The charged Higgs boson mass in the 2HDM: decoupling and CP violation

Maria Krawczyk$^{1,2}$ * and Dorota Sokolowska$^1$

1. Institute of Theoretical Physics, University of Warsaw
00-681 Warsaw, ul. Hoża 69, Poland

2. TH-Division, CERN, CH-1211 Genève 23, Switzerland

Mass range of the charged Higgs boson in the 2HDM with explicit and spontaneous CP violation is discussed. Constraints on $M_{H^\pm}$ in the CP conserving 2HDM(II) are shown.

1 The 2HDM potential and spontaneous symmetries breaking

The most general, invariant under gauge group $SU(2)_L \times U(1)_Y$ and renormalizable potential of the Two Higgs Doublet Model (2HDM) [2, 3, 4] is given by

$$V = \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)$$

$$+ \left[ \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + \lambda_6 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_7 (\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_1) + h.c. \right],$$

where $\lambda_1-7, m^2_{11}, m^2_{22} \in \mathbb{R}$ (by the hermicity of the potential), while in general $\lambda_5-7, m^2_{12} \in \mathbb{C}$. In the most general CP breaking form it has 14 parameters, however only 11 are independent, see e.g. [5, 6]. In the model there are five Higgs particles: three neutral $h_1, h_2, h_3$ (for CP conservation - two CP-even $h, H$ and one CP-odd $A$) and two charged Higgs bosons $H^\pm$.

1.1 $Z_2$ and CP symmetries

The $Z_2$ symmetry of the potential (1) is defined as the invariance of $V$ under the following transformation of doublets: $\Phi_1 \rightarrow -\Phi_1, \Phi_2 \rightarrow -\Phi_2$ or $\Phi_1 \rightarrow \Phi_1, \Phi_2 \rightarrow -\Phi_2$. If $Z_2$ (in either form) is a symmetry of the potential, then $m^2_{12} = \lambda_6 = \lambda_7 = 0$. The $Z_2$ symmetry is softly broken by the terms proportional to $m^2_{12}$.

General 2HDM allows for CP violation both explicitly and spontaneously [7, 8, 2]. The CP violation can be naturally suppressed by imposing a $Z_2$ symmetry on the Higgs potential.

1.2 Reparametrization transformation

A global unitary transformation which mix two doublets and change their relative phase does not change the physical content of 2HDM as discussed recently in [9], see also [3, 4, 2]. It is given by

$$\begin{pmatrix} \Phi_1^\prime \\ \Phi_2^\prime \end{pmatrix} = \mathcal{F} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}, \quad \mathcal{F} = e^{-i\rho_0} \begin{pmatrix} \cos \theta e^{i\rho/2} & \sin \theta e^{-i(\tau-\rho/2)} \\ -\sin \theta e^{-i(\tau-\rho/2)} & \cos \theta e^{-i\rho/2} \end{pmatrix}. \quad (2)$$

*Supported in part by EU Marie Curie Research Training Network HEPTOOLS, under contract MRTN-CT-2006-035505 and by FLAVIAnet contract No. MRTN-CT-2006-035482.

LCWS/ILC 2007
There are three reparametrization parameters - \( \rho, \theta, \tau \), and in addition \( \rho_0 \) parameter as an overall phase. If \( \theta = 0 \) there is no mixing of two dublets and the transformation becomes a global transformation of doublets with an independent phase rotations (rephasing):

\[
k = 1, 2 : \Phi_k \rightarrow e^{-i\rho}\Phi_k, \quad \rho_1 = \rho_0 - \frac{\rho}{2}, \quad \rho_2 = \rho_0 + \frac{\rho}{2}, \quad \rho = \rho_2 - \rho_1.
\]

The original form of the potential is recovered by the appropriate changes of phases of the following coefficients:

1.3 Explicit and spontaneous CP violation in 2HDM

CP violation may occur in 2HDM only if \( Z_2 \) symmetry is broken [8, 2, 3, 4, 9]. A necessary condition for an explicit CP violation in the Higgs potential \( V \) is an existence of complex parameters. However, if there exists a reparametrization leading to \( V \) with only real parameters (real basis), then there is no explicit CP violation in \( V \). A spontaneous CP breaking, by the vacuum state, is still possible [7, 8, 2].

In the simply analysis [14], which results we present here, only the potential with exact and softly broken \( Z_2 \) symmetry was considered, i.e. \( \lambda_{6,7} = 0 \). In studying 2HDM with an explicit CP conservation or violation the real vacuum representation [4] was applied. A spontaneous CP violation was discussed assuming the explicitly CP conserving \( V \).

1.4 Vacuum expectation values

The most general vacuum (extremum) state can be described by [8, 11, 12, 13, 14]

\[
\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} u \\ v_2 e^{i\xi} \end{pmatrix},
\]

where \( v_1, v_2, \xi, u \in \mathbb{R} \). By gauge transformation one can always make \( v_1 > 0 \). Below we will assume that \( v_2 \neq 0 \), with \( v^2 = v_1^2 + v_2^2 = (246 \text{ GeV})^2 \), and \( 0 \leq \xi < 2\pi \).

For vacuum with \( u \neq 0 \) the electric charge is not conserved and the photon becomes a massive particle ("charged vacuum"). If \( u = 0 \) then a "neutral vacuum" are possible. Depending on the value of \( \xi \) there may or may not be a spontaneous CP violation [8, 3, 12, 13]. The useful quantity is \( \nu = \frac{m_{12}^2}{2m_{12}v_2} \) (or \( \nu = \frac{m_{12}^2}{2m_{12}v_2} \)) [4], which here is taken to be positive.

1.5 Extremum conditions

For the extremum states (4) the first derivatives of the considered potential lead to the following set of extremum conditions:

\[
0 = u \left[ v_1 v_2 \cos \xi (\lambda_4 + \lambda_5) - m_{12}^2 \right], \quad 0 = u \left[ \lambda_2 (u^2 + v_2^2) + \lambda_3 v_1^2 - m_{22}^2 \right]
\]

\[
0 = v_2 \sin \xi \left[ 2\lambda_5 v_1 v_2 \cos \xi - m_{12}^2 \right], \quad 0 = v_2 \sin \xi \left[ v_1^2 (\lambda_3 + \lambda_4 - \lambda_5) + \lambda_2 (u^2 + v_2^2) - m_{22}^2 \right]
\]

\[
0 = v_1 \left[ v_2^2 (\lambda_5 \cos^2 2\xi + \lambda_4) + \lambda_1 v_1^2 + \lambda_3 (u^2 + v_2^2) - m_{11}^2 \right] - m_{12}^2 v_2 \cos \xi
\]

\[
0 = u v_1 v_2 \sin \xi (\lambda_4 - \lambda_5), \quad 0 = v_2 \cos \xi \left[ v_1^2 (\lambda_3 + \lambda_4 + \lambda_5) + \lambda_2 (u^2 + v_2^2) - m_{22}^2 \right] - m_{12}^2 v_1
\]

If \( u = 0 \) then above conditions are satisfied for an exact \( Z_2 \) symmetry (\( m_{12}^2 = 0 \)) when the only possible neutral vacuum state is the one which respects CP, i.e. with \( \sin \xi = 0 \), and for a broken \( Z_2 \) symmetry. In the latter case two neutral vacuum states are possible - without
and with CP violation, for $\sin \xi = 0$ and $\sin \xi \neq 0$, respectively. To get a real minimum of the potential the eigenvalues of the squared mass matrix have to be positive. We will assume in addition that positivity constraints hold guaranteeing stability of the vacuum [10].

1.6 Physical regions for CP conserving 2HDM

Expressions for masses of $H^\pm$ and $A$ for 2HDM with an explicit or a spontaneous CP conservation are as follows.

$Z_2$ symmetry broken If $Z_2$ symmetry is softly broken ($\nu \neq 0$), then the masses squared of $H^\pm$ and $A$ are given by:

$$M_{H^\pm}^2 = v^2 \left( \nu - \frac{1}{2} (\lambda_4 + \lambda_5) \right), \quad M_A^2 = v^2 (\nu - \lambda_5).$$

In order to have positive $M_{H^\pm}^2$ and $M_A^2$ inequalities $\lambda_5 + \lambda_4 < 2\nu$ and $\lambda_5 < \nu$ should hold.

Large masses for $H^\pm$ and $A$ (9) can arise from large $\nu$. In the limit $\nu \to \infty$ the decoupling is realized - $h$ is like the Higgs boson in the Standard Model, while $H^\pm, A, H$ are heavy and almost degenerate [3, 4].

Exact $Z_2$ symmetry The results for an exact $Z_2$ symmetry can be obtained from above expressions in the limit $\nu \to 0$. Then $\lambda_5 < 0$. Masses cannot be too large, as here they can arise only due to $\lambda'$s. However, large $\lambda'$s may violate tree-level unitarity constraints [15].

1.7 Physical regions for CP violating 2HDM

As it was mentioned above if the 2HDM potential breaks $Z_2$ symmetry then CP violation may be realized in the model. Note, that if CP is violated physical neutral Higgs states are $h_1, h_2, h_3$, without definite CP properties, while $h, H, A$ are useful but only auxiliary states.

Explicit CP violation If there is explicit CP violation all formulae derived for the CP conservation case (9 and beyond) hold after the replacements: $\lambda_5 \to \Re \lambda_5$ and $m_{12}^2 \to \Re m_{12}^2$. Note, that the decoupling can be realized here as well, with large $M_{H^\pm}^2$ arising from large $\nu$.

Spontaneous CP violation Spontaneous CP violation may appear if there is a CP breaking phase of the VEV, so $\sin \xi \neq 0$. From the extremum condition one gets that:

$$\cos \xi = \frac{m_{12}^2}{\lambda_5^2 v_1 v_2} = \frac{\nu}{\lambda_5^2},$$

from which it follows that $|\nu/\lambda_5| < 1$. The squared masses for $H^\pm$ and $A$ are given by the following expressions, see also [13]:

$$M_{H^\pm}^2 = \frac{v^2}{2} (\lambda_5 - \lambda_4), \quad M_A^2 = \frac{v^2}{\lambda_5} (\lambda_5^2 - \nu^2) = v^2 \lambda_5 \sin^2 \xi.$$

We see that they are quite different from the formulae for $M_{H^\pm}^2$ and $M_A^2$ discussed above. (Note, that although $A$ is no longer a physical state, positivity of $M_A^2$ still provides a good constraint since it gives at the same time a condition for positivity of squared masses of physical particles.) From the last expression for $M_A^2$ (11) it is easy to see that $\lambda_5$ have to be positive. Furthermore, squared masses (11) are positive if $\lambda_5 > \lambda_4$ and $\lambda_5 > \nu > 0$.

It is worth mentioning that the squared mass of $H^\pm$ does not depend on $\nu$ at all. Therefore, $M_{H^\pm}$ cannot be too large in 2HDM with CP violated spontaneously, for the same reason as in the discussed above case of exact $Z_2$ symmetry.

LCWS/ILC 2007
### Conclusion on possible vacuum states in 2HDM

Regions where various vacuum states (conserving or spontaneously violating CP) can be realized in 2HDM are mutually exclusive [10, 12, 13, 14]. The mass of charged Higgs boson may serve as a guide over various regimes of the 2HDM. Existence of heavy charged Higgs boson, with mass above 600-700 GeV [4, 14], would be a signal that in 2HDM $Z_2$ symmetry is violated, and CP can be violated only explicitly.

### Experimental constraints on the 2HDM(II) with CP conservation

Here we consider the CP conserving 2HDM, assuming that $Z_2$ symmetry is extended also on the Yukawa interaction, which allows to suppress the FCNC [16]. We limit ourself to constraints on the Model II of the Yukawa interaction, as in MSSM, see e.g. [17]. There are 7 parameters for the potential with softly breaking $Z_2$ symmetry: masses $M_h, M_H, M_A, M_{H^\pm}$, mixing angles $\alpha$ and $\tan \beta = v_2/v_1$, and parameter $\nu$.

Couplings (relative to the corresponding couplings of the SM Higgs) are as follows:

- to $W/Z$: $\chi_V = \sin(\beta - \alpha)$  
- to down quarks/charged leptons: $\chi_d = \chi_V - \sqrt{1 - \chi_V^2} \tan \beta - i\gamma_5 \tan \beta$  
- to up quarks: $\chi_u = \chi_V + \sqrt{1 - \chi_V^2} / \tan \beta - i\gamma_5 / \tan \beta$  

$H$ couples like $h$ with following replacements: $\sin(\beta - \alpha) \rightarrow \cos(\beta - \alpha)$ and $\tan \beta \rightarrow -\tan \beta$. For large $\tan \beta$ there are enhanced couplings to $d$--type fermions. Note, that coupling $\chi_{h,V,H^\pm} = \cos(\beta - \alpha)$ is complementary to the $\chi_V^h$.

Important constraints on mass of charged Higgs boson in 2HDM (II) are coming from the $b \rightarrow s\gamma$ and $B \rightarrow \tau\nu$ decays. The rate for the first process calculated at the NNLO accuracy in the SM [18], after a comparison with the precise data from BaBar and Belle, leads to the constraint: $M_{H^\pm} > 295$ GeV at 95 % CL for $\tan \beta > 2$. This limit together with the constraints from the tree-level analysis of $B \rightarrow \tau\nu$ [19] is presented in Fig.1 (Left).

The 2HDM analysis has been performed at the one-loop level for the leptonic tau decays [20]. The constraints are shown in Fig.1 (Right). Not only lower, but also in the non-decoupling scenario upper limits can be derived here. In contrast to the mentioned results from $b$ decays here the (one-loop) constraints depend on masses of neutral Higgs bosons.

### Acknowledgment

MK is grateful to Ilya Ginzburg and Rui Santos for important discussions.

### References

[1] Slides: [http://ilcagenda.linearcollider.org/contributionDisplay.py?contribId=454&sessionId=718&confId=1296](http://ilcagenda.linearcollider.org/contributionDisplay.py?contribId=454&sessionId=718&confId=1296)

[2] G. C. Branco, L. Lavoura and J. P. Silva, “CP violation,” Oxford, UK: Clarendon (1999) 511 p

[3] J. F. Gunion and H. E. Haber, Phys. Rev. D 67 (2003) 075019, Phys. Rev. D 72 (2005) 095002.

[4] I. F. Ginzburg and M. Krawczyk, Phys. Rev. D 72, 115013 (2005).

[5] L. Lavoura, Phys. Rev. D 50 (1994) 7089.

[6] E. Accomando et al., arXiv:hep-ph/0608079.

[7] T. D. Lee, Phys. Rev. D 8 (1973) 1226.

*LCWS/ILC 2007*
Figure 1: Left: Constraints from $B \rightarrow \tau \nu_{\tau}$ and $b \rightarrow s \gamma$ data on the charged Higgs boson mass as a function of $\tan \beta$ in 2HDM (II) [19]; Right: Limits from the leptonic $\tau$ decay for $M_h = 20$ GeV and $\chi^A_V = 0$ in 2HDM(II): tree-level exclusion of a region below the straight line $M_{H^\pm} \geq 1.71 \, \tan \beta$ GeV and one-loop exclusion of the region above the curve $\Delta \sim \tan \beta^2 \left[ \ln \frac{M_h}{M_{H^\pm}} + 1 \right]$. The excluded region lies on the right on the curves: bold for $M_A = M_{H^\pm}$, dotted for $M_A = 100$ GeV. Exclusion from $\tau \rightarrow e \nu_{\tau} \bar{\nu}_{\tau}$ is represented by dashed line [20].

[8] G. C. Branco, Phys. Rev. Lett. 44 (1980) 504. G. C. Branco, Phys. Rev. D 22 (1980) 2901.
[9] I. P. Ivanov, Phys. Lett. B 632 (2006) 360, Phys. Rev. D 75, 035001 (2007) [Erratum-ibid. D 76, 039902 (2007)], arXiv:0710.3490 [hep-ph];
M. Maniatis, A. von Manteuffel and O. Nachtmann, arXiv:0707.3344 [hep-ph];
C. C. Nishi, Phys. Rev. D 76, 055013 (2007), Phys. Rev. D 74, 036003 (2006).
[10] N. G. Deshpande and E. Ma, Phys. Rev. D 18, 2574 (1978).
[11] J. L. Diaz-Cruz and A. Mendez, Nucl. Phys. B 380, 39 (1992).
[12] A. Barroso, P. M. Ferreira and R. Santos, Phys. Lett. B 652, 181 (2007);
A. Barroso, P. M. Ferreira, R. Santos and J. P. Silva, Phys. Rev. D 74, 085016 (2006);
A. Barroso, P. M. Ferreira and R. Santos, Phys. Lett. B 632, 684 (2006), Phys. Lett. B 603, 219 (2004)
[Erratum-ibid. B 629, 114 (2005)];
J. Velhinho, R. Santos and A. Barroso, Phys. Lett. B 322, 213 (1994).
[13] I. F. Ginzburg and K. A. Kanishev, arXiv:0704.3664 [hep-ph].
[14] M. Krawczyk and D. Sokolowska - in preparation;
D. Sokolowska, Master Diploma, Dept. of Physics, U. of Warsaw, July 2007
[15] I. F. Ginzburg and I. P. Ivanov, Phys. Rev. D 72, 115010 (2005);
A. G. Akeryoyd, A. Arhrib and E. M. Naimi, Phys. Lett. B 490, 119 (2000)
[16] S. L. Glashow and S. Weinberg, Phys. Rev. D 15 (1977) 1958;
E. A. Paschos, Phys. Rev. D 15 (1977) 1966.
[17] M. Krawczyk, Acta Phys. Polon. B 33, 2621 (2002).
[18] M. Misia et al., Phys. Rev. Lett. 98, 022002 (2007).
[19] D. s. Du, arXiv:0709.1315 [hep-ph], M. Nakao, talk given at LP2007, Korea.
[20] M. Krawczyk and D. Temes, Eur. Phys. J. C 44, 435 (2005).

LCWS/ILC 2007