Bit Rate of Programs
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Abstract—A program can be considered as a device that generates discrete time signals, where a signal is an execution. Shannon information rate, or bit rate, of the signals may not be uniformly distributed. When the program is specified by a finite state transition system, algorithms are provided in identifying information-rich components. For a black-box program that has a partial specification or does not even have a specification, a bit rate signal and its spectrum are studied, which make use of data compression and the Fourier transform. The signal provides a bit-rate coverage for testing the black-box while its spectrum indicates a visual representation for execution’s information characteristics.

Index Terms—Shannon information, program, Lempel-Ziv compression, testing.

1 BIT RATE OF PROGRAMS

A program consumes an input, runs its instructions, and provides an output. The input and output can be encoded as strings. They are possibly interleaved (e.g., dollar?drink!dollar?drink!... observed in a soft-drink vending machine). An execution which is a sequence of instructions, again, can be encoded as a string. In this paper, we only consider programs that halt. That is, an execution is of finite but unbounded length. When the program is deterministic, there is only one execution on a given input. We consider an information theoretic [1] model of a black-box program as a channel:
• On the sender side of the channel, an input is fed into the channel;
• On the receiver side of the channel, an execution along with the corresponding output is obtained.

A technically more convenient way is to treat the channel as a transducer $T$ that maps a string $x$ (the input) to a string $y$ (an execution along with output on the execution. For now, we may ignore the output since it is encoded in the output instructions in the execution. Hence, $y$ is simply an execution on input $x$). $T$, as a mapping, is many-to-many in general. When the program is deterministic, $T$ must be many-to-one.

To avoid issues caused by granularity of executions, we understand an execution as a sequence of assembly instructions executed from the compiled code of the program. In particular, the input, as a string, is fed into the transducer/channel symbol by symbol (i.e., byte by byte), and therefore, the execution consumes the input also symbol by symbol (e.g., through load-byte instructions). We use $S_{\text{input}}(n)$ to denote the number of inputs with length $n$, and $S_{\text{exe}}(n)$ to denote the number of executions with length $n$. The input bit-rate $\lambda_{\text{input}}$ is defined as

$$\lambda_{\text{input}} = \lim_{n \to \infty} \frac{\log S_{\text{input}}(n)}{n}.$$  

When the limit does not exist, we take the upper limit, which always exists (for a finite input alphabet). By convention, $\log 0 = 0$. Throughout this paper, the logarithm is base 2. Similarly, the program (execution) bit-rate $\lambda_{\text{exe}}$ is defined as

$$\lambda_{\text{exe}} = \lim_{n \to \infty} \frac{\log S_{\text{exe}}(n)}{n}.$$  

The notions used in [1] and [2] come from a well-known fundamental formula proposed by Shannon [1] in defining a channel capacity which was later used by Chomsky and Miller [2] for describing a complexity of regular languages.

1.1 Intuitive explanations of the bit rates

Intuitively, the program bit rate, which is a nonnegative real number, measures “how many” executions are possible in the program. Clearly, for nontrivial programs, the number of executions should be simply infinite — even when the program is deterministic, considering the fact that the domain that the inputs are drawn from may be infinite. To avoid this problem, the notion defined in [2] is used, which is always a finite number since the alphabet of instructions has finite size. Theoretically, the program bit rate $\lambda_{\text{exe}}$ is exactly the number of bits per symbol needed when one losslessly compresses an average execution. In other words, a larger program bit rate makes the execution harder to compress, and therefore, contains more information (hence, intuitively, the program’s semantics is harder to comprehend). In terms of (white-box) software testing, its direct implication, according to the definition in [2], is that the program
is harder to test (i.e., “more” execution paths to cover, even though the total number of paths is infinite).

The input bit rate is similar. It characterizes “how many” inputs could possibly be fed into the program. Even though the number is usually infinite, the input bit rate, when calculated in (1), is always a finite number. A larger input bit rate indicates that an average input carries more information per symbol. By looking at the definition in (1), it also says that, in terms of (black-box) software testing, the program is harder to test since the “number” of inputs is also higher.

1.2 Deterministic programs

It can be shown that, for a deterministic program, the input bit rate is always greater or equal to the program bit rate (since an execution has to, as assumed, consume the entire input symbol by symbol, and there is only one execution per input); i.e.

\[ \lambda_{\text{input}} \geq \lambda_{\text{exe}}. \] (3)

This observation is interesting in many ways. Clearly, the inequality in (3) says that an average input carries more information than an average execution. In other words, the program, understood as a channel, adds redundancy to the information-rich (i.e., harder to understand) input so that the resulting execution is not so information-rich (i.e., easier to understand). At an abstract level, a program is to solve a problem, where an input is simply an instance of the problem to solve; this view can be found in any standard automata theory textbook. Therefore, (3) implies that, using a deterministic program to solve the problem, one has to “stretch” an instance (i.e., the input) by inserting redundancy and, as a result, the bit rate of the solution (i.e., the execution) is diluted and hence lower. The essential reason of this comes from the fact that a program, when compiled into assembly code can perform only extremely simple instructions per unit time. This is evidenced by the simplicity of an instruction set of a modern processor that the program eventually runs on, where its theoretical root comes from the equal (if not lower) simplicity of the instruction set in a Turing machine. Following this understanding, it is clear that:

Deterministic programming adds redundancy to inputs.

Suppose that one tries to understand the semantics of the program (i.e., figure out that the program indeed solves the problem that it intended to solve) by tracing its executions. Clearly, an execution represented at the assembly level has to be abstracted back to the source code level or even at the design level. In terms of the channel we mentioned earlier, it is a process of decoding an execution back to the input, which necessarily squeezes out all of the redundancy added by the program. Hence,

Deterministic program understanding removes redundancy in executions.

What if (3) does not hold? In this case, the program is simply not correct (does not solve the problem that it is intended to solve). The conclusion is reached without any testing nor structural analysis on the code.

1.3 Nondeterministic programs

Nondeterminism is common in concurrent programming, which makes the transducer corresponding to a program be a many-to-many mapping. In other words, an input may result in multiple or even infinite number of executions. Because of this, the inequality in (3) does not always hold, and in some cases,

\[ \lambda_{\text{input}} \leq \lambda_{\text{exe}}. \] (4)

In the light of the channel, noise is added during transmission and thus, on the receiver’s side, an execution is not only added with redundancy but also with noise. In the case when the information rate contained in the noise exceeds the decrement of information rate caused by redundancy, (4) is therefore possible. A nondeterministic program is known to be notoriously hard to design, develop, understand, and test. One reason of this, in the view of information theory, is that nondeterminism is interpreted as noise and hence makes the bit rate of a program higher:

Nondeterministic programming adds redundancy and noise to inputs.

A program with higher bit rate could be more information-efficient (per instruction, the execution
In this section, we consider a finite state transition possible assignments. Nondeterministic program understanding removes redundancy and noise in executions. This likely implies, in practice, a good program seeks a balance in the bit rate, which is not too high so that the code is still understandable and not too low as well so that executions still carry a reasonable amount of information.

1.4 Where are we heading?

Given these information-theoretic understandings of programs, the reader might bear the following doubt in mind: would this help research in software engineering in certain ways? The central idea of the understandings defines the notion of “bit rate” (or a measure of information quantity) of a program. However, it is left unanswered where the information is. We think that an answer to this question is a key to addressing the software engineering aspects of the idea. We do not intend to completely answer the question in this paper. Instead, in the subsequent two sections, we probe the answers from the following two angles, respectively:

- A program is a transition system, in particular, when one looks at a high-level design of the program. Information may not be uniformly distributed across the transition system. So, when this transition system is treated as a white-box, where is the information concentrated?
- When the program is treated as a black-box, its internal structure is not observable (i.e., the transition system as well as its number of states is unknown). However, when it runs, the execution (at the assembly level) can be observed. On an execution, the bit rate may not be uniform: when the execution passes through an information-rich component, the bit rate would be higher. Recall that bit rate of an execution is closely related to lossless compression as we mentioned earlier. Therefore, using an optimal universal compression algorithm (runs in linear time) like Lempel-Ziv in various ways, we can obtain a bit rate signal of the execution. Using signal analysis techniques, we can further use the spectrum of the signal to create a run-time “coverage” of the blackbox.

2 Bit rate of a program modeled as a finite state transition system

In this section, we consider a finite state transition system

\[ M = (Q, R, q_{\text{enter}}, q_{\text{exit}}) \]

where \( Q \) is a finite set of states, \( R \subseteq Q \times Q \) is a set of transitions and with \( q_{\text{enter}} \in Q \) being the entering state, and \( q_{\text{exit}} \in Q \) being the exit state. We often write a transition as \( q \rightarrow q' \) when \((q, q') \in R\). For technical simplicity, herein, we assume that there is only one entering state and only one exit state; when there are multiple entering states and exit states, the results in this section can be easily generalized.

For the given \( M \), a path is a sequence of states \( q_0 \cdots q_n \), for some \( n \) such that, for each \( 0 \leq i < n \), \( q_i \rightarrow q_{i+1} \). In this case, the path is of length \( n \), and we say that \( q_0 \) reaches \( q_n \), written \( q_0 \rightarrow q_n \). The information rate of \( M \) is defined as

\[ \lambda_M = \lim_{n \to \infty} \frac{\log S_M(n)}{n}, \tag{5} \]

where \( S_M(n) \) is the number of paths, with length \( n \), in \( M \) from \( q_{\text{enter}} \) to \( q_{\text{exit}} \). The rate can be efficiently and numerically computed as the Perron number of the adjacency matrix of the graph \( M \). Indeed, \( M \) is essentially a directed graph. Let \( Q' \subseteq Q \). We use \( M' \) to denote the subgraph that only keeps nodes in \( Q' \) and edges between nodes in \( Q' \) in \( M \). As usual, \( Q' \) is a strongly connected component (SCC) if \( Q' \) is maximal and satisfies, for every \( q, q' \in Q', q \sim q' \).

Notice that, in \( M \), a transition does not have a label. Doing this is purely due to technical convenience since the information rates studied in a moment depend only on path counts. Of course, one may associate a label with a transition \( q \rightarrow q' \) in \( M \); such a label, depending on the applications, can be interpreted as, e.g., an I/O event.

Consider a path \( \alpha \) from \( q_{\text{enter}} \) to \( q_{\text{exit}} \) in \( M \). As we mentioned earlier, an information-theoretic interpretation of \( \lambda_M \) is the amount of information carried on \( \alpha \) in terms of average number of bits per symbol. However, this amount of information is not necessarily uniformly distributed over \( \alpha \); some segment within \( \alpha \) may carry more information; i.e., with higher information density. This is because the path may pass through a subgraph that has a higher information rate. Identifying such a subgraph is practically meaningful, since when \( M \) is a design or even the code of a finite state program, the subgraph (particularly when containing most information) indicates a part of focus for testing. To do this, we need some more results.

Suppose that the subgraph \( M' \) is with its own entering state \( q'_{\text{enter}} \in Q' \) and exit state \( q'_{\text{exit}} \in Q' \), and that the subgraph is reachable; i.e., \( q_{\text{enter}} \sim q'_{\text{enter}} \) and \( q_{\text{exit}} \sim q'_{\text{exit}} \). We first observe that

\[ \lambda_{M'} \leq \lambda_M; \tag{6} \]

i.e., the information rate of the subgraph is at most that of the entire graph \( M \). This can be shown as follows. Consider a path \( \alpha \) that witnesses \( q_{\text{enter}} \sim q'_{\text{enter}} \) and a path \( \beta \) that witnesses \( q'_{\text{exit}} \sim q_{\text{exit}} \). Clearly, for every path \( \gamma \), from \( q'_{\text{enter}} \) to \( q'_{\text{exit}} \), with length \( n \) of the subgraph \( M' \), the “concatenated” path of \( \alpha, \gamma, \beta \),
written \( \alpha \gamma \beta \), is also a path of \( M \) from \( q_{\text{enter}} \) to \( q_{\text{exit}} \). This immediately gives
\[
\lim \frac{\log S_{M'}(n)}{|\alpha| + n + |\beta|} \leq \lim \frac{\log S_M(|\alpha| + n + |\beta|)}{|\alpha| + n + |\beta|} \leq \lambda_M.
\]
Noticing that the left hand side is
\[
\lim \log S_{M'}(n) = \lim \frac{\log S_{M'}(n)}{n} \cdot \frac{n}{|\alpha| + n + |\beta|} = \lambda_{M''},
\]
the result in (6) follows. (We should emphasize that the result is only asymptotic - generally doesn’t need to hold for finite \( n \)).

2.1 Finding an information rich component
Let \( \theta \) be a number in \([0, 1]\). From (6), there exists a reachable subgraph \( M' \), called a \( \theta \)-information rich component (\( \theta \)-IRC), which is minimal with the rate \( \lambda_{M'} \geq \theta \lambda_M \). Observe that a \( \theta \)-IRC is not necessarily unique.

Without loss of generality, let us assume that \( M \) is cleaned; i.e., every node is on a path from the entering state to the exit state. Also we assume that \( \lambda_M > 0 \) (otherwise, any subgraph is a \( \theta \)-IRC). The following is a straightforward algorithm to find a \( \theta \)-IRC in \( M \), which runs in worst-case time \( O(m \cdot \text{Rate}(m)) \), where \( m \) is the size (number of states + number of transitions) of \( M \), and \( \text{Rate}(m) \) is the time complexity, which is known efficient (polynomial time) in theory and in practice (as our experiments in [3] implemented in MATLAB), of numerically computing the information rate in (5) of a graph:

Alg 2.1
1. Initially, every edge in \( M \) is unmarked, and \( M'' = M \).
2. If there is no unmarked edge, goto 7.
3. Delete an arbitrary unmarked edge from \( M'' \) (but keep the nodes of the edge).
4. Compute the information rate of the resulting \( M'' \) (\( M'' \) is with the same entering and exist states as in \( M \)).
5. If \( \lambda_{M''} \geq \theta \lambda_M \), then goto 2.
6. Else (now the condition \( \lambda_{M''} < \theta \lambda_M \) holds when \( M'' \) is without the deleted edge) add the deleted edge back to \( M'' \) and mark the edge, goto 2.
7. (***) using Tarjan’s algorithm finding a SCC in \( M'' \), return the SCC (take arbitrary nodes in the SCC as the entering state and the exit state) as \( M' \).

We claim that the SCC \( M' \) returned from the algorithm is a \( \theta \)-IRC. First observe that \( \lambda_{M''} \geq \theta \lambda_M \) when in statement (***) a SCC is selected and marked as \( M' \), where \( M' \) is a subgraph of \( M'' \). In particular, every edge is marked in \( M'' \). Suppose that \( \lambda_{M'} < \lambda_{M''} \). Let \( \alpha \) be a simple path from the entering state to the exit state in \( M' \). The \( \alpha \) can be obtained by deleting (at least one) edges \( e \) from \( M' \). After deleting these edges, \( M'' \) is a new graph, denoted by \( M''' \). Because \( \lambda_{M'} < \lambda_{M''} \), it can be shown that \( \lambda_{M''} = \lambda_{M''} \geq \theta \lambda_M \). This violates the condition in line 5 of the algorithm when the edges \( e \) were marked. Due to the same reason, \( M' \) is minimal; i.e., dropping any edge from \( M' \) will make it with lower rate and hence be not a \( \theta \)-IRC anymore.

For example, Figure 1 shows a finite state transition system of Hilo’s image browser view model [4]. A \( \theta \)-IRC (with \( \theta \) being 0.79) of the transition system is shown in Figure 2. That is, the IRC concentrates 79% of the bit rate of the original transition system.

Now, consider two finite state transition systems \( M_1 \) and \( M_2 \) (with disjoint state sets). As usual, we use \( (M_1; M_2) \) to denote the sequential composition of the two (by connecting the exit state of \( M_1 \) with the entering state of \( M_2 \)). One can show that, if \( M'_1 \) (respectively \( M'_2 \)) is a \( \theta \)-IRC of \( M_1 \) (respectively \( M_2 \)), then \( M'_1 \) (respectively \( M'_2 \)) is also a \( \theta \)-IRC of the sequentially composed system \( (M_1; M_2) \).

The same result holds when \( M_1 \) and \( M_2 \) are composed nondeterministically as \( (M_1 \;\Box\; M_2) \) (by creating a new state with a transition to the entering state of \( M_1 \) and a transition to the entering state of \( M_2 \)).

When \( M_1 \) and \( M_2 \) are synchronously composed as \( (M_1 \parallel M_2) \), finding a \( \theta \)-IRC in the composed system is more difficult and deserves investigation. The IRC
informs a tester which parts of $M_1$ and $M_2$ shall be a focus for intensive testing. We start with definitions.

We pair some states in $M_1$ and some states in $M_2$ together, and put the pairs in a set $\Pi$. Each such pair $(s^1, s^2) \in \Pi$ is called a synchronization pair, whose intended meaning is that, when $M_1$ is at state $s^1$, and $M_2$ is at state $s^2$, in the synchronized system $(M_1 \parallel M_2)$, both shall move together by each firing a transition at the same time. Define $Q_1$ (respectively $Q_2$) to be the states of $M_1$ (respectively $M_2$) that do not appear in any pair of $\Pi$. Using the standard interleaving semantics of concurrency, mathematically, a path of $(M_1 \parallel M_2)$ is a word $\alpha$ in alphabet $Q_1 \cup \Pi \cup Q_2$ satisfying:

- when deleting symbols in $Q_2$ from $\alpha$, and deleting symbols in $Q_2$ from every synchronized pair appearing in $\alpha$, we obtain a path in $M_1$ from its entering state to its exit state; and,
- when deleting symbols in $Q_1$ from $\alpha$, and deleting symbols in $Q_1$ from every synchronized pair appearing in $\alpha$, we obtain a path in $M_2$ from its entering state to its exit state.

We use $\text{Paths}(M_1 \parallel M_2)$ to denote the set of all paths of $(M_1 \parallel M_2)$. $\text{Paths}(M_1 \parallel M_2)$ is a regular language, which can be accepted by a DFA (that is a graph, with $O(n_1 n_2)$ nodes, where $n_1$ and $n_2$ are the numbers of nodes in $M_1$ and in $M_2$, respectively), still denoted by $(M_1 \parallel M_2)$. Therefore, the information rate $\lambda_{(M_1 \parallel M_2)}$ of the DFA can be computed efficiently. Notice that the rate $\lambda_{(M_1 \parallel M_2)}$ could be much higher than the rates for $M_1$ and $M_2$, because of the nondeterministic interleaving between states in $Q_1$ and states in $Q_2$ in $\alpha$.

In order to define an IRC of $(M_1 \parallel M_2)$, we need more definitions. As before, we assume that $\lambda_{(M_1 \parallel M_2)} > 0$. Consider a subgraph $M'_1$ and a subgraph $M'_2$ of $M_1$ and $M_2$, respectively. Different from the previous definitions of IRCs, we require now that $M'_1$ (respectively $M'_2$) contains the entering state and the exit state of $M_1$ (respectively $M_2$). The difference comes from the fact that state pairs in $\Pi$ must be synchronized on a path. We call $(M'_1, M'_2)$ a $\theta$-IRC of $(M_1 \parallel M_2)$ if $\lambda_{(M'_1 \parallel M'_2)} \geq \theta \lambda_{(M_1 \parallel M_2)}$ and both $M'_1$ and $M'_2$ are minimal. A similar algorithm to find a $\theta$-IRC for $(M_1 \parallel M_2)$ is as follows.

Alg 2.2

1. Initially, every edge in $M_1$ and $M_2$ is unmarked, and $(M'_1, M'_2) = (M_1, M_2)$.
2. If there is no unmarked edge in both $M'_1$ and $M'_2$, return $(M'_1, M'_2)$.
3. Delete an arbitrary unmarked edge from either $M'_1$ or $M'_2$ (but keep the nodes of the edge).
4. Compute the information rate of the resulting $(M'_1, M'_2)$.
5. If $\lambda_{(M'_1 \parallel M'_2)} \geq \theta \lambda_{(M_1 \parallel M_2)}$, then goto 2.
6. Else put the deleted edge back and mark the edge, goto 2.

It is clear that the $(M'_1, M'_2)$ returned from the algorithm satisfies $\lambda_{(M'_1 \parallel M'_2)} \geq \theta \lambda_{(M_1 \parallel M_2)}$. In fact, it is also minimal. Otherwise, if one deletes an edge $e$ from, say $M'_1$, and still $\lambda_{(M'_1 \parallel M'_2)} \geq \theta \lambda_{(M_1 \parallel M_2)}$ holds, then this $e$ (being a marked edge) cannot be marked in step 6. Hence, the algorithm returns a $\theta$-IRC $(M'_1, M'_2)$. The algorithm has worst-case time complexity $O(m \cdot \text{Rate}(\mathcal{O}(m^2)))$, where $m$ is the maximal size of $M_1$ and $M_2$.

2.2 Information rich inputs to a finite state transition system

We now consider inputs to a finite state transition system $M$ defined earlier. Let $\Sigma$ be nonempty and finite alphabet. It is necessary now to associate a label $a \in \Sigma$ to a transition in $M$; as a result, the $M$ is a labeled finite state transition system. Notice that some transitions are labeled by null symbol ($\epsilon$) instead of a symbol in $\Sigma$ to indicate, e.g., it is an unobservable transition performing some internal actions. For technical simplicity, we deliberately ignore output symbols. In fact, one may also associate an output symbol on some of the transitions labeled with $\epsilon$, to indicate, on a path of $M$, an output sequence of symbols can be observed in response to the input sequence of symbols fed along the path. Adding such output symbols does not change any definition or algorithms in this section and hence, our simplification is without loss of generality.

So now, a path $\alpha$ is a state-symbol sequence $q_0 a_0 q_1 a_1 \cdots a_{n-1} q_n$, for some $n$, where $q_i \rightarrow q_{i+1}$ is a transition in $M$ (where $a \in \Sigma \cup \{\epsilon\}$), for each $0 \leq i < n$. The input word $a_0 \cdots a_n$ on the path is denoted by $w_\alpha$. In this section, we will identify a “minimal” set of inputs that causes a highest information rate of execution paths in $M$. The set is called a set of information rich inputs (IRI). This is practically meaningful for black-box testing, where an input in such a subset can be intuitively considered as one carrying the most information with respect to the transition system.

However, defining (not to say finding) an IRI is difficult, due to the fact that an IRI could be infinite and we need a feasible way to make it "minimal". One way to define an IRI is as follows. One can run Alg 2.1 on the graph of $M$ to identify a $\theta$-IRC in $M$. The IRC, by definition, has the information rate $\lambda_{M'} \geq \theta \lambda_M$, and it is minimal. The rate is measured on the paths walking inside the IRC. Hence, the input on each such path forms an IRI (from now on we call it $\theta$-IRI). There is a small problem here since a path in the IRC is not necessarily a complete path (from the entering state to the exit state of the original $M$). To fix this, we choose a simple path $\alpha$ from the entering state to a state $q$ in the IRC, and a simple path $\beta$ from

3. When composition, which is not studied in Section [2.2] is concerned, it is not generally a good idea to ignore output symbols. This is particularly true when the transition system is defined with a powerful semantics like in I/O automata [5].
a state \( p \) in the IRC to the exit state. We use \( \gamma \) to denote a path inside the IRC from state \( q \) to state \( p \). Then, we use \( w_{\alpha\gamma\beta} \) to denote the input word on the “concatenation” of \( \alpha, \gamma, \beta \). We now put all such \( w_{\alpha\gamma\beta} \) in a set, for all the \( \gamma \)'s (\( \alpha \) and \( \beta \) are fixed). The set is a subset of input words, and is defined as the \( \theta \)-IRI. Clearly, the \( \theta \)-IRI is regular, and it is accepted by the finite automaton specified by the \( \alpha \) (treated as a single-path graph), sequentially composed with the IRC, and then the \( \beta \) (treated again as a single-path graph). The whole computation takes the same worst case time as Alg 2.1. This \( \theta \)-IRI obtained from \( M \) is denoted by \( \theta \)-IRI\((M) \) and will be used below.

Suppose now that an input \( w \) is drawn from a domain specified by a regular language \( L \) on alphabet \( \Sigma \). Such a domain is used to restrict “valid” input to feed into the system. For instance, in an ATM banking system, multiple withdrawals for more than 300 dollars within a day are not allowed in many locations. This requirement makes some arbitrary deposit-withdraw sequences invalid. When this \( L \) is given but the finite state transition system is not given, we may still ask a similar question: what would be a “minimal” subset of \( L \) that contains the most information in \( L \)? Let \( M \) be a DFA to accept \( L \). Then, the minimal subset that we are looking for can be defined to be the \( \theta \)-IRI\((M) \) obtained in the preceding paragraph. This \( \theta \)-IRI, which will be used below, is denoted by \( \theta \)-IRI\((L) \).

It becomes rather complicated when we are given a finite state transition system \( M \) as well as input language \( L \). The difficulty now is that an IRI in \( L \) may not be the inputs carried on an IRC of \( M \). We first define an automata-theoretic construction of an NFA \( M' \) as follows. \( M' \) works on a path of \( M \). While reading the input, \( M' \) simulates \( M'' \) by feeding every input symbol on the path into \( M'' \). Meanwhile, \( M' \), by memorizing the graph \( M \) (which is finite), checks that the path is indeed a path of \( M \), from the entering state to the exit state. At the end of the path, \( M \) accepts if \( M'' \) enters its own accepting state. We use \( P(M'') \) to denote the set of paths accepted by \( M' \). Clearly, \( M' \) accepts exactly the paths of \( M \) (from the entering state to the exit state) that will “consume” an input from \( L \). The number of states in \( M' \) is \( O(nm) \) where \( n \) is the number of states in \( M \) and \( m \) is the number of states in \( M'' \). We shall notice that the rate \( \lambda_{P(M'')} \) is the maximal rate \( M \) running on input words drawn from \( L \). So, the desired \( \theta \)-IRI will be a minimal set of input words such that when \( M \) runs on these input words, its rate is at least \( \theta \lambda_{P(M'')} \). Since now \( M' \) only runs on inputs from \( L \), the desired \( \theta \)-IRI is simply the \( \theta \)-IRI\((P(M'')) \) defined in the previous paragraph. But this is not right: the inputs to \( M' \) are paths (instead of input words in \( L \)). We need project those paths into input words as follows. First, using the procedure in the last paragraph, we find \( \theta \)-IRI\((P(M'')) \). Second, since \( \theta \)-IRI\((P(M'')) \) is regular (see the previous paragraph), an NFA \( M''' \) can be constructed to “project” every path in \( \theta \)-IRI\((P(M'')) \) to the input on the path. The resulting set of inputs, denoted by \( \theta \)-IRI\((P(M'')) \) \( \downarrow \) \( \Sigma \) is the desired \( \theta \)-IRI, and can be specified by the \( M''' \) whose size is at most \( O(nm) \), where \( n \) is the size of \( M \) and \( m \) is the size of \( M'' \). The entire process of finding the desired IRI takes worst-case time \( O(nm \cdot \text{Rate}(O(nm))) \).

3 Bit rate and spectrum of a black-box

In the previous section, bit rate is measured over a specification as a finite state transition system, and therefore, one may select an “information rich” test case from the specification. Notice that by information is meant the information on the specification instead of on the black-box, due to the fact that, in practice, there is a gap between a specification and an implementation. A specification is not the code; it is a hint of the code at best. However, it is the code, instead of the specification, that runs on a CPU and may fail. Using specification-based testing, one can select a number of test cases from the specification and run them on a black-box. However, after running the test cases, the natural conclusion reached is that one really does not know how much information of the black-box (instead of the specification) has been covered. But is this conclusion necessarily true?

If the conclusion is not true, then the picture of black-box testing could change fundamentally in several ways. For instance, one could re-evaluate test cases generated in specification-based testing and see whether they have covered enough information of the unknown code. If not, some heuristic approaches could be used to re-generate test cases so that more information will be covered. Our belief, justified by the experimental results presented in this section, is that the conclusion above is not true in general. The heuristic approaches will also be applicable to guide random testing over a black-box \([4]\), \([7]\), \([8]\) so that test cases generated are not only based on the specification but also based on the information of the black-box. This even works when the specification is not complete or even not available.

As a motivating concept, consider the compression of a discrete-time signal. An effective compression algorithm, such as JPEG for images or Lempel-Ziv (LZ) for text, generates a bit-stream representative of the information rate in the signal. A signal with high (respectively, low) information rate is harder (easier) to compress. Common signals have non-uniform information rates as a function of time (or, for an image, spatial location). For example, Figure 3 shows a digital image where information (loosely, image activity) is concentrated in several relatively small areas. Hence, by monitoring the bit rate of the JPEG encoding bit-stream, one can roughly determine the information rate of different regions in the image. Furthermore,
very efficient data compression algorithms, such as LZ or adaptive arithmetic codes [9], include either implicit or explicit source modeling. That is, the encoding is performed conditioned on a context (of previously encountered and encoded source samples in the signal) and the required bit rate is reflective of the innovation in the signal, and hence, within the limits of the modeling, of the per-sample information rate of the signal.

Now, a black-box program can be viewed as a source device that emits an execution at run time, the execution corresponding to a sequence of assembly instructions. Analogous to the image source in Figure 3, the information rate of the execution carries valuable information about the source device (the program). An efficient data compression algorithm can be used to access this rate information, generating a signal to be analyzed and used to discover patterns in the software execution.

As before, $S_{exe}(n)$ denotes the number of executions (of the black-box under test) with length $n$. The bit rate of the black-box is defined, as before,

$$\lambda_{exe} = \lim_{n \to \infty} \frac{\log S_{exe}(n)}{n}.$$ 

However, it cannot be computed since the code is not available. Despite this, $\lambda_{exe}$ is exactly the average number of bits per symbol needed to losslessly encode an average execution. That is, $\lambda_{exe}$ is the average bit rate of an execution. Every individual execution sequence has a bit rate, which indicates how much information is carried by the execution. How can the bit rate of an execution be measured? The Lempel-Ziv data compression algorithm [10] (which is known to be universal) is one of the best and most commonly used compression algorithms, and is a good potential choice. Why? Let’s go back to the origin of this problem, and ask a broader question: what is the “essential” information in an execution? We may have different answers to this question from various perspectives. One important perspective is provided by information theory. In information theory, “non-essential” information is modeled as a form of statistical dependence, and is hence a form of redundancy. The purpose of data compression (an application of information theory) is to reduce redundancy for efficient representation. A sequence is more compressible if it contains more redundancy. Therefore, the rate of a compressed sequence is an information-theoretic indicator that can measure information rate in a sequence.

### 3.1 Summary of our approach

Our approach uses the following two initial steps. First, in Lempel-Ziv encoding of an execution (for a given test case), a bit-stream is produced. By looking up the dictionary formed in Lempel-Ziv encoding, the number of bits carried by each instruction, i.e., the instantaneous bit rate of the encoded execution sequence data, is obtained. Through dividing the encoded execution sequence data into consecutive blocks, we can calculate the average instantaneous bit rate for every consecutive block. This yields a “rate vs. time” characteristic for the encoding. Second, the rate vs. time characteristic is some signal (here simply called the bit rate signal), and hence it can be analyzed using existing signal processing methods, such as the Fourier transform, power spectrum estimation, smoothing filters, linear prediction analysis, etc. Focusing on the Fourier transform, in this step a frequency domain spectrum of the signal is computed, which is referred to as the (bit rate) spectrum of the execution.

This approach is illustrated through an example. Consider a software system bzip2 (a popular compression software package), which is treated as a black-box. When the black-box runs under an input (test case, which is a file to be compressed), we monitor its execution at the assembly level (this can be done by, for instance, an instruction set monitor of the CPU or even in a debugger gdb) and record a sequence of assembly instructions, that is the execution. Then, the sequence is compressed using Lempel-Ziv encoding and the (instantaneous) bit rate of the sequence, (the compression length of each instruction in the sequence), is calculated. After that, the sequence is parsed into consecutive segments of equal length and the average (instantaneous) bit rate of every segment is computed. As a result, a time-domain bit rate signal is generated, where the time index is the index of a block of instructions. Each segment serves, intuitively, as a “rate region” of the signal, analogous to the image regions in Figure 3.

Figures 4 (1-a)(2-a)(3-a) show three bit rate signals corresponding to, respectively, three executions (for three different test cases, denoted as test-case-1, test-case-2, and test-case-3, which are a PDF file, a WORD file, and a binary file) of the example black-box. The length of each execution is 500,000 assembly
instructions. For the figure, the execution instruction sequence is parsed into 1000 segments, each containing 500 assembly instructions.

The three bit rate signals look very irregular. Signal processing techniques can be used to apply a Fourier transform so that the three signals are, respectively, transformed into frequency-domain bit rate spectra, as shown in Figure 4 (1-b)(2-b)(3-b). (In order to smooth the spectra, a low-pass filter was also used.) In the following, we define a distance between bit-rate signals and explain the representation and meaning of the distance in time-domain and frequency-domain.

Let \( x_r(n) \) be a bit-rate signal - that is, a sequence of non-negative bit-rates for successively encoded blocks. The mean is \( m_{x_r} = \frac{1}{N} \sum_{i=1}^{N} x_r(i) \), and the mean-removed signal is \( x(n) = x_r(n) - m_{x_r} \). (Note that the bit-rate signals in (1-a),(2-a) and (3-a) are mean-removed signals.) Let \( X(k) \) denote the \( N \)-point discrete Fourier transform (DFT) of \( x(n) \), where \( x(n) \) is assumed to be of length \( N \). Define the \( \ell_2 \) norm of a discrete signal, \( x(n) \), as

\[
||x(n)||^2 = \sum_{i=1}^{N} |x(i)|^2.
\]

Then the norms of \( x(n) \) and its DFT, \( X(k) \) are related by

\[
||x(n)||^2 = \frac{1}{N} ||X(k)||^2. \tag{7}
\]

When comparing two signals \( x(n) \) and \( y(n) \), the norm of the error signal, \( x(n) - y(n) \) is given by \( ||x(n) - y(n)|| \). Let \( x_r(n) \) and \( y_r(n) \) be two bit-rate signals, with respective mean \( m_x \) and \( m_y \), and define the mean-removed bit-rate signals as \( x(n) = x_r(n) - m_x \) and \( y(n) = y_r(n) - m_y \). Then the norms of the difference between original bit-rate signals, and the mean-removed bit-rate signals, are related as

\[
||x_r(n) - y_r(n)||^2 = ||x(n) - y(n)||^2 + N(m_x - m_y)^2, \tag{8}
\]

where \( N \) is the length of the signals.

It is common to study the magnitude of a signal DFT, \( |X(k)| \), and this is often referred to as the (magnitude) spectrum. For power signals, \( |X(k)|^2 \) is often referred to as the power spectrum. From (7) the signal norm can be computed either from the time-domain signal, or from its DFT spectrum.

Now, when comparing two DFT signals, say \( X(k) \) and \( Y(k) \), it is common to plot their respective magnitude, \( |X(k)| \) and \( |Y(k)| \). However, the norm of the difference of magnitude signals is not the same as the norm of the difference of the two signals. That is, from (7)

\[
||x(n) - y(n)||^2 = \frac{1}{N} ||X(k) - Y(k)||^2, \tag{9}
\]

and this is generally not \( ||(X(k) - Y(k)||^2/N \). The operation of taking the magnitude of the DFT, namely, \( |X(k)| \), is generally information-lossy since it eliminates all phase information.

As a result, we use (8) to compute the distance. The corresponding distances between (1-a) and (2-a), (1-a) and (3-a), (2-a) and (3-a) are 15722.5, 58973.4 and 29331.2, respectively. Clearly, the above results show that test-case-1 and test-case-2 are more similar while test-case-1 (as well as test-case-2) is quite different from test-case-3. (Notice that the norms of (1-a), (2-a) and (3-a) are only 7664.3, 9056.7 and 27980.8, respectively. So the difference is significant.) The distance, introduced in (8) will yield a coverage indicator which will be described in detail in the following subsection. In addition, we also show the mean and variance of bit rate signals in (1-a), (2-a) and (3-a), which can be a useful auxiliary indicator to differentiate distinct signals.

Fig. 4. Three bit rate signals (1-a)(2-a)(3-a) for the same black-box under three test cases and the three corresponding spectra (1-b)(2-b)(3-b) of the three executions, respectively.
We also need to clarify the significance of using the Fourier transform in our approach. The distances between different signals can be computed in either the time-domain or the frequency domain since the Fourier transform is an invertible transform. It seems that everything can be done in the time-domain and the use of the Fourier transform is redundant. Why do we choose to use the Fourier transform in this approach? The main advantage of the Fourier transform is to provide an alternative, frequency domain, representation of a signal, revealing some signal characteristics not as readily apparent in the time domain. One example is human speech. As a time signal, voiced speech exhibits quasi-periodic behavior, while unvoiced speech appears noise-like. In the Fourier domain, voiced speech exhibits clear spectral peaks, with the pitch period of female speakers tending to be significantly smaller (higher frequency) than for male speakers. Additionally, the Fourier magnitude spectrum is time-shift invariant. The Fourier transform is used in this paper to provide an intuitive and visual representation of the frequency domain information in an execution.

In addition, we shall point out that a bit rate spectrum can be obtained efficiently in $O(n \log n)$ time from an execution (because the Lempel-Ziv algorithm is linear time and the (Fast) Fourier transform runs in $O(n \log n)$ time) and what we need is only an execution of the black-box instead of its source code (which is assumed unavailable).

### 3.2 Experiments

In this subsection, we will show a set of experiments to validate the usefulness of the bit rate spectrum of a black-box program, and analyze its implications.

#### 3.2.1 Subjects

In order to estimate and compare the bit rate of various programs, two classes of programs are chosen as subjects. One class is of large programs and the other class is of small programs. The experiments consist of two groups. Group 1 is designed to estimate the bit rate of a large program. In the first group, bzip2 is chosen as the subject and 10 distinct inputs are fed to bzip2. Group 2 is designed to estimate the bit rate of a small program, selected from a set of students’ programming assignments, which implement PRIME (i.e., checking whether a number is prime or not). Group 2 has ten different inputs fed to the small program. In our tests, execution sequences for bzip2 were selected with length around 500,000; i.e. sequences of 500,000 assembly instructions are generated for bzip2. However, considering the smaller size of PRIME programs, execution sequences of length around 100,000 were generated.

#### 3.2.2 Experimental Setting

The procedure of our experiments is presented in the following.

First, using a gdb script, we trace the execution of a program step by step (i.e. instruction by instruction) such that a sequence of assembly instructions is generated. In group 1, 10 different files, such as PDFs, pictures and binary executables, are fed to bzip2 as inputs so that 10 execution sequences are generated. In group 2, one specific program implementation for PRIME is chosen, and ten distinct inputs are fed to this implementation to generate 10 execution traces.

Second, we use the Lempel-Ziv algorithm to compress the execution trace. Meanwhile, we look up the dictionary (at the instruction level) generated by Lempel-Ziv algorithm to compute each instruction’s compression length, i.e. instantaneous bit rate.

Third, each execution trace is equally cut into 1000 blocks, say $B_i$, $1 \leq i \leq 1000$. Each block contains a sequence of consecutive instructions. The starting location of a block $B_{i+1}$ immediately follows the ending location of the previous block $B_i$. Using the instantaneous bit rate in previous step, the average (instantaneous) bit rate of each block, (i.e., the summation of compression length of all instructions in the block divided by the number of instructions in the block), is computed. Then, using the Fourier transform and the low-pass filter, the bit rate spectrum, which is a frequency-domain signal, is obtained.

#### 3.2.3 Results

Figures 5-8 show the spectra for executions in groups 1 and 2. We present our findings from the results in the following.

1. Although it is difficult to compare bit rate signals in the time domain, it looks simpler to compare these signals in the frequency domain. For instance, it is difficult to tell how different case 3 and case 5 in Figure 5 are from their time domain bit-rate signals. However, it is easy to see the two signals show quite different spectra in the frequency domain. Also, case 3 in Figure 5 and case 6 in Figure 5 have very similar looking time domain bit-rate signals, but their spectra appear very different.

2. The spectrum reflects behavioral characteristics of a program. For example, case 3 in Figure 5, the magnitude spectrum has strong spectral peaks in the normalized intervals $[0.045,0.055]$ and $[0.09,0.1]$. What is the intuitive explanation of this observation? Since the horizontal axis denotes frequency, the large magnitudes at a specific frequency implies some periodic tendencies in the bit-rate signal. We interpret this to mean that the information quantity of the execution has some roughly periodic aspects, and hence a concentration at certain frequencies. An almost flat spectrum indicates an almost uniform distribution of information along the execution, while a spectrum with several peaks suggest an uneven distribution.
Fig. 5. Cases 1-5 of group 1

Fig. 6. Cases 6-10 of group 1
Fig. 7. Cases 1-5 of group 2

Fig. 8. Cases 6-10 of group 2
3. Bit rate signals show a dynamic coverage of the black-box. Running a set of test cases on the same black-box, we accordingly obtain a set of executions as well as their bit rate signals. How much have the set of bit rate signals “covered”? We mathematically define a bit-rate coverage as follows. Intuitively, a set of two similar bit rate signals should cover less than a set of two bit rate signals that are not so similar. In other words, one can treat a set of signals as a set of points in a metric space, and ask how much the points “span” in the space. An ideal approach would be to use the Hausdorff content of the signal set. However, since the dimension of the metric space is unknown in practice, this approach is not suitable. Another approach is to borrow the idea of Borel cover by using finitely many ε-balls, in the metric space, to cover the points. Our preliminary studies show that it would end up with an exponential time algorithm to find such a cover. Herein, we propose a very intuitive and simple bit-rate coverage (indicator). Let $T$ be a set of test cases and, for each $t \in T$, $x_{r,t}(n)$ is the bit-rate signal of the execution under test case $t$. The bit-rate coverage is defined as

$$\text{Cover}(T) = \frac{1}{2} \sum_{t_1, t_2 \in T} \|x_{r,t_1}(n) - x_{r,t_2}(n)\|^2. \tag{4}$$

For example, in group 2, 10 distinct inputs (test cases) are fed to the same program to generate 10 executions. The inputs of the first five cases are prime numbers while the inputs of the last five cases are non-prime numbers. The bit-rate coverage of the first five cases is 136466.2 while the bit-rate coverage of the last five cases is 274102. The difference suggests that, obviously, the first five test cases (all prime numbers) have covered less than the last five test cases (all non-prime numbers). In other words, trying to extensively test the black-box, one should run more non-prime inputs than prime inputs. Notice that, this conclusion is drawn without inspecting the code of the black-box. This conclusion is consistent with our reading of the code: the control flow of a prime input is much simpler than that of a non-prime input.

From the bit-rate coverage, we can also define a relative bit-rate coverage as follows:

$$\text{Cover}(t|T) = \sum_{t' \in T} \|x_{r,t'}(n) - x_{r,t}(n)\|^2,$$

where $t \notin T \neq \emptyset$. For instance, in group 1, we can calculate

$$\text{Cover}\{\text{case9}\} \{\text{case1}, ..., \text{case8}\} = 113564.3$$

and

$$\text{Cover} \{\text{case10}\} \{\text{case1}, ..., \text{case8}\} = 113938.2.$$ 

This intuitively says that, within the group, case 9 covers almost the same additional amount of information as case 10.
which is consistent with what the industry views a new testing approach. Instead, our approaches will complement them. Different testing approaches are just different angles from which a tester looks at the system under test. We believe that our angle is a new one.

In [29], Lempel-Ziv algorithms are used to detect plagiarism in programming assignments. The use of Lempel-Ziv is to approximate Kolmogorov complexity, instead of Shannon information rate. Additionally, source programs are compressed in [29] while, in our experiments, execution sequences, i.e., run-time behaviors of source programs, are compressed.

What merits can our information-theoretic approach bring to software testing? The most desirable property of Shannon entropy is that the Shannon entropy of a discrete random variable remains unchanged after a one-to-one function is applied [13]. Such a characterization is of great importance, since it suggests a way to test a software system based on its internal meanings (i.e., semantics), instead of its appearance (i.e., syntax). For instance, suppose that two distinct test sets are selected from a graph modeling the system’s control flow, both with 75%, say, branch coverage. The adequacy degree does not differentiate the two sets. Or, in other words, each branch is born equal. This is not intuitively true: buttons on an LCD television do carry different “amounts of information” (e.g., a television with a failed power-on button is more “useless” than one with a bad volume button).

For the future work, many issues can be investigated. For instance, blocks in this paper are of the same size. Averaging the bit rate over a block serves as a low pass filter and hence a larger block size tends to result in a smoother spectrum. In the future, we need a study to determine advantages and disadvantages of different block sizes. Additionally, signal analysis techniques such as correlation and linear prediction can also be used to analyze the internal structural information of a black-box. The coverage developed in this paper is based on the $\ell_2$ norm; it is also worthwhile to develop other distance measures; e.g., spectral magnitude based distances that are invariant under signal time shift.

Currently, the bit rate analysis approach can only work for evaluating the bit-rate coverage of a given set of test cases. It is not a test case generation approach. To address the issue, we will implement a testing tool whose framework is sketched in Figure 9.

The testing framework mainly consists of three main components: Specification (e.g., requirement, design, etc.; this component is optional, i.e., the specification might be not available); Test case generator (TCG) to generate test cases on the (black-box) system under test (SUT); and Test driver that analyzes and outputs results.

When the specification of the SUT is available, TCG generates test cases according to the specification.
that it needs to be considered as a black-box.

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