Scenario-based decision-making for power systems investment planning

Jialin Liu1,2 and Olivier Teytaud1,3

1Inria TAO, LRI, UMR 8623 (CNRS - Univ. Paris-Saclay), Gif-sur-Yvette, France
2CSEE, University of Essex, Colchester, United Kingdom
3Google, Zürich, Switzerland
jialin.liu@essex.ac.uk

Abstract—The optimization of power systems involves complex uncertainties, such as technological progress, political context, geopolitical constraints. Negotiations at COP21 are complicated by the huge number of scenarios that various people want to consider; these scenarios correspond to many uncertainties. These uncertainties are difficult to modelize as probabilities, due to the lack of data for future technologies and due to partially adversarial geopolitical decision makers. Tools for such difficult decision making problems include Wald and Savage criteria, possibilistic reasoning and Nash equilibria. We investigate the rationale behind the use of a two-player Nash equilibrium approach in such a difficult context; we show that the computational cost is indeed smaller than for simpler criteria. Moreover, it naturally provides a selection of decisions and scenarios, and it has a natural interpretation in the sense that Nature does not make decisions taking into account our own decisions. The algorithm naturally provides a matrix of results, namely the matrix of outcomes in the most interesting decisions and for the most critical scenarios. These decisions and scenarios are also equipped with a ranking.

I. INTRODUCTION: DECISION MAKING IN UNCERTAIN ENVIRONMENTS

Planning in power systems relies on many uncertainties. Some of them, originating in nature or in consumption, can be tackled through probabilities (Pinson et al. 2013; RTE-ft 2008; Siqueira et al. 2006; Vassena et al. 2003); others, such as technology evolution, geopolitics or CO2 penalization laws, are somewhere between stochastic and adversarial:

Climate: The United Nations Climate Change Conference, COP21, aims at achieving a new universal agreement on climate agreement, which is an issue of cooperation and competition.

Uranium supply: India has been using imported enriched uranium from Russia since 2001. In 2004, Russia deferred to the Nuclear Suppliers’ Group and declined to supply further uranium for India’s reactors. The uranium supply was not resumed until the end of 2008 (after the refurbishment was finished). Now, Russia is already supplying the India’s first large nuclear power plant under a Russian-financed 3 billion contract; and in 2014, Russia agreed to help building 10 nuclear reactors in India.

Curtailment risk: Wind and solar curtailment may occur for several reasons including transmission congestion (or local network constraints), global oversupply and operational issues (Lew et al. 2013). Each type of curtailment occurs with different frequencies depending on the generation and electrical characteristics of the regional and local systems. Another example is the risk of terrorism in the congested traffic, which cannot be represented by any stochastic model.

Geopolitical implications: Affected by the dollar, geopolitical and other factors, at the beginning of 2008 the international crude oil prices rose sharply. Another example is the Ukraine Crisis, which made Europe consider seriously adjusting its energy policy to reduce its dependence on imported energy supply.

Handling such uncertainties is a challenge. For example, how should we modelize the risk of gas curtailment in Europe, and the evolution of oil prices? We discuss existing methodologies in Section II. Section III compares them. Section IV describes our proposed approach. In particular, Section IV-C summarizes our method. Experiments are provided in Section V. Section VI concludes.

II. STATE OF THE ART: DECISION WITH UNCERTAINTIES

The notations are as follows: $K$ is the number of possible policies. $S$ is the number of possible scenarios. $R$ is the matrix of rewards and the associated reward function ($R_{k,s} = R(k,s)$), i.e. $R(k,s)$ is the reward when applying policy $k \in K = \{1, \ldots, K\}$ in case the outcome of uncertainties is $s \in S = \{1, \ldots, S\}$. The reward function is also called a utility function or a payoff function. A strategy (a.k.a. policy) is a random variable $k$ with values in $K$. A mixed strategy is a probability distribution of possible policies; this is the general case of a strategy. A pure strategy is a deterministic policy, i.e. it is a mixed strategy with probability 1 for one element, others having probability 0. The exploitability of a (deterministic or randomized) strategy $k$ is

$$\left( \max_{k' \text{ stochastic}} \min_{s \in \{1, \ldots, S\}} \mathbb{E}_{k'} R(k', s) \right) - \min_{s \in \{1, \ldots, S\}} \mathbb{E}_k R(k, s).$$

(1)

We refer to the choice of $s$ as Nature’s choice. This does not mean that only natural effects are involved; geopolitics and technological uncertainties are included. $k$ is chosen by us. In fact, natural phenomena can usually be modeled with probabilities, and are included through random perturbations.
they are not the point in this work - contrarily to climate change uncertainties.

A. Scenario-based planning

Maybe the most usual solution consists in selecting a small set \( s_1, \ldots, s_M \) of possible \( s \), assumed to be most realistic. Then, for each \( s_j \), an optimal \( k_i \) is obtained. The human then checks the matrix of the \( R(k_i, s_j) \) for \( i \) and \( j \) in \( \{1, \ldots, M\} \). Variants of this approach are studied in scenario planning (Chaudry et al., 2014; Saisirarat et al., 2013; Schwartz, 1996; Feng, 2014) provides examples with more than 1000 scenarios. When optimizing the transmission network, we must take into account the future installation of power plants, for which there are many possible scenarios - in particular, the durations involved in power plant building are not necessarily larger than constants involved in big transmission lines. The scenarios involving large wind farms, or large nuclear power plants, lead to very specific constraints depending on their capacities and locations.

B. Wald criterion

The Wald criterion (Wald, 1939) consists in optimizing in the worst case scenario. For a maximization problem, the Wald-value is

\[
v = \max_{k \text{ pure strategy on } \{1, \ldots, K\}} \min_{s \in \{1, \ldots, S\}} R_{k,s},
\]

and the recommended policy is \( k \) realizing the max. We choose a policy which provides the best solution (maximal reward) for the worst scenario. Wald’s maximin model provides a reward which is guaranteed in all cases. Implicitly, it assumes that Nature will make its decision in order to bother us, and, in a more subtle manner, Nature will make its decision while knowing what we are going to decide. It is hard to believe, for example, that the ultimate technological limit of photovoltaic units will be worse if we decide to do massive investments in solar power. Therefore, Wald’s criterion is too conservative in many cases; hence the design of the Savage criterion.

C. Savage criterion

The Savage-value (Savage, 1951) is:

\[
v = \min_{k \text{ pure strategy on } \{1, \ldots, K\}} \max_{s \in \{1, \ldots, S\}} \text{regret}(k, s),
\]

where \( \text{regret}(k, s) = \max_{K' \in K} (R_{K',s} - R_{k,s}) \). The Savage criterion is an application of the Wald maximin model to the regret. Contrarily to Wald’s criterion, it does not focus on the worst scenario. Its interpretation is that we optimize the guaranteed loss compared to an anticipative choice (anticipative in the sense: aware of all future outcomes) of decision. On the other hand, Nature still makes its decision after us, and has access to our decision before making its decision - Nature, in this model, can still decide to reduce the technological progress of wind turbines just because we have decided to do massive investments in wind power.

D. Nash equilibria

The principle of the Nash equilibrium is that contrarily to what is assumed in Wald’s criterion (Eq. 3), there is no reason for Nature (the opponent) to make a decision after us, and to know what we have decided. The Nash-value \( v \) is

\[
v = \max_{k \text{ mixed strategy on } \{1, \ldots, K\}} \min_{s \in \{1, \ldots, S\}} \mathbb{E}_k R(k, s).
\]

As a mixed strategy is used, the fact that the maximum is written before the min does not change the result (v. Neumann, 1928): \( v \) is also equal to

\[
\min_{s \text{ r.v. on } S} - \max_{k \in K} \mathbb{E}_k R(k, s).
\]

where r.v. stands for “random variable”. The exploitability (Eq. 4) of a (possibly mixed) strategy \( k \) is equivalent to

\[
\text{Nash-value} = \min_{s \in S} \mathbb{E}_k R(k, s).
\]

A Nash strategy is a strategy with exploitability equal to 0. A Nash strategy always exists; it is not necessarily unique. A Nash equilibrium, for a finite-sum problem, is a pair of Nash strategies for us and for Nature respectively. In the general case, a Nash strategy is not pure. Criteria for Nash equilibria corresponds to Nature and us making decision privately, i.e. without knowing what each other will do. In this sense, it is more intuitive than other criteria.

E. Other decision tools

Other possible tools for partially adversarial decision making are multi-objective optimization (i.e. for each \( s \), there is one objective function \( k \mapsto R(k, s) \)) and possibilistic reasoning (Dubois and Prade, 2012). These tools rely intensively on human experts, a priori (selection of scenarios) or a posteriori (selection in the Pareto set).

III. COMPARISON BETWEEN VARIOUS DECISION TOOLS

Let us compare the various discussed policies, where \( K \) is the number of possible investment policies, \( S \) is the number of scenarios, \( K' \) is the number of displayed policies, \( S' \) is the number of displayed policies; we provide an overview in Table 1. We see that the Nash approach (at least with the algorithms reaching the bound mentioned in the table) has a lower computational cost and some advantages in terms of modeling; Nature makes its decision privately (which means we do not know the uncertainties), but not with access to our decisions. On the other hand, its output is stochastic, which might be a drawback for users.

IV. OUR PROPOSAL: NAISHUNCERTAINTYDECISION

Our proposed tool is as follows: (i) We use Nash equilibria, for their principled nature and (as discussed later) low computational cost in large scale settings. (ii) We compute the equilibria thanks to adversarial bandit algorithms, as detailed in the next section. (iii) We use sparsity, for (i) improving the precision (ii) reducing the number of pure strategies in our recommendation. The resulting algorithm has the following advantage:
It is fast; this is not intuitive, but Nash equilibria, in spite of the complex theories behind this concept, can be approximated quickly, without computing the entire matrix of \( R(k, s) \). A pioneering work in this direction was [Grigoriadis and Khachiyan (1995)]; within logarithmic terms and dependency in the precision, the cost is roughly the square root of the size of the matrix.

We believe that such outcomes are natural tools for including in platforms for simulating large scale power systems involving huge uncertainties.

### A. The algorithmic technology under the hood: computing Nash equilibria with adversarial bandit algorithms

For the computational cost issue for computing Nash equilibria, there exist algorithms reaching approximate solutions much faster than the exact linear programming approach [von Stengel (2002)]. Some of these fast algorithms are based on the bandit formalism. The Multi-Armed Bandit (MAB) problem [Auer et al., 1995] [Katehakis and Veinott Jr. (1983)]. Lai and Robbins (1985) is a model of exploration/exploitation trade-offs, aimed at optimizing the payoff. Let us define an adversarial multi-armed bandit with \( K \) arms and let \( K \) denote the set of arms. Let \( T = \{1, \ldots, T\} \) denote the set of time steps, with \( T \in \mathbb{N}^+ \) a finite time horizon. At each time step \( t \in T \), the algorithm chooses \( i_t \in K \) and obtains a reward \( R_{i_t, t} \). The reward \( R_{i_t, t} \) is a mapping \((K, T) \mapsto \mathbb{R}\).

The generic adversarial bandit is detailed in Algorithm 1. In the case of adversarial problems, when we search for a Nash equilibrium for a reward function \((k, s) \mapsto R(k, s)\), two bandit algorithms typically play against each other. One of them is Nature, and the other plays our role. At the end, our bandit algorithm recommends a (possibly mixed) strategy over the \( K \) arms. This recommended distribution is often the empirical distribution of play during the games against the Nature bandit.

Such a fast approximate solution can be provided by \( \text{Exp3} \) (Exponential weights for Exploration and Exploitation) [Auer et al., 2002a] and its \( \text{Exp3.P} \) variant [Auer et al. 2002b], presented in Algorithm 2. \( \text{Exp3} \) has the same efficiency as the Grigoriadis and Khachiyan method [Grigoriadis and Khachiyan (1995)] for finding approximate Nash equilibria, and can be implemented with two bandits playing one against each other, e.g. one for us and one for Nature. \( \text{Exp3.P} \) is not anytime: it requires the time horizon in order to initialize some input meta-parameters. Busa-Fekete et al. (2010) optimized Adaptive Boosting (AdaBoost), a popular machine-learning meta-algorithm, by the adversarial bandit algorithm \( \text{Exp3.P} \), and proposed two parametrizations of the algorithm, as detailed in Table 2. Bubeck and Cesa-Bianchi (2012) proved a high probability bound on the weak reward of \( \text{Exp3.P} \).

### B. Another ingredient under the hood: sparsity

Teytaud and Flory (2011) proposed a truncation technique on sparse problem. Considering the Nash equilibria for two-
player finite-sum matrix games, if the Nash equilibrium of the problem is sparse, the small components of the solution can be removed and the remaining submatrix is solved exactly. This technique can be applied to some adversarial bandit algorithm such as Grigoriadis’ algorithm (Grigoriadis and Khachiyan, 1996), Exp3 (Auer et al., 2002a) or Inf (Bubeck et al., 2009). The properties of this sparsity technique are as follows. Asymptotically in the computational budget, the convergence to the Nash equilibrium is preserved (Teytaud and Flory, 2011). The computation time is lower if there exists a sparse solution (Teytaud et al., 2014). The support of the obtained approximation has at most the same number of pure strategies and often far less (Teytaud and Flory, 2011). Essentially, we get rid of the random exploration part of the empirical distribution of play.

C. Overview of our method

We first give a high level view of our method, in Algorithm 3. All the algorithmic challenge is hidden in the tExp3P algorithm, defined later. We now present the computational engine tExp3P. We apply the truncation technique (Teytaud and Flory, 2011) to Exp3P. We present in Algorithm 4 the resulting algorithm, denoted as tExp3P.

V. EXPERIMENTS

We propose a simple model of investments in power systems. Our model is not supposed to be realistic; it is aimed at being easy to reproduce.

A. Power investment problem

We consider each investment policy, sometimes called action or decision, a vector $k = (C, F, X, S, W, P, T, U, N, A) \in \{0, 1\}^{10}$. A scenario is a vector $s = (Z, W, P, B, T, B, X, B, S, B, C, C) \in \{0, 1\}^{9}$. Detailed descriptions of parameters are provided in Tables IIIa and IIIb.

| NOTATION | PARAMETERS OF EXP3P |
|----------|----------------------|
| $\eta$   | $\gamma$             |
| Exp3P + p| 0.3                  |
| Exp3P + p| 0.15                 |
| Exp3P + p| $2\sqrt{\log \frac{2}{\epsilon}} \min(0.6, 2\sqrt{\frac{3k \log(3k)}{4\epsilon}})$ |

Let $S$ be the set of possible scenarios and $K$ be the set of possible policies. The utility function $R$ is a mapping $(K, S) \mapsto \mathbb{R}$. Given decision $k \in K$ and scenario $s \in S$, a reward can be computed by

$$R(k, s) = \frac{2}{3}(1 + \text{rand}) \cdot (N(1 - Z)/5$$

$$- \text{cost} \cdot (N + U + T + P + W + S + X + F + C) + 7XB \cdot X + W(1 + WB)(SB + \sqrt{S})/2 + 3(PB + SB) - 4C \cdot CC - F \cdot NT + S(1 - Z) + P \cdot Z + U \cdot UB + T \cdot (1 + TB - SB)(2) - F \cdot NT + A \cdot (1 + W + P - 2SB)),$$

where $\text{cost}$ is a meta-parameter. This provides a reward function $R(k, s)$, with which we can build a matrix $R$ of rewards. However, with a ternary discretization for each variable we get a huge matrix, that we will not construct explicitly - more precisely, it would be impossible to construct it explicitly with a real problem involving hours of computation for each $R(k, s)$. Fortunately, approximate algorithms can solve Nash equilibria with precision $\epsilon$ with $O(K \log(K)/\epsilon^2)$ requests to the reward function, i.e. far less than the quadratic computation time $K^2$ needed for reading all entries in the matrix. We do experiments on this investment problem and apply the algorithms described in Table III. We consider policies and scenarios in discrete domains: $K = \{0, \frac{1}{2}, 1\}^{10}$, $S = \{0, \frac{1}{2}, 1\}^{9}$. The reward matrix $R_{3 \times 10}^{10 \times 3}$ can be defined by $\forall i \in \{1, \ldots, 3^{10}\}, \forall j \in \{1, \ldots, 3^9\}, R_{i,j} = R(k_i, s_j)$, but the reward is noisy as previously mentioned, where $k_i$ denotes the $i^{th}$ policy in $K$ and $s_j$ denotes the $j^{th}$ scenario in $S$. Thus, each line of the

\textbf{Algorithm 3} The SNash (Sparse-Nash) algorithm for solving decision under uncertainty problems.

**Require:** A family $\{1, \ldots, K\}$ of possible decisions (investment policies).

**Require:** A family $\{1, \ldots, S\}$ of scenarios.

**Require:** A mapping $(k, s) \mapsto R_{k,s}$, providing the rewards.

Run tExp3P on the mapping $R$, get a probability distribution on $K$ and a probability distribution on $S$.

Output $k_1, \ldots, k_n$, the policies with positive probability and $s_1, \ldots, s_n$ the scenarios with positive probability. Emphasize the policy with highest probability. Output the matrix of $R(k_i, s_j)$ for $i \leq n$ and $j \leq p$.

\textbf{Algorithm 4} tExp3P, combining Exp3P and the truncation method. $\alpha$ is the truncation parameter.

**Require:** $R_{m,n}$, matrix defined by mapping $(i, j) \mapsto R_{i,j}$

**Require:** a time horizon (computational budget) $T \in \mathbb{N}^+$

**Require:** $\alpha$, truncation parameter

1. Run Exp3P during $T$ iterations; get an approximation $(p, q)$ of the Nash equilibrium if the Nash equilibrium of the problem exists.

2. $\zeta = \max_{i \in \{1, \ldots, n\}} (\text{tr}(p_i)^{10})$ compute the threshold for $p$

3. $\text{for } i \leftarrow 1 \text{ to } m \text{ do}$

4. if $p_i \geq \zeta$ then $p_i' = p_i$

5. else $p_i' = 0$

6. $\text{end if}$

7. $\text{end for} t$

8. $\text{for } i \leftarrow 1 \text{ to } n \text{ do}$

9. if $q_i \geq \zeta'$ then $q_i' = q_i$

10. else $q_i' = 0$

11. $\text{end if}$

12. $\text{end for} n$

13. $\zeta' = \max_{i \in \{1, \ldots, n\}} (\text{tr}(q_i)^{9})$ compute the threshold for $q$

14. $\text{for } i \leftarrow 1 \text{ to } m \text{ do}$

15. if $q_i \geq \zeta'$ then $q_i' = q_i$

16. else $q_i' = 0$

17. $\text{end if}$

18. $\text{end for}$

19. $\text{for } i \leftarrow 1 \text{ to } n \text{ do}$

20. if $q_i \geq \zeta'$ then $q_i' = q_i$

21. else $q_i' = 0$

22. $\text{end if}$

23. $\text{end for}$

24. return $p'$ and $q'$ as an approximate Nash equilibrium of the problem.
matrix is a possible policy and each column is a scenario, \( R_{i,j} \)
is the reward obtained by apply the policy \( k_i \) to the scenario \( s_j \). Experiments are performed for different numbers of time steps in the bandit algorithms, i.e. we consider \( T \) simulations for each \( T \in \{1, 2, 8, 10, 32, 128, 512, 2048\} \times [3^{10}/10] \). Thus when playing with the “theoretical” parametrization, for each \( T \), the input meta-parameters \( \eta \) and \( \gamma \) are different, as they depend on the budget \( T \). In the entire paper, when we show an expected reward \( R(k, s) \) for some \( s \) and for \( k \) learned by one of our methods, we refer to \( tExp \) and \( P(k, s) \) are played for 10000 randomly drawn pairs \((k_{i_n}, s_{j_n})\) i.i.d. according to the random variables \( i_n \) and \( j_n \) proposed by the considered policies. The performance is the average reward of these 10000 trials \( R(k_1, s_1), \ldots, R(k_{10000}, s_{10000}) \). There is an additional averaging, over learning. Namely, each learning (i.e. the sequence of \( Exp \) iteration for approximating a Nash equilibrium) is repeated 100 times. The meta-parameter \( cost \) is set to 1 in our experiments.

We use the parametrizations of variants of \( Exp3.P \) presented in Table IV [Teytaud and Flor[2011] proposed \( \alpha = 0.7 \) as truncation parameter in truncated \( Exp3.P \) and [Teytaud et al. [2014] used the same value. The sparsity level, as well as the performance, are given in Table IV[1]. We validate the good performance of \( \alpha = 0.7 \). However, the sparsity is better with higher values - but these higher values do not always provide better results than the original non-sparse bandit.

We observe that when the number of simulations is bigger than the cardinality of the search domain, i.e. the number of possible pure policies, then \( \alpha \approx 0.9 \) leads to better empirical mean reward against the uniform policy. Values between 0.5 and 1 are the best ones. When learning with few simulations \( 5905 = \lceil K/10 \rceil \), the non-truncated solutions and non-sparse solutions are as weak as a random strategy. Along with the increment of simulation times, the non-truncated solutions and non-sparse solutions become stronger, but still weaker than the truncated solutions. When we use the truncation, we get significant mean reward even with a small horizon, i.e. the \( tExp3.P + t \) succeeds in finding better and “purer” policies than \( Exp3 \).

1) The parameters of \( Exp3.P + t \): When learning with few simulations \( 5905 = \lceil K/10 \rceil \), the non-truncated solutions and non-sparse solutions are as weak as a random strategy. Along with the increment of simulation times, the non-truncated solutions and non-sparse solutions become stronger, but still weaker than the truncated solutions. Sparsity level “0.01” means that one and only one solution of the 100 learnings has one element above the threshold \( \zeta \), the other 99 solutions of the 99 learnings have no element above the threshold \( \zeta \). This situation is not far from the non-truncated or non-sparse case. If the solution is sparse, we get a better empirical mean reward even with a small horizon, i.e. the \( tExp3.P + t \) succeeds in finding better pure policies.

We see that truncated algorithms outperform their non-truncated counterparts, in particular, truncation clearly shows its strength when the number of simulations is small in front of the size of search domain.

B. A modified power investment problem

Now we modify the reward function as follows:

\[
R'(k, s) = R(k, s) + c \cdot ((X = XB) + (C = CC) + (NT = F) + (P = PB)),
\]

where \( cost \) and \( c \) are meta-parameters.

As presented in the previous section, we can build a matrix \( R' \) with the reward function \( R'(k, s) \). We do experiments on this modified investment problem and apply the algorithms described in Table II. We consider policies and scenarios in discrete domains as used in the previous section. The meta-parameters \( cost \) is set to 1 and \( c \) is set to \( \{1, 2, \ldots, 10\} \) in our experiments. The reward matrix is normalized in the experiments.

We present the results with \( c = 1 \) and \( c = 10 \) in Tables IV and VI in both testcases, \( \alpha = 0.9 \) does not provide good results when \( T = K \), however \( \alpha = 0.7 \) (recommended by previous works) is always better than the baseline, to which the truncation technique is not applied; for the testcase with \( c = 1 \), \( \alpha = 0.9 \) outperforms the other values of \( \alpha \) at most of time; when the budget is big, \( \alpha = 0.99 \) provides better results.

VI. CONCLUSION: NASH-METHODS HAVE COMPARATIVE ADVANTAGES FOR DECISION MAKING UNDER UNCERTAINTIES

We propose a new criterion (based on Nash equilibria) and a new methodology (based on adversarial bandits + sparsity) for decision making with uncertainty. Technically speaking, we tuned a parameter-free adversarial bandit algorithm \( tExp3.P + t \), for large scale problems, efficient in terms of performance itself, and also in terms of sparsity. \( tExp3.P + t \) performed better than \( Exp3.P \) without truncation. Moreover, \( tExp3.P + t \) with truncation parameter \( \alpha = 0.7 \), which is theoretically guaranteed [Teytaud and Flor[2011], got stable performance in the experiments.

From a user point of view, we propose a tool with the following advantages: (i) Natural extraction of interesting policies and critical scenarios. However, we point out that \( \alpha = 0.7 \) provides stable (and proved) results, but the extracted submatrix becomes easily readable (small enough) with larger values of \( \alpha \). (ii) Faster computational cost than the Wald or Savage classical methodologies. Our methodology only requires a mapping \( R : (k, s) \mapsto R(k, s) \), which computes the outcome if we use the policy \( k \) and the outcome is the scenario \( s \). Multiple objective functions can be handled: if we have two objectives (e.g. economy and greenhouse gas pollution), we can just duplicate the scenarios, one for which the criterion is economy, and one for which the criterion is greenhouse gas. Given a problem, the algorithm will display a matrix of rewards for different policies and for several scenarios (including, by the trick above, several criteria such as particular matter, greenhouse, and cost).

As a summary, we get a fast criterion, faster than Wald’s or Savage’s criteria, with a natural interpretation. The algorithm
naturally provides a matrix of results, namely the matrix of outcomes in the most interesting decisions and for the most critical scenarios. These decisions and scenarios are also equipped with a ranking.

REFERENCES

Peter Auer, Nicolò Cesa-Bianchi, Yoav Freund, and Robert E. Schapire. Gambling in a rigged casino: the adversarial multi-armed bandit problem. In Proceedings of the 36th Annual Symposium on Foundations of Computer Science, pages 322–331. IEEE Computer Society Press, Los Alamitos, CA, 1995.

Peter Auer, Nicolò Cesa-Bianchi, and Paul Fischer. Finite-time analysis of the multiarmed bandit problem. Machine learning, 47(2-3):235–256, 2002a.

Peter Auer, Nicolò Cesa-Bianchi, Yoav Freund, and Robert E. Schapire. The nonstochastic multiarmed bandit problem. SIAM Journal on Computing, 32(1):48–77, 2002b.

Sébastien Bubeck and Nicolò Cesa-Bianchi. Regret analysis of stochastic and nonstochastic multi-armed bandit problems. arXiv preprint arXiv:1204.5721, 2012.

Sébastien Bubeck, Rémi Munos, and Gilles Stoltz. Pure exploration in multi-armed bandits problems. In Algorithmic Learning Theory, pages 23–37. Springer, 2009.

Róbert Busa-Fekete, Balázs Kégl, et al. Fast boosting using adversarial bandits. In Proceedings of the 27th International Conference on Machine Learning, pages 143–150, 2010.

Modassar Chaudry, Nick Jenkins, Meysam Qadrani, and Jianzhong Wu. Combined gas and electricity network expansion planning. 113(C):1171–1187, 2014. URL http://EconPapers.repec.org/RePEc:eap:appene:v:113:y:2014:i:c:p:1171-1187

Didier Dubois and Henri Prade. Possibility theory. In Robert A. Meyers, editor, Computational Complexity, pages 2240–2252. Springer New York, 2012. ISBN 978-1-4614-1799-6. doi: 10.1007/978-1-4614-1800-9_139. URL http://dx.doi.org/10.1007/978-1-4614-1800-9_139

Yonghan Feng. Scenario Generation and Reduction for Long-term and Short-term Power System Generation Planning under Uncertainties. Theses, Iowa State University, 2014.

Michael D. Grigoriadis and Leonid G. Khachiyan. A sublinear-time randomized approximation algorithm for matrix games. Operations Research Letters, 18(2):53–58, Sep 1995.

Michael N Katehakis and Arthur F Veinott Jr. The multiarmed bandit problem: decomposition and computation. Mathematics of Operations Research, 12(2):262–268, 1987.

T.L. Lai and H. Robbins. Asymptotically efficient adaptive allocation rules. Advances in Applied Mathematics, 6:4–22, 1985.

Debra Lew, Lori Bird, Michael Milligan, Bethany Speer, Xi Wang, Enrico Maria Carlini, Ana Estanqueiro, Damian Flynn, Emilio Gomez-Lazaro, Nickie Menemenlis, et al. Wind and solar curtailment. In Int. Workshop on Large-Scale Integration of Wind Power Into Power Systems, pages 1–9, 2013.

Pierre Pinson. Renewable energy forecasts ought to be probabilistic. In WIPFOR seminar, 2013.

RTF-ft. Rte forecast team: Electricity consumption in France : Characteristics and forecast method, 2008.

Peerawat Saisirirat, Nuwong Chollacoop, Manida Tongroon, Yossapong Laoonual, and Jakapong Pongthananisawon. Scenario analysis of electric vehicle technology penetration in thailand: Comparisons of required electricity with power development plan and projections of fossil fuel and greenhouse gas reduction. Energy Procedia, 34(0):459 – 470, 2013. ISSN 1876-6102. doi: http://dx.doi.org/10.1016/j.egypro.2013.06.774. URL http://www.sciencedirect.com/science/article/pii/S1876610213010175

10th Eco-Energy and Materials Science and Engineering Symposium.

L. Savage. The theory of statistical decision. Journal of the American Statistical Association, 46:5567, 1951.

Peter Schwartz. The Art of the Long View: Paths to Strategic Insight for Yourself and Your Company. Random House, 1996. ISBN 0-385-26732-0.

TG Siqueira, Monica Zambelli, Marcelo Cicogna, Marinho G. Andrade, and Secundino M. Soares. Stochastic dynamic programming for long term hydrothermal scheduling considering different streamflow models. In Probabilistic Methods Applied to Power Systems, 2006. PMAPS 2006. International Conference on, pages 1–6. IEEE, 2006.

Olivier Teytaud and Sébastien Flory. Upper confidence trees with short term partial information. In Applications of Evolutionary Computation, pages 153–162. Springer, 2011.

Olivier Teytaud, David L. Saint-Pierre, Sylvie Ruette, Jialin Liu, and David Auger. Sparse binary zero-sum games. In Asian Conference on Machine Learning, volume 29, page 16, 2014.

J. v. Neumann. Zur theorie der gesellschaftsspiele. Mathematische Annalen, 100(1):295–320, 1928. ISSN 0025-5831. doi: 10.1007/BF01448847. URL http://dx.doi.org/10.1007/BF01448847

Stefano Vassena, Philippe Mack, P Rousseaux, C Druet, and Louis Wehenkel. A probabilistic approach to power system network planning under uncertainties. In IEEE Bologna Power Tech Conference Proceedings, 2003.

Bernhard von Stengel. Computing equilibria for two-person games. In R.J. Aumann and S. Hart, editors, Handbook of Game Theory, volume 3, pages 1723 – 1759. Elsevier, Amsterdam, 2002.

A. Wald. Contributions to the theory of statistical estimation and testing hypotheses. The Annals of Mathematics, 10(4): 299–326, 1939.
TABLE III: Parameters and descriptions of policy variables (vector $k$) and scenario (vector $s$) in power investment problem.

(a) Parameters and descriptions of policy variables (vector $k$) in power investment problem.

| $k \in \{0, \frac{1}{2}, 1\}$ | CORRESPONDING INVESTMENT |
|-------------------------------|---------------------------|
| C                             | Coal                      |
| F                             | Nuclear fission           |
| X                             | Nuclear fusion            |
| S                             | Supergrids                |
| W                             | Wind power                |
| P                             | PV units                  |
| T                             | Solar thermal             |
| U                             | Unconventional renewable  |
| N                             | Nanogrids                 |
| A                             | massive storage in Scandinavia |

(b) Parameters and descriptions of scenario (vector $s$) in power investment problem.

| $s \in \{0, \frac{1}{2}, 1\}$ | NATURE'S ACTION |
|-------------------------------|-----------------|
| Z                             | Massive geopolitical issues |
| WB                            | Wind power technological breakthrough |
| PB                            | PV Units breakthrough |
| TB                            | Solar thermal breakthrough |
| XB                            | Fusion breakthrough |
| UB                            | Unconventional renewable breakthrough |
| SB                            | Local storage breakthrough |
| CC                            | Climate change disaster |
| NT                            | Nuclear terrorism |

TABLE IV: In these tables, the result is averaged over 100 independent learnings. The standard deviation is shown after ±. “simul.” refers to “simulation number”, i.e. horizon. Top: Average sparsity level (over $3^{10} = 59049$ arms), i.e. number of pure policies in the support of the obtained approximation, of solutions provided by $Exp3.P + t$ in power investment problem. “non-truncated” means that all elements of the solution provided are below the threshold $\zeta$ (line 2 in Algorithm 3); “non-sparse” means that all elements of the solution provided are above the threshold $\zeta$. In both cases, we play with the original solution provided by $Exp3.P + t$. Middle: Empirical mean reward obtained using different truncation parameter $\alpha$. Bottom: Exploitation using different truncation parameter $\alpha$ for solutions provided by $Exp3.P + t$ in power investment problem. In this table, we exclude the solutions which can not be truncated, i.e. all elements are below or all elements are above the truncation threshold $\zeta$. If at least one solution is excluded, we show between parenthesis the number of solutions excluded from the 100 learning. When $\alpha = 1.0$, $\zeta = \max_{i \in \{1, \ldots, K\}} p_i$. Thus, the remaining policy (policies), after truncation, is the element with the highest frequency (i.e., except in the rare case of a tie, only one arm).

| $\alpha$ | $S/10^{10}$ simul. | K simul. | $\zeta/10^{10}$ simul. | $\zeta$ simul. | $\zeta$ simul. | $\zeta$ simul. | $\zeta$ simul. |
|----------|-------------------|---------|------------------------|----------------|----------------|----------------|----------------|
| 0.1      | 1873.70 ± 10.87   | 4621.87 ± 28.34 | non-truncated           | non-truncated  | non-truncated  | non-truncated  |
| 0.3      | 491.53 ± 7.74     | 959.32 ± 13.78  | 12140.13 ± 234.46      | 7577.95 ± 154.37 | 710.45 ± 11.28 | 320.43 ± 2.91 |
| 0.5      | 126.18 ± 3.71     | 216.63 ± 5.53   | 1502.24 ± 33.85        | 6847.42 ± 19.37 | 33.01 ± 1.14  | 10.16 ± 0.27  |
| 0.7      | 24.80 ± 1.23      | 36.69 ± 1.69    | 168.02 ± 6.94          | 63.84 ± 2.49   | 6.57 ± 0.27   | 2.59 ± 0.11   |
| 0.9      | 3.54 ± 0.23       | 7.33 ± 0.26     | 7.35 ± 0.49            | 5.12 ± 0.29    | 1.93 ± 0.09   | 1.17 ± 0.04   |

| $\alpha$ | $S/10^{10}$ simul. | K simul. | $\zeta/10^{10}$ simul. | $\zeta$ simul. | $\zeta$ simul. | $\zeta$ simul. | $\zeta$ simul. |
|----------|-------------------|---------|------------------------|----------------|----------------|----------------|----------------|
| 0.1      | 2.595 ± 0.005     | 2.174 ± 0.006 | -0.029 ± 0.004         | 1.050 ± 0.004  | 2.184 ± 0.005  | 4.105 ± 0.006  |
| 0.3      | 3.299 ± 0.010     | 3.090 ± 0.009 | 2.195 ± 0.017          | 3.892 ± 0.018  | 6.555 ± 0.008  | 6.822 ± 0.004  |
| 0.5      | 3.896 ± 0.016     | 3.779 ± 0.015 | 3.592 ± 0.016          | 5.275 ± 0.020  | 6.741 ± 0.007  | 6.853 ± 0.004  |
| 0.7      | 4.501 ± 0.030     | 4.454 ± 0.027 | 4.674 ± 0.022          | 6.101 ± 0.016  | 6.777 ± 0.007  | 6.858 ± 0.005  |
| 0.9      | 5.021 ± 0.058     | 5.149 ± 0.062 | 5.703 ± 0.040          | 6.536 ± 0.017  | 6.813 ± 0.007  | 6.873 ± 0.004  |
| Pure     | 4.883 ± 0.158     | 5.027 ± 0.143 | 5.709 ± 0.101          | 6.137 ± 0.163  | 6.413 ± 0.136  | 6.844 ± 0.028  |

| $\alpha$ | $S/10^{10}$ simul. | K simul. | $\zeta/10^{10}$ simul. | $\zeta$ simul. | $\zeta$ simul. | $\zeta$ simul. | $\zeta$ simul. |
|----------|-------------------|---------|------------------------|----------------|----------------|----------------|----------------|
| 0.5      | -5.560 ± 0.070    | -5.693 ± 0.058 | -5.725 ± 0.060         | -3.479 ± 0.061 | -0.576 ± 0.041 | 0.056 ± 0.024  |
| 0.7      | -4.028 ± 0.094    | -4.132 ± 0.094 | -4.038 ± 0.074         | -1.243 ± 0.032 | 0.010 ± 0.018  | 0.268 ± 0.011  |
| 0.9      | -2.012 ± 0.107    | -1.859 ± 0.115 | -1.369 ± 0.081         | -0.195 ± 0.028 | 0.272 ± 0.011  | 0.330 ± 0.003  |
| Pure     | -0.938 ± 0.078    | -0.971 ± 0.092 | -0.455 ± 0.060         | 0.182 ± 0.021  | 0.323 ± 0.005  | 0.333 ± 0.000  |
TABLE V: Results for reward matrix $R'$ computed with $c = 1$. In these tables, the result is the average value of 100 learnings. The standard deviation is shown after ±. “NT” means that the truncation technique is not applied; “non-sparse” means that all elements of the solution provided are above the threshold $\zeta$. **Top:** Average sparsity level (over $3^{10} = 59049$ arms), i.e. number of pure policies in the support of the obtained approximation, of solutions provided by $Exp3.P + t$ in power investment problem. **Middle:** Proxy exploitability (to be maximized) using different truncation parameter $\alpha$ for solutions provided by $Exp3.P + t$ in power investment problem. The proxy exploitability is the difference between the best robust score in the table, minus the robust score. **Bottom:** Robust score (to be minimized) using different truncation parameter $\alpha$ for solutions provided by $Exp3.P + t$ in power investment problem. The robust score is the worst of the scores against pure policies.

| $\alpha$ | $T = K$ | $T = 10K$ | $T = 50K$ | $T = 100K$ | $T = 500K$ | $T = 1000K$ |
|----------|---------|-----------|-----------|-----------|-----------|-----------|
| 0.1      | 13804.380 ± 52.015 | non-sparse | non-sparse | non-sparse | non-sparse | non-sparse |
| 0.3      | 2810.120 ± 59.083  | non-sparse | non-sparse | non-sparse | non-sparse | non-sparse |
| 0.5      | 395.920 ± 15.835   | non-sparse | non-sparse | non-sparse | non-sparse | non-sparse |
| 0.7      | 43.230 ± 2.624     | 58925.340 ± 26.821 | 55383.140 ± 150.057 | 46000.020 ± 277.653 | 9065.180 ± 159.610 | non-sparse |
| 0.9      | 3.600 ± 0.260      | 992940 ± 64.474   | 796500 ± 41.724     | 503600 ± 24.927     | 97670 ± 5.445     | 52632.820 ± 522.505 |

| $\alpha$ | $J = K$ | $J = 10K$ | $J = 50K$ | $J = 100K$ | $J = 500K$ | $J = 1000K$ |
|----------|---------|-----------|-----------|-----------|-----------|-----------|
| NT       | 4.92e-01 ± 6.849e-07 | 4.92e-01 ± 1.78e-06 | 4.95e-01 ± 4.016e-06 | 4.99e-01 ± 5.892e-06 | 5.23e-01 ± 1.404e-05 | 4.93e-01 ± 1.687e-06 |
| 0.1      | 4.94e-01 ± 5.73e-05  | 4.92e-01 ± 1.787e-06 | 4.95e-01 ± 4.016e-06 | 4.99e-01 ± 5.892e-06 | 5.22e-01 ± 1.404e-05 | 4.93e-01 ± 1.687e-06 |
| 0.3      | 5.00e-01 ± 1.397e-04 | 4.92e-01 ± 1.787e-06 | 4.95e-01 ± 4.016e-06 | 4.99e-01 ± 5.892e-06 | 5.22e-01 ± 1.404e-05 | 4.93e-01 ± 1.687e-06 |
| 0.5      | 5.059e-01 ± 2.27e-04 | 4.92e-01 ± 1.787e-06 | 4.95e-01 ± 4.016e-06 | 4.99e-01 ± 5.892e-06 | 5.24e-01 ± 5.491e-05 | 4.93e-01 ± 1.687e-06 |
| 0.7      | 5.054e-01 ± 1.327e-03 | 4.92e-01 ± 3.835e-06 | 4.96e-01 ± 3.896e-05 | 5.03e-01 ± 1.016e-04 | 5.31e-01 ± 9.573e-05 | 4.93e-01 ± 1.687e-06 |
| 0.9      | 4.28e-01 ± 6.92e-03  | 5.137e-01 ± 4.199e-04 | 5.151e-01 ± 5.067e-04 | 5.140e-01 ± 4.965e-04 | 5.487e-01 ± 9.413e-04 | 4.96e-01 ± 1.828e-04 |
| 0.99     | 3.63e-01 ± 8.19e-03  | 4.357e-01 ± 6.873e-03 | 4.612e-01 ± 5.380e-03 | 4.683e-01 ± 4.834e-03 | 5.24e-01 ± 3.302e-03 | 5.399e-01 ± 3.167e-03 |
| Pure     | 3.505e-01 ± 7.84e-03 | 3.946e-01 ± 7.181e-03 | 4.287e-01 ± 6.203e-03 | 4.489e-01 ± 5.410e-03 | 5.143e-01 ± 3.597e-03 | 4.837e-01 ± 5.558e-03 |

| $\alpha$ | $T = K$ | $T = 10K$ | $T = 50K$ | $T = 100K$ | $T = 500K$ | $T = 1000K$ |
|----------|---------|-----------|-----------|-----------|-----------|-----------|
| NT       | 1.369e-02 | 2.092e-02 | 1.946e-02 | 1.492e-02 | 2.669e-02 | 4.525e-02 |
| 0.1      | 1.109e-02 | 2.092e-02 | 1.946e-02 | 1.492e-02 | 2.669e-02 | 4.525e-02 |
| 0.3      | 5.485e-03 | 2.092e-02 | 1.946e-02 | 1.492e-02 | 2.669e-02 | 4.525e-02 |
| 0.5      | 0.000e+00 | 2.092e-02 | 1.946e-02 | 1.492e-02 | 2.669e-02 | 4.525e-02 |
| 0.7      | 4.328e-04 | 2.091e-02 | 1.854e-02 | 1.083e-02 | 1.707e-02 | 4.525e-02 |
| 0.9      | 7.778e-02 | 0.000e+00 | 0.000e+00 | 0.000e+00 | 0.000e+00 | 0.406e-02 |
| 0.99     | 1.425e-01 | 7.806e-02 | 5.385e-02 | 4.564e-02 | 2.456e-02 | 0.000e+00 |
| Pure     | 1.554e-01 | 1.191e-01 | 8.638e-02 | 6.503e-02 | 3.443e-02 | 5.573e-02 |
TABLE VI: Results for reward matrix $R'$ computed with $c = 10$. In these tables, the result is the average value of 100 learnings. The standard deviation is shown after ±. “NT” means that the truncation technique is not applied; “non-sparse” means that all elements of the solution provided are above the threshold $\zeta$. **Top:** Average sparsity level (over $3^{10} = 59049$ arms), i.e. number of pure policies in the support of the obtained approximation, of solutions provided by $\text{Exp}3.P + t$ in power investment problem. **Middle:** Proxy exploitability (to be maximized) using different truncation parameter $\alpha$ for solutions provided by $\text{Exp}3.P + t$ in power investment problem. The proxy exploitability is the difference between the best robust score in the table, minus the robust score. **Bottom:** Robust score (to be minimized) using different truncation parameter $\alpha$ for solutions provided by $\text{Exp}3.P + t$ in power investment problem. The robust score is the worst of the scores against pure policies.

| $\alpha$ | $T = K$ | $T = 10K$ | $T = 50K$ | $T = 100K$ | $T = 500K$ | $T = 1000K$ |
|----------|---------|-----------|-----------|-----------|-----------|-----------|
| NT       | $6394.625 \pm 84.308$ | non-sparse | non-sparse | non-sparse | non-sparse | non-sparse |
| 0.1      | $1337.896 \pm 40.491$ | non-sparse | non-sparse | non-sparse | non-sparse | non-sparse |
| 0.3      | $206.146 \pm 12.647$ | non-sparse | non-sparse | non-sparse | non-sparse | non-sparse |
| 0.5      | $25.583 \pm 2.045$ | non-sparse | non-sparse | non-sparse | non-sparse | non-sparse |
| 0.7      | $3.729 \pm 0.353$ | non-sparse | non-sparse | non-sparse | non-sparse | non-sparse |
| 0.9      | $1.208 \pm 0.072$ | non-sparse | non-sparse | non-sparse | non-sparse | non-sparse |

| $\alpha$ | $T = K$ | $T = 10K$ | $T = 50K$ | $T = 100K$ | $T = 500K$ | $T = 1000K$ |
|----------|---------|-----------|-----------|-----------|-----------|-----------|
| NT       | $4.994e-04 \pm 4.959e-04$ | $3.392e-03$ | $4.772e-03$ | $9.903e-03$ | $3.388e-03$ |
| 0.1      | $7.727e-04$ | $4.594e-04$ | $3.592e-03$ | $4.772e-03$ | $9.903e-03$ | $3.388e-03$ |
| 0.3      | $5.838e-04$ | $4.594e-04$ | $3.592e-03$ | $4.772e-03$ | $9.903e-03$ | $3.388e-03$ |
| 0.5      | $0.000e+00$ | $4.594e-04$ | $3.592e-03$ | $4.772e-03$ | $9.903e-03$ | $3.388e-03$ |
| 0.7      | $9.391e-05$ | $4.594e-04$ | $3.592e-03$ | $4.772e-03$ | $9.903e-03$ | $3.388e-03$ |
| 0.9      | $9.758e-03$ | $0.000e+00$ | $2.860e-03$ | $2.992e-03$ | $0.000e+00$ | $3.371e-03$ |
| 0.99     | $3.236e-02$ | $3.647e-03$ | $0.000e+00$ | $0.000e+00$ | $1.211e-02$ | $0.000e+00$ |
| Pure     | $3.848e-02$ | $3.204e-02$ | $2.559e-02$ | $2.562e-02$ | $1.463e-02$ | $3.103e-02$ |