Particle jumps between optical traps in a one-dimensional (1D) optical lattice

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Abstract. We address the problem of stochastic particle transitions between stable positions in a one-dimensional (1D) periodic potential profile. With respect to experimental realization, such stable positions are represented by the optical traps formed in an evanescent standing wave. The behaviour of sub-micrometre-sized particles in this ‘optical potential energy landscape’ is analysed theoretically and experimentally, and the emphasis is put on particle jumps between neighbouring optical traps. Our theoretical model assumes overdamped stochastic motion of a particle in a finite-depth potential well. Subsequently, the mean first passage time is utilized to express the new quantity called the mean optical trap escape time (MOTET), which describes the mean time of the particle escape to a neighbouring stable position (optical trap). Theoretical predictions of the MOTET are compared with the Monte-Carlo simulations and with the experimental results for similar parameters of the potential energy profile. This comparison reveals that the properties of the optical traps (trap stiffness and depth) can be obtained from the analysis of the MOTET for the experimentally observed particle jumps only if high-speed video microscopy is used and the surface–particle distance is known.

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1. Introduction

Thermal fluctuation-induced escape over a potential barrier is one of the key mechanisms involved in many physical, chemical and biological processes. It is not surprising that this topic has attracted attention for many years and has been deeply analysed in a number of scientific papers [1]–[11]. In many instances, control of the escape rate is the key mechanism by which to influence the speed or direction of the process driven by a stochastic force. Detailed introduction to such a diverse topic in its full depth is beyond the scope of this paper; instead, we want to address a somewhat narrower problem dealing with the motion of a solid particle in a liquid in the presence of an external potential. A thermally driven colloidal particle moving in a force field is an excellent soft-matter model system to study Brownian motion. In such a system, video microscopy is sufficient to observe the system behaviour and various types of potentials can be easily generated by external electric, magnetic or optical fields [12, 13].

In this context, the methods based on advanced optical micromanipulation with microparticles, in particular, [14]–[17] offer an efficient way to create complex potential energy profiles and enable the study of many exciting phenomena, such as stochastic resonances and optical ratchets (Brownian motors). In this paper, we focus on the classical steady-state problem of particle diffusion in a symmetric periodic potential from both the theoretical and experimental sides.

In an individual optical trap (optical tweezers), analysis of the random motion of a colloidal particle is used to calibrate the particle position sensor [14, 18], quantify the trap stiffness [19]–[21] or study the non-diffusive motion of the Brownian particle [22]. Consequently, the local viscosity of the fluid surrounding the particle can be determined [23, 24] or the random particle motion can map the accessible space in the so-called thermal noise imaging [25]–[27].

If multiple optical traps of finite depth are created simultaneously close to each other, thermal activation can be sufficiently strong to drive the particle over the potential barrier between neighbouring optical traps. Therefore, the particle stays confined in a certain stable position (optical trap) only for a limited time before transiting to another trap. In general, the rate of transitions can be characterized by the so-called mean first passage time (MFPT),
which quantifies how long the particle takes to jump over the barrier. This is equal to the so-called residence time, expressing how long the particle stays in a single selected stable position. The MFPT is mathematically obtained from the solution of the Fokker–Planck equation (FPE), describing the time evolution of the probability density distribution of particle positions and velocities [2] in a certain type of potential profile. The analytical form of the MFPT is known for the one-dimensional (1D) overdamped case [3]. If the potential barrier between neighbouring wells is deep enough, the MFPT is equivalent to the so-called Kramers time [1] for a double-well potential (i.e. one having two stable particle positions separated by a finite height energy barrier) that is widely used in physical chemistry [4]. So far, theoretical predictions for the MFPT have been compared with experimental observations based on the particle motion between two single-beam optical traps created by tightly focused laser beams (optical tweezers) [28]–[32].

A double-well system realized by optical tweezers was also employed to demonstrate the so-called stochastic resonances. Here, the probability of particle transition over the potential barrier is amplified due to the periodic modulation of trap depths with the period two times longer than the Kramers time [33]–[35].

More than two simultaneously existing optical traps can be created by spatially periodic arrays of the optical intensity maxima and minima—the so-called optical lattices. Optical lattices are frequently used for optical sorting where their potential energy landscapes serve to change the motional state of particles of different properties rather than confine them [36]. Several sorting schemes have already been experimentally demonstrated, based on the optical forces [37]–[44] or the optically induced dielectrophoretic forces [45, 46]. All of them are based on different probabilities of different types of particles overcoming the potential barriers of the underlying potential energy landscape. The smaller the particle, the stronger the influence of random particle motion (Brownian motion) on particle passage over the potential barrier and the greater the probability of the particle moving in an unwanted direction leading to erroneous sorting.

To date, there are a few published papers that mention the experimental observation of particle jumps in a 1D optical lattice made by interfering incident and retro-reflected beams near the surface [47]–[50], counter-propagating evanescent waves [40, 51] or a rotational array of optical traps [52]. However, neighbouring optical traps are separated by only half the trapping wavelength, which makes the observation of the jumps challenging. A novel technique for the detection of particle position with respect to a 1D optical lattice (including the travelling lattice) [53] enabled the first detailed analyses of particle behaviour in travelling periodic optical potential landscapes [54]. For a constant velocity of lattice translation, the travelling periodic potential is equivalent to the static periodic potential tilted upwards in the direction of lattice translation. This feature enables direct comparison with the theoretical predictions of stochastic models. A tilted 1D optical lattice was also created by the interference of co-propagating non-diffracting beams of different core diameters [55]. In this configuration, the equilibrium positions are separated by distances that are an order of magnitude larger in comparison with the counter-propagating beam lattice, and the tilt of the periodic potential stems from the radiation pressure of co-propagating beams. The dynamics of the jumps of bigger particles in such a potential were recorded by a fast CCD camera and analysed [55].

In this paper, we study the dynamics of particle motion in periodic potentials by using an experimental arrangement based on wide counter-propagating evanescent beams that create an evanescent standing wave (see figure 1(a)). The intensity of such a wave changes sinusoidally along the z-axis with a period close to one half the laser wavelength (standing wave fringes).
Figure 1. (a) Configuration of the studied system. Two counter-propagating coherent laser beams are incident on the prism–water boundary under an angle bigger than the critical one, thus creating an evanescent standing wave that is depicted in the horizontal and vertical sections. (b) Calculated potential energy profile of the force field above the surface (y–z section) for a polystyrene particle of 520 nm diameter and vacuum laser wavelength 532 nm. The potential energy profile along the z-direction is given by equation (5) for the trap depth $\Delta U_0 = 10 k_B T$, and the profile in the y-direction is calculated from the force equations given in [51].

The evanescent wave intensity decays exponentially with the distance from the surface along the y-axis with a decay length roughly equal to 1.5 laser wavelengths. In the direction of the x-axis, the wave intensity profile is Gaussian with a width in tens of laser wavelengths. We select the particle diameter so that it overlaps slightly more than three intensity maxima or minima in the z-direction and, therefore, the depth of the potential well along the z-axis (across the fringes) is about one order of magnitude lower compared with the other two directions (see figure 1(b)). Consequently, the particle motion is confined within a thin layer above the surface and further restricted to the vicinity of the beam axis [51]. However, the particle is allowed to move rather freely across the periodic fringes of the standing wave. This geometry represents a very good experimental approximation to the particle stochastic motion in a 1D periodic potential profile. The motion of an individual sub-micrometre-sized particle is recorded by a fast CCD camera and its positions are carefully analysed in a way that allows for comparison to the theory based on the statistics of particle jumps between optical traps. We introduce a new quantity that is independent of the initial particle position in the optical trap—the mean optical trap escape time (MOTET)—to perform this comparison assuming particle motion in an array of optical traps with a sinusoidal potential profile. We show how the analysis of particle jumps between neighbouring potential wells can provide information about the studied opto-stochastic system.

2. Mean first passage time

There exist two main and equivalent treatments of particle stochastic motion. One of them is based on the equation of motion with an added fluctuating force (the so-called Langevin force) [2]. This method is very useful for numerical simulations of various complex and even multi-dimensional problems [2], especially for overdamped systems (i.e. a particle moving in
a liquid) with the omitted inertial term. The second method uses the FPE simplified to the form of the so-called Smoluchowski equation that describes the development of the probability density function \( P(z, t) \) in time and space for an overdamped system. Generally, only some basic and mainly 1D cases can be treated analytically. More complicated configurations also need approximative or numerical procedures \([2, 3]\).

In this paper, we focus on the problem of particle residence time in one potential well, i.e. how long the particle takes to reach the trap boundaries located at points \( z = a \) and \( z = b \). Such a quantity, \( T(t) \), is frequently called the MFPT \([2, 3]\) as well as the escape or the residence time \([28]\). It denotes the mean time needed for the particle originally (at \( t = 0 \)) placed at \( z (a \leq z \leq b) \) to leave the region between \( a \) and \( b \). The general solution for an arbitrary 1D potential profile has been given by Gardiner \([3]\). Here, we will mention only the key points of the MFPT derivation that will later be employed during the processing of experimental data.

Let us assume that at time \( t = 0 \) the particle was located at \( z \) lying within the interval \((a, b)\), i.e. \( a \leq z \leq b \). This particle moves randomly in the potential energy landscape described by a function \( U(z) \) until it reaches any of the two boundaries \( a \) or \( b \) at time \( t \), and, consequently, it is removed from the system. Let us now consider a probability \( G(z, t) \) (not the probability density) that the particle is still within the interval \((a, b)\) at time \( t \), assuming it was at \( z \) at \( t = 0 \). Such a probability is equivalent to the probability that the particle residence time \( t_r \) is bigger than \( t \).

Using \( G(z, t) \), one can derive the mean time of the particle movement before it reaches one of the trap boundaries at \( a \) or \( b \), i.e. one obtains the MFPT \( T(z) \) as follows,

\[
T(z) = \int_0^\infty dt G(z, t). (1)
\]

Furthermore, an ordinary differential equation for the MFPT can be derived using equation \((1)\) and its solution can be found \([3]\),

\[
T(z) = \frac{\gamma}{k_B T} \left[ \left( \int_a^z \frac{dy}{\psi(y)} \right) \left( \int_b^x \frac{dx}{\psi(x)} \right) \int_a^x dx' \psi(x') \right. \\
- \left. \left( \int_z^b \frac{dy}{\psi(y)} \right) \int_z^x \frac{dx}{\psi(x)} \int_a^x dx' \psi(x') \right], (2)
\]

where \( \gamma \) is the Stokes viscous drag coefficient \((\gamma = 6\pi \nu R)\), where \( \nu \) is the dynamic viscosity of the immersion liquid and \( R \) is the radius of the spherical particle, \( k_B \) is the Boltzmann constant, \( T \) is the thermodynamic temperature and

\[
\psi(z) = \exp \left( -\frac{U(z)}{k_B T} \right). \quad (3)
\]

In certain experimental realizations, e.g. those using the double-well potential \([28], [30]–[32]\), the particle can leave the considered spatial interval \((a, b)\) only through one side. In this case, the particle is reflected back to the interval \((a, b)\) if it reaches the no-exit boundary \((a)\). One then obtains a simplified relation for the MFPT \( T(z) \), as follows \([3]\),

\[
T(z) = \frac{\gamma}{k_B T} \left[ \int_z^b \frac{dx}{\psi(x)} \int_a^x dx' \psi(x') \right], \quad (4)
\]

where it is assumed that the particle is reflected back at point \( a \) and it leaves the studied interval when it reaches \( b \) \((a < b)\). For the sake of clarity, it is useful to add the position of the absorbing boundary into the notation of equation \((4)\) in the following way: \( T(z) \equiv T(z \rightarrow b) \).
3. Mean optical trap escape time (MOTET) in a one-dimensional (1D) optical lattice

Let us assume that the particle moves in a periodic potential energy profile given by

$$U(z) = -\frac{\Delta U_0}{2} \cos\left(\frac{2\pi z}{L}\right),$$

where $L$ is the potential energy period and $\Delta U_0$ is the height of the energy barrier (the optical trap depth). Such a potential profile can be experimentally obtained by the interference of two counter-propagating waves forming a standing wave. The boundaries of an individual optical trap can be naturally located at $a = -L/2 = -\pi/(2k)$ and $b = L/2 = \pi/(2k)$, where $k$ corresponds to the size of the wave vector of interfering evanescent waves. Using the potential energy profile from equation (5) in equations (2) and (3) gives the MFPT of the particle initially placed at $z$. Figure 2 shows how selection of this starting position and the height of the energy barrier $\Delta U_0$ influences the MFPT of a polystyrene particle of 520 nm diameter. However, the results of figure 2 can be extended to any particle size because the MFPT is directly proportional to the particle radius $R$ through the drag coefficient $\gamma$ (see equation (2)). The particle is placed in water and illuminated by a standing wave of vacuum wavelength 532 nm (corresponding to 400 nm in water). Therefore, the trap boundaries are equal to $a = -100$ nm and $b = 100$ nm.

Figure 2 demonstrates that the MFPT depends on the initial position of the particle within the single trap and, therefore, the MFPT cannot be used as a unique quantity describing the optical trap without additional information about the particle position. However, for practical reasons it is desirable to define a single quantity, that is independent of the particle initial position within the selected optical trap. Let us call this quantity the mean optical trap escape time (MOTET) and denote it as $\bar{T}$. In order to obtain $\bar{T}$, the values of the MFPT $T(z)$ should be
weighted over all possible starting positions $z$ within the trap with probability density $P(z, 0)$ of particle occurrence at $z$ at time $t = 0$:

$$\bar{T} = \int_a^b T(z) P(z, 0) \, dz = \int_0^\infty dt \int_a^b G(z, t) P(z, 0) \, dz \equiv \int_0^\infty G(t) \, dt. \quad (6)$$

For the limiting case of infinitely deep potential energy wells (minimum at $z = 0$), the probability density approaches the delta function $\delta(z)$: $P_1(z, 0) \approx \delta(z)$ and one obtains $\bar{T} \approx T(0)$. Vice versa, for very shallow potential energy wells the probability density is almost constant: $P_2(z, 0) \approx 1/(b - a)$. Finally, it can be intuitively assumed that $P_3(z, 0) \propto P_s(z)$, where $P_s(z)$ is the Boltzmann distribution given by

$$P_s(z) = \frac{1}{\mathcal{N}} \exp\left(-\frac{U(z)}{k_B T}\right), \quad (7)$$

where $\mathcal{N}$ denotes the normalization constant. This selection of $P_3(z, 0)$ expresses the expectation that the observed system is in the thermodynamical equilibrium state at $t = 0$. Distribution $P_3(z)$ of the particle starting positions also implies that more frequent jumps will start from more populated places at the trap centre corresponding to lower values of the potential energy. Figure 3 compares the values of the MOTET $\bar{T}$ for all three initial probability densities $P(z, 0)$ mentioned above.

The Kramers time $T_K$ is another frequently used approximation that does not depend on the initial position of the particle in the potential well and corresponds to the MOTET $[1]$. It can be derived for the double-well potential profile using the MFPT equation (4) where the particle

---

**Figure 3.** Comparison of the MOTET $\bar{T}$ (see equation (6)) for all three approximations of the starting point probability densities $P_1(z, 0) - P_3(z, 0)$ and different trap depths $\Delta U_0$. The dashed curve denotes the frequently used approximation based on the Kramers time $T_K$ divided by 4 (see the explanation in the text). The inset shows the relative differences between $\bar{T}$ calculated for $P_1$, $P_2$, or $P_3$, respectively.
Figure 4. Particle moving in a tilted periodic potential (black line). The particle located at point $A$ can leave the well over the boundaries at $B_+$ or $B_-$ to the neighbouring wells placed at $C_+$ or $C_-$, respectively. Times needed to reach these points are symbolically shown to help in comparison with the Kramers time $T_K$.

starts in the potential well $A$, escapes over a potential barrier $B$ between both wells and ends in the deeper potential well $C$ [3, 4],

$$T_K = \frac{2\pi \gamma}{\sqrt{|U''(A)|/|U''(B)|}} \exp \left( \frac{U(B) - U(A)}{k_B T} \right),$$  

(8)

where $U''(A)$ and $U''(B)$ denote the second derivative of the potential energy profile at points $A$ and $B$, respectively. The Kramers time depends exponentially on the potential energy difference between points $A$ and $B$ and on the shape of the double-well potential near $A$ and $B$. Generally, it can be said that, if the height of the barrier is kept constant, it is harder to pass a flat-top barrier than a sharply peaked one and a flat-bottomed well confines the particle longer compared with a well with a sharper profile. The Kramers time approximation fails for the second derivatives far from ‘reasonable’ and for the hallow potential barrier fulfilling $U(B) - U(A) < k_B T$, and for the second derivatives is far from ‘reasonable’. In figure 4, we compare the Kramers time with the MFPT in a periodic tilted potential. The particle is located at the potential well near $A$. It can leave to one of the neighbouring traps $C_+$ or $C_-$ over the boundary at $B_+$ or $B_-$, respectively. Using equation (8), we obtain two different Kramers times, $T_{K+}$ and $T_{K-}$, for the particle moving to the neighbouring wells, i.e. from $A$ to $C_+$ or from $A$ to $C_-$, respectively. To compare the Kramers times with the MOTET, we must unify the position where the particle ends after the transition over the barrier. As the first step, we use equation (2) and look for the times $T_+ = T(A \rightarrow B_+)$ and $T_- = T(A \rightarrow B_-)$. These times are very close to one half of the times required to reach the neighbouring potential minima [2, 5], i.e. $T_{\pm} \simeq T_{K\pm}/2$. The time $T_B$ needed to reach any one of the two boundaries $B_+$ and $B_-$ can be expressed as the sum of the rates at which the particle reaches each of the two boundaries,

$$\frac{1}{T_B} = \frac{1}{T_+} + \frac{1}{T_-}.$$  

(9)

If the potential energy profile is symmetric, i.e. there is no tilt, the times $T_{\pm}$ are equal as well as the Kramers times $T_{K\pm}$. Therefore, the MOTET $T_B$ expressed in the approximation using the Kramers time is

$$T_B = T_{\pm}/2 \simeq T_K/4.$$  

(10)

This quantity is shown in figure 3 as the dashed curve. It diverges as the trap depth $\Delta U_0$ vanishes because the second derivative of the potential in equation (8) goes to zero. However, it is almost equal to the MOTET $\bar{T}$ calculated with $P(z) = P_1$ or $P(z) = P_3$ for trap depths above $5k_B T$. 

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4. Experimental evaluation of particle jumps between neighbouring optical traps

The goal of this section is to use an experimental record of particle motion in an almost 1D periodic potential profile for the comparison of experimentally observed MOTET $\bar{T}_e$ with the theoretical prediction of the MOTET $\bar{T}_t$ based on equations (2), (3) and (6). The first three subsections explain how we obtain the necessary parameters for equations (2), (3) and (6) from the experimental record in order to determine $\bar{T}_t$. The next subsection explains how to determine the MOTET $\bar{T}_e$ directly from the experimental record. The final discussion compares the obtained $\bar{T}_e$ and $\bar{T}_t$ with the results of the Monte-Carlo simulations (MCS) and stresses the overestimates of $\bar{T}_e$ due to the data under-sampling.

4.1. The experimental setup

The experimental arrangement is shown in figure 5. This was based on two independent interfering counter-propagating Gaussian beams that were focused on the top surface of the prism. Both beams overlapped there and created a spot 100 $\mu$m long and 10 $\mu$m wide with the standing wave fringes separated by 200 nm. These fringes created an array of optical traps near the surface [40, 51]. The angle of incidence of both beams was 62°, while the critical angle for the selected geometry (see figure 5) was 61.4°. This means that the intensity of the evanescent wave created above the surface decayed exponentially into water (the decay length was $\sim$ 600 nm).
Furthermore, the change in the laser power or the polarization of the incident waves enabled us to change the trap depth $\Delta U_0$ along the $z$-direction. Therefore, we were able to observe more frequent jumps between the optical traps while we still obtained very good particle localization in the other two dimensions. The use of evanescent waves for particle manipulation has several advantages over free space systems, such as easy sample access, background signal reduction and easy integration into opto-fluidic systems.

We placed a single polystyrene particle with diameter 520 nm into the interference light structure. A fast CCD camera detected the pattern of light scattered by the particle perpendicularly to the surface with a frame rate of 5000 fps over a time interval of 58 s. The positions of the particle centre taken with respect to the standing wave nodes and antinodes were determined using our novel detection method [53]. Figure 6 shows both the optical trap boundaries as dashed lines and the particle motion and jumps between the traps. Figure 7 shows the histogram of the particle positions in the $z$ direction and also the 2D histogram in the $x$–$z$ directions. These results illustrate that the particle jumped between 15 neighbouring optical traps and occupied each trap for periods of different lengths.

4.2. Evaluation of trap depths

Based on the results of figure 6, we identified the traps where the particle stayed longer than 1 s (including all its subsequent visits of the trap). Therefore, we obtained at least 5000 particle positions within each trap. We applied the Boltzmann distribution (7) together with equation (5) to fit these data and construct the probability density of particle occurrence in each of these traps. Here, $\Delta U_{0,i}$ is the only fitted parameter for the $i$th trap and is expressed in multiples of $k_B T$ ($T = 293$ K). The values found from fitting are shown in figure 8 and the blue error bars express their confidence intervals for 95% confidence level.

The average trap depth $\Delta U_0 = (5.20 \pm 0.12) k_B T$ was obtained from the whole record on the assumption that the particle moves between identical optical traps. This is denoted in figure 8.

\[^2\] We used the MATLAB functions \texttt{nlinfit} for fitting and \texttt{nlparci} for the confidence intervals.
Figure 7. The rate of particle occurrence in different $z$ positions (a) and 2D histogram of particle positions in the plane of the glass–water interface (b). The numbers above the peaks denote the number of each trap; zero corresponds to the first trap where the particle motion started ($t = 0$ s). Both figures correspond to the record shown in figure 6.

Figure 8. Found trap depths $\Delta U_0$, of the $i$th trap with the estimated confidence interval for 95% confidence level (blue error bars) and the global trap depth $\Delta U_0$ obtained from the whole record (dashed red line with error bar). The total time that the particle spent in each trap is indicated by the green curve that corresponds to the values on the right vertical axis.

as the red dashed line. Analyses of the record also provided the total time (including all particle ‘returns’) that the particle spent in each trap. This is shown in figure 8 as the green curve. The reason why the trap depths differ is mainly the non-uniform light intensity distribution of the two counter-propagating evanescent waves along their propagation $z$-axis [54].

4.3. When does the particle start to escape after the jump?

In section 3, we defined the MOTET as the mean time $\bar{T}$ that the particle needs to leave the optical trap. We assumed several initial probability densities of particle occurrence at time $t = 0$ with the Boltzmann distribution (7) expressing best the expectation that the system has developed into the thermodynamical equilibrium state within the single trap. This MOTET is
understood as the mean time of each event during which the particle randomly located in the optical trap at time $t = 0$ (with the Boltzmann distribution) moves by the Brownian motion within the trap and leaves this trap after a certain time period.

During the experiment, however, we recorded the continuous movement of a single particle that jumped between several optical traps. Clearly, such a continuous record of particle positions, especially with respect to particle passages over the trap boundaries, does not fulfill the requirement of the equilibrium Boltzmann distribution of the starting positions before the particle started to escape from the trap. Figure 9(a) shows the probability density of particle positions taken in the frame immediately after the particle passage between the optical traps. With respect to the CCD frame rate, this time period was shorter than $200 \mu s$. Obviously, this probability distribution is far from the Boltzmann distribution and it does not correspond to any of the other two probability densities taken into account in section 3.

In order to bring the experimental conditions closer to the theoretical assumptions, we can either modify the initial probability density of the particle positions in order to correspond to the experimental data or omit some experimental data recorded just after the jump event. The second option enables the particle to move for some time, even perform another jump and reach the equilibrium state characterized by the Boltzmann probability distribution. Since we have no theoretical description of the probability distribution after the jumps (see figure 9(a)), we used the second option. We found the probability density distribution of the particle positions to be closer to the Boltzmann one when we took the data later than 25 ms after the particle entered the trap (see figure 9(b)). During this period, the considered particle (diameter 520 nm) under the studied configuration (water viscosity $10^{-3}$ Pa s$^{-1}$ at 293 K) traveled 205 nm by free diffusion [2]. This distance is comparable to the extent of one optical trap and, therefore, the selected time period should be sufficiently long to establish the stationary Boltzmann distribution of particle positions over the optical trap. The red curve in figure 9(b) is not a fit, but it corresponds to the Boltzmann distribution $P_s$ calculated with the measured average trap depth $\Delta U_0 = 5.2k_B T$ (see equations (5) and (7)). The coincidence is fairly good. In order to verify the correctness of our choice of the waiting period of 25 ms, we have also considered shorter and longer periods of the omitted parts of the particle motion. Very short values of the dwell period after the jump led to probability densities of the particle initial positions that differed from the

**Figure 9.** Probability densities of particle positions immediately (less than 200 $\mu$s) after the jumps (a) and 25 ms after the jumps (b). The red curve in (b) is not a fit but shows the Boltzmann distribution $P_s$ calculated for the measured average trap depth $\Delta U_0 = 5.2k_B T$ using equations (5) and (7).
The probability $G(t)$ of the particle being in the optical trap at time $t$ calculated from the experimentally measured residence times taken from the data in figure 6 using equation (11). Equations (13) and (15) gave $\bar{T}_e = (190 \pm 20)$ ms for the confidence level 95%. The dashed curve shows the same quantity calculated from equation (14) for the trap depth $\Delta U_0 = 5.2k_B T$.

Boltzmann distribution. Longer values did not change the results discussed in the following sections in any way. However, the number of detected jumps decreased and the uncertainty of the obtained results increased.

4.4. Experimentally obtained MOTET $\bar{T}_e$

In this section, we describe how to determine the MOTET $\bar{T}_e$ directly from the measured record without the theoretical assumptions mentioned above. The last part of equation (6) allows us to obtain the function $G(t)$ directly from the experimentally obtained residence times $t_i$, $i = 1, \ldots, N$,

$$G(t) = \frac{1}{N} \sum_{i=1}^{N} H(t_i - t),$$

where $N$ is the number of particle escapes and $H(t)$ is the Heaviside step function,

$$H(t) = \begin{cases} 
0, & t < 0, \\
1, & t \geq 0.
\end{cases}$$

We employed equation (6) and integrated equation (11) over time to obtain the experimental value of the MOTET $\bar{T}_e$,

$$\bar{T}_e = \frac{1}{N} \sum_{i=1}^{N} t_i.$$  

In this procedure, we again used the data from figure 6. However, we assumed that all traps were identical and, therefore, we considered the process as the repeated escape from the same trap. Figure 10 shows the obtained function $G(t)$ that resembles an exponential distribution. This was also pointed out by Simon and Libchaber [28] based on the fact that the jumps of

Figure 10. The probability $G(t)$ of the particle being in the optical trap at time $t$ calculated from the experimentally measured residence times taken from the data in figure 6 using equation (11). Equations (13) and (15) gave $\bar{T}_e = (190 \pm 20)$ ms for the confidence level 95%. The dashed curve shows the same quantity calculated from equation (14) for the trap depth $\Delta U_0 = 5.2k_B T$. 

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the particle between the neighbouring traps are random and have small probability, i.e. one is observing a Poisson process having exponential probability density.

This intuitive conclusion can be compared with more exact results obtained by solving the following equation for the probability \( G(z, t) \) obtained from the Fokker–Planck/Smoluchowski equation [3],

\[
\frac{\partial}{\partial t} G(z, t) = -\frac{1}{\gamma} \frac{dU(z)}{dz} \frac{\partial}{\partial z} G(z, t) + \frac{k_B T}{\gamma} \frac{\partial^2}{\partial z^2} G(z, t).
\] (14)

We used the finite elements method (COMSOL Multiphysics) to solve this equation numerically for the trap depth \( \Delta U_0 = 5.2 k_B T \). We further integrated the solution over the \( z \) variable (see the last part of equation (6)) to obtain the profile of \( G(t) \). The calculated profile is depicted in figure 10 as the dashed curve for the considered experimental parameters.

Since the probability \( G(t) \) decays exponentially, the interval estimate of \( \bar{T}_e \) is given as [56]

\[
\frac{2N \bar{T}_e}{\chi^2_{\alpha/2}(2N)} \leq \bar{T}_e \leq \frac{2N \bar{T}_e}{\chi^2_{1-\alpha/2}(2N)},
\] (15)

where \( \chi^2(N) \) is the chi-square inverse cumulative distribution function and \( 1 - \alpha \) is equal to the given confidence level. We took the measured data from figure 6 and we again considered the jumps between the traps as the repeated escape from the same trap. This gave us \( N = 272 \) data points of the residence times \( t_i \). Equations (13) and (15) gave \( \bar{T}_e = (190 \pm 20) \text{ ms} \).

We applied the same procedure as above in the determination of the MOTET \( \bar{T}_{e,i} \) in the \( i \)th trap. We analysed only those traps that were occupied at least 20 times, and this condition was satisfied by five optical traps. The results are summarized in figure 11. The blue colour corresponds to the mean value of the MOTET \( \bar{T}_{e,i} \) and its error estimate at 95% confidence level that was calculated for the \( i \)th trap using the experimental residence times and equations (13) and (15). Moreover, the red colour denotes the MOTET \( \bar{T}_e \) calculated from all the residence times in the record and it corresponds to the value obtained from figure 10. The magenta colour corresponds to the MOTET \( \bar{T}_{e,i} \) calculated using equations (2), (3) and (6) from the experimentally obtained trap depths \( \Delta U_{0,i} \). The number of residence times in each trap is shown by the green curve with the corresponding scale on the right y-axis. The errors of \( \bar{T}_{e,i} \) are rather high because only a small number of residence times were taken in the corresponding trap. The values of \( \bar{T}_{e,i} \) and \( \bar{T}_{e,i} \) differ for different traps but they are comparable for the same single trap within the considered confidence level.

The experimentally obtained value of \( \bar{T}_e \) is compared with the theoretical prediction of the MOTET \( \bar{T}_i = (147 \pm 14) \text{ ms} \) obtained using equations (2), (3) and (6) for the global trap depth \( \Delta U_0 = (5.20 \pm 0.12) k_B T \). \( \bar{T}_e \) is approximately 30% bigger than the corresponding theoretical prediction \( \bar{T}_i \). This disagreement is discussed below.

4.5. Discussion

We performed additional measurements with different settings of the optical trap parameters and determined the trap depths \( \Delta U_0 \) and the MOTETs \( \bar{T}_e \) and \( \bar{T}_i \) for the whole record. The results are summarized in table 1. The ratio of \( \bar{T}_e \) and \( \bar{T}_i \) is slightly bigger than 2 for all the measurements except the first one corresponding to the deepest potential well \( \Delta U_0 \) based on the data in figure 6. Due to the complexity of the problem, there are several possible reasons for the disagreement between the theory and experiments. These include the too low camera

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Figure 11. The MOTET $\bar{T}_{\text{e},i}$ of the particle starting at the $i$th trap (the blue colour) obtained using equation (13) and the MOTET $\bar{T}_{\text{e}}$ obtained from the whole record (the dashed red line). The values of the MOTET $\bar{T}_{\text{e},i}$ calculated using equations (2), (3) and (6) for the trap depths $\Delta U_{0,i}$ given in figure 8 are denoted by the magenta colour and the value of the MOTET $\bar{T}_{\text{e}}$ obtained from the whole record is depicted by the magenta dashed line. The blue and magenta error bars corresponding to the same trap are shifted with respect to each other in order to increase the clarity of the depiction. Moreover, the number of jumps from each trap is shown by the green curve that corresponds to the values on the right vertical axis.

Table 1. The comparison of the MOTETs $\bar{T}_{\text{e}}$ and $\bar{T}_{\text{t}}$ for measurements having different average trap depths $\Delta U_0$. $N$ denotes the number of obtained residence times corresponding to $TC$ optical traps and $f$ denotes the camera frame rate used for the measurement.

| $N$ | TC | $f$ (fps) | $\Delta U_0$ ($k_B T$) | $\bar{T}_{\text{e}}$ (ms) | $\bar{T}_{\text{t}}$ (ms) |
|-----|----|----------|-----------------|----------------|----------------|
| 272 | 15 | 5000     | 5.20 ± 0.12     | 190 ± 20       | 147 ± 14       |
| 549 | 15 | 5000     | 3.00 ± 0.10     | 64 ± 6         | 27 ± 2         |
| 647 | 30 | 4000     | 2.80 ± 0.14     | 44 ± 4         | 24 ± 2         |
| 741 | 17 | 4000     | 2.10 ± 0.13     | 32 ± 2         | 15 ± 1         |

frame rate, systematic error between the 1D theoretical model and 3D experiment, and incorrect values of the drag coefficient near the surface.

Let us focus first on the camera frame rate and consider the situation when the particle is detected inside a particular optical trap at the first camera frame. Afterwards, the particle leaves the trap but returns back into it before the next frame is recorded. Therefore, such an escape event is not detected and the measured residence time $\bar{T}_{\text{e}}$ appears longer (as in our case). This example shows that the camera frame rate could influence the measured MOTET but does not show quantitatively how strong this influence would be. To get a quantitative insight, we artificially decreased the camera frame rate by omitting several subsequent frames in the experimental records and determined the corresponding MOTET $\bar{T}_{\text{e}}$. Figures 12(a)–(d) show...
Figure 12. Dependence of the MOTET $\bar{T}_e$ on the camera frame rate. The MOTETs $\bar{T}_e$ are obtained from the artificially decreased frame rates (blue) taken from the experimental records. The green colour denotes the MOTET $\bar{T}_{eMCS}$ obtained from the analyses of the MCS of the particle motion in the potential profile described by equation (5). The values of $\bar{T}_t$ are shown by the red lines. Four trap depths were considered: $\Delta U_0 = 5.2k_BT$ (the output laser power $P = 4.5 \text{ W}$) (a), $\Delta U_0 = 3.0k_BT$ ($P = 3.5 \text{ W}$) (b), $\Delta U_0 = 2.8k_BT$ ($P = 4.0 \text{ W}$) (c) and $\Delta U_0 = 2.1k_BT$ ($P = 1.5 \text{ W}$) (d).

these results in blue for four different trap depths. As the frame rate decreases, the obtained value of $\bar{T}_e$ increases. Consequently, the total number of detected jumps decreases and, therefore, the error of $\bar{T}_e$ strongly increases.

In order to understand this trend in more depth, we used the MCS [57] of the stochastic particle motion in the potential profile described by equation (5). The obtained particle positions were analysed using the same procedure as for the experimentally measured data. The particle motion was evolved in $5 \mu$s time steps but the presence of the particle in the trap was only verified in the intervals corresponding to the frame rates in figure 12. We obtained 250 000 different residence times and we used equation (13) to get the MOTET $\bar{T}_{eMCS}$. The results of these simulations are shown in figures 12(a)–(d) in green. They indicate a significant influence of the camera frame rates on $\bar{T}_{eMCS}$ and $\bar{T}_e$. Clear findings for all four cases are that both $\bar{T}_{eMCS}$ and $\bar{T}_e$ MOTETs follow the same trend with increasing camera frame rate and keep almost constant ratios $\bar{T}_e/\bar{T}_{eMCS}$ between them. These ratios seem to depend on the depths of potential wells $\Delta U_0$ and we found their average values to be equal to 1.0, 1.8, 1.3 and 1.5 for the data shown in figures 12(a)–(d), respectively. At high camera frame rates (higher than we could reach experimentally), $\bar{T}_{eMCS}$ always coincides with $\bar{T}_t$. This proves that different methods of determination of $\bar{T}_{eMCS}$ and $\bar{T}_t$ give the same results if the same physical effects are considered. The only unanswered question is why $\bar{T}_{eMCS}$ and $\bar{T}_e$ coincide only for the
deepest traps having $\Delta U_0 = 5.2 k_B T$. Therefore, we extended the MCS to two dimensions and we included the particle motion perpendicularly to the surface. The obtained $\bar{T}_{\text{eMCS2D}}$ values differed from $\bar{T}_{\text{eMCS}}$ by not more than 5%. Therefore, we do not think that particle motion in more dimensions is the source of the systematic error. The results from this paragraph suggest that very high frame rates are needed in order to exploit the analysis of the dynamics of particle jumps between optical traps for determining the properties of traps. Since these frame rates were not accessible in our experiments, we cannot compare the simulation results directly with the experimental $\bar{T}_{\text{e}}$. However, extrapolation of the experimental results to higher frame rates supports this conclusion. To avoid this under-sampling in future experimental setups, particle positions must be recorded with higher frequency—for example, using a quadrant position detector (QPD). However, the QPD detects particle position precisely only in a limited spatial volume and, therefore, the particle dynamics over a limited number of the optical traps could be detected. Another possibility is to use the immersion medium with a higher viscosity. This approach would slow down the particle stochastic motion and CCD-based video microscopy could be employed. However, much longer data acquisition times would be needed (of the order of hours) and this increases the demands on the stability of the experimental system.

Up to now, only the bulk drag coefficient $\gamma$ has entered the theoretical equations (2), (3) and (6) and has been used in the MCS. These equations show that the MOTET $\bar{T}_t$ depends linearly on this coefficient. However, if the particle moves close to the surface, hydrodynamic corrections for the surface proximity should be applied. Here the drag coefficient is not constant but depends on the distance $y$ between the surface and the particle centre [58], $\gamma(y) = 6\pi \nu R \gamma_F(y)$, where $\nu$ is the temperature-dependent dynamical viscosity of water, $R$ is the radius of the particle and $\gamma_F(y)$ is the hydrodynamic correction for the surface proximity given by Faxén’s law [58],

$$\gamma_F(y) = \left[ 1 - \frac{9}{16} \left( \frac{R}{y} \right) + \frac{1}{8} \left( \frac{R}{y} \right)^3 - \frac{45}{256} \left( \frac{R}{y} \right)^4 - \frac{1}{16} \left( \frac{R}{y} \right)^5 \right]^{-1}. \quad (16)$$

The distance of the particle from the surface is usually determined using the evanescent field illumination [59]. Even though we used evanescent standing waves for trapping, we could not use such an illumination for the determination of the particle–surface distance. The reason is varying intensity of the scattered light if the particle moves along the $z$-axis. Let us consider that the smallest gap between the particle and prism distance corresponds to the range 100–150 nm [59]. In this case, $\gamma_F$ varies from 1.72 (closer to the prism) to 1.57. Comparing these values with the ratios $\bar{T}_e/\bar{T}_{\text{eMCS}} \approx \gamma_F$, one can find very good coincidence in three out of the four studied cases. To quantify more precisely the viscosity correction $\gamma_F$, weaker evanescent waves of different laser wavelengths should be used in the upcoming experiment so that its exponential intensity decrease would determine the particle–surface distance.

The third effect entering the discussion is the temperature dependence of the viscosity $\nu$. With increasing temperature, its value decreases and, consequently, $\gamma$ decreases, too. This phenomenon could explain why the coincidence between $\bar{T}_e$ and $\bar{T}_{\text{eMCS}}$ for the deepest trap (figure 12(a)) is so good, even without the consideration of surface proximity. In this case, we used higher laser power to obtain a deeper potential well, which probably caused stronger heating of the prism surface and consequently higher temperature of water. Therefore, the decrease in the viscosity due to the heating could compensate for the increase in $\gamma_F$ and, as a result, $\gamma \approx \gamma_F \nu$ did not change. One notices immediately that $\bar{T}_e/\bar{T}_{\text{eMCS}} \approx \gamma_F = 1.8$ seems to be too high for the second deepest potential well presented in figure 12(b). However, in this
case the prism was thoroughly cleaned before the measurement and, therefore, lower heating of the impurities on the prism surface is expected. More precise determination of $\gamma$ requires the measurement of the distance between the particle and the surface and also the measurement of the local temperature that influences the viscosity. We expect that these additional corrections would improve the agreement between the theory and experiments.

5. Conclusions

We used the theoretical description of the particle MFPT to study the particle jumps between the neighbouring optical traps in a 1D periodic array of optical traps (optical lattice). We introduced a new quantity—the mean optical trap escape time (MOTET)—to define this time independently of the initial position of the particle in the optical trap. We demonstrated that, for optical traps deeper than $5k_B T$, this quantity is equivalent to the well-known Kramers time if the Boltzmann distribution is assumed for the initial particle positions in the trap.

The theoretical predictions were compared with experimental observations of the particle dynamics in two counter-propagating interfering evanescent waves. It turned out that there existed a significant (up to 100%) difference between the experimental and theoretical results, which, however, decreased with increasing camera frame rate used for the particle position sampling. These findings were corroborated by the Monte-Carlo dynamics simulations of the experiment that led to the agreement with the theoretical predictions for the frame rates in hundreds of thousands of frames per second. Such high frame rates were not accessible in the experiments. Despite identical trends in the dependence of the experimental and simulated values of the MOTET on the camera frame rate, quantitative agreement between the two values was only obtained for the deepest studied optical trap. For three out of four data sets having the lower depths of the optical traps, the agreement between the experiments and simulations was improved significantly by including the corrections of the Stokes drag coefficient for surface proximity by Faxén’s formula. In the last mismatched data set, we speculate that the water viscosity could be lowered by the increase in water temperature due to high-intensity laser heating. This decrease in water viscosity could compensate the hydrodynamic correction factor and, therefore, the Stokes drag coefficient would not change significantly. As a result, a higher discrepancy between the measurement and simulation would be expected.

Based on all these results, we conclude that the parameters of the optical trap (its potential depth) can be determined from studying the dynamics of the particle jumps between neighbouring optical traps only for very high camera frame rates accompanied by detection of the particle–surface distance. Either faster particle detection methods or a more viscous surrounding medium, together with a more stable experimental arrangement, should be used to fully confirm this conclusion experimentally.

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References

[1] Kramers H A 1940 Physica 7 284–304
[2] Risken H 1996 The Fokker–Planck Equation (Berlin: Springer)
[3] Gardiner C W 2004 Handbook of Stochastic Methods (Berlin: Springer)
[4] van Kampen N 2003 Stochastic Processes in Physics and Chemistry (Amsterdam: North-Holland)
[5] Hänggi P, Talkner P and Borkovec M 1990 Rev. Mod. Phys. 62 251–341
[6] Metzler R and Klafter J 2000 Phys. Rep. 339 1–77
[7] Reimann P 2002 Phys. Rep. 361 57–265
[8] Hänggi P and Marchesoni F 2005 Chaos 15 026101
[9] Pollak E and Talkner P 2005 Chaos 15 026116
[10] Selmeczi D, Tolić-Nørrelykke S F, Schäffer E, Hagedorn P H, Mosler S, Berg-Sørensen K, Larsen N B and Flyvbjerg H 2007 Acta Phys. Pol. B 38 2407–31
[11] Hänggi P and Marchesoni F 2009 Rev. Mod. Phys. 81 387–442
[12] Lowen H 2001 J. Phys. Condens. Matter 13 R415–32
[13] Babič D, Schmitt C and Bechinger C 2005 Chaos 15 026114
[14] Neuman K C and Block S M 2004 Rev. Sci. Instrum. 75 2787–809
[15] Grier D G 2003 Nature 424 810–6
[16] Dholakia K, Reece P and Gu M 2008 Chem. Soc. Rev. 35 42–55
[17] Jonáš A and Zemánek P 2008 Electrophoresis 29 4813–51
[18] Bussi M, Pesce G and Sasso A 2004 Opt. Commun. 230 357–68
[19] Florin E L, Pralle A, Stelzer E H K and Hörber J K H 1998 Appl. Phys. A 66 75–8
[20] Tolić-Nørrelykke I M, Berg-Sørensen K and Flyvbjerg H 2004 Comput. Phys. Commun. 159 225–40
[21] Berg-Sørensen K and Flyvbjerg H 2004 Rev. Sci. Instrum. 75 594–612
[22] Lukic B, Jeney S, Tischer C, Kulik A, Forro L and Florin E 2005 Phys. Rev. Lett. 95 160601
[23] Pralle A, Florin E L, Stelzer E H K and Hörber J K H 1998 Appl. Phys. A 66 71–3
[24] Pesce G, Sasso A and Fusco S 2005 Rev. Sci. Instrum. 76 115105
[25] Tischer C, Altmann S, Fisinger S, Hörber J, Stelzer E and Florin E L 2001 Appl. Phys. Lett. 79 3878–80
[26] Rohrbach A, Tischer C, Neumayer D, Florin E L and Stelzer E H K 2004 Rev. Sci. Instrum. 75 2197–210
[27] Jeney S, Stelzer E, Grubmüller H and Florin E L 2004 Chem. Phys. Chem. 5 1150–8
[28] Simon A and Libchaber A 1992 Phys. Rev. Lett. 68 3375–8
[29] McCann L I, Dykman M and Golding B 1999 Nature 402 785–7
[30] Seol Y, Stein D L and Visscher K 2009 Phys. Rev. Lett. 103 050601
[31] Wu D, Ghosh K, Inamdar M, Lee H J, Fraser S, Dill K and Phillips R 2009 Phys. Rev. Lett. 103 050603
[32] Hayashi Y, Ashihara S, Shimura T and Kuroda K 2008 Opt. Commun. 281 3792–8
[33] Gammaitoni L, Hänggi P, Jung P and Marchesoni F 1998 Rev. Mod. Phys. 70 223–87
[34] Wellens T, Shatokhin V and Buchleitner A 2004 Rep. Prog. Phys. 67 45–105
[35] Babic D, Schmitt C, Poborij I and Bechinger C 2004 Europhys. Lett. 67 158–64
[36] Dholakia K, MacDonald M P, Zemánek P and Čižmář T 2007 Methods Cell Biol. 82 467–95
[37] MacDonald M P, Spalding G C and Dholakia K 2003 Nature 426 421–4
[38] Ladvac K, Kasza K and Grier D G 2004 Phys. Rev. E 70 010901
[39] Milne G, Rhodes D, MacDonald M and Dholakia K 2007 Opt. Lett. 32 1144–6
[40] Čižmář T, Šiler M, Šery M, Zemánek P, García-Chávez V and Dholakia K 2006 Phys. Rev. B 74 035105
[41] Ricárdez-Vargas I, Rodríguez-Montero P, Ramos-García R and Volke-Sepúlveda K 2006 Appl. Phys. Lett. 88 121116
[42] Smith R L, Spalding G C, Dholakia K and MacDonald M P 2007 J. Opt. A: Pure Appl. Opt. 9 S134–8
[43] Jákl P, Čižmář T, Šery M and Zemánek P 2008 Appl. Phys. Lett. 92 161110
[44] Hayashi Y, Ashihara S, Shimura T and Kuroda K 2009 Appl. Opt. 48 1543–52
[45] Chiou P, Otha A and Wu M C 2005 Nature 436 370–2
[46] Lin W Y, Lin Y H and Lee G B 2010 *Microfluidics Nanofluidics* **8** 217–29
[47] Jákš A, Zemánek P and Florin E L 2001 *Opt. Lett.* **26** 1466–8
[48] Jákl P, Šerý M, Ježek J, Jánáš A, Liška M and Zemánek P 2003 *J. Mod. Opt.* **50** 1615–25
[49] Fujiwara H, Takasaki H, Hotta J I and Sasaki K 2004 *Appl. Phys. Lett.* **84** 13–15
[50] Jákš P, Šerý M, Ježek J, Liška M and Zemánek P 2007 *J. Opt. A: Pure Appl. Opt.* **9** S251–5
[51] Šíler M, Čižmář T, Šerý M and Zemánek P 2006 *Adv. Phys.* **55** 157–65
[52] Evstigneev M, Zvyagolskaya O, Bleil S, Eichhorn R, Bechinger C and Reimann P 2008 *Phys. Rev. E* **77** 041107
[53] Čižmář T and Zemánek P 2007 *Opt. Express* **15** 2262–72
[54] Šíler M, Čižmář T, Jánáš A and Zemánek P 2008 *New J. Phys.* **10** 113010
[55] Čižmář T, Kollárová V, Bouchal Z and Zemánek P 2006 *New J. Phys.* **8** 43
[56] Modarres M, Kaminskiy M and Krivtsov V 1999 *Reliability Engineering and Risk Analysis: a Practical Guide* (New York: Marcel Dekker)
[57] Šíler M 2009 Behavior of microparticles in different types of optical traps *PhD thesis* Masaryk University in Brno, [http://www.isibrno.cz/omitec/](http://www.isibrno.cz/omitec/)
[58] Happel J and Brenner H 1965 *Low Reynolds Number Hydrodynamics* (Englewood-Cliffs, NJ: Prentice-Hall)
[59] Prieve D 1999 *Adv. Colloid Interface Sci.* **82** 93–125