Design and analysis of three-dimensional m-sequence bipolar OCDMA spectral amplitude code

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Received: 21 May 2021 / Accepted: 15 December 2021 / Published online: 26 February 2022
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Abstract
This paper has been focused to utilize the m-sequence (matrix) code in spectral/time/spatial OCDMA domain in a bipolar mode (Three Dimensional modes). The scheme has elegantly incorporated the design of an encoder and decoder such that the system efficiently generates the code in three-dimensional bipolar mode. The decoder design is such that it effectively suppresses the multiple user interference (MUI) of three-dimensional code. System analyses are carried for defining the BER performance variation with users acceptability and received power. The observed results demonstrate that the cardinality of m- matrix three-dimensional bipolar code is larger than the existing 3D Pascal code, 2D-MDW, 3D-PDC, 2D-PDC, 1D-CWH and 1D-MS codes. The received power in proposed system is −5.8 dBm lesser than the 3D Pascal Code at 10⁻¹⁸ BER and the users in system more than 210% can be accomplished as compared to 3D Pascal code.

Keywords Optical code division multiple access(OCDMA) · m-sequence(matrix) code · Multiple access interference(MAI) · Phase induced intensity noise(PIIN) · Complementary wash hardamard code (CWH) · Perfect difference code (PD).two dimensional code (2D) Code · Three dimensional(3D) Code

1 Introduction
In the last few decades, the Optical code division multiple access technique has become a prominent technology in the simultaneous transmission of data by multiple users within the same bandwidth with high confidentiality of codes (Noshad and Jamshidi 2011; Kavehrad and Zaccarin 1995; Prucnal et al. 1986). The OCDMA system uses the codes to transmit the data of several users. Coding approach may utilize the time spreading, frequency hopping, spectral amplitude and spatial coding schemes (Anuar et al. 2000). A single code is assigned to a user for information processing. The aforementioned coding schemes are assigned to users either in unipolar or bipolar mode to enhance the security. In unipolar code, the code is assigned to the information ‘1’ bit,
and there is no code for null information (‘0’ bit). On the other side, the bipolar scheme applies the two separate codes for the data ‘0’ and ‘1’ bit (Park et al. 2004). This work aims to focus on the development of bipolar code so that it could enlarge the confidentiality of code against the eavesdroppers.

The multiple access interference (MAI) that arises due to the simultaneous transmission of multiple users in the channel degrades the OCDMA code performance. This MAI can be eliminated through the construction of codes with uniform in phase cross-correlation by the efficient decoding scheme at the receiver end (Ahmed et al. 2013; Wei et al. 2001).

The Multiservice code (MS) and Modified Double Weight codes (MDW) are based on the constant in phase cross-correlation in one-dimensional uni-polar mode (1D) (Alwee et al. 2004; Kakaeaa et al. 2014). The m- matrix and 1D-CWH code in the bipolar scheme also follows the in-phase cross-correlation (Huang and Hsu 2000). As seen, the performances of these one-dimensional coding schemes degrade in handling, a large number of users. Moreover, the designing of codes under such condition becomes sophisticated and not useful to tackle the problem of higher multiple access interference in the channel.

To alleviate the problem that emerges in 1-D schemes, 2-D codes such as 2D MM, 2D- DPD and 2D–MDW codes, the three dimensional code have been proposed (Yang and Huang 2003; Lin et al. 2005; Yeh et al. 2009: Arief et al. 2012). The reported three dimensional-PD and PASCAL exists only in uni polar mode operation (Cherifi et al. 2020). To develop a more secured system, a scheme should be adopted that can support a large number of users with the bipolar property. In this regard, this work focused to develop the code for three-Dimensional mode in bipolar domain. Moreover, a decoding scheme also projected to assuage the multiple inferences in the channel.

The paper explores the work into five sections. Proposed work has been explained in Sect. 1. The mathematical section covered in Sect. 2. Section 3 analysed the result and discussion part and the paper concluded with the conclusion.

### 1.1 Proposed approach

m-matrix three dimensional bipolar code is an enhancement of the one dimensional m-sequence (matrix) bipolar code (Park et al. 2004). The code construction achieved by extending the one dimensional Bipolar m-matrix code in spectral, time spreading and spatial domain. The code is expressed by the \((M \times N \times P, K_1, K_2, K_3, \lambda_a, \lambda_b)\). Where \(M\) is the number of wavelengths, \(N\) is temporal length and \(P\) is the spatial length. \(K_1, K_2\) and \(K_3\) are their corresponding weight numbers and \(\lambda_a, \lambda_b\) are the auto and cross correlations.

\[
K_1 = \frac{(m+1)}{2} = \frac{M}{2}, \quad K_2 = \frac{(n+1)}{2} = \frac{N}{2}, \quad K_3 = \frac{(p+1)}{2} = \frac{P}{2}
\]

\[M = m + 1, \quad N = n + 1, \quad P = p + 1\]

m-matrix three dimensional bipolar code can be constructed using the three code sequence \((m, n\) and \(p)\) of one dimensional Bipolar code (Ahmed et al. 2013). \(X = [x_0 x_1 \ldots \ldots x_{M-1}]\), \(X = [\bar{x}_0 \bar{x}_1 \ldots \ldots \bar{x}_{M-1}]\), \(Y = [y_0 y_1 \ldots \ldots y_{N-1}]\), \(Y = [\bar{y}_0 \bar{y}_1 \ldots \ldots \bar{y}_{N-1}]\) and \(Z = [z_0 z_1 \ldots \ldots z_{P-1}]\), \(Z = [\bar{z}_0 \bar{z}_1 \ldots \ldots \bar{z}_{P-1}]\) represents the three \((m, n\) and \(p)\)-sequence bipolar codes. As table.1 shows the basic three dimensional code construction of m-sequence code for bipolar modes of 000 dimensions similar way codes in other dimensions can be extended. M, N and P are the length of codes in one dimensional bipolar m, n and p-matrix code.
Table 1  Code construction of 3D bipolar code

|       |       |       |       |
|-------|-------|-------|-------|
| $Y_0 = [11100100]$ | $Z_0 = [1100]$ | $Y_0 = [00011011]$ | $Z_0 = [0011]$ |
| $X_0^T$ | $A_{0,0,0}$ | $\bar{X}_0^T$ | $\bar{A}_{0,0,0}$ |
| 1 | 11001000 11001000 00000000 00000000 | 0 | 00000000 00000000 00000000 00000000 |
| 1 | 11001000 11001000 00000000 00000000 | 0 | 00000000 00000000 00000000 00000000 |
| 1 | 00000000 00000000 00000000 00000000 | 0 | 00000000 00000000 00000000 00000000 |
| 0 | 11001000 11001000 00000000 00000000 | 0 | 00000000 00000000 00000000 00000000 |
| 0 | 00000000 00000000 00000000 00000000 | 1 | 00011011 00011011 |
| 1 | 00000000 00000000 00000000 00000000 | 1 | 00011011 00011011 |
| 0 | 00000000 00000000 00000000 00000000 | 1 | 00011011 00011011 |
| 0 | 00000000 00000000 00000000 00000000 | 1 | 00011011 00011011 |
For expression of cross correlation of this three dimensional bipolar code the characteristics matrix $A^{(d)}$ for $d \in (0, 1, ..., 7)$ are given by $A^{(0)} = X^T YZ$, $A^{(1)} = X^T YZ$, $A^{(2)} = \bar{X} YZ$, $A^{(3)} = \bar{X} YZ$. $A^{(4)} = X^T \bar{Y}Z$, $A^{(5)} = X^T \bar{Y}Z$, $A^{(6)} = X^T Y\bar{Z}$, $A^{(7)} = X^T Y\bar{Z}$. where $\bar{X}$, $\bar{Y}$ and $\bar{Z}$ denotes the complementary code sequence of X, Y and Z.

The cross correlation between the $A^{(d)}$ and $A_{g,h,l}$ is defined as

$$ R^{(d)} = \sum_{k=0}^{P-1} \sum_{j=0}^{M-1} a_{i,j,k} A^{(d)}(g, h, l) $$

(1)

In the similar manner cross correlation between the $A^{(d)}$ and $A_{g,h,l}$ is defined as

$$ \bar{R}^{(d)} = \sum_{k=0}^{P-1} \sum_{j=0}^{M-1} \sum_{i=0}^{N-1} a_{i,j,k}\bar{A}^{(d)}(g, h, l) $$

(2)

where $a_{i,j,k}$ is the (i,j,k)th entry of $A^{(d)}$, $A_{i,j,k}(g, h, l)$ is the (i,j,k)th entry of $A_{g,h,l}$ and $\bar{A}_{i,j,k}(g, h, l)$ is the (i,j,k)th entry of $A_{g,h,l}$ with $g = 0, 1, 2, ..., M - 1$, $h = 0, 1, 2, ..., N - 1$ and $l = 0, 1, 2, ..., P - 1$. The $A_{g,h,l}$ is code corresponds to ‘1’ bit and $A_{g,h,l}$ denotes the code corresponds to ‘0’ information bits for user $(g, h, l)$.

The cross correlation of $A^{(d)}$ and $A_{g,h,l}$ and $A^{(d)}$ and $A_{g,h,l}$, shown in Tables 2 and 3.

For analysing the MUI cancellation property in ‘1’ and ‘0’ information bit, Tables 2 and 3 incorporated. From Table 2, Where $R^{(3)}(g, h, l)$ has non zero value only when $g \neq 0 \land h \neq 0$. $R^{(3)}(g, h, l)$ is used to eliminate the influence caused by $A_{g,h,l}$ from $R^{(0)}(g, h, l)$, $R^{(1)}(g, h, l)$ and $R^{(2)}(g, h, l)$ with $g \neq 0 \land h \neq 0$. The new cross correlation is defined by

$$ R^{0}(g, h) - R^{1}(g, h) - R^{2}(g, h) + R^{3}(g, h) = \begin{cases} K_1 K_2 K_3 \frac{1}{2} & \text{for } g = 0 \land h = 0 \land l = 0 \\ K_1 K_2 K_3 \frac{1}{2} & \text{for } g = 0 \land h = 0 \land l \neq 0 \\ 0 & \text{otherwise} \end{cases} $$

(3)

Similarly from the Table 3, the new cross correlation is given by

| Table 2 | Cross correlation of 3D m-sequence bipolar code corresponds to information ‘1’ and ‘0’bit |
|--------|---------------------------------------------------------------|
| $R^{0}(g, h, l)$ | $R^{0}(g, h, l)$ | $R^{1}(g, h, l)$ | $R^{1}(g, h, l)$ | $R^{2}(g, h, l)$ | $R^{2}(g, h, l)$ | $R^{3}(g, h, l)$ | $R^{3}(g, h, l)$ |
| $g=0 \land h=0 \land l=0$ | $K_1 K_2 K_3$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $g=0 \land h \neq 0 \land l=0$ | $\frac{1}{2} K_1 K_2 K_3$ | $K_1 K_2 K_3$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $g \neq 0 \land h=0 \land l=0$ | $\frac{1}{2} K_1 K_2 K_3$ | 0 | 0 | 0 | $K_1 K_2 K_3$ | 0 | 0 | 0 |
| $g \neq 0 \land h \neq 0 \land l=0$ | $\frac{1}{2} K_1 K_2 K_3$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $g=0 \land h=0 \land l \neq 0$ | $\frac{1}{2} K_1 K_2 K_3$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $g=0 \land h \neq 0 \land l \neq 0$ | $\frac{1}{2} K_1 K_2 K_3$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $g \neq 0 \land h=0 \land l \neq 0$ | $\frac{1}{2} K_1 K_2 K_3$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $g \neq 0 \land h \neq 0 \land l \neq 0$ | $\frac{1}{2} K_1 K_2 K_3$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
Table 2 and Table 3 can be used to eliminate the MAI caused by ‘0’ information bits. In the similar manner the Table 3 defines the new cross correlation is given as

\[
R^4(g, h) - R^5(g, h) - R^6(g, h) + R^7(g, h) = \begin{cases} 
0 & \text{for } g = 0 \ h = 0 \ l = 0 \\
\frac{K_1K_2K_3}{2} & \text{for } g = 0 \ h = 0 \ l \neq 0 \\
0 & \text{otherwise}
\end{cases}
\]

(4)

The final MUI cancellation is performed with the Eqs. 3 and 4 then the new cross correlation equation is

\[
= \begin{cases} 
K_1 K_2 K_3 & \text{for } g = 0 \ h = 0 \ l = 0 \\
0 & \text{otherwise}
\end{cases}
\]

(5)

In the similar way, from the above mentioned cross correlation cancellation method, Table 2 and Table 3 can be used to eliminate the MAI caused by ‘0’ information bits. In the similar way, from the above mentioned cross correlation cancellation method, Table 2 and Table 3 can be used to eliminate the MAI caused by ‘0’ information bits. In Table 2, \( \overline{R}^5(g, h, l) \) is used to eliminate the influence caused by \( \overline{A}^0_{(g,h,l)} \) from \( \overline{R}^0(g, h, l) \). \( \overline{R}^5(g, h, l) \) and \( \overline{R}^7(g, h, l) \) with \( g \neq 0 \ h \neq 0 \). The new cross correlation is defined by

\[
\overline{R}^0(g, h) - \overline{R}^1(g, h) - \overline{R}^2(g, h) + \overline{R}^3(g, h) = \begin{cases} 
0 & \text{for } g = 0 \ h = 0 \ l = 0 \\
\frac{K_1K_2K_3}{2} & \text{for } g = 0 \ h = 0 \ l \neq 0 \\
0 & \text{otherwise}
\end{cases}
\]

(6)

Similar manner the Table 3 defines the new cross correlation is given as
Than from the Eqs. 6 and 7 the new cross correlation is given as

\[
\begin{align*}
\bar{R}^1(g,h) - \bar{R}^5(g,h) - \bar{R}^6(g,h) + \bar{R}^7(g,h) &= \begin{cases} 
K_1K_2K_3 & \text{for } g = 0 \ h = 0 \ l = 0 \\
\frac{K_1K_3}{2} & \text{for } g = 0 \ h = 0 \ l \neq 0 \\
0 & \text{otherwise}
\end{cases} \\
= \begin{cases} 
-K_1K_2K_3 & \text{for } g = 0 \ h = 0 \ l = 0 \\
0 & \text{otherwise}
\end{cases}
\end{align*}
\]
Fig. 2 Architecture of encoder design of 3D m-sequence Bipolar code

Fig. 3 Architecture of decoder design of 3D m-sequence Bipolar code
FBGt2 with equal gratings number to reflect back the similar spectral components for run trip delays compensation.

Initially optical source spectrum reaches to the any one encoder by the incoming information bit 0 or 1. If incoming information bit is 1 then upper encoder is selected otherwise lower encode is allocated for 0 bit. In selected encoder, the optical pulse spectrally encoded by the two set of FBGs which generates the code sequence $X_g$ ($\overline{X}_g$, in case of 0 bit). After the spectral encoding, optical delay lines provide the time spreading sequence $Y_h$ ($\overline{Y}_h$, in case of 0 bit). Then finally for spatial encoding purpose optical spectrum process to the splitters and switches which corresponds to the $Z_l$ (in case of 0 bit).

The Fig. 3 shows the structure of receiver. The receiver consists the two optical combiners, four correlators, four balanced detector with each having two FBGs and photodiode, and an integrator.

Optical combiner combines the incoming signal and first decorrelate the spatial code sequence $Z_l$ and complementary code sequence $\overline{Z}_l$. The output of the optical combiner process to the correlators which is a design of optical delay line corresponds to the code sequence $Y_h$ and $\overline{Y}_h$. The correlators 1 and correlator 3 decode the code sequence $Y_h$ and The correlator 2 and correlator 4 decode the code sequences $\overline{Y}_h$. All correlators acquires the K2 optical delay lines. Finally for decoding of spectral component, each balanced detector consists the two set of FBGs and two p-i-n diode. FBGr1 to FBGs8 follows the same number of gratings and it can be utilized to reflect back similar spectral components which corresponds to the $X_g$. Again in order to compensate the different round trip delays of the matched spectral components, the position of gratings in FBGs 0, 1, 2 and 4 are similar but contrary to that in FBGs 3, 5, 6 and 7.

The operation of receiver is described. At the receiver side, the upper combiner receives the signal corresponds to the spatial code $\overline{Z}_l$. The lower optical combiner is located for receiving $Z_l$ for providing the cancellation information.

Optical signal from two combiners, split in two. Upper combiner carries the signal to the 1 and 2 correlator and lower combiner carries the signal to the 3 and 4 correlator.

If $Y_h$ is [11101000] the length of delay line 1, 2, 3 and 4 are designed to delay the optical signal first, second, third and five times of the chip duration. In similarly the delay lines 5–8 are designed to provide the delay in the optical signal for four, six, seven and eight times of the chip duration represent the ‘0’ alignment. Each balanced detector consists the two set of FBGs. The function of balanced detector is explained as. The output signal from correlator 1 split in two part and signal delivered to the FBGr1 of the upper balanced detector and this signal reflects back to code corresponds to the 1 s code sequence. The matched signal is also passed to FBG2 for delay compensation and reaches to PD0. The unmatched section is processed to the PD2 via delay lines. The delay line is applied to compensate the delay due to FBGr1 and FBG2.

The signal portion matched to the time spreading, spatial and spectral code sequence of the receiver can reach to PD0, the output current is proportion to the $R^{(0)}(g, h, l)$ for ‘1’ information bit and $R^{(0)}(g, h, l)$ for ‘0’ information bit. Similarly The output current from PD1 to the PD7 are proportional to $R^{(1)}(g, h, l), R^{(2)}(g, h, l), R^{(3)}(g, h, l), R^{(4)}(g, h, l), R^{(5)}(g, h, l), R^{(6)}(g, h, l)$ and $R^{(7)}(g, h, l)$ for ‘1’ information bit and $\overline{R}^{(1)}(g, h, l), \overline{R}^{(2)}(g, h, l), \overline{R}^{(3)}(g, h, l), \overline{R}^{(4)}(g, h, l), \overline{R}^{(5)}(g, h, l), \overline{R}^{(6)}(g, h, l)$ and $\overline{R}^{(7)}(g, h, l)$ for ‘0’ information bit. So output current generated from four balanced detectors for ‘1’ information and ‘0’ information bit are proportional to $R^{(0)}(g, h, l) - R^{(2)}(g, h, l), R^{(1)}(g, h, l) - R^{(3)}(g, h, l), R^{(4)}(g, h, l) - R^{(6)}(g, h, l), R^{(5)}(g, h, l) - R^{(7)}(g, h, l)$ and $\overline{R}^{(0)}(g, h, l) - \overline{R}^{(2)}(g, h, l), \overline{R}^{(1)}(g, h, l) - \overline{R}^{(3)}(g, h, l), \overline{R}^{(4)}(g, h, l) - \overline{R}^{(6)}(g, h, l), \overline{R}^{(5)}(g, h, l) - \overline{R}^{(7)}(g, h, l)$. Finally
the generated output currents after eliminating the MUI is proportional to the 
\[ R^0(g, h) - R^1(g, h) - R^2(g, h) + R^3(g, h) - [R^4(g, h) - R^5(g, h) - R^6(g, h) + R^7(g, h)] \] and 
\[ \overline{R^0}(g, h) - \overline{R^1}(g, h) - \overline{R^2}(g, h) + \overline{R^3}(g, h) - [\overline{R^4}(g, h) - \overline{R^5}(g, h) - \overline{R^6}(g, h) + \overline{R^7}(g, h)] \] corresponds to the ‘1’ and ‘0’ information bit.

2 Mathematical analysis

Gaussian approximation is considered for analysis (Prucnal et al. 1986; Anuar et al. 2000). PIIN noise, thermal noise and shot noise are acquired for calculation. The SNR for electrical signal is the average signal power to noise power, SNR = [I/R]. The following assumptions are made.

1. Each light source spectrum is flat over the bandwidth \([V_o-\Delta V/2, V_o-\Delta V/2]\) where \(V_o\) is central frequency and \(\Delta V\) is the optical source bandwidth in Hertz.
2. Each power spectral component has an identical spectral width.
3. Each user has nearly equal power at the transmitter.
4. Each user bit stream is synchronized.

The power spectral density (PSD) of the received signals can be given

\[
r(f) = \frac{P_{sr}}{K_2K_3\Delta f} \sum_{w=1}^{W} d(w) \sum_{k=0}^{P-1} \sum_{j=0}^{N-1} \sum_{i=0}^{M-1} a_{ij,k}(W) \times F(f, i) \quad (9)
\]

For convenience analysis we define the \(F(f, i)\) as

\[
F(f, i) = \left\{ u\left[f - f_0 - \frac{\Delta f (-M + 2i)}{2M}\right] - u\left[f - f_0 - \frac{\Delta f (-M + 2i - 2)}{2M}\right]\right\} \quad (10)
\]

In case of information ‘1’ bit

The output current of photodiode PD0 to PD7 which suffer due to cross correlation between the \(A^{(d)}\) and \(A_{g,h,l}^{(d)}\) is define as

\[
I_0(f) = \Re \int_0^{\infty} G_0(f)df = \int_0^{\infty} \frac{\Re P_{sr}}{K_2K_3\Delta f} \sum_{w=1}^{W} d(w) R^0(i, j, l)F(f, i)
\]

From Eq. 10 and Table 2

\[
\Delta = \frac{K_1K_2K_3(W - 1)}{(MNP - 1)}
\]

\[
= \frac{\Re P_{sr}}{K_2K_3(MNP)} \left\{ \frac{K_1K_2}{2} + \frac{\Delta(N-1)}{2} + \frac{\Delta(M-1)}{2} + \frac{\Delta(M-1)(N-1)}{4} + \frac{\Delta(P-1)}{2} + \frac{\Delta(P-1)(N-1)}{4} + \frac{\Delta(M-1)(P-1)}{4} \right\}
\]

\[
I_1(f) = \Re \int_0^{\infty} G_1(f)df = \int_0^{\infty} \frac{\Re P_{sr}}{K_2K_3\Delta f} \sum_{w=1}^{W} d(w) R^1(i, j, l)F(f, i)
\]
\[
I_2(f) = \Re \int_0^\infty G_2(f)df = \int_0^\infty \frac{\mathcal{R}P_{sr}}{K_2K_3\Delta f} \times \sum_{w=1}^w d(w)R^2(i,j,l)F(f,i)
\]

\[
I_3(f) = \Re \int_0^\infty G_3(f)df = \int_0^\infty \frac{\mathcal{R}P_{sr}}{K_2K_3\Delta f} \times \sum_{w=1}^w d(w)R^3(i,j,l)F(f,i)
\]

\[
I_4(f) = \Re \int_0^\infty G_4(f)df
\]

\[
I_5(f) = \Re \int_0^\infty G_5(f)df = \int_0^\infty \frac{\mathcal{R}P_{sr}}{K_2K_3\Delta f} \times \sum_{w=1}^w d(w)R^5(i,j,l)F(f,i)
\]

\[
I_6(f) = \Re \int_0^\infty G_6(f)df = \int_0^\infty \frac{\mathcal{R}P_{sr}}{K_2K_3\Delta f} \times \sum_{w=1}^w d(w)R^6(i,j,l)F(f,i)
\]

\[
I_7(f) = \Re \int_0^\infty G_7(f)df = \int_0^\infty \frac{\mathcal{R}P_{sr}}{K_2K_3\Delta f} \times \sum_{w=1}^w d(w)R^7(i,j,l)F(f,i)
\]

The average photocurrent output at the receiver due to ‘1’information bit

\[
I_r = \int_0^\infty \{[G_0(f) - G_2(f)] - [G_1(f) - G_3(f)] - [G_4(f) - G_6(f)] - [G_5(f) - G_7(f)]\}
\]
Design and analysis of three-dimensional m-sequence bipolar…

In case of information ‘0’ bit

The output current of photodiode PD0 to PD7 which suffer due to cross correlation between the $A^{(d)}$ and $A_{g.h,f}$, is define as, from Eqs. 9 and 10 and Table 3.

$$I_r = |I_0(f) - I_2(f)| - |I_1(f) - I_3(f)| - |I_4(f) - I_6(f)| - |I_5(f) - I_7(f)|$$

$$\begin{align*}
I'_0(f) &= \mathcal{R} \int_0^\infty G'_0(f) df \\
&= \frac{\mathcal{R} \rho_{sr}}{K_2K_3M} \left\{ \frac{\Delta(M-1)(N-1)(P-1)}{8} \right\}
\end{align*}$$

$$\begin{align*}
I'_1(f) &= \mathcal{R} \int_0^\infty G'_1(f) df = \int_0^\infty \frac{\mathcal{R} \rho_{sr}}{K_2K_3M} \times \sum_{w=1}^{\infty} d(w) \overline{R}^1(i,j,l) F(f,i) \\
&= \frac{\mathcal{R} \rho_{sr}}{K_2K_3M} \left\{ \frac{\Delta(M-1)(P-1)}{4} + \frac{\Delta(M-1)(N-1)(P-1)}{8} \right\}
\end{align*}$$

$$\begin{align*}
I'_2(f) &= \mathcal{R} \int_0^\infty G'_2(f) df = \int_0^\infty \frac{\mathcal{R} \rho_{sr}}{K_2K_3M} \times \sum_{w=1}^{\infty} d(w) \overline{R}^2(i,j,l) F(f,i) \\
&= \frac{\mathcal{R} \rho_{sr}}{K_2K_3M} \left\{ \frac{\Delta(N-1)(P-1)}{4} + \frac{\Delta(M-1)(N-1)(P-1)}{8} \right\}
\end{align*}$$

$$\begin{align*}
I'_3(f) &= \mathcal{R} \int_0^\infty G'_3(f) df = \int_0^\infty \frac{\mathcal{R} \rho_{sr}}{K_2K_3M} \times \sum_{w=1}^{\infty} d(w) \overline{R}^3(i,j,l) F(f,i) \\
&= \frac{\mathcal{R} \rho_{sr}}{K_2K_3M} \left\{ \frac{\Delta(P-1)}{2} + \frac{\Delta(N-1)(P-1)}{4} + \frac{\Delta(M-1)(P-1)}{4} + \frac{\Delta(M-1)(N-1)(P-1)}{8} \right\}
\end{align*}$$

$$\begin{align*}
I'_4(f) &= \mathcal{R} \int_0^\infty G'_4(f) df = \int_0^\infty \frac{\mathcal{R} \rho_{sr}}{K_2K_3M} \times \sum_{w=1}^{\infty} d(w) \overline{R}^4(i,j,l) F(f,i) \\
&= \frac{\mathcal{R} \rho_{sr}}{K_2K_3M} \left\{ \frac{\Delta(M-1)(N-1)}{4} + \frac{\Delta(M-1)(N-1)(P-1)}{8} \right\}
\end{align*}$$

$$\begin{align*}
I'_5(f) &= \mathcal{R} \int_0^\infty G'_5(f) df = \int_0^\infty \frac{\mathcal{R} \rho_{sr}}{K_2K_3M} \times \sum_{w=1}^{\infty} d(w) \overline{R}^5(i,j,l) F(f,i) \\
&= \frac{\mathcal{R} \rho_{sr}}{K_2K_3M} \left\{ \frac{\Delta(M-1)}{2} + \frac{\Delta(M-1)(N-1)}{4} + \frac{\Delta(M-1)(P-1)}{4} + \frac{\Delta(M-1)(N-1)(P-1)}{8} \right\}
\end{align*}$$
The average photocurrent output at the receiver due to ‘0’ information bit

\[ I'_6(f) = \mathcal{R} \int_0^\infty G'_6(f)df = \int_0^\infty \frac{\mathcal{R}P_{sr}}{K_2K_3\Delta f} \times \sum_{i=1}^w d(w)R^6(i,j,l)F(f,i) \]

\[ = \frac{\mathcal{R}P_{sr}}{K_2K_3M} \left\{ \frac{\Delta(N-1)}{2} + \frac{\Delta(M-1)(N-1)}{4} + \frac{\Delta(M-1)(P-1)}{4} + \frac{\Delta(M-1)(N-1)(P-1)}{8} \right\} \]

\[ I'_7(f) = \mathcal{R} \int_0^\infty G'_7(f)df = \int_0^\infty \frac{\mathcal{R}P_{sr}}{K_2K_3\Delta f} \times \sum_{i=1}^w d(w)R^7(i,j,l)F(f,i) \]

\[ = \frac{\mathcal{R}P_{sr}}{K_2K_3M} \left\{ \frac{K_1K_3 + \Delta(N-1)}{2} + \frac{\Delta(M-1)}{2} + \frac{\Delta(M-1)(N-1)}{4} + \frac{\Delta(M-1)(P-1)}{2} + \frac{\Delta(N-1)(P-1)}{4} + \frac{\Delta(M-1)(P-1)}{4} \right\} \]

The average photocurrent output at the receiver due to ‘0’ information bit

\[ I'_r = \int_0^\infty \{ [G'_0(f) - G'_2(f)] - [G'_1(f) - G'_3(f)] \} - \{ [G'_4(f) - G'_6(f)] - [G'_5(f) - G'_7(f)] \} \]

\[ I'_r = [I'_0(f) - I'_2(f)] - [I'_1(f) - I'_3(f)] - [I'_4(f) - I'_6(f)] - [I'_5(f) - I'_7(f)] \]

The resulted current at the receiver end is given as

\[ I_{Bipolar} = I_r - I'_r \]

\[ I_{Bipolar} = \frac{2\mathcal{R}P_{sr}K_1}{M} \] (11)

Noise can be given as

\[ \langle \dot{i}^2 \rangle_{\text{noise}} = \langle \dot{i}^2_{\text{PIN}} \rangle + \langle \dot{i}^2_{\text{shot}} \rangle + \langle \dot{i}^2_{\text{thermal}} \rangle \]

\[ \dot{i}^2 = \dot{i}^2 B_r \tau_c + 2eB_rI_r + \frac{4K_tT_nB_r}{R_L} \] (12)

\[ \tau_c = \frac{\int_0^\infty [S^2(f)]df}{\left[ \int_0^\infty S(f)df \right]^2} \]

where \( G(f) \) is the power spectral density (PSD) of the light incident to the photodiode. Then PIIN noise is given as
Design and analysis of three-dimensional m-sequence bipolar...

\[
\langle i_{\text{PIN}}^2 \rangle = B_i R^2 \int_0^{\infty} \left( G_0^2(f) + G_1^2(f) + G_2^2(f) + G_3^2(f) + G_4^2(f) + G_5^2(f) + G_6^2(f) - 2G_0(f)G_1(f) - 2G_0(f)G_4(f) \right) \, df \\
+ 2G_0(f)G_5(f) + 2G_1(f)G_4(f) - 2G_2(f)G_4(f) - 2G_2(f)G_5(f) - 2G_3(f)G_4(f) - 2G_3(f)G_5(f) - 2G_4(f)G_5(f)
\]

Equation (13)

\[
\int_0^{\infty} G_0^2(f) df = \frac{P^2_{sr}}{k^2_{2}K^2_{2}f^2} \int_0^{\infty} \left[ \sum_{w=1}^{\infty} R^0(i,j,l)F(f,i) \right]^2 \, df = \frac{M_0^2}{K^1_{1}f R^2}
\]

In the similar way the remaining similar terms are given as.

\[
\int_0^{\infty} G_1^2(df) = \frac{M_1^2}{K^1_{1}f R^2}, \quad \int_0^{\infty} G_2^2(df) = \frac{M_2^2}{K^1_{1}f R^2}, \quad \int_0^{\infty} G_3^2(df) = \frac{M_3^2}{K^1_{1}f R^2},
\]

\[
\int_0^{\infty} G_4^2(df) = \frac{M_4^2}{K^1_{1}f R^2}, \quad \int_0^{\infty} G_5^2(df) = \frac{M_5^2}{K^1_{1}f R^2}, \quad \int_0^{\infty} G_6^2(df) = \frac{M_6^2}{K^1_{1}f R^2}, \quad \int_0^{\infty} G_7^2(df) = \frac{M_7^2}{K^1_{1}f R^2}
\]

now the another term can be given as

\[
\int_0^{\infty} G_0(f)G_1(df) = \frac{P^2_{sr}}{k^2_{2}K^2_{2}f^2} \int_0^{\infty} \left[ \sum_{w=1}^{\infty} R^0(i,j,l)F(f,i) \right] \times \left[ \sum_{w=1}^{\infty} R^0(i,j,l)F(f,i) \right] \, df = \frac{M_0^2}{K^1_{1}f R^2}
\]

As same as other similar terms are given as.

\[
\int_0^{\infty} G_0(f)G_4(df) = \frac{M_0^2}{K^1_{1}f R^2}, \quad \int_0^{\infty} G_0(f)G_5(df) = \frac{M_0^2}{K^1_{1}f R^2}, \quad \int_0^{\infty} G_0(f)G_6(df) = \frac{M_0^2}{K^1_{1}f R^2}, \quad \int_0^{\infty} G_0(f)G_7(df) = \frac{M_0^2}{K^1_{1}f R^2}
\]

\[
\int_0^{\infty} G_1(f)G_5(df) = \frac{M_1^2}{K^1_{1}f R^2}, \quad \int_0^{\infty} G_1(f)G_6(df) = \frac{M_1^2}{K^1_{1}f R^2}, \quad \int_0^{\infty} G_1(f)G_7(df) = \frac{M_1^2}{K^1_{1}f R^2}
\]

\[
\int_0^{\infty} G_2(f)G_6(df) = \frac{M_2^2}{K^1_{1}f R^2}, \quad \int_0^{\infty} G_2(f)G_7(df) = \frac{M_2^2}{K^1_{1}f R^2}, \quad \int_0^{\infty} G_2(f)G_7(df) = \frac{M_2^2}{K^1_{1}f R^2}
\]

\[
\int_0^{\infty} G_3(f)G_7(df) = \frac{M_3^2}{K^1_{1}f R^2}, \quad \int_0^{\infty} G_3(f)G_7(df) = \frac{M_3^2}{K^1_{1}f R^2}
\]

Then from Eq. 13 and above terms the PIIN noise now can be expressed as

\[
\langle i_{\text{PIN}}^2 \rangle = B_i R^2 \left[ \frac{I_1^2 + I_2^2 + I_3^2 + I_4^2 + I_5^2 + I_6^2 + I_7^2 - 2I_1I_4 - 2I_1I_6 + 2I_1I_4 + 2I_1I_6 - 2I_1I_4}{M_{1}} \right]
\]

\[
= \frac{-2I_1I_4 - 2I_1I_6 + 2I_1I_4 + 2I_1I_6 - 2I_1I_4 - 2I_1I_6}{M_{1}}
\]
\[
\langle \tilde{i}^2_{PIN} \rangle = \frac{MB_r}{K_1 \Delta f} \left( (I_0 - I_1 - I_4 + I_5)^2 + (I_2 - I_3 - I_6 + I_7)^2 \right) \quad (14)
\]

\[
\langle \tilde{i}^2_{shot} \rangle = 2eB_r I_{Total}
\]

\[
\langle \tilde{i}^2_{thermal} \rangle = \frac{4K_0 T_n B_r}{R_L}
\]

After placing all current in Eq. 14.

\[
\langle \tilde{i}^2_{shot} \rangle = \frac{2eB_r}{K_2 K_3 M} \left\{ \frac{K_1 K_2 K_3}{4} + \frac{2\Delta(N-1)}{2} + \frac{2\Delta(M-1)}{2} + \frac{4\Delta(M-1)(N-1)}{4} \right\}
\]

Then SNR is given as, from Eqs. 11, 12, 13 and 14

\[
SNR = \frac{\left( \frac{2R_P u K_1}{M} \right)^2}{\frac{MB_r}{K_1 \Delta f} \left( (I_0 - I_1 - I_4 + I_5)^2 + (I_2 - I_3 - I_6 + I_7)^2 \right) + 2eB_r (I_0 + I_1 + I_2 + I_3 + I_4 + I_5 + I_7) + \frac{4K_0 T_n B_r}{R_L}}
\]

Following parameter are considered during the analysis.

Photo detector quantum efficiency (η) 0.75.
Line-width broadband source (ΔV) 3.75 THz.
Operating wavelength (k_o) 1552 nm.
Data bit rate (R_b) 1Gbps.
Receiver noise temperature (T_n) 300 K.

3 Result and analysis

System capabilities are analyzed by observing the Bit error rate and SNR with the capacity of users and sustainable received power.

Fig. 4 Variation in BER with number of users
3.1 BER variation with users

Figure 4 shows the three dimensional m-sequence (matrix) bipolar code performance of BER by bit error rate variation with the number of simultaneous users for $M = 8$, $N = 16$, $P = 4$ ($m = 7$, $n = 15$, $p = 3$). The observed result in proposed System is much better and it can be verified by the outputs where the increment in BER for three dimensional m-matrix bipolar code is less as compare to the reported results. The users which can be accepted in system at $10^{-12}$ BER are 310. so system is capable to handle much more users than the 3D PASACAL code ($M = 8$, $N = 16$, $P = 4$), 3D-PD ($M = 7$, $N = 21$, $P = 3$), 2D-m matrix ($M = 127$, $N = 3$) code where number of users can be accommodated in 3D PASACAL code ($M = 8$, $N = 16$, $P = 4$), 3D-PD ($M = 7$, $N = 21$, $P = 3$), 2D-m matrix ($M = 127$, $N = 3$) are 117, 80 and 63 as shown in Fig. 4. For 120 users in system the measured BER is $10^{-37}$ and BER in other reported system such as 3D PASACAL ($M = 8$, $N = 16$, $P = 4$), 3D-PD ($M = 7$, $N = 21$, $P = 3$) and 2D-m matrix ($M = 127$, $N = 3$) Code are $10^{-14}$, $10^{-10}$ and $10^{-04}$.

Figure 5 discussed the BER performance criteria with sets of received power for constant users. For $10^{-12}$ BER, proposed system stands with $-5.2$ dBm less power than the 3D PASACAL code and $-7.3$ dBm lesser power as compare to 3D PD Code. If system is used at the $-15$ dBm received power then system shows the $10^{-21}$ BER and 3D PASACAL code and 3D PD give the $10^{-09}$ and $10^{-04}$ BER.
Current age technology demands the high data rate transmission so that system should support the high data rate transmission. As Fig. 6 shows that the system can acquire the 120 uses at 2.5Gbit/s data rate for $10^{-25}$ BER and for similar users and same (2.5Gbits/s) data rate the BER achieved in 3D PASCAL code is $10^{-07}$ and in 3D PD code is $10^{-02}$. For $10^{-24}$ BER in proposed system possible data rate is 2.5Gbits/s and in PASCAL code for similar BER, data rate is 0.5Gbits/s.

The analysis of spectral band-width is also a parameter in optical system. Analysis of spectral width variation with BER is shown in Fig. 7. For 4Thz Spectral Width, the BER $10^{-59}$ in 3D-m matrix bipolar code (M = 8, N = 16, P = 4) has been achieved and, in 3D PASCAL (M = 8, N = 16, P = 4), 3D-PD (M = 7, N = 21, P = 3) and 2D-m matrix (M = 127, N = 3) code was $10^{-14}$, $10^{-04}$ and $10^{-03}$. It has noticed that spectral width parameter shows the much improvement in suggested system as compare to published results.

4 Conclusion

The suggested approach elaborated the three-dimensional m-sequence (matrix) code in a bipolar mode is much better against the eavesdropping situation due to the property of bipolar coding system. Extension of one-dimensional m-sequence (matrix) bipolar code into spectral, time spreading and spatial domain was carried for the proposed three-dimensional method. Results are obtained for various numbers of users, received power, data rates and spectral width. The proposed work shows the larger cardinality and lower effective transmitted power than the existing three dimensional and two dimensional techniques. Moreover, it is seen that the lower effective received power ($-17.4$ dBm) is achieved to acquire the BER of $10^{-12}$ through the proposed system. Analysis of results shows that the system is capable of handling more than 240 users at $10^{-18}$ BER at the data rate of 1.0 Gbits/s and $-10$ dBm received power, which is much better than the reported methods.

Funding The authors have not disclosed any funding.

Declarations

Conflict of interest The authors have not disclosed any competing interests.
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