A Novel Approach to Non-linear Shock Acceleration
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First order Fermi acceleration at astrophysical shocks is often invoked as a mechanism for the generation of non-thermal particles. This mechanism is especially simple in the approximation that the accelerated particles behave like test particles, not affecting the shocked fluid. Many complications enter the calculations when the accelerated particles have a backreaction on the fluid, in which case we may enter the non-linear regime of shock acceleration. In this paper we summarize the main features of a semi-analytical approach to the study of the non-linearity in shock acceleration, and compare some of the results with previous attempts and with the output of numerical simulations.

1. Introduction

Shock acceleration has been studied carefully and a vast literature exists on the topic, including some recent excellent reviews [1,2]. Hence we do not try to provide here any extensive introduction to the problem, but we rather limit ourselves to summarize some of the open issues related to the backreaction of the accelerated particles onto the shocked fluid. The accelerated particles start to affect the fluid when their energy density becomes comparable to the kinetic energy density of the fluid. In this regime the test particle approximation breaks down, and the standard results of shock acceleration cannot be recovered. The only way to have a complete quantitative picture of this problem is to use numerical simulations [3–6], but it is useful to have a practical analytical tool to understand the physics behind the simulations and also to take into account the non linear effects also when these simulations are not available, which is unfortunately the rule rather than the exception.

Numerical simulations show that even when the fraction of particles injected from the plasma is relatively small, the energy channelled into these few particles can be close to the kinetic energy of the unshocked fluid, making the test particle approach unsuitable. The most visible effect is on the spectrum of the accelerated particles, which shows a peculiar hardening at the highest energies.

The need to have a theoretical understanding of the non-linear effects in particle acceleration fueled many efforts in finding some effective though simplified picture of the problem (see [8] for a discussion of these alternative approaches). We will compare our findings with those of Ref. [7].

In the present paper we summarize the results widely discussed in [8], where an approach was proposed that provides a very nice fit to the results of simulations and is in agreement with previous analytical calculations.

2. Non Linear Shock Acceleration

The distribution function of the particles accelerated at a planar infinite shock can be written in implicit form as [8]:

\[
 f_0(p) = \frac{N_{inj} q_s}{4\pi p_{inj}} \times \\
 \exp \left\{ - \int_{p_{inj}}^{p} \frac{dp}{p} \left[ \frac{3u_p}{u_p - u_2} + \frac{1}{u_p - u_2} \right] \right\},
\]

where we put \( u_p = u_1 + \frac{1}{f_0(p)} \int_0^\infty dx \left( \frac{4u}{dx} \right) f(x, p) \), \( q_s = \frac{3R_{sub}}{R_{sub} - 1} \), and \( R_{sub} = u_1/u_2 \) is the compression factor at the shock surface \([u_1 (u_2) is the fluid velocity upstream (downstream)]\). \( N_{inj} \) is the
number of density of particles injected at the shock, parametrized here as \(N_{\text{inj}} = \eta N_{\text{gas,1}}\), where \(N_{\text{gas,1}}\) is the gas density at \(x = 0^+\) (upstream). Particles are assumed to be injected at the shock surface with momentum \(p_{\text{inj}}\). Eq. (2) tells us that the spectrum of accelerated particles has a local slope given by \(Q(p) = -\frac{3\rho u_p}{u_g-u_p} \ln \rho \frac{d\rho}{dp} \). The problem of determining the spectrum of accelerated particles is then solved if the relation between \(u_p\) and \(p\) is found.

The thermodynamic properties of the shocked fluid are embedded in the usual conservation equations, including now the contribution from accelerated particles. The mass and momentum conservation equations read:

\[\rho_0 u_0 \rho_0 u_0^2 + P_{g,0} = \rho_0 u_p^2 + P_{g,p} + P_{\text{CR},p},\] (2)

where \(P_{g,0}\) and \(P_{g,1}\) are the gas pressures at the point \(x = +\infty\) and \(x = x_p\) respectively, and \(P_{\text{CR},p}\) is the pressure in accelerated particles at the point \(x_p\) (we used the symbol CR to mean cosmic rays, to be interpreted here in a very broad sense). In writing eqs. (2) and (3) we implicitly assumed that the average velocity \(u_g\) coincides with the fluid velocity at the point where the particles with momentum \(p\) turn around to recross the shock. Our basic assumption is that the diffusion is \(p\)-dependent and that therefore particles with larger momenta move farther away from the shock than lower momentum particles. At each fixed \(x_p\) only particles with momentum larger than \(p\) are able to affect the fluid. Since only particles with momentum \(p > p\) can reach the point \(x = x_p\), we can write \(P_{\text{CR},p} = \frac{4\pi}{3} \int_{p}^{p_{\text{max}}} dp v(p)f(p)\), where \(v(p)\) is the velocity of particles whose momentum is \(p\), and \(p_{\text{max}}\) is the maximum momentum achievable in the specific situation under investigation. In realistic cases, \(p_{\text{max}}\) is determined from geometry or from the duration of the shocked phase, or from the comparison between the time scales of acceleration and losses. Here we consider it as a parameter to be fixed a priori. From eq. (3) we can see that there is a maximum distance, corresponding to the propagation of particles with momentum \(p_{\text{max}}\) such that at larger distances the fluid is unaffected by the accelerated particles and \(u_p = u_0\). Assuming an adiabatic compression of the gas in the upstream region, we can write \(P_{g,p} = \rho_0 u_0^2 \left(\frac{\gamma g}{M_0^2}\right)^{\gamma g} = P_{g,0} \left(\frac{\gamma g}{M_0^2}\right)^{\gamma g}\), where we used the conservation of mass, eq. (2). The gas pressure far upstream is \(P_{g,0} = \rho_0 u_0^2/(\gamma g M_0^2)\), where \(\gamma g\) is the ratio of specific heats (\(\gamma g = 5/3\) for an ideal gas) and \(M_0\) is the fluid Mach number far upstream. Note that the adiabaticity condition cannot be applied at the shock jump, where the adiabaticity condition is clearly violated.

After some algebra (see [8] for the details), the conservation equations imply the following equation:

\[\ln(DU) + \ln\left[1 - \frac{1}{M_0^2}U^{-(\gamma g+1)}\right] = \ln\left[1 + \frac{N_{\text{inj}}}{3 \rho_0 u_0^2} v(p)f(p)\right] + 4 \ln\left(\frac{p}{p_{\text{inj}}}\right) - \int_{p_{\text{inj}}}^p dp \frac{3 R_{\text{tot}} U - 1}{R_{\text{sub}} - 1},\] (4)

where \(R_{\text{tot}} = u_0/u_2\), \(U(p) = u_p/u_0\), and we put \(DU = dU/d\ln p\). Solving this differential equation provides \(U(p)\) and therefore the spectrum of accelerated particles, through eq. (3).

The operative procedure for the calculation of the spectrum of accelerated particles is simple: we fix the boundary condition at \(p = p_{\text{inj}}\) such that \(U(p_{\text{inj}}) = u_1/u_0\) for some value of \(u_1\) (fluid velocity at \(x = 0^+)\). The evolution of \(U\) as a function of \(p\) is determined by eq. (3). The physical solution must have \(U(p_{\text{max}}) = 1\) because at \(p > p_{\text{max}}\) there are no accelerated particles to contribute any pressure. There is a unique value of \(u_1\) for which the fluid velocity at the prescribed maximum momentum \(p_{\text{max}}\) is \(u_{\text{max}} = u_0\). Finding this value of \(u_1\) completely solves the problem, since eq. (4) provides \(U(p)\) and therefore the spectrum of accelerated particles. Fig. 1 illustrates the distribution function (multiplied by \(p^4\)) for Mach number at infinity \(M_0 = 43, p_{\text{inj}} = 10^{-2}mc\) and \(\eta = 10^{-3}\) and for \(p_{\text{max}} = 10^3mc\) (solid line), \(p_{\text{max}} = 10^5mc\) (dotted line) and \(p_{\text{max}} = 10^7mc\) (dashed line), where \(m\) is the mass of the accelerated particles. The superimposed symbols show the corresponding results for the method in [8]. In Fig. 2 we plotted the
results of our method for another set of parameters (indicated in the figure) and compared these results with the output of numerical simulations reported in [7]. It can be easily seen that the agreement is impressive.

3. Conclusions

We report on a novel semi-analytical approach to non linear shock acceleration, which improves some previous attempts of other authors. This method is in good agreement with the previous approaches, and is also in impressive agreement with the results of numerical simulations on shock acceleration. An extensive sets of predictions of this approach and a more complete comparison with previous results are presented in [8]. The most important phenomenological consequence of the inclusion of the non linear effects in shock acceleration is the hardening of particle spectra, which may reflect in a corresponding hardening of the spectra of secondary particles (photons, electrons and neutrinos) generated in the interactions of the accelerated particles with the environment.

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