Is the Sullivan Process Compatible with QCD Chiral Dynamics?

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We calculate the leading non-analytic quark-mass dependence in the moments of isovector quark distributions using heavy-baryon chiral perturbation theory. The results differ from what has been obtained from the Sullivan process in which hard scattering occurs through the virtual pion cloud of the nucleon. Our results provide useful guidance in formulating meson-cloud models consistent with chiral dynamics and can be used to constrain the extrapolations of the existing lattice QCD results to the physical quark masses.

The nucleon (proton and neutron) has a quark sea. For a long time, the leading picture for this sea comes from the meson-cloud model in which the nucleon can virtually dissociate into mesons such as pions, kaons, etc., plus a baryon core. A concrete realization of this model can be found in lepton-nucleon deep-inelastic scattering in which the (Bjorken-like) virtual photon scatters deep-inelastically off the meson cloud. The Feynman diagram for this process is shown in Fig. 1 and is called the Sullivan process [1]. Many interesting results from the meson-cloud model have been obtained, which have shed important light on the dynamics of the quark sea.

| FIG. 1. The Sullivan process in which the virtual photon scatters off the meson cloud in the nucleon. |

A crucial question about the meson-cloud model and the associated Sullivan Process is to what extent the model can be justified from the underlying quantum chromodynamics (QCD) and its low-energy effective theories? Recently, A. Thomas et. al. pointed out the best place to check is the chiral properties of the model predictions [2]. They worked out, for the first time, the detailed leading non-analytic quark (pion) mass dependence of the parton distributions probed in the Sullivan process. This letter seeks to answer the above question by calculating the same type of chiral logarithms using the low-energy effective theory of QCD: the heavy-baryon chiral perturbation theory [3]. As it turns out, there are subtle differences between the traditional approach in calculating the Sullivan process and chiral perturbation theory. Our results can be used to improve upon the formulation of the Sullivan process and to study the quark mass dependence of the lattice QCD calculations.

We are interested in the moments of the quark distributions in the proton,

\[
\begin{align*}
\langle x^{n-1} \rangle_q &= \int_0^1 dx x^{n-1} (q(x) + (-1)^n \overline{q}(x)) , \\
\langle x^{n-1} \rangle_{\Delta q} &= \int_0^1 dx x^{n-1} (\Delta q(x) + (-1)^{n-1} \Delta \overline{q}(x)) , \\
\langle x^{n-1} \rangle_{\delta q} &= \int_0^1 dx x^{n-1} (\delta q(x) + (-1)^n \delta \overline{q}(x)) , \\
\end{align*}
\]

where \( q (\overline{q}) \) is the quark (antiquark) spin-averaged distribution, \( \Delta q (\Delta \overline{q}) \) the helicity distribution, and \( \delta q (\delta \overline{q}) \) the transversity distribution [4]. The variable (Feynman) \( x \) is the momentum fraction of the proton carried by a quark in the infinite momentum frame, and for simplicity we have suppressed the renormalization scale dependence.

In QCD, the moments are related to the forward matrix elements of the twist-two operators

\[
\begin{align*}
\mathcal{O}^{\mu_1 \cdots \mu_n} &= T \gamma_5 (\mu_1 D^{\mu_2} \cdots i D^{\mu_n}) q , \\
\mathcal{\overline{O}}^{\mu_1 \cdots \mu_n} &= \overline{T} \gamma_5 (\mu_1 D^{\mu_2} \cdots i D^{\mu_n}) q , \\
\mathcal{O}_T^{\mu_1 \cdots \mu_n} &= \overline{\sigma}^{\alpha} (\mu_1 \gamma_5 D^{\mu_2} \cdots i D^{\mu_n}) q , \\
\end{align*}
\]

through the relations,

\[
\begin{align*}
\langle PS \mid \mathcal{O}^{\mu_1 \cdots \mu_n} \mid PS \rangle &= 2 \langle x^{n-1} \rangle_q P^{\mu_1} \cdots P^{\mu_n} , \\
\langle PS \mid \mathcal{\overline{O}}^{\mu_1 \cdots \mu_n} \mid PS \rangle &= 2 \langle x^{n-1} \rangle_{\Delta q} M S^{\mu_1} P^{\mu_2} \cdots P^{\mu_n} , \\
\langle PS \mid \mathcal{O}_T^{\mu_1 \cdots \mu_n} \mid PS \rangle &= 2 \langle x^{n-1} \rangle_{\delta q} \times M S^{\alpha} P^{\mu_1} P^{\mu_2} \cdots P^{\mu_n} ,
\end{align*}
\]

where \((\cdots)\) and \([\cdots]\) denote, respectively, the symmetrization and antisymmetrization of the indices in between. \( \mid PS \rangle \) is the ground state of the nucleon with four-momentum \( P^\mu \) and polarization vector \( S^\mu (S^2 = -1) \). M is the nucleon mass. All tensors are trace free.
To calculate the leading non-analytic quark mass dependence of these moments in heavy-baryon chiral perturbation theory, we need to construct a chiral expansion of the quark operators in terms of the hadronic (pion and nucleon) operators with the identical symmetry properties. In this study, we focus on the isovector combinations of the quark densities, up minus down quarks. To leading order in chiral power counting, we have,

\[ \mathcal{O}_{\mu_1 \cdots \mu_n}^{u-d} \sim \bar{N} v^{(\mu_1 \cdots \mu_n)} (u \tau_3 u + u \tau_3 u^\dagger) N, \]

\[ \tilde{\mathcal{O}}_{\Delta u - \Delta d}^{\mu_1 \cdots \mu_n} \sim \bar{N} S^{(\mu_1 \cdots \mu_n)} (u \tau_3 u^\dagger + u \tau_3 u) N, \]

\[ \tilde{\mathcal{O}}_{\delta u - \delta d}^{\mu_1 \cdots \mu_n} \sim \bar{N} S^{(\mu_1 \cdots \mu_n)} (u \tau_3 u^\dagger - u \tau_3 u) N, \]

where \( N \) and \( \bar{N} \) are the nucleon fields, \( v^{\mu} \) is the nucleon four-velocity, and \( u = \exp(i\pi^a \tau^a / 2 f_\pi) \) with pion fields \( \pi^a \) and decay constant \( f_\pi = 93 \) MeV. \( \alpha, \beta \) and \( \gamma \) are unknown coefficients. For \( n = 1 \), there is also a leading pion operator in \( \mathcal{O}_{u-d}^{\mu_1 \cdots \mu_n} \), which has not been shown explicitly.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig2}
\caption{Feynman diagrams contributing to the leading chiral logarithms in the nucleon matrix elements of twist-two operators. The dashed lines represent pions.}
\end{figure}

The contributions to the leading chiral logarithms of the twist-two matrix elements come from three separate sources: first, the wave function renormalization of the nucleon states; second, the virtual-pion cloud as shown in the first diagram in Fig. 2; third, the pion tad-pole contribution as shown in the second diagram in Fig. 2. A straightforward calculation yields,

\[ \langle x^n \rangle_{u-d} = C_n \left\{ 1 - \frac{(3g_A^2 + 1) m^2}{4 \pi f_\pi^2} \ln \left( \frac{m_\pi^2}{m^2} \right) \right\}, \]

\[ \langle x^{n-1} \rangle_{\Delta u - \Delta d} = \tilde{C}_{n-1} \left\{ 1 - \frac{(2g_A^2 + 1) m^2}{4 \pi f_\pi^2} \ln \left( \frac{m_\pi^2}{m^2} \right) \right\}, \]

\[ \langle x^{n-1} \rangle_{\delta u - \delta d} = \tilde{C}_{n-1} \left\{ 1 - \frac{(4g_A^2 + 1) m^2}{2 \pi f_\pi^2} \ln \left( \frac{m_\pi^2}{m^2} \right) \right\}, \]

where \( C_n, \tilde{C}_n, \) and \( \tilde{C}_n \) are the corresponding moments in the chiral limit, \( g_A \) is the isovector axial charge in the same limit, and the renormalization scale \( \mu \) can be taken to be \( 4 \pi f_\pi \sim 1 \) GeV. The relative coefficients between the results in the chiral limit and the leading chiral-logarithms are seen to be fixed. For \( \langle 1 \rangle_{\Delta u - \Delta d} \), which is the physical isovector axial charge, our leading non-analytic correction agrees with the previous calculation \( g_A \). \( \langle 1 \rangle_{u-d} \) is the isospin charge which is protected from chiral corrections by the isospin symmetry. The next important chiral correction is of type \( m^2 / (4 \pi f_\pi^2) \) which is linear and analytical in quark masses. These corrections cannot be computed without additional parameters.

The leading non-analytical quark mass dependence of \( \langle x^n \rangle_{u-d} \) was calculated using the pion-cloud model (the Sullivan process) in ref. \( \cite{5} \). Instead of the pre-factor \( (3g_A^2 + 1) \) front of the chiral logarithm, they found \( 4g_A^2 \). Although in the \( g_A = 1 \) limit the two results agree, they are different in general. In particular, in the limit of a large number of colors \( (N_c) \), \( g_A \sim N_c \). \( \cite{5} \).

We find that the above discrepancy comes from the differences between the linear and non-linear formulations of chiral expansion. While the non-linear formulation used in our calculation has a simple and clear power counting scheme, the linear formulation does not. The Sullivan process is traditionally done in the linear sigma model with the following lagrangian

\[ L_{\text{linear-}} = \bar{\Psi} (i \partial \tau + g_{\pi NN} (\sigma + i \tau \tau \cdot \sigma \gamma_5)) \Psi + \ldots \]

where \( \Psi \) represents the nucleon field in the linear representation and \( g_{\pi NN} \) is the nucleon-pion coupling. The pure meson sector of the lagrangian has been omitted. After spontaneous symmetry breaking, the above model in principle yields a Goldberg-Treiman relation \( g_{\pi NN} = M / f_\pi \) with \( g_A = 1 \). In the literature \( \cite{3} \), however, \( g_{\pi NN} \) in the linear-\( \sigma \) model result is usually replaced by the full Goldberg-Treiman relation \( g_{\pi NN} = g_A M / f_\pi \).

To make the linear \( \sigma \) model result consistent with that of non-linear chiral expansion, one must add higher-dimensional operators to the lagrangian in Eq. \( \cite{5} \). As we have mentioned above, the interaction terms of different mass dimensions contribute to the same order of chiral power is the main disadvantage of the linear formulation. In the present example, we need to add the following dimension-five term \( \cite{5} \),

\[ L' = g' \bar{\Psi} \left( i \tau \tau \cdot \partial \sigma - \sigma \tau \cdot i \partial \tau \right) \gamma_5 + \tau \cdot (\partial \tau \cdot i \partial \tau) \Psi. \]

To restore the full Goldberg-Treiman relation, \( g' \sim g_A - 1 \). Setting \( g_A = 1 \) in the result of Ref. \( \cite{5} \) and adding the additional contributions generated from the above lagrangian, we find the chiral logarithms in the linear formulation coincide completely with the results above.

Our results can be used to constrain the quark mass dependence of the matrix elements calculated in lattice
QCD [11]. Indeed the isovector combinations of the moments are easier to obtain since they do not require a computation of expansive “disconnected” diagrams. The results can also be used to improve the meson-cloud model to the point where it becomes consistent with the chiral physics of QCD.

To summarize, we have calculated the leading non-analytic quark mass dependence of the twist-two, isovector quark matrix elements in the nucleon. The results indicate that the traditional approach in calculating the Sullivan process needs to be modified to be fully consistent with the chiral dynamics of QCD.

Note added in proof: After this work was completed, we learned a similar work by D. Arndt and M. J. Savage [12]. For the same quantities computed, both papers agree.

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