Flux calculations in an inhomogeneous Universe: 
weighting a flux-limited galaxy sample

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ABSTRACT

Many astrophysical problems arising within the context of ultra-high energy cosmic rays, very-high energy gamma rays or neutrinos, require calculation of the flux produced by sources tracing the distribution of galaxies in the Universe. We discuss a simple weighting scheme, an application of the method introduced by Lynden-Bell in 1971, that allows the calculation of the flux sky map directly from a flux-limited galaxy catalog without cutting a volume-limited subsample. Using this scheme, the galaxy distribution can be modeled up to large scales while representing the distribution in the nearby Universe with maximum accuracy. We consider fluctuations in the flux map arising from the finiteness of the galaxy sample. We show how these fluctuations are reduced by the weighting scheme and discuss how the remaining fluctuations limit the applicability of the method.

Key words: methods: miscellaneous, catalogues, large-scale structure of Universe, galaxies: luminosity function, cosmic rays

1 INTRODUCTION

Recent developments in multi-wavelength/multi-messenger observational techniques often make it desirable to calculate the angular distribution of a diffuse flux expected from sources with a given spatial distribution. The predicted flux distribution may then be used for source identification, estimation of the background, etc. Necessity for such a calculation arises in the context of ultra-high energy cosmic rays (UHECRs), neutrino physics, as well as gamma-ray astronomy.

If the sources are extragalactic, their space distribution can be derived from the matter distribution in the Universe. The latter can be inferred from galaxy surveys, e.g. [Adelman-McCarthy et al. (2008); Skrutskie et al. (2006); Jones et al. (2005)]. Good distance determination is required to reconstruct the spatial mass distribution. Special techniques have been developed to minimize the impact of distance errors and to suppress the short-scale noise (see, e.g., Erdogdu et al. (2006) and references therein).

The problem of flux calculation has a number of features that make it different from (and easier than) reconstruction of the full three-dimensional mass distribution: (i) only a two-dimensional projection of the three-dimensional distribution is needed; (ii) contributions of remote sources are suppressed by the geometrical factor $r^{-2}$ and, in many cases, by the flux attenuation due to interactions with the ambient matter; (iii) the smaller amplitude of inhomogeneities at larger scales makes the contribution of remote sources essentially isotropic; only the overall normalization of such an isotropic part has to be calculated. These simplifications result in weaker requirements on the quantity and quality of astronomical data in flux calculations, which makes it advantageous to by-pass the reconstruction of matter density and calculate the flux distribution directly from the galaxy catalogs. Accurate results may be achieved with substantially smaller input.

Both in the context of mass distribution and in flux calculations, a crucial requirement is completeness of the underlying galaxy catalog. That is, a volume-limited sample is needed which includes all galaxies of a certain kind within a given volume. On the contrary, a natural product of an astronomical survey is a flux-limited sample that contains all galaxies up to certain apparent magnitude as set by the instrumental sensitivity and observation time. Volume-limited samples may be obtained from a flux-limited sample by cutting away objects that are further than a given distance and dimmer than a certain absolute magnitude, chosen in such a manner that the resulting sample is complete.

To model adequately the source distribution in the
The sliding-box technique is essentially a method to deal with the fact that galaxy catalogs do not contain an infinite number of galaxies. The finiteness of a galaxy catalog unavoidably leads to fluctuations in flux predictions. The sliding-box technique strongly reduces these fluctuations by efficient use of the available data. Nevertheless, the remaining fluctuations may still be large enough to spoil the accuracy of flux maps modeled from a galaxy catalog. We will address this problem in detail and present a criterion for the applicability of the sliding-box method.

An an illustration we will apply the sliding-box technique to a subset of the 2 Micron All-Sky Redshift Survey (2MRS) [Huchra et al. 2009], a flux-limited sample of galaxies with observed $K_s$-magnitude $m \leq 11.25$ that contains measured redshifts for all but a few galaxies.\footnote{This subset was kindly provided to us by John Huchra. Tailored to model the distribution of galaxies in the field of view of a northern-hemisphere cosmic-ray experiment, it does not cover the galactic plane with $|b| < 10^\circ$, $b$ being the galactic latitude, nor the region with $\delta < -30^\circ$, $\delta$ being the declination in J2000 equatorial coordinates.} We would like to stress, however, that the technique is tailored for flux calculations, in keeping with the aim of the present study.

The rest of this paper is organized as follows. In section 2 we discuss the sliding-box technique and present an efficient implementation scheme. We also discuss the connection between the weights associated with the algorithm on one hand and the luminosity and selection functions on the other. Section 3 is concerned with the effect of fluctuations on model fluxes due to the finite size of a galaxy sample. In section 4 as an example, we apply the sliding-box method to model the flux of UHECR protons with energies above 60 EeV from the 2MRS sample. We summarize our work in section 5.

2 THE SLIDING-BOX WEIGHTING SCHEME

2.1 Combining two volume-limited samples

To illustrate the general idea of the sliding-box technique, consider a flux-limited sample that is complete to a given apparent magnitude $m_0$. On the $M-D$ plane, where $M$ is the absolute magnitude and $D$ is the distance, this sample occupies the populated region in Fig. 1. The apparent magnitude $m$ of a source is a function of its absolute magnitude and distance, $m = m(M, D)$. For a given absolute magnitude, $m$ increases with distance and reaches the limiting value $m_0$ at a distance $D$ satisfying $m(M, D) = m_0$. This determines the line $M_*(D)$, the boundary of the populated region in Fig. 1. Beyond this line the objects are too dim and the completeness of the sample cannot be guaranteed.

At a given distance $D_0$, only objects with absolute magnitude $M < M_0 = M_*(D_0)$ are sufficiently bright to be included in the flux-limited catalog. Galaxies that are closer than $D_0$ and brighter than $M_0$ form a volume-limited sample. These are objects in regions marked with $S_1$ and $S_2$ in Fig. 1. The completeness of this subsample follows from the completeness of the original flux-limited catalog.

It is clear from figure 1 that at small distances the volume-limited sample $S_1 + S_2$ contains only a fraction of available galaxies which may be insufficient to represent luminosity function. Our formulation of the scheme is tailored for flux calculations, in keeping with the aim of the present study.
accurately the details of the matter distribution. To improve the situation, one may construct a denser volume-limited sample corresponding to a smaller distance $D_1$ (the sample $S_2 + S_3$ on Fig. 1). When calculating the flux one may use galaxies from $S_2 + S_3$ at distances $D < D_1$ and galaxies from $S_1$ at $D_1 < D < D_0$. The luminosity of a given volume is determined by the number of galaxies in the sample $S_1 + S_2$ contained in that volume. At distances $D < D_1$, the same luminosity may be represented in a greater detail by galaxies from $S_2 + S_3$ provided they are assigned smaller “weight”, that is, luminosity per galaxy. If the galaxies in the sparse sample have a weight $w_0$ each, the galaxies in the dense sample $S_2 + S_3$ should be weighted with

$$w_1 = \frac{S_2}{S_2 + S_3} w_0. \quad (1)$$

Here and below we use the same letter to denote the sample and the number of galaxies in the sample. At distances $D < D_1$, the total weight in the sparse and dense samples is the same, $S_2 w_0$. The difference is that in the dense sample it is distributed among a larger number of galaxies, and hence the spatial distribution of matter is represented more accurately.

Several volume-limited samples may be combined in the same way. In the limit of an infinite number of nested volume-limited samples one arrives at the “sliding-box” weighting scheme described now. This scheme is essentially an implementation of the $C^*$-method, proposed by Lynden-Bell (1971), applied to distance $D$ and magnitude $M$.

### 2.2 Sliding-box scheme

Imagine a variable rectangular “sliding box” with one corner fixed at zero distance and minimum $M$ (the lower-left corner in figure 1) and the opposite corner moving along the line $M_\nu(D)$. At any given position the box defines a volume-limited sample. One starts at some maximum distance $D_{\text{max}}$; galaxies that are further than $D_{\text{max}}$ are disregarded, i.e. assigned a zero weight (this is the part of the catalog that is lost). The current weight is set to, say, 1. Now the free corner of the sliding box is moved towards smaller distances. Each time a galaxy exits the box through its vertical edge it is assigned the current weight. Each time a galaxy enters the box through the horizontal edge, the current weight is multiplied by $N/(N+1)$, where $N$ is the current number of galaxies in the box. When the procedure is finished, all the galaxies at $D < D_{\text{max}}$ have been assigned a weight.

The main asset of the sliding-box scheme is that the weight at a given scale $D$ is computed from a volume-limited sample corresponding to distances just slightly larger than $D$; this sample has the maximum available number of galaxies and hence the smallest fluctuations. To demonstrate the accuracy of the sliding-box method, consider a direct computational scheme in which the weight at distance $D$ is determined from a volume-limited sample up to $D$ and a volume-limited sample up to $D_{\text{max}}$ (i.e., without refining the weights at intermediate distance as is done in the sliding-box method). The direct scheme and the sliding-box method are equivalent in the limit of infinite galaxies in the original sample. Given a finite number of galaxies, however, large statistical fluctuations will show up in the direct computational scheme due to the sparsity of the volume-limited sample extending to $D_{\text{max}}$ at small distances. This can be seen clearly in figure 2 which shows the weights as a function of distance for all galaxies in the 2MRS sample computed by the sliding-box technique and by direct computation.

### 2.3 Relation to luminosity and selection functions

The weight defined by the sliding-box method is related to the galaxy luminosity distribution and to the selection function characterizing the flux-limited galaxy sample. The original construction by Lynden-Bell (1971) was, in fact, aimed at recovering the quasar luminosity function (see also Jackson 1974, Choloniewski 1987, Efstathiou et al. 1988). To clarify these relations we consider the problem in general terms. We assume in this section that all the samples are very large so that a statistical description applies. For simplicity, we also assume that the distribution of galaxies in luminosity is space-independent that is the full distribution factorizes into a spatial and a luminosity part. The number of galaxies with magnitudes between $M$ and $M + dM$ at distances between $D$ and $D + dD$ is then expressed as a product of two factors,

$$dN = \lambda(M)dM \cdot \nu(D)d^2D. \quad (2a)$$

Let $N(D, M)$ denote the total number of galaxies within distance $D$ and brighter than $M$. It factorizes into a product of two cumulative distributions:

$$N(D, M) = N(D)L(M), \quad (2a)$$

where

$$N(D) \equiv \int_0^D dDD^2 \nu(D); \quad (2b)$$

The flux non-uniformity arises at $z \ll 1$ where one can neglect the evolution of sources. The contribution from regions $z \gtrsim 1$ may be non-negligible or even dominant; however, this contribution is isotropic.
The total number of galaxies within distance $D < D_{\text{max}}$ may be expressed as follows:

$$L(M) \equiv \int_{-\infty}^{M} dM \lambda(M).$$  \hspace{1cm} (2c)

According to eq. (2), the weight function $w(D)$ at some distance $D < D_{\text{max}}$ may be expressed as follows:

$$w(D) = \frac{N_\text{sel}(D, M, (D_{\text{max}}))}{N(D, M_{\text{c}}(D))} = \frac{L(M_{\text{c}}(D_{\text{max}}))}{L(M_{\text{c}}(D))}$$  \hspace{1cm} (3)

where we have normalized the weights so that $w(D_{\text{max}}) = 1$. Equations (2) and (3) imply that

$$w(D)^{-1} \propto \int_{-\infty}^{M_{\text{c}}(D)} \lambda(M')dM'.$$  \hspace{1cm} (4)

This relation can be inverted to read

$$\lambda(M) \propto \frac{w'(D_{\text{c}}(M))}{w(D_{\text{c}}(M))^2} \frac{dD_{\text{c}}(M)}{dM},$$  \hspace{1cm} (5)

where $w'(D) = dw/dD$ and $D_{\text{c}}(M)$ is the inverse of $M_{\text{c}}(D)$ defined by $M_{\text{c}}(D_{\text{c}}(M)) = M$. Hence we can directly infer the galaxy luminosity distribution $\lambda(M)$ from the weight function $w(D)$ and vice versa.

Figure 3 shows the luminosity distribution for a volume-limited subsample of the 2MRS sample up to 30 Mpc derived from the weights, together with the distribution directly reconstructed from the data. The agreement is excellent.

Now consider the relation between the weights and the selection function $\phi_{\text{sel}}(D)$ in a flux-limited sample. The total number of galaxies within distance $D$ in an incomplete sample is expressed in terms of the selection function as follows,

$$n(D) = \int_{0}^{D} dDd^2\nu(D)\phi_{\text{sel}}(D).$$  \hspace{1cm} (6)

Making use of the general expressions in eqs. (2) one finds that

$$\phi_{\text{sel}}(D) = w(D)^{-1} \cdot L(M_{\text{c}}(D_{\text{max}})),$$  \hspace{1cm} (7)

where the last factor is just a normalization constant. The selection function derived from the weights as given in eq. (7) is shown in figure 4. The model curve is in fair agreement with the data, given that it is derived under the assumption of a homogeneous Universe.

3 ACCURACY OF FLUX PREDICTIONS

In this section we address fluctuations associated with the finite number of galaxies in flux maps modeled from a galaxy catalog. Although the sliding-box technique efficiently suppresses these fluctuations, they still pose a potential limitation to the applicability of the method. This makes it important to quantify them, as we do in the following.

First we briefly discuss the construction of flux maps from a galaxy catalog. We express the model flux from galaxy $i$ as follows:

$$F_i = \frac{F_0 w_i J(D_i)}{4\pi D_i^2},$$  \hspace{1cm} (8)

where $F_0$ is a normalization constant, $w_i$ is the weight assigned to galaxy $i$ by the sliding-box technique, $D_i$ is the galaxy distance, and $J(D)$ represents the fraction of the integral flux from a source at distance $D$ that survives attenuation by redshift and interaction with the ambient matter. The function $J(D)$ is different for UHECRs, neutrinos, and gamma rays; $J(D) = 1$ corresponds to no attenuation. To keep the discussion general we do not specify $J(D)$ at this point.

Generally speaking, the model flux for a given direction on the sky is constructed by adding and averaging the fluxes of individual sources close to the line of sight. This can be done in various ways, in particular by dividing the sky into bins or by employing a smearing routine that distributes single-source fluxes over (part of) the sky. For the discussion of fluctuations the precise method is not very important; the critical parameter is the solid angle $\Delta \Omega$ over which flux contributions of individual sources are averaged.

How large may fluctuations be in order not to spoil the accuracy of the flux map? The answer to this question depends clearly on the purpose of the map and can thus not be answered in general. A reasonable requirement, which we will pursue in the following, is that no significant contribution to the flux within the solid angle $\Delta \Omega$ should
The fractional flux \( f \) sample (\( \Omega = 4\pi \)) can be expressed in general terms. We first approximate \( \Delta F \simeq F \Delta \Omega / \Omega \), where \( F = \sum F_i \) stands for the total flux and \( \Omega \) is the solid angle occupied by the sample (\( \Omega = 4\pi \) in the case of complete sky coverage). Equation (9) then reduces to:

\[
\Omega \frac{F_i}{\Delta \Omega} \frac{\Delta F}{F} \ll 1.
\]

(10)

To satisfy this requirement it is necessary to have many sources in the solid angle \( \Delta \Omega \). However, this may be not sufficient because not all sources contribute the same flux at Earth. The situation is thus complicated by the dependence on distance \( D \). Introducing the fraction \( f(D) \) of the total flux produced by the sources closer than \( D \), the number of these sources in the sample \( n(D) \), equation (10) can be rewritten in the following way, defining the quantity \( \Upsilon \):

\[
\Upsilon \equiv \frac{\Delta \Omega}{\Omega} \frac{dn}{df} \gg 1.
\]

(11)

The number of sources \( n(D) \) is readily calculated from equation (2):

\[
n(D) = \int_0^D dD dJ_0 (M_\ast(D))
\]

The fractional flux \( f(D) \equiv F(D)/F(D_{\text{max}}) \), where

\[
F(D) = \frac{F_0}{4\pi} \int_0^D dD dJ_\ast(D) w(D) J(D),
\]

represents the total flux from sources closer than \( D \). Note the appearance of the weights \( w(D) \) assigned by the sliding-box method in the last equation. Neglecting deviations of \( \nu(D) \) from 1, which is reasonable on cosmological scales, we use the above equations to find that

\[
\frac{dn}{df} \simeq \frac{3N_\nu(D)}{J(D)} \int_0^{D_{\text{max}}} dDJ_\ast(D) \quad (\text{sliding box}),
\]

(12)

where \( N_\nu(D) = N(D, M_\ast(D)) \) is the number of galaxies in the volume-limited sample at distance \( D \) (cf. eqs. 2). We stress that equation (12) is valid for fluxes modeled using the sliding-box technique. For a single volume-limited subsample that is valid up to \( D_{\text{max}} \), a similar computation yields:

\[
\frac{dn}{df} \simeq \frac{3N_\nu(D_{\text{max}})D^2}{D_{\text{max}}^3 J(D)} \int_0^{D_{\text{max}}} dDJ(D) \quad (\text{vol. ltd.}),
\]

(13)

Comparing eqs. (12) and (13), we see that the number of sources contributing to a given flux fraction at distance \( D \) is increased by a factor \( N_\nu(D)/N_\nu(D_{\text{max}}) \). This factor is unity at \( D = D_{\text{max}} \) (where the sliding-box method offers no improvement), but may become very large at small distances.

Inserting equation (12) into (11) brings us to the final expression for our criterion of small fluctuations:

\[
\Upsilon \simeq \frac{3\Delta \Omega N_\nu(D)D_{\text{max}}}{\Omega} \int_0^{D_{\text{max}}} dDJ(D) \int_0^{D_{\text{max}}} dD J(D) \gg 1.
\]

(14)

The three factors in equation (14) respectively encode the dependence of \( \Upsilon \) on the angular scale, on the statistics of the flux-limited sample, and on the attenuation of the model flux. The last factor reduces to unity in the case of no attenuation. The equation has to hold for all values of \( D \); when it is violated \( \mathcal{O}(1) \) fluctuations in flux may occur in regions of angular size \( \Delta \Omega \) due to the contribution of a single source.

We now discuss some of the quantities entering equation (14). The number of galaxies \( N_\nu(D) \) becomes small at both very large and very small distances, potentially leading to large fluctuations. For large distances this can be prevented by considering only sources up to a maximum distance \( D_{\text{max}} \) and assuming an isotropic flux from sources beyond that distance. Alternatively, particle horizons may provide a natural maximum distance (see below). In the case of small distances, the number of nearby sources is small while their contribution may be important due to their proximity. Unlike the fluctuations at large distances which are due to our poor knowledge of the galaxy distribution at those scales, the fluctuations at small distances are physical and may represent the actual flux variations due to close sources. Their complete treatment may require a case-by-case study of the most nearby objects.

The last factor in equation (14) encodes the effect of flux attenuation, which can play an important role in modeling the flux of UHECRs and of very-high energy gamma rays. Focusing on the case of UHECRs, we show in figure 5 the flux attenuation factor \( J \) as a function of distance for UHECR protons. The attenuation factor is obtained using a numerical cosmic-ray propagation code described in Koers & Tinyakov (2008, 2009). For comparison the attenuation factor due to redshift only is also shown in the figure. In figure 6 we show the quantity

\[
A \equiv \frac{\int_0^{D_{\text{max}}} dDJ(D)}{D_{\text{max}}^3 J(D)}
\]

(15)

which accounts for flux attenuation in equation (14). Note that, as indicated in figure 5, the horizon for UHECR protons above 60 EeV is around 200 Mpc. Since sources beyond this distance do not contribute to the observed flux, the requirement \( \Upsilon \gg 1 \) should be satisfied automatically. We observe from figure 6 that \( A \) indeed blows up around 200 Mpc, which guarantees that \( \Upsilon \gg 1 \) for any value of \( \Delta \Omega \) or \( N_\nu \).

4 EXAMPLE: UHECR FLUX PREDICTIONS USING THE 2MRS CATALOG

In this section we apply the sliding-box technique to model the flux of UHECR protons with energies in excess of 60 EeV from sources tracing the distribution of matter in the Universe. In modeling the effect of flux suppression due to attenuation, we assume a power-law injection spectrum with index \( p = 2.2 \) extending to very high energies.

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For comparison we also model the flux distribution using a single volume-limited galaxy sample up to 250 Mpc.

The distribution of matter is modeled using a subset of the 2MRS galaxy sample. This subset does not cover the galactic plane with |b| < 10°, nor the region with δ < −30° (see footnote [1]). Due to these cuts the catalog covers 63% of the sky, so that the total field of view is Ω = 0.63 · 4π = 7.9 sr. In figure 6 we show the number of galaxies in a volume-limited sample \( N_V(D) \) of the 2MRS as a function of distance \( D \).

In actual flux computations, dividing the sky into bins of fixed size has disadvantages related to boundary effects and the arbitrariness of the binning scheme. These problems are avoided with an angular smearing routine, which essentially replaces the point-source flux of an individual source by a (Gaussian) probability distribution. Adopting a smearing routine, the flux in a given direction \( \vec{n} \) is computed as follows:

\[
\Phi(\vec{n}) = \sum_i \phi_i(\theta),
\]

where

\[
\phi_i(\theta) = \frac{F_i \exp(-\theta^2/\theta_s^2)}{\pi \theta_s^2}.
\]

Here \( F_i \) represents the flux from galaxy \( i \), \( \theta \) denotes the angle between the galaxy and the line of sight \( \vec{n} \), and \( \theta_s \) is the smearing angle.

In figure 6 we show model UHECR flux maps obtained with equation (10) using the full 2MRS sample with the sliding-box method (top panel) and, for comparison, using a volume-limited subsample extending to 250 Mpc (bottom panel). A comparison between the two panels demonstrates the significant increase in accuracy achieved with the sliding-box technique. In particular, the shot noise artefacts that are visible in the bottom panel are absent in the top panel.

We now consider the fluctuations associated with the finite number of galaxies in the 2MRS catalog. In figure 7 we show the quantity \( \Upsilon \) defined in eq. (14) as a function of \( D \) for the exemplary case of 60 EeV UHECR protons. For comparison we also show \( \Upsilon \) for the volume-limited sample (using eqs. (11) and (13)). Because 63% of the total flux is contained within the opening angle \( \theta_s \), the solid angle \( \Delta \Omega \) that enters in eq. (14) is related to \( \theta_s \) as follows:

\[
\Delta \Omega = \frac{2\pi(1 - \cos \theta_s)}{0.63} \approx 5 \theta_s^2.
\]

We observe from figure 8 that, for the sliding-box method, \( \Upsilon \gg 1 \) for distance between 5 and 250 Mpc. Hence the results of the previous section imply that fluctuations associated with the finiteness of the galaxy sample should be small. For the volume limited subsample, on the other hand, the figure indicates that strong fluctuations are to be expected.

The estimates on the strength of fluctuations can be verified through a direct computation of the ratio of individual flux contributions \( \phi_i \) to the total flux \( \Phi \). Sampling over many directions on the sky, we compute the total flux \( \Phi \) via equation (10) and keep track of \( \phi_{\text{max}} = \max \phi_i \), the largest individual contribution to \( \Phi \). The distribution of \( \phi_{\text{max}}/\Phi \) is shown in figure 10 for the flux maps shown in figure 8 i.e. for the case of UHECR protons with energy in excess of 60 EeV and smearing angle \( \theta_s = 3° \). As may be verified from figure 10 \( \phi_{\text{max}}/\Phi \ll 1 \) for the

\[\text{Figure 5.} \text{ Flux suppression factor } J \text{ as a function of distance for three different scenarios: redshift only, UHECR protons with energy above 40 GeV, and UHECR protons with energy above 60 GeV. In producing this figure we have assumed a power-law injection spectrum with index } p = 2.2 \text{ extending to very high energies.}\]

\[\text{Figure 6.} \text{ Flux attenuation factor } A \text{ defined in eq. (13) as a function of distance for the same scenarios as shown in figure 5.}\]

\[\text{Figure 7.} \text{ Size of a volume-limited sample of our 2MRS sample up to distance } D.\]
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Figure 8. Aitoff projection of the sky in galactic coordinates showing the model flux of UHECR protons above 60 EeV from sources tracing the distribution of galaxies up to 250 Mpc. The grayscale shows the relative flux on a logarithmic scale. (Areas in white are not covered by our subsample of the 2MRS catalog.) The top panel shows the flux constructed from the original flux-limited sample using the sliding-box technique; the bottom panel shows the flux constructed from a volume-limited subsample up to 250 Mpc. For both cases we have removed sources closer than 5 Mpc and smeared the flux distribution with $\theta_s = 3^\circ$.

Figure 9. $\Upsilon$ as a function of $D$ for the case of 60 EeV UHECR protons modeled using our subset of the 2MRS catalog. The figure applies to a smearing angle $\theta_s = 3^\circ$, corresponding to $\Delta \Omega = 0.01$ sr; the total field of view $\Omega = 7.9$ sr.

sliding-box method: no single source outshines the bulk. On the other hand, if we model the flux distribution from the volume-limited sample (bottom panel of figure S), we find that the distribution of $\phi_{max}/\Phi$ peaks near 1. In this case $O(1)$ fluctuations in the predicted flux due to a single source are common (which is also clear from the bottom panel of figure S), which means that the galaxy sample is too small to provide an accurate flux map.

Figure 10. Distribution of $F_{max}/F_{\Delta \Omega}$, i.e. the ratio of the largest individual contribution to the total flux within solid angle $\Delta \Omega$. This figure applies to the flux map shown in the top panel of figure 8.

5 SUMMARY

We have addressed the problem of flux calculation from sources tracing the galaxy distribution in the Universe. We have discussed a sliding-box weighting scheme, building on the work of Lynden-Bell (1971), that makes use of the information contained in a flux-limited galaxy catalog in the most efficient way. This scheme allows us to represent the distribution of matter up to large scales while representing the distribution of matter on small scales with maximum accuracy. The resulting weight function is related to the galaxy luminosity function and may be used to infer the latter under the assumption of its coordinate independence.

The sliding-box weighting scheme suppresses efficiently fluctuations due to the finite size of the sample at most distances except the largest and the smallest ones. We have presented estimates on the size of the remaining fluctuations. These estimates can be used to determine a maximum distance at which the catalog should be cut, or to find the minimum angular scale on which flux maps can be constructed accurately. We would like to stress that our estimates regard the size of fluctuations, and not their importance. For example, in a statistical test based on model flux distributions, fluctuations of order unity may be acceptable if the overall flux distribution shows very strong contrasts or when the angular scales of interest are much larger than the scale at which fluctuations occur.

An advantage of the sliding-box scheme is that it allows a straightforward generalization to the cases when the sources trace preferentially certain types of galaxies, or when the source luminosity is correlated with the galaxy type. Such effects may be accounted for by pre-weighting the galaxies in the catalog in a corresponding way and modifying accordingly the sliding-box weighting scheme.
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