Gödel’s Universe in a Supertube Shroud

Nadav Drukker, Bartomeu Fiol and Joan Simón

Department of Particle Physics, The Weizmann Institute of Science, 2 Herzl Street, Rehovot 76100, Israel

We demonstrate that certain supersymmetric Gödel-like universe solutions of supergravity are not solutions of string theory. This is achieved by realizing that supertubes are BPS states in these spaces, and under certain conditions, when wrapping closed timelike curves, some world-volume modes develop negative kinetic terms. Since these universes are homogeneous, this instability takes place everywhere in space-time. We also construct a family of supergravity solutions which locally look like the Gödel universe inside a domain wall made out of supertubes, but have very different asymptotic structure. One can adjust the volume inside the domain wall so there will be no closed timelike curves, and then those spaces seem like perfectly good string backgrounds.

I. INTRODUCTION.

Einstein’s equations in General Relativity have many solutions that seem unphysical. Some have curvature singularities, which generically lie beyond the regime of validity of the classical theory. Other solutions may have no singularities, but violate causality, by having closed timelike curves. Hawking [1] has presented arguments to support the Chronology Protection Conjecture: namely that when quantum mechanics is taken into account, backgrounds not having closed timelike curves do not develop them. Even if this is true, it seems to leave open the question of whether backgrounds having closed timelike curves to begin with have to be discarded or not.

The answer to such questions may not be found entirely within classical general relativity. One would like to know that the energy momentum tensor, serving as the source in Einstein’s equations corresponds to reasonable physical matter. Another direction to address the problem is to ask whether there is a way to make sense of physics on spaces with closed timelike curves in classical mechanics, quantum mechanics or quantum field theory.

Probably best of all is to ask those questions in string theory, which encompasses both general relativity and quantum physics. Just as string theory has managed to resolve certain curvature singularities, mainly timelike, by the addition of extra degrees of freedom, it is natural to wonder about the status of closed timelike curves in such a quantum theory of gravity. Are such backgrounds allowed in string theory? Does string theory realize the Chronology Protection conjecture? Some recent work in this direction is [2, 3, 4, 5, 6, 7].

In string theory, the existence of dynamical extended objects provides us with probes suited particularly well to study non-local issues like closed timelike curves. Instead of trying to quantize the string worldsheet theory in one of these backgrounds, finding its spectrum and adding interactions, our strategy will be to look for BPS states corresponding to extended branes in this vacuum and to analyze the dynamics of small fluctuations around them. For a specific subset of backgrounds having closed timelike curves we will find BPS states with a sickness in their effective action, signaling the invalidity of the solution.

Let us be more precise. There are many supergravity backgrounds having closed timelike curves. We will be interested in Gödel-like universes, which were first embedded in five dimensional supergravity [8]. Some other examples of similar metrics were found in [2, 9]. All these solutions are generalizations of a four dimensional metric studied by Som and Raychaudhuri [10], and are very similar to the original Gödel universe [11].

All these solutions describe rotating spaces, where one can choose the planes of rotation as well as some fluxes that make the solution supersymmetric. We will concentrate on a specific solution of type IIA supergravity with rotation in a single plane, since this is the simplest example of such a metric with closed timelike curves.

This solution [2] has a nontrivial metric in three dimensions, which we parameterize by the time coordinate $t$, and polar coordinates $r$ and $\phi$ in the plane. There is a NS-NS flux as well as RR 2-form and 4-form fluxes, and we label the extra direction in which there is flux by $y$. The metric and fluxes are

$$ds^2 = - (dt + cr^2 d\phi)^2 + dr^2 + r^2 d\phi^2 + dy^2 + \sum_{i=1}^{9} (dx^i)^2$$

$$H_3 = -2cr \, dr \wedge d\phi \wedge dy, \quad F_2 = -2cr \, dr \wedge d\phi, \quad F_4 = 2cr \, dt \wedge dr \wedge d\phi \wedge dy.$$ (1)

This one parameter solution preserves 8 supercharges and we shall assume positive $c$ without loss of generality. If we consider motion in the periodic $\phi$ direction at constant $r$, the corresponding spacetime curve becomes timelike for $r > 1/c$, hence the closed timelike curves.

Though this seems like a perfectly valid solution to the equations of supergravity, we will demonstrate that,
as it stands, it can’t be a valid solution of string theory. One way of realizing this fact is to notice that supertubes can exist in the above background. Supertubes are bound states of D0-branes and fundamental strings having non-vanishing D2-dipole moment. They can be realized on the worldvolume of a cylindrical D2-brane with electric and magnetic fields turned on, such that they induce a non-vanishing angular momentum that balances the tension which would naturally tend to collapse the tube. In our discussion, supertubes extend in the $y$ direction and wrap the angular direction $\phi$ at fixed radius $r$. It is not too surprising that this is the most interesting object to probe the above geometry with, since it has the correct rotational symmetry, and couples to all the background fields that were turned on. As will also be made clear below, this object preserves the same supersymmetries as the Gödel-like universe itself. Even though the tension of this supertube is just given by the sum of the D0-brane and fundamental string charges, and it can have an arbitrary size (by adjusting these charges), we will find that for a certain range of charges, some worldvolume modes develop a negative kinetic term whenever the radius satisfies $r \geq 1/c$, so that the supertube wraps a closed timelike curve. This sickness is much like that found by considering a probe brane in the repulsion background, which led to the enhançon mechanism. We interpret it as an instability of the vacuum itself, which indicates that these supertubes will want to condense.

The plan of the rest of the paper is the following. In the next section we shall briefly discuss the probe calculation of a supertube in our Gödel universe, and show that the world-volume theory may become sick at the radius where closed timelike curves appear. In Section 3, we describe a family of supergravity solutions, that locally look like the Gödel universe, but have a domain wall made out of smeared supertubes separating them from a space that does not have closed timelike curves asymptotically. The domain wall in these solutions can be thought of as a regulator, the initial solution arising in the limit where the domain wall is sent to infinity. One might think it possible to enlarge the region inside the domain-wall to get closed timelike curves, but the same problem we encountered in the full Gödel universe prevents this from happening.

We conclude with some discussion of the analogy to the enhançon mechanism and how similar problems show up for other Gödel universes of supergravity, invalidating them too.

II. SUPERTUBES IN A GÖDEL-LIKE UNIVERSE.

As mentioned in the introduction, the type IIA Sorn-Raychaudhuri solution has closed timelike curves. Thus, one may suspect that an extended probe would develop some sickness in this background. Our first goal is then to discuss the possible probes in this background. This task is simplified by the realization that the solution is a particular case of a family of IIA solutions discussed in the authors of the paper prove that in general these solutions admit D0 branes, fundamental strings and supertubes as probes that don’t break any further supersymmetry. Even better, they perform a probe analysis of these solutions, and show that when the solution presents closed timelike curves, the worldvolume theory on the supertube is ill-defined. Our computation is just a special case of theirs.

The supertube is a cylindrical D2-brane which is extended in the $y$ direction as well as the angular direction $\phi$ at fixed radius $r$ about the origin. The world-volume theory on the supertube is just that of a D2-brane in curved background, which includes the Dirac-Born-Infeld and Wess-Zumino terms

$$S = - \int e^{-\Phi} \sqrt{-\det(G + F)} - \int (C_3 + C_1 \wedge F),$$  \hspace{1cm} (2)$$

where $G$ is the pullback of the metric and $C_1$ and $C_3$ the pullbacks of the RR potentials. The field strength $F$ includes the gauge fields that are turned on in the brane and those induced by the NS field

$$F = F - B_2 = E \, dt \wedge dy + B \, dy \wedge d\phi.$$  \hspace{1cm} (3)$$

$E$ is the electric field and $B = -c^2 j$ the magnetic field, including the term induced by the background.

The equations of motion are solved by $E = 1$ (which is not critical in the presence of magnetic field) and constant $B > 0$, whereas the radius of the tube is fixed to be the angular momentum $J$, related to the D0 and fundamental string charge densities $q_0 = B$ and $q_s$ by $r^2 = J = q_0 q_s$. It is a simple exercise to show that the supertube preserves the same supersymmetries as the background, which is also clear from the analysis of the next section, where the metric is shown to belong to the family of metrics studied in the paper.

After adjusting the electric and magnetic fields as above, the Lagrangian density of the D-brane probe is simply the magnetic field $L = -B$. The supertube tension is just given by the sum of charges it carries, that of D0-branes and fundamental strings, as dictated by supersymmetry. Therefore we can seemingly make the tube as small or as large as we wish.

To verify that, we can start with a very small supertube and slowly increase the radius. There are a number of ways one could envision this happening. For instance, one could imagine a bath of D0-branes and fundamental strings surrounding the supertube, and allowing the supertube to change its radius by exchanging charges with

\footnotesize
1 The metric is actually homogeneous. The addition of the supertube breaks translational invariance, but preserves the rotation around the origin.

2 The positivity of the magnetic field is enforced by supersymmetry.
the bath, satisfying the BPS condition at all times. Perhaps more naturally, one can also consider fluctuations of the radius, in time and/or in the y direction, which don’t preserve the BPS condition. In this latter case, a non-zero potential would also appear.

This calculation was done in [13] for the background generated by a collection of supertubes, and is trivial to adapt it to our metric. The result is that the expansion of the Lagrangian at small velocities is

$$\mathcal{L} = \mathcal{L}_0 + \frac{1}{2} M r^2 + \ldots$$

where the dots can contain a potential term if the fluctuation doesn’t preserve the BPS condition. $\mathcal{L}_0$ is the static Lagrangian density and

$$M = \frac{(B - cr^2)^2 + r^2 - c^2r^4}{B},$$

is the mass of the excited mode. This is positive definite for $r < 1/c$, but for larger radii, it becomes negative for a certain range of charges.

How do we interpret this singularity? In similar computations, e.g. when we encounter that the metric in moduli space becomes negative we interpret it as meaning that the effective Lagrangian is not valid, because it is missing some light degrees of freedom. One then corrects the effective description by adding those extra degrees of freedom. An example of this is the enhançon mechanism [14]. It is not exactly clear how to apply this procedure to the case at hand. Recall that the Gödel universe [1] is homogeneous, so there are closed timelike curves through every point in space, and the sickness we found is not confined to a bounded region of spacetime. In addition we don’t have well defined asymptotic conditions, or well defined global charges that a new solution should reproduce (since it is not clear how to define charge in Gödel universes). What we can safely conclude is that the solution [1] is not a good string theory background.

### III. A DOMAIN WALL SOLUTION.

In the previous section we argued that the Gödel-type solution [1] can’t be a correct effective description of a string theory background. Nevertheless it is fairly easy to display supergravity solutions that locally are similar to Gödel, but are free of closed timelike curves. To do so, we start by recalling the family of IIA solutions considered in [13, 15, 19].

$$ds^2 = -U^{-1}V^{-1/2}(dt - A)^2 + U^{-1}V^{1/2}dy^2$$

$$+ V^{1/2} \sum_{i=2}^9 (dx^i)^2$$

$$B_2 = -U^{-1}(dt - A) \wedge dy + dt \wedge dy,$$

$$C_1 = -V^{-1}(dt - A) + dt,$$

$$C_3 = -U^{-1}dt \wedge dy \wedge A,$$

$$e^{-\Phi} = U^{-1/2}V^{3/4}.$$
At most, if $R = 1/c$ there are closed null curves on the shell itself, which might warrant further study.

If we place the shell at a larger radius there will be causality violating curves. It is therefore natural to think of the full Gödel solution as the limit when the domain wall is taken to infinity and the region outside discarded. This picture may provide a better laboratory for studying the problems of the Gödel universe. In particular, if the radius is only slightly larger than $1/c$, there is only a thin shell of closed timelike curves. So the sickness is not spread over the entire space, and it should be possible to describe a process by which the shell contracts to eliminate the region with closed timelike curves.

In any event, it seems like those spaces cannot be created. A calculation similar to the one carried out in section II will show that, if one tries to increase the size of the region inside the shell, so it exceeds $R = 1/c$, the same problems will appear, indicating that one cannot increase the radius of the shell further.

IV. DISCUSSION

One of the most striking uses of D-branes in string theory is as probes of spacetime, often exposing the limitations of supergravity, and providing a cure to some of its pathological solutions. The enhançon mechanism provides a beautiful example of this. Studying supertubes on the Gödel-like universe teaches us a similar lesson, that the naive supergravity solution is not valid. As opposed to the enhançon case, we find that the space is sick everywhere, and the question of how to “cure” it does not seem well posed.

Nevertheless, the domain wall solutions we discussed in section III locally look like Gödel universes, and present certain analogies with the enhançon mechanism: in both cases, we keep part of the original metric, and replace the problematic part (the repulsion singularity in one case, the region with closed timelike curves in the other) by another solution, separated by a shell formed by the probes themselves. Of course, an obvious difference is that in the enhançon case one keeps the asymptotic form of the metric and changes the interior, while in our case we do the opposite.

In this paper we concentrated on a specific Gödel universe solution of supergravity. Other solutions were studied in involving rotations in more planes. It is a simple check to see that all the solutions in type IIA and IIB suffer from the same problems we described. One has to take the same supertube, or in some cases the T-dual objects, and place them so that their circles follow the closed timelike curves, and the rest of the calculation is identical. Therefore all those spaces are not good string theory backgrounds. We expect this to be a general mechanism that eliminates many solutions with closed timelike curves in string theory.

Acknowledgements

We are very grateful to Ofer Aharony, Tom Banks, Micha Berkooz, Jacques Distler, Roberto Emparan, Willy Fischler, Surya Ganguli, Shinji Hirano, Eliezer Rabinovici and Kip Thorne for interesting discussions on related topics. We are particularly grateful to Roberto Emparan for pointing out a mistake in an earlier version of this paper. We would also like to thank the ITP at Stanford university and the organizers of the Stanford-Weizmann workshop, where this project started. BF would further like to thank the Santa Cruz Institute for Particle Physics at UCSC and JS would like to thank the Perimeter Institute and ITP at the University of Amsterdam for their hospitality. BF is supported through a European Community Marie Curie Fellowship, and also by the Israel-U.S. Binational Science Foundation, the IRF Centers of Excellence program, the European RTN network HPRN-CT-2000-00122 and by Minerva. JS is supported by the Phil Zacharia Postdoctoral Fellowship.

[1] S. W. Hawking, Phys. Rev. D 46, 603 (1992).
[2] E. K. Boyda, S. Ganguli, P. Hofava and U. Varadarajan, Phys. Rev. D 67, 106003 (2003) [arXiv:hep-th/0212087].
[3] B. Herdeiro, arXiv:hep-th/0302052.
[4] J. Järv and C. V. Johnson, Phys. Rev. D 67, 066003 (2003) [arXiv:hep-th/0211097].
[5] E. G. Gimon and A. Hashimoto, arXiv:hep-th/0304181.
[6] C. A. Herdeiro, arXiv:hep-th/0402062.
[7] R. Biswas, E. Keski-Vakkuri, R. G. Leigh, S. Nowling and E. Sharpe, arXiv:hep-th/0304241.
[8] J. P. Gauntlett, J. B. Gutowski, C. M. Hull, S. Pakis and H. S. Reall, arXiv:hep-th/0209114.
[9] T. Harmark and T. Takayanagi, arXiv:hep-th/0301206.
[10] M. M. Som and A. K. Raychaudhuri Proc. R. Soc. London A304, 81 (1968).
[11] K. Gödel, Rev. Mod. Phys. 21, 447 (1949).
[12] Mateos and P. K. Townsend, Phys. Rev. Lett. 87, 011602 (2001) [arXiv:hep-th/0103030].
[13] R. Emparan, D. Mateos and P. K. Townsend, JHEP 0107, 011 (2001) [arXiv:hep-th/0106012].
[14] C. V. Johnson, A. W. Peet and J. Polchinski, Phys. Rev. D 61, 086001 (2000) [arXiv:hep-th/9911161].
[15] J. P. Gauntlett, R. C. Myers and P. K. Townsend, Phys. Rev. D 59, 025001 (1999) [arXiv:hep-th/9809065].
[16] W. Bonnor, N. O. Santos and M. A. MacCallum, Class. Quant. Grav. 15, 357 (1998) [arXiv:gr-qc/9711011].
[17] W. Israel, Nuovo Cim. B 44S10 (1966) 1 [Erratum-ibid. B 48 (1967 NUCIA,B44,1.1966) 463].
[18] C. V. Johnson, R. C. Myers, A. W. Peet and S. F. Ross, Phys. Rev. D 64, 106001 (2001) [arXiv:hep-th/0105047].
[19] J. Gomis, T. Mateos, P. J. Silva and A. Van Proeyen, arXiv:hep-th/0304210.