Signatures of Wigner crystal of electrons in a monolayer semiconductor

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When the Coulomb repulsion between electrons dominates over their kinetic energy, electrons in two-dimensional systems are predicted to spontaneously break continuous-translational symmetry and form a quantum crystal1. Efforts to observe this elusive state of matter, termed a Wigner crystal, in two-dimensional extended systems have primarily focused on conductivity measurements on electrons confined to a single Landau level at high magnetic fields. Here we use optical spectroscopy to demonstrate that electrons in a monolayer semiconductor with density lower than $3 \times 10^{11}$ per centimetre squared form a Wigner crystal. The combination of a high electron effective mass and reduced dielectric screening enables us to observe electronic charge order even in the absence of a moiré potential or an external magnetic field. The interactions between a resonantly injected exciton and electrons arranged in a periodic lattice modify the exciton bandstructure so that an umklapp resonance arises in the optical reflection spectrum, heralding the presence of charge order11. Our findings demonstrate that charge-tunable transition metal dichalcogenide monolayers14 enable the investigation of previously uncharted territory for many-body physics where interaction energy dominates over kinetic energy.

The electronic properties of most metals and semiconductors at low temperatures can be described using the Fermi liquid theory. This is a consequence of the fact that in most material systems the typical kinetic energy of electrons exceeds the Coulomb interaction energy. Investigation of strong electronic correlations that emerge in the complementary regime—where the ratio $r_s$ of the Coulomb interaction to the kinetic energy well exceeds unity—has been a ‘holy grail’ of condensed-matter physics. A landmark state of matter that was predicted to appear in this latter limit is a Wigner crystal1 where electrons spontaneously break translational symmetry and form a periodic lattice. Quantum Monte Carlo calculations15 indicate that for the Wigner crystal to be the ground state of a two-dimensional electron system when the external magnetic field $B = 0$. Since $r_s = m_e^* e^2 / (4\pi \epsilon_0 \epsilon_r h^2 \sqrt{\mu_0 m_e})$, simple considerations show that high-quality materials with large $m^*_e$ low $n_e$ and small $c$ could yield the requisite $r_s$ values. Here, $n$, $\mu$, $\epsilon_r$, $\epsilon_0$ and $m^*_e$ denote the electron density, reduced Planck constant, vacuum permittivity, dielectric constant, elementary charge and effective electron mass, respectively.

The difficulty in simultaneously satisfying the above-mentioned stringent conditions has, with a few remarkable exceptions16–18, hindered the search for an electronic Wigner crystal state at $B = 0$. Instead, a majority of the experimental efforts2–12 so far have focused on two-dimensional electron systems in conventional semiconductors under large $B$ and at $n_e$ much lower than the Landau level degeneracy. In this limit the kinetic energy of electrons is quenched and the Coulomb interaction provides the only relevant energy scale.

Transition metal dichalcogenide (TMD) monolayers have the promise to overcome this conundrum for $B = 0$19: the combination of small $c$ together with large $m^*_e$ implies that $r_s > 40$ can be obtained for $\mu_e = 1 \times 10^{11}$ cm$^{-2}$. The stark contrast with conventional materials becomes even more prominent in the limit $B \neq 0$, where the strength of interactions is quantified by the ratio of the Coulomb interaction ($E_C$) to the cyclotron energy ($\hbar \nu$): $\kappa = E_C / \hbar \nu = m_e^* e^2 / (4\pi \epsilon_0 \epsilon_r \hbar B) = r_s \nu / 2$, with $l_0 = \sqrt{\hbar / (eB)}$ and $\nu$ denoting the magnetic length and Landau level filling factor, respectively. Since $\kappa \gtrsim 30$ for $B = 5$ T in TMD monolayers, interactions lead to strong Landau level mixing and it is no longer possible to treat electron motion as being restricted to a single Landau level.

Here, we report evidence for an electronic Wigner crystal in two different monolayer TMD devices. In contrast to the recently observed Mott–Wigner states in moiré superlattices20–25, the Wigner crystal states in our devices spontaneously break continuous translational symmetry. Both samples consist of a charge-tunable MoSe$_2$ monolayer that was encapsulated between hexagonal boron nitride (hBN) layers and covered with few-layer graphene serving as top and back transparent gate electrodes (see Methods). In the main text we focus on the device depicted in Fig. 1a for which $n_e$ was precisely calibrated using the Landau level fan chart of the Shubnikov–de Haas oscillations of the exciton linewidth26 (Methods).

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Signatures of a Wigner crystal at $B = 0$

We first concentrate on the fingerprints of the Wigner crystal phase measured at $B = 0$ in a dilution refrigerator at temperature $T = 80$ mK. Figure 1b shows representative top-gate voltage ($V_g$) evolution of the resonant reflectance contrast spectrum $(R - R_0)/R_0 = \Delta R/R_0$ of our device, where $R_0$ is the reflectance measured in the MoSe$_2$ monolayer region, and $R$ represents a background reflectance taken off the MoSe$_2$ flake (see Methods). At $V_g < 0$, when the MoSe$_2$ is charge neutral, the spectrum displays a single bare exciton ($X$) resonance. Upon electron doping (at $V_g > 0$), the excitons become dressed into exciton-polarons$^{27,28}$, owing to dynamical screening by the electrons. This leads to emergence of a second, red-shifted resonance, termed the attractive polaron. In parallel, the excitons transforms into a repulsive polaron that blueshifts with increasing $n_e$.

In search for signatures of the periodic charge order, we examine the limit of low $n_e$ in the spectral vicinity of the repulsive polaron transition, which, for simplicity, we will refer to as the exciton resonance. Notably, we observe a second, higher-energy resonance for low $n_e$. Owing to its small oscillator strength, this resonance is barely visible in $\Delta R/R_0$, but becomes prominent after differentiating $\Delta R/R_0$ with respect to $V_g$, or equivalently, $n_e$ (Fig. 1c). This weak resonance blueshifts faster than the exciton with increasing $n_e$, and becomes indiscernible at $n_e \gtrsim 3 \times 10^{11}$ cm$^{-2}$. At the same time, its energy splitting $\Delta E_{\text{ex}}$ from the main exciton transition, determined by fitting both resonances with dispersive Lorentzian spectral profiles (Methods), scales linearly with $n_e$ and extrapolates to zero at $n_e = 0$ (Fig. 1d).

These observations, in conjunction with recent work linking the appearance of high-energy excitonic umklapp resonances to the presence of electronic charge order in a Mott insulator state$^{13}$, allow us to conclude that electrons at low $n_e$ form a Wigner crystal. Even though there is no moiré potential in the single monolayer TMD sample we investigate, the charge order appearing in the Wigner crystal state introduces a periodic potential for the excitons, which in turn leads to the emergence of new bright resonances in the reduced excitonic Brillouin zone. These new transitions originate from the Bragg–umklapp scattering of the dark exciton states with momentum $k = k_X$ (where $k_X$ denotes the magnitude of the Wigner crystal reciprocal lattice vectors), which folds these states back to the light cone where they hybridize with the $k = 0$ exciton and thus acquire a finite oscillator strength (as qualitatively illustrated in Fig. 1e–g). In the relevant limit of weak exciton–electron interactions, the energy of such an umklapp resonance $\Delta E_{\text{ex}}$ is simply determined by the exciton kinetic energy $\hbar^2 k_X^2/2m_X$ at momentum $k = k_X$, where $m_X$ stands for the exciton mass. Because $k_X \propto 1/\sqrt{X}$, the umklapp energy increases linearly with $n_e$, whereas the Wigner crystal lattice constant is reduced, in full agreement with our experiments. Moreover, assuming a triangular lattice, the slope $\Delta E_{\text{ex}}/n_e = \hbar^2/k_X^2/3m_X$, extracted from the data in Fig. 1d, corresponds to $n_e = (1.1 \pm 0.2)m_e$ (see Methods for details; $h$, Planck constant; $m_e$, free electron mass) in agreement with $m_X = 1.3m_e$, revealed by previous experiments$^{13,29,30}$.

Temperature dependence

Figure 2a shows the temperature dependence of back-gate voltage ($V_g$) evolution of $d(\Delta R/R_0)/dV_g$. Notably, the umklapp signature becomes almost indiscernible by $T = 10$ K and disappears completely for $T = 17$ K,
while the main exciton transition remains only weakly affected. We associate this observation with a phase transition from a Wigner crystal to a liquid state. To determine the corresponding melting temperature $T_{\text{m}}$, we fix $n_e = 1.6 \times 10^{11} \text{ cm}^{-2}$ where the umklapp resonance is particularly well resolved and analyse the $T$ dependence of the average value of $d(\Delta R/R_0)/dV_e$ in a spectral window around the umklapp energy (see Fig. 2b). As shown in Fig. 2c, such umklapp amplitude initially exhibits a linear decrease with $T$ and then saturates at a constant background level for larger $T$. By tracing the crossover between these two regimes we extract $T_{\text{m}} = 11 \pm 1 \text{ K}$. Similar values of $T_{\text{m}}$ are also obtained for $n_e$ in the range $1.2-1.9 \times 10^{11} \text{ cm}^{-2}$ (Methods).

**Wigner crystal at magnetic fields**

The stability of the Wigner crystal at $T = 80 \text{ mK}$ could be further enhanced upon application of $B > 0$ giving rise to more pronounced Wigner crystal signatures, as revealed by Fig. 3a, showing the $V_e$ dependence of $d(\Delta R/R_0)/dV_e$ at $B = 6 \text{ T}$. Even though the umklapp resonances in both circular polarizations ($\sigma$) are more intense than their counterparts, the $\sigma$-polarized resonance is clearly stronger than that for $\sigma$. Moreover, both peaks exhibit almost identical energy for any given $n_e$. In stark contrast to the $k = 0$ exciton transitions exhibiting a large valley Zeeman effect $^{12}$: $\Delta E = g \mu_B B$ with $g = 4.3$ and $\mu_B$ representing the Bohr magneton (Fig. 3b). This striking disparity arises from the long-range electron–hole exchange interaction $^{16}$, which strongly mixes high-$k$ excitons in $K$ valleys, resulting in formation of longitudinal- and transverse-polarized exciton branches that are split by $\Delta E_{\sigma} (k) \approx |k|$ (Fig. 1e–g). Since $\Delta E_{\sigma} (k) > \Delta E_{\sigma}$ at $B = 6 \text{ T}$ even for $n_e = 1 \times 10^{11} \text{ cm}^{-2}$ (for estimation of $\Delta E_{\sigma}$, see Supplementary Information section 3), the Zeeman effect for the umklapp resonances is almost fully suppressed, as demonstrated by our calculation of the exciton dispersion in Fig. 3d. Consequently, experimentally determined splittings $\Delta E_{\sigma}$ between the umklapp and the exciton peaks in $\sigma$ polarization (Fig. 3c), despite exhibiting the same increasing slope with $n_e$ as in the $B = 0$ case, now extrapolate to $\Delta E_{\sigma}/n_e = 0$. These observations further support our identification of the umklapp resonance. The lower value of $\Delta E_{\sigma}$ as compared to $\Delta E_{\sigma}$ also implies stronger hybridization of the $\sigma$-polarized umklapp resonance with the $k = 0$ excitons, which in turn explains its higher intensity.

Figure 3a indicates that the Wigner crystal phase exists in a broad range of Landau level filling factors for $B = 6 \text{ T}$, and, in particular, at $\nu = 1$. This result is surprising, as integer quantum Hall states are non-degenerate and incompressible, and should be robust against Coulomb interactions. For TMDs, however, $k \gg 1$ for $B \lesssim 10 \text{ T}$: such large $k$, in conjunction with a possible disorder-induced suppression of the integer quantum Hall liquid, renders the Wigner crystal the ground state, even at $\nu = 1$.

The combination of the vanishing Zeeman splitting of the umklapp resonance and the large $\Delta E_{\sigma}$ of $k = 0$ excitons enables us to bring these two transitions into resonance by tuning the $B$ field such that $\Delta E_{\sigma}/n_e = 1.5 \times 10^{11} \text{ cm}^{-2}$ and $\Delta E_{\sigma}$ polarization (Fig. 3e, f). We note that the umklapp resonance for $n_e$ satisfying $\Delta E_{\sigma}/n_e = 1.5 \times 10^{11} \text{ cm}^{-2}$ is visible even in the bare $\Delta E_{\sigma}$ spectra (Fig. 3e). In turn, differentiation of $\Delta R/R_0$ with respect to the energy $E$ indicates the onset of an anticrossing between the umklapp and the main $\sigma$ resonance (Fig. 3f). The latter plot also reveals a pronounced asymmetry in the intensities of the involved transitions: at low $n_e < n_{\text{m}}$, both resonances are clearly visible, whereas for $n_e > n_{\text{m}}$, the umklapp resonance is no longer discernible. We tentatively attribute this observation to an electronic phase transition corresponding to the melting of the Wigner crystal with increasing $n_e$, which takes place at a low filling factor ($\nu = 0.5$), owing to reduced $d(\Delta R/R_0)/dV_e$ at $B = 14 \text{ T}$. Remarkably, however, the higher-energy resonance reappears in the $\sigma$-polarized spectrum for $n_e$ corresponding to $\nu = 1$ (see Fig. 3f). We attribute this effect to magnetoroton softening at a wave vector close to $k_{\text{m}}$, in the integer quantum Hall state. This interpretation is based on a Kallin–Halperin-type analysis of magnetoplasmon excitations in the integer quantum Hall liquid at $\nu = 1$ (see Fig. 4a and Supplementary Information section 1 for details). Although scattering off soft magnetorotons also leads to a peak in the exciton spectral function at an energy splitting close to $\Delta E_{\sigma}$, the resulting transition is considerably broader than the umklapp resonance associated with a Wigner crystal (Fig. 4b).

Dynamical dressing of excitons by soft rotons at $B = 0$ in the liquid phase near the critical point ($n_e \approx n_{\text{m}}$) also leads to a satellite peak in exciton spectrum (see Supplementary Information section 4), but at an energy that is almost twice as large as that of the umklapp resonance in the Wigner crystal phase ($n_e \ll n_{\text{m}}$). The accuracy in the determination of $\Delta E_{\sigma}$ in our $B = 0$ experiments clearly rules out a competing explanation based on a liquid state with short-range correlations, thereby providing a strong evidence for the Wigner crystal phase.

**Disorder and the correlation length**

Next, we discuss the role of disorder on Wigner crystalization. In general, random potential suppresses long-range crystalline order, turning the transition point between the liquid and Wigner crystal phases into an extended crossover region. In a clean system, the liquid phase for large $k$ possesses strong local charge correlations, even in the absence of long-range order. This allows the pinning potential from impurities to nucleate crystallites, which are not phase-locked with each other.

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**Fig. 2 | Phase transition between the Wigner crystal and electron liquid states at $B = 0$.** a, Back-gate-voltage evolution of the derivative of the reflectance contrast spectrum with respect to $V_e$, measured in a setup with a variable temperature insert at $B = 0$ and different temperatures of the sample: 4.5 K (left), 9.9 K (middle), 17.0 K (right). Black dashed lines indicate the fitted energy $E_\sigma$ of the main exciton resonance, and the green line represents the expected position of the umklapp peak $E_\sigma + k Z/2 m$, for $n_e = 1.3 m$, which is taken from refs. $^{11,12}$ because the precise umklapp energy was not extracted for these datasets. b, Cross-sections through colour plots (a) as well as through analogous datasets acquired at different temperatures at $V_e = -0.4 V$ corresponding to $n_e = 1.6 \times 10^{11} \text{ cm}^{-2}$. Temperature-dependent differentiated reflectance contrast $d(\Delta R/R_0)/dV_e$, averaged over a 0.8-meV-wide energy range around the umklapp resonance (marked by the shaded region in b; see Extended Data Fig. 6e for the definition of the error bars). The solid line represents the fit of the data points with a linear decrease at temperatures $T < T_{\text{m}}$ and constant background level at $T > T_{\text{m}}$, which corresponds to the Wigner crystal melting temperature of $T_{\text{m}} = 11 \pm 1 \text{ K}$.
resulting in static, short-range crystalline order. A possible attempt to describe the corresponding phase diagram is to employ the Hartree–Fock approximation (see Supplementary Information section 2). While this approach cannot quantitatively predict the transition between the integer quantum Hall state and Wigner crystal for \( \nu = 1 \), it does indicate where the liquid state develops strong local crystal correlations that can be turned into Wigner crystal islands by impurities; such Hartree–Fock predictions remain in qualitative agreement with the \((\nu, B)\) regions where we observe the umklapp signature in the experiment.

To obtain a more quantitative understanding of the range of crystal correlations, we developed a microscopic model neglecting phonon fluctuations and assuming a Gaussian disorder potential (see Supplementary Information section 6). This model allows us to extract the correlation length \( \xi_{\text{corr}} \) of the Wigner crystal from the lineshape of the umklapp resonance. Figure 4c shows a series of calculations, corresponding to different \( \xi_{\text{corr}} \)s, fitted to the differentiated \( \sigma^- \)-polarized reflectance contrast spectrum at \( B = 14 \) T and \( \nu = 0.34 \). Under these experimental conditions, the umklapp signature is strongly pronounced and calculated along the \( \Gamma - M \) and \( \Gamma - K \) paths in the Brillouin zone for a Wigner crystal with \( n_e = 2.1 \times 10^{11} \) cm\(^{-2} \) at \( B = 0 \) (left) and \( B = 6 \) T (right). For each field there are two bright main exciton states and two bright umklapp states, each of which originates from the band that is labelled by a coloured line around the \( \Gamma \) point. The plots on the right side illustrate the corresponding exciton spectral function, \( A(E) \) (see Supplementary Information section 3). e, \( f / V \), dependence of \( \Delta R / R_0 \) and \( d \Delta R / dV \) (meV\(^{-1} \)) acquired at \( B = 14 \) T and \( T = 80 \) mK in \( \sigma^- \) (left) and \( \sigma^+ \) (right) polarizations. Similarly to a, grey horizontal lines mark integer values of \( v \), and white (black) dashed (dotted) lines indicate the fitted energy of the co-polarized (cross-polarized) main exciton resonance in each map. Dashed circle indicates the area corresponding to the resonant enhancement of the umklapp peak in \( \sigma^+ \) polarization, and the dashed rectangle marks the region where the higher-energy peak reappears around \( v = 1 \).

**Discussion**

Our findings open up possibilities for studying strongly correlated electrons in previously uncharted territory. An interesting extension of our work would be to combine it with transport spectroscopy to measure Hall conductivity: such measurements will help to explore whether a Hall crystal that concurrently exhibits a unity Chern number and broken translational invariance exists in the vast parameter regime that can be studied in charge-tunable monolayer TMDs. Additional insight into the dynamical properties of Wigner crystals can be obtained from a.c. conductivity measurements, which can elucidate the role of disorder through measurements of the pinning.
Our calculations show that in the absence of disorder, the generically broad asymmetric lineshapes associated with the dynamical screening of excitons by softened magnetoroton contrast sharply with the delta-function umklapp peak of a Wigner crystal (Fig. 4b): consequently, we expect materials with reduced disorder to allow for an all-optical investigation of the transition from a quantum liquid to a quantum solid and thereby shed light on the conjectured existence of intermediate phases. Another exciting direction is the investigation of emerging spin order and its relation to the crystalline structure. Finally, we note that experiments monitoring the response of Rydberg excitons to varying frequency. Our calculations show that in the absence of disorder, the generically broad asymmetric lineshapes associated with the dynamical screening of excitons by softened magnetoroton contrast sharply with the delta-function umklapp peak of a Wigner crystal (Fig. 4b): consequently, we expect materials with reduced disorder to allow for an all-optical investigation of the transition from a quantum liquid to a quantum solid and thereby shed light on the conjectured existence of intermediate phases. Another exciting direction is the investigation of emerging spin order and its relation to the crystalline structure. Finally, we note that experiments monitoring the response of Rydberg excitons to varying frequency. 

**Online content**

Any methods, additional references, Nature Research reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at https://doi.org/10.1038/s41586-021-03590-4.
Device and experimental setup

The structure of the device studied in the main text is schematically depicted in Extended Data Fig. 1a. The device consisted of a charge-tunable MoSe₂ monolayer that was electrically contacted with a few-layer graphene flake, encapsulated between two hBN flakes, and finally embedded between the top and bottom few-layer graphene gates. The thicknesses of the hBN layers—which are particularly important for modelling of the spectral profiles of excitonic resonances—were determined by means of atomic force microscopy (AFM) to be \( t_\text{e} = 74 \pm 5 \) nm and \( t_\text{e} = 91 \pm 5 \) nm for the top and bottom layers, respectively (where the uncertainties represent the approximate surface roughness). All flakes were first mechanically exfoliated either from synthetic (HQ Graphene MoSe₂, NIMS hBN) or natural (graphene) bulk synthetic (HQ Graphene MoSe₂, NIMS hBN) or natural (graphene) bulk crystals onto Si/SiO₂ substrates using a backing grinding tape (Ultron Systems). The actual heterostructure was assembled with a standard, dry-transfer technique, in which the flakes were sequentially picked up and stacked together with sub-micron spatial precision using a hemispherical, polydimethylsiloxane (PDMS) stamp covered with a thin layer of polycarbonate (PC). A complete stack was then released on a transparent, sapphire substrate, and the residual PC layer was subsequently dissolved in chloroform. Finally, the device was contacted with Ti/Au electrodes that were fabricated using standard electron-beam lithography followed by metal evaporation. During all of the reported mK-temperature experiments, the electron density \( n_e \) in the MoSe₂ monolayer was controlled by applying a voltage \( V_g \) to the top gate, while keeping the MoSe₂ and the other gate grounded. For the temperature-dependent measurements carried out in a variable temperature insert, the density \( n_e \) was in turn tuned with a back-gate voltage \( V_b \) (here the MoSe₂ was again grounded, but the top gate was electrically floating). Notably, in both of these cases the sample exhibited virtually no electrical hysteresis when the gate voltage was swept upwards or downwards, as the difference in voltage values required to reach a given density in these two cases was not exceeding 0.1 V. Nonetheless, to avoid this small uncertainty, in all of our experiments the gate was always ramped in the same direction (from negative to positive values).

Extended Data Fig. 1b displays a simplified schematic of the experimental setup used in our mK-temperature measurements. The device was mounted inside a dilution refrigerator enabling us to reach a base temperature of 80 mK. The refrigerator was immersed in a liquid He bath cryostat equipped with a superconducting solenoid producing a magnetic field of up to 16 T in the direction perpendicular to the sample surface. The resonant reflection of the device was measured with the use of a broadband light-emitting diode (LED) with a centre wavelength of 760 nm and a linewidth of 20 nm. The LED light was first arbitrarily polarized using a linear polarizer as well as a set \( \lambda/2 \) and \( \lambda/4 \) waveplates. Then it was transmitted to the sample in a cross-polarized configuration, to avoid its uncontrolled bending or stretching that would cause random changes in the polarization configuration of the setup. Finally, to enhance the fidelity of the detected polarization, the incoming white light was always co-polarized to the currently analysed \( \sigma^- \) polarized response of the sample. This allowed us to suppress the cross-polarized signal by at least three orders of magnitude.

The above-discussed setup does not allow controllable increase of the sample temperature beyond \( T = 4 \) K. Consequently, to investigate the properties of the Wigner crystal at higher temperatures, the device was transferred to a different setup consisting of a home-built variable temperature insert (VTI) that was immersed in a liquid helium bath cryostat. The VTI had the form of a stainless steel tube featuring a very similar design to that used in our previous study. Specifically, it provided a free-space optical access to the sample being placed on an x-y-z piezo-electric attocube stage. The crucial difference of the present setup with respect to that of the previous study is that the sample was not directly mounted on the top piezo-electric stage, but there was an upper plate of the attocube thermal coupler ATC100 screwed in between of them. This plate was equipped with a precalibrated silicon-diode-based temperature sensor as well as a 50-Ω resistor allowing us to heat the sample with a power of up to 2 W. At the same time, the plate remained in direct thermal contact with the steel tube, thus providing efficient cooling of the device. The overall cooling power was further increased by filling the tube with a helium exchange gas, the pressure of which was carefully selected (at a level of a few \( \mu \)bar at 4 K) to allow the sample temperature to be tunable in a wide range between \( T = 4.5 \) K and \( T = 70 \) K by controlling a voltage applied to the heating resistor.

Details of normalization and differentiation of the reflectance spectra

As stated in the main text, to extract the reflectance contrast \( R \) from the measured white-light reflectance spectrum \( R \) of the MoSe₂ monolayer, we acquire a reference spectrum \( R_0 \) at a different spot in the sample region featuring all the layers (that is, hBN and graphene gates) except for the MoSe₂. To minimize systematic errors stemming from sample inhomogeneities or defocusing when travelling along the sample surface, the reference spot is selected as close as possible to the investigated MoSe₂ spot. The sample pair of such \( R \) and \( R_0 \) spectra is shown in Extended Data Fig. 2a. On the basis of these data we determine the reflectance contrast as \( R_g = R_g / R_0 \). The sample was mounted inside a dilution refrigerator enabling us to reach a base temperature of 80 mK. The refrigerator was constructed from a superconducting solenoid producing a magnetic field of up to 16 T in the direction perpendicular to the sample surface. The resonant reflection of the device was measured with the use of a broadband light-emitting diode (LED) with a centre wavelength of 760 nm and a linewidth of 20 nm. The LED light was first arbitrarily polarized using a linear polarizer as well as a set \( \lambda/2 \) and \( \lambda/4 \) waveplates. Then it was transmitted to the sample in a cross-polarized configuration, to avoid its uncontrolled bending or stretching that would cause random changes in the polarization configuration of the setup. Finally, to enhance the fidelity of the detected polarization, the incoming white light was always co-polarized to the currently analysed \( \sigma^- \) polarized response of the sample. This allowed us to suppress the cross-polarized signal by at least three orders of magnitude.

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To evaluate a derivative \( R'(E) = dR_g / dE \) of the reflectance contrast with respect to the energy (which is plotted in Figs. 3f, 4c), we apply a standard symmetric difference quotient method, which approximates a numerical derivative as a slope of the line connecting the two neighbouring data points \( R_n(E_n) = [R_n(E_n) - R_n(E_{n-1})]/E_{n-1} - E_n \), where the index \( n \) labels subsequent points. In principle, a similar method could be also applied for computing a derivative of \( R_g \) versus gate voltage as \( R'_g(V_g) = [R_g(V_g) \) versus gate voltage as \( R'_g(V_g) = [R_g(V_{g,n}) - R_g(V_{g,n-1})]/V_{g,n} - V_{g,n-1} \). However, owing to the small gate-voltage step used in our measurements, the signal differentiated in such a way is relatively noisy, as shown in Extended Data Fig. 2c for the case of zero-field reflectance contrast spectra. For this reason, in Figs. 1a, 2a, 3a we use a slightly different procedure, in which \( R'_g(V_g) \) is determined as a slope connecting not neighbouring, but next-neighbouring data points:

\[ R'_g(V_g) = \frac{R_g(V_{g,n}) - R_g(V_{g,n-1})}{V_{g,n} - V_{g,n-1}} \]
from abrupt changes of estimated based on the values reported in several pre-

γ

ν

\[ E = \frac{E_0 - E}{(E - E_0)^2 + \frac{1}{4} + A}, \]

where \( E \) denotes the photon energy, \( C \) represents a flat background, \( A > 0 \), \( E_0 \) and \( \gamma \) correspond, respectively, to the amplitude, energy, and linewidth of the resonance, and \( \varphi \) stands for interference-induced phase shift, which depends on both the energy and the amplitude of the resonance.

To extract the energies of the exciton and umklapp peaks from a given reflectance contrast spectrum, we first fit the spectral profile \( R_e(E) \) of the exciton peak with the aforementioned dispersive Lorentzian formula leaving the umklapp peak aside, which is justified owing to its very small oscillator strength. The result of such a fit performed for an example zero-field spectrum measured at \( V_t = 0.8 \) V is presented in Extended Data Fig. 3c. To determine the umklapp position, we subtract the fitted lineshape \( R_e(E) \) from the original data \( R_\gamma(E) \) and repeat this for each gate voltage, thus obtaining a voltage evolution of the corrected spectra \( R_e(E, V_t) - R_e(E, V_t) \). At this stage, to account for the presence of a residual smooth background in the data, we fit one of the corrected spectra obtained at the charge neutral region (where we do not expect to see an umklapp signature) with a phenomenological, Gaussian formula \( B(E) = A_\text{e} \exp \left[ -\frac{(E - E_0)^2}{2\sigma^2} \right] + C \). This fixed background is then subtracted from all the spectra measured at different voltages. The resulting top-gate voltage dependence of \( R_e(E, V_t) - R_e(E, V_t) - B(E) \) is displayed in Extended Data Fig. 3b, together with the corresponding raw reflectance contrast \( R_e(E, V_t) \) data in Extended Data Fig. 3a (for comparison). As seen, the above approach greatly reduces the nonlinear background around the umklapp peak, which makes it possible to fit its spectral profile. To this end, to avoid the spurious contribution from residuals of the exciton fitting (that appear around the exciton energy), we truncate the fitting range on the red side of the umklapp resonance to the energies \( E > E_\gamma + 5\gamma \), where \( E_\gamma \) and \( \gamma \) represent, respectively, the fitted energy and linewidth of the exciton resonance. Moreover, bearing in mind the relatively low intensity of the umklapp resonance, we also reduce the number of independent parameters in the dispersive Lorentzian formula, equation (1). Specifically, we set \( C = 0 \) (since the background was already subtracted) and fit the phase of the umklapp peak to \( \varphi = \varphi_0 \), where \( \varphi_0 = 2.5 \) rad represents the resonance phase at the umklapp energy in the limit of zero oscillator strength for our heterostructure. This value is determined based on the measured gate-voltage dependence of the phase of the attractive polaron resonance extrapolated to the zero-electron-density limit and further confirmed by the results of transfer-matrix simulations of our device reflectivity spectrum (assuming hBN thicknesses mentioned in Methods section ‘Device and experimental setup’ as well as a hBN refractive index of 2.1)\(^{44,45}\). Under the above assumptions, we fit the umklapp peak in the corrected spectra (as shown in Extended Data Fig. 3d for an example case of \( V_t = 0.8 \) V), which finally allows us to extract its energy as a function of the gate voltage, which is plotted in Figs. 1d and 3b, c for the cases of \( B = 0 \) and \( B = 6 \) T, respectively. We stress that owing to large number of steps involved in the above approach, it may be fraught with a systematic error originating, for example, from the uncertainty of assumed umklapp peak phase \( \varphi_0 \). This may in turn lead to a systematic error of the exciton mass determined based on the slope of the umklapp energy increase with the gate voltage. We account for this effect in the main text by assuming the mass uncertainty to be larger than its statistical uncertainty determined by the spread of the data points in Figs. 1d, 3b, c.

**Calibration of the electron density dependence on the gate voltage**

Given that the energy splitting between the umklapp and exciton transitions is governed by the Wigner crystal lattice constant that is evaluated based on the electron density \( n_e \), it is essential for the validity of our analysis to precisely establish a dependence of \( n_e \) on the gate voltage. In principle, this might be done by modelling the device as a parallel-plate capacitor\(^{26,46}\) with the geometrical capacitance per unit area \( C_{\text{geom}} = \epsilon_\text{BN} \frac{\epsilon_\text{hBN}}{h} t \) defined by the (top or bottom) hBN thickness \( t \) and its static out-of-plane dielectric constant \( \epsilon_\text{hBN} \). This procedure, however, may be fraught with a systematic error stemming from sizable uncertainty of the \( \epsilon_\text{hBN} \) constant.

To avoid the above difficulties, we utilize a more accurate approach, in which \( n_e(V_t) \) in the dilution refrigerator experiments is obtained based on the Shubnikov–de Haas oscillations of the exciton transition at high magnetic fields. As we demonstrated in our previous work\(^{26}\), under such conditions the linewidth \( \gamma \) of the exciton peak in \( \sigma \) polarization (in case of the electron doping) exhibits sharp minima each time the Landau level filling factor \( \nu \) takes on an integer value, as seen in Extended Data Fig. 4a, b for an example set of data taken at \( B = 16 \) T. This allows us to extract the exciton energies \( V_e(B) \) and \( V_e(B) \) corresponding to subsequent integer \( \nu \) by fitting the vicinity of each minimum in \( \gamma(V) \) dependence with a phenomenological, Gaussian profile. Bearing in mind that our Wigner crystal investigation from the main text is mostly focused on the low-doping density limit, we simplify the current analysis by restricting it to a regime in which the Fermi level does not exceed the valley Zeeman splitting\(^{31,44,47}\) of the conduction band, and hence where the electrons occupy the states in a single \( K \) valley. Extended Data Fig. 4c displays the values of \( V_e(B) \) obtained in the above-described way for all magnetic fields \( B \geq 8 \) T at which we observe resolvable Shubnikov–de Haas oscillations in the exciton linewidth. As expected, the voltages form a characteristic Landau level fan chart exhibiting linear increase with both \( v \) and \( B \), as confirmed by their perfect agreement with a set of linear dependencies of the form \( V_e(B) = V_{e,0} + \delta V_e \nu + 0.13 \text{V} \). Importantly, all of these dependencies have just two common fitting parameters: \( V_{e,0} = -0.13 \) V being the voltage that corresponds to the onset of filling the conduction band with electrons, and \( \delta = 0.12 \text{V T}^{-1} \) representing a change of the gate voltage that is required to fill a single Landau level in a unit magnetic field. Taking into account that the investigated Landau levels are fully spin- and valley-polarized, each of the above-introduced linear dependencies corresponds to an electron density of \( n_e(B, \nu) = \nu e B / h \). This finally allows us to extract the \( n_e(V) \) by linearly interpolating between subsequent integer filling factors:

\[
\left\{ \begin{array}{l}
n_e(V_t > V_{e,0}) = \frac{e}{hB} (V_t - V_{e,0}) = 2.00 \times 10^{11} \text{cm}^{-2} \text{V}^{-1} \times (V_t + 0.13 \text{V})
\end{array} \right.
\]

This expression was used to calibrate the electron density in Figs. 1, 3, 4. It is noteworthy that the capacitance \( C = \epsilon_\text{BN} \frac{\epsilon_\text{hBN}}{h} t \) in n=22 nm\(^{-2}\) of the device evaluated using the above \( n_e(V) \) dependence agrees well with that obtained within the parallel plate approximation \( C_{\text{geom}} = \epsilon_\text{hBN} n t = 0.42 \pm 0.1 \text{nm}^{-2} \) for \( t = 74 \pm 5 \) nm and \( \epsilon_\text{hBN} = 3.5 \pm 0.5 \text{nm}^{-2} \) estimated based on the values reported in several previous works\(^{48,46}\).

A similar approach is also used to determine \( n_e(V) \) in the temperature-dependent experiments carried out in the VT1-based
setup, during which the electron density was controlled by applying a back-gate voltage \( V_b \). In this case, instead of analysing a complete Landau level fan chart, a slope \( s_b = \Delta n_e/\Delta V_e = (eB/\hbar)\Delta V_e = 2.97 \times 10^8 \text{ cm}^{-2} \text{ V}^{-1} \) of the density increase with \( V_e \) is extracted based on the measured gate voltage change \( \Delta V_{el} = 1.14 \text{ V} \) that is needed to fill a single Landau level in a fixed magnetic field of \( B = 14 \text{ T} \). We note that this slope is slightly larger than that expected within the parallel-plate capacitor approximation \( \approx 1.0 \text{ V} \). For \( V_{el} = 1.0 \text{ V} \) stands for \( V_e \) corresponding to the onset of injecting itinerant electrons to the MoSe\(_2\) monolayer, which is extracted as a voltage at which the exciton resonance begins to blueshift. This formula was used to calibrate the electron density in Fig. 2.

**Stability of the Wigner crystal at elevated temperatures**

Here we extend our discussion of the temperature response of the electronic Wigner crystal in the main device. Extended Data Fig. 5 displays the comparison between the zero-magnetic-field top-gate voltage evolutions of the reflectance contrast derivative \( d(\Delta R_b)/dV_b \), acquired in the dilution refrigerator at \( T = 80 \text{ mK} \) and \( T = 4 \text{ K} \). The second dataset was measured after removing the \(^{3}\)He–\(^{4}\)He mixture from the mixing chamber of the refrigerator and filling its vacuum can with about 1 mbar of \(^{3}\)He exchange gas to facilitate the heat exchange between the sample and liquid helium bath inside the cryostat. The presented maps clearly demonstrate that while the umklapp signature is still visible at the same energies, it becomes sizably weaker upon rising the temperature. This is in stark contrast to the case of the main exciton peak, the amplitude of which remains almost unaffected by the temperature change. This finding further supports our conclusion that although the melting temperature of the Wigner crystal exceeds 4 K, the enhanced thermal fluctuations of the electrons around their lattice sites suppress the intensity of the exciton umklapp scattering off the electronic crystal at higher temperatures.

Extended Data Fig. 6a shows back-gate voltage evolutions of the zero-field reflectance contrast derivative \( d(\Delta R_b)/dV_b \), measured in the VTH-based setup at six different temperatures of the sample (three of these colour maps are presented in Fig. 2a). As discussed in the main text, the umklapp peak is getting progressively fainter upon increasing the temperature, and is already indiscernible in the map acquired at \( T = 12.5 \text{ K} \). Importantly, such a temperature rise only weakly affects the main exciton resonance, as its amplitude at \( T = 12.5 \text{ K} \) is found to be reduced by less than 15% of its initial value at \( T = 4.5 \text{ K} \). The gradual temperature-induced weakening of the umklapp transition is even better seen in Extended Data Fig. 6b–d displaying the cross-sections through the colour maps from Extended Data Fig. 6a at three selected electron densities of \( n_e = 1.2, 1.6 \) and \( 1.9 \times 10^{11} \text{ cm}^{-2} \). Although these datasets clearly indicate the melting temperature \( T_m \) of the electronic Wigner crystal (at which the umklapp disappears from the spectrum) to be around 10–12 K in the investigated density range, the quantitative analysis of the umklapp amplitude turns out to be hindered by the difficulty in fitting its lineshape in the differentiated spectra. This is due to the fact that this lineshape \( d(\Delta R_b)/dV_b \) is determined by gate-voltage dependencies of the amplitude, linewidth and energy of the umklapp resonance \( R_b \), out of which the former two dependencies are hard to predict theoretically. At the same time, attempts to directly fit a dispersive Lorentzian profile of the umklapp resonance in the bare \( \Delta R_b/R_b \) spectra after subtracting the main exciton profile (following the procedure described in Methods section ‘Determination of energies of the exciton and umklapp resonances’, which is used in our analysis of mK-temperature data) turn out to be inaccurate, owing to poor signal-to-noise ratio at higher temperatures.

For these reasons, we utilize different approaches to obtain the temperature-dependent umklapp amplitude. In the first one (used in the main text), the relative strength of the umklapp peak is extracted as an average value of differentiated reflectance signal \( d(\Delta R_b)/dV_b \) in the 0.8-meV-wide spectral window around the umklapp energy (the position of this window is assumed to be independent of the temperature for each doping density, as illustrated by shaded regions in Extended Data Fig. 6b–d). As shown in Extended Data Fig. 6e, the amplitude established in such a way exhibits a similar \( T \)-dependence irrespective of \( n_e \); it initially decreases linearly with \( T \) and then saturates for higher values of \( T \) at a constant background level, which is governed by the signal arising from the main exciton peak (and hence depends on the energy detuning of the umklapp resonance). By fitting the data with such a two-piece function we obtain the Wigner crystal melting temperature \( T_m = 11 \pm 1 \text{ K} \) corresponding to the onset of saturation (that is, disappearance of the umklapp signature) to be the same within the experimental uncertainties for all three investigated electron densities. However, as noted in the main text, we expect this melting temperature to eventually decrease once \( n_e \) is either further increased (thereby reducing the \( r \) parameter and hence the corresponding stability of the Wigner crystal) or further reduced (when the characteristic energy scale associated with the Wigner crystal formation is lowered). Unfortunately, it is not possible to perform similar quantitative analysis of the umklapp amplitude for these density values in the present experiments, due to either too-weak umklapp intensity (at high \( n_e \)) or sizable spectral overlap between the umklapp and the main exciton resonances (at low \( n_e \)).

Finally, we also demonstrate that quantitatively the same results are obtained when the umklapp amplitude is determined using a different procedure, in which the spectral profile of the umklapp peak \( \Delta R_b(E, V_b)/dV_b \) in the differentiated data is fitted with a phenomenological Lorentzian formula \( A_b(T) \gamma_b/2 \left( \gamma_b + \gamma_b/2 \right)^2 \) overlaid on a polynomial background \( P_{\text{bckg}}(E) \) capturing the contribution of the main exciton transition. In this second approach, the background profile \( P_{\text{bckg}}(E) \) is first fitted to the spectral profile at highest analysed temperature of \( T = 17.0 \text{ K} \) (where the umklapp signature is absent), and then kept fixed while the relative umklapp amplitude \( A_b(T) \) is being extracted by fitting the lower-temperature profiles (at a given \( n_e \)). For simplicity, the energy \( E_b \) and linewidth \( \gamma_b \) of this peak are assumed to be temperature-independent and fixed at the values obtained by fitting the profile at \( T = 4.5 \text{ K} \) where the umklapp signature is most prominent. The results of such a fitting procedure nicely reproduce the experimental data, as shown in Extended Data Fig. 6f for an example case of \( n_e = 1.6 \times 10^{11} \text{ cm}^{-2} \). Moreover, the umklapp amplitude \( A_b(T) \) extracted from these fits exhibits exactly the same temperature behaviour as that discussed before: it decreases linearly with \( T \) and reaches zero at higher values of \( T \) (Extended Data Fig. 6g). Importantly, the value of \( T_m = 11 \pm 1 \text{ K} \) determined by extrapolating the initial linear decrease to \( A_b = 0 \) precisely matches the value obtained using the first data analysis approach, which confirms the robustness of our conclusions.

The above-determined Wigner crystal melting temperature can also be used to extract a Lindemann parameter, \( \gamma \), for our system. We recall that the Lindemann criterion is a way of estimating the melting temperature of crystals, asserting that melting occurs when the root-mean-square fluctuations of the relative phonon coordinates exceed some fraction of the lattice constant \( a_0 \),

\[
\sqrt{\langle (u(r + a_W) - u(r))^2 \rangle} > \gamma a_0.
\]

where \( u \) is the phonon displacement, \( a_W \) is a vector connecting nearest-neighbour lattice sites at \( r \) and \( r + a_W \), and \( a_0 \) is the lattice constant, and \( \gamma \) is the Lindemann parameter. The above form is appropriate to two dimensions, as the relative fluctuations remain finite in the thermodynamic limit. Using the measured melting temperature of \( T_m = 11 \pm 1 \text{ K} \) we obtain \( \gamma = 0.3 \). Interestingly, this value is roughly a factor of two larger than that of the classical one-component plasma\(^3\), which is not so surprising, given the crude nature of the criterion. We note that in making the above estimate we used the two-dimensional
dielectric constant $\varepsilon_{\text{hBN}} = \sqrt{\varepsilon_{\text{hBN}}^\perp} \varepsilon_{\text{hBN}}^\parallel$, where $\varepsilon_{\text{hBN}}^\perp = 3.5$ and $\varepsilon_{\text{hBN}}^\parallel = 7$ denote, respectively, the out-of-plane and in-plane dielectric constants of hBN.

**Estimation of the lower bound for Wigner crystal correlation length**

As discussed in the main text and in Supplementary Information section 6, the umklapp resonance is expected to broaden in the presence of a finite electronic disorder in the crystalline state, which provides an opportunity to estimate the Wigner crystal correlation length based on the analysis of the umklapp lineshape. However, the experimental determination of such a lineshape is usually hindered by a small intensity of the umklapp peak and by its spectral overlap with the main exciton transition. Although under such conditions it is still possible to relatively precisely extract the energy of the umklapp resonance by subtracting the spectral profile of the main peak (following the procedure outlined in Methods section ‘Determination of energies of the exciton and umklapp resonances’ that was used in our analysis of the $B = 0$ and $B = 6$ T data), determination of the umklapp linewidth turns out to be more difficult and fraught with a large systematic uncertainty. For this reason, in our investigation of disorder-induced umklapp broadening we focus exclusively on $\sigma^-$-polarized spectra acquired at high magnetic field and at relatively low electron densities, where the umklapp peak can be resonantly enhanced by bringing it into close proximity with the main exciton transition, as seen in Extended Data Fig. 7a, b (as well as in Fig. 3e, f showing similar datasets). Specifically, for our analysis we select the spectrum obtained at $B = 14$ T and $v = 0.34$ (Extended Data Fig. 7c, d), where the umklapp peak displays a sizable spectral weight, but remains still relatively well separated from the main resonance (note that the umklapp appears on the red side of the main resonance owing to a large Zeeman splitting of the $k = 0$ exciton at the investigated magnetic field).

First, we fit the main and umklapp peaks in the spectrum differentiated with respect to the energy (where the umklapp resonance is better visible) with a sum $R_s(E) + R_c(E)$ of two differentiated dispersive Lorentzian spectral profiles (each given by equation (1)). This allows us to extract the amplitudes, linewidths, and energies of both resonances, as well as the phase of the main exciton (owing to a small umklapp intensity, its phase is fixed at $\phi_0 = 2.5$ rad corresponding to the zero-oscillator-strength limit of a resonance phase in our device, similarly as in Methods section ‘Determination of energies of the exciton and umklapp resonances’). We note that the splitting $\Delta E_0 = 1.4$ meV between the two resonances is found to be larger by about 0.8 meV than that theoretically expected for the investigated $n_e = 1.16 \times 10^{11}$ cm$^{-2}$ by a model from Supplementary Information section 3. The umklapp peak therefore appears in the experimental spectra to be slightly redshifted with respect to its theoretically predicted position (marked by the green dashed line in Extended Data Fig. 7a, b), but still exhibits similar blueshifting slope with increasing $n_e$ as that found at $B = 0$ and $B = 6$ T (for example, in Figs. 1d, 3c). Such a small deviation in the umklapp spectral position (not observed at lower fields) indicates that some of the assumptions of our theoretical model (for example, concerning the strength of the electron–hole exchange interaction, or symmetric Zeeman splitting of $\sigma^-$ and $\sigma^+$-polarized main exciton branches) are not fully valid, but the present experimental data does not allow us to provide a definitive explanation of the observed effect. Nevertheless, to account for its presence in our theoretical analysis of disorder-related effects in Supplementary Information section 6, the energy shift of the umklapp peak in equation (S55) was adjusted to match the actual umklapp position determined in the experiment.

The fit in Extended Data Fig. 7c evidences the umklapp resonance to be about two times narrower as compared to the main exciton transition. This is expected (in the relevant case of a weak disorder) since the exciton is broadened by both radiative and nonradiative decay processes, whereas the radiative broadening of the umklapp resonance is almost fully suppressed owing to its negligible spectral weight. To independently determine the radiative and nonradiative contributions to the linewidth of the main resonance, we fit its lineshape in the bare reflection spectrum with a transfer-matrix model assuming the following complex susceptibility of the MoSe$_2$ monolayer $\chi(\Omega) = -(\hbar / E_0) \gamma_{\text{rad}} / (E - E_0 + i \gamma_{\text{rad}} / 2)$, where $\gamma_{\text{rad}}$ and $\gamma_{\text{rad}}$ denote, respectively, radiative and nonradiative exciton decay rates. In these calculations we neglect the presence of the umklapp peak and use the same parameters (for example, hBN refractive index) as listed in Methods section ‘Determination of energies of the exciton and umklapp resonances’. As shown in Extended Data Fig. 7d, such a fit aptly reproduces the data allowing us to extract $\gamma_{\text{rad}} \approx 0.78$ meV. In the following we assume that this nonradiative decay rate is independent of the exciton momentum such that the umklapp resonance is subjected to the same broadening $\gamma_{\text{rad}}$ in addition to the broadening $\gamma_{\text{dis}}$ stemming from electronic disorder in the Wigner crystal phase (that does not affect the zero-momentum exciton). We also neglect any other possible sources of broadening of the umklapp resonance, which allows us to express its linewidth as $\gamma_{\text{umklapp}} = \gamma_{\text{rad}} + \gamma_{\text{dis}}$: this is the principal assumption of the analysis presented in this section, which cannot be verified experimentally.

Under the above assumptions we are able to compare the experimental data with the results of the theoretical model of disorder-induced umklapp broadening developed in Supplementary Information section 6. To this end, we first fix the correlation length $\xi_{\text{corr}}$ of the Wigner crystal and calculate the corresponding dependence of $\gamma_{\text{dis}}$ on the umklapp amplitude $a_0$ relative to the main peak, which is controlled by the strength $\lambda_{\text{exc}}$ of the exciton–electron interaction. On this basis we compute the linewidth $\gamma_{\xi_{\text{corr}}} = \gamma_{\text{rad}} + \gamma_{\text{dis}}$ and amplitude $A_0 = a_0 A_0$ of the umklapp peak that are plugged into the differentiated dispersive Lorentzian formula $R_s(E) + R_c(E)$, in which all other parameters are kept fixed at the values obtained from the fit in Extended Data Fig. 7c. Finally, such a formula $R_s(E) + R_c(E)$ is fitted to differentiated reflectance contrast spectrum with $\lambda_{\text{exc}}$ being the only fitting parameter. Its value is also constrained to be lower than the slope of density-dependent blueshift $\Delta E_s / \Delta n_s = 2 \times 10^{-15}$ meV cm$^{-2}$ of the main exciton resonance at $B = 14$ T, which corresponds to the upper bound for $a_0 \lesssim 5$% (note that the current value $\Delta E_s / \Delta n_s$ is about two times smaller as compared to the zero-field $\Delta E_s / \Delta n_s$ given in Supplementary Information section 3, which is most probably due to magnetic-field-induced changes in the dynamical screening of excitons by itinerant electrons).

The results of such a fitting procedure carried out for various $\xi_{\text{corr}}$ ranging from 3 to 14 Wigner crystal lattice constants $a_0$ are displayed in Extended Data Fig. 7e (and in Fig. 4c for the narrower energy range). Remarkably, the theoretical curves corresponding to larger Wigner crystal correlation lengths clearly better reproduce the experimental lineshape of the umklapp resonance, whereas those for low $\xi_{\text{corr}}$ fail to capture the umklapp. This analysis can be made more quantitative by computing the coefficient of determination $R^2(\xi_{\text{corr}}) = 1 - \left( \sum [y_i - f(\xi_{\text{corr}})]^2 / \sum [y_i - \bar{y}]^2 \right)$ based on the comparison between the fit and data points $y_i$ in the 2-meV-wide window around the umklapp energy (marked by shaded region in Extended Data Fig. 7e); here $\bar{y}$ denotes the average value of differentiated reflectance signal in the selected spectral window, and $f(\xi_{\text{corr}})$ represents the value of the fit corresponding to $\xi_{\text{corr}}$ evaluated at the energy of the $i$th data point. As shown in Extended Data Fig. 7f, the $R^2$ determined for the largest $\xi_{\text{corr}} = 14a_0$ is almost the same as that corresponding to unconstrained fit of the exciton data from Extended Data Fig. 7c (in the case of which $R^2$ is limited to 0.76 owing to the existence of deviations between the main exciton lineshape and the postulated simple dispersive Lorentzian spectral profile, which may be caused, for example, by exciton–electron interactions). At the same time, the value $R^2$ sharply drops when lowering $\xi_{\text{corr}}$ beyond $-5a_0$, allowing us to provide a rough lower bound for $\xi_{\text{corr}} \geq 5a_0$, corresponding to $\xi_{\text{corr}} \geq 160$ nm for the
investigated $n_e$. We stress that a similar result is also obtained based on our analysis of the reflectance spectrum acquired at similar $\nu$ at $B = 16$ T, which strengthens the generality of our conclusion. However, we note that given the number of aforementioned assumptions, the obtained lower bound for $\xi_{\text{corr}}$ might be fraught with a systematic error. At the same time we also emphasize that the employed theoretical model from Supplementary Information section 6 overestimates the role of disorder.

Reproducibility of the results on a different device

As stated in the main text, we observed similar signatures of the electronic Wigner crystal also for a second device, which was fabricated using the same technique as the main one. In general, this device featured a more complex structure, but in the present study we focused exclusively on the region where the layer arrangement was similar to the case of the main device, and consisted of a dual-graphene-gated MoSe$_2$ monolayer that was fully encapsulated between the two hBN layers. The thicknesses of those hBN flares (measured with AFM) were slightly smaller and yielded $t_g = 41 \pm 5$ nm and $t_{\text{e}} = 50 \pm 5$ nm, respectively, for the top and bottom layers. Unlike the case of the main sample, the optical measurements on the second sample were not carried out in a dilution refrigerator, but in a standard dipstick filled with about 0.3 mbar of helium exchange gas and immersed in a liquid helium bath cryostat, allowing us to reach the sample temperature of $T = 4$ K. The electron density $n_e$ was controlled by applying a top-gate voltage $V_t$ while keeping the back gate and the MoSe$_2$ monolayer grounded.

Extended Data Fig. 8a presents a gate-voltage evolution of the reflectance contrast spectra measured for the second sample at $B = 0$ in the spectral vicinity of the exciton resonance. The corresponding voltage dependence of the derivative of $R/W_r$ with respect to $V_t$ is shown in Extended Data Fig. 8b. The differentiated data clearly demonstrate the presence of an umklapp peak on the high-energy side of the exciton, which behaves in the same way as in the case of the main device: its detuning from the main exciton peak increases for larger electron densities, it appears to merge with the exciton in the limit of $n_e \rightarrow 0$, and becomes indiscernible at $n_e \gtrsim 4 \times 10^{12}$ cm$^{-2}$. Owing to lower intensity of the umklapp peak in the present experiments (partially stemming from elevated temperature of the sample), we find it difficult to directly fit its spectral profile using the same technique as for the main device. Instead, we attempt to estimate the umklapp spectral position as $E_x + \Delta E_{\text{UV}}$, where $E_x$ represents the exciton energy extracted from dispersive Lorentzian fit, while the $\Delta E_{\text{UV}} = \hbar^2 n_e/\sqrt{3} m_e$ denotes the exciton–umklapp splitting that is computed under assumption of the triangular Wigner crystal geometry and for the value of the exciton mass $m_x = m_e + m_h = 1.3m_e$ obtained from previous experiments on MoSe$_2$ monolayers.$^{13,29,30,56}$ The electron density $n_e$ is in turn determined within a parallel-plate capacitor approximation as $n_e/(V_t - V_0) C_{\text{geom}} e$, where $C_{\text{geom}} = \epsilon_0 \epsilon_r C_{\text{hBN}} / t_z = 0.76 \pm 0.22$ nF mm$^{-2}$ stands for the geometrical capacitance of the device, and $V_0 = 0.5$ V is the voltage corresponding to the onset of filling the conduction band with electrons (which is extracted from the reflectance data as $V_t$, at which the main exciton resonance starts to blueshift). Remarkably, the umklapp energy estimated in the above-defined way (marked with a green dashed line in Extended Data Fig. 8a, b) remains in a good agreement with the actual peak position. This finding further supports the identification of the umklapp peak and confirms that the investigated electronic Wigner crystallization is not specific to a single device, but is a general characteristic of TMD monolayers.

Data availability

The data that support the findings of this study are available at the ETH Research Collection (https://doi.org/10.3929/ethz-b-000478739).

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Acknowledgements C.K. and A.I. thank M. Knap for discussions. This work was supported by the Swiss National Science Foundation (SNSF) under grant no. 200021-178909/1. K.W. and T.T. acknowledge support from the Elemental Strategy Initiative conducted by MEXT, Japan, A3 Foresight by JSPS and CREST (grant no. PMUC15SF3) and JST. P.E.D., I.E. and E.D. were supported by Harvard-MIT CUA, AFOSR-MURI, Photonic Quantum Matter award FA95501601323, Harvard Quantum Initiative.

Author contributions T.S. and A.I. conceived the experiments. T.S. carried out the measurements and analysed the data. M.K. and Y.S. helped with the experimental setup and the data analysis. K.W. and T.T. grew the hBN crystals. A.P., P.B. and X.L. fabricated the samples. P.E.D., C.K. and I.E. did the theoretical modelling and carried out the calculations under the guidance of E.D.T.S., P.E.D. and A.I. wrote the manuscript. T.S., E.D. and A.I. supervised the project.

Competing interests The authors declare no competing interests.

Additional information Supplementary information The online version contains supplementary material available at https://doi.org/10.1038/s41586-021-03590-4.

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Peer review information Nature thanks the anonymous reviewers for their contribution to the peer review of this work.

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**Extended Data Fig. 1 | Device structure and the experimental setup.**

**a**, Illustration of the structure of the device investigated in the main text. **b**, Simplified schematic of the experimental setup used for magneto-optical, mK-temperature measurements. FLG, few-layer graphene; NA, numerical aperture.
Extended Data Fig. 2 | Normalization and differentiation of the reflectance data. 

a. Reflectance spectra acquired for the main device at \( T = 80 \text{ mK} \) at two different spots: one in the MoSe\(_2\) monolayer region (blue) and one off the MoSe\(_2\) monolayer (red). Both spectra were obtained at charge neutrality (\( V_t = -1 \text{ V} \)) and in the absence of the magnetic field.

b. The reflectance contrast spectrum \( R = \Delta R/R_0 \) determined based on the two spectra from a.

c, d. Colour plots showing zero-field top-gate voltage evolutions of the derivative of reflectance contrast \( R'_c = dR_c/dV_t \), with respect to the \( V_t \) (dashed lines mark the exciton and umklapp energies). The left panel presents a derivative evaluated numerically using standard, symmetric difference quotient method as \( R'_c(V_t) = [R_c(V_t+1) - R_c(V_t-1)]/[V_t+1 - V_t-1] \). The right panel shows a derivative of the same data obtained using the other method, in which \( dR_c/dV_t \) is computed as a difference quotient between symmetric data points separated not by two, but by four gate-voltage steps, that is, \( R'_c(V_t) = [R_c(V_t+2) - R_c(V_t-2)]/[V_t+2 - V_t-2] \). This method was used to plot the gate-voltage derivatives in Figs. 1–3.
Extended Data Fig. 3 | Fitting the exciton and umklapp peaks with dispersive Lorentzian spectral profiles. **a**, Colour map showing top-gate voltage evolution of the reflectance contrast $R_c$ spectra measured at $T = 80$ mK for the main device, at zero magnetic field, for low electron doping densities, and in the spectral vicinity of the exciton peak. **b**, Similar map presenting evolution of the reflectance contrast $R_c$ upon subtraction of the fitted exciton spectral profile $R_X$ as well as a smooth Gaussian-like background $B$ (independent of the voltage). The dashed lines in both panels indicate the energies of the exciton and umklapp peaks. **c, d**, Cross-sections through the maps in **a, b** at $V_t = 0.8$ V showing, respectively, bare and background-corrected reflectance contrast spectra. The solid lines indicate the fits to the experimental data with dispersive Lorentzian spectral profiles, based on which we determined the exciton and umklapp energies (marked by vertical dashed lines). In the case of the umklapp peak, the fitting was carried out only in the energy region covered by the data points shown in green, in order to avoid spurious contribution around the energy of the main exciton (of about 1,635.5 meV at the selected $V_t = 0.8$ V) that originates from the residual of the exciton resonance fitting.
Extended Data Fig. 4 | Calibration of the electron density for the dilution refrigerator experiments based on the Shubnikov–de Haas oscillations in the exciton linewidth. a, Colour map presenting \( \sigma^- \)-polarized reflectance contrast spectra measured at \( T = 80 \) mK for the main device as a function of the top-gate voltage at \( B = 16 \) T. b, Gate-voltage dependence of the exciton linewidth extracted from the data in a by fitting the exciton resonance with a dispersive Lorentzian spectral profile. Dashed lines in both panels indicate the positions of integer filling factors. c, Gate voltages \( V_t(\nu, B) \) corresponding to the positions of the exciton linewidth minima extracted from the reflectance measurements carried out at different magnetic fields. All of the presented data points were obtained in the regime where the Fermi level does not exceed the valley Zeeman splitting in the conduction band. Solid lines represent the fit of the data points with a set of linear dependencies corresponding to subsequent integer filling factors, which form a Landau level fan chart.
Extended Data Fig. 5 | Comparison of the zero-field Wigner crystal signatures at $T = 80$ mK and $T = 4$ K. a, b. Colour maps showing zero-field top-gate voltage evolution of the derivative of reflectance contrast spectra with respect to $V_t$ measured in the dilution refrigerator for the main device at two different temperatures: $T = 80$ mK (a) and $T = 4$ K (b). Black dashed lines mark the fitted energy $E_X$ of the exciton resonance, and the green lines indicate the expected position of the umklapp peak $E_X + \Delta E_U$ for $\Delta E_U = \hbar^2 \kappa_U / \sqrt{3} m_X$ corresponding to a triangular Wigner crystal and $m_X = 1.1 m_e$. 
Extended Data Fig. 6 | Melting of the Wigner crystal at elevated temperatures. a, Colour maps displaying back-gate voltage evolution of the derivative of reflectance contrast spectra with respect to $V_b$ measured in the VTI-based setup at zero magnetic field and different temperatures of the main device (as indicated). In each map, the black dashed lines mark the fitted energy $E_X$ of the main exciton peak, while the green line in the map at $T = 4.5\text{ K}$ represents the expected position of the umklapp resonance $E_h n_e/\sqrt{3} m_e$ corresponding to a triangular Wigner crystal and $m_e = 1.3 m_e$. 

cross-sections through the maps in a (as well as through a similar map measured at $T = 15.2\text{ K}$ at $V_b = -0.6\text{ V}$ (b), $V_b = -0.45\text{ V}$ (c), and $V_b = -0.35\text{ V}$ (d) corresponding to electron densities $n_e/10^{11}\text{ cm}^{-2}$ of, respectively, 1.2, 1.6 and 1.9. e, Relative amplitude of the umklapp transition determined as a function of the temperature at the average value of $d(\Delta R/R_0)/dV_b$ in a 0.8-meV-wide energy window around the umklapp energy (marked by shaded regions in b–d) at different electron densities (as indicated). The error bar for each data point is estimated as a standard error (that is, standard deviation of the mean) of all points within the aforementioned energy window for a given $n_e$ and $T$. The solid lines represent the fits of the temperature dependence for each $n_e$ with a linear decrease $a(T_m - T) + b$ at $T < T_m$ and a flat background level $b$ at $T > T_m$ with a melting temperatures $T_m = 11 \pm 1 \text{ K}$ of the Wigner crystal being found to be the same for all investigated values of $n_e$ within the experimental uncertainty. 
f, The same cross-sections as in c (determined at $V_b = -0.45\text{ V}$) together with the solid lines representing the fits of the umklapp lineshapes at different temperatures with a phenomenological Lorentzian profile overlaid on a (fixed) polynomial background. g, Temperature-dependent relative amplitude of the umklapp peak at $n_e = 1.6 \times 10^{11}\text{ cm}^{-2}$ extracted based on the Lorentzian fits from f (as well as based on similar fits of the data taken at different temperatures; the error bars correspond to standard errors increased by a temperature-independent term that is introduced to account for systematic uncertainty stemming from determination of the background profile). Solid line indicates a linear fit to the data points with non-zero amplitude corresponding to the same Wigner crystal melting temperature of $T_m = 11 \pm 1\text{ K}$. 
Extended Data Fig. 7 | Analysis of disorder-induced broadening of the umklapp peak. 

**a, b**, Colour maps showing top-gate voltage evolution of the differentiated reflectance contrast $d(\Delta R/R_0)/dE$ (a) and the bare reflectance contrast $\Delta R/R_0$ (b) acquired for the main device at $B = 14$ T, $T = 80$ mK, and in $\sigma^-$ polarization of detection (note that these results were obtained in a different measurement than that shown in Fig. 3e, f). Grey horizontal dashed lines mark the voltages corresponding to filling factors of $\nu = 0, 0.34,$ and 1. White (black) dashed (dotted) line indicates the fitted energy $E_{\text{X}^-}$ ($E_{\text{X}^+}$) of the $\sigma^-$-polarized ($\sigma^+$-polarized) main exciton resonance. The green dashed line represents the expected energy of the umklapp peak determined as $E = E_{\text{X}}(\sigma^+ + \Delta E_{\text{Z}} + \Delta E_{\text{U}})/2$, where $\Delta E_{\text{Z}} = \hbar^2 n_c/\sqrt{3} m_e + s k_w - s (2k_w)^2 + (\Delta E_2/2)^2$ represents the umklapp main exciton detuning calculated within the frame the model from Supplementary Information section 3 using the exciton mass of $m_e = 1.3 m_0$, the strength of the electron-hole exchange $s k_w = 300$ meV (with $k_w$ representing the valley momentum), and experimentally determined main exciton Zeeman splitting $\Delta E_2 = 3.6$ meV. 

**c, d**, Cross-sections through the maps in **a, b** at $V_t = 0.45$ V corresponding to the electron density $n_e = 1.16 \times 10^{11}$ cm$^{-2}$ and Landau level filling factor $\nu = 0.34$. The solid line in c shows the fit of the experimental data with a sum of two differentiated dispersive Lorentzian spectral profiles $R_{\text{X}^-}^c(E) + R_{\text{X}^+}^c(E)$ representing the main and umklapp resonances. The solid line in d in turn depicts the fit of the main exciton resonance in the bare reflectance spectrum with a transfer-matrix model, allowing us to extract the nonradiative broadening of the exciton $\gamma_{\text{X}^\text{rad}} = 0.78$ meV. 

**e**, Fits of the differentiated reflectance spectrum at $\nu = 0.34$ with the sum of two differentiated dispersive Lorentzians $R_{\text{X}^-}^c(E) + R_{\text{X}^+}^c(E)$ corresponding to different correlation lengths of the Wigner crystal (as indicated). For each fit the parameters of the main exciton resonance (that is, amplitude $A_X$, linewidth $\gamma_X$, energy $E_X$ and phase $\phi_X$) as well as the phase $\phi_0$ and the energy $E_0$ of the umklapp peak are fixed at the values obtained from the fit in c. The linewidth and amplitude of the umklapp resonance are assumed to be given by $\gamma = \gamma_{\text{X}^\text{rad}} + \gamma_{\text{U}^\text{dis}}$ and $A_U = a_U A_X$, where the disorder-induced broadening $\gamma_{\text{U}^\text{dis}}$ and the relative amplitude $a_U$ are fitted based on the predictions of the theoretical model from Supplementary Information section 6 describing the umklapp lineshape for a Wigner crystal with a given correlation length. For all fits $a_U$ is assumed to be smaller than 5%. 

**f**, The coefficient of determination $R^2$ evaluated for the fits corresponding to different Wigner crystal correlation lengths based on the data points in a 2-meV wide energy window around the umklapp resonance (marked by shaded region). The dashed line indicates the value of $R^2$ obtained in the same way for an unconstrained fit in c.
Extended Data Fig. 8 | Observation of zero-magnetic-field Wigner crystal signatures for the second device. a, Colour map showing reflectance contrast spectra measured as a function of the top-gate voltage $V_t$ for the second device. The data were acquired at $T = 4$ K and in the absence of the magnetic field. b, Gate-voltage evolution of the derivative of the spectra from a with respect to $V_t$. Black dashed lines in both panels indicate the energy of the exciton peak $E_X$ obtained by fitting its spectral profile with dispersive Lorentzian lineshape. Green lines mark the expected position $E_X + \Delta E_{\text{umklapp}}$ of the umklapp peak, where $\Delta E_{\text{umklapp}} = \hbar^2 n_e^2 / 12 m_e$ is computed under an assumption of a triangular Wigner crystal and for the value of exciton mass $m_e = 1.3 m_e$. 

Carbon