Warped Supersymmetric Grand Unification

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Abstract

We construct a realistic theory of grand unification in AdS$_5$ truncated by branes, in which the unified gauge symmetry is broken by boundary conditions and the electroweak scale is generated by the AdS warp factor. We show that the theory preserves the successful gauge coupling unification of the 4D MSSM at leading-logarithmic level. Kaluza-Klein (KK) towers, including those of XY gauge and colored Higgs multiplets, appear at the TeV scale, while the extra dimension provides natural mechanisms for doublet-triplet splitting and proton decay suppression. In one possible scenario supersymmetry is strongly broken on the TeV brane, in which case the lightest $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauginos are approximately Dirac and the mass of the lightest XY gaugino is pushed well below that of the lowest gauge boson KK mode, improving the prospects for its production at the LHC. The bulk Lagrangian possesses a symmetry that we call GUT parity. If GUT parity is exact, the lightest GUT particle, most likely an XY gaugino, is stable. Once produced in a collider, the XY gaugino hadronizes to form mesons, some of which will be charged and visible as highly ionizing tracks. The lightest supersymmetric particle is the gravitino of mass $\sim 10^{-3}$ eV, which is also stable if $R$ parity is conserved.
1 Introduction

Grand unification of all standard model forces is an extremely attractive idea that has been actively studied since it was first proposed in 1974 [1]. An important consequence of this idea is that it requires the unification of gauge interaction strengths at an extremely high energy [2]. When low energy supersymmetry is incorporated, the predicted value of $\sin^2 \theta_w$ agrees well with the experimentally observed value, strongly supporting the idea of supersymmetric grand unification [3]. It has recently been observed that the introduction of an extra dimension of size around the unified scale allows the construction of completely realistic grand unified theories preserving successful gauge coupling unification [4, 5, 6]. The doublet-triplet splitting problem is elegantly solved by orbifold boundary conditions [7], while problematic dimension four and five proton decay is absent because of the underlying $R$ symmetry structure of higher dimensional theories [4]. This framework also leads to a new level of precision for gauge coupling unification, improving the agreement between the prediction and data [5].

A common feature for all these theories is weak scale supersymmetry. The standard model particles are accompanied by $N = 1$ supersymmetric partners with masses around the TeV scale, and above this scale physics is effectively described by the minimal supersymmetric standard model (MSSM) up to an extremely high unified mass scale. This provides not only a solution to the gauge hierarchy problem but also a successful picture for electroweak symmetry breaking, which is triggered by radiative corrections to the Higgs mass squared parameter through the large value of the top Yukawa coupling [8]. Despite these remarkable successes, however, the paradigm is not free from difficulties. The absence of flavor violation beyond that of the standard model requires a specific pattern for supersymmetry breaking parameters, whose generation mechanism is not yet fully understood, and the failure to discover superparticles and the light Higgs boson at LEP seems to imply the necessity of some fine-tuning to obtain realistic electroweak symmetry breaking within the MSSM.

In this paper we study an alternative framework for implementing the idea of grand unification, which has the potential to alleviate the above problems of the conventional supersymmetric desert picture while preserving the desired features of grand unification. In this framework, $N = 2$ superpartners for the standard model particles arise at the TeV scale, together with their $SU(5)$ partners such as the XY gauge bosons and the colored Higgs particles. In fact, the theory becomes five dimensional above the TeV scale, which is characterized by the appearance of Kaluza-Klein (KK) towers for some of the standard model particles. Nevertheless, due to the conformal nature of the sector giving these KK towers, it is still meaningful to consider logarithmic unification of the three standard model gauge couplings. The possibility of such a theory was first suggested by Pomarol [9] in the setup of Randall and Sundrum for generating
the hierarchy between the weak and the Planck scales from the Anti-deSitter (AdS) warp factor [10]. Logarithmic evolution of the gauge couplings in AdS backgrounds was first discussed in Refs. [9, 11]. An effective field theory approach to gauge coupling evolution in AdS, based on gauge theory correlators whose external points are on the Planck brane, was obtained in Ref. [12]. Different methods of computing radiative corrections to the low-energy zero-mode gauge couplings were adopted in Ref. [13] using Pauli-Villars regularization, and in Ref. [14] using 4D effective supergravity. The issue of gauge coupling evolution in the AdS background has also been discussed recently in Refs. [15, 16, 17], and analyzed using deconstruction in [18]. However, a fully realistic theory of grand unification in AdS has not been constructed. Moreover, the successful prediction for gauge coupling unification has not explicitly been shown, although successful unification was anticipated in Ref. [9] based on heuristic arguments. In this paper we construct a realistic supersymmetric $SU(5)$ grand unified theory in a 5D truncated AdS background, which provides the same prediction for gauge coupling unification as the MSSM at the level of the leading logarithm. Since our theory is formulated in higher dimensions, we can employ various methods used to build realistic theories with unification scale extra dimensions, including mechanisms for obtaining doublet-triplet splitting and suppressing proton decay.

Our theory has the following features. (i) Successful gauge coupling unification is obtained: the predicted value for $\sin^2 \theta_w$ is the same as that of the MSSM at the leading-logarithmic level. (ii) A complete understanding of the MSSM Higgs sector is obtained; in particular, doublet and triplet components of the Higgs multiplets split automatically due to the boundary conditions while a large mass term for the Higgs doublets is forbidden by a $U(1)_R$ symmetry, which arises from the higher dimensional structure of the theory. A supersymmetric mass term for the Higgs doublets of order the weak scale is obtained through the AdS warp factor. (iii) There is no excessive proton decay: decays caused by exchange of the broken gauge bosons or the colored Higgs fields are suppressed while dangerous tree-level dimension four and five operators are forbidden by the $U(1)_R$ symmetry. (iv) There is a rich spectrum of new particles at the TeV scale, coming from the KK towers for the standard model fields and their supersymmetric and $SU(5)$ partners. These towers are approximately $SU(5)$ symmetric and (before supersymmetry breaking) also $N = 2$ supersymmetric. (v) Particularly interesting among these TeV states are the XY gauge bosons and gauginos, which we find may be produced at future hadron colliders, as suggested in Ref. [9]. The lightest of the $SU(5)$ partner states is stable (unless we break a certain bulk parity symmetry on the branes), and is most likely one of the XY gauginos. After produced, it hadronizes by picking up a quark, forming neutral and charged mesons. These meson states are sufficiently long lived so that they leave the detector without decaying, and the charged ones will be seen as highly ionizing tracks. (vi) The lightest supersymmetric particle (LSP) is the gravitino of mass $\sim 10^{-3}$ eV.
The theory is defined on a 5D warped spacetime with the metric

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2,$$

where $y$ is the coordinate for the extra dimension compactified on an $S^1/Z_2$ orbifold ($0 \leq y \leq \pi R$), and $k$ is the AdS curvature. We consider the scenario where hierarchy of the scales is generated by the AdS warp factor: $m_{\text{weak}} \ll k \sim M_{\text{Pl}}$ [10]. We take $k$ somewhat smaller than the 5D Planck scale $M_5$, so that the theory is perturbative between $k$ and $M_5$ and we can control the dynamics using the 5D AdS picture. (The 4D Planck scale, $M_{\text{Pl}}$, is given by $M_{\text{Pl}}^2 \simeq M_5^3 / k$). By choosing $kR \sim 10$, the weak scale is generated from the warp factor. In particular the fundamental mass scale on the $y = \pi R$ brane (TeV brane) is rescaled to the TeV scale, which also sets the masses for the KK towers of the bulk fields [19, 20, 21, 22]. In flat spaces it is known that a TeV scale extra dimension leads to new interesting possibilities for supersymmetry breaking and electroweak symmetry breaking [23, 24, 25, 26]. Locality in the extra dimension allows electroweak symmetry to be broken in a controllable and highly predictive way [24, 25], and the mass spectrum characteristic of higher dimensional supersymmetry breaking allows one to push the superpartner masses up to multi-TeV scales without fine-tuning [26]. An important feature of these theories is that the physics of supersymmetry and electroweak symmetry breaking is completely dominated by energies around the low-lying KK masses and is insensitive to physics at higher energies. Since our AdS theory appears five dimensional at a scale near the low-lying KK masses, we expect that the essential properties for electroweak breaking are not changed by making the spacetime AdS. For example, if we break supersymmetry on the TeV brane, the scale of KK masses is TeV [27], and we obtain a similar phenomenology to the corresponding theory with a TeV-scale flat extra dimension: the squarks and sleptons obtain finite and calculable masses at one loop, solving the supersymmetric flavor problem. This feature could be useful for building a theory of electroweak symmetry breaking without fine-tuning.

In section 2 we review gauge coupling running in an AdS background (we also present an alternative discussion of some basic features of theories on AdS in appendix A). In section 3 we construct our theory. Employing boundary condition breaking of the unified gauge symmetry, we find that the simplest theory is obtained with the following structure of the extra dimension: the TeV brane respects the full $SU(5)$ symmetry while the Planck brane respects only the standard model gauge symmetry. We present a fully realistic grand unified theory (GUT) in 5D truncated AdS space, including a discussion of supersymmetry breaking and electroweak symmetry breaking. We also discuss how some features of our theory, in particular the role of $SU(5)$ symmetry, manifest themselves as purely 4D mechanisms in the dual description suggested by the AdS/CFT correspondence. In section 4 we discuss various phenomenological issues, especially the possibility of producing GUT particles at future collider experiments. Conclusions
are drawn in section 5.

2 Gauge Coupling Evolution in AdS

We begin by reviewing the aspects of field theory in compactified AdS backgrounds that are relevant to understanding the evolution of bulk gauge couplings. Our discussion here is mainly a summary of the arguments presented in [12, 16]. A different discussion of some general properties of field theory in AdS, based on the 4D KK picture, is presented in appendix A. Readers familiar with the recent literature on gauge coupling evolution in compactified AdS may wish to skip directly to Eqs. (7, 9, 10), which contain the results that we employ in section 3.

First consider gauge fields compactified in flat 5D space. We would like to understand the extent to which gauge theory observables (for instance correlators of bulk fields) are calculable in an effective field theory context. For energies below the compactification scale all correlators reduce to the Green’s functions of the KK zero modes of bulk gauge fields. As long as the effective 4D gauge coupling is small enough, then it is possible to reliably calculate observables in a weakly coupled 4D description. However, as energies become larger than the compactification scale $1/R$, bulk field correlators acquire corrections that grow as powers of the external momenta in a given process.\footnote{If the 4D coupling is fixed to be order one, then the power law growth of correlators is saturated at a scale which is roughly a loop factor above the compactification scale. At such energies, effective field theory methods are no longer reliable (higher dimensional operators with any number of derivatives contribute equally) and predictivity is lost.}

At first sight, the situation in compactified AdS does not appear much different. Suppose one is interested in computing quantum corrections to the gauge coupling of the KK zero mode of a bulk gauge field. In compactified AdS, the low-lying KK excitations of bulk fields have masses of order the scale $T \equiv ke^{-\pi kR}$. 4D observers would interpret this scale as an effective compactification scale even though the true compactification radius, the proper distance between the branes, is much smaller, of order $k^{-1}$. Thus for the same reason as in flat space field theories, the zero-mode gauge coupling observable becomes strongly coupled at a scale near $T$. For instance, higher dimensional operators give rise to tree-level corrections to this quantity that grow as powers of $q^2/T^2$, where $q^2$ is the external momentum.

If effective field theory breaks down in AdS above the scale $T$, one is forced to conclude that the concept of coupling unification at a large scale, say of order $k$, is not meaningful. For instance, it would be impossible to verify whether gauge couplings, as defined by a suitable
gauge theory correlator, meet at a high scale, since such correlators are not well defined in a field theory context. However, as emphasized in [12, 16], this is not actually the case. While zero-mode correlators do become strongly coupled at a scale which is order $T$, there exist correlators of bulk fields that are weakly coupled up to much higher energies. What makes this possible is the warp factor of the AdS metric. General covariance implies that in terms of energy scales measured on the Planck brane (using the canonical flat metric $\eta_{\mu\nu}$), a correlator whose external points are located at a point $y$ in the bulk is perturbative up to an energy given by

$$E(y) \sim M_5 \exp(-ky),$$

where $M_5$ is the 5D Planck scale. In particular, Planck brane localized gauge theory correlators (such as the one in Fig. 1) can be calculated reliably in a field theory framework up to extremely high energies.

A priori, one would think that this observation is not useful for working out the consequences that GUT symmetry imply for 4D observers, which only have experimental access to the correlators of bulk field zero modes. However, the Planck correlators become indistinguishable from the Green’s functions of zero modes as the external momenta are lowered below the KK mass gap. It follows from this that there is a calculable relation between the UV couplings of the bulk theory (as defined through Planck observables) and the parameters measured at low energy. Consequently, high energy GUT symmetry in an AdS$_5$ background compactified by branes makes definite predictions for low energy data.

It turns out that the external momentum dependence of the one-loop corrections to the Planck brane gauge field two-point function is logarithmic for a wide range of energies smaller than $k$. To compute Planck correlators, it is most convenient to work out the relevant Feynman diagrams in
a mixed position/4D momentum basis for bulk field propagators and vertices. Then to calculate
the diagram of Fig. 1 one needs to convolve the boundary-to-bulk gauge boson propagators of
the external gauge fields (that is, propagators with one point on the Planck brane and one point
at an arbitrary location in the bulk) with the propagators of the bulk field appearing inside the
loop. At (Euclidean) external momenta much larger than $T$, the boundary-to-bulk gauge boson
propagators are, up to terms that are pure gauge, given by

$$D_p(z)_{\mu\nu} \sim \frac{(kz) K_1(pz)}{p K_0(p/k)} \eta_{\mu\nu},$$  \hspace{1cm} (3)

where we have used AdS conformal coordinates, related to those of Eq. (1) by $kz = \exp(ky)$
($K_{0,1}(x)$ are modified Bessel functions). Since $pz \gg 1$ for points far from the Planck brane, we
may use the asymptotic expansion $K_1(pz) \sim \sqrt{\pi/(2pz)} \exp(-pz)$ to see that, in general, the $z$
integral over the two internal vertices receives most of its support on the Planck brane. Roughly
speaking, it is then possible to replace $z$ integrals by an expression that involves only the internal
line propagators evaluated at the position of the Planck brane. At this point only the integrals
over 4D loop momentum remain. Since the number of loop integrations is reduced relative to
those of a 5D Feynman graph, one expects that the ultraviolet power counting of Planck brane
correlators is essentially that of a 4D theory, leading to logarithmic evolution of gauge couplings.
The computation outlined here has been performed explicitly for one-loop corrections due to
charged bulk scalars in Ref. [16]. In that case, one indeed finds that the gauge coupling defined
in terms of the Planck brane gauge field two-point function runs logarithmically as a function of
the external momentum (with a coefficient that is identical to the 4D beta function contribution
of a complex scalar field). Similar results are expected to hold in theories with a more realistic
matter content.

When worked out more explicitly [16], these arguments lead to a simple effective field theory
understanding of the large logarithms of UV scales that may arise in the computation of low
energy couplings in AdS backgrounds. According to this picture, one simply runs the effective
Planck brane $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge couplings from a high scale of order the curvature
scale $k$ down to energies of order $T$, and then matches this quantity to the coupling constants
measured by 4D observers. In order to make low energy predictions, one must make certain
assumptions regarding the values of the couplings at the UV matching scale. These initial
conditions depend on the specific mechanism by which GUT symmetry is broken in the UV.
In models in which the GUT group is broken by scalar vacuum expectation values (either in
the bulk or on branes) tree-level matching at the symmetry breaking scale implies equality of
the standard model high energy couplings. Alternatively, the bulk gauge symmetry could be
broken by orbifold boundary conditions. In this case, operators localized on the boundaries of
the space need not respect the underlying gauge symmetry of the theory. Consequently, one
cannot strictly claim that the gauge couplings of the UV theory are unified. Nevertheless, under reasonable assumptions about the high energy dynamics of the theory (see below), it is possible to argue that possible differences in the high scale standard model gauge couplings do not affect the low energy prediction. In this case, the leading one-loop logarithms are meaningful, and predictivity of the theory is maintained. In this paper we will only consider GUT models with the unified gauge symmetry broken by orbifold boundary conditions.

Of course, given the bulk field content of the 5D theory, it is possible to work out the low energy predictions of GUTs without recourse to effective field theory. Since in the end we are interested in evaluating loop corrections to the gauge couplings evaluated at the weak scale, we could always compute them in a non-decoupling scheme, in which KK excitations of arbitrarily high 4D mass are kept in the loops. As long as the external momentum is smaller than the KK mass gap, such a computation is insensitive to the uncalculable tree-level contributions of higher-dimension operators, and gives reliable results for gauge couplings evaluated at the weak scale. While the effective field theory picture based on renormalization group (RG) flows of couplings measured by Planck brane correlators is physically more intuitive, in the remainder of this paper we will employ the technically more direct non-decoupling approach for calculating loop corrections to 4D couplings.

To be more specific, we will define effective momentum dependent low energy gauge couplings in terms of the quadratic term in the 1PI action for the gauge field zero modes. It can be calculated by simply summing the KK modes of AdS bulk fields in loops with external zero-mode gauge bosons. Employing dimensional regularization, this has been done for scalar fields in [12, 15, 16] and for fermi fields and non-Abelian gauge bosons in [17] (charged scalars were also treated by the Pauli-Villars method in [13]). We now summarize the relevant aspects of these results. Begin by writing the classical gauge field action in 5D as

\[ S = -\frac{1}{4} \int d^4x \, dy \sqrt{-\mathbf{G}} \left[ \frac{1}{g_5^2} F_{MN} F^{MN} + \delta(y) \lambda_0^a F_{\mu\nu}^a F^{a\mu\nu} + \delta(y - \pi R) \lambda_\pi^a F_{\mu\nu}^a F^{a\mu\nu} \right], \quad (4) \]

where the index \( a \) runs over \( SU(3)_C, \, SU(2)_L \) and \( U(1)_Y \) (also, \( M, N \) run over the full set of 5D coordinates, while \( \mu, \nu \) only run over the 4D Poincare coordinates). The parameter \( g_5 \) is the 5D gauge coupling, with \( [g_5] = -1/2 \), and the dimensionless couplings \( \lambda_0^a, \lambda_\pi^a \) are the coefficients of brane-localized gauge field strength operators. Simple power counting indicates the couplings \( \lambda_0^a, \lambda_\pi^a \) receive logarithmically divergent corrections starting at one-loop, and therefore exhibit non-trivial RG flows. Because these logarithmic divergences are inherently short distance effects, they are not sensitive to the spacetime curvature, and consequently the RG equations for the boundary couplings take the exact same form as they do in flat spacetime [12, 16]. The coupling \( g_5 \) gets linearly divergent loop-corrections and therefore does not run. Thus given the parameters
of Eq. (4), one may express the low energy one-loop gauge couplings as

$$\frac{1}{g_a^2(q)} = \frac{\pi R}{g_5^2} + \lambda_0^a(\mu) + \lambda_a^a(\mu) + \frac{1}{8\pi^2} \tilde{\Delta}^a(q, \mu),$$  \hspace{1cm} (5)

where the first three terms are tree-level contributions, and $\tilde{\Delta}^a(q, \mu)$ represents the one-loop corrections (we give an explicit form for this term in the case of supersymmetric models below). The explicit dependence of $\tilde{\Delta}^a(q, \mu)$ on the subtraction scale $\mu$ cancels that of the running boundary couplings $\lambda_{0,\pi}^a(\mu)$ in such a way that the quantity $g_a^2(q)$ is independent of the renormalization scale.

In theories with unified gauge symmetry broken by orbifold boundary conditions, the boundary terms $\lambda_{0,\pi}^a$ do not have to respect the full unified symmetry of the bulk theory. Therefore, it is non-trivial that we can obtain a prediction for gauge coupling unification in these theories. However, the prediction for gauge coupling unification is recovered if the volume of the extra dimension is large compared with the cutoff scale of the theory [4]. In particular, if the theory is strongly coupled at the cutoff scale $\Lambda$, the size of uncalculable contribution from (potentially) $SU(5)$-violating brane terms is reliably estimated using naive dimensional analysis (NDA), giving a highly predictive class of higher dimensional theories [5, 28]. In flat spacetime, the NDA assumption implies that $\lambda_0^a(\Lambda) \simeq \lambda_0^a(\Lambda) \simeq 1/16\pi^2$, where the scale $\Lambda$ is the strong coupling scale estimated to be $\Lambda \sim 16\pi^3/g_5^2 = 16\pi^2/g_4^2 R \sim 16\pi^2/R$. Since the contribution from the bulk gauge coupling is given by $\pi R/g_5^2 = 1/g_4^2 = O(1)$, unknown $SU(5)$-violating contributions, which are encoded into $\lambda_{0,\pi}^a(\Lambda)$, are suppressed by a loop factor without a large logarithm. The boundary couplings at the scale $\mu$ are then given using the RG equations $d\lambda_{0,\pi}(\mu)/d\ln\mu = -\tilde{b}_{0,\pi}^a/8\pi^2$. The beta function coefficient $\tilde{b}_{0,\pi}^a$ is given by $1/4 (-1/4)$ of the one-loop coefficient of a 4D gauge coupling if the corresponding field satisfies even (odd) boundary conditions at $y = 0, \pi R$ [29].

In AdS, the situation is slightly more subtle. As previously mentioned, warping implies that a correlator whose endpoints are localized at a given position in the bulk space becomes strongly coupled at a scale given by Eq. (2). Clearly, then, the NDA scale is also correlated to bulk location. If we denote the NDA scale on the Planck brane by $\Lambda$, then we again expect $\lambda_0^a(\Lambda) \simeq 1/16\pi^2$. Likewise, $\lambda_0^a(\Lambda e^{-\pi k R}) \simeq 1/16\pi^2$. The scale $\Lambda$ is obtained using NDA as $\Lambda \sim 16\pi^2/R$, which we identify with the 5D Planck scale $M_5$. Rewriting $\lambda_{0,\pi}^a(\mu)$ using the RG equations $\lambda_0^a(\mu) = \lambda_0^a(\Lambda) + (\tilde{b}_0^a/8\pi^2) \ln(\Lambda/\mu)$ and $\lambda_0^a(\mu) = \lambda_0^a(\Lambda e^{-\pi k R}) + (\tilde{b}_0^a/8\pi^2) \ln(\Lambda e^{-\pi k R}/\mu)$ (recall that the RG equations for the boundary couplings are still the same as the flat space case even in the presence of background curvature), we ensure that we do not miss any of the potentially large non-universal logarithms that arise in the low energy prediction. Specifically, defining $\Delta^a(q,\Lambda) \equiv \tilde{\Delta}^a(q, \mu) + \tilde{b}_0^a \ln(\Lambda/\mu) + \tilde{b}_0^a \ln(\Lambda e^{-\pi k R}/\mu)$, we obtain

$$\frac{1}{g_a^2(q)} = \frac{\pi R}{g_5^2} + \lambda_0^a(\Lambda) + \lambda_0^a(\Lambda e^{-\pi k R}) + \frac{1}{8\pi^2} \Delta^a(q, \Lambda).$$  \hspace{1cm} (6)
The point is that, by rewriting in this way, we can explicitly see that (potentially) $SU(5)$-violating brane terms, $\lambda^a_s(\Lambda) \simeq \lambda^a_s(\Lambda e^{-\pi k R}) \simeq 1/16\pi^2$ are smaller than the $SU(5)$ invariant bulk term, $\pi R/g_s^2 \simeq 1/g_s^2 = O(1)$, by a one-loop factor (without a large logarithm), so that the first three terms are approximately $SU(5)$ symmetric:

$$\frac{1}{g_s^2(q)} \simeq (SU(5) \text{ symmetric}) + \frac{1}{8\pi^2} \Delta^a(q,\Lambda). \quad (7)$$

Therefore, once $\Delta^a(q,\Lambda)$ is given, we can obtain a prediction for gauge coupling unification.

In the following section we will need the result for the quantity $\Delta^a(q,\Lambda)$ for supersymmetric theories. The 5D gauge supermultiplet $\mathcal{V} = \{A_M, \lambda, \lambda', \sigma\}$ consists of a 5D vector field, $A_M$, two gauginos, $\lambda$ and $\lambda'$, and a real scalar, $\sigma$. For convenience, we decompose these fields into a 4D vector supermultiplet, $V(A_M, \lambda)$ and a 4D chiral supermultiplet $\Sigma((\sigma + i A_5)/\sqrt{2}, \lambda')$. Similarly, a 5D hypermultiplet $\mathcal{H}_\Phi = \{\phi, \phi^c, \psi, \psi^c\}$, which consists of two complex scalars, $\phi$ and $\phi^c$, and two Weyl fermions, $\psi$ and $\psi^c$, are decomposed into two chiral superfields $\Phi(\phi, \psi)$ and $\Phi^c(\phi^c, \psi^c)$. A hypermultiplet can have an arbitrary mass term in the bulk, which we parametrize by $c$. In component language,

$$S = \int d^4x \, dy \, \sqrt{-G} \left[ -\{|\partial_M \phi|^2 + m^2_\phi |\phi|^2\} - \{|\partial_M \phi^c|^2 + m^2_\phi^c |\phi^c|^2\} - \{i \bar{\Psi} \gamma^M D_M \Psi + im_\Psi \bar{\Psi} \Psi\} \right], \quad (8)$$

with $m_{\phi,\phi^c} = (c^2 \pm c - 15/4) k^2 + (3 \mp 2c) k [\delta(y) - \delta(y - \pi R)]$ and $m_\Psi = c k \epsilon(y)$, where $\Psi = \{\psi, \psi^{c\dagger}\}$ is a Dirac field [27]. The quantity $\Delta^a(q,\Lambda)$ also depends on the boundary conditions imposed on the 5D fields. Since the extra dimension is the line segment $y : [0, \pi R]$, the boundary conditions for a field $\varphi$ are specified by the two parities $p = \pm 1$ and $p' = \pm 1$ imposed at $y = 0$ and $y = \pi R$: $\varphi(-y) = \rho \varphi(y)$ and $\varphi(-y') = \rho' \varphi(y')$, where $y' \equiv y - \pi R$. We can now present the contributions to $\Delta^a(q,\Lambda)$ coming from gauge multiplets and hypermultiplets. These results are taken from Ref. [17]. Denoting the parity transformations by the subscript as $\varphi_{pp'}$, the contribution from the gauge multiplet is given by

$$\Delta^a(q,k)|_V = -T_a(V_{++}) \left[ 3 \ln \left( \frac{k}{q} \right) - \frac{3}{2} \ln \left( \frac{k}{T} \right) \right]$$

$$-\frac{3}{2} T_a(V_{+-}) \ln \left( \frac{k}{T} \right) + \frac{3}{2} T_a(V_{-+}) \ln \left( \frac{k}{T} \right)$$

$$+ T_a(V_{--}) \left[ \ln \left( \frac{k}{q} \right) + \frac{1}{2} \ln \left( \frac{k}{T} \right) \right], \quad (9)$$

where $T_a(V_{pp'})$ is the sum of the Dynkin index for the 5D gauge supermultiplet whose components in $V$ have $p$ and $p'$ parities at $y = 0$ and $\pi R$, respectively. In this equation, we have taken $q \ll T \ll k \sim \Lambda$ and dropped small scheme dependent constants which do not have a large
logarithm. Bulk hypermultiplets yield
\[
\Delta^a(q,k)|_{H_a} = T_a(\Phi^{++}) \left[ \ln \left( \frac{k}{q} \right) - c_{++} \ln \left( \frac{k}{T} \right) - \ln \left( \frac{e^{(1-2c_{++})\pi kR} - 1}{\pi (1 - 2c_{++})} \right) \right] \\
+ c_{+-} T_a(\Phi_{+-}) \ln \left( \frac{k}{T} \right) - c_{--} T_a(\Phi_{--}) \ln \left( \frac{k}{T} \right) \\
+ T_a(\Phi_{--}) \left[ \ln \left( \frac{k}{q} \right) + c_{--} \ln \left( \frac{k}{T} \right) - \ln \left( \frac{e^{(1+2c_{--})\pi kR} - 1}{\pi (1 + 2c_{--})} \right) \right],
\]
where \( T_a(\Phi_{pp'}) \) is the sum of the Dynkin index for the 5D hypermultiplets whose components in \( \Phi \) have \( p \) and \( p' \) parities at \( y = 0 \) and \( y = \pi R \). In the next section we construct realistic unified theories on AdS and show that they predict the same \( \sin^2 \theta_w \) as the 4D MSSM at the leading-log level.

3 The Theory

In this section we present our theory. We construct realistic 5D \( SU(5) \) models in which \( N = 2 \) supersymmetric KK towers for the standard-model and GUT particles appear at the TeV scale while logarithmic gauge coupling unification gives the same successful prediction for \( \sin^2 \theta_w \) as the MSSM at the leading-log level. In subsection 3.1 we present the basic structure of our theory, putting matter fields on the Planck brane. In subsection 3.2 we construct a more satisfactory model in which matter fields are located in the bulk but strongly localized to the Planck brane by the bulk masses. Supersymmetry breaking and electroweak symmetry breaking are discussed in subsection 3.3. We also discuss an alternative 4D picture of our theory, based on AdS/CFT duality, in subsection 3.4.

3.1 Basic structure of the theory

We consider supersymmetric 5D \( SU(5) \) gauge theory compactified on the orbifold \( S^1/Z_2 \) in the AdS space of Eq. (1). There are two different possibilities for the breaking of \( SU(5) \): breaking by the Higgs mechanism or by boundary conditions. Here we adopt boundary condition breaking, which provides natural mechanisms for obtaining doublet-triplet splitting and proton decay suppression. Note that proton decay is potentially problematic in unified theories on AdS because the mass of the lightest XY gauge bosons is in the TeV region \( \approx T \equiv k e^{-\pi kR} \).

\(^2\)This equation does not give correct gauge coupling values at \( q \lesssim T \) for \( c_{+-} > 1/2 \) or \( c_{-+} < -1/2 \), due to the appearance of extra light states with masses exponentially smaller than \( T \). An appropriate formula for these cases is obtained by replacing the second and third terms by \( T_a(\Phi_{+-})[2 \ln(k/q) - (1 + c_{+-}) \ln(k/T)] \) and \( T_a(\Phi_{--})[2 \ln(k/q) - (1 - c_{-+}) \ln(k/T)] \), respectively.
Using 4D $N = 1$ superfield language, in which the gauge degrees of freedom are contained in $V(A_{\mu}, \lambda)$ and $\Sigma((\sigma + iA_{3})/\sqrt{2}, \lambda')$, the boundary conditions for the 5D gauge multiplet are given by

$$\left( \begin{array}{c} \nu \\ \lambda \end{array} \right) \left( x^\mu, -y \right) = \left( \begin{array}{c} PV^{P-1} \\ -P\Sigma P^{-1} \end{array} \right) \left( x^\mu, y \right),$$

$$\left( \begin{array}{c} \nu \\ \lambda \end{array} \right) \left( x^\mu, -y' \right) = \left( \begin{array}{c} PV^{P'-1} \\ -P\Sigma P^{-1} \end{array} \right) \left( x^\mu, y' \right),$$

(11)

where $y' = y - \pi R$, and $P$ and $P'$ are $5 \times 5$ matrices acting on gauge space. There are three different choices for the boundary conditions that break $SU(5)$ down to $SU(3)_C \times SU(2)_L \times U(1)_Y$: $(P, P') = (1, B), (B, 1)$ and $(B, B)$ where $1 = \text{diag}(+ , + , + , + , + )$ and $B = \text{diag}(+, +, +, + , +, +, +, +, +)$. We first consider the case $(P, P') = (B, B)$. In this case the off-diagonal components of the $\Sigma$ field have even parity at both $y = 0$ and $y = \pi R (p = p' = 1)$. This gives massless fermions whose quantum numbers are those of the XY gauge bosons, so we do not consider this case further.\(^3\)

Next, we require that this gauge sector gives the same “beta functions” as the MSSM to reproduce the correct prediction for $\sin^2 \theta_w$. In the case of $(P, P') = (1, B)$, the $SU(3)_C \times SU(2)_L \times U(1)_Y$ vector multiplets, $V^{321}$, have the parities $(p, p') = (+, +)$ and the $SU(5)/(SU(3)_C \times SU(2)_L \times U(1)_Y)$ ones, $V^{XY}$, have $(p, p') = (+, -)$. Therefore, we obtain $(T_1, T_2, T_3)(V_{++}) = (0, 2, 3), (T_1, T_2, T_3)(V_{+-}) = (5, 3, 2)$ and $(T_1, T_2, T_3)(V_{--}) = (0, 0, 0)$. Using Eq. (9), and setting $k \sim \Lambda$, we find $(\Delta^1, \Delta^2, \Delta^3)(q, k)|_\nu \simeq (0, -6, -9) \ln(T/q) + (SU(5)$ symmetric). This means, in a sense, that the running caused by the gauge multiplet above the TeV scale is $SU(5)$ symmetric, and a value of $\sin^2 \theta_w$ at the weak scale is grossly incompatible with data.\(^4\)

Thus, we are finally left with the possibility $(P, P') = (B, 1)$. In this case $(T_1, T_2, T_3)(V_{++}) = (0, 2, 3), (T_1, T_2, T_3)(V_{+-}) = (5, 3, 2)$ and $(T_1, T_2, T_3)(V_{--}) = (0, 0, 0)$. And we find that

$$\left( \begin{array}{c} \Delta^1 \\ \Delta^2 \\ \Delta^3 \end{array} \right) (q, k)|_\nu \simeq \left( \begin{array}{c} 0 \\ -6 \\ -9 \end{array} \right) \ln \left( \frac{k}{q} \right) + (SU(5)$ symmetric).$$

(12)

This is exactly the relation we obtain in the MSSM, which gives a successful prediction for gauge coupling unification. We therefore see that the “geometry” of the fifth dimension is fixed by gauge coupling unification: the Planck brane is the “$SU(3)_C \times SU(2)_L \times U(1)_Y$ brane” and the TeV brane is the “$SU(5)$ brane”. A picture of this extra dimension is drawn in Fig. 2.

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\(^3\)It may be possible to construct a realistic theory with $(P, P') = (B, B)$ if we give masses for the XY gauginos (and scalars) coming from $\Sigma$, through supersymmetry breaking effects. Such a theory would give the correct $\sin^2 \theta_w$ prediction, since the contribution from $\Sigma$ (and colored Higgs triplets with $c \geq 1/2$) to the relative gauge coupling running shuts off above the TeV scale (see Eqs. (9, 10)).

\(^4\)This can also be understood [15, 16] in terms of the 4D conformal field theory (CFT) picture dual to the 5D AdS theory. The breaking of 5D $SU(5)$ by the boundary conditions on the TeV brane corresponds to breaking 4D $SU(5)$ symmetry by the strong CFT dynamics at the TeV scale. Since the breaking occurs at the TeV scale, the gauge coupling evolution above the TeV scale is completely $SU(5)$ symmetric.
Now we consider the Higgs sector of our model. We introduce two Higgs hypermultiplets in the bulk, transforming as 5 and $\bar{5}$ under $SU(5)$: $\{H, H^c\} (5) + \{\bar{H}, \bar{H}^c\} (\bar{5})$. (In our notation $H$ and $H^c$ transform as 5, and $\bar{H}$ and $\bar{H}^c$ transform as $\bar{5}$.) The boundary conditions for the $\{H, H^c\}$ fields are given by

$$\begin{pmatrix} H \\ H^c \end{pmatrix} (x^\mu, -y) = \eta_H \begin{pmatrix} BH \\ -BH^c \end{pmatrix} (x^\mu, y), \quad \begin{pmatrix} H \\ H^c \end{pmatrix} (x^\mu, -y') = \begin{pmatrix} H \\ -H^c \end{pmatrix} (x^\mu, y'),$$

(13)

and similarly for $\{\bar{H}, \bar{H}^c\}$. We choose the boundary conditions so that we have two Higgs doublet chiral superfields as zero modes: $\eta_H = \eta_{\bar{H}} = -1$, which gives $(T_1, T_2, T_3)(H_{++}) = (3/10, 1/2, 0)$, $(T_1, T_2, T_3)(H_{+-}) = (1/5, 0, 1/2)$ and $(T_1, T_2, T_3)(H_{-+}) = (T_1, T_2, T_3)(H_{-}) = (0, 0, 0)$ and the same for $\bar{H}$. We then find from Eq. (10) that the contribution from the Higgs multiplets are independent from the bulk hypermultiplet mass terms $c_H$ and $c_{\bar{H}}$ as long as $c_H \geq 1/2$ and $c_{\bar{H}} \geq 1/2$, which we assume to be the case. For these values of $c_H$ and $c_{\bar{H}}$, we obtain

$$\begin{pmatrix} \Delta^1 \\ \Delta^2 \\ \Delta^3 \end{pmatrix} (q, k)|_{H_H + H_{\bar{H}}} \simeq \begin{pmatrix} 3/5 \\ 1 \\ 0 \end{pmatrix} \ln \left( \frac{k}{q} \right) + (SU(5) \text{ symmetric}).$$

(14)

This is again the relation we find in the MSSM. Therefore, we conclude that our theory gives the same prediction for $\sin^2 \theta_w$ as the MSSM at the leading-log level, provided that the contribution from matter fields is $SU(5)$ symmetric.
Next we discuss the matter fields. We first consider a simple case where all the matter fields are localized on the Planck brane, postponing the case with bulk matter until the following subsection. Since the Planck brane respects only $SU(3)_C \times SU(2)_L \times U(1)_Y$, we introduce three families of quark and lepton chiral superfields $Q(3, 2)_{1/6}$, $U(3, 1)_{-2/3}$, $D(3, 1)_{1/3}$, $L(1, 2)_{-1/2}$ and $E(1, 1)_1$ on that brane. Here we have normalized hypercharge so that it matches standard convention: $Q_{EM} = T_3 + Y$ where $Q_{EM}$, $T_3$, and $Y$ are electric charge, the third-component of isospin, and hypercharge. The Yukawa couplings are written on the Planck brane as

$$S = \int d^4x \, dy \sqrt{-G} \delta(y) \left[ \int d^2\theta \left( y_u Q U H_D + y_d Q D \bar{H}_D + y_e L E \bar{H}_D \right) + \text{h.c.} \right],$$

where $H_D$ and $\bar{H}_D$ represent the doublet components of $H$ and $\bar{H}$, respectively. Note that with $c_H, c_{\bar{H}} \geq 1/2$ the wavefunctions for the Higgs fields, $H$ and $\bar{H}$, are either conformally flat or peaked at the Planck brane. Thus the resulting 4D Yukawa couplings do not receive disastrous exponential suppressions due to the profiles of the Higgs wavefunctions. Note also that since the gauge symmetry on the $y = 0$ brane is only $SU(3)_C \times SU(2)_L \times U(1)_Y$, we do not have unwanted $SU(5)$ mass relations such as $m_s/m_d = m_{\mu}/m_e$.

Matter fields localized on the Planck brane contribute to the running of the gauge couplings in the usual 4D way, since the physics is completely four-dimensional up to the scale $k$ on the $y = 0$ brane. Alternatively, this arises because the brane fields at $y = 0$ are elementary in the dual 4D CFT picture. Therefore, we obtain the usual 4D contribution from matter fields to the running of the gauge couplings, and the matter contribution to the “beta function” is $SU(5)$ symmetric. Together with the contributions from the gauge and Higgs fields, Eqs. (12, 14), we find that the prediction for gauge coupling unification (i.e. $\sin^2 \theta_w$) in our model is completely the same as the usual MSSM at the leading-log level. The scale of unification is around $10^{16}$ GeV, so $k$ and $\Lambda \sim M_5$ must fall near this scale, presumably with $k \sim 10^{16} - 10^{17}$ GeV and $\Lambda \sim M_5 \sim 10^{17} - 10^{18}$ GeV. Fortunately, with these values, one obtains roughly the correct size for the 4D Planck scale, $M_{Pl} \sim (M_5^2/k)^{1/2}$, so we do not have to introduce a new scale to explain the observed value of $M_{Pl}$.

Having demonstrated successful gauge coupling unification, we now turn to a discussion of phenomenological issues. First, we note that the masses for the XY bosons and colored Higgs

Technically, the running caused by a brane field can be understood as follows. Suppose we consider a real scalar field $\phi$. Then, regardless of where this field is located, $\Delta^a(q, \mu)$ in Eq. (5) is given by $\Delta^a(q, \mu) = (T_a(\phi)/6) \ln(\mu/q) + O(1)$. If this field is localized on the $y = 0$ brane, it causes the brane coupling $\lambda^a_\pm$ to run but not $\lambda^a_\mp$, so that we have $\lambda^a_\pm(\mu) = \lambda^a_\mp(\Lambda) + (T_a(\phi)/48\pi^2) \ln(\Lambda/\mu)$ and $\lambda^a_\pm(\mu) = \lambda^a_\mp(\Lambda e^{-\pi k R})$. This gives $\Delta^a(q, \Lambda) = (T_a(\phi)/6) \ln(\Lambda/q) + O(1)$, showing that brane fields at $y = 0$ contribute to the running exactly as in the 4D case. On the other hand, if $\phi$ is localized at $y = \pi R$, we get $\lambda^a_\pm(\mu) = \lambda^a_\mp(\Lambda)$ and $\lambda^a_\pm(\mu) = \lambda^a_\mp(\Lambda e^{-\pi k R}) + (T_a(\phi)/48\pi^2) \ln(\Lambda e^{-\pi k R}/\mu)$, so that $\Delta^a(q, \Lambda) = (T_a(\phi)/6) \ln(\Lambda e^{-\pi k R}/q) + O(1)$ and the running is absent above the scale $\Lambda e^{-\pi k R} \sim$ TeV. These results are consistent with the dual CFT interpretation, in which brane fields localized at $y = 0$ are elementary while those at $y = \pi R$ are composite fields arising from the strong TeV scale dynamics.
fields are both at the TeV scale \( \simeq T \) in our theory, so that there is potentially the danger of rapid proton decay. However, the wavefunctions for these fields vanish on the Planck brane, where matter fields live, so there is no direct coupling between the matter fields and the XY gauge bosons or colored Higgs fields. There may still be couplings of the \( y \) derivative of, e.g. the XY gauge fields to matter. Even if present, though, these terms are typically not problematic.\(^6\)

Consider an interaction term of the form

\[
S \sim \int d^4x \sqrt{-G} \delta(y) \frac{g_5}{M_5} \bar{\psi} \gamma^\mu (\partial_\mu A^{XY}_\mu) \psi,
\]

where \( \psi \) represents standard model fermions. In terms of a KK decomposition, the amplitude for exchange of the field \( A^{XY}_\mu \) is proportional to

\[
\left( \frac{g_5^2}{M_5^2} \right) \sum_n \left( \frac{Y_0(m_n/k)}{\sqrt{Y_1(m_n/k)/Y_0(m_n/T)}} \right)^2 - 1 \right)^{-1},
\]

where \( m_n \) are the masses of the XY gauge boson KK modes. Since for \( m_n \gg k \) each term in the sum is approximately \( n \)-independent (and of order \( T/k \)), we find that the amplitude diverges linearly with \( n \). This divergence is absorbed into a four-fermion counterterm in the tree-level Lagrangian, which means that the proton decay rate is not strictly calculable in this model. However, one can still estimate the rate: one expects that the coefficient of the four-fermi operator is of order \( 1/M_5^2 \) (this is what we would obtain, for instance, if we had performed the sum of XY gauge KK towers with masses up to \( M_5 \)). We can therefore be assured that the rate for a process such as \( p \rightarrow e^+ \pi^0 \) is guaranteed to be small enough for the model to be viable.

Alternatively, we could impose global baryon number on the theory, in which case the proton becomes completely stable. In any event, we find that our theory does not suffer from the problem of excessive proton decay, despite the presence of TeV scale XY gauge fields. On the other hand, their presence (and that of the colored Higgs fields) leads to the interesting possibility that such states will be discovered at future collider experiments such as the LHC. We discuss this issue in more detail in section 4.

### 3.2 Model with matter in the bulk

In the previous subsection we constructed a model in which the quarks and leptons are localized on the Planck brane, where only \( SU(3)_C \times SU(2)_L \times U(1)_Y \) is manifest. Strictly speaking, however, this model does not automatically give a successful prediction for gauge coupling unification. Because the matter fields need only respect \( SU(3)_C \times SU(2)_L \times U(1)_Y \), the normalization of hypercharge is not fixed: we can have an arbitrary value for \( \alpha \) in \( Q(3,2)_{\alpha/6} \), \( U(3,1)_{-2\alpha/3} \), \( D(\bar{3},1)_{\alpha/3} \), \( L(1,2)_{-\alpha/2} \) and \( E(1,1)_{\alpha} \). To obtain the successful prediction for gauge coupling unification, as well as Yukawa couplings to the Higgs fields as in Eq. (15), we must choose \( \alpha = 1 \);

\(^6\)There could also be tree-level proton decay operators on the Planck brane at dimension four and five, \( \delta(y)[QDL + UDD + QQQL + UUDE]_{\beta} \), which could cause overly rapid proton decay. These operators, however, can be forbidden by imposing a continuous \( U(1)_R \) symmetry whose charge assignment is given by \( V(0), \Sigma(0), H(0), H^c(2), H'^c(2), Q(1), U(1), D(1), L(1) \) and \( E(1) \) [4].
Table 1: The boundary conditions for the bulk fields under the orbifold reflections. Here, we have used the 4D $N = 1$ superfield language; $T^{(i)}_{Q,U,E}$ ($F^{(i)}_{D,L}$) are the components of $T^{(i)}$ ($F^{(i)}$) decomposed into irreducible representations of the standard model gauge group. The fields written in the $(p,p')$ column, $\varphi$, obey the boundary condition $\varphi(-y) = p \varphi(y)$ and $\varphi(-y') = p' \varphi(y')$.

| $(p,p')$ | gauge and Higgs fields | bulk matter fields |
|-------|----------------------|-------------------|
| $(+,+)$ | $V_{321}, H_D, H_D$  | $T_{U,E}, T_Q, F_D, F_L^q$ |
| $(-,-)$ | $\Sigma_{321}, H_D^c, H_D^c$ | $T_{U,E}, T_Q^c, F_D^c, F_L^c$ |
| $(-,+)$ | $V_{X}, H_T, H_T$ | $T_Q, T_{U,E}, F_L, F_D'$ |
| $(+,-)$ | $\Sigma_X, H_T^c, H_T^c$ | $T_Q^c, T_{U,E}^c, F_L^c, F_D^c$ |

but in our effective 5D theory there is no reason for choosing $\alpha$ to be this specific value.\(^7\) On the other hand, putting matter fields on the Planck brane does have some desired features: we naturally obtain 4D Yukawa couplings of order one, and any higher dimensional operators for the matter fields are suppressed by a large mass scale $M_5$ in 4D. If matter fields were localized on the $y = \pi R$ brane, we would have a series of higher dimensional operators suppressed only by $M_5 e^{-\pi k R} \sim T$ in 4D, which could cause phenomenological problems. Therefore, we want to construct a model which gives the appropriate hypercharge quantization while preserving the desired features of Planck brane matter. We will construct such a theory in this subsection.

The key idea is to put the matter fields in the bulk, but to localize them towards the Planck brane using the masses for the matter hypermultiplets, parametrized by $c$ (a similar construction has been considered in flat space in [31]). We first describe how to obtain standard-model quarks and leptons from the bulk fields. Since the bulk respects 5D supersymmetry, we must introduce matter as bulk hypermultiplets. We introduce two hypermultiplets, $\{T, T^c\} + \{T', T'^c\}$, in the 10 representation of $SU(5)$ and two hypermultiplets, $\{F, F^c\} + \{F', F'^c\}$, in the 5 representation of $SU(5)$. The boundary conditions for these fields are given as Eq. (13), but for the $T$ and $T'$ multiplets the matrix $B$ acts on both $SU(5)$ fundamental indices. Then, choosing $\eta_T = -\eta_{T'} = 1$ and $\eta_F = -\eta_{F'} = 1$, we find that a generation $Q, U, D, L, E$ arises from the zero modes of bulk fields as $T(U, E), T'(Q), F(D)$ and $F'(L)$. (These boundary conditions are shown explicitly in Table 1.) Therefore, by introducing three sets of $\{T, T^c\} + \{T', T'^c\} + \{F, F^c\} + \{F', F'^c\}$ with $\eta_T = \eta_F = -\eta_{T'} = -\eta_{F'} = 1$, we obtain three generations of standard-model quarks and leptons, $Q_i, U_i, D_i, L_i, E_i$ $(i = 1, \cdots, 3)$. Here we note an important difference from the previous Planck-brane matter case: since the matter fields come from bulk multiplets which are in $SU(5)$

\(^7\)One possibility for understanding this quantization, $\alpha = 1$, is to consider higher dimensional theories with a larger gauge group, as in the flat space case of Ref. [30].
representations, the hypercharges for the matter fields are correctly normalized ($\alpha$ is fixed to be $\alpha = 1$).

We next consider the profiles of the bulk matter fields in the extra dimension. The wavefunction for a bulk field depends on the bulk mass parameter $c$ of its hypermultiplet. In the present case, we have bulk mass parameters $c_T$, $c_T^c$, $c_F$, and $c_F^c$. Now, if we choose these $c$ parameters to be larger than $1/2$, which we will show is consistent with successful gauge coupling unification, we find that the wavefunctions for the zero modes are strongly peaked at the Planck brane by an exponential factor $e^{-(c-1/2)|y|}$. Therefore, by making bulk masses for matter hypermultiplets large, we recover the desired properties of Planck brane matter. In particular, various higher dimensional operators localized on the TeV brane are suppressed in 4D, since the matter (zero-mode) wavefunctions at $y = \pi R$ are exponentially suppressed. The Yukawa couplings are located on the $y = 0$ brane as in Eq. (15), which gives 4D Yukawa couplings without unwanted wavefunction suppressions. Note that these Yukawa couplings do not respect SU(5) so that we do not have unwanted SU(5) mass relations such as $m_s/m_d = m_\mu/m_e$.

The issue of gauge coupling unification is slightly more complicated in the bulk matter model than in the brane matter model; since the matter fields propagate in the bulk, we have to evaluate their contribution to the “beta functions” using Eq. (10). As in the case of the Higgs hypermultiplets, however, we find that effectively only the zero modes contribute to the relative running among the three gauge couplings, as long as bulk hypermultiplet masses, $c$’s, are larger than or equal to $1/2$. For instance, the contribution from the $\{T, T^c\}$ hypermultiplet is given by $(\Delta^1, \Delta^2, \Delta^3)(q,k)|_{\mathcal{H}_T} \simeq (7/5, 0, 1/2) \ln(k/q) + (SU(5) \text{ symmetric})$ for $c_T \geq 1/2$, which is the relation we obtain from $U$ and $E$ fields in 4D theories. Similarly, $\{T', T'^c\}, \{F, F^c\}$ and $\{F', F'^c\}$ hypermultiplets give relative runnings as if only $Q$, $D$ and $L$ contribute, respectively, for $c_{T'}, c_{F}, c_{F'} \geq 1/2$. Thus, choosing all matter $c$’s to be larger than or equal to $1/2$, we find that the contribution from bulk matter to the gauge coupling evolution is completely $SU(5)$ symmetric. Therefore, together with the contribution from the gauge and Higgs fields, Eqs. (12, 14), we find that the successful prediction for gauge coupling unification is obtained in the model with matter in the bulk. Incidentally, setting $c_H = c_H$ and $c_{T_i} = c_{T_i} = c_{F_i} = c_{F_i} = c_M$, we obtain a simple expression for $\Delta^a(q,k)$:

$$
\begin{pmatrix}
\Delta^1 \\
\Delta^2 \\
\Delta^3
\end{pmatrix}
(q,k) \simeq
\begin{pmatrix}
33/5 \\
1 \\
-3
\end{pmatrix}
\ln \left( \frac{k}{q} \right) +
\begin{pmatrix}
15/2 \\
15/2 \\
15/2
\end{pmatrix}
- c_H
\begin{pmatrix}
1 \\
1 \\
1
\end{pmatrix}
- c_M
\begin{pmatrix}
12 \\
12 \\
12
\end{pmatrix}
\ln \left( \frac{k}{T} \right),
$$

where the first term gives the MSSM beta functions. The last three terms are $SU(5)$ symmetric, and do not affect the prediction for gauge coupling unification.

Is proton decay a problem in this setup? Since the TeV brane respects $SU(5)$, proton decay could be induced by brane operators at $y = \pi R$, such as $\delta(y - \pi R)[T^T][T]_{\theta_2\theta_2}$. However,
Table 2: $U(1)_R$ charges for 4D vector and chiral superfields.

| $U(1)_R$ | $V$ | $\Sigma$ | $H$ | $H^c$ | $T$ | $T^c$ | $F$ | $F^c$ | $N$ | $N^c$ |
|-----------|-----|---------|-----|-------|-----|-------|-----|-------|-----|-------|

since matter wavefunctions on this brane are exponentially suppressed as $e^{-(c-1/2)\pi k R}$, we can suppress proton decay caused by these operators by taking the parameters $c$ to be large. Because the resulting effective dimension six operators are suppressed by $e^{-4(c-1/2)\pi k R}$, the condition $e^{-2(c-1/2)\pi k R} \lesssim T/k$ is enough to suppress proton decay sufficiently, leading to $c \gtrsim 1$. This is a very weak condition and could easily be satisfied within the parameter region where the effective theory makes sense, $c \lesssim M_5/k$. Higher order processes, such as loop processes involving higher KK towers, are also suppressed by the same exponential factor. The remaining sources of dangerous proton decay are tree-level dimension four and five operators located at the Planck brane. These operators are suppressed by imposing a $U(1)_R$ symmetry, which naturally arises from the structure of higher dimensional theories [4, 5]. The charge assignment for $U(1)_R$ is given in Table 2, where we have omitted the primed fields which have the same charges as unprimed fields ($N$ and $N^c$ represent right-handed neutrino fields; see below). This $U(1)_R$ symmetry forbids not only dimension four and five proton decay operators but also a large mass term for the Higgs doublets on the Planck brane, $\delta(y)[H\bar{H} + LH]_{\varphi^2}$, providing a complete solution to the doublet-triplet splitting problem. After supersymmetry is broken, $U(1)_R$ is broken to the $Z_2$ subgroup, which is the usual $R$ parity of the MSSM. In the models for supersymmetry breaking considered in the next subsection, this breaking turns out not to reintroduce the problem of proton decay.

An alternative possibility for suppressing proton decay is to impose a global baryon number symmetry on the model, in which case the proton becomes absolutely stable. The fact that we can impose this symmetry is somewhat non-trivial, since $U$ and $E$ originate from the same hypermultiplet, $\{T,T^c\}$. Thus, here we explicitly show that it is indeed possible. We consider a global $\tilde{U}(1)$ symmetry whose charge assignment is given by $\{T,T^c\}(2), \{T', T'^c\}(-3), \{F,F^c\}(4), \{F',F'^c\}(-1), \{H,H^c\}(1)$ and $\{\bar{H}, \bar{H}^c\}(-1)$ (for right-handed neutrino hypermultiplets $\{N,N^c\}(0)$, which we will introduce later to induce small neutrino masses). This symmetry allows all the desired operators such as Yukawa couplings and the supersymmetric mass term for the Higgs doublets (and Majorana masses for right-handed neutrinos). At first sight, $\tilde{U}(1)$ does not look like baryon number. However, by taking a linear combination $\tilde{U}(1) - 2U(1)_Y$, we find that various MSSM chiral superfields carry charges $Q(-10/3), U(10/3), D(10/3), L(0), E(0), H(0), \bar{H}(0), (N(0))$, which is exactly baryon number (multiplied by 10). We thus find that
imposing $U(1)$ symmetry completely forbids all baryon-number violating processes, such as the decay of the proton. Incidentally, if we impose baryon number, the bulk mass parameter $c$ for the matter fields can be smaller than $\simeq 1$; for example, the matter fields can have conformally flat wavefunctions, $c_M = 1/2$. We will return to this possibility briefly in the following subsection.

Small neutrino masses are obtained in our model by introducing right-handed neutrinos through the seesaw mechanism [32]. They could be either brane chiral multiplets on the $y = 0$ brane, $N$, or bulk hypermultiplets, $\{N, N^c\}$ with $\eta_N = 1$. In both cases, the Yukawa couplings and Majorana masses are located on the $y = 0$ brane

$$S = \int d^4x dy \sqrt{-G}\delta(y) \left[ \int d^2\theta \left( y_n L N H_D + \frac{\kappa}{2} N N \right) + h.c.\right], \quad (17)$$

where $\kappa$ is a dimensionful (dimensionless) parameter in the case of brane (bulk) right-handed neutrinos. Let us first consider the cases of brane $N$’s and bulk $N$’s with $c_N \geq 1/2$. In these cases, we obtain $O(1)$ Yukawa couplings and $O(k)$ right-handed Majorana neutrino masses after dimensional reduction to 4D (up to possible volume suppression factor). Therefore, we obtain light neutrino masses of the correct size, $O(v^2/k)$, through the seesaw mechanism, where $v$ is the vacuum expectation value for the electroweak Higgs field. Next we consider the case of bulk $N$’s with $c_N < 1/2$. (Note that the hypermultiplet masses, $c_N$, for the right-handed neutrinos are not constrained by the argument of gauge coupling unification.) In this case the 4D neutrino Yukawa couplings receive exponential suppressions, $e^{-(1/2-c)\pi kR}$, due to the wavefunction form of the zero-mode right-handed neutrinos. However, the wavefunction suppressions for Yukawa couplings are canceled by those of the Majorana masses, so that we obtain the correct size for the neutrino masses, $O(v^2/k)$, regardless of the value for $c_N$ (a similar observation has been made in flat space in [33]). The only requirement is $c_N \geq 0$ so that the contribution to Majorana masses from the TeV brane is negligible; $c_N < 0$ is possible if we introduce $U(1)_X (\subset SO(10)/SU(5))$ symmetry broken only on the Planck brane. This leads to the interesting possibility that the masses for the right-handed neutrinos are around the TeV scale while the light neutrino masses are correctly reproduced through the suppression of the Yukawa couplings, which is naturally correlated to the suppression of the Majorana masses by the wavefunction suppression of the $N$ zero modes.

Finally, we discuss a new feature of the bulk matter model, which is absent in the Planck-brane matter model. In the case of brane matter, there are no “GUT partners” for standard model matter fields because the gauge symmetry on the brane is only $SU(3)_C \times SU(2)_L \times U(1)_Y$. However, in the bulk matter case, there are GUT partners for the standard model

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8In the case of bulk $N$’s with $c_N \geq 1/2$, there could be additional wavefunction suppressions for both Yukawa couplings and Majorana masses. However, these suppression factors cancel each other in the seesaw mechanism and do not affect the masses for the light neutrinos.
matter fields. For instance, the standard-model $D$ has a GUT partner, which is a field in the $\{F,F^c\}$ hypermultiplet having the quantum numbers of the standard-model lepton $L$. (Note that this is different from the usual 4D GUTs where the GUT partner of the standard-model $D$ is the standard-model $L$.) Such GUT partners also arise from the $\{T,T^c\}, \{T',T'^c\}$ and $\{F',F'^c\}$ multiplets. Therefore, in addition to the GUT partners of the gauge and Higgs fields (i.e. the XY gauge bosons and the colored Higgs fields), we have GUT partners for the quarks and leptons around the TeV scale, whose quantum numbers are the same as the standard-model quarks and leptons.

3.3 Supersymmetry breaking and electroweak symmetry breaking

In this subsection we discuss supersymmetry breaking and electroweak symmetry breaking. Our purpose here is to illustrate some of the viable approaches that are available in our warped supersymmetric unified theory framework. There will clearly be many possibilities for realistic electroweak and supersymmetry breaking in our theory, and we will leave a more thorough investigation of such models for future work.

In our theory one attractive scenario has supersymmetry broken on the TeV brane, because in that case the natural scale for the mass splitting among the particles in a supermultiplet is TeV. Thus we consider a setup where supersymmetry is broken by the $F$-term vacuum expectation value of a superfield $Z$ located on the TeV brane. The required $F$ term is easily generated by introducing the TeV-brane superpotential $\delta(y - \pi R)Z^\dag Z^\alpha W^\alpha /M_5^2 \theta^2$. Since the mass scales on the TeV brane receive an exponential suppression, this leads to TeV-scale supersymmetry breaking without introducing a small parameter [27].

Supersymmetry breaking is felt by the gauge sector at tree level through the TeV-brane operator $\delta(y - \pi R)[Z W^\alpha W_\alpha /M_5^3 \theta^2]$, giving the gauginos masses of order TeV. Bulk hypermultiplets $\{\Phi,\Phi^c\}$ also couple to the supersymmetry breaking at tree level through operators like $\delta(y - \pi R)[Z^\dag Z \Phi^\dag \Phi/M_3^3 \theta^2 \bar{\theta}^2]$, so that scalar fields in a hypermultiplet with $c = 1/2$ also receive a mass shift of order TeV. On the other hand, the mass shift for scalar fields in a hypermultiplet with $c$ somewhat larger than 1/2 (say $c \gtrsim 0.7$) is negligibly small because the zero-mode wavefunction is peaked at the Planck brane. There are then essentially two options for the Higgs hypermultiplets. If their zero-mode wavefunctions are nearly conformally flat, $c_{H,\bar{H}} \simeq 1/2$, a supersymmetric mass ($\mu$ term) and a holomorphic supersymmetry-breaking mass ($\mu B$ term) for the Higgs fields can be generated by direct couplings to the $Z$ field, but the coefficients of operators like $\delta(y - \pi R)[Z^\dag Z H^\dag H/M_3^3 \theta^2 \bar{\theta}^2]$ may be required to be somewhat small ($\lesssim 10^{-2}$) to avoid large masses squared for the electroweak Higgs doublets. This issue, however, is sensitive to the precise values of the bulk mass parameters; for instance, taking $c_{H,\bar{H}}$ slightly larger than 1/2 gives a moderate suppression of the Higgs wavefunctions at the TeV brane, which could be used
to construct a model for electroweak symmetry breaking without small parameters. The other possibility is that the Higgs zero-mode wavefunctions are essentially Planck-brane localized, \( i.e. \ c_{H,R} \) sufficiently large so that their tree-level soft masses effectively vanish. For simplicity, we will focus our attention on this case below.

In the setup with the Higgs localized towards the Planck brane, a weak-scale \( \mu \) term cannot arise directly from a superpotential term on the TeV brane, since a supersymmetric Higgs mass on the TeV brane gives a 4D \( \mu \) term only of order \( Te^{-2(c_H-1/2)\pi kR} \). However, we can generate the desired size of the \( \mu \) term in the following way. We introduce a bulk hypermultiplet \( \{S, S^c\} \) with \( c_S = 1/2 \), which is singlet under the gauge group. Then we induce a vacuum expectation value for the \( S \) field by introducing a brane superpotential on the TeV brane: \( \delta(y-\pi R)[X(S^2-M^3)]_{g^2} \), where \( X \) is a brane field located at \( y = \pi R \) and we have omitted the overall coupling factor having the mass dimension \(-1\). Since all the mass scales on the TeV brane, including \( M \), are scaled by an exponential factor, \( e^{-\pi kR} \), we obtain a weak-scale expectation value for the \( S \) field in the 4D picture. Therefore, by introducing the Planck-brane superpotential \( \delta(y)[SH_D\bar{H}_D/M_5^{3/2}]_{g^2} \), where \( H_D \) and \( \bar{H}_D \) represent the Higgs doublet superfields, we generate a weak-scale \( \mu \) term without fine-tuning.\(^9\) (The triplet Higgses \( H_T \) and \( \bar{H}_T \) have vanishing wavefunctions at \( y = 0 \) and do not have a coupling of the form \( SH_T\bar{H}_T \) at \( y = 0 \)).

Note that we have assumed that the \( U(1)_R \) symmetry is strongly broken at the TeV brane: since the superpotential coupling \( SH_D\bar{H}_D \) on the Planck brane requires \( S \) to have a \( U(1)_R \) charge of \(+2\), the superpotential \( X(S^2-M^3) \) cannot be \( U(1)_R \) invariant. Nor do the TeV brane operators \( [M_5^2Z]_{g^2} \) and \( [ZW^aW^a/M_5]_{g^2} \) allow for consistent \( U(1)_R \) charge assignments. However, the strong \( U(1)_R \) breaking on the TeV brane does not cause phenomenological problems, such as rapid dimension five proton decay, because the wavefunctions of the triplet Higgs fields vanish on the Planck brane (the Higgs triplets can couple to matter fields through their \( y \) derivatives, but this produces a negligibly small proton decay rate).

We now describe a parameter region where a realistic phenomenology is obtained. At tree level, the gauginos and the scalar components of \( S \) obtain masses of order \( T \) (we assume the presence of the operator \( \delta(y-\pi R)[Z^iZ^jS/M_5^{3}g^2] \). One loop, this induces squark and slepton masses through gauge interactions, and negative Higgs masses squared through the \( SH_D\bar{H}_D \) interaction. These masses are finite and calculable (up to an exponentially suppressed TeV-brane localized counterterm contribution), since the matter and Higgs fields are effectively

\(^9\)This can be seen more precisely in the 5D picture. The TeV-brane term forces the scalar component of \( S \) to acquire a \( y \) dependent classical profile. By solving the bulk equations of motion, we obtain \( S(y) = \alpha e^{ky} + \beta e^{3ky} \). If we take into account the mass term for \( S \) on the Planck brane (as required by supersymmetry in AdS; see Eq. (8)) we find that \( \beta \) must vanish. (Alternatively, unbroken bulk supersymmetry requires this mode to be absent [34].) The TeV-brane potential gives \( S(\pi R) \sim M_5^{3/2} \), \( i.e. \ \alpha \sim e^{-\pi kR}M_5^{3/2} \), and therefore we obtain \( S(0) = \alpha \sim M_5^{3/2}e^{-kpR} \), producing a weak-scale \( \mu \) term through the \( S(0)H_D\bar{H}_D \) coupling.
localized to the Planck brane while the supersymmetry breaking is localized to the TeV brane.\(^\text{10}\) By choosing \(M\) and/or the coupling of \(S H_D \bar{H}_D\) somewhat small in units of the fundamental scale of the theory, we obtain realistic electroweak symmetry breaking. Moreover, since the squark and slepton masses are generated by gauge interactions, they are flavor universal, solving the supersymmetric flavor problem.

In the limit of \(F_Z \to \infty\), one of the XY gauginos becomes massless. Taking \(F_Z \sim M_5^2\), the mass of the lightest XY gaugino is still suppressed (see appendix B),

\[
m_{XY} \sim \frac{M_5}{F_Z R} T \sim \frac{1}{M_5 R} T.
\]  

(18)

Since we expect \(M_5 R \sim 16\pi^2\), this state can be much lighter than \(T\), which will be an important point when we come to consider production of GUT particles at collider experiments in section 4. In the same limit of strong supersymmetry breaking, the lightest 321 gauginos form pseudo-Dirac states with mass \([34]\)

\[
m_{321} \simeq \sqrt{\frac{2}{\pi k R}} T \simeq \frac{1}{4} T.
\]  

(19)

Thus, in this limit, the underlying bulk \(N = 2\) supersymmetry leaves an imprint on the lowest lying states, through the approximately Dirac nature of the lightest gauginos. In general the masses for the three MSSM gauginos are not degenerate despite the fact that their masses arise from an \(SU(5)\)-symmetric operator on the TeV brane, because the rescalings of the gauginos required to canonically normalize their kinetic terms vary according to the different values of the low energy gauge couplings and these rescalings give rise to different 4D masses for the \(SU(3)_C\), \(SU(2)_L\), and \(U(1)_Y\) gauginos. The non-universality is expected to persist for \(g_2^a F_Z R / M_5 \gg 1\), although the gaugino mass ratios in this case will not be the ones simply obtained by the 4D rescalings, \(m_{321,a} \propto g_2^a\), due to the distortion of the gaugino wavefunctions caused by the large brane masses.\(^\text{11}\)

In deriving the above results we have assumed that the operator \(\delta(y - \pi R)[ZW^a W_\alpha / M_5]_{a\beta}^2\) appears with order one coefficient. If instead this coefficient is tiny, the lightest XY gaugino mass approaches its value in the supersymmetric limit, \(m_{XY} \sim (3\pi/4)T\); meanwhile the lightest 321 gauginos are non-degenerate Majorana states with masses much less than \(T\). The real situation could lie between these two extreme cases. For instance, if the coefficient of \(ZW^a W_\alpha / M_5\) is of

\(^{10}\)In our unified models the radiative squark and slepton masses are not exactly the same as the case of non-unified theory given in Ref. [35] due to the contribution from the GUT particles such as the XY gauge multiplet, although we expect that qualitative features are unchanged from the non-unified case; for instance, the lightest of the squarks and sleptons will still be the right-handed sleptons.

\(^{11}\)In an earlier version of the paper, it was stated that the gaugino masses become universal in the limit \(g_2^a F_Z R / M_5 \gg 1\). However, the non-universality may in fact persist even in this limit because the effect from Planck-brane localized gauge kinetic terms does not disappear. This was noted by Chacko and Ponton in [36].
$O(0.1)$ (and $F_Z \sim M_\ast^2$), the 321 gaugino masses are not approximately Dirac. In fact, in the specific model with bulk $\{S,S^c\}$, the deviation from the Dirac limit is necessary to induce $\mu B$ through radiative corrections from gaugino loops.$^{12}$ It is interesting that the $XY$ gaugino mass is still small, $m_{XY} \sim 0.1 T$, in this parameter region.

We note in passing that there is a qualitatively different setup that one can consider, in which all quark and lepton hypermultiplets have conformally flat wavefunctions, with $c = 1/2$. Baryon number conservation must then be imposed to stabilize the proton, as described in the previous subsection. A remaining difficulty associated with matter in the bulk, the supersymmetric flavor problem, can be avoided if supersymmetry is broken by the vacuum expectation value for the auxiliary component of the radion field, $F_T$, leading to universal superpartner masses. A brane-localized $F_T$ can be generated by a constant superpotential $W$ on the TeV brane [34]. Supersymmetry breaking by $F_Z$ is also feasible, if some flavor symmetry (e.g. $U(3)^4$) is explicitly broken only on the Planck brane, where the Yukawa interactions are located. In this case as well, the squark and slepton masses generated on the TeV brane are flavor universal. The flavor symmetry has the additional virtue of forbidding higher-dimensional flavor-violating operators on the TeV brane involving the quark and lepton superfields alone. In this setup, the superpartner masses all arise at tree level, while, if we take $c > 1/2$ for the Higgs hypermultiplets, the soft masses for their scalars are loop suppressed relative to all superpartner masses, a feature that may improve the naturalness of the Higgs sector. The scenario with breaking by $F_Z$ is essentially a warped version of the model with a flat TeV-scale extra dimension presented in Ref. [25].

Here we stress again that there are many possible realizations for the Higgs sector in our theory, and the one presented above, using the $\{S,S^c\}$ bulk hypermultiplet, is just one simple example. In general, the scale $T$ is related to the electroweak vacuum expectation value, $v$, through the dynamics of electroweak symmetry breaking. In the case of $F_Z$ breaking with matter and Higgs localized to the Planck brane, the gauginos receive tree-level masses of order $T$, while the squarks, sleptons and the Higgs bosons receive masses at one loop. Therefore, the gaugino masses are significantly larger than the scalar masses and we expect $T$ to be in the multi-TeV region. However, the value for $T$ depends strongly on the Higgs sector, i.e. the sector of electroweak symmetry breaking, as well as the parameters of the model. For instance, if the $\mu$ term is generated in another way without introducing the bulk $\{S,S^c\}$ hypermultiplet, say through a singlet field on the Planck brane as in the NMSSM, radiative electroweak symmetry breaking is triggered at the two-loop level, giving somewhat larger values for $T$. On the other hand, in the model just mentioned with delocalized matter, all superpartners obtain tree-level masses, so that the gauginos and scalars have comparable masses and $T$ is likely to be smaller.

$^{12}$Alternatively, $\mu B$ can be generated by adding a TeV brane term such as $[Z^\dagger ZX]_{\theta^2\bar{\theta}^2}$, in which case Dirac gaugino masses are acceptable.
One general consequence of the model is that, provided that the supersymmetry breaking scale is generated by the warp factor, the lightest superparticle is the gravitino, with mass \( \sim T^2/M_{Pl} \), regardless of the details of the electroweak symmetry breaking sector [27, 35]. This mass is in the \( 10^{-3} \) eV range for \( T \sim 1 \) TeV. Unless we break \( R \) parity by brane-localized operators, the gravitino is absolutely stable, leading to missing energy signatures at collider experiments.

Finally, let us consider the important issue of the effect of supersymmetry breaking on gauge coupling unification. Because supersymmetry is broken on the TeV-brane, it is clear that supersymmetry breaking is caused by TeV-scale physics, since the fields located at \( y = \pi R \) can be viewed in the dual 4D picture as composite states of the (quasi-)CFT which becomes strong at the TeV scale. Indeed, as just mentioned, the gravitino mass is given by \( \sim T^2/M_{Pl} \), implying that supersymmetry breaking occurs at a TeV. Thus it is clear that supersymmetry breaking does not affect the prediction for gauge coupling unification at the leading-log level, since physics occurring at the TeV scale cannot affect the gauge coupling running above the TeV scale.

In fact, we can explicitly check that supersymmetry breaking does not affect gauge coupling unification. Without supersymmetry breaking, the KK towers for the gauginos are given as follows. For the 321 gauginos, we have a Majorana fermion with \( m_0 = 0 \) and a Dirac fermion at each KK level with \( m_n \simeq (n - 1/4)\pi T \) (\( n = 1, 2, \ldots \)); while for the XY gauginos, we have two Dirac fermions at each KK level with \( m_n \simeq (n - 1/4)\pi T \) (\( n = 1, 2, \ldots \)). Thus, in each KK level (\( n = 1, 2, \ldots \)) the gauginos form a complete \( SU(5) \) multiplet, and the relative gauge coupling running comes entirely from the zero-mode 321 gauginos, \( \tilde{g}, \tilde{w} \) and \( \tilde{b} \). When we turn on supersymmetry breaking, the masses for the gaugino KK towers shift. For simplicity, here we demonstrate our point in the large \( F_Z \) limit of the brane \( Z \) scenario. In this limit, the masses for the gaugino KK towers become the following. For the 321 gauginos, there is a Dirac fermion at each KK mass level given by \( m_n \simeq (n + 1/4)\pi T \) (\( n = 1, 2, \ldots \)) and \( m_0 \simeq T/4 \); for the XY gauginos, there are two Dirac fermions at each KK level of \( m_n \simeq (n + 1/4)\pi T \) (\( n = 1, 2, \ldots \)) and one Dirac fermion of mass \( m_0 \sim (M_5/F_Z R)T \). From this, we find that each KK mass level with \( n = 1, 2, \ldots \) forms a complete \( SU(5) \) multiplet, while the lowest level with \( n = 0 \) does not. In the lowest level, one Majorana degree of freedom from 321 and a Dirac fermion of XY are paired to complete the \( SU(5) \) multiplet (though their masses are different), which leaves one Majorana 321 degree of freedom unpaired. Therefore, we find that the unpaired 321 Majorana gauginos contribute exactly the same relative running of the gauge couplings above the TeV scale as in the unbroken supersymmetry case. A similar result can be obtained as well for the Higgs multiplets. In this way, we explicitly see that supersymmetry breaking indeed does not change the prediction for gauge coupling unification at the leading-log level.
3.4 Dual 4D picture

In this subsection we discuss how certain features of our 5D AdS theory can be understood from the point of view of the 4D picture implied by AdS/CFT duality [37]. As has been suggested in Refs. [38, 39, 40], a 5D theory in AdS truncated by branes corresponds to a 4D theory that includes a strongly coupled CFT explicitly broken in the UV (the dual of the Planck brane) by a coupling to 4D gravity, and in which the scale invariance is spontaneously broken in the IR (the TeV brane). The 4D CFT picture has also been discussed for bulk gauge fields in [39, 12, 15, 16] and for supersymmetry breaking in [35, 34].

In the 5D picture, we have introduced $SU(5)$ gauge symmetry in the bulk. In the 4D dual this $SU(5)$ is interpreted as a global symmetry of the CFT sector. Truncating the AdS space by the UV brane corresponds to gauging the global symmetry of the CFT. Since the Planck brane, which explicitly breaks the 5D $SU(5)$ gauge group, provides a UV definition of the theory, we expect that only an $SU(3)_C \times SU(2)_L \times U(1)_Y$ subgroup of the global $SU(5)$ symmetry is gauged. The spectrum of the theory thus consists of elementary gauge fields of $SU(3)_C \times SU(2)_L \times U(1)_Y$ and the states of the CFT sector. Since the IR dynamics of the CFT fully respects $SU(5)$, all composite states that arise from the low energy breaking of conformal invariance fall into complete $SU(5)$ multiplets. Also, the CFT has a mass gap (of order $T$), and no massless states with global $SU(5)$ quantum numbers arise from the low energy condensation of the CFT. This spectrum exactly matches to what we found in its AdS dual: the massless excitations (the gauge sector coupled to the CFT) comes in a 321 adjoint multiplet, while the massive states (which correspond to the KK excitations of AdS bulk fields) respect global $SU(5)$ and are therefore in complete $SU(5)$ multiplets.

Since our theory does not have an $SU(5)$ gauge symmetry, how can we expect to obtain a prediction for gauge coupling unification? In fact, the theory does not require the unification of three gauge couplings at high energies.\(^{13}\) Nevertheless it is still possible to make reliable predictions for the low energy value of the weak mixing angle, under the assumption that the 321 gauge couplings become strong at a scale comparable to the UV scale $\Lambda_{UV}$ where we define our theory. To see how this works, let us consider radiative corrections to the standard model gauge couplings. The CFT contributes equally to the running of all three couplings, since its field content is fully $SU(5)$ universal.\(^{14}\) There is also a contribution coming from the elementary

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\(^{13}\)This follows because the gauge coupling in the 4D picture corresponds to the coefficient of the Planck-brane correlator in the 5D picture, which at high energies is approximately given by the coupling of the boundary gauge kinetic term at $y = 0$, which does not respect $SU(5)$.

\(^{14}\)The contribution to the running due to pure CFT effects can be easily computed by the rules of AdS/CFT [39]. According to AdS/CFT, the two-point function of the currents that generate $SU(3)_C \times SU(2)_L \times U(1)_Y$ in the CFT is given, in the large $N$ limit, by simply evaluating the classical AdS gauge field propagator evaluated on the Planck brane. By setting $z = 1/k$ in Eq. (3) we see that because $K_0(q/k) \sim \log(q/k)$ for $q/k \ll 1$ the CFT...
gauge fields, that is, the 321 $N = 1$ gauge multiplet. Thus the low energy gauge couplings are given by

$$\frac{1}{g_a^2(q)} = \frac{1}{g_a^2(\Lambda_{UV})} + \frac{b_{\text{CFT}}}{8\pi^2} \ln\left(\frac{\Lambda_{UV}}{q}\right) + \frac{b_a}{8\pi^2} \ln\left(\frac{\Lambda_{UV}}{q}\right),$$

(20)

where $b_{\text{CFT}}$ and $b_a$ are the contributions to the beta function coefficients due to CFT and massless elementary fields, respectively. The first term represents the UV values of the gauge couplings, which in the absence of unified gauge symmetry are completely non-universal. The second term is the universal contribution from the CFT, which is asymptotically non-free ($b_{\text{CFT}} > 0$). Using $b_{\text{CFT}} \sim 8\pi^2/g_a^2 k$, the size of this term is of order $\pi R/g_a^2 \sim 1$ for $q \sim T$ (taking $\Lambda_{UV} \sim k$ and $T = k e^{-\pi k R}$). The third term is the contribution from elementary fields, of order $\ln(k/T)/16\pi^2$. Despite the assumption that the theory is strongly coupled in the UV ($g(\Lambda_{UV}) \sim 4\pi$), the observed gauge couplings at low energies, $q \sim T$, are weak $\sim O(1)$ due to the large asymptotically non-free contribution from the CFT. The difference between the gauge couplings comes from the first and third terms. Because of the logarithmic enhancement of the third term, however, we find that the values of the first term (the values of the UV couplings) are completely irrelevant. Therefore, although we do not have unification at high energies, we recover the prediction for $\sin^2 \theta_w$, which is given by the contribution from the elementary fields of the theory.

To complete the discussion, we also consider matter and Higgs fields. In the 4D picture matter fields localized strictly on the Planck brane correspond to spectator fields with no direct couplings to the CFT (they only interact with CFT fields through the 321 gauge interactions). Therefore, as we discussed previously, this setup does not provide quantization of hypercharge, so it is impossible to predict the weak mixing angle (unless additional assumptions about hypercharge normalization are made). On the other hand, we have seen that bulk matter provides the required quantization. The CFT understanding of this fact is as follows. According to AdS/CFT, a bulk field $\varphi$ is interpreted as a source of the corresponding CFT operator $O$:

$$\mathcal{L}_{4D} \sim \varphi O.$$  

(21)

Let us consider the Higgs field, as an example. Since the operator $O$ consists of CFT fields, it must be in a representation of $SU(5)$ (5 in this case). This means that $\varphi$ must also be in a representation of $SU(5)$ (5 in this case). Now, the UV truncation of AdS implies that in the dual description, the source $\varphi$ gets promoted to a dynamical field. However, since the UV brane respects only the 321 part of $SU(5)$, we expect that only a fraction of the components of $\varphi$ becomes dynamical. In the present Higgs case, we find that only the doublet component of $\varphi$ acquires a kinetic term. This field is interpreted as an elementary field in the 4D picture and corresponds to the zero mode of the Higgs hypermultiplet. (Note that this situation contributes a universal logarithm to the running of the 321 gauge couplings.)
quite similar to the case of the gauge fields, where only the 321 part of the $SU(5)$ adjoint representation is gauged (made dynamical.) In this way, we obtain an (elementary) Higgs doublet which is not in a complete $SU(5)$ multiplet but which has hypercharge still quantized according to $SU(5)$ normalization. The KK towers arise as composite states of the CFT and thus are $SU(5)$ symmetric. The same argument applies also to bulk matter. In general, in the 4D dual the zero modes of bulk fields are interpreted as elementary fields (for $c \geq 1/2$), while the KK modes are composite states of the CFT. Since all the elementary (zero-mode) fields contribute to the third term of Eq. (20), the coefficients $b_a$ are as in the MSSM.

We now see that our theory employs a completely different implementation of the grand unification idea than in conventional GUTs. In the usual 4D GUTs, $SU(5)$ is a gauge symmetry. The three gauge couplings are unified at the UV scale, but deviate in the IR due to the contribution of $SU(5)$-violating matter content in loops. On the other hand, in our theory, the three gauge couplings are completely unrelated in the UV. Nevertheless, if we assume that 321 gauge interactions are strong in the UV, we can recover the correct prediction for $\sin^2 \theta_w$, which is determined by the loop contributions of elementary (MSSM) fields. The low energy 321 couplings are weak due to a large $SU(5)$ symmetric CFT contribution. In this framework, the elementary fields do not have to be in a complete representation of $SU(5)$, so there is no doublet-triplet splitting problem (the $SU(5)$ gauge symmetry is simply absent and the normalization of hypercharge is determined by a global $SU(5)$). We here note that this scenario is different from that of Ref. [9], where the 5D $SU(5)$ gauge symmetry is broken by the vacuum expectation value of the GUT-breaking Higgs field localized on the Planck brane. In this case, the 4D dual field theory interpretation is more along the lines of conventional $SU(5)$ gauge symmetry broken by the Higgs mechanism at high energy, and consequently one must solve, for example, the doublet-triplet splitting problem to obtain a completely realistic theory.

Since the strong CFT sector is almost conformal above the TeV scale, the scale of KK excitations, e.g. the masses of the XY gauge bosons, is dynamically generated through dimensional transmutation. In our case, this strong CFT dynamics also induces supersymmetry breaking, and the MSSM gauginos feel the supersymmetry breaking at tree level because of the direct couplings between the gauge multiplets and the CFT sector (this determines the IR scale of the CFT to be TeV). The couplings between MSSM matter and the CFT sector are suppressed, so that squarks and sleptons obtain supersymmetry breaking masses only at the loop level, providing the solution to the supersymmetric flavor problem. In the 4D picture, the absence of direct couplings between the MSSM matter and the CFT sector is understood as a result of the conformal sequestering effect, which has been discussed in Ref. [41] in simple $N = 1$ supersymmetric gauge theories.
4 Phenomenology

In this section we summarize the main phenomenological features of the model. Consider first the limit of unbroken $N = 1$ supersymmetry. The spectrum consists of the field content of the MSSM at the massless level, accompanied by massive $N = 2$ towers for the vector multiplets and Higgs hypermultiplets. Thus at each gauge KK level there is a massive gauge boson, a Dirac gaugino, and a real scalar in the adjoint representation, while each Higgs KK level consists of a vector-like pair of complex scalars and a Dirac Higgsino. The $N = 2$ particle content of the KK towers is familiar from 5D supersymmetric models with a flat TeV-scale extra dimension. Because the $c$ parameters for the two Higgs hypermultiplets may be different, the two Higgs KK towers may be non-degenerate.

Since the $SU(5)$ gauge symmetry of the bulk is broken by boundary conditions, one might expect the massive KK towers for the 321 and XY vector multiplets to be shifted relative to one another. This shift, however, turns out to be very small. The mass eigenvalues, $m_n$, for the 321 and XY vector multiplets are determined by different equations,

$$\frac{J_0(m_n/k)}{Y_0(m_n/k)} = \frac{J_0(m_n/T)}{Y_0(m_n/T)} \quad \{V, \Sigma\}_{321}, \quad (22)$$

$$\frac{J_1(m_n/k)}{Y_1(m_n/k)} = \frac{J_0(m_n/T)}{Y_0(m_n/T)} \quad \{V, \Sigma\}_{XY}, \quad (23)$$

but these equations yield nearly identical non-zero eigenvalues (although only the 321 multiplet has a zero mode). In both cases the masses for the non-zero modes are given approximately by

$$m_n \simeq (n - 1/4)\pi T \quad (n = 1, 2, ...), \quad (24)$$

with the first excited states in $\{V, \Sigma\}_{321}$ being heavier than those of $\{V, \Sigma\}_{XY}$ by just $\sim 2\%$ (for $kR \sim 10$). Thus the massive vector multiplet towers are not only $N = 2$ symmetric but also approximately $SU(5)$ symmetric as well.

The same can be said for the Higgs hypermultiplets, $\{H, H^c\}$ and $\{\bar{H}, \bar{H}^c\}$. The mass eigenstates for the doublet and triplet hypermultiplets contained in $\{H, H^c\}$ and $\{\bar{H}, \bar{H}^c\}$ satisfy

$$\frac{J_{|c-1/2|}(m_n/k)}{Y_{|c-1/2|}(m_n/k)} = \frac{J_{|c-1/2|}(m_n/T)}{Y_{|c-1/2|}(m_n/T)} \quad \text{(doublets)}, \quad (25)$$

$$\frac{J_{c+1/2}(m_n/k)}{Y_{c+1/2}(m_n/k)} = \frac{J_{c-1/2}(m_n/T)}{Y_{c-1/2}(m_n/T)} \quad \text{(triplets)}, \quad (26)$$

where $c$ is equal to $c_H$ or $c_R$, depending on the hypermultiplet in question. For $c_H, c_R \geq 1/2$, the case suggested by gauge coupling unification, the massive doublet and triplet towers for a given hypermultiplet become approximately degenerate, with

$$m_n \simeq (n + c/2 - 1/2)\pi T \quad (n = 1, 2, ...). \quad (27)$$
From the point of view of AdS/CFT, the massive KK modes of the Higgs and vector multiplets are composite states in the CFT. The 5D AdS picture, however, suggests that the widths for these states are sufficiently narrow to be interpreted as particle states, as long as their masses are less than the rescaled 5D Planck scale, \( T' = M_5 e^{-\pi k R} \), which is assumed to be somewhat larger than \( T = k e^{-\pi k R} \). Thus the first few KK modes, at least, can be resolved as particle resonances at collider experiments.

When supersymmetry is broken, the \( N = 2 \) symmetry of the KK towers is spoiled. In the supersymmetric limit, the two Weyl fermions contained in an \( N = 2 \) vector multiplet form a Dirac gaugino that is degenerate with the gauge boson and adjoint scalar, but after supersymmetry breaking the Dirac gaugino splits into two non-degenerate Majorana gauginos, one of which is heavier than the gauge boson, and the other of which is lighter. We have seen that if supersymmetry is broken strongly by \( F_Z \), there is a light Dirac XY gaugino state, with a mass \( \sim (M_5/F_2 R) T \), while the lightest 321 gaugino becomes nearly Dirac, with a mass \( \sim \sqrt{2/(\pi k R)} T \simeq T/4 \). The rest of the gaugino KK towers are nearly \( SU(5) \) symmetric, with masses shifted above those of the massive gauge boson tower by \( (\pi/2) T \).

In the bulk Lagrangian a symmetry that we will call GUT parity is preserved. Fields from 321 vector multiplets and \( SU(2)_L \) doublet Higgs hypermultiplets have \((+)^\) GUT parity, while fields from XY vector multiplets and \( SU(3)_C \) triplet Higgs hypermultiplets have \((-)^\) GUT parity. If the quarks and leptons propagate in the bulk, the GUT parity can be chosen such that the quark and lepton chiral multiplets with \((++)^\) or \((--)^\) boundary conditions have \((+)^\) GUT parity while those with \((-+)^\) or \((+-)^\) boundary conditions have \((-)^\) GUT parity. In the case where the quark and lepton chiral multiplets are located on the Planck brane, we can assign \((+)^\) GUT parity for all these fields. Then, we find that the lightest particle with \((-)^\) GUT parity (or "LGP") will be stable, unless brane interactions induce its decay. Which GUT particle do we expect to be the LGP? Because \( c \geq 1/2 \) for the Higgs hypermultiplets, a comparison between Eqs. (24) and (27) tells us that the lightest XY gauge multiplet will be at least as light as the lightest colored Higgs multiplet. If multiplets containing quarks and leptons propagate in the bulk, their mass eigenvalues are also given by Eq. (27), again with \( c \geq 1/2 \), so the lightest XY gauge multiplet will also be at least as light as the lightest matter multiplet with \((-)^\) GUT parity. We have argued that when supersymmetry is broken, the mass of the lightest XY gaugino will be pushed below that of the corresponding gauge boson and adjoint scalar, so that the LGP will most likely be the XY gaugino (although this may depend on the details of the Higgs sector). It is natural to have \( R \) parity conservation in our model, in which case the LSP gravitino is also stable (the lightest XY gauge boson decays into an XY gaugino and a gravitino).

If baryon number conservation is not imposed there may be couplings of the \( y \) derivative of the XY gauge fields to the Planck brane, which would allow, for example, an XY gauge
boson to decay directly into standard model fermions. However, these couplings, required to
be tiny anyway to avoid rapid proton decay, are likely to be so small that the LGP will be
effectively stable for collider purposes. For example, we find that at the Planck brane, the
derivative of the light XY gauge boson KK mode wavefunction is proportional to $m_n^2/k$, leading
to a coupling to the brane suppressed by a factor $(m_n/k)^2 \sim 10^{-26}$. Similarly, if the quark
and lepton hypermultiplets propagate in the bulk, it is possible to have GUT-parity violating
interactions localized to the TeV brane. Since the wavefunctions for the quark and lepton zero
modes are already required to be peaked at the Planck brane (unless baryon number conservation
is imposed), these interactions also lead to a minuscule decay rate for the LGP.\(^{15}\)

Because the LGP (most likely the XY gaugino) is colored, it will hadronize after production
by forming a bound state with a quark. There are four mesons with almost degenerate masses:
$T^0 \equiv \tilde{X}_\uparrow \bar{d}, T^- \equiv \tilde{X}_\uparrow \bar{u}, T'^- \equiv \tilde{X}_\downarrow \bar{d}$ and $T''^- \equiv \tilde{X}_\downarrow \bar{u}$, where $\tilde{X}_\uparrow$ and $\tilde{X}_\downarrow$ are the isospin up and
down components of the XY gauginos, respectively. Since the colored triplet gauginos, $\tilde{X}_\uparrow$ and
$\tilde{X}_\downarrow$, have electrical charge $-1/3$ and $-4/3$, a neutral bound state is possible if the first kind binds
with $\bar{d}$. Although the electromagnetic splitting lowers the mass of this bound state relative to
the others, isospin violating effects raise its mass relative to bound states involving $\bar{u}$. Since both
effects cause order MeV shifts in the masses, it is unclear whether the LGP will be neutral, $T^0$,
or charged, $T^-$. In either case, the heavier states will undergo beta decay to produce the lightest
one. However, this process is slow enough that, if produced in a collider, these meson states will
traverse the entire detector without decaying. This causes observable signals; in particular, the
charged states will easily be seen by highly ionizing tracks.

The prospects for producing GUT particles at the LHC depend on the scale $T$. As mentioned
earlier, the value of $T$ depends on the details of the supersymmetry breaking and the Higgs sector
of the model. However, the lower bound on $T$ coming from precision electroweak constraints is
extremely mild, $T \gtrsim 200 \text{ GeV}$, when standard model fermion wavefunctions are peaked at the
Planck brane \cite{27, 42}. In 4D models with light vector leptoquarks, leptoquark pair production
at the LHC is dominated by the gluon-gluon initial state contribution, and the discovery reach
in masses is roughly $2 - 2.5 \text{ TeV}$ \cite{43} assuming that the leptoquark decays into a charged lepton
and a jet. In the 5D model considered here, the zero-mode gluons from the colliding protons can
counter to pair-produce the first XY KK mode by $s$-channel exchange of the zero-mode gluon,
by $t$-channel exchange of the first XY KK mode, or simply by the quartic coupling. No other
KK modes contribute in the intermediate states because of the flatness of the zero mode and

\(^{15}\)It is possible to consider scenarios where the LGP decays promptly. For instance, suppose that the wave-
functions for the $F_i$ and $F'_i$ are conformally flat, while the $T_i$ and $T'_i$ are strongly localized to the Planck brane.
Then the TeV-brane operator $\delta(y - \pi R)[F_i^† F'_i]_{g2\bar{u}_2}$ induces unsuppressed decays of XY gauge bosons to standard
model fermions, but since the wavefunctions for the $T$'s are extremely small at the TeV brane, proton decay can
be sufficiently suppressed.
the orthogonality relations for the KK modes. Thus the contribution to the production cross section from this initial state is the same as in 4D. We have argued that in the present model, the LGP would yield a distinctive signature if produced, so we expect a similar discovery reach as in the 4D case.

The lightest XY gauge boson has a mass $\simeq 2.4 T$, giving the LHC a reach in $T$ of about 1 TeV for XY gauge boson production. Crucially, the lightest XY gaugino may be much lighter than this (e.g. in the case of strong breaking by $F_Z$), so GUT particles might still be produced at the LHC for much larger values of $T$. This is an important point because a realistic superparticle spectrum may require larger values of $T$.

There is also a contribution to the XY pair-production cross section coming from a $q\bar{q}$ initial state, involving $s$-channel exchange of all of the standard model gauge boson KK modes. There are resonant enhancements of this contribution at center of mass energies corresponding to the masses of the 321 gauge boson KK modes. These enhancements might make the $q\bar{q}$ initial state the dominant source of pair production for certain values of the XY pair’s invariant mass squared, and might extend the reach of the LHC, but a more careful analysis would be required to determine whether this is the case.

The detection of the first KK excitation of the standard model gauge bosons was considered in Ref. [42], where it was estimated that Drell-Yan searches at the LHC would place a lower bound on the mass, $m_1 \simeq 2.4 T$, of about 4 TeV if no resonance is seen (here we assume that the quarks and leptons are effectively localized to the Planck brane). The prospects for producing the KK modes of the Higgs multiplets (and the KK modes of the quark and lepton multiplets, if they propagate in the bulk) depend on the various $c$ parameters, because the masses of the KK modes increase linearly with $c$. If $c$ is not much larger than 1/2, the production rate at the LHC will be comparable to that for the gauge KK modes, but as $c$ increases the production is suppressed.

5 Conclusions

In this paper we have studied an implementation of the grand unification idea that is an alternative to conventional 4D grand unified theories (GUTs). We have constructed a completely realistic unified theory in truncated AdS$_5$ compactified on an $S^1/Z_2$ orbifold ($0 \leq y \leq \pi R$), with the unified gauge symmetry broken by boundary conditions. We have found that a realistic model with successful gauge coupling unification is obtained with the following structure of the extra dimension: the TeV brane at $y = \pi R$ respects the full SU(5) symmetry while the Planck brane at $y = 0$ respects only $SU(3)_C \times SU(2)_L \times U(1)_Y$. The gauge supermultiplets propagate in the bulk, while Higgs and matter fields are strongly localized to the Planck brane (Higgs and
matter fields could instead have conformally flat wavefunctions if certain conditions are met). In the dual 4D picture, suggested by the AdS/CFT correspondence, the $SU(5)$ symmetry is realized as a global symmetry of the CFT and is not a fundamental gauge symmetry of the theory. Nevertheless, this global $SU(5)$ provides an explanation of charge quantization and the observed values of the standard model gauge couplings. In our theory the strong CFT also breaks 4D $N = 1$ supersymmetry and thus electroweak symmetry. This relates the fundamental scale on the TeV brane to the electroweak scale, determining the masses for the KK towers of the standard-model and GUT particles to be around the TeV scale. These states are composite states of the strong CFT, and they can be produced at future collider experiments if their masses are sufficiently low.

Our theory has the following features:

- Successful gauge coupling unification is preserved. The theory gives the same prediction for $\sin^2 \theta_w$ as the 4D MSSM, at the leading-logarithmic level.
- There is a complete understanding of the MSSM Higgs sector. The masses for the doublet and triplet components of the Higgs multiplets are automatically split by the boundary conditions, while a large mass term for the Higgs doublets is forbidden by a $U(1)_R$ symmetry arising from the higher dimensional structure of the theory. A weak-scale $\mu$ term is obtained without strong fine-tuning as the fundamental scale on the $y = \pi R$ brane is TeV.
- There is no excessive proton decay. If the matter fields are localized to the Planck brane, where the fundamental scale is near the 4D Planck scale and the wavefunctions for the colored Higgs and XY gauge fields vanish, proton decay caused by GUT particle exchange is strongly suppressed. Dangerous tree-level dimension four and five operators are also forbidden by the $U(1)_R$ symmetry. An alternative possibility is to impose a global baryon number symmetry, which is allowed by the structure of the theory, making the proton completely stable.
- An understanding of hypercharge quantization is obtained. Even if its zero modes are strongly localized to the Planck brane, bulk matter arises from $SU(5)$ multiplets and hypercharge is appropriately quantized for successful gauge coupling unification. If matter does not propagate in the bulk but is instead located on the Planck brane, additional dynamics, such as additional space dimensions and a larger unified group, are needed.
- Neutrino masses are obtained through the conventional seesaw mechanism. The desired masses for the light neutrinos are obtained regardless of the wavefunction profiles for the right-handed neutrinos.
- The theory predicts a rich set of new particles at the TeV scale. In particular, there are KK towers for the standard-model gauge and Higgs fields, together with towers for their
supersymmetric and GUT partners. (If matter propagates in the bulk, there are also matter KK towers together with their “GUT partners”.) These towers are approximately $SU(5)$ symmetric and, before supersymmetry breaking, also $N = 2$ supersymmetric.

- Supersymmetry is broken on the TeV brane. The gauge multiplet (and additional bulk multiplets with $c \simeq 1/2$) obtain supersymmetry breaking masses, at tree level, of order the weak scale, $T \equiv ke^{-\pi kr}$. Assuming Planck-brane localized matter and Higgs, squarks and sleptons obtain finite and calculable masses at one loop through gauge interactions. Since these masses are flavor universal, the supersymmetric flavor problem is solved. The Higgs bosons also receive soft masses through loop effects, but the details of electroweak symmetry breaking depend on specific features of the model, especially on the structure of the Higgs sector. The value of $T$ is generically expected to be in the TeV region or higher.

- Supersymmetry breaking is effectively induced at the scale $T$, so that there is little energy interval between the weak scale and the scale where soft supersymmetry breaking masses are generated. This naturally pushes up the masses of the gauginos compared with scalars and could ameliorate the (moderate) fine-tuning required in the MSSM to obtain realistic electroweak symmetry breaking. The lightest supersymmetric particle is the gravitino with mass of order $T^2/M_{Pl} \sim 10^{-3}$ eV. The gravitino is absolutely stable unless we break $R$ parity by brane localized interactions.

- The bulk Lagrangian possesses a symmetry that we call GUT parity, under which all the MSSM particles can be assigned (+) parity while their $SU(5)$ partners (−) parity. (Regardless of whether matter is in the bulk or on the brane, standard-model quarks and leptons, say $D$ and $L$, are not $SU(5)$ partners with each other.) Thus, if this parity is also preserved by brane interactions, the lightest particle with odd GUT parity (LGP) is stable. In most cases, the LGP is one of the XY gauginos. We find that it is possible for supersymmetry breaking effects to push the mass of the lightest XY gaugino well below $T$.

- The KK towers for the MSSM and GUT particles can be produced at collider experiments, if their masses are sufficiently small. After produced, they decay eventually into the LGP (most likely one of the XY gauginos) and the LSP (the gravitino of mass $\sim 10^{-3}$ eV). Since the XY gauginos are colored, they hadronize by picking up an up or down quark, making neutral or charged mesons $T^0 \equiv \tilde{X}_\uparrow \tilde{d}, T^- \equiv \tilde{X}_\uparrow \tilde{u}, T^+ \equiv \tilde{X}_\downarrow \tilde{d}$ and $T^{--} \equiv \tilde{X}_\downarrow \tilde{u}$, where $\tilde{X}_\uparrow$ and $\tilde{X}_\downarrow$ are the isospin up and down components of the XY gauginos, respectively. Among these mesons, the lightest one is either $T^0$ or $T^-$, to which the heavier states can decay through beta processes. However, the decay is slow enough that all the meson states are effectively stable for collider purposes, and the charged mesons will easily be seen because they leave highly ionizing tracks inside the detector.
It is remarkable that the properties of AdS allow a synthesis of Planck-cutoff and TeV-cutoff approaches to physics beyond the standard model. On one hand our theory reveals a variety of exotic phenomena at the TeV scale, as in models with a TeV-scale cutoff: higher dimensional physics shows up through the appearance of KK towers and unified physics is probed by the production of GUT particles. Supersymmetry breaking has intrinsically higher dimensional features, offering the possibility of natural electroweak symmetry breaking. Yet this new physics arises in such a way that certain quantities remain four dimensional (and perturbative) up to energies near the Planck scale. As a consequence, the theory retains many advantages of the high cut-off scale paradigm: in particular, successful gauge coupling unification is preserved and dangerous non-renormalizable operators are highly suppressed.

We have found that the theory has a rigid structure as far as the gauge symmetry breaking is concerned. However, the sector of supersymmetry and electroweak symmetry breaking is far less constrained. Since the detailed phenomenology of our theory, especially the discovery potential at future colliders, depends on this sector (through the precise value of $T$), it will be important to work out further details of the electroweak symmetry breaking sector. In this way we will be able to learn more about the spectrum of the supersymmetric and GUT particles predicted in the theory, beyond the generic features discussed above.

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Appendix A

In this appendix we discuss several properties of theories compactified on the AdS space, clarifying some confusing issues especially when viewed in terms of the 4D KK decomposed picture. We begin by considering the case of a theory compactified on the flat space of an orbifold $S^1/Z_2$; a line segment $y : [0, \pi R]$. Let us first consider how physics appears to an “observer” localized on a point in the extra dimension at $y = y_*$. As we know, the observer feels the effects of the extra dimension through the appearance of KK towers: when the relevant energy scale becomes
higher than $1/R$, the KK towers of the bulk field start participating in physical processes. This implies that, regardless of the location, $y_*$, of the observer, physics becomes five dimensional once the energy scale becomes larger than $1/R$.

What happens when we turn on an AdS curvature $k$ in this system? If $k$ is smaller than $1/R$, the physics picture is almost unchanged from the flat case, $k = 0$. However, once $k$ becomes larger than $1/R$, drastic changes occur. Suppose we consider $k \gtrsim 1/R$. Then, for the observer located at $y = y_*$, physics becomes five dimensional when the relevant energy scale becomes larger than $ke^{-\pi ky_*}$. How is this possible? Suppose we are living on the $y = \pi R$ brane ($y_* = \pi R$). Then we find that the KK states start participating in physical processes when the energy $E$ becomes larger than $ke^{-\pi kR}$. This implies that the mass of the lightest KK states is about $ke^{-\pi kR}$ and we can actually produce these states when $E > \sim ke^{-\pi kR}$. What happens if we are localized on the $y = 0$ brane ($y_* = 0$)? In this case, although the masses of the KK states start from $ke^{-\pi kR}$, their interactions to the particles on the $y = 0$ brane are extremely weak. Thus we do not see the effect of KK towers (i.e. the extra dimension) unless the energy reaches $k$. In fact, we can explicitly see that the wavefunctions of the KK towers for the bulk scalars are exponentially suppressed at $y = 0$, and their interactions with the fields on the $y = 0$ brane are extremely weak.

However, this is not the end of the story. If the observer on the $y = 0$ brane (Planck brane) interacted only with zero modes, the physics could not possibly be four dimensional up to an extremely high scale, $k$. To see this, suppose we put a $U(1)$ gauge field in the bulk and consider scattering two charged particles localized at $y = 0$. Then if we have only zero-mode gauge boson exchange in the $t$-channel, this leads to the following situation: once the energy $E$ becomes larger than $ke^{-\pi kR}$, the loops of the KK states give power-law corrections to the zero-mode gauge coupling through the vacuum polarization diagram (Fig. 3a) and the physics is no longer four dimensional even at energies far below $k$. However, this is not what happens. In contrast to the bulk scalar field, the wavefunctions for higher KK towers of the bulk gauge field are not
exponentially suppressed at $y = 0$. In fact, the wavefunction value at $y = 0$ goes like $1/(\pi k R n)^{1/2}$ for the $n$-th KK tower of the bulk gauge filed. This means that if we compute the scattering amplitude where the towers of the gauge field are exchanged in the $t$-channel, it is proportional to $\sum_n (1/n)$, which is logarithmic in $n$. This implies that all the KK towers contribute “equally” to the process, and we must sum up internal gauge KK bosons (Fig. 3b). In fact, by computing the amplitude given in Fig. 3b, the power corrections from the loops of the bulk field cancel among the diagrams with different KK levels for the gauge boson, and the strength of the scattering depends only logarithmically on the external energy. This is the meaning of the logarithmic running in AdS. There are two sources for this logarithmic dependence; one comes from the summation of the KK gauge towers, and the other from loop corrections to the gauge field propagators. The former is simply a classical effect in the AdS picture and universal for all the gauge groups in unified theories [9]. It is classical because the effect appears even in the tree-level diagrams with $t$-channel exchange of the gauge bosons, and is universal because it is determined only by the form of the KK gauge boson wavefunctions which is independent of the gauge group and matter content. On the other hand, the latter reflects the dynamics of a specific theory, the properties of the fields circulating in the loop. It is this latter effect that is relevant for gauge coupling unification, and which is obtained by the effective field theory calculation of Refs. [12, 16] based on the Planck-brane gauge two-point correlator.

The viewpoint of the Planck correlator leads to an extremely useful intuition about what happens in the theory on AdS. Suppose we compute the gauge two-point correlator with external points on the Planck brane (see Fig. 1 in section 2). Then, if we compute the contribution from the bulk scalar, we find that only the zero mode contributes to this correlator at high energies $E (\lesssim k)$, since an internal gauge boson propagator carries an exponential damping factor $\sim e^{-E|y|}$ and the wavefunctions for the higher KK states are peaked around the TeV brane. From this, we can conclude that the running of the Planck correlator is logarithmic up to the scale $k$. Above $k$, however, power-law corrections arise even for the Planck correlator, since the wavefunctions for the KK towers with 4D masses larger than $k$ are not suppressed at $y = 0$. This shows that the theory appears five dimensional above $k$ if we are living at $y = 0$.

The above consideration suggests that if some unified physics appears at the scale “$M_u$” that is lower than $k$, then it does make sense to talk about logarithmic unification and thus the prediction for gauge coupling unification, since then the uncalculable power-divergent corrections from the scale above $k$ are universal. (Here, “$M_u$” means the scale which appears as $M_u e^{-\pi k y_*}$ when viewed from the observer at $y = y_*$.) Two simple ways of achieving this situation are the case with a 5D unified symmetry broken by the vacuum expectation value of the Higgs field either in the bulk or on the brane, and the unified symmetry broken by boundary conditions in the extra dimension. In this paper we consider the latter case.
We now consider running down the “gauge coupling” \( \tilde{g} \) defined using the Planck correlator: the coefficient of the gauge two-point correlator whose external points are on the Planck brane. Since the Planck correlator is a four-dimensional quantity below \( k \), at low energies \( q \ll M_u \) we obtain
\[
1/g^2_a(q) = 1/g^2_a(M_u) + (b_a/8\pi^2)\ln(M_u/q) \quad \text{up to small non-log corrections,}
\]
where \( a \) represents \( SU(3)_C, SU(2)_L \) and \( U(1)_Y \), and \( b_a \) are numbers of order one, whose non-universal part can be unambiguously computed in the effective theory. Thus, assuming \( 1/g^2_a(M_u) \) is universal, we obtain a prediction for low-energy gauge couplings \( g_a(q) \). (The assumption that \( 1/g^2_a(M_u) \) is universal is not always justified; in the case where the unified gauge symmetry is broken by orbifold boundary conditions, there are uncalculable non-unified corrections from operators localized on the brane, although these contributions are suppressed by the volume of the extra dimension in certain circumstances [4].) Now, the quantities we are really interested in are the couplings \( g_a \) of our zero-mode gauge fields at low energies: energies lower than the scale of KK masses \( T \equiv ke^{-\pi kR} \). At sufficiently low energies \( q \ll T \), however, these zero-mode gauge couplings are the same as the couplings defined by the correlator on the Planck brane, since all the KK towers for the gauge bosons decouple. Therefore, we obtain the prediction for gauge coupling unification by setting \( 1/g^2_a(q) = 1/g^2_a(T) \) at \( q \ll T \).

Having seen that the sensitivity to the “unification scale” is logarithmic in the zero-mode gauge couplings, we can directly calculate the contribution to the zero-mode gauge couplings from the loops of the brane and bulk fields, setting the external momentum scale \( q \) much lower than \( T \). Since we are calculating the quantity \( g_a(q) \), which is equal to \( \tilde{g}_a(q) \) at \( q \ll T \), the answer is again given such that the non-universal correction is logarithmic. This type of calculation is employed in Refs. [12, 15, 16, 17], and we have summarized the essence of the calculation in section 2. Here we just clarify one issue which might be somewhat confusing. In section 2 we have estimated the boundary operators using NDA at the scale “\( \Lambda \)” where the theory becomes strong coupled. Here, the scale \( \Lambda \) is given by \( \Lambda \sim 16\pi^2/R \), since the 5D gauge coupling \( 1/g^2_5 \sim \Lambda/16\pi^3 \) must reproduce the 4D gauge coupling \( 1/g^2_4 = \pi R/g^2_5 \sim \Lambda R/16\pi^2 \), which is of order unity. One might think that the scale of strong coupling, \( \Lambda \), is one-loop larger than \( k \) (not \( 1/R \)), since the theory becomes higher dimensional only above \( k \) (viewed in terms of the Planck brane observer). However, due to the (classical) logarithmic running for \( \tilde{g}_a \), which gives
\[
\tilde{g}_a^2(k) \sim \ln(k/T)\tilde{g}_a^2(T) \sim (\pi kR)g_a^2(T) \sim kR,
\]
the strong coupling scale is determined as \( (\tilde{g}_a^2(k)/(16\pi^2)(\Lambda/k)) \sim (g_a^2(T)/(16\pi^2)(\Lambda R)) \sim 1 \), giving \( \Lambda \sim 16\pi^2/R \). Therefore, in any picture we obtain the answer that the scale \( \Lambda \), which we identify with the 5D Planck scale \( M_5 \), is one-loop higher than \( 1/R \). This shows that the ratio \( M_5/k \) is at most of \( O(10) \) since the \( kR \) must be \( \sim 10 \) to produce the hierarchy between the weak and the Planck scales.

36
Appendix B

We have argued that in the presence of strong supersymmetry breaking on the TeV brane, the lightest GUT particles are XY gauginos. In this appendix we derive the mass eigenvalue equation for these states and derive Eq. (18). We will use a notation in which the XY gauginos and their conjugate states, all contained in $V$, are represented by Weyl spinors $\lambda$ and $\eta$. These are accompanied by $\lambda'$ and $\eta'$ from $\Sigma$. With this notation the boundary conditions on the gauginos are $\lambda(-+), \lambda'(+-), \eta(-+),$ and $\eta'(+-),$ and the mass term generated on the TeV brane by the supersymmetry breaking is of the form $(F_Z/M_5)(\lambda\eta + \text{h.c.})$. The analogous calculation for the case with gauge-trivial boundary conditions was done in [34], which we follow here. After rescaling all fermions by a factor $e^{2ky}$, the equations of motion for the gauginos are

$$\frac{1}{g_5^2} \left[ i e^{ky} \bar{\sigma}^\mu \partial_\mu \lambda - (\partial_y - k/2) \bar{\lambda}' \right] - \frac{F_Z}{M_5} \delta(y - \pi R) \bar{\eta} = 0,$$

(28)

$$\frac{1}{g_5^2} \left[ i e^{ky} \bar{\sigma}^\mu \partial_\mu \lambda' + (\partial_y + k/2) \bar{\lambda} \right] = 0,$$

(29)

along with the same equations with $\lambda \leftrightarrow \eta$ and $\lambda' \leftrightarrow \eta'$. Looking for solutions of the form

$$\lambda = \sum_n f_\lambda^n(y) \lambda_n(x), \quad \lambda' = \sum_n f_{\lambda'}^n(y) \eta_n(x),$$

(30)

$$\eta' = \sum_n f_{\eta'}^n(y) \lambda_n(x), \quad \eta = \sum_n f_\eta^n(y) \eta_n(x),$$

(31)

yields the wavefunctions

$$f_\lambda^n(y) = \frac{e^{ky/2}}{N_n} \left[ J_1 \left( \frac{m_n}{k} e^{ky} \right) + b_\lambda(m_n) Y_1 \left( \frac{m_n}{k} e^{ky} \right) \right],$$

(32)

$$f_{\lambda'}^n(y) = \frac{e^{ky/2}}{N_n} \left[ J_0 \left( \frac{m_n}{k} e^{ky} \right) + b_{\lambda'}(m_n) Y_0 \left( \frac{m_n}{k} e^{ky} \right) \right],$$

(33)

again with identical equations with $\lambda \rightarrow \eta$ and $\lambda' \rightarrow \eta'$. Here $N_n$ are normalization constants.

The $b$ coefficients and the masses $m_n$ are determined by the boundary conditions. At the $y = 0$ boundary we have

$$f_\lambda^n \big|_{y=0} = f_\eta^n \big|_{y=0} = 0,$$

(34)

$$(\partial_y - k/2) f_{\lambda'}^n \big|_{y=0} = (\partial_y - k/2) f_{\eta'}^n \big|_{y=0} = 0,$$

(35)

giving

$$b_i(m_n) = - \frac{J_1(m_n)}{Y_1(m_n)},$$

(36)
for $i = \lambda, \lambda', \eta, \eta'$. Meanwhile the boundary conditions at $y = \pi R$ are

$$f_{n}^{\lambda'} \bigg|_{y=\pi R} = \frac{\pi g_{4}^{2} F_{Z} R}{2 M_{5}} f_{n}^{\lambda} \bigg|_{y=\pi R}, \quad \text{(37)}$$

$$f_{n}^{\eta'} \bigg|_{y=\pi R} = \frac{\pi g_{4}^{2} F_{Z} R}{2 M_{5}} f_{n}^{\lambda} \bigg|_{y=\pi R}, \quad \text{(38)}$$

where we have used $g_{5}^{2} = \pi R g_{4}^{2}$. Then the mass eigenvalues are determined by a single equation,

$$\left[ J_{0} \left( \frac{m_{n}}{T} \right) - J_{1} \left( \frac{m_{n}}{k} \right) Y_{0} \left( \frac{m_{n}}{T} \right) \right] = \frac{\pi g_{4}^{2} F_{Z} R}{2 M_{5}} \left[ J_{1} \left( \frac{m_{n}}{T} \right) - J_{1} \left( \frac{m_{n}}{k} \right) Y_{1} \left( \frac{m_{n}}{T} \right) \right]. \quad \text{(39)}$$

For $(M_{5}/F_{Z}) (2/\pi g_{4}^{2}) \ll 1$, there is thus a light Dirac XY gaugino whose mass is

$$m_{XY} \simeq \frac{4}{\pi g_{4}^{2}} \frac{M_{5}}{F_{Z} R} T. \quad \text{(40)}$$

Taking $(4/\pi g_{4}^{2})$ to be order one then gives Eq. (18).
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