Research on Frequency Hopping Synchronization Strategies based on TOPSIS Method

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Abstract. The synchronization of hopping frequency is a key step in frequency hopping communication systems. During system synchronization, the contradiction emerges between synchronization duration and credibility in the capture confirmation process after frequency acquisition, which make it difficult for the system to establish synchronization quickly and steadily. Thus, a solution is presented based on the classic Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) to solve this problem. Firstly, a mathematic model for recognition process is built by Markov chains. Secondly, TOPSIS is employed to make analysis and suggestions to the recognition strategies. Results show that the presented method can shorten the synchronization time, ensure the reliability of the synchronization systems, and achieve an optimal effect in frequency hopping capture confirmation process.

1. Introduction
Frequency-hopping communication is an important type of spread spectrum communication system. As an effective means of anti-jamming and anti-interception in modern communication field, it has been widely used in military and civil communication fields. The essential premise of frequency hopping communication system is that the system can establish and realize accurate frequency hopping synchronization. Therefore, the synchronization process is the key step of frequency hopping system. The speed of synchronization and the reliability of synchronization system will directly affect the performance of the whole frequency hopping communication system [1-2].

Frequency hopping synchronization consists of three processes: initial acquisition, capture validation and tracking [3]. Initial acquisition is the detection of a potential synchronization through correlation, acquisition confirmation needs to determine whether the synchronization acquisition is caused by interference or real acquisition, usually through successive detection to determine[4]; The final tracking phase is to complete the fine synchronization on the system frequency[5]. The core of acquisition and confirmation is to compare the frequency hopping between the two ends of the receiver and transceiver. At present, if the same number of frequency points exceed the predetermined threshold value, usually set to half of the total number of hops, the symbol system has completed the frequency hopping synchronization and can enter the frequency hopping system. Data transmission phase; conversely, re-enter the frequency capture phase [6-9]. In theory, the more frequency points are detected in this stage, the higher the reliability of frequency hopping synchronization is at the receiving end. However, in the actual situation, the system often has very strict requirements for the time required for frequency hopping synchronization. Therefore, the detection time and reliability of the verification process are a set of contradictions, which need to be considered in practice in order to ensure the comprehensive performance of the synchronization phase of the system.
In this paper, the mathematical model of frequency capture and confirmation process is established. The contradiction between detection time and reliability is regarded as a multi-attribute comprehensive decision making problem, which is called "multi-attribute comprehensive evaluation and decision method" in mathematical modeling. There are a large number of algorithms suitable for different application scenarios in this field. The most classical and widely used algorithms are the TOPSIS method and the grey correlation analysis. The idea of TOPSIS method is to calculate the Euclidean distance between each index of the to-be-assessed method and the ideal index, then make them have a weighed summation [10], whereas gray correlation analysis method regards the value of each index as a point on a curve and compares the curve of the to-be-assessed method with the ideal value curve. Therefore, when the evaluation index is relatively less, the evaluation results of the TOPSIS method is more accurate, and when the number of evaluation index is large, there are enough indexes to fit the curve accurately, so the evaluation result of the gray correlation analysis method is more accurate. In this study, four evaluation indexes are adopted, the number of which is small, so the TOPSIS method is used to further analyze, and through TOPSIS analysis, the optimal frequency capture confirmation scheme is selected from many options.

2. Mathematical Modelling of the Frequency Capture Confirmation Process
The frequency hopping frequency capture confirmation process can be mathematically abstracted into a mutual transformation problem in which a plurality of discrete states satisfy a certain condition. At the same time, when the system frequency hopping detection reaches a certain state, the state of the next moment depends only on the current state and is independent of the previous state. Stochastic processes for this feature are mathematically typically modeled using homogeneous Markov chain [11].

In homogeneous Markov chain, \( P_{ij}(n) \) indicates the n-step transition probability, and \( P(n) = \left( P_{ij}(n) \right) \) indicates n-step transition probability matrix. The sum of the row elements of this matrix is equal to 1, that is: \( \sum_{j=1}^{\infty} P_{ij}(n) = 1 \). When \( n \) is a value of 1, it is a one-step state transition probability:

\[
P_{ij} = P_{ij}(1) = P\{S_{m+1} = a_j | S_m = a_i\}
\]  \hspace{1cm} (1)

And the one-step transition probability matrix \( P = P(1) = \left( P_{ij} \right) \). In the homogeneous Markov chain situation, the n-step transition probability is completely determined by the one-step transition probability:

\[
P(n) = \left[ P(1) \right]^n = P^n
\]  \hspace{1cm} (2)

The Markov chain is used to mathematically model the frequency hopping synchronization confirmation process. The continuous acquisition confirmation mode is adopted, that is, the continuous detection of the L hops frequency in the K hops detection is determined to be passed. As shown in the following figure 1:

Figure 1. Continuous capture mode.
In figure 3, \( q \) is the false detection rate of detecting a whole hop in the system, \( p = 1 - q \). For a homogeneous discrete Markov chain, the one-step state transition probability matrix can describe the whole states of Markov chain [12]. For the continuous detection method, the matrix is denoted as \( \pi \):

\[
\pi = \begin{bmatrix}
q & 1-q & 0 & 0 & K & 0 \\
q & 0 & 1-q & 0 & K & 0 \\
q & 0 & 0 & 1-q & K & 0 \\
K & K & L & K & K & 0 \\
q & 0 & 0 & K & K & 1-q \\
0 & 0 & 0 & K & 0 & 1
\end{bmatrix}
\]

(3)

Especially,

\[
P(n) = P(0) * \pi^n
\]

(4)

The probability distribution \( P(0) \) of the initial stage of the continuous detection scheme is \((1, 0, 0, 0, 0, L, L)\), and the transition probability of the Markov chain at any time can be obtained according to the above formula.

According to the above analysis, it is necessary to use the Markov chain to solve the frequency acquisition confirmation probability. After the \( K \) hops, the frequency hopping receiver can successfully reach the probability of detecting the detection threshold \( L \), that is, the \( L \)th value in the \( P(K) \) probability vector, which is recorded as \( P_L \). In addition to the \( K \) hop and the detection threshold \( L \), the other one involved in calculating the \( P_L \) is the parameter \( q \) in the state transition probability matrix.

Through the mathematics software Mathematica operation, the relationship between \( P_L \) and the matrix parameter \( q \) in the case of different total hops \( K \) and detection threshold \( L \) is simulated as shown in figure 2 and figure 3.

Figure 2. The probability of continuously detect 3 hops in 5 hops and 10 hops.
Figure 3. The probability of continuously detect 5 hops in 5 hops and 10 hops.

It can be seen from the analysis results that when the total hop count \( K \) and the detection threshold \( L \) are given, the probability of successful detection decreases as the system false detection rate \( q \) increases. When \( L \) is constant, as the total number of hops \( K \) increases, the influence of the system false detection rate \( q \) on the successful pass detection decreases. In extreme cases, when the total number of hops is infinite, even if the false positive rate is very high, the system can finally reach the required threshold. For the same \( K \) and \( L = 5 \) and \( L = 3 \), the probability of successful detection is small. Obviously, continuous detection of 3 hops is simpler than continuous detection of 5 hops. At the same time, the simulation results are in line with the normal logical judgment, which fully proves the rationality of this mathematical model. The following further analysis will continue to adopt the mathematical model.

3. Strategy Analysis of the Frequency Capture Confirmation Process

3.1. Indicator Selection

After completing the mathematical modelling of the detection process, it is necessary to consider how to choose a better detection scheme which is to select the optimal combination of \( K \) and \( L \) values. Two issues should be considered here the first is to select which evaluation index, the second is how to use an algorithm to integrate these evaluation indexes.

Selection of frequency hopping evaluation indicators to be evaluated: the detection time \( t \), the capture probability \( p_{bh} \), the normalized average length \( B \) and the credibility index \( T \). The parameters used in the calculation of the above indicators include: the hopping speed \( v \), the signature length \( m \), the signature threshold \( n \), the error rate of baseband signal \( p_b \), the total detected hop count \( K \) and the detection threshold \( L \).

The concrete calculating idea is:

Detection time \( t \) is the total time required for the frequency hopping capture confirmation process:

\[ t = v \cdot K \] \hspace{1cm} (5)

The acquisition probability \( p_{bh} \) is the probability that the capture confirmation process can be successfully completed after the detection time \( t \), and is calculated by the Markov chain state transition matrix under different detection conditions.

The normalized average length \( B \) refers to the average step size from the initial state to the captured state in the Markov chain:
The credibility index $T$ represents the reliability of the results of the program to be evaluated:

$$T = \frac{n}{16 \frac{K_{\text{max}}}{K}} = \frac{nK^2}{16K_{\text{max}}(K - L)}$$  \hspace{1cm} (7)

### 3.2. Frequency Acquisition Confirmation Scheme using TOPSIS Method

The TOPSIS method is a more popular multi-index evaluation method. This method will favor ideal solution and negative ideal solution of all evaluation indexes, called the optimal solution and the worst solution. Then it will calculate the relative similarity degree of each method away from the ideal method that is the degree of each solution closed to the optimal solution and away from the worst solution. Finally, we'll get a sort of all solutions, and choose the best one.

The concrete algorithm steps of the TOPSIS method are shown below [13]:

- The vector decision method is used to find the canonical decision matrix, and the decision matrix for setting the multi-attribute decision problem is $A = (a_{ij})_{n \times m}$, where $a_{ij}$ is the attribute value of each scheme, and the normalized decision matrix is $B = (b_{ij})_{n \times m}$, where $b_{ij} = a_{ij} / \sqrt{\sum_{i=1}^{m} a_{ij}^2}$.

- Constitute a weighted normative matrix $C = (c_{ij})_{n \times m}$. The decision maker set the weight vector for each attribute $w = [w_1, w_2, w_3, K, K, w_n]^T$. And $c_{ij} = w_j b_{ij}$, $i = 1, 2, 3, K, m$, $j = 1, 2, 3, L, n$.

- Determine the positive ideal solution $c^*$ and the negative ideal solution $c^0$.

- Calculate the Euclidean distance of each solution between it and the positive ideal solution and negative ideal solution.

$$s_i^* = \sqrt{\sum_{j=1}^{m} (c_{ij} - c_{ij^*})^2} \quad i = 1, 2, 3, K, m$$  \hspace{1cm} (8)

$$s_i^0 = \sqrt{\sum_{j=1}^{m} (c_{ij} - c_{ij^0})^2} \quad i = 1, 2, 3, K, m$$  \hspace{1cm} (9)

- Calculate the queued index value of each method (called the comprehensive evaluation index)

$$f_i^* = \frac{s_i^0}{s_i^* + s_i^0} \quad i = 1, 2, 3, K, m$$  \hspace{1cm} (10)

- Arrange $f_i^*$ from large to small.

The following table 1 selects 10 groups of schemes and different combinations of K and L values, representing different hopping frequency synchronization confirmation schemes.

| Scheme Number | Signature Threshold | Total Hop Count | Detection Threshold |
|---------------|---------------------|-----------------|---------------------|
| SCH-1         | 10                  | 10              | 8                   |
According to the frequency hopping parameters of the 10 groups of schemes, 10 sets of indicators to be evaluated are calculated by the index calculation formula, and then normalized, as shown in Table 2:

| Scheme | Detection Time | Capture Probability | Normalized Average Length | Credibility Index |
|--------|----------------|---------------------|---------------------------|-------------------|
| SCH-1  | 0.4834         | 0.3437              | 0.5075                    | 0.5118            |
| SCH-2  | 0.3867         | 0.3400              | 0.3768                    | 0.3276            |
| SCH-3  | 0.3867         | 0.3447              | 0.2511                    | 0.1638            |
| SCH-4  | 0.3384         | 0.3402              | 0.3805                    | 0.5517            |
| SCH-5  | 0.3384         | 0.3436              | 0.2528                    | 0.1839            |
| SCH-6  | 0.2900         | 0.3384              | 0.2618                    | 0.2211            |
| SCH-7  | 0.2417         | 0.3386              | 0.1947                    | 0.1536            |
| SCH-8  | 0.1933         | 0.3330              | 0.1947                    | 0.1965            |
| SCH-9  | 0.1933         | 0.2055              | 0.3076                    | 0.2293            |
| SCH-10 | 0.1450         | 0.1694              | 0.3076                    | 0.3194            |

In MATLAB, the above data is brought into the TOPSIS algorithm for comprehensive evaluation to obtain the ranking results of the 10 groups of programs. The best to worst solutions are from left to right:

| Scheme Number | SCH-4 | SCH-8 | SCH-7 | SCH-6 | SCH-2 | SCH-7 | SCH-5 | SCH-10 | SCH-3 |
|---------------|-------|-------|-------|-------|-------|-------|-------|--------|-------|
| Weight        | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8      | 9     |

Take the top three schemes and re-list:

| Scheme Number | Signature Threshold | Total Hop Count | Detection Threshold |
|---------------|---------------------|-----------------|---------------------|
| SCH-4         | 11                  | 7               | 6                   |
| SCH-8         | 12                  | 4               | 3                   |
| SCH-7         | 13                  | 5               | 3                   |

According to the analysis, from the perspective of detection credibility, although the No. 4 scheme has lower requirements on the signature threshold than the No. 8 and No. 7 schemes, the requirements for detecting the hop count are more demanding, and it is necessary to continuously detect 6 hops in 7 hops in order to successfully capture the confirmation process. The No. 8 and No. 7 schemes increase the signature threshold, but reduce the total number of detected hops. It can be seen that the three preferred schemes are more focused on the credibility of the capture confirmation process, except that the No. 4 scheme focuses more on the credibility of the multi-hop combination test results, while the No. 8 and No. 7 schemes are more focused on 1 hop detection credibility (1 hop corresponds to a complete signature). At the same time, the total number of detected hops of these three preferred schemes is the difference.
schemes is basically at the intermediate level of 10 schemes, thus ensuring a fast capture confirmation. Therefore, the earlier ranking scheme considers the two factors of the length and reliability required for simultaneous confirmation.

Then take out the scheme in the last few places and re-list:

| Scheme Number | Signature Number | Total Hop Count | Detection Threshold |
|---------------|------------------|-----------------|---------------------|
| SCH-10        | 14               | 3               | 3                   |
| SCH-3         | 10               | 8               | 4                   |
| SCH-9         | 14               | 4               | 3                   |

It is analyzed that the requirements of the 10th and 9th schemes are very demanding, and the signature threshold is set to 14 hops in the 16-hop signature, and 3 hops must be detected in the detection of 3 hops and 4 hops respectively to pass the capture confirmation process. Such a strict setting will result in a greatly reduced probability that the receiving end will successfully capture the confirmation. The system will not be able to pass the confirmation process and then need to re-enter the initial capture phase, which will take longer synchronization time, so the solution ranks lower. And No. 3 The scheme is at the other extreme, the low requirements result in a much lower confidence in the validation process.

From the brief analysis of the evaluation results of the scheme, it can be clearly seen that in the frequency hopping synchronization acquisition confirmation process, the optimal scheme should ensure the high reliability of each hop detection result, and at the same time reduce the total detection hop count as much as possible. The strategy for ensuring the frequency hopping frequency capture confirmation process takes into account the reliability and synchronization duration.

4. Conclusion
In this paper, we use the Markov chain to mathematically model the capture confirmation process, then select 4 sets of indicators and use the TOPSIS algorithm for comprehensive evaluation of 10 sets of programs. It is then pointed out that in a particular frequency hopping communication system, time and credibility as evaluation indicator are more important than other indicators. Therefore, with the application of frequency hopping technology in military communications, mobile communications, and personal communication systems, this technology should receive more attention.

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