Incentivized Advertising: Treatment Effect and Adverse Selection

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Abstract

Incentivized advertising is a new ad format that is gaining popularity in digital mobile advertising. In incentivized advertising, the publisher rewards users for watching an ad. An endemic issue here is adverse selection, where reward-seeking users select into incentivized ad placements to obtain rewards. Adverse selection reduces the publisher’s ad profit as well as poses a difficulty to causal inference of the effectiveness of incentivized advertising. To this end, we develop a treatment effect model that allows and controls for unobserved adverse selection, and estimate the model using data from a mobile gaming app that offers both incentivized and non-incentivized ads. We find that rewarding users to watch an ad has an overall positive effect on the ad conversion rate. A user is 27% more likely to convert when being rewarded to watch an ad. However there is a negative offsetting effect that reduces the effectiveness of incentivized ads. Some users are averse to delayed rewards, they prefer to collect their rewards immediately after watching the incentivized ads, instead of pursuing the content of the ads further. For the subset of users who are averse to delayed rewards, the treatment effect is only 13%, while it can be as high as 47% for other users.

Keywords: online advertising, mobile, causal inference, bayesian estimation, endogenous selection
Mobile advertising, including video ads and banner ads in mobile devices, is a dominant segment of digital advertising. In the U.S., businesses spending on mobile advertising accounts for more than 50% of the total spending on digital advertising.\footnote{According to the 2017 Internet Advertising Revenue Report from PricewaterhouseCoopers, spending on mobile ads is $36.6 billions in 2016, while total spending on digital advertising is $72.5 billions.} The growth of mobile advertising is fueled by the widespread usage of mobile applications or apps \cite{Ghose:2014} – it is now commonplace to advertise on mobile apps.

Mobile advertising is also a fast evolving industry, where advertisers and publishers continuously innovate on ad formats, improve data tracking capabilities \cite{Goldfarb:2011a, Goldfarb:2011b} and optimize ad placements. In recent years, mobile publishers have widely adopted a new format of ad placement, called incentivized advertising. In an incentivized ad placement, publishers reward users for watching an ad. More generally, incentivized ad takes the form of rewarding a user for completing an action related to the ad.\footnote{WSJ (Jan 5, 2016), More Marketers Offer Incentives for Watching Ads} Incentivized advertising is also commonly known as reward advertising.

Incentivized ads first appeared among mobile gaming apps. Examples include ad placements where publishers reward users with in-game virtual items, additional game levels and lives, for viewing an ad, typically in a full-screen video format. One of the reasons for using incentivized advertising is to reduce annoyance towards ads, which is of particular concern in mobile advertising. Mobile devices have smaller screen sizes compared to personal computers, and as such it is more difficult to effectively advertise in mobile devices. For instance, conventional banner ads are very intrusive in mobile devices. Moreover,
mobile apps especially mobile gaming apps, rely on a continuous user’s experience, so that interstitial full-screen ads do not tend to work well.\(^3\)

Incentivized advertising allows the app developer to incorporate advertising into the game-play, for instance, by offering to revitalize an injured game character if the user watches an ad. Therefore incentivized ads allow for a more seamless transition between gameplay and ads, which improves the playability of the game and reduces the annoyance due to interruptions. Moreover, rewarding users to watch an ad could affect the mood of the users, and contribute to an overall positive perception towards the ads. For these reasons, incentivized advertising has become a popular format of advertising within mobile gaming apps. Various industry white papers have reported that incentivized advertising is well-received by users.\(^4\) It has even expanded beyond mobile gaming publishers.\(^5\)

Despite the increasing adoption of incentivized advertising, little is known about how incentivized advertising affects users’ behavior (on the other hand, we have known quite a bit about the effects of other important formats of online advertising, see Bart, Stephen, and Sarvary [2014], Bruce, Murthi, and Rao [2017], Manchanda, Dubé, Goh, and Chintagunta [2006]). To this end, we aim to study the causal effect of incentivized advertising by developing a treatment effect model with unobserved selection. Our goal is to understand and quantify the effect of incentivized advertising on user’s conversion rate as compared with non-incentivized advertising. That is, how much ad conversion rate changes as a

\(^{3}\)This is related to the topic of ‘viewability’ in advertising. c.f. The Economist (March 26, 2016). Invisible ads, phantom readers.

\(^{4}\)eMarketer (July 1, 2014): Want App Users to Interact with Your Ads? Reward Them

\(^{5}\)For example, the mobile music streaming app Spotify incentivizes users to watch a video ad with 30 minutes of ad-free music; the video streaming website Hulu incentivizes users to watch a longer video ad with an ad-free episode; the mobile operator Sprint rewards certain users with reduced phone bill for watching ads.
result of offering rewards to users for watching ads. From a managerial perspective, this model allows us to ask whether a publisher can obtain higher ad revenue using incentivized or non-incentivized ad placements.

We estimate this treatment effect model using a large impressions-level dataset from a publisher who uses both incentivized and non-incentivized ad placements. This publisher is a mobile gaming app, and incentivized ads take the form of rewarding a user with additional game levels if the user watches a full-screen video ad trailer about another app. The publisher uses CPI (cost-per-install)\(^6\) pricing for all its ads, so that the publisher is only paid whenever an ad leads to a conversion event, defined as the user installing the advertised app.

The main feature of our treatment effect model is that we allow and control for unobserved adverse selection. When the publisher rewards users for watching an ad, it causes an adverse selection effect where users who are reward-seeking self-select into incentivized ad placements to obtain rewards. In the presence of adverse selection, a user is not randomly assigned to either incentivized or non-incentivized ads, therefore it is important to control for adverse selection in order to properly assess the causal effect of incentivized advertising. If reward-seeking attitude is an observable characteristics, controlling for adverse selection is done using propensity score methods (Section 5).

When there is unobserved adverse selection, we develop and estimate a model where users can endogenously select into watching incentivized ads, and where watching incentivized ads then translates into users’ outcomes. This model has two outcomes, an

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\(^6\)In other forms of online advertising such as sponsored search advertising, it is more common for the publisher to be paid per clicks, see Ghose and Yang [2009], Hu, Shin, and Tang [2015], Rutz and Bucklin [2011], Yao and Mela [2011], Zhu and Wilbur [2011]).
intermediate outcome where the user can express an intention to install the advertised app, and a final outcome where the user decides to install the app. In our dataset, we observe both the intermediate and the final outcomes of the users. In the intermediate stage, the user chooses whether or not to click at the end of the ad, which redirects the user to the App Store. In the final stage, the user chooses whether to install the app that was advertised. Identification of the model requires a variable that enters into the selection equation but not the outcome equations, while estimation is implemented using Bayesian MCMC.

Our main result shows that rewarding users to watch an ad has a negative effect on the intermediate outcome (where the user clicks on the ads to proceed to the App Store). Our explanation is that some users are averse to delayed rewards, and therefore prefer to collect their rewards immediately after watching incentivized ads. As such, rewards have the negative effect of reducing the user’s intention to take any action that delays the rewards. The user prefers to collect the rewards immediately instead of going to the App store and installing a new app. We also find that users exhibit varying degrees of aversion to delayed rewards.

On the flip side, we find that rewarding users to watch an ad has a positive effect on install (the final outcome) conditional on clicking the ad (the intermediate outcome). This result is in line with common findings that giving out rewards induces positive effects on products adoption and purchases. In our context, when the publisher gives rewards to its users, they induce the users to perceive the publisher’s content more favorably. As such, an ad that is published in an incentivized ad placement is then perceived more favorably by the users, and elicited a more positive response. This particular finding...
has some basis in the consumer’s behavior literature, where researchers have found that consumers’ affective feelings of favorability toward the ad itself, is an important predictor of advertising effectiveness and response (Calder and Sternthal [1980], MacKenzie and Lutz [1989], MacKenzie et al. [1986], Mitchell and Olson [1981]; Shimp 1981). Their findings resonate with our explanation that rewarding users to watch an ad causes users to feel less ad annoyance, and consequently increases the ad conversion rate.

The overall causal effect of incentivized advertising depends on the interplay between the negative effect on clicking and the positive effect on installing conditional on clicking. For our particular publisher, we find that incentivized advertising has an overall positive effect on the ad conversion rate. A user is 27% more likely to install when served incentivized advertising compared to non-incentivized advertising. In terms of ad revenue, this effect is equivalent to a CPM (revenue per thousand of impressions) of $0.413. To give a sense of the industry (mobile ad networks) benchmarks, the average CPMs for the US and China are reported to be $7.00 and $2.70.\footnote{See http://ecpm.adtapsy.com/}

Our result highlights the benefits of targeting the placement of incentivized ads according to demographics. Rewards have a negative effect on \textit{Clicks} for the users who are averse to delayed rewards, and therefore the overall treatment effect on \textit{Install} is heterogenous according to user’s characteristics. We find that incentivized advertising is least effective when the device language is set to \textit{Russian} (effect size of 13\%), and most effective for Chinese languages (effect size of 47\%). Given the potential cost of giving rewards, the publisher should not use incentivized ads when the effect size is close to zero.
The rest of the paper is organized as follows. Section 2 describes the data and relevant industry background. Section 3 develops and estimate the model. Section 4 develops an alternative estimator using the propensity scores. Section 5 concludes. The appendix contains all figures and tables.

2. DATA AND INDUSTRY BACKGROUND

The dataset comes from a mobile gaming app. The genre of the app is classified as “Action” in the Android App Store (it is not available in iOS or other operating systems). The app relies on publishing ads to monetize its user base. It uses both incentivized and non-incentivized ad placements.

In the context of this publisher, we define an incentivized ad to be a video ad that rewards users after the ad has been played. While a non-incentivized ad is a video ad that does not reward users after the ad has been played. Every ad is either incentivized or non-incentivized. The rewards are tied to the game itself (in-app rewards). Typically, the rewards unlock additional levels in the game for the users.\(^8\)

The content of the ad consists of a short video trailer showing another mobile app. These ads are users-targeted – they show mobile apps that users are likely to download and install. The targeting and serving of these ads are operated by a platform. The platform shares a pre-specified amount of percentage revenue with the publisher.

Users are not allowed to skip the ads. At the end of the ad, the user can exit the ad by either clicking the ‘x’ button, or the user can click on ‘Install’ button. When the user

\(^8\text{Another kind of incentivized advertising provides rewards for users to install apps, but Apple has blocked applications with such ad formats since 2011. c.f. TechCrunch (April 2011) Apple Clamps Down On Incentivized App Downloads}
clicks on the ‘Install’ button, the user will be directed to the App Store where she can
download the advertised app.

We define Intermediate to be a binary variable indicating whether the user has expressed
an intention to install by clicking on the ‘install’ button at the end of the ad – whereby
the user would have a chance to review more information about the advertised app in the
App Store. Intermediate is an intermediate outcome. The final outcome is Install, which
is a binary variable indicating whether the user has downloaded the advertised app.

This particular platform operates on a cost-per-install (CPI) model, where an advertiser
only pays the publisher in the event that the user installs the advertiser’s app. CPI
advertising is growing rapidly. Spending on CPI campaigns increased by 80% from 2014
to 2015 and accounted for 10.3% of mobile advertising spend in 2015.\textsuperscript{9}

2.1. Adverse selection

Adverse selection is an issue endemic to incentivized advertising. Adverse selection here
means the following: users deliberately seek out incentivized ad placements, in order to
obtain rewards. For instance, users who know where and when in the game to find ad
placements that are incentivized could then seek them out. These reward-seeking users
have low intention to install new apps. Incentivized advertising becomes ineffective when
adverse selection is severe – users only watch ads to collect rewards and are not converted
to install. It remains an open question whether incentivized advertising is effective and
should be widely adopted by publishers.

\textsuperscript{9}eMarketer (December, 2015). Mobile Advertising and Marketing Trends Roundup
On the other hand, adverse selection also poses challenges to data analysis and causal inference. Whenever an ad is served, it appears as an observation in our dataset. Therefore in the presence of adverse selection, our sample of incentivized ads is self-selected and consists disproportionately of reward-seeking users. Since a user is not randomly assigned to either incentivized or non-incentivized ads, estimating the effect of incentivized advertising would be biased. If reward-seeking attitude is an observable characteristics, correcting for selection can be done using propensity score methods. This is accomplished in Section 5. More generally, we develop and estimate a model which allows and controls for unobserved adverse selection in Section 3.

2.2. Data and variable description

The dataset contains 365,847 observations generated from the publisher. The timeframe spans from May 1, 2016 to May 31, 2016. Each observation consists of an ad serving instance. An ad serving instance is also commonly called an impression.

Whenever an ad is served, it is recorded as a unit of observation in the database. Note that after the ad has been served, a user can choose not to watch or pay attention to the ad. The user can take some actions such as clicking or installing after the ad has been served, which we observed (outcome variables). We also observe some characteristics of the users (control variables).

Each row of the dataset corresponds to an impression, hence we say that we have impression-level dataset. Now a single user may be served multiple ads by the publisher. Although we have 365,847 impressions, there are 143,280 unique users. The median user
generated only 1 impression, while the average user generated 2.55 impressions (standard deviation of 3.26).

We now describe the treatment and the outcome variables. We also provide some summary statistics of these variables. Each variable is subscripted by $i$, which we refer to as impression $i$.

(1) Incentivized, $d_i$: a binary (zero or one) variable, where $d_i = 1$ indicates that the user is in the treatment group during impression $i$. The user has been served an incentivized ad. If $d_i = 0$, then the user is in the control group and has been served a non-incentivized ad. The mean of $d_i$ is 0.6898, i.e. 68.98% of all observations correspond to incentivized ads.

(2) Intermediate, $y^\tau_i$: a binary outcome variable indicating whether the user during impression $i$ has expressed intention to install by clicking on the ‘Install’ button at the end of the ad. This intention is credible in the sense that the user would then be redirected to the relevant page in the Android App Store for downloading of the advertised app. The mean of $y^\tau_i$ is 0.1344, that is, there are 49,179 clicks on ‘install’.

(3) Install, $y_i$: a binary outcome variable indicating whether the user during impression $i$ has downloaded the advertiser’s app to her mobile device from the Android App Store. The mean of $y_i$ is 0.0029, that is, there are 1,067 installs in total.

In addition to the treatment and outcome variables above, we now describe the control or covariate variables. These variables are the observable characteristics of the users.
(1) **Language:** the language used in the user’s mobile device. The top 5 languages by number of observations are: (1) Spanish (ES), 35.86%; (2) English (EN), 25.82%; (3) Portuguese (PT), 11.03%; (4) Russian (RU) 6.93%; (5) Chinese (ZH), 6.81%.

(2) **Country:** the country of the user based on device and time-zone setting. The top 5 countries by number of observations are: (1) India, 12.13%; (2) Mexico, 11.75%; (3) Brazil, 10.36%; (4) China, 7.40%; (5) Indonesia, 5.52%.

(3) **Region:** it is useful to group countries into geographical regions that are similar to each other. We classify countries into statistical subregions as defined by the United Nations. The top 10 subregions by number of observations are: (1) South America, 28.84%; (2) Central America, 15.33%; (3) Southern Asia, 14.68%; (4) South-Eastern Asia, 14.19%; (5) Eastern Asia, 7.94%; (6) Eastern Europe, 5.21%; (7) Western Asia (Middle East), 3.70%; (8) Northern America, 2.35%; (9) Central Asia, 1.97%; (10) Southern Europe, 1.88%. However some of these regions are highly correlated with languages. As such, we will not construct indicator variables for Eastern Asia (correlation of 0.91 with ZH), Central America (correlation of 0.55 with ES), and Central Asia (correlation of 0.50 with RU).

(4) **WiFi:** whether the device is connected via WiFi or mobile data when the ad request is sent to the intermediary. The average value of WiFi is 0.7804, that is, 78.04% of the users were on WiFi.

(5) **Device Brand:** the manufacturer of the user’s mobile device. Since this particular app operates on an Android platform, one of the most prominent brand, Apple, is not included here. The top 5 device brands by number of observations are: (1)
(6) **Device Volume**: a numeric value from $[0, 1]$ that describes the level of device volume when the device sends the intermediary with an ad request. The mean of *Device Volume* is 0.55, with a standard deviation of 0.30.

(7) **Screen Resolution**: the number of pixels (per million) of the user’s mobile device. It is computed by multiplying the number of pixels per horizontal line by the number of pixels per vertical line. A higher screen resolution means better visual quality. The mean is 0.857, while the standard deviation is 0.645.

(8) **Android Version**: an integer-valued variable from 1 to 8 indicating the version number of the Android mobile operating system. A higher number corresponds to a newer and more recent Android operating system. At the time of this dataset, the most recent Android version is Android 6.0 (code name: Marshmallow). The mean is 4.45 and the standard deviation is 0.61.

The characteristics of a user can change over time, for instance, a user could have different device volume settings at different time periods. Causal inference does not follow simply from comparing the outcome of a user for when she was served incentivized versus non-incentivized ads.

### 3. Treatment effect model with unobserved selection

How does rewarding users for watching an ad affect the subsequent action (Install) taken by the user? When users are randomly assigned incentivized (treatment) or non-incentivized ads (control), then the causal effect of incentivized ads can be determined...
by comparing the outcome of the treatment versus the control group. Here, we do not have the luxury of random assignment, and we must then control for the selection of reward-seeking users into the treatment group (i.e. adverse selection).

When adverse selection is solely attributed to the observable characteristics of the users, estimators based on propensity scores can be used to obtain the treatment effect of incentivized advertising. This is done in Section 5. Here, we undertake a more general treatment effect model that allows for unobserved selection. As a motivation, suppose that there is an unobserved variable $v_i$ that measures the degree of rewards-seeking behavior of user $i$. Users who are more reward-seeking are more likely be self-selected into the treatment group due to the rewards from incentivized ads. This is modeled as Equation 1 below, where $d_i = 1[x_{1i}\gamma + v_i + \epsilon_{1i} \geq 0]$. Here, $x_{1i}$ is a vector of observed characteristics of the user $i$, and $\gamma$ is a vector of unknown parameters.

The probability that the user $i$ then expresses the intention to install is $y^*_i = 1[u_i + \epsilon_{2i} \geq 0]$. Now, $u_i$ is the utility that a user $i$ enjoys from installing a new app. $\epsilon_{2i}$ is the unobserved taste of the users. If $\epsilon_{2i}$ and $v_i$ are correlated, then the assumption underlying the standard propensity score method (Section 5) is violated.\(^\text{10}\) In particular, it is likely that $v_i$ is negatively correlated with $\epsilon_{2i}$. That is, a more reward-seeking user is less likely to click on ‘install’, because the reward-seeking user would rather collect the rewards immediately instead of clicking on ‘install’ and going to the App store. We will take unobserved adverse selection as meaning that there is a negative correlation between $\epsilon_{1i}$ and $\epsilon_{2i}$.

\(^{10}\)Users’ outcome is no longer independent of their treatment assignment conditional on observables. Here, a user who has higher unobserved $v_i$ is more likely to be selected into $d_i = 1$, and subsequently affects the outcome $y^*_i$. 

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Conditional on clicking on ‘install’, the user’s probability of installing the app is given by $1[u_i + \epsilon_{3i} \geq 0]$, where $\epsilon_{3i}$ is the unobserved tastes that affect users at the App Store (when users could see more information about the app). As before, $u_i$ is the utility that the user enjoys from installing a new app.

### 3.1. Unobserved selection

Based on our preceding discussion, we can estimate a model incorporating unobserved adverse selection. The model is an endogenous treatment effect model with two layers of outcomes: the intermediate outcome and the final Install outcome. The model consists of three interdependent non-linear equations, as given below. Note that we have absorbed $v_i$ (the user’s reward-seeking attitude) into $\epsilon_{1i}$.

\[
\begin{align*}
    d_i &= 1[u_{1i} + \epsilon_{1i} \geq 0] \\
    y_i^\tau &= 1[\alpha_1 d_i + u_{2i} + \epsilon_{2i} \geq 0] \\
    y_i \mid (y_i^\tau = 1) &= 1[\alpha_2 d_i + u_{3i} + \epsilon_{3i} \geq 0] \\
    y_i \mid (y_i^\tau = 0) &= 0
\end{align*}
\]

Equation 1 is the selection equation, it determines when a user is selected into the incentivized ads treatment. Equations 2 and 3 are the outcome equations. Equation 2 determines when a user would express the intention to install (by clicking on “install”). Equation 3 determines when a user would install the advertised app after clicking on “install”. Equation 3 can be written more compactly as $y_i \mid y_i^\tau = y_i^\tau \cdot 1[\alpha_2 d_i + u_{3i} + \epsilon_{3i} \geq 0]$. 

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\( \alpha_1 \) and \( \alpha_2 \) measure the effect of incentivized advertising on the pair of outcomes intention and install. \((\epsilon_1i, \epsilon_2i, \epsilon_3i)\) are idiosyncratic preferences unobserved to us, but observed by the users. Crucially, we allow these errors to be correlated with each other. If they are uncorrelated, there is no unobserved selection effect and we can use propensity score methods. It is not feasible to use a two-stage plug-in procedure where we first estimate the selection equation then plug-in the estimates for \( d_i \). These equations must be estimated jointly. The joint distribution of \((\epsilon_1i, \epsilon_2i, \epsilon_3i)\) will be specified in the next section.

Now we parameterize the utilities as follows: (i) \( u_{1i} = x_{1i}\gamma \), (ii) \( u_{2i} = x_{2i}\beta \), and (iii) \( u_{3i} = w_1 \cdot (x_{2i}\beta) + w_2 \). Now \( x_{1i} \) and \( x_{2i} \) are vectors of covariates that are subsets of \( x_i \). The utility from installing a new app is \( u_{2i} = x_{2i}\beta \). This utility enters into the equations for both Intermediate and Install. We allow this utility to be scaled and translated by \( w_1 \) and \( w_2 \) when it enters into the equation for Install. The parameter \( w_1 \) allows the user to express curiosity or motives for information acquisition. For example, when \( 0 < w_1 < 1 \), then the user’s utility for the app is magnified during the Intermediate stage, and the user is more likely to click on the ad to find out more about the app in the App Store. At the Install stage, this amplification disappears, and the likelihood of installing the app would just depend on the actual utility for the app plus some noise that represents new information from the App Store.

This formulation of utilities is not crucial to the model. We parameterize the utilities in this manner in order to reduce the number of parameters to be estimated. Even with this structure, we have a high-dimensional set of parameters to be estimated. Almost all our covariates are indicator or categorical variables: whether a user is located at a certain
region, whether a user speaks a certain language, etc. For this reason, the formulation
\[ u_{3i} = w_1 \cdot (x_{2i}/\beta) + w_2 \] is helpful in reducing the number of parameters.

The pair of Equations 1 and 2 represents a standard approach for handling treatment
endogeneity in binary outcome models (see, e.g., Smith and Blundell [1986], Rivers and
Vuong [1988], or Wooldridge [2002] (Section 15.7). The outcome variable is modeled
as Equation 2, but it contains an endogenous treatment variable \( d_i \), which we model
as Equation 1. This endogeneity arises because of the correlation between \( (\epsilon_{1i}, \epsilon_{2i}, \epsilon_{3i}) \).

Our framework here differs from the standard approach in that we have an additional
outcome variable (Equation 3) that also depends on the endogenous treatment variable.
In a well-known study, Evans and Schwab [1995] estimates the pair of Equations 1 and 2
as a bivariate probit model.

3.2. Identification

In the frequentist setting, identification and estimation of the model relies on the presence
of an exclusion restriction – an instrumental variable that enters into the selection equa-
tion, but does not enter into the outcome equations (see Wooldridge [2002] and Evans
and Schwab [1995]). Now among the variables that are available to us in Section 2.2, it
is not clear \textit{a priori} whether we have an exogenous instrumental variable. Therefore we
follow the \textit{plausibly exogenous} approach of Conley, Hansen, and Rossi [2012], where we
place a near-zero prior on a plausibly exogenous variable. We then estimate the model
using Bayesian MCMC.

Specifically, we choose the variable \textit{Device Volume} as a plausible instrumental variable.
Let the coefficient on \textit{Device Volume} in Equation 2 be denoted by \( \gamma \), our prior for \( \gamma \) is
\( \gamma | \alpha_1 \sim N(0, \delta^2 \alpha_1^2) \). When \( \delta = 0 \), \textit{Device Volume} is a fully valid exclusion restriction in the frequentist sense. We set \( \delta = 0.25 \), which allows \textit{Device Volume} to have a small effect in the outcome equation, in particular, the effect of \textit{Device Volume} is proportionally smaller than the treatment effect \( \alpha_1 \). The idea is that \textit{Device Volume} enters into the selection equation, but only has a relatively small effect on the user’s eventual outcomes.

This is reasonable: the user’s device volume is recorded at the moment of ad servings. If the user’s volume setting is high, she will be less inclined to seek out and watch incentivized ads, hence \textit{Device Volume} affects selection (negatively). Now after the selection stage, the user is free to adjust her volume setting during the ad. Because users adjust their volumes during the ads, the pre-adjusted volume settings should not affect users’ outcomes. While the volume settings prevailed during the ads could affect users’ outcomes, this volume setting is different from the recorded volume settings, which should not affect users’ outcomes.

### 3.3. Scalable Estimation

A desideratum for our estimation procedure is that it must be scalable, in the sense that it must be suitable for impressions-level data. For some popular publisher, impressions-level data means billions of observations in a single day.\(^{11}\) Estimation entails calculating the likelihood for each impression and summing them up. Moreover, calculating the likelihood for each impression involves modeling the dependence between the unobservables in the selection and the outcome equations (due to adverse selection). We find that modeling the dependence between \((\epsilon_{1i}, \epsilon_{2i}, \epsilon_{3i})\) as a multivariate Gaussian is too slow in this setting,

\(^{11}\text{http://www.businessinsider.com/the-size-of-fbx-facebooks-ad-exchange-2012-11}\)
even though we only have over 350,000 impressions. The reason is: we need to compute
the CDF of a trivariate Gaussian as many times as there are impressions. Computing
each CDF of a trivariate Gaussian involves multi-dimensional integrations, which required
either Monte Carlo integration or numerical quadrature.\(^{12}\)

With this in mind, we now specify the distributions of \((\epsilon_{1i}, \epsilon_{2i}, \epsilon_{3i})\) that lead to a
tractable likelihood. The marginal distributions of \(\epsilon_{1i}, \epsilon_{2i}\) and \(\epsilon_{3i}\) are assumed to have
the standard logistic distributions. That is, \(\epsilon_{1i} \sim \text{Logistic}(0, 1)\), and the CDF of \(\epsilon_{1i}\) is
\[ Pr(\epsilon_{1i} \leq x) = \frac{1}{1+e^{-x}}. \]
Similarly, the marginal distributions of \(\epsilon_{2i}\) and \(\epsilon_{3i}\) are both assumed
to have the standard logistic distributions. Denote \(F_1(e_1), F_2(e_2), F_3(e_3)\) as the marginal
CDFs of \(\epsilon_{1i}, \epsilon_{2i}\) and \(\epsilon_{3i}\) respectively.

To model the dependence between \((\epsilon_{1i}, \epsilon_{2i}, \epsilon_{3i})\), the joint CDF of \((\epsilon_{1i}, \epsilon_{2i}, \epsilon_{3i})\) is formu-
lated as \(C(F_1(e_1), F_2(e_2), F_3(e_3))\). This is without loss of generality – any joint CDF of
\((\epsilon_{1i}, \epsilon_{2i}, \epsilon_{3i})\) can be written this way (Skylar’s Theorem). The function \(C\) is known as a
Copula. Conversely, when \(C\) satisfies some properties, then \(C(F_1(e_1), F_2(e_2), F_3(e_3))\) is a
valid joint CDF. The idea is to choose a copula that is more tractable than the multivari-
ate Gaussian. Copulas are used extensively in finance to model the dependence among
random variables, and recently, copulas have appeared in various marketing journals,
see Danaher and Smith [2011a,b], George and Jensen [2011], Kumar, Zhang, and Luo
[2014]. These papers also contain formal introductions of copulas and their applicability
in marketing.

\(^{12}\)For instance in MATLAB and R, the algorithm to calculate the CDF of a trivariate Gaussian employs
numerical quadrature techniques developed by Drezner and Wesolowsky (1989), and Genz (2004). For
higher dimensions, quasi-Monte Carlo integration algorithm is used.
We model the joint CDF of \((\epsilon_{1i}, \epsilon_{2i}, \epsilon_{3i})\) as \(\Pr(\epsilon_{1i} \leq e_1, \epsilon_{2i} \leq e_2, \epsilon_{3i} \leq e_3) = \left([F_1(e_1)^{-\theta} + F_2(e_2)^{-\theta} + F_3(e_3)^{-\theta} - 2]_+\right)^{-1/\theta}\). The notation \([x]_+\) means \(\max\{x, 0\}\), i.e. \([x]_+\) cannot be negative. \(F_1, F_2,\) and \(F_3\) are marginal CDFs of \(\epsilon_{1i}, \epsilon_{2i}\), and \(\epsilon_{3i}\) respectively. The parameter \(\theta \in [-1, \infty) \setminus \{0\}\) controls the dependence among the variables. This copula is known as the Clayton copula, where \(C(x, y, z; \theta) = ([x^{-\theta} + y^{-\theta} + z^{-\theta}]_+)^{-1/\theta}\). There is a one-to-one relationship between the parameter \(\theta\) and Kendall rank correlation coefficient \(\tau\) between the variables, given by \(\tau = \frac{\theta}{\theta + 2}\). Therefore, when \(\theta\) is negative, \(\epsilon_{1i}\) and \(\epsilon_{2i}\) are negatively correlated in the sense of having a negative rank correlation coefficient, which is indicative of unobserved adverse selection. When \(\tau\) is estimated to be close to zero, \((\epsilon_{1i}, \epsilon_{2i})\) are uncorrelated, and there is no unobserved adverse selection (we can then use standard propensity score methods). Another commonly used copula is the Gumbel copula, which is a multivariate extension of the familiar Gumbel distribution. We do not use the Gumbel copula because it restricts \(\tau\) to be positive.

Having formulated the joint distributions of \((\epsilon_{1i}, \epsilon_{2i}, \epsilon_{3i})\), we can then derive the likelihood for each impression \(i\) according to Equations 1 to 3. The log-likelihood of observing the data \((d_i, y_i^r, y_i, x_i)_{i=1}^n\) given \(\Theta\), the set of parameters to be estimated, is denoted as \(\mathcal{L}((d_i, y_i^r, y_i, x_i)_{i=1}^n|\Theta)\). There are 52 parameters to be estimated, and we will describe them in the next section. Due to the choice of our joint distribution, this log-likelihood function can be derived in closed-form. This log-likelihood function can be computed very quickly even when there is a large number of impressions because it does not involve numerical integration.

More importantly, the gradient of the log-likelihood function with respect to the parameters can also be computed with ease. Being able to easily compute the gradient of
the target distribution allows us to employ more efficient Markov Chain Monte Carlo algorithms such as Hamiltonian Markov Chain or Metropolis-adjusted Langevin algorithm (MALA) (Roberts and Tweedie [1996]). These MCMC methods are more suitable here compared to the plain random walk metropolis since we have a moderately large number of parameters. Our MCMC method will be based on MALA. Informally, MALA constructs a random walk that drifts in the direction of the gradient, and hence the gradient enables the random walk to move more efficiently towards regions of high-probability. It also has a Metropolis-Hastings accept/reject mechanism that improves the mixing and convergence properties of this random walk.

For the priors, we impose uninformative priors on all the parameters, except for the parameters corresponding to the instrument variable (Device Volume), and the scale parameter \( w_1 \). The uninformative prior for a parameter is given by the Gaussian distribution with a mean of zero and a standard deviation of 100. The scale parameter \( w_1 \) has a prior of \( N(0.5, 0.25) \). In order to restrict the copula dependence parameter \( \theta \) to be within \([-1, \infty)\), we apply the transformation \( \theta = f(\tilde{\theta}) = (\tilde{\theta} + 1)^2 - 1 \), and subsequently impose an uninformative prior of \( N(0, 100) \) on \( \tilde{\theta} \).

We ran the MALA Markov Chain 5,000 iterations. Despite such a small number of iterations, convergence occurred quickly, which is not surprising since we have employed a gradient-based MCMC algorithm. Specifically, using the diagnostic of Heidelberger and Welch individually on all parameters, we reject the null hypothesis of non-stationarity for all parameters when the first-half of the chain is discarded as burn-in samples. We report the posterior means and standard deviations after discarding the burn-in samples. This is done in the next section.
4. Parameter estimates and results

In total, there are 52 parameters to be estimated. We allow the treatment effect for Intermediate to vary over the main language groups, so that Equation (2) now becomes
\[ y^\tau_i = 1[(\alpha_1 z_i) d_i + x_i \beta + \epsilon_{2i} \geq 0], \]
where \( \alpha_1 z_i = a_0 + a_1 \times EN_i + a_2 \times ES_i + a_3 \times PT_i + a_4 \times RU_i + a_5 \times ZH_i \). The indicator variables \( EN_i, ES_i, PT_i, RU_i \) and \( ZH_i \) indicate whether the language setting of impression \( i \) is English, Spanish, Portuguese, Russian, or Chinese. These are the five major language groups covering over 86% of all impressions. We do not estimate for heterogeneous treatment effects in the Install stage because the number of impressions where both selection and install occurred is much smaller compared to the number of impressions where both selection and clicks occurred.

To summarize, there are 21 parameters to be estimated in the selection equation \( d_i = 1[x_i \gamma + \epsilon_{1i} \geq 0] \). We list these parameters and show their estimates in Table 1. There are 26 parameters to be estimated in the Intermediate outcome equation \( y^\tau_i = 1[(\alpha_1 z_i) d_i + x_i \beta + \epsilon_{2i} \geq 0] \). We describe these parameters and show their estimates in Table 2. There are 4 parameters to be estimated in the Install outcome equation \( y_i | y^\tau_i = y^\tau_i \cdot 1[\alpha_2 d_i + w_1 x_i \beta + w_2 + \epsilon_{3i} \geq 0] \). We list these parameters in Table 3. Finally, we also need to estimate the parameter \( \theta \) which controls the degree of dependence among the unobserved error terms.

In Section 5.3, we use the standard propensity score method to show that qualitatively similar results are obtained. While our model here controls for unobserved selection, the standard propensity score methods control only for observed selections.
4.1. Estimates of the selection equation

Let us elaborate on Table 1, which reports the posterior means and standard errors of the parameters in the selection equation, \( d_i = 1[x_i\gamma + \epsilon_i, \geq 0] \).

First, we see that \( \theta \), the dependence parameter of the copula is \(-0.353\). This translates to a Kendall rank correlation coefficient \( \tau \) between \( \epsilon_{1i} \) and \( \epsilon_{2i} \) of \( \tau = \frac{\theta}{\theta+2} = -0.214 \). This is an evidence for unobserved adverse selection. There is an unobserved user’s characteristic (degree of reward-seeking) that increases the likelihood of selection into treatment, and at the same time, decreases the likelihood of clicking on ‘install’.

Looking at the other coefficients in Table 1, we find that they support an adverse selection narrative. For instance, the coefficient on WiFi is positive – a user with WiFi internet connection is more likely to seek out the incentivized ad treatment. Users are less likely to seek out incentivized ad placements when connected to cellular networks, which are slower and costly.

The coefficient on Device Volume is negative. A user whose device’s volume is higher is less likely to seek out incentivized ad treatment. An explanation is that a user would experience more annoyance and discomfort from watching an ad when the volume is higher, and hence, she is more reluctant to seek out incentivized ads.

The coefficient on Screen Resolution is positive. A user who has a better visual experience is less averse to watching ads, and hence is more likely to seek out incentivized ad treatment. The coefficient on Android Version is also positive, suggesting that a user with a more recent Android operating system is more likely to seek out incentivized ad treatment.
Overall, the result from Table 1 shows evidence of adverse selection – users deliberately seek out incentivized ads to obtain rewards.

4.2. Estimates of the intermediate outcome equation

Now we examine the estimates for the Intermediate outcome equation, \( y_i = 1[(\alpha_1 z_i) d_i + x_i \beta + \epsilon_i] \geq 0 \). Table 2 reports the posterior means and standard deviations of the coefficients.

We find that the treatment effects vary according to the languages that were chosen by the users. The baseline treatment effect \( \alpha_1 \) is significantly negative. Moreover for the users who have chosen English, Spanish and Russian, the treatment effects are significantly negative and larger in magnitudes than the baseline. While the users who have chosen Portuguese and Chinese, the treatment effects are significantly positive.

The negative treatment effect is surprising, as it implies that incentivized ad decreases the probability of clicks compared to non-incentivized ads. That is, for a subset of users being exposed to incentivized ads, they are less likely to go beyond this intermediate step of clicking on the ads, compared to their counterparts in the control group (exposed to non-incentivized ads).

Our explanation is that rewards have negative distortionary effects in the intermediate stage because users prefer not to delay their rewards by clicking on ‘install’. These users are averse to delayed rewards. They would rather collect their rewards immediately rather than going to the App Store even though they are sufficiently interested in the advertised app. In the absence of rewards (setting \( d_i = 0 \)), these users would not be distracted away
by the rewards, and would actually be more likely to click on the ads and go to the App Store.

For the users whose device languages are Portuguese and Chinese, the treatment effect on the intermediate outcome is positive. The fact that rewards have a positive effect is somewhat less surprising. We will postpone the explanation to the next section when we discuss the final outcome equation.

4.3. Estimates of the final outcome equation

We see in Table 3 that $\alpha_2$, the treatment effect on Install (conditional on having clicked) is positive. Previously, we also see that during the intermediate stage, the treatment effect on clicks is positive for some users. Therefore, the overall treatment effect for these users are unambiguously positive.

We now offer an explanation for the positive treatment effects of incentivized ads on Intermediate and Install. Research in the consumer’s behavior literature (Calder and Sternthal [1980], MacKenzie and Lutz [1989], MacKenzie et al. [1986], Mitchell and Olson [1981]; Shimp 1981) suggests that a person’s affective state (moods and feelings) when he or she watches the ad is an important predictor of advertising effectiveness and purchase intention. The reward, which is given by the publisher, causes the user to perceive the publisher’s content more favorably, including the ads that are published therein. Therefore ad conversion is higher when users are being rewarded for watching the ads.

Note that the reward is unrelated to the advertiser’s content or product, therefore we can rule out the complementarity between rewards and the advertiser’s product. When
there is a complementarity, a user could is more interested in the advertiser's app when
she is also being rewarded.

Now for those users who experienced a negative treatment effect during the Intermediate
stage, the overall treatment effect is ambiguous. We will quantify the overall treatment
effect in the next section. Our estimation suggests that a user can both experience a
negative treatment effect during the Intermediate stage, but a positive treatment effect
during the final Install stage. This is not contradictory to our explanation. If the user
were to reach the final stage, the aversion to delayed rewards would diminish since there
is now a shorter time between Install and the collection of rewards.

4.4. Counterfactuals

In the previous section, we have seen that the overall treatment effects are ambiguously
signed for some users. Here, we would like to quantify the overall treatment effects. First,
we calculate the overall Average Treatment Effect (ATE) on Install implied by the model.
The ATE is calculated as follows: for each impression \(i\), we compute the probability
that the user would click on ‘install’ and eventually install, if the user were to be in the
treatment group, then minus the probability that the user would click on ‘install’ and
eventually install, if the user were to be in the control group. More precisely, we have:

\[
ATE = \frac{1}{n} \sum_{i=1}^{n} \left[ \Pr \left( (\alpha_1 z_i) d_i + x_i \beta + \epsilon_{2i} \geq 0, \quad \alpha_2 d_i + w_1 x_i \beta + w_2 + \epsilon_{3i} \geq 0 \right) \right]
- \Pr \left( x_i \beta + \epsilon_{2i} \geq 0, \quad w_1 x_i \beta + w_2 + \epsilon_{3i} \geq 0 \right).
\]
The ATE measures how much the overall unconditional *Install* rate would change as a result of comparing two counterfactual scenarios for every impression: (1) when the user’s impression is served an incentivized ad, and (2) when the user’s impression is served a non-incentivized ad. These changes in the *Install* rate are then averaged over all impressions to obtain the ATE.

Using the formula in Equation 4, the Average Treatment Effect implied by the model is 0.000795. This is a large magnitude given that the baseline install is 0.00292 (1,067 installs out of 365,847 ad serving). The ATE of 0.000795 represents an increase of 27%. Therefore, a user is 27% more likely to install when served incentivized advertising compared to non-incentivized advertising. Since the publisher is paid per-install, this represents a large increase in ad revenue for the publisher (as well as the platform who shares revenue with the publisher). We have proposed an explanation for why rewards have a positive effect on user’s behavior. There is a well-known link between a person’s affective state (moods and feelings) during ad exposure, and the subsequent purchase intention. Therefore being rewarded for watching an ad causes the user to feel less annoyed at advertising, which increases ad effectiveness and conversion rate.

How does this ATE translate to ad revenue? We can provide a back-of-the-envelope calculation. The average price per-install commanded by this publisher is $0.52. Hence this ATE translates to 0.000795 × $0.52 = $0.0004134, or $0.413 per thousands of impressions. Ad revenues are frequently measured in terms of CPM (revenue per thousands of impressions). To give a sense of the industry (mobile ad networks) benchmarks, the average CPMs for the US and China are reported to be $7.00 and $2.70.  

\[ \text{http://ecpm.adtapsy.com/} \]
While incentivized advertising has an overall positive effect, we saw previously that there is a negative countervailing effect. This negative countervailing effect enters in the intermediate stage. Rewards have negative distortionary effects in the intermediate stage when users prefer to collect their rewards immediately after watching the ads, instead of clicking on ‘install’ and going to the App Store. Moreover, this negative effect varies widely among users. Therefore we expect the effect of incentivized ads to be less for those users who are adverse to delayed rewards. To quantify this, we compute the treatment effects averaged locally according to users’ languages. When we calculate the (Local) Average Treatment Effects by languages of the users, we see that rewarding users to watch ads has the largest effect on users whose device language is Chinese. The treatment effects averaged over English, Spanish, Portuguese, Russian, Chinese users are respectively 0.000752, 0.000667, 0.000608, 0.000391, 0.00138. In terms of dollar amounts and CPM, the magnitudes of these treatment effects are $0.391, $0.347, $0.316, $0.203, and $0.718, respectively.

Another useful counterfactual from the perspective of the publisher is the Average Treatment Effect on the Treated. Suppose we had switched all incentivized ads to non-incentivized ads, what is the effect? This is more relevant to the publisher because it represents a counterfactual that the publisher can directly implement. The average treatment effect on the treated is computed by averaging Equation 4 over \( i \) such that \( d_i = 1 \), which amounts to 0.000724, or an equivalent CPM of $0.376. Moreover since there are 252,379 treated observations, this implies that the publisher would lose 183 installs.
We can also quantify the revenue impact of adverse selection. In the following counterfactual, we remove unobserved adverse selection, that is, we suppose that selection is independent of outcomes.\textsuperscript{14} Whether or not an impression is served an incentivized ad is independent of the actions that would be taken during the Intermediate and Install stages. This rules out reward-seeking users who self-select into watching incentivized ads but otherwise they are not interested in the ad itself. The revenue impact of unobserved adverse selection is calculated using Equation 5 below, which amounts to 0.000552, or $0.287$ CPM. Therefore, adverse selection negatively impacts publisher’s ad revenue.

\begin{equation}
\frac{1}{252,379} \sum_{i: d_i = 1} \left( \hat{Pr}[d_i = 1] \cdot \hat{Pr}[y_i^\tau = 1, y_i = 1] - \hat{Pr}[d_i = 1, y_i^\tau = 1, y_i = 1] \right)
\end{equation}

5. Estimating treatment effects using propensity scores

In this section, we estimate the treatment effect of incentivized advertising using propensity scores. We want to compare our previous results to other model-free approaches. Propensity score method can control for selection bias to the extent that selection is based on observables. Therefore it is not valid in the presence unobserved selection, which we have analyzed previously.

5.1. Estimation procedure

Identical to the previous data environment, we observe \((d_i, y_i^\tau, y_i, x_i)\) for the sample of impressions \(i = 1, \ldots, n\), where \(x_i\) is a vector of user’s covariates during impression \(i\).

\textsuperscript{14}We implicitly conditioned on observed covariates. Note that this is precisely the assumption that underlies standard propensity score methods.
Our estimation procedure consists of two steps. In the first step, we estimate the \textit{propensity scores}: \( \hat{p}_i = \Pr(d_i = 1|x_i) \), which is the probability that a user is served an incentivized ad during impression \( i \). We estimate the propensity scores using a Probit regression of \( d_i \) on the user’s covariates \( x_i \). Note that \( x_i \) must only contain pre-treatment covariates. Pre-treatment covariates are the user’s characteristics that could affect the user’s selection into treatment.

In the second step, we construct \( \hat{p}_i \), which are the fitted values of the Probit regression from the first-step. Then, we run the regression of \( y_i \) on \( 1, d_i, \hat{p}_i, d_i(\hat{p}_i - \mu_p) \) for \( i = 1, \ldots, n \), where \( \mu_p \) is the average value of \( \hat{p}_i \) across \( i = 1, \ldots, n \). This is the control function approach explained in Proposition 18.5 of \textit{Wooldridge [2002]}. Under some assumptions, the ATE on \textit{Intermediate} can be recovered as the coefficient on the regressor \( d_i \) when regressing \( y_i^* \) on \( 1, d_i, \hat{p}_i, d_i(\hat{p}_i - \mu_p) \) for \( i = 1, \ldots, n \), while the ATE on \textit{Install} can be obtained as the coefficient on the regressor \( d_i \) when regressing \( y_i \) on \( 1, d_i, \hat{p}_i, d_i(\hat{p}_i - \mu_p) \) for \( i = 1, \ldots, n \).

In addition, we can include higher order polynomial terms of the propensity scores in order to better control for selection bias (making sure to de-mean the propensity score term before constructing its interaction with \( d_i \)). Therefore we also regress \( y_i \) on \( 1, d_i, \hat{p}_i^2, \hat{p}_i^3, d_i(\hat{p}_i - \mu_p) \), for \( i = 1, \ldots, n \).

The assumptions needed are explained in Proposition 18.5 of \textit{Wooldridge [2002]}. We will briefly discuss the main assumption, which is the assumption of “ignorability of treatment” (\textit{Rosenbaum and Rubin [1983]}). This assumption is also known as selection on observables. Given observed covariates \( x \): \( d_i \) and \((y_{0i}, y_{1i})\) are independent conditional on \( x_i \). This assumption implies that \( \mathbb{E}[y_{0i}|x_i, d_i] = \mathbb{E}[y_{0i}|x_i] \) and \( \mathbb{E}[y_{1i}|x_i, d_i] = \mathbb{E}[y_{1i}|x_i] \).
There are other methods for estimating the ATE, relying on different assumptions. We find that these other methods deliver similar results. For instance, the ATE can be estimated as an Inverse Probability Weighted Estimator using the propensity scores. That is, \( ATE = \frac{1}{n} \sum_{i=1}^{n} \frac{y_i (d_i - \hat{p}(x_i))}{\hat{p}(x_i)(1-\hat{p}(x_i))} \) (see Proposition 18.3 of Wooldridge [2002]). One method to compute the ATE that does not rely on the propensity scores is \( \frac{1}{n} \sum_i \hat{r}(x_i) \), where \( \hat{r}(x) = \Pr[y_i = 1|x, d_i = 1] - \Pr[y_i = 1|x, d_i = 0] \).

5.2. First-stage adverse selection estimation

In the first stage, we estimate the propensity scores via a Probit regression. Specifically, the dependent variable is the binary treatment variable Incentivized, or \( d_i \). The covariates are Android Version, Wifi, Screen Resolution, Device Volume. We also control for the following fixed effects: Countries, Languages and Device Brands.

The result is given in Table 4. We find that the result is qualitatively similar to the result obtained from estimating the selection equation (see Section 4.1).

5.3. Second-stage treatment effect estimation

Using the first-stage propensity scores, we now estimate the average treatment effects (ATE). We show the result in Tables 5 and 6. Again, the results obtained here are qualitatively similar to the model-based results.

The ATE on Intermediate is significantly negative, while the ATE on Install is significantly positive. From Column 2 (Intermediate) of Table 5, the ATE on Intermediate is \(-0.0635\). This means that rewarding users to watch an ad reduces the probability that a user clicks on install by \(-0.0635\) on average. The baseline Intermediate is 0.1344, i.e.
49,179 clicks out of 365,847. An ATE of this magnitude represents almost 50% decrease in the probability that a user would click on install.

Now the ATE for Install is statistically significant at 0.00795 (Column 2 of Table 6). This is a large magnitude because the baseline Install is 0.0217 (i.e. 1,067 installs out of 49,179 clicks). Therefore an ATE of this magnitude represents 36.6% increase in Install. In another words, if users are rewarded for watching the ads, they are 36.6% more likely to install the advertised app at the App store.

Compounding the effect of Intermediate, the overall effect on Install is positive and significant. From Column 4 of Table 6, the overall ATE obtained using the propensity score method here is 0.00187, while the ATE obtained using the model that controls for unobserved selection is 0.000795. Hence, the propensity score method biases the ATE upwards.

5.4. Naive treatment effects

In the Appendix (Table 7), we show results without controlling for any selection bias. We use probit regressions to show how incentivized advertising is related to (i) the user’s probability of clicking ‘install’, and (ii) the user’s probability of installing. We control for all the user’s characteristics mentioned in the preceding section. However these regressions are not valid if there is a selection bias. We will not interpret these coefficients further.

6. Appendix

6.1. Tables and Figures
Table 1. Parameters appearing in the selection equation, \( d_i = 1[x_i \gamma + \epsilon_{1i} \geq 0] \). The variables that correspond to these parameters are detailed in Section 2.2.

| Parameter (Description)          | Estimates            |
|----------------------------------|----------------------|
| \( \theta \) (Dependence parameter of the copula) | -0.353 (0.00323)     |
| Device Volume                    | -0.0879 (0.00536)    |
| WiFi                             | 0.352 (0.00712)      |
| Android Version                  | 0.133 (0.00106)      |
| Screen Resolution                | -0.0172 (0.00174)    |
| Huawei Dummy                     | 0.0837 (0.0134)      |
| Lenovo Dummy                     | -0.0792 (0.00346)    |
| LG Dummy                         | 0.157 (0.00415)      |
| Motorola Dummy                   | 0.17 (0.00186)       |
| Samsung Dummy                    | 0.0141 (0.000779)    |
| EN (English Language Dummy)      | -0.183 (0.00179)     |
| ES (Spanish Language Dummy)      | 0.253 (0.00617)      |
| PT (Portuguese Language Dummy)   | 0.317 (0.00976)      |
| RU (Russian Language Dummy)      | 0.114 (0.0019)       |
| ZH (Chinese Language Dummy)      | -0.573 (0.0192)      |
| Parameter (Description)                  | Estimates (Standard Error) |
|-----------------------------------------|----------------------------|
| North America Dummy                     | 0.0571 (0.00441)           |
| South America Dummy                     | 0.18 (0.00206)             |
| South-East Asia Dummy                   | 0.00197 (0.00162)          |
| South Asia Dummy                        | -0.276 (0.00528)           |
| Middle East Dummy                       | -0.214 (0.0141)            |
| Southern and Eastern Europe Dummy       | 0.117 (0.00133)            |
| Constant                                | 0.0151 (0.000812)          |

Table 2. Parameters appearing in the Intermediate outcome equation, \( y_i^* = 1[ (\alpha_1 z_i d_i + x_i \beta + \epsilon_{2i}) \geq 0 ] \)
| Category               | Value         |
|------------------------|---------------|
| Device Volume          | -0.0132 (0.0223) |
| WiFi                   | -0.181 (0.00526) |
| Android Version        | -0.327 (0.000725) |
| Screen Resolution      | -0.0314 (0.000939) |
| Huawei Dummy           | -0.0413 (0.00175) |
| Lenovo Dummy           | -0.0505 (0.00447) |
| LG Dummy               | -0.0787 (0.0023) |
| Motorola Dummy         | -0.0388 (0.00102) |
| Samsung Dummy          | -0.026 (0.00141) |
| EN (English Language Dummy) | -0.0245 (0.00243) |
| ES (Spanish Language Dummy) | -0.145 (0.00583) |
| PT (Portuguese Language Dummy) | 0.0349 (0.00109) |
| RU (Russian Language Dummy) | -0.0917 (0.00214) |
| ZH (Chinese Language Dummy) | 0.00464 (0.00311) |
| North America Dummy    | -0.138 (0.00566) |
| South America Dummy    | -0.0577 (0.00194) |
| South-East Asia Dummy  | -0.115 (0.00379) |
| South Asia Dummy       | 0.0727 (0.0043) |
Table 3. Parameters appearing in the Install outcome equation, $y_i|y_i^T = y_i^T \cdot 1[\alpha_2 d_i + w_1 x_i \beta + w_2 + \epsilon_{3i} \geq 0]$

| Parameter (Description)     | Estimates  |
|-----------------------------|------------|
| $\alpha_2$ (Install treatment effect) | 0.141 (0.0074) |
| $w_1$ (Scale parameter)     | 0.00732 (0.000534) |
| $w_2$ (Constant)            | -0.199 (0.0015) |
|                        |      |
|------------------------|------|
| Incentivized           |      |
|                        |      |
| Android Version        | 0.117*** |
|                        | (0.00482) |
| Device Volume          | -0.217*** |
|                        | (0.00810) |
| Screen Resolution      | 0.0117*** |
| (millions of pixels)   | (0.00432) |
| WiFi                   | 0.556*** |
|                        | (0.00594) |
| Constant               | -1.148*** |
|                        | (0.0628) |

| $N$                    | 358,127 |
| Countries controlled:  | Yes (178 indicator variables) |
| Languages controlled:  | Yes (48 indicator variables)  |
| Device brands controlled: | Yes (10 indicator variables) |

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

**Table 4.** First-stage probit estimation of propensity scores.
|                      | (1)             | (2)             |
|----------------------|-----------------|-----------------|
| Intermediate         | -0.0611***      | -0.0635***      |
|                      | (0.00146)       | (0.00146)       |
| $\hat{p}(x)$         | -0.213***       | -0.452***       |
|                      | (0.00540)       | (0.0685)        |
| Incentivized $\times$ $\hat{p}(x)$ | 0.150***       | 0.0978***       |
|                      | (0.00680)       | (0.00754)       |
| $\hat{p}(x)^2$       |                 | 0.194           |
|                      |                 | (0.126)         |
| $\hat{p}(x)^3$       |                 | 0.0319          |
|                      |                 | (0.0717)        |
| Constant             | 0.317***        | 0.372***        |
|                      | (0.00338)       | (0.0113)        |
| \textit{N}           | 358128          | 358128          |

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 5. Regressions with propensity score to control for selection bias. The coefficient on \textit{Incentivized} shows the average treatment effect of incentivized advertising on the \textit{Intermediate} outcome.
|                  | (1)       | (2)       | (3)       | (4)       |
|------------------|-----------|-----------|-----------|-----------|
| Install          | 0.00724***| 0.00795***| 0.00800***| 0.00187***|
|                  | (0.00111) | (0.00112) | (0.00113) | (0.000196) |
| \( \hat{p}(x) \) | 0.000597  | 0.140***  | 0.136***  | 0.0527***  |
|                  | (0.00230) | (0.0302)  | (0.0238)  | (0.00796)  |
| Incentivized \( \times \hat{p}(x) \) | -0.00811* | -0.00242  | -0.00480  | -0.00369***|
|                  | (0.00488) | (0.00539) | (0.00497) | (0.00121)  |
| \( \hat{p}(x)^2 \) | -0.253*** | -0.295*** | -0.101*** |
|                  | (0.0662)  | (0.0586)  | (0.0160)  |
| \( \hat{p}(x)^3 \) | 0.136***  | 0.183***  | 0.0548*** |
|                  | (0.0427)  | (0.0412)  | (0.00972) |
| Constant         | 0.00802***| -0.0132***| -0.00786***| -0.00477***|
|                  | (0.00128) | (0.00378) | (0.00248) | (0.00112)  |
| \( N \)          | 48390     | 48390     | 48266     | 358128     |

Standard errors in parentheses

\* \( p < 0.10 \), \*\* \( p < 0.05 \), \*\*\* \( p < 0.01 \)

**Table 6.** Regressions with propensity score to control for selection bias. In Columns (1) to (3), we conditioned on \( \text{Intermediate} = 1 \). In Column (3), the first-stage propensity scores are computed using only the subset of data such that \( \text{Intermediate} = 1 \).
|                          | (1)       | (2)       | (3)       | (4)       | (5)       |
|--------------------------|-----------|-----------|-----------|-----------|-----------|
|                          | Intermediate | Intermediate | Install | Install | Install |
| Incentivized             | -0.408*** | -0.307*** | 0.0984*** | 0.227*** | 0.271*** |
|                          | (0.00541)  | (0.00625) | (0.0228)  | (0.0270) | (0.0396) |
| Device Volume            |           |           |           |           |           |
|                          | 0.100***   | 0.0433    | -0.0482   |           |           |
|                          | (0.00926)  | (0.0371)  | (0.0569)  |           |           |
| Android Version          |           |           |           |           |           |
|                          | 0.0850***  | 0.0358*   | -0.0177   |           |           |
|                          | (0.00543)  | (0.0216)  | (0.0331)  |           |           |
| Screen Resolution        |           |           |           |           |           |
|                          | -0.0411*** | -0.0224   | 0.0343    |           |           |
|                          | (0.00496)  | (0.0200)  | (0.0293)  |           |           |
| WiFi                     |           |           |           |           |           |
|                          | -0.106***  | -0.136*** | 0.0872**  |           |           |
|                          | (0.00696)  | (0.0259)  | (0.0427)  |           |           |
| \(N\)                    | 365847     | 358087    | 365847    | 340662    | 45724     |
| Marginal Effects         | -0.0867*** | -0.0636***| 0.00088***| 0.00201***| 0.00859***|
|                          | (0.0011)   | (0.0013)  | (0.00020) | (0.00024) | (0.00129) |
| Countries controlled:    | No        | Yes       | No        | Yes       | Yes       |
| Languages controlled:    | No        | Yes       | No        | Yes       | Yes       |
| Device brands controlled:| No        | Yes       | No        | Yes       | Yes       |

Standard errors in parentheses

* \(p < 0.10\), ** \(p < 0.05\), *** \(p < 0.01\)

Table 7. Probit regressions without controlling for selection bias. In the last column, we condition on Intermediate = 1.
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