The low-energy constants of the $\pi N$ system

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Abstract

Recent analyses of low-energy $\pi N$ experimental data have provided clear evidence for the violation of the isospin symmetry of the strong interaction. In the present work, it is shown that the single-charge-exchange reaction ($\pi^- p \rightarrow \pi^0 n$) is the culprit for the effect. Given the present experimental uncertainties, no evidence for isospin breaking was found in the two elastic-scattering processes for pion laboratory kinetic energies between 20 and 100 $MeV$. In agreement with most of the recent determinations, the value for the charged-pion coupling constant $f_{\pi^\pm pn}$, extracted herein, is ‘small’. The energy dependence of the $s$- and $p$-wave hadronic phase shifts, obtained from low-energy elastic-scattering data exclusively, as well as their values in tabular form (including meaningful uncertainties), are provided. Discrepancies with two ‘standard’ phase-shift solutions in the $s$-wave part of the interaction are seen; small differences may be observed in three of the $p$-wave channels.

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1 Introduction

All pion-nucleon ($\pi N$) measurements (differential cross sections, partial-total cross sections, total nuclear cross sections, and analyzing powers) in the three low-energy experimentally-accessible channels \(^1\) between (pion laboratory kinetic energies of) 20 and 100 $MeV$ were recently analyzed in the framework of a relativistic isospin-symmetric model \(^1\); due to the existence of discrepant measurements (outliers) in the input data base, robust statistics was implemented in the problem. Provided the correctness of the bulk of the experimental data and of the electromagnetic corrections applied to the scattering problem, it was concluded (in Ref. \(^1\)) that the isospin symmetry of the strong interaction is violated in the $\pi N$ system at low energies. The reproduction of the input data was subsequently investigated \(^2\); it was shown that, with the exclusion of about 14 % of the elastic-scattering measurements (mostly pertaining to the $\pi^+ p$ reaction) and the additional renormalization of seven (out of 40) $\pi^+ p$ data sets, the input data base becomes internally consistent.

The present article completes the research program set forward in Ref. \(^1\); its objectives may be summarized as follows:

a) After isospin-symmetry violation in the $\pi N$ system (at low energies) has been established, the interest should lie with the question of which reaction is responsible for the effect; the identification of the culprit is expected to lead to clearer ideas as to which physical processes are involved in the breaking.

b) The $\pi N$ model, used in Ref. \(^1\), constitutes the means for the extraction of new hadronic phase shifts (or amplitudes) from the experimental data on the exclusive basis of the low-energy information. Among others, this is essential since the dynamical structure of the $\pi N$ interaction might be energy-dependent; put in different words, it might be that hadronic symmetries, obeyed (by the $\pi N$ system) at high energies (above the $\Delta_{33}$ resonance), are not valid close to the $\pi N$ threshold (zero kinetic energy of the incident pion). Such an effect could remove some of the acclaimed ‘discrepancies’ between the theoretical predictions, obtained via dispersion relations mainly on the basis of high-energy data, and the low-energy measurements. Furthermore, reliable low-energy information (in the form of hadronic phase shifts) is expected to be of great interest in calculations conducted within the framework of the Chiral-Perturbation Theory.

c) The model, used herein, has already been established as a firm basis for the extraction of low-energy hadronic constants of the $\pi N$ system. In turn, such an extraction is essential because: i) The study of the $\pi N$ system yields information on constants which play a fundamental role in other research fields, e.g., in the $NN$ sector, where the $\pi N$ interaction comprises the microscopic input; the coupling constants, corresponding to the $\pi NN$ and $\pi N\Delta$ vertices, belong to this category. ii) In view of the conclusions of Ref. \(^1\), the discussion about some of the low-energy

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\(^1\)The low-energy $\pi N$ channels, which are susceptible to experimentation (at present), are the two elastic-scattering processes ($\pi^\pm p$) and the single-charge-exchange (SCX) reaction ($\pi^- p \rightarrow \pi^0 n$).
hadronic constants might only be meaningful if the corresponding input data base is specified. For instance, instead of talking about one coupling constant $g_{\pi NN}$, one might have to distinguish between $g_{\pi^+pn}$ (deduced from $\pi^+p$ data), $g_{\pi^-pn}$ (deduced from $\pi^-p$ data), and $g_{\pi^0pp}$ or $g_{\pi^0nn}$ (involved in the SCX reaction). Therefore, it comes as a natural consequence of the findings of Ref. [1] to investigate which of the model parameters are affected by the effect reported therein. iii) A legitimate question relates to the physical processes involved in the observed isospin-breaking effect: their identification might be enabled with the proper comparison of the various parameter values obtained during the fitting procedure.

In the context of the present work, two kinds of possible differences in the values of the model parameters are relevant. i) Differences between the results obtained in the single-channel analyses of the measurements in the two elastic-scattering reactions. In the following, these differences will constitute ‘type I’ effects; they are expected to create isospin breaking in the two elastic-scattering processes. ii) Differences between the results of the combined fits to the elastic-scattering measurements and the ones obtained from the fits to the SCX data. In the following, these differences are to be referred to as ‘type II’ effects.

It should be reminded that the model parameters (determined from the various fits to the low-energy $\pi N$ measurements) are:

- a) $G_\sigma$ and $\kappa_\sigma$ for the scalar-isoscalar $t$-channel exchange,
- b) $G_\rho$ and $\kappa_\rho$ for the vector-isovector $t$-channel exchange,
- c) $g_{\pi NN}$ and $x$, standing for the $\pi NN$ coupling constant and the pseudoscalar admixture in the $\pi NN$ vertex, respectively, and
- d) $g_{\pi N\Delta}$ and $Z$, the former denoting the $\pi N\Delta$ coupling constant, the latter being associated with the spin-$1/2$ admixture in the $\Delta$-isobar field.

It was observed in Ref. [1] that the analysis of low-energy $\pi N$ data cannot lead to the determination of the parameter $G_\rho$. For an unbiased analysis, this parameter was fixed at seven (equidistant) values between 30 and 60 GeV$^{-2}$; the interval chosen corresponds to the extreme cases found in the literature. All details about the $\pi N$ interaction model, used herein, may be found in Refs. [1] and [3]. The method of the analysis has been thoroughly described in Ref. [1]. With one exception (to be cited in Section 3), the input data base has been listed in Ref. [1].

# 2 The two elastic-scattering reactions

## 2.1 On isospin-symmetry violation

Firstly, type I effects were investigated. The following steps were carried out:
I. Combined fits to all low-energy elastic-scattering data were performed. From

\[\text{In Ref. [1], the measurements in the three low-energy reaction channels were analyzed separately leading to three classes of parameter values. A combined fit to the elastic-scattering data yielded a fourth class.}\]
these fits, Solution A (for the seven model parameters) was obtained. A strong
dependence of the parameters $G_\sigma$, $\kappa_\rho$, and $x$ on $G_\rho$ was observed in agreement with
the findings of Ref. [3].

II. Subsequently, separate fits to the data in the two elastic-scattering reactions were
to be performed. However, it is known (from Ref. [1]) that a seven-parameter (ex-
clusive) fit to the $\pi^+p$ measurements cannot be carried out because of the problem
of the large correlations among the model parameters. Therefore, it was decided
to fix some of the model parameters at the corresponding values of Solution A.
Evidently, a question arises: Which of the model parameters are less likely to be
influenced by a potential violation of isospin symmetry in the elastic scattering,
and, therefore, may be fixed? A clue may be obtained by recalling the fact that
the $\pi N$ interaction is so weak at low energies (as a consequence of the underlying
approximate chiral symmetry) that the tree-level version of the model provides a
satisfactory description of the $\pi N$ dynamics up to about 40 MeV (see Ref. [3]). At
the tree level, no isospin-breaking effect is expected in the scalar-isoscalar $t-$channel
graph; hence, $G_\sigma$ and $\kappa_\sigma$ may be fixed. Additionally, the coupling constant $g_{\pi NN}$,
obtained from $\pi^+p$ data, corresponds to the $\pi^+pn$ vertex, whereas the one, result-
ning from the $\pi^-p$ fits, relates to $\pi^-pn$; due to charge symmetry (which is a looser
constraint than isospin symmetry), these two coupling constants are expected to be
equal. An additional assumption may be that all the $\Delta-$isobar states involve the
same parameter $Z$. Therefore, it might be concluded that $\kappa_\rho$, $x$, and $g_{\pi N\Delta}$ could be
chosen as free parameters in these fits. If the analysis is carried out in this manner,
effects in $\kappa_\rho$ and $g_{\pi N\Delta}$ are observed (i.e., these two parameters come out different
in the two types of fits).

III. In order to establish the aforementioned effect (in $\kappa_\rho$ and $g_{\pi N\Delta}$), the influence
of the outliers has to be carefully examined. Although it is true that, in general, the
results from a robust fit are not sensitive to the treatment of the outliers, there exist
three reasons why such an investigation is necessary: a) Most of the discrepant data
are contained in the $\pi^+p$ reaction (which determines the isospin $-\frac{3}{2}$ amplitudes al-
most exclusively). b) The distribution of the normalized residual $z$ of the $\pi^+p$ data
is asymmetrical (see Fig. 3(a) in Ref. [1]). c) The outliers comprise a significant
amount of the whole $\pi^+p$ data base; this is an important observation, especially so
when combined with the fact that all these measurements lie systematically above
the bulk of the data (i.e., they populate the lower tail of the $z$ distribution). Due
to these reasons, an investigation of the stability of the solutions, obtained at step
II, with respect to the treatment of the discrepant data is pertinent. Repeating
the analysis after the data of Bertin et al. [4] and of Carter et al. [5] were removed
from the data base [1] showed that the aforementioned effect in $\kappa_\rho$ and $g_{\pi N\Delta}$ was
spurious; it was simply an artifact of the discrepant data. No statistically signif-
ificant effects in these two parameters were observed after the data of Refs. [4] and [5]
were excluded. In practice, this implies that both elastic-scattering processes may

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3The conclusion of Ref. [2] is that these data are way off any other low-energy $\pi^+p$ measurement.
be accounted for by a common set of parameter values. The solution, obtained after the two aforementioned data sets were excluded from the input data base, will be referred to as Solution B and will be exclusively used hereafter.

The first conclusion of this work is that, in the energy region, dealt with herein, and given the present experimental uncertainties, there is no evidence for isospin breaking in the two elastic-scattering reactions. This observation is consonant with Weinberg’s statement that isospin breaking in the πN system involves (at least) one neutral pion \(^6\).

At this point, one comment is prompt. The \(\rho^0 - \omega\) mixing mechanism \(^7\)-\(^8\) was proposed in the past as a potential means for isospin breaking in \(\pi N\) elastic scattering. Despite the fact that such effects have not been seen in the energy region of the present work, one should remark that (possible) deviations from isospin symmetry may be observed at places where the main parts of the interaction (i.e., the real parts of the \(s\)- and \(p\)-wave amplitudes) cancel each other; at these places, small effects are magnified. One of these regions was recently explored experimentally in Ref. \[^9\]. An analysis of more recent data, taken at the same kinematic region and accompanied by a significant improvement in the experimental systematic uncertainties, is expected to shed more light on this matter.

### 2.2 Low-energy information extracted from elastic-scattering data

Let us start with a discussion on each one of the model parameters. The results are concisely contained in Figs. 1 and 2 which show the \(G_\rho\) dependence of the model parameters fitted to the data in the optimization phase. The three values displayed (per \(G_\rho\) point) correspond to the cases where the (small) \(d\) and \(f\) waves: a) were fixed at the values of the most recent Karlsruhe solution (KA85) \[^10\], b) were fixed at the values of the VPI solution SM95 \[^1\], and c) were calculated with the \(\pi N\) interaction model \[^4\]. In all cases, the agreement among these three solutions is very satisfactory.

a) \(G_\sigma\) is a decreasing function of \(G_\rho\); it varies between (approximately) 41 and 27 \(\text{GeV}^{-2}\). Its large values signify the importance of the scalar-isoscalar interaction in the description of the low-energy \(\pi N\) dynamics.

b) \(\kappa_\sigma\) is found to be almost independent of \(G_\rho\); its average value is about two to three standard deviations away from zero. The smallness of this value is the reason why, prior to Ref. \[^1\] (i.e., prior to the introduction of the derivative coupling of the scalar-isoscalar to the pion field in the model), the \(\pi N\) data could also be accounted for by the model very successfully.

c) \(\kappa_\rho\) is a decreasing function of \(G_\rho\). In general, the values, obtained herein, are

\[^4\]It should be kept in mind that the \(s\)- and \(u\)-channel contributions of the \(d\) and \(f\) (baryon) resonances (to the \(K\)-matrix elements of the model) cannot be calculated due to the problem relating to the quantization of high-spin fields.
significantly smaller than the ones extracted with dispersion-relation techniques (which lie in the vicinity of 6, e.g., see Ref. [12]); the vector-meson-dominance prediction (3.7) is in agreement with the values of the present work, in the lower part of the interval corresponding to the $G_\rho$ variation. Brown and Machleidt [13] have argued that instead of comparing the $G_\rho$ and the $\kappa_\rho$ values in different analyses, one should rather quantify the overall strength of the $\rho$ coupling as

$$\left(g_{\rho NN}^{(V)}\right)^2 \frac{1+\kappa_\rho^2}{4\pi},$$

where $g_{\rho NN}^{(V)}$ is the vector coupling constant corresponding to the $\rho NN$ vertex. In this context, the value of the present work for the overall strength of the $\rho$ coupling (which is around 9) supports the weak-$\rho$ scenario; this result is in sharp contradiction to the conclusions of analyses achieved with dispersion-relation techniques.

d) $g_{\pi NN}$ is probably the hadronic constant with the longest history, a fact signifying its importance also outside the field of Pion Physics. A compilation of pre-meson-factory values may be found in Refs. [14]. The value of $g_{\pi NN}$, extracted in Ref. [15], was found to reproduce the $\pi N$ experimental data in the Karlsruhe-Helsinki (KH80) analysis [14] which was simultaneous to the first results from meson-factory experiments. The most recent literature values for $g_{\pi NN}$ are listed in Table 1 along with the values extracted in this work. One should remark that these values originate from analyses of $\pi N$ (Refs. [16], [17], [18], [19], and the present work), as well as $NN$ and $\overline{NN}$ data (the outcome of a long history of research from the Nijmegen group (e.g., see Refs. [20] and the articles cited therein), and the values of Refs. [21] and [22]). The result of Ref. [23] has not been included in the table because it was recently revised [24] and the new value is still considered to be preliminary [25]; this value is in agreement with Ref. [16]. To enable the easy comparison of the various items found in the literature, the values of the strong coupling constant are given in Table 1 in the form:

$$f_{\pi NN}^2 = \left(\frac{m_\pi}{2m}\right)^2 \frac{g_{\pi NN}^2}{4\pi},$$

where $m_\pi$ and $m$ denote the charged-pion and the proton mass, respectively. With the exception of the values of Refs. [16] and [22], general agreement is observed among the entries of Table 1.

e) $x$ is found to be $G_\rho$-dependent. In all cases, the pseudoscalar admixture in the $\pi NN$ vertex is small.

f) $g_{\pi N\Delta}$. The values of the present work are compatible with determinations based on the $\Delta$-isobar width (e.g., see Ref. [26]) and are totally inconsistent with the dispersion-relation result [27].

g) $Z$ has an average value of about $-0.39$. This value is compatible with the earlier determination of Ref. [3] and signifies the importance of the proper relativistic treatment of the $\Delta$-isobar.

Figures 3 and 4 show the energy dependence of the $s$- and $p$-wave hadronic phase shifts corresponding to the combined fits to the elastic-scattering data; the values
represent averages over the three options for the $d$ and $f$ waves (the sensitivity of the results on the choice of the $d$ and $f$ waves was checked and found to be small compared to the statistical uncertainties). To enable the straightforward application of the results of the present analysis, the $s$- and $p$-wave hadronic phase shifts in the low-energy domain are listed in Tables 2. The conclusions, drawn on the basis of Figs. 3 and 4 and Tables 2, may be summarized as follows:

a) The KA85 solution is incompatible with our results in the $s$-wave part of the interaction. Although, the SM95 solution does a better job, there is definitely some discrepancy (with our solution) in the $S_{31}$ phase shift (the disagreement being milder than in the case of the KA85). It has to be noted that the $S_{31}$ values, extracted in the present work, are in excellent agreement with the results of Ref. [28] which were exclusively based on the recent $\pi^+p$ differential-cross-section measurements.

b) Although the overall status in the $p$-wave part of the interaction seems to be satisfactory, a difference of about one degree between our results and the KA85 values was detected in the $P_{33}$ channel (close to the highest energy allowed in the present work); our solution is (once more) in excellent agreement with the values of Ref. [28]. All solutions agree well in the $P_{31}$ channel. A small, yet systematic, deviation from the SM95 $P_{13}$ phase-shift values was observed. Finally, there is a good agreement between our values in the $P_{11}$ channel and the SM95 solution, whereas, in this channel, the KA85 values are smaller by half a degree at (pion center-of-mass kinetic energy of) 60 MeV.

Finally, one should state that our results for the $p$-wave part of the interaction are in disagreement with the recent findings of Ref. [19]. The present article cannot support the statement of Ref. [19] that the hadronic phase shifts (and the corresponding scattering volumes) in the $P_{31}$ and $P_{13}$ channels are equal (see Figs. 3 and 4 and Tables 2). Additionally, we do not find a smaller value (e.g., than the value of Ref. [10]) for the scattering volume in the $P_{33}$ channel.

### 3 The SCX reaction

Based on the conclusions, drawn from the analysis of the measurements in the two elastic-scattering processes, one has to attribute the isospin-breaking effect, reported in Ref. [1], to the SCX reaction. Isospin breaking may enter the interaction in two ways; either as contributions from (isospin-breaking) diagrams which are not present in the model (external breaking) or as a difference in some of the coupling constants and/or vertex factors in the Feynman diagrams of the model (internal breaking). In Ref. [1], it was found that the model can account for the SCX reaction successfully. In practice, this observation implies that, if the breaking is external, the model contributions can successfully mimic the missing pieces. Since we do not possess a way to distinguish between internal and external effects, the parameter values, obtained from the fits to the SCX data, will not be taken too seriously though they do come out reasonable from the fits; only the amplitude (which
mainly depends on the experimental data and the electromagnetic corrections) will be assigned significance. Five-parameter fits to the SCX data were carried out \(^5\) and the SCX amplitude was successfully constructed. The experimental data of Ref. [29], which were recently finalized, were also included in the database; these measurements had not been available at the time the analysis of Ref. [1] was completed.

Table 3 shows the isospin-breaking effect in the s-wave part of the interaction (the effects in the p-wave component are not statistically significant) for three cases, corresponding to the three treatments of the d and f waves, in the form of the symmetrized ratio:

\[
ISB = 2 \frac{Re f_{\text{SCX}} - Re f_{\pi \pm p}^{\text{SCX}}}{Re f_{\text{SCX}} + Re f_{\pi \pm p}^{\text{SCX}}};
\]

\(f_{\text{SCX}}\) denotes the SCX amplitude extracted directly from SCX data and \(f_{\pi \pm p}^{\text{SCX}}\) stands for the SCX amplitude as predicted from the elastic-scattering data. (Evidently, in case that isospin symmetry holds, \(ISB\) should equal 0.) An energy dependence of \(ISB\) was not observed. The effect is large (about 7.5 \%) and statistically significant.

It was found that the model parameters \(x\) and \(Z\) come out different in elastic scattering and SCX; as seen in Fig. 5, the values, extracted from the fits to the SCX measurements, are systematically larger for both parameters. This implies that the s- and u-channel contributions are different in these two cases; the difference in \(x\) affects the nucleon graphs, the one in \(Z\) influences the graphs with a \(\Delta\)-isobar in the intermediate state. Of course, it is evident that the \(\rho NN\) vertex might also be different in elastic scattering and in SCX [30]; since the fits are performed at fixed \(G_\rho\) values, such a difference is expected to be transferred to \(x\) (since \(x\) is correlated with \(G_\rho\)). Unfortunately, one cannot distinguish between these internal effects. Finally, the question arises as to which role external effects (e.g., the \(\eta - \pi^0\) mixing mechanism [8]-[31]) play in this issue. This mechanism is expected to affect all channels; on top of the straightforward s- and u-channel modifications due to this mechanism, a Feynman diagram with an \(a_0(980)\) t-channel exchange might make large contributions since the dominant decay of \(a_0(980)\) is in the \(\eta \pi\) mode. An estimation of these effects is needed before one associates certain physical processes with the isospin breaking established in the low-energy \(\pi N\) scattering.

4 Conclusions

One of the aims of the present work was the identification of the reaction creating the isospin breaking \(^5\) in the \(\pi N\) system at low energies (pion laboratory kinetic

\(^5\)Notice that, as in Ref. [1], the scalar-isoscalar parameters were fixed at zero for the fits to the SCX data on the basis of the validity of the tree-level approximation at low energies; in such a case, the scalar-isoscalar interaction cannot produce a neutral pion in the final state (starting from a charged projectile).
energy between 20 and 100 $MeV$); the SCX reaction is the culprit for the effect. The source of the breaking may involve a difference in some coupling constants and/or vertex factors in the Feynman diagrams of the model or be exclusively due to missing (isospin-breaking) pieces. An estimation of the contributions, which are not included in the model, is necessary in order to associate certain physical processes with the effect.

In the energy region, dealt with herein, and given the present experimental uncertainties, there is no evidence for isospin breaking in the two elastic-scattering processes. The possibility of additional investigation of this issue, at kinematical regions where the main parts of the interaction cancel each other, is left open; such an investigation may be enabled after the finalization of the data of an improved version of the experiment of Ref. [9].

The model parameters, obtained from the combined fits to the elastic-scattering data, have been given (Figs. 1 and 2). The values of the charged-pion coupling constant, extracted herein, were found to be ‘small’ (Table 1). The energy dependence of the $s$- and $p$-wave hadronic phase shifts, obtained from the combined fits to the elastic-scattering data, has been provided (Figs. 3 and 4) and the corresponding values have been listed (Tables 2) ready for use since meaningful uncertainties are also quoted. It was found that the KA85 solution [10] is incompatible with our results in the $s$-wave part of the interaction. Although, the SM95 solution [11] does a better job, there is definitely some discrepancy (with our solution) in the $S_{31}$ phase shift. Small differences (among the solutions) may be observed in three $p$-wave channels. The present analysis cannot support the statements of Ref. [19] concerning the $p$-wave component of the interaction.

The stability of the aforementioned results on the treatment of the $d$ and $f$ waves has been investigated; three sources for their values (the model, Ref. [10], and Ref. [11]) have been assumed, leaving the results of this work intact. Finally, it should be mentioned that all the above results rely on the correctness of the bulk of the existing low-energy $\pi N$ experimental data. It is also assumed that the electromagnetic corrections of the NORDITA group [32], which have been exclusively used in this research program, are not largely erroneous.

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| Reference                                           | Data base | $f_{\pi NN}^2$ | Vertex type |
|----------------------------------------------------|-----------|----------------|-------------|
| This work (model $d$ and $f$ waves)                | $\pi^\pm p$ | $(76.6 \pm 1.1)10^{-3}$ | $\pi^\pm pn$ |
| This work (d and f waves of Ref. [10])             | $\pi^\pm p$ | $(75.5 \pm 1.2)10^{-3}$ | $\pi^\pm pn$ |
| This work (d and f waves of Ref. [11])             | $\pi^\pm p$ | $(74.5 \pm 1.2)10^{-3}$ | $\pi^\pm pn$ |
| [10]                                               | $\pi N$   | $(79 \pm 1)10^{-3}$  | $\pi NN$    |
| [11]                                               | $\pi N$   | $(77.1 \pm 1.4)10^{-3}$ | $\pi NN$    |
| [16]                                               | $\pi N$   | $(76 \pm 1)10^{-3}$  | $\pi NN$    |
| [17]                                               | $\pi^\pm p$ | $(75.6 \pm 0.7)10^{-3}$ | $\pi^\pm pn$ |
| [18]                                               | $np$      | $(74.8 \pm 0.3)10^{-3}$ | $\pi^\pm pn$ |
| [19]                                               | $pp$      | $(74.5 \pm 0.6)10^{-3}$ | $\pi^0 pp$  |
| [20]                                               | $\bar{p}p$ | $(73.2 \pm 1.1)10^{-3}$ | $\pi^\pm pn$ |
| [21]                                               | $np$      | $(75.7 \pm 0.8 \pm 1.3)10^{-3}$ | $\pi^\pm pn$ |
| [22]                                               | $pp$      | $(77.1 \pm 0.9 \pm 0.4)10^{-3}$ | $\pi^0 pp$  |
| [22]                                               | $\bar{p}p$ | $(71 \pm 2)10^{-3}$  | $\pi^\pm pn$ |

**Table 1:** The values of the present work for the strong coupling constant as obtained from the combined fits to the elastic-scattering data along with literature values.
| $T_\pi$ | $S_{31}$  | $P_{33}$  | $P_{31}$  |
|-------|----------|----------|----------|
| 20.0  | $-2.34 \pm 0.03$ | $1.30 \pm 0.01$ | $-0.23 \pm 0.01$ |
| 25.0  | $-2.73 \pm 0.03$ | $1.84 \pm 0.01$ | $-0.32 \pm 0.01$ |
| 30.0  | $-3.12 \pm 0.03$ | $2.46 \pm 0.02$ | $-0.41 \pm 0.01$ |
| 35.0  | $-3.50 \pm 0.03$ | $3.16 \pm 0.02$ | $-0.51 \pm 0.02$ |
| 40.0  | $-3.88 \pm 0.03$ | $3.93 \pm 0.02$ | $-0.62 \pm 0.02$ |
| 45.0  | $-4.27 \pm 0.03$ | $4.78 \pm 0.03$ | $-0.73 \pm 0.03$ |
| 50.0  | $-4.66 \pm 0.04$ | $5.72 \pm 0.03$ | $-0.84 \pm 0.03$ |
| 55.0  | $-5.05 \pm 0.04$ | $6.74 \pm 0.03$ | $-0.96 \pm 0.04$ |
| 60.0  | $-5.45 \pm 0.04$ | $7.86 \pm 0.03$ | $-1.08 \pm 0.04$ |
| 65.0  | $-5.85 \pm 0.05$ | $9.09 \pm 0.03$ | $-1.20 \pm 0.05$ |
| 70.0  | $-6.25 \pm 0.06$ | $10.42 \pm 0.04$ | $-1.32 \pm 0.06$ |
| 75.0  | $-6.66 \pm 0.06$ | $11.87 \pm 0.04$ | $-1.45 \pm 0.06$ |
| 80.0  | $-7.07 \pm 0.07$ | $13.46 \pm 0.04$ | $-1.58 \pm 0.07$ |
| 85.0  | $-7.49 \pm 0.08$ | $15.18 \pm 0.04$ | $-1.71 \pm 0.08$ |
| 90.0  | $-7.91 \pm 0.09$ | $17.05 \pm 0.05$ | $-1.84 \pm 0.09$ |
| 95.0  | $-8.34 \pm 0.11$ | $19.09 \pm 0.06$ | $-1.98 \pm 0.09$ |
| 100.0 | $-8.76 \pm 0.12$ | $21.31 \pm 0.08$ | $-2.11 \pm 0.10$ |

**Table 2(a):** The isospin-$\frac{3}{2}$ s- and p-wave hadronic phase shifts (in degrees) in the low-energy domain extracted from elastic-scattering data. Averages over the three options for the $d$ and $f$ waves (see text) are assumed. $T_\pi$ (in MeV) denotes the pion laboratory kinetic energy.
Table 2(b): The isospin−\(1/2\) s- and p-wave hadronic phase shifts (in degrees) in the low-energy domain extracted from elastic-scattering data. Averages over the three options for the \(d\) and \(f\) waves (see text) are assumed. \(T_\pi\) (in \(MeV\)) denotes the pion laboratory kinetic energy.

| \(T_\pi\) | \(S_{11}\)   | \(P_{13}\)   | \(P_{11}\)   |
|---------|-------------|-------------|-------------|
|   20.0  | 4.22 ± 0.02 | −0.17 ± 0.01| −0.36 ± 0.01|
|   25.0  | 4.70 ± 0.03 | −0.23 ± 0.01| −0.47 ± 0.02|
|   30.0  | 5.13 ± 0.03 | −0.30 ± 0.01| −0.58 ± 0.02|
|   35.0  | 5.53 ± 0.03 | −0.36 ± 0.02| −0.68 ± 0.03|
|   40.0  | 5.88 ± 0.04 | −0.43 ± 0.02| −0.77 ± 0.03|
|   45.0  | 6.21 ± 0.04 | −0.50 ± 0.02| −0.86 ± 0.04|
|   50.0  | 6.51 ± 0.05 | −0.57 ± 0.03| −0.93 ± 0.04|
|   55.0  | 6.78 ± 0.06 | −0.64 ± 0.03| −0.99 ± 0.05|
|   60.0  | 7.04 ± 0.07 | −0.71 ± 0.04| −1.04 ± 0.05|
|   65.0  | 7.28 ± 0.08 | −0.78 ± 0.04| −1.07 ± 0.06|
|   70.0  | 7.50 ± 0.10 | −0.85 ± 0.05| −1.09 ± 0.06|
|   75.0  | 7.70 ± 0.11 | −0.92 ± 0.05| −1.09 ± 0.07|
|   80.0  | 7.88 ± 0.13 | −0.99 ± 0.06| −1.07 ± 0.08|
|   85.0  | 8.05 ± 0.14 | −1.06 ± 0.06| −1.04 ± 0.08|
|   90.0  | 8.21 ± 0.16 | −1.12 ± 0.07| −0.99 ± 0.09|
|   95.0  | 8.35 ± 0.18 | −1.19 ± 0.08| −0.92 ± 0.10|
|  100.0  | 8.48 ± 0.20 | −1.25 ± 0.08| −0.83 ± 0.11|
Table 3: The isospin-breaking effect in the $s$-wave part of the interaction for three cases, corresponding to three treatments of the $d$ and $f$ waves, in the form of the symmetrized ratio given in Eq. (2). $T_\pi$ denotes the pion laboratory kinetic energy. The entry in the last column represents an average over the three energies of the table which are representative of the input data.
Figure 1: The $G_\rho$-dependence of the model parameters $G_\sigma$, $\kappa_\sigma$, $\kappa_\rho$, and $g_{\pi NN}$ for the combined fits to the elastic-scattering data and three treatments of the $d$ and $f$ waves; filled circles: the $d$ and $f$ waves are calculated with the model, open circles: the $d$ and $f$ waves are taken from Ref. [10], squares: the $d$ and $f$ waves are taken from Ref. [11].
Figure 2: The $G_\rho$-dependence of the model parameters $x$, $g_{\pi N\Delta}$, and $Z$ for the combined fits to the elastic-scattering data and three treatments of the $d$ and $f$ waves; filled circles: the $d$ and $f$ waves are calculated with the model, open circles: the $d$ and $f$ waves are taken from Ref. [10], squares: the $d$ and $f$ waves are taken from Ref. [11].
Figure 3: The isospin $-\frac{3}{2}$ s- and p-wave hadronic phase shifts corresponding to the combined fits to the elastic-scattering data (filled circles); averages over the three options for the $d$ and $f$ waves (see text) are assumed. $\epsilon$ stands for the pion center-of-mass kinetic energy. The open circles denote the KA85 solution [10], whereas the squares represent the SM95 values [11].
Figure 4: The isospin-$\frac{1}{2}$ $s$- and $p$-wave hadronic phase shifts corresponding to the combined fits to the elastic-scattering data (filled circles); averages over the three options for the $d$ and $f$ waves (see text) are assumed. $\epsilon$ stands for the pion center-of-mass kinetic energy. The open circles denote the KA85 solution [10], whereas the squares represent the SM95 values [11].
Figure 5: The $G_P$-dependence of the model parameters $x$ and $Z$ in Solution B (see text) and from the fits to the SCX measurements (filled and open circles, respectively). The $d$ and $f$ waves have been taken from Ref. [10]; the other two $d$- and $f$-wave treatments (see text) lead to almost identical pictures.