NLO QCD corrections to off-shell $t\bar{t}W^\pm$ production at the LHC: Correlations and Asymmetries

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ABSTRACT: Recent discrepancies between theoretical predictions and experimental data in multi-lepton plus $b$-jets analyses for the $t\bar{t}W^\pm$ process, as reported by the ATLAS collaboration, have indicated that more accurate theoretical predictions and high precision observables are needed to constrain numerous new physics scenarios in this channel. To this end we employ the NLO QCD computations with the full off-shell top quark effects included to provide theoretical predictions for the $\mathcal{R} = \sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$ cross section ratio at the LHC with $\sqrt{s} = 13$ TeV. Depending on the transverse momentum cut on the $b$-jet we obtain $1\% - 2\%$ theoretical precision on $\mathcal{R}$, which should help to shed some light on new physics effects that can reveal themselves only once sufficiently precise Standard Model theoretical predictions are available. Furthermore, triggered by these discrepancies we reexamine the top quark charge asymmetry and the charge asymmetries of the top quark decay products in the $t\bar{t}W^\pm$ production process. In the case of charge asymmetries, that are uniquely sensitive to the chiral nature of possible new physics in this channel, theoretical uncertainties below $15\%$ have been obtained. Additionally, for both cases the impact of the top quark modelling is scrutinised by the explicit comparison with the predictions in the narrow-width approximation.

KEYWORDS: NLO Computations, QCD Phenomenology, Heavy Quark Physics

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1 Introduction

The Large Hadron Collider (LHC) with the Run II energy of $\sqrt{s} = 13$ TeV has opened up the possibility of studying various top quark production and decay mechanisms at larger mass scales than previously explored in any experiment. The $t\bar{t}$ pair production associated with the $W^\pm$ gauge boson is among the most massive signatures that can be studied with high precision at the LHC. It is a key process to constrain top quark intrinsic properties, which might be modified in the presence of new physics. Moreover, the process can be used in the framework of the Standard Model Effective Field Theory (SMEFT), where the effects of potential new particles can be systematically included in terms of higher-dimensional operators. The latter are suppressed by a sufficiently large new physics energy scale $\Lambda$. The framework relies on the idea that new physics is too heavy to be directly produced and observed at the LHC, thus, only deviations from the Standard Model (SM) can be probed in various ATLAS and CMS top quark measurements. Compared with top quark pair production and single top quark production, the associated $t\bar{t}W^\pm$ process does not bring sensitivity to new operators, however, it helps to resolve blind directions in the SMEFT parameter space that occur in the current LHC fits. On top of that $t\bar{t}W^\pm$ can probe operators that are difficult to access in other channels. For example, since the $W^\pm$ gauge boson is radiated from the initial state, $t\bar{t}W^\pm$ is sensitive to a subset of the possible four-quark operators only. In the SM, $t\bar{t}W^\pm$ is dominated by quark-antiquark interactions, while $t\bar{t}$ is dominated by the $gg$ initial state. This means that relative to the SM contribution the four-quark operators would give sizeable effects in the $t\bar{t}W^\pm$ production process. Consequently, $t\bar{t}W^\pm$ production is often included in the global SMEFT analysis of LHC top quark measurements, see e.g. [1].
In addition, the $t\bar{t}W^\pm$ process play an important role in the studies of the top quark charge asymmetry denoted as $A_t^c$ [2]. Also in this case the lack of the symmetric $gg$ initial state and the emission of the $W^\pm$ gauge boson from the initial states contribute to a substantially larger top quark charge asymmetry than that measured in the $t\bar{t}$ process. Furthermore, the asymmetry of the top quark decay products, i.e the charged lepton ($A_\ell^c$) and the $b$-jet ($A_b^c$) are very large and already present at the LO due to the polarisation of the initial fermionic line by the $W^\pm$ emission. These asymmetries are an interesting playground for various beyond the SM (BSM) theories, as $A_t^c$, $A_\ell^c$ and $A_b^c$ are uniquely sensitive to the chiral nature of possible new physics that might directly affect such measurements.

Last but not least, $t\bar{t}W^\pm$ production is a background process in the multi-lepton final state with two same-sign leptons, accompanied by missing transverse momentum and $b$-jets [3–6]. Even though same-sign leptons are a relatively rare phenomenon in the SM, as they only appear in processes with a rather small cross section, they have been extensively exploited in various models of new physics. The same-sign lepton signature is present, among others, in models with supersymmetry, universal extra dimensions, top-quark partners and the extended Higgs boson sector [7–13]. Besides, same-sign leptons are considered a key feature in searches for heavy Majorana neutrinos as well as for $tt$ and $t\bar{t}$ resonances [14, 15].

Finally, the $pp \rightarrow t\bar{t}W^\pm$ process is the main background in SM measurements involving final states with multiple leptons and $b$-jets. This is the case, for example, for the measurement of the associated production of the SM Higgs boson with top quarks [16]. The $pp \rightarrow t\bar{t}W^\pm$ process has also played a crucial role in the announcement of the strong evidence of the production of four top quarks, a measurement, which has been recently performed by the ATLAS Collaboration [17].

The direct measurement of $pp \rightarrow t\bar{t}W^\pm$ production in multi-lepton final states has already been carried out at $\sqrt{s} = 13$ TeV by the ATLAS and CMS collaborations [18–20]. In the recent measurement of $t\bar{t}H$ and $t\bar{t}W^\pm$ production in multi-lepton final states [16] the resulting $t\bar{t}W^\pm$ normalisation has been found to be higher than the theoretical prediction provided by the multipurpose Monte Carlo (MC) generators, which are currently employed by the ATLAS collaboration. Apart from the $t\bar{t}W^\pm$ normalisation, a tension in the modelling of the final state kinematics in the phase space regions dominated by $t\bar{t}W^\pm$ production, has been observed. From the experimental point of view such an accurate study of $pp \rightarrow t\bar{t}W^\pm$ production in the same-sign lepton final state has become feasible thanks to the increasing amount of data collected at the LHC with $\sqrt{s} = 13$ TeV. This increased integrated luminosity has significantly raised the need for more precise theoretical predictions. The latter should include higher order QCD corrections both to the production and decays of top quarks and $W$ gauge bosons as well as incorporate $t\bar{t}$ spin correlations at the same level of accuracy.

The first calculations for the $pp \rightarrow t\bar{t}W^\pm$ process, that meet the mentioned conditions, have been carried out in the narrow-width approximation (NWA) within the MCfM framework [21]. The first full NLO QCD computations, which include complete top quark off-shell effects for the $pp \rightarrow t\bar{t}W^\pm$ process in the multi-lepton channel, have been recently presented in Ref. [22]. In these computations, obtained with the help of Helac-NLO, off-shell top quarks have been described by Breit-Wigner propagators, furthermore, double-,
well as non-resonant top-quark contributions along with all interference effects have been consistently incorporated at the matrix element level. Independent computations for $t\bar{t}W^\pm$ production have been obtained very recently within the MoCaNLO+Recola framework [23]. They not only confirmed the results presented in Ref. [22] but also performed a comparison between the full results and those obtained with the help of the double-pole approximation. We also note that continuous efforts have been devoted to improve the theoretical modeling of hadronic observables at NLO through matching with parton shower and multi-jet merging [24–26] as well as by incorporating soft gluon resummation effects with the next-to-next-to-leading logarithmic (NNLL) accuracy [27–30].

In Ref. [22] results at NLO QCD accuracy have been presented in the form of fiducial integrated and differential cross sections for two selected renormalisation and factorisation scale choices (a fixed and a dynamical ones) and three different PDF sets. Detailed studies of the scale dependence of the NLO predictions have been carried out together with calculations of the PDF uncertainties. Furthermore, the impact of the top quark off-shell effects on the $pp \to t\bar{t}W^\pm$ cross section has been examined by an explicit comparison with the results in the NWA. In the current paper we will move away from the technical aspects of higher order calculations and the estimation of the residual theoretical uncertainties and go towards more phenomenological studies for the $pp \to t\bar{t}W^\pm$ process. Specifically, the purpose of this paper is twofold. First, we would like to provide a systematic analysis of the two processes $pp \to t\bar{t}W^+$ and $pp \to t\bar{t}W^-$ in the multi-lepton decay channel to extract the most accurate NLO QCD predictions for the $R = \sigma_{t\bar{t}W^+}^{NLO}/\sigma_{t\bar{t}W^-}^{NLO}$ cross section ratio. Generally, the cross section ratios are more stable against radiative corrections than the absolute cross sections, assuming that the two processes are correlated. They have smaller theoretical uncertainties as various uncertainties tend to cancel in the cross section ratio. Consequently, such precise theoretical predictions have enhanced predictive power and should be used in indirect searches for new physics at the LHC. The second goal of the paper is to study separately the intrinsic properties of $t\bar{t}W^+$ and $t\bar{t}W^-$ production. More specifically, we shall use the state-of-the-art NLO QCD theoretical predictions for the $t\bar{t}W^\pm$ process to re-examine the top quark charge asymmetry and asymmetries of the top quark decay products both at the integrated and differential level. Likewise, in this case, the polarisation and asymmetry effects in the $pp \to t\bar{t}W^\pm$ production process can be employed to constrain new physics effects that might occur in this channel. Furthermore, for both the cross section ratio and the top quark (decay product) charge asymmetry, the impact of the modelling of top quark production and decays will be studied.

We note here, that the state-of-the art theoretical predictions at NLO in QCD with the complete top quark off-shell effects included are also available for other processes at the LHC. Such effects, for example, have been incorporated for $pp \to tf$ [31–34], $pp \to t\bar{t}j$ [35, 36], $pp \to t\bar{t}H$ [37], $pp \to t\bar{t}\gamma$ [38] and for $t\bar{t}Z(Z \rightarrow \nu\ell\nu\ell)$ [39]. Very recently they have also been incorporated for the $pp \to t\bar{t}b\bar{b}$ process [40].

The paper is organised as follows. In section 2 the Helac-NLO computational framework and input parameters used in our studies are briefly described. In section 3 correlations between $t\bar{t}W^+$ and $t\bar{t}W^-$ are examined. The results for the cross section ratio $R = \sigma_{t\bar{t}W^+}^{NLO}/\sigma_{t\bar{t}W^-}^{NLO}$ are provided in section 4. The integrated top quark charge asymmetry
as well as asymmetries of the top quark decay products are studied in section 5. Results for the differential and cumulative $A_{\ell \, c}$ asymmetry are provided in section 6. Finally, in section 7 the results are summarised and our conclusions are provided.

2 Computational Framework and Input Parameters

All our results both for the full off-shell and NWA computations have been obtained with the help of the HELAC-NLO Monte Carlo framework [41]. The calculation was performed using HELAC-1LOOP [42, 43] for the virtual corrections and HELAC-Dipoles [44, 45] for the real emission part. The integration over the phase space has been achieved with the help of KALEU [46]. In our studies we keep the Cabibbo-Kobayashi-Maskawa mixing matrix diagonal and neglect the Higgs boson contributions. Following recommendations of the PDF4LHC Working Group for the usage of PDFs suitable for applications at the LHC Run II [47] we employ the NNPDF3.0 PDF set [48]. In particular, we use NNPDF30-nlo-as-0118 with $\alpha_s(m_Z) = 0.118$ (NNPDF30-lo-as-0130 with $\alpha_s(m_Z) = 0.130$) at NLO (LO). The running of the strong coupling constant with two-loop accuracy at NLO is provided by the LHAPDF interface [49]. The number of active flavours is set to $N_F = 5$. We employ the following SM input parameters

\begin{align}
G_\mu &= 1.166378 \cdot 10^{-5} \text{ GeV}^{-2} , \\
m_t &= 172.5 \text{ GeV} , \\
m_W &= 80.385 \text{ GeV} , \\
\Gamma^{\text{NLO}}_W &= 2.09767 \text{ GeV} , \\
m_Z &= 91.1876 \text{ GeV} , \\
\Gamma^{\text{NLO}}_Z &= 2.50775 \text{ GeV} , \\
\Gamma^{\text{NLO}}_t &= 1.33247 \text{ GeV} , \\
\Gamma^{\text{NLO}}_{t, \text{NWA}} &= 1.35355 \text{ GeV} .
\end{align}

For the $W$ and $Z$ gauge boson widths we use the NLO QCD values as calculated for $\mu_R = m_W$ and $\mu_R = m_Z$ respectively. All other partons, including bottom quarks, as well as leptons are treated as massless particles. The LO and NLO top quark widths are calculated according to Ref. [33]. The top quark width is treated as a fixed parameter throughout this work. Its value corresponds to a fixed scale $\mu_R = m_t$. The electromagnetic coupling $\alpha$ is calculated from the Fermi constant $G_\mu$, i.e. in the $G_\mu$–scheme, via

\begin{equation}
\alpha_{G_\mu} = \frac{\sqrt{3}}{\pi} G_\mu m_W^2 \sin^2 \theta_W ,
\end{equation}

where $\sin^2 \theta$ is defined according to

\begin{equation}
\sin^2 \theta = 1 - \frac{m_W^2}{m_Z^2} .
\end{equation}

We use the kinematic-dependent factorisation and renormalisation scales $\mu_R = \mu_F = \mu_0$ with the central value $\mu_0 = H_T/3$ where $H_T$ is the scalar sum of all transverse momenta in the event, including the missing transverse momentum. The latter is constructed from the three neutrinos $\nu_e$, $\nu_\mu$ and $\nu_\tau$. The additional light jet, if resolved, is not included in the definition of $H_T$. In various comparisons we also use a fixed scale defined as $\mu_0 = m_t + m_W/2$. Jets are constructed out of all final-state partons with pseudo-rapidity $|\eta| < 5$.
via the anti-\(k_T\) jet algorithm \cite{50} with the separation parameter \(R = 0.4\). We require exactly two \(b\)-jets and three charged leptons, two of which are same-sign leptons. All final states have to fulfill the following selection criteria that mimic very closely the ATLAS detector response \cite{16}

\[
\begin{align*}
  p_{T,\ell} & > 25 \text{ GeV}, & p_{T,b} & > 25 \text{ GeV}, \\
  |y_\ell| & < 2.5, & |y_b| & < 2.5, \\
  \Delta R_{\ell\ell} & > 0.4, & \Delta R_{\ell b} & > 0.4,
\end{align*}
\]

where \(\ell\) stands for the charged lepton. We do not impose any restrictions on the kinematics of the additional light jet and the missing transverse momentum.

3 Correlations between \(ttW^+\) and \(ttW^-\)

We start with the NLO QCD differential cross sections for \(pp \rightarrow e^+\nu_e e^-\bar{\nu}_e e^+\nu_e b\bar{b} + X\) and \(pp \rightarrow e^-\bar{\nu}_e \mu^+\nu_\mu \mu^-\bar{\nu}_\mu e^+\nu_e b\bar{b} + X\). They are obtained for the LHC Run II energy of \(\sqrt{s} = 13\) TeV. For brevity, we will refer to these reactions as \(pp \rightarrow ttW^+\) and \(pp \rightarrow ttW^-\). We would like to understand similarities and potential differences between the two processes. We note that, at the leading order the production mechanism for \(ttW^+\) (\(ttW^-\)) is via the scattering of up-type quark (anti-quark) and the corresponding down-type anti-quark (quark), i.e. \(u\bar{d}\) and \(c\bar{s}\) for \(pp \rightarrow ttW^+\) as well as \(\bar{u}d\) and \(\bar{c}s\) for \(pp \rightarrow ttW^-\). The quark-gluon initial state opens up only at the NLO in QCD. Similarities in the production mechanisms and final states suggest that the two processes are correlated. To show this we examine the common features in the kinematics of the final states. Since we are interested in the shape differences/similarities only and because the fiducial cross section for \(pp \rightarrow ttW^-\) is about a factor of two smaller than the one for the \(pp \rightarrow ttW^+\) process we concentrate on the normalised NLO QCD differential cross sections.

In the following the collection of leptonic observables will be examined. In the \(pp \rightarrow ttW^\pm\) process same-sign charged leptons \(e^\pm e^\pm\) occur. In the case of final states with identical leptons the ordering in \(p_T\) has to be introduced to label the particles. To this end, we denote the first and the second hardest same-sign charged lepton as \(e_1^\pm\) and \(e_2^\pm\) respectively. In Figure 1 we present the NLO QCD differential cross sections for \(pp \rightarrow ttW^+\) and \(pp \rightarrow ttW^-\) as a function of the transverse momentum of \(e_1^\pm\) \((p_{T,e_1})\), the invariant mass of the \(e_1^\pm e_2^\pm\) system \((M_{e_1,e_2})\) and the scalar sum of the transverse momenta of the charged leptons available in the given process \((H_T^{lep})\). The latter is defined as

\[
H_T^{lep} = p_{T,\mu^\pm} + p_{T,e_1^\pm} + p_{T,e_2^\pm}.
\]

Also shown in Figure 1 is the distance in the azimuthal angle rapidity plane between \(e_1^\pm\) and \(e_2^\pm\) \((\Delta R_{e_1,e_2})\). All differential cross section shown are, indeed, very similar.

In the next step we look at the \(b\)-jet kinematics. The two \(b\)-jets are ordered according to their \(p_T\). The hardest \((b_1)\) and the softest \(b\)-jet \((b_2)\) kinematics are exhibited in Figure 2. We note here, however, that the charge identification of the \(b\)-jets is possible at the LHC, see e.g. \cite{51–54}. Thus, one can distinguish between \(b\)-jets initiated by \(b\) and \(\bar{b}\). In this work,
Comparison of the normalised NLO QCD differential cross sections for $pp \rightarrow t\bar{t}W^\pm$ in the multi-lepton final state at the LHC with $\sqrt{s} = 13$ TeV. The transverse momentum of the hardest same-sign lepton ($p_T, e_1$) and the invariant mass of the two same-sign leptons ($M_{e_1,e_2}$) are presented. Also given are the scalar sum of the transverse momenta of the leptons ($H^\text{lep}_T$) and the distance in the azimuthal angle rapidity plane between the two same-sign leptons ($\Delta R_{e_1,e_2}$). The NLO NNPDF3.0 PDF set is employed and $\mu_R = \mu_F = H_T/3$ is used.

Figure 1.

However, we do not perform such $b$-jet identification. We depict the NLO QCD differential cross sections as a function of the transverse momentum of $b_1$ ($p_T, b_1$), the invariant mass of the two $b$-jet system ($M_{b_1,b_2}$) and the distance in the azimuthal angle rapidity plane between $b_1$ and $b_2$ ($\Delta R_{b_1,b_2}$). Also presented in Figure 2 is the scalar sum of the transverse momenta of all the visible final states, denoted as $H^\text{vis}_T$. The latter is given by

$$H^\text{vis}_T = p_T, b_1 + p_T, b_2 + p_{T,\mu^\pm} + p_{T, e_1^\pm} + p_{T, e_2^\pm}.$$ (3.2)

An interesting comment can be made here. Namely, that the $b$-jets are preferably produced in back-to-back configurations. Hereby, $b$-jets come more often from top quark decays rather than from the $g \rightarrow b\bar{b}$ splitting. The latter configuration, which is produced in the off-shell case where no top-quark resonances are presented, would manifest itself in the enhancement close to $\Delta R_{b_1,b_2} \approx 0.4$. In the case of $b$-jet kinematics and for the $H^\text{vis}_T$ observable we can see similarities between $pp \rightarrow t\bar{t}W^+$ and $pp \rightarrow t\bar{t}W^-$. To summarise this part, as anticipated both $t\bar{t}W^+$ and $t\bar{t}W^-$ production processes are highly correlated. This fact will be exploited when the theoretical uncertainties due to the scale dependence for the $t\bar{t}W^+$ and $t\bar{t}W^-$ cross section ratio will be estimated. For
As in Figure 1 but for the transverse momentum of the hardest $b$-jet ($p_{T,b_1}$), the invariant mass of the two $b$-jets ($M_{b_1b_2}$), the scalar sum of the transverse momenta of the visible final states ($H_{T}^\text{vis}$) and the distance in the azimuthal angle rapidity plane between the two $b$-jets ($\Delta R_{b_1b_2}$).

Figure 2. As in Figure 1 but for the transverse momentum of the hardest $b$-jet ($p_{T,b_1}$), the invariant mass of the two $b$-jets ($M_{b_1b_2}$), the scalar sum of the transverse momenta of the visible final states ($H_{T}^\text{vis}$) and the distance in the azimuthal angle rapidity plane between the two $b$-jets ($\Delta R_{b_1b_2}$).

Both processes, our findings are not modified when the fixed scale choice $\mu_R = \mu_F = \mu_0 = m_t + m_W/2$ is used instead or when different PDF sets are employed.

So far, we have only used visual inspection to see whether two given one-dimensional normalised cross section distributions are similar or not. Even though this is an excellent place to start with, we would like to find a more quantitative approach to analyse the issue. In statistics literature several standard procedures exist for this task. Typically, the similarity of histograms is measured by a test statistic. The latter provides the quantitative expression of the distance between the two histograms that are compared. The smaller the distance the more similar are the compared histograms. There are several definitions of the test statistics in specialist literature on statistical methods. In the following we shall concentrate on the Kolmogorov-Smirnov test (KS-test) statistics. The purpose of the (two-sample) KS-test is to look for differences in the shape of two one-dimensional probability distributions. It is based on comparing two cumulative distribution functions (CDFs). The KS-test reports on the maximum difference between the two CDFs and calculates a $p$-value from that and the sample sizes. If the two tested histograms are indeed identical then they would have the same CDF. However, in reality two samples that are compared are randomly taken from their corresponding probability distributions. Therefore, even for the two truly identical histograms the corresponding CDFs will be slightly different. We can
use this fact to test the two distribution equality by comparing the KS-test statistic to 0. If the latter is significantly larger than 0 and close to 1, then we might conclude that the distributions are not equal and the two processes considered are not correlated. We continue with the differential cross section distribution for $pp \rightarrow t\bar{t}W^+$ and $pp \rightarrow t\bar{t}W^-$ as a function of the variable $x$, where $x$ for example is $x = p_{T,e_1}, M_{e_1e_2}$. When comparing both histograms, we use the same number of bins. We would like to verify the hypothesis that the two histograms are similar. To this end we calculate the KS-test statistics according to

$$KS_{\text{statistic}} = \sup_x |F_{n_1}^1(x) - F_{n_2}^2(x)|,$$  

where $F_{n_1}^1$ and $F_{n_2}^2$ are the CDFs, $n_1$ and $n_2$ are the sizes of first and second sample respectively and sup is the supremum function. We assume approximately 2000 events for $pp \rightarrow t\bar{t}W^+$ and about 1000 for $pp \rightarrow t\bar{t}W^-$, which correspond to the integrated LHC luminosity of $L = 500 \text{ fb}^{-1}$ including a lepton-flavour factor of 8. After finding the maximum distance, we use the following condition

$$\sqrt{n} \cdot KS_{\text{statistic}} > \lambda(\alpha),$$  

where

$$n = \frac{n_1 \cdot n_2}{n_1 + n_2},$$  

with $n_1 = 2000$, $n_2 = 1000$ and $\lambda(\alpha)$ is the threshold value that depends on the level of significance $\alpha$. It can be found from the following condition

$$\mathcal{P}\left(\sqrt{n} \cdot KS_{\text{statistic}} > \lambda(\alpha)\right) = 1 - Q_{KS}(\lambda(\alpha)) = \alpha,$$

where $\mathcal{P}$ denotes probability and $Q_{KS}(x)$ stands for the Kolmogorov-Smirnov distribution. We reject the hypothesis that the two distributions are similar if

$$\sqrt{n} \cdot KS_{\text{statistic}} > \lambda(\alpha),$$  

and accept it when

$$\sqrt{n} \cdot KS_{\text{statistic}} \leq \lambda(\alpha).$$

We would normally start to question the hypothesis of the similarity of the histograms only if we find a difference larger than $2\sigma$ (the $p$-value smaller than 0.0455). If the difference is smaller than $2\sigma$ (the $p$-value larger than 0.0455) then we assume that the two tested distributions are indeed similar. Results that differ more than $3\sigma$ (the $p$-value smaller than $0.0027$) can be directly translated into having enough evidence to reject the hypothesis, i.e. saying that there is a real difference between the two samples that are being studied. Note that the KS-test does not identify the source of the difference between histograms. It is a robust way of saying that there is a difference, however, the origin of such a difference must be identified by other means.

As the example in Figure 3 we present the distribution of the KS-test statistic for the following NLO QCD differential cross sections: $p_{T,e_1}$, $M_{ee}$, $H_T^{lep}$ and $\Delta R_{ee}$ for $pp \rightarrow t\bar{t}W^+$ and $pp \rightarrow t\bar{t}W^-$. The total number of tries is set to $N_{\text{tries}} = 1000$. All KS-test statistic
Figure 3. The distribution of the Kolmogorov-Smirnov test statistic (distance) for the null hypothesis of equality of the histogram shapes. NLO QCD differential cross section distributions for $pp \rightarrow t\bar{t}W^+$ and $pp \rightarrow t\bar{t}W^-$ in the multi-lepton final state are employed as a function of $p_T, e_{1e_2}, M_{e_1e_2}, H_T^{lep}$ and $\Delta R_{e_1e_2}$ for the LHC with $\sqrt{s} = 13$ TeV. Also shown are the distributions of the corresponding p-values. The total number of $N_{\text{tries}}$ is set to 1000.

values are distributed within the $0.01 - 0.07$ range, i.e. very close to zero, which suggests that $pp \rightarrow t\bar{t}W^+$ and $pp \rightarrow t\bar{t}W^-$ are indeed correlated. Also shown in Figure 3 are the
The distribution of $p$-values for the Kolmogorov-Smirnov test statistic for the NLO QCD differential cross section for $pp \rightarrow t\bar{t}W^+$ and $pp \rightarrow t\bar{t}W^-$ in the multi-lepton final state as a function of $H_{T}^{vis}$ for the LHC with $\sqrt{s} = 13$ TeV. A different number of bins is assumed for each plot, however, the integrated luminosity is kept fixed. Specifically, we use the following four cases: 5 bins (upper left), 10 bins (upper right), 20 bins (lower left) and 40 bins (lower right). The total number of $N_{\text{tries}}$ is set to 1000.

We would like to stress at this point, that for the higher integrated luminosity or when the number of bins increases, the sensitivity of the KS-test increases as well. As an example we present in Figure 4 the distribution of $p$-values for the KS-test statistic for the $H_{T}^{vis}$ observable for $pp \rightarrow t\bar{t}W^+$ and $pp \rightarrow t\bar{t}W^-$. We use four different values for the number of histogram bins, keeping the number of total events fixed for both processes. Specifically, we employ 5, 10, 20 and 40 bins respectively. We can observe that the percentage of $N_{\text{tries}}$ with the $p$-value close to 1 is getting lower as the number of bins increases.

We summarise this part by noting, that there are many test statistics for the comparison of the shapes of two one-dimensional histograms. The most popular are: the Pearson-$\chi^2$ test, the Anderson-Darling test or the Cramer-von-Mises test, see e.g. [55]. Each of these tests has its pros and cons and it is not possible to choose the one test that is the best for all applications. Overall, the more we know about what we really want to compare and test, the more reliable the test we can choose for our particular problem. We have examined...
all the above-mentioned tests and have decided to use the Kolmogorov-Smirnov test of the equality. The two sample KS-test assumes continuous distributions. It is one of the most general nonparametric\textsuperscript{1} tests for comparing two samples, as it is sensitive to differences in shape of the empirical cumulative distribution functions of the two samples. It is also the most robust test as it tests for any violation of the null hypothesis. However, it requires a relatively large number of data points in each bin. We further notice, that the KS-test is more sensitive to the regions near the peak of the tested distributions rather than to their tails. For the latter the Anderson-Darling test would do a better job. This observation is very useful in our case as for many dimensionful observables tails are usually plagued by larger statistical fluctuations and are, therefore, not really reliable for such comparisons.

4 Cross Section Ratios

Once we established that \( pp \to t\bar{t}W^+ \) and \( t\bar{t}W^- \) are correlated we can look at their ratio with the goal of increasing the precision of NLO QCD predictions for both processes. The fact that the processes are correlated is exploited when estimating the theoretical error for the cross section ratio. Specifically, the theoretical error for the cross section ratio is estimated by calculating

\[
R = \frac{\sigma_{t\bar{t}W^+}(\mu_1)}{\sigma_{t\bar{t}W^-}(\mu_2)},
\]

with \( \mu_1 = \mu_2 = \mu_0 \) where \( \mu_0 = m_t + m_W/2 \) or \( \mu_0 = H_T/3 \). Furthermore, only the following combinations are considered

\[
\left(\frac{\mu_1}{\mu_0}, \frac{\mu_2}{\mu_0}\right) = \{(2,2), (0.5,0.5)\}.
\]

In Tables 1 and 2 we present integrated fiducial cross sections at NLO in QCD for \( pp \to t\bar{t}W^+ \) and \( t\bar{t}W^- \) in the multi-lepton decay channel together with the theoretical uncertainties due to scale dependence. Also given is the \( R \) cross section ratio. First we examine the stability of \( R \) with respect to the \( p_{T,b} \) cut. To this end, we show results for four different values of the \( p_{T,b} \) cut. We observe very stable cross section ratio results both in terms of the central value and theoretical uncertainties. Furthermore, we notice that the scale choice does not play any role for such an inclusive observable. In the case of \( R \) systematic uncertainties \( \delta_{\text{scale}} \) and \( \delta_{\text{PDF}} \) have been added in quadrature. We point out, however, that the PDF uncertainties, which for \( pp \to t\bar{t}W^+ \) and \( pp \to t\bar{t}W^- \) are consistently at the 2\% level, cancel out in the ratio to 0.2\%. The final theoretical uncertainty is completely dominated by the scale dependence. The latter is at the 1\% - 2\% level. Such precise theoretical predictions have normally been obtained only once the NNLO QCD corrections are incorporated. Thus, \( R \) at NLO in QCD represents a very precise observable to be measured at the LHC.

In the next step we examine the impact of the top quark production and decay modelling on the cross section ratio. To this end we present results for the full NWA and for the NWA\textsubscript{LO}\textsubscript{decay} case. The former comprises NLO QCD corrections to the production and

\textsuperscript{1}The nonparametric test does not assume that data points are sampled from the Gaussian distribution or any other defined distribution for that matter.
Different values of the uncertainties as estimated from the scale variation and from the PDFs are listed as well. Four different values of the $p_T,b$ cut are used. The NNPDF3.0 PDF set is employed and $\mu_R = \mu_F = \mu_0$ where $\mu_0 = m_t + m_W/2$.

to the subsequent top quark decays, the latter NLO QCD corrections to the production of $t\bar{t}W^\pm$ and LO top quark decays. Should we use the NLO QCD results in the full NWA for the $pp \to t\bar{t}W^\pm$ process our findings for $\mu_0 = m_t + m_W/2$ would be as follows

$$\mathcal{R} = \frac{\sigma_{NWA, NLO}}{\sigma_{NLO, NWA}} = 1.81 \pm 0.04 \text{ (2%)}. \quad (4.3)$$

On the other hand for the dynamical scale choice $\mu_0 = H_T/3$ we would obtain

$$\mathcal{R} = \frac{\sigma_{NWA, NLO}}{\sigma_{NLO, NWA}} = 1.81 \pm 0.03 \text{ (2%)}. \quad (4.4)$$

We can observe that the full NWA approach does not modify either the value or the size of the theoretical error for the integrated cross section ratio. The latter result is not surprising taking into account that the impact of the top quark off-shell effects on the integrated fiducial $t\bar{t}W^\pm$ cross section is negligible. Furthermore, theoretical uncertainties for the full NWA and full off-shell case are similar independently of the scale choice [22].

Finally, we have employed the NWALOdecay case. For $\mu_0 = m_t + m_W/2$ we obtained

$$\mathcal{R} = \frac{\sigma_{NWA, NLO, \text{decay}}}{\sigma_{NLO, NWA, \text{decay}}} = 1.82 \pm 0.02 \text{ (1%)}, \quad (4.5)$$

whereas for $\mu_0 = H_T/3$ we can report

$$\mathcal{R} = \frac{\sigma_{NWA, NLO, \text{decay}}}{\sigma_{NLO, NWA, \text{decay}}} = 1.81 \pm 0.02 \text{ (1%)}. \quad (4.6)$$

---

Table 1. NLO QCD integrated fiducial cross sections for $pp \to t\bar{t}W^\pm$ in the multi-lepton final state at the LHC with $\sqrt{s} = 13$ TeV. Also shown are results for $\mathcal{R} = \sigma_{NLO}^{t\bar{t}W^+}/\sigma_{NLO}^{t\bar{t}W^-}$. Theoretical uncertainties as estimated from the scale variation and from the PDFs are listed as well. Four different values of the $p_T,b$ cut are used. The NNPDF3.0 PDF set is employed and $\mu_R = \mu_F = \mu_0$ where $\mu_0 = m_t + m_W/2$. 

| $p_T,b > 25$ GeV | $\sigma_{NLO}^{t\bar{t}W^+}$ $\delta_{\text{scale}}$ $\delta_{\text{PDF}}$ | $\sigma_{NLO}^{t\bar{t}W^-}$ $\delta_{\text{scale}}$ $\delta_{\text{PDF}}$ | $\mathcal{R}$ |
|------------------|-------------------------------|-------------------------------|-----------|
| NNPDF3.0         | [ab]                          | [ab]                          | $\mathcal{R}$ |
| $p_T,b > 30$ GeV | 123.2 $+6.3\,(5\%)$ $+2.1\,(2\%)$ $-8.7\,(7\%)$ $-2.1\,(2\%)$ | 68.0 $+4.8\,(7\%)$ $+1.2\,(2\%)$ $-5.5\,(8\%)$ $-1.2\,(2\%)$ | 1.81 $\pm 0.03 \text{ (2\%)}$ |
| $p_T,b > 35$ GeV | 113.1 $+5.4\,(5\%)$ $+1.9\,(2\%)$ $-7.8\,(7\%)$ $-1.9\,(2\%)$ | 62.3 $+4.2\,(7\%)$ $+1.1\,(2\%)$ $-4.9\,(8\%)$ $-1.1\,(2\%)$ | 1.81 $\pm 0.03 \text{ (2\%)}$ |
| $p_T,b > 40$ GeV | 102.6 $+4.7\,(5\%)$ $+1.7\,(2\%)$ $-6.8\,(7\%)$ $-1.7\,(2\%)$ | 56.3 $+3.7\,(7\%)$ $+1.0\,(2\%)$ $-4.4\,(8\%)$ $-1.0\,(2\%)$ | 1.82 $\pm 0.03 \text{ (2\%)}$ |
| $p_T,b > 50$ GeV | 92.0 $+4.0\,(4\%)$ $+1.6\,(2\%)$ $-6.1\,(7\%)$ $-1.6\,(2\%)$ | 50.3 $+3.3\,(6\%)$ $+0.9\,(2\%)$ $-3.9\,(8\%)$ $-0.9\,(2\%)$ | 1.83 $\pm 0.04 \text{ (2\%)}$ |
of the more centrally produced top antiquarks. This suggests that the initial state can be from valence, while the antiquarks are from the sea, the larger average distributions in the SM is therefore broader than that due to the scale dependence are higher in the former case, up to 11%–13% [22]. Yet in the cross section ratio these differences cancel out making \( \mathcal{R} = \sigma_{tW^+/tW^-}^{NLO} / \sigma_{tW^-}^{NLO} \) very precise and an extremely interesting theoretical observable to be measured at the LHC.

To conclude this part, we note that for the cross section ratio at NLO in QCD the residual perturbative uncertainties are reduced to 1%–2%. The theoretical uncertainties associated with the top quark modelling are negligible. This suggests that the \( \mathcal{R} = \sigma_{tW^+/tW^-}^{NLO} / \sigma_{tW^-}^{NLO} \) observable can be employed either for the precision SM measurements or to shed some light on possible new physics scenarios that might reveal themselves only once sufficiently precise theoretical predictions are available. In the case of the SM the \( \mathcal{R} \) observable, which is sensitive to the valence content of the proton, can be used to provide valuable input for the up and down quark parton distribution functions of the proton at higher values of \( x \) (the momentum fraction of the parton). In the case of BSM physics the presence of two same-sign leptons in the final state, a rare phenomenon at the LHC, constitutes an optimal signature for many new physics models from supersymmetry, supergravity and Majorana neutrinos to models with the modified Higgs boson sector. Given the final accuracy of \( \mathcal{R} \), it should be used to put more stringent constraints on the parameter space of these models.

### 5 Charge Asymmetries in \( t\bar{t}W^\pm \) Production

The pp initial state at the LHC is expected to produce top quark and antiquark rapidity distributions in \( t\bar{t} \) production that are symmetric about \( y = 0 \). However, since the quarks in the initial state can be from valence, while the antiquarks are from the sea, the larger average momentum-fraction of quarks leads to an excess of top quarks produced in the forward directions. The rapidity distribution of top quarks in the SM is therefore broader than that of the more centrally produced top antiquarks. This suggests that \( \Delta|y| = |y_t| - |y_{\bar{t}}| \), which

| \( \mu_0 = H_T/3 \) | \( \sigma_{tW^+}^{NLO} + \delta_{scale} + \delta_{PDF} \) | \( \sigma_{tW^-}^{NLO} + \delta_{scale} + \delta_{PDF} \) | \( \sigma_{tW^+/tW^-}^{NLO} \) |
|---|---|---|---|
| NNPDF3.0 | ab | ab |
| \( pT, b > 25 \text{ GeV} \) | 124.4 \( \pm 4.3\) (3%) +2.1 (2%) | 68.6 \( \pm 3.5\) (5%) +1.2 (2%) | 1.81 \( \pm 0.02\) (1%) |
| \( pT, b > 30 \text{ GeV} \) | 113.9 \( \pm 3.5\) (3%) +1.9 (2%) | 62.7 \( \pm 3.0\) (5%) +1.1 (2%) | 1.82 \( \pm 0.03\) (2%) |
| \( pT, b > 35 \text{ GeV} \) | 103.1 \( \pm 3.1\) (3%) +1.7 (2%) | 56.5 \( \pm 2.6\) (5%) +1.0 (2%) | 1.82 \( \pm 0.02\) (1%) |
| \( pT, b > 40 \text{ GeV} \) | 92.3 \( \pm 2.8\) (3%) +1.5 (2%) | 50.4 \( \pm 2.3\) (5%) +0.9 (2%) | 1.83 \( \pm 0.03\) (2%) |

Table 2. As in Table 1 but for \( \mu_0 = H_T/3 \).
is the difference between the absolute value of the top quark rapidity \(|y_t|\) and the absolute value of the anti-top quark rapidity \(|y_{\bar{t}}|\), is a suitable observable to measure the top quark charge asymmetry at the LHC. This asymmetry is nevertheless very small, see e.g. \([56, 57]\).

For the \(pp \rightarrow t\bar{t}W^\pm\) process the presence of the \(W^\pm\) gauge boson polares the initial quark line and in turn the \(t\bar{t}\) pair \([2]\). As a consequence the emerging top quark charge asymmetry is larger than that observed in \(pp \rightarrow t\bar{t}\) production. Furthermore, the lepton and b-jet charge asymmetries are very large and already present at the leading order. In the following we calculate afresh the top quark charge asymmetry in the \(t\bar{t}W^\pm\) process in the multi-lepton final state using the state-of-the-art NLO QCD calculations with the full top quark off-shell effects included. Additionally, the asymmetries for the top quark decay products, \(A^b_c\) and \(A^\ell_c\), will be examined. In this part of the paper, one of our main goals is to carefully assess the impact of the top quark modelling on \(A^b_c, A^\ell_c\) and \(A^b_c\). We start with asymmetries at the integrated level albeit in the fiducial regions of the phase space as defined in Section 2. For \(A^\ell_c\) we will additionally calculate the differential and cumulative asymmetry with respect to the following observables: \(p_T(\ell_1\ell_2)\), \(|y(\ell_1\ell_2)|\) and \(M(\ell_1\ell_2)\), where \(\ell_1, \ell_2\) stands for the charged leptons stemming from the top and anti-top quark decay respectively. For the two processes under consideration \(pp \rightarrow e^+\nu_e\mu^-\bar{\nu}_\mu e^+\nu_e b\bar{b}\) and \(pp \rightarrow e^-\bar{\nu}_e\mu^+\nu_\mu e^-\bar{\nu}_e b\bar{b}\) the reconstruction of the (anti-)top quark momentum through its decay products is carried out. As we are dealing with identical leptons in the final state, however, we must employ an additional mechanism to properly assign the positron (electron) and the corresponding neutrino \(\nu_e\) (anti-neutrino \(\bar{\nu}_e\)) to the correct top (anti-top) quark. In the case of \(t\bar{t}W^+\) production we use the following four different resonance histories (a similar procedure is applied in the \(t\bar{t}W^-\) case)

\[
\begin{align*}
   t &\rightarrow e_1^+ \nu_e, 1 \ b & & \text{and} & & \ell &\rightarrow \mu^- \bar{\nu}_\mu \bar{b}, \\
   t &\rightarrow e_1^+ \nu_e, 2 \ b & & \text{and} & & \ell &\rightarrow \mu^- \bar{\nu}_\mu \bar{b}, \\
   t &\rightarrow e_2^+ \nu_e, 1 \ b & & \text{and} & & \ell &\rightarrow \mu^- \bar{\nu}_\mu \bar{b}, \\
   t &\rightarrow e_2^+ \nu_e, 2 \ b & & \text{and} & & \ell &\rightarrow \mu^- \bar{\nu}_\mu \bar{b}.
\end{align*}
\]

(5.1)

These four resonant histories are not sufficient if NLO QCD calculations are considered. In the case of the subtracted real emission part we additionally take into account the extra light jet if resolved. Specifically, to closely mimic what is done on the experimental side only the light jet that passes all the cuts, that are also required for the two b-jets, is added to the resonance history. Thus, in such a case a total of twelve different resonant histories have to be considered. We compute for each history the following quantity, see Ref. \([58]\)

\[
Q = |M_t - m_t| + |M_{\bar{t}} - m_{\bar{t}}|,
\]

(5.2)

where \(M_t\) and \(M_{\bar{t}}\) are the (reconstructed) invariant masses of the top and anti-top quark respectively and \(m_t = 172.5\) GeV. For each phase space point we pick the history that minimises the \(Q\) value. In this way all the (anti-)top quark decay products are identified. They are employed in the definition of \(A^b_c, A^\ell_c\) and \(A^b_c\). To show how well such a reconstruction works in Figure 5 we display the reconstructed invariant mass of the top (anti-top) quark
Figure 5. Reconstructed invariant mass of the top quark and anti-top quark at NLO in QCD for \( pp \to t\bar{t}W^+ \) and \( pp \to t\bar{t}W^- \) in the multi-lepton final state. Results are given for the LHC with \( \sqrt{s} = 13 \) TeV. The NLO NNPDF3.0 PDF set is employed and \( \mu_R = \mu_F = \mu_0 \) where \( \mu_0 = H_T/3 \).

at NLO in QCD for the \( pp \to t\bar{t}W^+ \) (\( pp \to t\bar{t}W^- \)) process in the multi-lepton channel. Out of all twelve histories the four histories with the smallest \( Q \) value are shown. Clearly one can see that the reconstruction works very well.

Using the notation of Ref. [57, 59, 60] we define the top quark charge asymmetry as follows

\[
A_t^c = \frac{\sigma^+_{\text{bin}} - \sigma^-_{\text{bin}}}{\sigma^+_{\text{bin}} + \sigma^-_{\text{bin}}}, \quad \sigma^\pm_{\text{bin}} = \int \theta(\pm\Delta|y|) \theta_{\text{bin}} d\sigma,
\]

where \( \Delta|y| = |y_t| - |y_{\bar{t}}| \) and \( d\sigma \) is the differential fiducial \( t\bar{t}W^\pm \) cross section calculated at NLO in QCD. The binning function \( \theta_{\text{bin}} \) can take the values zero or one. Its purpose is to restrict to a given bin the kinematics of the \( t\bar{t}W^\pm \) process in one of the kinematic variables that is considered. The integrated asymmetry is obtained by setting \( \theta_{\text{bin}} = 1 \).

We note here that the charge-symmetric \( gg \) initial state, that is the dominant mechanism for \( t\bar{t} \) production at the LHC, is not present for \( t\bar{t}W^\pm \) production. Therefore, unlike for \( pp \to t\bar{t} \), it will not contribute to the denominator of Eq. (5.3) to dilute the asymmetry.

The LHC measurements for the top quark charge asymmetry in \( pp \to t\bar{t} \) production have been carried out in terms of rapidity as well as pseudorapidity differences, see e.g. [61–66]. Even though, the top quark charge asymmetry based on rapidity and pseudorapidity has the same features its value can differ quite substantially. Consequently, we shall provide results for \( A_t^c \) for both cases. In the case of the top quark decay products \( A^c_\ell \) and \( A^b_\ell \) are based on \( \Delta|y| = |y_\ell| - |y_{\bar{\ell}}| \) and \( \Delta|y| = |y_b| - |y_{\bar{b}}| \) respectively.

The top quark charge asymmetry can be visualised by superimposing the rapidity (or the pseudo-rapidity) of \( t \) and \( \bar{t} \) for the \( t\bar{t}W^\pm \) process. The same can be done separately for \( t\bar{t}W^- \). Similarly, we can plot together the top and anti-top quark decay products, \( b \) and \( \bar{b} \) as well as \( \ell_t \) and \( \ell_{\bar{t}} \). In Figure 6 we present such a comparison at the NLO QCD level for the \( t\bar{t}W^\pm \) process. We can observe that all spectra are symmetric about \( y = 0 \) (\( \eta = 0 \)), as it should be, and that the anti-top quark is more central with respect to the top quark. The same is visible for the \( b \)-jet. This can be directly translated into the positive value of \( A^c_\ell \) and \( A^b_\ell \). The situation is reversed for the charged leptons. In the later case the charged
Figure 6. Comparison of the rapidity and pseudo-rapidity distributions of the $t$ and $\bar{t}$ quarks at the NLO QCD level for $pp \to t\bar{t}W^+$ and $pp \to t\bar{t}W^-$ in the multi-lepton final state. Also shown are rapidity distributions of the charged leptons and $b$-jets from $t$ and $\bar{t}$ decays. Results are given for the LHC with $\sqrt{s} = 13$ TeV. The NLO NNPDF3.0 PDF set is employed and $\mu_R = \mu_F = \mu_0$ where $\mu_0 = H_T/3$ is used.

lepton from the top quark decay is more central, which will manifest itself in the negative value of $A^\ell_t$.

In Table 3 we present our findings for $A^t_c$, $A^\ell_c$ and $A^b_c$ at NLO QCD for $t\bar{t}W^+$ production in the multi-lepton channel at the LHC with $\sqrt{s} = 13$ TeV. Results are given for the fixed scale choice $\mu_R = \mu_F = m_t + m_W/2$. The top quark charge asymmetry calculated in terms of rapidities (pseudo-rapidities) is denoted as $A^{t,c,y}_c (A^{t,c,\eta}_c)$. We present the results with the full off-shell effects included as well as for the full NWA and for the NWA_{LO\,decay} case. For all three approaches theoretical uncertainties due to the scale dependence are also given. They are estimated by varying the renormalisation and factorization scales in $\alpha_s$ and PDFs up and down by a factor 2 around the central scale of the process $\mu_0$. We show theoretical predictions for the unexpanded and expanded version of the asymmetry. The expanded version of $A^i_c$ at NLO in QCD, where $i$ stands for $i = t, \ell, b$, is defined as

$$A^{i,c,\exp}_c = \frac{\sigma^{-}\LO}{\sigma^{+}\LO} \left( 1 + \frac{\delta\sigma^{-}\NLO}{\sigma^{-}\LO} - \frac{\delta\sigma^{+}\NLO}{\sigma^{+}\LO} \right). \quad (5.4)$$

where $\sigma^{\pm}$ stands for $\sigma^{\pm}_{\text{bin}} \pm \sigma^{\pm}_{\text{bin}}$ and $\delta\sigma^{\pm}_{\NLO}$ are the NLO contributions to the fiducial cross section. Furthermore, $\sigma^{\pm}_{\LO}$ are evaluated with NLO input parameters. In the case of
Table 3. Unexpanded and expanded $A_{t,c,y}^t$, $A_{c,\eta}^t$ and $A_{c,y}^b$ asymmetries at NLO in QCD for $pp \rightarrow t\bar{t}W^\pm$ in multi-lepton channel at the LHC with $\sqrt{s} = 13$ TeV. Various approaches for the modelling of the top quark production and decays are considered: the full off-shell case, the full NWA and the NWA$_{LO\text{decay}}$ case. Also given are Monte Carlo (in parenthesis) integration and theoretical errors. The NNPDF3.0 PDF set is employed and $\mu_R = \mu_F = \mu_0$ where $\mu_0 = m_t + m_W/2$.

| $t\bar{t}W^+$ | Off-shell | Full NWA | NWA$_{LO\text{decay}}$ |
|---------------|-----------|-----------|--------------------------|
| $\mu_0 = m_t + m_W/2$ |           |           |                          |
| $A_{t,c,y}^t$ [%] | 2.09(8) $^{+1.06 (51\%)}_{-0.70 (33\%)}$ | 1.68(4) $^{+1.00 (60\%)}_{-0.67 (40\%)}$ | 0.86(3) $^{+0.66 (77\%)}_{-0.43 (50\%)}$ |
| $A_{t,c,exp,y}^t$ [%] | 2.62(10) $^{+0.39 (15\%)}_{-0.34 (13\%)}$ | 2.19(4) $^{+0.38 (17\%)}_{-0.34 (16\%)}$ | 1.94(5) $^{+0.46 (24\%)}_{-0.32 (16\%)}$ |
| $A_{t,c,\eta}^t$ [%] | 3.10(8) $^{+1.21 (39\%)}_{-0.80 (26\%)}$ | 2.58(4) $^{+1.31 (51\%)}_{-0.75 (29\%)}$ | 1.16(4) $^{+0.71 (61\%)}_{-0.44 (38\%)}$ |
| $A_{t,c,exp,\eta}^t$ [%] | 3.70(10) $^{+0.46 (12\%)}_{-0.40 (11\%)}$ | 3.18(5) $^{+0.56 (18\%)}_{-0.34 (11\%)}$ | 2.25(5) $^{+0.51 (23\%)}_{-0.32 (14\%)}$ |
| $A_{b,c,y}^b$ [%] | 6.46(8) $^{+0.05 (0.8\%)}_{-0.05 (0.8\%)}$ | 6.18(4) $^{+0.13 (2.1\%)}_{-0.05 (0.8\%)}$ | 5.99(3) $^{+0.10 (1.7\%)}_{-0.01 (0.2\%)}$ |
| $A_{b,c,exp,y}^b$ [%] | 6.56(10) $^{+0.02 (0.3\%)}_{-0.07 (1.1\%)}$ | 6.28(4) $^{+0.03 (0.5\%)}_{-0.01 (0.1\%)}$ | 6.21(5) $^{+0.06 (1.0\%)}_{-0.01 (0.2\%)}$ |
| $A_{c,y}^t$ [%] | $-7.90(10)$ $^{+2.15 (27\%)}_{-1.39 (17\%)}$ | $-8.43(4)$ $^{+2.10 (25\%)}_{-1.37 (16\%)}$ | $-10.11(3)$ $^{+1.36 (9.4\%)}_{-0.95 (9.4\%)}$ |
| $A_{c,exp,y}^t$ [%] | $-7.00(12)$ $^{+1.60 (14\%)}_{-0.80 (11\%)}$ | $-7.52(4)$ $^{+0.95 (13\%)}_{-0.78 (10\%)}$ | $-8.23(5)$ $^{+1.01 (12\%)}_{-0.79 (9.6\%)}$ |

large differences between the LO and NLO asymmetries, which is true for the $pp \rightarrow t\bar{t}W^\pm$ process, it is necessary to expand $A_{t}^c$ to first order in $\alpha_s$ as the ratio in Eq. (5.3) generates contributions of $\mathcal{O}(\alpha_s^2)$ and higher, which are affected by the unknown NNLO contributions. Furthermore, in Table 3 we include in parenthesis the Monte Carlo (MC) integration errors to show that the latter are smaller than or at least similar in size to the theoretical errors from the scale dependence. Since the PDF dependence of the asymmetry is very small (at the per-mill level) we do not quote the PDF errors in our predictions.

In the following we analyse our findings that are presented in Table 3. We can first notice that the difference between the expanded and unexpanded asymmetries is rather moderate for the full off-shell and the full NWA case. We observe differences up to $25\% - 30\%$. For the NWA$_{LO\text{decay}}$ approach, on the other hand, they are much larger, even above $100\%$. Secondly, theoretical uncertainties due to the scale dependence are substantially reduced for the expanded version of the asymmetries. In the case of the off-shell predictions, for example, we obtained $51\%$, $39\%$ and $27\%$ uncertainties for $A_{t,c,y}^t$, $A_{c,\eta}^t$ and $A_{c,y}^b$ respectively. Once the expansion is introduced $15\%$, $12\%$, $14\%$ uncertainties are estimated instead. For the $A_{c,y}^b$ asymmetry no reduction is observed. Similar conclusions are obtained for the two other approaches. The two definitions, with unexpanded NLO asymmetry and with a consistent expansion in $\alpha_s$, give reasonably consistent results for the central scale, especially
in the case of the full off-shell and NWA results. However, for the theoretical uncertainties large differences are observed in all three cases. Similar effects have been noticed for $A^t_{c,y}$ and $A^t_{c,\text{exp},y}$ in the $pp \rightarrow t\bar{t}$ process at the LHC [57]. Once the next-to-next-to-leading order QCD corrections have been incorporated the difference between the expanded and unexpanded predictions has been reduced both for the central value and for the theoretical error underling the reliable theoretical control over these predictions. However, more important is the fact that the inclusion of the NNLO QCD corrections for $t\bar{t}$ has shown that theoretical uncertainties for the $A^t_{c,\text{exp},y}$ observable are not underestimated at the NLO QCD level. Based on this argument and in the absence of NNLO QCD predictions for $pp \rightarrow t\bar{t}W^\pm$ process we can conclude that our expanded asymmetry results might be safely employed in the comparisons with the LHC experimental data. We can also note that the results for the top quark charge asymmetry based on the rapidity differences differ by almost 50% from those calculated in terms of pseudo-rapidity differences. Only in the case of NWA_{LOdecay} is the difference smaller, i.e. of the order of 15% – 35%. Finally, we compare the full off-shell case with the full NWA for the expanded asymmetries. For $A^t_{c,\text{exp},y}$, $A^b_{c,\text{exp},y}$ and $A^b_{c,\text{exp},y}$ differences between central values are well below 1σ when compared with the respective theory uncertainties. This suggests that $A^t_{c,y}$, $A^b_{c,y}$ and $A^b_{c,y}$ are rather inclusive observables. For $A^b_{c,\text{exp},y}$, however, for which theoretical uncertainties of the order of 1% are estimated, we obtained a substantial 3.7σ difference. This points to the conclusion that once the theoretical uncertainties are sufficiently reduced, the sensitivity of the asymmetries to the top quark production and decay modelling will increase. For now, however, the full NWA description is sufficient to describe (at least) $A^t_{c,\text{exp},y}$, $A^t_{c,\text{exp},y}$ and $A^t_{c,\text{exp},y}$. In the next

| $t\bar{t}W^+$ | Off-shell | Full NWA | NWAL_{Odecay} |
|----------------|-----------|-----------|---------------|
| $\mu_0 = H_T/3$ |           |           |               |
| $A^t_{c,y}$ [%]   | 2.36(8)$^{+1.19}_{-0.77}$ (50%) | 1.93(5)$^{+1.23}_{-0.72}$ (64%) | 1.11(3)$^{+0.55}_{-0.53}$ (49%) |
| $A^t_{c,\text{exp},y}$ [%] | 2.66(10)$^{+0.38}_{0.34}$ (14%) | 2.20(5)$^{+0.45}_{-0.31}$ (20%) | 2.08(5)$^{+0.24}_{-0.40}$ (11%) |
| $A^t_{c,\text{exp},\eta}$ [%] | 3.46(9)$^{+1.41}_{-0.90}$ (41%) | 3.02(5)$^{+1.44}_{-0.93}$ (48%) | 1.42(4)$^{+0.59}_{-0.56}$ (11%) |
| $A^b_{c,y}$ [%]   | 6.48(9)$^{+0.04}_{-0.05}$ (0.6%) | 6.16(4)$^{+0.07}_{-0.01}$ (1.1%) | 6.05(3)$^{+0.02}_{-0.01}$ (0.3%) |
| $A^b_{c,\text{exp},y}$ [%] | 6.53(10)$^{+0.03}_{0.08}$ (1.2%) | 6.21(5)$^{+0.09}_{-0.05}$ (1.4%) | 6.23(5)$^{+0.02}_{-0.04}$ (0.3%) |
| $A^t_{c,\text{exp},y}$ [%] | -7.46(11)$^{+2.46}_{-1.55}$ (33%) | -7.94(4)$^{+2.45}_{-1.54}$ (31%) | -9.81(4)$^{+1.46}_{-1.03}$ (15%) |
| $A^t_{c,\text{exp},\eta}$ [%] | -6.93(13)$^{+1.01}_{-0.81}$ (14%) | -7.43(5)$^{+0.99}_{-0.70}$ (13%) | -8.14(5)$^{+1.00}_{-0.81}$ (12%) |

Table 4. As in Table 3 but for $\mu_0 = H_T/3$. 
step the NWA_{LO,decay} case is compared to the full off-shell case. The discrepancies between central values of the asymmetries increased for $A^{t}_{c,exp,y}, A^{t}_{c,y} \text{ and } A^{t}_{c,exp,y'}$ in some cases even up to 2$\sigma$, while it remained almost the same for $A^{b}_{c,exp,y'} (3.8\sigma)$. We can conclude that in the case of the top quark charge asymmetry and the asymmetries of the top quark decay products the NLO QCD corrections to the top quark decays play a crucial role. Overall, the inclusion of the complete off-shell effects for the $pp \to t\bar{t}W^{\pm}$ process increases the central values of the asymmetries while at the same time the theoretical errors are kept almost unchanged.

Our conclusions are not changed when the dynamical scale choice, $\mu_0 = H_T/3$, is employed instead. Results for $A^{t}_{c,y}, A^{t}_{c,\eta}, A^{t}_{c,y} \text{ and } A^{b}_{c,\eta}$ at NLO in QCD with $\mu_0 = H_T/3$ are shown in Table 4. When comparing to the theoretical predictions for $\mu_0 = m_t + m_W/2$ we can notice an overall agreement, within $0.1\sigma - 0.7\sigma$, between all central values of the asymmetries. In addition, similar theoretical uncertainties due to the scale dependence are estimated for both scale choices.

Our state-of-the-art results for the top quark charge asymmetry and for the charge asymmetries of the top quark decay products are summarised in Table 5 and Table 6. In both cases we provide the NLO QCD results for the expanded version of $A^{\ell}_{c,y}$, $A^{\ell}_{c,\eta}$ and $A^{b}_{c,\eta}$. They are calculated from the theoretical predictions, which include the full top quark off-shell effects. We present results for $pp \to t\bar{t}W^{+}$ and $pp \to t\bar{t}W^{-}$ in the multi-lepton channel at the LHC with $\sqrt{s} = 13$ TeV. We additionally present the combined results for the $pp \to t\bar{t}W^{\pm}$ process. Also in this case the results for the top quark charge asymmetry are given in terms of rapidities, $\Delta|y| = |y_t| - |y_{\bar{t}}|$, and pseudorapidities, $\Delta|\eta| = |\eta_t| - |\eta_{\bar{t}}|$. A comment on the difference in size of asymmetries for $pp \to t\bar{t}W^{+}$ and $pp \to t\bar{t}W^{-}$ is in order. The asymmetries are larger for $pp \to t\bar{t}W^{+}$ than for $pp \to t\bar{t}W^{-}$. However, otherwise they behave similarly. As pointed out in Ref. [2] this can be understood by applying an argument based on PDFs. At the LO the $t\bar{t}W^{+}$ process is produced predominantly via $ud$ whereas for $t\bar{t}W^{-}$ the $\bar{u}d$ subprocess is the most relevant one. The longitudinal momenta of the initial partons are on average $p_u > p_d > p_{\bar{u}} \approx p_{\bar{d}}$. In both cases the momentum of the top and anti-top quarks is connected to the momentum of the $q$ and $\bar{q}$ respectively. The large longitudinal momentum transferred to the top quark from the initial $u$ quark in the $t\bar{t}W^{+}$ case increases the corresponding $|y_t|$ value. Consequently, the charge asymmetry of the top quark is enhanced compared to the one calculated for $t\bar{t}W^{-}$. When analysing the combined results for $pp \to t\bar{t}W^{\pm}$ we can observe that the theoretical uncertainties due to the scale dependence reach up to 15%. The scale choice play no role here as for $\mu_0 = m_t + m_W/2$ and $\mu_0 = H_T/3$ similar results are obtained.

6 Differential and cumulative asymmetry

In this part of the paper we present predictions for differential $A^{\ell}_{c,y}$ asymmetry with respect to the following observables: transverse momentum of the two charged leptons, $p_T(\ell_t\ell_{\bar{t}})$, rapidity of the two charged leptons, $|y(\ell_t\ell_{\bar{t}})|$, and invariant mass of the two charged leptons, $M(\ell_t\ell_{\bar{t}})$, where $\ell_t\ell_{\bar{t}}$ originate from the $t\bar{t}$ pair. The differential results are given using the unexpanded definition from Eq. (5.3). We also present predictions for cumulative asymme-
\[ \mu_0 = m_t + m_W/2 \]

| \( A^l_{c,\text{exp},y} \) [\%] | \( t\bar{W}^+ \) | \( t\bar{W}^- \) | \( t\bar{W}^\pm \) |
|----------------|----------------|----------------|----------------|
| 2.62^{+0.39}_{-0.34} (15%) | 1.97^{+0.31}_{-0.25} (16%) | 2.40^{+0.37}_{-0.31} (15%) |
| 3.70^{+0.46}_{-0.40} (12%) | 1.31^{+0.32}_{-0.25} (24%) | 2.87^{+0.41}_{-0.35} (14%) |
| 6.56^{+0.02}_{-0.07} (0.3%) | 4.80^{+0.05}_{-0.05} (1.0%) | 5.93^{+0.03}_{-0.08} (0.5%) |
| -7.00^{+1.00}_{-0.80} (14%) | -5.68^{+0.78}_{-0.61} (14%) | -6.51^{+0.93}_{-0.74} (14%) |

Table 5. Expanded \( A^l_c \), \( A^l_c \) and \( A^b \) at NLO in QCD for \( pp \rightarrow t\bar{W}^+ \) and \( pp \rightarrow t\bar{W}^- \) in the multi-lepton channel at the LHC with \( \sqrt{s} = 13 \) TeV. Results are obtained with the full off-shell effects included. Also given are combined results for \( pp \rightarrow t\bar{W}^\pm \) and theoretical uncertainties. The NNPDF3.0 PDF set is employed and \( \mu_R = \mu_F = \mu_0 \) where \( \mu_0 = m_t + m_W/2 \).

| \( \mu_0 = H_T/3 \) | \( t\bar{W}^+ \) | \( t\bar{W}^- \) | \( t\bar{W}^\pm \) |
|----------------|----------------|----------------|----------------|
| 2.66^{+0.38}_{-0.34} (14%) | 2.05^{+0.33}_{-0.27} (16%) | 2.45^{+0.37}_{-0.31} (15%) |
| 3.81^{+0.46}_{-0.40} (12%) | 1.31^{+0.33}_{-0.26} (25%) | 2.94^{+0.42}_{-0.35} (14%) |
| 6.53^{+0.03}_{-0.08} (0.4%) | 4.80^{+0.06}_{-0.11} (1.2%) | 5.91^{+0.04}_{-0.09} (0.7%) |
| -6.93^{+1.01}_{-0.81} (14%) | -5.67^{+0.81}_{-0.63} (14%) | -6.46^{+0.95}_{-0.75} (15%) |

Table 6. As in Table 5 but for \( \mu_0 = H_T/3 \).

tries, that are closely related to the corresponding differential asymmetries. One can employ the same definition as in Eq. (5.3), however, this time for a given value of the kinematic variable for which we compute the asymmetry the bin ranges from zero to that value. Even though differential and cumulative asymmetries contain the same information, the latter one behaves better simply because it is more inclusive, i.e. the higher order corrections are distributed more uniformly over the whole kinematic range. In addition, the cumulative asymmetry should give the integrated one in the last bin assuming that the plotted range of the corresponding differential distribution covers the whole available phase space. In practice, we shall see that if the left-over phase space region is negligible the integrated asymmetry can be recovered very accurately. Let us note that differential asymmetries have been studied at the LHC for the \( pp \rightarrow t\bar{t} \) production process by both experimental collaborations ALTAS and CMS, see e.g. [65, 67].

In Figure 7 the \( p_T(\ell_1,\ell_2) \)-dependent differential and cumulative \( A^l_c \) asymmetry at NLO QCD for \( pp \rightarrow t\bar{W}^\pm \) in multi-lepton channel at the LHC with \( \sqrt{s} = 13 \) TeV is displayed. Various approaches for the modelling of the top quark production and decays are considered. Also given are theoretical uncertainties for the full off-shell case. For all approaches Monte
Figure 7. The $p_T(\ell_1\ell_2)$-dependent differential (left panel) and cumulative (right panel) $A_\ell^c$ asymmetry at NLO QCD for $pp \rightarrow t\bar{t}W^{\pm}$ in the multi-lepton channel at the LHC with $\sqrt{s} = 13$ TeV. Various approaches for the modelling of the top quark production and decays are considered. Also given are theoretical uncertainties for the full off-shell case. For all approaches Monte Carlo errors are provided for both differential and cumulative asymmetries. The NNPDF3.0 PDF set is employed and $\mu_R = \mu_F = \mu_0$ where $\mu_0 = H_T/3$.

Figure 8. As in Figure 7 but for $|y(\ell_1\ell_2)|$.

Carlo integration errors are provided for both differential and cumulative asymmetries. For the $p_T(\ell_1\ell_2)$-dependent differential asymmetry in all bins but the last the MC error is smaller than the theoretical one. In the last bin both uncertainties are comparable in size. The NNPDF3.0 PDF set is employed and $\mu_R$ as well as $\mu_F$ are set to the common value $\mu_R = \mu_F = \mu_0 = H_T/3$. For the differential $A_\ell^c$ asymmetry the difference between the full off-shell result and the full NWA case is in the $5\% - 30\%$ range depending on the bin, yet within theoretical uncertainties, that are of the order of $30\%$. We notice that this is not the case for the last bin where the top quark off-shell effects affect $A_\ell^c$ substantially. Specifically, they are above $60\%$. Also theoretical uncertainties increase in that bin and are of the order of $50\%$. The NWA$_{\text{LOdec}}$ case, on the other hand, is outside the scale dependence bands almost in the whole plotted range. The difference to the full off-shell approach is larger, even up to $70\%$. A similar effect is also visible for the cumulative asymmetry where a rather
constant 30% difference is noted for the NWA_{LOdecay} case. Finally, we note that the last bin of the cumulative $A^\ell_c$ asymmetry gives $A^\ell_c = -7.02(8)$ for the complete off-shell case, $A^\ell_c = -7.30(4)$ for the full NWA and $A^\ell_c = -8.94(3)$ for the NWA_{LOdecay} case, where in parentheses the MC error is displayed. All three results are indeed in perfect agreement within the MC errors with the corresponding results for the unexpanded leptonic charge asymmetry for the combined $pp \rightarrow t\bar{t}W^\pm$ process.

Similar observations can be made for the other two differential and cumulative $A^\ell_c$ asymmetries. The $|y(\ell t\ell\bar{t})|-$ and $M(\ell t\ell\bar{t})$-dependent versions of $A^\ell_c$ are exhibited in Figure 8 and Figure 9 respectively. Since in each case the $y$ axis is chosen to be the same for the differential and cumulative version of $A^\ell_c$ we can distinctly observe that the cumulative asymmetry behaves substantially better. Taking the differential $M(\ell t\ell\bar{t})$-dependent version of $A^\ell_c$ as an example we observe large, of the order of 100%, theoretical uncertainties at the tails. On the other hand, the cumulative $M(\ell t\ell\bar{t})$-dependent $A^\ell_c$ asymmetry has stable theoretical uncertainties of the order of 30% in the whole plotted range.

We summarise by noting, that several processes beyond the SM can alter $A_c$, see e.g. [2, 68–71], either with anomalous vector or axial-vector couplings or via interference with SM processes. Different models also predict different asymmetries as a function of the invariant mass and the transverse momentum, see e.g. [72]. Of course due to the much smaller cross section the $pp \rightarrow t\bar{t}W^\pm$ process will not replace the use of the asymmetries in $t\bar{t}$ production, however, it can provide a complementary tool as it is uniquely sensitive to the chiral nature of possible new physics that might manifest itself in this channel. This motivates our interest in the top quark charge asymmetry as well as the asymmetries of its decay products and their sensitivity to the top quark production and decay modelling. Furthermore, using our NLO QCD results with the full off-shell effects included, we are able to provide more precise theoretical predictions for $A^t_c, A^\ell_c$ and $A^b_c$ in the $t\bar{t}W^\pm$ production process at the LHC with $\sqrt{s} = 13$ TeV. Finally, having at hand the full theory with no approximations included we are able to study the real size of theoretical uncertainties due to the scale dependence. In other words to verify whether they are under- or overestimated in the presence of various approximations.
7 Summary

In this paper we provided more accurate theoretical predictions for high precision observables, which should be used to constrain numerous new physics scenarios in the $t\bar{t}W^\pm$ channel. We considered the $t\bar{t}W^\pm$ production process in the multi-lepton decay channel for the LHC Run II energy of $\sqrt{s} = 13$ TeV for which discrepancies in the overall normalisation and in the modelling of the top quark decays have been recently reported by the ATLAS collaboration. Without the need of including terms beyond NLO in the perturbation expansion in $\alpha_s$ we obtained $1\% - 2\%$ theoretical uncertainties due to the scale dependence for this process by calculating the following cross section ratio $R = \sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$. Such precision has been achieved as the two process are indeed highly correlated. We proposed a rather simple method to examine the correlation between the two processes. Furthermore, fully realistic NLO QCD calculations have been employed in our studies for both $t\bar{t}W^+$ and $t\bar{t}W^-$. Specifically, we use $e^+\nu_e\mu^-\bar{\nu}_\mu e^+\nu_e\bar{b}b$ and $e^-\bar{\nu}_e\mu^+\nu_\mu e^-\bar{\nu}_e\bar{b}b$ matrix elements in our NLO QCD calculations. They include all resonant and non-resonant top quark and $W$ gauge boson Feynman diagrams, their interference effects as well as off-shell effects of $t$ and $W$. We examined the fixed and dynamical scale choice for $\mu_R$ and $\mu_F$ to assess their impact on the cross section ratio. We noticed that the scale choice does not play any role for such an inclusive observable. In the next step we examined the impact of the top quark production and decay modelling on the cross section ratio. We observed that the full NWA approach does not modify either the value or the size of the theoretical error for the integrated cross section ratio. Even for the simplified version of the NWA, i.e. for the NWA_{LOdecays} case, no changes have been observed. Thus, the $R = \sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$ observable is very stable and insensitive to the details of the modelling of the top quark decay chain. As such, it can be safely exploited at the LHC either for the precision SM measurements or in searches for BSM physics. Because the $R$ observable is sensitive to the $u/d$ valence content of the proton it can be used, for example, to provide valuable input for the up and down quark parton distribution functions. In the case of new physics searches the presence of two same-sign leptons in the final state offers a very interesting signature, that has been highly scrutinised in many new physics models. The latter range from supersymmetry and supergravity to the more specific scenarios with the Majorana neutrinos and the modified Higgs boson sector. Given the final accuracy of $R$ and its insensitivity to the top quark production and decay modelling, the $R$ observable should be used at the LHC, for example to achieve more stringent constrain limits on the parameter space of these models.

In the second part of the paper we reexamined the top quark charge asymmetry and the charge asymmetries of the top quark decay products for the $t\bar{t}W^\pm$ process in the fiducial phase-space regions. Also in this case the state-of-the-art NLO QCD theoretical predictions were utilised. We presented predictions for the expanded and unexpanded asymmetries. As large differences have been observed only the expanded results should be used in practice. Furthermore, we have compared the full off-shell case with the results in the NWA. For $A_{c,exp,y}^L$, $A_{c,exp,\eta}^L$ and $A_{c,exp,\eta}^f$, for which theoretical uncertainties of the order of $15\%$ have been estimated, we have observed differences below $1\sigma$. This finding suggests a rather inclusive nature of these observables. One has to keep in mind, however, that the final
uncertainties on \( A_{\text{t,exp,y}} \), \( A_{\text{t,exp,\eta}} \) and \( A_{\ell,\text{exp,y}} \) are rather moderate. For \( A_{\text{b,exp}} \) on the other hand, for which theoretical uncertainties are very small, i.e. of the order of 1%, a 4\( \sigma \) difference has been obtained. We can conclude that once theoretical uncertainties due to the scale dependence are sufficiently reduced, the sensitivity of the charge asymmetries to the top quark modelling might indeed increase. With the theoretical uncertainties at hand, however, the full NWA description is perfectly adequate. In the next step the NWA\( _{\text{L,decay}} \) case has been compared to the full off-shell one. In this case the discrepancies between central values of the asymmetries have increased even up to 2\( \sigma \). The later fact indicates that NLO QCD corrections to the top quark decays play a crucial role here. Overall, the inclusion of the complete description for the \( pp \to t\bar{t}W^\pm \) process in the multi-lepton final state has increased the central values of the asymmetries keeping at the same time the theoretical errors unchanged. We reported on our state-of-the-art results for the top quark charge asymmetry and for the charge asymmetries of the top quark decay products separately for \( pp \to t\bar{t}W^+ \) and \( pp \to t\bar{t}W^- \) as well as for the combined \( pp \to t\bar{t}W^\pm \) process. The scale choice has played no role as for \( \mu_0 = m_t + m_W/2 \) and \( \mu_0 = H_T/3 \) similar results have been obtained for \( A_{\text{t,exp}} \), \( A_{\ell,\text{exp}} \) and \( A_{\text{b,exp}} \).

Furthermore, we presented predictions for the differential and cumulative \( A_{\ell,\text{exp}} \) asymmetry with respect to \( p_T(\ell_1,\ell_2) \), \( \mid y(\ell_1,\ell_2) \mid \) and \( M(\ell_1,\ell_2) \). The advantage of choosing \( A_{\ell,\text{exp}} \) lies in the fact that the measurements of the charged leptons are particularly precise at the LHC due to the excellent lepton energy resolution of the ATLAS and CMS detectors. We note here that for these studies the unexpanded version of \( A_{\ell,\text{exp}} \) has been examined. Depending on the bin the differences between the full off-shell results and the full NWA ones have been in the 5% – 30% range. However, this is well within theoretical uncertainties, that are of the order of 30%. On the other hand, large differences have been noticed for the NWA\( _{\text{L,decay}} \) case even up to 70%. Similarly for the cumulative asymmetry the NWA\( _{\text{L,decay}} \) curves are lying outside the uncertainty bands independently of the observable and the considered bin. We would like to add here that even though differential and cumulative asymmetries contain the same information, the latter behaves better simply because it is more inclusive. In other words, the higher order corrections are distributed more uniformly over the whole kinematic range.

Last but not least, several BSM physics scenarios can alter the top quark charge asymmetry. Thus, theoretical predictions for the \( A_{\text{t,exp}} \), \( A_{\ell,\text{exp}} \) and \( A_{\text{b,exp}} \) quantities should be as accurate as possible. Using our NLO QCD results with the full off-shell effects included not only are we able to provide the state-of-the-art theoretical predictions for \( A_{\text{t,exp}} \), \( A_{\ell,\text{exp}} \) and \( A_{\text{b,exp}} \) in the \( t\bar{t}W^\pm \) production process but we could also carefully examine the real size of theoretical uncertainties due to the scale dependence.

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