Theoretical Overview of Neutrino Properties

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I give an overview of some basic properties of massive neutrinos. The first part of this talk is devoted to three fundamental questions about three known neutrinos and to their flavor issues — the mass spectrum, mixing pattern and CP violation. The second part of this talk is to highlight a few hot topics at the frontiers of neutrino physics and neutrino astrophysics, including the naturalness and testability of TeV seesaw mechanisms at the LHC, effects of non-standard interactions on neutrino oscillations, flavor distributions of ultrahigh-energy cosmic neutrinos at neutrino telescopes, collective flavor oscillations of supernova neutrinos, flavor effects in thermal leptogenesis, the GSI anomaly and Mössbauer neutrino oscillations, and so on. I finally make some concluding remarks for the road ahead.

1. THE ROAD BEHIND

The history of neutrino physics can be traced back to the end of 1930, when Pauli wrote that famous letter to “radioactive ladies and gentlemen” in Tübingen and proposed a desperate remedy for the energy crisis observed in the beta decay. It is hard to count how many papers about neutrinos have been published since then. With the help of the SLAC-SPIRES HEP Database, I have made a search for papers relevant to neutrinos by inputting “find title NEUTRINOS and date XXXX”. It turns out that the first neutrino paper recorded in this HEP archive was the one of Cowan and Reines on the discovery of $\nu_e$ in 1956. Since then, more than 20000 papers on neutrinos have appeared in the literature. Figure 1 shows how the number of papers varies from 1956 to 2007. One can see some interesting peaks of the curve, which characterize some great moments in the (incomplete) history of neutrino physics. For example, the peak around 1968 was triggered by Davis’ discovery of the solar neutrino anomaly; the peak around 1987 was associated with Koshiba’s discovery of the supernova neutrinos; and the peak around 1998 was ascribed to the discovery of atmospheric neutrino anomaly in the Super-Kamiokande experiment. However, what does the sharp peak at the beginning of the 1990’s mean? I am afraid that this strange inflation in the number of neutrino papers might be triggered partly by the 17 keV neutrino episode and partly by the experimental establishment of $N_v = 3$ at the LEP. One may also identify two golden times in neutrino physics from this statistical picture: one is the period of 1976 — 1982 and the other is from 1998 to the present. I am wondering how long the second golden time can last.

The outline of my talk is as follows. In the first part, I am going to give an overview of some fundamental neutrino properties. They include the mass puzzle of neutrinos, the nature of massive neutrinos, the number of neutrino species, and the flavor issues of neutrino physics (i.e., the mass spectrum, mixing pattern and CP violation). In the second part of this talk, I shall highlight a few hot topics at the frontiers of neutrino physics and neutrino astrophysics, such as the naturalness and testability of TeV seesaw mechanisms at the LHC, effects of non-standard interactions on neutrino oscillations, flavor distributions of ultrahigh-energy cosmic neutrinos at neutrino telescopes, collective flavor oscillations of supernova neutrinos, flavor effects in thermal leptogenesis, the GSI anomaly and Mössbauer neutrino oscillations. Finally, I shall summarize my talk by making some concluding remarks for the road ahead.

2. THREE FUNDAMENTAL QUESTIONS

2.1. Question 1: Massless or Massive?

Three known neutrinos ($\nu_e$, $\nu_\mu$, $\nu_\tau$) are massless in the standard model (SM) as a straightforward consequence of its simple structure and renormalizability. On the one hand, the SM does not contain any right-handed neutrinos, and thus there is no way to write out the Dirac neutrino mass term. On the other hand, the SM conserves the $SU(2)_L$
gauge symmetry and only contains the Higgs doublet, and thus the Majorana mass term is forbidden. Although the SM accidently possesses the \((B - L)\) symmetry and “naturally” allows neutrinos to be massless, the vanishing of neutrino masses in the SM is not guaranteed by any fundamental symmetry or conservation law. Today we have achieved a lot of robust evidence for neutrino oscillations from solar, atmospheric, reactor and accelerator neutrino experiments \([1]\). The phenomenon of neutrino oscillations implies that at least two neutrinos must be massive and three neutrino flavors must be mixed. This is the first convincing evidence for new physics beyond the SM.

### 2.2. Question 2: Dirac or Majorana?

A pure Dirac mass term added into the SM is theoretically disfavored, unless the theory is built by introducing extra dimensions. Such a mass term in the renormalizable models of electroweak interactions would worsen the problem of large fermion mass hierarchy; it would violate ’t Hooft’s naturalness criterion, as a Majorana mass term of right-handed neutrinos is not forbidden by the SM gauge symmetry; and it would impose the contrived assumption of lepton number conservation on the theory. Hence most theorists believe that massive neutrinos are more likely to be Majorana particles and their salient feature is lepton number violation. If massive neutrinos are really Majorana particles, their masses must have a different origin in comparison with the masses of charged leptons and quarks.

The only experimentally feasible way to verify the Majorana nature of massive neutrinos is to observe the neutrino-less double-beta \((0\nu\beta\beta)\) decay. So far we have not obtained very convincing evidence for this lepton-number-violating process. Note that an uncontrovertible observation of the \(0\nu\beta\beta\) decay will definitely imply that massive neutrinos are Majorana particles \([2]\), but it may not uniquely point to neutrino masses and flavor mixing quantities.

It is worth emphasizing that a massive Dirac neutrino, given the SM interactions, can have a tiny (one-loop) magnetic dipole moment \(\mu_\nu \sim 3 \times 10^{-20} \mu_B (m_\nu/0.1 \text{ eV})\), where \(\mu_B\) is the Bohr magneton \([3]\). In contrast, a massive Majorana neutrino cannot have magnetic and electric dipole moments, because its antiparticle is just itself. Both Dirac and Majorana neutrinos can have transition dipole moments (of a size comparable with \(\mu_\nu\)) \([4]\), which may give rise to neutrino decays; scattering effects with electrons; interactions with external magnetic fields (red-giant stars, the sun, supernovae, and so on); and contributions to neutrino masses. Current experimental bounds on neutrino dipole moments are at the level of \(\mu_\nu < a \text{ few } \times 10^{-11} \mu_B\).
2.3. Question 3: Three Species or More?

It is well known that “three” is a mystically popular number in particle physics: three $Q = +2/3$ quarks; three $Q = -1/3$ quarks; three $Q = 0$ leptons; three $Q = 0$ neutrinos; three colors; and three forces in the SM. In this case, why do not we just consider three species of neutrinos and why do we consider to go beyond $N_{\nu} = 3$?

In the past one and a half decades, the main motivation for some theorists to speculate the existence of light sterile neutrinos was to account for the LSND anomaly together with solar and atmospheric neutrino oscillations. A global analysis of current neutrino oscillation data disfavors plain $(3+1)$, $(3+2)$ and even $(3+3)$ scenarios of active-sterile neutrinos was to account for the LSND anomaly together with solar and atmospheric neutrino oscillations. A global analysis of current neutrino oscillation data disfavors plain $(3+1)$, $(3+2)$ and even $(3+3)$ scenarios of active-sterile neutrinos were to account for the LSND anomaly together with solar and atmospheric neutrino oscillations. A global analysis of current neutrino oscillation data disfavors plain $(3+1)$, $(3+2)$ and even $(3+3)$ scenarios of active-sterile neutrinos were to account for the LSND anomaly together with solar and atmospheric neutrino oscillations. A global analysis of current neutrino oscillation data disfavors plain $(3+1)$, $(3+2)$ and even $(3+3)$ scenarios of active-sterile neutrinos were to account for the LSND anomaly together with solar and atmospheric neutrino oscillations. A global analysis of current neutrino oscillation data disfavors plain $(3+1)$, $(3+2)$ and even $(3+3)$ scenarios of active-sterile neutrinos were to account for the LSND anomaly together with solar and atmospheric neutrino oscillations. A global analysis of current neutrino oscillation data disfavors plain $(3+1)$, $(3+2)$ and even $(3+3)$ scenarios of active-sterile neutrinos.

The recent MiniBOONE experiment does not support the LSND result either. Conservatively speaking, it would be a big surprise if such exotic particles were really staying with us and in the universe.

A very large number of theorists are motivated by the elegant (type-I) seesaw mechanism to consider the existence of heavy Majorana neutrinos, because it is currently the most natural way to understand the origin of neutrino masses and why they are so tiny. If three known neutrinos really have three unknown partners whose masses are above or far above the Fermi scale, an exciting window will be open to new physics at high energy scales. In this case, however, the mixing between light and heavy neutrinos violates the unitarity of the $3 \times 3$ unitary neutrino mixing matrix and might result in some observable effects in the future precision neutrino oscillation experiments.

3. ISSUES OF NEUTRINO FLAVORS

There are three central concepts in flavor physics: mass, flavor mixing and CP violation [6]. Fogli et al. [7] have recently done a global analysis of current neutrino oscillation data and obtained the ranges of two neutrino mass-squared differences ($\delta m^2 \equiv m_2^2 - m_1^2$) and $\Delta m^2 \equiv (m_3^2 - (m_1^2 + m_2^2)/2)$) and three neutrino mixing angles ($\theta_{12}$, $\theta_{13}$ and $\theta_{23}$) in the standard parametrization of the $3 \times 3$ unitary neutrino mixing matrix $V$ [8], as listed in Table I.

| Parameter | $\delta m^2/10^{-5}$ eV$^2$ | $\sin^2 \theta_{12}$ | $\sin^2 \theta_{13}$ | $\sin^2 \theta_{23}$ | $\Delta m^2/10^{-3}$ eV$^2$ |
|-----------|----------------------------|----------------------|----------------------|----------------------|----------------------|
| Best fit  | 7.67                       | 0.312                | 0.016                | 0.466                | 2.39                 |
| 1σ range  | 7.48···7.83                | 0.294···0.331        | 0.006···0.026        | 0.408···0.539        | 2.31···2.50          |
| 2σ range  | 7.31···8.01                | 0.278···0.352        | < 0.036              | 0.366···0.602        | 2.19···2.66          |
| 3σ range  | 7.14···8.19                | 0.263···0.375        | < 0.046              | 0.331···0.644        | 2.06···2.81          |

3.1. Neutrino Mass Spectrum

Two mass-squared differences of three known neutrinos have been determined, to a good degree of accuracy, from current experimental data: $\Delta m^2_{21} \approx 7.7 \times 10^{-5}$ eV$^2$ and $\Delta m^2_{32} \approx \pm 2.4 \times 10^{-3}$ eV$^2$. The absolute neutrino mass scale remains unknown and may hopefully be determined in three experimental or observational ways: the single beta decay; the $0\nu\beta\beta$ decay; and the cosmological constraints. Considering the tight and loose constraints from cosmology together with the present neutrino oscillation results, Fogli et al. [7] have analyzed the parameter space of $\sum m_i$ and $m_{\beta\beta}$ as shown in Figure 2, where $m_{\beta\beta} = \sum (m_i V^2_{i\beta})$ is the effective mass of the $0\nu\beta\beta$ decay. One can see that the Heidelberg-Moscow claim for the $0\nu\beta\beta$ evidence [9] is compatible with the loose CMB data and, if finally confirmed by other experiments, would imply a near degeneracy of three neutrino masses.

Before the absolute neutrino mass scale is fixed, there remain two open questions: (1) is $m_3$ bigger or smaller than $m_1$ (i.e., normal or inverted hierarchy)? (2) can one neutrino mass ($m_1$ or $m_3$) be vanishing or vanishingly small? Question (1) requires an experimental answer in the near future, such as the long-baseline neutrino oscillation experiments with appreciable terrestrial matter effects [10], and question (2) depends on the special structure or symmetry of a realistic neutrino mass model, such as the minimal type-I seesaw mechanism with two heavy Majorana neutrinos [11] or the Friedberg-Lee symmetry of an effective Dirac or Majorana neutrino mass operator [12].
3.2. Flavor Mixing Pattern

In the standard neutrino phenomenology, the flavor mixing matrix of three neutrinos $V$ is assumed to be unitary and can be parametrized in terms of three rotation angles ($\theta_{12}, \theta_{13}, \theta_{23}$) and three CP-violating phases ($\delta, \rho, \sigma$). Current experimental constraints on $\theta_{12}, \theta_{13}$ and $\theta_{23}$ are given in Table I, but $\delta, \rho$ and $\sigma$ are entirely unrestricted. The upper bound on $\theta_{13}$ is $\theta_{13} < \theta_C \approx 13^\circ$ at the $3\sigma$ level, and a global analysis of all neutrino oscillation data yields $\sin^2 \theta_{13} = 0.016 \pm 0.010$ at the $1\sigma$ level [7]. How small $\theta_{13}$ is remains an open question. Reactor and accelerator neutrino oscillation experiments will hopefully answer this question in a direct way, but can they answer it before the global fit indirectly “predicts” the value of $\theta_{13}$?

We see that $\theta_{12}$ and $\theta_{23}$ are both large and close to two special numbers: $\theta_{12} \sim \arctan(1/\sqrt{2}) \approx 35.3^\circ$ and $\theta_{23} \approx 45^\circ$. This naive observation implies that the realistic neutrino mixing pattern might result from a certain underlying flavor symmetry (e.g., $S_3, S_4, A_4, Z_2, U(1)_F, \cdots$ [13]) and its spontaneous or explicit breaking. One may play games with a few small integers and their square roots to reconstruct the form of $V$ in an economical group language. A typical example is the so-called tri-bimaximal neutrino mixing pattern [14], whose three angles happen to be $\theta_{12} = \arctan(1/\sqrt{2}) \approx 35.3^\circ$, $\theta_{13} = 0^\circ$ and $\theta_{23} = 45^\circ$. So far many interesting possibilities of model building have been tried with the help of many flavor symmetries [15], and this situation seems to be promising on the one hand and giving rise to the uniqueness problem on the other hand.

3.3. CP and T Violation

If neutrinos are Majorana particles, the $3 \times 3$ unitary neutrino mixing matrix $V$ contains three CP-violating phases $\delta, \rho$ and $\sigma$. Among them, $\delta$ determines the strength of CP and T violation in neutrino oscillations, because both $P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$ and $P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)$ are proportional to the Jarlskog invariant $\mathcal{J} = \sin \theta_{12} \cos \theta_{12} \sin \theta_{23} \cos \theta_{23} \sin \theta_{13} \cos^2 \theta_{13} \sin \delta$ in vacuum. The Majorana phases $\rho$ and $\delta$, which have nothing to do with neutrino oscillations, are associated with the $0\nu\beta\beta$ decay. Note that $\delta$ itself is also of the Majorana nature, although it is usually referred to as the Dirac phase: one reason is that $\delta$ may appear in other lepton-number-violating processes, even if it can always be arranged not to appear in the $0\nu\beta\beta$ decay; and the other reason is that $\delta, \rho$ and $\sigma$...
A natural theoretical way to understand why 3 ν-masses are very small.

**Type-I:** SM + 3 right-handed Majorana ν’s (Minkowski 77; Yanagida 79; Glashow 79; Gell-Mann, Ramond, Slanski 79; Mohapatra, Senjanovic 79)

\[
M_\nu \approx -y^2 y_\nu^T M_R^{-1} y_\nu^T \]

**Type-II:** SM + 1 Higgs triplet (Magg, Wetterich 80; Schechter, Valle 80; Lazarides et al 80; Mohapatra, Senjanovic 80; Gelmini, Roncadelli 80)

\[
M_\nu \approx \lambda_\Delta Y_\Delta^T M_\Delta^{-1} \]

**Type-III:** SM + 3 triplet fermions (Foot, Lew, He, Joshi 89)

\[
M_\nu \approx -y^2 y_T^T M_T^{-1} y_T^T \]

Other variations or combinations (e.g., type-I + type-II in SO(10) GUT)

Figure 3: Three types of seesaw mechanisms to understand finite but tiny neutrino masses.

are actually entangled with one another in the renormalization-group running from one energy scale to another [16].

It is worth pointing out that \( J \) takes its maximal value \( J_{\text{max}} = 1/(6\sqrt{3}) \approx 9.6\% \) when \( \theta_{12} = \theta_{23} = 45^\circ \), \( \theta_{13} = \arctan(1/\sqrt{2}) \approx 35.3^\circ \) and \( \delta = 90^\circ \) hold. Here again the mysterious value 35.3° shows up. Note that such a special angle has a simple geometric explanation [17]: it corresponds to the angle formed by two unequal diagonals from the same vertex of a cube, and thus it might imply a certain flavor symmetry described by a simple group language. If \( \delta \) is large and \( \theta_{13} \) is not too small, it will be hopeful to observe leptonic CP and T violation in the future long-baseline neutrino oscillation experiments. Of course, terrestrial matter effects, which might fake the genuine signals of CP or T violation, have to be taken into account in a realistic long-baseline experiment.

Taking account of the lesson that we have learnt from the phenomenology of quark flavor mixing and CP violation, I stress that a known value of the smallest neutrino mixing angle \( \theta_{13} \) will be an important turning point to the era of precision measurements in experimental neutrino physics, towards the observation of CP violation, the test of unitarity of \( V \), and the search for possible new (expected or unexpected) physics in the long run.

### 4. SELECTED HOT TOPICS

#### 4.1. TeV Seesaws and Collider Signatures

The origin of finite but tiny neutrino masses is a big puzzle in particle physics. Among many theoretical and phenomenological ideas to solve this problem, the seesaw picture seems to be most natural and elegant at present. Three typical seesaw mechanisms are illustrated in Figure 3, and some other variations or combinations are possible.

The scale where a seesaw mechanism works is crucial, because it is relevant to whether this mechanism is theoretically natural and experimentally testable. Between Fermi and Planck scales, there might exist two other fundamental scales: one is the GUT scale at which strong, weak and electromagnetic forces can be unified, and the other is the TeV scale at which the unnatural gauge hierarchy problem of the SM can be solved or softened by new physics. Many theorists argue that the conventional seesaw pictures are natural because their scales (i.e., the masses of heavy degrees of freedom) are close to the GUT scale. If the TeV scale is also a fundamental scale, can we argue that the
TeV seesaws are also natural? In other words, we are reasonably motivated to speculate that possible new physics existing at the TeV scale and responsible for the electroweak symmetry breaking might also be responsible for the origin of neutrino masses. It is therefore interesting and meaningful to balance the “naturalness” and “testability” of TeV seesaws at the energy frontier set by the LHC.

Among many works done in the past two years about TeV seesaws (see Refs. [18, 19, 20] for an incomplete list), at least the following lessons can be learnt. First, lepton-number-violating ($\Delta L = 2$) like-sign dilepton events are clean collider signatures of heavy seesaw particles in most cases [21]. Typical signatures at the LHC include $pp \rightarrow W^{\pm} \rightarrow l^{\pm} N \rightarrow l^{\pm}_{\alpha} j j$ (type-I seesaw); $pp \rightarrow \gamma^* Z^* \rightarrow H^{++} H^{-}$ and $pp \rightarrow W^{\pm} \rightarrow H^{\pm} H^{\mp}$ with $H^{\pm} \rightarrow l^{\pm}_{\alpha} j j$ (type-II seesaw); and $pp \rightarrow W^{* \pm} \rightarrow T^{\pm} T^{0} \rightarrow l^{\pm}_{\alpha} l^{\pm}_{\beta} + Z W^{\mp}(\rightarrow 4j)$ (type-III seesaw). Second, the LHC signatures of heavy Majorana neutrinos and triplet fermions are likely to be decoupled from the parameters of light Majorana neutrino masses. Third, the level of fine-tuning in TeV seesaws to get a mass scale of 0.1 eV for light Majorana neutrinos could even be $10^{-10}$ — very unnatural? In addition, the mixing between light neutrinos and heavy particles in type-I and type-III seesaws may lead to observable unitarity violation of the $3 \times 3$ light neutrino mixing matrix [22].

### 4.2. Non-standard Interactions and Non-unitary Neutrino Oscillations

Non-standard interactions (NSIs) of neutrinos can be described by the effective four-fermion operators, $\mathcal{L}_{\text{NSI}} \approx 2\sqrt{2} G_F (\overline{\nu}_\alpha L) (\gamma^\mu \nu_\beta) (\overline{\nu}_\beta j L) \epsilon_{\alpha \beta}$ at low energy scales, after heavy degrees of freedom are integrated out. The magnitude of $\epsilon_{\alpha \beta}$ is expected to be $|\epsilon_{\alpha \beta}| \sim M_W^2 / M_{\text{NSI}}^2 \lesssim O(0.1)$, as constrained by current experimental data. The effects of NSIs can show up not only in the $0\nu\beta\beta$ decay and lepton-flavor-violating rare processes but also in neutrino oscillations (at the source, at the detector and in propagation) [22]. Future precision neutrino oscillation experiments might be able to probe such sub-leading new physics effects beyond the SM [24], e.g., through the measurements of interference terms in neutrino oscillations and new CP-violating phenomena.

The minimal unitarity violation in the neutrino sector can be regarded as a special NSI of neutrinos at low energies. In this scheme, only three light neutrino species are considered, and the sources of non-unitarity are allowed only in those terms of the SM Lagrangian which involve neutrinos [25]. TeV seesaws are therefore a good framework which can naturally accommodate non-unitary mixing of three light neutrinos. A careful analysis of current experimental data shows that the unitarity of the $3 \times 3$ light neutrino mixing matrix $V$ is good at the percent level, implying that possible non-unitary effects should at most be at the percent level. A salient feature of non-unitary neutrino oscillations is the “zero-distance” (near detector) effect: $P(\nu_\alpha \rightarrow \nu_\beta)_{L=0} = |(VV^\dagger)_{\alpha\beta}|^2 / (VV^\dagger)_{\alpha\alpha} (VV^\dagger)_{\beta\beta} \neq 0$ for $\alpha \neq \beta$; i.e., a flavor transition can take place even at the source before neutrino oscillations really develop. Another interesting consequence of non-unitary neutrino mixing is the new CP-violating effect in short- or medium-baseline $\nu_\mu \rightarrow \nu_\tau$ oscillations [26]. We find $P(\tau_\mu \rightarrow \tau_\tau) - P(\nu_\mu \rightarrow \nu_\tau) \approx 2 \sin \theta_{24} \sin \theta_{34} \sin(\delta_{24} - \delta_{34}) \sin(0.5 m_{32}^2 L / E)$ in vacuum, where $\theta_{ij}$ and $\delta_{ij}$ describe the new rotation angles and CP-violating phases due to the mixing between three light Majorana neutrinos and one heavy Majorana neutrino in a simplified seesaw scenario [22]. This non-trivial signature of CP violation can maximally be at the percent level, and it might be contaminated by terrestrial matter effects if the baseline length is sufficiently long [27].

### 4.3. Flavor Effects in Thermal Leptogenesis

Fukugita and Yanagida’s canonical idea of baryogenesis via leptogenesis [28] works in the type-I seesaw mechanism with two or more heavy Majorana neutrinos, where lepton number is violated at the tree level and direct CP violation occurs at the one-loop level of heavy Majorana neutrino decays. Two key points of thermal leptogenesis: (1) the CP-violating asymmetry between the out-of-equilibrium decay of the lightest heavy Majorana neutrino and its CP-conjugate process is partly converted into a net lepton number asymmetry; (2) the latter is finally converted into a net baryon number asymmetry via $(B - L)$-conserving but $(B + L)$-violating sphaleron processes, and thus the observed baryon number asymmetry of the Universe (i.e., $\eta_B \equiv n_B / n_\gamma \approx 6.1 \times 10^{-10}$ [8]) can naturally be interpreted.
Figure 4 illustrates why flavor effects might be important in leptogenesis. One can see that four temperature intervals are of interest in solving the Boltzmann equations which describe the evolution of the baryon number asymmetry $n_B / n_c$: (1) above $T \sim 10^{12}$ GeV, the leptonic Yukawa interactions are not in equilibrium — “unflavored” leptogenesis; (2) between $T \sim 10^9$ GeV and $T \sim 10^{12}$ GeV, the $\tau$-lepton Yukawa interactions are in equilibrium — $\tau$-flavored leptogenesis; (3) between $T \sim 10^5$ GeV and $T \sim 10^9$ GeV, both $\mu$- and $\tau$-lepton Yukawa interactions are in equilibrium — $(\mu + \tau)$-flavored leptogenesis; and (4) below $T \sim 10^5$ GeV, all the leptonic Yukawa interactions are in equilibrium — fully-flavored leptogenesis. A lot of interesting works on flavor effects in thermal leptogenesis have been done since 2004 [32]. The most striking consequence of such effects might be that it is possible to establish a direct relationship between the cosmological baryon number asymmetry and the CP violation at low energies via flavored leptogenesis in a class of seesaw models.

4.4. Flavor Distributions of UHE Cosmic Neutrinos

Now that neutrinos can oscillate from one flavor to another, it will be extremely interesting to detect the oscillatory phenomena of ultrahigh-energy (UHE) cosmic neutrinos produced from distant astrophysical sources. IceCube [33], a km$^3$-volume under-ice neutrino telescope, is now under construction at the South Pole and aims to observe the UHE neutrino oscillations. Together with the under-water neutrino telescopes in the Mediterranean Sea [34], IceCube has the potential to shed light on the acceleration mechanism of UHE cosmic rays and to probe the intrinsic properties of cosmic neutrinos. An immediate consequence of neutrino oscillations is that the flavor composition of cosmic neutrinos to be observed at the telescopes must be different from that at the sources [35]. By measuring the cosmic neutrino flavor distribution, one can either constrain the standard neutrino mixing parameters ($\theta_{12}$, $\theta_{13}$, $\theta_{23}$ and $\delta$) or
probe possible new physics beyond the SM (e.g., CPT violation, quantum decoherence, unitarity violation, neutrino decays, and active-sterile neutrino oscillations). A lot of attention has been paid to these possibilities.”

Let me make two brief remarks [37]. (1) For the conventional sources of UHE cosmic neutrinos (produced from the decays of charged pions arising from UHE pp and (or) pγ collisions) with \( \phi_e : \phi_\mu : \phi_\tau = 1 : 2 : 0 \), we can get the democratic flavor distribution \( \phi_e^T : \phi_\mu^T : \phi_\tau^T = 1 : 1 : 1 \) at a terrestrial neutrino telescope under the condition \(|V_{\mu i}| = |V_{\tau i}| \) (for \( i = 1, 2, 3 \)), which is equivalent to either \( \theta_{13} = 0 \) and \( \theta_{23} = \pi/4 \) (CP-conserving case) or \( \delta = \pm \pi/2 \) and \( \theta_{23} = \pi/4 \) (CP-violating case). (2) If there exist some contaminations to the conventional sources of UHE cosmic neutrinos or if the astrophysical sources are quite different from the conventional ones, one may adopt a generic parametrization of neutrino flavors at the sources: \( \phi_e : \phi_\mu : \phi_\tau = \sin^2 \xi \cos^2 \zeta : \cos^2 \xi \cos^2 \zeta : \sin^2 \zeta \). Both \( \xi \) and \( \zeta \) can in principle be measured at neutrino telescopes, although \( \mu \) is extremely difficult (if not impossible) in practice.

### 4.5. Collective Neutrino Flavor Transitions in Supernovae

Neutrinos streaming off a collapsed supernova core are so dense near the neutrino sphere that their nonlinear self-interactions are very significant [40] and can give rise to collective flavor transitions [41] in which neutrinos (or antineutrinos) of different energies almost have the same behaviors. The practical importance of such effects has recently been recognized and has triggered some intensive studies [42].

The condition for neutrino-neutrino scattering effects to be relevant can simply be explained in a two-flavor neutrino oscillation scenario, where \( \omega = \Delta m^2/(2E) \) is the vacuum oscillation frequency, \( \lambda = \sqrt{2} G_F n_e \) describes the ordinary MSW matter effects, and \( \mu = \sqrt{2} G_F (n_\nu + n_\overline{\nu}) \) denotes the self-interactions of neutrinos and antineutrinos. When \( \omega \lesssim \lambda \), the MSW matter effects are important; and when \( \omega \lesssim \mu \), neutrino-neutrino scattering effects are important. Note that it is a misconception that neutrino-neutrino scattering effects would be negligible even if \( \mu \ll \lambda \) holds.

Some interesting properties of collective supernova neutrino oscillations have been discussed in the literature [42]. Synchronized oscillations take place when the self-interactions of neutrinos and antineutrinos “glue” the neutrino flavor polarization vectors together such that they evolve in the same way. Bipolar oscillations occur in a neutrino gas with equal densities of neutrinos and antineutrinos (e.g., \( \nu_e \) and \( \overline{\nu}_e \)). For the inverted neutrino mass hierarchy with very small \( \theta_{13} \), the ensemble will undergo oscillations of the type \( \nu_e \overline{\nu}_e \rightarrow \nu_\mu \overline{\nu}_\mu \rightarrow \nu_e \overline{\nu}_e \rightarrow \cdots \), approximately with the “bipolar frequency” \( \kappa = \sqrt{2} \omega/\mu \gg \lambda \). For the normal neutrino mass hierarchy, the ensemble will perform small-amplitude harmonic oscillations with the frequency \( \kappa \), and thus nothing happens macroscopically. It has also been found that the collective effects can lead to a spectral split or stepwise swap of neutrino flavors, where a critical energy splits the transformed spectrum sharply into parts of almost pure but different flavors [43].

### 4.6. The GSI Anomaly and Mössbauer Neutrino Oscillations

The orbital electron capture decays of hydrogen-like \(^{140}\text{Pr}\) and \(^{142}\text{Pm}\) ions, similar to the \( p + e^- \rightarrow n + \nu_e \) process, have recently been measured at GSI Darmstadt by using the time-resolved Schottky mass spectrometry [44]. The experimental result, contrary to the expected pure exponential behavior, can be described by an exponential fit plus a superimposed oscillation at the 3.5σ level. Several authors have argued that this anomaly might be attributed to neutrino mixing in the final state [45], but their arguments seem to be unconvincing. As carefully analyzed by some other authors [46], the GSI anomaly cannot originate from neutrino mixing. If it finally survives, it might be associated with the properties of \(^{140}\text{Pr}\) and \(^{142}\text{Pm}\) ions themselves.

Some special interest has recently been paid to the possibility of measuring \( \theta_{13} \), the smallest neutrino mixing angle, by doing a Mössbauer neutrino oscillation experiment [47]. The basic idea of such an experiment is rather simple, as illustrated in Figure 5. If \(^3\text{H}\) and \(^3\text{He}\) are both embedded into a solid-state lattice (e.g., metal crystals), the recoilless emission and resonant absorption of \( \overline{\nu}_e \) neutrinos will in principle be possible [48]. We refer to the nearly monochromatic \( \nu_e \) beam with energy \( E = 18.6 \) keV in these reactions as the Mössbauer neutrinos, because the mechanism of their production and detection is quite similar to the Mössbauer effect of gamma rays.
Can Mössbauer neutrinos oscillate? An affirmative answer to this question has been given in Ref. [49], although there were some controversies [50]. Today we have many technical difficulties in realizing the oscillation of Mössbauer neutrinos, but in the long run it might not be impossible to do such a highly-statistical $\nu_e$-disappearance experiment to measure $\theta_{13}$ with a baseline of some ten meters, to determine the neutrino mass hierarchy in the absence of terrestrial matter effects, and to probe new physics beyond the standard picture of three active neutrino mixing.

### 4.7. Massive Neutrinos in GUTs and Unparticle Physics

The study of massive neutrinos in grand unified theories (GUTs) has been an important topic in theoretical particle physics. In particular, the type-(I+II) seesaw mechanism can naturally be realized in the SO(10) GUT with an intriguing unification of leptons and quarks. Babu’s talk in this conference is an excellent overview of neutrino masses and flavor mixing in a class of GUTs [51], and thus it allows me to have a good excuse to skip this topic here.

A brand-new but more or less exotic topic is associated with the applications of Georgi’s unparticle idea [52] to neutrino physics. A number of authors have considered the influence of unparticle physics on neutrino decays, neutrino scattering with electrons, and neutrino oscillations [53]. Current works on this subject are purely speculative.

It is a pity that I am unable to cover some other interesting topics in neutrino physics and neutrino astrophysics. For example, a lot of works done in the past two years (from ICHEP06 in Moscow to ICHEP08 in Philadelphia) focus on model building with the help of supersymmetries, flavor symmetries, GUTs or strings, extra dimensions, and some new ideas. A possible roadmap of neutrino mass models can be found in King’s talk at Neutrino 2008 [54].

### 5. THE ROAD AHEAD

We have known a lot about the properties of three known neutrinos, but there remain many things that we do not know. The present situation of model building seems quite messy, although we are more or less guided by some principles including the self-consistency, naturalness, simplicity and testability. As argued by Witten [55] at Neutrino 2000, “for neutrino masses, the considerations have always been qualitative, and, despite some interesting attempts, there has never been a convincing quantitative model of the neutrino masses”. The road ahead is therefore to establish the unique and quantitative theory of neutrino masses, flavor mixing and CP violation.

In his autobiographic book The Road Ahead [56], Bill Gates admits that “people often overestimate what will happen in the next two years and underestimate what will happen in ten”. My mild estimate is that new breakthroughs might be possible in the near future at three frontiers: the energy frontier set by the LHC, the intensity frontier associated with neutrino experiments; and the cosmic frontier for the study of dark matter and dark energy. The overlap of three frontiers is obvious, and it has much to do with the topics covered in this talk. I hope that new experimental discoveries may help to lead us to a fundamental neutrino theory. In particular, I hope that the LHC might tell us something behind three known neutrinos — either new particles or new symmetries, or both of them.
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