Photon impact factor in the next-to-leading order

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An analytic coordinate-space expression for the next-to-leading order photon impact factor for small-$x$ deep inelastic scattering is calculated using the operator expansion in Wilson lines.

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I. INTRODUCTION

It is well known that the small-$x$ behavior of structure functions of deep inelastic scattering is determined by the hard pomeron contribution. In the leading order the pomeron intercept is determined by the BFKL equation [1] and the pomeron residue (the $\gamma^*\gamma^*$-pomeron vertex) is given by the so-called impact factor. To find the small-$x$ structure functions in the next-to-leading order, one needs to know both the pomeron intercept and the impact factor. The NLO pomeron intercept was found many years ago [2] but the analytic expression for the NLO impact factor is obtained for the first time in the present paper.

We calculate the NLO impact factor using the high-energy operator expansion of T-product of two vector currents in Wilson lines (see e.g the reviews [3, 4]). Let us recall the general logic of an operator expansion. In order to find a certain asymptotical behavior of an amplitude by OPE one should

• Identify the relevant operators and factorize an amplitude into a product of coefficient functions and matrix elements of these operators

• Find the evolution equations of the operators with respect to the factorization scale

• Solve these evolution equations

• Convolute the solution with the initial conditions for the evolution and get the amplitude.

Since we are interested in the small-$x$ asymptotics of DIS it is natural to factorize in rapidity: we introduce the rapidity divide $\eta$ which separates the “fast” gluons from the “slow” ones.

As a first step, we integrate over gluons with rapidities $Y > \eta$ and leave the integration over $Y < \eta$ for later time, see Fig. 2. It is convenient to use the background field formalism: we integrate over gluons with $\alpha > \sigma = e^\eta$ and leave gluons with $\alpha < \sigma$ as a background field, to be integrated over later. Since the rapidities of the background gluons are very different from the rapidities of gluons in our Feynman diagrams, the background field can be taken in the form of a shock wave due to the Lorentz contraction. To derive the expression of a quark (or gluon) propagator in this shock-wave background we represent the propagator as a path integral over various trajectories, each of them weighed with the gauge factor $\text{Pexp}(ig \int dx \mu A^\mu)$ ordered along the propagation path. Now, since the shock wave is very thin, quarks (or gluons) do not have time to deviate in transverse direction so their trajectory inside the shock wave can be approximated by a segment of the straight line. Moreover, since there is no external field outside the shock wave, the integral over the segment of straight line can be formally extended to $\pm \infty$ limits yielding the Wilson-line operator

$$ U_\eta^\eta = \text{Pexp} \left[ ig \int_{-\infty}^{\infty} du p_1^\mu A_\mu^\eta (up_1 + x) \right], $$

$$ A_\mu^\eta (x) = \int d^4 k \, \theta(e^\eta - |\alpha_k|) e^{ik \cdot x} A_\mu(k) \quad (1) $$

where the Sudakov variable $\alpha_k$ is defined as usual, $k = \alpha k p_1 + \beta k p_2 + k_\perp$. (We define the light-like vectors $p_1$ and $p_2$ such that $q = p_2 - x_B p_2$ and $p_N = p_2 + \frac{m_N}{2} p_1$ where $p_N$ is the nucleon momentum). The structure of the propagator in a shock-wave background looks as follows (see Fig. 1):

[Free propagation from initial point $x$ to the point of intersection with the shock wave $z$]
\[ \text{Interaction with the shock wave described by the Wilson-line operator } U_z \]
\[ \text{Free propagation from point of interaction } z \text{ to the final point } y. \]

\[ \text{FIG. 1: Propagator in a shock-wave background} \]

The explicit form of quark propagator in a shock-wave background can be taken from Ref. [5]

\[ \langle \bar{\psi}(x)\psi(y) \rangle = \int d^4z \delta(z_{\perp}) \frac{(\hat{x} - \hat{y})}{2\pi^2(x-z)^4} p^2 U_{\perp} \frac{(\hat{x} - \hat{y})}{2\pi^2(x-z)^4} \]
where we use the notations \( x_\perp = p_\perp^2 x_\perp = \frac{\sqrt{z}}{2} x_\perp \), \( x_\perp = p_\perp^2 x_\perp = \frac{\sqrt{z}}{2} x_\perp \) (our metric is (1,1,1,1)). Note that the Regge limit in the coordinate space corresponds to \( x_\perp \to \infty, y_\perp \to -\infty \) while \( x_\perp, y_\perp \) are fixed, see the discussion in Refs. [6, 7].

As we mentioned above, the result of the integration over the rapidities \( Y > \eta \) gives the impact factor - the amplitude of the transition of virtual photon in two-Wilson-lines operators (sometimes called “color dipole”). The LO impact factor is a product of two propagators (2), see Fig. 2

\[ \text{FIG. 2: Impact factor in the leading order. Solid lines represent quarks.} \]

\[ \langle \bar{\psi}(x)\gamma^\mu \psi(x)\bar{\psi}(y)\gamma^\nu \psi(y) \rangle = \]
\[ = \frac{s^2}{2y^4x^2y^2} \int dz_1 dz_2 \frac{tr\{U_{z_1 + } U_{z_2 + }^\dag} \}
\times \frac{\partial^2}{\partial x^\mu \partial y^\nu} \left[ (\kappa \cdot \zeta_1)(\kappa \cdot \zeta_2) - \kappa^2(\zeta_1 \cdot \zeta_2) \right] + O(\alpha_s) \]

Here we introduced the conformal vectors [8, 9]

\[ \kappa = \frac{\sqrt{z}}{2x_\perp}, \]
\[ \zeta_i = \left( \frac{p_1}{s} + z_i^2 p_2 + z_i \right), \]
and the notation \( R = R^2 \equiv \frac{\kappa^2(\zeta_1 \cdot \zeta_2)}{2(\kappa \cdot \zeta_1)(\kappa \cdot \zeta_2)} \). The above equation is explicitly Möbius invariant. In addition, it is easy to check that \( \frac{\partial}{\partial x^\mu} \text{ (r.h.s)} = 0 \).

Our goal is the NLO contribution to the r.h.s. of Eq. (7), but first let us briefly discuss the three remaining steps of the high-energy OPE. The evolution equation for color dipoles has the form [5, 10]

\[ \frac{d}{d\eta} \text{tr}\{\hat{U}_{z_1} \hat{U}_{z_2}^\dag} \]
\[ = \frac{\alpha_s}{2\pi^2} \int d^2z_3 \frac{z_{12}^2}{z_{13} + z_{23}} \left[ \text{tr}\{\hat{U}_{z_1} \hat{U}_{z_2}^\dag} \right] tr\{\hat{U}_{z_3} \hat{U}_{z_2}^\dag} \]
\[ - N_c tr\{\hat{U}_{z_1} \hat{U}_{z_2}^\dag} \] + NLO contribution (5)

(To save space, hereafter \( z_1 \) stand for \( z_{1\perp} \) so \( z_{12}^2 \equiv z_{12\perp}^2 \) etc.) The explicit form of the NLO contributions can be found in Refs. [4, 11, 12] while the argument of the coupling constant in the above equation (following from the NLO calculations) is discussed in Refs. ([13, 14]).

The next two steps, solution of the evolution equation (5) with appropriate initial conditions and the eventual comparison with experimental DIS data are discussed in many papers (see e.g. [15]).

II. CALCULATION OF THE NLO IMPACT FACTOR

Now we would like to repeat the same steps of operator expansion at the NLO accuracy. A general form of the expansion of T-product of the electromagnetic currents in color dipoles looks as follows:

\[ (x - y)^4 T \{ \bar{\psi}_\mu \gamma_\mu \psi(x) \bar{\psi}_\nu \gamma_\nu \psi(y) \} \]
\[ = \int d^2z_1 d^2z_2 \frac{\gamma^\nu(z_1, z_2)}{z_{12}} \left( 1 + \frac{\alpha_s}{\pi} \right) tr\{\hat{U}_{z_1} \hat{U}_{z_2}^\dag} \]
\[ + \int d^2z_3 I_{12}^{NLO} \left( z_1, z_2, z_3; \eta \right) \]
\[ \times \left[ tr\{\hat{U}_{z_1} \hat{U}_{z_2}^\dag} tr\{\hat{U}_{z_3} \hat{U}_{z_2}^\dag} \right] \right) \] (6)

The structure of the NLO contribution is clear from the topology of diagrams in the shock-wave background, see Fig. 3 below. Also, the term \( \sim 1 + \frac{\alpha_s}{\pi} \) can be restored from the requirement that at \( U = 1 \) (no shock wave) one should get the perturbative series for the polarization operator \( 1 + \frac{\alpha_s}{\pi} + O(\alpha_s^2) \).

In our notations

\[ I_{12}^{LO} (z_1, z_2) = \frac{R^2}{\pi^6(\kappa \cdot \zeta_1)(\kappa \cdot \zeta_2)} \]
\[ \times \frac{\partial^2}{\partial x^\mu \partial y^\nu} \left[ (\kappa \cdot \zeta_1)(\kappa \cdot \zeta_2) - \frac{1}{2} \kappa^2(\zeta_1 \cdot \zeta_2) \right]. \] (7)

which corresponds to the well-known expression for the LO impact factor in the momentum space.

The NLO impact factor is given by the diagrams shown in Fig. 3. The calculation of these diagrams is similar to the calculation of the NLO impact factor for scalar
currents in $\mathcal{N} = 4$ SYM carried out in our previous paper [12]. The gluon propagator in the shock-wave background at $x_\mu > 0 > y_\nu$ in the light-like gauge $p_\mu^2 A_\mu = 0$ is given by [16, 17],

$$\langle \hat{A}_\mu(x) \hat{A}_\nu(y) \rangle_{x_\mu > 0 > y_\nu} = -i \int d^4z \delta(z_\nu)$$

with $\frac{1}{\sigma}$ can be either $\frac{1}{\sigma_{+e}}$ or $\frac{1}{\sigma_{-e}}$ which leads to the same result. (This is obvious for the leading order and correct in NLO after subtraction of the leading-order contribution, see Eq. (15) below).

The diagrams in Fig. 3a,b can be calculated using the conformal integral

$$\int d^4z \frac{\tilde{\xi} - \tilde{\eta}}{(x-z)^2} \frac{\tilde{\eta} - \tilde{y}}{(z-y)^2} = \frac{\pi^2}{x^2 y^2 (x-y)^2} \left[ \tilde{\xi} \tilde{\eta} - \tilde{\eta} \tilde{y} \right] - \mu \leftrightarrow \nu$$

which gives the 3-point $\psi \bar{\psi} F_{\mu\nu}$ Green function in the leading order in $g$. Using Eqs. (2), (8) and (9), performing integrals over $z_\nu$'s and taking traces one gets after some algebra the NLO contribution of diagrams in Fig. 3 in the form

$$I^{\text{Fig.3}}_{\mu\nu}(z_1, z_2, z_3) = I^{\mu\nu}_{1}(z_1, z_2, z_3) + I^{\mu\nu}_{2}(z_1, z_2, z_3)$$

where

$$I^{\mu\nu}_{1}(z_1, z_2, z_3) = \frac{\alpha_s}{4\pi^2} I^{\mu\nu}_{\text{LO}}(z_1, z_2, z_3)$$

and

$$I^{\mu\nu}_{2}(z_1, z_2, z_3) = \frac{\alpha_s}{16\pi^2} \frac{\mathcal{R}^2}{(z_1 \cdot z_2) (z_2 \cdot z_3)}$$

$$\times \left\{ \frac{(z_1 \cdot z_2)}{(z_1 \cdot z_3)} \frac{\partial^2}{\partial x^\mu \partial y^\nu} - \frac{(z_1 \cdot z_2)}{(z_2 \cdot z_3)} \frac{(z_2 \cdot z_3)}{(z_1 \cdot z_3)} \frac{\partial^2}{\partial y^\mu \partial x^\nu} \right\} \right\}$$

(recall that $z^2_{ij} = 2(z_i \cdot z_j)$ and $Z_i = \frac{4}{\alpha_s}(\gamma_3)$. We obtained this expression at $x_\mu > 0 > y_\nu$ but from the conformal structure of the result it is clear that this expression holds true at $x_\mu < 0 < y_\nu$ as well.

The integral over $\alpha$ in the r.h.s. of Eq. (11) diverges. This divergence reflects the fact that the contributions of the diagrams in Fig. 3 is not exactly the NLO impact factor since we must subtract the matrix element of the leading-order contribution. Indeed, the NLO impact factor is a coefficient function defined according to Eq. (6). To find the NLO impact factor, we consider the operator equation (6) in the shock-wave background (in the leading order $\langle \hat{U}_{z_3} \rangle = U_{z_3}$):

$$\langle T \{ \tilde{\psi} \gamma_\mu \psi(x) \tilde{\psi} \gamma_\nu \psi(y) \} \rangle_A$$

$$= \int \frac{d^2z_1 d^2z_2}{z_1^2} I^{\mu\nu}_{\text{LO}}(x, y; z_1, z_2) \langle \{ \hat{U}_{z_1} \hat{U}_{z_2} \} \rangle_A$$

$$= \int \frac{d^2z_1 d^2z_2}{z_1^2} d^2z_3 I^{\mu\nu}_{\text{NLO}}(x, y; z_1, z_2, z_3)$$

$$\langle \{ \hat{U}_{z_1} \hat{U}_{z_2} \rangle - N_c \langle \hat{U}_{z_3} \hat{U}_{z_2} \rangle \rangle$$

The NLO matrix element $\langle T \{ \tilde{\psi} \gamma_\mu \psi(x) \tilde{\psi} \gamma_\nu \psi(y) \} \rangle_A$ is given by Eq. (10) while the subtracted term is

$$\alpha_s \int \frac{d^2z_1 d^2z_2}{z_1^2} d^2z_3 I^{\mu\nu}_{\text{LO}}(x, y; z_1, z_2, z_3)$$

$$\times \langle \{ \hat{U}_{z_1} \hat{U}_{z_2} \rangle - N_c \langle \hat{U}_{z_3} \hat{U}_{z_2} \rangle \rangle$$

as follows from Eqs. (7) and (5). The $\alpha$ integration is cut from above by $\sigma = e^\eta$ in accordance with the definition of operators $\hat{U}^\eta$, see Eq. (1). Subtracting (14) from Eq. (10) we get

$$I^{\mu\nu}_{\text{NLO}}(z_1, z_2, z_3; \eta) = I^{\mu\nu}_{1}(z_1, z_2, z_3; \eta) + I^{\mu\nu}_{2}(z_1, z_2, z_3; \eta)$$

$$I^{\mu\nu}_{1}(x, y; z_1, z_2, z_3; \eta) = \frac{\alpha_s}{2\pi^2} I^{\mu\nu}_{\text{LO}}(z_1, z_2, z_3; \eta)$$

$$\times \left\{ \int_0^\infty \frac{d\sigma}{\alpha} e^{i\sigma z_3} - i \int_0^\infty \frac{d\alpha}{\sigma} e^{i\sigma z_3} \right\}$$

Note that one should expect the NLO impact factor to be conformally invariant since it is determined by tree diagrams in Fig. 3. However, as discussed in Refs. [4, 7,
We need to choose the "new rapidity cutoff" \(a\) so that the \(\{\text{tr}(\hat{U}_z \hat{U}_z^{\dagger})\}_a\) does not depend on energy. A suitable choice is given by \(a = -\kappa^2 + i\epsilon = -\frac{4z^2}{\pi(x-y)^2} + i\epsilon\) so that the \(\{\text{tr}(\hat{U}_z \hat{U}_z^{\dagger})\}_a\) depends only on \(z\) and not on \(x\).

Rewritten in terms of composite dipoles (16), the operator expansion (6) takes the form:

\[
T\{\hat{\psi}\gamma_\mu\hat{\psi}(x)\hat{\psi}\gamma_\nu\hat{\psi}(y)\} = \int d^2z_1 d^2z_2 \left[ I_{\text{LO}}^{\mu\nu}(z_1,z_2) + I_{\text{NLO}}^{\mu\nu}(z_1,z_2) \right] \text{tr}(\hat{U}_z \hat{U}_z^{\dagger})_a
\]

where \(a\) is an arbitrary constant. It is convenient to choose the rapidity-dependent constant \(a = ae^{-2}\eta\). The result is

\[
T_{\text{LO}}^{\alpha\beta}(x,y;z_1,z_2) = R^2 \frac{g^{\alpha\beta}(\zeta_1,\zeta_2)}{2\pi^4 (\kappa,\zeta_1)(\kappa,\zeta_2)}
\]

(see Eq. (7) and

\[
T_{\text{NLO}}^{\alpha\beta}(x,y;z_1,z_2) = \frac{\alpha_s}{4\pi^2} R^2 \left[ \zeta_1^\alpha \zeta_2^\beta + \zeta_1 \leftrightarrow \zeta_2 \right]
\]

\[
\times \left[ 4\text{Li}_2(1-R) - \frac{2\pi^2}{3} + \frac{2\ln R}{1-R} + 2\ln R - \frac{\ln R}{R} - \frac{4\ln R}{1-R} \right]
\]

\[
- \left( \ln \frac{1}{R} + \frac{1}{R} - 2 \right) \left( \frac{3}{2} \ln R - 2C \right)
\]

\[
- \zeta_1^\alpha \zeta_2^\beta \left[ \frac{2\ln R}{1-R} + \frac{2\ln R}{R} - \frac{1}{2R} \right]
\]

\[
+ \zeta_1^\alpha \zeta_2^\beta \zeta_1 \leftrightarrow \zeta_2 \left[ \frac{2\ln R}{1-R} + \frac{2\ln R}{R} - \frac{1}{2R} \right]
\]

\[
+ \left[ \zeta_1^\alpha \zeta_2^\beta + \zeta_1^\alpha \zeta_2^\beta + \zeta_1 \leftrightarrow \zeta_2 \right] \left[ \frac{2\ln R}{1-R} + \frac{2\ln R}{R} - \frac{1}{2R} \right]
\]

(22)
one should be careful when checking the electromagnetic gauge invariance in the next-to-leading order. The reason is that the composite dipole $\hat{U}^{\alpha}(z_1,z_2)$ depends on $x$ via the rapidity cutoff $a_0 = -\frac{4x+y}{s(x-y)^4}$ so from Eq. (20) we get

$$\frac{d}{dx} \frac{1}{s(x-y)^4} \frac{\partial \tilde{\epsilon}^{\alpha}}{\partial x} \frac{\partial \tilde{\epsilon}^{\beta}}{\partial y} = \frac{1}{s(x-y)^4} \cdot \int \frac{dz_1 dz_2}{(z_1 z_2)^4} \hat{U}_{a_0}(z_1,z_2) T^2_{\alpha \beta} \left( \frac{x}{z_1}, \frac{y}{z_2} \right)$$

Using the leading-order BFKL equation in the dipole form (linearization of Eq. (5))

$$d\hat{U}_a(z_1,z_2) = \frac{\alpha_s N_c}{4\pi^2} \int d^2 z_3 \frac{z_3^2}{z_2^2 z_3^2}$$

we obtain the following consequence of gauge invariance

$$\frac{d}{dx} \frac{1}{s(x-y)^4} \frac{\partial \tilde{\epsilon}^{\alpha}}{\partial x} \frac{\partial \tilde{\epsilon}^{\beta}}{\partial y} = \frac{\alpha_s N_c}{4\pi^2} \left[ \hat{U}_a(z_1,z_2) \right]$$

We have verified that the expression (22) satisfies the above equation.

IV. CONCLUSIONS AND OUTLOOK

We have calculated the NLO impact factor for the virtual photons both in the non-linear form (18) and with the linear (two-gluon) accuracy (20). Our results are obtained in the coordinate representation so the next step should be the Fourier transformation of Eq. (22) which would give the momentum-space impact factor convenient for phenomenological applications (and available at present only as a combination of numerical and analytical expressions[19]). The study is in progress.

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