Disc instabilities and semi-analytic modelling of galaxy formation

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ABSTRACT
The Efstathiou, Lake and Negroponte (1982) criterion cannot distinguish bar stable from bar unstable discs and thus should not be used in semi-analytic galaxy formation simulations. I discuss the reasons for this, illustrate it with examples and point out shortcomings in the recipes used for spheroid formation. I propose an alternative, although much less straightforward, possibility.

Key words: methods: numerical – galaxies: evolution – galaxies: haloes – galaxies: kinematics and dynamics – galaxies: structure.

1 INTRODUCTION
Technically, it is not yet possible to make full cosmological simulations which include both the dark matter and the baryons, down to the scale of individual galaxies. Yet the dark matter only simulations have reached very high resolutions (Springel et al., private communication) and it is crucial to find ways of exploiting them fully. A scheme involving resampling has been devised, in which a given object and its surrounding region are singled out and resimulated at high resolution (e.g. Katz & White 1993). Nevertheless, this technique can only give information on one, or at best a few objects. For this reason, semi-analytical models were introduced, which can give information on properties of galaxy populations. These models are tagged on to the dark matter only simulations and use ‘recipes’ to describe the evolution of the baryons. It is clear that the results will be useful only if the recipes in question are correct and adequately chosen. This is not an easy task, since these recipes need to include in a relatively simple way a fair fraction of the available information on many key astrophysical processes.

In their quest to have more spheroids, either bulges or ellipticals, some semi-analytic modellers are now considering disc instabilities. They use the Efstathiou, Lake & Negroponte (1982, hereafter ELN) criterion to distinguish between bar stable and unstable discs. Using this criterion, this information can be obtained very simply from the maximum rotational velocity and the disc mass and scale-length. Once a disc is found to be bar unstable, it is turned instantaneously in the model into an elliptical (e.g. Christodoulou, Shlosman & Tohline 1995) or, alternatively, half of its mass is turned into a bulge and this repeatedly until disc stability according to the ELN criterion is achieved (e.g. De Lucia & Helmi 2008). In this way, a considerable amount of mass is turned into a spheroid and several problems concerning the K-band luminosity function are alleviated (see Parry, Eke & Frenk 2008, for a discussion of the relative merit of the two approaches).

There are, however, a number of shortcomings in this modelling approach.

2 BAR FORMATION
Efstathiou, Lake and Negroponte (1982) ran a number of two-dimensional N-body simulations of a purely stellar disc in a rigid halo and, based on these, proposed a very simple criterion to distinguish bar stable from bar unstable discs. This reads

\[ R_{\text{ELN}} = \frac{V_{\text{m}}}{(M_{\text{D}}G/R_{\text{D}})^{1/2}}, \]

where \( V_{\text{m}} \) is the maximum rotational velocity, \( M_{\text{D}} \) is the mass of the disc, \( R_{\text{D}} \) is its scalelength and \( G \) is the gravitational constant. If \( R_{\text{ELN}} < 1.1 \), ELN propose that the disc is bar unstable, while it will be bar stable for values larger than that limit. This criterion was subsequently used by Mo, Mao & White (1998) and is sometimes referred to as the Mo, Mao & White criterion. The ELN criterion was derived for purely stellar discs. Christodoulou, Shlosman & Tohline (1995) rederived it for the case of purely gaseous discs, and found that the stability threshold is considerably lower. Thus, purely gaseous discs will be stable if \( R_{\text{ELN}} > 0.9 \). So far, however, this criterion has not been extended to the physically relevant case where both stars and gas are available in the same disc, even when no star formation or feedback is present. Our discussion will therefore, by necessity, follow this limitation.

Other variants of this criterion have also been proposed (see e.g. van den Bosch 1998). In all its forms, however, the ELN criterion is intimately related to the question of whether disks are self-gravitating in their inner parts, the so-called ‘maximum disc’ problem (see Bosma 2004 for a review).

The ELN criterion was derived more than 25 yr ago and since then our understanding of bar formation has advanced considerably. A number of criticisms of this criterion can be and have been made. First, the central concentration of the halo has not been fully taken into account. In the linear theory, if this was sufficiently high it would cut the swing amplifier cycle and thus stop any growth of the...
bar mode (Toomre 1981). In simulations, however, there can still be a tunneling through this barrier (e.g. Sellwood 1989), so that a bar may still grow. Thus, this criticism may not be particularly acute.

A more serious one is that the disc velocity dispersion is not taken into account. A few years after the ELN criterion was published, Athanassoula & Sellwood (1986) showed that velocity dispersion has an important influence on bar stability, which is, in fact, a function of both disc random motions and halo mass. They also presented a disc with no halo, which is stable because of its high-velocity dispersion. Although these results are based on two-dimensional simulations, i.e. have by necessity rigid haloes, they still have the advantage of stressing the effect of random motions on stability and thus underline an inadequacy of the ELN criterion.

A major problem of the ELN simulations is that they are two-dimensional. Such simulations have by definition rigid haloes, i.e. haloes which are represented by an external forcing and cannot respond to the evolution of the disc. This approximation was reasonable in 1982, due to the limited computer means available at the time. With the increase in computer power, however, high-resolution three-dimensional simulations with adequate self-consistent treatment of both the baryonic and the dark matter component have become the norm. Athanassoula (2002) compared the results of such fully self-consistent three-dimensional simulations to those of simulations with rigid haloes, and showed that the results obtained with the latter can be totally unreliable. In particular, she showed that rigid haloes can erroneously claim stability for cases which the live haloes show are unstable. She explained the difference by the fact that angular momentum exchange between the halo and the disc is de facto not existent in simulations with rigid haloes. On the contrary, in fully self-consistent simulations angular momentum is emitted by near-resonant material in the bar region and absorbed by near-resonant material in the halo and in the outer disc (Athanassoula 2003). This leads to a considerable growth of the bar and explains why strong bars can be seen in discs immersed in massive haloes (Athanassoula & Misiriotis 2002). All this complexity is, of course, not contained in the very simple ELN criterion or in the work by Ostriker & Peebles (1973), which come to the opposite conclusions, since they do not include the interaction between the disc and the halo.

To make the inadequacies of the ELN criterion clearer, I have chosen amongst my \(N\)-body simulations (Athanassoula 2003, 2007) six examples, three with \(R_{\text{ELN}}\) smaller than 1.1 and three with larger. The three first examples, shown in Fig. 1, have identical \(R_{\text{ELN}}\) values but different disc velocity dispersion. Since \(R_{\text{ELN}} = 0.89\), they should all be bar unstable by the ELN criterion, but Fig. 1 shows that this is not the case. The example on the left-hand panel has a very hot disc which makes it stable against bar formation, the middle one shows an average-sized bar, while the one on the right-hand panels shows a strong bar. The value of \(R_{\text{ELN}}\) chosen is nearly half way between the minimum allowed value (0.63, for a bare exponential disc with no halo) and the stability threshold of 1.1 and shows that even

**Figure 1.** The disc component of three simulations which, by the ELN criterion, should form a bar. The upper panels show the face-on views, the middle ones the edge-on, side-on view (i.e. with the line of sight along the bar minor axis) and the lower ones the edge-on, end-on view (i.e. with the line of sight along the bar major axis). In all the cases, the projected density of the disc is given by grey-scale and also by isocontours (spaced logarithmically) and the numerical value of \(R_{\text{ELN}}\) is given in the upper left-hand corner of the face-on views.
for $R_{\text{ELN}}$ values far from the stability threshold, the disc velocity dispersion can stabilize the disc for very long times, of the order of, or more than a Hubble time. This is true for even lower values of $R_{\text{ELN}}$. For example, as already stated, Athanassoula & Sellwood (1986) produced a model with no halo at all which was stable over a Hubble time. This is further enhanced in the case where strong central concentrations, and/or high-velocity dispersions in the halo component add their stabilizing effect to that of the disc velocity dispersion. Thus, a galaxy may be bar stable when the ELN criterion predicts instability.

The three examples given in Fig. 2 have values of $R_{\text{ELN}}$ equal to 1.22 for the simulation on the left-hand panel, and 1.19 for the two others. Thus, the ELN criterion predicts that all three are bar stable, since their $R_{\text{ELN}}$ values are larger than 1.1. Fig. 2 shows that one of the three cases is indeed bar stable, the second one forms a small inner bar and the third one a fair-sized bar. The edge-on views are also widely different, since the first two have only a disc component, while the third one shows a clear peanut bulge. So again, the ELN criterion is found faulty. This is due to the fact that the live halo helps bar growth by absorbing at its resonances the angular momentum that the inner disc can emit. Of course, there is a limit beyond which the disc would not be able to emit sufficient angular momentum, e.g. because it is not sufficiently massive and/or because it is too hot. In such cases, the angular momentum exchange within the galaxy would be limited by the emitters (or lack thereof) and no bar could grow within an astronomically relevant time even though the halo has responsive resonances (Athanassoula 2003). This limit, however, is very far from the ELN predictions, and, more important, does not depend only on the mass ratios, but also on the velocity dispersions of the various components.

To summarize, the ELN criterion should not be used in semi-analytic simulations. In cases where it predicts instability, the disc can still be stabilized by factors which $R_{\text{ELN}}$ does not take into account, such as a strong central concentration, strong random motions in the disc and halo, or, even better, a combination of all three. Conversely, in cases where the ELN criterion predicts stability, a bar can still form due to the destabilizing influence of the halo resonances. Shifting the threshold up or down will not solve the problem as long all the other stabilizing/destabilizing influences are not taken into account. Finally, note that in all cases, both bar stable and bar unstable, the disc is preserved, i.e. no elliptical galaxy is formed.

3 SUMMARY AND DISCUSSION

In the previous section, I showed that the ELN criterion is too simplistic to be able to describe a complex phenomenon such as bar formation and to distinguish bar stable from bar unstable discs. It thus cannot be used in semi-analytic calculations.

Further problems concern the subsequent evolution, after the bar has formed. No simulation has ever shown an unstable disc turn into an elliptical, or acquire a very massive classical bulge instantaneously. The bar can form a boxy/peanut bulge (see Figs 1 and 2) or a discy bulge, and the formation of a classical bulge from
either of those is not excluded.\(^1\) This classical bulge, however, would be much smaller than required by the semi-analytic models, and, furthermore, it would not form instantaneously since the bar needs some time to form, more time is necessary for the boxy/peanut, or discy bulge formation and yet more time is necessary for the putative conversion into a classical bulge. Thus, the second part of the De Lucia & Helmi (2008) model has shortcomings, but these may turn out to be quantitative rather than qualitative. On the contrary, the Bower et al. (2006) model has shortcomings at the qualitative level, since the disc cannot disappear, i.e. an elliptical could not form from a bar unstable disc without a merger. Both models, of course, have already problems in the first part, i.e. deciding whether a disc is bar unstable or not, since they both use the ELN criterion.

Can we replace the simple ELN recipe for handling disc instabilities by a better one? This task is not easy, since we need to include in this recipe information on disc stability, bar formation and evolution and the subsequent formation of the different types of bulges, all of which are complex, non-linear processes. I firmly believe that it is not possible to find one, or a few simple formulas, like the ELN criterion, which can describe all the necessary ingredients of these processes. It might thus be preferable to seek a solution intermediate between the full cosmological simulation including both the dark matter and the baryons at sufficient resolution, which will not be available in the foreseeable future, and the equally difficult task of finding appropriate recipes.

Computer simulations allow us today to routinely make \(N\)-body simulations which can describe the evolution of a single galaxy. In fact, a very large number are already available. Using their results instead of the recipes is a possibility well worth exploring. For example, for the problem at hand one would need a small library of \(N\)-body simulations with and without gas, describing bar formation and evolution. These simulations should cover, albeit very crudely, the necessary parameter space describing the halo, stellar disc and gas components. This would include not only the mass and scalelength ratios of the various components, but also the different amounts of random motion in the stellar disc and in the spheroids. From these simulations, one should extract all the necessary parameters, such as the times necessary for bar and peanut formation, the amount of mass in the boxy/peanut bulge or the discy bulge, etc.

This information would be assembled in some sort of table. Then at each time-step of the semi-analytic simulation, instead of checking the criterion or applying the recipe, one would extract from the above table the relevant information as a function of the properties of the galaxy at the time under consideration.

The above scheme is, of course, not a substitute for full scale cosmological simulations, starting \textit{ab initio}. It has also a number of shortcomings, the most important of which is that our knowledge about the velocity dispersions in disc galaxies is very limited. Nevertheless, it is a very important improvement with respect to the presently used scheme.

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