Ultrasonic nondestructive characterization of mortars by the width of the resonances

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Abstract. In this work, we study the width of the resonances of the ultrasound waves reflection coefficient backscattered by a plane structure of the mortar. We establish the relationship between this width with two parameters which are widely used in non-destructive characterization of cementitious materials namely the velocity and attenuation. Monitoring the hydration of three solutions of mortars produced with different sizes of sand grains shows that the experimental results confirmed the theoretical predictions. Linear correlations are established between the width of resonance and the two ultrasonic parameters.

1. Introduction

Techniques using the ultrasonic wave propagation are very suitable tools for non-destructive characterization of cementitious materials. The measured parameters are typically the speed which is related to the dynamic elasticity and attenuation which is related to the microstructure of the material. Several research studies have been conducted to a nondestructive characterization of the cementitious materials [1, 2, 3, 4]. Among these methods, measurements of ultrasonic velocity and attenuation which are proven in evaluating the mechanical and structural properties of these materials [5, 6, 7].

The resonant scattering theory [8, 9, 10] provides that the propagation of sound waves in the material generates resonances in its thickness, where each resonance is characterized by its frequency and width. We focus on the resonances of the reflection coefficient of a plane mortar structure that will be put in the vicinity of an isolated resonance in the form of a nuclear function known as the Breit-Wigner. Two parameters appear in the expression of this function, namely the frequency \( f_0 \) and its width \( \Gamma \). During its hydration, the mortar is transformed from the viscous state to the viscoelastic one and then to the solid elastic one. This transformation is accompanied by a change in velocity and ultrasonic attenuation but also the movements of resonance frequencies and variation in their widths. We seek in this work to highlight the behavior of the resonance width with respect to two parameters of the ultrasonic wave in a mortar during its hydration namely the velocity and the attenuation.

2. Materials and methods

2.1. Experimental device

The experimental device (figure 1) described in detail in a previous work [11], consists of a parallel sided container enclosing a thick layer of mortar \( d_m = 15 \text{mm} \), irradiated at normal incidence by an ultrasonic wave emitted by a transducer of central frequency 0.5MHz which acts as emitter and...
receiver. The recording of the signals reflected by the Plexiglas / mortar / glass structure, is 15min all three days.

**Figure 1.** Geometry of the problem and the path of incident and reflected signals

2.2. Techniques of measures

2.2.1. Theory

Assuming that the three layer structures; Plexiglas, mortar and glass are flat, and by placing in the context of the theory of linear systems, the signal reflected by the mortar layer can be written as a sum of the echo reflected by the interface plexiglas / mortar and all echoes that undergoes multiple reflections inside the material. The signal processing in the frequency plan allows to write the reflection coefficient as a series of Debye:

\[
R = R_p + T_p R_m T_m \exp(-j2k_m d_m) \times \sum_{n=0}^{\infty} (R_{m/g} R_{g/p} \exp(-j2k_m d_m)) ^ n
\]

Experimentally terms of order \( n \geq 1 \) in equation (1) are not visible during all handling. In this case the complex reflection coefficient of the mortar is then:

\[
R \cong R_p + T_p R_m T_m \exp(-j2k_m d_m)
\]

Where

- \( k_m = \frac{\omega}{v_m} - j \alpha_m \): The complex wave vector in the mortar.
- \( v_m \): The velocity of the wave in the mortar.
- \( \alpha_m \): The attenuation of the wave in the mortar.
- \( d_m \): The thickness of the mortar layer.

\( R_i/j, T_i/j \) are respectively the coefficients of reflection and transmission of the interface \( i/j \). The letters p, m, and g represent the Plexiglas, mortar and glass environments.

In the vicinity of an isolated resonance, the reflection coefficient can be written as a complex function of Breit-Wigner characterized by its frequency \( f_0 \), its width \( \Gamma \):

\[
R \cong A - \frac{B}{1 - \left( \frac{f-f_0}{\Gamma} \right)^2}
\]

Where:

- \( A = \frac{R_p}{m} \), \( B = T_p \frac{R_m}{g} \frac{T_m}{p} \exp(-2\gamma_m d_m) \) and \( \Gamma = \frac{v_m}{2\pi d_m} \)

2.2.2. Experimental measurements

The coefficient of experimental reflection of the mortar layer (figure 2) is calculated by using the ratio of Fourier transforms:
\[ R = R_m \times \frac{S_{r2}(f) + S_{r3}(f)}{S_{r2}(f)} \]  \hspace{1cm} (4)

Figure 2. A representation of the reflection coefficient modulus as a function of the frequency

The speed of ultrasounds in 1.5 cm thick mortar is obtained by measuring time delay \( \Delta t \) between the echoes \( S_{r2}(t) \) and \( S_{r3}(t) \) and attenuation, by using the reporting method spectra of these two echoes [11].

3. Results and discussion

3.1. Relationship between the width of resonance and ultrasonic velocity

Monitoring the hydration at temperature \( T = 42^\circ C \) for three mortars solutions made with grains of sand of different sizes (\( d = 250\mu m \), \( d = 315\mu m \) and \( d = 500\mu m \)), from the young age to curing for 70h, by measuring the ultrasonic velocity and the width of a single resonance mode (\( n=5 \)) is used to draw the curves shown in Figure 3. The three curves show that the evolution of the resonance width follows a linear law with velocity. The analysis of the parameters of the linear regression shows that the constant of proportionality \( c \approx 16.67 \text{ m}^{-1} \approx \frac{1}{4d_m} \) depends only on the thickness of the mortar. This result is in good agreement with the theoretical interpretation of equation (3). The figure shows also the sensitivity of the resonance width to the variation of the mortar microstructure; the higher the sand grain size, the higher the width of resonance.
Figure 3. Variations in the width of resonance depending on the velocity in mortars made with the sand grains of different sizes (d = 250µm, d=315µm and d= 500µm)

3.2. Variation of ultrasonic velocity and the width of resonance according to the attenuation
The curves shown in Figures 4 and 5 respectively illustrate the variations of the velocity and the width of resonance according to the attenuation in the three mortars solutions in the curing phase. The two figures show that this variation is linear, with negative slope. The effect of sand grains size on the evolution of the mortar is visible across the width of resonance: it is more important than when the grain diameter is small.

Figure 4. Variation of the velocity according to the attenuation in the mortar curing phase
The linear regression parameters for three mortar solutions are summarized in Table 1.

| Parameters of the linear regression | d=250µm | d=315µm | d=500µm |
|------------------------------------|---------|---------|---------|
| V(m/s)                             | 4562    | 4884    | 4746    |
| a                                  | 5.55E-3 | 6.19E-3 | 5.88E-3 |
| R²                                 | 0.99    | 0.99    | 0.97    |
| Γ(KHz)                             | 70.71   | 76.20   | 74.72   |
| b                                  | 5.50E-3 | 5.96E-3 | 5.94E-3 |
| R²                                 | 0.98    | 0.98    | 0.97    |

Our results show that the variations in the velocity of compression and the width of resonance follow a linear relationship with the attenuation. Similar results were observed in cementitious materials by other authors [15, 16, 17]. In the elastic and isotopes media at low porosity values, the velocity can be approximated by a linear law \( v = v_0(1 - bP) \), with \( P \) the medium porosity and \( v_0 \) the velocity of the wave at zero porosity and \( b \) an empirical parameter. In our case the linear regression is made in the mortar curing phase when the hydration reactions are completed, and the linear dependence of the compression velocity and the width of resonance with the attenuation can be explained by improved mechanical and structural properties of the material which acquired in the later stages of hydration a low porosity structure. In addition, the results of correlations can be equated with those of other authors if we assume that changes in the ultrasonic attenuation is linear with porosity in specific conditions which assume that the environments are elastic, isotopes and at low porosity values. We observe that the straight slopes obtained by linear regression are of the same order of magnitude \( a \approx b \approx 5.83.10^{-3}\text{Np}^{-1} \). This allows us to find the empirical relationship established in (paragraph 3.1) between the width of resonance and the velocity \( \frac{\Gamma}{
u} = \frac{\Gamma_0}{v_0} = c \).

3.3. Comparison of experimental results with theoretical predictions

Measuring the width of resonance using the resonances of the theoretical coefficient of reflection (equation (1)) makes it possible to highlight the contribution parameters of the ultrasonic wave and the mortar on the evolution of the width of resonance. The curves shown in Figure 6 show, respectively, that the width of resonance is proportional to the inverse of the thickness of the mortar and the ultrasonic velocity. This is consistent with the formula derived from the Breit-Wigner function and confirms experimentally the variation law established between the width of resonance and the velocity (section 3.1). The curve which shows the effect of the variation of the ultrasonic attenuation in the mortar on the resonance width by assuming that the ultrasonic velocity remains unchangeable, is carried on the Figure 7. We find that this variation follows a logarithmic law and that attenuation change does not affect much the resonance width except for lower values, which explains the very
low sensitivity of the resonance width to the attenuation variation of ultrasound in the mortar compared to its variation with velocity.

![Graph showing resonance width vs. inverse of the thickness of mortar and velocity](image)

**Figure 6.** Evolution of the width of resonance as a function of the inverse of the thickness of mortar (velocity = 3072 m/s, attenuation = 45.5 Np/m) and in function of the velocity (thickness of mortar d = 1.5 cm, attenuation = 45.5 Np/m)

![Graph showing resonance width vs. attenuation](image)

**Figure 7.** Evolution of the width of resonance as a function of the attenuation (thickness of mortar d = 1.5 cm, attenuation = 45.5 Np/m)

4. Conclusion

This study highlights a non-destructive testing technique of cementitious materials based on the measurement of resonance width of the reflection coefficient. The theoretical and experimental study of this parameter shows a linear dependence of the width of resonance with the ultrasonic velocity and with the inverse of the thickness of the mortar in all stages of hydration. A linear correlation with the attenuation is also observed in the curing phase. The sensitivity of the resonance width to variations in the mechanical and structural properties of the mortar shows that the method developed in this work, using the resonance width as characterization parameter, has an alternation with the different methods of existing non-destructive testing.
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