Time reversal invariance violation for high energy charged baryons in bent crystals

V.G. Baryshevsky

Research Institute for Nuclear Problems, Belarusian State University, 11 Bobruiskaya str., 220030, Minsk, Belarus

Abstract

Spin precession of channeled particles in bent crystals at the LHC gives unique possibility for measurements as electric and magnetic moments of charm, beauty and strange charged baryons and constants determining CP ($T_{\text{odd}}, P_{\text{odd}}$) violation interactions and $P_{\text{odd}}, T_{\text{even}}$ interactions of baryons with electrons and nucleus (nucleons). For a particle moving in a bent crystal a new effect caused by nonelastic processes arises: in addition to the spin precession around the direction of the effective magnetic field (bend axis), the direction of electric field and the direction of the particle momentum, the spin rotation to the mentioned directions also appears.

Crystal, bent crystal, charm baryon, electric dipole moment, spin rotation, parity violation, magnetic moment, CP violation.

1 Introduction

The spin rotation phenomenon of channelled particles, moving in a bent crystal, which was theoretically predicted in [1] and observed in [2–4], gives us the opportunity to measure anomalous magnetic moment of high energy short-lived particles. The appearance of beams with energies up to 7 TeV on LHC and further growth of particles’ energy and beams’ luminosity on FCC do essentially improve the possibility of using the phenomenon of spin rotation of the high energy particles in bent crystals and spin depolarization of such particles for measuring anomalous magnetic moments of positively charged, as well as neutral and negatively charged short-lived hyperons, and $\tau$-lepton [5–8]. The detailed analysis of conditions of the experiment on measuring magnetic dipole moment (MDM) of charm baryon $\Lambda_c^+$ on LHC, which has confirmed the possibility of measuring MDM of such baryon on LHC, was accomplished recently in [9]. Strong electric field affects the channelled particle in a bent crystal. As a consequence, the spin rotation phenomenon of the channelled particle allows to obtain information about the possible
value of the electric dipole moment of short-lived baryons, which elementary particles can obtain as a result of the violation of the T-invariance [10,11].

It should be noted, that besides electromagnetic interaction the channelled particle moving in a crystal experiences weak interaction with electrons and nuclei as well as strong interaction with nuclei. Mentioned interactions lead to the fact, that in the analysis of the particle’s spin rotation, caused by electric dipole moment interaction with electric field, both \(P_{\text{odd}}, T_{\text{even}}\) and \(P_{\text{odd}}, T_{\text{odd}}\) non-invariant spin rotation, resulting from weak interaction should be considered [12,13].

As obtained here, spin precession of channelled particles in bent crystals at the LHC gives unique possibility for measurements as electric and magnetic moments of charm, beauty and strange charged baryons and constants determining CP (\(T_{\text{odd}}, P_{\text{odd}}\)) violation interactions and \(P_{\text{odd}}, T_{\text{even}}\) interactions of baryons with electrons and nucleus (nucleons). A new effect arises for a particle moving in a bent crystal due to nonelastic processes, namely: along with the spin precession around three directions (the direction of the effective magnetic field (bend axis), the direction of electric field and the direction of the particle momentum) the spin rotation to the mentioned directions also appears.

### 2 Spin rotation and scattering particles in crystal

The spin rotation phenomenon for high-energy particles, moving in a bent crystal, as a result of quasi-classical motion of particles channelled in crystals, can be described by equations similar to those for motion of particles’ spin in the storage ring with the inner target [12,13]. The theory, which describes motion of the particle spin in electromagnetic fields in a storage ring, has been developed in many papers [14,15,23].

According to [14,15,23], the basic equation, which describes particle spin motion in an electromagnetic field, is the Thomas-Bargmann–Michel–Telegdi (T-BMT) equation. Refinement of the T-BMT equation, allowing us to consider the possible presence of the particle EDM, was made in [24,25].

Now let us consider a particle with spin \(S\) which moves in the electromagnetic field. The term “particle spin” here means the expected value of the quantum mechanical spin operator \(\hat{\mathbf{S}}\) (hereinafter the symbol marked with “hat” means a quantum mechanical operator). Further, movement of the high-energy particles in non-magnetic crystal will be considered. In this case magnetic field \(\mathbf{B} = 0\) and Lorentz-factor \(\gamma \gg 1\).
Spin motion is described by the Thomas–Bargmann–Michel–Telegdi equation (T-BMT) in this case as follows:

\[
\frac{d\vec{S}}{dt} = [\vec{S} \times \vec{\Omega}],
\]

(1)

\[
\vec{\Omega} = -\frac{e}{mc} a \left[ \vec{\beta} \times \vec{E} \right],
\]

(2)

where \( \vec{S} \) is the spin vector, \( t \) is the time in the laboratory frame, \( m \) is the mass of the particle, \( e \) is its charge, \( \gamma \) is the Lorentz-factor, \( \vec{\beta} = \vec{v}/c \), where \( \vec{v} \) denotes the particle velocity, \( \vec{E} \) is the electric field at the point of particle location in the laboratory frame, \( a = (g - 2)/2 \) and \( g \) is the gyromagnetic ratio (by definition, the particle magnetic moment \( \mu = (eg\hbar/2mc)S \), where \( S \) is the particle spin). The T-BMT equation describes the spin motion in the rest frame of the particle, wherein the spin is described by the three component vector \( \vec{S} \). In practice the T-BMT equation works well for the description of spin precession in the external electric and magnetic fields encountered in typical present–day accelerators. Study of the T-BMT equation allows us to determine the major peculiarities of spin motion in an external electromagnetic field. However, it should be taken into account that particles in an accelerator or bent crystal have an energy spread and move along different orbits. This necessitates one to average the spin–dependent parameters of the particle over the phase space of the particle beam. This is why one must always bear in mind the distinction between the beam polarization \( \vec{\xi} \) and the spin vector \( \vec{S} \). A complete description of particle spin motion can be made applying spin density matrices equation (in more details see [13, 26]).

If a particle possesses an intrinsic electric dipole moment, then the additional term, describing spin rotation induced by the EDM, should be added to (1):

\[
\frac{d\vec{S}_{\text{EDM}}}{dt} = \frac{d}{\hbar} \left[ \vec{S} \times \left\{ \vec{E} - \frac{\gamma}{\gamma + 1} \vec{\beta}(\vec{\beta} \vec{E}) \right\} \right],
\]

(3)

where \( d \) is the electric dipole moment of the particle.

As a result, the motion of particle spin due to the magnetic and electric dipole moments can be described by the following equation:

\[
\frac{d\vec{S}}{dt} = -\frac{e}{mc} a \left[ \vec{\beta} \times \left( \vec{\beta} \times \vec{E} \right) \right] + \\
+ \frac{d}{\hbar S} \left[ \vec{S} \times \left\{ \vec{E} - \frac{\gamma}{\gamma + 1} \vec{\beta}(\vec{\beta} \vec{E}) \right\} \right].
\]

(4)

Recall now, that electric field in a crystal is formed by atoms. Scattering on atoms leads to the fact, that the high-energy particle moving in a crystal
experience interaction from electric and magnetic fields. However it is not only the electromagnetic interaction that influence on the scattering. Particles also participate in strong and weak interactions with electrons and nuclei. The interactions, mentioned above, depend on the spin of colliding particles and therefore have effect on evolution of the spin of the particle moving in matter [13].

It would be recalled that the particle refractive index in matter formed by different scatterers has the form:

\[ n = 1 + \frac{2\pi N}{k^2} f(0), \]  

(5)

where \( N \) is the number of scatterers per \( cm^3 \) and \( k \) is the wave number of the particle incident on the target, \( f(0) \equiv f_{aa}(\vec{k}' - \vec{k} = 0) \) is the coherent elastic zero angle scattering amplitude. In this scattering, momentum of the scattered particle \( \vec{p}' = \hbar \vec{k}' \) (where \( \vec{k}' \) is a wave vector) equals to initial momentum \( \vec{p} = \hbar \vec{k} \). Atom (nucleus) that was in quantum state before interaction with the incident particle characterized by stationary wave function \( \Phi_a \) will stay in the same quantum state after interaction with the incident particle. If the energy of interaction between particle and a scatterer depends on spin of the particle, then scattering amplitude \( f(\vec{k}' - \vec{k}) \) can also depend on spin. As a consequence refractive index \( \hat{n} \) (symbol \( \hat{\ } \) means that mentioned magnitude is an operator in spin space of a particle) depends on spin as well [13].

If the matter is formed by different scatterers, then

\[ n = 1 + \frac{2\pi}{k^2} \sum_j N_j f_j(0), \]

(6)

where \( N_j \) is the number of j-type scatterers per \( cm^3 \), \( f_j(0) \) is the amplitude of the particle coherent elastic zero-angle scattering by j-type scatterer.

Let us consider a relativistic particle refraction on the vacuum-medium boundary (see [13]). The wave number of the particle in the vacuum is denoted \( k \). The wave number of the particle in the medium is \( \vec{k}' = kn \). As is evident the particle momentum in the vacuum \( \vec{p} = \hbar \vec{k} \) is not equal to the particle momentum in the medium. Therefore, the particle energy in the vacuum \( E = \sqrt{\hbar^2 k^2 c^2 + m^2 c^4} \) is not equal to the particle energy in the medium \( E_{med} = \sqrt{\hbar^2 k^2 n^2 c^2 + m^2 c^4} \).

The energy conservation law immediately requires the particle in the medium to have the effective potential energy \( U_{eff} \). This energy can be easily found from relation:

\[ E = E_{med} + U_{eff}, \]

(7)
i.e.

\[ U_{\text{eff}} = E - E_{\text{med}} = -\frac{2\pi\hbar^2}{m\gamma} N f(E, 0) = (2\pi)^3 NT_{aa}(\vec{k}' - \vec{k} = 0), \quad (8) \]

\[ f(E, 0) = -(2\pi)^2 \frac{E}{c^2\hbar^2} T_{aa}(\vec{k}' - \vec{k} = 0) = -(2\pi)^2 \frac{m\gamma}{\hbar^2} T_{aa}(\vec{k}' - \vec{k} = 0), \quad (9) \]

where \( T_{aa}(\vec{k}' - \vec{k} = 0) \) is the matrix element of the T-operator describing elastic coherent zero-angle scattering.

Let us remind that T-operator is associated with scattering matrix \( S \) \cite{15, 17}:

\[ S_{ba} = \delta_{ba} - 2\pi i \delta(E_b - E_a) T_{ba}, \quad (10) \]

where \( E_a \) is the energy of scattered particles before the collision, \( E_b \) - after the collision, matrix element \( T_{ba} \) corresponds to states \( a \) and \( b \) that refer to the same energy.

For the matter formed by different scatterers effective potential energy can be written as:

\[ U_{\text{eff}} = -\frac{2\pi\hbar^2}{m\gamma} \sum_j N_j f_j(E, 0). \quad (11) \]

Due to periodic arrangement of atoms in a crystal the effective potential energy is a periodic function of coordinates of a particle moving in a crystal \cite{13}:

\[ U(\vec{r}) = \sum_{\vec{\tau}} U(\vec{\tau}) e^{i\vec{\tau}\vec{r}}, \quad (12) \]

where \( \vec{\tau} \) is the reciprocal lattice vector of the crystal;

\[ U(\vec{\tau}) = \frac{1}{V} \sum_j U_j(\vec{\tau}) e^{i\vec{\tau}\vec{r}_j}, \quad (13) \]

here \( V \) is the volume of the crystal elementary cell, \( \vec{r}_j \) is the coordinate of the atom (nucleus) of type \( j \) in the crystal elementary cell.

\[ U_j(\vec{\tau}) = -\frac{2\pi\hbar^2}{m\gamma} F_j(\vec{\tau}), \quad (14) \]

According to \cite{14} effective potential energy \( U(\vec{\tau}) \) is determined by amplitude \( F_j(\vec{\tau}) = F_{jaa}(\vec{k}' - \vec{k} = \vec{\tau}) \). In contrast to the case of chaotic matter where effective potential energy is determined by the amplitude of elastic
coherent scattering \(f(\vec{k}' - \vec{k})\), here it is defined by the amplitude \(F(\vec{r})\) (see Annex and [13]), which can be written as:

\[
F_j(\vec{k}' - \vec{k}) = f_j(\vec{k}' - \vec{k}) - ik^4/4\pi \int f_j^* (\vec{k}'' - \vec{k}') f_j(\vec{k}'' - \vec{k}) d\Omega_{k''}. \tag{15}
\]

where \(d\Omega_{k''}\) means integration over all of the vector \(\vec{k}''\) directions, \(\mid \vec{k}' \mid = \mid \vec{k} \mid = \mid \vec{k}'' \mid\).

The occurrence of the amplitude \(F(\vec{k}' - \vec{k})\) instead of the amplitude of elastic coherent scattering \(f(\vec{k}' - \vec{k})\) in (10) is specified by the fact, that unlike of an amorphous matter, the wave elastically scattered in a crystal, due to rescattering by periodically located centers is involved in formation of a coherent wave propagating through the crystal.

As follows from (15):

\[
F_j(0) = f_j(0) - ik^4/4\pi \int f_j^* (\vec{k}'' - \vec{k}') f_j(\vec{k}'' - \vec{k}) d\Omega_{k''}. \tag{16}
\]

The integral in (16) is identical with the total cross-section of the elastic coherent scattering by nucleus (atom). According to optical theorem:

\[
Im f_j(0) = \frac{k}{4\pi} \sigma_{\text{tot}} = \frac{k}{4\pi} \sigma_{\text{elast}} + \frac{k}{4\pi} \sigma_{\text{nonelast}}. \tag{17}
\]

As we can see, unlike the matter where scatterers are spread chaotically in crystal, for the amplitude \(F_j(0)\) following expression is true

\[
F_j(0) = \tilde{f}_j(0) = f_j(0) - \frac{k}{4\pi} \sigma_{\text{elast}}. \tag{18}
\]

In other words, cross-section of elastic coherent scattering in crystal does not contribute to the imaginary part of the amplitude \(F_j(0)\). Imaginary part is determined by the cross-section of nonelastic processes (reaction cross-section) only:

\[
F_j(0) = Re F_j(0) + i Im F_j(0) = Re F_j(0) + i \frac{k}{4\pi} \sigma_{\text{nonelast}}. \tag{19}
\]

The situation is also similar for nonzero-angle scattering. It becomes clear when we use the equality that is correct for elastic scattering [16]:

\[
Im f_{\text{elast}}(\vec{k}' - \vec{k}) = \frac{k}{4\pi} \int f_{\text{elast}}^* (\vec{k}'' - \vec{k}') f_{\text{elast}}(\vec{k}'' - \vec{k}) d\Omega_{k''}. \tag{20}
\]

As a result, according to [15], we should subtract the elastic scattering contribution from the imaginary part of \(f(\vec{k}' - \vec{k})\). This can be evidently shown
if the interaction with scatterer can be considered in terms of the perturbation theory. In this case at the first Born approximation scattering amplitude \( f^{(1)}(\vec{k}' - \vec{k}) \) doesn’t have an imaginary part:

\[
\text{Im} f^{(1)}_{aa}(\vec{k}' - \vec{k}) = 0.
\]  

(21)

An imaginary part appears at the second Born approximation. Let’s remind that T-operator, determining the scattering amplitude (see (9)), satisfies the following equation [15, 17]:

\[
T = V + V_1 E - H_0 + i\eta T.
\]  

(22)

where \( V \) is the interaction energy, \( H_0 \) is the Hamilton operator of colliding systems located at great distance from each other.

As a result for the amplitude of the elastic coherent scattering \( f_{aa} \) with the accuracy up to second order terms over the interaction energy, we have:

\[
f_{aa}(\vec{k}' - \vec{k}) = -(2\pi)^2 \frac{m\gamma}{\hbar^2} \left( \Phi_{\vec{k}a} |V| \Phi_{ka} > + \Phi_{\vec{k'}a} |V \frac{1}{E_a(\vec{k}) - H_0 + i\eta} V| \Phi_{ka} > \right),
\]  

(23)

where \( \Phi_{ka} \) is an eigenfunction of Hamilton operator \( H_0 \),

\[
\Phi_{\vec{k}a} = \frac{1}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{r}} \Phi_a,
\]  

(24)

\( \Phi_a \) is a wave function of scatterer stationary states, \( H_0 \Phi_{\vec{k}a} = E_a(\vec{k}) \Phi_{\vec{k}a} \).

Using the completeness of the function \( \Phi_{\vec{k}a} \) and replacing ”1” in (23) by \( \sum_{k''b} |\Phi_{k'b}|^2 = 1, \) in (23) we obtain the sum over the intermediate states \( b \), which includes states with \( b = a \). This term contains the following expression:

\[
\frac{1}{E_a(\vec{k}) - E_a(\vec{k}'') + i\eta} - \frac{1}{E_a(\vec{k}) - E_a(\vec{k}'')} - i\pi \delta(E_a(\vec{k}) - E_a(\vec{k}'')).
\]  

(25)

The ” \( P \)” symbol in the real part of (25) means, that in (23) integrals containing the ” \( P \)” symbol are principal-value integrals.

This real part contribution to the first Born approximation is negligibly small therefore it will not be considered further. The imaginary unit in second term of (25), which is proportional to the \( \delta \) function, leads to occurrence of an imaginary part in amplitude \( f(\vec{k}' - \vec{k}) \). After substitution of the expression with \( \delta \) function into (23) it becomes obvious that the term in which sum \( b = a \)
is equivalent to the term subtracted from the amplitude $f_{aa}(\vec{k}' - \vec{k})$ in (15).

As a result only contributions caused by nonelastic processes and reactions with $b \neq a$ make contributions to the imaginary part of the amplitude in crystal. Further in expressions for $F$ amplitude $f_{aa}$ without the contribution of the elastic coherent scattering in the imaginary part will be considered. The second term in (15) also will not be written explicitly.

### 3 Effective potential energy of a spin-particle moving close to crystal planes (axes)

Suppose a high energy particle moves in a crystal at a small angle to the crystallographic planes (axes) close to the Lindhard angle. This motion determined by the plane (axis) potential $\tilde{U}(x)(\tilde{U}(\vec{r}))$, which can be determined from $\tilde{U}(\vec{r})$ by averaging over distribution of atoms (nuclei) in a crystal plane (axis). Similar result is obtained when all the terms with $\tau_y \neq 0, \tau_z \neq 0$ for the case of planes or $\tau_z \neq 0$ for the case of axes are removed from the sum (12).

As a consequence for the potential of periodically placed axes we can write:

$$U(\vec{r}) = \sum_{\vec{\tau}_z} U(\vec{\tau}_\perp, \vec{\tau}_z = 0) e^{i\vec{\tau}_\perp \vec{r}}$$

$z$-axis of the coordinate system is directed along the crystallographic axis. For the potential of a periodically placed planes we have:

$$U(x) = \sum_{\vec{\tau}_z} U(\tau_x, \tau_y = 0, \tau_z = 0) e^{i\tau_x x}$$

$y, z$-planes of the coordinate system are parallel to the chosen crystallographic planes family. Let’s remind that according to (13-14) the magnitude $U(\vec{r})$ is expressed in terms of the amplitude $F(\vec{r})$.

Since the amplitude $\hat{F}(\vec{k}' - \vec{k})$ depends on spin, the effective potential energy $\tilde{U}$ depends on a spin as well [13]. The magnitude $\tilde{U}$ is the operator in spin space of a particle incident on a crystal.

Elastic coherent scattering of a particle by an atom is caused by electromagnetic interaction of the particle with the atom electrons and nucleus as well as weak and strong nuclear interaction with electrons and nucleus. The general expression for an amplitude of elastic scattering of spin $\frac{1}{2}$ particles by a spinless or unpolarized nuclei can be written as:
\[ \hat{F}(\vec{q}) = A_{\text{coul}}(\vec{q}) + A_s(\vec{q}) + (B_{\text{magn}}(\vec{q}) + B_S(\vec{q}))\vec{\sigma}[\vec{n} \times \vec{q}] + \]
\[ + (B_{\text{we}}(\vec{q}) + B_{\text{wnuc}}(\vec{q}))\vec{\sigma}\vec{N}_w + \]
\[ + (B_{\text{EDM}}(\vec{q}) + B_{\text{T e}}(\vec{q}) + B_{\text{T nuc}}(\vec{q}))\vec{\sigma}\vec{q}, \]
\[ (28) \]

where \( \vec{q} = \vec{k}' - \vec{k}, \vec{n} = \frac{\vec{k}'}{k}, A_{\text{coul}}(\vec{q}) \) is the spin-independent part of the amplitude of elastic coulomb scattering of a particle by an atom (according to \( (26,27) \)) it leads to Coulomb potential energy of crystal planes and axes; \( A_s(\vec{q}) \) is the spin-independent part of a scattering amplitude, which is caused by strong interaction (similar contribution caused by weak interaction is negligibly small therefore it is omitted).

The spin-dependent amplitude, which is proportional to \( B_{\text{magn}}(\vec{q}) \), is determined by electromagnetic spin-orbit interaction. This amplitude is responsible for the effect of particle spin rotation in the electric field of a bent crystal, that is proportional to \((g - 2)\). The term proportional to \( B_s(\vec{q}) \) is responsible for the contribution of the spin-orbit strong interaction to a scattering process of a baryon by nucleus. This term leads to spin rotation caused by strong interaction.

The term proportional to the parity odd pseudo scalar \( \vec{\sigma}\vec{N}_w \) (unit vector \( \vec{N}_w = \frac{\vec{k} + \vec{k}'}{|k' + k|} \)) includes two contributions: a) Contribution to the amplitude proportional to \( B_{\text{we}}(\vec{q}) \), that describes elastic scattering caused by the parity violating weak interaction between baryon and electrons. b) Contribution to the amplitude proportional to \( B_{\text{wnuc}}(\vec{q}) \), that describes elastic scattering caused by the parity violating weak interaction between baryon and nucleus.

The term proportional to the time (T) violation (CP non-invariant) pseudo scalar \( \vec{\sigma}\vec{q} \) includes three contributions: a) Contribution proportional to \( B_{\text{EDM}}(\vec{q}) \), that describes elastic scattering of baryon with EDM by the atom's Coulomb field. This contribution leads to the term in \((4,26,27)\) describing baryon spin rotation in electric field of planes (axes) caused by EDM. b) Contribution proportional to \( B_{\text{T e}}(\vec{q}) \) describes possible short-distance T-non-invariant interaction between baryon and electrons. c) Contribution to the amplitude, which is proportional to \( B_{\text{T nuc}}(\vec{q}) \), describes scattering caused by T-non-invariant interaction between baryon and nucleons. This contribution also leads to spin rotation of a baryon moving in bent crystal.

Therefore it is also possible to examine T-non-invariant baryon-nucleon interaction (and short-range baryon-electron interaction) and obtain restrictions on the value of such interactions in experiments on measurements of charm and beauty baryons EDM.

Let’s express the amplitude \( \hat{F}(q) \) as Fourier transformation of function
\[ F(\vec{r}) : \]
\[ \hat{F}(\vec{q}) = \int \hat{F}(\vec{r}') e^{-i\vec{q}\vec{r}'} d^3r'. \]

\[ (29) \]

Considering mentioned above we can conduct summation of \( \tau_x \) and \( \tau_\perp \) in \( (26,27) \) using following expression:
\[ \sum_{\tau_x} e^{i\tau_x} = d_x \sum_l \delta(x - X_l), \]

\[ (30) \]

where \( d_x \) is the lattice period along axis \( x \); \( X_l \) are coordinates of \( l \) plane.

\[ \sum_{\tau_x,\tau_y} e^{i\tau_x,\tau_y} = d_x d_y \sum_l \delta(\vec{\rho} - \vec{\rho}_l), \]

\[ (31) \]

where \( \vec{\rho}_l \) is a coordinate of an axis, located in point \( \vec{\rho}_l \); \( d_x, d_y \) are lattice periods along axes \( x \) and \( y \).

As a result we obtain following expression for the effective potential energy of interaction between an incident particle and a plane (axis) (the lattice is assumed to consist of atoms of one kind):
\[ \hat{U}(x) = -\sum_{\tau_x} \frac{2\pi \hbar^2}{m\gamma V} \hat{F}(q_x = \tau_x, q_y = q_z = 0) e^{i\tau_x} = -\frac{2\pi \hbar^2}{m\gamma V d_y d_z} \hat{F}(x, q_y = q_z = 0), \]

\[ (32) \]

\[ \hat{U}(\vec{\rho}) = -\frac{2\pi \hbar^2}{m\gamma V} \sum_{\tau_x,\tau_y} \hat{F}(q_x = \tau_x, q_y = \tau_y, q_z = 0) e^{i\tau_x} = -\frac{2\pi \hbar^2}{m\gamma d_z} \hat{F}(\vec{\rho}, q_z = 0), \]

\[ (33) \]

\( d_z \) is the lattice period along the axis \( z \).

Let’s consider the expression for effective potential energy in detail. According to \( (26,32,33) \) contributions to the effective potential energy are caused by interactions of different types including short-range and long-range interactions. In the presence of several types of interaction, for describing their different contributions to the scattering amplitude as a result of their mutual influence on scattering process, it is convenient to separate scattering caused only by long-range interactions and present amplitude in following form:
\[ f(\vec{q}) = f_{\text{long}}(\vec{q}) + f_{\text{shortlong}}(\vec{q}), \]

\[ (34) \]
were used as an incident waves). For general scattering theory in the presence of several interactions see, for example, [15, 17].

The interactions mutual influence on a scattering amplitude can be easily followed with the help of perturbation theory. Let interaction energy $V$ a sum of several interactions: $V = \sum_i V_i$. Then at the first Born approximation scattering amplitude is a sum of scattering amplitudes caused by every interaction separately: $f = \sum_i f_i(V_i)$. But at the second Born approximation scattering amplitude is determined by squared interaction $V$, more precisely by the following expression (see [15–17])

$$V \frac{1}{E - H_0 - i\eta} V = \sum_p V_p \frac{1}{E - H_0 - i\eta} \sum_i V_i; \quad (35)$$

As we can see, equality (35) contains interference contributions to $f$ proportional to $V_p V_i$.

The coulomb amplitude, described by the first term in (28), leads to the usual expression for potential energy of interaction between a charged particle and a plane (axis).

The second term $A_s(\vec{q})$ is caused by short-range interaction. Amplitude $A_s(\vec{q})$ can be written as:

$$A_s(\vec{q}) = A_{nuc}(\vec{q})\Phi_{osc}(\vec{q}), \quad (36)$$

where $A_{nuc}(\vec{q})$ is the spin independent part of the amplitude of elastic scattering by the resting nucleus, $\Phi_{osc}(\vec{q})$ is the form-factor caused by nucleus oscillations in crystal.

Owing to the short-range kind of strong interactions amplitude $A_{nuc}(\vec{q})$ is equal to zero-angle scattering amplitude $A(0)$ in scattering angles range $\vec{q} \leq \frac{1}{kR_{osc}} \ll 1$.

Form-factor $\Phi_{osc}(\vec{q})$ has the form [16]:

$$\Phi_{osc}(\vec{q}) = \sum_n \rho_n <\varphi_n(r)|e^{-i\vec{q}\vec{r}}|\varphi_n(r)> = \int e^{-i\vec{q}\vec{r}}N_{nuc}(\vec{r})d^3r, \quad (37)$$

where $\varphi_n(r)$ is the wave function describing vibrational state of nuclei in crystal, summation $\sum_n \rho_n$ means statistical averaging with Gibbs distribution over vibrational states of nucleus in crystal. Lets remind, that squared form-factor $\Phi_{osc}(\vec{q})$ is equal to Debye-Waller factor, $N_{nuc}(\vec{r})$ is a probability density of vibrating nuclei detection in point $\vec{r}$, $\int N_{nuc}(\vec{r})d^3r = 1$.

As a result, according to (32), this contribution to effective potential plane energy can be written as:

$$U_{nuc}(x) = -\frac{2\pi\hbar^2}{m\gamma_d\gamma d_z}N_{nuc}(x)A_{nuc}(0), \quad (38)$$
where \( N_{\text{nuc}}(x) = \int \int N_{\text{nuc}}(x, y', z') dy'dz' \) is the probability density of vibrating nuclei detection in point \( x \) (in direction orthogonal to the chosen crystallographic plane).

Similarly, for the axis we have:

\[
U(\vec{\rho}) = -\frac{2\pi \hbar^2}{m\gamma d_z} N_{\text{nuc}}(\vec{\rho}) A_{\text{nuc}}(0), \tag{39}
\]

where \( N_{\text{nuc}}(\vec{\rho}) = \int N_{\text{nuc}}(\vec{\rho}, z') dz' \).

### 4 Effective potential energy determined by the anomalous magnetic moment

According to (28) the scattering amplitude, which is determined by baryons magnetic moment, has the form:

\[
\hat{F}_{\text{magn}}(q) = B_{\text{magn}}(q) \vec{\sigma}[\vec{n} \times \vec{q}]. \tag{40}
\]

Defining the scattering amplitude one could at first consider only magnetic scattering and its interference with coulomb scattering (see (34)), and then add the term caused by interference between magnetic and nuclear interactions.

For the first case perturbation theory can be used. In the first order of perturbation theory interference between the magnetic moment scattering by the coulomb field and the coulomb scattering of baryon electric charge by the coulomb field is absent. The amplitude \( \hat{F}^{(1)} \) can be written as:

\[
\hat{F}^{(1)}_{\text{magn}}(q) = i f_{\text{coul}}(q) \frac{\hbar}{mc} \left( g - \frac{2}{2} \right)^{1/2} \vec{\sigma}[\vec{n} \times \vec{q}], \tag{41}
\]

where \( f_{\text{coul}}(q) \) is the amplitude of coulomb scattering of a baryon by an atom in the first Born approximation; \( \vec{n} = \frac{\vec{k}}{E} \), \( m \) is the baryon mass.

It should be noted that amplitude \( F_{\text{magn}}(q) \) is a pure imaginary quantity. After substitution of (41) into (32) and summation over \( \tau_x \) one obtains the expression for effective interaction energy:

\[
\hat{U}_{\text{magn}}(x) = -\frac{e\hbar}{2mc} \frac{g - 2}{2} \vec{\sigma}[\vec{E}_{\text{plane}}(x) \times \vec{n}], \tag{42}
\]

where \( \vec{E}_{\text{plane}}(x) \) denotes the electric field, produced by the crystallographic plane in point \( x \). In axis case \( U_{\text{magn}}(\vec{\rho}) \) can be obtained by replacement of \( x \) by \( \vec{\rho} \) in (42) and \( \vec{E}_{\text{plane}}(x) \) by \( \vec{E}_{\text{axis}}(\vec{\rho}) \).
Using (42) and Heisenberg equations for spin operator, the motion equation for polarization vector (12) for the case of \(B = 0\) and \(\gamma >> 1\) can be obtained.

The equation (42) can be represented as:

\[
\hat{U}_{\text{magn}} = -\frac{e\hbar}{2mc} g - \frac{2}{2} E_{\text{plane}}(x) \vec{\sigma} \vec{N},
\]

where \(\vec{N} = [\vec{n}_x \times \vec{n}]\) is the unit vector, \(\vec{n}_x \perp \vec{n}\), direction of the unit vector \(\vec{n}\) is parallel to the crystallographic plane.

The expression (43) for the effective potential energy is purely real. However, the scattering amplitude \(\hat{F}(\vec{q})\) has both real and imaginary parts. Due to this fact, the effective potential energy \(\hat{U}\) also has real and imaginary parts.

In the second order of perturbation theory the amplitude \(\hat{F}(\vec{q})\) is not purely imaginary, but also has a real part. By means of (15, 23-25) the following expression for the contribution \(\tilde{F}^{(2)}(\vec{q})\) to the amplitude \(\hat{F}(\vec{q})\) can be obtained:

\[
\tilde{F}^{(2)}(\vec{q} = \vec{\tau}) = i \frac{k}{4\pi\hbar^2 c^2} \left\{ \Phi_a \left[ \int \varepsilon_\perp z \right] \right\}^2 d^2 r_\perp = \left\{ \int \Phi_a \left[ \int \varepsilon_\perp z \right] \right\}^2 d^2 r_\perp
\]

where \(\varepsilon_\perp z = \varepsilon_{\text{coul}}(\vec{r}_\perp, z) + \varepsilon_{\text{magn}}(\vec{r}_\perp, z)\),

\(\varepsilon_{\text{magn}}(\vec{r}_\perp, z) = -\mu_a \vec{\sigma}[\vec{E}(\vec{r}_\perp, z) \times \vec{n}]\), \(z\) axis of the coordinate system is directed along the unit vector \(\vec{n}\), \(\vec{n}\) is the unit vector directed along the particle momentum before scattering \(\hbar k\), \(\mu_a\) is the anomalous magnetic moment of the particle \(\mu_a = \frac{e\hbar}{2mc} \left( g - \frac{2}{2} \right)\).

When deriving (44), it was considered that the particle energy is much greater than the electrons’ binding energy in atoms and the atoms’ binding
energy in crystal. As a result it is possible at first to examine scattering by electrons and nuclei, which rest in points \( r_1, r_2, \ldots \), and then to average the result over electrons and nuclei positions with wave functions \(|\Phi_a\rangle\) (impulse approximation, for example see [15]). The line in (44) denotes such kind of averaging. The contribution caused by interference between magnetic and nuclear scattering, and possible contributions determined by the particle magnetic moment should complete the expression mentioned above. In the case of positively charged particles, moving far from the top of the potential barrier, the contribution caused by interactions with nuclei is suppressed and will not be considered in detail below. After substitution of (44) into (32) and summation over \( \tau \) the following expression for the contribution to the effective potential energy caused by the amplitude \( \tilde{f}_{\text{magn}}(\vec{x}) \) can be obtained:

\[
\hat{U}^{(2)}_{\text{magn}}(\vec{x}) = -\frac{i}{16} \int dy dz \left( g - \frac{2}{2}\right) \partial_x \delta V^2(\vec{x}) \vec{\sigma} \vec{N},
\]

where \( \vec{N} = [\vec{r}_x \times \vec{r}_y, \vec{r}_z] \), \( \vec{r}_x \perp \vec{r}, \vec{r}_z \) is the unit vector along axis \( x \), \( \delta V^2(\vec{x}) = \int \left[ \int V_{\text{coul}}(\vec{x}, y, z) \right]^2 - \left[ \int V_{\text{coul}}(\vec{x}, y, z) \right]^2 \] dy.

For the axisymmetric case:

\[
\hat{U}^{(2)}_{\text{magn}}(\rho) = -\frac{i}{16} \int dz \left( g - \frac{2}{2}\right) \partial_\rho \delta V^2(\rho) \vec{\sigma} \vec{n}_\rho \times \vec{n},
\]

where \( \vec{n}_\rho = [\rho \times \vec{n}, \vec{n}] \), \( \vec{n} \perp \vec{n}, \vec{n} \) is the unit vector, \( \delta V^2(\rho) = \int \left[ \int V_{\text{coul}}(\rho, z) \right]^2 - \left[ \int V_{\text{coul}}(\rho, z) \right]^2 \] dz.

Below the case of planar channeling will be considered. According to (43), (45) the interaction between anomalous magnetic moment and plane can be written as:

\[
\hat{U}_{\text{magn}}(x) = \hat{U}^{(1)}_{\text{magn}}(x) + \hat{U}^{(2)}_{\text{magn}}(x) = \left( g - \frac{2}{2}\right) \vec{\sigma} \left[ \vec{E}_{\text{plane}} \times \vec{n} \right] - \frac{i}{16} \int dy dz \left( g - \frac{2}{2}\right) \partial_x \delta V^2(\vec{x}) \vec{\sigma} \vec{N},
\]

where \( \vec{N} = [\vec{r}_x \times \vec{r}_y, \vec{r}_z] \), \( \vec{r}_x \perp \vec{n}, \vec{r}_z \) is the unit vector along axis \( x \), \( \delta V^2(\vec{x}) \) denotes such kind of averaging. The contribution caused by interference between magnetic and nuclear scattering, and possible contributions determined by the particle magnetic moment should complete the expression mentioned above. In the case of positively charged particles, moving far from the top of the potential barrier, the contribution caused by interactions with nuclei is suppressed and will not be considered in detail below. After substitution of (44) into (43) the following expression for the contribution to the effective potential energy caused by the amplitude \( f_{\text{magn}}(\vec{x}) \) can be obtained:

\[
\hat{U}^{(1)}_{\text{magn}}(x) = -\frac{e \hbar}{2 \mu_c} \left( g - \frac{2}{2}\right) \vec{\sigma} \left[ \vec{E}_{\text{plane}} \times \vec{n} \right] - \frac{1}{16} \int dy dz \left( g - \frac{2}{2}\right) \partial_x \delta V^2(\vec{x}) \vec{\sigma} \vec{N},
\]

where \( \vec{N} = [\vec{r}_x \times \vec{r}_y, \vec{r}_z] \), \( \vec{r}_x \perp \vec{n}, \vec{r}_z \) is the unit vector along axis \( x \), \( \delta V^2(\vec{x}) = \int \left[ \int V_{\text{coul}}(\vec{x}, y, z) \right]^2 - \left[ \int V_{\text{coul}}(\vec{x}, y, z) \right]^2 \] dy.

Similarly for the case of axis it can be obtained:

\[
\hat{U}^{(2)}_{\text{magn}}(\rho) = -\frac{e \hbar}{2 \mu_c} \left( g - \frac{2}{2}\right) \vec{\sigma} \left[ \vec{n}_\rho \times \vec{n} \right] - \frac{1}{16} \int dz \left( g - \frac{2}{2}\right) \partial_\rho \delta V^2(\rho) \vec{\sigma} \vec{n}_\rho \times \vec{n},
\]

and for the axisymmetric case:

\[
\hat{U}^{(2)}_{\text{magn}}(\vec{r}) = -\frac{e \hbar}{2 \mu_c} \left( g - \frac{2}{2}\right) \vec{\sigma} \left[ \vec{E}_{\text{plane}} \times \vec{n} \right] - \frac{1}{16} \int dy dz \left( g - \frac{2}{2}\right) \partial_x \delta V^2(\vec{x}) \vec{\sigma} \vec{N},
\]
It will be shown herein, that the first term leads to spin rotation around $\vec{N}$, whereas the second term results in spin rotation in direction of $\vec{N}$.

5 Effective potential energy $\hat{U}$ determined by spin-orbit interaction

According to (28) the part of the scattering amplitude caused by the strong spin-orbit interaction has the form:

$$\hat{F}_{\text{ssp-orb}}(\vec{q}=\vec{\tau}) = B_s(\vec{\tau}) \sigma[\vec{n} \times \vec{n}] \sigma_\tau.$$  \hfill (49)

The coefficient $B_s(\vec{\tau})$ can be expressed similar to (36) as follows:

$$B_s(\vec{\tau}) = B_{\text{nuc}}(\vec{\tau}) \Phi_{\text{osc}}(\vec{\tau}),$$  \hfill (50)

where $B_{\text{nuc}}(\vec{\tau})$ describes scattering by a resting nucleus, $\Phi_{\text{osc}}(\vec{\tau})$ is the form-factor determined by nucleus oscillations in crystal.

In the considered case, similar to the approach used when deriving (38), the short-range character of the nuclear forces and small (as compared with the amplitude of nucleus oscillations) nucleus radius enables assumption $B_{\text{nuc}}(\vec{\tau}) \approx B_{\text{nuc}}(0)$. It is important that the coefficient $B_{\text{nuc}}(0)$ has both real and imaginary parts:

$$B_{\text{nuc}}(0) = B'_{\text{nuc}} + iB''_{\text{nuc}}.$$  \hfill (51)

This is similar to the case of amplitude, which describes scattering of the magnetic moment by the atom (nucleus). To obtain the expression for the effective potential energy the summation over $\tau_x$ should be conducted in (32). The resulted expression is similar to that for $\hat{U}_{\text{magn}}$. For example, for the plane case:

$$\hat{U}_{\text{ssp-orb}} = -\frac{2\pi\hbar^2}{m\gamma d_y d_z} B_{\text{nuc}}(0) (-i)\sigma\vec{n} \frac{\partial}{\partial x} N_{\text{nuc}}(x) =$$

$$= -\frac{2\pi\hbar^2}{m\gamma d_y d_z} (B'' - iB') \frac{\partial N_{\text{nuc}}}{\partial x} \sigma[\vec{n} \times \vec{n}] =$$

$$= \frac{2\pi\hbar^2}{m\gamma d_y d_z} \frac{\partial N_{\text{nuc}}}{\partial x} B'' \vec{\sigma} \vec{N} - i \frac{2\pi\hbar^2}{m\gamma d_y d_z} B' \frac{\partial N_{\text{nuc}}}{\partial x} \vec{N}. \hfill (52)$$

Finally

$$\hat{U}_{\text{ssp-orb}} = -(\alpha_s + i\delta_s) \sigma \vec{N}, \hfill (53)$$
where

\[ \vec{N} = [\vec{n}_x \times \vec{n}], \]
\[ \alpha_s = -\frac{2\pi\hbar^2}{m\gamma d_y d_z} \frac{\partial N_{\text{nuc}}}{\partial x} B', \]
\[ \delta_s = \frac{2\pi\hbar^2}{m\gamma d_y d_z} B' \frac{\partial N_{\text{nuc}}}{\partial x}. \]

As is shown below, similar to the magnetic contribution, the first term, which is pure real, leads to rotation of spin around \( \vec{N} \), while the second pure imaginary term results in spin rotation in the direction parallel or antiparallel to vector \( \vec{N} \). Let us remind that the contribution determined by elastic scattering, which is described by the second term in (16), should be subtracted from the expression for the amplitude. However at high energies this contribution is negligibly small, in comparison with nonelastic contributions to the amplitude, and therefore can be omitted.

6 Effective potential energy \( \hat{U} \) determined by P-odd and T-even interactions

The next group of terms, which are proportional to \( B_w \), is determined by weak P-odd and T-even interactions. According to (28), corresponding terms in the scattering amplitude can be written as:

\[ \tilde{F}_w(\vec{q}) = (B_{we}(\vec{q}) + B_{wnuc}(\vec{q}))\vec{\sigma}\vec{N}_w. \]  \hspace{1cm} (54)

Contribution \( B_{we}(\vec{q}) \) caused by parity violating weak interaction between the baryon and electrons can be expressed as:

\[ B_{we}(\vec{q}) = \hat{B}_{we}(\vec{q})\Phi_e(\vec{q}), \]  \hspace{1cm} (55)

where \( \hat{B}_{we} \) is the coefficient defining baryon elastic scattering amplitude by resting electron \( \hat{f}_{we}(q) = \hat{B}_{we}\vec{\sigma}\vec{N}_w, \Phi_e(\vec{q}) = \int e^{-i\vec{q}\vec{r}}N_e(\vec{r})d^3r = Z, \) \( Z \) is the nucleus charge. Minor corrections caused by thermal oscillations of atoms centers of gravity will not be considered further. To take them into consideration one should multiply \( \Phi_e(\vec{q}) \) by \( \Phi_{osc}(\vec{q}) \), which is the form-factor defined by oscillations of atoms nucleus.

Term \( B_{wnuc}(\vec{q}) \) (see (51)), which caused by parity violating weak interaction between the baryon and nucleus can take the following form:

\[ B_{wnuc}(\vec{q}) = \hat{B}_{wnuc}(\vec{q})\Phi_{osc}(\vec{q}), \]  \hspace{1cm} (56)
where \( \tilde{B}_{\text{wnuc}} \) is the coefficient defining baryon elastic scattering amplitude by resting nucleus \( \tilde{f}_{\text{wnuc}} = \tilde{B}_{\text{wnuc}} \tilde{\sigma} \tilde{N}_w \).

Due to the short-range character of P-violating interactions coefficients \( \tilde{B}_{\text{we}}(\vec{q}) \approx \tilde{B}_{\text{we}}(0) \) and \( \tilde{B}_{\text{wnuc}}(\vec{q}) \approx \tilde{B}_{\text{wnuc}}(0) \) at angles \( \theta \approx \frac{\tau}{k} \ll 1 \). As a result following expressions can be obtained for the effective potential energy of interaction with a crystallic plane (axis), which is caused by P-violating interactions \( \hat{U}_w \)

\[ \hat{U}_w = \hat{U}_{\text{we}} + \hat{U}_{\text{wnuc}} \quad (57) \]

a) for the case of plane:

\[ \hat{U}_{\text{we}}(x) = -\frac{2\pi \hbar^2}{m\gamma dydz} \tilde{B}_{\text{we}}(0) N_e(x) \tilde{\sigma} \tilde{N}_w, \]
\[ \hat{U}_{\text{wnuc}}(x) = -\frac{2\pi \hbar^2}{m\gamma dydz} \tilde{B}_{\text{wnuc}}(0) N_{\text{nuc}}(x) \tilde{\sigma} \tilde{N}_w, \]
\[ N_{\text{e(nuc)}}(x) = \int N_{\text{e(nuc)}}(x,y,z) dydz. \quad (58) \]

b) for the case of axis:

\[ \hat{U}_{\text{we}}(\vec{\rho}) = -\frac{2\pi \hbar^2}{m\gamma dz} \left\{ \tilde{B}_{\text{we}}(0) N_e(\vec{\rho}) + \tilde{B}_{\text{wnuc}}(0) N_{\text{nuc}}(\vec{\rho}) \right\} \tilde{\sigma} \tilde{N}_w, \]
\[ N_{\text{e(nuc)}}(\vec{\rho}) = \int N_{\text{e(nuc)}}(\vec{\rho},z) dz. \quad (59) \]

Thus:

\[ \hat{U}_w(x) = \hat{U}_{\text{we}}(x) + \hat{U}_{\text{wnuc}}(x) = -(\alpha_w(x) + i\delta_w(x)) \tilde{\sigma} \tilde{N}_w, \quad (60) \]

where

\[ \alpha_w(x) = \alpha_{\text{we}}(x) + \alpha_{\text{wnuc}}(x), \]
\[ \delta_w(x) = \delta_{\text{we}}(x) + \delta_{\text{wnuc}}(x), \]

\[ \alpha_w(x) = \frac{2\pi \hbar^2}{m\gamma dydz} (\tilde{B}_{\text{we}}'(0) N_e(x) + \tilde{B}_{\text{wnuc}}'(0) N_{\text{nuc}}(x)), \]
\[ \delta_w(x) = \frac{2\pi \hbar^2}{m\gamma dydz} (\tilde{B}_{\text{we}}''(0) N_e(x) + \tilde{B}_{\text{wnuc}}''(0) N_{\text{nuc}}(x)). \]
7 Effective potential energy $\hat{U}$ determined by the electric dipole moment and other T-nonivariant interactions

Let us consider now the electric dipole moment and other T-nonivariant contributions to the spin rotation. According to (28), corresponding terms in the scattering amplitude can be written as:

$$\hat{F}(q) = (B_{EDM}(q) + B_{Te}(q) + B_{T\text{nucl}}(q))\hat{\sigma}\vec{q}. \quad (61)$$

Let us consider the term $\hat{F}_{EDM}(\vec{q}) = B_{EDM}(\vec{q})\hat{\sigma}\vec{q}$. The coefficient $B_{EDM}(q)$ has both real and imaginary parts $B_{EDM}(q) = B_{EDM}' + iB_{EDM}''$. By the approach used for deriving $\hat{F}_{magn}(q)$, for $\hat{F}_{EDM}(\vec{q})$ we obtain:

$$\hat{F}_{EDM}(\vec{q}) = -\frac{\hbar^2}{2\pi} V_{\text{coul}}(\vec{q})\hat{\sigma}\vec{q} +$$

$$+ \frac{k}{4\pi\hbar^2 c^2} \int e^{-i\vec{q}\cdot\vec{r}} \left\{ \left[ \int \vec{V}(\vec{r}, z) dz \right]^2 - \left[ \int \vec{V}(\vec{r}, z) dz \right]^2 \right\} d^2r, \quad (62)$$

where $\vec{V}(\vec{r}) = V_{\text{coul}}(\vec{r}) + V_{EDM}(\vec{r})$, $V_{EDM} = -D\hat{\sigma}\vec{E}$ is the energy of interaction between the electric dipole moment $D$ and the electric field $\vec{E}$, $D = ed$, $e$ is the electric charge of the particle.

Using (32), the following expression for the potential energy of interaction between the particle and the plane can be obtained:

$$\hat{U}_{EDM} = -edE_{pl}(x)\hat{\sigma}\vec{N}_T - i \frac{d}{2d}\frac{\partial}{\partial x} \delta V^2(x)\hat{\sigma}\vec{N}_T, \quad (63)$$

where the unit vector $\vec{N}_T$ is orthogonal to the plane, $E_{pl}(x) = E_x\vec{N}_T$.

Evidently, $\hat{U}_{EDM}$ can be written as:

$$\hat{U}_{EDM} = -(\alpha_{EDM} + i\delta_{EDM})\hat{\sigma}\vec{N}_T. \quad (64)$$

Similar to $\hat{U}_{magn}$, the energy $\hat{U}_{EDM}$ has both real and imaginary parts. The expression for $U_{magn}$ converts to $\hat{U}_{EDM}$ when replacing $\frac{g-2}{2} \rightarrow 2\frac{d}{\lambda_c} (\lambda_c = \frac{\hbar}{mc}$ is the Compton wave-length of the particle) and $\vec{N} \rightarrow \vec{N}_T$. Therefore:

$$\frac{U_{EDM}}{U_{magn}} = \frac{4d}{\lambda_c(g-2)} = \frac{ed}{\mu_A} = \frac{D}{\mu_A}, \quad (65)$$

18
As is expected, the above expression is equal to the ratio of electric dipole moment to anomalous magnetic moment.

Let’s remind that amplitude $\hat{F}_T(\hat{q})$ contains both terms caused by EDM and those determined by short-range T-noninvariant interactions between the baryon and electrons and nuclei $B_{Te}(\hat{q})$ and $B_{Tnuc}(\hat{q})$. Contribution caused by these terms should also be added to the effective potential energy of the interaction between baryon spin and nuclei of the crystal $\hat{U}_T(x)$:

$$\hat{U}_T(x) = \hat{U}_{EDM} + \hat{U}_{Te} + \hat{U}_{Tnuc} = -(\alpha_T(x) + i\delta_T(x))\hat{\sigma}\hat{\bar{N}}_T,$$  
(66)

where $\alpha_T = \alpha_{EDM} + \alpha_{Te} + \alpha_{Tnuc}$, $\delta_T = \delta_{EDM} + \delta_{Te} + \delta_{Tnuc}$.

Expressions for coefficients $\alpha_{Te(nuc)}$ and $\delta_{Te(nuc)}$ can be evaluated in terms of scattering amplitude by the following way. Let’s define the form-factor determined by electrons distributions in atom and nucleus oscillations.

$$B_{Te}(\hat{q}) = \tilde{B}_{Te}(\hat{q})\Phi_e(\hat{q}), B_{Tnuc}(\hat{q}) = \tilde{B}_{Tnuc}(\hat{q})\Phi_{osc}(\hat{q}),$$  
(67)

where $\Phi_e(\hat{q}) = \int e^{-i\hat{q}\hat{r}}N_e(\hat{r})d^3r$, $N_e(\hat{r})$ is electrons density distribution in atom, $\int N_e(\hat{r})d^3r = Z$, $Z$ is the nucleus charge, $\Phi_{osc}(\hat{q})$ is determined by (37), $\tilde{B}_{Te}$ is the coefficient defining baryons scattering amplitude by resting electron $\hat{f}_{Te} = \tilde{B}_{Te}(\hat{q})\hat{\sigma}\hat{\bar{q}}$, $\tilde{B}_{nuc}(\hat{q})$ is the coefficient defining baryons scattering amplitude by resting nucleus $\hat{f}_{nuc} = \tilde{B}_{nuc}(\hat{q})\hat{\sigma}\hat{\bar{q}}$. Let’s remind that in compliance with (15) the contribution caused by elastic coherent scattering should be subtracted from the amplitude $B_T$.

Due to the short-range character of T-noninvariant interactions coefficients $\tilde{B}_{Te}(\hat{q}) \simeq \tilde{B}_{Te}(0)$ and $\tilde{B}_{nuc}(\hat{q}) \simeq \tilde{B}_{nuc}(0)$ at angles $\vartheta \simeq \frac{\vartheta}{k} << 1$. As a result following expressions can be obtained:

$$\hat{U}_{Te}(x) = i\frac{2\pi\hbar^2}{m\gamma d_yd_z}\tilde{B}_{Te}(0)\frac{dN_e(x)}{dx}\hat{\sigma}\hat{\bar{N}}_T,$$

$$\hat{U}_{Tnuc}(x) = i\frac{2\pi\hbar^2}{m\gamma d_yd_z}\tilde{B}_{Tnuc}(0)\frac{dN_{nuc}(x)}{dx}\hat{\sigma}\hat{\bar{N}}_T,$$

$$N_{e(nuc)}(x) = \int N_{e(nuc)}(x,y,z)dydz,$$  
(68)

Coefficients $\tilde{B}_{Te}(0)$ and $\tilde{B}_{Tnuc}(0)$ are complex values:

$$\tilde{B}_{Te(nuc)}(0) = \tilde{B}_{Te(nuc)}' + i\tilde{B}_{Te(nuc)}''.$$

As a result we have:

$$\hat{U}_{Te(nuc)}(x) = -(\alpha_{Te(nuc)} + i\delta_{Te(nuc)})\hat{\sigma}\hat{\bar{N}}_T;$$  
(69)
where
\[ \alpha_{Te(nuc)} = \frac{2\pi\hbar^2}{m\gamma dydz} \tilde{B}_{Te(nuc)}(nuc) \frac{dN_e(nuc)(x)}{dx}, \delta_{Te(nuc)} = \frac{2\pi\hbar^2}{m\gamma dydz} \tilde{B}_{Te(nuc)}(nuc) \frac{dN_e(nuc)(x)}{dx}. \]

Thus in the experiment aimed to obtain the limit for the EDM value, the limits for the scattering amplitude, which is determined by T(CP)-noninvariant interactions between baryons and electrons, and nuclei, will be found as well. The obtained values of these amplitudes for different interaction types allows one to restore the values of corresponding constants, too. The simplest model for such a potential is the Yukawa potential [28]. Using it all the equations for \( \alpha_{Te(nuc)} \) can be obtained by replacement in (44) of \( V_{coul} + V_{EDM} \) by \( V_{coul} + V_T, V_T = -d_T \vec{r} \vec{\sigma} \vec{r} \). Here \( d_T \) is the interaction constant, \( \varkappa_T \sim \frac{1}{M_T} \), where \( M_T \) is the mass of heavy particles, exchange of which leads to interaction \( V_T \) [28]. It should be noted that constant \( d_T \) for interaction between the heavy baryon and the nucleon can be greater than that for nucleon-nucleon interaction. This effect can be explained by the reasoning similar to that explaining expected EDM growth for the heavy baryon. T-odd interaction mixes baryon stationary states with different parity more effectively that occurs due to probably smaller spacing between energy levels corresponding to these states.

8 P and CP violating spin rotation in bent crystals

Expressions for the energy of interaction between the baryon and the plane(axis) that were obtained above, allow us to find the equation describing evolution of the particle polarization vector in a bent crystal. The mentioned equations differ from those, which describe evolution of the spin in external electromagnetic fields in vacuum, by the presence of terms defining the contribution of P and T(CP) noninvariant interactions between electrons and nuclei to the spin rotation.

These equations can be obtained by the following approach [13]. The spin wave function \( |\Psi(t)\rangle \) meets the equation as follows:

\[ i\hbar \frac{\partial |\Psi(t)\rangle}{\partial t} = \hat{U}_{eff} |\Psi(t)\rangle. \]  

(70)

The baryon polarization vector \( \vec{\xi} \) can be found via \( |\Psi(t)\rangle \):

\[ \vec{\xi} = \frac{\langle \Psi(t) | \vec{\sigma} | \Psi(t) \rangle}{\langle \Psi(t) | \Psi(t) \rangle}. \]  

(71)
Thus the equation for spin rotation of a particle \((\gamma >> 1)\), which moves in a bent crystal, reads as follows:

\[
\frac{d\vec{\xi}}{dt} = -\frac{e(g - 2)}{2mc}\left[\vec{\xi} \times [\vec{n} \times \vec{E}]\right] - \frac{2}{\hbar}\delta_m\{\vec{N}_m - \vec{\xi}(\vec{N}_m\vec{\xi})\} - \\
- \frac{2}{\hbar}\alpha_{s0}\{\vec{\xi} \times \vec{N}_m\} - \frac{2}{\hbar}\delta_{s0}\{\vec{N}_m - \vec{\xi}(\vec{N}_m\vec{\xi})\} + \\
+ \frac{2ed}{\hbar}\{\vec{\xi} \times \vec{E}\} + \frac{2}{\hbar}\delta_{EDM}\{\vec{N}_T - \vec{\xi}(\vec{N}_T\vec{\xi})\} + \\
+ \frac{2}{\hbar}\alpha_{T e}(\vec{\xi} \times \vec{N}_T) + \frac{2}{\hbar}(\delta_{T e} + \delta_{T nuc})\{\vec{N}_T - \vec{\xi}(\vec{N}_T\vec{\xi})\} + \\
+ \frac{2}{\hbar}\alpha_{w}\{\vec{\xi} \times \vec{n}\} - \frac{2}{\hbar}\delta_{w}\{\vec{n} - \vec{\xi}(\vec{\xi}\vec{n})\}. \tag{72}
\]

Let us note that vector \([\vec{n} \times \vec{E}]\) is parallel to vector \(\vec{N}_m = [\vec{n} \times \vec{n}_x]\), vector \(\vec{E}\) is parallel to \(\vec{N}_T = \vec{n}_x\), \(\vec{n} = \frac{\vec{k}}{k}\) is the unit vector parallel to the direction of the particles momentum. Equation \((72)\) can be also expressed as:

\[
\frac{d\vec{\xi}}{dt} = -\left(\frac{e(g - 2)}{2mc}\right)E_x(x) + \frac{2}{\hbar}\alpha_{s0}(x)\{\vec{\xi} \times \vec{N}_m\} - \\
- \frac{2}{\hbar}\left(\delta_m(x) + \delta_{s0}(x)\right)\{\vec{N}_m - \vec{\xi}(\vec{N}_m\vec{\xi})\} + \\
+ \frac{2}{\hbar}\left(edE_x(x) + \alpha_{T e}(x) + \alpha_{T nuc}(x)\right)\{\vec{\xi} \times \vec{N}_T\} + \\
+ \frac{2}{\hbar}\left(\delta_{EDM}(x) + \delta_{T e}(x) + \delta_{T nuc}(x)\right)\{\vec{N}_T - \vec{\xi}(\vec{N}_T\vec{\xi})\} + \\
+ \frac{2}{\hbar}\alpha_{w}\{\vec{\xi} \times \vec{n}\} - \frac{2}{\hbar}\delta_{w}\{\vec{n} - \vec{\xi}(\vec{\xi}\vec{n})\}. \tag{73}
\]

According to \((73)\) baryon spin rotates about three axes \([29]\): the effective magnetic field direction \(\vec{N}_m||[\vec{n} \times \vec{E}]\), the electric field direction \(\vec{N}_T||\vec{E}\) and the direction of the momentum \(\vec{n}\).

Nonelastic processes in crystals cause appearance of different types of contributions in \((72,73)\): terms proportional to \(\delta\) lead to rotation of the polarization vector in directions of vectors \(\vec{N}_m\), \(\vec{N}_T\) and \(\vec{n}\). As is also seen from \((73)\), when an unpolarized beam enters a crystal polarization in direction of vectors \(\vec{N}_m\), \(\vec{N}_T\) and \(\vec{n}\) arises \([29]\).

Contributions to the equation \((73)\), which are caused by the interaction between baryon and nuclei, depend on distribution of nuclei density \(N_{nuc}(x)\)
(see terms proportional to $\alpha_{a0}(x), \delta_{a0}(x), \alpha_{T_{nuc}}(x), \delta_{T_{nuc}}(x)$). As a result, for positively charged particles, moving in the channel along the trajectories located in the center of the channel, such contributions are suppressed.

Thus, according to (73), when conducting and interpreting the experiments for measuring EDM, one should take into consideration the fact that measuring spin rotation provides information about the sum of contributions to T-noninvariant rotation. The stated rotation is determined by both EDM and short-range CP-noninvariant interactions. Nonelastic T-noninvariant processes lead to spin rotation in direction of $\vec{N}_T$ as well, which gives additional opportunities for EDM measurement.

Let us evaluate the most important new effects described by the equation (73) and consider the contribution to spin rotation caused by spin rotation in direction of $\vec{N}_m$. According to (49) coefficient $\delta_m$ has the following form:

$$\delta_m = \frac{1}{4d_yd_zmc^2} \left( g - \frac{2}{2} \right) \frac{\partial}{\partial x} \delta V^2(x) =$$

$$= \frac{1}{4d_yd_zmc^2} \left( g - \frac{2}{2} \right) \frac{\partial}{\partial x} \int \left\{ \left[ \int V_{coul}(x, y, z) dz \right]^2 - \left[ \int V_{coul}(x, y, z) dz \right] \right\} dy,$$

where $V_{coul}(x, y, z) = \sum V_e(x - x_i, y - y_i, z - z_i) - V_{nuc}(x - \eta_{fx}, y - \eta_{fy}, z - \eta_{fz})$, $x_i, y_i, z_i$ are the coordinates of the $i$-th electron in atom, $\eta_{fx}, \eta_{fy}, \eta_{fz}$ are the coordinates of nucleus. Let us choose the equilibrium point position of the oscillating nucleus as the origin of coordinates. The line denotes averaging of electrons and nuclei positions by electrons density distribution and nuclei oscillations; in other words, averaging with wave-functions of atoms in crystal. By means of these functions, the density distribution takes the form:

$$N(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_z, \vec{\eta}) = N_e(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_z, \vec{\eta})N_{nuc}(\vec{\eta}),$$

where $N_e$ is the density distribution of electrons in atom, $N_{nuc}(\vec{\eta})$ is the density distribution of nucleus oscillations.

Let us introduce the function $W(x, y) = \int V(x, y, z) dz$. From (74) we have:

$$W(x, y) = \sum_i \int V_e(x - x_i, y - y_i, z) dz - \int V_{nuc}(x - \eta_{fx}, y - \eta_{fy}, z) dz =$$

$$= \sum_i W_e(x - x_i, y - y_i) - W_{nuc}(x - \eta_{fx}, y - \eta_{fy})$$

(76)
\[ W^2(x, y) = \int \left[ \sum_i W_e(\vec{\rho} - \vec{\rho}_i) - W_{\text{nuc}}(\vec{\rho} - \vec{\eta}_\perp) \right]^2 \times 
\times N_e(\vec{\rho}_1 - \vec{\eta}_1, \ldots, \vec{\rho}_z - \vec{\eta}_\perp) N_{\text{nuc}}(\vec{\eta}_\perp) d^2\rho_1 d^2\rho_z d^2\vec{\eta}_\perp, \]  
(77)

where \( \vec{\rho} = (x, y) \), \( \vec{\eta}_\perp = (\eta_x, \eta_y) \), \( Z \) is the number of electrons in atom.

In other words:

\[ W^2(\vec{\rho}) = \int \left\{ \sum_i W_e(\vec{\rho} - \vec{\rho}_i) \right\}^2 - 2 \sum_i W_e(\vec{\rho} - \vec{\rho}_i) W_{\text{nuc}}(\vec{\rho} - \vec{\eta}_\perp) + W^2_{\text{nuc}}(\vec{\rho} - \vec{\eta}_\perp) \right\} N_e(\vec{\rho}_1 - \vec{\eta}_1, \ldots, \vec{\rho}_z - \vec{\eta}_\perp) N_{\text{nuc}}(\vec{\eta}_\perp) d^2\rho_1 d^2\rho_z d^2\vec{\eta}_\perp. \]
(78)

Averaging \( W^2(\vec{\rho}) \), provides appearing the expressions for both density distribution of a single electron in atom and those dependent on coordinates of two electrons in the atom, which describe pair correlations in electrons positions in atom. However, the influence of pair correlations will be ignored during the estimations. As a result the expression (78) can be represented as follows:

\[ W^2(\vec{\rho}) = \int d^2\eta_\perp \{ Z[\langle W^2_e(\vec{\rho}, \vec{\eta}_\perp)\rangle^2_e - \langle W_e(\vec{\rho}, \eta_\perp)\rangle^2_e] + Z^2 \langle W_e(\vec{\rho}, \vec{\eta}_\perp) \rangle^2_e - 2Z \langle W_e(\vec{\rho}, \vec{\eta}_\perp) \rangle W_{\text{nuc}}(\vec{\rho} - \vec{\eta}_\perp) + W^2_{\text{nuc}}(\vec{\rho} - \vec{\eta}_\perp) \}, \]  
(79)

where the function

\[ \langle W_e(\vec{\rho}, \vec{\eta}_\perp) \rangle_e = \int W_e(\vec{\rho} - \vec{\rho}') N_e(\vec{\rho}' - \vec{\eta}_\perp) d^2\rho' \]

\[ \langle W^2_e(\vec{\rho}, \vec{\eta}_\perp) \rangle_e = \int W^2_e(\vec{\rho} - \vec{\rho}') N_e(\vec{\rho}' - \vec{\eta}_\perp) d^2\rho' , \]

that means

\[ W^2(\vec{\rho}) = \int d^2\eta_\perp \{ Z[\langle W^2_e(\vec{\rho}, \vec{\eta}_\perp) \rangle_e - \langle W_e(\vec{\rho}, \eta_\perp) \rangle_e^2] + 
+ (Z \langle W_e(\vec{\rho}, \vec{\eta}_\perp) \rangle_e - W_{\text{nuc}}(\vec{\rho} - \vec{\eta}_\perp))^2 \} N_{\text{nuc}}(\vec{\eta}_\perp). \]
(80)

According to (74), the function \( \int \left[ W^2(\vec{\rho}) - W(\vec{\rho})^2 \right] dy \) determines the expression for \( \delta_m \). It should be noted that when fluctuations caused by
nuclei oscillations are neglected, only fluctuations, which are determined by distribution of electrons coordinates in atom, are left.

As a result, the following equation for $\delta_m$ can be obtained:

$$\delta_m = \frac{1}{4d_y d_z mc^2} \left( \frac{g - 2}{2} \right) \frac{\partial}{\partial x} \int \left\{ W^2(x, y) - W(x, y)^2 \right\} dy,$$  \hspace{1cm} (81)

where $W(\vec{\rho}) = \int \left\{ Z W_e(\vec{\rho}, \eta_{\perp}) - W_{nuc}(\vec{\rho} - \eta_{\perp}) \right\} N_{nuc}(\eta_{\perp}) d^2 \eta_{\perp}$.

The following estimation for the value $\delta_m$ can be obtained from (81): $\frac{1}{\hbar} \delta_m \sim 10^8 \div 10^9$ sec$^{-1}$. According to [10, 11] the charm baryon EDM can be as large as $d \sim 10^{-17}$. Spin rotation frequency $\Omega_{EDM}$ determined by such charmed baryon EDM is $\Omega_{EDM} \sim 10^6 - 10^7$ sec$^{-1}$. As a result, nonelastic processes caused by magnetic moment scattering can imitate the EDM contribution.

The contributions of P-odd and T-even rotation effect to the general spin rotation can be evaluated by the following way. Precession frequency $\Omega_w = \frac{2}{\hbar} \alpha_w$ is determined by the real part of the amplitude of baryon weak scattering by an electron (nucleus). This amplitude can be evaluated in the energy range of about W and Z bosons production and smaller by Fermi theory [27]:

$$ReB \sim G_Fk = 10^{-5} \frac{1}{m_p^2} k = 10^{-5} \frac{\hbar}{m_p c m_p \gamma} = 10^{-5} \lambda_{cp} \frac{m}{m_p \gamma},$$  \hspace{1cm} (82)

where $G_F$ is the Fermi constant, $m_p$ is the proton mass, $\lambda_{cp}$ is the proton Compton wavelength. For particles with energy from hundreds of GeV to TeV $ReB \sim G_F k = 10^{-16}$ cm.

For different particle trajectories in a bent crystal precession frequency $\Omega_w$ could vary in the range $\Omega_w \simeq 10^3 \div 10^4$ sec$^{-1}$. Therefore, when particle a passes 10 cm in a crystal, its spin undergoes additional rotation around momentum direction at angle $\vartheta_p \simeq 10^{-6} \div 10^{-7}$ rad. The effect grows for a heavy baryon as a result of the mechanism similar to that of its EDM growth (see the explanation for the growth of constant $d_T$ mentioned above).

Absorption caused by parity violation weak interaction also contributes to change of spin direction (see in [72, 73] the terms proportional to $\delta_w$). This rotation is caused by imaginary part of weak scattering amplitude and is proportional to the difference of total scattering cross-sections $\sigma_{\uparrow\uparrow}$ and $\sigma_{\downarrow\downarrow}$ [20].

This difference is proportional to the factor, which is determined by interference of coulomb and weak interactions for baryon scattering by an electron, and of strong (coulomb) and weak interactions for baryon scattering by
\[ \sigma_{\uparrow\uparrow} = \int |f_c(nuc) + B_{0w} \pm B_w|^2 d\Omega, \]

\[ \sigma_{\uparrow\uparrow} - \sigma_{\downarrow\uparrow} = 2 \int [(f_c(nuc) + B_{0w})B^* + (f_c(nuc) + B_{0w})^*B]d\Omega. \]

When baryon trajectory passes in the area, where collisions with nuclei are important (this occurs in the vicinity of potential barrier for positively charged particles), the value \( \frac{\delta}{\hbar} \delta_w \sim 10^6 \div 10^7 \text{sec}^{-1} \). Similar to the real part \( \text{Re}B \) for the case of heavy baryons the difference in cross-sections grows. Multiple scattering also contributes to spin rotation \[13, 26\]. Particularly, due to interference of weak and coulomb interactions the root-mean-square scattering angle appears changed and dependent on spin orientation with respect to the particle momentum direction \[29\].

When measuring MDM and T-odd spin rotation in a bent crystal, one can eliminate parity violating rotation by the following way. MDM and T-odd spin rotations, unlike P-odd spin rotations, depend on crystal turning at 180° around the direction of incident baryon momentum. Namely, P-odd effect does not change, while the sign of MDM and T-odd spin rotations does due to change of the electric field direction. Subtracting results of measurements for two opposite crystal positions on could obtain the angle of rotation, which does not depend on P-odd effect.

9 Conclusion

Besides electromagnetic interaction the channelled particle moving in a crystal experiences weak interaction with electrons and nuclei as well as strong interaction with nuclei. Mentioned interactions lead to the fact that in the analysis of the particle’s spin rotation, which is caused by electric dipole moment interaction with electric field, both \( P_{\text{odd}}, T_{\text{even}} \) and \( P_{\text{odd}}, T_{\text{odd}} \) non-invariant spin rotation resulting from weak interaction should be considered. As obtained here, spin precession of channelled particles in bent crystals at the LHC gives unique possibility for measurements of both electric and magnetic moments of charm, beauty and strange charged baryons, as well as constants determining CP \( (T_{\text{odd}}, P_{\text{odd}}) \) violation interactions and \( P_{\text{odd}}, T_{\text{even}} \) interactions of baryons with electrons and nucleus (nucleons). For a particle moving in a bent crystal a new effect caused by nonelastic processes arises: in addition to the spin precession around the direction of the effective magnetic field (bend axis), the direction of electric field and the direction of the particle momentum, the spin rotation to the mentioned directions also appears.

25
Let us consider scattering of a relativistic particle in crystal, formed by the set of \( N \) atoms. It should be reminded, that the Dirac equation describing scattering process can be transformed to the Schrödinger equation for the particle with relativistic mass \( M = m \gamma \), where \( m \) is the rest mass of the particle, \( \gamma \) is the Lorentz factor. The corresponding equation is

\[
\left( E_a - H(\xi_1 \ldots \xi_N) + \frac{A^2}{2m\gamma} \right) \psi(\vec{r}, \xi_1 \ldots \xi_N) = \sum_{i=1}^{N} V_i(\vec{r}, \xi_i) \psi(\vec{r}, \xi_1 \ldots \xi_N),
\]

where \( H(\xi_1 \ldots \xi_N) \) is the Hamiltonian of the scatters; \( \xi_1 \ldots \xi_N \) is the set of coordinates describing the first and other scatterers (\( \xi \) also includes spin variables); \( V_i(\vec{r}, \xi_i) \) is the energy of the interaction between the incident particle and the \( i \)-th scatterer; \( \vec{r} \) is the coordinate of the incident particle. If \( G(\vec{r}, \xi_1 \ldots \xi_N; \vec{r}', \xi_1' \ldots \xi_N') \) is the Green function of the operator

\[
E_a - H + \frac{A^2}{2m\gamma} \Delta_r
\]

then \[85\] can be written in the form

\[
\psi_a(\vec{r}, \xi_1 \ldots \xi_N) = \Phi_a(\vec{r}, \xi_1 \ldots \xi_N) + \int \int G(\vec{r}, \xi_1 \ldots \xi_N; \vec{r}', \xi_1' \ldots \xi_N') \sum_{i=1}^{N} V_i(\vec{r}', \xi_i') \psi_a(\vec{r}', \xi_1' \ldots \xi_N') d_3r' d_3\xi_1' \ldots d_3\xi_N'.
\]

[ \( \Phi_a \) are the eigenfunctions of the operator \( \left( -\frac{A^2}{2m\gamma} \Delta_r + H \right) \).] Taking into account that \[13\]

\[
\sum_i V_i(\vec{r}, \xi_i) \psi_a(\vec{r}, \xi_1 \ldots \xi_N) = T(\vec{r}, \xi_1 \ldots \xi_N) \Phi_a(\vec{r}, \xi_1 \ldots \xi_N),
\]

where \( T \) is the operator of scattering by \( N \) centers, the following equation can be derived for \( T \):

10 Annex
\[ T(\bar{r}, \xi_1 \ldots \xi_N) \Phi_a(\bar{r}, \xi_1 \ldots \xi_N) = \sum_{i=1}^{N} V_i(\bar{r}, \xi_i) \Phi_a(\bar{r}, \xi_1 \ldots \xi_N) \]
\[ + \sum_{i=1}^{N} V_i(\bar{r}, \xi_i) \int \int G(\bar{r}, \xi_1 \ldots \xi_N; \bar{\rho}, \bar{\eta}_1 \ldots \bar{\eta}_N) \]
\[ \times T(\bar{\rho}, \bar{\eta}_1 \ldots \bar{\eta}_N) \Phi_a(\bar{\rho}, \bar{\eta}_1 \ldots \bar{\eta}_N) d^3 \rho d^3 \eta_1 \ldots d^3 \eta_N. \]  
(88)

Let us introduce the notation \( T(\bar{r}, \xi_1 \ldots \xi_N) \Phi_a(\bar{r}, \xi_1 \ldots \xi_N) = T_a(\bar{r}, \xi_1 \ldots \xi_N). \) Then, it is convenient to introduce the operators \( T^i \), using the equalities:
\[ T^i_a(\bar{r}, \xi_1 \ldots \xi_N) = V_i(\bar{r}, \xi_i) \Phi_a(\bar{r}, \xi_1 \ldots \xi_N) \]
\[ + V_i(\bar{r}, \xi_i) \int \int G(\bar{r}, \xi_1 \ldots \xi_N; \bar{\rho}, \bar{\eta}_1 \ldots \bar{\eta}_N) \]
\[ \times T_a(\bar{\rho}, \bar{\eta}_1 \ldots \bar{\eta}_N) d^3 \rho d^3 \eta_1 \ldots d^3 \eta_N, \]  
(89)
i.e., \( T = \sum_i T^i \). The system (88) can be represented as
\[ T^i_a(\bar{r}, \xi_1 \ldots \xi_N) = t^i_a(\bar{r}, \xi_1 \ldots \xi_N) \]
\[ + t^i_a(\bar{r}, \xi_1 \ldots \xi_N) \int \int G(\bar{r}, \xi_1 \ldots \xi_N; \bar{\rho}, \bar{\eta}_1 \ldots \bar{\eta}_N) \]
\[ \times \sum_{l \neq i} T^l_a(\bar{\rho}, \bar{\eta}_1 \ldots \bar{\eta}_N) d^3 \rho d^3 \eta_1 \ldots d^3 \eta_N, \]  
(90)

where
\[ t^i_a(\bar{r}, \xi_1 \ldots \xi_N) = V_i(\bar{r}, \xi_i) \Phi_a(\bar{r}, \xi_1 \ldots \xi_N) \]
\[ + V_i(\bar{r}, \xi_i) \int \int G(\bar{r}, \xi_1 \ldots \xi_N; \bar{\rho}, \bar{\eta}_1 \ldots \bar{\eta}_N) \]
\[ \times t^i_a(\bar{\rho}, \bar{\eta}_1 \ldots \bar{\eta}_N) d^3 \rho d^3 \eta_1 \ldots d^3 \eta_N. \]

As is known [15, 17],
\[ G(\bar{r}, \xi_1 \ldots \xi_N; \bar{\rho}, \bar{\eta}_1 \ldots \bar{\eta}_N) = -\frac{m^2}{2\pi \hbar^2} \sum_b \varphi_b(\xi_1 \ldots \xi_N) \varphi^*_b(\bar{\eta}_1 \ldots \bar{\eta}_N) e^{ik_b|\bar{r} - \bar{\rho}|}, \]

27
where \( \varphi_b(\xi_1 \ldots \xi_N) \) are the eigenfunctions of the operator \( H(\xi_1 \ldots \xi_N) \);

\[
k_b^2 \equiv \frac{2m\gamma}{\hbar^2} \left( E_A + \frac{\hbar^2 k_a^2}{2m\gamma} - E_B \right) = \frac{2m\gamma}{\hbar^2} (E_a - E_B);
\]

\( E_A \) and \( E_B \) are the internal energies of the scattering system before and after the collision, respectively.

If the scatterers are independent of each other, the wave function \( \varphi_b(\xi_1 \ldots \xi_N) \) is represented as the product of the wave functions of the scatterers:

\[
\varphi_b(\xi_1 \ldots \xi_N) = \prod \varphi_b(\xi_i).
\]

In this case the direct substitution can verify that

\[
t^{(i)}(\vec{r}, \xi_1 \ldots \xi_N) = t^{(i)}(\vec{r}, \xi_i),
\]

where \( t^{(i)}(\vec{r}, \xi_i) \) is the operator of particle scattering by the \( i \)-th center in the absence of other centers.

Note now that the quantities \( T^{(i)}_a \) can be written as follows:

\[
T^{(i)}_a = \frac{1}{T^{(i)}_a(\vec{r}, \xi_1 \ldots \xi_N)} \mathcal{F}^{(i)}_a(\vec{r}, \xi_1 \ldots \xi_N),
\]

(91)

where

\[
\mathcal{F}^{(i)}_a(\vec{r}, \xi_1 \ldots \xi_N) = \Phi_a(\vec{r}, \xi_1 \ldots \xi_N) + \int \int G(\vec{r}', \xi_1 \ldots \xi_N; \vec{r}, \eta_1 \ldots \eta_N) \times \sum_{i \neq i} T^{(i)}_a(\vec{r}', \eta_1 \ldots \eta_N) d^3 \rho d^3 \eta_1 \ldots d^3 \eta_N.
\]

(92)

The first term in (92), being the function of \( \vec{r} \), describes the initial wave falling upon the \( i \)-th scatterer. The second term can be interpreted as the contribution to the wave incident on the \( i \)-th center that is due to scattering by other centers. Indeed, if the interaction of the incident particle with all the centers excepting for the \( i \)-th center equaled zero, then the second term would also equal zero.

Let us make use of the definition (87) and rewrite equation (86) for the wave function \( \psi_a \) in the form:

\[
\psi_a(\vec{r}, \xi_1 \ldots \xi_N) = \Phi_a(\vec{r}, \xi_1 \ldots \xi_N) + \int \int G(\vec{r}', \xi_1 \ldots \xi_N; \vec{r}, \eta_1 \ldots \eta_N) \sum_{i=1}^{N} t^{(i)}(\vec{r}', \xi_1' \ldots \xi_N') \times \mathcal{F}^{(i)}_a(\vec{r}', \xi_1' \ldots \xi_N') d^3 \rho d^3 \xi_1' \ldots d^3 \xi_N'.
\]

(93)

From (93) follows that the probability amplitude \( \psi_{ba}(\vec{r}) = \langle \varphi_b | \psi_a \rangle \) to find the particle at point \( \vec{r} \) and the system in state \( b \) satisfies the system of equations
\[ \psi_{ba}(\vec{r}) = e^{ik_a \vec{r}} \delta_{ba} - \frac{m \gamma}{2 \pi \hbar^2} \int \int \frac{e^{ik_b |\vec{r} - \vec{r}'|}}{|\vec{r} - \vec{r}'|} \varphi_b^* (\xi_1' \ldots \xi_N) \times \sum_t t^{(i)}(\vec{r}', \xi_1' \ldots \xi_N) \mathcal{F}_a^{(i)}(\vec{r}', \xi_1' \ldots \xi_N) d^3 \vec{r}' d^3 \xi_1' \ldots d^3 \xi_N. \] (94)

Transformation of (94) into a differential equation gives

\[ (\Delta_r + k_b^2) \psi_{ba}(\vec{r}) - \frac{2m \gamma}{\hbar^2} \int \int \varphi_b^* (\xi_1' \ldots \xi_N) \times \sum_{i=1}^N t^{(i)}(\vec{r}, \xi_1' \ldots \xi_N) F^{(i)}_a(\vec{r}, \xi_1' \ldots \xi_N) d^3 \xi_1' \ldots d^3 \xi_N = 0 \] (95)

or

\[ (\Delta_r + k_b^2) \psi_{ba}(\vec{r}) - \frac{2m \gamma}{\hbar^2} \sum_f \sum_{i=1}^N t_b^{(i)}(\vec{r}) F_{fa}^{(i)}(\vec{r}) = 0, \] (96)

where

\[ t_b^{(i)}(\vec{r}) = \int \varphi_b^* (\xi_1' \ldots \xi_N) t^{(i)}(\vec{r}, \xi_1' \ldots \xi_N) \varphi_a (\xi_1' \ldots \xi_N) d^3 \xi_1' \ldots d^3 \xi_N; \]

\[ \mathcal{F}_{fa}^{(i)}(\vec{r}) = \int \varphi_J^* (\xi_1' \ldots \xi_N) \mathcal{F}_{a}^{(i)}(\vec{r}, \xi_1' \ldots \xi_N) d^3 \xi_1' \ldots d^3 \xi_N. \]

Let us consider in more detail the equation describing the elastically scattered wave:

\[ (\Delta_r + k_a^2) \psi_{aa}(\vec{r}) - \frac{2m \gamma}{\hbar^2} \sum_{i=1}^N t_{aa}^{(i)}(\vec{r}) \mathcal{F}_{aa}^{(i)}(\vec{r}) \]

\[ - \frac{2m \gamma}{\hbar^2} \sum_{f \neq a} \sum_{i=1}^N t_{af}^{(i)}(\vec{r}). \mathcal{F}_{fa}^{(i)}(\vec{r}) = 0. \] (97)

The amplitude \( \mathcal{F}_{fa}^{(i)} \) in the third term (unlike \( \mathcal{F}_{aa}^{(i)} \)) appears only as a result of rescattering (see the general expression (92)). For this reason, under the conditions when the elastic scattering amplitude \( f \) is of the same order of magnitude as the inelastic scattering amplitude and much smaller than the distance between the scatterers, the third term in the relation \( f/R \) for correlated scatterers and \( (f/R)^2 \) for independent scatterers is smaller than the second term, and can be discarded.
As a result we have:

\[ \mathcal{F}_{aa}^{(i)}(\vec{r}) = \Phi_a(\vec{r}) + \frac{2m\gamma}{\hbar^2} \sum_{i \neq i} \int G_{aa}(\vec{r} - \vec{r}') t_{aa}^{(i)}(\vec{r}') \mathcal{F}_{aa}^{(i)}(\vec{r}') d^3r' \] (98)

\[ G_{aa}(\vec{r} - \vec{r}') = -\frac{1}{4\pi} \frac{e^{ik_a|\vec{r} - \vec{r}'|}}{|\vec{r} - \vec{r}'|} \] (99)

The system of equations (97,98) describes propagation of the coherent elastic scattered wave in crystal. Study of mentioned equations for the case of interaction between thermal neutrons and photons with crystals was conducted in [13, 30]. Specifically, it was shown that the effective potential energy of interaction between thermal neutrons, which possess de Broglie wavelength with the radius larger than the one of nucleus (S-scattering), and the amplitude of thermal oscillations of nucleus, is determined by more complex magnitude than scattering amplitude:

\[ F = \frac{f}{1 + ikf} \] (100)

The stated result for crystal, neglecting thermal oscillations of the nucleus, was obtained in [31]. The case of photons was described in [13, 30]. Magnitude \( F \) is the element of the reaction matrix \( K \) (for example see [15, 17]). Equations (96-97), derived in [13, 32], allow one to consider the case of fast particles with de Broglie wavelength comparable to or much smaller than atoms size and scattering amplitude of nuclei in crystals. It is important to notice, that for obtaining (12-15), the equation for \( \mathcal{F}_{aa}^{(i)} \) can be expressed as:

\[ \mathcal{F}_{aa}^{(i)}(\vec{r}) = \Psi_{aa}(\vec{r}) - \frac{2m\gamma}{\hbar^2} \int G_{aa}(\vec{r} - \vec{r}') t_{aa}^{(i)}(\vec{r}') \mathcal{F}_{aa}^{(i)}(\vec{r}') d^3r' \] (101)

As a result the equation for the coherent wave propagating through the crystal takes the form:

\[ (\Delta_r + k_a^2)\Psi_{aa}(\vec{r}) - \frac{2m\gamma}{\hbar^2} \sum_{i=1}^{N} t_{aa}^{(i)}(\vec{r}) \Psi_{aa}(\vec{r}) + \frac{2m\gamma}{\hbar^2} \sum_{i=1}^{N} t_{aa}^{(i)}(\vec{r}) \frac{2m\gamma}{\hbar^2} \int G_{aa}(\vec{r} - \vec{r}') t_{aa}^{(i)}(\vec{r}') \mathcal{F}_{aa}^{(i)}(\vec{r}') d^3r' = 0. \] (102)

Matrix elements of the scattering operator \( t^{(i)} \) in the equation (102) act as effective interaction. This interaction allows one to use the perturbation
theory while deriving (12.15) for both Coulomb, magnetic and even strong interaction with nuclei in consequence of averaging over nuclei oscillations in crystal. As a result, in the last term of (102) function \( F^{(i)}_a(\vec{r}) \) can be replaced with \( \Psi_{aa}(\vec{r}) \) so that the closed equality for function \( \Psi_{aa}(\vec{r}) \) can be obtained.

\[
(\Delta_r + k^2_a)\Psi_{aa}(\vec{r}) - \frac{2m\gamma}{\hbar^2} \sum_{i=1}^{N} t^{(i)}_{aa}(\vec{r}) \Psi_{aa}(\vec{r}) + \\
\frac{2m\gamma}{\hbar^2} \sum_{i=1}^{N} t^{(i)}_{aa}(\vec{r}) \frac{2m\gamma}{\hbar^2} \int G_{aa}(\vec{r} - \vec{r}') \cdot t^{(i)}_{aa}(\vec{r}') \Psi_{aa}(\vec{r}') d^3r' = 0. 
\]

(103)

Operators \( t^{(i)}_{aa} \) in equation (103) depend on the coordinates of position of the center of the atom \( R_i : t^{(i)}_{aa}(\vec{r}) = t^{(i)}_{aa}(\vec{r} - \vec{R_i}) \). Sums present in the equation (103), are periodical functions of \( \vec{r} \). Let us introduce Fourier expansion for operator \( t^{(i)}_{aa} \):

\[
t^{(i)}_{aa}(\vec{r}) = \frac{1}{(2\pi)^3} \int t_{aa}(\vec{q}) e^{i\vec{q}(\vec{r} - \vec{R_i})} d^3q 
\]

(104)

where \( t_{aa}(\vec{q}) \) does not contain index \( i \) when lattice consists of atoms of one kind.

Summation over \( \vec{R_i} \) in (103) and representation of the Green function (25) leads to the expression for effective potential energy of interaction (12.15).

References

[1] Baryshevsky V. G., Spin rotation of ultrarelativistic particles passing through a crystal, Pis'ma. Zh. Tekh. Fiz. 5, 3 (1979) 182–184.

[2] Baublis V. V., et al., First observation of spin precession of polarized \( \Sigma^+ \) hyperons channeled in bent crystals, LNPI Research Report, (1990–1991) E761 Collaboration (St. Petersburg) (1992) 24–26.

[3] Chen D., Albuquerque I. F. and Baublis V. V., First observation of magnetic moment precession of channeled particles in bent crystals, Phys. Rev. Lett. 69, 23 (1992) 3286–3289.

[4] Khanzadeev A. V., Samsonov V. M. Carrigan R. A. and Chen D., Experiment to observe the spin precession of channeled relativistic \( \Sigma^+ \) hyperons Nucl. Instr. Methods B 119, 1-2 (1996) 266–270.
[5] Baryshevsky V. G., Spin rotation and depolarization of high-energy particles in crystals at Hadron Collider (LHC) and Future Circular Collider (FCC) energies and the possibility to measure the anomalous magnetic moments of short-lived particles, (2015), arXiv:1504.06702.

[6] Baryshevsky V. G., *Phys. Lett. B* **757**, (2016) 426–429.

[7] Baryshevsky V. G., Depolarization of high-energy neutral particles in crystals and the possibility to measure anomalous magnetic moments of short-lived hyperons, (2016), arXiv:1608.06815.

[8] Baryshevsky V. G., Spin rotation and depolarization of high-energy particles in crystals at LHC and FCC energies. The possibility to measure the anomalous magnetic moments of short-lived particles and quadrupole moment of Ω-hyperon, *Nucl. Instr. Methods B* **402** (2017), 5–10.

[9] Bezhshyyko O. A., Burmistrov L., Fomin A. S., et all, Feasibility of measuring the magnetic dipole moments of the charm baryons at the LHC using bent crystals, *JHEP* **8** (107), (2017), arXiv:1705.03382.

[10] Botella F. J., Garcia Martin L. M., Marangotto D., et all, On the search for the electric dipole moment of strange and charm baryons at LHC, (2016), arXiv:1612.06769; *Eur. Phys J.C.* **77**, 181 (2017), DOI 10.1140/epjc/s10052-017-4679-y.

[11] Bagli E., Bandiera L., Cavoto G., et all, Electromagnetic dipole moments of charged baryons with bent crystals at the LHC, *Eur. Phys J.C.* (2017) 77:828, p. 1-19.

[12] Baryshevsky V. G., Rotation of particle spin in a storage ring with a polarized beam and measurement of the particle EDM, tensor polarizability and elastic zero-angle scattering amplitude, *J. Phys. G*, 35 (2008).

[13] Baryshevsky V. G., High-Energy Nuclear Optics of Polarized Particles, World Scientific Publishing Company, 640 p. (2012).

[14] Derbenev Ja. S. and Kondratenko, A. M. (1973). Polarization kinetics of particles in storage rings, *Zh. Eksp. Teor. Fiz.* **64**, 6, pp. 1918–1929 (*Sov. Phys. JETP* **37** p. 968).

[15] Goldberger, M. L. and Watson, R. M. (1984). *Collision Theory* (Wiley, New York).
[16] Berestetskii, V. B., Lifshitz, E. M. and Pitaevskii, L. P. (1982). Quantum Electrodynamics, in Landau, L. D. and Lifshitz, E. M. Course of Theoretical Physics Vol. 4., 2nd edn., (Butterworth-Heinemann).

[17] Davydov, A. S,(2010), Quantum Mechanics (BHV-Petersburg).

[18] Jackson, J. D. (1976). On understanding spin–flip synchrotron radiation and the transverse polarization of electrons in storage rings, Rev. Mod. Phys. 48, 3, pp. 417–433.

[19] Heinemann, K. and Hoffstaetter, G. H. (1996). Tracking algorithm for the stable spin polarization field in storage rings using stroboscopic averaging, Phys. Rev. E 54, 4, pp. 4240–4255.

[20] Mane, S. R., Shatunov, Yu. M. and Yokoya, K. (2005). Spin–polarized charged particle beams in high–energy accelerators, Rep. Prog. Phys. 68, 9, pp. 1997–2266.

[21] Hoffstaetter, G. H., (2000). Polarized Protons in HERA, LANL e-print arXiv:physics/0006007v1 [physics.acc-ph]; Nucl. Phys. A 666 pp. 203–213.

[22] Hoffstaetter, G. H. (2006). High Energy Polarized Proton Beams (A Modern View Series: Springer Tracts in Modern Physics, Vol. 218).

[23] Heinemann, K. and Barber, D. P. (2001). Spin transport, spin diffusion and Bloch equations in electron storage rings, Nucl. Instrum. Methods A 463, 1–2, pp. 62–67; Erratum Nucl. Instrum. Methods A 469, 2, p. 294 (2001).

[24] Ford, G. W., Hirt, C. W, (1961), Michigan Univ. Report unpublished.

[25] Mane, S. R., Shatunov, Yu. M. and Yokoya, K. (2005). Spin–polarized charged particle beams in high–energy accelerators, Rep. Prog. Phys. 68, 9, pp. 1997–2266.

[26] Baryshevsky V. G., Bartkevich A. R., Tensor polarization of deuterons passing through matter, J. Phys. G, 39, (2012).

[27] Leader E., Predazzi E., An introduction to Gauge Theories and the ”New Physics”, Cambridge University press, (1982).

[28] Gudkov V.P. Xiao-Gang He, McKeller B.H.J., CP-odd nucleon potential, Phys. Rev. C 47, 5, (1993), pp. 2365–2368.
[29] Baryshevsky V. G., On the search for the electric dipole moment of strange and charm baryons at LHC and parity violating (P) and time reversal (T) invariance violating spin rotation and dichroism in crystal, arXiv:1708.09799 [hep-ph], (2017).

[30] Baryshevsky V. G., Gurinovich A.A., Spontaneous and induced parametric and Smith-Parcell radiation from electrons moving in a photonic crystal built from the metallic threads, NIM B 252, (2006), pp. 92–101.

[31] Goldberger M.L., Seitz F., Theory of the refraction and the diffraction of neutrons by crystals, Phys. Rev. 71, (1947), pp. 294-310.

[32] Baryshevsky V. G., Nuclear Optics of Polarized Media, Belorussian State University Press, Minsk, (1976).