Comparison between UofC Model and Ionosphere-free Combination Model in PPP

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Abstract. GPS Navigation Satellite System is a satellite-based positioning system, whose accuracy and reliability depend on the number of observable satellites. The precise point positioning (PPP) in GPS navigation satellite system is a technique which can provide the centimeter-level positioning by a dual-frequency receiver and thus the research on precise point positioning is of great significance. In this paper, the ionosphere correction models of precise point positioning (PPP) are studied, including ionosphere-free combination model and UofC model. The static GPS positioning data obtained from the experiment is used and we compare the positioning accuracy of the two models. The result shows that the positioning precision using UofC model is decimeter grade, while the positioning precision using ionosphere-free combination model is centimeter level.

1. Introduction

GPS system is used widely in our daily life, especially in the field of positioning and measurement. With the fast developing of satellite positioning technique, receiver users expect not only more on accuracy, real-time and integrity of positioning, but also convenience and cost. However, in traditional GPS static positioning, the accuracy of the absolute positioning can be up to 20m, which cannot satisfy the demand of precision navigation and measurement[1]. The PPP technique provides a precision positioning method which can be used in the GPS navigation satellite system. Since the PPP approach was firstly proposed to realize the single station positioning with fixed precise orbit solutions and Doppler satellite observations by R.R. Anderle in early 1970’s, it has experienced the development from dual-frequency to single-frequency and to multi-frequency, from CPS single system to GNSS multi systems, from fuzziness floating-point solution to ambiguity fixed solution and from post processing to real-time processing. For GPS users, carrier phase and pseudo-range observations from one GNSS receiver are utilized, combined with the product of high accuracy satellite orbit and clock error correction, as well as model correction and parameter estimation, correcting errors from satellite, signal path and receiver[2]. And then, the high accuracy positioning can be realized. This process is called PPP technique.

Based on different processing methods towards ionosphere and clock errors of receivers, different PPP positioning models were proposed in these years. In early years, an ionosphere-free combination model was utilized, which used dual frequency pseudo-range and phase observation values, constituting the pseudo range observation equation and carrier phase observation equation of the ionosphere respectively[3][4][5]. In 2001, Gao Yang Professor proposed a UofC model to solve the problem of large pseudo-range ionospheric combination noise. UofC model used the phase observation equation of the dual frequency ionospheric combination and the two-half frequency observation equation with pseudo range carrier frequency[6].
In this paper, two methods of ionospheric correction are compared, UofC model and ionosphere-free combination model. By the means of comparing the two models, it is helpful when choosing ionosphere correction model.

2. PPP observation equations
In the technique of PPP, observations are pseudo-range and carrier-phase. Pseudo-range and carrier-phase observation equations in meters are:

\[ p_{i,k}^k = \rho_{i,k}^k + c \cdot (dt - \delta t)^k + T_{i,k}^k + I_{i,F}^k + d m_{i,F}^k + e_{i,F}^k \]  

\[ l_{i,k}^k = \rho_{i,k}^k + c \cdot (dt - \delta t)^k + T_{i,k}^k - I_{i,F}^k + \lambda \cdot N_{i,k}^k + \delta m_{i,F}^k + e_{i,F}^k \]  

Where:
\[ P, L \]: represent pseudo-range and carrier-phase observation in meters. Superscript \( k \) represents satellite, subscript \( i \) represents receiver, subscript \( F \) represents frequency;
\[ \rho = ||r_i - r_k|| \]: represents the three-dimensional distance of a signal from a satellite to a satellite navigation receiver antenna
\[ r_i = [x_i, y_i, z_i]^T \] and \[ r_k = [x_k, y_k, z_k]^T \] represent the position vectors of the receiver antenna and the satellite respectively.
\[ dt \] and \( \delta t \) represent clock correction of receiver and satellite respectively in seconds.
\[ T \]: tropospheric delay in meters
\[ I_{i,F} = \alpha T E C / f_k^2 \]: Ionospheric delay in meters, in which \( \alpha \) is a constant, \( T E C \) is total electron density of signal propagation path, \( f_k \) is signal frequency.
\[ d m \] and \( \delta m \) multipath error in pseudo-range and carrier-phase propagation channels
\[ N \]: phase ambiguity in cycles
\[ \lambda \]: carrier wavelength in m/cycle
\[ c \]: propagation speed of light in vacuum in m/s
\[ e \] and \( \varepsilon \): pseudo-range noise and carrier phase noise respectively in meters.

3. The parameters and error correction of PPP
Concentrating on analyze and compare the ionospheric correction models, two of the ionospheric correction methods are UofC model and ionosphere-free combination model.

3.1. UofC
Ionospheric delay in pseudo-range and carrier-phase observation equations is equal in quantity and opposite in direction. The model of UofC establishes thought pseudo-range and carrier-phase observations summing up in half-rations.
Mathematical expressions are:

\[ P_{(i,1)} = 1/2(P_{(i,1)} + I_{(i,1)}) = \rho_{i,1}^k + c(dt - \delta t)^k + T_{i,1}^k + I_{(i,1)}^k + d m_{i,1}^k + e_{i,1}^k \]  

\[ P_{(i,2)} = 1/2(P_{(i,2)} + I_{(i,2)}) = \rho_{i,2}^k + c(dt - \delta t)^k + T_{i,2}^k + I_{(i,2)}^k + d m_{i,2}^k + e_{i,1}^k \]  

\[ l_{i,(F)} = \frac{1}{(f_i - f_2)^2} (f_i^2 l_i - f_2^2 l_2) = \rho_{i,2}^k + c(dt - \delta t)^k + T_{i,2}^k + \frac{C(f_i, N_{i,k}^2 - f_2, N_{i,k}^2)}{f_i^2 - f_2^2} + \frac{f_i^2 e_{i,k}^2 - f_2^2 e_{i,k}^2}{f_i^2 - f_2^2} + \frac{f_i^2 e_{i,k}^2 - f_2^2 e_{i,k}^2}{f_i^2 - f_2^2} \]  

UofC model takes the advantage that ionospheric delay is equal in size and opposite in direction in pseudo-range and carrier-phase observation equations, so that it can eliminate the error. As for parameter estimation, it introduces an extra parameter of ambiguity.
3.2. Ionosphere-free combination model

Let $\alpha = \frac{f_1^2}{f_1^2 - f_2^2}$ and $\beta = \frac{f_2^2}{f_1^2 - f_2^2}$, then multiply $\alpha$, $\beta$ to observation equations of two frequency respectively, subtract each other, we can acquire equations as follows:

\[
L_s = \frac{1}{(f_1^2 - f_2^2)} (f_1^2 L_1 - f_2^2 L_2) = \rho^i + c(d_i - \delta^i_t) + T_t^i + \frac{\gamma(f_1 N_{i(1)}^t - f_2 N_{i(2)}^t) + f_1^2 \delta N_{i(1)}^t - f_2^2 \delta N_{i(2)}^t + f_1^2 \epsilon_{i(1)}^t - f_2^2 \epsilon_{i(2)}^t}{f_1^2 - f_2^2} (6)
\]

\[
P_e = \frac{1}{f_1^2 - f_2^2} (f_1^2 P_1 - f_2^2 P_2) = \rho^k + c(d_i - \delta^k_t) + T_t^k + \frac{f_1^2 \delta m_{i(1)}^t - f_2^2 \delta m_{i(2)}^t + f_1^2 \epsilon_{i(1)}^k - f_2^2 \epsilon_{i(2)}^k}{f_1^2 - f_2^2} (7)
\]

4. Static PPP experiment

In this section, the experiment of two models of PPP was done. Experiment was carried out on a top of building in Beijing, located in $39^\circ57' N, 116^\circ19' E$ on March 3th, 2018. It observed 12400 epochs, dual-frequency receiver used GPS L1 and L2 frequency. Satellite cut-off elevation angle was 10°. Troposphere delay model used tropospheric zenith delay model, receiver clock error correction looked as white noise approximately. In order to analyze the static positioning accuracy, we used observation data in RINEX format, meanwhile adopted precise ephemeris from IGS to correct the errors.

PPP result saved in RINEX format files in the version of 3.02, accompanied by precise ephemeris. Results were transformed into local Cartesian coordinate system, with three axes in E, N, U terms, which represent east, north and up respectively.

Dealing with ionosphere-free combination model, having converged, result is shown as figure 1:

![Figure 1. positioning errors dealt with ionosphere-free combination model](image-url)

While, dealing with UofC model, having converged, result is shown as figure 2:

![Figure 2. positioning errors dealt with UofC model](image-url)
In order to compare positioning errors from two models, results after convergence are dealt with 2σ standard deviation. Result of two methods are list as follows:

|                  | UofC                        | ionosphere-free combination |
|------------------|-----------------------------|-----------------------------|
| E(2σ)            | 0.1057                      | 0.0018                      |
| N(2σ)            | 0.1914                      | 0.0302                      |
| U(2σ)            | 0.0988                      | 0.0245                      |

From list 1, when positioning with ionosphere-free combination model, errors after convergence are within 4cm, which is centimeter-level. As for UofC model, errors after convergence are within 20cm, which is decimeter-level.

5. Conclusion

GPS is the first and most widely-used global satellite navigation system. PPP technique can provide centimeter-level positioning result. Based on the theoretical research on PPP, this paper compared the models of ionosphere-free combination and UofC. We did an experiment to compare the two models with dual-frequency GPS receiver, also using precise ephemeris from IGS. This experiment shows that, in static PPP experiment, PPP with ionosphere-free combination model is centimeter-level, while UofC model is decimeter-level. It also helps us with PPP when choosing models to eliminate ionosphere errors.

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