High Energy Neutrinos From
Superheavy Dark Matter Annihilation

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Abstract
Superheavy \((M > 10^{10} \text{ GeV})\) particles produced during inflation may be the dark matter, independent of their interaction strength. Strongly interacting superheavy particles will be captured by the sun, and their annihilation in the center of the sun will produce a flux of energetic neutrinos that should be detectable by neutrino telescopes. Depending on the particle mass, event rates in a cubic-kilometer detector range from several per hour to several per year. The signature of the process is a predominance of tau neutrinos, with a relatively flat energy spectrum of events ranging from 50 GeV to many TeV, and with the mean energy of detected tau neutrinos about 3 TeV.

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I. INTRODUCTION

It is usually assumed that the interactions of dark matter and ordinary matter are weak, at least as small as ordinary weak interactions. It is also often assumed that the dark matter is a thermal relic of the big bang. If it is a thermal relic particle, then its mass should be less than 340 TeV, the unitarity limit [1]. These considerations lead to the popular picture for relic dark matter of an electrically neutral particle, without strong interactions, and with mass less than a couple of hundred TeV. In this paper we explore a path less traveled, and assume that the dark matter is a nonthermal relic, it interacts strongly with normal matter, and it is very massive. We show that the clean signature of this possibility is a detectable flux of energetic neutrinos from the sun.

It has long been appreciated that if the dark matter is massive, say larger than a few TeV, it will behave effectively as dissipationless dark matter regardless of whether it has strong or electromagnetic interactions [2, 3]. However, since the upper limit to the mass of a thermal relic is a few hundred TeV, the window for very massive dark matter particles was thought to be not very wide.

The recent development of scenarios for nonthermal production of dark matter has opened the window to the possibility that the dark matter might be supermassive, independent of its interaction strength [4, 5, 6]. Of the many possibilities for producing supermassive dark matter, perhaps gravitational production is the most general [4, 5]. In this scenario, dark matter is produced by vacuum quantum fluctuations toward the end of inflation. The resulting particle density is independent of the interaction strength of the particle, which leads to the possibility that the dark matter may be electrically charged, strongly interacting, weakly interacting, or may have only gravitational interactions with normal matter.

A particularly promising mass range for gravitational production of dark matter is the mass scale of the inflaton, about $10^{12}$ GeV in chaotic inflation models. If the inflaton mass heralds a new mass scale, then it would be reasonable to imagine that there are other particles of similar mass. Furthermore, gravitational production of particles with a mass comparable to the inflaton mass naturally leaves behind a cosmologically interesting density
of dark matter today [4]. The particle content may include exotic quarks or other strongly interacting particles in the spectrum of new particles [7].

In this paper we will consider the case that the dark matter is strongly interacting and supermassive, a SIMPZILLA. Although our calculations will not be sensitive to whether the SIMPZILLA is electrically charged, there are arguments that suggest that the SIMPZILLA must be neutral [8, 9]. The possibility that the dark matter may be very massive and strongly interacting was recently discussed by Faraggi, Olive, and Pospelov [10]. In that paper they have a nice discussion of the particle physics motivations for the existence of a massive, stable, strongly interacting particle, and they point out that the sun and Earth may be the source of high-energy neutrinos from the annihilation of the particles.

In the next section we will calculate the trapping rate and annihilation rate of SIMPZILLAS in the sun (and Earth) as a function of the SIMPZILLA mass and interaction cross section. In Section III we will discuss the emergent spectrum of neutrinos from SIMPZILLA annihilation in the center of the sun. In Section IV we will calculate the event rate in cubic-kilometer underwater or underice neutrino detectors. Finally, the last section contains our conclusions.

A preview of our main conclusion is that we expect a detectable high-energy solar neutrino flux for much of the parameter range of interest.

II. CAPTURE AND ANNIHILATION RATE

Before launching into the details of the capture-rate calculation it is useful to make some extremely crude estimates. The first estimate is for the number of SIMPZILLAS that hit the sun.

Assuming the SIMPZILLAS comprise the local dark matter density of 0.3 GeV cm$^{-3}$, the local number density of SIMPZILLAS is $3 \times 10^{-13} \ M_{1_{2}}^{-1} \text{cm}^{-3}$, where $M_{12}$ is the SIMPZILLA mass in units of $10^{12}$GeV. Assuming a typical velocity of 240 km s$^{-1}$ for the SIMPZILLAS, the local flux is approximately $6 \times 10^{-7} M_{1_{2}}^{-1} \text{cm}^{-2} \text{s}^{-1}$. The surface area of the sun is about $6 \times 10^{22} \text{cm}^2$, and the product of the surface area and the flux, $4 \times 10^{16} M_{1_{2}}^{-1} \text{s}^{-1}$, is a crude estimate of the rate of SIMPZILLAS hitting the sun.
Now consider trapping of simpzillas in the sun. Assuming the simpzilla impacts the sun with the solar escape velocity of 600 km s\(^{-1}\), the kinetic energy of the simpzilla is about \(10^6 M_{\odot} \text{GeV}\). Suppose the simpzilla scatters with nucleons with a cross section of \(\sigma = 10^{-24} \text{cm}^2\) \([1]\), and in every collision suffers an energy loss of \(m_{\text{proton}} v^2 / 2 = 2 \times 10^{-6} \text{GeV}\). Through the center of the sun is about \(2 \times 10^{36} \text{cm}^{-2}\) of material, so there will be about \(2 \times 10^{12} \sigma_{-24}\) collisions with a total energy loss of \(4 \times 10^6 \sigma_{-24} \text{GeV}\). Since the initial kinetic energy is about \(10^6 M_{\odot} \text{GeV}\), for masses much less than \(10^{12} \sigma_{-24} \text{GeV}\), simpzillas with average velocity hitting the sun will be trapped, but if the simpzilla mass is much larger than that, only the low-velocity tail of the phase-space distribution will be captured.

These considerations determine the gross behavior of the dependence of the simpzilla trapping rate on the simpzilla mass and its interaction cross section. For a mass of about \(10^{12} \text{GeV}\), about \(10^{17}\) simpzillas hit the sun per second. For an interaction cross section of about \(10^{-24} \text{cm}^2\), most of those simpzillas are captured.

Now we turn to the details of the capture calculation. The capture of dark matter particles in the sun and Earth has been studied in detail \([12, 13]\); here we adapt the considerations to the capture of simpzillas by the sun. The capture rate of dark matter by the sun is given in terms of an integral over \(f(u)\), the phase-space density of dark matter in the solar neighborhood normalized such that \(\int_0^\infty f(u) \, du \, d^3 \mathbf{r} = \rho(R)\), where \(\rho(R)\) is the number density of particles at \(R\), some sufficiently large radius where the gravitational pull of the sun is negligible. The capture rate is given by

\[
\Gamma_C = 4\pi R^2 \int \left[ \frac{1}{4} f(u) \, u \, du \, d\sin^2 \theta \right] P(\theta, u), \tag{1}
\]

where \(\theta\) is the angle between the velocity of the particle and the normal to the surface at \(R\), and hence the expression inside the square bracket is the contribution to the inward flux at radius \(R\) from particles at velocity \(u\) and angle \(\theta\). Finally, \(P(\theta, u)\) gives the probability that particles with the given velocity and direction at some large radius \(R\) will be captured by the sun.

Writing the angular momentum per unit mass as \(J = Ru \sin \theta\), and defining \(w_\odot^2 = u^2 + v_\odot^2\)
where \( v_\odot \) is the escape velocity at the surface of the sun, we can rewrite the above as

\[
\Gamma_C = \pi \int_0^\infty f(u) u \, du \int_0^{R_\odot} \frac{dJ^2}{u^2} P(J, u),
\]

(2)

where \( R_\odot \) is the solar radius, and \( P(J, u) \) now gives the capture probability for a particle with angular momentum \( J \) and a velocity at infinity of \( u \).

The regime in which dark matter capture is often considered is that of weakly interacting particles, where the optical depth of the sun is much less than unity, i.e., \( n_N \sigma R_\odot \ll 1 \) where \( n_N \) is the number density of nuclei in the sun, and \( \sigma \) is the dark-matter—nucleon cross section. In this case \( P(J, u) \) is given by [12]:

\[
P(J, u) = \left( \int_0^{R_\odot} \frac{dJ^2}{u^2} \right)^{-1} \int_0^{R_\odot} dr \int_0^{rw} \frac{dJ^2}{u^2} \sqrt{1 - \left( \frac{J}{rw} \right)^2} \left( 1 - \frac{\mu u^2}{4w^2} \right) \Theta \left( \frac{4}{\mu} - \frac{u^2}{w^2} \right),
\]

(3)

where \( w \) now stands for \( u^2 + v^2 \) where \( v \) is the escape velocity at radius \( r \), and \( \mu = m_X/m_N \) where \( m_X \) is the mass of the dark matter particle (in our case, the SIMPZILLA) and \( m_N \) is the average nucleon mass. The first term in square brackets is a normalizing factor, the term \( 2n_N \sigma dr/\sqrt{1 - (J/rw)^2} \) gives the probability of collision, and the terms in the second line give the probability that the particle gets scattered into a bound orbit. The step function \( \Theta(x) \) equals 1 or 0 depending on whether its argument is positive or not. It is simple to show that a collision produces a fractional energy change \( \Delta E/E \) that is uniformly distributed between 0 and \( 4/\mu \) where we have taken the large \( \mu \) limit (otherwise, \( 4/\mu \) is replaced by \( 4\mu/(\mu + 1)^2 \)).

The above assumes that the dark matter particle typically suffers at most one collision in its passage through the sun. We are here interested in the opposite regime where \( n_N \sigma R_\odot \gg 1 \). For each collision, the fractional energy change is of the order of \( 1/\mu \). Therefore, after \( n_N \sigma R_\odot \) collisions in the sun, the SIMPZILLA would get captured if \( v_\odot^2 \lesssim (v_\odot^2 + u^2)(1 - 1/\mu)^n n_N \sigma R_\odot \), implying all particles with \( u < u^* \) get captured where \( u^* = v_\odot [1/(1 - 1/\mu)^n n_N \sigma R_\odot - 1]^{1/2} \). For capture by the sun, it adequate to approximate \( u^* \) by \( u^* = v_\odot / \sqrt{\mu/(n_N \sigma R_\odot) - 1} \) if \( \mu/n_N \sigma R_\odot > 1 \) or \( u^* = \infty \) if \( \mu/n_N \sigma R_\odot \leq 1 \). In other words, we approximate \( P \) in Eq. (2) as a step function.

Using the Maxwell–Boltzmann phase space distribution function for the SIMPZILLAS

\[
f(u) = 4(n_X/\sqrt{\pi u_{th}^3})(u/u_{th})^2 \exp(-u^2/u_{th}^2) \]

with \( \rho_X = 0.3(\text{GeV}/m_X)\text{cm}^{-3} \), we obtain
FIG. 1: The capture rate by the sun (solid curves) and Earth (dashed curves) for different values of the simpzilla–nucleon cross section. The three curves for the sun and for Earth correspond to, from top to bottom, $\sigma = 10^{-22}$, $10^{-24}$, and $10^{-26}$ cm$^2$.

From Eq. (2) the capture rate for simpzillas. The capture rate has two forms, depending on the efficiency of energy loss in the sun. This is parametrized by $q$, defined as

$$q \equiv \frac{\mu}{n_N \sigma R_\odot} = 20 \left( \frac{m_X}{10^{12} \text{GeV}} \right) \left( \frac{10^{-24} \text{cm}^2}{\sigma} \right) \left( \frac{R_\odot}{7 \times 10^{10} \text{cm}} \right)^2 \left( \frac{2 \times 10^{33} \text{g}}{M_\odot} \right). \tag{4}$$

If $q \leq 1$, the simpzilla will be efficient in losing energy in its passage through the sun, and the capture rate is

$$\Gamma_C = 10^{17} \text{s}^{-1} (1 + y^2) \left( \frac{10^{12} \text{GeV}}{m_X} \right) \left( \frac{u_{\text{th}}}{240 \text{ km s}^{-1}} \right) \left( \frac{R_\odot}{7 \times 10^{10} \text{cm}} \right)^2, \tag{5}$$
where
\[ y \equiv 2.5 \left( \frac{v_c}{600 \text{ km s}^{-1}} \right) \left( \frac{u_{th}}{240 \text{ km s}^{-1}} \right)^{-1}. \] (6)

On the other hand, if \( q > 1 \), only low velocity SIMPZILLAS are captured and the capture rate is
\[ \Gamma_C = 10^{17} \text{ s}^{-1} \left[ 1 + y^2 - \exp(-x^2)(1 + y^2 + x^2) \right] \left( \frac{10^{12} \text{ GeV}}{m_X} \right) \left( \frac{u_{th}}{240 \text{ km s}^{-1}} \right) \left( \frac{R_{\odot}}{7 \times 10^{10} \text{ cm}} \right)^2. \] (7)
where
\[ x \equiv \frac{y}{\sqrt{q - 1}}. \] (8)

The capture rate as a function of SIMPZILLA mass and cross section is shown in Fig. 1.

The captured SIMPZILLAS settle into the core of the sun on a time scale determined by their drift velocity, \( t_{\text{drift}} \sim r/v_{\text{drift}} \) where \( v_{\text{drift}} \) can be estimated by balancing gravity with the viscous drag due to scattering with nucleons \[ G\bar{\rho}r^3/m_X/r^2 \sim \sigma_N v_{\text{drift}} m_N (\bar{T}/m_N)^{1/2}. \]
Here, \( \bar{\rho} \) and \( \bar{T} \) is the typical mass density and temperature of the sun. The timescale is very short, of order 100 s \( (\sigma/10^{-24} \text{ cm}^2)(10^{12} \text{ GeV}/m_X)(\bar{T}/10^7 \text{ K})^{1/2} \).

The subsequent evolution of the collection of SIMPZILLAS at the core can be divided into two stages. The first stage is when \( N \), the total number of SIMPZILLAS, is less than \( N_{SG} \), the critical number necessary for the SIMPZILLAS to become self gravitating. In this first stage the SIMPZILLAS are supported by the thermal pressure of the surrounding plasma and the SIMPZILLA profile follows an isothermal distribution given by \( \rho_X(r) \propto \exp(-r^2/2r_*^2) \) where
\[ r_* = \left( \frac{3T_{\odot}}{4\pi G m_X \rho_{\odot}} \right)^{1/2} = 5000 \text{ cm} \left( 10^{12} \text{ GeV}/m_X \right)^{1/2}(T_{\odot}/10^7 \text{ K})^{1/2}(150 \text{ g cm}^{-3}/\rho_{\odot})^{1/2}, \]
and \( T_{\odot} \) and \( \rho_{\odot} \) are the core temperature and density respectively. The critical number is defined by \( 4\pi r_*^3 \rho_{\odot}/3 \sim N_{SG} m_X \), and is equal to
\[ N_{SG} \sim 10^{26} \left( \frac{10^{12} \text{ GeV}}{m_X} \right)^{5/2} \left( \frac{T_{\odot}}{10^7 \text{ K}} \right)^{3/2} \left( \frac{150 \text{ g cm}^{-3}/\rho_{\odot}}{1/2} \right). \] (9)

To determine if \( N \) ever exceeds \( N_{SG} \), we have to determine \( N_{EQ} \), the number of SIMPZILLAS in the sun when there is an equilibrium between annihilation and capture. If the size is given by \( r_* \) above, then \( N_{EQ} \) is found by equating the annihilation rate and the capture rate:
\[ 2(\sigma_A v)(N_{EQ})^2/(4\pi r_*^3/3) \sim \Gamma_C. \] Using the annihilation cross-section \( \langle \sigma_A v \rangle \lesssim 1/(m_X^2 v) \),
we obtain

$$N_{EQ} \sim 10^{30} \left( \frac{m_X}{10^{12} \text{GeV}} \right)^{1/2} \left( \frac{T_\odot}{10^7 \text{K}} \right)^{3/2} \left( \frac{150 \text{ g cm}^{-3}}{\rho_\odot} \right)^{3/2} \left( \frac{\Gamma_C}{10^{17} \text{ s}^{-1}} \right)^{1/2} \left( \frac{v}{10^{-9} c} \right)^{1/2}. \quad (10)$$

The above means that unless $m_X$ is significantly smaller than $10^{12} \text{ GeV}$, no equilibrium is reached for $N < N_{SG}$. With the capture rate given above, $N$ can reach $N_{SG}$ on a time-scale much shorter than the lifetime of the sun. The next stage is then set for the collapse of the simpzilla collection, whose final state will be determined by a number of factors.

First, the critical $N_{Chandra}$, beyond which the collection of simpzellias cannot be supported by degeneracy pressure, is given by balancing $Gm_X^2 N_{Chandra}/r \sim \alpha^{1/3} N_{Chandra}^{1/3}/r$:

$$N_{Chandra} \sim \alpha^{1/2} 10^{21} \left( \frac{m_X}{10^{12} \text{ GeV}} \right)^{-3}, \quad (11)$$

where $\alpha$ is the ratio of electron number density to simpzilla number density. This is initially a large number, i.e., the mass density of simpzellias and the mass density of nuclei are initially comparable, which implies $\alpha \sim m_X/m_N$. It is unclear how much $\alpha$ would be reduced in the collapse process. It depends on how effective simpzellias are in dragging along protons. If $\alpha \gtrsim 10^{10}(m_X/10^{12} \text{ GeV})$, $N_{grav}$ is smaller than $N_{Chandra}$, and so the collapse would result in a simpzilla collection supported by non-relativistic electron degeneracy pressure. If $\alpha \lesssim 10^{10}(m_X/10^{12} \text{ GeV})$, the configuration will collapse to a black hole, unless sufficient annihilation occurs along the way.

Let us consider the question of whether annihilation would halt an otherwise catastrophic collapse. Following [3], the fractional change in $N$ can be estimated by

$$\frac{\Delta N}{N} \sim N \int dt \frac{\langle \sigma_A v \rangle}{4\pi r^3(t)}. \quad (12)$$

Whether sufficient annihilation occurs or not depends on whether the integral is dominated by small $r$ (late times) or large $r$ (early times). The longer the configuration spends at small radii, or in other words, the slower the acceleration, the better the chance for annihilation to work against collapse. The relation $dr/dt \sim (r_0/r)^2(r_0/t_{drift})$ was used in Ref. [3], where $r_0$ is the initial radius and $t_{drift}$ is the drift-time given before (the timescale for collapse is set by viscous drag). A perhaps more reasonable limit to how fast a given shell can accelerate
is given by free fall: \( dr/dt \propto 1/r^{3/2} \). Integrating, one obtains:

\[
\frac{\Delta N}{N} \sim \frac{N \langle \sigma_A v \rangle t_{\text{drift}}}{2\pi r_0^{3/2} r_f^{3/2}},
\]

where we have taken the limit of \( r_f \ll r_0 \). Clearly, if \( r_f \) were sufficiently small, \( \Delta N/N \sim 1 \) can always be achieved. The only thing one has to make sure is that the required \( r_f \) is larger than the Schwarzschild radius. Using \( r_0 \sim r_* \) and \( N \sim N_{SG} \), it can be verified that

\[
r_f \sim 10^{-4} \text{ cm} \left( \frac{N}{10^{30}} \right)^{2/3} \left( \frac{\langle \sigma_A v \rangle}{10^{-32} \text{ cm}^3 \text{s}^{-1}} \right)^{2/3} \left( \frac{t_{\text{drift}}}{100 \text{ s}} \right)^{2/3} \left( \frac{5000 \text{ cm}}{r_0} \right)
\]

would do the job.

After shedding a fair fraction of the simpzillas, the equilibrium configuration should in principle be one where support is provided by non-relativistic electron degeneracy pressure:

\[
Gm_X^2 N/r \sim \alpha^{2/3} N^{2/3} / (2m_e r^2)
\]

where \( m_e \) is the electron mass. The above, together with the requirement of the balance of annihilation and capture, gives

\[
N_{EQ} \sim 10^{20} \alpha^{2/3} \left( \frac{\Gamma_C}{10^{17} \text{ s}^{-1}} \right)^{1/3} \left( \frac{10^{12} \text{ GeV}}{m_X} \right)^{4/3} \left( \frac{5 \times 10^{-4} \text{ GeV}}{m_e} \right) \left( \frac{v}{10^{-9} \text{c}} \right)^{1/3}.
\]

Comparing the above with \( N_{\text{Chandra}} \) shows that the final configuration is just barely stable, depending somewhat on the exact value of \( \alpha \) and \( m_X \). If not stable, then the configuration goes through another cycle of collapse and eventual halt by annihilation. It is curious that this cycle might go on indefinitely, in which case each collapse would be accompanied by enhanced annihilation and therefore a mild neutrino outburst (using for example \( \Delta N \sim 10^{20} \) and \( t_{\text{drift}} \sim 100 \text{ s} \) gives an annihilation rate of \( 10^{19} \text{ s}^{-1} \), not overwhelmingly larger than the capture rate of \( \sim 10^{17} \text{ s}^{-1} \)). We will assume for the rest of this paper that the annihilation rate is given by capture rate, or more precisely, \( 2\Gamma_A \sim \Gamma_C \).

Finally, there is the possibility that \( \alpha \) drops to a sufficiently small value later on that electron degeneracy pressure is irrelevant, and the simpzillas are supported instead by their own degeneracy pressure (or its analog if it were a boson; see e.g., Ref. [16]). In this case the expression equivalent to Eq. (11) is

\[
N_{\text{Chandra}} \sim 10^{21} (10^{12} \text{ GeV} / m_X)^3,
\]

(16)
and the equilibrium number will be

\[ N_{\text{EQ}} \sim 10^5 (\Gamma_C/10^{17} \text{s}^{-1})^{1/3} (10^{12} \text{GeV}/m_X)^{7/3}(v/10^{-9}c)^{1/3}, \]  
(17)

so that the final equilibrium configuration is clearly stable.

For completeness, we give the capture rate of SIMPZILLAS by Earth:

\[ \Gamma_C = 8 \times 10^{12} \text{s}^{-1} \left[ 1 + y^2 - \exp(-x^2)(1 + y^2 + x^2) \right] \left( \frac{10^{12} \text{GeV}}{m_X} \right), \]  
(18)

where now \( y = 0.04 \), and \( x \) is given by

\[ x \equiv y \left[ \frac{1}{(1 - \mu^{-1})N_{\text{coll}} - 1} \right]^{1/2}, \quad N_{\text{coll}} \equiv 2 \times 10^9 \left( \frac{\sigma}{10^{-24} \text{cm}^2} \right), \quad \mu \equiv 10^{12} \left( \frac{m_X}{10^{12} \text{GeV}} \right). \]  
(19)

The parameter \( N_{\text{coll}} \) is the average number of collisions the SIMPZILLA suffers. It may be identified with \( \mu/q \) in the case of the sun \[17\]. The expression for \( \Gamma_C \) is well approximated by the corresponding expressions in Eqs. (4) through (7) for \( y > 1 \), which applies for the case of the sun. For Earth we have to resort to this more complicated expression; however, it has simple limiting forms. For \( N_{\text{coll}}/\mu \gg 1 \), one can use [cf. Eq. (5)]

\[ \Gamma_C = 8 \times 10^{12} \text{s}^{-1} (1 + y^2) \left( \frac{10^{12} \text{GeV}}{m_X} \right), \]  
(20)

For \( N_{\text{coll}}/\mu \ll 1 \), the capture rate is well approximated by [cf. Eq. (7)]

\[ \Gamma_C = 8 \times 10^{12} \text{s}^{-1} x^2 y^2 \left( \frac{10^{12} \text{GeV}}{m_X} \right) \]  
(21)

with \( x = y/\sqrt{q - 1} \), where \( q = 5 \times 10^2 (m_X/10^{12} \text{GeV})(10^{-24} \text{cm}^2/\sigma) \). For most, but not all, of the parameters of interest, it is this last regime that is relevant for capture by Earth, which gives \( \Gamma_C \sim 4 \times 10^4 \text{s}^{-1} (10^{12} \text{GeV}/m_X)^2(\sigma/10^{-24} \text{cm}^2) \). The total number of SIMPZILLAS captured in the lifetime of Earth, \( t_E \sim 10^{17} \text{s} \), is then \( 4 \times 10^{21} (10^{12} \text{GeV}/m_X)^2(\sigma/10^{-24} \text{cm}^2) \).

We can compare this with the number of SIMPZILLAS in equilibrium using Eq. (10), with the replacements \( T_{\odot} \rightarrow T_{\oplus} \simeq 5000 \text{ K}, \rho_{\odot} \rightarrow \rho_{\oplus} \simeq 10 \text{ g cm}^{-3} \), and \( v \rightarrow 2 \times 10^{-11}c \), obtaining \( N_{\text{EQ}} \sim 5 \times 10^{18} (m_X/\text{GeV})^{1/2}(\Gamma_C/400 \text{ s}^{-1})^{1/2} \). Therefore, the SIMPZILLAS captured in Earth would be in equilibrium unless the mass or cross section is significantly different from what is assumed above. In general, the annihilation rate is given by:

\[ \Gamma_A = 2 \times 10^4 \text{s}^{-1} \left[ \frac{\Gamma_C}{4 \times 10^4 \text{s}^{-1}} \right] \tanh^2 \left[ 10^3 \left( \frac{10^{12} \text{GeV}}{m_X} \right)^{1/2} \left( \frac{\Gamma_C}{4 \times 10^4 \text{s}^{-1}} \right)^{1/2} \left( \frac{t_E}{10^{17} \text{s}} \right) \right]. \]  
(22)

The capture rate for Earth is shown in Fig. 1 along with the capture rate for the sun.
III. SIMPZILLA ANNIHILATION IN THE SUN

Take a simple picture where the SIMPZILLA annihilates and produces two quarks or two gluons, each of energy $m_X$ where $m_X$ is the mass of the SIMPZILLA. The quarks and gluons then fragment into high multiplicity jets of hadrons and secondary decay products.

Defining $x = E/E_{\text{jet}} = E/m_X$, one can take the fragmentation function for the total number of hadrons to be 

$$
\frac{dN_H}{dx} = ax^{-3/2}(1 - x)^2.
$$

(23)

Here $a$ is some constant that can be set by total energy considerations:

$$
1 = \int_0^1 x \frac{dN_H}{dx} dx = a \frac{16}{15},
$$

(24)

so $a = 15/16$.

The total number of hadrons produced in the fragmentation of the jet is

$$
N_H = \int_0^1 \frac{dN_H}{dx} dx = \frac{15}{8} \frac{1}{\sqrt{\epsilon}} \left[ 1 - \frac{8}{3} \sqrt{\epsilon} + 2\epsilon - \epsilon^2/3 \right].
$$

(25)

When calculating the total multiplicity, the cutoff for the integral should be $\epsilon = \Lambda_{\text{QCD}}/m_X$.

Using $\Lambda_{\text{QCD}} = 0.1$ GeV and defining $M_{12} = m_X/10^{12}$ GeV,

$$
N_H = \frac{15}{8} \left( \frac{m_X}{\Lambda_{\text{QCD}}} \right)^{1/2} = 6 \times 10^6 M_{12}^{1/2}.
$$

(26)

The final decay chain of the hadrons will contain all species of neutrinos.

We will be interested in the number of heavy quarks, bottom and top. If all quarks were light, then all flavors would be produced equally. However, because of the mass of the heavy quark, the number must be found by using $\epsilon = M_Q/m_X$ in Eq. (26) rather than $\epsilon = \Lambda_{\text{QCD}}/m_X$:

$$
N_Q \simeq N_H \sqrt{\frac{\Lambda_{\text{QCD}}}{M_Q}},
$$

(27)

where $M_Q$ is the mass of a typical meson containing the heavy quark: 2 GeV for the charm, 5 GeV for bottom, and 175 GeV for top. Therefore, per annihilation, one expects $N_{\text{charm}}/N_H = 0.23$, $N_{\text{bottom}}/N_H = 0.14$, and $N_{\text{top}}/N_H = 0.024$. This means that each annihilation into two jets produces $7.4 \times 10^6 M_{12}^{1/2}$ light hadrons, $2.8 \times 10^6 M_{12}^{1/2}$ charmed hadrons, $1.6 \times 10^5 M_{12}^{1/2}$ bottom hadrons, and $2.8 \times 10^5 M_{12}^{1/2}$ top hadrons.
FIG. 2: The spectrum of top hadrons produced by the fragmentation of quark and gluon jets from simpzilla annihilation (dotted line), and the $\nu_\tau$ spectrum produced by decay of top (dashed line). For the fragmentation spectrum, $E_{\text{min}} = 175$ GeV and $N_{\text{TOTAL}} = 2.8 \times 10^5 M_{12}^{1/2}$ per annihilation. For the decay spectrum, we will only be concerned with neutrinos above $E_{\text{min}} = 50$ GeV. The number of tau neutrinos above $E_{\text{min}}$ is $N_{\text{TOTAL}} = 10^4 M_{12}^{1/2}$ per annihilation, and half that for the other neutrinos.

It is also possible to estimate the spectrum of the heavy-quark hadrons:

$$\frac{E_{\text{min}}}{N_{\text{TOTAL}}} \frac{dN}{dE} \sim \frac{1}{2} \left( \frac{E}{E_{\text{min}}} \right)^{-3/2}, \quad (E > E_{\text{min}}),$$

where $E_{\text{min}}$ is the mass of the top quark, bottom quark, or charm quark.

The resulting $E^{-3/2}$ fragmentation spectrum for top hadrons is shown by the dotted line labeled “fragmentation” in Fig. 2. The minimum energy is approximately the mass of the top quark, 175 GeV.

We will assume that simpzilla annihilation occurs in a medium of density found in the center of the sun, $\rho \sim 200$ g cm$^{-3}$, or $n \sim 10^{26}$ cm$^{-3}$. Using an interaction cross sec-
tion of $10^{-24}\text{cm}^2$, the hadronic interaction length is about $10^{-2}\text{cm}$. Light and charmed hadrons scatter many times before decay and the resultant neutrinos will have very low energy. The $B$ lifetime is about $10^{-12}\text{s}$, so the decay length is $3 \times 10^{-2}(E/M_B)\text{cm}$, or $6 \times 10^{-3}(E/\text{GeV})\text{cm}$ using $M_B = 5\text{GeV}$. The ratio of the decay length to interaction length is $L_D/L_I = 0.6(E/\text{GeV})$. So for $E > 5\text{GeV}$ the $B$ will also scatter and lose energy before decay. It won’t completely stop after one scattering. Most of the cross section is diffractive production of low-energy crud and there should still be a leading $B$, so $B$ decay is a potential source of high-energy neutrinos.

However, top hadrons are a promising source. The top lifetime is short, and almost 100\% of the time decays as $t \rightarrow Wb$. The $W$ lifetime is $3 \times 10^{-25}(E/80\text{GeV})\text{s}$, which results in a decay length of about $10^{-16}(E/\text{GeV})\text{cm}$, which will be much less than the interaction length for even the most energetic tops. The $W$ then decays with a branching ratio of $1/3$ as $W \rightarrow l\nu_l$ (equally into $\tau\nu_\tau$, $\mu\nu_\mu$ and $e\nu_e$). Therefore it is reasonable to assume the top quark will produce energetic neutrinos before losing energy.

The spectrum of neutrinos produced in the chain $t \rightarrow W \rightarrow \nu$ is straightforward to calculate. A convenient analytic fit to the spectrum is given by

$$\frac{dN}{dE} = \mathcal{N} \frac{E + M_W}{\sqrt{(E + M_t)[(E + M_t)^2 - M_t^2][E + M_W^2 - M_W^2]}}.$$ \hspace{1cm} (29)

where $\mathcal{N}$ is a normalization factor. This spectrum is shown in Fig. 2. As expected, at energies larger than the top mass the $E^{-3/2}$ spectrum is recovered.

We will use the spectrum in Fig. 2 as the spectrum of neutrinos produced by SIMPZILLA annihilation. Since there are about $2.8 \times 10^5 M_{12}^{1/2}$ tops produced per annihilation, and 10\% of them make $\tau\nu_\tau$ followed by $\tau$ decay including a $\nu_\tau$, the total yield of $\nu_\tau$’s per annihilation is $5.6 \times 10^4 M_{12}^{1/2}$. Top (as well as $\tau$) decay also produces $\mu + \nu_\mu$ and $e + \nu_e$, but the electrons are absorbed and the muons are stopped before decay, so the yield of high-energy $\nu_\mu$ and $\nu_e$ per annihilation is about $3.8 \times 10^4 M_{12}^{1/2}$. Only about 20\% of the neutrinos will be produced with energy above 50 GeV, so the emission rate of tau neutrinos in the core is about $10^4 M_{12}^{1/2} \Gamma_A$ with a spectrum above 50 GeV shown in the figure. This, of course, is the emission rate in the core. We now turn to the propagation of the neutrinos through the sun.
IV. THE EMERGENT SPECTRUM OF NEUTRINOS

The total neutrino emission rate above 50 GeV from the core of the sun is 
\[ f_{\text{core}} = 10^4 M_1^{1/2} \Gamma_A \] 
for \( \nu_\tau \), and half that for \( \nu_\mu \) and \( \nu_e \). Here \( \Gamma_A = \Gamma_C / 2 \) is the SIMPZILLA annihilation rate. The core emission rate spectrum above 50 GeV for each neutrino species is [see Eq. (29)]

\[
\left( \frac{df}{dE} \right)_{\text{core}} = 333 \text{ GeV}^{3/2} \frac{f_{\text{core}}}{E_{\text{min}}} \times \frac{E + M_W}{\sqrt{[E + M_t][E + M_t]^2 - M_t^2]} \frac{[E + M_W]^2 - M_W^2} \Theta(E - E_{\text{min}}),
\]

where \( E_{\text{min}} = 50 \text{ GeV} \) and the theta-function vanishes for negative argument and is unity for positive argument. Of course \( df/dE \) has been normalized such that

\[
f_{\text{core}} = \int_{E_{\text{min}}}^{\infty} \left( \frac{df}{dE} \right)_{\text{core}} dE.
\]

But since the sun is opaque to energetic neutrinos, the emergent emission rate spectrum is not the same as the core emission rate spectrum. The emission rate spectrum of neutrinos that emerge unscathed is

\[
\left( \frac{df}{dE} \right)_{\text{unscattered}} = \left( \frac{df}{dE} \right)_{\text{core}} \exp \left( -\sigma(E) \int_0^{n(r)} n(r) \, dr \right),
\]

where \( n(r) \) is the radial dependence of the number density of the sun and \( \sigma_{\text{CC}}(E) \) is the energy-dependent charged-current cross section given in Table I. Also given in Table I is the neutral-current cross section.

The electrons and muons produced by charged-current interactions are rapidly thermalized, so the effect of charged-current interactions is effectively to remove \( \nu_\mu \) and \( \nu_e \) neutrinos above a transparency energy \( E_\kappa \) where \( \sigma(E_\kappa) \int n(r) \, dr \) becomes unity. Using

\[
n(r) = 1.4 \times 10^{26} \exp(-r/0.1 R_\odot) \text{ cm}^{-3}
\]

for the density profile and adopting the cross section for \( E < 10^4 \text{ GeV} \), the transparency energy is \( E_\kappa = 150 \text{ GeV} \), and the emission spectrum of \( \nu_\mu \) and \( \nu_e \) emergent from the sun is

\[
\left( \frac{df}{dE} \right)_{\text{emergent}} = \left( \frac{df}{dE} \right)_{\text{core}} \exp \left( -E/E_\kappa \right).
\]
TABLE I: The energy-dependent cross sections used to calculate the flux of neutrinos from the sun \[19\].

| Energy Range [GeV] | $\sigma_{NC} \text{ [cm}^2\text{]}$ | $\sigma_{CC} \text{ [cm}^2\text{]}$ |
|-------------------|----------------------------------|----------------------------------|
| $0 \leq E \leq 10^4$ | $2.0 \times 10^{-39} \left( \frac{E}{\text{GeV}} \right)$ | $6.6 \times 10^{-39} \left( \frac{E}{\text{GeV}} \right)$ |
| $10^4 \leq E \leq 10^5$ | $2.1 \times 10^{-38} \left( \frac{E}{\text{GeV}} \right)^{0.714}$ | $6.1 \times 10^{-38} \left( \frac{E}{\text{GeV}} \right)^{0.714}$ |
| $10^5 \leq E \leq 10^7$ | $0.3 \times 10^{-36} \left( \frac{E}{\text{GeV}} \right)^{0.462}$ | $1.0 \times 10^{-36} \left( \frac{E}{\text{GeV}} \right)^{0.462}$ |
| $10^7 \leq E \leq 10^{12}$ | $2.3 \times 10^{-36} \left( \frac{E}{\text{GeV}} \right)^{0.363}$ | $5.5 \times 10^{-36} \left( \frac{E}{\text{GeV}} \right)^{0.363}$ |

FIG. 3: The emergent emission rate spectrum of neutrinos from the sun. For $\nu_{\tau}$, $f_{\text{core}} = 10^4M_{12}^{1/2}\Gamma_A$, and for $\nu_{\mu}$ and $\nu_e$, half that value. In all cases, $E_{\text{min}} = 50$ GeV.
This emission rate spectrum is shown in Fig. 3.

The situation is different for tau neutrinos. The lifetime of the tau produced in a charged-current scattering is so short that it decays before significant energy loss. Since the decay of the tau includes a $\nu_\tau$, the effect of the scattering and subsequent decay is simply to reduce the energy of the incident $\nu_\tau$ to about 20% of its incident value. Therefore, an incident tau neutrino above the transparency energy will continually suffer tau production and decay interactions, but will not be removed from the flux of high-energy neutrinos. The process will continue until the tau-neutrino energy has been degraded to the transparency energy or below, then the tau neutrino will escape.

This process has been considered by Halzen and Saltzberg [20] for very energetic neutrinos propagating through Earth. They found that the emergent spectrum of neutrinos was very well described by a lognormal distribution centered on the transparency energy with a dispersion of 0.49 decades in energy. We will assume that the flux of scattered tau neutrinos above the transparency energy emerges in a lognormal distribution peaked at $E_\kappa = 150$ GeV with a dispersion of 0.49 decades in energy.

The emergent emission spectrum of tau neutrinos has two contributions. The first contribution is the unscattered emission spectrum, given by Eq. (32). The second contribution is the fraction of the original emission above the transparency energy which will emerge as a lognormal distribution centered on the transparency energy;

$$\left( \frac{df}{dE} \right)_{\text{scattered}} = \frac{f_{\text{core}}}{E_{\text{min}}} \mathcal{F} \exp \left[ -\frac{(\log E - \log E_\kappa)^2}{2\sigma^2} \right], \quad (35)$$

where $\sigma = 0.49$ and $\mathcal{F}$ is found by demanding that the integral of Eq. (35) results in the total number of neutrinos above 150 GeV that are scattered. (If the initial energy of the neutrino is below 150 GeV, a scattering will produce a neutrino below $E_{\text{min}}$.) The result is $\mathcal{F} = 4.6 \times 10^{-2}$.

So for tau neutrinos, the emergent neutrino spectrum is

$$\left( \frac{df}{dE} \right)_{\text{emergent}} = \left( \frac{df}{dE} \right)_{\text{unscattered}} + \left( \frac{df}{dE} \right)_{\text{scattered}}. \quad (36)$$

This spectrum is also shown in Fig. 3.
V. THE EVENT RATE

In the last section we calculated the emission rate and emission rate spectrum of neutrinos from the sun. In this section we will calculate the event rate in a suppositional underwater neutrino detector of approximate size of a cubic kilometer \[21\]. We will only consider the rate for “contained events,” where the neutrino converts inside the volume of the detector. Including “uncontained events” will not significantly alter our results because the muon range at the relevant energy range here is comparable to a kilometer. We will assume that the efficiency of detection is a step function: zero below 50 GeV and unity above 50 GeV.

The first step in calculating the event rate is the simple step of converting the emission rate spectrum calculated in the last section to a flux spectrum arriving at Earth. Since the sun-Earth distance is \(D = 1.5 \times 10^8\) km, the flux spectrum is

\[
\frac{dF}{dE} = \frac{1}{4\pi D^2} \left( \frac{df}{dE} \right)_{\text{emergent}} = 3.5 \times 10^{-18} \left( \frac{df}{dE} \right)_{\text{emergent}} \text{ km}^{-2}. \tag{37}
\]

The mean-free-path of neutrinos is much larger than the size of the detector, so the fraction of the incident neutrinos of energy \(E\) that convert inside the detector is \(n_{\text{ice}} \sigma_{CC}(E) L\) where \(n_{\text{ice}}\) is the number density of the target and \(L = 1\) km is the size of the detector. The event-rate spectrum is given simply by

\[
\frac{dR}{dE} = \frac{dF}{dE} [n_{\text{ice}} \sigma_{CC}(E) L] A \Theta(E - 50 \text{ GeV}), \tag{38}
\]

where \(A\) is the area of the detector, assumed to be 1 km\(^2\), and the \(\Theta\)-function represents the detector efficiency.

Since \(n_{\text{ice}} \sigma_{CC}(E) L = 4 \times 10^{-10} (E/1 \text{ GeV})\), we find for the event-rate spectrum

\[
\frac{dR}{dE} = 1.4 \times 10^{-27} \frac{E}{\text{GeV}} \left( \frac{df}{dE} \right)_{\text{emergent}} \Theta(E - 50 \text{ GeV}). \tag{39}
\]

The total event rate is the integral of the event-rate spectrum.

For muon or electron neutrinos, the total event rate can be found using Eq. (34) for the emergent emission spectrum, with result

\[
R_{\text{TOTAL}}(\nu_\mu, \nu_e) = 1.6 \times 10^{-22} M_{12}^{1/2} \Gamma_A. \tag{40}
\]
The event rate in a cubic-kilometer underice/underwater detector for different values of the SIMPZILLA–nucleon cross section: from top to bottom the curves are for $\sigma = 10^{-22}$, $10^{-24}$, and $10^{-26}$ cm$^2$. The upper solid curves are for tau neutrinos and the lower dashed curves are for muon and electron neutrinos. For comparison, the 90% C.L. upper limits on muon fluxes of nonatmospheric origin in the direction of Earth’s core or the sun is approximately $10^4$ km$^{-2}$ year$^{-1}$ [22].

This event rate is shown in Fig. 4, and the event-rate spectrum is shown in Fig. 5.

For tau neutrinos, the total event rate is found using Eq. (36) for the emergent emission spectrum. For tau neutrinos the total event rate is

$$R_{\text{TOTAL}}(\nu_\tau) = 1.1 \times 10^{-20} M_{12}^{1/2} \Gamma_A.$$

(41)

This event rate is also shown in Fig. 4, along with the event-rate spectrum in Fig. 5.

The contribution of scattered tau neutrinos dominates the event rate. While the $\nu_\mu$ and $\nu_e$ events would be peaked toward the lower energy of the detector and drop rapidly, the $\nu_\tau$ events would have a relatively flat spectrum extending from the lower limit of the detector.
FIG. 5: The spectrum of events for different neutrino species. The total event rate $R_{\text{TOTAL}} = 1.1 \times 10^{-20} M_{12}^{1/2} \Gamma_A$ is the sum of the electron-neutrino, muon-neutrino, and tau-neutrino rates. Also shown are the two contributions to the tau-neutrino events, from unscattered neutrinos and from scattered neutrinos [see Eq. (36)].

Out to about 1000 GeV.

The mean energies of the detected neutrinos are

$$\langle E \rangle = 197 \text{ GeV} \quad (\nu_\mu, \nu_e)$$

$$\langle E \rangle = 3454 \text{ GeV} \quad (\nu_\tau).$$

We close this section by remarking on the possibility of a detectable event rate from annihilation of simpzillas captured by Earth. The ratio of the event rate for neutrinos of solar origin to the event rate for neutrinos of terrestrial origin is shown in Fig. 6. For small mass or large interaction cross section the signal from the center of Earth may be larger than the solar signal. For $\sigma \lesssim 10^{-24} \text{ cm}^2$ the solar signal will dominate for simpzilla masses larger than about $10^9 \text{ GeV}$, while for $\sigma \lesssim 10^{-26} \text{ cm}^2$, the solar signal dominates for the
FIG. 6: The ratio of the event rates from solar and terrestrial neutrinos originating from SIMPZILLA annihilation. The ratio is a function of SIMPZILLA mass $m_X$ and scattering cross section $\sigma$ through the dependence of the capture rates on $m_X$ and $\sigma$.

VI. CONCLUSIONS

If the local dark matter is very massive and strongly interacting, it should collect in the sun and Earth in sufficient numbers that equilibrium will be maintained between the capture rate and the annihilation rate.

Annihilation or decay of very massive particles into hadronic channels in the solar core or at the center of Earth leads to the production of high-energy neutrinos. While electron and muon neutrinos above 150 GeV are mostly absorbed, energetic tau neutrinos will be emitted, and will be the signature of energetic jet fragmentation at the center of the sun.

For most of the range of parameter space, the event rate expected in kilometer-scale
FIG. 7: The shaded region above the jagged line is excluded by a variety of considerations as discussed in [3]. Slightly stronger, model-dependent constraints can be found in Refs. [3, 23]. The shaded region above the straight line would result in more than 10 events per year in a cubic-kilometer underice or underwater detector and should easily be able to be excluded.

neutrino detectors will be well within detection limits. Such detectors should be able to exclude (or confirm!) the possibility of SIMPZILLAS as dark matter.

In this paper we have only considered production of neutrinos through top quark production and decay. Another potential source of tau neutrinos is bottom quark production and decay. While this may increase the emission rate of tau neutrinos, the spectrum is expected to be the same.

It is important to note that there is essentially no background. For example consider the background from cosmic-ray produced neutrinos. To produce a 1 TeV neutrino, a center-of-momentum energy of around $\sqrt{s} = \sqrt{2m_{\text{proton}}E} = 10$ TeV is required, where $E$ is the cosmic-ray energy. Thus, a threshold energy of $E_{TH} = 5 \times 10^{16}$eV is required to produce a 1 TeV
neutrino. The cosmic-ray flux at high energies is approximately

\[ E^3 \frac{dF}{dE} = 10^{24.5} \frac{eV^2}{s \ sr \ m^2}, \]  

(43)

which when integrated to give the total flux of particles with energy greater than \( E \) gives

\[ F(E > E_{TH}) = \frac{1}{2E_{TH}^2} 10^{24.5} \frac{eV^2}{s \ sr \ m^2} = \frac{6.1}{yr \ deg^2 \ km^2}. \]  

(44)

Since the sun subtends about a square degree, this sets the scale of the background. For much of parameter space, the signal should be well above the background.

In Fig. 7 we present out results in the \( \sigma \) vs. \( m_X \) plane and compare then with other limits. Clearly our considerations greatly extends the excluded region.

Indirect detection of simpzillas through annihilation in the sun is complementary to the other idea for indirect detection: wimpzilla decay producing ultra-high energy cosmic rays [24, 25].

Finally, we comment on the possible role of neutrino oscillations. Neutrino oscillations will be important if

\[ \frac{\Delta m^2}{10^{-3}eV^2} \frac{10^2GeV}{E_\nu} \frac{L}{2 \times 10^9 cm} \gtrsim 1, \]  

(45)

where \( L \) is the path length. Since \( 2 \times 10^9 \) cm is \( R_\odot/35 \), if \( \Delta m^2 \) is greater than or of the order of \( 3 \times 10^{-5}eV^2 \), oscillations will occur in the sun. If \( \Delta m^2 \) is greater than or of order of \( 10^{-7}eV^2 \), then oscillations will be important during the neutrino’s transit to Earth. Oscillations between \( \nu_\tau \) and \( \nu_\mu \) or \( \nu_e \) in the sun will decrease the \( \nu_\tau \) emission rate, while oscillations in transit will change the flavor signature of the signal.

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