Modulino Dark Matter and the INTEGRAL 511 keV Line

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Abstract

In this paper we present a simple extension of the minimal supersymmetric standard model [MSSM] which “naturally” produces the INTEGRAL photon signal. The model can be embedded in an SU(5) grand unified theory [GUT] with gauge mediated SUSY breaking. The new ingredients are the addition of several MSSM singlets/moduli. While the masses of the singlets are at the weak scale, their mass splittings are suppressed by chiral symmetry breaking and naturally lie around an MeV. The decay of the heavier modulino to the lighter one with the associated production of electron - positron pairs explains the INTEGRAL signal. Finally, the detection of diffuse gamma rays from internal bremsstrahlung in the galactic halo would be a suggestive indication of dark matter decays associated with the 511 keV line, and is an unambiguous additional prediction of this model.
I. INTRODUCTION

The nature of dark matter remains one of the most pressing problems of cosmology and particle physics alike. Although many conventional dark matter candidates may be probed by particle colliders and direct-detection experiments, the most suggestive signals – particularly for dark matter candidates with extremely weak interactions – may come from indirect measurements. Indeed, the recent spate of astrophysical anomalies suggests that the most telling signatures of dark matter may come from the sky (for a comprehensive review, see [1] and references therein).

Although recent astrophysical anomalies measured by PAMELA, ATIC, FERMI, and HESS have drawn attention to dark matter physics above 100 GeV, there remain decisive signatures from much lower energies. The 511 keV line from the galactic center, as measured by the SPI spectrometer of the INTEGRAL satellite [2, 3, 4, 5], corresponds to a measured photon flux

$$\Phi_{\text{exp}} = (9.35 \pm 0.54) \times 10^{-4} \text{ph cm}^{-2}\text{s}^{-1}$$

with a large component from the galactic bulge. The emission is approximately spherically symmetric with a $\sim 6^\circ$ FWHM, potentially with additional unresolved components along the plane of the galaxy. The 511 keV line (with a 3 keV line width set by instrumental resolution) is dominated by $e^+ + e^-$ annihilations via positronium [6]. A variety of Standard Model processes have been suggested to explain the positron excess, including, e.g., production within Type Ia supernovae and low-mass X-ray binaries [4]. These explanations require some degree of speculative astrophysics in order to explain both the nature and intensity of the signal, and to date it remains unclear whether astrophysical explanations may suffice [5].

Thus far suggestions for origins of the 511 keV signal from physics beyond the Standard Model have largely focused around the annihilations or decays of light, MeV-scale dark matter (see, e.g.,[7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24]). These models confront a common challenge: additional MeV-scale sectors are not particularly natural and, if constituting cold dark matter, require no small degree of engineering to obtain acceptable relic abundance and phenomenology. A perhaps more natural scenario might involve TeV-scale dark matter candidates with MeV-scale splittings between components; the decays or annihilations of such dark matter may account for the 511 keV signal from
weak-scale hidden sectors \[25, 26, 27, 28, 29, 30\]. Nonetheless, the origin of the MeV-scale splitting among dark matter components itself requires further explanation.

In this paper we present a simple extension of the minimal supersymmetric standard model [MSSM] which “naturally” produces the INTEGRAL photon signal. The model can be embedded in an SU(5) grand unified theory [GUT] with gauge mediated SUSY breaking. The new ingredients are the addition of several MSSM singlets. While the masses of the singlets are at the weak scale, their mass splittings are suppressed by chiral symmetry breaking and naturally lie around an MeV.

II. MODEL

Consider a pair of gauge singlet chiral superfields \( \Phi_1, \Phi_2 \), coupled to the Higgs fields \( H_u, H_d \) and messengers \( X, \bar{X} \) via the superpotential interactions

\[
W = \lambda \Phi_1 H_u \bar{X} + \bar{\lambda} \Phi_2 H_d X
\]

and canonical kinetic terms. The \( X, \bar{X} \) are messengers of gauge mediation with the quantum numbers of \( H_u, H_d \), and assume a messenger mass \( M + F \theta^2 \) from supersymmetry breaking. In order to preserve unification, we may readily assume that the \( X \) and \( \bar{X} \) are part of complete \( 5 + \bar{5} \) messenger multiplets whose remaining components couple to the (heavy) Higgs triplets.\(^1\) We will be interested in messenger masses corresponding to intermediate-scale gauge mediation, with \( M \simeq 10^7 - 10^8 \) GeV. In order to obtain weak-scale soft masses for the fields of the Standard Model, this implies that the messenger fields experience SUSY-breaking around \( F \simeq 10^{13} \) GeV\(^2\) with \( \frac{F}{M} \simeq 10^5 \) GeV. The gauge singlets \( \Phi_1 \) and \( \Phi_2 \) can be thought of as moduli superfields whose only renormalizable couplings to the Standard Model are those listed above.\(^2\)

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\(^1\) We assume the GUT theory has a natural doublet-triplet splitting mechanism. In this case, in addition to the color triplets in the Higgs multiplets \( (T, \bar{T}) \) and the messenger triplets \( (X_T, \bar{X}_T) \), we have auxiliary triplets \( (T', \bar{T}') \) with the mass matrix in the basis \( (T, T', X_T) \) given by

\[
M_T = \begin{pmatrix}
0 & M_G & 0 \\
M_G & 0 & 0 \\
0 & 0 & M
\end{pmatrix}.
\]

Such a missing partner mechanism is natural in the context of orbifold GUTs.

\(^2\) Of course, one might imagine the presence of many additional light string-theoretic moduli in this theory with masses of order \( m_{3/2} \), subject to the usual cosmological constraints. However, in the cosmology...
Below the scale $M$, we can integrate out the messengers to give irrelevant interactions in the superpotential and Kähler potential of the form

$$W \supset -\frac{\lambda}{M} \Phi_1 \Phi_2 H_u H_d$$

$$K \supset \frac{|\lambda|^2}{M^2} \Phi_1^\dagger \Phi_1 H_u^\dagger e^{-2(g T_a V_a + \frac{1}{2} g' Y V_Y)} H_u + \frac{|\lambda|^2}{M^2} \Phi_2^\dagger \Phi_2 H_d^\dagger e^{-2(g T_a V_a + \frac{1}{2} g' Y V_Y)} H_d + ...$$

We are also interested in generating supersymmetric masses for the $\Phi_1$, $\Phi_2$ and a $\mu$ term for the Higgses, which we can assume to come from some Planck-suppressed operators in the superpotential:

$$W \supset \frac{S^2}{M_P} (\Phi_1^2 + \Phi_2^2) + \frac{\bar{S}^2}{M_P} H_u H_d + ZX \bar{X}$$

Here $S$, $\bar{S}$ obtain vacuum expectation values $\langle S \rangle \simeq \langle \bar{S} \rangle \simeq 10^{10}$ GeV to generate a $\mu$ term and supersymmetric mass terms of order 100 GeV. $Z$ is a hidden-sector singlet whose vev $\langle Z \rangle = M + F \theta^2$ breaks supersymmetry and imparts a mass to the messenger fields.

There arise a number of additional contributions to the masses of moduli and modulini when supersymmetry is broken by the $F$-term expectation value of $Z$. In addition to their supersymmetric masses $m^2 \sim |\langle S \rangle|^4/M_P^2$, the moduli $\phi_1, \phi_2$ obtain SUSY-breaking soft masses $m^2_{\phi_1} \simeq \frac{\lambda^2}{16\pi^2} F^2$, $m^2_{\phi_2} \simeq \frac{\bar{\lambda}^2}{16\pi^2} F^2$ of order 1 TeV (see Fig. 1). Such large masses lead the moduli to decay rapidly into modulini and MSSM fields well before BBN, rendering them both cosmologically safe and uninteresting.

The mass spectrum of the modulini $\psi_1, \psi_2$ is somewhat more interesting. The modulini obtain tree-level masses $m = \langle S^2 \rangle/M_P \simeq 10^2$ GeV from the VEV of $S$. In addition, a hard SUSY-breaking Dirac mass is generated radiatively from the $\mu$ term when supersymmetry is broken (see Fig. 1), and is of order $\delta m \simeq \frac{\lambda\bar{\lambda}}{16\pi^2} F^2 \mu$. The mass eigenstates are then $\psi_{A,B} = \frac{1}{\sqrt{2}} (\psi_1 \pm \psi_2)$ with masses $m_{A,B} = m \pm \delta m$. It is precisely this small splitting of the modulini mass eigenstates that will prove interesting for the 511 keV line.

This theory possesses a number of amusing continuous and discrete symmetries in addition to those of the Standard Model, including both a $U(1)_{PQ}$ symmetry and a $\mathbb{Z}_4$ symmetry under which $W \rightarrow -W$. This $\mathbb{Z}_4$ symmetry includes the familiar discrete $R$-parity that conventionally ensures the stability of the MSSM LSP and guards against proton decay. The discussed here these light string moduli are relatively uninteresting, and further use of “moduli” will generally denote the singlets $\Phi_1, \Phi_2$ except where otherwise noted.
FIG. 1: Diagrams generating a hard SUSY-breaking mass for $\psi_1\psi_2$ (left) and soft scalar masses for $\phi_1^*\phi_1, \phi_2^*\phi_2$ (right).

TABLE I: Fields & charge assignments. Note the fields $\chi + \bar{\chi}$ are a $5 + \bar{5}$ of SU(5) while the fields $X + \bar{X}$ are assumed to be the doublet members of a complete $5 + \bar{5}$.

|        | $SU(2)_L$ | $U(1)_Y$ | $U(1)_{PQ}$ | $Z_4$ |
|--------|-----------|-----------|-------------|--------|
| $H_u$  | □         | $\frac{1}{2}$ | 1           | 1      |
| $H_d$  | □         | $-\frac{1}{2}$ | 1           | $-1$   |
| $X$    | □         | $\frac{1}{2}$ | 0           | $i$    |
| $\tilde{X}$ | □       | $-\frac{1}{2}$ | 0           | $i$    |
| $Z$    | 1         | 0          | 0           | 1      |
| $\Phi_1$ | 1       | 0          | $-1$        | $i$    |
| $\Phi_2$ | 1       | 0          | $-1$        | $-i$   |
| $S$    | 1         | 0          | 1           | $-1$   |
| $\tilde{S}$ | 1      | 0          | $-1$        | $-1$   |
| $\chi$ | *         | *          | $\frac{1}{2}$ | 1      |
| $\bar{\chi}$ | *       | *          | $\frac{1}{2}$ | 1      |

appropriate charge assignments of the Higgs fields, messengers, and gauge singlets are shown in Table I. The $U(1)_{PQ}$ charges for quarks and leptons are $-\frac{1}{2}$, while the $Z_4$ quantum numbers are consistent with SU(5) with the 10 ($\bar{5}$) with charge $i$ ($-i$). Among other things, the expectation value $\langle \tilde{S} \rangle$ breaks the PQ symmetry of the theory, and generates an invisible axion with $f_a \simeq 10^{10}$ GeV. It is also worth noting that a tree-level Dirac mass for the $\Phi_{1,2}$ is forbidden by the symmetries of the theory.

The stability of the MSSM LSP is guaranteed by these symmetries. The stability of the
lightest modulino is guaranteed by a $\mathbb{Z}_2$ symmetry [moduli parity] where $\Phi_1, \Phi_2, X, \bar{X}$ are odd and all other fields are even. If a cosmological abundance of this modulino is generated in the early universe, it will remain as a component of dark matter in the present era.

III. COSMOLOGY

The thermal history of a theory with moduli coupled to the MSSM via modestly irrelevant operators is rather interesting. Scattering with the Higgs fields keeps the modulini in thermal equilibrium with the Standard Model down to relatively low temperatures. The cross-sections for Higgs-modulino scattering go like $\sigma v \sim \frac{1}{M^2}$, significantly smaller than weak-scale cross-sections. Modulini in thermal equilibrium freeze out when $n_{eq} \langle \sigma v \rangle \sim H$, which corresponds to a temperature $T_f \simeq m_\psi/8$. Their freezeout density is far too high due to the relative inefficiency of annihilations, and at freezeout corresponds to a thermal abundance $\Omega_\psi h^2 \simeq 10^6$. It is therefore necessary to dilute any initial thermal abundance of modulini. As is often the case for weakly-coupled fields with large equilibrium number density, it is therefore reasonable to imagine a scenario wherein the initial thermal abundance of modulini is diluted by inflation. The universe may then reheat after inflation to a temperature $T_R$ below the freezeout temperature of the modulini.

Of course, some thermal abundance of modulini will be produced by thermal scattering even if the reheating temperature is too low to produce modulini in equilibrium; the universe will be repopulated with modulini produced by scattering and decay processes of Standard Model fields in the thermal bath. It is therefore equally important that reheating not produce an excessive thermal abundance from scattering and decays of Standard Model particles.

It is relatively straightforward to compute the abundance of the modulini due to thermal production. The time evolution of the modulini number density $n_\psi$ (we need not distinguish between $\psi_A$ and $\psi_B$ here, since their mass splittings are sufficiently small to make their production rates virtually identical) is described by the Boltzmann equation,

$$\frac{dn_\psi}{dt} + 3Hn_\psi \approx 2\langle \sigma(hh \rightarrow \psi\psi)v \rangle n_h^2 + 2\langle \Gamma(h \rightarrow \psi\psi) \rangle n_h - \text{inverse}$$ \hspace{1cm} (6)

Assuming small density $n_\psi$, we can make the approximation $3Hn_\psi \approx 0$ and $n_{eq}^q \approx 0$. Then we have

$$\frac{dn_\psi}{dt} \approx 2\langle \sigma(hh \rightarrow \psi\psi)v \rangle n_h^2 + 2\langle \Gamma(h \rightarrow \psi\psi) \rangle n_h$$ \hspace{1cm} (7)
A thermal abundance of modulini is produced principally by Higgs scattering and decays; the scattering cross-sections into modulini are

\[
\sigma(h + h \rightarrow \psi_A + \psi_A)v = \frac{1}{256\pi} \left( \frac{s_\alpha^2 c_\alpha^2}{M^2} \right) \left( 1 - \frac{4m_A^2}{s} \right)^{3/2}
\]

(8)

\[
\sigma(h + h \rightarrow \psi_B + \psi_B)v = \frac{1}{256\pi} \left( \frac{s_\alpha^2 c_\alpha^2}{M^2} \right) \left( 1 - \frac{4m_B^2}{s} \right)^{3/2}
\]

(9)

Here \( s_\alpha, c_\alpha \) denote sin, cos of the angle determining the light neutral Higgs mass eigenstate, while \( s_\beta, c_\beta \) are the sin, cos of the angle \( \beta \) with \( \tan \beta = \langle H_0^u \rangle / \langle H_0^d \rangle \). For simplicity we have set \( \lambda = \bar{\lambda} = 1 \), although the more general case is straightforward. Assuming that \( m_h > 2m_A, 2m_B \), the decay rates are given by

\[
\Gamma(h \rightarrow \psi_A + \psi_A) = \frac{1}{256\pi} (s_\alpha s_\beta - c_\alpha c_\beta)^2 \frac{v^2}{M^2} m_h \left( 1 - \frac{4m_A^2}{m_h^2} \right)^{3/2}
\]

(10)

\[
\Gamma(h \rightarrow \psi_B + \psi_B) = \frac{1}{256\pi} (s_\alpha s_\beta - c_\alpha c_\beta)^2 \frac{v^2}{M^2} m_h \left( 1 - \frac{4m_B^2}{m_h^2} \right)^{3/2}
\]

(11)

Of course, if \( m_h < 2m_A, 2m_B \), the principle decays arise from other Higgs mass eigenstates such as the heavier CP even Higgs field \( H \); we will assume \( m_h > 2m_A, 2m_B \) for simplicity.

In order to solve the Boltzmann equation, we may define the modulino thermal production yield \( Y_\psi = n_\psi / s \), where \( s \) is the entropy density. The total yield is simply the sum of yields from scattering and decay, \( Y_\psi = Y_{\sigma hh}^\psi + Y_{\Gamma h}^\psi \), where

\[
Y_{\sigma hh}^\psi = \int_0^{T_R} dT \frac{2\sigma(hh \rightarrow \psi\psi)v^2 n_h^2}{sHT}
\]

(12)

\[
Y_{\Gamma h}^\psi = \int_0^{T_R} dT \frac{2\Gamma(h \rightarrow \psi\psi)n_h}{sHT}
\]

(13)

\[A \ posteriori, \] we will be interested in reheating temperatures on the order of a few GeV, so we can assume \( m_h \gg T_R \). In this case the Higgs is essentially nonrelativistic, with number density well approximated by

\[
n_h = 4 \left( \frac{m_h T}{2\pi} \right)^{3/2} \exp[-m_h/T]
\]

(14)

We also have \( s = (2\pi^2/45)g_\ast T^3 \) and \( H = \frac{\pi}{\sqrt{90}} \sqrt{g_\ast T^2/M_p} \). Here \( M_p \) is the reduced Planck mass, \( 2.43 \times 10^{18} \) GeV. Having determined the thermal yield \( Y_\psi \), we may readily determine the modulino relic abundance from thermal production via

\[
\Omega_\psi h^2 = m_\psi Y_\psi \frac{s(T_N)}{\rho_c h^2}.
\]

(15)
Reheating temperature $T_{RH}/(1\text{GeV})$

| 0.0001 | 0.001 | 0.01 | 0.1 | 1 | 10 | 100 |
|---|---|---|---|---|---|---|

Moduli abundance $\Omega\psi h^2$

FIG. 2: Relic abundance $\Omega\psi h^2$ as a function of reheating temperature $T_R$. From blue to red, lines correspond to $m_\psi = 10^{-1}, 1, 10, 50\text{ GeV}$ for $m_h = 150\text{ GeV}$ and $M = 10^8\text{ GeV}$.

At reheating temperatures of a few GeV, scattering processes are terribly inefficient – both because thermal energies are well below threshold, and because of the $n_h^2$ Boltzmann suppression – so that decay processes dominate over annihilations. Nonetheless, production via decays is still relatively effective and constrains $T_R \lesssim 10\text{ GeV}$ in order to avoid overproduction of modulini, as seen in Fig. 2.

A. Thermal inflation

Supergravity theories generally suffer from light moduli with masses of order the weak scale and long lifetimes. These light moduli tend to dominate the energy density of the universe and decay after the epoch of nucleosynthesis resulting in major cosmological problems \cite{31, 32}. Thermal inflation \cite{33} is a mechanism introduced to solve the cosmological moduli problem. The universe is assumed to go through an initial epoch of inflation followed by reheating thus solving the homogeneity, isotropy and flatness problems and generating the primordial density fluctuations. Then at a later time the universe undergoes another period of thermal inflation with of order 9 e-foldings of expansion, hence diluting the light moduli. The main ingredient in thermal inflation is a “flaton” field with mass $m$ of order $10^2$ to $10^3\text{ GeV}$ and a potential whose minimum is located at a scale $M_f$ of order $10^{10}$ to $10^{11}\text{ GeV}$. This field induces a period of late time inflation with the flaton field stuck at the origin in field space and dominating the energy density of the universe starting at a temperature $T$
of order $10^6$ to $10^7$ GeV. The inflation period then ends at the critical temperature $T_C \sim m$, followed by a period of entropy production ending with a reheat temperature $T_D \sim 1$ to $10^2$ GeV. The reheat temperature is safely above the epoch of nucleosynthesis. However, with such a low reheat temperature one must necessarily be concerned that the baryon number density has also been severely diluted. Fortunately, it has been shown \cite{34, 35} that a short period of Affleck-Dine leptogenesis via preheating near $T_C$ can generate a sufficient baryon number asymmetry.

It is remarkable that our model, for completely independent reasons, contains all the ingredients required by \cite{34, 35}. In fact, we can interpret the field $\bar{S}$ (Eqn. 5) as the flaton. It can couple to new fields $\chi(= 5) + \bar{\chi}(= \bar{5})$ that obtain mass of order $\langle \bar{S} \rangle = M_f$ via the interaction

$$W \supset S\chi\bar{\chi}. \quad (16)$$

It is this coupling which was shown to generate the negative mass squared for the flaton at the origin. Note that the $PQ$ symmetry is also broken at the scale $M_f = f_a$.

Finally at the reheat temperature $T_D$ we have already shown that we obtain a satisfactory abundance of modulinos through thermal processes. These are then available to be a major component of the dark matter of the universe and, as we now show, they can produce the INTEGRAL signal.

**B. Dark Matter**

While the modulini $\psi_A, \psi_B$ may obtain a significant cosmological abundance from reheating, it is by no means necessary for them to constitute the sole components of dark matter. Indeed, obtaining the measured dark matter relic abundance $\Omega h^2 \simeq 0.1$ for these modulini would involve a rather careful, and perhaps unnatural, tuning of the reheating temperature. A smaller abundance of modulini may still account for the observed 511 keV line, while allowing a broader range of reheating temperatures. The remaining dark matter relic abundance may be accounted for by additional dark matter candidates. Indeed, our theory possesses an abundance of such candidates. The gravitino mass in this theory may range from $m_{3/2} \simeq 10 \text{ keV} – 100 \text{ MeV}$ and comprise a component of dark matter as
The low reheating temperature following thermal inflation ensures that gravitinos are not overproduced by thermal processes, and may instead have an abundance close to the observed amount.

The theory also possesses an invisible axion and axino. The axino mass arises through supersymmetry breaking, and is highly dependent upon the details of the hidden sector. In models of gauge-mediated SUSY-breaking with the gravitino mass range discussed above, the axino mass may range from eV to GeV, and certainly may be lighter than the gravitino \[36, 37\]. In this case, the axino is likely to be the MSSM LSP. Provided that it is heavier than a few eV, the axino will be a warm or cold component of dark matter. With such low reheat temperatures, the thermal abundance of axinos is likely too small to account for a significant fraction of dark matter \[38\]. However, for masses above an MeV, the axinos may inherit a suitable freezeout relic abundance from the decay of heavier species such as the neutralino \[39\]. In such a scenario, the axino may naturally account for a significant fraction of the dark matter relic abundance without fine-tuning the reheating temperature. Of course, the axino may also be quite heavy in gauge mediation, leaving the axion or various light string moduli as additional candidates \[40\].

The precise spectra and abundances of additional dark matter candidates depends strongly on the details of the hidden sector and PQ symmetry breaking, but there remain a plethora of additional candidates that may obtain significant abundances from thermal production or the freezeout abundance of heavier species. Thus it is not unreasonable for the modulini $\psi_A, \psi_B$ to constitute part or all of the observed dark matter abundance without prohibitive fine-tuning of the reheating temperature after thermal inflation. Of course, whatever the dark matter candidate(s), the prospects for direct detection are somewhat bleak; modulino, axino, or gravitino dark matter are unlikely to produce a measurable nuclear recoil signal in direct detection experiments.

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3 The lower bound arising if $F = F_0$, i.e., the primordial SUSY-breaking scale is the same as that felt by the gauge messengers. Of course, it is entirely possible that $F \ll F_0$, in which case the upper bound comes from flavor constraints.
IV. DECAYS OF THE MODULINO: INTEGRAL

Thus far we have considered a model of gauge mediation incorporating additional singlets \( \Phi_1, \Phi_2 \) that couple to the MSSM via tree-level interactions with messengers and the Higgs fields \( H_u, H_d \). The fermionic modulini of these singlets obtain both supersymmetric and supersymmetry-breaking masses, which result in splittings of \( \mathcal{O}(\text{MeV}) \). The lightest of these modulini is stable, and both modulini may obtain a sensible cosmological abundance from thermal production after reheating. Due to the smallness of their mass splittings, it is conceivable that the decay of the heavier modulino into the lighter modulino may occur sufficiently slowly to explain the 511 keV excess measured by INTEGRAL.

Before we discuss the particulars of the decay process, it is worth considering how the decays of dark matter might explain the observed excess. The dark matter distribution in the galactic halo may be parameterized as a function of \( r \) by

\[
\rho(r) \propto \frac{1}{(r/a)^\gamma [1 + (r/a)^\gamma (\beta - \gamma)/\alpha]}
\]

for halo profile parameters \( \alpha, \beta, \gamma \); here \( a \) is the distance from the galactic center where the power law breaks. Near the galactic bulge, \( \rho(r) \propto (r/a)^{-\gamma} \). Following [9], the current SPI data with FWHM of 6° is well fit by a cusped profile with \( \gamma \approx 1.6 \), fairly consistent with results of high-resolution N-body simulations [41, 42].

Normalizing the halo profile to the local dark matter density, one thus finds, close to the galactic center,

\[
\rho(r) \simeq \frac{0.3 M_\odot}{\text{pc}^3} \frac{1}{(r/1 \text{ kpc})^{1.6}}
\]

The total mass within the 6° circle of the INTEGRAL signal is thus

\[
M_{\text{INT}} = \int_0^{425 \text{ pc}} \rho(r) 4\pi r^2 dr \simeq 9.1 \times 10^{65} \text{ GeV}.
\]

The number of gamma rays contributing to the 511 keV line per annihilated non-relativistic positron is given by \( 2(1 - 3f/4) \), where \( f = 0.967 \pm 0.022 \) is the positronium fraction; consequently, the decay rate producing positrons is 3.6 times larger than would be deduced from the gamma ray flux itself [43]. With this in mind, matching the rate of decays to the observed 511 keV flux yields

\[
\frac{M_{\text{INT}}}{m_{d\text{m}} \tau_{d\text{m}}} \sim 3.6 \times \frac{1}{2} \Phi_{\exp} 4\pi R_{GC}^2
\]
where $R_{GC} \simeq 2.5 \times 10^{22}$ cm is the distance of the Earth from the galactic center. This suggests the lifetime for dark matter decays in order to provide the INTEGRAL signal is

$$\tau_{dm} \simeq 7 \times 10^{22}/m_{dm}(\text{GeV}) \text{s.} \quad (21)$$

Of course, it is not strictly necessary for the decaying particle to comprise the entirety of dark matter in the galaxy; for a decaying particle with abundance $\Omega_\psi < \Omega_{dm}$, the lifetime required to explain INTEGRAL is instead

$$\tau_\psi \simeq 7 \times 10^{22} \left( \frac{\Omega_\psi}{\Omega_{dm}} \right) \frac{1}{m_\psi(\text{GeV})} \text{s.} \quad (22)$$

provided that the distribution of the decaying particle roughly tracks the dark matter distribution discussed above.

The INTEGRAL signal also places rather significant constraints on the injection energy of the positrons and electrons which produce the observed gamma rays. Taking into account energy losses as well as diffusion and delay, the annihilation of positrons produces gamma rays at or below 511 keV. However, the emission of gamma rays by positrons prior to annihilation may place a stringent constraint on the injection energy. Firstly, the internal bremsstrahlung radiation associated with positron production (a QED correction to the decay process producing positrons) conflicts with COMPTEL and EGRET diffuse gamma-ray constraints unless injection energies are $< \sim 20$ MeV [44]. Secondly, the inflight annihilation of relativistic positrons with electrons in the interstellar medium – a fate that may befall as many as 20% of energetic positrons – places a much more stringent constraint of $\lesssim 3$ MeV on the injection energy of individual positrons in order to avoid exclusion by COMPTEL and EGRET bounds [43]. It should be noted that this bound assumes monoenergetic injection of positrons, and may be slightly relaxed by the energy distribution of three-body phase space. Nonetheless, the limits from internal bremsstrahlung and inflight annihilation significantly proscribe the types of dark matter decays that could explain the observed signal from INTEGRAL. Positron injection energies of $\lesssim 3$ MeV from decaying dark matter allow two possibilities: that the dark matter mass itself is $\mathcal{O}(\text{MeV})$, or that dark matter of mass $\gtrsim \mathcal{O}(\text{MeV})$ is decaying into another state with a mass difference $\delta m \sim \mathcal{O}(\text{MeV})$. Our model is suggestive of the latter possibility: a modulino of mass $m_\psi \sim 100$ GeV (which may provide all or part of the dark matter in our galaxy) decaying in a nearly-degenerate modulino with a mass splitting of order $\sim \text{few MeV}$. 

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The leading contribution to the decay $\psi_A \rightarrow \psi_B + e^+ + e^-$ comes from the Kähler terms

$$K \supset \frac{1}{M^2} \Phi_A \Phi_A \Phi_H^1 H_u e^{-2(gT a V_a + \frac{1}{2} g' Y V_Y)} H_d + \frac{1}{M^2} \Phi_B \Phi_B H_d^1 e^{-2(gT a V_a + \frac{1}{2} g' Y V_Y)} H_d$$  \hspace{1cm} (23)

These Kähler terms result in interaction terms between the modulini and the $Z$ boson which, in the mass eigenbasis of the modulini, take the form

$$V \supset \frac{M_Z v}{4M^2} \left[ (s_2^2 - c_2^2) \psi_A \sigma^\mu \psi_A^\dagger Z^\mu + (s_2^2 - c_2^2) \psi_B \sigma^\mu \psi_B^\dagger Z^\mu + \psi_A \sigma^\mu \psi_B^\dagger Z^\mu + \psi_B \sigma^\mu \psi_A^\dagger Z^\mu \right]$$  \hspace{1cm} (24)

The smallness of the splitting between $\psi_A$ and $\psi_B$ constrains the $\psi_A$ to decay via an off-shell $Z$ into electrons and positrons. The decay rate for this process is given by

$$\Gamma(\psi_A \rightarrow \psi_B + e^+ + e^-) = \frac{1}{240\pi^2 s_w^2 m_W} \left[ \left( -\frac{1}{2} + s_w \right)^2 + s_w^4 \right] \frac{(\delta m)^5}{M^4}$$  \hspace{1cm} (25)

Using $\alpha(m_Z) \approx 1/128, s_w^2 \approx 0.231, m_W \approx 80.4 \text{ GeV}, v = 174 \text{ GeV}$, the lifetime of the $\psi_A$ is simply

$$\tau_{\psi_A} \approx 7.83 \times 10^{-20} \frac{M^4}{(\delta m)^5} \text{s}$$  \hspace{1cm} (26)

For $m_{\psi_A} \approx 50 \text{ GeV}$ and messenger scale $M \approx 10^8 \text{ GeV}$, the INTEGRAL signal may be matched for $\delta m \approx 20 \text{ MeV}$, assuming dark matter relic abundance for the $\psi_A$. Slightly smaller values of $M$ readily accommodate splittings $\delta m = 1 - 10 \text{ MeV}$ consistent with observational constraints. Of course, it is not necessary for the modulini to comprise the entirety of the dark matter relic abundance; moreover the INTEGRAL signal may be produced for a range of $m_{\psi_A}, \delta m, M,$ and $\Omega_{\psi} h^2$ (see Fig. 3).

One might also be interested in decays via superpotential terms such as

$$W \supset -\frac{\lambda}{M} \Phi_1 \Phi_2 H_u H_d$$  \hspace{1cm} (27)

which appears to generate decays into $e^+ e^-$ via Higgs exchange. However, this term results in an effective interaction of the form

$$\mathcal{L} \supset \frac{1}{4} \frac{\lambda}{M} \frac{m_e}{m_h^2} (s_\alpha^2 \tan \beta + s_\alpha c_\alpha) \psi_1 \psi_2 \bar{e} e.$$  \hspace{1cm} (28)

For the mass spectrum considered here, where the mass eigenstates $\psi_A, \psi_B$ are maximally-mixed combinations of the fields $\psi_1, \psi_2$, this interaction does not lead to a decay between the two mass eigenstates. For more general theories where this is not the case, the decay mediated via the Higgs results in a coincidentally similar rate to that via $Z$ bosons considered above, despite the different parametric dependence on $M$. 

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FIG. 3: Messenger mass required to explain the INTEGRAL signal as a function of reheating temperature (and hence abundance). The region of interest lies between the two blue lines; the upper line corresponds to $\delta m = 10$ MeV, the lower to $\delta m = 1$ MeV. These bounds are only weak functions of the modulino mass; the width of each blue lines indicates the variation between $m_{\psi} = 1 - 50$ GeV. The red dashed line denotes $\Omega_{\psi} h^2 = 0.1$ for $m_{\psi} = 50$ GeV (and $\Omega_{\psi} h^2 < 0.1$ to the left of the line), while the orange dashed line denotes $\Omega_{\psi} h^2 = 0.1$ for $m_{\psi} = 1$ GeV.

V. DIFFUSE GAMMA RAY PRODUCTION

Any theory giving rise to the 511 keV signal via dark matter decays or annihilations inevitably makes an additional prediction: potentially significant contributions to the isotropic diffuse photon background [iDPB] due to internal bremsstrahlung from the final-state electrons and positrons. Such an isotropic, diffuse gamma ray signal coming from the galactic halo would be a suggestive signal of dark matter decays.\textsuperscript{4}

The isotropic gamma-ray background has been measured at high galactic latitudes ($\gtrsim 10^\circ$) by COMPTEL (the Compton Imaging Telescope) and SMM (the Solar Maximum Mission) over ranges $0.8 - 30$ MeV \textsuperscript{45} and $0.3 - 7$ MeV \textsuperscript{46} (respectively), while INTEGRAL has measured a diffuse photon background over the range $5 - 100$ keV \textsuperscript{47}. The analysis of iDPB in the MeV region is made difficult by both instrumental and cosmic ray backgrounds and requires careful analysis. While the iDPB is well-explained at energies below a few hundred keV (where active galactic nuclei dominate) and above 10 MeV (where

\textsuperscript{4} As opposed to dark matter annihilation which would be confined predominantly to the galactic center.
blazars may account for the measured signal), there is currently no known astrophysical source capable of accounting for the entirety of the observed iDPB between 1 and 5 MeV.\textsuperscript{5} If anything, the observed iDPB between 1 and 5 MeV may be indicative of new physics such as the decay or annihilation of dark matter \cite{27}.

The internal bremsstrahlung signal from dark matter decays in the galactic halo may be readily estimated from \cite{44}, and falls well within the observed iDPB signal in the MeV range \cite{49}. However, future measurements of the isotropic gamma ray background, coupled with an improved understanding of astrophysical sources in the MeV range, may significantly improve these bounds. The detection of diffuse gamma rays from internal bremsstrahlung in the galactic halo would certainly be a suggestive indication of dark matter decays associated with the 511 keV line, and is an unambiguous additional prediction of this model.

VI. CONCLUSIONS

We have considered a relatively minimal extension of the MSSM with gauge mediated supersymmetry breaking, wherein additional MSSM gauge singlet modulinos may account for some or all of the observed dark matter. Natural SUSY breaking-induced mass splittings between the modulini are of the appropriate scale to provide both the decay rate and positron injection energy required to explain the 511 keV signal. The cosmology of such a model is no different from that required to solve the gravitino and moduli problems of gauge-mediated supersymmetry breaking, and coincidentally produces a suitable cosmological abundance of the modulini. While the lightest modulino is stable, the modulini need not comprise the entire dark matter of the galaxy; gravitinos or axinos may also constitute a sizable fraction of dark matter without undue tuning of the reheating temperature. Whatever the exact composition of dark matter, the prospects for direct detection are invariably bleak; the interactions of the modulini and axino are far too small to be measured above irreducible backgrounds in nuclear recoil experiments. Additional signatures of this scenario must come from the sky; internal bremsstrahlung from the decay of the heavier modulino inevitably produces diffuse gamma rays in the galactic halo that may be measured by future experiments.

\textsuperscript{5} Type Ia supernovae do contribute gamma rays below 5 MeV, but the most recent astrophysical data suggest that they cannot account for the entire spectrum \cite{48}.
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