Radiative stability of neutrino-mass textures

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Neutrino-mass textures proposed at high-scales are known to be unstable against radiative corrections especially for nearly degenerate eigen values. Within the renormalization group constraints we find a mechanism in a class of gauge theories which guarantees reproduction of any high-scale texture to low energies with radiative stability. We also show how the mechanism explains solar and atmospheric neutrino anomalies through the bimaximal texture at high scale.

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I. Introduction: A major challenge to particle physics at present is the theoretical understanding of experimentally observed neutrino anomalies. This has led to suggestions of many interesting models and mass textures with hierarchial or degenerate eigen values \[ \lambda \]. Whereas the observed mixing between quarks is small, experimental indications appear to favor maximal mixings in the neutrino sector. A possible mechanism to explain large neutrino mixings at low-energies starting from small high-scale mixings similar to quarks could be through radiative magnification for quasidegenerate neutrinos \[ \lambda \].

An outstanding problem with bimaximal neutrino-mass textures with degenerate eigen values is the instability of the masses and mixing angles due to radiative corrections which spoils their prospects for the neutrinoless-double-beta decay and the neutrino anomalies \[ \lambda \]. While investigating radiative stability the usual procedure has been to assume the bimaximal texture to be associated with a single 5-dim operator through see-saw mechanism with SM or MSSM as gauge theories at lower scales \[ \lambda \].

Using renormalization group constraints in this letter we show that it is possible to reproduce any high-scale mass texture at low scale \( \mu = M_Z \) with high degree of accuracy leading to stable evolution of the physically relevant Majorana-neutrino-mass matrix. The models where the mechanism operates consists of two component matrices contributing to the resultant Majorana-neutrino mass texture at the highest scale. It is quite interesting to note that the mechanism operates successfully in 2HDM and also with SM and MSSM in the presence of type II see-saw mechanism probing left-right model (LRM) and SO(10) GUT as prospective high-scale theories \[ \lambda \]. We also show how experimental data on solar and atmospheric neutrino anomalies are explained through the bimaximal texture.

II. The mechanism in 2HDM: In the SM, MSSM, and 2HDM, the Majorana-neutrino mass in the flavor basis originates from 5-dim operators generated at the lepton-number-breaking scale \( \mu \). But, unlike SM or MSSM, there are two neutrino-mass operators contributing to the neutrino-mass matrix due to the coupling of up type doublet with \( m^{(MSSM)} = -(1/4)K^{(MSSM)}v^2_u \). In a class of 2HDM there are two doublets, \( \Phi_u \) and \( \Phi_d \), with VEVs \( v_u/\sqrt{2} = v\sin\beta/\sqrt{2} \), \( v_d/\sqrt{2} = v\cos\beta/\sqrt{2} \). But, unlike SM or MSSM, there are two neutrino-mass operators, \( K_i^U \) and \( K^{H} \), and two matrices \( m^I \) and \( m^{II} \) which add up to generate the physically relevant Majorana-neutrino-mass matrix \[ \lambda \], \( m = -(1/4)\left(K^U v_u^2 + K^{H} v_d^2\right) \equiv m^I + m^{II} \). We use the renormalization scheme where the running of \( v_u, v_d \), and \( \tan \beta \) are ignored. Then the relevant RGEs and their one-loop solutions for \( \mu < M_N \) \( t = \ln \mu < t_0 \) are,

\[
\begin{align*}
16\pi^2 \frac{dm^I}{dt} &= \left\{-3g_2^2 + 2\lambda_2 + 2\text{Tr} \left( 3Y^U_Y^U \right) \right\} m^I + \frac{1}{2} \left( Y^E_Y^E m^I + m^I \left( Y^E_Y^E \right)^T \right) + 2\lambda_3 m^{II}, \\
16\pi^2 \frac{dm^{II}}{dt} &= \left\{-3g_2^2 + 2\lambda_1 + 2\text{Tr} \left( 3Y^D_Y^D + Y^E_Y^E \right) \right\} m^{II} - \frac{3}{2} \left( Y^E_Y^E m^{II} + m^{II} \left( Y^E_Y^E \right)^T \right) + 2\lambda_3 m^I, \\
m^I_{ij}(t) &= a^I_{ij}(t)m^I_{ij}(0), \quad a^I_{ij}(t) = I_{g_2}^\frac{1}{2} I_{r}^\frac{1}{2} \left( I_i I_j \right)^\frac{1}{2} R_{ij}, \\
m^{II}_{ij}(t) &= a^{II}_{ij}(t)m^{II}_{ij}(0), \quad a^{II}_{ij}(t) = I_{g_2}^\frac{1}{2} I_{r}^\frac{1}{2} \left( I_i I_j \right)^\frac{1}{2} R_{ij}.
\end{align*}
\]
$m_{ij}^{I,II}(t) = m_{ij}^{I,II}(t_0)$,

$I_l = \exp \left( \frac{1}{8\pi^2} \int_{t_0}^{t} h_l^2 dt \right), (l = e, \mu, \tau, top, b)$

$I_{g_k} = \exp \left( \frac{1}{8\pi^2} \int_{t_0}^{t} g_k^2 dt \right), (k = 1, 2)$

$I_{\lambda_k} = \exp \left( \frac{1}{8\pi^2} \int_{t_0}^{t} \lambda_k dt \right), (k = 1, 2)$

$R_{ij} = \exp \left[ \frac{1}{8\pi^2} \int_{t_0}^{t} (m^I m^{I^{-1}})_{ij} \lambda_3 dt \right],$

$R_{ij} = \exp \left[ \frac{1}{8\pi^2} \int_{t_0}^{t} (m^I m^{I^{-1}})_{ij} \lambda_3 dt \right]. \quad (3)$

Any texture at the highest scale for the physically relevant Majorana-neutrino-mass matrix

$m(0) = m^I(0) + m^{II}(0), \quad (4)$

never determines both the matrices $m^I(0)$ and $m^{II}(0)$. Given any element $m_{ij}(0)$ at $t_0$, one of the component elements, $m_{ij}^I(0)$ or $m_{ij}^{II}(0)$, remains completely undetermined at that scale. Then (4)-[3] show that the same matrix, $m^I(t)$ or $m^{II}(t)$, is undetermined at all lower scales too. This is in clear contrast to the cases in conventional analyses (CA) with SM or MSSM where there is only one $m(0)$ at $\mu = M_N$ and the texture gives all the elements of $m_{ij}(0)$ and $m_{ij}(t)$ [10, 11]. Now we impose the stability criterion that the texture is exactly reproduced at the lowest scale by demanding that

$m_{ij}(t_Z) = m_{ij}^I(t_Z) + m_{ij}^{II}(t_Z) \equiv m_{ij}(0). \quad (5)$

Since $a^I(t_Z)$ and $a^{II}(t_Z)$ are known in terms of the model parameters, solutions of (3) and [3] now determine both $m^I(0)$ and $m^{II}(0)$ in terms of the high-scale neutrino-mass texture, $m(0)$,

$m_{ij}^{I}(0) = m_{ij}(0) (a_{ij}^I(t_Z) - 1)/d_{ij},$

$m_{ij}^{II}(0) = m_{ij}(0) (1 - a_{ij}^{II}(t_Z))/d_{ij},$

$d_{ij} = a_{ij}^I(t_Z) - a_{ij}^{II}(t_Z). \quad (6)$

The components of the texture parameters, determined from the boundary conditions (3) and [3] are expected to guarantee reproduction of the high-scale texture at $M_Z$ when $m^I(t)$ and $m^{II}(t)$ are evolved through (3)-[3].

As an example we study RG evolution of the bimaximal texture with triply degenerate masses at $M_N \approx 10^{13}$ GeV [10].

$m(0) = \begin{pmatrix} 0 & 1/\sqrt{2} & 1/2 \\ 1/\sqrt{2} & 1/2 & -1/\sqrt{2} \\ 1/2 & -1/\sqrt{2} & 1/2 \end{pmatrix} m_0. \quad (7)$

Using, $\lambda_1 = 0.16, \lambda_2 = 1.13, \lambda_3 \approx -0.011 [10], and tan\beta = 40$ we obtain $l_{\lambda_3} \approx 0.95, l_{\lambda_3} \approx 0.6976, l_{\lambda_{top}} = 0.833213, l_\mu = 0.935023, l_\tau = 0.950882, l_\mu = 0.999832, l_\tau = 0.999999, l_{\mu_{Z}} = 0.478614$ at $\mu = M_Z$. We compute $a^I(t_Z)$ and $a^{II}(t_Z)$, and, hence, $m^I(0)$ and $m^{II}(0)$ shown in Table 1 as input parameters. The solutions for $m^I(t)$ and $m^{II}(t)$ are obtained through (3)-[3] and the elements of the Majorana-neutrino-mass matrix $m_{ij}(t)$ are obtained as their sum for all $t < t_0$. In Fig. 4 we have shown the radiative corrections for $m_{\mu\tau}(t)$. For comparison we have shown the results of the conventional analysis as SM (CA) and MSSM (CA) for which there is only one matrix at the highest scale. The maximum radiative correction of the matrix elements in 2HDM using the present mechanism is found to be only 3-4% as compared to 30-40% in the SM (CA) or MSSM (CA). Whereas the maximal corrections in SM (CA) or MSSM (CA) occur at $\mu = M_Z$, in our case they occur with substantially reduced magnitude at intermediate scales. Non-SUSY SM and 2HDM have been successfully embedded in SO(10) with single intermediate symmetries [13].

III. IMPLEMENTATION IN SM OR MSSM: We note that the present mechanism also operates in SM and MSSM if they originate from high-scale theories which predict two component matrices at $M_N$. The popular see-saw mechanism which has its natural origin in LRM and SO(10) contains the second contribution and leads to the two matrices in type II see-saw formula with $m^I = m^{SM}(m^{MSSM})$ and $m^{II} \approx f \nu^2/M_N (f \nu^2/M_N)$ when SM (MSSM) is obtained after symmetry breaking of LRM or SO(10) [4, 5, 6, 7, 8, 14, 15, 16]. Here $f$ is the Majorana type Yukawa coupling of the neutrino. The mechanism also operates in SM or MSSM when there are other types of contributions [13]. The problem of obtaining a specific texture for $m(0)$, or $m^I(0)$ and $m^{II}(0)$, may call for appending specific flavor symmetries to $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ or SO(10). Assuming such possibilities we derive the constraints on $m^I(0)$ and $m^{II}(0)$ resulting from the bimaximal texture for $m(0)$ and its radiative stability. The RG evolutions of the standard see-saw term is the same as in SM or MSSM as shown through $a^I(t)$ in [3]-[3] below. But those for $m^{II}(t)$ occur due to loop-mediation of the standard-weak-Higgs doublet and gauge bosons (plus superpartners) with the LH neutrinos alone. We derive them as

$16\pi^2 \frac{dm^{II}}{dt} = \left( c^{I(1)} g_1^2 + c^{I(2)} g_2^2 \right) m^{II}$

$+ c^{I(3)} \left[ (Y^T_E Y_E) m^{II} + m^{II} (Y^T_E Y_E)^T \right],$
\[ a_{ij}^{(t)}(t) = I_{g_1}^{a_1} I_{g_2}^{a_2} (I_i I_j)^{-\frac{t}{2}} , \]  

MSSM

\[ a_{ij}^{(t)}(t) = I_{g_1}^{a_1} I_{g_2}^{a_2} I_{\text{top}}^{a_3} (I_i I_j)^{\frac{t}{2}} , \]

\[ a_{ij}^{(t)}(t) = I_{g_1}^{a_1} I_{g_2}^{a_2} (I_i I_j)^{\frac{t}{2}} . \]  

Then using (4)-(6) we obtain the initial values of \( m^{t,11}(0) \) and, hence, solutions for \( m_{ij}(t) \) for \( \mu = M_Z - M_N \) exhibiting stability of all the matrix elements of \( m(t) \) under radiative corrections. The elements of the component matrices for the two cases are also shown in Table I. In der radiative corrections. The elements of the component matrices exhibit stability of all the matrix elements of \( m(t) \) under radiative corrections. The elements of the component matrices for the two cases are also shown in Table I. In Fig. we plotted \( m_{\tau \tau}(t) \) in comparison to conventional analyses. As against the maximal 30-40% radiative corrections in SM (CA) and MSSM (CA) occurring at \( \mu = M_Z \), they are only 3-4% in SM and 1.5% in MSSM which occur at intermediate scales in the present analysis. Among all the three models, the minimum radiative corrections upto 1.5% is found to occur in MSSM.

IV. Fitting the neutrino anomalies: When the bimaximal texture is exactly reproduced at \( M_Z \), one way to explain neutrino anomalies could be through threshold effects. But here ignoring threshold effects we show how the present mechanism permits matching of the observed solar (LAMSW) and atmospheric neutrino anomalies starting from the bimaximal texture with degenerate mass eigen values at the highest scale. Using quasidegenerate neutrinos with masses \( m_1 = -0.2 \) eV, \( m_2 = 0.200045 \) eV, \( m_3 = 0.2075 \) eV which are spread around \( m_0 = 0.2 \) eV, the mixing angles suitable for LAMSW with \( s_3 = 0.6946 \) and atmospheric neutrino oscillations with \( s_1 = 0.6950 \), it is straight forward to construct the mass matrix consistent with the experimental data

\[ m^{(e)}(t_Z) = \begin{bmatrix} -0.044716 & 0.722025 & -0.690426 \\ 0.722025 & 0.501718 & 0.477908 \\ -0.690426 & 0.477908 & 0.544506 \end{bmatrix} m_0. \]  

Although we have used \( s_2 = 0 \), the mechanism is found to work for other values consistent with CHOOZ bound. Similarly the mechanism also works with other values of \( m_0 \approx 0.1 - 1.0 \) eV. Within the RG-constraints, the high scale texture can match the experimentally observed anomalies provided \( m_{ij}(0) \) in (3) and (8) is replaced by \( m^{(e)}(t_Z) \) leading to

\[ m_{ij}^{(e)}(0) = \left( a_{ij}^{(t)}(t_Z)m_{ij}(0) - m^{(e)}(t_Z) \right) / d_{ij}, \]

\[ m_{ij}^{(t)}(0) = \left( m^{(e)}(t_Z) - a_{ij}^{(t)}(t_Z)m_{ij}(0) \right) / d_{ij}. \]  

In Fig. the curves 2HDM(e), MSSM(e) and SM(e) represent the result of fitting the data through the high scale bimaximal texture given in (3) and \( m^{(e)}(t_Z) \) given in (10) using 2HDM, MSSM and SM, respectively. We note that similar RG-stability also holds approximately for certain other elements depending upon the exact values of \( s_1 \) and \( s_3 \). But the radiative corrections are found to be larger if the difference between \( s_1 \) and \( s_3 \) is larger. Similar curves can be plotted for other elements also.

V. Conclusion: The present mechanism demonstrates how to evade RG-constraints on neutrino-mass textures in conventional analyses. It operates in a class of gauge theories leading to 2HDM, SM or MSSM where two component matrices contribute to the physically relevant Majorana-neutrino mass at the highest scale. Once a resultant texture is generated using suitable flavor symmetries at the highest scale, this mechanism determines the two unknown matrices at the highest scale which ensure its RG-stability at all lower scales or its matching with the experimental data. The mechanism can be applied to reproduce any high-scale texture at low energies with any desired degree of stability including higher order corrections in (3). It is quite interesting that the stability criteria operate in the presence of type II see-saw mechanism and probe into models including left-right gauge theories and \( SO(10) \) as prospective high-scale theories. The textures for component matrices derived from the stability condition sets considerable constraint on future model building with flavor symmetry.

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[1] R.N. Mohapatra, hep-ph/9910368, hep-ph/0008233; S.
TABLE I: Component matrices determined from RG stability criteria and also by matching the experimental data as denoted by (e). Here $m_0$ is a common factor.

| Model | $m^I(0)/m_0$ | $m^{II}(0)/m_0$ |
|-------|--------------|-----------------|
| 2HDM  | 0.000000, 0.220454, 0.146400 | 0.000000, 0.927561, 0.853507 |
|       | 0.146400, 0.103358, 0.058182 | 0.853507, 0.655695, 0.603538 |
| 2HDM (e) | 0.112140, 0.257854, 3.041252 | 0.112140, 0.964961, 2.334145 |
|       | 0.257854, 0.160003, -2.126483 | 0.964961, 0.660003, 1.626483 |
| MSSM  | 0.000000, 0.414613, 0.389877 | 0.000000, 0.292493, 0.317228 |
|       | 0.414613, 0.293120, -0.275628 | 0.292493, 0.206879, -0.224308 |
| MSSM (e) | 0.389877, -0.275628, 0.257691 | 0.317228, -0.224371, 0.242308 |
| SM    | 0.061597, 0.394061, 2.364023 | 0.061597, 0.313044, 1.656916 |
|       | 0.394061, 0.290752, -1.657126 | 0.313044, 0.209247, 1.157126 |
| SM (e) | 0.031932, -0.292451, 0.206794 | 0.031932, -0.292451, 0.206794 |
|       | -0.292451, 0.206794, -0.206800 | -0.292451, 0.206794, -0.206800 |
|       | 0.061597, 0.394061, 2.364023 | 0.061597, 0.313044, 1.656916 |
|       | 0.394061, 0.290752, -1.657126 | 0.313044, 0.209247, 1.157126 |
|       | 0.031932, -0.292451, 0.206794 | 0.031932, -0.292451, 0.206794 |
|       | -0.292451, 0.206794, -0.206800 | -0.292451, 0.206794, -0.206800 |
|       | 0.061597, 0.394061, 2.364023 | 0.061597, 0.313044, 1.656916 |
|       | 0.394061, 0.290752, -1.657126 | 0.313044, 0.209247, 1.157126 |
|       | 0.031932, -0.292451, 0.206794 | 0.031932, -0.292451, 0.206794 |
|       | -0.292451, 0.206794, -0.206800 | -0.292451, 0.206794, -0.206800 |

Barr and I. Dorsner, [hep-ph/0003058].
[2] R. Barbieri et al., [hep-ph/0001228], [hep-ph/9807235].
[3] K.S. Babu, J.C. Pati and F. Wilczek, [hep-ph/9812538].
[4] H. Georgi and S.L. Glashow, [hep-ph/9808293].
[5] C.H. Albright and S.M. Barr, [hep-ph/9806387].
[6] K.S. Babu and R.N. Mohapatra, Phys. Rev. Lett. 70, 2845 (1993); B. Brahmachari and R.N. Mohapatra, Phys. Rev. D 58, 015001 (1998); C.S. Aulakh, B. Baie, A. Melfo, A. Rasin and G. Senjanovic, [hep-ph/0004031].
[7] D. Caldwell and R.N. Mohapatra, Phys. Rev. D 48, 3259 (1993), Phys. Rev. D 50, 3477 (1994).
[8] A. Ioannissian and J. Valle, Phys. Lett. B 332, 93 (1994); J. Peltoniemi and J.W.F. Valle, Nucl. Phys. B 406, 409 (1993).
[9] K.R.S. Balaji, A.S. Dighe, R.N. Mohapatra, and M.K. Parida, Phys. Rev. Lett. 84, 5034 (2000); Phys. Lett. B 481, 33 (2000); K.R.S. Balaji, R.N. Mohapatra, M.K. Parida, and E.A. Paschos, Phys. Rev. D 63, 113002 (2001).
[10] K.S. Babu, C.N. Leung, and J. Pantaleone, Phys. Lett. B 319, 191 (1993); S. Antusch, et al., [hep-ph/0108005].
[11] J.Ellis and S. Lola, [hep-ph/0004279].
[12] E. Ma, [hep-ph/9904395], [hep-ph/9905381], [hep-ph/9906281], [hep-ph/9907353].
[13] M. Apollonio et al., [hep-ex/9907037].
[14] N. Haba et al., [hep-ph/9810471], [hep-ph/9905381], [hep-ph/9906281], [hep-ph/9907353].
[15] J.Ellis and S. Lola, [hep-ph/0004197].
[16] R.N. Mohapatra and G. Senjanovic, Phys. Rev. D 44, 912 (1991).
[17] R.N. Mohapatra and G. Senjanovic, Phys. Rev. D 23, 165 (1981).
[18] D. Chang, R.N. Mohapatra, and M.K. Parida, Phys. Rev. Lett. 52, 1072 (1984); Dae-Gye Lee, R.N. Mohapatra, M.K. Parida, and M. Rani, Phys. Rev. D 51, 229 (1995); M.K. Parida and A. Usmani, Phys. Rev. D 53, 3663 (1996).
[19] E. Ma and U. Sarkar, Phys. Rev. Lett. 80, 5716 (1998).
[20] M. Apollonio et al., [hep-ex/9907037].