An odd-number limitation of extended time-delayed feedback control in autonomous systems

Andreas Amann\textsuperscript{1,2} and Edward W. Hooton\textsuperscript{1}

\textsuperscript{1}School of Mathematical Sciences, University College Cork, Ireland
\textsuperscript{2}Tyndall National Institute, University College Cork, Ireland

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Abstract

We propose a necessary condition for the successful stabilisation of a periodic orbit using the extended version of time-delayed feedback control. This condition depends on the number of real Floquet multipliers larger than unity and is therefore related to the well-known odd-number limitation in non-autonomous systems. We show that the period of the orbit which is induced by mismatching the delay-time of the control scheme and the period of the uncontrolled orbit plays an important role in the formulation of the odd-number limitation in the autonomous case.

1 Introduction

By their nature, chaotic systems are extremely sensitive to external perturbations, which makes it difficult to predict their future evolution. However, this sensitivity also allows for the surprising possibility of controlling a chaotic system. As first demonstrated by Ott, Grebogi and Yorke \cite{1}, an external perturbation can be grafted in such a way that one of the unstable periodic orbits of the chaotic attractor becomes stable. This discovery has triggered a large research activity centred around the oxymoronic term \textit{chaos control} \cite{2, 3}.

A simple but highly efficient scheme of chaos control was introduced by Pyragas in \cite{4}. In order to stabilise a periodic orbit of period $\tau$ the original system is converted into a system of time-delayed differential equations by adding terms which involve the difference $x(t - \tau) - x(t)$. Here $x(t)$ denotes a point in the phase space of the original system. Control terms of this form vanish, whenever a periodic orbit with period $\tau$ is reached and therefore the Pyragas control scheme is automatically \textit{non-invasive}. The time-delayed feedback control was extended by Socolar et al. \cite{5} by employing additional control terms of the form $x(t - k\tau) - x(t - (k - 1)\tau)$ for integer values $k > 1$. Again, terms of this form vanish on a periodic orbit of period $\tau$, and therefore enable \textit{non-invasive} control. This \textit{extended time-delayed feedback control} (EDFC) scheme is important for practical applications, because it can significantly increase the range of periodic orbits, which can be stabilised \cite{6}.

Unfortunately, not all periodic orbits can be stabilised by time-delayed feedback control, which severely limits its applicability. In particular, in a non-autonomous system a hyperbolic periodic orbit with an odd number of Floquet multipliers larger than unity can never be stabilised by time-delay feedback control. This became known as the \textit{odd-number limitation} and was proved in \cite{7} for the original Pyragas control scheme and in \cite{8} for a fairly general class of EDFCs. While the proofs concerning non-autonomous systems presented in \cite{7, 8} are correct, the autonomous case was not treated correctly. In particular footnote 2 in \cite{8} claims that all results regarding the odd-number limitation can be proved for the autonomous case “with a slight revision”. More importantly, Theorem 2 in \cite{8} claims explicitly that the odd number limitation applies to the autonomous case. However, the proof of this theorem contains an error in the expansion of a determinant and is therefore wrong. The non-autonomous case was also discussed in \cite{9} and it was stated that the Pyragas method for non-autonomous systems should only be able to stabilise orbits with “finite torsion”, but not orbits with a single Floquet multiplier larger then unity. Although it was not explicitly claimed in \cite{9} that this statement can be extended to the autonomous case, the casual reader might not have noticed this subtle point. Based on \cite{2, 3, 6} there was therefore a general belief among members of the chaos-control community that the odd-number limitation also holds in the autonomous case.

This changed with the work by Fiedler et al. \cite{10}, who gave an example of a two-dimensional autonomous system, with precisely one Floquet multiplier larger than one, which can be stabilised using the original Pyragas control scheme. This immediately showed that the odd-number limitation does not hold for autonomous systems and opened up the exciting possibility that the time-delayed feedback control could be far more powerful than previously thought.

While \cite{10} provides a counter example for the original odd-number limitation, it raises the question, if the odd-number limitation is outright wrong, or if it holds at least under certain additional conditions. After
all, numerical and experimental evidence never showed any problem with the odd-number limitation before the publication of [10], which suggests that for many systems or control schemes the odd-number limitation might indeed be true. Motivated by this possibility we showed in our recent work [11] that a modified version of the odd-number limitation holds for Pyragas type control. Like the original odd-number limitation, our modified version also involves the number of Floquet multipliers greater than unity, but in addition also depends on an analytical expression which involves an integral of the control force along the desired periodic orbit. Interestingly, this analytical expression can also be obtained by studying the period of the orbit which is induced if the system is forced with a delayed feedback term where the delay time does not match the period of the orbit in the unforced system. Our modified version of the odd-number limitation correctly predicts the stability boundaries of the previous counter example presented in [10].

In the current work we generalise our previous results on the odd-number limitation of Pyragas control to the case of EDFC. While we closely follow the arguments laid out in [11] we intend to keep the presentation self-contained. The remainder of the paper is organised as follows: in Section 2 we introduce the notation and state the main theorem which is then proved in Section 3. In the final Section 4 we discuss the significance and practical implication of our results.

2 Statement of the theorem

We consider a dynamical system of the form

\[ \dot{x}(t) = f(x(t)) \]  

with \( x(t) \in \mathbb{R}^n \) and \( f : \mathbb{R}^n \to \mathbb{R}^n \), and assume that there exists a \( \tau \) periodic solution \( x^*(t) = x^*(t + \tau) \) of \( f \). With this periodic orbit we associate a (principal) fundamental matrix \( \Phi(t) \) which fulfills the matrix equation

\[ \dot{\Phi}(t) = Df(x^*(t))\Phi(t); \quad \Phi(0) = I, \]  

(2)

where \( Df(x^*(t)) \) denotes the Jacobian of \( f \) evaluated at the point \( x^*(t) \) along the periodic orbit. For \( t = \tau \) the fundamental matrix \( \Phi(\tau) \) is often called monodromy matrix, and the generalised eigenvalues \( \mu_1, \ldots, \mu_n \) of \( \Phi(\tau) \) are known as Floquet multipliers (or characteristic multipliers) of the periodic orbit \( x^*(t) \). Taking the time derivative of \( (1) \) and taking into account that the function \( f \) does not explicitly depend on time, we observe that

\[ \frac{d}{dt}x^*(t) = Df(x^*(t))x^*(t). \]

Comparing with \( (2) \) we can therefore identify \( \dot{x}^*(t) = \Phi(t)\dot{x}^*(0) \) and in particular note that

\[ \Phi(\tau)\dot{x}^*(0) = \dot{x}^*(\tau) = \dot{x}^*(0). \]

It therefore follows that the monodromy matrix \( \Phi(\tau) \) for a periodic orbit in an autonomous system has at least one eigenvalue equal to one, and we choose in the following \( \mu_1 = 1 \). As \( \Phi(\tau) \) is a real matrix, the set of the remaining Floquet multipliers \( \{\mu_2, \ldots, \mu_n\} \) is composed of either real numbers, or complex conjugate pairs of complex numbers. The significance of the Floquet multipliers lies in the fact that they allow us to characterise the periodic orbit in question. For example, if there exists at least one Floquet multiplier \( \mu_k \) such that \( |\mu_k| > 1 \), then the periodic orbit is unstable. If there exists at least one Floquet multiplier \( \mu_k \) with \( k > 1 \) such that \( |\mu_k| = 1 \), then the orbit is called non-hyperbolic. In the following we will assume that the the periodic orbit \( x^*(t) \) is hyperbolic, i.e. none of the Floquet multipliers other than \( \mu_1 \) is located on the unit circle. We also define the symbol \( m \) to denote the number of Floquet multipliers which are real and larger than one. For \( m > 0 \) the periodic orbit is unstable, however the converse is not true.

Following the ideas of [5] we now introduce an extended time-delayed feedback term by modifying the system \( (1) \) as follows

\[ \begin{align*}
\dot{x}(t) &= f(x(t)) + Ky(t - 0) \\
y(t) &= x(t - \tau) - x(t) + Ry(t - \tau)
\end{align*} \]  

(3)

where \( y(t) \in \mathbb{R}^n \), \( \tau \) is a positive parameter for the delay time. Both, the control matrix \( K \) and the memory matrix \( R \) are \( n \times n \) matrices, where \( R \) fulfills the condition

\[ \lim_{k \to \infty} R^k = 0. \]

The solution of \( (3) \) for \( t > 0 \) requires the knowledge of the two (left-continuous) functions \( x(t) \) and \( y(t) \) on the interval \( t \in (\tau, 0] \). Note that formally the system \( (3) \) is not a delay differential equation, since no time derivative of \( y(t) \) appears. For \( R = 0 \) we recover the traditional Pyragas control scheme. Instead of \( (3) \) many other essentially equivalent formulations of EDFC are used, for example

\[ \dot{x}(t) = f(x(t)) + K \left\{ \sum_{j=0}^{\infty} R^j [x(t - (j+1)\tau) - x(t - j\tau)] \right\}. \]

The system \( (3) \) possesses the obvious \( \tau \) periodic solution
$$x_\tau (t) = x^\ast (t)$$
$$y_\tau (t) = 0,$$  \hspace{1cm} (4)

however the stability of this solution may have been affected by the control scheme. In the following we
would like to gain some insight into the stability properties of this solution.

Before we formulate our main theorem, we slightly change the time-delay parameter $\tau$ appearing in (3) to a new value $\hat{\tau}$ as follows:

$$\dot{x}(t) = f(x(t)) + Ky(t - 0)$$
$$y(t) = x(t - \hat{\tau}) - x(t) + Ry(t - \hat{\tau})$$

If $\hat{\tau}$ is sufficiently close to $\tau$ we can assume that there exists a periodic solution $(x_\tau, y_\tau)$ for (3) which is close to the original solution (3). However, we expect that the period of this new solution will be in general different from both, $\tau$ and $\hat{\tau}$, and we denote this period by the new symbol $\tau^\ast$. Continuity requires that $\lim_{\tau \to \alpha} \tau^\ast (\hat{\tau}) = \tau$. We can now formulate our main theorem.

**Theorem:** Let $x^\ast (t)$ be a $\tau$-periodic orbit of (1) with $m$ real Floquet multipliers greater than unity and let $\tau^\ast (\hat{\tau})$ be the period of the induced periodic orbit of (3) with $\lim_{\tau \to \alpha} \tau^\ast (\hat{\tau}) = \tau$. Then the orbit $x^\ast (t)$ is an unstable solution of (3) if the condition

$$(-1)^m \lim_{\hat{\tau} \to \tau} \frac{\hat{\tau} - \tau}{\tau - \tau^\ast (\hat{\tau})} < 0,$$  \hspace{1cm} (6)

is fulfilled.

### 3 Proof of the theorem

In the proof we follow the techniques developed in [11, 7, 8], but we aim to keep the following presentation as self-contained as possible. We first formulate a lemma, which allows us to connect the linear stability of system (3) with the solution of a time dependent linear equation.

**Lemma 1:** If the equation

$$\frac{d}{dt} [\delta x(t)] = \left\{ Df(x^\ast (t)) + (\nu^{-1} - 1) K [1 - R\nu^{-1}]^{-1} \right\} \delta x(t),$$  \hspace{1cm} (7)

possesses a solution of the form

$$\delta x(t) = \nu \delta x(t - \tau)$$  \hspace{1cm} (8)

for real $\nu > 1$, then the periodic orbit $x^\ast (t)$ is an unstable solution of (3).

Proof: First note that because of the requirement $\lim_{R \to \infty} R^k = 0$, the eigenvalues of $R\nu^{-1}$ are contained in the unit circle of the complex plane, and the matrix $[1 - R\nu^{-1}]$ is indeed non-singular for all $\nu \geq 1$. It is then straightforward to check that the ansatz

$$x(t) = x^\ast (t) + \delta x(t)$$
$$y(t) = [1 - R\nu^{-1}]^{-1} (\nu^{-1} - 1) \delta x(t)$$

fulfills (3) to first order in $\delta x(t)$. Because of (8) we have therefore explicitly constructed an exponentially growing linear perturbation of the solution (3), which implies that $x^\ast (t)$ is an unstable solution of (3). This concludes the proof of Lemma 1.

We now have to show that under the conditions of the theorem there exists a $\nu > 1$ such that the linear equation (7) allows for a solution fulfilling (5). We first introduce the fundamental matrix for (7) via

$$\Psi_\nu(t) = \left\{ Df(x^\ast (t)) + (\nu^{-1} - 1) K [1 - R\nu^{-1}]^{-1} \right\} \Psi_\nu(t),$$  \hspace{1cm} (9)

$$\Psi_\nu(0) = I$$  \hspace{1cm} (10)

If we now find a $\nu > 1$ such that

$$\det (\nu I - \Psi_\nu(\tau)) = 0,$$  \hspace{1cm} (11)

then there exists a $\delta x_0 \in \mathbb{R}^n$ which fulfills $\Psi_\nu(\tau) \delta x_0 = \nu \delta x_0$. Using $\delta x(t) = \Psi_\nu(t) \delta x_0$ we can then construct a solution of (7) which is of the form required by (5). This motivates the introduction of the function $F$ via

$$F(\nu) = \det (\nu I - \Psi_\nu(\tau))$$  \hspace{1cm} (12)

and from the above discussion we can conclude that the proof of our theorem is complete, if we are able to show that there exists a $\nu_0 > 1$ such that $F(\nu_0) = 0$. For $\nu = 1$ we see from (9) that $\Psi_1(t) = \Phi(t)$. One of the eigenvalues of $\Phi(\tau)$ is however equal to unity, and we therefore find that $F(1) = 0$.

For the further discussion of $F(\nu)$ we write the fundamental matrix in the form

$$\Psi_\nu(t) = \Phi(t) \left[ I + (\nu^{-1} - 1) \int_0^t \Phi^{-1}(u) K [1 - R\nu^{-1}]^{-1} \Psi_\nu(u) du \right],$$  \hspace{1cm} (13)
which can be easily checked by direct differentiation. All the terms appearing in this expression for $\Psi_\nu (t)$ remain finite for $\nu \to \infty$, and therefore $\lim_{\nu \to \infty} \frac{\partial F (\nu)}{\partial \nu} = 1$, or in other words, $F (\nu)$ diverges as $\nu^n$ for large $\nu$. We can therefore summarise the discussion in the last two paragraphs in the following lemma.

**Lemma 2:** If for a given periodic orbit $x^* (t)$ the condition

$$\frac{\partial F (\nu)}{\partial \nu} \bigg|_{\nu=1} = F' (1) < 0$$

holds, then the orbit is an unstable solution of (3).

Proof: As $F (\nu)$ is continuous, $F (1) = 0$ and $F' (1) < 0$ there exists a $\nu_n > 1$ such that $F (\nu_n) < 0$. However as $F (\nu)$ diverges as $\nu^n$ for large $\nu$, it follows by the intermediate value theorem that there exists $\nu_c > 1$ with $F (\nu_c) = 0$. For this $\nu_c$ we can then construct the function $\delta x (t)$ required for Lemma 1 and the orbit is unstable. Therefore Lemma 2 is proved.

In view of Lemma 2 we now need to study the conditions for which $F' (1) < 0$ holds. Before we proceed it is now useful to introduce the matrix $W$, which diagonalizes $\Phi (\tau)$, i.e.

$$
\begin{pmatrix}
1 & 0 & 0 & \cdots & 0 \\
0 & \mu_2 & * & \cdots & \\
\vdots & \ddots & \ddots & \ddots & 0 \\
\vdots & & \ddots & \ddots & * \\
0 & \cdots & \cdots & 0 & \mu_n
\end{pmatrix}
= W^{-1} \Phi (\tau) W.
$$

Here the $*$ denotes either 0 or 1, depending on the Jordan block associated with a particular eigenvalue. This allows us to formulate

**Lemma 3:** A periodic orbit with $m$ Floquet multipliers greater than unity is an unstable solution of the EDFC scheme if the condition

$$(-1)^m \left( 1 + \int_0^\tau (W^{-1} \Phi (t))^{-1} K [1 - R]^{-1} \Phi (t) W \right) dt < 0$$

holds.

Proof: The idea of the proof is to show that (15) implies (14) and then use Lemma 2. To calculate the sign of $F' (1) = \lim_{\epsilon \to 0} F (1 + \epsilon) / \epsilon$ we expand $F (1 + \epsilon)$ to first order in $\epsilon$. We obtain

$$
F (1 + \epsilon) = \det [(1 + \epsilon) \mathbb{I} - \Psi_{1+, \tau} (\tau)] = \det \left[ W^{-1} ((1 + \epsilon) \mathbb{I} - \Psi_{1+, \tau} (\tau)) W \right] = \det [M^0 + \epsilon M^1] + O (\epsilon^2)
$$

where the matrices $M^0$ and $M^1$ collect the terms in the zeroth and first order of $\epsilon$. We obtain

$$
W^{-1} ((1 + \epsilon) \mathbb{I} - \Psi_{1+, \tau} (\tau)) W
= (1 + \epsilon) \mathbb{I} - W^{-1} \Phi (\tau) \left[ \mathbb{I} + \left( \frac{1}{1 + \epsilon} - 1 \right) \int_0^\tau \Phi^{-1} (u) K \left[ 1 - R \frac{1}{1 + \epsilon} \right]^{-1} \Psi_{1+, \tau} (u) du \right] W
= (1 + \epsilon) \mathbb{I} - W^{-1} \Phi (\tau) W + \epsilon \int_0^\tau W^{-1} \Phi^{-1} (u) K [1 - R]^{-1} \Phi (u) du W + O (\epsilon^2)
= \mathbb{I} - W^{-1} \Phi (\tau) W + \epsilon \left[ \mathbb{I} + \int_0^\tau W^{-1} \Phi^{-1} (u) K [1 - R]^{-1} \Phi (u) du \right] + O (\epsilon^2)
$$

and therefore

$$
M^0 = W^{-1} (\mathbb{I} - \Phi (\tau)) W =
\begin{pmatrix}
0 & 0 & 0 & \cdots & 0 \\
0 & 1 - \mu_2 & * & \cdots & \\
\vdots & \ddots & \ddots & \ddots & 0 \\
\vdots & & \ddots & \ddots & * \\
0 & \cdots & \cdots & 0 & 1 - \mu_n
\end{pmatrix}
$$

and

$$
M^1 = \mathbb{I} + \int_0^\tau W^{-1} \Phi^{-1} (u) K [1 - R]^{-1} \Phi (u) W du
$$

Because of the special form of $M^0$, only one term contributes to the determinant in (16) in first order of $\epsilon$,

$$
F (1 + \epsilon) = \det [M^0 + \epsilon M^1] + O (\epsilon^2) = \epsilon M^1_{11} \prod_{k=2}^n (1 - \mu_k) + O (\epsilon^2)
$$

For the sign of $F' (1)$ we therefore conclude

$$
\text{sgn} (F' (1)) = \text{sgn} \left( M^1_{11} \prod_{k=2}^n (1 - \mu_k) \right) = \text{sgn} \left( M^1_{11} (-1)^m \right)
$$

(17)
where in the last step we used that every real Floquet multiplier greater than unity contributes a minus sign to the product. Comparing with (15) we observe that if condition (15) is fulfilled then condition (13) is also fulfilled because of (14). Therefore Lemma 3 now follows by evoking Lemma 2.

As a final step in the proof of the main theorem, it now remains to be shown that (5) implies condition (13). We need to establish, how the detuning between the period \( \tau \) of the uncontrolled orbit and the delay time \( \tau \equiv \hat{\tau} \) of the EDFC scheme (7) influence the period \( \hat{\tau} \) of the orbit which is induced by this detuning. For the Pyragas control this problem was solved by Just et al. in [12] and for the case of EDFC by Novčenko and Pyragas in [13]. In our notation the result of equation (28) from [13] can be written in the form

\[
\hat{\tau} = \tau + (\hat{\tau} - \tau) \frac{M_1^1}{M_1^1} + O \left[(\hat{\tau} - \tau)^2\right]
\]

or

\[
M_1^1 = \frac{\hat{\tau} - \tau}{\tau - \tau}
\]

It therefore follows that the conditions (15) in Lemma 3 and the condition (6) in the main theorem are equivalent, and this concludes the proof of the theorem.

4 Discussion

Our main theorem provides a limitation on the applicability of EDFC, which involves the number of real Floquet multipliers (\( m \)) and a combination of the period of the uncontrolled system (\( \tau \)), a detuned delay time (\( \hat{\tau} \)) and the resulting period of the induced orbit (\( \hat{\tau} \)). Let us now have a closer look at the condition (6) appearing in the theorem. We see that \( m \) only appears in the form \((-1)^m\), which is negative for odd \( m \) and positive otherwise. The second factor involves a ratio \( r \) of the form

\[
r = \frac{\hat{\tau} - \tau}{\tau - \tau},
\]

and we are asked to evaluate the sign of this ratio in the limit where \( \hat{\tau} \) goes to \( \tau \). If this ratio was always positive, then our theorem would simply reduce to the statement of the old odd-number limitation, i.e. orbits with odd number \( m \) cannot be controlled. Our theorem now modifies this statement as follows: orbits with odd \( m \) can be stabilised, but only if the control terms are implemented in such a way that \( r \) is negative. We stress that our theorem only gives a necessary condition for control, i.e. a violation of condition (6) will not guarantee that an orbit will be stabilised. Intuitively the case of negative \( r \) seems to be unusual, as it implies that if we slightly increase the delay time \( \tau \) to a value larger than \( \tau \), then the period \( \hat{\tau} \) of the induced orbit needs to be even larger than \( \hat{\tau} \) itself. This intuitively strange situation might be one of the reasons, why the violation of the original odd-number limitation in the autonomous case was not observed earlier. In the case of the counter example given in [14], \( r \) is indeed negative, as was shown in [11].

We also remark that Lemma 3 provides useful insight in its own right. We can slightly rewrite the condition (6) as follows

\[
(-1)^m \left( 1 + \int_0^\tau \left(z^T(t) K [1 - R]^{-1} \dot{x}^*(t)\right) dt \right) < 0
\]

Here \( z^T(t) \) is the first row of the matrix \( W^{-1}\Phi(t)^{-1} \), and is also known as the dual vector of the zero mode. If we now write the control matrix with a scalar prefactor \( k \) in the form

\[
K = kK_0
\]

and introduce \( \kappa \) via

\[
-\kappa^{-1} = \int_0^\tau \left(z^T(t) K_0 [1 - R]^{-1} \dot{x}^*(t)\right).
\]

Then the condition (6) can be written in the form

\[
(-1)^m \left( 1 - \frac{k}{\kappa} \right) < 0.
\]

The period of the induced orbit \( \hat{\tau} (\hat{\tau}, k) \) now depends on the scalar coupling strength \( k \) and the delay time \( \hat{\tau} \) in the following way [12, 13],

\[
\hat{\tau} (\hat{\tau}, k) = \tau + \frac{k}{k - \kappa} (\hat{\tau} - \tau) + O \left[(\hat{\tau} - \tau)^2\right].
\]

This allows us to conveniently determine the parameter \( \kappa \) through

\[
\kappa = \lim_{k \rightarrow 0} \lim_{\hat{\tau} \rightarrow \tau} \frac{\hat{\tau} (\hat{\tau}, k) - \hat{\tau}}{\hat{\tau} (\hat{\tau}, k) - \hat{\tau}}
\]

If we focus our interest again to the case, where the stabilisation for odd \( m \) is possible, we can conclude from (19) that this requires a positive \( \kappa \). From (20) we can obtain the value and in particular the sign of \( \kappa \) from a “measurement” of \( \hat{\tau} \) in the low coupling regime. From (21) we observe that for positive \( \kappa \) the period of the induced orbit \( \hat{\tau} \) is always outside the interval \([\tau, \hat{\tau}]\) for all positive values of \( k \), and \( \hat{\tau} \) diverges at the point where (19) is first violated.
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References

[1] E. Ott, C. Grebogi, and J. A. Yorke, Phys. Rev. Lett. 64, 1196 (1990).

[2] E. Schöll and H. G. Schuster, eds., Handbook of Chaos Control, 2nd ed. (Wiley-VCH, Weinheim, 2007).

[3] S. Boccaletti, C. Grebogi, Y. C. Lai, H. Mancini, and D. Maza, Phys. Rep. 329, 103 (2000).

[4] K. Pyragas, Phys. Lett. A 170, 421 (1992).

[5] J. Socolar, D. Sukow, and D. Gauthier, Phys. Rev. E 50, 3245 (1994).

[6] O. Beck, A. Amann, E. Schöll, J. E. S. Socolar, and W. Just, Phys. Rev. E 66, 016213 (2002).

[7] H. Nakajima, Phys. Lett. A 232, 207 (1997).

[8] H. Nakajima and Y. Ueda, Physica D 111, 143 (1998).

[9] W. Just, T. Bernard, M. Ostheimer, E. Reibold, and H. Benner, Phys. Rev. Lett. 78, 203 (1997).

[10] B. Fiedler, V. Flunkert, M. Georgi, P. Hövel, and E. Schöll, Phys. Rev. Lett. 98, 114101 (2007).

[11] E. W. Hooton and A. Amann, Phys. Rev. Lett. (2012), in print.

[12] W. Just, D. Reckwerth, J. Möckel, E. Reibold, and H. Benner, Phys. Rev. Lett. 81, 562 (1998).

[13] V. Novičenko and K. Pyragas, Phys. Rev. E 86, 026204 (2012).