Flavor Mixing and Rare $B$ and $K$ Decays in the Minimal 331 Model

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Abstract. We analyze the implications of the minimal 331 model with respect to flavor physics. These new contributions stem from the tree level exchange of a flavor changing additional $Z'$ gauge boson. The observables under investigation are the mixing related observables $\epsilon_K$, $\Delta M_K$, $\Delta M_{d/s}$ and $\sin 2\beta$, which we use to constrain the parameter space, as well as the rare decays $K \rightarrow \pi \nu \bar{\nu}$, $K_L \rightarrow \pi^0 l^+ l^-$ and $B_{d/s} \rightarrow \mu^+ \mu^-$. We find that significant effects are still possible in the $K$ physics sector, while the $B$ physics sector is already rather strongly constrained. The exception is the $B_s - \bar{B}_s$ mixing phase $\beta_s$, which can be large.

1. Introduction
Within the standard model (SM) of particle physics, one of the important remaining questions is regarding the particle content of the model. This concerns, in particular, the fact that both quarks and leptons appear in three families, and is reflected in Rabi’s famous question ‘Who ordered that?’ upon the discovery of the muon. A possible answer to this question is given within the context of the 331 models, introduced in their minimal form almost 15 years ago by Frampton [1] as well as Pisano and Pleitez [2]. Here, the electroweak $SU(2)_L$ of the SM is extended to an $SU(3)_L$, which is then broken down in two steps to the $U(1)_{em}$ present in the SM. Additionally, the third generation of fermions is treated as an anti triplet instead of a triplet. Within this setup the requirements of anomaly cancellation and asymptotic freedom combined require the number of generation to be exactly three. For a thorough discussion of the main theoretical aspects of the model, see [3].

In the symmetry breaking process, one encounters (among others) an additional neutral $Z'$ boson, which turns out to have a flavor changing vertex due to the difference in coupling between the third and the first two generations. These tree level FCNCs are the by far dominant new contributions to several low energy observables such as meson mixing amplitudes and rare $K$ and $B$ decays. One also finds that these observables are generally the best suited to constrain and investigate this model, since many other observables, such as the electroweak precision observables, are only modified at loop level. Earlier analyses of flavor physics processes in the minimal 331 model can be found in [4, 5, 6]. In all of these earlier cases, however, CP violating effects have been neglected, which have now been taken into account in [7].
2. FCNC Vertices and Contributions to Observables

As mentioned above, the observables we study all receive contributions at tree level from the additional \( Z' \) boson, which has the following flavor changing vertex:

\[
\mathcal{L}_{\text{FCNC}} = \frac{g^W}{\sqrt{3} \sqrt{1 - 4 s_W^2}} [\overline{q}_i \gamma_L \tilde{V}_L] \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \tilde{V}_L d |Z'|^m. \tag{1}
\]

Here, the matrix \( \tilde{V}_L \) is the matrix used to diagonalize the down quark Yukawa coupling. The prefactor, on the other hand, just corresponds to the difference in first and third generation contributions, only. From this term, it follows that there are three relevant combinations of matrix elements that appear in the observables, which, incidentally, also correspond to the different types of flavor transitions discussed. These are the real and imaginary parts of \( \tilde{V}_{12} \tilde{V}_{13} \) (governing the \( K \) physics observables), \( \tilde{V}_{13} \tilde{V}_{31} \) (which governs \( B_d \) observables) and \( \tilde{V}_{32} \tilde{V}_{33} \), which, finally, appears in the \( B_s \) system.

From this vertex, one easily reads off the new contributions to a given observable, and all of the relevant expressions can be found in [7]. For example, the effective Hamiltonian for a particle antiparticle mixing amplitude is given by

\[
H_{\Delta s=2}^{\text{eff}, Z'} = \frac{G_F}{\sqrt{2}} \frac{1}{3} \frac{c_W^4}{1 - 4 s_W^2} \left( \frac{M_Z}{M_{Z'}} \right)^2 (\tilde{V}_{3i} \tilde{V}_{3j}^*)^2 (\overline{q}_j q_i)_{V-A} (\overline{q}_j q_i)_{V-A},
\]

with \( i, j \) corresponding to the appropriate combination according to the rules given above. Roughly speaking, one then finds that, in the \( K \) system, \( \epsilon_K \) constrains the imaginary part of this new contribution, with \( \Delta M_K \) constraining the real part. Analogously, in the \( B_s \) sectors, the mass differences constrain the absolute values, while the mixing phases constrain contributions orthogonal to the SM ones. In this context one should note that the angle \( \beta_s \) is still essentially unconstrained and can therefore be large. Fortunately, a first measurement has now become available, which is not very precise yet, but will hopefully be improved upon.

3. Numerical Analysis

The numerical analysis [7] proceeds now in two steps: First, we use the above mentioned observables to constrain the observable space. Theoretical uncertainties are taken into account by keeping the theoretical expressions fixed at their central values and inflating the corresponding experimental uncertainty. The parameters of the 331 model are then scattered in the ranges thereby allowed. Also, the CKM parameters obtained from a unitarity triangle constructed from tree level input only, since all other input may be polluted by new physics.

For the decays \( K^+ \rightarrow \pi^+ \nu \bar{\nu} \) and \( K_L \rightarrow \pi^0 \nu \bar{\nu} \), this results in the very peculiar pattern shown in Fig. 1. Effectively, one observes that only one of both decays may be enhanced strongly, with the other one rather close to its SM value. A similar signal is found in the lightest Higgs model with T-parity [8].

Next, it is interesting to analyze correlations between several \( K \) decay branching fractions. The ones between \( K_L \rightarrow \pi^0 \mu^+ \mu^- \), \( K_L \rightarrow \pi^0 e^+ e^- \), \( K_L \rightarrow \pi^0 \nu \bar{\nu} \) are shown in Figures 2 and 3. The most interesting feature is a relatively strong possible enhancement of \( \text{BR}(K_L \rightarrow \pi^0 e^+ e^-) \), which is not encountered in other models. In general, however, the coupling of the \( Z' \) to leptons is suppressed in the minimal 331 model, and it seems that stronger effects are expected in observables in which leptons do not appear.

Finally, we have also analyzed \( B \) decay branching fractions, but find that both \( B_{d/s} \rightarrow \mu^+ \mu^- \) decays cannot be strongly modified. A large enhancement, in particular, is ruled out.
Figure 1. A projection onto the $K_L \rightarrow \pi^0 \nu \bar{\nu}$-$K^+ \rightarrow \pi^+ \nu \bar{\nu}$ plane including the upper bounds from $\Delta M_K$ and $\epsilon_K$ for $M_{Z'} = 5$ TeV (red) and $M_{Z'} = 1$ TeV (blue).

Figure 2. Contour in the $K_L \rightarrow \pi^0 e^+ e^-$-$K_L \rightarrow \pi^0 \nu \bar{\nu}$ plane.

Figure 3. Analog to Fig. 2 in the $K_L \rightarrow \pi^0 e^+ e^-K_L \rightarrow \pi^0 \mu^+ \mu^-$ plane. A measurement of any two decays tests the minimal 331 model.

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