Visualisation of the large scale circulation in Rayleigh-Bénard convection using contactless inductive flow tomography

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Abstract

Rayleigh-Bénard (RB) convection plays an important role in geo- and astrophysics as well as in many metallurgical applications. At sufficiently high values of the Rayleigh number, a large scale circulation (LSC) is formed whose dynamics had turned out to be surprisingly rich. In this paper, the applicability of the contactless inductive flow tomography (CIFT) for the detection of the torsional mode of the LSC is investigated. CIFT enables the three-dimensional reconstruction of flow structures in liquid metals by applying one or more magnetic fields and measuring the flow induced perturbations of those fields outside the melt. Additionally, preliminary measurements of the flow induced magnetic field with a similar sensor arrangement will be presented.

Key words: flow measurement techniques, Rayleigh-Bénard convection

Introduction

Thermally driven flows of liquid metals play an important role for instance in the production of mono-crystalline silicon using the Czochralski (Cz) crystal growth method. The measurement of the velocity of the liquid silicon is a challenging task because the liquid is opaque and has a temperature of about 1500 °C. Therefore, non-contacting methods for the determination of the flow are needed. The contactless inductive flow tomography (CIFT) [1-2] is able to reconstruct the three dimensional structure of the flow by applying primary magnetic fields to the melt and by measuring the flow induced magnetic field perturbation outside the fluid volume. Based on these measurements, the flow field is reconstructed by solving a linear inverse problem. Appropriate regularization methods, similar to Tikhonov regularisation in combination with the L-curve technique, are necessary to mitigate the non-uniqueness problem.

In order to investigate the applicability of CIFT for Cz crystal growth, an existing modified Rayleigh-Bénard (RB) setup had been equipped with one pair of excitation coils and 20 magnetic field sensors for CIFT at one plane at mid-height of the cylinder [3]. One of the challenges for CIFT is the reliable measurement of the flow induced magnetic field which is about 4-5 orders smaller than the applied magnetic field. The reason for this is the low velocity in the order of 1 cm/s. In the previous study [3], the flow induced magnetic field was robustly measured for a variety of temperature differences between 2.3 K and 80.8 K over a period of 8 hours. It was demonstrated that dynamical features of the large scale circulation (LSC), like azimuthal rotations and cessations, as well as typical frequencies of the torsional/sloshing mode are well detectable. However, due to the lack of information over the height of the cylinder, it was not possible to distinguish between torsional modes [4] and sloshing modes [5,6]. Therefore, a new sensor arrangement has to be found that enables a robust reconstruction while using as few as sensors possible. The procedure is as follows: in the first step, the secondary field is computed outside the vessel for a numerically simulated time-dependent flow. In the second step, the flow is reconstructed using different sensor arrangements. The quality of the reconstructions is assessed by an error estimation. It turned out that an arrangement of 8 sensors in azimuthal direction for each of the 3 layers over the height (labelled as 8 × 3) is a good choice [7]. In this paper we will investigate if the selected sensor arrangement is able to detect the low velocity in the order of 1 cm/s. In the previous study [3], the flow induced magnetic field was robustly measured for a variety of temperature differences between 2.3 K and 80.8 K over a period of 8 hours. It was demonstrated that dynamical features of the large scale circulation (LSC), like azimuthal rotations and cessations, as well as typical frequencies of the torsional/sloshing mode are well detectable. However, due to the lack of information over the height of the cylinder, it was not possible to distinguish between torsional modes [4] and sloshing modes [5,6]. Therefore, a new sensor arrangement has to be found that enables a robust reconstruction while using as few as sensors possible. The procedure is as follows: in the first step, the secondary field is computed outside the vessel for a numerically simulated time-dependent flow. In the second step, the flow is reconstructed using different sensor arrangements. The quality of the reconstructions is assessed by an error estimation. It turned out that an arrangement of 8 sensors in azimuthal direction for each of the 3 layers over the height (labelled as 8 × 3) is a good choice [7]. In this paper we will investigate if the selected sensor arrangement is able to detect the flow structure of the torsional mode and compare it with a preliminary measurement.

After a short description of CIFT and the experimental setup, an analysis of the reconstruction for a temperature difference of 2.3 K is carried out. The paper concludes with a preliminary measurement of the flow induced magnetic field for a temperature difference of 30 K.

Contactless inductive flow tomography

CIFT relies on the following integral equation system which provides the flow induced magnetic field \( \vec{B} \) outside the fluid volume \( V \) for an applied magnetic field \( \vec{B}_0 \) under the assumption of insulating boundaries [1,2]:

\[
\vec{b}(\vec{r}) = \frac{\sigma \mu_0}{4\pi} \iint_V \left[ \frac{\vec{b}(\vec{r}') \times \vec{B}_0(\vec{r}')}{|\vec{r} - \vec{r}'|^3} \right] dV' - \frac{\sigma \mu_0}{4\pi} \iint_{S} \frac{\vec{b}(\vec{r}') \times \vec{b}(\vec{r})}{|\vec{r} - \vec{r}'|^3} dS'
\]  

(1)
\[
\varphi(\vec{s}) = \frac{1}{2\pi} \iiint_{V} \left[ \vec{v}(\vec{r}') \times \vec{B}_0(\vec{r}') \right] \cdot (\vec{s} - \vec{r}') \frac{dV'}{|\vec{s} - \vec{r}'|^3} \ - \frac{1}{2\pi} \iint_{S} \frac{\varphi(\vec{s}') \vec{n}(\vec{s}') \cdot (\vec{s} - \vec{s}')}{|\vec{s} - \vec{s}'|^3} dS'
\]  

(2)

In this equation system \(\varphi\) is the electric scalar potential and \(\vec{v}\) denotes the velocity of the liquid metal which is assumed to have uniform conductivity \(\sigma\) in the volume \(V\) with the surface \(S\). In the volume integrals, \(dV\) is the volume element and \(\vec{r}'\) the position vector in the volume. In the surface integrals, \(dS\) denotes a surface element and \(\vec{n}(\vec{s}')\) represents the normal vector of the surface at position \(\vec{s}'\).

In order to reconstruct the velocity field \(\vec{v}\) in the volume \(V\) this integral equation system has to be inverted. The intrinsic non-uniqueness problem of the emerging linear inverse problem, which mainly concerns the detailed depth-dependence of the velocity, is circumvented by utilizing the so-called Tikhonov regularization [1,8].

**Experimental setup**

The setup consists of a cylindrical column of aspect ratio one filled with GaInSn which is homogeneously heated from below through a copper plate (Fig. 1). In order to emulate the growing crystal of a Czochralski system, only partial cooling of the top is realized by means of a circular heat exchanger mounted concentrically within the upper lid. This partial cooling covers approximately 17% of the mean area modelling the growing crystal in a real puller. PID-controlled thermostats with a large reservoir supply the copper heat exchangers (heater and cooler) with coolant/heating fluid at high flow rate. Fig. 1(b) shows a photograph of the experimental cell having an aspect ratio of one (height \(H =\) diameter \(D = 87\) mm). The transparent side wall is made of a borosilicate glass pipe because of its low heat conductivity in order to approach an adiabatic boundary condition. The whole apparatus was embedded in mineral wool during the measurements to minimize lateral heat loss.

![Schematic sketch of the setup](image1)

![Photo of the cell](image2)

![Photo of the setup with instrumentation for CIFT](image3)

Fig. 1: Schematic sketch and photos of the modified Rayleigh-Bénard configuration.

The excitation magnetic field for CIFT is generated by a pair of circular coils which are fed by a current of 20 A generating a constant vertical field with approximately 4 mT in the fluid. Additionally, the coils are cooled in order to minimize temperature changes over time. The flow induced magnetic field is measured by 21 Fluxgate probes which are arranged in three layers over the height of the cylinder with 7 sensors in each layer (\(7 \times 3\)). A very stiff mounting of sensors with respect to the coils is necessary to avoid any magnetic field changes resulting from geometric deformations due to thermal expansion during the experiment. The flow in the container was simulated for a temperature difference of 2.3 K using the buoyant\(\text{BoussinesqPimpleFoam}\) solver of the finite volume library OpenFOAM over 3000 s [3]. No turbulence models were used.
Fig 2. Calculated flow induced magnetic field $\vec{b}$ for horizontal (a) and vertically (b) applied field and a sketch of the sensor arrangement $8 \times 3$ (c)

**Numerical simulations for a temperature difference of 2.3 K**

In order to enable a complete three-dimensional reconstruction, two homogeneous magnetic fields in horizontal and vertical direction with $\vec{B}_{0,1}(\vec{r}) = B_0 \hat{e}_x$ and $\vec{B}_{0,2}(\vec{r}) = B_0 \hat{e}_z$ were applied with $B_0 = 3$ mT. The flow induced magnetic field was calculated at a distance of 36 mm from the fluid which correspond to the smallest possible distance in the experimental setup (see also [3]). As an example, Fig. 2 displays the computed fields around the vessel for a time averaged LSC, which has a flywheel structure.

In order to investigate the quality of the reconstruction of the time dependent flow, the flow induced magnetic field was calculated according to Eq. (1) and (2) for each second of a selected time period between 1600 s and 1680 s of the flow simulation. We have chosen a sensor arrangement of $8 \times 3$ sensors. In order to identify the dynamics of the torsional flow, the direction and amplitude of the flow close to the top and the bottom of the cylinder is important. Therefore, we averaged the velocity in a plane close to the top and the bottom of the cylinder for each reconstruction. Fig. 3(a) shows the time dependent behaviour of the direction at the top and the bottom of the cylindrical vessel. It is evident that the direction of the flow at both locations is well reconstructed. Fig. 3(b) shows the time dependent behaviours of the amplitude of the flow. It is interesting to note that there is a slight discrepancy in the amplitude between the reconstruction and the original velocity. Numerical simulations suggest that a better agreement could only be achieved, if additional sensors would be added above the top and below the bottom of the vessel.

**Fig. 3:** Comparison of the reconstructed and the original velocity regarding the direction (a) and amplitude (b) of the mean velocity at the top and at the bottom of the cylindrical vessel
Measurements for a temperature difference of 30 K
For the preliminary measurements, an arrangement of 7 sensors evenly distributed in azimuthal direction on each of the 3 layers over the height were selected. As seen in Fig. 1(a), a spacer made of a ceramic material was used to allow for a complete covering of the height of the vessel. The measurements were conducted during the night in order to avoid magnetic field changes generated by any moving ferromagnetic parts or switching currents in the experimental hall. With this precaution we were able to measure the magnetic field over 8 hours with a drift of about 20 nT. Fig. 4, shows a preliminary measurement of the flow induced magnetic field for a temperature difference of 30 K. From the 21 magnetic field sensors we selected 3 sensors at one azimuthal position and at three different heights (see also Fig. 1(a)). In Fig. 3(a) it can be clearly seen that the magnetic field rises after generating the temperature gradient at about 2 hours and is maintained until the temperature gradient approaches zero again at about 11 hours. At the end of the experiments, all 3 sensors return approximately to zero, with an offset between 20 and 40 nT. In Fig. 4(b) a time interval of 80 seconds is shown which was taken at 7.2 h after starting the measurement. It could clearly be seen that there is a phase shift between sensor 0, which is close to the top, and sensor 2, which is close to the bottom. This indicates that the flow structure of torsional modes could be indeed visualized by CIFT.

Fig. 4: Measurement of the flow induced magnetic field for one experiment with a temperature difference of 30 K for 3 sensors (a) and a zoom into the measurement (b) at about 7.2 hours

Conclusions
The presented numerical and experimental investigations indicate the applicability of CIFT for the detection of the rich dynamics of the LSC of a Rayleigh-Bénard experiment using liquid metals. Beside the visualisation of azimuthal rotations and cessations of the LSC, the reconstruction of dynamics of torsional modes seems to be feasible, too. The next steps will be the reconstruction of the flow based on the measured induced magnetic fields and a comparison with ultrasound Doppler velocimetry measurements. In order to achieve a full three-dimensional reconstruction of the flow, a second pair of coils will be added to the experiment for generating a transversal magnetic field. The use of gradiometric induction coils in combination with an alternating primary magnetic field will reduce effects of fluctuations of the environmental magnetic field.

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