ON THE B AND J/ψ CROSS SECTION MEASUREMENTS

AT UA1 AND CDF

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Abstract

We analyse the implications of the measurement of $B$ and $J/ψ$ inclusive $p_t$ distributions performed in $pp$ collisions by the UA1 and CDF experiments.
1 Introduction

Heavy quark production in high energy hadronic collisions constitutes a fundamental arena for the study of perturbative QCD. The comparison of experimental data with the predictions of QCD provides a necessary check that the ingredients entering the evaluation of hadronic processes (partonic distribution functions and higher order corrections) are under control and can be used to evaluate the rates for more exotic phenomena or to extrapolate the calculations to even higher energies. The estimates of production rates for the elusive top quark rely on the understanding of heavy quark production properties within QCD. Following the initial encouraging agreement between the UA1 measurements of inclusive bottom-quark and $J/\psi$ $p_t$ distributions and the best available QCD calculations, the subject has acquired a new particular interest after the latest measurements by UA1 and CDF and after the recent studies of the small-$x$ behaviour of hadro-production cross-sections.

On the experimental side UA1 has confirmed with the most recent analyses of the $b$ sample the agreement between next-to-leading order (NLO) QCD and data, extending it to large values of $p_t$. At the same time, however, UA1 has found a significant disagreement between its latest $J/\psi$ $p_t$ spectrum and the available leading order (LO) QCD calculations. Independently, CDF preliminary and published results indicate a clear discrepancy with respect to theoretical expectations for normalization and shape of the $b$ and $J/\psi$ $p_t$ spectra.

On the theoretical side, studies have indicated that large corrections to the NLO evaluation of the $b$ cross section should be expected at very large energies. A precise quantitative definition of how large these energies should be for these effects to become dominant is however still missing, and the question of whether these effects can indeed justify the findings by CDF is still open.

In parallel, several measurements have improved the knowledge of the parton distribution functions (PDF) down to values of $x$ of the order of 0.01, and the relative new sets of parametrizations have now become available. Both these latest developments could have some bearing on the measurements of the $b$ and $J/\psi$ production rates at UA1 and CDF, because the values of $x$ probed by CDF are approximately a factor of 3 smaller than those explored by UA1. Therefore CDF is more sensitive than UA1 to the possible uncertainties of the extrapolation to small-$x$ of both PDF’s and partonic cross-sections.

Attempts have also been made to incorporate directly into the fits of gluon distributions the experimental information contained in the $b$ cross section measurements by UA1 and CDF. However when the new PDF measurements quoted above are included as an additional constraint, there seems not to be enough freedom to correct the theoretical prediction by the needed amount to produce a fully satisfactory agreement with CDF data.

In this letter we will reconsider these data and use the most recent inputs to shed, if possible, some more light on these problems.
2 The inputs of the theoretical calculations

We will start by discussing the ingredients of the calculation presented here and by comparing them to the analyses quoted previously.

$J/\psi$ production is evaluated by summing the result of inclusive $b$ production followed by $B \to J/\psi + X$ decays, direct $J/\psi$ production and production via radiative decays of $\chi$ states. These last two processes we will refer to as direct charmonium production, while the first one will be referred to as $b$-decays. The matrix elements for charmonium production are taken from the original calculations of ref [21], and therefore coincide with those used in previous studies ([1], from now on indicated by GMS).

As an improvement with respect to GMS we will use:

1. the full NLO matrix elements for the production of $b$ quarks before their decay into $J/\psi$;
2. the most recent inclusive $B \to J/\psi + X$ and $B \to \psi(2S) + X$ fragmentation spectra [22, 10];
3. the most recent parametrizations of PDF described in [17], exploring the dependence on the value of the 2-loop $\Lambda_4$ within the one standard deviation range $\Lambda_4 = 215 \pm 60$MeV.

This third point, in particular, will cause a significant drop in the prediction of the absolute rates for charmonium compared to GMS, because the PDF’s used in GMS had a much larger value of $\Lambda_4$ ($\Lambda_4^{1\text{-}\text{loop}} = 400$ MeV versus $\Lambda_4^{2\text{-}\text{loop}} = 215$ MeV as the central value of the new sets, corresponding to $\Lambda_4^{1\text{-}\text{loop}} = 140$ MeV.).

As was done in GMS we will smear the $b$-quark $p_t$ with a Peterson fragmentation function [23] before its decay into $\psi$’s:

$$\frac{dN}{dz} \propto \frac{z(1-z)^2}{[\epsilon z + (1-z)^2]^2},$$

with $\epsilon = 0.006$. Furthermore we will assign to the $b$-meson after fragmentation a mass of 5.28 GeV, regardless of the input $b$-quark mass.

The central values we use for the $b$-quark mass and the factorization/renormalization scale $\mu$ are $m_b=4.75$ GeV and $\mu^2 = m_b^2 = p_t^2 + m_b^2$ [1]. We will consider in the following the effect of changing these parameters. The numerical values of other parameters used in the calculations are contained in Tables 1 and 2.

\[\text{Notice that this prescription is slightly different from the one chosen in [3], where } \mu^2 = p_{t_{\text{min}}}^2 + m_b^2 = m_T^2 \text{ was used to estimate } \sigma(p_t > p_{t_{\text{min}}}). \text{ This difference amounts to an effect of the order of } -20\text{% at the smallest values of } p_t.\]
2.1 Discussion of the uncertainties in the calculations

Several factors contribute to the uncertainty of the calculations. We will discuss first those intrinsic to the perturbative approach and will consider later those related to external inputs such as structure functions, $\Lambda_{QCD}$ or potential models in the case of the charmonium production.

First we consider $b$ production. As is well known, the radiative corrections to $b$ production are rather large and extremely unstable under changes in factorization scale $\mu$, in particular at the larger CM energies. The standard way to establish how reliable the perturbative expansion is, is to vary the factorization scale $\mu$ within some range of the order of the hard energy scale relevant for the process considered. If the observed change in cross sections is large, one might think of selecting a best value for $\mu$ by fitting the measured rates. Since the dependence on $\mu$ varies as a function of the beam energy (indicating that the effect of yet higher order corrections is not the same at different energies) there is no reason a priori for which the expression for $\mu$ fitted at one value of the beam energy should be the same at different energies.

Therefore the uncertainty related to the choice of $\mu$ cannot be removed by performing independent measurements at different energies.

Likewise, there is no guidance on what is the proper range within which to allow $\mu$ to vary. If $\mu_0$ is the typical energy scale of a given process (say the transverse mass of a heavy quark or the $E_t$ of a jet), it is customary to vary $\mu_0/2 < \mu < 2\mu_0$. There are several indications, however, that when working at a fixed order in perturbation theory scales significantly smaller than the natural scale are needed to reproduce the data. As examples, we quote the cone-size dependence of the jet $E_t$ distributions in hadronic collisions [24] or the jet multiplicity distributions in $e^+e^-$ [25]. Wherever all-order calculations have become available, these indicate that the resummation of leading and sub-leading large logarithmic terms at any order in the perturbative expansion restores the insensitivity to $\mu$ and allows $\mu$ to be chosen of the order of the natural scale $\mu_0$ [26].

We therefore believe it is legitimate to push $\mu$ to values as low as possible, compatibly with the range allowed by the PDF parametrizations. In our case, we will consider the range $m_T/4 < \mu < m_T$, which will give us $\mu^2 > 5$ GeV$^2$ for all values of $p_t$ probed by the CDF data, and therefore does not include regions of $Q^2$ which are not under the control of the DIS data.

Similar considerations apply to the case of direct quarkonium production of $J/\psi$’s. Here the situation is even worse, because only the LO production processes are available and the expected $\mu$ dependence is even more significant. In GMS the effect of possible higher order terms was parametrized in terms of a constant $K$ factor, chosen to have a value of 2 to reproduce ISR data. As mentioned above, however, there is no reason a priori why this same value for $K$ should apply at the significantly higher energies used at UA1 and CDF. Once again, therefore, we will choose to probe the possible effects of higher order terms by selecting different values of $\mu$. In accordance with the choice made for the $b$ production, we will choose $m_T/4 < \mu < m_T$, where in this case $m_T$ represents the transverse mass of the quarkonium state. As will be shown later on, different values
of $\mu$ will not only change the absolute normalization but will also affect the shape of the $p_t$ distribution of $J/\psi$'s.

In addition to the above uncertainties, one should add the uncertainty in the evaluation of the parameters of the quarkonium states entering the estimate of their production cross section. As an example, we quote a recent study \cite{27} of $|R'(0)|^2$ – the first derivative of the wave function at the origin for P-wave states – indicating a value for $\chi_c$ states which is approximately 50% higher than what obtained from previous models \cite{28}. The values used in this paper are given in Table 1, and follow the old results of \cite{28}.

**2.2 Numerical Results**

We collect the results obtained for the $b$ and $J/\psi$ cross sections as a function of the various input parameters in a series of tables. As a standard reference we will use sets D0 and D– of the recent MRS PDF parametrization \cite{17}. We will use two values of $\Lambda_{QCD}$, $\Lambda_0^0=215$ MeV and $\Lambda_+^+ =275$ MeV, corresponding respectively to the central value and to one standard deviation above the central value obtained from the fit. Tables \ref{table:3} and \ref{table:4} contain the bottom quark $p_t$ distribution integrated above a given $p_t$ at 1.8 TeV and for the two extreme values of $\mu$, $\mu=m_T$ and $\mu=m_T/4$. The quark is required to satisfy $|y|<1$, to allow comparison with the CDF data. Tables \ref{table:6} and \ref{table:7} contain the same information, but at 630 GeV and with $|y|<1.5$, to allow comparison with UA1 data.

Several comments are in order. First of all notice that while the use of the more singular set of structure functions leads to larger values of the total cross sections ($p_t>0$) at 1.8 TeV, the opposite happens at 630 GeV. This is because the higher density of gluons at small $x$ described by set D– forces via momentum sum rules a depletion at larger values of $x$. Since UA1 is sensitive to larger values of $x$, an overall decrease in the total cross section is observed.

Notice also, on the other hand, that even at CDF the singular gluon parametrization D– will give a cross section smaller than the set D0 as soon as we consider transverse momenta of the $b$ above 10 GeV – which is the region where most of the CDF data are. Since above 10 GeV the shapes of the integrated $p_t$ distributions for the two parametrization D0 and D– are similar, this indicates that the measurement of the cross section for $b$ production in this region cannot be reliably used to extract via extrapolation to $p_t=0$ the total $b$ production cross section. For example, while the region $p_t > 10$ GeV represents 10% of the total cross section according to D0 and using $\Lambda_{QCD}=215$ MeV, the same region represents only 7% of the total according to D– and using $\Lambda_{QCD}=275$ MeV. A similar exercise at the UA1 energy indicates a more reliable extrapolation.

As already indicated in \cite{8}, the dependence on the value of the $b$ mass is not significant. In Table \ref{table:8} we show a comparison between the integrated $b$ $p_t$ distribution obtained using $m_b=4.5$ and $m_B=4.75$ GeV. The difference is of the order of 20% for the total cross section, but becomes negligible for $p_t>10$ GeV.

In Table \ref{table:9} we present the integrated $p_t$ distribution of $J/\psi$ mesons, calculated at CDF energy and divided into the direct quarkonium and $B$ decay contributions. The relative fraction due to $B$ decays, indicated by $f_B$, is also shown as a function of the $p_t$
threshold, and the dependence on $\Lambda_{QCD}$ and $\mu$ is studied by considering the central case of $\Lambda_{QCD}=215\,\text{MeV}$, $\mu=m_T$ and the extreme case of $\Lambda_{QCD}=275\,\text{MeV}$, $\mu=m_T/4$. A priori there is no reason why the same factorization scale should be used for the two contributions, as the two physics processes are entirely different. Furthermore the $B$ decay is evaluated at NLO, while as mentioned previously only the LO terms are available and included in the quarkonium term. Nevertheless we take here the value of $\mu$ for the two processes to be the same, in order to extract and indicative range of values for $f_B$. The value of $f_B$ plays an important role in the experimental determination of the $B$ cross section out of the measurement of the inclusive $J/\psi$ rate, and the range of values exhibited by the tables indicates what is the systematic uncertainty that one should expect in deriving $f_B$ from the theory. Parallel results for UA1 energy are shown in Table 9.

The most important thing to notice about these tables is the fact that the $B$ contribution only changes within a factor of 2 by changing $\mu$, while a variation ranging from a factor of 7 to 10, depending on $p_t$, is observed for the charmonium case. This indicates that the LO prediction for direct charmonium is very poor, and very large NLO corrections should be expected.

3 Comparison with the data and discussion

We will now compare UA1 and CDF data with the results obtained so far. For these comparisons we use the results obtained with the D0 PDF set and with the two different choices $(\mu, \Lambda_{QCD}) = (m_T, 215\,\text{MeV})$ and $(\mu, \Lambda_{QCD}) = (m_T/4, 275\,\text{MeV})$, which provide an acceptable upper and lower limit to the band of current theoretical uncertainty relative to the inputs discussed above.

Figs. 1 and 2 show the UA1 measurement of the inclusive $b$-quark and $B$ meson $p_t$ distribution integrated above a given threshold $p_t^{min}$ and with $|y_{b,B}| < 1.5$. The agreement between data and theory observed in ref.[8] is confirmed, even though the central value of the prediction has dropped by almost 50% as a consequence of the smaller value of $\Lambda_4$ in the central MRS fit compared to the central DFLM fit (215 versus 260 MeV). Also the $B$-meson spectrum is well consistent with what expected from a Peterson fragmentation model, as anticipated in [1]. Notice, however, that a priori there is no guarantee that the Peterson model should work for values of $p_t$ of the order of the $B$ mass, as in this region corrections to factorization could be significant.

Fig. 3 shows the inclusive $p_t$ differential distribution for $J/\psi$’s produced with $p_t > 5\,\text{GeV}$ and $|y| < 2$. We superimpose the contributions from direct charmonium production, $b$-decays and the sum of the two. The data fall all inside the theoretical band. Since as mentioned above there is no reason to expect the expression for $\mu$ to be the same for the two contributions – while for the sake of simplicity we imposed this in adding the separate terms in the figure – a better fit to the data could be obtained by choosing $\mu=m_T/4$ for the charmonium and $\mu=m_T$ for the $B$ production.

Fig. 4 shows the integrated $p_t$ distribution of $b$ quarks with $|y| < 1$ from CDF, compared to the results of the NLO calculation. The data are taken from published results.
as well as from recent public presentations [9, 10, 11]. As already observed in [9] there is a clear excess in the observed rate at small $p_t$. At larger values of $p_t$, in the region of the inclusive $b \to l + X$ measurements, the data are consistent with the upper extreme of the theoretical band. A similar feature is observed in the $\psi$ differential $p_t$ measurement, shown in fig. 3.

Equally worrisome is the comparison between theory and data in the case of the $p_t$ spectrum of the $\psi(2S)$, shown in fig. 4. As noted in [10, 11], the expected contribution from direct quarkonium production is heavily suppressed. We confirm this estimate, and verified that it remains true even allowing for the variation of $\mu$ within the $\mu_0/4 < \mu < \mu_0$ range.

Is it possible to explain the patterns observed by UA1 and CDF in a unified fashion by invoking generic small-x effects, either from PDF’s or from violation of factorization? Rather than studying this question by directly attempting to modify the gluon densities, as done in refs. [19, 20], we will address it here by considering the following quantity:

$$\sigma(x_g < x; p_t^b > p_t^{\text{min}}) = \int_{x_g}^{x} dx_g \frac{d\sigma(p_t^b > p_t^{\text{min}})}{dx_g}, \quad (3.1)$$

namely the contribution to the integrated $p_t$ distribution coming from partons with momentum fraction smaller than a given value of $x$. We plot this variable as a function of $x$ and for different values of $p_t^{\text{min}}(b)$ in Fig. 3 for UA1 and CDF. We only integrated over $b$ quarks within the regions of acceptance of the experiments, namely $|y_b| < 1.5$ for UA1 and $|y_b| < 1$ for CDF. Since the contribution to the cross-sections due to the $q\bar{q}$ and $qg$ initial states are negligible for the relevant regions of $p_t$ we are concerned with, we limited ourselves to the $gg$ process and normalized the curves to the value of 1 at $x = 1$. Therefore the plotted functions represent the fraction of cross-section due to gluons with $x_g < x$.

The first thing to notice is that the distribution corresponding to $p_t > 5$ GeV at UA1 lies between the curves for $p_t > 10$ and $p_t > 20$ GeV at CDF, consistently with the factor of 3 difference in beam energy. The second thing to notice is that at CDF energies the contribution to the cross section for $p_t > 10$ GeV from the region $x < 0.01$ is less than 20%. Furthermore no contribution at all comes from the region $x < 0.003$. We verified that different fits of the NMC and CCFR data, obtained in Ref. [18], give gluon densities which differ, over the relevant kinematic range, by no more than 10% from the MRSD0 set used here. Since all of these gluon parametrizations do not differ significantly from previous extrapolations, we conclude that the knowledge of the gluon density in the relevant region $0.1 > x > 0.01$ and $Q > 5$ is today rather solid. We therefore expect that only dramatic changes in the gluon densities in the region $0.003 < x < 0.01$ will lead to a change of a factor of 2 in the cross section integrated above $p_t = 10$ GeV.

Therefore while it is tempting to conjecture that the ignorance about the behaviour of the gluon densities at small-$x$ could explain the discrepancy between the overall rates measured by UA1 and CDF and the difference in slope of the CDF spectra compared to theory, we find no evidence that this assumption is justified. Rather, we find that the region $x_g < 0.01$ is marginal in the production of $b$ quarks or $\psi$’s passing the required acceptance and $p_t$ cuts imposed by the two experiments.
The effect of the small-x corrections to the partonic cross-section considered in \cite{12, 14} is more difficult to estimate. In fact these phenomena alter the kinematic connection between $p_t$ and $x$, since they predict that initial state gluons with a given momentum fraction $x$ can have a $p_t$ non negligible w.r.t. $xE_{\text{beam}}$. This is equivalent to having an intrinsic $p_t$ of the order of the scale of the hard process itself, namely $m_b$. As a result, the region with $x_g < 0.01$ could provide a significant contribution to the rate for $p_{tb} > 10$ GeV, thanks to the transverse momentum smearing induced by this sort of small-x primordial $p_t$. Even though it was found in ref.\cite{12} that these small-x effects can add at most 50% of the NLO contribution to the total $b$ cross section at 1.8 TeV, no explicit indication is given on the $p_t$ distribution of this additional 50%. Since the cross section observed experimentally ($p_{tb} > 8.5$ GeV) represents of the order of 10% of the total rate at NLO, we cannot exclude that the $p_t$ smearing induced by these effects be responsible for the factor of 2-3 discrepancy observed between data and NLO predictions. Notice that the hypothesis of a $p_t$ smearing would help understanding not just the rate deficiency, but also the apparent difference in shape between NLO and data. A quantitative statement regarding these possibilities will only come from more explicit studies along the lines of ref.\cite{14}.

While we await for more explicit calculations, it might be worth exploring some additional consequences of this scenario. In addition to trying to push the measurement of the $b$ cross section to even smaller values of $p_t$, it would be important to study correlations between the pair of $b$ quarks. NLO calculations exist for these correlations \cite{29}. If the small-x effects were to behave as indicated previously, we would expect to observe a flattening of the $\Delta \phi$ and $p_t^{bb}$ distributions w.r.t the NLO prediction. Here $\Delta \phi$ represents the difference in azimuth between the $b$ and the $\bar{b}$, and $p_t^{bb}$ represents the transverse momentum of the pair. The flattening would be caused by the additional intrinsic $p_t$ due to the gluon transverse momentum.

Measurements of the $\Delta \phi$ correlations have been performed by UA1 \cite{30}, indicating a good agreement with the NLO calculation \cite{29}. This result does not resolve the issue, though, because the agreement of the NLO $b$ cross section with the data suggests that the energy at UA1 is below the threshold for the possible onset of these new small-x phenomena.

4 Conclusions

After allowing for rather generous estimates of the theoretical uncertainties involved in the calculations currently available for $b$ and $J/\psi$ production in hadronic collisions, we conclude that the most worrisome points of discrepancy can be summarised as follows:

1. The production of direct charmonium both at CDF and UA1 is much more abundant than would be obtained from the LO calculation using a standard value of $\mu=m_T$. The $J/\psi$ can be explained by using $\mu=m_T/4$, which however gives a rate 8-10 times larger than for $\mu=m_T$, indicating a rather unstable perturbative expansion. However this is not sufficient to explain the rate of $\psi(2S)$ production.
2. $b$ production at CDF for values of $p_t$ around 10 GeV is significantly larger than can be accommodated by current estimates of the higher order effects or by possible structure function uncertainties. Using the extreme value of $\mu = m_T/4$ is not sufficient to explain all the data points, the discrepancy being still larger than a factor of 2. We cannot however exclude that the solution be in the large smearing induced by a small-$x$ intrinsic $p_t$ of the initial state gluons. At larger values of $p_t$ we believe that the consistency between data and theory is acceptable.

What other effects could be responsible for the remaining discrepancies? It should be noticed that the two points above might not be uncorrelated. In fact the absolute normalization of the two CDF points at lower $p_t$, coming from the measurement of inclusive $J/\psi$ and $\psi(2S)$ rates \cite{10}, relies on two assumptions: (i) that all of the $\psi(2S)$ come from $B$ decays, and (ii) that the $J/\psi$ fraction $f_B$ is known. The right hand side of Table 8 – which represents the choice of parameters which comes closer to representing the CDF $J/\psi$ spectrum – suggests a value for $f_B$ which is significantly smaller than the central value used by CDF (namely 37\% vs. 63$\pm$17\% for $p_t(\psi) > 6$ GeV \cite{11}). This would decrease the effective $b$ cross section by a factor of 50\%. In addition, the new processes responsible for the large $K$ factor apparent in $J/\psi$ production might affect $\psi(2S)$ production and could provide enough rate to reduce the $b$ rate extracted from the assumption that $B$ decays are the only source of $\psi(2S)$.

Notice that $f_B$ could in principle be extracted experimentally, for example by separating the direct $J/\psi$’s from those due to $B$ decays via the observation of the displaced vertex from which the $\psi$ originates – due to the long $B$ lifetime. UA1 measured $f_B$ by assuming that direct $J/\psi$’s are isolated while $J/\psi$’s from $B$ decays are not, and studying the isolation of the $J/\psi$’s in the data. This assumption however might not be correct if other production mechanisms were responsible for direct quarkonium production, such as for example gluon $\rightarrow J/\psi$ fragmentation \cite{31}.

It is very reasonable to expect that at some value of $p_t$ the dominant production mechanism for charmonium states will indeed be via gluon fragmentation. The main reason being that direct production as described by the LO mechanisms inhibits production at large $p_t$ via a form factor suppression (the probability that a charmonium bound state will hold together when produced directly in an interaction with a large virtuality scale is highly suppressed). The fragmentation functions for the creation of $S$-wave charmonium ($\eta_c$ and $J/\psi$) in a gluon shower have recently been calculated \cite{31} and work on the creation of $P$-wave states ($\chi$) is in progress \cite{32}. It will be interesting to use these calculations in order to extract the fragmentation contribution to charmonium production in the regions of $p_t$ explored experimentally, and verify whether these new processes can account at least in part for the large observed $K$ factor. The experimental detection of non-isolated $J/\psi$’s from a primary vertex – and therefore presumably not coming from $B$ decays – could provide a strong indication that these processes are indeed present.

Similar measurements of the decay-vertex position of the $\psi(2S)$ would provide evidence in favour or against the current belief that most of them come from $B$ decays. Once again the gluon fragmentation contribution to production of this charmonium state could turn out to be significant, and would manifest itself with a signal of non-isolated prompt $\psi(2S)$. 

8
Similarly interesting would be a separate measurement of the $\chi$ $p_t$ spectrum, which is expected to be dominated by direct production rather than $B$ decays. A preliminary measurement by CDF [11] reports $BR(\psi \to \mu^+\mu^-) \times \sigma(\chi_c \to \psi\gamma; p_t \chi > 7 \text{ GeV}; |\eta| < 0.5) = 3.2 \pm 0.3 \pm 1.2 \text{ nb}$. Both $\chi_1$ and $\chi_2$ are here included. This can be compared with the range $0.64 \text{ nb} < \sigma < 5.1 \text{ nb}$ obtained using the calculation described in the previous sections and the two extreme choices $(\mu, \Lambda_{QCD}) = (m_T/4, 215 \text{ MeV})$ and $(\mu, \Lambda_{QCD}) = (m_T/4, 275 \text{ MeV})$. Using the above cross section and using the inclusive $B \to \chi_c \ell$ branching ratio of $0.54 \pm 0.21\%$ [33], we estimate that only a fraction of the order of 10% or less – depending on $p_t$ – of the $\chi$’s come from $B$ decays. Since the production mechanisms for $\chi_1$ and $\chi_2$ are different even at LO [21], a separate measurement of the two states would be welcome, even though their closeness in mass makes it very hard to separate one from the other in practice.

It would also be interesting to evaluate the effects of resumming some of the leading and next-to-leading corrections to the evolution of the initial state, using the calculation of the $gg \to$ color-singlet NLO form factor calculated in ref. [34].

An experimental measurement of the production cross section and $p_t$ spectrum for $\Upsilon$ states would be very useful in understanding the quarkonium production mechanisms [35]. In this case, in fact, one would have at least three advantages: (i) the masses involved are larger and presumably both the non-relativistic approximation involved in the determination of the quarkonium wave function and the QCD perturbative expansion would work much more reliably than for charmonium; (ii) the signal does not have a contamination similar to the one due to $B$ decays; (iii) the $p_t$ spectrum could hopefully be extended to very small values of $p_t$, possibly even to $p_t=0$, thanks to the large mass of the $\Upsilon$ and the rather large momentum and easier detection of the decay muons.

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| $|R_{1S}(0)|^2$ | $|R_{2S}(0)|^2$ | $|R'_{1P}(0)|^2$ |
|----------------|----------------|----------------|
| 0.7            | 0.4            | 0.006          |

Table 1: Values of wave functions used in the evaluation of the charmonium cross sections.

\[
\begin{array}{|c|c|}
\hline
BR(\bar{B} \to \psi X) \times BR(\psi \to \mu^+\mu^-) & BR(\bar{B} \to \psi(2S)X) \times BR(\psi \to \mu^+\mu^-) \\
7.7 \times 10^{-4} & 3.6 \times 10^{-5} \\
\hline
\end{array}
\]

Table 2: Values of BR’s used in the evaluation of the $J/\psi$ cross sections.

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Table 3: Integrated bottom quark $p_t$ distribution at 1.8 TeV. $m_b=4.75$ GeV, $\mu=\mu_0$, $\Lambda_0 = 215$ MeV, $\Lambda_+ = 275$ MeV.

| $p_t^{\text{min}}$ (GeV) | MRSD0 |     | MRSD– |     |
|--------------------------|-------|-----|-------|-----|
|                          | $\Lambda_0$ | $\Lambda_+$ | $\Lambda_0$ | $\Lambda_+$ |
| 0                        | 1.14E+04  | 1.34E+04  | 1.33E+04  | 1.57E+04  |
| 5                        | 4.50E+03  | 5.22E+03  | 4.50E+03  | 5.27E+03  |
| 10                       | 1.05E+03  | 1.22E+03  | 9.45E+02  | 1.09E+03  |
| 15                       | 3.16E+02  | 3.56E+02  | 2.70E+02  | 3.11E+02  |
| 20                       | 1.15E+02  | 1.32E+02  | 9.69E+01  | 1.12E+02  |
| 25                       | 4.97E+01  | 5.58E+01  | 4.20E+01  | 4.75E+01  |
| 30                       | 2.40E+01  | 2.72E+01  | 2.06E+01  | 2.34E+01  |
| 40                       | 6.94E+00  | 7.85E+00  | 6.05E+00  | 6.80E+00  |
| 50                       | 2.60E+00  | 2.79E+00  | 2.16E+00  | 2.38E+00  |
| 59                       | 1.21E+00  | 1.30E+00  | 1.03E+00  | 1.14E+00  |

Table 4: Integrated bottom quark $p_t$ distribution at 1.8 TeV. $m_b=4.75$ GeV, $\mu=\mu_0/4$, $\Lambda_0 = 215$ MeV, $\Lambda_+ = 275$ MeV.

| $p_t^{\text{min}}$ (GeV) | MRSD0 |     | MRSD– |     |
|--------------------------|-------|-----|-------|-----|
|                          | $\Lambda_0$ | $\Lambda_+$ | $\Lambda_0$ | $\Lambda_+$ |
| 0                        | 2.17E+04  | 3.03E+04  | 2.69E+04  | 3.71E+04  |
| 5                        | 7.23E+03  | 9.39E+03  | 7.68E+03  | 1.00E+04  |
| 10                       | 1.83E+03  | 2.27E+03  | 1.66E+03  | 2.09E+03  |
| 15                       | 5.78E+02  | 7.11E+02  | 4.93E+02  | 6.10E+02  |
| 20                       | 2.23E+02  | 2.68E+02  | 1.83E+02  | 2.25E+02  |
| 25                       | 9.86E+01  | 1.17E+02  | 7.98E+01  | 9.75E+01  |
| 30                       | 4.84E+01  | 5.63E+01  | 3.89E+01  | 4.72E+01  |
| 40                       | 1.44E+01  | 1.65E+01  | 1.23E+01  | 1.38E+01  |
| 50                       | 5.28E+00  | 6.04E+00  | 4.48E+00  | 4.87E+00  |
| 59                       | 2.23E+00  | 2.49E+00  | 2.04E+00  | 2.27E+00  |
Table 5: Mass dependence of the integrated bottom quark $p_t$ distribution at 1.8 TeV. MRSD0 parton distributions, $\mu=\mu_0/4$, $\Lambda_0 = 215$ MeV, $\Lambda_+ = 275$ MeV.

| $p_{t_{\text{min}}}$ (GeV) | $\Lambda_0$ | $\Lambda_+$ |
|-----------------------------|------------|------------|
| $m_b=4.5$ GeV | $m_b=4.75$ GeV | $m_b=4.5$ GeV | $m_b=4.75$ GeV |
| 0 | 2.6E+04 | 2.2E+04 | 3.8E+04 | 3.0E+04 |
| 5 | 8.0E+03 | 7.2E+03 | 1.0E+04 | 9.4E+03 |
| 10 | 1.9E+03 | 1.8E+03 | 2.4E+03 | 2.3E+03 |
| 20 | 2.3E+02 | 2.2E+02 | 2.8E+02 | 2.7E+02 |

Table 6: Integrated bottom quark $p_t$ distribution at UA1. $m_b=4.75$ GeV, $\mu=\mu_0$, $\Lambda_0 = 215$ MeV, $\Lambda_+ = 275$ MeV.

| $p_{t_{\text{min}}}$ (GeV) | MRSD0 | MRSD– |
|-----------------------------|--------|--------|
| $\Lambda_0$ | $\Lambda_+$ | $\Lambda_0$ | $\Lambda_+$ |
| 0 | 6.016E+03 | 7.110E+03 | 5.584E+03 | 6.606E+03 |
| 5 | 1.838E+03 | 2.149E+03 | 1.582E+03 | 1.848E+03 |
| 10 | 2.953E+02 | 3.390E+02 | 2.482E+02 | 2.857E+02 |
| 15 | 6.353E+01 | 7.183E+01 | 5.378E+01 | 6.188E+01 |
| 20 | 1.776E+01 | 2.009E+01 | 1.566E+01 | 1.778E+01 |
| 25 | 6.148E+00 | 6.714E+00 | 5.271E+00 | 6.110E+00 |
| 30 | 2.419E+00 | 2.620E+00 | 2.143E+00 | 2.378E+00 |
| 40 | 4.654E–01 | 5.086E–01 | 4.616E–01 | 5.098E–01 |
| 50 | 1.312E–01 | 1.480E–01 | 1.324E–01 | 1.381E–01 |
| 59 | 4.672E–02 | 5.346E–02 | 4.242E–02 | 4.552E–02 |
Table 7: Integrated bottom quark $p_t$ distribution at 630 GeV. $m_b=4.75$ GeV, $\mu=\mu_0/4$, $\Lambda_0 = 215$ MeV, $\Lambda_+ = 275$ MeV.

| $p_t^{\min}$ (GeV) | MRSD0 $\Lambda_0$ | MRSD0 $\Lambda_+$ | MRSD– $\Lambda_0$ | MRSD– $\Lambda_+$ |
|---------------------|-------------------|-------------------|-------------------|-------------------|
| 0                   | 1.321E+04         | 1.833E+04         | 1.201E+04         | 1.647E+04         |
| 5                   | 3.619E+03         | 4.705E+03         | 3.069E+03         | 3.985E+03         |
| 10                  | 6.036E+02         | 7.479E+02         | 4.986E+02         | 6.202E+02         |
| 15                  | 1.310E+02         | 1.600E+02         | 1.123E+02         | 1.325E+02         |
| 20                  | 3.635E+01         | 4.344E+01         | 3.193E+01         | 3.666E+01         |
| 25                  | 1.184E+01         | 1.391E+01         | 1.092E+01         | 1.235E+01         |
| 30                  | 4.452E+00         | 5.139E+00         | 4.210E+00         | 4.575E+00         |
| 40                  | 8.034E–01         | 9.621E–01         | 7.467E–01         | 8.312E–01         |
| 50                  | 1.723E–01         | 1.828E–01         | 1.765E–01         | 1.579E–01         |
| 59                  | 4.111E–02         | 4.241E–02         | 4.878E–02         | 4.510E–02         |

Table 8: Integrated $\psi$ $p_t$ distribution from $B$ decays, from charmonium production ($\chi+\psi$) and relative $B$ fraction at 1.8 TeV. MRSD0, $\Lambda_0 = 215$ MeV, $\Lambda_+ = 275$ MeV. BR($J/\psi \rightarrow \mu^+\mu^-$) included.

| $p_t^{\min,\psi}$ (GeV) | $\Lambda_0$, $\mu=\mu_0$ | $\Lambda_+$, $\mu=\mu_0/4$ |
|-------------------------|---------------------------|------------------------------|
|                         | $\sigma_B$·BR (nb) | $\sigma_\chi$·BR (nb) | $f_B$ (%) | $\sigma_B$·BR (nb) | $\sigma_\chi$·BR (nb) | $f_B$ (%) |
| 3                       | 2.6E+00                  | 5.4E+00                    | 32       | 5.4E+00                  | 3.9E+01                   | 12       |
| 4                       | 1.7E+00                  | 1.9E+00                    | 46       | 3.4E+00                  | 1.5E+01                   | 19       |
| 5                       | 1.1E+00                  | 7.6E–01                    | 58       | 2.2E+00                  | 6.0E+00                   | 26       |
| 6                       | 6.7E–01                  | 3.4E–01                    | 66       | 1.4E+00                  | 2.8E+00                   | 33       |
| 8                       | 2.9E–01                  | 8.7E–02                    | 77       | 6.3E–01                  | 7.7E–01                   | 45       |
| 10                      | 1.4E–01                  | 2.9E–02                    | 83       | 3.2E–01                  | 2.6E–01                   | 55       |
| 12                      | 7.8E–02                  | 1.2E–02                    | 86       | 1.7E–01                  | 1.0E–01                   | 62       |
| 14                      | 4.4E–02                  | 4.9E–03                    | 89       | 1.0E–01                  | 5.0E–02                   | 67       |
| 16                      | 2.6E–02                  | 2.5E–03                    | 91       | 6.1E–02                  | 2.5E–02                   | 71       |
| 18                      | 1.7E–02                  | 1.3E–03                    | 92       | 3.9E–02                  | 1.3E–02                   | 75       |
| 20                      | 1.1E–02                  | 7.3E–04                    | 93       | 2.5E–02                  | 6.9E–03                   | 78       |
| 25                      | 4.2E–03                  | 1.9E–04                    | 95       | 9.4E–03                  | 1.9E–03                   | 82       |
| 30                      | 1.9E–03                  | 4.9E–05                    | 97       | 4.1E–03                  | 5.2E–04                   | 88       |
Table 9: Integrated $\psi p_t$ distribution from $B$ decays, from charmonium production ($\chi + \psi$) and relative $B$ fraction at 630 GeV. MRSD0, $\Lambda_0 = 215$ MeV, $\Lambda_+ = 275$ MeV. BR($J/\psi \rightarrow \mu^+\mu^-$) included.

| $p_t^{min,\psi}$ (GeV) | $\Lambda_0$, $\mu=\mu_0$ | $f_B$ (%) | $\Lambda_+$, $\mu=\mu_0/4$ | $f_B$ (%) |
|-------------------------|--------------------------|-----------|---------------------------|-----------|
|                         | $\sigma_B \cdot BR$ (nb) | $\sigma_\chi \cdot BR$ (nb) | $\sigma_B \cdot BR$ (nb) | $\sigma_\chi \cdot BR$ (nb) |
| 3                       | 3.0E+00                  | 3.4E+00   | 7.8E+00                  | 2.9E+01   | 21         |
| 4                       | 1.6E+00                  | 1.2E+00   | 4.0E+00                  | 1.1E+01   | 27         |
| 5                       | 8.3E-01                  | 4.7E-01   | 2.1E+00                  | 4.3E+00   | 33         |
| 6                       | 4.6E-01                  | 2.1E-01   | 1.2E+00                  | 1.9E+00   | 37         |
| 8                       | 1.6E-01                  | 5.1E-02   | 4.0E-01                  | 5.1E-01   | 43         |
| 10                      | 6.1E-02                  | 1.6E-02   | 1.6E-01                  | 1.7E-01   | 47         |
| 12                      | 2.7E-02                  | 6.2E-03   | 6.7E-02                  | 6.7E-02   | 50         |
| 14                      | 1.3E-02                  | 2.7E-03   | 3.1E-02                  | 2.9E-02   | 51         |
| 16                      | 6.6E-03                  | 1.2E-03   | 1.6E-02                  | 1.4E-02   | 53         |
| 18                      | 3.6E-03                  | 6.1E-04   | 8.4E-03                  | 7.2E-03   | 53         |
| 20                      | 2.0E-03                  | 3.2E-04   | 4.5E-03                  | 3.9E-03   | 53         |
| 25                      | 5.8E-04                  | 7.2E-05   | 1.2E-03                  | 9.2E-04   | 55         |
| 30                      | 2.0E-04                  | 1.8E-05   | 3.6E-04                  | 2.3E-04   | 61         |
Figure 1: Integrated $b$ $p_t$ distribution at 630 GeV: UA1 data \cite{8} versus NLO QCD. The $J/\psi$ point assumes a $B$ fraction in the inclusive $J/\psi$ sample of $31\pm12\%$ \cite{7}.

Figure 2: Integrated $B$ meson $p_t$ distribution at 630 GeV: UA1 data versus NLO QCD.

Figure 3: Differential $J/\psi$ $p_t$ distribution at 630 GeV: UA1 data versus different QCD contributions, as shown in the legend.

Figure 4: Integrated $b$ $p_t$ distribution at 1.8 TeV: CDF data \cite{11} versus NLO QCD. The $J/\psi$ point assumes a $B$ fraction in the inclusive $J/\psi$ sample of $63\pm17\%$ \cite{11}.

Figure 5: Differential $J/\psi$ $p_t$ distribution at 1.8 TeV: CDF data versus different QCD contributions, as shown in the legend.

Figure 6: Differential $\psi(2S)$ $p_t$ distribution at 1.8 TeV: CDF data versus total QCD. The different contributions from direct production and $B$ decays are labeled as in the previous Figure.

Figure 7: Contribution to the $b$ cross section above given $p_t$ thresholds as a function of the gluon momentum fraction $x$. 

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