Abstract

In this talk we review the status of the theoretical estimates for $CP$ violating asymmetries in non-leptonic hyperon decays.
1 – The Decay $\Lambda^0 \rightarrow p\pi^-$

The reaction $\Lambda^0 \rightarrow p\pi^-$ will be used as an example to set up the model independent formulation of the $\mathcal{C}\mathcal{P}$ odd observables in non-leptonic hyperon decays of the form $B_i \rightarrow B_f\pi$. In the $\Lambda^0$ rest frame, $\vec{\omega}_{i,f}$ will denote unit vectors in the directions of the $\Lambda$ and $p$ polarizations, and $\vec{q}$ the proton momentum. The isospin of the final state is $I = 1/2$ or $3/2$, and each of these two states can be reached via a $\Delta I = 1/2$ or $3/2$ weak transition respectively. There are also two possibilities for the parity of the final state. They are the $s$-wave, $l = 0$, parity odd state (thus reached via a parity violating amplitude); and the $p$-wave, $l = 1$, parity even state reached via a parity conserving amplitude.

We first perform a model independent analysis of the decay by writing the most general matrix element consistent with Lorentz invariance:

$$\mathcal{M} = G_F m^2_\pi (A - B\gamma_5) U_\Lambda.$$  \hfill (1)

This matrix element reduces to

$$A(B_i \rightarrow B_f\pi) = s + p\vec{\sigma} \cdot \vec{q}$$  \hfill (2)

In terms of these quantities one can compute the decay distribution, and the total decay rate. One finds that the decay is characterized by three independent observables: the total decay rate and two parameters that determine the angular distribution. The total decay rate is given by:

$$\Gamma = \frac{q^2 (E_P + M_P)}{4\pi M_\Lambda} G^2_F m^4_\pi \left(|s|^2 + |p|^2\right).$$  \hfill (3)

The angular distribution is proportional to:

$$\frac{d\Gamma}{d\Omega} \sim 1 + \gamma \vec{\omega}_i \cdot \vec{\omega}_f + (1 - \gamma) \vec{q} \cdot \vec{\omega}_i \vec{q} \cdot \vec{\omega}_f + \alpha \vec{q} \cdot (\vec{\omega}_i + \vec{\omega}_f) + \beta \vec{q} \cdot (\vec{\omega}_f \times \vec{\omega}_i),$$  \hfill (4)

where we have introduced the notation:

$$\alpha \equiv \frac{\text{Re}^* p}{|s|^2 + |p|^2}, \quad \beta \equiv \frac{\text{Im}^* p}{|s|^2 + |p|^2}, \quad \gamma \equiv \frac{|s|^2 - |p|^2}{|s|^2 + |p|^2}. \quad (5)$$

However, only two of these parameters are independent, since $\alpha^2 + \beta^2 + \gamma^2 = 1$. We will treat $\alpha$ and $\beta$ as the independent ones, although sometimes the parameters $\alpha$ and $\phi$, with $\beta = \sqrt{1 - \alpha^2} \sin \phi$ and $\gamma = \sqrt{1 - \alpha^2} \cos \phi$ are used instead.

The parameter $\alpha$ governs the $T$-even correlation between the proton momentum and the $\Lambda$ polarization, whereas $\beta$ governs the $T$-odd correlation involving the two polarization vectors and the proton momentum. I use $T$ to indicate the operation that reverses the sign of all momenta and spins in the reaction, and not the time reversal operation. The significance of this discrete symmetry is that operators that are even under it can only be used to construct $\mathcal{C}\mathcal{P}$ odd observables that require...
final state interactions, whereas those that are odd can be used to construct $\mathcal{CP}$ odd observables that do not vanish in the absence of final state interactions.

One way to interpret the parameter $\alpha$ follows from considering the angular distribution in the case when the final baryon polarization is not observed:

$$
\frac{d\Gamma}{d\Omega} = \frac{1}{16\pi^2 M_\Lambda} |\hat{q}| G_F^2 m_\pi^4 (E_P + M_P) \left( |s|^2 + |p|^2 \right) \left( 1 + \alpha \hat{q} \cdot \hat{\omega}_i \right).
$$

(6)

The polarization of the decay proton in the $\Lambda^0$ rest frame is given by:

$$
\vec{P}_p = \frac{1}{1 + \alpha \vec{P}_\Lambda \cdot \hat{q}} \left[ (\alpha + \vec{P}_\Lambda \cdot \hat{q}) \hat{q} + \beta \left( \vec{P}_\Lambda \times \hat{q} \right) + \gamma \left( \hat{q} \times \left( \vec{P}_\Lambda \times \hat{q} \right) \right) \right].
$$

(7)

From this expression we can relate $\beta$ to the proton polarization in the direction perpendicular to the plane formed by the $\Lambda$ polarization and the proton momentum. If the initial hyperon is unpolarized, $\alpha$ gives us the polarization of the proton:

$$
\vec{P}_p = \alpha \Lambda \hat{q}
$$

(8)

Since the proton polarization is not measured, the parameter $\beta$ is not useful for the reaction $\Lambda \to p\pi^-$. It is, however, useful for other hyperon decays, such as the chain:

$$
\Xi^- \to \Lambda^0 \pi^- \to p\pi^- \pi^- \pi^-
$$

(9)

where the second decay analyzes the polarization of the $\Lambda$ and allows one to observe the parameter $\beta_{\Xi}$.

## 2 – $\mathcal{CP}$-odd Observables

To construct $\mathcal{CP}$-odd observables we compare the reactions $\Lambda^0 \to p\pi^-$ and $\bar{\Lambda}^0 \to \bar{p}\pi^+$ in terms of the three independent observables. One can show that $\mathcal{CP}$ symmetry predicts that:

$$
\begin{align*}
\Gamma &= \Gamma \\
\bar{\alpha} &= -\alpha \\
\bar{\beta} &= -\beta
\end{align*}
$$

(10)

and, therefore, one can construct the following $\mathcal{CP}$-odd observables:

$$
\begin{align*}
\Delta &= \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} \\
A &= \frac{\alpha \Gamma + \bar{\alpha} \bar{\Gamma}}{\alpha \Gamma - \bar{\alpha} \bar{\Gamma}} \approx \frac{\alpha + \bar{\alpha}}{\alpha - \bar{\alpha}} + \Delta \\
B &= \frac{\beta \Gamma + \bar{\beta} \bar{\Gamma}}{\beta \Gamma - \bar{\beta} \bar{\Gamma}} \approx \frac{\beta + \bar{\beta}}{\beta - \bar{\beta}} + \Delta
\end{align*}
$$

(11)
One can show that in the low energy reaction \( p\overline{p} \rightarrow \Lambda\overline{\Lambda} \rightarrow p_f \pi^- \overline{p}_f \pi^+ \) it is possible to construct counting asymmetries that measure \( A \) and \( B \). If we label each particle’s momentum by the particle name, and denote by \( N_p^\pm \) the number of events with \((\vec{p}_i \times \vec{p}_\Lambda) \cdot \vec{p}_f\) greater or less than zero then:

\[
\overline{A} = \frac{N_p^+ - N_p^- + N_{\overline{p}}^+ - N_{\overline{p}}^-}{N_{\text{total}}} = P_{\Lambda} \alpha \Lambda A_{\Lambda}
\]  

(12)

Similarly, in the reaction \( p\overline{p} \rightarrow \Xi\overline{\Xi} \rightarrow \Lambda\pi^- \overline{\Lambda}\pi^+ \rightarrow p_f \pi^- \overline{p}_f \pi^+ \pi^+\), we can define \( N_p^\pm \) to be the number of events with \( P_{\Xi} \cdot (\vec{p}_f \times \vec{p}_\Lambda)\) greater or less than zero and construct the counting asymmetry:

\[
\overline{B} = \frac{N_p^+ - N_p^- + N_{\overline{p}}^+ - N_{\overline{p}}^-}{N_{\text{total}}} = \frac{\pi^8}{8} P_{\Xi} \alpha \Lambda \beta \Xi (A_{\Lambda} + B_{\Xi})
\]  

(13)

3 – Isospin Decomposition

To discuss the final state interaction phases it is convenient to analyze the final pion-nucleon system in terms of isospin and parity eigenstates. In that way we can have in mind the simple picture:

\[
\begin{align*}
\Lambda^0 &\rightarrow H_w \left( p\pi \right)_I \delta^I \rightarrow \left( p\pi \right)_I.
\end{align*}
\]  

(14)

At \( t = 0 \) the weak Hamiltonian induces the decay of the \( \Lambda^0 \) into a pion-nucleon system with isospin and parity given by \( I, \ell \). If there is \( \mathcal{CP} \) violation in this decay there will be a \( \mathcal{CP} \)-odd phase \( \phi^I_\ell \). This pion-nucleon system is an eigenstate of the strong interaction. Furthermore, at an energy equal to the \( \Lambda \) mass, it is the only state with these quantum numbers. The pion-nucleon system will then rescatter due to the strong interactions into itself, and in the process pick up a phase \( \delta^I_\ell \). This is an example of what is known as Watson’s theorem.

If we parameterize the amplitudes in non-leptonic hyperon decays as:

\[
s = \sum_I s_I e^{i(\delta^I_\ell + \phi^I_\ell)} \quad p = \sum_I p_I e^{i(\delta^I_\ell + \phi^I_\ell)},
\]  

(15)

then \( \mathcal{CPT} \) invariance of the weak Hamiltonian predicts:

\[
\bar{s} = \sum_I -s_I e^{i(\delta^I_\ell - \phi^I_\ell)} \quad \bar{p} = \sum_I p_I e^{i(\delta^I_\ell - \phi^I_\ell)},
\]  

(16)

whereas \( \mathcal{CP} \) invariance of the weak interactions predicts:

\[
\overline{s} = \sum_I -s_I e^{i(\delta^I_\ell + \phi^I_\ell)} \quad \overline{p} = \sum_I p_I e^{i(\delta^I_\ell + \phi^I_\ell)}.
\]  

(17)
From this we see that the $\phi^I_\ell$ phases violate $\mathcal{CP}$. We want to extract $\phi^I_\ell$ from the $\mathcal{CP}$-odd observables discussed in the previous section.

Introducing the notation used in the literature: \[6\]

$$
\begin{align*}
S(\Lambda_0^-) &= -\sqrt{2/3}S_{11} e^{i(\delta_1 + \phi^s_1)} + \sqrt{1/3}S_{33} e^{i(\delta_3 + \phi^s_3)} \\
P(\Lambda_0^-) &= -\sqrt{2/3}P_{11} e^{i(\delta_{11} + \phi^p_1)} + \sqrt{1/3}P_{33} e^{i(\delta_{33} + \phi^p_3)} \\
S(\Xi^-) &= S_{12} e^{i(\delta_2 + \phi^s_{12})} + \frac{1}{2}S_{32} e^{i(\delta_2 + \phi^s_{32})} \\
P(\Xi^-) &= P_{12} e^{i(\delta_{21} + \phi^p_{12})} + \frac{1}{2}P_{32} e^{i(\delta_{21} + \phi^p_{32})}
\end{align*}
$$

(18)

where $\Lambda_0^-$ refers to the reaction $\Lambda^0 \rightarrow p\pi^-$ and $\Xi^-$ refers to the reaction $\Xi^- \rightarrow \Lambda^0\pi^-$. The notation for the isospin amplitudes is $S_{ij} \equiv S_{2\Delta I,2I}$, $P_{ij} \equiv P_{2\Delta I,2I}$, the $s$-wave phases are denoted by $\delta_{2I}$ and the $p$-wave phases by $\delta_{2I,1}$.

It is useful to construct approximate expressions based on the fact that there are three small parameters in the problem:

- The strong rescattering phases are measured or estimated to be small. Experimentally we know that: \[8\]

$$
\begin{align*}
\delta_1 &\approx 6.0^\circ, \quad \delta_3 \approx -3.8^\circ, \quad \delta_{11} \approx -1.1^\circ, \quad \delta_{31} \approx -0.7^\circ
\end{align*}
$$

(19)

with all the errors on the order of $1^\circ$. For the $\Xi$ decays there are no experimental results. An early calculation within a model predicted $\delta_{21} = -2.7^\circ$ and $\delta_2 = -18.7^\circ$. \[8\] A recent calculation using chiral perturbation theory predicts instead $\delta_{21} = -1.7^\circ$ and $\delta_3 = 0$. \[8\] Clearly the resulting asymmetries will be completely different depending on which of these results is closer to the true scattering phases.

- The $\Delta I = 3/2$ amplitudes are much smaller than the $\Delta I = 1/2$ amplitudes. Experimentally we know that: \[10\]

$$
\begin{align*}
S_{33}/S_{11} &= 0.027 \pm 0.008, \quad P_{33}/P_{11} = 0.03 \pm 0.037 \\
S_{32}/S_{12} &= -0.046 \pm 0.014, \quad P_{32}/P_{12} = -0.01 \pm 0.04
\end{align*}
$$

(20)

- The $\mathcal{CP}$ violating phases are presumed to be small.

To leading order in all the small quantities one finds: \[11\]

$$
\begin{align*}
\Delta(\Lambda_0^-) &= \sqrt{2} \frac{S_{33}}{S_{11}} \sin(\delta_3 - \delta_1) \sin(\phi^s_3 - \phi^s_1) \\
A(\Lambda_0^-) &= -\tan(\delta_{11} - \delta_1) \sin(\phi^p_1 - \phi^p_1) \\
B(\Lambda_0^-) &= \cot(\delta_{11} - \delta_1) \sin(\phi^p_1 - \phi^p_1)
\end{align*}
$$

(21)
\[ \Delta(\Xi^-) = 0 \]
\[ A(\Xi^-) = -\tan(\delta_{21} - \delta_2) \sin(\phi_{12}^p - \phi_{12}^s) \]
\[ B(\Xi^-) = \cot(\delta_{21} - \delta_2) \sin(\phi_{12}^p - \phi_{12}^s) \]  

(22)

We can see in these expressions that \( \Delta \) arises mainly from an interference between \( \Delta I = 1/2 \) and \( \Delta I = 3/2 \) s-waves, and that it is suppressed by three small quantities. On the other hand, \( A \) arises as an interference of \( s \) and \( p \)-waves of the same isospin and, therefore, it is not suppressed by the \( \Delta I = 1/2 \) rule. Finally, we can see that \( B \) is not suppressed by the small rescattering phases. This is as we expected for a \( \mathcal{CP} \) odd observable that is also (naive)-\( T \) odd. The hierarchy \( B \gg A \gg \Delta \) emerges. The quantity \( \Delta(\Xi^-) \) vanishes because there is only one isospin final state in this decay.

This is as far as we can go in a model independent manner. If we want to predict the value of these observables within a model for \( \mathcal{CP} \) violation we take the value for the ratio of amplitudes and for the strong rescattering phases from experiment and we try to compute the weak phases from theory.

4 – Standard model calculation

In the case of the minimal standard model, the \( \mathcal{CP} \) violating phase resides in the CKM matrix. For low energy transitions, this phase shows up as the imaginary part of the Wilson coefficients in the effective weak Hamiltonian. In the notation of Buras [11],

\[ H^{\text{eff}}_W = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_i c_i(\mu) Q_i(\mu) + \text{hermitian conjugate} \]  

(23)

\( Q_i(\mu) \) are four quark operators, and \( c_i(\mu) \) are the Wilson coefficients that are usually written as:

\[ c_i(\mu) = z_i(\mu) + \tau y_i(\mu) \]
\[ \tau = -\frac{V_{ud}^* V_{ts}}{V_{us}} \]  

(24)

with the \( \mathcal{CP} \) violating phase being the phase of \( \tau \). Numerical values for these coefficients can be found, for example, in Buchalla et. al. [12]

The calculation would proceed as usual, by evaluating the hadronic matrix elements of the four-quark operators in Eq. (23) to obtain real and imaginary parts for the amplitudes, schematically:

\[ \langle p\pi | H^{\text{eff}}_W | \Lambda^0 \rangle | I \rangle = \text{Re} M^I_\ell + i \text{Im} M^I_\ell, \]  

(25)

and to the extent that the \( \mathcal{CP} \) violating phases are small, they can be approximated by

\[ \phi^I_\ell \approx \frac{\text{Im} M^I_\ell}{\text{Re} M^I_\ell}. \]  

(26)
At present, however, we do not know how to compute the matrix elements so we cannot actually implement this calculation. If we try to follow what is done for kaon decays, we would compute the matrix elements using factorization and vacuum saturation as a reference point, then define some parameters analogous to $B_K$ that would measure the deviation of the matrix elements from their vacuum saturation value. A reliable calculation of the “$B$” parameters would probably have to come from lattice QCD.

For a simple estimate, we can take the real part of the matrix elements from experiment (assuming that the measured amplitudes are real, that is, that $\mathcal{CP}$ violation is small), and compute the imaginary parts in vacuum saturation. This approach provides a conservative estimate for the weak phases because the model calculation of the real part of the amplitudes is much smaller than the experimental value. Of course, if we cannot predict the real part of the amplitude at all, we might question the reliability of the imaginary part as well.

There are many models in the literature that claim to fit the experimentally measured amplitudes. Without entering into the details of these models, it is obvious that to fit the data, the models must enhance some or all of the matrix elements with respect to vacuum saturation. Clearly, one would get completely different phases depending on which matrix elements are enhanced. It is not surprising, therefore, that a survey of these models yields weak $\mathcal{CP}$ phases that differ by an order of magnitude [13].

The approximate weak phases estimated in vacuum saturation are: [13]

\[
\begin{align*}
\phi_s^1 & \approx -3y_6 \text{Im}\tau \\
\phi_p^1 & \approx -0.3y_6 \text{Im}\tau \\
\phi_s^3 & \approx \left[ 3.56(y_1 + y_2) + 4.1(y_7 + 2y_8) \frac{m_\pi^2}{m_s(m_u + m_d)} \right] \text{Im}\tau
\end{align*}
\]

To get some numerical estimates we use the values for the Wilson coefficients of Buchalla et. al. [12] with $\mu = 1$ GeV, $\Lambda_{QCD} = 200$ MeV. Although quantities such as the quark masses that appear in Eq. 27 are not physical [14], we will use for an estimate the value $m_\pi^2/(m_s(m_u + m_d)) \sim 10$. For the quantity $\text{Im}\tau$ we use the current upper bound $\text{Im}\tau \leq 0.0014$. Putting all the numbers together yields:

\[
\begin{align*}
\Delta(\Lambda^0_\tau) & = \begin{cases} -1.4 \times 10^{-6} & \text{for } m_t = 150 \text{ GeV} \\
-9.1 \times 10^{-7} & \text{for } m_t = 200 \text{ GeV} \end{cases} \\
A(\Lambda^0_\tau) & = 3.7 \times 10^{-5} \\
B(\Lambda^0_\tau) & = 2.4 \times 10^{-3}
\end{align*}
\]

A poor man approach to the problem of the hadronic matrix elements consists of surveying several models. Combining this with a careful analysis of the allowed range for the short distance parameters that enter the calculation yields results similar to that of Eq. 28: that $A$ is in the range of “a few” $\times 10^{-5}$ and that $\Delta$ is almost two orders of magnitude smaller. The rate asymmetry exhibits a strong dependence on
the top-quark mass: for a certain value of $m_t$, the two terms in Eq. 27 cancel against each other. The angular correlation asymmetries, on the other hand, depend mildly on the top-quark mass. This is understood from the point of view that the most important effect of a large top-quark mass is to enhance electroweak corrections to the effective weak Hamiltonian. This is important for the $\Delta I = 3/2$ amplitudes but not for the $\Delta I = 1/2$ amplitudes.

5 – Other Models of $C\!\!P$ Violation

Other models of $C\!\!P$ violation contain additional short distance operators with $C\!\!P$ violating phases. [15, 16] Some of these have been analyzed in the literature fixing the strength of the $C\!\!P$ violating couplings from the parameter $\epsilon$ in kaon decays. [4]

A summary of those results is shown in Table 1, taken from a recent talk by He. [17]

Table 1. Sample of models of CP violation in hyperon decays.

| Decay | KM model | Weinberg Model | Left-Right Model |
|-------|----------|----------------|------------------|
| $\Delta (\Lambda^0_\pi^-)$ | $< 10^{-6}$ | $-0.8 \times 10^{-5}$ | 0 |
| $A(\Lambda^0_\pi^-)$ | $-(1 \sim 5) \times 10^{-5}$ | $-2.5 \times 10^{-5}$ | $-1.1 \times 10^{-5}$ |
| $B(\Lambda^0_\pi^-)$ | $(0.6 \sim 3) \times 10^{-4}$ | $1.6 \times 10^{-3}$ | $7.0 \times 10^{-4}$ |
| $\Xi$ decay | | | |
| $\Delta (\Xi^-_\pi^-)$ | 0 | 0 | 0 |
| $A(\Xi^-_\pi^-)$ | $-(1 \sim 10) \times 10^{-5}$ | $-3.2 \times 10^{-4}$ | $2.5 \times 10^{-5}$ |
| $B(\Xi^-_\pi^-)$ | $(1 \sim 10) \times 10^{-3}$ | $3.8 \times 10^{-3}$ | $-3.1 \times 10^{-4}$ |

An important point is that the numbers for the $\Xi^- \rightarrow \Lambda \pi^-$ decay were obtained using the early model calculation. [8] With the chiral perturbation theory numbers $A(\Xi^-)$ would be smaller by a factor of 10.

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