“Economic” forest growth models: the impact of phytomass removal

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Abstract. An ecological analogue of the economic growth model has been proposed to describe the growth processes of stands. The proposed model takes into account the growth of planting after the removal of phytomass (damage by insects, thinning, etc.). The function of the dependence of growth efficiency on the age of the stand and the volume of phytomass withdrawn describes the non-monotonic effects of changes in the plant phytomass after thinning. Data on the growth of plant phytomass after thinning is in good qualitative agreement with the proposed model.

1. Introduction

Growth tables and growth models of trees and forest stands usually describe the situation when the stands are not exposed to external influences. However, trees in the process of growth are often affected by external influences of various types. First of all, these are selective cutting, which lead to a sharp change in the density and amount of phytomass in the stands. In addition, during the growth of trees in the forest, an attack of insect pests, diseases, pollutants are possible. These damages lead to the death of part of the trees or to a change in the growth of the remaining trees. Assessing the risks of changes in the state of the forest and the rate of growth processes in the stands after external influences is very important for making right management decisions to determine the set of necessary steps to protect the stands. Understanding the mechanisms of response of trees to damage intensity allows planning protective measures (in the case of uncontrolled external influences) and assessing the required volume of selective cutting. The theoretical basis for understanding the processes occurring under various kinds of impacts on the forest are models of planting growth that take into account such external influences.

Forestry is focused on creating favorable conditions for improving the growth of forest stands. Clear cutting is a widespread method in forestry, and is often criticized for its negative effects [1]. Therefore, in many countries various types of thinning are used when only a part of trees is removed from the stand. With this approach, the complex structure and composition of the forest is preserved, which contributes to the conservation of biodiversity and the maintenance of the forest ecological functions [2-4].

In the “classical” models of tree growth in a stand, models are usually used in the form of ordinary differential or difference equations with variables such as the height and diameter of a tree trunk, the total phytomass of a tree or stand [5-7]. Typically, such models (models of Verhulst, Gompertz, Mitscherlich, Bertalanffy, etc.) describe the observed growth processes of the stands quite well.
However, when using such models to describe the impact of external influences on the forest (for example, damage by phylophagous insects, thinning,) fundamental problems arise due to discrepancies between the models and the observational data. To a large extent, these discrepancies are associated with the presence of only one steady state in the models. In the Verhulst model, the growth curve of the phytomass $M(t)$ is described by a linear differential equation:

$$\frac{dM(t)}{dt} = kM(t) \cdot (M_{\text{max}} - M(t))$$  \hspace{1cm} (1)

where the parameters $k$ is the growth rate, and $M_{\text{max}}$ is the maximum value of the plant phytomass. It is usually assumed that both parameters are constants.

Suppose, for some reason, part of the phytomass in the stands was removed (for example, during selective felling). What then should happen to the forest if its growth is described by equation (1). In figure 1 the phase portrait (1) in the \{M, dM/dt\} plane is shown.

![Figure 1. Phase portrait of equation (1).](image)

The removal of part of the phytomass at time $t_0$ will cause a displacement of the position of the point that characterizes the state of planting at time $M(t_0)$ (figure. 1). This will lead to a continued growth of “rejuvenated” stands. The result of the impact will be the state of the stands with the phytomass $M_{\text{max}}$ and this phytomass will be achieved later in comparison with the time of reaching this state for the stands, which is not affected. Thus, the efficiency of selective cutting can be reduced to obtaining an additional crop as a result of selective cutting. In this case, the intensity of phytomass removal will not affect the intensity of growth processes, since the growth parameter $k$ will not change and it will be possible to repeat the procedure of phytomass removal at any time. According to model (1), the stands can be imagined as invariably returning to its only stable state with $M = M_{\text{max}}$. Of course, such a model is not consistent with observational data and practices.

In addition, the phytomass in the stands has different functions. So, the trunk performs the supporting function and the function of conducting substances. Roots perform the function of absorbing nutrients and water from the soil, leaves (needles) - the function of synthesizing new phytomass during photosynthesis. Thus, in order to evaluate the response of trees to external influences, it is extremely important to know which phytomass components are affected. So, with the removal of only 3-5\% of the total plant phytomass during exposure to insects can lead to the death of the tree [8]. Whereas a much larger volume of phytomass removal in the ode of selective cutting will not lead to the destruction of the forest. Thus, if models of type (1) satisfactorily describe the growth curve of the stand in the absence of external influences, it is impossible to use models of type (1) to assess the response of the stand to external influences. Attempts to represent the parameter $k$ as a variable also will not lead to success, since in the models of type (1) the type of phytomass withdrawn does not differ. For example, the experiments with thinning indicate that both an increase and a decrease in the growth rate of phytomass are possible [9]. The absence of a reliable growth model for
the phytomass of forest stands that takes into account external influences does not allow us to assess the risk of deterioration of growth processes after impacts and to choose the optimal exposure regimes.

2. Model

In this paper, as an alternative to the “classical” plant growth models, we consider models that are similar to economic production models [10]. In classical economic models, the intensity of production depends on two independent variables - labor $L$ and capital $K$. The dependence of $Y$ on $L$ and $K$ was described by the Cobb-Douglas multiplicative equation:

$$ Y = A \cdot K^\alpha \cdot L^\beta $$

where coefficient $A$ characterizes production efficiency, the coefficients $\alpha$ and $\beta$ are the elasticity factors of production according to labor and capital.

When describing the growth of a forest stand from the point of view of production economics, the total phytomass $K$ of the non-photosynthetic components of phytomass (trunk, branches and roots) is introduced as an analog of capital, and the phytomass $L$ of the photosynthetic part of the stand phytomass - foliage or needles - as an analog of labor. Then we get an environmental analogue of the production model in the economy:

$$ M(t) = A \cdot K^\alpha(t-1) \cdot L^\beta(t-1) $$

where $M(t)$ is the phytomass produced at time $t$, $K(t-1)$ and $L(t-1)$ are, respectively, non-photosynthetic and photosynthetic plant phytomasses at time $(t-1)$, $A$ is the efficiency of phytomass synthesis for trees of a certain breed in a particular habitat under the given conditions.

As can be seen from (3), when the photosynthetic component is removed, the growth of the stand will stop, which is consistent with the change in the state of the stand, as it happens in real life, for example, after a forest is attacked by phyllophagous insects.

However, there are important differences between economic and environmental models. Unlike an economic model, where the dimensions of labor and capital are not interchangeable, in an environmental model, the produced phytomass can be used for the growth of the non-photosynthetic part of the phytomass, the growth of foliage (needles). This means that an additional condition for the distribution of the synthesized phytomass is necessary. In an economic model, the value of labor is introduced exogenously, and grows exponentially in time. In contrast, in the ecological analogue of the model, the volume of photosynthetic phytomass depends on the phytomass of other components, and is limited.

![Figure 2. Dynamics of phytomass growth (birch stands). Source of data according: Usolstev, [11]. 1 – inventory data, 2 – model.](image-url)

To test the suitability of the economic model for describing the growth of forest stands, we used data on the growth of stands, taking into account the information about the phytomass of fractions
(trunks, branches, roots, leaves or needles). As an example, we give a model of the growth dynamics of various types of birch forests [11]. For six types of birch habitats, the model coefficients were calculated (3). According to the calculations, \( \alpha + \beta = 1 \). This means that the growth of phytomass is described by the model with only two free parameters such as \( A \) and \( \alpha \). Figure 2 presents the data from field observations and model calculations.

A comparison of taxation data and model data (figure 3) indicates a very high degree of agreement between model and field data (the coefficient of determination between series of field and model data for all types of plantings is higher than 0.98).

![Figure 3. Comparison of field observations \( M_{\text{inv}} \) and model data \( M_{\text{mod}} \) for various types of birch forests (Source of data according: Usolstev, [11]).](image)

One of the factors that can influence the growth of the forest is thinning. It is a form of forest care where people remove unwanted trees that do not meet economic goals and negatively affect the growth and condition of the best and auxiliary trees. Thinning is carried out to create favorable conditions for the growth of the best trees of the main species. Thinning is aimed at the formation of highly productive high-quality forests and timely use of wood.

The question arises about the possibility of describing the dynamics of plant phytomass after thinning, depending on their intensity. Sennov [12] shows that the growth of stands after thinning is monotonously dependent on the age and intensity of thinning. Therefore, we propose to use the following model:

\[
M(t) = A(t, \rho) \cdot K^{\alpha} (t - 1) \cdot L^{1-\alpha} (t - 1)
\]

(4)

The effect of thinning on the growth of stands will be considered through a change in the efficiency coefficient \( A \) in model (3). Thinning affects the growth of stands through the ratio of the growth efficiency coefficient \( A_\epsilon \) after cutting to the value of this coefficient in the control \( A_0 \), depending on the age \( t \) of the stand and the relative intensity of thinning \( \rho \):

\[
\frac{A_\epsilon}{A_0} = \exp(-\mu \rho)^2 B \rho \exp(-\lambda \rho)^2
\]

Where \( \rho \) is the volume of phytomass removal:

\[
\rho = \frac{M_0 - M_\epsilon}{M_0}
\]
Then the curves of change in the values of the efficiency coefficient $A$ of the growth of phytomass after felling will look as follows (figure 4).

![Figure 4](image)

**Figure 4.** Model curves of change in the values of the efficiency coefficient $A$ of the growth of phytomass after felling. The age of stands $T1 \approx 20$-30 years; $T2 \approx 40$ years; $T3 \approx 50$ - 60 years.

Figure 4 demonstrates that thinning is effective at early ages (up to 40 years) with moderate (up to 30%) removal of part of the trees. Thinning is completely ineffective for forests over the age of 50-60. The proposed model allows us to assess the characteristic trajectories of phytomass changes during thinning for specific types of stands. The approach can be used by forest managers to make decisions while choosing the optimal strategy for thinning and assessing changes in the dynamics of planting phytomass after cutting.

3. Findings
An ecological analogue of the economic growth model has been proposed to describe the growth processes of stands. For calculations, data on the masses of fractions of plant phytomass are used.

The proposed model takes into account the growth of planting after the removal of phytomass (damage by insects, thinning, etc.).

The function of the dependence of growth efficiency on the age of the stand and the volume of phytomass withdrawn describes the non-monotonic effects of changes in the plant phytomass after thinning.

Data on the growth of plant phytomass after thinning is in good qualitative agreement with the proposed model.

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