I. INTRODUCTION

Various generalized statistical mechanics have been proposed to study the systems with long-range interactions [1–8]. Since the Bekenstein entropy is a non-extensive measure, and in fact, gravity is a long-range interaction, physicists are motivated to consider various generalized statistical mechanics for studying the cosmic evolution, and in general, the gravitational phenomena [3–34]. In this regard, the Tsallis entropy is a well-known entropy measure defined as

\[ S_T = \frac{1}{1-q} \sum_{i=1}^{W} (P_i^q - P_i) \]

for a system including W discrete states. In this definition, q is an unknown constant evaluated by fitting with experiments (observations) [2, 3]. The Rényi entropy is also another useful one-parameter generalized entropy measure [2, 3, 6] defined as

\[ S = \frac{1}{1-q} \ln \sum_{i=1}^{W} P_i^q. \]

Since the Bekenstein entropy is non-extensive, it has been proposed as a suitable candidate for the Tsallis entropy \( S_T = \frac{1}{q} \) [10, 21, 22], a claim proved by Majhi [23]. Combining \( S_T = \frac{1}{q} \) with the \( \delta \equiv 1 - q \) definition and also the above equations, one easily get [10, 21, 22, 30]. The generalized uncertainty principle (GUP) can also motivate physicists to consider the Rényi entropy instead of the Bekenstein entropy [33].

In Ref. [22], considering Eq. (3) as the entropy of the apparent horizon of the flat FRW universe \( (r = \frac{1}{H}) \), and applying the Clausius relation to the horizon, modified Friedmann equations has been obtained in the Rényi cosmology. Thereinafter, it has been shown that the obtained additional terms, compared to the standard Friedmann equations, may describe the current accelerated universe. Thus, based on this study, the probable non-extensive features of spacetime may be responsible for the current accelerated universe, a result motivating us to study the possibility of obtaining a primary inflationary era in the framework of Rényi cosmology obtained in Ref. [22].

In the standard inflationary model, a quantum field (inflaton) satisfies certain conditions, such as slow rolling, supports the inflationary era [35]. There are also another models for describing this era regardless of the inflaton [37–40]. In this paper, firstly, the power of Rényi cosmology in describing the current accelerated era in the absence of the integration constant obtained in Ref. [22] and also any dark energy fluid is investigated. We are also interested in studying the possibility of describing inflation in Rényi cosmology without considering an inflaton field. We present our analysis in the next section, and give a summary in the third section. In this paper units have been set so that \( G = \hbar = c = k_B = 1 \), where \( k_B \) denotes the Boltzmann constant and dot displays the time derivative.

II. ACCELERATION IN THE RÉNYI COSMOLOGY

Based on Ref. [22], which attributes the Rényi entropy instead of the Bekenstein entropy to the apparent horizon of a flat FRW universe, the Friedmann equations are modified as

\[ H^2 - \delta \pi \ln(\delta \pi + H^2) + C = \frac{8\pi}{3} \rho, \]
\[ H^2 + \frac{2}{3} \dot{H} - \delta \pi \ln(\delta \pi + H^2) - \frac{2 \dot{H}}{3(H^2 + \delta \pi)} + C = \frac{-8\pi}{3} p, \]

where \( C \) is a constant of integration, and ordinary energy momentum conservation law is obeyed by the energy source \( T^\mu_\nu = \text{diag}(-\rho, p, p, p) \) (filling the background) leading to \[ \dot{\rho} + 3H(\rho + p) = 0. \] (5)

Current cosmos

In Ref. [22], the authors did not consider the \( C = 0 \) case, and thus, in the limit \( \delta \to 0 \), their model reduces to the standard Friedmann equation in the presence of the cosmological constant. It means that the \( C \neq 0 \) case imposes the problems of the cosmological constant to the model. Hence, we will focus on the \( C = 0 \) case in our study. In fact, choosing this case, we run away from the phenomenological problems of the existence of a constant within the field equations [30].

Now, whenever \( p = 0 \), Eq. (5) leads to \( \rho = \rho_0 a^{-3} \), and thus, inserting this into Eq. (3) we arrive at

\[ H^2 - \delta \pi \ln(\delta \pi + H^2) = \frac{8\pi}{3} \rho_0(1 + z)^3, \] (6)

\[ H^2 + \frac{2}{3} \dot{H} - \delta \pi \ln(\delta \pi + H^2) - \frac{2 \dot{H}}{3(H^2 + \delta \pi)} = 0, \]

where \( \rho_0 \) and \( z \) denote the current value of the matter density and redshift, respectively. The relation \( 1 + z = \frac{1}{a} \) has also been employed. Eq. (6) leads to

\[ \frac{\dot{H}}{H^2} + 1 = -4\pi \rho_0(1 + z)^3, \] (7)

which clearly indicates that \( \dot{H} = 0 \) at \( z \to -1 \), and thus, we will face a de-Sitter universe in this limit. For deceleration parameter \( q \), one obtains \( q(z \to -1) \to -1 \) because

\[ q = -1 - \frac{\dot{H}}{H^2} = -1 + \frac{4\pi \rho_0(1 + z)^3}{H^2} (\frac{\delta \pi}{H^2} + 1). \] (8)

Moreover, using the definitions

\[ \rho_e = 3\delta \frac{8}{8} \ln(\delta \pi + H^2), \]

\[ p_e = -\rho_e - \frac{\dot{H}\delta}{4(H^2 + \delta \pi)}, \] (9)

given in [22], in order to rewrite Eq. (6), one can easily reach at [22]

\[ H^2 + \frac{2}{3} \dot{H} - \delta \pi \ln(\delta \pi + H^2) - \frac{2 \dot{H}}{3(H^2 + \delta \pi)} + C = \frac{-8\pi}{3} p_e, \]

In fact, in this way, we replaced a hypothetical fluid with energy density and pressure \( \rho_e \) and \( p_e \), respectively, by the modifications of the standard Friedmann equations originated from the differences between the Bekenstein and Rényi entropies. The dimensionless density parameters are also defined as

\[ \Omega_m = \frac{8\pi \rho_0}{3H^2}, \Omega_e = \frac{8\pi \rho_e}{3H^2}. \] (11)

The classical stability of the effective source [31] is evaluated by determining the sign of the square of sound speed given by

\[ v_s^2 = \frac{d \rho_e}{d \rho_e} - \frac{\dot{\rho}_e}{\rho_e} = w_e + \frac{\dot{w}_e}{\rho_e}, \] (12)

where \( w_e = \frac{\rho_e}{\rho_e} \) denotes the state parameter of the effective source [31]. In Figs. [1]-[2], the parameters \( q, \Omega_e, v_s^2 \) and \( w_e \) have been plotted, respectively, for the initial condition \( H(z = 0) = 67 s^{-1} \) and various values of \( \Omega_m(z = 0) = \Omega_{0m} \). In this situation, although the model is not always classically stable (the \( v_s^2 > 0 \) condition is not always satisfied), numerical results show that the acceptable behavior of \( \Omega_e, q \) and \( w_e \) can be obtained for \( \delta \gtrsim 118 \), depending on the value of \( \Omega_{0m} \).

As it has been obtained in Eq. (5), we have \( \dot{H} = 0 \) at \( z = -1 \) or equally \( H(z = -1) = \text{constant} \). Moreover, we

FIG. 1: \( q \) versus \( z \) for \( H(z = 0) = 67 s^{-1} \) and some values of \( \Omega_{0m} \). For the transition redshift \( z_t \) (at which the universe expansion phase is changed from the decelerated matter dominated phase to an accelerated expansion), we have \( z = 0.654, z = 0.771 \) and \( z = 0.873 \) for \( \Omega_{0m} = 0.3, \Omega_{0m} = 0.26 \) and \( \Omega_{0m} = 0.23 \), respectively.
\[ \Omega_e \text{ versus } z \text{ for } H(z = 0) = 67 s^{-1} \text{ and some values of } \Omega_0 m. \]

\[ v_z^2 \text{ versus } z \text{ for } H(z = 0) = 67 s^{-1} \text{ and some values of } \Omega_0 m. \]

\[ w_e \text{ versus } z \text{ for } H(z = 0) = 67 s^{-1} \text{ and some values of } \Omega_0 m. \]

have \( \rho_m = 0 \) at \( z = -1 \) meaning that proper values of \( \delta \) should lead to \( \Omega_e(z = -1) = 1 \), a condition that has been used to find some suitable values for \( \delta \) in plotting Fig. (2). The values of \( \delta \) obtained here (for \( C = 0 \)) are almost 100 times greater than those found in [22] where \( C \neq 0 \).

Inflation without inflaton

An interesting query is modelling the inflation in the absence of inflaton [37–40]. In the language of the Friedmann equations [41], it is equivalent to ask whether the non-extensive features of spacetime prompt the universe to experience an inflationary expansion or not. In order to answer this question, we write Eq. (4) in vacuum as

\[ H^2 - \delta \pi \ln(\delta \pi + H^2) = 0, \quad (13) \]

\[ H^2 + \frac{2 \dot{H}}{3 \left( \frac{\Delta}{\pi} + 1 \right)} - \delta \pi \ln(\delta \pi + H^2) = 0, \]

which are available at the same time, only if \( \dot{H} = 0 \). This result indicates that the probable non-extensive features of spacetime has theoretically enough power to describe the primary inflationary era. Such an era lasted almost \( 10^{-34} s \) leading to \( H_{inf} \approx 10^{34} s^{-1} \) for the Hubble parameter at this inflationary era [36]. Inserting this result into the first line of the above equation, one reaches \( \delta \approx 0.31 \).

It is worthwhile to mention here that this value of \( \delta \) cannot lead to a suitable behavior for the current universe which means that the value of \( \delta \) in the inflation era differs from that of the current universe.

III. SUMMARY AND CONCLUDING REMARKS

Considering the framework of Rényi cosmology, we showed that the probable non-extensive features of gravity formulated by the Rényi formalism may model the current accelerated universe in a classically unstable mode without employing an ambiguous fluid called dark energy such as a cosmological constant. It has also been obtained that the vacuum solution of Rényi cosmology provides a setting for the inflationary phase.

In one hand, gravity is described as the curvature of spacetime at the classical level. On the other, the generalized entropy formalisms such as that of Rényi are used due to the long-range nature of gravity (or equivalently the long-range nature of the spacetime curvature). Therefore, one may argue that if the spacetime, and in fact, its constituents satisfy the Rényi entropy bound instead of that of Bekenstein, in agreement with GUP.
then differences between the Rényi and Bekenstein entropies may be responsible for the description of the accelerated expansion of the universe. It is worthwhile mentioning that the values of $\delta$ in the current and primordial accelerated eras differ from each other in this model.

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