Destination-aided Wireless Power Transfer in Energy-limited Cognitive Relay Systems

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Abstract—This paper considers an energy-limited cognitive relay network where a secondary transmitter (ST) assists to forward the traffic from a primary transmitter (PT) to a primary receiver (PR), in exchange for serving its own secondary receiver (SR) in the same frequency. The multiple-antenna ST is assumed to be energy-constrained and powered by both information flow from source (PT) and dedicated energy streams from destinations (PR and SR), which is called destination-aided wireless power transfer (DWPT) scheme. Then, the relay processing matrix, cognitive beamforming vector and power splitter are jointly designed to maximize the rate of secondary users under the energy causality constraint and the constraint that the demanded rate of primary users is satisfied. For the perfect channel information state (CSI) case, by adopting semi-definite relax (SDR) technique and Charnes-Cooper transformation, the global optimal solution can be achieved. For the imperfect CSI case, S-procedure is used to transform the worst-case robust problem into a tractable semi-definite program (SDP). Simulation results reveal that our proposed DWPT scheme is greatly preferred for both perfect and imperfect CSI cases when ST is close to PR/SR.

Index Terms—Wireless power transfer, cognitive relay networks, beamforming design, power splitting, semi-definite program.

I. INTRODUCTION

Wireless power transfer, potentially enabling low-power cost systems to work self-sustainably, has attracted considerable attention recently. Since electromagnetic waves can carry both energy and information, simultaneous wireless information and power transfer (SWIPT) is first creatively proposed in [1]. In that paper, the same received radio frequency (RF) flow is ideally assumed to be extracted for both information decoding (ID) and energy harvesting (EH), which cannot be conducted in recent electric circuits. Then, two basic practical receiver architectures for SWIPT named time switching (TS) and power splitting (PS) are put forward [2]. TS switches the receiver between ID and EH in a time-division manner, while PS divides the received stream into two flows with one for ID and the other for EH. Since then, several techniques in wireless communications have been extended to SWIPT systems, including multiple-input multiple-output (MIMO) [2], [3], relay [4]–[7], cognitive radio [8], [9], and full duplex [10], [11]. In the SWIPT relay systems, energy-limited relays are able to assist the traffic from sources to destinations with the scavenged energy from sources as their transmission power. Based on the proposed TS or PS scheme, resource allocation and beamforming design [12]–[16] in relay systems are widely studied.

In cognitive relay networks, secondary users (SU) are encouraged to relay primary users’ (PU) messages for accessing the licensed spectrum to send their own information to secondary receivers (SR). It is a win-win strategy especially when the direct links between primary transmitters (PT) and primary receivers (PR) suffer from severe fading. With the EH ability, energy-limited secondary transmitters (ST) can be strongly stimulated by both the information and energy cooperation from PT to ST, which is investigated in [8]. In that paper, the ST is powered by PT first, and then uses the harvested energy as transmission power to forward PT’s information to PR as well as to send its own information to SR. The SU rate maximization problem is considered subject to the PU rate demand constraint and the energy causality constraint. It has been found that, the SU-PU rate region can be enlarged with the energy cooperation from PT to ST.

However, for the uplink transmission in sensor networks or other low-power networks, the power budget of PT is strictly restricted by its battery capacity, while the destinations are information access points, which usually have constant power supply. To better explore the system performance for this practical scenario, power of PR and SR is fully exploited in this paper. In particular, a destination-aided wireless power transfer (DWPT) scheme is proposed for a cognitive relay system, where the energy-limited ST is not only powered by PT, but also assisted by energy transfer from PR and SR.

In [4], a relay node powered by both source and destination is investigated for a simple three-node SWIPT relay system. The system rate maximization problem is studied with the energy causality constraint. It is worth pointing out that, our considered DWPT scheme for cognitive relay system is a more general scenario as compared with [4]. If ST is a pure relay and does not send message to SR (of course, the rational SR also does not transfer energy to ST), our considered scenario will degrade into a three-node relay system in [4]. If ST is a pure transmitter and does not forward the traffic from PT to PR (in this case, PR also does not transfer energy to ST), our considered scenario will become a two-node wireless powered communication network in [12], where the downlink energy
transfer and uplink information transmission are assumed.

To be specific, this paper studies a cognitive relay network where a PT, a PR, an energy-limited ST and a SR are included. The ST is assumed to have multiple antennas and other users all have a single antenna. In the first phase, PT transmits information flow to ST. At the same time, PR and SR also send dedicated wireless RF energy stream to ST to further enhance the scavenged energy at ST. Based on the PS scheme, the received RF flow at ST can be split for EH and ID. In the second phase, ST assists to relay the traffic from PT to PR with the amplify-and-forward (AF) protocol and also sends its own information to SR. The main contributions of this work are listed as follows.

1) A DWPT scheme for the cognitive relay system is first proposed and investigated, which fully exploits the power of destinations. With this scheme, ST can extract energy from both information flow from PT and dedicated energy flows from destinations (PR and SR). The relay processing matrix, cognitive beamforming vector and power splitter are jointly optimized to maximize the SU rate under the energy causality constraint and the constraint that minimal PU rate demand is guaranteed.

2) Under the assumption that ST perfectly knows all the channel state information (CSI), both the optimal and low-complexity suboptimal solutions are given. Since the problem is non-convex, iterative approaches are presented. With given power splitter, to achieve the optimal relay matrix and cognitive beamforming vector, we first derive lower dimensional structures for them, and then adopt the Charnes-Cooper transformation and semi-definite relax (SDR) method. To reduce the complexity, matrix decomposition, zero forcing (ZF) scheme, and dual method are jointly employed to derive the closed-form solution. Then, the optimal power splitters for both optimal and suboptimal algorithms are found via bisection.

3) Under the assumption that ST imperfectly knows the channel links from ST to PR and ST to SR, a worst-case robust solution is proposed in an iterative manner. With fixed power splitter, to find the relay matrix and cognitive beamforming vector, some matrix lemmas and S-procedure are used to transform the robust problem into a tractable semi-definite program (SDP). Then, the optimal power splitter is found via one-dimensional search.

4) For comparison, the energy harvesting cognitive radio system without destinations’ power transfer in [8] is also considered. Simulation results reveal that, when ST is close to PR/SR, our proposed DWPT scheme is greatly preferred for both perfect and imperfect CSI cases.

The remainder of the paper is organized as follows. In Section II, system model and problem formulation are introduced. In Section III, we present both optimal and suboptimal solutions to the SU rate maximization problem with the perfect CSI. In Section IV, we further state a worst-case robust algorithm for the problem with the imperfect CSI. The simulation results are presented and discussed in Section V. Finally, Section VI concludes the paper.

**Notation:** Bold lower and upper case letters are used to denote column vectors and matrices, respectively. The superscripts $H^T$, $H^*$ and $H^H$ is standard transpose, conjugate and (Hermitian) conjugate transpose of $H$, respectively. $\| \cdot \|_2$ and $\| \cdot \|_F$ refer to the Euclidean norm and the Frobenius norm, respectively. $\text{Rank}(W)$, $\text{Tr}(W)$ and $\text{vec}(W)$ denote the rank, trace and vectorization of matrix $W$, respectively. $W \succeq 0 \preceq 0$ means that matrix $W$ is positive semidefinite (negative semidefinite). $\otimes$ and $\circ$ are Kronecker product and Hadamard product, respectively. Matrix $E$ represents diag$(1, 1, \ldots, 1)$.

**II. SYSTEM MODEL AND PROBLEM FORMULATION**

Considering a cognitive relay network where a ST assists to forward the traffic from a PT to a PR, in exchange for serving its own SR in the same frequency, as illustrated in Fig. 1. The ST is equipped with $M$ antennas while other users have a single antenna. We assume that the ST is energy-limited, and thus powered by PT as well as PR and SR to enhance the harvested energy, which is called DWPT scheme in this paper. Assume that the entire communication time slot, which consists of two equal phases, is normalized to be 1.

In the first phase, PT transmits information signal $x_p$ with power $P_{PT}$. Meanwhile, PR and SR respectively send energy signals $x'_p$ and $x'_s$ with their corresponding power, $P_{PR}$ and $P_{SR}$. Suppose that different signals are statistically independent. The observation at ST is expressed as

$$y_r = g x_p + h_p x'_p + h_s x'_s + n_r,$$

where $g$, $h_p$ and $h_s$ are $M \times 1$ channel vectors from PT to ST, PR to ST, and SR to ST, respectively; $n_r \sim CN(0, \sigma_r^2 I_M)$ is the received noise vector at ST. To concurrently process the information decoding and the energy harvesting, the practical PS receiver architecture is adopted at ST. In particular, the received signal is split into two streams, one for ID and one for EH, with the relative power ratio of $\rho$ and $1 - \rho$, respectively. The signal stream for EH and the harvested energy at ST are respectively given by

$$y_r^{EH} = \sqrt{1 - \rho} (g x_p + h_p x'_p + h_s x'_s + n_r)$$

$$P_r^{EH} = (1 - \rho) (P_{PT} \|g\|_2^2 + P_{PR} \|h_p\|_2^2 + P_{SR} \|h_s\|_2^2 + \sigma_r^2),$$

where $0 < \xi \leq 1$ is the energy conversion efficiency. Let $n_c \sim CN(0, \sigma_c^2 I_M)$ denote the $N \times 1$ circuit noise vector caused by the signal frequency conversion from RF to baseband and hence the other stream for ID is given by

$$y_r^{ID} = \sqrt{\rho} (g x_p + h_p x'_p + h_s x'_s + n_r) + n_c.$$
During the second phase, ST uses the AF protocol to relay the traffic from PT to PR and also superimposes its own message \(x_s\) to SR with \(E[|x_s|^2] = 1\). Denote \(F \in \mathbb{C}^{M \times M}\) and \(w \in \mathbb{C}^{M \times 1}\) as the relay processing matrix and the cognitive beamforming vector respectively, the transmit signal at ST is

\[
x_r = Fy^{D}_s + wx_s
\]

with average power

\[
P_{ST}(F, w, \rho) = \rho(P_{PT} ||Fg||^2_2 + P_{PR} ||Fh||^2_2 + P_{SR} ||Fx||^2_2
\]

\[
+ \sigma_r^2 ||F||^2_F + \sigma_c^2 ||F||^2_F + ||w||^2_2.
\]

With the perfect CSI at ST, we assume that PR/SR can successfully cancel its self-interference \(x'_p/x'_r\). Hence, the received signals at PR and SR are respectively expressed as

\[
y_p = \sqrt{\rho}h_p^H Fg + \sqrt{\rho}h_p^H Fh_s x_s + h_p^H w x_s
\]

\[
+ \sqrt{\rho}h_p^H Fx_r + h_p^H Fc + n_p,
\]

\[
y_s = \sqrt{\rho}h_s^H Fg + \sqrt{\rho}h_s^H Fh_s x_s + h_s^H w x_s
\]

\[
+ \sqrt{\rho}h_s^H Fx_r + h_s^H Fc + n_p,
\]

where \(n_p \sim \mathcal{CN}(0, \sigma_p^2)\) and \(n_s \sim \mathcal{CN}(0, \sigma_s^2)\) are additive Gaussian white noises (AWGNs) at PR and SR, respectively. The received signal to interference plus noise ratios (SINRs) at PR and SR are respectively given by

\[
\Gamma_p(F, w, \rho) = \frac{\rho P_{PT} ||h_p^H Fg||^2}{\rho P_{SR} ||h_p^H Fh_s||^2 + (\rho \sigma_p^2 + \sigma_s^2) ||h_p^H F||^2 + ||h_p^H w||^2 + \sigma_p^2},
\]

\[
\Gamma_s(F, w, \rho) = \frac{||h_s^H w||^2}{\rho P_{PR} ||h_p^H Fg||^2 + P_{PR} ||h_p^H Fh_p||^2 + (\rho \sigma_p^2 + \sigma_s^2) ||h_p^H F||^2 + \sigma_p^2}.
\]

In this paper, we focus on the joint design of relay matrix \(F\), cognitive beamforming vector \(w\) and power splitter \(\rho\) to maximize the achieved rate of SU under the constraint that the rate demand of PU, \(R_p^{min}\), is met. The optimization problem is formulated as (P1)

\[
\max_{F, w, 0 \leq \rho \leq 1} \Gamma_s(F, w, \rho)
\]

s. t. \(\Gamma_p(F, w, \rho) \geq \Gamma_p^{min}\), \(P_{ST}(F, w, \rho) \leq P_r^{EH}\),

where \(\Gamma_p^{min} = 2R_p^{min} - 1\). Constraint (11c) is to guarantee that the transmission power at ST is not more than its harvested energy.

III. SOLUTIONS TO DWPT WITH PERFECT CSI

In this section, we suppose that all the channel knowledge is perfectly known at ST. Under this assumption, both the optimal and low-complexity suboptimal solutions to problem P1 are proposed.

A. Optimal Solution to DWPT

In this subsection, we present the optimal solution to problem P1. Given fixed \(\rho\), optimal \(F\) and \(w\) are obtained, and then the optimal \(\rho\) is found via bisection. In what follows, we first focus on the design of optimal \(F\) and \(w\) with fixed \(\rho\).

**Proposition 1**: Define two QR decompositions as \([h_s, h_p] = V_1R_1\) and \([h_s, h_p, g] = U_2R_2\), where \(V_1 \in \mathbb{C}^{M \times 2}\), \(U_2 \in \mathbb{C}^{M \times 3}\) are orthonormal matrices and \(R_1 \in \mathbb{C}^{2 \times 2}\), \(R_2 \in \mathbb{C}^{3 \times 3}\) are upper triangular matrices. Then, the optimal relay matrix and cognitive beamforming vector have the following structures:

\[
F = V_1 A U_2^H, \quad w = V_1 b,
\]

where \(A \in \mathbb{C}^{2 \times 3}\) and \(b \in \mathbb{C}^{2 \times 1}\) are optimization variables.

**Proof**: Please see Appendix A. ■

If \(M \geq 3\), with Proposition 1, the \(M^2\) unknowns in \(F\) and \(M\) unknowns in \(w\) are respectively reduced to \(2 \times 3\) and unknowns. This greatly reduces the computational complexity of the beamforming design. If \(M \leq 2\), we optimize \(F\) and \(w\) directly.

Define \(\hat{h}_s = V_1^H h_s\), \(\hat{h}_p = V_1^H h_p\), \(\hat{h}_s = U_2^H h_s\), \(\hat{h}_p = U_2^H h_p\) and \(\hat{g} = U_2^H g\). With Proposition 1, the problem P1 can be reformulated as (given fixed \(\rho\)) (P2)

\[
\max_{A, b} \rho(P_{PT} ||\hat{h}_p^H A \hat{g}||^2 + P_{PR} ||\hat{h}_s^H \hat{A} \hat{h}_p||^2 + (\rho \sigma_p^2 + \sigma_s^2) ||\hat{h}_s^H A||^2 + \sigma_p^2, \quad \rho(P_{PT} ||\hat{h}_p^H \hat{A} \hat{h}_p||^2 + P_{PR} ||\hat{A} \hat{h}_s||^2 + P_{SR} ||\hat{A} \hat{h}_s||^2)
\]

s. t.

\[
\rho(P_{PT} ||\hat{A} \hat{h}||^2 + P_{PR} ||\hat{A} \hat{h}||^2 + P_{SR} ||\hat{A} \hat{h}||^2) \geq \Gamma^{min}_p,
\]

\[
\rho(P_{PT} ||\hat{A} \hat{h}||^2 + P_{PR} ||\hat{A} \hat{h}||^2 + P_{SR} ||\hat{A} \hat{h}||^2) \geq \Gamma^{min}_p
\]

To solve this problem effectively, we use equations

\[
\vec{c}(\vec{A} \vec{X} \vec{B}) = (\vec{B}^T \otimes \vec{A}) \vec{c}(\vec{X})
\]

\[
\vec{c}(\vec{T} \vec{X}_1 \vec{X}_2) = \vec{c}(\vec{T} \vec{X}_1)^T \vec{c}(\vec{X}_2)
\]

to further rewritten problem (P2) as (P2.1)

\[
\max_{a, b} \frac{b^H \hat{H} \hat{b}}{a^H \hat{C}_p a + \sigma_s^2}
\]

s. t.

\[
\frac{a^H \hat{D}_p a + b^H \hat{H} \hat{b}}{a^H \hat{E}_p a + b^H \hat{b}} \leq P_r^{EH},
\]

where

\[
B_p = \rho(\sigma_p^2 I + P_{PR} \hat{G}^T + P_{PR} \hat{H}^T_p) \otimes \hat{h}_s + \sigma_s^2 I \otimes \hat{h}_s
\]

\[
C_p = \rho P_{PT} \hat{G}^T \otimes \hat{h}_p,
\]

\[
D_p = \rho(\sigma_p^2 I + P_{SR} \hat{H}_p^T) \otimes \hat{h}_p + \sigma_p^2 I \otimes \hat{h}_p
\]

\[
E_p = \rho((P_{PT} \hat{G}^T + P_{PR} \hat{H}_p^T + P_{SR} \hat{H}_p^T) \otimes I_{2 \times 2} + \sigma_p^2 I_{6 \times 6})
\]

and
\[ a = \text{vec}(A), \hat{H}_s = \hat{h}_s\hat{H}_s, \hat{H}_p = \hat{h}_p\hat{H}_p, \]
\[ \hat{H}_s = \hat{h}_s\hat{H}_s, \hat{H}_p = \hat{h}_p\hat{H}_p, \quad G = \hat{g}\hat{g}^H. \]

Before solving problem \( P2.1 \), we first analyze its feasible condition, which can be obtained by finding the maximum \( \Gamma_{\text{min}}^{\text{ps}} \). Setting \( b = 0 \), the optimization problem is given as (\( P2.2 \))

\[
\begin{align*}
\max_{a} & \quad \frac{a^H C_p a}{a^H D_p a + \sigma_p^2} \\
\text{s. t.} & \quad a^H E_p a \leq P_r^{\text{EH}}.
\end{align*}
\]
(23a)

It is easy to verify that, at the optimum, the power constraint (23b) is active, i.e., \( a^H E_p a = P_r^{\text{EH}} \). With this equation, problem \( P2.2 \) can be equivalently written as (\( P2.3 \))

\[
\max_{\|a\|^2 = 1} \frac{a^H C_p a}{a^H D_p a + \sigma_p^2 / P_r^{\text{EH}} a}
\]
(24)

which is a generalized Rayleigh quotient [13]. The optimal \( a^* \) of problem \( P2.3 \) is equal to the dominant generalized eigenvector of the matrix pair \((C_p, \frac{D_p + \sigma_p^2 / P_r^{\text{EH}}}{P_r^{\text{EH}}} E_p)\). And the achieved optimal value of problem \( P2.3 \), \( \Gamma_{\text{min}}^{\text{ps}} \), is the largest generalized eigenvalue of the same matrix pair. Thus, the feasible condition is \( \Gamma_{\text{min}}^{\text{ps}} \leq \Gamma_{\text{p}} \).

Within the feasible region, we then resort to the SDR technique and Charnes-Cooper transformation [14] to solve problem \( P2.1 \). Introducing \( \hat{A} = a^H, \hat{B} = b^H \) and applying SDR technique, problem \( P2.1 \) can be relaxed as (\( P2.4 \))

\[
\begin{align*}
\max_{\hat{A}, \hat{B}} & \quad \text{Tr}(\hat{H}\hat{B}) \\
\text{s. t.} & \quad \text{Tr}(C_p\hat{A}) - \Gamma_{\text{min}}^{\text{p}} \{ \text{Tr}(D_p\hat{A}) + \text{Tr}(\hat{H}_p\hat{B}) \} \geq \Gamma_{\text{p}}^{\text{min}} \sigma_p^2, \\
& \quad \text{Tr}(E_p\hat{A}) + \text{Tr}(\hat{B}) \leq P_r^{\text{EH}}, \\
& \quad \hat{A} \succeq 0, \hat{B} \succeq 0,
\end{align*}
\]
(25a)

which is a linear fractional quasi-convex problem. From Charnes-Cooper transformation, we define \( A = q\hat{A}, \hat{B} = q\hat{B}(q > 0) \) and rewrite the problem \( P2.4 \) as (\( P2.5 \))

\[
\begin{align*}
\max_{\hat{A}, \hat{B}} & \quad \text{Tr}(\hat{H}\hat{B}) \\
\text{s. t.} & \quad \text{Tr}(B_p\hat{A}) + \sigma_q^2 q = 1, \\
& \quad \text{Tr}(C_p\hat{A}) - \Gamma_{\text{min}}^{\text{p}} \{ \text{Tr}(D_p\hat{A}) + \text{Tr}(\hat{H}_p\hat{B}) \} \geq \Gamma_{\text{p}}^{\text{min}} \sigma_p^2 q, \\
& \quad \text{Tr}(E_p\hat{A}) + \text{Tr}(\hat{B}) \leq P_r^{\text{EH}} q, \\
& \quad \hat{A} \succeq 0, \hat{B} \succeq 0, q > 0,
\end{align*}
\]
(26a)

which is a convex SDP and can be efficiently solved by convex optimization solvers, e.g., CVX [15].

Remark 1: More importantly, According to the Theorem 2.3 in [16], the optimal solution to problem \( P2.5 \) always satisfies \( \text{Rank}(A^*) + \text{Rank}(\hat{B}^*) \leq 3 \), since the number of generalized constraints is 3. For the nontrivial case where \( A^* \neq 0, \hat{B}^* \neq 0 \), we have \( \text{Rank}(A^*) = 1, \text{Rank}(\hat{B}^*) = 1 \). So the SDR problem is tight and thus the optimal \( a^* \) and \( b^* \) for problem \( P2.1 \) can be obtained.

So far, the optimal solution to problem \( P1 \) with fixed \( \rho \) is derived. In the sequel, we focus on the finding of optimal \( \rho \).

Proposition 2: Define the objective value of problem \( P2.5 \) as a function of \( \rho \), i.e., \( h(\rho) \). Then, \( h(\rho) \) is concave in \( \rho \) and its optimal value can be obtained via bisection.

Proof: Let \( \theta_1, \theta_2 \) and \( \theta_3 \) denote the dual variables of the corresponding constraints in problem \( P2.5 \), respectively. Then the Lagrangian function of problem \( P2.5 \) is given by

\[
\mathcal{L}(\hat{A}, \hat{B}, q, \theta_1, \theta_2, \theta_3, \rho) = \text{Tr}(Q_1\hat{A}) + \text{Tr}(Q_2\hat{B}) + q_3,
\]
(27)

where

\[
\begin{align*}
Q_1 &= -\theta_1 B_p + \theta_2 C_p - \theta_2 \Gamma_{\text{min}}^{\text{p}} D_p - \theta_3 E_p, \\
Q_2 &= \hat{H}_s - \theta_2 \Gamma_{\text{p}}^{\text{min}} \hat{H}_p - \theta_3 I, \\
q_3 &= \theta_1 \sigma_q^2 q - \theta_2 \Gamma_{\text{min}}^{\text{p}} \sigma_p^2 q + \theta_3 q P_r^{\text{EH}}.
\end{align*}
\]
(28)

The Lagrangian dual function is given by

\[
\begin{align*}
g(\theta_1, \theta_2, \theta_3, \rho) &= \max_{\hat{A} \succeq 0, \hat{B} \succeq 0, q > 0} \mathcal{L}(\hat{A}, \hat{B}, q, \theta_1, \theta_2, \theta_3, \rho).
\end{align*}
\]
(31)

Since \( P2.5 \) is a convex problem and satisfies the Slater’s condition, the strong duality holds [17]. Thus,

\[
h(\rho) = \min_{\theta_1, \theta_2, \theta_3 \geq 0} g(\theta_1, \theta_2, \theta_3, \rho).
\]

From (27)-(30), we can observe that only \( B_p, C_p, D_p, E_p \) in \( Q_1 \) and \( P_r^{\text{EH}} \) in \( q_3 \) are related with \( \rho \). And all of these terms are linear in \( \rho \), such that \( g(\theta_1, \theta_2, \theta_3, \rho) \) is a linear function with respect to \( \rho \). Accordingly, it is easily verified that \( h(\rho) \) is a point-wise minimum of a family of affine function and hence concave in \( \rho \) [17]. Therefore, its maximum can be found through a one-dimensional search, such as bisection. This completes the proposition.

As analyzed before, we have \( h(\rho) = \mathcal{L}(A^*, \hat{B}^*, q^*, \theta_1^*, \theta_2^*, \theta_3^*, \rho) \), where \( A^*, \hat{B}^*, q^* \) are the optimal primary variables and \( \theta_1^*, \theta_2^*, \theta_3^* \) are the optimal dual variables for a given \( \rho \), respectively. With (27), (17)-(20) and (3), the gradient of \( h(\rho) \) can be expressed as

\[
\frac{dh(\rho)}{d\rho} = \text{Tr} \left( (\theta_1 B_p + \theta_2 C_p) - \theta_2 \Gamma_{\text{min}}^{\text{p}} D_p \right),
\]
(32)

where

\[
\begin{align*}
B_{p1} &= (\sigma_q^2 I + P_{PR} \hat{G}^T \otimes \hat{H}_s), \\
C_{p1} &= P_{PR} \hat{G}^T \hat{H}_p, \\
D_{p1} &= (\sigma_q^2 I + P_{SR} \hat{H}_s^T \otimes \hat{H}_p), \\
E_{p1} &= (P_{PR} \hat{G}^T + P_{PR} \hat{H}_p^T \otimes I_{2 \times 2} + \sigma_q^2 I_{6 \times 6}), \\
P_{r1}^{\text{EH}} &= \xi (P_{PR} \|g\|_2^2 + P_{PR} \|b\|_2^2 + P_{SR} \|\hat{h}\|_2^2 + \sigma_q^2).
\end{align*}
\]
(33)

(34)

(35)

(36)

(37)

Above all, problem \( P1 \) can be solved in two steps: (i) Given any \( 0 < \rho \leq 1 \), we first solve the Problem \( P2.5 \) to obtain \( h(\rho) \); (ii) Then, we use the bisection method to find optimal \( \rho \) by using the gradient of \( h(\rho) \). Repeat these two procedures until problem converges. Detailed steps of proposed algorithm are
outlined in Algorithm 1. It is worth pointing out that the global optimization solution to the problem $\mathcal{P}1$ can be achieved by Algorithm 1.

**Algorithm 1** Optimal solution to problem $\mathcal{P}1$ with perfect CSI

1. Initialize $\rho_{min}, \rho_{max}$ and tolerance $\delta_\rho$;
2. while $\rho_{max} - \rho_{min} > \delta_\rho$ do
3. $\rho \leftarrow (\rho_{min} + \rho_{max})/2$;
4. Solve problem $\mathcal{P}2.5$ via CVX to obtain $\hat{A}^*, \hat{B}^*, q^*, \theta^*; \theta_2^*$; and $\theta_3^*$;
5. Calculate $\frac{d\rho}{dp}$ according to (32);
6. if $\frac{d\rho}{dp} \geq 0$ then
7. $\rho_{min} \leftarrow \rho$;
8. else
9. $\rho_{max} \leftarrow \rho$;
10. end if
11. end while
12. return $\hat{A}^* = \hat{A}^*/q^*, \hat{B}^* = \hat{B}^*/q^*$;
13. return $a^*$ and $b^*$ via eigenvalue decomposition (EVD) of $\hat{A}^*$ and $\hat{B}^*$, respectively;

**B. Suboptimal Solution to DWPT**

Although the optimal solution to problem $\mathcal{P}1$ is obtained, the complexity is high due to the adoption of standard tool box, CVX. In this subsection, we present a low-complexity suboptimal solution, where the closed-form of $\mathbf{F}$ and $\mathbf{w}$ is derived with given $\rho$. Similar to the former subsection, optimal $\rho$ is found via bisection. In the following, we put the emphasis on the design of $\mathbf{F}$ and $\mathbf{w}$.

For simplicity, we decompose $\mathbf{F}$ as $\mathbf{F} = f_t^h f_r^H$ [13], where $f_t$ is the transmit beamforming vector and $f_r$ is receiver filter at ST. Without loss of generality, we further suppose that $\|f_t\|_2^2 = 1$. According to the property of matrix norm $\|f_t f_r^H\|_F = \|f_t\|_2 \|f_r^H\|_2$, the original problem $\mathcal{P}1$ is converted as (with fixed $\rho$) $(\mathcal{P}3)$

$$\max_{f_t, f_r, w} \begin{align*}
\frac{\|h_t^H f_t^2\|_2^2}{(\rho P_{PR} |f_t^H h_p|^2 + P_{TR} |f_r^H h_p|^2) + (\rho \sigma_r^2 + \sigma_s^2) + \sigma_s^2}
\end{align*}$$

(38)

s. t.

$$\begin{align*}
\rho P_{PT} |f_t^H h_t|^2 &\geq \Gamma_{opt}^r, \\
\|f_t^2\|_2 (\rho P_{PR} |f_t^H h_p|^2 + P_{PR} |f_r^H h_p|^2 + P_{SR} |f_t^H h_s|^2) &\geq \Gamma_{opt}^r, \\
\|f_t^2\|_2 (\rho P_{PR} |f_t^H h_p|^2 + P_{PR} |f_r^H h_p|^2 + P_{SR} |f_t^H h_s|^2) &\geq \Gamma_{opt}^r,
\end{align*}$$

(39)

which is non-convex due to coupling variables $f_t$ and $f_r$.

To tackle this difficulty effectively, we first design $f_r$ with given $f_t$ and $w$. The ZF method is used here. Suppose that $h_t^H f_t = 0$ and $h_r^H f_r = 0$, we have $f_r = \hat{V} f_r$, where $\hat{V}$ is the orthogonal basis for null space of $h_t$. Then, we can derive from the singular value decomposition (SVD) method [14].

To satisfy the SINR of PR, we further assume that $f_r$ is aligned to the same direction of $\hat{V}^H g$. Together with $\|f_r\|_2^2 = 1$, we have

$$f_r = \hat{V} \frac{\hat{V}^H g}{\|\hat{V}^H g\|_2}.$$
Similarly, (47c) can be equivalently reformulated as
\[
\text{s. t. } \quad I + b \lambda_2^H h_s^H \geq e \lambda_1^H h_p^H ,
\]
where \( \lambda_1, \lambda_2 \) are dual variables and \( e = \frac{1}{c d e^2} \). From the constraint (41b), we know that, if \( \| h_p^H f_t \|_2^2 \rightarrow \infty, \)
\[
\frac{1}{c d e^2} \rightarrow \infty .
\]
Since the non-trivial case, where the problem \( P3.1 \) is feasible, is considered, we have
\[
\Gamma_{st} \leq \frac{\| h_p^H f_t \|_2^2}{c d e^2 + \| h_p^H w^H \|_2^2 + \sigma^2_p} \leq \frac{1}{c} .
\]
(48)
Thus, \( c \Gamma_{st} < 1 \), i.e., \( e > 0 \) must hold. From (42) and (45), we also have \( b > 0 \) and \( d > 0 \).

Since \( I + \lambda_1 h_p^H h_s^H > 0 \), constraint (47b) can be rewritten as
\[
I \geq \lambda_2^H f_t \left( I + \lambda_1 h_p^H h_s^H \right)^{-1} h_s^H .
\]
Due to the fact that \( \text{Rank} \left( \left( I + \lambda_1 h_p^H h_s^H \right)^{-1} h_s^H \right) = 1 \), this matrix only has one nonnegative eigenvalue, that is, \( h_s^H \left( I + \lambda_1 h_p^H h_s^H \right)^{-1} h_s \). Hence, (49) is equivalent to
\[
\Gamma_{st} \geq \lambda_2 h_s^H \left( I + \lambda_1 h_p^H h_s^H \right)^{-1} h_s .
\]
Similarly, (47c) can be equivalently reformulated as
\[
1 \geq e \lambda_1 h_p^H \left( I + \frac{b \lambda_2}{d} h_s^H h_s^H \right)^{-1} h_s .
\]
To maximize the objective of problem \( P3.3 \), i.e., \( \sigma_p^2 \lambda_1 + \sigma_s^2 \lambda_2 \), one can easily verify that, at the optimum, two constraints (50) and (51) are both active, i.e.,
\[
\Gamma_{st} \geq \lambda_2 h_s^H \left( I + \lambda_1 h_p^H h_s^H \right)^{-1} h_s \quad \text{and} \quad 1 \geq e \lambda_1 h_p^H \left( I + \frac{b \lambda_2}{d} h_s^H h_s^H \right)^{-1} h_s .
\]
Then, using the matrix inversion lemma
\[
\left( A + u v^H \right)^{-1} = A^{-1} - A^{-1} u v^H A^{-1} / 1 + v^H A^{-1} u ,
\]
we have
\[
\lambda_2 = \frac{\Gamma_{st} \left( I + \lambda_1 \| h_p^H \|_2^2 \right)}{\| h_s^H \|_2^2 + \lambda_1 \left( \| h_s^H \|_2^2 \| h_p^H \|_2^2 - \| h_s^H h_p^H \|_2^2 \right)} ,
\]
(55)
\[
\lambda_1 = \frac{1}{e} + \frac{b \lambda_2}{c d e^2} \| h_s^H \|_2^2 .
\]
(56)
As mentioned before, the optimal objective value of problem \( P3.2 \) is \( P_r^{EH} \). Due to the strong duality, the objective value of its dual problem must satisfy
\[
\sigma_p^2 \lambda_1 + \sigma_s^2 \lambda_2 = P_r^{EH} .
\]
Combining this power equation (57) with (56), a quadratic function in terms of \( \lambda_2 \) is derived. That is,
\[
f(\lambda_2) = \alpha \lambda_2^2 + \beta \lambda_2 + \gamma = 0 ,
\]
where
\[
\alpha = \frac{b}{c d e^2} \| h_s^H \|_2^2 P_r^{EH} ,
\]
\[
\beta = \sigma_p^2 \| h_p^H \|_2^2 P_r^{EH} \frac{b}{c d e^2} \| h_s^H \|_2^2 \| h_p^H \|_2^2 - \| h_s^H h_p^H \|_2^2 ,
\]
(59)
\[
\gamma = \frac{\sigma_p^2}{e} P_r^{EH} \| h_p^H \|_2^2 .
\]
(60)
\( \gamma < 0 \) is obtained from (48). Note that \( \gamma = 0 \) is the trivial case where the SU rate is zero. Therefore, \( f(\lambda_2) = 0 \) always has one unique real root, which can be derived based on the roots formula of the quadratic equation. \( \lambda_1^* \) and \( \Gamma_{st}^* \) can then be obtained based on (55) and (56). Thus, the optimal value of problem \( P3.1 \) is achieved.

Meanwhile, the closed-form of \( f_t^* \) and \( w^* \) is given by
\[
f_t^* = \sqrt{p_f^*} f_t^* = \sqrt{p_f^*} \left( \frac{I + \frac{b \lambda_2}{d} h_s^H h_p^H}{\| I + \frac{b \lambda_2}{d} h_s^H h_p^H \|_2^2} \right) h_p^H ,
\]
(62)
\[
w^* = \sqrt{p_w^*} w^* = \sqrt{p_w^*} \left( \frac{I + \lambda_1 h_p^H h_s^H}{\| I + \lambda_1 h_p^H h_s^H \|_2^2} \right) h_s^H ,
\]
(63)
where the power \( p_f^*, p_w^* \) can be easily obtained from
\[
\begin{align*}
\frac{p_f^* \| h_p^H f_t^* \|_2^2}{e \sigma^2} & = \Gamma_{pt}^*, \\
p_w^* + \sigma^2 & = \frac{p_f^* + p_w^* \| h_p^H f_t^* \|_2^2 + \sigma^2}{e \sigma^2} .
\end{align*}
\]
(64)

To summarize, the proposed low-complexity suboptimal solution to problem \( P1 \) consists of two steps: (i) With a given \( 0 < \rho \leq 1, \) \( f_t \) can be first derived based on (40). Then, \( f_t \) and \( w^* \) are obtained based on (62) and (63), respectively. Note that the expression of \( f_t \) is independent of \( f_t \) and \( w^* \), so it is not necessary to optimize \( f_t \) and \( f_t \) (or \( f_t \), \( w^* \)) iteratively. (ii) Similar to the optimal solution in subsection A, the optimal \( \rho \) can be found via bisection. Repeat these two procedures until problem converges.

IV. ROBUST SOLUTION TO DWPT WITH IMPERFECT CSI

Until now, we assume that all channel state information is perfectly known at ST. In practice, the knowledge of channel \( g \) can be directly estimated by ST, whereas the CSI of \( h_p/h_s \) depends on the quantized feedback from PR/SR. As a result, the level of channel uncertainty is much higher in \( h_p \) and \( h_s \). For this reason, the perfect CSI of \( g \) and imperfect CSI of \( h_p \) and \( h_s \) are considered in this section [19].

The imperfect \( h_p \) and \( h_s \) are respectively modeled as
\[
\hat{h}_p = h_p + \Delta h_p , \quad \Delta h_p = \left\{ \Delta h_p : \| \Delta h_p \|_2^2 \leq \varepsilon_p^2 \right\}
\]
and (65)
\[
\hat{h}_s = h_s + \Delta h_s , \quad \Delta h_s = \left\{ \Delta h_s : \| \Delta h_s \|_2^2 \leq \varepsilon_s^2 \right\}
\]
(66)
where \( \tilde{h}_p \) and \( \tilde{h}_s \) are the estimated CSI; \( \Delta h_p \) and \( \Delta h_s \) are the channel error vectors; \( \varepsilon_p^2 \) and \( \varepsilon_s^2 \) are radii of the channel error uncertainty regions.

With the imperfect CSI, the harvested energy at ST (i.e., (3)) and the transmission power at ST (i.e., (6)) can be respectively rewritten as

\[
\tilde{P}_r^{EH} = \xi (1 - \rho) \left( P_{PR} \left\| \tilde{h}_p + \Delta h_p \right\|_2^2 + P_{SR} \left\| \tilde{h}_s + \Delta h_s \right\|_2^2 \right) + P_{PT} \left\| g \right\|_2^2 + \sigma_r^2 \) \quad \text{and} \quad (67)
\]

\[
\tilde{P}_{ST}(F, w, \rho) = \rho \left( P_{PT} \left\| Fg \right\|_2^2 + P_{PR} \left\| F(\tilde{h}_p + \Delta h_p) \right\|_2^2 \right) + P_{SR} \left\| F(\tilde{h}_s + \Delta h_s) \right\|_2^2 + \sigma_r^2 \Vert F \Vert_F^2 + \Vert w \Vert_2^2. \quad (68)
\]

It is worth pointing out that, with the imperfect \( h_p \) and \( h_s \), the self-interference of the received signal at PR/SR cannot be cancelled completely. Actually, only the estimated part of self-interference can be removed. Consequently, the received signals at PR and SR are respectively given as

\[
\tilde{y}_p = \sqrt{\rho} \left( \tilde{h}_p + \Delta h_p \right)^H Fg x_p \quad \text{(Desired signal)}
\]

\[
+ \sqrt{\rho} \left( \Delta h_p^H F(\tilde{h}_p + \Delta h_p) + \tilde{h}_p^H F \Delta h_p \right) x'_p \quad \text{(Residual self-interference)}
\]

\[
+ \left( \tilde{h}_p + \Delta h_p \right)^H \left( \sqrt{\rho} n_r + n_c \right) + n_p \quad \text{and}
\]

\[
\tilde{y}_s = \left( \tilde{h}_s + \Delta h_s \right)^H w x_s \quad \text{(Desired signal)}
\]

\[
+ \sqrt{\rho} \left( \Delta h_s^H F(\tilde{h}_s + \Delta h_s) + \tilde{h}_s^H F \Delta h_s \right) x'_s \quad \text{(Residual self-interference)}
\]

\[
+ \sqrt{\rho} \left( \tilde{h}_s + \Delta h_s \right)^H F \left( \sqrt{\rho} n_r + n_c \right) + n_s. \quad \text{Noise}
\]

Hence, from (69), the SINR at PR is

\[
\tilde{\Gamma}_p = \frac{\mu_1}{\mu_2 + \mu_3 + \mu_4}, \quad (71)
\]

where

\[
\mu_1 = \rho P_{PT} \left\| \tilde{h}_p + \Delta h_p \right\|_2^2, \quad (72)
\]

\[
\mu_2 = \rho P_{PR} \left\| \Delta h_p^H F(\tilde{h}_p + \Delta h_p) + \tilde{h}_p^H F \Delta h_p \right\|_2^2, \quad (73)
\]

\[
\mu_3 = \rho P_{SR} \left\| \tilde{h}_p + \Delta h_p \right\|_2^2 + \left( \tilde{h}_p + \Delta h_p \right)^H w \right\|_2^2 \quad \text{and}
\]

\[
\mu_4 = (\rho \sigma_r^2 + \sigma_s^2) \left\| \left( \tilde{h}_p + \Delta h_p \right)^H F \right\|_2^2 + \sigma_s^2. \quad (75)
\]

Similarly, from (70), the SINR at SR is

\[
\tilde{\Gamma}_s = \frac{\eta_1}{\eta_2 + \eta_3 + \eta_4}, \quad (76)
\]

where

\[
\eta_1 = \left\| \left( \tilde{h}_s + \Delta h_s \right)^H w \right\|_2^2, \quad (77)
\]

\[
\eta_2 = \rho P_{SR} \left\| \Delta h_s^H F(\tilde{h}_s + \Delta h_s) + \tilde{h}_s^H F \Delta h_s \right\|_2^2, \quad (78)
\]

\[
\eta_3 = \rho P_{PR} \left\| \left( \tilde{h}_s + \Delta h_s \right)^H F(\tilde{h}_p + \Delta h_p) \right\|_2^2 \quad (79)
\]

\[
+ \rho P_{PT} \left\| \left( \tilde{h}_s + \Delta h_s \right)^H Fg \right\|_2^2 \quad \text{and}
\]

\[
\eta_4 = (\rho \sigma_r^2 + \sigma_s^2) \left\| \left( \tilde{h}_s + \Delta h_s \right)^H F \right\|_2^2 + \sigma_s^2. \quad (80)
\]

Accordingly, the worst-case SU rate maximization problem subject to PU rate constraint and energy causality constraint is formulated as (P4)

\[
\begin{align*}
\max_{F, w, \rho} & \quad H \quad \text{s. t.} \quad \mu_1 \geq \Gamma_p, \quad \forall \Delta h_p \in B_p, \forall \Delta h_s \in B_s, \quad (82a) \\
\tilde{P}_{ST}(F, w, \rho) & \leq \tilde{P}_r^{EH}, \quad \forall \Delta h_p \in B_p, \forall \Delta h_s \in B_s. \quad (82c)
\end{align*}
\]

Referring to (77), this max-min problem can be equivalently rewritten as (P4.1)

\[
\begin{align*}
\max_{F, w, \rho} & \quad t \quad \text{s. t.} \quad \eta_1 \geq t (\eta_2 + \eta_3 + \eta_4), \quad \forall \Delta h_p \in B_p, \forall \Delta h_s \in B_s, \quad (82b) \\
\mu_1 & \geq \Gamma_p (\mu_2 + \mu_3 + \mu_4), \quad \forall \Delta h_p \in B_p, \forall \Delta h_s \in B_s, \quad (82c)
\end{align*}
\]

where \( t \) is a introduced nonnegative parameter. Given fixed \( \rho \) and \( t \), generally speaking, this kind of robust problem can be solved by the SDR technique and S-procedure [20]. Nevertheless, different from the beamforming vectors design for the simple downlink broadcast scenario in [20], our considered problem involves two hops relay transmission and the design of relay matrix \( F \). To tackle the difficulty caused by \( F \), the matrix properties described in Lemma 1 [22] are applied to transform the \( F \) related terms into our desired expressions.

**Lemma 1:** Define \( f = \text{vec}(F) \), we have

\[
\Delta z^T Fg = \Delta z^T (g^T \otimes I) f, \quad (83)
\]

\[
g^T F \Delta z = \Delta z^T (I \otimes g^T) f, \quad (84)
\]

\[
z^T FF^H \Delta z^* = z^T (1^T \otimes I) (E \otimes ft)^H (1 \otimes I) \Delta z^* \quad \text{and} \quad (85)
\]

\[
z^T FT F^* \Delta z^* = z^T (1^T \otimes I) (E \otimes \text{vec}(F^T) \text{vec}(F^T)^H) (1 \otimes I) \Delta z^*, \quad (86)
\]

where \( P \) is the permutation matrix and \( \text{vec}(F^T) = P \text{vec}(F) = Pf \).
where uncertainties. Hence, the ZF scheme is used to force the former \( \eta_1 = \Delta h^H A_s \tilde{W} A_s^H \Delta h + 2 \Re \{ \tilde{h}_s^H \tilde{W} A_s^H \Delta h \} + \tilde{h}_s^H \tilde{W} \tilde{h}_s \),

\[
\begin{align*}
\Delta h & = \begin{bmatrix} \Delta h_p \\ \Delta h_s \end{bmatrix}, \quad A_p = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad A_s = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \text{and thus} \\
\Delta h_p^H = \Delta h^H A_p, \quad \Delta h_s^H = \Delta h^H A_s.
\end{align*}
\]

Note that \( \eta_3 \) involves terms of both \( \tilde{h}_s^H F \tilde{h}_p \) and \( \Delta h_p^H F \Delta h_p \). However, it is difficult to tackle the product of these two terms, which is the second order of channel uncertainties. Hence, the ZF scheme is used to force the former term to zero, i.e., \( \tilde{h}_s^H F = 0 \), which is equivalent to

\[
\text{Tr} \left( (I \otimes \tilde{h}_s^H)^H F \right) = 0.
\]

Then, using Lemma 1, we have

\[
\eta_2 + \eta_3 + \eta_4 = \Delta h^H K_1 \Delta h + 2 \Re \{ k_2^H \Delta h \} + k_5,
\]

where

\[
K_1 = \rho P_{SR} A_s (\Phi_1 + \Phi_2)^H (\Phi_1 + \Phi_2) A_s^H + \rho P_{PR} (A_s \Phi_1 + A_p \Phi_2)^H (A_s \Phi_3 + A_p \Phi_1^H + A_p \Phi_2^H + A_s \Psi_1 A_s^H + A_s \Psi_2 A_s^H),
\]

\[
k_2^H = \tilde{h}_s^H \Psi_1 A_s^H + \tilde{h}_s^H \Psi_2 A_s^H,
\]

\[
k_3^H = \tilde{h}_s^H (\Psi_1 + \Psi_2) + \sigma_r^p,
\]

\[
\Phi_1 = \tilde{h}_s^T \otimes I, \quad \Phi_2 = I \otimes \tilde{h}_s^T, \quad \Phi_3 = \tilde{h}_s^T \otimes I,
\]

\[
\Psi_1 = \rho P_{PR} (g^T \otimes I) (g^T \otimes I)^H,
\]

\[
\Psi_2 = (\rho \sigma_r^p + \sigma_g^p) (I \otimes (E \otimes F))(I \otimes I),
\]

In (89), terms including the third or higher order of channel uncertainties are ignored due to their small values.

Substituting (87) and (89) into (82c), we can obtain

\[
\Delta h^H (A_s \tilde{W} A_s^H - (K_1) \Delta h + 2 \Re \{ (\tilde{h}_s^H \tilde{W} A_s^H - tk_2^H) \Delta h \}) + \tilde{h}_s^H \tilde{W} \tilde{h}_s - tk_4 \geq 0.
\]

So far, the first constraint (82b) is reformulated as (88) and (96).

In the similar way, the second constraint (82c) can be rewritten as

\[
\text{Tr} \left( (I \otimes \tilde{h}_p^H \tilde{h}_p^H)^H F \right) = 0 \quad \text{and}
\]

\[
\Delta h^H (A_p \Psi_1 A_p^H - \Gamma_p^H K_3) \Delta h + 2 \Re \{ \tilde{h}_p^H \Psi_1 A_p^H - \Gamma_p^H k_4 \} \geq 0,
\]

where

\[
K_3 = \rho P_{PR} A_p (\Phi_4 + \Phi_5)^H (\Phi_3 + \Phi_4) A_s^H + \rho P_{SR} (A_s \Phi_1 + A_s \Phi_4)^H (A_s \Phi_3 + A_p \tilde{W} A_s^H + A_p \Psi_2 A_s^H),
\]

\[
k_4^H = \tilde{h}_p^H \tilde{W} A_p^H + \tilde{h}_p^H \Psi_2 A_p^H,
\]

\[
k_p^H = \tilde{h}_p^H (\tilde{W} + \Psi_2 \tilde{h}_s + \sigma_r^p),
\]

\[
\Phi_4 = (I \otimes \tilde{h}_p^H)\).
\]

The third constraint (82d) can be equivalently converted as

\[
\Delta h^H K_5 \Delta h + 2 \Re \{ k_6^H \Delta h \} + k_7 \geq 0,
\]

where

\[
K_5 = P_{PR} A_p \Psi_4 A_p^H + P_{SR} A_s \Psi_4 A_s^H,
\]

\[
k_6^H = P_{PR} \tilde{h}_p^H \Psi_4 A_p^H + P_{SR} \tilde{h}_s^H \Psi_4 A_s^H,
\]

\[
k = P_{PR} \tilde{h}_p^H \Psi_4 \tilde{h}_p + P_{SR} \tilde{h}_s^H \Psi_4 \tilde{h}_s + P_{PR} \tilde{t} \Psi_4 + (\rho \sigma_r^2 + \sigma_g^2) \text{Tr}(\tilde{F}) + \text{Tr}(\tilde{W}) - (1 - \rho) \sigma_r^2,
\]

\[
\Psi_3 = (I \otimes (E \otimes \tilde{F})) (I \otimes I),
\]

\[
\Psi_4 = \rho \Psi_3 - (1 - \rho) I.
\]

Next, we rely on the S-Procedure to further transform the re-expressions of constraints (82b)-(82d) into their corresponding tractable linear matrix inequality (LMI) forms.

**Lemma 2 (S-procedure [17]):** Given Hermitian matrices \( \tilde{A}_i \in \mathbb{C}^{N \times N} \) and \( \tilde{b}_i \in \mathbb{C}^{N \times 1} \), \( i = 1, 2, 3, 4, 5, 6, 7, 8, 9 \) for which the functions \( f_i(x) = x^H \tilde{A}_i x + 2 \Re \{ \tilde{b}_i^H x \} + \tilde{c}_i \). Then, \( f_1(x) \geq 0 \) and \( f_2(x) \geq 0 \) imply \( f_3(x) \geq 0 \), if and only if there exist \( x_1 \geq 0 \) and \( x_2 \geq 0 \) such that

\[
\begin{bmatrix}
\tilde{A}_3 & \tilde{b}_3 \\
\tilde{b}_3^H & x_1
\end{bmatrix} \geq 0,
\]

provided that there exists a vector \( \tilde{x} \) with \( f_1(\tilde{x}) > 0 \) and \( f_2(\tilde{x}) > 0 \).

In this paper, we can take

\[
f_1(\Delta h) = -\Delta h^H A_p A_p^H \Delta h + \varepsilon_p^2 \geq 0 \quad \text{and}
\]

\[
f_2(\Delta h) = -\Delta h^H A_s A_s^H \Delta h + \varepsilon_s^2 \geq 0.
\]

For the first constraint (82b), by employing Lemma 2, the LMI form of (96) is given as

\[
\begin{bmatrix}
T_s + \varsigma_1 A_s A_p^H + \varsigma_2 \tilde{A}_s^H & A_s \tilde{W} \tilde{h}_s - tk_2 \\
\tilde{h}_s^H \tilde{W} A_s^H - tk_3 & t_s - \varsigma_1 \varepsilon_p^2 - \varsigma_2 \varepsilon_s^2
\end{bmatrix} \geq 0,
\]

where \( T_s = A_s \tilde{W} A_s^H - tk_2 \), \( t_s = \tilde{h}_s^H \tilde{W} \tilde{h}_s - tk_3 \) and \( \varsigma_1, \varsigma_2 \) are introduced variables.

Similarly, for the secondary constraint (82c), (98) can be transformed as

\[
\begin{bmatrix}
T_p + \varsigma_3 A_p A_p^H + \varsigma_4 \tilde{A}_p^H & A_p \tilde{h}_p - \Gamma_p^H k_4 \\
\tilde{h}_p^H \tilde{W} A_p^H - \Gamma_p^H k_5 & t_p - \varsigma_3 \varepsilon_p^2 - \varsigma_4 \varepsilon_p^2
\end{bmatrix} \geq 0,
\]

where \( T_p = A_p \tilde{h}_p - \Gamma_p^H k_4 \), \( t_p = \tilde{h}_p^H \tilde{W} \tilde{h}_p - \Gamma_p^H k_5 \) and \( \varsigma_3, \varsigma_4 \) are introduced variables.

And for the third constraint (82d), (103) can be rewritten as

\[
\begin{bmatrix}
-K_5 + \nu_1 A_s A_s^H - k_9 \\
k_6^H - k - \nu_1 \varepsilon_p - \nu_2 \varepsilon_s
\end{bmatrix} \geq 0,
\]

where \( \nu_1 \) and \( \nu_1 \) are introduced variables.
Therefore, with given \( t \) and \( \rho \), problem \( \mathcal{P}4.1 \) can be re-expressed as a feasible SDR problem (\( \mathcal{P}4.2 \))

\[
\begin{align*}
\max_{\mathbf{F}, \mathbf{W}} & \quad 0 \\
\text{s. t.} & \quad (88), (97), (112) - (114), \\
& \quad s_{x1} \geq 0, s_{x2} \geq 0, s_{p1} \geq 0, s_{p2} \geq 0, \nu_1 \geq 0, \nu_2 \geq 0, \\
& \quad \mathbf{F} \succeq 0, \mathbf{W} \succeq 0,
\end{align*}
\]

which is convex and can be effectively solved by off-the-shelf solvers, such as CVX [15]. The optimal \( t \) can be found via bisection and the optimal \( \rho \) can be achieved via the exhaustive search. Hence, the algorithm 2 for the robust scheme is listed as below. What is noteworthy is that the Gaussian randomization method [23] can be employed to extract the rank-one solution if the rank of obtained solution is greater than one.

**Algorithm 2** Robust solution to problem \( \mathcal{P}4 \) with imperfect CSI

1. Initialize a step size \( \rho_s \) for \( \rho \);
2. for \( \rho = 0 \); \( \rho \); : 1 do
3. Initialize \( t_{\min}, t_{\max} \) and tolerance \( \delta_t \);
4. while \( t_{\max} - t_{\min} > \delta_t \) do
5. \( t \leftarrow (t_{\min} + t_{\max})/2 \);
6. Check the feasibility of problem \( \mathcal{P}4.2 \) to via CVX;
7. if it is feasible then
8. \( t_{\min} \leftarrow t \);
9. else
10. \( t_{\max} \leftarrow t \);
11. end if
12. end while
13. end for
14. Find the maximum \( t \) as \( t^* \) and its related \( \rho^* \);
15. return \( \mathbf{F}^*, \mathbf{W}^*, \rho^* \) and \( t^* \);

V. SIMULATION RESULTS

In this section, we evaluate the performance of our proposed DWPT scheme via computer numerical simulations. For simplicity, the received noise power is \( \sigma_r^2 = \sigma_n^2 = \sigma_p^2 = \sigma_\nu^2 = 1 \) mW. Unless otherwise specified, other simulation parameters are set as follows. Assume that ST has \( M = 4 \) antennas and the transmission power at PT, PR and SR is \( P_{PT} = 30 \) dBm, \( P_{PR} = P_{SR} = 30 \) dBm. The energy conversion efficiency is \( \xi = 50\% \) and the minimal rate requirement of PU is \( R^\text{min}_p = 2.5 \) bps/Hz. As described in Fig. 2, we consider a simple scenario where locations of PT, ST, PR and SR are \((-5, 0), (0, 0), (5, -1) \) and \((5, 1) \) in coordinates, respectively. The distance unit is in meters and the path-loss exponent is 2. All channel entries are independently generated from i.i.d Rayleigh fading with their respective average power values.

For comparison, the energy harvesting cognitive radio system without power transfer from destinations (i.e., PR and SR) [3] is also considered, which is labeled as ‘w/o destination-aided’. Except for Fig. 3, the simulation results are achieved over 500 independent channel realizations.

**A. Performance Evaluation for the Perfect CSI case**

At first, the achievable SU-PU rate regions are characterized in Fig. 3 for different schemes. A specific channel realization is randomly chosen as \( \mathbf{g} = [-0.2694 - 0.0123i, -0.2221 - 0.0584i, -0.1695 + 0.2270i, -0.1823 + 0.1044i]^T \), \( \mathbf{h}_p = [-0.0762 - 0.1064i, 0.0060 + 0.2268i, 0.0962 - 0.3864i, 0.0037 + 0.0652i]^T \) and \( \mathbf{h}_s = [-0.0036 + 0.0617i, -0.1718 - 0.0510i, 0.0218 - 0.1389i, 0.0480 + 0.2174i]^T \). It is observed that both optimal and suboptimal DWPT schemes achieve significantly larger rate region than the ‘w/o destination-aided’ scheme due to the destinations’ energy transfer. In addition, the optimal DWPT scheme always outperforms the suboptimal DWPT scheme. This is owing to the fact that the spatial degrees of freedom for the suboptimal scheme are slightly reduced by the decomposition of relay matrix \( \mathbf{F} \) and the ZF design of receiver filter \( \mathbf{f}_r \). Moreover, the SU rate of the low-complexity suboptimal DWPT scheme closely approaches to that of the optimal DWPT scheme when \( R^\text{min}_p \leq 1.5 \) bps/Hz. This is because that when the value of \( R^\text{min}_p \) is small, the allocated power for relay matrix \( \mathbf{F} \) is extremely low such that the suboptimal design of \( \mathbf{F} \) has little effect on the SU rate.

In Fig. 4, the impact of transmission power at PR and SR on the achievable SU rate is investigated with different energy conversion efficiencies \( \xi \). We assume that transmission
observed that our proposed DWPT scheme is greatly preferred between ST and D, PT and D. From Fig. 6, it can be easily found that, the SU rate performance of the suboptimal scheme is gradually approaching to that of the optimal scheme with the increasing number of antennas at ST, especially when $M \geq 8$. This is mainly because that, for the suboptimal scheme with ZF-based receiver filter $f$, the spatial degrees of freedom loss caused by the ZF-based receiver filter $f$, can cancel the strong interferences and approximately achieve the optimal performance.

Fig. 5 plots the achievable SU rates versus the number of antennas at ST for different schemes with $R_{p}^{\text{min}} = 2.5 \text{ bps/Hz}$. For all schemes, as expected, more antennas employed by ST will result in better SU rate performance. In addition, it can be easily found that, the SU rate performance of the suboptimal scheme is gradually approaching to that of the optimal scheme with the increasing number of antennas at ST, especially when $M \geq 8$. This is mainly because that, for the suboptimal scheme, the spatial degrees of freedom loss caused by the ZF-based receiver filter $f$, can be improved as $M$ increases.

To illustrate the impact of ST’s location on the SU rate in Fig. 6, we assume that ST can move along the X-coordinate axis from PT to D, a virtual node situated at (5, 0) as plotted in Fig. 2. $d_{ST-D}$ and $d_{PT-D}$ respectively denote distances between ST and D, PT and D. From Fig. 6, it can be easily observed that our proposed DWPT scheme is greatly preferred when ST is close to PR and SR, since the amount of harvested energy at ST is effectively enhanced. Besides, it is of interest to find that, with the ST’s movement from PT to D (i.e., $d_{ST-D}/d_{PT-D}$ from 0.9 to 0.1), the achieved SU rates for all three schemes first decrease and then increase. And the worst point is moving to PT when transmission power of PR and SR is increasing. More curiously, the worst point of the ‘w/o destination-aided’ scheme occurs when $d_{ST-D}/d_{PT-D}$ ratio is 0.5 rather than 0.1. This is mainly due to the fact that, the harvested energy at ST is not enough to offset the severe path-loss between ST and PR/SR when ST is in the middle location.

In addition to the performance evaluation of achievable SU rate, we also investigate the outage performance of PU in Fig. 7. The outage will occur when the required rate demand of PU cannot be guaranteed. That is, the considered problem is infeasible. ‘ST (0, 0)’ and ‘ST (3, 0)’ in Fig. 7 respectively
mean that ST is situated at (0, 0) and (3, 0) in Fig. 2. From Fig. 7, we can observe that the outage probability of PU is declining with the increase of transmission power at PR and SR. Combining Fig. 4 and Fig. 7, it is noted that when ST is located at (0, 0) and $P_{PR} = P_{SR} \geq 30$ dBm, the growth trend of SU rate is very evident and the outage probability of PU is close to zero. This reveals that, not only SU but also PU can benefit from our proposed DWPT scheme. Furthermore, the proposed optimal DWPT scheme achieves better outage performance than the suboptimal scheme, and the gap between them reduces as transmission power at PR and SR increases. The reason behind this phenomenon is similar to Fig. 4. Besides, it can also be found that the outage performance of ‘ST (0, 0)’ unexpectedly outperforms that of ‘ST (3, 0)’ for both optimal and suboptimal DWPT schemes when $P_{PR} = P_{SR} \geq 30$ dBm. This is mainly because that the interference at PR is stronger when ST is located at (3, 0). However, this slight worse outage performance of PU does not affect the improvement of SU rate when ST moves from (0, 0) to (3, 0) (i.e., $d_{ST-D}/d_{PT-D}$ from 0.5 to 0.2) according to Fig. 6.

B. Performance Evaluation for the Imperfect CSI case

This subsection shows the performance evaluation for the imperfect CSI case. For simulation, we assume that the radii of channel uncertainty regions for $h_{p}$ and $h_{s}$ are equal to each other, i.e., $\varepsilon_{p} = \varepsilon_{s} = \varepsilon$. Other simulation parameters are the same as the perfect CSI case.

The achieved worst-case SU rates versus transmission power at PR and SR are characterized in Fig. 8 for different levels of channel uncertainty. The rate demand of PU is set as $R_{p}^{\text{min}} = 2.5$ bps/Hz. As can be seen, for both perfect and imperfect CSI cases, the larger value of transmission power at destinations, the better rate performance SU has. Furthermore, Observing from this figure, we can see that with the increase of channel uncertainty level, the achieved performance in terms of the worst-case SU rate is deteriorated.

In Fig. 9, the impact of ST’s location on the worst-case robust SU rate is presented for different schemes. We assume that the rate demand of PU is $R_{p}^{\text{min}} = 2.5$ bps/Hz and the transmission power at each nodes is $P_{PR} = P_{SR} = 30$ dBm. Similar to the perfect CSI case, we can observe that our proposed robust DWPT scheme is also more preferred for all different channel uncertainty levels when ST is closer to destinations. Nevertheless, note that when ST is much closer to PT (i.e., $d_{ST-D}/d_{PT-D} \rightarrow 0.9$), the SU rates of the DWPT scheme for both perfect and imperfect CSI cases will fall to that of the ‘w/o destination-aided’ scheme. In this case, it is not necessary for destinations to assist power to ST.

VI. CONCLUSION

This paper has proposed a DWPT scheme for a cognitive relay network, where the multiple-antenna energy-limited ST first harvests the energy sent by PT as well as PR and SR, and
then relays the traffic from PT to PR and also serves SR. The relay process matrix, cognitive beamforming vector and power splitter have been jointly optimized to maximize the SU rate with the energy causality constraint and the constraint that the rate requirement of PU is met. Both the perfect and imperfect CSI scenarios have been investigated. For the former case, the global optimal and low-complexity suboptimal solutions have been presented. For the latter case, a worst-case robust algorithm has been proposed. It has been demonstrated in the simulation that our proposed DWPT scheme is greatly preferred when ST is close to PR and SR. Thus, the location-based relay selection scheme could be our future work.

APPENDIX A
PROOF OF PROPOSITION 1

Without loss of generality, similar to [24], F can be expressed as

\[
F = \begin{bmatrix} V_1 & V_1^T \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} U_2^H \\ V_2^H \end{bmatrix}
\]

\[= V_1 A U_2^H + V_1 B U_2^H + V_2^T C U_2^H + V_2^T D U_2^H \tag{116}\]

where \( V_1^T \in \mathbb{C}^{M \times (M - 2)} \), \( U_2^H \in \mathbb{C}^{M \times (M - 3)} \), \( A \in \mathbb{C}^{2 \times 3} \), \( B \in \mathbb{C}^{2 \times (M - 3)} \), \( C \in \mathbb{C}^{(M - 2) \times 3} \) and \( D \in \mathbb{C}^{(M - 2) \times (M - 3)} \).

Obviously, \( V_1^H V_1^T = 0 \), \( U_2^H U_2^H = 0 \). Thus, we have

\[
\begin{bmatrix} h_s^H \\ h_p^H \end{bmatrix} V_1^T = 0, \quad U_2^H U_2^H = 0.
\]

Substituting (116) into terms related to \( F \) in problem \( P1 \), we know that \( B, C \) and \( D \) do not affect \( \| h_s^H F g \|_2^2 \), \( \| h_p^H F h_p \|_2^2 \) and \( \| h_p^H F h_p \|_2^2 \). In addition, \( C \) and \( D \) have no impact on \( \| h_s^H F \|_2^2 \) and \( \| h_p^H F \|_2^2 \). As a result, from (9) and (10), it is observed that \( C \) and \( D \) have no effect on SINRs of PR and SR. However, from (6), they increase the transmission power at ST. Hence, the optimal choice of \( C \) and \( D \) is \( C = 0 \) and \( D = 0 \). Besides, both SINRs of PR and SR are increased if we set \( B = 0 \). Therefore, \( F = V_1 A U_2^H \).

Similarly, \( w \) can be expressed as

\[
w = \begin{bmatrix} V_1 & V_1^T \end{bmatrix} \begin{bmatrix} b \\ c \end{bmatrix} = V_1 b + V_1^T c, \tag{118}\]

where \( b \in \mathbb{C}^{2 \times 1} \) and \( c \in \mathbb{C}^{(M - 2) \times 1} \). Note that \( c \) has no impact on \( \| h_s^H w \|_2^2 \), \( \| h_p^H w \|_2^2 \), and thus does not affect SINRs at PR and SR. But it increases the transmission power at ST. Hence, \( c = 0 \) and \( w = V_1 b \). This completes the proof. \( \blacksquare \)

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