Dirac tensor with heavy photon

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Abstract

For the large-angles hard photon emission by initial leptons in process of high energy annihilation of $e^+e^- \to$ to hadrons the Dirac tensor is obtained, taking into account the lowest order radiative corrections. The case of large-angles emission of two hard photons by initial leptons is considered. This result is being completed by the kinematics case of collinear hard photons emission as well as soft virtual and real photons and can be used for construction of Monte-Carlo generators.

Key words: tensor, photon emission

1 Introduction

The problem of precise knowledge of the cross section of annihilation $e^+e^- \to$ hadrons caused by the long staying problem of theoretical estimation of muon anomalous magnetic moment $g - 2$ [1]:

$$a^{\text{hadr}}_\mu = \left( \frac{g - 2}{2} \right)_\mu = \frac{1}{3} \left( \frac{\alpha}{\pi} \right)^2 \int_4^{\infty} \frac{ds}{s} R(s) K^{(1)}(s),$$

$$R(s) = \frac{\sigma^{e\bar{e} \to \text{hadr}}}{\sigma^{ee \to \mu\bar{\mu}}}, \quad K^{(1)}(s) = \int dx \frac{x^2(1 - x)}{x^2 + \rho(1 - x)}, \quad \rho = \frac{s}{m^2_{\mu}}.$$

Extraction of cross-section $e^+e^- \to \gamma^* \to \text{hadrons}$ from experimental data is one of the main problem of modern experimental physics. The Monte-Carlo
programs creation which takes into account the emission of real photons by
the initial leptons is the motivation of this paper.

Dirac tensor (cross-symmetry partner of Compton tensor) i.e. the bilinear
combination of the currents of hard photon emission averaged on leptons spin
states and summed on photon polarization states takes the contribution on
Born level and the ones arising from 1-loop correction. Infrared divergences
are parametrized by the introduction "photon mass" $\lambda$. In the final answer
it is removed in a usual way by adding the contribution from additional soft
photon emission.

We don't consider photon emission by the final charged particles as well as
the effects of charge-add interference of emission of virtual or real photon
emission from leptons and hadrons. So the Dirac tensor obtained in such way
is universal.

The paper is organized as follow. In the part 2 the relation of Dirac tensor with
cross section of the radiative annihilation of lepton pair to hadros is clarified.
We put the Born level expression for Dirac tensor and rerive the general form
of radiative correction to it by using the symmetry relation.

In part 3 we obtain the contribution arising from mass operator of positron
and vertex function for the case when positron and photon are on mass shell.
In the part 4 we consider the contribution from vertex function for the case
of electron on mass shell and the box-type Feynman amplitude with electron,
positron and one of photons on mass shell.

In section 5 we analyze the total result for Dirac tensor, adding the emission of
additional soft photon contributions, which provide the infrared divergences
free final result. Limiting case of almost collinear hard photon emission is
considered and some numerical estimation are given.

We put the form of hadronic tensor for several final states: $\gamma^* \rightarrow \pi^+\pi^-$, $\mu^+\mu^-$,
$\rho^+\rho^-$. In Appendixes A and B the details of calculation are presented. In
Appendix C the contribution to Dirac tensor for the case of two hard photon
emission is given.

2 General analysis

The Born level matrix element of hard photon emission by initial leptons in
process of annihilation $e^+$, $e^-$ to hadrons through the single virtual photon
intermediate state

\[ e^+(p_+) + e^-(p_-) \rightarrow \gamma^*(q) + \gamma(p_1) \rightarrow \gamma(p_1) + h(q) \]  

(2)
has a form (see Fig. 1)

\[
M = \frac{(4\pi\alpha)^{3/2}}{q^2}\bar{v}(p_+)O^{(B)}_{\rho}(p-)H_\rho(q),
\]

\[
O^{(B)}_{\rho} = \gamma_{\rho} \frac{\hat{p}_- - \hat{p}_1}{-\chi_-} \hat{e} + \frac{-\hat{p}_+ + \hat{p}_1}{-\chi_+} \gamma_{\rho},
\]

where \(\hat{e}(p_1)\) is polarization vector of the real photon. The \(H_\rho(q)\) is the current describing the conversation of virtual photon with momentum \(q\) to hadronic state. We will restrict ourselves by kinematics conditions of large-angles scattering.

\[
s = 2p_+p_-, \quad \chi_\pm = 2p_1p_\pm, \quad p_1^2 = 0, \quad p_\pm^2 = m^2,
\]

\[
s - \chi_+ - \chi_- = q^2, \quad q^2 > 0, \quad s \sim q^2 \sim \chi_+ \sim \chi_- \gg m^2.
\]

In expressions below we put \(m = 0\) everywhere except the denominators of loops integrals.

Cross section can be expressed in terms of the summed on spin states of the module of matrix element square:

\[
\sum_{\text{spin}} |M|^2 = (4\pi\alpha)^3 \frac{4B_{\rho\rho_1}H_{\rho\rho_1}}{(q^2)^2},
\]

\[
B_{\rho\rho_1} = \frac{1}{4} \text{Tr} \hat{p}_+ O_\rho \hat{p}_- \hat{O}_{\rho_1}, \quad H_{\rho\rho_1} = \sum_{\text{spin}} H_\rho(q) H^*_{\rho_1}(q).
\]

The differential cross-section can be written as:

\[
d\sigma^{e^+e^\to\gamma X} = \frac{1}{8s} \sum_{\text{spin}} |M|^2 \frac{d^3p_1}{2\omega(2\pi)^3} d\Gamma_f,
\]

\[
d\Gamma_f = (2\pi)^4 \delta^4 \left( p_+ + p_- - p_1 - \sum_f q_i \right) \prod_f \frac{d^3 q_i}{2\varepsilon_i(2\pi)^3}.
\]

For differential hard photon cross section we obtain

\[
\frac{\omega_1 d\sigma^{e^+e^\to\gamma X}}{d^3p_1d\Gamma_f} = \frac{2\alpha^3}{s(q^2)^2} H_{\rho\rho_1} B_{\rho\rho_1},
\]
where

\[
B_{\rho\rho_1} = B_g \tilde{g}_{\rho\rho_1} + B_+ \tilde{\rho}_{+\rho_1} + B_- \tilde{\rho}_{-\rho_1} + B_+(\tilde{\rho}_+ \tilde{\rho}_+_{\rho\rho_1}),
\]

\[
(p_+p_-)_{\rho\rho_1} = p_+p_- - p_+p_- = p_+p_-.
\]

The quantities with the "tilde" are defined as

\[
\tilde{g}_{\rho\rho_1} = g_{\rho\rho_1} - \frac{1}{q^2} q^2 q_{\rho\rho_1}, \quad \tilde{\rho}_{\pm\rho_1} = \rho_{\pm\rho_1} - q_{\rho_1} q_{\rho_1}. \tag{8}
\]

In the Born approximation (see Fig. 1) we have

\[
B_g^B = \frac{1}{\chi + \chi_-} (2q^2 + \chi_+^2 + \chi_-^2), \quad B_+^B = B_-^B = \frac{4q^2}{\chi + \chi_-}, \quad B_\pm^B = 0. \tag{9}
\]

For \(q^2 = 0\) we reproduce the Dirac cross-section of \(e^- e^+ \rightarrow \gamma \gamma\):

\[
\frac{d\Gamma}{dO_1} = \frac{2\alpha^2 \chi_+^2 + \chi_-^2}{s - \chi + \chi_-}. \tag{10}
\]

Below we concentrate on calculation of one-loop radiative correction to Dirac tensor.

Let us now show that in considering of the corrections one can restrict only by half of full set of Feynman diagrams for considering process (2). So we put \(O_\rho = O_\rho^- + O_\rho^+\) separating the contribution of emission from electron leg \(O_\rho^-\) and positron one \(O_\rho^+\) (see Fig. 1 for the Born case and Fig. 2 for the 1-loop corrections).

One can show that using the cyclic property of the trace as well as the mirror property

\[
\text{Tr} \hat{a}_1 \hat{a}_2 \cdots \hat{a}_{2n} = \text{Tr} \hat{a}_2 \cdots \hat{a}_1, \hat{a}_1.
\]

that the total contribution to the Dirac leptonic tensor can be written as:

\[
\text{Tr} \hat{p}_+ O_\rho^{(1)} \hat{p}_- \hat{O}_\rho^B + \text{Tr} \hat{p}_+ O_\rho^B \hat{p}_- \hat{O}_\rho^{(1)} = (1 + \Delta_{\rho\rho_1})(1 + \mathcal{P}) \text{Tr} \hat{p}_+ O_\rho^+ \hat{p}_- \hat{O}_\rho^B. \tag{11}
\]

Here the exchange operations acting as

\[
\Delta_{\rho\rho_1} F_{\rho\rho_1} = F_{\rho_1\rho}, \\
\mathcal{P} F(p_+, p_-, p_1) = F(-p_-, -p_+, -p_1), \\
\mathcal{P} F(s, q^2, \chi_+, \chi_-) = F(s, q^2, \chi_-, \chi_+) \equiv \tilde{F}. \tag{12}
\]

Here and below we imply only the real part of leptonic tensor.
3 One-loop corrections. Real photon vertex and self energy contribution

The virtual correction of lowest order is described by 8 Feynman diagrams shown on the Fig. 2.

![Feynman diagrams](image)

Fig. 2. Diagrams contributing in 1-loop level.

Let us distinguish contribution of FD Fig. (2(e-h)) to 3 classes

\[
\text{Tr } \hat{p}_+ O^+_{\rho} p_- \hat{O}^B_{\rho_1} = T_{\rho\rho_1}^{\text{box}} + T_{\rho\rho_1}^{\text{verx}} + T_{\rho\rho_1}^{\Sigma}
\]

(13)

with \(T^{\text{box}}\) and \(T^{\text{verx}}\) correspond to Fig. (2 (h,g)) and \(T^{\Sigma}\) to Fig. (2 e,f).

Consider first the contribution to matrix element arising from Feynman diagram Fig. 2 e,f.

Matrix element of FD Fig. (2 (e)) contain the mass operator of electron \(\Sigma(\hat{p})\).

In kinematics conditions of our problem \((\chi_+ \gg m^2)\) we obtain [3]:

\[
M_e = \frac{\alpha}{2\pi} \frac{3}{2} + \frac{1}{2} \left( l_+ - l_\lambda \right) \bar{v}(p_+) \hat{e} \left( \frac{-\hat{p}_+ + \hat{p}_1}{-\chi_+} \right) \gamma_\rho u(p_-),
\]

(14)

\[
l_\pm = \ln \frac{\chi_\pm}{m^2}, \quad l_\lambda = \ln \frac{m^2}{\lambda^2},
\]

where \(\lambda\) is the so-called "photon mass”.

Matrix elements of FD Fig. (2 (f)) contain the vertex function with real photon [3].

\[
M_f = \frac{\alpha}{4\pi} \bar{v}(p_+) \int \frac{d^4k}{i\pi^2} \frac{\gamma^\lambda (-\hat{p}_+ - \hat{k}) \hat{e} (-\hat{p}_+ + \hat{p}_1 - \hat{k}) \gamma^\lambda ((-\hat{p}_+ + \hat{p}_1) \gamma_\rho \frac{1}{-\chi_+} u(p_-).}
\]

(13)

We use here the notations (see eq. (A.1))
Using the relevant loop integrals, obtained in [2] (see Appendix) we have a matrix elements of FD Fig. (2(f)) which contain the vertex function with real photon [3]

\[ M_f = \frac{\alpha}{2\pi} \bar{v}(p_+) [\frac{1}{\chi_+} (l_+ - \frac{1}{2}) \hat{p}_1 \hat{e} + \hat{e} (l_\lambda - \frac{3}{2}) \frac{-\hat{p}_1 + \hat{p}_1}{-\chi_+} \gamma_\rho u(p_-)] . \] (15)

As a result we obtain the infra-red free and gauge-invariant expression:

\[ M_e + M_f = \frac{\alpha}{\pi} \Phi_+ \bar{v}(p_+) \hat{p}_1 \hat{e} \gamma_\rho u(p_-), \quad \Phi_+ = \frac{1}{2\chi_+} (l_+ - \frac{1}{2}). \] (16)

Inserting this expression to the relevant part of \( O_+^\rho \) we obtain for \( T_{\rho\rho_1}^\Sigma \)

\[ T_{\rho\rho_1}^\Sigma = -\Phi_+ \text{Tr} \hat{p}_+ \hat{p}_1 \gamma_\lambda \gamma_\rho \hat{p}_- \left[ \frac{1}{\chi_+} \gamma_\lambda (\hat{p}_- - \hat{p}_1) \gamma_\rho_1 + \frac{1}{\chi_+} \gamma_\rho_1 (-\hat{p}_+ + \hat{p}_1) \gamma_\lambda \right]. \] (17)

where we used relation \( p_\mu \rho = (p_+ + p_-)_\rho \) keeping mind the gauge invariance of hadronic tensor \( q_\mu H_{\mu\nu} = 0 \). Expression (17) leads to the form for contributions of diagrams Fig. (2(e,f)):

\[ T_{\rho\rho_1}^\Sigma = -4\Phi_+ [2p_- \rho - p_\rho_1 (\frac{q^2}{\chi_-} - 1) + 2p_- \rho_+ p_\rho_1 (\frac{s}{\chi_-} - 1) - (s - \chi_-) g_{\rho\rho_1}]. \] (18)

Applying the operation \( 1 + \Delta_{\rho\rho_1} \) and \( 1 + \mathcal{P} \) we obtain the full result:

\[ (1 + \Delta_{\rho\rho_1}) \times (1 + \mathcal{P}) T_{\rho\rho_1}^\Sigma = g_{\rho\rho_1} + B_{-\rho\rho_1} + B_{+\rho\rho_1} + B_{+\rho\rho_1}^\Sigma (\hat{p}_+ \rho - \hat{p}_- \rho_1), \] (19)

with

\[ B_g = \frac{4}{\chi_- - \chi_+} [sc - \chi_-^2 - \chi_+^2]l_s + T_g^\Sigma, \quad B_+ = -\frac{8}{\chi_- - \chi_+} [q^2 - \chi_-] l_s + T_+^\Sigma, \]

\[ B_- = -\frac{8}{\chi_- - \chi_+} [q^2 - \chi_+] l_s + T_-^\Sigma, \quad B_{+\rho\rho_1}^\Sigma = -\frac{4}{\chi_- - \chi_+} [q^2 + s] l_s + T_{+\rho\rho_1}^\Sigma, \]

\[ c = \chi_+ + \chi_- . \] (20)

Here we use the notation

\[ T_g^\Sigma = -2 \frac{s - \chi_-}{\chi_+} [1 + 2l_{sp}] - 2 \frac{s - \chi_+}{\chi_-} [1 + 2l_{sm}], \]

\[ T_-^\Sigma = \frac{4}{\chi_- - \chi_+} [q^2 - \chi_-] [1 + 2l_{sp}], \quad T_+^\Sigma = \frac{4}{\chi_- + \chi_+} [q^2 - \chi_+] [1 + 2l_{sm}]. \]
\[ T_{\Sigma}^{\chi} = \frac{2}{\chi+\chi^-}[s-\chi^-][1+2l_{sp}] + \frac{2}{\chi+\chi^-}[s-\chi^+][1+2l_{sm}], \]
\[ l_{sp} = l_s - l_+, \quad l_{sm} = l_s - l_. \] (21)

4 Vertex and box type diagram contributions

Contribution of Fig. (2(g,h)) can be written as

\[ T_{\rho\rho_1}^{\text{box}} + T_{\rho\rho_1}^{\text{vert}} = \frac{S_1}{\chi^-} + \frac{S_2}{\chi^-} - \frac{C_1}{\chi^-} - \frac{C_2}{\chi^+}, \] (22)

with

\[ S_1 = \int \frac{1}{4} \frac{d^4k}{i\pi^2} \text{Tr} \hat{B}_\rho \hat{p}_- \gamma_\eta (\hat{p}_- - \hat{p}_i) \gamma_{\rho_1}, \]
\[ S_2 = \int \frac{1}{4} \frac{d^4k}{i\pi^2} \text{Tr} \hat{B}_\rho \hat{p}_- \gamma_\eta (-\hat{p}_+ + \hat{p}_i) \gamma_{\rho_1}, \]
\[ C_1 = \int \frac{1}{4} \frac{d^4k}{i\pi^2} \text{Tr} \hat{V}_\rho \hat{p}_- \gamma_\eta (\hat{p}_- - \hat{p}_i) \gamma_{\rho_1}, \]
\[ C_2 = \int \frac{1}{4} \frac{d^4k}{i\pi^2} \text{Tr} \hat{V}_\rho \hat{p}_+ \gamma_\eta (-\hat{p}_+ + \hat{p}_i) \gamma_{\rho_1}, \]

Using the loop integrals listed in Appendix A we obtain:

\[ T_{\rho\rho_1}^{\text{box}} + T_{\rho\rho_1}^{\text{vert}} = D_g g_{\rho\rho_1} + D_- p_+ \rho p_{-\rho_1} + D_+ p_+ \rho p_{+\rho_1} + D_+ p_+ p_{-\rho_1} + D_- p_+ p_{+\rho_1}, \] (24)

Applying the interchange operator \(1+\Delta_{\rho\rho_1}\) and \(1+\mathcal{P}\), \(\tilde{D}(\chi_+, \chi_-) = \mathcal{PD}(\chi_-, \chi_+),\)
and rearranging the gauge-invariance we put it in the form:

\[ (1 + \Delta_{\rho\rho_1})(1 + \mathcal{P})(T_{\rho\rho_1}^{\text{box}} + T_{\rho\rho_1}^{\text{vert}}) = B_{g}^{V_B} g_{\rho\rho_1} + B_{-}^{V_B} p_{-\rho} \tilde{p}_{-\rho_1} + B_{+}^{V_B} p_+ \rho p_{+\rho_1} + B_{+}^{V_B} \tilde{p}_+ \tilde{p}_{+\rho_1}, \]

where

\[ B_{g}^{V_B} = 2(\tilde{D}_g + D_g), \quad B_{-}^{V_B} = 2(D_- + \tilde{D}_-), \quad B_{+}^{V_B} = 2(D_+ + \tilde{D}_+), \quad B_{+}^{V_B} = 2(D_+ + \tilde{D}_+). \] (25)

7
Here we could see by construction that

\[ B^V_B = \tilde{B}^V_B, \quad B^V_B = \tilde{B}^V_B, \quad B^V_B = \tilde{B}^V_B, \quad (26) \]

and in explicit form \( B^V_B \) are:

\[ B^V_B = \frac{4sc - 8s^2}{\chi - \chi_+} l_s + \frac{2\chi^2 + 2\chi_+^2 - 4sc + 4s^2}{\chi - \chi_+} [l_s^2 + 2(l_s - 1)l_\lambda - l_\lambda] + T^V_B, \]

\[ B^V_B = \frac{8\chi_+ - 8s}{\chi - \chi_+} l_s + \frac{8q^2}{\chi - \chi_+} [l_s^2 + 2(l_s - 1)l_\lambda - l_\lambda] + T^V_B, \]

\[ B^V_B = \frac{8\chi_+ - 8s}{\chi - \chi_+} l_s + \frac{8q^2}{\chi - \chi_+} [l_s^2 + 2(l_s - 1)l_\lambda - l_\lambda] + T^V_B, \]

\[ B^V_B = \frac{4(s + q^2)}{\chi - \chi_+} l_s + T^V_B, \]

where the expressions \( T^V_B \) contains non-leading terms.

These quantities contain the ultra-violet cut off logarithm \( L = \ln \frac{\Lambda^2}{m^2} \) which is eliminated by standard regularization procedure \([3]\) \( L \to 2l_\lambda - 9/2 \).

Collecting the leading terms which contains the large logarithm \( l_s \) and infrared one \( l_\lambda \) we obtain:

\[
(B^V_B + B^\Sigma_B)_{leading} = 2B^B_g(l_s^2 + 2(l_s - 1)L_\lambda - 3l_s),
\]

\[
(B^V_B + B^\Sigma_B)_{leading} = (B^V_B + B^\Sigma_B)_{leading} = 2B^B_g(l_s^2 + 2(l_s - 1)L_\lambda - 3l_s),
\]

\[
(B^V_B + B^\Sigma_B)_{leading} = 0.
\]

\[
(27)
\]

5 Discussion, explicit form of tensor structures.

The infrared divergences constrained in contribution of virtual photon emission canceled when takes into account the emission of additional soft photon (center-of mass of \( e^+e^- \) initial is implied)

\[
\delta_{soft} = \delta_{soft} d\sigma_B, \quad \frac{d\sigma}{d\sigma_B} = \frac{4\pi \alpha}{16\pi^3} \int \frac{d^3k}{w} (-\frac{p_-}{p_-} + \frac{p_+}{p_+})^2, \quad w < \Delta \epsilon \ll \sqrt{s}/2, \quad (28)
\]

where \( w = \sqrt{k^2 + \lambda^2} \). Using the standard integrals we obtain

\[
\delta_{soft} = \frac{\alpha}{\pi} [(l_s - 1)(l_\lambda + 2\ln \frac{\Delta E}{E}) + \frac{l_s^2}{2} - \frac{\pi^2}{3}]. \quad (29)
\]

Summing all contributions we find Dirac tensor:

\[
\]
\[ B_{\rho \rho_1} = (B^B_{g} \tilde{g}_{\rho \rho_1} + B^B_{+} \tilde{p}_+ \tilde{p}_{\rho_1} + B^B_{-} \tilde{p}_- \tilde{p}_{\rho_1}) \]
\[ \times (1 + \frac{\alpha}{\pi} (l_s - 1)(\frac{3}{2} + 2\ln \frac{\Delta E}{E}) + \frac{\alpha}{\pi} (\frac{\pi^2}{3} + \frac{3}{2})) \]
\[ - \frac{\alpha}{4\pi} [T_g \tilde{g}_{\rho \rho_1} + T_- \tilde{p}_- \tilde{p}_{\rho_1} + T_+ \tilde{p}_+ \tilde{p}_{\rho_1} + T_-(\tilde{p}_+ \tilde{p}_-)]_{\rho \rho_1}. \]  

(30)

Quantities \( T_i = T_i^\Sigma + T_i^{VB} \) are free from infrared singularities and do not contain large logarithms. Quantities \( T_i^\Sigma \) are given in (21). Quantities \( T_i^{VB} \) are given in Appendix B.

Expressions for \( T_i \) contains in addition nonphysical singularities \( \chi^\pm_2, \chi^\pm_3 \). Nevertheless one can be convinced in cancelation of terms proportional \( \chi^3_\pm \) with structure G (see eq. (B.2)) and terms \( \chi^2_\pm \) in convolution of \( T_i \). To be defined let us consider the case of small values of \( \chi_- \), \( m^2 \ll \chi_- \ll s \sim q^2 \sim \chi_+ \). It corresponds to the kinematics \( p_1 = yp_- \). In this case non-leading terms containing poles could be put in the form:

\[ T_g \tilde{g}_{\rho \rho_1} + (T_- + \tilde{g}^2T_+ - 2\tilde{g}T_- \tilde{p}_- \tilde{p}_{\rho_1}), \quad \tilde{g} = 1 - y, \quad y = \frac{\chi_+}{s} \]

(31)

and this combination contains only the lowest order pole \( \chi_-^{-1} \). The Dirac tensor in the limit \( m^2 \ll \chi_- \ll s \) has a form:

\[ B_{\rho \rho_1}^{lim} = B_{\rho \rho_1}^{B,lim} (1 + \frac{\alpha}{\pi} (l_s - 1)(\frac{3}{2} + 2\ln \frac{\Delta E}{E}) + \frac{\alpha}{\pi} (\frac{\pi^2}{2} - \frac{\pi^2}{3})) \]
\[ - \frac{\alpha}{4\pi} T_g^{lim} (\tilde{g}_{\rho \rho_1} + \frac{4\tilde{g}}{s} \tilde{p}_- \tilde{p}_{\rho_1}), \]  

(32)

where

\[ B_{\rho \rho_1}^{B,lim} = \frac{1 + \tilde{g}^2}{xy} (\tilde{g}_{\rho \rho_1} + \frac{4\tilde{g}}{s} \tilde{p}_- \tilde{p}_{\rho_1}), \quad \tilde{x} = 1 - x, \quad x = \frac{\chi_-}{s}, \]
\[ T_g^{lim} = \frac{1}{xy} \left( 16 - 18y + 10y^2 + 4(1 + \tilde{g}^2) \left( \ln \tilde{g} \ln \frac{y}{x} - \text{Li}_2 \left( \frac{1}{1 - y} \right) - \frac{\pi^2}{2} \right) \right) \]
\[ - 8\tilde{g}y \ln y + (-12 + 20y - 14\tilde{g}^2) \ln \tilde{g} \right), \]

(33)

Values of \( xyT_g^{lim} \) as a function of \( y \) and \( x = 0.1 \) are presented at Fig. 3.

Obtained formula have a power accuracy as well as we systematically omit the terms of order \( m^2/s \) compared to ones of order of unity.

Similar properties have a cross-channel-Compton tensor with one real and another virtual (space-like) photon [4], where terms of order \( m^2/s \) was taken into account.
6 Samples of hadronic tensors.

Hadronic tensor is the summed on spin states of bilinear combination of matrix elements $M_\rho M_\rho^*$, where the current $M_\rho$ describes the conversion of heavy time-like photon to some set of hadrons. For the case of creation of a pair of charged pseudoscalar mesons ($\pi^+\pi^-$, $K^+K^-$, ...) we have

$$H_{\rho\rho_1}^{p^+,p^-} = (p_+^s - p_-^s) \rho (p_+^s - p_-^s) \rho_1, \quad q = p_+^s + p_-^s.$$  

For conversion to pair of charged spin 1/2 fermions $\gamma \rightarrow \mu^+(p_+) + \mu^-(p_-)$ we have

$$H_{\mu\mu_1}^{\mu^+,\mu^-} = 4[p_+^m p_-^m + p_+^m p_-^m - \frac{q^2}{2}\rho_{\mu\mu_1}].$$  

(34)

For creation of a pair of charged vectors mesons $\rho^+\rho^-$, $K^{*+}K^{*-}$ one obtaines

$$H_{\rho\rho}^{q^+,q^-} \approx q^2(8 - 2\eta)(q_{\rho\rho_1} - \frac{q_\rho q_{\rho_1}}{q^2})$$

$$+ (q_+ - q_-) \rho (q_+ - q_-) \rho_1(3 - 5\eta + \frac{9}{4}\eta^2),$$

$$\eta = \frac{q^2}{m_\rho^2}, \quad q = q_+ + q_-.$$  

(35)

The gauge invariance requirement $H_{\rho\rho_1} q_\rho = H_{\rho\rho_1} q_\rho = 0$ is fulfilled.
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A One-loop Feynman integrals

In this section we perform the result of calculation of 4-fold integrals, associated with one-loop Feynman diagram. Here and below we imply only real part of integrals. The denominators of integrals defined as

\[(0) = k^2 - \lambda^2, \]
\[(2) = (p_+ - k)^2 - m^2 + i0 = k^2 - 2p_-k + i0, \]
\[(\bar{2}) = (-p_+ - k)^2 - m^2 + i0, \]
\[(q) = (p_1 - p_+ - k)^2 - m^2 + i0. \]  

(A.1)

The four denominator scalar integral

\[I_{02\bar{2}q} = \int \frac{1}{(0)(2)(\bar{2})(q)}, \quad dk = \frac{d^4k}{i\pi^2} \]  

(A.2)
has the form

\[ I_{02q} = \frac{1}{s\chi_+} \left[ l_q^2 - 2l_+l_s - l_s l_l + 2\text{Li}_2 \left( 1 - \frac{q^2}{s} \right) - \frac{5\pi^2}{6} \right], \quad (A.3) \]

where the logarithms was denoted in (2) and

\[ l_q = \ln \frac{q^2}{m^2}, \quad l_s = \ln \frac{s}{m^2}. \quad (A.4) \]

For the tree and two denominator scalar integrals

\[ I_{ijk} = \int \frac{d^4k}{r} \quad (A.5) \]

with \( r = (ij), (ijk), (ijkl) \) where \( i, j, k, l = (0), (2), (\bar{2}), (q) \), we have following expressions

\[ I_{02q} = -\frac{1}{2\chi_+} \left[ l_q^2 + \frac{2\pi^2}{3} \right], \quad I_{022} = \frac{1}{2s} \left[ l_s^2 + 2l_s l_l - \frac{4\pi^2}{3} \right], \]

\[ I_{22q} = -\frac{1}{2(s - q^2)} \left[ l_q^2 - l_s^2 \right], \]

\[ I_{02q} = \frac{1}{\chi_+ + q^2} \left[ l_q(l_q - l_+) + \frac{1}{2}(l_q - l_+)^2 + 2\text{Li}_2 \left( 1 + \frac{\chi_+}{q^2} \right) - \frac{3\pi^2}{2} \right]. \quad (A.6) \]

Two denominator scalar integrals are

\[ I_{02} = L + 1, \quad I_{2q} = L - l_q + 1, \quad I_{0q} = L - l_+ + 1, \]

\[ I_{02} = L + 1, \quad I_{22} = L - L_s + 1, \quad I_{2q} = L - 1. \]

The vector integrals can be defined as

\[ I^\mu_r = \int \frac{d^4k k^\mu}{r} = a^+_r q^\mu_r + a^-_r q^-_r + a^1_r p^\mu_r. \quad (A.7) \]

For the vector integrals with two denominators we have (imaginary part is neglected):

\[ a^1_{2q} = a^1_{2q} = -a^1_{2q} = -\frac{1}{2} \left( L - l_q + \frac{1}{2} \right), \quad a^1_{0q} = -a^1_{0q} = \frac{1}{2} \left( L - l_+ + \frac{1}{2} \right), \]

\[ a^-_{22} = -a^+_{22} = -\frac{1}{2} \left( L - l_s + \frac{1}{2} \right), \quad a^1_{2q} = -\frac{1}{2} a^1_{2q} = \frac{1}{2} \left( L - \frac{3}{2} \right), \]

\[ a^1_{02} = \frac{1}{2} L - \frac{1}{4}, \quad a^+_{02} = -\frac{1}{2} L + \frac{1}{4}. \quad (A.8) \]
and the coefficients for the vector integrals with three denominators are

\[ a_{02q} = \frac{1}{a} \left( \chi_+ l_{02q} + \frac{2 \chi_+ l_+ + q^2 - \chi_+ l_q}{a} \right), \quad a_{02q}^+ = -a_{02q}^1 = \frac{1}{a} (l_+ - l_q), \]

\[ a_{02q}^1 = \frac{1}{\chi_+} (l_+ - l_+ + 2), \quad a = \chi_+ + q^2, \]

\[ a_{02q}^+ = -l_{02q} - \frac{1}{\chi_+} l_+, \quad a_{02q}^2 = -a_{02q}^1 = \frac{1}{s} l_s, \]

\[ a_{22q}^1 = \frac{1}{c} (l_s - l_q), \quad a_{22q}^+ = -l_{22q} + \frac{1}{c} (l_s - l_q), \]

\[ a_{22q}^1 = \frac{s}{c} I_{22q} + \frac{1}{c} (l_q + 2) - \frac{2s}{c^2} (l_s - l_q), \quad c = s - q^2 = \chi_+ + \chi_. \quad (A.9) \]

Finally, the coefficient of the vector integral with 4 denominators has the form

\[ a^+ = \frac{\chi_+}{d} \left( \chi_+ A + \chi_- B + sC \right), \quad a^+ = \frac{\chi_-}{d} \left( \chi_+ A - \chi_- B + sC \right) \]

\[ a^- = \frac{\chi_+}{d} \left( -\chi_+ A + \chi_- B + sC \right), \quad d = 2s \chi_+ \chi_, \]

\[ A = I_{22q} - I_{02q}, \quad B = I_{02q} - I_{22q}, \]

\[ C = I_{02q} - I_{02q} - \chi_+ I_{02q}. \quad (A.10) \]

The second rank tensor integrals can be parameterized in the form

\[ I^{\mu \nu}_r = \int \frac{d^4 k}{(2\pi)^4} \frac{k^\mu k^\nu}{r} = \left[ a^q g + a^{11} p_1 p_1 + a^{++} q_+ q_+ + a^{--} q_-- q_- + a^{1+} (p_1 q_+ + q_+ p_1) \right. \]

\[ + a^{1-} (p_1 q_- + q_- p_1) + a^{+-} (q_+ q_- + q_- q_+) \right] \quad (A.11) \]

The coefficients for tensor integral with four denominators are (we suppressed the index 02q)

\[ a^{1+} = \frac{1}{\chi_+} (A_6 + A_7 - A_{10}), \quad a^{1-} = \frac{1}{s} (A_2 + A_6 - A_{10}), \]

\[ a^{11} = \frac{1}{\chi_-} (A_3 - s a^{1+}), \quad a^{1+} = \frac{1}{s} (A_3 - \chi_- a^{1+}), \]

\[ a^q = \frac{1}{2} (A_{10} - A_2 + \chi_+ a^{1+}), \quad (A.12) \]

with

\[ A_1 = a_{22q}^1 - a_{02q}^1, \quad A_2 = a_{22q}^2, \quad A_3 = a_{22q}^2 - a_{02q}^1, \]

\[ A_6 = a_{02q}^1 - a_{22q}^1, \quad A_7 = a_{02q}^1 - \chi_+ a^{1}, \quad A_8 = a_{02q}^1 - a_{02q}^1 - \chi_+ a^-. \]
\[ A_4 = a_{02q}^1 - a_{22q}^1, \quad A_9 = a_{02q}^+ - a_{22q}^+ - \chi a^+, \]
\[ A_5 = a_{02q}^- - a_{22q}^-, \quad A_{10} = I_{22q}. \]  
(A.13)

For the tensor integrals with three denominators \( I_{02q}^{\mu \nu} \) we have coefficients

\[ a_{02q}^g = \frac{1}{4} L + \frac{3}{8} - \frac{q^2}{4a} l_q - \frac{\chi}{4a} l_+, \]
\[ a_{02q}^{++} = \frac{1}{2a} \left[ \frac{\chi}{a} (l_+ - l_q) - 1 \right], \]
\[ a_{02q}^{++} = a_{02q}^{11} = \frac{1}{a} (l_q - l_+), \]
\[ a_{02q}^{--} = \frac{1}{2a} \left[ \frac{\chi^2}{2} l_q + \frac{3\chi^2}{2} l_+ - \frac{(q^2)^2 + 4q^2 \chi - 3\chi^2}{2a} l_q - \frac{q^2 + 3\chi}{2} \right]. \]  
(A.14)

The coefficients entering into the tensor integral \( I_{022}^{\mu \nu} \) are

\[ a_{022}^g = \frac{1}{4} (L - l_s) + \frac{3}{8}, \quad a_{022}^{++} = a_{022}^{--} = \frac{1}{2a} (l_+ - 1), \quad a_{022}^{+ -} = -\frac{1}{2s}, \]  
(A.15)

and the coefficients for the tensor integral \( I_{02q}^{\mu \nu} \) are

\[ a_{02q}^g = \frac{1}{4} (L - l_+) + \frac{3}{8}, \quad a_{02q}^{1+} = \frac{1}{\chi} \left( l_+ - \frac{5}{2} \right), \]
\[ a_{02q}^{11} = \frac{1}{2\chi} (-l_+ + 2), \quad a_{02q}^{++} = I_{02q} + \frac{1}{2\chi} (3l_+ - 1). \]  
(A.16)

In the case of the tensor integral \( I_{22q}^{\mu \nu} \) they have the form

\[ a_{22q}^g = \frac{1}{2} \left[ \frac{1}{2} L + \frac{3}{4} - \frac{s}{2c} l_s + \frac{q^2}{2c} l_q \right], \quad a_{22q}^{--} = -\frac{1}{2c} (l_q - l_s), \]  
\[ a_{22q}^{++} = I_{22q} + \frac{3}{2c} (l_q - l_s), \]
\[ a_{22q}^{1+} = \frac{1}{c} \left[ -\frac{1}{2} + \frac{s}{2c} l_s - \frac{s}{2c} l_q \right], \]
\[ a_{22q}^{11} = \frac{1}{c} \left[ -\frac{5}{2} - s I_{22q} + \frac{5s}{2c} l_s - \frac{2q^2 + 3s}{2c} l_q \right], \]
\[ a_{22q}^{1+} = \frac{1}{c} \left[ 4s - q^2 + s^2 I_{22q} - \frac{3s^2}{c} l_s + \frac{3s^2 - (q^2)^2 + 4sq^2}{2c} l_q \right]. \]  
(A.19)

Formula (2.15) form [2] contains a misprint.
B Explicit form of coefficients of non-leading tensor structures

\[ T^V_B = (1 + \mathcal{P}) \left[ a_0 + a_1 l_{sq} + a_2 l_{qp} + a_4 l_{sp} + a_6 l_{sq} l_{sp} + a_8 l_{sq}^2 ight. 
+ a_9 \text{Li}_2 \left( 1 - \frac{q^2}{s} \right) + a_{10} \text{Li}_2 \left( 1 + \frac{\chi_-}{q^2} \right) - 4 \frac{s q^2}{\chi_-^2} G \right] , \quad (B.1) \]

where

\[
\begin{align*}
  a_0 &= \frac{\pi^2}{3} \left[ -10 \frac{\chi_+}{\chi_-} + 22 \frac{s}{\chi_-} - 4 \frac{s^2}{\chi_+ \chi_-} \right] + 8 \frac{\chi_+}{\chi_-} - 16 \frac{s}{\chi_-} + 8 \frac{s^2}{\chi_+ \chi_-}, \\
  a_1 &= \frac{4}{c} \frac{s}{\chi_-} + 10 \frac{\chi_+}{\chi_-} - 20 \frac{s}{\chi_-} + 8 s^2, \\
  a_2 &= \frac{6}{q^2 + \chi_+}, \\
  a_4 &= -4 \frac{\chi_-}{\chi_+} - \frac{4 \chi_+}{\chi_-} + \frac{4 s}{\chi_-} + \frac{4 s}{\chi_-}, \\
  a_6 &= -4 \frac{\chi_-}{\chi_+} + \frac{8 s}{\chi_-} + \frac{4 s}{\chi_-} - \frac{8 s^2}{\chi_+ \chi_-}, \\
  a_8 &= \frac{2 \chi_+}{\chi_-} - \frac{6 s}{\chi_-} + \frac{4 s^2}{\chi_+ \chi_-}, \\
  a_9 &= -4 \frac{\chi_+}{\chi_-} + \frac{12 s}{\chi_-} - \frac{8 s^2}{\chi_+ \chi_-}, \\
  a_{10} &= \frac{\chi_+}{\chi_-} - \frac{4 s}{\chi_-} - \frac{8 s}{\chi_-} + \frac{8 s^2}{\chi_+ \chi_-}, \\
  G &= \text{Li}_2 \left( 1 - \frac{q^2}{s} \right) + \text{Li}_2 \left( 1 + \frac{\chi_-}{q^2} \right) + l_{sq} l_{sp} - \frac{1}{2} l_{sq}^2 + \frac{\pi^2}{6}. \quad (B.2)
\end{align*}
\]

Note that in the limit \( \chi_- \to 0 \) quantity \( G \) turns to zero.

\[
\begin{align*}
  T^V_+ &= \mathcal{P} T^V_B, \\
  T^V_- &= b_0 + b_1 l_{sq} + b_2 l_{qp} + b_3 l_{sp} + b_4 l_{sq} l_{sp} + b_6 l_{sq} l_{sp} + b_8 l_{sq}^2 \\
  &\quad + b_9 \text{Li}_2 \left( 1 - \frac{q^2}{s} \right) + b_{10} \text{Li}_2 \left( 1 + \frac{\chi_-}{q^2} \right) + b_{11} \text{Li}_2 \left( 1 + \frac{\chi_+}{q^2} \right) \\
  &\quad - 8 \frac{(s - \chi_+)^3}{\chi_+ \chi_-^3} G + \frac{s^2}{\chi_+^3} \left( 1 - \frac{s}{\chi_-} \right) \mathcal{P} G, \quad (B.3)
\end{align*}
\]
where

\[
\begin{align*}
 b_0 &= \frac{16}{c^2} \left(1 - s + \frac{s^2}{\chi^2} \right) - \frac{32}{\chi^+} - \frac{48}{\chi^-} + \frac{60s}{\chi^+ \chi^-} - \frac{16s^2}{\chi^+ + \chi^-}, \\
 b_1 &= \frac{16s}{\chi^+} \left(1 + \frac{s}{\chi^-} + \frac{s^2}{\chi^2} \right) - \frac{32}{\chi^+} + \frac{8s}{\chi^-} + \frac{64s^2}{\chi^+ + \chi^-} - \frac{32s^2}{\chi^+ \chi^-} + \frac{8s^3}{\chi^2 \chi^2}, \\
 b_2 &= \frac{1}{q^2 + \chi^+} \left( -8\chi^+ + 4\frac{\chi^2}{\chi^-} + 4s \right) + \frac{1}{q^2 + \chi^+} \left( 4 + 4\frac{\chi^+}{\chi^-} - \frac{8\chi^2}{\chi^2} - 4s \right), \\
 b_3 &= -\frac{4}{\chi^+} - \frac{4}{\chi^-} - \frac{4s}{\chi^2} + \frac{8s^2}{\chi^+ + \chi^-}, \\
 b_4 &= \frac{16}{\chi^+} + \frac{12}{\chi^-} + \frac{24\chi^2}{\chi^2} - \frac{12s}{\chi^2} + \frac{24s^2}{\chi^+ + \chi^-}, \\
 b_5 &= \frac{8}{\chi^+} + \frac{8}{\chi^-} - \frac{8s}{\chi^2} + \frac{8s^2}{\chi^+ + \chi^-}, \\
 b_6 &= \frac{16}{\chi^+} + \frac{16}{\chi^-} + \frac{8\chi^2}{\chi^2} - \frac{16s}{\chi^2} + \frac{16s^2}{\chi^+ + \chi^-}, \\
 b_7 &= -\frac{12}{\chi^+} - \frac{12}{\chi^-} - \frac{4\chi^2}{\chi^2} - \frac{12s}{\chi^2} + \frac{8s^2}{\chi^+ + \chi^-}, \\
 b_8 &= \frac{24}{\chi^+} + \frac{4}{\chi^-} + \frac{8}{\chi^2} - \frac{24s}{\chi^2} + \frac{16s^2}{\chi^+ + \chi^-}, \\
 b_9 &= -\frac{8}{\chi^+} - \frac{8}{\chi^-} + \frac{8s}{\chi^2} - \frac{8s^2}{\chi^+ + \chi^-}, \\
 b_{10} &= -\frac{16}{\chi^+} - \frac{16}{\chi^-} - \frac{8\chi^2}{\chi^2} - \frac{16s}{\chi^2} + \frac{16s^2}{\chi^+ + \chi^-}.
\end{align*}
\]

\[
T_{+}^{V_B} = (1 + \mathcal{P}) \left( c_0 + c_1 l_{sq} + c_3 q_{qm} + c_4 l_{sm} + c_6 l_{sq} l_{sm} + c_8 l_{sq}^2 q_{sm} + c_9 l_{iq} \left( 1 - \frac{q^2}{s} \right) + c_{10} l_{iq} \left( 1 + \frac{\chi^-}{q^2} - \frac{8s(s - \chi^+)^2}{\chi^+ \chi^-} \right) \right),
\]

where

\[
c_0 = \frac{8}{c} \left( 1 - \frac{s^2}{\chi^+ + \chi^-} \right) - \frac{4}{q^2 + \chi^-} - \frac{6}{\chi^-} + \frac{10s}{\chi^+ + \chi^-}.
\]
\[ + \frac{\pi^2}{3} \left[ -\frac{4}{\chi} - \frac{4\chi_+}{\chi^2} + \frac{4s^2}{\chi + \chi^2} \right], \]

\[ c_1 = \frac{-8s}{c^2} \left( 1 - \frac{s^2}{\chi + \chi_-} \right) + \frac{1}{c} \left( 8 - \frac{12s^2}{\chi + \chi_-} \right) \]

\[ + \frac{8}{\chi_-} - \frac{8\chi_+}{\chi_-^2} \frac{16s}{\chi_+ - \chi_-} + \frac{24s^2}{\chi^2} - \frac{24s^2}{\chi + \chi_-} + \frac{4s^3}{\chi^2 + \chi^-}, \]

\[ c_3 = \frac{-4\chi_-}{(q^2 + \chi_-)^2} + \frac{12\chi_-}{\chi_+(q^2 + \chi_-)}, \]

\[ c_4 = \frac{8\chi_-}{\chi_+^2} - \frac{8}{\chi_+} + \frac{4}{\chi_-} - \frac{16s}{\chi_-^2} \frac{8s}{\chi_+ - \chi_-} + \frac{8s^2}{\chi^2}, \]

\[ c_6 = -\frac{8\chi_-}{\chi_+^2} - \frac{8}{\chi_+} + \frac{8s^2}{\chi_-^2}, \]

\[ c_8 = \frac{4}{\chi_-} + \frac{4\chi_+}{\chi^2} - \frac{4s^2}{\chi + \chi_-}, \]

\[ c_9 = -\frac{8}{\chi_-} - \frac{8\chi_+}{\chi_-^2} + \frac{8s^2}{\chi_+^2}, \]

\[ c_{10} = \frac{8\chi_-}{\chi_+^2} + \frac{8}{\chi_+} - \frac{8s^2}{\chi + \chi_-}. \]

\section*{C Two hard photon large-angles emission by the initial leptons}

Cross section of 2 photon emission by the initial leptons masses

\[ e^+(p_+) + e^-(p_-) \rightarrow \gamma(p_1) + \gamma(p_2) + \text{hadr}(q) \quad (C.1) \]

has a form

\[ \frac{d\sigma^{2\gamma}}{d\Gamma_h} = \frac{1}{2!} \frac{\alpha^4}{2\pi^2 s} \frac{H_{\rho \rho_1} O^{(2)}_{\rho \rho_1} d^2p_1 d^2p_2}{(q^2)^2} \omega_1 \omega_2, \quad \omega_1, \omega_2 < \Delta \varepsilon, \quad (C.2) \]

where the factor 1/2! takes into account the identity of final-state hard photons. The relevant contribution to lepton tensor is

\[ Q_{\rho \rho_1}^{(2)} = \frac{1}{4} \text{Tr} \rho_+ O_{12\rho}^{\sigma \eta} p_- \bar{O}_{12\rho}^{\sigma \eta}, \quad (C.3) \]

\[ O_{12\rho}^{\sigma \eta} = \gamma_{\rho} \frac{\hat{p} - \hat{p}_1 - \hat{p}_2}{d_{-12}} \left( \gamma_{\eta} \frac{\hat{p}_1 - \hat{p}}{d_{-1}} \gamma_{\sigma} + \gamma_{\sigma} \frac{\hat{p}_2 - \hat{p}}{d_{-2}} \gamma_{\eta} \right) \]

\[ + \left( \gamma_{\eta} \frac{\hat{p}_1 + \hat{p}_2}{d_{+2}} \gamma_{\sigma} + \gamma_{\sigma} \frac{\hat{p}_1 + \hat{p}_2}{d_{+1}} \gamma_{\eta} \right) \frac{\hat{p}_2 + \hat{p}_1 + \hat{p}_2}{d_{+12}} \gamma_{\rho}, \quad (C.4) \]
\[
\frac{1}{d_{-1}d_{+2}} \gamma^\sigma (\hat{p}_+ + \hat{p}_2) \gamma_\rho (\hat{p}_- - \hat{p}_1) \gamma^\eta \\
+ \frac{1}{d_{-2}d_{+1}} \gamma^\rho (\hat{p}_- + \hat{p}_1) \gamma_\rho (\hat{p}_- - \hat{p}_2) \gamma^\sigma ,
\]

and

\[
d_{-12} = (p_- - p_1 - p_2)^2 - m^2; \\
d_{-1} = (p_- - p_1)^2 - m^2; \\
d_{-2} = (p_- - p_2)^2 - m^2; \\
d_{+12} = (-p_+ + p_1 + p_2)^2 - m^2; \\
d_{+1} = (-p_+ + p_1)^2 - m^2; \\
d_{+2} = (p_+ + p_2)^2 - m^2.
\]

Tensor \( Q^{(2)}_{\rho \rho} \) obey the gauge invariance \( Q^{(2)}_{\rho \rho} q_\rho = Q^{(2)}_{\rho \rho} q_\rho = 0 \) and can be put on the form

\[
Q^{(2)}_{\rho \rho} = A_g \tilde{g}_{\rho \rho} + [A_- \tilde{p}_- \tilde{p}_- + A_+ \tilde{p}_+ \tilde{p}_+ + A_{11} \tilde{k}_1 \tilde{k}_1 + A_{++} (\tilde{p}_+ \tilde{p}_- + \tilde{p}_+ \tilde{p}_+) + A_{--} (\tilde{p}_- \tilde{p}_- + \tilde{p}_+ \tilde{p}_-)]_{\rho \rho},
\]

coefficients \( A_i \) can be obtained in the standard way: constructing the values

\[
B_g, B_{11}, B_{++}, B_{--}, ... = Q_{\rho \rho} [g_{\rho \rho}, p_1 p_1 \rho_1, p_+ p_+ \rho_1, ...]
\]

and solving the set of 7 linear equations.