Decoherence effects in a three-level system when coupled to classical environment driven by Gaussian process

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We investigate the information and coherence protection in a three-level quantum system when subjected to a classical fluctuating field described by a Gaussian process. This random field is further investigated in both noiseless and noisy regimes. The noisy regimes include fractional Gaussian, Gaussian, Ornstein Uhlenbeck, and power-law noise. We find that the Ornstein Uhlenbeck noise has a reduced destructive nature toward coherence and information initially encoded in the three-level system. Based on our findings, the proper fixing of the noisy parameters to certain provided values can contribute to optimal extended coherence and information survival. In the single qutrit system, because of all Gaussian noises, monotonic decay with no revivals has been observed. Using purity and von-Neumann entropy, we discovered that a single qutrit system outperforms systems with multiple qubits or qutrits in terms of coherence and information preservation. The fluctuating nature of the local random fields is completely lost, as evidenced by a comparison of noisy and noiseless situations. We found this entirely dependent upon the Gaussian and Markovian properties of the included noises.

Keywords: Coherence, three-level system, classical fluctuating field, Gaussian process, purity

I. INTRODUCTION

Quantum information processing and quantum computing have shown considerable development in quantum research in recent years [1–5]. Quantum coherence remained one of the most active research parts of quantum information sciences and has been investigated vastly [6, 7]. The preservation of coherence for a quantum system guarantees successful transmission and higher efficiency in practical quantum information processing. The term ‘quantum coherence’ refers to the concept of super-positioning, which is central to quantum physics and quantum computing. Quantum coherence considers a scenario in which an object’s wave feature is divided into two and the two waves coherently overlap with one another. Quantum coherence is a crucial physical resource in quantum computation and quantum information processing, as well as a need for entanglement and other types of quantum correlations [8–12].

The entangled and coherent states are not physically disconnected from their surroundings in a practical sense. Due to dephasing effects, connecting such quantum systems to these surroundings leads to a loss of coherence and entanglement [13]. This can be caused by a variety of environmental characteristics, such as the random mobility of particles and certain different disorders, among others. According to the defect and the type of system and type of system-environment interaction involved, these faulty environments produce a variety of noises. The classical realm

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of the environment is preferred in this case because it allows for a more thorough analysis of the dynamics of quantum systems with a large number of degrees of freedom. Coherence dynamics for decreasing the degrading effects have been investigated under various noisy conditions for a range of quantum systems, both theoretically and experimentally [14–20].

We present a comprehensive investigation of the coherence preservation for a three-level system under various Gaussian noises in this respect. Fractional Gaussian noise ($\mathcal{F}G_n$), Gaussian noise ($G_n$), Ornstein Uhlenbeck noise ($\mathcal{O}U_n$), and Power-law noise ($\mathcal{P}L_n$) are among the noise types. The typical random motion of the particles produces these kinds of noises, which can deteriorate the entanglement and coherence [21–23]. The primary intent of the study will be to develop efficient methods for preventing the deteriorating effects caused by the corresponding Gaussian noise. Besides, the comparative coherence dynamics under various noises will also be thoroughly investigated. We use two metrics to examine coherence preservation for a single qutrit entangled state: purity and Von Neumann entropy [24]. This will provide exact solutions to avoid coherence and, as a result, entanglement losses for the practical implementation of quantum information processing. Most importantly, we address the optimal fixing of the parameters of the current local noises. Finding this optimal configuration will be the key to obtaining the required qualitative dynamics and preservation. Quantum system dynamics are genuinely concerned with the system’s phase. To determine the noise’s damaging consequences, an average of the noise phases will be obtained once the time evolved density matrix has been computed. The quantum system’s dynamics will be performed using the time unitary operation. The energy state of the qutrit system under the classical fluctuating field is described using the stochastic Hamiltonian. In addition, we examine the qualitative behaviour of the system’s phase under noisy and noiseless conditions. The existence of a pure noiseless configuration is an ideal example, however, will be useful to estimate the dissipation power and other characteristics of the local environments in order to decohere an entangled state. Differentiating between noiseless and noisy classical channels could help with quantum mechanical circuit design and long-term coherence and entanglement preservation. In addition, a brief preservation description of the single qutrit system will be examined in contrast to other quantum systems. This will be the key criterion in whether or not the single qutrit system should be used in the practical implementation of the quantum mechanical protocols.

The paper is organized as: In Sec.II, physical model for the dynamics of the three-level system in the classical fluctuating field under distinct noises will be given. In Sec.III, results obtained for the dynamics of the system and for the Gaussian noisy effects over the coherence will be discussed. Here, we also intend to give a brief detailed difference between the dynamics of the qutrit system under noisy as well as noiseless classical channels. Sec.IV will explain the conclusion from the present investigations.

II. MODEL AND DYNAMICS

The physical model of a single three-level maximally entangled qutrit state is discussed in this section [25]. This is subjected to local external fields characterized by a Gaussian kind of noise with Hamiltonian of energy splitting $\varepsilon_0$ and is defined by [24]:

$$H_{qt}(t) = \varepsilon_0 S_z + \omega \eta(t) S_x,$$
FIG. 1: The current configuration model shows the coupling of a three-level system \( qt \) exposed to a classical fluctuation field \( \mathcal{E}L(t) \). The system-environment coupling strength \( \omega \) is shown by the blue-reddish wavy lines, while the noise’s influence is represented by the yellowish glow in the qutrit. The brownish wavy lines indicate the system dynamics as characterized by the stochastic parameter \( \eta(t) \) of the coupled environment, with decreasing amplitude suggesting dephasing effects induced by Gaussian noises.

where \( S_z \) and \( S_x \) are Pauli spin operators acting on the sub-space of the single qutrit system. \( \eta(t) \) is the stochastic parameter and \( \omega \) is the coupling strength between the system and classical field. The time evolution of the three-level system can be done by using the time evolution operator and is given as [26]:

\[
U_{qt}(t) = T \frac{1}{\hbar} \exp[-i \int_0^t H_{qt}(s) ds],
\]

where \( T \) is the time ordering operator. On commuting of \( [H_{qt}(t1), H_{qt}(t2)] \), the two included times can be neglected and Eq.(2) takes the form as:

\[
U_{qt}(t) = \exp[-i \int_0^t H_{qt}(s) ds],
\]

where \( \hbar \) is set to 1 for the current configuration. Further, Eq.(3) can simply be written as \( U_{qt}(t) = \exp[-i \phi_{qt}(t)] \). Here, \( \phi_{qt}(t) \) is the phase of the system and reads as \( \phi_{qt}(t) = \int_{t_o}^t [\omega \eta(s)] ds \). If the qutrit system here is initially prepared in the state \( \rho_o \) then the time evolved density matrix of this state is given by [26]:

\[
\rho_{qt}(t) = U_{qt}(t) \rho_o U_{qt}(t)^\dagger
\]

To find out the noisy effects, we define various local Gaussian noises. In this proceeding, we evaluate the application of the fractional Gaussian (\( F\mathcal{G}_n \)), Gaussian (\( \mathcal{G}_n \)), Ornstein Uhlenbeck (\( O\mathcal{U}_n \)) and power-law noise (\( P\mathcal{L}_n \)). The corresponding autocorrelation functions that relate the noise phases with the stochastic field and system are given by [23, 26]:

\[
K_{F\mathcal{G}_n}(t - t') = \frac{|t'|^{2H} - |t - t'|^{2H} + |t|^{2H}}{2},
\]

where \( H \) is the Hurst parameter.
\[ K_{G_n}(t - t', \gamma, \Gamma) = \frac{\Gamma \gamma e^{-\gamma^2(t-t')^2}}{\sqrt{\pi}}, \quad (6) \]

\[ K_{\mathcal{OU}_n}(t - t', g) = \frac{g e^{-g|t-t'|}}{2}, \quad (7) \]

\[ K_{\mathcal{PL}_n}(t - t', \chi, \Gamma, \alpha) = \frac{[\alpha - 1]\chi\Gamma}{2[\chi|t - t'| + 1]^2}. \quad (8) \]

In the case of \( \mathcal{F}_G_n \), \( H \) is known as the Hurst exponent and ranges between 0 and 1. To generate classical noise for the stochastic process, one has to include \( \beta \)-function which reads as [26]:

\[ \beta_{ab}(t) = \int_0^t \int_0^t K(s - s' dsds'). \quad (9) \]

The \( \beta \)-function for the \( \mathcal{F}_G_n \) can be obtained by putting the auto-correlation function from Eq.(5) into Eq.(9) and is given as [26]:

\[ \beta_{\mathcal{F}_G_n} = \frac{\pi^{2(H+1)}}{2(H + 1)}. \quad (10) \]

where, we assumed \( t = \tau \). For the \( G_n \), we assume \( \gamma = g\Gamma \) and \( t = \frac{\tau}{\Gamma} \) and by putting the autocorrelation function from Eq.(6) into Eq.(9), we can get [23]:

\[ \beta_{\mathcal{G}_N} = \frac{1}{g} \frac{e^{-g^2\tau^2} - 1}{\sqrt{\pi}} + \text{Erf}[g\tau](g\tau), \quad (11) \]

where, \( \text{Erf}[x] \) is the error function for the normal distribution of the normalized Gaussian distribution function and is defined as \( \text{Erf}[\tau] = \frac{2}{\sqrt{\pi}} \int_0^\tau \text{Exp}[-t^2] dt \). For the \( \mathcal{OU}_n \), the relative \( \beta \)-function can be obtained by inserting Eq.(7) into Eq.(9) and is given as [26]:

\[ \beta_{\mathcal{OU}_n}(t) = g\tau + e^{-g\tau} - \frac{1}{g}, \quad (12) \]

where, \( g \) is the inverse of the auto-correlation time \( \tau \). Similarly, upon assuming the dimension-less noisy quantities \( g = \frac{1}{\sqrt{\chi}} \) and \( \tau = \Gamma t \) and inserting the auto-correlation function given in Eq.(8) for \( \mathcal{PL}_n \) into Eq.(9), we obtain the \( \beta \)-functions of the corresponding noise as [23]:

\[ \beta_{\mathcal{PL}_n}(t) = \frac{g\tau(\alpha - 2) - 1 + (1 + g\tau)^{2-\alpha}}{g(\alpha - 2)}. \quad (13) \]

The final density matrix of the system is averaged over the corresponding noise phase. For the system phase \( \phi \), this average is defined as \( \langle e^{i\phi\hat{L}(t)} \rangle = e^{i\varphi_i(t)} \) and \( \varphi_i(t) = -\frac{1}{2} n^2 \beta_{ab}(t) \). Here, \( \varphi_i(t) \) is the superimposed noise phase over the phase of the system with \( \beta_{ab}(t) \) represent the \( \beta \)-function of the noise. The time evolved density matrix of the system is given by [26]:

\[ \rho_{ql}(t) = \left\langle U_{ql}(t) \rho_0 U^\dagger_{ql}(t) \right\rangle \varphi_i(t). \quad (14) \]

The initial density matrix for the three-level maximal entangled qutrit system is defined by \( \rho_o = \frac{(1-r)I_{3x3} + r|\psi\rangle\langle \psi|}{\sqrt{3}} \) [26]. Here, \( x \) represent the initial purity of the system and ranges as \( x \in \{0,1\} \) and \( \psi = \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + |2\rangle) \).
A. Coherence measures

1. Purity

Purity ($P_r(t)$) is used to measure the degree of quantum coherence preserved by a quantum system. $P_r(t)$ for the final density matrix of the system is defined as [24]:

$$P_r(t) = Tr[\rho qt(t)]^2.$$  (15)

$P_r(t)$ for a three-level system ranges from 0 to $\frac{1}{n}$, where $n$ denotes the system’s dimensionality. The state of being pure and coherent is represented by the upper bound, while the state of being entirely mixed and decoherent is occurring at the lower bound.

2. Von Neumann entropy

Von Neumann entropy ($V_e(t)$) will be used to explain how coherence, information and the amount of correlations between sub-quantum systems declines over time. In general, this metric is used to assess the degree of disorder, information loss and coherence decay between the system and environment connected. This measure for a time developed density matrix can be computed as follows [24]:

$$V_e(t) = -Tr[\rho qt(t) \log \rho qt(t)].$$  (16)

$V_e(t) = 0$ indicates the state to be coherent with no information loss. Any other value will represent the corresponding amount of coherence and information loss for the three-level system.

III. RESULT AND DISCUSSION

In this section, the major results for the dynamics of a three-level maximally entangled qutrit system under Gaussian noises originated from a classical fluctuating field will be given. Besides this, the results for the time evolution of the purity and von Neumann entropy from Eqs.(15) and (16) will be explored, respectively. The dynamics of the system are both studied under noiseless and noisy conditions to distinguish the capacity of the current noises to dephase the entangled single qutrit system.

A. Noiseless classical field

The dynamics of the single qutrit system, when subjected to a stochastic field without introducing noise, are covered in this section. This ideal situation will be utilised to separate the stochastic fields’ original role in the exclusion of any noise. In both noiseless and noisy classical fields, this can estimate the qualitative dynamics of the single qutrit system. The structure of the density matrix given in Eq.(18) illustrates that the system is coherent and entangled. As the diagonal and off-diagonal elements of the matrix are non zero.

Fig.2 shows the fluctuation rate for the time evolved density matrix state given in Eq.(4) for the single qutrit system when coupled to the classical field. Note that the superposition of the noise
FIG. 2: Shows the fluctuation rate for the single qutrit system when subjected to classical fluctuating field with noiseless channels for $\omega = 0.5$ (a) and 1 (b) with $\eta(t) = 1$ against time evolution parameter $t = 15$.

phase over the system phase is not applied. From the current result, this kind of environment has a random character and strongly supports fluctuations. These fluctuations are the main reasons for the dynamics and the preservation of entanglement and coherence. As in most of the previous results, whenever the fluctuations caused in the time evolved density matrix vanish, the entangled state becomes separable [24, 26–29]. Thus, these fluctuations in the systems under unwanted noises can be helpful to preserve the quantum correlations and coherence encoded initially in a quantum state. Here, the fluctuations are directly controlled by the parameter $\omega$ and $\eta(t)$. As one can note that the number of revivals are greater for $\omega = 1$ in Fig.(b) and lesser for $\omega = 0.5$ in Fig.(a). The amplitude of the fluctuations is completely independent of the $\omega$ as well as of $\eta(t)$ and only depends upon the type of system involved. This optimal setting for controlling fluctuations can lead to engineer and design circuits and protocols for required results [30–32].

B. Classical field with Gaussian noises

This section includes the application of the noises of Gaussian nature over the time evolved density matrix of the single qutrit system given in Eq.(14). In the current case, the noise phase is superimposed over the system’s phase. One can note that the diagonal and off-diagonal elements of the matrix become changes from that given in Eq.(18), however, have not vanished. This comprehends that, unlike the bipartite and tripartite quantum systems, the current three-level state remained coherent and entangled under the presence of noises. Thus, proving itself a better resource to store information than bipartite and tripartite quantum systems showing greater loss [24, 27, 29, 33].

Fig.3 shows the dynamics of the final density matrix of the single qutrit system under local Gaussian noises originating from the stochastic field. The current results represent the time evolution of the superimposed noise phases over the system phase. By comparing Figs.2 and 3, one can easily determine the dominant degrading character of the classical noises for the fluctuations. After the first death, the fluctuations are fully destroyed due to the Gaussian noisy classical field. This means that quantum information processing employing local Gaussian noisy fields will be a deli-
FIG. 3: Upper panel: Time evolution of the phase of the single qutrit system when subjected to the classical field generating fractional Gaussian noise (a) when $H = 0.5$ and Gaussian noise (b) when $g = 1$ against evolution parameter $\tau = 3$. Bottom panel: Time evolution of the phase of the single qutrit system when subjected to the classical field generating Ornstein Uhlenbeck noise (c) $g = 1$ and power-law noise (d) when $g = 1$ with $\alpha = 5$ against evolution parameter $\tau = 3$.

cate operation with a high likelihood of being deficient. The quantum correlations and coherence preservation capability in classical fields with non-Gaussian disturbances have been showed to have this feature as well, as seen in [24, 27, 29]. Each noise and its associated parameters has a unique ability to suppress fluctuations in a time frame. There is a significantly greater and faster decay in the given Hurst exponent ($H$) range. For both high and low values of other noisy parameters, the loss is substantially smaller in the latter scenarios when compared to $FG_n$. It’s important to note that the decay behaviour of $FG_n$ is markedly different, and there has been no evidence of a considerable increase in decay when the parameter $H$ is increased. This is in direct contrast to the later included noises when the slopes change towards higher decay as the noisy parameters are increased. Besides, the $FG_n$ is followed by the $PL_n$ in producing greater dephasing effects. As seen, the fluctuations are readily diminished in a quick time. However, unlike the $FG_n$, $G_n$, and $OU_n$, the fluctuation interval in the case of $PL_n$ is characterized by two noise parameters. The saturation values for all the included noises remained constant, implying a comparable relevant Gaussian character. Based on the existing findings, it is possible to predict that the coherence and information decay caused by these noises will be monotonic rather than having revivals.
Here, we briefly analyze for the dynamics of the single qutrit system under \( \mathcal{FG}_n \) arising from the classical field. The impact of the current noise is applied by taking the average of the final density matrix in Eq.\((14)\) over the noise phase with \( \beta \)-function from Eq.\((10)\).

Fig.4 explores the dynamics of the purity \((a)\) and von Neumann entropy \((b)\) for a single qutrit system under the local \( \mathcal{FG}_n \). By comparing Figs.\(2, 3\) and \(4\), the destructive nature of the \( \mathcal{FG}_n \) towards the revivals and preservation of the coherence and information is enough obvious. Purity and coherence, because of \( \mathcal{FG}_n \), attain final saturation values after enduring maximum decay. It’s important to recall that present saturation levels only reflect a small partial loss of coherence, information, and, as a result, entanglement. This is both startling and counterintuitive to most previous results for systems with multiple qubits or qutrits \([24, 26, 28, 29]\). No revivals have been seen, implying that there will be no entanglement sudden death (ESD) or birth (ESB) revivals for the current system under \( \mathcal{FG}_n \). This means that once information is lost, this noise does not facilitate the repeated interchange of information between the system and the environment. One might deduce \([26, 34]\), that the information degradation is irreversible and cannot be reversed. The dynamics of the bipartite and tripartite state under \( \mathcal{OU}_n \) is like this monotonic qualitative decay, however, with different decay levels or reaching complete separability \([26, 34]\). Another unexpected finding is that increasing \( H \) for a given time limit causes the green slopes to shift towards the red end. The supportive nature of the parameter \( H \) is interpreted in this way for the subsequent decay of coherence and information. This contradicts nearly all the prior results for the noisy parameters, for example, all those given in \([24, 26, 28, 34]\). The maxima and minima coincide with all parameter values, showing that both measures reveal a single saturation level. As a result, the findings show a high level of agreement and forecast validity. Here, the parameters do not completely control the loss, but the noise phase has a significant impact. The preservation time is only minimally affected by adjusting parameters, and any choice of \( H \) does not guarantee that the coherence and information will be protected against decay.
| Measure       | Preservation time | Remarks                                                                 |
|--------------|-------------------|-------------------------------------------------------------------------|
| Purity       | $\tau = 1.75$ and $1.6$ | For increasing $H$, the decay rate becomes smaller, however, the death points occurred at near regions. |
|              | for $H = 0.1$ and $0.9$ | under $FG_n$ while undergoing partial decay.                            |
| Von Neumann entropy | $\tau = 1.75$ and $1.5$ | For increasing $H$, the rise in Von Neumann entropy becomes later, however, the death points occurred at near regions under $FG_n$ undergoing partial decay. |
|              | for $H = 0.1$ and $0.9$ | states are same but the entire qualitative behavior differs greatly. For two qubit state, the slopes seems more damped in comparison under $FG_n$ showing complete loss of $Q_C$. |
|              | reaching $S.L = 0.9$. | quantum negativity, quantum discord and saturation levels correspondingly. |
| Concurrence  | For two qubit states under $F_{G_n}$ and for $H = 0.9$, $\tau = 1.5$ | The two qubit states, showed either sudden death or complete loss for different values of the noise parameters have been observed [33]. |

TABLE I: Shows the comparison of the time preservation of the purity, coherence under $F_{G_n}$ with the previous investigations. This table explores the preservation capacity and the description of the detrimental effects produced by the current and other previously investigated noises. Here, $QN$, $QD$ and $S.L$ represents quantum negativity, quantum discord and saturation levels correspondingly.

2. Classical field with $G_n$

Performing the average of the final density matrix given in Eq.(14) over the noise phase with $\beta$-function given in Eq.(11) result in the dynamics of the single qutrit system under $G_n$ noise. Fig.5 shows the dynamics of the purity $(a)$ and von Neumann entropy $(b)$ for a single qutrit system when subjected to classical field generating Gaussian noise when $g = 1$ (green), $3$ (blue), $10$ (red) against evolution parameter $\tau = 2$.

![FIG. 5: Time evolution of the purity (a) and von Neumann entropy (b) for the single qutrit system when subjected to classical field generating Gaussian noise when $g = 1$ (green), $3$ (blue), $10$ (red) against evolution parameter $\tau = 2$.](image)

when subjected to the classical field with $G_n$. The system’s time evolution is investigated further for various $g$ values in relation to $\tau$. By comparing Figs.2, 3 and 5, one can deduce the dominating deteriorating character of the $G_n$ to lower preservation capacity and vanishing revival feature of the environments. Due to $G_n$, the purity and coherence encoded in the system initially showed monotonic decline rather than any $ESD$ and $ESB$. As a result, the classical random field with $G_n$
does not allow information from the environment to flow back to the system. As a result, one can deduce that the degradation produced by $G_n$ is irreversible. The observed qualitative behaviour for bipartite and tripartite states is consistent with prior results under $OU_n$ [26, 34]. However, in these circumstances, the quantitative analysis is becoming progressively distinct. When $g$ increases, the slopes move from the green to the red end. For large values of $g$, this means a higher occurrence of purity and coherence decay. The decay that occurred due to $FG_n$ is incompatible with this behaviour. It is important to note that under $G_n$, the coherence and information are not lost and reach a final saturation level after maximal decay. This property of the single qutrit system, which shows partial loss rather than complete loss, is a useful resource that contradicts most previous findings [26, 27, 33, 34]. According to both measures, the saturation levels for each value of the noise parameter meet at the same height, implying that the decay rate is the same. The measures’ maxima and minima are comparable, showing that the results are consistent. In addition, the $G_n$ noise phase is less decoherent towards the coherence and information decay than the $FG_n$ noise phase.

| Measure          | Preservation time | Remarks                                           |
|------------------|-------------------|---------------------------------------------------|
| Purity           | $r = 2$ and $1.3$ for $g$ and $S.L = 0.944$. | The monotonic decay rate becomes faster for increasing $g$. Unlike $FG_n$, the death points are distant due to $G_n$, however, undergoes partial decay. |
| Von Neumann entropy | $r = 2$ and $1.3$ for $g$ and $S.L = 0.13$. | The monotonic rise in Von Neumann entropy becomes faster for increasing $g$. Unlike $FG_n$, the death points are distant due to $G_n$, however, undergoes partial decay. |
| QN and QD        | $r = 3$ and $10$ in $de$ and $c$ under $OU_n$ and $S.L = 0$ and $0.2$ respectively. | Here, the two qubit state preserved $c$ for long interaction time with evident $ESD$ and $ESB$ revivals showing back-flow mechanism of information processing [35]. |
| QN and QD        | $r \geq 40$ under Brown and pink noises for different values of the parameters with single qutrit system under Gaussian noises tend to have greater capacity of $Q_c$ preservation and strongly showed $ESD$ and $ESB$ phenomenon within the non-Markovian regime bistable fluctuator. [36]. |

TABLE II: Shows the comparison of the time preservation of the purity, coherence under $G_n$ with the previous studies. Here, $ce$ and $de$ represent common and different system-environment coupling. This table explores the preservation capacity and the description of the detrimental effects produced by the current and other previously investigated noises.

3. Classical field with $OU_n$

This section involves the noisy effects due to $OU_n$ by averaging the final density matrix given in Eq.(14) over the noise phase having the $\beta$-function from Eq.(11).

Fig.6 shows the dynamics of the purity ($a$) and Von Neumann entropy ($b$) for the single qutrit system when coupled to the classical field generating $OU_n$. The degrading quality of the $OU_n$ can simply be shown by comparing the initial purity and coherence with the latter. The loss caused by the current noise has resulted in monotonous functions over time with no revivals. As a result, the $ESD$ and $ESB$ phenomena, as well as the backflow of information from the environment to the system, are not facilitated by this noise. This result contradicts the findings of bipartite and
FIG. 6: Time evolution of the purity (a) and von Neumann entropy (b) for the single qutrit system when subjected to classical field generating Ornstein Uhlenbeck noise when $g = 1$ (green), 3 (blue), 10 (red) against evolution parameter $\tau = 2$.

hybrid qubit-qutrit state dynamics given in [26, 28]. The initial encoded purity, coherence, and

| Measure                  | Preservation time | Remarks                                                                 |
|--------------------------|-------------------|------------------------------------------------------------------------|
| Purity                   | $\tau \geq 2$ and $1.4$ for $g = 1$ and $5$ reaching $S.L = 0.944$. | The monotonic decay rate becomes faster for increasing $g$. Unlike, $F_G$, the death points are farther due to $G_n$, however, undergoes partial decay. |
| Von Neumann entropy      | $\tau \geq 2$ and $1.25$ for $g = 1$ and $5$ reaching $S.L = 0.13$. | The monotonic rise in Von Neumann entropy becomes faster for increasing $g$. Unlike $F_G$, the death points are farther due to $G_n$, however, undergoes partial decay. |
| Von Neumann entropy      | $7 \leq \tau \leq 12$ for three qubit $GHZ$ and $W$-type state under $OU_n$ in $ce$ and $de$ for $g = 10^{-4}$ with $0.58 \leq S.L \leq 1.6$. | For both states, no entanglement revivals were seen. $Q_c$ were preserved for long duration however, the $S.L$ suggests that the $Q_c$ decay is enough higher than encountered for the single qutrit system under the same noise [34]. |
| $GMQD$                   | $\tau = 1.5$ for qutrit-qutrit system under global dephasing noise and $\tau = 3$ under local depolarizing noise with $S.L = 0$. | Qutrit-qutrit state under global dephasing as well as local depolarizing noises has shown complete decay of $Q_c$. The decay here is exponential and has no revivals [39]. |

TABLE III: Shows the comparison of the time preservation of the purity, coherence under $OU_n$ with the previous studies. This table explores the preservation capacity and the description of the detrimental effects produced by the current and other previously investigated noises. Here, $ce$, $de$ and $GMQD$ represents common, different system-environment coupling and geometric measure of quantum discord respectively.

information are not fully lost, and the saturation threshold is reached. The measures’ maximum and minimum values are comparable, and there is a single saturation level for all $g$ values. As a result, the findings are consistent and valid. By comparing Figs. 2, 3 and 6, it is easy to determine that this noise has a dominant character to suppress oscillation and preservation capacity of the system. We noticed that as compared to the previously investigated systems in [34, 37, 38], the single qutrit system has exhibited superior preservation capacity. Aside from that, raising the noise parameter $g$ causes the purity, coherence and information to decay faster. As seen, the slopes move towards the red end with increasing values of $g$, suggesting greater degradation. However,
by limiting $g$ as minimal as possible, the optimal smaller decay can be produced. In contrast to $\mathcal{F}G_n$, the present noise phase has shown to have a lower deteriorating character for the memory properties of the system, as the preservation time encountered in the current case is longer.

4. Classical field with $\mathcal{PL}_n$

To evaluate the degrading effects of the $\mathcal{PL}_n$ over the dynamics of the single qutrit state, we perform an average of the final density matrix given in Eq. $(14)$ over the noise phase with $\beta$-function given in Eq.$(13)$.

Fig. 7 shows the time evolution of the purity ($a,c$) and von Neumann entropy ($b,d$) for the single qutrit system when subjected to classical field generating power-law noise when $g = 1$ (green), 3 (blue), 10 (red) with $\alpha = 3$. Bottom panel: Time evolution of the purity ($a$) and Von Neumann entropy ($b$) for the single qutrit system when subjected to classical field generating power-law noise when $\alpha = 3$ (green), 5 (blue) and 10 (red) when $g = 0.5$ against evolution parameter $\tau = 2$.

![Image](image.png)

FIG. 7: Upper Panel: Time evolution of the purity ($a$) and von Neumann entropy ($b$) for the single qutrit system when subjected to classical field generating power-law noise when $g = 1$ (green), 3 (blue), 10 (red) with $\alpha = 3$. Bottom panel: Time evolution of the purity ($a$) and Von Neumann entropy ($b$) for the single qutrit system when subjected to classical field generating power-law noise when $\alpha = 3$ (green), 5 (blue) and 10 (red) when $g = 0.5$ against evolution parameter $\tau = 2$.

In the current case, the dynamics of the system is investigated under two different noisy parameters, namely $g$ (upper panel) and $\alpha$ (bottom panel). By comparing Figs. 2, 3 and 7, the dissipative capability of the $\mathcal{PL}_n$ in terms of two noisy parameters to disappear revivals and lower the initial encoded coherence and information can be validated. Large values of $g$ have degraded purity, coherence, and information
TABLE IV: Shows the comparison of the time preservation of the purity, coherence under $\mathcal{PL}_n$ with the previous studies. Moreover, this table explores the preservation capacity as well as the description of the detrimental effects produced by the current and other previously investigated noises.

More than the parameter $\alpha$. For $g$, the slopes for purity and von Neumann entropy reach saturation values faster than for $\alpha$. Aside from the decaying nature of the $\mathcal{PL}_n$, the smaller partial loss rather than complete decay cannot be overlooked. This directly opposes most of the prior findings for various quantum systems [26, 34, 37, 40, 41]. All slopes for various noise parameter values reach a single saturation level, although at different intervals. As a result, there appears to be a strong link between the measures for demonstrating consistency and agreement in the results. As the values of both parameters increase, the slopes move from green to red, implying that the decay rate increases. In current cases, this qualitative behaviour is comparable to that of the $\mathcal{OU}_n$; nevertheless, the decay levels encountered for the single qutrit system differ significantly from those previously investigated [26, 34]. There was no evidence of ESD or ESB, and both noisy parameters showed a monotonic reduction. As a result, there is no way for information from the environment to flow back into the system, contradicting the findings of [19, 24, 28, 42, 43]. Because of this noise, purity, coherence and information are permanently lost rather than experiencing periodic transitory deterioration.

C. Relative dynamics

Fig. 8 evaluates the time evolution of the purity ($a$) and von Neumann entropy ($b$) for the single qutrit system under the presence of $\mathcal{G}_n$ in green, $\mathcal{OU}_n$ in blue, and $\mathcal{PL}_n$ in red slopes. We mainly focus on protecting the purity, coherence and information for a large duration. Proceeding to this, we have set the noise parameter $g = 10^{-3}$ in non-dashed and $10^{-2}$ in dashed slopes for $\tau = 50$. Note that the $\mathcal{FG}_n$ is excluded from the current study due to its discrete nature (where $0 < H > 1$). We discovered that the current quantitative behaviour of purity and von Neumann
FIG. 8: Time evolution of the purity (a) and von Neumann entropy (b) for single qutrit system under Gaussian (green), Ornstein Uhlenbeck (blue) and power-law noise (red) stemming from the classical field. Here, $g = 10^{-3}$ non-dashed and $g = 10^{-2}$ (dashed) against evolution parameter $\tau = 50$.

| Noise | $g = 10^{-3}$ | $g = 10^{-2}$ | $g = 1$ | $g = 5$ |
|-------|---------------|---------------|---------|---------|
| $G_n$ | $\tau = 46$  | $\tau = 18$  | $\tau = 2$ | $\tau = 1.3$ |
| $OU_n$| $\tau = 50$  | $\tau = 20$  | $\tau \geq 2$ | $\tau = 1.4$ |
| $PL_n$| $\tau = 32$  | $\tau = 12$  | $\tau = 2$ | $\tau = 1.3$ |

TABLE V: The preservation time for the coherence and information under the Gaussian process is shown in this table. These are stated in terms of multiple $g$ values, making it easier to construct quantum protocols that need precision and accuracy. Under the current noise circumstances, the purity saturation level stayed at 0.944 and the Von Neumann entropy saturation threshold remained at 0.13. When compared to the other quantum systems explored previously, these saturation values show the least degradation.

entropy differs significantly from that observed when $g$ is large. The preservation duration of the phenomenon is substantially longer for minor values of $g$, as shown. In [34], this was also seen for a maximal entangled tripartite state under $OU_n$, however, with different decay levels. The qualitative degradation behaviour is monotone, as it was in the prior situations. As a result, there are no entanglement revivals in any proper adjustment of the noisy parameters for these noises.

In comparison, $OU_n$ followed by $G_n$ has had less degrading impacts on the purity, coherence, and information survival over a long period of time. Finally, the $PL_n$ is found to be the most harmful to the dynamics of the purity, coherence and information, with saturation levels reaching earlier, especially for large $g$ values. The single qutrit system’s quantitative degradation is partial, which contradicts most of the earlier results, for example, those given in [19, 26, 33, 34, 43]. Most significantly, we found the decay rate to be greatly regulated by altering the values of $g$, which directly increases as this parameter is increased. Regardless of the preservation duration and parameter values, all noises can induce a similar amount of decay. This strongly suggests the relevance between the Gaussian nature of the noises. As shown, following maximum decay, the slopes under all the noises remained at the same elevation level.
IV. CONCLUSION

When subjected to a classical fluctuating field, we explored the dynamics of purity and coherence in a single qutrit system. The classical fields are supposed to be driven by a pure Gaussian process that generates fractional Gaussian ($\mathcal{FG}_n$), Gaussian ($\mathcal{G}_n$), Ornstein Uhlenbeck ($\mathcal{OU}_n$) and power-law noise ($\mathcal{PL}_n$). A relationship between the current and previously examined quantum systems, noises and noise parameters has been established. The coherence and information retention capability of a single qutrit as well as other systems with many qubits and qutrits has also been examined. Hereafter, we distinguish between local fields with noisy and noiseless characters. Finally, by deploying purity and von Neumann entropy, the amount of pureness, coherence and information preserved by the single qutrit system have been computed.

Our findings show that classical random fields are good resources for the dynamics of quantum correlations, coherence, and information, and they can be related to prior findings in [28, 36, 37, 42, 43]. By contrasting Figs.2 and 3, one can easily distinguish between the qualitative behaviour of the coherence and information dynamics, as well as the related preservation interval, in noiseless and noisy classical fields. When compared to other quantum systems, the single qutrit system has a prominent advantage of having less coherence and information loss with higher purity under Gaussian noises. Other quantum systems, on the other hand, have demonstrated more degradation when subjected to Gaussian and non-Gaussian noise, as addressed in [29, 44, 45].

Except for the $\mathcal{FG}_n$, all the noises have displayed the same degrading behaviour regarding purity, coherence, and information. For the current Gaussian noises, we discovered that as the noise parameters were increased, the decay rate increased. Conversely, coherence and information become more robust as $H$ increases, but only at initial intervals. This qualitative behaviour of $H$ differs from all the previously examined noise parameters. Additionally, the saturation levels in each case were consistent. This suggests the relevance between the noise phases for having the same Gaussian nature and causing an equal amount of decay. The same result can be seen for the dynamics of tripartite state under coloured noise in [46, 47], but with different decay amounts. Most significantly, the preservation time remained greater in the case of $\mathcal{OU}_n$ for small $g$ and under $\mathcal{PL}_n$ for small $\alpha$. In the case of measures, purity and von Neumann entropy were discovered to be in perfect agreement. The maxima and minima of both measures are perfectly concordant across comparable periods. Both procedures produced the same preservation time and saturation values. As a result, purity and von Neumann entropy are trustworthy metrics that assess the purity, coherence, and information initially encoded in a quantum system.

Finally, we discovered that under the current Gaussian noises, coherence and information suffer a less but rapid loss strongly depending upon the noise parameters values. Non-Gaussian noises, on the other hand, are observed to lose coherence and quantum correlations later [29, 37, 46, 47]. The preservation time under Gaussian noises remained sufficiently small for the single qutrit and other quantum systems. To minimise this decline, the Gaussian noisy parameters should be kept as low as possible. In particular, for $g = 10^{-3}$ and $g = 10^{-2}$ the coherence, information and, in turn, quantum correlations can be preserved for enough long interaction time. Except for the Hurst exponent $H$ of $\mathcal{FG}_n$, this will assure an increase in time preservation of quantum correlations and coherence. For extended memory features resulting in smaller coherence, quantum correlations, and information losses, we strongly recommend using the single qutrit entangled state over other quantum systems.
V. APPENDIX

This section gives the details of the numerical simulations done for the time evolution of the single qutrit system. We first give the time evolved density matrix of the three-level system by using time unitary operator from Eq. (3) and is given as:

\[
U_{\eta t}(t) = e^{-it\epsilon}
\begin{bmatrix}
\cos\left(\frac{\phi}{2}\right)^2 & -i\sin[\phi] & \frac{1}{2}(1 + \cos[\phi]) \\
-i\sin[\phi] & \cos[\phi] & -i\sin[\phi] \\
\frac{1}{2}(1 + \cos[\phi]) & -i\sin[\phi] & \cos\left(\frac{\phi}{2}\right)^2
\end{bmatrix}.
\] (17)

Hereafter, by using Eq. (4), the time evolved density matrix for the single qutrit system reads as:

\[
\rho_{\eta t}(t) = \frac{1}{12}
\begin{bmatrix}
3 + \cos[2\phi] & 4 + i\sqrt{2}\sin[2\phi] & 3 + \cos[2\phi] \\
4 - i\sqrt{2}\sin[2\phi] & 6 - 2\cos[2\phi] & 4 - i\sqrt{2}\sin[2\phi] \\
3 + \cos[2\phi] & 4 + i\sqrt{2}\sin[2\phi] & 3 + \cos[2\phi]
\end{bmatrix},
\] (18)

where, \(\phi\) is the phase of the system and is defined by \(\phi = -i\omega\eta(t)\omega t\) where \(\omega = 2\). Next, by using Eq. (14), we can get the final density matrix for the single qutrit system under Gaussian process as:

\[
\rho_f(t) = \frac{1}{12}
\begin{bmatrix}
\frac{1}{3} (3 + e^{-2\beta_{ab}(t)}) & \frac{1}{3} - e^{-2\beta_{ab}(t)} & \frac{1}{3} (3 + e^{-2\beta_{ab}(t)}) \\
\frac{1}{3} (3 + e^{-2\beta_{ab}(t)}) & \frac{1}{3} - e^{-2\beta_{ab}(t)} & \frac{1}{3} (3 + e^{-2\beta_{ab}(t)})
\end{bmatrix}
\] (19)

where \(\beta_{ab}(t)\) is defined in Eq. (9). Next, the numerical results for the purity and von Neumann entropy for the time evolved density matrix can be given by using the Eq. (15) and (16) and reads as:

\[
D(t) = -\frac{1}{6} \left(3 - e^{-4\beta_{ab}(t)}\sqrt{e^{4\beta_{ab}(t)} + 8e^{8\beta_{ab}(t)}}\right) \log \left[\frac{1}{6} \left(3 - e^{-4\beta_{ab}(t)}\sqrt{e^{4\beta_{ab}(t)} + 8e^{8\beta_{ab}(t)}}\right) + 8e^{8\beta_{ab}(t)}\right]
\]

\[
-\frac{1}{6} \left(3 + e^{-4\beta_{ab}(t)}\sqrt{e^{4\beta_{ab}(t)} + 8e^{8\beta_{ab}(t)}}\right) \log \left[\frac{1}{6} \left(3 + e^{-4\beta_{ab}(t)}\sqrt{e^{4\beta_{ab}(t)} + 8e^{8\beta_{ab}(t)}}\right) + 8e^{8\beta_{ab}(t)}\right]
\] (20)

and

\[
P_r(t) = \frac{1}{18} \left(17 + e^{-4\beta_{ab}(t)}\right).
\] (21)

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