Optimality of intuitionistic fuzzy fractional transportation problem of type-2

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\textbf{ABSTRACT}

In this paper, we formulate an effective method to find an optimal solution of trapezoidal intuitionistic fuzzy fractional transportation problem (TIFFTP) of type-2. The proposed method achieve its goal successively when compared to the existing methods (Gupta and Anupum). Trapezoidal ranking method is used, which is based on the area of both membership and non membership parts of the numbers. An illustrative example is provided to demonstrate the feasibility of this method.

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1. Introduction

In today’s highly competitive market, many organizations trying to find better ways to create and deliver value to customers become stronger. How and when to send the products safely to the customers in the quantities with minimum cost become more challenging. To meet this challenging, transportation models provide a powerful framework. It is a well-known optimization problem in operational research and was first developed by Hitchcock (1941). The main aim of transportation problem is to determine the shipping schedule which minimizes the total shipping cost while satisfying supply limit and demand requirements. The classical transportation problem refers to a special class of linear programming problems. Fractional programming is a special case of non-linear programming developed by Swarup in (1966), which plays an important role in game theory, logistics, stochastic process etc. The fractional transportation problem (FTP) plays an important role in logistics and supply management for reducing cost and improving service. In real life, there are many diverse situations due to uncertainty in judgment, lack of evidence, etc. Fuzzy transportation problem is more appropriate to model and solve the real world problems. In any organization optimization of resource is very much handled by the application of mathematical programming. It deals with situations where a ratio between two mathematical functions is either maximized or minimized. Thus fractional programming problem is one among the most difficult problems in the field of optimization.

The transportation models or problems are primarily concerned with the optimal (best possible) way in which a product produced at different factories or plants (called supply origins) can be transported to a number of warehouses or customers (called demand destinations). The objective in a transportation problem is to fully satisfy the destination requirements within the operating production capacity constraints at the minimum possible cost. Whenever there is a physical movement of goods from the point of manufacturer to the final consumers through a variety of channels of distribution (wholesalers, retailers, distributors etc.), there is a need to minimize the cost of transportation so as to increase profit on sales. The fractional transportation problem is to transport various amounts of a single homogeneous commodity that are initially stored at various origins, to different destinations in such a way that the total fractional transportation cost is minimum.

The concept of derivative is the key idea of calculus. It shows the sensitivity to change of a function...
interpreted the instantaneous velocity (Boyer & Newton with a physical viewpoint of derivative interpretation was suggested by Newton. The fractional order calculus was unexplored in engineering, because of its complexity and self-sufficiency nature and the fact that it does not have a fully acceptable geometrical or physical interpretation.

Fractional calculus is a field of mathematics study that grows out of the traditional definitions of calculus integral and derivative operators in much the same way fractional exponents is an outgrowth of exponents with integer value. It is a generalization of classical calculus. Whereas the fractional transportation problem plays an important role in logistics and supply management for reducing cost and improving service. In the real world, however, the parameters in the models are seldom known exactly and have to be estimated. In this the cost coefficients and right-hand sides are represented by fuzzy parameters. Intuitively, when the parameters in the fractional transportation problem are fuzzy numbers, the derived objective value should be also a fuzzy number. Based on Zadeh’s extension principle, a pair of two-level mathematical programs is formulated to calculate the fuzzy objective value of the fractional transportation with fuzzy parameters. By applying the dual formulation of linear fractional programming and variable substitution techniques, the two-level mathematical programs are transformed into ordinary one-level linear programs to solve. Here the main objective is to minimise the cost of distributing a product from a number of sources or origins to a number of destinations. Fractional calculus allows integrals and derivatives of any positive order. In fractional calculus fractional derivative is defined.

Fractional derivative is to model phenomena in fractal media. It can be considered a branch of mathematical physics that deals with integro-differential equations. This topic has gained popularity and importance in the past few decades in diverse field of science and engineering. It may be important to point out that the first application of fractional calculus was made by Abel (1802-1829) in the solution of an integral equation that arises in the formulation of the tautochronous problem. It has wide and fruitful applications in various fields such as mechanics, electricity, chemistry, biology, economics, notably control theory, and signal and image processing. Major topics include anomalous diffusion, vibration and control, continuous time random walk, Levy statistics, fractional Brownian motion, fractional neutron point kinetic model, power law, Riesz potential, fractional derivative and fractals, fractional phase-locked loops, fractional variational principles, fractional transforms, fractional wavelet, fractional predator prey system, soft matter mechanics, fractional signal and image processing, singularities analysis and integral representations for fractional differential systems, special functions related to fractional calculus, non-Fourier heat conduction, acoustic dissipation, geophysics, relaxation, creep, viscoelasticity, rheology, fluid dynamics, Chaos etc., Thus we can say that fractional calculus is therefore an excellent set of tools for describing the memory and hereditary properties of various materials and processes. It is a subject that has gained considerably popularity and importance in the past few decades in diverse fields of science and engineering. Recently, researchers have been used fractional calculus for probing viscoelasticity of materials with a high precision. Therefore, we can say that fractional calculus is a powerful tool for modeling complex phenomenon.

To a layman, most physical problems can be expressed in terms of mathematical formulations called differential equations; the differential equation’s aim is to analyze, understand, and predict the future of a physical problem. One of the most used differential operators was that developed in the 17th century by Isaac Newton and Gottfried Leibniz, but this failed to model complex real-world problems. The concept of nonlocal operators called fractional derivatives and integrals was suggested by Bernhard Riemann and Joseph Liouville with the aim to capture more complex phenomena.

The idea of intuitionistic fuzzy set was introduced by Atanassov (1986, 1999) with vagueness or uncertainty. The main advantage of intuitionistic fuzzy sets is that both the degree of membership and non-membership of each elements are included in the set. In recent years, it plays a vital role in decision making in fuzzy environment. It is a tool in modelling real life problems like financial services, sales analysis, product marketing, planning, manufacturing, transportation etc., In intuitionistic fuzzy environment, ranking plays an efficient role in decision making. Ranking fuzzy numbers is one of the fundamental problems of fuzzy arithmetic. In the present study, we formulated a transportation problem in which transportation costs are trapezoidal intuitionistic fuzzy numbers, supply and demands are taken as numerical values, this type of problem is called as “Intuitionistic fuzzy transportation problem of type-2”. This paper is motivated by Gupta and Anupum (2017), it will be very useful in solving transportation problem having uncertainty as well as in prediction of the transportation cost. It reduces computational work as well as time. While comparing with the existing paper (Gupta & Anupum, 2017), this paper propose an effective method to find
2. Literature review

A lot of researchers have been studied the fractional transportation problem and interval type-2 fuzzy numbers which is one way or the other relates to this paper. Gupta (Aggarwal & Gupta, 2016; Gupta & Anupum, 2017) proposed an efficient method for solving intuitionistic fuzzy transportation problem of type-2 and also developed a new method for solving unbalanced intuitionistic fuzzy transportation problem via new ranking method based on signed distance. Zadeh (1965, 1996) explained Fuzzy Sets and proposed a theory of fuzzy systems. Zimmermann (1978) proposed Fuzzy Sets and proposed a theory of fuzzy systems. Kour, Mukherjee, and Basu (2017) found out a method to solve intuitionistic fuzzy transportation problem using linear programming. Kumar and Hussain (2015, 2014) developed a method to solve unbalanced intuitionistic fuzzy transportation problem. Deli and Çağman (2015) discussed intuitionistic fuzzy parametrized soft set theory and its decision making. Gani et al. (Gani & Abbas, 2013; Gani & Mohamed, 2015) discussed a method of ranking generalized trapezoidal intuitionistic fuzzy numbers and also developed a new method for solving intuitionistic fuzzy transportation problem. Aggarwal and Gupta (2014) proposed a novel algorithm for solving intuitionistic fuzzy transportation problem via new ranking method. Chakraborty, Jana, and Roy (2015) discussed some arithmetic operations on generalized intuitionistic fuzzy number and its applications to transportation problem. Kaur and Kumar (2011) proposed a new method for solving fuzzy transportation problems using ranking function. Rani and Gulati (Kumar & Hussain, 2015; Rani, Gulati, & Kumar, 2014; Rani & Gulati, 2014) presented a method for unbalanced transportation problem in fuzzy environment and imprecise environment. Singh and Gupta (2014) developed a new approach for solving cost minimization balanced transportation problem under uncertainty. Huan (2018, 2014) explained fractal calculus and its geometrical explanation. Podlubny (1998) proposed an introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications. Ross (1977) explained about fractional calculus. Elkanzi, Qambar, and Suliman (2019) explained safety mapping for the transportation of raw sewage by trucks in Bahrain. Singh et al. (Singh & Garg, 2016; Singh & Yadav, 2016) discussed a new approach for solving intuitionistic fuzzy transportation problem of type-2 and also developed distance measures between type-2 intuitionistic fuzzy sets and their application to multicriteria decision making process. The fractional derivative of the exponential function obtained by Liouville in 1832, and the fractional derivative of power function got by Riemann in 1847 and Liouville in 1832, and the fractional derivative of power function got by Riemann in 1847.
\((a, a+\delta_1)\) respectively. \(\gamma_i\) and \(\delta_i\) are the left and right spreads of \(\mu^a_{x_i}(x)\) and \(\nu^a_{x_i}(x)\).

**Definition 3.3** A trapezoidal intuitionistic fuzzy number is denoted by \(\tilde{A}^l = (a_1, a_2, a_3, a_4, (a'_1, a'_2, a'_3, a'_4))\) where \((a'_1 \leq a_1 \leq a_2 \leq a_3 \leq a_4 \leq a'_4)\) with membership and non-membership functions are defined as follows:

\[
\mu_{x_i}^a(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1}, & \text{for } a_1 \leq x \leq a_2 \\
1, & \text{for } a_2 \leq x \leq a_3 \\
\frac{a_4-x}{a_4-a_3}, & \text{for } a_3 \leq x \leq a_4 \\
0, & \text{otherwise.}
\end{cases}
\]

\[
\nu_{x_i}^a(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1}, & \text{for } a_1 \leq x \leq a_2 \\
1, & \text{for } a_2 \leq x \leq a_3 \\
\frac{a_4-x}{a_4-a_3}, & \text{for } a_3 \leq x \leq a_4 \\
0, & \text{otherwise.}
\end{cases}
\]

**Definition 3.4** [9] Let \(\tilde{A}^l = ((a_1, a_2, a_3, a_4, (b_1, b_2, b_3, b_4); \omega_{AB}^a, \mu_{AB}^a)\) and \(\tilde{B}^l = ((c_1, c_2, c_3, c_4, (d_1, d_2, d_3, d_4); \omega_{AB}^b, \mu_{AB}^b)\) are the two generalized trapezoidal intuitionistic fuzzy numbers. The arithmetic operations are shown below:

(i) Addition: \(\tilde{A}^l + \tilde{B}^l = ((a_1 + c_1, a_2 + c_2, a_3 + c_3, a_4 + c_4); (b_1 + d_1, b_2 + d_2, b_3 + d_3, b_4 + d_4); \omega_{AB}^a, \mu_{AB}^a)\) where \(\omega = \min(\omega_{AB}^a, \omega_{AB}^b)\) and \(\mu = \max(\mu_{AB}^a, \mu_{AB}^b)\).

(ii) Subtraction: \(\tilde{A}^l - \tilde{B}^l = ((a_1 - c_1, a_2 - c_2, a_3 - c_3, a_4 - c_4); (b_1 - d_1, b_2 - d_2, b_3 - d_3, b_4 - d_4); \omega_{AB}^a, \mu_{AB}^a)\) where \(\omega = \min(\omega_{AB}^a, \omega_{AB}^b)\) and \(\mu = \max(\mu_{AB}^a, \mu_{AB}^b)\).

(iii) Multiplication: \(\tilde{A}^l \times \tilde{B}^l = ((a_1 \times c_1, a_2 \times c_2, a_3 \times c_3, a_4 \times c_4); (b_1 \times d_1, b_2 \times d_2, b_3 \times d_3, b_4 \times d_4); \omega_{AB}^a, \mu_{AB}^a)\) where \(\omega = \min(\omega_{AB}^a, \omega_{AB}^b)\) and \(\mu = \max(\mu_{AB}^a, \mu_{AB}^b)\).

(iv) Scalar Multiplication: \(\lambda \tilde{A}^l = ((\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4); (\lambda b_1, \lambda b_2, \lambda b_3, \lambda b_4); \omega_{AB}^a, \mu_{AB}^a)\) if \(\lambda > 0\).

\(\lambda \tilde{A}^l = ((\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4); (\lambda b_1, \lambda b_2, \lambda b_3, \lambda b_4); \omega_{AB}^a, \mu_{AB}^a)\) if \(\lambda < 0\).

(v) Division: \(\tilde{A}^l / \tilde{B}^l = \left(\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \frac{a_4}{b_4}; \omega_{AB}^a, \mu_{AB}^a\right)\) where \(\omega = \min(\omega_{AB}^a, \omega_{AB}^b)\) and \(\mu = \max(\mu_{AB}^a, \mu_{AB}^b)\).

### 4. Problem formulation

#### 4.1. Fractional transportation problem (FTP)

The FTP is the problem of minimizing q interval valued objective functions with interval cost. When the objective functions coefficients \(a_i = \left(a_i, a_i+\delta_i\right)\), \(A_i\) is the source parameters, \(B_j\) is the destination parameter and \(C_{ij}\) is the conveyance parameter, which are in the form of interval, where \(A_i = [s_{iL}, s_{iU}], i = 1, 2, \ldots, m\) and \(B_j = [t_{jL}, t_{jU}], j = 1, 2, \ldots, n\), are interval valued of source and destination. The formulation is

\[
\text{Minimize } Z^q(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} \left| C_{ij}^q \right| x_{ij} + \alpha q = 1, 2, \ldots, Q
\]

subject to

\[
\sum_{j=1}^{n} x_{ij} = A_i = [s_{iL}, s_{iU}], i = 1, 2, \ldots, m
\]

\[
\sum_{i=1}^{m} x_{ij} = B_j = [t_{jL}, t_{jU}], j = 1, 2, \ldots, n
\]

\(x_{ij} \geq 0, \forall i, j\).

The balanced condition is a necessary and sufficient condition for the existence of a feasible solution

\[
|C_{ij}^q, C_{ji}^q| \leq |D_{ij}^q|, \quad q = 1, 2, \ldots, Q,
\]

\[
p^q = \frac{|p^q|}{|D_{ij}^q|} = \frac{|C_{ij}^q|}{|C_{ji}^q|}
\]

is an interval representing the uncertain cost for the transportation problem. When the feasible solutions are uncertain cost for the transportation problem. Uncertainty specifically concerns right hand side constraints and objective functions. When the set of feasible solutions is uncertain, we identify a class of linear programs for which these classical approaches are no longer relevant. However it is possible to compute the worst optimum solution. By the definition the equivalent multi-objective deterministic transportation problem is given as

\[
\text{Minimize } Z^q(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} p^q_{ij} x_{ij} + \sum_{i=1}^{m} \sum_{j=1}^{n} p^q_{ij} x_{ij}
\]

subject to

\[
\sum_{j=1}^{n} x_{ij} = s_{iL}, \sum_{i=1}^{m} x_{ij} = s_{iU}
\]

\[
\sum_{j=1}^{n} x_{ij} = t_{jL}, \sum_{i=1}^{m} x_{ij} = t_{jU}
\]

\(x_{ij} \geq 0, \forall i, j\).

#### 4.2. Intuitionistic fuzzy fractional transportation problem of type-2

The proposed method is the best method to find the optimal solution of an intuitionistic fuzzy fractional transportation problem having supply and demand which are real numbers and transportation cost \(\gamma_j^q\) \((i = 1, 2, \ldots, m); \quad (j = 1, 2, \ldots, n)\) from \(i^{th}\) source to \(j^{th}\) destination, taken as intuitionistic fuzzy fractional transportation problem represented in Table 1. Here the optimal solutions are calculated separately for both the numerator and denominator, and then divide both the results, then we arrive the optimal solution of an intuitionistic fuzzy fractional transportation problem of type-2.
Table 1. Intuitionistic fuzzy fractional transportation problem of type-2.

| $D_1$ | $D_2$ | $D_3$ | $D_4$ | $s_i$ |
|-------|-------|-------|-------|-------|
| $c_{i1}$ | $c_{i2}$ | ... | $c_{in}$ | $s_i$ |
| $d_{i1}$ | $d_{i2}$ | ... | $d_{in}$ |

Figure 1. Trapezoidal Intuitionistic Fuzzy Transportation Problem (TIFTP).

4.3. Ranking method

Let $\tilde{A}^i = (a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4; \omega, \mu)$. In this $A = [a_2 = a_3 = b_2 = b_3]$. Now the membership function is defined as $S(\gamma_A) = \omega_0, \mu_0 = (\frac{2a_2 + 7a_3 + 7a_4}{18}, \frac{7a_4}{18})$. Similarly non-membership function is defined as $S(\nu_A) = \omega_0, \mu_0 = (\frac{2b_2 + 7b_3 + 7b_4}{18}, \frac{7b_4}{18})$. Then by using these we define the rank as follows:

$$\mathcal{R}(A) = \frac{\omega_A S(\gamma_A) + \mu_A S(\nu_A)}{\omega_A + \mu_A}$$

In ranking method, for comparing two ranking generalized intuitionistic trapezoidal fuzzy numbers $\tilde{A}^i$ and $\tilde{B}^j$, there are some orders defined as follows:

(i) $\tilde{A}^i > \tilde{B}^j$ if $\mathcal{R}(\tilde{A}^i) > \mathcal{R}(\tilde{B}^j)$,
(ii) $\tilde{A}^i > \tilde{B}^j$ if $\mathcal{R}(\tilde{A}^i) > \mathcal{R}(\tilde{B}^j)$,
(iii) $\tilde{A}^i = \tilde{B}^j$ if $\mathcal{R}(\tilde{A}^i) = \mathcal{R}(\tilde{B}^j)$.

Graphical representation for trapezoidal intuitionistic fuzzy transportation problem (TIFTP) is given in Figure 1, here $A = [a_2 = a_3 = b_2 = b_3]$.
5. Proposed method

5.1. Algorithm

The steps required to find out the optimal solution is written as follows:

**Step 1: Row reduced form**
Select the minimum intuitionistic fuzzy number from each row of intuitionistic fuzzy cost matrix of intuitionistic fuzzy fractional transportation problem of type-2 and subtract it from each intuitionistic fuzzy numbers of their corresponding column.

**Step 2: Column reduced form**
Select the minimum intuitionistic fuzzy number from each column of intuitionistic fuzzy cost matrix of intuitionistic fuzzy fractional transportation problem of type-2 and subtract it from each intuitionistic fuzzy numbers of their corresponding column.

**Step 3: Intuitionistic fuzzy zero centered value**
Here check whether that each row and column has at least one intuitionistic fuzzy number whose rank is zero. Otherwise, repeat from Step 1 and Step 2, or we can calculate the intuitionistic fuzzy zero centered value i.e. \( \frac{\hat{c}_{ij}}{\hat{d}_{ij}} \) for each cell having zero rank value. Here \( \frac{\hat{c}_{ij}}{\hat{d}_{ij}} \) is defined as follows:

\[
\frac{\hat{c}_{ij}}{\hat{d}_{ij}} = \frac{\text{Sum of intuitionistic fuzzy around the cell having zero rank value}}{\text{Number of intuitionistic fuzzy cost added having non-zero rank value}}
\]

**Step 4: Allocation**
Firstly choose a cell \( \{i, j\} : \max \{R_{ij}\} \} \), and then allot the maximum possible quantity to this cell. Then delete either the \( i^{th} \) row or \( j^{th} \) column, whose quantity is fully allocated.

**Step 5: Iterations**
Now again repeat Step 3 and Step 4 until all the allocations has made.

**Step 6: Optimal solution and Intuitionistic fuzzy optimal value**

The solution obtained from Step 5, is the required optimal solution \( \{x_{ij}\} \) and the intuitionistic fuzzy optimal value is \( \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\hat{c}_{ij}}{\hat{d}_{ij}} \otimes x_{ij} \).

6. Numerical example

**Example:** An intuitionistic fuzzy fractional transportation problem of type-2 with four suppliers i.e. \( S_1, S_2, S_3 \) and \( S_4 \) for each numerator and denominator, then four destinations i.e. \( D_1, D_2, D_3 \) and \( D_4 \) for each numerator and denominator respectively, which are represented in Table 2, which is solved by using the zero centered method.

**Solution:** Here the intuitionistic fuzzy fractional transportation problem of type-2 solved by finding the optimal solution and intuitionistic fuzzy optimal value for the numerator and denominator separately.

Firstly, we found out the values for the numerator. The steps are mentioned as follows.

**Step 1: Row reduced form for numerator**
By using the Step 1 the intuitionistic fuzzy fractional transportation problem of type-2 given in Table 3, which we transformed into the row reduced form as given in Table 4.

**Step 2: Column reduced form for numerator**
Now the problem in Table 4 is transformed into column reduced form and are given in Table 5.

**Step 3: Intuitionistic fuzzy Zero Centered value for numerator**
In Table 5, it is clear that the cells (1–3) and (4) is having zero rank value. Now we found out the intuitionistic fuzzy zero centered value corresponding to these cells are as follows,
**Step 4: First allocation for numerator** From the above calculations, \( R_{33} = 4.321 \) is the largest value obtained in ranking, therefore we can allocate the maximum possible quantity that is 12 to the cell \((c_{ij}, i = j = 3)\) and delete the destination \( D_{35} \). This is given in Table 6. Since the third row doesn’t have any intuitionistic fuzzy number whose rank is zero, therefore subtracting the intuitionistic fuzzy number which is having minimum ranking value from third row. Now we obtained new reduced table which is shown in Table 7. Again apply Step 3 and Step 4 of the above proposed method, now after all the allocations are made as shown in Table 8.

**Step 5: Optimal solution and Intuitionistic fuzzy optimal value for numerator**

The optimal solution for numerator obtained in Step 4 is,

\[
x_{11} = 2, x_{13} = 12, x_{21} = 15, x_{31} = 1, x_{32} = 12, x_{41} = 2, x_{44} = 8.
\]

Thus the numerator for the intuitionistic fuzzy transportation problem of type-2, given in Table 8 is calculated as,

\[
2 \otimes [(4, 5, 5, 7), (2, 5, 5, 9); 0, 6, 0.3] \oplus 12 \otimes [(5, 7, 7, 10) (3, 7, 7, 12); 0, 9, 0.5] \oplus 15 \otimes [(4, 6, 6, 10) (2, 6, 6, 12); 0, 5, 0.3] \oplus 1 \otimes [(7, 10, 10, 13) (6, 10, 10, 15); 0, 7, 0.2] \oplus 12 \otimes [(10, 13, 13, 15) (8, 13, 13, 18); 0, 8, 0.6] \oplus 2 \otimes [(6, 10, 10, 12) (5, 10, 10, 13); 0, 6, 0.1] \oplus 8 \otimes [(8, 10, 10, 12) (7, 10, 10, 14); 0, 7, 0.5] = [(331, 450, 450, 597), (238, 450, 450, 711); 0.5, 0.6].
\]

Graphical representation of optimal solution for intuitionistic fuzzy fractional transportation problem of type-2 for numerator is given in Figure 2.

Next, we found out the values for the denominator. The tables are given as follows.

The steps are mentioned as follows for the denominator.

**Step 1: Row reduced form for denominator**

By using the Step 1 the intuitionistic fuzzy fractional transportation problem of type-2 given in Table 9, which we transformed into the row reduced form as given in Table 10.

**Step 2: Column reduced form for denominator**

Now the problem in Table 10 is transformed into column reduced form and are given in Table 11.

**Step 3: Intuitionistic fuzzy zero centered value for denominator**

In Table 11, it is clear that the cells (1–4) and (1, 4) is having zero rank value. Now we found out the intuitionistic fuzzy zero centered value corresponding to these cells are as follows.
Step 4: First allocation for denominator

From the above calculations, $\mathcal{R}_{13} = 2.1268$ is the largest value obtained in ranking, therefore we can allocate the maximum possible quantity that is 7 to the cell $(d_{ij}, i = j = 3)$ and delete the destination $D_3$. This is given in Table 12. Here, since the third row doesn’t have any intuitionistic fuzzy number whose rank is zero, therefore subtracting the intuitionistic fuzzy number which is having minimum ranking value from third row. Now we obtained new reduced table which is shown in Table 13.

Again apply Step 3 and Step 4 of the above proposed method, now after all the allocations are made as shown in Table 14.

Step 5: Optimal solution and Intuitionistic fuzzy optimal value for denominator

The optimal solution for denominator obtained in Step 4 is,

$x_{11} = 6, x_{14} = 6, x_{22} = 11, x_{24} = 3, x_{33} = 7, x_{34} = 4, x_{41} = 12.$

Thus the numerator for the intuitionistic fuzzy transportation problem of type-2, given in Table 14 is calculated as

Thus, now we found out the optimal solutions for both the numerator and denominator, and finally we can calculate the trapezoidal intuitionistic fuzzy fractional transportation problem of type-2 by dividing the numerator and denominator calculated above. That is after calculations we got the optimal solution for intuitionistic fuzzy fractional transportation problem of type-2 is as written below,

Graphical representation of optimal solution for trapezoidal intuitionistic fuzzy fractional transportation problem of type-2 for denominator is given in Figure 3.

Thus, now we found out the optimal solutions for both the numerator and denominator, and finally we can calculate the trapezoidal intuitionistic fuzzy fractional transportation problem of type-2 by dividing the numerator and denominator calculated above. That is after calculations we got the optimal solution for intuitionistic fuzzy fractional transportation problem of type-2 is as written below,

Thus, now we found out the optimal solutions for both the numerator and denominator, and finally we can calculate the trapezoidal intuitionistic fuzzy fractional transportation problem of type-2 by dividing the numerator and denominator calculated above. That is after calculations we got the optimal solution for intuitionistic fuzzy fractional transportation problem of type-2 is as written below,

Graphical representation of optimal solution for trapezoidal intuitionistic fuzzy fractional transportation problem of type-2 for denominator is given in Figure 3.

7. Discussion and comparison

This paper is motivated by Gupta and Anupum (2017) in Figure 5, it will be very useful in solving transportation problem having uncertainty. In this paper there is no need to find the initial basic feasible solution,
but the proposed method directly gives the optimal solution. It reduces computational work as well as time. While comparing with the existing paper (Gupta & Anupum, 2017), our proposed method shows an effective method to find intuitionistic fuzzy optimal solution of intuitionistic fuzzy fractional transportation problem of type-2. For this we calculated the numerator and denominator separately to find the effective optimal solution. The method adopted here is simple and achieve its goal speedily and accurately for trapezoidal intuitionistic fuzzy fractional transportation problem [TIFFTP] of type-2, as compared to the existing methods in the literature. Here also, there is no need to find the initial basic feasible solution separately for numerator and denominator, but gives the optimal solution. Also in proposed method the authors found out ranking for generalized trapezoidal intuitionistic fuzzy fractional transportation problem of type-2. This method can rank several types of generalized intuitionistic fuzzy numbers and also crisp numbers which are considered to be a special case of fuzzy numbers. In our research we have attempted for the first time an effective method for solving trapezoidal intuitionistic fuzzy fractional transportation problem of type-2. This method can be used in several areas like fuzzy environment, intuitionistic fuzzy environment etc.

Interest in Fractional Calculus for many years was purely mathematic, and it is not hard to see why. Fractional calculus is used to model physical and engineering processes that are found to be best described by fractional differential equations. The models of fractional derivative are used for accurate modelling of those systems which require accurate modelling of damping. Only the very basic concepts regarding fractional order calculus were addressed here, and yet it is evident that the study fractional calculus opens the mind to entirely new branches of thought. It fills in the gaps of traditional calculus in ways that as of yet, no one completely understands. In these fields, several analytical and numerical methods with their applications to new problems have been proposed in recent years. Thus “Fractional Calculus and its Applications in Applied Mathematics and Other Sciences” is devoted to study the recent works in the above fields of fractional calculus done by the leading researchers. The overall goal of this is to show that fractional calculus is not just a mathematical framework which can only be empirically introduced to curve fit the experimental observations. Rather, it has an inherent connection to real physical processes which needs to be explored more. We hope that the results obtained here may benefit the scientific communities of fractional calculus, seismology, non-Newtonian rheology, sediment acoustics etc. Here by using intuitionistic fuzzy set we transform a vague pattern classification problem into a precise, well defined optimization problem. Thus, intuitionistic fuzzy set have essentially higher describing possibilities than fuzzy sets.

Comparison between the existing paper and the proposed paper is given below.

8. Conclusion

In this paper an effective method is used to find out the optimal solution of trapezoidal intuitionistic fuzzy fractional transportation problem [TIFFTP] of type-2. Here in this case there is no need to find the initial basic feasible solution. This proposed method

| Table 7. | New reduced table for numerator. |
|---|---|---|---|---|---|
| S<sub>1</sub> | S<sub>2</sub> | S<sub>3</sub> | S<sub>4</sub> | D<sub>1</sub> | D<sub>2</sub> | D<sub>3</sub> | D<sub>4</sub> | S<sub>i</sub> |
| [-8,1,1,11][-1,16,18][0,5,0,3] | [-8,0,0,8][-14,0,14][0,5,0,1] | [-5,5,12][-5,10,19][0,4,0,5] | 14 |
| [-13,0,0,13][-20,0,0,20][0,5,0,3] | [-7,2,2,13][-2,16,19][0,5,0,4] | [-7,5,5,13][-5,14,16][0,5,0,8] | 15 |
| [-25,0,0,25][-38,0,0,38][0,5,0,5] | [-21,3,3,24][-3,19,28][0,5,0,6] | [-21,4,4,26][-4,32,36][0,5,0,5] | 1 |
| [-12,0,0,13][-18,0,0,18][0,5,0,3] | [-7,3,3,12][-3,12,17][0,5,0,5] | [-10,0,0,10][-14,0,14][0,6,0,5] | 2 |
| 20 | 12 | 8 |

| Table 8. | Optimal solution for intuitionistic fuzzy fractional transportation problem of type-2 for numerator. |
|---|---|---|---|---|---|
| S<sub>1</sub> | S<sub>2</sub> | S<sub>3</sub> | S<sub>4</sub> | D<sub>1</sub> | D<sub>2</sub> | D<sub>3</sub> | D<sub>4</sub> | S<sub>i</sub> |
| [2, 4, 5, 7, 9][0,6,0,3] | [1, 3, 4, 7, 11][0,5,0,1] | [3, 5, 7, 10, 12][0,9,0,5] | 14 |
| [2, 4, 6, 10, 12][0,5,0,3] | [3, 7, 8, 13, 14][0,6,0,4] | [4, 8, 10, 17, 18][0,7,0,6] | 15 |
| [6, 7, 10, 13, 15][0,7,0,2] | [8, 10, 13, 15, 18][0,8,0,6] | [6, 7, 10, 14, 15][0,8,0,5] | 13 |
| [5, 6, 10, 12, 13][0,6,0,1] | [8, 9, 13, 15, 18][0,8,0,5] | [13, 15, 17, 18, 20][0,9,0,3] | [7, 8, 10, 12, 14][0,7,0,5] | 10 |
| 20 | 12 | 12 | 8 |

Figure 2. TIFFTP - Numerator.
Table 9. Intuitionistic fuzzy fractional transportation problem of type-2 for denominator.

| D_1           | D_2           | D_3           | D_4           | s_i |
|---------------|---------------|---------------|---------------|-----|
| [(1, 2, 4, 5, 8);0.4,0.1] | [(3, 5, 6, 9, 11);0.3,0.1] | [(4, 5, 7, 10, 12);0.4,0.2] | 12  |
| [(4, 6, 9, 12);0.6,0.3]  | [(2, 4, 7, 10, 13);0.4,0.1] | [(7, 9, 12, 14, 16);0.3,0.1] | 14  |
| [(5, 7, 8, 10, 11);0.4,0.2] | [(2, 4, 6, 7);0.5,0.3]  | [(4, 5, 9, 10, 13);0.2,0.1] | 11  |
| [(1, 2, 4, 5, 9);0.7,0.5]  | [(5, 6, 8, 10, 12);0.6,0.5] | [(7, 9, 11, 13, 15);0.4,0.1] | 12  |
| d_j           |               |               |               |     |
| 18            | 11            | 7             | 13            |     |

Table 10. Row reduced form for denominator.

| D_1           | D_2           | D_3           | D_4           | s_i |
|---------------|---------------|---------------|---------------|-----|
| [(1, 2, 4, 5, 8);0.4,0.1] | [(3, 5, 6, 9, 11);0.3,0.1] | [(4, 5, 7, 10, 12);0.4,0.2] | 12  |
| [(4, 6, 9, 12);0.6,0.3]  | [(2, 4, 7, 10, 13);0.4,0.1] | [(7, 9, 12, 14, 16);0.3,0.1] | 14  |
| [(5, 7, 8, 10, 11);0.4,0.2] | [(2, 4, 6, 7);0.5,0.3]  | [(4, 5, 9, 10, 13);0.2,0.1] | 11  |
| [(1, 2, 4, 5, 9);0.7,0.5]  | [(5, 6, 8, 10, 12);0.6,0.5] | [(7, 9, 11, 13, 15);0.4,0.1] | 12  |
| d_j           |               |               |               |     |
| 18            | 11            | 7             | 13            |     |

Table 11. Column reduced form for denominator.

| D_1           | D_2           | D_3           | D_4           | s_i |
|---------------|---------------|---------------|---------------|-----|
| [(1, 2, 4, 5, 8);0.4,0.1] | [(3, 5, 6, 9, 11);0.3,0.1] | [(4, 5, 7, 10, 12);0.4,0.2] | 12  |
| [(4, 6, 9, 12);0.6,0.3]  | [(2, 4, 7, 10, 13);0.4,0.1] | [(7, 9, 12, 14, 16);0.3,0.1] | 14  |
| [(5, 7, 8, 10, 11);0.4,0.2] | [(2, 4, 6, 7);0.5,0.3]  | [(4, 5, 9, 10, 13);0.2,0.1] | 11  |
| [(1, 2, 4, 5, 9);0.7,0.5]  | [(5, 6, 8, 10, 12);0.6,0.5] | [(7, 9, 11, 13, 15);0.4,0.1] | 12  |
| d_j           |               |               |               |     |
| 18            | 11            | 7             | 13            |     |

Table 12. First allocation for denominator.

| D_1           | D_2           | D_3           | D_4           | s_i |
|---------------|---------------|---------------|---------------|-----|
| [(1, 2, 4, 5, 8);0.4,0.1] | [(3, 5, 6, 9, 11);0.3,0.1] | [(4, 5, 7, 10, 12);0.4,0.2] | 12  |
| [(4, 6, 9, 12);0.6,0.3]  | [(2, 4, 7, 10, 13);0.4,0.1] | [(7, 9, 12, 14, 16);0.3,0.1] | 14  |
| [(5, 7, 8, 10, 11);0.4,0.2] | [(2, 4, 6, 7);0.5,0.3]  | [(4, 5, 9, 10, 13);0.2,0.1] | 11  |
| [(1, 2, 4, 5, 9);0.7,0.5]  | [(5, 6, 8, 10, 12);0.6,0.5] | [(7, 9, 11, 13, 15);0.4,0.1] | 12  |
| d_j           |               |               |               |     |
| 18            | 11            | 7             | 13            |     |

Table 13. New reduced table for denominator.

| D_1           | D_2           | D_3           | D_4           | s_i |
|---------------|---------------|---------------|---------------|-----|
| [(1, 2, 4, 5, 8);0.4,0.1] | [(3, 5, 6, 9, 11);0.3,0.1] | [(4, 5, 7, 10, 12);0.4,0.2] | 12  |
| [(4, 6, 9, 12);0.6,0.3]  | [(2, 4, 7, 10, 13);0.4,0.1] | [(7, 9, 12, 14, 16);0.3,0.1] | 14  |
| [(5, 7, 8, 10, 11);0.4,0.2] | [(2, 4, 6, 7);0.5,0.3]  | [(4, 5, 9, 10, 13);0.2,0.1] | 11  |
| [(1, 2, 4, 5, 9);0.7,0.5]  | [(5, 6, 8, 10, 12);0.6,0.5] | [(7, 9, 11, 13, 15);0.4,0.1] | 12  |
| d_j           |               |               |               |     |
| 18            | 11            | 7             | 13            |     |

Table 14. Optimal solution of Intuitionistic fuzzy fractional transportation problem of type-2 for denominator.

| D_1           | D_2           | D_3           | D_4           | s_i |
|---------------|---------------|---------------|---------------|-----|
| [(1, 2, 4, 5, 8);0.4,0.1] | [(3, 5, 6, 9, 11);0.3,0.1] | [(4, 5, 7, 10, 12);0.4,0.2] | 12  |
| [(4, 6, 9, 12);0.6,0.3]  | [(2, 4, 7, 10, 13);0.4,0.1] | [(7, 9, 12, 14, 16);0.3,0.1] | 14  |
| [(5, 7, 8, 10, 11);0.4,0.2] | [(2, 4, 6, 7);0.5,0.3]  | [(4, 5, 9, 10, 13);0.2,0.1] | 11  |
| [(1, 2, 4, 5, 9);0.7,0.5]  | [(5, 6, 8, 10, 12);0.6,0.5] | [(7, 9, 11, 13, 15);0.4,0.1] | 12  |
| d_j           |               |               |               |     |
| 18            | 11            | 7             | 13            |     |

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Figure 3. TIFFTP - Denominator.

Figure 4. TIFFTP of type-2.
will directly gives the optimal solution of an intuitionistic fuzzy fractional transportation problem (TIFTP) of type-2. This type of problem will be very useful in solving transportation problem. Also this method achieves its goals speedly as compared to the existing methods.

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No potential conflict of interest was reported by the author(s).

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**Figure 5.** Comparison Table.
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