Testing $\kappa$-Poincaré with neutral kaons

Giovanni AMELINO-CAMELIA$^1$ and Franco BUCCCELLA$^2$

Theory Division, CERN, CH-1211, Geneva, Switzerland

ABSTRACT

In recent work on experimental tests of quantum-gravity-motivated phenomenological models, a significant role has been played by the so-called “$\kappa$” deformations of Poincaré symmetries. Sensitivity to values of the relevant deformation length $\lambda$ as small as $5 \cdot 10^{-33} m$ has been achieved in recent analyses comparing the structure of $\kappa$-Poincaré symmetries with data on the gamma rays we detect from distant astrophysical sources. We investigate violations of CPT symmetry which may be associated with $\kappa$-Poincaré in the physics of the neutral-kaon system. A simple estimate indicates that experiments on the neutral kaons may actually be more $\lambda$-sensitive than corresponding astrophysical experiments, and may already allow to probe values of $\lambda$ of order the Planck length.

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$^1$Marie Curie Fellow of the European Union (address from February 2000: Dipartimento di Fisica, Università di Roma “La Sapienza”, Piazzale Moro 2, Roma, Italy)

$^2$On leave of absence from Dipartimento di Scienze Fisiche, Università di Napoli, Mostra D’Oltremare, Pad. 19, Napoli, Italy.
It has been recently realized [1, 2, 3] that observations of gamma rays from distant astrophysical sources can be used to set stringent bounds on the length parameter that characterizes “κ-Poincaré”, one of the most studied scenarios for the dimensionful quantum deformation of Poincaré symmetries, which has been developed most notably in Refs. [4, 5, 6]. This realization has attracted significant attention because the level of sensitivity of planned gamma-ray observatories (see, e.g., Refs. [7, 8]) should be sufficient to explore values of the length parameter as small as the Planck length ($L_p \sim 1.6 \cdot 10^{-35}$ m). Only very few other experimental contexts [9, 10, 11, 12] can achieve this type of “Planckian sensitivity”, and such sensitivity levels with respect to κ-Poincaré are especially meaningful in light of the fact that it appears likely [10, 13, 14, 15, 16, 17] that the space-time foam of quantum gravity would induce a dimensionful (although, of course, not necessarily “κ”) deformation of Poincaré symmetries.

We are here interested in exploring the consequences of the fact that [18] the κ-Poincaré deformation would also induce a corresponding deformation of CPT invariance. There is a wide literature on the idea that ordinary CPT invariance might be violated by quantum-gravity effects (see, e.g., Refs. [9, 10, 19, 20, 21, 22]), but usually the relevant quantum-gravity scenarios are not developed to the point of allowing a definite prediction for the effects of violation of ordinary CPT invariance, and one is led to the use of multi-parameter phenomenological approaches. As observed in Ref. [18], in the κ-Poincaré framework it appears plausible that one should arrive at a definite prediction for all CPT violating effects parametrized only by the single length parameter $\lambda$ characterizing the underlying non-commutative “κ-Minkowski” space-time [3]

$$[x^j, t] = i\lambda \frac{x^j}{c}, \quad [x^j, x^k] = 0 \quad (1)$$

where $c$ is the conventional speed-of-light constant and $j, k = 1, 2, 3$.

Such a single-parameter description of CPT violation within the κ-Poincaré framework however still requires [18] the development of several mathematical tools on the non-commutative κ-Minkowski space-time. Since it appears likely that a long time will be necessary for these technical developments to mature, in this Letter we present a simple heuristic estimate of κ-Poincaré-induced CPT violation in the neutral-kaon system. As already emphasized in Ref. [18], the neutral-kaon system, with its delicate balance of mass/length scales, can provide a natural context for setting bounds on $\lambda$ through tests of CPT invariance. It is in fact well established [9, 19, 20, 21, 22] that observable properties of the neutral-kaon system are extremely sensitive to any deviation from ordinary CPT invariance.

Our heuristic estimate is based on the κ-deformed dispersion relation [1, 4, 5, 6, 18]

$$c^4 M^2 = \frac{\hbar^2 c^2}{\lambda^2} \left( e^{\frac{\hbar c}{\lambda E}} + e^{-\frac{\hbar c}{\lambda E}} - 2 \right) - c^2 \vec{p}^2 e^{-\frac{\hbar c}{\lambda E}} \simeq E^2 - c^2 \vec{p}^2 + \frac{\lambda c E}{2\hbar} \vec{p}^2 \quad , \quad (2)$$

where $\hbar$ is the Planck constant, $M, \vec{p}$ and $E$ respectively denote the mass, the momentum and the energy of the particle, and, since available data already imply [3]

3Note that in some of the related literature the length parameter we denote by $\lambda$ is written as $\hbar \kappa^{-1}$ (see, e.g., Refs. [1, 2, 3]) or $\hbar c E^{-1}_{QG}$ (see, e.g., Refs. [1, 2, 3]). Following Ref. [18], we adopt the notation to emphasize the forefront role played by the κ-Minkowski space-time in our considerations. In particular, a requirement of duality with respect to κ-Minkowski singles out the realization of κ-Poincaré which is considered here and in Refs. [1, 2, 3].
\( \lambda < 5 \cdot 10^{-33} m \), on the right-hand side we used the fact that in all contexts of possible interest the particles will satisfy \( E \ll \hbar c / \lambda \). Clearly, solving for \( E \) in Eq. (2) one does not recover the ordinary result (with its traditional two solutions of equal magnitude and opposite sign); instead, one finds that the two solutions \( E_+ \), \( E_- \) are given by

\[
E_\pm \simeq \frac{\lambda c}{2\hbar} p^2 \pm \sqrt{c^4 M^2 + c^2 p^2}.
\]

Since it is anyway quite natural for quantum gravity to violate CPT invariance [9, 19, 20, 21, 22] and Eq. (3) provides not exactly opposite solutions for \( E_+ \) and \( E_- \), as a heuristic argument we conjecture that in the phenomenological Hamiltonian describing neutral-kaon dynamics in presence of CPT violation within otherwise ordinary quantum mechanics

\[
H = \left( \frac{(M + \frac{1}{2} \delta M) - \frac{1}{2} i(\Gamma + \frac{1}{2} \delta \Gamma)}{M_{12} - \frac{1}{2} i\Gamma_{12}} \left( M - \frac{1}{2} \delta M \right) - \frac{1}{2} i(\Gamma - \frac{1}{2} \delta \Gamma) \right),
\]

the parameter \( \delta M \) could take a value of order

\[
|\delta M| \sim \frac{E_+ - E_-}{E_+ + E_-} 2M \simeq \frac{\lambda c}{\hbar} \sqrt{c^4 M^2 + c^2 p^2}.
\]

We also make an heuristic estimate of the parameter \( \delta \Gamma \) using the observation that a difference \( \delta M \) in the masses would induce a corresponding difference for the rate into two pions, which gives the main contribution to the width of the opposite-strangeness kaons. The amplitude is proportional to \( M^2 - M^2_\pi \) (it should vanish in the SU(3) limit [23, 24])

\[
\Gamma \sim \sqrt{M^2 - 4M^2_\pi(M^2 - M^2_\pi)^2}.
\]

and (if \( \delta \Gamma \) is exclusively due to \( \delta M \)) this leads to the estimate

\[
|\delta \Gamma| / \Gamma \sim \frac{3M^4 - 7M^2 M^2_\pi - 8M^4_\pi}{M^4 - 5M^2 M^2_\pi + 4M^4_\pi} \frac{\delta M}{M} \simeq \frac{4 \delta M}{M},
\]

which according to (5) corresponds to

\[
|\delta \Gamma| \sim \frac{4 \lambda c}{\hbar} \sqrt{c^4 M^2 + c^2 p^2}.
\]

Based on the structure of our estimate (7), the fact that \( \Gamma / M \ll 1 \) and the fact that in precision measurements on the neutral-kaon system \( \delta M \) competes with the quantity \( M_{KL} - M_{KS} \sim 3.5 \cdot 10^{-15} GeV \) and \( \delta \Gamma \) competes with \( \Gamma \sim 7.4 \cdot 10^{-15} GeV \) we conclude that the bounds on \( \lambda \) that can be derived from our estimate (8) of \( \delta \Gamma \) are

\footnote{The possibility of CPT violation outside quantum mechanics has also been considered in the literature [21], and it is not implausible [18] that it might turn out to be the proper way to describe CPT violation in the \( \kappa \)-Poincaré framework; however, for the present preliminary and heuristic analysis we shall only consider the possibility of CPT violation within quantum mechanics.}
necessarily much less stringent than the corresponding bounds obtainable from our estimate (\(\delta M\)). We restrict our attention to (\(\delta M\)) in the following.

A prominent feature of our estimate (5) is that, for given \(|\lambda|\), it predicts a \(|\delta M|\) which is an increasing function of \(|\vec{p}|\), quadratic in the non-relativistic limit and linear in the ultra-relativistic one. Therefore among experiments achieving comparable \(\delta M\) sensitivity the ones studying more energetic kaons are going to lead to more stringent bounds on \(\lambda\).

Meson factories in \(e^+e^-\) rings, such as DAΦNE \([25, 26]\), can provide strong tests of CPT \([10]\). In particular, the KLOE experiment is expected to reach \([26]\) sensitivity to values of \(\delta M\) around \(3 \cdot 10^{-18} GeV\), which, using our estimate (4) and the fact that the kaons have momenta of about \(110 MeV\) at the \(\phi\) resonance, corresponds to sensitivity to values of \(|\lambda|\) around \(6 \cdot 10^{-32} m\).

Of course, the argument for \(\kappa\)-Poincaré-induced CPT violation we have considered for the neutral-kaon system would also apply to the other analogous neutral-meson systems and, despite the lower precision reachable for \(\delta M_B\) at beauty factories \([27, 28]\), sensitivity comparable to the one of KLOE could be achieved as a result of the boost with respect to the laboratory, which is a consequence of the asymmetric setup of these factories. At BaBar \([27]\) electrons with momenta \(\sim 9 GeV\) colliding with positrons with momenta \(\sim 3 GeV\) produce B’s with momenta \(\sim 2.8 GeV\) and this, for the expected sensitivity \([29]\) to \(\delta M_B \sim 5 \cdot 10^{-15} GeV\), provides sensitivity to values of \(|\lambda|\) around \(1.4 \cdot 10^{-31} m\).

A better sensitivity is given by the CPLEAR experiment \([30]\), which studied the neutral kaons produced in the reactions

\[
\begin{align*}
pp &\rightarrow K^+\pi^-\bar{K}^0, & pp &\rightarrow K^-\pi^+K^0,
\end{align*}
\]

with the strangeness of the neutral kaon being tagged as the opposite of the corresponding charged kaon. They find \([30]\) \(|\delta M| < 7.5 \cdot 10^{-18} GeV\) and this corresponds to the bound \(|\lambda| < 1.2 \cdot 10^{-32} m\), taking into account that the CPLEAR reactions happen at rest so that the neutral-kaon energy is around \(2M_{\text{proton}}/3\), which corresponds to a momentum of order 0.4 GeV. By neglecting \(\delta\Gamma\) and considering only the contribution of the \(\pi\pi\) decay channel to the Bell-Steinberger unitarity relationship \([31]\)

\[
\left[ \frac{1}{2}(\Gamma_{K_S} + \Gamma_{K_L}) + i(M_{K_L} - M_{K_S}) \right] < K_S|K_L> = \Sigma_f A^*(K_S \rightarrow f)A(K_L \rightarrow f) \quad (10)
\]

CPLEAR obtains \([30]\) the bound \(|\delta M| < 4.4 \cdot 10^{-19} GeV\) and this corresponds to the bound \(|\lambda| < 7 \cdot 10^{-34} m\).

Very stringent bounds on \(\lambda\) can be derived from the data of the NA31 \([32]\) and the E731 \([33]\) experiments by assuming the approximate equality

\[
M_{\bar{K}^0} - M_{K^0} \simeq 2(M_{K_L} - M_{K_S})|\eta| \left( \frac{2}{3}\phi^{+-} + \frac{1}{3}\phi^{00} - \phi_0 \right)
\]

\[
\sin\phi_0 \quad (11)
\]

where the phases \(\phi^{+-}\) and \(\phi^{00}\) are defined by

\[
\frac{A(K_L \rightarrow \pi^{+(0)}\pi^{-(0)})}{A(K_S \rightarrow \pi^{+(0)}\pi^{-(0)})} = |\eta^{+-}(00)| e^{i\phi^{+-}(00)}, \quad (12)
\]
and $\phi_0$ is the superweak phase

$$\phi_0 = \tan^{-1} \frac{2(M_{K_L} - M_{K_S})}{\Gamma_{K_S} - \Gamma_{K_L}} = 43.5^\circ \pm 0.1^\circ, \quad (13)$$

which also follows from unitarity by considering only the $\pi\pi$ decay channel on the right-hand side of Eq. (10) and neglecting any other source of CPT violation. The experimental results [32]

$$\phi^{+-} = 46.9 \pm 2.2^\circ, \quad \phi^{00} = 47.1 \pm 2.8^\circ, \quad (14)$$

and [33]

$$\phi^{+-} = 42.4 \pm 1.4^\circ, \quad \phi^{00} - \phi^{+-} = -1.6 \pm 1.2^\circ, \quad (15)$$

lead to the bounds $|\delta M| < 5 \cdot 10^{-18}GeV$ and $|\delta M| < 2.5 \cdot 10^{-18}GeV$ respectively. Taking into account that the kaon beam has an average momentum of 100$GeV/c$ in NA31 and ranges from 40$GeV/c$ to 150$GeV/c$ in E731 (which we take to correspond to a reference momentum of 80$GeV/c$) one obtains the bounds $|\lambda| < 2 \cdot 10^{-35}m$ from NA31 data and

$$|\lambda| < 1.2 \cdot 10^{-35}m \quad (16)$$

from E731. This “conditional” bound on $\lambda$ (conditional in the sense that it relies on an heuristic analysis which might turn out to be unreliable when tested within a more rigorous study) obtained by analyzing the neutral-kaon system goes even beyond the Planck length and is significantly more stringent than the previous bound $\lambda < 5 \cdot 10^{-33}m$, which was obtained by comparing [1, 2, 3] data on the gamma rays we detect from distant astrophysical sources with the structure of the $\kappa$-Poincaré deformed dispersion relation. Our analysis therefore provides motivation for more rigorous analyses of the implications of $\kappa$-Poincaré for the neutral-kaon system; in fact, if our heuristic estimate was confirmed by such more rigorous studies one could conclude that precision measurements on the validity of CPT in the neutral-kaon system are a better probe of the $\kappa$-Poincaré deformation than the astrophysical observations previously considered in the literature.

This possible “competition” between astrophysical and neutral-kaon-related bounds on $\lambda$ is made possible by the fact that, as already emphasized in Ref. [18], the $\kappa$-Poincaré framework can lead to phenomenological models in which the magnitude of all new effects could be related directly and calculably to the single parameter $\lambda$, while in other quantum-gravity-motivated formalisms the evaluation of the magnitude of the effects directly from the original theory turns out to be too difficult and one can only make phenomenological models [10, 14, 21] to parametrize the magnitude of the effects, with independent parametrizations for each of the effects.

Looking beyond our relation (5), which is only a conjecture in the framework of $\kappa$-Poincaré, and also in light of the fact that $\kappa$-Poincaré is of course only one specific

The fact that the sample of the NA31 and E731 data is rather homogenous (both measuring the phases $\phi^{+-}$ and $\phi^{00}$ and using kaons with momenta of roughly the same order) can provide motivation for obtaining a limit on $\lambda$ by combining the results of the two experiments. By properly taking into account the slightly different kaon-momentum scales of the two experiments we find the combined limit $|\lambda| < 7 \cdot 10^{-36}m$. The fact that the central values of the NA31 and of the E731 data would correspond to values of $\lambda$ with opposite signs is one of the factors that contribute to rendering the combined limit significantly (roughly a factor 2) more stringent than (16).
candidate for quantum-gravity-deformed symmetries, it is worth observing that in any quantum-gravity theory predicting a non-vanishing $\delta M/M$ and predicting an effect that is linear in a characteristic length scale $\lambda$ (to be possibly identified with the Planck length) one would be able to write a relation of the type $\delta M/M \sim \lambda E^*/\hbar$, where $E^*$ carries dimensions of an energy (from now on we set $c = 1$ to simplify formulas). Our conjecture within the $\kappa$-Poincaré framework corresponds to the case $E^* = \vec{p}^2/E$, but in general $E^*$ will be given by some combination of $M$, $|\vec{p}|$ and $E$. For the simple possibilities $E^* = E$, $E^* = |\vec{p}|$, $E^* = M$, in addition to the possibility $E^* = \vec{p}^2/E$ we already considered, we report in Table 1 the limits on $\lambda$ which can be obtained (using unitarity) from the quoted experiments. Notice that, apart from the case $E^* = M$ (where the best bound is set by CPLEAR), E731 always sets the best bound on $\lambda$ and this bound is more than two orders of magnitude better than the astrophysical bound $\lambda < 5 \cdot 10^{-33} m$ (the kaons of E731 and NA31 are relativistic and therefore do not distinguish between the scenarios $E^* = E$, $E^* = |\vec{p}|$ and $E^* = \vec{p}^2/E$, as it is evidently also true for the gamma rays used for the astrophysical bound). Even the bounds set by CPLEAR are more stringent than the astrophysical bound.

In closing, we observe that the study of neutral kaons and other neutral mesons might also prove useful for the exploration of some of the open conceptual issues associated with deformations of spacetime symmetries, and in particular with $\kappa$-Poincaré. Most notably, above our estimates were obtained by considering the momentum of the particles in the laboratory frame of reference (which also made sense for the comparison with the astrophysical bounds where a corresponding assumption was made), but the frame dependence inherently associated with $\kappa$-Poincaré renders this choice quite significant. One could for example imagine that the “$\kappa$-Poincaré-preferred frame” would be set by the particular state of the quantum-gravity foam [16, 34, 35, 36] realized in the experiment. If effects of the type here considered were eventually detected it seems likely that the controlled environment of neutral-meson experiments might prove more useful than astrophysical experiments for the investigation of these delicate issues.

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Table 1: Limits on $\lambda$ (in meters) that can be obtained (using unitarity) from DAΦNE, BABAR, CPLEAR, NA31 and E731 for the cases $E^* = p^2/E$, $E^* = E$, $E^* = |p|$ and $E^* = M$.

|       | $E^* = p^2/E$ | $E^* = E$ | $E^* = |p|$ | $E^* = M$ |
|-------|---------------|-----------|------------|-----------|
| DAΦNE | $6 \cdot 10^{-32}$ | $2.8 \cdot 10^{-33}$ | $1.3 \cdot 10^{-32}$ | $2.8 \cdot 10^{-33}$ |
| BABAR | $1.4 \cdot 10^{-31}$ | $3.6 \cdot 10^{-32}$ | $7.2 \cdot 10^{-32}$ | $4 \cdot 10^{-32}$ |
| CPLEAR | $7 \cdot 10^{-34}$ | $2.8 \cdot 10^{-34}$ | $4.4 \cdot 10^{-34}$ | $3.6 \cdot 10^{-34}$ |
| NA31  | $2 \cdot 10^{-35}$ | $2 \cdot 10^{-35}$ | $2 \cdot 10^{-35}$ | $4 \cdot 10^{-33}$ |
| E731  | $1.2 \cdot 10^{-35}$ | $1.2 \cdot 10^{-35}$ | $1.2 \cdot 10^{-35}$ | $2 \cdot 10^{-33}$ |