Twisted acceleration-enlarged Newton-Hooke Hopf algebras

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Abstract

Ten Abelian twist deformations of acceleration-enlarged Newton-Hooke Hopf algebra are considered. The corresponding quantum space-times are derived as well. It is demonstrate that their contraction limit $\tau \to \infty$ leads to the new twisted acceleration-enlarged Galilei spaces.
1 Introduction

The two Newton-Hooke cosmological algebras $NH_{\pm}$ were introduced in the framework of classification of all kinematical groups [1]. Both algebras contain the characteristic cosmological time scale $\tau$ which can be interpreted in terms of the inverse of Hubble’s constant for the expanding universe ($NH_+$) or associated with the ”period” for the oscillating case ($NH_-$). For time parameter $\tau$ approaching infinity we get the Galilei group $\mathcal{G}$ acting on the standard (flat) nonrelativistic space-time.

Recently, there were proposed two acceleration-enlarged Newton-Hooke algebras $\tilde{NH}_{\pm}$ (see [2], [3]), which contain, apart from rotation $(M_{ij})$, boost $(K_i)$ and space-time translation $(P_i, H)$ generators, the additional ones denoted by $F_i$, responsible for constant acceleration. Of course, if all generators $F_i$ are equal zero we obtain the Newton-Hooke algebras $NH_{\pm}$ [1] (see also [4]-[6]), while for time parameter $\tau$ running to infinity we get the acceleration-enlarged Galilei group $\tilde{\mathcal{G}}$ proposed in [7].

In this article we discuss the role which can be played by the acceleration-enlarged Newton-Hooke symmetries in a context of noncommutative geometry. The suggestion to use noncommutative coordinates goes back to Heisenberg and was formalized by Snyder in [8]. Recently, there were also found formal arguments based mainly on Quantum Gravity [9], [10] and String Theory models [11], [12] indicating that space-time at Planck scale should be noncommutative, i.e. it should have a quantum nature.

Recently, the Abelian (Reshetikhin) twist deformations (see [13]-[15]) of the (ordinary) Newton-Hooke Hopf algebras $U_0(NH_{\pm})$ have been proposed in [16]. It has been shown that the corresponding quantum space-times are periodic or expanding in time for $U_0(NH_-)$ or $U_0(NH_+)$ algebras respectively. Besides, it was also demonstrated that for cosmological time parameter $\tau$ approaching infinity, we get the twisted Galilei quantum groups and the corresponding canonically, Lie-algebraically and quadratically deformed nonrelativistic space-times [17], [18].

In this article we consider ten Abelian twist deformations of the acceleration-enlarged Newton-Hooke Hopf algebras $U_0(\tilde{NH}_{\pm})$. In such a way we investigate the impact of the cosmological time $\tau$ as well as the impact of the additional generators $F_i$ on the structure of quantum space. Particularity, we demonstrate that due to the presence of parameter $\tau$, the corresponding space-times can be periodic or expanding in time for $U_0(\tilde{NH}_-)$$ or $U_0(\tilde{NH}_+)$ Hopf algebras respectively. Moreover, we also provide twisted acceleration-enlarged Galilei quantum groups and the corresponding space-times, as the $\tau \to \infty$ limit of the considered acceleration-enlarged Newton-Hooke Hopf structures. Surprisingly, the obtained in such a way acceleration-enlarged Galilei quantum spaces provide (due to the presence of generators $F_i$) the new cubic and quartic type of space-time noncommutativity, i.e. they take the form

$$\left[ x_{\mu}, x_{\nu} \right] = i\alpha_{\mu\nu}^{\rho_1...\rho_n} x_{\rho_1} ... x_{\rho_n}, \quad (1)$$

1 By ”ordinary” we mean the Newton-Hooke algebra without additional $F_i$ generators.

2 The Newton-Hooke Hopf algebras $U_0(NH_{\pm})$ are given by algebraic commutation relations for $NH_{\pm}$ groups, supplemented by the trivial coproduct sector $\Delta_0(a) = a \otimes 1 + 1 \otimes a$.

3 $x_0 = ct.$
with $n = 3$ and $4$ respectively\(^4\).

It should be noted that two kinds of such obtained space-times appear to be quite interesting. First of them (see formula (33)) provides the deformation parameter $\beta$ with dimension $[\beta] = [\text{acceleration} \times \text{acceleration}]$, while the second one (see formula (36)) provides the parameter $\beta'$ with dimension $[\beta'] = [\text{acceleration}]$. Such a result indicates that there can appear a direct link between noncommutativity and the intensively studied in the last time, so-called MOND model \(^{19}\), which assumes that there exist in nature observer independent acceleration parameter $a_0$. Consequently, it looks sensible to consider the simple classical (Newtonian) models associated with the above acceleration-enlarged quantum space-times. In the case of ordinary twisted Galilei symmetry such investigations have been performed in \(^{20}\) and \(^{21}\) respectively.

Finally, it should be mentioned that considered acceleration-enlarged Newton-Hooke Hopf algebras and the corresponding quantum space-times play a special role. By the proper contractions limit ($\tau \to \infty$ or $F_i \to 0$) of such structures one can derive (or reproduce) the quantum spaces associated with: the twisted acceleration-enlarged Galilei Hopf algebras, the twist-deformed Newton-Hooke quantum groups \(^{16}\), and twisted Galilei Hopf algebras \(^{17, 18}\). For this reason, the considered spaces can be treated as a ”source” for other Abelian twist-deformed nonrelativistic space-times.

The paper is organized as follows. In second section ten Abelian classical $r$-matrices for twisted acceleration-enlarged Newton-Hooke Hopf algebras are considered. The corresponding ten quantum space-times are provided in section 3, while their $\tau \to \infty$ contractions to the acceleration-enlarged Galilei spaces are discussed in section 4. Finally, two contractions leading to the well known (ordinary) Newton-Hooke and Galilei space-times are mentioned in section 5. The final remarks are presented in the last section.

### 2 Twisted acceleration-enlarged Newton-Hooke Hopf algebras

In accordance with Drinfeld twist procedure \(^{13-15}\), the algebraic sector of twisted acceleration-enlarged Newton-Hooke Hopf algebra remains undeformed

\[
\begin{align*}
[M_{ij}, M_{kl}] &= i (\delta_{il} M_{jk} - \delta_{jl} M_{ik} + \delta_{jk} M_{il} - \delta_{ik} M_{jl}) , \\
[H, P_i] &= \pm \frac{i}{\tau^2} K_i , \\
[M_{ij}, K_k] &= i (\delta_{jk} K_i - \delta_{ik} K_j) , \\
[M_{ij}, P_k] &= i (\delta_{jk} P_i - \delta_{ik} P_j) , \\
[M_{ij}, H] &= [K_i, K_j] = [K_i, P_j] = 0 , \\
[K_i, H] &= -i P_i , \\
[P_i, P_j] &= 0 ,
\end{align*}
\] (2)

\(^4\)There was considered in the literature only canonical ($n = 0$), Lie-algebraic ($n = 1$) and quadratic ($n = 2$) type of space-time noncommutativity.

\(^5\)MOND model in a simple way explains the movies of galactic’s arms at long distance scale. However, the proper modification of Newton equation as well as the acceleration parameter $a_0$ are introduced into model without any principal (theoretical) rules.
\[[F_i,F_j] = [F_i,P_j] = [F_i,K_j] = 0 \ , \ [M_{ij},F_k] = i (\delta_{jk} F_i - \delta_{ik} F_j) \ , \]
\[[H,F_i] = 2i K_i \ , \]

while the coproducts and antipodes transform as follows
\[ \Delta_0(a) \to \Delta(a) = \mathcal{F} \circ \Delta_0(a) \circ \mathcal{F}^{-1} \ , \ S.(a) = u \cdot S_0(a) \cdot u^{-1} , \]

with \( \Delta_0(a) = a \otimes 1 + 1 \otimes a \), \( S_0(a) = -a \) and \( u = \sum f_{(1)} S_0(f_{(2)}) \) (we use Sweedler’s notation \( \mathcal{F} = \sum f_{(1)} \otimes f_{(2)} \)). Present in the commutation relations \( [2] \) parameter \( \tau \) denotes the characteristic for Newton-Hooke algebra cosmological time scale (in the limit \( \tau \to \infty \) we get the acceleration-enlarged Galilei Hopf structure \( \mathcal{U}_0(\hat{G}) \)). Besides, it should be noted, that the twist factor \( \mathcal{F} \in \mathcal{U}(\hat{NH}_\pm) \otimes \mathcal{U}(\hat{NH}_\pm) \) satisfies the classical cocycle condition
\[ \mathcal{F}_{12} \cdot (\Delta_0 \otimes 1) \mathcal{F} = \mathcal{F}_{23} \cdot (1 \otimes \Delta_0) \mathcal{F} , \]

and the normalization condition
\[ (\epsilon \otimes 1) \mathcal{F} = (1 \otimes \epsilon) \mathcal{F} = 1 , \]

with \( \mathcal{F}_{12} = \mathcal{F} \otimes 1 \) and \( \mathcal{F}_{23} = 1 \otimes \mathcal{F} \).

It is well known, that the twisted algebra \( \mathcal{U}(\hat{NH}_\pm) \) can be described in terms of so-called classical \( r \)-matrix \( r \in \mathcal{U}(\hat{NH}_\pm) \otimes \mathcal{U}(\hat{NH}_\pm) \), which satisfies the classical Yang-Baxter equation (CYBE)
\[ [[ r_\cdot, r_\cdot ]] = [ r_{12}, r_{13} + r_{23} ] + [ r_{13}, r_{23} ] = 0 \ , \]

where symbol \([ [ \cdot, \cdot ] ]\) denotes the Schouten bracket and for \( r = \sum_i a_i \otimes b_i \)
\[ r_{12} = \sum_i a_i \otimes b_i \otimes 1 \ , \ r_{13} = \sum_i a_i \otimes 1 \otimes b_i \ , \ r_{23} = \sum_i 1 \otimes a_i \otimes b_i \ . \]

In this article we consider ten Abelian twist-deformations of acceleration-enlarged Newton-Hooke Hopf algebra, described by the following \( r \)-matrices:

1) \[ r_{\beta_1} = \frac{1}{2} \beta_{kl}^1 F_k \wedge F_l \quad [ \beta_{kl}^1 = -\beta_{lk}^1 ] \ , \]
2) \[ r_{\beta_2} = \frac{1}{2} \beta_{kl}^2 F_k \wedge P_l \quad [ \beta_{kl}^2 = -\beta_{lk}^2 ] \ , \]
3) \[ r_{\beta_3} = \frac{1}{2} \beta_{kl}^3 K_k \wedge F_l \quad [ \beta_{kl}^3 = -\beta_{lk}^3 ] \ , \]
4) \[ r_{\beta_4} = \beta_4 F_m \wedge M_{kl} \quad [ m, k, l - \text{fixed}, \ m \neq k, l ] \ , \]

\[ a \wedge b = a \otimes b - b \otimes a. \]
5) \[ r_{\beta_5} = \frac{1}{2} \beta_{5}^{kl} P_k \wedge P_l \quad [ \beta_{5}^{kl} = -\beta_{5}^{lk} ] , \]  
6) \[ r_{\beta_6} = \frac{1}{2} \beta_{6}^{kl} K_k \wedge P_l \quad [ \beta_{6}^{kl} = -\beta_{6}^{lk} ] , \]  
7) \[ r_{\beta_7} = \frac{1}{2} \beta_{7}^{kl} K_k \wedge K_l \quad [ \beta_{7}^{kl} = -\beta_{7}^{lk} ] , \]  
8) \[ r_{\beta_8} = \beta_{8} K_m \wedge M_{kl} \quad [ m, k, l - \text{fixed}, \quad m \neq k, l ] , \]  
9) \[ r_{\beta_9} = \beta_{9} P_m \wedge M_{kl} \quad [ m, k, l - \text{fixed}, \quad m \neq k, l ] , \]  
10) \[ r_{\beta_{10}} = \beta_{10} M_{ij} \wedge H . \]  

Due to Abelian character of the above carriers (all of them arise from the mutually commuting elements of the algebra), the corresponding twist factors can be get in a standard way [13]-[15], i.e. they take the form 

\[ F_{\beta_k} = \exp (i r_{\beta_k}) \quad ; \quad k = 1, 2, ..., 10 . \]  

Let us note that first four matrices include acceleration generators \( F_i \), while the next six factors are the same as in the case of Galilei and ordinary Newton-Hooke Hopf algebra, considered in [17] and [16] respectively. Of course, for all deformation parameters \( \beta_i \) approaching zero the discussed above Hopf structures \( U_{\beta_i}(\hat{N}H_{\pm}) \) become classical, i.e. they become undeformed.

### 3 Quantum acceleration-enlarged Newton-Hooke space-times

Let us now turn to the deformed space-times corresponding to the twist-deformations 1)-10) discussed in pervious section. They are defined as the quantum representation spaces (Hopf modules) for quantum acceleration-enlarged Newton-Hooke algebras, with action of the deformed symmetry generators satisfying suitably deformed Leibnitz rules [22], [23], [24].

The action of generators \( M_{ij}, K_i, P_i, H \) and \( F_i \) on a Hopf module of functions depending on space-time coordinates \( (t, x_i) \) is given by

\[ H \triangleright f(t, \overline{x}) = i \partial_t f(t, \overline{x}) , \quad P_i \triangleright f(t, \overline{x}) = i C_\pm \left( \frac{t}{\tau} \right) \partial_i f(t, \overline{x}) , \]  
\[ M_{ij} \triangleright f(t, \overline{x}) = i (x_i \partial_j - x_j \partial_i) f(t, \overline{x}) , \quad K_i \triangleright f(t, \overline{x}) = i \tau S_\pm \left( \frac{t}{\tau} \right) \partial_i f(t, \overline{x}) , \]

and

\[ F_i \triangleright f(t, \overline{x}) = \pm 2 i \tau^2 \left( C_\pm \left( \frac{t}{\tau} \right) - 1 \right) \partial_i f(t, \overline{x}) , \]
with \( C_+\left[\frac{t}{\tau}\right] = \cosh \left[\frac{t}{\tau}\right] \), \( C_-\left[\frac{t}{\tau}\right] = \cos \left[\frac{t}{\tau}\right] \), \( S_+\left[\frac{t}{\tau}\right] = \sinh \left[\frac{t}{\tau}\right] \), \( S_-\left[\frac{t}{\tau}\right] = \sin \left[\frac{t}{\tau}\right] \).

Moreover, the \(*\)-multiplication of arbitrary two functions is defined as follows

\[
f(t, \tau) \ast_{\beta_i} g(t, \tau) := \omega \circ \left( \mathcal{F}_{\beta_i}^{-1} \triangleright f(t, \tau) \otimes g(t, \tau) \right)
\]

where symbol \( \mathcal{F}_{\beta_i} \) denotes the twist factor (see (17)) corresponding to the proper acceleration-enlarged Newton-Hooke Hopf algebra and \( \omega \circ (a \otimes b) = a \cdot b \).

In such a way we get ten quantum space-times

1) \( [t, x_a] \ast_{\beta_1} = 0 \), \( [x_a, x_b] \ast_{\beta_1} = 4i\beta_1^k l^4 \left( C_\pm \left( \frac{t}{\tau} \right) - 1 \right)^2 (\delta_{ak}\delta_{bd} - \delta_{al}\delta_{bk}) \)

2) \( [t, x_a] \ast_{\beta_2} = 0 \), \( [x_a, x_b] \ast_{\beta_2} = \pm i\beta_2^k l^2 \left( C_\pm \left( \frac{t}{\tau} \right) - 1 \right) C_\pm \left( \frac{t}{\tau} \right) (\delta_{ak}\delta_{bd} - \delta_{al}\delta_{bk}) \)

3) \( [t, x_a] \ast_{\beta_3} = 0 \), \( [x_a, x_b] \ast_{\beta_3} = \pm i\beta_3^k l^3 \left( C_\pm \left( \frac{t}{\tau} \right) - 1 \right) S_\pm \left( \frac{t}{\tau} \right) (\delta_{ak}\delta_{bd} - \delta_{al}\delta_{bk}) \)

4) \( [t, x_a] \ast_{\beta_4} = 0 \)

\[
[x_a, x_b] \ast_{\beta_4} = \pm 4i\beta_4 l^2 \left( C_\pm \left( \frac{t}{\tau} \right) - 1 \right) \left[ \delta_{ma}(x_k\delta_{bd} - x_l\delta_{bk}) - \delta_{mb}(x_k\delta_{al} - x_l\delta_{ak}) \right]
\]

5) \( [t, x_a] \ast_{\beta_5} = 0 \), \( [x_a, x_b] \ast_{\beta_5} = i\beta_5^k l^2 C_\pm \left( \frac{t}{\tau} \right) \left( \delta_{ak}\delta_{bd} - \delta_{al}\delta_{bk} \right) \)

6) \( [t, x_a] \ast_{\beta_6} = 0 \), \( [x_a, x_b] \ast_{\beta_6} = i\beta_6^k l^2 C_\pm \left( \frac{t}{\tau} \right) S_\pm \left( \frac{t}{\tau} \right) (\delta_{ak}\delta_{bd} - \delta_{al}\delta_{bk}) \)

7) \( [t, x_a] \ast_{\beta_7} = 0 \), \( [x_a, x_b] \ast_{\beta_7} = i\beta_7^k l^2 S_\pm \left( \frac{t}{\tau} \right) (\delta_{ak}\delta_{bd} - \delta_{al}\delta_{bk}) \)

8) \( [t, x_a] \ast_{\beta_8} = 0 \)

\[
[x_a, x_b] \ast_{\beta_8} = 2i\beta_8 l S_\pm \left( \frac{t}{\tau} \right) \left[ \delta_{ma}(x_k\delta_{bd} - x_l\delta_{bk}) - \delta_{mb}(x_k\delta_{al} - x_l\delta_{ak}) \right]
\]

9) \( [t, x_a] \ast_{\beta_9} = 0 \)

\[
[x_a, x_b] \ast_{\beta_9} = 2i\beta_9 C_\pm \left( \frac{t}{\tau} \right) \left[ \delta_{ma}(x_k\delta_{bd} - x_l\delta_{bk}) - \delta_{mb}(x_k\delta_{al} - x_l\delta_{ak}) \right]
\]

10) \( [t, x_a] \ast_{\beta_{10}} = 2i\beta_{10} \left( \delta_{ia} x_j - x_i \delta_{ja} \right) \), \( [x_a, x_b] \ast_{\beta_{10}} = 0 \),

associated with matrices 1)-10), respectively.
Let us note that due to the form of functions \( C_{±}[\tau] \) and \( S_{±}[\tau] \) the spatial noncommutativities 1)-9) are expanding or periodic in time respectively. Moreover, all of them introduce classical time and quantum spatial directions. The last type of space-time noncommutativity provides the quantum time and classical spatial variables. It should be also noted that spaces 1), 2), 4), 5), 7) and 9) are invariant with respect time reflection \( t → -t \), while space-times 1), 3), 5), 7) and 10) - with respect \( \vec{x} → -\vec{x} \) transformation.

Of course, for all deformation parameters \( \beta_i \) approaching zero, the above quantum space-times become commutative.

### 4 Twisted acceleration-enlarged Galilei Hopf algebras - the \( \tau → ∞ \) limit

In this section we provide twisted acceleration-enlarged Galilei Hopf algebras \( U_{\beta_i}(\hat{G}) \) and corresponding quantum space-times, as the \( \tau → ∞ \) limit of Hopf structures discussed in previous sections. In such a limit the commutation relations (2) become \( \tau \)-independent, i.e. we neglect the impact of the cosmological time scale \( \tau \) on the structure of the considered Hopf algebras.

First of all, we perform the contraction limit \( \tau → ∞ \) of the formulas (2) and (7)-(16). Consequently, the corresponding classical \( r \)-matrices remain the same as (7)-(16), while the algebraic sector of all considered \( U_{\beta_i}(\hat{G}) \) algebras takes the form

\[
\begin{align*}
[M_{ij}, M_{kl}] &= i (\delta_{il} M_{jk} - \delta_{jl} M_{ik} + \delta_{jk} M_{il} - \delta_{ik} M_{jl}) , \\
[M_{ij}, K_k] &= i (\delta_{jk} K_i - \delta_{ik} K_j) , \\
[M_{ij}, H] &= [K_i, K_j] = [K_i, P_j] = 0 , \\
[M_{ij}, P_k] &= i (\delta_{jk} P_i - \delta_{ik} P_j) , \\
[F_i, F_j] &= [F_i, P_j] = [F_i, K_j] = 0 , \\
[M_{ij}, F_k] &= i (\delta_{jk} F_i - \delta_{ik} F_j) , \\
[H, F_i] &= 2i K_i .
\end{align*}
\]

The corresponding coproduct sectors can be get by application of the formulas (3) and (17).

Let us now turn to the corresponding quantum nonrelativistic space-times. One can check (see \( \tau → ∞ \) limit of the formulas (22)-(31)) that they look as follows:

\[
\begin{align*}
1) \quad [t, x_a]_{\star_{\beta_1}} &= 0 , \\
[x_a, x_b]_{\star_{\beta_1}} &= i \beta_1^{kl} t^4 (\delta_{ak} \delta_{bl} - \delta_{al} \delta_{bk}) , \\
2) \quad [t, x_a]_{\star_{\beta_2}} &= 0 , \\
[x_a, x_b]_{\star_{\beta_2}} &= \frac{i}{2} \beta_2^{kl} t^2 (\delta_{ak} \delta_{bl} - \delta_{al} \delta_{bk}) ,
\end{align*}
\]

\(^7\)It should be noted that the commutation relations (33)-(42) can be also derived with use of the formula (21) and differential representation of acceleration-enlarged Galilei generators.

from physical point of view. Already mentioned in Introduction both quantum spaces appear to be quite interesting in the case of $U^{\beta}(G)$. The second contraction is given by $F_i \rightarrow 0$ and $\tau \rightarrow \infty$ limit, and provides the twist-deformed $U^{\beta_i}(G)$ Galilei Hopf algebras, proposed (together with corresponding quantum spaces) in the articles [17] and [18].

5 Twisted Newton-Hooke (and Galilei) Hopf algebras - the $F_i \rightarrow 0$ (and $\tau \rightarrow \infty$) limit

It should be noted, that apart of provided in pervious section contraction limit $\tau \rightarrow \infty$ of $U^{\beta_i}(\hat{NH}_\pm)$ Hopf algebras, there exist two other contractions. First of them is defined by $F_i \rightarrow 0$ limit and leads to the twisted $U^{\beta_5}(\hat{NH}_\pm), \ldots, U^{\beta_{10}}(\hat{NH}_\pm)$ Newton-Hooke Hopf algebras and corresponding quantum space-times, introduced in paper [16]. The second contraction is given by $F_i \rightarrow 0$ and $\tau \rightarrow \infty$ limit, and provides the twist-deformed $U^{\beta_i}(G), \ldots, U^{\beta_{10}}(G)$ Galilei Hopf algebras, proposed (together with corresponding quantum spaces) in the articles [17] and [18].
6 Final remarks

In this article we consider ten Abelian twist-deformations of acceleration-enlarged Newton-Hooke Hopf algebras. Besides, we demonstrate that as in the case of ordinary twist-deformed Newton-Hooke Hopf algebra, the corresponding spaces can be periodic and expanding in time for $U_{\beta_i}(\widehat{NH}_-) \text{ and } U_{\beta_i}(\widehat{NH}_+)$ quantum groups respectively. In $\tau \to \infty$ limit we also discover new twisted acceleration-enlarged Galilei Hopf algebras and ten quantum space-times (33)-(42).

It should be noted that present studies can be extended in various ways. First of all, one can find the dual Hopf structures $D_{\beta_i}(\widehat{NH}_{\pm})$ with the use of FRT procedure [25] or by canonical quantization of the corresponding Poisson-Lie structures [26]. Besides, as it was already mentioned in Introduction, one should ask about the basic dynamical models corresponding to the acceleration-enlarged Newton-Hooke and Galilei space-times (22)-(31) and (33)-(42). Finally, one can also consider more complicated (non-Abelian) twist deformations of acceleration-enlarged Newton-Hooke Hopf algebras, i.e. one can find the twisted coproducts, corresponding noncommutative space-times and dual Hopf structures. Such problems are now under consideration.

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