Attractive polaron formed in doped nonchiral/chiral parabolic system within ladder approximation

Chen-Huan Wu *
College of Physics and Electronic Engineering, Northwest Normal University, Lanzhou 730070, China
August 28, 2019

We investigate the properties of attractive polaron formed by a single impurity dressed with the particle-hole excitations in a three-dimensional (3D) doped (extrinsic) parabolic system. Based on the single particle-hole variational ansatz, we study the pair propagator, self-energy, and the non-self-consistent medium $T$-matrix. The non-self-consistent $T$-matrix discussed in this paper contains only the open channel since we don’t consider the shift of center-of-mass due to the resonance (e.g., induced by the magnetic field). Besides, since we focus on the low-density regime of the majority particles, the effective Fermi wave vector is small. The scattering form factor is discussed in detail for the chiral case and compared to the non-chiral one. The effects of the bare coupling strength, which is momentum-cutoff-dependent, are also discussed. We found that the pair propagator and the related quantities, like the self-energy, spectral function, induced effective mass, and residue (spectral weight), all exhibit different features in the low-momentum regime and the high one, which also related to the polaronic instabilities as well as the many-body fluctuation and nonadiabatic/adiabatic dynamics. The pair-propagator and the energy relaxation time at finite temperature are also explored.

Keywords: Attractive polaron; non-self-consistent medium $T$-matrix; Retarded self-energy; Pair propagator; Chiral system; Ladder approximation

1 Introduction

The polaron formed in the cold-atom systems has been intensely studied where a lattice model is absent similar to the Frohlich model, but the polaron formed in a lattice model (atomic lattice or the crystal lattice) has been less studied, especially the topological[1, 2] or chiral systems. In this paper, we focus on the properties of fermion polaron formed in a three-dimensional (3D) doped parabolic system in the crystal lattice. Considering the effect of impurities (in coherent background), the polaron as an excited quasiparticle in the population/spin-imbalanced Fermi gases, BEC, or the topological insulator [2, 3, 4], are important when the many-body effect is been taken into account. Unlike the gapless Dirac system where the electron-electron interaction usually gives rise to the renormalization of quasiparticle velocity[5, 6], for systems with quadratic dispersion, the electron-electron interaction usually gives rise to the effective mass renormalization[7], which will be discussed in this paper, and we note that the effective mass approximation[8, 9, 10] is applicable for the polaron in small momentum regime. Besides, since the spin rotation is missing in our system in the presence of $\delta$-type impurity field, the spin structure is fixed, and the interacting spins between impurity and majority particle are usually

*chenhuanwu1@gmail.com
opposite in direction (due to the Pauli principle) which provides the opportunity to form the Cooper pair and the strongly bound dimer in superfluid. That also implies the low-temperature which with weaker spin relaxation is beneficial to the formation of polarons, that can also be seen from the finite temperature pair propagator as presented in this paper. We present in Appendix. A a brief discussion of the variational approach in mean-field approximation which is valid in the weak interacting regime with non-too-low density[11], for the polaron dressed by the partially polarized excitations-cloud, however, the mean-fields approximation sometimes overestimates the interaction effect[12] in the strongly interacting regime (with tightly bound dimers) where the self-trapping, soliton, and breather are harder to formed than that in the weakly bound pairs (e.g., the BCS superfluid state in the non-Fermi-liquid picture). The stable repulsive polarons are most likely to be found in the side away from the Feshbach resonance (for gases) where polaron energy is large and positive (with $1/k_Fa \gg 1$; $a_{\psi\phi}$ is the scattering length related to the impurity-majority interaction, which is produced by the attractive potential and $k_F$ is the fermi wave vector), while in strong scattering region ($1/k_Fa \lesssim 1$), the tightly-bound molecule (within Fermi-liquid picture) could also be found which with binding energy $E_b = \hbar^2/2a^2\tilde{m} > 0$[9, 13], as experimentally realized in Ref.[14, 15]. However, in most cases the repulsive polaron is thermodynamically unstable in the strong interaction region even at low-temperature. While for the attractive polaron, whose eigenenergy is negative, it can be observed in the solid state systems[13, 16, 17, 18, 19, 20] including the topological materials, through the, e.g., substrate-related polaronic effect. Besides, the effects of both the intrinsic and extrinsic (like the electric field-induced Rashba) spin-orbit coupling on the polaron have also been explored[16, 17, 18, 19], and it is found that the spin-orbit coupling is also related to the polaron-molecule transition[21]. The quasiparticle residue $Z$ (spectral weight), as an important property of the polarons, which can be measured by the Rabi oscillations[22], is finite even at zero-energy state for the parabolic system or normal Dirac semimetals, but it vanishes at zero-energy state for the Weyl semimetals which with higher dispersion[23, 24] (i.e., the multi-Weyl semimetal).

In this paper, we will focus on the weak-interacting (short-range) regime in nonadiabatic configuration, where the quasiparticle spectral weight is away from zero and the polaron is well-defined (and thus long-lived). While the long-range Coulomb interaction, which will gives rise to the adiabatic effect and thus accelerates the electrons, is been ignored here due to the screening effect. For example, by including a Thomas-Fermi screening wave vector ($k_{TF} = me^2/\epsilon$) into the Coulomb interaction (i.e., the static part of the particle-hole bubble), the interaction range will becomes shorter, and it satisfies $a_B k_F \ll 1$[25, 26] ($a_B = k_F^{-1}$) at small density. In two-body problem ($k_F \rightarrow 0$), the feature of screened polaron will becomes more obvious in the non-relativistic limit, which with a slower motion compared to the intrinsic Dirac electron (relativistic), and that also gives rise to the nonadiabatic dynamics. Due to the nonadiabatic character, the Matsubara Green’s function is fully momentum- and frequency-dependent in renormalization group flow, which is different from the bare Coulomb interaction in the clean Dirac system with the instantaneous nature. The vertex function as well as the pair propagator is also fully momentum- and frequency-dependent.

For the attractive fermionic polaron formed through the attractive interaction between the impurity with the electron-hole pairs excited in a doped parabolic system in weak coupling regime, its polaronic effects are investigated by using the non self-consistent $T$-matrix approach (consist of the undressed propagators, and thus the particle/energy conserving is broken) within the ladder approximation. The accuracy of non-self-consistent method used here has been verified[27, 28]. Note that, however, the energy of polaron obtained here will shows deviation from the DFT result (for unit cell)[29] since the Coulomb energy are ignored, also, the non-self-consistent treatment will results in deviations from the real total energy[28]. We consider the low-doping case of the parabolic system so that the interband electron-hole excitation can
be created by the impurity, and not be suppressed by the phase space restriction. Thus the system discussed in this paper (no matter the chiral or nonchiral one) will exhibits a little Fermi liquid charater, which can be seen from the power-law behavior in frequency of the self-energy imaginary part. Our theory can also be applied to the massive (doped) Dirac or Weyl systems where the band structure in low-energy regime is gapped.

In Sec.2, we introduce the model we studying. The Hamiltonian of the nonchiral or chiral impurity model are presented. In Sec.3, we present the expressions of the non-self-consistent T-matrix, part-propagator, and self-energy. The interacting Hamiltonian are presented, where we can see that the polaron problem can be related to the Schrodinger-type eigenvalue problem subjected to a wave functional constraint, which is similar to Ref.\[29, 30\]. Besides, the validity for non-self-consistent T-matrix in studying the many-body effect are also discussed in this section. In Sec.4, we present the main results of this paper, where the single-channel BCS model as well as the variational wave function are introduced, and the pair propagator-related quantities (including the self-energy and spectral function) are derived for the chiral and non-chiral cases for zero-temperature case. The induced effective masses and the residue are also calculated for different coupling strength. The scattering form factor here is different from the one with single-particle propagation. The polaronic stability and the many-body fluctuation are also discussed. In Sec.5, we discuss the pair propagator and the relaxation time at finite temperature which exhibit some differences compared to the zero-temperature case.

2 Theories

Different to the non-Fermi-liquid picture in the Dirac/Weyl semimetal, the impurities of the Fermi gases are mobile as widely studied\[13, 14, 31\], while for the immobile one the Kondo effect as well as the indirect exchange interactions are considered. For such mobile impurity, the \[\delta\]-type impurity field (within Born approximation) is valid in measuring the effect of impurity-interaction. However, that also cause the vanishing of the spin rotation as well as the intrinsic spin current during the impurity-majority scattering (collision) process. Then, the fixed spin structure guarantees that there exists only the singlet pairing between the impurity and majority particles, otherwise the triplet pairing exists as discussed in Ref.\[21\]. In Fermi gases or the dilute BEC, the interactions between impurity and majority component, and that between the majority particles needed to taken into account, such problems are usually dealt by the non-self-consistent many-body T-matrix (i.e., the ladder approximation). Here we consider the interactions between the fermions (bath) and the single fermionic impurity. The difference compared to the normal solid state system\[32, 33, 34\] is that we are taking the mobile impurity into account and note that we still focus on the single impurity problem since the Fermi polaron is well defined (with symmetry and easy-to-identify spectral function) in the single-impurity-limit\[13\]. At first, we write the microscopic euclidean action as

\[
S = \int d\tau d^3r \{ \psi^\dagger [\partial_\tau + H_0(k)] \psi + \frac{1}{2} \sum_{\alpha=x,y,z} (\partial_\alpha \phi)^2 + \frac{1}{2} g_{\psi\psi} \psi^\dagger \psi^\dagger \psi + g_{\psi\phi} \psi^\dagger \psi^\dagger \phi \},
\]

(1)

where \(\psi\) and \(\phi\) denote the majority and impurity field, respectively. \(g_{\psi\psi}\) and \(g_{\psi\phi}\) are the intraspecies (only around the impurity) and interspecies coupling, respectively, for the mobile particles. For parabolic chiral system (doped), the noninteracting Hamiltonian of the impurity reads

\[
H_0 = \frac{|p|^2}{2m} \hat{p} \cdot \sigma + hv_x p_x \sigma_z - \mu,
\]

\[
= \frac{p_x^2}{2m} \sigma_x + 2 \frac{p_x p_y}{2m} \sigma_y - \frac{p_y^2}{2m} \sigma_x + hv_x p_x \sigma_z - \mu.
\]

(2)
Note that here \( p \) can be replaced by other momenta to represent the other particles (like the majority component). While for the nonchiral parabolic system, the Hamiltonian can be obtained by removing the term \( \mathbf{p} \cdot \sigma \) in above equation or replacing it by a (spin) Pauli matrix \( \sigma_z \). We assume the longitudinal term involving \( p_z \) is small, and thus the eigenenergy can be approximated as \( \frac{p^2}{2m} - \mu \). Due to the chirality (as can be seen, the momentum is locked with the Pauli matrices acting on the band space), the eigenvectors can be written as [24] (after ignoring the \( p_z \)-term)

\[
|p\rangle = e^{i p \cdot r} \frac{1}{\sqrt{S}} \left( \frac{1}{\lambda e^{i 2 \phi \lambda}} \right),
\]

where \( \phi = \arctan \frac{p_y}{p_x} \) is the polar angle of \( p \). \( \lambda = \pm 1 \) correspond to the electron and hole states, respectively.

In this paper, we only consider the interspecies interaction, \( g_{\psi \phi} \), which describes the bare attractive contact interaction strength. The contact potential (with Guassian broadening) which with only the short-range attractive/repulsive interaction is used here since the size of pair is much larger than the range of potential, thus the long-range Coulomb repulsion is reduced here. While for relativistic particles in, e.g., the 3D Dirac/Weyl system, the mass term is necessary to form the polaron. We here use the coupling constants of the impurity-fermion sea mixture system, \( g_{\psi \psi}^{-1} = \frac{m_0}{\text{tr} \sigma_a \sigma_d} \), \( g_{\psi \phi} = \left[ \frac{2 \pi \hbar^2}{m_0^*} \right]^{-1} \), where \( m_0 = m_\psi m_\phi / (m_\psi + m_\phi) \) is the renormalized mass, superscript \( b \) denotes the bare coupling. \( \Lambda \) is the momentum cutoff. \( a_{\psi \phi} \) is the impurity-majority scattering length. The \( g_{\psi \phi} \) here follows the general definition of the background coupling constant which related to the background scattering length \( a_{\psi \phi} \).

Indeed, the interaction effect of the polaron system requires the investigation of the effective mass in contrast to the semimetal system, especially in the strong interacting regime with the obvious renormalization effect and the Fermi-liquid feature.

## 3 T-matrix and the self-energy

The ladder approximation (non-perturbative) is applicable not only for the imbalanced Fermi gases or nuclear physics, but also for the solid state systems with finite effective mass. Further, for large-species Fermion system, the ladder approximation is similar to the leading order \( 1/N \)-expansion. In thermodynamic limit with \( N \to \infty \), the self-energy which describes the pairing fluctuation becomes zero [35] which implies that the interaction between impurity and majority particles vanishes (i.e., without the polaron). In the strong interacting case, the self-energy effect as well as the resummation of ladder diagrams (for the forward scattering) are important to be considered. The non-self consistent \( T \)-matrix, which does not contains the self-consistency of the Green’s function, describes the fluctuations in \( s \)-wave cooper channel. Firstly, we can write the \( T \)-matrix between the single impurity and the majority component as

\[
T(p + q, \omega + \Omega) = \left[ \frac{\tilde{m}}{2\pi \hbar^2 a_{\psi \phi}} + \Pi(p + q, \omega + \Omega) \right]^{-1},
\]

where \( q \) is the momentum of the majority particle, \( p \) is the momentum of the impurity. Note that for the \( T \)-matrix here, we only consider the closed channel scattering (i.e., the bare case), and without consider the interchanging as well as the spin/valley degrees of freedom. The term \( (p + q) \) can be treated as the center-of-mass momentum. \( \Omega \) is the Fermionic frequency since we assume the zero-temperature limit, similarly, \( \omega \approx \varepsilon_{\mu\uparrow} = \frac{k^2(p^2)}{2m_\phi} - \mu_\downarrow = \frac{k^2(q^2)}{2m_\phi} - \text{Re} \Sigma(p = 0, \omega = 0) \) is the Bosonic frequency where \( \Sigma(p, \omega) \) is the impurity self-energy as stated below. Here \( \mu_\uparrow \neq \mu_\downarrow \) due to the spin-imbalance. Note that this \( T \)-matrix is non-self-consistent, which with the bare impurity propagator and the majority propagator as diagrammatically shown by the Bethe-Salpeter equation (see, e.g., Ref. [36]). While the self-consistent \( T \)-matrix requires the dressed
impurity propagator which containing the impurity self-energy effect, and it is more suitable to apply when take into account an infinite number of the majority particles, in which case its statistical properties emerge including the imbalance between the two majority species[37]. The Bethe-Salpeter equation about the non-self-consistent many-body $T$-matrix reads

$$T(p + q, \square; p + q - k') = V_0(p + q, \square; p + q - k')$$

$$+ \sum_k V_0(p + q, \square; k)G_0^\psi(p + q - k)G_0^\psi(\square + k)T(p + q - k, \square + k; p + q - k - k')$$  \hspace{1cm} (5)

where $V_0$ are the bare impurity-majority interactions, specially, $V_0(p + q, \square; k)$ is the interaction induced by the polarization operator (consist of the two bare Green’s functions; see Appendix. B). $k$, $k'$ are the relative momentum. $G_0^\psi$ and $G_0^\phi$ are the bare Fermionic and Bosonic Green’s function, respectively, as presented below. The symbol $\square$ can be omitted, but we retain it here for the integrity of the above equation. In the absence of the center-of-mass momentum $(p + q = 0)$, the Bethe-Salpeter equation reduced to the Lippmann-Schwinger equation

$$T(k_1, k_2; \omega) = V_0(k_1, k_2) + \sum_{k_3} V_0(k_1, k_3) \frac{1}{\omega + i0 - 2\varepsilon_{k_3}} T(k_3, k_2, \omega).$$  \hspace{1cm} (6)

The impurity-majority pair propagator (unrenormalized) in non-self consistent $T$-matrix approximation reads

$$\Pi(p + q, \omega + \Omega) = \int \frac{d^3k}{(2\pi)^3} \int \frac{d\nu}{2\pi} G_0^\psi(-\Omega + \nu, k - q)G_0^\psi(\omega + \Omega - \nu, p + q - k),$$  \hspace{1cm} (7)

where $G_0^\psi(-\Omega + \nu, k - q) = [i\nu - i\Omega - \frac{k^2}{2m_e} + \mu_\uparrow]^{-1}$ is the noninteracting (in the absence of a condensate and the long-range Coulomb interaction) majority particle (Fermion) propagator and $G_0^\psi(\omega + \Omega - \nu, p + q - k) = [i\omega + i\Omega - i\nu - \frac{(p + q - k)^2}{2m_\uparrow} + \mu_\downarrow]^{-1}$ is the bare impurity propagator (not the scalar-field one). In this majority particle propagator we ignore the perturbation from the single impurity to the Fermi sea.

We at first discuss the self-energy of polaron in the general Fermi sea, which is usually referred to as the polaronic binding energy[12] or the molecule binding energy[30, 13]. This self-energy can be obtained by the following impurity-majority interaction Hamiltonian

$$H_{\text{int}} = \begin{pmatrix} g_{\psi\phi} |\Psi_\psi|^2 & 0 \\ 0 & g_{\psi\phi} |\Psi_\phi|^2 \end{pmatrix},$$  \hspace{1cm} (8)

and the related the interaction energy

$$\varepsilon_{\psi\phi} = n_F \int d^3R |\Psi(R)| \left[ \frac{-\hbar^2 \nabla_R^2}{2m_e} + U_{\psi\phi} \right] |\Psi(R)|,$$  \hspace{1cm} (9)

where $\Psi(R) = (\Psi_\psi(R), \Psi_\phi(R))$ is the normalized wave function. $n_F = \int \frac{d^3q}{(2\pi)^3} \int \frac{d\Omega}{2\pi} G_F(q, \Omega) e^{i0+\Omega}$ is the numerical density of the Fermions, where $G_F(q, \Omega)$ is the dressed (full) Green’s function unlike the above ones, and gives the actual Fermion dispersion. There exists the constrains

$$4\pi n_F \int_0^\Lambda dRR^2 |\Psi(R)| \Psi^\dagger(R) = I,$$  \hspace{1cm} (10)

$$|\Psi(R \geq \Lambda_c)| = 1,$$

where $I$ is the identity matrix. Base on the many-body scattering theory at low-temperature, where we consider only the s-wave scattering, the $T$-matrix in Fermi (or Bose) gases is usually
self-consistent, i.e., it’s a two-channel $T$-matrix \[^{38,39}\] while in our model, the $T$-matrix contains only the open channel (i.e., the bare one) which is non-self-consistent.

To study the many-body effect, the non-self-consistent $T$-matrix is similar but not exactly like the leading order $1/N$ expansion, since it ignores the dynamical screening effect. For this reason, the non-self-consistent $T$-matrix is more like the leading-order loop expansion within GV approximation rather than the leading order $1/N$ expansion within GW approximation. It’s also found that the static screening (GV) to the Coulomb interaction can be a good approximation for the dynamical screening in the low doping regime, especially for carriers frequency of the order of binding energy\[^{10}\]. The related studies are also reported in Refs.\[^{35,27,41}\]. Besides, the validity of the non-self-consistent $T$-matrix in studying the BCS-BEC crossover has also been verified \[^{42,43}\].

In our model, the self-energy about the interaction between mobile impurity and the bath reads

$$
\Sigma(p, \omega) = \sum_{q<k_F} \frac{N_F(\Omega)}{2\pi \hbar^2 a_{\phi} } + \Pi(p+q, \omega+\Omega),
$$

(11)

and the numerator can be replaced by $\theta(k_F - q)$ at zero-temperature limit, where $\theta$ stands the step function. Note that this self-energy expression describes only the region around the single impurity (the attractive polaron) which is small but not localized (since the impurity is mobile). It’s different to the Fermi gases that, the self-energy of the impurity does not contain the condensate density as well as the condensate-related spin fluctuation and the pair propagator contains the chiral factor $F_{\lambda\lambda'} (\lambda, \lambda' = \pm 1)$ (the wave function overlap) which suppresses the backscattering and is absent in the 2D electron gas. The chiral factor here is indeed observable in the polarons formed in the Dirac system\[^{44}\]. While for 2D electron gas, $F_{\lambda\lambda'} = 1$ and contains only the intraband contribution, except at a quantum Hall setup with strong magnetic filed as report in Ref.\[^{45}\]. We can also see that, in the narrow gap limit, the pair propagator reduced to the well known dynamical polarization, and $T = \Pi^{-1}$. In the surface of Dirac system, since away from the condensed phase, the condensate density vanishes but the related pairing fluctuations remain as long as $g\psi\phi \neq 0$, i.e., the pairing instability exists (especially when the spin-orbit coupling turns on\[^{21}\]) even in the case of $g\psi\psi = 0$.

4 Pair propagator and related quantities at zero-temperature limit

To describe the polaronic dynamics, we use the following BCS-type many-body Hamiltonian

$$
H = \sum_k \epsilon_{k\uparrow} c^\dagger_{k\uparrow} c^\dagger_{k\uparrow} + \sum_p \epsilon_{p\downarrow} c^\dagger_{p\downarrow} c^\dagger_{p\downarrow} + \frac{1}{N} \sum_{k,p,q} g_q \langle p - q | p \rangle \langle k + q | k \rangle c^\dagger_{p-q\downarrow} c_{k+q\uparrow} c_{k\uparrow} c^\dagger_{p\downarrow},
$$

(12)

where $N = S/s_0$ is the total number of the unit cell where $S$ is the total area and $s_0$ is the area of the unit cell.

Within one-particle-hole approximation, which is valid according to the Monte Carlo calculation and the experimental results due to the destructive interference in the presence of more than one particle-hole part, the variational wave function reads\[^{46,47}\]

$$
|\psi\rangle = \psi_0 c^\dagger_{p\downarrow}|0\rangle_\uparrow + \sum_{k>k_F, q<k_F} \psi_{kq} c^\dagger_{p-q\downarrow} c^\dagger_{k\uparrow} c_{k\uparrow} |0\rangle_\uparrow,
$$

(13)

where $|0\rangle_\uparrow = \Pi_{k<k_F} c^\dagger_{k\uparrow} |\text{vac}\rangle$ is the ground state of the majority particles\[^{48}\] and $|\text{vac}\rangle$ is the vacuum electron state. We focus on the coherence case, where the masses of impurity and the majority particle are comparable, and thus decoherence effect\[^{49}\] is weak while the nonadiabatic dynamics is dominating. The first term in the right-hand-side of above equation describes the
free impurity which assumed is that totally delocalized. $k$ is the momentum of a majority-particle scattered out of fermi surface, and $q$ is the momentum of a majority-particle before scattering. Through the normalization condition $\langle \psi | \psi \rangle = 1$, we have, after minimizing the total energy,

$$\psi_{kq} = \psi_0 \frac{T(p + q, \omega + \Omega)}{\omega - \varepsilon_{p+q-k} - \varepsilon_{k} + \varepsilon_{q}},$$

$$\psi_0 = \frac{1}{\sqrt{1 + \sum_{k > k_F, q > k_F} (\frac{\psi_{kq}}{\psi_0})^2}}. \quad (14)$$

At zero-temperature limit, the above renormalized pair propagator can be written as

$$\Pi(p + q, \omega + \Omega) = -\int \frac{d^3k}{(2\pi)^3} \frac{1 - N_F(\varepsilon_k)}{\omega - i0 - \Omega + \varepsilon_{k+q-k} - \varepsilon_{k} + \varepsilon_{q}} F_{\lambda\lambda'}$$

$$= -\int \frac{d^3k}{(2\pi)^3} \frac{N_F(\varepsilon_k)}{\omega + i0 + \Omega - \varepsilon_{k} + \varepsilon_{k+q-k}} F_{\lambda\lambda'}$$

$$= -\int \frac{4\pi}{(2\pi)^3} \int_{k_F}^{\Lambda} \frac{-k^2\theta(k-k_F)}{\omega + i0 + \varepsilon_{q} - \varepsilon_{k} - \varepsilon_{k+q-k}} F_{\lambda\lambda'} dk, \quad (15)$$

where $N_F$ is the Fermi-distribution function and it appears only in the presence of nonzero center-of-mass momentum. Note that instead of using a term $\frac{2i}{k^2}$ to make the pair propagator convergent even in the ultraviolet limit [38, 50] (renormalized pair propagator), we here use the momentum cutoff $\Lambda$ similar to the literatures [9, 26], and the value of momentum cutoff is setted as 3 eV which is the same as the graphene-like systems [51]. Interestingly, it is also found that, for open channel $T$-matrix (which is what we focus on throughout this paper), the open channel shift due to the medium effect (i.e., the integral for open channel propagator over the scattering wave vector [39, 10]) just equals to this term ($\sum_k \frac{2i}{k^2}$ in the vacuum limit (with zero center-of-mass momentum and zero impurity frequency)). Base the above expression of the pair propagator, we can obtain that, the polaron self-energy increases with the increasing mass term or coupling parameter $g_{\psi\phi}$, however, there is an exception: when the intrinsic spin-orbit coupling (not the extrinsic one) is presented, then the increase of $g_{\psi\phi}$ will reduces the self-energy since it will greatly reduces the mass [16]. As we can see, although the chiral factor $F_{\lambda\lambda'}$ is contained, it has $F_{\lambda\lambda'} = 1$ for the non-chiral systems (like the 2D electron gas) or the systems which are dominated by the backscattering (like the bilayer Dirac system [52]).

Different to the retarded polarization function (density-density correlation) in one-loop approximation which only describes the scattering of one kind of particle (like the electron) due to the interaction (like the Coulomb interaction), the pair propagator describes both the scatterings of the impurity and the majority particle (electron-hole pair here). The second term in above trial wave function corresponds to the excited state where the impurity scattered from state with intrinsic momentum $p$ to the final state with momentum $p + q - k$, i.e., the scattering wave vector is $q - k$; while the majority particle (electron-hole pair) scattered from the initial state with momentum 0 (does not exis yet until created by the impurity) to the final state with momentum $k - q$, i.e., the scattering wave vector is $k - q$. Note that here we assume the scattering wave vectors are small and thus the inter valley scattering can be ignored. We can see that the scattering wave vectors for impurity and majority particles are opposite in direction. Thus for such a pair propagator, the spinor wave function overlap (form factor) according to Eq.3 reads

$$F_{\lambda\lambda'} = \langle (p + q - k)|p\rangle\langle k - q|0\rangle$$

$$= \frac{1}{2} (e^{i\phi_{p+q-k}\lambda'} \lambda e^{-i\phi_{p+q-k}\lambda'}) (e^{i\phi_{p}\lambda} \lambda e^{-i\phi_{p}\lambda}), \quad (16)$$
where the $\theta$ is the angle between the initial impurity wave vector $p$ and the scattering wave vector $(q - k)$. Apparently such form factor is a little different to the one which is widely seen in the chiral solid system\[24, 53\]. Through this form factor, we can clearly see the difference between the pair propagator and the one loop diagrams where the two propagators describe the same particle before and after scattering, respectively. For intraband scattering ($\lambda\lambda' = 1$),

$$
F_{\lambda\lambda'} = \cos(\phi_{p+q-k} - \phi_p)
= \frac{p + (q - k)\cos\theta}{\sqrt{p^2 + (q - k)^2 + 2p(q - k)\cos\theta}}
\approx 1 - \frac{\sin^2\theta}{2p^2}(q - k)^2.
$$

(17)

where $\theta$ is the angle between initial momentum $p$ and the scattering one $q - k$. For interband scattering ($\lambda\lambda' = -1$),

$$
F_{\lambda\lambda'} = i\sin(\phi_{p+q-k} - \phi_p)
\approx i\sqrt{1 - [1 - \frac{\sin^2\theta}{2p^2}(q - k)^2]^2}.
$$

(18)

Next we only focus on the nonchiral and intraband chiral cases. For the polaron formed in parabolic systems with nonrelativistic interacting particles, the renormalized interacting strength can be represented in another representation

$$
g^{-1}_{\psi\phi} = \frac{1 + g_b \sum_{k = k_F} A_{k\psi} \frac{1}{2\epsilon_k}}{g_b}.
$$

(19)

where the bare coupling $g_b$ is tunable and it tends to zero when let $\Lambda \rightarrow \infty$.

Obviously, as showed in Fig.1(a), the above scattering form factor (Eq.(16)) corresponds to the pair propagator of $\Pi_{11}$, while for the particle-hole propagator up to second order\[10\] $\Pi_{22}$ as showed in the Fig.1(b), we will see that the value of its form factor is the same as the one corresponds to $\Pi_{11}$, i.e.,

$$
F'_{\lambda\lambda'} = \langle (p + q - k)|p\rangle\langle k - q|0\rangle = F'_{\lambda'\lambda} = \langle (p + q - k)|p\rangle\langle 0|k - q\rangle.
$$

(20)

Note that we assume the direction of initial wave vector of the majority particle is along the final one, i.e., $k - q$. But for the case that the direction of initial wave vector of the majority particle is the same with the impurity before scattering, i.e., $p$, and the scattering wave vector $q - k \gg p$ is mainly along the $x$-direction in the momentum space, then the form factor $F_{\lambda\lambda'}(F'_{\lambda\lambda'})$ equals zero, which means that for chiral system the pair propagator vanishes, which is the case for helical system like $Bi_2Se_3$\[3\].

In Fig.2-3 and Fig.4-5, we show the pair propagator and the corresponding self-energy, respectively, for nonchiral and chiral cases. By comparing the chiral case to the nonchiral one, we can see that, when the angle $\theta$ (between the initial impurity wave vector and the scattering wave vector) is nonzero, both the pair propagator and self-energy diverges away from the nonchiral ones, and such divergence locates mainly in the low-momentum region. Thus the chiral effect leads to the instability to both the pair propagator and self-energy in small-$p$ region. For low initial momentum of impurity. In Fig.4-5, we can also see that, for stronger bare coupling $|g_b|$, the polaron has a lower self-energy. Since we consider the polaron formed in weak-coupling regime, the final value of self-energy obtained here at large momentum agrees with the result of perturbation theory: $\Sigma(p, \omega) \propto g_b n$ (the total particle-number density $n$ is setted as 1 here). That is also in agreement with the mean-field result for the homogeneous condensate\[27\]. Note that during the calculation of pair propagator and self-energy, the direction of $p$ is unfixed, and
thus we don’t integrate over the all possible angle \( \theta \) (between \( p \) and \( q - k \)), as obviously can be seen from the figures. The suppression of marginal fermi liquid character can be seen from the self-energy as a function of \( \omega \) as shown in lower-panel of Fig.5, where we can see that the imaginary part of the self-energy diverges away from the intrinsic case \( |\text{Im}\Sigma| \sim \omega^2 \) at large value of \( \omega \). Through the imaginary part of the lower-panel of Fig.5, we can also see that the marginal fermi liquid character is suppressed while the fermi liquid character is rised with the increasing \( g_b \). The increase of \( g_b \) also gives rise to the intraband single-particle excitation.

We note that, in Fermi-liquid picture with strong screening effect, the Fermion interaction is dominated by the short-ranged one, i.e., the Hubbard interaction, then the strong spin fluctuation as well as the particle-hole fluctuation are possible to build the bipolaron[54]. The quasiparticle properties can also be detected by the spectral function, which measures the propability of exciting or removing a (quasi)particle at a certain momentum. Next we consider the particle spectral function containing the many-body effect,

\[
A(p, \Omega) = -\frac{1}{\pi} \frac{|\text{Im}\Sigma(p, \omega)|}{(\Omega - \text{Re}\Sigma(p, \omega) - \frac{\hbar p^2}{2m_\phi} + \mu)\Omega + (\text{Im}\Sigma(p, \omega))^2}.
\]  

(21)

From Fig.6-7, we can see that, except for zero frequency, the spectral functions exhibit symmetry feature, and the peaks decrease with the increase of \( |g_b| \). From the intensity plot of spectral function (Fig.8), we can see that the spectral function exhibits similar dispersion with the parabolic impurity (before scattering) in low-momentum region, but exhibits linear dispersion in the large-\( p \) region. Also, we find that the width of spectral function becomes very narrow when close to zero-momentum. Further, by comparing to the dash-line in the center inset of Fig.8, we can easily see that the dispersion of original impurity \( (\frac{p^2}{2m_\phi} - \mu) \) is been lowered by the polaronic effect (negative interaction energy), and, in the mean time, the effective mass is also increased due to the decrease of dispersion slope (compared to the original one).

At zero energy limit, the quasiparticle residue of the usual Dirac Fermions remains finite (here we assuming a noninteracting initial state) and thus the coefficients \( \psi_0 \) and \( \psi_kq \) remain finite too, while for the multi-Weyl semimetal, the residue vanishes at zero-energy and then \( \psi_0 = \psi_kq = 0 \). For zero momentum \( (q = 0) \) with the lowest dispersion, the impurity self-energy becomes,

\[
\Sigma(p, \omega) = \frac{1}{S} \sum_{\mu_\uparrow} \frac{m}{2\pi\hbar^2a_{\upsilon\phi}} \frac{1}{\Pi(\omega + \Omega, p \Omega + i0 + \mu)} + \frac{1}{\Pi(\omega + \Omega, p \Omega + i0 + \mu_\uparrow)},
\]  

(22)

with \( \Pi(\omega + \Omega, p) = \Pi(\omega + \Omega, p = 0) \), and \( S \) is the volume of the space where all the chemical potential around the polaron are taken into account. Now the center of mass is just \( p \). That’s also agree with the results of the Fröhlich polaron model[19, 55] in long-wavelength limit which with the strong electron-phonon coupling as well as the observable optical excitations[20].

For simplicity, we further set \( \mu_\uparrow = 0 \) and the Fermi frequency \( \Omega = 1 \) (which is possible in the zero-temperature limit), then the polaron self-energy becomes \( \Sigma(p, \omega) = T(p, \omega)G_{\psi}(0, 0) \) where \( G_{\psi}(0, 0) = 1 \) The self-energy is shown in Fig.7(a). Then by substituting the above self-energy to the Eq.(22), we obtain the corresponding particle spectral function as shown in the Fig.7(b), where we set \( \omega > 0 \) \( (\omega > \mu_\uparrow) \) to make sure the spectral function here describes only the particle states. Through Fig.7(b), the total density of states can be obtained by integrating over the \( p \)-axis, while the occupation probability[56] can be obtained by integrating over the \( \omega \)-axis.

In Fig.9, we show the effective masses induced by the polaronic effect and the quasiparticle residue, which read

\[
\Delta m^* = (\frac{\partial^2 \Sigma(p, \omega)}{\partial p^2})^{-1},
\]

\[
Z = \frac{1}{1 - \text{Re}\partial_\omega \Sigma(p, \omega)} \bigg|_{\omega=E(p)},
\]

(23)
where \( E(p) \) is determined self-consistently by the equation

\[
E(p) = \frac{p^2}{2m_\downarrow} - \mu_\downarrow + \text{Re}\Sigma(E(p), p),
\]

(24)

after the expansion coefficients of the polaron trial wave function are obtained by performing the variational minimization. In Fig. 9, we plot the induced effective mass and the quasiparticle residue as a function of momentum. In small momentum region \((p < 4)\), both the induced effective mass and residue exhibit unusual behavior (due to the instabilities of the slow polaron with strong nonadiabatic dynamics). In this region, the interaction may very strong and thus the residue could be very low, and the induced effective mass may even becomes negative. In the large momentum region \((p > 4)\), the polaron becomes relatively stable, then the power law behavior of \( \Delta m^* \) and the logarithmic behavior of \( Z \) can be seen. We can also see that, for stronger attractive interaction (i.e., for larger \(|g_b|\)), the polaron has a higher effective mass \((m^* = m_\downarrow + \Delta m^*)\), which is consistent with the result of Ref. [10, 14], and the residue asymptotically approaches one more slowly. From the effective masses, we can easily see that the positive interaction (repulsive polaron) induces stronger instability compares to the negative one in the low-momentum region (especially for \( \omega = 0 \)). In stable regime, the induced effective masses \( \Delta m^* \) increase more and more fast with the increasing initial impurity momentum \( p \). That is in consistent with the result of Ref. [27], which, in addition, shows that the polaron is possible to changes to molecule if the \( p \) keeps increasing. In large \( p \) region, the many-body fluctuation is supressed, and the interaction effect is faded (while the adiabatic effect is enhanced). When the residue reaches one, the impurity will acts like a free particle. Note that here the coupling strength is mainly controlled by the parameter \( g_b \), but also affected by the value of initial momentum \( p \) as shown in the figure.

5 Pair propagator and relaxation time at finite temperature

For the case of finite temperature, we introduce the fermionic Matsubara frequencies read

\[
\Omega = (2n + 1)\pi T, \quad \nu = (2n' + 1)\pi T \quad (n, n' \text{ are integer numbers}),
\]

which are discrete variables. At finite temperature, the pair propagator can be written as

\[
\Pi(p + q, \omega + \Omega) = \int \frac{d^3k}{(2\pi)^3} \frac{T}{V} \sum_{n'} G_0^\psi(\nu, k) G_0^\phi(\omega + \Omega - \nu, p + q - k),
\]

(25)

where we consider the single band model and regard the Green’s functions as the only eigenvalue of the matrix. Here we define

\[
G_0^\psi(\nu, k) = \frac{1}{i\nu - \frac{k^2}{2m_\psi} + \mu_\uparrow},
\]

\[
G_0^\phi(\omega + \Omega - \nu, p + q - k) = \frac{1}{i\omega + i\Omega - i\nu - \frac{(p+q-k)^2}{2m_\phi} + \mu_\downarrow}.
\]

(26)

The summation over Matsubara frequencies \((i\nu)\) can be calculated as

\[
\sum_{\nu} G_0^\nu(\nu, k) G_0^\phi(\omega + \Omega - \nu, p + q - k) = \sum_{n'=-\infty}^{\infty} \frac{1}{i(2n'+1)\pi T - a - i(2n'+1)\pi T - b} = \frac{\tanh \frac{b}{2T} + \tanh \frac{a}{2T}}{2T(a + b)} = \frac{\tanh \frac{b}{2T} + \tanh \frac{a}{2T}}{2T(a + b)},
\]

(27)
where \( a \equiv \varepsilon k \uparrow = \frac{k^2}{2m_\psi} - \mu \uparrow \), \( b \equiv -i\omega - i\Omega + \frac{(p+q-k)^2}{2m_\phi} - \mu \downarrow \)), and the above result can also be rewritten as

\[
\sum_\nu G_0^\nu(\nu, k)G_0^\nu(\omega + \nu, p + q - k) = \frac{e^{a+b} - 1}{T(a + b)(e^{a/T} + 1)(e^{b/T} + 1)} = \frac{N_F(a)N_F(b)}{N_B(a + b)(a + b)},
\]

where \( N_F(x) = 1/(e^{x/T} - 1) \) is the Bose distribution function and \( N_F(x) = 1/(e^{x/T} + 1) \) is the Fermi distribution function. For small \( a \) and \( b \), i.e., in the limit of small energy and small frequency, we approximate \( \tanh(x) \approx x - \frac{x^3}{3} + O(x^5) \), then the pair propagator (nonchiral) becomes

\[
\Pi(p + q, \omega + \Omega) = \frac{4\pi}{(2\pi)^3} \int_{k_F}^\Lambda k^3 dk \left[ \frac{\tanh \frac{a}{2T} + \tanh \frac{b}{2T}}{2T(a + b)} \right]
= \frac{4\pi}{(2\pi)^3} \int_{k_F}^\Lambda k^2 dk \left[ \frac{1}{4T^2} - \frac{1}{3} \left( \frac{1}{2T} \right)^4 \frac{1}{a + b} \right]
= \frac{4\pi}{(2\pi)^3} \int_{k_F}^\Lambda k^2 \left[ \frac{1}{4T^2} - \frac{1}{3} \left( \frac{1}{2T} \right)^4 (a^2 - ab + b^2) \right]
= \frac{1}{8\pi^2 T^2} \frac{(\Lambda - k_F)^3}{3} - \mathcal{F},
\]

where

\[
\mathcal{F} = \frac{1}{6\pi^2} \frac{1}{(2T)^4} \int_{k_F}^\Lambda k^2 (a^2 - ab + b^2) dk
= \frac{1}{6\pi^2} \frac{1}{(2T)^4} \frac{k^3}{420m_\psi^2 m_\phi^2} \left[ 35m_\psi^2 (4c^2 m_\phi^2 + 2cm_\phi ((p + q)^2 - 2dm_\phi)) \right.
+ ((p + q)^2 - 2dm_\phi)^2) + 21k^2 m_\psi (2cm_\phi (m_\psi - 2m_\phi) + 2dm_\phi (m_\phi - 2m_\psi) + 6m_\psi - m_\phi)(p + q)^2)
- 105km_\psi (p + q)(cm_\phi - 2dm_\phi + (p + q)^2) + 15k^4 (m_\psi^2 - m_\phi m_\psi + m_\phi^2)
- \left. 35k^3 m_\psi (2m_\psi - m_\phi)(p + q) \right|_{k_F}^\Lambda.
\]

where we define \( c \equiv \mu \uparrow \), \( d \equiv i\omega + i\Omega + \mu \downarrow \). As shown in the Fig.10, the momentum- or energy-dependence of the polarization function decreases with the increase of temperature. At high-enough temperature, both the imaginary and real part of the polarization function become constant, and thus we can suspect that the induced effective mass will become infinite (self-trapped polaron) at high enough temperature while the residue will equals to one in the same case.

The ladder approximation (by summing over the ladder diagrams which correspond to the forward scattering) results in accurate results of the pair propagator and self-energy, and it also agrees with the Quantum Monte-Carlo calculation as well as the experimental results. The single-channel \( T \)-matrix (which contains the pair propagator) introduces the tunable \( s \)-wave scattering length to the manipulation of the behavior of a single impurity embedded to a fermi sea, which describes the scattering between a pair of atoms with up and down spins, respectively, and within the center of mass frame with energy \( \varepsilon = \omega - (p + q)/2(m_\psi + m_\phi) + \mu \uparrow + \mu \downarrow \). At finite temperature and for the configuration that the number density of the bose impurity is much
lower than the fermions (without the effect of three-atom loss (the Efimov trimers))\textsuperscript{31,38,37},
the pair propagator can also be written as \textsuperscript{30,31}
\[ \Pi(p + q, \omega + \Omega) = \sum_k \frac{1 - N_F(\varepsilon_{k\uparrow}) - N_F(\varepsilon_{p+q-k\downarrow})}{\omega + i0 - \varepsilon_{p+q-k\downarrow} - \varepsilon_{k\uparrow} + \varepsilon_{q\uparrow}}, \] (31)
where \( p \) and \( q \) correspond to the momentum of impurity and hole respectively, \( \omega \) and \( \Omega \) correspond to the frequency of impurity and hole respectively. For pairing mechanism, this expression is definitely important, e.g., for the pairing instability\textsuperscript{57,37,58,59} and the resonantly enhanced correlation, and its real part and imaginary part are easy to obtain by firstly replacing the imaginary frequencies in denominator with the analytical continuation and then using the Dirac identity (for retarded functions) \( \lim_{\eta \to 0} \frac{1}{\pi \pm i \eta} = \frac{1}{x} \mp i \pi \delta(x) \). We can see that the factor \( F_{\lambda \lambda'} \) is in fact related to the angle between the wave vectors of polaron (coherently dressed by the particle-hole excitations of majority part) and the electron with momentum \( k \). And this term is unnecessary in the three (or two)-dimensional electron (or hole) gases, it is nonzero only when the eigenstates at different wave vectors have overlap (corresponds to the two statistical functions in the numerator), which for three-dimensional system (consider the longitudinal wave vector \( k_z \) reads
\[ \Psi_+ = \left( e^{i\theta_0 \cos\theta_z \frac{1}{2}} \sin\theta_z \right), \]
\[ \Psi_- = \left( e^{i\theta_0 \sin\theta_z \frac{1}{2}} - \cos\theta_z \right), \] (32)
where the indices ± denote the sign of band energy (i.e., the conduction band and valence band), and \( \theta_\parallel = \arctan k_y/k_x, \theta_\perp = \arctan \sqrt{k_x^2 + k_y^2}/k_z \). Then the overlap factor reads \( F_{\lambda \lambda'} = \frac{1 \pm (\cos\theta_\perp \cos\theta'_\perp - \sin\theta_\perp \sin\theta'_\perp \sin\theta \sin\theta')}{2} \) where \( \theta'_\perp = \arctan \sqrt{k_x^2 + k_y^2}/k_z' \). That is clearly different from the ones appear in two-dimensional system\textsuperscript{60}. For the calculation in main text, we use the two-dimensional chiral factor \( F_{\lambda \lambda'} = \frac{1 \pm \cos k}{2} = \frac{1}{2} \left( 1 \pm \frac{k \pm \cos a}{\sqrt{k^2 + q^2 + 2kq \cos a}} \right) \) due to the nature of weak-chirality of the system we discussed. For another case of three-dimensional system, at long-wavelength limit \( (k_z \to 0) \) and with isotropic dispersion, such approximation is also applicable as shown in, e.g., Ref.\textsuperscript{61}. When the vertex correction is not taken into account, for the case of inversed frequency \( \Pi(p + q, -\omega - \Omega) \), we can use the identity \( \Pi(p + q, -\omega - \Omega) = \Pi^*(p + q, \omega + \Omega) \), i.e., \( \text{Re} \Pi(p + q, -\omega - \Omega) = \text{Re} \Pi(p + q, \omega + \Omega), \text{Im} \Pi(p + q, -\omega - \Omega) = -\text{Im} \Pi(p + q, \omega + \Omega) \). Further, when the chirality (from the Weyl system) appears, the causality relations are studied in Ref.\textsuperscript{62}.

Then the self-energy at finite temperature can be obtained as
\[ \Sigma(p, \omega) = \frac{T}{V} \int_{0}^{k_F} \frac{d^3q}{(2\pi)^3} \sum_{\nu} T(p + q, \omega + \Omega) G^\psi_{\nu}(q, \Omega), \] (33)
where \( G^\psi_{\nu}(q, \Omega) \) can also be replaced by \( G^\psi_{\nu}(q, \Omega) \) which contains the self-energy term when consider the self-energy effect as done in Ref.\textsuperscript{35} with strong scattering strength. In addition, we discuss the case when consider the ladder vertex correction, where summation over Matsubara frequency can be done by using the method of contour integral (in optical limit)\textsuperscript{63},
\[ \frac{T}{V} \sum_{\nu'} G^\psi_{\nu'}(\nu) G^\phi(\omega + \Omega - \nu) \Gamma(\nu, \omega + \Omega - \nu) = -\int_{C} \frac{dz}{2\pi i} G^\psi(z) G^\phi(\omega + \Omega - z) \Gamma(z, \omega + \Omega - z), \] (34)
where \( \Gamma(\nu, \omega + \Omega - \nu) \) denotes the vertex function.
At finite temperature where the $s$-wave scattering is still dominating, the low-energy excitations induced by quantum fluctuation has a more significant effect on the properties of polaron compared to the thermal excitations especially for the case of small-chemical potential, like the particle-hole parts (especially at low dimension [13, 64]) or the phonon-like (Fröhlich type) excitations. For chiral system at finite temperature, the transport relaxation time (in ladder diagram) of impurity due to the scattering by the electron-hole pair contains a $(1 - \cos \theta)$ term, which supresses the forward scattering ($\theta = 0$) and exists as long as the elastic scattering is involved in the scattering event. This term cannot be found in the quasiparticle relaxation approximation which supresses the forward scattering (diagram) of impurity due to the scattering by the electron-hole pair contains a $(1 - \cos \theta)$ term, which supresses the forward scattering ($\theta = 0$) and exists as long as the elastic scattering is involved in the scattering event. This term cannot be found in the quasiparticle relaxation time in single bubble diagram, and it together with the chiral term determines the scattering cross section [52]. While for the gapless Dirac system, like the intrinsic graphene, both the forward and backward scattering are supressed, as calculated by literatures [52, 65, 66]. The Boltzmann transport theory gives the following inversed relaxation time (in second order Born approximation)

$$
\frac{1}{\tau_p'} = \frac{2\pi}{\hbar} \sum_{k,q} (1 - \cos \theta) W_{p+q-k,p},
$$

where $p' = p + q - k$ is the wave vector of impurity after scattering, and $\theta = \phi_{p+q-k} - \phi_p$ is the angle between wave vectors before and after scattering.

$$
W_{p+q-k,p} = (1 - N_F(\varepsilon_{k-q})) \delta(\omega - \varepsilon_{p+q-k\downarrow} - \varepsilon_{k\uparrow} + \varepsilon_{q\uparrow}) |g_b(k - q)|^2 |p + q - k| |p| \tag{36}
$$

is the transition rate and can be approximated as the imaginary part of self-energy $\text{Im} \Sigma(p, \omega)$ and the from factor can be found in Eq.(16). Then the relaxation time can be obtained as

$$
\frac{1}{\tau_p} = \frac{2\pi}{\hbar} \sum_{k,q} (1 - \cos(\phi_{p+q-k} - \phi_p))(1 - N_F(\varepsilon_{k-q}))
$$

\begin{align*}
\delta(\omega - \varepsilon_{p+q-k\downarrow} - \varepsilon_{k\uparrow} + \varepsilon_{q\uparrow}) |g_b(k - q)|^2 \cos^2(\phi_{p+q-k} - \phi_p) \\
\approx \frac{2\pi}{(2\pi)^3} \int_{k_F}^{\Lambda} k^2 dk \int_0^\pi \sin \Phi d\Phi \int_0^{2\pi} d\phi_{p+q-k} (1 - N_F(\varepsilon_{k-q})) \\
\delta(\omega - \varepsilon_{p+q-k\downarrow} - \varepsilon_{k\uparrow} + \varepsilon_{q\uparrow}) |g_b(k - q)|^2 \cos^2(\phi_{p+q-k} - \phi_p) \\
= \frac{4\pi}{(2\pi)^3} \int_{k_F}^{\Lambda} k^2 dk \int_0^{2\pi} d\phi_{p+q-k} (1 - N_F(\varepsilon_{k-q})) \\
\delta(\omega - \varepsilon_{p+q-k\downarrow} - \varepsilon_{k\uparrow} + \varepsilon_{q\uparrow}) |g_b(k - q)|^2 \cos^2(\phi_{p+q-k} - \phi_p),
\end{align*}

where we assume the case of satric hole $(q = 0)$ for simplicity. Also, we assume the scattering wave vector is larger than the initial impurity wave vector $(k > p)$ so that the integral of $\phi_{p+q-k}$ can over the whole range. Unlike the above pair propagator or self-energy, the variation of scattering angle is integrated (with the momentum $k$), and thus the transition rate here is scattering angle-independent. $g_b(k - q)$ is the scattering wave vector-dependent bare interaction vertex (kernel). Note that in case of electron-phonon interaction, where both the electron-phonon polaron and the electron-ion polaron are produced with the emergent electronic screening and lattice screening [29], the interaction vertex here will dependent on both the scattering wave vector and the initial wave vector (like $p$) [67]. While for the case of multi-impurity (Eq.(22)), the above relaxation time should be rewritten as

$$
\frac{1}{\tau_p} = \frac{2\pi}{\hbar} \sum_{q,k} (1 - \cos \theta) W_{p+q-k,p} \left( \frac{1 - N_F(p + q - k)}{1 - N_F(p)} \right), \tag{38}
$$

which can be obtained through the following relation in large-impurity momentum regime,

$$
1 - N_F(\varepsilon_{k\uparrow}) - N_F(\varepsilon_{p+q-k\downarrow}) = (1 - N_F(\varepsilon_{k-q\uparrow})) \left( \frac{1 - N_F(p + q - k)}{1 - N_F(p)} \right) \tag{39}
$$
Also here the existence of factor \( \frac{1-N_F(p+q-k)}{1-N_F(p)} \) implies the scattering is inelastic \([68, 69, 70]\) in multi-impurity case. The impurity scattering angle \( \theta \) here is defined as the angle between \( p \) and \( p+q-k \), which can be written as \( \theta = \arccos(\frac{n_{p+q-k}}{n_p}) \) where \( n_p \) is the direction projection, and it is related to the scattering wave vector by \( q-k = 2p_F\sin(\theta/2) \).

### 6 Summary

In this paper, we discuss the polaron formed in a doped chiral/nonchiral parabolic system. The method reported here can also be applied to the Dirac systems with finite effective mass. In the numerical simulations, we studied the effect of bare coupling as well as the instability of the pair propagation and spectral function with a small-renormalized effective mass and chemical potential. The many-body effect is also analyzed through the study of spectral function. Although the \( T \)-matrix approximation here takes into account the pairing interaction with the leading instability even in the presence of weak intraspecies interactions (the \( p \)-wave interaction), it is indeed a nonperturbation theory which is evidenced by the absence of the self-consistency (i.e., the Coulomb induced exchange self-energy is the Hartree-Fock type and in lack of the dynamical dielectric function), thus the energy is unconserving here and the quasiparticle weight is lower than the one in random phase approximation (RPA) theory (with dynamical screening) or the partial self-consistent theory (with static screened interaction or dielectric function)\([28]\). Recently, the formation and properties of the attractive polaron formed in a two-dimensional semi-Dirac system is reported in Ref. \([36, 71]\), where the anisotropic effective masses distribution takes an important role, and we approximate the dispersion as the parabolic anisotropic one similar to the plasmon-polaron formed by the phosphorene locates on polar substrates\([72]\). Besides, the \( p \)-wave scattering of the polaron system is also been studied in topological superfluid and the weak-coupled BEC recently\([73, 4]\). The attractive polaron as a quasiparticle can be observed experimentally through the momentum-resolved photoemission spectroscopy\([13]\).

For Bose-Hubbard model in superfluid phase, the Bose polaron with a spin impurity can be created by the off-resonance laser and microscope objective\([75, 76]\). Specially, at half-filling (\( \mu = 0 \)) where the electron density equals 1 and the on-site Hubbard \( U \) is much larger than the mobility of impurity, the spin impurity is localized and in this case the non-self-consistent \( T \)-matrix approximation has high accuracy due to the weak dynamical screening effect from the carriers. In one-dimensional geometry, this experiment also provides a platform to explore the other polaronic physics like the propagation velocity affected by the self-trapping effect, which implies that the polaronic effect can emergent also in the superconductors or the Mott insulators\([64]\). The self-trapping effect will becomes more obvious at finite temperature due to the emergent electron-phonon coupling\([71]\). While at low-temperature limit (e.g., \( < 1\mu K \)) the magnetic or electric trapping can be applied to the molecule cloud or the hyperfine states (can be treated as the species as we discuss in above) of the alkali atoms, to design the quantum memory setups in the quantum circuit\([77]\). For solid state like the Dirac system, in the presence of, e.g., the separable \( s \)-wave potential\([78]\), the \( s \)-wave scattering as well as the elastic scattering can be treated as dominating at low-temperature limit, and the two-body Lippmann-Schwinger equation is still valid in obtaining the coupling parameters and the \( T \)-matrix. In fact for impurity and the particle-hole part (excited by the quantum fluctuation) with energies similar to the same (gapped) Dirac cone, the scattering can be treated as the intravalley one, which can help us to deal with the multichannel problem.
For Bose polaron in BEC, the method of mean-field approximation is valid in the presence of the weak on-site Boson-Boson or Boson-Fermion (impurity) interaction, i.e., the dilute BEC, and certainly, the physical parameters like the lattice parameter or the interaction strength can be controlled by the Feshbach technique, and the strong-interaction regime can also reached by this method. Here we use the variational approach base on Gaussian variational ansatz and the Lagrangian optimization. The variational approach can be generalized by the differential of the matrix element
\begin{equation}
\frac{\partial \langle \Psi | H - E | \Psi \rangle}{\partial (i \psi^*)} = 0,
\end{equation}
where $\Psi$ is the trial wave function including the interaction effect, $H$ is the effective Hamiltonian of the discussing system, $E$ is the Lagrange multiplier which gives the local minimal energy, $\psi$ is the real components. We make the mean-field approximation to the Grassmann field which written as $c_j$ at site $j$, then the Lagrangian reads
\begin{equation}
L = \sum_j i \frac{\partial H^{MF}}{\partial (ic_j^*)} c_j^* - H^{MF},
\end{equation}
where $H^{MF}$ is the mean-field Hamiltonian, the Grassmann field is treated as a dynamical Gaussian profile as
\begin{equation}
c_j = \sqrt{\frac{2}{r \sqrt{\pi}}} \exp\left[ -\frac{(j - c)^2}{r^2} + ik(j - c) \right],
\end{equation}
where $c$ and $k$ is the coordinate and momentum of the center of wave package, respectively, $r$ is the width of wave package.

Then base on the Euler-Lagrangian relation
\begin{equation}
\frac{\partial L}{\partial c} - \frac{d}{dt} \frac{\partial L}{\partial c} = -m \ddot{c} = 0,
\end{equation}
we can obtain
\begin{equation}
c = \sin k e^{-\frac{1}{2r^2} t},
m = \frac{1}{\hbar^2} \cos k e^{-\frac{1}{2r^2}},
\end{equation}
In the absence of the external potential, the effective Hamiltonian is independent of $c$,
\begin{equation}
\frac{\partial H^{MF}}{\partial c} = 0,
\end{equation}
\begin{equation}
\frac{\partial H^{MF}}{\partial c} = \left[ -A \frac{r}{r^2} - \cos k \frac{1}{r^3} \right] \frac{r^3}{\sin k e^{-\frac{1}{2r^2}}},
\end{equation}
where the effective coupling parameter $A = U/(4J \sqrt{\pi})$ as a ratio between the on-site interaction $U$ and the tunneling strength $J$, the mean-field Hamiltonian here reads $H^{MF} = \frac{4}{r} - \cos e^{-\frac{1}{2r^2}}$. Here we note that the critical value of effective coupling parameter $A$ for the self-trapping, soliton, and breather are not continued during the BEC-BCS crossover, unlike the attractive self-energy beyond the Hartree-Fock approximation. In the strong interacting case, the electron may become self-trapped and with localized wave package characterized by a diverging effective mass $m$.  

15
References

[1] Grusdt F, Yao N Y, Demler E A. Topological polarons, quasiparticle invariants, and their detection in one-dimensional symmetry-protected phases. Physical Review B, 2019, 100(7): 075126.

[2] Camacho-Guardian A, Goldman N, Massignan P, et al. Dropping an impurity into a Chern insulator: A polaron view on topological matter. Physical Review B, 2019, 99(8): 081105.

[3] Shvonski A, Kong J, Kempa K. Plasmon-polaron of the topological metallic surface states. Physical Review B, 2019, 99(12): 125148.

[4] Qin F, Cui X, Yi W. Polaron in a $p + ip$ Fermi topological superfluid. arXiv preprint arXiv:1901.02766, 2019.

[5] Elias D C, Gorbachev R V, Mayorov A S, et al. Dirac cones reshaped by interaction effects in suspended graphene. Nature Physics, 2011, 7(9): 701.

[6] Wu C H. Electronic properties of the parabolic Dirac system. Physics Letters A, 2019, 383(15): 1795-1805.

[7] Sarma S D, Hwang E H, Tse W K. Many-body interaction effects in doped and undoped graphene: Fermi liquid versus non-Fermi liquid. Physical Review B, 2007, 75(12): 121406.

[8] Devreese J T, Alexandrov A S. Fröhlich polaron and bipolaron: recent developments. Reports on Progress in Physics, 2009, 72(6): 066501.

[9] Parish M M, Levinsen J. Highly polarized Fermi gases in two dimensions. Physical Review A, 2013, 87(3): 033616.

[10] Christensen R S, Levinsen J, Bruun G M. Quasiparticle properties of a mobile impurity in a Bose-Einstein condensate. Physical review letters, 2015, 115(16): 160401.

[11] Ćwik J A, Reja S, Littlewood P B, et al. Polariton condensation with saturable molecules dressed by vibrational modes. EPL (Europhysics Letters), 2014, 105(4): 47009.

[12] Li W, Sarma S D. Variational study of polarons in Bose-Einstein condensates. Physical Review A, 2014, 90(1): 013618.

[13] Koschorreck M, Pertot D, Vogt E, et al. Attractive and repulsive Fermi polarons in two dimensions. Nature, 2012, 485(7400): 619.

[14] Scazza F, Valtolina G, Massignan P, et al. Repulsive Fermi polarons in a resonant mixture of ultracold Li 6 atoms. Physical review letters, 2017, 118(8): 083602.

[15] Jørgensen N B, Wacker L, Skalmstang K T, et al. Observation of attractive and repulsive polarons in a Bose-Einstein condensate. Physical review letters, 2016, 117(5): 055302.

[16] Mogulkoc A, Modarresi M, Kandemir B S. Spin-dependent polaron formation in pristine graphene. The European Physical Journal B, 2015, 88(2): 49.

[17] Modarresi M, Mogulkoc A, Roknabadi M R, et al. Possible polaron formation of zigzag graphene nanoribbon in the presence of Rashba spinorbit coupling. Physica E: Low-dimensional Systems and Nanostructures, 2015, 66: 303-308.

[18] Ding Z H, Zhao Y, Xiao J L. The properties of strong coupled bound polaron in monolayer graphene. Superlattices and Microstructures, 2016, 97: 298-302.

[19] Mogulkoc A, Mogulkoc Y, Rudenko A N, et al. Polaronic effects in monolayer black phosphorus on polar substrates. Physical Review B, 2016, 93(8): 085417.

[20] Falck J P, Levy A, Kastner M A, et al. Optical excitation of polaronic impurities in La 2 CuO 4+y. Physical Review B, 1993, 48(6): 4043.

[21] Yi W, Zhang W. Molecule and polaron in a highly polarized two-dimensional Fermi gas with spin-orbit coupling. Physical review letters, 2012, 109(14): 140402.
[22] Kohstall C, Zaccanti M, Jag M, et al. Metastability and coherence of repulsive polarons in a strongly interacting Fermi mixture. Nature, 2012, 485(7400): 615.

[23] Wang J R, Liu G Z, Zhang C J. Breakdown of Fermi liquid theory in topological multi-Weyl semimetals. Physical Review B, 2018, 98(20): 205113.

[24] Wu C H. Electronic properties of the Dirac and Weyl systems with first-and higher-order dispersion in non-Fermi-liquid picture. Physics Letters A, 2019: 125876.

[25] Miserev D, Klinovaja J, Loss D. Exchange intervalley scattering and magnetic phase diagram of transition metal dichalcogenide monolayers. Physical Review B, 2019, 100(1): 014428.

[26] Sidler M, Back P, Cotlet O, et al. Fermi polaron-polaritons in charge-tunable atomically thin semiconductors. Nature Physics, 2017, 13(3): 255.

[27] Rath S P, Schmidt R. Field-theoretical study of the Bose polaron. Physical Review A, 2013, 88(5): 053632.

[28] Holm B, von Barth U. Fully self-consistent GW self-energy of the electron gas. Physical Review B, 1998, 57(4): 2108.

[29] Sio W H, Verdi C, Ponc S, et al. Ab initio theory of polarons: Formalism and applications. Physical Review B, 2019, 99(23): 235139.

[30] Yu Z Q, Zhang S, Zhai H. Stability condition of a strongly interacting boson-fermion mixture across an interspecies Feshbach resonance. Physical Review A, 2011, 83(4): 041603.

[31] Fratini E, Pieri P. Mass imbalance effect in resonant Bose-Fermi mixtures. Physical Review A, 2012, 85(6): 063618.

[32] Jian S K, Jiang Y F, Yao H. Emergent spacetime supersymmetry in 3D Weyl semimetals and 2D Dirac semimetals. Physical review letters, 2015, 114(23): 237001.

[33] Yang B J, Moon E G, Isobe H, et al. Quantum criticality of topological phase transitions in three-dimensional interacting electronic systems. Nature Physics, 2014, 10(10): 774.

[34] Han S E, Lee C, Moon E G, et al. Emergent Anisotropic Non-Fermi Liquid at a Topological Phase Transition in Three Dimensions. Physical review letters, 2019, 122(18): 187601.

[35] Enss T. Quantum critical transport in the unitary Fermi gas. Physical Review A, 2012, 86(1): 013616.

[36] Wu C H. Attractive fermi polaron in a semi-Dirac system within ladder approximation. arXiv preprint arXiv:1901.07881, 2019.

[37] Pietilä V, Pokker D, Nishida Y, et al. Pairing instabilities in quasi-two-dimensional Fermi gases. Physical Review A, 2012, 85(2): 023621.

[38] Massignan P. Polarons and dressed molecules near narrow Feshbach resonances. EPL (Europhysics Letters), 2012, 98(1): 10012.

[39] Bruun G M, Jackson A D, Kolomeitsev E E. Multichannel scattering and Feshbach resonances: Effective theory, phenomenology, and many-body effects. Physical Review A, 2005, 71(5): 052713.

[40] Yong C K, Utama M I B, Ong C S, et al. Valley-dependent exciton fine structure and Autler-Townes doublets from Berry phases in monolayer MoSe 2. Nature materials, 2019: 1-6.

[41] Nikolić P, Sachdev S. Renormalization-group fixed points, universal phase diagram, and 1 N expansion for quantum liquids with interactions near the unitarity limit. Physical Review A, 2007, 75(3): 033608.

[42] Tsuchiya S, Watanabe R, Ohashi Y. Single-particle properties and pseudogap effects in the BCS-BEC crossover regime of an ultracold Fermi gas above T c. Physical Review A, 2009, 80(3): 033613.

[43] Haussmann R, Punk M, Zwerger W. Spectral functions and rf response of ultracold fermionic atoms. Physical Review A, 2009, 80(6): 063612.

[44] Kandemir B S. Possible formation of chiral polarons in graphene. Journal of Physics: Condensed Matter, 2012, 25(2): 025302.
[45] Bocquillon E, Freulon V, Parmentier F D, et al. Electron quantum optics in ballistic chiral conductors. Annalen der Physik, 2014, 526(1-2): 1-30.

[46] Combescot R, Giraud S. Normal state of highly polarized Fermi gases: full many-body treatment. Physical review letters, 2008, 101(5): 050404.

[47] Chevy F. Universal phase diagram of a strongly interacting Fermi gas with unbalanced spin populations. Physical Review A, 2006, 74(6): 063628.

[48] Combescot R, Recati A, Lobo C, et al. Normal state of highly polarized Fermi gases: simple many-body approaches. Physical review letters, 2007, 98(18): 180402.

[49] Visuri A M, Kinnunen J J, Baarsma J E, et al. Decoherence of an impurity in a one-dimensional fermionic bath with mass imbalance. Physical Review A, 2016, 94(1): 013619.

[50] Massignan P, Zaccanti M, Bruun G M. Polaron, dressed molecules and itinerant ferromagnetism in ultracold Fermi gases. Reports on Progress in Physics, 2014, 77(3): 034401.

[51] Chae J, Jung S, Young A F, et al. Renormalization of the graphene dispersion velocity determined from scanning tunneling spectroscopy. Physical review letters, 2012, 109(11): 116802.

[52] Adam S, Sarma S D. Boltzmann transport and residual conductivity in bilayer graphene. Physical Review B, 2008, 77(11): 115436.

[53] Efimkin D K, Lozovik Y E, Sokolik A A. Collective excitations on a surface of topological insulator. Nanoscale research letters, 2012, 7(1): 163.

[54] Camacho-Guardian A, Ardila L A, Pohl T, et al. Bipolarons in a Bose-Einstein condensate. arXiv preprint arXiv:1804.00402, 2018.

[55] Grusdt F, Shechadilova Y E, Rubtsov A N, et al. Renormalization group approach to the Fröhlich polaron model: application to impurity-BEC problem. Scientific reports, 2015, 5: 12124.

[56] Hassaneen K S A. Spectral functions of nuclear matter using self-consistent Greens function approach based on three-body force. The European Physical Journal Plus, 2018, 133(11): 484.

[57] Cui X, Zhai H. Stability of a fully magnetized ferromagnetic state in repulsively interacting ultracold Fermi gases. Physical Review A, 2010, 81(4): 041602.

[58] Adachi K, Ikeda R. Dimensionality-induced BCS-BEC crossover in layered superconductors. Physical Review B, 2018, 98(18): 184502.

[59] Pekker D, Babadi M, Sensarma R, et al. Competition between pairing and ferromagnetic instabilities in ultracold Fermi gases near Feshbach resonances. Physical review letters, 2011, 106(5): 050402.

[60] Culcer D, Winkler R. External gates and transport in biased bilayer graphene. Physical Review B, 2009, 79(16): 165422.

[61] Ahn S, Hwang E H, Min H. Collective modes in multi-Weyl semimetals. Scientific reports, 2016, 6: 34023.

[62] Zhou J, Chang H R. Dynamical correlation functions and the related physical effects in three-dimensional Weyl/Dirac semimetals. Physical Review B, 2018, 97(7): 075202.

[63] Mahan G D. Many-particle physics[M]. Springer Science & Business Media, 2013.

[64] Endres M, Cheneau M, Fukuhara T, et al. Observation of correlated particle-hole pairs and string order in low-dimensional Mott insulators. Science, 2011, 334(6053): 200-203.

[65] Hwang E H, Sarma S D. Screening-induced temperature-dependent transport in two-dimensional graphene. Physical Review B, 2009, 79(16): 165404.

[66] Iurov A, Gumbs G, Huang D. Temperature-and frequency-dependent optical and transport conductivities in doped buckled honeycomb lattices. Physical Review B, 2018, 98(7): 075414.

[67] Marchand D J J, de Filippis G, Cataudella V, et al. Sharp transition for single polarons in the one-dimensional Su-Schrieffer-Heeger model. Physical review letters, 2010, 105(26): 266605.
[68] Kim S, Woo S, Min H. Vertex corrections to the dc conductivity in anisotropic multiband systems. Physical Review B, 2019, 99(16): 165107.

[69] Gunst T, Markussen T, Stokbro K, et al. First-principles method for electron-phonon coupling and electron mobility: Applications to two-dimensional materials. Physical Review B, 2016, 93(3): 035414.

[70] Hwang E H, Sarma S D. Acoustic phonon scattering limited carrier mobility in two-dimensional extrinsic graphene. Physical Review B, 2008, 77(11): 115449.

[71] Wu C H. Complex polaron formed on surface of two-dimensional lattice system in weak coupling regime. arXiv preprint arXiv:1906.06359, 2019.

[72] Saberi-Pouya S, Vazifehshenas T, Salavati-fard T, et al. Anisotropic hybrid excitation modes in monolayer and double-layer phosphorene on polar substrates. Physical Review B, 2017, 96(11): 115402.

[73] Kinnunen J J, Wu Z, Bruun G M. Induced p-wave pairing in Bose-Fermi mixtures. Physical review letters, 2018, 121(25): 253402.

[74] Cazalilla M A, Ho A F, Ueda M. Ultracold gases of ytterbium: Ferromagnetism and Mott states in an SU (6) Fermi system. New Journal of Physics, 2009, 11(10): 103033.

[75] Fukuhara T, Kantian A, Endres M, et al. Quantum dynamics of a mobile spin impurity. Nature Physics, 2013, 9(4): 235.

[76] Weitenberg C, Endres M, Sherson J F, et al. Single-spin addressing in an atomic Mott insulator. Nature, 2011, 471(7338): 319.

[77] Rabl P, DeMille D, Doyle J M, et al. Hybrid quantum processors: molecular ensembles as quantum memory for solid state circuits. Physical review letters, 2006, 97(3): 033003.

[78] Gaul C, Domnguez-Adame F, Sols F, et al. Feshbach-type resonances for two-particle scattering in graphene. Physical Review B, 2014, 89(4): 045420.
Figure 1: Pair propagators $\Pi_{11}$ (a) and particle-hole-like propagator $\Pi_{22}$ (b). (c) Polar plot of the scattering form factor $F_{\lambda\lambda}(q-k, \Delta\phi)$ (Eq.(16)) for different values of scattering wave vector $(q-k)$. The impurity is setted as $p = 1$. 
Figure 2: Real part (left) and imaginary part (right) of the pair propagator in chiral and nonchiral case with $\theta = 0$ as a function of the impurity momentum $p$. The rows from top to bottom correspond to the Bosonic frequency (impurity) $\omega = 0, 1, 2$, respectively. The momentum cutoff $\Lambda$ is setted as 1 and the chemical potential is setted as 0.1. The masses of impurity, electron, and hole are setted as the same for realize the coherent and nonadiabatic configuration.

Figure 3: The same as Fig.1 but for $\theta = \pi/4$. 
Figure 4: Real part (left) and imaginary part (right) of the self-energy in chiral and nonchiral case with $\theta = 0$ as a function of the impurity momentum $p$. The rows from top to bottom correspond to the Bosonic frequency (impurity) $\omega = 0, 1, 2$, respectively. The momentum cutoff $\Lambda$ is set as 3 eV (which is large enough for the system we discuss) and the chemical potential is set as 0.1 eV. The blue circle and the red lines correspond to the case of $g_b = -0.5$ for nonchiral and chiral cases, respectively, and the dash lines correspond to the case of $g_b = -0.8$. We can see that the real part of the self-energy is always negative, which agrees with the attractive feature of the polaron.
Figure 5: The same as Fig.4 but for $\theta = \pi/4$. The lower panel shows the self-energy as a function of frequency $\omega$, which is presented to show the divergence from the marginal fermi liquid character.
Figure 6: The momentum- and frequency-dependent polaron spectral function in chiral and nonchiral case with $\theta = 0$.

Figure 7: The same as Fig.5 but for $\theta = \pi/4$. 
Figure 8: Intensity plot of spectral function in $\omega - p$ plane. For comparison, the inset in the center shows the dispersion of the noninteracting impurity as a function of momentum $p$. 
Figure 9: Induced effective masses by the polaronic effect for attractive polaron with negative $g_b$ (a) and repulsive polaron with positive $g_b$ (b). (c) is the quasiparticle residue $Z$ as a function of impurity momentum for different bare coupling constant $g_b$ in nonchiral doped parabolic system. For zero coupling $g_b = 0$, the induced effective mass vanishes and the residue equals one. $\Delta m^*(0)$ denotes the induced effective mass in static case ($p = 0$).

Figure 10: Pair propagator at finite temperature as a function of the impurity momentum (left panel) and frequency (right panel). Related parameters are setted the same as Fig.2.