One of the tasks that imply increasing the milk productivity of cows is to create optimal maintenance conditions that ensure the increased use of the genetic potential of cattle based on the implementation of engineering and technological solutions.

A mathematical model has been built that links the technical and technological parameters of the vacuum system of milking equipment, namely, the value of the working vacuum P, the pulsation frequency n, the ratio of pulsation cycles, and the tension strength of milking rubber F_t to cows’ milk yield rate V. The range of milking plant operating parameters for milking in the milk line has been determined, at which the milk yield rate is maximum: P = 52 kPa, n = 55.6–38.8 min⁻¹, δ = 0.59–0.64, F_t = 59.3–60.4 H. Under these parameters, the milk yield rate is V = 1.48–1.53 l/min.

The results of the multifactor experiment have helped construct an adequate mathematical model of the second order, which confirms the theoretical dependence of the influence of the technical and technological parameters of the vacuum system of milking equipment on milk yield rate and the airflow of the milking machine. Analysis of the mathematical model has made it possible to establish the rational structural and technological parameters for the vacuum system of a milking machine: the value of the working vacuum, P = 50.6 kPa; pulsation frequency, n = 55.9 min⁻¹; the ratio of pulsation cycles and the tension force of milking rubber F_t = 64.8 H. Under these parameters, the milk yield rate is maximum: V = 1.47–1.52 l/min; the airflow consumption of the milking machine is Q = 2.49 m³/h. The mathematical model built fully reveals the influence of technical and technological parameters of milking equipment on the efficiency of machine milking. Owing to this, the issue related to the rational choice of equipment is resolved.

Keywords: milking equipment, milking machine, teat rubber, vacuum system, milk yield rate

1. Introduction

Machine milking of cows is a technological process in which a controlling element (milking machine) interacts with the animal’s body. This interaction (milking) occurs 2–4 times a day for 5–6 minutes over an extended period [1].

Machine milking is inferior to manual in terms of efficiency but it facilitates the work of operators and improves...
productivity. Along with this, such milking of cows makes it possible to obtain clean, good-quality milk as it does not come into contact with the external environment [2–4].

In [5, 6], it was established that the effectiveness and completeness of milk yield when milking cows depends not only on the reflex activity of the animal’s body but also on the technical characteristics of the milking equipment. Deviation from norms and violation of the rules of operation of milking equipment lead to a disruption in the functioning of the mammary gland of cattle.

In this regard, it is a relevant task to investigate conceptual approaches to the issue of dairy cattle breeding. Its relevance is due to the trends in the development of milk production and the course of technological processes; necessitating the development of new methodological techniques for defining the system of indicators and recommendations in the areas of resource potential selection.

Thus, the current study is necessary to determine the influence of technical and technological parameters of milking equipment on the efficiency of machine milking.

This approach could make it possible to eliminate various violations in the management of machine milking and obtaining high yields. That would improve the productivity of milking equipment and the quality of the products obtained. Along with this, that could make it possible to reveal the mechanism of interaction between the milking plant and an animal, which is both of theoretical and practical interest.

2. Literature review and problem statement

As noted in work [7], the productivity of labor and maintaining the productivity of cows at the optimal level during lactation depend on the milking equipment used. Study [8] reports on the complexity of the machine milking process given the fact that the effectiveness and completeness of milk release depend on the reflex activity of the body. Paper [9] proved that the efficiency of milk yield depends on the technical characteristics of milking equipment, that is, technical and technological parameters.

Work [10] argues that the productivity of the cow and the quality of milk depend on many factors, including the quality and completeness of milk release. That, in turn, directly depends on the applied milking equipment. In [11], it is noted that the ideal system of milk release is the natural biological system «cow-calf». However, its main «drawback» is the rapid saturation of the calf, which does not contribute to the growth of the productivity of the cow. Thus, a certain increase in animal productivity is ensured by the use of milking equipment [12]. It should be noted that one of the most important reserves for increasing the milk productivity of cows is the use of milking machines with the best and optimal technical characteristics for a given group of cows. That is such devices that can support the reflex of milk release during milking at a fairly high level.

The effectiveness of machine milking is influenced by a variety of factors that can be divided into the following groups:

– the technical characteristics of milking machines;
– the technological indicators of the process;
– the breeding and genetic indicators of the udder and milk yield.

A significant influence on the process of machine milking is exerted by the technical and technological parameters of milking plants. They change widely in the cases of various violations of the technical condition of the equipment [13, 14]. Violations and drastic changes in the technical parameters of milking plants during the milking process significantly affect the speed of milk yield and increase the time of cattle milking, which is emphasized in work [15].

Study [16] proved that systematically diagnosing the technical condition of milking systems, as well as the timely and high-quality elimination of detected defects, could reduce the disease of cows with mastitis, increase the productivity of cattle by 3–5 %. Other researchers hold the same opinion [17].

According to the authors of work [18], the key role in the efficiency of milking belongs to fluctuations in a working vacuum under the milking of the udder. The reasons, as noted in [19], are the insufficient intersection of vacuum pipelines, synchronous operation of pulsators, and the low performance of vacuum pumps.

At the same time, milking cows under a stable working vacuum contributes to creating a stable milk reflex in cows.

Vacuum is a pressure drop, acting on the teat sphincter on both sides: excessive milk pressure from the inside and vacuum outside in the milking cup space under the teat. The higher this difference, the faster, under other equal conditions, milk is released from the udder. At the same time, the higher the vacuum under the teat, the greater the injury effect of vacuum on the udder tissues. Along with this, there is a greater likelihood of irritation of the mammary gland and disease of cows for mastitis. As practice has shown, the optimal vacuum value for different devices is in the range of 45–53 kPa [20].

Paper [21] states that there are milking machines that copy the act of sucking or manual milking. However, many years of practice have shown that for the introduction of effective machine milking, mechanical copying of the main parameters of the act of sucking and manual milking [22] was insufficient.

One of the main indicators that characterize the operation of milking machines is the pulsation frequency, the ratio of cycles.

The pulsator performs the function of the regulatory part of the milking machine, which sets the mode of its operation. This is a complex pneumatic system consisting of a series of chambers, the volume and pressure in which vary significantly, and throttle tubes of different lengths and diameters connecting those chambers. The calculation of such a system comes down to determining the time of ending and filling the control chamber with air [23].

The main working body interacting with the cow’s rubber during machine milking is teat rubber [24]. Paper [25] emphasizes the need for milking machines teat rubber to correspond to one group. However, the issues related to the establishment of the effects of rubber on the process of milk production [26] remained unresolved. The reason may be objective difficulties associated with the fact that biological objects that are extremely varied are studied.

Most milking machines operate at liquefaction of 42–53 kPa, but, in some designs, this range is much wider (33.3–91.3 kPa). To carry out effective milking, it is necessary to maintain stable working liquefaction. Its instability leads to a violation of the stereotype of milking, deterioration of the milk release reflex, an increase in the time spent on cattle milking, a decrease in productivity [27].

Study [28] found that fluctuations in the working rarefaction in a vacuum system lead to mastitis disease in up to 32 % of cows. Along with this, there is an irritation of the
mammary gland by 23–30 %, a 23 % decrease in milk productivity, and a 25 % reduction in the lactation period.

However, the cited works do not fully reveal the impact of the technical and technological parameters of milking equipment on the efficiency of machine milking and, accordingly, ways to address those issues.

Thus, as stated in [29], an option to overcome the corresponding difficulties in establishing the interaction of milking equipment with cattle is to conduct appropriate research.

In order to eliminate various violations in the management of machine milking and to obtain high yields, it is necessary to establish the effect of technical and technological parameters of milking equipment on the efficiency of machine milking [30].

Thus, resolving the issue related to improving the efficiency of machine milking requires studying, revising, and improving key provisions and elements in the system of technological measures and technical means, which is of both scientific and practical interest.

3. The aim and objectives of the study

The purpose of this study is to establish the influence of technical and technological parameters of milking equipment with upper milking line and milking machines of simultaneous action on the efficiency of machine milking. Results to be obtained could make it possible to link the technical and technological parameters of the vacuum system of milking equipment with the upper milking line and milking machines of simultaneous action to the milk yield rate, which would ensure an effective milking process.

To accomplish the aim, the following tasks have been set:
– to conduct a theoretical study into the influence of technical and technological parameters of milking equipment with upper milking line and milking machines of simultaneous action on the efficiency of machine milking;
– to experimentally investigate the impact of technical and technological parameters of the vacuum system of milking equipment with upper milking line and milking machines of simultaneous action to the milk yield rate, which would ensure an effective milking process;
– to establish the adequacy of the mathematical model to the derived patterns.

4. The study materials and methods

Experimental studies were carried out at TOV «Dolynska», Berdiansk region, Zaporizhzhia oblast (Ukraine) using the UDM-type milking plant (Fig. 1), which complied with the requirements of ISO 5707. The plant was equipped with terminals for connecting a set of instrument equipment to determine its technical and technological parameters in accordance with ISO 3918.

To measure the milk yield rate, an individual milk meter produced by VAT Bratslav (Ukraine) was used. To measure the technical and technological parameters of milking equipment, a calibrated set of instrument equipment that meets the requirements of ISO 6690 was used. The tension force of the teat rubber was determined using the PCE FG 200 dynamometer (an error of 0.01 N) according to GOST 270-75.

The duration of milking was measured by a stopwatch.

To investigate the impact of the technical parameters of the vacuum system of milking equipment on milk yield rate, a group of healthy animals in the amount of 10 heads was chosen. Animals were of the same breed and age, the diet and motion over the study period were the same.

Our study into determining the dependences of technical parameters of the vacuum system of milking equipment on milk yield rate was carried out using the method of mathematical planning of a multifactor experiment. The method makes it possible to build the following second-order regression equation [31]:

\[ y = a_0 + \sum_{i=1}^{k} a_i x_i + \sum_{i=1}^{k} a_{ij} x_i x_j + \sum_{i=1}^{k} a_{i} x_i^2, \]

where \( a_{0}, a_{i}, a_{ij}, a_{ii} \) are the regression coefficients; \( x_i, x_j \) are the independent variable factors.

To construct regression equations, it is necessary to encode research factors using the following transformation:

\[ X_i = \frac{A_i - A_{ii}}{\varepsilon}, \]

where \( X_i \) is the value of the factor in an encoded form (it takes the value from –1 to +1), \( A_i \) is the natural value of the factor, \( A_{ii} \) is the natural value of the factor corresponding to the encoded value of 0, \( \varepsilon \) is the natural value of the variation interval of the factor.

Our studies employed the following factors: the working pressure of a vacuum system \( P \) (48–52 kPa), pulsation frequency \( n \) (50–65 min \(^{-1}\)), the ratio of pulsation cycles \( \delta \) (0.5–0.7), the tension force of teat rubber \( F_{t} \) (50–70 N). The operating pressure of the vacuum system was determined using a vacuum regulator. The ratio of pulsation cycles and the pulsation frequency were adjusted using the pulsator throttle channel screw. The tension force of the teat rubber was adjusted according to the level of the scaling belt.

The criteria for assessing our study were a milk yield rate \( V \), l/min; and air consumption of the milking machine \( Q \), m\(^3\)/h.

The milk yield rate \( V \) was calculated according to the following formula:

\[ V = \frac{q}{t}, \]

where \( q \) is the amount of milk obtained during milking, l; \( t \) is the milking time, min.

The research was carried out based on the Box-Benkin orthogonal plan of the second order for 4 factors (BB\(_4\)). The total number of experiments was 27.
The study results were processed using the software «Wolfram Mathematica». Regression equations were determined for each of the optimization criteria. Regression coefficients were determined using generally accepted formulas for D-optimal research plans [32]. The mathematical model in an encoded form was built on the basis of the derived regression coefficients.

The levels of factors variance were chosen based on the analysis of the effect of failures of the vacuum system of milking equipment on the technological process of machine milking.

The reproducibility of the experiments (10 repetitions) was tested using the Cochrane criterion, which was calculated according to the ratio of maximum variance to the sum of all variances. The calculated value of Cochrane’s criterion was compared to a tabular value [33]. In this case, the variance is homogeneous under the condition \( \sigma_x < \sigma_{table} \). In the case of \( \sigma_x > \sigma_{table} \), the experiment is not accurate and requires smaller intervals in the variation of factors, an increase in the number of repetitions, and an increase in the accuracy of the measuring equipment.

A Student criterion was used to calculate the confidence boundaries of the random error of measurement results. Comparing the calculated value of the Student criterion with a tabular value makes it possible to determine the significance of regression coefficients.

The hypothesis on the adequacy of the established models was tested using the Fisher criterion [32].

Regression equations [33] produce optimal parameters for research factors when determining the extremes of the corresponding criteria, which was carried out by the method of canonical transformation of the mathematical model.

Based on the derived regression equations, we used the software package «Wolfram Mathematica» to construct the response surfaces at the fixed values of research factors.

In the case of differences in the optimal values of research factors for two or more criteria, a compromise problem was solved. Its essence is to analyze the ranges of optimal values of factors, taking into consideration the most significant optimization criteria. Mathematically, this problem is solved by solving a system of corresponding equations.

### 5. Results of establishing the influence of technical and technological parameters of milking equipment on the efficiency of machine milking

#### 5.1. A theoretical study into the influence of technical and technological parameters of milking equipment on the efficiency of machine milking

The volumetric speed of milk release from the cow’s udder, \( V \), when using a milking machine, is represented in the form of the following dependence:

\[
V = q \cdot n \cdot N_c, \tag{4}
\]

where \( V \) is the volumetric speed of milk release, \( m^3/s \), \( q \) is the milk release through sphincters, \( m^3/cycle \), \( n \) is the pulsation rate of the milking machine, cycle/s, \( N_c \) is the number of yields.

A volumetric speed of milk release is a function that depends on the structural and technological parameters of the biotechnical system «animal – milking plant». The study of dependence (4) in the disclosed form makes it possible to determine the nature and significance of the influence of the structural and technological parameters of the milking unit on the efficiency of milk production.

For further theoretical research, accept the following assumptions:

- pressure in the volume of the udder during the milking period is a constant value;
- the volume of the teat and sphincter in the open state takes the shape of a cylinder.

For further calculations, we adopt the following designations (Fig. 2):

- \( P_0 \) is the milk pressure in the volume of the udder, Pa;
- \( P \) is the pressure in the under-teat chamber, Pa;
- \( d \) is the sphincter diameter, m;
- \( v \) is the milk speed in a teat channel, m/s;
- \( L \) is the length of the teat, m;
- \( Z_0, Z \) are the geometric heads, m.

![Fig. 2. Scheme of interaction between the teat rubber and teat](image)

According to Fig. 2, a Bernoulli equation for the cross-section 0-0 takes the following form:

\[
Z_0 + \frac{P_0}{\rho g} = Z + \frac{P}{\rho g} + \frac{v^2}{2g}, \tag{5}
\]

where \( \rho \) is the milk density, kg/m\(^3\), \( g \) is the acceleration of free fall, m/s\(^2\).

In equation (5), move the known parameters to the left-hand part, and the unknown ones – to the right-hand part of the following equality:

\[
Z_0 - Z = \frac{P_0}{\rho g} - \frac{P}{\rho g} = \frac{v^2}{2g}, \tag{6}
\]

since \( Z_0 - Z = L \), we have:

\[
\frac{P_0 - P}{\rho g} + L = \frac{v^2}{g}. \tag{7}
\]

Thus, we have found the flow rate of milk from the resulting equation (7) in the form of a function of the diameter of
the sphincter $d$ and the maximum milk flow rate $Q_{\text{max}}$. The average milk flow rate will be:

$$v = \frac{4Q_{\text{max}}}{\pi d^2},$$

(8)

By substituting the value of (8) in equation (7) and, upon transforming, we have:

$$Q_{\text{max}} = \frac{\pi d^2}{4} \sqrt{\frac{P_b - P + pgL}{\rho}}.$$  

(9)

Assume the milk flow rate $Q$ through a sphincter per a single cycle of sucking varies according to the parabolic curve depending on time (Fig. 3):

$$Q = at^2 + bt + c,$$  

(10)

where $a$, $b$, $c$ are the coefficients.

![Fig. 3. Dependence plot of the milk flow rate through a teat’s sphincters during the sucking time cycle](image)

To determine the coefficients in equation (10), we use the initial conditions:

$$t = 0 \Rightarrow Q = 0 \Rightarrow c = 0,$$

$$t = t_0 = \frac{t_c}{2} \Rightarrow Q = \frac{at_c^2}{2} + b\frac{t_c}{2} = Q_{\text{max}},$$

$$t = t_d \Rightarrow Q = at_d^2 + bt_d = 0,$$

where $t_d$ is the maximum sucking time cycle when the process of milk release stops, $s$; $t_0$ is the time of the greatest milk release through a sphincter, $s$.

The joint solution of the above equations makes it possible to determine their coefficients. Substituting them into equation (10), we obtain:

$$Q = -4\frac{Q_{\text{max}}}{t_d} t^2 + 4\frac{Q_{\text{max}}}{t_d} t = 4\frac{Q_{\text{max}}}{t_d} t \cdot (t_d - t^2).$$

(11)

Derive the integral in equation (11) for time and determine the total amount of milk released in one sucking cycle:

$$q = \int_0^t Q dt = \frac{4Q_{\text{max}}}{t_d^3} \left[ t \cdot \frac{t^2}{2} \right]_0 =$$

$$= 4\frac{Q_{\text{max}}}{t_d^3} \left( \frac{1}{2} t_c^2 t_d - \frac{1}{3} t_0^3 \right),$$

(12)

Taking into consideration the determined ratio of cycles $\delta$ and pulsation frequency $n$:

$$\delta = \frac{t_c}{t_p}, \quad n = \frac{1}{t_p + t_c}.$$  

(13)

Substituting (13) in (12), we obtain:

$$q = 4\frac{Q_{\text{max}}}{t_d^3} \delta^2 \left( \frac{1}{2} t_0^2 - \frac{1}{3} \frac{\delta}{n(1+\delta)} \right).$$  

(14)

Substituting the value of (9) in equation (14), we finally obtain:

$$q = \frac{4\pi d^2}{t_d^3} \sqrt{\frac{P_b - P + pgL}{\rho}} \delta^2 \left( \frac{1}{2} t_0^2 - \frac{1}{3} \frac{\delta}{n(1+\delta)} \right).$$

(15)

Consider the balance of forces of the ring of teat rubber acting on the teat. From one side, there is force $F = P_d D_L$. From the other side, the circumferential force in the ring, arising from the tension of the teat rubber $F = \sigma_r (D_2 - \Delta D - D_1) L$. The average length of the cross-section of the teat rubber before tension is $C_b = \pi/2(D_1 + D_2)$, and, after the tension, $C_i = \pi/2(D_2 - \Delta D + D_1)$. According to the Hook law for teat rubber:

$$\sigma_r = E_i \frac{C_0 - C_1}{C_0} = E_i \frac{\Delta D}{D_1 + D_1}.$$  

Regrouping the aforementioned identities, we finally have:

$$P = \frac{E_i \Delta D}{D_1 + D_1}.$$  

(16)

where $\Delta D$ is the absolute deformation of the teat rubber, $m$; $E_i$ is Young’s modulus of teat rubber; $P_b$, $D_2$, $D_1$ are the outer and inner diameters of the teat rubber, respectively, $m$.

On the other hand, based on the equilibrium equation, the projections of all forces acting on the ring take the form $P_d L + F = P d L$. The circumferential force in the teat is $F = \sigma_r (d_i - d) L$. According to the Hook law for a teat:

$$\sigma_r = E_i \frac{d_i}{d}.$$  

By regrouping the aforementioned identities, we finally have:

$$P = \frac{P d - E_i \frac{d_i}{d} (d_i - d)}{d_i},$$

(17)

where $d_i$ is the teat diameter, $m$; $E_i$ is the teat’s Young’s module, Pa.
The absolute deformation of teat rubber $\Delta D$ can be represented as follows:

$$\Delta D = \varepsilon \cdot D_2 = \mu \frac{F \cdot D_2}{E \cdot \frac{1}{4}(D_2^4 - D_1^4)},$$  \hspace{1cm} (18)

where $\varepsilon$ is the relative deformation, $\mu$ is the Poisson coefficient, $F$ is the teat rubber tension force, $N$.

Taking into consideration equations (16) to (18), we have an expression for calculating the inner diameter of the teat sphincter:

$$d = \frac{(E - P)d_1}{2E_c} - d_1 \sqrt{\frac{(P - E)^2}{4E^2} + \frac{4\mu F \cdot D_2}{\pi D_1 E_c (D_1 + D_2)(D_2^4 - D_1^4)}} \left(\frac{D_2 - D_1}{\frac{4\mu F \cdot D_2}{\pi D_1 E_c (D_2^4 - D_1^4)}} - \frac{D_2 - D_1}{\frac{4\mu F \cdot D_2}{\pi D_1 E_c (D_2^4 - D_1^4)}}\right).$$

Substituting the values of equations (15) in equation (4), we finally receive an expression for a milk yield rate:

$$V = N \cdot \frac{\pi}{2L} \left(\frac{P_b - P + \rho g L}{\rho n(1+\delta)} \left[\frac{1}{2} \delta + \frac{1}{3} \delta \right] \right)^2 \times \sqrt{\frac{(E - P)d_1}{2E_c} - d_1 \sqrt{\frac{(P - E)^2}{4E^2} + \frac{4\mu F \cdot D_2}{\pi D_1 E_c (D_1 + D_2)(D_2^4 - D_1^4)}} \left(\frac{D_2 - D_1}{\frac{4\mu F \cdot D_2}{\pi D_1 E_c (D_2^4 - D_1^4)}} - \frac{D_2 - D_1}{\frac{4\mu F \cdot D_2}{\pi D_1 E_c (D_2^4 - D_1^4)}}\right)}.$$  \hspace{1cm} (19)

Thus, the milk yield rate depends on the value of a working vacuum, the ratio of the cycles, the number of pulsations of the milking machine, and the tension force of teat rubber.

Substituting into the resulting theoretical formula (20) the empirical coefficients [34–36] and the design parameters of the milking equipment, we determine the range of operating parameters for milking equipment, at which the milk yield rate is maximum: $P = 52$ kPa, $n = 57.6–58.8$ min$^{-1}$, $\delta = 0.59–0.64$, $F_H = 59.3–60.4$ N. Plots of the effect of each parameter on the milk yield rate at the fixed values of other parameters are shown in Fig. 4.

$$N = 4,$$

$$P_b = 7−10$$ kPa,

$$L = 0.069−0.072$$ m,

$$\rho = 1.027$$ kg/m$^3$,

$$g = 9.8$$ N/kg,

$$t_d = 0.39−0.46$$ s,

$$D_1 = 0.020−0.025$$ m,

$$D_2 = 0.035−0.041$$ m,

$$E_s = 18−22$$ kPa,

$$E = 320−350$$ kPa,

$$\mu = 0.75.$$

One can see (Fig. 4) the dependence of milk yield rate on the value of a working vacuum, the ratio of the cycles, the number of pulsations of the milking machine, and the tension force of teat rubber.

![Fig. 4. The theoretical dependence of milk yield rate $V$: $a$ - on a working vacuum $P$; $b$ - on the pulsation rate $n$; $c$ - on the force of tension of teat rubber $F_H$; $d$ - on the ratio of cycles $\delta$](image)

![Fig. 4. The theoretical dependence of milk yield rate $V$: $a$ - on a working vacuum $P$; $b$ - on the pulsation rate $n$; $c$ - on the force of tension of teat rubber $F_H$; $d$ - on the ratio of cycles $\delta$](image)
5.2. Results of the experimental study into the influence of the technical and technological parameters of a vacuum system on the efficiency of machine milking

Our experimental study was carried out according to the Box-Behnken plan of the second order for 4 factors. At the same time, we employed the mathematical apparatus for planning a multifactor experiment according to the D-optimal plan. The following factors of research were chosen: the working pressure of a vacuum system ($x_1$), frequency ($x_2$), and the ratio of pulsations of the milking machine ($x_3$), as well as the tension force of teat rubber ($x_4$). The optimization criteria are the milk yield rate $y_1$ and the air consumption of the milking machine $y_2$.

Based on the results of our theoretical study, a mathematical model of the influence of the investigated factors on the effectiveness of machine milking [37] was built.

The influence of research factors on milk yield rate in the form of a regression equation in an encoded form is as follows:

$$y_1 = 1.395 + 0.186x_1 - 0.068x_1^2 + 0.032x_2 + 0.005x_1x_2 - 0.132x_1^2 - 0.015x_3 + 4.6 - 10^{-5}x_4x_3 + 0.115x_3 - 0.104x_3^2 + 0.023x_4 + 0.0025x_3x_4 - 8.0 - 10^{-4}x_4x_3 + 5.767 - 10^{-5}x_4x_1 - 0.177x_4^2.$$ \hspace{1cm} (21)

At a 95 % confidence level for equation (21), variances are homogeneous, and the Cochrane criterion value $G = 0.06364 > G_{0.05}(9, 26) = 0.0958$.

The variance of the adequacy of the regression equation $S_{ad} = 0.0522$, the variance of error $S_E = 0.0659$; the Fisher criterion $F = 1.04 < F_{0.05}(16, 234) = 1.687$; the model is adequate. According to the Student’s criterion, coefficients at the following terms of the regression equation are significant: $x_1$, $x_2$, $x_3$, $x_4$, $x_2x_4$, $x_3x_4$, $x_2^2$, $x_3^2$, $x_4^2$. Based on the above, regression equation (21) takes the following form:

$$y_1 = 1.395 + 0.186x_1 - 0.068x_1^2 + 0.032x_2 + 0.005x_1x_2 - 0.132x_1^2 - 0.015x_3 + 0.115x_3x_4 - 0.104x_3^2 + 0.023x_4 + 0.0025x_3x_4 - 0.177x_4^2.$$ \hspace{1cm} (22)

In a decoded form, model (22) is as follows:

$$V = -57.17 + 1.76P - 1.69 - 10^{-3}P^2 + 1.65 - 10^{-4}n + 3.3 - 10^{-4}Pn - 2.34 - 10^{-5}n^2 + 3.538 + 1.53 - 10^{-3}\delta - 10.418^2 + 2.08 - 10^{-3}F_H + 1.30 - 10^{-3}PF_H - 1.76 - 10^{-3}F_H^2.$$ \hspace{1cm} (23)

where $V$ is the milk yield rate, 1/min, $P$ is the working pressure of a vacuum system, kPa, $n$ is the pulsation rate of the milking machine, min$^{-1}$, $\delta$ is the ratio of pulsations, $F_H$ is the tension force of teat rubber, N.

We have built the regression equation for the influence of research factors on the air consumption of the milking machine:

$$y_2 = 2.311 + 0.185x_1 + 0.00094x_2^2 + 0.151x_2 + 0.01x_1x_2 - 0.0053x_2^2 + 0.065x_3 + 0.005x_3x_2 + 0.005x_3x_4 - 0.066x_3^2 - 0.229x_3 - 0.015x_3x_4 - 0.018x_3x_4 - 0.0027x_4^2.$$ \hspace{1cm} (24)

At a 95 % confidence level for equation (24), variances are homogeneous, and the Cochrane criterion value $G = 0.0611 > G_{0.05}(9, 26) = 0.0958$.

The variance of adequacy of the regression equation $S_{ad} = 0.002589$; the variance of error $S_E = 0.0628$; the Fisher criterion $F = 1.04 < F_{0.05}(16, 234) = 1.709$; the model is adequate.

According to the Student criterion, coefficients at the following terms of the regression equation are significant: $x_1$, $x_2$, $x_3$, $x_4$, $x_1x_3$, $x_2x_3$, $x_1x_4$, $x_2x_4$.

Based on this, regression equation (24) takes the form:

$$y_2 = 2.311 + 0.185x_1 + 0.151x_2 + 0.01x_1x_2 + 0.065x_3 - 0.229x_3 - 0.015x_3x_4 - 0.018x_3x_4.$$ \hspace{1cm} (25)

In a decoded form, model (25) takes the following form:

$$Q = -2.88 + 6.07 - 10^{-2}P + 7.64 - 10^{-3}n + 6.7 - 10^{-4}P + 1.04 - 10^{-5}\delta + 3.42 - 10^{-3}F_H - 7.5 - 10^{-4}PF_H - 2.33 - 10^{-3}F_H^2.$$ \hspace{1cm} (26)

where $Q$ is the air consumption of the milking machine, m$^3$/h.

Analyzing equation (23), it can be argued that the milk yield rate is influenced by all the above factors. Namely, the working pressure of a vacuum system, the frequency, and ratio of pulsations of the milking machine and the tension force of teat rubber. At the same time, with an increase in the working pressure of the vacuum system, the milk yield rate increases, and when varying the values, the pulsation frequency of the milking machine, the ratio of pulsation cycles, and the tension force of teat rubber, the milk yield rate is optimal:

$$V = \left( \frac{P = 52 \text{kPa; } n = 58.6 \text{ min}^{-1}}{\delta = 0.6; } F_H = 60.7 \text{ N} \right) = 1.541 \text{/min}.$$ \hspace{1cm} (27)

Analyzing equation (26) reveals that the air consumption of the milking machine depends on the factors linearly. Obviously, the optima according to the criteria of milk yield rate and the air consumption of the milking machine do not coincide. Thus, to find the optimal values of factors, it is necessary to solve the compromise problem of optimum search for two criteria. We shall build two-dimensional dependences of the milk yield rate and the air consumption of the milking machine on factors (Fig. 5).

The compromise problem was to determine the optimal values of research factors in minimizing the air consumption of the milking machine at the maximum value of milk yield rate:

$$\begin{align*}
Q(P, n, \delta, F_H) & \rightarrow \min; \\
V(P, n, \delta, F_H) & \rightarrow \max;
\end{align*}$$ \hspace{1cm} (28)

$$48 \leq P \leq 52; 50 \leq n \leq 65; 0.5 \leq \delta \leq 0.7; 50 \leq F_H \leq 65.$$ \hspace{1cm} (28)

Taking the ratio of air consumption to the milk yield rate, convert problem (28) to the form:

$$\left( \frac{Q(P, n, \delta, F_H)}{V(P, n, \delta, F_H)} \right) \rightarrow \min.$$ \hspace{1cm} (29)

Equation (29) was solved using the software package Wolfram Mathematica, which has made it possible to establish the optimal parameters and operational modes for a vacuum system of milking equipment:

$$P = 50.6 \text{kPa; } n = 55.9 \text{ min}^{-1}; \delta = 0.58; F_H = 64.8 \text{ N; } V_{\max} = 1.52 \text{ l/min; } Q_{\min} = 2.19 \text{ m}^3/\text{h}.$$ \hspace{1cm} (30)
Fig. 5. The impact exerted on the milk yield rate $V$ and the air consumption of the milking machine $Q$ by: 
- $a$ — a working vacuum $P$; 
- $b$ — pulsation rate $n$; 
- $c$ — the force of tension of teat rubber $F_H$; 
- $d$ — the ratio of cycles $\delta$

The graphic interpretation of dependence (23) is represented in Fig. 6–11.

![Graphs showing the relationship between various parameters and milk yield rate](attachment:image1)

Fig. 6. The effect of pulsation rate $n$ and the ratio of pulsation cycles $\delta$ on the milk yield rate $V$

![Graph showing the effect of pulsation rate and teat rubber force on milk yield rate](attachment:image2)

Fig. 7. The effect of pulsation rate $n$ and the tension force of teat rubber $F_H$ on the milk yield rate $V$

![Graph showing the effect of working vacuum and pulsation frequency on milk yield rate](attachment:image3)

Fig. 8. The effect of the ratio of pulsation cycles $\delta$ and the tension force of teat rubber $F_H$ on the milk yield rate $V$

![Graph showing the effect of working vacuum and pulsation frequency on milk yield rate](attachment:image4)

Fig. 9. The effect of a working vacuum $P$ and pulsation frequency $n$ on the milk yield rate $V$

At these optimal values of technical and technological parameters (30), the milking unit UDM consumes a power of $W=3.97$ kW. At the same time, the specific electricity
consumption by the milking plant during milking is \( E = 0.095 \text{ kW\-h/cow} \). Compared to the technical and technological parameters recommended by the manufacturer, there is a 14.8% reduction in electricity consumption.

5.3. Adequacy of the mathematical model of forecasting the resource of milking equipment

Using the built mathematical model of the influence of the technical and technological parameters of a vacuum system of milking equipment on the efficiency of machine milking (23), we tested the correspondence of the obtained experimental data to theoretical prerequisites. To this end, we matched the theoretical (20) and experimental (23) milk rate values depending on the operating pressure of the vacuum system, the frequency and ratio of pulsations of the milking machine, and the tension force of teat rubber.

The results of the correlation of theoretical and experimental values of milk yield rate are shown in Fig. 12.

Fig. 12 demonstrates that the experimental data are within the corridor of the theoretical ones. The correlation between the theoretical and experimental data is 0.953.

6. Discussion of results of establishing the influence of the technical and technological parameters of milking equipment on the efficiency of machine milking

One of the areas of increasing the efficiency of dairy farming is to improve the process of milking cows, which implies the development and use of milking equipment that most fully meets the zoo-technical, veterinary, physiological, and engineering requirements [38].

In works [5, 17], it is noted that milking equipment is a component that comes into close contact with the cow’s rudder during milking and milk. The effectiveness of the milking process depends on its technical parameters. The advantages of our research compared to those specified above are the establishment of the influence of the technical and technological parameters of milking equipment on the efficiency of machine milking.
Thus, at the initial stage, theoretical studies of the influence of technical and technological parameters of milking equipment on the efficiency of machine milking were carried out. The scheme of interaction of the teat rubber with a teat (Fig. 2) and the plot of the dependence of milk flow rate through a teat’s sphincters during the sucking time cycle (3) were built. The range of operating parameters for milking equipment has been determined, at which the milk yield rate is maximum: \( P = 52 \text{kPa} \), \( n = 57.6–58.8 \text{min}^{-1} \), \( \delta = 0.59–0.64 \), \( F_{\text{H}} = 59.3–60.4 \text{N} \). The influence of each parameter on the rate of milk production at the fixed values of other parameters is represented graphically (Fig. 4).

Experimental studies, with the help of the software package «Mathematica», solved problems (29), which led us to the optimal parameters and modes of operation of the vacuum system of milking equipment.

At these optimal values of the technical and technological parameters (30), the milking unit UDM consumes a power of \( W = 3.97 \text{KW} \). At the same time, the specific electricity consumption by the milking plant during milking is \( E = 0.095 \text{kw/h cow} \). Compared to the technical and technological parameters recommended by the manufacturer, there is a 14.8% reduction in electricity consumption.

As regards the adequacy of the mathematical model of forecasting the resource of milking equipment, Fig. 12 shows that the experimental data are within the corridor of the theoretical ones. The correlation between the theoretical and experimental data is 0.953.

The mathematical model constructed fully reveals the influence of technical and technological parameters of milking equipment on the efficiency of machine milking. Given this, the task of the rational choice of equipment is resolved.

Our research was made possible by the use of innovative equipment (Fig. 1). That has made it possible to conduct a set of experiments with specific results.

The results of the current research are consistent with earlier studies reported by other authors [24, 26], and complement them. A significant difference in the methodological plan of our research was that there was an opportunity to study a wide range of indicators – from technical to technological.

Along with this, due to the extremely high variability of the design parameters of milking equipment, there are difficulties in fully resolving the issue of absolute establishment of the influence of technical and technological parameters of the equipment on the efficiency of machine milking. This remains an unresolved issue in the general technological link of milk production at dairy complexes.

Promising are the research aimed at modeling the process of moving a milk and air mixture in the milking machine.

### 7. Conclusions

1. As a result of theoretical studies into the influence of the technical and technological parameters of milking equipment with an upper milk line and milking machines of simultaneous action on the efficiency of machine milking, a mathematical model has been built. The model linked the value of a working vacuum \( P \), the pulsation frequency \( n \), the ratio of pulsation cycles \( \delta \), and the force of tension of milk rubber \( F_{\text{H}} \) to the milk yield rate \( V \).

2. As a result of theoretical studies, the range of operating parameters for the milking plant UDM was determined, at which the milk yield rate is maximum: \( P = 52 \text{kPa} \), \( n = 57.6–58.8 \text{ min}^{-1} \), \( \delta = 0.59–0.64 \), \( F_{\text{H}} = 59.3–60.4 \text{N} \). At these parameters, the milk yield rate is \( V = 1.48–1.53 \text{/min} \).

3. As a result of experimental studies, a mathematical model has been built on the influence of the technical and technological parameters of a vacuum system of milking equipment with an upper milk line and milking machines of simultaneous action on the efficiency of machine milking. The analysis of the mathematical model has made it possible to define the rational structural and technological parameters for a vacuum system of the milking machine UDM. These include the value of a working vacuum \( P = 50.6 \text{kPa} \), the pulsation rate \( n = 55.9 \text{ min}^{-1} \), the ratio of pulsation cycles \( \delta = 0.58 \), and the tension force of milk rubber \( F_{\text{H}} = 64.8 \text{N} \). At these parameters, the milk yield rate is maximal: it is \( V = 1.47–1.52 \text{/min} \), and the air consumption by the milking plant is \( Q = 2.19 \text{m}^3/\text{h} \).

The adequacy of the mathematical models built was tested based on the calculation of variances of adequacy and error of regression equations, followed by determining the Fisher criterion at a 95% level of confidence \( F = 1.04 < F_{0.05(16, 234)} = 1.687 \). We have compared the obtained theoretical and experimental dependences by determining the correlation coefficient, which is 0.953.

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