Renormalization of the Cabibbo-Kobayashi-Maskawa Quark Mixing Matrix

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We have investigated the present renormalization prescriptions of Cabibbo-Kobayashi-Maskawa (CKM) matrix. When considering the prescription which is formulated with reference to the case of zero mixing we find it doesn’t satisfy the unitary condition of the bare CKM matrix. After added a delicate patch this problem can be solved at one-loop level. In this paper We generalize this prescription to all loop levels and keep the unitarity of the bare CKM matrix, simultaneously make the amplitude of an arbitrary physical process involving quark mixing convergent and gauge independent. We also find that in order to keep the CKM counter terms gauge independent the unitarity of the bare CKM matrix must be preserved.

11.10.Gh, 12.15.Lk, 12.15.Hh

I. INTRODUCTION

Since the exact examination of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [1–6] has been developed quickly, the renormalization of CKM matrix becomes very important. This was realized for the Cabibbo angle in the standard model (SM) with two fermion generations in a pioneering paper by Marciano and Sirlin [7] and for the CKM matrix of the three-generation SM by Denner and Sack [8] more than a decade ago. In recent years many people have discussed this issue [9–11], but a completely self-consistent scheme to all loop levels has been not obtained. In this paper we try to solve this problem and give some instructive conclusion.

In general, a CKM matrix renormalization prescription needs to satisfy three criterions, as Diener has declared [12]:

1. In order to make the transition amplitude of any physical process involving quark mixing ultraviolet finite, the CKM counterterm must cancel out the ultraviolet divergence left in the loop-corrected amplitudes. On the other hand it must include proper infrared divergence for the sake of infrared finiteness of the final scattering cross-section including soft quanta emission.

2. It must guarantee the transition amplitude of any physical process involving quark mixing gauge parameter independent [13], which is a fundamental requirement.

3. SM requires the bare CKM matrix \( V^0 \) is unitary,

\[
\sum_k V^0_{ik} V^0_{jk} = \delta_{ij}
\]  

with \( i, j, k \) the generation index and \( \delta_{ij} \) the unit matrix element. If we split the bare CKM matrix element into the renormalized one and its counterterm

\[
V^0_{ij} = V_{ij} + \delta V_{ij}
\]

and keep the unitarity of the renormalized CKM matrix, the unitarity of the bare CKM matrix requires

\[
\sum_k (\delta V_{ik} V^*_{jk} + V_{ik} \delta V^*_{jk} + \delta V_{ik} \delta V^*_{jk}) = 0
\]

Until now there are many papers discussing this problem. The modified minimal subtraction (\( \overline{MS} \)) scheme [14,15] is the simplest one, but it introduces the \( \mu^2 \)-dependent terms which are very complicated to be dealt with. In the on-shell renormalization scheme, however, there isn’t still an integrated CKM renormalization prescription. The early prescription [8] used the \( SU_L(2) \) symmetry of SM to relate the CKM counterterm with the fermion wave-function renormalization constants (WRC) [16]. Although it is a delicate and simple prescription, it reduces the physical amplitude involving quark mixing gauge dependent\(^1\) [17–19]. A remedial method of this prescription is to replace the

\(^1\)This is easy to be understood since the \( SU_L(2) \) symmetry of SM has been broken by the Higgs mechanism
on-shell fermion WRC in the CKM counterterms with the ones calculated at zero momentum [17]. Another remedial method [19] is to rearrange the off-diagonal quark WRC in a manner similar to the pinch technique [20].

In the following section we discuss a CKM renormalization prescription which, after some modification satisfies the three criterions. Next we generalize this prescription to n-loop level, simultaneously keep the satisfaction of the three criterions. In section 4 we discuss the relationship between the unitarity of the bare CKM matrix and the gauge independence of the CKM counterterms. Lastly we give our conclusions.

II. ONE-LOOP RENORMALIZATION OF CKM MATRIX

Different from the idea of Ref. [8], another idea is to formulate the CKM renormalization prescription with reference to the case of zero mixing. This has been done in Ref. [21,12] at one loop level. The main idea is to renormalize the transition amplitude of W gauge boson decaying into up-type and down-type quarks to make it equal to the amplitude of the same process which is in the case of no quark generation mixing. In order to elaborate this idea we firstly introduce the one loop decaying amplitude of $W^+ \rightarrow u_i d_j$ [21]

$$T_1 = A_L[V_{ij}(F_L + \frac{2g}{g} + \frac{1}{g}\delta Z_W + \frac{1}{2}\delta Z_{uL} + \frac{1}{2}\delta Z_{dL}) + 2\sum_{k\neq j} V_{ik}\delta Z_{kj} + \sum_{k\neq i} V_{ik}\delta Z_{ik} + \delta Z_{ij}] + \frac{1}{g}(ARF_R + BLGL + BRGR),$$

with $g$ and $\delta g$ the SU(2) coupling constant and its counterterm, $\delta Z_W$ the W boson WRC, $\delta Z_{uL}$ and $\delta Z_{dL}$ the left-handed up-type and down-type quark’s WRC [10,22], and

$$A_L = \frac{\alpha}{\sqrt{2}} \frac{\nu_i(p_1)\gamma_L V_{ij}^*(q - p_1)}{\gamma_L V_{ij}^*(q - p_1)} ,$$

$$B_L = \frac{\alpha}{\sqrt{2}} \frac{\nu_i(p_1)\gamma_R V_{ij}^*(q - p_1)}{\gamma_R V_{ij}^*(q - p_1)} .$$

where $\varepsilon^\mu$ is the W boson polarization vector, $\gamma_L$ and $\gamma_R$ are the left-handed and right-handed chiral operators, and $M_W$ is the W boson mass. Similarly, replacing $\gamma_L$ with $\gamma_R$ in the above equations we can define $A_R$ and $B_R$ respectively. $F_{L,R}$ and $G_{L,R}$ are four form factors. The main idea of Ref. [21] is to choose the CKM counterterm to make the amplitude $T_1$ similar as the amplitude of W boson decaying into leptons, which has no fermion generation mixing. This idea is reasonable since such a renormalized amplitude will be ultraviolet finite and gauge independent [21]. After introducing the proper CKM counterterm the decaying amplitude of $W^+ \rightarrow u_i d_j$ will be changed to such form [21]:

$$T_1 = V_{ij}(F_L + \frac{\delta g}{g} + \frac{1}{g}\delta Z_W + \frac{1}{2}\delta Z_{uL} + \frac{1}{2}\delta Z_{dL}) + A_RF_R + BLGL + BRGR$$

where the subscript "[l]" denotes the quantity is obtained by replacing CKM matrix elements with unit matrix elements. So it is easy to obtain [21]

$$\delta V_{ij} = -\frac{1}{2} \sum_k [\delta Z_{ik} V_{kj} + V_{ik} \delta Z_{kj}] + \frac{1}{2} V_{ij} [\delta Z_{ij} + \delta Z_{ij}]$$

But in fact such a CKM counterterm doesn’t make the decaying amplitude $T_1$ ultraviolet finite when $i \neq j$. It is easy to calculate the ultraviolet terms of $T_1$ using this CKM counterterm, as follows,

$$T_1|_{UV-divergence} = \frac{\alpha V_{ij}}{32\pi M_W s_W} (m_{d,i}^2 - m_{d,j}^2 + m_{u,i}^2 - m_{u,j}^2)$$

with $\alpha$ the fine structure constant, $s_W$ the sine of the weak mixing angle $\theta_W$, $m_{d,i}$ and $m_{d,j}$ the down-type quark’s masses, $m_{u,i}$ and $m_{u,j}$ the up-type quark’s masses, and $\Delta = 2/(D-4) + \gamma_E - \ln(4\pi) + \ln(M_W^2/\mu^2)$ (D is the space-time dimensionality, $\gamma_E$ is the Euler’s constant and $\mu$ is an arbitrary mass parameter). This result shows that when $i \neq j$ the decaying amplitude of $W^+ \rightarrow u_i d_i$ will be ultraviolet divergent. We argue that the origination of this error comes from our one-sided knowledge about the difference between the two cases of having and not having fermion generation mixing. In the case of no fermion generation mixing only the same fermions as the external-line fermions can appear at the fermion line that connects with the external-line fermions in the Feynman diagrams of $W^+ \rightarrow u_i d_j$. This is because if there is no generation mixing only one generation fermions can appear at a fermion line in a Feynman
The same restraint is also suitable for the fermion’s mass counterterms and WRC which appear at the fermion line that connects with the external-line fermions. Of course no CKM matrix element appears at the fermion line that connects with the external-line fermions. On the other hand, if there has fermion generation mixing there is no such restraint. At one-loop level the difference between these two cases is at the fermion’s WRC. Different from the result of Eq.(24) in Ref. [21], the amplitude $T_1$ will be the following form in the case of no quark generation mixing,

$$T_1 = V_{ij}[A_L(F_L + \frac{\delta g}{g} + \frac{1}{2}\delta Z_{W} + \frac{1}{2}\delta Z_{W}^{ill}_{m,i \rightarrow m,j} + \frac{1}{2}\delta Z_{W}^{dL}_{m,i \rightarrow m,u,i} + A_R F_R + B_L G_L + B_R G_R]$$

So the CKM counterterm is obtained compared with Eq.(4) and Eq.(6)

$$\delta V_{ij} = -\frac{1}{2}\sum_k \delta Z_{ik}^{UL} V_{kj} + \frac{1}{2} \sum_{i \neq j} \sum_{m,k \neq m,i} \sum_{m,j \neq m,k} \left[ \frac{1}{2} \sum_{i \neq j} \sum_{m,k \neq m,i} \sum_{m,j \neq m,k} \right]$$

Our calculation has shown this CKM counterterm is gauge independent and makes the physical amplitude $T_1$ convergent.

As mentioned in section 1, the CKM renormalization prescription should keep the unitarity of the bare CKM matrix. Now we check this point. At one-loop level, only four diagrams need to be considered when we calculate the CKM counterterm in Eq.(7), as shown in Fig.1.

\[FIG. 1. Quark’s self-energy diagrams that contribute to the CKM counterterm in Eq.(7).\]

We have used the software packages FeynArts [23] to draw the Feynman diagrams and generate the corresponding Feynman amplitudes, and used the software packages FeynCalc [24] to calculate these Feynman amplitudes. We can obtain the analytical results of $\delta V_{ij}$ because the quark’s self-energy functions are very simple. In order to check the unitary condition of Eq.(3) analytically, we use the Taylor’s series: ($m_{quark}/M_W^2)^n$ to expand $\delta V_{ij}$. The one and two order results are shown as follows:

$$\delta V_{ij}^{(1)} = \left[ \frac{2}{\delta V_{ij}^{(1)}} \left[ \sum_{k \neq j} V_{ik} V_{kj} m_{u,k}^2 + \sum_{i \neq j} V_{ij} V_{ij}^* m_{u,k}^2 \right] + \sum_{i \neq j} V_{ij} V_{ij}^* m_{u,k}^2 \right]$$

$$\delta V_{ij}^{(2)} = \left[ \frac{2}{\delta V_{ij}^{(2)}} \left[ \sum_{k \neq j} V_{ik} V_{kj} m_{u,k}^2 + \sum_{i \neq j} V_{ij} V_{ij}^* m_{u,k}^2 \right] + \sum_{i \neq j} V_{ij} V_{ij}^* m_{u,k}^2 \right]$$

where the superscript "(1)" and "(2)" denote the one order and two order results about the series $m_{quark}/M_W^2$. Here we have used the dimensional regularization [25] and $R_\xi$-gauge [26]. Replacing $\delta V$ with $\delta V^{(1)} + \delta V^{(2)}$ in Eq.(3), we find at one-loop level they satisfy the unitary condition.

But when we consider the three order result of $\delta V$ about the series $m_{quark}/M_W^2$, we find it doesn’t satisfy the unitary condition, as shown below:

$$\sum_k (\delta V_{ki}^{(3)} V_{kj} + V_{ki}^{*} \delta V_{kij}) = \sum_k \left[ \sum k \sum_{m,k \neq m,i} \sum_{m,j \neq m,k} \left[ \frac{9}{12\pi M_W^2} \left[ \frac{1}{2} \sum_{i \neq j} \sum_{m,k \neq m,i} \sum_{m,j \neq m,k} \right] \right] \right] \neq 0$$

3
which shows that $\delta V$ doesn’t comply with the unitary criterion. We can estimate the deviation of $\sum_k (\delta V^*_k V_k + V^*_k \delta V_k)$ from 0. We argue that only the terms that contain the largest series of $m^2/M^2_W$ are important. Calculating to five order results of $\delta V$ about the series $m^2_{\text{quark}}/M^2_W$, we find the largest deviation of $\delta V + V^* \delta V$ from 0 is proportional to $\alpha |V_{ji}|^2 m^2_{ij}/(v^2 W^2 M^1_W) \sim 10^{-7}$, which is very small compared with the present measurement precision of the CKM matrix elements. Thus in actual calculations this deviation can be neglected.

Since $\delta V$ doesn’t comply with the unitary criterion, Diener has put forward an amended prescription, to shift $\delta V_{ij}$ as

$$\delta \delta V_{ij} = \frac{1}{2} (\delta V_{ij} - \sum_{k,l} V_{kb} \delta V^*_h V_{lj}) \tag{11}$$

It is easy to check that $\delta \delta V$ satisfies the unitary criterion at one-loop level. On the other hand, $\delta \delta V$ is gauge independent since $\delta V$ is gauge independent, which makes the physical amplitude of $W^+ \to u_i \bar{d}_j$ gauge independent [17,12]. Because the ultraviolet divergence of $\delta V$ satisfies the unitary criterion [8], it is easy to prove that $\delta \delta V$ has the same ultraviolet divergence as $\delta V$ [12]. So the new CKM counterterm $\delta \delta V$ satisfies the three criterions mentioned in section 1 at one-loop level.

### III. N-LOOP RENORMALIZATION OF CKM MATRIX

All of the mentioned prescriptions are only applied to one loop level. A suitable prescription for higher loop level is still not present. In view of the delicacy of Eq.(11), we want to follow the ideas of Ref. [21] and [12] to generalize them to be suitable for higher loop level.

As we know when there is no quark generation mixing the amplitude of $W^+ \to u_i \bar{d}_j$ doesn’t need CKM renormalization. In other words, in the case of no quark generation mixing the amplitude of $W^+ \to u_i \bar{d}_j$ is gauge independent and ultraviolet finite after introducing accurate physical parameter’s counterterms except for the CKM renormalization counterterm. Based on this point we want to choose the CKM counterterm to make the amplitude of $W^+ \to u_i \bar{d}_j$ equal to the amplitude of the same process that is in the case of no quark generation mixing. Thus the CKM counterterm will be equal to the difference between the two amplitudes of $W^+ \to u_i \bar{d}_j$ in the two different cases. Since the reason of introducing CKM counterterm is because of the existence of this difference, it is reasonable to make the CKM counterterm represents this difference.

Now our task is to find this difference. Although at an arbitrary loop level the Feynman diagrams of $W^+ \to u_i \bar{d}_j$ are very complex, it is still very clearly that the difference between the two cases of having quark generation mixing and zero-mixing (without quark generation mixing) only occurs at the fermion line that connects with the fermion external-lines. According the discussion in section 2, the zero-mixing amplitude of $W^+ \to u_i \bar{d}_j$ can be obtained by modifying the amplitude of $W^+ \to u_i \bar{d}_j$ as

1. Do such treatment only at the fermion line that connects with the fermion external-lines: change the CKM matrix elements to unit matrix elements and CKM counterterms to zero; if there have odd number CKM matrix elements, leave a CKM matrix element unchanged (because the unit matrix element is a $\delta$ function, the remaining CKM matrix element must be $V_{ij}$); then do such replacement: $m_{u,j} \to m_{u,i}, m_{d,j} \to m_{d,i}$.

2. After the first step, change the fermion’s mass counterterms and WRC appearing at the fermion line which connects with the fermion external-lines by the first step.

In order to determine the n-loop level CKM counterterm $\delta V_n$ we construct the n-loop level amplitude of $W^+ \to u_i \bar{d}_j$ as follows (where only the n-loop level counterterms are listed for convenience)

$$T_n = A_L [F_{Ln} + V_{ij} (\delta g_n/g + \frac{1}{2} \delta Z_{Wn}) + \frac{1}{2} V \delta Z u L_n + \frac{1}{2} V \delta Z d L_n + \delta V_n] + A_R F_{Rn} + B_L G_{Ln} + B_R G_{Rn} \tag{12}$$

where the added denotation ”n” represents the n-loop level result. According to the above discussion we require this amplitude equal to the amplitude of the same process in the case of zero-mixing:

$$T_n = A_L [F_{Ln}[l] + V_{ij} (\delta g_n/g + \frac{1}{2} \delta Z_{Wn} + \frac{1}{2} \delta Z u L_n + \frac{1}{2} \delta Z d L_n) + A_R F_{Rn} + B_L G_{Ln} + B_R G_{Rn} \tag{13}$$

where the footnote ”[l]” represents the new meaning: changing the quantity according to the two steps we have listed. So we can obtain the n-loop CKM counterterm by comparing Eq.(12) and (13)
\[ \delta V_n = F_{Ln[i]} + \frac{1}{2} V_{ij} (\delta Z_{uLn[i]} + \delta Z_{dLn[i]}) - F_{Ln} - \frac{1}{2} (\delta Z_{uLn} V + V \delta Z_{dLn}) \]  

(14)

Here we argue that such CKM counterterm will contain the subdivergences when they appear at the right hand side of Eq.(14).

We have known that such CKM counterterm doesn’t satisfy the unitary criterion at one-loop level. It needs to be modified. Here we introduce a new set of quantities: \( \delta V_1, \cdots, \delta V_n \), the real CKM counterterm which satisfy the unitary criterion to n-loop level. Our aim is to construct \( \delta V_n \) through \( \delta V_n, \delta V_{n-1}, \cdots, \delta V_1 \), or equivalently, through \( \delta V_n, \delta V_{n-1}, \cdots, \delta V_1 \). Here we require that \( \delta V_n \) is obtained by using \( \delta V_{n-1}, \cdots, \delta V_1 \) as the lower loop level CKM counterterms in Eq.(14). Now the unitary criterion of Eq.(3) becomes

\[
\begin{align*}
\delta V_1 V^\dagger + V \delta V_1^\dagger &= 0, \\
\delta V_2 V^\dagger + V \delta V_2^\dagger &= -\delta V_1 \delta V_1^\dagger, \\
\delta V_3 V^\dagger + V \delta V_3^\dagger &= -\delta V_1 \delta V_2^\dagger - \delta V_2 \delta V_1^\dagger, \\
\cdots, \\
\delta V_n V^\dagger + V \delta V_n^\dagger &= -\delta V_1 \delta V_{n-1}^\dagger - \delta V_2 \delta V_{n-2}^\dagger - \cdots - \delta V_n \delta V_1^\dagger, \\
\cdots
\end{align*}
\]  

(15)

In order to solve these equations, we introduce a set of symbols \( B_n \)

\[ B_n = 0, \quad B_n = \sum_{i=1}^{n-1} -\delta V_i \delta V_{n-i}^\dagger. \]  

(16)

Obviously \( B_n \) satisfies

\[ B_n = B_n^\dagger \]  

(17)

Assuming that we have obtained the counterterms \( \delta V_1, \delta V_2, \cdots, \delta V_{n-1} \) and \( \delta V_n \), the n-loop level CKM counterterm \( \delta V_n \) can be obtained in this way

\[ \delta V_n = \frac{1}{2} (\delta V_n - V \delta V_n^\dagger V + B_n V) \]  

(18)

Using induction it is easy to see that such CKM counterterms will satisfy the unitary criterion to n-loop level.

The next step is to test whether the new CKM counterterm \( \delta V_1 + \delta V_2 + \cdots + \delta V_n \) satisfies the two other criterions: make the physical amplitudes convergent and gauge independent. We can use induction to prove this point. The one-loop level result has been proven in section 2. We only need to prove that if \( \delta V_1 + \cdots + \delta V_{n-1} \) satisfies the two criterions to \( n - 1 \) loop level, \( \delta V_1 + \cdots + \delta V_n \) will satisfy the two criterions to n-loop level. To do so we only need to prove the divergent and gauge-dependent part of \( \delta V_n \) equal to the divergent and gauge-dependent part of \( \delta V_n \), since the latter contains the exact n-loop level divergent and gauge-dependent terms. Based on the renormalizability and predictability of SM, we can predict that the divergent and gauge-dependent part of \( \delta V_n \) must satisfy the unitary criterion at n-loop level,

\[ \delta V_n^{DG} V^\dagger + V \delta V_n^{DG\dagger} = B_n^{DG} \]  

(19)

where the superscript "DG" denotes the divergent or gauge dependent part of the quantity. This is because if not so the unitary condition of the bare CKM matrix will require the divergent and gauge dependent part of the real CKM counterterm different from \( \delta V_n \) thus will reduce the physical amplitude of \( W^+ \rightarrow u_i d_j \) divergent and gauge dependent. Using Eq.(18) and (19) we obtain

\[ (\delta V_n^{DG} - \delta V_n^{DG\dagger}) V = \frac{1}{2} (B_n^{DG} - V \delta V_n^{DG\dagger} V^\dagger - V \delta V_n^{DG\dagger}) = 0 \]  

(20)

This identity manifests that

\[ \delta V_n^{DG} = \delta V_n^{DG\dagger} \]  

(21)

i.e. \( \delta V_n \) contains the same divergent and gauge dependent terms as \( \delta V_n \). So we have proven the CKM counterterm \( \delta V_1 + \delta V_2 + \cdots + \delta V_n \) satisfies the three criterions to n-loop level.

Now we have obtained the suitable CKM counterterm to n-loop level. \( \{ \delta V_1, \delta V_2, \cdots, \delta V_n, \cdots \} \) constructs a series. Here we list the results of \( \delta V_1, \delta V_2, \delta V_3 \) and \( \delta V_4 \).
\[ \delta V_1 = \frac{1}{2} (\partial V_1 - V \partial V_1' V) , \]
\[ \delta V_2 = \frac{1}{2} (\partial V_2 - V \partial V_2' V) + \frac{1}{8} (\partial V_1 V \partial V_2 + V \partial V_1 V \partial V_2' V - V \partial V_2 V \partial V_1' V - \partial V_1 V \partial V_2' V) , \]
\[ \delta V_3 = \delta V_3' V + \frac{1}{8} (\partial V_1 V \partial V_2 + V \partial V_1 V \partial V_2' V + V \partial V_2 V \partial V_1' V + \partial V_1 V \partial V_2' V) , \]
\[ \delta V_4 = \frac{1}{2} (\partial V_1 - V \partial V_1' V) + \frac{1}{8} (\partial V_1 V \partial V_2 + V \partial V_1 V \partial V_2' V + V \partial V_2 V \partial V_1' V - \partial V_1 V \partial V_2' V) , \]
\[ \frac{2}{\alpha A} \left( \sum_{k \neq i} \delta Z_{ikl} V_{ik} + \sum_{k \neq j} V_{ik} \delta Z_{lij} \right) + \frac{V_1}{2} \sum_{k,l} \left( \frac{1}{2} \frac{\partial \delta Z_{ikl}}{\partial V_{ik}} + \frac{\partial \delta Z_{ijl}}{\partial V_{ij}} + \frac{\partial \delta Z_{jkl}}{\partial V_{jkl}} \right) + \frac{1}{2} \left( \sum_{k \neq i} \delta Z_{ikl} V_{ik} + \sum_{l,m,k \neq i} \frac{\partial \delta Z_{ijkl}}{\partial V_{lm}} \right) + \sum_{k \neq j} V_{ik} \delta Z_{ijkl} + \sum_{l,m,k \neq j} V_{ik} \frac{d \delta Z_{ijkl}}{d V_{lm}} \right) A_L \]

We guess our CKM renormalization prescription doesn’t break the present symmetries of SM, e.g. Ward-Takahashi identity, since it only changes the value of CKM matrix elements from \( V_{ij} \) to \( V_{ij} + \delta V_{ij} \).}

**IV. RELATIONSHIP BETWEEN THE UNITARITY AND GAUGE INDEPENDENCE OF CKM MATRIX**

It has been proven using Nielsen identities [27] that any physical parameter’s counterterm must be gauge independent [13,28]. The CKM matrix elements are physical parameters so their counterterms must be gauge independent. At one-loop level this conclusion has been proven [17]. When considering higher loop level case a concrete problem will arise that whether the choice of \( \delta V_1 \) on the gauge independence of the lower loop level CKM counterterms will affect the gauge independence of the higher loop level CKM counterterms? As we know at one-loop level one can choose the gauge-independent convergent part of the CKM counterterm freely. Will the different choices change the gauge independence of the CKM counterterm at two loop level? In order to clarify this problem we express the amplitude of \( W^+ \rightarrow u_i d_j \) as

\[ T(V^0) = T(V + \delta V) = T(V) + T'(V) \delta V + \frac{1}{2} T''(V)(\delta V)^2 + \cdots \]

where the superscript \( t \) denotes the partial derivative with respect to CKM matrix of the quantity. To two loop level, this equation becomes

\[ T_2(V^0) = T_2(V) + T_1'(V) \delta V_1 + \delta V_2 A_L \]

with \( T_2(V) \) and \( T_1(V) \) the 2-loop and 1-loop amplitudes of \( W^+ \rightarrow u_i d_j \) which don’t contain CKM counterterms. In order to find the effect of the choice of \( \delta V_1 \) on the gauge independence of \( \delta V_2 \) we need to calculate \( T_1'(V) \delta V_1 \) analytically. From Eq. (4), since \( F_2 \) and \( G_L,R \) are gauge independent and don’t contain CKM matrix element, only the terms in the first bracket of Eq. (4) need to be considered. Using the fact that the terms in the first bracket of Eq. (4) is gauge independent [17], we have

\[ T_1'(V) \delta V_1 |_{\xi} = \frac{\alpha A}{2 \pi} \sum_{k \neq i} (\delta V_{ikl} V_{ik} + \sum_{k \neq j} V_{ik} \delta Z_{ijkl}) + \sum_{k \neq i} (\delta V_{ikl} V_{ik} + \sum_{k \neq j} V_{ik} \delta Z_{ijkl}) + \sum_{k \neq i} V_{ik} \delta Z_{ijkl} + \sum_{l,m,k \neq i} \frac{d \delta Z_{ijkl}}{d V_{lm}} + \sum_{k \neq j} V_{ik} \delta Z_{ijkl} + \sum_{l,m,k \neq j} V_{ik} \frac{d \delta Z_{ijkl}}{d V_{lm}} \right) A_L \]

where the subscript “1” of \( \delta V_1 \) has been omitted and the subscript “\( \xi \)” on the left hand side of this equation denotes the gauge dependent part of the quantity. Omitting the imaginary parts of the quark’s self energies (because they are gauge independent), we obtain

\[ T_1'(V) \delta V_1 |_{\xi} = \frac{\alpha A}{2 \pi} \sum_{k \neq i} (\delta V_{ikl} V_{ik} + \sum_{k \neq j} V_{ik} \delta Z_{ijkl}) + \sum_{k \neq i} (\delta V_{ikl} V_{ik} + \sum_{k \neq j} V_{ik} \delta Z_{ijkl}) + \sum_{k \neq i} V_{ik} \delta Z_{ijkl} + \sum_{l,m,k \neq i} \frac{d \delta Z_{ijkl}}{d V_{lm}} + \sum_{k \neq j} V_{ik} \delta Z_{ijkl} + \sum_{l,m,k \neq j} V_{ik} \frac{d \delta Z_{ijkl}}{d V_{lm}} \right) A_L \]

\[ (\xi W M_{\delta u_i} - m_{d_j}) \arctan \frac{-\xi W M_{\delta u_i} + 2 \xi W m_{d_j} M_{W} - m_{d_j} - m_{u_i} \ln |\xi W|}{\xi W M_{\delta u_i} + 2 \xi W m_{d_j} M_{W} - m_{d_j} + m_{u_i} \ln |\xi W|} + 2 \xi W m_{d_j} M_{W} - m_{d_j} + m_{u_i} \ln |\xi W| \]

\[ (\xi W M_{\delta u_i} - m_{d_j}) \arctan \frac{-\xi W M_{\delta u_i} + 2 \xi W m_{d_j} M_{W} - m_{d_j} - m_{u_i} \ln |\xi W|}{\xi W M_{\delta u_i} + 2 \xi W m_{d_j} M_{W} - m_{d_j} + m_{u_i} \ln |\xi W|} + 2 \xi W m_{d_j} M_{W} - m_{d_j} + m_{u_i} \ln |\xi W| \]
with $\xi_W$ the W boson gauge parameter. It can be seen that if $\delta V_1$ satisfies the unitary criterion, the gauge dependent part of $T'_1(V)\delta V_1$ will be equal to zero. On the other hand, from Eq.(24) we can see that the gauge dependent part of $\delta V_2$ is determined by the gauge dependent part of $T_2(V)$ and $T'_1(V)\delta V_1$ since $T_2(V_0)$ is gauge independent. So we know as long as $\delta V_1$ satisfies the unitary criterion, no matter how to change the value of $\delta V_1$ will not affect the gauge independence of $\delta V_2$. On the contrary, if $\delta V_1$ doesn’t satisfy the unitary criterion, changing the value of $\delta V_1$ will change the gauge dependent part of $\delta V_2$, thus will make $\delta V_2$ gauge dependent. Therefore we can draw a conclusion that only if keep the unitarity of the bare CKM matrix in the CKM matrix renormalization prescription the renormalized CKM matrix and its counterterm will be gauge independent.

V. CONCLUSION

In summary, we have investigated the present CKM matrix renormalization prescriptions and found all of them are only suitable for one loop level. In this paper, we have checked the prescription in Ref. [21] and found it doesn’t satisfy the unitary criterion of the bare CKM matrix. There is also an error in this prescription. The correct one-loop CKM counterterm should be the form of Eq.(7). Then we generalize the prescription in Ref. [21] and [12] to make it suitable for any loop level and comply with the unitary criterion of the bare CKM matrix. The concrete results are shown in Eq.(18) and Eqs.(22). Our prescription also makes the amplitude of an arbitrary physical process involving quark mixing convergent and gauge independent, as required. Lastly We point out that only if the CKM renormalization prescription keeps the unitarity of the bare CKM matrix the renormalized CKM matrix and its counterterm will be gauge independent.

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