Photoproduction of $J/\psi$ and $\psi(2S)$ in proton–proton 
ultraperipheral collisions at the LHC

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Using the framework of leading order perturbative QCD along with the CTEQ6L1 
gluon distribution of the proton and high-energy HERA data on the $\gamma p \to J/\psi p$ and $\gamma p \to \psi(2S)p$ cross sections along with the corresponding H1 fit, we obtain a good 
description of the rapidity dependence of the cross sections of photoproduction of $J/\psi$ or $\psi(2S)$ mesons in proton–proton ultraperipheral collisions (UPCs) measured 
by the LHCb collaboration at the LHC. Within the same framework we also make 
predictions for the $pp \to ppJ/\psi$ and $pp \to pp\psi(2S)$ UPC cross sections at $\sqrt{s_{NN}} = 8$ 
and 14 TeV. We show that the possible contribution of the $p \to \Delta\gamma$ transition to 
the photon flux discernibly increases the $pp \to ppV$ UPC cross section and thus can 
affect the theoretical interpretation of results.

Ultraperipheral collisions (UPCs) of protons (ions) correspond to the situation when 
they pass each other at the impact parameter $|\vec{b}|$ larger than the sum of the hadron radii 
so that the strong interaction between the hadrons is suppressed and they interact electro-
magnetically via emission of quasi-real photons. Thus, UPCs at the LHC give a possibility 
to study photon–proton, photon–nucleus and photon–photon reactions at unprecedentedly 
high energies [1].

Recently the LHCb collaboration at the LHC published results of an updated analysis 
of exclusive photoproduction of $J/\psi$ and $\psi(2S)$ vector mesons in proton–proton UPCs [2] 
at $\sqrt{s_{NN}} = 7$ TeV, which supersedes and improves on the first measurement of this process 
made by LHCb [3]. The new data has much smaller experimental errors, which are of the 
order of 5% for $J/\psi$ and of the order of 15 – 20% for $\psi(2S)$, which allows one to better 
constrain and to some degree distinguish among various theoretical approaches, see the

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discussion in [2]. The improved accuracy of the data also requires a more accurate theoretical treatment, which should include effects that may contribute at the level of approximately 10%. One such effect is the possibility for each proton to transform into a Δ while emitting a quasi-real photon in the $p \rightarrow \Delta \gamma$ transition [4]. Since the final proton and products of the Δ decay travel essentially along the beam pipe and, hence, are not detected by the LHCb detector, both $p \rightarrow \Delta \gamma$ and $p \rightarrow p\gamma$ transitions contribute to the photon flux entering the calculation of the $pp \rightarrow ppV$ UPC cross section.

The aim of this note is twofold. First, we show that the leading order perturbative QCD provides the good description of the rapidity dependence and—to some extent—of the normalization of the $pp \rightarrow ppV$ cross section, where $V$ stands for $J/\psi$ or $\psi(2S)$, measured by the LHCb collaboration [2]. Second, we show how the additional $p \rightarrow \Delta \gamma$ contribution to the photon flux affects the predicted $pp \rightarrow ppV$ cross section.

The cross section of photoproduction of $J/\psi$ and $\psi(2S)$ vector mesons in proton–proton UPCs reads:

$$\frac{d\sigma_{pp \rightarrow ppV}(y)}{dy} = \begin{array}{c} + \quad r_+ N_{\gamma/p}(y)(1 + \delta(y))\sigma_{\gamma p \rightarrow Vp}(y) + r_- N_{\gamma/p}(-y)(1 + \delta(-y))\sigma_{\gamma p \rightarrow Vp}(-y), \\
- \quad r_- N_{\gamma/p}(y)(1 + \delta(y))\sigma_{\gamma p \rightarrow Vp}(y) - r_+ N_{\gamma/p}(-y)(1 + \delta(-y))\sigma_{\gamma p \rightarrow Vp}(-y), \end{array}$$  \hspace{1cm} (1)

where $V$ stands for $J/\psi$ or $\psi(2S)$; $N_{\gamma/p}(y) = \omega dN_{\gamma/p}(\omega)/d\omega$ is the photon flux of the proton; $y = \ln(2\omega/M_V)$ is the vector meson rapidity, where $\omega$ is the photon energy in the reaction (laboratory) reference frame; $\sigma_{\gamma p \rightarrow Vp}$ is the cross section of $J/\psi$ [$\psi(2S)$] photoproduction on the proton; $r_+$ and $r_-$ are absorptive corrections given by the rapidity gap survival probabilities; $\delta(y)$ takes into account a possible additional contribution to the photon flux due to the $p \rightarrow \Delta$ transition. Below we consider each ingredient of Eq. (1) in detail.

The expression for the photon flux of a fast moving proton (ion) is well known from quantum electrodynamics, see, e.g. [5]. In practical applications, one often uses approximate expressions reproducing the exact result with a few percent accuracy. In this note, we use the photon flux produced by a point-like (PL) particle passing a target at a minimal impact parameter $b_{\text{min}}$, which can be calculated analytically with the result:

$$N_{\gamma/p}(\omega) = \frac{2\alpha_{\text{em}}}{\pi} \left[ YK_0(Y)K_1(Y) - \frac{Y^2}{2} \left( K_1^2(Y) - K_0^2(Y) \right) \right],$$  \hspace{1cm} (2)

where $\alpha_{\text{em}}$ is the fine-structure constant; $K_0$ and $K_1$ are modified Bessel functions; $Y = \omega/\gamma_L b_{\text{min}}$, where $\gamma_L$ is the proton Lorentz factor and $b_{\text{min}} = 0.7 \text{ fm}$ [6].

In the leading logarithmic approximation of perturbative QCD, the high-energy $\gamma p \rightarrow Vp$
cross section is proportional to the gluon density of the proton \( G_p(x, \mu^2) \) squared:

\[
\frac{d\sigma_{\gamma p \rightarrow Vp}}{dt}(W_{\gamma p}, t = 0) = C(\mu^2)[\alpha_s(\mu^2)xG_p(x, \mu^2)]^2,
\]

where \( W_{\gamma p} \) is the invariant photon–proton center of mass energy; \( \mu \) is the factorization scale; \( x = M_V^2/W_{\gamma p}^2 \) is the light-cone momentum fraction associated with the two-gluon exchange; \( C(\mu^2) \) determines the cross section normalization, which depends on the wave function of the final charmonium state and approximations used in the calculation of the strong \( \gamma p \rightarrow Vp \) amplitude.

In our approach, we treat Eq. (3) phenomenologically and determine the values of \( \mu^2 \) and \( C(\mu^2) \) from comparison to the high-energy HERA data on \( J/\psi \) \([8–11]\) and \( \psi(2S) \) \([12, 13]\) photoproduction on the proton.

In particular, in the \( J/\psi \) case, we find \([14]\) that Eq. (3) with \( \mu^2 \approx 3 \text{ GeV}^2 \) and \( C(\mu^2) = F^2(\mu^2)(1 + \eta^2)R_g^2\pi^3\Gamma_{ee}M_{J/\psi}^3/(48\alpha_{em}\mu^8) \), where \( F^2(\mu^2 = 3 \text{ GeV}^2) = 0.48 \) for the CTEQ6L1 leading-order gluon density of the proton \([15]\), provides the good description of the \( W_{\gamma p} \) dependence of the \( \gamma p \rightarrow J/\psi p \) cross section and its normalization at \( W_{\gamma p} = 100 \text{ GeV} \). In the expression for \( C(\mu^2) \) \([16, 17]\), \( \Gamma_{ee} \) is the \( J/\psi \rightarrow e^+e^- \) decay width; \( \eta \) is the ratio of the real to the imaginary parts of the \( \gamma p \rightarrow J/\psi p \) scattering amplitude and \( R_g \) is the skewness factor taking into account the off-forward nature of this amplitude. The factors of \( \eta \) and \( R_g \) are calculated from the \( x \) dependence of \( xG_p(x, \mu^2) \) at small \( x \), see details in \([14]\).

To convert the differential cross section of Eq. (3) to the \( t \) integrated cross section, we used the conventional exponential parameterization of the \( t \) dependence of the \( \gamma p \rightarrow J/\psi p \) cross section:

\[
\sigma_{\gamma p \rightarrow J/\psi p}(W_{\gamma p}) = \frac{1}{B_{J/\psi}(W_{\gamma p})} \frac{d\sigma_{\gamma p \rightarrow J/\psi p}}{dt}(W_{\gamma p}, t = 0),
\]

where \( B_{J/\psi} = 4.5 + 0.4 \ln(W_{\gamma p}/90 \text{ GeV}) \), which is consistent with the HERA results \([8–11]\). In the \( \psi(2S) \) case, the measured \( W_{\gamma p} \) dependence of the diffractive \( \gamma p \rightarrow \psi(2S)p \) cross section is found to be similar or possibly somewhat steeper than that for the \( J/\psi \) cross section \([12, 13]\). Since the proton gluon density has an increasingly steeper \( x \) dependence at small \( x \) as one increases the factorization scale \( \mu \), the \( W_{\gamma p} \) dependence of \( \gamma p \rightarrow \psi(2S)p \) can be accommodated by Eq. (3) with \( \mu^2 \approx 4 \text{ GeV}^2 \) \([18]\). In addition, using the result of the H1 analysis \([12]\) on the ratio of the \( \psi(2S) \) and \( J/\psi \) photoproduction cross sections on the proton, \( \sigma_{\gamma p \rightarrow \psi(2S)p}/\sigma_{\gamma p \rightarrow J/\psi p} = 0.166 \pm 0.007(\text{stat.}) \pm 0.008(\text{sys.}) \pm 0.007(\text{BR}) \) on the interval
40 < \gamma p < 150 \text{ GeV}, we fix the normalization of the \sigma_{\gamma p \rightarrow \psi(2S)p} cross section as:

\sigma_{\gamma p \rightarrow \psi(2S)p}(\gamma p = 100 \text{ GeV}) = 0.166 \sigma_{\gamma p \rightarrow J/\psi p}(\gamma p = 100 \text{ GeV}), \quad (5)

where \sigma_{\gamma p \rightarrow J/\psi p}(\gamma p) is calculated using Eqs. (3) and (4). To sum up, the \gamma p dependence of the \sigma_{\gamma p \rightarrow \psi(2S)p}(\gamma p) cross section is calculated using Eq. (3) evaluated at \mu^2 \approx 4 \text{ GeV}^2 and its normalization is fixed by Eq. (5).

For the absorptive corrections \( r_+ \) and \( r_- \) for \( J/\psi \) and \( \psi(2S) \) photoproduction in proton–proton UPCs, we used the results of [19] and [20], respectively. On average, \( r_+ \approx 0.8 \).

Equation (1) also takes into account the possibility for each proton to transform into a \Delta while emitting a quasi-real photon in the the \( p \rightarrow \Delta \gamma \) transition. The photon flux associated with the \( p \rightarrow \Delta \gamma \) transition is [4]:

\[ N_{\gamma/p \rightarrow \Delta}(\omega) = \frac{\alpha_{\text{em}} \mu^*}{4\pi} \frac{\mu^2}{9m_p^2} \left( \frac{M + m_p}{2m_p} \right)^2 \left[ \frac{1}{t_{\text{min}}} \left( \ln \frac{t_{\text{min}}}{\Lambda^2 + t_{\text{min}}} + \frac{11}{6} - \frac{2t_{\text{min}}}{\Lambda^2 + t_{\text{min}}} \right) \ln \frac{t_{\text{min}}}{\Lambda^2 + t_{\text{min}}} \right] \left( \frac{1}{3(\Lambda^2 + t_{\text{min}})^3} \right) \], \quad (6)

where \( m_p \) and \( M \) are proton and \( \Delta \) masses, respectively; \( \Lambda^2 = 0.71 \text{ GeV}^2; \mu^* = 9.42; t_{\text{min}} \) is the minimal momentum transfer, \( t_{\text{min}} = (\omega/\gamma_L)^2 + (M - m_p)^2/\gamma_L^2 + 2\omega(M - m_p)/\gamma_L \).

In Eq. (1), \( \delta \) denotes the ratio of the photon fluxes due to the \( p \rightarrow \Delta \gamma \) and \( p \rightarrow p\gamma \) transitions:

\[ \delta(y) = \frac{N_{\gamma/p \rightarrow \Delta}(\omega)}{N_{\gamma/p}(\omega)}. \quad (7) \]

In the LHC kinematics, the values of \( \delta \) range from a few percent at small \( \omega \) to \( \delta \approx 0.1 \) for \( \omega = O(10 \text{ GeV}) \) and to \( \delta \approx 0.2 \) for \( \omega = O(100 \text{ GeV}) \).

Our results for \( J/\psi \) and \( \psi(2S) \) photoproduction in proton–proton UPCs at the LHC at \( \sqrt{s_{NN}} = 7 \text{ TeV} \) are shown and compared to the LHCb data [2] in Figs. 1 and 2 respectively. In the figures, we present \( d\sigma_{pp \rightarrow ppV}(y)/dy \) of Eq. (1) as a function of the vector meson rapidity \( y \). We show predictions corresponding to \( \sigma_{\gamma p \rightarrow Vp} \) calculated using Eqs. (3), (4) and (5) with the CTEQ6L1 gluon density (curves labeled “CTEQ6L1”) and also parameterized in the following simple form obtained in the H1 analyses [11, 12] (curves labeled “H1 fit”):

\[ \sigma_{\gamma p \rightarrow J/\psi p}^{H1 \text{ fit}}(W_{\gamma p}) = N \left( \frac{W_{\gamma p}}{90 \text{ GeV}} \right)^\lambda, \]

\[ \sigma_{\gamma p \rightarrow \psi(2S)p}^{H1 \text{ fit}}(W_{\gamma p}) = 0.166N \left( \frac{W_{\gamma p}}{90 \text{ GeV}} \right)^\lambda, \quad (8) \]
FIG. 1: The cross section of $J/\psi$ photoproduction in proton–proton UPCs at $\sqrt{s_{NN}} = 7$ TeV as a function of the $J/\psi$ rapidity $y$. The theoretical predictions labeled by “CTEQ6L1” and “H1 fit” are compared to the LHCb data [2].

where $N = 81 \pm 3$ nb and $\lambda = 0.67 \pm 0.03$.

As follows from the preceding, in our analysis we explored two viable possibilities for the $W_{\gamma p}$ dependence of the $\sigma_{\gamma p \to \psi(2S)p}$ cross section: our leading order pQCD analysis corresponds approximately to $\sigma_{\gamma p \to \psi(2S)p} \propto W_{\gamma p}^{0.9}$, while we used the result of the H1 fit corresponding to $\sigma_{\gamma p \to \psi(2S)p} \propto W_{\gamma p}^{0.67}$.

In each case we consider two sets of theoretical curves: the thick upper-lying curves correspond to the calculation including both $p \to p\gamma$ and $p \to \Delta\gamma$ transitions in the flux of equivalent photons and the thin lower-lying curves correspond to the calculation where we set $\delta(y) = 0$.

To obtain the experimental values for the $d\sigma_{pp \to ppV}(y)/dy$ cross section, we divided the published values of the differential cross section times the corresponding branching ratio [2] by the acceptance in each bin of $y$ and by the $V \to \mu^+\mu^-$ branching ratio. For the latter, we used $\text{Br}(J/\psi \to \mu^+\mu^-) = (5.93 \pm 0.06)\%$ and $\text{Br}(\psi(2S) \to \mu^+\mu^-) = (7.7 \pm 0.8) \times 10^{-3}$ [21]. The experimental errors have been added in quadrature.

One can see from Fig. 1 that the theoretical description of the $y$ dependence of the data
FIG. 2: The cross section of $\psi(2S)$ photoproduction in proton–proton UPCs at $\sqrt{s_{NN}} = 7$ TeV as a function of the $\psi(2S)$ rapidity $y$. The theoretical predictions labeled by “CTEQ6L1” and “H1 fit” are compared to the LHCb data.

is good. In addition, the leading order pQCD formalism employing the CTEQ6L1 gluon density also reproduces correctly the normalization of the data in the $\delta(y) \neq 0$ case. The H1 fit corresponding to the systematically larger $\sigma_{\gamma p \rightarrow J/\psi p}$ cross section overestimates the normalization of the $pp \rightarrow ppJ/\psi$ cross section in the $\delta(y) \neq 0$ case but agrees with the data much better in the $\delta(y) = 0$ case.

Turning to the $\psi(2S)$ case, one can see from Fig. 2 that both the leading order pQCD framework and the H1 fit reproduce the $y$ dependence of the $pp \rightarrow pp\psi(2S)$ cross section. As to the normalization, the calculation with $\delta(y) = 0$ agrees with the data better than the result of our calculation, when we also include the $p \rightarrow \Delta \gamma$ transition.

Table I summarizes our predictions for the $pp \rightarrow ppJ/\psi$ and $pp \rightarrow pp\psi(2S)$ cross sections integrated over the rapidity range $2 < y < 4.5$ taking into account the LHCb acceptance and multiplied by the corresponding branching ratios for the two-muon decay, $\sigma_{pp \rightarrow ppV \rightarrow pp\mu^+\mu^-} (2 < \eta_{\mu^\pm} < 4.5)$. 
TABLE I: Predictions for the $pp \rightarrow ppJ/\psi$ and $pp \rightarrow pp\psi(2S)$ cross sections integrated over the rapidity range $2 < y < 4.5$ taking into account the LHCb acceptance and multiplied by the two-muon branching ratio, $\sigma_{pp \rightarrow ppV \rightarrow pp\mu^+\mu^-} (2 < \eta_{\mu^+\mu^-} < 4.5)$.

| Model       | $J/\psi$ [pb] | $\psi(2S)$ [pb] |
|-------------|---------------|-----------------|
| CTEQ6L1    | 298           | 7.9             |
| CTEQ6L1, $\delta(y) = 0$ | 268           | 7.0             |
| H1 fit     | 272           | 7.1             |
| H1 fit, $\delta(y) = 0$ | 311           | 6.7             |

The values in Table I should be compared to LHCb result [2]:

\[
\sigma_{pp \rightarrow J/\psi \rightarrow \mu^+\mu^-} (2 < \eta_{\mu^+\mu^-} < 4.5) = 291 \pm 7 \pm 19 \text{ pb},
\]
\[
\sigma_{pp \rightarrow \psi(2S) \rightarrow \mu^+\mu^-} (2 < \eta_{\mu^+\mu^-} < 4.5) = 6.5 \pm 0.9 \pm 0.4 \text{ pb},
\]

where the first uncertainty is statistical and the second one is systematic. In the $J/\psi$ case, the best agreement between our predictions and the experimental value is found for the calculation using the CTEQ6L1 gluon distribution with $\delta(y) \neq 0$ and the H1 fit with $\delta(y) = 0$. In the $\psi(2S)$ case, the best agreement between the experiment and theory is found in the case, when the contribution of the $p \rightarrow \Delta \gamma$ transition is omitted ($\delta(y) = 0$).

Figures 3 and 4 show our predictions for the cross section of $J/\psi$ and $\psi(2S)$ photoproduction, respectively, in proton–proton UPCs at $\sqrt{s_{NN}} = 8$ TeV (left) and $\sqrt{s_{NN}} = 14$ TeV (right). In these figures, different curves correspond to different assumptions explained in text and used already in Figs. 1 and 2.

Our analysis of high-energy exclusive photoproduction of $J/\psi$ and $\psi(2S)$ mesons on the proton, which is the underlying process in photoproduction of these mesons in proton–proton UPCs, is based on the two-gluon ladder exchange reaction mechanism in the leading logarithmic approximation [7] and, hence, is similar in the spirit to the analyses of Refs. [19, 20]. However, our approaches differ in implementation and details. In Ref. [19], combing the $\gamma p \rightarrow J/\psi p$ HERA data and the $pp \rightarrow ppJ/\psi$ LHCb data, the leading and next-to-leading order gluon distributions in the proton have been determined. Using the obtained gluon distribution, predictions for $pp \rightarrow pp\psi(2S)$ were made in [20]. In our approach, we use the leading order CTEQ6L1 gluon distribution and choose the factorization scale $\mu$ to reproduce the $W_{\gamma p}$ dependence of the $\gamma p \rightarrow J/\psi p$ and $\gamma p \rightarrow \psi(2S)p$ cross sections measured at HERA.
FIG. 3: The cross section of $J/\psi$ photoproduction in proton–proton UPCs at $\sqrt{s_{NN}} = 8$ TeV (left) and $\sqrt{s_{NN}} = 14$ TeV (right) as a function of the $J/\psi$ rapidity $y$. Different curves correspond to different theoretical calculations explained in text.

FIG. 4: The cross section of $\psi(2S)$ photoproduction in proton–proton UPCs at $\sqrt{s_{NN}} = 8$ TeV (left) and $\sqrt{s_{NN}} = 14$ TeV (right) as a function of the $\psi(2S)$ rapidity $y$. Different curves correspond to different theoretical calculations explained in text.

As a result, we find that $\mu^2 \approx 3$ GeV$^2$ for $J/\psi$ and $\mu^2 \approx 4$ GeV$^2$ for $\psi(2S)$, which are somewhat higher than the leading-order values of $\mu^2 = M_{J/\psi}^2/4 = 2.4$ GeV$^2$ for $J/\psi$ and $\mu^2 = M_{\psi(2S)}^2/4 = 3.4$ GeV$^2$ for $\psi(2S)$ used in \cite{19, 20}. While this difference in the $\mu^2$ values is not large, it affects the $y$ dependence of the predicted $pp \to ppV$ UPC cross section.

Also, while we fix the normalization of the $\gamma p \to \psi(2S)p$ cross section using the HERA data, which in turn determines the normalization of the $pp \to pp\psi(2S)$ cross section, it is
predicted in \[20\] and, hence, can be used to assess the accuracy of the used framework in the $\psi(2S)$ case.

Photoproduction of $J/\psi$ and $\psi(2S)$ in proton–proton UPCs was also considered in the $k_t$-factorization approach \[22, 23\]. Using different models for the unintegrated gluon distribution, it was found \[23\] that the rapidity dependence and normalization of the $pp \to ppJ/\psi$ and $pp \to pp\psi(2S)$ cross sections measured by the LHCb collaboration is best described by an unintegrated gluon distribution including nonlinear effects and by the eikonal absorption factor of the order of 0.7. Note that this corresponds to much stronger absorption than that used in our analysis and in the analyses of Refs. \[19, 20\]. At the same time, since the simple H1 parameterization, see Eq. (8), leads to the similarly good description of the LHCb data, a good agreement with the LHCb data within the $k_t$-factorization approach precludes drawing a definite conclusion about an onset of saturation \[23\].

Photoproduction of $J/\psi$ in proton–proton UPCs was also considered in the dipole approach \[24\]. The resulting predictions are in a broad agreement with the LHCb data, see the discussion in \[2\].

In summary, we showed that the framework of leading order perturbative QCD, where we used as an example the CTEQ6L1 gluon distribution of the proton, provides the good description of the rapidity dependence of the cross sections of photoproduction of $J/\psi$ or $\psi(2S)$ mesons in proton–proton UPCs measured by the LHCb collaboration at the LHC. In addition, constraining the normalization of the $\gamma p \to J/\psi p$ and $\gamma p \to \psi(2S)p$ cross sections by the high-energy HERA data, allows us also to correctly reproduce the normalization of the $pp \to ppV$ cross section. A similarly good description of the LHCb data on the $pp \to ppV$ cross section is obtained using the H1 fit to the $\gamma p \to J/\psi p$ and $\gamma p \to \psi(2S)p$ cross sections. Using the same framework, we also made predictions for the $pp \to ppJ/\psi$ and $pp \to pp\psi(2S)$ UPC cross sections for $\sqrt{s_{NN}} = 8$ and 14 TeV. We showed that the contribution of the $p \to \Delta \gamma$ transition to the flux of equivalent photons discernibly increases the $pp \to ppV$ UPC cross section and thus can affect its theoretical interpretation in the situation, when the final hadron is not detected as is the case for the LHCb detector.

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