Dark Matter as Dense Color Superconductor

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We discuss a novel cold dark matter candidate which is formed from the ordinary quarks during the QCD phase transition when the axion domain wall undergoes an unchecked collapse due to the tension in the wall. If a large number of quarks is trapped inside the bulk of a closed axion domain wall, the collapse stops due to the internal Fermi pressure. In this case the system in the bulk, may reach the critical density when it undergoes a phase transition to a color superconducting phase with the ground state being the quark condensate, similar to BCS theory. If this happens, the new state of matter representing the diquark condensate with a large baryon number \( B \geq 10^{20} \) becomes a stable soliton-like configuration. Consequently, it may serve as a novel cold dark matter candidate.

1. Introduction

The presence of large amounts of non-luminous components in the Universe has been known for a long time. In spite of the recent advances in the field, the mystery of the dark matter/energy remains: we still do not know what it is. The main goal of this talk is to argue that the dark matter could be nothing but well-known quarks which however are not in the “normal” hadronic phase, but rather in some “exotic”, the so-called color superconducting (CS) phase.

This is a novel phase in QCD when light quarks form the condensate in diquark channels, and it is analogous to Cooper pairs of electrons in ordinary superconductors described by BCS theory. There existence of CS phase in QCD represents our first crucial element for our scenario to work. The study of CS phase received a lot of attention last few years. It has been known that this regime may be realized in nature in neutron stars interiors and in the violent events associated with collapse of massive stars or collisions of neutron stars, so it is important for astrophysics. The goal of this talk is to argue that such conditions may occur in early universe during the QCD phase transition, so it might be important for cosmology as well.

The force which squeezes quarks in neutron stars is gravity; the force which does a similar job in early universe during the QCD phase transition is a violent collapse of a bubble formed from the axion domain wall. If number of quarks trapped inside the bulk is sufficiently large, the collapse stops due to the internal Fermi pressure. In this case the system in the bulk may reach the critical density when it undergoes a phase transition to CS phase with the ground state being the diquark condensate. We shall call the configuration with a large number of quarks in color superconducting phase formed during the QCD phase transition as the QCD-ball. Therefore, an existence of the axion domain wall represents our second crucial element for our scenario to work. As is known, the axion is one of the favorite candidates for the cold dark matter, see recent reviews. In the present scenario the axion field plays the role of squeezer rather than dark matter itself.

As will be shown below, once such a configuration is formed, it will be extremely stable soliton like particle. The source of the stability of the QCD-balls is related to the fact that its mass \( M_B \) growth as \( M_B \sim B^{8/9} \) for the large baryon (quark) charge \( B \) and becomes smaller than the mass of a collection of free separated nucleons with the same baryon charge. The region of the absolute stability of the QCD-balls is determined by inequality \( m_N > \frac{\partial M_B}{\partial B} \) which can be always satisfied for very large \( B > B_c \). However, even for sufficiently large \( B \) but smaller than \( B_c \), i.e. \( 1 \ll B < B_c \), we still expect that QCD-balls...
will be very long-lived metastable states. This is because the elementary excitations carrying the baryon charge in CS phase and hadronic phase are very different: in CS phase they carry color charge along with baryon charge; in hadronic phase they are color singlets. Therefore, if sufficiently large number of quarks is trapped inside the axion bubble during its shrinking, it may result in formation of an absolutely stable (or long lived, metastable) QCD-ball with the ground state being a diquark condensate. Such QCD-balls, therefore, may serve as the cold dark matter candidate and contribute to $\Omega_{DM} \simeq 0.3$.

Strictly speaking, the QCD-balls being the baryonic configurations, would behave like non-baryonic dark matter. In particular, QCD-balls, in spite of their QCD origin, would not contribute to $\Omega_{b}\hbar^{2} \simeq 0.02$ in nucleosynthesis calculations because the QCD-balls would complete the formation by the time when temperature reaches the relevant for nucleosynthesis region $T \sim 1 \text{MeV}$. Once QCD-balls are formed, their baryon charge is accumulated in form of the diquark condensate, rather than in form of free baryons, and in such a form the baryon charge is not available for nucleosynthesis.

2. QCD-balls

Crucial for our scenario is the existence of a squeezer, axion domain wall which will be formed during the QCD phase transition. We assume that the standard problem of the domain wall dominance is resolved in some way as discussed previously in the literature [3]. We also assume that quarks which are trapped inside of the axion bubble, can not easily escape the bulk when the bubble is shrinking. In different words, the axion domain wall is not transparent due to the QCD sandwich structure of the wall as discussed in [4, 5]. The collapse is halted due to the Fermi pressure. Therefore, we assume that a large number of quarks remains in the bulk of volume $V$ when the Fermi pressure cancels the surface tension. Exactly this condition determines a typical radius $R_{0}$ of the QCD-ball at the moment of formation. The pressure due to the surface tension is $P_{\sigma} = \frac{\sigma}{R}$, while the fermi pressure $P_{f}$ in the bulk can be easily estimated from the thermodynamical potential for the non-interacting relativistic fermi gas, $\Omega = -P_{f}V$ with $\Omega$ given by:

$$\Omega = gV \int_{0}^{\mu} \frac{(p - \mu)d^{3}p}{(2\pi)^{3}} = -\frac{gV \mu^{4}}{24\pi^{2}} = -P_{f}V,$$  \hspace{1cm} (1)

Here $\mu$ is the Fermi momentum of the system to be expressed in terms of the fixed baryon charge $B$ trapped in the bubble;

$$B = gV \int_{0}^{\mu} \frac{d^{3}p}{(2\pi)^{3}} = \frac{2g}{9\pi} \mu^{3} R^{3}, \quad \mu = \left( \frac{9\pi B}{2gR^{3}} \right)^{\frac{1}{3}},$$

$g \simeq 2N_{c}N_{f} = 18$ is the degeneracy factor for massless degrees of freedom. The condition $P_{\sigma} \simeq P_{f}$ determines a typical radius of the QCD-ball with a fixed initial baryon charge $B$ at the moment of formation:

$$P_{f} \simeq P_{\sigma} \Rightarrow \frac{g}{24\pi^{2}} \left( \frac{9\pi B}{2gR^{3}} \right)^{\frac{1}{3}} \simeq \frac{2\sigma}{R} \Rightarrow$$

$$R_{0} \simeq \frac{cB^{4/3}}{8\pi\sigma}, \quad c = \frac{3}{4} \left( \frac{9\pi}{2g} \right)^{1/3} \sim 0.7.$$  \hspace{1cm} (2)

Here $\sigma \simeq f_{a}m_{\pi}f_{\pi}$ is the axion domain wall tension with $f_{a} \sim (10^{10} - 10^{12})\text{GeV}$ being constrained by the axion search experiments. As the first approximation, we neglected many contributions in $\Omega$ which might be important numerically but can not drastically change our main results. In particular, we neglected the quark-quark interaction on the Fermi surface, which brings the system into superconducting phase for relatively large $\mu \geq 400\text{MeV}$ [6]. The corresponding contribution gives some correction to the energy of order $(\frac{A_{QCD}}{\mu})^{4}$ with $\Delta \sim 100\text{MeV}$ is the superconducting gap, and can be neglected for the asymptotically large $\mu$. We also neglected the so-called bag constant contribution which is a phenomenological way to simulate the confinement. Parametrically this contribution is also suppressed as $(\frac{A_{QCD}}{\mu})^{4}$ but in reality might be important. Let us emphasize: we are not attempting to solve the problem of equilibrium between CS and hadronic phases [7]. Rather, we are attempting to estimate

\footnote{Such a problem has been recently discussed in ref. [8] where the interface region between nuclear matter and CFL (color flavor locking) superconducting phase was analyzed.}
the initial size $R_0$ of the QCD ball which is formed due to the collapse of the axion domain wall halted by the fermi pressure. Other contributions, such as quark-quark interaction or the bag constant, may change the numerical estimates for $R_0$ however they can not replace the main player of the game, the fermi pressure which is responsible for the stopping of the collapse.

Once the QCD ball with radius $R_0$ is formed, the bulk of this object being in CS phase will be quite stable with respect to the decay on the free nucleons if some conditions are met, see below. The stability remains intact even if the original axion domain wall would decay: the configuration remains stable due to the CS phase itself, and not because of the support from the axion domain wall. The only role the axion domain wall plays in our scenario is the squeezing the matter at the very first instant in order to bring the large number of quarks into the CS phase. The later stages of the evolution of the QCD ball will determine the precise structure of this configuration at the equilibrium, including the interface region separating the CS phase with nuclear matter phase; we do not expect, though, that this evolution would considerably change our estimates for $R_0$ presented above.

Once a typical radius $R_0$ for a QCD ball is estimated \( \square \), the total energy $E$ (fermion energy + axion domain wall energy) and baryon number density $n$ of the QCD-ball can be also estimated with the result

$$E \approx 4\pi \sigma R_0^2 + gV \frac{\mu^4}{8\pi^2} \approx \frac{3}{2} (8\pi c^2)^{1/3} B^{8/9} \sigma^{1/3}$$

$$n = \frac{B}{V} \approx \frac{6\sigma}{c} B^{-1/3} \quad (3)$$

The most important consequence of this result is the behavior $E \sim B^{8/9}$ which implies that for sufficiently large $B$ the QCD-ball becomes an absolutely stable object. It happens when $\frac{m_N}{3} > \frac{\partial E}{\partial B}$, i.e.$\square$

$$B > B_c \approx (2^9 \pi c^2 \frac{\sigma}{m_N})^{3/2} \approx 10^{33}, \quad n_c \approx 9n_0 \quad (4)$$

where $n_0$ is the nuclear saturation density, $n_0 \approx 0.16 (fm)^{-3}$. Numerically, $B_c \approx 10^{33}$ corresponds to stabilization radius $R_c \approx 10^{12} GeV^{-1}$ and mass $E_c \approx 10^{13} GeV$. Here we use $f_\pi \approx 10^{10} GeV$ which corresponds to $\sigma \approx 1.8 \cdot 10^8 GeV^{-3}$. As we mentioned in Introduction, we expect that the QCD-balls with much smaller $B$ can live long enough to serve as a dark matter. Therefore, in what follows we use a more realistic value for $B_{\text{exp}} \approx 10^{20}$ which follows from the experimental bounds on masses and fluxes of nontopological solitons, see below.

Few remarks are in order. First, we note that there is an upper limit on value $\tilde{B}$ above which the QCD-balls can not be formed at the very first instant during the collapse of the domain walls. This follows from the expression \( \mathbb{B} \) for the baryon number density which decreases when $B$ increase. Therefore, at some point, the density $n$ reaches the magnitude where the phase transition between nuclear matter and CS phase occurs. Numerically it happens at $\tilde{B} \sim 10^2 B_c$. However, it does not preclude from formation of a larger object at the later stages of the QCD-ball evolution.

As our next remark, we would like to mention some similarity between the QCD-balls (which is the subject of this letter) and Q-balls \( \mathbb{Q} \) which are nontopological solitons associated with some conserved global $Q$ charge. In both cases, a soliton mass as function of $Q$ has behavior, similar to our eq. \( \mathbb{Q} \), and therefore, it may become a stable configuration for relatively large $Q$ charge. Therefore, an effective scalar field theory with some specific constraint on potential (when $Q$ ball solution exists) is realized for QCD in high density regime. The big difference, of course, is that underlying theory for QCD-balls is well known: it is QCD with no free parameters, in huge contrast with the theory of Q-balls. Formal similarity becomes even more striking if one takes into account that the ground state of the CFL phase in QCD is determined by the diquark condensate with the following time dependence $\langle \Psi \Psi \rangle \sim \exp(i2\mu t)$, with $\Psi$ being the original QCD quark fields, and $\mu$ being the chemical potential of the system, see formula (40) from ref. \( \mathbb{Q} \). As is known, such time-dependent phase is the starting point in construction of the Q balls \( \mathbb{Q} \).

Finally, we want to constraint the QCD-ball pa-
rameters from available data. We follow \[3\] and assume that a typical cross section of a neutral QCD-ball with matter is determined by their geometrical size, \(\pi R_B^2\). We also assume that the QCD-balls is the main contributor toward the dark matter in the Galaxy, in which case their flux \(F\) should satisfy

\[
F < \frac{\rho_{DM} v}{4\pi M_B} \sim 7.2 \cdot 10^5 \text{GeV cm}^{-2} \text{sec}^{-1} \text{sr}^{-1},
\]

(5)

where \(\rho_{DM}\) is the energy density of the dark matter in the Galaxy, \(\rho_{DM} \approx 0.3 \text{GeV cm}^{-3}\), and \(v \sim 3 \cdot 10^{-3} c\) is the Virial velocity of the QCD-ball. We identify \(M_B\) in the expression \[2\] with the total energy \(E\) of the QCD ball at rest with given baryon charge \(B\). Different experiments discussed in \[3\] give the following bound on the flux of neutral soliton-like objects: \(F < \text{const.} \cdot 10^{-16} \text{cm}^{-2} \text{sec}^{-1} \text{sr}^{-1}\). This translates to the following lower limit of the neutral QCD-ball mass \(M_B^{\text{exp}}\) and baryon charge \(B\),

\[
M_B^{\text{exp}} > 2 \cdot 10^{21} \text{GeV}, \quad B^{\text{exp}} > 1.6 \cdot 10^{20}.
\]

(6)

These critical bounds are well below the critical line of the absolute stability of the QCD-balls, given by eq. \[4\]. However, as we mentioned above, we expect that QCD-balls will be very long-lived metastable states even if their baryon charge is below the critical limit \[4\]. Therefore, we consider experimental constraint \[3\], rather than a theoretical stability bound \[4\] as a more realistic bound on \(M_B\), \(B\) for the QCD-balls.

We have seen that the observed relation \(\Omega_B \sim \Omega_{DM}\) within an order of magnitude finds its natural explanation in this scenario: both contributions are originated from the same physics at the same instant during the QCD phase transition. As is known, this fact is extremely difficult to explain in models that invoke a dark matter candidate not related to baryons. We have also seen that without adjusting any parameters the baryon density inside the QCD-ball falls exactly into appropriate region of the QCD phase diagram where color superconductivity takes place. The scenario is no doubt lead to important consequences for cosmology and astrophysics, which are not explored yet. In particular, some unexplained events, such as Centauro events, or even the Tunguska-like events (when no fragments or chemical traces have ever been recovered), can be related to the very dense QCD balls. If this is the case, the arrival directions should correlate with the dark matter distribution and show the halo asymmetry. Also: recent discovery\[10\] suggests that the matter in some stars could be even denser than nuclear matter. It could be also related to the very dense QCD balls. Therefore, the “exotic”, dense color superconducting phase in QCD, might be much more common state of matter in the Universe than the “normal” hadronic phase we know. In conclusion, qualitative as our arguments are, they suggest that the dark matter could be originated at the QCD scale.

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