2s \sqrt{\text{det}\partial_A X^\mu \partial_B X_\mu}, \]
(15)
where \( \Lambda^{-1} \to \infty \) is an ultraviolet cutoff, \( \partial_A = \partial / \partial z^A \), and \( \text{det}\partial_A X^\mu \partial_B X_\mu = -X'^2 + [\dot{X} \times X']^2 \) in the chosen gauge. Up to an overall factor, the last equality in Eq. (15) is recognized to be the NG action.

Coming back to the case of large but finite \( m_v \), one should first make the integration over momenta neglecting exponentially small terms and taking into account the fact that only nearby segments of the string contour give an appreciable contribution to the integral over the arc length. One obtains the action of a single vortex in the form
\[ S_{\text{vortex}} = \frac{2\pi}{g^2} \ln \frac{m_H}{m_v} \int dt \int d\sigma \left\{-m_v^2 X'^2 + m_v^2 [\dot{X} \times X']^2 / c_0^2 \right\}. \]
(16)
The energy of the magnetic field is not enhanced in the London limit and by this reason it is neglected. One can see that in comparison with the first line of Eq. (15), the contribution of the kinetic part of the string action in Eq. (16) is enhanced by the factor $1/c_0^2$, where

$$c_0^2 = \frac{4m_V^2}{O(1)m_H^2} \ln \frac{m_H}{m_V} \ll 1$$

is the velocity squared characterizing the classical string motion in the case of finite masses of the bosonic fields, and the unknown numerical factor $O(1)$ reflects the ignorance of the true Higgs field profile. This enhancement can be understood qualitatively. The vortex motion demands the rearrangement of the Higgs field condensate at distances $\sim m_H^{-1}$ and by this reason demands the large, $\propto m_H^2$, energy, resulting in a large effective kinetic mass of the vortex segment. Here the mass of the Higgs boson $m_H$ appears as the natural upper limit of the integration over momentum.

Let us show that the physical motions of the string characterized by the action (16) are indeed slow. To this end one should consider the circular string loop and find its time to shrink. The equation of the contour is

$$X(\sigma, t) = a \left( e_x \cos \frac{\sigma}{a} + e_y \sin \frac{\sigma}{a} \right),$$

where $a \equiv a(t)$ is the time dependent loop radius, $e_{x,y}$ being the unit vector in the corresponding direction. Then the Lagrangian obtained from the Eq. (16) becomes

$$L = 2\pi a \varepsilon_v \left( -1 + \frac{\dot{a}^2}{c_0^2} \right),$$

with

$$\varepsilon_v = \frac{2\pi m_V^2}{g^2} \ln \frac{m_H}{m_V}$$

being the energy per unit length. The equation of motion, with initial conditions in the form $\dot{a}(0) = 0$ and $a(0) = R$, is solved through the equation of the energy conservation in the process of collapse,

$$E = 2\pi R \varepsilon_v = 2\pi a \varepsilon_v \left( 1 + \frac{\dot{a}^2}{c_0^2} \right).$$

One obtains an equation determining implicitly the dependence of the loop radius $a$ on time,

$$R \left( \frac{\pi}{2} - \arcsin \sqrt{\frac{a}{R}} \right) + \sqrt{a(R-a)} = c_0 t.$$  

One can see from Eq. (19) that the time of the shrinkage is $t_\ast = R\pi/2c_0 \gg R$.

On the other hand, by direct application of the Hamiltonian formalism to the action (16) subjected to the constrains $X'^2 = 1$ and $X' \cdot \dot{X} = 0$, one can show that the string stabilized by some means (say, by rotation in the plane of the loop) possesses elastic waves travelling with phase velocity $c_0$.

Let us consider finite mass corrections to the magnetic helicity (11) of the string configuration, with the further goal of calculating the time derivative of this quantity. The purpose of this study is twofold. First, the role of configurations with a nonzero magnetic
helicity is intensively discussed in the connection with processes at the epoch of the electroweak phase transition. The dynamics of such configurations affects, in the view of the anomaly equation, the dynamics of some fermionic charges, in particular, the baryon and/or lepton numbers \cite{4}. Second, a number of papers have appeared recently \cite{12–14}, where the dynamics of the twisting and writhing numbers (see below) of the curve is discussed from the geometrical point of view. In the meantime, the gauge string in the coupled Higgs and vector field system should be governed by the corresponding field equations. In this respect it would be interesting to compare the results inferred from the field equations to the results inferred from pure geometrical approach of Refs. \cite{12–14}.

The representation of the magnetic helicity $h_A$ in terms of the space Fourier components of the gradient of the singular phase of the Higgs field, Eq. (10), found in \cite{15} is useful. One has

$$h_A = \int d^3x \mathbf{A} \cdot (\nabla \times \mathbf{A}) = \int \frac{d^3k}{(2\pi)^3} \frac{i\mathbf{k} \cdot [\mathbf{v}(\mathbf{k}, t) \times \mathbf{v}^*(\mathbf{k}, t)]}{(k^2 m_V^2 + 1)^2} =$$

$$= \left(\frac{2\pi}{g}\right)^2 \int \frac{d^3k}{(2\pi)^3} \left(\frac{m_V^2}{k^2 + m_V^2}\right)^2 \sum_{a,b} \oint \oint d\sigma_a d\sigma_b \exp[-i\mathbf{k} \cdot (\mathbf{X}_a - \mathbf{X}_b)]$$

$$\times i\mathbf{k} \cdot [\mathbf{X}'_a \times \mathbf{X}'_b]/k^2,$$

which is nonzero only for configurations that are not invariant under the space inversion. The shorthand notation $\mathbf{X}_{a,b} \equiv \mathbf{X}_{a,b}(\sigma_{a,b}, t)$ is used hereafter. Terms with $a \neq b$, after the momentum integration, give the linking number

$$L[a, b] = \frac{1}{4\pi} \oint d\sigma_a \oint d\sigma_b \mathbf{X}_{ab} \cdot \mathbf{X}'_{ab}/|\mathbf{X}_{ab}|^3$$

of two contours \cite{16,17}, with the corrections suppressed exponentially as $\exp(-m_V|\mathbf{X}_a - \mathbf{X}_b|)$. The contribution of the typical term with $a = b$, after momentum integration, reads

$$h_A(a = b) \propto W[a] - \frac{1}{4\pi} \oint d\sigma_1 \oint d\sigma_2 \mathbf{X}_{12} \cdot \mathbf{X}'_{12} / |\mathbf{X}_{12}|^3 \left(1 + m_V|\mathbf{X}_{12}| + \frac{1}{2} m_V^2|\mathbf{X}_{12}|^2\right)$$

$$\exp(-m_V|\mathbf{X}_{12}|),$$

where $\mathbf{X}_{12} \equiv \mathbf{X}_a(\sigma_1) - \mathbf{X}_a(\sigma_2)$ refers to the same contour $a$, and

$$W[a] = \frac{1}{4\pi} \oint d\sigma_1 \oint d\sigma_2 \mathbf{X}_{12} \cdot \mathbf{X}'_{12} / |\mathbf{X}_{12}|^3$$

is the writhing number of the contour $a$ \cite{15}. The $m_V$-dependent term in Eq. (21) is evaluated with the help of the expansion Eq. (14) to give

$$\delta h_A(a = b, \text{mass correction}) \propto -\frac{1}{2\pi m_V^2} \oint d\sigma \mathbf{X}'_a \cdot [\mathbf{X}'_a \times \mathbf{X}''_a].$$

In the case of sufficiently smooth contours the latter can be represented as $-T[a]/(m_V R)^2$, where

$$T[a] = \frac{1}{2\pi} \oint d\sigma \mathbf{X}' \cdot [\mathbf{n} \times \mathbf{n}']$$
is the twisting number \([16–18]\) of the contour \(a\) whose normal vector is \(n\) and the radius of curvature is \(R\). Thus, the part of the twist contribution to the helicity originating from the finite width of the vortex is suppressed as \((R m_v)^{-2}\), and the resulting expression for the helicity can be written as \([3, 15]\)

\[
h_A = \Phi_0^2 \left\{ \sum_a W[a] + 2 \sum_{a<b} L[a, b] \right\}. \tag{22}
\]

The rate of the helicity change in the course of the contour evolution can be evaluated explicitly by taking the time derivative of the right hand side of Eq. (20). With the help of the relation

\[
\frac{\partial}{\partial t} \oint d\sigma [k \times X'] \exp(-i k \cdot X) = i \int d\sigma k \times (k \times [\dot{X} \times X']) \exp(-i k \cdot X),
\]

which can be verified by a straightforward calculation, one finds

\[
\dot{h}_A = \Phi_0^2 \int \frac{d^3 k}{(2\pi)^3} \left( \frac{m_v^2}{k^2 + m_v^2} \right)^2 \sum_{a b} \oint d\sigma_a \oint d\sigma_b (\dot{X}_a - \dot{X}_b) [X'_a \times X'_b] \exp(-i k \cdot X_{a b}) \times \exp(-i k \cdot X_{a b})
\]

\[
= \Phi_0^2 \frac{m_v^3}{8\pi} \sum_{a b} \oint d\sigma_a \oint d\sigma_b (\dot{X}_a - \dot{X}_b) [X'_a \times X'_b] \exp(-m_v |X_{a b}|). \tag{23}
\]

It is clear that terms with \(a \neq b\) give an exponentially small correction \(\propto \exp(-m_v |X_a - X_b|)\). This is natural, since the analogous terms in the expression for \(h_A\) give a contribution to the linking number \(L[a, b]\) known to be the topological invariant. The contribution of the terms with \(a = b\) is calculated with the help of the expansion (14) taken at \(t_1 = t_2\) and \(\sigma_2 = \sigma_a + z\). One obtains

\[
\dot{h}_A = \Phi_0^2 \frac{m_v^3}{8\pi} \sum_a \oint d\sigma_a \dot{X}_a \cdot [X'_a \times X''_a] \int_{-\infty}^{+\infty} dz (-z^2) \exp(-m_v |z|)
\]

\[
= \frac{\Phi_0^2}{2\pi} \sum_a \oint d\sigma_a \dot{X}_a \cdot [X'_a \times X''_a]. \tag{24}
\]

The contribution to the time derivative turns out to be independent of \(m_v\). Note that the time derivative of helicity calculated in Eq. (24) originates from the intrinsic dynamical contour motion. Indeed, in the case of translational motion with constant velocity one can show, with the help of the Frenet equations, that the right hand side of Eq. (24) vanishes.

The above change of helicity is identified as due to the change of the writhing number. Hence, Eq. (24), after dividing by \(\Phi_0^2\), gives the time derivative of the writhing number; see Eq. (22). The presented derivation can be compared to the earlier derivation \([12]\) obtained by using geometrical means. The equation found here coincides exactly with the equation obtained in Ref. \([12]\).

One can write Eq. (24) in terms of an infinitesimal deformation of the contour \(\delta X\) as

\[
\delta W[a] = \frac{1}{2\pi} \oint d\sigma \delta X \cdot [X' \times X''] . \tag{25}
\]
The transversal deformation preserving the gauge condition $X'^2 = 1$ should look like $\delta X = \delta X_b b$, where $b$ is the unit vector of binormal. This can be verified with the help of the Frenet equations. After an extensive use of these equations, one can write Eq. (25) in the form

$$\delta W[a] = -\frac{1}{2\pi} \oint d\sigma \kappa \delta X'_b,$$

where $\kappa$ is the curvature of the contour. On the other hand, an infinitesimal variation of the twisting number can be written as the following chain of equations:

$$\delta T[a] = \frac{1}{2\pi} \oint d\sigma = \frac{1}{2\pi} \oint d\sigma (b \cdot n') = \frac{1}{2\pi} \oint d\sigma \{ (\delta b \cdot n') - (\delta n \cdot b') \}$$

$$= \frac{1}{2\pi} \oint d\sigma \{ \delta b (-\kappa X' + \tau b) + \tau n \cdot \delta n \} = -\frac{1}{2\pi} \oint d\sigma \kappa (X' \cdot \delta b).$$

The orthonormality property of the three vectors $n, b,$ and $X'$ is used. This property permits one to write finally that

$$\delta T[a] = \frac{1}{2\pi} \oint d\sigma \kappa \delta X'_b = -\delta W[a].$$

Hence, the dynamical evolution of the vortex string in the Abelian Higgs model obeys the conservation law $W[a] + T[a] = S[a] = \text{const}$, where the integration constant $S[a]$ is known as the self-linking number of a curve $a$, with the property being the topological invariant [9,16–19]. Hence topological invariance is obtained here, in fact, from the field equations of the Abelian Higgs model, without the assumption of conserved helicity. The present conclusion is important, because an earlier derivation [17] of the above topological conservation law was based on the conservation of helicity. The latter is valid only approximately, in the limit of an infinite conductivity of the medium [11,17,20].

In conclusion, let us discuss the meaning of the results obtained here. First, the NG string action used so far in the description of various physical processes involving cosmic strings is not so universal as one might think, since the specific sectors of the parameter space of an underlying gauge model may give a different type of action. In the meantime, the time derivative of the writhing number is independent of the mass of the vector boson and coincides with the expression [12] found in a completely different situation. This signals the universal dynamics of the writhing number of the objects spreading from cosmic strings to polymer chains. Second, the physical meaning of Eq. (24) is the following. The dynamics of the $W^\pm$ condensates [4] and the electroweak plasma [5] in the electroweak phase transition is helicity preserving. The same situation takes place in magnetohydrodynamics [11,17,20]. In all these cases the change of helicity is due to the finite conductance of a medium and, consequently, is dissipative. Only in this case does a nonzero scalar product $E \cdot H$ contribute to the time derivative of the magnetic helicity, provided there are no objects that are not invariant under space inversion. The situation when such objects are present is considered in the present work. These are gauge strings with the parity-noninvariant contours evolving in accordance with the field equations. The above scalar product is nonzero in this case. Using Eqs. (3) and (9) one can show that Eq. (24) can be alternatively obtained from a direct evaluation of the right hand side of the equation $\dot{h}_A = -2 \int d^3x E \cdot H$. This demonstrates that magnetic helicity can change even in the case of an infinite plasma conductivity, since the electric field strength has a nonzero projection onto the magnetic field strength for the
closed mirror-noninvariant string. The change is dissipationless, and the sign of the right hand side of Eq. (24) can be arbitrary.

ACKNOWLEDGMENTS

I am indebted to Dr. Renzo L. Ricca for bringing some relevant references, especially Ref. [17], to my attention. The study of that paper has promoted a clarification of some points in the present work.
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