One-Loop Perturbative Corrections to non(anti)commutativity Parameter of $\mathcal{N} = \frac{1}{2}$ Supersymmetric $U(N)$ Gauge Theory

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Abstract

Perturbative corrections to $\mathcal{N} = \frac{1}{2}$ supersymmetric $U(N)$ gauge theory at one-loop order are studied. It is shown that whereas the quantum corrections to $\mathcal{N} = 1$ sector of the theory are not affected by the $C$-deformation, the non(anti)commutativity parameter $C$ receives one-loop perturbative corrections. These perturbative corrections are computed by performing an explicit one-loop calculation of the three and four-point functions of the theory. The running of the non(anti)commutativity parameter $C$ is also studied using an appropriate Callan-Symanzik equation.

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1 Introduction

In the past few years noncommutative field theories have been studied extensively, mainly due to their realization in the string theory [1]. To be more precise, by wrapping the branes with non-zero constant background $B_{\mu\nu}$ field the corresponding low energy effective gauge theory is deformed to a noncommutative supersymmetric gauge theory in such a way that those (bosonic) directions in which the $B_{\mu\nu}$ field is defined become noncommutative. The noncommutativity parameter can then be given in terms of the finite $B_{\mu\nu}$ background field [2]-[6].

Recently it has been shown that noncommutative superspace is also realized in the string theory by turning on a constant graviphoton field strength $F^{\alpha\beta}$ [7, 8, 9], which now changes the anticommutation relation between Grassmanian (fermionic) variables of the superspace. This deformation is in such a way that the anticommuting coordinates $\theta$ form a Clifford algebra [10]-[19]

$$\{\theta^\alpha, \theta^\beta\} = 2\alpha^2 F^{\alpha\beta} = C^{\alpha\beta}. \quad (1)$$

Starting from an $\mathcal{N} = 1$ supersymmetric gauge theory, the half of the supersymmetry is therefore broken by this deformation and we are left with an $\mathcal{N} = 1/2$ supersymmetric gauge theory [9]. Note that since the anticommutation relation of $\bar{\theta}$ remains undeformed, $\bar{\theta}$ is not the complex conjugate of $\theta$ and this is only possible in the Euclidean space $\mathbb{R}^4$, where the ordinary space-time coordinates $x_\mu$ turn out to be noncommutative. In fact one has

$$[x^\mu, \theta^\alpha] = iC^{\alpha\beta} \sigma^{\mu}_{\beta\delta} \bar{\theta}^\delta, \quad [x^\mu, x^\nu] = \bar{\theta} \bar{\theta} C^{\mu\nu}, \quad (2)$$

where $C^{\mu\nu} = C^{\alpha\beta} \epsilon^{\beta\gamma} \sigma^{\mu\nu}_{\alpha\beta\gamma}$. The chiral coordinates $y^\mu = x^\mu + i\theta^\alpha \sigma^{\mu}_{\alpha\beta} \bar{\theta}^\beta$, however, can be taken to be commutative

$$[y^\mu, y^\nu] = [y^\mu, \theta^\alpha] = [y^\mu, \bar{\theta}^\alpha] = 0. \quad (3)$$

One of the consequences of the non(anti)commutation relation (1) in the superspace is that the products of superfields as functions of $\theta$ are now to be ordered. This can be imposed by a novel $*$-product defined by

$$f(\theta) \ast g(\theta) = f(\theta)e^{-\frac{C^{\alpha\beta}}{2} \partial_\alpha \partial_\beta} g(\theta). \quad (4)$$

Replacing all the ordinary products with the above $*$-product, one may proceed by studying a supersymmetric field theory in this non(anti)commuting superspace, taking into account that this deformed supersymmetry algebra admits well-defined representations. Namely, one can define chiral and vector superfields much similar to the ordinary $\mathcal{N} = 1$ supersymmetry [9]. For example, the vector multiplet in Wess-Zumino gauge is given by [9]

\footnote{The noncommutativity of the $x$ space due to the RR fields is also studied in [20].}
\[ V(y, \theta, \bar{\theta}) = -\theta \sigma^\mu \bar{\theta} A_\mu + i \bar{\theta} \theta \bar{\lambda} - i \bar{\theta} \theta \alpha \left( \lambda_\alpha + \frac{1}{4} \epsilon_{\alpha \beta} C^{\beta \gamma} \sigma^\mu_{\gamma \bar{\gamma}} \{ \bar{\lambda}, A_\mu \} \right) + \frac{1}{2} \theta \bar{\theta} \bar{\theta} \bar{\theta} (D - i \partial \mu A^\mu). \] (5)

One can also define the corresponding superfield strength tensor \( W_\alpha \) and thereby give

\[
\int d^2 \theta \ Tr \ W \ast W = \int d^2 \theta \ Tr \ WW(C = 0) - i C^{\mu \nu} \Tr (F_{\mu \nu} \bar{\lambda} \bar{\lambda}) + \frac{|C|^2}{4} \Tr (\bar{\lambda} \bar{\lambda})^2
\]

\[
\int d^2 \bar{\theta} \ Tr \ \bar{W} \ast \bar{W} = \int d^2 \bar{\theta} \ Tr \ \bar{W} \bar{W}(C = 0) - i C^{\mu \nu} \Tr (F_{\mu \nu} \bar{\lambda} \bar{\lambda}) + \frac{|C|^2}{4} \Tr (\bar{\lambda} \bar{\lambda})^2
\]

+ total derivative ,

which can be used to define the Lagrangian of \( \mathcal{N} = 1/2 \) supersymmetric \( U(N) \) gauge theory.

Recently various field theoretical aspects of the non(anti)commuting superspace have been studied in [21]- [37]. In particular, the renormalizability of non(anti)-commutative gauge theory with \( \mathcal{N} = 1/2 \) supersymmetry has been studied in [32]. Using an explicit dimensional analysis, the authors in [32] show that this theory is renormalizable to all order of perturbation theory.

In this paper, an explicit one-loop perturbative calculation of \( \mathcal{N} = 1/2 \) supersymmetric \( U(N) \) gauge theory is performed. It is shown that whereas the quantum corrections to \( \mathcal{N} = 1 \) sector remain unaffected by the \( C \)-deformation, the non-(anti)commutativity parameter \( C \) itself receives one-loop quantum corrections. An explicit one-loop calculation of the three and four-point functions of the theory in the \( C \)-deformed sector is carried out to calculate these corrections explicitly. In Section 2, after giving the full action of the theory including the pure gauge part, the gauge fixing and the ghost parts, the Feynman rules for propagators and vertices are presented. Using these Feynman rules an explicit one-loop calculation of the undeformed \( \mathcal{N} = 1 \) sector and the \( C \)-deformed sector of the theory is performed in Section 3. We have shown that the corresponding renormalization constant \( Z_C \) is always given by the inverse renormalization constant \( Z_g \) which corresponds to the coupling constant \( g \) of the theory. To find the one-loop correction to \( C^2 \) and to check the relation \( Z_{C^2} = Z_C^2 \), suggested first in [32], an explicit one-loop calculation of the \( \bar{\lambda} \) four-point function of the theory is performed in Section 4. In Section 5, we derive an appropriate Callan-Symanzik differential equation for the renormalized three-point function \( \Gamma_{A \bar{\lambda} \bar{\lambda}} \). The running of the non(anti)commutativity parameter \( C \) with the RG-scale \( \mu \) is then calculated explicitly by solving the corresponding Callan-Symanzik \( \gamma_C \)-function for \( C \). We have shown that the product of \( C(\mu)g(\mu) \) is an RG-invariant. Here \( g(\mu) \) is the standard running coupling constant of the ordinary \( U(N) \) theory. The last section is devoted to conclusion.
The following conventions are used in this paper. For $U(N)$ elements we use capital indices while for $SU(N)$ elements small indices are used so that

$$\text{Tr}(t^A t^B) = \frac{1}{2} \delta^{AB}, \quad [t^A, t^B] = i f^{ABC} t^C, \quad \{t^A, t^B\} = d^{ABC} t^C.$$ (7)

In this notation we have $t^0 = \frac{1}{\sqrt{2N}}$, $d^{0AB} = \frac{\sqrt{2}}{N} \delta^{AB}$. We further use [38, 39]

$$f^{IAJ} f^{JBK} f^{KCI} = -\frac{N}{2} f^{ABC}, \quad d^{IAJ} f^{JBK} f^{KCI} = \frac{N}{2} d^{ABC} d_A f_B f_C,$$ (8)

and

$$f^{IAJ} f^{JLK} d_K d_L = \frac{1}{2} f_A f_B \left[ f_C f_D \left( \delta^{AC} \delta^{BD} - \delta^{AB} \delta^{CD} + \delta^{AD} \delta^{BC} \right) 
+ \frac{N}{4} d_C d_D \left( f^{ABX} f^{CDX} - f^{ADX} d^{CDX} - d^{ABX} d^{BCX} \right) \right].$$ (9)

where $f_A = 1 - \delta_{0A}$, $d_A = 2 - f_A$. Note also that $f^{iaj} f^{jbi} = -N \delta^{ab}$.

2 \( \mathcal{N} = \frac{1}{2}, \ U(N) \) SYM theory; Feynman Rules

Following our notation from the previous section, the classical action for $\mathcal{N} = \frac{1}{2}$ supersymmetric $U(N)$ gauge theory is given by

$$S_{\text{gauge}} = \int d^4 x \text{Tr} \left[ -\frac{1}{2} F_{\mu\nu} A^A - 2i \bar{\lambda} \tilde{\sigma}^\mu D_\mu \lambda + D^2 + 2igC^{\mu\nu} A^A \lambda \bar{\lambda} + g^2 |C|^2 (\bar{\lambda} \lambda)^2 \right],$$ (10)

with

$$F_{\mu\nu}^A = \partial_\mu A_\nu^A - \partial_\nu A_\mu^A + gf^{ABC} A_\mu^B A_\nu^C,$$

$$D_\mu \lambda^A = \partial_\mu \lambda^A + gf^{ABC} A_\mu^B \lambda^C.$$ (11)

In the superspace formalism the gauge fixing action is given by

$$S_{\text{GF}} = -\frac{1}{4\xi} \int d^4 \theta \text{Tr} (D^2 V * \bar{D}^2 V),$$ (12)

that in terms of the components of the vector superfield reads

$$S_{\text{GF}} = -\frac{1}{\xi} \text{Tr} \left( D^2 + (\partial_\mu A^\mu)^2 - 2i \bar{\lambda} \tilde{\sigma}^\mu \partial_\mu \lambda + \frac{i}{2} g C^{\mu\nu} \partial_\mu A_\nu (\bar{\lambda} \lambda) \right).$$ (13)

\footnote{In comparison with [9] we have rescaled the components of $V$ by $-2g$.}
Together with the gauge invariant action (10), we have
\[ S_{\text{gauge}} + S_{\text{GF}} = \]
\[ = \int d^4x \text{Tr} \left[ -\frac{1}{2} F^{\mu\nu} F_{\mu\nu} - \frac{1}{\xi}(\partial_{\mu} A^\mu)^2 + (1 - \frac{1}{\xi})D^2 - 2i\bar{\lambda} \bar{\sigma}^\mu (\mathcal{D}_\mu - \frac{1}{\xi} \partial_\mu) \lambda \right. \]
\[ + 2igC^{\mu\nu} (F_{\mu\nu} - \frac{1}{4\xi} \partial_{\mu} A_{\nu}) \bar{\lambda} \bar{\lambda} + g^2 |C|^2 (\bar{\lambda} \bar{\lambda})^2 \].
\[ (14) \]
This action can be used to read the Feynman rules. In the super-Fermi-Feynman gauge, \( \xi = 1 \), the Feynman rules are given by
\[ p \]
\[ \bar{\lambda}_\alpha^A \quad \lambda^B_\alpha \]
\[ \frac{\delta^{AB} \sigma_{\alpha \dot{\alpha}} p_\mu}{p^2}, \]  
\[ (15) \]
\[ k \]
\[ A^A_\mu \quad A^B_\nu \]
\[ - \frac{\eta_{\mu \nu} \delta^{AB}}{k^2}, \]
\[ (16) \]
\[ \bar{\lambda}^\dot{\alpha} \quad \lambda^A_\alpha \quad \bar{\lambda}^\dot{\alpha} \quad \lambda^A_\alpha \]
\[ i g f^{ABC} \bar{\sigma}^\mu \dot{\alpha} \alpha, \]
\[ (17) \]
\[ A^A_\mu, k \quad A^B_\nu, p \quad A^C_\rho, q \]
\[ 2ig^2 C^{\mu\nu} \epsilon^{\dot{\alpha}\dot{\beta}} d^{ABL} f^{LCD}, \]
\[ (20) \]
\[ \bar{\lambda}^\dot{\alpha} \quad \lambda^A_\alpha \quad \bar{\lambda}^\dot{\alpha} \quad \lambda^A_\alpha \]
\[ g^2 |C|^2 \left( \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon^{\dot{\gamma}\dot{\delta}} d^{ABM} d^{MCD} - \epsilon^{\dot{\alpha}\dot{\gamma}} \epsilon^{\dot{\beta}\dot{\delta}} d^{ACM} d^{MBD} + \epsilon^{\dot{\alpha}\dot{\gamma}} \epsilon^{\dot{\beta}\dot{\gamma}} d^{ADM} d^{MBC} \right). \]
\[ (21) \]
Further, the ghost field action in the superfield formalism is given by
\[ S_{\text{ghost}} = 2 \text{Tr} \int d^4 \theta (\mathcal{C}' + \bar{\mathcal{C}}') L_{-gV} \left[ (\mathcal{C} + \bar{\mathcal{C}}) + \coth(-gV) (\mathcal{C} - \bar{\mathcal{C}}) \right], \]
\[ (22) \]
where \( L_X Y = [X,Y] \) and all products are understood as star-product. We use the following notations for the components of the ghost superfields

\[
\mathcal{C} = c + \sqrt{2} \theta \zeta + \theta F, \quad \mathcal{C}' = b + \sqrt{2} \theta \eta + \theta F' \\
\bar{\mathcal{C}} = \bar{c} + \sqrt{2} \bar{\theta} \bar{\zeta} - 2i \bar{\theta} \sigma^\mu \bar{\theta} \partial_\mu \bar{c} + \bar{\theta} \bar{\theta} (\bar{F} + i \sqrt{2} \theta \sigma^\mu \partial_\mu \bar{\zeta} + \theta \theta \partial^2 \bar{c}), \\
\bar{\mathcal{C}}' = \bar{b} + \sqrt{2} \bar{\theta} \bar{\eta} - 2i \bar{\theta} \sigma^\mu \bar{\theta} \partial_\mu \bar{b} + \bar{\theta} \bar{\theta} (\bar{F}' + i \sqrt{2} \theta \sigma^\mu \partial_\mu \bar{\eta} + \theta \theta \partial^2 \bar{b}) .
\]

After integrating out the auxiliary field, the quadratic (kinetic) term of the ghost action reads

\[
2 \text{ Tr } \int d^4 \theta \left( \mathcal{C}' \mathcal{C} - \mathcal{C} \mathcal{C}' \right) = -2 \text{ Tr } \left( c \partial^2 \bar{b} + b \partial^2 \bar{c} + i \zeta \sigma^\mu \partial_\mu \bar{\eta} + i \eta \sigma^\mu \partial_\mu \bar{\zeta} \right) .
\]

The interaction terms of the ghost action have two different parts. The first one arises from those terms including one vector superfield

\[
-2g \text{ Tr } \int d^4 \theta (\mathcal{C}' + \bar{\mathcal{C}}') [V, \mathcal{C} + \bar{\mathcal{C}}] = g f^{ABC} \left[ \left( \partial^\mu \bar{b}^A \right. \right. c^B + \partial^\mu \bar{b}^A b^B \left. \right. \right] A_\mu^C - \bar{b}^A c^B \partial^\mu A_\mu^C \\
+ \left( \frac{i}{2} \zeta^A \sigma^\mu \bar{\eta}^B + \frac{i}{2} \bar{\eta}^A \sigma^\mu \zeta^B \right) A_\mu^C + \frac{\sqrt{2}}{2} g f^{ABC} \left( (b + \bar{b}) A_\mu^C \right) f^C_{\alpha} \\
- \frac{\sqrt{2}}{2} g f^{ABC} \left( (b + \bar{b}) \zeta^B + (\bar{c} + c) \eta^B \right) \bar{\lambda}^C \\
+ \frac{\sqrt{2}}{2} g f^{ABC} \sigma^\mu_{\alpha \alpha} C^{\alpha \beta} \left( \partial_\mu \bar{b}^A \zeta_\beta + \partial_\mu \bar{c}^A \eta_\beta \right) \bar{\lambda}^C ,
\]

where \( f^C_{\alpha} = \lambda^C_{\alpha} - \frac{2}{3} \xi_{\alpha \beta \gamma} c^\beta \sigma^\gamma_{\gamma \alpha} d^{CDE} \bar{\lambda}^D A_\mu^E \). Further, for our purpose, the relevant terms of the second part of the ghost action, including two vector superfields read

\[
\frac{2}{3} g^2 \text{ Tr } \int d^4 \theta (\mathcal{C}' + \bar{\mathcal{C}}') [V, [\mathcal{C} - \bar{\mathcal{C}}]] = \frac{2}{3} g^2 \text{ Tr } \left[ - i \frac{\sqrt{2}}{4} (b + \bar{b}) \{ F_{1 \alpha}, \zeta^\alpha \} \\
+ i \frac{\sqrt{2}}{4} \eta_\alpha \{ F^1_{\alpha}, (c - \bar{c}) \} - \eta_\alpha \{ B_1, \zeta^\alpha \} + \frac{|C|^2}{4} \eta_\alpha \bar{\lambda}^\alpha \zeta^\alpha \bar{\lambda}^\alpha \right. \\
+ i \frac{\sqrt{2}}{2} C^{\alpha \beta} \sigma^\mu_{\alpha \alpha} \left( (b + \bar{b}) (A_\mu \zeta_\beta \bar{\lambda}^\alpha - \bar{\lambda}^\alpha \zeta_\beta A_\mu) \right) \\
\left. + \eta_\beta A_\mu (c - \bar{c}) \bar{\lambda}^\alpha + \eta_\beta \bar{\lambda}^\alpha (c - \bar{c}) A_\mu \right] .
\]

with

\[
B_1 = -\frac{1}{8} |C|^2 \bar{\lambda} \lambda, \quad \text{and} \quad F^1_{\alpha} = C^{\alpha \beta} \sigma^\mu_{\beta \beta} [A_\mu, \bar{\lambda}^\alpha].
\]

Using the kinetic part of the ghost action (24) the propagators read

\[
\frac{k}{b^A, c^A, \bar{b}^B, \bar{c}^B} = - \frac{\delta_{AB}}{k^2} .
\]
The relevant vertices can be read from the interaction parts of the action (25) and (26):
\[ \zeta^A_\alpha \quad \eta^B_\beta \quad \bar{\lambda}^C_\bar{\alpha} \quad \bar{\lambda}^{D\bar{\beta}} \to \frac{1}{24} g^3 |C|^2 \epsilon^{\alpha\beta\bar{\epsilon}\bar{\gamma}} \left\{ -i f^{ACM} d^{BDM} + i f^{BCM} d^{ADM} \\
- i f^{ADM} d^{BCM} + i f^{BDM} d^{ACM} \\
- d^{ADM} d^{BCM} + d^{BDM} d^{ACM} \right\}, \quad (38) \]

where \( A^a = (b, c, \bar{b}, \bar{c})^a \), \( \Psi^c_\alpha = (\zeta, \eta, \zeta, \eta)_c^\alpha \) and \( B^a = (\bar{c}, \bar{b})^a \), \( \Phi^c_\alpha = (\eta, \zeta)_c^\alpha \).

### 3 One-loop Perturbative Corrections

In this section, we will calculate the one-loop perturbative corrections to the undeformed \( N = 1 \) part and the \( C \)-deformed part of the theory separately. Due to the additional vertices more diagrams are to be considered comparing to the ordinary \( N = 1 \) supersymmetric \( U(N) \). However, the tadpole diagrams from both undeformed and \( C \)-deformed sectors vanish identically due to the antisymmetry properties of \( f^{abc} \) and \( C^{\mu\nu} \). We will show that the quantum corrections of \( N = 1 \) part of the theory are not affected by \( C \)-deformation, whereas \( C \) receives one-loop perturbative corrections.

#### 3.1 \( N = 1 \) Sector

In this sector, the standard field theory results for one-loop perturbative corrections to \( N = 1 \) supersymmetric \( U(N) \) gauge theory can be used (for example see [40]). Evaluating in particular the vertex function of the theory using \( D \)-dimensional regularization, the renormalization constant \( Z_1 \) is given by

\[ Z_1 = Z_{A\bar{A}A} = 1 - \frac{N g^2}{16 \pi^2} \frac{2}{\epsilon}. \quad (39) \]

Further evaluating the self energy and the vacuum polarization tensor, the wave function renormalization constant \( Z_i \), \( i = 2, 3 \) for \( \lambda^a, \bar{\lambda}^a \) and \( A^a_\mu \) can be determined and read

\[ Z_2 = Z_{\lambda\bar{\lambda}} = 1 - \frac{N g^2}{16 \pi^2} \frac{2}{\epsilon}, \]
\[ Z_3 = Z_{AA} = 1 + \frac{3 N g^2}{16 \pi^2} \frac{2}{\epsilon}. \quad (40) \]

Here \( \epsilon = 4 - D \) is the regulator. Combining the above renormalization constants in the standard way, the gauge coupling renormalization constant \( Z_g \) can be determined which is

\[ Z_g = \frac{Z_1}{Z_2 Z_3^{1/2}} = 1 - \frac{3}{2} \frac{N g^2}{16 \pi^2} \frac{2}{\epsilon}. \quad (41) \]
Using now the relation $g_0 = g \mu^{\epsilon/2} Z_\mu$ and the fact that the bare coupling constant $g_0$ does not depend on the renormalization scale $\mu$, $\frac{d}{d\mu} g_0 = 0$, the $\beta$ function of the theory can be obtained as

$$
\beta(g(\mu)) \equiv \mu \frac{\partial g(\mu)}{\partial \mu} = -\frac{3Ng^3}{16\pi^2}.
$$

(42)

Solving this differential equation for the running coupling we arrive at

$$
g^2(\mu) = \frac{g^2(\mu_0)}{\frac{3Ng^2(\mu_0)}{8\pi^2} \ln \frac{\mu}{\mu_0} + 1},
$$

(43)

where $\mu_0$ is a fixed energy.

### 3.2 $C$-Deformed Sector

The non(anti)commutativity parameter $C^{\mu\nu}$ appears in two different terms of the $C$-deformed part of the action. In the first term including the three fields interaction $A_\mu \bar{\lambda} \bar{\lambda}$, it appears as a rank two tensor contracted with $F^{\mu\nu}$ and in the second term containing a four $\bar{\lambda}$ interaction, it appears as a determinant. In the following subsection we will study the one-loop perturbative corrections corresponding to both terms separately.

#### 3.2.1 One-loop Correction to $C$; $A_\mu \bar{\lambda} \bar{\lambda}$ Three-Point Function

In this section, we will study in detail those one loop graphs including only one new vertex arising from the $C$-deformed part of the action. Starting with the two point function $\bar{\lambda} \bar{\lambda}$, the only possible diagram is the self-energy diagram from figure 1, with internal gluon and ghost fields. Using the Feynman rules given in (16)-(21), one finds

$$
\Gamma^{AB}_{\bar{\alpha}\bar{\beta}} \sim ig \, d^{ALC} f^{BLC} \, C^{\xi\mu} \, \epsilon^{\dot{\alpha}\delta} \, \sigma^\rho_{\gamma\delta} \, \bar{\sigma}^{\dot{\beta}\gamma} \, \int \frac{d^4k}{(2\pi)^4} \, \frac{k_\rho (p - k)_\xi}{k^2(p - k)^2} = 0,
$$

(44)

as expected. Here we have used the relation $C^{\xi\mu} \bar{\sigma}^{\dot{\alpha}\gamma} \sigma^{\rho}_{\gamma\delta} = -C^{\xi\rho} \delta^{\dot{\alpha}}_{\delta}$ and the antisymmetry property of $C^{\alpha\rho}$. Similarly one can also evaluate the two point function with ghost field in the loop. But, such a self energy diagram vanishes too, due to the same symmetry properties of $C^{\mu\nu}$ as above. Hence the two-point function $\bar{\lambda} \bar{\lambda}$ do not receive any perturbative correction at one-loop order. Therefore the theory survives this first check of its renormalizability, at least at one-loop order.

The next step would be to consider the $C$-deformed $A \bar{\lambda} \bar{\lambda}$ three-point function of the theory. Evaluating these three-point functions could indicate whether $C$ receives any one-loop correction or not. All the relevant diagrams, containing only matter fields, are depicted in figure 2.
Explicit calculation shows that the graphs (b,c,d), figure 2, vanish identically taking into account the corresponding diagrams with crossed external legs. Equivalently, we can consider two different diagrams for each of the graphs b,c, and d, and move within the loop first in the clockwise direction and then in the counterclockwise direction. After our convention for the Feynman rule (18), moving from $A_\mu$ to $\bar{\lambda}_{\dot{\alpha}}$ and then to $\bar{\lambda}_{\dot{\beta}}$ picks a plus sign and moving in the other direction, i.e. from $A_\mu$ to $\bar{\lambda}_{\dot{\beta}}$ and then to $\bar{\lambda}_{\dot{\alpha}}$, a minus sign. Adding both contributions one can show that the diagrams (b,c), and (d) of figure 2 vanish. We are therefore left with diagram (a) from figure 2. To evaluate this graph we consider three inequivalent situations depending on the position of the new $C$-deformed vertex in the graph. Let us label the vertices of the graph with (1,2,3) as shown in figure 3. Let us also indicate the position of the new vertex with an index “c” on the label of the vertices. We will find therefore three different situations corresponding to $(1,c,2,3)$, $(1,2,c,3)$ and $(1,2,3,c)$.

Using the Feynman rules (16)-(21) one can proceed to evaluate these graphs. As a sample calculation let us compute the graph corresponding to $(1,c,2,3)$

\[
\Gamma^{ABC,\dot{\alpha}\dot{\beta},\mu}_{(a)} = -2 \times \frac{1}{3} \times \frac{3}{2} g^3 f^{ICJ} d^{IJK} f^{KBI} C^\rho_{\gamma}\sigma^\mu_{\dot{\gamma}} \bar{\sigma}^\lambda_{\dot{\gamma}} \bar{\sigma}^\nu_{\dot{\lambda}} C^\rho_{\delta}\sigma^\lambda_{\delta} \sigma^\nu_{\delta} \frac{k_\rho}{(2\pi)^4} \frac{(k + p_1)_\lambda (k - p_2)_\kappa}{k^2 (k + p_1)^2 (k - p_2)^2}.
\]

The factor 2 comes from two different directions one can move in the loop and 1/3 is the symmetry factor. Using the conventions (8) and the fact that $C^\rho_{\gamma}\sigma^\mu_{\dot{\gamma}} \bar{\sigma}^\lambda_{\dot{\gamma}} \bar{\sigma}^\nu_{\dot{\lambda}} = -C^\rho_{\gamma}\delta^\delta_{\dot{\gamma}}$, one finds

\[
\Gamma^{ABC,\dot{\alpha}\dot{\beta},\mu}_{(a)} = -N \frac{g^3}{16\pi^2} d^{ABC} d_A f_B f_C C^\rho_{\gamma}\sigma^\mu_{\dot{\gamma}} \bar{\sigma}^\lambda_{\dot{\gamma}} \bar{\sigma}^\nu_{\dot{\lambda}} (p_2)_\kappa \int \frac{d^4k}{(2\pi)^4} \frac{k_\rho (k + p_1)_\lambda}{k^2 (k + p_1)^2 (k - p_2)^2}.
\]

The divergent part of the integral is $\frac{1}{4\pi^2} \frac{\delta^{\rho\kappa}}{2}$. Therefore we obtain

\[
\Gamma^{ABC,\dot{\alpha}\dot{\beta},\mu}_{(a)} = \frac{1}{8} \frac{N g^3}{16\pi^2} d^{ABC} d_A f_B f_C \epsilon^{\dot{\alpha}\dot{\beta}} C^\mu_{\nu}(p_2)_{\nu} \frac{2}{\epsilon} + \text{finite terms}.
\]

Adding the same contribution with $p_1 \leftrightarrow p_2$ to this result, the final result for the divergent part of $(1,c,2,3)$ reads

\[
(1,c,2,3) : \Gamma^{ABC,\dot{\alpha}\dot{\beta},\mu}_{(a)} = \frac{1}{8} \frac{N g^3}{16\pi^2} d^{ABC} d_A f_B f_C \epsilon^{\dot{\alpha}\dot{\beta}} C^\mu_{\nu} q_{\nu} \frac{2}{\epsilon},
\]

\[\text{Note that moving in two different directions within the loop in the diagram corresponding to (1,c,2,3), is equivalent to crossing the external legs of the diagram corresponding to (1,2,c,3).}\]
where \( q = p_1 + p_2 \). Similarly the diagrams corresponding to two other situations (1, 2c, 3) and (1, 2, 3c) can be evaluated explicitly. We find that

\[
(1, 2c, 3) : \quad \Gamma_{(a)}^{ABC, \alpha \beta, \mu} = \frac{1}{8} \frac{Ng^3}{16\pi^2} d_{ABC} d_B f_A f_C \epsilon^{\alpha \beta} C_{\mu \nu} q_{\nu} \frac{2}{\epsilon},
\]

\[
(1, 2, 3c) : \quad \Gamma_{(a)}^{ABC, \alpha \beta, \mu} = -\frac{1}{2} \frac{Ng^3}{16\pi^2} d_{ABC} d_B f_A f_C \epsilon^{\alpha \beta} C_{\mu \nu} q_{\nu} \frac{2}{\epsilon}.
\]  

(49)

Let us now continue with the relevant ghost diagrams containing only one \( C \)-deformed vertex and contributing to the three point function \( A_\mu \bar{\lambda} \bar{\lambda} \) (see figure 4). As in the previous case each graph includes two different situations corresponding to the position of the new \( C \)-deformed vertex in the graph. Using the ghost Feynman rules one can evaluate these graphs as well. The results are

\[
\begin{align*}
&\text{(a)} \quad \left\{ 
\begin{aligned}
(1, 2) : \quad &\Gamma_{(a)}^{ABC, \alpha \beta, \mu} = \frac{1}{4} \frac{Ng^3}{16\pi^2} d_{ABC} d_B f_A f_C \epsilon^{\alpha \beta} C_{\mu \nu} q_{\nu} \frac{2}{\epsilon}, \\
(1, 2c) : \quad &\Gamma_{(a)}^{ABC, \alpha \beta, \mu} = \frac{1}{4} \frac{Ng^3}{16\pi^2} d_{ABC} d_B f_A f_C \epsilon^{\alpha \beta} C_{\mu \nu} q_{\nu} \frac{2}{\epsilon}.
\end{aligned}
\right.
\]

\[
\begin{align*}
&\text{(b)} \quad \left\{ 
\begin{aligned}
(1, 2) : \quad &\Gamma_{(b)}^{ABC, \alpha \beta, \mu} = \frac{1}{4} \frac{Ng^3}{16\pi^2} d_{ABC} d_B f_A f_C \epsilon^{\alpha \beta} C_{\mu \nu} q_{\nu} \frac{2}{\epsilon}, \\
(1, 2c) : \quad &\Gamma_{(b)}^{ABC, \alpha \beta, \mu} = \frac{1}{4} \frac{Ng^3}{16\pi^2} d_{ABC} d_B f_A f_C \epsilon^{\alpha \beta} C_{\mu \nu} q_{\nu} \frac{2}{\epsilon}.
\end{aligned}
\right.
\]  

(50)

Having these results in hand, we are now ready to compute the counterterms and thereby the corresponding one-loop renormalization constant corresponding to the deformation parameter \( C \). In fact looking at the action we see that the \( A_\mu \bar{\lambda} \bar{\lambda} \) term has different \( U(1) \) and \( SU(N) \) components

\[
i \sqrt{\frac{2}{N}} g \epsilon^{\alpha \beta} C_{\mu \nu} \partial_\mu A_\nu^0 \bar{\lambda}_\alpha \bar{\lambda}_\beta + i \sqrt{\frac{2}{N}} g \epsilon^{\alpha \beta} \delta^{ab} C_{\mu \nu} \partial_\mu A_\nu^a \bar{\lambda}_\alpha \bar{\lambda}_\beta + i g \epsilon^{\alpha \beta} \epsilon^{ab} \partial_\mu A_\nu^a \bar{\lambda}_\alpha \bar{\lambda}_\beta + i \sqrt{\frac{2}{N}} g \epsilon^{\alpha \beta} \delta^{ab} C_{\mu \nu} \partial_\mu A_\nu^a \bar{\lambda}_\alpha \bar{\lambda}_\beta.
\]  

(51)

Adding the different contributions (48), (49) and (50) from the diagrams of figure 3 and 4 corresponding to different \( U(1) \) and \( SU(N) \) couplings we arrive at the following counterterms for the five possible combinations of \( A_\mu^C - \bar{\lambda}^A - \bar{\lambda}^B \) in (51). Whereas the first term including \((U(1))^3\) coupling receives no quantum correction, all the other counterterms can be given by

\[
\begin{align*}
A_\mu^0 \bar{\lambda}_\alpha \bar{\lambda}_\beta : & \quad -\frac{g^2 N}{16\pi^2} \frac{2}{\epsilon}, \\
A_\mu^c \bar{\lambda}_\alpha \bar{\lambda}_\beta : & \quad +\frac{g^2 N}{16\pi^2} \frac{2}{\epsilon}, \\
A_\mu^c \bar{\lambda}_\alpha \bar{\lambda}_\beta : & \quad +\frac{g^2 N}{16\pi^2} \frac{2}{\epsilon}, \\
A_\mu^c \bar{\lambda}_\alpha \bar{\lambda}_\beta : & \quad +\frac{1}{2} \frac{g^2 N}{16\pi^2} \frac{2}{\epsilon}.
\end{align*}
\]  

(52)
Adding now the counterterm action to the original one and comparing the resulting expression with the bare action the value of one-loop perturbative correction to $C$ can be determined. Using the relation $C_{0}^{\mu\nu} \equiv Z_C C_{0}^{\mu\nu}$ between the bare parameter $C_{0}^{\mu\nu}$ and the renormalized parameter $C^{\mu\nu}$, one finds immediately from the first uncorrected term including $(U(1))^3$ coupling that $Z_C = Z_g^{-1}$. Using further the results from the one-loop computations of the undeformed $\mathcal{N} = 1$ sector and the $C$-deformed sector (52), one finds the corresponding corrections to the other terms of the action including two or three $SU(N)$ couplings

$$
A^0 \tilde{\lambda}^a \tilde{\lambda}^b : \quad \left(1 - \frac{g^2 N}{16\pi^2} \frac{2}{\epsilon}\right) = Z_2,
$$

$$
A^c \tilde{\lambda}^a \tilde{\lambda}^b : \quad \left(1 + \frac{g^2 N}{16\pi^2} \frac{2}{\epsilon}\right) = (Z_2 Z_3)^{1/2},
$$

$$
A^c \tilde{\lambda}^a \tilde{\lambda}^b : \quad \left(1 + \frac{g^2 N}{16\pi^2} \frac{2}{\epsilon}\right) = (Z_2 Z_3)^{1/2},
$$

$$
A^c \tilde{\lambda}^a \tilde{\lambda}^b : \quad \left(1 + \frac{1}{2} \frac{g^2 N}{16\pi^2} \frac{2}{\epsilon}\right) = (Z_2^2 Z_3)^{1/2}. \quad (53)
$$

Comparing again each term with the bare action, all these contributions cancel and we are left with the conclusion that $Z_C = Z_g^{-1}$ for all $U(1)$ and $SU(N)$ couplings. The gauge coupling renormalization constant $Z_g$ is determined from the undeformed $\mathcal{N} = 1$ sector of the theory (41). Using this result, the one-loop quantum correction to the non(anti)commutativity parameter $C$ reads

$$
C_{0}^{\mu\nu} = \left(1 + \frac{3}{2} \frac{g^2 N}{16\pi^2} \frac{2}{\epsilon}\right) C^{\mu\nu} \equiv Z_C C^{\mu\nu}. \quad (54)
$$

### 3.2.2 One-loop Correction to $C^2; \tilde{\lambda}$ Four-Point Function

The non-trivial $\tilde{\lambda}$ four-point functions containing only matter fields and at most two $C$-deformed vertices are depicted in figure 5. They lead to one-loop perturbative corrections to the coupling $|C|^2$. Explicit calculations show that the graphs (a,b,c) are zero when their corresponding crossed graphs are taken into account. The situation is very similar to that in three-point functions mentioned in the previous section. Therefore as far as the matter fields are concerned the corrections come from graph (d) in figure 5. More precisely, there are two different situations for this graph. They are given in figure 6. To evaluate the graphs in figure 6 we have to consider different inequivalent situations for each graph depending on the position of the new $C$-dependent vertex in the graph. We obtain

$$
(1_c, 2, 3, 4) : \quad \Gamma^{ABCD, \dot{\alpha} \dot{\beta} \dot{\gamma} \dot{\delta}}_{(6a)} = 0, \quad (1, 2_c, 3, 4_c) : \quad \Gamma^{ABCD, \dot{\alpha} \dot{\beta} \dot{\gamma} \dot{\delta}}_{(6a)} = 0,
$$

$$
(1_c, 2, 3_c, 4) : \quad \Gamma^{ABCD, \dot{\alpha} \dot{\beta} \dot{\gamma} \dot{\delta}}_{(6b)} = 0, \quad (1, 2_c, 3, 4_c) : \quad \Gamma^{ABCD, \dot{\alpha} \dot{\beta} \dot{\gamma} \dot{\delta}}_{(6b)} = 0. \quad (55)
$$
The remaining nonzero graphs are all equal. For instance, using the Feynman rules from (16)-(21), the Feynman integral corresponding to the \((1c, 2c, 3, 4)\) situation is given by

\[
\Gamma_{(a)}^{ABCD,\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}} = \frac{9}{4} g^2 d^4AJ d^4BK f^KCL f^LDI C^\lambda\mu C^\rho(\tilde{\sigma}^\nu\sigma^\chi\epsilon)^{\dot{\lambda}\dot{\alpha}}(\tilde{\sigma}_\nu\sigma_\chi\epsilon)^{\dot{\gamma}\dot{\beta}} \\
\times \int \frac{d^4k}{(2\pi)^4} \frac{k_\lambda k_\rho(k+p_2)_\kappa(k-p_1)_\chi}{k^2(k-p_1)^2(k+p_2)^2(k-p_1-p_4)^2}. \tag{56}
\]

Taking into account all the symmetry factors and summing over all different configurations, the divergent part of the above expression is given by

\[
\Gamma^{ABCD,\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}} = -\frac{1}{16\pi^2} \left(\frac{8 \times 4 \times 2}{4!}\right) \frac{9}{8} C^2 g^4 \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon^{\dot{\gamma}\dot{\delta}} d^4AJ d^4BK f^KCL f^LDI \frac{2 \epsilon}{\epsilon}. \tag{57}
\]

In particular, for the situation where two of \(\bar{\lambda}\)'s carry \(U(1)\) index and the others \(SU(N)\) index the above expression reads

\[
\Gamma^{ab00,\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}} = \frac{3}{8\pi^2} C^2 g^4 \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon^{\dot{\gamma}\dot{\delta}} \delta^{ab} \frac{2 \epsilon}{\epsilon}. \tag{58}
\]

This is the one-loop correction to \(\frac{1}{N!} g^2 |C|^2 \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon^{\dot{\gamma}\dot{\delta}} \delta^{ab} \bar{\lambda}^0 \bar{\lambda}^a \bar{\lambda}^b\) term in the action arising from the matter fields in \((U(1))^2 \times (SU(N))^2\) combination.

Now what concerns the ghost field contributions to the \(\bar{\lambda}\) four-point function, explicit calculations show that all diagrams depicted in figure 7, containing two \(C\)-deformed vertices vanish due to the antisymmetry property of \(C^{\mu\nu}\) two-form, and therefore (58) is the only one-loop correction to the \(\bar{\lambda}\) four-point function for the \((U(1))^2 \times (SU(N))^2\) coupling. To determine the corresponding correction to the action (counterterm), a factor \(-1/24\) must be multiplied with the above \(\bar{\lambda}\) four-point correction (58). This factor arises from 24 different contractions which are to be performed to obtain the four \(\bar{\lambda}\) interaction vertex. Equivalently, the above correction (58) can be compared with the four \(\bar{\lambda}\) vertex and leads to the counterterm to \(\bar{\lambda}^4\) part of the action.

Next adding the part of the original and the counterterm action containing \(\bar{\lambda}^0 \bar{\lambda}^a \bar{\lambda}^b\) and comparing the resulting expression with the bare action containing the same \(\bar{\lambda}\) combination, we conclude that \(Z_{C^2} = Z_g^{-2}\). Using further the one-loop results from previous section \(Z_C = Z_g^{-1}\), we arrive at the conclusion that

\[
Z_{C^2} = Z_C^2. \tag{59}
\]

This results was also suggested in [32], where it was supposed to be correct in all order of perturbation theory due to some consistency arguments. Using the value of \(Z_g\) from (41) we arrive at

\[
C_0^2 = \left(1 + 3 \frac{g^2 N}{16\pi^2} \frac{2 \epsilon}{\epsilon}\right) C^2 \equiv Z_{C^2} C^2. \tag{60}
\]
4 Running $C^{\mu\nu}$ and $C^2$

As next it is interesting to study the running of the non(anti)commutativity parameter $C^{\mu\nu}$ and the related $|C|^2$ with the renormalization scale $\mu$. Let us consider the bare three-point function $\Gamma^{0}_{A\bar{\lambda}\lambda}$ consisting of one gauge and two $\bar{\lambda}$ fermion fields. The Callan-Symanzik differential equation for the corresponding renormalized three-point function, $\Gamma^{R}_{A\bar{\lambda}\lambda}$, can be given using the fact that the bare three-point function, $\Gamma^{0}_{A\bar{\lambda}\lambda}$, is independent of the renormalization scale $\mu$. In other words, one has

$$\Gamma^{0}_{A\bar{\lambda}\lambda} = Z_{3}^{-1/2}Z_{2}^{-1} \Gamma^{R}(p_i, g(\mu), C^{\mu\nu}(\mu); \mu), \quad \mu \frac{d}{d\mu} \Gamma^{0}_{A\bar{\lambda}\lambda} = 0 , \quad (61)$$

which leads to the following differential equation for the renormalized three-point function $\Gamma^{R}_{A\bar{\lambda}\lambda}$

$$\left( \mu \frac{\partial}{\partial \mu} \ln Z_{3}^{-1/2} + 2 \mu \frac{\partial}{\partial \mu} \ln Z_{2}^{-1/2} + \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} + C^{\mu\nu} \gamma_{C}(g) \frac{\partial}{\partial C^{\mu\nu}} \right) \Gamma^{R}_{A\bar{\lambda}\lambda} = 0 . \quad (62)$$

Here $\beta(g)$ is the ordinary $\beta$-function of the theory (42) and $\gamma_{C}$ is defined by

$$C^{\mu\nu} \gamma_{C}(g) \equiv \mu \frac{\partial}{\partial \mu} C^{\mu\nu}(\mu) . \quad (63)$$

To see how the non(anti)commutativity parameter $C^{\mu\nu}$ runs with the renormalization scale $\mu$, it is useful to find a general expression for $\gamma_{C}$ in terms of $Z_{C}$. Using (54) we arrive at

$$C^{\mu\nu} \gamma_{C}(g) = \mu \frac{\partial}{\partial \mu} \left( Z_{C}^{-1} \right) C^{\mu\nu}_{0} = C^{\mu\nu} Z_{C} \mu \frac{\partial}{\partial \mu} Z_{C}^{-1} , \quad (64)$$

which means that

$$\gamma_{C} = -\mu \frac{\partial}{\partial \mu} \ln Z_{C} = -\beta(g) \frac{\partial}{\partial g} \ln Z_{C} . \quad (65)$$

Using our previous results for $Z_{C}$, we obtain

$$\gamma_{C} = + \frac{3Ng^{2}}{16\pi^{2}} . \quad (66)$$

It is now easy to solve this equation and study the behavior of non(anti)commutativity parameter in terms of the scale $\mu$. Indeed, using the fact that the gauge coupling runs as $g^{2}(\mu) = \frac{8\pi^{2}}{3N \ln(\mu/\Lambda_{U(N)})}$, with $\Lambda_{U(N)} = \mu_{0} e^{-\frac{8\pi^{2}}{3Ng^{2}(\mu_{0})}}$, we get

$$\ln \frac{C^{\mu\nu}(\mu)}{C^{\mu\nu}(\mu_{0})} = \frac{1}{2} \int_{\mu_{0}}^{\mu} \frac{d\mu}{\ln \Lambda_{U(N)}} \ln \left( \ln \frac{\mu/\Lambda_{U(N)}}{\ln(\mu_{0}/\Lambda_{U(N)})} \right)^{1/2} , \quad (67)$$

We note that $\Lambda_{U(N)}$ is RG invariant, i.e. $\Lambda_{U(N)} = \mu_{0} e^{-\frac{8\pi^{2}}{3Ng^{2}(\mu_{0})}} = \mu e^{-\frac{8\pi^{2}}{3Ng^{2}(\mu)}}$. 

13
which leads to

\[ C^{\mu \nu}(\mu)g(\mu) = C^{\mu \nu}(\mu_0)g(\mu_0) . \]  

We arrive therefore at

\[ C^2(\mu)g^2(\mu) = C^2(\mu_0)g^2(\mu_0) = \text{const} . \]  

Alternatively, the same relation can be obtained by looking for an appropriate Callan-Symanzik for the renormalized \( \bar{\lambda} \) four-point function of the theory and solving the equation for running \( C^2(\mu) \) using the fact that \( Z_C^2 = (Z_C)^2 \). This leads to \( \gamma_C^2 = 2\gamma_C \) and consequently the same result from (69) can be obtained.

An immediate consequence of the equation (69) is that another RG invariant, \( \Lambda_C \), can be defined which is related to \( C^2 \) at a fixed energy and can be expressed as a function of the well-known RG invariant \( \Lambda_{U(N)} \)

\[ \Lambda_C = \frac{1}{C^2(\mu_0)} \ln \left( \frac{\mu_0}{\Lambda_{U(N)}} \right) = \frac{1}{C^2(\mu)} \ln \left( \frac{\mu}{\Lambda_{U(N)}} \right) . \]  

One can show that \( \Lambda_C \) is always positive for \( \mu_0 \gg \Lambda_{U(N)} \). This means that \( C^2 \) grows linearly with \( \ln(\mu/\Lambda_{U(N)}) \) with the slope \( \Lambda_C^{-1} \). Using further the fact that the running coupling \( g^2(\mu) = \frac{8\pi^2}{3N\ln(\mu/\Lambda_{U(N)})} \), we obtain

\[ C^2(\mu)g^2(\mu) = \frac{8\pi^2}{3N\Lambda_C} \equiv \eta^2 = \text{const}. \]  

Due to the positivity of \( \Lambda_{U(N)} \), the constant \( \eta \) is always positive. In fact \( \eta \) is the value of the gauge coupling at the energy where the running coupling constant and non(anti)commutativity parameter have the same value. If we plot \( C^2 \) and \( g^2 \) in one and the same diagram as a function of \( \ln \mu \), both curves intersect at one point, say \( \mu_c \), and we obtain \( g(\mu_c) = \eta \).

5 Conclusion

We have studied perturbative corrections to pure \( \mathcal{N} = \frac{1}{2} \) supersymmetric \( U(N) \) gauge theory at one loop order. We have used the fact that the corresponding action to this theory can be separated into two parts: the first part preserves standard \( \mathcal{N} = 1 \) supersymmetry and the second part is the \( C \)-deformed part and breaks the supersymmetry to \( \mathcal{N} = \frac{1}{2} \). We have shown that \( \mathcal{N} = 1 \) part does not receive any \( C \)-dependent corrections and can therefore be treated as the standard \( \mathcal{N} = 1 \) supersymmetric \( U(N) \) gauge theory. Explicit one-loop calculation of \( A\bar{\lambda}\lambda \) three-point

\[ \text{8We do not consider the case where } \mu_0 = \Lambda_{U(N)}, \text{ because in this case } g(\mu_0 = \Lambda_{U(N)}) \to \infty, \text{ and this would break our perturbative considerations. Further the case } \mu_0 < \Lambda_{U(N)} \text{ would end up with an imaginary gauge coupling and this breaks the unitarity of the theory.} \]
function and $\bar{\lambda}$ four-point function show however that the non(anti)commutativity parameter $C^{\mu\nu}$ and $C^2$ receive one-loop corrections. As a result we have $Z_C = Z_g^{-1}$ with $Z_g$ the standard gauge coupling renormalization constant and consequently

$$Z_C^2 = (Z_C)^2 = 1 + 3 \frac{N g^2}{16\pi} \frac{2}{\epsilon} . \quad (72)$$

Using this correction we found the running of the non(anti)commutativity parameter as a function of the renormalization scale $\mu$

$$C^2(\mu) = \frac{1}{\Lambda_C} \ln \left( \frac{\mu}{\mu_0} \right) + C^2(\mu_0) . \quad (73)$$

where $\Lambda_C$ is the new RG invariant scale.

Finally we note that since the $\mathcal{N} = 1$ sector of the theory remains unaffected by the $C$-deformation, the standard anomaly of the theory would be the same as before. Nevertheless one would expect to get a $C$-dependent corrections for those anomalies which whole supersymmetry of the theory ($\mathcal{N} = 1/2$) is involved. In particular the Konishi anomaly can also be studied along [41] (see also [16][42].

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6 Figures

![Figure 1](image)

![Figure 2](image)

![Figure 3](image)
Figure 7
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