Why replica symmetry breaking does not occur below six dimensions in Ising spin glasses

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The leading term for the average height of the barriers which could separate pure states in Ising spin glasses is calculated using instanton methods. It is finite in dimensions \( d < 6 \). Replica symmetry breaking requires that the barriers between pure states are infinite in the thermodynamic limit, as finite barriers allow thermal mixing of pure states. As a consequence, a replica symmetry broken phase cannot survive when \( d < 6 \). However, for \( d > 6 \) no similar instanton solution exists.

The nature of the ordered phase of spin glasses has been controversial for decades. The standard calculational methods such as the renormalization group and mean-field theory are in disagreement with each other: The picture which derives from mean-field theory, which is valid for infinite dimensional systems, is that of broken replica symmetry (RSB) \([1–5]\). This is contradicted by the results of real-space renormalization group (RG) and scaling calculations \([6–8]\), which favor in low dimensions an ordered phase with replica symmetry \([9–13]\). Recent calculations using the strong disorder renormalization group suggest that the spin glass phase is replica symmetric for \( d \leq 6 \) \([10, 14]\). Real-space RG calculations are ad hoc, and it is hard to convince supporters of RSB with them. What is needed is a calculation within the replica framework which shows why RSB will go away below six dimensions. To date, there have been controversial hints \([15, 16]\) within the replica framework that six might be the special dimension below which RSB might not exist, but see \([17–19]\).

In the RSB state there are many pure states present whose free energies differ by \( O(1) \). These states are separated by high barriers. Numerical studies \([20, 21]\) of the Sherrington-Kirkpatrick (SK) model \([22]\) and other arguments \([23, 24]\) suggest that these barriers depend on the number of spins \( N \) in that model as \( N^{1/3} \). It is vital to the whole RSB picture that the barriers between the pure states become infinite in the thermodynamic limit i.e. as \( N \to \infty \). If they are finite, thermal fluctuations will mix the pure states together and the RSB picture cannot then apply. The basic argument of this paper is that the barriers between the pure states of the RSB state would be finite when \( d < 6 \), thereby causing the whole RSB picture to fall apart.

We start from the Edwards-Anderson model \([25]\) defined on a \( d \)-dimensional cubic lattice with the Hamiltonian

\[
H = -\sum_{ij} J_{ij} S_i S_j,
\]

where the summation is over only nearest-neighbor bonds and the random couplings \( J_{ij} \) are chosen from the standard Gaussian distribution of unit variance and zero mean. The Ising spins take the values \( S_i \in \{ \pm 1 \} \) with \( i = 1, 2, \ldots, N \). From the expression for the partition function associated with Eq. (1) one can derive \([26–29]\) the replicated and bond-averaged functional in the fields \( Q_{\alpha \beta} \) which is believed cap-
up to constants the free energy functional becomes

\[
F[\{R_{\alpha\beta}\}]/k_B T = \int d^d r \left[ -\frac{1}{2} \sum_{\alpha<\beta} R_{\alpha\beta}^2 + \frac{1}{2} \sum_{\alpha<\beta} (\nabla R_{\alpha\beta})^2 \\
- w Q \sum_{\alpha<\beta<\gamma} (R_{\alpha\beta} R_{\alpha\gamma} + R_{\alpha\gamma} R_{\beta\gamma} + R_{\alpha\gamma} R_{\beta\gamma}) \\
- w \sum_{\alpha<\beta<\gamma} R_{\alpha\beta} R_{\alpha\gamma} R_{\beta\gamma} \right]. \tag{6}
\]

The quadratic terms are not diagonal in the replica indices. To deal with this it is useful to first introduce the following propagators in terms of the Fourier components \( R_{\alpha\beta}(q) \)

\[
G_1(q) = \langle R_{\alpha\beta}(q) R_{\alpha\beta}(-q) \rangle, \\
G_2(q) = \langle R_{\alpha\beta}(q) R_{\alpha\gamma}(-q) \rangle, \quad \beta \neq \gamma, \\
G_3(q) = \langle R_{\alpha\beta}(q) R_{\alpha\beta}(-q) \rangle, \quad \alpha, \beta \neq \gamma, \delta. \tag{7}
\]

Then, following Ref. [29] the quadratic form is readily diagonalized in terms of three linear combinations of \( G_1, G_2 \) and \( G_3 \):

\[
G_B \equiv G_1 + 2(n - 2)G_2 + \frac{1}{2} (n - 2)(n - 3) G_3 = (q^2 + \tau)^{-1} \\
G_A \equiv G_1 + (n - 4)G_2 - (n - 3) G_3 = (q^2 + 2wQ)^{-1} \\
G_R \equiv G_1 - 2G_2 + 3G_3 = (q^2 + nwQ)^{-1}. \tag{8}
\]

All three of these propagators are of the form \((q^2 + m_\alpha^2)^{-1}\), with the mass of the breather mode given by \(m_B^2 = \tau\), that of the ‘anomalous’ mode by \(m_A^2 = 2wQ\) and that of the replicon mode by \(m_R^2 = nwQ\). In the limit of \(n \to 0\) the breather and the anomalous masses become equal while the replicon mass goes to zero. Stability of course requires that all the \(m_\alpha^2\) be non-negative. Thus at Gaussian order the replica symmetric solution has marginal stability. (If we had retained the quartic terms in the Hamiltonian density the replicon mode would have become unstable at Gaussian order). To see the apparent instability of the replica symmetric state in the absence of the quartic term it is necessary to go to one loop order and calculate the self-energies of the propagators. The replicon self-energy \(\Sigma_R(q)\) is defined via

\[
G_R = (q^2 + nwQ - \Sigma_R(q))^{-1}. \tag{9}
\]

To one-loop order the calculation of \(\Sigma_R(q)\) is straightforward [29, 30]; \(\Sigma_R(0)\) is given by [29]

\[
\Sigma_R(0) \approx \frac{4w^2\tau^2}{N} \sum_\mathbf{q} \frac{1}{q^4(q^2 + \tau)^2}. \tag{10}
\]

In the large \(N\) limit the sum over the wavevectors \(\mathbf{q}\) in Eq. (10) can be converted to an integral. For \(d > 8\) the integrals will exist with a cutoff at \(q = \Lambda\), where \(\Lambda \sim 1/\alpha\) and \(\alpha\) is the lattice spacing. Then \(\Sigma_R(0) \sim w^2\tau^2\). For \(4 < d < 8\), \(\Sigma_R(0)\) does not require an upper cutoff and \(\Sigma_R(0) \sim w^2\tau(d-4)/2\). It is useful to define the coupling constant \(g_R^2\) of the cubic theory

\[
g^2 = \frac{w^2}{\tau^3(d-4)/2}. \tag{11}
\]

Then in terms of \(g^2\), \(\Sigma_R(0) \sim \tau g^2\) as \(g^2 \to 0\), while higher term in the loop expansion make \(\Sigma_R(0) = \tau f(g^2)\). The function \(f(g^2)\) has a (weak-coupling) series expansion in \(g^2\). Then according to Eq. (9) in the limit \(n \to 0\) the replica symmetric solution appears to be perturbatively unstable. To proceed further one needs the Parisi RSB scheme. However, the extent of replica symmetry breaking is vanishingly small as \(g^2 \to 0\), as the breakpoint \(x_1 \sim g^2[17]\). It is possible that if the series for \(f(g^2)\) could be summed to all orders the replica symmetric solution might then prove to be stable when \(d < 6\). In Ref. [31] an example of a situation where the replica symmetric theory was found to be stable when the series was summed to all orders was explicitly constructed, even though it was unstable in low order perturbation theory as \(n \to 0\). Alas no argument for this possibility has been found for the series for \(f(g^2)\) below six dimensions.

Because it cannot be demonstrated that the replica symmetric solution becomes stable when the perturbative expansion is taken to all orders, we shall adopt another approach and show that the barriers between the putative RSB pure states would be finite if pure states existed below six dimensions. To determine barriers one looks for instanton solutions of the Euler-Lagrange equations associated with Eq. (6) [32–34]. The instanton procedure for calculating barrier heights is usually used when there are states which are metastable: indeed for RSB there are states with a range of free energies which differ by \(O(1)\) from each other and between which one can envisage transitions.

We can only find instanton solutions when \(d < 6\) in the (massive) breather and anomalous sectors. The anomalous sector can be spanned by a variable \(\rho_\alpha\) with the constraint that \(\sum_\alpha \rho_\alpha = 0\), while the breather sector requires just a single scalar to describe it, so that the combined breather and anomalous sector can be described by a new field \(\phi_\alpha\), with \(\alpha = 1, \ldots, n\) [30]. In this sector the functional of Eq. (6) becomes just the the effective Hamiltonian

\[
\hat{H}/k_B T = \int d^d r \left[ \sum_{\alpha=1}^n \left( \frac{1}{2} (\nabla \phi_\alpha)^2 + \frac{2wQ}{3} \phi_\alpha^2 - \frac{w}{3} \phi_\alpha^3 \right) + \frac{n}{2} \left( \frac{wQ}{2} \phi_\alpha^2 + O(\phi_\alpha^2 \phi_\beta + \phi_\alpha \phi_\beta^2) \right) \right]. \tag{12}
\]

If the quadratic form in \(\phi_\alpha\) is diagonalized there are \((n - 1)\) eigenvalues with mass \(m_A^2\) and a single eigenvalue with mass \(m_R^2\). The mass difference of order \(nwQ\) between the breather mode and that of the anomalous mode will produce interesting effects at higher order in \(g^2\) [35–37]. We shall just look now for the instanton solutions in the \(\phi_\alpha\) variables, and assume that it too is replica symmetric so that \(\phi_\alpha = S\) for all \(\alpha\). One can ignore the terms in Eq. (12) involving two replica
indices as they will make a contribution of order $n^2$. $S(r)$ is the instanton solution in

$$\dot{H} = nK,$$

(13)

where

$$\frac{K}{k_BT} = \int d^t\tau \left[ \frac{1}{2} (\nabla S)^2 + \frac{\tau}{2} S^2 - \frac{w}{3} S^3 \right],$$

(14)

(in which the limit $n \to 0$ has been taken).

It is convenient to scale out the coefficients $\tau$ and $w$ by the variable change

$$S(r) = \frac{\tau}{w} P(r), \quad x = r/\sqrt{\tau},$$

(15)

so that distances $x$ are measured in units of the mean-field correlation length, that is, $x = r/\xi$, where $\xi = 1/\sqrt{\tau}$. Then Eq. (14) can be rewritten as

$$K = \frac{\tau^{3-d/2}}{w^2} H = \frac{1}{g^2} H,$$

(16)

and $H$ is given by

$$\frac{H}{k_BT} = \int d^t x \left[ \frac{1}{2} (\nabla P)^2 + \frac{1}{2} P^2 - \frac{1}{3} P^3 \right],$$

(17)

on setting $n = 0$. The instanton is the spatially varying solution which makes $H$ in Eq. (17) stationary and is the solution of the Euler-Lagrange equation

$$\nabla^2 P = P - P^3.$$

(18)

Assuming that the solution has spherical symmetry, that is $R(r) = R(r)$, the stationarity equation reduces to

$$\frac{d^2 P}{dx^2} + \frac{d - 1}{x} \frac{d P}{dx} = P - P^3,$$

(19)

with boundary conditions $P(x \to \infty) \to 0$ and $dP/dx = 0$ as $x \to \infty$, so that at large distance from the origin, (which is the center of the instanton), the replica symmetric spatially uniform mean-field solution is recovered.

The solution of Eq. (19) with these boundary conditions is most easily understood by means of the “mechanical analogue” (see Ref. [33]). The mechanical analogue consists of interpreting $P$ as a particle position and $x$ as time. The particle is moving in the potential $V[P]$ (see Fig. 1) which is given by

$$V[P] = \frac{1}{2} P^2 + \frac{1}{3} P^3,$$

(20)

subject to a “viscous” damping force $-\frac{d}{dx} \frac{dP}{dx}$. Fig. 1 shows that to solve the mechanical problem one has to choose the initial point on the curve, $P_0$, so that the particle can roll down the slope with sufficient speed so that it can overcome the viscous damping force and reach the origin with zero speed. This problem is readily solved numerically which is just as well as no analytical solution can be found.

The numerical solutions show that the initial value of $P$ at $x = 0$, $P_0$, has to become larger and larger as $d \to 6$ to achieve a solution: $P_0 \sim 1/(6-d)$ suggesting that $d = 6$ is indeed a special dimension for the instanton solution. In fact it is readily demonstrated [38–41] that it is only in dimensions $d < 6$ that there exists an instanton solution of finite action.

To show this, we first multiply both sides of Eq. (18) by $P$ and use the identity $\nabla \cdot (P \nabla P) = P \nabla^2 P + (\nabla P)^2$ the integral over all space of $\nabla \cdot (P \nabla P)$ vanishes by Gauss’s theorem and the imposed boundary conditions at $x = \infty$ so that

$$\int d^d x [(\nabla P)^2 + P^2 - P^3] = 0.$$

Using Eq. (17) one deduces that

$$\frac{H}{k_BT} = \frac{1}{6} \int d^d x P^3.$$

(22)

As $P > 0$ it follows that $H > 0$.

Next we obtain a second identity from studying dilatations of the solution $P(x)$ [40, 41]. Define $P_\lambda(x) = \lambda^2 P(\lambda x)$ so that

$$H(P_\lambda)/k_BT = \lambda^{6-d} \int d^d x \left[ \frac{1}{2} (\nabla P)^2 + \frac{1}{2 \lambda^2} P^2 - \frac{1}{3} P^3 \right]$$

$$= \lambda^{6-d} H(P)/k_BT + \frac{1}{2} \lambda^{6-d} \left( \frac{1}{\lambda^2} - 1 \right) \int d^d x P^2.$$  

(23)

Thus

$$\frac{dH(P_\lambda)}{d\lambda} = (6-d)H(P) - k_BT \int d^d x P^2,$$

(24)

at $\lambda = 1$.

Since $P_\lambda$ is a solution which makes $H$ stationary when $\lambda = 1$,

$$(6-d)H(P) = k_BT \int d^d x P^2.$$

(25)
We have already established that $H > 0$ and as the integral of $P^2$ over space must be positive, this equation implies that there can only be an instanton solution for $d \leq 6$.

The instanton solution should not be a stable solution of Eq. (18) as it needs to correspond to a saddle point. To study its stability against a deviation $\psi(x)$ from the solution $P(x)$ of Eq. (18), requires solving the Schrödinger-like eigenvalue equation,

$$-\nabla^2\psi + \psi - 2P(x)\psi = \epsilon\psi.$$  \hspace{1cm} (26)

Stability would require that all the eigenvalues $\epsilon > 0$. However, the lowest eigenvalue of this equation, (the nodeless s-wave solution), has a negative eigenvalue, indicating that the instanton is unstable. The negative eigenvalue actually contributes to the prefactor $\Gamma_0$ in the transition rate $\Gamma \sim \Gamma_0 \exp(-B/k_B T)$ out of the initial state due to thermal fluctuations [32]. An upper bound on the negative eigenvalue of $-3/(6 - d)$ can be obtained by setting $\psi \propto P(x)$. The next eigenvalues up are the $p$-wave modes which are $d$-fold degenerate and are null eigenvalues: they correspond to translations of the instanton. Thus there is one unstable downward direction so the instanton solution is a saddle point of the functional $H$ of Eq. (17).

The instanton energy is equal to the barrier which has to be overcome to escape from the initial state. It is finite as $d \to 6$ from below and scales as $1/g^2$ according to Eq. (16). Numerical work indicates that $H/k_B T$ of Eq. (17) approaches a number $\approx 40.8$ as $d \to 6$, so that

$$B/k_B T \sim 40.8/g^2, \quad g^2 \to 0.$$ \hspace{1cm} (27)

For $d > 6$ there is no instanton solution of finite action (energy), so that RSB should be stable in these dimensions, while for $d < 6$ the existence of instantons of finite action implies that the barriers between pure states are finite, which would mean that RSB should not exist in these dimensions. Thermal fluctuations will cause the pure states to mix together.

There are loop corrections in ascending powers of $g^2$ to the non-perturbative leading term for the barrier height $B$ in Eq. (27), arising from several sources, such as the coupling to the replicon fields and the differences between the breather and anomalous mode masses, but dealing with them will be hard, harder than extending the loop expansion around the spatially constant mean-field solution beyond Gaussian order, which has yet to be done. The calculation though is fraught with conceptual difficulties; assuming that spin glasses are replica symmetric for $d < 6$, one would find that any calculation to a finite number of loops would appear to contradict that assumption and require RSB. It is the device of studying the limit $g^2 \to 0$ which has opened up a window to some progress. For example, the fluctuation addition to the coefficient $y$ of the quartic term, i.e. $y\alpha_4^4$ in Eq. (2) is of order $w^4\tau^{d/2-4}$ [26, 42]. When $4 < d < 8$ the fluctuation term dominates the bare Landau-Ginzburg coefficient $y$ provided $g^4 > y\tau^{d/2-2}$, which is possible for any chosen value of $g^2$ by making $\tau$ small enough. The fluctuation contribution to the quartic term gives a contribution $g^2P^4$ in the effective rescaled Hamiltonian of Eq. (17), which is negligible compared to the terms $P^2/2 - P^4/3$ in the limit $g^2 \to 0$. A quartic term which is not vanishingly small would have had important consequences, just as it does for the instantons in spinodal nucleation theory [38, 39]. In the present calculation the term in $g^4P^4$ gives a simple perturbative correction to the barrier height – a term of $O(1)$ on the right hand side of Eq. (27).

The SK model is the $d \to \infty$ limit of spin glasses and has a free energy landscape which is well-understood [43] from solutions of the mean-field equations of Thouless, Anderson and Palmer (TAP) [44]. The solutions of low free energy correspond to the pure states, and at finite $N$ there is a path from each minimum of the free energy functional to a saddle point (there is an associated saddle point for every minimum [24, 43, 45]). The saddle point has just one negative eigenvalue. The barrier is then the difference in free energy between the saddle point free energy and that of its associated minimum and this is thought to vary as $N^{1/3}$ [24]. We believe that the replica symmetric instanton in the breather-anomalous sector provides some similarity with the TAP landscape picture. Thus if we restored the replica index $\alpha$ to $\psi$, in Eq. (26), there would have been $n$ identical negative eigenvalues of the functional $H$ of Eq. (12), one for each of the $n$ copies in the replicated system, which mimics the single negative eigenvalue at the saddle point for the TAP equations, (which are not replicated) [24, 43, 45].

One difference with the barriers of the SK model is that in that model of order $N$ spins are involved in the escape from a pure state [24, 43, 45], whereas in the instanton solution for $d < 6$ the changes are localized over a finite region whose size is the correlation length $\xi$ (see Eq. (15)). It follows from Eq. (3) that the free energy density is of order $\sim \tau^3/w^2$ and so a modification of this by a spatial variation (as in the instanton) over a region of size $\xi = 1/\sqrt{\tau}$ will have a total energy cost of $\sim (\tau^3/w^2)\xi^d \sim 1/g^2$. It is thus very natural that the barrier height should be finite and vary as $1/g^2$. But this requires the existence of an instanton solution, which is only possible for $d < 6$.

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