Does $K_L - K_S$ mass difference constraints or claims new physics beyond the Standard Model?

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The ratio $\Delta m_K/m_K$ within the standard model with 3 generations is calculated as a function of the CP nonconserving phase $\delta_{13}$ and the quark masses $m_c, m_t$ assuming the current values of the Cabibbo-Kobayashi-Maskawa mixing angles. We have found that varying $\delta_{13}$ and $m_c$ within the allowed range, not all the values for the top quark mass fit the experimental value for that ratio.
The absence of $\Delta S \neq 0$ neutral currents led to the so called Glashow-Iliopoulos-Maiani (GIM) mechanism \cite{GIM} in the context of the $SU(2)_L \otimes U(1)_Y$ gauge theory \cite{SU2U1} with four quarks (two generations). The GIM mechanism is also implemented in the 3 generations case \cite{3generations}. However, the natural (independent of mixing angles) flavor conservation is a characteristic of the standard model without heavy quarks \cite{natural}, i.e., $m_q^2/M_W^2 \ll 1$. On the other hand, if there are quarks with mass $m_q^2/M_W^2 \approx 1$ or greater, the requirements for the natural flavor conservation in the neutral currents to order $\alpha G_F$ break down, and it is necessary to impose the restriction that the mixing angles between ordinary and superheavy quark sectors must be very small \cite{natural}.

It is well known that the $K_L-K_S$ mass difference was used to estimate the mass of the $c$ quark even before its discovery \cite{Kmass}. Since then, this was used to argue that any additional contribution to this mass difference coming from new particles, usually present in models beyond the standard model, cannot be much bigger than the contribution of the $c$ quark. This means that the observation of even a tiny flavor changing neutral currents (FCNC) effect would imply new physics beyond the standard model. Since then, it has been mandatory to study the top quark contribution to rare processes in kaon \cite{Kaon} and beauty \cite{Beauty} mesons. In particular the $K_L-K_S$ mass difference ($\Delta m_K = m_{K_L} - m_{K_S}$) and $CP$ non–conservation were used to limit the range of the Cabibbo-Kobayashi-Maskawa (CKM) mixing angles $\theta_{2,3}$ and phase parameter $\delta_{13}$. These processes in addition with $B$-$\bar{B}$ mixing, and the ratio $\Gamma(W)/\Gamma(Z)$ extracted from $p\bar{p}$ colliders were used to fit the top quark mass \cite{topmass}

$$35 \text{ GeV}/c^2 \lesssim m_t \lesssim 55 \text{ GeV}/c^2.$$ \hfill (0.1)

At present, however, we know that \cite{topmass}

$$m_t > 108 \text{ GeV}/c^2.$$ \hfill (0.2)

On the other hand, from radiative corrections it was obtained that \cite{topmass}

$$m_t \leq 200 \text{ GeV}/c^2;$$ \hfill (0.3)

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assuming that the Higgs bosons mass $m_H$ is not larger than 1 TeV. With such a value of $m_t$ we see, as it was stressed in Ref. [3], that the suppression of FCNC to order $\alpha G$ requires that the mixing angles between the $t$ quark and the lighter quarks must be very small.

In this work we will show that either: i) we do not know the correct CKM parameters yet, or ii) not all values of $m_t$ can accommodate the $K_L-K_S$ mass difference. The last point is probably valid for other rare processes but we will not address this issue here [12].

We will assume that CKM mixing angles have the present values: $s_{12} = 0.218$ to 0.224, $s_{23} = 0.032$ to 0.054, and $s_{13} = 0.002$ to 0.0077 [13], next we calculate $\Delta m_K/m_K$ for several values of $\delta_{13}$, $m_c$ and $m_t$. We did this not only for the sake of simplicity but because, as it is usually believed, whereas the tree level decays are sensitive mainly to the hierarchy of weak couplings, the radiative corrections in decays and transitions depend mainly on the hierarchy of quark masses. In the box diagrams usually the $m_c$ entering in the calculation is the running $c$ quark mass. Here we will allow this mass to run upon the range 1.3–1.7 GeV/$c^2$ which is consistent with the experimental value [13].

In fact, it has been shown that $m_t$ is almost independent of the CKM parameters [14,15]. It means that only when the heavy quark masses become determined from scattering processes and/or spectroscopy we could look for their effects in rare decays, mixing and $CP$ violation in $K$, $B$- and $D$- mesons.

Of course, this is not easy to be implemented because the top quark has not been discovered yet but it is known that its mass must satisfy the lower bound given in Eq. (0.2). However, in the future, we expect that the top quark mass can be well determined in processes different from those in which the weak couplings are determined. For the reasons above we will assume, as we said before, that we already know the mixing angles in the CMK matrix but we allow to vary the phase $\delta_{13} = \delta$ [16] and $m_t$ for a given mass of the $c$ quark. With these free parameters we study the contributions to the $K_L-K_S$ mass difference. In particular, we will compare this quantity with its experimental value.

Neglecting long distance contributions, one has that the ratio is given by [17,18]
\[
\frac{\Delta m_K}{m_K} = \frac{2}{3} \kappa B_K f_K^2 \frac{G_F}{\sqrt{2}} \bar{E}(x_i, x_j), \tag{0.4}
\]

where \( \kappa = \alpha (4\pi \sin^2 \theta_W)^{-1} \) and

\[
\bar{E}(x_i, x_j) = \left[ (Re \lambda_c)^2 - (Im \lambda_c)^2 \right] E(x_c, x_c) \eta_1 + \left[ (Re \lambda_t)^2 - (Im \lambda_t)^2 \right] E(x_t, x_t) \eta_2
+ \left[ (Re \lambda_c^2)(Re \lambda_t^2) - (Im \lambda_c)^2(Im \lambda_t)^2 \right] E(x_c, x_t) \eta_3, \tag{0.5}
\]

where \( \lambda_i = V_{is}^\ast V_{id}, V_{ij} \) are the CKM matrix elements in the parametrization of Maiani \[13,19\] and \( x_i = m_i^2 / M_W^2 \). The QCD corrections, which depend weakly on the top mass, are in the \( \eta_i \) coefficients \[20\].

First we will use \( \eta_i = 1, i = 1,2,3 \). Considering the free quark model is interesting because it is the cleaner calculated part. It is important to recall, that it was in the free-quark case that the value of \( m_c \) was predicted. At the end of this work we will comment the QCD corrections.

In Eq. (0.4) the “bag” parameter \( B_K \) depends on the calculation of the amplitude of the transition \( \bar{K}^0 \leftrightarrow K^0 \). We will use \( B_K = 1 \) and \( f_K = 161 \pm 1 \) MeV for the kaon decay constant \[14\].

The functions \( E(x_i, x_j) \) in Eq. (0.5) were calculated by Inami and Lim \[17\] and by Buras \[18\] and they are given by

\[
E(x_i, x_i) = x_i \left[ \frac{1}{4} + \frac{9}{4}(1 - x_i)^{-1} - \frac{3}{2}(1 - x_i)^{-2} \right] + \frac{3}{2} \frac{[x_i/(x_i - 1)^2]^3 \ln x_i,} {x_i(1 - x_i)^{-1}} \tag{0.6}
\]

\[
E(x_i, x_j) = x_ix_j \left\{ (x_i - x_j)^{-1} \left[ \frac{1}{4} + \frac{3}{2}(1 - x_j)^{-1} - \frac{3}{4}(1 - x_j)^{-2} \right] \ln x_i + x_i \leftrightarrow x_j - \frac{3}{4} [(1 - x_j)(1 - x_i)]^{-1} \right\} \tag{0.7}
\]

The \( u \) quark contribution in Eqs. (0.4) is rearranged into the heavy quark contributions \[17\].

We use the CMK matrix elements given in Ref. \[13\] which correspond to the central values \( \sin \theta_{12} = 0.221, \sin \theta_{23} = 0.043 \) and \( \sin \theta_{13} = 0.036 \). We recall that at present it is an open question what the form of the unitary triangle is, i.e, if \( \pi/2 \leq \delta \leq \pi \) or \( 0 \leq \delta \leq \pi/2 \). For this reason, we treat this phase as a free parameter.
In Fig. 1 we give the calculated values of $\Delta m_K/m_K$ in terms of the top quark mass for different values of the charm quark mass and the $\delta$ parameter. The error in $f_K$ is not shown in the figure. The experimental value $\Delta m_K/m_K = 7.08 \times 10^{-15}$ \cite{13} is shown as a horizontal dashed line.

We see from Fig. 1 that only the following values make the standard model consistent with the experimental value of $\Delta m_K/m_K$:

$$m_c = 1.6 \text{ GeV}/c^2, \quad m_t \simeq 157 \text{ GeV}/c^2, \quad \delta \lesssim \pi.$$ \hspace{1cm} (0.8)

For lower (higher) values of the quark masses there is a deficit (excess) on the $\Delta m_K/m_K$.

For instance, for $m_c = 1.3 \text{ GeV}/c^2$, $\delta = \pi/4$ and $m_t \approx 110 \text{ GeV}/c^2$ we have $\Delta m_K/m_K \simeq 0.62(\Delta m_K/m_K)^{\text{exp}}$, a deficit of $\sim 38\%$, whereas for $m_c = 1.7 \text{ GeV}/c^2$, $\delta = \pi$ and $m_t \approx 110 \text{ GeV}/c^2$ we have $\Delta m_K/m_K \simeq 1.08(\Delta m_K/m_K)^{\text{exp}}$ that is, an excess of about 8\%.

We have repeated our analysis varying the Cabibbo-Kobayashi-Maskawa parameters within the allowed range \cite{13}, however, in this case the numerical results almost do not vary and the main feature remains: within the standard model scenario for all current values allowed for the CKM parameters, only the masses of the quarks $m_c$ and $m_t$ and $\delta$ given in Eq. (0.8) fit the experimental data of the $K_L-K_S$ mass difference.

Of course, strong interaction corrections short ($\eta_i$ coefficients) and long distances effects, should be included. Although strong corrections depend very weakly on $m_t$ in earlier papers rather low values for $m_t$ were used \cite{18,20}. For large top quark mass the $\eta_i$ coefficients were calculated in Ref. \cite{20}. For example, for $\Lambda_{QCD} = 200 \text{ MeV}$, $m_c = 1.7 \text{ GeV}/c^2$, $m_t = 200 \text{ GeV}/c^2$ one has $\eta_1 = 0.82$, $\eta_2 = 0.35$ and $\eta_3 = 0.62$ and we obtain $\Delta m_K/m_K = 6.40 \times 10^{-15}$; for $\Lambda_{QCD} = 250 \text{ GeV}$, $m_c = 1.7 \text{ GeV}/c^2$, $m_t = 200 \text{ GeV}/c^2$, $\eta_1 = 0.89$, $\eta_2 = 0.62$ and $\eta_3 = 0.34$ we obtain $\Delta m_K/m_K = 6.91 \times 10^{-15}$. It means that in order to obtain a consistent value for this mass difference $m_t > 200$, GeV/c$^2$. This lower bound would be larger if we assumed values for $B_K$ smaller than 0.80 \cite{12}.

Our objective was just to put forward the narrow window for the standard model parameters in processes like the $K_L-K_S$ mass difference. The main point, we would like to
stress, in this work is that the allowed values for $m_t$ from loop effects not necessarily will coincide with the correct value of $m_t$ coming from scattering processes and/or spectroscopy and in this case all constraints in the physics beyond the standard model, coming from the $\Delta m_K/m_K$, should be revisited. That is, if the top quark mass is discovered with a mass which do not fit the experimental value of $\Delta m_K/m_K$ and if the improved knowledge of the mixing angles confirm the current values then, it will imply that there is a new physics beyond the standard model which contributes positively or negatively to the $K_L-K_S$ mass difference. It means that this parameter instead of constraints new physics may claim for it.

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FIGURES

FIG. 1. $\Delta m_K/m_K$ versus $m_t$ for the given values of the phase $\delta$ and $m_c$. The dashed line denotes the experimental value \cite{13}.