The realization of the virtual synchronous generator based on the park model

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Abstract. In this paper, a kind of virtual synchronous generator control algorithm is proposed that can accurately simulate the external characteristic of a synchronous generator. This algorithm based on the synchronous generator park mathematical model considers the motor salient pole effect and the influence of damper windings. Using two-step Adams method to solve the mathematical model of the synchronous motor for current of the motor's port, the converter is controlled to track the current so that the output status values have the same port characteristics with the actual motor. Simulation results verify that in many typical working conditions such as the synchronous motor starting, the output voltage mutating or the frequency changing, the converter controlled in the above algorithm can mimic a real synchronous generator.

1. Introduction

In the distributed generation and the micro grid technology, the grid-connected inverter has played an irreplaceable role as an important interface that connects the renewable energy unit and the grid. The control strategy of the inverter is always the key point of the study. However, because of the fast response speed, traditional control methods can hardly participate in the regulation of the grid and are unable to provide barely any inertia for the micro grid that relatively lacking stability or the necessary voltage and frequency support[1-4]. If a control strategy is able to make a grid-connected inverter has the same external characteristic like a synchronous generator, the traditional power system related control strategies can be easily transplanted in micro grid or other active distributed grid. This control strategy is so called the virtual synchronous generator (VSG) strategy.

Leuven University has put forward a kind of low-order VSG algorithm [5] that can simulate the characteristics of rotor inertia and the frequency modulation process. However, this algorithm only considers the motion equation of the motor which ignores the excitation adjustment mechanism. To further reflect the features, German Clausthal industrial university puts forward the VISMA scheme[6] that uses the third-order model of the motor considering the stator electric equation as well as the rotor mechanical equation. By controlling the filter inductance current, the inverter thus achieve the purpose of port characteristics simulations. Although this simplified model that neglects the salient pole effect and the influence of damper windings meets the demand for most steady circumstances, the dynamic process does differ a lot when the actual operating condition changes. Since the damper windings maintain the flux constant and suppress the rotor oscillation in dynamic variation, the effects should be taken into account. In order to mimic the damper effect, literatures[7-10] introduce the damper
coefficient. Although this parameter is equivalent to damper windings in terms of impact, it can’t match the actual motor coefficients. Thus, the transient characteristics between the VSG model and the actual motor are inconsistent.

To accurately simulate the transient characteristics of the actual synchronous generator, this paper proposes a virtual synchronous generator algorithm based on the synchronous generator full-order model. This method takes the salient-pole effect and the impacts of damper windings into account so that the output characteristics of the inverter match the actual synchronous generator in steady state as well as in transient process. Therefore, an inverter controlled in the VSG strategy is able to provide grid-connected interface in a micro grid and simulate the synchronous motor fault model.

2. synchronous generator modelling

In a synchronous generator, a dc current is applied to the rotor winding, which produces a rotor magnetic field. The rotor of the generator is then turned by a prime mover and produces a rotating magnetic field within the machine. This rotor magnetic field induces a three-phase set of voltages in the stator windings.

The synchronous generator operation satisfies the following equations (1)-(3)[11].

\[
E_A = K\psi \omega
\]

\[
V_\phi = E_A - jX_s I_A - R_A I_A
\]

\[
T_m - T_e = J \frac{d\omega}{dt}
\]

where \(E_A\) is the armature voltage, \(K\) is the proportional factor, \(\psi\) is the magnetic flux, \(V_\phi\) is the stator output voltage, \(X_s\) is the synchronous reactance, \(R_A\) is the synchronous resistance, \(\delta\) is the phase angle between \(E_A\) and \(V_\phi\), \(T_m\) is the mechanical torque, \(T_e\) is the electromagnetic torque.

Commonly, synchronous generator model has two forms[12]: one is the park model based on the prototype circuit parameters and takes flux as the state variable; the other is the practical model based on the practical parameters and takes the electric potential as the state variable. This article chooses the park model.

2.1. Flux equations

The fluxes in stator and rotor can be governed by equation (4) where \(\psi_{dq0s}\), \(\psi_{dq0r}\), \(I_{dq0s}\) and \(I_{dq0r}\) refer to the stator flux, the excitation and the rotor flux, the stator current and the rotor current in damper windings in dq axis respectively.

\[
\begin{bmatrix}
\psi_{dq0s} \\
\psi_{dq0r}
\end{bmatrix} =
\begin{bmatrix}
L_{11} & L_{12} \\
L_{12}^T & L_{22}
\end{bmatrix}
\begin{bmatrix}
-I_{dq0s} \\
I_{dq0r}
\end{bmatrix} +
\begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\psi_{dq0s} \\
\psi_{dq0r}
\end{bmatrix} +
p
\begin{bmatrix}
\psi_{dq0s} \\
\psi_{dq0r}
\end{bmatrix}
\]

2.2. Voltage equations

Due to the presence of various types of power losses and voltage drops, the output voltage in stator can be expressed in equations (2). Through park transformation, the stator voltage equation can be described in equation (5) where \(U_{dq0s}\) is the stator voltage while \(U_{dq0r}\) refers to the excitation voltage and the rotor damper voltage sags in dq axis.

\[
\begin{bmatrix}
U_{dq0s} \\
U_{dq0r}
\end{bmatrix} =
\begin{bmatrix}
R_s & 0 \\
0 & R_s
\end{bmatrix}
\begin{bmatrix}
-I_{dq0s} \\
I_{dq0r}
\end{bmatrix} +
\begin{bmatrix}
\omega_t \Gamma & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\psi_{dq0s} \\
\psi_{dq0r}
\end{bmatrix} +
p
\begin{bmatrix}
\psi_{dq0s} \\
\psi_{dq0r}
\end{bmatrix}
\]

Take the flux formula (4) into voltage equation (5), then equation (6) can be found.

\[
\begin{bmatrix}
-I_{dq0s} \\
I_{dq0r}
\end{bmatrix} =
\begin{bmatrix}
L_{11} & L_{12} \\
L_{12}^T & L_{22}
\end{bmatrix}
\begin{bmatrix}
R_s & 0 \\
0 & R_s
\end{bmatrix}
\begin{bmatrix}
-I_{dq0s} \\
I_{dq0r}
\end{bmatrix} +
\begin{bmatrix}
L_{11} & L_{12} \\
L_{12}^T & L_{22}
\end{bmatrix}
\begin{bmatrix}
U_{dq0s} \\
U_{dq0r}
\end{bmatrix}
\]

2.3. Rotor motion equation
The synchronous generator rotor motion equation has already shown in (3) and two more equation (7), (8) should be added:

\[
\frac{d\delta}{dt} = \omega - 1
\]  

\[
T_e = \psi_{d}i_q - \psi_{q}i_d
\]  

Thus, the rotor motion equation is reorganized in equation (9).

\[
\begin{bmatrix}
\frac{d\omega}{dt} \\
\frac{d\delta}{dt}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 \\
1 & 0
\end{bmatrix} \begin{bmatrix}
\omega \\
\delta
\end{bmatrix} +
\begin{bmatrix}
\frac{T_m-T_e}{J} \\
-1
\end{bmatrix}
\]  

where $T_m$ and $T_e$ are the mechanical torque and electromagnetic torque per unit values respectively. $\omega$ is the actual generator mechanical speed p.u value, $J$ refers to the inertia while $\delta$ is the phase difference angle between the excitation no-load EMF and the terminal voltage.

### 3. Realization of the synchronous generator mathematical model

Since the mathematical model has been determined by the equations above, it needs to be transformed into available numerical model for computer calculating. The key procedure is to select the appropriate state variables to equivalent convert the mathematical model into state equation form.

#### 3.1. Establishment of the state equation

According to formula (6) and (9), an eight-order motor state equation model can be found using $i-\omega-\delta$ as the state variables. Evidently, the synchronous generator is a high order nonlinear time-varying system with the terminal voltage $u$ and the mechanical torque $T_m$ as inputs. All the variables in the formulas are calculated in the form of the unitary value, and the time variable $t$ and the motion inertia $J$ need to be processed in non-dimension way.

#### 3.2. Numerical algorithm selection

To obtain the solutions of the motor state equations, it is necessary to choose a proper numerical algorithm. The commonly used numerical algorithms are Euler method, trapezoidal method and Adams method[13]. Comparing the three methods that Euler method is quite simple but less precision, trapezoidal method has high calculation precision with a large number of computations. Relatively, as a typical linear multistep method, Adams method acquires a balance between precision and computations. Therefore, two step Adams explicit method is used in this paper. The two step Adams method is described as equation (10):

\[
\begin{align*}
x_{m+1} &= x_m + h \left( 3F_m - F_{m-1} \right) / 2 \\
F_m &= f(t_m, x_m)
\end{align*}
\]  

(10)

where $x_m$ means the state variable at the moment $t_m$ and $F_m$ describes the differential value of $x_m$ at time $t_m$. Parameter $h$ represents the calculation step and in this paper it is selected to 100us. Take formula (10) into equation (6) and (9), and then equation (11) is obtained.

\[
\begin{align*}
F^k_i &= -L^1 \cdot R_x \cdot i_k + L^1 \cdot u_k \\
i_{k+1} &= i_k + 0.5h * (3F^k_i - F^{k-1}_i) \\
F^k_{\omega} &= \frac{T_m - T_e}{J} \\
\omega_{k+1} &= \omega_k + 0.5h * (3F^k_{\omega} - F^{k-1}_{\omega}) \\
F^k_\delta &= \omega_{k+1} - 1 \\
\delta_{k+1} &= \delta_k + 0.5h * (3F^k_\delta - F^{k-1}_\delta)
\end{align*}
\]  

(11)
In the above equations, \( F_i \) expresses the differential values of state variables at the moment \( k \) and \( L, R_k \) are the inductance and resistance matrix respectively.

### 4. Virtual synchronous generator control scheme

![Figure 1. VSG scheme diagram.](image)

Now that the synchronous generator mathematical model has established and the numerical solution has determined. The inverter VSG control scheme is then obtained in fig 1. The main circuit of the diagram is a three-phase bridge inverter circuit with \( L_f \) as the filter inductor. The port current reference values in \( dq \)-axis \( i_{dq0ref} \) are computed through motor mathematical model using grid voltage \( V_{grid} \) and the input power \( P_m \). Then the reference values are transformed into abc static axis as the current controller instructions. Inductance current \( I_{abc} \) are controlled by the current controller to track the instructions so the inverter finally has the same port characteristics as true synchronous generator does. Conventional generators are equipped with speed controller and excitation regulator to improve the stability and operation of the generator, therefore constrains mentioned above are introduced to make sure the EMF value is regulated when mechanical power changing.

### 5. Simulation results

To validate the feasibility of the proposed algorithm, simulations are executed in matlab/Simulink environment. The algorithm is operated in a s-function block and the topology of the main inverter circuit part is the same with the VSG scheme shown in Figure 1. For comparison, a 8.1kVA three-phase synchronous generator model of matlab own in PSB(powersystem blockset) block is also operating. The parameters of this model are exhibited in Table 1.

**Table 1.** The parameters of synchronous generator.

| Parameters/p.u | values    |
|---------------|-----------|
| stator resistance \( R_s \) | 0.08201   |
| excitation resistance \( R_{id} \) | 0.06117   |
| damper winding resistance in d-axis \( R_{id} \) | 0.1591    |
| damper winding resistance in q-axis \( R_{iq} \) | 0.2416    |
| armature reaction inductance in d-axis \( L_{md} \) | 1.728     |
| armature reaction inductance in q-axis \( L_{mq} \) | 0.823     |
| stator winding leakage inductance \( L_{1s} \) | 0.072     |
| excitation winding leakage inductance \( L_{1f} \) | 0.1801    |
| damper winding leakage inductance in d-axis \( L_{1d} \) | 0.1166    |
| damper winding leakage inductance in q-axis \( L_{1q} \) | 0.1615    |
Under the circumstance that the VSG is connected to the grid, when the input mechanical power varies the motor rotating speed and the output stator current curves are exhibited in Figure 2. The red line is the matlab synchronous generator model while the blue line is the inverter controlled in the VSG algorithm.

Obviously during the whole mutation process, the output of the inverter controlled in VSG algorithm has the same response curve as the actual motor.

**Figure 2.** Dynamic responses when Mechanical power mutation.

**Figure 3.** dynamic responses when grid voltage mutation.
Figure 4. dynamic responses when grid frequency mutation.

More simulations are executed to explore the port characteristics of the VSG if other variables changing. Figure.3 shows when the grid voltage mutates at the time 0.4s the output stator current and rotating speed also follow the motor model. And so does the grid frequency mutation shown in the figure 4. Compare these three Figures, it’s evident that the VSG controlled inverter and the real synchronous generator model have the same characteristics of both the dynamic response and the stable value if under the same conditions. Thus, the practicability of the proposed VSG control algorithm has been verified.

6. Conclusion

This paper proposes a realization method of the virtual synchronous generator based on park model. The accurate model is achieved through the solution of the motor full-order equations. All the numerical variables are computed through initial values and port constraints. Simulations have proved the VSG dynamic responses are equal to the real synchronous generator with the same parameters. Although due to the deviation of the initial values, at the motor start period the two curves are not strictly coincidence, this algorithm does have some practical guiding significances.

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