LIMITS ON NEUTRINO MASSES FROM LARGE-SCALE STRUCTURE

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Massive neutrinos have a detectable effect on cosmological structure formation, in particular on the large-scale distribution of galaxies. Adding Hot Dark Matter to the now-standard ΛCDM model leads to a worse fit to large-scale structure and CMB anisotropy data. This results in a limit on the mass of the most massive neutrino of 4 eV, assuming a power-law primordial power spectrum.

1 What Can Cosmology Tell Us About Neutrino Masses?

There is now evidence for neutrino oscillations from SuperKamiokande (\(\Delta m^2 \approx 10^{-3} \text{ eV}^2\)), the solar neutrino problem (\(\Delta m^2 \approx 10^{-5} \text{ eV}^2\)) and LSND (\(0.1 \text{ eV}^2 \leq \Delta m^2 \leq 20 \text{ eV}^2\)). While these oscillations only test the difference in squared masses, they give evidence that the mass of at least one (and probably at least three) neutrino species is non-zero. Laboratory limits from tritium beta decay rule out the possibility of an electron neutrino more massive than 4.4 eV. Present cosmological bounds on the masses of other neutrino species are stricter than those from laboratory experiments; a 45 eV neutrino would lead to \(\Omega_\nu = 1\), so for a universe at less than critical density the neutrinos must all be lighter than this. The exception to this is if a neutrino is so massive (\(> 1 \text{ MeV}\)) that it was non-relativistic during freeze-out, i.e. Cold Dark Matter (CDM).

Each model of structure formation predicts Cosmic Microwave Background (CMB) anisotropy and large-scale structure inhomogeneities. Massive neutrinos lead to slightly different predictions for CMB anisotropies and significantly different predictions for large-scale structure. The CMB radiation power spectrum is given by \(C_\ell = \int_k C_\ell(k) P_p(k)\) and the matter power spectrum by \(P(k) = T^2(k) P_p(k)\), where \(C_\ell(k)\) and \(T(k)\) are transfer functions predicted by a given model of structure formation and \(P_p(k)\) is the primordial power spectrum of density perturbations that originated in the early universe. Massive neutrinos alter these transfer functions, so for a given \(P_p(k)\) we can test a model with massive neutrinos by comparing its predictions to the observed \(C_\ell\) and \(P(k)\). The effect is to exponentially damp the matter transfer.
function, \( T(k) \), on scales smaller than the neutrino free-streaming scale\(^6\),

\[
    k \simeq 0.026 \left( \frac{m_{\nu}}{1\text{eV}} \right)^{1/2} \Omega_m^{1/2} h \ Mpc^{-1},
\]

which is equal to the horizon size when the neutrinos become non-relativistic. For mostly Cold Dark Matter and a fraction of Hot Dark Matter (massive neutrinos), the damping is no longer exponential but still quite significant. The effect on the CMB is more subtle; relativistic neutrinos increase the radiation density before decoupling, which affects the shape of the acoustic peaks in the CMB angular power spectrum.

2 Approach

We assume here that \( \Lambda \text{CDM} \) is the correct model of structure formation and that the primordial power spectrum is well-described by a power-law. We start with a version of \( \Lambda \text{CDM} \) which is in good agreement with observations of Type Ia supernovae, the cluster baryon fraction, the primordial deuterium abundance, and Hubble’s constant, with \( \Omega_m = 0.4 \), \( \Omega_b = 0.04 \), and \( h = 0.7 \). However, this \( \Lambda \text{CDM} \) model is not a great fit to large-scale structure data\(^7\), so we investigate whether adding a Hot Dark Matter component will improve the fit.

Our data compilation includes COBE and smaller-scale CMB anisotropy detections, measurements of \( \sigma_8 \) from galaxy clusters\(^8\), a measurement of the matter power spectrum from peculiar velocities\(^9\), redshift-space matter power spectra from SSRS2+CfA2\(^10\), LCRS\(^11\), PSCz\(^12\), and APM Clusters\(^13\), and a real-space matter power spectrum derived from the APM angular correlation function\(^14\). Using all available CMB and large-scale structure data gives us information on intermediate scales, which helps to differentiate between variations in the primordial power spectrum and the reduction in small-scale power caused by massive neutrinos. We analyze this data compilation using the methods of Gawiser & Silk\(^7\).

Even given all of this data, we need to assume something about the primordial power spectrum. We have tried using Harrison-Zel’dovich (scale-invariant, \( P_p(k) = Ak \)) and scale-free (\( P_p(k) = Ak^n \)) primordial power spectra. For inflationary models, the most reasonable parameterization is \( \log P_p(k) = \log A + n \log k + \alpha (\log k)^2 + ... \) with successive terms decreasing in importance in the slow-roll regime. Unfortunately, an unconstrained primordial power spectrum \( P_p(k) \) can easily mimic the effect that massive neutrinos

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\(^6\)Smaller values of \( \Omega_m \) will lead to tighter limits on \( \Omega_c \).
have on the matter transfer function $T(k)$, making it nearly impossible to limit the neutrino mass.

3 Results

The results presented here were first determined by Gawiser[16]. We find that as HDM is added, the combined fit to CMB and large-scale structure deteriorates. This occurs because adding HDM reduces the power on physical scales shorter than the neutrino free-streaming length, which degrades the fit to large-scale structure data (see Figure 1). For $\Omega_\nu = 0.05$, a blue tilt of the primordial power spectrum ($n = 1.3$) is necessary to counteract the damping of small-scale perturbations by free-streaming of the massive neutrinos. Even with this best-fit value of $n$, the fit to the data is worse than with no HDM, because CMB observations disfavor such a high value of $n$. For a higher HDM fraction, an even higher value of $n$ is preferred ($n = 1.5$ for the $\Omega_\nu = 0.1$ model of Figure 1), leading to an even worse fit to the CMB data.

Our limits on the neutrino mass are based upon an attempt to search the reasonable parameter space around this fiducial model to produce the best fit possible to the data for a given neutrino mass. Since disagreement with CMB data is the main problem once a blue tilt is considered, we have tried to alleviate this by adding a significant tensor component or early reionization. Each of these effects reduces the small-scale CMB power relative to COBE scales, which helps to reconcile $n > 1$ with the CMB data. However, no parameter combination helps enough to make $\Lambda$CHDM a better fit than the fiducial $\Lambda$CDM model, and this allows us to set an upper limit on the sum of the neutrino masses, $\Sigma m_\nu = 94\Omega_\nu h^2$ eV. An upper limit on $\Omega_\nu$ leads to an upper limit on the mass of the most massive neutrino. If the mass is split between multiple nearly-equal-mass neutrinos, the limit on the sum of the masses is tighter because, for example, two 1 eV neutrinos depress the power spectrum more than one 2 eV neutrino because they both become non-relativistic later.

If $\Lambda$CDM is right, and $H_0$ is about 65 $h^{-1}$Mpc and $n = 1$, then $\Omega_\nu \leq 0.05, m_\nu \leq 2$ eV.

If $n$ can vary, $\Omega_\nu \leq 0.1, m_\nu \leq 4$ eV.

If $P_p(k)$ is not a power-law (non-inflationary or a complicated two-field inflationary model), then all bets are off.

This is compatible with the recent claim by Croft et al. [17] that the Lyman $\alpha$ forest power spectrum plus COBE limits the neutrino mass to 5 eV or less. Our method appears more robust as the Lyman $\alpha$ forest power spectrum has an uncertain normalization and covers a narrower range of scales.
than our large-scale structure compilation; moreover, the origin of the Lyman α forest is not yet well understood. Fukugita et al. [18] use only COBE and $\sigma_8$ and assume $n = 1$, yielding $\sum m_\nu \leq 3$ eV. This is highly model-dependent because the primordial power spectrum is nearly degenerate with neutrino free-streaming when structure formation is only probed at two different spatial scales. ACHDM has also been explored by Valdarnini et al. [19] and Primack & Gross [20] with different analysis methods and significantly smaller data compilations.

Hu et al. [6] discuss the future prospects of including the Sloan Digital Sky Survey (SDSS) $P(k)$ in a method similar to that used here; they expect to detect or limit $m_\nu$ down to 0.5 eV. Cooray [21] gives an expected future limit from measuring $P(k)$ with surveys of weak gravitational lensing of $m_\nu \leq 3$eV.

4 Conclusions

The currently-favored ΛCDM model does not prefer the addition of a Hot Dark Matter component. This leads to an upper limit on the mass of the most massive neutrino of 4eV if a power-law primordial power spectrum is assumed. This limit is comparable to tritium beta-decay limits on the electron neutrino mass, and it should improve considerably with data from SDSS and the MAP satellite. Our results are already in conflict with the portion of the parameter space compatible with LSND [3] that requires a mass difference greater than 16 eV$^2$.

In evaluating these results and other work on this subject, the reader is encouraged to check what assumptions have been made about cosmological models, the selection and normalization of data, and the primordial power spectrum. Restrictive assumptions can lead to tight but meaningless limits on the neutrino mass.

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Figure 1. Comparison of large-scale structure data compilation with ΛCDM (solid), ΛCHDM with Ω_ν = 0.1 and n = 1.0 (dotted), and ΛCHDM with Ω_ν = 0.1 and n = 1.5 (dashed). Corrections for redshift distortions and non-linear evolution are not shown but were used to get quantitative results on scales k \leq 0.2.

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