Identification of the Shale Production Model

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Abstract. The technique of appraisal have been proposed for the evaluation of the recoverable reserves according to production schedules for the total wells number of the same calendar year into operation. NPV formula representation and necessary conditions for optimality have been proposed. Recoverable reserves and rate of production (production decline) have been identified for the wells put into operation in 2012 – 2017. As a trend, the initial production rate of wells and recoverable reserves grew from year to year, as well as investments in one well. The ties between the length of horizontal wellbores multiplied by the volume of injected proppant and production parameters have been studied. The choice of production technology depends on the actual oil price. With an increase in the oil price, optimally coordinated increase both the recoverable reserves and the rate of production at the well.

1. Introduction
In [1], the task is to determine the recoverable reserves of shale deposits. Paper [2] has plots showing actual data for shale production. This allows for the identification of the mathematic model parameters [3]. These plots are digitized in increments of 1 month. Plots were digitized in the Paint software with the use of integrated vertical and horizontal rulers with 1-pixel scale intervals.

2. Oil production free fall model
The theory is that the production of a single shale hydrocarbon well is at its maximum on the first day (month) after its putting into operation. If the production method remains the same, it leads to a production-free fall in the future. For total wells production, which has been put into operation during the same calendar year, free fall starts at the end of the current calendar year.

Each plot shows data for wells put into operation during the same calendar year (2012-2018). The model assumes that the wells are put into operation evenly throughout the year. Each point on the plot represents overall month production, divided by the number of producing wells (at the end of the month), divided by the number of days in the months $q_i$.

Let’s plot a free fall trend for the production schedule [2] of the wells put into operation in 2012 – 2017 years.

To implement that, we shall plot logarithmic plots $\ln(q_i)$ starting from the 13-th month after putting the first well into operation, i.e. since the moment of the free fall beginning for all the wells of this calendar year. Let’s use EXCEL instruments to plot linear trends of these plots (figure 1).
Figure 1. Trends of production logarithms, their formulas and dispersion index $R^2$

As for wells put into operation in 2014 – 2017, the linear trend on a logarithmic scale closely corresponds with actual data ($R^2 > 0.93$). For wells put into operation in 2012 – 2013, this correspondence is worse ($R^2 > 0.86$).

Let’s build an exponent from the formula

$$y = -mx + q, \quad q(t) = q(12) \cdot e^{-m(t-12)}, \quad t \geq 12,$$

where $q(12)$ – average production rate at the year’s end [BOE/day], $t$ is measured in months, and production decline parameter $m$ is in units per month. The trend parameters (1) are shown in Table 1, where $V(12)$ – estimates of residual recoverable reserves.

Table 1. Parameters of production free fall trends.

| Year of putting into operation | Production rate at the year’s end, BOE/day | Production decline $m$, %/month | Production decline $m$, %/year | $m$, per year | $V(12)$, thous. BOE |
|-------------------------------|------------------------------------------|---------------------------------|---------------------------------|---------------|-------------------|
| 2012                          | 100                                      | 1.6%                            | 19%                             | 18%           | 189               |
| 2013                          | 100                                      | 1.4%                            | 17%                             | 16%           | 217               |
| 2014                          | 133                                      | 1.9%                            | 23%                             | 21%           | 211               |
| 2015                          | 160                                      | 2.6%                            | 31%                             | 27%           | 189               |
| 2016                          | 289                                      | 4.5%                            | 54%                             | 42%           | 196               |
| 2017                          | 284                                      | 4.4%                            | 53%                             | 41%           | 197               |

Production rates at the year’s end significantly lower than maximum actual production rates. It means that production declines significantly during the first year. Instantaneous value $m$ is tied with production decline during 1 year by the formula $m = 1 - \exp(-m)$. The year decline in production during following years is not lower than 16% per year, which is significantly higher than in the production of conventional oil (4% in average).
If high decline rates remain the same, then production rates will lower in course of time and it makes it possible to consider production in the infinite interval of time without major errors. If we will take decline rate \( m \) as a constant value, we can use formula (1) for evaluation of recoverable reserves \( V(t) \) at the moment \( t \)

\[
q(t) = q(12) \cdot e^{-m(t-12)}, \quad t \geq 12 ,
\]

where 30.4 – the average number of days in a month, \( m \) in \([1/\text{month}]\).

As a result, the production decline rate \( m \) in model (1) will be in coincidence with the rate of recovery \( m \).

3. Oil production model
Let’s use an oil (gas) production model for a single well, based on two characteristics:
- residual recoverable reserves \( V(t) \),
- production rate \( m \).

These values depend on the field’s properties, as well as on production technologies (well design and methods of reservoir stimulation).

We’ll consider that instantaneous oil production rate \( q(t) \) of well in \( t \) moment of time

\[
q(t) = -\frac{dV}{dt},
\]

is proportional to residual reserves \( V(t) \)

\[
q(t) = mV(t),
\]

where recovery rate \( m \) is constant. Calculation of the obtained equation

\[
\frac{dV}{dt} = -m \cdot V, \quad V(0) = V_1,
\]

is written as

\[
V(t) = V_1 \cdot e^{-m(t-t_0)}, \quad t \geq t_0,
\]

where \( V_1 \) – initial recoverable reserves for a single well,
\( t_0 \) – the moment of production start at the well.

Then production rate \( q(t) \) equals

\[
q(t) = m \cdot V_1 \cdot e^{-m(t-t_0)},
\]

production per time interval \([t_1, t_2], t_1 \geq t_0\), \( Q(t_1, t_2) \) equals

\[
Q(t_1, t_2) = V(t_1) - V(t_2) = V_1 \cdot [e^{-m(t_1-t_0)} - e^{-m(t_2-t_0)}].
\]

4. Identification of the model
The Source of actual data for the model identification is the same plot as in [2].

Identification has its goal in finding values for \( V_1 \) and \( m \) parameters, and it makes it possible to move theoretical value \( g_{th}(i) \) as close to actual value \( g_{i}(i) \) on the plot as it possible. It will need function minimization
where \( i_k \) – the number of actual month in the chart.

Our end goal is to bring theoretical \( NPV \) for one shale well to forward to actual \( NPV \) values. It needs a close approximation of total discounted revenue. Therefore multipliers \( d_i \) were chosen as weight parameters in (7), these multipliers are used in the reduction of revenue to the initial moment.

It should be noted that instantaneous discount factor \( E \) is tied with the annual \( E_y \) factor by the function:

\[
E = \ln(I + E_y).
\]

We’ll take in calculations \( E_y=10 \% \) per year, then \( E=9.5 \% \).

Values of \( V \) and \( m \) parameters, which are minimum for the (7) function, were found in the EXCEL software with the “Table” instrument. The most complete data is available for the 2012 year (78 months). For other years of putting wells into operation, there are data for 67, 55, 43, 31, 19 and 6 months. In order to have a possibility to match results of various years let’s implement identification of the 2012 year model in accordance with the same time interval (Table 2).

For 6 and 19 months, there are good approximation qualities, for larger intervals it is bad. Nevertheless, reduction to (3) - (6) model has sense, cause for this model optimization task can be solved more easily.

A greater number of years in the interval results in a lower estimate of the recovery factor \( m \) and a greater estimate of recoverable reserves \( V_1 \) (Table 2).

Table 2 also shows total discounted production and adjustment factors which can help to transform \( x \)-months model into the model with \( x = 78 \).

Table 3 shows models identified by the first year.

| Year of putting into operation | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 (six points) |
|-------------------------------|------|------|------|------|------|------|-------------------|
| Production rate at the end of the year, BOE/day | 140  | 116  | 143  | 176  | 288  | 294  | 303               |
| Production decline m, %/year | 372  | 468  | 535  | 475  | 340  | 382  | 450               |
| Production decline my, per year, % | 97.6 | 99  | 99.5 | 99.2 | 96.7 | 97.8 | 98.9              |
| \( V(0) \) recoverable reserves, thous. BOE | 629  | 513  | 631  | 780  | 1310 | 1320 | 1348              |
| \( V(12) \), thous. BOE | 165  | 109  | 117  | 162  | 373  | 338  | 296               |
Table 4. Trend parameters for all actual data.

| Year of putting into operation | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
|-------------------------------|------|------|------|------|------|------|------|
| Production rate at the end of the year, BOE/day | 187  | 176  | 227  | 255  | 374  | 333  | 304  |
| Production decline m, %/year | 123  | 74   | 108  | 151  | 145  | 284  | 450  |
| After correction, %/year     | 123  | 73   | 100  | 121  | 91   | 114  | 155  |
| V (0) recoverable reserves, thous. BOE | 1184 | 1518 | 1541 | 1465 | 2180 | 1560 | 1348 |
| After correction             | 1184 | 1538 | 1616 | 1669 | 2887 | 2642 | 2459 |
| Number of months             | 79   | 67   | 55   | 43   | 31   | 19   | 6    |

5. Economical model

[2] has plots for the ratio of shale oil production $q_i(i)$ by months to the volume of injected proppant $P$, multiplied by the length of horizontal wellbores, i.e. the author considers $P \times L$ value as a good measure of action to the reservoir. Let’s take those investments $K_i$ into the reservoir stimulation are proportional to the degree of $P \times L$

$$K_p \sim (P \times L)^b.$$

We’ll consider that $P$-value is proportional to recoverable reserves $V_i$ of the well with the use of selected technology. We’ll consider that the initial production rate of the well $m \times V_i$ is proportional to the $L$ value. As a result, we will have the formula for investments

$$K_p = b \cdot m^b \cdot V_i^2,$$  \hspace{1cm} (8)

where $b$ – specific investments (values are not known).

For evaluation of $P \times L$ value, plots $\frac{q_i(i)}{P \times L}$ were digitized and dependencies between points on plots were calculated (Table 5).

Table 5. Average actual investments into the reservoir.

| Year of putting into operation | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
|-------------------------------|------|------|------|------|------|------|------|
| Multiplication P×L, MMlbs×Mft | 45   | 58   | 74   | 75   | 97   | 118  | 143  |

Figure 2 shows the correspondence between $P \times L$ and $m \cdot V_i^2$.

![Figure 2. Dependence of m-$V_i^2$ value on P-L.](image-url)
From year to year technology of the reservoir stimulation has been changing, production rates of wells have been increasing but investments have been growing as well. Let’s draw a formula for NPV

\[ NPV = \frac{(p-c)mV_1}{E+m} - K_v - b \cdot m^s \cdot V_1^{2s}, \]

where \( p \) – product cost,
\( c \) – specific operating costs,
\( K_v \) – investments for drilling a vertical (inclined) well from the surface to the reservoir.

6. Necessary conditions for optimization
Let’s write out the necessary conditions of the minimum NPV.

\[ \frac{\partial NPV}{\partial m} = \frac{(p-c)EV_1}{(E+m)^2} - b \cdot s \cdot m^{s-1} \cdot V_1^{2s} = 0, \]
\[ \frac{\partial NPV}{\partial V_1} = \frac{(p-c)m}{E+m} - 2b \cdot m^s \cdot s \cdot V_1^{2s-1} = 0. \]

We need to exclude bracket \((p-c)\) from the system of two equations and to get a single equation, tying optimal values \( m \) and \( V_1 \). If oil’s price \( p \) rises, then values of recovery rate \( m \) and recoverable reserves \( V_1 \) will rise. Far less grows wells’ production rate. The volume of injected proppant \( P \) and horizontal wellbores length \( L \) grow too. But in case of price \( p \) decrease, it is necessary to return to a cheaper technology, i.e. to decrease \( P \) and \( L \) values.

7. Conclusions
In order to increase the cost efficiency of shale oil production it is necessary not to only optimize the initial production rate of the well, but recoverable reserves as well. Recoverable reserves can be identified with the use of the oil production decline plot.

The model should establish a link between the investment in the implementation of the technology and the production parameters. Intermediate parameters may be represented by horizontal wellbores’ length and volume of injected proppant.

Production model refinement needs publication of actual data, natural and economic as well, including data on operational costs.

The establishment of an optimal correlation between production rate and recoverable reserves can be chosen as a goal, in other words: correlation between horizontal wellbores’ length and injected proppant volume. The following step shall be the determination of the dependence of the best technologies on the price of oil.

8. References
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