Stability of two-fluid partially ionized slow-mode shock fronts

B. Snow and A. Hillier

Centre for Geophysical and Astrophysical Fluid Dynamics, University of Exeter, Exeter, EX4 4QF, UK

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ABSTRACT

A magnetohydrodynamic (MHD) shock front can be unstable to the corrugation instability, which causes a perturbed shock front to become increasingly corrugated with time. An ideal MHD parallel shock (where the velocity and magnetic fields are aligned) is unconditionally unstable to the corrugation instability, whereas the ideal hydrodynamic (HD) counterpart is unconditionally stable. For a partially ionized medium (for example, the solar chromosphere), both HD and MHD species coexist and the stability of the system has not been studied. In this paper, we perform numerical simulations of the corrugation instability in two-fluid partially ionized shock fronts to investigate the stability conditions, and compare the results to HD and MHD simulations. Our simulations consist of an initially steady two-dimensional parallel shock encountering a localized upstream density perturbation. In MHD, this perturbation results in an unstable shock front and the corrugation grows with time. We find that for the two-fluid simulation, the neutral species can act to stabilize the shock front. A parameter study is performed to analyse the conditions under which the shock front is stable and unstable. We find that for very weakly coupled or very strongly coupled partially ionized system the shock front is unstable, as the system tends towards MHD. However, for a finite coupling, we find that the neutrals can stabilize the shock front, and produce new features including shock channels in the neutral species. We derive an equation that relates the stable wavelength range to the ion-neutral and neutral-ion coupling frequencies and the Mach number. Applying this relation to umbral flashes gives an estimated range of stable wavelengths between 0.6 and 56 km.

Key words: hydrodynamics – instabilities – MHD – Shock waves – Sun: chromosphere.

1 INTRODUCTION

Magnetohydrodynamic (MHD) shock waves are a fundamental feature of astrophysical systems, occurring across a range of physical systems, e.g. the solar atmosphere (Beckers & Tallant 1969; Hollweg 1982) and molecular clouds (Draine, Roberge & Dalgarno 1983). The formation mechanism behind astrophysical shocks is as varied and results as a consequence of a number of physical processes, such as magnetic reconnection (Petschek 1964; Yamada, Kulsrud & Ji 2010) and wave steepening (Suematsu et al. 1982). For a hydrodynamic (HD) system, sonic shocks can exist as a transition from above to below the sound speed. In MHD, there are three characteristic wave speeds (slow, Alfvén, and fast), which leads to a wealth of possible shock transitions (Tidman & Krall 1971). Here, we study the stability of slow-mode shock fronts in partially ionized systems.

For a steady-state shock, the MHD shock jumps can be described in terms of the conservative quantities sufficiently upstream and downstream of the shock. It can be shown that MHD systems support a variety of shock transitions (broadly categorized as slow, intermediate, and fast) as well as contact and tangential discontinuities. If a non-zero component of magnetic field exists normal to the steady shock ($B_\perp \neq 0$), then jumps in the tangential magnetic field imply jumps in the tangential velocity. As such, vorticity ($\nabla \times \mathbf{v}$) can exist at an MHD shock front. For an MHD contact discontinuity, where there is zero mass flux across the interface, only jumps in the density are permitted. As such, there is no magnetic field jump to support a jump in tangential velocity; hence, vorticity cannot exist at a steady-state MHD contact discontinuity.

In an HD system, the magnetic field is zero and hence the shock jumps are easier to compute. With regard to vorticity, the HD system has the opposite behaviour to an MHD system; vorticity can exist across an HD contact discontinuity, but not an HD shock (since a transverse velocity is no longer supported by a magnetic field; Hayes 1957). This has consequences for instabilities such as the Richtmyer–Meshkov instability that results in an HD shock contact discontinuity being unstable; however, the instability is suppressed in MHD (Wheatley, Santaney & Pullin 2009). For the corrugation instability (where a shock front is perturbed), the opposite can be true where the instability grows in HD but is suppressed in HD.

The stability of shocks to the corrugation instability depends on the type of shock. Fast MHD shocks are categorically stable provided the adiabatic index $\gamma < 3$ (Gardner & Kruskal 1964), whereas parallel MHD shocks (where the velocity and magnetic fields are parallel to the shock front) are always unstable to the corrugation instability (Stone & Edelman 1995). The stability of other types of shocks (e.g. switch-off) depends on the angle of the magnetic field (relative to the shock front) and the Alfvén Mach number (Lessen & Deshpande 1967; Édel’ Man 1989; Stone & Edelman 1995). Here, we focus on a slow-mode (parallel) shock that is categorically unstable for MHD, and stable for HD.

* E-mail: b.snow@exeter.ac.uk

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The stability of shock fronts to perturbations is important for compressible turbulent astrophysical systems, such as the interstellar medium (Elmegreen & Scalo 2004) and solar atmosphere (Reardon et al. 2008), where slow-mode shocks can encounter a non-uniform medium and develop instability. This may make slow-mode shocks more difficult to identify than fast-mode shocks due to the corrugated shock front (Park & Ryu 2019). Similar results are present in medium (Elmegreen & Scalo 2004) and solar atmosphere (Reardon 2011, 2020).

In addition to MHD and HD systems, shocks regularly occur in warm plasma, where the medium is only partially ionized, such as the solar chromosphere and molecular clouds. These partially ionized mediums represent a challenging area of study, specifically for instabilities, where the general behaviour is neither MHD-like or HD-like. Partially ionized mediums can be modelled using two-fluid (neutral, ion + electron) equations that allow for coupling and decoupling of the two species, and non-MHD-like behaviour. Two-fluid interactions in interface instabilities (where the initial magnetic field is parallel to the jump) can suppress small-scale features (Popescu Braileanu et al. 2020) and allow cross-field transport of momentum and energy (Hillier 2019).

The stability of two-fluid shocks to the corrugation instability has not been studied and likely depends on the level of coupling of the plasma and neutral species. A consequence of partial ionization on shocks is that the shock wave has a finite width that is determined by the physical parameters of the system. Hillier, Takasao & Nakamura (2016) extensively studied the two-fluid effects in a switch-off slow-mode shock. Within the finite width of the shock, additional shock transitions can occur due to the species decoupling (Snow & Hillier 2019). One can hypothesize that under the extreme conditions of the finite width being zero or infinity, the plasma should reduce to MHD-like behaviour. However, in the more realistic finitely coupled regimes the behaviour is undetermined, which forms the basis of the study presented here.

In this paper, we study the stability of two-fluid slow-mode shock fronts using two-dimensional (2D) numerical simulations. We investigate the consequences and stability of the shock front for different levels of collisional coupling and different ionization fractions. We find that the shock front can be stable or unstable depending on the wavelength of the perturbation relative to the finite width of the shock. This may have consequences for observations by giving an expected range at which partially ionized slow-mode shocks can be stable.

2 CORRUGATION INSTABILITY SCHEMATIC

In this paper, we focus on specifically the parallel slow-mode shock, where the flow is aligned with the magnetic field on either side of the shock. This type of shock is unconditionally unstable in the MHD model (Stone & Edelman 1995), and stable in the HD model, to the corrugation instability. In this section, we present the basic schematic for the HD stability and MHD instability for a 2D shock front encountering an upstream density perturbation, and make conjectures about the potential stability or instability expected for a partially ionized system.

2.1 MHD

An ideal MHD shock is unstable to the corrugation instability under certain conditions. For a parallel MHD shock, where the velocity and magnetic fields are parallel to the shock front on either side of the shock (i.e. \( \mathbf{v} = [v_x, 0, 0], \mathbf{B} = [B_z, 0, 0] \) in the shock frame), the shock front is always unstable to the corrugation instability (Stone & Edelman 1995). The steady-state shock jump conditions can be written as

\[
[\rho v_x]_d^u = 0, \\
[B_z]_d^u = 0, \\
\rho v_x \left[ B_x \right]_d^u = B_z \left[ B_y \right]_d^u, \\
\rho v_x \left[ B_y \right]_d^u = B_x \left[ v_y \right]_d^u, \\
\left[ \rho v_x^u + P + \frac{1}{2} B_x^2 \right]_d^u = 0, \\
\rho v_x \left[ \frac{1}{2} (v_y^u + v_x^u) + \frac{P}{(\gamma - 1) \rho} + \frac{P}{\rho} + \frac{B_x^2}{\rho} \right]_d^u = B_z \left[ v_y \right]_d^u,
\]

where the above notation relates conserved quantities upstream (superscript \( u \)) and downstream (superscript \( d \)) of the shock front as

\[
[Q]_d^u = Q^u - Q^d
\]

for any conserved quantity \( Q \). From these equations (particularly equation 3), it can be seen that a jump in the tangential component of the velocity can be supported across a steady-state shock front by a jump in the tangential magnetic field component.

When the shock front corrugates, it generates a tangential magnetic field component and hence a tangential velocity. This velocity is continuous across the interface and hence the flow is directed behind the peaks; see Fig. 1. The flow pattern increases the pressure behind the peaks and decreases it behind the troughs. As such, the peaks are pushed up and the troughs are pushed down, increasing the magnitude of the corrugation and leading to growth of the instability. This is shown by the schematic in Fig. 1.

2.2 Hydrodynamics

An ideal HD shock is stable to the corrugation instability. In an HD system, tangential velocity is continuous across the shock front. Therefore, at the distorted shock front the flow is refracted away from the ‘peaks’. The flow then enhances the pressure behind the ‘troughs’, and decreases the pressure behind the ‘peaks’; see Fig. 2. As such, the pressure provides a restoring force whereby the peaks are pushed down and the troughs are pushed up. The pressure restoring force...
therefore acts to mitigate the corrugation of the shock front and the system stabilizes, returning to a flat shock front. A schematic of this is shown in Fig. 2 where a high-pressure region forms behind the troughs.

2.3 Interpolating these results to partially ionized plasma (PIP)

In the two-fluid partially ionized case, the schematic is not as straightforward. The ionized species behaves like an MHD system, whereas the neutral species is HD. As such, one may consider that the isolated neutral species tends towards stability, whereas the isolated plasma species tends towards being unstable. The stability of the bulk (plasma + neutral) PIP shock depends on the balance of these forces, and the time-scales on which the species interact.

The coupling between the ionized and neutral species is governed by the collisional coefficient \( \alpha_c \) (Hillier et al. 2016) that is discussed further in Section 3. In the extreme of \( \alpha_c = 0 \), the system is fully decoupled, and the ionized species will be unstable. At the other extreme of \( \alpha_c = \infty \), the system is fully coupled and MHD-like; hence, one would also expect the system to be unstable. In the infinitely coupled regime, interactions can allow the system to be unstable or stable. The finitely coupled regime is the focus of our paper.

Similarly, one has to consider the perturbation wavelength relative to the finite width of the shock. Consider a fixed size perturbation of wavelength \( \lambda \), that is parallel to the shock front, at different levels of coupling. For an infinitely coupled system, the finite width of the shock approaches zero and hence the perturbation wavelength is much larger than the finite width of the shock. As such, one may expect that the system will behave similar to a bulk-MHD model. At the other extreme of coupling tending to zero, the finite width of the shock becomes large. The upstream perturbation in this case will encounter the plasma shock front first and therefore also behave like an MHD simulation. Between these two extremes, we expect the system to be finitely coupled. As such, the stability of a partially ionized shock can be connected to the wavelength of the perturbation relative to the finite shock width.

3 METHODS

The two-fluid equations governing the behaviour of a neutral species (subscript \( n \)) and a charge-neutral electron + ion species (subscript \( p \)) are

\[
\frac{\partial \rho_n}{\partial t} + \nabla \cdot (\rho_n v_n) = 0,
\]

\[
\frac{\partial \rho_p}{\partial t} + \nabla \cdot (\rho_p v_p) = 0.
\]

\[
\frac{\partial e_n}{\partial t} + \nabla \cdot [e_n (v_n + P_n) n] = -\alpha_c \rho_n \rho_p (v_n - v_p).
\]

\[
\frac{\partial e_p}{\partial t} + \nabla \cdot [e_p (v_p + P_p) p] = -\alpha_c \rho_n \rho_p (v_n - v_p).
\]

Figure 2. HD schematic for flow entering a distorted shock front. The colour map shows the gas pressure. Velocity vectors are overplotted as the black lines, with the \( v_p \) velocity multiplied by a factor of 30 to emphasize the flow.

where \( T_n \) and \( T_p \) are the neutral and plasma temperatures, respectively. The constant \( \alpha_0 \) can be specified and governs the coupling between the two species. We study a hydrogen-only system.

We use a fourth-order central difference solver with artificial viscosity to limit the numerical oscillations around the shock front. It was found that this solver is far less susceptible to the numerical Carbunkle instability (which forms due to a curved feature forming on a Cartesian grid; Elling 2009) than the first-order HLLD solver used in previous shock studies with the (PIP) code (Snow & Hillier 2019, 2020, 2021). The fourth-order scheme in the (PIP) code has been used successfully in prior studies of partially ionized plasma dynamics (Hillier 2019; Murtas, Hillier & Snow 2021).

3.1 Initial conditions

The initial conditions used in this paper are in the shock frame, where the shock is stationary at \( x = 0 \), with the inflow coming from the upstream conditions \( x > 0 \), and the post-shock region downstream \( x < 0 \). This has the advantage of allowing us to increase the resolution of the system since no space is required for the shock to propagate into.

A sonic Mach 2 parallel shock is specified analytically (see Appendix A), where ‘parallel’ refers to the initial flow being aligned with the magnetic field. Here, the shock is in the \( x \)-direction and \( v_p = v_n = -B_z \) initially and corresponds to a slow-mode shock. The pressure and density are specified such that we have a thermal equilibrium and the initial plasma-beta \( \beta = 0.1 \) in the upstream medium.
where $x$, $v$, $\rho$ are therefore defined as the bulk pressure and density. The neutral and plasma parameters are defined in the context of the MHD shock thermally coupled, the plasma conditions sufficiently upstream of the shock front and described by the MHD shock jump equations (Snow & Hillier 2019, 2021). As such, the bulk (plasma + neutral) density, pressure, velocity, and magnetic field jumps are the same as the MHD case; see Appendix A. The partial pressures and densities are calculated using the neutral fraction and the bulk pressure and density. The neutral and plasma parameters are therefore defined as

$$
\rho_\alpha = (1 - \xi_n) \rho_t, \quad \rho_n = \xi_n \rho_t, \quad \rho_p = \frac{2 \xi_n}{\xi_n + 2 \xi_p} \rho_t,
$$

where a subscript $t$ denotes the MHD value and $\xi_n$ and $\xi_p$ are the neutral and plasma fractions, respectively. In this formation, the MHD simulation is effectively the infinitely coupled case where the system behaves like a bulk (plasma + neutral) fluid. At the other extreme of $\alpha = 0$, the fluids are completely decoupled and the plasma will behave MHD-like, free of any interactions with the neutral species. The shock front is allowed to numerically stabilize before encountering the upstream density perturbation.

3.2 Boundary conditions

Upper and lower ($y = 0, 10$) boundary conditions are set to periodic. The left and right boundaries ($x = -25, 25$) are specified as damping layers using a similar formulation as Felipe, Khomenko & Collados (2010). Several waves are generated from the initial shock wave relaxing and the interactions with the density perturbation. This damping layer effectively removes these unwanted waves from the system.

$$
U = U_0 + (1 - D(x))(U - U_0)
$$

$$
D(x) = \frac{D_0}{\Delta x} \left( \frac{x - x_D}{W} \right)^2,
$$

where $U$ is the instantaneous evolved vector of conserved variables and $U_0$ is the equilibrium quantity. The initial quantities $U_0$ are determined analytically using the shock jump conditions. The perturbation to be damped is then $U - U_0$, which is gradually damped over $W = 20$ grid cells. $x_D$ is the $x$ value at the end of the damping region. The factor $D_0$ scales the maximum damping to be 0.3. We find that this boundary condition is sufficient to remove unwanted perturbations from system and prevents reflections from the boundary. The boundaries are also placed far from the shock front to further minimize their impact on the results.

4 REFERENCE MODEL

4.1 MHD simulation

For the MHD system, the shock front encounters the density perturbation, corrugates, and then the shock front perturbation grows with time; see Fig. 4. The initial perturbation of the shock front allows the MHD model, Fig. 5, the integrated enstrophy around the shock front as

$$
\varepsilon = \int_0^{10} \int_{x_s - 4}^{x_s + 4} (v \times v)^2 \, dx \, dy,
$$

where $x_s$ is the location of the shock front. The integral limits capture the full extent of the domain in the $y$-direction, and the horizontal window is chosen to capture the full extent of the corrugation in the $x$-direction. The vorticity is non-zero at the distorted shock front, and zero elsewhere. As such, the integrated enstrophy allows us to determine if the corrugation is growing or decaying with time.

For the MHD model, Fig. 5, the integrated enstrophy around the shock is close to zero initially, as it should be for a uniform shock front. When the shock front encounters the density enhancement, there is a sudden rise in the enstrophy as the shock front corrugates. As time increases, the corrugation continues to grow, as seen in
Figure 4. Time series showing the evolution of the density around the shock front in the MHD (left), and P\textsuperscript{IP} (plasma centre, neutral right). Snapshots are taken at times $t = 5, 7, 10, 15$, and $30$ non-dimensional times from top to bottom.

Fig. 4, and hence the integrated enstrophy grows. As time increases towards infinity, the growth of the instability will saturate, as seen in Stone & Edelman (1995); however, this has not yet occurred in time frame studied here.

A 1D shock front can be extracted from the 2D simulation by taking the displacement at each height $y$, with the location of the shock determined by the maximum density gradient in the $x$-direction. Plotting the shock front through time shows the growth of the corrugation, as shown in Fig. 6. The initial disturbance warps the shock front. As time increases, the displacement grows across all frequencies. This shows the expected result that the MHD simulation is unstable to the corrugation instability.

4.2 P\textsuperscript{IP} simulation

Here, we present an initial two-fluid P\textsuperscript{IP} simulation where the neutral fraction is set as $\xi_n = 0.9$ and a coupling coefficient of $\alpha_0 = 1$, giving a mostly neutral medium that is relatively weakly coupled on the time-scales considered here. This choice of coupling coefficient and initial conditions means that on average upstream collisions occur on a time-scale of unity. The bulk (neutral + plasma) density and pressure are the same as in the MHD case; see Appendix A. For a two-fluid system, the bulk (neutral + plasma) shock jumps reduce to the MHD shock jumps sufficiently upstream and downstream of the shock (Snow & Hillier 2019). However, within the finite width of the shock, drift velocities form between the two species and substructure exists. The role of this substructure on the stability of the shock front is investigated here.

Fig. 4 shows the plasma and neutral densities through time for this fiducial P\textsuperscript{IP} simulation. Initially, the shock front is uniform in the $y$-direction, as with the MHD case; however, the two-fluid effects produce a finite width to the shock, which is highlighted in Fig. 7. When the shock front encounters the density enhancement, corrugations occur in both the plasma and neutral shock fronts. These corrugations initially grow in the plasma and decay in the neutrals. Finally, the distortions decay in both species and the shock front stabilizes. This can be seen in Fig. 5, where there is a rise in corrugation, followed by a decay towards a stable shock front.

For this simulation, the plasma shock is at the leading edge, followed by the neutral shock; see Fig. 7. As such, the plasma species encounters the density perturbation first, and the instability grows in an MHD-like manner initially. At this time, there is also a slight corrugation of the neutral shock front from the encountered perturbation. However, as time advances, the displacement of the
The shock front decreases in both the neutral and plasma species and the shock front stabilizes.

The schematic for flow entering the initially distorted shock is given in Fig. 8 and can be compared to the schematics for MHD (Fig. 1) and HD (Fig. 2). The flow entering the plasma shock is slightly corrugated in the same way as the MHD case. The \(v_y\) magnitude is far less in the PP case than that in the MHD case due to the resistance provided by the neutral species. In the neutral species, the flow is directed towards the troughs, creating a high-pressure region that suppresses the instability. In this regime with a mostly neutral medium (\(\xi_n = 0.9\)) and a weakly coupled system (\(\alpha_0 = 1\)), the interactions between the plasma and neutral species are sufficient to suppress the corrugation instability and stabilize the shock front. This is further shown in the evolution of the shock front displacement with time, Fig. 6, where the plasma shock front distorts initially and then stabilizes. The neutral shock front distorts slightly after encountering the density enhancement but is quickly stabilized. Physically, the neutral species is trying to stabilize the shock front, whereas the plasma species is trying to go unstable. Here, the overall physics is dominated by neutral species since medium is mostly neutral (hence, the plasma-neutral coupling is prevalent) and the perturbation wavelengths in the \(y\)-direction are equally affected by the coupling (this is discussed further in Section 5).

The finite width of the shock front here is \(\approx 0.6\), which is smaller than parallel perturbation wavelengths that are in the range of \(\lambda_c = 1-10\) non-dimensional units. The ratio of the shock width to the perturbation wavelength should govern the behaviour of the system. For a perturbation much larger than the finite width, the system should behave like a fully coupled MHD simulation. For a perturbation much smaller than the finite width, the simulation should behave like a fully decoupled simulation, also MHD-like, since on short time-scales the perturbation will only encounter the plasma shock front. Between these two limits, the finite coupling leads to non-MHD-like behaviour.

### 5 Parameter Study

#### 5.1 Coupling coefficient

Changing the coupling coefficient changes the finite width of the shock, with larger coupling leading to a narrower finite width (Hillier et al. 2016). For the two-fluid \(M = 2\) shock investigated in this paper, with an upstream plasma-\(\beta = 0.1\), the finite width of the shock is a function of the coupling coefficient, as shown in Fig. 9 from a series of 1D numerical simulations. An analytical estimate is also shown, which is calculated as the difference in propagation speeds of the isolated fluids, divided by the coupling frequency, i.e.

\[
W = \frac{c_{p\text{m}} - c_{n\text{m}}}{\alpha c_\beta},
\]

for the plasma and neutral sound speeds \(c_{p\text{m}}\) and \(c_{n\text{m}}\). The analytical and numerical results for the shock width pair up very well, and scale well in the coupling regime investigated here. Also, changing the neutral fraction does not greatly affect the shock width and this estimate remains a good fit to the 1D simulation data.

At the limit of \(\alpha_c = \infty\) and 0, the system is MHD with the fluid acting as a bulk (neutral + plasma) system or a completely...
decoupled system, respectively. Between these limits, we expect two-fluid effects to become important, as seen in Section 4.2. Here, we investigate the consequences of changing the coupling coefficient on the stability of the shock front.

Fig. 10 shows a time evolution of the shock front for different levels of coupling, and two MHD simulations that represent the infinitely coupled and fully decoupled extremes. Fig. 11 shows the corresponding integrated enstrophy around the shock front through time for these cases. One can see that in the MHD simulations, the corrugation grows with time and persists throughout the simulation. The growth of the $\alpha = 0$ case saturates near the end of the simulation, whereas the $\alpha = \infty$ case continues to grow. In both cases, however, the shock front has become unstable and the corrugation is a consistent feature of the system. In the \textit{pp} simulations, the initial perturbation distorts the shock front (as with the MHD cases); however, the perturbation is seen to stabilize to varying degrees depending on the coupling coefficient. For the $\alpha = 1$ case, all frequencies are damped at roughly the same rate, with the shock front tending to a flat shock front. For $\alpha = 10$, the moderate coupling has a severe effect in rapidly damping the perturbation, quickly leading to a stable/flat shock front. At $\alpha = 100$, the perturbation grows and has saturated; however, the smaller frequencies have been damped from the system and only larger frequencies exist.

The integrated enstrophy around the shock is shown in Fig. 11(a) where the time zero corresponds to the shock encountering the perturbation. The corresponding time derivative at time $\tau = 10$ is shown in Fig. 11(b). The MHD ($\alpha = 0$) saturates during the simulation and hence the time derivative is very close to zero, and in fact slightly negative. It can be seen from Fig. 10 that the perturbed shock front remains distorted with time and the growth rate being close to zero is due to saturation of the instability. For a weakly coupled system ($\alpha = 0.1$), the system is unstable and growing; however, the rate is very small, implying that it is approaching saturation. At the end time, the simulation is comparable to the MHD ($\alpha = 0$) case. The perturbation width for $\alpha = 0.1$ is approximately the same size as the finite width of the shock. As the coupling increases ($\alpha = 1, 10$), the system becomes stable as the neutrals start to influence the plasma. This is seen clearly in Fig. 10 where these cases approach an undisturbed state at late times. At strong coupling ($\alpha = 100$), the system begins to act like an ensemble bulk MHD ($\alpha = \infty$) system and the perturbation persists through time. From Fig. 11(b), it can be seen that the growth rate here is very close to zero and likely saturated for $\alpha = 100$. The structure of the perturbed shock front in this case is also slightly different from the MHD $\alpha = \infty$ case; only the low-frequency components are present in the perturbation, and it is much smoother than the MHD case.

The wavelengths that persist in the shock front vary for different coupling coefficients; see Fig. 12 that shows the normalized shock front displacement. For weak coupling ($\alpha = 1$), the wavelengths are all damped fairly uniformly, as seen in Fig. 6. For moderate coupling ($\alpha = 10$), the system stabilizes very rapidly and only the longest wavelengths persist. This is shown in Fig. 12 where only the fundamental mode is present. For strong coupling ($\alpha = 100$), the shorter scales are damped, and several longer scales persist. It is known that there are several scales on which the neutral and plasma species interact (Hillier 2019). On the largest scales, the neutrals and plasma species are coupled. On intermediate length-scales, the neutrals are decoupled from the plasma, and on the smallest length-scales both species are decoupled from each other. These scales are determined using the collision coupling frequencies:

$$v_{in} = \rho_n c_{in} \xi_n \alpha_c,$$

(30)

where $v_{in}$ and $v_{in}$ are the ion-neutral and neutral-ion collisional frequencies, respectively. As $\alpha_c$ increases, the coupling frequencies increase. One can separate the coupling with wavelength into three frequency bands. For low frequencies, the neutrals and plasma are well coupled. For intermediate frequencies, the neutrals decouple from the plasma (and hence the magnetic field); however, the plasma is still coupled to the neutrals. For high frequencies, both species decouple. For the simulations performed here, increasing the coupling coefficient effectively decreases the wavelength at which the neutrals decouple from the plasma. This can be used to explain the damping of short wavelengths in the $\alpha_n = 100$ case.

The approximate coupling frequency at which the system becomes unstable can be estimated by equating the coupling frequency to the frequency of the wavelength. For the lower limit, the frequency at which the plasma decouples from the neutrals for the largest wavelength is

$$\alpha_c \min \nu_p^u = 2\pi \frac{|v^u - v_p^u|}{\lambda_{10}^{\max}}.$$ 

(32)

Similarly, the upper limit is provided by the frequency at which the neutral decoupled from the plasma using the smallest wavelength:

$$\alpha_c \max \nu_p^u = 2\pi \frac{|v^u - v_p^u|}{\lambda_{10}^{\min}}.$$ 

(33)

The velocity jump in our simulation is $|v^u - v_p^u| \approx 1.1$, the upstream densities are $\rho_n^u = 0.1$, $\rho_p^u = 0.9$, and our wavelengths are in the range $\lambda_{10} \in Z : \lambda_{10} \in [1, 10]$, with $\lambda_{10}^{\max} = 10$ and $\lambda_{10}^{\min} = 1$. As such, we estimate that the upper and lower limits on stability of our system are $\alpha_c^{\min} \approx 0.754$ and $\alpha_c^{\max} \approx 69.1$, which are shown by the dashed lines on Fig. 11. One can see that these approximations are a reasonably good fit to the simulation results, and all simulations between $\alpha_c^{\min} < \alpha_c < \alpha_c^{\max}$ are stable to the corrugation instability.

We note that a time-scale argument exists for the weakly coupled systems. On the time-scales considered here, the corrugation evolves and reaches saturation; however, as time advances, the plasma will experience more collisions from the neutral species. As such, as time tends to infinity, the saturated, corrugated \textit{pp} shocks may further evolve due to a gradual influence of the neutral species.

5.2 Ionization fraction

Simulations thus far used a neutral fraction of $\xi_n = 0.9$, meaning that the medium is mostly neutral particles. Here, we investigate changing the neutral fraction by looking at $\xi_n = 0.99, 0.5$, and 0.1. The $\xi_n = 0.1$ case is mostly ionized. The $\xi_n = 0.99$ case is dominated by the neutral species. The $\xi_n = 0.5$ is an interesting case where the system is equally ionized and neutral. A time series showing the plasma and neutral densities at different times is shown in Fig. 13.

The $\xi_n = 0.99$ case behaves fairly similar to the $\xi_n = 0.9$ case studied previously. Both the plasma and neutral shock fronts corrugate slightly when they encounter the upstream density perturbation. As time advances, the shock front stabilizes. Since this simulation is dominated by the neutral species, the stabilizing forces from the neutral species are stronger than the destabilizing forces of the plasma. The corrugation of the plasma shock front increases between times $t = 2$ and 5. However, the neutral coupling then leads to a stabilization of the shock front. This is comparable to the results of the $\xi_n = 0.9$ case in Section 4.2.
Figure 10. Time series of the shock front for different levels of collisional coupling and two MHD simulations for the extremes of zero and infinite coupling.

For a neutral fraction of $\xi_n = 0.5$, the system is equally neutral and ionized. The evolution of the shock front through time is shown in Fig. 13. The shock front is unstable to the corrugation instability here and the shock front distortion grows with time. Interestingly, this simulation forms far longer fingers than what has been seen in the MHD or PIP simulations in Section 4.2. Between the elongated fingers, the neutral species also develops shock channels, as seen by the sharp density structures.

In the MHD case, the flow is directed behind the peaks (see Fig. 1). For the PIP $\xi_n = 0.5$ case, the neutral species resists this pattern and the plasma flow is directed towards the troughs instead; see Fig. 14. As time advances, this forces the troughs to extend further back and creates the elongated fingers seen in Fig. 13. In the neutral species, similar behaviour occurs where the flow is directed towards the troughs, resulting in an increased neutral pressure and the formation of shocks due to the converging flow (Fig. 14). When the plasma can exert a strong influence on the neutrals, these shock channels can form as flow pattern of the neutral species is dominated by the plasma motion.

The $\xi_n = 0.1$ case is dominated by the plasma species, and, as such, the corrugation of the plasma shock front grows, similar to the MHD model. The behaviour of the neutral species is mostly determined through the coupling with the plasma species. The result is that the neutral flow is channelled towards the troughs, as with the $\xi_n = 0.5$ case. However, here the neutral species constitutes far less of the bulk medium and hence there is very little feedback from the neutrals to the plasma.

6 DISCUSSION

6.1 Perturbation length-scale

One can re-frame the effect of different collisional coefficients in terms of the length-scale relative to the finite width of the shock. The initial case studied in Section 4.2 has a finite shock width of approximately $W = 0.68$ and an initial parallel perturbation wavelengths in the range $\lambda_\parallel = [1, 10]$; therefore, the wavelength of the perturbation is larger than the finite width of the shock. The perturbation wavelength relative to the finite width of the shock is an important parameter. The simulations in Section 5 showed that when the perturbation wavelength was between 10 and 1000 times larger than the finite width, the neutral interactions stabilized the shock front.

The finite width of the parallel slow shock considered here can be estimated using equation (29). Rewriting this in terms of bulk fluid properties,

$$c_{s,\parallel}^2 = \frac{\gamma P_\parallel}{\rho_\parallel} = \frac{\gamma P_\parallel}{\rho_\parallel} \frac{\chi_i}{\xi_i}$$

$$c_{s,\parallel}^2 = \frac{\gamma P_\parallel}{\rho_\parallel} = \frac{\gamma P_\parallel}{\rho_\parallel} \frac{\chi_n}{\xi_n}$$

$$\chi_i = \frac{2\xi_i}{\xi_n - 2\xi_i}$$

$$\chi_n = \frac{\xi_n}{\xi_n - 2\xi_i}$$
The ion-neutral collisional frequency in the chromosphere varies for visualization purposes. The bulk sound speed in the chromosphere is approximately 8 km s\(^{-1}\).

\[
\lambda_{\text{sp}} = \frac{\lambda_{\text{max}}}{W_{\text{shock}}} = 10 \div \text{the finite width of the shock}.
\]

\[
d\log e / dt \approx 0.04,
\]

\[
0.01 \times 10^3 \ldots 10^6 \text{ s}^{-1}.
\]

\[
\lambda_{\text{max}} = \frac{2 \pi c_s}{\lambda_{\text{max}}^{\parallel}} \left( r - 1 \right),
\]

\[
\alpha_c \min \rho_p = \frac{2 \pi c_s}{\lambda_{\text{min}}^{\parallel}} M^2 (r - 1),
\]

\[
\alpha_c \max \rho_p = \frac{2 \pi c_s}{\lambda_{\text{max}}^{\parallel}} M^2 (r - 1),
\]

\[
r = \frac{M^2 (r - 1)}{2 + M^2 (r - 1)},
\]

\[
\frac{M^2 (r - 1)}{2 + M^2 (r - 1)}.
\]

\[
\text{in the range of } 10^2 \text{–} 10^6 \text{ s}^{-1} \text{ (Popescu Braileanu et al. 2019).}
\]

\[
\text{in Section 5, it is shown that perturbations } 1000 \text{ times larger than finite width were unstable.}
\]

\[
\text{Therefore, we expect that in the partially ionized solar chromosphere, slow-mode shock fronts will be stable to perturbations of the order of } 4\text{–}4000 \text{ m. Physically, this means that the perturbations that we are able to observe should result in unstable shock fronts. Below our observational limits, the partially ionized shocks should stabilize.}
\]

\[
\text{The analytical approximation for the stability range of a partially ionized parallel shock (given by equations (32 and 33) is a reasonable approximation to the simulation results. This formulation can be written for a parallel slow-mode shock of an arbitrary sonic Mach number } M \text{ as}
\]

\[
\alpha_c \min \rho_p = \frac{2 \pi c_s M}{\lambda_{\text{max}}^{\parallel}} (r - 1),
\]

\[
\alpha_c \max \rho_p = \frac{2 \pi c_s M}{\lambda_{\text{min}}^{\parallel}} (r - 1),
\]

\[
\text{for compressible ratio } r = \rho^2 / \rho_p^2 \text{, where } r > 1 \text{ for a shock. Using}
\]

\[
\text{the relationship between } r \text{ and } M \text{ for a parallel slow-mode shock:}
\]

\[
r = \frac{M^2 (y - 1)}{2 + M^2 (y - 1)},
\]

\[
\frac{M^2 (y - 1)}{2 + M^2 (y - 1)}.
\]

\[
\text{Therefore, given the upstream plasma and neutral densities, the upstream sound speed, the shock Mach number, and the range of perturbation wavelengths upstream of the shock, one can estimate the coupling frequencies at which the shock may become unstable. These coupling frequencies can then be compared to the expected range of frequencies for the medium (e.g. Popescu Braileanu et al. 2019).}
\]

\[
\text{The results can be used to approximate the stability of shocks in a solar sunspot, where the magnetic field is fairly straight and propagating shocks are regularly observed in the form of umbral flashes (Beckers & Tallant 1969), which have average lifetimes of around } 44.2 \text{ s (Nelson et al. 2017). Umbral flashes have Mach numbers between } M = 1 \text{ and 1.7 (Anan et al. 2019) and using the}
\]

\[
\text{the coupling frequencies in the upper chromosphere (e.g. see fig. 2 in Popescu Braileanu et al. 2019) of } v_{\text{in}} = 3 \times 10^2, v_{\text{in}} = 3 \text{ s}^{-1} \text{ with a typical sound speed of } c_s = 8 \text{ km s}^{-1} \text{ gives a stable range of wavelengths between approximately } 0.6 \text{ and } 56 \text{ km (for } M = 1.7).}
\]

\[
\text{For the parallel slow-mode shock investigated here, the shock width has very little dependence on the ionization fraction; see Fig. 9. However, it is known that for switch-off shocks, the finite width has a dependence on the neutral fraction and plasma- } \beta \text{ (Hillier et al. 2016). Also, the physical width of the shocks studied in this paper is much smaller than the } < 300 \text{ km shocks in Snow & Hillier (2020). As such, one would expect that the stability range changes for different shock types. Further work is needed to calculate this stable range for different types of partially ionized shocks.}
\]

\[
\text{6.2 Slow-mode shocks in turbulence}
\]

\[
\text{In MHD simulations of turbulence, fast-mode shocks are identified more readily than slow-mode shocks (Park & Ryu 2019). The}
\]

\[
\frac{\gamma P_0}{\rho_0} \left( \frac{X_n}{\xi} - \frac{X_0}{\xi} \right).
\]

\[
\text{The bulk sound speed in the chromosphere is approximately } 8 \text{ km s}^{-1}.
\]

\[
\text{The ion-neutral collisional frequency in the chromosphere varies}
\]
potential reason for this is that the slow-mode shocks are unstable to the corrugation instability; hence, the shock front deforms and is difficult to identify. However, analysing the MHD simulation of the corrugation instability by using an automated shock detection method (based on the SHOCKFIND algorithm developed by Lehmann, Federrath & Wardle 2016), we see that the slow-mode shocks are still detected over the vast majority of the shock front; see Fig. 15. In face, due to the increased length of the shock, approximately 10 times more points are detected in the corrugated shock front at time $t = 30$ than in the initial stable shock front. This implies that the corrugation instability should lead to a greater detection of slow-mode shocks, contrary to the claim of Park & Ryu (2019). Further study is needed to determine the effect of the corrugation instability on the presence of shocks for different levels of corrugation and Mach numbers.

If the corrugation instability is reducing the detection of slow-mode shocks (as suggested by Park & Ryu 2019), then in a partially ionized system, we may detect statistically more slow-mode shocks in a two-fluid turbulence than MHD turbulence. Here, we show that for a partially ionized medium, the neutral species can act to stabilize the system. In 3D, we would expect that the neutral species could act to stabilize the corrugation growth in the out-of-plane direction, resulting in a stable 3D partially ionized shock. A future study will investigate the stability of 3D partially ionized shocks.

6.3 Extension to 3D shocks

Stone & Edelman (1995) show that a wider range of shock types are unstable in 3D MHD, since perturbations are allowed to grow in the out-of-plane direction and MHD shocks that are stable in 2D can be unstable in 3D. Here, we see that for an unstable 2D shock in a PIP, the neutral species can act to stabilize the system. In 3D, we would expect that the neutral species could act to stabilize the corrugation growth in the out-of-plane direction, resulting in a stable 3D partially ionized shock. A future study will investigate the stability of 3D partially ionized shocks.

7 CONCLUSIONS

In this paper, we have investigated the stability of slow-mode shocks in partially ionized plasma to the corrugation instability. As the shock front encounters a density perturbation, the shock front distorts, leading to the corrugation instability. For an MHD system, a parallel slow-mode shock is always unstable, whereas an HD shock is usually stable. For the partially ionized cases presented here, we show that the instability can be stable or unstable depending on the system parameters.

In the reference case ($\alpha_c = 1$, $\xi_n = 0.9$), the corrugation grows in the plasma species initially, and then the coupling with the neutral...
species leads to stabilization of the shock front. The stability of the shock front is dependent on the coupling coefficient. At large coupling ($\alpha_c \gg 100$), the system is unstable to the corrugation instability and behaves like a bulk MHD system. At the other extreme ($\alpha < 0.1$), the system is again unstable and MHD-like, behaving like a fully decoupled plasma. Between these limits ($0.1 < \alpha_c < 100$), the neutral interactions with the plasma species lead to a stabilization of the shock front to the corrugation instability.

The stability range can be rewritten in terms of the perturbation width relative to the finite width of the shock, and determines the stability of the shock front. When the perturbation width is of the same order (or smaller) than the finite width of the shock, the system is unstable and MHD-like. Similarly, when the perturbation is 1000 times larger than the finite width, the system is again unstable and MHD-like. Between these limits, the two-fluid interactions become important and the neutral species can stabilize the shock front.

In conclusion, two-fluid interactions can lead to stabilization of MHD parallel slow-mode shock fronts to the corrugation instability. Further work is needed to determine the stability criteria of different shock types.

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DATA AVAILABILITY

The simulation data from this study are available from BS upon reasonable request. The (P\textsuperscript{IP}) code is available at https://github.com/AstroSnow/PIP.

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APPENDIX A: PARALLEL SHOCKS

A parallel shock is where both the velocity and magnetic field vectors are parallel shock front in both the upstream and downstream states,
Table A1. Analytically determined upstream and downstream states for a parallel slow-mode shock.

|    | $\rho$ | $P$ | $v_x$ | $v_y$ | $v_z$ | $B_x$ | $B_y$ | $B_z$ |
|----|--------|-----|-------|-------|-------|-------|-------|-------|
| Upstream | 1 | 1/γ | $M$ | 0 | 0 | $B_0$ | 0 | 0 |
| Downstream | $r$ | $r/\gamma$ | $M/r$ | 0 | 0 | $B_0$ | 0 | 0 |

i.e. $v = (v_x, 0, 0)$, $B = (B_z, 0, 0)$. Using these, the shock equations reduce to

\[
[r v_x]_d = 0
\]

\[
[r v_x v_x + P + B^2/2]_d = 0
\]

\[
[\frac{\gamma \cdot P}{\gamma - 1} + \frac{v_x^2}{2}]_d = 0
\]

\[
[B_x]_d = 0.
\]

The compression across the shock can be defined as $\rho d / \rho u = r$. This leads to the relation that $v_x^d / v_x^u = 1/r$ (from equation A1).

Introducing an upstream Mach number $M = v_x^u / V_{\text{sound}}^u$, where $V_{\text{sound}}^u = \sqrt{\gamma P_u / \rho_u}$ is the upstream sound speed, one can determine the upstream velocity as $v_x^u = M \sqrt{\gamma P_u / \rho_u}$. One can express the downstream velocity and density in terms of upstream quantities, namely $\rho d = r \rho u$ and $v_x^d = v_x^u / r$. Equation (A2) becomes

\[
\rho d M^2 \gamma \frac{P^u}{\rho u} + P^d = r \rho u \left[ \gamma \frac{P^u}{\rho u} + P^u \right]^\frac{1}{r^2}
\]

Finally, $r$ can be determined using equation (A3) and some algebra as

\[
r = \frac{M^2 (\gamma + 1)}{2 + M^2 (\gamma - 1)}.
\]

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