MULTIDIMENSIONAL COSMOLOGY WITH ANISOTROPIC FLUID: ACCELERATION AND VARIATION OF $G$

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A multidimensional cosmological model describing the dynamics of $n + 1$ Ricci-flat factor-spaces $M_i$ in the presence of a one-component anisotropic fluid is considered. The pressures in all spaces are proportional to the density: $p_i = w_i \rho_i$, $i = 0, \ldots, n$. Solutions with accelerated expansion of our 3-space $M_0$ and small enough variation of the gravitational constant $G$ are found. These solutions exist for two branches of the parameter $w_0$. The first branch describes superfluid matter with $w_0 > 1$, the second one may contain phantom matter with $w_0 < -1$, e.g., when $G$ grows with time.

1. Introduction

There are many hot spots in the gravitational interaction $[1]$. Among them one may point out such problems as acceleration of the Universe expansion, description and detection of strong field objects (black holes, wormholes etc.) and gravitational waves, near-zone experiments, such as equivalence principle tests, second-order tests, rotation and torsion effects of general relativity etc. Within the last block, we want to stress a special role of experiments to measure the value of the gravitational (or Einstein) constant and its possible variations. These experiments already belong to new generation ones since they are testing not only the gravitational interaction but also some predictions of unified models and theories. A special role in these activities is played by space experiments, and this role will increase in the future. Modern cosmology and its observational part already became an arena for testing predictions of high energy physics. All this leads to a fundamental role of gravity in the present investigations, it is still a missing link in unified theories. Fundamental physical constants, relations between them and their possible variations are a reflection of the situation with unification $[1][2][3][4]$. We will be mainly interested in the gravitational constant and its possible variations. There are three problems connected with the Newtonian gravitational constant $G$ $[1]$: 

1. Absolute value of $G$. 
2. Possible time variations of $G$. 
3. Possible range variations of $G$, or new interactions (forces).

The oldest one is the problem of possible temporal variation of $G$, which arose due to papers of Milne (1935) and Dirac (1937). In Russia, these ideas were developed in the 60s and 70s by K.P. Staniukovich $[5][6][7]$ who was the first to consider simultaneous variations of several fundamental constants.

Our first calculations based on general relativity with a perfect fluid and a conformal scalar field $[6]$ gave $G/G$ at the level of $10^{-11} - 10^{-13}$ per year. Our calculations in string-like $[7]$ and multidimensional models with perfect fluid $[8]$ gave the level $10^{-12}$, those based on a general class of scalar-tensor theories $[11]$ and simple multidimensional model with $p$-branes $[10][12]$ gave for the present values of cosmological parameters $10^{-13} - 10^{-14}$ and $10^{-13}$ per year, respectively. Similar estimations were made by Miyazaki within Machian theories $[13]$ giving for $G/G$ the estimate $10^{-13}$ per year and by Fujii — on the level $10^{-14} - 10^{-15}$ per year $[14]$. Analysis of one more multidimensional model with two curvatures in different factor spaces gave an estimate on the level $10^{-12}$ $[15]$. Here we continue our studies on variation of $G$ in another multidimensional cosmological model.

2. The model

We consider a cosmological model describing the dynamics of $n$ Ricci-flat spaces in the presence of a 1-component “perfect-fluid” matter $[10]$. The metric of the model

$$ g = -\exp[2\gamma(t)]dt \otimes dt + \sum_{i=0}^{n} \exp[2\alpha^i(t)]g^i \quad (2.1) $$

is defined on the manifold

$$ M = \mathbb{R} \times M_0 \times \ldots \times M_n, $$

(2.2)

where $M_i$ with the metric $g^i$ is a Ricci-flat space of dimension $d_i$, $i = 0, \ldots, n$; $n \geq 2$. The multidimensional Hilbert-Einstein equations have the form

$$ R^M_N - \frac{1}{2} \delta^M_N R = \kappa^2 T^M_N, \quad (2.3) $$

where $\kappa^2$ is the gravitational constant, and the energy-momentum tensor is adopted as

$$ (T^M_N) = \text{diag}(-\rho, p_0 \delta^{00}, \ldots, p_n \delta^{nn}), $$

(2.4)

describing, in general, an anisotropic fluid.
We put pressures of this “perfect” fluid in all spaces to be proportional to the density,
\[ p_i(t) = (1 - u_i/d_i)p(t), \]
where \( u_i = \text{const}, \ i = 0, \ldots, n. \) We also put \( \rho > 0. \)

We impose also the following restriction on the vector \( u = (u_i) \in \mathbb{R}^{n+1}: \)
\[ \langle u, u \rangle_s < 0. \]  
(2.6)

Here, the bilinear form \( \langle \ldots \rangle_s : \mathbb{R}^{n+1} \times \mathbb{R}^{n+1} \to \mathbb{R} \) is defined by the relation
\[ \langle u, v \rangle_s = G^{ij}u_iv_j, \]
(2.7)

\( u, v \in \mathbb{R}^{n+1}, \) where
\[ G^{ij} = \frac{\delta^{ij}}{d_i} + \frac{1}{2 - D} \]
(2.8)
are components of the matrix inverse to the matrix of the minisuperspace metric \( [20, 21] \)
\[ G_{ij} = d_i\delta_{ij} - d_id_j. \]  
(2.9)

In (2.8), \( D = 1 + \sum_{i=0}^{n} d_i \) is the total dimension of the manifold \( M \) (2.2).

The restriction (2.6) reads
\[ \langle u, u \rangle_s = \sum_{i=0}^{n} \frac{(u_i)^2}{d_i} + \frac{1}{2 - D} \left( \sum_{i=0}^{n} u_i \right)^2 < 0. \]  
(2.10)

3. Solutions with power-law scale factors

Here, we consider a special family of "power-law" solutions from \([16, 17]\) with the metric written in the synchronous time parametrization
\[ g = -dt_s \otimes dt_s + \sum_{i=0}^{n} a_i^2(t_s)g^i. \]  
(3.1)

Solutions with a power-law behaviour of the scale factors take place for
\[ \langle u^{(\Lambda)} - u, u \rangle_s \neq 0. \]  
(3.2)

Here and below the vector
\[ u_s^{(\Lambda)} = 2d_i \]
(3.3)
corresponds to the \( \Lambda \)-term fluid with \( p_i = -\rho \) (vacuum-like matter).

In this case, the solutions are determined by the metric (3.1) with the scale factors
\[ a_i = a_i(t_s) = A_i t_s^{\nu_i}, \]
and the density
\[ \kappa^2 \rho = -\frac{2\langle u, u \rangle_s}{\langle u^{(\Lambda)} - u, u \rangle_s^{2/3}}. \]  
(3.5)

Here
\[ \nu_i = 2u_i/\langle u^{(\Lambda)} - u, u \rangle_s \]  
(3.6)
where \( u^i = G^{ij}u_j \) and \( A_i \) are positive constants, \( i = 0, \ldots, n. \)

The model under consideration was integrated in [16] for \( \langle u, u \rangle_s < 0. \) The solutions from [16] were generalized in [17] to the case when a massless minimally coupled scalar field was added. Families of exceptional solutions with power-law and exponential behaviours of the scale factors in terms of synchronous time were singled out in [17] and correspond to a constant value of the scalar field: \( \varphi = \text{const.} \) When the scalar field is omitted, we are led to solutions presented above (in [16] these solutions were originally written in the harmonic time parametrization). It may be verified that the exceptional solutions with power-law dependence of scale factors are also valid when the restriction (2.6) is omitted. Moreover, it may be shown that for \( \langle u, u \rangle_s = 0 \) the power-law solutions coincide with the vacuum Kasner-like solution from [22]. In this case, the matter source vanishes since \( \rho = 0 \) in (5.5).

4. Acceleration and variation of \( G \)

In this section, the metric \( g^0 \) is assumed to be flat, and \( d_0 = 3. \) The subspace \( (M_0, g^0) \) describes "our" 3-dimensional space and \( (M_i, g^i) \) internal factor-spaces.

We are interested in solutions with accelerated expansion of our space and small enough variations of the gravitational constant obeying the present experimental constraints, see [10]:
\[ |\dot{G}/(GH)|(t_{s0}) < 0.1, \]
where
\[ H = \frac{\dot{a}_0}{a_0} \]
(4.2)
is the Hubble parameter. We suppose that the internal spaces are compact. Hence our 4-dimensional constant is (see [8])
\[ G = \text{const} \cdot \prod_{i=1}^{n} (a_i^{-d_i}). \]  
(4.3)

We will use the following explicit formulae for the contravariant components:
\[ u^i = G^{ij}u_j = \frac{u_i}{d_i} + \frac{1}{2 - D} \sum_{j=0}^{n} u_j, \]
and the scalar product reads
\[ \langle u^{(\Lambda)} - u, u \rangle_s = -\frac{2\langle u, u \rangle_s^2}{\langle u^{(\Lambda)} - u, u \rangle_s^{2/3}} \]
(4.5)

4.1. Power-law expansion with acceleration

For solutions with power-law expansion, an accelerated expansion of our space takes place for
\[ \nu^0 > 1. \]  
(4.6)
For $D = 4$, when internal spaces are absent, we get
\[ \nu^0 = 2/(6 - u_0), \]
\[ \langle u^{(A)} - u, u \rangle_* = \frac{1}{6}(u_0 - 6)u_0 \neq 0, \] (4.7) (4.8)
which implies $u_0 \neq 0$ and $u_0 \neq 6$ (here $\langle u, u \rangle_* = -\frac{1}{6}u_0^2 < 0$). The condition $\nu^0 > 1$ is equivalent to $4 < u_0 < 6$, or, equivalently,
\[ -\rho < p < -\rho/3, \] (4.9)
which agrees with the well-known result for $D = 4$. (We note that recently special 5-dimensional power-law solutions (e.g., with acceleration) were considered in [29].)

For power-law solutions we get
\[ \frac{\dot{G}}{G} = -\frac{1}{\nu^0} \sum_{j=1}^{n} \nu^id_i, \quad H = \frac{a_0}{a_0} = \frac{\nu^0}{t_s}, \] (4.10)
and hence
\[ \frac{\dot{G}}{G} / (GH) = -\frac{1}{\nu^0} \sum_{j=1}^{n} \nu^id_i \equiv \delta. \] (4.11)

The constant parameter $\delta$ describes variation of the gravitational constant and, according to (3.1),
\[ |\delta| < 0.1. \] (4.12)

It follows from the definition of $\nu^i$ in (3.6) that
\[ \delta = -\frac{1}{u^0} \sum_{i=1}^{n} u^id_i, \] (4.13)
or, in terms of covariant components (see (4.4))
\[ \delta = -\frac{(D - 4)u_0 - 2}{3(5 - D)u_0 + \sum_{i=1}^{n} u_i}. \] (4.14)

Thus the relations (4.2), (4.6), (4.12), (4.13) and the constraint (3.2) determine a set of parameters $u_i$ compatible with the acceleration and tests on G-dot.

In what follows we will show that these relations do really determine a non-empty set of parameters $u_i$ describing the equations of state.

### 4.1.1. The case of constant $G$

Consider the most important case $\delta = 0$, i.e., when the variation of $G$ is absent: $\dot{G} = 0$.

Indeed, there is a tendency of lowering the upper bound on $\dot{G}$. Moreover, according to arguments of [19], $\delta < 10^{-4}$. This severe constraint just follows from the identity
\[ \dot{G}/G = \dot{\alpha}/\alpha \] (4.15)
that takes place in multidimensional models. Here $\alpha$ is the fine structure constant.

**Isotropic case.** First we consider the isotropic case when the pressures coincide in all internal spaces. This takes place when
\[ u_i = vd_i, \quad i = 1, \ldots, n. \] (4.16)
For pressures in internal spaces we get from (2.5)
\[ p_i = (1 - v)\rho, \quad i = 1, \ldots, n. \] (4.17)
Then we get from (2.10) and (4.5)
\[ \langle u, u \rangle_* = \frac{1}{2 - D}\left[-\frac{1}{4}(d - 1)u_0 + 2du_0v - 2dv^2\right], \] (4.18)
\[ \langle u^{(A)} - u, u \rangle_* = \frac{1}{2 - D}\left[2u_0 + dv \right. \\
\[ \left. + \frac{1}{3}(d - 1)u_0^2 - 2du_0v + 2dv^2\right]. \] (4.19)

Here and in what follows we denote $d = D - 4$.

For $\delta = 0$, we get in the isotropic case
\[ v = u_0/2, \] (4.20)
or, in terms of pressures,
\[ p_i = (3p_0 - p)/2, \quad i = 1, \ldots, n. \] (4.21)

Substituting (4.20) into (4.18) and (4.19) we get
\[ \langle u, u \rangle_* = -u_0^2/6, \] (4.22)
\[ \langle u^{(A)} - u, u \rangle_* = u_0(u_0 - 6)/6. \] (4.23)

Remarkably, we obtain the same relations as in $D = 4$ case (see the remark above). For our solution, we should put $u_0 \neq 0$ and $u_0 \neq 6$.

Using (4.10) and (4.20) we get $u^0 = -u_0/6$ and $a_i = 0$ for $i > 0$, hence $\nu_i = 0$ for $i = 1, \ldots, n$, i.e., all internal spaces are static.

The metric (3.1) reads in our case
\[ g = -dt_s \otimes dt_s + A_0^2t_s^2g^0 + \sum_{i=1}^{n} A_i^2g^i, \] (4.24)
where $A_i$ are positive constants, and
\[ \nu^0 = 2/(6 - u_0). \] (4.25)

We see that the power $\nu^0$ is the same as in case $D = 4$. For the density we get from (3.5)
\[ \kappa^2\rho = \frac{12}{(u_0 - 6)^2t_s^2}. \] (4.26)

Thus the equations of state (2.5), with relations (4.10) and (4.20) imposed, lead to the solution (4.24)–(4.20) with a Ricci-flat (e.g., flat) 3-metric and $n$ static internal Ricci-flat spaces. For
\[ 4 < u_0 < 6 \] (4.27)
or, equivalently, $-\rho < p_0 < -\rho/3$, we get an accelerated expansion of “our” 3-dimensional Ricci-flat space.

**Isotropic case.** Consider the anisotropic (w.r.t. internal spaces) case with $\delta = 0$, or, equivalently (see (4.14))
\[ (D - 4)u_0 = 2 \sum_{i=1}^{n} u_i. \] (4.28)
This implies
\[ \langle u^{(A)} - u, u \rangle_* = \frac{1}{6}u_0(u_0 - 6) - \Delta, \]  
(4.29)
\[ \langle u, u \rangle_* = -\frac{1}{2}u_0^2 + \Delta, \]  
(4.30)
where
\[ \Delta = \sum_{i=1}^{n} \frac{u_i^2}{d_i} - \frac{1}{d} \left( \sum_{i=1}^{n} u_i \right)^2 \geq 0, \quad d = D - 4. \]  
(4.31)

The inequality in (4.31) can be readily proved using the well-known Cauchy-Schwarz inequality:
\[ \left( \sum_{i=1}^{n} b_i^2 \right) \left( \sum_{i=1}^{n} c_i^2 \right) \geq \left( \sum_{i=1}^{n} b_i c_i \right)^2. \]  
(4.32)
Indeed, substituting \( b_i = \sqrt{d_i} \) and \( c_i = u_i / \sqrt{d_i} \) into (4.32), we get (4.31). The equality in (4.32) takes place only when the vectors \( (b_i) \) and \( (c_i) \) are linearly dependent, that for our choice reads: \( u_i / \sqrt{d_i} = \nu v \) where \( v \) is constant. Thus \( \Delta = 0 \) only in the isotropic case (4.19). In the anisotropic case we get \( \Delta > 0 \).

In what follows we will use the relation
\[ \langle u^{(A)} - u, u \rangle_* = \frac{1}{6}(u_0 - u_0^+)(u_0 - u_0^-), \]  
(4.33)
where
\[ u_0^\pm = 3 \pm \sqrt{9 + 6\Delta} \]  
(4.34)
are roots of the quadratic trinomial (4.29) obeying
\[ u_0^- < 0, \quad u_0^+ > 6 \quad \text{for} \quad \Delta > 0. \]  
(4.35)
It follows from (4.29) that \( u^0 = -u_0/6 \) and hence
\[ \nu^0 = -\frac{2u_0}{u_0(u_0 - 6) - 6\Delta} \]  
(4.36)
(here \( u_0 \neq u_0^\pm \)).

The function \( \nu^0(u_0) \) is monotonically increasing: (i) from 0 to \( +\infty \) in the range \(( -\infty, u_0^-) \); (ii) from \( -\infty \) to \( +\infty \) in the range \(( u_0^-, u_0^+) \); (iii) from \( -\infty \) to 0 in the range \(( u_0^+, +\infty) \). This behaviour of \( \nu^0(u_0) \) in each of the three ranges simply follows from the relation
\[ \frac{d\nu^0}{du_0} = \frac{2(u_0^2 + 6\Delta)}{(u_0 - u_0^-)^2(u_0 - u_0^+)^2} > 0. \]  
(4.37)

The accelerated expansion of our space takes place when \( \nu^0 > 1 \), or, equivalently, when either
\[ (A) \quad u_0 \in (u_0^*, u_0^-), \quad \text{or} \]  
(4.38)
\[ (B) \quad u_0 \in (u_0^*, u_0^+), \]  
(4.39)
where
\[ u_0^* = 2 \pm \sqrt{4 + 6\Delta}. \]  
(4.40)
In terms of the parameter \( w_0 \),
\[ p_0 = w_0^0, \quad w_0 = 1 - u_0/3, \]  
(4.41)
these two branches read:
\[ (A) \quad w_0^- = \sqrt{1 + \frac{2}{3}\Delta} < w_0 < \frac{1}{3} + \frac{2}{3}\sqrt{1 + \frac{2}{3}\Delta} = w_0^*, \]  
(4.42)
\[ (B) \quad w_0^+ = -\sqrt{1 + \frac{2}{3}\Delta} < w_0 < \frac{1}{3} - \frac{2}{3}\sqrt{1 + \frac{2}{3}\Delta} = w_0^*, \]  
(4.43)
The first branch (A) describes superfluff matter \( (u_0 > 1) \) with negative density. Indeed, \( \rho < 0 \) follows from (3.30) and \( \langle u, u \rangle_* > 0 \), see (4.30).

The second branch (B) corresponds to matter with a broken weak energy condition (since \( w_0 < -\frac{1}{3} \)) and positive density (since \( \langle u, u \rangle_* < 0 \)). This matter is phantom (i.e., \( w_0 < -1 \)) when \( \Delta \geq 2 \). For \( \Delta < 2 \) the interval \( w_0^+, w_0^- \) contains both “phantom” \( (w_0 < -1) \) and “non-phantom” points \( (w_0 > -1) \).

4.1.2. The case of varying \( G \)
Now we consider another important case \( \delta \neq 0 \), i.e., when \( G \neq 0 \). In what follows, we use the observational bound (4.12): \( |\delta| < 0.1 \), stating the smallness of \( \delta \).

Using (4.14), we get
\[ \sum_{i=1}^{n} u_i = \frac{1}{2}dbu_0, \]  
(4.44)
where \( d = D - 4 \) and
\[ b = b(\delta) = \frac{1 + \delta(1 - d)/(3d)}{1 - \delta/2}. \]  
(4.45)
For the scalar product we get from (4.44)
\[ \langle u^{(A)} - u, u \rangle_* = \frac{1}{6}Au_0^2 - Bu_0 - \Delta, \]  
(4.46)
\[ \langle u, u \rangle_* = -\frac{1}{6}Au_0^2 + \Delta, \]  
(4.47)
where \( \Delta \) was defined in (4.31) (see (2.10) and (4.50)),
\[ A = \frac{1}{d + 2} \left( 1 + \frac{d}{2}b \right)^2 - \frac{d}{4}b^2 - \frac{1}{3}, \]  
(4.48)
\[ B = \frac{1}{d + 2}(2 + db). \]  
(4.49)
Using (4.49), we obtain the explicit formulae
\[ A = A(\delta) = 1 - \frac{(d + 2)^2\delta^2}{12d(1 - \delta/2)^2}, \]  
(4.50)
\[ B = B(\delta) = \frac{1 - \delta/3}{1 - \delta/2}. \]  
(4.51)
It should be noted that, due to \( |\delta| < 0.1 \), \( A \) is positive, \( A > 0 \), and close to unity: \( |A - 1| < \frac{1}{3}10^{-2} \).

For the contravariant component \( u^\Delta \) we get from (4.44) and (4.44):
\[ u^0 = -Cu_0/6, \]  
(4.52)
where
\[ C = C(\delta) = 3B - 2 = 1/(1 - \delta/2). \]  
(4.53)
It follows from (4.47) and (4.52) that (see (3.6))

$$\nu^0 = -\frac{2Cu_0}{A u_0^2/6 - Bu_0 - \Delta}. \quad (4.54)$$

Here $u_0 \neq u_0^\pm$ where

$$u_0^\pm = u_0^\pm(\delta) = \frac{1}{A}(3B \pm \sqrt{9B^2 + 6A\Delta}) \quad (4.55)$$

are roots of quadratic trinomial (4.46).

In what follows we will use the identity

$$\nu^0 - 1 = -\frac{Au_0^2 - 4u_0 - 6\Delta}{Au_0^2 - 6Bu_0 - 6\Delta}. \quad (4.56)$$

**Isotropic case.** Let us consider the isotropic case (4.16). Then we obtain from (4.44)

$$v = dbu_0/2. \quad (4.57)$$

or, in terms of pressures

$$p_i = \frac{1}{2}[3b\rho_0 + (2 - 3b)\rho], \quad i = 1, \ldots, n. \quad (4.58)$$

For scalar products we get

$$\langle u, u \rangle_s = -Au_0^2/6, \quad (4.59)$$

$$\langle u^{(A)} - u, u \rangle_s = u_0(Au_0 - 6B)/6. \quad (4.60)$$

For our solution, we should put $u_0 \neq 0$ and $u_0 \neq 6B/A$. The metric (3.1) reads in our case

$$g = -dt_s \otimes dt_s + A(t_s)u_0^2g^0 + t_s^2 \sum_{i=1}^n A_i^2g_i, \quad (4.61)$$

where $A_i$ are positive constants,

$$\nu^0 = -\frac{2C}{Au_0 - 6B}, \quad (4.62)$$

$$\nu = \nu^i = \frac{2\delta}{d(1 - \delta/2)(Au_0 - 6B)}. \quad (4.63)$$

$$i = 1, \ldots, n. \quad (4.64)$$

The last formula follows from (3.6) and

$$u^i = \frac{u_0\delta}{6d(1 - \delta/2)}. \quad (4.64)$$

We see that the power $\nu^0$ does not coincide, for $\delta \neq 0$, with that in case $D = 4$.

For the density, since $A > 0$, we get from (4.5)

$$\kappa^2 \rho = \frac{12A}{(Au_0 - 6B)^2t_s^2} > 0. \quad (4.65)$$

The accelerated expansion condition for our 3D space, $\nu^0 > 1$, reads in this case

$$4A(\delta) < u_0 < \frac{6B(\delta)}{A(\delta)}. \quad (4.66)$$

or, equivalently, in terms of $w_0$ (4.11) ($p_0 = w_0\rho$)

$$w_0^+(\delta) = 1 - \frac{2B(\delta)}{A(\delta)} < w_0 < 1 - \frac{4}{3A(\delta)} = w_0^+(\delta). \quad (4.67)$$

For $\delta > 0$, we get an isotropic contraction of the whole internal space $M_1 \times \ldots \times M_n$. In this case, $w_0^+(\delta) < -1$, and hence phantom matter may occur with the equation of state close to the vacuum one since

$$w_0^+(\delta) + 1 = -\frac{\delta(1 + \delta/d)}{3[1 - \delta + (d - 1)\delta^2/(6d)]}. \quad (4.68)$$

For small $\delta$ we have $w_0^+(\delta) = -1 - \delta/3 + O(\delta^2)$.

For $\delta < 0$ we get an isotropic expansion of the whole internal space. Then $w_0^+(\delta) > -1$, and phantom matter does not occur. In both cases $w_0^+(\delta) < -1/3$ and $w_0^+(\delta) + 1/3 = O(\delta^2)$.

**Anisotropic case.** Now we consider the anisotropic case $\Delta > 0$ when $\delta \neq 0$. Using (4.60), we obtain

$$\nu^0 - 1 = -\frac{(u_0 - u_0^+)(u_0 - u_0^-)}{(u_0 - u_0^-)(u_0 - u_0^+)} \quad (4.69)$$

where $u_0^\pm = u_0^\pm(\delta)$ were defined in (4.55) and

$$u_0^\pm = u_0^\pm(\delta) = 2 \pm \sqrt{4 + 6A(\delta)\Delta}. \quad (4.70)$$

Accelerated expansion of our 3-dimensional space takes place when $\nu^0 > 1$, or, equivalently, when either

- (A) $u_0 \in (u_0^-(\delta), u_0^+(\delta))$,
- (B) $u_0 \in (u_0^+(\delta), u_0^-)$. \quad (4.71)

In terms of the parameter $w_0$ ($p_0 = w_0\rho$, $w_0 = 1 - \frac{\rho}{\rho}$) these two branches read:

- (A) $w_0^-(\delta) < w_0 < w_0^-(\delta)$, \quad (4.72)
- (B) $w_0^+(\delta) < w_0 < w_0^+(\delta)$. \quad (4.74)

where

$$w_0^+(\delta) = 1 - u_0^+(\delta)/3, \quad (4.75)$$

$$w_0^+(\delta) = 1 - u_0^+(\delta)/3. \quad (4.76)$$

For small $\delta$ we have

$$w_0^+(\delta) = w_0^+(0) - \delta \left(1 + \frac{3}{\sqrt{9 + 6\Delta}}\right) + O(\delta^2). \quad (4.77)$$

$$w_0^+(\delta) = w_0^+(0) + O(\delta^2). \quad (4.78)$$

Thus for small $\delta$ the lower and upper bounds on $w_0$ have a small deviation from those obtained for $\delta = 0$. For small $\delta$, the upper bounds shift only by $O(\delta^2)$, while the lower bounds shift by $O(\delta)$.

The first branch (A) describes superfluff matter $w_0 > 1$ since $w_0^-(\delta) > 1$ due to $u_0^+(\delta) < 0$. It may be shown that the density is negative in this case since $\langle u, u \rangle_s > 0$.

For branch (B) we get for the upper bound

$$w_0^+(\delta) < -1/3. \quad (4.79)$$

due to $u_0^+(\delta) > 4$. For the lower bound we find that

$$w_0^-(\delta) < -1 \quad (4.80)$$

only if

$$\Delta > 6[A(\delta) - B(\delta)] = -\delta/(1 - \delta/2)^2. \quad (4.81)$$

This is a condition on the appearance of “phantom” matter. For $\delta > 0$ this inequality is valid, but for $\delta < 0$ it is satisfied only for a big enough value of anisotropy parameter $\Delta$, see (4.81).
5. Conclusions

We have considered multidimensional cosmological models describing the dynamics of \( n + 1 \) Ricci-flat factor spaces \( M_i \) in the presence of a one-component anisotropic fluid with pressures in all spaces proportional to the density: \( p_i = w_i \rho_i, \) \( i = 0, \ldots, n \). Solutions with accelerated expansion of our 3-dimensional space \( M_0 \) and small enough variation of the gravitational constant \( G \) were found. These solutions exist for two branches of the parameter \( w_0 \): (A) from (4.73) and (B) from (4.74). Branch (A) describes superstiff matter with \( w_0 > 1 \) while branch (B) may contain phantom matter with \( w_0 < -1 \). The second branch obviously contains phantom matter states when (i) \( \delta > 0 \), i.e., \( G \) increases with time, or (ii) \( \delta = 0 \) and \( \Delta > 0 \), i.e., if \( G \) is constant and the expansion (or contraction) of the factor spaces is anisotropic. If (iii) \( \delta < 0 \), i.e., \( G \) decreases with time, phantom matter appears if the anisotropy parameter \( \Delta \) is big enough (see (4.81).)

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