New Renormalization Scheme of Vacuum Polarization in QED

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(Dated: January 22, 2009)

We examine the vacuum polarization contribution in the renormalization scheme of QED. Normally, the quadratic divergence term is discarded under the condition that the counter term of the Lagrangian density should be gauge invariant. Here, it is shown that the whole contribution of the photon self-energy should not be considered for the renormalization procedure. In fact, the finite contribution of the renormalization in the vacuum polarization is shown to give rise to the hyperfine splitting energy which disagrees with the experimental observation in hydrogen atom. For the treatment of the vacuum polarization, we present a new renormalization scheme of the photon self-energy diagram.

PACS numbers: 11.10.Gh, 12.38.Gc, 11.15.Ha

I. INTRODUCTION

In the renormalization procedure of QED, one considers the vacuum polarization which is the contribution of the self-energy diagram of photon

\[ \Pi^{\mu\nu}(k) = ie^2 \int \frac{d^4p}{(2\pi)^4} \text{Tr} \left[ \gamma^\mu \frac{1}{p - m} \gamma^\nu \frac{1}{p - k - m} \right]. \]  

(1.1)

This integral obviously gives rise to the quadratic divergence (\( \Lambda^2 \) term). However, when one considers the counter term of the Lagrangian density which should cancel this quadratic divergence term, then the counter Lagrangian density violates the gauge invariance since it should correspond to the mass term in the gauge field Lagrangian density. Therefore, one has to normally erase it by hand, and in the cutoff procedure of the renormalization scheme, one subtracts the quadratic divergence term such that one can keep the gauge invariance of the Lagrangian density. Here, we should notice that the largest part of the vacuum polarization contributions is discarded, and this indicates that there must be something which is not fully understandable in the renormalization procedure. Physically, it should be acceptable to throw away the \( \Lambda^2 \) term since this infinite term should not be connected to any physical observables. Nevertheless we should think it over why the unphysical infinity appears in the self-energy diagram of photon.

On the other hand, the quadratic divergence term disappears in the treatment of the dimensional regularization scheme. Here, we clarify why the quadratic divergence term does not appear in the dimensional regularization treatment. That is, the treatment of the dimensional regularization employs the mathematical formula which is not valid for the evaluation of the momentum integral. Therefore, the fact that there is no quadratic divergence term in the dimensional regularization is simply because one makes a mistake by applying the invalid mathematical formula to the momentum integral. This is somewhat surprising, but now one sees that the quadratic divergence is still there in the dimensional regularization, and this strongly indicates that we should reexamine the effect of the photon self-energy diagram itself.

This paper is organized as follows. In the next section, we discuss the momentum integral in the vacuum polarization contributions, and the integration rotated into the Euclidean space is presented. In section 3, we treat the dimensional regularization and discuss the problem in the application of the gamma function which appears in the momentum integral. In section 4, we show the calculated result of the hyperfine splitting energy in hydrogen atom which is corrected from the finite contribution of the vacuum polarization. In section 5, we discuss the physical reason why
one should not consider the vacuum polarization diagram in the renormalization procedure in QED. Finally, section 6 summarizes what we clarify in this paper.

II. MOMENTUM INTEGRAL WITH CUTOFF $\Lambda$

In this section, we briefly review the standard renormalization scheme of the vacuum polarization diagram. The evaluation of the photon self-energy diagram is well explained in the textbook of Bjorken and Drell \[1\], and therefore we describe here the simplest way of calculating the momentum integral. The type of integral one has to calculate can be summarized as

$$\int d^4p \frac{1}{(p^2 - s + i\varepsilon)^n} = i\pi^2 \int_0^{\Lambda^2} w dw \frac{1}{(w - s + i\varepsilon)^n} \text{ with } w = p^2 \quad (2.1)$$

where $i$ appears because the integral is rotated into the Euclidean space and this corresponds to $D = 4$ in the dimensional regularization as we will see it below.

A. Photon Self-energy Contribution $\Pi^\mu\nu(k)$

The photon self-energy contribution $\Pi^\mu\nu(k)$ can be easily evaluated as

$$\Pi^\mu\nu(k) = \frac{4ie^2}{(2\pi)^4} \int_0^1 dz \int d^4p \left[ \frac{2p_\mu p_\nu - g_{\mu\nu}p^2 + sg_{\mu\nu} - 2z(2 - z)(k_\mu k_\nu - k^2 g_{\mu\nu})}{(p^2 - s + i\varepsilon)^2} \right]$$

$$= \frac{\alpha}{2\pi} \int_0^1 dz \int_0^{\Lambda^2} dw \left[ \frac{w(w - 2s)g_{\mu\nu} + 4z(1 - z)w(k_\mu k_\nu - k^2 g_{\mu\nu})}{(w - s + i\varepsilon)^2} \right] \quad (2.2)$$

where $s$ is defined as $s = m^2 - z(1 - z)k^2$. This can be calculated to be

$$\Pi^\mu\nu(k) = \Pi^{(1)}_{\mu\nu}(k) + \Pi^{(2)}_{\mu\nu}(k)$$

where

$$\Pi^{(1)}_{\mu\nu}(k) = \frac{\alpha}{2\pi} \left( \Lambda^2 + m^2 - \frac{k^2}{6} \right) g_{\mu\nu} \quad (2.3a)$$

$$\Pi^{(2)}_{\mu\nu}(k) = \frac{\alpha}{3\pi} (k_\mu k_\nu - k^2 g_{\mu\nu}) \left[ \ln \left( \frac{\Lambda^2}{m^2} \right) - 6 \int_0^1 dz z(1 - z) \ln \left( 1 - \frac{k^2}{m^2} z(1 - z) \right) \right]. \quad (2.3b)$$

Here, the $\Pi^{(1)}_{\mu\nu}(k)$ term corresponds to the quadratic divergence term and this should be discarded since it violates the gauge invariance when one considers the counter term of the Lagrangian density. The $\Pi^{(2)}_{\mu\nu}(k)$ term can keep the gauge invariance, and therefore one can renormalize it into the new Lagrangian density.

B. Finite Term in Photon Self-energy Diagram

After the renormalization, one finds a finite term which should affect the propagator change in the process involving the exchange of the transverse photon $A$. The propagator $\frac{1}{q^2}$ should be replaced by

$$\frac{1}{q^2} \Rightarrow \frac{1}{q^2} \left[ 1 + \frac{2\alpha}{\pi} \int_0^1 dz z(1 - z) \ln \left( 1 - \frac{q^2 z(1 - z)}{m^2} \right) \right] \quad (2.4)$$

where $q^2$ should become $q^2 \approx -q^2$ for small $q^2$. It should be important to note that the correction term arising from the finite contribution of the photon self-energy diagram should affect only for the renormalization of the vector field $A$. Since the Coulomb propagator is not affected by the renormalization procedure of the transverse photon (vector field $A$), one should not calculate its effect on the Lamb shift where the propagator is, of course, the static Coulomb propagator.
III. DIMENSIONAL REGULARIZATION

Before discussing the physical effect of the renormalization of the photon self-energy contribution, we examine the dimensional regularization method [2, 3] which is commonly used in evaluating the momentum integral in self-energy diagrams. In the evaluation of the momentum integral which appears in the photon self-energy diagram, one often employs the dimensional regularization where the integral is replaced as

$$\int \frac{d^4 p}{(2\pi)^4} \rightarrow \lambda^{4-D} \int \frac{d^D p}{(2\pi)^D}$$

where $\lambda$ is introduced as a parameter which has a mass dimension in order to compensate the unbalance of the momentum integral dimension. This is the integral in the Euclidean space, but $D$ is taken to be $D = 4 - \epsilon$ where $\epsilon$ is an infinitesimally small number.

A. Photon Self-energy Diagram with $D = 4 - \epsilon$

In this case, the photon self-energy $\Pi_{\mu\nu}(k)$ can be calculated to be

$$\Pi_{\mu\nu}(k) = i\lambda^{4-D} \epsilon^2 \int \frac{d^D p}{(2\pi)^D} \text{Tr} \left[ \gamma_\mu \frac{1}{p - m} \gamma_\nu \frac{1}{p - k - m} \right]$$

$$= \frac{\alpha}{3\pi} (k_\mu k_\nu - g_{\mu\nu} k^2) \left[ \frac{g}{\epsilon} + \text{finite term} \right]$$

where the finite term is just the same as eq.(2.3). In eq.(3.2), one sees that the quadratic divergence term ($\Pi_{\mu\nu}^{(1)}(k)$) is missing. This is surprising since the quadratic divergence term is the leading order contribution in the momentum integral, and whatever one invents in the integral, there is no way to erase it unless one makes a mistake.

B. Mathematical Formula of Integral

Indeed, in the treatment of the dimensional regularization, people employ the mathematical formula which is invalid for the integral in eq.(3.2). That is, the integral formula for $D = 4 - \epsilon$

$$\int d^D p \frac{p_\mu p_\nu}{(p^2 - s + i\epsilon)^n} = i\pi^\frac{n}{2} (-1)^{n+1} \frac{\Gamma(n - \frac{1}{2}D - 1)}{2\Gamma(n)} \frac{g_{\mu\nu}}{s^n - \frac{D}{2} - 1}$$ \quad (for \quad n \geq 3) \quad (3.3)$$

is only valid for $n \geq 3$ in eq.(3.3). For $n = 3$, the integral should have the logarithmic divergence, and this is nicely avoided by the replacement of $D = 4 - \epsilon$. However, the $n = 2$ case must have the quadratic divergence and the mathematical formula of eq.(3.3) is meaningless. In fact, one should recover the result of the photon self-energy contribution $\Pi_{\mu\nu}(k)$ of eqs.(2.3) at the limit of $D = 4$, apart from the $(2/\epsilon)$ term which corresponds to the logarithmic divergence term.

C. Reconsideration of Photon Self-energy Diagram

In mathematics, one may define the gamma function in terms of the algebraic equations with complex variables. However, the integral in the renormalization procedure is defined only in real space integral, and the infinity of the integral is originated from the infinite degrees of freedom in the free Fock space [4]. Therefore, one sees that the disappearance of the quadratic divergence term in the evaluation of $\Pi_{\mu\nu}(k)$ in the dimensional regularization is not due to the mathematical trick, but simply due to a simple-minded mistake. In this respect, it is just accidental that the $\Pi_{\mu\nu}^{(1)}(k)$ term in the dimensional regularization vanishes to zero. Indeed, one should obtain the same expression of the $\Pi_{\mu\nu}^{(1)}(k)$ term as eq.(2.3a) when one makes $\epsilon \rightarrow 0$ in the calculation of the dimensional regularization. This strongly suggests that we should reconsider the photon self-energy diagram itself in the renormalization procedure.
IV. PROPAGATOR CORRECTION OF PHOTON SELF-ENERGY

In order to examine whether the inclusion of the photon self-energy contribution is necessary for the renormalization procedure or not, we should consider the effect of the finite contribution from the photon self-energy diagram. As we see, there is a finite contribution of the transverse photon propagator to physical observables after the renormalization of the photon self-energy. The best application of the propagator correction must be the magnetic hyperfine splitting of the ground state (1s\textsubscript{1/2} state) in the hydrogen atom since this interaction is originated from the vector field \( A \) which gives rise to the magnetic hyperfine interaction between electrons and nucleus.

A. Lamb Shift Energy

In some of the textbooks \[1\], the correction term arising from the finite contribution of the photon self-energy diagram is applied to the evaluation of the Lamb shift energy in hydrogen atom, and it is believed that the finite correction term should give the Lamb shift energy of \(-27 \text{ MHz}\) in the 2s\textsubscript{1/2} state in hydrogen atom. However, this is not a proper application since only the renormalization of the vector field \( A \) should be considered. This is closely connected to the understanding of the field quantization itself. One sees that the second quantization of the electromagnetic field should be made only for the vector field \( A \), and this is required from the experimental observation that photon is created from the vacuum of the electromagnetic field in the atomic transitions. Therefore, it is clear that the renormalization becomes necessary only for the vector field \( A \).

In fact, the Coulomb propagator is not affected by the renormalization procedure of the transverse photon since the \( A_0 \) term is exactly solved from the constraint equation. In this respect, the Lamb shift energy of the 2s\textsubscript{1/2} state in hydrogen atom should be understood without the finite correction term of the propagator from the vacuum polarization. However, it may well be difficult to calculate the Lamb shift energy to a high accuracy since it involves the bound state wave function of the hydrogen atom. At the accuracy of \(10^{-3}\) level, one has to carefully consider the reduced mass effect which is normally neglected in the Lamb shift calculations \[1\]. In addition, if one should avoid the problem of the logarithmic divergence in the Lamb shift calculation, one has to carry out the fully relativistic calculations. However, in this case, one has to face the difficulty of the negative energy states in hydrogen atom, and this is a non-trivial task.

B. Magnetic Hyperfine Interaction

The magnetic hyperfine interaction between electron and proton in hydrogen atom can be written with the static approximation in the classical field theory as

\[
H' = - \int j_e(r) \cdot A(r) d^3r
\]

(4.1)

where \( j_e(r) \) denotes the current density of electron, and \( A(r) \) is the vector potential generated by proton and is given as

\[
A(r) = \frac{1}{4\pi} \int \frac{J_p(r')}{|r-r'|} q^3 r'
\]

(4.2)

where \( J_p(r) \) denotes the current density of proton. The hyperfine splitting of the ground state in the hydrogen atom can be calculated as

\[
\Delta E_{hf} = \langle 1s\textsubscript{1/2}, F ; I : F | H' | 1s\textsubscript{1/2}, I ; F \rangle
\]

(4.3)

where \( I \) and \( F \) denote the spins of proton and atomic system, respectively. This can be explicitly calculated as

\[
\Delta E_{hf} = (2F(F + 1) - 3) \frac{\alpha g_p}{3M_p} \int_0^{\infty} F^{(1s)}(r) G^{(1s)}(r) dr
\]

(4.4)

where \( g_p \) and \( M_p \) denote the g-factor and the mass of proton, respectively. \( F^{(1s)}(r) \) and \( G^{(1s)}(r) \) are the small and large components of the radial parts of the Dirac wave function of electron in the atom. In the nonrelativistic approximation, the integral can be expressed as

\[
\int_0^{\infty} F^{(1s)}(r) G^{(1s)}(r) dr \approx \frac{(m_e \alpha)^3}{m_c}
\]

(4.5a)
where \( m_e \) is the mass of electron and \( m_r \) denotes the reduced mass defined as

\[
m_r = \frac{m_e}{1 + \frac{m_e}{M_p}}.
\]  

(4.6)

It should be noted that, in eq.(4.5), \( m_e \) appears in the denominator because it is originated from the current density of electron. Therefore, the energy splitting between \( F = 1 \) and \( F = 0 \) atomic states in the nonrelativistic limit with a point nucleus can be calculated from Eq.(4.4) as

\[
\Delta E_{hfs}^{(0)} = \frac{8\alpha^4 m_3}{3m_e M_p}.
\]  

(4.7)

C. QED Corrections for Hyperfine Splitting

There are several corrections which arise from the various QED effects such as the anomalous magnetic moments of electron and proton, nuclear recoil effects and relativistic effects. We write the result

\[
\Delta E_{hfs}^{(QED)} = \frac{4g_0\alpha^4 m_3}{3m_e M_p} (1 + a_e) \left( 1 + \frac{3}{2} \alpha^2 \right) (1 + \delta_R)
\]  

(4.8)

where \( a_e \) denotes the anomalous magnetic moment of electron. The term \( (1 + \frac{3}{2} \alpha^2) \) appears because of the relativistic correction of the electron wave function

\[
\int_0^\infty F^{(1s)}(r) G^{(1s)}(r) dr = \frac{(m_e a_e)^3}{m_e} \left( 1 + \frac{3}{2} \alpha^2 + \cdots \right).
\]  

(4.5b)

The term \( \delta_R \) corresponds to the recoil corrections and can be written as

\[
\delta_R = \alpha^2 \left( \ln 2 - \frac{5}{2} \right) - \frac{8\alpha^3}{3\pi} \ln \alpha \left( \ln \alpha - \ln 4 + \frac{281}{480} \right) + \frac{15.4\alpha^3}{\pi}.
\]  

(4.9)

Now the observed value of \( \Delta E_{hfs}^{(exp)} \) is found to be

\[
\Delta E_{hfs}^{(exp)} = 1420.405751767 \text{ MHz}.
\]

Also, we can calculate \( \Delta E_{hfs}^{(0)} \) and \( \Delta E_{hfs}^{(QED)} \) numerically and their values become

\[
\Delta E_{hfs}^{(0)} = 1418.83712 \text{ MHz}, \quad \Delta E_{hfs}^{(QED)} = 1420.448815 \text{ MHz}.
\]

Therefore, we find the deviation from the experimental value as

\[
\frac{\Delta E_{hfs}^{(exp)} - \Delta E_{hfs}^{(QED)}}{\Delta E_{hfs}^{(0)}} \simeq -30 \text{ ppm}.
\]  

(4.10)

D. Finite Size Corrections for Hyperfine Splitting

In addition to the QED corrections, there is a finite size correction of proton and its effect can be written as

\[
\Delta E_{hfs}^{(FS)} = \Delta E_{hfs}^{(0)}(1 + \varepsilon)
\]  

(4.11)

where the \( \varepsilon \) term corresponds to the Bohr-Weisskopf effect

\[
\varepsilon \simeq -m_e \alpha R_p
\]  

(4.12)

where \( R_p \) denotes the radius of proton. It should be noted that the perturbative treatment of the finite proton size effect on the hyperfine splitting overestimates the correction by a factor of two. Now, the calculated value of \( \varepsilon \) becomes

\[
\varepsilon \simeq -17 \text{ ppm}.
\]

Therefore, the agreement between theory and experiment is quite good.
E. Finite Propagator Correction from Photon Self-energy

The hyperfine splitting of the $1s_\downarrow$ state energy including the propagator correction can be written in terms of the momentum representation in the nonrelativistic limit as

$$\Delta E^{(V_p)}_{hfs} = (2F(F+1) - 3) \frac{16 \alpha^5 m_r^4}{3\pi m_e M_p} \int_0^\infty \frac{q^2 dq}{(q^2 + 4(m_r \alpha)^2)^2} \left(1 + M^R(q)\right)$$

$$\equiv \Delta E^{(0)}_{hfs}(1 + \delta_{vp}). \quad (4.13)$$

$M^R(q)$ denotes the propagator correction and can be written as

$$M^R(q) = \frac{2\alpha}{\pi} \int_0^1 dz z(1-z) \ln \left(1 + \frac{q^2}{m_e^2} z(1-z)\right). \quad (4.14)$$

We can carry out numerical calculations of the finite term of the renormalization in the photon self-energy diagram, and we find

$$\delta_{vp} \simeq 18 \text{ ppm} \quad (4.15)$$

which tends to make a deviation larger between theory and experiment of hyperfine splitting in hydrogen atom. This suggests that the finite correction from the photon self-energy contribution should not be considered for the renormalization procedure.

V. NEW RENORMALIZATION SCHEME OF PHOTON SELF-ENERGY IN QED

The difficulty of the photon self-energy must be connected to the field quantization itself since we do not fully understand it yet. It is clear that the field quantization is required from experiment, and the field quantization procedure itself must be well justified. However, this does not mean that we understand it theoretically within the framework of quantum field theory. For the quantization of the vector field $A(r,t)$, one has to first fix the gauge since otherwise one cannot determine the gauge field $A(r,t)$ which depends on the choice of the gauge. Then, one can quantize it and accordingly one can quantize the Hamiltonian of the electromagnetic field. In this case, one can calculate the contributions of the photon self-energy diagram which, in fact, give rise to the quadratic divergence. Since this divergence is too difficult to handle, people assumed that it should be discarded since it is indeed the same term as the mass term of the electromagnetic field Hamiltonian. Therefore, it is required that this quadratic divergence term should be discarded by the condition that the renormalized Hamiltonian should be gauge invariant. However, one can easily notice that this requirement is somewhat incomprehensible since one has already fixed the gauge before the field quantization.

A. Intuitive Picture of No Renormalization of Photon Self-energy

Now, we should start from the equations of motion for the gauge field as well as the fermion field

$$\Box A(x) = e \bar{\psi}(x) \gamma \psi(x) \quad (5.1a)$$

$$(i\partial_\mu \gamma^\mu - m) \psi(x) = e A_\mu(x) \gamma^\mu \psi(x) \quad (5.1b)$$

where $x$ denotes $x = (t, r)$. These equations of motion can be solved perturbatively by assuming that the coupling constant $e$ is small, which is, indeed, an observed fact. Here, one sees that the vector field $A(x)$ should not be influenced by the renormalization procedure since there is no term which involves the vector field $A(x)$ in the right hand side of eq.(5.1a). On the other hand, the fermion field $\psi(x)$ should be affected by the perturbative evaluation since the interaction term contains the fermion field $\psi(x)$ in the right hand side. Therefore, the contribution of the fermion self-energy diagram should be renormalized into the fermion mass and the wave function $\psi(x)$. 

B. Integral Equations

To see it more explicitly, we can convert eqs.(5.1) into the following integral equations

\[ A(x) = A_0(x) - e \int \frac{e^{iqx}}{q^2} \left( \bar{\psi}(q) \gamma \psi(q) \right) \frac{d^4q}{(2\pi)^4} \]  
\[ \psi(x) = \psi_0(x) - e \int \frac{e^{iqx}}{q - m} \left( \bar{A}(q) \cdot \gamma \right) \psi(q) \frac{d^4q}{(2\pi)^4} \]

where \( \bar{\psi}(q) \) and \( \bar{A}(q) \) denote the fields in momentum representation. Here, \( A_0(x) \) and \( \psi_0(x) \) denote the free state solutions which satisfy

\[ \Box A_0(x) = 0, \quad (i\partial_{\nu} \gamma^\mu - m) \psi_0(x) = 0. \]  

C. S-matrix Evaluation

If one carries out the calculation of the photon self-energy diagram in the S-matrix method, then one finds that the contribution has the quadratic divergence. However, the process is not physical one, and therefore one should just keep them as the mathematical effects. This can sometimes happen to the S-matrix evaluation since the contributions of some of the diagrams include the processes which cannot be realized as the physical process. In fact, the photon self-energy diagrams should be unphysical since the photon after the interaction does not change its quantum state at all. This situation may become physically detectable if the photon could be found in the bound state. However, as one knows, there is no possibility that photon can be bound. Therefore, there is no chance to observe any effects of the photon self-energy contributions, contrary to the fermion self-energy contributions where fermions can be found as a bound state which is completely different from the free Fock space evaluations.

VI. CONCLUSIONS

The renormalization procedure in QED is most successful and reliable in field theory calculations. In particular, the treatment of the fermion self-energy is well established, and the finite effect after the renormalization procedure is successfully compared to the experimental observation of the Lamb shift energy of the 2s\( _\downarrow \) state in hydrogen atom. Further, the vertex correction is well calculated to a high accuracy and is compared to the experimental observation of the magnetic moment of electron \((g - 2)\) experiment, and the agreement between theory and experiment is surprisingly good. For the above two contributions which have the logarithmic divergences, there is no conceptual problem since both of the diagrams are related to the physical processes. Further, the physical meaning of the renormalization is well justified even though there appears some infinity. This is connected to the fact that the logarithmic divergence can never get to a real infinity in physical space, and therefore the procedure of the renormalization of the logarithmic infinity has no intrinsic difficulty.

On the other hand, the vacuum polarization contribution is completely different from the above two effects. It has the quadratic divergence which is impossible to handle for the renormalization procedure. Therefore, before the renormalization of the vacuum polarization diagram, one has to discard the photon self-energy contribution at the level of the calculation of \( \Pi^{\mu\nu}(k) \). This treatment itself cannot be justified within the renormalization scheme of QED. Therefore, people had to stick to the gauge invariance of the calculated result in order to find any excuse of discarding the quadratic divergence term even though the calculated result is obtained by having fixed the gauge.

In this sense, there must have been many physicists who had an uneasy feeling on the treatment of the photon self-energy diagram. The principle we should take in physics is that any theoretical frameworks must be connected to physical observables. The self-energy of fermion cannot be related to physical observables as far as one works within the free Fock space of QED. However, as one knows, fermions can become a bound state which is not found in the free Fock space, and therefore the physical effect of the self-energy of fermion after the renormalization procedure can be detected as the Lamb shift energy in hydrogen atom. On the other hand, there is no detectable effect of the self-energy of photon since photon can be found always as a free state which is indeed within the free Fock space. Therefore, from the beginning, physically there is no need of renormalization of self-energy of photon even though mathematically the S-matrix evaluation gives rise to the infinite contributions from the self-energy of photon diagram.
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