Effect of lumped mass on the structural dynamic modifications of a structure

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Abstract. The identification of the effect due to structural changes using modal data is very challenging in the structural dynamic modifications (SDM) study. In this paper, the aim of this study is to identify the effect of physical changes of a structural system based on the natural frequencies and mode shapes. A mild steel thin plate structure was used in the study. A mass was attached to the thin plate to alter the dynamic characteristics of the thin plate structure. Experimental modal analysis (EMA) and the normal modes analysis were performed in two stages to the thin plate structure, without mass and the mass is attached to the thin plate structure. The normal modes of the thin plate structure were obtained using finite element (FE) analysis. The results of the natural frequencies and mode shapes between the thin plate and thin plate with SDM were compared. It is concluded that the impact of structural changes due to SDM will change the modal parameters of the structure.

1. Introduction

Beams are widely used in engineering field especially in mechanical and civil engineering applications. Beams come in a broad variety of forms for various uses and capabilities. Beams are historically examples of construction or civil engineering structural features, such as vehicle frames, aircraft parts, computer frames and other mechanical or structural devices, which include beam assemblies intended to support lateral loads. Beams are distinguished by their form of reinforcement, profile (cross-sectional shape), equilibrium requirements, length and content.

The cumulative result of all forces operating on the beam is to create shear forces and bending moments within the beam, which, in essence, cause internal pressures, strains and deflections of the beam. The loads added to the beam result in response forces at the support points of the beam. Throughout architecture, beams are categorized in many forms on the basis of supports, comprising of purely fixed beam, cantilever beam and continuous beam.

Structural dynamic modifications strategies can be characterized as methods by which the dynamic behavior of the system is enhanced by anticipating the changed behavior induced by the introduction of modifications, such as lumped mass, rigid linkages, dampers, supports, or by changes in the design parameters of the structures themselves [1], [2]. Modification of any structure to enhance its natural frequencies is necessary in many places as the structural response is highly affected by its natural
Structural dynamic modifications are strategies for achieving the functional properties of the changed structure owing to structural improvements without the requirement for a repetitive theoretical approach or experiments [4], [5].

This paper focuses on the SDM study in the experimental and numerical frameworks. Modal testing and normal modes analysis are used to obtain natural frequencies and mode shapes. The comparison of natural frequencies and mode shapes between the thin plate and thin plate with SDM are conducted to investigate the effect of SDM.

2. Materials and methods

2.1. Finite element modelling

A structure used in the SDM was made from a thin steel plate with the length of 520 mm, 60 mm width and 1.5 mm of thickness. The finite element (FE) modelling was carried out using MSC PATRAN software. The FE model of the thin plate was constructed using CQUAD4 shell elements with a size of 5 mm as shown in Figure 1a. CQUAD4 element is significantly used for meshing the FE model of thin structure [6], [7], [8]. The material properties of mild steel were assigned for the plate using nominal parameter values such as Young’s modulus, $E = 200$ GPa, Poisson’s ratio, $\nu = 0.30$ and density, $\rho = 7900$ kg/m$^3$.

SDM is implemented in this study, by introducing a block with a solid mass of 100 gram which was glued to the thin plate. Figure 1b shows the FE model of the thin plate with SDM. The rigid body element of RBE2 was used for modelling a rigid glued type connection between the block mass and the thin plate. The weight of the block mass was modelled by concentrated mass element connection using CONM2.

![Figure 1. FE model (a) thin plate without mass (b) thin plate with SDM](image-url)
2.2. Normal modes

The equation of motion for free vibration of undamped system [9], [10] is described by equation (1).

\[ M \ddot{x}(t) + Kx(t) = 0 \]  

(1)

where \( K \) and \( M \) are all symmetric matrices of stiffness and mass. Meanwhile, \( x \) and \( \ddot{x} \) are the vector of displacements and accelerations. For the analysis of eigenvalue problem [11], [12],[13] equation (2) was used to obtain the natural frequencies and mode shapes of the thin plate.

\[ K \phi_i = \lambda_i M \phi_i \]  

(2)

where \( \lambda_i \) are eigenvalues (natural frequencies) and \( \phi_i \) are the corresponding eigenvectors (mode shapes). MSC Nastran Solution 103 Normal Modes was used for performing normal modes analysis. The common algorithm which is known as Lanczos method was utilised for computing the free vibration modes of the thin plate. The eigenvalues can be calculated using equation (3). The eigenvalues are related to natural frequencies \( f_i \) that can be expressed by equation (4).

\[ \lambda_{n,i} = \sqrt{\frac{K}{M}} \]  

(3)

\[ f_i = \frac{\lambda_i}{2\pi} \]  

(4)

2.3. FE natural frequencies and mode shapes

In this study, the natural frequencies and mode shapes of the first five elastic modes of the thin plate structure is calculated by using normal modes analysis as shown in equation 3. Table 1 lists the FE natural frequencies of the thin plate (column II) and thin plate with SDM (column III). The first five FE natural frequencies of thin plate without mass and with SDM are compared for the validation purposes. The results show that the FE natural frequencies of the thin plate without mass were calculated 28.53 Hz, 78.79 Hz, 147.69 Hz, 155.85 Hz and 256.70 Hz. Meanwhile, for the thin plate with SDM, the FE natural frequencies were 28.88 Hz, 76.60 Hz, 136.74 Hz, 154.89 Hz and 254.15 Hz.

The natural frequencies of the thin plate with SDM were particularly observed lower than the natural frequencies of the thin plate without mass. The reduction of natural frequencies was influenced by the weight introduced by the block mass which affected and changed the total mass of the thin plate structure. Mode 1 showed less significant effect of the SDM. A small change of natural frequency is based on the location of the block mass which is at the nodal point of the structure for mode 1 configuration.

Based on equation 3, the eigenvalues are depending on the stiffness and mass properties. The increment of the parameter mass with respect to the initial mass will decrease the eigenvalue. In this case, the additional mass introduced by the block affects the eigenvalues where the natural frequencies decreased to lower values.
Table 1. Results of FE natural frequencies between the thin plate without mass and thin plate with SDM

| Mode | FE Natural Frequencies Thin Plate without Mass (Hz) | FE Natural Frequencies Thin Plate with SDM (Hz) |
|------|---------------------------------------------------|-----------------------------------------------|
| 1    | 28.53                                             | 28.88                                         |
| 2    | 78.79                                             | 76.60                                         |
| 3    | 147.69                                            | 136.74                                        |
| 4    | 155.85                                            | 154.89                                        |
| 5    | 256.70                                            | 254.15                                        |

Figure 2 shows the FE mode shapes of the thin plate without mass and thin plate with SDM. Two types of elastic modes were obtained for the dynamic behaviour. Four elastic modes which were mode 1, mode 2, mode 4 and mode 5 were bending modes while mode 3 was a torsion mode. For the thin plate without mass, the bending and torsion mode shapes were in symmetrical characteristic. Meanwhile, for the thin plate with SDM, the bending and torsion mode shapes were asymmetry. This is due to the effect of mass introduced on the thin plate and asymmetry in mass placement on the thin plate.
3. Results and discussion

3.1. Test set up
EMA was performed to validate the results obtained from the numerical analysis. Figure 3, shows the experimental setup for impact testing. EMA was performed on the two case studies: namely, 1) thin plate without mass, and 2) thin plate with SDM. The measurement of natural frequencies and mode shapes was conducted with free-free boundary conditions in order to minimise any constraint. The free-free boundary conditions were achieved by using very soft springs for the suspension. The impact hammer was used for applying light impulse excitation and the dynamic response in the acceleration unit was measured using three 10 mV/g sensitivity of miniature uniaxial accelerometers. These light accelerometers contributed to no mass loading issue during the measurement. The data was stored and analysed using LMS Test.Lab software.

Figure 2. Comparison of FE mode shapes between the thin plate without mass and thin plate with SDM.
3.2. Test procedures
The mode of interest are the first five elastic modes. Point 2 which is located at the edge of the thin plate was assigned as a reference point since this point covered the dynamic response for all five elastic modes. During the impact testing, the thin plate was impacted at the same degree of freedom (DOF) in the Z-axis. The dynamic response covered 42 DOFs and the measurement of all the measuring points was in the Z-axis.

Roving accelerometer was used in EMA. Reference point 2 was used as a fixed location for accelerometer no. 1 while accelerometer no. 2 and no. 3 were roved around the measuring points. The frequency resolution was 0.5 Hz and the rigid body modes were neglected. For each measurement point, the dynamic response in term of frequency response function (FRF) was recorded by ten measurements. The average FRF was considered for the determination of natural frequencies of the thin plate and thin plate with SDM.

3.3. EMA natural frequencies and mode shapes
The EMA natural frequencies were determined from a set of 42 measured frequency response functions in Z-axis which were acquired during an impact test. Rigid body modes were also excluded in the analysis. Table 2 contains the EMA natural frequencies of the thin plate without mass (column II) and thin plate with SDM (column III). The EMA natural frequencies of the thin plate without mass were 26.69 Hz, 73.26 Hz, 136.17 Hz, 144.36 Hz and 239.54 Hz. Meanwhile, for the thin plate with SDM, the EMA natural frequencies were 27.13 Hz, 71.61 Hz, 123.54 Hz, 142.89 Hz and 235.64 Hz. The first five EMA natural frequencies were less than 240 Hz.

The EMA natural frequencies were compared between the thin plate without mass and thin plate with SDM. A clear indication that the EMA natural frequencies measured from the thin plate with SDM were lower than EMA natural frequencies measured from the thin plate without mass. The reduction of EMA natural frequencies for most of the vibration modes of the thin plate with SDM was occurred because of the introduction of additional mass which affected the natural frequencies. Mode 1 showed less significant effect of the SDM because the location of the block mass was at the nodal point of the thin plate structure.
Table 2. Results of EMA natural frequencies between the thin plate without mass and thin plate with SDM

| Mode | EMA Natural Frequencies Thin Plate without Mass (Hz) | EMA Natural Frequencies Thin Plate with SDM (Hz) |
|------|---------------------------------------------------|--------------------------------------------------|
| 1    | 26.69                                            | 27.13                                            |
| 2    | 73.26                                            | 71.61                                            |
| 3    | 136.17                                           | 123.54                                           |
| 4    | 144.36                                           | 142.89                                           |
| 5    | 239.54                                           | 235.64                                           |

Figure 4 shows the EMA mode shapes of the thin plate without mass and thin plate with SDM. The bending modes were mode 1, mode 2, mode 4 and mode 5; and the torsion mode was mode 3. For the thin plate without mass, the bending and torsion mode shapes were in symmetrical shape. Meanwhile, for the thin plate with SDM, the bending and torsion mode shapes were asymmetry due to the effect of mass placement on the thin plate. These EMA findings validate the comparison results of natural frequencies and mode shapes obtained from the numerical analysis of the thin plate without mass and thin plate with SDM.
4. Conclusion

Structural dynamic modifications based on modal approach using experimental and numerical analysis were presented in this paper. Utilising free-free boundary conditions in the test set up and finite element modelling of the thin plate, the natural frequencies for the first five elastic modes were less than 260 Hz. From the comparison of results between the EMA and FE analysis, the natural frequencies between the thin plate without mass and thin plate with SDM, the natural frequencies of the thin plate with SDM showed some reductions. Significant changes of the mode shapes between the thin plate without mass and thin plate with SDM was obtained from the mode shape comparison. This finding shows the importance of the additional mass and location of the lumped mass to the eigenvalues and eigenvectors of the structure for the structural dynamic modifications.

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