Analysis of the Tensile Strength of Copy Paper with a Center Notch Based on Four Strength Theories

by

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The tensile strength of a copy paper with a center notch was measured. During the test, the loading direction coincided with the cross direction (CD) or the machine direction (MD) of the paper sheet, and the configuration and dimension of the notch were varied. The relationship between the tensile strength and notch dimension was fitted using the following four theories: uniform stress criterion (USC), point stress criterion (PSC), average stress criterion (ASC), and fracture mechanics theory (FMT). Although the effect of loading direction was significant on the relationship between the tensile strength and notch dimension, the effect of notch configuration was not so significant. When the loading direction coincided with the CD of the paper, the PSC and ASC methods were superior to the other two methods for fitting the relationship. In contrast, the PSC and FMT methods were superior to the other two methods for fitting the relationship when the loading direction coincided with the MD.

Key words:
Average stress criterion, (ASC), center slit (CS) specimen, fracture mechanics theory (FMT), hole-slit (HS) specimen, open-hole (OH) specimen, point stress criterion (PSC), tensile strength, uniform stress criterion (USC)

1 Introduction

For effective utilization of paper products such as packaging and containers, it is necessary to determine the mechanical properties reliably. The tensile strength of paper containing notches is one of the important properties because notches are easily introduced in paper products because of their thinness and the failure caused by the notch is easily induced. Additionally, there are many occasions to use paper with notches such as notebooks, calendars, etc. These practical utilizations enhance the importance of predicting the strength properties of notched paper.

When a material contains notches, its strength is usually reduced because of the stress concentration near the notch edge, and the reduction is enhanced with the increase of the notch size. For composite laminates with an open-hole notch, the dependence of the strength on the hole size has been examined. Several semiempirical strength models have been proposed, and their validity has been frequently examined1-7). The methods presented in these examples are also effective to analyze the tensile strength of paper with a notch, and there are several examples examining the tensile strength of paper material with a notch8,9). However, the notch configuration was often restricted to an open-hole, the diameter of which is very restricted, so the effect of notch configuration and dimension has not been discussed in detail. Additionally, the notch strength should be analyzed based on the fracture mechanics theory when the notch is a sharp crack. In the aforementioned examples, however, analyses based on the fracture mechanics were not conducted at all.

In this study, tension tests were performed on samples of copy paper with a notch. The loading direction, notch configuration, and notch dimension were varied during the tests, and their effects were investigated and analyzed based on the following four methods: (a) uniform stress criterion (USC), (b) point stress criterion (PSC), (c) average stress criterion (ASC), and (d) fracture mechanics theory (FMT), the details of which are described below.

2 Materials and Methods

2.1 Materials

Commercial copy paper (Type 6200, Ricoh Company, Ltd.) was used in this study. Rectangular specimens with length and width of 210 and 50 mm, respectively, were cut from the copy paper. The width and length directions were defined as the x- and y-directions, respectively, and the y-direction of the specimen always coincided with either the MD or CD. The CM and MC systems in the tension tests, the details of which are described below, were defined as those where the CD and MD coincided with the loading direction (y-direction), respectively. The grammage, thickness, and density of the test samples were 71.2 ± 0.8 g/m², 88 ± 1 µm, and 815 ± 15 kg/m³, respectively. In the present study, the thickness of the samples was measured according to JIS...
P8118-1998 using a micrometer (measuring range = 0-25 mm, 0.01 mm divisions, PPM-25, Mitsutoyo, Kawasaki, Japan) with a constant pressure of 50 kPa applied by a pair of flat circular ground faces with a diameter of 14.3 mm. The specimens were prepared in a room maintained at a constant temperature of 20 °C and 65% relative humidity (RH). Five specimens were prepared for one test condition.

After the initial property measurements, the tension test specimens were prepared. Figure 1 shows the diagrams of the test specimens. To examine the configuration of the notch on the tensile strength, three kinds of specimens, i.e., (a) open-hole (OH) specimen, (b) center slit (CS) specimen, and (c) hole-slit (HS) specimen were prepared. In the OH specimen, a circular hole with a diameter of 2a was cut at the center of the specimen using a punch set in a manual press (Fig. 1a). In the CS specimen, a slit with a length of 2a was cut at the center of the specimen using a scalpel as shown in Fig. 1b. In the HS specimen, a pair of circular holes with a diameter of 2 mm was cut along the centerline so that the distance between the outer edges of the holes was 2a. Then, a slit was cut between the inner edge of the holes using a scalpel as shown in Fig. 1c. The notch length 2a was varied from 5 to 45 mm at intervals of 5 mm, and the effect of the notch length was examined. As shown in Figure 1, the sections gripped at the top and bottom of the specimen had a length of 30 mm.

In addition to the notched specimens described above, notch-free specimens were prepared to measure the unnotched strength. To reduce the stress concentration imposed by the grips, emery cloth tabs (grit = 150, thickness = 0.6 mm) were bonded to each end of the notch-free specimen using double-sided tape with a thickness of 0.086 mm (Nice tack, Nichiban Co., Ltd. Tokyo, Japan). By bonding the emery cloth tabs, the notch-free specimen was effectively failed at the location far from the grips.

2.2 Tension Tests

When the tensile load is applied to the specimen without any supports, an out-of-plane deformation around the notch is often induced when a tensile load is applied to a notched paper because of the Poisson’s effect. To reduce the out-of-plane deformation around the notch, the specimen was sandwiched between Teflon® (PTFE) and polycarbonate (PC) plates. The dimensions of PTFE and PC were 145 × 80 × 4 mm³ and 50 × 80 × 4 mm³, respectively. The PTFE plate was set on the top surface of the bottom grip, and a pair of clamps was used for sandwiching the specimen. Using these plates, the out-of-plane deformation around the notch and frictional forces between the specimen and plates were effectively reduced.

The tensile load was applied to the specimen using an Instron-type universal test machine (Shimadzu Autograph AG-100KNG, Shimadzu Co., Ltd. Kyoto, Japan) at a crosshead speed of 1 mm/min until the load markedly decreased. The nominal tensile stress $\sigma_{nom}$ was obtained from the following equation:

$$\sigma_{nom} = \frac{F_y}{WT}$$  \hspace{1cm} (1)

where $W$ and $T$ are the width and thickness of the specimen, respectively. The maximum load was designated $P_{max}$ and the nominal tensile strength $F_{y nom}$ was obtained by substituting $P_{max}$ into Eq. (1).

In the notch tension tests previously conducted, the ratio of the notch length to the specimen width 2a/W was varied from 0 to 0.4. This range was selected to reduce the effects of finite-width and length in analyzing the $F_{y nom}$-2a/W relationship based on the point stress criterion (PSC). In this study, however, the value of 2a/W was varied from 0 to 0.9 to examine the effect of the notch thoroughly in a wide range.

It should be noted that every notch was cut at the mid-length symmetrically about the mid-width of the specimen. When cutting the notch eccentrically, the effect of the location of the notch might influence the tensile strength. Further research should be conducted on the effect of the location of the notch.

3 Results and discussion

Figure 2 shows the relationship between the nominal tensile strength ($F_{y nom}$) and notch length (2a) calculated using Eq. (1). In the CM system, the $F_{y nom}$ value decreases rather linearly as the 2a value increases. In the MC system, the $F_{y nom}$-2a relationship tends to be concave in the 2a range from 0 to 20 mm, which corresponds to 0 ≤ 2a/W ≤ 0.5; then, it tends to be rather convex in the 2a range larger than 20 mm (0.5 ≤ 2a/W ≤ 0.9). Therefore, the anisotropy of the paper significantly influences the $F_{y nom}$-2a relationship. In contrast, the effect of the notch configuration is not so significant on the $F_{y nom}$-2a relationship.
The dependence of the $F_{n}^{\text{nom}}$ value on the $2a$ value can be described using four criteria: (a) uniform stress criterion (USC), point stress criterion (PSC), average stress criterion (ASC), and fracture mechanics theory (FMT), the details of which are demonstrated in Fig. 3.

(a) Uniform stress criterion (USC)

When the critical stress is insensitive to the notch, the tensile stress is uniformly distributed in the intact area of the specimen as shown in Fig. 3a. Therefore, the $F_{n}^{\text{nom}}$ value can be predicted using the unnotched tensile strength $F_{0}$ as follows:

$$F_{n}^{\text{nom}} = 1 - 2a \frac{2a}{W} F_{0}$$  \hspace{1cm} (2)

(b) Point stress criterion (PSC)

The PSC was originally proposed for predicting the strength of an open-hole sample\(^1\). When a material with a finite width of $W$ contains a circular hole with a diameter of $2a$ at the mid-width, the stress distribution along the $x$-axis $\sigma_{x}(x,0)$ is approximately given as follows\(^2\): \hspace{1cm} (3)

$$\sigma_{x}(x,0) = \left\{ 2 \left[ \frac{2a}{W} \right] - \left( \frac{2a}{W} \right)^{2} \right\} \sigma_{0}^{\text{nom}}$$

As shown in Figure 3b, failure is assumed to occur when the $\sigma_{x}(x,0)$ value attains the unnotched strength at a characteristic distance ahead of the notch edge in the $x$-direction as shown in Figure 5a. When the characteristic distance is defined as $\Delta_{\text{USC}}$, the following equation is obtained:

$$\sigma_{x}(a + \Delta_{\text{USC}},0) = F_{0}$$  \hspace{1cm} (4)

From Eqs. (3) and (4), the $F_{n}^{\text{nom}}$ value is derived as follows:

$$F_{n}^{\text{nom}} = \left\{ \frac{2 - \left( \frac{2a}{W} \right)^{2} - \left( \frac{2a}{W} \right)^{4} - \left( \frac{2a}{W} \right)^{6}}{2 + \frac{2a}{W} + 3 \frac{a}{W}} \right\} F_{0}$$  \hspace{1cm} (5)

where $\lambda = a/(a + \Delta_{\text{USC}})$. The value of $\Delta_{\text{USC}}$ is determined by regressing the $F_{n}^{\text{nom}}-2a$ relationship into Eq. (6) using the method of least squares. The PSC method is only applicable for the OH specimen because the tensile stress ahead of the notch does not distribute as that shown in Figure 5b for the CS and HN specimens. In this study, however, the PSC method was also applied to the CS and HN specimens as well as the OH specimen.

(c) Average stress criterion (ASC)

Similar to the PSC, the ASC was also proposed for predicting the strength of an open-hole sample\(^1\). In the ASC, failure is assumed to occur when the average of the normal stress in the $y$-direction between the notch edge and a certain distance from the notch edge, defined as $\Delta_{\text{ASC}}$, attains the unnotched strength as shown in Fig. 3c. Therefore,

$$\frac{1}{\Delta_{\text{ASC}}} \int_{\Delta_{\text{ASC}}}^{\infty} \sigma_{y}(y,0) dy = F_{0}$$  \hspace{1cm} (6)

From Eqs. (3) and (6), the $F_{n}^{\text{nom}}$ value is derived as follows\(^2\):

$$F_{n}^{\text{nom}} = \left\{ \frac{2 - \left( \frac{2a}{W} \right)^{2} - \left( \frac{2a}{W} \right)^{4} - \left( \frac{2a}{W} \right)^{6}}{(1 + \xi) \left( 1 + 2 \xi \right)} \right\} F_{0}$$  \hspace{1cm} (7)

where $\xi = a/(a + \Delta_{\text{ASC}})$. The value of $\Delta_{\text{ASC}}$ is also determined by regressing the $F_{n}^{\text{nom}}-2a$ relationship into Eq. (7) using the method of least squares. Similar to the PSC method, the ASC method is applicable for the OH specimen alone. In this study, however, the ASC method was also applied to the analyses using the CS and HN specimens.

(d) Fracture mechanics theory (FMT)

It is valid to analyze the failure of a sample with a sharp crack based on fracture mechanics theory. Rigorously, the linear-elastic fracture mechanics (LEFM) theory is only applicable in the case of a brittle material; as a result, the LEFM theory may not be appropriate for a material with a blunt notch, such as a circular hole\(^2\). Nevertheless, the LEFM concept has often been shown to be effective for predicting the strength properties of a material with a blunt notch, such as a circular hole\(^1\). Therefore, the FMT method was also applied to the analyses using the OH and HN specimens as well as the CS specimen.

The fracture in paper often propagates after the onset of nonlinearity\(^1,11,18,19\). Therefore, the Mode I $J$-integral $J_{I}$ is effective to characterize the fracture mechanics property after the onset of nonlinearity. The $J_{I}$ value can be partitioned into...
linear elastic and nonlinear components, i.e., $J_L$ and $J_{NL}$, as follows:

$$J_L = J_{NL}$$ (8)

The linear elastic component can be derived using the elastic constants and Mode I stress intensity factor $K_i$ of the material as follows:

$$J_L = \frac{K_i^2}{\sqrt{2\pi}E_E} \sqrt{\frac{E_E}{E_E} + \frac{1}{2} \frac{E_L}{G_{xy}}} = c_iK_i^2$$ (9)

In the tension test of the specimen with a center notch with a length of $2a$, $K_i$ is obtained as follows:

$$K_i = \frac{x_{nom}}{\sqrt{\pi a} f \left( \frac{2a}{W} \right)}$$ (10)

where $f(2a/W)$ is the crack geometry factor approximated as:

$$f \left( \frac{2a}{W} \right) = \left[ 1 - 0.025 \left( \frac{2a}{W} \right)^2 \right] + 0.060 \left( \frac{2a}{W} \right)^4$$ (11)

In contrast, the nonlinear component $J_{NL}$ is obtained from the $P-\delta$ diagram. The $P-\delta$ relationship is approximated using the Ramberg-Osgood function as follows:

$$\delta = \frac{P}{M} + k \left( \frac{P}{M} \right)^n$$ (12)

where $M$, $k$, and $n$ are the parameters determined from the least squares regression method. Using these parameters, $J_{NL}$ is determined as follows:

$$J_{NL} = \frac{\beta J_L = 2nk \left( \frac{P_{nom}}{M} \right)^{n-1} J_L}{\frac{2a}{W}}$$ (13)

From Eqs. (8)-(13), $J_L$ is obtained as follows:

$$J_L = (1 + \beta)J_L = c_i(1 + \beta) \left[ \sigma_{\text{nom}} \sqrt{\pi a} f \left( \frac{2a}{W} \right) \right]$$ (14)

Crack propagates when $J_L$ attains the critical value $J_c$, which is regarded to be a material constant. In a previous study, however, the $J_c$ value obtained from Eq. (14) significantly decreased as the $2a$ value increased. To reduce the dependence of the $J_c$ value on the $2a$ value, it is effective to introduce an additional crack length $2A_{\text{MT}}$ as shown in Fig. 3d; and therefore, Eq. (14) can be modified as follows:

$$J_c = c_i(1 + \beta) \left[ \sigma_{\text{nom}} \sqrt{\pi (a + A_{\text{MT}})} f \left( \frac{2a + 2A_{\text{MT}}}{W} \right) \right]$$ (15)

From Eq. (16), the $\sigma_{\text{nom}}$ value is derived as follows:

$$\sigma_{\text{nom}} = \frac{J_c}{\pi c_i(1 + \beta)(a + A_{\text{MT}}) f \left( \frac{2a + 2A_{\text{MT}}}{W} \right)}$$ (16)

Fig. 3 Determination of the nominal tensile strength $F_{\text{nom}}$ based on the USC (a), PSC (b), ASC (c), and FMT (d).
toughness values except for those obtained from the CS specimens, which contains a sharp notch.

Figure 4 shows the dependence of the Mode I critical \( J \)-integral normalized by the density \( J_c/\rho \) and the value of \( \beta \) on the notch length \( 2a \) and notch length normalized by the sample width \( 2a/W \). The relationships shown in Figure 4 are obtained from the tension tests using the CS specimens, but the tendencies shown in this figure are also applicable to those obtained using the OH and HS specimens. As shown in Figures 4a and b, the dependence of \( J_c/\rho \) on \( 2a \) is not entirely reduced even when introducing the value of \( \Delta_{MT} \). Therefore, the \( J_c/\rho \) value listed in Table 3 was calculated without using the data obtained from the unnotched specimen (\( 2a = 0 \) mm).

Additionally, Figs. 4c and d indicate that the value of \( \beta \) decreases as the \( 2a \) value increases; then, it tends to be constant. Considering this phenomenon, the \( \beta\cdot 2a/W \) relationship was regressed into the following relationship from the method of least squares:

\[
\beta = A_1 + A_2 \exp (-A_3 \cdot \frac{2a}{W}) \quad (17)
\]

The values of \( A_1, A_2, \) and \( A_3 \) corresponding to each specimen are listed in Table 3.

Table 3 Parameters used for fitting the \( F_\gamma^\text{nom-2a} \) relationship.

| System | Specimen | CM | MC |
|--------|----------|----|----|
| \( \Delta_{SC} \) (mm) | 5.20 | 3.81 | 3.48 | 2.36 | 1.95 | 1.89 |
| \( \Delta_{CS} \) (mm) | 18.6 | 12.2 | 11.1 | 6.50 | 7.81 | 7.50 |
| \( \Delta_{MT} \) (mm) | 2.23 | 2.79 | 1.85 | 2.95 | 3.41 | 2.89 |
| \( L_0/\rho \) (J/m/kg) | 11.0 | 5.79 | 5.38 | 12.8 | 9.16 | 9.16 |
| \( A_1 \) | 2.30 | 0.47 | 0.80 | 13.5 | 0.35 | 17.2 |
| \( A_2 \) | 8.33 | 3.51 | 6.54 | 12.5 | 3.28 | 16.5 |
| \( A_3 \) | 10.4 | 5.75 | 8.21 | 0.046 | 18.2 | 0.044 |

Figure 2 also shows the comparison between the \( F_\gamma^\text{nom-2a} \) relationships obtained from the USC, PSC, ASC, and FMT methods. In the CM system, the PSC and ASC methods are superior to the other two methods. For the OH specimen, the linearity in the \( F_\gamma^\text{nom-2a} \) relationship is significant, so the USC method is applicable in this case. Although the FMT method is expected to be applicable for the CS specimen, it is not effective when \( 2a = 0 \) and 5 mm. As shown in Figure 4a, it is difficult to correct the \( J_0 \) value by introducing the correction notch length \( \Delta_{MT} \) alone in these notch lengths. When the \( J_0 \) value is determined applicable in the short crack range, however, the fitness of the FMT method will be improved. Because there are few examples examining the fracture mechanics properties of paper materials with a short crack, further research should be conducted to reveal the applicable method to obtain the \( J_0 \) value in the short crack range more accurately. In contrast, the fits of the \( F_\gamma^\text{nom-2a} \) relationships obtained from the PSC and FMT methods are superior to the other two methods in the MC system. According to the notch strength analysis of a bolted joint of timber by Daudeville and Yasumura\(^\text{20}\), the ASC was superior to the PSC. Nevertheless, the results obtained in this study indicate that the \( F_\gamma^\text{nom-2a} \) relationship fitted using the PSC method is superior to that using the ASC method. Additionally, the FMT method is particularly accurate for fitting the \( F_\gamma^\text{nom-2a} \) relationship except for the case where \( 2a = 45 \) mm. In this notch length, the crack geometry factor \( f[(2a + 2\Delta_{MT})/W] \) cannot be obtained because the value of \( 2a + 2\Delta_{MT}/W \) exceeds 1. This obstacle will be reduced when the crack geometry factor is appropriately determined. Further research should also be conducted to reduce this obstacle.

From the experimental results above, the PSC method is more applicable than the other three methods for fitting the \( F_\gamma^\text{nom-2a} \) relationship of copy paper accurately in a wide range of notch dimension and various notch configurations. Nevertheless, the fit by the FMT method will be improved after several obstacles are overcome.

4 Conclusions

The tensile strengths of a copy paper with a center notch in the cross direction and machine direction were measured. During the test, the configuration and length of the notch were varied. The effect of loading direction was significant on the relationship between the tensile strength and notch length, whereas that of the notch configuration was not so significant. The relationship between the tensile strength and notch length was analyzed using four strength theories. In the CM system, the PSC and ASC methods were superior to the other two methods for fitting the tensile strength-notch length relationship. In contrast, the PSC and FMT methods were superior to the other two methods for fitting the relationship in the MC system.
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