Superconducting qubit storage and entanglement with nanomechanical resonators

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We describe a quantum computational architecture based on integrating nanomechanical resonators with Josephson junction phase qubits, with which we implement single- and multi-qubit operations. The nanomechanical resonator is a GHz-frequency, high-quality-factor dilatational resonator, coupled to the Josephson phase through a piezoelectric interaction. This system is analogous to one or more few-level atoms (the Josephson qubits) in a tunable electromagnetic cavity (the nanomechanical resonator). Our architecture combines the best features of solid-state and cavity-QED approaches, and may make possible multi-qubit processing in a scalable, solid-state environment.

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The lack of scalable qubit architectures, with sufficiently long quantum-coherence lifetimes and a suitably controllable entanglement scheme, remains the principal roadblock to building a large-scale quantum computer. Superconducting devices exhibit robust macroscopic quantum behavior. Recently, there have been exciting demonstrations of long-lived Rabi oscillations in current-biased Josephson junctions, subsequently combined with a two-qubit coupling scheme, and in parallel, demonstrations of Rabi oscillations and Ramsey fringes in a Cooper-pair box. These accomplishments have generated significant interest in the potential for Josephson-junction-based quantum computation. Coherence times $\tau_c$ up to $5 \mu$s have been reported in the current-biased devices, with corresponding quality factors $Q_c = \tau_c \Delta E / h$ of the order of $10^5$, yielding sufficient coherence to perform many logical operations. Here $\Delta E$ is the qubit energy level spacing.

In this paper, we describe an architecture in which ultrahigh-frequency resonators coherently couple two or more current-biased Josephson junctions, where the superconducting “phase qubits” are formed from the energy eigenstates of the junctions. We show that the system is analogous to one or more few-level atoms (the Josephson junctions) in a tunable electromagnetic cavity (the resonator), except that here we can individually tune the energy level spacing of each atom, and control the electromagnetic interaction strength.

Our implementation uses large-area current-biased Josephson junctions, with capacitance $C$ and critical current $I_0$, a circuit model is shown in Fig. 1. The lowest two quasi-bound states in a local minimum, $\mathcal{E}_\text{min}$, with $s < 1$, the potential $U(\delta)$ has metastable minima, separated from the continuum by a barrier $\Delta \mathcal{U} = U(\delta_{\text{max}}) - U(\delta_{\text{min}}) \to (4\sqrt{2}/3)\mathcal{E}_\text{J}(1-s)^3/2$ for $s \to 1^-$, as shown in Fig. 4. The curvature $U''(\delta)$ defines the small-amplitude plasma frequency $\omega_p^2 \equiv |U''(\delta_{\text{min}})| / M = \omega_{p0} (1-s^2)^{1/4}$, with $\omega_{p0} = \sqrt{2eI_0/hC} = \sqrt{2\mathcal{E}_\text{J}/h}$. The Hamiltonian for the junction phase difference is $H_\text{J} = P^2/2M + U(\delta)$, with $P = -i\hbar d/d\delta$ the momentum operator. The junction’s zero-voltage state corresponds to the phase “particle” trapped in one of the metastable minima.

The lowest two quasi-bound states in a local minimum,
\[ |0\rangle \text{ and } |1\rangle, \text{ define the phase qubit. State preparation is typically carried out with } s \text{ just below unity, in the range } s = 0.95 - 0.99, \text{ where } U(\delta) \text{ is strongly anharmonic, and for which there are only a few quasibound states.} \]

The anharmonicity allows state preparation from a classical radiofrequency (rf) field, as then the frequency of the classical field can be set to couple to only the lowest two states. In our scheme, by contrast, single quanta are exchanged between the junction and the resonator, so anharmonicity is not necessary; we find it convenient to work with \( s \) between 0.5 and 0.9.

We focus here on coupling a single resonator to two Josephson qubits; extensions to larger systems will be considered in later work. The two-junction circuit is shown in Fig. 2. The disk-shaped element is the nanomechanical resonator, consisting of a single-crystal piezoelectric disc sandwiched between two metal plates, and the junctions are the crossed boxes on either side of the resonator, interrogated by high-impedance circuits.

The nanomechanical resonator is designed with a fundamental thickness resonance frequency \( \omega_0/2\pi \approx 1 - 10 \text{ GHz, with quality factor } Q \sim 10^5 - 10^6. \) Piezoelectric dilatational resonators with resonance frequencies in this range, and quality factors of \( 10^3 \) at room temperature, have been fabricated from sputtered AlN. Single-crystal AlN can also be grown by chemical vapor deposition.

A resonator, with a diameter \( d = 1.16 \mu m \) and thickness \( b = 0.5 \mu m \), such resonators can be used to coherently store a qubit state prepared in a current-biased Josephson junction, return it to that junction, or transfer it to another junction, as well as entangle two or more junctions. These operations are performed by tuning the energy level spacing \( \Delta E \) into resonance with \( \hbar \omega_0 \), generating electromechanical Rabi oscillations.

Referring to Fig. 2, the total bias current of junction 1 is \( I_{dc1} + I_{res} \), where \( I_{res} \) is the current through the resonator from that junction. A simple model for the resonator allows us to write \( I_{res} = C_{res}(V + \hbar \omega_{33} U) \), where \( C_{res} \) is the resonator geometric capacitance, \( \hbar \omega_{33} \) the relevant piezoelectric coupling constant.

\[ V \text{ the rate of voltage change, and } U \text{ the rate of change of the mechanical strain. The current } I_{res} \text{ is partly due to the capacitance } C_{res} \text{ and partly due to the piezoelectrically-coupled strain } U. \]

\[ C_{res}, \text{ in parallel with the junction capacitance } C, \text{ renormalizes the mass } M \text{ to } M = \hbar^2 C/4e^2, \text{ where } C = C_{res} + C. \]

With the resonator coupled to the superconducting phase through the voltage \( V \), the Hamiltonian for the combined junction-resonator system is \( \delta H = H_1 + H_{res} + \delta H \). Here \( \delta H = h \omega_0 a a^\dagger \) is the Hamiltonian of the isolated resonator, where we have quantized the resonator displacement field with creation (destruction) operators \( a^\dagger (a) \), and only included the fundamental dilatational mode.

\[ \delta H = \frac{C_{res} \hbar \omega_{33}}{2e(1 - \eta)} \delta U \delta = ig(a - a^\dagger)\delta, \]

where \( \eta = 0.054 \) and the coupling constant \( g \) is

\[ g = \frac{\hbar^{3/2} C_{res} \hbar \omega_0}{(1 - \eta)e \sqrt{\rho \pi bd^2/4}}. \]

For our model resonator \( g = 0.820 \mu eV \).

In the junction eigenstate basis, the junction Hamiltonian is \( H_1 = \sum_m c_m^\dagger c_m \), with creation (destruction) operators \( c_m^\dagger (c_m) \) acting on the phase qubit states. The interaction Hamiltonian is

\[ \delta H = ig \sum_{m m'} \langle m | \delta | m' \rangle c_{m}^\dagger c_{m'}^\dagger (a - a^\dagger). \]
The eigenstates of the noninteracting Hamiltonian \( H_0 = H_J + H_{\text{res}} \) are \( |mn\rangle \equiv |m\rangle_J \otimes |n\rangle_{\text{res}} \), with energies \( E_{mn} = \epsilon_m + \hbar \omega_0 n \), where \( n \) is the resonator occupation number. An arbitrary state can be expanded as \( |\Psi(t)\rangle = \sum_{mn} c_{mn}(t) |mn\rangle \exp(-iE_{mn}t/\hbar) \).

The full Hamiltonian is equivalent to a few-level atom in an electromagnetic cavity. The cavity “photons” are phonons, which interact with the “atoms” (here the Josephson junctions) via the piezoelectric effect. This analogy allows us to adapt quantum-information protocols developed for cavity-QED to our architecture.

We show that we can coherently transfer a qubit state from a junction to a resonator, using the adiabatic approximation combined with the rotating-wave approximation (RWA) of quantum optics [28]. We assume that the bias current \( s \) changes slowly on the time scale \( \hbar/\Delta E \), and work at temperature \( T = 0 \). The RWA is valid when \( \Delta E \) and \( \hbar \omega_0 \) are close on the scale of \( \hbar \omega_0/\Omega_{\text{res}} \), and when the interaction strength \( g \ll \Delta E \). At time \( t = 0 \), we prepare the resonator in the state \( |0\rangle_{\text{res}} \). In the RWA, neglecting relaxation, we obtain the amplitude evolution

\[
\begin{align*}
\hbar i \partial_t c_{0n} &= -ig \sqrt{n} \langle 0 | [\hat{\delta}, 1] | 1 \rangle e^{i\omega_d t} c_{1,n-1} \\
\hbar i \partial_t c_{1n} &= ig \sqrt{n+1} \langle 1 | [\hat{\delta}, 1] | 0 \rangle e^{-i\omega_d t} c_{0,n+1},
\end{align*}
\]

where \( \omega_d \equiv \omega_0 - \Delta E/\hbar \) is the resonator–qubit detuning. We integrate to find the reduced density matrices \( \rho_J(t) \) (in the qubit subspace) and \( \rho_{\text{res}}(t) \) (in the zero- and one-phonon resonator subspace). The junction phase is initially prepared in the pure state \( \alpha |0\rangle_J + \beta |1\rangle_J \), corre-
To pass a qubit state $\alpha|0\rangle + \beta|1\rangle$ from junction 1 to junction 2, the state is loaded into the first junction and the bias current changed to bring the junction into resonance with the resonator for half a Rabi period. This writes the state $\alpha|0\rangle + \beta|1\rangle$ into the resonator. After the first junction is taken out of resonance, the second junction is brought into resonance for half a Rabi period, passing the state to the second junction. We have simulated this operation numerically, assuming two identical junctions coupled to the resonator described above. The results are shown in Fig. 4 and Table I, where $c_{m_1m_2}$ is the probability amplitude (in the interaction representation) to find the system in the state $|m_1m_2\rangle$, with $m_1$ and $m_2$ labeling the states of the two junctions.

We can prepare an entangled state of two junctions by bringing the first junction into resonance with the resonator for $1/4$th of a Rabi period [30], which, according to our RWA analysis, produces the state $(|100\rangle - |010\rangle)/\sqrt{2}$. After bringing the second junction into resonance for half a Rabi period, the state of the resonator and second junction are swapped, leaving the system in the state $(|100\rangle - |010\rangle)/\sqrt{2}$ with a probability of 0.987, where the resonator is in the ground state and the junctions entangled, as demonstrated in Fig. 5. Using the cavity-QED analogy, it will be possible to transfer the methodology developed for the standard two-qubit operations, in particular controlled-NOT logic, to this system, using mostly existing technology and demonstrated techniques.

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