Amplitude Quantization Techniques of Aperture Distribution in Phased Array Antennas

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Abstract. In phased array antennas, the high side-lobe levels due to the amplitude quantization often need to be reduced to meet some requirements. In order to develop the digitalization of antenna arrays in phased array radars, the amplitude quantization of aperture distribution in array antennas is studied. In this paper, the random technique and a novel appropriate random technique are presented and used to reduce the side-lobe levels, and the side-lobe levels in the amplitude quantized array pattern are calculated. The reduction of side-lobe level due to amplitude quantization are introduced using the improving method and confirmed with computations, where the related parameters are modified and numerous patterns are computed and compared. The quasi-optimum patterns are also obtained and given.

1. Introduction
In the early days of research and development of the phased array antennas, the phase quantization topics were studied if the digital phase shifters are used.

For the phase quantization studies, several papers can be referred [1-5]. However, for the amplitude quantization studies, it is likely to be paid more attentions. In this paper, we present the amplitude quantization techniques in phased array antennas. The results here can be applied to the DDS design.

2. Reduction of Sidelobe Level Due To Amplitude Quantization For Phased Array Antennas
Consider a linear antenna array of N elements with inter element spacing d. Each element is connected to a single attenuator with digital amplitude — controlled attenuating sections, which correspond to various bits.

For example, 0 bit means the amplitude at each element only has two values 0 and 1. It implies the element can be fed with equal amplitude current or not. In fact, this array is often called a thinned array antenna. 1 bit corresponds to that each element can have amplitude of 3 values i.e. 0, 1, 2; 2 bit means the amplitude includes 5 values i.e.0, 1, 2, 3, 4, 1, etc.

The far-field factor of the digital amplitude quantized linear array antenna is expressed as

\[ E(u) = \sum_{n=1}^{N} A \exp j(2n - 1)u \]
where \( u = k d (\sin \theta - \sin \theta_0)/2 \) is the generalized coordinate in far field pattern for the array (\( k = \lambda/2\pi \) is the wave number, \( \theta \) is the azimuthal angle, \( \theta_0 \) is the pointing direction of the main beam in the array pattern), \( A_n \) is the quantized amplitude.

We can assume

\[
A_n = an + s\delta \quad (0 \leq \delta < 1)
\]

where \( a_n \) is the amplitude of a desired aperture distribution at the \( n \)-th element, \( \delta \) is the amplitude quantization unit, \( S \) is the fractional factor for the amplitude quantization.

The amplitude quantization error is

\[
A_N^\delta = A_n - an = (an + s\delta) - an = sn\delta.
\]

The far-field error due to the amplitude quantization is

\[
\delta E(u) = \sum_{n=1}^{N} sn\delta \exp j[(2n-1)u].
\]

The relative mean energy of the parasitic radiation is the mean square value

\[
(SLL) = \frac{\delta E(u)}{E(0)} = \frac{1}{N^2} \left( \frac{\delta E(u)}{\delta E(0)} \right)^2 = \frac{1}{3N^2} \left( \frac{\delta}{2} \right)^2.
\]

Let’s say \( \delta = \frac{1}{2p} \), where \( p \) is the bit number of the amplitude quantization. Then the relative mean energy of the parasitic radiation becomes

\[
(SLL) = \frac{1}{3N^2} \left( \frac{1}{2p+1} \right)^2 = \frac{1}{3N^2(2p+1)}
\]

or in decibels

\[
(SLL)_{dB} = 10 \log(SLL) = -10\log N - 6p - 11
\]

### 3. Further Reduction of SLL Using Appropriate Random Amplitude Technique

In order to reduce the level of the parasite side-lobes due to the amplitude quantization in phased array antenna patterns, now, the appropriate random technique are introduced and described as follows.

We assume

\[
A_n = m\delta + s\delta \quad (0 \leq s < 1)
\]

where \( m = \left\lfloor \frac{a_n}{\delta} \right\rfloor \)

Then we can have

\[
A_n^\delta = \begin{cases} 
m\delta & \text{if } 0 \leq s \leq c \\
(m + f(s))\delta & \text{if } c < s < 1 - c \\
(m + 1)\delta & \text{if } 1 - c \leq s \leq 1
\end{cases}
\]

where \( f(s) = 0 \) or \( 1 \) according to the applied random amplitude quantization technique.

Now, the expectation of the pattern with random amplitudes of elements is

\[
\langle E(u) \rangle = \sum_{n=1}^{N} \langle A_n \rangle \exp j[(2n-1)u].
\]
4. Conclusion
When $c = 0.5$, the pattern corresponds to the case of the ideal amplitude distribution. From the computation results, it shows the random amplitude technique (it is the case when $c = 0$) can effectively suppress the amplitude quantization side-lobes in phased array antennas. When $0 < c < 0.5$, it corresponds to the case that the appropriate random amplitude techniques are applied to calculations. This technique can further reduce the parasite amplitude quantization side-lobes in these patterns. (shown in Fig. 1-6).

In fact, 1 to 8 bits are usually used in the phased array antennas. However, 9 to 15 bits are often used in Direct Digital Synthesizer (DDS) to reduce the spur, since its mathematical model is similar to the phased array antennas. We may use particle swarm technique, genetic algorithm technique for further optimization.

Fig. 1-6. Top: the probability of SLL of the quantized pattern. Middle: the calculated quasi-optimum quantized distribution over array aperture. Bottom: the pattern of the corresponding aperture distribution.

Fig. 1 bit = 0

Fig. 2 bit = 1

Fig. 3 bit = 2
Fig. 4 bit = 10

Fig. 5 bit = 11

Fig. 6 bit = 12
Table 1. Minimum results of maximum slm among 5000 experiences for various bit values and various proximate factor c values.

| c    | 0.0      | 0.1      | 0.2      | 0.3      | 0.4      | 0.5      |
|------|----------|----------|----------|----------|----------|----------|
| bit=0| -21.310  | -21.588  | -21.687  | -18.517  | -15.002  | -13.359  |
| bit=1| -26.408  | -26.699  | -26.715  | -26.222  | -22.667  | -19.169  |
| bit=2| -31.805  | -32.195  | -32.319  | -32.795  | -27.808  | -24.915  |
| bit=3| -37.251  | -37.449  | -37.559  | -38.255  | -37.670  | -34.135  |
| bit=4| -42.303  | -42.526  | -42.848  | -43.279  | -43.747  | -41.808  |
| bit=5| -47.692  | -47.947  | -49.129  | -49.372  | -49.603  | -46.128  |
| bit=6| -53.883  | -54.451  | -54.706  | -55.279  | -52.456  |          |
| bit=7| -59.096  | -59.190  | -59.387  | -60.227  | -60.504  | -58.212  |
| bit=8| -64.876  | -65.049  | -65.672  | -66.226  | -66.555  | -62.817  |
| bit=9| -70.450  | -70.544  | -71.197  | -71.103  | -71.797  | -69.093  |
| bit=10| -75.884 | -76.177  | -77.318  | -77.503  | -78.017  | -75.940  |
| bit=11| -81.829 | -82.085  | -82.769  | -83.840  | -83.816  | -81.704  |
| bit=12| -88.394 | -87.377  | -87.949  | -88.393  | -88.910  | -85.541  |

References

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