Constraining parameters of effective field theory of inflation from Planck data

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Abstract

The Cosmic Microwave Background can provide information regarding physics of the very early universe, more specifically, of the matter-radiation distribution of the inflationary era. Starting from the effective field theory of inflation, we use the Goldstone action to calculate the three point correlation function for the Goldstone field, whose results can be directly applied to the field describing the curvature perturbations around a de Sitter solution for the inflationary era. We then use the data from the recent Planck mission for the parameters $f_{\text{equil}}^\text{NL}$ and $f_{\text{orthog}}^\text{NL}$ which parametrize the size and shape of non-Gaussianities generated in single field models of inflation. Using these known values, we calculate the parameters relevant to our analysis, $f_{\text{NL}}^{\varphi}\pi^3$, $f_{\text{NL}}^{\varphi}\left(\partial_i\pi\right)^2$ and the speed of sound $c_s$ which parametrize the non-Gaussianities arising from two different kinds of generalized interactions of the scalar field in question.

1 Introduction

The Cosmic Microwave Background (CMB) is an important probe of physics of the early universe. We study the phenomenon of primordial inflation and its observational signatures on the CMB spectrum (for review see [1, 2, 3]). In standard inflationary scenarios, the universe should be very close to a Gaussian random field. The two point correlation function and its Fourier transform, the angular power spectrum for most models of inflation give similar prediction of a scale invariant, adiabatic, Gaussian spectrum. Therefore, in order to distinguish between the competing models of inflation and to constrain the parameters common to these models, we look at the bispectrum predictions from these models, and deviations from Gaussianity. Such a calculation of three-point function was first done by Maldacena in [4]. The theory of single field slow roll inflation achieves accelerated expansion by means of a scalar field slowly rolling down a potential. It predicts non-Gaussianities that should be too small for observation. However, other models of inflation predict larger departures from
the Gaussian spectrum, and have their characteristic types of non-Gaussianities. Any observation of large non-Gaussianity from the Planck mission thus allows us to constrain these models. Moreover, the angular bispectrum has different shapes, and the different models of inflation show peaks for distinct shapes. As elaborated in [5][6] and references therein, various models of primordial non-Gaussianity are known as local, equilateral, orthogonal or folded models in literature. Different aspects of physics of the early universe appear in different shapes of the three point function.

- **Local Non-Gaussianity** appears in multi-field models of inflation due to interactions which operate on superhorizon scales.

- **Equilateral Non-Gaussianity** includes single field models with non-canonical kinetic term such as k-inflation or Dirac-Born-Infeld inflation models [13, 14, 15, 16] characterized by more general higher derivative interactions of the inflaton field such as ghost inflation and models arising from effective field theories.

- **Folded Non-Gaussianity** include single field models with non-Bunch-Davies vacuum.

- **Orthogonal Non-Gaussianity** may be generated in single field models of inflation with a non-canonical kinetic term or with general higher derivative interactions. The orthogonal form is constructed in such a way that it is nearly orthogonal to both local and equilateral forms.[7]

The plan of this article is as follows: In the next section we review the effective field theory model of inflation. After that we explicitly calculate 2-point and 3-point function in section 3 and finally conclude in section 4.

## 2 The Effective Field Theory of inflation

Cheung, Creminilli, Fitzpatrick, Kaplan and Senatore have used the effective field theory approach in [8] to describe the theory of fluctuations around an inflating cosmological background. While the inflaton field $\phi$ is a scalar under all diffeomorphisms, the perturbation $\delta\phi$ is a scalar only under spatial diffeomorphisms, and transforms non-linearly with respect to time diffeomorphisms,

$$t \rightarrow t + \xi^0(t, \vec{x})$$

$$\delta\phi \rightarrow \delta\phi + \dot{\phi}_0 \xi^0$$

We can describe the perturbations during inflation directly around the time-evolving vacuum where the time diffeomorphisms are non-linearly realised. In unitary gauge, the most generic Lagrangian with broken time diffeomorphisms and unbroken spatial diffeomorphisms around a flat FRW with Hubble parameter $H(t)$ is given by [8],

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_{pl}^2 R + M_{pl}^2 \dot{H} g^{00} - M_{pl}^2 (3H^2 + \dot{H}) + \frac{M_3^2(t)}{2!} (g^{00} + 1)^2 \right]$$
As elaborated by Senatore, Smith and Zaldariagga, in inflation there is a physical clock that controls the end of inflation, so that time translations are spontaneously broken, and there is a Goldstone boson associated with the symmetry breaking. The Lagrangian of the Goldstone boson is highly constrained by the symmetries of the problem, in this case the fact that spacetime is approximately de Sitter, and \( \frac{\dot{H}}{H} \ll 1 \). The Goldstone boson, \( \pi \) can be thought of as being equivalent, in standard models of inflation driven by a scalar field, to the perturbations in the scalar field \( \delta \phi \). The relation valid at linear order is \( \pi = \frac{\delta \phi}{\dot{\phi}} \), where \( \dot{\phi} \) is the speed of the background solution. The Goldstone boson is related to the standard curvature perturbation \( \zeta \) by the relation, \( \zeta = -H\pi \), which is valid at linear order and leading order in the generalised slow roll parameters. The most general Lagrangian for the Goldstone boson is given by \cite{8,9,12}. The general action for the perturbation \( \pi \) is

\[
S_{\pi} = \int d^4x \sqrt{-g} \left[ -\frac{M_{pl}^2}{c_s^2} \left( \dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2} \right) + M_{pl}^2 \frac{\dot{H}}{a^2} \left( 1 - \frac{1}{c_s^2} \right) \left( \dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right) - \frac{4}{3} M_{pl}^4 \dot{\pi}^3 + \ldots \right]
\]

To arrive at the Lagrangian for the Goldstone mode from the most generic Lagrangian in unitary gauge, one performs a time diffeomorphism with parameter \( \xi^0(t, x) \) and promotes the parameter to a field, \( \pi(t, x) \), which shifts under time diffeomorphisms, \( \pi(t, \vec{x}) \rightarrow \pi(t, \vec{x}) + \xi^0(t, \vec{x}) \). This scalar, \( \pi \), is the Goldstone mode which non-linearly realises time diffeomorphisms and describes the scalar perturbations around the FRW solution \cite{8}.

### 3 Calculation of Non-Gaussianities

We use the action described above in Eq. (1) here to calculate the bispectrum.

**Solution of quadratic action** Considering only up to the quadratic terms the action is,

\[
S_2 = \int d^4x (-M_{pl}^2) \dot{H} \left( a^3 \frac{\dot{\pi}^2}{c_s^2} - a (\partial_i \pi)^2 \right)
\]

The Euler-Lagrange equation for the field \( \pi \) becomes,

\[
\frac{\partial}{\partial t} \left( \frac{2a^3M_{pl}^2 \dot{H}}{c_s^2} \dot{\pi} \right) - 2M_{pl}^2 \dot{H} a \nabla^2 \pi = 0
\]

Decomposing \( \pi \) into momentum modes using Fourier transform,

\[
\pi(\tau, \vec{x}) = \int d^3k \tilde{\pi}_k(\tau) e^{i\vec{k}\cdot\vec{x}}
\]
we get
\[ \frac{\partial}{\partial t} \left( 2a^3 M_{pl}^2 \dot{H} \frac{\pi_k}{c_s^2} \right) + 2M_{pl}^2 \dot{H} ak^2 \pi_k = 0 \]

Now defining the Mukhanov-Sasaki variable
\[ v_k = z \pi_k \]
where
\[ z = a \sqrt{-2 \dot{H} M_{pl} c_s} \]
and transforming everything to conformal time \( \tau \), such that \( dt = a d\tau \) we get
\[ v''_k + \left( k^2 c_s^2 - \frac{z''}{z} \right) v_k = 0 \]
where \( ' \) represents derivative with respect to conformal time. Now the strongest time dependence is contributed by scale factor \( a \) so if we take \( \dot{H} \) and \( c_s \) to be varying slowly, we can write \( \frac{z''}{z} = \frac{\dot{a}''}{a} \). For perfect de Sitter space \( a = -\frac{1}{H \tau} \) so \( \frac{\dot{a}''}{a} = \frac{1}{\tau^2} \).
So the equation reduces to
\[ v''_k + \left( k^2 c_s^2 - \frac{2}{\tau^2} \right) v_k = 0 \]
which has a solution
\[ v_k = \frac{-i(1 + ikc_s \tau)}{\sqrt{2k^2 c_s^2 \tau}} e^{-ikc_s \tau} \]
For perturbation \( \pi \),
\[ \pi_k = \frac{v_k}{z} = \frac{i(1 + ikc_s \tau)}{2 \sqrt{\epsilon c_s k} M_{pl}} e^{-ikc_s \tau} \]
where \( \epsilon = -\frac{\dot{H}}{H^2} \) is the usual slow roll parameter and we have used \( a = -\frac{1}{H \tau} \).
Differentiating it with respect to \( \tau \) we get
\[ \pi'_k = \frac{i}{2 \sqrt{\epsilon k c_s M_{pl}}} \left( k c_s^2 \tau e^{-ikc_s \tau} \right) \]
Now we need to think of cubic terms in the action as the perturbation to quadratic action. In other words cubic terms will contribute to interaction part of the Hamiltonian. Upto the cubic order we have \( H_{int} = - L_{int} \) so the interaction Hamiltonian in leading order becomes
\[ H_{int}(t) = - \int d^3 x a^3 \left[ M_{pl}^2 \dot{H} \left( 1 - \frac{1}{c_s^2} \right) \left( \dot{\pi}^3 - \dot{\pi} (\partial_i \pi)^2 \right) - \frac{4}{3} M_{pl}^4 \dot{\pi}^3 \right] \]
This has two terms: one is \( a^3 \dot{\pi}^3 \) and the other one is \( a\dot{\pi} (\partial_i \pi)^2 \). We will calculate the correlation function corresponding to each separately.
**Calculation for $a^3\pi^3$ term** We can expand $\pi$ into creation and annihilation operators as

$$\pi = \pi_ka_k + \pi_k^*a_k^\dagger$$

Weinberg’s in-in formalism [10] gives us a very nice way to calculate correlation functions. According to it any correlation function ($\langle W(t) \rangle$) is given by

$$\langle W(t) \rangle = \sum_{N=0}^\infty i^N \int_{t_0}^{t_1} dt_N \int_{t_0}^{t_N} dt_{N-1} \ldots \int_{t_0}^{t_2} dt_1 \langle [H_{\text{int}}(t_1), [H_{\text{int}}(t_2), \ldots [H_{\text{int}}(t_N), W(t)]\ldots] \rangle.$$  

If we consider only upto first order in interaction we get

$$\langle W(t) \rangle = i \int_{t_0}^{t_1} dt \langle [H_{\text{int}}(t), W(t)] \rangle$$

We will take $t_0$ to be the beginning of the universe which will become $-\infty$ once we convert everything to conformal time. We do the integral upto $\tau = 0$.

$$\langle \pi(k_1, 0)\pi(k_2, 0)\pi(k_3, 0) \rangle = -i \int_{-\infty}^0 d\tau \int d^3x \langle [\pi(k_1, 0)\pi(k_2, 0)\pi(k_3, 0), \pi'0(\tau, x)] \rangle \quad (6)$$

Now again decomposing $\pi'$ into Fourier modes

$$\langle \pi(k_1, 0)\pi(k_2, 0)\pi(k_3, 0) \rangle = -i(2\pi)^3 \int_{-\infty}^0 d\tau \int dk_4 \int dk_5 \int dk_6 \langle [\pi(k_1, 0)\pi(k_2, 0)\pi(k_3, 0), \pi'(k_4, 4)\pi'(k_5, 5)\pi'(k_6, 6)] \rangle \delta^3(k_4 + k_5 + k_6)$$

Now using Wick contraction and considering only connected diagrams

$$\langle \pi(k_1, 0)\pi(k_2, 0)\pi(k_3, 0) \rangle = -6i(2\pi)^3 \delta^3(k_1 + k_2 + k_3) \int_{-\infty}^0 d\tau \langle \pi_{k_1}(0)(\pi'_{k_1}(\tau))^* \rangle$$

Putting values of $\pi_k$ and $\pi_k'$ from Eq. 3 and 4 we get

$$\langle \pi(k_1, 0)\pi(k_2, 0)\pi(k_3, 0) \rangle = -\frac{3ic_s^3}{32\pi^3 k_1 k_2 k_3 M_{\text{pl}}^2}(2\pi)^3 \delta^3(k_1 + k_2 + k_3)$$

$$\int_{-\infty}^0 d\tau \left[ e^{i(k_1 + k_2 + k_3)c_s \tau} - e^{-i(k_1 + k_2 + k_3)c_s \tau} \right]$$

or putting $a = -\frac{1}{H\tau}$ we get

$$\langle \pi(k_1, 0)\pi(k_2, 0)\pi(k_3, 0) \rangle = +\frac{3ic_s^3}{32\pi^3 H k_1 k_2 k_3 M_{\text{pl}}^2}(2\pi)^3 \delta^3(k_1 + k_2 + k_3)$$

$$\left[ \int_{-\infty}^0 \tau^2 e^{i(k_1 + k_2 + k_3)c_s \tau} d\tau - \int_{-\infty}^0 \tau^2 e^{-i(k_1 + k_2 + k_3)c_s \tau} d\tau \right]$$
Now this integral doesn’t converge. But we can get rid of divergences using a slightly imaginary time axis while doing integration as follows:

\[
\langle \pi(k_1, 0)\pi(k_2, 0)\pi(k_3, 0) \rangle = + \frac{3i\nu^3}{32\epsilon^3Hk_1k_2k_3M_{pl}^3} (2\pi)^3 \delta^3(k_1 + k_2 + k_3) \left[ \int_{-\infty}^{0} \tau^2 e^{i(k_1+k_2+k_3)c_\tau d\tau} - \int_{-\infty}^{0} \tau^2 e^{-i(k_1+k_2+k_3)c_\tau d\tau} \right]
\]

which gives

\[
\langle \pi(k_1, 0)\pi(k_2, 0)\pi(k_3, 0) \rangle = - \frac{3}{8\epsilon^3Hk_1k_2k_3K^2M_{pl}^3} (2\pi)^3 \delta^3(k_1 + k_2 + k_3)
\] (7)

where \( K = k_1 + k_2 + k_3 \).

**Calculation for \( a\bar{\pi}(\partial_\tau)^2 \) term**  

Now doing this calculation for \( a\bar{\pi}(\partial_\tau)^2 \) term, we get

\[
\langle \pi(k_1, 0)\pi(k_2, 0)\pi(k_3, 0) \rangle = -i \int_{-\infty}^{0} a d\tau \int d^3x \langle [\pi(k_1, 0)\pi(k_2, 0)\pi(k_3, 0), \pi'(\partial_\tau)^2(\tau, x) \rangle \rangle \] (8)

\[
\langle \pi(k_1, 0)\pi(k_2, 0)\pi(k_3, 0) \rangle = -i (2\pi)^3 \int_{-\infty}^{0} a d\tau \int d^4k \int d^5k \int d^6k \langle [\pi(k_1, 0)\pi(k_2, 0)\pi(k_3, 0), \pi'(k_4, \tau)\pi(k_5, \tau)\pi(k_6, \tau) \rangle \rangle \delta^3(k_4 + k_5 + k_6)
\]

Now using Wick contraction and considering only connected diagrams

\[
\langle \pi(k_1, 0)\pi(k_2, 0)\pi(k_3, 0) \rangle = i (2\pi)^3 \delta^3(k_1 + k_2 + k_3) \left[ \int_{-\infty}^{0} a d\tau \left[ 2k_2.k_3(\pi_{k_1}(0))(\pi'_{k_2}(\tau))^*(\pi_{k_3}(0))(\pi_{k_3}(\tau))^* + 2k_1.k_3(\pi_{k_1}(0))(\pi_{k_2}(\tau))^*(\pi_{k_3}(0))(\pi_{k_3}(\tau))^* + 2k_1.k_2(\pi_{k_1}(0))(\pi_{k_2}(\tau))^*(\pi_{k_2}(0))(\pi_{k_3}(\tau))^* \right] \right] + C.C.
\]

Now since \( k_1, k_2 \) and \( k_3 \) form the sides of a triangle, \( 2k_2.k_3 = (k_1^2 - k_2^2 - k_3^2) \) and similarly for other two permutations. Now we put the values of \( \pi_k \) and \( \pi'_k \) from Eq. 3 and 4 and integrate. After some algebra we get,

\[
\langle \pi(k_1, 0)\pi(k_2, 0)\pi(k_3, 0) \rangle = - \frac{1}{32\epsilon^4\epsilon_{pl}^2 M^3 M_{pl}^3} (2\pi)^3 \delta^3(k_1 + k_2 + k_3) \left( K^6 + 12M^2 - 4KL \right) \left( M - 4K^2 L^2 + 11M^3 - 3LK^4 \right)
\] (9)

where

\[
K = k_1 + k_2 + k_3
\] (10)

\[
L = k_1k_2 + k_2k_3 + k_3k_1
\] (11)

\[
M = k_1k_2k_3
\] (12)
**Total Bispectrum:** We can get total bispectrum by adding the above two terms multiplied by the coefficients in the original action. This gives:

\[
\langle \pi(k_1,0)\pi(k_2,0)\pi(k_3,0) \rangle = - (2\pi)^3 \delta^3(k_1 + k_2 + k_3) \left[ - M_{pl}^2 \dot{H} \left( 1 - \frac{1}{c_s^2} \right) - \frac{4}{3} M_3^4 \right] \frac{3}{8 \epsilon_k k_1 k_2 k_3 K^3 M_{pl}^5} (K^6 + 12 M^2 - 4 K L M - 4 K^2 L^2 + 11 M K^3 - 3 L K^4)
\]

In the effective field theory model this \(\pi\) can be related to \(\tau\) that remains unchanged after horizon crossing as

\[
\zeta(t, x) = - H \pi(t, x)
\]

So,

\[
\langle \zeta(k_1,0)\zeta(k_2,0)\zeta(k_3,0) \rangle = - H^3 \langle \pi(k_1,0)\pi(k_2,0)\pi(k_3,0) \rangle
\]

or

\[
\langle \zeta(k_1,0)\zeta(k_2,0)\zeta(k_3,0) \rangle = (2\pi)^3 \delta^3(k_1 + k_2 + k_3) \left[ F_\pi^3(k_1, k_2, k_3) + F_{\pi(\partial\pi)^2}(k_1, k_2, k_3) \right]
\]

where

\[
F_\pi^3(k_1, k_2, k_3) = - \left( M_{pl}^2 \dot{H} \left( 1 - \frac{1}{c_s^2} \right) - \frac{4}{3} M_3^4 \right) \frac{3H^2}{8 \epsilon_k k_1 k_2 k_3 K^3 M_{pl}^5}
\]

and

\[
F_{\pi(\partial\pi)^2}(k_1, k_2, k_3) = \left( 1 - \frac{1}{c_s^2} \right) \times \frac{H^2 \dot{H}}{32 \epsilon_k c_s^2 (M^3 M_{pl}^4)} (K^6 + 12 M^2 - 4 K L M - 4 K^2 L^2 + 11 M K^3 - 3 L K^4)
\]

**Power Spectrum:** Let’s now define power spectrum \(\Delta_\zeta\) as

\[
\langle \zeta(k_1,0)\zeta(k_2,0) \rangle = (2\pi)^3 \delta^3(k_1 - k_2) \frac{\Delta_\zeta(k)}{k_1^3}
\]

Also

\[
\langle \zeta(k_1,0)\zeta(k_2,0) \rangle = H^2 (2\pi)^3 \delta^3|k_1 - k_2|^2
\]

or

\[
\langle \zeta(k_1,0)\zeta(k_2,0) \rangle = H^2 (2\pi)^3 \delta^3(k_1 - k_2) \frac{1}{4ek^4 c_s M_{pl}^2}
\]

So

\[
\Delta_\zeta(k) = \frac{H^2}{4ek^4 c_s M_{pl}^2}
\]
Experimental constraints: We define $f_{NL}$ as
\[
f_{NL} = \frac{5}{18} \frac{F(k,k,k)}{\Delta \zeta(k)^2}
\]

For $\dot{\pi} (\partial_i \pi)^2$ term
\[
f_{NL}^{\dot{\pi} (\partial_i \pi)^2} = \frac{85}{324} \left( 1 - \frac{1}{c_s^2} \right)
\]

and for $\dot{\pi}^3$ term
\[
f_{NL}^{\dot{\pi}^3} = \frac{15}{243} \left( c_s^2 - 1 - \frac{4 s^2 M_4^4}{3 M_{pl}^2 H} \right)
\]

Now this shape doesn’t match equilateral or orthogonal shape. It is a combination of both. The exact relation between $f_{NL}^{\dot{\pi} (\partial_i \pi)^2}$, $f_{NL}^{\dot{\pi}^3}$ and $f_{NL}^{\text{equil.}}$, $f_{NL}^{\text{orthog.}}$ have been given in [12] as
\[
\begin{pmatrix}
  f_{NL}^{\text{equil.}} \\
  f_{NL}^{\text{orthog.}}
\end{pmatrix}
= \begin{pmatrix}
  1.040 & 1.210 \\
  -0.03951 & -0.1757
\end{pmatrix}
\begin{pmatrix}
  f_{NL}^{\dot{\pi} (\partial_i \pi)^2} \\
  f_{NL}^{\dot{\pi}^3}
\end{pmatrix}
\]

Inverting these relations, we get
\[
f_{NL}^{\dot{\pi} (\partial_i \pi)^2} = 1.3022 f_{NL}^{\text{equil.}} + 8.9682 f_{NL}^{\text{orthog.}}
\]
\[
f_{NL}^{\dot{\pi}^3} = -0.2928 f_{NL}^{\text{equil.}} - 7.7082 f_{NL}^{\text{orthog.}}
\]

Planck [11] sets the constraints as $-86 < f_{NL}^{\text{equil.}} < 54$ and $-67 < f_{NL}^{\text{orthog.}} < -1$ (68 % CL statistical) which implies constraints as
\[
-712 < f_{NL}^{\dot{\pi} (\partial_i \pi)^2} < 61 \quad (15)
\]
\[
-8.103 < f_{NL}^{\dot{\pi}^3} < 541 \quad (16)
\]

This gives for speed of sound
\[
c_s > 0.019 \quad (17)
\]

4 Conclusion

We use the effective field theory model of inflation and analytically calculate the 2-point and the 3-point functions for the field perturbations. Then we use the latest Planck data [11] to constrain various parameters of the model.

This similar calculation can be found in [9] where authors have used WMAP data to constrain the parameters. Our final form of three point function matches with that. Moreover here we show all the calculation steps explicitly and use the latest Planck data. Also the constraint on speed of sound $c_s$ was found by Planck collaboration in [11] as $c_s > 0.021$ for DBI model.
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References

[1] D. Baumann, Lecture notes on "The Physics of Inflation", DAMTP.

[2] V. F. Mukhanov, "Physical Foundations of Cosmology" (Cambridge University Press, Cambridge, England, 2005).

[3] J. L. Cervantes-Cota, G. Smoot, "Cosmology today-A brief review", AIP Conf. Proc. Vol. 1396, pp. 28-52 (2011), arXiv:1107.1789 [astro-ph.CO].

[4] J. Maldacena, "Non-Gaussian features of primordial fluctuations in single field inflationary models", JHEP 0305 (2003) 013, arXiv: 0210603 [astro-ph].

[5] D. Regan and D. Munshi, "Principal Components of CMB non-Gaussianity", MNRAS, Volume 448, Issue 3, p.2232-2244, arXiv:1407.0402 [astro-ph.CO].

[6] D. Babich, P. Creminelli and M. Zaldarriaga, The shape of non-Gaussianities, JCAP 0408 (2004) 009 [arXiv:astro-ph/0405356].

[7] E. Komatsu, "Hunting for primordial non-Gaussianity in the cosmic microwave background", Class.Quant.Grav. 27: 124010, 2010, arXiv:1003.6097 [astro-ph].

[8] C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan and L. Senatore, The Effective Field Theory of Inflation, JHEP 0803 (2008) 014 arXiv:0709.0293 [hep-th].

[9] Senatore, Smith, Zaldariagga, "Non-Gaussianities in single field inflation and their optimal limits from the WMAP five year data", JCAP 1001:028 (2010) arXiv:0905.3746 [astro-ph].

[10] Steven Weinberg, "Quantum contributions to Cosmological Correlations", Phys.Rev.D72:043514 (2005), arXiv:0506236 [hep-th].

[11] Planck Collaboration ," Planck 2015 results. XVII. Constraints on primordial non-Gaussianity", arXiv:1502.01592 [astro-ph.CO].
[12] C. Cheung, A. L. Fitzpatrick, J. Kaplan and L. Senatore, On the consistency relation of the 3-point function in single-field inflation, JCAP 0802 (2008) 021, [arXiv:0709.0295] [hep-th].

[13] M. Alishahiha, E. Silverstein, D. Tong, "DBI in the Sky", Phys.Rev. D70 (2004) 123505, [arXiv:0404084][hep-th].

[14] K. Koyama, G. W. Pettinari, S. Mizuno, C. Fidler, "Orthogonal non-Gaussianity in DBI Galileon: prospect for Planck polarisation and post-Planck experiments", Class.Quant.Grav. 31 (2014) 125003, [arXiv:1303.2125] [astro-ph.CO].

[15] J. Zhang, Y. Cai, Y. Piao, "Preheating in a DBI Inflation Model", [arXiv:1307.6529][hep-th].

[16] J. M. Weller, C. Bruck, D. F. Mota, "Inflationary predictions in scalar-tensor DBI inflation", JCAP 06 (2012) 002, [arXiv:1111.0237] [astro-ph.CO].

[17] E. A. Lim, Lectures on "Advanced Cosmology : Primordial non-Gaussianities and detection of stochastic gravitational waves", DAMTP.