A NOTE ON GREEN FUNCTORS WITH INFLATION

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ABSTRACT. This note is motivated by the problem to understand, given a commutative ring $F$, which $G$-sets $X$, $Y$ give rise to isomorphic $F[G]$-representations $F[X] \cong F[Y]$. A typical step in such investigations is an argument that uses induction theorems to give very general sufficient conditions for all such relations to come from proper subquotients of $G$. In the present paper we axiomatise the situation, and prove such a result in the generality of Mackey functors and Green functors with inflation. Our result includes, as special cases, a result of Deligne on monomial relations, a result of the first author and Tim Dokchitser on Brauer relations in characteristic 0, and a new result on Brauer relations in characteristic $p > 0$. We will need the new result in a forthcoming paper on Brauer relations in positive characteristic.

1. Introduction

The Burnside ring $B(G)$ of a finite group $G$ is, as a group, the free abelian group on the set of isomorphism classes of transitive $G$-sets. Any transitive $G$-set is isomorphic to a set of cosets $G/H$ for some $H \leq G$, so we may write elements of $B(G)$ as formal $\mathbb{Z}$-linear combinations of symbols $[G/H]$. Let $A$ be a field of characteristic $p \geq 0$. The representation ring $R_A(G)$ of a finite group $G$ over $A$ is, as a group, the free abelian group on the set of isomorphism classes of indecomposable $A[G]$-modules. For every finite group $G$ there is a natural homomorphism $B(G) \to R_A(G)$, which sends the isomorphism class represented by a $G$-set $X$ to the isomorphism class of the $A[G]$-module $A[X]$ with a canonical $A$-basis given by the elements of $X$, and with $G$ acting by permutations on this basis. Let $K_A(G)$ denote the kernel of this homomorphism. It is easy to see that $K_A(G)$, as a subgroup of $B(G)$, only depends on the characteristic of $A$, and we refer to elements of $K_A(G)$ as Brauer relations of $G$ in characteristic $p$.

It is an old problem, with many applications in number theory and geometry, to understand the structure of $K_A(G)$ for all finite groups $G$. See e.g. [1, §1] for a brief overview of the history of the problem and of some of the applications. The most efficient and, from the point of view of number theoretic and geometric applications, the most useful way of giving a complete characterisation of $K_A(G)$, not just as an abstract group, but with an explicit description of generators, is to view $K_A(G)$ as a Mackey functor with inflation. We briefly explain informally what this means, and refer to Section 2 for the formal discussion.

If $H$ is a subgroup of a finite group $G$, then Brauer relations of $H$ can be induced to Brauer relations of $G$. Moreover, if $G$ is a quotient of a finite group $G$, then Brauer relations of $G$ can be lifted to Brauer relations of $G$.
Let \( \text{Imprim}_{K_A}(G) \) be the subgroup of \( K_A(G) \) generated by all relations that are induced from proper subgroups or lifted from proper quotients, and let \( \text{Prim}_{K_A}(G) \) be the quotient \( K_A(G)/\text{Imprim}_{K_A}(G) \). If one can give, for every finite group \( G \), generators of \( \text{Prim}_{K_A}(G) \), then one obtains a list of Brauer relations with the property that all Brauer relations in all finite groups are \( \mathbb{Z} \)-linear combinations of inductions and lifts of relations in this list.

In [1] the structure of \( \text{Prim}_{K_A}(G) \) has been completely determined, in the above sense, in the case when \( A \) has characteristic 0. The following theorem was a crucial step towards that result. If \( q \) is a prime number, then a group is called \( q \)-quasi-elementary if it has a normal cyclic subgroup of \( q \)-power index. A group is called quasi-elementary if it is \( q \)-quasi-elementary for some prime number \( q \).

**Theorem 1.1** ([1], Theorem 4.3). Let \( G \) be a finite group that is not quasi-elementary. Then:

(a) if all proper quotients of \( G \) are cyclic, or if there exists a proper quotient that is not quasi-elementary, then \( \text{Prim}_{K_3}(G) \) is trivial;

(b) if \( q \) is a prime number such that all proper quotients of \( G \) are \( q \)-quasi-elementary, and at least one of them is not cyclic, then \( \text{Prim}_{K_q}(G) \cong \mathbb{Z}/q\mathbb{Z} \), and is generated by any element of \( K_q(G) \subseteq B(G) \) of the form \( [G/G] + \sum_{H \leq G} a_H [G/H] \), \( a_H \in \mathbb{Z} \);

(c) if \( q \) and \( q' \) are distinct prime numbers such that there exists a proper quotient of \( G \) that is non-cyclic \( q \)-quasi-elementary, and a proper quotient that is non-cyclic \( q' \)-quasi-elementary, then \( \text{Prim}_{K_q}(G) \) is trivial.

Deligne [7] had proven a similar result on relations between monomial representations, see Theorem 5.1 below.

The main motivation for this paper is to understand \( \text{Prim}_{K_A}(G) \) when \( A \) has positive characteristic. To that end, we prove the following characteristic \( p \) analogue of Theorem 1.1, which will be used in a forthcoming paper to give a characterisation of \( \text{Prim}_{K_p}(G) \). If \( p \) and \( q \) are prime numbers, then a group is called \( p \)-hypo-elementary if it has a normal \( p \)-subgroup with cyclic quotient, and it is called a \((p,q)\)-Dress group if it has a normal \( p \)-subgroup with \( q \)-quasi-elementary quotient.

**Theorem 1.2.** Let \( G \) be a finite group that is not a \((p,q)\)-Dress group for any prime number \( q \). Then:

(a) if all proper quotients of \( G \) are \( p \)-hypo-elementary, or if there exists a proper quotient that is not \((p,q)\)-Dress group for any prime number \( q \), then \( \text{Prim}_{K_p}(G) \) is trivial;

(b) if \( q \) is a prime number such that all proper quotients of \( G \) are \((p,q)\)-Dress groups, and at least one of them is not \( p \)-hypo-elementary, then \( \text{Prim}_{K_p}(G) \cong \mathbb{Z}/q\mathbb{Z} \), and is generated by any element of \( K_p(G) \subseteq B(G) \) of the form \( [G/G] + \sum_{H \leq G} a_H [G/H] \), \( a_H \in \mathbb{Z} \);

(c) if \( q \) and \( q' \) are distinct prime numbers such that there exists a proper quotient of \( G \) that is not \( p \)-hypo-elementary but a \((p,q')\)-Dress group, and a proper quotient that is not \( p \)-hypo-elementary but a \((p,q')\)-Dress group, then \( \text{Prim}_{K_p}(G) \) is trivial.
To prove part (b) of Theorem 1.2 we prove an induction theorem for \((p,q)\)-Dress groups, which we believe to be of independent interest. It is a characteristic \(p\) analogue of the main Theorem of [8].

**Theorem 1.3.** Let \(p\) and \(q\) be prime numbers, let \(G\) be a \((p,q)\)-Dress group, and let \(a\) be an integer. Then there exists an element in \(K_{p,q}(G)\) of the form \(a[G/G] + \sum_{H \leq G} a_H [G/H]\), \(a_H \in \mathbb{Z}\) if and only if \(q|a\).

In fact, we deduce Theorems 1.1 and 1.2, as well as Deligne’s theorem on monomial relations, as special cases of a general result on kernels of morphisms between Green functors with inflation. This formalism, which is a mix of axiomatisations that have appeared in the literature many times before, see e.g. [12] and [4], will be introduced in Section 2. In Section 3 we recall the concepts of primordial groups for a Mackey functor. Our main theorems on kernels of morphisms of Green functors will be proven in Section 4. Section 5 is devoted to concrete applications, and it is there that we prove Theorems 1.1, 1.2, and 1.3.

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Our rings are always assumed to be associative, with a unit element. Let \(R\) be a commutative ring. By an \(R\)-algebra we mean a ring \(A\) equipped with a map \(R \to Z(A)\), where \(Z(A)\) denotes the centre of \(A\). If \(p\) is a prime ideal of \(R\), then \(R_p\) denotes the localisation of \(R\) at \(p\). Given a finite group \(G\) and a prime number \(p\), we denote the largest normal \(p\)-subgroup of \(G\) by \(O_p(G)\), and the smallest normal subgroup of \(p\)-power index by \(O^p(G)\). We also define \(O^p(G)\) to be \(G\). If \(\pi\) is a set of prime numbers, and \(n\) is an integer, then we denote by \(n\pi'\) the largest positive integer dividing \(n\) that is coprime to all \(p \in \pi\). In this paper, \(R\) will always denote a domain.

## 2. Mackey and Green functors with inflation

One can find many variations on the theme of Mackey functors in the literature. The axiomatisation that we need is very similar to those of [12, 4].

**Definition 2.1.** A **global Mackey functor with inflation** (MFI) over \(R\) is a collection \(\mathcal{F}\) of the following data.

- For every finite group \(G\), \(\mathcal{F}(G)\) is an \(R\)-module;
- for every monomorphism \(\alpha: H \hookrightarrow G\) of finite groups, \(\mathcal{F}_\alpha(\alpha): \mathcal{F}(H) \to \mathcal{F}(G)\) is a covariant \(R\)-module homomorphism (which we think of as induction);
- for every homomorphism \(\epsilon: H \to G\) of finite groups, \(\mathcal{F}_\epsilon(\epsilon): \mathcal{F}(G) \to \mathcal{F}(H)\) is a contravariant \(R\)-module homomorphism (which we think of as restriction when \(\epsilon\) is a monomorphism, and as inflation when \(\epsilon\) is an epimorphism);

satisfying the following conditions.
(MFI 1) Transitivity of induction: for all group monomorphisms \(U \overset{\beta}{\rightarrow} H \overset{\alpha}{\rightarrow} G\), we have \(\mathcal{F}_s(\alpha\beta) = \mathcal{F}_s(\alpha)\mathcal{F}_s(\beta)\).

(MFI 2) Transitivity of restriction/inflation: for all group homomorphisms \(U \overset{\beta}{\rightarrow} H \overset{\alpha}{\rightarrow} G\), we have \(\mathcal{F}^*(\alpha\beta) = \mathcal{F}^*(\beta)\mathcal{F}^*(\alpha)\).

(MFI 3) For all inner automorphisms \(\alpha : G \rightarrow G\), we have \(\mathcal{F}^*(\alpha) = \mathcal{F}_s(\alpha) = 1\).

(MFI 4) For all automorphisms \(\alpha\), we have \(\mathcal{F}_s(\alpha) = \mathcal{F}^*(\alpha^{-1})\).

(MFI 5) The Mackey condition: for all pairs of monomorphisms \(\alpha : H \rightarrow G\) and \(\beta : K \rightarrow G\), we have

\[
\mathcal{F}^*(\beta)\mathcal{F}_s(\alpha) = \sum_{g \in \alpha(H) \cap \beta(K)} \mathcal{F}_s(\phi_g)\mathcal{F}^*(\psi_g),
\]

where \(\phi_g\) is the composition

\[
\phi_g : \beta(K)^g \cap \alpha(H) \overset{c_g}{\rightarrow} \beta(K) \cap \alpha(H) \overset{\alpha^{-1}}{\rightarrow} K,
\]

\(c_g\) denoting conjugation by \(g\), and \(\psi_g\) is the composition

\[
\psi_g : \alpha(H) \cap \beta(K)^g \overset{\alpha}{\rightarrow} \alpha(H) \overset{\alpha^{-1}}{\rightarrow} H.
\]

(MFI 6) Commutativity of induction and inflation: whenever there is a commutative diagram

\[
\begin{array}{ccc}
H & \overset{\alpha}{\rightarrow} & G \\
\downarrow{\epsilon} & & \downarrow{\delta} \\
\hat{H} & \overset{\beta}{\rightarrow} & \hat{G},
\end{array}
\]

where \(\epsilon, \delta\) are epimorphisms, and \(\alpha, \beta\) are monomorphisms, we have

\[
\mathcal{F}^*(\delta)\mathcal{F}_s(\beta) = \mathcal{F}_s(\alpha)\mathcal{F}^*(\epsilon).
\]

We will often use the following more intuitive notation: if \(\mathcal{F}\) is an MFI, and \(\alpha : H \rightarrow G\) is a monomorphism, we will write \(\text{Res}_{G/H}\) for \(\mathcal{F}^*(\alpha)\), and \(\text{Ind}_{G/H}\) for \(\mathcal{F}_s(\alpha)\). Usually, the suppressed dependence on \(\alpha\) and \(\mathcal{F}\) will not cause any confusion. Similarly, if \(\epsilon : G \rightarrow \hat{G}\) is an epimorphism with kernel \(N\), we will write \(\text{Inf}_{G/N}\) for \(\mathcal{F}^*(\epsilon)\).

**Definition 2.2.** A Green functor with inflation (GFI) over \(R\) is an MFI \(\mathcal{F}\) over \(R\), satisfying the following additional conditions.

(GFI 1) For every finite group \(G\), \(\mathcal{F}(G)\) is an \(R\)-algebra.

(GFI 2) For every homomorphism \(\alpha : H \rightarrow G\) of finite groups, \(\mathcal{F}^*(\alpha)\) is a homomorphism of \(R\)-algebras.

(GFI 3) Frobenius reciprocity: for every monomorphism \(\alpha : H \hookrightarrow G\) and for all \(x \in \mathcal{F}(H), y \in \mathcal{F}(G)\), we have

\[
\text{Ind}_{G/H}(x) \cdot y = \text{Ind}_{G/H}(x \cdot \text{Res}_{G/H}(y)),
\]

\[
y \cdot \text{Ind}_{G/H}(x) = \text{Ind}_{G/H}(\text{Res}_{G/H}(y) \cdot x).
\]

**Definition 2.3.** A morphism from an MFI (respectively GFI) \(\mathcal{F}\) to an MFI (respectively GFI) \(\mathcal{G}\) is a collection \(r\) of \(R\)-module (respectively \(R\)-algebra) homomorphisms \(r_G : \mathcal{F}(G) \rightarrow \mathcal{G}(G)\) for each finite group \(G\), commuting in the obvious way with \(\mathcal{F}_s, \mathcal{F}^*, \mathcal{G}_s, \mathcal{G}^*\).

**Definition 2.4.** Let \(\mathcal{F}\) be a GFI over \(R\). A (left) module under \(\mathcal{F}\) is an MFI \(\mathcal{M}\) over \(R\), satisfying the following conditions.
(MOD 1) For every group $G$, $\mathcal{M}(G)$ is an $R$-linear (left) $\mathcal{F}(G)$-module, i.e. there is a map $\mathcal{F}(G) \times \mathcal{M}(G) \to \mathcal{M}(G)$ factoring through $\mathcal{F}(G) \otimes_R \mathcal{M}(G)$.

(MOD 2) For every homomorphism $\epsilon: H \to G$, and for any $x \in \mathcal{F}(G)$, $y \in \mathcal{M}(G)$, we have
$$\mathcal{M}^*(\epsilon)(x \cdot y) = \mathcal{F}^*(\epsilon)(x) \cdot \mathcal{M}^*(\epsilon)(y).$$

(MOD 3) For every monomorphism $\alpha: H \hookrightarrow G$ and for all $x \in \mathcal{F}(H)$, $y \in \mathcal{M}(G)$, we have
$$\mathcal{F}_*(\alpha)(x) \cdot y = \mathcal{F}_*(\alpha)(x \cdot \mathcal{M}^*(\alpha)(y)).$$

Example 2.5. The following are examples of GFIs over $\mathbb{Z}$.

(a) The Burnside ring functor $B$: for a finite group $G$, $B(G)$ is the free abelian group on isomorphism classes $[X]$ of finite groups, modulo the relations $[X \sqcup Y] - [X] - [Y]$ for all finite $G$-sets $X$, $Y$, and with multiplication defined by $[X] \cdot [Y] = [X \times Y]$. Here, $B_*$ is the usual induction of $G$-sets, and $B^*$ is inflation/restriction of $G$-sets.

(b) The representation ring functor $R_F$ over a given field $F$: for a finite group $G$, $R_F(G)$ is the free abelian group on isomorphism classes $[V]$ of finitely generated $F[G]$-modules, modulo the relations $[U \oplus V] - [V] - [U]$, and with multiplication defined by $[U] \cdot [V] = [U \otimes_F V]$, with diagonal $G$-action on the tensor product. As in the previous example, $(R_F)_*$ is induction of modules, and $(R_F)^*$ is inflation/restriction.

(c) The monomial ring functor $M$: for a finite group $G$, $M(G)$ is the free abelian group on symbols $[H, \lambda]$, as $H$ runs over representatives of conjugacy classes of subgroups of $G$, and $\lambda$ runs over isomorphism classes of complex 1-dimensional representations of $H$, and with multiplication defined by
$$[H, \lambda] \cdot [K, \chi] = \sum_{g \in H \cap K} [gH \cap K, \text{Res}_{gH \cap K} \lambda \cdot \text{Res}_{gH \cap K} \chi].$$

If $\alpha: U \to G$ is a monomorphism, $[H, \lambda] \in M(U)$, and $[K, \chi] \in M(G)$, then
$$M_*(\alpha)([H, \lambda]) = [\alpha(H), \lambda \circ \alpha^{-1}],$$
$$M^*(\alpha)([K, \chi]) = \sum_{g \in \alpha(U) \setminus G/K} [\alpha^{-1}(\alpha(U) \cap gK), \text{Res}_{gK/(\alpha(U) \cap gK)} \chi \circ \alpha].$$

Every GFI is a module under itself, called the (left) module. We also have the obvious notions of sub-MFIs, sub-GFIs, and submodules.

Definition 2.6. A left ideal of a GFI is a sub-MFI that is also a submodule of the left regular module.

Definition 2.7. Let $r: \mathcal{F} \to \mathcal{G}$ be a morphism of MFIs over $R$. Its kernel $\mathcal{K}$ is defined as follows: for every finite group $G$, we define $\mathcal{K}(G) = \ker(r(G)): \mathcal{F}(G) \to \mathcal{G}(G)$; for every homomorphism $\epsilon: H \to G$ of groups, we define $\mathcal{K}^*(\epsilon) = \mathcal{F}^*(\epsilon)|_{\mathcal{F}(G)}$; and for every monomorphism $\alpha: H \to G$ of groups, we define $\mathcal{K}_*(\alpha) = \mathcal{F}_*(\alpha)|_{\mathcal{F}(H)}$. The image of a morphism is defined
analogously. Let $\mathcal{F}$ be a sub-MFI (respectively an ideal) of the MFI (respectively GFI) $\mathcal{G}$. The quotient $\mathcal{Q} = \mathcal{G}/\mathcal{F}$ is defined as follows: for every finite group $G$, we define $\mathcal{Q}(G) = \mathcal{G}(G)/\mathcal{F}(G)$; for every homomorphism $\epsilon: H \rightarrow G$, we define $\mathcal{Q}^*(\epsilon) = \mathcal{G}^*(\epsilon) \mod \mathcal{F}(H)$; and for every monomorphism $\alpha: H \rightarrow G$, we define $\mathcal{Q}_*^*(\alpha) = \mathcal{G}_*(\alpha) \mod \mathcal{F}(G)$.

The proof of the following is routine and will be omitted.

**Lemma 2.8.** (a) Let $r: \mathcal{F} \rightarrow \mathcal{G}$ be a morphism of MFIs over $R$. Then its kernel is a sub-MFI of $\mathcal{F}$, and its image is a sub-MFI of $\mathcal{G}$.
(b) Let $r: \mathcal{F} \rightarrow \mathcal{G}$ be a morphism of GFIs over $R$. Then its kernel is an ideal of $\mathcal{F}$, and its image is a sub-GFI of $\mathcal{G}$.
(c) Let $\mathcal{F}$ be a sub-MFI of an MFI $\mathcal{G}$. Then the quotient $\mathcal{G}/\mathcal{F}$ is an MFI.
(d) Let $\mathcal{F}$ be an ideal of a GFI $\mathcal{G}$. Then $\mathcal{G}/\mathcal{F}$ is a GFI.

**Example 2.9.** The following are some motivating examples for this work.
(a) There is a GFI morphism $m'_C: M \rightarrow R_C$, sending, for every finite group $G$, a symbol $[H, \lambda] \in M(G)$ to $\text{Ind}_{G/H} \lambda \in R_C(G)$. The kernel of $m'_C$ was investigated by, among many others, Langlands [10], Deligne [7], Snaith [11], Boltje [3], and Boltje–Snaith–Symonds [5].
(b) Let $F$ be a field. There is a GFI morphism $m_F: B \rightarrow R_F$, which maps, for every finite group $G$, a $G$-set $X$ to the permutation module $F[X]$ over $F$. Its kernel $K_F$ is the MFI of Brauer relations over $F$. In [1], an explicit description of generators of this MFI is given in the case when $F$ is a field of characteristic 0. The primary motivation for this note is to give a similarly explicit description when $F$ is a field of positive characteristic.

### 3. Primordial groups

If $S$ is an $R$-algebra, and $\mathcal{F}$ an MFI (respectively GFI) over $R$, then $S \otimes_R \mathcal{F}$, defined in the obvious way, is an MFI (respectively GFI) over $S$. If $R = \mathbb{Z}$, then we will suppress any mention of $R$, and will just say “…$\mathcal{F}$ is a MFI (respectively GFI)”.

Throughout the rest of the paper, $Q$ will denote the field of fractions of $R$. For a prime ideal $\mathfrak{p}$ of $R$, we will write $\mathcal{F}_\mathfrak{p}$ for $R_\mathfrak{p} \otimes_R \mathcal{F}$, and $\mathcal{F}_Q$ for $Q \otimes_R \mathcal{F}$.

**Notation 3.1.** Let $\mathcal{F}$ be an MFI, and let $\mathcal{X}$ be a class of groups closed under isomorphisms. For every finite group $G$, we define the following $R$-submodules of $\mathcal{F}(G)$:

\[
\mathcal{I}_{\mathcal{F}, \mathcal{X}}(G) = \sum_{H \leq G, H \in \mathcal{X}} \text{Ind}_{G/H} \mathcal{F}(H),
\]
\[
\mathcal{I}_\mathcal{F}(G) = \sum_{H \leq G} \text{Ind}_{G/H} \mathcal{F}(H),
\]
\[
\mathcal{K}_{\mathcal{F}, \mathcal{X}}(G) = \bigcap_{H \leq G, H \in \mathcal{X}} \ker(\text{Res}_{G/H} \mathcal{F}(G)),
\]
\[
\mathcal{K}_\mathcal{F}(G) = \bigcap_{H \leq G} \ker(\text{Res}_{G/H} \mathcal{F}(G)).
\]
Definition 3.2. Let $\mathcal{F}$ be an MFI and let $G$ be a finite group. We say that $G$ is \textit{primordial} for $\mathcal{F}$ if either $G$ is trivial, or $\mathcal{F}(G) \neq I_\mathcal{F}(G)$. We denote the class of all primordial groups for $\mathcal{F}$ by $\mathcal{P}(\mathcal{F})$.

Remark 3.3. Let $\mathcal{F}$ be an MFI.

(a) Suppose that $\mathcal{X}$ is a class of finite groups that is closed under isomorphisms and under taking subgroups, with the property that for every finite group $G$, we have $\mathcal{F}(G) = I_{\mathcal{F},\mathcal{X}}(G)$. Then it is shown in [13, Theorem 2.1] that $\mathcal{X}$ contains the closure of $\mathcal{P}(\mathcal{F})$ under taking all subgroups.

(b) Suppose that $\mathcal{F}$ is a GFI. Then it follows from axiom (GFI 3) that $G$ is primordial for $\mathcal{F}$ if and only if $1_{\mathcal{F}(G)} \notin I_{\mathcal{F}}(G)$. It easily follows from this and from axioms (GFI 2) and (MFI 6) that $\mathcal{P}(\mathcal{F})$ is closed under quotients.

Example 3.4. (a) Every finite group is primordial for the Burnside ring functor $B$, and also for $B_Q$. Indeed, no non-zero multiple of the identity element of $B(G)$ can be contained in the image of induction from proper subgroups. Similarly, every finite group is primordial for the monomial ring functor $M$.

(b) Recall from Example 2.9 (b) the representation ring functor $R_C$. Brauer’s induction theorem [2, Theorem 5.6.4] implies that $\mathcal{P}(R_C)$ is contained in the class of elementary groups, i.e. of direct products of finite cyclic groups by $p$-groups. Moreover, it is a theorem of Green [9] that in fact $\mathcal{P}(R_C)$ consists precisely of the elementary groups.

(c) Recall from Example 2.9 (a) the GFI morphism $m'_C : M \rightarrow R_C$ from the monomial ring functor to the complex representation ring functor. It follows from Brauer’s induction theorem that $(m'_C)_G$ is surjective for every finite group $G$, so by the previous example, $\mathcal{P}(\operatorname{Im} m'_C)$ consists precisely of the elementary groups.

(d) Recall from Example 2.9 (b) the GFI morphism $m_Q : B \rightarrow R_Q$. Let $q$ be a prime number. Solomon’s induction theorem implies that $\mathcal{P}(\operatorname{Im}(m_Q)_q)$ is contained in the class of $q$-quasi-elementary groups, i.e. of semidirect products $C \rtimes U$, with $C$ finite cyclic and $U$ a $q$-group. Moreover, it is a theorem of Dokchitser [8] that if $G$ is $q$-quasi-elementary, then the trivial character of $G$ is not in the image of induction of trivial characters from proper subgroups, so $\mathcal{P}(\operatorname{Im}(m_Q)_q)$ is precisely the class of all $q$-quasi-elementary groups.

(e) Let $m_Q$ be as above. It follows from Artin’s induction theorem [2, Theorem 5.6.1] that $\mathcal{P}(\operatorname{Im}(m_Q)_Q)$ is the class of finite cyclic groups.

(f) Let $p$ be a prime number, and let $m_{R_p} : B \rightarrow R_p$ be as in Example 2.9 (b). Given a prime number $q$, define a finite group $U$ to be a $(p,q)$-\textit{Dress group} if $U/O_p(U)$ is $q$-quasi-elementary. Dress’s induction theorem [11, Theorem 9.4] implies that $\mathcal{P}(\operatorname{Im} m_{R_p})$ is contained in the class of $(p,q)$-Dress groups. We will show in Theorem 5.3 that the trivial representation of a $(p,q)$-Dress group is not in the image of induction of trivial representations from proper subgroups, so in fact, $\mathcal{P}(\operatorname{Im} m_{R_p})$ is precisely the class of all $(p,q)$-Dress groups.
Lemma 4.1. Let $m : \mathcal{F} \to \mathcal{G}$ be a morphism of GFIs over a ring $R$ with kernel $\mathcal{K}$, and let $G \in \mathcal{P}(\mathcal{F})$. Then the following are equivalent:

(i) the group $G$ is not primordial for $\text{Im} \, m$;

(ii) for each proper subgroup $H$ of $G$, there exists an element $x_H \in \mathcal{F}(H)$ such that $x = 1_{\mathcal{F}(G)} + \sum_{H \leq G} \text{Ind}_{G/H}(x_H)$ is a non-zero element of $\mathcal{K}(G)$.

Proof. By Remark 3.3(b) $G$ is not primordial for $\text{Im} \, m$ if and only if

$$m_G(1_{\mathcal{F}(G)}) \in \sum_{H \leq G} \text{Ind}_{G/H}(m_H(\mathcal{F}(H))) = m_G \sum_{H \leq G} \text{Ind}_{G/H}(\mathcal{F}(H)).$$

This is equivalent to the existence of elements $x_H \in \mathcal{F}(H)$ for $H \leq G$ such that $x = 1_{\mathcal{F}(G)} + \sum_{H \leq G} \text{Ind}_{G/H}(x_H) \in \mathcal{K}(G)$. Since $G$ is primordial for $\mathcal{F}$, Remark 3.3(b) implies that any such element $x$ must be non-zero. □

Definition 4.2. Let $G$ be a finite group, let $\mathcal{F}$ be a GFI over $R$, and let $\mathcal{M}$ a module under $\mathcal{F}$. Let $D(G)$ be an $R$-subalgebra of the centre of $\mathcal{F}(G)$.

Define the set of $D$-imprimitive elements of $\mathcal{M}(G)$ by

$$\text{Imprim}_{\mathcal{M}, D}(G) = D(G) \cdot \left( \sum_{H \leq G} \text{Ind}_{G/H}(\mathcal{M}(H)) + \sum_{1 \neq N \leq G} \text{Inf}_{G/N}(\mathcal{M}(G/N)) \right).$$

This is an $R$-submodule of $\mathcal{M}(G)$. Define the $D$-primitive quotient of $\mathcal{M}(G)$ to be the quotient of $R$-modules

$$\text{Prim}_{\mathcal{M}, D}(G) = \mathcal{M}(G) / \text{Imprim}_{\mathcal{M}, D}(G).$$

When $D(G)$ is generated by $1_{\mathcal{F}(G)}$ over $R$, we will drop it from the notation.

Notation 4.3. For the rest of the section, we put ourselves in the following situation. We fix a morphism $m : \mathcal{F} \to \mathcal{G}$ of GFIs over a domain $R$ with the property that $\mathcal{F}(H)$ is $R$-torsion free for all finite groups $H$, and we let $\mathcal{K}$ denote its kernel. Recall from Lemma 2.8 that $\mathcal{K}$ is an ideal of $\mathcal{F}$. Further, we fix a finite group $G$, and an $R$-subalgebra $D(G)$ of the centre of $\mathcal{F}(G)$.

We make the following assumptions:

- the $R$-module $\mathcal{F}(G)$ is generated by $\mathcal{I}_\mathcal{F}(G)$ and $D(G)$, and
- all quotients of $G$ are primordial for $\mathcal{F}_Q$.

Lemma 4.4. Let $\mathcal{M}$ be any module under $\mathcal{F}$, and let $x$ be any element of $\mathcal{M}(G)$. Then the $R$-submodule of $\mathcal{M}(G)$ generated by $D(G) \cdot \mathcal{I}_\mathcal{M}(G)$ and $D(G) \cdot x$ is an $\mathcal{F}(G)$-submodule.

Proof. Let $\Theta$ be an element of the $R$-module $D(G) \cdot \mathcal{I}_\mathcal{M}(G) + D(G) \cdot x$, and let $\alpha \in \mathcal{F}(G)$. If $\alpha = \text{Ind}_{G/H} y$ for some $y \in \mathcal{F}(H)$, where $H$ is a proper subgroup of $G$, then by property (MOD 3), $\alpha \cdot \Theta = \text{Ind}_{G/H}(y \cdot \text{Res}_{G/H} \Theta) \in \mathcal{I}_\mathcal{M}(G)$. If, on the other hand, $\alpha \in D(G)$, then $\alpha \cdot \Theta \in D(G) \cdot \mathcal{I}_\mathcal{M}(G) + D(G) \cdot x$ by definition. Since $\mathcal{F}(G)$ is assumed to be generated by $\mathcal{I}_\mathcal{F}(G)$ and by $D(G)$, it follows that $\alpha \cdot \Theta \in D(G) \cdot \mathcal{I}_\mathcal{M}(G) + D(G) \cdot x$ for all $\alpha \in \mathcal{F}(G)$. □
Thus $K$ is in $x$. Theorem 4.6. Suppose that there is a non-trivial normal subgroup such that $G/N$. Let $y \in \mathbb{K}(G)$. Lemma 4.5 implies that $I$ is an ideal of $F$. Since we have $x \in D(G) \cdot x \subseteq I$, it follows that $y \cdot x \in I$. Also,

$$y \cdot x - y = \sum_{H \leq G} y \cdot \text{Ind}_{G/H}(x_H) = \sum_{H \leq G} \text{Ind}_{G/H}(\text{Res}_{G/H}(y) \cdot x_H)$$

is in $I_K(G)$, and therefore in $I$. It follows that $y = y \cdot x + (y - y \cdot x) \in I$. Thus $\mathbb{K}(G) \subseteq I$, and the proof is complete. □

**Theorem 4.6.** Suppose that there is a non-trivial normal subgroup $N$ of $G$ such that $G/N$ is not primordial for $\text{Im} \ m$. Then $\text{Prim}_{K,D}(G)$ is trivial.

**Proof.** By Lemma 4.1 applied to the quotient $G/N$, there exists a non-zero $z = 1_{F(G/N)} + \sum_{H/N \leq G/N} \text{Ind}_{G/(H/N)}(x_H) \in \mathbb{K}(G/N)$. Since $N$ is non-trivial, the inflation $x = \text{Inf}_{G/N} z$ is contained in $\text{Im} \text{Prim}_{K,D}(G)$. It follows from Lemma 4.4 that $\mathbb{K}(G) = (D(G) \cdot I_K(G) + D(G) \cdot x)_R \subseteq \text{Im} \text{Prim}_{K,D}(G)$, as claimed.

**Theorem 4.7.** Suppose that $G$ is non-trivial, and that $\text{Prim}_{K,D}(G)$ is non-trivial. Then $G$ is an extension of the form $1 \rightarrow S^d \rightarrow G \rightarrow H \rightarrow 1$, where $S$ is a finite simple group, and $H$ is primordial for $\text{Im} \ m$.

**Proof.** By the existence of a chief series, there exists a normal subgroup of $G$ that is isomorphic to $S^d$, where $S$ is a finite simple group, and $d \geq 1$ is an integer. By Theorem 4.6 the quotient $G/S^d$ is primordial for $\text{Im} \ m$. □

**Assumption 4.8.** In addition to the assumptions of Notation 1.3 we now assume that:

- the ring $R$ is a Euclidean domain;
- for every normal subgroup $N$ of $G$, the inflation map $\text{Inf}_{G/N} : F(G/N) \rightarrow F(G)$ is injective;
- the subalgebra $D(G)$ is generated by $1_{F(G)}$ over $R$, so that the $R$-module $F(G)$ is generated by $I_F(G)$ and 1. From now on, we will drop $D(G)$ from the notation.

**Theorem 4.9.** Suppose that $G$ is not primordial for $\text{Im} \ m$, and that all proper quotients of $G$ are primordial for $\text{Im} \ m$. Then $\text{Prim}_K(G)$ is generated by the image of any element of the form $x = 1_{F(G)} + \sum_{H \leq G} \text{Ind}_{G/H} x_H \in \mathbb{K}(G)$, and has the following structure:

(a) if for all non-trivial normal subgroups $N$ of $G$, the quotient $G/N$ is primordial for $(\text{Im} \ m)_Q$, then $\text{Prim}_K(G)$ is isomorphic to $R$

(b) if there exists a non-zero prime ideal $p$ of $R$ such that for all prime ideals $q \neq p$ of $R$, there exists a proper quotient $G/N$ that is not primordial for $(\text{Im} \ m)_Q$, then $\text{Prim}_K(G) \cong R/p^n$, where $n$ is the smallest positive integer with the property that $a1_G \in I_{\text{Im} \ m}(G)$ for some $a \in p^n$.
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(c) if for every non-zero prime ideal \( p \) of \( R \) there exists a non-trivial normal subgroup \( N \) of \( G \) such that the quotient \( G/N \) is not primordial for \( (\text{Im} m)_p \), then \( \text{Prim}_K(G) \) is trivial.

Proof. By Lemma 4.5 the quotient \( \text{Prim}_K(G) \) is generated by any \( x \in K(G) \) of the form \( x = 1_{F(G)} + \sum_{H \leq G} \text{Ind}_{G/H} x_H \), where \( x_H \in F(H) \). Since by assumption \( G \) is primordial for \( F_Q \), Remark 3.3(b) implies that \( ax \notin I_K(G) \) for any non-zero \( a \in R \). It also follows from the same remark and from the assumptions 4.3 and 4.5 that any element of \( K \) can be uniquely written as \( a1_{F(G)} + y \), where \( a \in R \) and \( y \in I_F(G) \). We deduce that the annihilator \( A \subseteq R \) of \( x + \text{Imprim}_F(G) \in \text{Prim}_F(G) \) is generated, as an \( R \)-module, by all those \( a \in R \) for which there exists a non-trivial normal subgroup \( N \) of \( G \) and an element \( x' = a1_{F(G/N)} + y' \in K(G/N) \), where \( \text{Inf}_{G/N} y' \in I_F(G) \). Moreover, we then have \( \text{Prim}_K(G) \cong R/A \).

If all proper quotients \( G/N \) are primordial for \( (\text{Im} m)_Q \), then by Remark 3.3(b) we have \( A = 0 \), which proves part (a).

To prove part (b), suppose that there exists a non-zero prime ideal \( p \) of \( R \) such that for each prime ideal \( q \neq p \) of \( R \), there exists a proper quotient \( G/N \) that is not primordial for \( (\text{Im} m)_q \). Then Lemma 4.1 applied to the map \( F_q \to q_A \) and to a proper quotient \( G/N \notin P((\text{Im} m)_q) \), implies that for every prime ideal \( q \neq p \), there exists \( a \in A \) that is not in \( q \). Since \( R \) is a Euclidean domain, this implies that \( A = p^n \) for some integer \( n \geq 0 \). Since no proper quotient is primordial for \( \text{Im} m \), we have \( 1 \notin A \), so \( n \) is non-zero, which proves part (b).

Finally, to prove part (c), suppose that for every non-zero prime ideal \( p \) of \( R \) there exists a non-trivial normal subgroup \( N \) of \( G \) such that the quotient \( G/N \) is not primordial for \( (\text{Im} m)_p \). Then by the same argument as above, for every non-zero prime ideal \( p \) there exists \( a \in A \) that is not in \( p \). Since \( R \) is a Euclidean domain, it follows that \( 1 \in A \), which proves part (c). \( \square \)

5. Applications

In this section we explicate the results of Section 4 in the case of monomial relations and of Brauer relations. The main new results are on Brauer relations in positive characteristic, but we also show how to derive some known results on monomial relations and on Brauer relations in characteristic 0 from the formalism of Green functors. In particular, we prove Theorems 1.1, 1.2, and 1.3 from the introduction.

Theorem 5.1 (Deligne–Langlands, [7]). Let \( K'_C \) be the kernel of the morphism of GFIs \( m'_C : M \to R_C \) as in Example 2.9(a). Let \( G \) be a finite group, and let \( N \) be a non-trivial normal subgroup such that \( G/N \) is not elementary. Let \( D(G) \) be generated over \( \mathbb{Z} \) by symbols \( [G, \lambda] \), as \( \lambda \) runs over isomorphism classes of 1-dimensional representations of \( G \). Then \( \text{Prim}_{K'_C, D}(G) \) is trivial.

Proof. By Brauer’s induction theorem 2 Theorem 5.6.4 and by 9, the primorial groups for \( \text{Im} m'_C \) are precisely the elementary groups, and every group is primordial for \( M_Q \) (see Example 3.4(a)). It easily follows that the assumptions of Notation 4.3 are satisfied for this morphism of GFIs and this choice of \( D(G) \). The result therefore follows from Theorem 4.6 \( \square \)
Theorem 5.2 (Bartel–Dokchitser, [1]). Let $K_Q$ be the kernel of the morphism of GFIs $m_Q$: $B \to R_Q$ as in Example 2.9(b), and let $G$ be a finite group that is not quasi-elementary. Then:

(a) if all proper quotients of $G$ are cyclic, or if there exists a proper quotient that is not quasi-elementary, then $\text{Prim}_{K_Q}(G)$ is trivial;

(b) if $q$ is a prime such that all proper quotients of $G$ are $q$-quasi-elementary, and at least one of them is not cyclic, then $\text{Prim}_{K_Q}(G) \cong \mathbb{Z}/q\mathbb{Z}$, and is generated by any element of $K_Q(G) \subseteq B(G)$ of the form $[G/G] + \sum_{H \leq G} a_H [G/H]$, $a_H \in \mathbb{Z}$;

(c) if $q$ and $q'$ are distinct primes such that there exists a proper quotient of $G$ that is non-cyclic $q$-quasi-elementary, and a proper quotient that is non-cyclic $q'$-quasi-elementary, then $\text{Prim}_{K_Q}(G)$ is trivial.

Proof. Every finite group is primordial for $B$. The morphism of GFIs $m_Q$ therefore satisfies the assumptions of [1, 3] and [1, 3]. By Artin’s Induction Theorem [2, Theorem 5.6.1], $P((\text{Im } m_Q)_Q)$ is the class of cyclic groups. Let $q$ be a prime number. By Solomon’s Induction Theorem and by [8], $P(\text{Im } m_Q)$ is the class of quasi-elementary groups, and $P((\text{Im } m_Q)_Q)$ is the class of $q$-quasi-elementary groups. Moreover, if $U$ is a non-cyclic $q$-quasi-elementary group, then by [8], there exists an element of $K_Q(U) \subseteq B(U)$ of the form $q[U/U] + \sum_{H \leq U} a_H [U/H]$. The result therefore follows from Theorems 4.6 and 4.9.

Fix a prime number $p$. The rest of the section is devoted to the kernel $K_{\mathbb{F}_p}$ of the morphism of GFIs $m_{\mathbb{F}_p}$: $B \to R_{\mathbb{F}_p}$ as in Example 2.9(b).

First, we prove Theorem 1.3, which is a characteristic $p$ analogue of the main result of [8]. We recall the statement.

Theorem 5.3. Let $q$ be a prime number, let $G$ be a $(p,q)$-Dress group, and let $a$ be an integer. Then $a[G/G] \in \mathcal{I}_{\text{Im } m_{\mathbb{F}_p}}(G)$ if and only if $q | a$.

Proof. Since $G$ is a $(p,q)$-Dress group, it is an extension of a $q$-group $U$ by a normal $p$-hypo-elementary subgroup $N = P \rtimes C$, where $P$ is a $p$-group and $C$ is cyclic of order coprime to $pq$.

First we prove that if $a[G/G] \in \mathcal{I}_{\text{Im } m_{\mathbb{F}_p}}(G)$, then $q | a$. Suppose that there exist integers $a_H$ for $H \leq G$ such that

$$a_{\mathbb{F}_p}[G/G] = \sum_{H \leq G} a_H \mathbb{F}_p[G/H] \in \mathbb{F}_p(G),$$

where the sum runs over representatives of conjugacy classes of subgroups of $G$, and where $\mathbb{F}_p[G/H] \in R_{\mathbb{F}_p}(G)$ denotes the linear permutation module $\text{Ind}_{G/H} 1_H$ over $\mathbb{F}_p$. By restricting to the normal $p$-hypo-elementary subgroup $N$, we find that

$$a_{\mathbb{F}_p}[N/N] = \sum_{H \leq G} a_H \sum_{g \in G/HN} \mathbb{F}_p[N/N \cap gHg^{-1}].$$

By Conlon’s Induction Theorem [6, Lemma 81.2], $p$-hypo-elementary groups are primordial for $\text{Im } m_{\mathbb{F}_p}$, so the coefficient of $\mathbb{F}_p[N/N]$ on the right hand side of equation 5.4 must be equal to $a$:

$$a = \sum_{N \leq H \leq G} a_H \cdot \#(G/H).$$
But for every $H \leq G$ that contains $N$, the quantity $\#(G/H)$ is divisible by $q$, so $a$ is divisible by $q$, as claimed.

Now we show that $q[G/G] \in \mathcal{I}_{\text{Im }m_{\mathcal{F}_p}}(G)$. First, we treat a special case: assume that $P$ is the trivial group, so that $G \cong C \times U$ is non-cyclic $q$-quasi-elementary, where $C$ is cyclic of order coprime to $pq$. Assume further that either $p \neq q$, or $U$ acts faithfully on $C$. By [3], there exists an element $x = q[G/G] + \sum_{H \leq G} a_H [G/H] \in K_{\mathcal{F}_p}(G)$. By Artin’s Induction Theorem [2, Theorem 5.6.1], this is equivalent to the statement that there exists an $x \in K_{\mathcal{F}_p}(G)$ as above such that for all cyclic subgroups $H \leq G$, we have $f_H(x) = 0$, where $f_H : B(G) \to \mathbb{Z}$ is defined on a $G$-set $X$ as the number of fixed points $\#X^H$. But under the hypotheses on $G$, the cyclic subgroups of $G$ are precisely the $p$-hypo-elementary subgroups of $G$. By Conlon’s Induction Theorem [1, Lemma 81.2], the above statements are therefore equivalent to the existence of an element $x = q[G/G] + \sum_{H \leq G} a_H [G/H] \in K_{\mathcal{F}_p}(G)$, as required.

Now, we deduce the general case. Given a non-$p$-hypo-elementary $(p, q)$-Dress group $G$, let $\tilde{G} = G/P$. This is a non-cyclic $q$-quasi-elementary group, $\tilde{G} = C \times U$, where $U$ is a $q$-group, and $C$ is cyclic of order coprime to $pq$. Let $K$ be the kernel of the action of $U$ on $C$. If $K = U$ and $p = q$, then $\tilde{G} \cong C \times U$, and $G$ is $p$-hypo-elementary, contradicting the assumptions. Otherwise, $\tilde{G} = G/K$ is as in the special case above, so there exists an element $x = q[\tilde{G}/G] + \sum_{H \leq G} a_H [G/H] \in K_{\mathcal{F}_p}(G)$. Taking the inflation of $x$ to $G$ yields the desired element of $K_{\mathcal{F}_p}(G)$, and the proof is complete. □

**Corollary 5.5.** Let $q$ be a prime number. Then $\mathcal{P}(\text{Im }m_{\mathcal{F}_p})$ is the class of $(p, q)$-Dress groups.

**Proof.** By Dress’s Induction Theorem in the version as stated in [1, Theorem 9.4], and by Remark [3, a] all primordial groups for $\text{Im }m_{\mathcal{F}_p}$ are $(p, q)$-Dress groups. The reverse inclusion follows from Theorem 5.3. □

**Theorem 5.6.** Let $G$ be a finite group that is not a $(p, q)$-Dress group for any prime number $q$. Then:

(a) if all proper quotients of $G$ are $p$-hypo-elementary, or if there exists a proper quotient that is not a $(p, q)$-Dress group for any prime number $q$, then $\text{Prim}_{\mathcal{F}_p}(G)$ is trivial;

(b) if $q$ is a prime number such that all proper quotients of $G$ are $(p, q)$-Dress groups, and at least one of them is not $p$-hypo-elementary, then $\text{Prim}_{\mathcal{F}_p}(G) \cong \mathbb{Z}/q\mathbb{Z}$, and is generated by any element of $K_{\mathcal{F}_p}(G) \subseteq B(G)$ of the form $[G/G] + \sum_{H \leq G} a_H [G/H]$, $a_H \in \mathbb{Z}$;

(c) if $q$ and $q'$ are distinct prime numbers such that there exists a proper quotient of $G$ that is not $p$-hypo-elementary but a $(p, q')$-Dress group, and a proper quotient that is not $p$-hypo-elementary but a $(p, q')$-Dress group, then $\text{Prim}_{\mathcal{F}_p}(G)$ is trivial.

**Proof.** The morphism of GFIs $m_{\mathcal{F}_p}$ clearly satisfies the assumptions of 4.8 and 4.8. By Conlon’s Induction Theorem [1, Lemma 81.2], $\mathcal{P}(\text{Im }m_{\mathcal{F}_p})$ is the class of $p$-hypo-elementary groups. Let $q$ be a prime number. By Corollary 5.5, $\mathcal{P}(\text{Im }m_{\mathcal{F}_p})$ is the class of $(p, q)$-Dress groups, and $\mathcal{P}(\text{Im }m_{\mathcal{F}_p})$...
is the class of all groups that are \((p, q')\)-Dress groups for some prime number \(q'\). Moreover, if \(U\) is a non-\(p\)-hypo-elementary \((p, q)\)-Dress group, then by Theorem 5.3, there exists an element of \(K_{F_p}(U) \subseteq B(U)\) of the form \(q[U/U] + \sum_{H \leq U} a_H [U/H]\). The result therefore follows from Theorems 4.6 and 4.9.

\[\square\]

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