Sum rule for transport in a Luttinger liquid with long range interaction in the presence of an impurity.

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We show that the non-linear dc transport in a Luttinger liquid with interaction of finite range in the presence of an impurity is governed by a sum rule which causes the charging energy to vanish.

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A fundamental problem of electron transport in quantum coherent systems is tunneling through a potential barrier [1]. Since the discovery that interactions play here a crucial role [2] considerable theoretical effort has been devoted to the study of the transport in one dimensional (1D) interacting electron systems described by a Luttinger model [3,4] with a potential barrier [5]. It has been shown by renormalization group techniques [6] and by conformal field theory [7] that the current is suppressed at low bias voltage $U$. The current-voltage characteristic is given by $I(U) \propto U^{2/g-1} \ (U \rightarrow 0)$, with the interaction constant $g$ ($<1$ for repulsive interaction). This implies that even an infinitesimally weak scattering potential is completely insulating. For a tunnel junction connecting two Luttinger systems, it has been shown that a finite range of the interaction induces linear behavior at sufficiently high bias voltage, $I(U) = R_t^{-1}(U-U_c)$ (tunnel resistance $R_t$) [8], with the “charging energy” $E_c \equiv eU_c$ proportional to the interaction potential at zero distance.

In this paper, we consider a potential barrier of the height $U_b$ in a Luttinger liquid. The charging energy can be related to the difference of the spectral functions $J(\omega)$ and $J_0(\omega)$ of the elementary excitations with ($g < 1$) and without ($g = 1$) interaction, respectively, via the sum rule ($\hbar = 1$)

$$E_c = \int_0^\infty d\omega \frac{J(\omega) - J_0(\omega)}{\omega} = \int_0^\infty d\omega Z(\omega). \quad (1)$$

The “impedance function”, $Z(\omega)$, is asymptotically given for large and weak impurity potential

$$Z(\omega) = \begin{cases} \frac{2\omega^2}{\pi} \text{Re} \left[ \frac{1}{\sigma(\omega)^{-1} - \sigma_0(\omega)^{-1}} \right] & (U_b \rightarrow \infty) \\ \frac{2\omega^2}{\pi} \text{Re} \left[ \frac{1}{\sigma(\omega) - \sigma_0(\omega)} \right] & (U_b \rightarrow 0) \end{cases} \quad (2)$$

with the retarded conductivities $\sigma(\omega) = \sigma(x = x', \omega)$ and $\sigma_0(\omega)$ of the interacting and the non-interacting electron system, respectively, without impurity. We will show that for arbitrary form of the interaction potential of a finite range $E_c = 0$ in both asymptotic limits. From this, and the results of extensive numerical calculations, we infer that the charging energy of an impurity in a Luttinger liquid is always zero, independently of the height of the barrier. The physical reason for this is the conservation of the total number of excitations, as one can see from eqs. (1) and (2) and will be discussed below in more detail.

We consider the Hamiltonian $H \equiv H_0 + H_b + H_U$ where the spinless Luttinger Hamiltonian $H_0$ corresponds to the spectrum $\omega_k = v(k)|k|$ of Bosonic pair excitations with the charge sound velocity $v(k) = v_F(1 + \bar{V}(k)/\pi v_F)^{1/2}$, $v(0) \equiv v_F/g$, with $\bar{V}(k)$ the Fourier transform of the interaction potential of the range $\alpha^{-1}$, and $v_F$ Fermi velocity [9]. The Hamiltonian of the localized potential barrier at $x = 0$, $H_b = U_b \cos[2\pi \theta(x=0)]$, is non-linear in the phase field $\theta(x)$ which describes the electron density fluctuations $\rho(x) = \rho_0 + \partial_x \theta(x)$.

The term due to the applied voltage is $H_U = e \int_{-\infty}^\infty dx \, U(x) \rho(x)$ and the current

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\[ j(x) \equiv -e \dot{\vartheta}(x). \]  

Since we consider the stationary limit, the average current \( I(x) \) is independent of the position. We evaluate the current at \( x = 0 \). The reduced density matrix is then calculated as usual by averaging over the bulk modes at \( x \neq 0 \).

The result allows to identify the dissipative kernel of the Bosonic excitations and the effective driving force. Using imaginary time formalism the effective Euclidean action for the “particle”, \( \vartheta(x = 0, \tau) \equiv \vartheta(\tau) \) is [10]

\[ S[\vartheta] = \int_0^\beta d\tau \left[ U_b \cos(2\pi \vartheta(\tau)) + F \vartheta(\tau) \right] - \frac{1}{2} \int_0^\beta d\tau d\tau' \vartheta(\tau) K(\tau - \tau') \vartheta(\tau'). \]  

Here, the external force \( F \equiv e \int_{-\infty}^\infty dx E(x) \equiv U \) is independent of the spatial shape of the electric field \( E(x) = -\partial_x U(x) \).

The kernel \( K(\tau - \tau') \) corresponds to the inverse of the propagator of the Bosonic excitations in imaginary time

\[ \tilde{K}(\omega_n) = 2e^2 \frac{\omega_n}{\sigma(\omega_n)}. \]  

The conductivity without impurity potential at the Matsubara frequencies \( \omega_n \) is [11]

\[ \sigma(x, \omega_n) = \frac{2e^2 e^2}{\pi^2} \int_0^\infty dk \frac{\omega_n \cos(kx)}{\omega_n^2 + \omega_k^2}. \]  

In order to evaluate the average current we use the real time path integral formulation. It can be applied straightforwardly in the above two limiting cases of strong and weak scatterer. For a strong barrier, the minima of \( H_b \) are very deep, and the variable \( \vartheta(x = 0) = n \) \((n \text{ integer})\) is discrete. This reflects that charge is transferred through the barrier only in integer units with a tunneling probability amplitude \( \Delta \equiv \Delta(U_b) \).

In lowest order, only paths connecting neighboring minima contribute to the current

\[ I(U) = \frac{ie\Delta^2}{2} \int_{-\infty}^\infty dt \sin(eUt)e^{-W(t)} \]  

with

\[ W(t) = \int_0^\infty d\omega \frac{J(\omega)}{\omega^2} \left[ (1 - \cos(\omega t)) \coth \left( \frac{\beta \omega}{2} \right) + i \sin(\omega t) \right] \]  

(\( \beta \) inverse temperature). The spectral density \( J(\omega) \) is related to the analytic continuation of the above kernel \( \tilde{K}(\omega_n) \)

\[ J(\omega) = -\frac{1}{\pi} \text{Im} \left[ \tilde{K}(-i\omega + \delta) \right] = \frac{2e^2}{\pi} \Re e \frac{\omega}{\sigma(\omega)}. \]  

In order to avoid introducing a cutoff in eq. [3], we add and subtract in the exponential of eq. [3] \( W_0(t) \), the kernel for \( g = 1 \). Thus, one can formally rewrite the current into a form which resembles the semi-classical result [12],

\[ I(U) = \frac{1 - \exp(-\beta eU)}{eR_t} \int_{-\infty}^\infty dE dE' f(E)f(-E')P(E - E' + eU) \]  

with \( f(E) \) the Fermi function and \( R_t \equiv 2\omega_{\text{max}}^2/\pi e^2 \Delta^2 \gg 2\pi/e^2 \) the tunneling resistance (\( \omega_{\text{max}} \) cutoff frequency).

The function

\[ P(E) = \frac{1}{2\pi} \int_{-\infty}^\infty dt e^{iEt} e^{-[W(t) - W_0(t)]} \]  

has in the semi-classical theory the meaning of a probability for an excitation with energy \( E \). Here, in the microscopic model, this is no longer the case: it can assume negative values.

At zero temperature eq. [10] becomes

\[ I(U) = \frac{1}{eR_t} \int_0^\infty dE (eU - E)P(E). \]
For small bias voltage we find

\[ I(U) \propto \frac{U}{R_t} \left( \frac{eU}{\lambda} \right)^{2/g-2}. \] (13)

This result is well known, except that here an intrinsic cutoff parameter \( \lambda(\propto \alpha) \) appears which reflects the decay of \( Z(\omega) \) for \( \omega \to \infty \) due to the finite range of the interaction, \( \alpha^{-1} \).

In the limit of large voltage, only small times contribute to the integral in eq. (11). By expanding the kernel eq. (8), \( W(t) \approx \int_0^\infty d\omega J(\omega)/\omega \), and evaluating \( P(E) \) from eq. (11), one finds \( I(U) = (eU - E_c)/eR_t + O(1/U) \), where \( E_c \) is given by eq. (1) and (2) for \( U_b \to \infty \). By rewriting \( \sigma(\omega) - \sigma_0(\omega) \equiv -\sigma_0(\omega)^{-1} \sum_{n=0}^{\infty} [\sigma(\omega)/\sigma_0(\omega) - 1]^n \), and using that \( \lim_{\omega_{\text{max}} \to \infty} \sigma_0(\omega) = \text{const.} \), one can straightforwardly show via Fourier transformation that \( \Re \int_0^\infty d\omega [\sigma(\omega) - \sigma_0(\omega)]^n = 0 \), since \( \hat{\sigma}(t), \hat{\sigma}_0(t) \propto \Theta(t) \) (\( \Theta \) Heavyside function) [13]. This implies \( E_c = 0 \).

We evaluated the \( I(U) \) for various forms of the interaction potentials, which were obtained by projecting a 3D screened Coulomb repulsion to a quasi-1D quantum wire [11]. We found that the charging energy always vanishes for high barrier within the numerical errors (\(< 10^{-9}\) ), as long as the range of the interaction is finite. Figure shows the result for the “Luttinger limit” \( V(x) = V_0 \alpha e^{-\alpha|x|} \). It is defined by the requirement that the screening length \( \alpha^{-1} \) is much smaller than the effective width of the wire. The non-analytical small-voltage behavior which depends strongly on the interaction constant \( g \equiv v_F/v(0) \) is clearly depicted. For large voltage all of the curves tend asymptotically to the non-interacting limit, \( I(U) = U/R_t \), in accordance with the above analytic finding. It is important to notice that for the vanishing of the charging energy it is necessary to have a point of inflection in \( I(U) \). The existence of the latter is guaranteed by the fact that the asymptotic limits of \( J(\omega) \) for small and large frequency behave as

\[ J(\omega) = 2\omega \begin{cases} \frac{1}{g} + O \left( \frac{\omega}{\alpha} \right)^2 & (\omega \to 0) \\ 1 - \frac{\omega}{2\pi} & (\omega \to \infty) \end{cases} \] (14)

Here, \( V_\omega \equiv \tilde{V}(\omega/v_F) \).

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For small potential barrier, the current can be evaluated by using the well known duality argument [14,15,16]. The calculation to second order gives

\[ I(U) = G_0 U - \frac{1}{2} g e U^2 \rho b \int_{-\infty}^{\infty} dt \sin (geUt) e^{-\tilde{W}(t)} \] (15)

FIG. 1. The current-voltage characteristic of a high potential barrier in a Luttinger liquid with an interaction potential of finite range \( \alpha^{-1} \) for different strengths of the interaction, \( g^{-2} \).
with the renormalized dc conductance of the Luttinger wire, \( G_0 = g e^2 / 2 \pi \). The function \( \tilde{W}(t) \) has the same form as \( W(t) \), however with a spectral density \( \tilde{J}(\omega) \) that is related to \( J(\omega) \) by the duality transformation \[ \tilde{J}(\omega) = \frac{(2\pi\omega)^2 J(\omega)}{|K(\omega)|^2} = \frac{2\pi}{e^2} \omega \text{Re} \sigma(\omega). \] (16)

At \( T = 0 \) one obtains now

\[ I(U) = G_0 U - \frac{1}{\varepsilon R_b} \int_0^u dE (eU - E) \tilde{P}(E) \] (17)

where \( \tilde{P}(E) \) is given again by eq. (11), but with \( \tilde{W}(t) \) instead of \( W(t) \) and with the resistance of barrier given by \( R_b = 2\omega_{max}^2 / \pi e^2 g U_b^2 \).

At sufficiently high voltage, the current becomes now

\[ I(U) = (G_0 - R_b^{-1})U + \frac{E_c}{\varepsilon R_b}, \] (18)

and always

\[ E_c = \frac{2\pi}{e^2} \text{Re} \int_0^\infty d\omega (\sigma(\omega) - \sigma_0(\omega)) = 0. \] (19)

The latter equality can easily been seen by evaluation of the frequency integral using eq. (16). One observes that that the vanishing of the charging energy is nothing but a consequence of the conservation of the number of elementary excitations, independently of the form of the interaction.

For small bias, but \( eU/\lambda \gg (U_b/\lambda)^{1/2 (1-g)} \), one finds

\[ I(U) - G_0 U \approx \frac{U}{R_b} \left( \frac{eU}{\lambda} \right)^{2(g-1)}. \] (20)

When assuming that for very small voltage the correct result was, as described above, \( I(U) \propto U^{2/g-1} \), a point of inflection must exist also for small barrier in the current-voltage curve. The point of inflection indicates the crossover from the suppression of the current due to the coupling to the bath of bulk modes of the Luttinger wire to the behavior at high voltage where interaction becomes unimportant, in this model.

From the asymptotic results for very high and small potential barrier that have been confirmed for several different forms and strengths of the interaction potential by numerical evaluation \[ 13 \], we conclude that any potential barrier in a Luttinger model of 1D interacting electrons gives always zero charging energy, as long as the interaction potential is of finite range.

This is also suggested by results obtained for the model of a tunnel junction connecting two semi infinite Luttinger systems \[ 13 \]. Here, we found \( E_c = 0 \) if the strength of the interaction between electrons located at different sides of the junction, \( V_{12} \), is the same as for electrons on the same side, \( V_{11} \). A non-zero charging energy is however obtained when \( V_{12}/V_{11} < 1 \). Generalization of the present scattering model to such an interaction potential yields the same result. This shows also that the approximations involved in both models, namely using a tunneling Hamiltonian in the former, and the “instantaneous jump approximation” when evaluating the path integral in the latter, are equivalent.

In the tunnel junction model, the physics is more clearly displayed: the absence of the charging effect at higher voltage is related to the presence of the interaction between electrons left and right of the barrier. It is only for zero range interaction that the current persists to be suppressed up to the bias voltage which corresponds to the cutoff frequency \( U_{max} = \omega_{max}/e \). Any finite interaction range will eventually, for sufficiently large bias voltage (but smaller than the cutoff voltage), yield the crossover to Ohm’s law \( I(U) = U/R \). As a consequence, the analogue to the classical “capacitance” of the system described by the present microscopic model diverges.

Generalizing the model of a tunnel junction between two Luttinger systems with zero range interaction (with cutoff) to include many \( \langle N \rangle \) transport channels \[ 17 \] yielded \( g(N) \rightarrow 1 \) for \( N \rightarrow \infty \), such that the suppression of the current for small bias vanishes in this limit. Since the interaction becomes small with increasing \( N \), we expect that our above, somewhat intriguing result will remain essentially unchanged when including the influence of many channels.

This should render the “capacitance of a tunnel junction” proportional to the area of the junction \( (\propto N) \) to be unobservable as long as the interaction between electrons on different sides is assumed the same as between those on the same side of the junction.
It is indeed well known that it is very difficult to observe experimentally Coulomb blockade using a single tunnel junction due to the presence of a large shunt capacitance \[18,19\]. Our above result offers an explanation for the microscopic source of this shunt capacitance: the interaction between electrons left and right of the impurity.

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