Natural Quintessence with Gauge Coupling Unification

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ABSTRACT

We show that a positive accelerating universe can be obtained simply by the dynamics of a non-abelian gauge group. It is the condensates of the chiral fields that obtain a negative power potential, below the condensation scale, and allow for a quintessence interpretation of these fields. The only free parameters in this model are $N_c$ and $N_f$ and the number of dynamically gauge singlet bilinear fields $\phi$ generated below the condensation scale. We show that it is possible to have unification of all coupling constants, including the standard and non standard model couplings, while having an acceptable phenomenology of $\phi$ as the cosmological constant. This is done without any fine tuning of the initial conditions. The problem of coincidence (why the universe has only recently started an accelerating period) is not solved but it is put at the same level as what the particle content of the standard model is.

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In the past few years different observations have lead to conclude that the universe is flat and filled with an energy density with negative pressure, a cosmological constant [1],[2]. The cosmological constant is perhaps best understood, from an elementary particle point of view, as the contribution from a scalar field that interacts with all other fields only gravitationally, i.e. quintessence [3]. Recent observational results constrain the class of potentials since they require an energy density $\Omega_{\phi o} = 0.7 \pm 0.1$ with an equation of state parameter $w_{\phi o} = p_{\phi o}/\rho_{\phi o} \leq -0.6$, where the subscript $o$ refers to present day quantities [1],[2],[4].

In this work we show that a non-abelian gauge group with $N_c$ the number of colours and $N_f$ that of chiral fields leads to an acceptable quintessence potential. We show that the only degrees of freedom are precisely the simple choice of $N_c$, $N_f$ and the number of dynamically generated bilinear fields which set the scale of condensation and the power in the potential of the scalar field responsible for present day acceleration of the universe. Of course, we are not able to determine from first principles the values of $N_c, N_f$ but they are at the same footing as the choice of gauge groups and matter content of the standard model.

The model is quite simple, we start with a non-abelian gauge group at a high energy scale (could be the unification scale of the standard model gauge groups) with massless matter fields and we let it evolve to lower scales. By lowering the energy scale, the gauge coupling constant becomes large and all fields become strongly interacting at the condensation scale $\Lambda_c$. Below this scale there are no more free elementary fields, chiral nor gauge fields, similar to what happens with QCD and we are left with gauge singlets bilinear fields $\phi^2 \equiv <Q\bar{Q}>$ (the square in $\phi$ is to give the field a mass dimension one). We use Affleck-Seiberg superpotential [5] to determine the form of the scalar potential $V$ in terms of $\phi$ (related work can be seen in [6]). Afterwards, we solve Einstein’s General Relativity equations in a Friedmann–Robertson–Walker flat metric and determine the cosmological evolution of $\phi$. We show that a positive accelerating universe at present time with $\Omega_{\phi o} \simeq 0.7$ and $w_{\phi o} < -0.6$ is possible. We will bear in mind that the second restriction can be set in terms of an effective equation of state parameter $w_{eff} \equiv \int da \Omega(a)w_{\phi}(a)/\int da \Omega(a) < -0.7$ [4].

Furthermore, we constrain the model to have the same unification scale and gauge coupling as the standard model gauge groups. This is by all means not a necessary condition but it gives a very interesting model. We could think of this model as coming from string theory after compactifying the extra dimensions. The gauge coupling is unified for all gauge groups, the standard and non standard model gauge groups, at the compactification scale which is, in this case, also the unification scale. We then allow all fields to evolve cosmologically. Since at the beginning all fields are massless they behave as radiation until a gauge group becomes strongly coupled and there is a phase transition. Below this scale the particles charged under the strongly coupled gauge group condense while the other fields still evolve as radiation. Finally, we take into account the matter domination period and determine today’s relevant cosmological quantities.

Let us begin by writing the scalar potential for a non-abelian $SU(N_c)$ gauge group with $N_f$
(chiral + antichiral) massless matter fields. The superpotential is given by
\[ W = (N_c - N_f)(\frac{\Lambda^b_{det < Q\bar{Q} >}}{N_c - N_f})^{1/(N_c - N_f)} \]
and the scalar potential in global supersymmetry is
\[ V = |W_\phi|^2, \text{ with } W_\phi = \partial W/\partial \phi, \text{ giving} \]
\[ V = (2N_f)^2 \Lambda_c^{4+n} \phi^{-n} \tag{1} \]
where we have taken \( det < Q\bar{Q} > = \Pi_{j=1}^{N_f} \phi_j^2 \), \( n = 2^{N_c+N_f}/N_c-N_f \) and \( \Lambda_c \) is the condensation scale of the gauge group \( SU(N_c) \). We have taken \( \phi \) canonically normalized, however the full Kahler potential \( K \) is not known and for \( \phi \simeq 1 \) other terms may become relevant \( \bar{\phi} \) and could spoil the runaway and quintessence behaviour of \( \phi \). Expanding the Kahler potential as a series power \( K = |\phi|^2 + \Sigma_i a_i |\phi|^{2i}/2i \) the canonically normalized field \( \phi' \) can be approximated\( \phi' = (K_\phi^0)^{1/2}\phi \) and eq.(1) would be given by \( V = (K_\phi^0)^{-1}|W_\phi|^2 = (2N_f)^2 \Lambda_c^{4+n} \phi^{-n} (K_\phi^0)^{(n/2)-1} \). For \( n < 2 \) the exponent term of \( K_\phi^0 \) is negative so it would not spoil the runaway behaviour of \( \phi \).

In terms of the evolution of the gauge coupling constant we have
\[ \Lambda_c = \Lambda_0 e^{\frac{1}{2n_0 b_0}} \tag{2} \]
with \( \Lambda_0, g_0 \) the energy scale and coupling constant at a high energy scale where the gauge group is weakly coupled and \( b_0 = (3N_c - N_f)/16\pi^2 \) the one-loop beta function. We would like to take \( \Lambda_0 \) as the unification scale \( \Lambda_{gut} \approx 10^{16} GeV \) and \( g_0 \) as the unification coupling \( g_{gut} = \sqrt{4\pi/25.7} \).

The presence of the field \( \phi \) with potential eq.(1) begins only at the condensation scale \( \Lambda_c \). We can relate the scale \( \Lambda_c \) to the Hubble constant using \( H^2 = \rho/3 \simeq \Lambda_c^4/3 \) giving \( \Lambda_c \simeq (3H^2)^{1/4} \) where we have set the reduced Planck mass to one (i.e. \( m_P^2 = 8\pi G = 1 \)). By dimensional analysis we set the initial condition for \( \phi \) to be \( \phi_i = \Lambda_c \) which is the natural choice.

The cosmological evolution of inverse power potential has been studied in \( \bar{\phi}, \bar{\phi} \). The equations to be solved, for a spatially flat Friedmann–Robertson–Walker Universe, in the presence of a barotropic fluid, which can be either radiation or matter given by an energy density \( \rho_\gamma \), are given by \[ x_N = -3x + \sqrt{3} \lambda y^2 + \frac{3}{2} x y [2x^2 + \gamma_\gamma (1 - x^2 - y^2)] \]
\[ y_N = -\sqrt{3} \lambda x y + \frac{3}{2} y [2x^2 + \gamma_\gamma (1 - x^2 - y^2)] \]
\[ H_N = -\frac{3}{2} H [2x^2 + \gamma_\gamma (1 - x^2 - y^2)] \tag{3} \]
where \( N \) is the logarithm of the scale factor \( a, N \equiv \ln(a), f_N \equiv df/dN \) for \( f = x, y, H \) and \( \gamma_\gamma = 4/3, 1 \) for radiation or matter fields respectively and \( \lambda(N) \equiv -V'/V \). We have defined the variables \( x \equiv \dot{\phi}/\sqrt{6}H, y \equiv \sqrt{\gamma}/\sqrt{3}H \). In terms of \( x, y \) one has \( \Omega_\phi = x^2 + y^2 \) and the

\[ \text{The canonically normalized field } \phi' \text{ is defined as } \phi' = g(\phi, \bar{\phi} )\phi \text{ with } K_\phi^0 = (g + \phi g_\phi + \bar{\phi} g_{\bar{\phi}})^2 \]
equation of state for quintessence field is given by \( w_\phi = (x^2 - y^2)/(x^2 + y^2) \). Generic solutions to eqs. (3) can be found in [11], [12]. Notice that all model dependence in eqs. (3) is through the quantity \( \lambda(N) \). Using the potential given in eq. (1) we have \( \lambda = n \phi_i = n H_i y_i - 1/2 (H y)^2/n^{1/4} \) where \( i \) stands for initial conditions.

Since \( \Lambda_c \ll m_p \) and \( \phi_i = \Lambda_c \) the initial value of \( \lambda \) is very large \( \lambda_i = n m_p/\phi_i \gg 1 \) and this has interesting consequences.

From eqs. (3), the evolution of \( \Omega_\phi \) is to drop rapidly, \( \Omega_\phi \ll 1 \), in about 3 e-folds, i.e. \( \delta N \simeq 3 \), regardless of its initial value and it remains very small for a large period of time (see fig. (2)). These properties are due to the fact that \( \lambda_i \) is large. The evolution of \( \phi \) enters a scaling regime with \( \lambda = constant \) and \( \Omega_\phi \ll 1 \) during all this period. The scaling regime ends when \( x = O(1/10) \) and \( \Omega_\phi \) becomes also of the order of 0.1.

In this work we have determined \( w_{eff} \) for different values of \( n \) and we have concluded that for \( \Omega_{\phi i} \leq 0.25 \) one needs \( n < 2.7 \) for \( w_{eff} \) to be smaller than \( -0.7 \) as required [4]. In Figure 1 we show \( n \) as a function of \( w_{eff} \) assuming \( \Omega_{\phi o} = 0.7 \) and \( h_o = 0.7 \), where the Hubble constant is given by \( H = 100 h \text{ km/Mpc sec} \). This result constrains many inverse power models. In fact for \( N_c > N_f \) one has \( n > 2 \). It is important to point out that we find a decreasing value of \( w_{\phi o} \) with decreasing value of \( n \) in contrast with [3]. The main difference may be that in our models the value of \( \Omega_\phi = 0.7 \) is reached before \( w_\phi \) joins the tracker solution.

Figure 1: Restriction on \( n \) from the upper limit \( w_{eff} \leq -0.7 \).

Furthermore, to avoid any conflict with the standard big bang nucleosynthesis (NS) results one requires \( \Omega_{\phi}(NS) < 0.1 \) at the energy scale of NS [15], i.e. \( E_{NS} = 0.1 - 10 \text{ MeV} \). The condition of not spoiling the NS results rules out the values of \( n \) between \( 1.2 < n < 2.1 \) for models with \( \Omega_{\phi i} > 0.1 \). This is because for those values of \( n \) the initial value \( H_i \) lies within the value of the Hubble parameter at NS and \( \Omega_\phi \) is not yet smaller than 0.1. Of course we could start with a small value of \( \Omega_\phi \) but we would loose our democratic choice of initial conditions. These constraints would leave a window as small as \( 2.1 < n < 2.7 \) for \( N_c > N_f \). However, if we insist deriving \( \Lambda_c \) from eq. (2) with \( \Lambda_0, g_0 \) the unification scale and coupling, the above conditions constrains the models even more. In fact for \( N_c > N_f \) (\( n > 2 \)) there
are no models available that satisfy all constraints: \( \Omega_{\phi_0} = 0.7, \ w_{\text{eff}} < -0.7 \) and \( \Lambda_c \) given by eq.(2) with \( \Lambda_0 = \Lambda_{\text{gut}} \) and \( g_0 = g_{\text{gut}} \). A full analysis of all cases will be presented elsewhere [16].

In order to have \( \Lambda_0 = \Lambda_{\text{gut}} \) and \( g_0 = g_{\text{gut}} \) we require the number of dynamically bilinear fields \( \phi \tilde{Q} \) to be different from \( N_f \). Some of these fields may be fixed at their condensate constant vacuum expectation value (v.e.v.) with \( < \phi \tilde{Q} > = \Lambda_c^2 \) or we could have a gauge group with unmatching number of chiral and antichiral fields.

Here, we will present the case of an \( SU(3) \) gauge group with \( N_f = 6 \) chiral fields in the chiral and antichiral representation and we will assume that only one bilinear field \( \phi^2 = Q \tilde{Q} \) becomes dynamical with all other condensates remaining constant with a v.e.v. equal to \( \Lambda_c \). Notice that this gauge group is self dual (\( \tilde{N}_c = N_f - N_c = 3 \) with \( N_f \) flavours) under Seiberg’s duality transformation[4][17].

The potential generated in this case is

\[
V = 4\Lambda^{4+n}\phi^{-n} \tag{4}
\]

with \( n = 2\left(1 + \frac{2}{N_c-N_f}\right) = 2/3 \) and \( \phi_i = \Lambda_c \). Using eq.(2) with \( 16\pi^2b_0 = 3N_c - N_f = 3 \) one has \( \Lambda_c = 4 \times 10^{-8} \text{GeV} \), which is well below NS energy scale. Notice that \( n < 2 \) so the noncanonically terms in \( K \) will not spoil the quintessence behaviour of \( \phi \) and the mass is \( m \simeq H_0 \) so it is cosmologically fine [13].

Solving eqs.(3) with the potential given in eq.(4) and initial conditions \( \Omega_i = 0.25 \) and \( H_i = (4\Lambda_c^4/3y_i^2)^{1/2} = 1 \times 10^{-33} \text{GeV} \) gives for \( h_0 = 0.7 \) the values \( \Omega_{\phi_0} = 0.7 \) and an equation of state parameter \( w_{\phi_0} = -0.97 \) (with an effective \( w_{\text{eff}} = -0.98 \)). We see that the present day value of the parameters agrees with the analysis of recent observations [4] and there is no conflict with nucleosynthesis, since during nucleosynthesis the \( SU(3) \) gauge group was not strongly coupled and all those fields were massless and behaved as radiation at that epoch. The choice of initial conditions is not very sensitive and we took it as \( \Omega_{\phi_0} = 0.25 \) to be democratic with the standard model gauge groups. A variation of 40% in the initial value of \( \Omega_{\phi_0} \) gives still a final result within the range of \( h_0 = 0.7 \pm 0.1 \) and \( \Omega_{\phi_0} = 0.7 \pm 0.1 \). Finally, we show in Figure 2 the evolution of \( \Omega_\phi \) and \( w_\phi \) as a function of \( N \).

To conclude, we have shown that starting with a non-abelian gauge group with a gauge coupling constant unified with the standard model gauge couplings at the unification scale, a gauge singlet bilinear field \( \phi \), arising due to non-perturbative effects of to the strongly interacting non-abelian gauge group at the condensation scale, gives an acceptable phenomenology for the cosmological constant and it is therefore a natural candidate for quintessence.

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\textsuperscript{4}It is important to point out that even though it has been argued that for \( N_f > N_c \) there is no non-perturbative superpotential \( W \) generated [3] this is not always the case [4].
Figure 2: Evolution of $\omega_\phi$ and $\Omega_\phi$ (dotted and solid lines respectively). The vertical line represents the point for which $\Omega_\phi = \Omega_{\phi_0} = 0.7$ and $h = h_0 = 0.7$. The lower dot marks $\Omega_\phi = 0.6$ while the upper one stands for $\Omega_\phi = 0.8$.

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