AN IMPROVED METHOD TO TEST THE DISTANCE–DUALITY RELATION

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\textbf{ABSTRACT}

Many researchers have performed cosmological-model-independent tests for the distance–duality (DD) relation. Theoretical work has been conducted based on the results of these tests. However, we find that almost all of these tests were perhaps not cosmological-model-independent after all, because the distance moduli taken from a given type Ia supernovae (SNe Ia) compilation are dependent on a given cosmological model and Hubble constant. In this Letter, we overcome these defects and by creating a new cosmological-model-independent test for the DD relation. We use the original data from the Union2 SNe Ia compilation and the angular diameter distances from two galaxy cluster samples compiled by De Filippis et al. and Bonamente et al. to test the DD relation. Our results suggest that the DD relation is compatible with observations, and the spherical model is slightly better than the elliptical model at describing the intrinsic shape of galaxy clusters if the DD relation is valid. However, these results are different from those of previous work.

\textit{Key words:} distance scale -- galaxies: clusters: general -- supernovae: general

\textit{Online-only material:} color figures

1. INTRODUCTION

The luminosity distance, $D_L$, and the angular diameter distance, $D_A$, are both fundamental observations in astronomy. They satisfy an important relationship known as the distance–duality (DD) relation (Ellis 2007), which can be expressed as

\begin{equation}
\frac{D_L}{D_A}(1 + z)^{-2} = 1, \quad (1)
\end{equation}

where $z$ is the cosmological redshift. This equation is always valid if and only if the following three conditions are satisfied (Ellis 1971):

1. cosmological models are based on Riemannian geometry,
2. photons travel along a null geodesic, and
3. the photon number is conserved.

The DD relation is violated when all of the above conditions are not satisfied. Some non-metric theories do not meet condition 1 or condition 2. The DD relation is also violated if condition 3 is not satisfied (Basset & Kunz 2004). The sensitivity of the detector, dust, and an exotic theory (e.g., photon decay) can cause non-conservation of the photon number. Therefore, it is necessary to test whether this relation is valid in the real universe.

The parameter $\eta(z)$ was introduced by previous authors to test the DD relation (Holanda et al. 2010), i.e.,

\begin{equation}
\frac{D_L}{D_A}(1 + z)^{-2} = \eta(z), \quad (2)
\end{equation}

where the DD relation holds when $\eta(z) = 1$. The way to accomplish a cosmological-model-independent test for the DD relation is to use values of both $D_L$ and $D_A$ from observations (Holanda et al. 2010). It is important to note that the methods of measuring $D_L$ and $D_A$ should be cosmology-independent and should also be independent of each other.

Generally, the $D_A$ data are obtained from galaxy cluster samples. Based on observations of the Sunyaev–Zeldovich effect (SZE) and X-ray surface brightness from galaxy clusters, the intrinsic sizes of galaxy clusters can be measured, which can enable us to derive the angular diameter distance of a galaxy cluster, $D_A^\text{cluster}$ (Reese et al. 2002). Moreover, the $D_L$ data can be obtained from type Ia supernovae (SNe Ia) sample compilations. The DD relation can be tested by plugging these data into Equation (2) (De Berbardis et al. 2006).

However, using this method to test the DD relation is inappropriate. Uzan et al. (2004) pointed out that the SZE and X-ray techniques are related to the DD relation, which means that observations of the angular diameter distance, $D_A^\text{cluster}$, and the true angular diameter distance, $D_A$, have the following relation:

\begin{equation}
\frac{D_A^\text{cluster}}{D_A} = \eta^2, \quad (3)
\end{equation}

The above equation means that $D_A^\text{cluster} = D_A$ only when the DD relation holds ($\eta = 1$), but their equation is slightly different from ours because of the different definitions of $\eta$ used. Therefore, $D_A^\text{cluster}$ should not be put directly into Equation (2) to test the DD relation. Holanda et al. (2010) plugged Equation (3) into Equation (2) to get $D_A^\text{cluster}(1 + z)^2/D_L = \eta(z)$. Then, they used SNe Ia data $D_L$ and $D_A^\text{cluster}$ from two galaxy cluster samples compiled by De Filippis et al. (2005) and Bonamente et al. (2006) to constrain $\eta(z)$, assuming $\eta(z) = 1 + \eta_1 z$ and $\eta(z) = 1 + \eta_2 z/(1 + z)$. Their results show that the DD relation can be satisfied at $2\sigma$ CL for an elliptical model and cannot be satisfied for a spherical model even at $3\sigma$ CL. In subsequent work, Li et al. (2011) and Meng et al. (2012) came to the conclusion that the DD relation is satisfied at the $1\sigma$ CL for an elliptical model, and it cannot be satisfied even at the $2\sigma$ for a spherical model. Thus, these authors concluded that an elliptical model is better than a spherical model. Lima et al. (2011) suggested that the deviation of $\eta$ from 1 may indicate a break with fundamental physical theories.

However, we suggest that the SNe Ia data cannot also be put directly into Equation (2) to constrain $\eta(z)$, because the distance modulus, $\mu$, of SNe Ia data depends on a cosmological model and selection of the Hubble constant, $H_0$. Therefore, previous work may need to be improved.
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In this Letter, we make a slight change to the distance estimate procedure of SALT2 (Guy et al. 2007) then we perform an improved cosmological-model-independent test of the DD relation.

This Letter is organized as follows. Section 2 briefly introduces the approach for obtaining the distance moduli of SNe Ia. In Section 3, we briefly describe the SNe Ia data (Amanullah et al. 2010) and the angular diameter distance data (De Filippis et al. 2005; Bonamente et al. 2006). In Section 4, we propose a new cosmological-model-independent method for testing the DD relation and note our results. Finally, the discussions and conclusions are given in Section 5.

2. A BRIEF INTRODUCTION TO THE APPROACH FOR OBTAINING THE DISTANCE MODULI OF SNe Ia

In astronomy, astronomers use SNe Ia as secondary standard candles to measure luminosity distance because the peak luminosities of light curves (the graphs of luminosity as a function of time) of all SNe Ia are nearly identical. In other words, their peak absolute magnitudes $M_{\text{max}}$ are nearly identical. Assuming a Cepheid variable and a SN Ia share the same host galaxy, one can use the Cepheid variable to measure the luminosity distance $D_L$ of the host galaxy. Combining the peak magnitudes $m_{\text{max}}$ of the SN Ia with the formula for the distance modulus:

$$u = 5 \log D_L - 5 = m_{\text{max}} - M_{\text{max}},$$

the peak absolute magnitude $M_{\text{max}}$ of an arbitrary SN Ia can be easily obtained, because every SN Ia has almost the same $M_{\text{max}}$. Therefore, the luminosity distance of an arbitrary SN Ia can be obtained if its $m_{\text{max}}$ is known. However, the peak luminosities of SNe Ia are not exactly the same because they are related to the shapes and colors of the light curves (Guy et al. 2005) and the extinction effects the magnitudes $m_{\text{max}}$. So Equation (4) needs to be modified. Many fitters (SALT (Guy et al. 2005), SALT2 (Guy et al. 2007), MLCSC2K2 (Jha et al. 2007)) have been proposed to parameterize the light curves of SNe Ia and their distance moduli can be obtained.

We take the light-curve fitter SALT2 as an example to illustrate the process for obtaining the distance modulus (Guy et al. 2007). Guy et al. (2007) modified Equation (4) by adding perturbations of shapes and colors to get

$$\mu_B(\alpha, \beta, M_B) = m_{\text{max}}^B - M_B + \alpha x - \beta c,$$

where $m_{\text{max}}^B$ is the rest-frame peak magnitude of the $B$ band, $x$ is the stretch factor, which describes the effects of the shapes of light curves on $\mu$, and $c$ is the color parameter, which represents the influences of the intrinsic color and reddening due to dust on $\mu$. These three parameters can be obtained by fitting the light curves of SNe Ia. Thus, they are independent of the cosmological model. The absolute magnitude $M_B$, $\alpha$, and $\beta$ are nuisance parameters, which will be fitted by minimizing the residuals in the Hubble diagram given by the cosmological model. For example, Amanullah et al. (2010) used the abovementioned method and a $\chi^2$ minimization to constrain $\Omega_M$, $\omega$ and to obtain the Union2 compilation. The formula for the $\chi^2$ minimization is

$$\chi^2(\alpha, \beta, M_B) = \sum_{\text{SNe}} \left[ \frac{\mu_B(\alpha, \beta, M_B; z) - \mu_{\text{theory}}(\Omega_M, \Omega_\omega, \omega; z)}{\sigma_{\text{total}}} \right]^2.$$

where $\mu_{\text{theory}}(\Omega_M, \Omega_\omega, \omega; z)$ is obtained from the $\omega$CDM model. The best-fit values of $\alpha$, $\beta$, and $M_B$ are obtained by minimizing $\chi^2$, and then, $\mu$ is obtained. Obviously, $\mu$ is strongly dependent on the $\omega$CDM model because its value is obtained from Equation (6). One point should be noted, which is that $\mu_{\text{theory}}(\Omega_M, \Omega_\omega, \omega; z)$ contains a constant term $5 \log H_0$. For example, the formula of $D_L$ for a flat $\Lambda$CDM model reads

$$D_L = \frac{c(1+z)}{H_0} \int_0^z \frac{dz'}{[\Omega_M(1+z')^3 + (1-\Omega_M)]^{1/2}},$$

where $c$ is the speed of light and $H_0$ is the Hubble constant. One can easily see the constant term $5 \log H_0$ by putting this equation into Equation (4).

Therefore, $M_B$ is degenerate with $H_0$ because they are constants and have the same status in Equation (6). Using the method of minimizing $\chi^2$, Amanullah et al. (2010) obtained the best-fit values of $M_B = 5 \log H_0$, $\alpha$, and $\beta$ and they chose $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ to get $M_B$ and $\mu$. It is obvious that $\mu$ in the Union2 SNe Ia compilation is dependent on the choice of $H_0$ and the $\omega$CDM model. The distance moduli $\mu$ of other SNe Ia samples (e.g., Constitute (Hicken et al. 2009), Davis07 (Davis et al. 2007)) are obtained in a similar way. It is important to note that the arbitrary selection of $H_0$ does not affect the restriction on $\Omega_M$ and $\omega$, because they used $m_{\text{max}}^B$, $c$, and $x$ rather than $\mu$ to constrain $\Omega_M$ and $\omega$.

Thus, it is inappropriate to directly use the distance moduli $\mu$ of SNe Ia samples to test the DD relation. In this Letter, we overcome $\mu$ and directly use $m_{\text{max}}^B$, $x$, and $c$ from the Union2 sample to test the DD relation. By marginalizing $M_B$, $\alpha$, and $\beta$, one can obtain the probability distribution of $\eta$.

3. SAMPLES

In order to test the DD relation, we need both $D_L$ and $D_A^{\text{cluster}}$ data from a cosmological-model-independent measurement. For $D_L$, we use the Union2 SNe Ia data (Amanullah et al. 2010), which contain 557 well-measured SNe Ia. For $D_A^{\text{cluster}}$, we employ the SZE and X-ray observations of two galaxy cluster samples: an elliptical model sample (De Filippis et al. 2005) and a spherical model sample (Bonamente et al. 2006). The elliptical model sample was compiled by De Filippis et al. (2005) with an isothermal elliptical $\beta$ model, which contains 18 galaxy clusters compiled by Reese et al. (2002) and 7 galaxy clusters compiled by Mason et al. (2001). Assuming that the distribution of cluster plasma and dark matter is hydrostatic equilibrium and spherical geometry, Bonamente et al. (2006) (see their Table 2) compiled a spherical model sample that includes 38 galaxy clusters. In principle, giving a $D_A^{\text{cluster}}$, one should select a $D_L$ of the SN Ia data point that shares the same redshift $z$ as the given data point of $D_A^{\text{cluster}}$ to get $\eta$. However, the above condition usually cannot be satisfied in reality. So we use the selection criteria, as in Holanda et al. (2010), $\Delta z = |z_{\text{SNe}} - z_{\text{cluster}}| < 0.005$ to select a SN Ia point. If there are at least two points of SNe Ia data that satisfy this criterion for a given $D_A^{\text{cluster}}$, we select the SN Ia data whose $\Delta z$ is the smallest. The selection criterion can be satisfied for all galaxy cluster data except for the cluster CL J1226.9 +3332 from the spherical model sample (Bonamente et al. 2006), which just gives the $\Delta z = 0.005$. We keep this cluster data point in our analysis.
4. NEW TEST FOR THE DD RELATION AND RESULTS

Combining Equation (2) with Equation (3), we get
\[
D_L = \eta(z)^{-1}D_A^{\text{cluster}}(1+z)^2,
\]
then, we define
\[
\mu_{\text{cluster}}(\eta; z) = 5 \log [\eta(z)^{-1}D_A^{\text{cluster}}(1+z)^2] - 5,
\]
which is the distance modulus of a galaxy cluster data point. Because \(D_A \simeq D_L\) when \(z \to 0\), we parameterize \(\eta(z)\) in the following form as in Holanda et al. (2010),
\[
\eta(z) = 1 + \eta_0 z,
\]
where \(\eta_0\) is a constant. We only use this functional form in our analysis because other forms of \(\eta(z)\) can approximate this form via a Taylor expansion when \(z < 1\) (the redshifts of all data points in our analysis are smaller than 1). Now, we use a \(\chi^2\) minimization to constrain \(\eta_0\),
\[
\chi^2(\alpha, \beta, M_B, \eta_0) = \sum_i \left[ \mu_i(\alpha, \beta, M_B; z_i) - \mu_{\text{cluster}}(\eta_0; z_i) \right]^2,
\]
where \(\mu_i(\alpha, \beta, M_B; z_i)\) of a SN Ia comes from Equation (5), \(\mu_{\text{cluster}}(\eta_0; z_i)\) of the galaxy cluster is given by Equation (9), and the uncertainty \(\sigma_{\text{total}}^2(z_i)\) is given by
\[
\sigma_{\text{total}}^2(z_i) = \sigma_m^2(z_i) + \sigma_x^2(z_i) + \sigma_c^2(z_i) + \left[ \frac{5}{\ln 10} \frac{\delta_D(z_i)}{D_A^{\text{cluster}}} \right]^2,
\]
where \(\sigma_m, \sigma_x, \sigma_c, \) and \(\delta_D\) are the errors of \(m_B^{\text{max}}, x, c, \) and \(D_A^{\text{cluster}}\) respectively.

Inserting data points \((m_B^{\text{max}}, x, c, \sigma_m, \sigma_x, \sigma_c)\) from the SNe Ia Union2 and data points \((D_A^{\text{cluster}}, \delta_D)\) of the galaxy cluster samples into Equation (11), we obtain \(\chi^2(\alpha, \beta, M_B, \eta_0)\). Then, the joint probability density of these parameters can be obtained, 
\[
P(\alpha, \beta, M_B, \eta_0) = A \exp(-\chi^2/2),
\]
where \(A\) is a normalized coefficient, which makes \(\int \int A \exp(-\chi^2/2) \, d\alpha \, d\beta \, dM_B \, d\eta_0 = 1\). By integrating over \(\alpha, \beta, \) and \(M_B, \) the probability distribution function of \(\eta_0\) is obtained, i.e., 
\[
P(\eta_0) = \int \int A \exp(-\chi^2/2) \, d\alpha \, d\beta \, dM_B.
\]

We adopt an iterative method to calculate \(P(\eta_0)\) with step size 0.01 for all parameters. In principle, we should calculate all the values of \(\chi^2(\alpha, \beta, M_B, \eta_0)\) for these parameters in a \(3\sigma\) interval instead of an infinite interval and then we get \(\chi^2, P(\alpha, \beta, M_B, \eta_0)\), and \(P(\eta_0)\), with \(P(\eta_0) \propto \sum_i \sum_j \sum_k P(\alpha(i), \beta(j), M_B(k), \eta_0)\), where \(i, j, k\) run over all the data points for \(\alpha, \beta, \) and \(M_B\) in a \(3\sigma\) interval with a step size of 0.01 respectively.

The next step is to use the equation, \(\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}\) to constrain the one-dimensional CL of a parameter with 1 and 4 levels (Press et al. 1992), where \(\chi^2_{\text{min}}\) is the minimum of \(\chi^2\). For example, if we want to calculate the 1\(\sigma\) and 2\(\sigma\) CL of \(\eta_0\), we just need to find the data points of \(\eta_0\) that satisfy \(\Delta \chi^2 \leq 1\) and \(\Delta \chi^2 \leq 4\), respectively.

Using the above procedure, combining the Union2 SNe Ia sample (Amanullah et al. 2010) and the elliptical model sample (De Filippis et al. 2005) to calculate \(\chi^2\), we get best-fitting values of \(M_B, \alpha, \beta, \) and \(\eta_0\) of \(-19.45, -0.03, 4.05, \) and 0.18, respectively. Then, we constrain \(\alpha, \beta, M_B\) by integrating over these to get the likelihood distribution of \(\eta_0\) and obtain \(\eta_0 = 0.16^{+0.56}_{-0.39}\) at the 1\(\sigma\) CL and \(\eta_0 = 0.16^{+1.31}_{-0.70}\) at the 2\(\sigma\) CL.

![Figure 1](https://example.com/figure1.png)

**Figure 1.** (a) Likelihood distribution function (LDF) of \(\eta_0\) for an elliptical model. (b) LDF of \(\eta_0\) for a spherical model. (A color version of this figure is available in the online journal.)
In our analysis, the best fitting values of \( M_B, \alpha, \) and \( \beta \) are \(-19.45, -0.03, 4.05 \) and \(-19.38, 0.34, 4.13 \) for the elliptical model and the spherical model, respectively. Because the difference in the best-fitting values of parameters shift the peak of the probability distribution of \( \eta \), our result will inevitably be different from that of Li et al. (2011). Their result shows that the DD relation can be satisfied for an elliptical model at \( 1\sigma (\eta_0 = -2.22^{+0.11}_{-0.13}) \) but cannot be satisfied for a spherical model even at \( 2\sigma (\eta_0 = -2.07^{+0.13}_{-0.12}) \). Moreover, we constrain the parameters \( M_B, \alpha, \) and \( \beta \) by integrating over them to plot \( P(\eta_0) \). Obviously, constraining these parameters will broaden the profile of \( P(\eta_0) \). Therefore, this operation may make the DD relation hold at \( 1\sigma \) by expanding the \( 1\sigma CL \) of \( \eta_0 \). Hence, because of the difference in the best-fitting values parameters and the constraints applied, we get a different conclusion, namely, that the DD relation is compatible with observations at \( 1\sigma \).

Furthermore, we plot the two-dimensional contour of \( \eta_0 \) versus \( \alpha, \eta_0 \) versus \( \beta, \) and \( \eta_0 \) versus \( M_B \) for the two galaxy cluster samples (see Figure (2)) to see whether there is a degeneracy effect on the DD relation. In Figure (2), the red and blue contours are derived from the spherical model and the elliptical model respectively. "\(^{\circ}\)" indicates the best fitting values of \( \alpha, \beta, M_B \) \((-19.31, 0.12, 2.51) \) by assuming \( \eta_0 = 0 \) in the Union2 sample.

The best values of planes \((\eta_0, \alpha), (\eta_0, \beta), \) and \((\eta_0, M_B)\) are \((0.02, 0.34), (0.02, 4.17), (0.01, -19.39) \) for red contours and \((-0.15, -0.04), (0.19, 4.13), (0.16, -19.46) \) for blue contours, respectively. There is no distinct degeneracy effect on \( \eta_0 \) because all the planes have similar intervals for \( \eta_0 \) at \( 1\sigma \) and \( 2\sigma \). With the same operation on \( \eta_0 \), we found that at the \( 1\sigma CL \), parameters \( M_B, \alpha, \) and \( \beta \) respectively are \(-19.37^{+0.14}_{-0.16}, 0.34^{+0.06}_{-0.08}, 4.19^{+0.58}_{-0.62} \) for the spherical model, and \(-19.42^{+0.20}_{-0.24}, -0.04^{+0.10}_{-0.09}, 4.35^{+1.20}_{-1.73} \) for the elliptical model samples. In the Union2 SNe Ia sample (Amanullah et al. 2010), they are \(-19.31^{+0.014}_{-0.014}, 0.12^{+0.007}_{-0.007}, 2.51^{+0.007}_{-0.007} \) respectively. Comparing these three sets of data, one finds that only the confidence level of \( M_B \) is compatible. For \( \alpha, \) these samples fail to be compatible with each other at \( 1\sigma \). For \( \beta, \) the two galaxy cluster samples have \( \beta \sim 4.10 \) which is larger than the 2.51 of the SNe Ia Union2 sample; one can see these conclusions in Figure (2). The reasons why these three sets of data are different from each other are worth considering.

5. DISCUSSION AND CONCLUSIONS

Holanda et al. (2010) proposed a cosmological-model-independent method of testing the DD relation. Using this method, much research (Nair et al. 2011, 2012; Yang et al. 2013) has been done. However, we contend that the method used in these previous studies may depend on the selection of \( H_0 \) and a cosmological model. Different choices of \( H_0 \) will lead to different constraints on \( \eta_0 \), and one cannot eliminate this effect by constraining \( H_0 \) because \( M_B \) is degenerate with \( H_0 \).

Therefore, we improve the method and perform a new test for the DD relation. In the previous method (Holanda et al. 2010), \( D_L \) was directly obtained from the distance moduli \( \mu \) of SNe Ia. However, in our analysis, \( D_L \) is not provided by the distance modulus \( \mu \) but by the original data \( m_B, x, c \). In this way, we do not need a given cosmological model or any information about \( H_0 \). Hence, our test is independent of both the cosmological model and \( H_0 \).

Our results show that the DD relation can be satisfied at \( 1\sigma CL \) for both an elliptical model and a spherical model, and that the spherical model is slightly better than the elliptical model if the DD relation is valid. These results, however, are different from those of previous work (Holanda et al. 2010; Li et al. 2011; Meng et al. 2012). In previous studies, the DD relation at most can be satisfied at \( 1\sigma CL \) only marginally for an elliptical model sample, and the DD relation can barely be satisfied at \( 3\sigma CL \) for a spherical model sample. Thus, previous authors concluded that the elliptical model is better than the spherical model in describing the intrinsic shape of galaxy clusters, and some work (Lima et al. 2011; Nair et al. 2012) used the deviation of \( \eta(z) \) from 1 to search for new physics. However, from Figure 1, one can see clearly that the DD relation is compatible with both an elliptical model and a spherical model at the \( 1\sigma CL \). Thus, the conclusions obtained by previous authors should be treated with caution.

Furthermore, we note that the best-fitting values of parameters \( M_B, \alpha, \) and \( \beta \) in different samples are different. Only \( M_B \) is compatible across different samples. This at least means the luminosity distances from these samples are similar because the stretch factor \( x \) and the color factor \( c \) experience perturbation. Even \( M_B \) in Union2 depends on \( H_0 \). For \( \alpha, \) the elliptical model sample gives the smallest values of \( \alpha \), and the spherical gives the largest values of \( \alpha \). For \( \beta, \) the two galaxy cluster samples have \( \beta \sim 4.10 \) which is larger than the 2.51 in the Union2 SNe Ia sample. This means that the color factor has a larger effect on \( H_0 \) in our analysis than in standard cosmological analysis. It is possible that these differences are produced by some unknown physical effects, inaccuracy, or too few data points for the galaxy cluster samples. Nevertheless, using more galaxy cluster samples or other methods which are independent of the
cosmological model to constrain $\alpha$, $\beta$, and $M_B$, and researching what cause these differences may be worth studying in the future.

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