Application of ESMD Method to the Runoff Process in the Arid and Semi-Arid Area

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Abstract. The ESMD method was good at finding the optimal adaptive global mean fitting curve, which was superior to the common least-square method and running-mean approach. It was the new development of the well-known Hilbert-Huang transform and could be used effectively in the science researches and engineering applications associated with data analysis from atmospheric and oceanic sciences, informatics and so on. Take the runoff process of West Liaohe River as an example, the test on the observational data indicated that this approach was very feasible.

1. Introduction

The ESMD (extreme-point symmetric mode decomposition) method could be seen as a new alternate of the well-known Hilbert-Huang transform (HHT) method, and it was anticipated as an effective usage in the fields of atmospheric and oceanic sciences, informatics, economics, ecology, medicine and seismology etc[1-2]. Now the corresponding paper was available at http://arxiv.org/abs/1303.6540 with a protection of software by the national copyright administration of China [3]. For a given data, its frequency analysis should be done on the oscillating part. Hence, to clear away the global mean curve was the first and foremost problem. In fact, only when the global mean curve was an optimal one, can the remainder signal be seen as actual oscillations caused by a series of wave fluctuations. After a large number of statistic research, it was found that models' frequency and amplitude generated by ESMD method are in normal distribution when the amount of data was appropriate. From the statistical standpoint, the coefficient of skew of the statistical sample proved that the sample statistical distribution can be handled as a normal distribution curve. From the variance analysis, with the change of the variance, the curve of the sample statistical distribution trends also accorded with the characteristics of normal distribution [4]. The ESMD method also offered a good adaptive approach for finding the optimal global mean fitting curve. It was superior to the commonly used least-square method (requires a priori function form) and running-mean approach (lacks of theoretical base and different choices of time-window and weight coefficients may result in different curves). This advantage is good for the runoff process.

2. Decomposition Algorithm for ESMD Method

There are several adjacent equal extreme points, we only choose the first one as a representative. Program code is exploited on the scilab platform and the algorithm is as follows [5]:
Step 1: Find all the local extreme points (maxima points plus minima points) of the data $Y$ and numerate them by $E_i$ ($i=1, 2, \ldots, n$);

Step 2: Connect all the adjacent $E_i$ with line segments and mark their midpoints by $F_i$ with $i=1, 2, \ldots, n-1$.

Step 3: Add a left and a right boundary midpoints $F_0$ and $F_n$ through a certain approach (Figure 1).

Step 4: Construct $p$ interpolating curves $L_1, \ldots, L_p$ ($p \geq 1$) with all these $n+1$ midpoints and calculate their mean value by $L^* = (L_1 + \ldots + L_p)/p$.

Step 5: Repeat the above four steps on $Y-L^*$ until $|L^*| \leq \varepsilon$ ($\varepsilon$ is a permitted error) or the sifting times attain a present maximum number $K$. At this time, we get the first mode $M_1$.

Step 6: Repeat the above five steps on the residual $Y-M_1$ and get $M_2, M_3, \ldots$ until the last residual $R$ with no more than a certain number of extreme points.

Step 7: Change the maximum number $K$ on a finite integer interval $[K_{\text{min}}, K_{\text{max}}]$ and repeat the above six steps. Then calculate the variance $\sigma^2$ of $Y-R$ and plot a figure with $\sigma/\sigma_0$ and $K$, here $\sigma_0$ is the standard deviation of $Y$.

Step 8: Find the number $K_0$ which accords with minimum $\sigma/\sigma_0$ on $[K_{\text{min}}, K_{\text{max}}]$. Then use this $K_0$ to repeat the previous six steps and output the whole modes. At this time, the last residual $R$ is actually an optimal AGM curve.

There are several questions associated with this algorithm. In the following we explain them one by one.

3. The Time-Series of Runoff Analysis
The decomposition of the runoff process (Figure 2), the feature and modelling method of each component was studied (Figure 3). There were three models. The cycle of Mode 1 is four years. The cycle
of Mode2 is six years. The cycle of Mode3 is twelve years. The result of Mode1 and Mode2.
Development tendency of Runoff process was decrease – increase – decrease, which means runoff
process of 1970-1987 and 2000-2012 at the bottom of its cycle. Besides, the results reflect the
complexity of the runoff change.

![Figure 2. The Time-Series of Runoff for West Liaohe River](image)

![Figure 3. The decomposition result for the wind data given by EMD method with 156 sifting times.](image)
There are two parts for “extreme-point symmetric mode decomposition (ESMD)” method. The first part is the decomposition approach which yields a series of intrinsic mode functions (IMFs) together with an optimal “adaptive global mean (AGM)” curve, the second part is the “direct interpolating (DI)” approach which yields instantaneous amplitudes and frequencies for the IMFs together with a time-varying figure for the total energy. It can be seen as a new alternate of the well-known “Hilbert-Huang Transform (HHT)” method.

The ESMD method has three superiorities in data analysis [6]: 1) it is good at finding the changing trend. It can not only extract the annual changing trend from the observation sequence of a few years, but also draw out the climate changing trend; 2) it is good at abnormal diagnosis which is helpful for exploring the problem of climatic anomaly due to the ability of finding the abnormal time and frequency from the decomposed modes; 3) it is good at time-frequency analysis and energy variation analysis. With the abandon of Hilbert transform which has many defects, the data-based direct interpolating approach is developed to compute the total energy and instantaneous frequencies at any time scales. It follows from the above features that the ESMD method has a good application prospect in the research of climate change. The test on the observational data indicates that this approach is very feasible. The ESMD method has its unique role in the research and engineering application of data processing involved in various research fields. According to the principle of ESMD data analysis method, it is generally suitable for the more symmetrical data about the midpoint in every half period. For asymmetric (peak-valley asymmetry) data, the midpoint selected in this method is unrepresentative. The local mean value per half period is used to replace the midpoint of maximum and minimum in the original algorithm, so that the asymmetric data can be better reflected. The numerical simulation proves the method’s feasibility and effectiveness [7].

4. Conclusion
The ESMD method is the development of Hilbert-Huang transform, making up for the defect that the number of screening is difficult to determine and the trend function of decomposed is too rough. The ESMD method not only intuitively reflects the time-varying of modal amplitude and frequency, but also clearly reflects the change of the total energy. The given time-frequency distribution is more intuitive sense. What’s more, the method has a unique advantage in the analysis of frequency change. But there still exist many problems, such as the problems about mode changes, what features obtained about the
modals which disposed by ESMD method, the rationality about using the reckoned average frequency and amplitude to estimate the change of the mode, etc., this topic will be used to solve these problems with a lot of actual data. In addition, the amount of date changed, whether the conclusion is reasonable. What different effects will occur when the number of screening is changed? All of these will be worth further research.

Acknowledgements
This work was financially supported by Fund of National Key R&D Program of China (2017YFC0403604, 2016YFC0401005, 2017YFC0403505, 2016YFA0601703), the Basic Scientific Research Business Expense of Yellow River Institute of Hydraulic Research (HKY-JBYW-2020-04, HKY-JBYW-2018-12), and the National Natural Science Foundation of China (51479222, 51109138).

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