New scenario of high-energy particle collisions near static
wormholes

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We suggest a new scenario of high energy particle collisions in the background of
a static wormhole. Particle 1 falls from the right infinity, decays to particles 2 and 3.
Particle 3 escapes to the left infinity, while particle 2 bounces back and collides with
particle 4 moving from the right infinity. If a wormhole is on the verge of forming
the horizon (but the horizon does not form), the energy in the center of mass frame
becomes unbounded. Thus one can probe the other side of a wormhole remaining
completely on our side of it.

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I. INTRODUCTION

During last decade, a lot of efforts was devoted to description of high energy collisions
in the region of the strong gravitation field. This was stimulated by the observation about
possibility to obtain an indefinitely large energy $E_{\text{c.m.}}$ in the centre of mass frame of two
colliding particles [1] (see also earlier works [2] - [4]). The corresponding observations were
made for rotating black holes. Meanwhile, later, similar results were obtained for another
strongly gravitating objects. Thus, the unbounded energies $E_{\text{c.m.}}$ were found for processes
near naked singularities and wormholes. In the present article, it is the latter case which we
are interested in.

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For the first time, the corresponding observations were made in \[5\] for a particular type of wormholes, so-called Teo wormholes \[6\]. They are necessarily rotating, the corresponding space-time does not have asymptotically flat region. Later, it was observed \[7\] that high energy collisions can be realized even for static wormholes (for example, if two Schwarzschild-like wormholes are glued by means of "cut and past" technique, see e.g. Sec. 15.2.1 of \[8\]). In doing so, two particles come from opposite mouths and meet near the throat. In the present work, we suggest an alternative scenario. We show that an indefinitely large energy \(E_{\text{c.m.}}\) can occur even if both particles are sent from the same side of the throat. However, this requires two-step process.

All scenarios connected with using wormholes for obtaining unbounded \(E_{\text{c.m.}}\) share the same features. The lapse function near the throat should be small. This leads to indefinite growth of the curvature invariants (say, the Kretschmann scalar \(K\)) there. Meanwhile, one can reconcile large \(E_{\text{c.m.}}\) and \(K\) remaining below the Planckian scale by choosing the parameters of the system accordingly \[9\].

We use the geometric system of units in which fundamental constants \(G = c = 1\).

II. BASIC FORMULAS

Let us consider the spherically symmetric metric

\[
d s^2 = -f d\tau^2 + \frac{d\rho^2}{f} + r^2(\rho)d\omega^2, \quad \quad d\omega^2 = d\theta^2 + \sin^2 \theta d\phi^2, \quad (1)
\]

where we used a so-called quasiglobal coordinate \(\rho\) (see, e.g. Sec. 3.3.2 of \[10\]). Motion of free particles occurs in the plane which we choose to be the equatorial one \(\theta = \frac{\pi}{2}\). Equations of motion read

\[
 m\dot{\tau} = \frac{E}{f}, \quad (2)
\]

\[
 m\dot{\rho} = \sigma P, \quad (3)
\]

\[
 m\dot{\phi} = \frac{L}{r^2}, \quad (4)
\]

where dot denotes differentiation with respect to the proper time \(\tau\), \(E\) being the conserved energy, \(L\) conserved angular momentum, \(\sigma = \pm 1\) depending on the direction of motion,

\[
P = \sqrt{E^2 - N^2 \tilde{m}^2}, \quad (5)
\]
\[ \tilde{m}^2 = m^2 + \frac{L^2}{r^2}. \] (6)

The forward-in-time condition \( \dot{t} > 0 \) is satisfied, provided \( E > 0 \).

If two particles 1 and 2 collide, one can define the energy in the center of mass frame according to

\[ E_{c.m.}^2 = -(m_1 u_{1\mu} + m_2 u_{2\mu})(m_1 u_1^\mu + m_2 u_2^\mu) = m_1^2 + m_2^2 + 2m_1m_2\gamma. \] (7)

Here, \( u^\mu \) is the four-velocity, subscript label particles, \( \gamma = -u_{1\mu}u_2^\mu \) is the Lorentz factor of relative motion. Using (2) - (4) one obtains

\[ m_1m_2\gamma = \frac{E_1E_2 - \sigma_1\sigma_2P_1P_2}{f} - \frac{L_1L_2}{r^2}. \] (8)

In what follows we consider the manifold to be a wormhole. For simplicity, we assume that the function \( r(\rho) \) has one minimum at \( \rho = \rho_0 \), so

\[ r \geq r_0 \equiv r(\rho_0). \] (9)

We restrict ourselves by pure radial motion \( L = 0 \) since this simplified case captures the main features of the phenomenon under discussion. Then,

\[ \dot{\rho} = \sigma p, \] (10)

\[ p = \sqrt{\varepsilon^2 - f}, \] (11)

where \( \varepsilon = \frac{E}{m} \),

\[ \gamma = \frac{\varepsilon_1\varepsilon_2 - \sigma_1\sigma_2p_1p_2}{f}. \] (12)

### III. SCENARIO

Let us consider the following scenario. Particle 1 has the energy \( E_1 > m \) and starts it motion, say, from the right infinity. In some point it decays to two particles 2 and 3. We assume that particle 2 has the energy \( E_2 < m \), whereas particle 3 has \( E_3 > m, \sigma_3 = -1 \). Then, particle 3 escapes to the left infinity. Meanwhile, particle 2 has the turning point \( r_2 \), where \( p_2 = 0 \), its position is given by

\[ f(r_2) = \varepsilon_2^2. \] (13)
We assume that $f$ is a monotonic function of $r$ in each half-space, so there is one value of $r_2$ but there are two turning points in terms of $\rho$ in which $r(\rho) = r_2$. It is also clear that $f$ attains its minimum $f_0$ at point $\rho_0$, $f_0 = f(r(\rho_0))$.

Particle 2 oscillates between both turning points. Let it collide in point $\rho_0$ with one more particle 4 having (for simplicity) the same mass that comes from infinity, $\epsilon_4 > 1$, $\sigma_4 = -1$. We choose the moment of collision in such a way that particle 2 moves from the left to the right, so $\sigma_2 = +1$. From (12), we have

$$\gamma = \frac{\epsilon_4 \epsilon_2 + p_4(\rho_0)p_2(\rho_0)}{f}. \quad (14)$$

Now, we consider configurations with small $f_0 \ll \epsilon_2 < \epsilon_4$. Then, $p_2(\rho_0) \approx \epsilon_2$, $p_4(\rho_0) \approx \epsilon_4$,

$$\gamma \approx \frac{2\epsilon_4 \epsilon_2}{f}. \quad (15)$$

When $f \to 0$, $\gamma$ grows unbounded, and so does $E_{c.m.}$

**IV. DISCUSSION**

There are few scenarios of high energy particle collisions in which unbounded $E_{c.m.}$ is obtained in head-on collisions. The key point of such scenarios is to obtain somehow a particle that moves in the opposite direction (with respect to another particle that falls from infinity) and arrange collision in the point where the lapse function is very small. This can be realized (i) near white holes [11], (ii) in the background of a naked singularity [12], (iii) in the background of a wormhole. In case (ii) there is a two-step scenario in which a particle bounces back from an indefinitely high potential barrier and meets a new particle coming from infinity. In case (iii), there are two options. One of them (iii-a) consists in that two particles comes from opposite mouths [7]. Meanwhile, in our scenario (iii-b) all particles participating in the process, start in our universe.

Thus in our scenario we can probe the other side of a wormhole starting the experiment on our side of it and remaining only there.

In the present work we considered the process in which unbounded $E_{c.m.}$ is achieved. A separate question remains, to what extent our type of scenario allows a wormhole to serve as a source of ultra-energetic particles with high energy $E$ at infinity.
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