New Real-Time Estimation Method for Inertia Properties of STSAT-3 using Gyro Data

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A satellite’s inertia properties can be changed after launch due to consumption of propellant, deployment of solar panels, collision with space debris, sloshing, etc. For reliable and efficient satellite control, it is necessary to predict accurate information regarding the satellite’s inertia properties. In this note, we suggest a new real time method to estimate the inertia properties. First of all, we filter the noise of the gyro data with the extended Kalman filter. Then we estimate the inertia properties using the recursive least squares algorithm. For an improved estimation result, we combine the angular acceleration method and the angular velocity method. We have verified the performance of the suggested method through the case of STSAT-3, a Korea Science Technology Satellite, which has already been developed.

Nomenclature

\( I \): inertia matrix of satellite
\( J \): inertia vector to be estimated
\( u \): reaction wheel control torque
\( \omega \): angular velocity
\( y \): measurement vector
\( \Omega \): matrix consisting of the angular velocities
\( \times \): vector cross product

1. Introduction

The process of inertia properties estimation consists of two steps. The first step is obtaining the filtered data of angular rates using the filtering algorithm, and then second step is obtaining the inertia properties using the estimation algorithm from the filtered data. The Kalman filter and the extended Kalman filter are the most popular methods used for filtering the gyro angular velocity measurement. The least squares, recursive least squares, and Kalman filter can be applied to the estimation of inertia properties. For applying the estimation algorithm, it is necessary to reformulate Euler’s equation of motion or the total angular momentum of the satellite. While the extended Kalman filter, Kalman filter, and recursive least squares methods can be applied to real-time estimation, the least squares method cannot.

The reformulation of Euler’s equations of motion is commonly used for the regressor model in the estimation algorithm. Palimaka and Burlon presented the application of the weighted least squares method for the estimation of the diagonal inertia matrix.1) Zhao et al. applied the Kalman filter for the estimation of inertia properties.2) Lee and Wertz developed the estimation methodology for the inertia properties using the least squares method, and reformulated the total angular momentum of a satellite.3) Yang et al. showed that the extended Kalman filter was the best method for reducing the noise. They also estimated the diagonal inertia matrix using the least squares method.4) Kim et al. developed a combined method for the diagonal inertia matrix estimation. This method consisted of the extended Kalman filter and the least squares method.5)

In this note, we suggest a new estimation method for the constant terms of the inertia matrix. The extended Kalman filter is applied to reduce the noise of gyro angular velocity measurement, and then the recursive least squares method is applied to estimate the inertia properties. For the recursive least squares method, we reformulated Euler’s equations of motion for the regressor model. Numerical simulation is performed to demonstrate the improvement in the estimation result.

2. Inertia Properties Estimation

The equations of motion for a rigid satellite under control input can be written as follows:

\[
\dot{\omega} = -J^{-1} \omega \times I \omega + J^{-1} u
\]  
(1)

\[
y = \omega
\]  
(2)

The estimation of inertia properties requires a different form, which constructs the elements of the inertia matrix.\(^{4-6}\)

First, we define the matrix \( \Omega \) and the vector \( J \). We assumed \( J \) is a constant during the integration interval.

\[
\Omega = \begin{bmatrix}
\omega_1 & 0 & 0 & \omega_2 & \omega_3 & 0 \\
0 & \omega_2 & 0 & \omega_3 & 0 & \omega_1 \\
0 & 0 & \omega_3 & 0 & \omega_1 & \omega_2
\end{bmatrix}
\]  
(3)

\[
J = \begin{bmatrix}
I_{xx} & I_{xy} & I_{xz} & I_{yy} & I_{yz} & I_{zz}
\end{bmatrix}^T
\]  
(4)

For the angular acceleration method, Eq. (1) is reformulated as follows:

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*Received 19 August 2014; final revision received 6 January 2015; accepted for publication 11 March 2015.
and angular velocity method as follows:

\[ \dot{\omega} + \omega \times \Omega = \frac{u - \omega \times \Omega}{J} \]

We obtain the regression equation of the angular velocity method from integrating Eq. (5):

\[ \int u \, dt = \left[ \dot{\omega} + \int \omega \times \Omega \, dt \right] \]

For solving this problem, we attempted to combine the angular acceleration method and the angular velocity method. From Eqs. (5) and (6), we obtain the angular acceleration and angular velocity method as follows:

\[ \begin{bmatrix} u \\ \int u \, dt \end{bmatrix} = \begin{bmatrix} \dot{\omega} + \omega \times \Omega \\ \Omega + \int \omega \times \Omega \, dt \end{bmatrix} \]

We reduce the noise of gyro angular velocity measurement using the extended Kalman filter. From the filtered data, we estimate the inertia properties using the recursive least squares algorithm. We estimate the inertia properties of the satellite under two assumptions: the inertia properties are constant, and \( u \) is the only command torque applied to the satellite. We consider the simulation model STSAT-3, which has already been developed.

We simulate the gyro angular velocity measurements of STSAT-3 and the noise of this data is filtered out by the extended Kalman filter before estimating the inertia properties. Four reaction wheels of 5 mNm are mounted on the satellite in a tetrahedron configuration. Four reaction wheels of 5 mNm are used to generate the command torque \( [1 \ 1 - 2] \times 10^{-3} \) Nm. We assumed that all four wheels ran during the entire simulation time. The simulation conditions are shown in Table 1.

The tendency of convergence upon the time and the improvement in the estimated results are shown in Figs. 1–3 and Tables 2–4, respectively. The estimation error of the angular acceleration and angular velocity method is within 1%.
But the estimation error of the angular acceleration method and that of the angular velocity method are within about 7% and 5%, respectively. Figures 4 and 5 represent the three-dimensional trajectory of the moment of inertia and the product of inertia that are obtained by using the angular acceleration and angular velocity method.

3. Conclusions

In this note, a new real-time method is suggested to estimate the inertia properties. To reduce the noise in gyro angular velocity measurement, we applied the extended Kalman filter, and then the recursive least squares algorithm is applied to the real-time estimation of inertia properties. To verify the performance of this suggested method, numerical simulation was performed for the case of STSAT-3. When we combined the angular acceleration method and the angular velocity method, the estimation results were more accurate than the estimation results that only used either the angular acceleration method or the angular velocity method.

Future work will involve the real-time estimation of time varying inertia properties that could be caused by consumption of propellant, collision with space debris, etc.

Acknowledgments

This work was supported by Global Surveillance Research Center (GSRC) program funded by the Defense Acquisition Program Administration (DAPA) and the Agency for Defense Development (ADD).

References

1) Palimaka, J. and Burlton, B. V.: Estimation of Spacecraft Mass Properties using Angular Rate Gyro Data, Proc. of AIAA/AAS, 1992, pp. 21–26.
2) Zhao, Y., Zhang, D., Tian, H., and Li, N.: Mass Property Estimation for Mated Flight Control, Proc. of Computer Modeling and Simulation, 2009.
3) Lee, A. Y. and Wertz, J. A.: In-Flight Estimation of the Cassini Spacecraft’s Inertia Tensor, Spacecraft and Rocket J., 39 (2008), pp. 459–462.
4) Yang, S., Cheon, D., Lee, S., and Oh, H.: On-orbit Estimation of Dynamic Properties for STSAT3, Proc. of ICASS, 2008, pp. 459–462.
5) Kim, D., Yang, S., Cheon, D., Lee, S., and Oh, H.: Combined Estimation Method for Inertia Properties of STSAT-3, Mechanical Science and Technology J., 24 (2010), pp. 1737–1741.
6) Tanygin, S. and Williams, T.: Mass Property Estimation using Coasting Maneuvers, Guidance, Control, and Dynamics J., 20 (1997), pp. 625–632.
7) Simon, D.: Optimal State Estimation, John Wiley & Sons, Hoboken, New Jersey, United States, 2006, pp. 84–92.

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