Complexity Comparison between Two Optimal-Ordered SIC MIMO Detectors Based on Matlab Simulations

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Abstract—This paper firstly introduces our shared Matlab source code that simulates the two optimal-ordered SIC detectors proposed in [1] and [2]. Based on our shared Matlab code, we compare the computational complexities between the two detectors in [1] and [2] by theoretical complexity calculations and numerical experiments. We carry out theoretical complexity calculations to obtain the worst-case complexities for the two detectors in [1] and [2]. Then from the theoretical worst-case complexities, we make the conclusion that the detector proposed in [2] requires $9N^2$ more floating-point operations (flops) than the detector proposed in [1], where $N$ is the number of transmit antennas. Our numerical experiments also show that the detector in [2] requires more worst-case and average complexities than the detector in [1].

Index Terms—MIMO, optimal ordered SIC detectors, fast algorithms, complexity comparison.

I. INTRODUCTION

FOR multiple input multiple output (MIMO) systems, two optimal ordered successive interference cancellation (SIC) detectors have been proposed in [1] and [2], respectively. In [3], it has been shown that both optimal-ordered SIC detectors proposed in [1] and [2] require the same $O(MN^2 + N^3)$ complexity, where $N$ and $M$ are the numbers of transmit and receive antennas, respectively. This paper firstly introduces our shared Matlab source code, which simulates the two detectors in [1] and [2]. In our shared Matlab code, each statement requiring floating-point operations (flops) is followed by a statement to count its flops, while the statement to count the flops is obtained manually. Then the execution of the Matlab code can generate the exact total complexity (in the number of flops) for each detector.

Our shared Matlab code is utilized in this paper to compare the complexities of the two detectors in [1] and [2] by numerical experiments. On the other hand, based on the above-mentioned statements to count the flops, we carry out theoretical complexity calculations to obtain the worst-case $O(MN^2 + N^3)$ and $O(MN + N^2)$ complexities for the two detectors in [1] and [2]. Then we compare the theoretical complexities of the two detectors in [1] and [2], to determine one with a lower complexity.

II. MATLAB CODE AND COMPLEXITY CALCULATIONS FOR THE DETECTORS PROPOSED IN [1] AND [2]

In our shared Matlab code, each statement requiring flops is followed by a statement to count the required complex multiplication, complex additions, real multiplications, real additions, real divisions, real square root operations, and multiplications between a real number and a complex number, which are denoted as cm, ca, rm, ra, rdiv, rsqrt and rcm, respectively. We set cm=6, ca=2, rm=1, ra=1, rdiv=1, rsqrt=1 and rcm=2, to compute the number of flops.

Based on the above-mentioned statements to count the required complexities, we carry out theoretical complexity calculations to obtain the worst-case $O(MN^2 + N^3)$ and $O(MN + N^2)$ complexities for the two detectors in [1] and [2], in the numbers of complex multiplications, complex additions and flops. Notice that we do the best to convert the complexities into the numbers of complex multiplications and complex additions, and only the complexities that cannot be converted (into the numbers of complex multiplications and complex additions) are measured by the numbers of flops.

Though most statements in our shared Matlab code can be obtained directly from the relevant algorithms described in [1] and [2], there are still some statements in our shared Matlab code which cannot be obtained directly from the relevant descriptions in [1] and [2]. Then in this section, we will also explain the algorithms for some statements in our shared Matlab code, which have not been described in detail in [1] and [2].

In what follows, the computational complexity of $j$ complex multiplications and $k$ complex additions will be denoted as $[j,k]$, which is simplified to $[j]$ if $j=k$. Moreover, notice that

\[
\left\{ \begin{array}{l}
\sum_{i=1}^{N} i = \frac{N(N+1)}{2} = \frac{N^2}{2} + \frac{N}{2} \\
\sum_{i=1}^{N} i^2 = \frac{N(N+1)(2N+1)}{6} = \frac{N^3}{3} + \frac{N^2}{2} + \frac{N}{6}
\end{array} \right.
\]

will be utilized to compute the $O(MN^2 + N^3)$ and $O(MN + N^2)$ complexities.

A. Matlab Code and Complexity Calculations for the Detector Proposed in [1]

The optimal-ordered SIC detector proposed in [1] consists of steps N1-N7, where step N1 includes sub-steps N1-a, N1-

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1As in [1], flops in this paper means real flops.
b, N1-c and N1-d. In the shared Matlab code, sub-step N1-a has been omitted for simplicity. Based on the statements to count the complexities in our Matlab code to implement the detector in [1], we calculate the worst-case \( O(MN^2 + N^3) \) and \( O(MN + N^2) \) complexities for the steps/sub-steps of the detector in [1], and the corresponding results are given in Table I. Among the steps/sub-steps listed in Table I, steps N2 and N3 will be further described in this subsection, since some details about steps N2 and N3 have not been covered in [1].

In step N3, the permuted inverse Cholesky factor \( F_n \) is block upper-triangularized by a unitary transformation to obtain \( F_{n-1} \) for the next iteration, as shown in equation (9) of [1], i.e.,

\[
F_n \Sigma = \begin{bmatrix} F_{n-1} & u_{n-1} \\ 0_{n-1} & \lambda_n \end{bmatrix}, \tag{2}
\]

where \( \Sigma \) is a unitary transformation, \( u_{n-1} \) is an \((n-1) \times 1\) column vector, and \( \lambda_n \) is a scalar. In [1], the unitary transformation \( \Sigma \) in (2) is performed by a sequence of Givens rotations, and the fast complex Givens rotation in (4) is utilized. Thus in the shared Matlab code, the fast complex Givens rotation described by Algorithm 3 in [4] is utilized, which computes the Givens matrix \[
\begin{bmatrix} c & s \\ -\bar{s} & c \end{bmatrix}
\]
by 22 flops,

\[2\text{N}^2\text{flops and } 2(\frac{5}{2}M \text{N}^2 + \frac{5}{2}N^3 + \frac{1}{2}M \text{N}^2 - \frac{3}{8}N^2 + \frac{1}{2}M \text{N}^2 + \frac{1}{4}M \text{N}^2 - \frac{3}{8}N^2) = 4M \text{N}^2 + 6\text{N}^3 + 12M \text{N} + \frac{5}{2}N^2 \]
i.e., 15 real multiplications, 5 real additions, 1 real division and 1 real square root operation. In the \( n^{th} \) iteration, \( n = 1, 2, \ldots, 1 \) iteration, \( n-1 \) Givens rotations are required in the worst case, and then it requires \( 22(n-1) \approx 22n \) flops to compute \( n-1 \) Givens matrices, as shown in line 10 of Table I.

\[ \left| F_{n-1}(j,:) \right|^2 = \left| F_n(j,:) \right|^2 - \left| u_{n-1}(j) \right|^2, \tag{3} \]

where \( j = 1, 2, \ldots, n-1 \), and \( \bullet \) denotes the squared length of a vector or the squared absolute value of a number. In our opinion, (3) was first implied by equation (7) in [5], which is the same as (2). Moreover, (3) can also be regarded as a special case of equation (7) (A-34) on [6] p. 120, where the variables are defined in (A-28) and computed by (A-27) with \( k = 0 \).

From (3), it can be seen that in the \( n^{th} \) iteration of the iterative detection phase, step N2 obtains the squared length of all the \( n-1 \) rows in \( F_{n-1} \) by \( 2(n-1) \) real multiplications and \( 2(n-1) \) real additions, which are equivalent to \((n-1)/2 \approx n/2\) complex multiplications and the same
number of complex additions,\(^5\) as shown in line 9 of Table I. On the other hand, in the first iteration with \(n = N\), it requires about \(N^2/4\) complex multiplications and the same number of complex additions to compute the initial squared length of all the \(N\) rows in the triangular \(F_N\), as shown in line 8 of Table I.

Table I also gives the total number of worst-case \(O(MN^2 + N^3)\) and \(O(MN + N^2)\) flops required by the detector in \(\text{[1]}\), which is

\[4MN^2 + 6N^3 + 12MN + \frac{17}{2}N^2.\]  

**B. Matlab Code and Complexity Calculations for the Detector Proposed in \(\text{[2]}\)**

In \(\text{[2]}\), the proposed optimal-ordered SIC detector consists of steps 1-3 and 10-19 in Table I, and the details of step 2 in Table I are described in Table II. Based on the statements to count the complexities in our Matlab code to implement the detector in \(\text{[2]}\), we compute the worst-case \(O(MN^2 + N^3)\) and \(O(MN + N^2)\) complexities for the steps of the detector in \(\text{[2]}\), and give the corresponding results in Table II. Table II includes the complexities of steps 1, 11, 13 and 15 in Table I of \(\text{[2]}\), and the complexities of steps 2, 3, 5, 12, 13 and 18 in Table II of \(\text{[2]}\), since the complexities of those steps are \(O(MN^2 + N^3)\) and \(O(MN + N^2)\). Among the steps listed in Table II, steps 2, 3, 5, 12, 13 and 18 in Table II of \(\text{[2]}\) will be further described in this subsection, since some details about those steps have not been covered in \(\text{[2]}\).

In the left column on \(\text{[2]}\) p. 4630], \(\text{[7]}\) has been cited to describe the details about the implementation of the Cholesky factorization and the back-substitution (to compute the inverse of the Cholesky factor). Then in our shared Matlab code, Algorithm 4.2.1 (GaxpyCholesky) on \(\text{[7]}\) p. 164] is utilized to implement the Cholesky factorization for step 2 in Table II of \(\text{[2]}\), and Algorithm 3.1.2 (Row-Oriented Back Substitution) on \(\text{[7]}\) p. 107] is utilized to implement the back-substitution for step 3 in Table II of \(\text{[2]}\).

The Givens rotation in steps 12 and 13 in Table II of \(\text{[2]}\) is obtained by exchanging the columns of the conventional Givens rotation in \(\text{[7]}\), as mentioned in the third paragraph of the left column on \(\text{[2]}\) p. 4628]. Then \(\text{[7]}\) was cited again in lines 3-5 of the right column on \(\text{[2]}\) p. 4630], to claim that in step 13 in Table II of \(\text{[2]}\), a conventional Givens rotation on a \((j+1) \times 2\) matrix with complex entries requires a complexity of

\[O(2j + 2).\]  

\(\text{[2]}\)
Accordingly, to implement the Givens rotation utilized in [2], we should follow the complex Givens rotation introduce in [2], which can be written as [7] equation (5.1.12) on p. 244
\[
\begin{bmatrix}
c & s \\
-s^* & c
\end{bmatrix}^H
\begin{bmatrix}
u \\
v
\end{bmatrix} =
\begin{bmatrix}
r \\
0
\end{bmatrix}.
\]
(6)

In (6), \((\cdot)^*\) represents conjugate, and the real \(c\) and the complex \(s\) satisfy
\[
\begin{align}
c &= \cos(\theta) \\
s &= \sin(\theta)e^{j\phi}.
\end{align}
\]
(7a)
(7b)

In [5], [1] has been cited to claim that the complexity of [5] should be revised into the complexity of
\[
[3j + 3, j + 1].
\]
(8)

In Appendix A, we will verify the complexity of [8]. Our shared Matlab code computes the Givens matrix
\[
\begin{bmatrix}
c & s \\
-s^* & c
\end{bmatrix}
\]
and the corresponding result \(r\) in (6) by the conventional Givens rotation algorithm in [2], which will be introduced in Appendix B. Appendix B also gives the conclusion that the conventional Givens rotation algorithm in [7] computes \(c\), \(s\), and \(r\) in (6) by 20 real multiplications, 5 real additions, 4 real division and 3 real square root operation, which can be counted as 32 flops.

Table II also gives the total number of worst-case \(O(MN^2 + N^3)\) and \(O(MN + N^2)\) flops required by the detector in [2], which is
\[
4MN^2 + 6N^3 + 12MN + \frac{35}{2}N^2.
\]
(9)

III. EXPERIMENT AND DISCUSSION

Firstly, let us compare (4) and (9), which are the total numbers of worst-case \(O(MN^2 + N^3)\) and \(O(MN + N^2)\) flops required by the detectors proposed in [1] and [2], respectively. It can be seen that both optimal-ordered SIC detectors proposed in [1] and [2] require the same dominant complexity, i.e., the \(O(MN^2 + N^3)\) complexity of
\[
4MN^2 + 6N^3
\]
(10)
flops. On the other hand, the detector proposed in [2] requires \(9N^2\) more flops than the detector proposed in [1].

From Table I and Table II, it can be seen that the computation of the Givens matrices in [2] requires \(16N^2 - 11N^2 = 5N^2\) more flops than the corresponding computation in [1]. Accordingly, the detector proposed in [2] requires \(9N^2 - 5N^2 = 4N^2\) more flops than the detector proposed in [1], if we neglect the difference in the number of flops caused by the different algorithms in [1] and [2] to compute the Givens matrices, by assuming that the same algorithm is utilized in [1] and [2] to compute the Givens matrices.

Assume \(N = M\). For different number of transmit/receive antennas, we apply our shared Matlab code to count the worst-case and average flops of the optimal-ordered SIC detectors proposed in [1] and [2]. The results are shown in Fig. 1. As in [2] and [1], the maximum number of Givens rotations are assumed to count the worst-case flops. To count the average flops, we simulate 10000 random channel matrices \(H\), and for fair comparison, we do not permute the columns of \(H\). From Fig. 1, it can be seen that the detector proposed in [2] requires more worst-case and average flops than the detector proposed in [1], which is consistent with the theoretical flops calculation.

Fig. 2 shows the numbers of worst-case flops obtained by our shared Matlab code and those computed by [3], [9] and [10]. From Fig. 2, it can be seen that the numbers of worst-case flops obtained by our shared Matlab code are very close to the theoretical numbers of worst-case \(O(MN^2 + N^3)\) and \(O(MN + N^2)\) flops computed by [3] and [9], and are clearly larger than the theoretical numbers of worst-case \(O(MN^2 + N^3)\) flops computed by [10].
IV. Conclusion

In this paper, we introduce our shared Matlab source code that simulates the two optimal-ordered SIC detectors proposed in [1] and [2]. We also explain some algorithms utilized in our shared Matlab code, which have not been described in detail in [1] and [2].

Based on our shared Matlab code, we compare the computational complexities between the two detectors in [1] and [2] by theoretical complexity calculations and numerical experiments. We carry out theoretical complexity calculations to obtain the worst-case $O(MN^2 + N^3)$ and $O(MN + N^2)$ complexities for the two detectors in [1] and [2], from which we make the conclusion that the detector proposed in [2] requires 9N^2 more flops than the detector proposed in [1]. Our numerical experiments show that the detector in [2] requires more worst-case and average flops than the detector in [1], and the numbers of worst-case flops obtained by our shared Matlab code are very close to the theoretical numbers of worst-case $O(MN^2 + N^3)$ and $O(MN + N^2)$ flops.

Finally c and s in the Givens matrix $\begin{bmatrix} c & s \\ -s^* & c \end{bmatrix}$ are obtained by

\[
\begin{align*}
  c &= c_0 \\
  s &= s_0 e^{i\phi}.
\end{align*}
\]

Now an efficient algorithm to compute the result $r$ in (6) is still required, which has not been given in [7]. $e^{i\phi} = e^{i(\beta - \alpha)}$ in (14) can be substituted into (15b) to obtain

\[
s = s_0 e^{i(\beta - \alpha)}.
\]

Then substitute (16), (15a) and (12) into (6) to obtain

\[
r = (c_0 r_u - s_0 r_v) e^{-i\alpha}.
\]

From (15), we can deduce

\[
r_0 = c_0 r_u - s_0 r_v,
\]

and from (12a), we can deduce

\[
e^{-i\alpha} = u/r_u.
\]

Then (18) and (19) can be substituted into (17) to obtain

\[
r = (r_0/r_u) u.
\]

A real Givens rotation computed by equation (5.1.8) on [7] p. 240] requires 4 real multiplications, 1 real additions, 1 real division and 1 real square root operation [7]. Then the conventional complex Givens rotation on [7] p. 244, which includes the above-described 3 real Givens rotations, (14), (15) and (20), totally requires 20 real multiplications, 5 real additions, 4 real division and 3 real square root operation to compute $c, s$ and $r$ in (6).

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\[\text{\footnotesize\textsuperscript{1}}\text{When implementing equation (5.1.8) in [7], one real division is reduced at the cost of adding two real multiplications, since division and square root are the most expensive real floating point operations.}\]