COMPRESSING COLOUR IMAGES WITH JOINT INPAINTING AND PREDICTION

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ABSTRACT
Inpainting-based codecs store sparse, quantised pixel data directly and decode by interpolating the discarded image parts. This interpolation can be used simultaneously for efficient coding by predicting pixel data to be stored. Such joint inpainting and prediction approaches yield good results with simple components such as regular grids and Shepard interpolation on grey value images, but they lack a dedicated mode for colour images. Therefore, we evaluate different approaches for inpainting-based colour compression. Inpainting operators are able to reconstruct a large range of colours from a small colour palette of the known pixels. We exploit this with a luma preference mode which uses higher sparsity in YCbCr colour channels than in the brightness channel. Furthermore, we propose the first full vector quantisation mode for an inpainting-based codec that stores only a small codebook of colours. Our experiments reveal that both colour extensions yield significant improvements.

Index Terms— inpainting, image compression, colour, prediction, vector quantisation

1. INTRODUCTION
In order to compress colour images, contemporary transform-based codecs have dedicated colour modes. For instance, JPEG [1] and JPEG2000 [2] use chroma subsampling in YCbCr space, guided by the idea that structural information is visually more important than colour. Discarding some of the information of the chroma channels Cb and Cr allows them to store more accurate transform coefficients for a brightness representation of the image.

Inpainting-based approaches [3, 4, 5, 6] rely on an entirely different concept. They create sparsity directly in the spatial domain. On colour images, these codecs usually rely on coarse quantisation of RGB values. Interestingly, inpainting operators tend to fill in gaps not only in the spatial domain, but also in the co-domain of colour values. Despite this property, little research has been invested into colour modes for inpainting-based codecs. In particular, the potential of techniques that create sparse colour palettes such as vector quantisation has remained unexplored so far.

1.1. Our Contribution
In the present paper, we introduce and evaluate different concepts for inpainting-based image compression. To this end, we propose two new colour modes for the recent RJIP codec which performs regular grid coding with joint inpainting and prediction [5].

Our first mode adapts a state-of-the-art colourisation-based concept [7] in YCbCr mode to the RJIP setting: We dedicate a higher budget to the image structure than to colour and augment it with efficient post-processing specifically tailored to fast Shepard interpolation [8]. Additionally, we propose the first full vector quantisation mode for inpainting-based compression. Our evaluation on the Kodak database [9] reveals that both colour modes outperform the original RGB mode significantly.

1.2. Related Work
While there are various inpainting-based codecs that can compress colour images [10, 3, 4, 5, 6], the only dedicated colour mode so far is the luma preference (LP) mode for the R-EED codec [7]. It relies on the core idea to dedicate a higher budget to the luma channel of YCbCr space than to the colour channels. We adapt this concept to the setting of RJIP. In contrast to our approach, R-EED relies on a more complex inpainting method [11] and a tree-based subdivision scheme to select and store positions of known data [6]. In a broader sense, the LP mode resembles the chroma subsampling of JPEG [1].

Our second colour mode uses vector quantisation, a concept that was already described by Shannon [12] in his influential early work on information theory. Due to the large amount of research activity in the early 80s and 90s for compression of both visual and audio data, a full review is beyond the scope of this work. We refer the reader to the comprehensive monograph of Gersho and Grey [13] instead.

More recent works that deal with lossy compression and vector quantisation are rare. Venkateswaren and Ramana Rao [14] quantise wavelet coefficients from different sub-bands with vector clustering, while Somasundaram and Rani [15] focus solely on more efficient vector quantisation with a modified k-means clustering. For compression with neural networks, vector quantisation is enjoying increasing popularity [16]. However, there is it not applied to colour values, but more generically to image features or network parameters to be stored. Zhou et al. [17] combine vector quantisation with inpainting, but in contrast to our work, they quantise blocks of grey value data instead of individual colour values.

Even though they do not deal with colour, the work of Hoeltgen et al. [18] comes close in spirit to our own research. They assess how clustering techniques affect inpainting-based reconstruction from sparse data. However, they only use scalar quantisation. Their work confirms, together with the findings of Celebi [19], that the k-means clustering algorithm by Lloyd [20] is one of the best clustering techniques for quantisation. Consequentially, we rely on k-means clustering for our vector mode. We will discuss these methods in more detail in later sections.

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1.3. Organisation of the Paper

In Section 2, we give a short review of the RJIP codec. Based on this foundation, we propose two novel colour extensions: a luma preference mode in Section 3, and a vector mode in Section 4. Finally, we evaluate these new approaches in Section 5 and conclude our paper with a summary and an outlook on future work in Section 6.

2. REVIEW: INPAINTING-BASED COMPRESSION WITH SCALAR QUANTISATION

2.1. Inpainting

At their core, all inpainting-based image compression codecs rely on interpolation from sparse image data. For a typical inpainting problem in RGB space, the colour image $f : \Omega \rightarrow \mathbb{R}^3$ is only known at a few locations, the inpainting mask $K \subseteq \Omega$. The missing parts of the image domain $\Omega$ need to be reconstructed during decoding.

For the Shepard interpolation in RJIP [5], computing an unknown pixel value $u_c(x_i)$, $x_i \in \Omega \setminus K$ for each channel $c \in \{R, G, B\}$ comes down to a simple weighted averaging of the known data according to

$$u_c(x_i) = \frac{\sum_{x_j \in K} w(x_j - x_i) f_c(x_j)}{\sum_{x_j \in K} w(x_j - x_i)}.$$  

(1)

Following Achanta et al. [21], we use a truncated Gaussian $w$ with standard deviation $\sigma = \sqrt{(m \cdot n)/(\pi |K|)}$ for a discrete $m \times n$ image where $|K|$ is the number of mask pixels.

2.2. Data Selection and Storage

Each inpainting-based codec requires an adequate strategy to select and encode the inpainting mask. RJIP stores the known data on a regular grid with grid size parameter $h$. This approach is fast and generates little overhead, but results in many colour values that need to be stored. RJIP compensates for this fact by joint inpainting and prediction: While decompressing the image, it performs a partial inpainting whenever a new mask point is decompressed and uses this to predict the remaining mask points. Thus, only prediction errors need to be stored.

For the colour data corresponding to the mask positions, existing inpainting-based codecs use uniform scalar quantisation in each channel: They map the 8 bit colour values to a reduced range $[0, \ldots, q - 1]$ by partitioning the tonal (i.e. colour value) domain into $q$ subintervals of equal length, limiting the amount of different colours for the known data to $q^3$. These quantised values are then stored with a suitable entropy encoder. RJIP relies on finite state encoding (FSE) [22], a fast coder similar to arithmetic coding.

For a given compression ratio, RJIP chooses the parameters $h$ and $q$ such that the best reconstruction quality for the desired file size is obtained.

2.3. Tonal Optimisation

RJIP benefits from tonal optimisation in post-processing. For an image with $|K|$ mask points, it performs iterative random walks over all $3|K|$ R, G, or B values, adjusting them to a higher or lower quantisation level in case this yields a lower inpainting error. Even though this introduces a bias to the sparse stored data, it can increase the overall reconstruction quality in the large unknown areas significantly. We need to adapt this step for vector quantisation.

3. RJIP WITH LUMA PREFERENCE MODE

3.1. File Size Budget Distribution

As a new colour mode for RJIP we consider a luma preference mode in the YCbCr colour space. This technique is an application of image colourisation that has been successfully used in the R-EED codec [7]. Its core idea is to store more data for the luma channel $Y$ to reconstruct the image colour accurately and fill in the colour information in the $Cb$ and $Cr$ channels from very sparse masks.

To this end, we distribute the total file size budget $B$ between $Y$ and $CbCr$ according to a new parameter, the luma factor $f$ such that $B_y = f \cdot B_{CbCr}$. The compression pipeline itself remains the same as in Section 2. On a regular mask, we employ uniform quantisation, joint inpainting and prediction, and FSE encoding. However, we use a separate mask for the $Y$ channel and a joint mask for the $Cb$ and $Cr$ channels. The respective grid sizes $h$ and quantisation parameters $q$ are chosen such that the budget constraints are fulfilled. Afterwards, we perform tonal optimisation for all channels.

3.2. Tonal Optimisation

For tonal optimisation, we do not use the random walk trial-and-error approach of RJIP described in Section 2. Instead, we propose a direct, more efficient algorithm. We exploit that the truncated weights in Eq. (1) limit the influence of each pixel to a local area. In the following, given a current pixel value $u_i^{new}$ at $x_i \in K$, we want to find $u_i^{old}$, such that it minimises the mean squared error (MSE).

To shorten notation, we define the value accumulation map $v$ as the numerator of Eq. (1) and write $w_{i,j}$ for $w(x_j - x_i)$ for weights at positions $x_j$ from a neighbourhood $N_i$ relative to its centre $x_i$. Then the new error after changing $u_i^{old}$ to $u_i^{new}$ is given by

$$e(u_i^{new}) = \sum_{x_j \in N_i} \left( f_j = \frac{w_j + w_{i,j} (u_i^{new} - u_i^{old})}{w_j} \right)^2,$$

(2)

where

$$w_j = \sum_{x_i \in N_i} w_{i,j},$$

Since the optimal new tonal value $u_i^{new}$ should minimise $e(\cdot)$, we solve $\frac{d}{du_i} e(u_i^{new}) = 0$ for $u_i^{new}$ and obtain

$$u_i^{new} = \frac{\sum_{x_j \in N_i} \frac{G_{i,j}}{w_j} \left( f_j = \frac{v_j - G_{i,j} u_i^{old}}{w_j} \right)}{\sum_{x_j \in N_i} \frac{G_{i,j}}{w_j}}.$$  

(3)

Instead of testing neighbouring quantisation levels, we can now compute the locally optimal value directly. However, note that we need to project these unconstrained solutions onto the set of admissible quantisation values after each computation. Iterating these steps eventually converges to a fully optimised inpainting mask.

4. RJIP WITH VECTOR QUANTISATION

As a second alternative to the LP mode from Section 2 we propose the first full vector quantisation mode for inpainting-based colour image compression. In the following we describe the corresponding modifications to the compression pipeline of RJIP.
Fig. 4. Colour histograms for the original image *kodim07* [9] and the reconstructed image with Shepard inpainting. The histograms (created with Colour Inspector 3D [23]) show each bin as a ball with radius proportional to the number of contained colours. Inpainting can reconstruct a good approximation to the original histogram from only a few quantised colours present in the inpainting mask. Here, the image is compressed to a ratio of 50:1 and the mask has only 92 different colours.

### 4.1. Clustering with k-means

Experimental evaluations have shown that the simple k-means algorithm for clustering by Lloyd [20] is still one of the most successful techniques for vector quantisation [19]. Given an initial set of colour vectors \( v_1, ..., v_n \), k-means clustering aims to partition \( V \) into \( k \) disjoint clusters \( C_1, ..., C_k \subset V \) such that they minimise the cumulative squared Euclidean distance of the points in each cluster \( C_\ell \) to the corresponding cluster centre \( \mu_\ell \).

To achieve this goal, the k-means algorithm randomly selects \( k \) cluster centres.

Alternating assignment and update steps refine this initialisation iteratively. The assignment step maps all colours to the cluster with the nearest mean value. In the subsequent update, the mean values are set to the centroids of the newly assigned clusters. Note that there are many more sophisticated initialisation strategies than the random one, e.g. k-means++ [24], histogram-based approaches [25, 26, 27, 28, 29], and iterative subdivision methods [30]. Even though the initialisation method impacts the quantisation error, we experienced empirically that it does not impact the final compression performance significantly. The tonal and cluster post-processing steps described in the following paragraphs ensure that we achieve good final results regardless of the initialisation.

A representative histogram of an image after vector quantisation can be seen in Fig. 1(c). The same figure also reveals that inpainting can approximate such a histogram well from data that is simultaneously sparse in the spatial domain and the colour space after coarse vector quantisation.

### 4.2. Cluster Post-Processing

For tonal optimisation, we use our improved method from Section 4.2 to find optimal unconstrained values. After each iteration, we project them onto the closest cluster centre w.r.t. Euclidean distance. However, for vector quantisation we consider an additional post-processing step.

Just as the original colour values do not necessarily yield the best inpainting result, the k-means clusters are not necessarily optimal w.r.t. reconstruction quality. As another post-processing step, we can check if moving a cluster centre to a neighbouring vector from \( \{0, ..., 256\}^3 \) decreases the MSE and each such global change affects all known pixels with the same label. Thus, we optimise the quantisation codebook for final reconstruction quality and can simultaneously compensate for suboptimal initialisations of the k-means algorithm. Even though the overall quantitative gain is negligible for lower compression ratios, it becomes significant for higher compression ratios.

### 4.3. Storing Colour Palettes and Entropy Coding

In contrast to uniform scalar quantisation, we need to store the colour palette used by the encoder. In the header we first save the number \( q \leq 256 \) of quantised colours with 1 byte and the cluster centres themselves with 1 byte per channel. The position of known data points is stored as in standard RJIP, but their value is now represented by a cluster centre index \((0, ..., q − 1)\) from the stored codebook.

Finally, we apply entropy coding. Unfortunately, the joint prediction and inpainting from Section 4 yields unsatisfactory results in the vector case. While inpainting can still predict the colour value of neighbouring known data, the codebook labels are not tied to the distance between vectors in the colour space. Thus, an accurate prediction of the colour itself can still yield a high error in the label prediction. Therefore, we replace this step with Prediction by Partial Matching (PPM) [31].

The original version of PPM uses the linear sequence of previously encoded symbols contexts for deriving conditional probabilities for entropy coding. However, this approach discards the 2-D relations of image pixels. Instead, we iterate over all mask pixel and use their three direct neighbours that have already been encoded as contexts for deriving conditional probabilities. Therefore, we replace this step with prediction of the colour itself can still yield a high error in the label prediction.

5. EXPERIMENTS

On the well-known Kodak image database [9], we compare the RGB scalar mode of the original RJIP [5] to our new colour modes: RGB with vector quantisation and YCbCr luma preference mode (LP) with scalar quantisation. We use the MSE over all RGB channels as our error measure. For LP mode, we choose the luma factor \( f \in \{0.5, 0.6, 0.7, 0.8, 0.9\} \) that minimises the MSE.

Fig. 5 shows that both our new colour modes for RJIP consistently outperform the scalar RGB mode by a large margin. This also results in a significant improvement of visual quality, which is illustrated by Fig. 2 and Fig. 3. On average over the whole database, both colour modes yield a similar error. However, a more detailed analysis shows that both modes have distinct advantages on different types of image content.
Fig. 2. RJIP compression results (RGB-MSE) for *kodim20* (768 × 512 pixels) with a compression ratio of 20:1. Both our colour modes outperform RGB RJIP significantly. On images with low and moderate amount of texture, vector quantisation also outperforms LP mode considerably.

Fig. 3. RJIP compression results (RGB-MSE) for *kodim13* (768 × 512 pixels) with a compression ratio of 50:1. Both our colour modes outperform RGB RJIP significantly. On images with a high amount of texture, LP mode outperforms vector quantisation considerably.

Fig. 4. On the full Kodak database, both our new colour modes for RJIP outperform RGB scalar RJIP consistently and yield a similar error on average.

For images with low amounts of texture, vector quantisation is the better choice and can reduce the MSE by up to 54% compared to scalar LP as shown in Fig. 2. This yields a noticeable increase in visual quality due to higher RGB mask densities. However, on heavily textured images such as the one in Fig. 3, LP mode yields better results. Here, more data in the luma channel allows a more accurate reconstruction of the image structure.

In general, vector quantisation is computationally faster than the LP mode. Even though vector quantisation is more complex than uniform scalar quantisation due to k-means clustering, it does not require to optimise the luma factor \( f \). This yields a runtime reduction of up to 50%.

6. CONCLUSION

Our two new colour modes for RJIP offer a significant visual and quantitative improvement over the standard RGB mode. Our evaluation of the vector quantisation mode reveals that inpainting-based image compression can reconstruct a wide range of colours from a sparse codebook. On highly textured images, a scalar luma preference mode can reproduce the image structure more accurately. In future research we want to investigate our colour modes on data with many channels, e.g. hyperspectral imaging [33].

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