TRANSPORT OF TOROIDAL MAGNETIC FIELD BY THE MERIDIONAL FLOW AT THE BASE OF THE SOLAR CONVECTION ZONE

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ABSTRACT

In this paper we discuss the transport of toroidal magnetic field by a weak meridional flow at the base of the convection zone. We use the differential rotation and meridional flow model developed by Rempel and incorporate feedback of a purely toroidal magnetic field in two ways: directly through the Lorentz force (magnetic tension) and indirectly through quenching of the turbulent viscosity, which affects the parameterized turbulent angular momentum transport in the model. In the case of direct Lorentz force feedback, we find that a meridional flow with an amplitude of around 2 m s$^{-1}$ can transport a magnetic field with a strength of 20–30 kG. Quenching of turbulent viscosity leads to deflection of the meridional flow from the magnetized region and a significant reduction of the transport velocity if the magnetic field is above equipartition strength.

Subject headings: Sun: interior — Sun: magnetic fields — Sun: rotation

1. INTRODUCTION

Flux-transport dynamos have proven to be successful for modeling the evolution of the large-scale solar magnetic field (Choudhuri et al. 1995; Dikpati & Charbonneau 1999). In a flux-transport dynamo the equatorward propagation of the magnetic activity belt (butterfly diagram) is a consequence of the meridional flow into a dynamo model and allow for the feedback of the Lorentz force on the meridional flow. In order to be able to address this question, it is necessary to incorporate a model for the solar differential rotation and meridional flow at the base of the solar convection zone inferred from studies of rising magnetic flux tubes (Choudhuri & Gilman 1987; Fan et al. 1993; Schüssler et al. 1994; Caligari et al. 1995, 1998) is around 100 kG and thus orders of magnitude larger than the equipartition field strength estimated from a meridional flow velocity of a few m s$^{-1}$. Therefore, it is crucial for flux-transport dynamos to address the feedback of the Lorentz force on the meridional flow.

In order to be able to address this question, it is necessary to incorporate a model for the solar differential rotation and meridional flow into a dynamo model and allow for the feedback of the Lorentz force on differential rotation and meridional flow. Differential rotation and meridional flow have been addressed in the past through mainly two approaches: three-dimensional full spherical shell simulations (Glatzmaier & Gilman 1982; Gilman & Miller 1986; Miesch et al. 2000; Brun & Toomre 2002) and axisymmetric mean field models (Kitchatinov & Rüdiger 1993, 1995; Rüdiger et al. 1998; Küker & Stix 2001). While the three-dimensional simulations have trouble reproducing a consistent large-scale meridional flow pattern (poleward in the upper half of the convection zone), as it is observed by helioseismology (Braun & Fan 1998; Haber et al. 2002; Zhao & Kosovichev 2004), such a flow is a common feature in most of the mean field models.

In this paper we present a first step toward building a dynamical dynamo model by focusing primarily on the transport of a prescribed toroidal magnetic field by meridional circulation. We do not attempt, in this very first model, to solve the dynamo equations in order to generate the magnetic fields. To this end we use the mean field model for differential rotation and meridional circulation described by Rempel (2005) and extend it by incorporating the magnetic tension resulting from a purely toroidal magnetic field at the base of the solar convection zone. In this paper we assume that the toroidal magnetic field consists of a homogeneous layer. We briefly discuss in § 2 how an intermittent field structure would change the results. In a future paper we will present a full axisymmetric mean field dynamo simulation.

2. MODEL

In this paper we use the differential rotation/meridional circulation model of Rempel (2005) and add the Lorentz force of a toroidal magnetic field. For the details of the model we refer the reader to Rempel (2005). The equations we solve are

\[ \frac{\partial \psi_1}{\partial r} = -\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \psi_0 \right) - \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \psi_0 \psi_0 \right) \] (1)

\[ \frac{\partial \psi_2}{\partial t} = - \frac{v_r}{r} \frac{\partial \psi_2}{\partial r} - \frac{v_\theta}{r} \frac{\partial \psi_2}{\partial \theta} + \frac{v_r v_\theta}{r} \frac{\partial}{\partial \theta} \left( \psi_0 \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left( \nu_0 \psi_0 \right) + \frac{B^2}{\mu_0 \psi_0 r} \] (2)

\[ \frac{\partial \Omega_1}{\partial t} = -\frac{v_r}{r^2} \frac{\partial}{\partial r} \left[ r^2 \left( \Omega_0 + \Omega_1 \right) \right] - \frac{v_\theta}{r \sin \theta} \frac{\partial}{\partial \theta} \left[ \sin^2 \theta \left( \Omega_0 + \Omega_1 \right) \right] + \frac{F_\theta}{\psi_0 r} \sin \theta \] (3)

\[ \frac{\partial s_1}{\partial t} = -\frac{v_r}{r} \frac{\partial s_1}{\partial r} - \frac{v_\theta}{r} \frac{\partial s_1}{\partial \theta} + \frac{v_r v_\theta}{r} \frac{\partial}{\partial \theta} \left( s_1 \right) + \frac{\gamma \delta}{H_p} + \gamma - 1 \frac{Q}{\psi_0} + \frac{1}{\psi_0 Q} \nabla \cdot (\kappa \psi_0 T_0 \nabla s_1) \] (4)
The term \( p_1 \) denotes the pressure perturbation with respect to the reference state, \( p_0 \); and \( s_1 \) is the entropy perturbation, \( s_1 = p_1/p_0 - \gamma \theta_1/\theta_0 \) (made dimensionless with the heat capacity \( c_p \)), where \( \theta_0 \) denotes the density of the reference state. We use a reference state corresponding to a polytropic atmosphere with gravity varying \( \sim r^{-2} \). Since our model describes the convection zone and overshoot region where the deviation from adiabaticity is small (\( \nabla - \nabla_{ad} \ll 1 \)) we use an adiabatic polytrope for the reference state. However, small perturbations from adiabaticity are considered in the entropy equation (eq. [5]) through the third term on the right-hand side. We use values for \( \delta = \nabla - \nabla_{ad} \sim -10^{-4} \) below \( \rho = 0.725 R_e \) and \( \delta = 0 \) above. Different profiles of \( \delta \) influence primarily the differential rotation profile (more specifically, the deviation from the Taylor-Proudman state); however, the influence on the meridional flow, which is of primary interest here, is very weak. Here \( \Omega_0 \) denotes the core rotation rate; \( \kappa_t \) in equation (5) denotes the turbulent convective heat diffusivity, and \( Q \) denotes the viscous heating. We write the pressure/buoyancy term in equation (2) assuming small deviations from adiabaticity (\( \nabla - \nabla_{ad} \ll 1 \)). \( F_r, F_p, \) and \( F_\theta \) denote the viscous stress, including a parameterization of turbulent angular momentum transport (\( \Lambda \)-effect; Kiticatinov & Rüdiger 1995). The turbulent angular momentum transport is the driver for the differential rotation and the meridional flow in this model. The exact form of these terms is discussed in detail in Rempel (2005).

In this model we do not include the induction equation, which would be required to address the nonlinear evolution of the field transported by the meridional flow. As a first step toward the full problem, we solve here for a stationary solution similar to Rempel (2005), but include the magnetic tension force of a toroidal magnetic field. This allows determination of the field strength up to which an equatorward transport of toroidal magnetic field is possible if the feedback of the magnetic tension force on the meridional flow is included. For reasons of simplicity, we omit the magnetic buoyancy term in equation (2) (fifth term on the right-hand side) and focus this discussion only on the magnetic tension force. Magnetic buoyancy has been addressed in great detail by Moreno-Insertis et al. (1992) and Rempel et al. (2000).

In this study we assume a homogeneous toroidal field at the base of the convection zone. Alternatively, the magnetic field could be highly intermittent, leading to the following two complications: the Lorentz force is dependent on the structure of the field in detail and cannot be expressed just by the mean field alone; and the advection velocity of the mean field is not given by the mean flow, since field-free plasma can flow around magnetic elements. In that sense, for the case of an intermittent field two extreme scenarios are possible:

1. The field is highly intermittent and couples only weakly through the drag force of individual flux elements to the mean flow. In this case the equatorward transport of field by the meridional flow is mainly dependent on the strength of the coupling between mean field and mean flow (similar to the behavior of individual flux tubes, as discussed by Rempel 2003); the change of the meridional flow pattern by the feedback of the Lorentz force is a secondary effect.

2. The coupling between mean flow and mean field is strong. In this case equatorward transport of field by the meridional flow is dependent on the change of the meridional flow pattern caused by the Lorentz force.

Although we assume a homogeneous field in this study, our results also apply to an intermittent field in the case of strong coupling, discussed above. Independent of the particular way the field couples to the fluid, momentum conservation requires that the bulk force acting on the bulk flow is given by the mean Lorentz force. However, the relation between the mean Lorentz force and the mean field strength can be more complicated, as we illustrate in the following very simple example.

We assume that the toroidal field consists of magnetic flux elements with a total filling factor \( f \) and field strength \( B_0 \). The mean tension force \( \langle F \rangle \) is proportional to \( f B_0 \), while the mean field strength is given by \( \langle B \rangle = f B_0 \); therefore \( \langle F \rangle \sim \langle B \rangle \langle f \rangle \). This means that for a given tension force the assumption of an intermittent field requires a mean field a factor of \( f^{-1/2} \) smaller than the homogeneous case; however, the field strength of individual flux elements would be a factor of \( f^{-1/2} \) larger.

In this paper we evaluate under which conditions an equatorward meridional flow at the base of the convection zone can transport a magnetic field equatorward. To this end we compute at a fixed latitude the following effective transport velocity for a given magnetic field configuration:

\[
 v_{\eta}^\text{eff} = \frac{\int v_{\eta} B_r dr}{\int B_r dr},
\]

where \( v_{\eta}^\text{eff} > 0 \) means a transport of magnetic flux toward the equator. In the case of intermittent field \( v_{\eta}^\text{eff} \), it is an upper estimate for the transport capability. Explicitly calculating the coupling between mean flow and mean field is beyond the scope of this paper and most likely highly dependent on the exact field configuration.

3. NUMERICAL RESULTS

In this section we use models for the differential rotation and meridional flow that are very similar to model 1 in Rempel (2005). Parameters that differ from those used in Rempel (2005) are summarized in Table 1. We have chosen the parameters such...
that we have three models with meridional flow velocities varying by a factor of 4, while the turbulent diffusivity is unchanged (models 1, 2, and 3), and three models with a turbulent diffusivity varying by a factor of 4, while the meridional flow speed is roughly the same (models 1, 4, and 5). Figure 1 shows the flow profile of \( v_\theta \) at 30° latitude for the models listed in Table 1.

Figure 2 summarizes the results obtained with reference models 1, 2, and 3 (varying flow speed, but same turbulent viscosity). It turns out that the changes of the meridional flow and differential rotation are largely independent of the reference model (within a few percent variation). Therefore, we plot in Figures 2a and 2b the change of meridional flow and differential rotation for reference model 1. In each case we add magnetic field with a maximum strength of 1, 2, 3, and 4 T. The magnetic field is located at 30° latitude and has a radial width of 0.05 \( R_\odot \), and a parabolic profile in \( B^2 \) that is centered at 0.75 \( R_\odot \). In latitude we use a Gaussian profile with a width of about 15°. For the case with 4 T field strength, the change of meridional flow speed has a maximum amplitude of around 7 m s\(^{-1}\) and is therefore larger than the meridional flow to be expected at the base of the convection zone (the observed surface flow speed is around 10–20 m s\(^{-1}\)), which requires a return flow of only 1–2 m s\(^{-1}\) at the base of the convection zone due to mass conservation.

A consequence of the meridional flow change induced by the magnetic tension force is the formation of a prograde jet within the magnetized region shown in Figure 2b, which partially compensates the magnetic tension due to the Coriolis force. The amplitude of the jet that would be required for a complete compensation is given by (see § 4 for a derivation)

\[
\frac{\Omega'_1}{\Omega_0} = \frac{1}{2} \left( \frac{v_A}{\Omega_0 r \sin \theta} \right)^2,
\]

with the Alfvén speed \( v_A = B/(\mu_0 \rho_0)^{1/2} \). For a field of 4 T at \( r = 0.75 R_\odot, 30^\circ \) latitude, and value of \( \rho_0 = 150 \) kg m\(^{-3}\)), this yields \( \Omega'_1/\Omega_0 \approx 0.025 \), which means that for the case shown in Figure 2 the jet compensates about 70% of the magnetic tension force. (For the other field strengths shown, this ratio is about the same.)

The interesting result is that the prograde jet forms independent of whether the magnetic field is transported equatorward or poleward (angular momentum conservation would only provide a prograde jet for a poleward movement). This is caused by the fact that the reference state (without any influence of magnetic field) is characterized by an equilibrium between turbulent angular momentum transport (including dissipative terms and nondissipative terms “\( \Lambda \)-effect”) and the angular momentum transport of the meridional flow (the divergence of the total angular momentum flux has to be zero for a stationary solution). The magnetic tension force reduces the equatorward flow speed at the base of the convection zone, disturbing this balance and therefore forcing a change of the differential rotation. Formally, this result can be understood as follows. Let \( F^0_\Lambda, F^0_\nu, \) and \( F^0_\mu \) be the angular momentum fluxes in the reference state due to turbulent nondissipative angular momentum transport (\( \Lambda \)-effect), turbulent dissipation, and meridional flow, respectively. Then, the stationary reference state is characterized by \( \nabla \cdot (F^0_\Lambda + F^0_\nu + F^0_\mu) = 0 \). The presence of a magnetic field changes the meridional flow, leading to a perturbation \( F^1_\mu \), which changes the differential rotation and leads to a perturbation of the dissipative angular momentum flux \( F^1_\nu \). Since the \( \Lambda \)-effect remains unaffected in this case, the new stationary equilibrium requires \( \nabla \cdot (F^0_\Lambda + F^1_\nu + F^0_\mu) = 0 \). Since for all cases considered \( F^0_\mu \) is always poleward-directed in the magnetized region, this requires the formation of a prograde jet, although \( F^0_\mu + F^1_\mu \) can be still equatorward-directed.
fig. 3.—Changes of meridional flow (top panels) and differential rotation (bottom panels) using reference models 4, 1, and 5 (left to right). An increase of turbulent viscosity (left to right) while keeping the flow speed constant leads to a spread of the sidelobes and a reduction of the prograde flow within the magnetized region. The line styles indicate different magnetic field strengths, as in Fig. 2. The effective transport velocity is only marginally affected (see Fig. 5).

Moreno-Insertis et al. (1992) and Rempel et al. (2000) discussed the formation of prograde jets required for equilibrium states of the toroidal field at the base of the convection zone. In their studies a poleward movement of the magnetic field was always required, since turbulent angular momentum transport and meridional flow were not considered in the reference state. The magnetic field strength of around 10 T they considered is always required, since turbulent angular momentum transport and meridional flow were not considered in the reference state. The critical field strength up to which an equatorward transport of the magnetic field is possible by around 10%.

While the changes of the meridional flow are independent of the meridional flow speed in the reference model, the question whether magnetic field is still transported equatorward depends on the total flow speed. Figure 2c shows the effective transport velocity according to equation (9) for the three cases. In the case of a meridional flow of around 1.5 m s⁻¹, magnetic fields up to about 2.2 T can be transported equatorward, while a 6 m s⁻¹ flow could transport up to 4.5 T equatorward. Note that the critical field strength scales roughly with the square root of the meridional flow velocity. This is different from the case of individual flux tubes coupling to the flow through the drag force, which results in $B \sim v^n$.

Figure 3 shows the result for reference models 1, 4, and 5, which have roughly the same flow velocity, but a variation in the turbulent viscosity by a factor of 4. Although the amplitude of the prograde jet changes significantly, the transport capability of the meridional flow (see Fig. 5) is only marginally affected. In model 4, with a value of $\nu_t = 2.5 \times 10^9$ m² s⁻¹, the jet compensates around 90% of the curvature force (Fig. 3, left panels), whereas in model 5, with a value of $\nu_t = 10^9$ m² s⁻¹, the jet compensates only around 50% of the curvature force (Fig. 3, right panels). This does not strongly affect the transport capability of the meridional flow, since at the same time of increasing turbulent viscosity the transport of magnetic field through viscous drag increases (a typical value of the Reynolds number on scales of the magnetic field is of the order of 1 due to the small meridional flow velocity and the large turbulent viscosity). We discuss this in detail in § 4.

Figure 4 shows the result for different radial widths of the magnetic field. In all cases we use model 1 as the reference model. Figures 4a and 4d show results for a width of 0.05 $R_\odot$, Figures 4b and 4e for a width of 0.035 $R_\odot$, and Figures 4c and 4f for a width of 0.025 $R_\odot$. A comparison with Figure 3 shows clearly that a reduction of the width of the magnetic field is equivalent to an increase of the viscosity, as expected from a simple scaling argument.

Figure 5 shows the effective transport velocity for the cases shown in Figure 3 (Fig. 5a) and Figure 4 (Fig. 5b). Changes of the turbulent viscosity (by a factor of 4) and the width of the magnetic field (by a factor of 2) considered here influence the critical field strength up to which an equatorward transport of flux is possible by around 10%.

4. ANALYTIC ESTIMATE

Let $v_x$, $v_y$, and $\Omega_z$ be a stationary solution of the differential rotation problem without a magnetic field, and $v'_x$, $v'_y$, and $\Omega'_z$ be perturbations around that state caused by the presence of the magnetic field. If we only consider the $\theta$-component of the momentum equation, we can estimate the balance among Coriolis force, magnetic tension, and viscous stress by

$$2\Omega_0\Omega'_z \sin \theta \cos \theta - \frac{B^2}{\mu_0 \partial \Phi / \partial r} \cot \theta - c_{d,\nu} \frac{v'_y}{d^2} = 0,$$

where $d$ denotes a length associated with the radial width of the magnetic field, and $c_{d,\nu}$ is a coefficient, which takes care of the more complicated flow structure. The formulation given here relates to the more general formulation of the drag force $\sim \omega \nu^2 / (2d)$.
through the assumption $c_w \sim c_d/Re$, with the turbulent Reynolds number $Re = v t/c_d$. Since the large-scale flow pattern in combination with the large turbulent viscosity is a flow with a small turbulent Reynolds number $1$, it can be expected that the meridional flow behaves like a highly viscous fluid. This is justified as long as the scale of the flow field is larger than the typical turbulence scale. In this limit typical values for $c_d$ should be of the order of $10$ (see textbooks on fluid dynamics for Stokes law). For small magnetic flux tubes a formulation with $c_w = \text{const}$ rather than $c_d = \text{const}$ is more valid. We show later that for a reasonable choice of $c_d$ a good agreement between this analytic scaling analysis and the numerical result can be achieved.

We emphasize that drawing parallels between laminar, viscous laboratory flows and highly turbulent astrophysical flows with a low turbulent Reynolds number is speculative. However, it is unavoidable when applying the mean field approach, leading to the parameterization of large turbulent viscosities.

The balance between angular momentum transport and viscous dissipation yields

$$-2\Omega_0 \frac{v_\theta}{r} \cot \theta - \nu_t \frac{\Omega'_i}{d^2} = 0.$$  \hfill (12)

We did not introduce here an additional free parameter in front of the diffusive loss term, since this prefactor, $~1$, can be easily absorbed into the definition of the length scale $d$. Combining both equations gives a relation between $B$ and $v'_\theta$:

$$B^2 = -\mu_0 \theta_0 r \tan \theta \left[ (2\Omega_0 \cos \theta)^2 \frac{\nu_t}{d^2} + c_d \frac{\nu_t}{d^2} \right] v'_\theta.$$  \hfill (13)

An equatorward transport of the magnetic field requires $|v'_\theta| \ll |v_\theta|$, which yields an upper limit for magnetic field strength of

$$B^2 \sim \mu_0 \theta_0 r \tan \theta \left[ (2\Omega_0 \cos \theta)^2 \frac{\nu_t}{d^2} + c_d \frac{\nu_t}{d^2} \right] v_\theta.$$  \hfill (14)

The importance of the two terms in the brackets depends on the value of the turbulent viscosity $\nu_t$ and the width of the magnetic field $d$. For a given width, the first term is important for low viscosity, the second one for high viscosity. Both are of the same importance if

$$\nu_t = \nu_{\text{crit}} = \frac{2}{c_d} \frac{\Omega_0 d^2 \cos \theta}{12},$$  \hfill (15)
Equation (14) can be written together with equation (15) in the form

$$B \sim 2^{1/2} \varepsilon_d \frac{1}{4} \sqrt{\mu_0 \rho_0 \Omega_0 r \sin \theta v_{t}} \left[ \frac{\nu_{t} \Omega_0}{v_{t}} \right]^{1/2} \frac{\nu_{t}}{v_{t}^{1/2}}. \quad (16)$$

Equation (15) clearly shows the relation $B \sim v_{t}^{1/2}$, as suggested by Figure 2c.

Inserting equation (13) into equation (12) yields, for the perturbation of $\Omega$,

$$\frac{\Omega_1}{\Omega_0} = \frac{1}{2} \left( \frac{\nu_{t}}{\Omega_0 r \sin \theta} \right)^{2} \left[ 1 + \left( \frac{\nu_{t}}{v_{t}^{1/2}} \right) \right]^{-1}. \quad (17)$$

In the case of very low viscosity, the perturbation of $\Omega$ is given by

$$\frac{\Omega_1}{\Omega_0} = \frac{1}{2} \left( \frac{\nu_{t}}{\Omega_0 r \sin \theta} \right)^{2}, \quad (18)$$

which is exactly the value required to balance the magnetic tension through the Coriolis force. In the case of large viscosity the Coriolis force is unimportant, and the magnetic field is dragged by the meridional flow through viscous coupling.

In order to compare these estimates to the numerical results, we use model 1, which is shown in Figures 2a and 2b. As already mentioned earlier, the jet amplitude is sufficient to compensate for $70\%$ of the curvature stress, which yields, according to equation (17), $\nu_{t} \sim 0.65 \nu_{t}^{\text{crit}}$. With a value of $\nu_{t} \sim 2.5 \times 10^{8} \text{ m}^2 \text{s}^{-1}$ at $0.75 \, R$, (roughly half the convection zone value) and $\Omega_0 = 2.7 \times 10^{8} \text{ s}^{-1}$, equation (15) yields $d = c_d^{(1)} 0.014 \, R_0$. Using a value of $c_d = 10 \, 10 \, 0.025 \, R_0$, which is half of the assumed width of the magnetic field. With $c_d = 10$, $\nu_{t} \sim 0.65 \nu_{t}^{\text{crit}}$, $\rho_0 = 150 \, 10 \, 0.75 \, R_0$, and $v_{t} \sim 2.5 \, 2.8 \, \text{ m} \, \text{s}^{-1}$, equation (16) yields $B \sim 2.8 \, \text{ T}$, which is very close to the numerical result.

Since equation (16), as a function of $\nu_{t}^{\text{crit}}$, has a local minimum for $\nu_{t} = \nu_{t}^{\text{crit}}$, the dependence of $B$ on $\nu_{t}^{\text{crit}}$ is expected to be rather weak, whereas $\Omega_1 / \Omega_0$ shows a much stronger dependence, as found in Figures 3 and 4. In more detail, equation (17) does not exactly reflect the scaling of $\Omega_1 / \Omega_0$ with $\nu_{t}^{\text{crit}}$ and $d$ found in Figures 3 and 4, which suggests that $c_d$ depends on $\nu_{t}^{\text{crit}}$ and $d$ itself; however, the tendency is indicated correctly.

5. SOLUTIONS WITH QUENCHED VISCOSITY

So far, we have discussed the back-reaction of toroidal field on the meridional flow through the magnetic curvature force. Since the equipartition field strength at the base of the convection zone is around 1 T (based on mixing-length models) the magnetic field also quenches the turbulent viscosity to some extent. Since this also changes the turbulent angular momentum flux, an influence on the meridional flow is expected. In order to demonstrate this effect, we use a very weak field of 1 T field strength, which has nearly no influence on the meridional flow through the tension force, and include a quenching of the viscosity given by

$$\nu_{t}^{\text{crit}} = \nu_{t} \left[ 1 + \left( \frac{B}{B_{\text{eq}}} \right)^{2} \right]^{-1}. \quad (19)$$

Note that $\nu_{t}$ scales in our model the viscous stress and the turbulent angular momentum transport ($\Lambda$-effect). Figure 5a shows the results for the values $B_{\text{eq}} = B_{\text{max}}$, $B_{\text{eq}} = B_{\text{max}}/\sqrt{3}$, and $B_{\text{eq}} = B_{\text{max}}/\sqrt{7}$ that correspond to a quenching by a factor of 2, 4, and 8, respectively. A comparison with the results presented in Figures 2–4 shows that the back-reaction through quenching of viscosity has, at least for weak field, a much stronger effect than the direct feedback through magnetic tension. Whereas in the case of feedback through magnetic tension the meridional flow moves around the magnetized region on both sides (below and above), in the case of quenching the meridional flow closes above the magnetized region. The changes of the differential rotation (Fig. 5b) are around 1 order of magnitude lower than in Figures 2–4, and no distinct jet forms.

This results from the fact that the change in the viscosity alters the parameterized turbulent angular momentum transport, which is the indirect driver for the meridional flow. In our model the meridional return flow is localized in the region where the turbulent viscosity shows the strongest radial gradient. The quenching of the viscosity therefore moves the return flow upward, as is clearly indicated in Figure 6c. However, the transport of magnetic flux is not switched off completely in this case.
The magnetic field we assumed is centered at $r = 0.75 R_\odot$ and has a width of 0.05 $R_\odot$, which means that the upper half is still in a region of considerable flow speed. For example, the effective transport velocity is only reduced by a factor of 2 when the turbulent viscosity is quenched by a factor of 8.

6. IMPLICATIONS FOR SOLAR DYNAMO MODELS

This investigation shows as a very robust result that the transport capability of the meridional flow is mainly determined by the flow velocity. For a flow velocity of the return flow at the base of the convection zone of about 2.5 m s$^{-1}$, the maximum field strength that can be transported is around 3 T (30 kG). This field strength is found to be rather insensitive to the turbulent viscosity or width of the magnetic band. Inspecting equation (16) shows that a larger value for this field strength would require either a very small or a very large value of $\nu_s/\nu_{cmt}$, or a very large value of $c_g$. Since the meridional return flow is located roughly where $\nu_s$ shows the strongest radial gradient at the base of the convection zone, the value of $\nu_s$ in that region will reflect more convection zone values rather than very small overshoot values. The value of $\nu_{cmt}$ is mainly influenced through the effective thickness $d$. Values much larger than the value used in this investigation are not feasible, since the magnetic layer would have a larger extent in radius than the meridional return flow. For very small values of $d$ an increase in the magnetic field strength that can be transported is expected. This is not too surprising, since for magnetic flux tubes the influence of the drag force is anti-proportional to the diameter. Rempel (2003) showed that flux tubes with a diameter of less than 100 km can be transported equatorward even if the field strength is around 10 T; however, the magnetic flux associated with these flux tubes is orders of magnitude smaller than the flux of a typical sunspot. We want to emphasize that for flux tubes the use of $c_g \sim \nu/\nu^2$ gives a different scaling of $B \sim v/d^{1/2}$ compared to the $B \sim (\nu/v)^{1/2}/d$ scaling derived from equation (16) in the limit of small values for $d$.

Feedback through quenching of turbulent viscosity (also affecting the $\lambda$-effect, which is $\sim \nu_s$ in our model) leads to a significant modification of the meridional flow if the field strength exceeds equipartition. Since typical mixing-length estimates for $B_{eq}$ at the base of the convection zone are around 1 T, this happens in the same field strength range at which the direct feedback through magnetic tension becomes important, too. Therefore, if we consider both effects together, the effective transport velocities are modified, but the results do not change dramatically.

To summarize, the result that the consideration of the magnetic curvature force limits the magnetic field strength that can be transported toward the equator to about 3 T (30 kG) seems to be very robust within the framework of the mean field model used in this investigation. Current kinematic solar flux-transport dynamo models rely on the equatorward transport of toroidal field at the base of the convection through the meridional flow (see the comparison of solutions with and without meridional flow in Dikpati & Charbonneau [1999] and Dikpati & Gilman [2001]). The meridional flow velocity at the base of the convection zone assumed in most of these models is around 1–2 m s$^{-1}$, which is close to the values considered in this paper. That value is also consistent with the observed surface flow of around 10–20 m s$^{-1}$ and a dynamo period around 22 years. The main consequence of the work presented here for flux-transport dynamos can be summarized as follows. Any field exceeding a few T (10 kG) cannot be transported toward the equator through a meridional flow with an amplitude of a few m s$^{-1}$. If the solar dynamo produces a stronger field (e.g., 100 kG, as inferred from studies of rising magnetic flux tubes; Choudhuri & Gilman 1987; Fan et al. 1993; Schüssler et al. 1994; Caligari et al. 1995, 1998), the field must get amplified locally through induction effects. The stronger field could be in an equilibrium, as discussed by Moreno-Insertis et al. (1992), Rempel et al. (2000), and Rempel & Dikpati (2003), which would prevent further poleward movement, but an equatorward transport is not possible.

First attempts to include the feedback on meridional flow and differential rotation in a “dynamic” dynamo model (Rempel et al. 2005) showed that flux-transport dynamos work with toroidal field strengths up to around 30 kG; however, additional constraints apply through the observed limits on the amplitude of torsional oscillations (Howe et al. 2004; Rempel et al. 2005) if the feedback on differential rotation is also included.

The problem addressed in this paper is not limited to flux-transport dynamos. Any dynamo faces the problem that the toroidal field will start moving toward the pole due to the magnetic tension force if the field strength is large enough. If the propagation of the magnetic activity is not an advection effect, but rather a classic dynamo wave, the wave has to compete with the poleward movement induced by the magnetic tension force. Since the changes of the meridional flow shown in Figure 2 are only weakly dependent on the meridional flow speed of the reference state, they also give an estimate of how large the poleward movement would be if no meridional flow is present.

We have shown that the formation of a prograde jet is an unavoidable consequence when including the magnetic tension force. Recently, Christensen-Dalsgaard et al. (2004) tried to detect jets associated with the toroidal magnetic field in the tachocline. The detection limit they found is of the order of 2–4 nHz, which is around 1% of the rotation rate. The jets we see in this study have an amplitude of around 1.5% for the strongest field we considered (4 T). A magnetic field of 2 T or less leads to the formation of jets with less than 0.5% amplitude. These jets are therefore at or below the detection limit of current helioseismic techniques. If the magnetic field has a more complicated intermittent and also nonaxisymmetric structure, it is likely that the amplitude of the prograde jet is lower than predicted by our axisymmetric model.

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