A Three-stage Multiple Criteria Decision Making Model Based on AHP-TOPSIS and AFS Concept Description

Lili Tao
School of Information Science & Engineering, Dalian Ocean University, Dalian, China.
Email: taolili1986@163.com

Abstract. In this paper, we propose an integrated model to solve the multi-criteria decision making problem. Firstly, AHP method is applied to calculate the weight value of each attribute on all the alternatives; secondly, TOPSIS method is used to get the preference value based on the weights calculated in the first step; finally, AFS concept description algorithm is employed to give the semantic descriptions of performance characteristics for all the alternatives. Compared with other models, this model can not only consider the comprehensive preference value, but also consider the specific performance characteristic of the alternative, which makes the decision more scientific. The supplier selection problem is applied to explain the application ability of this model.

1. Introduction
Multiple attribute decision making (MCDM) is an important research topic in the field of modern decision science. Its theory and method are widely used in many fields such as economy, management and military affairs, such as investment decision-making, factory location selection, weapon system performance evaluation, comprehensive ranking of economic benefits, etc. With the development of economy, science and technology and society, the problem of MCDM is becoming more and more complex so that it can not adapt to this kind of change just by the decision maker's personal knowledge and experience. Therefore, it is necessary to study the method of multi-attribute decision making.

In the past research, many models have been proposed to deal with the multi-criteria decision making problems. For instance, Toloo and Nalchigar [1] proposed a new model for finding most BCC-efficient DMU by integrated DEA. Yue [2] applied TOPSIS method for group decision making. Chen et al. [3] applied DEA to compute Banking efficiency in China. Shih et al. [4] made an extension of TOPSIS for group decision making. Garg [5] made a special issue on Pythagorean fuzzy set and its extensions in decision-making process. Among these methods, AHP and TOPSIS are more classic.

In depth study of existing techniques, it is obvious that almost all the MCDM methods have a common defect that there are not some semantic interpretations of decision results. In order to solve above issue, we try to integrate AHP, TOPSIS and AFS concept description to build a three-stage model. At first, AHP method is used to compute the weight of each attribute of all the alternatives. Then, the TOPSIS method is used to rank all the alternatives according to the weights of attributes. At last, we provide semantic descriptions of performance characteristics for all the alternatives by AFS concept description algorithm so that the decision maker can make the best choice on the basis of their own preferences on the attributes through comparing the semantic descriptions.
2. Knowledge of AFS Concept Description

2.1. AFS Structures of the Data

Definition 1 [6,7]. Let \( X, M \) be sets and \( 2^M \) be the power set of \( M \). Let \( \tau : X \times X \to 2^M \), \((M, \tau, X)\) is called an AFS structure if \( \tau \) satisfies the following axioms:

\[
\text{AX1: } \forall (x_1, x_2) \in X \times X, \tau(x_1, x_2) \subseteq \tau(x_i, x_j)
\]

\[
\text{AX2: } \forall (x_1, x_2), (x_3, x_3) \in X \times X, \tau(x_1, x_2) \cap \tau(x_3, x_3) \subseteq \tau(x_4, x_5).
\]

In practical applications, If \( \tau : X \times X \to 2^M \) is defined as follows:

\[
\tau(x, y) = \{ m \mid m \in M, \ x \geq_m y \} \in 2^M
\]

Then \((M, \tau, X)\) is an AFS structure.

Definition 3 [8]. Let \( X \) and \( M \) be sets, \((M, \tau, X)\) be an AFS structure and \((M, \sigma, \mu)\) be a measure space, where \( \mu \) is a finite and positive measure. The membership function of \( \eta \) is defined as follows: for any \( x \in X \),

\[
\mu_{\eta}(x) = \sup_{i \in I} \frac{A_i^\eta(x)}{\mu(X)},
\]

where \( A_i^\eta(x) = \{ y \in X | \tau(x, y) \supseteq A_i \} = \{ y \in X | x \geq_m y, \forall m \in A_i \} \).

In this paper, let \( \sigma = 2^X \), for \( V \in 2^X \), \( \mu(V) = |V| \) (|V| is the number of elements in \( V \)). Then (2) can be converted into:

\[
\mu_{\eta}(x) = \sup_{i \in I} \frac{|A_i^\eta(x)|}{|X|}.
\]

2.2. The Algorithm of AFS Concept Description

(1) Let \( \varepsilon \geq 0 \), \( B_{x_i}^\varepsilon \) defined as follows:

\[
B_{x_i}^\varepsilon = \{ m \in EM \mid \mu_m(x_i) \geq \mu_{\eta}(x_i) - \varepsilon \}.
\]

(2) \( A_{x_i}^\varepsilon \) defined as follows:

\[
A_{x_i}^\varepsilon = \{ \prod_{m \in A} \mu_m(x_i) \geq \mu_{\eta}(x_i) - \varepsilon, A \subseteq B_{x_i}^\varepsilon \}
\]

(3) The best description of \( x_i \) defined as follows:

\[
\xi_{x_i} = \arg \min_{\zeta \in A_{x_i}} \left\{ \sum_{x \in X, x \neq x_i} \mu_{\zeta}(x) \right\}
\]

3. A Three-stage Model Based on AHP-TOPSIS and AFS Concept Description

Now, we represent each step of the model.

3.1. Analytic Hierarchy Process

First of all, the AHP method is used to compute the weight of each attribute, the details are as followings:
(1) Define the comparison matrix. It usually use 1 to 9 as its scale.
(2) Compute the weight.
   a. Add all the elements of each column of the comparison matrix to get the sum.
   b. Divide each element in the comparison matrix by the sum of its column.
   c. Compute the average value of all elements in each row.
(3) Test the consistency.
   The consistency index $CI = \frac{\lambda_{\text{max}} - n}{n-1}$, and the consistency ratio $CR = \frac{CI}{RI}$, where $n$ is the number of the attribute.
   If the value of $CR$ exceeds 0.10, the consistency is not passed and we need to redefine the comparison matrix.

3.2. TOPSIS Method
Next, we use TOPSIS method to calculate the preference value of all the alternatives. In our study, the weights of all the attributes have been computed by AHP method. The detailed steps of TOPSIS are as follows:

(1) The raw data is put into a matrix, as follows:

$$C = \begin{pmatrix}
  x_{11} & \cdots & x_{1n} \\
  \vdots & \ddots & \vdots \\
  x_{m1} & \cdots & x_{mn}
\end{pmatrix}$$

Where $m$ represents the number of attributes and $n$ represents the number of alternatives.

(2) Standardize the matrix $C$ and defined as $R$:

$$R = (r_{ij})_{m \times n}; r_{ij} = \frac{x_{ij}}{\sum_i x_{ij}^2}, i=1,\ldots,m; j=1,\ldots,n.$$ 

(3) Compute the weighted matrix of $R$ and defined as $V$, $V = RW$, where $W$ is the weights of attributes, calculated as:

$$v_{ij} = r_{ij} \times w_j, \sum_{j=1}^n w_j = 1$$

(4) Find the ideal and negative ideal solutions: ideal $A^* = (v_{1}^*, v_{2}^*, \ldots, v_{m}^*, v_{n}^*)$, negative $A^- = (v_{1}^-, v_{2}^-, \ldots, v_{m}^-, v_{n}^-)$, where

$$v_{j}^* = \begin{cases}
  \max(v_{ij}), & j \in J^+ \\
  \min(v_{ij}), & j \in J^-
\end{cases} (j=1,2,\ldots,n), \quad
v_{j}^- = \begin{cases}
  \min(v_{ij}), & j \in J^+ \\
  \max(v_{ij}), & j \in J^-
\end{cases} (j=1,2,\ldots,n),$$

Where $J^+$ represent beneficial attributes and $J^-$ represent non-beneficial attributes.

(5) Compute Euclid distance:

$$S_j^* = \sqrt{\sum_{i=1}^n (v_{ij} - v_{ij}^*)^2} (i = 1,2,\ldots,m), \quad S_j^- = \sqrt{\sum_{i=1}^n (v_{ij} - v_{ij}^-)^2} (i = 1,2,\ldots,m).$$

(6) The preference values are calculated as follows:

$$C_i^* = \frac{S_j^-}{S_j^* + S_j^-}.$$ 

(7) Sort all the alternatives based on $C_i^*$. 


3.3. Find the Semantic Descriptions for all the Alternatives

Finally, we find the descriptions of all the alternatives by the algorithm of AFS concept description.

4. Numerical Example

In our research, a supplier selection problem is applied to illustrate the proposed model.

A factory need to select the most suitable supplier for its important parts. There are 4 attributes from 6 alternative suppliers, which are beneficial attributes $(a_1, a_2)$ and non-beneficial attributes $(b_1, b_2)$. It is shown in Table 1.

| Supplier | $a_1$ (%) | $a_2$ (%) | $b_1$ (RMB) | $b_2$ (%) |
|----------|-----------|-----------|-------------|-----------|
| $s_1$    | 0.81      | 0.97      | 355         | 0.83      |
| $s_2$    | 0.75      | 0.89      | 289         | 0.92      |
| $s_3$    | 0.74      | 0.87      | 295         | 0.90      |
| $s_4$    | 0.95      | 0.94      | 307         | 0.79      |
| $s_5$    | 0.89      | 0.99      | 492         | 0.89      |
| $s_6$    | 0.99      | 0.85      | 338         | 0.80      |

Initially, we calculate the weigh values of all the attributes applying AHP. Based on experience of experts, the comparison matrix is as follows:

$$
\begin{align*}
1 & 1 & 1:3 & 1:5 \\
1 & 1 & 1:3 & 1:4 \\
3:1 & 3:1 & 1 & 1:2 \\
5:1 & 4:1 & 2:1 & 1
\end{align*}
$$

and then compute the weight values as follows:

$$
W_{a_1} : 0.1015; W_{a_2} : 0.1076; W_{b_1} : 0.2906; W_{b_2} : 0.5562.
$$

In what follows, the TOPSIS method is applied to compute the preference value and the results are expressed in Table 2.

| Supplier | Preference value |
|----------|------------------|
| $s_4$    | 0.928975433      |
| $s_6$    | 0.810230015      |
| $s_1$    | 0.714844523      |
| $s_3$    | 0.700223724      |
| $s_2$    | 0.673423084      |
| $s_5$    | 0.137813945      |

Finally, the semantic descriptions of all the suppliers according to AFS concept description algorithm are stated as follows:

$\zeta_{s_1} = a_2 b_1$ represents that attribute $a_2$ and attribute $b_1$ of supplier 1 are both strong.

$\zeta_{s_2} = b_1 b_2$ represents that attribute $b_2$ of supplier 2 is strong and attribute $b_1$ of supplier 2 is not strong.

$\zeta_{s_3} = a_1^+$ represents that attribute $a_1$ of supplier 3 is not strong.

$\zeta_{s_4} = b_2^+$ represents that attribute $b_2$ of supplier 4 is not strong.
\[ \zeta_{s_5} = a_2b_1 \] represents that attribute \( a_2 \) and attribute \( b_1 \) of supplier 5 are both strong.

\[ \zeta_{s_6} = a_1a_2' \] represents that attribute \( a_1 \) of supplier 6 is strong and attribute \( a_2 \) of supplier 6 is not strong.

On the basis of Table 2, it is obvious that the best is \( s_4 \), followed by \( s_6, s_1 \) and \( s_3 \). Moreover, based on above semantic descriptions, if the decision maker pays attention to attribute \( b_2 \), then \( s_4 \) is the optimal choice; if the decision-maker pays attention to attribute \( a_2 \), then \( s_1 \) can be selected, which can make the decision making basis more sufficient and the decision making result more reasonable.

5. Conclusion
A three-stage model is proposed for MCDM problem in this research. At first, AHP method is used to calculate the weight of each attribute on all the alternatives. And then, TOPSIS method is used to get the preference value based on the weights calculated in the first step. At last, AFS concept description algorithm is employed to give the semantic descriptions of performance characteristics for all the alternatives which can make the optimal choice based on their own preferences on the attribute, so that the decision results are more reasonable. The supplier selection problem is applied to explain the application ability of the proposed model.

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