Tristability in viscoelastic flow past side-by-side microcylinders

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Viscoelastic flows through microscale porous arrays exhibit complex path-selection and switching phenomena. However, understanding this process is limited by a lack of studies linking between a single object and large arrays. Here, we report experiments on viscoelastic flow past side-by-side microcylinders with variable intercylinder gap. With increasing flow rate, a sequence of two imperfect symmetry-breaking bifurcations forces selection of either one or two of the three possible flow paths around the cylinders. Tuning the gap length through the value where the first bifurcation becomes perfect reveals regions of bi- and tristability in a dimensionless flow rate-gap length ‘phase’ diagram.

Since the advent of microfluidics in the early 2000s [1, 2], geometries with length-scales $\ell \sim O(100 \, \mu m)$ have become a vital tool in experimental fluid dynamics. At the microscale, viscoelastic fluids (with properties between viscous liquids and elastic solids) can flow with negligible inertia (Reynolds number $Re \sim \ell \ll 1$), but high elasticity (Weissenberg number $Wi \sim \ell^{-1} \gg 1$) [2]. In such flows, elasticity becomes the dominant source of nonlinearity, leading to instabilities [3–9], and time-dependency that impact widespread processes ranging from jet fragmentation [10–11] to hemodynamics [12, 13] and porous media flows [14–20]. In particular, the path selection and switching phenomena in viscoelastic porous media flow is considered of fundamental importance in processes such as enhanced oil recovery, groundwater remediation, filtration, and drug delivery [14–19].

Porous media are frequently modeled by ordered and disordered arrays of microfluidic circular cylinders [13–19]. Flow past a single circular cylinder in a channel is an archetypal problem in fluid dynamics, and a ‘benchmark’ for studying viscoelastic flows. The stagnation point downstream of a cylinder is a location where streamline curvature combines with strong velocity gradients; conditions that render viscoelastic base flows prone to instability and downstream fluctuations [21–24]. For fluids with a shear-rate-dependent viscosity (i.e., shear-thinning), the perturbation to the base flow can lead to a steady symmetry-breaking flow bifurcation where the viscoelastic fluid selects a preferred path around one, or other, side of the cylinder [5, 7, 23]. This behavior has clear relevance to understanding transport through porous arrays, but the interaction with neighboring array elements is lacking. Building ‘bottom-up’ complexity towards more realistic model systems, it is natural to consider two cylindrical objects either aligned in the flow direction, or positioned side-by-side in a channel. Viscoelastic flow past two (or more) objects aligned on the flow axis is a well studied problem (e.g., Refs. 8, 23, 26, and 27). However, although equally important, the case of two objects positioned transverse to the flow has received scant attention, with only one numerical study conducted at high Reynolds number [28]. To date, creeping viscoelastic flow past side-by-side cylinders has not been studied.

In this Letter, we present microfluidic experiments of a viscoelastic shear-thinning fluid flowing past two microcylinders transverse to the primary flow direction (Fig. 1) and show that the resulting non-linear flow behavior at high $Wi$ is significantly influenced by the spacing of the cylinders. We show that, due to a combination of super-critical bifurcations that occur as $Wi$ is varied, multiple stable flow states are possible in a given geometry. This is the first study of low-Re viscoelastic flow in such a geometry and serves as a fundamental contribution towards understanding deterministic path-selection in porous media flow.

The model viscoelastic fluid is a well-studied aqueous wormlike micellar (WLM) solution consisting of 100 mM cetylpyridinium chloride (CPyCl) and 60 mM sodium salicylate (NaSal) [29, 30]. At 24°C (ambient laboratory temperature), the entangled WLM solution has a zero shear viscosity $\eta_0 = 47$ Pa s, exhibits a stress-plateau (shear-banding region [31]), and in small-amplitude oscillation is well-described by a single-mode Maxwell model with relaxation time $\lambda = 1.7$ s (Fig. S1 [32]).

Microfluidic channels (Fig. 1) were fabricated in fused silica by selective laser-induced etching [33]. The eleven channels used all have a rectangular cross-section with width $W = 400 \, \mu m$ transverse to the flow (y-direction) and height $H = 2000 \, \mu m$ in the neutral (z) direction. Each channel contains two cylinders of radius $R = 20 \, \mu m$ equally spaced either side of the primary flow (x) axis. The intercylinder separation $L_1$ is varied between chan-

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1}
\caption{Schematic diagram of the $x-y$ plane of the microfluidic channels. Flow is left-to-right at volumetric rate $Q$.}
\end{figure}
nals in the range $10^7 < L_1 < 147 \, \mu m$. The spacing between the cylinders and the channel sidewalls is $L_2 = (W - L_1 - 4R)/2$, and we define a dimensionless gap ratio $G = L_1/(L_1 + L_2)$. This parameter in principle spans $0 < G < 1$, where $G = 0$ implies the two cylinders are touching at the channel centerline, while $G = 1$ implies the cylinders are touching opposite channel walls. The channels used span a range $0.50 \leq G \leq 0.62$, which encompasses the full range of flow behavior.

Flow is driven by syringe pumps (Cetoni GmbH) programmed to impose quasistatic variations in the volumetric flow rate $Q$, hence average flow velocity $U = Q/WH$, and Weissenberg number $Wi = \lambda U/R$. Quantitative spatially-resolved flow fields are obtained using micro-particle image velocimetry ($\mu$-PIV, TSI Inc., MN [34, 33]). At each imposed $Wi$, the motion of a low concentration of fluorescent seeding particles (2 \( \mu m \) diameter) is captured at the channel half-height ($z = 0$ plane) using an inverted microscope (Nikon Ti) with a 5\( \times \), NA = 0.15 numerical aperture objective lens and a high speed camera (Phantom Miro) working in frame-straddling mode at 25 Hz. Cross-correlation between images yields velocity vectors $\mathbf{u} = (u, v)$. Since the flows examined are all time invariant, data are ensemble averaged over a 6 s sampling window. Due to shear-localization at the channel walls, the flow profile is essentially plug-like over most of the channel cross-section [6]. Therefore, the shear-rate near the cylinders is small and we define $Re = \rho UR/\eta_0$, where $\rho = 1000 \, kg \, m^{-3}$ is the fluid density. In all experiments, $Re \lesssim 10^{-4}$.

Flow fields representative of those observed as $Wi$ is varied are shown in Fig. 2 using two channels with contrasting $G$. Fig. 2a-c and Fig. 2d-f illustrate the behavior for 'small' and 'large' $G$, respectively. Irrespective of $G$, for low $Wi < Wi_1 \approx 15$ (Figs. 1d,1), elastic and inertial forces are small and the flow is dominated by the viscous force. Flow is approximately symmetric about $x = 0$ and $y = 0$, and fluid passes through all three available gaps. For 'small' $G = 0.500$, as $Wi$ exceeds $Wi_1$ (Fig. 2b), elasticity dominates and the system undergoes a first transition from the low-$Wi$ symmetric state to a diverging 'D' state where the fluid avoids the gap between the cylinders and flows symmetrically around their sides. The velocity field is qualitatively similar to that observed for viscoelastic shear-thinning fluids flowing around a single cylinder [4, 27]. For a Newtonian fluid, two objects appear as one when the ratio of separation to radius, $L_1/R < 0.2$ [36], whereas here the ratio is much greater at $L_1/R > 5$. Further increasing $Wi$, the system undergoes a second transition at $Wi_2 \approx 50$ to an asymmetric-diverging 'AD' state in which the fluid selects a single preferred path either above ($y > 0$, Fig. 2c1), or below ($y < 0$, Fig. 2c2) the pair of cylinders. This randomly chosen bias is also similar to that observed for viscoelastic shear-thinning fluids flowing around a single cylinder [4, 7, 25].

For 'large' $G = 0.603$, the first transition at $Wi > Wi_1$ results in a converging 'C' flow state where the fluid flows preferentially between the cylinders, avoiding the gaps at their sides (Fig. 2a). In contrast to the small-$G$ case, as $Wi$ increases there is no second transition at $Wi_2$ and the C state is maintained until the flow eventually becomes time-dependent at $Wi \gg Wi_1$. The nature of the time-dependence is qualitatively similar to that previously reported for a single cylinder [4] and will not be discussed further here. We note that results for small and large $G$ in Fig. 2 using a shear-thinning, but non-shear-banding viscoelastic polymer solution, show analogous flow behavior see Figs. S2 and S3 [32].

We quantify the critical flow behavior using two dimensionless flow asymmetry parameters $I'$ and $I''$:

$$I' = \frac{1}{2}(\bar{u}_+ + \bar{u}_-) - \bar{u}_0$$

and

$$I'' = \frac{\bar{u}_+ - \bar{u}_-}{\bar{u}_+ + \bar{u}_- + \bar{u}_0}. \quad (1)$$

Here, $\bar{u}_+$, $\bar{u}_-$, and $\bar{u}_0$ are the average values of $u$ in the upper, lower, and intercylinder gaps, respectively (see Fig. 1). $I'$ serves as the order parameter to quantify the first transition from the low-$Wi$ symmetric state to either the D or C states (Fig. 2a,c). $I' = 0$ when the average flow through the upper and lower gaps equals the flow through the center. Transition to the D state results in $I' > 0$, since $\bar{u}_0$ decreases. Transition to the C state results in $I' < 0$, since $\bar{u}_0$ increases. $I''$ serves as the order parameter to quantify the second transition between the D and the AD states. $I'' = 0$ in the D state, since $\bar{u}_+ = \bar{u}_-$ (Fig. 2b). In the AD state, fluid flows preferentially through either the upper or lower gap (Fig. 2c1,c2), resulting in $I'' > 0$ or $I'' < 0$, respectively.
FIG. 3. Flow asymmetry parameters $I'$, $I''$, and $I = I' + I''$ vs $Wi$ for microfluidic channels with (a-c) $G = 0.500$, (d-f) $G = 0.603$, and (g-i) $G = 0.588$. Symbols indicate the qualitative flow states observed at high $Wi$ for a given experiment. (∆, ▽): AD state with $I'' > 0$ or $I'' < 0$, respectively, and (□): C state. The solid and dotted black curves are fits of a 4th-order Landau potential to the data (see main text). Colored backgrounds in (c),(f), and (i) delineate the various flow states.

The asymmetry parameters $I'$, $I''$ are shown vs $Wi$ in Fig. 3 for various values of $G$ and are fitted with a quartic (double-well) Landau-type potential minimized as:

$$Wi = Wi_c (g c^2 + h c^{-1} + 1),$$  

where $Wi_c = Wi_1$ or $Wi_2$ is the critical Weissenberg number for the bifurcation, and the order parameter $c$ can be either $I'$ or $I''$. In all the fits, the growth rate coefficient $g$ is order unity, and the asymmetric term in $h$ quantifies system imperfections that bias a transition to a favored branch. The phenomenological Landau model for equilibrium phase transitions has long been found to provide a good description of bifurcation phenomena in driven nonequilibrium systems including Newtonian and viscoelastic flows [4, 6, 37, 38]. Eq. 2 describes forward (supercritical) pitchfork bifurcations without hysteresis.

For small $G = 0.500$, the first transition in $I'$ (Fig. 3b) occurs at $Wi_1 \approx 13$, and is a slightly imperfect ($h \approx -0.016$) supercritical pitchfork bifurcation where the favored branch ($I' > 0$) gives diverging (D) flow. The unfavored ($I' < 0$) branch was never observed, but its hypothetical existence is indicated in Fig. 3b by the dotted line. With increasing $Wi$, $I' \rightarrow -1$, implying that almost no fluid passes between the two cylinders (as qualitatively evident from Fig. 2b). The second transition in $I''$ (Fig. 3b) from the D state to the asymmetric diverging (AD) state, occurs at $Wi_2 \approx 44$. The imperfection in this second bifurcation is very small ($h \approx -0.0013$). The favored branch gives $I'' > 0$, but the unfavored $I'' < 0$ branch can also be reached and followed by initiating the flow at a high $Wi$ and subsequent quasistatic reduction. The complete bifurcation diagram showing the total asymmetry $I = I' + I''$ vs $Wi$ for $G = 0.500$ is shown in Fig. 3. The first bifurcation results in $I \rightarrow 1$. The second bifurcation splits $I$ into two branches, $I \rightarrow 1.5$ or $I \rightarrow 0.5$.

The behavior for $G = 0.603$ is shown in Fig. 3f. In this case, the first bifurcation occurs at $Wi_1 \approx 20$ (Fig. 3f). The symmetric term in Eq. 2 is positive ($h \approx 0.033$), resulting in a preferred transition from symmetric to converging (C) flow. With increasing $Wi$, $I' \rightarrow -1$, indicating that nearly all of the fluid passes between the cylinders (see Fig. 2f). Since $h$ is relatively large, the negative $I'$ branch is strongly preferred. The positive $I''$ branch (dotted line in Fig. 3f) is never observed experimentally. When the system selects the C state at the first transition, a second bifurcation is not observed, and $I'' \approx 0$ for all $Wi$ (Fig. 3f). The complete bifurcation diagram for $G = 0.603$ is shown in Fig. 3f:

The data shown in Figs. 2 and 3f demonstrate two disparate flow behaviors that are sensitive to the value of $G$. The first bifurcation to either the D or C states is well described as a supercritical pitchfork bifurcation quantified by $I'$. The two states are different branches of the same bifurcation and the value of $G$ determines which branch is selected by changing the sign of the asymmetric term ($h$) in Eq. 2. This implies the existence of a specific intermediate value of $G$ at which the bifurcation of $I'$ should be perfect ($h = 0$) and the D or C states are equally likely.

By examining a range of intermediate values $0.56 <$
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