SIGNIFICANCE OF NEGATIVE ENERGY STATES IN QUANTUM FIELD THEORY (1)

SHI-HAO CHEN

Abstract. We suppose that there are both particles with negative energies described by $\mathcal{L}_W$ and particles with positive energies described by $\mathcal{L}_F$. $\mathcal{L}_W$ and $\mathcal{L}_F$ are independent of each other before quantization, dependent on each other after quantization and symmetric, and $\mathcal{L} = \mathcal{L}_W + \mathcal{L}_F$. From this we present a new quantization method for QED. That the energy of the vacuum state is equal to zero is naturally obtained. Thus we can easily determine the cosmological constant according to data of astronomical observation, and it is possible to correct nonperturbational methods which depend on the energy of the ground state in quantum field theory.

1. Introduction

There are the following five problems to satisfactorily solve in the conventional quantum field theory (QFT).

1. The issue of the cosmological constant.
2. The problem of divergence of Feynman integrals with loop diagrams.
3. The problem of the origin of asymmetry in the electroweak unified theory.
4. The problem of triviality of $\phi^4$-theory.
5. The problems of dark matter and the origin of existence of huge cavities in cosmos.

In brief, we present a consistent QFT without divergence, give a fully method evaluating Feynman integrals (see the second and third papers), give possible solutions to the five problems in a unified basis to reexplain the physics meanings of negative energies.

There is an inconsistency in the conventional QFT.

As is well known, before redefining a Hamilton $H$ as a normal-ordered product, the energy $E_0$ of the vacuum state is divergent. Because we may arbitrarily choose the zero point of energy in QFT, we can redefine $E_0$ to be zero. This is, in fact, equivalent to demand

$$\{a_{p^s}, a_{p^s}^+\} = \{b_{p^s}, b_{p^s}^+\} = [c_{k^\lambda}, c_{k^\lambda}^+] = 0,$$

in the conventional QFT. But in fact these commutation relations are equal to 1, and in other cases, e.g. in propagators, they must also be 1. Thus the conventional QFT is not consistent.

Divergence of Feynman integrals with loop diagrams seems to have been solved by introducing the bare mass and the bare charge or the concepts equivalent to...
them. But both bare mass and charge are divergent and unmeasured, thus QFT is still not perfect. In order to overcome the shortcomings, people have tried many methods. For example, G. Scharf attempted to solve the difficulty by the causal approach [1]. Feynman integrals with loop diagrams are not divergent in some supersymmetric theories. But the supersymmetry theory lacks experiment foundations. In fact, there should be no divergence and all physical quantities should be measurable in a consistent theory.

According to the given generalized electroweak unified models which are left-right symmetric before symmetry spontaneously breaking, asymmetry is caused by symmetry spontaneously breaking. In such models there must be many unknown particles with massive masses. Such models are troublesome and causes many new problems. Hence the origin of asymmetry in the electroweak unified theory should still be explored.

In order to introduce the present theory, we first discuss measurement of an energy. For a measurable energy, in fact, there is the following conjecture.

**Conjecture 1.** Any measurable energy $E$ of a physical system must be a difference between two energies $E_i'$ and $E_j'$ belonging respectively to two states $|E_i'\rangle$ and $|E_j'\rangle$, i.e.,

$$E = \langle E_i' | H | E_i' \rangle - \langle E_j' | H | E_j' \rangle = E_i' - E_j',$$

(1.1)

where $|E_j'\rangle$ is such a state into which $|E_i'\rangle$ can transit by radiating the energy $E$.

It is seen from the conjecture 1 that in order to determine a measurable energy $E$ we must firstly determine two states $|E_j'\rangle$ and $|E_i'\rangle$, and only when $|E_i'\rangle$ can transit into $|E_j'\rangle$ by radiating the energy $E$, $E$ is measurable. Let $E_{min}'$ be the minimal energy of a system, an energy $E$ of the system can be defined as $E = E_j' - E_{min}'$. It is obvious that $E \geqslant 0$. $E_{min}' = 0$ is necessary. According to the general relativity

$$m_g = m_i = E,$$

(1.2)

where $m_g$ and $m_i$ are the gravitational mass and the inertial mass corresponding to $E$, respectively. If $E_{min}' \neq 0$, $E_{min}'$ will cause a gravitational effect. Thus $E_{min}'$ is measurable (let the cosmological constant be given). If $E_{min}'$ is a measurable energy, $E_{min}' > 0$. If $E_{min}' > 0$, there must be another state $|E_{min}''\rangle$ with its energy $E_{min}'' < E_{min}'$, thus $|E_{min}''\rangle$ is no longer the ground state. Hence we should have

$$E_{min}' = 0,$$

(1.3)

But before Hamiltonian operator is defined as a normal-ordered product, the energy $E_0 = E_{min}'$ is divergent, thus the conventional QFT is inconsistent with the general theory of relativity and cause the issue of cosmological constant. Of course, if the general theory of relativity is not considered, $E_{min}'$ is unmeasured and undetermined.

It is seen from the above mentioned that the conventional QFT is not perfect and should be corrected.

We introduce the present theory as follows.

1. A new Lagragian density

The physics basis of the present theory is to reexplain the physics meanings of negative energies. The relativistic theory is very perfect, and existence of negative
energies is its essential character. Any existence must depend on its existing conditions. As an ancient Chinese philosopher said, nothing in nothing is just some existence; existence in existence is just some nothing. We think that positive energies and negative energies depend on each other, not only are negative energies not a difficulty, but have profound physical meanings. Existence of antiparticles is only, in fact, a result of particle-inversion symmetry, and do not reveal the essence of negative energies. For example, a pure neutral particle or a pure neutral world also has its negative-energy states. From this, we think that existence of positive energies and negative energies imply that there are two sorts of matter in form and the two sorts of matter are symmetric. \[2\] supposes that the two sorts of matter possess all positive energies and try to solve the problems above. In fact, this is not necessary. Because $L_F$ and $L_W$ are independent of each other, there is no interaction and mutual transformation of the two sorts of matter. Thus no contradiction will appear if there is matter with negative energy. On the basis of the conjecture 1, in relativistic quantum mechanics (RQM) frame we present the conjecture 2. In contrast with \[2\], from the conjecture we will see that there are particles with negative energies as well as particles with positive energies.

**Conjecture 2.** Any particle can exist in two sorts of forms —— $|E^+,q,1\rangle$ and $|E^-,\bar{q},0\rangle$ or $|E^+,\bar{q},1\rangle$ and $|E^-,q,0\rangle$, where $|E^-,q,0\rangle$ ($|E^-,\bar{q},0\rangle$) is a vacuum state with the potential energy $E^-$ and the quantum number $-q$ ($-\bar{q}$) relative to the ground state $|E^-,q,1\rangle$ ($|E^-,\bar{q},1\rangle$). The two sorts of existing forms can transform from one to another. $\mathcal{L} = \mathcal{L}_F + \mathcal{L}_W$, $\mathcal{L}_F$ describing $|E^\pm,q,1\rangle$ and $\mathcal{L}_W$ describing $|E^\pm,\bar{q},1\rangle$ are independent of each other before quantization. Particles described by $\mathcal{L}_F$ ($F$-particles) and the particles described by $\mathcal{L}_W$ ($W$-particles) are symmetric.

We explain the conjecture 2 as follows.

The bases of the conjecture are that

A. there are always positive solutions $|E^+,q,1\rangle$ and negative solutions $|E^-,q,1\rangle$ for any particle in RQM, $E^+$ and $E^-$ are strict symmetric in relativistic theory, and the property $E^+ > 0$ and $E^- < 0$ is relativistic invariant, where $q$ denotes the quantum number of a particle, $E^\pm = \pm \sqrt{p^2 + m^2}$ and 1 denotes the number of particle;

B. analogously to the Dirac’s hole theory, $|E^-,\bar{q},0\rangle$ as $|E^+,q,1\rangle$ is regarded as an existing state with $E^+$ and $q$;

C. any particle is the same in existing form. According to the Dirac’s hole theory, $e^+$ and $e^-$ are different in existing form, i.e., $e^+$ exists as a hole and $e^-$ exists as a particle. According to the conjecture, $e^+$ exists in $|E^-,e^-,0\rangle$ or $|E^+,e^+,1\rangle$, and $e^-$ exists in $|E^-,e^+,0\rangle$ or $|E^+,e^-,1\rangle$, i.e., $e^+$ and $e^-$ are the same in existing form.

The physical meanings of $E^+$ and $E^-$ are different in essence. If positive energies and negative energies are respectively conservational, \{|$E^+,q,1\rangle$\} and \{|$E^-,q,1\rangle$\} will correspond to two different sorts of particles. Of course, positive energies and negative energies are not respectively conservational in RQM frame. But according to the conjecture 1, we will see that any particle exists in two sorts of forms.

Because only when there is $|E^-,q,0\rangle$, can a particle $q$ transit to the states with $E^-$ and $q$ and $|E^-,q,1\rangle$ can come into being. Hence when there is $|E^-,q,0\rangle$, the ground state of the system is not $|0\rangle$, but $|E^-,q,1\rangle$. $|E^-,q,0\rangle$ is different from $|0\rangle$. When $|E^-,q,0\rangle$ becomes the final state $|E^-,q,1\rangle$ by some way, according
to the conjecture 1 the energy and the additive quantum number released by the system are respectively

\[(1.4) \quad E^- \cdot 0 - E^- \cdot 1 = -E^- = E^+,\]

\[(1.5) \quad q \cdot 0 - q \cdot 1 = -q \equiv q.\]

Let there be \(|E^+, q, 1\rangle\) whose ground state is \(|E', q', n\rangle\) (e.g., \(|E', q', n\rangle = |0\rangle\), \(|E^-, \bar{q}, 1\rangle\) etc.). When \(|E^+, q, 1\rangle\) becomes the final state \(|E', q', n\rangle\), the energy and the additive quantum number released by the system are respectively

\[(1.6) \quad (E^+ \cdot 1 + nE') - nE' = E^+;\]

\[(1.7) \quad (q \cdot 1 + nq') - nq' = q.\]

It is seen that like existence of \(|E^-, q, 0\rangle\), existence of \(|E^-, q, 0\rangle\) is not unconditional, but conditional, i.e., \(|E^-, q, 0\rangle\) should be regarded as an excited state. \(|E^-, q, 0\rangle\) is equivalent to \(|E^+, \bar{q}, 1\rangle\) with the same ground state \(|E^-, q, 1\rangle\). But the existing forms of \(|E^+, q, 1\rangle\) and \(|E^-, q, 0\rangle\) are different. \(|E^+, q, 1\rangle\) exists as a particle, and \(|E^-, q, 0\rangle\) exists as ‘nothing in nothing’, and is analogous to a hole in Dirac’s theory. It should be emphasized that because the number of particles \(n = 0\), the properties (energy and quantum number) of the \(|E^-, q, 0\rangle\) cannot directly be determined by Lagrangian density, but can only be determined by the particle state which \(|E^-, q, 0\rangle\) transforms into (i.e., \(|E^-, q, 1\rangle\) and \(|E^+, \bar{q}, 1\rangle\), see below).

We regard \(|E^-, q, 0\rangle\) whose ground state is \(|E^-, q, 1\rangle\) as such a vacuum state with potential energy \(-E^-\) and quantum number \(-q\). \(|E^-, q, 0\rangle\) is a vacuum state different from \(|0\rangle\) whose ground state is still \(|0\rangle\). We cannot measure the potential, but can only measure an energy of a particle. We suppose that \(|E^-, q, 0\rangle\) and a particle state can transform from one to another by the following process.

\[(1.8) \quad |E^-, q, 0\rangle \overset{\text{w}}{\Rightarrow} |E^-, q, 1\rangle |E^+, \bar{q}, 1\rangle.\]

The ground state of the right-side in (1.8) is still \(|E^-, q, 1\rangle\), hence from (1.1) we obtain the energy and quantum number of the right-side to be respectively

\[(1.9) \quad (E^- + E^+) - E^- = -E^- = E^+,\]

\[(1.10) \quad (q + \bar{q}) - q = \bar{q}.\]

The process does not destroy any conservation law, hence it may occur.

In RQM frame, \(|E^+, \bar{q}, 1\rangle\) cannot be described by \(L_F\) describing \{|E^+, q, 1\rangle\}, a Lagrangian density describing \{|E^+, q, 1\rangle\} must be different from \(L_F\), and it is denoted by \(L_W\). Because \(E^+\) and \(E^-\) are strict symmetric in relativistic theory, \(|E^+, q, 1\rangle\) and \(|E^-, q, 1\rangle\) must be symmetric, \(|E^+, q, 1\rangle\) and \(|E^-, q, 1\rangle\) are also symmetric, hence \(|E^+, \bar{q}, 1\rangle\) and \(|E^+, q, 1\rangle\) must be symmetric. Thus we suppose that \(L_F\) and \(L_W\) are symmetric, independent of each other before quantization and \(L = L_F + L_W\). Because \(L_F\) and \(L_W\) are symmetric, there are states \{|E^+, \bar{q}, 1\rangle\} of the particle \(\bar{q}\) and the process

\[(1.11) \quad |E^-, \bar{q}, 0\rangle \overset{\text{w}}{\Rightarrow} |E^-, \bar{q}, 1\rangle |E^+, q, 1\rangle.\]

Thus, both \(q\) and \(\bar{q}\) have two sorts of existing forms, i.e., \(q\) exists in forms \(|E^+, q, 1\rangle\) and \(|E^-, \bar{q}, 0\rangle\) (their grounds are all \(|E^-, \bar{q}, 1\rangle\) ), \(\bar{q}\) exists in forms \(|E^+, \bar{q}, 1\rangle\) and
\[ |E^-, q, 0\rangle\] (their grounds are all \(|E^-, q, 1\rangle\)), and \(q\) and \(\overline{q}\) are symmetric and the two sorts of existing forms can transform from one to another.

From the conjecture 1 we see that only when all physics quantities of the ground state of a physics system are invariant, is it convenient to discuss evolution of the system, and we can compare physics quantities of two states or two systems. Hence we present the following conjecture for the ground state of a system.

**Conjecture 3.** All physics quantities of the ground state of a system are always invariant.

According to the conjecture, there are processes analogous to the following process

\[(1.12) \quad |E_1^-, e^-, 0\rangle + |E_2^-, e^-, 1\rangle \rightarrow |E_1^+, e^+, \gamma, 1\rangle + |E_2^-, e^-, 1\rangle ,\]  
but there are not such processes analogous to

\[(1.13) \quad |E_1^+ + E_2^+, \gamma, 1\rangle + |E_2^-, e^-, 1\rangle \rightarrow |E_2^+, e^-, 1\rangle .\]

We will prove in another paper that in RQM frame from conjectures 2 and 3 we can obtain the same results as those obtained from conventional RQM.

In contrast with the symmetry of a particle and its antiparticle, the symmetry is a strict symmetric which is not destroyed in any interaction, on the other hand, even \(q\) is a pure neutral particle, i.e., \(q = \overline{q}\), \(|E^+, \overline{q}, 1\rangle\) and \(|E^+, q, 1\rangle\) cannot be regarded as the same particle since \(q\) is in \(\mathcal{L}_F\), \(\overline{q}\) is in \(\mathcal{L}_W\) and \(\mathcal{L}_F\) and \(\mathcal{L}_W\) are independent of each other. \(q\) in \(\mathcal{L}_F\) and \(\overline{q}\) in \(\mathcal{L}_W\) are two freedoms. In contrast with the Dirac’s hole theory in which the electron sea with the infinite negative energy is necessary, in the present theory, \(e^+\) and \(e^-\) are the same existing form and the electron sea is no longer necessary.

Of course, if only in RQM frame to discuss matter, \(\mathcal{L}_W\) is unnecessary since we cannot obtain new measurable results by \(\mathcal{L}_F + \mathcal{L}_W\). But after quantization, \(\mathcal{L}_F\) and \(\mathcal{L}_W\) will be dependent on each other, and the present theory is different from the conventional QFT. The discussion above and the conjecture 1-3 may be regarded as a transition.

In QFT frame, fields become field operators. We will see that energies \(E^+\) determined by \(\mathcal{L}_F\) are all positive, energies \(E^-\) determined by \(\mathcal{L}_W\) are all negative, and it is unnecessary to consider states analogous to \(|E^-, q, 0\rangle\). Thus, in QFT the ground state of the world described by \(\mathcal{L}_F\) is still the vacuum state \(|0\rangle\), the state with the highest energy of the world described by \(\mathcal{L}_W\) is also the vacuum state \(|0\rangle\). Because there is no coupling between the fields in \(\mathcal{L}_F\) and the fields in \(\mathcal{L}_W\), the energy \(E^+\) determined by \(\mathcal{L}_F\) and the energy \(E^-\) determined by \(\mathcal{L}_W\) are respectively conservational, and a \(F\)-particle and a \(W\)-particle cannot transform from one to another in fact by interaction determined by \(\mathcal{L}\). Thus, in QFT the conjectures 1 and 3 are not necessary, and the conjecture 2 become the following form.

**Conjecture A.** Any particle can exist in two sorts of states —— \(F^-\mid E^+, q, 1\rangle\) described by \(\mathcal{L}_F\) and \(W^-\mid E^+, q, 1\rangle\) described by \(\mathcal{L}_W\). \(\mathcal{L} = \mathcal{L}_F + \mathcal{L}_W\), \(\mathcal{L}_F\) and \(\mathcal{L}_W\) are independent of each other before quantization and dependent on each other after quantization. The particles described by \(\mathcal{L}_F\) and the particles described by \(\mathcal{L}_W\) are symmetric.

According to the conjecture, every particle in \(\mathcal{L}_F\) is accordant with a particle in \(\mathcal{L}_W\), and the properties of the two particles are the same, e.g., there are two
sorts of electrons, i.e., a $F-$electron with a positive energy and a $W-$electron with a negative energy. That $\mathcal{L}_F$ and $\mathcal{L}_W$ are independent of each other implies that there is no coupling between fields in $\mathcal{L}_F$ and fields in $\mathcal{L}_W$, but after quantization, $\mathcal{L}_F$ and $\mathcal{L}_W$ will be dependent on each other. Thus, the two sorts of energies corresponding to $\mathcal{L}_F$ and $\mathcal{L}_W$ are respectively conservational, a real particle in $\mathcal{L}_F$ cannot transform into a real particle in $\mathcal{L}_W$, and vice versa. But the two sorts of virtual particles can transform from one to another.

We call the conjecture A the F-W (fire-water) symmetry conjecture. We may also call conjecture A the L-R (left-right) symmetry conjecture since $\mathcal{L}_F$ and $\mathcal{L}_W$ describe respectively the left-hand world (matter world) and right-hand world (dark-matter world) and $\mathcal{L}_F + \mathcal{L}_W$ is left-right symmetry (see paper 3), or $E^+ - E^-$ symmetry conjecture since $E^\pm$ is the basis of the conjecture.

From conjecture A we can obtain possible solutions to the five problems above.

2. Transformation operators and a new method quantizing fields.

Because particles can exist in the two sorts of forms, we can define transformation operators which transform a F-particle into a W-particle or a W-particle into a F-particle, and can quantize fields by the transformation operators replacing creation and annihilation operators in the conventional QFT. Thus it is necessary that $g_f$ and $m_{ef}$ respectively become operators $G_F$ and $M_F$ to be determined by $S_w$, and $g_w$ and $m_{ew}$ respectively become operators $G_W$ and $M_W$ to be determined by $S_f$, here $S_w$ and $S_f$ are the scattering operators respectively determined by $\mathcal{L}_W$ and $\mathcal{L}_F$. $G_F$ and $M_F$ multiplied by field operators $\psi$ and $A_\mu$ become the coupling coefficient $g_f(p_2,p_1)$ and mass $m_{ef}(p)$ determined by scattering amplitude $\langle W_f \mid S_w \mid W_i \rangle$, and $G_W$ and $M_W$ multiplied by field operators $\psi$ and $A_\mu$ become $g_w(p_2,p_1)$ and $m_{ew}(p)$ determined by scattering amplitude $\langle F_f \mid S_f \mid F_i \rangle$. Thus after quantization, $\mathcal{L}_F$ and $\mathcal{L}_W$ will be dependent on each other.

3. Two sorts of corrections.

In the conventional QED, there are two sort of parameters, e.g., the physical charge and the bare charge, and one sort of corrections originating $S$ equivalent to $S_f$. In contrast with the given QED, there are only one sort of parameters defined at so-called subtraction point $q_2$, $q_1$ and $q'$, i.e., $g_f(q_2,q_1) = g_w(q_2,q_1) = g_0$ and $m_{ef}(q) = m_{ew}(q) = m_0$, and two sorts of corrections originating from $S_w$ and $S_f$ to scattering amplitudes, $g_0$ and $m_0$. Thus $\mathcal{L}_F$ and $\mathcal{L}_W$ together determine the loop-diagram corrections. When n-loop corrections originating from $S_f$ and $S_w$ are simultaneously considered, the integrands causing divergence in $\langle F_f \mid S_f \mid F_i \rangle$ or $\langle W_f \mid S_w \mid W_i \rangle$ will cancel each other out, hence all Feynman integrals are convergent, e.g.,

$$g_f^{(1)}(p_2,p_1) = g_{ff}^{(1)}(p_2,p_1) + g_{fw}^{(1)}(q_2,q_1)$$

is finite and $g_f^{(1)}(q_2,q_1) = 0$, where $g_{ff}$ originates from $S_f$ and $g_{fw}$ originates from $S_w$, and the superscript (1) denotes 1-loop correction. Thus it is unnecessary to introduce counterterms and regularization. We give a complete Feynman rules to evaluate Feynman integrals by the new concepts (see second and third papers).

It should be pointed that in the meaning of perturbation theory, because the coupling coefficients and masses will be corrected by n-loop diagrams, we cannot give a absolutely precise $\mathcal{L}_F$ and $\mathcal{L}_W$ in the prime, and can only give the precise $\mathcal{L}_F^{(0-loop)}$ and $\mathcal{L}_W^{(0-loop)}$ at the subtraction point or approximate to tree diagrams.
Of course, by such $L^{(0-loop)}_F$ and $L^{(0-loop)}_W$ we can obtain scattering amplitudes approximate arbitrary n-loop diagrams.

4. $\langle 0 | H | 0 \rangle \equiv E_0 = E_{0F} + E_{0W} = 0$ is naturally derived, thereby we can easily determine the cosmological constant according to data of astronomical observation, and it is possible to correct nonperturbational methods which depend on the energy of the ground state in QFT.

When $\psi$ and $A_\mu$ in $L_F$ and $\overline{\psi}$ and $\partial_\mu$ in $L_W$ are regarded as the classical fields or the coupling coefficients $g_f$ and the electromagnetic masses $m_{ef}$ in $L_F$, $g_w$ and $m_{ew}$ in $L_W$ are regarded as constants, $L_F$ and $L_W$ will be independent of each other. In this case, except $E_0 = 0$, all results obtained by the present theory will be the same as those obtained by the conventional theory.

5. Generalizing the present theory to the electroweak unified theory, we will see a possible origin of symmetry breaking. According to this model, the world is symmetric on principle (i.e., $L = L_W + L_F$ is symmetric), but the world observed by us is asymmetric since $L_W$ or $L_F$ is asymmetric. In this model there is no unknown particle with a massive mass (see the third paper).

6. Because there is no interaction between the two sorts of matter by a given quantizable field. Only possibility is that there is repulsion or gravitation of the two sorts of matter. Because the sort of matter described by $L_W$ is one new sort of matter, it is impossible from theory to determine that there is what sort of interaction. We can only determine the new sort of interaction from data of astronomical observation. If the new interaction is repulsion, it is possible that the new interaction is the reason for cosmos expansion. If the new interaction is gravitation, it is possible that the new sort of matter is the candidate for dark matter. It is also possible that there is new and more important relationship between the two sorts of matter.

7. The new QFT will also give a possible solution for the problem of triviality of $\varphi^4$-theory.

8. Two sorts of transformation.

According to the present theory, there are two sorts of transformation.

The first sort of transformation must correspond to a coupling term of field operators, e.g., $ig_\varphi \overline{\psi} \gamma_\mu A_\mu \psi$ determines the transformation $e^+ + e^- \rightarrow \gamma + \gamma$. The sort of transformation is measurable.

The second sort of transformation is defined by the transformation operators as $| a_p s \rangle \not\equiv | a_p s \rangle$ and cannot correspond to any coupling term of quantizable fields. The processes determined by the sort of transformation, e.g.,

$$| a_p s \rangle \not\equiv | a_p s \rangle$$

cannot be measured. The sort of transformation can only be potential and is realized by the virtual-particle processes. Existing reasons of the sort of transformation are that by it we can eliminate divergence of $E_0$ and Feynman integrals with loop diagrams, explain the left-right asymmetry and some phenomena of the cosmos, and so on.

By the conventional creation and annihilation operators in the given QFT we can also obtain the similar results, provided we suppose $L = L_F + L_W$ and that $g_f$ and $m_f$ are determined by $S_w$ and $\varphi_w$ and $m_w$ are determined by $S_f$ (of course, in this case this conjecture is not natural). It is also possible to obtain the same results but that both F-particles and W-particles possess positive energies, provided $L_F$ and $L_W$ are independent of each other before quantization.
The present theory contains three parts. The first part takes QED as example to illuminate the method to reconstruct QFT, and give the solutions of the issue of the cosmological constant and the problem of divergent Feynman integrals in QED. The first part is the present paper which is composed of the part and the following two parts. The second part discusses the problem of triviality of $\varphi^4$-theory. The third part discusses the problem of the origin of asymmetry in the electroweak unified theory in detail.

Quantization for free fields will be discussed in the present paper. The outline of this paper is as follows. In section 2, we construct the Lagrangian density of free fields. In section 3, quantization for free fields is carried out. In section 4, we discuss the meanings of $E_0 = 0$. Section 5 is a conclusion.

### 2. Lagrangian density and equations of motion for free fields

There must be positive-energy resolutions and negative-energy resolutions for a classical relativistic equation of motion, and positive-energy solutions and negative-energy solutions are symmetric. On the other hand, we think that only equations of motion are insufficient in order to determine the complete properties of a relativistic system. A complete Lagrangian density is very necessary. From this, we present the following conjecture.

**Conjecture:** There are both particles with negative energies described by $\mathcal{L}_W$ (W-particle) and particles with positive energies described by $\mathcal{L}_F$ (F-particle), the total Lagrangian density is

\[
\mathcal{L}_0 = \mathcal{L}_F + \mathcal{L}_W,
\]

(2.1)

$\mathcal{L}_F$ and $\mathcal{L}_W$ are independent of each other and symmetric.

Field operators are composed of transformation operators which transform a F-particle into a W-particle or a W-particle into a F-particle. Such transformation may occur for virtual particles. But because $\mathcal{L}_F$ and $\mathcal{L}_W$ are independent of each other, such transformation cannot occur for realistic particles.

We suppose the Lagrangian densities $\mathcal{L}_F$ and $\mathcal{L}_W$ for the free Dirac fields and the Maxwell fields to be respectively

\[
\mathcal{L}_F = -\bar{\psi}_0(x)\left(\gamma_\mu \partial_\mu + m\right)\psi_0(x) - \frac{1}{2} \partial_\mu A_{0\nu} \partial_\nu A_{0\mu},
\]

(2.2)

\[
\mathcal{L}_W = \bar{\psi}_0(x)\left(\gamma_\mu \partial_\mu + m\right)\psi_0(x) + \frac{1}{2} \partial_\mu A_{0\nu} \partial_\nu A_{0\mu}.
\]

(2.3)

(2.2) and (2.3) imply the Lorentz gauge to be already fixed. The difference between (2.1) and the corresponding Lagrangian density in the given QED is $\mathcal{L}_W$ which describes motion of particles existing in the other form. We call $\psi$, $A_\mu$, $\bar{\psi}$ and $\bar{A}_\mu$ the F-electron field, the F-photon field, and the W-electron field, the W-photon field, respectively. The conjugate fields corresponding to them are respectively
\[
\pi_{0\psi} = \frac{\partial L_0}{\partial \dot{\psi}_0} = i\psi_0^+,
\]
\[
\pi_{0\mu} = \frac{\partial L_0}{\partial \dot{A}_{0\mu}} = \dot{A}_{0\mu},
\]
\[
\pi_{0\psi} = \frac{\partial L_0}{\partial \dot{\psi}_0} = -i\psi_0^+.
\]
\[
\pi_{0\mu} = \frac{\partial L_0}{\partial \dot{A}_{0\mu}} = -\dot{A}_{0\mu}.
\]

From the Noether's theorem, we have
\[
H_0 = H_{F0} + H_{W0},
\]
\[
H_{F0} = \int d^3x \{\psi_0^+ \gamma_4 (\gamma_j \partial_j + m) \psi_0 + \frac{1}{2} (\dot{A}_{0\mu} \dot{A}_{0\mu} + \partial_j A_{0\nu} \partial_j A_{0\nu})\},
\]
\[
H_{W0} = -\int d^3x \{\psi_0^+ \gamma_4 (\gamma_j \partial_j + m) \psi_0 + \frac{1}{2} (\dot{A}_{0\mu} \dot{A}_{0\mu} + \partial_j A_{0\nu} \partial_j A_{0\nu})\}.
\]
\[
Q = Q_F + Q_W,
\]
\[
Q_F = \int d^3x \psi_0^+ \dot{\psi}_0,
\]
\[
Q_W = -\int d^3x \dot{\psi}_0^+ \psi_0,
\]

where \(j = 1, 2, 3\), a repeated index implies summation over all values of the index. The Euler-Lagrange equations of motion derived from Hamilton's variational principle are
\[
i\frac{\partial}{\partial t} \dot{\psi}_0 = \hat{H}_0 \psi_0,
\]
\[
\square A_{0\mu} = 0,
\]
\[
i\frac{\partial}{\partial t} \dot{\psi}_0 = \hat{H}_0 \psi_0,
\]
\[
\square \dot{A}_{0\mu} = 0,
\]

where \(\hat{H}_0 = \gamma_4 (\gamma_j \partial_j + m)\). It is seen from (2.14)-(2.17), (2.9), (2.10), (2.12), and (2.13) that although \(L_{F0} \neq L_{W0}, Q_F \neq Q_W\) and \(H_{0F} \neq H_{0W}\), the equations satisfied by \(\dot{\psi}_0\) and \(\dot{A}_{0\mu}\) are the same as those satisfied by \(\psi_0\) and \(A_{0\mu}\), respectively.
This implies that for a relativistic physical system, only equations of motion are insufficient for corrective description of all properties of the system. A complete Lagrangian density is very necessary.

When $\psi_0$, etc., are regarded as the classical fields and

$$\partial_\mu A_{0\mu} = \partial_\mu A_{0\mu} = 0,$$

(2.18)

$\psi$ and $\psi$ can be expanded in terms of the complete set of plane-wave solutions

$$\frac{1}{\sqrt{V}} u_{ps} e^{ipx}, \quad \frac{1}{\sqrt{V}} v_{ps} e^{-ipx}, \quad s = 1, 2,$$

(2.19)

where $px = px - E_p t$, $E_p = \sqrt{p^2 + m^2}$, and the complete set of plane-wave solutions to (2.15) and (2.17) is

$$\frac{1}{\sqrt{2\omega_k V}} c_{k}^{\lambda} e^{\pm ikx},$$

(2.20)

where $kx = kx - \omega_k t, \omega_k = k$ $= 1, 2$. To get a completeness relation, it is necessary to form a quartet of orthonormal 4-vectors$^{[3]}$.

$$e_k = (e_k^1, 0), \quad e_k^2 = (e_k^2, 0), \quad e_k^3 = - \frac{1}{k} \frac{1}{k} [k + \eta (k\eta)] \eta,$$

$$\eta = (0, 0, 0, i), \quad e_k^4 = i\eta, \quad e_k^{1/2}, \quad k = 0.$$

(2.21)

Moreover, all four vectors are normalized to 1, i.e.,

$$e_k^{\lambda} e_k^{\lambda'} = \delta_{\lambda\lambda'}, \quad \sum_{\lambda=1}^4 e_{k\mu}^{\lambda} e_{k\nu}^{\lambda} = \delta_{\mu\nu}.$$

3. Quantization for free fields

We now regard $\psi_0$ etc. as field operators. $\psi_0, A_{0\mu}, \psi_0$, and $A_{0\mu}$ as the solutions of the equations of the quantum fields (2.14) – (2.17) can also be expanded in terms of the complete sets (2.19) and (2.20), respectively, only the expanding coefficients are all operators. Thus we have

$$\psi_0 (x) = \frac{1}{\sqrt{V}} \sum_{ps} \{ \bar{a}_{ps} \xi_{ps} \leq a_{ps} (t) \mid u_{ps} e^{ipx} + \mid b_{ps} (t) \geq d\eta^+_{ps} (\bar{b}_{ps} \mid v_{ps} e^{-ipx}) \},$$

(3.1)

$$A_{0\mu} (x) = \frac{1}{\sqrt{V}} \sum_k \frac{1}{\sqrt{2\omega_k}} \sum_{\lambda=1}^4 c_{k}^{\lambda} \{ \bar{j}_{k} \leq c_{k\lambda} (t) \mid e^{ikx} + \mid \bar{c}_{k\lambda} (t) \geq \bar{j}_{k} \leq e^{-ikx} \},$$

(3.2)

$$\psi_\mu (x) = \frac{1}{\sqrt{V}} \sum_{ps} \{ \bar{b}_{ps} \eta_{ps} \leq b_{ps} (t) \mid u_{ps} e^{ipx} + \mid b_{ps} (t) \geq d\eta^+_{ps} (a_{ps} \mid v_{ps} e^{-ipx}) \},$$

(3.3)
where $|\alpha\rangle$ and $\langle\alpha|$ are respectively a state ket and a state bra which do not change as time $t$. $\langle\alpha|t\rangle$ and $|\alpha\rangle t$ are operators changing as time $t$. $|\alpha\rangle$ act on the state $\langle\alpha|$ or $|\alpha\rangle t$ act on the state $\langle\alpha|t\rangle$. We call $|\alpha\rangle$ a F-electron, $|\alpha\rangle t$ a F-positron and a F-photon state ket, respectively; $\langle\alpha|$ a F-phantom, $\langle\alpha| t$ a W-electron, $\langle\alpha| t$ a W-positron and a W-photon state ket, respectively. We can also name the state bars and the operators in similar method. We call $|\alpha\rangle$ and $|\alpha\rangle t$ a W-imaginary current and a F-imaginary current, respectively. It should be pointed out that $A_{0\mu}$, $(\bar{A}_{0\mu})$, in fact, may be written out in W-state and F-operator (F-state and W-operator) as well as the form of $\psi_0 (\bar{\psi}_0)$, but in this case, it is not convenient to discuss the Hamiltonian equations.
3.1. Properties and multiplication rules of the transformation operators.
We define inner products of the states, products of \( j_k \), etc., and commutation relations of operator \( |a_{ps}\rangle \geq \) etc. as follows.

\[
\langle \alpha_{ps} | \cdot | \beta_{p's'} \rangle = \langle \alpha_{ps} | \beta_{p's'} \rangle = \delta_{\alpha\beta}\delta_{pp'}\delta_{ss'},
\]

\[
\langle \gamma_{k\lambda} | \cdot | \gamma_{k'\lambda'} \rangle = \begin{cases} 
\delta_{kk'}, & \lambda = 1, 2, 3, \\
-\delta_{kk'}\delta_{\lambda\lambda'}, & \lambda = 4 
\end{cases}
\]

\[
\langle \beta_{ps} | \cdot | \gamma_{k\lambda} \rangle = \langle \gamma_{k\lambda} | \cdot | \beta_{ps} \rangle = 0,
\]

\[
\langle 0 | 0 \rangle = 1,
\]

\[
\langle 0 | \alpha \rangle = \langle 0 | \gamma \rangle = \langle 0 | \beta_{ps} | \cdot | \gamma_{k\lambda} \rangle = 0.
\]

\[
\hat{j}_k \hat{j}_{-k'} = \hat{i} \hat{i}_{-k} = \delta_{kk'}, \quad \hat{j}_k \hat{i}_{k'} = 0,
\]

where \( \alpha, \beta = a, b, g, h \) and \( \gamma = c, \xi \). Two single-fermion states are anticommutative; two boson states or a fermion state and a boson state are commutative, i.e.,

\[
\begin{align*}
| \alpha_{ps} \rangle | \beta_{p's'} \rangle &= -| \beta_{p's'} \rangle | \alpha_{ps} \rangle, \\
Tr | \alpha_{ps} \rangle \langle \beta_{p's'} | &= -\langle \beta_{p's'} | \alpha_{ps} \rangle = -\delta_{\alpha\beta}\delta_{pp'}\delta_{ss'},
\end{align*}
\]

\[
\langle \alpha_{ps} \rangle | \gamma_{k\lambda} \rangle = | \gamma_{k\lambda} \rangle | \alpha_{ps} \rangle, \quad Tr | \gamma_{k\lambda} \rangle \langle \alpha_{ps} | = \langle \alpha_{ps} | \gamma_{k\lambda} \rangle = 0.
\]

The operators \( \hat{b}_{ps} \geq \) or \( \hat{c}_{k\lambda} \geq \) etc. satisfy the same anticommutation or commutation relations as those for creation and annihilation operators in the known QED.

\[
\begin{align*}
\{ \langle \beta_{p's} (t) | \cdot | \beta_{p's'} (t) \rangle \} &= \delta_{\beta\beta'}\delta_{pp'}\delta_{ss'}, \\
\{ \langle \beta_{p's} (t) | \cdot | \beta_{p's'} (t) \rangle \} &= \{ | \beta_{ps} (t) \rangle \geq, | \beta_{p's'} (t) \rangle \geq \} = 0,
\end{align*}
\]

\[
\begin{align*}
\{ \langle \gamma_{k\lambda} (t) | \cdot | \gamma_{k'\lambda'} (t) \rangle \} &= \begin{cases} 
\delta_{\gamma\gamma'}\delta_{kk'}\delta_{\lambda\lambda'}, & \lambda = 1, 2, 3, \\
-\delta_{\gamma\gamma'}\delta_{kk'}\delta_{\lambda\lambda'}, & \lambda = 4 
\end{cases}, \\
\{ \langle \gamma_{k\lambda} (t) | \cdot | \gamma_{k\lambda} (t) \rangle \} &= \{ \langle \gamma_{k\lambda} (t) | \cdot | \gamma_{k\lambda} (t) \rangle \} = 0,
\end{align*}
\]

\[
\begin{align*}
\{ J, | \sigma \rangle \} &= \{ J, \langle \sigma | \} = \{ J, | \sigma (t) \rangle \} = \{ J, \langle \sigma (t) | \} = 0.
\end{align*}
\]
where $J = j_k, j_{\bar{k}}, \sigma = a, b, c, \bar{a}, \bar{b}, \bar{c}.$

We define the action of an operator on a state as follows.

\[
\langle \beta_{ps} | \beta'_{p's'} \rangle \equiv \langle 0 | \delta_{\beta \beta'} \delta_{pp'} \delta_{ss'}. \]
\[
(3.24)
\]

\[
\langle \gamma_{\kappa \lambda} | \gamma'_{\kappa' \lambda'} \rangle = \left\{ \begin{array}{ll}
0 & \delta_{\gamma \gamma'} \delta_{kk'} \lambda \lambda' = 1, 2, 3, \\
-0 & \delta_{\gamma \gamma'} \delta_{kk'} \lambda \lambda' = 4,
\end{array} \right.
\]
\[
(3.25)
\]

\[
\langle \beta_{ps} | \beta'_{p's'} \rangle = \langle \gamma_{\kappa \lambda} | \beta_{ps} \rangle = \langle \beta_{ps} | \gamma_{\kappa \lambda} \rangle,
\]
\[
\langle 0 | \beta_{ps} \rangle = \langle 0 | \gamma_{\kappa \lambda} \rangle = \langle 0 | J = J | 0 \rangle = 0. \quad (3.26)
\]

The transformation operators have the following properties and observe the following multiplication rules.

(1). Because a transformation operator is a whole, the order of its two parts cannot exchange, e.g., $b_{ps} \not= (b_{ps} |$ and $\tau_{k\lambda} \not= j_{-k} \langle j_{-k} |$ cannot be written as $\langle b_{ps} |$ $b_{ps} \rangle$ and $j_{-k} \rangle$.

(2). Because a transformation operator contains a Grassman number and a state, when a physics-quantity operator is constructed by the transformation operators, it is necessary to integrate over the Grassman numbers and to trace with respect to states in product of the transformation operators.

According to the rules above we easily obtain the charge operators and the Hamiltonian operators.

\[
Q_F = \int d^3x Tr \int \bar{\psi}_0 \psi_0
\]
\[
= \int Tr \sum_p \{ | a_{ps}(t) \rangle \rangle \delta_{\xi_{ps}} \langle \alpha_{ps} | \xi_{ps} \langle a_{ps} | \langle \alpha_{ps} | a_{ps}(t) | \}
\]
\[
+ | \langle \beta_{ps} \rangle \rangle \delta_{\eta_{ps}} \langle b_{ps} | \langle \alpha_{ps} | a_{ps} | \langle \alpha_{ps} | a_{ps}(t) | \}
\]
\[
= Tr \sum_p \{ | a_{ps}(t) \rangle \rangle \delta_{\alpha_{ps}} \langle \alpha_{ps} | | a_{ps}(t) | \}
\]
\[
- | \langle \beta_{ps} \rangle \rangle \delta_{b_{ps}} \langle b_{ps} | \langle \alpha_{ps} | a_{ps} | \langle \alpha_{ps} | a_{ps}(t) | -1 \}
\]
\[
(3.27)
\]

\[
Q_W = - \int d^3x Tr \int \bar{\psi}_0 \psi_0
\]
\[
= - \sum_p \{ | \beta_{ps}(t) \rangle \rangle \delta_{\beta_{ps}} \langle b_{ps} | \langle \alpha_{ps} | a_{ps} | \langle \alpha_{ps} | a_{ps}(t) | -1 \}
\]
\[
(3.28)
\]
Similarly to $Q_F$ and $Q_W$, we have

$$H_{F0} = \sum_{ps} \omega_p \| a_{ps}(t) \rangle \langle a_{ps}(t) | + | b_{ps}(t) \rangle \langle b_{ps}(t) | - 1|$$

$$+ \sum_k \omega_k \sum_{j=1}^3 \left( c_{kj}(t) \rangle \langle c_{kj}(t) | - | c_{kj}(t) \rangle \langle c_{kj}(t) | + \frac{1}{2}\right).$$

(3.29)

$$H_{W0} = - \sum_{ps} \omega_p \| b_{ps}(t) \rangle \langle b_{ps}(t) | + | a_{ps}(t) \rangle \langle a_{ps}(t) | - 1|$$

$$- \sum_k \omega_k \sum_{j=1}^3 \left( \omega_{kj}(t) \rangle \langle \omega_{kj}(t) | - | \omega_{kj}(t) \rangle \langle \omega_{kj}(t) | + \frac{1}{2}\right).$$

(3.30)

It is seen from (3.27)-(3.30) that both energy and charge of the vacuum state are zero, the energies of the F-states are all positive and the energies of the W-states are all negative, i.e.,

(3.31) \hspace{1cm} E_0 = \langle 0 | H_0 | 0 \rangle = Q_0 = \langle 0 | Q | 0 \rangle = 0,

(3.32) \hspace{1cm} \langle F | H_0 | F \rangle = \langle F | (H_{F0} + \sum_{ps} \omega_p - \frac{1}{2} \sum_k \omega_k) | F \rangle > 0,

(3.33) \hspace{1cm} \langle W | H_0 | W \rangle = \langle W | (H_{W0} - \sum_{ps} \omega_p + \frac{1}{2} \sum_k \omega_k) | W \rangle < 0.

It should be pointed out that $E_0 = 0$ and $Q_0 = 0$ do not depend on existence of negative energies, in fact, provided fields are quantized by the transformation operators, (3.31) can be obtained[2]. (3.33) does not imply that the masses of the W-particles are negative, oppositely, it is known from (2.10) that the masses of the W-particles are positive.

The energies and charges of particles can also be written as

(3.34) \hspace{1cm} H_{F0} = \int d^3x : \psi^+ \gamma_4 (\gamma_j \partial_j + m) \psi + \frac{1}{2} \left( A_{\mu} A_{\mu}' + \partial_j A_{\mu}' \partial_j A_{\mu}' \right) :;

(3.35) \hspace{1cm} H_{W0} = - \int d^3x : [\psi^+ \gamma_4 (\gamma_j \partial_j + m) \psi + \frac{1}{2} (A_{\mu}' A_{\mu}' + \partial_j A_{\mu}' \partial_j A_{\mu}')] :;

(3.36) \hspace{1cm} Q = \int d^3x : \psi^+ \psi : - \int d^3x : \psi^+ \psi : ;

where

(3.37) \hspace{1cm} \psi'_0 (x) = \frac{1}{\sqrt{V}} \sum_{ps} \left( \langle \psi_{ps}(t) | u_{ps} e^{ipx} + | b_{ps}(t) \rangle \langle b_{ps}(t) | e^{-ipx} \right),

(3.38) \hspace{1cm} A'_{\mu} (x) = \frac{1}{\sqrt{V}} \sum_k \frac{1}{\sqrt{2\omega_k}} \sum_{\lambda=1}^4 \langle c_{k\lambda}(t) | e^{ikx} + | c_{k\lambda}(t) \rangle e^{-ikx} ,

(3.39) \hspace{1cm} \psi'_0 (x) = \frac{1}{\sqrt{V}} \sum_{ps} \left( \langle b_{ps}(t) | u_{ps} e^{ipx} + | a_{ps}(t) \rangle \langle a_{ps}(t) | e^{-ipx} \right).
(3.40) \[ A'_\mu (x) = \frac{1}{\sqrt{V}} \sum_k \frac{1}{\sqrt{2\omega_k}} \sum_{\lambda=1}^4 \epsilon^\mu_{\lambda k} \left( \langle \mathbf{c}_{k\lambda} (t) \right| e^{ikx} + \langle \mathbf{u}_{k\lambda} (t) \right| e^{-ikx} \),

(3.41) \[ \pi'_{\mu} = i\psi'_{0(x)} = \frac{i}{\sqrt{V}} \sum_{ps} \left( \langle a_{ps} (t) \right| u_{ps}^+ e^{-ipx} + \langle b_{ps} (t) \right| v_{ps}^+ e^{ipx} \),

(3.42) \[ \pi'_{0\mu} = A'_{0\mu} (x) = -\frac{i}{\sqrt{V}} \sum_k \sqrt{\omega_k} \sum_{\lambda=1}^4 \epsilon^\lambda_{\mu k} \left( \langle c_{k\lambda} (t) \right| e^{ikx} - \langle \bar{c}_{k\lambda} (t) \right| e^{-ikx} \),

(3.43) \[ \pi'_{0\mu} = -i\psi'_{0} (x) = -\frac{i}{\sqrt{V}} \sum_{ps} \left( \langle b_{ps} (t) \right| u_{ps}^+ e^{-ipx} + \langle a_{ps} (t) \right| v_{ps}^+ e^{ipx} \),

(3.44) \[ \pi'_{\mu} = -A'_{0\mu} = \frac{i}{\sqrt{V}} \sum_k \sqrt{\omega_k} \sum_{\lambda=1}^4 \epsilon^\lambda_{\mu k} \left( \langle c_{k\lambda} (t) \right| e^{ikx} - \langle \bar{c}_{k\lambda} (t) \right| e^{-ikx} \),

where the double-dot notation \( \cdots \) is known as normal ordering. An operator product is in normal ordered form if all operators as \( | \sigma \rangle \) stand to the left of all operators as \( | \leq \sigma \rangle \). In contrast with the given QED, (3.31)-(3.33) are the inferences of the multiplication rules, and are not definition.

3.2. Subsidiary condition. After the Maxwell field is quantized, the Lorentz condition (2.18) is no longer applicable. From (3.38) and (3.40) we have

(3.45) \[ (\partial_\mu A'_{0\mu})^+ = \frac{i}{\sqrt{V}} \sum_k \frac{|k|}{\sqrt{2\omega_k}} \left( \langle \mathbf{c}_{k3} \right| - i \langle \mathbf{c}_{k4} \right| e^{ikx} \),

(3.46) \[ (\partial_\mu A'_{0\mu})^- = \frac{i}{\sqrt{V}} \sum_k \frac{|k|}{\sqrt{2\omega_k}} \left( \langle \mathbf{c}_{k3} \right| - i \langle \mathbf{c}_{k4} \right| e^{ikx} \).

Thus we define the subsidiary condition to be

(3.47) \[ (\partial_\mu A'_\mu)^+ \left| c_p \right\rangle = 0,

(3.48) \[ (\partial_\mu A'_\mu)^- \left| \xi_p \right\rangle = 0,

\( | c_p \rangle \) and \( | \xi_p \rangle \) are known as F-physics state ket and W-physics state ket, respectively. From (3.47) – (3.48) we obtain

(3.49) \[ \left| c_p \right\rangle = \left| \mathbf{c}_T \right\rangle \{ 1 + \sum_k f (k) \left| c_{pk} \right\rangle + \cdots + \sum_{k_1 \cdots k_n} f (k_1 \cdots k_n) \left| c_{pk_1} \cdots c_{pk_n} \right\rangle + \cdots \},

(3.50) \[ \left| \xi_p \right\rangle = \left| \xi_T \right\rangle \{ 1 + \sum_k f (k) \left| \xi_{pk} \right\rangle + \cdots + \sum_{k_1 \cdots k_n} f (k_1 \cdots k_n) \left| \xi_{pk_1} \cdots \xi_{pk_n} \right\rangle + \cdots \},
where $|c_T\rangle$ and $|\varphi_T\rangle$ are states containing only transverse photons, and

$$|c_{pk}\rangle = |c_{k3}\rangle + i |c_{k4}\rangle,$$

(3.51)

$$|\varphi_{pk}\rangle = |\varphi_{k3}\rangle + i |\varphi_{k4}\rangle.$$  

(3.52)

From (2.21), (3.11), (3.22), (3.23) and (3.29)-(3.33) we obtain

$$\langle c_{pk} | c_{pk'} \rangle = \langle \varphi_{pk} | \varphi_{pk'} \rangle = \langle c_{p} | H_{0} | c_{p'} \rangle = \langle \varphi_{p} | H_{0} | \varphi_{p'} \rangle = 0,$$

(3.53)

$$\langle c_{p} | c_{p'} \rangle = \langle c_{T} | c_{T'} \rangle, \quad \langle \varphi_{p} | \varphi_{p'} \rangle = \langle \varphi_{T} | \varphi_{T'} \rangle,$$

(3.54)

$$\langle c_{p} | H_{0} | c_{p} \rangle = \langle c_{T} | H_{0} | c_{T} \rangle,$$

(3.55)

$$\langle \varphi_{p} | H_{0} | \varphi_{p} \rangle = \langle \varphi_{T} | H_{0} | \varphi_{T} \rangle.$$  

(3.56)

3.3. Eigenstates of $H_{0}$. It is easily seen from (3.16) and (3.17) that there is at most only one particle in a fermion state denoted by $p_{s}$, and there may be many particles in a boson state denoted by $k_{\lambda}$. Thus, a state in which there are $n$ F-photons or $n$ W-photons can be represented by

$$|n_{k\lambda}\rangle = \frac{1}{\sqrt{n!}} |c_{k\lambda}\rangle^{n},$$

(3.57)

$$|\varphi_{k\lambda}\rangle = \frac{1}{\sqrt{n!}} |\varphi_{k\lambda}\rangle^{n},$$

(3.58)

From (3.11) and (3.25) we have

$$\langle n_{k\lambda} | n_{k\lambda} \rangle = \langle \varphi_{k\lambda} | \varphi_{k\lambda} \rangle = \begin{cases} 1, & \lambda = 1, 2, 3 \\ (-1)^{n}, & \lambda = 4 \end{cases},$$

(3.59)

$$\langle c_{k\lambda} | n_{k\lambda} \rangle = \begin{cases} \sqrt{n} |(n-1)_{k\lambda}\rangle, & \lambda = 1, 2, 3 \\ -\sqrt{n} |(n-1)_{k\lambda}\rangle, & \lambda = 4 \end{cases},$$

(3.60)

$$\langle \varphi_{k\lambda} | n_{k\lambda} \rangle = \begin{cases} \sqrt{n} |(n-1)_{k\lambda}\rangle, & \lambda = 1, 2, 3 \\ -\sqrt{n} |(n-1)_{k\lambda}\rangle, & \lambda = 4 \end{cases},$$

(3.61)
3.4. The equations of motion. From (3.1) -(3.4), (3.6), (3.8), (3.29) and (3.30), we have

\[ i \frac{\partial \psi_0}{\partial t} = [\psi_0, H] = \hat{\psi}_0, \]

\[ i \frac{\partial \psi}{\partial t} = -[\psi, H] = \hat{\psi}_0, \]

\[ \frac{\partial A_0}{\partial t} = -i[A_0, H], \quad \frac{\partial A_0}{\partial t} = -i[A_0, H] = \nabla^2 A_0, \]

\[ \frac{\partial A_0}{\partial t} = i[A_0, H], \quad \frac{\partial A_0}{\partial t} = i[A_0, H] = \nabla^2 A_0. \]

Since \([H_0, H] = [H_0, W] = 0\), \(H_0\) and \(W\) are the constants of motion. Thus we have

\[ \psi_0(x, t) = e^{iH_0 t} \psi_0(x, 0) e^{-iH_0 t}, \]

\[ \psi(x, t) = e^{-iW_0 t} \psi_0(x, 0) e^{iW_0 t}, \]

\[ A_0(x, t) = e^{iH_0 t} A_0(x, 0) e^{-iH_0 t}, \]

\[ \Delta_0(x, t) = e^{-iW_0 t} \Delta_0(x, 0) e^{iW_0 t}. \]

As seen the equations (3.62)–(3.65) are consistent with (2.14)–(2.17), respectively.

4. The physical meanings of that the energy of the vacuum state is equal to zero.

It is seen from (3.31) that both energy and charge of the vacuum state are equal to zero. (3.31) is not relative to the definition for multiplication of transformation operators. (3.31) holds water provided \(H_0\) is composed of transformation operators. In contrast with the given QED, (3.31) is derived without application of the normal ordered product of \(H_0\). According to the given QED, before redefining \(H_0\) as normal-ordered products \(E_0 \neq 0\). After redefining \(H_0\) as normal-ordered products, \(E_0 = 0\). But this only transfers the divergence difficulty of the energy of the ground state. Because we may arbitrarily choose the zero point of energy in quantum field theory, we can redefine the ground-state energy to be zero. But in the theory of gravitation, if \(E_0 \neq 0\), \(E_0\) will have gravitational effect. Hence we are not at liberty to redefine \(E_0 = 0\). Thus the knotty problem of the cosmological constant arises in the given QFT and the relativistic theory of gravitation. In the present theory \(E_0 = 0\). Hence the density of the energy of the vacuum state \(\rho_{vac} = 0\). Thus, from the equation of gravitation field

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda g_{\mu\nu} = -8\pi G (T_{\mu\nu} - \rho_{vac} g_{\mu\nu}), \]
and data of astronomical observation we can easily determine the cosmological constant $\lambda$.

We secondly simply discuss the correction originating from $E_0 = 0$ to a nonperturbational method in quantum field theory.

When one evaluates the energy of a system by a nonperturbational method, e.g., a Hartree-type approximation\cite{5}, it is necessary to subtract the zero-point energy $E_0^{[6]}$. According to the given quantum field theory $E_0 \neq 0$, while according to the present theory $E_0 = 0$, hence we will obtain different results in nature.

We will discuss the two knotty problems above in detail in other papers.

5. Conclusions

We suppose that there are both particles with negative energies described by $L_W$ and particles with positive energies described by $L_F$, and $L_W$ and $L_F$ are independent of each other and symmetric. From this we present a new Lagrangian density $\mathcal{L} = L_W + L_F$ and a new quantization method for QED. That the energy of the vacuum state is equal to zero is naturally obtained. Thus we can easily determine the cosmological constant according to data of astronomical observation, and it is possible to correct nonperturbational methods which depend on the energy of the ground state in quantum field theory.

6. Acknowledgement

I am very grateful to professor Zhan-yao Zhao for best support and professor Zhao-yan Wu for helpful discussions.

References

[1] G. Scharf, Finite Quantum Electrodynamics, (1995) Second Edition, Springer-Verlag.
[2] Shi-hao Chen, Quantum Field Theory Without Divergence, hep-th/0203220.
[3] David Lurie, Particles and Fields, (1968), New York, London, Sydney.
[4] Yu Yun qiang, An Introduction to General Relativity (Second Edition), (1997), Peking University, Beijing.
[5] T. Kinoshita and Y. Nambu, Phys. Rev. 94, 598 (1954); A.L. Fetter and J.D. Walecka, Quantum Theory of Many-Particle Systems, (1971), McGraw-Hill, New York.
[6] Shau-Jin Chang and Jon A. Wright, (1975), Phys. Rev. D, 12, 1595.

Institute of Theoretical Physics, Northeast Normal University, Changchun 130024, China.

E-mail address: shchen@nenu.edu.cn