Simulation of Consensus Model of Deffuant et al on a Barabási-Albert Network

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Abstract: In the consensus model with bounded confidence, studied by Deffuant et al. (2000), two randomly selected people who differ not too much in their opinion both shift their opinions towards each other. Now we restrict this exchange of information to people connected by a scale-free network. As a result, the number of different final opinions (when no complete consensus is formed) is proportional to the number of people.

Keywords: Scale free networks, sociophysics, opinion formation, Axelrod model.

1 Introduction

Partially motivated by a model of Axelrod \cite{Axelrod97}, the model of Deffuant et al. \cite{Deffuant00} simulates the building of a consensus, or the lack of consensus, out of many initially random opinions. Other consensus models are those of Krause and Hegselmann \cite{Krause02}, Sznajd \cite{Sznajd00} (for a review see \cite{Hegselmann02}), and Galam \cite{Galam02}; they were summarized recently in \cite{Hegselmann02} and are part of sociophysics \cite{Galam02} or sociodynamics \cite{Hegselmann02}, which belong to the wider field of interdisciplinary applications of statistical physics methods \cite{Hegselmann02}.

The Deffuant model is the one where the largest number of people was simulated so far \cite{Deffuant00} so that the statistics was best and this is the reason why we selected this model for the present study. It assumes that everybody can talk with everybody else with the same probability, similarly to random graphs, but with sites living in the continuum. (So the model was not considered on random graphs in the sense of Erdős and Rényi \cite{Erdos60}.) In
this unrealistic limit, analytical approximations work well [11]. The opposite
limit of people restricted to a square lattice was also simulated, with interaction
between close neighbours (like nearest neighbours) only, but may apply
better to trees in an orchard than to human beings. Real social connections
may lie in between, with few people having lots of friends and many people
having few friends to talk with. Everybody is still connected with everybody
but only indirectly over a short link of mutual friends. The best studied
model for these types of connections are the scale-free networks of Barabási
and Albert [12] where the number of people having \( k \) friends decays as \( 1/k^3 \).

The effect of network topologies on the dissemination of culture [1] or on
the spreading of information will be studied in future work [13].

The next section defines the two models, with directed and undirected
bonds in a Barabási-Albert network, while section 3 gives the results and
section 4 the conclusions.

2 Models

The Barabási-Albert network starts with a small number \( m \) (\( m = 3 \) in our
simulations) of sites (agents, people) all connected with each other. (We
varied \( m = 3 \) also to \( m = 2, 4 \) and 5 and observed similar test results
for 100 runs each.) Then a large number \( N \) of additional sites is added as
follows: Each new site selects \( m \) of the already existing sites as friends, with
a probability proportional to the number of friends this already existing site
had before. When the new site \( A \) has selected an already existing site \( B \) as
friend, this selection increases for both \( A \) and \( B \) the number of friends by
one. In the usual undirected Barabási-Albert model, later \( A \) can talk with \( B \)
and \( B \) can talk with \( A \). In the simpler directed version, \( A \) initiates a talk with
\( B \), but \( B \) initiates talks only with those \( m \) people whom \( B \) had selected as
friends. Thus in this directed version, everybody has a fluctuating number of
people connected with him, but asks only one of exactly \( m \) people for advice
at a time, and these are the people the new site had selected when joining
the network. (One may think of a hierarchy of bosses and underlings.) For
the undirected case no such distinction between friends and people to talk
with is needed; the connection network of friends then is constructed as in
[14].

Once the network has been constructed, we start the consensus process
of Deffuant et al.. Everybody gets a random number \( S \) between zero and
one as initial opinion. Then for each iteration, every site $A$ is updated once by selecting randomly one site $B$ from the sites connected with $A$. In the undirected case the selection is taken from all $k$ sites who had selected $A$ as friend or whom $A$ has selected as friends. In the directed case the selection is made only from the $m$ sites which $A$ had selected as friends. If then the opinions $S_A$ and $S_B$ differ by more than a constant confidence bound $\epsilon$ between zero and one, $A$ and $B$ refuse to discuss and do not change their opinion. Therefore $\epsilon$ may be interpreted as a measure for the tolerance of people to other opinions. Otherwise both move closer to the position of the other by an amount $\delta = \mu(S_A - S_B)$ with $\mu = 0.3$ in our simulations, i.e. $A$ takes the opinion $S_A - \delta$ and $B$ the opinion $S_B + \delta$. The parameter $\mu$ characterizes the flexibility in changing the opinion. After sufficiently many iterations (unfortunately much more than the $\sim 10^2$ in the “random” network [2]) no opinion moves by more than $2 \times 10^{-8}$: a fixed point in the space of opinions is approximated. (For different $\mu$ this small value needs to be adapted.) Hundred samples of this type were averaged over. The opinions are then placed in bins of width $10^{-6}$ and are counted by checking which bins are occupied and do not have the lower neighbouring bin occupied. In this way, the total number of fixed opinions is found. The “directed” Fortran program is available from the authors.

3 Results

Figure 1 shows the undirected and figure 2 the directed case. For $\epsilon$ larger than about 0.4 a full consensus is reached; only one opinion survives. For smaller $\epsilon$, no consensus is reached and the number $F$ of fixed opinions increases with decreasing $\epsilon$. When the number $N$ of people increases, the $F$ for small $\epsilon$ also increases $\propto N$ for large $N$. This increase is the crucial difference to the random version without Barabási-Albert restriction, when $F$ is independent of $N$ for large $N$. Thus we plot in figures 1 and 2 the scaled excess number

$$F_E = (F - 1)/N$$

versus $1/\epsilon$: in the random case in the sense of Deffuant et al. (without specified network topology) $F$ roughly equalled $1/\epsilon$, while now $F \sim N/\epsilon$. The remaining mild $N$-dependence of the scaled excess number which is seen in Figs. 1 and 2 is interpreted as a finite size effect getting weaker for larger system sizes $N$, cf. the figures.
Figure 1: Scaled excess number $F_E = (F - 1)/N$ of different opinions in the undirected network, with $N$ between 100 and 20000 given in the headline.

Figure 3 shows the enormous fluctuations in the number $t_c$ of iterations needed to reach the fixed point, similarly to the Snajd model [4]. This feature is understood from the fact that reaching a certain fixed point, e.g. that of consensus, is a collective property of the $N$ agents which cannot be obtained as an average over subsystems. Values of such kind of collective quantities may strongly fluctuate like those of individual ones, even for $N \to \infty$, i.e. they are not self-averaging. However, the behaviour near $\epsilon = 0.1$ for $N = 5000$ is not understood.

4 Conclusions

Our condition for the confidence bound $\epsilon > 0.4$ to allow a complete consensus is about the same on the Barabási-Albert network as it was in the usual random case [2]. But when no consensus is formed because $\epsilon < 0.4$ is too small, then our number of different opinions is proportional to the number
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Figure 3: Fluctuations in the time $t_c$ to reach fixed points, in Monte Carlo steps per site, for $N = 100$ and 5000 for the directed case. We plot the standard deviation $\sigma$ of the logarithm, defined through $\sigma^2 = \langle (\ln t_c)^2 \rangle - \langle \ln t_c \rangle^2$.

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