Large Scale-Small Scale Duality and Cosmological Constant

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Abstract

We study a model of quantum cosmology originating from a classical model of gravitation where a self interacting scalar field is coupled to gravity with the metric undergoing a signature transition. We show that there are dual classical signature changing solutions, one at large scales and the other at small scales. It is possible to fine-tune the physics in both scales with an infinitesimal effective cosmological constant.
1. Introduction

The question of signature transition in classical and quantum gravity has been the subject of intense investigations during the past few years. These investigations have led to, basically, two approaches to the subject, discrete and continuous. The continuous approach has been studied by a number of authors. Of particular interest to the present work is the model adopted by Dereli and Tucker [1] in which a self interacting scalar field is coupled to gravity. In this model, Einstein’s field equations, coupled to the scalar field, are solved such that the scalar field and the scale factor are considered as dynamical variables, giving rise to cosmological solutions with degenerate metrics, describing transition from Euclidean to Lorentzian domains. A quantum cosmological version of this model was used to derive the wavefunction of the universe by solving the corresponding Wheeler-DeWitt equation [2]. This was achieved by adopting a new choice of variables through which Einstein’s classical equations of motion arise from an anisotropic constrained oscillator-ghost-oscillator Hamiltonian. A family of Hilbert subspaces were then derived in which states are identified with non dispersive wave packets peaking in the vicinity of classical loci with parametrizations corresponding to metric solutions of Einstein’s equations that admit a continuous signature transition.

In the approach detailed below we use the notations in [1] and [2] with their results and pay attention to duality transformations on the couplings in the present scalar field potential such that the classical signature changing cosmology transforms to its “dual”, but the quantum cosmology remains unchanged. As a result of [2], a remarkable correlation should be seen between this self dual quantum cosmology and dual classical cosmologies. Based on this correlation, we can introduce the “large scale-small scale” duality interrelated with the “small coupling-large coupling” duality in a way that dual classical cosmologies, one at large scales and the other at small scales, may have exactly a same infinitesimal effective cosmological constant. It was suggested [4] that a large distance-small distance connection should exist to shift the cosmological constant to zero and wormholes provide such a connection. Therefore, such a fine-tuning in the present model will be of particular interest since it may explain that: why the cosmological constant should be small from the microscopic point of view if it is small from the cosmic viewpoint [4]. Moreover the fine-tuning mechanism is related, somehow, to the existence of quantum wormholes in the model.

2. Dual classical cosmologies

Consider the Einstein-Hilbert action

\[ I = \int \sqrt{|g|} \left[ \frac{1}{16\pi G} R + L_{\text{matter}} \right] d^4x \]  

(1)

where \( L_{\text{matter}} = \frac{1}{2} \partial_\phi \partial^\phi - U(\phi) \) is the real scalar field Lagrangian (assume that \( \phi \) is homogeneous and only depends on time). We choose the chart \( \{ \beta, x^1, x^2, x^3 \} \) and parametrize the metric as [1]

\[ g = -\beta d\beta \otimes d\beta + \frac{R^2(\beta)}{[1 + \frac{3}{4} r^2]^2} \sum_i dx^i \otimes dx^i \]  

(2)
where \( r^2 = \sum x_i x^i \), \( R(\beta) \) is the scale factor with \( k = \{-1, 0, 1\} \) representing open, flat or closed universes and \( \beta \) is the lapse function with the hypersurface of signature change at \( \beta = 0 \). For \( \beta > 0 \), the cosmic time can be written as \( t = \frac{2}{3} \beta^{3/2} \). The scalar curvature \( \mathcal{R} \) is then given by

\[
\mathcal{R} = 6 \left[ \frac{\ddot{R}}{R} + \frac{k + \dot{R}^2}{R^2} \right]
\] (3)

where \( . \) denotes the derivatives with respect to \( t \) and the units is taken such that \( 3\pi G = 1 \).

Making the transformations

\[
X = R^{3/2} \cosh(\alpha \phi) \\
Y = R^{3/2} \sinh(\alpha \phi)
\] (4) (5)

where \( -\infty < \phi < +\infty \), \( 0 \leq R < \infty \) and \( \alpha^2 = \frac{3}{8} \), the corresponding effective Lagrangian form is obtained

\[
2\alpha^2 \mathcal{L} dt = \left\{ -\dot{X}^2 + \dot{Y}^2 + \frac{9k}{4} (X^2 - Y^2)^{1/3} - 2\alpha^2 (X^2 - Y^2) U(\phi(X, Y)) \right\} dt. \] (6)

Applying the Einstein equations, one can see that the Hamiltonian \( \mathcal{H} \) corresponding to \( \mathcal{L} \), must vanish identically giving a “zero energy condition” to be imposed on the solutions of the field equations. Concentrating on cosmologies with \( k = 0 \), we take the potential as

\[
2\alpha^2 (X^2 - Y^2) U(\phi(X, Y)) = a_1 X^2 + (a_2 - a_1^{-1}) Y^2 + 2b XY
\] (7)

where \( a_1, a_2 \) and \( b \) are assumed to be dimensionless constant parameters \( 1 \). As we shall see, this particular choice of the potential \( 2 \) leads to what we shall call “duality transformations”. Variation of the action with respect to the dynamical variables \( X \) and \( Y \) now gives

\[
\ddot{X} = a_1 X + b Y \] (8)

\[
\ddot{Y} = -b X + (a_1^{-1} - a_2) Y \] (9)

for which the “zero energy” condition are to be imposed. Writing the potential in terms of the physical couplings \( \lambda \), \( m^2 \) and \( b \) we obtain \( 3 \)

\[
U(\phi) = \lambda + \frac{1}{2\alpha^2} m^2 \sinh^2(\alpha \phi) + \frac{1}{2\alpha^2} b \sinh(2\alpha \phi)
\] (10)

where \( \lambda = U \mid_{\phi=0} = \frac{a_1}{2\alpha^2} \), \( m^2 = \frac{\partial^2 U}{\partial \phi^2} \mid_{\phi=0} = a_1 + a_2 - a_1^{-1} \) and \( b \) are the bare cosmological constant, positive mass square and coupling constant respectively. The minimum of the potential occurs when \( |2b/m^2| < 1 \). Equations (8) and (9) may be decoupled into normal modes \( \alpha = (u, v) \) with “zero energy” solutions for \( \lambda_+ , \lambda_- < 0 \) given by \( 2 \)

\[
v = 2A_0 \cos \left[ \frac{1}{r} \cos^{-1} \left( \frac{r u}{2A_0} \right) \right], \quad |u| \leq \frac{2A_0}{r}
\] (11)

\[
v = 2A_0 \cosh \left[ \frac{1}{r} \cosh^{-1} \left( \frac{r u}{2A_0} \right) \right], \quad |u| > \frac{2A_0}{r}
\]

\( ^1 \)Such an assumption is reasonable in the plank units

\( ^2 \)This potential is the same as that of Ref. 1 except a little difference in the coefficient of \( Y^2 \).
where $\lambda_{\pm}$ are the eigenvalues of the matrix $\begin{pmatrix} a_1 & b \\ -b & a_1^{-1} - a_2 \end{pmatrix}$ given by

$$\lambda_{\pm} = \frac{a_1 + a_1^{-1} - a_2}{2} \pm \sqrt{\left(\frac{a_1 + a_1^{-1} - a_2}{2}\right)^2 - (1 - a_1 a_2 + b^2)}$$

with $r = \frac{\sqrt{\lambda_{\pm}}}{\lambda_{\pm}}$, $0 < r < 1$, $A_0$ an arbitrary real constant and $\epsilon = \pm 1$ indicating two ways to satisfy the Hamiltonian constraint $\mathcal{H} = 0$.

An interesting feature of this model is that one can find a class of transformations on the space of parameters $\{a_1, a_2, b\}$ which would leave the eigenvalues $\lambda_{\pm}$ invariant. These transformations can be written as

$$a_1 \rightarrow a_1^{-1}$$
$$a_2 \rightarrow a_2$$
$$b^2 \rightarrow b^2 + (a_1^{-1} - a_1)a_2.$$  

(13)

In terms of the physical couplings $\lambda$, $m$ and $b$, the eigenvalues are

$$\lambda_{\pm} = \frac{3\lambda}{4} - \frac{m^2}{2} \pm \frac{1}{2}\sqrt{m^4 - 4b^2}.$$  

(14)

It is seen that although the classical loci (11) on the configuration space $(u, v)$ remains unchanged under (13), the corresponding solutions $R(\beta)$ and $\phi(\beta)$ change, since $X(\beta)$ and $Y(\beta)$ are related to $u(\beta)$ and $v(\beta)$ by the decoupling matrix which changes under (13) (see (15, 20). In terms of the physical couplings, (13) can be rewritten as

$$\lambda \rightarrow \tilde{\lambda} \equiv \frac{1}{4\alpha^4}\lambda^{-1}$$
$$m^2 \rightarrow \tilde{m}^2 \equiv m^2 - \frac{4\alpha^2\lambda^2 - 1}{\alpha^2\lambda}$$
$$b^2 \rightarrow \tilde{b}^2 \equiv b^2 + m^2[(2\alpha^2\lambda)^{-1} - 2\alpha^2\lambda] + [(2\alpha^2\lambda)^{-1} - 2\alpha^2\lambda]^2.$$  

(15)

Therefore, if we define (15) as “duality” transformations, then we have two sets of solutions for $R(\beta)$ and $\phi(\beta)$ corresponding to dual sets of physical couplings. We interpret the new couplings as dual bare cosmological constant, dual mass square and dual coupling constant respectively.

The behavior of the physical couplings under the duality transformations merits some discussion. As is discussed in [1], in order to have signature transition, both eigenvalues $\lambda_{\pm}$ must be negative and equation (14) gives

$$\lambda < \frac{4}{3}\left[\frac{m^2}{2} - \frac{1}{2}\sqrt{m^4 - 4b^2}\right].$$  

(16)

However, this does not guarantee that the dual potential has also a minimum and a positive mass square. In order to have both we take

$$\frac{m^2}{2} - \frac{1}{2}\sqrt{m^4 - 4b^2} \leq 1.$$  

(17)
One can find a suitable choice of couplings satisfying (17) so as to make the dual potentials $U(\phi)$ and $\tilde{U}(\tilde{\phi})$ have the same properties in a way that once $|2b/m^2| < 1$, then $|2\tilde{b}/\tilde{m}^2| < 1$, ensuring that both dual potentials have minimum (see below). In the case of a small cosmological constant the dual transformations map the desired small values of the bare cosmological constant $\lambda$, positive mass square $m^2$ and coupling constant $b$ to large values of the corresponding dual couplings, $\tilde{\lambda}$, $\tilde{m}^2$ and $\tilde{b}$. It then follows that two different classical cosmologies, one with small couplings $\{\lambda, m^2, b\}$ and the other with large dual couplings $\{\tilde{\lambda}, \tilde{m}^2, \tilde{b}\}$ exhibit the same signature dynamics on the configuration space $(u, v)$.

It is straightforward to show that the effect of “small $\rightarrow$ large” couplings transformations on the space of physical solutions $R(\beta)$ and $\phi(\beta)$ may cause a “large $\rightarrow$ small” scales transformations for a suitably chosen set of couplings. To see, one may recover $R$ from $X$ and $Y$ as

$$R = (X^2 - Y^2)^{\frac{1}{3}}$$

on the other hand, we have the following change of variables

$$\xi = S\alpha$$

where $\xi = (X, Y)$ and

$$S = \begin{pmatrix}
-\frac{m^2 - \sqrt{m^4 - 4b^2}}{2b} & -\frac{m^2 + \sqrt{m^4 - 4b^2}}{2b} \\
1 & 1
\end{pmatrix}$$

(20)

Now, by choosing the set of small couplings as \[3\]

$$\lambda \simeq 0 \quad m^2 \ll 1 \quad b \simeq 0$$

(21)

with the assumption that $m^2 \gg b, \lambda$ we find from (18-20) that $R$ may have a desired large value due to the adjustable large value of the matrix element $|S_{12}| \simeq \frac{m^2}{b} \gg 1$. On the other hand, taking the dual set of couplings through (15) we find the set of large couplings $\{\tilde{\lambda}, \tilde{m}^2, \tilde{b}\}$ for which the corresponding decoupling matrix (20) leads through (18,19) to a small value for $R$. This is because $\tilde{m}^4 - 4\tilde{b}^2 = m^4 - 4b^2 \ll 1$ and $\tilde{m}^2 \sim \tilde{b} \sim \lambda^{-1} \gg 1$, so $|\tilde{S}_{11}| \sim |\tilde{S}_{12}| \sim 1$.

This is a remarkable step towards solving the cosmological constant problem in the context of the present model. The cosmological constant plays two roles in physics. The first one, as a coupling constant in microscopic physics, has its origin in short distance physics whereas the other role, as a macroscopic parameter, controls the large scale behavior of the universe. There is no explanation of why the cosmological constant is so small that the universe can be flat and big enough. A direct connection between the large scale physics and the small scale physics should exist by which the microscopic physics be fine-tuned with good precision so that the large scale structure of space-time can look as it is observed today. One possible connection is due to the wormholes which provide such a large scale - small scale relation.

\[3\]Here $m$ as a reasonably small mass may be the planck mass $M_{pl}$, or another small energy density related to some spontaneous symmetry-breaking scale such as $M_{SUSY}$ or $M_{weak}$. On the contrary, consistent with equations (16) and (17), the two other couplings $\lambda$ and $b$ are assumed to be very small compared to $m^2$ namely, $m^2 \gg b, \lambda$. 
Peoples have considered the effects of wormholes in the Euclidean path integral of quantum gravity and shown that the effect is modifying the cosmological constant and providing a probability distribution concentrating at zero value of the cosmological constant. In particular, Hawking proposed that quantum fluctuations in space-time topology at small scales may play an important role in shifting the cosmological constant to zero.

The appearance of “large scale-small scale” duality interrelated with “small coupling-large coupling” duality in the present model is a good chance to obtain a fine-tuning mechanism to have an infinitesimal effective cosmological constant at both large and small scales. For the sake of definiteness we define the minima of the potentials \( U(\phi) \) and \( \tilde{U}(\tilde{\phi}) \) respectively

\[
\Lambda_{\text{eff}} = \lambda + \frac{m^2}{4\alpha^2} \left( \sqrt{1 - \frac{4b^2}{m^4}} - 1 \right), \quad (22)
\]

\[
\tilde{\Lambda}_{\text{eff}} = \tilde{\lambda} + \frac{\tilde{m}^2}{4\alpha^2} \left( \sqrt{1 - \frac{4\tilde{b}^2}{\tilde{m}^4}} - 1 \right), \quad (23)
\]

as dual effective cosmological constants in the form of the sum of two terms, the bare cosmological constant and the contribution of mass scale of the scalar field. Thanks to the duality transformations (15) we find the remarkable result

\[
\Lambda_{\text{eff}} = \tilde{\Lambda}_{\text{eff}} \quad (24)
\]

showing the self-duality of the effective cosmological constant. We now choose the set of suitably small couplings \( \{\lambda, m^2, b\} \) corresponding to the large scale \( R(\beta) \)

\[
\lambda \simeq 0 \quad m^2 \ll 1 \quad b \simeq 0 \quad (25)
\]

with \( b \ll m^2 \). Therefore, according to (22), the effective cosmological constant at large scales becomes infinitesimal, namely

\[
\Lambda_{\text{eff}} \simeq 0. \quad (26)
\]

At small scales \( \tilde{R}(\tilde{\beta}) \), however, due to duality transformations (15) the dual couplings \( \{\tilde{\lambda}, \tilde{m}^2, \tilde{b}\} \) are then very large and we have really a significant potential \( \tilde{U}(\tilde{\phi}) \) with strong couplings which are anticipated to affect essentially the value of the cosmological constant giving rise to the well known cosmological constant problem. However, as is shown in (24), the duality transformations are such that at small scale we have exactly the same infinitesimal effective cosmological constant

\[
\tilde{\Lambda}_{\text{eff}} = \Lambda_{\text{eff}} \simeq 0. \quad (27)
\]

Therefore, both \( \Lambda_{\text{eff}} \) at large scales and \( \tilde{\Lambda}_{\text{eff}} \) at small scales can be fine-tuned to zero by the same procedure including duality transformations. In other words, these duality transformations provide a direct connection between the large and small scales and set the cosmological constant, at both scales, to zero.

In the next section, we will see that the origin of these dual signature changing classical cosmologies with a self-dual effective cosmological constant is in a self dual Wheeler-DeWitt equation with dual solutions obeying the wormhole boundary conditions.
3. Self Dual Quantum Cosmology - Dual Quantum Wormholes

In the previous section we have shown that it is possible to find dual classical cosmologies on the $(R, \phi)$ configuration space corresponding to a unique classical cosmology defined on the $(u, v)$ configuration space. On the other hand it is shown [2] that the corresponding Wheeler-Dewitt equation in terms of variables $(u, v)$ is

$$\left\{ \frac{\partial^2}{\partial u^2} - \frac{\partial^2}{\partial v^2} - \omega_1^2 u^2 + \omega_2^2 v^2 \right\} \Psi(u, v) = 0 \quad (28)$$

where $\omega_1^2 = -\lambda_+, \omega_2^2 = -\lambda_-$. It has oscillator-ghost-oscillator solutions belonging to the Hilbert space $\mathcal{H}(m_1, m_2)(L^2)$ as

$$\Psi^{(m_1, m_2)}(u, v) = \sum_{l=0}^{\infty} c_l \Phi^{(m_1, m_2)}_l(u, v) \quad (29)$$

with $m_1, m_2 \geq 0$ and $c_l \in \mathbb{C}$. The basis solutions $\Phi^{(m_1, m_2)}_l(u, v)$ are separable as

$$\Phi^{(m_1, m_2)}_l(u, v) = \alpha_{m_2+(2m_2+1)l}(u) \beta_{m_1+(2m_1+1)l}(v) \quad (30)$$

with normalized solutions

$$\alpha_n(u) = \left( \frac{\omega_1}{\pi} \right)^{1/4} \frac{\exp(-\omega_1 u^2)}{\sqrt{2^n n!}} H_n(\sqrt{\omega_1} u)$$

$$\beta_n(v) = \left( \frac{\omega_2}{\pi} \right)^{1/4} \frac{\exp(-\omega_2 v^2)}{\sqrt{2^n n!}} H_n(\sqrt{\omega_2} v) \quad (31)$$

where $H_n(x)$ are Hermite polynomials. Obtaining these solutions requires a quantization condition [2]

$$\frac{\lambda_+}{\lambda_-} = \left( \frac{2m_1 + 1}{2m_2 + 1} \right)^2 \quad (32)$$

which is imposed on the couplings in the scalar field potential. For a given pair of $(m_1, m_2)$ it is shown [2] by graphical analysis that the absolute value of the solutions (29) have maxima in the vicinity of classical loci (11) admitting a signature transition. This defines a definite correlation between classical loci and quantum solutions on the space of the $(u, v)$ variables. On the other hand, we have already shown that there are duality transformations on the couplings in a given scalar field potential giving rise to a dual potential such that the solutions of field equations on the $(R, \phi)$ configuration space transform to dual solutions $(\tilde{R}, \tilde{\phi})$ whereas the solutions on the $(u, v)$ configuration space remain unchanged. Applying the quantum cosmology discussed above to this picture, it turns out that for any pair of $(m_1, m_2)$ defining a distinct quantum cosmology in terms of the variables $(u, v)$, we may correspond dual classical solutions $(R, \phi)$ and $(\tilde{R}, \tilde{\phi})$ admitting signature transition from a Euclidean to a Lorentzian space-time. This is because starting separately with dual classical systems $U(\phi)$ and $\tilde{U}(\tilde{\phi})$ tends to the same quantization condition and the same quantum cosmology concentrated on both over the configuration space $(u, v)$. 

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In this respect, for a given pair \((m_1, m_2)\) we have a quantum cosmology with dual predictions: a large scale cosmology \(R(\beta)\) and an small scale cosmology \(\tilde{R}(\beta)\), both having an infinitesimal effective cosmological constant. Time independence of quantum cosmology \(\beta\) would let us to choose two different parametrization of classical loci (11) in terms of classical time \(\beta\) so that the large scale cosmology is an old signature changing universe and the small scale one being the microscopic fluctuation in space-time signature with a short life-time.

According to a common belief \([\text{4}]\), the microscopic fluctuations in space-time topology or the microscopic wormholes may play an important role in shifting the cosmological constant to zero. This idea may be realized, somehow, in the present model for two reasons: firstly, the phenomenon of dynamical space-time topology change may be accompanied by a dynamical signature change of space-time metric and in the present model ( see (23) ) we see that at small scale \(\tilde{R}(\beta)\), the large bare cosmological constant \(\tilde{\lambda}\) can only shift to zero if there exist a signature changing mechanism \([\text{1}]\); secondly, there are quantum wormholes corresponding to the matter field potentials \(U(\phi)\) and its dual \(\tilde{U}(\tilde{\phi})\). In both cases, it is easy to show that the resulting quantized model of signature transition may give exponential wave functions ( one with large variable \(R\) and the other with small variable \(\tilde{R}\) ) satisfying the Hawking-Page boundary conditions for the existence of quantum wormholes \([\text{3}]\). Indeed, because the Wheeler-DeWitt equation is independent of the lapse function \(\beta\), the Euclidean regime as well as the Lorentzian one is already included in the formalism. So the exponential solutions are to be expected as well as oscillating solutions. One should then interpret the large scale quantum wormholes as macroscopic and the small scale ones as microscopic. Therefore, the existence of microscopic quantum wormholes in this model in which the cosmological constant can be tuned to zero at small scale, may be a realization of the common belief mentioned above.

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\(^4\)Wheeler-DeWitt equation as zero energy schrodinger equation gives rise to time independent solutions.

\(^5\)A classical scalar field potential as \(U(\phi) = \frac{1}{2}m^2 \sinh^2 \frac{2}{3} \phi\) is also introduced by Hawking and Page \([\text{3}]\) giving rise to quantum wormholes. That potential is the special form of the present one \(U(\phi)\) with \(b = 0\).

\(^6\)The conditions are: Wave function should decay exponentially for large scale factor and that be well behaved when the scale factor goes to zero. An attempt to obtain the quantum wormholes in the context of classical signature change \([\text{3}]\) shows that for any choice of perfect fluid matter, there are no quantum wormhole solutions while it may be there some classical wormholes.
We have shown that in the context of signature transition in classical and quantum cosmology based on the models proposed by Dereli et al. one may find, by appropriate choice of the couplings in the scalar field potential, a set of duality transformations such that relate the dual classical solutions to each other, while a same quantum cosmology is concentrated equally on both of them. In this respect we may have a quantum cosmology correlated with dual classical predictions. The nice property of duality transformations, based on this correlation between quantum and classical cosmologies, makes an interrelation between “large scale-small scale” duality and “small coupling-large coupling” duality. Thanks to this interrelation we have shown that it is possible to fine-tune the physics at both large and small scales to have exactly a same infinitesimal effective cosmological constant. In other words: A choice of an infinitesimal cosmological constant at microscopic scales implies an infinitesimal cosmological constant at cosmic scales.

The interesting features of this model are:

1) There is no semi-classical approximation in relating the solutions of Wheeler-DeWitt equation to dual classical solutions [2], hence we are not worried about the breakdown of any semi-classical approximation at small scales except planck scale at which neither Einstein nor Wheeler-DeWit equations make sense. On the other hand, there is an alternative possibility: if one assumes this Large scale-small scale duality potentially fundamental, then it should connect the typical size of the universe to a dual “natural” size which is not necessarily of planck size. This will let us to avoid the need for a quantum gravity domain. Recently, such ideas has been the subject of intense investigations [3].

2) Potentially, there is an evidence of topology change through the existence of quantum wormholes in the context of classical signature change.

3) It is seen that the wave packets, constructed by the quantum wormhole solutions of the Wheeler-DeWitt equation peak in the vicinity of dual classical cosmologies with a nice property: having a self-dual fine-tunable effective cosmological constant. The fact that a self-dual quantum cosmology peaks about a self-dual fine-tunable effective cosmological constant is of particular importance since it explains that the resolution of the cosmological constant problem may be in the quantum cosmological considerations.

4) The fine-tuning of effective cosmological constant at large and small scales could lead to a deeper understanding concerning the origin of the arrow of time at large and small scales, since the fine-tuning mechanism here is based on the signature transition “from” Euclidean (no time) “to” Lorentzian (beginning of time) domains.

5) From equation (24) we find that for a chosen set of couplings \{\lambda, m^2, b\} and its dual set \{\tilde{\lambda}, \tilde{m}^2, \tilde{b}\} a self-dual effective cosmological constant is attributed and these dual sets may correspond to dual typical scales. Therefore, one may fine-tune the corresponding effective cosmological constant for any dual scales emerging from dual sets \{\lambda, m^2, b\} and \{\tilde{\lambda}, \tilde{m}^2, \tilde{b}\}. 

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