Ricci curvature non-minimal derivative coupling cosmology with field re-scaling

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In this letter, cosmology of a simple NMDC gravity with \(\xi R_{\mu\nu} \phi^\mu \phi^\nu\) term and a free kinetic term is considered in flat geometry and in presence of dust matter. A logarithmic field transformation \(\phi' = \mu \ln \phi\) is proposed phenomenologically to ensures domination of the NMDC term at small field values. Assuming slow-roll approximation, equation of motion, scalar field solution and potential are derived as function of kinematic variables. The field solution and potential are found straightforwardly for power-law, de-Sitter and super-acceleration expansions.

I. INTRODUCTION

Various observations such as supernova type Ia (SNIa) [1–10], large-scale structure surveys [11, 12], cosmic microwave background (CMB) anisotropies [13–16] and X-ray luminosity from galaxy clusters [15, 17, 18] found recent cosmic acceleration and yet to be known form of dark energy is believed to drive the acceleration [19–22]. As far as the combined CMB data is concerned, the data favors the universe with a cosmological constant and a dark matter, that is the \(\Lambda\)CDM model. Severe inconsistency in the value of the observed value of \(\Lambda\) and the prediction of the quantum field theory motivates attempt to find alternative theory to cope with the present small value of dark energy. Amongst these attempts are dynamical scalar field models, for examples, quintessence [23] and k-essence [24–26] and modified gravity such as braneworlds, \(f(R)\), scalar-tensor theories (see e.g. [27, 28] for reviews). When the scalar field is non-minimally coupling to the gravity part, the acceleration is possible. Coupling function \(f(\phi, \phi, \phi')\) can be motivated in scalar quantum electrodynamics or in gravitational theory of which Newton’s constant is a function of the density [29]. Capozziello [30] found that other possible coupling terms apart from \(R_{\mu\nu} \phi^\mu \phi^\nu\) and \(R^\mu^\nu \phi_{\mu\nu}\) are not necessary. These terms can be lower energy limits of higher dimensional theories or Weyl anomaly of \(\mathcal{N}=4\) conformal supergravity [31, 32]. The scalar derivative term coupling to the gravitational sector is known as non-minimal derivative coupling-NMDC and theory with such term, e.g. \(\kappa_1 R_{\mu\nu} \phi^\mu \phi^\nu\) and \(\kappa_2 R^\mu^\nu \phi_{\mu\nu}\), without \(V(\phi)\) nor \(\Lambda\) could give de Sitter expansion [33] and this fact attracts cosmological consideration. NMDC models with these coupling terms were studied with further modifications [34–36]. A special case of when there are two terms with two coupling constants related by \(\kappa = -2\kappa_2\) gives rise to a theory of two NMDC terms making the Einstein tensor, \(G^{\mu\nu}\) coupled to the scalar field derivative term as seen in literatures [37–51]. Recent review of the studies in this area can be seen in Ref. [52].

In this letter, we consider the simplest NMDC model with \(\xi R_{\mu\nu} \phi^\mu \phi^\nu\) as the only NMDC coupling term and a free kinetic term. We propose modification to the model to enhance the NMDC term in the small field value range so that the NMDC term can phenomenologically be dominant at small field value. Introducing field transformation to the model the NMDC term could be effective at late time (for the quadratic or Higgs potential) or at inflation (for runaway potential, e.g. exponential potential). Hence the model can be either considered as dark energy model or inflationary model. We study here cosmological field equation of this model in a flat FLRW universe. We give a correction to a previous similar result. Assuming power-law, de Sitter and super-acceleration expansions, the scalar field solutions and scalar potential functions are found.

II. COSMOLOGY OF THE NMDC GRAVITY COUPLING TO RICCI CURVATURE

In this work, we consider the action,

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G} R - \frac{1}{2} \partial^\mu \phi' \partial_\mu \phi' - \frac{1}{2} \xi R \partial^\mu \phi' \partial_\mu \phi' - V(\phi') \right] + S_m. \tag{1}
\]

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The Lagrangian density has dimension of \( m^4 \), the scalar field \( \phi' \) has dimension of \( m^2 \) and the Ricci curvature has the dimension of \( \text{length}^{-2} = m^2 \). A transformation \( \phi' \to \phi = \mu \ln \phi \) is proposed here phenomenologically so that the domination of the NMDC terms is enhanced for small field value as seen later. The field \( \phi \) is dimensionless. The mass dimension of the field prior to the transformation (\( \phi' \)) is with the constant \( \mu \). Result of the transformation is an action with re-scaling field in all terms of the field function,

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G} R - \frac{1}{2} \left( \frac{\mu^2}{\phi^2} \right) \partial_{\mu} \phi \partial^{\mu} \phi - \frac{1}{2} \xi R \left( \frac{\mu^2}{\phi^2} \right) \partial_{\mu} \phi \partial^{\mu} \phi - V(\phi) \right] + S_m. \tag{2}
\]

All terms need to have dimension of \( m^4 \). Hence, in this theory, the coupling constant \( \xi \) has dimension of \( m^{-2} \) or \( \text{length}^2 \). Previously the simplest \( \xi R\phi, \mu \phi^\mu \) NMDC model was studied and the scalar potential was derived assuming power-law expansion [53]. The model with free kinetic term and the two NMDC terms: \( \kappa_1 R\phi, \mu \phi^\mu \) modulating with field re-scaling factors \( 1/\phi^2 \) was phenomenologically proposed also by Granda in Ref. [54]. Having the re-scaling factor in his model, the coupling constant, \( \kappa_1 \) and \( \kappa_2 \) become dimensionless. When both of the NMDC terms are dominant, they play the role of dark matter, giving dust expansion solution. In order to obtain present acceleration, the potential is needed in the theory. In our model, modulation with field re-scaling \( 1/\phi^2 \) is achieved by the field transformation and we have only one NMDC term, \( \xi R\phi, \mu \phi^\mu \) in our theory. Varying the action (2) in the metric formalism, we obtain

\[
T_{\mu \nu} = \frac{\mu^2}{\phi^2} \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu \nu} \left( \frac{\mu^2}{\phi^2} \nabla_\rho \phi \nabla^\rho \phi \right) - g_{\mu \nu} V(\phi')
+ \xi \left[ \left( R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R \right) \left( \frac{\mu^2}{\phi^2} \nabla_\rho \phi \nabla^\rho \phi \right) + \xi R \left( \frac{\mu^2}{\phi^2} \nabla_\mu \phi \nabla_\nu \phi \right) \right.
- g_{\mu \nu} \nabla_\rho \nabla^\rho \left( \frac{\mu^2}{\phi^2} \nabla_\sigma \phi \nabla^\sigma \phi \right) + \xi \nabla_\mu \nabla_\nu \left( \frac{\mu^2}{\phi^2} \nabla_\rho \phi \nabla^\rho \phi \right) \right]. \tag{3}
\]

Consider a flat, homogeneous and isotropic universe, the metric is of FLRW type,

\[
ds^2 = -dt^2 + a^2(t)dx^2 \tag{4}
\]

The (00) and (ii) components of the Einstein equation give the field equations,

\[
H^2 = \frac{8\pi G}{3}(\rho_\phi + \rho_m), \quad 2\dot{H} + 3H^2 = -8\pi G(P_\phi + P_m). \tag{5}
\]

Combining these two field equations, therefore

\[
\dot{H} = -4\pi G(P_{\text{tot}} + \rho_{\text{tot}}), \tag{6}
\]

where \( P_m, P_\phi \) are the pressure of the matter and scalar field, \( \rho_{\text{tot}}, \rho_m, \rho_\phi \) are respectively the energy density of matter and scalar field, \( P_{\text{tot}} = P_m + P_\phi \) and \( \rho_{\text{tot}} = \rho_m + \rho_\phi \). We consider dust matter here, hence \( P_m = 0 \). The (00) and (ii) components of tensor \( T_{\mu \nu} \), are as follows

\[
\rho_\phi = T_{00} = \frac{\mu^2}{2} \frac{\phi^2}{\phi^2} + V(\phi') + 3\mu^2 \xi \left[ 2H + 3H^2 \phi^2 \phi^2 + 2H \phi \phi \phi^3 - 2H \phi^3 \phi^3 \right], \tag{7}
\]

and

\[
P_\phi = T_{11} = \frac{\mu^2}{2} \frac{\phi^2}{\phi^2} - V(\phi') + \mu^2 \xi \left[ (2H + 3H^2) \phi^2 \phi^2 - 4H \phi \phi \phi^3 + 4H \phi^3 \phi^3 \right.
+ 2 \left( \phi \phi \phi^3 + \phi \phi \phi^3 - 5 \phi^2 \phi \phi^4 + 3 \phi^4 \phi^4 \right) \right]. \tag{8}
\]

Our results give minor correction to the result given in Ref. [53]. Varying the action (1) with respect to another dynamical variable \( \phi' \) to obtain Euler-Lagrange equation of the NMDC field, and transforming \( \phi' \) to \( \phi \), hence

\[
\ddot{\phi} + 3H \dot{\phi} - \frac{\phi^2}{\phi} \ddot{\phi} - \frac{\phi^2}{\phi} \xi \left( 6H + 12H^2 \right) + 6\xi \dot{H} \phi + 6\xi H \left( 7H + 6H^2 \right) \phi + \xi \left( 6H + 12H^2 \right) \phi + \frac{\phi^2}{\phi} \frac{d}{d\phi} V(\phi) = 0. \tag{9}
\]
Under slow-roll assumption of \(0 \ll |\dot{\phi}| \ll 1\) and \(|\ddot{\phi}| \ll |\dot{\phi}| \ll |\dddot{\phi}|\) we neglect the terms with \(\dddot{\phi}^2, \dddot{\phi}^2\), \(\dddot{\phi}^2\), \(\dddot{\phi}^2\) and \(\dddot{\phi}^2\) hence the pressure and density are

\[
\rho_\phi \simeq \frac{\mu^2}{2} \frac{\phi^2}{\phi^2} + V(\phi) + 3\mu^2 \xi \left(2\dot{H} + 3H^2\right) \frac{\dot{\phi}^2}{\phi^2} + 2H \frac{\dot{\phi}}{\phi^2} \phi.
\]  

(10)

and

\[
P_\phi \simeq \frac{\mu^2}{2} \frac{\ddot{\phi}^2}{\phi^2} - V(\phi) + \mu^2 \xi \left(2\dot{H} + 3H^2\right) \frac{\dot{\phi}^2}{\phi^2} - 4H \frac{\dot{\phi}}{\phi^2} \phi.
\]  

(11)

Using these approximated quantities, the Eq. (6) is

\[
\dot{H} \simeq -\frac{8\pi G}{2} \left[\frac{\mu^2}{2} \frac{\phi^2}{\phi^2} + 4\mu^2 \xi \left(2\dot{H} + 3H^2\right) \frac{\dot{\phi}^2}{\phi^2} + 2\mu^2 \xi H \frac{\dot{\phi}}{\phi^2} \phi + \rho_m\right].
\]  

(12)

Consider linear approximation of the the NMDC field equation,

\[
\dddot{\phi} \approx -3H \ddot{\phi} - \left(\frac{\phi^2}{\mu^2}\right) \frac{d}{d\phi} V(\phi).
\]  

(13)

Hence Eq. (12) is approximated as

\[
\dot{H} \simeq -4\pi G \left[\left(1 + 8\xi \dot{H} + 6\xi H^2\right) \frac{\mu^2}{2} \frac{\phi^2}{\phi^2} - 2\xi H \frac{\dot{\phi}}{\phi^2} V(\phi) + \rho_m\right],
\]  

(14)

which recovers the standard GR form when \(\xi \to 0\), that is \(\dot{H} = -4\pi G \left(\dddot{\phi}^2 + \rho_m\right)\). The Eq. (14) is a combination of two field equations. The other way of expressing \(\dot{H}\) is to consider the time derivative of the Friedmann equation,

\[
\dot{H} = \frac{4\pi G}{3H} (\dot{\rho}_\phi + \dot{\rho}_m),
\]  

(15)

The term \(\dot{\rho}_\phi\) can be found from Eq. (10). Using approximation (13),

\[
\dot{\rho}_\phi \simeq 3 \left[\frac{\phi^2}{\phi^2} \mu^2 \left(-H - 12\xi \dot{H} + 2\xi \ddot{H} - 18\xi H^3\right) - 6\xi \frac{\dot{\phi}}{\phi^2} V(\phi) \left(\dddot{H} + H^2\right)\right],
\]  

(16)

which reduces to GR case, \(\dot{\rho}_\phi = -3H \dddot{\phi}^2\), as \(\xi \to 0\). Using this \(\dot{\rho}\) in the Eq. (15) to obtain

\[
\dot{H} \simeq -4\pi G \left[\left(1 + 12\xi \dddot{H} + 2\xi \dddot{H} - 18\xi H^3\right) \frac{\phi^2}{\phi^2} \mu^2 + 6\xi \frac{\dot{\phi}}{\phi^2} \frac{dV}{d\phi} \left(\dddot{H} + H^2\right) + \rho_m\right]
\]  

(17)

or rewritten as

\[
\frac{dV}{d\phi} \simeq \frac{-H \dddot{H} - 4\pi G \left(\frac{\dddot{H} + 12\xi \dddot{H} H - 2\xi \dddot{H} - 18\xi H^3}{H} \right) \frac{\phi^2}{\phi^2} \mu^2 + 4\pi G \dot{\rho}_m \dddot{H} H}{24\pi G \xi \dddot{H} + \dddot{H} + H^2}.
\]  

(18)

This expression of \(dV/d\phi\) comes from the time derivative of the Friedmann equation (15). We use it in the other \(\dot{H}\) equation (14) so that

\[
\mu^2 \frac{\phi^2}{\phi^2} \dddot{\phi}^2 \simeq \frac{-6\dddot{H}^2 - 8H^2 \dddot{H} - 8\pi G \dot{\rho}_m \left(3\dddot{H} + 4H^2\right)}{8\pi G \left[3H + 4H^2 + \xi \left(36H^4 + 54H^2 \dddot{H} + 24H^2 - 2H \dddot{H}\right)\right]}.
\]  

(19)

Since \(\dot{\rho}_m = \rho_{m,0} a^{-3}\), hence Eq. (19) can be integrated to give a scalar field solution,

\[
\phi(t) \simeq \exp \left\{\frac{1}{H} \int dt \left[\frac{-6\dddot{H}^2 - 8H^2 \dddot{H} - 8\pi G \dot{\rho}_m(a) \left(3\dddot{H} + 4H^2\right)}{8\pi G \left[3H + 4H^2 + \xi \left(36H^4 + 54H^2 \dddot{H} + 24H^2 - 2H \dddot{H}\right)\right]}\right]^{1/2}\right\}
\]  

(20)
Using the Friedmann equation, the scalar field density (10), the linear approximation in Eq. (13) and Eq. (14), the scalar potential is derived,

\[
V(a, H, \dot{H}, \ddot{H}) \simeq \frac{6\dot{H}}{8\pi G} + \frac{3H^2}{8\pi G} + 4\rho_m(a) + \frac{1}{2} \left( 5 + 54\xi H^2 + 36\xi \dot{H} \right)
\times \left[ -6\dot{H}^2 - 8H^2\ddot{H} - 8\pi G\rho_m(a) \left( 3\dot{H} + 4H^2 \right) \right].
\]

(21)

If exact form of \(a = a(t)\) is known, one can straightforwardly derive the scalar field solution and the scalar potential.

### III. CASE STUDY: EXPLICITLY KNOWN FORMS OF EXPANSION

The cosmic system in the late universe where the ingredients are dark energy and dark matter could be viewed as the presence of the NMDC field with dust matter fluid. In this section we assume the known form of expansion so that the scalar field solution and the scalar potential can be derived straightforwardly. The expansions considered are of power-law, de-Sitter and super-acceleration types.

#### A. Power-law expansion

We consider an expansion function, \(a \propto t^p\) where \(p > 0\). To find the exact scalar field solution of this case, we need to neglect small contribution of \(\rho_m\) in order to obtain exact solution,

\[
\phi(t) \simeq \phi_0 \exp \left[ \frac{\sqrt{2p}}{\mu \sqrt{8\pi G}} \right] \arcsinh \left( \frac{t}{\sqrt{p} \alpha} \right),
\]

(22)

where \(\alpha^2 = \frac{36p^2 - 54p + 20}{4p^3} \simeq 9p - \frac{27}{4}\) and \(\chi = \mu \sqrt{8\pi G / \sqrt{2p}}\). The scalar potential is found,

\[
V(\phi) \simeq \frac{3(p - 2)}{8\pi G \xi \alpha^2 \sinh^2[\chi \ln(\phi/\phi_0)]} + \frac{5\alpha^2 \sinh^2[\chi \ln(\phi/\phi_0)] + 54p - 36}{8\pi G \xi \alpha^4 \sinh^2[\chi \ln(\phi/\phi_0)] \{ \sinh^2[\chi \ln(\phi/\phi_0)] + 1 \}}.
\]

(23)

#### B. de-Sitter expansion

The de-Sitter expansion, \(a \propto e^{H_0 t}\) is assumed and we keep the dust matter contribution in the solution here. The scalar solution is

\[
\phi(t) \simeq \phi_0 \exp \left[ -\frac{2 \rho_{m,0} \exp(-3H_0 t)}{3 \mu H_0 \sqrt{1 + 9 \xi H_0^2}} \right],
\]

(24)

with the potential

\[
V(\phi) \simeq \frac{3H_0^2}{8\pi G} + \frac{27}{8} H_0^2 \mu^2 (1 + 6\xi H_0^2) \left[ \ln \left( \frac{\phi}{\phi_0} \right) \right]^2.
\]

(25)

#### C. Super-acceleration expansion

We assume super-acceleration expansion, \(a = a_0 [(t_s - t)/(t_s - t_0)]^q\) where \(q < 0\) and \(t_s\) is the future singularity. Neglecting dust matter density, one can find the solution

\[
\phi(t) = \phi_0 \exp \left[ -\left( \frac{\sqrt{2q}}{\mu \sqrt{8\pi G}} \right) \arcsinh \left( \frac{t_s - t}{\sqrt{q} \beta} \right) \right],
\]

(26)

where \(\beta^2 = \frac{36q^2 - 54q + 20}{4q^3} \simeq 9q - \frac{27}{4}\) and the potential,

\[
V(\phi) = \frac{3(q - 2)}{8\pi G \xi \beta^2 \sinh^2[\chi \ln(\phi/\phi_0)]} + \frac{5\beta^2 \sinh^2[\chi \ln(\phi/\phi_0)] + 54q - 36}{8\pi G \xi \beta^4 \sinh^2[\chi \ln(\phi/\phi_0)] \{ \sinh^2[\chi \ln(\phi/\phi_0)] + 1 \}},
\]

(27)

which looks similar to the power-law case.
CONCLUSIONS

In this letter, we have derived cosmological field equations of a proposed phenomenological NMDC model containing $\xi R_{\phi} \mu \phi^\mu$ term with a logarithm field transformation $\phi' = \mu \ln \phi$ in a flat universe. The transformation giving modulating $1/\phi^2$ factor ensures domination of the NMDC term at small field values. This work modifies the action as well as gives minor corrections of the results given in Refs. [53] and [54]. Assuming slow-roll approximation, we derive scalar field equation of motion. In presence of dust matter, scalar field solution and scalar potential are derived as function of kinematic variables $a, H, H'$ and $H''$. Therefore, the field solution, $\phi(t)$ and the potential, $V = V(\phi)$ can be found straightforwardly for explicit forms of the expansion function $a(t)$. Dynamically significant expansion functions: power-law, de Sitter and super-acceleration expansions are assumed and scalar field solutions and potential are derived. Further dynamical analysis can be investigated in future with the aim to test the effect of the NMDC term at small field value, $\phi \ll 1$ for a chosen potential.

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