Deciphering the Kinematic Structure of the Small Magellanic Cloud through Its Red Giant Population

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Abstract

We present a new kinematic model for the Small Magellanic Cloud (SMC), using data from the Gaia Data Release 2 catalog. We identify a sample of astrometrically well-behaved red giant (RG) stars belonging to the SMC and cross-match with publicly available radial velocity (RV) catalogs. We create a 3D spatial model for the RGs, using RR Lyrae for distance distributions, and apply kinematic models with varying rotation properties and a novel tidal expansion prescription to generate mock proper motion (PM) catalogs. When we compare this series of mock catalogs to the observed RG data, we find that a combination of moderate rotation (with a magnitude of ~10–20 km s\(^{-1}\)) at 1 kpc from the SMC center, inclination between ~50 and 80°, and a predominantly north-to-south line-of-nodes position angle of ~180° and tidal expansion (with a scaling of ~10 km s\(^{-1}\) kpc\(^{-1}\)) is required to explain the PM signatures. The exact best-fit parameters depend somewhat on whether we assess only the PMs or include the RVs as a qualitative check, leaving some tension remaining between the PM and RV conclusions. In either case, the parameter space preferred by our model is different from previously inferred rotational geometries, including from the SMC HI gas, and from the RG RV-only analyses and new SMC PM analyses, which conclude that a rotation signature is not detectable. Taken together this underscores the need to treat the SMC as a series of different populations with distinct kinematics.

Unified Astronomy Thesaurus concepts: Proper motions (1295); Stellar motion (1615); Galaxy kinematics (602); Galaxy dynamics (591); Small Magellanic Cloud (1468); Galaxy interactions (600); Magellanic Clouds (990)

1. Introduction

More than a decade ago, our paradigm for the Magellanic Clouds shifted with the measurement of proper motions (PMs) for the Large and Small Magellanic Clouds (LMC and SMC, respectively). PM measurements of the Clouds made using the Hubble Space Telescope (HST; Kallivayalil et al. 2006a, 2006b) in conjunction with orbital integrations of their orbits with the Milky Way (MW) revealed a stronger preference for the first infall scenario (Besla et al. 2007), supported by follow-up PM analysis shortly thereafter (Piatek et al. 2008). Since then, both the systemic properties and internal dynamics of the LMC have been well studied (e.g., Olsen et al. 2011; Kallivayalil et al. 2013; van der Marel & Kallivayalil 2014; Platais et al. 2015). However, its smaller companion, the SMC, has proven significantly more challenging to fully characterize, in large part due to its complicated interaction history with the LMC.

In particular, the question of rotation in the SMC has remained frustratingly inconclusive. Stanimirović et al. (2004) produced one of the first detections of a gradient in the radial velocities (RVs) of the HI in the SMC, finding that the gradient was well fit by a rotating disk, with a maximum velocity of 60 km s\(^{-1}\), a line-of-nodes position angle (LON PA) of 40°, and an inclination angle out of the plane of the sky of 40°. An examination of RVs of the red giant (RG) population of the SMC by Harris & Zaritsky (2006) did not reproduce this, instead finding conclusive evidence for rotation in the SMC. Interestingly, an RV study of the OBA stars in the SMC (Evans & Howarth 2008) did find evidence for rotation in the stars (with a velocity gradient of ~26 km s\(^{-1}\) deg\(^{-1}\), an LON PA of 126, and no reported inclination), albeit with a different geometry than that of the HI, creating a complicated picture of rotation in the SMC. Two more recent studies have reinforced this disagreement between the type of tracer and the resulting inferred rotation. Dobbie et al. (2014) conducted a spatially broader study of the RG stars than Harris & Zaritsky (2006). With the larger sample, they found a measurable gradient in the RVs, inferring an observed rotation curve with velocity rising at a rate of 20–40 km s\(^{-1}\), potentially larger than their measurement of the internal dispersion of the SMC (~26 km s\(^{-1}\)), an LON PA between 120° and 130°, consistent with that of Evans & Howarth (2008), and a moderate inclination angle, between 25° and 70°.

For HI, Di Teodoro et al. (2019) provided the highest-resolution measurements of the gas in the SMC to date. Using the new data, they found a plane of rotation consistent with that originally inferred by Stanimirović et al. (2004; maximum circular velocity of 55 km s\(^{-1}\), LON PA of ~66°, and inclination angle of ~50°), with improved errors and constraints on the uncertainty in the model fit. Intriguingly, a slight preference for this rotational geometry was also found in an RV-only analysis of OB stars by Lamb et al. (2016), at odds with other stellar measurements. However, for all of these works, a key uncertainty lies in only measuring motion in one direction, leaving open the possibility for PM measurements (that complete the 3D velocity vector) to potentially begin to resolve these tensions.

The first PM insights about the internal kinematics of the SMC began with Kallivayalil et al. (2006b), where they measured PMs for five different fields in the inner regions of
the SMC with HST, enabling a measurement of the center-of-mass motion, but not strong constraints on the internal velocity field. Kallivayalil et al. (2013) improved the precision for these five fields, but the spatial distribution of the fields prevented further analysis into the question of rotation. van der Marel & Sahlmann (2016) used PMs from the Tycho-Gaia Astrometric Solution Catalog (Lindgren et al. 2016), a combination of Gaia Data Release 1 (Gaia Collaboration et al. 2016a, 2016b) and the Hipparcos Tycho-2 Catalog (Høg et al. 2000), to add another eight PM data points to constrain the velocity field of the SMC. Their resulting analysis found inconclusive evidence for rotation, limited in part by the relatively small number of data points.

Zivick et al. (2018) attempted to expand this data set, using HST to measure PMs for another 28 fields, sampling a broader region around the SMC, but kept their analysis for rotation restricted to the plane of the sky as well. While the measurement of a rotation signal proved inconclusive, the analysis did reveal coherent motions in the southeastern portion of the SMC moving radially outwards, in the direction of the Magellanic Bridge and LMC, potentially indicative of tidal expansion. Finally, the release of Gaia Data Release 2 (DR2) (Gaia Collaboration et al. 2018b) vastly expanded the catalog of PMs for the SMC, opening up for the first time the ability for the PM data to constrain the inclination and LON PA of the rotation model in the SMC.

The initial analysis of the SMC (Gaia Collaboration et al. 2018c) found a weak rotation signal throughout the SMC (∼8 km s$^{-1}$ at 3 kpc from SMC center) at a large inclination angle (∼75°) and an LON PA (∼0°) different from that measured from the RG stars in Dobbie et al. (2014) and the H1 in Di Teodoro et al. (2019). This departure from RV-inferred parameters continued with an analysis of the young, massive stars (which should trace the H1 kinematics) in Murray et al. (2019), where they combined Gaia PMs with RVs from spectroscopic surveys and found kinematics inconsistent with coherent disk rotation. De Leo et al. (2020) also found kinematics inconsistent with rotation, this time in the RG population, using Gaia PMs and an expanded RV catalog for the RGs, but they note evidence for significant tidal fragmentation in the RG population. Grady et al. (2020) also observed this tidal stripping in the RGs as well, finding trends not only in PM space from the SMC toward the LMC, but in metallicity space as well.

Given the unusual nature of the SMC and its interaction history with the LMC, constraining the presence of rotation within the SMC could provide us with a window into its original form. The magnitude of rotation and its orientation, contrasted against the internal dispersion, may help us to understand the SMC’s progress in possibly transitioning from a dwarf irregular to a dwarf spheroidal galaxy (Besla et al. 2012). Further, these constraints on rotation in turn become a new requirement for any simulations between the LMC and SMC, furthering our understanding of both the Magellanic system and potentially their joint history with the MW.

In simulations, we have perfect knowledge of the 6D phase-space information of the motions and locations of particles, but creating a comparable observational data set is challenging. Gaia DR2, however, now provides us with an opportunity to synthesize all of the observational efforts aimed at understanding the SMC structure and to begin to piece together a more holistic picture of the SMC. Here, we use a forward-modeling approach, creating a mock SMC data set that we transform into observational kinematic space, with a view to match the Gaia data. In addition to the tens of thousands of PMs in the Gaia DR2 Catalog, we cross-match with publicly available RV studies, bringing us closer to producing a truly 3D observational data set for the stars.

In Section 2, we describe our selection of Gaia DR2 data, both for astrometric quality and SMC membership, as well as cross-matching with existing RV catalogs. Section 3 first details the creation of our spatial model of the SMC and the kinematic model applied to create a mock data catalog. We then outline our methodology for assessing the fit of a given model to the data. In Section 4, we discuss the parameter space explored for our models and present the best-fitting class of models. Finally, in Section 5, we summarize our efforts in modeling the SMC and forecast ahead to future work for further improving our understanding of the SMC internal kinematics.

2. Data Selection

2.1. Gaia DR2 Selection

For our analysis, we select all stars from the Gaia DR2 catalog within roughly 5° from the optical center of the SMC using pygacs.\footnote{https://github.com/Johannes-Sahlmann/pygacs} We then apply a series of initial cuts to jointly select both astrometrically well-behaved stars and stars likely to belong to the SMC. To remove MW foreground stars, we require all stars in our sample to have a parallax < 0.2 mas and a proper motion within 3 mas yr$^{-1}$ of the SMC systemic motion ($\mu_\alpha = -0.82 \pm 0.1$ mas yr$^{-1}$ and $\mu_\delta = -1.21 \pm 0.03$ mas yr$^{-1}$ from Zivick et al. (2018), where $\mu_\alpha \equiv \mu_\alpha \cos \delta$ and $\mu_\delta \equiv \mu_\delta$ in Gaia-provided quantities).

Next, we apply the following cut to the renormalized unit weight error (RUWE) as described in the Gaia technical note GAIA-C3-TN-LU-LL-124-01, available through the Gaia Data Processing and Analysis Consortium Public Documents page (https://www.cosmos.esa.int/web/gaia/public-dpac-documents):

$$\frac{\sqrt{\chi^2/(N - 5)}}{u_0(G, C)} < 1.40,$$

(1)

which uses the following Gaia properties:

$$N \equiv \text{astrometric}_n\_\text{good}_\text{obs}_\text{al},$$
$$\chi^2 \equiv \text{astrometric}_\text{chi2}_\text{al},$$
$$u_0 \equiv \text{Normalization factor}(G, C),$$
$$G \equiv \text{phot}\_g\_\text{mean}_\text{mag},$$
$$C \equiv \text{bp}\_\text{rp}.$$
The zero point is set to the best-fit center for the RG population (the red ×), described in Section 2. The inferred kinematic center from H1 gas is marked by the blue star, and the geometric center derived from the RR Lyrae population is marked by the green square. The SMC Wing can be seen on the eastern (left) side of the SMC, heading toward the Magellanic Bridge and the LMC. We also note the empty patch on the right side of the figure as the location of 47 Tuc, which has been removed from our sample by our astrometric criteria.

Gaia Collaboration et al. (2018a), as follows:

\[ 1.0 + 0.015(G_{BP} - G_{RP})^2 < E < 1.3 \]
\[ + 0.06(G_{BP} - G_{RP})^2, \]

which uses the Gaia properties:

\[ G_{BP} \equiv \text{phot\_bp\_mean\_mag}, \]
\[ G_{RP} \equiv \text{phot\_rp\_mean\_mag}, \]
\[ E \equiv \text{phot\_bp\_rp\_excess\_factor}. \]

As we are concerned primarily with the bright, astrometrically well-behaved stars, and to provide another check to avoid MW contamination, we select stars brighter than \( G < 18 \), leading to the final source densities in Figure 1. We note the conspicuous absence of 47 Tuc on the right side of Figure 1 as an example of the power of Gaia DR2 to remove potential contamination from spatially coincident sources.

With this initial astrometric selection, we move to further isolate the SMC stars. Examining the color–magnitude diagram (CMD) in Figure 2, we identify two clear SMC stellar tracks in our sample: the main sequence (MS, marked in blue) and the red giants (RG, red). What we have defined as the RG sequence includes both standard RG branch stars as well as a population of carbon stars (the more redward extension at \( G \sim 16 \)). However, prior work has shown that the kinematics of the carbon stars appears consistent with the standard RG stars (van der Marel et al. 2002), so to improve our statistics, we use both populations as our “RG” sample. The SMC red clump is also observable near the bottom of the CMD, but due to worsening astrometric performance at \( G > 18 \), we do not examine it further in this work. We also present all sources with a cross-matched RV (see Section 2.2 for more details) in green.

We do note the presence of a likely third stellar sequence, a group of red supergiants (RSG) located just blueward of the RG branch, that contains a number of the RV measurements (indicated by the green points overlaid on the CMD), Rudimentary isochrone fits to the RSG sequence find a good agreement to an age \( \sim 150 \) Myr. As this age is consistent with the most recent time of peak interaction between the LMC and SMC (Zivick et al. 2018; Joshi & Panchal 2019; Martinez-Delgado et al. 2019), which may introduce complex kinematic signatures into the data, we choose to leave a more thorough study of this intermediate-age population to a future effort.

We next consider the spatial structure of these populations in the top panel of Figure 3. The MS and RG populations show a marked difference in their structure. The MS stars occupy an irregular distribution, roughly tracing the main optical body of the SMC with minimal presence in the SMC halo, and with numerous smaller clumps embedded within the larger structure. The RG population, however, appears to be approximately azimuthally isotropic, with only a radial dependence for its spatial density. This difference extends into the PM space as well (Figure 4), where the center of the RG PM distribution, fit with a Gaussian, is markedly offset from the bulk of the MS PMs. The distribution of the PMs also appears different, with the RGs having a smoother distribution while the MS stars have clear asymmetries toward the left (eastward) side of the figure. Given the smoother distribution of the RGs in both spatial and kinematic spaces and the wealth of literature on the structure of older stars in the SMC (e.g., Subramanian & Subramaniam 2012; Jacyszyn-Dobrzeniecka et al. 2017), we choose to only concentrate on the RGs for our current analysis.

2.2. RV Cross-matching

To attempt to provide full 3D velocity information for our selected stars, we look to cross-match our catalog with existing
exceeding any sources that may have been double counted, we find over 4000 sources common to our DR2 RG selection.

After a further examination of this RV catalog, we ultimately choose not to include it in our quantitative data-to-model evaluation. As will be more fully explained in Section 3.2, the introduction of the tidal expansion component adds a significant distance-dependent effect for a given stellar motion. In a regime like the Gaia PMs with over 100,000 stars, we expect to be sampling the full distance distribution, a necessary requirement for comparing our 3D spatial model to the data. However, given the comparably sparser sampling of the RV stars, we cannot be sure that requirement is met. As such, we limit the RV catalog to providing a qualitative check on the consistency and predictions of our models.

3. Data Analysis

3.1. Observational Data

Before any analysis of the internal kinematics of the SMC RGs can begin, we must first deal with correcting for viewing perspective effects, which cause the 3D motion of the SMC to project differently as we change our line of sight. However, this correction requires assumptions regarding the location (in distance and on the sky) and magnitude of the 3D motion. As Gaia affords us the opportunity to only examine individual stellar populations (as opposed to averaging over multiple populations), we derive the systemic properties for the perspective corrections directly from the RG data.

For this, we fit two 2D Gaussians to the spatial and PM distributions of the RGs, finding a center of \((\alpha, \delta)\) \((J2000) = (13^\circ04, -73^\circ10)\) and a systemic RG PM of \((\mu_\alpha, \mu_\delta) = (0.67 \text{ mas yr}^{-1}, -1.20 \text{ mas yr}^{-1})\). The spatial center is marked in Figure 1, and the systemic PM is marked in Figure 4. The center generally agrees well with previous analyses of the older stellar structure in the SMC (e.g., Jacyszyn-Dobrzeniecka et al. 2017), and the PM value is in line with other Gaia-based measurements of the RG systemic motion (e.g., De Leo et al. 2020). For a distance to the center, we use the estimates from Jacyszyn-Dobrzeniecka et al. (2017) for a distance modulus of \((m - M) = 18.91\).

With these three values in hand, we can turn to examining the PMs in the frame of the SMC. However, accurately determining the PMs in the frame of the SMC requires more than simply subtracting the systemic PM of the SMC from each star. As the total motion of the SMC is in truth a 3D vector, how that vector will project into the plane of the sky, and radial components will change as a function of position on the sky. This difference is often referred to as the “viewing perspective” correction and is vital to account for when examining relative PMs over a large area of sky, as in the case of the SMC. For a full explanation and treatment of the geometry involved, we refer the reader to van der Marel et al. (2002), in particular Equations 8, 9, and 13. With the viewing perspective correction in hand, we subtract it and the systemic PM from each individual PM to calculate the “residual” PM. We then convert these residual PMs into a Cartesian frame as outlined in Gaia Collaboration et al. (2018c). For display, the 100,000 stars are binned every 0.5° and the average residual PM is calculated. The resulting averaged residual SMC RG PMs are displayed in Figure 5 with the top panel displaying the full SMC window and the bottom panel a zoomed-in plot of the SMC core.
As other prior analyses have noted, there is a clear outward motion, particularly on the eastern side of the SMC toward the direction of the Magellanic Bridge. However, apparent rotation can be seen in the center of the SMC, with the averaged residual vectors displaying a distinct counterclockwise pattern in the inner 2° of the SMC. To investigate this further, we decompose the residual PM of each star into radial ($\mu_{\text{rad}}$) and tangential ($\mu_{\theta}$) components, relative to the assumed RG center. Figure 6 displays $\mu_{\theta}$ (which should be correlated with rotation) as a function of radius for all RG stars. For easier interpretation, we average $\mu_{\theta}$ every 0.1° along with the calculated error of the mean in Figure 6, marked by the orange circles and error bars. In this space, coherent rotation should appear as a nonzero $\mu_{\theta}$ (above or below zero depending on the direction of rotation). When we examine the data, we see that this is, in fact, the case, with $\mu_{\theta}$ continuing to rise in magnitude over the full radius, though the peak $\mu_{\theta}$ value (around $\sim$0.03 mas yr$^{-1}$, which corresponds to $\sim$9 km s$^{-1}$ at 60 kpc) stays roughly the same between 0.7° and just past 2° before beginning a significant incline. While this is not a large value, it does appear to be statistically significant across the full radius.

Returning to the more obvious feature in the outward eastern motions, the idea of tidal expansion or disruption has often been invoked in trying to understand this structure in the residual SMC PMs. To place this idea into context, we calculate the relative velocity between the LMC and SMC, $v_{\text{rel}}$. We use the new systemic motion for the RG population and take care to perform this subtraction in 3D Galactocentric coordinate space before transforming $v_{\text{rel}}$ back into the observed frame (where we find its components in our Cartesian frame to be ($\mu_{\alpha}$, $\mu_{\delta}$, $v_{\phi}$) = (0.43 mas yr$^{-1}$, −0.21 mas yr$^{-1}$, −36.6 km s$^{-1}$)). The PM components of $v_{\text{rel}}$, which we will refer to as $\mu_{\text{rel}}$, are displayed as the dark red vector at (0, 0) in Figure 5.

Immediately we see that the direction of $\mu_{\text{rel}}$ agrees with the general direction of the residual PMs in addition to the minor axis of the SMC. Combined with the hint of coherent rotation in the interior of the SMC (observed in the bottom panel of Figure 5), we propose that the internal velocity field of the SMC can be approximately modeled by two mechanisms: a cylindrical rotation and a linearly increasing tidal component. To fully understand this parameter space, we attempt to build a 3D model of the SMC, as described in the following section.

We do note the potential complications introduced by systematic Gaia errors that have a specific spatial frequency, in
particular, the Scanning Law (see the Gaia technical note GAIA-C3-TN-LU-LL-124-01). We address this potential concern in two key ways. First, we primarily focus on bright stars ($G < 18$) that are less affected by this systematic. Second, we structure our ultimate comparisons of data to our models (as described in later sections) around a large spatial coverage significantly larger than the spatial scale of the systematic error fluctuations.

3.2. Mock Data Creation

To create the mock SMC data set, we make the assumption that the RG population can be approximately modeled using a simple Gaussian distribution for each spatial dimension. For the dimensions in the plane of the sky, we use the values from the earlier 2D Gaussian fit to the RG Gaia data. We find a standard deviation of approximately 1.3 kpc for the longer axis and 1.0 kpc for the shorter axis with a rotation of $\sim 55^\circ$. For the line-of-sight (LOS) depth, we refer to Jacyszyn-Dobrzeniecka et al. (2017) for their measurements of the RR Lyrae distances and axial ratios in the SMC. Additionally, as has been shown, the range of LOS distances varies in the SMC as a function of spatial position. To account for this, we vary the Gaussian mean of the distance distribution as a function of $X$ and $Y$, which is able to reproduce the observed tilt in the SMC (with the southeastern corner being the closest to the observer and the northwestern corner being the furthest, as seen in the top-right panel of Figure 7 for the case of $X$ versus $Z$).

We then apply a random subsampling to the central region of our model to reflect the loss in astrometrically well-behaved stars observed in the RG Gaia data. At this point, we note that in the Gaia data there appear to be two different stellar components: a dense core, relatively well fit by a 2D Gaussian, and an offset halo of stars (offset as it appears to extend significantly farther to the northeast which can be clearly seen in the top-left panel of Figure 7). As this may represent a separate component of the SMC with unique kinematic properties, we choose to only model the dense core of the SMC, resulting in the final spatial model seen in the bottom-left panel of Figure 7. Additionally, as the projection of stellar distances into the $X/Y$ space can be nonobvious, we have binned the stars across the face of the SMC to demonstrate the 3D nature of our SMC, shown in the bottom right panel of Figure 7. With respect to the $X/Y$ space of our SMC, we do observe that our modeling does not quite capture the apparent boxiness of the observed data (top-left panel of Figure 7). However, as we are focused on developing a simple physical intuition for SMC kinematics, we believe the model to be appropriate for our purposes.

With a 3D spatial model complete, we turn to assigning velocities to each of our mock stars. For this, we adopt the formalism outlined in van der Marel et al. (2002, hereafter vdM02) to describe observations of a rotating solid body whose center of mass (COM) has some nonzero 3D motion. For more description, we refer the reader to that work. Here we will provide an outline of the key concepts used in creating the kinematic model, including the mechanisms described earlier. The kinematics for each star in our model can be described with the following:

\[
\begin{pmatrix}
  v_1 \\
  v_2 \\
  v_3 \\
\end{pmatrix}_\text{CM} = \begin{pmatrix}
  v_1 \\
  v_2 \\
  v_3 \\
\end{pmatrix}_\text{int} + \begin{pmatrix}
  v_1 \\
  v_2 \\
  v_3 \\
\end{pmatrix}_\text{pn} + \begin{pmatrix}
  v_1 \\
  v_2 \\
  v_3 \\
\end{pmatrix}_\text{tidal},
\]

where the different velocity components are: $v_{CM}$, the COM motion of the SMC; $v_{int}$, the precession and nutation of the velocity plane of the SMC; $v_{int}$, the internal rotation and dispersion of the SMC; and $v_{tidal}$, the tidal expansion due to the interactions with the LMC.

For the initial $(x, y, z)$ frame, we orient it such that that $0, 0, 0$ is located at the dynamical center of the SMC, with the positive $z$-axis pointing in the direction of the observer, the positive $x$-axis toward the west, and the positive $y$-axis pointing north. We first assign $(x, y, z)$ coordinates for each star in our SMC model then use these coordinates to calculate angular coordinates in the frame of the observer ($\rho, \phi$), following Equation (2) from vdM02. These coordinates inform the decomposition of the different velocity Cartesian components into the $(v_1, v_2, v_3)$ frame (hereafter the $v_i$ frame).

To calculate $v_{CM}$, the model is created with an initial kinematic center $(\alpha_0, \delta_0, D_0)$ and associated motion $(\mu_\alpha, \mu_\delta, v_z)$. Using these quantities, the transverse velocity ($v_t$) and direction of the transverse velocity ($\Theta_t$) measured east over north) are calculated. Combined with the star’s angular coordinates, we use Equation (13) from vdM02 to calculate the velocities for each star given its angular position on the sky.

For $v_{int}$, we assume a linearly increasing rotation curve, with a maximum velocity of $V_0$ at a scale radius of $R_0$, after which the rotation flattens out, as described in Equation (4):

\[
V(R') = \begin{cases} 
R' - V_0 & R' < R_0 \\
R_0 & R' \geq R_0
\end{cases}
\]

For simplicity, we assume cylindrical rotation, where $R'$ is only dependent on the $x'$ and $y'$ coordinates of the star in the rotating frame, denoted by $\ell$. While this likely represents an oversimplification of rotation in older, at least partially dispersion-supported, systems, we choose this prescription for consistency with other prior analyses of SMC rotation. For the plane of rotation, we define an inclination $i$ and an LON PA $\theta$ to describe the orientation of the plane with respect to the internal spatial frame of the galaxy. We then convert the internal Cartesian vectors to the $v_i$ frame using Equation (21) from vdM02. Once in the $v_i$ frame, we add a velocity “kick” to each component, randomly drawn from a normal distribution using 26 km s$^{-1}$ (from Dobbie et al. 2014) as a standard deviation.

As a brief aside related to the internal motions, we note that we are aiming to only model the observable motions of the SMC. As such, we are not considering asymmetric drift when reporting our final rotation curves. For the SMC, the known dispersion in the RG population of $>20$ km s$^{-1}$ would require significant corrections in order to model the rotation curve predicted by the underlying gravitational potential. However, the observational impact of asymmetric drift on older stellar populations, the flattening of the rotation curve beyond a certain scale radius due to random kicks and perturbations experienced by individual stars, mirrors the model we apply for rotation. As such, our modeled rotation can be understood as the curve measured for non-drift-corrected motion, just as one would measure in the raw Gaia data as well.

For $v_{pn}$, knowledge of the time dependency of $d\theta/dt$ and $di/dt$ is required. For the SMC, there are no known constraints for $d\theta/dt$, so we choose to not include it in the modeling. Attempts
have been made at constraining $d\ell/dt$ in the SMC (e.g., Dobbie et al. 2014; Di Teodoro et al. 2019), but the parameter space is still fairly unconstrained. Additionally, for an object the size of the SMC, contributions from even a fairly large $d\ell/dt$ would only account for up to a few km s$^{-1}$ in the RV component, significantly smaller than our uncertainties from other factors. As such, we choose to not include $v_{\text{pn}}$ in our final calculation.

For $v_{\text{tide}}$, given the complex gravitational interactions between the SMC and LMC, we do not attempt a rigorous numerical treatment. Instead, we use the expectation that tidal expansion should occur along the direction of relative motion between the two bodies to create a prescription for the individual stars, as seen in numerical simulations of dispersion-supported systems on plunging orbits into larger hosts (e.g.,

![Figure 7](image-url)
Peñarrubia et al. 2009). In this scenario, stars are evacuated out of the satellite across all locations, eventually forming wide tidal tails on either side of the satellite, rather than only through the Lagrange points into thin, kinematically cold tails, as observed in tidally disrupted globular clusters in the MW. As described earlier in Section 3.1, we calculate the relative velocity, \( v_{rel} \), between the LMC and SMC to determine the direction of the tidal component, which for simplifying purposes we will assume to be fixed for every star. For the magnitude of the motion for an individual star, we use the distance from the SMC center along \( v_{rel} \), \( d_{rel} \), in conjunction with a scaling ratio between 0 and 15 km s\(^{-1}\) kpc\(^{-1}\), \( v_{scale} \) (the possible range based on the relative motions in the Magellanic Bridge in Zivick et al. 2019), giving the final tidal contribution for each star, displayed below in Equation (5).

\[
v_{tadal} = d_{rel} \cdot v_{scale} \cdot \frac{v_{rel}}{|v_{rel}|}.
\]  

With all of the components combined together in the \( v_i \) frame, we then convert these physical velocities (km s\(^{-1}\)) into observed motions (mas yr\(^{-1}\)) for \( \mu_x \) and \( \mu_y \) using Equation (9) from vdM02. We add a random kick, selected from a Gaussian distribution with a standard deviation of 0.15 mas yr\(^{-1}\), calculated using the RG PM errors in Section 2, to the measured \( \mu_x \) and \( \mu_y \) to reflect the uncertainties in the Gaia DR2 catalog. This results in a final catalog for our model of stars, with each one possessing (R.A., decl., \( \mu_x \), \( \mu_y \), \( v_{LoS} \)) comparable to the data.

### 3.3. Comparison Quantification

To quantify our data, we require that our model be able to reproduce both the magnitude of the residual motions and the location of the residual motions. To that end, we construct a set of spatial versus kinematic comparisons. As we have chosen to only focus on the Gaia PMs, ignoring the RVs due to the uncertainties in the underlying distance distribution of the sample, and as we do not have distance information for individual stars, we are limited to four comparisons: \( \mu_x \) versus \( x \), \( \mu_y \) versus \( y \), \( \mu_x \) versus \( y \), and \( \mu_y \) versus \( y \).

For a choice of the \( x/y \) frame, we adopt the SMC’s shape as a natural frame, assigning the \( x \)-axis to align with the major axis and the \( y \)-axis to align with the minor axis of the SMC RG distribution (as seen by the dashed black lines in Figure 5). After rotating the positions and PMs of the RGs into this new frame, we bin the PMs along the spatial axis, using the Freedman–Diaconis (FD) rule to set the bin width (Freedman & Diaconis 1981) to avoid overfitting the data. We set the bin widths using the observational data but limit the comparison between model and data to only bins containing model data.

For each bin, we calculate the average of the residual PMs and the variance in the bin. Two examples of this spatial-kinematic scheme can be seen in Figure 8, which displays only the Gaia RG data. Then, we calculate a \( \chi^2 \) value between the mock data from each proposed kinematic model and the Gaia RG data for each of the four spatial-kinematic permutations. Bins with fewer than five stars are excluded from this calculation to avoid unintentionally biasing the comparisons toward sparsely sampled outer regions of the SMC model. To compare between different models, the four separate \( \chi^2 \) values for a given model are summed to produce a single statistic that is then minimized. Figure 9 demonstrates this comparison, with the mock data from an example model overplotted on the underlying data (each with binned averages and the standard error of the mean), with the spatial range being narrower than the data-only spatial-kinematic distributions from Figure 8 (due to our focus on the SMC core and not the extended halo structure).

### 4. Model-to-data Comparison Results

In exploring the parameter space, we choose to focus our analysis on four key parameters of the model: inclination angle \( i \), the position angle of the LON \( \Theta \), the rotation velocity \( V_0 \), and the tidal expansion scale factor \( v_{scale} \). Because we are utilizing a simple 3D model for the SMC and a forward-modeling approach aimed at building physical intuition for the SMC kinematics, we simply test the parameter space in discrete steps for each of the parameters.

For the remainder of the parameters, we choose to keep the distance \( D_0 \), the proper motion \( (\mu_W, \mu_N) \), center \( (\alpha_0, \delta_0) \), the...
For the distance and systemic velocity, both have already been better established for the RG population in prior studies (e.g., Dobbie et al. 2014; Jacyszyn-Dobrzeniecka et al. 2017). For the center and systemic PM, as we have already measured them based on the data, we choose not to test them to avoid introducing unexpected biases in our $\chi^2$ calculation. For the rotation scale radius, we choose to approximate it by using the radius at the initial peak of $\mu_\theta$ versus radius in Figure 6, at roughly $\sim 1^\circ$ (which corresponds to $\sim 1$ kpc at a distance of 60 kpc). Given the relatively flat behavior of $\mu_\theta$ (with small perturbations) for much of the inner radius until the large increase in the outer region, consistent with the effects of tidal expansion dominating, we think this scale radius is appropriate for our analysis. Finally, as described earlier, given the large degree of uncertainty and relatively small effect of the precession/nutation, $d\mu/dt$, we do not include it.

The parameter space step gradation for each parameter is as follows:

1. $i$: every 10° from 0° to 90° (rotation in plane of sky versus edge-on rotation).
2. $\Theta$: every 10° from 0° to 360°.
3. $V_0$: every 5 km s$^{-1}$ from 0 to 30 km s$^{-1}$, which probes the lower to upper range of previous inferred values for the SMC stellar component.
4. $v_{\text{scale}}$: every 5 km s$^{-1}$ kpc$^{-1}$ from 0 km s$^{-1}$ kpc$^{-1}$ to 15 km s$^{-1}$ kpc$^{-1}$.

After creating a mock set of data for each permutation of the above parameter space, the summed $\chi^2$ values were compared. As this was not a continuous exploration of the parameter space but a discrete one, we aim to identify a class of best-fitting models and examine the commonalities among them to build our understanding of the SMC. For this, we bin the $\chi^2$ values using the FD rule and calculate the gradient across the bins, choosing the largest positive gradient as the cutoff for the best-fitting class of models. Using this criterion results in roughly 430 models selected out of the over 12,000 tested.

Across this class of best-fitting models, we find a need for both a nonzero rotation ($V_0$ between 15 and 25 km s$^{-1}$, with the assumed $R_0$ of 1 kpc, placing the rotation likely below the measured dispersion of the SMC RGs but notably nonzero), at a relatively high inclination ($i$ between 70° and 90°) with an LON oriented from north to south ($\Theta$ around 150°–230°), and a nonzero tidal expansion component ($v_{\text{scale}}$ near 10 km s$^{-1}$ kpc$^{-1}$). The median parameter values from the best-fit class and comparisons to literature values can be found in Table 1. Estimates on the uncertainty in the model fitting can be found in Table 2, where the errors have been estimated using the median absolute deviation (MAD). Figure 10 shows the residual vector plot for the median parameters of the best-fit class of models, which displays similar key characteristics to
the Gaia data: large and roughly linear motion in the eastern portion of the SMC and a weak signal of rotation within the inner 1 degree of the spatial distribution.

We can see this in the spatial-kinematic comparisons in Figure 9, where our model is able to capture the behavior across much of the SMC, including asymmetric behavior in the $y$ versus $\mu_Y$ space. This asymmetry similarity can be seen in the residual vector plots as well (Figure 10), where the vectors on the eastern side display significantly larger magnitudes than the vectors on the western side. We primarily attribute this behavior to a combination of two different factors.

First, our distance-dependent prescription for tidal expansion combined with the relative LMC–SMC vector and the 3D structure of the SMC lead to a maximizing of the tidal contribution. This is due to the southeastern–northwestern axis having the largest LOS depth, which coincides with the direction of the relative motion (as displayed in the top-right panel of Figure 7), leading to larger tidal contributions than if the two directions were perpendicular (rather than roughly parallel) to each other. That they have this alignment may speak to the evolution of the SMC and its interactions with the LMC, but more detailed analyses and comparisons to simulation work of the Clouds will be required to be able to properly evaluate the significance of this alignment.

Second, this large distance gradient in the LOS depth results in the observed PMs (regardless of underlying physical mechanisms) between the two sides being significantly different (~15% for stars sitting ±5 kpc from the center of the SMC, ~30% at ±10 kpc, which begins to occur even a mere 2° from the center of the SMC as seen in the top-right panel of Figure 7). Given the magnitude of the underlying velocities, these fractional differences can result in significant differences in their projection into our observed space.

We do note some deviations from the observations, particularly on the western side of the SMC. Tied into the above discussion of the asymmetry, our model predicts slightly larger motions than observed around 2° from the center of the SMC. We also note that there appears to be a region in the southwest that has predominantly western residual PMs, in contrast to the same region in our model that predicts a northwestern motion. In both instances, one potential explanation could be a foreground feature that would lead to a different average distance (and thus a different tidal contribution).

However, on the whole, we do appear to capture the general behavior of the SMC, especially the eastern portion that appears to be well described by our model, capturing the rotation signal layered on top of the large tidal motions, which appears as the large vectors slowly changing in direction as a function of azimuthal position. As a final note on the comparisons, we note that because our model for the SMC only aimed to capture the central spatial density of the SMC
Figure 10. Residual PM vector plots for the Gaia RG stars (orange–red, left) and the best-fit model (purple, right). The frame and vectors are displayed in the same manner as Figure 5. The Gaia-measured residual PMs have been limited to only the bins that have measured PMs in the mock SMC data, due to the model only including a single 2D Gaussian function that does not extend fully into the halo of the SMC. We note that the noisier signal in the model at $\sim3^\circ$ from the center, compared to the Gaia data, results from a sparsity of stars (due to the model being constructed only to match the central SMC region) rather than an underlying physical mechanism.

Figure 11. (Left column) Binned and averaged residual radial velocities (RVs) for publicly available data, (middle column) our model-generated mock data subsampled to the same spatial density as the public data, (right column) and our full model-generated mock data. The model corresponding to the median values of the best-fit class of models is displayed in the top row of the middle and right columns, and the model with the smallest $V_0 \sin i$ from the best-fit class is displayed on the bottom row of the middle and right columns. An ellipse fit to the edges of the full mock data has been overlaid on the plots to help guide the eye in comparing spatial locations between the plots. Gaps in the spatial coverage of the available RVs can be seen, and the impact of the distance dependence in the tidal expansion prescription can be observed in the apparent outlier values at the edges of the mock data (where stars may lie significantly in front or behind the assumed SMC center).
and not the more diffuse “halo” of stars (easiest seen in Figure 7), the noise in our model gets significantly worse near the edges where it can be only one or two stars providing the signal, unlike in the Gaia RG data where there are many more stars per bin at ~3° from the SMC center.

As a final check on our best-fit class of models, we compare our predicted RVs to the cross-matched RV catalog. Given the degeneracy between $V_0$ and $i$ (where larger $V_0$ can be consistent with PM constraints given a large enough $i$), for a more complete comparison to the RV catalog, we also display the model from the best-fit class with the lowest $V_0$ sin $i$ as an example of predicted RVs for a slower-rotating, more face-on inclination model fails to reproduce the fast-rising $i/\alpha$ as a function of radius that we see in the data (in Figure 6), placing some amount of tension on results from PMs and results from RVs.

As one interesting note in the construction of the model, the apparent LON PA in the slower-rotating, lower-inclination model would stretch from the northeast to the southwest. However, these mock data were generated using an LON PA of $\theta = 170^\circ$, which would normally appear from north–northwest to south–southwest. The reason for this apparent shift lies in the underlying distance distribution of the SMC and the interplay between that distribution and our tidal prescription, where stars on the eastern side of the SMC are likely to be closer to the observer and receive a tidal contribution in the negative relative RV direction (which can be seen in the actual RV data on the central-eastern edge of the SMC). Similarly, stars in our model in the southwest region tend to lie behind the assumed SMC center and receive a tidal contribution in the opposite direction, enhancing the expected rotation component there.

This leads to the first of two noticeable areas of deviation between the models and the data. In the RV data, the southwest region appears to have stars at both extremes of the relative RVs there. With this tension, it is possible that our underlying 3D model of the SMC does not accurately capture the distance distribution of the RGs here, with more stars sitting either near the SMC center or in front of the SMC center, leading to less tidal contribution and a smaller relative RV signal. Examining the relative PMs from Figure 9 may appear to support this interpretation as the average PMs there are pointing toward the southeast, which would suggest the tidal component is playing a significant role and thus those stars would need to lie in front of the SMC center. However, whether this is simply an incorrectly modeled, but still smooth, distribution of stars or a more discrete substructure that is sitting in front of the SMC, we are currently unable to constrain.

The second notable area is in the northwest region of the SMC, where there is a clear trend in the RV data of a positive relative RV. Our model, due to focusing on the central core of the SMC, does not fully cover the sampled area here, but it is clear from the trends in our models that this large, positive RV would not be predicted. Unlike the southwest region, where it may be a relatively small correction to the distance assumption (as our model does capture some of the expected RV scatter there), the northwest region appears to be a substantial deviation. Perhaps the most straightforward explanation would be the presence of a stellar substructure that breaks with the apparently smooth RG spatial distributions. However, more work will be required, perhaps with the aid of simulations, in better understanding the type of structure that could lead to this signal in the northwest.

Underscoring all of these comparisons is the clear need for more RV measurements in the SMC. Already we can see that the RVs can help us in constraining a likely value for $V_0$ sin $i$ and in identifying potentially interesting substructure in the SMC. An expanded catalog will allow us to go further and better map the full kinematic structure of the SMC, including better constraining the LON PA of the rotation plane, which we can see poses a significant challenge, given its complicated interactions with the tidal expansion of the SMC.

Despite these complications, all of our best-fit class of models appear to be converging on the need for both a nonzero tidal expansion component and a nonzero coherent rotation curve in order to explain the observed PMs and RVs, placing this work in contrast to some of the most recent SMC Gaia efforts. Of De Leo et al. (2020), Murray et al. (2019), and Gaia Collaboration et al. (2018c), only the last study found any need for rotation (see values in Table 1). While the preferred inclination angle is similar to our median value, their preferred rotation curve is noticeably slower than ours, and their LON PA is nearly opposite to ours. However, this is not necessarily surprising, given both the differences in the assumed systemic properties (PM, kinematic center, and distance) and that their sample includes both young and old stars, which will have very different kinematic histories. Thus, it becomes difficult to fully assess this as an apples-to-apples comparison.

Similarly, the results of Di Teodoro et al. (2019) and Murray et al. (2019) are both focused on understanding the kinematics of the gas of the SMC, making a direct comparison challenging. Given the turbulent interaction history between the SMC and LMC, the dynamics of the SMC gas could have very likely been perturbed beyond a simple rotating disk, and these perturbations would have then transferred to the young stars examined in Murray et al. (2019). That the older, collisionless stars appear to disagree with the inferred kinematics of the gas and the youngest, most recently formed stars in the SMC avoid placing these in conflict as well.

The most direct comparison in the literature to our work is De Leo et al. (2020), who also focused on SMC RGs with a combination of RV and PM information. There they concluded that the magnitude of the tidal disruption occurring prevented...
any potential rotation from being detected. Certainly, in looking at the residual PMs on a large scale (like in the top panel of Figure 5), it would not appear that there is any obvious rotation. However, as we have demonstrated, with a careful removal of viewing perspective effects and simultaneously fitting for both rotation and tidal effects, there is a real rotation signature across the whole of the SMC. Taken together, these differing conclusions underscore the power of our forward-modeling technique and the need for a variety of analytical approaches to fully investigate the complexities of the SMC.

5. Conclusions and Discussion

With the release of Gaia DR2, the amount of kinematic information available for the Magellanic system expanded exponentially. Beyond PM information, novel spectroscopic surveys of the Magellanic Clouds have added new RV and metallicity information, bringing us closer to a more complete understanding of the Clouds. New combinations of these different parameter spaces will offer the opportunity to constrain the Clouds’ interactions and underlying physical properties. To help set the foundation for these more complex models and analyses of the SMC, we have presented a novel approach to disentangling the PMs of the SMC RG population.

We create a mock SMC that includes a tidal expansion and a rotation component for its stars and use existing frameworks for transforming 3D velocities for resolved stellar systems into observable quantities \(v_{dM02}\) to predict the PM distribution of the Gaia RGs. The result is a model capable of providing a physically intuitive explanation to the otherwise unusual-looking PM structure of the SMC, one which needs both a coherent rotation and tidal expansion to fully capture the PM behavior. Rotation alone is insufficient to explain the observed PM behavior in the SMC, and invoking tidal expansion of the stars, similar to the kinematics observed in simulations of dispersion-supported satellite systems on plunging orbits, offers a possible explanation. However, to model this effect, we are required to work with the velocities in true 3D space as the 3D structure of the SMC will drastically affect the implied PM distribution due to the tidal expansion and geometric projection effects. Therefore, this forward-modeling approach is crucial.

Using the data-derived systemic properties, we find a preference for kinematic models with a moderate rotation curve \((\sim 20 \text{ km s}^{-1})\), an LON pointing from the north to the south \((\Theta \sim 190^\circ)\), a large inclination angle \((\sim 80^\circ)\), and a tidal scale factor of \(10 \text{ km s}^{-1} \text{ kpc}^{-1}\). However, there exists a degeneracy between the magnitude of the rotation and the inclination of the rotation plane, with larger inclinations leading to preferring larger rotation velocities. To break this degeneracy and fully constrain the rotation curve, we will require more RV measurements to provide full 3D motions for a larger portion of our PM catalog.

Already, we have seen the ability of the currently available RVs to place some constraints on the kinematics, with the actual \(V_0 \sin i\) likely being smaller than the one in our median model. However, the slower-rotating, low-inclination model tested does struggle to replicate the fast-rising tangential PM component seen in the PMs, suggesting that the reality potentially lies somewhere in the middle of these two models. Beyond the PM and RV tensions, there appear to be significant deviations in the RV data that will require more sophisticated 3D modeling of the SMC and potentially allowing for spatial substructure to explain some of the more unusual signals, the significance of which will be clarified as the total RVs available continue to increase. Accordingly, we look toward those future RV surveys and data releases which will allow us to build a more complete picture of the SMC kinematic structure. In particular, improving RG sampling in the outer regions of the SMC (farther than \(\sim 3^\circ\) from the SMC center) will be a crucial constraint on the rotation plane of the SMC, both increasing the number densities in the already observed southwest and northeast outer regions in addition to filling in the sparse southeast and northwest regions.

The combination of the existence of coherent stellar rotation and a clear offset in the systemic RG properties (center, bulk PM) from previously measured studies with a mixed stellar sample suggests the need for a reassessment of how we understand the SMC. To this, we propose a new interpretation similar to the Bullet Cluster, where in the last interaction with the LMC, the SMC gas was slowed and pulled toward the LMC, while the RG stars passed through collision free, kinematically separating from the newly forming stars. Recent work for the LMC has revealed similar population-dependent kinematics, potentially due to interactions with the SMC, providing new constraints on understanding this system (Wan et al. 2020). Moving forward, we underscore the necessity of identifying distinct SMC stellar populations and treating their kinematics separately, which may provide a better handle on constraining the LMC–SMC interaction, as opposed to measuring averaged systemic SMC properties over multiple stellar populations. Future work will be necessary to properly characterize the kinematics of different stellar populations from each other, including modifying existing prescriptions or introducing new ones capable of capturing the behavior of younger stars born from turbulent gas.

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