Ultrarelativistic heavy ion collisions
Theoretical overview

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Abstract. This is a short review of some theoretical aspects of the physics of ultra-relativistic heavy ion collisions. I review the main properties of the QCD phase diagram and recent developments in the physics of high gluon densities in the hadronic wavefunctions at high energy. Then I comment salient results obtained at RHIC.

1. Introduction
Ultra-relativistic heavy ion collisions allow us to study the densest and hottest forms of matter that can be created in the laboratory, and to address fundamental questions related to the state of matter at temperatures and densities which may be considered as “extreme” on hadronic scales. On such scales, the dynamics is controlled by Quantum Chromodynamics (QCD) and the asymptotic freedom leads us to expect that, at high temperature and density, matter is a weakly interacting system of the QCD elementary degrees of freedom, the quarks and the gluons.

Much theoretical work has been devoted to understanding the properties of such a quark-gluon plasma in static, thermal equilibrium. For such conditions, lattice calculations provide reliable information “from first principles”. Such calculations have long been limited to equilibrium properties at vanishing chemical potential. However, recently, various techniques have been proposed that allow calculations at finite, albeit small, chemical potential; also, algorithms based on the maximum entropy method are being developed to reconstruct real time information from Euclidean correlators, and hence access in particular the plasma transport properties.

Aside from the issues concerning the state of matter at large density, there is another fundamental question that is playing an increasingly important role in the study of heavy ion reactions: that of the structure of the wavefunction of a nucleus at high energy. It turns out that this question also leads us to the consideration of high density gluonic systems where nonlinear QCD effects can play an important role. The study of dense systems of partons offers exciting challenges, and indeed much theoretical developments have been taking place in this area during the last years.

The asymptotic form of the wavefunction is of course directly relevant to the study of heavy ion collisions, as it determines the “initial conditions” for the creation of matter and its subsequent evolution. But the knowledge of the wavefunction is not enough to explain how the partons initially present in the wavefunctions as virtual entities turn into active degrees

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of freedom constituting matter with high energy density, leading eventually to a quark-gluon plasma. Much work is presently devoted to this question.

The two fundamental questions mentioned above, concerning the state of matter at high temperature or density and the form of the wavefunction at high energy, focus on extreme situations where simplicity can emerge, allowing for explicit calculations to be done from first principles. However the consideration of such idealized situations is clearly not enough to understand nuclear collisions. There, the field is largely driven by experiment, and theoretical efforts aim at getting a correct interpretation of the data. It is however encouraging to observe that as the collision energy increases, parton degrees of freedom play an increasingly important role in the collision dynamics, bringing the discussion closer to these fundamental issues alluded to earlier. This is particularly so for the observables that are directly sensitive to the early stages of the collisions. These are the observables on which I shall focus in the last part of this talk. In the next two sections, I say a few more words about the QCD phase diagram and on the hadronic wave functions with high gluon densities.

2. The QCD phase diagram and the Quark-Gluon Plasma

Much work on the phase diagram has been done since the original suggestions that, because of asymptotic freedom, hadronic matter should turn into free quark matter at sufficiently high density [1, 2].

A schematic picture of this phase diagram is presented in Fig. 1. Reliable information on the vertical axis (vanishing baryon chemical potential) is available from lattice gauge calculations (see for instance [3]). Recently various extrapolation techniques have been used to extend these calculations to finite baryon density, and yield evidence for a tri-critical point at some finite value of the baryon chemical potential [4].

At small temperature, but large baryon chemical potential, the phase diagram exhibits a rich structure dominated by the phenomena associated with color superconductivity [5]. It is not
clear whether such phenomena have any chance to be observed in heavy ion collisions, but they
could play an important role in the physics of compact stars, in particular they could affect the
cooling of neutron stars.

Lattice calculations show that, as expected, the thermodynamical functions of QCD matter
turn into those of free massless particles at asymptotically high temperatures. Analytical weak
coupling techniques are capable of reproducing those lattice results down to temperatures of the
order of $3T_c$ [6]. This gives support to a simple quasiparticle picture: at temperatures higher
than $\sim 3T_c$, the dominant effects of the interactions can be absorbed in a renormalization of
quasiparticle properties, leaving a weak residual interaction between the quasiparticles. In the
vicinity of $T_c$, it is still unclear what the relevant degrees of freedom are. Effective theories
in terms of Polyakov loops are being studied with encouraging results [7, 8, 9, 10]. There are
also arguments, inspired by strong coupling calculations in string theories, according to which
Coulomb bound states could play an important role [11]. It has also been suggested that the
reduction of the pressure from the free gas limit above $T_c$ is a strong coupling effect, analogous to
that found in the strong coupling regime of supersymmetric Yang-Mills theories [12, 13]. Such
theories have also been used to estimate the viscosity [14].

There is a word of caution that I would like to insert at this point. There is no doubt that
interactions play an essential role near $T_c$. However, while supersymmetric Yang-Mills theories
have nice properties, and offer in particular new tools to calculate in strong coupling, they differ
from QCD in some essential aspects: the coupling constant does not run, and no phase transition
occurs. Furthermore, we have indications from lattice calculations (from the measurement of
$\epsilon - 3P$, [15]) that the physics near $T_c$ is not conformal invariant. It is therefore unclear whether
the strong coupling regime of supersymmetric Yang-Mills theories really describes the physics
of QCD near its phase transition. In fact, recent QCD calculations using dimensional reduction
techniques seem to indicate that the effective coupling constant may not be so large even close
to $T_c$ [16], at least not large enough to totally invalidate weak coupling calculations (with proper
resummations, such as done in [6] for instance).

3. High gluon density in nuclear wavefunctions at high energy

When two nuclei collide at very high energy, they do not see each other as collections of nucleons
bound together by nuclear forces. Rather, the relevant degrees of freedom involved in the early
stages of a collision at sufficiently high energy are partons, mostly gluons, whose density grows
as the energy increases (i.e., when $x$, their momentum fraction, decreases). This growth of
the number of gluons in the hadronic wave functions is a phenomenon which has been well
established at HERA [17]. One expects however that it should eventually “saturate” when non
linear QCD effects start to play a role.

The existence of such a saturation regime has been predicted long ago [18, 19], together with
estimates for the typical transverse momenta where it could occur in heavy ion collisions [20].
But it is only during the last decade that equations providing a dynamical description of the
saturated regime have been obtained [21, 22] and [23, 24, 25, 26, 27]. A remarkable feature which
emerges from the solution of these equations is that the dense, saturated system of partons to be
found in hadronic wave functions at high energy has universal properties, the same for all hadrons
or nuclei. This suggests that the early stages of hadronic collisions at sufficiently high energies
are governed by universal wave functions whose properties could, in principle, be calculated from
QCD.

The momentum scale $Q_s$ that characterizes the onset of saturation is called the saturation
momentum [28]. Partons with transverse momentum $Q > Q_s$ are in a dilute regime; those with
$Q < Q_s$ are in the saturated regime. One may obtain an estimate of $Q_s$ by looking at the
scale at which QCD non linear effects become important. This occurs when field fluctuations
are such that gluon kinetic energies become comparable to their interaction energies, that is
when \( (\partial A)^2 \sim \alpha_s (A^2)^2 \), where \( A^2 \) denotes the magnitude of the fluctuations of the gauge fields. The relevant dynamics is in the transverse plane, and the magnitude of the gradient is fixed by the transverse momentum \( Q \); thus saturation sets in when \( Q^2 \sim \alpha_s A^2 \). The magnitude of the gauge field fluctuations, \( A^2 \), can be estimated from the particle number density in the transverse plane, i.e. \( A^2 \sim \rho \), with \( \rho \) given by

\[
\rho \sim \frac{xG(x,Q^2)}{\pi R^2},
\]

where \( R \) is the radius of the hadron, \( x \) the momentum fraction of the considered gluons and \( G(x,Q^2) \) the gluon structure function. The condition \( Q^2 \sim \alpha_s A^2 \) translates then into an equation for the saturation momentum

\[
Q_s^2 \sim \alpha_s(Q_s^2) \frac{xG(x,Q_s^2)}{\pi R^2}.
\]

Note that at saturation, naive perturbation theory breaks down, even though \( \alpha_s(Q_s) \) may be small if \( Q_s \) is large: the saturation regime is a regime of weak coupling, but large density.

The saturation momentum increases as the gluon density increases. This may come from an increase of the gluon structure function as \( x \) decreases. The increase of the density may also come from the additive contributions of several nucleons in a nucleus. In large nuclei, one expects \( xG_A(x,Q_s^2) \propto A \), and hence \( Q_s^2 \propto \alpha_s A^{1/3} \), where \( A \) is here the number of nucleons in the nucleus. Thus, the saturation regime sets in earlier (i.e., at lower energy) in collisions involving large nuclei than in those involving protons. In fact, the parton densities in the central rapidity region of a Au-Au collision at RHIC are not too different from those measured in deep inelastic scattering at HERA. There is however one important difference: while at HERA these densities result from evolution with energy, at RHIC there is little evolution, at least in the central rapidity region, and the large densities result mostly from the additive contributions of the participant nucleons. Of course, one has the possibility at RHIC to explore various situations. In particular the study of \( dA \) collisions in the fragmentation region of the deuteron gives access to a regime where final state interactions play a minor role and where quantum evolution could be significant. Indeed, exciting results have been obtained in this regime, as we shall mention later.

There is another interesting feature of the saturation regime. Consider the number of partons occupying a small disk of radius \( 1/Q_s \) in the transverse plane. This is easily estimated by combining eqs. (1) and (2); one finds this number to be proportional to \( 1/\alpha_s \), a large number if \( \alpha_s \) is small. In such conditions of large numbers of quanta, classical field approximations may become relevant to describe the nuclear wave-functions. This observation is at the basis of the McLerran-Venugopalan model [29]. The color glass formalism provides a more complete physical picture. It relies on the separation of the degrees of freedom into “hard”, Lorentz contracted, and frozen color sources flying along the light-cone, and low \( x \) partons which are described by classical gauge fields \( A^\mu(x) \) determined by solving the Yang-Mills equations with the source given by the frozen partonic configuration. An average over all acceptable configurations must be performed in order to calculate observables. In this average, the weight of a given configuration is a functional \( W_{x_0}[\rho] \) of the density \( \rho \) of color sources which depends on the separation scale \( x_0 \) between the modes which are treated as frozen sources, and those which are treated as fields. As one lowers this separation scale, more and more modes are included among the frozen sources, and therefore the functional \( W_{x_0} \) evolves with \( x_0 \) according to a renormalization group equation, which leads to the non linear evolution equations alluded to earlier, namely the Balitsky-Kovchegov equation [21, 22] and the JIMLWK equation [23, 24, 25, 26, 27].

As a final remark, oriented towards the future, one may note that many of the phenomena uncovered at RHIC and related to the early stages of the collisions, should become more clearly
visible at the LHC. There, with center of mass energies of 5.5 TeV per nucleon-nucleon collision collisions, the typical value of $x$ at mid-rapidity will be about $5 \times 10^{-4}$ and values as small as $10^{-5}$ could be reached at forward rapidities. The corresponding values of $Q_s^2$ range from 5 GeV$^2$ to 14 GeV$^2$. Such large values of the saturation momentum make the coupling constant $\alpha_s$ smaller than at RHIC, giving firmer grounds to the weak coupling expansion used in the color glass condensate framework. A corollary of this is that the gluon occupation number in the saturation region will be larger, making the classical description also more justified.

4. Some RHIC discoveries

Experiments done at RHIC complement those performed at the CERN SPS at lower energy. In the last few years, they have produced a lot of new results.

It is useful to keep in mind, when discussing the experimental data, that the “signatures” that have been proposed as diagnostics of quark-gluon plasma formation in the early days of the development of the field, are mostly based on the simple picture of non interacting particles. While this picture is expected to be correct at asymptotically high temperatures, as we have recalled earlier, it certainly does not reflect entirely the physics at temperatures around the transition, which are likely the temperatures achieved in present experiments. From that perspective, it is perhaps not too surprising to find sizeable interaction effects in the matter produced at RHIC, even if it is presumably dominated in the early stages by partonic degrees of freedom.

I shall focus mostly on the probes which are sensitive to the densest and hottest states of matter, as these are those which depend more directly on the partonic structure of the initial wave functions, and hence are more likely to be related to quantities that can be calculated directly from QCD with minimal phenomenological input.

- **Large energy densities** are achieved [30]. Of course, in an expanding system such as that produced in heavy ion collisions, the value of the energy density depends on the time at which it is measured. But even the most conservative estimates suggest that for the first couple of fm/c after the collision, the energy density exceeds that needed to turn hadronic matter into a quark-gluon plasma. It is also significant that the multiplicities of produced particles are smaller than had been anticipated by most models [31], suggesting that coherence effects in particle production are presumably important [32].

- **Collective behavior** is observed in a spectacular fashion through the so-called elliptic flow (see for instance [33] and [34]). This phenomenon occurs for non central collisions: the collision zone is then spatially asymmetric, with roughly the shape of an almond. If matter thermalizes and expands hydrodynamically, then the expansion will dominantly take place along the directions of the largest gradients, that is in the collision plane [35]. This is what has been observed. From a careful comparison of data with hydrodynamical calculations, it has been concluded that this phenomenon implies that the produced matter thermalizes at very early times [36]. Understanding how short thermalization times could be obtained from QCD remains a theoretical challenge [37, 38]. For a recent suggestion emphasizing the role of plasma instabilities, see [39, 40].

It is worth emphasizing at this point that hydrodynamics, while it can be considered as rather successful, in particular when it is combined with a cascade description of the late stages of the expansion [41], does not perfectly account for all the data. One can even argue that some data on the elliptic flow provide evidence for incomplete thermalization on the time scale needed to establish the elliptic flow, a time of order $\bar{R}$ with $\bar{R}$ a measure of the size of the interaction region in the transverse plane [42]. Without going into details, let us just mention that one may regard the quantity $(1/S)(dN/dy)$, where $S$ is the area of overlap of the two nuclei in the transverse plane and $dN/dy$ the multiplicity density, as a measure of the average number of collisions undergone by a particle at a time $t \sim \bar{R}$. If this is so, one would expect the elliptic flow,
under appropriate conditions [42], to be independent of this parameter if the hydrodynamical regime is attained. Data clearly indicate the contrary [43]. A more systematic discussion of this issue is presented in [42].

- **High transverse momentum particles are suppressed.** A suppression by a factor 5, relative to the extrapolation based on independent nucleon-nucleon collisions, was observed in central Au-Au collisions for momenta up to $p_T \sim 10$ Gev [44, 45]. This, together with the disappearance of jet-like correlations between particles produced in opposite azimuthal directions [46], confirms the role of strong final state interactions of the produced hard particles with the surrounding matter, and favors the interpretation in terms of “jet quenching” [47, 48, 49, 50, 51]. This interpretation receives further support from the d-Au collisions: there final state interactions play a minor role and no jet quenching is observed in the central rapidity region [52, 53].

- **Hints of gluon saturation.** The last discovery that I shall mention concerns the production of particles in d-Au collisions at forward rapidity, that is in the fragmentation region of the deuteron [54]. There, one does not expect final state interactions to play a major role. The suppression of particle production observed by the BRAHMS collaboration [54] may constitute evidence for initial state effects, caused perhaps by the phenomenon of gluon saturation [55, 56] (a more complete discussion is presented in [57]).

A more comprehensive discussion of RHIC data and the status of their present theoretical interpretation can be found in [58].

5. Conclusions
We have now clear evidence that the matter produced in ultrarelativistic heavy ion collisions has an energy density which is larger, on the relevant time scales, by one or two orders of magnitude than the energy density of ordinary nuclear matter. The required conditions for the formation of the quark-gluon plasma are therefore clearly met.

Data on the elliptic flow, on “jet quenching”, and many other, provide evidence for strong interaction effects in the produced matter, pointing out, although indirectly, for the increasingly important role of partonic degrees of freedom at RHIC.

It is sometimes considered as a “surprise” that the matter produced in nucleus-nucleus collisions does not behave as a gas of weakly interacting quarks and gluons, as would be the case for the quark-gluon plasma at high temperatures: but the temperatures probed at RHIC may simply not be high enough for that. Thus, in the same way as present lattice data indicate that the transition from hadronic matter to the quark-gluon plasma may be a smooth cross over rather than a sharp phase transition, it is not unconceivable that the “discovery” of the quark-gluon plasma may go through a smooth path, forcing us on the way to learn many of its intricate properties. But beautiful data are here, and more will come, to guide us in this endeavor.

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