RICHTMYER–MESHKOV-TYPE INSTABILITY OF A CURRENT SHEET IN A RELATIVISTICALLY MAGNETIZED PLASMA

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ABSTRACT

The linear stability of a current sheet that is subject to an impulsive acceleration due to shock passage with the effect of a guide magnetic field is studied. We find that a current sheet embedded in relativistically magnetized plasma always shows a Richtmyer–Meshkov-type instability, while the stability depends on the density structure in the Newtonian limit. The growth of the instability is expected to generate turbulence around the current sheet, which can induce the so-called turbulent reconnection, the rate of which is essentially free from plasma resistivity. Thus, the instability can be applied as a triggering mechanism for rapid magnetic energy release in a variety of high-energy astrophysical phenomena such as pulsar wind nebulae, gamma-ray bursts, and active galactic nuclei, where the shock wave is thought to play a crucial role.

Key words: instabilities – magnetic fields – relativistic processes – shock waves – turbulence

1. INTRODUCTION

The release of magnetic energy is believed to be essentially important in high-energy astrophysical phenomena such as pulsar wind nebulae, gamma-ray bursts, and active galactic nuclei (e.g., Kennel & Coroniti 1984; Giannios et al. 2009; Lyubarsky 2010). However, the detailed physics of the dissipation mechanism of the magnetic field is still unclear. Theoretical studies have suggested that the instability of a current sheet or a turbulent environment substantially enhances the rate of magnetic reconnection independent of plasma resistivity (Lazarian 1999; Kowal et al. 2009). Takamoto et al. (2012) also showed that the turbulent stretching of a current sheet quickly leads to the dissipation of the magnetic field in a turbulent eddy in a few eddy turnover times regardless of plasma resistivity.

Recently, the current sheet embedded in relativistically magnetized plasma under the effect of secular acceleration was found to be unstable (Lyubarsky 2010). Here, the relativistically magnetized plasma means that the magnetic energy density is larger than the rest-mass energy density. Even though the instability found by Lyubarsky is a Rayleigh–Taylor (or Kruskal–Schwarzschild)-type instability, even when the system is isochoric it can still be unstable since the magnetic and the thermal energies play the role of inertia in relativity. The Richtmyer–Meshkov instability, which is induced by impulsive acceleration due to shock passage (Richtmyer 1960; Nishihara et al. 2010), is a counterpart of the Rayleigh–Taylor instability, suggesting that the shock–current sheet interaction in relativistically magnetized plasma can be unstable as a counterpart of Lyubarsky’s instability. Since shock waves are essential ingredients in high-energy astrophysical phenomena, the shock–current sheet interaction is quite ubiquitous.

For these reasons, we study the linear stability of a relativistic current sheet that is subject to an impulsive acceleration due to the shock passage. The organization of this paper is as follows. In Section 2, we provide the unperturbed zeroth-order state of the current sheet. Then, in Section 3, linear stability analysis is performed and the master equation that governs instability growth is derived. In Section 4, basic properties of the instability are studied based on the solution of the master equation. Finally, in Section 5, we discuss the implications of the instability.

2. ZEROTH-ORDER STATE

In this paper, we use the unit system where the speed of light $c = 1$ and magnetic permeability $\mu = 1$. We consider the static initial current sheet to be separating magnetized media with oppositely oriented $z$-component magnetic field. In the left and right sides of the current sheet (henceforth regions 1 and 3, respectively), the magnetic field is in the $y-z$ plane $B_1(3) = (0, B_{y1(3)}, B_{z1(3)})$, where $B_{z1} = -B_{z3}$. The $y$-component magnetic field, the so-called guide field, is constant across the current sheet $(B_{y1} = B_{y2} = B_{y3} = B_y)$. Inside the current sheet, where the $z$-component magnetic field is dissipated, the total pressure is balanced with the external plasma such that $p_2 + B_{z2}^2/2 = p_{1(3)} + B_{y1(3)}^2/2 + B_{z1(3)}^2/2$. If the plasma in region 1 or 3 is relativistically magnetized and cold ($B_{z1} \gg p_1, p_3$), the ratio of the inertia (or total enthalpy) of media 1 and 2 is $w_2/w_1 = (\rho_2 + \gamma_2 p_2)/\gamma_2 (\rho_2 - 1 + B_{y2}^2/2)/(\rho_1 + \gamma_1 p_1)/\gamma_1 (\rho_1 - 1 + B_{y1}^2/2 + B_{z1}^2)/(2, B_{y2}^2 + B_{z1}^2)/B_{y1}^2 + B_{z1}^2) \approx (2, B_{y2}^2 + B_{z1}^2)/(B_{y1}^2 + B_{z1}^2)$, where the adiabatic index $\gamma = 4/3$ is used in the last expression for the relativistically hot plasma of region 2 ($p_2 \ll p_2 \sim O(B_1^2)$). In particular, when there is no guide field, the ratio becomes $w_2/w_1 \approx 2$. This indicates that the current sheet is heavier than the external plasma, which is the essence of the instability shown below.

Let us consider the propagation of a fast relativistic magnetohydrodynamic (RMHD) shock wave in region 1 toward the $+x$ direction. In the relativistically magnetized cold plasma ($B_1^2 > |p_1| > p_1$), the shock velocity is close to the speed of light ($v_{sh} \simeq 1$). Because the fast characteristic speed is close to the light velocity, the fast shock causes only a small jump in the fluid velocity (also in the density and magnetic field strength as well). According to the jump condition of the RMHD shock for the current sheet, we obtain

$$v \simeq v_{sh}^2 - 2(1 - v_{sh}) B_1^2/\rho_1$$

Solving Equation (1), we see that the post-shock velocity can be subsonic ($V < c_s = 1/\sqrt{3}$), provided $B_1^2/\rho_1 > v_{sh}^2 (1 - c_s v_{sh})/(2(1 + c_s)(1 - v_{sh}))$. Thus, for instance, even in the cases of $v_{sh} = 0.9, 0.99$, and 0.999, the post-shock velocity is subsonic for the medium with $B_1^2/\rho_1 > 1.23, 13.3, 134$, respectively.
the perturbed flow. In the above expression, we have omitted the $z$-component velocity and magnetic field perturbations due to the fact that they are decoupled from the other variables and only describe the propagation of the Alfvén wave in the $y$-direction. Because the shock crossing time of the current sheet can be much smaller than the timescale of the perturbations, the force term can be modeled as

$$F(t, x, y) = \left( W_2 + B^2_{z,2} + B^2_{z,3} - W_1 - B^2_{z,1} - B^2_{z,1} \right) \times \left[ H[x] - H[x + \xi_A(y, t)] \right] V \delta(t)$$

$$+ \left( W_3 + B^2_{z,3} + B^2_{z,3} - W_2 - B^2_{z,2} - B^2_{z,2} \right) \times \left[ H[x + \Delta] - H[x + \Delta - \xi_B(y, t)] \right] V \delta(t).$$

where $H(t)$ is the Heaviside function and $\delta(t)$ is the Dirac delta function (see also Wheatley et al. 2005). The regions for the perturbations that are subject to the acceleration compared to the zeroth-order state are illustrated as shaded regions in Figure 1. In this formulation, we have to be careful about the following points: In Wheatley et al. (2005), where the Richtmyer–Meshkov instability (the instability of a single contact surface) was studied, the speed of the contact surface after the shock passage is chosen as $V$ in Equation (8), which is slightly different from the post-shock velocity. Thus, if we consider the case with only interface A, the speed of the contact surface (interface A) between shocked regions 1 and 2 must be used as $V$ in Equation (8), which is obtained by solving an appropriate Riemann problem (see, e.g., Giacomazzo & Rezzolla 2006). This suggests that for the stability of interface A in the short-wavelength limit (see Equation (29) below), the use of the speed of the shocked contact surface as $V$ in Equation (8) rather than the post-shock velocity seen in Equation (1) would give a more accurate growth rate. For the finite thickness current sheet, the speed of interfaces A and B after the shock passage approaches the post-shock velocity $V$ of Equation (1) asymptotically with time. Thus, since the shock crossing time of the current sheet is much smaller than the growth timescale of the instability, the post-shock velocity of Equation (1) is more appropriate as the degree of impulsive acceleration $V$ in Equation (8). Another point we have to keep in mind is the neglect of the effect of a reflection shock (or a rarefaction wave) that is generated when the incident shock hits interface B, which causes complex dynamics in the current sheet. However, direct numerical simulations of the (Newtonian and unmagnetized) Richtmyer–Meshkov instability of a fluid layer of finite thickness showed that the impulsive model can indeed work well, although it may cause an error of about a factor of two or less in the growth rate (Mikaelian 1996).

Assuming perturbations of the form $q(x, y, t) = \tilde{q}(x, t) \exp(iky)$ and taking the temporal Laplace transformation $(\tilde{q}[x, s] = \int_0^\infty \tilde{q}[x, t] \exp[-s \cdot t] \, dt)$ of Equations (2)–(7) outside of the forced region give

$$\partial_x \tilde{v}_x + ik \tilde{v}_y = 0,$$

$$\partial_t \tilde{v}_x = B^\gamma \partial^\gamma_p,$$

$$\partial_t \tilde{v}_y = -B^\gamma \partial^\gamma_v,$$

$$\partial_x \tilde{b}_x + \partial_y \tilde{b}_y = 0,$$

where $W = \rho + \gamma P / (\gamma - 1)$ is the specific enthalpy and $F(t, x, y)$ is the force term that represents the acceleration of

![Figure 1. Schematic of the system under consideration. Shaded areas are the regions for the perturbations that are subject to the acceleration compared to the zeroth-order state.](image-url)
\[ \partial_x \tilde{b}_x + i k \tilde{b}_y = 0, \]  
(14)

where \( k \) is the wavenumber of the perturbation and \( s \) is the variable associated with the temporal Laplace transformation. Note that these equations describe the perturbations outside the forced regions. The effect of the force term is taken into account when the perturbations in the different regions are connected across the forced regions (when deriving Equations (24) and (25)). Eliminating \( \tilde{p}, \tilde{b}_x, \) and \( \tilde{b}_y \) in Equation (10) by using Equations (11)–(13) and (9), we obtain the following ordinary differential equation for \( \tilde{v}_x \):

\[
(\phi D_x^2 - \chi k^2) \tilde{v}_x = 0, 
\]
(15)

\[
\phi = B_y^2 k^2 + s^2 (B_x^2 + W), 
\]

\[
\chi = B_y^2 (k^2 + s^2) + s^2 (B_x^2 + W), 
\]

which has the general solution of the form

\[
\tilde{v}_x = \alpha \exp(\sqrt{\chi/\phi} k x) + \beta \exp(-\sqrt{\chi/\phi} k x). 
\]
(16)

Other variables are expressed by using \( \tilde{v}_x \), e.g.,

\[
\tilde{p} = -s (W + B_y^2) \partial_x \tilde{v}_x / k^2, 
\]

\[
\tilde{b}_y = -B_y \partial_x \tilde{v}_x / s. 
\]
(18)

Because the perturbations must vanish at \( x = \pm \infty \), \( \beta_1 \) (\( \beta \) in region 1) and \( \alpha_3 \) (\( \alpha \) in region 1) should be null coefficients. We write \( \tilde{v}_x \) in each region as follows:

\[
\tilde{v}_{x,1} = \alpha_1 e^{\sqrt{\chi/\phi} k x}, 
\]

\[
\tilde{v}_{x,2} = \alpha_2 e^{\sqrt{\chi/\phi} k (x-\Delta)} + \beta_2 e^{-\sqrt{\chi/\phi} k x}, 
\]

\[
\tilde{v}_{x,3} = \beta_3 e^{-\sqrt{\chi/\phi} k (x-\Delta)}. 
\]
(21)

Let us consider the junction conditions for the perturbations at the interfaces. At the moment, we have four undetermined amplitudes of the perturbations, \( \alpha_1, \alpha_2, \beta_2, \) and \( \beta_3 \), indicating that a total of four junction conditions are necessary to determine them. Two of them are the continuity of the velocity normal to the interfaces, which leads to \( \tilde{v}_{x,1}(0, s) = \tilde{v}_{x,2}(0, s) \) and \( \tilde{v}_{x,2}(\Delta, s) = \tilde{v}_{x,3}(\Delta, s) \) to the first order of the perturbed variables. Substituting Equations (19)–(21) into the conditions, we obtain

\[
\alpha_1 = \alpha_2 \exp(-\sqrt{\chi/\phi} k \Delta) + \beta_2, 
\]

\[
\alpha_2 + \beta_2 \exp(-\sqrt{\chi/\phi} k \Delta) = \beta_3. 
\]
(23)

The remaining two conditions, which describe the force balance at each interface, are obtained by integrating Equation (3) with respect to \( z \) across each inhomogeneous region. After the temporal Laplace transformation of the integrated equations, to the first order of the perturbed variables, we obtain

\[
(\tilde{p}_2[0, s] + B_{y,2} \tilde{b}_{y,2}[0, s]) - (\tilde{p}_1[0, s] + B_{y,1} \tilde{b}_{y,1}[0, s]) = (W_{tot,2} - W_{tot,1}) V \tilde{\xi}_{A,0}, 
\]

\[
(\tilde{p}_3[\Delta, s] + B_{y,3} \tilde{b}_{y,3}[\Delta, s]) - (\tilde{p}_2[\Delta, s] - B_{y,2} \tilde{b}_{y,2}[\Delta, s]) = (W_{tot,3} - W_{tot,2}) V \tilde{\xi}_{B,0}. 
\]
(25)

where \( \tilde{\xi}_0 \) is the initial amplitude of the interface, and we have defined the total enthalpy \( W_{tot} \equiv W + B_x^2 + B_y^2 \). In the above expression, we have made use of the fact that \( \tilde{v}_x \) is continuous across the interface and have also used the case of \( \tilde{b}_y \) from Equation (12). Substituting Equations (19)–(21) into Equations (24) and (25) via Equations (17) and (18), we obtain two conditions with \( \alpha_1, \alpha_2, \beta_2, \) and \( \beta_3 \). Solving these two conditions, Equations (22) and (23), with respect to the four coefficients, we get

\[
\alpha_1 = \frac{A \tilde{\xi}_{A,0} + B \tilde{\xi}_{B,0}}{C} k s V (W_{tot,2} - W_{tot,1}), 
\]

\[
\beta_3 = \frac{-A \tilde{\xi}_{B,0} + B \tilde{\xi}_{A,0}}{C} k s V (W_{tot,2} - W_{tot,1}), 
\]

\[
A = \left[ W_{tot,2} + W_{tot,1} - 2 B_x^2 + (W_{tot,2} - W_{tot,1}) e^{-2 k \Delta} \right] s^2 + 2 B_y^2 k^2, 
\]

\[
B = 2 \left( W_{tot,2} - B_x^2 \right) s^2 + 2 B_y^2 k^2 \right] e^{-k \Delta}, 
\]

\[
C = \left[ (W_{tot,2} + W_{tot,1} - 2 B_x^2) s^2 + 2 B_y^2 k^2 \right]^2 - (W_{tot,2} - W_{tot,1}) e^{-2 k \Delta} \Delta^4, 
\]

where we have used the zeroth-order conditions \( W_{tot,3} = W_{tot,1} \) and \( B_y = B_{y,1} = B_{y,2} = B_{y,3} \), and again assumed slow motion of the perturbed flows compared to the speed of light: \( \chi/\phi \approx 1 \).

The coefficients \( \alpha_1 \) and \( \beta_3 \) are equivalent to the Laplace transformation of the interface velocities \( \mathcal{L}[\partial_t \tilde{h}(t)] \) and \( \mathcal{L}[\partial_t \tilde{b}_y(t)] \), respectively. The inverse Laplace transformation of Equations (26) and (27) yield the temporal differential equations for the two interfaces:

\[
\frac{d}{dt} \tilde{\xi}_A(t) = (W_{tot,2} - W_{tot,1}) k V H(t) \times \left[ \mu (1 + e^{-k \Delta}) \tilde{\xi}_{A,0} - \tilde{\xi}_{B,0} \right] \cos(2 \sqrt{B_y} k t) + v (1 - e^{-k \Delta}) \tilde{\xi}_{A,0} + \tilde{\xi}_{B,0} \right] \cos(2 \sqrt{B_y} k t) \right], 
\]

\[
\mu^{-1} = 2 \left[ W_{tot,2}(1 - e^{-k \Delta}) + W_{tot,1}(1 + e^{-k \Delta}) - 2 B_x^2 \right], 
\]

\[
v^{-1} = 2 \left[ W_{tot,2}(1 + e^{-k \Delta}) + W_{tot,1}(1 - e^{-k \Delta}) - 2 B_x^2 \right], 
\]

where the equation for interface B is obtained by changing the overall sign of the right-hand side of Equation (28) and by interchanging the subscripts A and B.

4. PROPERTIES OF SOLUTION

4.1. Short-wavelength Limit

Let us first consider perturbations that are small compared to the thickness of the current sheet, which leads to independent evolutionary equations for the two interfaces. Taking the limit of \( k \Delta \to \infty \) in Equation (28) and integrating it with respect to \( t \), we obtain

\[
\tilde{\xi}_{A,B}(t) = \tilde{\xi}_{A,B}(0) \pm A V k t \tilde{\xi}_{A,B}(0) \sin(t/\tau), 
\]

\[
A = \frac{W_{tot,2} - W_{tot,1}}{W_{tot,2} + W_{tot,1} - 2 B_x^2}, 
\]

\[
\tau = (W_{tot,2} + W_{tot,1} - 2 B_x^2)^{-1}/(\sqrt{2} B_y k), 
\]

where \( A \) and \( \tau \) are the Alfvén speed and the Alfvén time, respectively.
where the plus (minus) sign in the second term of the right-hand side of Equation (29) is for interface A (B), and \( \tau \) represents the lateral Alfvén crossing time. Equation (29) shows that, as long as \( t \ll \tau \) (and \( A \neq 0 \)), the interface deformation grows linearly with time and with the velocity of

\[
\dot{\xi} = A V k \dot{\xi}(0) e^{k y}.
\]  

(32)

For \( t > \tau \), the interface oscillates at the Alfvén frequency \( \omega \sim B_y k / W_{\text{tot}}^{1/2} \) due to the magnetic tension force. In the case where the guide field is absent \( (B_y = 0) \), the interface grows perpetually with this speed. In the Newtonian limit \( (\rho \gg P \text{ and } B^2) \), \( A \) reduces to the Atwood number \( A = (\rho_2 - \rho_1)/(\rho_2 + \rho_1) \), and the growth velocity (Equation (32)) exactly reproduces that of the original Richtmyer–Meshkov instability (Richtmyer 1960). In addition, when \( B_y = 0 \), the condition \( A > 0 \) reduces to the criterion for Lyubarsky’s instability. Thus, the coefficient \( A \) can be regarded as the generalized Atwood number. It is noteworthy that, as opposed to the Rayleigh–Taylor instability (including Lyubarsky’s instability), the Richtmyer–Meshkov-type instability can grow even when the Atwood number is negative, i.e., when interface B is also unstable. The generalized Atwood number of the current sheet in the relativistically magnetized plasma \( (B_{\text{tot}}^2 \gg W_1 \text{ and } B_2 \gg \rho_2) \) is \( A = 1/3 \), indicating that the interface is always unstable with respect to the shock passage. However, in the Newtonian limit, the current sheet can be unstable provided that its density is different from the external plasma (in the case of the isothermal Harris current sheet, the Atwood number is \( A = (1 + 2 \beta)^{-1} \), where \( \beta \) is the ratio of the thermal to magnetic pressure in region 1).

4.2. Long-wavelength Limit

Next, we consider the large-scale perturbations compared to the thickness of the current sheet \( (k \Delta \ll 1) \). The Taylor expansion of Equation (28) with regard to \( k \Delta \) results in

\[
\frac{d}{dt} \dot{\xi}_A(t) = -\frac{d}{dt} \dot{\xi}_B(t)
\]

(33)

\[
= \frac{W_{\text{tot},2} - W_{\text{tot},1}}{W_{\text{tot},1} - B_y^2} \nabla^2 k \dot{\xi}_{\Delta,0} \cos(t/\tau) H(t),
\]

(34)

\[
\tau = \left( \frac{W_{\text{tot},1} - B_y^2}{B_y^2} \right)^{1/2} / (B_y k),
\]

where we have used the fact that the initial amplitudes \( \dot{\xi}_{\Delta,0} \) and \( \dot{\xi}_{B,0} \) must be equal in this limit. Equation (33) shows that the two interfaces move in opposite directions. The evolutionary sequence of the interfaces for the long-wavelength perturbation is shown in left panel of Figure 2. The growth in the opposite phase is not surprising because the generalized Atwood number \( \mathcal{A} \) has opposite signs at the two interfaces.

4.3. General Properties

In the general case including \( k \Delta \sim 1 \), as is also the case in both short- and long-wavelength limits, the following are true: (1) The interfaces can be unstable if the generalized Atwood number \( \mathcal{A} \) is nonzero. (2) When \( t \ll \tau \) or \( B_y = 0 \), where \( \tau \sim \sqrt{W_{\text{tot}}/B_y} k \) is the lateral Alfvén crossing time, the interfaces grow with a constant speed of \( \sim \mathcal{A} k V \xi_{0} \). (3) When \( t > \tau \), the interfaces oscillate with the frequency \( \sim 1/\tau \). Interestingly, Equation (28) suggests that, for \( B_y = 0 \) (or \( t \ll \tau \)), the growth speed of interface A \( (d\xi_A/dt) \) becomes zero in the special case of \( \dot{\xi}_{\Delta,0}/\dot{\xi}_{B,0} = [(W_{\text{tot},2} + W_{\text{tot},1}) + (W_{\text{tot},2} - W_{\text{tot},1}) e^{-2\Delta k \xi_0}) ] \sim r_c \) and that of interface B becomes zero in the case of \( \dot{\xi}_{\Delta,0}/\dot{\xi}_{B,0} = r_c^{-1} \), where \( r_c \) is always larger than unity. Furthermore, when the ratio of the initial amplitudes is in the range of \( r_c > \dot{\xi}_{\Delta,0}/\dot{\xi}_{B,0} > r_c^{-1} \), the ratio of the growth speeds of interfaces A and B is negative (out-of-phase growth). When the initial ratio is larger than \( r_c \) or smaller than \( r_c^{-1} \), including negative values, the ratio of the growth speeds is positive (in-phase growth). The reason for this is as follows: Since the generalized Atwood number \( \mathcal{A} \) has opposite signs at the two interfaces, the interfaces at the same \( y \) are accelerated in opposite (same) directions when the initial interfaces are in phase (out of phase). However, when the magnitudes of the initial amplitudes are very different, the motion of the interface with a minor initial amplitude is dragged toward the interface with the major initial amplitude, because the volume of the accelerated regions depends upon the initial amplitude (see Figure 1). The evolutionary sequences of the in- and out-of-phase growths are shown in the left and right panels of Figure 2, respectively.

5. DISCUSSION

Finally, we discuss implications of the instability. A recent particle-in-cell simulation of a relativistically magnetized pulsar wind has shown that shock propagation through the strips of an opposite magnetic field polarity induces magnetic reconnection (Sironi & Spitkovsky 2011). The triggering mechanism of the magnetic reconnection could be the instability studied in this article.

In addition, recent MHD simulations have shown that the growth of the Richtmyer–Meshkov instability induced by a (relativistic) shock propagating through an inhomogeneous density medium generates turbulence in its nonlinear stage (Giacalone & Jokipii 2007; Inoue et al. 2009, 2011, 2012; Beresnyak et al. 2009). Because the instability in the relativistic current sheet found in this study is also a Richtmyer–Meshkov-type instability, it is quite reasonable to expect the development of turbulence.

The instability evolves into the nonlinear stage if the amplitude of the interface can grow to \( \xi \sim 1/k \) before the magnetic tension force begins to suppress growth \( (t \lesssim \tau) \). Using the growth velocity (Equation (32)), the above condition is reduced to

\[
\frac{V}{V_{A,y}} \gtrsim (A k \xi_0)^{-1},
\]

(35)

where \( V_{A,y} \equiv B_y / \sqrt{W_{\text{tot}}} \) is the lateral Alfvén velocity. Note that if there is no guide field \( (B_y = 0) \), which is plausible at least in the pulsar wind, the instability can always go into its nonlinear stage. As in the case of the Richtmyer–Meshkov instability, the growth of the instability is not exponential with time but is linear with time (Richtmyer 1960), indicating that the timescale of the evolution depends upon the initial amplitude of the perturbation. If we consider the initial perturbation of scale \( k \gtrsim \Delta^{-1} \) with amplitude \( \xi_{B,0} = \xi_{\Delta,0} = \xi_0 \), in which the instability is the most influential, the instability becomes nonlinear \( (\xi k \sim 1) \) after the following elapsed time from the shock passage:

\[
t \sim (A V k \xi_0 k^2)^{-1}.
\]

(36)

where we have estimated the timescale based on Equation (28). This suggests that, if the initial amplitude is much smaller than
the wavelength of perturbation ($\lambda = k^{-1} \ll \xi_0$), it takes a very long time for the instability to evolve into the nonlinear stage. Fortunately, however, we can expect a large initial amplitude of the perturbation due to the tearing mode instability, relativistic drift kink instability (Zenitani & Hoshino 2007), and Lyubarsky’s instability, which can grow prior to the shock passage. It was suggested by Lyubarsky (2010) that Lyubarsky’s instability can grow in pulsar winds, gamma-ray burst jets, and active galactic nuclei at least on scales of $\lambda \lesssim \Delta$, although it may not fully dissipate in the magnetic field, especially in pulsar winds. Thus, we can expect an excitation of turbulence quickly (on the order of crossing time $\lambda V^{-1}$ for $\xi_0 \sim \lambda$) after the shock passage. Note that, in Lyubarsky’s instability, either interface A or B can be unstable depending on the orientation of the acceleration in the most unstable scales of $\lambda \lesssim \Delta$. In contrast, our instability can grow in both interfaces. Therefore, even if Lyubarsky’s instability is active, our instability, which agitates the current sheet as a whole is necessary, in order to excite “turbulence.”

Once the current sheet becomes turbulent, the induction of the so-called turbulent reconnection (Lazarian & Vishniac 1999; Kowal et al. 2009) and the effect of the turbulent stretching of the current sheet (Takamoto et al. 2012) can rapidly dissipate the magnetic field regardless of plasma conductivity. The excited turbulent reconnection would then evolve through the generation of more turbulence by positive feedback of the reconnection outflows. Therefore, the instability found in this study can be applied as a triggering mechanism of rapid magnetic energy release in a variety of high-energy astrophysical phenomena such as pulsar wind nebulae, gamma-ray bursts, and active galactic nuclei, where the shock wave is thought to play a crucial role. Different from secular instabilities such as the tearing mode instability and Lyubarsky’s instability, a sudden onset of our instability due to the shock passage and the following turbulent reconnection may be preferred, especially in intermittent phenomena such as gamma-ray bursts (see, e.g., Zhang & Yan 2011 for a recent model).

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REFERENCES

Beresnyak, A., Jones, T. W., & Lazarian, A. 2009, ApJ, 707, 1541
Giacalone, J., & Jokipii, J. R. 2007, ApJ, 663, 41
Giacomazzo, B., & Rezzolla, L. 2006, J. Fluid Mech., 562, 223
Giannios, D., Uzdensky, D. A., & Begelman, M. C. 2009, MNRAS, 395, L29
Inoue, T., Asano, K., & Ioka, K. 2011, ApJ, 734, 77
Inoue, T., Yamazaki, R., & Inutsuka, S. 2009, ApJ, 695, 825
Inoue, T., Yamazaki, R., Inutsuka, S., & Fukui, Y. 2012, ApJ, 744, 71
Kennel, C. F., & Coroniti, F. V. 1984, ApJ, 283, 694
Kowal, G., Lazarian, A., Vishniac, E. T., & Otmanowska-Mazur, K. 2009, ApJ, 700, 63
Lazarian, A., & Vishniac, E. T. 1999, ApJ, 517, 700
Lazebny, Y. 2010, ApJ, 725, L234
Mikaelian, K. O. 1996, Phys. Fluids, 8, 1269
Nishihara, K., Wounchuk, J. G., Matsuoka, C., Ishizaki, R., & Zhakhovsky, V. V. 2010, Phil. Trans. R. Soc. A, 368, 1769
Richtmyer, R. D. 1960, Commun. Pure Appl. Math., 13, 297
Sironi, L., & Spitkovsky, A. 2011, ApJ, 741, 39
Takamoto, M., Inoue, T., & Inutsuka, S. 2012, ApJ, 755, 76
Wheatley, V., Pullin, D. I., & Samtaney, R. 2005, Phys. Rev. Lett., 95, 125002
Zenitani, S., & Hoshino, M. 2007, ApJ, 670, 702
Zhang, B., & Yan, H. 2011, ApJ, 726, 90