The Meaning of Elements of Reality and Quantum Counterfactuals – Reply to Kastner.

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Abstract

This paper is the answer to the paper by Kastner [ *Found. Phys.* to be published, quant-ph/9807037] in which she continued the criticism of the counterfactual usage of the Aharonov-Bergman-Lebowitz rule in the framework of the time-symmetrized quantum theory, in particular, by analyzing the three-box “paradox”. It is argued that the criticism is not sound. Paradoxical features of the three-box example has been explained. It is explained that the elements of reality in the framework of time-symmetrized quantum theory are counterfactual statements and, therefore, even conflicting elements of reality can be associated with a single particle. It is shown how such “counterfactual” elements of reality can be useful in the analysis of a physical experiment (the three-box example). The validity of Kastner’s application of the consistent histories approach to the time-symmetrized counterfactuals is questioned.

1 Elements of Reality

Quantum theory teaches us that the concepts of “reality” developed on the basis of the classical physics are not adequate for describing our world. A new language with concepts which are appropriate is not developed yet and this is probably the root of numerous controversies regarding interpretation of quantum formalism. It seems to me that philosophers of science can make a real contribution for progress of quantum theory through developing of an appropriate language. A necessary condition for a success of this wisdom is that physicists and philosophers will try to understand each other. I hope, that the resolution of the current controversy about the time-symmetrized quantum theory (TSQT) will contribute to such understanding.
I took part in the development of the TSQT [1, 2, 3] and I believe that this is an important and useful formalism. It already helped us to find several peculiar quantum phenomena tested in laboratories in the world [4, 5]. In the framework of the TSQT I have used terms such as “elements of reality” [3, 7] in a sense which seems to be radically different from the concept of reality considered by philosophers and, apparently, this is the main reason for the current controversy.

I define that there is an element of reality at time \( t \) for an observable \( C \), \( \text{“} C = c \text{“} \) when it can be inferred with certainty that the result of a measurement of \( C \), if performed, is \( c \). Frequently, in such a situation it is said that the observable \( C \) has the value \( c \). It is important to stress that both expressions do not assume “ontological” meaning for \( c \), the meaning according to which the system has some (hidden) variable with the value \( c \). I do not try to restore realistic picture of classical theory: in quantum theory observables do not possess values. The only meaning of the expressions: “the element of reality \( C = c \)” and “\( C \) has the value \( c \)” is the operational meaning: it is known with certainty that if \( C \) is measured at time \( t \), then the result is \( c \).

Clearly, my concept of elements of reality has its roots in “elements of reality” from the Einstein, Podolsky, and Rosen paper (EPR) [8]. There are numerous works analyzing the EPR elements of reality. My impression that EPR were looking for an ontological concept and their “criteria for elements of reality” is just a property of this concept. I had no intention to define such ontological concept. I apologize for taking this name and using it in a very different sense, thus, apparently, misleading many readers. I hope to clarify my intentions here and I welcome suggestions for alternative name for my concept which will avoid the confusion.

I consider elements of reality as counterfactual statements. Even if at time \( t \) the system undergoes an interaction with a measuring device which measures \( C \), the truth of “\( C = c \)” is ensured not by the final reading of the pointer of this measurement, but by a counterfactual statement that if another measurement, with as short duration as we want, is performed at time \( t \), it invariably reads \( C = c \).

2 The three-box example

The actual story:

(i) A macroscopic number \( N \) of particles (gas) were all prepared at \( t_1 \) in a superposition of being in three separated boxes:

\[
|\psi_1\rangle = \frac{1}{\sqrt{3}}(|A\rangle + |B\rangle + |C\rangle),
\]

with obvious notation: \(|A\rangle\) is the state of a particle in box \( A \), etc.

(ii) At later time \( t_2 \) all the particles were found in another superposition (this is extremely rare event):

\[
|\psi_2\rangle = \frac{1}{\sqrt{3}}(|A\rangle + |B\rangle - |C\rangle).
\]

(iii) In between, at time \( t \), weak measurements of a number of particles in each box, which are, essentially, usual measurements of pressure in each box, have been performed.
The readings of the measuring devices for the pressure in the boxes $A$, $B$ and $C$ were
\begin{align*}
p_A &= p, \\
p_B &= p, \\
p_C &= -p,
\end{align*}
where $p$ is the pressure which is expected to be in a box with $N$ particles.

I am pretty certain that this “actual” story never took place because the probability for successful post-selection (ii) is of the order of $3^{-N}$; for a macroscopic number $N$ it is too small for any real chance to see it happens. However, given that the post-selection (ii) does happen, I am safe to claim that (iii) is correct, i.e., the measurements of pressure at the intermediate time with very high probability yielded the results (3).

The description of this example in the framework of the time symmetrized quantum formalism is as follows. Each particle at time $t$ is described by the two-state vector
\begin{equation}
\langle \psi_2 | \psi_1 \rangle = \frac{1}{3} (\langle A \rangle + \langle B \rangle - \langle C \rangle) \left( |A \rangle + |B \rangle + |C \rangle \right),
\end{equation}
The system of all particles (signified by index $i$) is described by the two-state vector
\begin{equation}
\langle \Psi_2 | \Psi_1 \rangle = \frac{1}{3^N} \prod_{i=1}^{i=N} (\langle A \rangle_i + \langle B \rangle_i - \langle C \rangle_i) \prod_{i=1}^{i=N} (|A \rangle_i + |B \rangle_i + |C \rangle_i)
\end{equation}

The ABL formula for the probabilities of the results of the intermediate measurements yields, for each particle,
\begin{align*}
P_A &= 1, \\
P_B &= 1, \\
P_A + P_B + P_C &= 1.
\end{align*}
Or, using my definition, for each particle there are three elements of reality: the particle is inside box $A$, the particle is inside box $B$, the particle is inside boxes $A$, $B$ and $C$.

A theorem in the TSQT (Ref. [3], p. 2325) says that a weak measurement, in a situation in which the result of a usual (strong) measurement is known with certainty, yields the same result. Thus, from (6) it follows:
\begin{align*}
(P_A)_w &= 1, \\
(P_B)_w &= 1, \\
(P_A + P_B + P_C)_w &= 1.
\end{align*}
Since for any variables, $(X + Y)_w = X_w + Y_w$ we can deduce that $(P_C)_w = -1$.

Similarly, for the “number operators” such as $\mathcal{N}_A \equiv \sum_{i=1}^{i=N} P_A^{(i)}$, where $P_A^{(i)}$ is the projection operator on the box $A$ for a particle $i$, we obtain:
\begin{align*}
(N_A)_w &= N, \\
(N_B)_w &= N, \\
(N_C)_w &= -N,
\end{align*}
In this rare situation the “weak measurement” need not be very weak: a usual measurement of pressure is a weak measurement of the number operator. Thus, the time-symmetrized formalism yields surprising result (3): the pressure measurement in box $C$ is negative! Its value equals minus the pressure measured in the boxes $A$ and $B$.

The analysis of “elements of reality” in this example which are clearly counterfactual statements (in actual world the measurements, results of which are quoted in (6), have not been performed) yields a tangible fruit: a shortcut for calculation of the expected outcome of an actual measurement. This outcome is surprising and paradoxical. Indeed, a usual device for measuring an observable which has only positive eigenvalues yields a negative value, the weak value in this rare pre- and post-selected situation.

There are other paradoxical aspects discussed in relation to this example. The first paradoxical issue which was discussed [11] reminds contextuality. Consider an observable $X$ which tells us the location of the particle: is it in box $A$, $B$, or $C$. The eigenstate of this observable corresponding to finding the particle in $A$ is identical to the eigenstate of the projection operator on $A$: $|X = A\rangle = |P_A = 1\rangle$. However, in this example there is no elements of reality $X = A$ (if we measure $X$ by opening all boxes at time $t$ we have only the probability $1/3$ to find the particle inside box $A$) in spite of the fact that $P_A = 1$ is an element of reality. Finally, the paradoxical aspect of the three-box example which was analyzed by Kastner I shall analyze in the next section.

3 Kastner’s analysis of the three-box example.

In the three-box example there are two elements of reality for the same particle: “the particle is inside box $A$”, and “the particle is inside box $B$”. Kastner [12] considers this situation as a paradox which she resolves by rejecting the legitimacy of my concept of elements of reality. She does not mention at all my resolution of the “paradox”. Elements of reality are counterfactual statements. To be more explicit, “the particle is inside box $A$” means that if the particle is searched in box $A$ (and if it is not searched in box $B$!) then it is certain that the particle would be found in box $A$. Obviously, the two elements of reality cannot be considered together. Each element of reality assumes that antecedent of the counterfactual statement, which is the other element of reality, is false. Thus, both elements of reality exist separately, but we should not conclude from this that there is an element of reality consisting of the union of the elements of reality: the antecedent “the particle is searched in $A$ and it is not searched in $B$ and the particle is searched in $B$ and it is not searched in $A$” is logically inconsistent. The fact that we cannot consider the union of elements of reality does not make the whole exercise empty. We still can consider consequences of all true elements of reality together. In particular, in the three-box example the consequences of elements of reality (3) are the statements about weak values (7) and weak measurements which yield these weak values can be performed together.

Kastner finds elements of reality “the particle is inside box $A$”, and “the particle is

\footnote{This example answers the criticism of Mermin [3] quoted by Kastner [10] in the context of my work. According to this criticism the elements of reality I defined are “rubbish – they have nothing to do with anything”.
}
inside box $B$” to be “highly peculiar and counterintuitive”. This is indeed so, especially
because there is no element of reality “the particle is inside box $A$ and inside box $B$”, as
it explained above. This peculiar situation is an example of the failure of the “product
rule” for pre- and post-selected elements of reality [6]. From $A = a$ and $B = b$ does not
follow $AB = ab$. The element of reality “the particle is inside box $A$ and inside box $B$
corresponds to the definite value of the product of projection operators: $P_A P_B = 1$. But
in the three-box example $P_A P_B = 0$, in spite of the fact that $P_A = 1$ and $P_B = 1$.

Kastner’s main objection is that the elements of reality “the particle is inside box $A$”,
and “the particle is inside box $B$” cannot be interpreted as applying to an individual
system because “being found in box $A$ and being found in box $B$ are mutually exclusive
states of affairs”. She does not take into account that “elements of reality” are just
counterfactual statements. She does not pay attention on the word “instead” in my
writings which she herself quotes in her paper: “If in the intermediate time it was searched
for in box $A$, it has to be found there with probability one, and if, instead, it was searched
for in box $B$, it has to be found there too with probability one...”

For demonstration that Kastner’s criticism is unfounded, let me repeat here an ex-
ample of a per-selected only situation [13] in which we attribute “mutually exclusive”
properties to an individual system.

Consider a system of two spin-$\frac{1}{2}$ particles prepared, at $t_1$, in a singlet state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2).$$

We can predict with certainty that the results of measurements of spin components of the
two particles fulfill the following two relations:

$$\{\sigma_{1x}\} + \{\sigma_{2x}\} = 0,$$  \hspace{1cm} (10)

$$\{\sigma_{1y}\} + \{\sigma_{2y}\} = 0,$$  \hspace{1cm} (11)

where $\{\sigma_{1x}\}$ signifies the result of measurement of the spin $x$ component of the first
particle, etc. The relations (10) cannot be tested together: the measurement of $\sigma_{1x}$ disturbs
the measurement of $\sigma_{1y}$ and the measurement of $\sigma_{2x}$ disturbs the measurement of $\sigma_{2y}$ (not
necessarily in the same way). According to the standard approach to quantum theory we
accept that there are two matters of fact: “the outcomes of the spin $x$ components for
the two particles have opposite values” and “the outcomes of the spin $y$ components for
the two particles have opposite values” in spite of the fact that the statements represent
“mutually exclusive states of affairs”. If the spin $x$ components have been measured at
time $t$, we know that $y$ components of spin were not measured at time $t$. Note that if
they were measured at a later time, after the spin $x$ component measurement, then the
outcomes might not fulfill the equation (11). According to Kastner’s line of argumenta-
tion the application of statements (10,11) which I named “generalized elements of reality”
(because they are not just about the values of observables, but about relations between
these values) to a single quantum system should also be rejected. However, physicists
do not reject such statements. There are innumerable works analyzing counterfactuals
related to incompatible measurements on a single system of correlated spin-$\frac{1}{2}$ particles.
Similarly, Kastner’s argumentation is not valid for the three-box example.
4 Quantum counterfactuals

I will try here to clarify my statements which were criticized in Section 4 of Kastner’s paper [12].

First, the meaning of the quotation from my work “indeterminism is crucial for allowing non-trivial time-symmetric counterfactuals” is just the following. Time-symmetric counterfactuals are related to time-symmetric background conditions, i.e. the state of the system is fixed both before and after the time about which the counterfactual statement is given. In a deterministic theory everything is fixed by conditions at a single time and, therefore, no novel (non-trivial) features can appear in the time-symmetric approach.

In order to clarify the meaning of my continuation: “Lewis’s and other general philosophical analyses are irrelevant for the issue of counterfactuals in quantum theory” let me quote Lewis’ “system of weights or priorities” for similarity relation of counterfactual worlds [14]:

\begin{align*}
(1) & \text{ It is of the first importance to avoid big, widespread, diverse violations of [physical] law.} \\
(2) & \text{ It is of the second importance to maximize the spatio-temporal region throughout which perfect match of particular facts prevails.} \\
(3) & \text{ It is of the third importance to avoid even small, localized, simple violations of law.} \\
(4) & \text{ It is of little or no importance to secure approximate similarity of particular fact, even in matters that concern us greatly.}
\end{align*}

(Lewis, p. 47)

This priorities might be helpful in the analysis of the truth value of a widely discussed counterfactual: “If Nixon had pressed the nuclear war button, the world would be very different”. The purpose of the priorities is to “resolve the vagueness of counterfactuals”. In physics context, however, the counterfactuals are not vague. (At least, I hope that counterfactuals I have defined, are not vague.) The truth value of quantum counterfactuals can be calculated from the equations of quantum theory. The above priorities cannot help in deciding the truth value of the counterfactual “the outcomes of the spin \( y \) components measurement at time \( t \) for the two particles have opposite values” in the world in which the two spin-\( \frac{1}{2} \) particles were prepared, at \( t_1 < t \), in a singlet state (9) and the spin \( y \) components were measured at time \( t \), instead. Priorities (1) and (3) are not relevant because violations of physical laws are not considered. The counterfactual worlds are different from the actual world not because of “miracles”, i.e., violations of physical laws, but because different measurements on the system are considered. And the question about how it was decided which measurement to perform, is not under discussion. Priorities (2) and (4) are not relevant because quantum theory fixes everything. In particular, there is perfect match before the time of the measurement, \( t \), and, in general, there cannot be arranged the perfect match after \( t \).

We do not have the freedom of interpretation in the framework of quantum counterfactuals after defining the similarity criteria. For the case of pre-selected counterfactuals it is simply the identity of quantum description of the system before the measurement and this is not controversial. For time-symmetrized counterfactuals there is no consensus. I have
my definition. Its advantage that it yields the standard definition as a particular case for pre-selected only situation and it allows us to analyze and derive useful results for pre- and post-selected quantum systems. I am aware of other proposals [13, 16]. Each proposal should be judged according to consistency and usefulness for the purpose it has been defined. The success or failure of various definitions of similarity criteria of counterfactuals in exact sciences is not measured by maximizing priorities (1)-(4), but by its effectiveness in the framework of a particular theory. The priorities (1)-(4) are relevant outside the framework of exact sciences, where we have no laws which determine unambiguously the truth values of counterfactual statements.

Contrary to Kastner’s writing I never claimed that Lewis’ theory is not applicable in an indeterministic universe. On the contrary, I have used Lewis’ framework of possible worlds for defining counterfactuals in quantum theory. I only claimed that most parts of Lewis’ analysis is irrelevant because counterfactuals in the context of quantum theory are of very specific form and the majority of aspects discussed in the general philosophical literature on counterfactuals are not present in the quantum case. To make things even more clear I will add another quotation from Lewis’ writings [14] with an example of argumentation for which I cannot find any counterpart in the analysis of quantum counterfactuals:

Jim and Jack quarreled yesterday, and Jack is still hopping mad. We conclude that if Jim asked Jack for help today, Jack would not help him. But wait: Jim is a prideful fellow. He never would ask for help after such a quarrel; if Jim were to ask Jack for help today, there would have to have been no quarrel yesterday. In that case Jack would be his usual generous self. So if Jim asked Jack for help today, Jack would help him after all. ...

(Lewis, p. 33)

Kastner continues by criticizing my definition of time-symmetrized counterfactual regarding results of a measurement performed on pre- and post-selected quantum system:

If it were that a measurement of an observable $A$ has been performed at time $t$, $t_1 < t < t_2$, then the probability for $A = a_i$ would be equal to $p_i$, provided that the results of measurements performed on the system at times $t_1$ and $t_2$ are fixed.

Her criticism [17] regarding “problematicity” of the fixing requirement is answered in another paper [13]. The latter was also criticized by Kastner [10]. She claims that fixing the results of measurements at $t_1$ and $t_2$ is “ad hoc gerrymndering” which relies on accidental similarity of individual facts”. But these facts are the physical assumptions in the pre- and post-selected situations for analysis of which the above concept of time-symmetrized counterfactuals has been introduced. Disregarding these facts is similar to deciding that there have been no quarrel between Jim and Jack even so the counterfactual statement starts with “Jim and Jack quarreled yesterday ...”. The definitions in physics have no ambiguity which might allow such free reading of the text.

In the present paper Kastner criticizes the syntax of the definition, in particular, that it reflects “a confusion between the non-counterfactual and counterfactual usage of the ABL rule”. In fact, I feel very unsure about the grammatical correctness of tenses in my definition. Also, I was not able to find exact philosophical definition according to which
one can decide if a certain statement is "counterfactual". However, it seems to me that the meaning of my definition is unambiguous and the name counterfactual is appropriate in the context of situations this definition was applied for. For example, in the three-box example described above, the definition is applied when it is known that in the actual world the observable $A$ (e.g. $P_A$) has not been measured.

Kastner suggests two possible "usages" of my definition. The difference, apart from using various tenses (the difference between which is beyond my linguistic understanding) is that only the second one includes the word "instead". This word is essential. According to my understanding it is implicit in every counterfactual statement, but maybe it is helpful to state it explicitly, modifying the definition to:

If it were that a measurement of an observable $A$ has been performed at time $t$, $t_1 < t < t_2$, instead of whatever took place at time $t$ in the actual world, then the probability for $A = a$ would be equal to $p_i$, provided that the results of measurements performed on the system at times $t_1$ and $t_2$ are fixed.

I hope this clarifies my definition and makes its meaning unambiguous, even so grammatically it might not be perfect. Again, Kastner’s arguments presented in her other paper [17] that this usage of my definition is “generally incorrect” have been answered in detail elsewhere [13]. Here I want only to comment on Kastner’s concluding sentence in which she writes: “[Vaidman’s] definition, as it stands, is grammatically incorrect in a way that reflects its lack of clarity and rigor with respect to the physically crucial point concerning which measurement has actually taken place”. According to my definition of time-symmetrized counterfactuals the measurement performed at time $t$ is not “the physically crucial point”, on the contrary, it plays no role in calculating the truth value of the counterfactual statement; I have noted this feature of my definition in the paper [18] which Kastner criticized. The counterfactual statement is about the counterfactual world in which at time $t$ some action was performed instead of the measurement which was performed in the actual world. Thus, the question which measurement has been actually performed is clearly irrelevant. The result of the measurement in the actual world does not add any information either, because in the framework of standard quantum theory to which the time-symmetrized formalism is applied, the results of measurements at $t_1$ and $t_2$ (which are fixed by definition) yield a complete description of the system at time $t$.

5 What does it mean: probability of a history?

I want to add a comment about a connection to the consistent histories approach [19] advocated by Kastner and presented in the Appendix to her paper. Following Cohen [20], Kastner claims that the counterfactual usage of the ABL rule is valid only for cases corresponding to “consistent” histories. Since for my counterfactuals the ABL rule is valid always, I find this approach to be an unnecessary limitation which prevents to see interesting results.

In addition, I have to admit that I was never been able to understand the meaning of a basic concept in the consistent history approach: probability of a history. A particular history associates set of values of observables in a sequential set of times. If the meaning
of probability is the probability for this set to be the results of the measurements of these observables at the appropriate times, then this is a well defined question in the framework of standard quantum theory. (The corresponding formula is given in the ABL paper [1].) Apparently, the meaning is something different. Indeed, in the example considered by Kastner, she uses the following expression:

“What is the probability that the system is in state $C_k$ at time $t_1$, given that it was preselected in state $D$ and post-selected in state $F$?”

What is the meaning of “the system is in state $C_k$? In this example the system (up to known unitary transformation) is in state $D$. This is a standard quantum state evolving towards the future. In the framework of the TSQT one can also associate with the system at time $t_1$ the backward evolving state $F$, and to say that the system is described by the two-state vector $\langle F | D \rangle$. However, from the text of Kastner’s paper it is obvious that she considers something different. She writes: “we consider a framework in which the system has some value $C_k$ associated with an arbitrary observable”. As I mentioned in Section 1, quantum observables do not possess values. Thus, I cannot understand the meaning of Kastner’s sentence: “... we cannot use the ABL rule to calculate the probability of any particular value of either $A$ or $B$ at time $t_1$...” because “probability of a value” is not defined.

In this paper I have clarified the meaning of the concepts from the time-symmetrized quantum formalism: quantum counterfactuals and elements of reality (which are particular quantum counterfactuals). I have answered recent criticism of these concepts in this journal by Kastner [12]. Kastner has claimed that the three-box example is a paradox arising from an invalid counterfactual usage of the ABL rule. I have argued here that if one adopts my definition of quantum counterfactuals, the ABL rule is valid. Peculiarities of this example do not represent a true paradox, but the unusual features of pre- and post-selected elements of reality, such as the failure of the product rule [6].

Current controversy can be added to the list of examples which led Bell to suggest abandoning the usage of the word “measurement” in quantum theory [21]. However, I do not think that abstaining from using problematic concepts is the most fruitful approach. I believe that physical and philosophical concepts which are vague and ambiguous should continue be under discussion until the concepts and the structure of the physical theory will be clear. I hope that current discussion brings us closer to constructing solid foundations for quantum theory.

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