Neutrino texture saturating the \(CP\) asymmetry

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We study a neutrino mass texture which can explain the neutrino oscillation data and also saturate the upper bound of the \(CP\) asymmetry \(\epsilon_1\) in the leptogenesis. We consider the thermal and nonthermal leptogenesis based on the right-handed neutrino decay in this model. A lower bound of the reheating temperature required for the explanation of the baryon number asymmetry is estimated as \(O(10^8)\) GeV for the thermal leptogenesis and \(O(10^9)\) GeV for the nonthermal one. It can be lower than the upper bound of the reheating temperature imposed by the cosmological gravitino problem. An example of the construction of the discussed texture is also presented.

I. INTRODUCTION

The discovery of the neutrino masses [1] gives a large impact to the study of particle physics and astroparticle physics. In particular, it presents an interesting approach to the study of the origin of the baryon number \((B)\) asymmetry in the universe, which is one of the important questions in these fields. The leptogenesis [2] based on the \(B - L\) violation due to the neutrino masses is considered to be the most promising scenario for the generation of the \(B\) asymmetry. During the past few years, the leptogenesis based on the \(CP\) asymmetric decay of the heavy right-handed neutrinos [3] whose existence is required by the seesaw mechanism [4] has been extensively studied [5,6].

Since the intermediate scale is generally necessary for the seesaw mechanism, it seems to be natural to consider the leptogenesis in the supersymmetric framework to guarantee the stability of that scale against the radiative correction. However, if we consider it in such a framework, a crucial problem called the cosmological gravitino problem is caused in relation to the generation of the right-handed neutrinos. If the reheating temperature \(T_R\) required to produce a sufficient amount of the right-handed neutrinos is a high value such as \(T_R \simeq 10^8\) GeV, the gravitino can be produced too much and its late time decay may disturb the nucleosynthesis [7].\(^1\) Thus, the production mechanism of the heavy right-handed neutrinos is the important ingredient for this problem. Several solutions for this difficulty have been proposed by now \([9,10]\).

On the other hand, the \(CP\) asymmetry [3] in the decay of the right-handed neutrinos is another crucial factor which plays the essential role to determine the generated lepton number \((L)\) asymmetry. Its magnitude depends on the structure of the neutrino mass matrix, which is severely constrained to explain the neutrino oscillation data [1]. From a viewpoint of the leptogenesis, it is favorable that the neutrino mass matrix can realize the maximum value of the \(CP\) asymmetry \([3,11]\) or enhance its value \([12,13]\). Thus, it is important for the quantitative study of the leptogenesis to construct such a concrete model for the neutrino mass matrix as done in \([5,14,15]\) and to proceed the investigation based on it. In this paper we present an example of the neutrino mass matrix and estimate the reheating temperature required to produce the sufficient \(B\) asymmetry.

The paper is organized as follows. In Sec. II we present a neutrino mass texture and discuss its phenomenological features. An example for its construction is discussed in Appendix A. In Sec. III we apply this model to both the thermal and nonthermal leptogenesis. We discuss the lower bound of the reheating temperature required for the production of the sufficient \(B\) asymmetry. Section IV is devoted to the summary.

II. NEUTRINO MASS TEXTURE

We consider the minimal supersymmetric standard model (MSSM) extended with gauge singlet chiral superfields \(N_i\) which correspond to three generation right-handed neutrinos. An effective superpotential for the neutrino sector is assumed as follows:

\[
W = \sum_{i,j=1}^{3} \left( h_{ij}^{\nu} N_i H_2 L_j + \frac{1}{2} \mathcal{M}_{ij} N_i N_j \right),
\]

(1)

where \(L_j\) and \(H_2\) are the lepton doublet and Higgs doublet chiral superfields, respectively. In this paper we use the same notation for both a superfield and its component fields. The right-handed neutrino mass matrix \(\mathcal{M}\) and the Dirac mass matrix \(m_D\) induced from the first term in this superpotential are assumed to take the form as\(^2\)

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\(^1\)If the gravitino is the lightest superparticle as in the gauge mediated supersymmetry breaking, there is no gravitino problem even in the case of \(T_R = O(10^{10})\) GeV [8].

\(^2\)This model can be considered as a simple extension of the model in [16]. It is realized by adding a right-handed neutrino \(N_1\) to the original one in such a way that \(N_1\) weakly couples to \(N_2\) alone.
where the charged lepton mass matrix is considered to be diagonal. Although each element of $\mathcal{M}$ is supposed to be real, $m_D$ is assumed to be a complex matrix. If the hierarchical structure

$$m, M_1 \ll M_2 \ll M_3$$

is assumed in the right-handed neutrino sector, the eigenvalues $M_i$ of the mass matrix $\mathcal{M}$ can be approximated to

(a) $\tilde{M}_1 (\equiv M_1)$, $M_2$, $M_3$, (for $m^2 < M_1 M_2$),

(b) $\tilde{M}_1 (\equiv M_2 \sin^2 \xi)$, $M_2$, $M_3$, (for $m^2 > M_1 M_2$),

where we use $\tilde{M}_i \approx M_i (i = 2, 3)$ and $\sin \xi \approx m / M_2$. These two cases are studied in the following part. The structure of $\mathcal{M}$ and $m_D$ can be effectively realized by imposing a suitable symmetry on the superpotential at the high energy scales. We give such an example for the construction of $\mathcal{M}$ and $m_D$ in Appendix A.

If we change $N_i$ into the $\mathcal{M}$ diagonal basis $\tilde{N}_i$, the Dirac neutrino mass matrix is transformed into

$$\bar{m}_D = \tilde{h}^* (H^2) = \begin{pmatrix} -a \sin \xi & -a' \sin \xi & 0 \\ a \cos \xi & a' \cos \xi & 0 \\ 0 & b & b' \end{pmatrix}.$$ (5)

Applying the seesaw mechanism to these matrices, we can obtain the light neutrino mass eigenvalues and the Maki-Nagagawa-Sakata (MNS) matrix. Here, for simplicity, we put $a' = \sqrt{2} a$ and $b' = b$\(^3\) and then the light neutrino mass eigenvalues are found to be

$$m_1 = 0, \quad m_2 \approx \frac{2|a|^2}{M_2}, \quad m_3 \approx \frac{2|b|^2}{M_3}.$$ (6)

The MNS matrix has the bi-large mixing form such as

$$U \approx \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} \cos \theta & \frac{1}{2} \cos \theta \\ \frac{1}{2} & -\frac{1}{2} \cos \theta & \frac{1}{2} \cos \theta \end{pmatrix}.$$ (7)

where we neglect the CP phase in this expression.

Now we can compare these results with the present experimental data. Since the neutrino mass eigenvalues are assumed to be hierarchical, the analysis for the neutrino oscillation experiments requires [1]

$$2|a|^2 \approx \frac{\Delta m_{sol}^2}{M_2} \approx (7 \times 10^{-5} \text{ eV}^2)^{1/2},$$

$$2|b|^2 \approx \frac{\Delta m_{atm}^2}{M_3} \approx (2 \times 10^{-3} \text{ eV}^2)^{1/2},$$ (8)

and $\sin \theta$ should satisfy

$$\sin \theta \approx \frac{|a|^2 / M_2}{\sqrt{2} |b|^2 / M_3} \approx \frac{1}{\sqrt{2}} \sqrt{\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2}} \approx 0.1.$$ (9)

This is consistent with the constraint $\sin \theta < 0.16$ which is imposed by the CHOOZ experiment [17]. The effective mass for the neutrinoless double $\beta$-decay is estimated in this model as

$$m_{ee} \approx \frac{1}{2} \sqrt{\Delta m_{sol}^2 \sin^2 \theta + \Delta m_{atm}^2 \cos^2 \theta} \approx 2 \times 10^{-3}.$$ (10)

It seems to be difficult to reach such a value in the next generation experiment.

The lepton flavor violating processes such as $\mu \rightarrow e \gamma$ can constrain the model. It has been suggested that these processes could impose the strong constraint because of the renormalization effect on the soft supersymmetry (SUSY) breaking parameters due to the off-diagonal Yukawa couplings. It can be very severe even in the case of the universal SUSY breaking in the gravity mediation scenario [18]. Here, in order to find the conservative condition, we consider the universal soft SUSY breaking in the gravity mediation. The branching ratio of the flavor changing process $\ell_i \rightarrow \ell_j \gamma$ is estimated by taking account of the one-loop contribution as [18]

$$Br(\ell_i \rightarrow \ell_j \gamma) \approx \frac{\alpha^3 \tan^2 \beta}{G_F m_{\ell}^3} \left| \frac{1}{8 \pi} (3 m_0^2 + A_0^2) (\tilde{h}^* \tilde{h})_{ij} \ln \frac{M_X}{M} \right|^2,$$ (11)

where $m_0$, $A_0$, and $M_\ell$ represent the soft scalar mass, the SUSY breaking $A$ parameter, and the relevant slepton mass, respectively. $M_X$ stands for the unification scale and $M$ is the right-handed neutrino mass scale. Since $\tilde{M}_1$ is irrelevant to the light neutrino masses as shown in Eq. (6), $M$ is appropriate to be taken as $M_2$ in the present model.

If we assume $m_0 = m_\ell = A_0$ and use Eq. (5) for the Yukawa couplings $\tilde{h}^*$, each branching ratio for $\mu \rightarrow e \gamma$ and $\tau \rightarrow e \gamma$ is estimated as\(^4\)

\(^3\)We adopt these relations in the following study. Under this assumption the model contains seven real parameters. We can lose these strict equalities without changing largely the qualitative results given in this paper.

\(^4\)The decay $\tau \rightarrow e \gamma$ is automatically forbidden as a result of the present texture of the neutrino mass matrix.
\[ Br(\mu \rightarrow e \gamma) \approx 3 \times 10^{-31} \frac{M_3^2}{m_0^2} (\ln \frac{M_X}{M_2})^2 \tan^2 \beta \]

\[ \leq 1.2 \times 10^{-11}, \]

\[ Br(\tau \rightarrow \mu \gamma) \approx 4 \times 10^{-30} \frac{M_3^2}{m_0^2} (\ln \frac{M_X}{M_2})^2 \tan^2 \beta \]

\[ \leq 1.1 \times 10^{-6}. \]

If we take \( M_X = 10^{16} \) GeV and \( m_0 = 100 \) GeV, for example, the present experimental bounds can be satisfied for \( M_2 \leq 10^{11} \) GeV and \( M_1 \leq 10^{13} \) GeV even in the case of large \( \tan \beta \) such as \( \tan \beta \approx 50 \). This means that no lepton flavor violating decays \( \ell_i \rightarrow \ell_j \gamma \) contradict with the present model as far as the universal SUSY breaking is assumed even in the gravity mediation scenario.

A remarkable feature of the model is that there are no constraints on \( M_1 \) and \( \sin \xi \) from the neutrino oscillation data and other present available experiments. If we apply this neutrino mass texture to the leptogenesis, \( M_1 \) and \( \sin \xi \) may be constrained to explain the \( B \) asymmetry. In the next section we focus our study on this point.

### III. APPLICATION TO LEPTOGENESIS

The decay of the heavy right-handed neutrinos can produce the \( B - L \) asymmetry.\(^5\) Then it may explain the \( B \) asymmetry in the universe since the sphaleron interaction can convert a part of the \( B - L \) asymmetry into the \( B \) asymmetry \([19]\). The \( L \) asymmetry or the \( B - L \) asymmetry induced through this decay is produced as a result of the CP asymmetry caused by the interference between the tree and one-loop diagrams.

The CP asymmetry which appears in the \( \tilde{N}_i \) decay can be generally expressed as \([3]\)

\[ \epsilon_i = \frac{1}{\sum_j \Gamma(\tilde{N}_j \rightarrow L_j H_2)} - \frac{1}{\sum_j \Gamma(\tilde{N}_i \rightarrow L_i \bar{H}_2)} \]

\[ = \frac{1}{\sum_j \Gamma(\tilde{N}_j \rightarrow L_j H_2)} \sum_k \text{Im} \left[ \left( \tilde{h}^r \tilde{h}^{r+} \right)_{ik} \right]^2 \left( \frac{M_3^2}{M_i^2} \right)^2 \]

\[ \left( \frac{M_3^2}{M_i^2} \right)^2 \]

where \( f(x) \) contains the contributions from both the vertex correction and the self-energy correction, and it has an expression

\[ f(x) = \sqrt{x} \left[ \ln \left( \frac{1 + x}{x} \right) + \frac{2}{x - 1} \right]. \]

Applying this formula to our model in which the hierarchical structure for the right-handed neutrino masses is assumed, we obtain

\[ \epsilon_1 = - \frac{1}{8 \pi} \sqrt{\Delta m_{12}^2 \tan^2 \beta} \sin 2\chi, \]

\[ \epsilon_2 = - \frac{1}{8 \pi} \sqrt{\Delta m_{13}^2 \tan^2 \beta} \sin 2\chi, \]

\[ \epsilon_3 = - \frac{1}{16 \pi} \sqrt{\Delta m_{23}^2 \tan^2 \beta} \left( \frac{M_1}{M_2} \right) \ln \left( \frac{M_3}{M_1} \right) \sin 2\chi, \]

where \( \langle H_2 \rangle = v \sin \beta \) and \( \chi \equiv \arg(a^* b) \). The formulas in Eq. (15) show that \( \epsilon_3 \) can be much smaller than \( \epsilon_{1,2} \). The interesting point of this result is that \( \epsilon_3 \) is almost equal to the expression which saturates its upper bound given in \([11]\).\(^6\) Thus, the present texture for the neutrino masses seems to be favorable to induce the \( B \) asymmetry.

By using these expressions for \( \epsilon_i \), the \( L \) asymmetry resulting from the decay of the right-handed neutrinos can be estimated. If we put the excess of the number density of the right-handed neutrino \( \tilde{N}_i \) from the equilibrium one as \( n_i \) and the entropy density in the comoving volume at the latest \( \tilde{N}_i \) decay as \( s \), the produced \( L \) asymmetry through this decay can be expressed as

\[ n_L = \frac{n_L}{s} \approx \sum_{i=1}^{3} n_i \epsilon_i \kappa_i, \]

where \( \kappa_i \) represents the washout effect which depends on the strength of the Yukawa couplings \( \tilde{h} \) in Eq. (5). The sphaleron interaction which is in the thermal equilibrium at the temperature \( 10^2 \) GeV \( \leq T_{\text{ph}} \leq 10^{12} \) GeV converts a part of the \( B - L \) asymmetry into the \( B \) asymmetry in such a way as \( n_B/s = -\langle 8/15 \rangle (n_L/s) \) in the MSSM case \([19,20]\). Thus \( n_L/s \) should satisfy \( |n_L/s| \approx 10^{-10} \) to realize the observed value \( n_B/s \approx (0.6-1) \times 10^{-10} \).

Since the number density \( n_i \) of the right-handed neutrino \( \tilde{N}_i \) depends on its generation mechanism, we need to fix it for the quantitative estimation of \( n_L/s \). In the following study we consider both the thermal and nonthermal scenarios for their generation. In the nonthermal leptogenesis, we mainly consider that the right-handed neutrinos couple to the inflaton directly and then the inflaton decay produces the right-handed neutrinos nonthermally.

### A. Thermal leptogenesis

Since the right-handed neutrino masses are supposed to be hierarchical in the present model, the \( L \) asymmetry is

\(^5\) Since we consider the neutrino mass texture in the context of supersymmetry, the superpartners of the right-handed neutrinos also contribute to the generation of the \( B - L \) asymmetry. Although we take them into account in the numerical analysis, we focus our attention on the standard model sector in the following qualitative discussion.

\(^6\) A model with this feature has been discussed in \([14]\) already. However, it seems not to have a satisfactory structure for the explanation of the neutrino oscillation data.
As is found from Eqs. (15) and (19), the decay width of $\tilde{N}_i$ can be estimated as

$$\Gamma_{\tilde{N}_i} \approx \frac{3}{16\pi} \sqrt{\Delta m_{\text{sol}}^2 M_2 M_1} \sin^2 \beta \sim \left( \frac{M_1}{10^7 \text{ GeV}} \right)^3 \left( \frac{M_2}{10^{10} \text{ GeV}} \right)^2 \sin^2 \xi,$$

$$\Gamma_{\tilde{N}_2} \approx \frac{3}{16\pi} \sqrt{\Delta m_{\text{sol}}^2 M_2^2} \cos^2 \beta \sim 10^{10} \left( \frac{M_2}{10^{10} \text{ GeV}} \right)^2 \cos^2 \xi,$$

$$\Gamma_{\tilde{N}_\ell} \approx \frac{1}{8\pi} \sqrt{\Delta m_{\text{sol}}^2 M_2^2} \sim 10^9 \left( \frac{M_3}{10^{13} \text{ GeV}} \right)^2,$$ (17)

where these decay widths are given in the unit GeV. This shows that $\Gamma_{\tilde{N}_1} < \Gamma_{\tilde{N}_2} < \Gamma_{\tilde{N}_\ell}$ is satisfied. The asymmetry generated by the decay of $\tilde{N}_2$ is washed out through the $L$ violating scattering mediated by the thermal $\tilde{N}_1$ etc. and then $\kappa_{2,3} \ll 1$.

If we use the thermal number density of the relativistic particle for $n_1$ and $s = \frac{2}{28} g_s T^3$ in Eq. (16), the L asymmetry generated through the $\tilde{N}_1$ decay is expressed as

$$\frac{n_1}{s} = \frac{1}{g_s} \varepsilon_1 \kappa_1,$$ (18)

where $g_s$ is a degree of freedom for the relativistic particles at this period and $g_s \sim 200$ in the MSSM. In this expression we should note that $\kappa_1$ includes also the efficiency factor to generate $\tilde{N}_1$ in the thermal bath other than the washout effect since we suppose that there are no thermal right-handed neutrinos initially.

As is well known in the thermal leptogenesis [6], there is an important quantity $\tilde{m}_1$ which is related to $\kappa_1$ and then the strength of the relevant Yukawa couplings of $\tilde{N}_1$. It controls how many $\tilde{N}_1$ are produced in the thermal equilibrium and also how much $L$ asymmetry is washed out. In the present model $\tilde{m}_1$ is expressed as

$$\tilde{m}_1 = (\tilde{h}^* \tilde{h}^\dagger)_{11} \frac{v^2 \sin^2 \beta}{M_1}$$

$$= \begin{cases} \frac{3}{2} \sqrt{\Delta m_{\text{sol}}^2 M_2^3 \sin^2 \xi} & \text{for (a),} \\ \frac{3}{2} \sqrt{\Delta m_{\text{sol}}^2 M_1^3 \sin^2 \xi} & \text{for (b).} \end{cases}$$ (19)

As is found from Eqs. (15) and (19), $\varepsilon_1$ and $\tilde{m}_1$ can be independent from each other because of the freedom of $\sin \xi$. We may expect that there is generally some correlation between these parameters from their definitions (13) and (19). However, the special texture can make them independent in the present model. This feature may cause a substantial influence on the reheating temperature required for the leptogenesis.

If we use the formula in Eq. (15), the $CP$ asymmetry $\varepsilon_1$ required from the $B$ asymmetry in the universe is estimated as

$$|\varepsilon_1| \approx 10^{-8} \times \left( \frac{M_1}{10^8 \text{ GeV}} \right) \approx 10^{-8},$$ (20)

where $g_s \sim 200$ is used. In this estimation the maximum $CP$ phase $|\sin 2\chi| \sim 1$ and $\sin \beta = 1$ are also assumed. From this condition we obtain $M_1 \approx 10^8$ GeV for the lower bound of the $\tilde{N}_1$ mass, which is the ordinary result in the out of equilibrium decay of the thermally produced $\tilde{N}_1$. On the other hand, the effective mass $\tilde{m}_1$ is estimated as

$$\frac{\tilde{m}_1}{10^{-2} \text{ eV}} \approx \begin{cases} 1 & \text{for (a),} \\ \frac{M_3^3}{M_1^2} \sin^2 \xi & \text{for (b).} \end{cases}$$ (21)

While $\tilde{m}_1$ takes a fixed value in case (b), there is the freedom $\sin \xi$ to tune $\tilde{m}_1$ into the desirable value in case (a). Equation (21) shows that the efficiency factor can be in the favorable region for the leptogenesis in case (a) but it seems to be larger than the favorable value in case (b) [5,6,15].

The necessary condition for the out of equilibrium decay of $\tilde{N}_1$ is given by $H > \Gamma_{\tilde{N}_1}$. If we use Eq. (17) for this condition, the successful leptogenesis requires that the temperature $T$ at the period of the $\tilde{N}_1$ decay should satisfy

$$T > T_{\text{min}} \approx M_1 M_2 \sin \xi.$$ (22)

In case (b) we find $T_{\text{min}} \approx \tilde{M}_1$ and then $T_R \approx \tilde{M}_1$ should be satisfied. Thus we expect the similar result for the efficiency factor to the previous works [5,6]. On the other hand, since case (a) is realized for $M_1 > M_2 \sin^2 \xi$, we find that $T < \tilde{M}_1$ could be consistent with the condition for the out of equilibrium decay. Such a situation seems to be realized for a sufficiently small $\sin \xi$ without conflicting with the neutrino oscillation data. The small $\sin \xi$ can also make the washout effect negligible. In this case, however, the sufficient number of $\tilde{N}_1$ may not be produced due to the Boltzmann suppression in the thermal equilibrium. We need to solve a set of Boltzmann equations numerically for the quantitative study of the relation between $T_R$ and $M_1$.

We study these points by solving numerically a set of Boltzmann equations for the MSSM presented in [5]. This analysis includes, other than the standard model processes, the decay of the right-handed neutrinos into a slepton and a Higgsino, the decay of the scalar partners of the right-handed neutrinos into a lepton and a Higgsino or into a slepton and a Higgs boson, the $2 \to 2$ scattering involving one scalar right-handed neutrino, and so on. The last process is known to be very effective in bringing the

\footnote{This assumption for $\sin \beta$ brings no crucial difference as far as $\tan \beta$ is in the interesting region such as $1 \leq \tan \beta \leq 50$.}
right-handed neutrinos and their superpartners into the thermal equilibrium. Through this process, in particular, the superpartner of \( N_1 \) is expected to play an important role in the washout of the \( B - L \) asymmetry at \( T \gtrsim M_1 \). If we use Eq. (8) and assume \(| \sin 2\chi | = 1\), the model parameters in this calculation are \( M_{1,2,3} \) and \( \sin \xi \). As an initial condition for the Boltzmann equations, we assume that both the number density of \( \tilde{N}_i \) and the \( L \) asymmetry are zero at \( z_0 = 0 \). A dimensionless parameter \( z \) is defined as \( z = M_1 / T \). The temperature corresponding to \( z_0 \) may be considered to correspond to the reheating temperature \( T_R \).

In Fig. 1 we give a solution of the Boltzmann equations with \( z_0 = 0.01 \) and the input parameters such as \( M_1 = 10^9 \) GeV, \( M_2 = 10^{10} \) GeV, \( M_3 = 10^{13} \) GeV, and \( \sin \xi = 0.02 \). This corresponds to case (a) with \( T_R = 10^{11} \) GeV. Case (b) cannot yield the sufficient \( L \) asymmetry. This can be understood as follows. In this case \( \sin \xi \) is required to take a rather large value to realize \( M_1 < M_2 \sin^2 \xi \) satisfying both constraints such as \( M_1 \gtrsim 10^8 \) GeV and \( M_3 \lesssim 10^{13} \) GeV, which are imposed by the previously discussed phenomenological constraints. Such a \( \sin \xi \) makes the \( \tilde{N}_1 \) Yukawa couplings larger and then the washout effect becomes effective. This is also suggested by Eq. (21).

In Fig. 2 we show the \( L \) asymmetry \( |Y_L| \) as a function of \( \sin \xi \), where \( Y_L \) stands for the total \( L \) asymmetry which is the sum of \( Y_{ij} \) in the standard model sector and \( Y_{L_i} \) in its superpartner sector. In the left panel, we plot \( |Y_L| \) for the various values of \( M_1 \). In the right panel, \( |Y_L| \) is plotted for the various values of \( z_0 \). If we use Eqs. (21) and (22), we find that the input parameters adopted to draw Fig. 2 give \( m_1 \approx 10^{-3} - 10^{-4} \) eV and \( T_R > 10^8 \) GeV. By using this kind of figure, we can search the lower bounds of \( M_1 \) and \( T_R \) required to explain the observed \( B \) asymmetry for the fixed \( M_{2,3} \). We practice this analysis changing the values of \( M_{2,3} \) within the allowed region discussed in the previous part. As the result, for the explanation of the \( B \) asymmetry based on the present model, we find that the lower bounds of \( M_1 \) and \( T_R \) can be estimated as

\[
M_1 > 5 \times 10^8 \text{ GeV}, \quad T_R > 6 \times 10^8 \text{ GeV},
\]

and also \( \sin \xi \) should be \( O(10^{-2}) \).

Although the obtained lower bound of the reheating temperature is comparable with the lowest value discussed in other neutrino mass matrix models where \( T_R \approx 10^{8-10} \) GeV is usually suggested, we cannot make it much lower. Since the lower bound of the mass eigenvalue \( M_1 \) is determined by Eq. (20), this result seems not to be avoided in the thermal leptogenesis as far as we do not assume the degeneracy among the masses of the right-handed neutrinos. Recently, in [21] the upper bound for the reheating temperature required from the gravitino problem is estimated as \( 10^{5-7} \) GeV if the gravitino has the mass in the range \( 10^{2-3} \) GeV. If we do not suppose the light gravitino scenario and we follow this bound, the present model is unable to be reconciled with the gravitino problem. We need to consider the initial \( \tilde{N}_1 \) to be yielded in another way. As such a possibility, we study the nonthermal leptogenesis in the next part.

B. Nonthermal leptogenesis

In this subsection we consider that the right-handed neutrinos \( \tilde{N}_i \) are produced through the decay of the infla-
ton. This kind of model has been discussed in [9]. The interaction between the inflaton superfield \( \Phi \) and \( \tilde{N}_i \) is assumed to be given by the superpotential

\[
W = \sum_{i=1}^{3} \lambda_i \Phi \tilde{N}_i^2.
\]  

(24)

After the inflation ends, the inflaton \( \phi \) starts to oscillate and decays to reheate the universe into the temperature \( T_R \). A part of its oscillation energy \( \rho \) of the inflaton is converted into \( \tilde{N}_i \) through its decay at \( H \approx \Gamma_\phi \). The decay width \( \Gamma_\phi \) of the inflaton \( \phi \) can be expressed as

\[
\Gamma_\phi = \sum_{i=1}^{3} \frac{\lambda_i^2}{4\pi} m_\phi + \cdots,
\]  

(25)

where the ellipses stand for the contribution from other decay modes of the inflaton and we assume them to be negligible. The coupling constants \( \lambda_i \) are constrained in such a way as

\[
\sum_{i=1}^{3} \lambda_i^2 \approx 10^{-18} \left( \frac{10^{13} \text{ GeV}}{m_\phi} \right)^2 \left( \frac{T_R}{10^6 \text{ GeV}} \right)^2,
\]  

(26)

which is derived from the condition \( H \approx \Gamma_\phi \). From this we find that the couplings between the right-handed neutrinos and the inflaton can be small enough not to affect the inflaton potential. The inflaton mass \( m_\phi \) can depend on the assumed inflaton model. However, it should satisfy \( m_\phi > \tilde{M}_i \) to guarantee the inflaton decay into the right-handed neutrinos \( \tilde{N}_i \).

If we use \( B_i \) to denote the branching ratio for the decay \( \phi \to \tilde{N}_i^2 \), we have the energy relation \( \rho B_i = \tilde{M}_i n_i \) where \( \rho = \frac{\tilde{M}_i}{\pi} g_* T_R^4 \). Thus the nonthermally generated number density \( n_i \) of \( \tilde{N}_i \) can be written as

\[
\frac{n_i}{s} = \frac{3T_R B_i}{4\tilde{M}_i}.
\]  

(27)

As we mentioned before Eq. (15), \( \epsilon_3 \) is much smaller than \( \epsilon_{1,2} \). In the case dominated by \( B_i \), the produced \( L \) asymmetry is expected to be very small unless \( T_R \) is rather large. Then we concentrate our study into the case dominated by \( B_{1,2} \). Since we find \( \epsilon_1/\tilde{M}_1 = \epsilon_2/\tilde{M}_2 \) from Eq. (15), the \( L \) asymmetry generated through the immediate decay of \( \tilde{N}_{1,2} \) is estimated by using Eq. (16) as

\[
\frac{n_L}{s} \approx \frac{3}{16\pi} \frac{\sqrt{\Delta m_{31}^2 T_R}}{v^2 \sin^2 \beta} (\kappa_1 B_1 + \kappa_2 B_2) \sin^2 2\chi \\
\approx 10^{-10} \left( \frac{T_R}{10^6 \text{ GeV}} \right) (\kappa_1 B_1 + \kappa_2 B_2) \sin^2 2\chi.
\]  

(28)

Equation (28) suggests that \( B_1 \) should be larger than \( T_R \) to explain the \( B \) asymmetry in any case. Moreover, we can find that the \( \tilde{M}_i \) dependence of \( n_L/s \) is confined into the washout factor \( \kappa_i \). If we consider that \( \tilde{M}_i \) is larger than \( T_R \), the washout effect due to \( \tilde{N}_i \) is expected to be suppressed by the Boltzmann factor. Thus for the larger \( \tilde{M}_i \) in such a region, \( n_L/s \) becomes insensitive for the change of the values of \( \tilde{M}_i \).

The estimation of the \( L \) asymmetry in Eq. (28) is justified only if \( \tilde{N}_i \) decays into the light fields immediately after its production [9]. This requires that \( H \approx \Gamma_\phi \leq \Gamma_{\tilde{N}_i} \) should be satisfied for all \( \tilde{N}_i \) which have the substantial branching ratio \( B_i \). Since the inflaton decay width can be estimated by using \( H \approx \Gamma_\phi \) as

\[
\Gamma_\phi \approx \frac{3g_*^{1/2} T_R^3}{M_{pl}},
\]  

(29)

we can write the condition for the justification of Eq. (28) by applying Eqs. (26) and (29) to \( \Gamma_\phi \triangleq \Gamma_{\tilde{N}_i} \) in the form

\[
\left( \frac{T_R}{10^6 \text{ GeV}} \right)^2 \leq 10^6 \left( \frac{\tilde{M}_i}{10^7 \text{ GeV}} \right) \left( \frac{M_2}{10^{10} \text{ GeV}} \right)^2 \sin^2 2\chi.
\]  

(30)

Since this condition can be easily satisfied for the desirable values of \( T_R, M_1, \) and \( M_2 \), we find that Eq. (28) can be validated in our interested case. However, it is also possible that the immediate decay condition is not satisfied for \( \tilde{N}_1 \). This occurs for \( \Gamma_{\tilde{N}_i} < \Gamma_\phi < \Gamma_{\tilde{N}_i} \) in the case of \( B_1 \approx B_2 \). In that case we should take account that the \( L \) asymmetry produced through the \( \tilde{N}_2 \) decay may be washed out by the late entropy release due to the \( \tilde{N}_1 \) decay other than by the usual thermal washout. This effect is discussed in Appendix B. We should also check that the condition (30) can be consistent with the above mentioned condition \( m_\phi > M_{\tilde{N}_i} \). This consistency can be easily checked by applying Eq. (26) to \( \Gamma_\phi \).

In order to study the relation between \( T_R \) and \( \tilde{M}_i \), it is useful to classify the situation into three cases: (i) \( T_R \cong M_1 \), (ii) \( M_1 \cong T_R \cong M_2 \), and (iii) \( M_2 \cong T_R \). Among these three cases, both cases (i) and (ii) can easily satisfy condition (30). On the other hand, case (iii) satisfies it only for \( M_2 \cong 10\tilde{M}_i \sin^2 2\chi \), which requires \( \sin 2\chi \gg 0.1 \). Thus only cases (i) and (ii) seem to be promising for the leptogenesis consistent with the low reheating temperature. In fact, \( T_R \approx 10^6 \text{ GeV} \) may be allowed in these cases with \( B_{1,2} = O(1) \) and \( \sin 2\chi = O(1) \) if \( \kappa_1 \) or \( \kappa_2 \) can be \( O(1) \).

The washout effect is expected to be mainly caused by the \( L \) violating interactions due to the thermal \( \tilde{N}_i \). Since there is the Boltzmann suppression for these processes in case (i), their substantial washout effect cannot be expected. On the other hand, in case (ii) \( \tilde{N}_i \) can contribute to the washout of the \( L \) asymmetry since there is no large Boltzmann suppression. To escape this situation the smallness of the \( \tilde{N}_i \) Yukawa couplings is required. This may be realized for the small \( \sin 2\chi \) case.

To take account of the washout effect quantitatively, we need to solve the Boltzmann equations numerically by

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\(^8\)In the construction of the mass matrices presented in Appendix A, we find that \( B_2 \gg B_1 \) is satisfied if the inflaton has no global charges. The nonthermal leptogenesis for a type of model with \( B_2 \gg B_1 \) has been discussed in [22]. However, we consider the general case here.

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In this case, as the input parameters we use $M_1 = 10^7\text{ GeV}$, $M_2 = 5 \times 10^6\text{ GeV}$, $M_3 = 10^{13}\text{ GeV}$ for the black symbols and $M_1 = 10^3\text{ GeV}$, $M_2 = 10^6\text{ GeV}$, $M_3 = 10^{13}\text{ GeV}$ for the white symbols. The typical feature in these cases is that the $\sin\xi$ value can be smaller compared with the thermal case since we need not to produce $\tilde{N}_i$ thermally.

In both panels of Fig. 4, the larger $|Y_{L_i}|$ is realized for the larger $B_2$ since the washout effect is smaller compared with the smaller $B_2$ case. If we make $M_j$ larger keeping $M_i$ fixed in these figures, $|Y_{L_i}|$ becomes a little bit larger but it seems to reach almost the upper bound in this setting. In the left panel the condition (30) is satisfied only for $\sin\xi \gtrsim 10^{-3}$. In the case of $B_2 = 1$, however, this condition should be replaced by $\Gamma_{\phi} \lesssim \Gamma_{\tilde{N}_i}$ and it is satisfied for all regions of $\sin\xi$ in this figure. In the case of $B_1 = B_2 = 0.5$, for $\sin\xi < 10^{-3}$ we should take account of the additional washout effect to the $L$ asymmetry produced through the $\tilde{N}_2$ decay, which is discussed in Appendix B. It introduces the suppression factor $\sqrt{M_1 M_2 \sin\xi / T_R}$. It takes a value smaller than $O(10^{-1})$ in the present case. On the other hand, the $L$ asymmetry produced by the late $\tilde{N}_1$ decay cannot be a sufficient amount because of the low reheating temperature. Thus, we find that the models with the substantial $B_1$ cannot explain the $B$ asymmetry for $\sin\xi < 10^{-3}$. The Yukawa couplings of $\tilde{N}_1$ and $\tilde{N}_2$ are proportional to $\sin\xi$ and $\cos\xi$, respectively. This fact affects the behavior of $|Y_{L_i}|$ as the function of $\sin\xi$. In fact, the figures show a slight increase in the case of $B_2 = 1$ and a decrease in the case of $B_1 = 1$ when the $\sin\xi$ value becomes larger.

In the right panel the condition (30) is satisfied only for $\sin\xi > 0.1$. The situation is the same as the left panel in the case $B_2 = 1$. Thus in case (ii) only the model with $B_2 \sim 1$ can have the possibility to explain the $B$ asymmetry. The magnitude of $|Y_{L_i}|$ becomes smaller for the larger $\sin\xi$ even in the case of $B_2 = 1$. Since $\tilde{N}_1$ can be produced thermally, the large $\sin\xi$ makes the washout effect more effective. This feature explains the $|Y_{L_i}|$ behavior against $\sin\xi$ in the right panel.

We practice this kind of analysis changing the input parameters within the allowed region. As a result of such study, we find that the lower bound of the required reheat-
ing temperature to explain the $B$ asymmetry is $T_R \approx 3 \times 10^6$ GeV for both cases (i) and (ii).

Finally, we briefly comment on another possibility for the nonthermal leptogenesis [10]. In the early universe the scalar potential of the sneutrino $\tilde{N}_1$ may be flat enough to deviate largely from its potential minimum. If this happens during the inflation, the condensate of $\tilde{N}_1$ starts to oscillate at $H \approx M_1$ and decays at $H \approx \Gamma_{\tilde{N}_1}$. This oscillation may dominate the energy density of the universe at a certain time after the reheating due to the inflaton decay because of its behavior as a matter. We assume that it is the case here.

Since its energy density is expressed by $\rho_{\tilde{N}_1} = M_1^2 |\tilde{N}_1|^2$, the $\tilde{N}_1$ number density $n_1$ is estimated as $M_1^2 |\tilde{N}_1|^2$. Thus the ratio of the $L$ asymmetry produced through its decay to the entropy density is estimated as

$$n_L/s = \frac{2M_1 |\tilde{N}_1|^2}{s} \epsilon_1 \kappa_1 = \frac{3}{2} \frac{T_R}{M_1} \epsilon_1 \kappa_1,$$ (31)

where in the last equality we use the above mentioned assumption $\rho_{\tilde{N}_1} = \frac{3}{2} g s T_R^4$ for the energy density. If we use Eq. (15) for the $CP$ asymmetry $\epsilon_1$, we obtain the similar result for $n_L/s$ to the previous example. However, in this case $\Gamma_\phi > \Gamma_{\tilde{N}_1}$ should be satisfied and this condition imposes $T_R > \sqrt{10M_1 M_2 \sin \xi}$. Thus the expected $L$ asymmetry is estimated as

$$n_L/s \approx 10^{-10} \frac{T_R}{10^6 \text{ GeV}} \approx 10^{-10} \frac{\sqrt{M_1 M_2 \sin \xi}}{10^5 \text{ GeV}},$$ (32)

where we assume $|\sin 2\chi| = 1$ and $\kappa_1 = 1$. This relation gives a constraint for the undetermined parameters in the present neutrino mass texture based on the $B$ asymmetry. The condition $\Gamma_\phi > \Gamma_{\tilde{N}_1}$ also requires us to consider the different type of the inflation from the previous nonthermal example.

Since $\kappa_1 \approx 1$ is validated only for the case $T_R < \tilde{M}_1$, Eq. (32) cannot be applied to case (b) in which $T_R \approx \tilde{M}_1$ follows. We can obtain a sufficient amount of the $L$ asymmetry by taking $\tilde{M}_1$ large enough to make $\epsilon_1$ large but keeping $\sin \xi$ small enough to make $T_R < \tilde{M}_1$. Thus, in case (a) a low reheating temperature like $T_R \approx 10^6$ GeV can be enough to produce the required $B$ asymmetry by setting $\tilde{M}_1$ and $\sin \xi$ suitably. To realize such a low reheating temperature, for example, the mass parameters in the neutrino sector may be taken as

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\[M_1 = 10^8 \text{ GeV}, \quad M_2 = 10^{10} \text{ GeV}, \quad M_3 = 10^{13} \text{ GeV}, \quad \sin \xi = 10^{-4}.\] (33)

Since the effective mass $\tilde{m}_1$ is estimated as $\tilde{m}_1 \sim 10^{-8}$ eV, the washout effect is completely negligible as expected. We have no gravitino problem in this case since the reheating temperature realized by the decay of the $\tilde{N}_1$ condensate is sufficiently low comparable to the one given in [21].

**IV. SUMMARY**

We have proposed the neutrino mass matrices in the framework of the MSSM extended with the three generation right-handed neutrinos. These mass matrices can realize the bi-large mixing among the neutrino flavors and explain the neutrino oscillation data. It can also saturate the upper bound of the $CP$ asymmetry $\epsilon_1$ which appears in the leptogenesis. Although this model is composed of a rather restricted number of parameters, it can make the $CP$ asymmetry $\epsilon_1$ and the effective neutrino mass $\tilde{m}_1$ independent. We have applied this model to the thermal and nonthermal leptogenesis and studied the influence of this feature on the reheating temperature, which is crucial for the cosmological gravitino problem.

In the thermal leptogenesis, our neutrino mass texture seems not to be able to make the reheating temperature required from the explanation of the $B$ asymmetry low enough to be consistent with the gravitino problem. However, it seems to be able to realize a value near the lower bound of the reheating temperature obtained in the thermal leptogenesis framework. In the nonthermal case we have found that the low reheating temperature consistent with the gravitino problem can be sufficient for the successful leptogenesis. Even in that case the parameters in the neutrino mass matrices can be consistent with the neutrino oscillation data.

As shown in this study, some kinds of neutrino mass texture can be constrained by the leptogenesis. It may be worthy to proceed with a lot of study based on the concrete neutrino model to clarify the relation among the neutrino mass texture, the leptogenesis, and the gravitino problem.

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**APPENDIX A**

In this appendix, we present an example of the construction for the assumed texture of the neutrino mass matrix. We consider the Froggatt-Nielsen-type global flavor symmetry $U(1)^5$ [25] and an additional discrete $Z_2$ symmetry in the lepton sector. The charge assignment of the chiral superfields for the symmetry $U(1)^5 \times Z_2$ is assumed as

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9The sneutrino $\tilde{N}_1$ can be an inflaton itself as discussed in [23]. However, we do not consider this possibility since the reheating temperature is too high to be reconciled with the gravitino problem in this case.

10This oscillation may start during the inflation ($\Gamma_\phi < \tilde{M}_1$) or after the inflation ($\Gamma_\phi > \tilde{M}_1$).

11During this oscillation, the flat direction may store the $L$ asymmetry due to the Affleck-Dine mechanism as discussed in [24]. However, we do not consider this possibility here.
\[ N_1(1, 1, 0, 1; +), \quad N_2(1, 0, 1, 1; +), \]
\[ N_3(1, 1, 0, 1; +), \quad L_1(0, 0, -1, 0; +), \quad L_3(0, -1, 0, -1; +). \quad (A1) \]

The Higgs chiral superfield \( H_3 \) is neutral for this symmetry. In order to realize the hierarchical structure of the mass matrices, we introduce the following several chiral superfields which are singlet for the standard gauge groups:

\[ \phi_1(-1, -1, 0, 0; -), \quad \phi_2(-1, 0, -1, 0; -), \]
\[ \phi_3(-2, -2, 0, 0; +), \quad \chi_1(-1, 0, 0, 0; +), \]
\[ \chi_2(0, -1, 0, 0; +), \quad \chi_3(0, 0, -1, 0; +), \]
\[ \eta(0, 0, 0, -1; +). \quad (A2) \]

If we assume that the scalar components of these superfields get vacuum expectation values defined by 12

\[ \epsilon_i \equiv \frac{\langle \phi_i \rangle}{M_{\text{pl}}}, \quad \delta \equiv \frac{\langle \chi_i \rangle}{M_{\text{pl}}}, \quad \zeta \equiv \frac{\langle \eta \rangle}{M_{\text{pl}}}, \quad (A3) \]

we can obtain both the right-handed Majorana mass matrix and the Dirac mass matrix as follows:

\[ \mathcal{M} = M_3 \begin{pmatrix} \epsilon_1^2 \zeta^2 / \epsilon_3 & \epsilon_1 \epsilon_2 \zeta^2 / \epsilon_3 & \epsilon_2^2 / \epsilon_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad m_D = v_2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & \delta & 0 \\ 0 & 0 & \delta \end{pmatrix}, \quad (A4) \]

where \( M_3 \equiv M_{\text{pl}} \epsilon_3 \) and the order one coefficients are abbreviated. The difference between cases (a) and (b) should be considered to be explained by these coefficients.

We can check that these mass matrices can consistently realize the texture assumed in the text. Comparing \( m_D \) in Eq. (2) with that in Eq. (A4) and also using Eq. (8), we find

\[ \delta \sim 0.1 \left( \frac{M_3}{10^{13} \text{GeV}} \right)^{1/2}, \quad \zeta \sim 0.4 \left( \frac{M_2}{M_3} \right)^{1/2}, \quad \epsilon_3 = \frac{M_3}{M_{\text{pl}}}, \quad (A5) \]

Applying this result to \( \mathcal{M} \) in Eqs. (2) and (A4), we obtain

\[ M_1 \sim \left( \frac{\epsilon_1}{\epsilon_2} \right)^2 M_2, \quad \epsilon_3 \sim 0.16 \epsilon_2, \quad \sin \xi \sim \epsilon_1 / \epsilon_2. \quad (A6) \]

If we take \( \delta \sim 0.1, \zeta \sim 0.03, \) and \( \epsilon_2 \approx 30 \epsilon_1, \) for example, we find

\[ M_1 \sim 10^8 \text{GeV}, \quad M_2 \sim 10^{10} \text{GeV}, \quad M_3 \sim 10^{13} \text{GeV}, \quad \sin \xi \sim 0.03. \quad (A7) \]

This suggests that (A4) can realize the mass matrices assumed in the text.

We can consider the couplings of the right-handed neutrinos \( \tilde{N}_i \) to the inflaton \( \phi \), which is assumed to be neutral under the present global symmetry. The lowest order superpotential allowed by this symmetry is written as

\[ W = c_1 \frac{\phi^2 \eta^2}{M_{\text{pl}}^4} \phi N_1^2 + c_2 \frac{\phi^2 \eta^2}{M_{\text{pl}}^4} \phi N_2^2 + c_3 \frac{\phi^2}{M_{\text{pl}}^3} \phi N_3^2, \quad (A8) \]

where the coefficients \( c_{1,2,3} \) are assumed to be \( O(1) \). Thus the coupling constants \( A_i \) are estimated as

\[ \lambda_1 = c_1 \epsilon_1^2 \zeta^2, \quad \lambda_2 = c_2 \epsilon_1^2 \zeta^2, \quad \lambda_3 = c_3 \epsilon_1. \quad (A9) \]

We find that these couplings can be consistent with the low value of \( T_R \) if \( m_\phi \approx 10^{12} \text{GeV} \) is assumed. In this case \( B_2 \gg B_1 \) can be satisfied since \( B_3 = 0 \) is realized for \( M_3 \approx 10^{13} \text{GeV} \).

**APPENDIX B**

In the nonthermal leptogenesis, the \( L \) asymmetry produced through the \( N_2 \) decay may be washed out in a different way compared with the thermal case. If \( \Gamma_{\tilde{N}_2} < \Gamma_{\phi} = \Gamma_{\tilde{N}_2} \) is satisfied, \( \tilde{N}_2 \) decays into the light particles at the time \( t_2 \) immediately after the inflaton decays into \( \tilde{N}_2 \). The decay products of \( \tilde{N}_2 \) behave as the radiation and its energy density decreases as \( \rho_{\tilde{N}_2} \propto a^{-3} \) where \( a \) is the cosmological scale parameter. On the other hand, \( \tilde{N}_1 \) decays at the time \( t_1 \) after the completion of the inflaton decay. If \( \tilde{N}_1 \) behaves as a matter because of \( T_R < M_1 \), its energy density decreases as \( \rho_{\tilde{N}_1} \propto a^{-3} \). Then the cosmological energy density may be dominated by the \( \tilde{N}_1 \) energy at least as far as \( B_1 \) and \( B_2 \) are the same order. The additional washout can occur in such a case.

The cosmological energy density \( \rho(t_2) \) and the temperature \( T_2(t_2) \) of its decay products can be expressed as

\[ \rho(t_2)B_2 = \frac{\pi^2}{30} g_*^2(t_2) T_2^4(t_2), \quad H(t_2) = \frac{\rho(t_2)}{3M_{\text{pl}}^2} \approx \frac{\Gamma_\phi}{\Gamma_{\tilde{N}_1}}. \quad (B1) \]

Taking account of these, we can find that the temperature \( T_2 \) of the decay products of \( \tilde{N}_2 \) satisfies the relation such as

\[ \left( \frac{T_2(t_2)}{T_2(t_1)} \right)^4 = \left( \frac{a(t_1)}{a(t_2)} \right)^4 = \left( \frac{H(t_2)}{H(t_1)} \right)^{8/3} = \left( \frac{\Gamma_\phi}{\Gamma_{\tilde{N}_1}} \right)^{8/3}. \quad (B2) \]

From this relation we obtain

\[ T_2(t_2) = T_2(t_1) \left( \frac{\Gamma_\phi}{\Gamma_{\tilde{N}_1}} \right)^{2/3}. \quad (B3) \]

Now we can estimate the entropy production of the late decay of \( \tilde{N}_1 \). Since \( H(t_1) = \rho(t_1)/3M_{\text{pl}}^2 = \Gamma_{\tilde{N}_1} \) is satisfied, the ratio between the entropy density \( s_{e}(t_1) \) before the \( \tilde{N}_1 \) decay and the entropy density \( s_{e}(t_1) \) after the decay can
By using the expression of each decay width, we find that $\kappa_2$ is written as

$$\frac{s_{\ell}(t_1)}{s_{\sigma}(t_1)} = \frac{g_{\sigma}(t_2)T_2^3(t_1)}{g_{\sigma}(t_1)T_1^3(t_1)} \approx \frac{T_2(t_2)}{T_1(t_1)} \left( \frac{\Gamma_{\sigma}}{\Gamma_\phi} \right)^2 \approx \left( \frac{\Gamma_{\sigma}}{\Gamma_\phi} \right)^{1/2}. \quad (B4)$$

$$\kappa_2 \approx \sqrt{\frac{M_{T_2} M_{T_1}}{T_R}} \sin \xi, \quad (B5)$$

where $\kappa$ is the usual thermal washout effect due to the $L$ violating scattering mediated by the right-handed neutrinos and so on.

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