Quantum communication in the spin chain with multiplespin exchange interaction

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Abstract

The transmission of quantum states in the anisotropic Heisenberg XXZ chain model with three-spin exchange interaction is studied. The average fidelity is used to evaluate the state transfer. It is found out that quantum communication can be enhanced by the anisotropic coupling and multiple spin interaction. Such spin model can reduce the time required for the perfect state transmission where the fidelity is unity. The maximally entangled Bell states can be generated and separated from the whole quantum systems.

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I. INTRODUCTION

Quantum communication is regarded as an important element of quantum information processing operations [1]. In any scenario, quantum states can be transferred from one location to another location through a quantum channel. The quantum channel is often composed of the qubits linearly connected by Heisenberg interactions. One qubit carries useful quantum information in the form of a finite state $|\psi\rangle$ while the other qubits are initialized into a certain state. The whole system evolves such that another qubit ends up in the state $|\psi\rangle$. This method does not require the qubit couplings to be switched on and off. Therefore, it is suitable to implement in solid-state quantum systems [2, 3].

Currently there are many schemes in low-dimensional quantum spin models [4–11]. Much more attention was paid to the investigation of the chains with only nearest-neighboring spin exchange interaction. In condensed matter physics, multiple spin exchange interactions are often present in many real quasi-one-dimensional magnets, especially the oxides of transition metals. Some theoretical models with three-spin interactions are extensively used for the description of the magnetic properties of real magnets [12–15]. These models are necessary because they can provide the possibility to compare experimental data with exact solutions for one-dimensional models. They often exhibit some quantum phase transitions [16, 17]. Therefore, it is of great interest to study the effects of multiple spin exchange interactions on quantum communication.

In this paper, the anisotropic Heisenberg XXZ chain model with three-spin interaction serves as the quantum channel for the transfer of arbitrary single-qubit quantum state. In experiments, a simple three-spin chain model with effective three-body interactions can be prepared by the optical lattices of equilateral triangles [18]. The simplest case of a three-qubit spin chain is investigated here because the quantum-state transfer over longer distance can be generated by extending this chain to the spin network according to the method in [6]. The exact dynamics of this model is analytically given in Sec. II. The general expression of the average fidelity for quantum transmission can also be obtained when a certain initial state is chosen. Finally, a brief discussion concludes the paper.
II. QUANTUM STATE TRANSFER

The quantum channel in this protocol is the anisotropic Heisenberg three-spin chain with three-body exchange interactions. This multiple spin interaction corresponds to the coupling between next-nearest neighbors adjusted by the middle spin \[14, 18\]. The theoretical model is extensively investigated in the field of quantum phase transitions. The Hamiltonian of the chain in the open boundary condition can be expressed as

\[
H = \frac{J}{4} \left[ \sum_{i=1}^{N-1} (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \Delta \sigma_i^z \sigma_{i+1}^z) + \omega (\sigma_{i-1}^x \sigma_i^z \sigma_{i+1}^y - \sigma_{i-1}^z \sigma_i^x \sigma_{i+1}^y) \right] \tag{1}
\]

where \(\sigma_i^\alpha (\alpha = x, y, z)\) is the spin operator of the \(i\)th qubit in the chain. The parameter \(J\) is the nearest-neighbor Heisenberg coupling and \(\Delta\) denotes the anisotropy. Here \(\omega\) represents the relative strength of three-body exchange interaction. For the convenience, the Planck constant \(\hbar\) is assumed to be one and \(|1\rangle, |0\rangle\) are the eigenvectors of \(\sigma_z\) with the corresponding eigenvalues \(\pm 1\). In the following discussion, the simplest case of \(N = 3\) is analytically studied. The propagator \(\hat{U}(t)\) is used to describe the process of quantum-state transfer,

\[
\hat{U}(t) = \sum_j \exp(-iE_j t) |\varphi_j\rangle \langle \varphi_j|.
\tag{2}
\]

Here \(E_j\) and \(|\varphi_j\rangle\) include all the eigenvalue of the Hamiltonian \(H\) and the corresponding eigenvector respectively. Because the Hamiltonian of the system satisfies the symmetric property of \([H, S^z] = 0\) where the symmetric operator \(S^z = \frac{1}{2} \sum_i \sigma_i^z\) is the \(z\)-component of the total spin, all of the eigenvalues and the corresponding degenerate eigenvectors can be written as,

\[
E_0 = J\Delta; |\varphi_0\rangle = |111\rangle, |\bar{\varphi}_0\rangle \\
E_{k=1,2,3} = \frac{J}{2} \eta_k; |\varphi_k\rangle = a_k |100\rangle + b_k |010\rangle + c_k |001\rangle, |\bar{\varphi}_k\rangle.
\tag{3}
\]

In the above equation, \(|\bar{\varphi}\rangle\) is the degenerate eigenvector which is the spin flipped state of \(|\varphi\rangle\). The coefficients satisfy \(a_k = \sqrt{1 - b_k^2} e^{-i\beta_k}\), \(c_k = a_k^*\) and \(b_k = \frac{\eta_k \omega^2}{(\eta_k^2 - \omega^2)^2 + 2(\eta_k^2 + \omega^2)}\) where \(\tan \beta_k = \frac{\omega}{\eta_k}\). Here \(\eta_k (k = 1, 2, 3)\) belong to the three real roots of the equation of \(\eta^3 + 2\Delta \eta^2 - (2 + \omega^2) \eta - 2 \Delta \omega^2 = 0\). For the special case of \(\Delta = 0\), \(\eta_k = 0, \pm \sqrt{2 + \omega^2}\). In the product Hilbert space of \(\{|111\rangle, |110\rangle, |101\rangle, |100\rangle, |011\rangle, |010\rangle, |001\rangle, |000\rangle\}\), the propagator
can be written as

\[
\hat{U}(t) = \begin{pmatrix}
e^{-iJ\Delta t} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \tau_1 & \tau_4 & 0 & \tau_6 & 0 & 0 \\
0 & \tau_2 & \tau_5 & 0 & \tau_4 & 0 & 0 \\
0 & 0 & 0 & \tau_1 & 0 & \tau_2 & \tau_3 & 0 \\
0 & \tau_3 & \tau_2 & 0 & \tau_1 & 0 & 0 & 0 \\
0 & 0 & 0 & \tau_4 & 0 & \tau_5 & \tau_2 & 0 \\
0 & 0 & 0 & \tau_6 & 0 & \tau_4 & \tau_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{-iJ\Delta t}
\end{pmatrix}.
\]  

(4)

Here the elements are obtained by

\[
\tau_1 = \sum_{k} e^{-iE_k t} a_k c_k, \tau_2 = \sum_{k} e^{-iE_k t} a_k b_k, \tau_3 = \sum_{k} e^{-iE_k t} a_k^2, \tau_4 = \sum_{k} e^{-iE_k t} b_k c_k, \tau_5 = \sum_{k} e^{-iE_k t} b_k^2 \text{ and } \tau_6 = \sum_{k} e^{-iE_k t} c_k^2.
\]

In the recent experiments of nuclear magnetic resonance [19–22], the three-body exchange interactions can be controlled by the selection of sequences of radio-frequency pulses. By the method in Ref. [1], we can initialize the whole system of three spins. The anisotropy can be controlled by the selection of sequences of radio-frequency pulses. The fidelity for a pure input state \(|\Psi(0)\rangle = |\psi\rangle_A \otimes |00\rangle_{BC}\) where \(|\psi\rangle = \cos \frac{\theta}{2} |0\rangle_A + \sin \frac{\theta}{2} e^{i\phi} |1\rangle_A\), (0 \(\leq\theta \leq\pi, 0 \leq \phi \leq 2\pi\)) is an arbitrary single-qubit state. In this scheme, this quantum state need be transferred from qubit \(A\) to qubit \(C\). To evaluate the efficiency of quantum communication, we use the average fidelity,

\[
F_A(t) = \frac{\int_{0}^{2\pi} d\phi \int_{0}^{\pi} F(t) \sin \theta \, d\theta}{4\pi}
\]

(5)

According to Ref. [1], the fidelity for a pure input state \(F(t) = \{\text{Tr}[(\rho_0)^{1/2} \rho(t)(\rho_0)^{1/2}]\}^2 = \text{Tr}[\rho(t)\rho_0]\). Here \(\rho_0 = |\psi\rangle\langle\psi|\) and \(\rho(t) = \text{Tr}_{AB}[\hat{U}(t)|\Psi(0)\rangle\langle\Psi(0)|\hat{U}^\dagger(t)]\) where the symbol TrAB describes the trace over the qubits \(A, B\). It is found out that the average fidelity for the case of \(|\Psi(0)\rangle = |\psi\rangle_A \otimes |00\rangle_{BC}\) can be given by

\[
F_A(t) = \frac{1}{3} \left[1 + |\tau_6|^2 + \text{Re}(e^{iJ\Delta t}\tau_6)\right] + \frac{1}{6}(|\tau_1|^2 + |\tau_4|^2).
\]

(6)

Here Re denotes the real part of a complex number. The numerical results of the average fidelity can be illustrated in Fig. 1(a) when the anisotropy \(\Delta\) is varied. It is clearly shown that the curve of \(F_A\) changes periodically with the time for \(\Delta = 0\). Compared to the special case of \(\Delta = 0\), the anisotropy \(\Delta = 1\) can increase the maximal value of \(F_A\). It is proven that the anisotropy can enhance the efficiency of the quantum-state transfer. It is interesting to study the quantum transfer in the case of \(N > 3\). When the initial state of the system is
$|1\rangle_A \otimes |0\cdots0\rangle$, the fidelity can be calculated and shown in Fig. 1(b). The values of the fidelity can be increased after the long time but the maximal value is always less than one. The efficiency of the quantum communication is degraded with the increase of the number of spins.

In order to demonstrate the effects of the multiple spin exchange interactions on quantum communication, we mainly discuss the special case of $\Delta = 0$. From the expression of the propagator, it is found out that there is the characteristic time $t_c$ where the elements satisfy $\tau_1(t_c) = \tau_4(t_c) = 0$, $\tau_6(t_c) = -1$ and $\tau_3(t_c) = \tau_5(t_c)$. By the analytical method, we can deduce the finite expression of characteristic time,

$$ t_c = \frac{(4n + 2)\pi + 2 \arctan(\omega \sqrt{\omega^2 + 2})}{J \sqrt{\omega^2 + 2}}, (n = 0, 1, 2, \ldots). \quad (7) $$

Owing to the periodic property of the characteristic time, the average fidelity $F_A$ evolves periodically like the curve in Fig. 1. We numerically calculate the minimum of the characteristic time $t_c(n = 0)$ with the variation of the multiple spin exchange interaction $\omega$ in Fig. 2. It is clearly seen that the values of $t_c(n = 0)$ slightly increase and then decrease with increasing the relative strength of the multiple spin exchange interaction. When $\omega = \omega_0 \approx 0.6$, the value of the characteristic time is maximal. When the initial state is chosen as $|\Psi(0)\rangle = |\psi\rangle_A \otimes |00\rangle_{BC}$, the state at $t_c$ can be expressed as $|\Psi(t_c)\rangle = |00\rangle_{AB} \otimes (-\sigma_z^C|\psi\rangle_C)$. Though the average fidelity $F_A(t_c)$ cannot arrive at the maximal value of unity, the quantum state $|\psi\rangle$ can also be constructed after we apply the local operation $\sigma_z^C$ on the qubit $C$. At the characteristic time, the perfect quantum-state transmission of $F_A = 1$ can also be implemented with no local quantum operations. For example, if the whole system is initialized into $|\Psi(0)\rangle = |\psi\rangle_A \otimes |01\rangle_{BC}$, the total state at the characteristic time $t_c$ can be obtained by

$$ |\Psi(t_c)\rangle = \frac{1}{\omega^2 + 1} \left[ (\omega^2 - 1)|10\rangle_{AB} + 2i\omega|01\rangle_{AB} \right] \otimes |\psi\rangle_C \quad (8) $$

In this condition, the time required for the perfect quantum communication can be shortened by increasing the multiple spin exchange interaction $\omega > \omega_0$. Meanwhile, it is seen that qubits $A$ and $B$ are entangled and separated from qubit $C$ at the characteristic time $t_c$. According to Ref. [23-25], the concurrence $C$ can be used to measure the entanglement of the pure state in qubits $A$ and $B$,

$$ C = 2 \max\{0, \frac{2\omega(\omega^2 - 1)}{(\omega^2 + 1)^2} \} \quad (9) $$
The influence of the multiple spin exchange interactions $\omega$ on the entanglement $C$ is clearly shown in Fig. 3. The values of $C$ can arrive at the maximal value of one when $\omega \approx 0.42$ or 2.41. In this condition, the pure state of qubits $A$ and $B$ is the maximally entangled Bell state which is very useful in other tasks of quantum information processing. When the values of $\omega$ are chosen as 0 and 1, the pure state corresponds to the separate state $|10\rangle_{AB}$ and $|01\rangle_{AB}$ respectively. The entanglement will be extinguished if the values $\omega \rightarrow \infty$. In association with the property of the characteristic time, it is found out that the time for the achievement of perfect quantum communication is very short in the condition of $\omega \approx 2.41$ where the maximally entangled state can be generated and separated from the system.

III. DISCUSSION

An arbitrary single-qubit quantum state is transferred through the quantum channel of anisotropic Heisenberg three-spin chain with multiple spin exchange interaction. The propagator for the quantum-state transfer is analytically deduced. It is found out that the anisotropy can improve the efficiency of quantum communication. For the special case of $\Delta = 0$, there exists the characteristic time where the perfect quantum communication can be carried out. Under the influence of the multiple spin exchange interaction, the characteristic time can be reduced by the increase of $\omega$. The entangled pure state is generated and separated from the quantum system. This resource is usefully implemented in other tasks of quantum information processing.

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Figure caption

Figure 1
The solid lines denote the case of the anisotropy $\Delta = 0$ and the dashed lines represent the case of $\Delta = 1$. (a): The average fidelity $F_A$ is plotted as a function of time $t$ when $J = 1, \omega = 1$; (b): The fidelity $F$ in the chain with $N = 6$ is calculated when $J = 1, \omega = 1$.

Figure 2
The minimal value of the characteristic time $t_c(n = 0)$ is plotted with the variation of the multiple spin exchange interaction $\omega$ for the case of $J = 1$.

Figure 3
The entanglement $C$ at the characteristic time $t_c$ is plotted as a function of the multiple spin exchange interaction $\omega$ if the initial state is chosen as $|\Psi(0)\rangle = |\psi\rangle_A \otimes |01\rangle_{BC}$. 
Fig. 1
Fig. 2
Fig. 3