Light Four-Quark States and New Observations by BES

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Four-quark states are discussed within the constituent quark model. Incompleteness of existed studies of four-quark state with QCD sum rule is analyzed. The masses of diquark cluster were determined by QCD sum rules, and light four-quark states masses were obtained in terms of the diquark. The four-quark state possibility of the newly observed near-threshold $p\bar{p}$ enhancement, $X(1835)$, $X(1812)$ and $X(1576)$ by BES is discussed.

§1. Four-quark states in the constituent quark model

After Gell-Mann\(^1\) first conjectured the existence of multi-quark states, four-quark states were predicted to exist in a consistent description of the hadron scattering amplitudes.\(^2\) In the constituent quark model, four-quark states may consist of a $qq$ diquark and a $q\bar{q}$ anti-diquark (tetraquark), or of two $q\bar{q}$ clusters due to the spatial extension of the quarks. Study of four-quark states was extensively performed in many constituent quark models in late 1970’s and revived with recent experimental\(^3\) and theoretical\(^4\) developments. Four-quark states were initially regarded as hadron molecules and charmonium atoms.\(^5\) Subsequently, it was studied in MIT bag model,\(^6\) color junction model,\(^7\) potential model,\(^8\)--\(^11\) relativistic quark model,\(^12\) effective Lagrangian,\(^13\) QCD sum rules\(^14\)--\(^18\) and other models.\(^20\) A review on exotica could be found in Ref. 21).

A four-quark state is a many-body system, in which quarks have many degrees of freedom such as color, flavor and spin etc. The correlation of these degrees of freedom among quarks is complicated, and so far the exact correlation is not clear. According to Ref. 22), both the $(qq)(\bar{q}\bar{q})$ and the $(q\bar{q})(q\bar{q})$ four-quark state may result in the same color, flavor and spin representation. $(qq)(\bar{q}\bar{q})$ may mix with $(q\bar{q})(q\bar{q})$. Furthermore, flavor Crypto-exotic four-quark states may mix with normal $q\bar{q}$ mesons. That is to say, intrinsic color, flavor configurations could not be distinguished without the establishment of some special observable. Unfortunately, no such an observable has been definitely constructed, and experimentally observed meson may be $|meson> = |q\bar{q}> + |(qq)(\bar{q}\bar{q})> + |(q\bar{q})(q\bar{q})> + \cdots$

Of course, the correlation of color, flavor and spin in hadrons is inter-related since their total correlation has to obey various symmetry constraints. In the tetraquark $(qq)(\bar{q}\bar{q})$, according to Ref. 21), two quarks correlate antisymmetrically in color.
flavor and spin, separately, and the diquark is in a "good" diquark configuration $|qq,3_F,3_C,0\rangle$. The tetraquark state consisting of "good" diquark may be the most suitable object to study the behaviour of quark confinement\(^{22}\) though quark dynamics in four-quark state is still not clear.

§2. SVZ sum rules and four-quark state

QCD sum rules\(^{23}\) are a semi-phenomenological nonperturbative method of relating fundamental parameters of QCD Lagrangian and vacuum to parameters of hadrons. In terms of suitable interpolating currents, correlators are constructed. On one hand, the correlator can be expressed with parameters of QCD and vacuum through the operator product expansion (OPE). On the other hand, the correlator can be expressed with parameters of hadrons through the dispersion relation.

To perform the analysis of a sum rule, the choice of a suitable interpolating current is important. In the study of four-quark states, different currents were used. In Ref. 14), a $(\bar{q}q)(\bar{q}q)$ current was used; In Ref. 15), both $(q\bar{q})^2$ and $(qq)(\bar{q}\bar{q})$ currents were analyzed; In Ref. 16), a $(cq)(\bar{q}q)$ current was used; In Ref. 17), a $(cu)(\bar{s}\bar{u})$ current was used; In Ref. 18), a $(ud)(\bar{s}\bar{s})$ current was used. These investigations are interesting and may give some hints to our understanding of four-quark states. However, they are not complete.

First, some of the conclusions related to the diquark concept based on these studies are not definitive. Diquarks may be the reality in constituent quark models, but a diquark interpretation is not meaningful in the framework of QCD sum rules. In view of the sum rule approach, the constituent quark structures of multi-quark states are difficult to detect through the couplings of interpolating currents to hadrons.\(^{19}\)

This point is much more easy to be realized in other ways. The Fierz transformation will turn the $(\bar{q}q)(qq)$ current into the $(\bar{q}q)(qq)$ current and vice-versa, and these two kinds of currents can mix with each other under renormalization. Therefore, it may not be meaningful to talk about diquarks in the framework of sum rules based on local interpolating currents.

Furthermore, correlators of tetraquark/four-quark interpolating currents have only been calculated to leading order in $\alpha_s$.\(^{14}-^{18}\) To make the sum rule analyses reliable and predictable, suitable interpolating currents (for example, mixed currents $((qq)(\bar{q}q)+(qq)(\bar{q}q))$) and $\alpha_s$ corrections (involving four-loop calculations and the renormalization of the dimension-six composite operators) are required.

It may be possible to proceed with the study of four-quark states in another way. We can determine the mass of a diquark cluster via QCD sum rules, and subsequently construct four-quark states in terms of these diquark constituents.

§3. Light tetraquark state and new observations by BES

We first describe the results of an updated sum-rule study of the mass of $J^P = 0^+$ "good" diquark. The flavor $(sq)$ "good" diquark current was taken as that given in Refs. 24) and 25), and with the input parameters,\(^{22}\) the most "suitable" $m_{qq}$ and $m_{sq}$ are determined: $m_{qq} \sim 400$ MeV ($s_0 = 1.2$ GeV\(^2\)) and $m_{sq} \sim 460$ MeV.
Once the determined masses are regarded as the constituent masses of diquark, it is possible to construct a four-quark provided that the quark dynamics in hadrons is known. Here hadron masses were obtained from constituent quarks as in Refs. 6) and 20).

The mass of the $L = 0$ tetraquark state is

$$M \approx 2m_{[qq]} - 3(k_{\eta q})_{\frac{3}{2}},$$

with $(k_{\eta q})_{\frac{3}{2}} = 103$ MeV, $(k_{\eta q})_{\frac{5}{2}} = 64$ MeV. The masses of $0^{++}$ tetraquark states $[\bar{q}q][qq]$, $[\bar{q}q][sq]$ and $[\bar{s}q][sq]$ are therefore found to be $\sim 490$ MeV, $\sim 610$ MeV and $\sim 730$ MeV, respectively. Taking into account the decay features, in the approximation we used, it is reasonable to identify $f_0(600)$ (or $\sigma$), $f_0(980)$, $a_0(980)$ and the unconfirmed $\kappa(800)$ as the $0^{++}$ light tetraquark states.

Similarly, the mass of the $1^{--}$ ($L = 1$) and the $2^{++}$ ($L = 2$) tetraquark state is

$$M_{4q} = m_d + m_{\bar{d}} + B_{dd}\frac{L(L+1)}{2}.$$  \hspace{1cm} (3.1)

In the following, $B_{dd}$ is denoted as $B'_{q}$, $B'_{1s}$ and $B'_{2s}$ ($B'_{q} > B'_{1s} > B'_{2s}$) with zero, one and two strange quarks, respectively. Masses of the $1^{--}$ orbital excited $[\bar{q}q][qq]$, $[\bar{s}q][qq]$ and $[\bar{s}q][sq]$ are respectively determined by $\sim 490 + B'_{q}$ MeV, $\sim 610 + B'_{1s}$ MeV and $\sim 730 + B'_{2s}$ MeV. Theoretical estimates of the masses of these tetraquark states are not given for the sensitivity of $B_{dd}$ to $\Lambda_{QCD}$.

So far, no four-quark state has been confirmed experimentally. In the low energy region, there are several four-quark candidates in experiments, including $f_0(600)$ (or $\sigma$), $f_0(980)$, $a_0(980)$ and the unconfirmed $\kappa(800)$. They have a long history of being interpreted as four-quark states.\(^{6,26}\)

In addition to these candidates, some recent observations by BES Collaboration such as the near-threshold $p\bar{p}$ enhancement,\(^{27}\) $X(1835)$,\(^{28}\) $X(1812)$,\(^{29}\) and $X(1576)$\(^{30}\) were interpreted as four-quark candidates.\(^{22,31-35}\)

In our analyses, the $p\bar{p}$ enhancements, $X(1835)$ and $X(1812)$ are unlikely to be tetraquark states. $X(1576)$ is likely to be the $1^{--}$ tetraquark state, which may be the $[\bar{s}q](sq)$ “exotic” orbital excited tetraquark state.\(^{22}\) It may be the first orbital excitation of $a_0(980)$ if its isospin is $I = 1$, and it may be the first orbital excitation of $f_0(980)$ if its isospin is $I = 0$. If this suggestion is true, other $1^{--}$ orbital excited tetraquark states corresponding to $[\bar{q}q](qq)$ ($\sim 1400$ MeV) and $[\bar{s}q](qq)$ ($\sim 1500$ MeV) are expected.

§4. Conclusions and discussion

Incompleteness of previous study of four-quark states with QCD sum rules has been discussed. The masses of diquarks have been determined through an updated QCD sum rule analysis, and the resulting masses of some tetraquark states have been constructed in the constituent quark model.

Based on these analyses, it is reasonable to identify $f_0(600)$ (or $\sigma$), $f_0(980)$, $a_0(980)$ and the unconfirmed $\kappa(800)$ as the $0^{++}$ light tetraquark states. The new
observations by BES are unlike to be the light four-quark states except that $X(1576)$ may be an “exotic” first orbital excited $(sq)(ar{s}ar{q})$ tetraquark state.

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