Mapping the proton drip line from
\[ Z = 31 \text{ to } Z = 49 \]

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November 21, 2018

Abstract

The structure of proton drip line nuclei in the \( 60 < A < 100 \) mass range is studied with the Relativistic Hartree Bogoliubov (RHB) model. For the elements which determine the astrophysical rapid proton capture process path, the RHB model predicts the location of the proton drip-line, the ground-state quadrupole deformations and one-proton separation energies at and beyond the drip-line. The results of the present theoretical investigation are compared with available experimental data. For possible odd-Z ground state proton emitters, the calculated deformed single-particle orbitals occupied by the odd valence proton and the corresponding spectroscopic factors are compared with predictions of the macroscopic-microscopic mass model.

PACS: 21.60.Jz, 21.10.Dr, 21.10.Jx, 26.50.+x, 27.50.+e
1 Introduction and outline of the relativistic Hartree-Bogoliubov model

The structure of nuclei at the proton drip line in the mass region $60 < A < 100$ is important for the process of nucleosynthesis during explosive hydrogen burning. The exact location of the proton drip line determines a possible path of rapid proton capture process. The path of the rp-process lies between the line of $\beta$-stability and the drip line, and it is a very complicated function of the physical conditions, temperature and density, governing the explosion [1]. The input for rp-process nuclear reaction network calculations includes the nuclear masses, or proton separation energies of the neutron deficient isotopes, the proton capture rates, their inverse photodisintegration rates, the $\beta$-decay and electron capture rates. In a recent extensive analysis [2], the influence of nuclear structure on the rp-process between Ge and Sn at extreme temperature and density conditions, has been studied with a number of theoretical models for the nuclear masses, deformations, reaction rates and $\beta$-decay rates.

In addition to its importance for astrophysical processes, the information about the exact location of the drip line, as well as the proton separation energies beyond the drip line, are essential for studies of ground state proton radioactivity [3]. No examples of ground state proton emitters below $Z = 50$ have been reported so far, and therefore theoretical studies might provide important information for future experiments in this region.

The proton drip line has been fully mapped up to $Z=21$, and possibly for odd-Z nuclei up to In [3]. In Ref. [4] the proton drip line has been mapped up to $A = 70$ by calculating Coulomb energy differences between mirror nuclei within the framework of the nuclear shell model. The structure of
proton drip line nuclei around the doubly magic $^{48}$Ni has been studied with various self-consistent mean field models in Ref. [5], and the proton capture reaction rates in the mass range $23 \leq A \leq 43$ have been calculated with the nuclear shell model in Ref. [6]. In a recent series of articles [7, 8, 9, 10], we have applied the Relativistic Hartree Bogoliubov (RHB) model [11, 12, 13] in the study of ground state properties of proton rich nuclei. In general, the calculated properties have been found in excellent agreement with available experimental data, and with the predictions of the macroscopic-microscopic mass model. In the present study we apply the RHB model in the description of the structure of nuclei along the proton drip line from $Z = 31$ to $Z = 49$. We will determine the location of the proton drip-line, the ground-state quadrupole deformations and one-proton separation energies at and beyond the drip-line, the deformed single-particle orbitals occupied by the odd valence proton in odd-Z nuclei, and the corresponding spectroscopic factors.

A detailed discussion of the relativistic Hartree-Bogoliubov theory can be found, for instance, in Ref. [9]. For completeness we include a description of the essential features of the RHB model. In Section II we present and discuss the results of the analysis the proton drip line from $Z = 31$ to $Z = 49$. Section III contains the summary.

In the framework of the relativistic mean field theory nucleons are described as point particles that move independently in mean fields which originate from the nucleon-nucleon interaction. The theory is fully Lorentz invariant. Conditions of causality and Lorentz invariance impose that the interaction is mediated by the exchange of point-like effective mesons, which couple to the nucleons at local vertices. The single-nucleon dynamics is de-
scribed by the Dirac equation

\[
\left\{ -i \alpha \cdot \nabla + \beta (m + g_\sigma \sigma) + g_\omega \omega^0 + g_\rho \sigma_3 \rho_3^0 + e \left( \frac{1 - \tau_3}{2} \right) A^0 \right\} \psi_i = \varepsilon_i \psi_i. \tag{1}
\]

\( \sigma, \omega, \) and \( \rho \) are the meson fields, and \( A \) denotes the electromagnetic potential. \( g_\sigma, g_\omega, \) and \( g_\rho \) are the corresponding coupling constants for the mesons to the nucleon. The lowest order of the quantum field theory is the mean-field approximation: the meson field operators are replaced by their expectation values. The sources of the meson fields are defined by the nucleon densities and currents. The ground state of a nucleus is described by the stationary self-consistent solution of the coupled system of the Dirac (1) and Klein-Gordon equations:

\[
\begin{align*}
\left[ -\Delta + m_\sigma^2 \right] \sigma(r) &= -g_\sigma \rho_3(r) - g_2 \sigma^2(r) - g_3 \sigma^3(r) \tag{2} \\
\left[ -\Delta + m_\omega^2 \right] \omega^0(r) &= g_\omega \rho_3(r) \tag{3} \\
\left[ -\Delta + m_\rho^2 \right] \rho_3(r) &= g_\rho \rho_3(r) \tag{4} \\
-\Delta A^0(r) &= e \rho_p(r), \tag{5}
\end{align*}
\]

for the sigma meson, omega meson, rho meson and photon field, respectively.

Due to charge conservation, only the 3rd-component of the isovector rho meson contributes. The source terms in equations (2) to (5) are sums of bilinear products of baryon amplitudes, and they are calculated in the no-sea approximation, i.e. the Dirac sea of negative energy states does not contribute to the nucleon densities and currents. Due to time reversal invariance, there are no currents in the static solution for an even-even system, and therefore the spatial vector components \( \omega, \rho_3 \) and \( A \) of the vector meson fields vanish.

The quartic potential

\[
U(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4 \tag{6}
\]
introduces an effective density dependence. The non-linear self-interaction of the $\sigma$ field is essential for a quantitative description of properties of finite nuclei.

In addition to the self-consistent mean-field potential, pairing correlations have to be included in order to describe ground-state properties of open-shell nuclei. For nuclei close to the $\beta$-stability line, pairing can been included in the relativistic mean-field model in the form of a simple BCS approximation. For more exotic nuclei further away from the stability line, however, the BCS model presents only a poor approximation. In particular, in order to correctly reproduce density distributions in nuclei close to the drip lines, mean-field and pairing correlations have to be described in a unified framework: the Hartree-Fock-Bogoliubov model or the relativistic Hartree-Bogoliubov (RHB) model. In the unified framework the ground state of a nucleus $|\Phi>\rangle$ is represented by the product of independent single-quasiparticle states. These states are eigenvectors of the generalized single-nucleon Hamiltonian which contains two average potentials: the self-consistent mean-field $\hat{\Gamma}$ which encloses all the long range particle-hole ($ph$) correlations, and a pairing field $\hat{\Delta}$ which sums up the particle-particle ($pp$) correlations. In the Hartree approximation for the self-consistent mean field, the relativistic Hartree-Bogoliubov equations read

$$
\begin{pmatrix}
\hat{h}_D - m - \lambda \\
-\hat{\Delta}^* \\
\end{pmatrix}
\begin{pmatrix}
U_k(\mathbf{r}) \\
V_k(\mathbf{r}) \\
\end{pmatrix}
= E_k \begin{pmatrix}
U_k(\mathbf{r}) \\
V_k(\mathbf{r}) \\
\end{pmatrix},
$$

(7)

where $\hat{h}_D$ is the single-nucleon Dirac Hamiltonian (1), and $m$ is the nucleon mass. The chemical potential $\lambda$ has to be determined by the particle number subsidiary condition in order that the expectation value of the particle number operator in the ground state equals the number of nucleons. The column vectors denote the quasi-particle spinors and $E_k$ are the quasi-particle
energies. The pairing field $\hat{\Delta}$ in (7) is defined

$$\Delta_{ab}(r, r') = \frac{1}{2} \sum_{c,d} V_{abcd}(r, r') \sum_{E_k > 0} U^*_{ck}(r) V_{dk}(r'),$$

(8)

where $a, b, c, d$ denote quantum numbers that specify the Dirac indices of the spinors, $V_{abcd}(r, r')$ are matrix elements of a general two-body pairing interaction. The RHB equations are solved self-consistently, with potentials determined in the mean-field approximation from solutions of Klein-Gordon equations for the meson fields. The current version of the model [9] describes axially symmetric deformed shapes. The Dirac-Hartree-Bogoliubov equations and the equations for the meson fields are solved by expanding the nucleon spinors $U_k(r)$ and $V_k(r)$, and the meson fields in terms of the eigenfunctions of a deformed axially symmetric oscillator potential. A simple blocking procedure is used in the calculation of odd-proton and/or odd-neutron systems. The blocking calculations are performed without breaking the time-reversal symmetry.

The input parameters of the RHB model are the coupling constants and the masses for the effective mean-field Lagrangian, and the effective interaction in the pairing channel. In most applications we have used the NL3 effective interaction [14] for the RMF Lagrangian. Properties calculated with NL3 indicate that this is probably the best effective interaction so far, both for nuclei at and away from the line of $\beta$-stability. For the pairing field we employ the pairing part of the Gogny interaction

$$V^{pp}(1, 2) = \sum_{i=1,2} e^{-(r_1 - r_2)/\mu_i} (W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau),$$

(9)

with the set D1S [15] for the parameters $\mu_i, W_i, B_i, H_i$ and $M_i$ ($i = 1, 2$). This force has been very carefully adjusted to the pairing properties of finite nuclei all over the periodic table. In particular, the basic advantage of the
Gogny force is the finite range, which automatically guarantees a proper cut-off in momentum space.

2 The proton drip line from $Z=31$ to $Z=49$

In this section, we use the relativistic Hartree-Bogoliubov model to map the proton drip line from $Z = 31$ to $Z = 49$. As in our previous studies of proton-rich nuclei [7, 8, 9, 10], the NL3 effective interaction is used in the mean-field Lagrangian, and pairing correlations are described by the pairing part of the finite range Gogny interaction D1S. We analyze the structure of proton drip line nuclei in the $60 < A < 100$ mass range: the location of the proton drip-line, the ground-state quadrupole deformations and one-proton separation energies at and beyond the drip-line, the deformed single-particle orbitals occupied by the odd valence proton in odd-Z nuclei, and the corresponding spectroscopic factors.

In Fig. [1] we display the section of the chart of the nuclides along the proton drip line in the region $31 \leq Z \leq 49$. The calculation predicts the last bound isotopes for each element. Nuclei to the left are proton unstable. For odd-Z nuclei the proton drip line can be compared with available experimental data. For $Z = 31$ and $Z = 33$ the calculated drip line nuclei $^{61}$Ga and $^{65}$As, respectively, are in agreement with experimental data reported in Refs. [16, 17]. These two nuclei are on the on the rp-process path proposed by Champagne and Wiescher [1]. In Ref. [18] evidence was reported for the existence of $^{60}$Ga, but of course the observation of an isotope does not necessarily imply that the nucleus is proton bound, but rather that its half-life is longer than the flight time through the fragment analyzer.

For $Z = 35$ the RHB calculation predicts that the last proton bound
isotope is $^{70}$Br. The isotope $^{69}$Br is calculated to be proton unbound in most mass models (see also Fig. 2). Experimental evidence for $^{69}$Br was reported in Ref. [16], but no evidence for this isotope was found in the experiment of Ref. [18], and it was deduced that $^{69}$Br is proton unbound with a half-life shorter than about 100 ns.

For $Z = 37$ the experiment of Ref. [16] confirms that $^{74}$Rb is the last proton bound nucleus, in agreement with the result of the present calculation. For $Z = 39, 41, 43$ the lightest isotopes observed in the experiment of Ref. [19] are $^{78}$Y, $^{82}$Nb and $^{86}$Tc, respectively. While for Nb and Tc these results correspond to the drip line as calculated in the present work, for Y the RHB model predicts that the last proton bound nucleus is $^{77}$Y. This isotope would then be the heaviest $T_z = -\frac{1}{2}$ nucleus, in contrast to the suggestion of Ref. [16] that $^{69}$Br is most likely the highest observable odd-Z $T_z = -\frac{1}{2}$ nucleus.

The calculated odd-Z drip line nuclei $^{90}$Rh and $^{94}$Ag were observed in the experiment reported in Ref. [20], and experimental evidence for $^{98}$In was reported in Ref. [21].

For even-Z nuclei in this region, it was not possible to compare the calculated proton drip line with experimental data. While for the odd-Z elements most of the last proton bound nuclei lie on the $N = Z$ line, with just few $T_z = -\frac{1}{2}$ nuclei, the proton drip line for even-Z elements is calculated to be at $T_z = -3$, or even at $T_z = -\frac{7}{2}$. The only exception is the drip line nucleus $^{84}$Ru with $T_z = -2$. Nuclei with such extreme values of $T_z$ are virtually impossible to produce in experiments, and since they lie so far away from the rp-process path, the even-Z proton drip line nuclei in this mass region play no role in the process of nucleosynthesis during explosive hydrogen burning.
We have, however, compared our calculated proton drip line with the predictions of other, well known and frequently used mass models \cite{22, 23, 24, 25}. The comparison is illustrated in Fig. 2, where the mass number of the first proton unbound nucleus along each isotopic chain is plotted as a function of the atomic number. We notice that, with the exception of the classical mass formula by Hilf et al. \cite{22}, all models agree on the location of the proton drip line for odd-Z nuclei (see also Fig. 9 in Ref. \cite{2}). The only significant difference is $^{77}$Y which, in contrast to the mass models of Refs. \cite{23, 24, 25}, is predicted to be the heaviest proton bound $^{77}_Z$ nucleus by the present RHB/NL3 calculation. This difference could be important because, if $^{77}$Y were bound and therefore located on the rp-process path, $^{76}$Sr would not be a waiting point nucleus. For even Z-nuclei, the theoretical models differ in their predictions for the location of the proton drip line. As it is shown in Fig. 2, the differences are especially pronounced for Sr, Zr, Ru and Pd, and they reflect the different treatment of pairing correlations and deformation effects. In fact, the combined effect of pairing correlations and nuclear deformation is responsible for the large difference between the $T_z$ values for even-Z and odd-Z nuclei at the proton drip line. The calculated ground-state quadrupole deformations of the last proton bound nuclei are shown in Fig. 3. For $Z \leq 33$ the drip line nuclei are moderately deformed, between $34 \leq Z \leq 41$ the odd-Z drip line nuclei are highly deformed, and for $Z > 41$ (protons in the $g_{9/2}$ orbital) the drip line enters a region of spherical nuclei. Between $34 \leq Z \leq 41$ one notices that the odd-Z nuclei are much more deformed than their even-Z neighbors on the drip line. The reason is that pairing correlations are strongly reduced in odd-Z nuclei, and as a result the nucleus is driven toward larger deformations. Much stronger pairing in
even-Z nuclei results in almost spherical shapes, which in turn shift the drip line to extremely low values $T_z \approx -3$. One could also say that the strong reduction of pairing in odd-Z nuclei causes the drip line to lie at $T_z = -\frac{1}{2}$ or $T_z = 0$. We have verified that the blocking of odd proton orbitals is essential for the correct description of the drip line in odd-Z nuclei. Without blocking, the calculated drip line in Fig. 1 is shifted to the left, to the position of the drip line of even-Z nuclei.

An important issue in future experimental studies of proton rich nuclei in this mass region is the possible observation of ground state proton emission. The structure and decays modes of nuclei beyond the proton drip-line represent one of the most active areas of experimental and theoretical studies of exotic nuclei with extreme isospin values. In the last few years many new data on ground-state and isomeric proton radioactivity have been reported in the region $51 \leq Z \leq 83$. In particular, measured half-lives and transition energies, as well as calculated transition rates, have shown that the ground state proton emitters are spherical in the regions $51 \leq Z \leq 55$ and $69 \leq Z \leq 83$ [3], and strongly deformed in the region of light rare-earth nuclei $57 \leq Z \leq 67$ [26, 27]. In Refs. [8, 9, 10] we have applied the RHB (NL3+D1S) model in the description of ground-state properties of proton-rich odd-Z nuclei in the region $53 \leq Z \leq 71$. We have calculated proton separation energies, ground-state quadrupole deformations, single-particle orbitals occupied by the odd valence proton, and the corresponding spectroscopic factors. The results were found to be in excellent agreement with experimental data, and we were also able to predict several new ground state proton emitters. And while the relatively high potential energy barrier enables the observation of ground state proton emission from medium-heavy and heavy nuclei, no examples of ground state
proton radioactivity have been discovered so far below $Z = 50$. The reason is, of course, the low Coulomb barrier. Nuclei beyond the proton drip line in this region exist only as short lived resonances. For a typical rare-earth nucleus the window of proton energies, i.e. the $Q_p$ values for which ground state proton emission can be directly observed is about $0.8 \text{–} 1.7 \text{ MeV}$ [28]. For lower $Q_p$ values the total half-life will be completely dominated by $\beta^+$ decay; higher transition energies result in extremely short proton-emission half-lives which cannot be observed directly. In an early study of the phenomena of proton and two-proton radioactivity [29], Goldansky has used an approximate formula to calculate the half-life of a ground state proton emitter

$$\log T_{1/2}(\text{sec}) \approx 0.43Z^{2/3}f(x) - 22,$$

(10)

where $x$ is the ratio of the proton transition energy to the height of the Coulomb barrier. For $x \ll 1$

$$f(x) \approx 0.6\left(\frac{1}{2} \pi x^{-1/3} - 2\right).$$

(11)

For the interval $T_{1/2} = 10 \text{–} 10^{-4} \text{ sec}$, the corresponding energy range of the emitted protons is calculated: $0.2 \text{–} 0.3 \text{ MeV}$ (for $Z=30$) and $0.35 \text{–} 0.5 \text{ MeV}$ (for $Z=40$).

In Figs. 4 and 5 the one-proton separation energies

$$S_p(Z, N) = B(Z, N) - B(Z-1, N),$$

(12)

are displayed for the odd-$Z$ nuclei $31 \leq Z \leq 49$, as function of the number of neutrons. In both figures the energy window extends beyond the proton drip line, in order to include those nuclei for which a direct observation of ground-state proton emission is in principle possible on the basis of calculated
separation energies. For the best candidates, the ground-state properties calculated with the RHB (NL3+D1S) model are displayed in Table 1.

The table includes the one-proton separation energies $S_p$, the quadrupole deformations $\beta_2$, the deformed single-particle orbitals occupied by the odd valence proton, and the corresponding theoretical spectroscopic factor. The spectroscopic factor $u^2_{\Omega}$, which results from the self-consistent treatment of pairing in the RHB model, is defined as the probability that the deformed odd-proton orbital is empty in the daughter nucleus with even proton number. In addition to the isotopes with one-proton separation energies in the energy range $\approx -0.1$ to $-0.5$ MeV, we have also included a few nuclei with higher transition energies ($^{72}$Rb, $^{88}$Rh and $^{96}$In). The reason is that, for these odd-odd nuclei, we cannot expect the RHB mean-field model to predict the one-proton separation energies with high accuracy. The model does not include any residual interaction between the odd proton and the odd neutron. Such an additional interaction, which could be represented for instance by the surface delta-force, will increase the binding energy. As a result, the proton transition energy would have a lower value, maybe in the energy range of observable ground state proton emission.

In Table 1 we also compare the results of the present calculations with the predictions of the finite-range droplet (FRDM) mass model: the projection of the odd-proton angular momentum on the symmetry axis and the parity of the odd-proton state $\Omega^p_\pi$ [23], the one-proton separation energy [25], and the ground-state quadrupole deformation [24]. In general, the two models predict very similar quadrupole deformations at the drip line. Nuclei with $Z \leq 41$ display pronounced quadrupole deformations, and spherical nuclei at the drip lines are found for $43 \leq Z$. The only notable difference is $^{73}$Rb.
Both models predict the same one-proton separation energy, but the nucleus is calculated to be oblate in the RHB model ($\beta_2 = -0.34$), while a prolate shape is obtained in FRDM ($\beta_2 = 0.37$). Correspondingly, the two models differ in the prediction of the odd-proton orbital occupied in $^{73}$Rb. In fact, the predictions for the deformed orbitals occupied by the odd proton differ in many cases. For the two As isotopes beyond the drip line, the RHB calculation predicts the $1/2^-[310]$ proton orbital, while a $3/2^-$ orbital is occupied in the FRDM. In the RHB calculation for $^{85}$Tc and $^{89}$Rh the odd proton is found in the $9/2^+[404]$ Nilsson orbital, while a $3/2^+$ and a $5/2^+$ states are, respectively, predicted by the FRDM. The calculated proton energies beyond the drip line are, on the average, higher in the FRDM. With the exception of $^{69}$Br, which is calculated to be proton bound in the FRDM, this model predicts more negative separation energies in the region of deformed nuclei $Z \leq 41$. These divergences might be related to the different descriptions of the effective spin-orbit single-nucleon potential.

The self-consistent ground-state quadrupole deformations for odd-Z nuclei $31 \leq Z \leq 47$, at and beyond the proton drip-line, are shown in Fig. 6. The Ga and As isotopes are moderately deformed, the Br nuclei are oblate, and a transition from prolate to oblate shapes is observed at the drip line in Rb. The Y isotopes are strongly prolate deformed, shape transitions are predicted in Nb, and the isotopes of Tc, Rh and Ag are essentially spherical at the drip line.

3 Summary

In this study we have analyzed the structure of the proton drip line in the mass region $60 < A < 100$. The theoretical model that we have used is based
on the relativistic Hartree-Bogoliubov theory. This framework provides a unified and self-consistent description of mean-field and pairing correlations, which is especially important for applications to exotic nuclei far from the valley of $\beta$-stability. In addition, the relativistic model includes the important isospin dependence of the spin-orbit term of the effective single-nucleon potential. The NL3 effective interaction has been used for the mean-field Lagrangian, and pairing correlations have been described by the pairing part of the finite range Gogny interaction D1S. This particular combination of effective forces in the $ph$ and $pp$ channels has been used in most of our applications of the RHB theory.

The proton drip line nuclei in the mass range $60 < A < 100$ determine the astrophysical rapid proton capture process path. Their properties are, therefore, crucial for a correct description of the process of nucleosynthesis during explosive hydrogen burning. In the present analysis we have calculated the location of the proton drip-line, the ground-state quadrupole deformations and one-proton separation energies at and beyond the drip-line, the deformed single-particle orbitals occupied by the odd valence proton in odd-Z nuclei, and the corresponding spectroscopic factors. For odd-Z nuclei, the predicted location of the proton drip line is found in excellent agreement with available experimental data, and with results calculated with various macroscopic-microscopic mass models. The even-Z nuclei at the drip line are not accessible in experiments, and therefore we have compared the RHB (NL3+D1S) results with the predictions of various mass models.

We have also addressed the important issue of ground state proton radioactivity below $Z = 50$. In this region nuclei beyond the drip line exist only as short lived resonances, and no examples of ground state proton emit-
ters have been discovered so far. It is expected that emitted protons could be observed in the range of transition energies: \(0.2 - 0.3\) MeV (for \(Z=30\)) and \(0.35 - 0.5\) MeV (for \(Z=40\)). The present RHB model calculations have shown that a number of odd-Z nuclei can be expected below \(Z = 50\), with negative one-proton separation energy in this energy windows. For these possible ground state proton emitters we have also determined the Nilsson orbitals occupied by the odd valence proton and the corresponding spectroscopic factors. These results can be used in calculations of partial proton decay half-lives, and therefore in comparison with future experimental data.

ACKNOWLEDGMENTS

This work has been supported in part by the Bundesministerium für Bildung und Forschung under project 06 TM 979, by the Deutsche Forschungsgemeinschaft, and by the Gesellschaft für Schwerionenforschung (GSI) Darmstadt.
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Figure 1: The proton drip line in the region $31 \leq Z \leq 49$. On this section of the chart of the nuclides the last bound isotopes for each element are indicated. Nuclei to the left are predicted to be proton unstable by the present RHB (NL3+D1S) calculation.

Figure 2: The proton drip line in the region $35 \leq Z \leq 49$. The mass number of the first proton unbound nucleus along each isotopic chain is plotted as a function of the atomic number. The results of the present RHB (NL3+D1S) calculation are compared with the predictions of various mass models.

Figure 3: Calculated ground-state quadrupole deformations for the nuclei at the proton drip line $31 \leq Z \leq 49$.

Figure 4: Predictions of the RHB model for the one-proton separation energies of odd-Z nuclei $31 \leq Z \leq 39$, at and beyond the drip-line.

Figure 5: Same as in Fig. [1] but for the odd-Z isotopes $41 \leq Z \leq 49$.

Figure 6: Self-consistent ground-state quadrupole deformations for odd-Z nuclei $31 \leq Z \leq 47$, at and beyond the proton drip-line.
Table 1: Candidates for odd-Z ground-state proton emitters in the region of nuclei with $31 \leq Z \leq 49$. The results of RHB calculation for the one-proton separation energies $S_p$, quadrupole deformations $\beta_2$, and the deformed single-particle orbitals occupied by the odd valence proton are compared with predictions of the macroscopic-microscopic mass model. All energies are in units of MeV; the RHB spectroscopic factors are displayed in the sixth column.

| N  | $S_p$ | $\beta_2$ | $p$-orbital | $u^2$ | $\Omega_p$ | $S_p$ | $\Omega_p$ | $\beta_2$ |
|----|------|---------|-------------|------|----------|------|----------|---------|
| $^{63}$As | 30 | -0.45 | 0.22 | 1/2$^-$[310] | 0.61 | 3/2$^-$ | -1.36 | 0.22 |
| $^{64}$As | 31 | -0.12 | 0.23 | 1/2$^-$[310] | 0.66 | 3/2$^-$ | -0.22 | 0.23 |
| $^{68}$Br | 33 | -0.26 | -0.28 | 9/2$^+$[404] | 0.80 | 9/2$^+$ | -0.33 | -0.32 |
| $^{69}$Br | 36 | -0.10 | -0.29 | 9/2$^+$[404] | 0.78 | 9/2$^+$ | 0.09 | -0.32 |
| $^{72}$Rb | 35 | -0.83 | -0.37 | 7/2$^+$[413] | 0.83 | 7/2$^+$ | -0.80 | -0.38 |
| $^{73}$Rb | 36 | -0.31 | -0.34 | 7/2$^+$[413] | 0.83 | 3/2$^+$ | -0.31 | 0.37 |
| $^{75}$Y | 36 | -0.56 | 0.42 | 5/2$^+$[422] | 0.92 | 5/2$^+$ | -1.57 | 0.41 |
| $^{76}$Y | 37 | -0.03 | 0.41 | 5/2$^+$[422] | 0.84 | 5/2$^+$ | -0.57 | 0.41 |
| $^{81}$Nb | 40 | -0.10 | 0.49 | 1/2$^+$[431] | 0.12 | 1/2$^+$ | -1.00 | 0.46 |
| $^{84}$Tc | 41 | -0.55 | -0.21 | 5/2$^+$[413] | 0.90 | 5/2$^+$ | -0.76 | -0.22 |
| $^{85}$Tc | 42 | -0.34 | -0.02 | 9/2$^+$[404] | 0.73 | 3/2$^+$ | -0.66 | 0.05 |
| $^{88}$Rh | 43 | -0.87 | 0.04 | 7/2$^+$[413] | 0.51 | 5/2$^+$ | -0.70 | 0.05 |
| $^{89}$Rh | 44 | -0.38 | 0.01 | 9/2$^+$[404] | 0.56 | 5/2$^+$ | -0.50 | 0.05 |
| $^{92}$Ag | 45 | -0.50 | 0.02 | 7/2$^+$[413] | 0.39 | 7/2$^+$ | -0.67 | 0.05 |
| $^{93}$Ag | 46 | -0.11 | 0.04 | 9/2$^+$[404] | 0.49 | 7/2$^+$ | -0.49 | 0.05 |
| $^{96}$In | 47 | -0.91 | 0.04 | 9/2$^+$[404] | 0.19 | 9/2$^+$ | -0.38 | 0.05 |
| $^{97}$In | 48 | -0.37 | 0.02 | 9/2$^+$[404] | 0.21 | 9/2$^+$ | -0.34 | 0.05 |
mass number of first p-unbound isotope

atomic number
The image shows a graph plotting proton separation energy (MeV) against neutron number. The graph includes data points for elements Ga, As, Br, Rb, and Y. The neutron number values range from 28 to 42, and the proton separation energy values range from -2.0 to 4.0 MeV. The graph visually represents the energy levels for different isotopes as a function of neutron number.
