A cascaded model of spectral distortions due to spectral response effects and pulse pileup effects in a photon-counting x-ray detector for CT

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(Received 25 June 2013; revised 8 February 2014; accepted for publication 11 February 2014; published 11 March 2014)

Purpose: Energy discriminating, photon-counting detectors (PCDs) are an emerging technology for computed tomography (CT) with various potential benefits for clinical CT. The photon energies measured by PCDs can be distorted due to the interactions of a photon with the detector and the interaction of multiple coincident photons. These effects result in distorted recorded x-ray spectra which may lead to artifacts in reconstructed CT images and inaccuracies in tissue identification. Model-based compensation techniques have the potential to account for the distortion effects. This approach requires only a small number of parameters and is applicable to a wide range of spectra and count rates, but it needs an accurate model of the spectral distortions occurring in PCDs. The purpose of this study was to develop a model of those spectral distortions and to evaluate the model using a PCD (model DXMCT-1; DxRay, Inc., Northridge, CA) and various x-ray spectra in a wide range of count rates.

Methods: The authors hypothesize that the complex phenomena of spectral distortions can be modeled by: (1) separating them into count-rate independent factors that we call the spectral response effects (SRE), and count-rate dependent factors that we call the pulse pileup effects (PPE), (2) developing separate models for SRE and PPE, and (3) cascading the SRE and PPE models into a combined SRE+PPE model that describes PCD distortions at both low and high count rates. The SRE model describes the probability distribution of the recorded spectrum, with a photo peak and a continuum tail, given the incident photon energy. Model parameters were obtained from calibration measurements with three radioisotopes and then interpolated linearly for other energies. The PPE model used was developed in the authors’ previous work [K. Taguchi et al., “Modeling the performance of a photon counting x-ray detector for CT: Energy response and pulse pileup effects,” Med. Phys. 38(2), 1089–1102 (2011)]. The agreement between the x-ray spectra calculated by the cascaded SRE+PPE model and the measured spectra was evaluated for various levels of deadtime loss ratios (DLR) and incident spectral shapes, realized using different attenuators, in terms of the weighted coefficient of variation (COV_w), i.e., the root mean square difference weighted by the statistical errors of the data and divided by the mean.

Results: At low count rates, when DLR < 10%, the distorted spectra measured by the DXMCT-1 were in agreement with those calculated by SRE only, with COV_w’s less than 4%. At higher count rates, the measured spectra were also in agreement with the ones calculated by the cascaded SRE+PPE model; with PMMA as attenuator, COV_w was 5.6% at a DLR of 22% and as small as 6.7% for a DLR as high as 55%.

Conclusions: The x-ray spectra calculated by the proposed model agreed with the measured spectra over a wide range of count rates and spectral shapes. The SRE model predicted the distorted, recorded spectra with low count rates over various types and thicknesses of attenuators. The study also validated the hypothesis that the complex spectral distortions in a PCD can be adequately modeled by cascading the count-rate independent SRE and the count-rate dependent PPE. © 2014 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution 3.0 Unported License. [http://dx.doi.org/10.1118/1.4866890]

Key words: computed tomography, photon counting, spectral response, pulse pileup
1. INTRODUCTION

State-of-the-art multislice computed tomography (CT) reconstructs three-dimensional images from intensity profiles of transmitted x rays. Most clinical CT systems use conventional energy-integrating detectors which measure only the intensity of x-ray beams but do not resolve the energy spectrum of x-ray photons. The energy spectrum, however, would provide additional information about the materials in the attenuating object. Dual-energy CT systems have aimed at extracting some of the energy information to improve tissue identification. The dual-energy CT systems available in the clinic today also use energy-integrating detectors, and their ability to resolve the energy dependence of the measured line integrals is limited by overlap between high and low energy projection data which are acquired by either using two different kVp settings or with a two-layered detector. Photon-counting detectors (PCDs) with energy discrimination, a different type of x-ray detector, allow for the counting of x-ray photons in multiple energy bins simultaneously, opening up the opportunity to improve and advance CT imaging methods. The potential clinical merits of PCD-CT have been discussed in the literature and include reduced noise and dose, improved contrast, better tissue identification, and novel applications in quantitative and molecular CT imaging.

However, the energy spectra recorded by PCDs can be distorted which negatively impacts the quality and accuracy of the reconstructed images. The distortions emerge from both multiple interactions of a single x-ray photon with the detector (count-rate independent effects) and multiple coincident photons, which results in several sets of such multiple interactions (count-rate dependent effects). Both effects are explained in more detail in the following two paragraphs.

An incident x-ray photon interacting with the detector can create electrical charges in one or several locations in the sensor material through the photoelectric effect, Compton scatter, and the reabsorption of K-escape photons (the photoelectric effect without re-absorption of K-escape photons creates charges in only one location). Also, the charge clouds grow in size due to the diffusion effect and Coulomb force as they approach the anode of the PCD, where they arrive quasicoincidentally and generate an integrated pulse. If the charges are created near a pixel boundary, the charge cloud may be split and be detected by multiple pixels at lower energies. K-escape photons may be reabsorbed in a different pixel so that some charge is lost and a peak is created at the incident photon energy reduced by the K-absorption energy. Due to charge sharing and K-escape reabsorption, the energy measured in a pixel can also depend on the incident energy spectrum of the neighboring pixels. The result of all these effects is a probability distribution for a detector pixel to measure an incident photon energy $E_0$ and recorded at an energy larger than that of any of the original photon energies. We call this peak pulse pileup. If the detector uses a unipolar pulse shape with a long tail, then the energy can also be measured at higher energies if the positive tail of a preceding photon signal is added to the current pulse. Similarly, if the detector uses a bipolar pulse shape with a long tail, then the energy can be measured at lower energies if the negative tail of a preceding photon signal is added to the current pulse. We call this tail pulse pileup. In addition, high photon fluxes can cause dynamic changes of the detector energy response through saturation from polarization effects.

As the development of PCDs advances, it is expected that the level of distortions will be reduced. But due to limitations from physics and technology, spectral distortions cannot be eliminated completely. Since quantitative, tissue specific spectral CT imaging with PCDs requires accurate measurements of line integrals in multiple energy bins, the spectral distortions discussed above need to be either corrected or compensated for.

Corrections to the measured spectra could be obtained from, e.g., (computationally expensive) Monte Carlo simulations. However, it would be very challenging or impossible in the general case to fully recover the transmitted spectra from the measurements with a small number of energy bins (we define the transmitted spectrum as the spectrum after transmission through a given set of attenuators).
Compensation techniques, on the other hand, do not need to recover the transmitted spectrum, and instead obtain estimations of the attenuators directly from the recorded energy bin data. There are two approaches for compensation techniques: measurement-based and model-based. Measurement-based compensation techniques use calibration measurements to take into account the spectral distortions. In order to be accurate, they require an extensive number of measurements to calibrate both SRE and PPE, and the finite number of measurements needs to be interpolated nonlinearly. In contrast, model-based compensation methods could use a small number of parameters to model spectral distortions for many different combinations of attenuators and count rates and require only an initial set of measurements to parameterize the model.

We have proposed such model-based compensation algorithms and tested their performance on simulated data previously, separately for SRE and PPE. When applied to real data from a PCD, these compensation approaches require an accurate model of the spectral distortions of the PCD caused by both SRE and PPE. As we outlined above, the SRE and PPE are complex phenomena. There is currently no single model which can calculate the distorted, recorded spectrum of PCDs due to the integrated effects of SRE and higher order PPE at a large range of count rates. In the literature, there are reports on measurements of PCD distortions and some reports on models for SRE (Refs. and 25) or PPE. present a model of both SRE and PPE, but PPE includes only peak pileup to first order. In this study, we propose a model for spectral distortions of PCDs resulting from the combination of SRE and higher-order PPE, including both peak and tail pileup with a realistic approximation of the measured pulse shape.

This paper is organized as follows. In Sec. 2, we introduce the PCD model and its parameters. In Sec. 3, we outline the methods to evaluate the model using data from a PCD. In Sec. 4, we present the results of the evaluation and discuss relevant issues in Sec. 5. In the Appendix, we provide a recipe-like description on how to use the PCD model.

2. CASCADED MODEL

The spectral distortions described in Sec. 1 result from a combination of phenomena when photons interact with a PCD. The photon detection process becomes very complex, especially at high count rates, as a number of charge clouds will interact with each other in the electric field of the sensor and the pulses from the charge clouds processed by the electronic circuit of the PCD will overlap. Even at low count rates the multiple charge clouds created from a single photon can generate multiple quasi-coincident pulses at the anode of the PCD, resulting in a pulse pileup-like effect. An accurate model of these complicated phenomena, even if successfully developed, would likely be very complex.

We propose that the spectral distortions in a PCD can be adequately described by a cascaded model using the following two hypotheses:

1. The spectral distortions can be separated into two factors: count-rate independent factors, which we call the SRE, and count-rate dependent factors, which we call the PPE.
2. The overall spectral distortions (at both low and high count rates) can be modeled by a cascaded SRE+PPE model by first modeling the SRE and then using the spectrum distorted by SRE as input to the PPE model.

In the following, we describe the proposed cascaded SRE+PPE model in more detail. First, we define the probability distribution function (PDF) of the x-ray spectrum transmitted through a given set of attenuators as the probability for photons at energy $E$ incident on the detector. The number of counts per energy for an incident rate $a_i$ in a time interval $\Delta t$ is then given by $n_i(E) = a_i \cdot \Delta t \cdot S_i(E)$ and is referred to as spectrum in the following. The process in which $S_i(E)$ is distorted by the detector and recorded as a distribution $S_{PCD}(E)$,

$$S_i(E) \xrightarrow{PCD} S_{PCD}(E).$$

is modeled by the cascaded effects of the spectral response and pulse pileup

$$S_i(E) \xrightarrow{SRE} S_{SRE}(E) \xrightarrow{PPE} S_{SRE+PPE}(E).$$

This can also be written in a functional form as

$$S_{SRE+PPE} = \Omega_{PPE}(S_{SRE}) = \Omega_{PPE}(\Omega_{SRE}(S_i)).$$

Here, $\Omega_{SRE}(S)$ is a distortion operation that models the SRE and $\Omega_{PPE}(S)$ is a distortion operation that models the PPE. The explicit dependence on energy and count rate was dropped in Eq. (3) to simplify the notation. Notice that $S_{SRE+PPE}(E)$ is not a PDF because counts are lost due to PPE. Ideally, the modeled spectrum $S_{SRE+PPE}(E)$ is equal to the recorded spectrum $S_{PCD}(E)$ within statistical variations.

In Secs. 2A and 2B, we introduce two models, one for count-rate independent SRE and the other for count-rate dependent PPE.

2.A. Count-rate independent spectral distortions (SRE model)

The count-rate independent distortions are modeled by parameterizing the spectral response of the detector for the incident photon energy $E_0$ (Fig. 1) using a function $D_{SRE}(E; E_0)$. $D_{SRE}(E; E_0)$ describes the fraction of photons detected at energy $E$ if the incident photon energy was $E_0$ and is given by the weighted sum of two functions

$$D_{SRE}(E; E_0, w(E_0), \lambda(E_0)) = w(E_0) \cdot D_G(E; E_0, \lambda(E_0)) + (1 - w(E_0)) \cdot D_I(E; E_0),$$

where $w(E_0)$ is a weighting parameter, also called photo peak ratio, and $D_G(E; E_0, \lambda(E_0))$ models the photo peak with a truncated Gaussian distribution centered at $E_0$ and with width...
\[ \sigma(E_0) \text{ that depends on a model parameter } \lambda(E_0) \]
\[ D_G(E; E_0, \lambda(E_0)) = \frac{f_G(E; E_0, \lambda(E_0))}{\int_{-\infty}^{\infty} f_G(E; E_0, \lambda(E_0)) dE}, \]
\[ f_G(E; E_0, \lambda(E_0)) = \begin{cases} \frac{1}{\sqrt{2\pi} \sigma(E_0)} \exp\left(-\frac{(E - E_0)^2}{2\sigma^2(E_0)}\right) & \text{if } |E - E_0| \leq 3\sigma(E_0), \\ 0 & \text{otherwise}, \end{cases} \]
\[ D_T(E; E_0) \text{ models the low-energy continuum tail and is a constant function with a smooth transition to zero at } E_0 \]
\[ D_T(E; E_0) = \frac{f_T(E; E_0)}{\int_{-\infty}^{\infty} f_T(E; E_0) dE}, \]
\[ f_T(E; E_0) = \begin{cases} \frac{1}{E_0 - E_{\min}} & \text{if } E_{\min} \leq E \leq vE_0, \\ \frac{1}{E_0 - E_{\min}} g\left(\frac{(E - vE_0)}{((1 - v)E_0)}\right) & \text{if } vE_0 < E \leq E_0, \\ 0 & \text{otherwise}, \end{cases} \]

where \( g(x) = 2x^3 - 3x^2 + 1 \) smoothly transitions from 1 to 0 in the range \( vE_0 < E \leq E_0 \), and \( v \) determines the width of the transition range. The transition was chosen to start at 80\% of \( E_0 \), hence \( v = 0.8 \), and the minimum energy of the continuum tail, \( E_{\min} \), is fixed to zero. The electronic noise is not part of the photon signal and is not modeled in this study.

The form of \( DSRE \) for a fixed \( v \) is determined by two parameters: \( w(E_0) \) for the ratio between \( D_G \) and \( D_T \) and \( \lambda(E_0) \) for the scaling of the width of the photo peak \( D_G \). Ideally, the two parameters \( w(E_0) \) and \( \lambda(E_0) \) are measured at all desirable energies \( E_0 \). In practice, however, both \( w(E_0) \) and \( \lambda(E_0) \) smoothly change with the energy and there is only a small number of radioisotopes in the diagnostic energy range that can be used for calibration measurements. Thus, we assume a simple linear energy dependence for \( w(E_0) \) and \( \lambda(E_0) \) and interpolate the parameter values for arbitrary energies \( E_0 \) from measured values at different incident energies.

Finally, the mean recorded counts at energy \( E \) according to the spectrum distorted by SRE, \( n_{SRE}(E) \), are calculated as
\[ n_{SRE}(E) = a_t \Delta t \Omega_{SRE}(S_t(E)) = a_t \Delta t \int_0^{\infty} S_t(E_0) D_{SRE}(E; E_0, w(E_0), \lambda(E_0)) dE_0, \]

where \( a_t \) is the count rate incident onto the detector and \( \Delta t \) is an acquisition time period.

In general, the continuum tail from Compton scatter, K-escape, charge sharing, and other effects may require a more complex representation than a simple constant function. Further, the energy dependence of the model parameters may not be linear. For the DXCMT-1 detector used in this study, these approximations worked well but for other detectors the parameterization of the spectral response and parameter interpolation may have to be refined.

2.B. Count-rate dependent spectral distortions (PPE model)

The model for count-rate dependent distortions has been described in detail in our previous publications.\(^7,10\) For convenience, we repeat here the main elements of the PPE model.

In the presence of photon pulse pileup, the mean recorded counts per keV at energy \( E \), \( n_r(E) \), are modeled by the product of three conditional probabilities:
1. \( \Pr(\text{rec}|a_t \tau) \), the probability of events being recorded,
2. \( \Pr(m|\text{rec}) \), the probability of \( m \)th order pulse pileup given that the event was recorded,
3. \( \Pr(E|m) \), the probability of events being recorded at an energy \( E \) with \( m \)th order pulse pileup including both peak and tail pileup effects.

The recorded counts \( n_r(E) \) are then given by
\[ n_r(E) = a_t \Delta t \Pr(\text{rec}|a_t \tau) \sum_{m=0}^{\infty} \Pr(m|\text{rec}) \Pr(E|m)) \]
\[ = a_t \Delta t \Omega_{PPE}(a_t, S(E)), \]

where \( a_t \) is the count rate incident onto the detector, \( \Delta t \) is an acquisition time period, \( \tau \) is the detector deadtime, and \( m \) is the order of the pulse pileup. The probabilities \( \Pr(\text{rec}|a_t \tau) \) and \( \Pr(m|\text{rec}) \) can be straightforwardly calculated from the distribution function of time intervals between consecutive events and the probability of \( m+1 \) photons hitting the detector during the deadtime \( \tau \).\(^8\) The third probability, \( \Pr(E|m) \), has a more complicated expression,\(^7\) involving the actual pulse
shape, the PDF of the recorded spectrum in the absence of pulse pileup, \( S(E) = S_{\text{SRC}}(E) \) for the cascaded model, and the probability distribution of the time intervals between incident photons.

3. EVALUATION METHODS

The evaluation of the cascaded model is based on four sets of measurements: The first two sets, A and B, are used to calibrate the PCD and to determine the model parameters for the SRE model and the PPE model, respectively. The last two sets, C and D, are used to evaluate the proposed cascaded SRE+PPE model with x-ray spectra recorded with various attenuators at low count rates (for data set C) and at higher count rates (for data set D).

3.A. The PCD and energy calibration

The photon-counting detector (model DXMCT-1; DxRay, Inc., Northridge, CA) that was used to assess our PPE model,\(^7\) was also used to acquire spectral data in this study. The design specifications and properties of the DXMCT-1 have been described in detail in the literature\(^{29,30}\), so we summarize here only its most important features.

The DXMCT-1 is a CdTe detector consisting of two blocks with 16×16 pixels each. The pixel size is approximately 1 × 1 mm\(^2\) on a 3 mm thick crystal. The signal from each detector pixel is processed by an amplifier, shaper, digital-to-analog converter, and two comparators, each followed by a digital counter. The two thresholds for the comparators can be adjusted independently, and the total number of comparators and counters is 1024 (two counters for each of the 512 pixels). In this text, we denote the path from the sensor to a counter as a detector “channel.” The counters count signals above an adjustable threshold value. For all recorded spectra, counts in energy bins were obtained for each pixel by subtracting the counts in the counter with the higher threshold from the counts in the counter with the lower threshold (resulting in two-sided energy bins).

The detector can be operated in two modes. The first mode uses fixed comparator thresholds and would be used for CT data acquisitions in two energy bins. The second mode, which is used in this study, is the “spectral sweeping mode” where the thresholds are automatically changed from a low to a high value and events are counted for each threshold for a time period \( \Delta t \).

The detector measures the energy of an incident photon as a pulse height \( H \) in millivolts. The same calibration procedure as in Taguchi et al.\(^7\) was used to relate the pulse height to the photon energy. The nonlinear relationship between the known incident photon energy \( E \) and the pulse height \( H \) at the photo peak was modeled by a function \( H(E) \):

\[
H(E) = c_1 - c_2 \exp(-c_3 E),
\]

with three parameters \( c_1, c_2, c_3 \) that were determined separately for each channel using radioisotope data. Once the calibration curves \( H(E) \) were known, the detector output at a pulse height \( H \) could be converted to counts at energy \( E \). Details of the radioisotopes are given in Sec. 3.B.1 and Subsection 3 of the Appendix.

3.B. Model parameters

This section describes the procedures to obtain the parameters for the models introduced in Sec. 2. The measurements \( A \) provided data to parameterize the spectral response functions [parameters \( w_i \) and \( \lambda_i \) in Eq. (4)] at \( N_D \) different incident photon energies, \( i = 1, \ldots, N_D \) and also for the energy calibration of the detector as discussed in Sec. 3.A. The measurements \( B \) provided data to estimate the deadtime \( \tau \) and a proportionality factor \( k_0 \) that relates the tube current \( I \) to the incident count rate \( a_0 \). The parameters \( w_i, \lambda_i, \tau, \) and \( k_0 \) were estimated separately for each of the 1024 channels.

3.B.1. Model parameters for count-rate independent effects (SRE model)

Data set \( A \) consisted of spectral data from three radioisotopes: \(^{241}\text{Am} \) \( (E_{\gamma} = 59.5 \text{ keV}), \) \(^{109}\text{Cd} \) \( (E_{\gamma} = 88 \text{ keV}), \) and \(^{99m}\text{Tc} \) \( (E_{\gamma} = 140 \text{ keV}) \). Details of the radioisotopes are given in Subsection 3 of the Appendix. The spectra were acquired by placing the radioisotopes directly in front of the detector and operating the detector in spectral sweeping mode.

Using the three radioisotope spectra, three sets of model parameters \( w_i \) and \( \lambda_i, i = 1, 2, 3 \), for each channel were obtained by performing a least-square fit of the function \( D_{\text{SRE}}(E; E_0, w(E_0), \lambda(E_0)) \) from Eq. (4) to the radioisotope spectra for fixed \( E_0 = E_{\gamma} \). The spectra were truncated below an energy threshold \( E_{\text{cut}} \) to eliminate the influence of secondary peaks on the model fit \( (E_{\text{cut}} = 28 \text{ keV for } ^{241}\text{Am}, 45 \text{ keV for } ^{109}\text{Cd}, \) and 40 keV for \(^{99m}\text{Tc} \)). The mean and standard deviation of the parameter sets \( w_i \) and \( \lambda_i \) at the three discrete energies 59.5, 88, and 140 keV for \(^{241}\text{Am}, ^{109}\text{Cd}, \) and \(^{99m}\text{Tc} \), respectively, were linearly interpolated in the range from 1 to 240 keV in steps of 1 keV.

3.B.2. Model parameters for count-rate dependent effects (PPE model)

Data set \( B \) was acquired with a clinical x-ray tube at various tube currents. A similar procedure as in our previous publication\(^7\) was used to jointly estimate the deadtime \( \tau \) and a scale factor \( k_0 \) by fitting the analytic function of the expected recorded count rate for a paralyzable detector, \( a_r \), to the data\(^7,9\):

\[
a_r = a_0 \exp(-a_0 \tau),
\]

with \( a_0 = \int S_0(E) \exp\left(-\int_{\text{path}} \mu(\tilde{x}, E) d\tilde{x}\right) dE, \)

where \( S_0(E) \) is the PDF of the source spectrum exiting the x-ray tube and \( \mu(\tilde{x}, E) \) are the linear attenuation coefficients of the attenuators and the second integral is calculated along
the beam path. The scale factor $k_0$ relates the tube current $I$ to the count rate $a_0$ exiting the x-ray tube,

$$a_0 = k_0 \ I,$$  \hspace{1cm} (15)

with $[I] = \text{mA}$ and $[k_0] = \text{photons per second per mm}^2/\text{mA}$. In practice, the SRE and detective quantum efficiency (DQE) must be taken into account and the count rate must be measured above an energy threshold $E_{th}$ to avoid noise. Equation (13) needs to be modified accordingly

$$a_r(E_{th}) = a_r \exp(-a_r \tau) \frac{\int_{E_{th}}^{\infty} S_{\text{SRE}}(S_t(E)) \, dE}{\int_{0}^{\infty} S_{\text{SRE}}(S_t(E)) \, dE},$$  \hspace{1cm} (16)

with $S_t(E) = S_0(E) \exp \left( - \int_{\mu} \mu(x, E) \, ds \right) \int S_0(E)$

$$\times \exp \left( - \int_{\mu} \mu(x, E) \, ds \right) \, dE.$$  \hspace{1cm} (17)

We did not determine the DQE of the detector specifically. Instead, the DC component of the DQE, $\text{DQE}(u = 0)$, were $u$ is the spatial frequency, was absorbed in the scale factor $k_0$. This approach does not take into account a possible dependence of the DQE on the transmitted spectrum and incident count rate.

A clinical x-ray tube (the model used in SOMATOM Emotion CT scanners, Siemens Healthcare, Forchheim, Germany) was used in this experiment with a minimum tube current of 2 mA. The detector was placed 1098 mm away from the x-ray tube and an aluminum filter with thickness $d_{\text{Al}} = 31.5$ mm was placed in front of the x-ray tube to reduce the count rates further, hence $\exp(-\int_{\mu} \mu(x, E) \, ds) = \exp(-\mu_{\text{Al}}(E)d_{\text{Al}})$ in Eqs. (14) and (17). The recorded count rates for $E_{th} = 40$ keV were measured for ten different tube currents $I = 2, 5, 10, 20, 30, 40, 50 70, 80, 100$ mA. The reduction of counts due to the threshold $E_{th}$ can be substantial. For the 31.5 mm Al filter, the ratio $a_r(E_{th} = 40 \text{ keV})/a_r(E_{th} = 0 \text{ keV})$, estimated using the model, over all channels was $(77 \pm 3)\%$ with a minimum of 62% and a maximum of 84%.

The mean and standard deviations of $\tau$ and $k_0$ were calculated from all channels and the quality of the fit was assessed by calculating the coefficient of variation, COV.

No flat-field corrections were applied since they assume a linear behavior of the detector. A large number of flat-field measurements would be required to approximate the actual nonlinear dependence of the detector readout on the incident count rate and transmitted spectrum. Instead, channel-to-channel variations were corrected by the channel-specific estimation of all model parameters: Variations in energy thresholds were taken into account by the channel-by-channel energy calibration. Variations in detector homogeneity and gain were included in the scale factor $k_0$ that relates the tube current to the count rate. Energy response fluctuations were included in the parameters of the SRE model. Variations in electronics affecting pulse-pileup were accounted for by the individual measurement of the detector deadtime $\tau$.

3.C. Test 1: Distorted, recorded spectrum at low count rates

The cascaded model was evaluated with spectral data recorded at low count rates (data set C). The purpose of this evaluation step was to assess the following two aspects. First, we evaluated if the SRE model and the interpolation of the parameters, which were obtained from a small number of radioisotope spectra, was able to model a wide range of continuous x-ray spectra. Second, we also evaluated if the mean distorted recorded spectrum could be calculated by the proposed cascaded model at low count rates once the x-ray spectrum $n_0(E)$ incident onto the attenuators and the attenuator materials with their thicknesses were known.

Different spectra shapes were realized by placing uniform attenuators of various materials and thicknesses in front of the x-ray tube operated at a fixed tube voltage of 120 kV. The list of attenuators is given in Table I. The estimated level of pulse pileup is shown in the third column and is expressed in terms of the deadtime-loss ratio,

$$\text{DLR} = 1 - a_r/a_t,$$

$$= 1 - \exp(-a_r \tau) \quad \text{[for a paralyzable detector]},$$  \hspace{1cm} (18)

which measures the fraction of counts lost due to the finite detector speed.

The PDF $S_0(E)$ of the source spectrum $n_0(E)$ was based on the TASLMIP model$^{31}$ for an x-ray tube with a 8° tungsten anode and modified to fit the measured spectra better (see Subsection 2 of the Appendix). The source spectrum was calculated as

$$n_0(E) = k_0 \ I \ \Delta t \ S_0(E).$$  \hspace{1cm} (19)

| Attenuator | Thickness (mm) | DLR (%) | $a_t$ (Mcounts/s/mm$^2$) | COV$_W$ (SRE+PPE) (%) | COV$_W$ (SRE,DL) (%) | COV$_W$ (n,DL) (%) |
|-----------|---------------|---------|-------------------------|------------------------|-----------------------|--------------------|
| PMMA      | 234           | 2.2     | 0.4                     | 2.2                    | 2.2                   | 9.1                |
| Al        | 26            | 3.5     | 0.6                     | 2.4                    | 2.5                   | 12.3               |
| W         | 0.11          | 7.4     | 1.3                     | 3.2                    | 5.3                   | 25.3               |
| Sn        | 0.85          | 1.7     | 0.3                     | 3.7                    | 3.9                   | 15.4               |
| Bi + Al   | 0.073 + 13    | 5.2     | 0.9                     | 2.9                    | 3.8                   | 12.7               |
| 20% Magnevist | 48             | 3.4     | 0.6                     | 2.4                    | 2.7                   | 10.4               |
The energy-dependent linear attenuation coefficients of material \( j \), \( \mu_j(E) \), were calculated using \texttt{xraylib}.\textsuperscript{32} The PDF \( S_j(E) \) of the transmitted spectrum \( n_j(E) \) was then calculated from \( n_0(E) \), \( \mu_j(E) \), and the thickness of the uniform attenuator material, \( d_j \)

\[
n_i(E) = n_0(E) \exp \left( -\sum_{j=1}^{N_a} \mu_j(E) d_j \right),
\]

\[
S_i(E) = n_i(E) \int_0^\infty n_i(E) dE,
\]

with \( N_a \) being the number of attenuator materials. The recorded spectra (in units of counts per energy) distorted with SRE only, \( n_{\text{SRE}}(E) \), were calculated according to Eq. (10), and the spectra distorted with SRE+PPE, \( n_{\text{SRE+PPE}}(E) \), were calculated using the cascaded model

\[
n_{\text{SRE+PPE}}(E) = a_i \Delta t \Omega_{\text{PPE}} (\Omega_{\text{SRE}} (S_i(E)))
\]

\[
= k_0 t \left( \int S_0(E) \exp \left( -\sum_{j=1}^{N_a} \mu_j(E) d_j \right) dE \right) \Delta t \times \Omega_{\text{PPE}} (\Omega_{\text{SRE}} (S_i(E))).
\]

The spectra measured by the DXMCT-1, \( n_{\text{PCD}}(E) \), were smoothed using a shift-variant moving-average filter (see Subsection 3 of the Appendix), rebinned, and compared to three model spectra: \( n_{\text{SRE+PPE}}(E) \) estimated by the proposed cascaded model, \( n_{\text{SRE,DL}}(E) \) estimated by the SRE model only and scaled by the deadtime losses, and \( n_{\text{DL}}(E) \) the transmitted spectrum scaled by the deadtime losses

\[
n_{\text{SRE,DL}}(E) = n_{\text{SRE}}(E) \cdot \exp (-a_i \tau),
\]

\[
n_{\text{DL}}(E) = n_i(E) \cdot \exp (-a_i \tau).
\]

We evaluated the models using \( n_{\text{SRE,DL}}(E) \) and \( n_{\text{DL}}(E) \) rather than \( n_{\text{SRE}}(E) \) and \( n_i(E) \) in order to focus on the changes in the spectrum \textit{shapes} due to SRE and PPE. The changes in average count rates [given by the exponential factor in Eqs. (18) and (23)] are well established in the literature.\textsuperscript{7,9,10}

The agreement between the data and the models was measured above an energy threshold \( E_{\text{eval}} \) by calculating the weighted COV that takes into account the statistical error of the measured data

\[
\text{COV}_w = \text{RMSD}_w / \bar{n}_{\text{PCD}};
\]

\[
\text{RMSD}_w = \sqrt{\frac{\sum_{l=\text{eval}}^{N_{\text{eval}}} (n_{l,\text{eval}} - n_{l,\text{model}})^2 / \sigma_{l,\text{PCD}}^2}{\sum_{l=\text{eval}}^{N_{\text{eval}}} 1 / \sigma_{l,\text{PCD}}^2}}.
\]

Here, \( \bar{n}_{\text{PCD}} \) is the mean of the spectral counts from the PCD above the threshold \( E_{\text{eval}} \), \( n_{l,\text{PCD}} \) and \( n_{l,\text{model}} \) are the counts in energy bin \( l \) for the PCD and any of the three models mentioned above, respectively, and \( \sigma_{l,\text{PCD}} \) is the standard deviation of the detector counts in bin \( l \) over multiple noise realizations. The threshold \( E_{\text{eval}} \) for the evaluation was set to 40 keV to avoid the effect of the electronic noise in the data and the sums run from bin \( l_{\text{eval}} \) that corresponds to \( E_{\text{eval}} \) to the bin \( N_a \) in the spectra that corresponds to the upper energy of the range used for the evaluation (150 keV).

3.D. Test 2: Distorted, recorded spectrum at higher count rates

The cascaded model was evaluated on spectral data recorded at higher count rates, where the combined effect of SRE and PPE was severe (data set D). The purpose of this evaluation step was to assess if the cascaded model was able to predict the mean distorted, recorded spectrum at various count rates once the x-ray spectrum \( n_0(E) \) incident onto an attenuator and the attenuator materials with their thicknesses were known.

The spectra were acquired using the same setup as described in Sec. 3.C. The tube current was changed to achieve different levels of PPE with each of three different attenuators: PMMA, aluminum, and tungsten with gadolinium dleumine as gadolinium contrast agent (Magneswet, Bayer HealthCare Pharmaceuticals, Montville, NJ). Details are given in Table II. The use of thinner PMMA and aluminum filters compared to Test 1 was the result of practical limitations. The lowest possible tube current setting was 2 mA and relatively thick filters were needed in Test 1 to avoid pulse pileup. For Test 2, the thick filters would not allow to explore the range of large deadtime losses (DLR > 50%) even at the highest tube current. It was also not practical in our experimental setup to change the distance between x-ray tube and detector. For these reasons, we chose thinner filters for Test 2.

The three model spectra \( n_{\text{SRE+PPE}}(E) \), \( n_{\text{SRE,DL}}(E) \), and \( n_{\text{DL}}(E) \) were calculated and compared to the measured spectra as described in Sec. 3.C.

4. EVALUATION RESULTS

4.A. Energy calibration

The mean and the standard deviation of the energy calibration parameters, calculated over all channels, were \( c_1 = (709.6 \pm 12.4) \text{ mV} \), \( c_2 = (813.6 \pm 53.5) \text{ mV} \), and \( c_3 = (0.0337 \pm 0.0011) \text{ keV}^{-1} \). Since the three parameters were estimated from three radioisotope data points, the mean COV of the energy calibration fits was calculated using two additional radioisotope data points (the 81 keV peak of \(^{133}\text{Ba} \) and the 122 keV peak of \(^{57}\text{Co} \)). The mean COV was \( (3.5 \pm 1.0\%) \), showing that the energy dependence was described well by the function in Eq. (12) for all channels.

4.B. Model parameters

The model parameters were obtained for all 1024 channels of the DXMCT-1 but not all channels were used for the model evaluation later on. Noisy and dead pixels were excluded and channels for which the model parameters were outliers (more than five standard deviations different from the average parameter value), leaving 834 channels for the evaluation. In the following, we give the mean values and standard deviations for the model parameters obtained from those 834 channels.
TABLE II. COVW, averaged over all channels, showing the agreement between data and the three models at higher count rates for three attenuators.

| Attenuator | DLR (%) | \( a_k \) (Mcounts/s/mm²) | COVW (\( n_{SRE+PPE} \)) (%) | COVW (\( n_{SRE,DL} \)) (%) | COVW (\( n_{DL} \)) (%) |
|-----------|---------|-----------------|----------------|----------------|----------------|
| PMMA (141 mm) | 1.1 | 0.2 | 1.5 | 1.7 | 8.3 |
| | 7.8 | 1.4 | 3.3 | 5.3 | 19.2 |
| | 13.2 | 2.4 | 5.2 | 8.9 | 26.9 |
| | 22.1 | 4.2 | 5.6 | 13.9 | 33.8 |
| | 37.5 | 7.8 | 6.8 | 24.7 | 51.7 |
| | 55.2 | 13.3 | 6.7 | 36.3 | 67.5 |
| | 74.3 | 22.6 | 14.0 | 42.4 | 68.2 |
| Al (13 mm) | 7.9 | 1.4 | 3.5 | 5.4 | 16.0 |
| | 17.6 | 3.2 | 5.7 | 11.5 | 29.4 |
| | 27.6 | 5.4 | 9.0 | 18.9 | 43.3 |
| | 46.0 | 10.2 | 12.5 | 31.1 | 63.3 |
| | 65.4 | 17.6 | 12.8 | 35.8 | 67.7 |
| | 79.7 | 26.5 | 23.6 | 35.6 | 55.6 |
| W (0.11 mm) | 1.0 | 0.2 | 1.9 | 2.1 | 11.7 |
| | 6.3 | 1.1 | 3.5 | 4.9 | 26.3 |
| | 12.4 | 2.2 | 5.8 | 9.9 | 40.1 |
| | 20.8 | 3.9 | 8.7 | 16.6 | 52.4 |
| | 31.5 | 6.3 | 10.5 | 25.5 | 71.4 |
| | 44.5 | 9.8 | 11.6 | 36.3 | 92.5 |
| | 59.7 | 15.1 | 10.8 | 45.5 | 104.6 |

The weight \( w(E_0) \) and width parameter \( \lambda(E_0) \) for the SRE model were \( w(60\text{ keV}) = 0.62 \pm 0.05 \) and \( \lambda(60\text{ keV}) = (0.53 \pm 0.02) \text{ keV}^{1/2} \) for \(^{241}\text{Am} \), \( w(88\text{ keV}) = 0.64 \pm 0.08 \) and \( \lambda(88\text{ keV}) = (0.53 \pm 0.06) \text{ keV}^{1/2} \) for \(^{109}\text{Cd} \), and \( w(140\text{ keV}) = 0.54 \pm 0.03 \) and \( \lambda(140\text{ keV}) = (1.13 \pm 0.10) \text{ keV}^{1/2} \) for \(^{99m}\text{Tc} \). The increase of \( \lambda \) for \(^{99m}\text{Tc} \), i.e., the fact that \( \lambda \) is not constant, indicates that the energy resolution does not follow a simple \( \sigma \propto \sqrt{E} \) relation [see Eq. (7) and Chap. 4 of Ref. 9], and must be caused by uncertainties of the nonlinear relation between energy and pulse height, Eq. (12). An example of the SRE model compared to DXMCT-1 data with the \(^{109}\text{Cd} \) source is shown in Fig. 2. The continuum part \( D_S(E; E_0) \) of \( D_SRE(E; E_0, w(E_0), \lambda(E_0)) \) was fitted to energies down to 0 keV although counts in the spectrum below \( E_{cut} \) were set to zero. This approach balanced out the slightly elevated counts observed in the energy range between \( E_{cut} \) and the photo peak and gave good agreement between data and model at low count rates.

The function \( D_{SRE}(E; E_0, w(E_0), \lambda(E_0)) \) for one channel is shown in Fig. 3 together with examples of the SRE model for fixed \( E_0 \). The linear interpolation of the model parameters \( w \) and \( \lambda \) for the calculation of \( D_{SRE}(E; E_0, w(E_0), \lambda(E_0)) \) for arbitrary energies \( E \) was tested with the same additional radioisotopes (\(^{133}\text{Ba} \) and \(^{57}\text{Co} \)) used in Sec. 4A. Figure 4 shows that within the statistical errors the two additional isotopes agree with the linear interpolation for the parameter \( w \). The parameter \( \lambda \) for \(^{133}\text{Ba} \) (the point at 81 keV) agrees well with the linear interpolation between \(^{241}\text{Am} \) (59.5 keV) and \(^{109}\text{Cd} \) (88 keV). The point for \(^{57}\text{Co} \) (122 keV) is below the linear interpolation. This is expected from the nonlinear behavior between energy and pulse-height described above. However, the deviation from the assumed linear behavior above 88 keV is not expected to pose a problem as the model is not very sensitive to the value \( \lambda \). The more important parameter is \( w \), the fraction of counts in the photo peak versus the counts in the continuum, which is described by linear interpolation.

The mean and the standard deviation of the parameters for the PPE model, calculated over all channels, were \( \tau = (60.2 \pm 5.4) \text{ ns} \) and \( k_0 = (2.2 \pm 0.2) \text{ Mcounts/s/(mm² mA)} \), where Mcounts/s stands for million counts per second. The standard
deviation of $\tau$ over all channels was 9.0%, indicating that it is advantageous to determine $\tau$ separately for each channel. The COV, calculated using data up to a tube current of 70 mA was $(3.6 \pm 1.6)\%$, indicating good agreement between the count rates measured with the DXMCT-1 and the paralyzable model of the recorded count rates [Eq. (13)].

4.C. Test 1: Distorted, recorded spectrum at lower count rates

Figure 5 shows the recorded spectra for six different attenuators for a typical channel and the three model spectra $n_{\text{SRE+PPE}}(E)$, $n_{\text{SRE,DL}}(E)$, and $n_{\text{DL}}(E)$. A “typical” channel is defined as a channel with model parameters close to the mean values over all channels. The count rates were low with DLRs generally below 5%, except tungsten with 7.2%. The agreement between data and model, expressed as the weighted COV defined in Eq. (24) and averaged over all channels is given in Table I. Several observations can be made: (#1) The SRE model $n_{\text{SRE,DL}}(E)$ and the SRE+PPE model $n_{\text{SRE+PPE}}(E)$ described the features of various measured spectra, which included K-edges, for energies above the noise (Fig. 5). (#2) As can be seen from both the figures and the table, there was a large discrepancy between the recorded spectra $n_{\text{PCD}}(E)$ and the scaled transmitted spectra $n_{\text{DL}}(E)$. For example, the average COV$_W$ for the transmitted tungsten spectrum was 25.3% compared to 3.2% for the spectrum calculated by the SRE+PPE model. (#3) The difference between $n_{\text{SRE,DL}}(E)$ and $n_{\text{SRE+PPE}}(E)$ was small as expected at low count rates (the curves for these two models in Fig. 5 are almost identical) and the recorded spectra could be estimated adequately by the simpler SRE model. (#4) The blurring effect of the SRE on distinct features in the transmitted spectrum were demonstrated by the K-edges of tungsten (70 keV), gadolinium in Magnevist (50 keV), and bismuth (90 keV) (arrows in Fig. 5). The dip in the spectra from the K-edges was visible in the tungsten and gadolinium spectra, but it was almost unnoticeable in the bismuth spectrum with a K-edge at a rather high energy. This is due to the smaller effect of the bismuth enhancement as can be seen in the transmitted spectrum compared to tungsten or gadolinium (Fig. 5).

4.D. Test 2: Distorted, recorded spectrum at higher count rates

The recorded spectra for a typical channel at different levels of pulse pileup are shown in Figs. 6 and 7 for PMMA and tungsten plus Magnevist, respectively. The values of COV$_W$ for the three models, averaged over all channels, are given in Table II for PMMA, aluminum, and tungsten plus Magnevist. The calculations of the model included pulse pileup up to order $m = 5$. 

Fig. 3. The spectral response function $D_{\text{SRE}}(E; E_0, u(E_0), \lambda(E_0))$ for one channel (left) and examples of three SRE parameterizations for fixed input energy $E_0$ (vertical profiles of the 2D function, right).

Fig. 4. The model parameters (a) $w$ and (b) $\lambda$ as a function of energy. Two additional isotopes (circle markers) were used to cross-check the linearity assumption.
FIG. 5. Spectra at low count rates for six different attenuators for the DXMCT-1, the model, and the transmitted spectrum incident onto the detector for a typical channel. Error bars are shown for every third data point. The level of pulse pileup was small and is expressed as the DLR in the figures. The COV$_W$ is given for the SRE$_{+}$PPE model. Notice that the DLR and COV$_W$ are given for the shown channel only and are not averaged over all channels. The black arrows indicate the K-edges of W, Sn, Gd, and Bi.

One can make several observations for the PMMA spectra (Fig. 6): (#1) When the count rate increased, the fraction of counts in the medium energy range (50–80 keV) decreases and the fraction of counts at higher energies (>80 keV) increased. This is because multiple coincident photons in the medium energies are piled up and counted as one event at higher energies. (#2) The scaled spectra with SRE only, $n_{SRE,DL}(E)$, modeled the measured spectra much better than the scaled transmitted spectrum, $n_{DL}(E)$, but the scaled SRE model overestimates counts in the medium energy range and underestimated counts at high energies. (#3) The spectrum, $n_{SRE+PPE}(E)$, estimated by the cascaded SRE+PPE model, predicts the distortions at higher count rates. (#4) The quantitative agreement (Table II) of the cascaded model with the measured spectra yields COV$_W$ = 6.8% for DLR = 37.5% and COV$_W$ = 14.0% for high count rates with a DLR of 74.3%. Table II also shows the COV$_W$ with only the SRE model and with no model (transmitted spectrum with no detector distortions). For a DLR of 37.5%, COV$_W$ was 24.7% for $n_{SRE,DL}(E)$ and 51.7% for $n_{DL}(E)$, compared to 6.8% for $n_{SRE+PPE}(E)$. (#5) The quantitative agreement indicates that the cascaded model accurately predicts both the shape of the distorted spectra and the loss of counts due to PPE. (#6) The spectra at low energies (20–50 keV) are increasingly affected by electronic noise at higher count rates as observed previously. (#7) At high count rates, the increase in counts above 120 keV due to pulse pileup is accurately described by the cascaded model.
The observations for the tungsten plus Magnevist spectrum (Fig. 7 and Table II) are: (#8) The change of the spectra with higher count rates is similar to that described in (#1). (#9) The quantitative agreement is $\text{COV}_W = 10.5\%$ for DLR = 31.5\% and $\text{COV}_W = 10.8\%$ for a DLR as high as 59.7\%. In comparison, the COV$_W$ at this count rate in Table II is 45.5\% for $n_{\text{SRE,DL}}(E)$ and 104.6\% for $n_{\text{DL}},(E)$. (#10) The dip in the spectra at 70 keV due the tungsten K-edge is overestimated by the cascaded model. (#11) The COV$_W$ at low count rates is similar to PMMA but increases faster than PMMA at higher count rates. This behavior may indicate some limitations of the PPE model that appear more clearly in spectra with distinct features such as K-edges and which are discussed in Sec. 5. (#12) At the highest count rates, the spectra measured with the DXMCT-I for PMMA and tungsten become similar, in particular at higher energies (this is also true for other attenuators). This indicates that it may be challenging to estimate the attenuators with material decomposition techniques\textsuperscript{19,20} if the deadtime losses are too high.

5. DISCUSSION AND CONCLUSIONS

We have developed a cascaded model for spectral distortions in PCDs. The model needs only seven parameters ($\omega$, $\lambda$, $c_1$, $c_2$, $c_3$, $\tau$, and $k_0$) to describe the complex spectral distortion mechanism of SRE and PPE. The model parameters were estimated separately for each channel. The variations of the model parameters were relatively large, for example, the standard deviation was 12.5\% for $\omega$(88 keV), 11.3\% for $\lambda$(88 keV), and 9.0\% for $\tau$. These variations emphasize the importance of modeling each channel separately. They can be caused by sensor inhomogeneities, ASIC-to-ASIC variations, and differences in the electronic processing chain of each channel. Estimating all model parameters channel-by-channel also has the advantage that no separate flat-field corrections are needed.

The model was evaluated using x-ray spectra recorded with the PCD at several count rates. Several spectra were used, including spectra with one or two K-edges. The spectra recorded at low count rates (small DLR) were described with sufficient accuracy by the SRE model alone (i.e., without the PPE part of the cascaded model). The COV$_W$’s for the SRE-only model were less than 4\% for DLR < 7.4\% and for the SRE+PPE model they were less than 4\% for DLR < 10.5\%. At higher count rates, the PPE had to be taken into account and the measured spectra were described accurately by the cascaded SRE+PPE model. The COV$_W$’s were less than 11\% for DLR < 32\% and still less than 24\% for DLR as high as 80\%. In comparison, the COV$_W$’s at a DLR of 80\% were 36\% for the scaled SRE model and 56\% for the scaled transmitted
FIG. 7. Tungsten plus Magnevist spectra at higher count rates for the DXMCT-1, the models, and the transmitted spectrum incident onto the detector for a typical channel. Error bars are shown for every third data point. The COVW is given for the SRE + PPE model. Notice that the DLR, COVW, and $a_i$ are given for the shown channel only and are not averaged over all channels.

The differences between the cascaded model and the simpler SRE model and the transmitted spectra would be even larger if the counts had not been scaled by the deadtime losses due to PPE.

The evaluation results validate our initial hypotheses that the complex spectral distortions in a PCD can be modeled by cascading count-rate independent SRE and count-rate dependent PPE. The results show also that the spectral response distortions of a continuous x-ray spectrum can be modeled by parameterizing the detector spectral response for just a small number of calibration measurements using monochromatic radioisotope data.

The proposed model is general, not specific to the PCD used in this study, and can easily be adapted to other detectors. For example, the approximation of the low-energy continuum with a constant function could be refined to reflect the peaks from K-escape photons. The pulse shape is an input to the PPE model and can be easily adjusted as well.

Now that a cascaded PCD model for SRE and PPE is available, it may be possible to compensate for spectral distortions of PCDs by integrating the PCD model as a part of the forward imaging process. Either the imaged object or the projection data (line integrals) can then be estimated accurately by compensating for the distortion effect using, for example, a maximum likelihood approach. The imaging chain for this compensation approach is identical to the setting used in Secs. 3.C and 3.D. We shall leave this to future work.

The study has several limitations. The current model includes only the signal from incident photons but no electronic noise. We believe this is the major cause of the disagreement between the model and data at low energies (below approximately 30 keV at low count rates) and even at higher energies with increased count rates. We plan to add electronic noise to the model in the future after a proper study of the noise properties. Some of the model parameters were estimated from calibration measurements that took several hours or days to acquire (spectra from radioisotopes). The measurements of the x-ray spectra at different count rates were acquired over two days. The repeatability of the calibration over this time span was not studied so that potential biases over time could not be accounted for. The model parameters of the spectral response function were estimated for three photon energies and were then interpolated linearly. However, the energy dependence of the parameter $\lambda$ may be nonlinear (Fig. 4) resulting in biases in the model spectrum distorted by SRE. The data/model agreement may be improved further by small, careful adjustments of the incident model spectrum to resemble the true incident spectrum more closely. The effect of scatter was not considered but reduced by
using a collimator and placing the attenuators close to the x-ray tube. Further, some of the probabilities calculated for the PPE model assume that the detector is paralyzable with a fixed deadtime. The true detection mechanism, however, is a pulse-height analyzer which does not have a fixed deadtime. For computational reasons, the calculation of photon pileup was limited to order \( m = 5 \), which may underestimate the blurring of the spectrum at very high count rates. The SRE+PPE model does not include distortions from polarization of the sensor material. However, the model is able to describe the data up to the highest count rates shown in Figs. 6 and 7, indicating that polarization is negligible in the DXMCT-1 detector at these count rates. The increasing deviation between data and model at even higher count rates (Table II) may partially be attributed to polarization effects and requires further study. Finally, future work is needed to determine whether the accuracy of the model is sufficient for a specific application.

ACKNOWLEDGMENTS

This work was supported in part by National Institutes of Health Grant No. R44 EB012379. The authors thank C. Szeles for discussion of the polarization model.

APPENDIX: MODELING DETAILS

1. Recipe for the use of the cascaded PCD model

This section provides a recipe-like list of instructions to obtain the inputs and parameters for the cascaded SRE+PPE model. Also see the notes at the end of the instruction list.

(Step 1) Parameterize the pulse shape of the PCD. Obtain \( b_2 \), and \( t_1/\tau, t_2/\tau, t_3/\tau \) in Fig. 2 and Sec. II of Ref. 10.

(Step 2) Relate energy \( E \) to pulse height \( H \) by obtaining the \( H(E) \) calibration function for each channel of the PCD. In particular, estimate \( c_1, c_2, c_3 \) in Eq. (12) in Sec. 3.A (see Notes 2 and 7).

(Step 3) Obtain the spectral responses at a finite number of input energies \( E_{0,i}, i = 1, \ldots, N_E \), using radioisotopes. Estimate \( w(E_{0,i}) \) and \( \lambda(E_{0,i}) \), \( i = 1, \ldots, N_E \) in Eqs. (4)–(7) in Sec. 2.A. The same radioisotopes as in Step 2 can be used after plotting the recorded spectra as a function of energy using the previously obtained calibration function \( H(E) \) (see Note 7).

(Step 4) Obtain the spectral response function \( D_{SRE}(E; E_0) \) of Eq. (4) for \( 0 \leq E_0 \leq E_{0\text{max}} \) and \( 0 \leq E \leq E_{\text{max}} \). \( E_{\text{max}} \) is the highest possible input energy such as the tube voltage, e.g., 120 keV, and \( E_{\text{max}} \) must be larger than the upper tail of the photo peak spread centering at \( E_{0\text{max}} \).

(Step 5) Estimate \( \tau \) and \( k_0 \) [Eqs. (13) and (15)] by measuring the count rates at different tube current values using x rays at fixed tube voltage, Sec. 3.B.2 (see Notes 3 and 7).

(Step 6) Obtain the source spectrum \( S_0(E) \) exiting the x-ray tube. \( S_0(E) \) should include the attenuation by the glass window and any other internal material. The spectrum can be obtained by directly measuring it or, more conveniently, by using tabulated spectra or simulation software (see Note 4).

(Step 7) Improve the source spectrum \( S_0(E) \) if not yet accurate enough. Select an attenuator with known attenuation (e.g., PMMA or aluminum), choose a tube current that results in DLR below 5% [calculate DLR from Eq. (18)], and measure \( n_{\text{PEC}}(E) \). Evaluate Eqs. (10) and (22) to obtain \( n_{\text{SRE}}(E) \) and \( n_{\text{SRE}+\text{PPE}}(E) \) and verify that \( \text{COV}_W < 5\% \). Modify \( S_0(E) \) carefully to reduce \( \text{COV}_W \) to a level acceptable for the purpose of the study (see Note 5).

(Step 8) Improve the agreement between data and model further if necessary by modifying \( D_{SRE}(E; E_0) \) (see Note 6).

Note 1: Do not change the geometry during the calibration process. Perform the above procedures without moving the PCDs.

Note 2: This can be accomplished using the known photo peak energy of radioisotopes or the endpoint of x-ray spectra at set kVp values. Record spectral data for several radioisotopes (for example, \( ^{241}\text{Am}, {^{109}}\text{Cd}, {^{99m}}\text{Tc} \)) and/or various x-ray tube voltage settings. In case of radioisotopes, determine the pulse height value of the photo peak. In case of x-ray spectra, determine the pulse height value of the high-energy endpoint. Plot the pulse height values against the corresponding energies and fit the calibration function \( H(E) \). Note also that the relationship can be linear and different from Eq. (12).

Note 3: Choose an energy threshold \( E_{0\text{th}} \) that is as low as possible but avoid counts from electronic noise. Pay attention to the deadtime losses and make sure to include data at very low count rates as well. Use an attenuator to reduce the count rate if needed but notice that all measurements have to be done with the same attenuation.

Note 4: Examples are the TASMIP spectra\(^{31}\) for tungsten anodes, the spectrum processor in IPEM Report 78,\(^{32,33}\) and the SpekCalc software.\(^{34}\)

Note 5: See in Subsection 2 of the Appendix how the original simulated spectrum was modified for this study.

Note 6: The simple form of \( D_{SRE}(E; E_0) \) [Eqs. (5)–(9)] worked well for the PCD used in this study but may have to be refined for other detectors. In particular, pay attention to the continuum tail and its continuation into the noise floor and the transition region between continuum and photo peak. The interpolation scheme to obtain \( D_{SRE}(E; E_0) \) from a finite set of measurements at \( E_{0,i} \) may also need to be modified.

Note 7: Calibration measurements for the model parameters should be performed before and after the spectral measurements. If a time-dependency is observed, then it should be parameterized (for example, assuming a linear change over time) and the values at the actual measurement times should be used as model parameters.
2. The generated x-ray spectrum

The generated 120 kVp x-ray spectrum was based on the TASMIP model and filtered with 0.5 mm aluminum but modified to match the spectra measured by the DXMCT-1 better. A comparison between the original TASMIP spectrum and the modified spectrum is shown in Fig. 8. In particular, we scaled the spectrum in three regions: we reduced the amplitude of the Kβ peak by 30%, increased the amplitude of the Kα peak by 10%, and increased the high energy tail in the region between 80 and 100 keV by 12%. The spectrum was then renormalized. The level of scaling in these three regions was determined empirically by comparing the agreement between data and model for 141 mm PMMA, 0.85 mm Sn, and 0.11 mm W + 48 mm Magnevist as attenuators at three different tube currents (corresponding to approximately DLR = 2%, 12%, 30%). The scaling that minimized the sum of COVw over the nine spectra was chosen for the final generated x-ray spectrum.

3. Radioisotopes

The radioisotopes listed in Table III were used to calibrate the detector and to estimate the SRE model parameters. The isotope sources were acquired from Eckert & Ziegler Isotope Products, Inc. (E&Z). The 99mTc source was acquired from Cardinal Health, Inc. (C. H.).

4. Smoothing of DXMCT-1 data

The spectral data were recorded using the “spectral-sweeping” mode of the DMCT-1 where the energy thresholds are swept through in small, equidistant increments. Conversion of the thresholds to energies with the nonlinear calibration curve from Eq. (12) results in nonuniform energy bins which pose a problem when the data are averaged along the energy-axis to reduce statistical variations. To avoid distortions from averaging, each spectrum n(E) was filtered using a moving average filter with three kernel sizes ks = 1, 5, and 9 bins to obtain the spectra nks=1(E), nks=5(E), nks=9(E). The smoothed spectra were calculated as a weighted summation of nks=1, nks=5, and nks=9

\[ n_{\text{smooth}}(E) = \beta_1(n_{ks=1}(E)) + \beta_5(n_{ks=5}(E)) + \beta_9(n_{ks=9}(E)), \]

where the weights \( \beta_1, \beta_5, \) and \( \beta_9 \) were chosen according to Fig. 9.

The smoothed DXMCT-1 spectra were linearly resampled into equidistant bins of 1 keV width before calculating the weighted coefficient-of-variation, COVw, in order to avoid giving higher significance to the more densely sampled low energy range of the spectra.

| Isotope | Energy used for calibration (keV) | Half life | Type | Manufacturer/product number | Activity at start of calibration (mCi) | Acquisition time duration (h) |
|---------|---------------------------------|-----------|------|------------------------------|----------------------------------------|-------------------------------|
| 241Am   | 60                              | 432.2 yr  | D disk (1 in. ø) | E&Z/GF-241-D | 0.010                                 | 122                           |
| 133Ba   | 81                              | 10.5 yr   | D disk (1 in. ø) | E&Z/GF-133-D | 0.082                                 | 62                            |
| 109Cd   | 88                              | 462.6 d   | D disk (1 in. ø) | E&Z/GF-109-D | 0.017                                 | 24                            |
| 57Co    | 122                             | 271.8 d   | D disk (1 in. ø) | E&Z/GF-057-D | 0.365                                 | 7                             |
| 99mTc   | 140                             | 6.0 h     | Vial, 0.4 ml      | C. H./TCO409  | 14.9                                  | 22                            |
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