Alternating Current Electrothermal Flow for Energy Efficient Thermal Management of Microprocessor Hot Spots

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Abstract

In this paper, we bring an innovative concept of an efficient, reliable cooling technology of microelectronic devices. We present an electrokinetic transport mechanism, known as Alternating Current Electrothermal Flow (ACET), where we demonstrate that the peak temperature of multiple hot spots can go down well below the safe operating range. The heat source, required to drive the ACET cooling mechanism, is obtained from the heat flux given off by the hot-spot which otherwise causes detrimental damage to the devices creating elevated high die temperature. The effect of parameters such as the applied potential, heat transfer coefficient on the top wall and thermal conductivity which directly change the flow actuation features and heat transfer characteristics are investigated systematically. The results showed that beyond a threshold value of the applied potential the heat transfer rate is not increased significantly.

1 Introduction

Thermal design of microelectronic equipment plays a crucial role in the electronic industry [1], [2]. For the safety and proper functionality of microelectronic components, it is important to remove the heat generated within the electronic component so that the component temperatures are kept within the safe operating limits. Due to the recent technological advancements, microelectronic devices have to efficiently control numerous functions simultaneously, resulting in increased heat dissipation levels within a reduced confinement [3], [4]. The resulting high heat fluxes may lead to a commensurate elevation of component temperature, resulting in the formation of hot spots in the microelectronic devices[5], [6]. Thus, effectiveness and reliability in heat removal process are the pivotal issues of a compact thermal management system[7], [8].

Elevated non-uniform temperature distributions raise various problems for proper functioning of the micro-devices in extended time periods. For instances, MOS transistor drive capability, electromigration and interconnect delay gets destroyed at high temperature [9]. In addition, nonuniform heat flux generated from the hot spots creates physical stress leading to reduce reliability. Various cooling techniques have been proposed over the years which may be broadly classified into three categories, namely passive [10], active [11], and active adaptive [12] processes. Passive cooling process does not involve any external energy source for operation. Examples include passive heat sinks, thermosyphons, heat spreaders and heat pipes. In contrast, active microcoolers use external agency such as pump, fan etc. to create the cooling mechanism. On the other hand, active adaptive microcooling processes employ a closed-loop feedback system to control the hot spot temperature and enable dynamic cooling for the devices. Passive cooling and conventional active cooling suffer from many intrinsic drawbacks [13] which led to the development of new methodologies targeting high heat flux dissipating areas. Some prominent approaches in this regard have been investigated in the literature, such as thermoelectric cooling [14] and electrowetting [15].

Thermoelastic cooling technology is an active model based cooling. However, It involves various problems, such as contact parasitic resistances, high cost, low energy conversion efficiency and additional thermal resistances [16], [17]. On the other hand, electrowetting (active adaptive model-based) also involves some limitations, such as its pumping capacity for heat transfer is low[18] and it requires high voltage, complex electrode designs, complicated operational algorithms (sequential ON/OFF) to operate the system. To circumvent the limitations stated above we adopt an energy efficient hot spot targeted liquid cooling driven by alternating current electrothermal forces. The electrothermal driven fluid flow arises in presence of local variation of electrical conductivity and permittivity[19], [20], [21]. Inhomogeneity in variation of conductivity and permittivity is generated due to temperature variation is occupied into the conducting fluid. The source of the thermal energy may be an external, for example, strong illumination[22] or may be internally evoked Joule heating[23]. Externally applied thermal is much effective because we can generate pre-defined temperature gradient. In a similar way, presence of hot spots in the flow path increases the effectiveness of the electrothermal mechanism. The thermal energy which needs to be dissipated from the hot spots, itself triggers the fluid over the hot spots and keeps peak temperature below a safe limit. Best of our knowledge, electrothermal based microcooling of hot spots have not been studied so far.
In the present study, we demonstrate that the unwanted thermal energy dissipated by the hot spot is used as the power source to execute the ACET process to maintain the peak temperature below the safe operating limit. In this paper, ACET mechanisms is employed effectively for fluid pumping\cite{24},\cite{25},\cite{26} and mixing\cite{27},\cite{28},\cite{29} that can be effectively utilized for hot spot cooling. The active cooler based electrothermal micropumping is highly energy saving for modern days applications of thermal management system for microelectronics packages.

![Figure 1: Schematic of the AC electrothermal based microcooling. Electrodes are shown in the inset. The hot spots are shown with red color.](image)

2 Materials and Methods

2.1 Description of the microcooler

The focus of the present article is to investigate an ACET-driven microcooler system for hot spots cooling where a conducting cooling fluid in absence of the influence of any external energy is driven by ACET actuator. The physical system of the scenario is shown in Fig. 1. The microcooler consists of three hot spots placed on the channel floor. Three pairs of electrodes are oriented on the floor before the hot spots and three pairs of electrodes next to the hot spots to drive the fluid. The height and length of the channel are $H$ and $L=500\,\mu m$, respectively. The $x$ and $y$ coordinates run along the channel length and height, respectively. The lengths of the various geometrical parameters shown in the figure are of the one pair of electrodes are $d_s = 6\,\mu m$, $d_i = 36\,\mu m$, $e = 22\,\mu m$, and $h = 20\,\mu m$. The channel is filled with a conducting fluid. An AC field is inputted on the electrodes. Joule heat is occupied over the electrode pairs due to the application of an electric field in the conducting solution. On the other hand, heat emerged from the hot spots generates a very sharp temperature gradient. Inhomogeneity in the local gradient of permittivity and electrical conductivity due to the thermal field causes a net pumping along the channel length. In addition, Dean vortexes make a significant role to decrease the hot spots temperature and allow to operate in the safe operating limit.

2.2 Numerical methods

Here we present the transport equations which govern the transport velocity and temperature rise in the domain. The ACET actuated fluid flow involves simultaneous coupling of the electric field, temperature field, flow field, and ACET forces. Considering quasi-electrostatic field (i.e., negligible magnetic field effect \cite{30},\cite{31}) the electrostatic field $E$ can be written in the term of electrical potential $\varphi$ by

$$
E = -\nabla \varphi, \quad (1)
$$

$$
\nabla \cdot (\sigma \nabla \varphi) = 0. \quad (2)
$$

where $\sigma$ is the electrical conductivity. As discussed previously gradients in electrical properties generate the electrothermal forces. The properties are function of temperature in the form of $\varepsilon(T) = \varepsilon_0 (T_0) \left(1 + \alpha (T - T_0)\right)$ and $\sigma(T) = \sigma_0 (T_0) \left(1 + \beta (T - T_0)\right)$. Here, $\varepsilon$ is the permittivity of the fluid. $T_0$ is the reference temperature. For fluid media used in the study values of gradients are, $\alpha = -0.004\,K^{-1}$ and $\beta = 0.02\,K^{-1}$ \cite{19}.

A small amount of Joule heat arises due to the application of AC electric field. On the other hand, large amount of heat is evolved from the hot spots. The temperature distribution for the Joule heat and heat from hot spots heat can be characterized by the following energy equation:

$$
\rho C_p \frac{DT}{Dt} = \nabla \cdot \left( k \nabla T \right) + \sigma |E|^2, \quad (3)
$$

where $T$ is the temperature. $\rho$ and $k$ are the fluid density and thermal conductivity, respectively. $C_p$ is the specific heat capacity. $V$ is the fluid velocity. The term $\sigma |E|^2$ is the Joule heat aroused from the electrical field within the fluid domain. $D / Dt$ denotes the material derivative.

In a microscale transport system, one may consider flow to be laminar and incompressible. The flow field in the form of Navier-Stokes equation is expressed by Eq. (4), (5). The fluid motion is developed by the body forces originating from electrothermal forces ($F_e$) . The set of Navier-Stokes equation thus takes the form

$$
\nabla \cdot \mathbf{V} = 0 \quad (4)
$$

$$
\rho \frac{DV}{Dt} = -\nabla p + \nabla \left[ \mu \left( \mathbf{V} \mathbf{V}^T \right) \right] + F_e \quad (5)
$$

where $\mu$ is the fluid viscosity, $p$ is the pressure. The time-averaged electrothermal force $F_e$ can be written as \cite{19}:

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Other parameters are the function of temperature only, hence, one can write \( \nabla E = (\partial E / \partial T) \nabla T \) and \( \nabla \sigma = (\partial \sigma / \partial T) \nabla T \).

To characterize the heat transfer effectiveness we define a parameter, effectiveness of the microcooler \( \varepsilon_{\text{ACET}} \) which is expressed by:

\[
\varepsilon_{\text{ACET}} = \frac{T_{\text{max, no cooler}} - T_{\text{max}}}{T_{\text{max, no cooler}} - T_{\text{ext}}}.
\]

where \( T_{\text{max, no cooler}} \) is the maximum temperature in the domain when ACET cooler is deactivated and \( T_{\text{max}} \) is for activation condition of the cooler. \( T_{\text{ext}} \) is the temperature of the fluid before entering the channel. The effectiveness varies from 0 to 1. The maximum effectiveness (\( \varepsilon = 1 \)) is obtained for the case of \( T_{\text{max}} = T_{\text{ext}} \).

The values of physical properties of the conducting fluid (KCl in the present study) assumed to be constant throughout the analysis and have the values are: \( \rho = 1000 \text{kg/m}^3, \quad C_p = 4.184 \text{J/kgK}, \quad \mu = 0.00108 \text{Pa s}, \quad \varepsilon = \varepsilon_0 \varepsilon_r = 7.08 \times 10^{-10} \text{C/Vm}, \) where \( \varepsilon_0 \) is permittivity of vacuum and \( \varepsilon_r \) is relative permittivity of the liquid. RMS (root mean square) voltage (\( \pm \varphi \varepsilon_{\text{rms}} \)) is applied across the electrode pair at a constant frequency of 100 kHz. Other boundaries are treated as electrically insulating. Inflow boundary condition \( -n \cdot (-k\nabla T) = \rho (\Delta h_m - \Delta h_{\text{ext}}) \mathbf{V} \cdot \mathbf{n} \) is taken at the inlet of the microchannel, where \( \Delta h_m - \Delta h_{\text{ext}} = \int_{T_{\text{ext}}}^{T} C_p dT \), \( (\Delta h_m - \Delta h_{\text{ext}}) \) is the enthalpy difference between two sides of the inlet. \( T_i \) is the inlet temperature. \( \mathbf{n} \) is the unit normal vector along the x-direction. We consider \( T_{\text{ext}} = 298 \text{K} \). The hot spots heat flux is maintained at \( 10^6 \text{W/m}^2 \). The top wall experiences an effective heat transfer coefficient of \( h_e \). Ambient temperature is set as 298 K. At outlet \( \partial T / \partial x = 0 \) is set. Other boundaries are treated as insulating. To obtain the velocity field no-slip condition is imposed on the solid boundaries and zero pressure is set at the inlet and outlet.

To avoid numerical errors and oscillations in the numerical simulations grid independence study has been performed for mesh (triangular) sizes \( 3 \times 10^{-6} - 4 \times 10^{-6} \). We have highlighted data of the maximum temperature in the domain for different mesh sizes in Fig. 2. It is found that above a mesh size of \( 3.25 \times 10^{-6} \) the change in temperature is very small (<0.005%). Therefore, in our simulations, we have used mesh size \( 3.25 \times 10^{-6} \) whose corresponding number of elements is 87480. Similar grid independent study is performed for benchmarked results and mesh size of \( 2.75 \times 10^{-6} \) is found to be optimum value above which results are independent of mesh size.

![Figure 2: Grid independence study: Above a maximum element size of \( 3.25 \times 10^{-6} \) (corresponding number of elements 87480) the change in maximum temperature is negligible and hence, this size is taken for all the simulations. The parameters used in the results are \( V_{\text{rms}} = 12 \text{Volt}, \) electrical conductivity \( \sigma = 2 \text{mS/m}, \) thermal conductivity \( k = 0.6 \text{W/mK}, \) heat transfer coefficient \( h_e = 500 \text{W/m}^2\text{K}, \) and channel height \( H = 600 \mu\text{m} \).](image)

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![Figure 3: Variation of the maximum temperature \( T_{\text{max}} \) of hot spots and ACET effectiveness (\( \varepsilon_{\text{ACET}} \)) with applied voltage (\( \varphi \)). We consider two the effects for two different electrical conductivities \( \sigma = 2 \text{and} 10 \text{mS/m} \). Other parameters are channel height \( H = 600 \mu\text{m} \), heat transfer coefficient \( h_e = 25 \text{W/m}^2\text{K}, \) thermal conductivity \( k = 0.6 \text{W/mK} \).](image)
3 Results and discussions

In this section, effect of AC potential on the pairs of electrodes is investigated. At the outset important point to be mentioned that an effective microcooler should not allow the hot spots temperatures to go beyond acceptable limits. The basic understanding of the electrothermal mechanism states that fluid velocity increases with increasing voltage. Accordingly convective heat transfer should increase with increasing signal voltage. However, at the same time increased field strength and current increase the internal Joule heating. From the energy equation, it can be shown that produced Joule heat is proportional with the square of the electric field. Thus, with increasing voltage generated Joule heat tries to increase the temperature in the system. Therefore, increase in applied voltage simultaneous reduces the overall temperature by increasing fluid velocity and increases the overall temperature owing to increased Joule heating. These two events influence the overall temperature dramatically as can be seen in Fig. 3 where variation of maximum temperature and effectiveness of the cooling are shown with voltage for two different conductivities \( \sigma = 2, 10 \text{ mS/m}. \) The other parameters are mentioned in the figure caption. The trend of variation reflects that the maximum temperature drastically falls at lower voltage and it is gradually decreases at higher value of voltage. At higher voltage, adverse effect of Joule heating does not allow higher efficiency of reduction in temperature. One can observe in the inset plot that ACET effectiveness of hot spot cooling first sharply increases and then a slow increase is seen. Finally, effectiveness is almost saturated at higher voltage (~10-12 V). Importantly, reduction in maximum temperature in the range \( (\rho < 8 \text{Volt}) \) is not effective since the value of the temperature is high and it crosses the allocable maximum temperature limit (say 370 K). However, at higher range cooling temperature goes in the safe temperature range. For \( \sigma = 10 \text{ mS/m} \) the adverse effect of voltage is deep. For this case, temperature is increased at high voltage (~14 V).

![Figure 4: Effect of heat transfer coefficient \( (h_c) \) on the variation of maximum temperature of hot spots. The parameters used in the results are AC potential \( V_{max} = 12 \text{Volt}, \) thermal conductivity \( k = 0.6 \text{W/mK}, \) electrical conductivity \( 2 \text{mS/m}, \) and channel height \( H = 600 \mu \text{m}. \)](image)

Figs. 4 and 5 shows the variation of the peak temperature with heat transfer coefficient experienced at the top wall due to flow of the ambient fluid and fluid thermal conductivity. From Fig. 4 it is clear that the peak temperature decreases with increasing heat transfer coefficient which is obvious. However, within a wide range of \( h_c \), the reduction in temperature (i.e., \( \Delta T \)) is only 2K. This implies that the local heat transfer from the hot spots is largely insensitive to the environmental conditions. The other important parameter which can directly affect the hot spot temperature is the fluid thermal conductivity \( (k) \). Fig. 5 shows the variation of the maximum temperature of the hot spots with the fluid conductivity. The peak temperature sharply decreases in the range of 0.5-0.8 and gently decreases in the other range of conductivity. In the context of our microcooler design, the range \( k = 0.5 - 0.8 \text{W/mK} \) of fluids are available. For example, conductivity of water (water as a base fluid) may be raised to 0.73 W/mK with the addition of 4% aluminum oxide (\( \text{Al}_2\text{O}_3 \)) [32].

4 Conclusions

Two-dimensional numerical results are shown for on-demand, location-specific, hot spots cooling for power microelectronic devices. The flow is actuated by alternating current electrothermal mechanism in a channel consisting of six pairs of electrodes and three hot spots. The new technique of thermal management removes the inherent limitations of existing methodologies of hot spots cooling. Our microcooler is highly energy saving because of the fact that accumulated heat of hot spots itself triggers the fluid flow. The effect of parameters such as applied potential, heat transfer coefficient on the top wall and thermal conductivity which directly change the flow actuation features and heat transfer characteristics are investigated. The results are
presented in the form of maximum temperature in the domain and electrothermal effectiveness. We found that heat transfer rate is invariant with heat transfer coefficient and thereby heat transfer rate is purely electrothermal driven based. The result also showed that above a threshold value of applied voltage electrothermal mechanism is inefficient to cool the hot spots.

We believe that interferences drawn from our studied will be helpful for further studied to design an efficient microcooler for site-specific localized hot spots cooling for power microelectronic devices.

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