STRING THERMALIZATION
AT A BLACK HOLE HORIZON

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Susskind has recently shown that a relativistic string approaching the event horizon of a black hole spreads in both the transverse and longitudinal directions in the reference frame of an outside observer. The transverse spreading can be described as a branching diffusion of wee string bits. This stochastic process provides a mechanism for thermalizing the quantum state of the string as it spreads across the stretched horizon.

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1 Introduction

Hawking’s information paradox [1], i.e. the question whether black hole evaporation can be reconciled with the unitary evolution of states in quantum theories, has received considerable attention in recent years. An attractive resolution to the paradox is that the information about the quantum state of collapsing matter be encoded in the outgoing Hawking radiation [2],[3]. This requires a kinematic description of matter at high energies which differs radically from the one offered by conventional local quantum field theory. In the reference frame of a distant observer the infalling matter must give up all information about its quantum state to the emitted Hawking radiation as it approaches the event horizon. There is, however, no invariant local signature of the presence of an event horizon: the curvature remains smooth there, and an observer in free fall who enters a large black hole will not experience any discomfort upon crossing the horizon. The principle of black hole complementarity [4] states that these seemingly contradictory viewpoints are equally valid. Reference [4] put forward a phenomenological description of black hole evolution in terms of a quantum mechanical membrane, the “stretched horizon”, which absorbs the energy and quantum information of incoming matter, thermalizes it, and eventually radiates it back as Hawking radiation.

These ideas are at odds with the usual semi-classical approach to black hole physics, where one considers quantum evolution of local fields in a (semi-)classical background geometry. Analysis of several gedanken experiments, designed to test black hole complementarity, indicates, however, that it does not contradict any known laws of physics, and that the conflict can be traced to unwarranted assumptions about short-distance physics which are implicitly made in the usual semi-classical approach [3].

String theory is widely believed to provide a consistent short-distance description of matter and gravity and Susskind has recently argued that relativistic strings in fact exhibit precisely the kinematic behavior required to implement black hole complementarity [4]. Following this work, we study relativistic strings approaching the event horizon of a black hole. We show that the transverse spreading of the string configuration simulates a stochastic process. More specifically, we obtain a Langevin equation describing the coarse-grained transverse evolution of the string and interpret it in terms of a branching diffusion of string bits. The infalling matter is thus very efficiently thermalized as it is absorbed into the stretched horizon.

Let us begin by briefly reviewing the arguments for the spreading of strings falling into a black hole. The key observation is that, due to zero-point fluctuations of string modes, the size and shape of strings are sensitive to the time resolution used [7],[8]. The shorter the time over which the oscillations of a string are averaged the larger is its spatial extent.
Consider a string configuration in free fall approaching a black hole event horizon. An observer at rest far away from the black hole measures asymptotic time, but because of the increasing redshift, a unit of asymptotic time corresponds to an ever shorter time interval in the free-fall frame. The distant observer is therefore using a shorter and shorter resolution time to describe the string configuration and, once it passes within a proper distance of order the string scale from the event horizon, the string begins to spread both in the longitudinal and transverse directions. The longitudinal spread is sufficiently rapid to cancel out the longitudinal Lorentz contraction caused by the black hole geometry. Meanwhile, the spread in the transverse directions causes the configuration to cover the entire horizon area in a time which is short compared to the black hole lifetime. The stretched horizon is thus made out of the strings in the infalling matter which forms the black hole.

On the other hand, this spreading effect is not present in the free-fall frame, where there is no redshift to enhance the time resolution, and from the point of view of an infalling observer there is no stretched horizon, in line with the principle of black hole complementarity.

A natural question to ask is in what sense does an asymptotic observer “see” the string spreading. First of all, any direct observation involving the scattering of test strings off the infalling matter near the event horizon, which then propagate to infinity, requires an enormous center of mass energy due to the gravitational redshift. The results of such scattering experiments would therefore be consistent with the notion that the infalling string has spread out. In the absence of such high energy scattering experiments, distant observers can only make measurements on the outgoing Hawking radiation, and the question is whether measurements will uncover subtle correlations amongst the outgoing quanta. The mathematical description of the string spreading suggests the existence of physical processes capable of producing such correlations.

Since the stringy stretched horizon is formed from the infalling matter itself, it efficiently absorbs the quantum information contained in that matter. Our results below illustrate how the string spreading process also thermalizes the stretched horizon. This comes about because a time-dependent cutoff is being used on the scalar fields which give the transverse location of the string. As time goes on, new modes emerge below the cutoff, and the random phases of the different modes lead to a classical stochastic evolution of the transverse fields. The physics of the process is quite analogous to that of field evolution in the rapidly expanding background geometry of an inflationary universe; and we shall draw upon the cosmology literature in our analysis. For technical reasons our discussion is limited to strings approaching the horizon of a classical black hole in the limit of large mass. Therefore we cannot at present establish, although we find it very plausible, that the stringy stretched horizon re-emits the original information encrypted in apparently thermal Hawking radiation.
2 Infalling string near the horizon

The string spreading effects in which we are interested take place on a short timescale compared to the black hole lifetime \[t\] and we therefore consider a static classical geometry. Furthermore, the spreading takes place in a thin layer (whose proper thickness is of order the string scale \(\sim \frac{1}{g_{\text{Pl}} M}\)) outside the event horizon and for a macroscopic black hole this region is well approximated by Rindler space. The Schwarzschild line element can be written
\[
  ds^2 = -\frac{2M}{r} e^{-(r/2M-1)} dU dV + r^2 d\Omega^2 ,
\]
where the Kruskal variables \(U, V\) are defined in terms of the original Schwarzschild coordinates through
\[
  -\frac{U}{V} = e^{t/2M} , \quad -U V = 16M^2 \left( \frac{r}{2M} - 1 \right) e^{(r/2M-1)} .
\]
As the horizon is approached its spherical shape is well approximated locally by a planar surface, so near \(r = 2M\) we can write,
\[
  ds^2 = -dU dV + dX^2 .
\]
The null coordinates \(U, V\) extend into the black hole interior and are appropriate for an observer passing through the horizon in free fall. They are related to the null coordinates \(u, v\) of an asymptotic observer through,
\[
  \frac{U}{4M} = -e^{-u/4M} , \quad \frac{V}{4M} = e^{v/4M} .
\]
The usual description of Rindler space is obtained by rescaling all the coordinates to absorb the factors of \(4M\), but we prefer to keep these factors explicit.

Rindler space is isomorphic to a slice of flat Minkowski space, where we can easily discuss free string propagation in light-cone gauge. Consider an infalling string described by transverse coordinates \(X^i(\sigma, \tau)\), with \(i = 1, 2\), and some internal degrees of freedom depending on the string model being used. In the free-fall frame the light-cone gauge condition is \(\tau = U/4M\). The internal degrees of freedom decouple from the transverse coordinates which satisfy a free wave equation,
\[
  \left[ \frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial \sigma^2} \right] X^i(\sigma, \tau) = 0 .
\]
The solution can be expressed by the usual sum over modes of oscillation, but, unless the infinite sum is cut off in some way, this leads to ill-defined expressions for quantities such as the average transverse area occupied by the string in the quantum theory \[7,8\]. Introducing a cutoff on the mode expansion corresponds physically to employing finite time resolution so that mode oscillations above a given frequency average out. The string wave-function extends over a transverse area which grows logarithmically with better resolution time while the average length of string projected onto the transverse directions grows linearly \[8\]. This means that the string density will increase at the center of the distribution, and eventually we can no longer neglect the effect of string interactions there. Our calculations, which are at the level of free string theory, will give a good description of the spreading process at the center early on and, as it turns out, remain valid at the outer edges of the spreading string configuration.

Now we want to consider the string evolution in the reference frame of a distant fiducial observer, whose retarded time, \( t = u \), is related to the worldsheet parameter time through

\[
\tau = -e^{-t/4M}.
\]  

(6)

As asymptotic time goes on, a fixed resolution in \( t \) thus corresponds to an exponentially improving resolution in \( \tau \), so that in the reference frame of the distant observer, the string wave-function will spread with time to occupy a transverse area proportional to \( t \). By using the constraint equations of light-cone string theory one can show that the string also spreads in the longitudinal direction and that the longitudinal spreading is rapid enough to balance the Lorentz contraction due to the black hole metric near the event horizon \[6\].

Let us look at the transverse evolution of the string in more detail. The following discussion closely parallels the treatment of scalar perturbations in models of inflationary cosmology (see for example \[9,10,11\]). In asymptotic time the equation of motion (5) of the field \( X^i(\sigma, \tau(t)) \) becomes

\[
\left[ \frac{\partial^2}{\partial t^2} + \frac{1}{4M} \frac{\partial}{\partial t} - \left( \frac{e^{-t/4M}}{4M} \right)^2 \frac{\partial^2}{\partial \sigma^2} \right] X^i(\sigma, t) = 0. 
\]  

(7)

Acting along the lines of \[10\], we split both the field \( X^i \) and its conjugate momentum \( \dot{X}^i \) (\( \dot{\equiv} \frac{\partial}{\partial \tau} \)) into a slowly-varying, classical part and fast-varying, quantum part

\[
X^i(\sigma, t) = x^i(\sigma, t) + x^i_f(\sigma, t),
\]

\[
\dot{X}^i(\sigma, t) = v^i(\sigma, t) + v^i_f(\sigma, t). 
\]  

(8)
The quantum field $x^i_j$ can be expressed as a sum over modes in the $(\tau, \sigma)$ frame, provided the frequency cutoff, which separates the fast modes from the slow ones, is chosen to reflect the exponentially improving resolution in \(\tau\): \(^3\)

$$x^i_j(\sigma, t) = \sum_{n=1}^{\infty} W(n + \frac{\epsilon}{\tau}) \left[ \frac{c^i_n}{\sqrt{n}} x_n^+ + \frac{\tilde{c}^i_n}{\sqrt{n}} x_n^- + h.c. \right], \quad (9)$$

where we have defined $x_n^\pm = \sqrt{\frac{\epsilon}{2}} e^{-in(\tau \pm \sigma)}$, and \(\epsilon\) is some constant. For a filter function we could use $W(n + \frac{\epsilon}{\tau}) = \theta(n + \frac{\epsilon}{\tau}) = \theta(n - \epsilon e^{4M})$; then an asymptotic observer would indeed include only modes of exponentially higher frequency in the definition of the quantum part of the field. In our calculations we will use a filter where the step of the theta function is smeared a little, in order to avoid unphysical effects associated with a sharp edge cutoff in two dimensions.

The expansion for $v^i_j$ is similar:

$$v^i_j(\sigma, t) = \sum_{n=1}^{\infty} W(n + \frac{\epsilon}{\tau}) \left[ \frac{c^i_n}{\sqrt{n}} \dot{x}_n^+ + \frac{\tilde{c}^i_n}{\sqrt{n}} \dot{x}_n^- + h.c. \right]. \quad (10)$$

With the above definitions, the long-wavelength fields, $x^i$ and $v^i$, will evolve nontrivially in asymptotic time due to the continual feeding in of modes from the quantum parts. In order to obtain an equation for this evolution, we substitute $X^i = x^i + x^i_f$ and $\dot{X}^i = v^i + v^i_f$ into the field equation (7). First, let us write

$$\dot{x}^i_f = \sum_{n>0} W(n + \frac{\epsilon}{\tau}) \left[ \frac{c^i_n}{\sqrt{n}} \dot{x}_n^+ + \frac{\tilde{c}^i_n}{\sqrt{n}} \dot{x}_n^- + h.c. \right] - \eta^i(\sigma, t) \quad (11)$$

where $\eta^i(\sigma, t)$ is defined as

$$\eta^i(\sigma, t) \equiv -\frac{1}{4M} \sum_{n>0} \frac{\epsilon}{\tau} W'(n + \frac{\epsilon}{\tau}) \left[ \frac{c^i_n}{\sqrt{n}} x_n^+ + \frac{\tilde{c}^i_n}{\sqrt{n}} x_n^- + h.c. \right], \quad (12)$$

and similarly,

$$\dot{v}^i_f = \sum_{n>0} W(n + \frac{\epsilon}{\tau}) \left[ \frac{c^i_n}{\sqrt{n}} \ddot{x}_n^+ + \frac{\tilde{c}^i_n}{\sqrt{n}} \ddot{x}_n^- + h.c. \right] - \xi^i(\sigma, t) \quad (13)$$

\(^5\)Our conventions are $[c^i_n, c^j_m] = \delta_{nm} \delta^{ij} = [\tilde{c}^i_n, \tilde{c}^j_m]$, all other commutators zero.
where \( \xi^i(\sigma, t) \) is defined as
\[
\xi^i(\sigma, t) \equiv -\frac{1}{4M} \sum_{n>0} \frac{\epsilon}{\tau} W'(n + \frac{\epsilon}{\tau}) \left[ \frac{c_n^i}{\sqrt{n}} \dot{x}_n^+ + \frac{\tilde{c}_n^i}{\sqrt{n}} \dot{x}_n^- + h.c. \right]. \tag{14}
\]

Upon making these substitutions, the field equation for \( X^i \) reduces to two coupled equations for the long-wavelength fields:
\[
\begin{align*}
\dot{x}^i &= v^i + \eta^i, \\
\dot{v}^i &= -\frac{1}{4M} v^i + \frac{\tau^2}{(4M)^2} \frac{\partial^2}{\partial \sigma^2} x^i + \xi^i. \tag{15}
\end{align*}
\]

Near the horizon \( \tau \to 0 \) and so the spatial derivative term becomes negligible. In order to know which of the other terms are important, we must study the quantum noise functions \( \eta \) and \( \xi \) as defined in eq.(12) and eq.(14).

Let us first consider what to use for the filter function. Were we to use a simple step-function, \( W = \theta(n + \frac{\epsilon}{\tau}) \), we would have \( -\frac{\epsilon}{\tau} W'(n + \frac{\epsilon}{\tau}) = \delta(n + \frac{\epsilon}{\tau} + 1) \). Instead, we use a smooth approximation to this \( \delta \)-function, a gaussian function of \( n \) centred about \( \frac{\epsilon}{|\tau|} \) and with width \( \frac{\beta \epsilon}{|\tau|} \) where \( \beta \) is a small number:
\[
-\frac{\epsilon}{\tau} W'(n + \frac{\epsilon}{\tau}, \beta) = \frac{1}{\sqrt{2\pi \beta}} \exp \left[ -\frac{1}{2} \left( \frac{n \tau}{\epsilon} + 1 \right)^2 \right]. \tag{16}
\]

Given a filter function of this form we can compute the various noise correlation functions and commutators. We first obtain the \( \eta \) correlator:
\[
\langle \eta^i(1) \eta^j(2) \rangle = \frac{1}{(4M)^2} \sum_{m,n>0} \frac{1}{2\pi \beta^2} \exp \left[ -\frac{1}{2\beta^2} \left( \frac{m \tau_1}{\epsilon} + 1 \right)^2 - \frac{1}{2\beta^2} \left( \frac{n \tau_2}{\epsilon} + 1 \right)^2 \right] \times
\]
\[
\times \langle 0 \left| \frac{c_n^i}{\sqrt{n}} x_n^+(1) + \frac{\tilde{c}_n^i}{\sqrt{n}} x_n^-(1) + h.c. \right| \frac{c_m^j}{\sqrt{m}} x_m^+(2) + \frac{\tilde{c}_m^j}{\sqrt{m}} x_m^-(2) + h.c. \right| 0 \rangle. \tag{17}
\]

The expectation value is
\[
\langle 0 | \cdots | 0 \rangle = \frac{\alpha'}{n} \delta_{\mu \nu} \delta_{\tau_1 \tau_2} e^{-i n (\tau_1 - \tau_2)} \cos[n(\sigma_1 - \sigma_2)], \tag{18}
\]

so that the double sum in eq.(17) reduces to a single sum. The identity
\[
\left( \frac{n \tau_1}{\epsilon} + 1 \right)^2 + \left( \frac{n \tau_2}{\epsilon} + 1 \right)^2 = 2 \left( \frac{n \tau}{\epsilon} + 1 \right)^2 + \frac{n^2}{2\epsilon^2} (\tau_1 - \tau_2)^2, \tag{19}
\]
where \( \tilde{\tau} = \frac{1}{2}(\tau_1 + \tau_2) \), allows us to rewrite the correlation function as

\[
\langle \eta^i(1) \eta^j(2) \rangle = \frac{\alpha'}{2} \frac{\delta^{ij}}{(4M)^2} \sum_{n>0} \frac{1}{\pi \beta^2 n} \exp\left[-\frac{1}{\beta^2} \left( \frac{n \tilde{\tau}}{\epsilon} + 1 \right)^2 \right] \exp\left[-\frac{n^2}{4 \beta^2 \epsilon^2} (\tau_1 - \tau_2)^2 \right] \times
\]

\[
\times e^{-in(\tau_1-\tau_2)} \cos[n(\sigma_1 - \sigma_2)] .
\]  

(20)

One of the gaussian factors gives a \( \delta \)-function in time for small \( \beta \). Near the horizon, where \( \tilde{\tau} \rightarrow 0 \), the remaining sum is well approximated by an integral and we obtain:

\[
\langle \eta^i(1) \eta^i(2) \rangle \simeq \frac{\alpha'}{2} \frac{\delta^{ij}}{4M} \delta(t_1 - t_2) \cos\left(\frac{\epsilon \Delta \sigma}{\tau}\right) \exp\left[-\frac{\beta^2}{4} \left(\frac{\epsilon \Delta \sigma}{\tau}\right)^2 \right] .
\]  

(21)

Had we used a sharp step-function as our filter function, \( i.e. \) taken \( \beta = 0 \), we would have obtained a purely oscillatory spatial dependence for the noise correlator, an unphysical artefact of using an abrupt cutoff.

Computing the \( \xi \) correlator involves the same steps and we find that

\[
\langle \xi^i(1) \xi^j(2) \rangle \simeq -\left(\frac{\epsilon}{4M}\right)^2 \langle \eta^i(1) \eta^j(2) \rangle .
\]  

(22)

We can also calculate the various commutators of \( \eta \) and \( \xi \) at the same level of approximation, with the results \([\eta^i(1), \eta^j(2)] = 0 = [\xi^i(1), \xi^j(2)]\) and

\[
[\eta^i(1), \xi^j(2)] = -\frac{i \epsilon}{4M} \langle \eta^i(1) \eta^j(2) \rangle .
\]  

(23)

For a macroscopic black hole \( M \gg \epsilon \), so from (22) and (23) we see that only the \( \eta^i \) noise term is important in eq.(15). The fact that the commutators of the fields are small as compared with their correlators shows that \( x^i \) and \( v^i \) become effectively classical.

The noise \( \xi^i \) in the momentum equation is negligible in the \( \epsilon \ll M \) limit and can be dropped. The equation for \( v^i \) is then solved by \( v^i = -\tau v_0^i \), which means that near the horizon where \( \tau \rightarrow 0 \) we may ignore \( v^i \) altogether.

Thus the coupled equations (15) reduce to a single, classical, Langevin equation for the transverse position \( x^i \), with quantum noise \( \eta^i \):

\[
\dot{x}^i = \eta^i .
\]  

(24)

The time-dependence in the correlator (21) assures us that the noise \( \eta^i \) is white, and the spatial dependence tells us that wee bits of string of parameter length \( \Delta \sigma \simeq |\tau|/(\beta \epsilon) \) evolve independently with time.
3 Branching diffusion of string bits

An important insight from the physics of the inflationary universe is that the Langevin equation of the previous section can be given an interpretation in term of a branching diffusion process \[12\]. Indeed, for any given point of the string, equations (21) and (24) tell us that the value of the slowly-varying field \(x^i(\sigma, t)\) experiences a Brownian motion which is essentially unaffected by anything lying outside the correlation length \(\Delta \sigma \approx |x|/\beta \epsilon\). This resembles the independence of the scalar field evolution in different Hubble domains in the case of chaotic inflation. The correlation length decreases exponentially with asymptotic time and therefore the number of such independent bits of the string increases exponentially:

\[
N \sim \frac{2\beta \epsilon}{|\tau|} = 2\beta \epsilon \exp(t/4M) .
\] (25)

We can describe the process as a branching diffusion where every bit of the string diffuses independently of all others and splits into two bits with intensity \(\rho(x)\), which is determined by matching the growth of the number of bits in eq. (25). The two bits then diffuse away from their “birthplace”, independent of each other, and split in their turn, and so on.

The diffusion of a given bit of string is governed by the Fokker-Planck equation:

\[
\frac{\partial}{\partial t} P(x^i, t) = \frac{\varsigma^2}{2} \delta_{ij} \frac{\partial}{\partial x^i} \frac{\partial}{\partial x^j} P(x^i, t) .
\] (26)

where \(P(x^i, t)\) is the normalized probability of finding that bit at \((x^i, t)\). On the stretched horizon of a finite-mass black hole, \(P \to \text{const}\) at late times.

The coefficient of diffusion can be read off the correlator of the noise, \(\varsigma^2 = \frac{\alpha'}{8M}\), and so the effective temperature of this diffusion is, according to the asymptotic observer, given by \(T \sim \frac{1}{M}\).

This branching diffusion picture has implications for various properties of the infalling string and these can be checked against previous results \[4,8\]. Consider first the mean square transverse position,

\[
\langle x^i x^i \rangle = 2\varsigma^2 t = \frac{\alpha'}{4M} t .
\] (27)

\[\text{This particular ratio of 2:1 of the number of “daughter” bits per “parent” corresponds to a specific time interval of effective temporal averaging. However, as in the case of chaotic inflation [12], this parameter drops out from the equations describing the average number of string bits.}\]
This is the transverse spread, linear in asymptotic time, pointed out by Susskind [6].

The equation for the average number of branching bits $N$ at a time $t$ is the Kolmogorov-Fokker-Planck equation (see [12] for a derivation in the case of chaotic inflation), which in our case turns out to be simply

$$\frac{\partial}{\partial t} N = \frac{\varsigma^2}{2} \delta^{ij} \frac{\partial}{\partial x^i} \frac{\partial}{\partial x^j} N + \rho(x) N . \tag{28}$$

Choosing the branching intensity to be the constant $\rho = 1/4M$, we get, asymptotically,

$$N \sim \exp\left(\frac{t}{4M}\right) . \tag{29}$$

which does indeed match the behavior (25).

This gives us the average time between splitting as

$$t_{spl} \sim \frac{1}{\rho} = 4M . \tag{30}$$

Therefore, using (27), we see that the average transverse distance between two adjacent string bits is

$$l_* \sim \sqrt{\frac{\alpha'}{4M} t_{spl}} \sim \sqrt{\alpha'} , \tag{31}$$

which finally gives us the average transverse length of the string:

$$\mathcal{L} = l_* N \sim \sqrt{\alpha'} \exp\left(\frac{t}{4M}\right) . \tag{32}$$

These considerations can be extended in a straightforward manner to the branching diffusion process which describes string spreading across the horizon of a higher dimensional analog of a Schwarzschild black hole. In $D$ dimensions, the Hawking temperature in Planck units of such a black hole is given by $2\pi T_D = f(D) \left(\frac{1}{M}\right)^{\frac{1}{D-3}}$ where $f(D)$ depends only on $D$ [13]. Thus, we simply make the replacement $\frac{1}{4M} \rightarrow 2\pi T_D$ in expressions like (29). In eq.(27) we must also replace $2 \rightarrow (D - 2)$, since we are summing over $i$. The average transverse distance between adjacent string bits then becomes:

$$l_* \sim \sqrt{\alpha'(D - 2)} , \tag{33}$$
where $D$ is the number of extended spacetime dimensions.

We can also estimate the average extrinsic curvature per unit length of the string using the branching diffusion picture. Consider at time $t_0$ a piece of a string; we may think of it as composed of wee bits of string separated by typical distance $l_*$. After a time $\Delta t \sim t_{spl}$ each bit will have branched into two, with the new bits again being separated by a length $l_*$ of string. In this discretized picture the string looks like a zigzag, see Fig.1(a).

We would now like to find the analogue of the average curvature along the string. Imagine smoothing out each corner of a zigzag; this is shown in Figure 1(b). Then the average extrinsic curvature for a piece of zigzag is

$$\langle R \rangle = \int dl \frac{1}{R} = \int d\phi \frac{1}{R} = \theta.$$  \hfill (34)

In this picture, then, we are interested in the average value of the angle $\theta$. However, to follow the evolution of $\theta$ during a branching process, we need only four vectors, as can be seen from
Figure 1(c). Thus, the average angle can depend only on a four-dimensional subspace of the $(D-2)$-dimensional transverse space. Therefore, for $D \geq 6$ the average angle $\theta_*$ is constant as a function of $D$, and the extrinsic curvature per unit length is

$$r = \frac{\langle R \rangle}{l_*} \sim \frac{\theta_*}{l_*} \sim \frac{1}{\sqrt{\alpha'(D-2)}}. \quad (35)$$

The leading $D$ dependence of both the average length in (33) and the extrinsic curvature in (35) agree with what was found by Karliner et al. [8]. We have emphasized the application to black hole physics but branching diffusion evidently provides a useful physical picture in its own right of how the size and shape of strings develop as a function of mode cutoff.

4 Discussion

In four spacetime dimensions the string will spread to cover the area of the black hole horizon in a time

$$t_S \sim g^2 M^3, \quad (36)$$

where $M$ and $t$ are measured in Planck units and $g$ is the string coupling strength [6]. This is a short time compared to the black hole lifetime if the string is weakly coupled. In the branching diffusion picture the relation (36) easily follows from eq. (27).

Another important timescale is that on which the volume density of string at the centre of the distribution becomes $O(1/g^2)$ in string units. At this point string interactions can no longer be ignored and our calculations, which are all at the level of free string theory, no longer apply. The proper thickness of the stretched horizon remains of order one in string units while the average area occupied increases linearly with time. At the same time the average number of string bits grows exponentially so the time $t_I$ at which string interactions become important at the center of the distribution of string bits is:

$$t_I \sim M \ln \left( \frac{1}{g^2} \right). \quad (37)$$

For a macroscopic black hole this timescale is very much shorter than the spreading time $t_S$. An attractive possibility, pointed out in [6], is that string interactions prevent the volume density from exceeding $O(1/g^2)$ and that the central density will level off at that value. This

\footnote{If $D > 4$ the lifetime $t_L \sim M^{\frac{D-1}{2\alpha'}}$ is always long compared to $t_S \sim g^2 M^{\frac{1}{1-\alpha'}}$.}
volume density corresponds to an $O(1)$ area density of string bits on the stretched horizon measured in Planck units, the value suggested by the Bekenstein-Hawking entropy. At any rate, the high string density in the center does not affect the branching diffusion at the fringe of the distribution so our estimate (27) of the transverse spreading rate remains unchanged.

The aim of the present paper was to provide some additional support for the view that a stretched horizon made out of strings has a key role to play in resolving the black hole information paradox. There remain many open questions within this approach; for example, how to extend the calculations from Rindler space to finite mass black hole geometries. Another important issue to address is the underlying causal properties of string theory which allow the spread of information across the horizon, see e.g. the forthcoming work of Lowe et al. [14].

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