Modal and frequency response characteristics of vehicle suspension system using full car model

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Abstract. This paper involves simulated studies of a vehicle suspension system for modal and frequency response characteristics. Ride comfort and car handling are two important considerations for finalizing the design of the vehicle suspension. However, both of these are conflicting to each other. Ride comfort and car handling depend on dynamic characteristics of the vehicle (i.e. natural frequencies, mode shapes, frequency response functions etc.) too among other parameters. Hence it is very important to carry out modal examination of the vehicle suspension system to find out the natural characteristics of the system. The same has been attempted in this work using full car model having seven degree of freedoms. The degrees of freedom include pitching, bouncing and rolling motion along with motion of the un-sprung masses. The software tools namely ANSYS Workbench, MATLAB, and SIMULINK environment has been employed to discover the system’s modal and frequency response features. The vehicle’s natural frequencies and mode shapes were evaluated and bode plots are drawn to explore the system’s frequency response features. The results produced in different software tools have also been compared and discovered to be in close agreement with the results present in the literature.

Keywords. Natural Frequency, Modal Analysis, Frequency Response, Ride Comfort.

1. Introduction
A Suspension system is a complicated system of vibration, which can be conveniently modelled with various degrees of freedom.
Suspension system is intended to separate the car body from the inputs of the highway. Suspension system serves a dual objective - to contribute to the road holding and handling of the vehicle.
It is important from the point of view of vehicle stability during braking, driving pleasure and comfort, and isolation from road bumps, vibration and noise, etc.
Today, there are many challenges to automobile companies in the cutting edge competition to survive and excel in the market. The competition among automobile companies has forced them to seek better alternative strategies in suspension systems through research studies. One of the performance demands is sophisticated suspension systems aimed at improving ride comfort and handling while improving riding skills and smooth driving performance. The suspension system aims at ensuring a smooth ride
and maintaining car control over rough roads or in the event of sudden brakes. Larger suspension travel and lower damping in wheel bounce mode result in improved riding convenience [1]. Suspensions systems may be classified on various basis. On the basis of control, it is characterized into three types, namely (i) Passive suspension systems, (ii) Semi active suspension system, and (iii) Active suspension system. Most of the cars are hang in passive way, where the suspension system includes basically springs and dampers. In passive system, the component features, i.e. the spring constant values and the damping coefficients, are fixed. The disadvantage of the passive suspensions system is that their performance remains effective only over a certain frequency range. In the early 1970s, semi-active suspensions were suggested and can be almost as efficient as full-active suspension to enhance the quality of car behavior. A semi-active system is a combination of passive spring element and a controllable damper element. It enables automatic adjustment/control of the damping coefficient based on control strategy. The controllable damper works using external energy by means of an integrated controller and a set of sensors. The semi-active suspension may continue to function under a passive condition if the control system fails. An active suspension system enables variation/adjustment of damper element as well as spring element. Passive devices that can only dissipate energy, while the force actuator in active systems can add and dissipate energy from the system. Unlike the passive suspension, the active suspension can enhance the efficiency of the suspension mechanism over a wide spectrum of frequencies. Aly and Salem [2] addressed the use of smart technology to regulate a continually variable suspension system for automotive damping. An active suspension system was suggested in their research to enhance riding convenience. Many studies have been carried out on suspensions system models to make it function efficiently by optimizing its parameters.

For many years, active and semi-active suspensions were used to enhance the stability and handling of all vehicle kinds. Bose, Daimler Chrysler, Land Rover, and Delphi have effectively used these kinds of devices as factory or aftermarket application. Controlling or helping to stabilize the roll, pitch, and yaw of a car through the suspension would be particularly useful for high-gravity vehicles such as SUVs, trucks, and ATV.

Griffin [3] has discussed the human response to vibration excitation discussing about both whole body vibration and hand-transmitted vibration using analytical as well as experimental studies while considering various aspects like psychology, physiology etc. The author reports that the human whole-body fundamental Natural frequency ranges from 3 Hz to 7 Hz.

Jazar RN [4] has categorized passenger cars in his book ‘Vehicle Dynamics Theory and Application,’ a vehicle intended to carry ten or fewer people depending on their size and weight. The author claims that for 205/50R15 passenger car tire the value of $k = 200000$ N / m is a good approximation, and for X31580R22.5 truck tire $k = 1200000$ N / m is a good approximation. Gillespie [5] in his book ‘fundamental of vehicle dynamics’ explained the stiffness ratio, $r_t$ (ratio of tire stiffness to suspension stiffness) which is the main criterion for deciding the car suspension stiffness. The stiffest suspension, $r_t =5$, would be indicative of sports vehicles and performance vehicles whereas the softest, $r_t =20$, would be relate to luxury vehicles with air suspensions. Figure 1 shows the stiffness ratios for most commercial vehicles. However, in practice it is not possible to use the entire range of performance shown in the figure 1 shaded region. The low damping concentrations lead to suspension strokes that are beyond the range available for most passenger vehicles, and low damping is also inadequate to regulate wheel hop oscillations that compromise the behaviour of road holding.
Gillespie also described that a soft suspension helps in ride isolation as the sprung mass acceleration was found to be minimum for lower values of natural frequencies. In the case considered, the sprung mass acceleration was found to be minimum at 1 Hz natural frequency. However, other practical considerations restrict the natural frequencies in the range from 1 to 1.5 Hz for most vehicles. The natural frequency in performance cars, where vehicle handling is more important than ride comfort, is between 2 to 2.5 Hz. Correct estimation of such frequencies is very important. Hence, it becomes necessary to model the vehicle suspension system accurately. This paper, therefore, discusses modelling of the vehicle suspension system through developing own code (MATLAB in this work) and using ready software tools (in this work ANSYS workbench environment). Bode plots are also drawn with the SIMULINK model to study frequency response of the system. The results produced in this way have also been compared and found to be in close agreement with the result present in the literature.

2. Modelling of the suspension system

Figure 2 shows the complete car model considered in this work. Vehicle wheels and Suspension System are regarded as spring-Damper element. $M_{U1}, M_{U2}, M_{U3}, M_{U4}$ denotes the car’s un-sprung masses while $M_S$ denotes the car’s sprung mass. $Z_{U1}, Z_{U2}, Z_{U3}$, and $Z_{U4}$ denote the un-sprung masses vertical displacement while $Z_S$ denotes the sprung mass vertical displacement. $\phi$ and $\psi$ denote the degree of freedom corresponding to pitching and rolling motion respectively. $K_{S1}, K_{S2}, K_{S3}, K_{S4}$ and $C_{S1}, C_{S2}, C_{S3}, C_{S4}$ indicates the Suspension System Spring and damping variables, Similarly $K_{W1}, K_{W2}, K_{W3}, K_{W4}$ and $C_{W1}, C_{W2}, C_{W3}, C_{W4}$ are the Spring and damping variables of four tires of car. a and b indicates distance of front and back tires from sprung mass center of gravity (CG), whereas c and d indicates distance of right and left tire from the Sprung mass CG. The seven coordinates which describe the vibrating system are $Z_S, \phi, \psi, Z_{U1}, Z_{U2}, Z_{U3}$, and $Z_{U4}$.

In this work the un-damped natural frequencies of full car model are analysed. Hence, the damping parameters of front suspension ($C_{S1}, C_{S2}$), rear suspension ($C_{S3}, C_{S4}$), and four wheels ($C_{W1}, C_{W2}, C_{W3}, C_{W4}$) of a vehicle are taken as zero.
The values of various vehicle parameters for numerical simulations are considered, as shown in Table 1 which are close to the practical passenger car.

**Table 1. Vehicle Parameters.**

| S. No | Model Parameter     | Value   | Unit  |
|-------|---------------------|---------|-------|
| 1     | \( M_S \)           | 1243.4  | kg    |
| 2     | \( M_{U1},M_{U2},M_{U3},M_{U4} \) | 62.8    | kg    |
| 3     | \( K_{S1},K_{S2} \)  | 17000   | N/m   |
| 4     | \( K_{S3},K_{S4} \)  | 20000   | N/m   |
| 5     | \( C_{S1},C_{S2},C_{S3},C_{S4} \) | 0       | N-s/m |
| 6     | \( K_{W1},K_{W2},K_{W3},K_{W4} \) | 200000  | N/m   |
| 7     | \( C_{W1},C_{W2},C_{W3},C_{W4} \) | 0       | N-s/m |
| 8     | \( a, b \)          | 1.21, 1.59 | m     |
| 9     | \( c = d \)         | 0.8     | m     |
| 10    | \( I_{XX} \)        | 940.85  | Kg-m² |
| 11    | \( I_{YY} \)        | 265.57  | Kg-m² |

2.1. **Equations of motion**

The motion equations for a full car model of seven degrees of freedom, as shown in Figure 2, were extracted and written in the following matrix as follows:

Where \( M \) is mass Matrix and \( K \) is stiffness matrix.

\[
M \ddot{Z} + KZ = 0
\]
2.2. Ansys Workbench model

The block diagram based approach in “Ansys Workbench” software have been used for modal analysis of the Suspension System to extract natural characteristics. The use of line body elements (spring) enables accurate modelling of the system with lesser mobility as compare to the use solid elements. The sprung mass has three mobility: first, vertical translation in Z direction (bouncing): Second, rotation about X direction (pitching): third, rotation about Y direction (rolling). Whereas, each un-sprung mass is provided with one mobility i.e. the translation in Z direction. It may be noted that the degrees of freedom, which are not considered, have been constrained in the software tool. The fully developed Ansys workbench model is shown in figure 3. Similar analysis has also been carried out by developing a code in the MATLAB environment and the results have been listed in the next section.
2.3. MATLAB code for modal analysis

Modal assessment is used to evaluate the natural frequencies, damping variable and mode shapes of the system [6, 7]. A MATLAB code has also been created for the seven degrees of freedom model. The code can be used to compute the Eigen-values and Eigen-vectors of the suspension systems. Eigenvalues give information about natural frequencies and damping in various modes, whereas eigenvectors provide information about the mode shapes. The “eig” command has been used for this purpose [8].

\[ [V, D] = \text{eig} (K, M) \]

generates a diagonal matrix \( D \) of Eigenvalues and a complete matrix \( V \) with the corresponding Eigen vectors in columns such that \( K*V = M*V*D \).

The results as obtained through ANSYS Workbench and MATLAB are also correlated and validated in the next section.

3. Result and discussion

3.1. Natural Frequencies

The natural frequencies obtained in ANSYS Workbench and using MATLAB code are shown in Table 2 along with percentage difference. It may be noted that the results as computed through the two different approaches are in close agreement with each other as well as the results presented in the literature and their percentage error is almost negligible. This also establishes the validation of the work. Hence, any of the two approaches can be reliable employed for the modal analysis of the system.

| Mode | Frequency (Hz) | Difference ( % ) |
|------|----------------|------------------|
|      | Ansys workbench | Matlab           |
| 1    | 1.1349          | 1.1349           | 0.0              |
| 2    | 1.9448          | 1.9449           | 0.005            |
| 3    | 2.0278          | 2.0283           | 0.024            |
| 4    | 9.3631          | 9.3631           | 0.0              |
| 5    | 9.3639          | 9.3639           | 0.0              |
| 6    | 9.4340          | 9.4382           | 0.0              |

3.2. Mode shapes

The analysis of the mode shapes figure 4, shows the relative displacement of the seven vibrating coordinates \( (Z_s, \Theta, \tilde{\Theta}, Z_{u1}, Z_{u2}, Z_{u3}, \text{and } Z_{u4}) \) at different natural frequencies, it can be clearly observed that first mode corresponds to bouncing motion, second mode corresponds to pitching motion and third mode corresponds to rolling motion of the car. The bounce center, pitch center, and the roll center of vehicle affects vehicle handling and ride comfort. Further, the mode shapes (eigenvectors) may also be used to compute the pitch, bounce and roll centers of the car. It can be seen that the un-sprung masses \( M_{u1} \) and \( M_{u2} \) move in opposite direction in the fourth mode whereas these mass move in same direction in the fifth mode. Similarly, \( M_{u3} \) and \( M_{u4} \) move in opposite direction in the sixth mode and in the same direction in the seventh mode.
3.3. Frequency response (Bode diagram)
Figure 5 shows the SIMULINK model for the complete vehicle suspension system of 7-dof. Bode plots are used to determine system response at different frequencies. The Simulink model is employed to study the Bode plots for the system. Figure 6 shows the Bode plot for the system. The peaks, at the natural frequencies as listed in Table 2, are clearly visible in the plot. The acceleration values at these peaks is also mentioned in the plot. The plot shows distinct peaks at the first three natural frequencies corresponding to bouncing, pitching and rolling motion. Next four modes are close modes and are not clearly distinguishable in the bode plot. These are the modes corresponding to the un-sprung masses.

Figure 5. SIMULINK diagram for seven degree of freedom Full Car model.
4. Conclusion
The work presented studies in modal and frequency response features of a vehicle using seven degrees of freedom full car model. The results have been computed using two different approaches, i.e. first writing a code and second using a software tool. The results obtained by the two approaches are found to be in close agreement. Hence, it is concluded that any of the two approaches can be reliably employed for the modal analysis of the system. System’s Mode shapes are also studied in the seven modes. Such mode shapes may be helpful in locating the pitching, bouncing and rolling centers of the vehicle. Bode plot has been studied for the study to find out the force vibration characteristics of the system. The peaks are observed at the natural frequencies of the vehicle in the plot.

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