Smooth Polynomial Approach for Microwave Imaging in Sparse Processing Framework

TUSHAR SINGH\(^1,2\), DARKO M. NINKOVIC\(^1\), (Graduate Student Member, IEEE),
BRANKO M. KOLUNDZIJA\(^1,2\), (Fellow, IEEE),
AND MARIJA NIKOLIC STEVANOVIC\(^1\), (Member, IEEE)

\(^1\)School of Electrical Engineering, University of Belgrade, 11120 Belgrade, Serbia
\(^2\)WIPL-D d.o.o., 11073 Belgrade, Serbia

Corresponding author: Tushar Singh (tushar.singh@wipl-d.com)

This work was supported by the EMERALD Project funded by the European Union’s Horizon 2020 Research and Innovation Program under the Marie Skłodowska-Curie Grant under Agreement 764479.

ABSTRACT We developed a novel qualitative imaging algorithm based on a polynomial approximation of the unknown contrast and sparse (\(l_1\)) regularization. Contrary to previously published results, we defined polynomial basis functions on subdomains that divide the investigation domain. Moreover, we formulated constraints that ensure the continuity of the contrast on subdomain borders. We showed that the proposed algorithm improved imaging resolution, particularly in multiple target scenarios. We demonstrated that partitioning the investigation domain together with contrast continuity formulation enhanced the numerical stability and reduced the computation time. The obtained results were significantly less sensitive to the regularization parameter values than those obtained using the standard polynomial approximation. Namely, smaller domains allow lower polynomial orders, which are numerically more favorable. Continuity constraints reduce the search space and mitigate the occurrence of false solutions. Another contribution of this study is a novel strategy for regularization parameter selection. We considered different figures of merit and numerical scenarios to study the influence of various parameters involved in the imaging process, such as the polynomial order and number of subdomains. An extensive analysis proved the robustness of the approach against noise. The proposed algorithm was designed for two-dimensional geometry. However, generalization to three-dimensional space is straightforward. The algorithm can also be used with other types of regularization such as the \(l_2\) regularization. Potential applications include medical microwave imaging, in which high resolution and noise immunity are vital features.

INDEX TERMS Inverse scattering, microwave imaging, polynomial approximation, sparse processing.

I. INTRODUCTION

Microwave imaging (MWI) has many practical applications such as ground-penetrating radar (GPR) [1], [2], through-the-wall-imaging (TWI) [3], [4], medical diagnostics [5], [6], and non-destructive evaluation (NDE) [7], [8]. Among others, the advantages of MWI are the utilization of non-ionizing radiation, portability, and affordability. However, solving MWI problems is difficult due to inherent non-linearity and ill-posedness [9]. MWI algorithms cope with non-uniqueness and false solutions caused by complex scattering mechanisms.

Incorporating prior knowledge about the target (e.g., its physical properties) reduces the ambiguity and moderates the occurrence of false solutions [10]. For example, the \(l_1\) regularization or sparse processing, is an efficient tool for the imaging of electrically small targets [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24]. However, in the case of electrically large targets, sparse processing yields images that often appear as collections of point scatterers [17], [21], [24]. In this case, the alternative solution is to utilize basis functions for which the unknown variables (field sources or contrast) are sparse [25].
Irrespective of the applied regularization type, expanding the contrast or field sources into a series of basis functions reduces the search space and mitigates the solution’s non-uniqueness [10]. Different basis functions have been utilized in the literature to reduce the number of unknowns. In [26], the unknown permittivity distribution inside a cylindrical scanner was modeled using the Zernike polynomials; the expansion coefficients were obtained through the nonlinear optimization. In [27], the Fourier–Jacobi (FJ) basis functions were incorporated into the particle swarm optimization (PSO) in order to retrieve the unknown permittivity profile. In [28], the contrast source inversion (CSI) algorithm was improved by utilizing the wavelet expansion for the unknown sources and contrast. The Haar wavelets have been integrated into a microwave imaging algorithm based on Born approximation [25]. Wavelets have been applied to model the inhomogeneous breast tissue in a nonlinear inverse scattering problem [29], [30]. Another application of wavelets in microwave medical imaging was found in [31], where the Haar wavelet expansions have been combined with the Born iterative method (BIM). Gaussian basis functions were also considered for medical MWI applications [32], [33]. B-spline modeling was proposed in [34] to reduce the problem complexity. Truncated cosine basis functions and B-spline were utilized in [35]. Spherical harmonics expansion, together with sparse processing, was utilized for complex-shaped target imaging in [17], [21], and [24].

Here, we propose a novel microwave imaging algorithm that combines sparse processing with the polynomial approximation of the contrast. Unlike in [26], [27], [28], [29], [30], [31], [32], [33], [34], and [35], where the support of the basis functions covered the whole search space, we defined the polynomials on partitions of the search space. In this way, we achieved better accuracy when imaging multiple targets of different sizes or contrast. Namely, if the polynomials are defined on a single domain, their order has to be very high, which causes numerical instability and strong dependence on the regularization parameter value. To ensure contrast continuity on the domain borders, we formulated additional constraints, which were included in the optimization algorithm.

The algorithm was defined for the two-dimensional (2D) geometry, but the extension to the three-dimensional (3D) geometry would be straightforward. To study the effect of the polynomial order and number of domains on the image reconstruction, we considered several figures of merit, such as the Dice similarity coefficient (DSC) and Matthews correlation coefficient (MC) [36], [37].

The sparse processing brings quality to the proposed algorithm by suppressing the artifacts and noise while representing the contrast with polynomials overcomes the drawback of the sparse processing to produce pixelated images. We showed that the latter feature may be significant for target recognition in scenarios with pronounced multiple scattering as it increases the reconstructed portion of the target. In the proposed method, all data are processed jointly, which is important when working with noisy measurements.

In contrast, many sparse processing algorithms dealing with electrically large targets utilize separate groups of measurements associated with close transmitters. Thus, they obtain the final image by combining the partial results [17], [21], [24], [38].

We have also compared the performance of the presented algorithm to that of the truncated singular value decomposition (TSVD), considered the gold standard in qualitative imaging [39], [40], [41], [42]. The advantages of the proposed approach compared to the TSVD are high-resolution, robustness against noise, and better performance in scenarios with pronounced multiple scattering and a limited number of measurements. In addition, the polynomial approximation of the contrast allows presenting images with arbitrary pixel density without increasing the number of unknowns. The downside is the longer computational time compared to that of the TSVD. The linear sampling method (LSM) [43], [44], [45], also widely used for qualitative imaging, yielded poor results for considered scenarios due to the insufficient number of transmitters (the rank of the multistatic matrix was smaller than the number of significant singular values); hence, we have not included those results.

Another contribution of this paper is a novel procedure for selecting the regularization parameter. Typically, L-curve is utilized for this purpose [46]. In [47], a more precise and demanding method has been proposed, which uses the derivative of the L-curve. Here, we described a simple method that infers the optimal value of the regularization parameter based on the function \( f(\gamma) = \Delta S_i \), where \( \Delta S_i \) is the length of the segment of the L-curve between two adjacent values of the regularization parameter (\( \gamma_i, \gamma_{i+1} \)).

The paper is organized as follows. The theoretical background of the inverse scattering was given in Section II. The polynomial expansion was described in Section III. In Section IV, we described the optimization function. In Section V, we gave brief description of the methods used for comparison. The results obtained for various scenarios were reported in Section VI. Performance analyses, including investigating the robustness against noise and computational time study, were explained in Section VII. Some final remarks were given in the Conclusion section.

II. STANDARD MICROWAVE IMAGING FORMULATION

Fig. 1 illustrates an imaging scenario consisting of elongated objects with constant cross-sections along the z-axis and a circular array of sensors. In this quasi-2D scenario (we consider real antennas instead of infinitely long current sources and targets with finite lengths) the approximate expression for the differential transmission coefficient between the \( i \)th and the \( j \)th antenna due to the presence of scatterers is given by [48]:

\[
\Delta s_{ij} \approx -\frac{j\omega}{2a_i a_j} \int_{V'} \Delta \varepsilon ( \mathbf{r}' ) \ E_i ( \mathbf{r}', \mathbf{r}_i ) \cdot E_i^* ( \mathbf{r}', \mathbf{r}_j ) \, dV',
\]

(1a)
\[ \begin{aligned}
\Delta s_{ij} &= s_{ij} - s_{ij}^0, \quad i, j = 1, \ldots, M, \\
\Delta \mathbf{r}' &= \mathbf{r}'(\mathbf{r}') - \mathbf{r}'(\mathbf{r}'),
\end{aligned} \tag{1b} \]
\[ \Delta \varepsilon \mathbf{r}' = \varepsilon \mathbf{r}'(\mathbf{r}'), \tag{1c} \]

where \( s_{ij} \) and \( s_{ij}^0 \) are the transmission coefficients computed when the scatterer is present and absent, respectively, \( \varepsilon \) is the permittivity of the scatter, \( \varepsilon_b \) is the permittivity of the background medium (a vacuum in our case), \( \Delta \varepsilon \) is the contrast, \( \mathbf{E}_i \) is the incident electric field vector, \( \mathbf{r}' \) is position vector of a point inside the scatterer, \( \mathbf{r}_i \) is the incident power wave at the \( i \)th port, \( \mathbf{r}_i \) is the position vector of the \( i \)th antenna, \( M \) is the size of the array, and \( \omega \) is the angular frequency. When the reconstruction is performed in only one cut, we have
\[ \Delta s \propto \int_{S'} \Delta \varepsilon \mathbf{r}' \mathbf{E}_i(\mathbf{r}', \mathbf{r}_i) \cdot \mathbf{E}_i(\mathbf{r}', \mathbf{r}_j) \, dS', \tag{2} \]

where \( S' \) is the union of objects’ cross-sections. For finite length targets, we cannot determine the exact permittivity distribution due to the lack of measurement data in other cuts. We can estimate the target image, with pixel intensities approximately proportional to the unknown contrast.

After discretizing (2), the measurement model becomes
\[ \Delta \mathbf{s} = \mathbf{Lk}, \tag{3} \]

where \( \mathbf{L} \) is the system matrix (4), as shown at the bottom of the page, \( \mathbf{t}_i \) is the location of the center of \( i \)th pixel, \( i = 1, \ldots, L \), \( \Delta \mathbf{s} \) is the known measurement vector
\[ \Delta \mathbf{s} = [\Delta s_{1,1} \Delta s_{1,2} \cdots \Delta s_{M,M}]^T, \tag{5} \]

and \( \mathbf{k} \) is the unknown vector whose elements are proportional to the contrast of the related pixel
\[ \mathbf{k} = [\Delta \varepsilon_1 \Delta \varepsilon_2 \cdots \Delta \varepsilon_L]^T. \tag{6} \]

III. CONTRAST APPROXIMATION

A. POLYNOMIAL APPROXIMATIONS

Without loss of generality, we assume a rectangular imaging space, divided into \( N_x \times N_y \) smaller domains, as illustrated in Fig. 2. In each domain, the unknown dielectric contrast is approximated by 2D polynomial basis functions of the generic form
\[ K(x, y) = \sum_{k=0}^{Q} \sum_{l=0}^{Q} c_{kl} x^l y^k, \tag{7} \]

where \( Q \) is the polynomial order, \( c_{kl}, \quad k, l = 0, \ldots, Q \), are the polynomial coefficients, and \( x \) and \( y \) are local coordinates, \(-1 \leq x, y \leq 1\). To apply (7), we consider a uniform grid of \( n_x \times n_y \) matching points, where \( n_x \) and \( n_y \) are the numbers of points along the \( x \)-axis and \( y \)-axis, respectively (Fig 3).

In the domain \((m, n), \quad m = 1, \ldots, N_x, \quad n = 1, \ldots, N_y \), the dielectric contrast at the matching points relates to the unknown polynomial coefficients through a matrix relation
\[ \mathbf{k}_{m,n} = \mathbf{Hc}_{m,n}, \tag{8} \]

where
- \( \mathbf{k}_{m,n} \) is a \((n_x \times n_y) \times 1\) vector containing the unknown contrast values in the domain \((n, m)\).
- \( \mathbf{c}_{m,n} \) is a \((Q + 1)^2 \times 1\) vector containing the unknown polynomial coefficients in the considered domain, and
- \( \mathbf{H} \) is the \((n_x \times n_y) \times (Q + 1)^2\) transformation matrix.

The order of the matching points in \( \mathbf{k}_{m,n} \) is defined by the red curve from Fig. 3. Thus, the Cartesian coordinates of the \( i \)th

\[
\mathbf{L} = \begin{bmatrix}
\mathbf{E}_i(t_1, r_1) \cdot \mathbf{E}_i(t_1, r_1) & \cdots & \mathbf{E}_i(t_L, r_1) \cdot \mathbf{E}_i(t_L, r_1) \\
\mathbf{E}_i(t_1, r_1) \cdot \mathbf{E}_i(t_1, r_2) & \cdots & \mathbf{E}_i(t_L, r_1) \cdot \mathbf{E}_i(t_L, r_2) \\
\vdots & \ddots & \vdots \\
\mathbf{E}_i(t_1, r_M) \cdot \mathbf{E}_i(t_1, r_M) & \cdots & \mathbf{E}_i(t_L, r_M) \cdot \mathbf{E}_i(t_L, r_M)
\end{bmatrix},
\tag{4}
\]
matching point are:

\[
\begin{align*}
    x_i &= -1 + \frac{2 \text{ mod } (i - 1, n_x)}{n_x - 1}, \quad i = 1, \ldots, n_x \cdot n_y, \\
    y_i &= -1 + \frac{2 \text{ mod } (i - 1, n_x)}{n_y - 1}, \quad i = 1, \ldots, n_x \cdot n_y,
\end{align*}
\]

where \(\text{mod}(a, n)\) denotes \(a\) modulo \(n\), and \(\lfloor \cdot \rfloor\) denotes the floor function. Thus, according to (7)–(10), the elements of the transformation matrix \(H\) are calculated as

\[
H(i, j) = x_i^\text{mod}(i-j, Q+1) \cdot y_j^{-\lfloor \frac{1}{Q+1} \rfloor},
\]

where \(i = 1, \ldots, n_x \cdot n_y\) and \(j = 1, \ldots, (Q + 1)^2\).

In the whole search space, the unknown contrast is related to all polynomial coefficients as

\[
k = H_{\text{tot}} c,
\]

where

- \(k\) is a \(N_x \cdot N_y \cdot n_x \cdot n_y \times 1\) vector containing the unknown contrast in the whole search space,
- \(c\) is a \(N_x \cdot N_y \cdot (Q + 1)^2 \times 1\) vector containing all unknown polynomial coefficients,
- \(H_{\text{tot}}\) is a block diagonal transformation matrix of the size \(N_x \cdot N_y \cdot n_x \cdot n_y \times N_x \cdot N_y \cdot (Q + 1)^2\).

The vectors \(k\) and \(c\) are obtained by concatenating the vectors \(k_{m,n}\) and \(c_{m,n}\), \(m = 1, \ldots, N_y\), \(n = 1, \ldots, N_x\), respectively, in the order defined by the red curve in Fig. 2. The total transformation matrix is

\[
H_{\text{tot}} = \begin{bmatrix}
    H & 0 & 0 & 0 \\
    0 & H & 0 & 0 \\
    0 & 0 & \ddots & 0 \\
    0 & 0 & 0 & H
\end{bmatrix}.
\]

When (12) is inserted into (3), we obtain the final system of equations relating the measurements and the polynomial coefficients

\[
\Delta s = L k = L H_{\text{tot}} c.
\]
or in the matrix form
\[ H^+_m \mathbf{c}_{m,n} = H^-_m \mathbf{c}_{m+1,n}, \]  
(24)
where \( H^+_m \) and \( H^-_m \) are \( n_x \times (Q + 1)^2 \) matrices
\[ H^+_m(i,j) = x_i \mod (j-1,Q+1) \cdot \left[ j-1 \atop \sigma \right], \]  
(25)
\[ H^-_m(i,j) = x_i \mod (j-1,Q+1) \cdot (-1) \left[ j-1 \atop \sigma \right] \]  
(26)
and
\[ x_i = -1 + \frac{2(i-1)}{n_x - 1}, \quad i = 1, \ldots, n_x \]  
(27)
is the \( x \)-coordinate of the \( i \)th matching point on the boundary between domains \((m, n)\) and \((m + 1, n)\). To impose the continuity along all boundaries parallel to \( x \)-axis, we set the condition
\[ H_x \mathbf{c} = 0, \]  
(28)
where we define a \( N_x \cdot N_y \cdot n_x \times N_y \cdot N_y \cdot (Q + 1)^2 \) matrix \( H_x \) as
\[ H_x = \begin{bmatrix}
H^{(1,1)}_x & H^{(1,2)}_x & \cdots & H^{(1,N_y,N_y)}_x \\
H^{(2,1)}_x & H^{(2,2)}_x & \cdots & H^{(2,N_y,N_y)}_x \\
\vdots & \vdots & & \vdots \\
H^{(N_y-N_y,1)}_x & H^{(N_y-N_y,2)}_x & \cdots & H^{(N_y-N_y,N_y,N_y)}_x
\end{bmatrix} \]  
(29)
\[ H^{(i,j)}_x = \begin{cases}
H^+_x, & \text{if } i = j \text{ and } i \leq (N_y - 1) \cdot N_x, \\
-H^-_x, & \text{if } i = j - N_x \text{ and } i \leq (N_y - 1) \cdot N_x, \\
0, & \text{otherwise.}
\end{cases} \]  
(30)

IV. OPTIMIZATION

We find the solution of the system of equations (14), with constraints (20) and (28), by solving the following optimization problem:
\[ \hat{\mathbf{c}} = \min_{\mathbf{c}} \left\{ \| \Delta \mathbf{s} - L \mathbf{H}_\text{tot} \mathbf{c} \|_2^2 + \gamma \| L \mathbf{H}_\text{tot} \mathbf{c} \|_1 \right\} \]  
subject to: \( H_x \mathbf{c} = 0 \) and \( H_y \mathbf{c} = 0 \),
(31)
where \( \gamma \) is the regularization parameter that balances between the \( l_2 \) norm of the error
\[ \chi_L = \| \Delta \mathbf{s} - L \mathbf{H}_\text{tot} \mathbf{c} \|_2^2, \]  
(32)
and \( l_1 \) norm of the solution
\[ \chi_L = \| L \mathbf{H}_\text{tot} \mathbf{c} \|_1. \]  
(33)
In addition, we normalize the system matrix \( L \) by dividing its columns with their square norms. In this way, the range of useful values of the regularization parameter is set to \( 0 \leq \gamma \leq 1 \) [24].

We note that the \( l_2 \) norm can also be utilized instead of the \( l_1 \) norm in (31). In this way, the applicability of the method can be further extended to arbitrary targets.

To select the optimal value of \( \gamma \), the L-curve [46] is typically utilized. The L-curve is obtained by plotting the solution norm versus the error norm, \( \chi_L (\chi_L) \), for a discrete set of values of the regularization parameter. The optimal value of the regularization parameter is usually associated to the knee of the L-curve. Instead of the L-curve, we propose the following approach (S-curve). First, we define a variable
\[ \Delta s = \sqrt{\| \Delta \chi_L \|^2 + \| \Delta \chi_L \|^2}, \]  
(34)
where
\[ \Delta \chi_L = \chi_L (\gamma) - \chi_L (\gamma - \Delta \gamma), \]  
(35)
\[ \Delta \chi_L = \chi_L (\gamma) - \chi_L (\gamma - \Delta \gamma), \]  
(36)
and \( \Delta \gamma \) is the increment of \( \gamma \). To infer the optimal value of the regularization parameter we used the plot \( \Delta s (\gamma) \), as we detailed in Section VI.

For the optimization (31), we utilized the convex programming package CVX [49]. Once the polynomial coefficients have been found, the unknown vector, proportional to the contrast, is determined as
\[ \hat{\mathbf{k}} = \mathbf{H}_\text{tot} \hat{\mathbf{c}}. \]  
(37)

To assess the quality of the approximation, we considered different quality measures such as the accuracy parameter (ACC), the Dice similarity coefficient (DSC), and Matthews correlation coefficient (MC), all described in [36] and [37]. Those metrics are defined for binarized images, which are obtained by setting the pixels whose value is above the adopted threshold to one, and setting to zero remaining pixels. The accuracy parameter (ACC), Dice similarity coefficient (DSC), and Matthews correlation coefficient (MC) are defined, respectively, as:
\[ ACC = \frac{TP + TN}{2 \cdot TP}, \]  
(38)
\[ DSC = \frac{TP + FP + FN}{2 \cdot TP}, \]  
(39)
\[ MC = \frac{TP \cdot TN - FP \cdot FN}{\sqrt{(TP + FP) (TP + FN) (TN + FP) (TN + FN)}}, \]  
(40)

In the above expressions TP stands for true positive, TN for true negative, FP for false positive, and FN for false negative.

The selection of the threshold primarily influences the maximal value of the metrics and, to a smaller extent, their dependence on the regularization coefficient. To obtain fair comparisons, we utilized the threshold values that maximized the metrics. (We were not interested in the absolute value of the metrics; the goal was to compare the results obtained with different polynomial orders and different values of the regularization parameter.)

We have also considered the root-mean-square error between the estimated (normalized) contrast vector and the (normalized) true contrast:
\[ E = \frac{\| \mathbf{k}_n - \mathbf{k}_n \|_2}{\sqrt{N}_\text{tot}}, \]  
(41)
where \( N_{\text{tot}} = N_x \cdot N_y \cdot n_z \cdot n_y, \) \( \hat{k}_n = \hat{k} / \max(\hat{k}), \) and \( \hat{k}_n = k / \max(k). \) To make this error metrics compatible to previous quality metrics, we defined the variable \( C_x = 1 - E. \)

V. COMPARISON METHODS

For comparison purposes, the TSVD and standard sparse processing were used. For the sake of completeness, we will briefly describe the implemented algorithms, defined in the same way as in [24].

A. TRUNCATED SINGULAR VALUE DECOMPOSITION

In the TSVD, the target image is obtained from (42) and (43), as shown at the bottom of the page, where \( L \) is the system matrix, \( \Delta s \) is the known measurement vector (defined in (5)), and \( f \) is the unknown vector (target image). The TSVD solution of the system is

\[
f = \sum_{i=1}^{i_{\text{max}}} \frac{1}{\sigma_i} \left( u_i^H \cdot \Delta s \right) v_i, \tag{44}
\]

where \( u_i, v_i \) are the singular vectors of matrix \( L, \) \( \sigma_i \) are the corresponding singular values, and \( i_{\text{max}} \) is the truncation index, obtained from the condition \( 10 \log_{10}(\sigma_{i_{\text{max}}}/\sigma_1) < -20 \text{ dB}. \)

B. STANDARD SPARSE PROCESSING (SSP)

In contrast to the TSVD, partial systems of equations are formed for each transmitter. When the \( i \)-th antenna is transmitting, the system of equations reads (45)–(47), as shown at the bottom of the page, where \( L^{(i)} \) is the partial system matrix, \( e^{(i)} \) is the corresponding measurement vector, and \( f^{(i)} \) is the unknown vector. The data obtained from \( P \) transmitting antennas are processed together. When \( P = 2, \) the resulting system of equations becomes

\[
\begin{bmatrix}
  e^{(i)} \\
  e^{(i+1)} \\
\end{bmatrix} \approx
\begin{bmatrix}
  L^{(i)} \\
  L^{(i+1)} \\
\end{bmatrix}
\begin{bmatrix}
  f^{(i)} \\
\end{bmatrix}. \tag{48}
\]

The minimization function is

\[
\hat{f}^{(i)} = \min_{f^{(i)}} \| e - Lf^{(i)} \|_2^2 + \gamma \| f^{(i)} \|_1, \tag{49}
\]

and the final image is obtained as the superposition of the partial results \( \hat{f}^{(i)}. \) For the regularization parameter, we used the value slightly smaller than the knee-curve value.

VI. NUMERICAL EXAMPLES

In this section, we presented imaging results obtained for several numerical scenarios. In all cases, the array response was computed using the full-wave electromagnetic solver WIPL-D Pro [50].

A. IMAGING CONFIGURATION 1

In the first example, we investigated the effect of the polynomial order on the reconstruction quality with a fixed number of subdomains. An additional point of interest was the selection of the regularization parameter. We compared two approaches for regularization parameter selection: the established L-curve and the proposed S-curve method.

The imaging objects were two identical dielectric cylinders made of a dielectric with the relative permittivity \( \varepsilon_r = 5, \) as shown in Fig. 5. The cylinders had elliptical cross-sections, where the lengths of the semi-minor and semi-major axes were \( a = 1 \text{ cm} \) and \( b = 2 \text{ cm}, \) respectively. The centers of the cylinders were at \((-3 \text{ cm}, 0, 0)\) and \((3 \text{ cm}, 0, 0)\).
The antenna array consisted of 18 half-wavelength dipoles, uniformly distributed on a circle of the radius \( R = 0.6 \) m. The operating frequency of the array was \( f = 3 \) GHz. We assumed that one antenna transmitted at a time, while all antennas were receiving signals. The obtained data were corrupted by the additive white Gaussian noise (AWGN), with the signal-to-noise ratio \( \text{SNR} = 10 \) dB.

The search space was a square area of the size \(-0.2 \leq x, y \leq 0.2\) m, divided into \( N_x = N_y = 5 \) domains. In each domain there were \( n_x \times n_y = 20^2 \) matching points (or 100\(^2\) matching points in the whole search space).

To study the influence of the polynomial order on the reconstruction accuracy, we utilized ACC, DSC, MC, and \( C_\varepsilon \) metrics. We computed those metrics for polynomials of the order \( 2 \leq Q \leq 6 \), utilizing regularization parameter values from the range \( 0.01 \leq \gamma \leq 1 \), with a step \( \Delta \gamma \) = 0.01. The obtained data indicated that the accuracy increased monotonically with polynomial degree up to \( Q = 4 \). Higher-order polynomials, \( Q = 5, 6 \), yielded a better solution only in a narrow range of values of the regularization parameter; otherwise, overfitting occurred.

Table 1 gives the statistics for \( C_\varepsilon \) and Table 2 gives the statistics for MC coefficient. The latter was computed using 40\% binarization threshold.

**Table 1.** Statistical data for \( C_\varepsilon \) coefficient versus polynomial order.

| \( Q \) | Max(\( C_\varepsilon \)) | Mean(\( C_\varepsilon \)) | Median(\( C_\varepsilon \)) |
|------|----------------|----------------|----------------|
| 2    | 0.87           | 0.84           | 0.84           |
| 3    | 0.91           | 0.88           | 0.89           |
| 4    | 0.93           | 0.89           | 0.90           |
| 5    | 0.92           | 0.90           | 0.89           |
| 6    | 0.93           | 0.90           | 0.91           |

**Table 2.** Statistical data for MC coefficient versus polynomial order.

| \( Q \) | Max(MC) | Mean(MC) | Median(MC) |
|------|--------|----------|------------|
| 2    | 0.51   | 0.42     | 0.43       |
| 3    | 0.58   | 0.49     | 0.50       |
| 4    | 0.66   | 0.55     | 0.55       |
| 5    | 0.65   | 0.49     | 0.50       |
| 6    | 0.72   | 0.54     | 0.53       |

The regularization parameter value has a significant influence on the imaging results. Typically, the optimal value of the regularization parameter is associated with the knee of the L-curve [46]. Fig. 6a shows the L-curve and the knee-point (red circle), computed for \( Q = 4 \). In Section IV, we defined the S-curve, which is more sensitive to the values of the regularization parameter. Fig. 6b shows the corresponding S-curve, denoted as \( \Delta s \), in which we distinguish three intervals: \( \gamma < 0.1 \) characterized by a steep slope, \( 0.1 < \gamma < \gamma_{knee} \) characterized by a small ripple, and a flat part for \( \gamma > \gamma_{knee} \). (The knee-point of the L-curve is easily identified on the S-curve as the last point before \( \Delta s \) drops down to zero.) Fig. 6b also shows all considered quality metrics.

Fig. 7 shows the reconstruction results computed for \( Q = 4 \) for several characteristic values of the regularization parameter. Fig. 7a shows the image obtained for \( \gamma_1 = 0.1 \), which was the regularization parameter value at the end of the first interval on the S-curve. Targets’ footprints were distorted with artifacts due to the insufficient value of the regularization parameter. Fig. 7b shows the image obtained for \( \gamma_2 = 0.35 \), which maximized the quality metrics. The targets were clearly visible and the artifacts were suppressed. Visually similar results were obtained for \( \gamma_3 = \gamma_{knee} = 0.43 \), as shown in Fig. 7c. (Small difference in the values of quality metrics computed for \( \gamma_2 \) and \( \gamma_3 \) was due to the binarization). Finally, Fig. 7d, shows the image obtained for \( \gamma_4 = 0.6 \), related to the vertical part of the L-curve (or horizontal part of the S-curve). The targets were easily identified but the image was not sharp due to the large value of the regularization parameter.
Fig. 7. Imaging results $N_x = N_y = 5, n_x = n_y = 20, Q = 4$: (a) $\gamma_1 = 0.1$, (b) $\gamma_2 = 0.35$, (c) $\gamma_3 = 0.43$, and (d) $\gamma_4 = 0.6$.

Fig. 8. S-curve and the quality metrics: $N_x = N_y = 5, n_x = n_y = 20$, and $Q = 6$. (a) $\gamma_1 = 0.25$, (b) $\gamma_2 = 0.43$, (c) $\gamma_3 = 0.5$, and (d) $\gamma_4 = 0.6$.

Fig. 9. Imaging results $N_x = N_y = 5, n_x = n_y = 20, Q = 6$: (a) $\gamma_1 = 0.25$, (b) $\gamma_2 = 0.43$, (c) $\gamma_3 = 0.5$, and (d) $\gamma_4 = 0.6$.

Fig. 10. Reconstruction results: (a) standard sparse processing (logarithmic scale) and (b) TSVD (linear scale).

In this subsection, we studied the role of the polynomial order on the quality of imaging of closely-spaced sparse targets. Numerical simulations showed that increasing the degree of polynomial approximation improves the resolution but only to a certain degree, after which overfitting may occur.

**B. IMAGING CONFIGURATION 2**

In the following two examples, we considered the array comprising 240 half-wavelength dipoles, uniformly distributed on a circle of the radius $R = 1.76$ m, as shown in Fig. 11. Out of 240 antennas, only 8 operated as transmitters (denoted as red circles in Fig. 11).

The angular distance between adjacent transmitters was $\Delta \varphi_{\text{TX}} = 45^\circ$ and the angular distance between the adjacent receivers was $\Delta \varphi_{\text{RX}} = 1.5^\circ$. The operating frequency was $f = 6$ GHz. The unknown targets were dielectric cylinders with circular cross-sections shown in Fig. 12.

As the second example, we considered the cylinders from Fig. 12a. The focus of this study was the influence on the number of the subdomains on the performance of the algorithm. The centers of the cylinders were at $(-3.12$ cm, $2.34$ cm, 0) and $(-1.56$ cm, $-1.56$ cm, 0).
Their radii were \( R_1 = 1.17 \text{ cm} \) and \( R_2 = 2.34 \text{ cm} \), and their relative permittivity were \( \varepsilon_{r1} = 2.32 \) and \( \varepsilon_{r2} = 1.47 \).

The search space was a square of the size \( 25 \text{ cm} \times 25 \text{ cm} \) with \( 64^2 \) matching points in the whole investigation domain. We considered subdivisions of the search space into: \( N_x = N_y = 4 \), \( N_x = N_y = 2 \), and \( N_x = N_y = 1 \) domains.

The experiments showed that for \( N_x = N_y = 4 \), \( n_x \times n_y = 16^2 \), the polynomials with the order \( Q \leq 2 \) could not resolve two targets. Also, increasing the polynomial order beyond \( Q > 4 \) did not improve the accuracy. Table 3 and Table 4 show the corresponding statistics for MC and \( C_e \) coefficients.

Fig. 13a and Fig. 13b show the obtained images computed for \( \gamma_1 = 0.15 \) (the small-ripple region on the S-curve) and \( \gamma_2 = 0.5 \) (horizontal part of the S-curve), respectively. The cylinders were easily identified and more intensive color referred to the cylinder with a larger contrast.

![FIGURE 11. Sensor array.](image)

![FIGURE 12. Targets from the (a) second and (b) third example.](image)

![FIGURE 13. Imaging results \( N_x = N_y = 4, n_x = n_y = 16, Q = 3 \): (a) \( \gamma_1 = 0.15 \), and (b) \( \gamma_2 = 0.5 \).](image)

![FIGURE 14. Imaging results \( N_x = N_y = 2, n_x = n_y = 32, Q = 8 \): (a) \( \gamma_1 = 0.15 \) and (c) \( \gamma_2 = 0.5 \).](image)

![FIGURE 15. Imaging results \( N_x = N_y = 1, n_x \times n_y = 64^2 \), and \( Q = 16 \). As expected, if the number of domains is decreased, the order of the polynomial approximation has to be increased. Finally, if there was only one domain, \( N_x = N_y = 1 \), \( n_x \times n_y = 64^2 \), the polynomial order had to be at least \( Q = 16 \). This caused severe instabilities, which manifest as strong dependence on the regularization parameter value. As an illustration, Fig. 15 shows the images computed for two consecutive values of the regularization parameter. The numerical instability is also observed on the L-curve (Fig. 16a) and S-curve (Fig. 16b).

![FIGURE 16. Imaging results \( N_x = N_y = 1, n_x \times n_y = 2^2, Q = 8 \): (a) \( \gamma_1 = 0.41 \) and (b) \( \gamma_2 = 0.42 \).](image)

To conclude this part of the study, we showed in Fig. 17 the results obtained using the standard sparse processing and the TSVD method. As before, the sparse processing produced pixelated image, while the TSVD image was characterized by artifacts.
In the last example, we analyzed the scenario with pronounced multiple scattering (Fig. 12b). The Cartesian coordinates of the centers of the cylinders were (2.54 cm, 1.36 cm, 0) and (−2.35 cm, 1.56 cm, 0), respectively. The radii of the cylinders were \( R_1 = 1.95 \text{ cm} \) and \( R_2 = 1.76 \text{ cm} \). The relative permittivities were \( \varepsilon_{r1} = 3.44 \) and \( \varepsilon_{r2} = 2.99 \), respectively. The search space was divided into \( N_x = N_y = 4 \) domains, with \( n_x \times n_y = 16^2 \) matching points in each domain.

As in previous examples, we computed quality metrics for different polynomial orders in the whole range of regularization parameter values. The coefficient \( C_\varepsilon \) was almost independent of the polynomial order, while the best results for ACC, MC, and DSC metrics were obtained for \( Q = 2 \). Table 5 and Table 6 show the results for selected quality metrics.

Fig. 18 shows the S-curve and quality metrics computed for \( Q = 2 \). Fig. 19 shows the obtained images, computed for:

\[
\begin{align*}
\gamma_1 &= 0.06, \\
\gamma_2 &= 0.10, \\
\gamma_3 &= 0.24, \text{ and } \\
\gamma_4 &= 0.5.
\end{align*}
\]

As in previous examples, we computed quality metrics for different polynomial orders in the whole range of regularization parameter values. The coefficient \( C_\varepsilon \) was almost independent of the polynomial order, while the best results for ACC, MC, and DSC metrics were obtained for \( Q = 2 \). Table 5 and Table 6 show the results for selected quality metrics.

Fig. 18 shows the S-curve and quality metrics computed for \( Q = 2 \). Fig. 19 shows the obtained images, computed for:

\[
\begin{align*}
\gamma_1 &= 0.06, \\
\gamma_2 &= 0.10, \\
\gamma_3 &= 0.24, \text{ and } \\
\gamma_4 &= 0.5.
\end{align*}
\]

VOLUME 10, 2022
Fig. 21 shows the results obtained using the sparse processing and the TSVD in which actual target shape is hardly recognizable.

Compared to the previous example (Fig. 12a), this reconstruction was more difficult due to the larger dimensions of the cylinders and the more significant contrast. By keeping a low-order approximation, we estimated the number and shape of the targets, which was impossible with the TSVD and standard sparse processing.

VII. PERFORMANCE ANALYSIS

A. ROBUSTNESS AGAINST NOISE

To study influence of the noise on the performance of the algorithm, we computed images of the target from Fig. 5 using different signal-to-noise ratios (SNR). We calculated SNR with respect to the power of differential signal.

Fig. 22 shows the obtained results. Two targets were discernable even in the case of severe noise. For $\text{SNR} < -5$ dB, the image quality started to deteriorate significantly.

Fig. 23 shows the results obtained using the TSVD, which were much more sensitive to noise. (The truncation threshold used to regularize TSVD was set to $-15$ dB.)

We have also calculated the statistical parameters of MC coefficient (MC), using regularization parameter values $0.1 \leq \gamma \leq 0.5$, with a $\Delta \gamma = 0.01$ step. The results shown in Table 7 supported previous discussion. Namely, the calculated values of MC coefficients were stable for $\text{SNR} \geq -5$ dB.

We observed similar behavior in the other two examples. Thus, we omitted those results.

B. COMPUTATION TIME

At a given frequency, the image resolution primarily depends on the number of unknown coefficients, $N_x N_y (Q + 1)^2$. 

| $\text{SNR [dB]}$ | Max(MC) | Mean(MC) | Median(MC) |
|-------------------|---------|-----------|-------------|
| 10                | 0.66    | 0.55      | 0.55        |
| 5                 | 0.65    | 0.54      | 0.55        |
| 0                 | 0.66    | 0.57      | 0.56        |
| -5                | 0.61    | 0.52      | 0.54        |
| -10               | 0.33    | 0.30      | 0.31        |
We showed that when the number of subdomains is decreased, the order of polynomial approximation has to be increased to keep similar resolution. Moreover, our analysis indicated that using large polynomial orders on a single domain causes numerical instability, which was particularly pronounced for sparse targets imaged with small number of measurements.

In this section, we studied the influence of the number of unknowns on the computation time. To that end, we considered the reconstruction of the target from Fig. 5 using three different divisions of the investigation domain: \(N_x = N_y = 5\), \(N_x = N_y = 2\), \(N_x = n_y = 50\), \(Q = 10\), and \(N_x = N_y = 1\), \(n_x = n_y = 100, Q = 16\).

Fig. 25 shows the results of the analysis. The shortest computation time was observed for the largest number of subdomains, \(N_x = N_y = 5\).

Fig. 26 shows the representative images for all three cases. The respective computational times, were \(t = 24\) s (625 unknowns), \(t = 125\) s (484 unknowns), and \(t = 255\) s (289 unknowns). In the last case \((N_x = N_y = 1)\), the results diverged.

**FIGURE 24.** Computation times versus the number of unknowns for different subdivisions of the imaging domain (target from Fig. 5).

**FIGURE 25.** Target images obtained for (a) \(N_x = N_y = 5, n_x = n_y = 20, Q = 4\), (b) \(N_x = N_y = 2, n_x = n_y = 50, Q = 10\), and (c) \(N_x = N_y = 1, n_x = n_y = 100, Q = 16\).

**FIGURE 26.** Computation times versus the number of unknowns for different subdivisions of the imaging domain (target from Fig. 12a).

Fig. 26 shows the computation times for targets from Fig. 12a. Again, the shortest time was observed for the largest number of domains, \(N_x = N_y = 4\). However, the difference among three cases was smaller compared to the previous example. The computation times for 256 unknowns were \(t = 46\) s \((N_x = N_y = 4)\), \(t = 59\) s \((N_x = N_y = 2)\), and \(t = 120\) s \((N_x = N_y = 1)\).

Generally, boundary constraints on subdomain borders effectively reduce the number of unknowns and, consequently, lessen the computation time and improve the stability. The computation times of the TSVD were of the order of seconds on the same computer. Therefore, higher resolution and immunity to noise were paid by longer execution times.

**VIII. CONCLUSION**

We have developed an algorithm that combines the polynomial approximation of the contrast and sparse processing. Unlike the previously published results, the support of the basis functions stretches over subdomains that divide the search space. To preserve the continuity of the contrast across subdomain borders, we formulated constraints, which were included in the optimization function. The algorithm’s performance was tested in several numerical scenarios using different quality metrics.

Another contribution of this work was a novel procedure for selecting the regularization parameter. The regularization parameter value, associated with the knee of the L-curve, typically used as the optimal, suppresses weak scatterers and leaves the strongest scatterer in the image. Instead, we defined the S-curve, which provides information on the lengths of the L-curve segments associated with an increment of the regularization parameter value. We showed that by selecting the regularization parameter from the S-curve region characterized by small ripples improves the image accuracy. Moreover, the shape of the S-curve indicated the sensitivity of the solution to the regularization parameter value. Namely, the larger the region with small ripples, the more stable was the solution of the inverse problem.

Extensive testing demonstrated the robustness of the proposed method against noise. Accurate images were obtained under \(SNR\) as low as \(-5\) dB for the array with a smaller number of sensors. Noise suppression and high resolution
are the qualities stemming from the $l_1$ regularization, whose application was made possible by partitioning the investigation space. At the same time, the polynomial approximation of the contrast eliminated the pixelated nature of images and, thus, extended the range of application of sparse processing.

REFERENCES

[1] M. Salucci, G. Oliveri, and A. Massa, “GPR prospecting through an inverse-scattering frequency-hopping multifocusing approach,” IEEE Trans. Geosci. Remote Sens., vol. 53, no. 12, pp. 6573–6592, Dec. 2015.

[2] K. Ren and R. J. Burkholder, “Identification of hidden objects in layered media with shadow projection near-field microwave imaging,” IEEE Geosci. Remote Sens. Lett., vol. 15, no. 10, pp. 1590–1594, Oct. 2018.

[3] S. Sadeghi, K. Mohammadpour-Aghdam, R. Faraji-Dana, and R. J. Burkholder, “A DORT-uniform diffraction tomography algorithm for through-the-wall imaging,” IEEE Trans. Antennas Propag., vol. 68, no. 4, pp. 3176–3183, Apr. 2020.

[4] R. Cicchetti, S. Pisa, E. Piuzzi, E. Pittella, P. D’Atanasio, and O. Testa, “Numerical and experimental comparison among a new hybrid FT-music technique and existing algorithms for through-the-wall radar imaging,” IEEE Trans. Microw. Theory Techn., vol. 69, no. 7, pp. 3372–3387, Jul. 2021.

[5] L. Crocco, I. Karanasiou, M. James, and R. Conceicão, “Emerging Electromagnetic Technologies for Brain Diseases Diagnostics, Monitoring and Therapy,” Switzerland: Springer, 2016.

[6] R. Conceicão, A. Abubakar, “A contrast source inversion method in the wavelet domain,” Inverse Problems, vol. 29, no. 2, Feb. 2013, Art. no. 025015.

[7] R. Scapaticci, I. Catapano, and L. Crocco, “Wavelet-based adaptive multiresolution inversion for quantitative microwave imaging of breast tissues,” IEEE Trans. Antennas Propag., vol. 60, no. 8, pp. 3717–3726, Aug. 2012.

[8] R. Scapaticci, P. Kosmas, and L. Crocco, “Wavelet-based regularization for robust microwave imaging in medical applications,” IEEE Trans. Biomed. Eng., vol. 62, no. 4, pp. 1195–1202, Apr. 2015.

[9] L. Guo and A. M. Abbosh, “Microwave imaging of nonsparsely distributed Born iterative method with wavelet transform and block sparse Bayesian learning,” IEEE Trans. Antennas Propag., vol. 63, no. 11, pp. 4877–4888, Nov. 2015.

[10] D. W. Winters, J. D. Shea, P. Kosmas, B. D. Van Veen, and S. C. Hagness, “Three-dimensional microwave breast imaging: Dispersive dielectric properties estimation using patient-specific basis functions,” IEEE Trans. Med. Imag., vol. 28, no. 7, pp. 969–981, Jul. 2009.

[11] D. W. Winters, B. D. Van Veen, and S. C. Hagness, “A sparsity regularization approach to the electromagnetic inverse scattering problem,” IEEE Trans. Antennas Propag., vol. 58, no. 1, pp. 145–154, Jan. 2010.

[12] A. Baussard, E. L. Miller, X. Li, and D. Premel, “Adaptive B-spline approach for inverse scattering problems,” in Proc. IEEE Antennas Propag. Soc. Int. Symp. Dig. Held Conjunct USNC/URSI North Amer. Radio Sci. Meeting, vol. 66, no. 7, Jun. 2003, pp. 772–775.

[13] A. Semmani, I. T. Rekanos, M. Kamyab, and M. Moghaddam, “Solving inverse scattering problems based on truncated cosine Fourier and cubic B-spline expansions,” IEEE Trans. Antennas Propag., vol. 60, no. 12, pp. 5914–5923, Dec. 2012, doi: 10.1109/TAP.2012.2214751.

[14] A. A. Taha and A. Hanbury, “Metrics for evaluating 3D medical image segmentation: Analysis, selection, and tool,” BMC Med. Imag., vol. 15, no. 1, Dec. 2015, doi: 10.1186/s12880-015-0068-X.

[15] A. Yago, M. Cavagnaro, and L. Crocco, “Deep learning-enhanced qualititative microwave imaging: Rationale and initial assessment,” in Proc. 15th Eur. Conf. Antennas Propag. (EuCAP), Mar. 2021, pp. 1–5.

[16] G. Oliveri, P. Rocca, and A. Massa, “A Bayesian-compressive-sampling-based inversion for imaging sparse scatterers,” IEEE Trans. Geosci. Remote Sens., vol. 49, no. 10, pp. 3993–4006, Oct. 2011.

[17] J. D. Shea, B. D. Van Veen, and S. C. Hagness, “A TSVD analysis of microwave inverse scattering for breast imaging,” IEEE Trans. Biomed. Eng., vol. 59, no. 4, pp. 936–945, Apr. 2012.

[18] R. O. Mays, N. Behdad, and S. C. Hagness, “A TSTD analysis of the impact of polarization on microwave breast imaging using an enclosed array of miniaturized patch antennas,” IEEE Antennas Wireless Propag. Lett., vol. 14, pp. 418–421, 2015.
TUSHAR SINGH was born in Prayagraj, India, in 1990. He received the B.Tech. degree from the Madan Mohan Malaviya University of Technology, Gorakhpur, India, the M.Tech. degree from the Indian Institute of Technology, BHU, Varanasi, India, and the Ph.D. degree in microwave engineering from the School of Electrical Engineering, University of Belgrade, Belgrade, Serbia, in 2018. He started his professional career as an Assistant Professor at KIIT University Bhubaneswar, India, from 2014 to 2018. He joined EU Horizon 2020 EMERALD Project, in 2018, under the host WIPL-D, Belgrade, as a Marie-Curie Fellow. His research interests include medical microwave imaging, numerical modeling, and simulations and antenna design.

DARKO M. NINKOVIC (Graduate Student Member, IEEE) was born in Belgrade, Serbia, in 1995. He received the B.Sc. and M.Sc. degrees in electrical engineering from the School of Electrical Engineering, University of Belgrade, Belgrade, in 2018 and 2019, respectively, where he is currently pursuing the Ph.D. degree. His current research interests include optimization algorithms applied to the electromagnetic problems, microwave imaging, and electromagnetic compatibility.

MARIJA NIKOLIC STEVANOVIC (Member, IEEE) received the B.Sc., M.Sc., and Ph.D. degrees from the University of Belgrade, Belgrade, Serbia, in 2000, 2003, and 2007, respectively, and the Ph.D. degree from Washington University in St. Louis, St. Louis, MO, USA, in 2011, all in electrical engineering. In 2011, she joined the School of Electrical Engineering, University of Belgrade, as a Teaching Assistant, and was promoted to an Associate Professor, in 2017. Her research interests include inverse scattering, medical microwave imaging applications, numerical electromagnetics, and antenna design. Since 2013, she has been the Serbian Representative at the Management Committees of COST actions devoted to medical applications of EM fields (MiMed on microwave imaging and MyWAVE on therapeutic applications of electromagnetic waves).