Experimental research of nonlinear damage diagnosis using ARMA/GARCH method

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Abstract. Nonlinear damage such as fatigue crack often occurs in some engineering structures, which brings challenges in structural damage diagnosis. A nonlinear damage diagnosis method based on ARMA/GARCH model is proposed to extract the nonlinear characteristic caused by cracks. First, the background description of ARMA/GARCH model is given and a damage diagnosis index based on this model is proposed to detect nonlinear damage location. The main advantage of ARMA/GARCH model is that this hybrid model can well simulate structural time domain acceleration response and effectively extract the nonlinear damage feature. An experiment of three-storey frame structure, which utilized a mechanism constituted of a bumper and a center column to model breathing crack, is studied. The results show that the damage diagnosis index based on ARMA/GARCH model exhibits good accuracy.

Keywords: damage diagnosis; crack; time series model; acceleration response; nonlinearity

1. Introduction

Structural damage diagnosis has received much attention in recent twenty years. The feasibility of vibration-based damage diagnosis is that the damage usually changes the structural characteristics like stiffness, which are reflected in the measured vibration signals of the structure [1]. Time-domain analysis method is an important damage diagnosis method, which can construct time series model to simulate acceleration response data. Time-domain analysis method has attracted attention in recent years and damage is diagnosed by extracting damage features from the constructed time series models[2].

Time series models can be divided into two categories: linear models and nonlinear models. Linear models include AR model, ARX model and ARMA model etc., which can be utilized to address the linear system problem. Liu et al [3] presented a non-probabilistic damage diagnosis approach based on coefficient matrix of AR model to find diagnosis elements and utilized Mahalanobis distance of the diagnosis elements to identify damage. Damage such as fatigue crack usually possesses time domain nonlinear characteristic caused by opening and closing of crack due to dynamic loading [4]. Nonlinear models include ARCH, GARCH etc., which can be used to address the nonlinear system problem. Pham et al. [5] proposed a hybrid model of ARMA and GARCH to diagnose the machine fault state using measurement vibration signals. Cheng et al. [6] employed a hybrid AR/ARCH model to detect nonlinear damage sites of an eight-storey shear structure and proposed a second-order-variance index. In this paper, a nonlinear damage diagnosis method based on ARMA/GARCH model is proposed to identify the nonlinear characteristic caused by cracks.
2. ARMA/GARCH Model and its application to damage diagnosis

2.1. ARMA/GARCH Model

ARMA models can be utilized to describe structural acceleration time series and evaluate conditional mean. The ARMA \((p, q)\) model for conditional mean can be written as

\[
y_t = c + \sum_{i=1}^{p} \varphi_i y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t
\]

in which \(y_t\) is the time series data, \(c\) is a constant, \(p\) is the number of autoregressive order, \(\varphi_i\) is autoregressive parameter, \(q\) is the number of moving average order, \(\theta_j\) is moving average parameter, \(\varepsilon_t\) is the residual. Generally, the residual is assumed to be zero mean and constant variance. However, for some practical time domain dynamic responses, the residual does not meet the homoscedastic assumption of constant variance. The time-varying variance is named as conditional variance. The conditional variance \(\sigma_t^2\) of the residual \(\varepsilon_t\) is given by

\[
\sigma_t^2 = \text{Var}_{t-1}(\varepsilon_t) = E_{t-1}\left(\varepsilon_t^2\right)
\]

A GARCH \((r, m)\) model for the conditional variance of residual \(\varepsilon_t\) is

\[
\sigma_t^2 = s + \sum_{i=1}^{r} B_i \sigma_{t-i}^2 + \sum_{j=1}^{m} A_j \varepsilon_{t-j}^2
\]

where \(s, B_i(i=1,\ldots,r)\) and \(A_j(j=1,\ldots,m)\) are the parameters of GARCH model. The values of these parameters are constrained by

\[
s > 0, B_i \geq 0(i = 1,\ldots,r), A_j \geq 0(j = 1,\ldots,m)
\]

The first three constraints ensure that the conditional variances are positive, and the fourth constraint ensures the covariance stationarity. Thus, the hybrid ARMA/GARCH model can be described as follows:

\[
y_t = c + \sum_{i=1}^{p} \varphi_i y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t \quad \varepsilon_t \sim N(0, \sigma_t^2)
\]

\[
\sigma_t^2 = s + \sum_{i=1}^{r} B_i \sigma_{t-i}^2 + \sum_{j=1}^{m} A_j \varepsilon_{t-j}^2
\]

\[
\varepsilon_t = \sigma_t z_t, \quad z_t \sim N(0,1)
\]

2.2. Nonlinear damage diagnosis

Here, we consider two states: undamaged state and damaged state. The mainly steps applying the ARMA/GARCH model to calculate nonlinear damage index are summarized as follows:

Firstly, acquire time series acceleration responses under undamaged and damaged states.

Secondly, construct the ARMA models using structural acceleration responses, estimate parameters in the ARMA models and obtain the residual sequences \(\{\varepsilon_t^U\}\) and \(\{\varepsilon_t^D\}\) under undamaged and damaged states, respectively.

Thirdly, test heteroscedastic effect of the residual sequences \(\{\varepsilon_t^U\}\) and \(\{\varepsilon_t^D\}\). Then, construct the GARCH models using the residual sequences and obtain the conditional variance sequences \(\{\sigma_t^{2U}\}\) and \(\{\sigma_t^{2D}\}\) of the residual sequences and further obtain the variance of the conditional variance sequences under undamaged and damaged states.
At last, establish the damage index based on ARMA/GARCH model. For undamaged structure, the acceleration response of a structure can be approximately regarded as linear data and can be adequately captured by the ARMA model. The residual sequences generated from ARMA model are considered as approximate white noise processes. Thus, the fluctuation of the conditional variances sequence obtained from the GARCH model is close to 0. On the other hand, when there is nonlinear damage occurring, the conditional variances sequences will vary with time. The variance of the conditional variances sequences is much greater than 0. Therefore, for an $n$-storey structure, the damage index based on ARMA/GARCH model is proposed as follows:

$$c_i = \frac{s_i \left\{ \sigma_i^{2_o} \right\} - s_i \left\{ \sigma_i^{2_v} \right\}}{\sum_{i=1}^{n} s_i \left\{ \sigma_i^{2_o} \right\} - s_i \left\{ \sigma_i^{2_v} \right\}}$$ (7)

3. Experiment
An experiment of three-storey frame structure was carried out to verify the effectiveness of the proposed method. The frame structure with sensors [7] is shown in Fig. 1. A mechanism constituted of a bumper and a center column is utilized to simulate breathing crack and it will induce nonlinear behaviors when the center column contacts the bumper mounted on the second floor. The distance between the bumper and the center column is adjustable. Thus, the greater the distance is, the smaller the nonlinear damage becomes.

Accelerometers are installed on each floor to record time series data. The channels of sensors and parameters are listed in Table 1. The acceleration time series data of the 1st-storey, 2nd-storey and 3rd-storey of the structure are the measurement data of Channels 3, 4 and 5. The damaged scenarios are listed in Table 2. The greater the gap distance is, the smaller the nonlinear damage degree becomes. Therefore, from damage scenarios 1 to 6, the nonlinear damage degree gradually decreases. If the distance is great enough and the center column doesn’t contact the bumper in the vibration process, we think that there isn’t nonlinear damage behavior. Thus, scenarios 5 and 6 can be regarded as weak nonlinearity problem. The damage scenario 6 considers the disturbance of additional mass, which can simulate the model error. The acceleration responses in undamaged state are depicted in Fig. 2, and the acceleration responses in damaged scenario 1 are depicted in Fig. 3. Akaike information criteria are adopted for order determination of the hybrid model. Here, for the hybrid model, $p$ is 30, $q$ is 0, $r$ is 0, and $m$ is 5.
Fig. 2. Acceleration responses in undamaged State.

Fig. 3. Acceleration responses in damaged scenario 1.

Table 1. Sensor channels and parameters

| Channels | Sensor       | Parameter         | Sensitivity  |
|----------|--------------|-------------------|--------------|
| Channel 1 | Load cell    | PCB 208 C03 SN 22569 | 2.2 mV/N    |
| Channel 2 | Accelerometer| PCB 336C SN 10099 | 1000 mV/g   |
| Channel 3 | Accelerometer| PCB 336C SN 10120 | 1000 mV/g   |
| Channel 4 | Accelerometer| PCB 336C SN 9916  | 1000 mV/g   |
| Channel 5 | Accelerometer| PCB 336C SN 10100 | 1000 mV/g   |
### Table 2 Damage scenarios of three-storey frame structure

| Damaged scenarios | Description                   |
|-------------------|-------------------------------|
| Scenario 1        | Gap is 0.05 mm                |
| Scenario 2        | Gap is 0.10 mm                |
| Scenario 3        | Gap is 0.13 mm                |
| Scenario 4        | Gap is 0.15 mm                |
| Scenario 5        | Gap is 0.20 mm (weak nonlinearity) |
| Scenario 6        | Gap (0.20 mm) and mass (1.2 kg) at the base (weak nonlinearity) |

For damaged scenarios 1-3, acceleration response signals are used to realize nonlinear damage diagnosis. The diagnosis results for damaged scenarios 1-3 are shown in Fig. 4. It can be seen that the calculation values of the 2nd and 3rd storeys are obviously higher than those of the 1st storey, which means that the third inter-storey stiffness next to the 2nd and 3rd storeys is the damage location. The index values of two storeys next to damaged inter-storey stiffness decrease along with the gap distance increasing from scenario 1 to scenario 3 (which means that the degree of nonlinear damage decreases). Therefore, the proposed damage index has the capacity of detecting damage severity.

For damaged scenarios 4-6, the damage index based on ARMA/GARCH model is applied to nonlinear damage extraction. The diagnosis results for damaged scenarios 4-6 are shown in Fig. 5. We can see that the calculation values of the 2nd and 3rd storeys are still higher than those of the 1st storey, which means that the third inter-storey stiffness is the damage location. The index values of the 2nd and 3rd storeys decrease along with the gap distance increasing from scenario 4 to scenario 5, which also implies the method has the potential to quantify the degree of nonlinear damage. In addition, from scenario 6 we can find that the disturbance of additional mass also reduces the diagnosis accuracy.

![Fig. 4. Nonlinear damage diagnosis results for scenarios 1-3.](image-url)
Fig. 5. Nonlinear damage diagnosis results for scenarios 4-6.

4. Conclusions
This paper proposed a damage index based on ARMA/GARCH model to identify nonlinear damage. The basic theories and main procedure are described in this paper and a three-storey frame structure experiment is employed to explore the effectiveness of the proposed method. The results showed that the damage index has good performance to locate the nonlinear damage and has the potential to reflect the changed degree of nonlinear damage caused by adjustable gap distance. In addition, due to the fact that the damage index is only rely on the output acceleration response data, it has the possibility to combine with continuous online monitoring to monitor the health condition of structure in real time.

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5. References
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