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Does flattening the curve make a difference? An investigation of the COVID-19 pandemic based on an SIR model

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ABSTRACT

We use a susceptible-infective-removed (SIR) model to examine the impacts of different isolation measures to combat the COVID-19 pandemic. The model predicts that strong isolation measures in the early stage of the pandemic can not only delay the time for the number of infections and deaths to reach the peak but also greatly reduce the cumulative number of infections and deaths. We verify the model predictions by using the simulation and the data of the COVID-19 cases. The results are independent of the joint distribution of the fatality rate and the initial number of active cases.

1. Introduction

The World Health Organization (WHO) declared the novel coronavirus (COVID-19) outbreak a global pandemic on March 11, 2020. The COVID-19 has presented a severe threat to the health and safety of people all over the world due to its rapid spread and potential harm. Several studies have analyzed the pandemic evolvement and spread pattern in some countries and recommended protective measures to prevent the virus transmission (Riou and Althaus et al., 2020; Shao & Wu, 2020; Tang et al., 2020; Li et al., 2020; Zhan et al., 2020).

Mathematical models are important tools to forecast the spread of infectious diseases. The classical susceptible-infective-removed (SIR) model of Kermack and McKendrick et al. (1927) has been widely used to analyze the dynamics of the COVID-19 spread. Calafiore et al. (2020) develop a modified SIR model for contagious disease and identify the parameters using the pandemic data. Simha et al. (2020) predict the evolution of COVID-19 infections using a stochastic SIR model and derive the parameters of the model based on the European country data. These parameters of the SIR models are used to project the COVID-19 spread in other countries. Similar

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modeling work has been conducted by Atkeson (2020), Bastos and Cajuieiro (2020), and Postnikov (2020). However, it is particularly difficult to predict the fatality rate of the pandemic due to the lack of widespread testing and reliable data in the initial phase of epidemic progression (Atkeson, 2020).

The outbreak of COVID-19 has prompted a wide range of policy responses from governments around the world. These responses, which vary greatly from voluntary individual quarantine to large-scale forced lockdown, can be broadly divided into two categories based on their stringency levels. The first one is a blocking strategy, characterized by strictly controlling the epidemic transmission. It has been adopted by countries including China, Thailand, and Singapore. While these measures may save lives, they incur a substantial cost including a skyrocketing unemployment rate and sharply declined economic activities. The other one is a mitigation strategy with a focus on the pandemic intervention. It has been adopted by countries such as the United States, Japan, France, and Italy. According to a survey of 14,276 participants in 14 countries (Devlin & Connaughton, 2020), while most people approve of their countries’ responses to the COVID-19, there are increasing divisions between those people with favorable and unfavorable views of measures such as stay-at-home orders and lockdowns. A natural question thus follows: which strategy is associated with the lower spread of COVID-19 cases?

We seek to examine the potential impact of government responses with different stringency levels on the spread of the COVID-19 using a SIR model. The model helps to compare the impact of the two isolation measures under the two parameters combined with a high fatality case and a low fatality case derived from the key statistics of the COVID-19 global pandemic. Empirical results based on the simulation and the data of the COVID-19 cases show that the stricter the initial isolation measures are, the fewer the cumulative deaths will be, and the longer it will take for the infectious disease to reach a steady state. The results are independent of the joint distribution of the fatality rate and the initial number of active cases.

The remainder of the paper is organized as follows. In Section 2, we present the SIR model and the key assumptions. In section 3, we verify our model predictions by using the simulation and the data of the COVID-19 cases. We conclude in Section 4.

2. The SIR model

The SIR Epidemic Model is derived under several strong assumptions. Some studies have extended this basic model in several directions by relaxing some assumptions. In this paper, we employ the simplest models described by Kermack and McKendrick (1927). After first dividing a country’s population into three groups: the susceptible, infected, and resistant, we use $S(t)$, $I(t)$, and $R(t)$ to denote the number of the susceptible, infected, and resistant populations at the time $t$, respectively, with the sum equal to the total population, denoted by $N$. In the basic SIR model,$^4$ the susceptible individuals have never been infected and have no immunity. Once they have the disease, they move into the infected state. The infected individuals can spread the disease to the susceptible individuals in the infectious period. They enter the recovered state afterward. The resistant individuals either recover or die from the disease, and hence, they have been removed from the possibility of infection.

There are some important assumptions for the simple SIR Model (Weiss, 2013):

1. The total population is large and closed, and its size remains constant.
2. The outbreak is short-lived.
3. No natural birth or natural death occurs.
4. Recovery from infection confers lifetime immunity.

The transfer diagram is depicted in Fig. 1.

The SIR model is the following system of quadratic ordinary differential equations (ODEs):

$$\begin{align*}
dS/dt &= -\beta SI \\
dI/dt &= \beta SI - \gamma I \\
dR/dt &= \gamma I
\end{align*}$$

(1)

whereas the disease transmission rate is denoted as $\beta(t)$ and the recovery rate as $\gamma(t)$. As the total population size is denoted by $N$, we have $R(t) = N - S(t) - I(t)$. The basic reproduction number is denoted as $R_0$, indicating the transmissibility of a virus within a particular population. Therefore, the relation between $R_0$ and transmission rate $\beta(t)$ is $\beta = R_0 e^{-\gamma t}$. We denote the fatality rate of the disease by $\nu$. That is, $\nu$ is a fraction of the infectives who become resistant, as they have died. Thus, the cumulative deaths are given by $D = \nu R$. The death rate per unit time is given by:

$$dD/dt = \nu dR/dt$$

(2)

The initial conditions of the model are $R(0) = 0$, $I(0) = I_0$, $S = 1 - I_0$. Some important model properties include:

\footnote{It should be noted that there are several typical epidemic models including SI, SIS, SEIR, and SIR. According to the transmission characteristics of an epidemic, we use the SIR model because SI and SIS models do not include the resistant populations. In addition, the exposed populations in the SEIR model are difficult to be observed from clinical data.}
(1) It is easy to prove that the disease always dies out. \( I(\infty) = 0 \) for all initial conditions, without having a formula for \( I(t) \). \( I(t) = 0 \) is called a steady state.

(2) If infectives are greater than 0, then if and only if \( S_n \leq 1/R_e \). Thus, the number of infectives decreases with time and the infection process reaches a steady state with infectives to be 0.

(3) If \( R_e > 1 \), then \( I(t) \) starts increasing, reaches its maximum, and then decreases to zero as \( t \to \infty \). We call this scenario of increasing numbers of infected individuals an epidemic. \(^5\)

3. Simulation

At the beginning of the COVID-19 infection and before an infected person is capable of transmitting the virus to another person, there is a period of time known as the exposed or latency period. Much of the uncertainty here is how long the incubation period of COVID-19 will be. According to the Joint Report of China and the World Health Organization (WHO, 2020),\(^7\) the COVID-19 infected individuals generally develop symptoms, including mild respiratory symptoms and fever, on an average of 5–6 days after infection (with a mean of 5–6 days and a range of 1–14 days). In our simulation, we set \( \gamma = 1/14 \).

From the SIR model, we can estimate \( R_e \) in the earliest phase of the epidemic assuming that the number of susceptible is close to the total population and that the parameter \( \gamma \) can be inferred from clinical data. For the purposes of the description here, \( R_e \) is set to 2.5 and 1.25 to refer to different isolation measures. The smaller \( R_e \) is, the stricter the isolation control measures are. For the sake of description, \( R_e = 1.25 \) is referred to as strong isolation measures and \( R_e = 2.5 \) is referred to as mild isolation measures. The following computational experiments are based on the assumption that the normalized transmission rate \( R_e \) is held constant.

Considering that the fatality rate is difficult to estimate from clinical data in the initial phase of the epidemic, we conduct two computational experiments to analyze the uncertainty over the joint distribution of the fatality rate and the initial number of active cases.

(1) High fatality rate – low initial number of active cases \( I_0 = 330 \) \( \nu = 0.01 \)

(2) Low fatality rate – high initial number of active cases \( I_0 = 3300 \) \( \nu = 0.001 \)

3.1. High fatality case

Under the parameter setting of the high fatality rate and the low initial number of active cases, we compare the influence of the two isolation measures on the number of active infections, the cumulative number of infections, and the cumulative deaths, as shown in Fig. 2.

Fig. 2 shows that these two alternative isolation measures configurations have dramatically different implications for the cumulative number of infections and deaths (see (B) and (C)). That is, 1.2 million to 1.3 million people would die if the continuous strong isolation measures are taken at the initial phase of the epidemic, but the cumulative death toll would be as much as 3 million if mild isolation measures are taken (see (C) in Fig. 2). According to the epidemic transmission mechanism, the difference in cumulative death toll under different isolation measures stems from the difference in cumulative infection number \( (R + I) \). As shown by (D), the fractional deaths change by day under the strong isolation measures reaches a steady state earlier compared with the other measures.

In the initial stage, we set \( R_e = 1.25 \) if the strong isolation measures are implemented. For example, an average infected person would transfer the COVID-19 virus to 1.25 people, which makes the spread of COVID-19 far less severe than the situation of other measures (i.e. \( R_e = 2.5 \)). After the first batch of infected people obtain immunity and return to normal life, they would become the

\(^5\) If \( R_e < 1 \), then \( I(t) \) starts decreasing, an infected could averagely infect no more than one person during her or his infection period, which means that the outbreak will die out naturally and will not cause a pandemic.

\(^6\) See Theorem 2.1 in Weiss (2013).

\(^7\) Report of the WHO-China on COVID-19 (2020). Source: http://www.nhc.gov.cn/xcs/yqlkdt/202002/87fd92510d094e4b9bad597608f5cc2c/files/fa3ab9461d0540c294b9982ac22af64d.pdf.
barrier between the virus and the susceptible and are able to protect the susceptible to a certain extent. When the immune population reaches a certain proportion (such as 60%) in the total population, then the epidemic situation will become the so-called “herd immunity”.\footnote{Horton, Richard. “Offline: COVID-19 – a reckoning.” The Lancet 395.10228 (2020): 935. https://www.thelancet.com/pdfs/journals/lancet/PIIS0140-6736(20)30669-3.pdf.}

In addition, no matter what isolation measures are implemented in the initial stage, the epidemic will eventually reach a steady state based on epidemiological characteristics. That is, the new death toll will become zero. However, under the same initial conditions, the time from the outbreak to the steady state of the COVID-19 under the strong isolation measures is much longer than that under the mild isolation measures. In about 200 days after the outbreak of the epidemic, the death change by day is to reach the steady state under the mild isolation measures ($R_e$ =2.5), while it takes about 800 days to reach the steady state for another measure ($R_e$ =1.25). This result shows that stringent isolation measures can prolong the time of infection and death peak, which will win valuable time for medical resources supplement and vaccine development. However, mild isolation measures can achieve a steady state quickly in the natural transmission state without vaccine intervention. Essentially, the decision becomes a trade-off between lives and time.

3.2. Low fatality cases

We now examine the model implications for the cumulative number of infections ($R + I$), the number of new infections ($I$), and deaths ($D$) under the assumption of low fatality cases, as shown in Fig. 3 below.

We observe that active infections (A) and cumulative cases (B) behave quite similarly across the two-fatality scenarios. The peak of deaths (C) and the time needed to reach a steady state are quite different under the two isolation measures. The comparison indicates that the control effect of the two different isolation measures separately on the COVID-19 is similar in the case of different fatalities of the epidemic. That is, under the two fatality cases, the time required to reach a steady state under strong isolation measures is always
longer than that in the other case, but the deaths will be greatly reduced due to continuous strong isolation measures (see Table 1).

When the governments around the world fight against the Covid-19 spread, life and time are the two most important considerations. Governments are facing a trade-off between time and life when implementing policies (i.e., stringent vs. loose containment and isolation policies) to slow down the virus spread. Specifically, the advantage of stringent isolation measures is that they can reduce the cumulative number of infections and deaths, but the economic activities cannot be carried out normally and such policies will prolong the time of economic recovery. The loose isolation measures have a mild impact on social and economic activities, but they will cause a high spread rate and more fatality. Therefore, governments have to determine which type of measures are needed.

4. Empirical analysis

The above simulation results can be verified by empirical data. In this part, we select Singapore, Japan, and Thailand as group-1 countries. These countries have implemented strict prevention and isolation measures, and adopted the prevention and control quarantine for confirmed, suspected, and positive cases in order to block the spread of the virus. In contrast, other countries like the United States, France, and Italy do not emphasize the early detection of all cases, the isolation treatment of mild cases, and the check and self-isolation of close contacts. Therefore, we select these countries as group-2 that take the mild isolation measures to compare with group-1. To illustrate the results in Section 3, we plot Fig. 4 to show the cumulative infections and deaths of the COVID-19 of these two groups of countries for 85 consecutive days (from February 28, 2020 to May 22, 2020). In Panel (A) and (B), we show the raw data of Cases Confirmed and of Deaths. We plot the log of the cumulative number of confirmed cases and deaths in Panel (C) and (D).

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10 As mentioned above, China has implemented stringent policies to contain the pandemic. However, we have not included China’s empirical data for a couple of reasons. First, the Wuhan Epidemic Prevention and Control Headquarters revised data about the number of actively infected and cumulative deaths on April 17, 2000. This means that the clinical data published before April 17, 2000 are not consistent with those after the date. Second, China probably has the most stringent policies in the world to deal with the pandemic.

11 Data Source: World Health Organization (WHO) Coronavirus Disease (COVID-19) Dashboard https://covid19.who.int.
We omit data from the first 60 days, as the initial number of deaths and infected cases is quite low. We observe that the cumulative infections and deaths of group-1 are significantly lower than that of group-2, which is consistent with the experimental analysis.

The global pandemic has severely impacted many countries’ economies, including the United States, China, and India (Açıkgöz & Günay, 2020; Kanitkar, 2020). The economic recovery differs for the countries that have implemented different prevention and control measures. In particular, countries with strong isolation measures have suffered more severe economic losses. For example, as one of the representative countries taking very strong isolation measures, China’s Purchasing Managers’ Index (PMI) has decreased by 28.6% from January to February in 2020. In contrast, the United States is one of the countries taking very loose isolation measures, and its PMI has dropped by 18.3% from January to March in 2020 (Barua, 2020).

5. Conclusion

The model simulation reveals that different isolation and prevention strategies adopted by the government at the early stage of the

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Table 1
Comparison of two computational experiments.

|                      | High fatality case : \( I_0 = 330 \), \( v = 0.01 \) | low fatality case : \( I_0 = 3300 \), \( v = 0.001 \) |
|----------------------|---------------------------------------------------|-----------------------------------------------------|
|                      | Number of deaths | Steady state time | Number of deaths | Steady state time |
| Strong isolation measures \( R_e = 1.25 \) | Close to 1.2 million | 800 days | Close to 0.12 million | 800 days |
| Mild isolation measures \( R_e = 2.5 \)     | Close to 3 million | 200 days   | Close to 0.3 million | 200 days   |

Fig. 4. Cross-country comparison of the COVID-2019 against time in days from Feb. 28, 2020 to May 22, 2020.

12 The Purchasing Managers’ Index (PMI) is an index of the prevailing direction of economic trends in the manufacturing and service sectors.
Data availability

Data will be made available on request.

Author statement

Please accept this as a formal statement that the authors of manuscript titled “Does flattening the curve make a difference? An investigation of the COVID-19 pandemic based on an SIR model” are coauthored by Hong Qiu, Qian Wang, Qun Wu, and Hongyong Zhou. We certify that all authors have seen and approved the final version of the manuscript being submitted. We warrant that the article is our original work, hasn’t received prior publication and isn’t under consideration for publication elsewhere.

Data availability

Data will be made available on request.

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