Square-Root Divided Difference Attitude Filter for SOTM Using Low Cost Sensors

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Abstract. In order to isolate the antenna pointing from disturbance of vehicle, an attitude stabilization method based on the MEMS inertial sensors and single baseline GPS is studied. Taking quaternion and drift of gyroscopes as state vector, a square-root DDF is used to combine the information from the inertial sensors with the heading attitude information derived from GPS. Aiming to different update frequency between accelerometers and GPS, two measurement equations with two different update frequencies are chosen. Then, with the estimated attitude information, a strap-down stabilization method for angular position is used to isolate disturbance of vehicle. The results show that DDF achieves good estimation accuracy, which can meet the demand for the attitude stabilization of mobile communication.

1. Introduction
The satcom reception system needs to keep uninterrupted communication between the satellite and the earth station while the carrier platform is in the rapid motion. Thus, attitude stabilization is key to isolate the line of sight from disturbances caused by vehicle motion[1]. High-performance gyros can derive precise attitudes under high dynamic conditions. However, high quality gyros are too expensive for wide applications. With the advent of MEMS technology, the low cost and small-size sensors are suitable for vehicular application.

However, MEMS-based sensors suffer from high noise, drift, and scale factor errors. In order to improve the accuracy and reliability of the attitude estimation methods when dealing with MEMS sensors, fusion algorithms using two or more types of sensors are implemented[2-3]. One method is identifying and eliminating noise from the frequency domain, without considering the statistical properties of the signal, such as the Complementary Filter[4]. The other is using optimal filter in the time domain, such as the Extended Kalman Filter [5], the Sigma Kalman Filter[6], and so on.

In this paper, an attitude stabilization method based two GPS/MIMU is investigated. Aim to different update frequency between accelerometers and GPS, a square-root divided difference filter with two different update frequencies is chosen.

2. Attitude determination

2.1. Sensors model
To MEMS gyroscope, it is necessary to minimize drift or bias with aiding sensors. Therefore, in this paper, we propose a new model as shown in figure 1. The gyroscopes are fused with accelerometers and two GPS sensors for tilt and heading angle measurement by SRDDF. Taking quaternion and drift
of gyroscopes as state vector, gyro drift is feedback to correct gyro output. Finally, the corrected angular rate and the estimated attitude angle as the system output.

\[ u = \omega + \Delta \omega + m_1 \]

\[ \Delta \omega = -\Delta \omega / \tau + m_2 \]

Where, \( u \) is the gyro output, \( \omega \) is the angular rate relative to the reference frame, \( \Delta \omega = [\Delta \omega_x, \Delta \omega_y, \Delta \omega_z] \) is the gyro drift bias, \( \tau \) is the for the relevant time constant, \( m_1 \) for the gyro random drift noise, \( m_2 \) for the gyro random walk noise.

Defined state variables \( x = [q, \Delta \omega]^T \), according to equation (1)-(3) the state equation can be expressed as:

\[ \dot{x} = f(x, u) + v \]

where,

\[ f(x) = \begin{bmatrix} 0.5\Omega(u - \Delta \omega)q \\ -\Delta \omega / \tau \end{bmatrix}, \quad v = \begin{bmatrix} m_1^T \\ m_2^T \end{bmatrix} \]

is white noise sequence of independent zero mean random Gaussian, \( v \sim N(0, Q) \).

(2) Measurement Equation
As for the slow update frequency of GPS information, measurement equation can divided into two parts. One is the tilt angle from accelerometers (roll angle $\phi$ and pitch angle $\theta$). The other is the yaw angle $\psi$ from double GPS. According to the relationship between euler angles and quaternion, the measurement models can be expressed as:

$$y_m = g_w(x_m) + w_m$$  \hspace{1cm} (5)

When the accelerometer information is updating, $m$ is 1; When GPS yaw information updates, $m$ is 2.

$$g_w(x) = \begin{bmatrix} a \tan \left( \frac{2 \left( x(3)x(4) + x(1)x(2) \right)}{1 - 2 \left( x(2)^2 + x(3)^2 \right)} \right) \\ a \sin \left( \frac{2 \left( x(1)x(3) - x(2)x(4) \right)}{1 - 2 \left( x(3)^2 + x(4)^2 \right)} \right) \end{bmatrix}, \quad y_i = \begin{bmatrix} \phi \\ \theta \end{bmatrix}^T$$

$$y_2 = [\psi], \quad w_m \text{ is independent zero mean Gaussian random white noise}, \text{ that is } w_m \sim N(0, R_m).$$

(3) SRDDF algorithm
Discrete measurement equation can be obtained from the formula (4) and (5).

$$\begin{align*}
\hat{x}_{k+1} &= f(x_k) + v_k \\
y_{mk} &= g_m(x_k) + w_{mk} \quad m = 1, 2
\end{align*}$$ \hspace{1cm} (6)

SRDDF algorithm is as follows:
1) Initialization
$$\hat{x}_0 = E[x_0]$$ \hspace{1cm} (7)
$$S_x = \text{chol} \left( E \left[ (x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T \right] \right)$$ \hspace{1cm} (8)
$$S_c = \sqrt{Q} \cdot S_w = \sqrt{R_w}$$ \hspace{1cm} (9)

2) Calculation of Sigma Point
$$\chi_{k-1} = \begin{bmatrix} \hat{x}_{k-1} \\ \hat{x}_{k-1} + \sqrt{(n+\xi)}S_{n,n,k-1} \\ \hat{x}_{k-1} - \sqrt{(n+\xi)}S_{n,n,k-1} \end{bmatrix}$$ \hspace{1cm} (10)
Where, $\sqrt{(n+\xi)}$ is the scaling factor, $n$ is the dimension, and $\xi$ is the scaling parameter.

3) State estimation
Time update:
$$\begin{align*}
\chi_{k,k-1} &= f(x_{k-1}, k-1) \\
\hat{x}_k &= \sum_{r=0}^{2n} \alpha_r^{(m)} \chi_{r,k,k-1} \\
S_{x_k} &= q_r \begin{bmatrix} \sqrt{\alpha_r^{(1)}} (x_{1,k,k-1} - x_{1,n+2,k,k-1}) \\ \sqrt{\alpha_r^{(2)}} (x_{1,n+k,k-1} + x_{n+12,k,k-1} - 2x_{0,k,k-1}) \end{bmatrix} S_r \end{align*}$$ \hspace{1cm} (11)

Where, $q_r \{ \}$ represents the matrix singular decomposition.
Measurement update:
$$\begin{align*}
y_{m,k} &= g_m(x_{k,k-1}) \\
\hat{y}_m &= \sum_{i=0}^{2n} \alpha_r^{(m)} y_{i,m,k-1} \\
S_{y_k} &= q_r \begin{bmatrix} \sqrt{\alpha_r^{(1)}} (y_{1,m,k-1} - y_{1,n+2,m,k-1}) \\ \sqrt{\alpha_r^{(2)}} (y_{1,n+2,m,k-1} + y_{n+12,m,k-1} - 2y_{0,m,k-1}) \end{bmatrix} S_r \end{align*}$$ \hspace{1cm} (12)
\[ P_{n} = \sum_{i=0}^{2n} \sqrt{h^2} S_{n}^{i} \left[ y_{n} - y_{n+i} \right]^{T} \]  

(17) 

\[ K = \left( P_{n} / \tilde{S}_{n} \right) \tilde{S}_{n}^{-1} \]  

(18) 

\[ \hat{x}_{n} = \hat{x}_{n} + K \left( y_{n} - \hat{y}_{n} \right) \]  

(19) 

\[ U = KS_{n}^{-1} \]  

(20) 

\[ S_{n} = \text{cholupdate} \left( S_{n}^{-1} U - 1 \right) \]  

(21) 

Where, \( \text{cholupdate} \{ S, U, \pm v \} \) expresses that the matrix \( S \) is update by cholesky, which is equal to calculate \( \text{chol} \{ SS^{T} \pm \sqrt{v} UU^{T} \} \). The corresponding weights of sigma points meet:

\[ \omega_{i}^{0} = \frac{h^2 - n}{h^2}, \quad \omega_{i}^{2n} = \frac{1}{2h^2}, \quad i = 1, \ldots, 2n \]

\[ \omega_{i}^{(1)} = \frac{1}{4h^2}, \quad \omega_{i}^{(2)} = \frac{h^2 - 1}{4h^2}, \quad i = 1, \ldots, 2n \]

Where, \( h \) is the step size, and when noise following distribution of Gaussian, \( h^2 = 3 \) is the best value.

4) Gyro error correction

Gyro drift error \( \Delta \hat{\omega} \) obtained by SRDDF filter is used to correct output of gyro \( \hat{\omega} \).

5) Cycle

Repeat the above steps (2)-(4).

3. Experiment and Analysis

In order to validate the algorithm under dynamic vehicle, we perform the experiment through a car. The sampling frequency of IMU is 100Hz, and output frequency of GPS receiver module is 10Hz. The baseline length is 1.8m. Meantime, attitude information collected from high-precision AHRS act as reference information. Simulation results are as follows.

In Figure 2, the curve of vehicle attitude angle obtained from SRDDF estimation is shown. We can find that the estimated attitude follow with the reference value well in the direction of roll and heading. Due to the acceleration of vehicle, there is a estimation bias in the 80s-120s. The maximum error is less than 1° in all conditions.

Figure 2. Attitude estimation using SRUKF

Table 1 shows the estimate the error from SRDDF compared with the EKF algorithm. From Table 1, we can find SRDDF filtering algorithm is better than EKF in terms of the accuracy.

| Filter | Statistics | \( \phi \) | \( \theta \) | \( \psi \) |
|--------|------------|-----------|-----------|-----------|
| SRDDF  | std(°)     | 0.2414    | 0.6424    | 0.0887    |
| EKF    | std(°)     | 0.2897    | 0.9111    | 0.1135    |
4. Conclusions
In this paper, a square-root DDF is developed for attitude estimation of broadband mobile satellite communications. The filter fuses the gyros with the aiding of accelerometers and two GPS. Aim at the different measurement periods, the paper introduces different measurement period to integrate various types of information for SRDDF filter. Experimental results have shown that the proposed method has a good performance.

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References
[1] Zhao, J. W., Gao, F. F., Wu, Q. H. (2018) Beam tracking for UAV mounted SatCom on-the-Move with massive antenna array. IEEE J. SEL. AREA. COMM., 36: 363-375.
[2] Sasani, S., Asgari, J., Amiri-simkooei, A. R. (2016) Improving MEMS- IMU/GPS integrated systems for land vehicle navigation applications. GPS SOLUT., 20: 89-100.
[3] Wu, J., Zhou, Z.B., Chen, J.J. (2016) Fast complementary filter for attitude estimation using low-cost MARG sensors. IEEE SENS. J., 16:6997-7007.
[4] Wang, Y.L., Soltani, M., Hussain, D. M. A. (2017) An attitude heading and reference system for marine satellite tracking antenna. IEEE T. IND. ELECTRON.,64: 3095-3104.
[5] E.J.Lefferts, E.L.Markley and M.D.Shuster, Kalman filtering for spacecraft attitude estimation, Journal of Guidance, Control and Dynamic, no.5, pp.417-429, 1982.
[6] S.Julier, The spherical simplex unscented tranformation, Proc. Amer. Control Conf., Denver, Colorado, pp.2430-2434, 2003.
[7] M. Nørgaard, N. K. Poulsen, O.Ravn, New developments in state estimation for nonlinear systems, Automatica, vol.36, no.11, pp.1627-1638,2000.