Remote transduction of high frequency torque sensors

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Nanophotonic cavity optomechanical devices sensitively probe mechanical resonator motion, enabling a wide range of breakthroughs in sensing and quantum nanomechanics. However, not all nanomechanical resonances can be efficiently transduced. In particular, challenges arise with detection of resonators whose geometry or material composition is not compatible with integration within optical cavities most sensitive to mechanical motion. Here we overcome these limitations using coupled oscillators, demonstrating a system that efficiently converts high frequency twisting motion of a nanodisk to vibrations of a photonic crystal cavity. Optomechanical readout of the cavity enables measurement of the nanodisk’s torsional resonances with sensitivity $9.1 \times 10^{-22} - 2.4 \times 10^{-19}$ Nm/$\sqrt{\text{Hz}}$ for a mechanical frequency range of 10–800 MHz. The nanodisk can be dressed with magnetic nanostructures or metasurfaces without affecting the cavity’s optical properties, making the system suited for magnetometry and structured light sensing.

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The enhanced optomechanical interaction between light and mechanical resonances in nanophotonic cavities allows ultra-precise measurement of force and displacement [11,9], enabling sensors including accelerometers [10], ultrasound detectors [11], mass spectrometers [12,13], and atomic force measurement devices [13]. Cavity optomechanical devices can also sensitively detect torque [15–19], leading to advances in torque magnetometry [20,21], spin detection of electrons [22], and measurement of photon spin and orbital angular momentum [17,19]. Ideally, mechanical resonators in these systems are integrated directly within the optical cavity to maximize transduction strength: the conversion of mechanical motion to an optical response. However, many of these applications require metallic or magnetic materials that degrade the cavity’s properties if they interact with its optical field. For example, torque magnetometers incorporate permalloy nanostructures [20], while mechanical transducers of microwave fields incorporate metal [23]. Challenges also arise with resonators whose geometry precludes their integration directly within nanoscale cavities such as photonic crystals and whispering gallery mode resonators whose optical confinement relies upon precisely engineered geometry that can not be perturbed, or whose mechanical resonance spatial symmetry results in vanishing optomechanical coupling to a cavity’s optical field.

Previously, these challenges have been addressed using mechanical resonators whose interaction with the cavity optical field is limited to regions not patterned with lossy material [20], or with resonators that only weakly perturb the optical cavity volume [21], enabling sensitive transduction of MHz frequency mechanical resonances. Here we demonstrate a new approach that couples two mechanical resonators—one actuated by an external signal, and one integrated within an optical cavity—allowing individual optimization of the system’s actuation and readout properties. We show that through resonant mechanical coupling, a photonic crystal readout cavity can sensitively monitor motion of a high mechanical frequency nanodisk positioned far from the cavity’s optical mode. The nanodisk’s resonances are suited for actuation by sources of torque commonly encountered in magnetometry [20] and structured light [19], and can be designed to reach frequencies $> 700$ MHz, nearly two orders of magnitude larger than previous optomechanical torque sensors [20,22]. The photonic crystal cavity allows sensitive measurement of the nanodisks’ mechanical resonances via their resonant mechanical coupling to its mechanical modes. This enables detection of high frequency nanodisk modes, creating a path towards resonant measurement of nanomagnetic dynamics [24], enhancement of the Einstein-de Haas effect [25,26], studies of spin and orbital angular momentum of photons and electrons [19,22,27], and transduction of radiofrequency signals [24,28,29].

The coupled resonator system is illustrated in Fig. 1(a) and an example of a fabricated device is shown in Fig. 1(b). A suspended silicon nanodisk anchored to an unpatterned silicon-on-insulator (SOI) chip is connected to a photonic crystal nanobeam that forms one half of an optomechanical zipper cavity [30,31]. The second half of the zipper cavity is separated by a small gap from the coupled resonators and anchored to the unpatterned region of the chip; it is labeled as the auxiliary nanobeam in Fig. 1(b). The nanodisk can be mechanically driven by sources of torque ($\tau$), for example from nanomagnetic structures used in torque magnetometry [20,24,25], or by angular momentum transfer from struc-
mirror region to $a_c = 350 \text{nm}$ at the center of the cavity. The resulting cavity supports fundamental bonded and anti-bonded optical modes shown in Figs. 1(c) and 1(d), respectively, with predicted radiation loss limited $Q_o > 10^7$ in the 1500 nm wavelength band. Mechanical motion of the nanobeams can change the gap separating them, shifting the cavity mode frequencies. The frequency shift per unit displacement is quantified by optomechanical coupling coefficients $g_{OM}/2\pi \approx 85 \text{GHz/nm}$ and $\tilde{g}_{OM}/2\pi \approx 2 \text{GHz/nm}$ between the fundamental in-plane mechanical flexural resonance of the nanobeams and the bonded and anti-bonded optical cavity modes, respectively. The larger optomechanical coupling of the bonded mode arises from its higher field concentration in the gap, making it more susceptible to vibrations of the nanobeams. As a result, we focus on the bonded mode in this study.

In the absence of coupling between the nanodisk and the photonic crystal nanobeam, torque actuated motion of the nanodisk is not transduced by the cavity. However, transduction becomes possible when mechanical coupling is present. Intuitively, this transduction will be enhanced if the nanodisk motion is resonant with a nanobeam mode. This coupled oscillator approach to transduction of the ‘remote’ and nominally dark nanodisk resonances is described by the model illustrated in Fig. 1(e). Motion of the readout nanobeam connected to the nanodisk is converted to an optical signal through its dispersive cavity optomechanical coupling quantified by $g_{OM}$. The dynamics of coupled nanobeam and readout nanobeam are described by equations of motion for the displacement amplitudes $x_d$ and $x_b$ of the nanodisk and readout nanobeam resonances, respectively:

$$
\ddot{x}_d = -\omega_d^2 x_d - \gamma_d \dot{x}_d + \sqrt{\frac{m_b}{m_d}} g_M^2 x_b + \frac{\tau_t}{I_d} f(t), \\
\ddot{x}_b = -\omega_b^2 x_b - \gamma_b \dot{x}_b + \sqrt{\frac{m_d}{m_b}} g_M^2 x_d,
$$

where $\omega_d$, $\gamma_d$, $m_d$, and $\omega_b$, $\gamma_b$, $m_b$, are the frequency, damping rate, and mass, respectively, of each mechanical resonator. These equations include an external torsional drive $f(t)$ with an amplitude $\tau_t$ applied to the nanodisk, whose moment of inertia $I_d$ is described by an effective mass $m_{\text{eff}}$ and effective radius $r_{\text{eff}}$ that is determined from the moment of inertia of the device, which relates the device’s angular velocity to its energy, and is defined in [22]. Note that for generality an arbitrary drive is used here, where $f(t)$ is a real function. The mechanical susceptibilities $\chi_{d,b}(\omega) = (\omega_d^2 - \omega^2 - i\gamma_d, \omega) - 1$ and mechanical coupling $g_M$ determine how effectively this applied torque is converted into motion of the readout nanobeam. The nanobeam amplitude can be derived by Fourier transforming Eqs. 1 and solving for $x_b$ to obtain,

$$
x_b(\omega) = \frac{g_M^2}{\sqrt{\frac{m_b}{m_d}} (\chi_d(\omega) \chi_b(\omega) - 1) - g_M^2} \frac{\tau_t}{I_d} f(\omega).$$

Figure 1: (a) Finite element simulation of an in-plane hybridized torsional resonance of the device. (b) SEM image of the device. Finite difference time domain simulation of electric field profile of the zipper cavity: (c) fundamental bonded mode with $\lambda = 1559 \text{nm}$, $Q_o = 1.8 \times 10^7$ and (d) fundamental anti-bonded mode with $\lambda = 1530 \text{nm}$, $Q_o = 9.2 \times 10^7$. (e) The torque sensing device (represented inside the yellow box) can be modeled as a coupled oscillator system where the nanodisk and readout nanobeam are mechanically coupled. The readout nanobeam forms one half of a photonic crystal zipper cavity. Motion of both the readout nanobeam and the auxiliary nanobeam of the cavity are optomechanically coupled to the cavity mode. Dynamics of each mode are described by frequency $\omega$ and damping rate $\gamma$. Mechanical coupling between the nanodisk and readout nanobeam resonances is determined by $g_{OM}$. The cavity optomechanical coupling is described by $g_{OM}$.

tured optical fields [19]. Its design addresses limitations of previous photonic crystal torque sensors that detect cantilever-like motion of split-beam cavities [15], extending both the frequency range and nature of mechanical resonances that can be routinely measured. This is accomplished by resonantly coupling a wide spectrum of high frequency mechanical resonances of the nanodisk to mechanical resonances of the zipper cavity. The zipper cavity combines high optical quality factor $Q_o$ with large optomechanical coupling. It is formed following the fabrication process in Ref. [15]. The suspended photonic crystal nanobeams (width 500 nm, thickness 220 nm) are separated by a gap 70 nm and patterned with holes of radius 110 nm. The hole spacing in each nanobeam tapers quadratically from $a_o = 400 \text{nm}$ in the cavity’s
Equation 2 shows that the resonantly driven transduced amplitude is enhanced,

$$|x_b(\omega_m)| = \frac{g_M^2}{\sqrt{m_m} \sqrt{\gamma_d \gamma_b \omega_m^2 + g_M^2}} \frac{\tau_i}{I_d}, \quad (3)$$

when the nanodisk and readout nanobeam are resonant, i.e., ($\omega_d = \omega_b = \omega_m$). Here, for simplicity we have chosen the drive component to be unity. They also show that the mechanical mode frequencies become renormalized, as expected for a coupled system, with a frequency difference $\Delta \omega^2 \approx 2g_M^2$. The coupling strength $g_M$ depends on the device geometry and modes of interest. It can be determined by calculating the mode spectrum of the coupled resonators for varying device parameters such that the nanodisk and nanobeam modes are tuned through resonance. An example of this approach is shown in Fig. 2. We first show in Fig. 2(a) the mechanical frequencies of the individual components of the device for varying length $l_s$ of the support beams anchoring the suspended nanodisk. We see that the nanobeam’s in-plane flexural resonance frequencies remain constant, as expected since the nanobeam geometry is not affected by the support length, while the resonance frequencies of the nanodisk’s in-plane twisting modes decrease quadratically with increasing $l_s$. In-plane twisting modes can be efficiently actuated by torque and for simplicity are referred to below as torsional modes.

At certain $l_s$ values, the mechanical resonances of the uncoupled nanodisk and readout nanobeam are resonant and the resonance frequencies are renormalized by the mode coupling. This is shown in Fig. 2(b), which plots the full device’s simulated mode spectrum for varying $l_s$. The anti-crossing behaviour at points where the nanodisk and readout nanobeam modes are resonant is a signature of coupling between them. This mechanical coupling is the key feature that makes transduction of torsional actuation of nanodisk to in-plane flexural motion of readout nanobeam possible. Figure 2(c) shows $g_M$ values extracted from the anti-crossing widths as a function of $l_s$ for the first three harmonics of the nanodisk’s twisting resonances. The trend of increasing $g_M$ with increasing mechanical frequency suggests that higher frequency nanodisk resonances have better overlap with the nanobeam resonances at their clamping point. Similar increases in coupling between modes with increasing frequency are predicted when considering phononic clamping loss in nanomechanical resonators [33].

Optically detecting displacement of the nanobeam from torque applied to the nanodisk is dependent on the optomechanical coupling strength between the hybridized mode and the photonic crystal cavity. To predict the optomechanical coupling for a given torque, we numerically calculate the mechanical displacement of the full device per unit applied external torque and the resulting torsional optomechanical frequency shift $\Delta \omega \propto \frac{1}{g_M^2 g_{OM}}$. This shift, which captures both the device’s cavity optomechanical coupling and the mechanical mode coupling, is shown in Fig. 2(d) for $l_s$ swept through a typical anti-crossing, and the simulated mechanical displacement profiles at several points within the anti-crossing are shown Fig. 2(e). For the lower frequency hybridized mode, $\Delta \omega_s$ is maximized at the center of the anti-crossing (II), and it decreases away from this point (I, IV). The higher frequency hybridized mode does not offer an enhancement at the anti-crossing (III) due to its displacement profile: it has a node in the center of the nanobeam where the optical cavity field is localized. For our cavity design, nanobeam modes with an even number of anti-nodes have small $\Delta \omega_s$ compared to modes with an odd number of anti-nodes.

To demonstrate the efficacy of the mode coupling effects discussed above, we experimentally study a range of devices with varying $l_s$. We optically measure the optomechanically transduced mechanical modes of our fabricated devices using a dimpled optical fiber taper waveguide [34] to evanescently couple light into and out of the cavity. Figure 3(a) shows the transmission profile of an optical cavity mode from a typical device. The measured mode has $Q_o \approx 160,000$ and a wavelength in the 1500 nm wavelength range. In a sideband unresolved system where the cavity field instantaneously follows the nanobeam’s mechanical motion, the change in the cavity’s optical intensity response from an applied torque is $\propto \tau(\omega_o) \Delta \omega_o dH/d\omega_o$. Here $\omega_o$ is the frequency of the probe laser input to the cavity, and $dH/d\omega_o$ is the slope of the cavity mode lineshape. Due to the device’s high sensitivity arising from its high-$Q_o$, large $\Delta \omega_o$, and low mass and associated large mechanical susceptibility (see Eq. (2)), thermomechanical driven motion of the mechanical resonances can be observed. Figure 3(b) shows a typical mechanical spectrum of a device ($l_s = 5.5 \mu m$). A spectrum analyzer is used to convert the temporal fluctuations in fiber-taper transmission to a power spectral density. Peaks in the spectrum correspond to thermally driven motion of the nanobeam’s mechanical resonances. Comparing the frequencies of the peaks in the spectrum with simulated values, we identified the hybridized torsional modes shown in Figure 3(b) at frequencies that range from 10.44 – 784.6 MHz. Following the process in [15], one can extract the torque sensitivity of these measurements, finding $9.1 \times 10^{-22} - 2.4 \times 10^{-19} \text{Nm/}\sqrt{\text{Hz}}$ depending on the mechanical mode. Note that these measurements were made at room temperature and in ambient pressure. Operating in vacuum we expect the mechanical quality factors to increase, which will enhance the torque sensitivity [15].

To further study the hybridization between the nanodisk and nanobeam resonances, the optomechanical spectrum was measured for devices with varying $l_s$. Figure 4 shows this measurement for $l_s$ swept from 1 $\mu m$ to 10 $\mu m$ in increments of 1 $\mu m$. As shown by the red trend line, the torsional peaks shift with the expected anti-crossing pattern predicted by the theory discussed in Figs. 2(a) and 2(b). Note that each measurement involves a unique device, and the contribution to the opti-
Figure 2: (a) Individual simulated mechanical in-plane flexural resonances of readout nanobeam (magenta) and torque actuated mechanical resonances of nanodisk (orange, green, cyan - for first three resonances respectively, and blue - for rest higher order resonances) as a function of the length of nanodisk’s support beams $l_s$. See the legend on the top. (b) Simulated eigenfrequency of the in-plane hybridized torsional resonances of the device as a function of $l_s$. (c) Calculated mechanical coupling strengths $g_M$ of a typical device for first three harmonics of nanodisk’s resonances. (d) A zoom in of the region highlighted by a red box in (b) showing the calculated optomechanical frequency shift per unit applied torque as indicated by the color scale. (e) The displacement profiles of four mechanical resonances with different $l_s$, labeled I-IV in (d). The dashed grey line suggests the displacement profile at the center of the readout nanobeam, where the optical cavity is localized.

cal transduction related to the optical cavity mode and its coupling to the fiber taper will vary between devices.

To compare the measured results with theory more quantitatively, Fig. 5 plots the frequencies of the hybridized torsional resonances measured in Fig. 4 as a function of support length, together with the simulated values. The experimentally observed frequency dependence on support length is in good agreement with the simulated values, with variations most likely attributed to fabrication imperfections. Most crucially, the width of the observed anticrossings closely match the predicted values. It is worth noting that peaks corresponding to nanobeam flexural resonances with an even numbers of anti-nodes do not appear in the spectrum. This can be understood from the fact that optomechanical couplings for these modes is near zero due to their odd symmetry around the center of the optical cavity.

The device demonstrated here shows how the high sen-
Figure 3: (a) Measured optical transmission profile of the torque sensing device. (b) Optomechanical spectrum of the device in ambient conditions. Peaks correspond to thermally driven Brownian motion of the device’s mechanical resonances. Peaks associated with torsional modes of the disk are identified.

Figure 4: Measured mechanical spectrum of the devices for varying support length. The signal strength is color coded. The red dashed line is guide to the eye that follows the moving torsional peaks.

Figure 5: Measured mechanical frequencies of the devices with varying support length (blue). The small red dots are simulated eigenfrequencies of the devices.

Sensitivity of nanoscale optomechanical cavities can be harnessed to detect motion of nanomechanical resonators not integrated directly within the optical cavity. By coupling remote and optically dark mechanical resonances to resonators integrated within a photonic crystal nanocavity, mechanical resonances with a wide range of frequencies, mode profiles and symmetries can be sensitively transduced. Since the nanomechanical resonator does not interact directly with the cavity field, it can be decorated with optically lossy metals or magnetic materials. This is of particular interest for advancing the performance and frequency range of optomechanical torque magnetometers used to probe nanomagnetic properties of permalloy. To this end, the device has demonstrated torque sensitivity of $9.1 \times 10^{-22} - 2.4 \times 10^{-19} \text{Nm/} \sqrt{\text{Hz}}$ for a mechanical frequency range of $10 - 800 \text{MHz}$. Previous
nanoscale cavity optomechanical torque sensing devices \cite{15, 18} achieve similar or lower sensitivities, but were limited to frequencies below 10 MHz. This sensitivity can be improved by orders of magnitude by performing the experiment in ultra high vacuum and cryogenic temperatures where mechanical quality factors in silicon devices can reach $Q_m > 10^8$ \cite{35}. Operation at higher (> GHz) frequencies may be possible using this approach by combining torque actuated mechanical resonators with optomechanical crystal cavities and waveguides \cite{30}.

In the near future, the device’s ability to operate at frequencies $> 100$ MHz will enable high frequency torque magnetometry studies of nanomagnetic systems, opening the door to observation of the Einstein-de Haas effect \cite{25, 26}, as well as resonant coupling between nanomagnetic spin dynamics and the optomechanical system. When integrated with magnetic materials it will also become possible to generate effective couplings between light and magnetic spins, with applications in development of hybrid quantum systems \cite{37}.

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\begin{thebibliography}{99}
\bibitem{1} M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Rev. Mod. Phys. \textbf{86}, 1391 (2014). URL \url{http://link.aps.org/doi/10.1103/RevModPhys.86.1391}
\bibitem{2} J. Xia, Q. Qiao, G. Zhou, F. S. Chau, and G. Zhou, Applied Sciences \textbf{10}, 7080 (2020).
\bibitem{3} X. Sun, J. Zhang, M. Poot, C. Wong, and H. Tang, Nano Lett. \textbf{12}, 2299 (2012).
\bibitem{4} E. Gavartin, P. Verlot, and T. Kippenberg, Nat. Nano. \textbf{7}, 509 (2012).
\bibitem{5} J. Gomis-Bresco, D. Navarro-Urrios, M. Oudich, S. El-Jallal, A. Griol, D. Puerto, E. Chavez, Y. Penneec, B. Djafari-Rouhani, F. Alzina, et al., Nature communications \textbf{5}, 1 (2014).
\bibitem{6} C. P. Ho, P. Pitchappa, B. W. Soon, and C. Lee, Optics express \textbf{23}, 10598 (2015).
\bibitem{7} K. Han, K. Zhu, and G. Bahl, Applied Physics Letters \textbf{105}, 014103 (2014).
\bibitem{8} B.-B. Li, L. Ou, Y. Lei, and Y.-C. Liu, Nanophotonics \textbf{10}, 2799 (2021).
\bibitem{9} X. Liu, W. Liu, Z. Ren, Y. Ma, B. Dong, G. Zhou, and C. Lee, International Journal of Optomechatronics \textbf{15}, 120 (2021).
\bibitem{10} A. G. Krause, M. Winger, T. D. Blasius, W. Lin, and O. Painter, Nat. Photon. \textbf{6}, 768 (2012).
\bibitem{11} S. Basiri-Esfahani, A. Armin, S. Forstner, and W. P. Bowen, Nature communications \textbf{10}, 1 (2019).
\bibitem{12} M. Sansa, M. Defoort, A. Brenac, M. Hermouet, L. Baniard, A. Faïn, M. Gely, C. Masselon, I. Favero, G. Jourdan, et al., Nature Communications \textbf{11}, 1 (2020).
\bibitem{13} J.-J. Li and K.-D. Zhu, Applied Physics Letters \textbf{101}, 141905 (2012).
\bibitem{14} Y. Liu, H. Miao, V. Aksyuk, and K. Srinivasan, Opt. Express \textbf{20}, 18268 (2012).
\bibitem{15} M. Wu, A. C. Hryciw, C. Healey, D. P. Lake, H. Jayakumar, M. R. Freeman, J. P. Davis, and P. E. Barclay, Phys. Rev. X \textbf{4}, 021052 (2014). URL \url{http://link.aps.org/doi/10.1103/PhysRevX.4.021052}
\bibitem{16} J. G. Huang, H. Cai, Y. Gu, L. Chin, J. Wu, T. Chen, Z. Yang, Y. Hao, and A. Liu, Applied Physics Letters \textbf{111}, 111102 (2017).
\bibitem{17} L. He, H. Li, and M. Li, Science advances \textbf{2}, e1600485 (2016).
\bibitem{18} P. Kim, C. Doolin, B. Hauer, A. MacDonald, M. Freeman, P. Barclay, and J. Davis, Appl. Phys. Lett. \textbf{102}, 053102 (2013).
\bibitem{19} H. Kaviani, R. Ghobadi, B. Behera, M. Wu, A. Hryciw, S. Vo, D. Fattal, and P. Barclay, Optics Express \textbf{28}, 15482 (2020).
\bibitem{20} M. Wu, N. L.-Y. Wu, T. Firdous, F. F. Sani, J. E. Losby, M. R. Freeman, and P. E. Barclay, Nature nanotechnology \textbf{12}, 127 (2017).
\bibitem{21} P. Kim, B. Hauer, T. Clark, F. F. Sani, M. Freeman, and J. Davis, Nature Communications \textbf{8}, 1355 (2017).
\bibitem{22} P. Kim, B. Hauer, C. Doolin, F. Souris, and J. Davis, Nature communications \textbf{7}, 1 (2016).
\bibitem{23} W. Jiang, C. J. Sarabalis, Y. D. Dahmani, R. N. Patel, F. M. Mayor, T. P. McKenna, R. Van Laer, and A. H. Safavi-Naeini, Nature Communications \textbf{11}, 1 (2020).
\bibitem{24} G. Hajsalem, J. E. Losby, G. de Oliveira Luiz, V. T. Sauer, P. E. Barclay, and M. R. Freeman, New Journal of Physics \textbf{21}, 095005 (2019).
\bibitem{25} J. E. Losby, V. T. Sauer, and M. R. Freeman, Journal of Physics D: Applied Physics \textbf{51}, 483001 (2018).
\bibitem{26} K. Mori, M. Dunsmore, J. Losby, D. Jenson, M. Belov, and M. Freeman, arXiv preprint arXiv:2005.08406 (2020).
\bibitem{27} T. Delord, P. Huillery, L. Nicolas, and G. Hétet, Nature \textbf{580}, 56 (2020).
\bibitem{28} T. Bagci, A. Simonsen, S. Schmid, L. G. Villanueva, E. Zeuthen, J. Appel, J. M. Taylor, A. Sørensen, K. Usami, A. Schiesser, et al., Nature \textbf{507}, 81 (2014).
\bibitem{29} A. Pearson, K. Khosla, M. Mergenthaler, G. A. D. Briggs, E. Laird, and N. Ares, Scientific reports \textbf{10}, 1 (2020).
\bibitem{30} J. Chan, M. Eichenfeld, R. Camacho, and O. Painter, Opt. Express \textbf{17}, 3802 (2009).
\bibitem{31} M. Eichenfeld, R. Camacho, J. Chan, K. J. Vahala, and O. Painter, Nature \textbf{459}, 550 (2009).
\bibitem{32} B. Hauer, C. Doolin, K. Beach, and J. Davis, Annals of Physics \textbf{339}, 181 (2013).
\bibitem{33} I. Wilson-Rae, Phys. Rev. B \textbf{77}, 245418 (2008), URL \url{https://link.aps.org/doi/10.1103/PhysRevB.77.245418}
\end{thebibliography}
[34] C. P. Michael, M. Borselli, T. J. Johnson, C. Chrystal, and O. Painter, Opt. Express 15, 4745 (2007).

[35] G. S. MacCabe, H. Ren, J. Luo, J. D. Cohen, H. Zhou, A. Sipahigil, M. Mirhosseini, and O. Painter, Science 370, 840 (2020), https://www.science.org/doi/pdf/10.1126/science.abc7312, URL https://www.science.org/doi/abs/10.1126/

[36] K. Fang, J. Luo, A. Metelmann, M. H. Matheny, F. Marquardt, A. A. Clerk, and O. Painter, Nature Physics 13, 465 (2017).

[37] M. Wallquist, K. Hammerer, P. Rabl, M. Lukin, and P. Zoller, Phys. Scr. T 137, 014001 (2009).