A Hexapod Design for All-sky Sidereal Tracking

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Abstract

In this paper, we describe a hexapod-based telescope mount system intended to provide sidereal tracking for the Fly’s Eye Camera project—an upcoming moderate, 21"/pixel−1 resolution all-sky survey. By exploiting such a kind of meter-sized telescope mount, we get a device that is both capable of compensating for the apparent rotation of the celestial sphere and the same design can be used independently from the actual geographical location. Our construction is the sole currently operating hexapod telescope mount performing dedicated optical imaging survey with a sub-arcsecond tracking precision.

Key words: instrumentation: miscellaneous – surveys – techniques: photometric – telescopes

Online material: color figures

1. Introduction

One of the most challenging parts of astronomical instrumentation is providing a mount on which the optical system of the telescope is installed. Basically, such a mount provides both a way to point the telescope toward a certain celestial position and to compensate the apparent rotation of the celestial sphere. In modern observational astronomy, the most commonly used equatorial and alt-azimuth-type mounts have two-stage serial kinematics. Equatorial mounts are more frequently used in autonomous or remotely operated sites while azimuth mechanics is the standard solution for the ground-based telescopes with the largest apertures.

In order to accurately quantify the motion of such telescope mechanics, pointing models are constructed. Such mathematical models characterize how the various mount axes should be actuated to attain the desired celestial positions (what are usually defined in first equatorial coordinate system: hour angle and decl, for the current epoch). Pointing models that exist for equatorial (Spillar et al. 1993; Buie 2003) and alt-azimuth mechanics (see e.g., Granzer et al. 2012), as well as mathematical constructions, can be generated for both types of serial kinematics (Pál et al. 2016). In addition, accurate pointing models can also be derived if only the attitude (orientation) of mount axes are known in respect to the vertical direction (Méaszáros et al. 2014). However, this type of pointing feedback can only be exploited in the case of equatorial mounts.

Time-domain astronomy is a novel approach in astronomical research that recently became due to advances in instrumentation. These advances include the possibility of autonomous operation of large telescopes, the employment of mosaic CCD detector arrays, as well as the development of huge computer databases and parallel data-processing systems. The Panoramic Survey Telescope And Rapid Response System (Pan-STARRS; Kaiser et al. 2002) or the (currently under construction) Large Synoptic Survey Telescope (LSST; Ivezić et al. 2008) project are prominent examples of initiatives exploiting these features. In these projects, it is possible to map the visible sky within a cadence of a few days and detect variations from this timescale up to several years of the planned operations. These projects are also noticeable in their large etendue.

With increasing technical capabilities, it is now possible to observe astronomical objects and phenomena on a wide range of timescales from seconds to years. These kinds of research are only possible with surveys that not only run for years, but also have frequent temporal sampling.

The goal of the Fly’s Eye project is to provide a continuous, 21"/pixel−1 resolution all-sky survey in multiple optical passbands by imaging the entire visible sky above the altitude of 30° with a cadence of a few minutes. The scientific purpose of the Fly’s Eye device is to perform time-domain observations by covering a time range of nearly six magnitudes with an imaging cadence of a few minutes up to several years, i.e., the planned length of the operations. With this moderate resolution, individual stellar sources can be resolved with a sufficient signal-to-noise ratio in the magnitude range of r = 7–15. The scientific goals of such a simultaneous all-sky survey as Fly’s Eye are induced by the continuous monitoring of the bright-end (r ≤ 15) regime of astrophysical variations and transients. Such signals can either be a small-amplitude variation of a
brighter source (where the signal-to-noise ratio is limited by the photon noise or by systematics) or a larger-amplitude variation of a fainter sources (where the signal-to-noise ratio is limited by the background/readout noise or by confusion).

Regarding the science domains, this magnitude regime covers (a) brighter minor planets in the Solar System, including the homogeneous multicolor sampling of the brightest $\sim 10,000$ objects or the flyby of near-Earth asteroids; (b) stellar astrophysics, including multicolor photometry of pulsating and rotating variables, active stars, eclipsing multiple stellar systems, transiting extrasolar planets; and (c) detection and observations of extragalactic phenomena such as brighter supernovae, optical counterparts of GRBs.

In the Fly’s Eye design, the simultaneous surveying of the visible sky is performed by a mosaic array of 19 wide-field camera units where each of the cameras has a field of view of 26° in diameter. These cameras are assembled on a single, hexapod-based telescope mount. In addition, the total étendue of such a configuration is comparable to that of Pan-STARRS (i.e., $\sim 30$ deg$^2$ m$^2$).

Similar initiatives for all-sky surveying include the PASS project (Deeg et al. 2004). This project has a similar mosaic setup (i.e., it uses 15 cameras to cover the sky above $h > 30$°), but the cameras are fixed to the ground. The Evryscope design (Law et al. 2015), like its prototype concept (Law et al. 2012), features a single-axis setup responsible for sidereal tracking and employs 24 wide-field lenses. Wide-field survey projects that have a comparable field of view to the Fly’s Eye were also successful in finding transiting extrasolar planet candidates (see, e.g., Pepper et al. 2007; Bieryla et al. 2015 and the references therein). More specific details of the key scientific projects and design concepts can be found in Pál et al. (2013) and Vida et al. (2014). These goals overlap with the goals of the Evryscope project (see also the detailed paper of Law et al. 2015).

Hexapods are only seldomly applied in astronomical instrumentation. One of the most prominent usages of such a device is to provide an alignment and focusing mechanism of secondary mirrors of large telescopes (see, e.g., Geijo et al. 2006). The instrument named “Hexapod Telescope” (Chini 2000) also exploits a similar mechanism for telescope pointing. This instrument is capable of performing optical spectroscopy. Probably the most spectacular instrument driven by a hexapod is the AMIBA mount (Koch et al. 2009), a standalone radio telescope system. Radio interferometer arrays also apply to hexapods (Huang et al. 2011).

In this paper, we describe our design of a small, meter-sized hexapod mount dedicated to the all-sky Fly’s Eye survey instrument. In Section 2, we detail how the mount itself is built and controlled. Section 3 describes the basic equations needed for the computations regarding hexapod motion control. In Section 4, we describe the computations and algorithms that provide the sidereal tracking itself. These computations include the methods needed for automatic alignment calibration. In Section 5, we summarize our results.

2. The Hexapod Mount

Hexapods, also known as Steward platforms, employ parallel kinematics to provide a complete, 6 degrees of freedom (dof) spatial motion. As its name implies, the motion of a hexapod is controlled via six legs by altering their lengths. We note here that in the literature about hexapods, these legs are also called actuators, struts, or jacks.

These six legs connect the so-called base of the hexapod with the payload platform. By extending or retracting the legs, the payload platform will move wrt the base. For instance, simultaneously increasing the lengths of all of the legs would move the payload upward while it would not rotate or offset horizontally. Throughout this paper, the points that attain the connection between the legs and the base/platform are called control points and usually realized as a kind of joint (for instance, ball joint or universal joint). Figure 1 displays a sketch of these basic components of the hexapods.

In the following, we describe the kinematic properties of generic hexapods in more details. This description is followed by the presentation of our hexapod designed for the Fly’s Eye project.

2.1. Hexapods at a Glance

One of the most relevant characteristics of a hexapod is the domain of the six spatial motions in which the payload platform is able to move (wrt the hexapod base and/or the reference frame). At first glance, the sizes of these domains (i.e., the total displacement and rotation of the platform) are defined by the size of the base/payload platforms, the locations of the control points on the base/payload platforms, and the minimum and maximum lengths of the legs. The difference between the maximum and minimum lengths of the legs is also referred to as travel length. More precisely, domain sizes depend on the driving mechanism employed in the legs as well as the type of joints.

As a rule of thumb, we can say that the size of the displacement domain for the payload platform is roughly equal to the travel length, while the size of the rotation domain in radians is roughly equal to the ratio of the travel length to the characteristic size of the platforms:

$$\pm S_{\text{platform}} \Delta \rho \approx \Delta L. \quad (1)$$

Here, $S_{\text{platform}}$ denotes the platform size, while $\Delta \rho$ stands for the rotation domain size and $\Delta L$ is the travel length.

Although any kind of hexapod can be constructed with the topology shown in Figure 1, practical designs almost always bear various symmetries. Namely, all of the legs are equivalent by considering their mechanical design and minimum/maximum strokes, while the arrangement of the
six control points also shows a triangular ($D_3$) symmetry. This arrangement showing the $D_3$ symmetry is in fact a planar hexagon with two alternating side lengths, while all of the internal angles are 120°. Hence, this geometry can be quantified by two numbers: the shorter and longer side lengths. Depending on the application, the home position of a particular hexapod is defined either at the state when all of the legs are retracted or when the legs are in middle position (i.e., halfway between fully extended and the fully retracted position). Considering these types of symmetries, hexapods can be characterized by $1 + 2 \times 2 = 5$ geometric parameters at its home position: the length of the legs, and the shorter and longer side lengths of the control point hexagons (both on the base and payload).

More precise qualitative characterization is needed when considering hexapods with a particular combination of a leg mechanism and a joint type. In the following, we describe the implications of using a ball-screw-driven electromechanical actuator combined with two universal joints on both ends of it. First of all, one has to ensure that this type of hexapod can unambiguously be assembled. In addition, enabling the ball screw threads to rotate in their bearing housing (i.e., to be driven by a motor or any external mechanism) allows us to control the hexapod.

One should note here that hexapods employing ball-screw-driven actuators and universal joints suffer from a side effect called screw rotation error (see e.g., Koch et al. 2009, Appendix B). This is implied by the fact that the sole internal dof of the actuator screw thread is coupled with a rotation of the payload platform. This rotation effectively changes the length of the leg (i.e., the distance between the control points) while the screw thread is not driven at all. Since the length of the leg is usually measured indirectly by encoders placed on the screw thread (see Koch et al. 2009, or this paper, Section 2.4), this feedback yields a need for correction. While this side effect can either be treated as an error, knowing the orientation of the universal joints wrt the platforms helps us to properly quantify it.

### 2.2. Payload

The Fly’s Eye design contains 19 camera + filter wheel + lens units mounted on a support frame, hence the expected weight of the payload is approximately 60 kgs. The CCD detectors have a pixel size of $9 \, \mu m \times 9 \, \mu m$, the imaging resolution is $4k \times 4k$, and we employ lenses with a focal length of $f = 85 \, mm$ and an f-number of $f/1.2$. This
optical and detector setup yields an effective resolution of 
21″ pixel\(^{-1}\).

The characteristic size of the payload (i.e., the structure that supports these 19 camera units) is in the range of a meter. From these parameters, one can estimate the principal moments of inertia as well as the forces acting on each actuator on average. The resolution and repeatability of the actuators are implied by the desired sidereal tracking (and/or pointing) precision and accuracy. Our requirement for the precision of the sidereal tracking is defined as 0.1 pixels, i.e., 2″. This is equivalent to 10 microradians (note that 1″ ≡ π/648,000 radians). Therefore, the characteristic size of the instrument (see also Equation (1)) implies a precision of ≈10 μm on each leg by expecting this sidereal tracking precision of 0.1 pixels.

Since our goal is to perform sidereal tracking during the exposures and re-adjust the hexapod platform between subsequent images, the rotational domain is roughly equivalent (in radians) to the exposure time multiplied by Earth’s sidereal angular rotation frequency. Hence, actuator travel length can be relatively small compared to the characteristic size of the hexapod. A great advantage of the small travel length during operations is that all actuators are pushed, yielding no backlash in the actuators.

### 2.3. Geometry

As we described in Section 2.1, the geometry of a hexapod bearing similar actuators and a \(D_3\) symmetry in both platforms can fully be characterized by five independent geometric parameters at its home position. Due to our intended usage (i.e., performing sidereal tracking), we declared the hexapod home position when all of the actuators are in their middle position.

The construction for our hexapod (both as a computer-aided design model as well as its fully built version) is shown in the panels of Figure 2. These images show that our choice for the hexapod contains a similar base and payload platform. The adjacent sides of the hexagons defined by the control points on both platforms have lengths of 97.41 and 742.59 mm by design. The lengths of the legs were set to 510.00 mm at the home position. Our choice for linear actuators was a model with a net (safe) travel length of ∼100 mm. Therefore, the control point distances \(l_i\) (1 ≤ \(i\) ≤ 6) can be varied in the domain of 460 mm ≤ \(l_i\) ≤ 560 mm. The full rotation and displacement motion domains of our hexapod design as implied by the geometry described here are summarized in Table 1.

| Direction          | Notation | – limit | + limit | Unit |
|--------------------|----------|---------|---------|------|
| Roll               | \(P\)    | –9.5    | +9.5    | deg  |
| Pitch              | \(\Pi\)  | –9.1    | +9.3    | deg  |
| Yaw                | \(\Omega\)| –9.2    | +9.2    | deg  |
| Left–right         | \(X\)    | –66     | +72     | mm   |
| Forward–backward   | \(Y\)    | –75     | +75     | mm   |
| Up–down            | \(Z\)    | –78     | +70     | mm   |

Note. The direction domains are defined around the home position (where the length of the legs are 510 mm). The motion domain limits for the rotations and displacements are rounded to the nearest tenth of a degree or millimeter, respectively.
2.4. Hardware and Electronics

As we mentioned in Section 2.1, our hexapod employs legs with two universal joints on both ends, while the leg itself is formed by a high-precision ball-screw-driven linear actuator. The thread of the actuator is driven indirectly using a worm gear mechanism with a speed ratio of 25:1. The pitch of the ball screw thread is 4 mm; hence, one revolution in the driven axis is equivalent to 0.16 mm travel length. In order to precisely control the motion, we employed stepper motors with 200 steps. One full step therefore corresponds to 0.8 \( \mu \text{m} \) of stroke or retract.

The motors are driven by a high-precision controller featuring sine-cosine microstepping down to 1/16 stepping as well as a digital motor current control circuit. A direct feedback from motor motion is provided by a full-turn Hall-sensor-based encoder with a 12-bit resolution (i.e., one full revolution is divided into 4096 ticks). This resolution is comparable to the 3200 microsteps per revolution provided by the motor controller. The actuators are also protected by redundant limit switches.

The motor controller as well as the Hall-encoder is driven by an on board microcontroller unit (MCU). The firmware of this MCU provides for each of the legs an embedded control, and therefore the legs are capable of performing motion in a fully autonomous manner. The firmware allows motor positioning with constant speed as well as ramping up with constant acceleration and constant jerk. In other words, strokes can be controlled up to a cubic function of the time. The duration of motion sequences can be commanded in the units of 1/256 s, while the coefficients of the cubic motion are commanded in units of motor microsteps.

The timing resolution of the motion control is 10–15 \( \mu \text{s} \). The actual resolution depends on the load of the MCU, i.e., in practice, it depends on the polynomial order of the currently running motion sequence. During sidereal tracking, the typical stroke/retract speeds are in the range of \( \lesssim 0.03 \text{ mm s}^{-1} \), which is equivalent to \( \lesssim 0.2 \text{ turn s}^{-1} \) in the driven shaft (see above for the actual values for thread pitch and gear ratios). Since one turn in the driven shaft is divided into 3200 motor microsteps, sidereal tracking implies a \( \lesssim 600 \text{ microstep}^{-1} \) stepping frequency. Hence, the jitter in the motor speed is less than 1% by considering microsteps (and less than 0.1% if we consider full motor steps). In addition, the corresponding motor control frequency of 60–100 kHz is also comparable to the pulse width modulation frequency of the motor driver circuit (which is set to \( \sim 35 \text{ kHz} \)).

In addition to the aforementioned features, the embedded electronics also support a microelectromechanical accelerometer system (MEMS accelerometer) on each leg using a similar data acquisition scheme as described in Mészáros et al. (2014). Since the tilt of the legs vary as the hexapod moves, these accelerometers provide an independent means of retrieving the state of the instrument. Due to its complexity, the mathematics of this hexapod pointing model based on such accelerometers are going to be described in a separate paper.

2.5. Control Subsystem

The microcontrollers are connected to an RS485 bus system in a similar fashion used in our former projects (Mészáros et al. 2014). The RS485 bus system allows the user to upload the coefficients of the elementary cubic functions independently for the six legs, while the actual motion can be started simultaneously with a broadcast RS485 command. The MCUs are capable of storing a queue with eight such cubic motion sequences in total, while new sequences can be uploaded during motion (if the queue has not been fully loaded). This implementation allows a smooth, continuous, and fully traceable hexapod motion control. The MCU firmware also includes features regarding the synchronization of the motion (which is needed because of the different primary internal clock frequencies). The topology of the electronics are displayed in the block diagrams of Figure 3. In Figure 4, we show the main loops running on the MCU firmware as well as the main communication channels between peripherals, the RS485 bus system, and the higher-level control channels. The higher-level control, in practice, is a TCP/IP-based protocol, implemented in a separate hardware (coined as MDSNACD, multi-drop serial network access and control daemon). It should be noted that in the MCU core, some of the peripheral communications run constantly (polling and/or driving the RS485 bus, reading the magnetic encoder, motor step control, etc.) while other peripherals are driven only on demand (such as reading the thermometer values, accessing the MEMS accelerometer). However, the external protocol principle is always master-slave, independently whether the addressed part is running in a loop or only on demand. In other words, this implementation yields a fully autonomous operation but on the other hand, the hexapod itself (i.e., the six legs, one by one) must be polled in order to figure out the current status.

3. Hexapod Motion

The hexapod is used primarily for sidereal tracking during exposures in an all-sky survey instrument. Since a hexapod is capable of performing arbitrary rotations (within its allowed rotation domain), it is an ideal instrument to use it for arbitrary geographical locations and without the need to be perfectly aligned with the principal directions (i.e., unlike a conventional equatorial mount that needs to be aligned properly with the celestial pole).

In order to quantify motions of a hexapod system, first, let us investigate the “small motion” limit of this construction (with the geometry described above). Let us quantify the attitude
of the platform by the orthogonal transformation matrix $O$, then let us introduce the roll, pitch, and yaw angles $P$, $\Pi$, and $\Omega$ as

$$O = \exp \left( \begin{array}{ccc} 0 & -\Omega & \Pi \\ \Omega & 0 & -P \\ -\Pi & P & 0 \end{array} \right)$$

while the offset vector between the base and platform centers is $\Delta = (X, Y, Z)$. Note that in its home position, corresponding to the geometry described in Section 2.3, the roll, pitch, and yaw angles are $P = \Pi = \Omega = 0$, the horizontal displacement offsets are $X = 0$ and $Y = 0$, while the distance between the centers of the platform reference points is $Z = 348.35$ mm.

During routine operations with a hexapod, including the normal astronomical image acquisition sequences with the all-sky survey instrument, we have to compute the actuator lengths $\ell_k (k=1,...,6)$ as the function of the attitude and offset parameter vector $(P, \Pi, \Omega, X, Y, Z)$ and vice versa. The former computation can be deduced with simple vector arithmetics. Using the notations introduced earlier, the actuator lengths, we got

$$\ell_k = \| \Delta + O \cdot j^p_k - j^B_k \|,$$

where $\| \cdot \|$ denotes the Euclid norm (length) of the vectors. Here, the vectors $j^p_k$ and $j^B_k$ are constants and define the control point offsets for the actuator $k$ wrt the payload platform and hexapod base, respectively. Due to the presence of the aforementioned screw rotation error, the true actuator length $\ell_k$ will slightly differ from the commanded actuator length.

Both the qualitative and the quantitative behavior of hexapod motion can well be analyzed via the derivative matrix

$$L = \left( \begin{array}{c} \ell_1 \\ \ell_2 \\ \ell_3 \\ \ell_4 \\ \ell_5 \\ \ell_6 \end{array} \right)$$

This matrix tells us how the lengths of the hexapod legs alter as one varies the payload attitude and/or displacement and vice versa, i.e., how the attitude and displacement changes when some of the legs are actuated. The hexapod does not suffer from any parametric singularity if the determinant of $L$ differs from zero at any arbitrary but allowed values for the $(P, \Pi, \Omega, X, Y, Z)$ parameter vector.

For the geometry used in the Fly’s Eye design (see above, in Section 2.3), the matrix $L$ at the hexapod’s home position is going to be

$$L \approx \begin{bmatrix} +0.033 & +0.287 & +0.254 & -0.254 & -0.287 & -0.033 \\ -0.312 & +0.127 & +0.185 & +0.185 & +0.127 & -0.312 \\ -0.307 & +0.307 & -0.307 & +0.307 & -0.307 & +0.307 \\ +0.365 & -0.730 & +0.365 & +0.365 & -0.730 & +0.365 \\ -0.633 & 0 & +0.633 & -0.633 & 0 & +0.633 \\ +0.683 & +0.683 & +0.683 & +0.683 & +0.683 & +0.683 \end{bmatrix}$$

where, for simplicity, the roll, pitch, and yaw angles $P$, $\Pi$, and $\Omega$ are measured in milliradians instead of radians. This matrix implies some of the expected characteristics of the qualitative motion of the hexapod. For instance, the last row is constant, meaning that a vertical offset of $1$ mm in the payload platform position needs a stroke of $\approx 0.683$ mm in all of the legs. However, one should note that a transformation that essentially swaps the $X$ and $Y$ directions is not a member of the $D_3$ symmetry group. Hence, the lines in the above matrix corresponding to the $X$ and $Y$ directions (or similarly, roll and...
pitch rotations do not show any (implied) correlation between the respective elements.

In order to see that the determinant of \( L \) differs from zero, let us compute the matrix \( L^T L \). Here, \((\cdot)^T\) denotes matrix transposition. This computation yields

\[
L^T L \approx \begin{pmatrix}
+0.295 & 0 & 0 & 0 & +0.279 & 0 \\
0 & +0.295 & -0.279 & 0 & 0 & 0 \\
0 & 0 & +0.565 & 0 & 0 & 0 \\
0 & -0.279 & 0 & +1.600 & 0 & 0 \\
+0.279 & 0 & 0 & 0 & +1.600 & 0 \\
0 & 0 & 0 & 0 & +2.799 & 0
\end{pmatrix},
\]

which clearly shows that the determinants of \( L^T L \) and hence that of \( L \) are non-zero.

The inverse problem related to Equation (3) is the computation of the payload displacement offset and the attitude \( \text{wrt} \) the hexapod base once the actuator lengths are known. This problem can also be effectively computed in an iterative manner by involving the Newton–Raphson algorithm and the derivatives given by \( L \). The effectiveness of this algorithm is due to the fact that \( L^T L \) (and hence \( L \)) varies only slightly in the full domain of \( \ell_i \). Namely, if we denote the derivative matrix of Equation (4) at the home position (where \( \ell_i = 510 \text{ mm} \)) by \( L_0 \), then the rms of the elements in the matrix \( L^T L - L_0^T L_0 \) is around 0.07, i.e., less, by a factor of \( \sim 4-40 \), than the elements of the matrix in Equation (6). We tested this algorithm by applying it to a real hexapod where the actuator travel length was also \( \sim 100 \text{ mm} \), but both the base and payload platform were smaller, yielding \( \pm 30^\circ \) of rotations in the roll, pitch, and yaw directions. In this case, the rms of the elements in \( L^T L - L_0^T L_0 \) were less than a factor of 2 than the elements in Equation (6) (this test was related to the hexapod described in Hirt et al. 2014). This iterative Newton–Raphson method is also capable of converging for geometries with even larger pitch and roll rotations (up to \( 70^\circ \)). However, it should be noted that there is an intrinsic singularity in this matrix when the yaw rotations are close to \( \pm 60^\circ \), corresponding to the cases when the \( Z^+ \) axis and one (or more) of the legs become coplanar instead of being skewed. In this case, the rank of \( L^T L \) decreases and hence the hexapod cannot be controlled.

4. Sidereal Tracking

Since hexapods are capable of performing arbitrary rotations, this type of telescope mount can be used without any

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Figure 4. Simple flowchart showing the main structure of the MCU firmware as well as the communication channels between the peripherals (including the connection with the RS485 bus system). One of the most notable feature of the structure is the lack of interrupt requests: all of the peripherals are accessed in a multiplexed form, keeping the duration of a single processing cycle relatively constant.

(A color version of this figure is available in the online journal.)
modifications at any geographical location as well as the installation procedure does not require precise alignment. For instance, expecting that the device is installed precisely at the poles, the apparent rotation of the celestial sphere is compensated by a purely yaw rotation, i.e., when $P = \Pi = 0$ and $d\Omega/dt = 2\pi/P_{\text{sidereal}}$ (where $P_{\text{sidereal}}$ is the sidereal rotation period of Earth). Similarly, considering a device installed at the equator, the celestial sphere would perform a purely roll rotation, i.e., when $\ell = \Pi = 0$, while the two other angles are zero throughout the motion. If the hexapod is installed at temperate latitudes, then the motion would be a combination of roll and yaw rotations. If the hexapod is not precisely aligned, corrections with small respective angular speeds are needed in all of the three principal directions.

As a foreword, we note here that the algorithm detailed in this section works if the conditions of the presented linear treatment meets with the accuracy of the initial alignment of the hexapod. In practice, our experience was that the employment of standard tools (e.g., classic bubble leveler, digital compass based on MEMS magnetometers embedded in a smartphone, etc.) yields a precision comparable to or less than one degree. This accuracy is sufficient to apply our linear algorithm and yield the sub-arcsecond tracking precision. However, we should also note that Equations (2) and (3) work on the full hexapod motion parameters is to involve the sky itself as a reference frame. First, let us assume that the hexapod, located at the geographical latitude $\varphi_0$, is roughly oriented and aligned to the cardinal directions as well as the base is nearly horizontal. In this case, the speed of the leg $k$ at a linear approximation is need to be

$$\frac{d\ell_k}{dt} = n_{\text{sidereal}} \left( \cos \varphi_0 \frac{\partial \ell_k}{\partial P} + \sin \varphi_0 \frac{\partial \ell_k}{\partial \Omega} \right), \quad (7)$$

where $n_{\text{sidereal}} = 2\pi/P_{\text{sidereal}}$, the sidereal angular frequency of Earth, and the partial derivatives $\partial \ell_k/\partial P$ and $\partial \ell_k/\partial \Omega$ can be taken from Equations (4) and (5).

Equation (7); see Section 2.5 above. Since the real placement of the hexapod is not perfect, the tracking that employs the speeds defined by this equation would also not be perfect. If one obtains a series of images, the drift as well as the field rotation can easily be measured by finding the plate solutions for these images. We note here that the hexapod also acts as a field rotator device, hence plate solution should also include information about the field rotation. Here, these “series” should mean at least two images; however, the longer the series are, the more accurate the determination of the plate drift and the apparent field rotation. Both the plate drift as well as the field rotation has a unit of radians per second, i.e., the actual apparent drift and/or field rotation between the subsequent images first must be converted to radians as well as divided by the image cadence. If the series contains more than two images, these parameters are derived by simple linear least squares regression. Let us denote these three angular speeds by $\omega_1$, $\omega_2$, and $\omega_3$. For instance, if all three of these parameters are obtained using a pair of images with a time separation of $\Delta T$, these angular speeds are computed as

$$\omega_1 = \frac{S \Delta x}{\Delta T}, \quad (8)$$
$$\omega_2 = \frac{S \Delta y}{\Delta T}, \quad (9)$$
$$\omega_3 = \frac{\Delta \varphi}{\Delta T}. \quad (10)$$

where the (linear part of the) differential plate transformation $(x, y) \rightarrow (x', y')$ between the pairs is described as

$$(x', y') = \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} + \begin{pmatrix} \cos \Delta \varphi & -\sin \Delta \varphi \\ \sin \Delta \varphi & \cos \Delta \varphi \end{pmatrix} \begin{pmatrix} x - x_0 \\ y - y_0 \end{pmatrix}. \quad (11)$$

$S$ is the plate scale (in units of arcseconds per pixel), and $x_0$ and $y_0$ are the field centroid coordinates (usually half of the image size dimensions). If tracking is nearly fine, then these values of $\Delta x$, $\Delta y$ (field center offsets) and $\Delta \varphi$ (field rotation) are usually small—some pixels or some tens of pixels, or equivalently some arcminutes—so the linear differential astrometric transformation is still reasonable.

In the next step, we perturb Equation (7) with adequately large offsets $A_1$, $A_2$, and $A_3$ as

$$\frac{d\ell_k^{(A)}}{dt} = n_{\text{sidereal}} \left[ \cos \varphi_0 + A_1 \frac{\partial \ell_k}{\partial P} + (\sin \varphi_0 + A_2) \frac{\partial \ell_k}{\partial \Omega} + A_3 \frac{\partial \ell_k}{\partial \Omega} \right] \quad (12)$$

In a similar fashion, as described above, the corresponding $\omega^{(A)}_m$ drift and field rotation speeds can also be retrieved by exactly the same procedure. It can be considered that the apparent drift and field rotation vanish if the parameters $A_1$, $A_2$, and $A_3$ are set
to satisfy the equation
\[ \omega_m + \sum_{k=1}^{3} A_k \frac{\partial \omega_m^{(A)}}{\partial A_k} = 0 \] (13)

for all \( m = 1, 2, \) and \( 3 \). Using real measurements on the stellar field seen by a camera mounted on the hexapod, the partial derivative \( \frac{\partial \omega_m^{(A)}}{\partial A_k} \) can be measured numerically by involving the formerly obtained series of test images. Namely, this derivative can be approximated as
\[ \frac{\partial \omega_m^{(A)}}{\partial A_k} \approx \frac{\omega_m^{(A_k)} - \omega_m}{A_k}. \] (14)

Here, \( \omega_m^{(A_k)} \) corresponds to a drift and field rotation speed derived from an image series when the hexapod was moved according to Equation (12) and \( A_k \) is non-zero while the other two \( A_k \) values are zero. All in all, four image series should be taken in total, corresponding to the speeds \( \omega_m^{(A_1)}, \omega_m^{(A_2)}, \) and \( \omega_m^{(A_3)} \). We note here that the derivatives of Equation (14) can even better be approximated using symmetric first derivatives instead of this one-sided numerical derivative. Namely, this approximation is
\[ \frac{\partial \omega_m^{(A)}}{\partial A_k} \approx \frac{\omega_m^{(+A_k)} - \omega_m^{(-A_k)}}{2A_k}. \] (15)

In order to obtain the derivatives in this way, six image sequences are needed to be taken instead of four.

One of the most important properties of the above procedure is that it does not need any kind of assumption from the underlying hexapod. Namely, the procedure also compensates for the screw rotation error and even we do not have to precisely know the hexapod geometry as well. In addition, we need to use at least three hexapod legs during the calibration procedure, i.e., the number of involved hexapod legs can be decreased until the corresponding linear combination of the \( L \) matrix components have a rank of 3 (otherwise, Equation (13) would also be singular). This fact has an important consequence that can be highly relevant during the operation of an autonomous and/or remote observatory such as the Fly’s Eye device: the sidereal tracking can be attained even if one, two, or three of the hexapod legs are stuck (due to, for instance, mechanical or electronic failures).

Moreover, in the case of astronomical imaging, the need for only 3 dofs out of the 6 allows us to vary the displacement of the hexapod payload independently from its rotation. This can be used to program the hexapod at different displacement offsets in order to avoid the wearing of the actuator’s ball screws at certain positions.

4.2. Absolute Calibration

The above presented algorithm can be extended to attain an absolute positioning of the hexapod wrt the celestial reference frame. In order to accomplish this step, one has to obtain the apparent coordinates of the stellar field seen by the cameras. After deriving the astrometric solution of the field in the J2000 system, the equatorial coordinates (R.A. and decl.) need to be computed after correcting for precession, nutation, aberration, and refraction using the standard procedures (Meeus 1998; Wallace 2008). These coordinates then need to be compared with the local sidereal time and the geographical latitude. In a similar fashion, the field rotation with respect to the J2000 reference frame should also be converted to the field rotation wrt the apparent north direction.

It should be noted that the conversion between the differences in the celestial coordinates or field rotations and the respective roll, pitch, and yaw offsets must be done with caution due to the singularity in the parameterization of the the celestial coordinate systems. Namely, let us define the matrix function \( A(\alpha, \delta, \rho) \) as
\[ A(\alpha, \delta, \rho) = \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \delta & -\sin \delta & 1 \\ 0 & 1 & 0 \\ \sin \delta & \cos \delta & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \rho & -\sin \rho \\ 0 & \sin \rho & \cos \rho \end{pmatrix}. \] (16)

where the angles \( \alpha, \delta, \) and \( \rho \) correspond to the R.A., decl., and field rotation. For simplicity, let us denote the field centroid coordinates (as obtained from the J2000 astrometric solution; see the details above) with the same symbols. Let us denote the local sidereal time (derived from UT1) by \( \vartheta \). Then, it can be shown that the matrix
\[ O = A(\alpha, \delta, \rho)^T A(\vartheta, \varphi_0, 0) \] (17)
would be close to unity, and the vector invariant of its logarithm (i.e., the inverse of Equation (2)) tells us the respective roll, pitch, and yaw offsets that needed to be commanded to the hexapod. In practice, this vector invariant can be computed as follows. Let us define
\[ x = O_{32} - O_{23}, \] (18)
\[ y = O_{13} - O_{31}, \] (19)
\[ z = O_{21} - O_{12}, \] (20)
\[ r = \sqrt{x^2 + y^2 + z^2}, \] and (21)
\[ t = \text{Tr}(O) = O_{11} + O_{22} + O_{33}. \] (22)

Then, the components of the logarithm of \( O \) are going to be either
\[ \begin{pmatrix} P \\ \Omega \end{pmatrix} = \arg(t - 1, r) \begin{pmatrix} x \\ y \\ z \end{pmatrix} \] (23)
or identically zero when \( x = y = z = 0 \).
4.3. Field Tests

In order to test both the constructed hexapod as well as the previously described algorithm, we attain a series of tests with a single camera mounted on the hexapod payload platform (see also the right panel of Figure 2 for the actual setup).

During the determination of the numerical derivatives needed by Equation (13), we mounted an f/1.8, f = 85 mm lens to a small-format camera having a pixel resolution of 4 k x 4 k and a pixel size of 9 µm x 9 µm. This is an equivalent setup to the final construction of the Fly’s Eye mosaic system with the exception that Fly’s Eye camera units are using an f/1.2 lens instead of f/1.8. During these test runs, the lens was focused using the spare test ports and pins found on the hexapod control electronics.

Four image series have been taken according to the one-sided numerical derivative needed by the approximation in Equation (14). All of the image series contain four individual frames acquired with 20 s of exposure time. In total, this procedure required 4 x 4 frames, while the gross time was roughly 10 minutes in total. However, during these experiments, the overall duty cycle was not ideal (for instance, the repositioning of the hexapod between each series was not performed in parallel with the image readout). Based on these series of images, we were able to obtain the field drift and field rotation parameters $\omega_m$ and $\omega_m^f$. These parameters were derived using the actual frame data (exposure durations and observation time instances as found in the respective FITS header keywords) as well as the differential astrometric solutions obtained with the aid of the FITSH package (Pál 2012). Once we derived the $A_k$ coefficients, we used these leg speeds (defined by Equation (12)) to program the hexapod motion and obtain images with significantly longer exposure times comparable to the proposed cadence of the Fly’s Eye device. One of the small stamps and the corresponding point-spread function (PSF) are depicted in Figure 5. One can clearly see that the procedure works smoothly and is adequate for the intended goals of the Fly’s Eye all-sky survey. We repeated this experiment several times by deliberately stirring the hexapod as well as conducting similar tests on subsequent nights. We note here that the aforementioned 20 s of exposure time is an optimal value due to the following. If the exposure time is shorter, then the both the signal-to-noise ratio of the stars are larger and the implied cadence would also make the numerical derivatives to be less precise. If we increase the exposure time, then the imperfect initial alignment would yield stellar trails that make the source extraction considerably difficult.

In order to better characterize the quality of the sidereal tracking, we performed an experiment by replacing the f/1.8, f = 85 mm lens by an f/8, f = 800 mm catadioptric optics. The usage of the latter type of lens yields a resolution of 2/3 pixel$^{-1}$, i.e., roughly 9.4 times finer than the intended Fly’s Eye plate scale. Once the $A_k$ coefficients were properly derived using the f = 85 mm setup, the lens was replaced and a ~3 minute long sequence was taken with exposure times of 30 s. Since this setup yields a smaller field of view, we obtain only the field centroid drifts. Our tests show that even with this finer resolution, the rms deviation of the frame sequences was always less than 0.3 pixels, i.e., 0"7.

Figure 5. Left panel: a small stamp showing a region of $64 \times 64$ pixels, i.e., approximately $23' \times 23'$ area of the sky. Image has been acquired during an exposure of 130 s, using an f/1.8, f = 85 mm lens and the hexapod for sidereal tracking. Right panel: the point-spread function of the stellar profiles shown on the stamp of the left panel. The PSF is clearly symmetric at its core; the wings are due to the aberration of the lens. Note that if the sidereal tracking by the hexapod would completely be turned off, the lengths of the stellar trails during this 130 s long exposure would be ~61 pixels long, i.e., comparable to the size of the stamp itself.
According to the data acquisition strategy of the Fly’s Eye device (Pál et al. 2013), frames are going to be synchronized to Greenwich sidereal time. Hence, frames in approx. every 23 hr and 56 minutes should show exactly the same field (neglecting the effect of precession and nutation). Since the hexapod would make several hundreds of individual movements (including tracking, re-positioning, and homing) between two frames with a cadence of exactly one sidereal day, even a few days long such series will provide us information about the longer-term repeatability of the hexapod configuration. During our initial test runs, we performed this analysis as well. We found that the rms deviation between these corresponding frames is around 0.10–0.12 pixels for the \( f = 85 \) mm lens. This deviation was obtained for a week long of consecutive run (i.e., the data acquisition was not interrupted by bad weather, but the system was shut down during daytime). While the primary purposes of this hexapod mount and the classical telescope mounts are different, we can conclude that this rms of \( \sim 2'' \) is well comparable to large, meter-class telescope pointing residuals (see, e.g., Pál et al. 2016 and the references therein). We note here that during this one-week run, the re-calibration procedure (described in Section 4.1) was not performed at all, i.e., the same calibration constants \( A_1, A_2, \) and \( A_3 \) were involved while commanding the hexapod. Therefore, we can expect that more frequent attitude re-calibration decreases this residual, even on the order of fractions of arcseconds (see in the following).

4.4. Laboratory Tests

Further laboratory tests were also performed using a very high resolution tilometer (HRTM) manufactured by Lippmann Geophysikalische Messgeräte (L-GM). Such sensors are used in geodetics, more specifically, in astrogeodetic measurements (see, e.g., Hirt & Seeber 2008). These sensors provide a white noise output for pitch and roll tilts in the range of \( 0.01 - 0.02 / \sqrt{\text{Hz}} \) and can even be used\(^4\) for metrology, i.e., to quantify and validate devices capable of altering attitude. We performed repeatability tests with our hexapod when such an HRTM was mounted on the payload platform. Based on these quick tests, we conclude that the resolution in the commanded attitude of this hexapod is in the range of \( \leq 0.1'' \), and the long-term variations due to the ambient temperature changes are in the range of \( \sim 1''/\degree \). Due to the parallel nature of the hexapod and the fact that actuation on all of the legs have an effect on the roll and pitch angles (see the first two rows in the matrix \( \mathbf{L} \) in Equation (5)), such tests can be used to characterize the behavior of each leg nearly independently—even if only 2 out of the 6 dofs of the hexapod can be measured directly by such a tilometer device. However, one should be aware that temperature inhomogeneities in the device itself could play an even more significant role in the precise response of the device to temperature changes. In order to accurately characterize this dependence, we built and mount thermometers in all of the leg control electronics (see Figure 3, where these thermometers are connected to the I^2C bus) as well as in many points of the hexapod payload platform (including the 19 camera units in the final design). We expect that the frequent re-calibration (see Section 4.2) in parallel with thermal data acquisition will provide sufficient information to perform sub-arcsecond accuracy during routinely observations.

All in all, we can conclude that our hexapod-based sky tracking system is safely capable of providing sub-arcsecond tracking precision. In addition, the hexapod based on this construction is capable of performing measurements at the precision level needed by astrogeodetics (see also Hirt et al. 2014 for more details about such applications).

5. Summary

This paper described how a hexapod can be used as a part of an all-sky survey instrument to perform sidereal tracking. We show how the astrometric information provided by the images can be exploited in order to attain sub-arcsecond tracking precision. As a hexapod provides a complete, 6 dof motion control, the survey instrument based on this type of parallel kinematics can be used without any modifications at arbitrary geographical location and without the need of precise alignment. We also demonstrated the mathematical background of the fault tolerant capabilities of a hexapod-based astronomical telescope mount. These capabilities are among the major advantages of using a hexapod in a fully autonomously and remotely operated astronomical instrument since the sky tracking can continuously be performed even if one, two, or three of the hexapod legs are stuck, most likely because of an electronic or mechanical failure. Our mount design presented here forms the core part of the Fly’s Eye project, an ongoing moderate resolution all-sky time-domain variability survey.

The benefits of a hexapod with respect to a conventional telescope mount are the following. First, conventional mounts are needed to be extremely precisely aligned to the cardinal directions—and if not, even in the framework of an isotropic pointing model (Spillar et al. 1993; Buie 2003; Pál et al. 2015), the required corrections have a complexity that is comparable to that of a hexapod. In addition, either an imprecisely aligned equatorial mount or an alt-az mount require a field rotation mechanism for compensating these errors. While in the case of a small field-of-view telescope, this third stage of serial kinematics providing the field rotation compensation is significantly smaller than the drivers of the two main axes, it is not true for an all-sky instrument. In other words, off-the-shelf field rotators cannot be applied at all for an all-sky device. Another benefit of using hexapods instead of conventional mounts is the fact that the characteristic alignment of the payload platform is always horizontal in its home position. In

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\(^{4}\) Beyond their intended purposes, such as leveling, seismology, or the measurement of lunisolar tides.
the case of an equatorial mount, it is not true and even this direction depends on the actual geographical location of the installation. Therefore, the same design cannot be used at different locations if the payload is a camera assembly performing all-sky surveying. In the case of alt-az mounts, the preferred direction of an all-sky camera (i.e., looking toward the zenith) coincides with the gimbal lock position of the telescope mount (even considering a field rotation mechanism); hence, alt-az mounts are intrinsically not capable of performing such a tracking. Of course, while the issues mentioned above can be overcome (see, e.g., Law et al. 2015, where a conventional equatorial mount is used to support a mosaic camera system), hexapods can provide a cost-effective and elegant way of long-term maintenance of all-sky instrumentation that also features precise sidereal tracking.

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