$B_d^0 - \bar{B}_d^0$ mixing in the left-right supersymmetric model

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We analyze $B_d^0 - \bar{B}_d^0$ mixing in a fully left-right supersymmetric model. We give explicit expressions for all the chargino, gluino, gluino-neutralino and neutralino amplitudes involved in the process. We calculate the mass difference $\Delta m_d$ and CP asymmetry $a_{J/\psi K_s}$ in both the constrained case (where the only flavor violation comes from the Cabibbo-Kobayashi-Maskawa matrix) and the unconstrained case (including soft supersymmetry breaking terms). The constrained case does not contain any new information beyond the supergravity-inspired MSSM. In the unconstrained case, the main contribution to $B_d^0 - \bar{B}_d^0$ and the CP asymmetry is due to either gluino diagrams, if the dominating flavor mixing arises in the down squark sector, or chargino diagrams, if the dominant flavor mixing comes from the up squark sector. We include numerical results and compare this analysis with the ones performed in other models.

PACS number(s): 12.60.Jv, 13.25.Hw, 14.40.Nd

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1 Introduction

Flavor changing neutral currents (FCNC) and charge parity (CP) violating phenomena are some of the best probes for physics beyond the Standard Model (SM). All existing measurements so far are consistent with the SM predictions involving the Cabibbo-Kobayashi-Maskawa (CKM) matrix as the only source of flavor violation.

In the SM, FCNC are absent at tree level, appear at one loop level, but they are effectively suppressed by the Glashow-Iliopulos-Maiani (GIM) mechanism and small CKM angles. In supersymmetric models, there is no similar mechanism to suppress the loop contributions to either flavor or CP violating phenomena. Experimental studies of flavor physics, especially in B decays, appear essential for the understanding of the mechanism for supersymmetry breaking. With increased statistical power of experiments at B factories, rare B decays will be measured very precisely.

CP violation arises in the SM from complex Yukawa couplings in the charged current, leading to a physical phase in the CKM matrix. In supersymmetric extensions of the SM there are additional sources of CP violation, due to the presence of new phases in the supersymmetric Lagrangian. CP violation was observed first in the kaon system [1]. Recently both BaBar [2] and BELLE [3] collaborations have provided clear evidence for CP violation in the B-system, although at present the experimental errors are relatively large. Even if these observed CP asymmetries agree, within errors, roughly with the SM prediction, there is still considerable space available for new physics, and supersymmetry particularly. However, new phases introduced by supersymmetry must be small, as constrained by electric dipole moments of the neutron, electron, and mercury atom [4].

It appears that the newly measured CP asymmetry, $a_{J/\Psi K_s}$, in the decay $B \rightarrow J/\Psi K_s$ requires large supersymmetric contributions to $B_d^0 - \overline{B}_d^0$ mixing. The asymmetry is defined
as follows

\[ a_{J/ΨK_s}(t) = \frac{\Gamma(B_d(t) \to J/ΨK_s) - \Gamma(\bar{B}_d(t) \to J/ΨK_s)}{\Gamma(B_d(t) \to J/ΨK_s) + \Gamma(\bar{B}_d(t) \to J/ΨK_s)} = -a_{J/ΨK_s} \sin(\Delta m_{B_d} t). \]  

(1)

BaBar and BELLE have announced the following results

\begin{align*}
    a_{J/ΨK_s} & = 0.59 \pm 0.14 \pm 0.05 \text{ (BaBar)}, \\
    a_{J/ΨK_s} & = 0.99 \pm 0.14 \pm 0.06 \text{ (BELLE)}. 
\end{align*}

(2)

The present world average is \( a_{J/ΨK_s} = 0.79 \pm 0.12 \) [5]. In the SM, \( a_{J/ΨK_s} \) is related to the inner angle of the unitarity triangle

\[ a_{J/ΨK_s}^{SM} = \sin 2\beta; \quad \beta = \arg(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}}). \]  

(3)

The experimental results agree well with the SM prediction, so this asymmetry can be a good probe to explore new physics and put stringent constraints on the models.

Rare B decays involving loop induced flavor changing neutral transitions are sensitive to the properties of the internal heavy particles, making them particularly suitable as probes of physics beyond the SM. It can be expected that there are considerable new physics contributions to \( B_0^0 - \bar{B}_d^0 \) mixing. However, these contributions appear small in a supergravity-inspired MSSM [6]. Therefore \( B_0^0 - \bar{B}_d^0 \) provides for excellent opportunities to test physics beyond MSSM.

\( \Delta B = 2 \) decays had been studied in the framework of supersymmetric models with a universal soft supersymmetry breaking terms [7]. It was shown that non-universal realizations of SUSY could give large contributions to \( \Delta F = 2 \) observables [8], making them distinguishable from the MSSM. It becomes possible in this scenario to discover SUSY indirectly in precision measurements of B-physics.

Although some attempts have been made to reconcile \( \Delta B = 2 \) with right-handed \( b \)-quark decays [9], a complete analysis of the \( B_0^0 - \bar{B}_d^0 \) mixing in a fully left-right supersymmetric model is still lacking. In our previous work [10, 11], we analyzed the \( \Delta B = 1 \)
processes in the context of the left-right supersymmetric model and found new contributions. We also found that these processes place tight bounds on supersymmetric flavor violation parameters. We extend this work here to $\Delta B = 2$ processes with the hope of adding one more piece to the puzzle of $B$ physics.

The Left-Right Supersymmetric (LRSUSY) models [12, 13], based on the group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, incorporate the advantage of supersymmetry within a natural framework for allowing neutrino masses through the seesaw mechanism [14]. LR-SUSY models can be embedded in a supersymmetric grand unified theory such as $SO(10)$ [15]. They would also appear in model building realistic brane worlds from Type I strings. This involves left-right supersymmetry, with supersymmetry broken either at the string scale $M_{SUSY} \approx 10^{10-12}$ GeV, or at $M_{SUSY} \approx 1$ TeV, the difference having implications for the gauge unification [16].

In this paper we study all contributions of the LRSUSY model to the $B_d^0 - \overline{B}_d^0$ mixing at one-loop level. The process can be mediated not only by left- and right-handed $W$ bosons and charged Higgs bosons as in nonsupersymmetric case, but also by charginos, neutralinos and gluinos. The structure of the LRSUSY model provides significant contributions from the right-handed squarks and an enlarged gaugino-higgsino sector with right-handed couplings, which are not as constrained as the right-handed gauge sector in left-right symmetric models. We anticipate that these would contribute a large enhancement of the mass difference and CP asymmetry and would constrain the parameter space of the model.

The paper is organized as follows. In Sec. II, we review the main features of LRSUSY and give the supersymmetric contributions to the $\Delta B = 2$ process. In Sec. III, we present the numerical analysis and compare the calculation with experimental results, to constrain the parameters of the model. We reach our conclusions in Sec. IV.
2 The analytic formulas

The minimal supersymmetric left-right model is based on the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The matter fields of this model consist of three families of quark and lepton chiral superfields transforming as the adjoint representations of the groups. The Higgs sector consists of the bidoublet and triplet Higgs superfields:

$$
\Phi_1 = \begin{pmatrix}
\Phi_{11}^0 & \Phi_{11}^+ \\
\Phi_{12}^- & \Phi_{12}^+
\end{pmatrix}, \quad
\Phi_2 = \begin{pmatrix}
\Phi_{21}^0 & \Phi_{21}^+ \\
\Phi_{22}^- & \Phi_{22}^+
\end{pmatrix}
$$

$$
\Delta_L = \begin{pmatrix}
\frac{1}{\sqrt{2}} \Delta_L^- & \Delta_L^0 \\
\Delta_L^0 & -\frac{1}{\sqrt{2}} \Delta_L^-
\end{pmatrix}, \quad
\delta_L = \begin{pmatrix}
\frac{1}{\sqrt{2}} \delta_L^+ & \delta_L^{++} \\
\delta_L^0 & -\frac{1}{\sqrt{2}} \delta_L^+
\end{pmatrix}
$$

$$
\Delta_R = \begin{pmatrix}
\frac{1}{\sqrt{2}} \Delta_R^- & \Delta_R^0 \\
\Delta_R^0 & -\frac{1}{\sqrt{2}} \Delta_R^-
\end{pmatrix}, \quad
\delta_R = \begin{pmatrix}
\frac{1}{\sqrt{2}} \delta_R^+ & \delta_R^{++} \\
\delta_R^0 & -\frac{1}{\sqrt{2}} \delta_R^+
\end{pmatrix}.
$$

The bidoublet Higgs superfields appear in all LRSUSY and serve to implement the $SU(2)_L \times U(1)_Y$ symmetry breaking and to generate the CKM mixing matrix. Supplementary Higgs representations are needed to break left-right symmetry spontaneously: either doublets or triplets would achieve this, but the triplet Higgs $\Delta_L, \Delta_R$ bosons have the advantage of supporting the seesaw mechanism. Since the theory is supersymmetric, additional triplet superfields $\delta_L, \delta_R$ are needed to cancel triangle gauge anomalies in the fermionic sector. The symmetry is broken spontaneously to $U(1)_{em}$. There are three different stages of symmetry breakdown. At the first stage only discrete parity is broken.

In the second stage of symmetry breaking, LRSUSY is broken down to the MSSM at $M_R$ by the vevs of the neutral triplet Higgs bosons $\langle \Delta_R \rangle \neq 0, \langle \delta_R \rangle \neq 0$. The final stage of symmetry breakdown takes place at electroweak scales $M_L$ and MSSM is broken down to $U(1)_{em}$ through bidoublet vevs $\kappa_1, \kappa_2 \neq 0$. In addition, supersymmetry can be broken at any scale between $M_R$ and $M_L$.

The most general superpotential involving these superfields is

$$
W = Y_Q^{(i)} Q^T \Phi_i \tau_2 Q^c + Y_L^{(i)} L^T \Phi_i \tau_2 L^c + i(Y_{LR} L^T \tau_2 \delta_L L + Y_{LR} L^{cT} \tau_2 \Delta_R L^c)
$$
where $W_{NR}$ denotes (possible) non-renormalizable terms arising from higher scale physics or Planck scale effects [17]. The presence of these terms insures that, when the SUSY breaking scale is above $M_{W_R}$, the ground state is R-parity conserving [18]. In addition, the potential also includes well-known $F$-terms, $D$-terms as well as soft supersymmetry breaking

$$
\mathcal{L}_{soft} = \left[ A^i_Q Y^Q (i) \tilde{Q}^T \Phi_i i \tau_2 \tilde{Q}^c + A^i_L Y^L (i) L^T \Phi_i i \tau_2 \tilde{L}^c + i A_{LR} Y_{LR} (L^T \tau_2 \delta_L \tilde{L} + L^T \tau_2 \Delta_R \tilde{L}^c) 
\right. \\
\left. + (m^2_\Phi)_{ij} \Phi_i^\dagger \Phi_j \right] + \left[ (m^2_L)_{ij} \tilde{l}_i^T \tilde{l}_j + (m^2_R)_{ij} \tilde{l}_R^T \tilde{l}_R \right] \\
- M^2_{LR} [Tr(\Delta_R \delta_R) + Tr(\Delta_L \delta_L) + h.c.] - [B \mu_{ij} \Phi_i \Phi_j + h.c.] 
$$

(6)

These parts of the Lagrangian are responsible for flavor violation in lepton and quark decays in general, and in the B system in particular.

The contributions of the left-right supersymmetric model to the $B_q^0 - \bar{B}_q^0$ $(q = d, s)$ mixing are given by the effective Hamiltonian

$$
\mathcal{H}_{eff}^{B=2} = \sum_i [C_i(\mu)Q_i(\mu) + \tilde{C}_i(\mu)\tilde{Q}_i(\mu)]. 
$$

(7)

where the relevant operators entering the sum are

$$
Q_1 = \tilde{q}^\alpha \lambda^\beta \gamma_\mu \lambda^\gamma b^\alpha \lambda^\beta, \\
\tilde{Q}_1 = \tilde{q}^\alpha \lambda^\beta \gamma_\mu \lambda^\gamma b^\alpha \lambda^\beta, \\
Q_2 = \tilde{q}^\alpha \lambda^\beta \gamma_\mu \lambda^\gamma b^\alpha \lambda^\beta, \\
\tilde{Q}_2 = \tilde{q}^\alpha \lambda^\beta \gamma_\mu \lambda^\gamma b^\alpha \lambda^\beta, \\
Q_3 = \tilde{q}^\alpha \lambda^\beta \gamma_\mu \lambda^\gamma b^\alpha \lambda^\beta, \\
\tilde{Q}_3 = \tilde{q}^\alpha \lambda^\beta \gamma_\mu \lambda^\gamma b^\alpha \lambda^\beta, \\
Q_4 = \tilde{q}^\alpha \lambda^\beta \gamma_\mu \lambda^\gamma b^\alpha \lambda^\beta, \\
\tilde{Q}_5 = \tilde{q}^\alpha \lambda^\beta \gamma_\mu \lambda^\gamma b^\alpha \lambda^\beta. 
$$

(8)
Figure 1: Leading box diagrams contributing to $B_d^0 - \bar{B}_d^0$ mixing.

The Wilson coefficients $C_i$ and $\tilde{C}_i$ are initially evaluated at the electroweak or soft supersymmetry breaking scale, then evolved down to the scale $\mu$. In the SM and constrained SUSY models, $\tilde{Q}_i$ contributions are generally suppressed by $O(m_q/m_b)$ compared with the contributions from $Q_i$. However this is not the case in generic SUSY models such as non-universal models, or in left-right models. Because of left-right symmetry, we must consider all contributions from both chirality operators. The $B_d^0 - \bar{B}_d^0$ mixing is mediated through the box diagrams Fig. 1. Below we give a comprehensive list of all box diagram contributions.

### 2.1 The Standard Model and Left-Right Symmetric Contributions

The SM and left-right symmetric contributions to the $\Delta B = 2$ transitions have been discussed before and are well-known [19]. We include them below for completeness.

The SM can only generate contribution to $C_1$ by the $t - W$ box. The contribution due to $W_R$ in the left-right symmetric model can be obtained from the SM by replacing $L$ with $R$, so contribute to $\tilde{C}_1$. The $W_L - W_R$ mixing can contribute to $C_4$.

$$C_4^{SM} = \frac{(\alpha_W)^2}{4M_{W_L}}(K_{t\bar{q}}K_{tb})^2 x_{W_L} A(x_{W_L}),$$  \hspace{1cm} (9)
\[ \tilde{C}_{1}^{LR} = \left( \frac{\alpha_W}{4 M_{W_L}} \right)^2 (K_{tq}^* K_{tb})^2 x_{tW_R} A(x_{tW_R}) \]  
\[ C_{4}^{LR} = \left( \frac{\alpha_W}{4 M_{W_L}} \right)^2 2 x_{W_L W_R} (K_{tq}^* K_{tb})^2 \sqrt{x_{W_L} x_{q' W_L}} [(1 + x_{tW_L} x_{q' W_L} x_{W_L W_R})] \]

where \( K_{ij} \) is the CKM matrix and \( q' = u(c) \) for \( q = d(s) \). We set in all our formulas \( x_{ab} = m_a^2/m_b^2 \) and all functions can be found in [11].

### 2.2 The Charged Higgs Contribution

These are the contributions from the left-right symmetric model. We include contributions from both the left and right handed quark sector, assuming the same CKM matrix in both.

\[ C_{1}^{H^-} = \left( \frac{\alpha_W}{4 M_{W_L}} \right)^2 (K_{tq}^* K_{tb})^2 \{ x_{tW_L} x_{tH} \cot^4 \frac{1}{4} G(x_{tH}, x_{tH}) \]  
\[ + 2 \cot^2 \beta x_{tW_L}^2 [F'(x_{tW_L}, x_{tW_L}, x_{H W_L}) + \frac{1}{4} G'(x_{tW_L}, x_{tW_L}, x_{H W_L})] \}, \]  
\[ \tilde{C}_{1}^{H^-} = C_{1}^{H^-} (L \to R), \]  
\[ C_{2}^{H^-} = -\left( \frac{\alpha_W}{4 M_{W_L}} \right)^2 (K_{tq}^* K_{tb})^2 x_{bW} x_{tW_L} x_{tH} \{ F(x_{tH}, x_{tH}) \]  
\[ + 2 \cot \beta F'(x_{tW_L}, x_{tW_L}, x_{H W_L}) \}, \]  
\[ \tilde{C}_{2}^{H^-} = C_{2}^{H^-} (L \to R). \]  

### 2.3 The Chargino Contribution

\[ C_{1}^{\tilde{\chi}^-} = \frac{\alpha_W^2}{16} \sum_{h,k=1}^{6} \sum_{i,j=1}^{5} \frac{1}{m_{\tilde{\chi}_j}} (G_{UL}^{ij} - H_{UR}^{ij}) (G_{UL}^{*ikq} - H_{UR}^{*ikq}) \]  
\[ (G_{UL}^{iib} - H_{UR}^{iib}) (G_{UL}^{jihq} - H_{UR}^{jihq}) G'(x_{\tilde{u}_h \tilde{\chi}_j}, x_{\tilde{u}_h \tilde{\chi}_j}, x_{\tilde{\chi}_j}), \]  
\[ \tilde{C}_{1}^{\tilde{\chi}^-} = C_{1}^{\tilde{\chi}^-} (L \leftrightarrow R), \]  
\[ C_{3}^{\tilde{\chi}^-} = -\frac{\alpha_W^2}{4} \sum_{h,k=1}^{6} \sum_{i,j=1}^{5} \frac{1}{m_{\tilde{\chi}_j}} (G_{UR}^{ij} - H_{UL}^{ij}) (G_{UL}^{*ikq} - H_{UR}^{*ikq}) \]  
\[ (G_{UR}^{iib} - H_{UL}^{iib}) (G_{UL}^{jihq} - H_{UR}^{jihq}) \sqrt{x_{\tilde{\chi}_j}} F'(x_{\tilde{u}_h \tilde{\chi}_j}, x_{\tilde{u}_h \tilde{\chi}_j}, x_{\tilde{\chi}_j}), \]
\[ C_3^- = C_3^-(L \leftrightarrow R), \]
\[ C_4^- = \frac{\alpha_W^2}{4} \sum_{i,j=1}^{6} \sum_{h,k=1}^{6} \frac{1}{m_{\chi_j}^2} (2H_{UR}^i H_{UL}^j G_{UL}^{*ijk} \Gamma_{UR}^{jhq} + 2H_{UL}^i H_{UR}^{jhh} G_{UL}^{*ihq} \Gamma_{UR}^{jikq}) \]
\[ - G_{UR}^{jhh} G_{UL}^{*ikq} G_{UL}^{*ihq}) \Gamma_{UR}^{jikq}) \]
\[ C_5^- = \frac{\alpha_W^2}{4} \sum_{i,j=1}^{6} \sum_{h,k=1}^{6} \frac{1}{m_{\chi_j}^2} \left[ (4G_{DR}^{jhh} G_{UL}^{*ikq} G_{UL}^{*ihq} - G_{UL}^{jhh} G_{UL}^{*ihq} G_{UL}^{*ikq}) \right] \]
\[ \Gamma_{DR}^{jikq}) \]
\[ G(x_{\tilde{u}_h \tilde{\chi}_j^-}, x_{\tilde{u}_h \tilde{\chi}_j^-}, x_{\tilde{\chi}_i \tilde{\chi}_j^-}) - (4G_{DR}^{jhh} G_{UL}^{*ikq} G_{UL}^{*ihq} - G_{UL}^{jhh} G_{UL}^{*ihq} G_{UL}^{*ikq}) \]
\[ 2 \sqrt{x_{\tilde{\chi}_i \tilde{\chi}_j^-} F'(x_{\tilde{u}_h \tilde{\chi}_j^-}, x_{\tilde{u}_h \tilde{\chi}_j^-}, x_{\tilde{\chi}_i \tilde{\chi}_j^-})}. \]
\[ (13) \]

There are no chargino contributions to \( C_2 \) and \( \tilde{C}_2 \) because of the color structure of the chargino box diagram. We have included in our expressions all the terms for left-right mixing, which are dominated by \( \delta_{a,LR(RL),13} \). The mixing matrices \( G_{UL}, G_{UR}, H_{UL} \) and \( H_{UR} \) are defined in [11].

2.4 The Gluino Contribution

\[ C_1^\tilde{g} = \frac{\alpha_s^2}{2m_{\tilde{g}}^2} \sum_{h,k=1}^{6} \Gamma_{DL}^{\alpha g} \Gamma_{DL}^{\alpha g} \Gamma_{DL}^{\alpha g} \frac{11}{9} G(x_{\tilde{u}_h \tilde{g}}, x_{\tilde{d}_k \tilde{g}}) - \frac{1}{9} F(x_{\tilde{u}_h \tilde{g}}, x_{\tilde{d}_k \tilde{g}}), \]
\[ \tilde{C}_1^\tilde{g} = \tilde{C}_1^\tilde{g}(L \leftrightarrow R), \]
\[ C_2^\tilde{g} = -\frac{\alpha_s^2}{2m_{\tilde{g}}^2} \sum_{h,k=1}^{6} \Gamma_{DR}^{\alpha g} \Gamma_{DR}^{\alpha g} \Gamma_{DL}^{\alpha g} \frac{17}{18} F(x_{\tilde{u}_h \tilde{g}}, x_{\tilde{d}_k \tilde{g}}), \]
\[ \tilde{C}_2^\tilde{g} = \tilde{C}_2^\tilde{g}(L \leftrightarrow R), \]
\[ C_3^\tilde{g} = \frac{\alpha_s^2}{2m_{\tilde{g}}^2} \sum_{h,k=1}^{6} \Gamma_{DR}^{\alpha g} \Gamma_{DR}^{\alpha g} \Gamma_{DL}^{\alpha g} \frac{1}{6} F(x_{\tilde{u}_h \tilde{g}}, x_{\tilde{d}_k \tilde{g}}), \]
\[ \tilde{C}_3^\tilde{g} = \tilde{C}_3^\tilde{g}(L \leftrightarrow R), \]
\[ C_4^\tilde{g} = -\frac{\alpha_s^2}{2m_{\tilde{g}}^2} \sum_{h,k=1}^{6} \{ \Gamma_{DR}^{\alpha g} \Gamma_{DL}^{\alpha g} \Gamma_{DR}^{\alpha g} \frac{1}{3} G(x_{\tilde{u}_h \tilde{g}}, x_{\tilde{d}_k \tilde{g}}) + \frac{7}{3} F(x_{\tilde{u}_h \tilde{g}}, x_{\tilde{d}_k \tilde{g}}) \}
\]
These terms include the box and the crossed diagrams, as well as chirality conserving $\delta_{d,LL(RR),13}$ and chirality flipping $\delta_{d,LR(RL),13}$ mixing. Again we use the two-point functions and the mixing elements $\Gamma_{DL}$, $\Gamma_{DR}$ as in [11].

### 2.5 The Neutralino Contribution

\[
C_{1}^{\tilde{\chi}^0} = \frac{\alpha^2}{4} \sum_{h,k=1}^{6} \sum_{i,j=1}^{9} \frac{1}{m_{\tilde{\chi}^0}^{2}} \left[ G_{0DL}^{jhb} G'_{0DL}^{*ikq} G_{0DL}^{*jhb} G_{0DL}^{*ikq} \left( x_{\tilde{d}_k x_j^0}, x_{\tilde{d}_h x_j^0}, x_{\tilde{e}_l x_i^0} x_{\tilde{e}_l x_i^0} \right) - G_{0DL}^{jhb} G'_{0DL}^{*ikq} G_{0DL}^{*jhb} G_{0DL}^{*ikq} \left( x_{\tilde{d}_k x_j^0}, x_{\tilde{d}_h x_j^0}, x_{\tilde{e}_l x_i^0} x_{\tilde{e}_l x_i^0} \right) \right],
\]

\[
\tilde{C}_{1}^{\tilde{\chi}^0} = C_{1}^{\tilde{\chi}^0} (L \leftrightarrow R),
\]

\[
C_{2}^{\tilde{\chi}^0} = \frac{\alpha^2}{2} \sum_{h,k=1}^{6} \sum_{i,j=1}^{9} \frac{1}{m_{\tilde{\chi}^0}^{2}} \left[ (H_{0DL}^{jhh} H'_{0DL}^{*ikq} G_{0DL}^{*jhh} G_{0DL}^{*ikq} - G_{0DR}^{jhh} G_{0DR}^{*ikq} G_{0DL}^{*ikq}) \left( x_{\tilde{d}_k x_j^0}, x_{\tilde{d}_h x_j^0}, x_{\tilde{e}_l x_i^0} x_{\tilde{e}_l x_i^0} \right) \right],
\]

\[
\tilde{C}_{2}^{\tilde{\chi}^0} = C_{2}^{\tilde{\chi}^0} (L \leftrightarrow R),
\]

\[
C_{3}^{\tilde{\chi}^0} = \frac{\alpha^2}{2} \sum_{h,k=1}^{6} \sum_{i,j=1}^{9} \frac{1}{m_{\tilde{\chi}^0}^{2}} \left[ (H_{0DL}^{jhh} H'_{0DL}^{*ikq} G_{0DL}^{*jhh} G_{0DL}^{*ikq} - H_{0DL}^{jhh} H_{0DL}^{*ikq} G_{0DL}^{*jhh} G_{0DL}^{*ikq}) \left( x_{\tilde{d}_k x_j^0}, x_{\tilde{d}_h x_j^0}, x_{\tilde{e}_l x_i^0} x_{\tilde{e}_l x_i^0} \right) \right] + G_{0DR}^{jhh} G_{0DR}^{*ikq} G_{0DL}^{*ikq} G_{0DL}^{*ikq},
\]

\[
\tilde{C}_{3}^{\tilde{\chi}^0} = C_{3}^{\tilde{\chi}^0} (L \leftrightarrow R),
\]

\[
C_{4}^{\tilde{\chi}^0} = \frac{\alpha^2}{4} \sum_{h,k=1}^{6} \sum_{i,j=1}^{9} \frac{1}{m_{\tilde{\chi}^0}^{2}} \left[ G'_{0DL}^{jhb} (x_{\tilde{d}_k x_j^0}, x_{\tilde{d}_h x_j^0}, x_{\tilde{e}_l x_i^0} x_{\tilde{e}_l x_i^0}) \left( 2H_{0DL}^{jhh} H_{0DL}^{*ikq} G_{0DL}^{*jhh} G_{0DL}^{*ikq} \right) + 2H_{0DL}^{jhh} H_{0DL}^{*ikq} G_{0DL}^{*jhh} G_{0DL}^{*ikq} \right],
\]

\[
\tilde{C}_{4}^{\tilde{\chi}^0} = C_{4}^{\tilde{\chi}^0} (L \leftrightarrow R),
\]

\[
C_{5}^{\tilde{\chi}^0} = \frac{\alpha^2}{4} \sum_{h,k=1}^{6} \sum_{i,j=1}^{9} \frac{1}{m_{\tilde{\chi}^0}^{2}} \left[ (4G_{0DR}^{jhh} G_{0DR}^{*ikq} G_{0DL}^{*jhh} G_{0DL}^{*ikq} - G_{0DR}^{jhh} G_{0DR}^{*ikq} G_{0DL}^{*ikq}) \left( x_{\tilde{d}_k x_j^0}, x_{\tilde{d}_h x_j^0}, x_{\tilde{e}_l x_i^0} x_{\tilde{e}_l x_i^0} \right) - G_{0DL}^{jhh} G_{0DL}^{*ikq} G_{0DL}^{*jhh} G_{0DL}^{*ikq} \right) \left( x_{\tilde{d}_k x_j^0}, x_{\tilde{d}_h x_j^0}, x_{\tilde{e}_l x_i^0} x_{\tilde{e}_l x_i^0} \right) \right] + 2G_{0DL}^{jhh} G_{0DL}^{*ikq} G_{0DL}^{*jhh} G_{0DL}^{*ikq} \left( x_{\tilde{d}_k x_j^0}, x_{\tilde{d}_h x_j^0}, x_{\tilde{e}_l x_i^0} x_{\tilde{e}_l x_i^0} \right) \left( x_{\tilde{d}_k x_j^0}, x_{\tilde{d}_h x_j^0}, x_{\tilde{e}_l x_i^0} x_{\tilde{e}_l x_i^0} \right) \right],
\]

(15)

The mixing elements for gauginos and higgsinos $G_{0DL}$, $G_{0DR}$, $H_{0DL}$, $H_{0DR}$ and the relevant functions are defined in [11].
2.6 The Gluino-Neutralino Contribution

\[
C_{1}^{\tilde{\chi}^0} = \frac{\alpha_s \alpha_W}{6 \abs{g}^2} \sum_{h,k=1}^{6} \sum_{i=1}^{9} \left[ 3 \Gamma_{DL}^{*\kappa q} \Gamma_{DL}^{*\kappa q} G_{0DL}^{*\kappa q} G_{0DL}^{*\kappa q} G^{*} \left( x_{\tilde{d}_b \tilde{g}}, x_{\tilde{d}_b \tilde{g}}, x_{\tilde{\chi}_{\tilde{q}}^0} \right) \right] \\
C_{2}^{\tilde{\chi}^0} = \frac{\alpha_s \alpha_W}{4 \abs{g}^2} \sum_{h,k=1}^{6} \sum_{i=1}^{9} \left[ 2 \Gamma_{DL}^{*\kappa q} \Gamma_{DL}^{*\kappa q} H_{0DL}^{\kappa q} H_{0DL}^{\kappa q} + \Gamma_{DR}^{\kappa q} \Gamma_{DR}^{*\kappa q} G_{0DL}^{*\kappa q} G_{0DL}^{*\kappa q} \right] \\
C_{3}^{\tilde{\chi}^0} = C_{2}^{\tilde{\chi}^0} (L \leftrightarrow R), \\
C_{4}^{\tilde{\chi}^0} = \frac{\alpha_s \alpha_W}{2 \abs{g}^2} \sum_{h,k=1}^{6} \sum_{i=1}^{9} \left[ 2 \Gamma_{DL}^{*\kappa q} \Gamma_{DL}^{*\kappa q} H_{0DL}^{\kappa q} H_{0DL}^{\kappa q} - \Gamma_{DR}^{\kappa q} \Gamma_{DR}^{*\kappa q} G_{0DL}^{*\kappa q} G_{0DL}^{*\kappa q} \right] \\
C_{5}^{\tilde{\chi}^0} = \frac{\alpha_s \alpha_W}{4 \abs{g}^2} \sum_{h,k=1}^{6} \sum_{i=1}^{9} \left[ 2 \Gamma_{DL}^{*\kappa q} \Gamma_{DL}^{*\kappa q} H_{0DL}^{\kappa q} H_{0DL}^{\kappa q} - \Gamma_{DR}^{\kappa q} \Gamma_{DR}^{*\kappa q} G_{0DL}^{*\kappa q} G_{0DL}^{*\kappa q} \right] \\
(16)
\]

2.7 Hadronic Matrix Elements

We follow the notations and parameterizations of Ref. [20]. The hadronic matrix elements in the vacuum insertion approximation (VIA) [21] are given by

\[
\langle B_{d}^{0}|Q_{1}|B_{d}^{0})_{VIA} = \frac{2}{3} m_{B_{d}}^{2} f_{B_{d}}^{2}, \\
\langle B_{d}^{0}|Q_{2}|B_{d}^{0})_{VIA} = -\frac{5}{12} \left( \frac{m_{B_{d}}}{m_{b} + m_{d}} \right)^{2} m_{B_{d}}^{2} f_{B_{d}}^{2}, \\
11
\]
\[ \langle B_d^0|Q_3|B_d^0\rangle_{VIA} = \frac{1}{12} \left( \frac{m_{B_d}}{m_b + m_d} \right)^2 m_{B_d}^2 f_{B_d}^2, \]
\[ \langle B_d^0|Q_1|B_d^0\rangle_{VIA} = \left[ \frac{1}{12} + \frac{1}{2} \left( \frac{m_{B_d}}{m_b + m_d} \right)^2 \right] m_{B_d}^2 f_{B_d}^2, \]
\[ \langle B_d^0|Q_5|B_d^0\rangle_{VIA} = \left[ \frac{1}{4} + \frac{1}{6} \left( \frac{m_{B_d}}{m_b + m_d} \right)^2 \right] m_{B_d}^2 f_{B_d}^2 \]
\[ \text{(17)} \]

where \( m_{B_d}, m_b \) and \( m_d \) are the mass of the \( B_d \) meson, \( b \) and \( d \) quark respectively. The expressions for \( \tilde{Q}_{1-3} \) are same as those of \( Q_{1-3} \).

To take into account renormalization effects, we define the \( B \) parameters as

\[ \langle B_d^0|Q_i(\mu)|B_d^0\rangle_{VIA} = \langle B_d^0|Q_i|B_d^0\rangle_{VIA} B_i(\mu), \quad i = 1, \ldots, 5 \]
\[ \text{(18)} \]

where the numerical values of the renormalization functions and masses at the \( m_b \) scale are

\[ m_b(m_b) = 4.6 \text{ GeV}, \quad m_d(m_b) = 5.4 \text{ GeV}, \]
\[ B_1(m_b) = 0.87(4)^{+5}_{-4}, \quad B_2(m_b) = 0.82(3)(4), \]
\[ B_3(m_b) = 1.02(6)(9), \quad B_4(m_b) = 1.16(3)^{+5}_{-7}, \]
\[ B_5(m_b) = 1.91(4)^{+22}_{-7}. \]
\[ \text{(19)} \]

The coefficients at the scale of \( m_b \) are given by

\[ C_r(m_b) = \sum_i \sum_s (b_i^{(r,s)} + \eta c_i^{(r,s)}) \eta^a C_s(M), \]
\[ \text{(20)} \]

where \( \eta = \alpha_s(M)/\alpha_s(m_b) \) and we have chosen \( M = (m_b + m_d)/2 \). The numerical coefficients \( a_i, b_i^{(r,s)}, c_i^{(r,s)} \) can be found in Ref. [20].

Putting all the above together, we can calculate the mass difference \( \Delta m_d \) and CP asymmetry \( a_{J/\psi K_s} \). The off-diagonal element of the \( B_d \) mass matrix can be written as

\[ M_{12}(B_d) = \frac{\langle B_d^0|H_{eff}^{B=2}|B_d^0\rangle}{2m_{B_d}}, \]
\[ \text{(21)} \]
We define

\[ \Delta m_{B_d} = 2|M_{12}(B_d)|, \]
\[ a_{J/\psi K_s} = \sin 2\beta_{eff}, \]

where \( 2\beta_{eff} = \arg M_{12}(B_d) \).

3 Numerical Analysis

We are interested in analyzing the case in which the supersymmetric partners have masses around the weak scale, so we will assume relatively light superpartner masses. We include in our numerical estimates the SM and (non-supersymmetric) LRM contributions, and for these we constrain the lightest Higgs mass to be 115 GeV [22].

At first, we assume the only source of flavor violation to come from the CKM matrix. This scenario is related to the minimal flavor violation scenario in supergravity. This restricted possibility of flavor violation could set important constraints on the parameter space of LRSUSY.

We then allow, in the second stage of our investigation, for new sources of flavor violation coming from the soft breaking terms. In the MSSM, this scenario is known as the unconstrained MSSM and there the gluino contribution dominates. We investigate this in LRSUSY, and we restrict all allowable LL, LR, RL and RR flavor mixings between the first and third squark family.

3.1 The constrained case

By the constrained LRSUSY model, we mean the scenario in which the only source of flavor violation comes from the quark sector, through the CKM matrix, which we assume to be the same for both the left and right handed sectors. Our choice does not favor one
handedness over the other, and has the added advantage that no new mixing angles are introduced in the quark matrices.

We choose all parameters as independently free parameters, with the numerical results compared with experiments directly. All trilinear scalar couplings in the soft supersymmetry breaking Lagrangian are assumed to be universal: $A_{ij} = A\delta_{ij}$ and $\mu_{ij} = \mu\delta_{ij}$, and we fix $A$ to be 100 GeV and $\mu = 200$ GeV throughout the analysis. We also set $M_L = M_R = 500$ GeV.

As the SM expectations fit the experimental data, there is no explicit deviation from the SM found in our analysis with the values for the parameters. In Fig. 2 we show the dependence of the mass difference $\Delta m_d$ as a function of $\tan \beta$. Although $\Delta m_d$ shows a slight decrease with $\tan \beta$, it is not sensitive to $\tan \beta$, because $\tan \beta$ affects mostly the contribution of the higgsino components of charginos. For $\tan \beta$ greater than 2, $\Delta m_d$ is always below the experimental bound.
3.2 The unconstrained case

When supersymmetry is softly broken, there is no reason to expect that the soft parameters would be flavor blind, or that they would violate flavor in the same way as in the SM. The unconstrained LRSUSY model, similar to the unconstrained MSSM, allows for new sources of flavor violation among generations. In the process $B \to X_s \gamma$ [10] and $B \to X_s l^+ l^-$ [11] we allowed for flavor violations between the second and third families in the down squark mass matrix only. Here we consider the effects of flavor violations between the first and third generation in both the up and down squark mass matrix.

We parameterize all the unknown soft breaking parameters coming mostly from the scalar mass matrices using the mass insertion approximation [23]. In this framework we choose a basis for fermion and sfermion states in which all the couplings of these particles to neutral gauginos are flavor diagonal. Flavor changes in the squark sector arise from the non-diagonality of the squark propagators. The normalized flavor mixing parameters used are

$$
\delta_{q,LL,ij} = \frac{(m^2_{q,LL})_{ij}}{m^2_0}, \quad \delta_{q,RR,ij} = \frac{(m^2_{q,RR})_{ij}}{m^2_0},
$$

$$
\delta_{q,LR,ij} = \frac{(m^2_{q,LR})_{ij}}{m^2_0}, \quad \delta_{q,RL,ij} = \frac{(m^2_{q,RL})_{ij}}{m^2_0},
$$

where $m^2_0$ is the average squark mass and $(m^2_{q,AB})_{ij}$ are the off-diagonal elements which mix squark flavor for both left- and right- handed squarks with $q = u, d$, and $A, B = L, R$. We diagonalize squark mass matrices numerically, which is valid even when the parameters are not perturbative.

We keep our analysis general, but to show our results, we select only one possible source of flavor violation in the squark sector at a time, and assume the others vanish. All diagonal entries in the squark mass matrix are set equal and we study the mass difference $\Delta m_d$ as a function of the common value $m^2_0$ and the relevant off-diagonal
Figure 3: $\Delta m_d$ as a function of $Re \delta_{d,LL,13}$ with other mass insertion terms switched off. The solid(dashed) line corresponds to $m_0/m_\tilde{g} = 200/200$ ($200/400$) GeV.

In Fig. 3, we show the variation of $\Delta m_d$ as a function of $Re \delta_{d,LL,13}$, for $m_0 = 200$ GeV. As the down squarks contribute in graphs with exchange of gluinos, we give two values to the mass of gluinos. For $m_\tilde{g} = 200$ GeV, the range of $Re \delta_{d,LL,13}$ is found to be $(-0.02, 0.01)$, while for $m_\tilde{g} = 400$ GeV, the range is much wider, which is $(-0.14, 0.02)$. Although we have chosen $\delta_{d,LL,13}$ as representative, very similar constraints are obtained for $\delta_{d,RR,13}$.

Even though $\Delta m_d$ is a CP conserving quantity, squark mass matrices can be complex. The imaginary parts of the squark mass mixing give rise to the CP asymmetry $a_{J/\Psi K_s}$. Although one could restrict both $a_{J/\Psi K_s}$ and $\Delta m_d$ from constraints on $Im \delta_{d,LL,13}$, we found it more convenient to analyze $\Delta m_d$. The present world-average CP asymmetry has a large uncertainty, and any constraints from $\Delta m_d$ yield $a_{J/\Psi K_s}$ within experimental bounds. We keep the real and imaginary parts of various mass insertions as independent parameters. In Fig. 4, we show $\Delta m_d$ as a function of $Im \delta_{d,LL,13}$ for $m_0 = 200$ GeV. For
$m_\tilde{g} = 200$ GeV, the range of $\text{Im}\ \delta_{d,LL,13}$ is found to be (-0.03, 0.04), while for $m_\tilde{g} = 400$ GeV, the range is much wider, which is (-0.15, 0.13). Again, the constraints obtained for $\text{Im}\ \delta_{d,RR,13}$ are very similar.

We proceed with an analysis of the chirality flipping flavor mixing parameters. In Fig. 5, we show $\Delta m_d$ as a function of $\text{Re}\ \delta_{d,LR,13}$ for $m_0 = 200$ GeV and two values of $m_\tilde{g}$. For $m_\tilde{g} = 200$ GeV, the range of $\text{Re}\ \delta_{d,LR,13}$ is found to be (-0.02, 0.01), while for $m_\tilde{g} = 400$ GeV, the range is a little wider, which is (-0.03, 0.02). Similar constraints are obtained for $\delta_{d,RL,13}$. In Fig. 6 we show the corresponding variation of $\Delta m_d$ with $\text{Im}\ \delta_{d,LR,13}$. The range of the chirality flipping parameter $\text{Im}\ \delta_{d,LR,13}$ is more restrictive than the chirality conserving $\text{Im}\ \delta_{d,LL,13}$ (both for the real and imaginary parts). Comparing the real and imaginary parts of each mass insertion, one can note that the real part is more constrained.

If we turn off the down squark mass insertion, the $B_d^0 - \overline{B}_d^0$ mass mixing will be dominated by up squark mass insertions coming from diagrams with charginos in the
Figure 5: $\Delta m_d$ as a function of $Re \delta_{d,LR,13}$ with other mass insertion terms switched off. The solid(dashed) line corresponds to $m_0/m_\tilde{g} = 200/200$ ($200/400$) GeV.

Figure 6: $\Delta m_d$ as a function of $Im \delta_{d,LR,13}$ with other mass insertion terms switched off. The solid(dashed) line corresponds to $m_0/m_\tilde{g} = 200/200$ ($200/400$) GeV.
Figure 7: $\Delta m_d$ as a function of $Re\delta_{u,LL,13}$ with other mass insertion terms switched off. The solid(dashed) line corresponds to $m_0/m_\tilde{g} = 200/200$ (300/200) GeV.

Figure 8: $\Delta m_d$ as a function of $Im\delta_{u,LL,13}$ with other mass insertion terms switched off. The solid(dashed) line corresponds to $m_0/m_\tilde{g} = 200/200$ (300/200) GeV.
loop. We analyze these restrictions next. In Fig. 7, we show the dependence of $\Delta m_d$ as a function of $\text{Re}\ \delta_{u,LL,13}$ for a fixed gluino mass $m_{\tilde{g}} = 200$ GeV. We show the variation for two values of the universal scalar mass $m_0$. For $m_0 = 200$ GeV, the range of $\text{Re}\ \delta_{u,LL,13}$ is found to be (-0.16, 0.30), while for $m_0 = 300$ GeV, the range is narrower, (-0.08, 0.20). Allowing for imaginary parts of the flavor mixing only, we show $\Delta m_d$ as a function of $\text{Im}\ \delta_{u,LL,13}$ for $m_{\tilde{g}} = 200$ GeV in Fig. 8. For $m_0 = 200$ GeV, the range of $\text{Im}\ \delta_{u,LL,13}$ is found to be (-0.56, 0.44), and for $m_0 = 300$ GeV, the range is almost same, which is (-0.58, 0.46).

In Fig. 9, we show $\Delta m_d$ as a function of $\text{Re}\ \delta_{u,RL,13}$ for $m_{\tilde{g}} = 200$ GeV, when only flavor violating, chirality flipping mass insertions in the up squark sector are non-zero. For $m_0 = 200$ GeV, the range of $\text{Re}\ \delta_{u,RL,13}$ is found to be (-0.30, 0.20), while for $m_0 = 300$ GeV, the range is a little wider, (-0.40, 0.20). And finally in Fig. 10, we show $\Delta m_d$ as a function of $\text{Im}\ \delta_{u,RL,13}$ for $m_{\tilde{g}} = 200$ GeV. For $m_0 = 200$ GeV, the range of $\text{Im}\ \delta_{u,RL,13}$ is found to be (-0.56, 0.50), while for $m_0 = 300$ GeV, the range is much wider, (-0.88, 0.82), almost covering the whole parameter space.

We note that the conditions on the mass insertion in the up squark sector are much less restrictive. This is due to the fact that these restrictions come from the chargino contributions, which are smaller than the combined gluino, gluino-neutralino and neutralino contribution. For the down squark mass insertions, the chirality flipping mass insertions are more restricted than the chirality conserving parts; while for the up squark mass insertions, the chirality conserving mass insertions are slightly more restricted than the chirality flipping parts.
Figure 9: $\Delta m_d$ as a function of $\text{Re}\delta_{u,RL,13}$ with other mass insertion terms switched off. The solid (dashed) line corresponds to $m_0/m_\tilde{g} = 200/200$ (300/200) GeV.

Figure 10: $\Delta m_d$ as a function of $\text{Im}\delta_{u,RL,13}$ with other mass insertion terms switched off. The solid (dashed) line corresponds to $m_0/m_\tilde{g} = 200/200$ (300/200) GeV.
4 Conclusions

We have studied $B_d^0 - \overline{B}_d^0$ mixing in the fully left-right supersymmetric model. Explicit expressions for all the chargino, gluino, gluino-neutralino and neutralino amplitudes involved in the process are given. We have calculated the mass difference and CP asymmetry in both the constrained case (where the only flavor violation comes from the CKM matrix) and the unconstrained case, including soft supersymmetry breaking mass insertions in both the up and down squark sectors. For the constrained LRSUSY model, we find that the contributions to $B_d^0 - \overline{B}_d^0$ is small, in agreement with previous results from a supergravity-inspired MSSM [6].

Throughout our analysis for the unconstrained case, we found the gluino and chargino to dominate the supersymmetric contributions; the neutralino and gluino-neutralino contributions are smaller. If the dominant flavor mixing comes from the up-squark sector, the chargino contribution dominates for large $\delta_{u,13}$ and quickly saturates the LRSUSY contribution. If the only source of flavor mixing arises from the down-squark region, the gluino contribution dominates for most of the parameter space. For small $|\delta_{d,13}|$ (of $\mathcal{O}(10^{-3})$), the chargino contribution is the largest, while for the rest of the parameter space the gluino dominates. In this case, the chargino contribution is negligible, as even the neutralino contribution becomes larger for $|\delta_{d,13}| > \mathcal{O}(10^{-2})$. If both sources of flavor violation are present, the gluino contribution will saturate the experimental bound faster, justifying most of the previous analysis which looked at the gluino contributions only. However, we stress the importance of analyzing both the up squark and down squark sources of flavor violation for a complete picture of the $B_d^0 - \overline{B}_d^0$ mixings.

Comparisons with other models show that some general features are similar: the chirality flipping mass insertions are more restricted than the chirality conserving ones and the down squark mixings are more restricted than the up squark mixings [20]. From
our analysis it appears that both $Re(Im) \delta_{d,LL,13}$ and $Re(Im) \delta_{d,LR,13}$ are more restricted in the LRSUSY. The same is true for $Re(Im) \delta_{u,LL,13}$ and $Re(Im) \delta_{u,RL,13}$. A more general conclusion escapes us because a comparable comprehensive analysis of mass insertions in the unconstrained MSSM does not exist for $B_d^0 - \bar{B}_d^0$ mixing.

FCNC and CP violating phenomena in B physics are promising candidates for indirect SUSY signals and complementary to the direct searches. Thus efforts to improve the theoretical precision in various SUSY scenarios are necessary. The present analysis is useful in restricting all mass insertions, taken as independent parameters, in a supersymmetric model with left-right symmetry. The dependence on the details of the model, beyond the requirement of left-right symmetry, (such as the triplet Higgs structure) is minimal and negligible. This analysis restricts further the FCNC CP conserving and CP violating parameters in the squark sector of LRSUSY, and provides complementary information to the one extracted from $B \to X_s \gamma$ and $B_s \to X_s l_1 l_2$.

Acknowledgements

This work was funded in part by NSERC of Canada (SAP0105354).

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