Thermoelectric generator in endoreversible approximation: 
the effect of heat-transfer law under finite physical dimensions constraint

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We revisit the optimal performance of a thermoelectric generator within the endoreversible approximation, while imposing a finite physical dimensions constraint (FPDC) in the form of a fixed total area of the heat exchangers. Our analysis is based on the linear-irreversible law for heat transfer between the reservoir and the working medium, in contrast to Newton’s law usually assumed in literature. The optimization of power output is performed with respect to the thermoelectric current as well as the fractional area of the heat exchangers. We describe two alternate designs for allocating optimal areas to the heat exchangers. Interestingly, for each design, the use of linear-irreversible law yields the efficiency at maximum power in the well-known form, $2\eta_C/(4 - \eta_C)$, earlier obtained for the case of thermoelectric generator under exoreversible approximation, i.e. assuming only the internal irreversibility due to Joule heating. On the other hand, the use of Newton’s law yields Curzon-Ahlborn efficiency.

I. INTRODUCTION

The real-world energy converters perform under finite-size and finite-time constraints on the resources. In recent years, finite-time thermodynamics [1] has been popular in the study of irreversible processes. Finite physical dimensions thermodynamics (FPDT) is another approach, which considers, for example, the physical size of heat exchanger between heat reservoir and working substance, to study irreversible processes in actual devices. This approach was started by Chambadal [2] in 1957, followed by Novikov [3] and further illustrated by other authors [4–6]. For instance, Chambadal and Novikov started with a steady-state heat engine which is simultaneously in contact with hot and cold reservoirs. It was coupled to the hot reservoir through a finite heat transfer conductance and in perfect contact with the cold reservoir. Its efficiency at maximum power (EMP) comes out in the now well-known form, known as Curzon-Ahlborn (CA) efficiency:

$$\eta_{CA} = 1 - \sqrt{\theta},$$

(1)

where $\theta = T_c/T_h$ is the ratio of cold to hot bath temperatures. This EMP is independent of any other model parameters like the Carnot efficiency $\eta_C = 1 - \theta$. The exact efficiency was reproduced in an elegant way by assuming the so-called endoreversible approximation where working substance is internally reversible [7, 8] and the only irreversibilities arise due to non-ideal contacts with the heat reservoirs.

In this work, we focus on the steady-state energy converter, working on the principle of thermoelectricity, which is a paradigmatic model to study the effect on performance due to different sources of irreversibility [9]. We find the optimal power output of thermoelectric generator (TEG) in the presence of finite physical dimensions constraint (FPDC). Here, in addition to optimizing the power output with respect to electric current, we also optimize with respect to the fractional area of a heat exchanger. With this step, it will be shown that the maximum power output should be at a proper selection of the area of the heat exchangers, in addition to an optimal value of the electric current. This selection is an important step in thermal optimization, as finiteness of the total heat transfer area is a relevant constraint in the overall design of the energy converter [5].

Another objective of this study is to examine the effect of heat transfer law between the working substance and reservoirs on the performance of thermoelectric generator. In particular, we investigate the endoreversible model based on linear-irreversible law of heat transfer. The results are compared with the usual results based on Newton’s law of heat transfer.

This paper is organized as follows: In Section II, we describe the Constant Properties model of thermoelectric generator along with the finite physical dimensions constraint. In Section III, power optimization is performed using two different heat transfer laws; the variables to optimize are the electric current and the fractional area of the heat exchangers. In Section IV, we discuss an alter-
FIG. 1: Schematic of a TEG consisting of two legs of thermoelectric material of n- and p-type, which are connected electrically in series and thermally in parallel. $T_{hM}(T_{c,M})$ is local temperature of thermoelectric material on the hot (cold) end. In the endoreversible approximation, only the irreversibility due to non-ideal thermal contacts with the reservoirs are considered. $U_h$ and $U_c$ are the heat transfer coefficients of heat exchangers, with areas $A_h$ and $A_c$ on the hot and cold sides, respectively.

inate design to constrain the areas of the heat exchangers and discuss its optimal properties. Section V is devoted to a discussion of the results. We end the paper with Section VI, presenting our conclusions.

II. THERMOELECTRIC GENERATOR MODEL

Thermoelectricity is a non-equilibrium phenomenon, studied within the framework of Onsager-Callen theory [10, 11]. The various thermoelectric effects emerge from the coupling between the gradients of temperature and electrochemical potential. Within Constant Properties model (CPM) [12], the thermoelectric material (TEM) is considered to be a one-dimensional, homogeneous substance of length $L$, with given values of internal resistance $R$, heat transfer conductance $K$, and Seebeck coefficient $\alpha$. Further, let $I$ denote the constant value of electric current flowing through the TEM (see Fig. 1). Then, on the basis of Onsager formalism and Domenicali’s heat equation [13, 14], thermal currents at the end points of TEM are written as follows.

$$\dot{Q}_h = \alpha T_{hM}I + K(T_{hM} - T_{cM}) - \frac{1}{2} RI^2,$$

$$\dot{Q}_c = \alpha T_{cM}I + K(T_{hM} - T_{cM}) + \frac{1}{2} RI^2.$$  

In the above equations, the first term corresponds to convective heat flow, where $T_{hM} < T_h$ ($T_{cM} > T_c$) is the local temperature of TEM at hot (cold) end. The second term takes into account heat leakage across the TEM, and the last term is the fraction of Joule heat received by each reservoir, which is equally distributed in case of CPM (see also [15]). Since, we are mainly interested in the efficiency at maximum power, we shall ignore the parasitic heat leaks which reduce the efficiency and consider only the so-called strong-coupling assumption ($K \approx 0$) [10]. The thermal currents are modified as follows:

$$\dot{Q}_h = \alpha T_{hM}I - \frac{1}{2} RI^2,$$

$$\dot{Q}_c = \alpha T_{cM}I + \frac{1}{2} RI^2.$$  

There are two further limiting operations of a TEG. In the so-called endoreversible approximation, only external irreversibility due to finite rate of heat exchange between reservoir and TEM is considered. Thus, setting $R = 0$ (when there is no Joule heating), thermal currents are written as

$$\dot{Q}_h = \alpha T_{hM}I,$$

$$\dot{Q}_c = \alpha T_{cM}I.$$  

In the following, we consider the problem of optimization of power output within endoreversible approximation which is given by the following condition:

$$\frac{\dot{Q}_h}{T_{hM}} = \frac{\dot{Q}_c}{T_{cM}}.$$  

Due to Eqs. (6) and (7), each term in the above equation is equal to $\alpha I$. Thus, the rate of entropy injection at the hot end of TEM and the rate of its removal at the cold end of the material are the same, implying that the process of energy conversion is assumed to be reversible.

We model the flow of heat between a reservoir and the TEM through the heat exchanger. Let $f(T_i, T_{iM})$ represent a general form of the heat transfer law, whereby the heat flux through the heat exchanger is given by

$$\dot{Q}_i = K_i f(T_i, T_{iM}).$$  

where $i = h, c$ and $K_i$ is the generalized thermal conductance of the heat exchanger at the hot or cold end, defined as the product of the heat transfer coefficient ($U_i$) and the area of heat exchanger ($A_i$), i.e. $K_i = U_iA_i$. Under FPDT, finite dimensions of, say, heat exchangers are recognized as optimizable variables [2, 3, 6, 17-20] in the presence of finite rates of heat transfer. Thus, the total heat transfer area to be allocated on the hot and cold sides of the energy conversion system is constrained: $A_T = A_h + A_c$. The performance of TEG will be additionally optimized subject to a given total area of the heat exchangers.

III. POWER OPTIMIZATION

A. Step 1: Optimization over the electric current

In the following, we perform the analysis using the heat transfer law based on linear-irreversible framework. Usually, in literature, Newton’s law for heat transfer is employed for simplicity and analytic solution. As we will
and the working medium. The heat flux entering the inverse temperatures between, say, the reservoir flux at a thermal contact is proportional to the difference see, the present model is also exactly solvable. According with arrows) of EMP are obtained for \( K_o = K_c/K_h \). Then, the EMP is evaluated to be

\[
\eta^* = 1 - \frac{\sqrt{(K_o + 1)(K_o + \theta^2)} + \theta - K_o}{1 + \theta}.
\] (16)

For a given value of \( \theta \), EMP is a monotonically decreasing function of \( K_o \), as depicted in Fig. 2. In particular, EMP is bounded between two limiting values. For, \( K_c < < K_h \), or, in the limit \( K_o \to 0 \), we have \( \eta^* \to \eta C/(2 - \eta C) \). In the opposite limit, when \( K_o \to \infty \), \( \eta^* \to \eta C/2 \). Interestingly, for \( K_o = \theta \), the form of EMP is simplified to \( \eta C \). Further, the series expansion of the above EMP for small temperature differences, or \( \eta C < < 1 \), is given by:

\[
\eta^* \sim \frac{\eta C}{2} + \frac{\eta C^2}{4(1 + K_o)} + O[\eta C^3].
\] (17)

The above series, for \( K_o = 1 \), is given by: \( \eta^* \sim \eta C/2 + \eta C^2/8 + ... \), which shows the same universality up to second order that is found for strong-coupling heat engines having a left-right symmetry [21].

### B. Step 2: Optimization over the area constraint

Now, the ratio \( K_o = K_c/K_h \equiv (U_c/U_h)(A_c/A_h) \) suggests that the parameter \( K_o \) may be tuned by choosing materials with different ratios of heat transfer coefficients \( (U_c/U_h) \), or by varying the allocation of areas \( (A_c/A_h) \). Thus, for the given set of materials (fixed \( U_c/U_h \), there may be a constraint of a fixed total area to be allocated to the heat exchangers. This constitutes an example of the finite physical dimensions constraint (FPDC) mentioned earlier, which we analyze in the following.

It is convenient to define the ratios \( u = U_c/U_h \) and \( x = A_h/A_T \). Note that \( x \) is the fraction of the total area allocated to the heat exchanger at the hot end. The maximum power output, Eq. (15), can then be written in a dimensionless form as:

\[
P^* = \left( \frac{2T_c}{U_hA_T} \right) P^* = \sqrt{\{(1-x)u + x\}^2 \{1-x\}} - \{1-x\}u + x\theta \theta.
\] (18)

In the second step, we optimize the power output with respect to \( x \), for a given value of \( u \) and the total area \( A_T \). The optimal fraction of the area is found to be

\[
x^* = \frac{\sqrt{(1+\theta)} + 2u}{2(1+\sqrt{u})(1+\theta)}.
\] (19)

The relative fraction of optimal areas is depicted in Fig. 3. The doubly-optimized power, \( \hat{\mathcal{P}} = \mathcal{P}(x) \), is

\[
\hat{\mathcal{P}} = \frac{u(1-\theta)^2}{2(1+\sqrt{u})(1+\theta)}.
\] (20)

The corresponding EMP is evaluated to be

\[
\hat{\eta} = \frac{\eta C}{2 - \gamma \eta C}.
\] (21)
where $\gamma = (1 + \sqrt{u})^{-1}$, which has been obtained in different scenarios $^{22}$ $^{26}$. For $u \to 0$, the EMP reaches the upper bound discussed earlier and the optimal fraction of area on the hot side follows $x \to 0$. On the other hand, for $u \to \infty$, the EMP reaches the lower bound and the optimal fraction of area on the hot side follows $x \to 1$.

For $u = 1$, the heat exchanger of the same material is to be used on the hot and cold sides. The EMP is then simplified to

$$\hat{\eta} = \frac{2\eta_C}{4 - \eta_C}. \quad (22)$$

The above expression also exhibits the universality up to second order, as mentioned below Eq. (17). Here, upon the second step of power optimization, the left-right symmetry manifests via the equality of heat transfer coefficients $(U_h = U_c)$ on the hot and cold sides. However, the corresponding optimal ratio of areas is given as: $(A_h/A_c)_{u=1} = (3 + \theta)/(1 + 3\theta)$, which implies that $K_o = (1 + 3\theta)/(3 + \theta)$. The foregoing case makes it apparent that the second-order universality of EMP may be manifested by more general choices of $K_o$, and not simply for $K_o = 1$, as mentioned in Section III.A.

C. Comparison with Newton’s law

Next, we employ Newton’s law for the finite rate of heat transfer between TEM and heat reservoirs, such that

$$\hat{Q}_h = K_h'(T_h - T_{hM}), \quad (23)$$

$$\hat{Q}_c = K_c'(T_cM - T_c), \quad (24)$$

where the thermal conductance $K'_i \equiv U'_i A_i$ and $U'_i$ is the corresponding heat transfer coefficient. Then, applying the flux-matching condition on both hot and cold sides of TEM, we obtain explicit expressions of the thermal currents

$$\hat{Q}_h = \frac{\alpha T_h K_h'I}{K_h' + \alpha I}, \quad (25)$$

$$\hat{Q}_c = \frac{\alpha T_c K_c'I}{K_c' - \alpha I}. \quad (26)$$

Optimizing the power output with respect to $I$, the optimal current is

$$I^* = \frac{K_h' K_c'}{\alpha (K_h' + \sqrt{\theta} K_c')} (1 - \sqrt{\theta}). \quad (27)$$

The optimal power output is given by

$$P^* = \frac{K_h' K_c' T_h}{(K_h' + K_c')} \left(1 - \sqrt{\theta}\right)^2, \quad (28)$$

and the corresponding hot flux is

$$Q_h^* = \frac{K_h' K_c' T_h}{(K_h' + K_c')} \left(1 - \sqrt{\theta}\right). \quad (29)$$

Thereby, the EMP is equal to $\eta_{CA}$. So, when the power output is optimized with respect to $I$ using Newton’s law, the EMP is independent of the heat transfer coefficients.

In the next step, we incorporate the finite physical dimensions constraint in the form of a fixed total area $A_T$, and rewrite the power output, Eq. (28), as

$$P = \frac{P^*}{A_T T_h U_h''} = \frac{x(1-x)u'}{(1-x)u' + x} \left(1 - \sqrt{\theta}\right)^2, \quad (30)$$

where $u' = U_h'/U_c'$. The power output may be further optimized with respect to $x$, obtaining the optimum at $x = \sqrt{u'/(1 + \sqrt{u'})}$. The doubly-optimized power is given by:

$$\hat{P} = \frac{u'}{(1 + \sqrt{u'})^2} \left(1 - \sqrt{\theta}\right)^2. \quad (31)$$

Thus, even though the power can be doubly optimized while using Newton’s law, the EMP does not change upon the inclusion of the finite physical dimensions constraint.

IV. AN ALTERNATE DESIGN

In the above, the total area $A_T$ is arbitrary, which may be decided from the cost of materials, or alternately, from the design constraint. As a case study, we analyze a design for the heat exchangers based on the two-leg configuration of TEG. The areas of cross-section of the $n$-type and $p$-type legs can be $A_n$ and $A_p$, respectively.
The area of a heat exchanger on each (hot or cold) side is set equal to the area of cross-section of the leg of TEM: \( A = A_n + A_p \). The power output is optimized w.r.t \( I \) and \( y = A_p/A \), yielding the EMP as

\[
P^* = \frac{2T_c P^o}{U_n} = \sqrt{(1 - \theta^2)(1 - v^2)y + (1 + v)(v + \theta^2)} - \{y(1 - v)(1 - \theta) + v + \theta\}.
\]

where \( v = U_p/U_n \) and \( y = A_p/A \).

Then, in the second step, the above power output is optimized w.r.t \( y \), obtaining the optimum at

\[
\hat{y} = \frac{3(v - \theta) + v\theta - 1}{4(v - 1)(1 + \theta)}.
\]

Now, since \( y \) represents a fraction of the area, we must have \( 0 \leq \hat{y} \leq 1 \). For a given value of \( \theta \), this constrains the permissible range of \( v \) values, as shown in Fig. 5. There are two regimes:

i) For \( v < 1 \), the allowed range of \( v \) is

\[
0 \leq v \leq \frac{1 + 3\theta}{3 + \theta} = v_1.
\]

ii) For \( v > 1 \), the allowed range is

\[
v_2 = \frac{3 + \theta}{1 + 3\theta} \leq v \leq \infty.
\]

It implies that in the range \([v_1, v_2]\), there is no physically allowed optimal solution of \( y \), which also includes the value \( v = 1 \). As \( \theta \to 1 \), this range shrinks and both \( v_1 \) and \( v_2 \) approach the value of unity (note that \( v_2 = 1/v_1 \)).

The doubly optimized power output is evaluated as

\[
\hat{P} = \frac{(1 + v)(1 - \theta)^2}{4(1 + \theta)}.
\]

Remarkably, the EMP for this problem is the same as Eq. (22). Also, the optimal value of \( K_o = K_c/K_h \), after the above optimization, is given from Eq. (32) as:

\[
\hat{K}_o = \frac{\hat{y} + v(1 - \hat{y})}{v\hat{y} + (1 - \hat{y})}.
\]

Upon using Eq. (34) in the above, we get \( \hat{K}_o = (1 + 3\theta)/(3 + \theta) \), which is consistent with the findings of Section III.B.

Finally, for the case of Newton’s law, when the power output is optimized with respect to \( y \), the optimal point is obtained at \( \hat{y} = 1/2 \). Thus, the optimal areas \( A_n \) and \( A_p \) come out to be equal at the doubly optimized power. The EMP remains at its CA-value.

**V. DISCUSSION**

We have investigated the problem of power optimization in a thermoelectric generator where the working medium is modelled within the Constant Properties model. As a tractable model, we have focused on the endoreversible approximation in the tight-coupling regime.
Thereby, the internal dissipation due to Joule heating and the heat leakage have been neglected. Usually in literature, Newton’s law is employed to model the finite rate of heat transfer through the heat exchangers. We have investigated the problem using the linear-irreversible law based on the difference of inverse temperatures. When the power output is optimized with respect to the electric current, a closed form expression for efficiency is obtained (Eq. (16)) that depends only on the ratio of thermal conductances of the heat exchangers ($K_p = K_c/K_h$) apart from the ratio of reservoir temperatures ($\theta$). As a second step of the optimization strategy, we impose a finite physical dimensions constraint in terms of a fixed total area of the heat exchangers, given that the materials on hot and cold sides can be different. Under this constraint, we further optimize the power, which yields an optimal allocation of the heat exchanger areas. The EMP corresponding to the doubly optimized power depends on the ratio of heat transfer coefficients ($u = U_c/U_h$), apart from the ratio of temperatures. Assuming equal coefficients ($u = 1$) on hot and cold sides, the EMP shows universal features for small temperature differences.

We have also studied an alternate design for the areas of heat exchangers based on two materials (with heat transfer coefficients $U_p$ and $U_n$), where the total constrained area is the total area of cross-section of the two legs of the thermoelectric module ($A = A_p + A_n$). Interestingly, the double optimization of power yields the EMP which is independent of the heat transfer coefficients. However, the optimal allocation of areas depends on the ratio $v = U_p/U_n$.

For the purpose of comparison, a similar analysis is performed based on Newton’s law of heat transfer. The EMP in this case is the well-known CA value, which is independent of the heat transfer coefficients. Here too, the relative areas of the heat exchangers can be moved to optimize the power output in the second step. The optimal areas of heat exchangers are found to be equal in this case.

In literature, there is an intense discussion on the occurrence of universal expressions of efficiency [8,25,27,29,30]. In the context of thermoelectric generators, the exoreversible approximation is based on the presence of internal irreversibility ($R \neq 0$) while assuming ideal thermal contacts with the reservoirs, that yields the following relations:

\[
\dot{Q}_h = \alpha T_h I - \frac{1}{2} RI^2, \\
\dot{Q}_c = \alpha T_c I + \frac{1}{2} RI^2, \\
P = \alpha I (T_h - T_c) - RI^2.
\]

The optimization of power with respect to $I$ yields the EMP as $2\eta_C/(4 - \eta_C)$. In the present work, we have analyzed endoreversible model based on the linear-irreversible law for the heat exchange with reservoirs and performed a double optimization of the power output, first over $I$ and secondly by imposing the area constraint. Thus, we come to obtain the same EMP within the endoreversible model as obtained above for the exoreversible model. Interestingly, this efficiency is also obtained in discrete endoreversible heat engines based on linear-irreversible law [22]. On the other hand, the endoreversible model using Newton’s law yields CA efficiency—with or without the area constraint.

VI. CONCLUSIONS

We have considered optimization of the power output of a thermoelectric generator based on FPDT, which allows the engineer/designer to allocate optimal areas to the heat exchangers, apart from an optimal value of thermoelectric electric current. The approach has been earlier applied to various industrial devices, power plants and cooling systems. The present application to a thermoelectric device shows the utility of FPDT for this class of energy conversion devices. In particular, our analysis also highlights the comparison between linear-irreversible and Newton’s laws in thermoelectric engines and provides a toy model to analyze the interplay of different forms of the efficiency in these devices.

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