An Optimized Bidirectional Quantum Teleportation Scheme with the use of Bell states

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Abstract

Bidirectional quantum teleportation scheme is a two-way quantum communication process, in which two parties simultaneously receive each other information. Recently, in paper [1] (Zhou et al., IEEE Access, 7, 44269 (2019)), a six-qubit cluster state has been used to teleport three-qubit entangled state (Alice) and arbitrary single-qubit unknown quantum state (Bob) bidirectionally. In experimental point of view, the preparation and the maintenance of cluster type entangled state is very difficult and costly. So, in this paper we have designed an optimal scheme for bidirectional quantum teleportation scheme and shown that proposed scheme requires optimized amount of quantum resources.

Keywords: Bidirectional quantum teleportation. Optimal quantum resource. Controlled NOT (CNOT). Unitary. Measurement.

1 Introduction

In 1993 [2], Bennett et al., proposed a quantum teleportation scheme for a single-qubit unknown quantum state using two-qubit maximally entangled state. There are many variant of quantum teleportation (QT) scheme, in which one variant of QT i.e. bidirectional quantum teleportation (BQT) attract the attention of the researchers because the beauty of this scheme is, two parties (Alice and Bob) can get each other information simultaneously. Later on, various papers on BQT have been proposed [1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16] (and references therein). In most of the papers, some are mentioned in Table 1 they have used highly amount of quantum resources (multi-qubit entangled states) without giving much attention to the experimental point of view. Experimental realization and maintenance of such type of multi-qubit entangled states is very difficulty and costly. So, these type of schemes
demand optimized amount of quantum resource, which can be prepared easily. In all multi-qubit entangled state as a quantum resource, Bell state attract much attention because Bell state is a two-qubit entangled state which is minimum qubit entangled state and the experimental realization of two-qubit entangled state is easier and less prone to decoherence as compare with the other multi-partite entangled states. Keep all these points in mind, I have designed optimal scheme of BQT for n-qubit quantum state by using only two Bell states.

The structure of the paper as follows: Section 2, optimized bidirectional scheme for n-qubit quantum state is shown with an example (shown in subsection 2.1). Finally, paper is concluded.

2 Optimized Bidirectional quantum teleportation

Alice’s $|\psi_n\rangle$ and Bob’s $|\phi_n\rangle$ n-qubit quantum state of the form of

$$|\psi_n\rangle = A_1 |\alpha_1 \alpha_2 \ldots \alpha_n\rangle + A_2 |\bar{\alpha}_1 \bar{\alpha}_2 \ldots \bar{\alpha}_n\rangle$$

(1)

$$|\phi_n\rangle = B_1 |\beta_1 \beta_2 \ldots \beta_n\rangle + B_2 |\bar{\beta}_1 \bar{\beta}_2 \ldots \bar{\beta}_n\rangle$$

(2)

Here, $|A_1|^2 + |A_2|^2 = 1$ and $|B_1|^2 + |B_2|^2 = 1$. Alice sends $|\psi_n\rangle$ to Bob and Bob sends $|\phi_n\rangle$ to Alice by using entangled quantum resource. Here, as an optimized BQT scheme, only two Bell states as a quantum resource is used. To teleport $|\psi_n\rangle$ and $|\phi_n\rangle$ to each other by using optimized amount of quantum resource, Alice and Bob applied some of CNOT operations. After that teleported state $|\psi_n\rangle$ and $|\phi_n\rangle$ converted into $|\psi_n\rangle'$ and $|\phi_n\rangle'$ of the form of

$$|\psi_n\rangle' = A_1 |\alpha_1\rangle + A_2 |\bar{\alpha}_1\rangle \otimes |00\ldots\rangle_{n-1}$$

(3)

$$|\psi_n\rangle' = |\psi\rangle'' \otimes |00\ldots\rangle_{n-1}$$

$$|\phi_n\rangle' = B_1 |\beta_1\rangle + B_2 |\bar{\beta}_1\rangle \otimes |00\ldots\rangle_{n-1}$$

(4)

The obtained quantum states $|\psi\rangle''$ and $|\phi\rangle''$ contains complete information of $|\psi_n\rangle'$ and $|\phi_n\rangle'$ and the remaining $(n - 1)$ qubits are in $|0\rangle$, which are registered qubits. Therefore, the task is reduced to the teleportation of $|\psi_n\rangle'$ and $|\phi_n\rangle'$ into the teleportation of $|\psi\rangle''$ and $|\phi\rangle''$. We know that teleportation of single qubit quantum state of the form of $\alpha |0\rangle + \beta |1\rangle$ required one Bell state which is shown in 1993 by Bennett, so for $|\psi\rangle''$ required one Bell state and other one Bell state required for $|\phi\rangle''$. Consequently, two Bell states are required for our proposed scheme. We can understand easily proposed n-qubit bidirectional scheme with an example which is shown in below subsection 2.1.
two-Bell states

1. (2016) [4] a₀(|00⟩ + a₁|11⟩)
   b₀(|00⟩ + b₁|11⟩)
   ¼(|000000⟩ + |001111⟩ + |111000⟩ + |111111⟩)
   two-Bell states

2. (2016) [5] a₀(|00⟩ + a₁|11⟩)
   b₀(|00⟩ + b₁|11⟩)
   ¼(|000000⟩ + |001111⟩ + |111000⟩ + |111111⟩)
   two-Bell states

3. (2017) [7] a₀(|00⟩ + a₁|11⟩)
   b₀(|00⟩ + b₁|11⟩)
   ¼(|000000⟩ + |001111⟩ + |111000⟩ + |111111⟩)
   two-Bell states

4. (2020) [9] a₀(|00⟩ + a₁|11⟩)
   b₀(|00⟩ + b₁|11⟩)
   ¼(|000000⟩ + |001111⟩ + |111000⟩ + |111111⟩)
   three-Bell states

5. (2019) [1] a₀(|00⟩ + a₁|11⟩)
   b₀(|00⟩ + b₁|11⟩)
   ¼(|000000⟩ + |001111⟩ + |111000⟩ + |111111⟩)
   two-Bell states

6. (2020) [8] a₀(|00⟩ + a₁|11⟩)
   b₀(|00⟩ + b₁|11⟩)
   ¼(|000000⟩ + |001111⟩ + |111000⟩ + |111111⟩)
   two-Bell states

7. (2020) [4] a₀(|00⟩ + a₁|11⟩)
   b₀(|00⟩ + b₁|11⟩)
   ¼(|000000⟩ + |001111⟩ + |111000⟩ + |111111⟩)
   two-Bell states

Table 1: In column 4, different type of multi-qubit quantum state as a resource is used to teleport different type of unknown quantum states as mentioned in column 3 in previous papers (column 2). According to our optimized scheme only some of Bell states is required for the same teleported state (column 3).

2.1 Proposed scheme with an example

In 2019, Zhou et al., proposed a bidirectional scheme [1] by using six-qubit cluster state of the form of 1/2(|000000⟩ + |001111⟩ + |111000⟩ + |111111⟩)_{123456}. In this scheme, Alice possesses three-qubit entangled state |φ⟩_{ABC} = a₀|000⟩ + a₁|111⟩, which is to be teleported to Bob and Bob possesses an arbitrary single qubit state |φ⟩_{D} = b₀|0⟩ + b₁|1⟩, which is to be teleported to Alice. They have used highly amount of quantum resource (six-qubit cluster state) and this proposed [1] bidirectional scheme can be done by using our proposed scheme.

**Improvement of Zhou et al. scheme [1] by using our proposed scheme:**

Alice wants to teleport three-qubit quantum state |φ⟩_{ABC} = a₀|000⟩ + a₁|111⟩ to Bob and Bob wish to teleport single qubit state |φ⟩_{D} = b₀|0⟩ + b₁|1⟩ to Alice. According to our optimized scheme as discussed in Section 2, Alice applies some CNOT operations on the last two-qubits in teleported three-qubit entangled state as shown in Fig. [1]

|φ⟩_{ABC} = a₀|000⟩ + a₁|111⟩

|φ⟩_{a₁a₂a₃} = (a₀|0⟩ + a₁|1⟩){a₁} ⊗ |00⟩_{a₂a₃} = |Ω⟩_{a₁} ⊗ |00⟩_{a₂a₃}

(5)

Here, |φ⟩_{ABC} is replace to |φ⟩_{a₁a₂a₃} which corresponds to Alice’s qubit and |φ⟩_{D} is replace with |φ⟩_{b₁} which corresponds to Bob’s qubit. In Eq. [5] |Ω⟩_{a₁} holds the complete information of three-qubit entangled state |φ⟩_{a₁a₂a₃}, whereas last two qubits a₂ and a₃ are registered qubits. The task is the teleportation of |Ω⟩_{a₁} state. Now, Alice sends the unknown single-qubit state to Bob and vice versa.
The quantum channel for this scheme is the tensor product of two Bell states as

\[
|\chi\rangle_{A_1B_1A_2B_2} = |\psi^+\rangle \otimes |\psi^+\rangle = \left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right)_{AB} \otimes \left(\frac{|0000\rangle + |0011\rangle + |1100\rangle + |1111\rangle}{2}\right)_{A_1B_1A_2B_2} \tag{6}
\]

Now, the bidirectional scheme for three qubit and single qubit unknown quantum states is:

\[
|\rho\rangle_{a_1b_1A_1B_1A_2B_2} = |\Omega\rangle_{a_1} \otimes |\varphi\rangle_{b_1} \otimes |\chi\rangle_{A_1B_1A_2B_2} \tag{7}
\]

The complete bidirectional teleportation process of above quantum state we can find in the Ref. [3], but the difference is only with the choice of quantum channel, according to our channel, at the last of the protocol process, we do not need to apply quantum controlled phase gate operation as used in Ref. [3]. After the bidirectional teleportation of both quantum states \(|\Omega\rangle_{a_1}\) and \(|\varphi\rangle_{b_1}\) simultaneously to Bob and Alice, Bob uses the knowledge of the unitary (CNOT operations) Alice has applied, he prepares \(a_2\) and \(a_3\) qubits in \(|0\rangle\). Then Bob applies CNOT operations to reconstruct the Alice’s three-qubit quantum state \(|\varphi\rangle_{a_1a_2a_3}\).

To reconstruct unknown quantum state Alice and Bob have to apply some unitary operations. In above optimized scheme, two copies of Bell state \(|\psi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}\) is used for the teleportation of single qubit states. In given Table 2, combinations of Bell states as a quantum channel, measurement operators and correspondingly unitary operations to reconstruct unknown quantum state is mentioned.

3 Conclusion

In this work, I have optimized the quantum resource for bidirectional quantum teleportation scheme and shown that only Bell states is required for the teleportation of any unknown quantum states of the form of as shown in Eqs. 1 and 2. It is also shown that some of previous
| S.no. | Combination of Bell states | Measurement Operators | Unitary Operations |
|-------|---------------------------|-----------------------|------------------|
| 1.    | $|\psi^+\rangle \otimes |\psi^+\rangle$ | $|\psi^+\rangle, |\psi^+\rangle$ | $I, I$ |
|       |                           | $|\psi^+\rangle, |\psi^-\rangle$ | $I, Z$ |
|       |                           | $|\psi^+\rangle, |\phi^+\rangle$ | $I, X$ |
|       |                           | $|\psi^+\rangle, |\phi^-\rangle$ | $I, iY$ |
|       | $|\psi^-\rangle, |\psi^+\rangle$ | $Z, I$ |
|       | $|\psi^-\rangle, |\psi^-\rangle$ | $Z, Z$ |
|       | $|\psi^-\rangle, |\phi^-\rangle$ | $Z, iY$ |
|       | $|\phi^+\rangle, |\psi^+\rangle$ | $X, I$ |
|       | $|\phi^+\rangle, |\psi^-\rangle$ | $X, Z$ |
|       | $|\phi^+\rangle, |\phi^+\rangle$ | $X, X$ |
|       | $|\phi^+\rangle, |\phi^-\rangle$ | $X, iY$ |
|       | $|\phi^-\rangle, |\psi^+\rangle$ | $iY, I$ |
|       | $|\phi^-\rangle, |\psi^-\rangle$ | $iY, Z$ |
|       | $|\phi^-\rangle, |\phi^+\rangle$ | $iY, X$ |
|       | $|\phi^-\rangle, |\phi^-\rangle$ | $iY, iY$ |
| 2.    | $|\psi^-\rangle \otimes |\psi^-\rangle$ | $|\psi^+\rangle, |\psi^+\rangle$ | $Z, Z$ |
|       | $|\psi^+\rangle, |\psi^-\rangle$ | $Z, I$ |
|       | $|\psi^+\rangle, |\phi^+\rangle$ | $Z, iY$ |
|       | $|\psi^+\rangle, |\phi^-\rangle$ | $Z, X$ |
|       | $|\psi^-\rangle, |\psi^+\rangle$ | $I, Z$ |
|       | $|\psi^-\rangle, |\psi^-\rangle$ | $I, I$ |
|       | $|\psi^-\rangle, |\phi^+\rangle$ | $I, iY$ |
|       | $|\psi^-\rangle, |\phi^-\rangle$ | $I, X$ |
|       | $|\phi^+\rangle, |\psi^+\rangle$ | $iY, Z$ |
|       | $|\phi^+\rangle, |\psi^-\rangle$ | $iY, I$ |
|       | $|\phi^+\rangle, |\phi^+\rangle$ | $iY, iY$ |
|       | $|\phi^+\rangle, |\phi^-\rangle$ | $iY, X$ |
|       | $|\phi^-\rangle, |\psi^+\rangle$ | $X, Z$ |
|       | $|\phi^-\rangle, |\psi^-\rangle$ | $X, I$ |
|       | $|\phi^-\rangle, |\phi^+\rangle$ | $X, iY$ |
|       | $|\phi^-\rangle, |\phi^-\rangle$ | $X, X$ |
Table 2: Combination of Bell states as quantum channel resource used in bidirectional teleportation scheme with their measurement operators and corresponding unitary operations applied by Alice and Bob. Whereas, 

\[
|\psi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}, \quad |\psi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}, \quad |\phi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}} \quad \text{and} \quad |\phi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}.
\]

Unitary operations 

\[
I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \text{and} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

| Bell State Combination | Measurement Operators | Unitary Operations |
|-----------------------|-----------------------|-------------------|
| $|\phi^+\rangle \otimes |\phi^+\rangle$ | $|\psi^+\rangle, |\psi^+\rangle$ | $X, X$ |
| $|\psi^+\rangle, |\psi^-\rangle$ | $X, IY$ |
| $|\psi^+\rangle, |\phi^+\rangle$ | $X, I$ |
| $|\psi^+\rangle, |\phi^-\rangle$ | $X, Z$ |
| $|\psi^-\rangle, |\psi^+\rangle$ | $iY, X$ |
| $|\psi^-\rangle, |\psi^-\rangle$ | $iY, iY$ |
| $|\psi^-\rangle, |\phi^+\rangle$ | $iY, I$ |
| $|\psi^-\rangle, |\phi^-\rangle$ | $iY, Z$ |
| $|\phi^+\rangle, |\psi^+\rangle$ | $I, X$ |
| $|\phi^+\rangle, |\psi^-\rangle$ | $I, iY$ |
| $|\phi^+\rangle, |\phi^+\rangle$ | $I, I$ |
| $|\phi^+\rangle, |\phi^-\rangle$ | $I, Z$ |
| $|\phi^-\rangle, |\psi^+\rangle$ | $Z, X$ |
| $|\phi^-\rangle, |\psi^-\rangle$ | $Z, iY$ |
| $|\phi^-\rangle, |\phi^+\rangle$ | $Z, iI$ |
| $|\phi^-\rangle, |\phi^-\rangle$ | $Z, Z$ |
| $|\phi^+\rangle, |\psi^+\rangle$ | $Z, X$ |
| $|\phi^+\rangle, |\psi^-\rangle$ | $Z, iY$ |
| $|\phi^+\rangle, |\phi^+\rangle$ | $Z, Z$ |
| $|\phi^+\rangle, |\phi^-\rangle$ | $Z, iI$ |
| $|\phi^-\rangle, |\psi^+\rangle$ | $I, iY$ |
| $|\phi^-\rangle, |\psi^-\rangle$ | $I, X$ |
| $|\phi^-\rangle, |\phi^+\rangle$ | $I, Z$ |
| $|\phi^-\rangle, |\phi^-\rangle$ | $I, I$ |
work on BQT have been used multi-qubit quantum states as a quantum channel (mentioned in Table 1), which is not required according to this proposed scheme. As well as, in Table 2 also provided the measurement operators and corresponding their unitary operations for the combination of Bell states. This work is very useful in the future to teleport bidirectionally any n-qubit unknown quantum state with the use of optimal resource. Because the cost and the maintenance of quantum resource of any scheme is the main concern.

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