Dynamic magnetic susceptibility and electrical detection of ferromagnetic resonance

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Abstract

The dynamic magnetic susceptibility of magnetic materials near ferromagnetic resonance (FMR) is very important in interpreting the dc voltage obtained in its electrical detection. Based on the causality principle and the assumption that the usual microwave absorption lineshape of a homogeneous magnetic material around FMR is Lorentzian, the general forms of the dynamic magnetic susceptibility of an arbitrary sample and the corresponding dc voltage lineshapes of its electrical detection were obtained. Our main findings are as follows. (1) The dynamic magnetic susceptibility is not a Polder tensor for a material with an arbitrary magnetic anisotropy. The two off-diagonal matrix elements of the tensor near FMR are not, in general, opposite to each other. However, the linear response coefficient of the magnetization to the total radio frequency (rf) field (the sum of the external and internal rf fields due to precessing magnetization is a quantity which cannot be measured directly) is a Polder tensor. This may explain why the two off-diagonal susceptibility matrix elements were always wrongly assumed to be opposite to each other in almost all analyses. (2) The frequency dependence of dynamic magnetic susceptibility near FMR is fully characterized by six real numbers, while its field dependence is fully characterized by seven real numbers. (3) A recipe of how to determine these numbers by standard microwave absorption measurements for a sample with an arbitrary magnetic anisotropy is proposed. Our results allow one to unambiguously separate the contribution of the anisotropic magnetoresistance to the dc voltage signals from the anomalous Hall effect. With these results, one can reliably extract the information of spin pumping and the inverse spin-Hall effect, and determine the spin-Hall angle. (4) In the case that resonance frequency is not sensitive to the applied static magnetic field, the field dependence of the matrix elements of dynamic magnetic susceptibility, as well as the dc voltage, may have another non-resonance broad peak. Thus, one should be careful in interpreting the observed peaks.

Keywords: ferromagnetic resonance, dynamic magnetic susceptibility matrix, electrical detection

(Some figures may appear in colour only in the online journal)
1. Introduction

Ferromagnetic resonance (FMR) is an important phenomenon, which can be used to probe the magnetic properties of ferromagnetic materials, and has many other applications [1–14]. Microwave absorption is usually measured in traditional FMR experiments, and the absorption lineshape around FMR is a Lorentzian function of microwave frequency for a fixed static magnetic field, when the inhomogeneity broadening of the FMR peak of a magnetic material is much smaller than that from the Gilbert damping. The peak position and peak width can be used to determine magnetic anisotropy and the Gilbert damping coefficient [15, 16]. In recent years, the electrical detection of FMR, in which a dc voltage is measured on a sample around it, has become very popular due to the high accuracy of dc voltage measurements, in which only a micron sample size is needed [17–33]. This technique has been used by many groups [19–25] in recent years to extract spin pumping and the spin-Hall angle that measures the strength of both the spin-Hall effect (SHE) and the inverse spin-Hall effect (ISHE). However, the experimentally extracted material parameters show a large discrepancy for the same materials with similar experimental setups. For example, the spin–Hall angle of Pt obtained by different groups differs by two orders of magnitude [21–23] due to the different interpretations of the dc voltage signal.

The dc voltage in the electrical detection of FMR can come from two sources. One is the generalized Ohm’s law, in which the well-known anisotropic magnetoresistance (AMR) and the anomalous Hall effect (AHE) couple the magnetization motion with the electric current (see the discussion below) [17, 18, 27]. This coupling between the magnetization and microwave-induced electric current results in the so-called spin rectification effect [27], which gives rise to a detectable dc voltage inside a magnetic layer or a multilayer at FMR. The other dc voltage source is the ISHE: a spin current converts into a transverse charge current via ISHE inside a multilayer, and the charge current, in turn, gives rise to a dc voltage. In the popular spin-Hall-angle experiments [19–21, 24, 25], the spin current comes from spin pumping by the magnetization precession at FMR [34, 35]. To use the electrical detection of FMR as a probe of material properties, one needs to separate the different dc voltage sources. A common method is the symmetry analysis of dc voltage lineshapes. The ISHE contribution to dc voltage is normally assumed to have a symmetric lineshape [19–25], resembling microwave absorption curves. The AMR contribution to dc voltage can be symmetric [25], antisymmetric [21] or asymmetric [23, 24], although resistance is normally linked to energy dissipation. On the other hand, the AHE contribution can be antisymmetric [19, 25] or vanishing small [21, 23, 24], because the Hall effect does not necessarily involve energy dissipation, depending on the material properties and experimental setup. So far, most analyses [19–21, 23, 24, 27] assume that the static response of magnetization is always along the external magnetic field, so that the microwave-induced magnetization precession is around the external static magnetic field. Furthermore, the two off-diagonal matrix elements of the dynamic magnetic susceptibility are assumed to be opposite to each other. However, both assumptions are questionable for layer/multilayer samples. Interfacial spin-orbit interactions and/or other interactions may modify the magnetic anisotropy of a film [36–38]. As we shall see below, the dynamic magnetic susceptibility of a sample with non-zero magnetic anisotropy is not a Polder tensor in general. For a given experimental setup, the dc voltage due to AMR and AHE is very sensitive to the dynamic magnetic susceptibility, since the spin rectification effect comes from the phase lag of magnetization precession and radio frequency (rf) current. Thus, the dynamic magnetic susceptibility at FMR is a central quantity.

In this work, we obtain general expressions of the dynamic magnetic susceptibility matrix of an arbitrary magnetic material near FMR, based on the causality principle and the fact that the microwave absorption at FMR of a homogeneous magnetic material—either single crystal or polycrystal—whose inhomogeneity broadening of the FMR peak is much smaller than that from damping, is Lorentzian. The dynamic magnetic susceptibility is not a Polder tensor in general. It becomes a Polder tensor when the sample is isotropic or uniaxial with the static magnetic field along its easy axis. The matrix is fully characterized by a few constants that can be determined by traditional microwave absorption experiments. Interestingly, the linear response coefficient of the magnetization to the total rf field, the sum of the applied external rf field and the internal rf field due to the magnetization precession, is a Polder tensor. However, this coefficient is not the dynamic magnetic susceptibility, and cannot be measured directly. Furthermore, under a fixed rf, the susceptibility matrix elements may have multiple peaks in the field at a fixed microwave frequency, in contrast to a single peak in a frequency at a fixed field. The multiple field peaks come from the non-monotonic behavior of the total effective magnetic field to the externally applied static magnetic field. With the knowledge on the dynamic magnetic susceptibility matrix, it is possible to separate the contributions of the AMR to the dc voltage from that of the AHE. We also show that another broad peak may appear in the susceptibility matrix elements if the resonance frequency is not very sensitive to the externally applied magnetic field. In turn, a broad peak might also appear in the field-dependence of the microwave absorption curves as well as the dc voltage lineshape. This peak is not from the resonance, and its shape is not Lorentzian. The paper is organized as follows. In section 2 we first reformulate the generalized Ohm’s law for a magnetic metal, and explain how a dc voltage can come out of the spin rectification at FMR. Then we derive the expression of the dynamic magnetic susceptibility matrix, both as a function of the microwave frequency or as a function of the static magnetic field. The experimental method of determining the matrix is then given. With the dynamic magnetic susceptibility matrix, we analyze the dc voltage originating from the AMR and the AHE. Section 3 uses several examples to verify the general form of the dynamic magnetic susceptibility matrix obtained in section 2. We show how multiple FMR peaks in a field at a fixed frequency can come from the non-monotonic behavior of the effective field in an applied field, and how a non-resonance peak can arise when
the resonance frequency (or effective field) is not sensitive to the externally applied static magnetic field. The conclusion is given in section 4, followed by the acknowledgements.

2. Theoretical analysis

2.1. The generalized Ohm’s law and spin rectification

The electrical detection of FMR is based on the generalized Ohm’s law in a polycrystalline ferromagnetic metal [17, 18, 27]

\[
E = \rho_i J + \frac{\Delta \rho}{M^2}(\mathbf{J} \cdot \mathbf{M})\mathbf{M} - R_0 \mathbf{J} \times \mathbf{H} - R_i \mathbf{J} \times \mathbf{M},
\]

(1)

where \( \mathbf{M}, \mathbf{H} \) and \( \mathbf{J} \) are respectively the magnitude of magnetization \( \mathbf{M} \), the magnetic field and the electric current density, \( \rho_i \) is the longitudinal resistivity when \( \mathbf{M} \) and \( \mathbf{J} \) are perpendicular to each other. \( \Delta \rho = \rho_i - \rho_l \) is the difference between \( \rho_l \) and the longitudinal resistivity \( \rho_i \) when \( \mathbf{M} \) is parallel to \( \mathbf{J} \), and the \( \Delta \rho \)-term is called AMR. \( R_0 \) and \( R_i \) describe respectively the ordinary and anomalous Hall effects.

Equation (1) is in fact the most general linear response of the electric field of a polycrystalline magnet to an applied electric current density. It can be understood by the following reasoning. For simplicity, let us assume that there is no external field \( (\mathbf{H} = 0) \). For the linear response of the electric field \( \mathbf{E} \) to an electric current density \( \mathbf{J} \), the most general expression of \( \mathbf{E} \) must be \( \mathbf{E} = \tilde{\rho}(\mathbf{M})\mathbf{J} \), where \( \tilde{\rho}(\mathbf{M}) \) is a rank-2 Cartesian tensor that depends on \( \mathbf{M} \), since \( \mathbf{M} \) is the only available vector. It is well known that a Cartesian tensor of rank 2 can be decomposed into the direct sum of a scalar of function of \( \mathbf{M} \)—a vector that is a function of \( \mathbf{M} \) multiplying \( \mathbf{M} \), and a traceless symmetric tensor of a function of \( \mathbf{M} \) multiplying \( \mathbf{M} - \mathbf{M}^2/3 \). Thus, the most general expression of \( \mathbf{E} \) is

\[
E = (\rho_i + \Delta \rho/3)\mathbf{J} + R_0 \mathbf{M} \times \mathbf{J} + (\Delta \rho/2\mathbf{M}(\mathbf{M} - \mathbf{M}^2/3)) \times \mathbf{J}.
\]

This is exactly equation (1), and no new physics is obtained from this reasoning. However, things are very different if we carry out a similar analysis for a polycrystalline magnetic film lying in the \( xy \)-plane. Although the polycrystalline sample is isotropic in the film plane, \( z \) is an available vector and \( \tilde{\rho}(\mathbf{M}, z) \) should be a function of both \( \mathbf{M} \) and \( z \). Since three vectors \( \mathbf{M}, z, \mathbf{M} \times z \) and three traceless symmetric tensors \( (\mathbf{M} \times \mathbf{M} - \mathbf{M}^2, \mathbf{M} \times \mathbf{M} - \mathbf{M} \times \mathbf{M} \times \mathbf{M}, \mathbf{M} \times \mathbf{M} - \mathbf{M} \times \mathbf{M} \times \mathbf{M}) \) can be constructed out of \( \mathbf{M} \) and \( z \), we have, with similar reasoning to what we used earlier for a polycrystalline magnetic bulk,

\[
E = (\rho_i + \Delta \rho/3)\mathbf{J} + (R_1(\mathbf{M} + \rho_i z) + R_2(\mathbf{M} \times z) \times \mathbf{J} + [(\Delta \rho/2\mathbf{M}(\mathbf{M} - \mathbf{M}^2/3)) M(\mathbf{M} - \mathbf{M}^2/3)) \times \mathbf{J} + R_1(\mathbf{M} \times \mathbf{M} - \mathbf{M}^2) \times \mathbf{M} \times \mathbf{J} + R_2(\mathbf{M} \times \mathbf{M} - \mathbf{M} \times \mathbf{M} \times \mathbf{M}) \times \mathbf{M} \times \mathbf{J} + R_0(\mathbf{M} \times \mathbf{M} - \mathbf{M} \times \mathbf{M} \times \mathbf{M}) \times \mathbf{M} \times \mathbf{J})- R_2(\mathbf{M} \times \mathbf{M} - \mathbf{M} \times \mathbf{M} \times \mathbf{M}) \times \mathbf{M} \times \mathbf{J},
\]

where \( R_1 \equiv R_{11}, R_2 \equiv R_{12} \), and \( R_{12} \equiv (2/3)R_{11}\). The \( \rho_i \)-term may be interpreted as the spin-Hall term if the sign of \( \rho_i \) for spin-up electrons is opposite to that for spin-down electrons. It is known that spin-orbit interactions can lead to a term like this. Obviously, \( \rho_i \) contributes to the longitudinal resistivity along the \( z \)-direction. Interestingly, three new terms are obtained: (1) A current perpendicular to the film induces an electric field in the \( \mathbf{M} \) direction (the \( R_2 \)-term). (2) A current along the \( \mathbf{M} \) direction generates an electric field in the \( z \) direction (the \( R_1 \)-term). (3) The \( R_3 \)-term says that the resistance of the film depends on \( M_z \) linearly. Of course, the coefficients of these terms depend, in principle, on \( M_z \). It should be pointed out that the interfacial effects are very common in physics [39]. Generally speaking, all new terms should exist in polycrystalline magnetic films. Of course, their values depend on microscopic interactions that lead to these terms. These terms are not the subjects of this study, although they do deserve a careful and in-depth investigation [40]. We shall only concentrate on AMR and AHE in equation (1).

Let us come back to the dc voltage in the electrical detection of FMR, in which the electric current density \( \mathbf{J} = \text{Re}(\text{je}^{-i\omega t + \phi_1}) \) comes from the rf electric field only, where \( \omega \) and \( \phi_1 \) are respectively the microwave frequency and the phase lag of the electric current with the electric field. \( \phi_1 \) is a material parameter that depends on the complex electric conductivity. The rf magnetic field \( \text{Re}(\text{he}^{-i\omega t + \phi_2}) \) exerts a torque on \( \mathbf{M} \), resulting in a magnetization precession around a static magnetization \( \mathbf{M}_0 \) that is determined by the external static magnetic field and the magnetic anisotropy. \( \phi_2 \) is the phase difference between the rf electric field and \( \mathbf{h} \) inside the sample, and its value depends on the particular experimental setup and the material parameters like the dielectric constant. For example, \( \phi_2 = \phi_1 \) in those experiments [21, 22] in which the rf magnetic field is generated by the rf electric current. However, in other experiments [19, 23, 25] with microwave cavities, \( \phi_2 \) differs from \( \phi_1 \). Since both \( \phi_1 \) and \( \phi_2 \) are material parameters and depend on the experimental setup, we will treat them as input parameters.

The magnetization \( \mathbf{M} = \mathbf{M}_0 + \text{Re}(\text{me}^{-i\omega t + \phi_2}) \) contains a small component precessing at frequency \( \omega \). Phase \( \phi_2 \) is added in our definition of \( \mathbf{m} \) for convenience. Using \((\ldots)\) to denote the time average, the dc voltage is given by

\[
U = \langle \mathbf{E} \rangle \cdot \mathbf{I},
\]

(2)

where \( \mathbf{I} \) is the displacement vector between two electrode contact points on the sample surface. Since \( \langle \text{Re}(\text{me}^{-i\omega t + \phi_2}) \rangle = 0 \) and \( \langle \mathbf{J} \rangle = 0 \), the dc voltage comes from the terms in equation (1) that contain \( \langle \text{Re}(\text{je}^{-i\omega t + \phi_1}) \cdot \text{Re}(\text{me}^{-i\omega t + \phi_2}) \rangle \) and \( \langle \text{Re}(\text{je}^{-i\omega t + \phi_1}) \times \text{Re}(\text{me}^{-i\omega t + \phi_2}) \rangle \), resulting in the spin rectification. \( \langle \mathbf{E} \rangle \) reads as

\[
\langle \mathbf{E} \rangle = \frac{\Delta \rho}{2M^2} \text{Re}(\mathbf{j}^* \cdot \mathbf{m}(\phi_1 - \phi_2)\mathbf{M}_0 + (\mathbf{j}^* \cdot \mathbf{M}_0)\mathbf{me}^{i(\phi_2 - \phi_1)})
\]

\[
- \frac{R_0}{2} \text{Re}(\mathbf{j}^* \cdot \mathbf{h}e^{i\phi_1}) - \frac{R_0}{2} \text{Re}(\mathbf{j}^* \cdot \mathbf{m}e^{i(\phi_2 - \phi_1)}),
\]

(3)

where the first and the second terms on the right-hand side come from the AMR and the last two terms arise respectively from the ordinary and anomalous Hall effects. It is convenient to define a Cartesian coordinate system \((x, y, z)\), where the \( z \)-axis is along \( \mathbf{M}_0 \). The ordinary Hall effect can be neglected since it is much smaller than the AMR and AHE in a ferromagnetic metal [17, 41]. By substituting equation (3) into equation (2), the AMR and AHE contributions to the dc voltage are
\[ U_{\text{AMR}} = \frac{\Delta \mu}{2M} \text{Re}[\{\mathbf{j} \cdot \mathbf{m}\}] + f_c'(\mathbf{m} \cdot \mathbf{1}) e^{i(\phi_1 - \phi_2)}, \] (4)

\[ U_{\text{ARE}} = -\frac{R_i}{2} \text{Re}(\mathbf{j} \times \mathbf{m}) \cdot e^{i(\phi_1 - \phi_2)}. \] (5)

Once the phase difference \( \phi_1 - \phi_2 \) between \( j \) and \( h \) is given by the experimental setup and material parameters, the dc voltage depends on how \( m \) responds to \( h \), or the dynamic magnetic susceptibility near FMR.

2.2. The universal form of the dynamic magnetic susceptibility matrix

It is important to obtain the dynamic magnetic susceptibility \( \chi \), because the dc voltage in the electrical detection of FMR depends on \( \chi \). Under microwave radiation, magnetization precession is governed by the Landau–Lifshitz–Gilbert (LLG) equation [42]:

\[ \frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t}, \] (6)

where \( \mathbf{H}_{\text{eff}} \) is the effective field, which includes the applied static magnetic field \( \mathbf{H} \), the rf magnetic field \( \text{Re}(\mathbf{h} e^{-i(\psi + \phi_2)}) \) and the anisotropy field; \( \alpha \) is the Gilbert damping coefficient. This is a nonlinear equation that has a lot of interesting physics—including spin wave emission by the magnetic domain wall motion [43]—and can only be solved for a few special cases [44]. However, if the amplitude of \( h \) is small enough, the linear response of \( m \) to \( h \) can be obtained from the linearized LLG equation and takes the following form,

\[
\begin{pmatrix}
\begin{pmatrix}
\chi_{xx} & \chi_{xy} & 0 \\
\chi_{yx} & \chi_{yy} & 0 \\
0 & 0 & 0
\end{pmatrix}
\end{pmatrix}
\begin{pmatrix}
h_x \\
h_y \\
h_z
\end{pmatrix} = \begin{pmatrix}
\mathbf{m}_x \\
\mathbf{m}_y \\
\mathbf{m}_z
\end{pmatrix}
\]

(7)

The structure of equation (7) comes from the fact that \( m \) is perpendicular to \( M_0 (m_z = 0) \), and \( h \) does not exert any torque on \( M \), so that \( m \) does not depend on \( h \).

The system energy change rate can be written as

\[ \frac{dc}{dt} = \nabla M \varepsilon \cdot \frac{\partial \mathbf{M}}{\partial t} - \mu_0 M \cdot \frac{\partial \mathbf{H}_{\text{eff}}}{\partial t}. \] (8)

where \( \varepsilon \) is the energy density of the system, and \( \mathbf{H}_{\text{eff}} \equiv \mathbf{H} + \text{Re}(\mathbf{h} e^{-i(\psi + \phi_2)}) \) is the total applied field. Equation (6) can also be cast as

\[ \frac{\partial \mathbf{M}}{\partial t} = -\frac{1}{1 + \alpha^2} \mathbf{M} \times \mathbf{H}_{\text{eff}} - \frac{\alpha \gamma}{(1 + \alpha^2)M} \mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{\text{eff}}). \] (9)

Substituting \( \nabla M \varepsilon = -\mu_0 \mathbf{H}_{\text{eff}} \) and equation (9) into equation (8), one obtains

\[ \frac{dc}{dt} = -\frac{\alpha \gamma \mu_0}{(1 + \alpha^2)M} |\mathbf{M} \times \mathbf{H}_{\text{eff}}|^2 - \mu_0 M \cdot \mathbf{H}_c. \] (10)

The first term on the right-hand side of equation (10) is the energy dissipation due to the motion of magnetization, while the second term describes the energy release/absorption rate from the microwave. Since the total system energy cannot increase or decrease indefinitely, the time average of \( \varepsilon \) should be zero, \( \langle \varepsilon \rangle = 0 \), so that the average microwave absorption rate \( I \) (by the sample) should be equal to the energy dissipation rate [45]. \( I \) reads

\[ I = -\mu_0 (\mathbf{M} \cdot \mathbf{H}_c) = \frac{\mu_0 \alpha \gamma}{2} \text{Im}(\mathbf{m} \cdot \mathbf{h}), \] (11)

and by substituting equation (7) into equation (11), \( I \), in terms of \( \chi \), becomes

\[ I = \frac{\mu_0 \alpha \gamma}{2} \{h_x h_y \chi''_{xx} + \text{Re}(h_x h_y)(\chi''_{xy} + \chi''_{yx}) + \text{Im}(h_x h_y)(\chi''_{xy} - \chi''_{yx}) + h_x h_y \chi''_{yy} \}, \] (12)

where \( \chi''_{jk} \) is given by

\[ \chi''_{jk} = \frac{1}{\pi} \left( \frac{\omega}{\omega - \omega_0} + \frac{1}{\omega + \omega_0} \right). \] (13)

The counterpart of \( L(\omega, \omega_0, \Gamma) \) in the Kramers–Kronig relations is the function \( D(\omega, \omega_0, \Gamma) \),

\[ D(\omega, \omega_0, \Gamma) = \frac{1}{\pi} \frac{\omega - \omega_0}{(\omega - \omega_0)^2 + (\frac{\Gamma}{2})^2}. \] (14)

where \( \omega_0 \) denotes the resonance frequency and \( \Gamma \) is the linewidth, which is a positive number. \( D \) can be both positive and negative, and is more extended than \( L(\omega, \omega_0, \Gamma) \). It should be pointed out that for inhomogeneous magnetic polycrystals, the inhomogeneity leads to a sum of many Lorentzian energy absorption curves at a different peak position, and the resulting absorption curve can be very different from a Lorentzian function [8, 9]. Thus, the result presented here is only valid for homogeneous magnetic material, such as single crystals or polycrystals, whose inhomogeneity broadening of the FMR peak is much smaller than that due to the Gilbert damping. Since the energy absorption lineshape should be Lorentzian for an arbitrary \( h \), each term on the right-hand side of equation (12) should have the form of equation (13) with the same resonance frequency \( \omega_0 \) and linewidth \( \Gamma \).
where the four real numbers $C_i$ ($i = 1, 2, 3, 4$) are determined by the material parameters and experimental setup. A factor of $\pi \Gamma/2$ is introduced in order to make $C_i$ ($i = 1, 2, 3, 4$) dimensionless. By using the Kramers–Kronig relations, one can write

\[
\begin{align*}
\chi''_x(\omega) &= -C_1 \frac{\pi \Gamma}{2} D(\omega, \omega_0, \Gamma), \\
\chi''_y(\omega) + \chi''_z(\omega) &= -2C_2 \frac{\pi \Gamma}{2} D(\omega, \omega_0, \Gamma), \\
\chi''_x(\omega) - \chi''_y(\omega) &= 2C_3 \frac{\pi \Gamma}{2} D(\omega, \omega_0, \Gamma), \\
\chi''_y(\omega) &= -C_4 \frac{\pi \Gamma}{2} D(\omega, \omega_0, \Gamma),
\end{align*}
\]

(15)

From equations (15) and (16), $\tilde{\chi}(\omega)$ should have the general form of

\[
\tilde{\chi}(\omega) = \frac{\pi \Gamma}{2} \left[ L(\omega, \omega_0, \Gamma) + iD(\omega, \omega_0, \Gamma) \right] \tilde{C},
\]

(17)

where

\[
\tilde{C} = \begin{pmatrix}
iC_1 & C_3 + iC_4 & 0 \\
-C_3 + iC_2 & iC_4 & 0 \\
0 & 0 & 0 
\end{pmatrix}.
\]

(18)

Thus, $\tilde{\chi}(\omega)$ of an arbitrary magnet is completely determined by six numbers: $\omega_0, \Gamma, C_1, C_2, C_3$ and $C_4$. Interestingly, the two off-diagonal matrix elements of $\tilde{\chi}(\omega)$ are not opposite to each other in general, because $C_2$ is only zero when the material has a certain special symmetry or for a special coordinate (see the discussion in section 2.3).

In terms of phase $\psi \equiv \cot^{-1}[−D/L2(DL)] = \cot^{-1}[\omega_0 − \omega]/\Gamma$, $m$ relates to $\mathbf{h}$ as

\[
m = \begin{pmatrix}
|x_{xx}| & |x_{xy}| e^{-i\cot^{-1}\frac{C_3}{C_1}} & 0 \\
|x_{xy}| e^{i\cot^{-1}\frac{C_3}{C_1}} & |x_{yy}| & 0 \\
0 & 0 & 0
\end{pmatrix} \text{e}^{i\omega t}.
\]

(19)

When the microwave frequency $\omega$ is swept through the FMR region at a fixed static magnetic field, the phase $\psi$ changes from $0^\circ$ to $180^\circ$ and is $90^\circ$ at FMR. This property of $\psi$ is universal for an arbitrary system.

### 2.2.2. The field dependence of $\tilde{\chi}$

In most experiments, the microwave frequency was fixed and the applied static magnetic field was swept. Thus, it is also useful to obtain the field dependence of $\tilde{\chi}$. One cannot directly apply the causality principle here, and we shall use equation (17). Notice that each of $\omega_0, \Gamma$ and $\tilde{C}$ is a function of $H$, and $\tilde{\chi}$ can be expressed as

\[
\tilde{\chi}(H) = \frac{\pi \Gamma(H)}{2} \left[ L(\omega, \omega_0(H), \Gamma(H)) + iD(\omega, \omega_0(H), \Gamma(H)) \right] \tilde{C}(H).
\]

(20)

Unlike the frequency sweep, where $\mathbf{M}_0$ is always in the same direction, $\mathbf{M}_0$ changes its direction during a field sweep (in which the direction of $\mathbf{H}$ does not change). However, one does not expect $\omega_0$, $\Gamma$ or $\tilde{C}$ to change much if the FMR peak width is narrow. Thus, one can replace $\mathbf{M}_0$, $\Gamma$ and $\tilde{C}$ by their values at the resonance field $H_0$ where $\omega_0(H_0) = \omega$,

\[
\begin{align*}
\mathbf{M}_0(H) &\approx \mathbf{M}_0(H_0), \\
\Gamma(H) &\approx \Gamma(H_0), \\
\tilde{C}(H) &\approx \tilde{C}(H_0).
\end{align*}
\]

(21)

Similarly, one can expand $\omega_0(H)$ around $H_0$ to the linear order in $H - H_0$,

\[
\omega_0(H) \approx \omega + \beta (H - H_0).
\]

(22)

Substituting equations (21) and (22) into equation (20), one has

\[
\tilde{\chi}(H) = \frac{\pi \Gamma_1}{2} \left[ L(H, H_0, \Gamma_1) + i\frac{-\beta}{|\beta|} D(H, H_0, \Gamma_1) \right] \tilde{C}(H_0),
\]

(23)

where $\Gamma_1$ is the linewidth of the field:

\[
\Gamma_1 = \frac{\Gamma(H_0)}{|\beta|}.
\]

(24)

It is not surprising to see that $\Gamma_1$ is large if $\omega_0(H)$ does not change much around $H_0$ (small $\beta$), while it is smaller for a larger $\beta$. Similar to the frequency dependence of $\tilde{\chi}$, the field dependence of $\tilde{\chi}$ around FMR is fully determined by six numbers: $H_0, \Gamma_1, C_1, C_2, C_3$ and $C_4$. However, the sign in front of the $D$ function in equation (23) is decided by the sign of $\beta$ (‘+’ when $\omega_0$ decreases with $H$ and ‘−’ otherwise). In terms of phase $\psi \equiv \cot^{-1}[−D/L2(DL)] = \cot^{-1}[\omega_0 − \omega]/\Gamma$, the $\beta$-sign decides whether $\psi$ changes from $180^\circ$ to $0^\circ$ or from $0^\circ$ to $180^\circ$ when the field is swept up through the resonance region. The sign also relates to the sign of the dc voltage as well as whether the observed signal—like $dI/dH$ in traditional microwave absorption measurement—increases or decreases with $H$. This is in contrast to the frequency dependence of $\tilde{\chi}$, as shown in equation (17), where the sign before the $D$ function is always positive so that the phase $\psi$ always increases from $0^\circ$ (far below the resonance frequency $\omega$) to $180^\circ$ far above the resonance frequency $\omega$. Obviously, $\tilde{\chi}(H)$ is not a Polder tensor in general. In contrast, $\tilde{\chi}(H)$ may deviate from equation (23) when the approximation of equation (21) is not good. As a result, changes to the non-resonance broad peak and symmetry in elements of $\tilde{\chi}(H)$ can occur. Explicit examples will be discussed in section 3.2.

#### 2.3. The Polder or non-Polder tensors

In order to understand why the Polder tensor is always used in the experimental analyses, we want to see which linear response coefficient is a Polder tensor. Assume that $H_K(M)$ is
the anisotropy field; then, to the first order in \( \mathbf{m} \), the effective field is

\[
\mathbf{H}_{\text{eff}} = \mathbf{H} + \mathbf{H}_K(\mathbf{M}) + \nabla M \mathbf{H}_K|_{\mathbf{M}} \mathbf{m} e^{-i(\omega t + \phi)} + \mathbf{h}_0 e^{-i(\omega t + \phi)}.
\]  

(25)

Substituting equation (25) and \( \mathbf{M} = \mathbf{M}_0 + \mathbf{m} e^{-i(\omega t + \phi)} \) into equation (6), one has, up to the linear term in \( \mathbf{m} \),

\[
-i \omega \mathbf{m} = - \gamma \mathbf{m} \times \mathbf{H}_{\text{dc}} - \gamma \mathbf{M}_0 \times \mathbf{h}_\text{ac} - i \omega \frac{\alpha}{M} \mathbf{M}_0 \times \mathbf{m}.
\]  

(26)

where \( \mathbf{H}_{\text{dc}} \) is the effective dc field

\[
\mathbf{H}_{\text{dc}} = \mathbf{H} + \mathbf{H}_K(\mathbf{M}_0)
\]  

(27)

and \( \mathbf{h}_\text{ac} \) is the effective ac field, which is the sum of the applied rf field \( \mathbf{h} \) and the internal rf field due to the precessing magnetization,

\[
\mathbf{h}_\text{ac} = \mathbf{h} + \nabla M \mathbf{H}_K|_{\mathbf{M}_0} \mathbf{m}.
\]  

(28)

\( \mathbf{m} \) in terms of \( \mathbf{h}_\text{ac} \) is, by solving equation (26),

\[
\mathbf{m} = \begin{pmatrix} \chi_L & -i\chi_T \\ i\chi_T & \chi_L \end{pmatrix} \mathbf{h}_\text{ac} = \chi_P \cdot \mathbf{h}_\text{ac},
\]  

(29)

where

\[
\chi_L = \frac{\omega \lambda M}{\omega^2 - \omega_T^2}, \quad \chi_T = \frac{\omega \lambda M}{\omega^2 - \omega_L^2}
\]  

(30)

with \( \omega_M = \gamma M \) and \( \omega_T = \gamma H_{\text{dc}} - i \omega \). \( \chi_P \) is the so-called Polder tensor \([2, 5]\) which is often assumed to be the dynamic magnetic susceptibility matrix. The Polder tensor \( \chi_P \) is the linear response coefficient of the ac field, \( \mathbf{m} = \chi_P \mathbf{h}_\text{ac} \). Different from \( \chi \), which is the linear response coefficient of \( \mathbf{m} \) to the applied rf field \( \mathbf{m} = \chi \mathbf{h} \) and measurable, \( \chi_P \) is not measurable, because \( \mathbf{h}_\text{ac} \) contains both the applied rf field \( \mathbf{h} \) and an oscillating anisotropy field due to the precessing magnetization. Obviously \( \mathbf{h}_\text{ac} \) cannot be measured directly. If one defines matrix \( \tilde{\mathbf{W}} \) as \( \tilde{\mathbf{W}} = \begin{pmatrix} \chi_P & \frac{\gamma}{\omega} \chi_T \end{pmatrix} \), then \( \tilde{\chi} \) relates to \( \chi_P \) as \( \tilde{\chi} = \tilde{\mathbf{W}} \cdot \chi_P \). \( \tilde{\chi} \) is simple and can be fully determined by a few constants, even though it is not a true measurable quantity, while \( \chi \) is much more complicated and is sensitive to the magnetic anisotropy. This may be the reason why \( \chi_P \) is so often taken as the dynamic magnetic susceptibility matrix, even though the real one is \( \tilde{\chi} \).

It would be interesting to know when \( \tilde{\chi} \) is a Polder tensor, i.e. \( \chi_{xy} = - \chi_{yx} \) and \( \chi_{xx} = \chi_{yy} \) or \( C_2 = 0 \) and \( C_1 = C_4 \), according to equation (18). Let us first understand the physical meanings of \( C_1 \sim C_4 \). Consider six different \( \mathbf{h} \) of amplitude \( h \) shown in figure 1: along the x-direction \( \mathbf{h} = h \hat{x} \) (a); the y-direction \( \mathbf{h} = h \hat{y} \) (b); the direction \( \eta \) angle with the x-axis \( \mathbf{h} = h(\cos \eta \hat{x} + \sin \eta \hat{y}) \) (c); the direction \( \pi - \eta \) angle with the x-axis \( \mathbf{h} = h(\cos \eta \hat{x} - \sin \eta \hat{y}) \) (d); the right-hand circularly polarized field \( \mathbf{h} = \frac{h}{\sqrt{2}}(\hat{x} + i \hat{y}) \) (e); and the left-hand circularly polarized field \( \mathbf{h} = \frac{h}{\sqrt{2}}(\hat{x} - i \hat{y}) \) (f). If one denotes the microwave absorption intensities around FMR in figures 1(a)–(f) by \( I_\alpha \sim I_f \), from equations (12), (15) and (16), one has \( I_\alpha = \frac{0.5 \mu_0 \mu_0 \omega C_1 L(\omega, \omega_0, \Gamma)}{C_3} \) and \( I_f = \frac{0.5 \mu_0 \mu_0 \omega C_1 L(\omega, \omega_0, \Gamma)}{C_3} \), proportional to \( C_1 \) and \( C_4 \) respectively. Similarly, \( I_\alpha - I_f = \frac{0.5 \mu_0 \mu_0 \omega C_1 L(\omega, \omega_0, \Gamma)}{C_3} \) is proportional to \( C_2 \) and \( I_f - I_f = \frac{0.5 \mu_0 \mu_0 \omega C_1 L(\omega, \omega_0, \Gamma)}{C_3} \) measures \( C_3 \). Obviously, \( C_1 \) and \( C_4 \) are always positive whereas \( C_2 \) and \( C_3 \) can be positive, negative or zero, depending on the experimental setup and the choice of coordinate. Thus, one has \( \chi_{xy} = - \chi_{yx} \) if \( C_2 \) is zero (or \( I_\alpha = I_f \)), and vice versa.

The condition for \( \chi_{xy} = - \chi_{yx} \) can be obtained from the exact solution of \( \tilde{\chi} \) for an arbitrary energy landscape \( \varepsilon(\mathbf{M}) \). By solving equation (26), \( \tilde{\chi} \) is

\[
\tilde{\chi} = \frac{1}{|T|^2 - P Q} \begin{pmatrix} P & T & 0 \\ T^* & Q & 0 \\ 0 & 0 & 0 \end{pmatrix}.
\]  

(31)

with

\[
P = - \frac{1}{\mu_0} \frac{\partial^2 \varepsilon}{\partial M^2 \partial M^2} |_{\mathbf{M}_0} - \frac{\mathbf{H}_{\text{dc}}}{M} + i \frac{\omega}{\gamma M},
\]  

\[
Q = - \frac{1}{\mu_0} \frac{\partial^2 \varepsilon}{\partial M^2 \partial M^2} |_{\mathbf{M}_0} - \frac{\mathbf{H}_{\text{dc}}}{M} + i \frac{\omega}{\gamma M},
\]  

\[
T = \frac{1}{\mu_0} \frac{\partial^2 \varepsilon}{M M M M} |_{\mathbf{M}_0} + i \frac{\omega}{\gamma M}.
\]  

(32)

Figure 1. The different polarizations of the applied microwave magnetic field \( \mathbf{h} \). \( \mathbf{h} \) is linearly polarized in (a)–(d) and circularly polarized in (e) and (f). The z-axis is along \( \mathbf{M}_0 \) and \( 0 < \eta < \pi/2 \) (in (c) and (d)).

- (a) Positive \( \mathbf{h} \); \( \mathbf{h} = h \hat{x} \) (b) Negative \( \mathbf{h} \); \( \mathbf{h} = h \hat{y} \) (c) Positive \( \mathbf{h} \); \( \mathbf{h} = h(\cos \eta \hat{x} + \sin \eta \hat{y}) \) (e) Negative \( \mathbf{h} \); \( \mathbf{h} = h(\cos \eta \hat{x} - \sin \eta \hat{y}) \) (d) Positive \( \mathbf{h} \); \( \mathbf{h} = h(\cos \eta \hat{x} + \sin \eta \hat{y}) \) (f) Negative \( \mathbf{h} \); \( \mathbf{h} = h(\cos \eta \hat{x} - \sin \eta \hat{y}) \)
Thus, one can have \( \chi_{xy} = -\chi_{yx} \) only when
\[
\frac{\partial^2 \varepsilon}{\partial M_x \partial M_y} \bigg|_{M_0} = 0. \tag{33}
\]

This could happen in several cases. If the system has rotational symmetry about \( M_0 \) so that the energy absorption rate of a linearly polarized microwave does not depend on the direction of \( \mathbf{h} \) in the xy-plane, then one has \( C_2 \propto I_c = I_0 = 0 \) and \( C_1/C_4 = I_c/|\mathbf{h}_0| = 1 \). Thus, \( \chi_{xy} = -\chi_{yx} \) and \( \chi_{xx} = \chi_{yy} \), and \( \chi \) is a Polder tensor. An obvious example for this case is an isotropic system without magnetic anisotropy, so that \( M_0 \) is along \( \mathbf{H} \). \( \chi_{xy} = -\chi_{yx} \) if the free energy density is symmetric about the yz- or xz-plane, i.e. \( \varepsilon(M_x, M_y, M_z) = \varepsilon(-M_x, M_y, M_z) \) or \( \varepsilon(M_x, M_y, M_z) = \varepsilon(-M_x, -M_y, -M_z) \). Taking the derivatives with respect to \( M_x \) and \( M_y \), on both sides, one has
\[
\frac{\partial^2 \varepsilon}{\partial M_x \partial M_y} \bigg|_{M_0} = -\frac{\partial^2 \varepsilon}{\partial M_y \partial M_x} \bigg|_{M_0} = 0.
\]

For an arbitrary magnetic material, it is also possible to make \( \chi_{xy} = -\chi_{yx} \) by properly choosing the \( x \) and \( y \) axes. To prove this statement, let \( \chi \) be characterized by \( C_1 \sim C_4 \), be a non-Polder tensor in the original \( xy \)-coordinate. One can choose a new \( x'y' \)-coordinate that relates to the \( xy \)-coordinate by the rotation matrix (around the \( z \)-axis)
\[
\mathbf{R} = \begin{pmatrix} \cos \delta & -\sin \delta & 0 \\ \sin \delta & \cos \delta & 0 \\ 0 & 0 & 1 \end{pmatrix}.
\]

In this new coordinate, the susceptibility matrix is \( \mathbf{R} \chi \mathbf{R}^T \) characterized by new \( C_1' \sim C_4' \). \( C_1' \sim C_4' \) relate to \( C_1 \sim C_4 \) by
\[
\begin{align*}
C_1' &= C_1 \cos^2 \delta - C_2 \sin 2\delta + C_4 \sin^2 \delta, \\
C_2' &= \frac{C_1 - C_2}{2} \sin 2\delta + C_2 \cos 2\delta, \\
C_3' &= C_3, \\
C_4' &= C_4 \sin^2 \delta + C_2 \sin 2\delta + C_4 \cos^2 \delta. 
\end{align*}
\]

If one chooses \( \delta = 0.5 \arctan \frac{2C_4}{C_1 - C_4} \), \( C_1' = 0 \) so that the off-diagonal elements are opposite to each other in the new coordinate. Interestingly, \( C_3 \) does not change while \( C_1, C_2 \) and \( C_4 \) depend on the coordinate change. This is not surprising, because \( C_3 \) relates to the microwave absorption rate difference between the right-hand and left-hand circularly polarized rf fields which rotate around the \( z \)-axis, so that the choice of \( x \)- and \( y \)-axes does not matter to the energy absorption, or to \( C_3 \).

Naïvely, one may not have expected the traditional microwave absorption experiments to have been able to measure a complex number, such as a susceptibility matrix element, because only Lorentzian lineshapes can be measured. However, the above discussions clearly show that the traditional microwave absorption experiments are capable of completely determining the dynamic magnetic susceptibility matrix.

2.4. The determination of the matrix \( \hat{\chi} \)

One of the achievements in section 2.2 is that the dynamic magnetic susceptibility can be fully characterized by a few parameters. This is important because \( \hat{\chi} \) expressed in terms of magnetic anisotropy, like equations (31) and (32), is not that useful, since the exact form of the magnetic anisotropy is hard to obtain in reality, if not impossible. A natural question is how to experimentally determine these parameters. From the early discussion of the meaning of \( C_1 \sim C_4 \), a recipe, which is based on traditional FMR measurements and allows the full determination of these parameters can be obtained. The idea is to apply different \( \mathbf{h} \), as shown in figure 1 so that one can associate \( C_1 \sim C_4, \gamma_0 \omega_0 \) (or \( H_0 \) and \( \Gamma \) (or \( \Gamma_l \)) with the peak-height, peak-position, and peakwidth of the microwave absorption spectrum. Our recipe is:

\textit{Step I}  Use the usual magnetic measurement to locate the direction of \( M_0 \) along which the \( z \)-axis is chosen, and a \( xyz \)-Cartesian coordinate can be assigned.

\textit{Step II}  Conduct the usual FMR microwave absorption experiment by using a microwave whose rf magnetic field \( \mathbf{h} \) is along the \( x \)-axis. Either frequency \( \omega \) or field \( H \) is swept, so that the resonance \( \omega_0 \) (or \( H_0 \)), together with the linewidth \( \Gamma \) (or \( \Gamma_l \)) can be obtained from the microwave absorption curve. As discussed earlier (figure 1), \( C_1 \) is given by the peak-height \( I_0 \) of the microwave absorption curve as \( C_1 = 2I_0/(\mu_0 H_0^2) \).

\textit{Step III}  Repeat step II but use \( \mathbf{h} = h\hat{y} \), \( C_4 \) is given by the peak-height \( I_0 \) as \( C_4 = 2I_0/(\mu_0 H_0^2) \).

\textit{Step IV}  Repeat step II but use \( \mathbf{h} = h(\cos \eta \hat{x} + \sin \eta \hat{y}) \) first to obtain the peak-height \( I_{yx} \) of the microwave absorption curve, then use \( \mathbf{h} = h(\cos \eta \hat{x} - \sin \eta \hat{y}) \) to obtain the peak-height \( I_{xy} \). \( C_2 \) is then given by \( C_2 = (I_x - I_y)/(\mu_0 H_0^2 \sin 2\eta) \).

\textit{Step V}  Repeat step II but use \( \mathbf{h} = h(\cos \eta \hat{x} - i \sin \eta \hat{y}) \) first to obtain the peak-height \( I_{xy} \) of the microwave absorption curve, then use \( \mathbf{h} = h(\cos \eta \hat{x} + i \sin \eta \hat{y}) \) to obtain the peak-height \( I_{yx} \). \( C_3 \) is then given by \( C_3 = (I_x - I_y)/(2\mu_0 H_0^2) \).

According to equation (17), \( \hat{\chi}(\omega) \) is fully determined from the above steps. For \( \hat{\chi}(H) \), one needs to determine whether \( \beta|\beta| = -1 \) or \( \beta|\beta| = 1 \), corresponding to the increase or decrease of \( \omega_0 \) with \( H \). Step V involves circularly polarized microwaves that may be difficult to obtain in experiments. However, one should notice that the recipe is not unique. One may use other easily obtained microwaves to determine \( C_3 \).

2.5. The lineshape of dc voltage

With the general form of the dynamic magnetic susceptibility matrix \( \hat{\chi} \), we can compute the AMR and AHE contributions to the dc voltage and study the symmetry of the dc voltage lineshapes. Substituting equation (7) into equations (4) and (5), \( U_{\text{AMR}} \) and \( U_{\text{AHE}} \) in terms of \( \hat{\chi} \) are
\[
U_{\text{AMR}} = -\frac{\Delta \rho}{2M} \text{Re}[(j_i^{xy} h_i^{xy} \chi_{xy} + j_i^{yx} h_i^{yx}) e^{i(\varphi_i - \varphi_j)}], \tag{36}
\]
\[
U_{\text{AHE}} = R_i \frac{\text{Re}(\epsilon_{ijk} j_i^{xy} h_i^{yx}) e^{i(\varphi_i - \varphi_j)}}{2}, \tag{37}
\]
where the subscript indices \( i, j, k \) and \( l \) can be \( x, y \) and \( z \). Throughout this paper, \( \epsilon_{ijk} \) is the Levi–Civita symbol, and the Einstein summation convention is used. Whether a matrix
element of $\chi$ is involved in dc voltage depends on the applied microwave fields and experimental setup. Substituting equations (17) and (18) into equations (36) and (37), the frequency dependence of dc voltage can be expressed in terms of Lorentzian and $D$ functions,

\[
U_{\text{AMR}}(\omega) = A_1 \frac{\pi \Gamma}{2} L(\omega, \omega_0, \Gamma) + A_2 \frac{\pi \Gamma}{2} D(\omega, \omega_0, \Gamma),
\]

\[
U_{\text{AHF}}(\omega) = A_3 \frac{\pi \Gamma}{2} L(\omega, \omega_0, \Gamma) + A_4 \frac{\pi \Gamma}{2} D(\omega, \omega_0, \Gamma),
\]

where the relative intensities $A_i \sim A_4$ are

\[
A_1 = -\frac{\Delta \rho}{2M} \text{Re}[\langle f^j h^l j, C_{ij} + f^j g^l j, C_{ij} \rangle e^{i(\phi_j - \phi_l)}],
\]

\[
A_2 = \frac{\Delta \rho}{2M} \text{Im}[\langle f^j h^l j, C_{ij} + f^j g^l j, C_{ij} \rangle e^{i(\phi_j - \phi_l)}],
\]

\[
A_3 = \frac{R}{2} \text{Re}(\langle \epsilon_{jk} f^j h^l j, C_{ij} e^{i(\phi_j - \phi_l)} \rangle),
\]

\[
A_4 = -\frac{R}{2} \text{Im}(\langle \epsilon_{jk} f^j h^l j, C_{ij} e^{i(\phi_j - \phi_l)} \rangle),
\]

where the subscript indices $i, j, k$ and $l$ are $x, y$ and $z$. $C_{ij}$ is the element of the $i$th row and the $j$th column of matrix $\tilde{C}$ defined in equation (18). The way to find the field dependence of dc voltage is similar to that for $\chi_{\text{eff}}(\omega)$, $A_1 \sim A_4$ are functions of $H$, therefore, from equations (38) and (39), one has

\[
U_{\text{AMR}}(H) = A_1(H) \frac{\pi \Gamma(H)}{2} L(\omega, \omega_0(H), \Gamma(H)) + A_2(H) \frac{\pi \Gamma(H)}{2} D(\omega, \omega_0(H), \Gamma(H)),
\]

\[
U_{\text{AHF}}(H) = A_3(H) \frac{\pi \Gamma(H)}{2} L(\omega, \omega_0(H), \Gamma(H)) + A_4(H) \frac{\pi \Gamma(H)}{2} D(\omega, \omega_0(H), \Gamma(H)).
\]

The above expressions do not mean that $U_{\text{AMR}}(H)$ and $U_{\text{AHF}}(H)$ are linear combinations of Lorentzian and $D$ functions because the $A_i$ ($i = 1, \ldots, 4$) are also functions of $H$. However, if the approximations of equation (21) are valid, i.e.

\[
A_i(H) \approx A_i(H_0) \quad (i = 1, 2, 3, 4),
\]

then $U_{\text{AMR}}(H)$ and $U_{\text{AHF}}(H)$ are linear combinations of the Lorentzian and $D$ functions,

\[
U_{\text{AMR}}(H) = A_i(H_0) \frac{\pi \Gamma}{2} L(H, H_0, \Gamma)
\]

\[
- \frac{\beta}{|\beta|} A_2(H_0) \frac{\pi \Gamma}{2} D(H, H_0, \Gamma),
\]

\[
U_{\text{AHF}}(H) = A_3(H_0) \frac{\pi \Gamma}{2} L(H, H_0, \Gamma)
\]

\[
- \frac{\beta}{|\beta|} A_4(H_0) \frac{\pi \Gamma}{2} D(H, H_0, \Gamma).
\]

The symmetric dc voltage lineshapes are from the Lorentzian terms, and the antisymmetric components are determined by the $D$ terms. $A_1 \sim A_4$ are linear combinations of $C_1 \sim C_4$, whose coefficients depend on magnetic anisotropy and the experimental setup, and their values determine the relative weights of the symmetric and antisymmetric components.

According to equation (40), the phase difference $\phi_1 - \phi_2$ between the rf current density and rf magnetic field is important for the dc voltage. Several groups have found experimental evidence that the phase difference relates to many factors like experimental setup, material parameters, microwave frequency, etc [23, 27]. Therefore, $\phi_1$ and $\phi_2$ are taken as input parameters in the following discussions.

3. Verification and discussion

In this section, we use a biaxial model to verify the universal expressions of the dynamic magnetic susceptibility matrix and dc voltage lineshape obtained in the last section. Both $\chi(\omega)$ and $\chi(H)$ are compared with the exact expression obtained from the LLG equation (6), as well as with numerical simulations of the LLG equation. [47] The universal expressions of dc voltage due to the AMR and AHE effects near FMR are compared with the exact numerical results based on the LLG equation and the generalized Ohm’s law of equation (1).

Our model system, which mimics popular experimental setups, is shown in figure 2. A biaxial magnetic film lies in the $x'y'$-plane with the length $f$ along the $x'$-direction. The $x'$- and $z'$-axes are respectively the easy and hard axes of the film. For simplicity, the rf magnetic field $b$ and the rf electric field (or rf electric current $j$) are along the $x'$- and $y'$-axes, respectively. A moving $x'y'z'$-coordinate is defined as follows. The $z$-axis is along $M_0$ and the $y$-axis is in the $x'y'$-plane. The $\theta$ and $\phi$ are the polar and azimuthal angles of the external static field $H$ in the $x'y'z'$-coordinate. Therefore, once $M_0$ is determined, the $z$-axis is $\hat{z} = \sin \theta \cos \phi \hat{x}' + \sin \theta \sin \phi \hat{y}' + \cos \theta \hat{z}'$, and the $x'$- and $y'$-axes are determined by $\hat{x}' = \cos \theta \hat{x}' + \sin \theta \hat{y}' - \sin \theta \hat{z}'$ and $\hat{y}' = -\sin \phi \hat{x}' + \cos \phi \hat{y}'$. The effective field of the biaxial model is

\[
H_{\text{eff}} = H + K_{12} z \hat{z}' + K_{23} z' \hat{z}' + \text{Re}(he^{-i(\phi_1 + \phi_2)}),
\]

where $K_{12}$ and $K_{23}$ are respectively the dimensionless easy axis anisotropy coefficient and the hard axis anisotropy coefficient. $K_{23}$ is mainly from the shape anisotropy for a soft magnetic film, so we set $K_{23} = 1$ to acknowledge this fact in the study.

According to equation (26), the linearized LLG equation in the present case becomes

\[
-i \omega \mathbf{m} = -\gamma \mathbf{m} \times (H + K_{12} M_0 \hat{x}' - K_{23} M_0 \hat{z}') - \gamma M_0 \times (h + K_{12} M_0 \hat{x}' - K_{23} M_0 \hat{z}') - \frac{\omega}{M} M_0 \times \mathbf{m}
\]

\[
-\gamma M_0 \times (h + K_{12} M_0 \hat{x}' - K_{23} M_0 \hat{z}') - \frac{\omega}{M} M_0 \times \mathbf{m}
\]

\[
\chi_{xy} = T'(l|^2 - P Q), \quad \chi_{xy} = T(l|^2 - P Q), \quad \chi_{yy} = Q(l|^2 - P Q).
\]

For our biaxial model, $P, Q$ and $T$ are
In the first example, we set \( K_1 = 0.0 \) (\( K_2 = 1 \)), \( \theta_H = 50^{\circ} \), \( \phi_H = 90^{\circ} \) and \( \phi_1 - \phi_2 = -90^{\circ} \). \( \mathbf{M}_0 \) is in the \( y^\prime z^\prime \)-plane with the polar angle of \( \theta = 28.7^{\circ} \), which is non-collinear with \( \mathbf{H} \). Obviously, the system is symmetric about the \( y^\prime z^\prime \)-plane so that \( \chi_{xx} = \chi_{yy} \). FMR occurs at \( H = 1052.8 \) mT for \( \omega = 50 \) GHz. The resonance frequency \( \omega_0 \) increases with \( H \). The exact results of both the real and imaginary parts of \( \chi_{xx} \) and \( \chi_{xy} \) are squares in figures 3(a)–(d). Figures 3(a) and (b) are \( \chi \) as functions of \( H \) at a fixed frequency \( \omega = 50 \) GHz, while figures 3(c) and (d) are as functions of \( \omega \) at a fixed field \( H = 1052.8 \) mT. The black curves are the universal expressions of equations (23) (for figures 3(a) and (b)) and (17) (for figures 3(c) and (d)) with fitting parameters of \( \Gamma = 0.87 \) GHz, \( \Gamma_1 = 17.4 \) mT, \( \beta |/|= 1, C_1 = 137.4, C_2 = 0.0, C_3 = 204.6 \) and \( C_4 = 304.8 \). The curves in this example are either Lorentzian or \( D \) functions according to equations (23) and (17). The perfect agreement demonstrates the validity of the universal expressions. This simple example shows that \( \mathbf{M}_0 \) and \( \mathbf{H} \) are not collinear with each other, even in a film made of zero-anisotropy (\( K_1 = 0 \)) magnetic materials, in contrast to the popular assumption that \( \mathbf{M}_0 \) is always along \( \mathbf{H} \) for soft magnetic films such as permalloy and yttrium iron garnet.

The dc voltage is computed from the LLG equation and the generalized Ohm’s law. According to equations (36) and (37), the dc voltage due to the AMR involves only \( \text{Im}(\chi_{xy}) \), which is a Lorentzian function in \( H \), while the dc voltage due to the AHE involves only \( \text{Im}(\chi_{zy}) \), which is a \( D \) function. According to equation (40), the above fitting values of \( C' \) result in \( A_1 = 73.3 \) and \( A_2 = 0.0 \) in units of \( \Delta \rho jhlM \), and \( A_3 = 0.0 \) and \( A_4 = -49.2 \) in units of \( R_jhl \), where \( j \) and \( h \) are respectively the amplitudes of the rf current density and the rf magnetic field. The \( H \)-dependence of the predicted dc voltage due to the AMR and AHE are plotted in figures 4(a) and (b) as black curves that perfectly describe the exact results (squares) from the LLG equation and the generalized Ohm’s law.

To show the importance of a small non-zero magnetic anisotropy, we introduce a small \( K_1 = 0.2 \) to the first example while keeping all other parameters unchanged. Due to the finite
easy axis anisotropy $K_1$, $M_0$ is pushed out of the $y'z'$-plane if $H$ is not excessively large, and clearly non-collinear with $H$. The system is not symmetric about either the $xz$- or $yz$-plane. Different from the first example, $\chi_{xy}$ and $\chi_{yx}$ are not opposite to each other. The exact results of both the real and imaginary parts of $\chi_{xy}$ and $\chi_{yx}$ are not shown around the first peak are asymmetric. The second example is also plotted as the black dashed curve in the figure. The system is not symmetric about either the $xz$- or $yz$-plane.

The direction of $M_0$ varies with $H$ unless $M_0$ is collinear with $H$ at high values of $H$. As a result, the effective field also varies with $H$. The effective field is not a monotonic function of $H$. Thus, $\omega_0(H)$ is not a monotonic function of $H$ either, as shown by the blue dotted curve in figure 5 [3, 4, 14, 26, 32, 33]. At microwave frequency $\omega = 50$ GHz, there are two FMR peaks, or two values of $\omega$ that satisfy $\omega_0(H) = \omega$, as shown by the red solid line of $\omega_0(H) = 50$ GHz in figure 5, which crosses the blue dotted curve at $H = 875.8$ mT and $H = 1317.0$ mT. For a comparison, the monotonic $\omega_0 - H$ behavior of the first example is also plotted as the black dashed curve in the figure. At the first peak ($H_0 = 875.8$ mT), $M_0$ lies with $\theta = 44.0^\circ$ and $\phi = 33.2^\circ$, which is non-collinear with $H$. The second peak occurs at $H_0 = 1317.0$ mT, and $M_0$ lies in the $y'z'$-plane with $\theta = 18.2^\circ$. The blue curves in figures 3(a)–(d) are the universal expressions of equations (23) for (a) and (b), and (17) for (c) and (d). For the curves in figures 3(a) and (b), the fitting parameters around the first (second) peak are $\Gamma_1 = 9.2$ mT, $\beta_1|\beta_1| = -1$, $C_1 = 83.0$, $C_2 = 39.5$, $C_3 = 168.1$ and $C_4 = 359.2$ ($\Gamma_2 = 6.0$ mT, $\beta_2|\beta_2| = 1$, $C_1 = 118.9$, $C_2 = 0.0$, $C_3 = 196.1$ and $C_4 = 323.2$). For the curves in figures 3(c) and (d), the fitting parameters are $\Gamma = 1.06$ GHz and the same $C$-values as those for the first peak in figures 3(a) and (b). Different from the first example, the real and imaginary parts of $\chi_{xy}$ and $\chi_{yx}$ (not shown) around the first peak are asymmetric functions. The perfect agreement verifies the validity of the universal expressions.

The numerical results of the dc voltage due to the AMR and AHE, obtained from the LLG equation and the generalized Ohm’s law, are denoted by the triangles in figures 4(a) and (b), respectively. Both FMR peaks generate dc voltage signals. According to equation (40) and using the fitting parameters found earlier for each peak, the $A$ are $A_1 = -13.8$, $A_2 = 35.1$ (in units of $\Delta\rho_{jhl}/M$), $A_3 = -24.8$ and $A_4 = -31.9$ (in units of $R_{jhl}$) for the first FMR peak, and $A_1 = 49.7$, $A_2 = 0.0$ (in units of $\Delta\rho_{jhl}/M$), $A_3 = 0.0$ and $A_4 = -30.1$ (in units of $R_{jhl}$) for the second FMR peak. The predicted dc voltage due to the AMR and AHE (blue curves in figures 4(a) and (b)) perfectly describe the exact numerical results (triangles). For the first peak, since each of $\mathbf{j}$, $\mathbf{h}$ and $\mathbf{l}$ is out of the $xz$- or $yz$-plane, the dc voltage due to both the AMR and AHE depends on $\text{Im}(\chi_{xy})$, $\text{Im}(\chi_{yx})$ and $\text{Im}(\chi_{yz})$ according to equations (36) and (37). $\text{Im}(\chi_{xx})$ and $\text{Im}(\chi_{yx})$, $\text{Im}(\chi_{yy})$ are symmetric (antisymmetric) about the FMR peak. Consequently, the dc voltage line shapes due to both the AMR and AHE are asymmetric. For the second peak, since $\mathbf{j}$ is in the $xz$-plane and $\mathbf{h}$ and $\mathbf{l}$ are along the $y$-axis, the dc voltage due to the AMR relates only to $\text{Im}(\chi_{xy})$, according to equation (36), and the voltage due to the AHE relates only to $\text{Im}(\chi_{yx})$ according to equation (37). $\text{Im}(\chi_{yy})$ is symmetric (antisymmetric) about the FMR peak. Consequently, the dc voltage line shapes about the second peak have the same symmetry as those in the first example.
The field dependence of the dc voltage due to the AMR contribution (a) and the AHE contribution (b) at $\omega = 50$ GHz. The model parameters are $K_1 = 0, K_2 = 1, \theta_H = 5.0^\circ, \phi_H = 90.0^\circ$, and $\phi_1 - \phi_2 = 90^\circ$ (squares); or $K_1 = 0.2, K_2 = 1, \theta_H = 5.0^\circ, \phi_H = 90.0^\circ$, and $\phi_1 - \phi_2 = 90^\circ$ (triangles); or $K_1 = 0.2, K_2 = 1, \mathbf{H}$ along the $z'$-axis and $\phi_1 - \phi_2 = 90^\circ$ (circles); or $K_1 = 0.1, K_2 = 1, \theta_H = 90.0^\circ, \phi_H = 87.0^\circ$, and $\phi_1 - \phi_2 = 30^\circ$ (cross). The symbols are the exact results from the LLG equation and the generalized Ohm’s law and the solid curves are the universal expressions of equations 44 (a) and 45 (b) with the $A$ given by equation (40) and the fitting parameters for the corresponding curves in figure 3. The curves are vertically offset (from the top down) 254, 175, 138, 80, 0 (a); 200, 120, 70, 30, 0 (b) for a better view.

In the third example, we set $K_1 = 0.2$ ($K_2 = 1$), $\mathbf{H}$ along the $z'$-axis and $\phi_1 - \phi_2 = 90^\circ$. $\mathbf{M}_0$ lies in the $x'y'z'$-plane so that the system is symmetric about the $x'z'$-plane (also the $xz'$-plane). Like that in the first example, $\chi_{x'y'}$ and $\chi_{xz'}$ are opposite to each other. The exact results of both the real and imaginary parts of $\chi_{x'y'}$ and $\chi_{xz'}$ are circles in figures 3(a)–(d). The $H$ dependences at a fixed frequency of $\omega = 50$ GHz are in figures 3(a) and (b), and the $\omega$ dependences at a fixed field of $H = 985.3$ mT are in figures 3(c) and (d). Two FMR peaks appear in figures 3(a) and (b), while there is only one peak in figures 3(c) and (d). The two peaks are due to the non-monotonicity of the effective magnetic field in $H$, resulting in multiple solutions (of $H$) which satisfy $\omega(H) = \omega$ [3, 4, 14, 26, 32, 33]. The non-monotonic behavior of $\omega_0$ versus $H$ is shown by the gray dashed-dotted curve in figure 5. In the current case of $\omega = 50$ GHz, there are two resonance fields (985.3 mT and 1407.3 mT). The first peak occurs at $H_0 = 985.3$ mT, and $\mathbf{M}_0$ lies in the $x'y'z'$-plane with $\theta = 34.9^\circ$, which is non-collinear with $\mathbf{H}$. The second peak occurs at $H_0 = 1407.3$ mT, and $\mathbf{M}_0$ is along the $z'$-axis, which is collinear with $\mathbf{H}$. The red curves in figures 3(a)–(d) are the universal expressions of equation (23) for (a) and (b), and equation (17) for (c) and (d). For the curves in figures 3(a) and (b), the fitting parameters around the first (second) peak are $\Gamma_1 = 8.3$ mT, $\beta_1/\beta = -1$, $C_1 = 147.5$, $C_2 = 0.0$, $C_3 = 208.5$ and $C_4 = 294.7$ ($\Gamma_1 = 4.6$ mT, $\beta_1/\beta = 1$, $C_1 = 294.8$, $C_2 = 0.0$, $C_3 = 208.4$ and $C_4 = 147.4$). For the curves in figures 3(c) and (d), the fitting parameters are $\Gamma = 0.85$ GHz, and the same C-values as those for the first peak in figures 3(a) and (b). Like the first example, each curve around FMR is either a Lorentzian or $D$ function. The validity of the universal expressions is shown by the perfect agreement.

The exact numerical results of the dc voltage due to the AMR and AHE are computed from the LLG equation and the generalized Ohm’s law, and they are denoted by the circles in figures 4(a) and (b), respectively. Interestingly, only one peak appears, corresponding to the first peak in the $\chi$. The second FMR peak does not generate any dc voltage signals because $\mathbf{M}_0$ is along the $z'$-axis, and the $x'y'z'$-coordinate and the $xyz$-coordinate are coincident with each other. Thus, $l_z = 0, j_z = 0$ and $\mathbf{j} \times \mathbf{m}$ are perpendicular to $\mathbf{l}$, and equations (4) and (5) predict $U_{AMR} = U_{AHE} = 0$. According to equation (40) and using the fitting parameters found earlier for the first peak, the $A$ are $A_1 = 0.0$ and $A_2 = -49.1$, in units of $\Delta \phi/\eta M$, and $A_3 = 34.8$ and $A_4 = 0.0$, in units of $R \phi/\eta$. The predicted dc voltage due to the AMR and AHE (red curves in figures 4(a) and (b)) again agrees perfectly with the exact results (circles) around the first FMR peak. Since $\mathbf{j}$ is along the $y$-axis and $\mathbf{h}$ and $\mathbf{l}$ are in the $xz$-plane in this case, the dc voltage due to the AMR depends on $\text{Im}(\chi_{xy})$ according to equation (36) and the voltage due to the AHE depends only on $\text{Im}(\chi_{xz})$ according to equation (37). $\text{Im}(\chi_{xy})(\text{Im}(\chi_{xz}))$ is antisymmetric (symmetric) about the FMR peak. Consequently, the dc voltage lineshapes have the opposite symmetry to the first example. In summary, these examples show that the dc voltage lineshapes can
change from symmetric to antisymmetric, or even vanish due to magnetic anisotropy.

3.2. The case of $\mathbf{H}$ in the $xy$-plane

In the fourth example, we set $K_1 = 0.1$ ($K_2 = 1$), $\theta_H = 90.0^\circ$, $\phi_H = 87.0^\circ$ and $\phi_1 - \phi_2 = -90^\circ$. $\mathbf{M}_1$ lies in the $xy$-plane in this case, so that the system is symmetric about the $xy$-plane (also the $yz$-plane). With this symmetry, $\chi_{xx}$ and $\chi_{yy}$ are opposite to each other. The exact results of both real and imaginary parts of $\chi_{xx}$ and $\chi_{yz}$ are crosses in figures 3(a)–(d) in which figures 3(a) and (b) (figures 3(c) and (d)) are $H$-dependent ($\omega$-dependent) at a fixed frequency of $\omega = 50$ GHz (field of $H = 58.3$ mT). Two FMR peaks appear in figures 3(a) and (b), while there is only one peak in figures 3(c) and (d). For the same reason as that for the second example, the two peaks are due to the non-monotonicity of the effective magnetic field in $H$, resulting in multiple solutions (of $H$) which satisfy $\omega_0(H) = \omega$ [3, 4, 14, 26, 32, 33]. The non-monotonic behavior of $\omega_0$ versus $H$ is shown by the green dashed-dotted-dotted curve in figure 5. The first peak occurs at $H_0 = 58.3$ mT, and the corresponding $\mathbf{M}_0$ lies in the $xy$-plane with $\phi = 33.9^\circ$, which is non-collinear with $\mathbf{H}$. The second peak occurs at $H_0 = 166.5$ mT, at which $\mathbf{M}_0$ lies in the $xy$-plane with $\phi = 82.5^\circ$, which is also non-collinear with $\mathbf{H}$. The green curves in figures 3(a)–(d) are the universal expressions of equation (23) for (a) and (b) and equation (17) for (c) and (d). For the curves in figures 3(a) and (b), the fitting parameters around the first (second) peak are $\Gamma_1 = 5.2$ mT, $\beta_1 = 27.2$, $\beta_2 = 0.0$, $C_3 = 106.3$ and $C_4 = 415.0$ ($\Gamma_2 = 4.8$ mT, $\beta_1 = 1$, $C_1 = 24.5$, $\beta_2 = 0.0$, $C_1 = 101.2$ and $C_2 = 417.7$). For the curves in figures 3(c) and (d), the fitting parameters are $\Gamma = 1.66$ GHz and the same $C$-values as those for the first peak in figures 3(a) and (b). Similar to that of the first example, each curve around FMR is either a Lorentzian or $D$ function. The perfect agreement verifies the validity of the universal expressions.

The exact numerical results of the dc voltage due to the AMR and AHE, obtained from the LLG equation and the generalized Ohm’s law, are denoted by the crosses in figures 4(a) and (b), respectively. Both the FMR peaks generate dc voltage signals in this case. According to equation (40) and using the fitting parameters found earlier for the first (second) peak, the $A$ are $A_1 = -21.7$ and $A_2 = 37.7$ in units of $\Delta \rho j HL / M$, and $A_3 = -25.7$ and $A_4 = -14.9$ in units of $R j HL / (A_1 = 100.1$ and $A_2 = -173.3$ in units of $\Delta \rho j HL / M$, and $A_3 = -43.4$ and $A_4 = -25.1$ in units of $R j HL$). The predicted dc voltage due to the AMR and AHE (gray curves in figures 4(a) and (b)) perfectly describes the exact results (stars). Since $j$, $h$, and $I$ are all in the $yz$-plane and $\phi_1 - \phi_2 = -30^\circ$, the dc voltage around each FMR peak due to the AMR relates to both $\text{Re}(\chi_{yy})$ and $\text{Im}(\chi_{xx})$ according to equation (36), and the voltage due to the AHE relates to both $\text{Re}(\chi_{xx})$ and $\text{Im}(\chi_{yy})$ according to equation (37). $\text{Re}(\chi_{xx})$ and $\text{Im}(\chi_{yy})$ (Re($\chi_{yy}$) and Im($\chi_{xx}$)) are symmetric (antisymmetric) about the FMR peak. Therefore, the dc voltage line shapes in this configuration are asymmetric.

3.3. Non-resonance peaks

So far, we have demonstrated the validity of the universal expressions of equations (23) and (17) for $\chi$, and equations (36) and (37) for dc voltage near FMR in a number of examples, as long as the approximation of equation (21) is good. When equation (21) does not hold, some of the matrix elements of $\chi$ may not follow equation (23), and an extra non-resonance broad peak can even appear in some of the matrix elements near a true FMR peak. To see a clear example, we set $K_1 = 0.0$ ($K_2 = 1$), $\theta_H = 3.0^\circ$, $\phi_H = 90.0^\circ$ and $\omega = 27$ GHz. $\mathbf{M}_0$ lies in the $yz$-plane, and the system is symmetric about the $yz$-plane (also the $xz$-plane) so that $\chi_{xx}$ and $\chi_{yz}$ are opposite to each other. Figure 6(a) shows how the FMR resonance frequency $\omega_0$ changes with $H$. Around the relative low frequencies ($\approx 27$ GHz), $\omega_0$ is not very sensitive to $H$ with a very small $\beta = 0.02$ GHz mT$^{-1}$. For both the Lorentzian function $L$ and the corresponding $D$ function, which decay as a power-law, the resonance region should be several peak widths whose value is $\Gamma \approx 1$ GHz in the current example. This means that the field in the resonance region can vary from about 300 mT ($\omega_0 \sim 23$ GHz) to 1000 mT ($\omega_0 \sim 31$ GHz). In this field range, $C_1$ increases by more than ten times, as shown in figure 6(b), and the direction of $\mathbf{M}_0$, around the $y'$-axis at 300 mT, moves to near the $z$-axis at 1000 mT (figure 6(c)). Thus, one should not expect our $\chi$ expression of equation (23) to be good anymore.

The exact results of both the real and imaginary parts of $\chi_{xx}$, $\chi_{yy} = -\chi_{xx}$ and $\chi_{yz}$ as functions of $H$ at the fixed frequency $\omega = 27$ GHz (corresponding to point A in
figure 6(a)) are squares in figures 6(d)–(f). The FMR peak occurs at $H_0 = 490.4$ mT, and $M_0$ lies in the $yz'$-plane with $\theta = 61.8^\circ$, which is non-collinear with $H$. The black curves in figures 6(d)–(f) are the universal expression of equation (23) with the fitting parameters of $K_1 = 0.0$, $\theta = 3.0^\circ$, and $\phi = 90.0^\circ$. Points A and B are respectively the resonance and non-resonance peaks of $\text{Re}(\chi_{xx})$. (b) The $H$ dependence of $C_1$. (c) The $H$ dependence of $M_{x0}'$, $M_{y0}'$ and $M_{z0}'$. (d)–(f) The real and imaginary parts of $\chi_{xx}$, $\chi_{yx}$ (or $-\chi_{xy}$) and $\chi_{yy}$ as functions of $H$. The exact results are denoted by symbols: squares for $\omega = 27$ GHz and red for $\omega = 60$ GHz. The top curves are vertically offset 36 (d), 220 (e) and 1000 (f) for a better view.

Figure 6. The model parameters are $K_1 = 0.0$ ($K_2 = 1.0$), $\theta = 3.0^\circ$ and $\phi = 90.0^\circ$. (a) The $H$ dependence of $\omega_0; \omega_0$ does not change much in the range of $400$ mT $< H < 900$ mT. At $\omega = 27$ GHz (blue arrowed line), $\beta = 0.02$ GHz mT$^{-1}$. Points A and B are respectively the resonance and non-resonance peaks of $\text{Re}(\chi_{xx}(H))$. (b) The $H$ dependence of $C_1$. (c) The $H$ dependence of $M_{x0}'$, $M_{y0}'$ and $M_{z0}'$. (d)–(f) The real and imaginary parts of $\chi_{xx}$, $\chi_{yx}$ (or $-\chi_{xy}$) and $\chi_{yy}$ as functions of $H$. The exact results are denoted by symbols: squares for $\omega = 27$ GHz and red for $\omega = 60$ GHz. The top curves are vertically offset 36 (d), 220 (e) and 1000 (f) for a better view.
depending on the magnetic anisotropy and experimental setup. It is already known that interfacial interaction can change the magnetic anisotropy [36–38] in multilayer structures. Thus the dc voltage signal, including its shape and symmetry, can be very different for a multilayer film and for a single magnetic layer.

4. Conclusions

In conclusion, using the causality principle and the assumption that the frequency dependence of the usual microwave absorption lineshape of a homogeneous magnetic material around an FMR is Lorentzian, the universal form of the dynamic magnetic susceptibility matrix is found. The dynamic magnetic susceptibility is not, in general, the Polder tensor that was widely assumed in the literature. The non-Polder tensor of the dynamic magnetic susceptibility has important effects on the dc voltage lineshape in the electrical detection of an FMR. It is also found that the linear response coefficient of the magnetization to the total local rf field (the sum of the applied external rf field and the internal rf field due to the precessing magnetization, which is a quantity that cannot be measured directly) is a Polder tensor. This finding may explain why the Polder tensor was so widely misused in previous analyses. Although the dynamic magnetic susceptibility and dc voltage near the FMR should depend, in principle, on the magnetic anisotropy, we show that they are fully determined by a few parameters instead of a function. These parameters can be experimentally determined by traditional microwave absorption experiments. Our results provide a reliable way of extracting the dc voltage induced by spin pumping and the ISHE. Furthermore, the non-monotonic behavior of the total effective magnetic field on the external field may lead to multiple FMR peaks in fields at a constant microwave frequency, while there is only a single frequency peak at a fixed static field. We also point out that the insensitivity of the resonance frequency to the magnetic field may lead to another broad peak, which does not correspond to FMR. This raises an alarm regarding the proper interpretation of detected FMR field peaks, especially those with broad peaks and a large deviation from our universal expressions.

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