Impact of Geometry Distribution of GNSS Station and Satellite on ERP Determination Precision

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Abstract. To directly describe the relationship between the ERP determination precision and the geometry distribution of GNSS station and satellite, a new dilution of precision factor (DOP) is suggested and named as EDOP. According to the hypothesis of smaller EDOP value corresponding to higher ERP solution precision, the minimum requirement and optimal distribution conditions of station and satellite are obtained to estimate the ERP using GNSS technology. The minimum requirement of ERP solvable are that there are at least two stations and two satellites, and they are not in the same plane. The optimal distribution conditions of station and satellite are that 1) the precision of ERP estimation is highest, when the stations are evenly distributed at the intersection of a sphere and a cone with a cone angle of 109.4712°, and the rotations of the distribution or additions of multiple distributions are still optimal; 2) the higher coverage density of satellites, the better precision of ERP estimation. To verify the above deductions, two experimental schemes are designed and real GNSS data is processed. The results show that 1) the hypothesis of EDOP is correct, where smaller EDOP value corresponds to better precision of ERP solution; 2) the linear functions can be adopted to describe the relation between the ERP solution precision and the EDOP value; 3) the solution precision of pole motion is more sensitive to EDOP variation than that of UT1-UTC.

Introduction

The Earth orientation parameter (EOP) are the five time-varying rotational angles consisting of the pole motion coordinates in the terrestrial reference frame (xp and yp), rotation about the polar axis (UT1-UTC), motion of the pole in the celestial frame (precession and nutation) [1]. EOP are the conventional representations for the transformation of the International Terrestrial Reference Frame (ITRF) into the International Celestial Reference Frame (ICRF). For satellite applications, which are relatively insensitive to small offsets in the celestial pole, the subset of EOP angles xp, yp and UT1-UTC, together with the duration of the day (LOD, equivalent to the time rate of change of UT1-UTC), are often considered separately as the Earth rotation parameters (ERP) [2].

The precise ERP are needed to track deep-space spacecraft and compute real-time satellite orbits. Because the errors in timing and polar motion estimates directly map into the spacecraft angular position errors. An error in UT1-UTC of 0.1 ms produces an error of 7 nrad in spacecraft right ascension, corresponding to a position error at Mars of 1.6 km. And the position errors of satellite orbit in the x-axis, y-axis, and z-axis directions are caused by the errors of yp and UT1-UTC, xp and UT1-UTC, xp and yp, respectively. The error of UT1-UTC has more great effects on the orbit position accuracy than that of xp and yp [3].

Each of the modern space-geodetic techniques of lunar laser range (LLR), satellite laser ranging (SLR), very long baseline interferometry (VLBI), and the global navigation satellite system (GNSS) is able to determine the ERP. But each technique has its own unique strengths and weakness in this regard. VLBI is the only technique able to determine all EOP components, but a continuous monitoring of Earth rotation by VLBI is at present not feasible, due to economic and logistic factors. Therefore, it is difficult to determine to the variations of nutation and UT1-UTC [4]. On the contrary, the satellite technique is unable to observe UT1-UTC and nutation offsets but is sensitive
to variations of these quantities. And GNSS technique appear to be highly competitive with those from other techniques and have the potential to generate continuous ERP series with a time resolution of 2 hours and even 15 minutes, due to a widespread station distribution, a low equipment cost and a high data-sampling rate [5].

Though the observation and computing methods are different, the basic principles of these techniques w.r.t the ERP determination are similar. It can be generalized as the ERP are determined by measuring the relative positions or movements between the station vectors linked with the terrestrial coordination system and the observed source vectors linked with the inertial coordination system. Therefore, the precision of ERP measurement is closely related to the distributions of station vectors and source vectors. Malkin et al. [6] investigated the impact of VLBI network distribution on the precision of ERP estimation. Coulot et al. [7] studied the impact of core SLR stations selection on the stability of ERP series w.r.t. ITRF2005. Sosnica et al. [8] analyzed the effects of SLR satellite distribution on the ERP determination precision, in which the different SLR satellite and their combinations (e.g. LAGEOS-1/2, Starlette, Stella and AJISAI) are tested.

In this paper, two questions of ERP determination using GNSS technique are addressed. First, what are the optimal distribution conditions of station and satellite for ERP solution using GNSS technology? Second, what are function relations between the geometry distribution of GNSS station and satellite and ERP solution precision?

The Method for Determining ERP Precision

The fundamental equation for the GNSS observation can be written as the simple form:

$$\rho(t) + \Delta\rho(t) = \bar{\rho}(t)$$

where $t$ is observation epoch in terrestrial time (TT), $\rho(t)$ is the range between the satellite and station, $\Delta\rho(t)$ is the distance error of the observation, and $\bar{\rho}(t)$ is the true value of the distance.

The term “EOP” comprises a set of five parameters, namely, the nutation offsets in $\Delta\epsilon$ and $\Delta\phi$, the pole coordinates $x_p$ and $y_p$, and UT1-UTC. With the term “ERP” we denote the subset $\{x_p, y_p, UT1-UTC\}$ in this contribution. The precession and nutation are taken as the known parameters in our discussion, which are not estimated but are from the results of a priori model (e.g. IAU 2000A). The ERP parameters are taken as unknown parameters to be estimated. Because of the rotation matrices are the implicit functions of the $(x_p, y_p, UT1-UTC)$, the rotation matrices need to be linearized w.r.t. the corresponding parameters respectively, if using the linear algebra method to estimate ERP. Using the Taylor series expansion to the first order, (1) can be written as

$$\rho(t) + \Delta\rho(t) = \bar{\rho}_0(t) + \frac{\partial\bar{\rho}(t)}{\partial \theta} \Delta \theta(t)$$

where, the symbol “$\Delta$“ means the correction and the subscript “0” is the initial value, $\theta=[\theta, \theta, \theta]^T$, $\theta, \theta$ and $\theta$ indicates $x_p, y_p$ and $UT1-UTC$. If the related initial values are given, $\frac{\partial\bar{\rho}(t)}{\partial \theta_0}$ can be expressed as

$$\frac{\partial\bar{\rho}(t)}{\partial \theta_0} = \frac{1}{\bar{\rho}_0(t)} \begin{bmatrix} x(t) - x(t) & y(t) - y(t) & z(t) - z(t) \\ -z(t) & 0 & y(t) \\ x(t) & -y(t) & 0 \end{bmatrix}$$

The linear observation equations of the ERP determination can be obtained once the initial values of the ERP and the observation time are provided. Assuming there are $n$ GNSS stations and each station tracks $m$ satellites at epoch $t$, the observation equation can be formulated as

$$A(t) \Delta \theta(t) = L(t), \quad W(t)$$

$$\begin{bmatrix} \alpha & \beta \\ \beta & \gamma \end{bmatrix}$$
where, \( W(t) \) is the weighting matrix. \( A(t)=[A_1^T(T),\cdots,A_n^T(T)]^T \) and \( L(t)=[L_1^T(T),\cdots,L_n^T(T)]^T \), are the coefficient matrix and observation vector, respectively. The subscripts “1…i…n” denote the station number. The elements \( A_i(t) \) of matrix \( A(t) \) and \( L_i(t) \) of vector \( L(t) \) for the \( i \)th station with \( m \) tracked satellites at epoch \( t \) are given as:

\[
A(t) = \begin{bmatrix}
\chi^o(t) - \chi^i(t) & \chi^y(t) - \chi^i(t) & \chi^z(t) - \chi^i(t) \\
\frac{\chi^o(t)}{\rho^o(t)} & \frac{\chi^y(t)}{\rho^o(t)} & \frac{\chi^z(t)}{\rho^o(t)} \\
\vdots & \vdots & \vdots \\
\frac{\chi^o(t)}{\rho^o(t)} & \frac{\chi^y(t)}{\rho^o(t)} & \frac{\chi^z(t)}{\rho^o(t)} \\
\chi^o(t) - \chi^i(t) & \chi^y(t) - \chi^i(t) & \chi^z(t) - \chi^i(t)
\end{bmatrix}
\]

From (5), it follows that the coefficient matrix, \( A(t) \), of the ERP consists of two parts: 1) the direction cosines matrix (i.e., \( B(t) \)) is composed of the direction cosines from the satellite to station and 2) the coordinate matrix (i.e., \( C(t) \)) of the stations.

If \( k \) epochs are observed and the unknown parameters are invariant during this period, the total coefficient matrix \( A \) can be written as

\[
A = \begin{bmatrix}
A^T(1) & \cdots & A^T(t) & \cdots & A^T(k)
\end{bmatrix}^T
\]  

(6)

In terms of the least squares criterion, the inverse matrix \( Q \) of the parameter weight matrix of the estimated ERP produces

\[
Q = (A^T W A)^{-1} = (\sum_{i=1}^{n} \sum_{i=1}^{m} C_i^T(t) B_i^T(t) W_i(t) B_i(t) C_i(t))^{-1}
\]

(7)

Then the precision of the \( j \)th element of the estimated parameter is

\[
p[j] = \sigma_0 \sqrt{Q[j][j]}
\]

(8)

where \( j \) is the element index of a vector or a matrix, \( \sigma_0 \) is the so-called standard deviation (or sigma), \( p[j] \) is the \( j \)th element of the precision vector, \( Q[j][j] \) is the \( j \)th diagonal element of the cofactor matrix \( Q \).

To directly describe the precision of a group of unknowns without knowledge of \( \sigma_0 \), a so-called dilution of precision factor (DOP) is usually used in GNSS measurements. The DOP is defined as the square root of the sum of some diagonal elements of the quadratic matrix \( Q \). In this study, this definition is similarly used to describe the precision of ERP determination, where the corresponding DOP is termed EDOP. The calculation formula of EDOP is as follows:

\[
EDOP = \sqrt{\sum_{j=1}^{3} Q[j][j]}
\]

(9)

It should be noted that the DOP is usually used for the instantaneous measurement. But the EDOP is indicative of a statistic for the continuous measurement. Generally, smaller EDOP value corresponds to higher precision of ERP solution. However, this deduction has to be verified by the real data processing since it is first suggested.

**Optimal Station and Satellite Distribution Conditions**

To simplify the description of the effect of the station distribution on the ERP solution, the observation weighing allocation and the satellite geometric distribution are not considered, assuming that
\[ B^T W B \approx \eta E \]  
(10)

where \( \eta \) is a random constant and \( E \) is a unit matrix. To validate the applicability of the assumption made in (10), 24-h GPS data were collected from the BJFS IGS station and IGS precise ephemeris on Jan. 1, 2013 and processed. The calculation result is as follows:

\[
\begin{bmatrix}
1 & 0.01 & 0.03 \\
0.01 & 0.96 & 0.05 \\
0.03 & 0.05 & 0.93
\end{bmatrix}
\approx
8836
\]

(11)

The result confirms that the matrix \( B^T W B \) is similar to a scaled unit matrix, which further shows that \( B^T W B \) will become increasingly similar to a \( \eta \) times unit matrix as the number of stations increases [9]. Therefore, for the case of \( n \) tracking stations, the \( A^T W A \) matrix can be approximated to

\[ A^T W A = k \eta \sum_{i=1}^{n} C_i(t) C_i(t) = k \eta \begin{bmatrix}
\sum_{i=1}^{n} (x_i(t))^2 + \sum_{i=1}^{n} (y_i(t))^2 & -\sum_{i=1}^{n} x_i(t) y_i(t) & -\sum_{i=1}^{n} z_i(t) y_i(t) \\
-\sum_{i=1}^{n} x_i(t) y_i(t) & \sum_{i=1}^{n} (y_i(t))^2 + \sum_{i=1}^{n} (z_i(t))^2 & -\sum_{i=1}^{n} z_i(t) z_i(t) \\
-\sum_{i=1}^{n} z_i(t) y_i(t) & -\sum_{i=1}^{n} z_i(t) z_i(t) & \sum_{i=1}^{n} (x_i(t))^2 + \sum_{i=1}^{n} (y_i(t))^2
\end{bmatrix} \]

(12)

According to the hypothesis of smaller EDOP value corresponding to higher ERP solution precision, the determinant of matrix \( A^T W A \) should be as large as possible to obtain high precision ERP estimation. Based on the theory that the determinant of a matrix is maximized if its non-diagonal elements are zero and the diagonal elements are equal [10], it follows that

\[
\begin{cases}
\sum_{i=1}^{n} x_i(t) y_i(t) = \sum_{i=1}^{n} z_i(t) y_i(t) = \sum_{i=1}^{n} z_i(t) x_i(t) = 0 \\
\sum_{i=1}^{n} (x_i(t))^2 = \sum_{i=1}^{n} (y_i(t))^2 = \sum_{i=1}^{n} (z_i(t))^2
\end{cases}
\]

(13)

To understand the above conditions clearly, we construct a matrix, \( H \), whose row vectors are the Cartesian coordinates of each station, i.e.

\[
H = \begin{bmatrix}
x_1(t) & y_1(t) & z_1(t) \\
: & : & : \\
x_n(t) & y_n(t) & z_n(t)
\end{bmatrix}
\]

(14)

If any two column vectors of \( H \) are orthogonal with each other and the norms of all column vectors are equal, the \( H \) matrix will satisfy the conditions in (13). Then, the station location distribution from the \( H \) matrix is optimal for ERP determination. For example, in Figure 1(a-b), the ERP will not be solvable if only one station is used because \( A^T W A \) is zero. For the case of two stations, the best ERP are computed when the vectors formed from these two stations to the geocenter are orthogonal. However, for three stations, the best ERP are computed when the vectors of three stations to the geocenter are mutually orthogonal [11].

To simplify the description of the optimal station distribution when more than 3 stations are used, the station coordinates are expressed using the spherical coordinate system, and the radius vectors of all stations to the geocenter are assumed to be the Earth’s radius. Then, the coordinates of the \( i \)th station are reformulated as

\[
\sum_{i=1}^{n} (x_i(t))^2 + \sum_{i=1}^{n} (y_i(t))^2 + \sum_{i=1}^{n} (z_i(t))^2
\]

(12)
\[ x'_i(t) = r \sin \theta_i \cos \varphi_i \]
\[ y'_i(t) = r \sin \theta_i \sin \varphi_i \]
\[ z'_i(t) = r \cos \theta_i \]

(15)

Substituting (15) into (13) yields

\[
\begin{align*}
\sum_{i=1}^{n} r^2 \sin^2 \theta_i \cos \varphi_i \sin \varphi_i &= \sum_{i=1}^{n} r^2 \cos \theta_i \sin \theta_i \sin \varphi_i = \sum_{i=1}^{n} r^2 \cos \theta_i \sin \theta_i \cos \varphi_i = 0 \\
\sum_{i=1}^{n} r^2 \sin^2 \theta_i \cos^2 \varphi_i &= \sum_{i=1}^{n} r^2 \cos \theta_i \sin \theta_i \sin^2 \varphi_i + \sum_{i=1}^{n} r^2 \cos \theta_i \cos \theta_i \sin \varphi_i = 0
\end{align*}
\]

(16)

If there are \( n \) stations that are evenly distributed at the intersection of a sphere and a cone, as shown in Fig. 1(c), some useful equations can be obtained

\[
\begin{align*}
\sum_{i=1}^{n} \sin \varphi_i &= \sum_{i=1}^{n} \cos \varphi_i = 0 \\
\theta_i &= \gamma
\end{align*}
\]

(17)

where \( \gamma \) is one half of the cone angle. Then, (16) can be expressed as

\[
\begin{align*}
\sum_{i=1}^{n} r^2 \sin^2 \theta_i \cos \varphi_i \sin \varphi_i &= \frac{1}{2} r^2 \sin^2 \gamma \sum_{i=1}^{n} \sin 2\varphi_i = 0 \\
\sum_{i=1}^{n} r^2 \cos \theta_i \sin \theta_i \sin \varphi_i &= \frac{1}{2} r^2 \sin 2\gamma \sum_{i=1}^{n} \sin \varphi_i = 0 \\
\sum_{i=1}^{n} r^2 \cos \theta_i \sin \theta_i \cos \varphi_i &= \frac{1}{2} r^2 \sin 2\gamma \sum_{i=1}^{n} \cos \varphi_i = 0 \\
r^2 \sin^2 \gamma \sum_{i=1}^{n} \cos^2 \varphi_i &= r^2 \sin^2 \gamma \sum_{i=1}^{n} \sin^2 \varphi_i \Rightarrow \frac{n}{2} = \frac{n}{2} \\
\sum_{i=1}^{n} r^2 \cos^2 \theta_i &= \sum_{i=1}^{n} r^2 \sin^2 \theta_i \sin^2 \varphi_i \Rightarrow \tan^2 \gamma = 2 \Rightarrow \gamma = 54.7356^\circ
\end{align*}
\]

(18)

It follows from (18) that the optimal ERP solution is obtained when \( n \) stations are evenly distributed at the intersection of a sphere and a cone with a cone angle of 109.4712°. However, this distribution is just one of the optimal distributions for the ERP determination when \( n > 3 \). It can be further demonstrated that the rotations of the distribution or additions of multiple distributions are still optimal, as shown in Fig. 1(d). The Appendixes provide a proof version to be referenced.

Figure 1. Optimal station distribution conditions for ERP determination

On the other hand, if the geometry distribution of the stations is given, the matrix \( C^T C \) is fixed. Then the determinant of matrix \( A^T W A \) will be just affected by the matrix \( B^T W B \) based on the (7). If the matrix \( W \) is a unit matrix, the matrix \( B^T W B \) can be expressed as
\[ B^T W B = B^T B = \begin{bmatrix} \sum_{j=1}^{n} \sum_{i=1}^{m} \left( \frac{(\Delta x^{l,i}(t))^2}{(\rho_{ji}^l(t))^2} \right) & \sum_{j=1}^{n} \sum_{i=1}^{m} \frac{\Delta x^{l,i}(t) \Delta y^{l,i}(t)}{(\rho_{ji}^l(t))^2} & \sum_{j=1}^{n} \sum_{i=1}^{m} \frac{\Delta x^{l,i}(t) \Delta z^{l,i}(t)}{(\rho_{ji}^l(t))^2} \\ \sum_{j=1}^{n} \sum_{i=1}^{m} \frac{\Delta y^{l,i}(t) \Delta x^{l,i}(t)}{(\rho_{ji}^l(t))^2} & \sum_{j=1}^{n} \sum_{i=1}^{m} \left( \frac{\Delta y^{l,i}(t))^2}{(\rho_{ji}^l(t))^2} \right) & \sum_{j=1}^{n} \sum_{i=1}^{m} \frac{\Delta y^{l,i}(t) \Delta z^{l,i}(t)}{(\rho_{ji}^l(t))^2} \\ \sum_{j=1}^{n} \sum_{i=1}^{m} \frac{\Delta z^{l,i}(t) \Delta x^{l,i}(t)}{(\rho_{ji}^l(t))^2} & \sum_{j=1}^{n} \sum_{i=1}^{m} \frac{\Delta z^{l,i}(t) \Delta y^{l,i}(t)}{(\rho_{ji}^l(t))^2} & \sum_{j=1}^{n} \sum_{i=1}^{m} \left( \frac{\Delta z^{l,i}(t))^2}{(\rho_{ji}^l(t))^2} \right) \end{bmatrix} \]

where \( \Delta x^{l,i}(t) = x_i^l(t) - x_i^r(t) \), \( \Delta y^{l,i}(t) = y_i^l(t) - y_i^r(t) \), \( \Delta z^{l,i}(t) = z_i^l(t) - z_i^r(t) \). It follows from (19) that the ERP will not be solvable if the stations and satellites are in the same plane because one of \( \Delta x^{l,i}(t) \), \( \Delta y^{l,i}(t) \) or \( \Delta z^{l,i}(t) \) is zero. Therefore, the minimum requirements of ERP solution are that there are at least two stations and two satellites, and they are not in the same plane. When the station distribution is given, the more satellites observed by the station network during the observation period, the better precision of ERP determination, because the diagonal elements of matrix \( B^T W B \) are the sum of squared direction cosines of the satellites relative to the stations.

**Experimental Results and Analysis**

To describe the relationship between the ERP determination precision and the geometry distribution of GNSS station and satellite, a new dilution of precision factor is defined and named as EDOP in this contribution. Based on the hypothesis of smaller EDOP corresponding to higher precisions of ERP estimation, the optimal geometric distribution conditions of station and satellite are obtained. However, is this hypothesis correct? What is the functional relation between the ERP solution precision and EDOP value? Which is the most sensitive parameter of ERP to the variation of EDOP value?

To address these questions, two experimental schemes were designed, and real GNSS data were processed. In scheme 1, the number of satellite is 32 and they all belong to GPS satellite. The number of station is 50 in all experiments, but the distribution of stations varies among the fifty experiments (see Fig. 2). In scheme 2, the number of station is 50 and the distribution of station is fixed in all experiments. But the number of satellite varies among the 48 experiments, where the nine GLONASS satellites are used firstly and then each time increases a new GLONASS or GPS satellite until all satellite are used (see Fig. 3). Fig. 2 and Fig. 3 show the distribution locations of station and the tracks of sub-satellite point corresponding to the maximum, medium and minimum EDOP values in all experiments of scheme 1 and scheme 2, respectively.

![Figure 2. The station distribution locations corresponding to the maximum, medium and minimum EDOP values in all experiments of scheme 1.](image-url)
One month of GNSS data from these stations were gathered and processed (from 2016.05.01 to 2016.05.31), and one day of observational data were obtained as a solution session. The average precision of all days is taken as the final solution precision of each experiment. Highly precise GNSS data-processing software, which was developed by Dr. Maorong Ge of the German Research Centre for Geosciences [12], was used to process the experimental data. The predicted values of IERS Bulletin A were used as the initial values of the ERP. The broadcast ephemeris was used to generate the initial orbit. The initial station coordinates and their precisions are from the IGS and iGMAS SINEX files. The parameter configurations are provided in Table 1.

Because the data quality has a significant effect on the precision of GNSS solution, the quality of all data has to be checked firstly for evaluating the different station network and satellite constellation contributed to the enhancement of the solution. The TEQC software, developed by UNAVCO [13], was used to test all data quality. The statistical results of data qualities are listed in the Table 2. DI denotes the data integrity; MP1/MP2 and SN1/SN2 are the multipath errors and the signal-to-noise ratio at the L1/L2 carrier frequency, respectively; CSR is the cycle slip ratio. The formula for these indicators and the ranges of normal values can be found in Zhang et al. [14]. The results show that the data qualities have almost the same level and can satisfy the general requirements of GNSS data quality. One cause may be the data are all from the IGS and iGMAS stations, which were constructed referring to high standard.

### Table 1. Parameter configurations for GNSS data processing.

| Parameter Name          | Configuration | Parameter Name          | Configuration               |
|-------------------------|---------------|-------------------------|-----------------------------|
| Obs. Data Type          | LC+PC         | Solid Earth Tide        | SUN MOON                    |
| Obs. Eq. Mode           | Undifferent   | Ocean Tide              | Yes                         |
| Weighing                | Elevation     | Point Mass              | SUN MOON MERC VENU MARS     |
| Obs. Interval           | 300 sec.      | Solar Radiation         | JUPI SATU URAN NEPT PLUT   |
| Estimator               | LSQ           |                         |                             |
| Rec. ISB/IFB            | Auto+CON      | Atm. Drag               | NONE                        |
| Rec. Sat. PCV           | A_E_E         | Relativity              | Yes                         |
| Remove Bias             | Yes           | Variation               | Yes                         |
| Orb. Ref. Frame         | CRS           | Tide Displacement       | SOLID_FREQ_POLE_OCEAN       |
| Amb. Fixing             | Round         | Est. ERP                | Xp_Yp_dXp_dYp_UT1_DUT1      |
| Baseline Limit          | 3500 km       | ERP constraint          | 3.0_3.0_0.3_0.3_0.0001_0.02 |
| Min. Time               | 900 sec.      | Par. To be Est.         | Sat. Coor., Vel. & BERN Par.|
| Est. Sat. Orb.          | Yes           | Orb. Par. constraint    | 10 m 0.1m/s 0.1m (F1-F9)    |
| ZTD Model               | PWC: 120      | Sat. Clk. constraint    | 5000 m                      |
| ZTD Gradients           | PWC: 1440     | Rec. Clk. constraint    | 9000 m                      |
| Gravity Model           | EGM 8         | Sta. Pos. constraint    | Using Solu. Precision of IGS SINEX |
Table 2. Statistical results of data quality assessment of scheme 1 and 2.

| Standards | DI | MP1 | MP2 | SN1 | SN2 | CSR |
|-----------|----|-----|-----|-----|-----|-----|
| scheme 1  | MIN | 95.80 | 0.03 | 0.04 | 6.43 | 4.42 | 0.02 |
|           | MAX | 100.00 | 0.29 | 0.32 | 50.85 | 46.29 | 4.22 |
|           | AVE | 97.19 | 0.21 | 0.27 | 42.17 | 34.88 | 1.49 |
| scheme 2  | MIN | 95.20 | 0.05 | 0.09 | 6.26 | 4.31 | 0.02 |
|           | MAX | 100.00 | 0.28 | 0.41 | 50.63 | 46.05 | 4.81 |
|           | AVE | 96.27 | 0.21 | 0.36 | 41.77 | 34.34 | 2.48 |

The final ERP products of the IGS were taken as the reference values to evaluate the ERP solution precisions of each experiment, which have the precisions of 25-40 µas for PM, and 8-20 µs for UT1-UTC [15]. The Fig. 4 and 5 show the ERP solution precisions (RMS) in all experiments of scheme 1 and 2, as well as the functional relations between the ERP solution precisions and the EDOP values. The Fig. 6 and 7 show the variation of the EDOP value and the volume of polyhedrons consisting of the satellites relative to the stations with number of satellites in the experiments of scheme 2. The Table 3 is the statistical results of ERP solution precisions and the corresponding EDOP values in the all experiments.

Table 3. Statistical results of ERP solution precisions and EDOP values in the scheme 1 and 2.

|         | EDOP | Xp (µas) | Yp (µas) | UT1-UTC (µs) |
|---------|------|----------|----------|--------------|
| scheme 1| MIN  | 1.38E-09 | 40       | 124          | 159          |
|          | MIDDLE | 1.62E-09 | 203      | 806          | 204          |
|          | MAX   | 2.16E-09 | 773      | 2472         | 246          |
| scheme 2| MIN  | 1.09E-09 | 6        | 28           | 143          |
|          | MIDDLE | 1.47E-09 | 56       | 328          | 170          |
|          | MAX   | 2.97E-09 | 1361     | 1811         | 255          |

According to the above experimental results, some conclusions can be drawn. (1) The EDOP can be used to evaluate the ERP solution precision, and the smaller EDOP value, the higher precision of ERP determination. (2) The wider the distribution of stations and more satellites, the smaller EDOP values. (3) The linear functions can be adopted to describe the relationships between the precisions.
of ERP estimation and the EDOP values. However, it is not a linear relation between the number of satellite and EDOP values. The variation of EDOP is insignificant when the number of satellite is more than 50 (see Fig. 5, 6 and 7). (4) The solution precision of pole motion is more sensitive to the EDOP variation than that of UT1-UTC.

Conclusions

Precise determination of the ERP is one of the most important issues for geodetic reference frame maintenance, satellite orbit determination and timing service. Therefore, it has attracted significant attention from researchers in the geodesy, geodynamic and astronomy communities. In this paper, a new dilution of precision factor of ERP solution is suggested. Based on the hypothesis of smaller EDOP corresponds to higher precisions of ERP estimation, the optimal geometric distribution conditions of station and satellite are investigated. To verify the hypothesis, the real GNSS data is tested. The results show the hypothesis is correct and the linear functions can be used to describe the relationships between the ERP solution precisions and EDOP values.

However, some problems persist, which should be researched further. For instance, there are in fact strong time dependencies due to correlations with parameters that have not been considered in this contribution. Therefore, the stricter optimal distribution conditions of station and satellite should be developed where the space and time factors are considered together. Therefore, they should be considered together when determining the optimal station or satellite distribution.

Whatever, the methods and conclusions of this contribution are helpful for achieved the final results of optimal geometry distribution of station and satellite for ERP determination. And they can be used to investigate the optimal combination of multi-GNSS for ERP estimation, as well as the optimal design of global GNSS tracking network.

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References

[1] Ray J, Kouba J and Altamimi Z, Is there utility in rigorous combinations of VLBI and GPS Earth orientation parameters, J. J Geod, 79 (2005) 505-511.

[2] Mireault Y, Kouba J and Ray J, IGS Earth Rotation Parameters, J. GPS Solut 3(1) (1999) 59-72.

[3] Ye X, Guo H, Yang J, Li C and Liu C, Analysis of BeiDou satellite orbit prediction based on ERP prediction errors impact, J. Lecture Notes in Electrical Engineering 340 (2015) 77-83.

[4] Englich S, Cerveira P J M, Weber R and Schuh H, Determination of Earth rotation variations by means of VLBI and GPS and comparison to conventional models, J. Vermessung und Geoinformation, 2 (2007) 104-112.

[5] Sibois A, GPS-based sub-hourly polar motion estimates: strategies and applications. PhD Thesis, University of Colorado, 2011.

[6] Malkin Z, On comparison of the Earth orientation parameters obtained from different VLBI networks and observing programs, J. J Geod 83 (2009) 547-556.

[7] Coulot D, Pollet A, Collilieux X and Berio P, Global optimization of core station networks for space geodesy: application to the referencing of the SLR EOP with respect to ITRF, J. J Geod 84 (2010) 31-50.
[8] Sošnica K, Jäggi A, Thaller D, Beutler G and Dach R, Contribution of Starlette, Stella, and AJISAI to the SLR-derived global reference frame, J. J Geod 88 (2014) 789-804.

[9] Xue S, Yang Y, Dang Y and Chen W, Dynamic positioning configuration and its first-order optimization, J. J Geod 88 (2014) 127-143.

[10] Serre D, Matrices theory and applications. Springer Verlag, New York, 2010, pp. 55-56.

[11] Wang Q, Dang Y and Xu T, The method of Earth rotation parameter determination using GNSS observations and precision analysis, J. Lecture Notes in Electrical Engineering 243 (2013) 247-256.

[12] Li X, Ge M Dai X, Ren X, Fritsche M and Wickert J, Accuracy and reliability of multi-GNSS real-time precise positioning: GPS, GLONASS, BeiDou, and Galileo, J. J. Geod 89 (2015) 607-635.

[13] Estey L and Meertens C, TEQC: The multi-purpose toolkit for GPS/GLONASS data, J. GPS Solutions, 3(1) (1999) 42-49.

[14] Zhang Y, Ding X, Han X and Su L, Evaluation and analysis on the data quality of GNSS continuously operating reference stations in Shanxi province, J. Plateau Earthquake Research, 26(2) (2014) 64-68.

[15] Rebischung P, Altamimi Z, Ray J and Garayt B, The IGS contribution to ITRF2014, J. J Geod 90 (2016) 611-630.