Measurement of the slope parameter $\alpha$ for the $\eta \rightarrow 3\pi^0$ decay at KLOE

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Abstract

We report a preliminary measurement of the slope parameter $\alpha$ for the $\eta \rightarrow 3\pi^0$ decay carried out with KLOE at DAΦNE; where $\alpha$ is the parameter describing the energy dependence of the square of the matrix element for this decay. By fitting the event density in the Dalitz plot with a collected statistic of $420 \text{ pb}^{-1}$ we determine $\alpha = -0.027 \pm 0.004 \text{ (stat)} \pm 0.006 \text{ (syst)}$. This result is consistent with current chiral perturbation theory calculations within the unitary approach.

1. Introduction

With a branching ratio of $32\%$ the $\eta \rightarrow 3\pi^0$ decay is a major decay mode of the $\eta$ despite the fact that is a G–parity forbidden transition. Neglecting a small electromagnetic transition (Sutherland’s theorem [1]) this decay is due almost exclusively to the isospin breaking part of QCD:

$$\mathcal{L}_I = -\frac{1}{2}(m_u - m_d)(\bar{u}u - \bar{d}d)$$

(1)

and provides a nice way to determine the up-down quark mass difference.

Many theoretical issues [2], [3] refer to the study of the charged and neutral decay mode of $\eta \rightarrow 3\pi$, and in particular to the Dalitz plot parameters of these decays.

The Dalitz plot distribution of the $\eta \rightarrow 3\pi^0$ decay is conventionally described in terms of one kinematical variable:

$$z = \frac{2}{3} \sum_{i=1}^{3} \left( \frac{3E_i - m_\eta}{m_\eta - 3m_{\pi^0}} \right)^2 = \frac{\rho^2}{\rho^2_{MAX}}$$

(2)

where $E_i$ denotes the energy of the i-th pion in the $\eta$ rest frame and $\rho$ is the distance from a point on the Dalitz plot to its center. $\rho_{MAX}$ is the maximum value of $\rho$. For the decays into three identical particles, it is possible to use a symmetrical Dalitz plot where the event density is described by a single quadratic slope parameter $\alpha$, which represents the difference from pure phase space:

$$|A_{\eta \rightarrow 3\pi^0}(z)|^2 \sim 1 + 2\alpha z.$$ 

(3)

The lowest order predictions of Chiral Perturbation Theory quote a zero value for $\alpha$. The event density in the Dalitz plot must be uniform. A non zero value is instead expected by dispersive calculation [4] where the effect of $\pi – \pi$ rescattering is included. Lately a theoretical result [5].
obtained in the chiral unitary approach based on the Bethe–Salpeter equation, provide for \( \alpha \) the value \(-0.031\). There are three previous experimental determination of \( \alpha \): the GAMS 2000 group \[6\] quoted \( \alpha = -0.022 \pm 0.023 \) based on \( 5 \times 10^4 \) events; the Crystall Barrel Collaboration obtained \( \alpha = -0.052 \pm 0.020 \) from a sample of \( 10 \times 10^4 \) events and finally the Crystal Ball \[8\] result, \( \alpha = -0.031 \pm 0.004 \) based on \( 10^6 \) events. This was the first statistically significant measurement of \( \alpha \) which agree very well with the most recent theoretical result. In this paper we report on a new precise measurement of the Dalitz plot parameter for the \( \eta \rightarrow 3\pi^0 \) decay.

3. Dalitz plot of \( \eta \rightarrow \pi^0 \pi^0 \pi^0 \) decay

At KLOE the \( \eta \) meson is produced in the process \( \phi \rightarrow \eta \gamma \) where the recoil photon \( (E_\gamma = 363 \text{ MeV}) \) is monochromatic and easily selected. Thus to select the final state we require to have seven prompt clusters in the event. After applying a kinematic fit, which requires the energy–momentum conservation, we look for the best pairing of photons into \( \pi^0 \) by constructing a pseudo–\( \chi^2 \) variable for each of the 15 possible pairs:

\[
\chi_j^2 = \sum_{i=1}^{3} \left( \frac{m_{j,\pi^0} - M_{\pi^0}}{\sigma_{m_{\pi^0}}} \right)^2 \quad j = 1, 2, \ldots, 15.
\]

where

- \( m_{j,\pi^0} \) is the invariant mass of the \( i^{th} \pi^0 \), in correspondence of the \( j^{th} \) combination;
- \( M_{\pi^0} \) is the \( \pi^0 \) mass, \( (M_{\pi^0} = 134.98 \text{ MeV}/c^2) \);
- \( \sigma_{m_{\pi^0}} \) is the resolution on \( m_{\pi^0} \).

The chosen pair is that one which minimize the \( \chi^2 \). A second kinematic fit constraining the \( \pi^0 \) mass has been performed, improving the resolution on \( z \) by a factor two. The Monte Carlo, MC, \( z \) distribution at generation (pure phase space), see fig[4] and after reconstruction, see fig[2] shows that resolution effects are not negligible for this analysis. Three samples with different efficiency, \( \varepsilon \) and purity on pairing, \( P \):

- Low Purity \( P = 75.4\% \) \( \varepsilon = 30.3\% \)
- Medium Purity \( P = 92.0\% \) \( \varepsilon = 13.6\% \)
- High Purity \( P = 97.6\% \) \( \varepsilon = 4.3\% \)

have been analyzed by cutting on the difference of the two lowest value of \( \chi^2 \).

4. Measurement of the slope parameter \( \alpha \)

In order to estimate \( \alpha \) an unbinned likelihood function is built by convoluting the event density

2. DAΦNE and KLOE

The DAΦNE e\(^+\)e\(^-\) collider operates at a total energy \( W = 1020 \text{ MeV} \), the mass of the \( \phi(1020) \)-meson. Approximately \( 3 \times 10^6 \phi \)-mesons are produced for each pb\(^{-1}\) of collected luminosity. Since 2001, KLOE has integrated a total luminosity of about 2.5 fb\(^{-1}\). Results presented in this paper are based on data collected on 2001 – 2002 only and correspond to about 420 pb\(^{-1}\). The KLOE detector consists of a large cylindrical drift chamber, DC, surrounded by a lead/scintillating-fiber electromagnetic calorimeter, EMC. The drift chamber \[9\], is 4 m in diameter and 3.3 m long. The momentum resolution is \( \sigma(p_T)/p_T \sim 0.4\% \). Two track vertices are reconstructed with a spatial resolution of \( \sim 3 \text{ mm} \). The calorimeter \[10\], composed of a barrel and two endcaps, covers 98\% of the solid angle. Energy and time resolution are \( \sigma(E)/E = 5.7%/\sqrt{E[\text{GeV}]} \) and \( \sigma(t) = 57 \text{ ps}/\sqrt{E[\text{GeV}]} \) \( \oplus \) 100 ps. A superconducting coil around the detector provides a 0.52 T magnetic field. The KLOE trigger \[11\], uses calorimeter and drift chamber information. For the present analysis only the electromagnetic calorimeter (EMC) signals have been used. Two local energy deposits above threshold, \( E_{\text{th}} \) > 50 MeV for the barrel and \( E_{\text{th}} \) > 150 MeV for the endcaps, are required.
with the resolution function and correcting for the probability of wrong photon pairing in $\pi^0$'s. Using a sample with high purity and fitting in the range $0-1$, we found the preliminary result [14]:

$$\alpha = -0.014 \pm 0.004 \, \text{(stat)} \pm 0.005 \, \text{(syst)}, \quad (5)$$

where the systematics has been evaluated by varying the analysis cuts, the fit range and measuring the maximum observed variation of $\alpha$ with respect to the samples with different purity.

We observed a relevant dependence of $\alpha$ on the fitting range. Moreover, the low purity sample shows two different slopes (see fig [3] in the Data–MC ratio of the $z$ distribution. With a dedicated simulation we have realized that this is essentially due to a different value of invariant mass of three pions system ($\pi^0 \pi^0 \pi^0$) in the Monte Carlo generator, $M_\eta = 547.30 \text{ MeV}/c^2$, with respect to the one recently measured [13] by our experiment:

$$M_\eta = 547.822 \pm 0.005_{\text{stat}} \pm 0.069_{\text{syst}} \text{ MeV}/c^2. \quad (6)$$

As a consequence, the accessible phase space on data is larger than the one on Monte Carlo simulation. A new evaluation of $\alpha$ has been obtained after a kinematic fit with an additional constraint on the $\eta$ mass, see Table 1.

| Range  | Low Pur. $\alpha \cdot 10^{-3}$ | Med. Pur. $\alpha \cdot 10^{-3}$ | High Pur. $\alpha \cdot 10^{-3}$ |
|--------|---------------------------------|----------------------------------|-------------------------------|
| (0,1)  | $-30 \pm 2$                     | $-31 \pm 3$                     | $-26 \pm 4$                   |
| (0,0.8)| $-26 \pm 2$                     | $-28 \pm 3$                     | $-22 \pm 5$                   |
| (0,0.7)| $-26 \pm 3$                     | $-27 \pm 4$                     | $-23 \pm 5$                   |
| (0,0.6)| $-30 \pm 4$                     | $-31 \pm 4$                     | $-20 \pm 6$                   |

Table 1
Fitted results for the slope parameter $\alpha$ for the kinematic fit with the $\eta$ mass constrained at $M_\eta = 547.822 \text{ MeV}/c^2$.

In this way, good stability with respect to the range of fit is observed, and the linearity of the
Data–MC ratio of the \( z \) distribution has been recovered, (see fig.4). In order to give the final result, the phase space area where the \( z \) distribution is flat, is used as fit region \((z \in [0, 0.7])\). The systematic uncertainties on \( \alpha \) are summarized in Table 2. We have first taken into account the correction for the Data–MC discrepancy in the photon energy resolution, RES, the effect of the fit range. We have then determined the \( \alpha \) variation with respect to other purity samples and the error due to the combinatorial background evaluation. We have also tested the \( \alpha \)–dependence on the chosen value of the \( M_\eta \) used in the fit constraint.

5. Results

We quote as preliminary result for the slope parameter \( \alpha \) the one obtained with the Medium Purity sample, \((\text{about } 650000 \eta \rightarrow 3\pi^0 \text{ decays})\). The result including the statistical uncertainty from the fit and the evaluated systematic error is:

\[
\alpha = -0.027 \pm 0.004 \, (\text{stat}) ^{+0.004}_{-0.006} \, (\text{syst}) \tag{7}
\]

with: \( \chi^2/\text{ndf} = 13.72/17 \). The result is within errors compatible with the result from Crystall Ball [8] based on \( 10^6 \) events of:

\[
\alpha = -0.031 \pm 0.004 \tag{8}
\]

The our measurement of \( \alpha \) also agrees with the calculations from the chiral unitary approach [5].

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