Passivity-based Control of Voltage Source Converter Based On Mixed Sensitivity $H_\infty$

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Abstract. Considering the uncertainty of the control object, this paper aims to design the passivity-based control of voltage source converter based on mixed sensitivity $H_\infty$. Firstly, the plant model of VSC converter has been deduced and proved by simulation. Then the controllable of plant has been analyzed. Furthermore, based on mixed sensitivity $H_\infty$ theory, the performance weight function and passivity-based control have been designed. Finally, proved by simulation comparison, this control has more accurate and robust dynamic performance than the traditional ac voltage control.

1. Introduction

Depending on the passive converter, low harmonic content and other characteristics, the VSC (voltage source converter) has become one of the important technical means to solve the problems, such as passive network power supply, variable frequency motor drive, electronic transformers. To ensure the normal operation of the load, the VSC must provide stable AC voltage and power support. Due to the various types and operating conditions, the loads carried by the converter have strong volatility and uncertainty, which puts forward strict requirements on the robustness of the converter's AC voltage controller. How to ensure the stability of the AC voltage control performance under the load uncertainty is one of the important issues in the design and engineering application of the VSC control.

Several researches on AC voltage controller of voltage source converter have been reported in literature[4]-[7]. Literature [4] designed an AC voltage controller under unbalanced faults according to the passive control law of MMC converter. A double closed-loop controller containing an inner fast current loop and an outer voltage loop is proposed for the control of the converter station in[7]. Reference[6] designed a constant AC voltage controller based on the steady state mathematical model of the converter. All of the above AC voltage controllers are based on proportional-integral controllers, which mainly rely on experience or manual debugging. Although the structure is simple and easy to engineering applications, it cannot guarantee accurate robust design requirements.

In this paper, the passivity-based control of VSC is designed to optimize the dynamic performance, robust stability and robust performance of the converter station, considering the uncertainty of the control object. Firstly, the plant model of VSC converter has been deduced and its controllable has been analyzed. Then, based on mixed sensitivity $H_\infty$ theory, the performance weight function and
passivity-based control have been designed. Finally, proved by simulation comparison, this control has more accurate and robust dynamic performance than the traditional ac voltage control.

2. Object model

Equivalent structure diagram of voltage source converter supplying power to passive load is shown in Fig.1, where \( u_s \angle \delta^\circ \) is the AC voltage phasor of passive load access point, \( R+jX \) is the equivalent impedance of reactor and transformer, \( u_c \angle \eta^\circ \) is the AC side voltage phasor of converter, \( i_c \angle \mu^\circ \) is the AC side current phasor of converter, \( i_s \angle \gamma^\circ \) is the AC current phasor of passive network, \( C \) is the equivalent capacitance of AC filter. \( U_d \) is the DC side voltage of converter, \( i_d \) is the DC current flowing into the converter, \( C_s \) is the equivalent capacitance of AC filter and VSC, \( U_d \) is the DC side voltage of converter, \( i_d \) is the DC current flowing into the converter, \( C_s \) is the equivalent capacitance of AC filter and VSC.

Due to the diverse types of passive network loads, which cannot be set uniformly, and focusing on the converter AC voltage controller in this paper, the passive load is taken as an external input and not included in the object model.

In the paper, the plant model including AC side of converter, AC filter and VSC, has been deducted and proved by simulation.

![Figure 1. Equivalent structure diagram of voltage source converter supplying power to passive load](image)

2.1. AC side of converter and AC filter

In the synchronous rotating coordinate system based on passive load access point voltage \( \dot{u}_s \), the AC side of converter and filter can be represented mathematically as (1). \( i_{sd} \) and \( i_{sq} \) are the \( dq \) components of \( i_s \) in this coordinate system. Similarly, other physical quantities are defined in this way.

\[
\begin{align*}
L \frac{di_{sd}}{dt} &= -Ri_{sd} - \omega_L i_{sq} + u_{sd} - u_{sd} \\
L \frac{di_{sq}}{dt} &= -Ri_{sq} + \omega_L i_{sd} + u_{sq} - u_{sq} \\
C_s \frac{du_{sd}}{dt} &= -\omega_L C_s u_{sq} + i_{sd} - i_{sd} \\
C_s \frac{du_{sq}}{dt} &= \omega_L C_s u_{sq} + i_{sq} - i_{sq}
\end{align*}
\]

After linearizing (1) and selecting a static operating point for Taylor series expansion, and then taking first order term, the small-signal modeling of the AC side of converter and AC filter is written as (2). \( \Delta i_{sd} \) and \( \Delta i_{sq} \) are the \( dq \) components of \( i_s \) disturbance quantity. Similarly, other physical quantities are defined in this way.
The control block diagram of AC side of converter and AC filter is figured after Laplace transform and sorting out equation (2).

\[
\begin{align*}
L \frac{d\Delta i_{sd}}{dt} &= -R\Delta i_{sd} - \omega_L L\Delta i_{sq} + \Delta u_{sd} - \Delta u_{sd} \\
L \frac{d\Delta i_{sq}}{dt} &= -R\Delta i_{sq} + \omega_L L\Delta i_{sd} + \Delta u_{sq} - \Delta u_{sq} \\
C_s \frac{d\Delta u_{sd}}{dt} &= -\omega_s C_s \Delta u_{sd} + \Delta i_{sd} - \Delta i_{sd} \\
C_s \frac{d\Delta u_{sq}}{dt} &= \omega_s C_s \Delta u_{sd} + \Delta i_{sq} - \Delta i_{sq}
\end{align*}
\]

(2)

2.2. Voltage source converter

When constructing a VSC model, it is generally regarded as an ideal voltage source on the AC side, and equivalent to a controlled current source on the DC side. In the case of neglecting the power loss, the equivalent structure of VSC model is shown in Figure 3, and represents mathematically as (3) and (4). Where \( P_{ac} \) is the active power received by the converter.

\[
\begin{align*}
P_{dc} &= \frac{3}{2} (u_{sd} i_{sd} + u_{sq} i_{sq}) \\
C \frac{dU_d}{dt} &= \frac{P_{dc}}{U_d} + i_d
\end{align*}
\]

(3)

\[
\begin{align*}
\Delta P_{dc} &= \frac{3}{2} (\Delta u_{sd} i_{sd} + \Delta u_{sq} i_{sq} + u_{sd} \Delta i_{sd} + u_{sq} \Delta i_{sq}) \\
C \frac{d\Delta U_d}{dt} &= \frac{\Delta P_{dc}}{U_d} - \frac{P_{dc}}{U_d^2} \Delta U_d + \Delta i_d
\end{align*}
\]

(4)

Since the converter is the actuator of the system control signal, the corresponding relationship between the modulated signal output by the controller and the voltage on the AC side of the converter
can be expressed by equation (5). \( T_\delta \) means the switching cycle, \( M \) is modulation, \( S \) is carrier amplitude. According to the principle of pulse width modulation, 
\[
M = \frac{u_c}{(U_{d^*} / 2)} = \frac{u_c^*}{S},
\]
thus 
\[
K_{PWM} = \frac{U_{d^*}}{2S}.
\]
When linearizing equation (5), the DC voltage \( U_d \) is not a steady-state value, that is,
\[
\begin{align*}
\frac{du_{cd}}{dt} &= \frac{1}{T_\delta} \left( \frac{U_d}{2S} u_{cd}^* - u_{cq}^* \right) \\
\frac{du_{cq}}{dt} &= \frac{1}{T_\delta} \left( \frac{U_d}{2S} u_{cq}^* - u_{cq}^* \right)
\end{align*}
\]
(5)
\[
\begin{align*}
\frac{d\Delta u_{cd}}{dr} &= \frac{1}{T_\delta} \left( \frac{U_d}{2S} \Delta u_{cd}^* + \frac{u_{cd}^*}{2S} \Delta U_d - \Delta u_{cd} \right) \\
\frac{d\Delta u_{cq}}{dr} &= \frac{1}{T_\delta} \left( \frac{U_d}{2S} \Delta u_{cq}^* + \frac{u_{cq}^*}{2S} \Delta U_d - \Delta u_{cq} \right)
\end{align*}
\]
(6)

The converter small-signal model is synthesized by equations (4) and (6), and its object block diagram is shown in Figure 4.

2.3. Object model and verification

The object model of the VSC station which supplies power to the passive network is constituted of equations (2), (4), and (10), and its object control block diagram is shown in Figure 5. As can be seen, the object model is a seven-order system, taking modulation signal as input, passive network AC current and DC network current as disturbance input, and grid point voltage as output. Where \( G_{d}, G_{q}, G_{UPac}, G_{Uad} \) are written by equations (7)-(10).

\[
G_d = \frac{u_d}{U_{d^*}}
\]
(7)
\[
G_q = \frac{u_q}{U_{d^*}}
\]
(8)
\[
G_{UPac} = \frac{U_{d^*}}{U_{d^*} C_s + P_{ac}}
\]
(9)
In order to verify the correctness of the VSC station object model, the electromagnetic transient model of the double-ended flexible DC transmission system which supplies power to the passive network is established in PSCAD/EMTDC, as shown in Figure 1. The AC voltage controller generally used in this model is shown in Figure 5. At the same time, its object model and controller model are constructed in Matlab. The system parameters in the model are shown in Table 1.

| parameter                                      | value     |
|-----------------------------------------------|-----------|
| AC side of converter and filter               |           |
| Equivalent inductance of connected reactor    | 0.05      |
| Equivalent resistance of connected reactor    | 0.8       |
| Equivalent capacitance of AC filter           | 0.00005   |
| DC voltage                                    | 60        |
| DC side capacitor                             | 0.005     |
| Controller setting value                      |           |
| D-axis component of AC voltage on inverter    | 20        |
| Q-axis component of AC voltage on inverter    | 0         |

To simulate the system disturbance, the command value of the AC voltage d-axis component is set to step from 20kV to 22kV at 0.2s for 0.1s. The comparison of electromagnetic transient simulation and small-signal model simulation waveform is shown in Figure 6, where the dotted line is the electromagnetic transient simulation waveform, and the solid line is the small signal model simulation model derived in this paper. Figure 6(a)-(d) are the d-axis and q-axis components of the passive network AC current $i_s$ and AC voltage $u_s$ disturbances.

Comparing Figure 6(a)-(d), we can see that the VSC object model for power supply to the passive network deduced in this paper can be well matched with electromagnetic transient simulation and has good accuracy, thereby laying the foundation for control strategy research.
3. Controllability analysis of controlled objects

Some inherent characteristics of the object, such as the right half-plane zero point, the right half-plane pole, the delay link, and the phase lag, will directly restrict the design of the controller. Literature [8] summarizes these restrictions as:

1) When the object contains time delay $\theta$, the system closed-loop bandwidth must be less than $1/\theta$.

2) When the object has the right-half plane zero point $z$, the system closed-loop bandwidth must be less than $z/2$, if desiring to achieve strict control at the low frequency band.

3) When phase lag, the closed-loop bandwidth of the system should be less than $\omega_u$, where $\omega_u$ is the frequency when the phase lag of the object is -180°.

4) When the object has an open-loop unstable real pole at $s=p$, the system is stable if high feedback gain is required, it is demanded that closed-loop bandwidth should be greater than $2p$. 
In this section, the load of the passive network is set to decrease gradually from 1p.u., 0.75p.u., 0.5p.u., and 0.25p.u. to analyze the controllable of the object, under the parameters shown in Table 1, and then obtain the zero point and pole distribution of object model. After calculation, the zero point and pole distribution is figured in Fig.7, which shows that the object has 7 poles in the left half plane, and has no zero point. As the passive network load changes, all the poles remain on the left half plane. Therefore, it can be judged that the object model is stable, and has no restriction on the controller.

4. Mixed sensitivity $H_\infty$

Mixed sensitivity $H_\infty$ control is one of the important methods of multivariable controller design. Its main feature is that it can deal with the uncertainty of the model well. For example, the $H_\infty$ controller can realize the closed-loop stability and good closed-loop dynamic performance of the system under the conditions such as object perturbation, external disturbance and sensor failure.

$$\begin{align*}
S(s) &= [I + G_n(s)K(s)]^{-1} \\
T(s) &= G_n(s)K(s)[I + G_n(s)K(s)]^{-1} \\
X(s) &= K(s)[I + G_n(s)K(s)]^{-1}
\end{align*}$$

Where $S(s)$ is defined as the sensitivity function, reflecting the closed-loop transfer function from the disturbance signal to the object output. $T(s)$ is defined as the complementary sensitivity function,
reflecting the closed-loop transfer function from controller input to object output. \(X(s)\) is unnamed currently, mainly reflecting the closed-loop transfer function from the object input to the object output.

The maximum singular value \(\bar{S}(s)\) of the sensitivity function \(S(s)\) determines the system's anti-disturbance ability and command tracking ability. The smaller the maximum singular value of the sensitivity function is, the smaller the response of the system output to the disturbance signal and the control error are. However, in a normal dynamic system, the maximum singular value of the sensitivity function cannot be kept small in the entire frequency band. In order to ensure system performance, it is generally chosen to make the maximum singular value of the sensitivity function tend to 0 within the required bandwidth. A specific weight function \(W_p(s)\) is set to satisfy the following relationship with \(\bar{S}(s)\):

\[
\bar{S}(s) \leq W_p^{-1}(s)
\]

As long as the maximum singular value of the sensitivity function is smaller than the amplitude of the weight function in any frequency band, the anti-interference performance and control accuracy of the system in a specific frequency range can be guaranteed. The above performance can be achieved by solving equation (13) to find a satisfactory controller \(K(s)\).

\[
\left\| W_p(j\omega)S(j\omega) \right\| \leq 1
\]

Where \(W_p(s)\) is the diagonal weight function matrix with the weight function \(W_p(s)\) as the diagonal element.

Similarly, the maximum singular value of the complementary sensitivity function \(T(s)\) can be used to measure the stability margin of the system when measuring the multiplicative uncertainty of the involved object. The smaller the maximum singular value of the complementary sensitivity, the larger the minimum instability multiplicative perturbation, and therefore, the greater the robust stability margin of the system. The controller can also be set to satisfy:

\[
\left\| W_T(j\omega)T(j\omega) \right\| \leq 1
\]

Where \(W_T(s)\) is a diagonal weight function matrix with the weight function \(W_T(s)\) as the diagonal element. In order to ensure the closed-loop stability and closed-loop dynamic performance of the system under the condition of object perturbation, the shaping principles of the sensitivity function and the complementary sensitivity function should be integrated, and the controller \(K\) that satisfies equations (13) and (14) simultaneously should be obtained. This problem above is usually called mixed sensitivity H\(_\infty\) control.

The basic criterion for setting the weight function is to make \(W_p(s)\) have low-pass characteristics and \(W_T(s)\) have high-pass characteristics to ensure that the crossover frequency of the weight function \(W_p(s)\) is lower than the crossover frequency of \(W_T(s)\). Only in this way can the closed-loop bandwidth of the system be in between of \(W_p(s)\) and \(W_T(s)\). The transfer functions above are:

\[
\begin{align*}
W_p(s) &= \frac{s}{M_s + \omega_p} \\
W_T(s) &= \frac{s + \omega_p}{A_T s + \omega_T}
\end{align*}
\]

Where \(M_s\) and \(M_T\) respectively represent the peak limit of the sensitivity function and the complementary sensitivity function, which are mainly used to measure the degree of deterioration of the system performance under the most severe conditions. That means \(M_s\) and \(M_T\) are related to the robustness and dynamic performance of the system. The larger the peak value is, the worse the system robustness and dynamic performance are. Usually, it is required that \(M_s<2\) and \(MT<1.25\). \(\omega_p\) and \(\omega_T\) respectively represent the bandwidth frequency of the sensitivity function and complementary
sensitivity function. $A$ and $AT$ represent the maximum steady-state tracking error, which is usually chosen to approach 0.

5. Case system controller design

According to the analysis in section 2, the mathematical object model of the passive load supply VSC station under the parameters in Table 1 does not have right half-plane zero point, right half-plane pole, delay link, and phase lag, thus there is no restriction on the controller design. The weight function is set as (16) in order to realize the dynamic performance and robustness requirements of passive control.

$$
\begin{align*}
W_p(s) &= \frac{s/1.9 + 200}{s + 200 \times 0.0001} \\
W_f(s) &= \frac{s + 500 / 1.2}{0.001s + 500}
\end{align*}
$$

The parameters of the sensitivity function are peak limit $M_s=1.9$, crossover frequency $\omega_B=200\text{rad/s}$, maximum steady state error $A=0.0001$. The parameters of complementary sensitivity function are peak limit $M_T=1.2$, crossover frequency $\omega_T=500\text{rad/s}$, maximum steady state error $A_T=0.001$.

Integrating the object model in the first section and the above weight function, and calling the $\text{Mixsyn}$ function in the Matlab robust control algorithm package, the $H_\infty$ passive controller can be obtained as shown in Figure 9, and its transfer function matrix is written as (17).

![Block diagram of passive controller based on mixed sensitivity $H_\infty$](image)

$$
\begin{align*}
K_{dd} &= \frac{291.5(s + 0.02)}{s^2 + 0.04s + 0.0004} \\
K_{dq} &= \frac{253.2(s + 0.02)}{s^2 + 0.04s + 0.0004} \\
K_{qd} &= \frac{253.2(s + 0.02)}{s^2 + 0.04s + 0.0004} \\
K_{qq} &= \frac{291.5(s + 0.02)}{s^2 + 0.04s + 0.0004}
\end{align*}
$$

6. Simulation comparison

In this paper, a double-ended flexible DC transmission system, which supplies power to passive loads, is constructed in PSCAD/EMTDC simulation environment to verify the effectiveness of the mixed sensitivity $H_\infty$ controller, as shown in Figure 1. The system parameters are shown in Table 1. In order to simulate the perturbation of the object, set the passive load decreased gradually from 1 p.u. to 0.25 p.u., where 1 p.u. = 40MW.

At the 0.2 seconds, set the d-axis component reference value of the system AC voltage stepping from 20kV to 22kV in 0.1s. As shown in Figure 10 and Figure 11, two dynamic response characteristics of system controlled amount $u_{sd}$ and $u_{sq}$ is compared separately, when using $H_\infty$ controller and diagonal PI control.

Where, figure 10(a) and Figure 10(b) are the waveforms of the d-axis component $u_{sd}$ and the q-axis component $u_{sq}$ of the passive network AC voltage when using $H_\infty$ controller. Figures 11(a) and 11(b)
are the waveforms of the d-axis component \( u_{sd} \) and the q-axis component \( u_{sq} \) of the passive network AC voltage when using PI control.

![Figure 10. Simulation waveform of mixed sensitivity H∞ passive control](image)

![Figure 11. Simulation waveform of Diagonal PI AC voltage control](image)

As figure shows, H∞ controller can track the change of command value quickly and accurately, and no matter how the passive load changes, its dynamic response characteristics remain unchanged basically, which shows strong robustness. However, the PI controller's dynamic response speed is slow, and its dynamic performance changes with different passive loads, which shows weakly robustness. Hence, it is proved that the H∞ AC voltage controller designed in this paper is superior to the PI controller in terms of dynamic performance and robustness.

7. Conclusion

In this paper, it is aimed to design the passivity-based control of VSC based on mixed sensitivity H∞, considering the uncertainty of the control object. Firstly, the plant model of VSC converter has been deduced and proved by simulation. Then the controllable of plant has been analyzed. Furthermore, based on mixed sensitivity H∞ theory, the performance weight function and passivity-based control have been designed. Finally, proved by simulation comparison, this control has more accurate and robust dynamic performance than the traditional diagonal PI ac voltage control.
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