Quantum Gravity and Renormalization: The Tensor Track

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Abstract

We propose a new program to quantize and renormalize gravity based on recent progress on the analysis of large random tensors. We compare it briefly with other existing approaches.

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1 Introduction

Quantization of gravity is still in debate. Nature keeps us away from direct experiments. Mathematics gives only a few hints. The key problem seems the ultraviolet fluctuations of space-time near the Planck scale. They cannot be renormalized through traditional field theoretic methods.

Risks are enormous in attacking this problem, among which mathematical hyper-sophistication is perhaps not the least. Existing approaches have been developed for decades, intimidating de facto the newcomer. Nevertheless we propose here still another road to quantize gravity.
It is born of closely related programs namely group field theory (GFT) \cite{1,2,3}, non commutative quantum field theory (NCQFT) \cite{4}, matrix models \cite{5} and dynamical triangulations \cite{6,7}, and also of many discussions with members of the loop quantum gravity (LQG) \cite{8} community. But it is now sufficiently distinct from all of these to deserve a name and a presentation of his own. For the moment let us call it tensor field theory, or simply TFT.

The program is based on a simple physical scenario and on a new mathematical tool.

The physical scenario, called geometrogenesis or emergence of space-time, is certainly not new: it has been discussed extensively in various quantum gravity approaches \cite{6,7,2,9,10,11,12,13,14}. At the big bang (which could perhaps more accurately be called the big cooling), space-time would condense from a perturbative or dilute pre-geometric phase to an effective state which is large and geometric. This state is better and better described by general relativity plus matter fields as the universe cools further. Our particular trend in this scenario is to emphasize the role of the renormalization group (RG) as the guiding thread throughout the pre-geometric phase as well as afterwards \textsuperscript{1}.

The new mathematical tool is the recently discovered universal theory of random tensors and their associated $1/N$ expansions \cite{15,16,17,18,19} \textsuperscript{2}. It seems sufficiently simple and general to likely describe a generic pre-geometric world.

The program consists in exploring this physical scenario with this new tool in the most conservative way, that is using quantum field theory and the renormalization group. Only the ordinary axiom of locality of quantum field interactions has to be abandoned and replaced by its tensor analog.

The first step of this program has been performed. Renormalizable tensor models have been defined \cite{20}. They provide random tensors with natural quantum field extensions equipped with their genuine renormalization group. They are higher rank analogs of the models of Grosse-Wulkenhaar type \cite{21,22} and of their matrix-like renormalization group. The next steps in TFT consist in a systematic analysis of the models in this class, their symmetries, flows, phase transitions and symmetry breaking patterns, as was

\textsuperscript{1}In particular we do not consider, as in \cite{9} that the emergence of time means the end of determinism; we would prefer to think that as long as there is a RG flow, there is an (extended) notion of determinism.

\textsuperscript{2}The theory considers general, ie unsymetrized, tensors and uses a combinatorial device called color to follow their indices, hence its initial name of colored tensor theory \cite{24}.
done for local quantum field theories in the 60’s and 70’s and was started for NCQFTs in the last decade. We hope to discover theories which lead to a large effective world with coordinate invariance, hence likely to bear an Einstein-Hilbert effective action. Since continuum limits can restore broken symmetries (such as rotation invariance), this hope does not seem completely unrealistic.

Let us precise the relationship of TFT with all its parents. TFT can in particular include the study of renormalizable GFT models, which are similar to combinatorial models but with an additional gauge invariance. It should be relatively easy to define such renormalizable GFT models by slight modifications of [20], since combinatorial and GFT models share the same 1/N expansion [16]. However exclusive attention to GFT models and their richer geometric content could be premature until we have better understood the simpler combinatorial models and their phase transitions. This is why we consider TFT not just a subprogram of GFT.

Matrix models provided the first controlled example of the geometrogenesis scenario [25, 26]. Shortly thereafter, tensor models were advocated as their natural higher dimensional generalization [27, 28, 29]. But since the 1/N expansion was missing, the program evolved over the years into the mostly numerical study of dynamical triangulations. TFT uses the same concepts than dynamical triangulations but revives the methods through the emphasis on renormalization and on the 1/N expansion. We hope and expect in the future rich interactions between analytic studies and numerical simulations in the development of TFT. TFT should also benefit as much as possible from all the expertise gained in the study of matrix models and of NCQFTs, as TFT is a direct attempt to generalize them to higher rank tensors.

The reader wont find here discussion of asymptotic safeness scenarios [30], although they share with TFT the use of RG as a main investigation tool, their approach is quite different. But we do include in the last section brief comments on the relationship between TFT and two other current approaches to quantum gravity, namely string theory and LQG. For LQG we have included a contribution to the evolving debate on whether and how to introduce renormalization in this field.

This paper is neither a review nor meant to be technical. It builds upon

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3This invariance has been studied in (colored) group field theories in [23]. See also the interesting remark about renormalization and diffeomorphism invariance at the end of section II in [2].

4We thank C. Rovelli for his recent invitation to lecture in Marseille on this subject.
a previous similar one [31], but with many changes incorporated. Indeed the last year has seen significant mathematical progress in the line of thought we advocated.

2 Why TFT?

Einstein understood gravity as a physical theory of geometry. Wheeler summarized his theory in the famous sentence: matter tells space how to curve, space tells matter how to move. General relativity was founded following the principle of general invariance under change of coordinates.

Feynman in an incredibly bold step reduced quantization to randomization. Instead of computing a classical deterministic history, quantization sums over all possible such histories with an action-depending weight. In modern simplified notations, it means that quantum expectations values are similar to statistical mechanics averages, possibly with an additional factor $i$

$$< A > = \frac{1}{Z} \int A(\phi) e^{-iS(\phi)} d\phi, \quad Z = \int e^{-iS(\phi)} d\phi$$

(1)

where $Z$ is a normalization factor. In the Euclidean formulation, which corresponds physically to a finite temperature, even the additional $i$ factor is absent and functional integral quantization reduces to probability theory.

If we try to join the two main messages of Einstein and Feynman we get the equation

$$\text{Quantum Gravity} = \text{Random Geometry.}$$

(2)

There is quite a wide agreement on this equation among all the main schools on quantum gravity, although neither string theory nor LQG take it as their starting point. The problem is that geometry is rich. Especially in three or four dimensions there does not seem to be a unique way to put a canonical probability theory on it.

We can take inspiration from another outstanding idea of Feynman, namely to represent quantum histories by graphs. We feel that it is perhaps not sufficiently emphasized in the graph theory community that quantum field theory and Wick’s theorem provide a canonical measure for graphs by counting graphs through pairings of half-lines into lines.\footnote{Counting graphs in this way is called zero-dimensional QFT by theoretical physicists.}
instance the two connected graphs you can create by joining four vertices of coordination 4. The first picture has canonical weight 24. The second picture has canonical weight 72. Hence the second picture is three times more probable than the first.

Graphs are among the simplest pre-geometric objects hence should play a leading role in a theory of quantum gravity. This was advocated in graphity [10]. However there remains a missing link between graphity and geometry, which can be borrowed from matrix models and dynamical triangulations. Since simplicial geometry is dual to stranded graph theory, we can import the canonical notion of probability theory from graphs to triangulations and declare it the correct randomization of geometry at the simplicial level. Combining graphs according to quantum field theory rules we arrive to random tensors. With the help of the 1/N expansion, the canonical interactions of these random tensors were identified in [32]. Finally adding a natural non trivial propagator to allow for dynamics and renormalization we arrive at TFT [20].

We hope that after having fixed the quantization of this most basic level of geometry, more complicated geometric structures such as differential and Riemannian aspects will follow from the renormalization group. Expressed the other way around we could consider the Planck scale as the limit at which local, continuum physics ceases to apply and has to be substituted by tensor physics. In this picture tensor physics is the more fundamental level, just as QCD is more fundamental than nuclear physics.

The 1/N expansion provides the tensor program with a new analytic tool. We could wonder how general is this tool and address now this question.

## 3 1/N Expansion and universality

There is a strong link between the universal character of the central limit theorem in probability theory and the existence of a 1/N expansion in corresponding field theoretic perturbative expansions. The initial discovery of the central limit theorem predates by more than a century the invention of the corresponding vector 1/N expansion in field theory. For random matrices
universality of the Wigner-Dyson laws \[33\] was followed rather soon by the \(1/N\) expansion of 't Hooft and followers \[34\]. But in the case of tensors in fact the \(1/N\) field theoretic method slightly predates universality! We have now three versions of the central limit theorem, for independent identically distributed (iid) vectors, matrices and tensors. All three are closely related to the corresponding \(1/N\) expansion. This has been clarified in \[15\], to which we refer for details. Here we just provide a sketch of this relationship.

The \(1/N\) expansion is a field theoretic method to find the leading stranded Feynman graphs at fixed perturbative order which arise in the perturbation expansion of an interacting quantum field theory. Such a theory incorporates natural interactions perturbing a Gaussian measure. So it seems at the beginning a very particular type of non-Gaussian measure.

A theorem of Kolmogorov essentially relates any probability law to its moments, that is to polynomial expectation values. Hence convergence to Gaussianity in the central limit theorem can be done by studying convergence of polynomial moments, or of their cumulants (connected parts). Consider large random vectors of size \(N\), matrices of size \(N^2\) or tensors of size \(N^3\) and so on. Their coefficients are iid with a fixed atomic law. We can restrict easily to an even centered such atomic law. What distinguishes matrices or tensors from just longer vector-like lists of variables is that we are not interested in the same observables. To take into account covariance or contravariance under change of basis, we are interested only in the statistical behavior at large \(N\) of certain trace invariants. Universality then reduces to prove that in the expectation value of any fixed such cumulant, any atomic moment of order 4 or more is washed out at large \(N\). Hence only the second moments of the atomic law survive and influence the value of the large \(N\) limit distribution.

The proof follows from a graphic representation. Any four moment of the atomic distribution (or higher) is an unlikely coincidence in the cumulant expansion of any moment of the tensor in the large \(N\) limit. It turns out to be washed out for the same reason than the \(1/N\) expansion computes the leading graphs in the corresponding field theoretic problem. Hence the \(1/N\) expansion can be considered a grinding machine to erase information from higher moments in atomic laws.

Because of this universality, we know that we are on solid ground if we base the pre-geometric phase of quantum gravity on the \(1/N\) tensor-expansion. Even if at extreme trans-Planckian scales the bare atomic laws for the very large tensors that in TFT would describe our "pre-universe"
remain forever unknown, even if they are discrete, such as throwing coins or dices, we should end up with the same limiting flow.

But in physics there is another, still richer, notion of universality than the central limit theorem. It is the RG theory of phase transitions and of their critical indices. We know that the RG is another powerful grinding machine which erases information and creates universality. Only a few marginal or relevant operators can emerge out of it. To close the loop we shall now show that RG is also intimately linked to 1/N expansions and central limit theorems. In particular it exhibits a parallel hierarchy of scalar, vector, matrix and tensor types\textsuperscript{6}. TFT interprets this hierarchy as that of the increasingly difficult quantum gravity problem in zero, one, two and more than two dimensions.

4 Renormalization

4.1 Essential Features

The essential features needed for renormalization are an action $S$, a scale decomposition, a notion of locality and a power counting theorem. As we shall see the notions of scale and locality can be adapted to quite exotic contexts (vectors, matrices, tensors).

The scale decomposition separates the ultraviolet ("high fluctuation") scales of the fluctuation fields from the infrared scales of the "background field". At fixed attributions for the scales of the lines of a graph, some subgraphs play an essential role. They are the connected subgraphs whose internal lines all have higher scale index than all the external lines of the subgraph. Let’s call them the "high" subgraphs.

The renormalization recipe is to check that the divergent high subgraphs satisfy the locality requirement. If by power counting their local divergent parts are of the form of the initial action, then the theory is renormalizable.

In ordinary scalar just renormalizable theories such as $\phi^4$ the power counting is summarized in a formula for the divergence degree $\omega$ such as

$$\omega = 4 - N$$

where $N$ is the number of external legs.

\textsuperscript{6}Except for the fact that there is no 1/N expansion for scalars.
Renormalization does not "pull infinities under the rug" but has a deep physical meaning. An external observer (which has only access to low momenta) has no choice but to measure effective constants which are the sum of bare constants plus high momentum radiative corrections. The parameters of the model change with scale but not the structure of the model itself. Renormalization generates a flow between the bare action and renormalized or effective action which depends on the observation scale. This flow is computed recursively, step by step, by the renormalization group.

4.2 Renormalization Group

Let us start by a classic citation [35]:

*The renormalization group approach is a strategy for dealing with problems involving many length scales. The strategy is to tackle the problem in steps, one step for each length scale. In the case of critical phenomena, the problem, technically, is to carry out statistical averages over thermal fluctuations on all size scales. The renormalization group approach is to integrate out the fluctuations in sequence starting with fluctuations on an atomic scale and then moving to successively larger scales until fluctuations on all scales have been averaged.*

It is now recognized that RG governs the standard model, hence all known physics\(^7\). RG is the only known way to understand and organize logically divergencies in quantum field theory, statistical mechanics and condensed matter.

An elementary step to compute the RG flow is made of two basic sub-steps. A functional integration over a slice of "fluctuation fields" $\phi_f$ is followed by the computation of a logarithm to define the effective action $S'(\phi_b)$ for the background field

$$I(\phi_b) = \int d\phi_f e^{-S(\phi_f+\phi_b)} = e^{-S(\phi_b)} \rightarrow S'(\phi_b) = -\log I(\phi_b).$$

The RG flow is non trivial because these two sub-steps do not commute\(^8\). An essential feature of this flow is that it is directed. Scales and locality match this orientation of the RG arrow, and cannot work the other way.

\(^7\) Except perhaps quantum gravity; but quantum gravity is still unknown physics and TFT precisely postulates that RG also applies to it.

\(^8\) A rescaling step is often added but this is technical rather than fundamental.
around. In this way there is a deep analogy between the renormalization group and the second law of thermodynamics. The RG flow is directed and irreversible, because its coarse graining erases information. Ultimately it depends on an external observer who must average with a probability law over the data he cannot access. The key physical analogies between time, scale and temperature are at the core of the paradigm to view the evolution of the universe as a RG cooling trajectory.

4.3 A hierarchy of Renormalization Group and 1/N Expansions

We saw already that there is a parallel between the hierarchy of central limit theorems in probability theory and the hierarchy of 1/N expansion expansions in quantum field theory. There is also an associated hierarchy of renormalization group types: scalar, vector, matrix, tensors. They can be distinguished by their different notions of locality and the different power counting formulas to which they lead to.

We have already discussed scalar-type RG: ordinary quantum field theories such as $\phi^4_4$ or Yang Mills theories are in this category. In the just renormalizable case, divergence degrees are simply formulas in the number of external legs such as $\omega = 4 - N - 8g - 4(B - 1)$.

Vector-type renormalization group occurs e.g. in condensed matter when approaching the Fermi surface $[36]$. The most divergent graphs are not the 2 and 4 point functions but the 2 and 4-point functions made of bubble chains at zero external momenta. They clearly form linear chains, hence are associated to one dimensional spaces, or one-dimensional quantum gravity from the point of view of geometrogenesis.

The epitome for matrix-type renormalization group is the Grosse-Wulkenhaar model. Here the degree of divergence is given by a formula such as

$$\omega = 4 - N - 8g - 4(B - 1)$$

where $g$ is the genus and $B$ the number of faces broken by external legs. Only planar 2 and 4 point graphs whose external legs all arrive on the same external face do diverge. They satisfy the locality principle (also called Moyality) which simply states that the matrix equivalent of local operators are traces of powers of the matrix.
In tensor-like renormalization group the formulas are still more complicated and involve sums over genera of jackets of the graph and of its boundary [20]. The locality principle follows again from the identification of the right trace invariants in [32].

We know the renormalization group type can change along a given RG trajectory at a phase transition point. For instance at the BCS transition in condensed matter, the RG type changes form vector to scalar. There is therefore no reason the RG cannot change from tensor to lower-rank type at geometrogenesis.

4.4 The Case for the Phase Transition

Can we escape the geometrogenesis phase transition along the tensor program? The short answer is no, for several compelling reasons.

First of all, phase transitions and the formation of bound states is perhaps the most general and generic aspect of all physics. No interacting physical system when developed over many scales is free of such transitions; for instance in condensed matter even initially repelling electrons do form bound states. It would be extremely strange, even almost unbelievable if the theory of quantum gravity was the only physical theory not to exhibit such a phenomenon.

Then two additional arguments apply more specifically to the tensor program. The first has been forcefully argued in eg [12]. In any GFT or in any tensor theory that tries to build space-time out of triangulations, the initial vacuum is not a particular space but no space at all. Hence it cannot be the final product, namely the large universe in which we live. This means there has to be a phase transition along the way.

A more mathematical argument relies again on the renormalization group. Only phase transitions allow a change in the RG power counting and type. But we need such a change to solve the apparent contradiction between the perfectly renormalizable pre-geometric theory that we envision as fundamental and the Einstein-Hilbert action on flat space, well-known to be non-renormalizable in the ultraviolet regime. In a phase transition the expansion point in the action changes, hence also the Hessian around this point. It means that the propagators, the particles and statistics after the transition can be completely different than before. For instance in the BCS phase transition we go from a Fermionic renormalizable model with a vector-type RG (around the Fermi surface) to a Bosonic model (the Cooper pair).
with a scalar-type RG. We need geometrogenesis for the initial tensor-type RG to morph into a lower-rank RG types that will govern the more ordinary flows of matter and gravity (i.e. the local metric field) after space-time has developed.

The phase transition scenario is already seen both in matrix and in the simplest tensor models analyzed so far [25, 37].

A last word about trans-Planckian physics. Gravity could have a true and absolute physical cutoff at the Planck scale. But most physicists agree that quantum gravity effects start around the Planck scale. If there is also an absolute UV cutoff at Planck scale there would be no real regime for quantum gravity to develop, except in an extremely limited sense of a few scales. If there is a trans-Planckian regime with many scales we think it should be appropriately called the quantum gravity regime, although it would be pre-geometric rather than geometric.

5 Other Approaches

5.1 String Theory

String theory is a world of its own. Lauded as the ultimate theory of everything it has also attracted criticism (see [38] for a recent example). We cannot even try to scratch this subject here. However since it remains by far the main stream approach to quantum gravity today we nevertheless include a few brief and casual remarks on the most likely possible contact points between string theory and the tensor program. It seems that there are at least two main such contact points, namely dualities and the AdS-CFT/holography correspondence.

Dualities lie at the heart of string theory but also play an essential role in NCQFT. The Grosse-Wulkenhaar model is based on Langmann-Szabo duality, which seems responsible for its conformal fixed point. We should study which dualities also occur in which tensor models.

The AdS-CFT correspondence shows how string theory can reduce to quantum field theory on a boundary, and the holography principle extends this to any bulk theory of quantum gravity. The idea remains mostly limited to supersymmetric models. TFT should certainly benefit from this beautiful circle of ideas, for instance from the possibility of identifying the radial direction in AdS-CFT with the RG scale. There are some preliminary
glimpses of a possible holographic nature of the boundary of colored tensor graphs.

Another more superficial similarity between tensors and strings is the nonlocal character of their interactions. Non-locality in string interactions is due to their extended character, whether the non-locality of tensors looks simpler but more abstract and radical, corresponding to the gluing rules of simplicial geometry.

String theory now includes branes and just as matrix models quantize the string world sheet, one can hope that tensor models can provide an appropriate quantization of the branes world volume.

Other superstring features such as supersymmetry or the Kaluza-Klein/Calabi-Yau ideas seem quite orthogonal to the tensor program. Supersymmetry of course could be included in tensor models but its key property to soften or cancel divergencies is not a prerequisite for TFT. Calabi-Yau compactification lead to fascinating mathematics. But tailoring a particular Calabi-Yau space so friendly to superstrings that moving in it they generate the standard model seems truly orthogonal to TFT.

String theory main successes relied on the spectacular development in the second half of the XXth century of key mathematical tools to analyze two-dimensional physics: conformal theory, integrability and random matrices. It is likely that the generalization of these tools to higher dimensions will bring new perspectives. Our hope is for the grinding machine of the renormalization group to provide a sturdy rather than elegant universe.

5.2 Loop Quantum Gravity and Renormalization

Several reviews discuss the pros and cons of LQG and spin foams \cite{38,39,40,41} but they do not focus on the specific question of renormalization. The development of GFT and TFT as more independent programs requires to clarify which of their aspects are compatible with LQG (see \cite{42} for a related discussion).

LQG, in its covariant version, expresses physical quantities through spin foam amplitudes, which contain sums over the discrete values $j$ of spin representations of $SU(2)$, or of another related Lie group. A basic tenet in LQG is that there is no external background geometry, hence the Lie group is “all you have”. All known spin foam models, starting with the simplest one (the Ponzano-Regge spin foams in three dimensions) contain divergencies; the high spin sums do not always converge. The corresponding
power counting is now understood in some detail [43, 44, 45].

In LQG high spins are interpreted as large geometries, hence the large spin divergencies are called infrared divergencies. The discretization at small \( j = 0, 1... \) is interpreted as an ultraviolet cutoff on the theory at the Planck scale. Ultraviolet finiteness in LQG is hailed, together with discretization of lengths and areas, as a major result of the theory. Also geometrogenesis is not part of mainstream LQG; the preferred cosmological scenario is a bounce, based on the analysis of models with a few degrees of freedom.

We claim that the divergencies of LQG cannot be renormalized without major changes to this picture, for two main reasons.

First in LQG as it exists today, there is no widely agreed action relating all spin foams. LQG is therefore only a first quantized theory of gravity. The list of spin foams to sum and their exact combinatoric weights for a given computation have to be fixed before any RG treatment. Of course individual divergent amplitudes can be regularized in many ways to produce finite answers. For instance they can be omitted (regularized to 0). But this is definitely not renormalization.

Second, the direction of the RG cannot be changed at will. As explained above, the RG flow is always directed from ultraviolet towards infrared. RG averages and erases information on fine details of the theory, not the converse. In LQG as in GFT there is no background, the Lie group is all one has to sum upon. So it is only on that Lie group that one can average in order to renormalize. RG can then only flow from large (ultraviolet) spins to smaller spins, not the converse. Locality, even in the extended tensor sense, does not work the other way around [20].

On the more positive side, any quantum gravity theory should make contact with physics, and produce an effective world which might include aspects of LQG. In particular geometrogenesis in TFT or GFT should occur at a still giant value of the tensor momenta or spins \( j \), to account for the enormous number of degrees of freedom that exist in our universe. Models of spin foams or GFTs that incorporate the geometric data of general relativity as large \( j \) asymptotics [46, 47, 48, 49, 50] might therefore be very important to understand how geometrogenesis leads to our world.

In short the issue of ultraviolet versus infrared in LQG is not just about names. To include renormalization, LQG must first adopt a second-quantized formalism such as GFT\(^9\). Even then, divergencies in LQG can

\(^9\)We are still far from this situation. Although spin foams were rather early identified
be renormalized only as ultraviolet divergencies. This has far-reaching consequences such as geometrogenesis.\textsuperscript{10}

6 Conclusion

The directed renormalization group flow is the modern avatar of the second law of thermodynamics. Therefore I would like to conclude this paper by paraphrasing the famous words of Sir Arthur Eddington in ”The Nature of the Physical World” (1927):

\textit{The flow of the renormalization group holds, I think, the supreme position among the laws of Nature. If someone points out to you that your pet theory of the universe is in disagreement with Maxwell’s equations - then so much the worse for Maxwell’s equations. If it is found to be contradicted by observation - well, these experimentalists do bungle things sometimes. But if your theory is found to be against the flow of the renormalization group I can give you no hope; there is nothing for it but to collapse in deepest humiliation.}

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\textsuperscript{10}Geometrogenesis suggests that bounce is unlikely but cannot rule it out completely for the current big-bang. Something could have heated the universe close to the boiling point and it could have re-cooled.
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