MASS NUMBER DEPENDENCE OF THE SKYRME-FORCE-INDUCED NUCLEAR SYMMETRY ENERGY

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The global mass dependence of the nuclear symmetry energy $a_{\text{sym}}(A)$ and its two basic ingredients due to the mean-level spacing $\varepsilon(A)$ and effective strength of the isovector mean-potential $\kappa(A)$ is studied within the Skyrme-Hartree-Fock model. In particular, our study determines the ratio of the surface-to-volume contributions to $a_{\text{sym}}(A)$ to be $r_{S/V} \sim 1.6$ and reveals that after removing momentum-dependent effects by rescaling $\varepsilon$ and $\kappa$ with isoscalar and isovector effective masses, respectively, one obtains $\varepsilon^* \approx \kappa^*$.

1. Introduction
The most common route in constructing effective microscopic nuclear models starts from the nuclear equation of state in infinite nuclear matter and use such parameters like isoscalar saturation density $\rho_0$, the volume binding energy $a_V$, the incompressibility parameter $K_\infty$, and the asymmetry energy $a_{\text{sym}}^{(V)}$ as primary constraints for these models. All these values are extrapolated, however, from the studies of finite nuclei, in particular from the semi-empirical mass formulas:

$$\frac{E}{A} = -a_V + \frac{a_S}{A^{1/3}} + \ldots + \left[ a_{\text{sym}}^{(V)} - \frac{a_{\text{sym}}^{(S)}}{A^{1/3}} + \ldots \right] \left( I^2 + \lambda \frac{I}{A} \right) + \ldots,$$

(1)

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where \( I \equiv |N - Z|/A \) while \( a_S \) and \( a_{sym}^{(S)} \) are coefficients defining contributions from the surface energy and the surface part of the nuclear symmetry energy (NSE), respectively.

Concerning the isovector terms in finite nuclei which are subject of this work there is at present no consensus concerning the magnitude, \( \lambda \), as well as origin of the Wigner, \( \sim I \), term. The Weizsäcker mass formula (LD) sets \( \lambda \equiv 0 \). The LD of Möller et al.\(^1\) admits only a volume-type Wigner term, which is not consistent with Eq. (1). For this term it gives \( \lambda \approx 0.975 \) i.e. a value close to the one used in shell-model inspired mass formulas which are using \( \sim T(T + 1) \) (i.e. \( \lambda \equiv 1 \)) where \( T = |T_z| = |N - Z|/2 \). Another controversy exists concerning the surface contribution to the NSE. The values of the surface-to-volume ratio \( r_{S/V} = a_{sym}^{(S)}/a_{sym}^{(V)} \) quoted in the literature vary strongly. For example, Danielewicz\(^2\) estimates it to be \( 2.0 \leq r_{S/V} \leq 2.8 \), the mass formula of Ref.\(^1\) yields \( r_{S/V} \approx 1.6 \) while the hydrodynamical-type models that include properly polarization of the isovector density predict \( r_{S/V} \approx 2.3 \).

The main objective of this work is to study various aspects of the NSE emerging within the Skyrme-Hartree-Fock (SHF) model. This work supplements our earlier studies on this subject\(^4,5,6,7\) to which we refer our readers for details.

2. The Skyrme-force induced nuclear symmetry energy and its mass number dependence

In Ref.\(^4\) we have demonstrated that the SHF symmetry energy behaves rather unexpectedly according to the formula:

\[
E_{sym}^{(SHF)} = \frac{1}{2} \varepsilon(A, T_z) T^2 + \frac{1}{2} \kappa(A, T_z) T(T + 1),
\]

where \( \varepsilon(A, T_z) \approx \varepsilon(A) \) and \( \kappa(A, T_z) \approx \kappa(A) \) are fairly independent on \( T_z \), at least for \( T_z \geq 8 \). Hence, for further quantitative analysis of the mass dependence of the NSE we use the mean values of \( \varepsilon(A) \) and \( \kappa(A) \). These averages over \( T_z \) at fixed \( A \) are calculated using the following restricted set of nuclei: \( T_z \geq 4 \) for \( A = 20 \); \( T_z \geq 6 \) for \( A = 24 \); and \( T_z \geq 8 \) for \( 28 \leq A \leq 128 \). By using a restricted set of nuclei we smooth out both \( \varepsilon(A) \) and \( \kappa(A) \) curves in order to diminish the influence of shell structure. The \( \varepsilon(A) \) and \( \kappa(A) \) are interpreted as the mean-level spacing at the Fermi energy in iso-symmetric nucleus and the effective strength of the isovector mean-potential, respectively.

The relation (2) holds extremely well within the SHF model except for the SkO parameterization\(^8\) for which we observe an enhancement in the linear part of the second term in Eq. (2). For the SkO the second term becomes \( \sim \kappa T(T + \lambda) \) with \( \lambda > 1 \), as illustrated in Fig. 1. The value of \( \lambda \) is however much weaker than \( \lambda_{RMF} \approx 1 + \varepsilon/\kappa \) found recently in relativistic mean-field (RMF)\(^6\). It should be mentioned that the SkO is characterized by an exceptionally strong isovector strength of the spin-orbit term, inspired by RMF, having opposite sign compared to standard SF parametrization.
Fig. 1. Values of $\kappa(A, T_z)$ calculated using SLy4 (circles) and SkO (squares). The calculated points mark the percentage of calculated nuclei for which $\kappa(A, T_z)$ deviates from the mean-value $\bar{\kappa}(A)$ by less than three percent versus $T_z$. Two different scenarios are assumed for the linear contribution to $\sim \kappa T_T \left( T + \lambda \right)$ term with $\lambda = 1$ (filled symbols) and $\lambda (\equiv \lambda_{RMF}) = 1 + \varepsilon / \bar{\kappa}$ (open symbols). Note that while for the SLy4 $\lambda = 1$, for the SkO it is half way between $1 < \lambda < \lambda_{RMF}$.

Fig. 2. The isoscalar effective mass scaled values of $\bar{\varepsilon}^*(A) \equiv m_A^2 A \bar{\varepsilon}(A)$ (stars) and $\bar{\kappa}(A)$ (circles) calculated using the shf method with SLy4 parametrization. Open symbols denote $\bar{\varepsilon}^*(A)$ and $\bar{\kappa}(A)$ averaged over all the calculated nuclei. Filled symbols mark smoothed values of $\bar{\varepsilon}^*(A)$ and $\bar{\kappa}(A)$ calculated using a restricted set of data. Vertical arrows indicate major shell gaps. Note the strong influence of shell structure on $\bar{\varepsilon}^*(A)$ and the smooth behavior of $\bar{\kappa}(A)$.

The global mass dependence of the two components of the symmetry energy, $\bar{\varepsilon}$ and $\bar{\kappa}$ is shown in Fig. 2. The figure reveals several universal features which appear to be independent of the type of the SF parametrization including: (i) strong
dependence of $\bar{\varepsilon}(A)$ on kinematics (shell effects); (ii) almost no dependence of $\bar{\kappa}(A)$ on kinematics; (iii) clear surface ($\sim 1/A^{4/3}$) dependence reducing the dominant volume term ($\sim 1/A$) in both $\bar{\varepsilon}(A)$ and $\bar{\kappa}(A)$.

Indeed, the values of $\bar{\varepsilon}(A)$ show characteristic kinks close to double-(semi)magic $A$-numbers. These kinks are magnified when all the calculated nuclei are used (no smoothing) to compute $\bar{\varepsilon}(A)$, but without affecting qualitatively the overall profile of the curve. On the other hand, $\bar{\kappa}(A)$ is almost perfectly smooth with barely visible traces of shell structure. It confirms our earlier conclusion \(^4\) that the gross features of the Skyrme isovector mean potential can be almost perfectly quantified by a smooth curve parametrized by a small number of global parameters.

Fig. 3. Histogram showing the deviation between neutron skin thickness estimated using the simple hydrodynamical formula (3) taken for $r_{S/V} = 1.6$ and the results of microscopic SHF calculations. Note, that the deviation is of the order of $\pm 3\%$. Note also, that the centroid of the histogram is slightly shifted to the right what indicates that $r_{S/V}$ should be slightly larger $r_{S/V} \sim 1.65$.

The SHF models yield \(^7\) $r_{S/V} \sim 1.6$ in accordance with the LD ratio \(^1\). There are several observables which are strongly sensitive to $r_{S/V}$. These include in particular the static dipole polarizability (SDP) and neutron skin thickness. Using the hydrodynamical model of \(^3\) one can derive simple expressions for both the SDP and the neutron skin thickness. It appears that for $r_{S/V} \sim 1.6$ these simple expressions yield results that are very consistent both with the data (for SDP) and the microscopic SHF calculations (neutron skin thickness), see \(^7\). For example, the hydrodynamical formula for the neutron skin thickness [in the lowest order expansion in $1/A$] reads:

$$
\frac{\delta r^2}{\langle r^2 \rangle} \approx \frac{N - Z}{A} \left\{ 1 + \frac{2}{3} \frac{r_{S/V}}{A^{1/3}} - \ldots \right\},
$$

(3)
The interesting feature of Eq. (3) is that it does not depend on the bulk nse coefficient but only on the ratio $r_{S/V}$. This formula can easily be cross-checked with our microscopic SHF calculations and the results are depicted in the form of a histogram in Fig. 3.

Fig. 4. The correlation between the effective mass scaled volume $\varepsilon^*_V$ and $\kappa^*_V$ (open dots) and the surface $\varepsilon^*_S$ and $\kappa^*_S$ expansion coefficients. Note, that except for the SIII and SkM* interactions, the expansion coefficients are equal.

The most striking result of our analysis is the near-equality of $\bar{\varepsilon}^* \approx \bar{\kappa}^*$ [where $\bar{\kappa}^* \equiv \bar{\kappa} \frac{m_1}{m}$ and $m_1^*$ denotes the standard isovectorial effective mass] occurring for all modern parameterizations, see Fig. 4. Indeed, $\bar{\varepsilon}^*$ differs from $\bar{\kappa}^*$ only for old parameterizations like the SIII and SkM*. This result confirms the rather loose claims often appearing in the textbooks that "the kinetic energy $[\varepsilon_F G]$ and the isovector mean-potential contribute to the $a_{\text{sym}}$ in a similar way" is indeed correct but only after disregarding non-local effects. To our knowledge, it has never been discussed why this apparently independent quantities should be similar.

3. Summary

The global mass dependence of the nse strength $a_{\text{sym}}(A)$, $\bar{\varepsilon}(A)$, and $\bar{\kappa}(A)$ is studied in detail within the SHF theory. Our study enables us to establish the surface-to-volume ratio of $a_{\text{sym}}(A)$, $r_{S/V} \approx 1.6$. This value is in agreement with the LD of Ref. 1 and is consistent with simple hydrodynamical estimates for the SDP and neutron skin thickness. Our study also reveals an almost linear correlation between $C^0_1$ and $(\bar{m}_1)^{-1}$, see Fig. 5, and a striking similarity between $\bar{\varepsilon}^* \approx \bar{\kappa}^*$. The latter near equality indicates that the contribution to $a_{\text{sym}}$ due to the mean-level spacing and due to the mean-isovector potential are similar but only after disregarding non-local effects. Whether this is a fundamental property of the nuclear mean field is
an open question that requires further studies.

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