Probe of New Physics using Precision Measurement of the Electron Magnetic Moment

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Abstract

The anomalous magnetic moment of the electron is determined experimentally with an accuracy of $2.8 \times 10^{-13}$ and the uncertainty may decrease by an order of magnitude in the future. While the current data is in excellent agreement with the standard model, the possible future improvement in the error in $\Delta a_e = a_e^{\text{exp}} - a_e^{\text{theory}}$ has recently drawn interest in the electron anomalous magnetic moment as a possible probe of new physics beyond the standard model. In this work we give an analysis of such physics in an extension of the minimal supersymmetric standard model with a vector multiplet. In the extended model the electroweak contribution to the anomalous magnetic moment of the electron include loop diagrams involving in addition to the exchange of W and Z, the exchange of charginos, sneutrinos and mirror sneutrinos, and the exchange of neutralinos, sleptons and mirror sleptons. The analysis shows that a contribution to the electron magnetic moment much larger than expected by $m_e^2/m_\mu^2$ scaling of the deviation of the muon anomalous magnetic moment over the standard model prediction, i.e., $\Delta a_\mu = 3 \times 10^{-9}$ as given by the Brookhaven experiment, can be gotten within the MSSM extension. Effects of CP violating phases in the extended MSSM model on the corrections to the supersymmetric electroweak contributions to $a_e$ are also investigated. The analysis points to the possibility of detection of new physics effects with modest improvement on the error in $\Delta a_e = a_e^{\text{exp}} - a_e^{\text{theory}}$.

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1 Introduction

The anomalous magnetic moment of the electron \(a_e = (g - 2)/2\) is one of most accurately determined quantities experimentally. Thus the most recent determination of it gives the value \[1\]

\[a_e^{\text{exp}} = 115\,965\,218\,073\,(2.8) \times 10^{-13}.\] (1)

In the standard model the contribution to the magnetic moment of the electron arises from several sources so that (for a review see \[2\])

\[a_e^{\text{SM}} = a_e^{\text{QED}} + a_e^{\text{EW}} + a_e^{\text{had}}\] (2)

where \(a_e^{\text{QED}}\) involves purely QED loop corrections \([3, 4, 5]\), \(a_e^{\text{EW}}\) contains the electroweak corrections involving the \(W\) and \(Z\) loops, and \(a_e^{\text{had}}\) contains the hadronic corrections \([7, 8]\). Recently, it has been emphasized \([9]\) that in comparison of the theory prediction and experiment one must use in the theory prediction the value of \(\alpha\) obtained in independent experiment rather than by equating \(a_e^{\text{SM}}(\alpha) = a_e^{\text{exp}}\). Thus the analysis of \([9]\) uses a determination of \(\alpha\) given by \(^{133}\text{Cs}\) data \([10]\) and by \(^{87}\text{Rb}\) data \([11, 12, 13, 14]\) (for a review see \([15]\))

\[\alpha^{(133}\text{Cs}) = 1/137.036\,000\,0 (11)\] (3)

\[\alpha^{(87}\text{Rb}) = 1/137.035\,999\,049 (90)\] (4)

Using the above the analysis of \([9]\) estimates

\[a_e^{\text{SM}} = 115\,965\,218\,178\,(0.6)(0.4)(0.2)(7.6) \times 10^{-13}.\] (5)

where the numbers in the parentheses are as follows: (0.6) refers to the uncertainty in the four loop QED coefficient, (0.4) refers to the uncertainty in the five loop QED co-efficient, (0.2) is the error in the hadronic contribution, and (7.6) arises from the error in the determination of \(\alpha\) in \(^{87}\text{Rb}\). Combining the errors in quadratures one finds \([9]\) that the uncertainty \(\delta \Delta a_e\), where \(\Delta a_e = (a_e^{\text{exp}} - a_e^{\text{SM}})\), is given by

\[\delta \Delta a_e = 8.1 \times 10^{-13}\] (6)

Very recently contributions of the positronium to \(a_e\) have been computed \([16, 17]\). The contribution is computed to be \(a_e^P = 1.32(\alpha/\pi)^5\) as is of the same order of \(\alpha\) as the five-loop perturbative QED contribution. Numerically the contribution is \(a_e^P = 0.89 \times 10^{-13}\). Thus inclusion of this result
would not have a significant effect on the constraint of Eq.\ref{eq:6}. We look now at the implications of Eq.\ref{eq:6} in view of the current status of the anomalous magnetic moment of the muon. Thus the Brookhaven experiment indicates a $\sim 3.5\sigma$ deviation from the standard model prediction, i.e., one has for $\Delta a_\mu$ the result \cite{18,19}

$$\Delta a_\mu = (287 \pm 80) \times 10^{-11} \quad (7)$$

Scaling the result of Eq.\ref{eq:7} to the case of the electron by using the naive scaling factor of $m_e^2/m_\mu^2$ one gets a correction of size $(0.6 \pm 0.2) \times 10^{-13}$ which is an order of magnitude smaller than the result of Eq.\ref{eq:6}. The above discussion indicates that if there are new physics effects larger than those given by naive scaling, they would be susceptible to discovery with modest improvements in the error $\delta \Delta a_e$.

In this work we carry out a detailed analysis of corrections to the anomalous magnetic moment of the electron in extensions of MSSM with a vector multiplet (For a non-supersymmetric analysis see also \cite{20,9}). The analysis will include contributions from the W and Z boson loops, as well as corrections from charginos, sneutrinos and mirror sneutrinos, from neutralinos and sleptons and mirror sleptons. It will be shown that the new physics corrections here can be far in excess of those implied by scaling and are of a size that could be detectable in modest improvement in $\delta \Delta a_e$. We also investigate the dependence of the anomalous magnetic moment of the electron on CP phases arising from the supersymmetric contributions from the exchange of the vectorlike multiplet. In previous analyses within MSSM the supersymmetric correction to the anomalous magnetic moment of the muon was found to be sensitive to CP phases in a significant way \cite{21} and we could have similar large CP dependent effects for $\Delta a_e(\text{EW})$ in the analysis based on the MSSM extension.

The outline of the rest of the paper is as follows: In Section 2 we discuss the MSSM extension with a vectorlike multiplet. Here we define the notation labeling the extra vectorlike particles, give their transformation properties under the SM gauge group and give the superpotential for the extended model. The D terms and the soft terms allowed in the model are discussed. In Section 3 the interactions of leptons-sneutrinos (mirror sneutrinos)-charginos in the mass diagonal basis are given. These interactions are used in the computation of the left diagram of Fig. 1. In Section 4 the interactions of leptons-sleptons (mirror sleptons)-neutralinos in the mass diagonal basis are given. These interactions are used in the computation of the right diagram of Fig. 1. In Section 5 the interactions of the W and Z bosons that are needed in the computation of the loop diagram of Fig. 2.
are discussed. In Section 6 an analytic analysis is given of the neutralino exchange contributions using the interactions of Section 3 and chargino exchange contribution using the interaction of Section 4. Here an analytic analysis is also given of the exchange contributions of the W and Z bosons using the interactions of Fig. 2. A detailed numerical analysis is given in Section 7 for the electroweak contribution contribution to the electron anomalous magnetic moment in the model. Here the dependence of electroweak contribution to the anomalous magnetic moment of the electron on supersymmetric CP phases is also investigated. It is shown that modest improvements in the current errors in $\Delta a_e$ can begin to probe the possible new physics contributions. Further, a relative comparison of the electroweak contributions to the anomalous magnetic moments of $e, \mu, \tau$ is also given. Conclusions are given in Section 8. Further details on the mass matrices for the sleptons and mirror sleptons are given in Section 9.

2 MSSM Extension with a vector leptonic multiplet

Vector like multiplets arise in a variety of unified models [22] some of which could be low lying. They have been used recently in a variety of analyses [23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33]. In the analysis below we will assume an extended MSSM with just one leptonic vector multiplet. The addition of a vector multiplet keeps the model anomaly free. Before proceeding further we define the notation and give a very brief description of the extended model and a more detailed description can be found in the previous works mentioned above. Thus the extended MSSM has contains a vectorlike multiplet with the transformations under $SU(3)_C \times SU(2)_L \times U(1)_Y$ as given below

$$\psi_{iL} \equiv \begin{pmatrix} \nu_{iL} \\ l_{iL} \\ \nu_{\bar{i}L} \end{pmatrix}$$

where the last entry on the right hand side column is the value of the hypercharge $Y$ defined so that $Q = T_3 + Y$. These leptons have $V - A$ interactions. We can now add a vectorlike multiplet where we have a fourth family of leptons with $V - A$ interactions whose transformations can be gotten from Eq.(8) by letting $i$ run from 1-4. A vectorlike lepton multiplet also has mirrors and so we consider these mirror leptons which have $V + A$ interactions. Its quantum numbers are given by

$$\begin{pmatrix} \nu_{\bar{i}L} \\ l_{\bar{i}L} \end{pmatrix}$$

(1, 1, 0).
where the basis vectors in which the mass matrix is written is given by

$$\chi^c \equiv \begin{pmatrix} E_L^c \\ N_L^c \end{pmatrix}$$

$$\begin{pmatrix} (1, 2, \frac{1}{2}) \\ (1, 1, -1) \end{pmatrix}$$

$$\begin{pmatrix} (1, 1, 0) \end{pmatrix} \quad (9)$$

Interesting new physics arises when we allow mixings of the vectorlike generation with the three ordinary generations. Thus the superpotential of the model allowing for the mixings among the three ordinary generations and the vectorlike generation is given by

$$W = -\mu_{\psi \chi} \hat{H}_1 \hat{H}_2 \psi^\dagger \chi + f_1 \hat{H}_1 \hat{H}_2 \psi^\dagger \nu_\tau \mu + f_2 \hat{H}_1 \chi^\dagger \tilde{N} \tilde{L} + f_2' \hat{H}_1 \chi^\dagger \tilde{L} \tilde{E}$$

$$+ h_1 H_1^1 \hat{H}_2 \hat{H}_1 \psi^\dagger \mu + h_1' H_1^1 \hat{H}_2 \hat{H}_1 \psi^\dagger \nu_\mu + h_2 H_1^2 \hat{H}_2 \hat{H}_1 \psi^\dagger \nu_\tau + h_2' H_1^2 \hat{H}_2 \hat{H}_1 \psi^\dagger \nu_{\tau \mu}$$

$$+ f_3 \hat{H}_1 \psi^\dagger \tilde{L} \tilde{N} + f_3' \hat{H}_1 \psi^\dagger \tilde{E} \tilde{L} + f_5 \tilde{N} \tilde{L} + f_5' \tilde{E} \tilde{L} + f_5'' \tilde{L} \tilde{L}, \quad (10)$$

where $\hat{H}_1$ implies superfields, $\hat{L}_L$ stands for $\hat{L}_3L$, $\hat{V}_\mu$ stands for $\hat{V}_2L$ and $\hat{V}_\tau L$ stands for $\hat{V}_1L$. The mass terms for the neutrinos, mirror neutrinos, leptons and mirror leptons arise from the term

$$\mathcal{L} = -\frac{1}{2} \partial^2 W \psi \psi^\dagger + \text{H.c.} \quad (11)$$

where $\psi$ and $A$ stand for generic two-component fermion and scalar fields. After spontaneous breaking of the electroweak symmetry, $\langle H_1^1 \rangle = v_1/\sqrt{2}$ and $\langle H_2^2 \rangle = v_2/\sqrt{2}$, we have the following set of mass terms written in the 4-component spinor notation so that

$$-\mathcal{L}_m = \xi_R^T (M_f) \xi_L + \eta_R^T (M_f) \eta_L + \text{H.c.}, \quad (12)$$

where the basis vectors in which the mass matrix is written is given by

$$\xi_R^T = (\bar{\nu}_{\tau R} \tilde{N}_R \bar{\nu}_{\mu R} \bar{\nu}_{e R})$$

$$\xi_L^T = (\nu_{\tau L} N_L \nu_{\mu L} \nu_{e L})$$

$$\eta_R^T = (\tau_R \tilde{E}_R \bar{\mu}_R \bar{e}_R)$$

$$\eta_L^T = (\tau_L E_L \mu_L e_L) \quad (13)$$

and the mass matrix $M_f$ is given by

$$M_f = \begin{pmatrix} f_1 v_2/\sqrt{2} & f_5 & 0 & 0 \\ -f_3 & f_2 v_1/\sqrt{2} & -f_4 & -f_3' \\ 0 & f_5' & h_1' v_2/\sqrt{2} & 0 \\ 0 & 0 & 0 & h_2' v_2/\sqrt{2} \end{pmatrix}. \quad (14)$$
The mass matrix is not hermitian and thus one needs bi-unitary transformations to diagonalize it. We define the bi-unitary transformation so that

\[ D_R^{\nu} (M_f) D_L^{\nu} = \text{diag}(m_{\psi_1}, m_{\psi_2}, m_{\psi_3}, m_{\psi_4}). \]  

(15)

Under the bi-unitary transformations the basis vectors transform so that

\[
\begin{pmatrix}
\nu_{\tau R} \\
N_R \\
\nu_{\mu R} \\
\psi_4 R
\end{pmatrix} = D_R^{\nu} \begin{pmatrix}
\psi_{1 R} \\
\psi_{2 R} \\
\psi_{3 R} \\
\psi_{4 R}
\end{pmatrix}, \quad
\begin{pmatrix}
\nu_{\tau L} \\
N_L \\
\nu_{\mu L} \\
\nu_{e L}
\end{pmatrix} = D_L^{\nu} \begin{pmatrix}
\psi_{1 L} \\
\psi_{2 L} \\
\psi_{3 L} \\
\psi_{4 L}
\end{pmatrix}.
\]  

(16)

In Eq. (15) \( \psi_1, \psi_2, \psi_3, \psi_4 \) are the mass eigenstates for the neutrinos, where in the limit of no mixing we identify \( \psi_1 \) as the light tau neutrino, \( \psi_2 \) as the heavier mass eigen state, \( \psi_3 \) as the muon neutrino and \( \psi_4 \) as the electron neutrino. A similar analysis goes to the lepton mass matrix \( M_\ell \) where

\[
M_\ell = \begin{pmatrix}
f_1 v_1 / \sqrt{2} & f_4 & 0 & 0 \\
1 & f_2 v_2 / \sqrt{2} & f_3 & f_3'' \\
0 & f_4' & h_1 v_1 / \sqrt{2} & 0 \\
0 & f_4'' & 0 & h_2 v_1 / \sqrt{2}
\end{pmatrix}.
\]  

(17)

Next we consider the mixing of the charged sleptons and the charged mirror sleptons. The mass squared matrix of the slepton - mirror slepton comes from three sources: the F term, the D term of the potential and the soft susy breaking terms. Using the superpotential of Eq. (10) the mass terms arising from it after the breaking of the electroweak symmetry are given by the Lagrangian

\[
\mathcal{L} = \mathcal{L}_F + \mathcal{L}_D + \mathcal{L}_{\text{soft}},
\]  

(18)

where \( \mathcal{L}_F \) is deduced from Eq. (10) and is given in the Appendix, while the \( \mathcal{L}_D \) is given by

\[
\begin{align*}
-\mathcal{L}_D &= \frac{1}{2} m_2^2 \cos^2 \theta_W \cos 2\beta \{ \bar{\nu}_{\tau R} \tilde{\nu}_{\tau L}^* - \tilde{\tau}_L \tilde{\tau}_L^* + \bar{\nu}_{\mu L} \tilde{\nu}_{\mu L}^* - \tilde{\mu}_L \tilde{\mu}_L^* + \bar{\nu}_{e L} \tilde{\nu}_{e L}^* - \tilde{e}_L \tilde{e}_L^* \\
&\quad + \bar{E}_R \tilde{E}_R^* - \tilde{N}_R \tilde{N}_R^* \} + \frac{1}{2} m_Z^2 \sin^2 \theta_W \cos 2\beta \{ \bar{\nu}_{\tau R} \tilde{\nu}_{\tau L}^* + \tilde{\tau}_L \tilde{\tau}_L^* + \bar{\nu}_{\mu L} \tilde{\nu}_{\mu L}^* + \tilde{\mu}_L \tilde{\mu}_L^* + \bar{\nu}_{e L} \tilde{\nu}_{e L}^* + \tilde{e}_L \tilde{e}_L^* - \bar{E}_R \tilde{E}_R^* - \tilde{N}_R \tilde{N}_R^* + 2 \bar{E}_L \tilde{E}_L^* - 2 \tilde{\tau}_R \tilde{\tau}_R^* - 2 \tilde{\mu}_R \tilde{\mu}_R^* - 2 \tilde{e}_R \tilde{e}_R^* \}.
\end{align*}
\]  

(19)

For \( \mathcal{L}_{\text{soft}} \) we assume the following form

\[
\begin{align*}
-\mathcal{L}_{\text{soft}} &= \tilde{M}_L^2 \psi_{L R} \tilde{\psi}_{L L}^* + \tilde{M}_R^2 \tilde{\psi}_{L L} \tilde{\psi}_{R R} + \tilde{M}_{\mu L}^2 \tilde{\psi}_{L L} \psi_{R R} + \tilde{M}_{\mu L}^2 \tilde{\psi}_{L R} \tilde{\psi}_{R L} + \tilde{M}_{\mu L}^2 \tilde{\psi}_{L L} \tilde{\psi}_{R L}^* + \tilde{M}_{\mu L}^2 \tilde{\psi}_{L R} \tilde{\psi}_{R L}^* + \tilde{M}_{\mu L}^2 \tilde{\psi}_{L L} \tilde{\psi}_{R L} + \tilde{M}_{\mu L}^2 \tilde{\psi}_{L R} \tilde{\psi}_{R L}^* \\
&\quad + \tilde{M}_L^2 \tilde{\psi}_{L L} \tilde{\psi}_{L L} + \tilde{M}_R^2 \tilde{\psi}_{L R} \tilde{\psi}_{R R} + \tilde{M}_L^2 \tilde{\psi}_{L R} \tilde{\psi}_{L L} + \tilde{M}_R^2 \tilde{\psi}_{L L} \tilde{\psi}_{R R} + \tilde{M}_L^2 \tilde{\psi}_{L L} \tilde{\psi}_{R R} + \tilde{M}_R^2 \tilde{\psi}_{L R} \tilde{\psi}_{R L} \\
&\quad + \epsilon_{ij} \{ f_1 A_{\tau R} H_1^i \tilde{\psi}_{L R} \tilde{\psi}_{R L} - f_2 A_{\tau R} H_2^i \tilde{\psi}_{L R} \tilde{\psi}_{R L} + h_1 A_{\mu L} H_1^i \tilde{\psi}_{L L} \tilde{\psi}_{R R} - h_1 A_{\mu L} H_1^i \tilde{\psi}_{L R} \tilde{\psi}_{R L} \\
&\quad + h_2 A_{\mu L} H_2^i \tilde{\psi}_{L L} \tilde{\psi}_{R R} - h_2 A_{\mu L} H_2^i \tilde{\psi}_{L R} \tilde{\psi}_{R L} + f_2 A_{N L} H_1^i \tilde{\psi}_{L L} \tilde{\psi}_{R R} - f_2 A_{N L} H_2^i \tilde{\psi}_{L R} \tilde{\psi}_{R L} \\
&\quad + f_2 A_{N L} H_2^i \tilde{\psi}_{L L} \tilde{\psi}_{R R} - f_2 A_{N L} H_2^i \tilde{\psi}_{L R} \tilde{\psi}_{R L} + f_2 A_{N L} H_2^i \tilde{\psi}_{L L} \tilde{\psi}_{R R} - f_2 A_{N L} H_2^i \tilde{\psi}_{L R} \tilde{\psi}_{R L} + f_2 A_{N L} H_2^i \tilde{\psi}_{L R} \tilde{\psi}_{R R} - f_2 A_{N L} H_2^i \tilde{\psi}_{L R} \tilde{\psi}_{R L} + f_2 A_{N L} H_2^i \tilde{\psi}_{L R} \tilde{\psi}_{R L} - f_2 A_{N L} H_2^i \tilde{\psi}_{L R} \tilde{\psi}_{R L} + f_2 A_{N L} H_2^i \tilde{\psi}_{L R} \tilde{\psi}_{R L} \}.
\end{align*}
\]  

(20)
3 Interactions of leptons, scalar neutrinos and charginos

In this section we discuss the interactions in the mass diagonal basis involving charged leptons, sneutrinos and charginos. Thus we have

$$-\mathcal{L}_{\tau-\tilde{\nu}-\chi} = \sum_{i=1}^{2} \sum_{j=1}^{8} \bar{\tau}_i (C_{\alpha ij}^{L} P_L + C_{\alpha ij}^{R} P_R) \tilde{\chi}_i \tilde{\nu}_j + \text{H.c.,}$$

such that

$$C_{\alpha ij}^{L} = g(-\kappa_{\tau} U_{i2}^{*} D_{R1a}^{\tau*} \tilde{D}_{i1j}^{\nu} - \kappa_{\mu} U_{i2}^{*} D_{R3a}^{\tau*} \tilde{D}_{i5j}^{\nu} - \kappa_{e} U_{i2}^{*} D_{R4a}^{\tau*} \tilde{D}_{i7j}^{\nu} + U_{i1}^{*} D_{R2a}^{\tau*} \tilde{D}_{i4j}^{\nu} - \kappa_{N} U_{i2}^{*} D_{R2a}^{\tau*} \tilde{D}_{i2j}^{\nu})$$

(22)

$$C_{\alpha ij}^{R} = g(-\kappa_{\nu} V_{i2}^{*} D_{L1a}^{\tau*} \tilde{D}_{i3j}^{\nu} - \kappa_{\mu} V_{i2}^{*} D_{L3a}^{\tau*} \tilde{D}_{i6j}^{\nu} - \kappa_{e} V_{i2}^{*} D_{L4a}^{\tau*} \tilde{D}_{i8j}^{\nu} + V_{i1}^{*} D_{L2a}^{\tau*} \tilde{D}_{i4j}^{\nu}$$

$$+ V_{i1}^{*} D_{L4a}^{\tau*} \tilde{D}_{i7j}^{\nu} - \kappa_{E} V_{i2}^{*} D_{L2a}^{\tau*} \tilde{D}_{i4j}^{\nu}),$$

with

$$(\kappa_{N}, \kappa_{\tau}, \kappa_{\mu}, \kappa_{e}) = \frac{(m_{N}, m_{\tau}, m_{\mu}, m_{e})}{\sqrt{2} m_W \cos \beta},$$

(24)

$$(\kappa_{E}, \kappa_{\nu}, \kappa_{\mu}, \kappa_{e}) = \frac{(m_{E}, m_{\nu}, m_{\mu}, m_{e})}{\sqrt{2} m_W \sin \beta}.$$
where

\[ \alpha_{Ei} = \frac{g m_x X_{4i}^*}{2m_W \sin \beta} ; \]

\[ \beta_{Ei} = e X_{1i}^* + \frac{g}{\cos \theta_W} X_{2i}^* \left( \frac{1}{2} - \sin^2 \theta_W \right) \]  \hspace{1cm} (29)

\[ \gamma_{Ei} = e X_{1i}^* - \frac{g \sin^2 \theta_W}{\cos \theta_W} X_{2i}^* ; \]

\[ \delta_{Ei} = -\frac{g m_x X_{4i}}{2m_W \sin \beta} \]  \hspace{1cm} (30)

and

\[ \alpha_{\tau i} = \frac{g m_\tau X_{3i}}{2m_W \cos \beta} ; \]

\[ \alpha_{\mu i} = \frac{g m_\mu X_{3i}}{2m_W \cos \beta} ; \]

\[ \alpha_{e i} = \frac{g m_e X_{3i}}{2m_W \cos \beta} \]  \hspace{1cm} (31)

\[ \delta_{\tau i} = -\frac{g m_\tau X_{3i}^*}{2m_W \cos \beta} ; \]

\[ \delta_{\mu i} = -\frac{g m_\mu X_{3i}^*}{2m_W \cos \beta} ; \]

\[ \delta_{e i} = -\frac{g m_e X_{3i}^*}{2m_W \cos \beta} \]  \hspace{1cm} (32)

and where

\[ \beta_{\tau i} = \beta_{\mu i} = \beta_{e i} = -e X_{1i}^* + \frac{g}{\cos \theta_W} X_{2i}^* \left( -\frac{1}{2} + \sin^2 \theta_W \right) \]  \hspace{1cm} (33)

\[ \gamma_{\tau i} = \gamma_{\mu i} = \gamma_{e i} = -e X_{1i}^* + \frac{g \sin^2 \theta_W}{\cos \theta_W} X_{2i}^* \]  \hspace{1cm} (34)

Here \( X' \) are defined by

\[ X_{1i}' = X_{1i} \cos \theta_W + X_{2i} \sin \theta_W \]  \hspace{1cm} (35)

\[ X_{2i}' = -X_{1i} \sin \theta_W + X_{2i} \cos \theta_W \]  \hspace{1cm} (36)

where \( X \) diagonalizes the neutralino mass matrix, i.e.,

\[ X^T M_{\chi^0} X = \text{diag}(m_{\chi^0_1}, m_{\chi^0_2}, m_{\chi^0_3}, m_{\chi^0_4}). \]  \hspace{1cm} (37)

5 Interaction of leptons and mirrors with W and Z bosons

In addition to the computation of the supersymmetric loop diagrams, we compute the contributions arising from the exchange of the W and Z bosons and the leptons and the mirror leptons in the loops. The relevant interactions needed are given below. For the W boson exchange the interactions that enter are given by
\[ -\mathcal{L}_{\tau W \psi} = W_\rho^\dagger \sum_{i=1}^{4} \sum_{\alpha=1}^{4} \bar{\psi}_i \gamma^\rho [C_{L_{i\alpha}}^W P_L + C_{R_{i\alpha}}^W P_R] \tau_\alpha + \text{H.c.} \tag{38} \]

where

\[ C_{L_{i\alpha}}^W = \frac{g}{\sqrt{2}} [D_{L_{1i}}^{\nu*} D_{L_{1\alpha}}^\nu + D_{L_{3i}}^{\nu*} D_{L_{3\alpha}}^\nu + D_{L_{4i}}^{\nu*} D_{L_{4\alpha}}^\nu] \tag{39} \]

\[ C_{R_{i\alpha}}^W = \frac{g}{\sqrt{2}} [D_{R_{2i}}^{\nu*} D_{R_{2\alpha}}^\nu] \tag{40} \]

For the Z boson exchange the interactions that enter are given by

\[ -\mathcal{L}_{\tau Z} = Z_\rho \sum_{\alpha=1}^{4} \sum_{\beta=1}^{4} \bar{\tau}_\alpha \gamma^\rho [C_{L_{\alpha\beta}}^Z P_L + C_{R_{\alpha\beta}}^Z P_R] \tau_\beta \tag{41} \]

where

\[ C_{L_{\alpha\beta}}^Z = \frac{g}{\cos \theta_W} \{ x(D_{L_{1\alpha}} D_{L_{1\beta}}^\dagger + D_{L_{2\alpha}} D_{L_{2\beta}}^\dagger + D_{L_{3\alpha}} D_{L_{3\beta}}^\dagger + D_{L_{4\alpha}} D_{L_{4\beta}}^\dagger) \\
- \frac{1}{2} (D_{L_{1\alpha}} D_{L_{1\beta}}^\dagger + D_{L_{2\alpha}} D_{L_{2\beta}}^\dagger + D_{L_{3\alpha}} D_{L_{3\beta}}^\dagger + D_{L_{4\alpha}} D_{L_{4\beta}}^\dagger) \} \tag{42} \]

and

\[ C_{R_{\alpha\beta}}^Z = \frac{g}{\cos \theta_W} \{ x(D_{R_{1\alpha}} D_{R_{1\beta}}^\dagger + D_{R_{2\alpha}} D_{R_{2\beta}}^\dagger + D_{R_{3\alpha}} D_{R_{3\beta}}^\dagger + D_{R_{4\alpha}} D_{R_{4\beta}}^\dagger) \\
- \frac{1}{2} (D_{R_{2\alpha}} D_{R_{2\beta}}^\dagger) \} \tag{43} \]

where \( x = \sin^2 \theta_W \).

6 An analytical computation of the anomalous magnetic moment

Using the interactions given in Section 3 the chargino contribution arises from the left diagram of Fig. 1. It is given by

\[ a_\alpha^{+} = \sum_{i=1}^{8} \frac{m_{\tau_\alpha}}{16 \pi^2 m_{\chi_i}^2} \text{Re}(C_{\alpha ij}^L C_{\alpha ij}^*) F_3 \left( \frac{m_{\chi_i}^2}{m_{\tau_\alpha}^2} \right) \]

\[ + \sum_{i=1}^{8} \frac{m_{\tau_\alpha}^2}{96 \pi^2 m_{\chi_i}^2} \left[ |C_{\alpha ij}^L|^2 + |C_{\alpha ij}^R|^2 \right] F_4 \left( \frac{m_{\chi_i}^2}{m_{\tau_\alpha}^2} \right), \tag{44} \]
Figure 1: The diagrams that contribute to the leptonic ($\tau_\alpha$) magnetic dipole moment via exchange of charginos ($\chi^-_i$), sneutrinos and mirror sneutrinos ($\tilde{\nu}_j$) (left diagram) inside the loop and from the exchange of neutralinos ($\chi^0_i$) sleptons and mirror sleptons ($\tilde{\tau}_j$) (right diagram) inside the loop.

where the form factors $F_3$ and $F_4$ are given by

$$F_3(x) = \frac{1}{(x-1)^3} \left[ 3x^2 - 4x + 1 - 2x^2 \ln x \right]$$

and

$$F_4(x) = \frac{1}{(x-1)^4} \left[ 2x^3 + 3x^2 - 6x + 1 - 6x^2 \ln x \right]$$

Using the interactions given in Section 4 the neutralino contribution arises from the right diagram of Fig. 1. It is given by

$$a_{\chi^0_\alpha} = \sum_{i=1}^{4} \sum_{j=1}^{8} \frac{m_{\tau_\alpha}}{16\pi^2m_{\chi^0_i}} \text{Re}(C'_{\alpha ij}C'^*_{\alpha ij}) \left( \frac{m^2_{\tilde{\tau}_j}}{m^2_{\chi^0_i}} \right) F_1 \left( \frac{m^2_{\tilde{\tau}_j}}{m^2_{\chi^0_i}} \right)$$

$$+ \sum_{i=1}^{2} \sum_{j=1}^{8} \frac{m^2_{\tau_\alpha}}{96\pi^2m^2_{\chi^0_i}} \left[ |C'_{\alpha ij}|^2 + |C'^*_{\alpha ij}|^2 \right] F_2 \left( \frac{m^2_{\tilde{\tau}_j}}{m^2_{\chi^0_i}} \right),$$

where the form factors are

$$F_1(x) = \frac{1}{(x-1)^3} \left[ 1 - x^2 + 2x \ln x \right]$$

and

$$F_2(x) = \frac{1}{(x-1)^4} \left[ -x^3 + 6x^2 - 2x - 6x \ln x \right]$$

The anomalous magnetic moments are known to exhibit a sharp dependence on the CP phases \[21, 34\]. The dependence of $a_e$ on CP phases will be exhibited in the numerical analysis to follow.
The contributions to the lepton magnetic moment from the W and Z exchange arise from the diagrams of Fig. 2. Using the interactions given in Section 5, the contribution arising from the W exchange diagram (the left diagram of Fig. 2) is given by

\[ a^W_{\tau\alpha} = \frac{m_{\tau\alpha}^2}{16\pi^2 m_W^2} \sum_{i=1}^{4} \left[ |C_{W_Li\alpha}|^2 + |C_{W_Ri\alpha}|^2 \right] F_W \left( \frac{m_{\psi_i}^2}{m_W^2} \right) + \frac{m_{\psi_i}}{m_{\tau\alpha}} \text{Re}(C_{W_Li\alpha}^* C_{W_Ri\alpha}) G_W \left( \frac{m_{\psi_i}^2}{m_W^2} \right), \]  

(50)

where the form factors are given by

\[ F_W(x) = \frac{1}{6(x - 1)^4} \left[ 4x^4 - 49x^3 + 18x^2 \ln x + 78x^2 - 43x + 10 \right] \]  

(51)

and

\[ G_W(x) = \frac{1}{(x - 1)^3} \left[ 4 - 15x + 12x^2 - x^3 - 6x^2 \ln x \right] \]  

(52)

Using the interactions given in Section 5, the contribution arising from the Z exchange diagram (the right diagram of Fig. 2) is given by

\[ a^Z_{\tau\alpha} = \frac{m_{\tau\alpha}^2}{32\pi^2 m_Z^2} \sum_{\beta=1}^{4} \left[ |C_{Z_{L\beta\alpha}}|^2 + |C_{Z_{R\beta\alpha}}|^2 \right] F_Z \left( \frac{m_{\tau\beta}^2}{m_Z^2} \right) + \frac{m_{\tau\beta}}{m_{\tau\alpha}} \text{Re}(C_{Z_{L\beta\alpha}}^* C_{Z_{R\beta\alpha}}) G_Z \left( \frac{m_{\tau\beta}^2}{m_Z^2} \right), \]  

(53)

where

\[ F_Z(x) = \frac{1}{3(x - 1)^4} \left[ -5x^4 + 14x^3 - 39x^2 + 18x^2 \ln x + 38x - 8 \right] \]  

(54)

and

\[ G_Z(x) = \frac{2}{(x - 1)^3} \left[ x^3 + 3x - 6x \ln x - 4 \right]. \]  

(55)
We now show that the standard model result \[35\] can be gotten in the limit when the off diagonal elements in the neutrino and lepton mass matrices are set to zero. The W boson contribution is obtained in this case where the couplings are \( C_{Wi\alpha} = \frac{g}{\sqrt{2}} \) for \( i = \alpha \) and zero otherwise and \( C_{Ri\alpha} = 0 \).

In this limit, the form factor \( F_W(0) = \frac{5}{3} \) and one gets
\[
a_W^{\tau\alpha} = \frac{5g^2m_{\tau\alpha}}{96\pi^2m_W^2} \tag{56}
\]

Using the relation that \( G_F = \frac{\pi\alpha_{em}}{\sqrt{2}m_W^2\sin^2\theta_W} \), one gets the well known W boson contribution to the lepton \( \tau\alpha \)
\[
a_W^{\tau\alpha} = \frac{5G_F m_{\tau\alpha}}{12\sqrt{2}\pi^2} \tag{57}
\]
where \( \alpha = 3 \) for the case of muon and \( \alpha = 4 \) for the case of electron.

To recover the Z boson contribution in the standard model limit we set
\[
C_{Li\beta\alpha} = \frac{g}{\cos\theta_W} \left( x - \frac{1}{2} \right),
\]
\[
C_{Ri\beta\alpha} = \frac{g}{\cos\theta_W} x
\]
(58)

for the case of \( \alpha = \beta \) and are set to zero otherwise. The form factors in this limit are given by \( F_Z(0) = -\frac{8}{3} \) and \( G_Z(0) = 8 \). In this case one finds for the Z contribution, the well known Standard Model result
\[
a_Z^{\tau\alpha} = \frac{G_F m_{\tau\alpha}}{2\sqrt{2}\pi^2} \left[ -\frac{5}{12} + \frac{4}{3} \left( \sin^2\theta_W - \frac{1}{4} \right)^2 \right]. \tag{59}
\]

7 Numerical analysis and results

In this section we present a detailed numerical analysis of the effect of the extra vectorlike generation on the magnetic moment of the electron. We will also study the effects of CP phases on the electron magnetic moment. The analysis is done under the Brookhaven constraint on the anomalous magnetic moment of the muon, i.e., the constraint of Eq.(7). In Table 1, we give a comparative analysis for the values of the electron anomalous magnetic moment for the case where no mixing occurs between generations and the case where such mixing takes place. For case (i) in Table 1 the couplings \( f_3, f'_3, f''_3, f_4, f'_4, f''_4, f_5, f'_5 \) and \( f''_5 \) are all set to zero and this represents the case of no mixing between the generations. The upper two rows exhibit the chargino and neutralino contributions to \( a_e \) while the next two rows give the standard model contribution arising from \( W \) and \( Z \).
Case of no mixing | Case of mixing
---|---
Chargino contribution | $4.31 \times 10^{-14}$ | $5.86 \times 10^{-13}$
Neutralino contribution | $9.65 \times 10^{-16}$ | $6.93 \times 10^{-16}$
W boson contribution | $9.09 \times 10^{-14}$ | $1.02 \times 10^{-13}$
Z boson contribution | $-4.59 \times 10^{-14}$ | $-3.89 \times 10^{-14}$
$\Delta a_e$(EW) total | $8.91 \times 10^{-14}$ | $6.50 \times 10^{-13}$

Table 1: An exhibition the relative contributions to the electron magnetic dipole moment arising from chargino exchange, neutralino exchange, W boson exchange and Z boson exchange and their sum for the case when (i) there is no mixing among generations and for the case when (ii) mixing occurs. The common parameters for the two cases are $m_E = 250$, $m_0 = m_{\tilde{\nu}} = 650$, $|A_0| = 520$, $|A_{\tilde{\nu}}| = 650$, $|\mu| = 101$, $|m_1| = 600$, $|m_2| = 200$, $\theta_{A_0} = 1.2$, $\theta_{A_{\tilde{\nu}}} = 2.8$, $\theta_1 = 2.5$, $\theta_2 = 1.5$, $\theta_\mu = 0.5$, $m_N = 212$ and $\tan \beta = 15$. For case (i), the couplings $f_3 = f_{\tilde{\nu}}''$, $f_4 = f_4'' = f_5 = f''_5 = f''_5 = f_5 = 0$. For case (ii), the $f$ couplings are non-zero and have the values $|f_3| = 7 \times 10^{-8}$, $|f_{\tilde{\nu}}'| = 5 \times 10^{-8}$, $|f_{\tilde{\nu}}''| = 8 \times 10^{-9}$, $|f_4| = 10$, $|f_4''| = 120$, $|f_5| = 8.11 \times 10^{-2}$, $|f_5| = 9.8 \times 10^{-2}$, $|f_{\tilde{\nu}}''| = 4 \times 10^{-2}$ and their phases are $\theta_{f_3} = 0.3$, $\theta_{f_{\tilde{\nu}}'} = 0.2$, $\theta_{f_4'} = 0.6$, $\theta_{f_4''} = 1.4$, $\theta_{f_5'} = 1.1$, $\theta_{f_5''} = 0.5$, $\theta_{f_{\tilde{\nu}}''} = 1.9$, $\theta_{f_{\tilde{\nu}}'} = 0.5$ and $\theta_{f_{\tilde{\nu}}''} = 0.7$. All masses are in GeV and phases in rad. Comparison of case (i) and case (ii) indicate a very significant increase for case (ii) overall. The last row gives the total electroweak contribution to the anomalous magnetic moment which is sum of the four contributions in rows 1-4. It is seen that the total contribution for case (ii) is 7.3 times larger than for case (i).
Figure 3: A display of the electron anomalous magnetic moment as a function of $\theta_\mu$, the phase of $\mu$, in the range $[-\pi, \pi]$. The three curves correspond to $m_0^\nu = 650$, $|A_0^\nu| = 650$, $|A_0| = 520$ (solid curve), $m_0^\nu = 660$, $|A_0^\nu| = 655$, $|A_0| = 530$ (dotted curve) and $m_0^\nu = 675$, $|A_0^\nu| = 660$, $|A_0| = 540$ (square dotted curve). Other parameters have the values $m_N = 212$, $m_E = 250$, $m_0 = 650$, $|m_1| = 600$, $|m_2| = 240$, $|\mu| = 104$, $\tan \beta = 15$, $|f_3| = 7 \times 10^{-8}$, $|f_3'| = 5 \times 10^{-8}$, $|f_3''| = 8 \times 10^{-9}$, $|f_4| = |f_4'| = 10$, $|f_4''| = 90$, $|f_5| = 8.11 \times 10^{-2}$, $|f_5'| = 9.8 \times 10^{-2}$, $|f_5''| = 4 \times 10^{-2}$, $\theta_{A_0} = 1.2$, $\theta_{A_0^\nu} = 2.8$, $\theta_1 = 2.5$, $\theta_2 = 1.5$, $\theta_{f_3} = 0.3$, $\theta_{f_3'} = 0.2$, $\theta_{f_3''} = 0.6$, $\theta_{f_4} = 1.4$, $\theta_{f_4'} = 1.1$, $\theta_{f_4''} = 0.5$, $\theta_{f_5} = 1.9$, $\theta_{f_5'} = 0.5$ and $\theta_{f_5''} = 0.7$. All masses are in GeV and phases in rad.

It is known that the supersymmetric electroweak correction to the anomalous magnetic moment is sensitive to CP phases. This was demonstrated for the case of the supersymmetric electroweak contributions to the anomalous magnetic moment of the muon in [21]. Here we exhibit this sensitivity for the case of the electroweak contributions to the electron anomalous magnetic moment. Thus Figure 3 displays the total electroweak contribution to the anomalous magnetic moment of the electron as a function of $\theta_\mu$ which is the phase of $\mu$ that appears in the chargino and neutralino mass matrices and in the slepton and sneutrino mass matrices. Over the interval $[-\pi, \pi]$, the electroweak correction to the anomalous magnetic moment of the electron shows a pronounced peak for a value of $\theta_\mu = 0.3$ rad. For the three sets of values considered, the peak values stretch from $\sim 3.7 - 6.9 \times 10^{-13}$. It should be noted that the variation in $a_e$ comes from the supersymmetric sector, and mainly from the chargino contribution. This is so because the neutralino contribution is relatively small, typically an order of magnitude smaller than the chargino exchange contribution. Because of this the variation of $a_e$ with CP phases is dominated by the chargino contributions. We note also that the $W$ and $Z$ contributions are not affected by the phases.
Figure 4: A display of the electroweak contribution to the anomalous magnetic moment of the electron as a function of $\theta_{A_0^\nu}$, the phase of $A_0^\nu$, in the range $[-\pi, \pi]$. The three curves correspond to (i) $m_0^\nu = 650$, $\theta_\mu = 0.3$ (solid curve), (ii) $m_0^\nu = 660$, $\theta_\mu = 1.3$ (dotted curve) and (iii) $m_0^\nu = 670$, $\theta_\mu = 2.3$ (square dotted curve). Other parameters have the values $m_N = 212$, $m_E = 250$, $m_0 = 650$, $|m_1| = 600$, $|m_2| = 240$, $|\mu| = 103$, $\tan \beta = 15$, $|A_0^\nu| = 660$, $|A_0| = 520$, $|f_3| = 7 \times 10^{-8}$, $|f_4^\nu| = 5 \times 10^{-8}$, $|f_5^\nu| = 8 \times 10^{-9}$, $|f_4| = |f_4^\nu| = 10$, $|f_5| = 90$, $|f_5^\nu| = 8.11 \times 10^{-2}$, $|f_0^\nu| = 9.8 \times 10^{-2}$, $|f_0| = 4 \times 10^{-2}$, $\theta_{A_0} = 1.2$, $\theta_1 = 2.5$, $\theta_2 = 1.5$, $\theta_{f_3} = 0.3$, $\theta_{f_5} = 0.2$, $\theta_{f_5^\nu} = 0.6$, $\theta_{f_4} = 1.4$, $\theta_{f_4^\nu} = 1.1$, $\theta_{f_4^\nu} = 0.5$, $\theta_{f_5} = 1.9$, $\theta_{f_5^\nu} = 0.5$ and $\theta_{f_5^\nu} = 0.7$. All masses are in GeV and phases in rad.

Figure 4 exhibits the variation of electroweak contribution to the anomalous magnetic moment of the electron $\Delta a_e^{(EW)}$ as a function of $\theta_{A_0^\nu}$, the phase of the trilinear coupling $A_0^\nu$, where in our analysis we have assumed that $A_{\nu_e} = A_{\nu_\mu} = A_{\nu_\tau} = A_N = A_0^\nu$ and $m_0^2 = M_N^2 = M_{\nu_e}^2 = M_{\nu_\mu}^2$ in the sneutrino mass matrix (see Appendix). Note that $m_0^2 = M_{\tau L}^2 = M_{E}^2 = M_{\tau L}^2 = M_{\chi}^2 = M_{\mu L}^2 = \tilde{M}_\mu^2 = \tilde{M}_{\tau L} = \tilde{M}_e^2$ and $A_0 = A_\tau = A_E = A_\mu = A_e$ in the slepton mass matrix (see Appendix). As can be seen from Fig. 4 the variation is very substantial with $\Delta a_e^{(EW)}$ varying in the range $\sim 7 \times 10^{-14} - 6 \times 10^{-13}$ which is an order of magnitude variation. As for the case of Fig. 3 the source of variation is the chargino exchange contribution once again. This is so because the chargino exchange diagram contains the sneutrino mass matrix in the loop which has a strong $A_0^\nu$ dependence.
Figure 5: A display of the electroweak contribution to the anomalous magnetic moment of $e, \mu$ as a function of $m_0^\tilde{\nu}$ in the range 650-750 GeV. Panel (a) gives the electron anomalous magnetic moment and panel (b) gives the muon anomalous magnetic moment. The curves correspond to $\tan \beta = 13$ (lowermost curve), $\tan \beta = 14$ (square dotted curve), $\tan \beta = 15$ (dotted curve), and $\tan \beta = 16$ (solid curve). Other parameters have the values $m_N = 212$, $m_E = 250$, $m_0 = 650$, $|m_1| = 600$, $|m_2| = 200$, $|\mu| = 104$, $|A_0^\tilde{\nu}| = 650$, $|A_0| = 520$, $|f_3| = 7 \times 10^{-8}$, $|f_4'| = 5 \times 10^{-8}$, $|f_5''| = 8 \times 10^{-9}$, $|f_4| = |f_4'| = 10$, $|f_5| = 9.01 \times 10^{-2}$, $|f_5'| = 9.8 \times 10^{-2}$, $|f_5''| = 4 \times 10^{-2}$, $\theta_{A_0} = 1.2$, $\theta_{A_0^\tilde{\nu}} = 2.8$, $\theta_1 = 2.5$, $\theta_2 = 1.5$, $\theta_\mu = 1.0$, $\theta_{f_3} = 0.3$, $\theta_{f_3''} = 0.2$, $\theta_{f_4} = 1.4$, $\theta_{f_4'} = 1.1$, $\theta_{f_5} = 0.5$, $\theta_{f_5'} = 1.9$, $\theta_{f_5''} = 0.5$ and $\theta_{f_5'''} = 0.7$. All masses are in GeV and phases in rad.

Figure 5 exhibits the variation of the electroweak contribution to the anomalous magnetic moment of the electron and of the muon as a function of $m_0^\tilde{\nu}$ over the range 650 - 750 GeV. For parametric curves corresponding to $\tan \beta = 13, 14, 15, 16$ (from bottom to top) are shown for each of the panels. A comparison of panel (a) with panel (b) shows that $a_e$ exhibits a much larger sensitivity to $m_0^\tilde{\nu}$. Figure 6 exhibits the variation of the electroweak contribution to the anomalous magnetic moment of the electron (panel (a)), of the muon (panel(b)) and of the tau (panel (c)) vs $\theta_1$, which is the phase of $m_1$, over the range $[-\pi, \pi]$. It seen that the variation is smooth in all cases as expected. However, the size of variation in each case is small as can be seen, for example, by comparing the range of variation in panel (a) in Fig. 6 with the range of variation in Fig. 3. The reason for this smallness is easily understood. Thus the parameter $m_1$ enters in the neutralino mass matrix and as discussed earlier the contribution from the neutralino exchange diagram to the electroweak contribution to the anomalous magnetic moment of the electron is relatively small which explains the relative smallness of the variation of $\Delta a_e(\text{EW})$ with $\theta_1$. Similar results hold for the variation of $\Delta a_\mu(\text{EW})$ and $\Delta a_\tau(\text{EW})$ with $\theta_1$. Regarding $a_\tau$ we note that the standard model
predicts \[36\]

\[ a_{\tau}^{\text{SM}} = 117721(5) \times 10^{-8} . \]  \hspace{1cm} (60)

The current experimental result is \[37\]

\[ a_{\tau}^{\text{EXP}} = -0.018(17) , \]  \hspace{1cm} (61)

while the analysis of \[38\] constraints the range of new physics so that

\[-0.007 < \Delta a_{\tau}^{\text{NP}} < 0.005 , \]  \hspace{1cm} (62)

where $\Delta a_{\tau}^{\text{NP}}$ refers to the new physics contribution. Future experiments \[39\] in high luminosity B factories are likely to significantly improve the limits in Eq.\[62\]. However, it is unlikely that the improvements in the measurement of the tau anomalous magnetic moment at the level needed to check on the contributions of panel (c) in Fig. \[6\] can be achieved in experiment in the very near future. Thus, $a_e$ gives the best hope for the test of new physics.
Figure 6: A display of the electroweak contribution to the anomalous magnetic moment of the \(e, \mu, \tau\) as a function of \(\theta_1\), the phase of \(m_1\), in the range \([-\pi, \pi]\). Panel (a) gives the electroweak contribution to the anomalous magnetic moment of the electron, panel (b) gives it for the muon and panel (c) gives it for the tau. The curves correspond to (i) \(|A_0| = 520, \theta_{A_0} = 1.2, |m_2| = 350\) (solid curve), (ii) \(|A_0| = 330, \theta_{A_0} = 1.9, |m_2| = 351\) (dotted curve), and (iii) \(|A_0| = 130, \theta_{A_0} = 2.6, |m_2| = 352\) (lowermost curve). Other parameters have the values \(m_N = 212, m_E = 250, m_0 = m_0^\tilde{\nu} = 650, |m_1| = 600, |\mu| = 104, |A_0^\tilde{\nu}| = 640, \tan \beta = 15, |f_3| = 7 \times 10^{-8}, |f_3'| = 5 \times 10^{-8}, |f_3''| = 8 \times 10^{-9}, |f_4| = |f_4'| = 10, |f_5'| = 90, |f_5''| = 8.11 \times 10^{-2}, |f_5'''| = 9.8 \times 10^{-2}, |f_5'''| = 4 \times 10^{-2}, \theta_2 = 0.1, \theta_{A_0} = 2.8, \theta_\mu = 0.3, \theta_{f_3} = 0.3, \theta_{f_3'} = 0.2, \theta_{f_3''} = 0.6, \theta_{f_4} = 1.4, \theta_{f_4'} = 1.1, \theta_{f_4''} = 0.5, \theta_{f_5} = 1.9, \theta_{f_5'} = 0.5\) and \(\theta_{f_5''} = 0.7\). All masses are in GeV and phases in rad.

8 Conclusion

The magnetic moment of the electron is one of the most precisely determined quantities in physics with an error in \(a_e^\text{exp}\) of \(\delta a_e^\text{exp} = 2.8 \times 10^{-13}\). The theory predictions for \(a_e\) have also been done with a high accuracy. However the error in theory prediction is significantly larger than the experimental error giving a total error in the difference in experiment minus theory of \(\delta \Delta a_e \simeq 8 \times 10^{-13}\). This error
is much larger than the new physics effects predicted by scaling if one extrapolates the discrepancy between experiment and theory for the muon anomalous magnetic moment. Thus the Brookhaven experiment gives \((a_\mu^{\text{exp}} - a_\mu^{\text{theory}}) = (287 \pm 80) \times 10^{-11}\) and if one uses the scaling factor of \(m_e^2/m_\mu^2\) the new physics effect in \(a_e\) would be of size \((.6 \pm .2) \times 10^{-13}\) which is an order of magnitude smaller than the current error in \(\delta \Delta a_e\). However, much larger new physics effects can occur if naive scaling law is violated. In this work we have shown that such violations do occur in extensions of MSSM with a vectorlike multiplet. Thus we have computed the effect of both the non-supersymmetric as well as the supersymmetric loop corrections to the anomalous magnetic moment of the electron in the MSSM extension with a vectorlike multiplet. We have shown that effects as large as factors of five or more can occur in the MSSM extension over what one expects from scaling. We have also investigated the effect of CP phases on the correction from the new physics sector. The largeness of the correction opens the possibility that such effects could be discerned even with modest further improvement in reducing the error in \(\Delta a_e\).

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9 Appendix: Further details on the scalar mass squared matrices

In this Appendix we give further details of the structure of the slepton mass matrices. The mass terms arising from the superpotential are given by

\[
\mathcal{L}_F^{\text{mass}} = \mathcal{L}_C^{\text{mass}} + \mathcal{L}_N^{\text{mass}},
\]  

(63)
where $\mathcal{L}_C^{\text{mass}}$ gives the mass terms for the charged leptons while $\mathcal{L}_N^{\text{mass}}$ gives the mass terms for the neutrinos. For $\mathcal{L}_C^{\text{mass}}$ we have

$$
-\mathcal{L}_C^{\text{mass}} = \left( \frac{v_2^2|f_2|^2}{2} + |f_3|^2 + |f_3''|^2 \right) \bar{E}_R E_R^* + \left( \frac{v_2^2|f_1|^2}{2} + |f_4|^2 + |f_4''|^2 \right) \bar{E}_L E_L^* + \left( \frac{v_3^2|h_1|^2}{2} + |f_5|^2 + |f_5''|^2 \right) \bar{\nu}_R \nu_R^* + \left( \frac{v_3^2|h_2|^2}{2} + |f_6|^2 + |f_6''|^2 \right) \bar{\nu}_L \nu_L^* + \left( \frac{v_4^2|h_3|^2}{2} + |f_7|^2 + |f_7''|^2 \right) \bar{\nu}_\tau \nu_\tau^* + \left( \frac{v_4^2|h_4|^2}{2} + |f_8|^2 + |f_8''|^2 \right) \bar{\nu}_\bar{\tau} \nu_{\bar{\tau}}^* + \left( \frac{v_5^2|h_5|^2}{2} + |f_9|^2 + |f_9''|^2 \right) \bar{\nu}_\mu \nu_\mu^* + \left( \frac{v_5^2|h_6|^2}{2} + |f_10|^2 + |f_10''|^2 \right) \bar{\nu}_\mu^* \nu_\mu^* R - \frac{h_2^2 v_2 v_3}{\sqrt{2}} \bar{E}_R E_R^* + H.c. \right) \right) (64)
$$

For $\mathcal{L}_N^{\text{mass}}$ we have

$$
-\mathcal{L}_N^{\text{mass}} = \left( \frac{v_2^2|f_2|^2}{2} + |f_3|^2 + |f_3''|^2 \right) \bar{N}_R \tilde{N}_R^* + \left( \frac{v_3^2|h_1|^2}{2} + |f_4|^2 + |f_4''|^2 \right) \bar{N}_L \tilde{N}_L^* + \left( \frac{v_4^2|h_2|^2}{2} + |f_5|^2 + |f_5''|^2 \right) \bar{N}_\tau \tilde{N}_\tau^* + \left( \frac{v_5^2|h_3|^2}{2} + |f_6|^2 + |f_6''|^2 \right) \bar{N}_\bar{\tau} \tilde{N}_{\bar{\tau}}^* + \left( \frac{v_6^2|h_4|^2}{2} + |f_7|^2 + |f_7''|^2 \right) \bar{N}_\mu \tilde{N}_\mu^* + \left( \frac{v_6^2|h_5|^2}{2} + |f_8|^2 + |f_8''|^2 \right) \bar{N}_\mu^* \tilde{N}_\mu^* R - \frac{h_3^2 v_2 v_3}{\sqrt{2}} \bar{N}_R \tilde{N}_R^* + H.c. \right) \right) \right)
$$
We define the scalar mass squared matrix $M^2_{ij}$ in the basis $(\tilde{\tau}_L, \tilde{E}_L, \tilde{\tau}_R, \tilde{E}_R, \tilde{\mu}_L, \tilde{\mu}_R, \tilde{e}_L, \tilde{e}_R)$. We label the matrix elements of these as $(M^2_{ij})_{ij} = M^2_{ij}$ where the elements of the matrix are given by

\[
M^2_{11} = \tilde{M}^2_{\tau_L} + \frac{v_2^2|f_1|^2}{2} + |f_3|^2 - m_Z^2 \cos 2\beta \left( \frac{1}{2} - \sin^2 \theta_W \right),
\]

\[
M^2_{22} = \tilde{M}^2_E + \frac{v_2^2|f_2|^2}{2} + |f_4|^2 + |f'_4|^2 + m_Z^2 \cos 2\beta \sin^2 \theta_W,
\]

\[
M^2_{33} = \tilde{M}^2_{\tau_R} + \frac{v_2^2|f_1|^2}{2} + |f_4|^2 - m_Z^2 \cos 2\beta \sin^2 \theta_W,
\]

\[
M^2_{44} = \tilde{M}^2_{E_L} + \frac{v_2^2|f_2|^2}{2} + |f_3|^2 + |f_3'|^2 + |f''_3|^2 + m_Z^2 \cos 2\beta \left( \frac{1}{2} - \sin^2 \theta_W \right),
\]

\[
M^2_{55} = \tilde{M}^2_{\mu_L} + \frac{v_2^2|h_1|^2}{2} + |f'_3|^2 - m_Z^2 \cos 2\beta \left( \frac{1}{2} - \sin^2 \theta_W \right),
\]

\[
M^2_{66} = \tilde{M}^2_{\mu_R} + \frac{v_2^2|h_1|^2}{2} + |f_4'|^2 - m_Z^2 \cos 2\beta \sin^2 \theta_W,
\]

\[
M^2_{77} = \tilde{M}^2_{E_R} + \frac{v_2^2|h_2|^2}{2} + |f''_3|^2 - m_Z^2 \cos 2\beta \left( \frac{1}{2} - \sin^2 \theta_W \right),
\]

\[
M^2_{88} = \tilde{M}^2_{\tau_R} + \frac{v_2^2|h_2|^2}{2} + |f''_3|^2 - m_Z^2 \cos 2\beta \sin^2 \theta_W,
\]

\[
M^2_{12} = M^2_{21} = \frac{v_2 f_2 f'_3}{\sqrt{2}} + \frac{v_1 f_4 f'_1}{\sqrt{2}},
\]

\[
M^2_{13} = M^2_{31} = \frac{f_1^*(v_1 A^*_\tau - \mu v_2)},
\]

\[
M^2_{14} = M^2_{41} = 0, M^2_{15} = M^2_{51} = f_3^* f'_3,
\]

\[
M^2_{16} = M^2_{61} = 0, M^2_{17} = M^2_{71} = f''_3 f_3^*, M^2_{18} = M^2_{71} = 0, M^2_{23} = M^2_{32} = 0,
\]

\[
M^2_{24} = M^2_{22} = \frac{f_2^2 (v_2 A^*_E - \mu v_1), M^2_{25} = M^2_{22} = \frac{v_2 f_2 f'_3}{\sqrt{2}} + \frac{v_1 h_1 f'_4}{\sqrt{2}},}
\]

\[
M^2_{26} = M^2_{62} = 0, M^2_{27} = M^2_{72} = \frac{v_2 f_3 f'_3}{\sqrt{2}} + \frac{v_1 h_1 f'_4}{\sqrt{2}}, M^2_{28} = M^2_{62} = 0,
\]

\[
M^2_{34} = M^2_{33} = \frac{v_1 f_4 f''_3}{\sqrt{2}} + \frac{v_1 f_4 f'_3}{\sqrt{2}}, M^2_{35} = M^2_{53} = M^2_{33} = 0, M^2_{36} = M^2_{63} = 0, M^2_{37} = M^2_{73} = 0,
\]

\[
M^2_{47} = M^2_{74} = 0, M^2_{48} = M^2_{84} = \frac{v_2 f'_2 f''_3}{\sqrt{2}} + \frac{v_1 f''_4 f'_3}{\sqrt{2}},
\]

\[
M^2_{56} = M^2_{65} = \frac{h_1^* (v_1 A^*_\tau - \mu v_2), M^2_{57} = M^2_{75} = f''_3 f'_3^*, M^2_{58} = M^2_{85} = 0, M^2_{67} = M^2_{76} = 0,
\]

\[
M^2_{68} = M^2_{86} = f''_4 f'_4^*, M^2_{78} = M^2_{87} = \frac{h_1^* (v_1 A^*_\tau - \mu v_2)}{\sqrt{2}}
\]
Here the terms \( M_{11}^2, M_{13}^2, M_{31}^2, M_{33}^2 \) arise from soft breaking in the sector \( \tilde{\tau}_L, \tilde{\tau}_R \), the terms \( M_{55}^2, M_{66}^2, M_{65}^2, M_{66}^2 \) arise from soft breaking in the sector \( \tilde{\mu}_L, \tilde{\mu}_R \), the terms \( M_{77}^2, M_{78}^2, M_{87}^2, M_{88}^2 \) arise from soft breaking in the sector \( \tilde{e}_L, \tilde{e}_R \) and the terms \( M_{22}^2, M_{24}^2, M_{42}^2, M_{44}^2 \) arise from soft breaking in the sector \( \tilde{E}_L, \tilde{E}_R \). The other terms arise from mixing between the staus, smuons and the mirrors. We assume that all the masses are of the electroweak size so all the terms enter in the mass squared matrix. We diagonalize this hermitian mass squared matrix by the unitary transformation \( \tilde{D}^T M_\tau^2 \tilde{D} = \text{diag}(M_{\tau_1}^2, M_{\tau_2}^2, M_{\tau_3}^2, M_{\nu_1}^2, M_{\nu_2}^2, M_{\nu_3}^2, M_{\tau_7}^2, M_{\tau_8}^2) \). For a further clarification of the notation see [32].

The mass\(^2\) matrix in the sneutrino sector has a similar structure. In the basis \((\tilde{\nu}_L, \tilde{N}_L, \tilde{\nu}_{\tau R}, \tilde{\nu}_{\mu L}, \tilde{\nu}_{\mu R}, \tilde{\nu}_{e L}, \tilde{\nu}_{e R})\), we can write the sneutrino mass\(^2\) matrix in the form \( (M_\tilde{\nu}^2)_{ij} = m_{ij}^2 \) where

\[
\begin{align*}
    m_{11}^2 &= \tilde{M}_{\tilde{\nu}_L}^2 + m_{\nu_e}^2 + |f_3|^2 + \frac{1}{2} m_\chi^2 \cos 2\beta, \\
    m_{22}^2 &= \tilde{M}_{\tilde{\nu}_\tau}^2 + m_{\nu_\tau}^2 + |f_5|^2 + |f_5'|^2, \\
    m_{33}^2 &= \tilde{M}_{\tilde{\nu}_\tau}^2 + m_{\nu_e}^2 + |f_5|^2, \\
    m_{44}^2 &= \tilde{M}_\chi^2 + m_{N}^2 + |f_3|^2 + |f_3'|^2 + |f_3''|^2 - \frac{1}{2} m_\chi^2 \cos 2\beta, \\
    m_{55}^2 &= \tilde{M}_{\tilde{\mu}_L}^2 + m_{\nu_\mu}^2 + |f_3'|^2 + \frac{1}{2} m_\chi^2 \cos 2\beta, \\
    m_{66}^2 &= \tilde{M}_{\tilde{\mu}_\tau}^2 + m_{\nu_\mu}^2 + |f_5'|^2, \\
    m_{77}^2 &= \tilde{M}_{\tilde{\nu}_L}^2 + m_{\nu_e}^2 + |f_3'|^2 + \frac{1}{2} m_\chi^2 \cos 2\beta, \\
    m_{88}^2 &= \tilde{M}_{\tilde{\nu}_\tau}^2 + m_{\nu_e}^2 + |f_5'|^2, \\
    m_{12}^2 &= m_{21}^2 = \frac{v_2 f_5 f_1'^*}{\sqrt{2}} - \frac{v_1 f_2 f_3'^*}{\sqrt{2}}, \\
    m_{13}^2 &= m_{31}^2 = \frac{f_1'^*}{\sqrt{2}} (v_2 A_{\nu_e}^* - \mu v_1), m_{14}^2 = m_{41}^2 = 0, \\
    m_{15}^2 &= m_{51}^2 = f_3' f_3'^*, m_{16}^2 = m_{61}^2 = 0, \\
    m_{17}^2 &= m_{71}^2 = f_3' f_4'^*, m_{18}^2 = m_{81}^2 = 0, \\
    m_{23}^2 &= m_{32}^2 = 0, m_{24}^2 = m_{42}^2 = \frac{f_2'^*}{\sqrt{2}} (v_1 A_N^* - \mu v_2), m_{25}^2 = m_{52}^2 = -\frac{v_1 f_2 f_3'}{\sqrt{2}} + \frac{h_1' v_2 f_5'^*}{\sqrt{2}}, \\
    m_{26}^2 &= m_{62}^2 = 0, m_{27}^2 = m_{72}^2 = -\frac{v_1 f_2 f_3'^*}{\sqrt{2}} + \frac{h_2' v_2 f_5'^*}{\sqrt{2}}, \quad (66)
\end{align*}
\]
\[ m_{28}^2 = m_{82}^2 = 0, m_{34}^2 = m_{43}^2 = \frac{v_1 f_2^* f_5}{\sqrt{2}} - \frac{v_2 f_1^* f_3^*}{\sqrt{2}}, \]

\[ m_{35}^2 = m_{53}^2 = 0, m_{36}^2 = m_{63}^2 = f_5 f_5^*, m_{37}^2 = m_{73}^2 = 0, m_{38}^2 = m_{83}^2 = f_5 f_5^*, m_{45}^2 = m_{54}^2 = 0, \]

\[ m_{46}^2 = m_{64}^2 = \frac{h_1^* v_2 f_3^*}{\sqrt{2}} + \frac{v_1 f_2 f_5^*}{\sqrt{2}}, m_{47}^2 = m_{74}^2 = 0, \]

\[ m_{48}^2 = m_{84}^2 = \frac{v_1 f_2 f_5^* f_5^{''*}}{\sqrt{2}} - \frac{v_2 h_2 f_3^* f_3^{''*}}{\sqrt{2}}, m_{56}^2 = m_{65}^2 = \frac{h_1^*}{\sqrt{2}}(v_2 A_{\nu_e}^* - \nu v_1), \]

\[ m_{57}^2 = m_{75}^2 = f_3 f_3^* f_3^{''*} m_{58}^2 = m_{85}^2 = 0, m_{67}^2 = m_{76}^2 = 0, \]

\[ m_{68}^2 = m_{86}^2 = f_5^* f_5^{''*}, m_{78}^2 = m_{87}^2 = \frac{h_2^*}{\sqrt{2}}(v_2 A_{\nu_e}^* - \nu v_1). \] (67)

As in the charged slepton sector here also the terms \( m_{11}^2, m_{13}^2, m_{31}^2, m_{33}^2 \) arise from soft breaking in the sector \( \tilde{\nu}_L, \tilde{\nu}_R \), the terms \( m_{55}^2, m_{56}^2, m_{65}^2, m_{66}^2 \) arise from soft breaking in the sector \( \tilde{\nu}_\mu L, \tilde{\nu}_\mu R \), the terms \( m_{77}^2, m_{78}^2, m_{87}^2, m_{88}^2 \) arise from soft breaking in the sector \( \tilde{\nu}_e L, \tilde{\nu}_e R \) and the terms \( m_{22}^2, m_{32}^2, m_{44}^2 \) arise from soft breaking in the sector \( \tilde{N}_L, \tilde{N}_R \). The other terms arise from mixing between the physical sector and the mirror sector. Again as in the charged lepton sector we assume that all the masses are of the electroweak size so all the terms enter in the mass\(^2\) matrix. This mass\(^2\) matrix can be diagonalized by the unitary transformation \( \tilde{D}^{\nu} M_\nu^2 \tilde{D}^{\nu} = \text{diag}(M_{\nu_1}^2, M_{\nu_2}^2, M_{\nu_3}^2, M_{\bar{\nu}_1}^2, M_{\bar{\nu}_2}^2, M_{\bar{\nu}_3}^2, M_{\bar{\nu}_4}^2, M_{\bar{\nu}_5}^2, M_{\bar{\nu}_6}^2, M_{\bar{\nu}_7}^2, M_{\bar{\nu}_8}^2) \).

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