Parametric Design on Solving the Maximum Deformation of Traction Helical Gear on Locomotive

Haichun Ding¹,*, Lei Lei²,ᵃ, Meichao Qin²,ᵇ and Guang Wang¹,ᶜ

¹Shanghai Branch, TianDi Science & Technology Co., Ltd. Shanghai, China
²School of Mechanical Engineering, Dalian JiaoTong University, Dalian, China

*Corresponding author e-mail: dhchaian666@163.com, ‡leileidalian@163.com,
ᵇqinmeichao.dl@crrcgc.cc, ‰1253437676@qq.com

Abstract. The helical gear for locomotive traction has a popular application in the locomotive transmission system due to its excellent transmission performance. The maximum amount of deformation needs to be taken into consideration when performing the tooth profile modification on helical gear. The tooth profile design and corresponding calculation process tend to be tedious since the variation of the contact ratio of gear might lead to the change of logarithm in meshing tooth. In this paper, starting from the effect of the contact ratio on the logarithm of meshing teeth, the mesh-stiffness of helical gear can be solved by cross-section deformation formula based on the similarity of helical gear and spur gear. And then the general calculation formula governing the variation on the length of contact line is derived regarding the moment when gear tooth just engages. On the basis of actual production requirements, the solution equation for inter-tooth load distribution coefficient is derived. As a result, the general mathematical model for maximum deformation of helical gear which can perform parameterized calculation is established. The theory provides a significant theoretical guide in the field of tooth profile modification on locomotive traction helical gear.

1. Introduction
With the prosperous market of high-speed train, traction gear which plays a critical role in the operation of high-speed stability of the train is becoming a popular topic in the field of railway. Compared with spur gear, the helical gear has significant advantage in the transmission stability due to its continuity of meshing stiffness. The critical issue of the research of the maximum deformation of the helical gear is to determine the load distribution and meshing stiffness of gears. The meshing stiffness of gear has become a major problem in the field of dynamic excitation due to its time-dependent property and analytical complexity. Especially for helical gear, considering the meshing style of its spatial spiral distribution and nonlinear profile, the complexity of solving the meshing stiffness has reached a whole new level. Integration and the finite element analysis are two commonly used methods that based on the theory of elastic deformation. However, these two methods are not suitable for parameters optimization design due to the complexity of calculation process and requirement of high configuration computer, which could prolong the product development cycle and increase the time costs. Up until now, there is no general algorithm for the maximum deformation of
the helical gear, the variation of the contact ratio in design process and the corresponding change of logarithm in meshing tooth tends to make the calculation procedure more complicated. In this paper, an efficient and convenient parameterization method was proposed to calculate the maximum amount of deformation based on logarithm of meshing tooth with the consideration of individual differences. The calculation of the maximum deformation with the contact ratio lying in arbitrary effective interval is realized. The accuracy of this theory is verified with a certain type of locomotive traction helical gear, corresponding software package is developed based on Visual Basic. This algorithm provides a significant theoretical guide in the field of tooth profile modification on traction helical gear.

2. Parametric Design of Maximum Deformation during the Engagement of Helical Gear

According to the formula of gear deformation, the main factors that affect the amount of deformation include loadings, working face-width and meshing stiffness. Even for the same helical gear, the number of teeth that simultaneously participate in the engagement during the transmission might be different. This difference tends to affect the inter-tooth load distribution coefficient and gradually the calculation of gear deformation. As for different gears, the diversity of those contact ratios might lead to the fact that the values of contact ratio fall into different intervals of adjacent natural numbers. Correspondingly there might be a large difference in the amount of deformation. In order to facilitate the realization of the parametric design, it becomes a critical issue to find patterns that rules the gear deformation calculation.

2.1. The Influence of the Total Contact Ratio of Helical Gear on the number of Meshing Teeth

2.1.1. Comparison and connection between transverse contact ratios at different intervals. The spatial distribution pattern of helical gears determines that it is necessary to study the contact ratio from a multi-dimensional point of view. When the transverse contact ratio of helical gears is between \( n \) and \( n+1 \), the actual line of engagement can be decomposed into \( 2n+1 \) section. When the meshing process is locates in \( n+1 \)-tooth zone of action, the length of these intervals can be expressed:

\[
L_{c(n-n+1,n+1)} = (\varepsilon_a - n)p_{bt} \tag{1}
\]

The length of remaining intervals is expressed as Eq. (2):

\[
L_{c(n-n+1,n)} = (n+1-\varepsilon_a)p_{bt} \tag{2}
\]

Where \( p_{bt} \) is defined as transverse base pitch.

According to the basic characteristics of actual line of action, it can be known that there is a base pitch between every meshing point. Therefore, for subsequent solution of load distribution between teeth, it can be simplified as the corresponding distribution of points on line of action. The load distribution at any meshing point on line of action can be determined by the proportion of the relationship after finding the position of the corresponding point.

2.1.2. The result of the joint action of transverse ratio and overlap ratio. For helical gear, transverse ratio \( \varepsilon_a \) is one of main parameters that determine the number of teeth which mesh simultaneously in plane of action. It is also necessary to take the influence of overlap ratio \( \varepsilon_{ol} \) into consideration. In order to clearly and intuitively study the meshing process and characteristics of helical gear, the plane of action of the helical gear is spread in the direction of the base circle. As shown in Fig.1, at this moment, the total length of the line of contact is at its minimum value.

The length and width of rectangle \( ABCD \) in Fig.1 are transverse base pitch \( p_{bt} \) and axial pitch \( p_{ba} \) respectively. The diagonal line \( BD \) of the rectangular \( ABCD \) is the line of contact of the latter pair of
gears, the point E represents the boundary condition of the actual zone of action. In most cases, the transverse contact ratio and axial contact ratio are not integers and the vertex E of zone of action falls into the area of rectangular $ABCD$; The vertex E falls on the rectangular edge only when the transverse contact ratio or overlap contact ratio are integers. The relative position of point E and diagonal $BD$ affects the length of the line of contact of gear pair. And it is characterized by the intersection of diagonal $BD$ in rectangular $ABCD$ and the zone of action $EFGH$. The length of the line of contact is determined by the sum of the fractional parts of transverse contact ratio and axial contact ratio. As shown in Fig.1(a), the sum of the fractional part $\alpha \epsilon$ of transverse ratio $\alpha$ and the fractional part $\beta \epsilon$ of axial ratio $\beta$ is less than 1, that is, $[\alpha \epsilon] + [\beta \epsilon] < 1$, it is known from the characteristics of the distribution of helix that the vertex E of plane of action can only fall within the area of the triangle $\triangle BCD$. As shown in Fig. 1(b), the vertex E of plane of action goes beyond the diagonal line $BD$ and falls within the area of triangle $\triangle ABD$. At this moment, the common part of the dash line $BD$ and plane of action is also meshing pair.

![Figure 1. Diagram of the zone of action and the variation of length of line of action of helical gear.](image1.png)

2.2. The Establishment of the Mathematical Model of the Line of Contact that Meshing Simultaneously and the Total Width of Working Face

There may be multiple pairs of gears meshing simultaneously in helical gear during transmission process. The length of the line of contact of each gear pair may also be different. It need to determine the worst condition of average loading of helical gear in the process of meshing transmission as for the profile modification of helical gear. The schematic diagram of the variation of the line of contact in area of action is shown in Fig. 2. This is the moment when the length of the total contact line is at its minimum value.

![Figure 2. Zone of action of helical gear.](image2.png)
The special case of total contact length is extended to the general situation. The starting point of engagement is defined as the origin of coordinates. A general formula for calculating the length for contact line of helical gears with single pair of teeth is obtained.

\[ L(i) = \begin{cases} 
  \frac{xb}{\varepsilon \beta \cos \beta_k}, & 0 \leq x_i \leq \varepsilon_p \beta_k \\
  \frac{b \cos \beta_k}{\varepsilon \beta \cos \beta_k}, & \varepsilon_p \beta_k < x_i \leq \varepsilon_p \beta_t \\
  \frac{(c \cdot \beta_k - x_i)b}{\varepsilon \beta \cos \beta_k}, & \varepsilon_p \beta_t < x_i \leq \varepsilon_p \beta_u \\
  0, & \text{other} 
\end{cases} \quad (3) \]

Where: \( x_i \) is the distance between projections of the point of contact line on meshing zone and starting point.

To solving the time-varying meshing stiffness of helical gears treated by slicing conveniently, the equivalent tooth profile method is used; the mathematical model can be simplified appropriately. The node position is taken as the coordinate origin, assuming that the right side of the origin is positive and the left side is negative. The simplified mathematical model is as follows:

\[ L(i) = \begin{cases} 
  \frac{pe - x_i}{\sin \beta_k}, & b \cdot \tan \beta_k \leq x_i \leq pe \\
  \frac{b \cdot \cos \beta_k}{\sin \beta_k}, & -ap \leq x_i \leq b \cdot \tan \beta_k \\
  \frac{b \cdot \tan \beta_k - [ap + x_i]}{\sin \beta_k}, & (b \cdot \tan \beta_k + ap) \leq x_i \leq -ap \\
  0, & \text{other} 
\end{cases} \quad (4) \]

Where: \( pe \) is the distance between nodes and meshing termination points, \( ap \) is the distance between nodes and the initial point of meshing.

In order to make the result more accurate, the influence of meshing stiffness to load distribution should be considered in follow-up. The formula for calculating the total length of contact line is expressed as Eq. (5):

\[ L_L = \sum_{i=1}^{n} L(i) \quad (5) \]

2.3. Calculation Flow of Time-dependent Meshing Stiffness

2.3.1. Method for the calculation of meshing stiffness. Time-dependent meshing stiffness of traction gear is the core theory for strength calculation and dynamics study of locomotive transmission system. The accuracy of stiffness calculation has a great influence on the load capacity of gear and load distribution coefficient between teeth. For the time being, when calculating time-dependent meshing stiffness of gear, the common method is firstly to solve the deformation of a single gear. The key idea for solving the deformation of gear by numerical method is material mechanics method, which models gear teeth as the structure of cantilever beam and sequentially solves the quantity of each part. The representative formula is Ishikawa formula.

2.3.2. Intrinsic relationship of meshing stiffness between helical gear and spur gear. Due to the existence of helical angle, the inclination of line of contact in meshing process and the transmission characteristics of non-uniform distribution of load, it is essentially determined that helical gear cannot be referenced to spur gear which is simplified into plane model with two-dimensional to conduct stiffness analysis. However, considering that formation mechanism of profile in helical gear is similar to that of spur gear, it is also an effective way to divide helical gear along the direction of face width into a number of spur gear with same transverse parameters and thin thickness. Since the thickness of
helical gear is thin enough, it can be approximated as spur gear, as shown in Fig.3. In this way, helical gear can be considered as a gear form composed of a series of spur gears, each small spur gear is ahead of the previous one in the phase position, and the formula for $\theta$ is expressed as Eq. (6):

$$\theta = \frac{AB}{B \tan \beta} = \frac{B \tan \beta}{N \beta}$$  \hspace{1cm} (6)

Where: $r_b$ is base circle radius of driving gear.

When $N$ approaches to infinity, a relatively accurate helical gear can be formed by superposition of each segment of small spur gears. It can be seen from the time-varying and periodicity of meshing stiffness in spur gear that, the meshing stiffness of helical gear has similar properties. In order to solve the total meshing stiffness of helical gear, the following equation which expressed as Eq. (7) is taken into account, where $n$ represents the number of meshing teeth participating in meshing simultaneously.

$$k = \sum_{j=1}^{N} \left[ \frac{1}{k_{v,1}} + \frac{1}{k_{v,2}} + \frac{1}{k_{v,3}} + \frac{1}{k_{v,4}} + \frac{1}{k_{v,5}} + \frac{1}{k_{v,6}} \right]$$  \hspace{1cm} (7)

Where: $i$ is Meshing teeth with number; 1, 2 is Suffix which represent driving gear and driven gear, $k_{v,i}$ is Bending stiffness of rectangular part in Ishikawa formula, $k_{v}$ is Bending stiffness of trapezoidal part, $k_c$ is contact stiffness, $k_s$ is Shear stiffness, $k_G$ is Elastic inclined stiffness of matrix.

**Figure 3.** Schematic diagram of phase difference model of the slicing of helical gear.

By combining the theory of deformation of variable cross-section in cantilever beam with the concept of calculus summation in higher mathematics, time-varying meshing stiffness of helical gear can be solved by the method based on numerical analysis. This way not only can significantly improve the efficiency in solving the time-varying meshing stiffness of helical gear and guarantee the accuracy in calculation of the meshing stiffness model, but also facilitate the realization of the parametric design.

2.4. Calculation of Load Distribution Coefficient between Helical Gear Teeth

Since there may be multiple pairs of gears meshing in simultaneously in helical gear during transmission process, the time-varying of length of single-tooth line of contact determines that it is not always distributed along direction of full-face width. The inter-tooth distribution of load at a certain moment is not representative and it cannot completely reflect the distribution state of load during entire meshing process. Therefore, load distribution coefficient needs to be calculated at different meshing positions of contact. The calculated results are fitted by the curve equation, which can directly reflect load distribution of the helical gear during the entire meshing process.

According to the formula of double-tooth zone of action extended to the general situation, the general formula of load distribution coefficient between teeth before modification is deduced.
\[
\phi = -\frac{\sum_{j} k_{ij} F_j}{\sum_{j} k_{ij}}
\]

(8)

2.5. **Solution for the Maximum Deformation of Helical Gear during the Process of Transmission**

The load distribution coefficient between the teeth is analyzed on the stage of just meshing in of helical gear as the research object. Since the length of line contact is zero when the first pair of gear just enters the meshing, it can be approximately considered that this gear pair has not yet come into contact with each other and the load is zero. Analysis on contact strength would not be necessary at this moment. Then pattern of load distribution coefficient between the teeth of helical gear is further studied, when the position where the maximum deformation is reached, and corresponding working face width is known. And the value of the load and meshing stiffness is determined. The equation of the maximum deformation during the transmission process of the helical gear is expressed as Eq. (9):

\[
\delta = \frac{F_n}{bc}
\]

(9)

Where: \(b\), \(F_n\) and \(c\) are the working face width at the position where the deformation is largest, the magnitude of load and meshing stiffness respectively.

3. **Program Developments and Example Verification**

The prerequisite for the calculation of load distribution coefficient of helical gear is to find the interval of natural number where the contact ratio lies and to determine upper and lower bounds of the number of the meshing tooth. Considering that angle of load action might change through trigonometric function with the position of base pitch as the boundary point when calculating stiffness, the key step in solution process is to find the relative position of meshing point and node of base pitch.

When the meshing stiffness of all gear pairs of helical gear has been solved, distribution of load on gears can be solved according to the conservation equation of forces and compatibility equation of deformation. The amount of maximum deformation can be calculated according to the formula of deformation ultimately, system development flow of calculating maximum amount of deformation of gear teeth is shown as in Fig.4, and Fig.5 is the solution program developed by VB.

![Figure 4](image-url)
3.1. Theory Analysis of the Case

In this paper the variation pattern of the length of line of contact is studied from the case of helical gear of a certain type of locomotive. Table 1 shows the basic design parameters of helical gear.

| Heading                      | Driving       | Driven       |
|------------------------------|---------------|--------------|
| Module m                     | 9.4832        |              |
| Pressure angle $\alpha$      | 22.5          |              |
| Helix angle $\beta$          | 7             |              |
| Face-width b                 | 127           | 127          |
| Number of teeth $Z$          | 16            | 91           |
| Center distance $\alpha$     | 515           |              |
| Modification coefficient     | 0.333         | 0.0803       |

Transverse ratio $\varepsilon_\alpha$ / Overlap ratio $\varepsilon_\beta$ / Total contact ratio $\varepsilon$ = 1.441/0.520/1.961

It can be seen from the table that the value of transverse overlap ratio of traction gear of 4400HP locomotive is 1.441, axial overlap ratio is 0.52. The total contact ratio turns out to be 1.961. Since the fractional part of transverse ratio $[\varepsilon_\alpha]$ is 0.441 and the fractional part of axial overlap ratio $[\varepsilon_\beta]$ is 0.520, the sum of them is less than 1. Then at the moment when gear just entered the meshing, although it is double-tooth zone of action, the gear pair which just entered the meshing is still in the point contact state, and the assumption here is that there is no contact force between teeth. Therefore, all loads are carried by the front pair of gear, as shown in Fig. 6.

3.2. The Definition of Material Properties and Settings of Contact Pairs

Before static analysis, it is necessary to define the material intrinsic properties of gear pair. Alloy 18CrNiMo7-6 is used as the gear material. The material properties can be determined by modulus of elasticity, Poisson's ratio and density. The speed of driving gear is 125.7r/min and the transmitted power is 129.63kW. According to formula of torque, the transmission torque is 9846.2N·m.
3.3. Analysis of the Contact of Tooth

The finite element analysis of contact on gear teeth is carried out and equivalent stress and strain are obtained. The contour plot of equivalent stress and equivalent strain are shown in Fig. 7.

According to the principle of meshing in gear, when the meshing is at the point C as shown in Fig. 6, the driving gear is at the worst moment of average line load. From the theoretical calculation, the deformation of driving gear is $4.765 \times 10^{-3}$ mm and the deformation of driven gear is $3.918 \times 10^{-3}$ mm. As shown in Fig. 7 (b), Finite element analysis indicates that the relative deformation of driving gear is $4.863 \times 10^{-3}$ mm, the error is 2.066% compared with the result from theoretical calculation. The relative deformation of driven gear is $3.782 \times 10^{-3}$ mm and the error is 3.469%, which is in the acceptable range.

It is verified that this theory could provide reliable guide for solving the maximum relative deformation of helical gears.

4. Conclusion

Based on the study of the influence of contact ratio on the numbers of meshing teeth, variation pattern of the length of line of contact and the solution of meshing stiffness of helical gear by the method of slicing, a general solution method for maximum amount of deformation in helical gear is proposed in this paper. The flow of system development for calculating the maximum deformation between teeth...
is presented. The results obtained from theoretical calculation are compared with results obtained from finite element simulation. It is concluded that the relative deformation of driving gear and driven gear are within the desirable range, verifying that that theory provides an effective assistance for studying the maximum deformation of helical gear.

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