Kinetic Theory Based Modeling of Type II Core Collapse Supernovae

Terrance Strother
Department of Physics and Astronomy and National Superconducting Cyclotron Laboratory, Michigan State University East Lansing, MI 48842, USA
E-mail: strothe6@pa.msu.edu

Wolfgang Bauer
Department of Physics and Astronomy and National Superconducting Cyclotron Laboratory, Michigan State University East Lansing, MI 48842, USA
E-mail: bauer@pa.msu.edu

Abstract. Motivated by the success of kinetic theory in the description of observables in intermediate and high energy heavy ion collisions, we use kinetic theory to model the dynamics of core collapse supernovae. The specific way that we employ kinetic theory to solve the relevant transport equations allows us to explicitly model the propagation of neutrinos and a full ensemble of nuclei and treat neutrino-matter interactions in a very general way. With these abilities, our preliminary calculations have observed dynamics that may prove to be an entirely new neutrino capture induced supernova explosion mechanism.

1. Introduction
Hydrodynamic models have been the tool of choice for supernova modeling for many decades. Most insights the field of supernova science have been made by hydrodynamic based calculations, however a great deal more must be done before these simulations can be regarded as complete. A truly complete hydrodynamic simulation would in principle have to model the dynamics of multiple fluids with strongly time dependent viscosities to model the propagation of a full ensemble of nuclei and neutrinos in the full six-dimensional phase space in a completely general way. No hydrodynamic simulation is currently capable of this. Since state-of-the-art hydrodynamic calculations already strain the capabilities of high performance supercomputers, there may be a long wait before more complex models that simulate the dynamics of hundreds of different species of nuclei and neutrinos in a general way in the full six-dimensional phase space. This motivates us to move away from the traditional hydrodynamic approach that we are familiar with and draw from other disciplines of physics in an attempt to circumvent this technological roadblock. It turns out that the field nuclear collision modeling is an ideal candidate for this purpose.

Intermediate and high energy nuclear collisions have been very accurately modeled by simulations that made use of transport theories based on a semiclassical implementation of kinetic theory [1, 2, 3]. Given the similarities between the requirements that must be satisfied by simulations of nuclear collisions and supernovae [4, 5, 6], such as the ability to model particle...
production, shock wave formation, collective deflection, as well as the interplay between regular and chaotic collective dynamics, it is tempting to implement these types of kinetic theory based approaches to model the physics and astrophysics of supernova explosions. This is the aim of our work.

As discussed in our previous work [7], the full potential of our code will be realized when it is run large multiprocessor installations. Once it is fully parallelized, it will be capable of efficiently calculating all desired statistical distributions in the full three-dimensional coordinate space while propagating test particles in the full six-dimensional phase space. However, for testing purposes, we also want to provide ways to implement our ideas on a single processor. The preliminary calculations discussed here were performed exactly for that purpose and were simulations of the collapse and early stages of the explosion of a non-rotating spherically symmetric core [8]. While test particles were still propagated in the full six-dimensional phase space, due to statistical limitations imposed by working on a single processor [7], we assumed that all statistical distributions were spherically symmetric. The algorithms employed to calculate the spherically symmetric statistical distributions are discussed in detail in our previous work [7]. Furthermore, our weak nuclear reaction network was only partially activated during these preliminary simulations, and no strong nuclear reactions (i.e. fission and fusion channels) were modeled. Since our approach to modeling these reactions is entirely new, we have elected to implement and test them in stages. The remainder of the weak and strong nuclear reaction networks will be activated and tested after the ongoing parallelization process is complete.

The output generated by this series of single processor test calculations is well understood and consistent with the physics included in the model thus far, however not all of the results were expected. The emergence of what may prove to be a new supernova explosion mechanism was observed in all test calculations. This potential new explosion mechanism is discussed in sections 6 and 7.

2. Equations of Motion

The one-body transport equation for the baryon phase space density \( f_b(xp) \) is given by [9]

\[
\frac{\partial f_b(xp)}{\partial t} + \frac{\Pi^i}{E_b(p)} \nabla_i f_b(xp) - \frac{\Pi^\mu}{E_b(p)} \nabla_\mu U_\mu(x) \nabla_p f_b(xp) + \frac{M_b^\nu}{E_b(p)} \nabla_i U_s \nabla_i f_b(xp) = I_{bb}(xp) + I_{b\nu}(xp)
\]

for the particular state \( b \) of the baryon. The left hand side of (1) describes the temporal changes in the baryon phase space density due to interactions due to the interactions of the nucleons with the mean field vector and scalar potentials \( U_\mu \) and \( U_s \). The two source terms on the right hand side of (1) are collision integrals that that represent the effects that correlations due to the two-body baryon-baryon and baryon-neutrino collisions have the baryon phase space density respectively. For any neutrino species, the transport equation simplifies to an equation of motion that contains only the streaming and baryon-neutrino collision terms since there are no mean field contributions and the possible effects of neutrino-neutrino collisions are neglected,

\[
\frac{\partial f_\nu(xk)}{\partial t} + \frac{k \cdot \nabla^x}{E_\nu(k)} f_\nu(xk) = I'_{b\nu}(xk)
\]

We want to point out, however, that neutrino oscillations (vacuum and/or matter-induced) between different lepton flavors can be explicitly included. It is through the baryon-neutrino collisions that the baryon and neutrino phase space densities can affect one another and the transport equations are coupled by the source terms that represent these effects.
3. Test Particle Method

To numerically solve the coupled baryon and neutrino transport equations (1) and (2), the so-called test particle method is used [10]. Instead of fully discretizing the relevant six-dimensional phase space and calculating the phase space densities in each grid cell in every time step, the test particle method only follows the initially occupied phase space cells in time and represent them by a finite number of imaginary test particles. These imaginary test particles are propagated in a way that models the physical evolution of the phase space. They interact with one another via mean field one-body potentials and scatter with realistic cross sections.

3.1. Test Particle Method Applicability

The test particle approach was originally used to model individual intermediate and high energy nuclear collisions [10]. In that microscopic system, there were many more test particles than there were physical particles. If we wish to use the test particle approach to model a macroscopic system containing a very large number of physical particles, it is clearly impossible to have the number of test particles, $N_{tp}$, exceed the number of physical particles, $N_{phys}$. Computational limitations require in such cases that $N_{phys}/N_{tp} \gg 1$.

The test particle approach can still be applicable in cases where $N_{phys}/N_{tp} \gg 1$, so long as $N_{tp}$ is sufficiently large to capture the gross dynamics of the macroscopic system’s phase space. The ratio $N_{phys}/N_{tp}$ effectively determines a scale cutoff of sorts below which details cannot be resolved. When $N_{phys}/N_{tp}$ becomes sufficiently large, some truly microscopic phenomena become impossible to directly simulate with test particles. Therefore it must be established that these unresolvable details do not impact the gross phase space dynamics and/or can be taken into account indirectly. This can be accomplished with convergence tests. In cases where convergence tests fail, we will have to implement multi-scale methods. This can be the case, for example, in the presence of thermodynamic instabilities or strongly chaotic dynamics. However, we have not reached this point as of now.

3.2. Test Particle Method Formalism

The test particle method formally approximates the phase space density with a sum over delta functions

$$f(\vec{r}, \vec{p}, t) = \sum_{i=0}^{N_{tp}} \delta^3(\vec{r} - \vec{r}_i(t)) \delta^3(\vec{p} - \vec{p}_i(t))$$

Insertion of this approximation of the phase space density into the transport equations (1) and (2) yields simple semi-classical first-order linear differential equations of motion for the centroid coordinates of each test particle. The fact that the baryon and neutrino transport equations are solved in an identical fashion means that the treatment of neutrino dynamics is on equal footing with that of baryons. This is a significant advantage that the test particle approach has over traditional hydrodynamic models.

4. Matter Test Particles

Matter test particles are used to propagate a full ensemble of nuclei and free baryons. It is through the propagation of an entire ensemble of nuclei and free baryons that our calculations are able to retain full knowledge of nuclear composition everywhere in the core at all times. This is superior to the standard tracking of the abundances of free protons, neutrons, alpha particles, and a “representative heavy nucleus” employed by most hydrodynamic calculations. Nuclear structure effects significantly impact electron and neutrino capture rates [11], and many of these effects can be missed if one speaks of a “representative heavy nucleus” instead of an ensemble of nuclei.
4.1. Matter Test Particle Properties

An exhaustive description of matter test particle properties can be found in our previous works [12, 13]. Here it suffices to say the following. Matter test particle explicitly represents a fixed number of nuclei of a given species and a variable numbers of free protons and neutrons. They also implicitly represent the number of electron required to render them charge neutral as well as electron-positron pairs. Each matter test particle has its own set of nuclear properties and temperature. Initially the nuclear properties and temperature of each matter test particle are determined by the initial conditions of the progenitor [8]. At later times, a matter test particle temperature can be changed by thermal mixing and both its nuclear properties and temperatures can be changed by weak and strong nuclear reactions.

4.2. Matter Test Particle Equations

After the insertion of the delta function approximation of the baryon phase space density into equation (1), it is found that the centroid of each matter test particle is subject to three mean field forces: a gravitational force, a nucleonic force, and a force generated by the pressure exerted by the local matter and radiation gas. Matter test particles can also scatter with one another. The equations of motion for the centroid coordinates of the matter test particles are given by the following first order differential equations

\[
\frac{d}{dt}\vec{p}_j = \vec{F}_{G,j} + \vec{F}_{nuc}(\vec{r}_j) + \vec{F}_{gas}(\vec{r}_j) + \vec{C}(\vec{p}_j)
\]

\[
\frac{d}{dt}\vec{r}_j = \frac{\vec{p}_j}{\sqrt{m^2 + p^2_j/c^2}}
\]

\[
j = 1, \ldots, N
\]

where \(\vec{F}_{G,j}\) is the gravitational force acting on the \(j^{th}\) matter test particle, \(\vec{C}(\vec{p}_j)\) symbolizes the effects that two-body collisions with other matter test particles have on the \(j^{th}\) matter test particle’s momentum, and \(N\) is the number of matter test particles used to model the core and is constant. The evaluations of the spherically symmetric forces \(\vec{F}_{nuc}(\vec{r}_j)\), \(\vec{F}_{gas}(\vec{r}_j)\), and \(\vec{F}_{G,j}\) are discussed in our previous works [12, 14, 15] respectively. The way we model test particle scattering is explained at length in our previous works [13, 16]. Here we simply state that test particle scatterings are modeled relativistically and semi-classically in a way similar to those used in the simulation of heavy ion collisions [17].

5. Neutrino Test Particles

Unlike the net baryon number, the number of neutrino test particles is not constant. Neutrino test particles can be created and destroyed. The latter process can be induced by a weak interaction or by a neutrino test particle escaping the core. The number of neutrino test particles therefore varies greatly at different times during the simulation. The only neutrino production mechanism included in our preliminary test calculations is electron capture by nuclei and free protons. It is clear how to include other neutrino production mechanisms into our model as well as how to model the presence of all flavors of neutrinos and anti-neutrinos, but we have elected to proceed incrementally when constructing the weak reaction network for this completely new model. Subroutines that model other neutrino production process such as weak decay and pair production have been written, but they will not be integrated into the code until the ongoing parallelization process is complete.

All present model of neutrino flavor oscillations indicate that these are strongly suppressed in the core [13, 18], only the presence of electron neutrinos is simulated by our test calculations.
(However, there are indications [19] that baryon densities are sufficiently low ($\approx 1 \text{ kg/cm}^3$) just outside the iron core and may have a significant impact on the relative isotope abundance of heavy nuclei produced in the final stages of the supernova explosion.) Neutrino test particles are assumed to be massless, move at the speed of light, and subject to no mean-field-type forces. The only way they can interact with other test particles is through scattering with or being captured by matter test particles. Thus the propagation of neutrino test particles between weak reaction sites is quite simple. Merely multiplying the intermediate unit momentum vector of a neutrino test particle by the speed of light and propagation time determines its new location. No complicated numerical method of approximating the solutions to differential equations is required. This light speed propagation does put limits in on the time step size. To realistically model the propagation of neutrino test particles within the constraints of our coordinate space cells, the time step size should be no larger than $10^{-5} \text{ s}$.

5.1. Neutrino Test Particle Creation
Since we do not explicitly simulate the presence of electrons, we rely upon electron capture rate tables to model the production of test particles representing neutrinos produced via electron capture. Because our simulation models the propagation of a full ensemble of nuclei, electron capture rates are needed for many rare isotopes far from the valley of beta stability. Currently no table exists that can satisfy our input needs in this regard. Until more comprehensive electron capture rate tables become available, we must extrapolate existing tabulated rates to the drip lines. For preliminary test calculations, the widely available Fuller-Fowler-Newman (FFN) table [20] is used as the source for electron capture rates. We extrapolate the rates from the FFN table to the nuclei not included in their table that we include in our simulation, and then reduce all entries by an order of magnitude. Our motivation for doing this is as follows. More recent calculations of weak reaction rates using new shell models of the distribution of Gamow-Teller strength have resulted in an improved and often reduced estimate of its strength compared to those the FFN calculations yielded using extrapolations of the known experimental rates and a simple single-state representation of this resonance [21]. The difference is often an order of magnitude or more, so this table serves as a reasonable estimate of the rates.

5.2. Neutrino Test Particle Propagation
Beam attenuation arguments are ideally suited for calculating neutrino-matter interaction probabilities for neutrinos represented by a neutrino test particle. The probability that the neutrinos represented by a neutrino test particle that moves from location $\vec{x}_1$ to $\vec{x}_2$ is taken to be given by

$$P_{int} = 1 - \exp \left[ - \int_{\vec{x}_1}^{\vec{x}_2} \sum_i \sigma_i(\vec{x}) n_i(\vec{x}) d\vec{x} \right]$$

(5)

where the sum over $i$ runs over all of the interaction channels available to the neutrino test particle between $\vec{x}_1$ and $\vec{x}_2$, $n_i$ is the local number density of the $i^{th}$ species of particle, electrons, free baryons, or nuclei, and $\sigma_i$ is the average effective interaction cross section corresponding to the $i^{th}$ interaction channel. The $n_i$’s are readily calculable and the $\sigma_i$’s are interpolated from tables [22]. Once the interaction probability (5) is calculated, a simple Monte-Carlo algorithm determines if the neutrino test particle interacts with matter. Interaction channels are selected by constructing relative probabilities out of weighted average effective cross sections and using another simple Monte-Carlo algorithm to choose a channel. The significant appeal that this very general way of modeling neutrino-matter interactions is that it applicable everywhere in the core. This is a major advantage this kinematic model has over its hydrodynamic counterparts. Typical hydrodynamic treatments of neutrinos are sufficient only in the extremely short and long mean free path limits and are quite problematic at intermediate values [23, 24].
6. New Dynamics

All of our test simulations have observed new dynamics that may prove to be an entirely new neutrino capture induced supernova explosion mechanism. The early stages of the collapse calculated by our code unfold identically to the accepted picture of supernova collapse. Electron capture rates initially slowly increased as the collapse progressed and at later times rapidly increased and eventually led to rapid deleptonization of the inner core. Not long after the inner region of the core begins to rapidly deleptonize, an unexpected phenomenon is observed in each test calculation. It is always found that neutrino captures at intermediate radii inside the inner region of the core deposit a large amount of electrons in a narrow spherical shell centered about a radius of approximately 45-55 km containing neutron-rich matter with a density on the order of 0.1 nuclear matter density. This radially localized accumulation of electrons alters the electron number density gradient in such a way that substantially increases the outward pressure exerted by the electron gas in the region just outside the radius at which the most electrons are produced. The resultant electron gas pressure profile generates an outward explosion of matter at radii of approximately 55 km. Matter inside this region always collapses inward and formed a proto-remnant.

This neutrino capture induces explosion mechanism substantially differs from the accepted bounce mechanism in many ways. The density in the region where the outward explosion of matter forms was always found to be on the order of $10^{-3}$ nuclear matter density while the central densities at that time were consistently found to be on the order of 0.1 nuclear matter density. This is a significant deviation from the accepted picture of bounce in which the initial outward explosion of matter is generated by the pressure exerted by matter at supernuclear densities accumulated in a small volume near the origin. Thus the role of the nuclear equation of state if fundamentally different in this scenario. Rather then directly dictating the how the outward explosion of matter is formed by determining the maximum density the central region can assume, it indirectly influences the neutrino capture induced explosion by governing the rate at which electron capture occur that power the explosion.

To gain further insight into the dynamics of the new explosion mechanism our calculations are consistently yielding thus far, we study two plots of the electron fraction, average radial beta, and density of matter in the spherical shells the core is divided into for the purpose of calculating statistical distributions. The plot displayed in figure 1 corresponds to the time time the outward explosion of matter begins shortly after the spike develops and the matter between the spike's peak and the outer edge of the rapidly deleptonized inner region of the core where the electron fraction is depressed has clearly decelerated. It is important to point out that nowhere in the core have we achieved supernuclear densities. The highest density at this point in our reference calculation only slightly more than 0.36 nuclear matter density and more importantly at the radius at which the outward explosion of matter forms the density is on the order of $10^{-3}$ nuclear matter density. All of or calculations yield results similar to this. The maximum density in the core when the explosion begins is always well below nuclear matter density and the density at the radius where it is always on the order of $10^{-3}$ nuclear matter density.

The plot displayed in figure 2 corresponds to a time when the outward explosion of matter is well underway. The matter at radii near the peak in the spike of the electron fraction distribution in fully releptonized by this time and a large outward moving density wave is seen beginning its separation from the dense proto-remnant. As the proto-remnant “gently” assumes the the configuration of minimum energy, supernuclear densities are achieved in its center. Typically maximum densities are on the order of 1.5 nuclear matter density.

7. Analysis of New Dynamics

For the spike in the electron fraction to be created, it must be that in the narrow shell in which the spike forms, neutrino capture strongly dominates electron capture. Recall that in all
calculations the spike always formed in a region containing neutron-rich matter with a density on the order of $10^{-3}$ nuclear matter density. The importance of this is fourfold. Obviously the farther away a nucleus is from the valley of beta stability and the closer it is to the neutron drip line, the lower its electron capture rate is. Furthermore, since it is energetically favorable for very neutron-rich nuclei to capture neutrinos and move towards the valley of stability, the neutrino capture Q-value becomes positive and large. This increases the bare capture cross section [22] and therefore the neutrino capture probability increases in the region. Additionally at densities on the order of $10^{-3}$ nuclear matter density none of the electron capture rates are insurmountably large and neutrino captures are not likely to be forbidden by the Pauli Exclusion Principle. This sets the stage for neutrino capture dominance.

Recall that the region of the core comprised of neutron-rich matter with a density approximately equal to $10^{-3}$ nuclear matter density where the spike in the electron fraction distribution forms is centered about a radius roughly 50 km from the origin. To its interior is a
region of hotter denser matter in which the degeneracy of the electron gas suppresses neutrino capture rates and electron capture rates are strongly dominant. The energy of the neutrinos escaping this hotter denser region increases as it continues to contract as does the rate at which it emanates neutrinos. Thus the region where the electron fraction spike forms is bathed in an increasingly intense flux of increasingly energetic neutrinos. For the reasons stated in the previous paragraph, neutrino capture rates in this region quickly outpace electron capture rates and the local electron fraction rises. Not only can neutrino captures push neutron-rich nuclei back along isobars towards the valley of stability, as the energies of the neutrinos propagating through the region increase, they can push some nuclei past the valley of stability into proton-rich territory. In addition to this leptonization mechanism, it turns out free neutrons are highly abundant in this region and they capture a large amount of neutrinos as well. The heavy presence of free neutrons is the result of numerous electron captures by neutron drip line nuclei in the region. Through these two mechanisms, the region can become hyper-leptonized. As it does, it absorbs nearly all of the high energy neutrinos from the flux bathing it. This deprives outlying regions of the opportunity to become hyper-leptonized and is the reason that only the base of the outer edge of the electron fraction spike thickens after it is formed. The tallest part of the electron fraction spike remains narrow and radically alters the radial derivative of the electron number density distribution. This in turn divides the core into a proto-remnant and ejecta in the fashion described in section 6.

8. Summary
The preliminary results generated by our kinetic theory based model and presented here are encouraging. The neutrino capture driven mechanism that it is uniquely poised to observe is robust enough to launch explosions in all test simulations conducted so far using three different nucleon potentials, four electron capture rate tables generated with the FFN table using different extrapolation techniques, and three different numbers of spherical shells used to calculate statistical distributions. We do not yet regard this work as complete, but we are confident that we are working in the right direction with our approach. The ability of the test particle method to treat the dynamics of baryons and neutrinos on equal footing, explicitly model the propagation of neutrinos in a general way that is applicable in all regions of the core at all times, and explicitly model the propagation of a full ensemble of nuclei are all significant steps forward. Without all of these assets that only our code possesses, it would not be capable of generating a neutrino capture driven explosion. They each play a critical role in its realization.

Before we can contend that this picture is complete, in addition to completing the parallelization process and activating the three-dimensional subroutines, there is still more physics that needs to be added to our model. In particular fusion and several additional weak reactions must be built in. It is clear how to accomplish all of this with the test particle approach. We are confident that this new explosion mechanism will survive the inclusion the aforementioned physics. The additional degrees of freedom introduced when three-dimensional distributions are calculated may destabilize the region in which electron are deposited by neutrino capture as it may be unstable to convection. An entropy analysis can confirm if this is a possibility. However even if convection does destabilize this region, the fact that convection begins at only $\sim 1100$ ms after the collapse begins at a radius on the order of 50 km would be a significant discovery in and of itself. The future of this work looks extremely bright, and it promises to provide a new intellectual bridge between nuclear physics and astrophysics. In particular, it has the capability to serve as the nexus for efforts made at nuclear physics laboratories such as NSCL and FRIB and laboratories that study neutrino physics like DUSEL, the Deep Underground Science and Engineering Laboratory. As this model advances, we hope to see its kinetic theory based approach establish itself as the new standard for supernova calculations.
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