SOFT INTERACTIONS

A Donnachie
Department of Physics and Astronomy, University of Manchester

P V Landshoff
DAMTP, University of Cambridge

Existing data on total cross sections, on elastic scattering at small, moderate and large values of \( t \), and on diffraction dissociation, reveal a surprisingly simple phenomenology, but they throw up many questions for the LHC to answer.

Total cross-sections

It is common ground that soft interactions are described by Regge theory [1], but the details are still controversial. Our own attitude [2] is to adopt the simplest approach as far as the data allow. In its crudest form, this approach results in a fit to total cross-section data with just two fixed powers, as is shown in figure 1. While most fits to the existing data predict a cross-section at LHC energy not far from 110 mb, and this agrees with a cosmic-ray measurement [3], the conflict between the two Tevatron data points [4] in figure 1 raises the question whether it might be considerably larger. An explanation for such a larger values, if it were to be found, would be readily available: the BFKL pomeron [5].

Elastic scattering – small \( t \)

In the fits shown in figure 1, the term \( s^{\epsilon} \), with \( \epsilon \approx 0.08 \), is said to be associated with the exchange of the soft pomeron. Although the fits work well with constant \( \epsilon \), it cannot really be constant. This is because one must take account not only of the exchange of a single pomeron, but also of more than one. It may be that single-pomeron exchange yields a fixed power \( s^{\epsilon_0} \) (though even this is not certain), but including the the exchange of two pomerons reduces the effective power and makes it vary with \( s \). The fact that the data fit well to constant \( \epsilon \) suggests that its variation with \( s \) is very slow, so that the contribution to the total cross-section from the exchange of two or more pomerons is rather small and \( \epsilon_0 \) is only a little greater than 0.08. This is the approach that we have taken, though others [6] disagree: for them, \( \epsilon_0 \) is significantly greater than 0.08 and multiple exchanges are not small.

Fits [7] to elastic scattering data suggest that the pomeron trajectory is linear in \( t \):

\[
\alpha(t) = 1 + \epsilon_0 + \alpha't
\]  

(1)

The value \( \alpha' = 0.25 \) GeV\(^2\) was extracted from data more than 20 years ago [8].
and remains unchanged today. It has a direct effect on the forward exponential slope $b$ in elastic scattering: if single-pomeron exchange alone is included then

$$b = b_0 - 2\alpha' |t| \log \alpha's$$  \hspace{1cm} (2)

The extent to which $b$ deviates from linear dependence on $\log \alpha's$ is therefore a measure of how important are the multiple exchanges.

Note that, even if the exchange of two pomerons is indeed small in the total cross-section, and therefore also in elastic scattering at very small $t$, we know that this cannot be so at larger $t$. We cannot calculate the magnitude of the contribution from this exchange, but we do know its general properties. It is flatter than single-pomeron exchange, and as $s$ increases it steepens half as quickly. But at $t = 0$ it rises twice as fast as single-pomeron exchange. So, as $s$ increases the point where the two are equal moves to lower and lower $t$. See figure 2. One consequence of this is that the shape of the differential cross-section, as a function of $t$, changes with increasing energy. It happens that, at Tevatron energy, the two contributions combine in such a way that a fit $e^{-b|t|}$ with $b$ independent of $t$ is quite good [9], though this is not true at either lower or higher energies: at low energies $d\sigma/dt$ curves downwards in a log/linear plot, while at very high energies it should curve upwards.
Figure 2: contributions to $\frac{d\sigma}{dt}$ from single and double pomeron exchange. The arrows indicate how they change as the energy increases.

**Elastic scattering – medium $t$**

At ISR energies, the $pp$ elastic scattering differential cross-section has a dip [10]. This dip is around $|t| = 1.4 \text{ GeV}^2$ and moves inwards slowly as the energy increases. It is particularly deep at $\sqrt{s} = 31 \text{ GeV}$. In $\bar{p}p$ scattering the dip is much less pronounced [11], or perhaps even not there at all. The fact that $pp$ and $\bar{p}p$ scattering behave differently in the dip region tells us that there is a significant $C = -1$ exchange that contributes there. Indeed, we have argued [7] that such an exchange is important in order to produce a dip at all. This is because the phase of the single-pomeron-exchange contribution to the amplitude at $t$-values near the dip is very different from that of double exchange. Our belief is [7] that the imaginary parts of the single and double exchanges cancel at the dip but then some other contribution is needed to cancel the real part of the amplitude, and the $C = -1$ exchange is the obvious candidate for this – it even has the right sign. It is rather fortuitous that there is a dip at all, since it needs simultaneous cancellation of the real and imaginary parts, and it seems that at CERN $\bar{p}p$ collider energy the “dip” has moved so far inwards that this simultaneous cancellation no longer occurs and the dip has been replaced by a mere shoulder [12] (it will be interesting to check in $pp$ scattering at RHIC that the same is true). The LHC data will provide a check on whether single and double pomeron exchanges are the dominant ones, or whether multiple exchanges of pomerons are important too. According to figure 2, the $t$-value at which the magnitudes of the single and double exchanges are equal continues to move inwards as the energy increases, until it reaches a
range of values where the phases of the single and double exchanges are nearly equal. We estimate \[7\] that this will be true at the LHC, and therefore that again there will be a noticeable dip – somewhere near \(|t| = 0.5 \text{ GeV}^2\).

**Elastic scattering – large \(t\)**

At ISR energies, the \(pp\) elastic differential cross section has a strikingly simple behaviour \[13\] \[10\] for \(|t|\) greater than about \(3 \text{ GeV}^2\): it is energy-independent and behaves at \(|t|^{-8}\). This is consistent with simple three-gluon exchange \[14\]. This same mechanism is likely to be the \(C = -1\)-exchange mechanism that helps to give the dip seen at smaller \(t\) in the ISR experiments.

It will be interesting to see whether this energy-independence at large \(t\) survives at the LHC. It may even be replaced with something more dramatic \[15\]. It may be that 3-gluon exchange is replaced with the exchange of the exchange of three BFKL pomeron, in which case the large-\(t\) differential cross section would \(\text{rise}\) rapidly with \(\sqrt{s}\).

![Figure 3: differential cross-section for \(pp\) elastic scattering, with the fit 0.09\(|t|^{-8}\)](image)

**Diffraction dissociation**

In diffraction dissociation, an extremely fast proton appears in the final state, so that necessarily there is a large rapidity gap. Interest in such events has been revived with the discovery at HERA that they occur even in deep in-
elastic electron scattering at high $Q^2$, which has made it important to try to understand the puzzling features [16] of the corresponding $pp$ and $\bar{p}p$ data.

There is more than a suspicion [17] that these puzzling features may not be real, but rather result from the way the data have been parametrised. It is important that experimentalists present their complete data, and not just the results of their own fits to them [18].

For an event to be classified as being diffraction dissociation, the fractional momentum loss $\xi$ of the fast proton should be less than a few percent. The magnitude of $\xi$ may be calculated from the invariant mass of the system $X$ of fragments of the other initial-state particle: $\xi = M_X^2/s$. Instead of $\xi$, the notation $x\rho$ is often used. If $\xi$ is small enough, the exchanged object should be the pomeron. If pomeron exchange is described by a simple pole in the complex $\ell$-plane, it should factorise [19]:

$$d^2\sigma_{Ap}/dtd\xi = F_{P/p}(\xi, t) \sigma^{PA}(M_X^2, t) \quad \text{with} \quad F_{P/p} = \frac{9\beta_0^2|F_1(t)|^2}{4\pi} \xi^{1-2\alpha(t)} \quad (3)$$

where $\beta_0$ is the coupling strength of the pomeron to a quark and $F_1(t)$ is the proton’s elastic form factor.

One issue in pomeron physics is whether the pomeron trajectory has on it particles, as do other Regge trajectories. If so, the popular belief is that these particles should be glueballs, and indeed there is [20] a $2^{++}$ candidate for the first of these, at a mass a little less than 2 GeV. But even if there are particles on the pomeron trajectory, when it is exchanged near $t = 0$ the pomeron cannot be said to be a particle. Nevertheless, the factorisation (3) – if it is valid – makes pomeron exchange very similar to particle exchange: the factor $\sigma^{PA}(M_X^2, t)$ may be thought of as the cross-section for pomeron-$A$ scattering. When its subenergy $M_X$ is large, it should have much the same power behaviour as the hadron-hadron total cross-sections: a combination of $(M_X^2)^{0.08}$ and $(M_X^2)^{-0.45}$ with $t$-dependent coefficients. The relative mix of these two contributions must be determined by experiment and there are not yet enough good data for this have been done definitively. In triple-Regge-exchange language, this amounts to saying that the relative strengths of the PPP and PPR terms is not well known.

But there are other complications: the exchange may not be the pomeron. Simple pomeron exchange may be contaminated in two ways. If $\xi$ is not small enough, one must add in a contribution from $\rho, \omega, f, a$ exchange, or even $\pi$ exchange when $t$ is close to 0. Parametrisations [21] [19] of ISR, CERN collider and HERA [22] data suggest that contributions from nonleading exchanges are large. In triple-Regge language, these are the PRP, PRR, RRP and RRR terms. It is evident that a great deal of data will be needed to fix them all well.

If one integrates (3) down to some fixed $M_X^2$, the resulting cross-section for diffraction dissociation behaves as $s^{2\alpha_0}$, and so unless something else intervenes
it would become larger than the total cross-section [23] [16]. As $s$ increases at fixed $M^2_X$, one is probing larger and larger values of $1/\xi$, so one expects that the same happens as in the total cross-section: the exchange of two pomerons becomes important and moderates the rising contribution from single exchange. But the simplest assumption is that this matters only at very small $\xi$.

Notice that the theory leads us to expect that adding all these exchanges should give us all the nonleading powers of $1/\xi$: there should be no other appreciable “background”. Note also that adding together all the exchanges will break factorisation.

**Semisoft processes**

Despite the success of this simple picture of the pomeron in describing a wide range of soft hadronic data, problems do arise when it is applied to processes in which there is a hard scale, even if that scale is still rather soft. The most obvious is deep inelastic scattering. At small $x$ and small $Q^2$ the structure function $\nu W_2$ is governed by Regge theory [24] giving a sum of terms $(1/x)^{\alpha(0)-1}$. The NMC data [25] at moderate $Q^2$ and not-too-small $x$ show that $\nu W_2$ contains such Regge terms: a slowly varying term from soft pomeron exchange and close to $\sqrt{s}$ from $f$, $a$ exchange. A simple fit [26] to the small-$x$ NMC data analogous to the fits [2] to the total hadronic cross sections gives an excellent description of the data and the predictions of the fit are in remarkably good agreement with the subsequent E665 data [27] over the same $Q^2$ range but at smaller values of $x$ than used in the original fit. However at yet smaller $x$ comparison with the HERA data [28] fails for values of $Q^2$ not much above 0.5 GeV$^2$. The implications are either that perturbative QCD is applicable at this surprisingly small value of $Q^2$ or that the non-perturbative processes are more complex than the simple picture allows.

The latter view is one adopted, for example, by Maor [29] in the context of the eikonal model. The principal effect is to make significant changes in the relative energy dependence of the total, elastic and diffractive dissociation cross sections as the energy scale at which screening effects become appreciable differs in each case. It is thus possible to fit simultaneously the total cross section and the CDF diffraction dissociation data [18]. Further, it is possible to take a large value for $\epsilon$ which is reduced to an effective low value in hadron hadron and in photon hadron reactions by the strong unitarity corrections. However for virtual photons these corrections start to diminish at some moderate value of $Q^2$ and so can incorporate the transition seen in deep inelastic scattering [30].

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