Extended Brown-Rho Scaling Law with QCD
Sum Rules

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Abstract

The scalar and vector self-energies obtained through QCD sum rules are introduced
in Quantum Hadrodynamics (QHD) equations. The results indicate that the ratios
of coupling constants to respective meson mass have a very small dependence on
density. Then, an extended Brown-Rho scaling law is conjectured.

Key words: Nuclear Matter, QCD sum rules and Self-energies
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In recent years the density dependence of meson masses and meson-baryon
coupling constants have received increasing interest [1–4]. Medium modification
of hadron properties can be seen as a manifestation of quark substructure.
One of the principal reasons to study that topic is the success of Quantum
Hadrodynamics (QHD) [5,6] in the relativistic description of nuclear phenomena.
QHD has large success in the explanation of properties of infinite nuclear
matter as well as finite nuclei. Several calculations of nuclear structure using
QHD and its extensions were made obtaining a good agreement with experimental data [6,7].

QHD describes the N-N (nucleon-nucleon) interaction through mesons exchange (π, σ, ω, ρ, etc.). However, for the purposes of this work, only the
simplest QHD model, namely QHD-I, which includes nucleons (ψ) coupled
with sigma (σ) and omega (ω) mesons, will be considered. In spite of the pion
to be the principal component of the N-N interaction, it does not play any
role in the model because nuclear matter is an isotropic system with parity
conservation. Analyzing the model, it is possible to verify that the real part
of the scalar term (σ-meson) is typically of the order of several hundred MeV
attractive while the real part of the time component of the vector term ($\omega$-meson) is typically of the order of several hundred MeV repulsive. However, the energies involved in problems of nuclear structure are only of a few tens of MeV. An energy of that order is obtained in QHD models due to a large cancellation between the scalar and vector pieces. In fact, scalar mesons produce a long range attractive potential which saturate due to the presence of a repulsive short range potential produced by vector mesons. This process of saturation can be controlled through an appropriate choice of coupling constants. However, there are strong indications that such constants, as well as meson masses, change with the density [1,2]. So, the success of QHD, which uses fixed values for these amounts, must be explained.

Recent results of the effective field theory (EFT) [6,8–12] have verified that QHD models are consistent with symmetries of Quantum Chromodynamics (QCD). Motivated by that, it will be very interesting to make a connection between QHD and QCD with the intention of studying coupling constants and meson masses in the medium. The natural way to make that connection take into account the nucleon self-energy. That idea was applied by Brockmann and Toki [2] to connect Relativistic Bruckner-Hartree-Fock Theory and Relativistic Hartree Approach. However, as it is known, the energies involved in nuclear matter problems are very lower than the energy scale of perturbative QCD. Thus, it is necessary to use a nonperturbative procedure. The indicated is the well-known method of QCD sum rules, that was introduced by Shifman, Vainshtein, and Zakharov in the late 1970’s [13]. The QCD sum rule approach was successfully used in a lot of different problems, since hadron properties to particle decay. For the purposes of this work, the most interesting use of QCD sum rules is the possibility of describing properties of a nucleon in the medium. Indeed, QCD sum rules succeed to obtain the nucleon self-energy in terms of quarks and gluons degree of freedom [14–18]. On the other hand, self-energies obtained in QHD models are dependent on coupling constants and meson masses. The objective of this work is to study these hadronic parameters of a more fundamental point of view.

To study meson-nucleon coupling constants and meson masses in nuclear matter, it is first necessary to solve the nuclear ground state within Relativistic Hartree Approximation. The energy density in the QHD-I model can be written as [5]

$$\varepsilon = \frac{1}{2} m_s^2 \Sigma_s^2 + \frac{1}{2} m_v^2 \Sigma_v^2 + \frac{1}{(2\pi)^3} \int_0^{k_F} d^3 k \sqrt{|k|^2 + M^*^2},$$

where $M^*$ is the effective nucleon mass in matter, $k_F$ is the Fermi momentum, $\gamma$ is the spin-isospin degenerescence, $g_s$ and $g_v$ are, respectively, the $\sigma$-N and the $\omega$-N coupling constants and $m_s$ and $m_v$ are, respectively, the $\sigma$- and $\omega$-meson.
masses. The self-energies $\Sigma_{(s)}$ and $\Sigma_{(v)}$ can be obtained through “tadpole” Feynman diagrams resulting in

$$\Sigma_{(s)} = M^* - M = -\frac{g_s^2}{m_s^2} \rho_s$$

and

$$\Sigma_{(v)}^\mu = \delta^{\mu0} \frac{g_v^2}{m_v^2} \rho_B ,$$

Here $M$ is the free nucleon mass with value $M \approx 939$ MeV. The baryon and the scalar densities are, respectively, given by:

$$\rho_B = \frac{\gamma}{6\pi^2} k_F^3$$

and

$$\rho_s = \frac{\gamma M^*}{4\pi^2} \left[ k_F E_F^* - M^* 2 \ln \left( \frac{k_F + E_F^*}{M^*} \right) \right]$$

The self-energies (2) and (3) can also be obtained through QCD sum rules. The principal ingredient of this method is the time-ordered correlation function, defined by

$$\Pi_{\alpha\beta}(q) \equiv i \int d^4x \, e^{iq \cdot x} \langle 0 | T [ \eta_\alpha(x) \bar{\eta}_\beta(0) ] | 0 \rangle ,$$

where $|0\rangle$ is the physical nonperturbative vacuum state and $\eta_\alpha(x)$ is an interpolating field with the quantum numbers of a nucleon. In agreement with Ref. [18], we have for the proton field

$$\eta(x) = \epsilon_{abc} \left[ t \left( u_a^T C \gamma_5 d_b \right) u_c + \left( u_a^T C d_b \right) \gamma_5 u_c \right] ,$$

where $u$ and $d$ are the up and down quark fields, $a$, $b$, $c$ are color indices, $T$ means transpose, $C$ is the charge-conjugation matrix and $t$ is an arbitrary real parameter. The OPE for the correlation function (6) can be generated through the following expansion for the quark propagator [19,20]

$$S_{ab}(x) = \langle 0 | T [ q_a(x) \bar{q}_b(0) ] | 0 \rangle$$

$$= i \frac{\delta_{ab}}{2\pi^2 x^4} - \frac{\delta_{ab}}{12} \langle \bar{q}q \rangle_{vac} + \cdots ,$$

$$\cdots$$
where $\langle \bar{q}q \rangle_{\text{vac}}$ is the free quark condensate, which can be determined from the Gell-Mann-Oakes-Renner relation,

$$2m_q \langle \bar{q}q \rangle_{\text{vac}} = -m^2_\pi f^2_\pi \left(1 + O(m^2_\pi)\right).$$ (9)

Here, $m_\pi = 138$ MeV is the pion mass, $f_\pi = 93$ MeV is the pion decay constant and $m_q = (m_u + m_d)/2$ is the average quark mass.

The same correlation function, Eq. (6), can be described through a phenomenological ansatz inspired in the nucleon Green function. With the fundamental state of the nuclear matter as vacuum and the interacting propagator in the phenomenological side, we obtain the following sum rules [18]

$$M^*_N = -\left(\frac{7 - 2t - 5t^2}{5 + 2t + 5t^2}\right) \frac{M^2_B \langle \bar{q}q \rangle_{\rho_B}}{16\pi^2 M^4_B + \frac{2}{3} E_q \langle q^1 q \rangle_{\rho_B}},$$ (10)

$$\Sigma^{QCD}_{(v)} = \frac{M^2_B \langle q^1 q \rangle_{\rho_B}}{3 \rho_B} M^4_B + \frac{1}{2} E_q \langle q^1 q \rangle_{\rho_B}.$$ (11)

where $M^*_B$ represent the Borel mass with value near $M^2_B \simeq 1$GeV [21], $\langle \bar{q}q \rangle_{\rho_B}$ represents the quark density, $\langle \bar{q}q \rangle_{\rho_B}$ is the in-medium quark condensate [22] and $E_q = -\Sigma_{(v)} + \sqrt{q^2 + M^2_{N}}$, with $|q| = 270$ MeV (i.e., approximately the Fermi momentum). Here, it was not take into account the continuous contribution. The expression (10), with $t = -1$, is a generalization of the Ioffe’s formula [21] to finite density. The explicit dependence on the choice of the parameter $t$, that means, on the choice of the interpolating field, limits the significance of the Ioffe’s formula to a qualitative role at best. A more realistic value for $t$ can be obtained in order to do minimal the contributions coming from continuum model and high dimensional operators in OPE [24]. This better value will be considered later. Calculating the ratio of finite-density to zero-density sum rules, we obtain

$$\frac{M_N}{M_N} = \frac{\langle \bar{q}q \rangle_{\rho_B}}{\langle \bar{q}q \rangle_{\text{vac}}} \left(\frac{1}{1 + 16\pi^2 F_q \rho_N} \right),$$ (12)

$$\frac{\Sigma^{QCD}_{(v)}}{M_N} = \frac{5 + 2t + 5t^2}{7 - 2t - 5t^2} \left(\frac{1}{1 + 16\pi^2 F_q \rho_N} \right)^2 \frac{2}{\langle \bar{q}q \rangle_{\text{vac}}} \rho_N.$$ (13)

Comparing expressions (2) and (12), we have for the scalar and vector sectors, respectively,

$$g^2_s m^2_s = -\frac{M_N}{\rho_s} \left[\langle \bar{q}q \rangle_{\rho_B} \left(\frac{1}{1 + 16\pi^2 F_q \rho_B} \right) - 1\right]$$ (14)
and
\[
g_s^2 m_s^2 = g_v^2 m_v^2 = -M_N \left[ \frac{5 + 2t + 5t^2}{7 - 2t - 5t^2} \right] \frac{2}{\langle \bar{q}q \rangle_{\text{vac}}} \frac{1}{1 + 16\pi^2 \frac{E_q}{M_B} \rho_B} . \tag{15}
\]

To analyze the expressions (12-15) is necessary to know the in-medium quark condensate, which can be written as \[22\]
\[
\langle \bar{q}q \rangle_{\rho_B} = \left( 1 - \frac{\sigma_N \rho_B}{m_{\pi}^2 f_{\pi}^2} + \cdots \right) \langle \bar{q}q \rangle_{\text{vac}} , \tag{16}
\]
where \( \langle \bar{q}q \rangle_{\text{vac}} \) is given by Eq.(9) and the sigma term is estimated in Ref. [23] as being \( \sigma_N \approx 45 \pm 10 \text{ MeV} \). The expression (16) is only been worth for low densities and considering those conditions we can write in a rude way the following approaches: \( M_B^4 \gg E_q \rho_B \) and \( \rho_B \sim \rho_s \). Therefore, the scalar sector is written as
\[
\Sigma_{QCD}^{(s)} = -\frac{\sigma_N M_N}{m_{\pi}^2 f_{\pi}^2} \rho_B , \tag{17}
\]
\[
g_s^2 m_s^2 = \frac{\sigma_N M_N \rho_B}{m_{\pi}^2 f_{\pi}^2} \rho_s \approx \frac{\sigma_N M_N}{m_{\pi}^2 f_{\pi}^2} . \tag{18}
\]

With the Ioffe’s choice \( t = -1 \), we obtain for the vector sector,
\[
\Sigma_{QCD}^{(v)} = \frac{8m_q M_N}{m_{\pi}^2 f_{\pi}^2} \rho_B , \tag{19}
\]
\[
g_v^2 m_v^2 = \frac{8M_N m_q}{m_{\pi}^2 f_{\pi}^2} . \tag{20}
\]

The principal conclusion obtained from equations (18) and (20) is that the ratios of coupling constants to respective meson mass are constants for low densities. Some values for these ratios are presented in table 1. The first line

| \( g_s^2/m_s^2 \) | \( g_v^2/m_v^2 \) |
|-----------------|-----------------|
| QHD-I           | 3.029 \( 10^{-4} \) | 2.222 \( 10^{-4} \) |
| QCDSR           | 2.793 \( 10^{-4} \) | 1.964 \( 10^{-4} \) |

Table 1

Ratios of coupling constants to respective meson mass. In the first line are presented the values used in Ref.[5] and in the second line the values obtained with equations (18) and (20) using Ioffe’s interpolating field \( t = -1 \).

of that table presents the ratios used in Ref. [5], which were adjusted so that
QHD-I reproduces the saturation properties of the nuclear matter. On the other hand, when the self-energies (17) and (19) and the ratios (18) and (20) (low density limit) are applied in Eq.(1), and we require the usual saturation condition for nuclear matter, namely \( \varepsilon/\rho_B - M = 15.75 \text{ MeV} \) at normal nuclear matter density \( (k_F = 1.42 \text{ fm}^{-1}) \), we obtain the values presented in the second line of table 1. We can observe that the QCDSR results are similar to the respective QHD-I values. However, this is not the most important result. The in-medium behavior of \( g_s^2/m_s^2 \) and \( g_v^2/m_v^2 \), named scalar and vector ratios, respectively, are the principal subject to be studied here.

Despite the fact of we have obtained the equilibrium properties of nuclear matter, the effects despised in the low density approximation must be included. For that, it is necessary to abandon Eq.(16) for the condensate and to use the familiar Nambu-Jona-Lasinio (NJL) model, which gives us the following expression [22] for the quark condensate:

\[
\langle \bar{q}q \rangle_{\rho_B} = -\frac{N_c M_q}{\pi^2} \frac{A_{NJL}}{k_{F(q)}} \int dp \frac{p^2}{\sqrt{M_q^2 + p^2}}
\]

and

\[
M_q = m_q - 2G_{NJL} \langle \bar{q}q \rangle_{\rho_B}.
\]

The Fermi momentum of quarks, \( k_{F(q)} \), is related to the Fermi momentum of nucleons through the density: \( \rho_q = 3\rho_B \), where \( \rho_q \) is the quark density. Here \( N_c \) is the color number, \( A_{NJL} \) is a cutoff, \( G_{NJL} \) is a coupling constant, \( m_q \) is the current quark mass and \( M_q \) is the constituent quark mass, which is dynamically generated by a partial restoration of chiral symmetry. With an appropriate choice of these parameters the NJL model gives us good results for the \( \pi \)-meson mass, \( f_\pi \), constituent quark mass, condensate, etc. A good choice is \( A_{NJL} = 900 \text{ MeV} \) and \( G_{NJL} = 3.54/A_{NJL}^2 \), for which the saturation point of nuclear matter is reproduced. The in-medium behavior of the relevant quantities \( g_s^2/m_s^2 \) and \( g_v^2/m_v^2 \), are presented on figure 1. The scalar [Eq. (14)] and the vector [Eq.(15)] ratios are represented by the solid and the dashed lines, respectively. In these calculation, a more realistic nucleon interpolating field \( (t = -1.1) \) [24] has been used.

In summary, the expressions (17-20) are valid just for the low density region. However, some insight can be obtained from these first results: the ratios of coupling constants to respective meson masses are constants in the low density limit. Actually, the principal conclusion obtained in this work can be summarized by figure 1, where the scalar and the vector ratios are presented as functions of the density. We can observe that they have a small dependence.
Fig. 1. The ratios $g_s^2/m_s^2$ (solid line) and $g_v^2/m_v^2$ (dashed line) calculated with Eqs.(14) and (15), respectively

on density and the vector ratio decrease when the density is enlarged. These results corroborate with the observation made in Ref.[4], where the nucleon flow probing higher density requires that $g_v/m_v$ be independent of density at low densities and decrease slightly at high densities. The results here obtained indicate that $(g_s/m_s)_{\text{vac}} \approx (g_s/m_s)_{\rho_B}$ and we can write

$$
\frac{(m_s)_{\rho_B}}{(m_s)_{\text{vac}}} \approx \frac{(g_s)_{\rho_B}}{(g_s)_{\text{vac}}},
$$

(23)

The same result is valid for the vector ratio. In other words, coupling constants must change with the same rate of the respective meson mass, in order to keep the ratios unaffected. Therefore, the fact of the ratios remain constants can give us an explanation of how QHD models, that use parameters independent of density, have success to explain the bulk properties of nuclear matter. The same is not true for finite nuclei, where the scalar and vector ratios do not appear explicitly on equations. Thus, the dependence on density of coupling constants and meson masses must be important in these cases.

Equation (23) and its equivalent for the vector sector, permit one to extend the Brown-Rho (BR) scaling law [1] to include coupling constants in the following way:

$$
\frac{M_N^*}{M_N} \approx \frac{f_\pi^*}{f_\pi} \approx \frac{m_s^*}{m_s} \approx \frac{m_v^*}{m_v} \approx \frac{g_s^*}{g_s} \approx \frac{g_v^*}{g_v},
$$

(24)
where the asterisks denote in-medium quantities.

Finally, it is necessary to say that these results are valid at low densities. For nuclear matter in extreme conditions, the ratios should change [4] and QHD-I must present bad results. Besides, in these calculations the contributions of gluon condensates and other high terms were not included in the OPE. However, we know that the contributions of these terms for the sum rule are very small. The use of an extended version of the sum rules (10) and (11) as well as the importance of scaling for finite nuclei, are left for future works.

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