Model-based Balancing Method of Rotors using Differential Evolution Algorithm

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Abstract. As a major source of undesired vibration, unbalance is a ubiquitous phenomenon for many types of rotor system. To suppress the unbalance vibration and avoid the complicated and dangerous process of taking trial weights during the balancing process, a model-based balancing method of rotors is proposed in this paper. The rotor model is built to analyse the dynamic characteristics of the unbalance rotor, an unbalance identification method is presented and the optimization objective function of the unbalance vectors is given, then the differential evolution algorithm is employed to acquire the optimization solution. Finally, the accuracy of the rotor model is verified by modal tests, and the performance of the proposed method is evaluated in balancing experiment, results reflect its effectiveness.

1. Introduction
Due to materials anisotropic, installation eccentricity, wear and other factors, the unbalance problem is unavoidable. Since even a slight unbalance can cause severe vibration of the rotor under high-speed operation condition, which leads to great impact on the quality of the rotating machinery [1]. Therefore, how to suppress the unbalance vibration becomes a key problem of rotor system.

Extensive work has been carried out on the identification of unbalance over the past several decades[2], and the trial weights is needed in the traditional methods. In order to improve the efficiency, the dynamics models are introduced into the balancing procedure. Based on this view point, Jalan et al.[3] described a model-based fault diagnosis of a rotor–bearing system for misalignment and unbalance under steady-state. Deepthikumar et al.[4] applied the polynomial curve for eccentricity distribution with finite element modeling to identify the distributed unbalance. Khulief et al.[5] developed a low-speed unbalance identification method relies on knowledge of the modal characteristics of the rotor. Tiwari et al.[6] proposed an identification method of bearing dynamic parameters and unbalance states using an unified finite element model. Wang et al.[7] proposed a novel unbalance identification strategy by the measurement point vector method, which based on the numerical rotor model. Moreover, in view of the convenience and flexibility of implementation, scholars also focus on the balancing methods based on intelligent algorithm. The genetic algorithm[8], the sequential quadratic programming[9], artificial neural network[10], least angle regression technique[11] and fuzzy logic[12] are employed in the optimization process of the balancing method.

However, it should be noted that the accuracy of the model-based methods are heavily dependent on the model accuracy, and identification results only based on intelligent algorithms is valid only in current condition, when the rotor working state changes, the results may not be effective. Hence, scholars research on the methods which integrate the dynamics models and the intelligent algorithms
to improve the efficiency and accuracy of the identification results. Zhang et al. [13] identified four types of faults including imbalance based on rotor dynamics and fuzzy support vector machine. Saldarriaga et al. [14] introduced genetic algorithms into the unbalance identification process, in which the response is obtained by a finite element model. Morais et al. [15] identified the unbalance distribution in linear and non-linear conditions through pseudorandom optimization method and the finite element model. Oke et al. [16] applied evolutionary algorithms to suppress the unbalance vibration of a rotor model based on simulation analysis.

Although remarkable balancing effects are obtained by above approaches, there are still some issues that can be worth studying. For example, only the amplitude is used for constructing the optimization objective function, and do not make full use of the rotor vibration information in some approaches, which may result in the identified unbalance vector being an equivalent solution rather than a true solution. In addition, there are some methods that have only been analyzed by simulation and have not been verified by actual experiments. Therefore, the contribution presented in this paper is an alternative balancing method for rotating machinery, aiming at making full use of the rotor space vibration information to construct the objective function and obtain the optimal solution, the model accuracy and the effectiveness of the proposed method are verified by experiment.

2. Unbalance identification method

The dynamics model of the rotor-bearing system can be divided into two parts, the finite element model of the rotor and the journal bearing. Assumed that the FE model may have $N$ nodes, the radial displacement and rotation along the radial coordinates of a single element in the generalized coordinates can be expressed as

$$\delta_n = \left[ y_n + i z_n, \theta_{iy}, i \theta_{iz}_n \right]^T, \quad (n=1, 2, L, N),$$

where $y_n$ and $z_n$ are the displacement of $Y$ and $Z$ axis, $\theta_{iy}$ and $\theta_{iz}$ are the rotation angle of $Y$ and $Z$ axis, respectively. The dynamic differential equation of the rotor is shown as

$$M \ddot{\delta} + C \dot{\delta} + K \delta = F^e + F^b$$

where $M$ is the mass matrix, $K$ is the stiffness matrix, $C$ is the damping and gyroscopic matrix. $F^e$ represents the unbalance force. $F^e$ stands for the oil film force which defines as $F^e = \{F^e_y, F^e_z, 0\}^T$. $F^b$ is the component of oil film force in different directions, which can be obtained as

$$\left\{ \begin{array}{c}
F^b_y \\
F^b_z
\end{array} \right\} = \sigma A \left[ \begin{array}{ccc}
3y & 3z & 2(\delta \cos \alpha - y \sin \alpha) \\
-2\cos \alpha & -2\sin \alpha & 1-\frac{\tan^{-1}(z \cos \alpha - y \sin \alpha)}{2} \\
-\sin \alpha & \cos \alpha & \frac{\pi}{2} + \tan^{-1}(\frac{z \cos \alpha - y \sin \alpha}{y \cos \alpha + z \sin \alpha})
\end{array} \right] \left[ \begin{array}{c}
2(1-y^2-z^2)^{3/2} \cos \alpha + z \sin \alpha
\\
2(1-y^2-z^2)^{3/2} \sin \alpha
\end{array} \right]$$

where $A = [(y-2z)^2+(z+2y)^2]/(1-y^2-z^2)$, $\alpha$ is the initial dynamic boundary angle, $\sigma$ is the Sommerfield correction coefficient, $\sigma = \kappa_0 RL / (2c_0)^3$, and $c_0$ is the radial clearance, $\kappa_0$ is the viscosity of the oil film, $\omega$ is the rotating speed.

If the simulated unbalance response curve matches the measured curve accurately, the $F$ obtained by Equation (1) can be considered as equal to that acquired by experiment. That means, the real $F$ can be identified by the comparison analysis of the response curves obtained by simulation and experiment, respectively. Since the oil film force $F^b$ can be calculated, the unbalance force $F^e$ can be identified by $F - F^b$. Based on this viewpoint, the optimization variables can be set as the amplitude $U$ and phase $\pi$ of the unbalance vector, and the sub objective functions can be built as

$$f_1 = \left[ v_{\max}(\phi) - v_{\min}(\phi) \right] - \left[ v_{\max}(\phi) - v_{\min}(\phi) \right]_o$$

$$f_2 = \left[ v_{\max}(\phi) - v_{\min}(\phi) \right] - \left[ v_{\max}(\phi) - v_{\min}(\phi) \right]_o$$

$$f_3 = \left[ \chi_{\phi}(\phi) - \chi_{\phi}(\phi) \right], f_4 = \left[ \chi_{\phi}(\phi) - \chi_{\phi}(\phi) \right]_o$$

where $v$ is the unbalance vibration response at rotating frequency, the subscripts $ep$ and $su$ represent experimental and simulated data, respectively, the subscript $i$ represents measuring plane, and the
subscript $j$ represents the measuring direction, $j=1$ represents $Y$ direction and $j=2$ represents $Z$ direction. $\varphi$ represents the phase of the unbalance vibration response, and $\varphi_{ep}$ can be obtained by vibration response and the signal of phase datum mark[17].

Consequently, the general objective function of the unbalance vector can be given as

$$
\min F_{obj} = \sum_{i}^{4} \lambda_i F_i
$$

subject to \( \sum_{i=1}^{4} \lambda_i = 1, \ U_i \leq (0, U_{\text{max}}), \ \Pi_p \subseteq (-\pi, \pi) \quad p = 1, 2, ..., P \)

where $\lambda_i$ is the weight coefficient, $U_{\text{max}}$ represents the upper limit of the correction mass, $F_i$ is the decreasing semi-trapezoidal fuzzy distribution function, if $\eta_{\text{min}} < f_i < \eta_{\text{max}}$, $F_i = (f_i - \eta_{\text{min}})(\eta_{\text{max}} - \eta_{\text{min}})$, and $\eta_{\text{min}}$ and $\eta_{\text{max}}$ is the maximum and minimum value in the feasible region of $f_i$.

3. Procedure of the balancing method

DE algorithm[18, 19] is a random parallel direct search algorithm, which simulates the biological evolution process based on a stochastic model. This algorithm has strong global convergence ability and robustness. During the operation of DE algorithm, the population size remains constant, and the main steps are basically the same as other evolutionary algorithms, including mutation, crossover and selection. The mutation individual $x_i^{+} = x_i^{*} + H(x_i^{*} - x_r^{*})$, where $x_i^{*}, x_r^{*} \in [1, 2, L, NP]$ are two different individuals, and $x_i^{*}$ is the best individuals at current generation. $H$ is the self-adaptive scaling factor which can be set as $2^H$, where $\lambda = e^{(G + r - G) / G}$, and $H_0$ is the initial factor, and $G$ represents the maximum and current evolutional generation, respectively, $1, 2, L = L\{G\}$. In the crossover procedure, if $j_{\text{rand}} / D \leq CR$ or $j = j_{\text{rand}}$, the trial individual $u_j^{+}$ can be set as $x_j^{+}$, otherwise, $u_j^{+}$ can be set as $x_j$, where $j$ represents the space dimension, $j = 1, 2, L, D$, $j_{\text{rand}} \in [1, 2, L, D]$, $CR$ is the crossover probability, $CR \in [0, 1]$. Then, the $x_j^{+}$ would be selected from the trial individual $u_j^{+}$ and the parent individuals $x_j$ based on the comparison result between $f(u_j^{+})$ and $f(x_j)$.

The DE algorithm is employed to acquire the optimum parameters of the $U$ and $\Pi$, and the flow chart of the unbalance optimization solution process is shown as Figure 1.

![Figure 1](image-url)
4. Verification and discussion

The experimental system shown in Figure 2 consists of rotor test platform, data acquiring and processing device, photoelectric and displacement sensors, force hammer, and motor controller. The photoelectric and displacement sensors are fixed on the radial axis, and the output analog signals generated by sensors are transformed into digital signals by data acquisition device.

The finite element model of the rotor system is established based on 50 elements and 50 nodes with 5-DOF,. The mass disk units are located at the 21th and 33th nodes, the measuring plane are located at the 18th and 36th nodes, and the journal bearing units are located at 9th and 45th node, respectively. The modal test is carried out on the experimental platform to analyze the accuracy of the model, and the measured FRFs of the rotor is shown in Figure 3.

According to the excitation and response curve, the structural damping ratio can be identified as 0.033, the first three order nature frequencies can be calculated as 62.5Hz, 190Hz and 590Hz in modal experiment, respectively. Meanwhile, that solved by Equation (1) are 62.3Hz, 189Hz and 565Hz, respectively. It is obviously that the error between simulation and experiment of the first-order nature frequency are 0.32%, 0.53% and 4.24%, respectively, which reflects the accuracy of the model.

Figure 2. Experimental rotor test system

Figure 3. The measured FRFs of the rotor

Figure 4. Evolution process for the unbalance vector: (a) Amplitude; (b) Phase
The balancing experiments are performed. The balancing speed is set to 2400 r/min, the measuring plane are on 18 and 36 nodes, and the sampling frequency is set to 10 kHz. The initialization parameters in Figure 1 are set as: \(NP=30, \ D=4, \ H_0=0.5, \ CR=0.3\). The weight coefficients are set as: \(\lambda_1=\lambda_2=0.15, \ \lambda_3=\lambda_4=0.35\). The evolution curve of the unbalance vector with the objective function value is shown in Figure 4. It can be seen that the amplitude of the unbalance vector on both disks are converged after iterations, and the convergence of the unbalance amplitude and phase on two disks can be observed in Figure 4. Then, the optimal solution of the unbalance vector can be obtained. In here, the identified unbalance vector at the 21th and 33th nodes that represent the mass disks are 133.3g.mm \(-107.7^\circ\) and 46.60g.mm \(172.9^\circ\), respectively.

The rotor startup process are acquired and indicated as Figure 5(a), in which the green solid and blue solid lines represent the vibration response before and after the balancing procedure, respectively. For safety consideration, the experimental speed did not reach the first critical speed before balancing, after adding the counterweight, the rotor system can pass the first order critical smoothly, and the amplitude of the vibration is only about 43.5μm when it reaches the first critical speed. It can be seen that the proposed method can effectively identify the unbalance, and the residual vibration below and beyond the first critical speed is very small after the balancing procedure, which proves the effectiveness of the method. Moreover, the first critical speed in this experiment is about 3714r/min, the error between the test result and Equation (1) is 0.62%, this indicates a good agreement between theoretical analysis and experimental test.

![Figure 5. The vibration information before and after the balancing procedure: (a) full speed range (b) at 2400r/min; (c) at 4800r/min](image)

The 3D holospectrum [20] of vibration at 2400r/min and 4800r/min on the measuring plane are shown as Figure 5(b)-(c). The axis orbit at both sides are reduced significantly at 2400 r/min, and after passing the critical speed, the residual vibration of the rotor at 4800 rpm is also very small. Overall, the Figure 5 show that the proposed dynamic balancing procedure is effective, which means that the unbalance mass and phase solved by the dynamics model and the DE algorithm are accurate.

5. Conclusions
In this paper, a model-based unbalance identification method of rotors has been proposed to tackle the unbalance problem of the rotor system. The rotor model with hydrodynamic bearings is built, the optimization function of unbalance are constructed, and then, the DE algorithm employed to identify the optimum results. The experiment results show that the unbalance vectors can be identified by collecting vibration data only at operating speed without trial weights, and the vibration amplitudes of the rotor around the first order critical speed are reduced greatly after adding the counterweights.

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