Cosmological Phase Transitions in a Brane World

Stephen C. Davis*, Warren B. Perkins†
Department of Physics, University of Wales Swansea,
Singleton Park, Swansea, SA2 8PP, Wales

Anne-Christine Davis‡ and Ian R. Vernon§
Department of Applied Mathematics and Theoretical Physics,
Centre for Mathematical Sciences,
University of Cambridge, Cambridge, CB3 0WA, UK.

I. INTRODUCTION

Recently there has been considerable interest in the novel suggestion that the physical universe is embedded in higher dimensions with standard model particles confined to a 3-brane and gravity propagating in the extra dimensions [1–4]. Randall and Sundrum [5] have even suggested that the extra dimension could be non-compact. Cosmological evolution in these extra dimension scenarios has been investigated. The Friedmann equation was shown to contain important deviations from the usual 4-dimensional case [6], giving rise to increased expansion at early times and a corresponding non-standard temperature-time relation. In section II we review brane world cosmologies, displaying the differences to the usual picture.

If the brane world scenario is correct then it needs to describe the world we live in. In this paper we investigate some of the implications of the brane world scenario for processes occurring in the early universe. In particular, the brane cosmology needs to reproduce standard model physics at low energies and also lead to cosmological structure formation. Fundamental to both are phase transitions in the early universe. Since the underlying cosmology is changed in the brane picture, cosmological phase transitions could also change. In particular, in section III we investigate first order phase transitions in brane cosmologies. The increased expansion at early times can have dramatic consequences, which we elucidate. We show that a higher bubble nucleation rate is required for the phase transition to complete, which could lead to more supercooling in some cases.

Whilst inflation in brane world scenarios has been investigated [7], the impact of the increased expansion on the cosmology of topological defects has yet to be investigated. We consider the possibility that the five dimensional Planck mass is just above the grand unified theory (GUT) scale, and consider the effect of cosmological GUT phase transitions. Such transitions can give rise to topological defects. We examine whether or not the properties of such resulting defects are modified in a brane world scenario. Moving to a brane world picture introduces many novel features to defect cosmology. There are two broad classes of effect that need to be considered. On the microscopic scale there are changes to the physics which determines the properties of individual defects. While on the macroscopic scale, modifications to the Friedmann equation change the evolution of defect networks. A full treatment of brane world defects would require a consistent solution representing a defect on the brane. Given that heavy defects may produce strong gravitational backreactions on the brane, in general such a calculation lies outside the scope of the low-energy effective field theory formalism [8]. Some possible changes in defect microstructure due to the modifications of...
the gravitational interaction on small scales are considered in [4]. In this paper we consider the evolution of standard defects with the brane world Friedmann equation in section III. We consider the effect the increased expansion has on the evolution of cosmic strings and on the monopole and domain wall densities.

The increased expansion rate in the brane world scenario will lead to different freeze-out temperatures for particle interactions. This will change the abundance of particles produced at early times. An example of this is GUT baryogenesis, which we examine in section V. Depending on the fundamental parameters, the increased expansion rate can lead to a suppression of the resulting baryon asymmetry. Our conclusions are summarised in section VI.

II. BRANE WORLD COSMOLOGIES

The recent interest in brane worlds and extra dimensions has been inspired mostly by [4], in which one of the extra dimensions is larger than the others and our universe is a positive tension 3-brane embedded in a five-dimensional bulk. The matter fields are confined to the brane and following [9] we consider a bulk space which contains only a cosmological constant, with energy density $\rho_C$.

In this case the effective Friedmann equation on the brane is

$$\left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{4\pi}{3M_5^2}\right) \rho_B + \left(\frac{4\pi}{3M_5^2}\right)^2 \rho_B - \frac{k}{a^2} + \frac{C}{a^4} + \frac{F^2}{\rho_B a^8},$$

(1)

where $M_5$ is the fundamental five-dimensional Planck mass, $\rho_B$ is the energy density of the brane fields, and $a(t)$ is the value of the scale factor on the brane. $C$ and $F$ are integration constants; the former is the dark energy term [10], while the latter term is due to a $Z_2$ breaking term in the metric [10]. While the energy density is $Z_2$ symmetric across the brane, the solution to Einstein’s equations need not be. If we insist on a $Z_2$ symmetric bulk solution, $F = 0$ as in [9].

To obtain the standard cosmology at late times we take $\rho_B = \rho + \rho_\lambda$, where $\rho_\lambda = 3M_5^6/(4\pi M_4^2)$ and $\rho$ are the energy densities corresponding to the brane tension and matter/radiation on the brane. To obtain a vanishing effective cosmological constant on the brane the bulk cosmological constant must be tuned to cancel the brane tension, leading to $\rho_B = -4\pi\rho_\lambda^2/(3M_5^3)$. The resulting Friedmann equation is

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3M_4^2} \rho + \left(\frac{4\pi}{3M_5^2}\right)^2 \rho_B - \frac{k}{a^2} + \frac{C}{a^4} + \frac{F^2}{(\rho + \rho_\lambda)^2 a^8}.$$ (2)

To agree with the standard cosmology the $F$ [10] and $C$ [9] terms in Eq. (2) must both be small; this is fully discussed in [10]. Throughout the rest of this paper we will assume they are negligible, and take $k = 0$.

As in the standard case, radiation dominates matter at early times. Initially in the brane picture the $\rho^2$ term dominates Eq. (2) and we have $a \sim t^{1/4}$. After $t = t_n = M_4^2/(8M_5^3)$ the $\rho$ term becomes dominant and $a \sim (t + t_n)^{1/2}$, as in the standard cosmology.

During the initial period of $\rho^2$ dominance, the Friedmann equation leads to a non-standard temperature-time relation,

$$t = \frac{45}{8\pi^3 g_* T} M_5^3.$$ (3)

This epoch ends when $t = t_n$ and $T = T_n = [45M_5^3/(g_\ast, M_4^3)]^{1/4}$. For $T < T_n$ the more familiar form is recovered,

$$t = \frac{3\sqrt{5}}{4\pi^{3/2} g_*^{1/2}} \frac{M_4}{T^2} - t_n.$$ (4)

The standard temperature-time relation is Eq. (1) with $t_n = 0$. As the Hubble parameter in this model is much greater at a given temperature than in the standard case, the universe initially cools far more rapidly. This could have dramatic consequences on cosmological phase transitions, which we investigate in subsequent sections.

III. PHASE TRANSITIONS IN BRANE COSMOLOGY

If the brane world scenario is to describe the Universe we live in then it must reproduce standard model physics. It must also give rise to a mechanism for structure formation. In this section we consider phase transitions in a brane
world; in particular first order phase transitions. In a cosmological setting, these could be modified because of the revised Friedmann equation.

During a phase transition, bubbles of true vacuum will nucleate and expand. Assuming they expand at the speed of light, the fraction of space remaining in the false vacuum at time $t$ is

$$p(t) = \exp \left\{ -\frac{4\pi}{3} \int_0^t dt_1 a^3(t_1) \Gamma(t_1) \left[ \int_{t_1}^t \frac{dt_2}{a(t_2)} \right]^3 \right\}. \tag{5}$$

The three factors in the integral are respectively the red shift, the bubble nucleation rate, and the volume at time $t$ of a bubble which formed at time $t_1$. The probability per unit time and volume that a critical size bubble will nucleate can be approximated by $\Gamma(T) = \nu T^4 \theta(T_c - T)$. The parameter $\nu$ will depend on the expansion rate as well as the details of the scalar potential, thus it will be different in the two models.

The number of bubbles nucleated by time $t$ is given by

$$n_{\text{bubble}} = a^{-3(t)} \int_0^t dt_1 a^3(t_1) \Gamma(t_1) p(t_1). \tag{6}$$

### A. Standard Cosmology

Evaluating the integrals in Eq. (5) in the standard cosmology gives

$$p(T) = \exp \left\{ -\frac{25\nu}{3\pi g_*} \left( \frac{3M_4}{2\pi} \right)^4 \left( \frac{1}{T} - \frac{1}{T_c} \right)^4 \right\}. \tag{7}$$

If the end of the phase transition is taken to be when $p = 1/2$, this will occur at $T = T_*$ given by

$$\frac{1}{T_*} = \frac{1}{T_c} + \frac{1.5g_*^{1/2}}{M_4 \nu^{1/4}}. \tag{8}$$

For a GUT transition $T_c \ll M_4$ and so unless $\nu$ is very small $T_* \approx T_c$.

Evaluating Eq. (8) reveals that the number of bubbles nucleated rapidly tends to $0.9\nu^{3/4}T^3$ after the end of the transition.

### B. Brane Cosmology

Using Eq. (3) to evaluate Eq. (5) gives, assuming $T_c > T_B$,

$$p(T) = \exp \left\{ -\frac{\pi \nu}{3} \left( \frac{15M_5^3}{2\pi^3 g_*} \right)^4 \left( \frac{1}{T^3} - \frac{1}{T_c^3} \right)^4 \right\}, \tag{9}$$

and so this time the transition ends when

$$\frac{1}{T_*^3} = \frac{1}{T_c^3} + \frac{3.7g_*}{M_5^3 \nu^{1/4}}. \tag{10}$$

If $T_c$ is close to $M_5$, then even for moderate values of $\nu$ the second term of Eq. (10) can be the dominant one. In this case the transition will be significantly slower and involve a greater temperature drop. The number of bubbles nucleated, Eq. (6), has the same large $t$ behaviour as in the standard cosmology.
C. Avoiding Vacuum Domination

If the phase transition goes too slowly, it is possible that false vacuum energy will dominate the radiation before the transition finishes. The Universe will then start to inflate, and never stop.

We will consider the simple Higgs model with an effective potential

$$V(\phi, T) = \frac{\lambda}{4} \left[ \phi^2 + \frac{T^2}{4} - \frac{T_c^2}{4} \right]^2. \quad (11)$$

The false vacuum energy density (when $\phi = 0$) is then

$$\rho_{FV} = \frac{\lambda}{64} (T_c^2 - T^2)^2 p(T). \quad (12)$$

To avoid false vacuum domination we need $\rho_{FV} < \rho_{\text{radiation}}$; hence,

$$\frac{15\lambda}{32\pi^2 g_*} p(T) < \left( \frac{T}{T_c} \right)^4. \quad (13)$$

Vacuum domination will only occur if $\nu \ll (T_c/M_4)^4$, i.e. if the second term of Eq. (8) is the most significant. Using this assumption, the bound on $\nu$ for the standard cosmology is

$$\nu \geq 0.1 g_\star \lambda \left( \frac{T_c}{M_4} \right)^4. \quad (14)$$

In brane cosmology, assuming $\nu \ll (T_c/M_5)^{12}$, the corresponding bound is

$$\nu \geq 4 \times 10^{-3} g_\star \lambda^3 \left( \frac{T_c}{M_5} \right)^{12} \approx 8 \times 10^{-3} g_\star \lambda \left( \frac{T_c}{M_4} \right)^4 \left( \frac{\lambda T_c^4}{g_* T_\text{B}^4} \right)^2. \quad (15)$$

Thus unless $T_c < (g_\star/\lambda)^{1/4} T_\text{B}$ phase transitions in the brane cosmology require a higher nucleation rate to complete successfully. However, the faster expansion of the universe during the brane era could allow smaller bubbles to survive, thus increasing the bubble nucleation rate and weakening the bounds on the underlying parameters of the theory. Of course if $T_c < T_\text{B}$ the phase transition will happen when brane effects are not significant, and the bound is given by Eq. (14).

IV. DEFECTS IN BRANE COSMOLOGY

A natural result of cosmological phase transitions are topological defects. If, after a phase transition, the vacuum manifold has non-trivial homotopy groups, topological defects will form in brane cosmology, just as they do in standard cosmology. As in the normal case, defects can have potentially useful (and sometimes disastrous) cosmological implications. For example, GUT scale cosmic strings can lead to a realistic scenario for structure formation. However, magnetic monopoles and domain walls rapidly dominate the energy density of the universe. We examine whether or not the properties of such resulting defects are modified in a brane world scenario. To evaluate their properties we need to find the initial defect density and then determine how the density evolves.

For a first order phase transition the initial correlation length of the defects is easily determined from the arguments in the previous section,

$$\xi \sim \frac{1}{\lambda T_c}, \quad (16)$$

Next we consider the evolution of defects in brane cosmology.
A. Shadowing versus Scaling

An immediate consequence of the modified brany evolution is the different relationship between scaling (i.e. a fixed number of defects per horizon volume) and shadowing (i.e. defect density remaining a fixed fraction of the dominant energy density).

For scaling defects in either model we have

\[ \rho_{\text{string}} \propto \frac{ct}{(ct)^3} \propto t^{-2}, \quad \rho_{\text{wall}} \propto \frac{(ct)^2}{(ct)^3} \propto t^{-1}. \]  

(17)

If the dominant energy density varies as \( a^{-w} \), we saw above that \( a \propto t^{1/w} \) in the brane era and \( a \propto t^{2/w} \) in the normal picture. Thus in the brane era we have

\[ \rho_{\text{dominant}} \propto t^{-1}, \]  

(18)

while in the normal picture we have

\[ \rho_{\text{dominant}} \propto t^{-2}. \]  

(19)

In the standard picture, scaling strings shadow the dominant energy density, while in the brane era scaling walls shadow the dominant energy density.

B. Monopoles

At formation \( n_{\text{monopole}} \sim n_{\text{bubble}} \). Red-shifting gives \( n_{\text{monopole}} \sim T^3 \) at later times, thus in the absence of annihilation, both cosmologies would predict the same monopole number in the current universe. While monopole annihilation could look very different in brane cosmology if the brane era were persistent, the limited duration of the brane epoch curtails annihilation.

As for any 2-body annihilation process, the number density of monopoles relative to photons, \( r_M = n_M/n_\gamma \) is governed by

\[ \frac{dr_M}{dt} = -\beta_M n_\gamma (r_M^2 - r_{M,eq}^2), \]  

(20)

where \( \beta_M \) parameterises the monopole annihilation rate and \( r_{M,eq} \) is the equilibrium monopole to photon ratio.

Let us assume that \( r_{M,eq} \) rapidly drops to zero and set \( n_\gamma = \alpha T^3 \). We can take the temperature-time relationship to be \( T = Ct^{-1/w} \), where \( C, \alpha \) and \( w \) are constants. If the monopoles form at time \( t_f \) at a density \( r_{M,f} \), integration gives

\[ r_M^{-1} = \beta_M \alpha C^3 t_f^{1-3/w} - \frac{t_f^{1-3/w}}{1-3/w} + r_{M,f}^{-1}. \]  

(21)

In the standard picture, for a radiation dominated Universe, \( w = 2 \) and at large times we have the standard freeze out picture with

\[ r_M^{-1}\big|_{t,\infty} = 2\beta_M \alpha C^3 t_f^{-1/2} + r_{M,f}^{-1}. \]  

(22)

Clearly, freeze out only occurs for \( w < 3 \), for \( w > 3 \), \( r_M \) decays with time. [For \( w = 3 \), \( r_M \) decays like \( 1/\log(t) \) at late times.] Thus naively things look very different in the brane era. Here \( w = 4 \) and

\[ r_M^{-1} = 4\beta_M \alpha C^3 (t_f^{1/4} - t_f^{1/4}) + r_{M,f}^{-1}. \]  

(23)

At large times, \( r_M \propto t^{-1/4} \), instead of freeze out, the monopole density continues to decay and there would appear to be no monopole problem. However, this result does not survive the inclusion of constants and the inevitable termination of the brane era.

Let the brane era persist well beyond the GUT time, then

\[ r_M^{-1} \approx 4\beta_M \alpha C^3 t_f^{1/4} + r_{M,f}^{-1}. \]  

(24)
If we denote the time and temperature of the brane-normal transition by \( t_B \) and \( T_B \), we have
\[
t_f = t_B \left( \frac{T_B}{T_f} \right)^w .
\] (25)

If we now look at the monopole density at \( t_B \), in the brane model we have
\[
r^{-1}_M \simeq 4\beta M^2 T_B^3 t_B + r^{-1}_M .
\] (26)

While in the normal model we have
\[
r^{-1}_M |_{t.o.} = 2\beta M^2 T_B^3 t_B \left( \frac{t_B}{t_f} \right)^{1/2} + r^{-1}_M .
\] (27)

Given that \( t_B \gg t_f \), we see that the annihilation in the normal model is far more efficient than in the brane model: the transient nature of the brane era and the constants conspire to over turn the naive expectations from the proportionality terms. Thus, the brane world scenario does not solve the usual monopole problem associated with cosmological phase transitions.

C. Cosmic Strings

At early times in their evolution cosmic strings experience a significant damping force from the background radiation density. For strings the damping force is the dominant effect when \( T^3/T_c^2 > H \), in which case \( \xi \propto T_c T^{-3/2} t^{1/2} \) \[3\].

In the standard cosmology the correlation length is
\[
\xi \sim \frac{T_c M^{1/2}}{T^{5/2}} \sim \frac{T_c}{M^{1/4} T^{5/4}},
\] (28)
and \( \rho_{\text{string}} = T_c^2/\xi^2 \). This continues until \( T \sim T_c^2/M_4 \), after which the evolution will start to approach a scaling solution (\( \xi \sim t \)).

During the initial period of non-standard evolution in the brane cosmology the string evolution is always friction dominated. This continues until \( T \sim T^2_c/M_4 \), as in the standard cosmology.

For \( T > T_B \) the string density satisfies
\[
\xi \sim \frac{T_c}{M^{3/2} T^{7/2}} \sim \frac{T_c}{M^{3/4} t^{7/4}}.
\] (29)

Using the relationship between \( T_B \), \( M_5 \) and \( M_4 \), we find that the correlation length at \( T = T_B \) in the brane cosmology is of the same order as in the standard cosmology. For \( T < T_B \) the correlation length evolves as in the standard case, thus the string density at the end of friction domination is the same in both pictures.

D. Domain Walls

The early evolution of domain walls can also be friction dominated. The correlation length is \( \xi \sim v t \), where \( v \) is the speed of the walls. During friction domination the wall tension and friction will of the same order. This determines the speed of the walls: \( v^2 \sim T_c^3 / (t T^4) \) \[3\]. Now \( \rho_{\text{wall}} \sim T^2_c/\xi \) so \( \rho_{\text{wall}}/\rho_{\text{rad}} \sim v \). This means that when the walls become relativistic they will also start to dominate the energy density of the Universe.

In the standard cosmology the velocity of the walls is
\[
v \sim \frac{\rho_{\text{wall}}}{\rho_{\text{rad}}} \sim \frac{T_c^{3/2}}{M_4^{1/2} T}.
\] (30)

Hence the walls dominate the Universe at \( T = (T_c/M_4)^{1/2} T_c \).

During the early brane cosmology,
\[
v \sim \frac{\rho_{\text{wall}}}{\rho_{\text{rad}}} \sim \left( \frac{T_c}{M_5} \right)^{3/2} = \text{const.}
\] (31)

Thus the domain wall energy density initially scales like radiation. After \( T = T_B \) they will behave as in the standard picture. As in the case with monopoles, the brane world scenario does not stop domain walls dominating the energy density of the universe, despite naive expectations.
V. GUT BARYOGENESIS

In this section we investigate the modifications of the usual picture of baryogenesis which result from the brane world Friedmann equation. In general there are three things which are required for successful baryogenesis [14]: (1) baryon number violation, (2) $C$ and $CP$ violation and (3) departure from thermal equilibrium. The third requirement can be illustrated with a simple generic model in which the baryon asymmetry is produced by the decay of GUT bosons ($X, \bar{X}$) [15]. At high temperatures ($T \gtrsim m_X$) the $X$-bosons behave relativistically and so $n_X = n_{\bar{X}} \simeq n_{\gamma}$ in equilibrium.

If the $X$-bosons are still in thermal equilibrium for $T \lesssim m_X$, $n_X = n_{\bar{X}} \simeq \left(\frac{m_X}{T}\right)^{3/2} \exp\left(-\frac{m_X}{T}\right) \ll n_{\gamma}$, and so when they eventually decay they will produce exponentially few baryons. On the other hand if the $X$-bosons decouple before $T \sim m_X$ there will be $n_{\gamma}$ of them to decay into baryons. In terms of the possible annihilation and decay processes, baryogenesis arises from the single particle decay of $X$ and $\bar{X}$'s rather than $X\bar{X}$ annihilation.

The interactions which determine the effectiveness of the above mechanism are the decays and inverse decays of $X$-bosons, and $2 \leftrightarrow 2$ $X$-mediated $B$-nonconserving scatterings between baryons. For $T \lesssim m_X$ rates of these processes are, respectively,

$$\Gamma_D \simeq \alpha m_X, \quad (32)$$

$$\Gamma_{ID} \simeq \alpha m_X \left(\frac{m_X}{T}\right)^{3/2} \exp\left(-\frac{m_X}{T}\right), \quad (33)$$

and

$$\Gamma_{BNC} \simeq A\alpha^2 \frac{T^5}{m_X^4}, \quad (34)$$

where $\alpha$ measures the coupling strength of the $X$-boson and $A$ is a large numerical factor which accounts for the number of scattering channels. If the inverse decays or the $2 \leftrightarrow 2$ baryon scatterings are still significant for $T \lesssim m_X$ the final baryon asymmetry will be suppressed.

It is convenient to define the parameters $\chi = m_X/T$ and $K = \Gamma_D/H|_{\chi=1}$. In the standard cosmology,

$$K \simeq \frac{\alpha M_4}{g^{1/2}m_X}, \quad (35)$$

while in the brane cosmology,

$$K \simeq \frac{\alpha M_4^2}{g^*, m_X^3}. \quad (36)$$

The ratios of the interaction rates to the Hubble parameter are then

$$\Gamma_{ID}/H \simeq K \chi^{w+3/2} e^{-\chi}, \quad (37)$$

$$\Gamma_{BNC}/H \simeq K A\alpha \chi^{w-5}, \quad (38)$$

where $w = 2$ for the standard cosmology and $w = 4$ for the brane cosmology.

If $K \ll 1$ then $\Gamma_{ID}/H < 1$ and $\Gamma_{BNC}/H < 1$ at $\chi = 1$, thus the $X$-bosons decouple when they are relativistic and the baryon asymmetry ($B$) will be maximal. If each $X$ decay produces a mean net baryon number $\epsilon$, then the final baryon number to entropy ratio produced when $K \ll 1$ will be

$$B = \frac{n_b - n_{\bar{b}}}{g_* n_{\gamma}} \simeq \frac{\epsilon}{g_*}. \quad (39)$$

The $C$ and $CP$ violation parameter $\epsilon$ is of order $\alpha^N$, where $N \geq 1$ since this is not a tree level process.

This baryon asymmetry can be damped if either the inverse decays or the baryon non-conserving scatterings persist beyond $\chi = 1$. If $1 \lesssim K \lesssim K_C$ (where $K_C$ is a theory dependent constant), the inverse decays will still be significant for $\chi > 1$. This continues until the inverse decays freeze out with $\Gamma_{ID}/H \simeq \chi$ at $\chi = \chi_f$. Approximate integration of the Boltzmann equation in this case gives
\[ B \simeq \frac{\epsilon}{g_* K \chi_f^{-1}}. \]  

(40)

For large \( K \), \( \chi_f \) has a slow, logarithmic dependence on \( K \) and the baryon asymmetry falls roughly as the inverse of \( K \).

If \( K \geq K_C \) the \( B \)-nonconserving scatterings will provide the dominant damping mechanism. The freeze out for these interactions is determined by \( \Gamma_{BNC}/H \simeq \chi \). In this case,

\[ B \simeq \frac{\epsilon}{g_* \chi_f^2} \exp\left( \frac{-6 - w}{5 - w} \chi_f \right). \]  

(41)

Thus for large \( K \) the baryon asymmetry is exponentially suppressed as expected. \( K_C \) is determined by the value of \( K \) that gives simultaneous freeze out of both the inverse decays and the baryon non-conserving scatterings.

To compare the two cases we will consider typical GUT parameters: \( g_* = 200 \), \( A = 5000 \), gauge coupling strength \( \alpha_G = 1/45 \) and Higgs coupling strength \( \alpha_H = 10^{-3} \).

In the standard cosmology these parameters give

\[ K \sim \begin{cases} 
10^{-3} M_4/m_X & \text{gauge} \\
10^{-4} M_4/m_X & \text{Higgs} 
\end{cases}, \quad K_C \sim \begin{cases} 
120 & \text{gauge} \\
15000 & \text{Higgs} 
\end{cases}. \]  

(42)

Assuming \( m_X \gtrsim 10^{14} \text{ GeV} \), there is no damping of \( B \) in the Higgs boson mediated case, while in the gauge boson mediated case \( B \) is power law damped. The \( CP \) violation required to obtain the observed baryon density, \( B \sim 10^{-10} \), is given by

\[ \epsilon \sim \begin{cases} 
10^{-7}(10^{16} \text{ GeV}/m_X) & \text{gauge} \\
10^{-8} & \text{Higgs} 
\end{cases}. \]  

(43)

The corresponding values of \( K \) and \( K_C \) in the brane cosmology are

\[ K \sim \begin{cases} 
(M_5/20m_X)^3 & \text{gauge} \\
(M_5/60m_X)^3 & \text{Higgs} 
\end{cases}, \quad K_C \sim \begin{cases} 
1 & \text{gauge} \\
60 & \text{Higgs} 
\end{cases}. \]  

(44)

Thus in the gauge boson mediated case, unless \( m_X \) is within an order of magnitude of \( M_5 \), the baryon asymmetry will be exponentially suppressed. In the Higgs boson case \( m_X \) must be within two orders of magnitude of \( M_5 \) to give significant baryogenesis. This change renders GUT baryogenesis particularly sensitive to an early brane era. If \( m_X \sim 10^{14} \text{ GeV} \), baryogenesis occurs in the brane era for \( M_5 \) as high as \( 10^{16} \text{ GeV} \).

However, this result was obtained using typical standard cosmology GUT values. If \( M_5 \) were far lower than the usual Planck scale, then the GUT scale would also have to be reduced. \( \alpha \) would also have to be substantially smaller in order to avoid breaking the experimental bounds on the proton lifetime. This would result in a lower \( \epsilon \), further constraining the model.

VI. CONCLUSIONS

In this paper we have considered the implications of brane world models for various processes which occur in the early universe. Due to the modified Friedmann equation, the rate of expansion of the Universe is increased at early times. The relation between temperature and time is also changed. This has important phenomenological consequences.

As discussed in section III, first order phase transitions require a higher nucleation rate in order to complete, which could result in more supercooling. Indeed, if the nucleation rate is not high enough, the Universe becomes dominated by the false vacuum and the transition does not complete.

Processes that rely on interactions freezing out are also sensitive to the enhanced expansion rate. For example, this has important consequences for GUT baryogenesis. As shown in section III, unless the mass of the relevant GUT particle is within two orders of magnitude of the fundamental Planck scale, the baryon excess is exponentially suppressed. More generally, the abundance of any species that freezes out during the brane epoch will be affected.

Defect evolution is also modified during the brane epoch. However, due to the transient nature of this phase, the current defect densities are largely unchanged. This suggests that the usual mechanism for defect inspired structure formation is largely unchanged. It also suggests that, despite the increased expansion rate at early times, the usual monopole problem associated with GUT models remains.
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