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**Title:** Total internal reflection based super-resolution imaging for sub-IR frequencies

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Total internal reflection based super-resolution imaging for sub-IR frequencies - Supplementary Information

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1 Introduction

In this supplementary material further details on the experimental design will be discussed, along with a deeper analysis of the images that are presented in the accompanying paper. Future works are explored, including thickness and optical parameter extraction from images, and imaging through opaque media.

2 Parameters of the system

2.1 Details of experimental setup

Since imaging in the mm-wave range is still a challenge we employ all-optical modulation (known as photomodulation), using a visible or near-IR light source to modulate the low frequency radiation source in a photo-active medium (such as silicon). We use light from a 4.7 W, 623 ± 5 nm LED, which travels up through the transparent prism to the silicon wafer. This allows photoexcitation of the silicon wafer while keeping the top surface of the silicon free for samples. Using a digital micromirror device (DMD) from Vialux, we pattern the optical beam in order to locally modulate the photoconductivity in the silicon wafer. A light guide is used for homogeneous illumination of the DMD, and projection of the patterned light to our modulator occurs using a series of achromatic lenses (focal lengths 10 cm and 7.5 cm) and mirrors. We generate masks of spatially varying photoconductivity, which are imprinted on to the reflected mm-wave beam, providing the spatial information needed to construct an image using a single detector. We use a set of Hadamard masks, similar to [1, 2], which can significantly boost the signal compared to conventional raster scanning [3]. As shown below, this approach provides good signal to noise in images while avoiding the complex image reconstruction calculations associated with compressive imaging techniques [3], allowing us to both measure and reconstruct the image in a few seconds. More details on the image reconstruction are given in section 3.1.

In the main text we refer to a numerical aperture of the imaging system, based on the focusing of the mm-wave beam on the silicon wafer. The beam has a radius of approximately
6 cm, and is focused to a spot of approximately 2 cm over a distance of approximately 35 cm. This gives us a spread in incident angles, forming the focused spot, of around 3.3°, equating to a numerical aperture of around 0.06.

The majority of images shown in this paper are defined by $64 \times 64$ pixels, requiring $2 \times 64^2 = 8192$ masks to be generated, and the mm-wave signal collected for each one. We collect the measurements at a frame rate of 2 kHz, so each image is recorded in 4 seconds. Images presented below are an average of 100 sequential image measurements. Images with smaller numbers of pixels or fewer averages can be collected in shorter times, and are presented in section 3.3 for comparison.

### 2.2 Wafer thickness

![Simulated modulation assuming 140 GHz radiation incident on a wafer with a 75 μs effective charge carrier lifetime and photoexcited by a 623 nm pump source of intensity 200 Wm⁻². The peaks in modulation are periodic with the wafer thickness, and suggest a Fabry-Perot cavity resonance in the wafer plays an important role in the modulation.](image)

**Fig. S 1.** Simulated modulation assuming 140 GHz radiation incident on a wafer with a 75 μs effective charge carrier lifetime and photoexcited by a 623 nm pump source of intensity 200 Wm⁻². The peaks in modulation are periodic with the wafer thickness, and suggest a Fabry-Perot cavity resonance in the wafer plays an important role in the modulation.

We mention in the main text that the thickness of the wafer is very important when designing the imaging system. Fig. S1 shows the modulation calculated for an incident wave of 140 GHz at various angles for various wafer thicknesses. Two clear peaks are shown with a separation of around 320 μm at the critical angle, as shown by the white arrow. The refractive index of silicon is 3.42, therefore the wavelength of the mm-wave beam inside the silicon is 625 μm. The peaks are separated by almost one half-wavelength, and shift to higher frequencies as the angle of the incident beam is increased. These are both behaviours that are characteristic of a Fabry-Perot resonance inside the silicon.
Fig. S2. (a) Calculated modulation and (b) penetration length of evanescent fields at the silicon-air boundary as a function of both angle of incidence and frequency for a 390 µm thick wafer. The white dashed lines represent the critical angle above which total internal reflection occurs. The red circles highlight the set of parameters chosen for the experimental measurements.

2.3 Frequency and incident angle

In Fig. S2 (a) we plot the modulation as a function of both incident angle and frequency. We see that the modulation for all frequencies with incident angles below $\theta_{\text{crit}}$ (white dotted line) is small, but for larger angles the modulation is highly frequency dependent. We observe a peak in the modulation just above 140 GHz, which varies with the thickness of the silicon wafer, and is attributed to a resonant Fabry-Perot cavity mode inside the wafer (see Fig. S1). Near this resonant feature, and for TE polarisation with an angle of incidence around 82°, we predict a modulation depth of > 99% with an optical illumination intensity of just 220 Wm$^{-2}$, achievable with a commercial LED source.

While matching the frequency to the silicon wafer thickness is beneficial to achieve the largest signals, one must also consider the thickness of the sample. When there is a dielectric medium above the silicon with a refractive index lower than that of the prism, the penetration length of the evanescent fields (L) will determine the depth of material that can be probed.
We plot

\[ L = \frac{\lambda}{4\pi \sqrt{n_{\text{silicon}} \sin^2 \theta_{\text{silicon}} - n_{\text{sample}}^2}} \]  

(S1)

as a function of frequency and incident angle in the prism in Fig. S2 (b), assuming a sample index of 1. \( \theta_{\text{silicon}} \) is the angle of refraction in the silicon, and is related to the angle of incidence in the prism, \( \theta_{\text{prism}} \) via Snell’s Law: \( \theta_{\text{silicon}} = \arcsin \left( \frac{n_{\text{prism}} \sin \theta_{\text{prism}}}{n_{\text{silicon}}} \right) \). For all frequencies and all angles below \( \theta_{\text{crit}} \) there are no evanescent fields at the silicon-air boundary. Above \( \theta_{\text{crit}} \) the penetration length is larger for lower frequencies and drops off towards higher frequencies. Thus, if we are interested in probing only the surface of a low-index sample, higher frequencies are more suitable. However, if one would like to probe the bulk of a sample, or even determine the thickness of layers, lower frequencies mean longer penetration of the fields into a sample. Hence, in practice, there is a trade off between the incident angles and frequencies which give the largest signals, and those that probe the bulk of the material in the half-space above the wafer. In the main text, we image tissue layers of a few hundred micron thickness, with refractive indices that are close to or higher than that of the prism. In this case the mm-wave beam is totally internally reflected from the sample-air boundary on the side furthest from the silicon, and instead the absorption and scattering of propagating waves inside the sample provide the contrast in the image. The red circles in Figs. S2 (a) and (b) represent the parameters selected for the experimental measurements - a frequency of 140 GHz (chosen from a limited selection of available sources and the proximity to the wafer cavity resonance) and incident angle of 50° (corresponding to a penetration depth of around 300 \( \mu m \) in air).

The experimentally measured modulation of the TE polarised beam is around 63.5%, while for the TM polarised beam the modulation is around 23.6%, compared to 44% for TE and 21% for TM found in the simulation. The value for the TM polarisation is more closely matched to the simulation results than that for the TE polarisation. The most likely causes of this difference are the spread in angles in the incident beam, which is around 3 - 4°, and possibly an underestimation of the incident angle of the mm-wave beam on a similar scale. Small variations in angle would have a larger effect on the TE polarisation, which matches with what we observe.

2.4 Measuring the effective charge carrier lifetime

In order to measure the effective charge carrier lifetime of the wafer used for the photomodulator, the transmission through the wafer at 60 GHz was measured as a function of time as a photoexciting light source was modulated. Note that the lifetime of the charge carriers is a property of the wafer and will not change as a function of frequency. The wafer was oriented at a 45° angle to a 1 ms TTL modulated photoexciting source (a collimated 4.8 W SOLIS-623C LED from Thorlabs with an output wavelength of 623 nm). The 60 GHz source was an Anritsu Vectorstar MS4647B Vector Network Analyser in CW mode with a Flann 25240 25 dB standard gain horn antenna, which was also oriented at 45° to the wafer, and orthogonal to the photoexciting light. The magnitude of the signal transmitted through the wafer was detected using a Sage Millimeter SFD-503753-15SF-P1 waveguide detector
connected to an oscilloscope triggered from the modulated light source. The waveform of both switch on and switch off events were recorded for a range of photoexciting intensities. Fig. S3 shows the transmission through the wafer as a function of time (blue dots) and an exponential curve fitted to the data. The photoexciting source is switched on at time = 0, and the time elapsed when the signal has fallen to 1/e of the original is taken to be the lifetime of the wafer, as marked by the black dashed lines.

### 2.5 Effect of charge carrier distribution

Throughout the analysis in the main text we assume that the modulation from the silicon wafer could be predicted reasonably well by assuming that the distribution of charge carriers is uniform across the wafer thickness. Under this assumption the modulation depth is proportional to the excess charge carrier density, $\Delta n$, which is dependent only on the effective lifetime of the carriers, $\tau_{\text{eff}}$, the excitation function, $G$ (proportional to the intensity of the incident light) and the thickness of the silicon wafer, $d$, according to $\Delta n = \tau_{\text{eff}} G / d$.

For modulators in a transmission geometry this approximation is very good, as the wave will travel through the entire wafer and be subject to the total amount of charge carriers. However in the total internal reflection geometry this approximation is not necessarily valid. Instead it may be more appropriate to separate the recombination of charge carriers into bulk, $\tau_b$ and surface $\tau_s$ contributions, which are related to the effective charge carrier lifetime via $1/\tau_{\text{eff}} = 1/\tau_b + 1/\tau_s$. However for comparison to the experiment we would need accurate measurements of bulk and surface lifetimes, and this is not a straightforward measurement to perform. Instead we can measure the effective lifetime (see Fig. S3) and use this value in the simulation. It is still pertinent to check that the assumption of effective lifetime is valid.
**Fig. S 4.** Simulated modulation of a 390 µm thick wafer with an effective lifetime of 75 µs. The red line assumes a uniform homogeneous charge carrier distribution, each of the other lines are calculated assuming different combinations of bulk and surface lifetimes that each lead to an effective lifetime of 75 µs. The inset shows a zoomed-in section that illustrates the small deviation in modulation for each wafer.

Fig. S4 shows the calculated modulation for wafers of various bulk, $\tau_b$, and surface, $\tau_s$, charge carrier lifetime. The distribution of charge carriers in silicon was found using a diffusion calculation, assuming illumination from one side. More details can be found in reference [4]. The red line represents a wafer with homogeneous charge carrier distribution, and assumes an effective charge carrier lifetime of 75 µs. The following lines show the modulation expected from wafers that each have an effective charge carrier lifetime of 75 µs, but which have different bulk and surface lifetimes. The charge carrier distribution will be most inhomogeneous in wafers with large bulk lifetimes and high surface recombination rates, which correspond to low surface lifetimes. This is the case where we would expect the maximum deviation in modulation from the homogeneous distribution. However for all lifetime combinations investigated here (which cover the typical values found in undoped silicon wafers), the difference in modulation between homogeneous and non-homogeneous distribution does not exceed 5%. We can therefore say that our approximation still leaves us with a reasonably valid model for the modulation, although we must be aware that there is a possibility the simulation will predict modulations slightly higher than those measured in experiment. It is also worth keeping in mind that if the wafer thickness was reduced this approximation becomes worse as the surface recombination rate becomes more dominant in determining the effective charge carrier lifetime.
3 Parameters for imaging

3.1 Orthonormal single-pixel computational imaging

Without photomodulation of the silicon wafer, the total internal reflection geometry results in the entire underside of the sample being immersed in a mm-wave field when the source is active. Inhomogeneities in the complex refractive index of the sample result in a spatial variation in the level of scattering of the evanescent field. Therefore, the intensity of totally internally reflected mm-wave radiation spatially varies in a sample dependent manner. In order to reconstruct an image of this spatial variation using only a single-element detector, we sequentially project a series of $N$ patterns, each of which probes a different subset of the spatial information of the sample. In each case, the detector records the totally internally reflected signal $a_n$, which is proportional to the level of spatial overlap (i.e. dot product) between the scattering profile of the sample and the $n^{th}$ displayed 2D pattern $H_n$. If these patterns are chosen to be orthonormal, then the resulting image $I$ can be reconstructed from a sum of the patterns, each weighted by the measured overlap with the sample:

$$I = \sum_{n=1}^{N} a_n H_n,$$

where $I$ represents a 2D image containing $N$ pixels. In this case the patterns are drawn from the orthonormal Hadamard basis, similar to [1,2], which, significantly boosts the signal compared to a set of patterns equivalent to conventional raster scanning, as shown in [3]. In practice, use of the fully sampled Hadamard basis requires the display of $2N$ patterns to ensure that the patterns are orthogonal, as described in [5].

Correct implementation of Hadamard masks for imaging entails the following: displaying a positive mask (with entries of 0 and 1) followed by the corresponding negative mask (i.e. with the positions of 1 and 0 swapped), and subtracting the signal obtained in each case. This emulates the display of masks with entries of -1 and 1, which are then orthogonal as desired. Sampling using Hadamard masks is beneficial when the dominant source of noise in the measurement is detector noise, as each Hadamard masks transmits 50% of the signal, and so the level of signal arriving at the detector for each measurement is increased in comparison with a raster scanning masking approach, where only a fraction of the mask is transmissive.

To collect the data we use a custom-built code written in Python, which controls the data acquisition and reconstructs the images from the collected data.

The code first generates the set of masks and sends this to the on-board memory on the digital micromirror device (DMD). The mm-wave source is continuous, so is always incident on the detector. The first mask in the set is projected onto the DMD, and the detector sends a voltage, proportional to the intensity of the beam, to an arduino, which acts as an analog-to-digital converter. The arduino waits for a specified time period (e.g. 300 $\mu$s) after the mask is projected, to allow the charge carriers in the silicon to reach a steady state distribution. It then collects the data over a specified time period (e.g. 600 $\mu$s), performs box-car averaging on the collected data, then sends the signal to the computer where it is stored, and the second mask in the set is projected onto the DMD. Each even-numbered mask is the inverse (negative) of the previous mask (positive).
Once the reflection from each mask has been recorded, the difference is found between the positive and negative masks, and this measurement is combined with the corresponding mask to reconstruct the image.

3.2 Number of pixels

![Image Comparison](image)

**Fig. S 5.** A comparison between images with 32 × 32 pixels (left panels) and 64 × 64 pixels (right panels), and a larger (top panels) and smaller (bottom panels) field of view.

When taking mm-wave images in the near-field we are able to select the field of view and the number of pixels of the image. Fig. S5 shows four images taken of the same porcine tissue sample with a larger (top row) and smaller (bottom row) field of view, and either 32 × 32 (left) or 64 × 64 (right) pixels. The darker areas are filaments of fat running through an area of protein.

Increasing the number of pixels does improve the clarity and resolution of the images, however this also leads to an increase in imaging time by a factor equal to twice the number of additional pixels. Reducing the field of view while maintaining the same number of pixels does not affect the time taken to collect images, and does allow finer structures in the sample to be seen more clearly. However reducing the field of view is done by decreasing the area of the digital micromirror device (DMD) that is used, and therefore reduces the portion of the mm-wave beam that is modulated and leads to an increase in the background signal. In extreme cases it will also limit the achievable resolution, as there needs to be as
many micromirrors as pixels in the selected DMD region. Therefore when considering any application, careful thought should be given to the required speed and acceptable resolution, in order to decide on the most appropriate number of pixels.

### 3.3 Measurement time

**Fig. S 6.** A selection of images taken of the same porcine tissue sample (optical image at top) taken with different imaging parameters. Rows contain images with the same frame rate, and columns contain images with the same averages. In the very top row the images have $64 \times 64$ pixels, all others have $32 \times 32$ pixels.

Another parameter of the imaging system that can be tuned is the time over which the mm-wave signal is collected for each frame. Fig. S6 shows an array of images taken of the same porcine tissue sample at various frame rates and averages, all with $32 \times 32$ pixels apart from the top row which has $64 \times 64$ pixels, as indicated.

The signal to noise ratio is drastically improved by either increasing the number of averages taken, or decreasing the frame rate and increasing the signal collection time accordingly. We can investigate which of these methods is best for reducing the noise, as this depends
on the nature of the noise itself. Taking only a few averages of signals collected over a long
time with a low frame rate will effectively eliminate noise that is quickly varying. However
if the noise in the system is varying slowly with time, it is more effectively minimised by
increasing the number of averages taken for each frame.

In this system, when comparing two images that have equivalent total collection times
(e.g. 20 averages at 2 kHz and 5 averages at 0.5 kHz), it can be seen that the high frame
rate/high averaging option gives a signal to noise ratio that is slightly better than the low
frame rate/low averaging option, although they are both very similar.

3.4 Polarisation

Fig. S 7. (a) Calculated reflection at 140 GHz from a 390 μm thick silicon wafer with a
75 μs charge carrier lifetime on a half-space of prism material (TPX polymer), as a function
of incident angle for both TE and TM polarisations. In the dark state there is no pho-
toexcitation, and in the bright state a 623 nm optical beam is incident on the wafer with
an intensity of 220 W/m². (b) Comparison between images taken with the same imaging
parameters but with different polarisations, represented by the arrows above each image.

One other parameter at our disposal is the polarisation of the mm-wave beam we are
using. The calculation presented in Fig. S7 (a) shows that, for an air sample, much larger
modulation is expected for TE polarisation than for TM. This should lead to greater contrast
in images, and improved signal-to-noise ratios.

To illustrate this, Fig. S7 (b) shows two images of porcine tissue taken with identical measurement parameters, but with opposite linear polarisations. It is clear that the contrast and signal-to-noise ratio are greatly improved for the TE polarisation compared to the TM.

4 Analysis of images

In this section we will analyse some possible sources of artefacts arising from the imaging process, such as diffraction of mm-wave beam from the masks and the use of Hadamard masks. We will also discuss in more detail the interpretation of the information held in each image, including the extraction of material parameters and thickness of samples. Finally, we will present evidence of imaging objects hidden behind a layer of biological tissue.

4.1 Reference image

Fig. S8 shows an example of an image of the mm-wave spot taken with no sample in place - in other words, an image of the modulation, \(MD(x, y)\). Each image presented in the paper is normalised to a reference image such as this one, taken with the same collection parameters, directly before the sample is positioned on the wafer. The spot is not entirely uniform, and this normalisation removes artefacts in images that arise from the inhomogeneity of the spot. These inhomogeneities are caused by reflections within the prism and the wider experimental set-up, as well as variation in the charge carrier lifetime across the wafer.

4.2 Lateral resolution

There are three factors that may play a role in the determining the overall resolution of
Fig. S 9. Images of 1951 USAF standard resolution target, taken with a TE polarised 140 GHz beam, using a 390 µm thick silicon wafer with a 75 µs charge carrier lifetime and a 623 nm optical beam incident on the wafer with an intensity of 220 W/m². All images were taken with a frame rate of 2 kHz, and the plots presented are averages of 10 images with 64 x 64 pixels. (a) shows an image of element 2 in group -2, with a line width and spacing of 1.8 mm. (b) shows an image of element 6 in group -2, with a line width and spacing of 1.1 mm.

the imaging system; the resolution of the optical images, the size of the pixels in the digital micromirror device, and the blurring of patterns caused by the diffusion of charge carriers. We discuss these below.

The optical resolution of the optical images projected onto the silicon is the first limit to consider. With a re-imaging lens of focal length 7.5 cm placed around 10 cm from the imaging plane (i.e. the modulator), we expect an optical resolution of the pump of approximately 2.5 µm.

The mask pixel resolution will also limit the maximum resolution. The DMD used is the Vialux-V700, which contains an array of 1024 x 768 micromirrors, each of which is around 13 µm across. This is projected to give a field of view of 2.5 x 2.0 cm, with a magnification of approximately 1.8. The smallest mask resolution that we project has 64 x 64 mask pixels – i.e. a single mask pixel is a ‘super-pixel’ consisting of 12 x 12 micromirrors, such that each mask pixel is 13 µm x 1.8 x 12 ~ 280 µm across. We are therefore not approaching image resolutions where the size of the micromirrors will be a limiting factor.

Mask blurring due to carrier diffusion sets the current resolution limit of the images. Fig. 4(a) in the main text shows that the diffusion length of the carriers with a charge carrier lifetime of 75 µs is around 350 µm. This carrier-diffusion blurring factor is orders of magnitude greater than the optical resolution with which the patterns are projected, and so we can safely neglect this as it plays no role in determining the resolution of our imaging system. The diffusion of carriers has the effect of convolving each mask pattern with a Gaussian function which decays to 1/e of its maximum value approximately 350 µm from
the edge of the pixel. Therefore we expect a lateral resolution of around $700\,\mu m + 280\,\mu m$. This agrees with the resolutions observed in our experimental images: In Fig. 3(c) we show that we can resolve thin veins of fat in protein that are approximately 1 mm thick, close to the limit in resolution.

In order to test the lateral resolution of our imaging system more thoroughly, we image the standard 1952 USAF resolution target. We find that, when the polarisation is perpendicular to the metallic edges of the targets, we can easily resolve the targets down to group -2, element 6, which corresponds to a resolution smaller than the strip width and spacing of 1.1 mm. This is in reasonable agreement with our tissue images in Fig. 3(c), as well as the estimate of resolution limit we make from charge diffusion alone (700 µm).

A target was made by selectively etching away copper on a flexible mylar sheet, leaving copper of 18 µm width in the regions indicated by shading in the figure below. To reach the maximum resolution, the mm-wave beam is TE polarised, i.e. the electric field is perpendicular to the plane of incidence, and the strips are oriented perpendicular to the direction of the electric field. Fig. S 9(a) shows the largest target in group -2, with 1.8 mm wide strips and spacing, which is easily resolved by our imaging system. Fig. S 9(b) shows the smallest target we were able to easily resolve with strips widths and spacing of 1.1 mm.

### 4.3 Diffraction from mask

A fundamental part of our imaging technique requires periodic structures in the conductivity of the silicon to be created. For some of the masks in the Hadamard set, these can be of a similar size to the wavelength. Therefore for these masks there will be some diffraction of the mm-wave beam that is reflected off this structured surface. It could be suggested that this will lead to artefacts in the images, and so is worth looking at in more detail.

Fig. S10 (a) shows how the mm-wave signal measured by the detector changes as a function of angle. For the measurement, the source detector and mm-wave lens were positioned as they were in the imaging system. The source was modulated at 1 kHz and the signal from the detector recorded with a lock-in amplifier for each angle as the source was rotated about the exit aperture. It is seen from Fig. S10 (a) that the signal has significantly reduced to less than 20% at angles greater than $\pm 10^\circ$ from normal incidence.

Figs. S10 (b) - (d) show how this angular response compares to typical angles of diffraction from a selection of Hadamard masks, shown in the inset in each plot. To obtain these, a 2-dimensional fast Fourier transform was taken of the electric field at the surface of each mask, assuming that it is projected over an area of 25 mm$^2$, and for a frequency of 140 GHz. A slice is taken through the centre of the 2-dimensional plot for comparison. Only masks with 1-dimensional structure have been presented here for simplicity, but this technique can be applied to all masks, as diffraction along the y-axis has the same behaviour as diffraction along the x-axis.

Panel (b) in Fig. S10 shows the Fourier transform of a mask with relatively coarse structure, which is one of very few that will produce diffracted orders within the acceptance angle of the detector. In this case, the first diffracted order will be detected when imaging. However, as the masks become finer and more sub-wavelength in structure, such as those in panels (c) and (d), the beams are diffracted at higher angles and will not be recorded by the detector. We can say that the majority of masks have a structure that is sufficiently
Fig. S 10. (a) shows the measured angular response of the detector. (b), (c) and (d) show the calculated diffraction pattern of a 140 GHz beam that is travelling through the prism material and reflected from a Hadamard mask shown in the inset. Coarse structured masks lead to some diffracted orders falling inside the detection range of the detector, but most masks have fine structures and the diffracted beams cannot be detected in the set-up.

It is also worthwhile considering the fact that the total power that can be diffracted into any of the higher orders is very limited. Ignoring the diffusion of charge carriers, and the less than 100% modulation, the masks can be approximated as a square-profile amplitude grating. The diffraction efficiency depends on the duty cycle of the grating, which will vary for different masks, and is highest for a 50% duty cycle grating. According to calculations performed here [6], at most 10.1% of the total power can be diffracted into the first order mode, and will decrease for duty cycles other than 50% and for less than 100% modulation, and further due to the smearing of the square profile due to charge carrier diffusion. Taking into account the angles and efficiency of diffraction, for the majority of the masks used in the imaging process, collection of diffracted beams is not a likely cause of artefacts in the images that we collect. The masks that may be affected by diffraction are those with very coarse structure, and if this was a significant effect we would clearly see a coarse structure over the images, and we do not.
4.4 Simulation of imaging

Another possible cause of unusual features in the images could be artefacts arising from the use of Hadamard masks themselves. In order to investigate this, a simulation of the imaging was created based on the analytical methods described in [4].

A transfer matrix code is used to calculate the reflection from an infinitely large area of several different stacks that are possible in the imaging system, i.e. combinations of illuminated or dark silicon, and protein or fat samples. These values are assigned to each pixel of a mask, and for mask pixels that cover regions with both fat and protein, linear interpolation between reflection coefficients is used. An arbitrary binary pattern of fat and protein regions is drawn. This is then overlaid with a Hadamard matrix, and the material in each pixel is used to select the reflection coefficients for dark and illuminated silicon. The reflection coefficients for each pixel are added and normalised to the area of the pixel to find the total reflected signal from that mask, which is then used in the reconstruction in the same way as in the experiment to find the value of $\delta r_0$. $\delta r_0$ is the difference in reflection when the modulator is on (illuminated) and off (dark), similar to the modulation depth, $MD$, but with a sample in place, and differs from the experimental image values by a factor of $MD$, due to the image normalisation. This is repeated for all masks in the set, and to obtain a true comparison with experiment, the matrices are separated into positive (i.e. in the Hadamard matrix +1s become 1s and -1s become 0s) and negative masks (i.e. +1s become 0s and -1s

Fig. S 11. Results of simulation of imaging a Siemens star. The original object is in panel (a), (b) shows an image taken with a $64 \times 64$ raster scan, (d) shows an image taken with a $32 \times 32$ pixel Hadamard scan, and (c) shows an image taken with a $64 \times 64$ pixel Hadamard scan.
become 1s) in the simulation as well.

This model is a very simple approximation of what is happening in the imaging. It does not take into account any effects due to diffraction of the mm-wave beam by the mask or the sample, and any resonances in the plane of the sample will not be reproduced, as each pixel is assumed to be infinitely large and not interacting with the neighbouring pixels. However it is still a useful tool for interpreting images and predicting the effects of sample index and thickness.

Fig. S 12. Plots of simulated $\delta r_0$ as a function of sample thickness and incident angle of the mm-wave beam. All results are calculated at 140 GHz mm-wave frequency for a 390 µm thick silicon wafer with a charge carrier lifetime of 75 µs, and an incident photoexcitation wavelength of 623 nm and power of 220 Wm$^{-2}$ when illuminated. The samples in each column have the same real permittivity, and in each row the imaginary permittivity is the same.

Fig. S11 shows the result of a simulation of a fat and protein sample. The thickness of the sample is 700 µm, and the material parameters for fat and protein are the same as in
the main text, along with all other experimental parameters. The object being imaged, (a),
is a Siemens star made of arms of fat (cream) and protein (pink). The image in panel (b)
was taken using a simulated raster scan, (c) using a lower resolution simulated Hadamard
scan, and (d) using a simulated Hadamard scan with the same resolution as (b). It is
clear that the magnitude of signals collected is much larger when using Hadamard masks
over Raster scanning, and also that the resolution is reduced when the number of pixels is
reduced. However, in all images there are no features or artefacts that arise from the use of
Hadamard masks.

4.5 Sample losses and critical coupling

It is explained in the main text of the paper that a change in sign of an image is due to a
change in sign of the $\delta r_0$. This is proposed to be due to meeting the condition for critical
coupling, where absorption of the mm-wave beam will be most efficient. This is achieved
when the damping coefficients for radiative and non-radiative loss mechanisms are equal [7],
and is due to a Fabry-Perot resonance in the thickness of the sample. This condition, rather
counter-intuitively, can be reached by reducing the losses in the silicon wafer, and lead to a
situation where more of the mm-wave is absorbed when the silicon is not illuminated. If this
is a correct interpretation, we expect that the losses in the sample will play an important role
in reaching this condition. We propose that it will also depend on a resonance determined
by the thickness and index of the sample, which provides the radiative loss component. In
this section we explore the parameter space, and look at how the $\delta r_0$ changes with real and
imaginary parts of the permittivity of the sample.

Fig. S12 shows the $\delta r_0$ as a function of the incident angle and sample thickness for a
range of real and imaginary permittivities. The sample that is closest to the experimental
example in the main text is (c). Comparing (a) and (b), we see that changing the real part
of the permittivity changes the sample thickness at which the resonant condition is met, and
shifts the resonant peak in $\delta r_0$ to lower thicknesses for higher indices, as well as reducing
the angular dispersion. These are both features of a Fabry-Perot type cavity resonance in
the sample. However, due to the low losses in the sample in (a) and (b), we do not observe
any negative $\delta r_0$ below very high incident angles. Increasing the loss, as in (c) and (d),
introduces a range of thicknesses where the $\delta r_0$ is negative, followed by a range of sample
thicknesses that give positive $\delta r_0$. Again the real part of the permittivity effects the thickness
and angle where this is observed. Finally, in (e) and (f), the losses are increased again, and
the magnitude of the positive $\delta r_0$ drops in general, while there is still a region where negative
$\delta r_0$ can be observed.

4.6 Variation in tissue properties

There are varying values for the permittivity of biological tissues reported in the literature.
For the analysis conducted in this work we use values reported by Gabriel et al [8], where the
complex permittivities are $\epsilon_{\text{fat}} = 2.89 + 0.64i$ and $\epsilon_{\text{protein}} = 8.63 + 11.20i$. Fig. S13 (a) and
(b) show the $\delta r_0$ calculated as a function of incident angle and sample thickness for layers of
protein and fat respectively, assuming these permittivities. However, there are wide ranges
of values reported for the complex permittivities of fat and protein in the literature [8–12].
Fig. S 13. Calculated $\delta r_0$ as a function sample thickness and incident angle for protein (panels (a) and (c)) and fat (panels (b) and (d)) samples, with permittivities taken from reference [8] (panels(a) and (b)), [9] (panel (C)) and [10] (panels (d)). All results are calculated at 140 GHz mm-wave frequency for a 390 $\mu$m thick silicon wafer with a charge carrier lifetime of 75 $\mu$s, and an incident photoexcitation wavelength of 623 nm and power of 220 W/m$^2$ when illuminated.

As an example, Bowman et al [9] report a different value for the permittivity of protein, $\epsilon_{\text{protein}} = 8.28 + 5.10i$. Note in particular the discrepancy between the losses in values reported by both references. Fig. S13 (c) shows the calculated $\delta r_0$ as a function of sample thickness and incident angle for protein, assuming the material properties from Bowman et al [9]. The effect of the lower losses given by Bowman et al is evident from the general increase in the magnitude of the $\delta r_0$, as well as the stronger dependence on the sample thickness, as there is less absorption of the mm-wave beam.

It is also worth noting that in reference [10], Ashworth et al state a real part of the permittivity of fat that is just below that of our prism ($\epsilon_{\text{prism}} = 2.5 > \epsilon_{\text{fat}} = 2.29 + 1.05i$). In this case, the contrast mechanism for the imaging changes slightly, as the mm-wave is now reflected from the silicon-sample boundary at high incident angles. Fig. S13 (d) shows the $\delta r_0$ calculated using this permittivity. By comparison with Fig. S13 (b) we see that the magnitude of the $\delta r_0$ decreases, as the imaginary part of the permittivity given by Ashworth et al is higher. In addition, a band of large negative $\delta r_0$ at high angles is seen. This is indicative of evanescently decaying fields inside the sample, which are very sensitive to the material parameters, but not to the thickness.
4.7 Extracting parameters

![Figure S14: (a) Calculated $\delta r_0$ as a function sample thickness and incident angle, for a sample with permittivity $\epsilon = 3.5 + 0.525i$. (b) Calculated $\delta r_0$ as a function of the real part of the sample permittivity and incident angle, for a 500 $\mu$m thick sample with imaginary permittivity $\epsilon_i = 0.525$. (c) and (d) Calculated (blue line, blue circles show the points chosen for fitting) $\delta r_0$ for a sample with material parameters listed in blue inset box, and the $\delta r_0$ calculated using the fitted material parameters in the yellow inset box (yellow circles). The grey shaded region shows the range of angles below the critical angle.

As mentioned in the main text, the ability to extract the permittivity and thickness of a sample from an image would open the door to many applications. Fig. S12 also gives us another insight, as we can see that the real and imaginary parts of the sample permittivity change the $\delta r_0$ in independent ways. In other words, in any of the plots in Fig. S12, any vertical slice of data taken will not be identical to the same slice in any other, with particularly large contrast at higher angles. This suggests that, if we were able to collect enough information, it is possible to determine the complex permittivity of a sample. Furthermore, Fig. S14 (a) and (b) show that the permittivity and thickness of the sample affect the $\delta r_0$ in non-identical ways. We can show that it is in fact possible to extract the complex permittivity and thickness of a sample from the $\delta r_0$.

Figs. S14 (c) and (d) show the results of two such extractions from simulated data, for two different samples. The inset blue box contains the expected parameters, and the blue circles on the plot correspond to the $\delta r_0$ calculated from that sample at 6 different angles. The $\delta r_0$ was then used in a non-linear least-squares fitting algorithm, and the parameters that we found as a result of the fitting are in the yellow box inset in each plot. The fitted $\delta r_0$ is also shown by the yellow circles.
In both cases the fit is very good, and we are able to extract accurate values for the thickness, and real and imaginary parts of the permittivity simultaneously. While six data points have been given to this fitting routine, in theory only three points are needed to extract the three parameters. Adding more data reduces the chances of finding a local minimum, which is always possible. In addition, if fitting to experimental data, noise can introduce some uncertainty and means local minima are less easy to avoid. However more complex and robust fitting routines can be used to help avoid this, such as genetic algorithms, simulated annealing or swarm optimisation.

4.8 Imaging through tissue

![Image](image.png)

**Fig. S 15.** Visible (bottom row) and mm-wave (top row) images of the same sample of porcine tissue. (a) shows the sample in isolation, and in (b) a rigid strip of metal was placed on the far side of the sample, covering regions of both fat and protein. The metal is visible in the mm-wave image through fatty tissue but not through the protein due to the high mm-wave absorption. Both $64 \times 64$ pixel mm-wave images were taken at 140 GHz using TE polarisation, and are averages of 100 images that took 4.1 seconds each to collect.

We present one final demonstration of near-field imaging using a TIR photomodulator, in which we image a metal object on the far side of a thin porcine tissue sample. This is shown in Fig. S15, using the same experimental parameters as described in the main text. Fig. S15 (a) shows the porcine tissue with only air on the top side, whereas in Fig. S15 (b) a rigid metal strip (shaded region in optical image) has been placed behind the tissue, on the opposite side to the modulator. In the region where the metal strip is behind the protein it cannot be observed, as the mm-wave is heavily absorbed within the protein, and the TIR wave is not strongly reflected due to the large real permittivity of protein. However
the metal is clearly visible as an increase in the reflected mm-wave in the fatty tissue region, seen by comparison to Fig. S15 (a). This demonstrates that it is possible to see through a thin layer of fat and observe objects behind it using a TIR mm-wave imaging technique.

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