Light-cone gauge approach to arbitrary spin fields, currents and shadows

R R Metsaev

Department of Theoretical Physics, P N Lebedev Physical Institute, Leninsky prospect 53, Moscow 119991, Russia

E-mail: metsaev@lpi.ru

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Abstract
Totally symmetric arbitrary spin fields in AdS space, conformal fields, conformal currents, and shadow fields in flat space are studied. Light-cone gauge formulations for such fields, currents and shadows are obtained. Use of the Poincaré parametrization of AdS space and ladder operators allows us to treat fields in flat and AdS spaces on an equal footing. Light-cone gauge realization of relativistic symmetries for fields, currents and shadows is also obtained. The light-cone gauge formulation for fields is obtained by using the gauge invariant Lagrangian which is presented in terms of modified de Donder divergence, while the light-cone gauge formulation for currents and shadows is obtained by using the gauge invariant approach to currents and shadows. This allows us to demonstrate explicitly how the ladder operators entering the gauge invariant formulation of fields, currents and shadows manifest themselves in the light-cone gauge formulation for fields, currents and shadows.

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1. Introduction
The light-cone gauge formalism [1] provides a systematic and self-contained way to study many problems of field and string theories. For example, we mention the construction of superfield formulation for some supersymmetric theories [2] and light-cone gauge string field theory [3, 4]. Another application of the light-cone gauge formalism is a construction of interaction vertices in the theory of higher-spin fields [5–7]. Some attractive applications of the light-cone gauge formalism to theories like QCD may be found in [8].
In this paper, we develop a light-cone gauge approach to fields in AdS space, conformal fields, conformal currents, and shadow fields in flat space\(^1\). Our approach to the light-cone gauge formulation of fields, currents and shadows can be summarized as follows.

(i) To obtain a light-cone gauge formalism for fields, we use the Lagrangian gauge invariant formulation of AdS fields developed in [16, 17] and the gauge invariant Lagrangian ordinary-derivative formulation of conformal fields in flat space developed in [18, 19]. Our representation of the gauge invariant Lagrangian is based on the use of ladder operators and modified de Donder divergence. The massless and massive fields in AdS\(_{d+1}\) space are considered by using the Poincaré parametrization of AdS\(_{d+1}\), while the conformal fields in flat space \(R^{d-1,1}\) are considered by using the Cartesian parametrization of \(R^{d-1,1}\). In our gauge invariant approach, we use the double-traceless tensor fields of the Lorentz algebra \(so(d−1,1)\) and the \(so(d−1,1)\) symmetries are manifestly realized both for AdS fields and conformal fields, while, in our light-cone gauge approach, we use traceless tensor fields of the \(so(d−2)\) algebra and the \(so(d−2)\) symmetries are manifestly realized both for light-cone gauge AdS fields and conformal fields. We note that it is the use of the ladder operators that allows us to develop the light-cone gauge formulation of fields in AdS space and conformal fields in flat space on an equal footing.

(ii) To develop a light-cone gauge formalism for currents and shadows, we use the gauge invariant approach to currents and shadows developed in [20–23]. This gauge invariant approach turns out to be convenient for the derivation of the light-cone gauge approach to currents and shadows.

Our paper is organized as follows. In section 2, we review the Lagrangian gauge invariant formulation of massless and massive fields in flat and AdS spaces and conformal fields in flat space. The representation for the gauge invariant Lagrangian in terms of the modified de Donder divergence discovered in [16–19] is discussed. In section 3, we review the gauge invariant approach to currents and shadows. In section 4, we describe \(so(d−1,1)\) covariant realization of the relativistic symmetries of fields, currents and shadows. In section 5, we develop the light-cone gauge formulation of fields, currents and shadows. First, we describe the field contents appearing in our light-cone gauge formulation. After this, we present our result for light-cone gauge action for fields and 2-point vertices for currents and shadows. In section 6, we discuss the light-cone gauge realization of relativistic symmetries for fields, currents and shadows. In section 7, we briefly discuss some potentially interesting applications of our results. In the Appendix, we summarize our conventions and the notation.

\(^1\) For the first time, the light-cone gauge approach to AdS fields, currents and shadows was developed in [9, 10]. Application of the light-cone approach in [9] to the various AdS field dynamical systems and the study of AdS/CFT correspondence may be found in [11–14]. Other interesting applications of the formalism in [9] to the study of AdS/CFT correspondence may be found in [15]. The advantage of the light-cone approach to AdS field dynamics we develop in this paper as compared to the one in [9] is that the light-cone gauge action obtained in this paper leads to decoupled equations of motion which are easily solved in terms of Bessel functions. The advantage of the light-cone approach to currents and shadows in this paper as compared the one in [10] is that the light-cone gauge generators of conformal algebra obtained in this paper are polynomial with respect to derivatives \(\partial_i, \partial^i\). The light-cone gauge formulation of conformal fields developed in this paper has not been discussed in earlier literature.
2. Gauge invariant approach to fields

We obtain our light-cone gauge formulation by using the Lagrangian gauge invariant approach. We start therefore with a review of the gauge invariant metric-like Lagrangian approach to totally symmetric fields in flat and AdS spaces. We discuss the dynamics of the following totally symmetric fields:

(i) massless and massive spin-$s$ fields in $R^{d-1,1}$;
(ii) massless and massive spin-$s$ fields in AdS$_{d+1}$;
(iii) conformal spin-$s$ field in $R^{d-1,1}$.

We use the following parametrizations of flat space $R^{d-1,1}$ and AdS$_{d+1}$ space,

\[ ds^2 = dx^a dx^a, \quad \text{for flat space}, \]

\[ ds^2 = \frac{1}{z^2} \left( dx^a dx^a + dz dz \right), \quad \text{for AdS space}. \]

The manifest symmetries of line elements in (2.1), (2.2) are described by the Lorentz algebra $so(d-1,1)$. It is the use of the manifest $so(d-1,1)$ symmetries that allows us, among other things, to treat fields in flat and AdS spaces on an equal footing. We now discuss the field contents.

**Field contents.** To discuss the Lagrangian description of the above-mentioned fields we use totally symmetric double-traceless tensor fields of the $so(d-1,1)$ algebra. The field contents we use are presented in table 1. To simplify the presentation of gauge invariant action we use oscillators $a^a$, $\alpha^a$, $\zeta^a$, $\upsilon^{\oplus}$, $\upsilon^{\ominus}$ and introduce the corresponding ket-vector which are also presented in table 1.

Concerning the field contents in table 1, the following remarks are in order.

(i) The Lagrangian description of massless spin-$s$ field in $R^{d-1,1}$ with the field content given in table 1 was developed in [25].

(ii) The Lagrangian description of massive spin-$s$ field in $R^{d-1,1}$ with the field content given in table 1 was discussed in [26]. Below we discuss the presentation of the Lagrangian for a massive field in terms of de Donder divergence which was obtained in [20].

(iii) The Lagrangian description of a massless spin-$s$ field in AdS$_{d+1}$ with the field content given in table 1 was discussed in [16]. For the first time, the Lagrangian description of a massless field in AdS$_{d+1}$, $d = 3$, was obtained in [27] by using the totally symmetric double-traceless tensor field of the Lorentz algebra $so(d,1)$. The $so(d-1,1)$ tensorial components of the $so(d,1)$ tensor field in [27] are not double-traceless and this tensor field is related to our gauge fields given in table 1 by the invertible transformation described in [16].

(iv) The Lagrangian description of a massive spin-$s$ field in AdS$_{d+1}$ with the field content given in table 1 was discussed in [17]. In [26], a massive field in AdS$_{d+1}$ is described by the set of fields involving double-traceless tensor fields of the Lorentz algebra $so(d,1)$.

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2 For massless and massive fields in flat space, derivation of the light-cone gauge approach from the Lagrangian BRST approach may be found in [24].

3 In earlier literature, the derivation of the Lagrangian for a massive field by using the dimensional reduction may be found in [28, 29]. The BRST approach to massive fields in flat space is studied in [30].

4 A frame-like approach to massless AdS fields was discussed in [31]. For arbitrary $d$, metric-like a gauge invariant formulation of massless AdS fields was discussed in [9, 32]. A discussion of various Lagrangian formulations of higher-spin field dynamics in terms of unconstrained fields may be found in [33–39].
Table 1. Field contents and the corresponding ket-vectors entering the gauge invariant formulation of fields, currents and shadows. Fields with \( s' \geq 4 \) are totally symmetric double-traceless tensor fields of \( so(d-1, 1) \) algebra. \( \phi^{\text{albho}}_{\alpha\phi} = 0 \), \( s' \geq 4 \), i.e., \((\alpha^s)^{a}\rangle | \phi \rangle = 0\). For notion of \( \lambda \in [s - s']['2] \) and oscillator algebra see appendix. In the table, algebraic constraints express the homogeneity properties of ket-vectors \( | \phi \rangle \) with respect to the oscillators.

| Field content | Ket-vector \( | \phi \rangle \) and algebraic constraints |
|---------------|-----------------------------------------------|
| Massless spin-\( s \) Field in \( \mathbb{R}^{d-1,1} \) | \( \phi^{ai} \), \( s' = 0, 1, \ldots, s \) | \( \sum_{i=0}^{s} \frac{z^{-s} \alpha^{a_1} \ldots \alpha^{a_s} \phi^{ai}}{s!} |0\rangle \), \( \lambda \in [s - s']['2] \) |
| Massive spin-\( s' \) Field in \( \mathbb{R}^{d-1,1} \) | \( \phi^{ai} \), \( s' = 0, 1, \ldots, s \) | \( \sum_{i=0}^{s} \frac{z^{-s} \alpha^{a_1} \ldots \alpha^{a_s} \phi^{ai} \lambda a_{i+1} \lambda a_{i+2} \lambda a_{i+3} \lambda a_{i+4} \phi^{ai}}{s!} |0\rangle \), \( \lambda \in [s - s']['2] \) |
| Massless spin-\( s' \) Field in \( AdS_{d+1} \) | \( \phi^{ai} \), \( s' = 0, 1, \ldots, s \) | \( \sum_{i=0}^{s} \frac{z^{-s} \alpha^{a_1} \ldots \alpha^{a_s} \phi^{ai} \lambda a_{i+1} \lambda a_{i+2} \lambda a_{i+3} \lambda a_{i+4} \phi^{ai}}{s!} |0\rangle \), \( \lambda \in [s - s']['2] \) |
| Massive spin-\( s' \) Field in \( AdS_{d+1} \) | \( \phi^{ai} \), \( s' = 0, 1, \ldots, s \) | \( \sum_{i=0}^{s} \frac{z^{-s} \alpha^{a_1} \ldots \alpha^{a_s} \phi^{ai} \lambda a_{i+1} \lambda a_{i+2} \lambda a_{i+3} \lambda a_{i+4} \phi^{ai}}{s!} |0\rangle \), \( \lambda \in [s - s']['2] \) |
| Conformal spin-\( s' \) Field in \( \mathbb{R}^{d-1,1} \) | \( \phi^{ai} \), \( s' = 0, 1, \ldots, s \) | \( \sum_{i=0}^{s} \frac{z^{-s} \alpha^{a_1} \ldots \alpha^{a_s} \phi^{ai} \lambda a_{i+1} \lambda a_{i+2} \lambda a_{i+3} \lambda a_{i+4} \phi^{ai}}{s!} |0\rangle \), \( \lambda \in [s - s']['2] \) |
| Canonical spin-\( s' \) Current in \( \mathbb{R}^{d-1,1} \) | \( \phi^{ai} \), \( s' = 0, 1, \ldots, s \) | \( \sum_{i=0}^{s} \frac{z^{-s} \alpha^{a_1} \ldots \alpha^{a_s} \phi^{ai} \lambda a_{i+1} \lambda a_{i+2} \lambda a_{i+3} \lambda a_{i+4} \phi^{ai}}{s!} |0\rangle \), \( \lambda \in [s - s']['2] \) |
| Canonical spin-\( s' \) Shadow in \( \mathbb{R}^{d-1,1} \) | \( \phi^{ai} \), \( s' = 0, 1, \ldots, s \) | \( \sum_{i=0}^{s} \frac{z^{-s} \alpha^{a_1} \ldots \alpha^{a_s} \phi^{ai} \lambda a_{i+1} \lambda a_{i+2} \lambda a_{i+3} \lambda a_{i+4} \phi^{ai}}{s!} |0\rangle \), \( \lambda \in [s - s']['2] \) |
| Anomalous spin-\( s' \) Current in \( \mathbb{R}^{d-1,1} \) | \( \phi^{ai} \), \( s' = 0, 1, \ldots, s \) | \( \sum_{i=0}^{s} \frac{z^{-s} \alpha^{a_1} \ldots \alpha^{a_s} \phi^{ai} \lambda a_{i+1} \lambda a_{i+2} \lambda a_{i+3} \lambda a_{i+4} \phi^{ai}}{s!} |0\rangle \), \( \lambda \in [s - s']['2] \) |
| Anomalous spin-\( s' \) Shadow in \( \mathbb{R}^{d-1,1} \) | \( \phi^{ai} \), \( s' = 0, 1, \ldots, s \) | \( \sum_{i=0}^{s} \frac{z^{-s} \alpha^{a_1} \ldots \alpha^{a_s} \phi^{ai} \lambda a_{i+1} \lambda a_{i+2} \lambda a_{i+3} \lambda a_{i+4} \phi^{ai}}{s!} |0\rangle \), \( \lambda \in [s - s']['2] \) |

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The $so(d-1,1)$ tensorial components of the tensor fields in [26] are not double-traceless. The fields in [26] are related to our fields given in table 1 by the invertible transformation described in [17].

(v) The ordinary-derivative Lagrangian description of conformal spin-$s$ field in $R^{d-1,1}$ with the field content given in table 1 was discussed in [18, 19].

**Action and Lagrangian.** The gauge invariant action for fields in flat and AdS spaces that we have found is given by

$$S = \int d^{d+1}x ~ \mathcal{L}, \quad \text{for fields in } R^{d-1,1},$$

$$S = \int d^{d+1}x ~ dz ~ \mathcal{L}, \quad \text{for fields in } AdS_{d+1},$$

(2.3)

where the Lagrangian is given by

$$\mathcal{L} = \frac{1}{2} \langle \phi | \mu (\Box - \mathcal{M}^2) | \phi \rangle + \frac{1}{2} \langle \mathcal{L}_\phi | \mathcal{L}_\phi \rangle,$$

(2.4)

$$\mathcal{L} \equiv \hat{\alpha} \partial - \frac{1}{2} \hat{\alpha} \partial \hat{\alpha}^2 - \hat{e}_1 \Pi^{1,2} + \frac{1}{2} \hat{e}_2 \hat{\alpha}^2,$$

(2.5)

$\Box \equiv \partial^a \partial^a$, $| \mathcal{L}_\phi \rangle \equiv \langle \mathcal{L}_\phi |$ and expressions $\hat{\alpha}^2$, $\hat{\alpha}^2$, $\Pi^{1,2}$ are defined in the appendix. The bra-vectors $\langle \phi |$, $| \mathcal{L}_\phi \rangle$ are defined as follows $\langle \phi | \equiv (\langle \phi |)^\dagger$, $\langle \mathcal{L}_\phi | \equiv (| \mathcal{L}_\phi \rangle)^\dagger$. Operators $\mathcal{M}^2$, $\hat{e}_1$, $\hat{e}_2$ appearing in (2.4), (2.5) are referred to as ladder operators in this paper. Explicit expressions for the ladder operators are given in table 2. From (2.4), (2.5) and table 2, we see that Lagrangians for various fields are distinguished only by the ladder operators.

The following remarks are in order.

(i) We refer to the quantity $\mathcal{L} | \phi \rangle$ as modified de Donder divergence. From table 2, we see that, only for massless field in flat space, $\hat{e}_1 = 0$, $\hat{e}_2 = 0$. This implies that, only for massless field in flat space, the $\mathcal{L} | \phi \rangle$ coincides with the standard de Donder divergence. From (2.4), (2.5), it is clear that many complicated terms contributing to the Lagrangian are collected into the $\langle \mathcal{L}_\phi | \mathcal{L}_\phi \rangle$-term. Thus, we see that it is the use of the modified de Donder divergence that allows us to simplify significantly a structure of the Lagrangian.

(ii) The representation for the Lagrangian in (2.4) is valid for all theories whose symmetry algebras involve the Poincaré algebra. The corresponding Lagrangians are distinguished by the ladder operators $\mathcal{M}^2$, $\hat{e}_1$, $\hat{e}_2$. Namely, from (2.4), we see that the dependence of the kinetic operator on the oscillators $\alpha^a$, $\hat{\alpha}^a$ and the flat derivative $\partial^a$ takes the same form for massless and massive fields in flat and AdS spaces and conformal fields in flat space. In other words, the kinetic operators for the just-mentioned fields are distinguished only by the ladder operators $\mathcal{M}^2$, $\hat{e}_1$ and $\hat{e}_2$. Thus we see that it is use of the ladder operators that allow us to treat AdS fields and conformal fields on an equal footing.

(iii) For a massive field in flat space, a representation of the gauge invariant Lagrangian in terms of modified de Donder divergence (2.4) was obtained in [20], while, for massless and massive fields in AdS space, such a representation for the Lagrangian was

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5 A frame-like approach to massive AdS fields was discussed in [40, 41]. A BRST approach to massive fields is studied in [42–44]. A discussion of massive AdS fields via various dimensional reduction techniques may be found in [45–48].

6 In the framework of AdS/CFT correspondence, a recent discussion of conformal fields may be found in [49, 50].

7 Applications of the standard de Donder–Feynman gauge condition to the problems of higher-spin gauge fields may be found in [51–53]. Recent discussion of the modified de Donder gauge may be found in [54].
Table 2. For fields, the ladder operators $\mathcal{M}_2, e_1, \bar{e}_1$ enter Lagrangian (2.4) and gauge transformation (2.6). For currents and shadows, the ladder operators $e_1, \bar{e}_1$ enter differential constraint (3.7) and gauge transformation (3.22). In the table, $m$ is a mass parameter of massive fields.

| Fields                          | $\mathcal{M}_2$ | $e_1$   | $\bar{e}_1$ |
|--------------------------------|-----------------|---------|-------------|
| Massless Spin-$s$  Field in $\mathbb{R}^{d-1,1}$ | 0              | 0       | 0           |
| Massive spin-$s$  Field in $\mathbb{R}^{d-1,1}$ | $m^2$           | $me_\zeta \bar{e}_\bar{\zeta}$ | $-me_\zeta \bar{e}_\bar{\zeta}$ |
| Massless spin-$s$  Field in $\text{AdS}_{d+1}$ | $-\partial_\zeta^2 + \frac{1}{(\zeta^2 - \frac{1}{4})}$ | $-a^2 e_\zeta T_\nu + \frac{1}{\nu} \partial_\zeta \bar{e}_\bar{\zeta}$ | $-T_\nu + \frac{1}{\nu} e_\zeta \bar{e}_\bar{\zeta}$ |
| Massive spin-$s$  Field in $\text{AdS}_{d+1}$ | $-\partial_\zeta^2 + \frac{1}{(\zeta^2 - \frac{1}{4})}$ | $-\zeta e_\nu T_{\nu-1} - a^2 e_\zeta T_\nu + \frac{1}{\nu} \partial_\zeta \bar{e}_\bar{\zeta}$ | $-T_\nu + \frac{1}{\nu} e_\zeta \bar{e}_\bar{\zeta}$ |
| Conformal spin-$s$ Field in $\mathbb{R}^{d-1,1}$ | $\nu^a_\nu \bar{v}^a_\bar{v}$ | $\zeta e_\nu \bar{v}^a_\bar{v}$ | $-\nu^a_\nu e_\zeta \bar{v}^a_\bar{v}$ |
| Canonical spin-$s$ Current in $\mathbb{R}^{d-1,1}$ | —       | $a e_\zeta$ | $e_\zeta a$ |
| Canonical spin-$s$ Shadow in $\mathbb{R}^{d-1,1}$ | —       | $a e_\zeta$ | $e_\zeta a$ |
| Anomalous spin-$s$  Current in $\mathbb{R}^{d-1,1}$ | —       | $\zeta e_\nu + a^2 e_\zeta$ | $e_\zeta \bar{e}_\bar{\zeta} - r e_\zeta a$ |
| Anomalous spin-$s$  Shadow in $\mathbb{R}^{d-1,1}$ | —       | $\zeta e_\nu + a^2 e_\zeta$ | $-e_\zeta \bar{e}_\bar{\zeta} - r e_\zeta a$ |
Table 2. (Continued.)

| Fields | $A^f$ | $e_1$ | $\tilde{e}_1$ |
|--------|-------|-------|-------|
| $e_c$  | $\left( \frac{2 s + d - 4 - N_c}{2 s + d - 4 - 2 N_c} \right)^{1/2}$ | $e_\alpha$ | $\left( \frac{2 s + d - 4 - N_\alpha}{2 s + d - 4 - 2 N_\alpha} \right)^{1/2}$ |
| $r_c$  | $\left( \frac{(s + d - 4) / s - 1}{2 (s + d - 4) / s - N_c} \right)^{1/2}$ | $r_\alpha$ | $\left( \frac{(s + d - 4) / s - 1}{2 (s + d - 4) / s - N_\alpha} \right)^{1/2}$ |

\[ N_c = \zeta^2, \quad N_\alpha = \alpha \tilde{a}^2 \]

\[ \kappa \equiv \sqrt{m^2 + \left( s + \frac{d - 4}{2} \right)^2} \]
obtained in [16, 17]. For a conformal field in flat space, Lagrangian (2.4) was obtained in [18, 19].

Gauge symmetries. We now discuss gauge symmetries of the Lagrangian given in (2.4). The gauge transformation parameters involved in gauge transformations of gauge fields are presented in table 3. We note that all gauge transformation parameters are traceless totally symmetric tensors of the Lorentz algebra $so(d - 1, 1)$.

The following remarks are in order.

(i) The gauge symmetries of a massless spin-$s$ field in $R^{d-1,1}$ with the gauge transformation parameters given in table 3 were discussed in [25].

(ii) The gauge symmetries of a massive spin-$s$ field in $R^{d-1,1}$ with the set of gauge transformation parameters given in table 3 were discussed in [26].

(iii) The gauge symmetries of a massless spin-$s$ field in AdS$_{d+1}$ with the set of gauge transformation parameters given in table 3 were discussed in [16]. This is to say that in [27], the gauge symmetries of a massless field in AdS$_{d+1}$, $d = 3$, are described by a gauge transformation parameter that is a totally symmetric traceless tensor field of the Lorentz algebra $so(d, 1)$. The $so(d - 1, 1)$ tensorial components of this gauge transformation parameter are not traceless. The gauge transformation parameter in [27] is related to our gauge transformation parameters given in table 3 by the invertible transformation described in [16].

(iv) The gauge symmetries of a massive spin-$s$ field in AdS$_{d+1}$ with the set of gauge transformation parameters given in table 3 were discussed in [17]. This is to say that in [26], the gauge symmetries of a massive field in AdS$_{d+1}$ are described by gauge transformation parameters that are totally symmetric traceless tensor fields of the Lorentz algebra $so(d, 1)$. The $so(d - 1, 1)$ tensorial components of the gauge transformation parameters in [26] are not traceless. Gauge transformation parameters in [26] are related to our gauge transformation parameters given in table 3 by the invertible transformation described in [17].

(v) The gauge symmetries of the conformal spin-$s$ field in $R^{d-1,1}$ with the set of gauge transformation parameters given in table 3 were introduced in [18, 19].

Using a representation of the field contents and gauge transformation parameters in terms of the ket-vectors $|\phi\rangle$ and $|\xi\rangle$, we now note that the gauge transformations can entirely be presented in terms of these ket-vectors. This is to say that the representation for gauge transformations found in [16–19] is given by

$$\delta|\phi\rangle = G|\xi\rangle,$$

where the ladder operators $e_1$ and $\bar{e}_1$ are given in table 2. For massless and massive fields in $R^{d-1,1}$, gauge transformations in (2.6) coincide with the ones in [25, 26]. For massless and massive fields in AdS$_{d+1}$, the gauge transformations in [26, 27] can be cast into the form given in (2.6) (see [16, 17]).

Summarizing the discussion of the gauge invariant formulation of fields, we note that our gauge invariant formulation allows us to demonstrate explicitly how the ladder operators $e_1$, $\bar{e}_1$ appearing in the gauge invariant Lagrangian (2.5) manifest themselves in the gauge transformations (2.6).
**Table 3.** Gauge transformation parameters and the corresponding ket-vectors entering gauge transformation of fields, currents and shadows. Gauge transformation parameters with \( s' \geq 2 \) are totally symmetric traceless tensor fields of \( so(d - 1, 1) \) algebra, \( \xi^{a_1 \ldots a_{d-1}} = 0, s' \geq 2 \).

| Fields | Gauge transformation parameters | Ket-vector \( |\xi\rangle \) |
|--------|---------------------------------|--------------------------|
| Massless spin-s | \( \xi^{a_1 \ldots a_{d-1}} \) | \( \frac{1}{(s - 1)!} \alpha^{a_1 \ldots a_{d-1}} \xi^{a_1 \ldots a_{d-1}} |0\rangle \) |
| Field in \( R^{d-1,1} \) | \( s = 0, 1, \ldots, s - 1 \) | \( \sum_{i=0}^{s-1} \xi^{a_1 \ldots a_i} \langle a_1 \ldots a_i |0\rangle \) |
| Massive spin-s | \( \xi^{a_1 \ldots a_{d-1}} \) | \( \sum_{i=0}^{s'} \xi^{a_1 \ldots a_i} \langle a_1 \ldots a_i |0\rangle \) |
| Field in \( R^{d-1,1} \) | \( s = 0, 1, \ldots, s - 1 \) | \( \sum_{i=0}^{s-1} \alpha^{a_1 \ldots a_i} \xi^{a_1 \ldots a_i} \langle a_1 \ldots a_i |0\rangle \) |
| Massless spin-s | \( \xi^{a_1 \ldots a_{d-1}} \) | \( \sum_{i=0}^{s-1} \alpha^{a_1 \ldots a_i} \xi^{a_1 \ldots a_i} \langle a_1 \ldots a_i |0\rangle \) |
| Field in \( AdS_{d+1} \) | \( s = 0, 1, \ldots, s - 1 \) | \( \sum_{i=0}^{s-1} \alpha^{a_1 \ldots a_i} \xi^{a_1 \ldots a_i} \langle a_1 \ldots a_i |0\rangle \) |
| Massive spin-s | \( \xi^{a_1 \ldots a_{d-1}} \) | \( \sum_{i=0}^{s-1} \alpha^{a_1 \ldots a_i} \xi^{a_1 \ldots a_i} \langle a_1 \ldots a_i |0\rangle \) |
| Field in \( AdS_{d+1} \) | \( s = 0, 1, \ldots, s - 1 \) | \( \sum_{i=0}^{s-1} \alpha^{a_1 \ldots a_i} \xi^{a_1 \ldots a_i} \langle a_1 \ldots a_i |0\rangle \) |
| Conformal spin-s | \( \xi^{a_1 \ldots a_{d-1}} \) | \( \sum_{i=0}^{s-1} \alpha^{a_1 \ldots a_i} \xi^{a_1 \ldots a_i} \langle a_1 \ldots a_i |0\rangle \) |
| Field in \( R^{d-1,1} \) | \( s = 0, 1, \ldots, s - 1 \) | \( \sum_{i=0}^{s-1} \alpha^{a_1 \ldots a_i} \xi^{a_1 \ldots a_i} \langle a_1 \ldots a_i |0\rangle \) |
| Canonical spin-s | \( \xi^{a_1 \ldots a_{d-1}} \) | \( \sum_{i=0}^{s-1} \alpha^{a_1 \ldots a_i} \xi^{a_1 \ldots a_i} \langle a_1 \ldots a_i |0\rangle \) |
| Current in \( R^{d-1,1} \) | \( s = 0, 1, \ldots, s - 1 \) | \( \sum_{i=0}^{s-1} \alpha^{a_1 \ldots a_i} \xi^{a_1 \ldots a_i} \langle a_1 \ldots a_i |0\rangle \) |
| Magnetic spin-s | \( \xi^{a_1 \ldots a_{d-1}} \) | \( \sum_{i=0}^{s-1} \alpha^{a_1 \ldots a_i} \xi^{a_1 \ldots a_i} \langle a_1 \ldots a_i |0\rangle \) |
| Shadow in \( R^{d-1,1} \) | \( s = 0, 1, \ldots, s - 1 \) | \( \sum_{i=0}^{s-1} \alpha^{a_1 \ldots a_i} \xi^{a_1 \ldots a_i} \langle a_1 \ldots a_i |0\rangle \) |
| Fields                     | Gauge transformation parameters | Ket-vector $|\xi\rangle$ |
|---------------------------|---------------------------------|----------------|
| Anomalous spin-$s$ Current in $R^{d-1,1}$ | $\xi_{c,\,\lambda}^{s_{c},\,s_{c}'}$ | \[s' = 0, 1, \ldots, s - 1, \lambda \in [s - 1 - s']_2\] |
|                           | $\sum_{s' = 0}^{s - 1} \sum_{\lambda \in [s - 1 - s']_2} \xi_{c,\,\lambda}^{s_{c},\,s_{c}'} (\sum_{\zeta=0}^{\lambda} \sqrt{s'!} \left( \frac{s' - 1 - s' + \lambda}{2} \right)^{s'!} )^{\frac{s - 1 - s' + \lambda}{2} } = \xi_{c,\,\lambda}^{s_{c},\,s_{c}'} |0\rangle$ |
| Anomalous spin-$s$ Shadow in $R^{d-1,1}$ | $\xi_{s,\,\lambda}^{s_{s},\,s_{s}'}$ | \[s' = 0, 1, \ldots, s - 1, \lambda \in [s - 1 - s']_2\] |
|                           | $\sum_{s' = 0}^{s - 1} \sum_{\lambda \in [s - 1 - s']_2} \xi_{s,\,\lambda}^{s_{s},\,s_{s}'} (\sum_{\zeta=0}^{\lambda} \sqrt{s'!} \left( \frac{s' - 1 - s' + \lambda}{2} \right)^{s'!} )^{\frac{s - 1 - s' + \lambda}{2} } = \xi_{s,\,\lambda}^{s_{s},\,s_{s}'} |0\rangle$ |
3. Gauge invariant approach to currents and shadows

As we obtain our light-cone gauge approach to currents and shadows by using the gauge invariant approach developed in [20–23], we now review the gauge invariant approach in [20–23].

Let us start with a brief recall of some basic notions of CFT. Fields of CFT can be separated into two groups: currents and shadows. In this paper, a field having Lorentz algebra spin \( s, s \geq 1 \) and conformal dimension \( \Delta = s + d - 2 \) is referred to as a canonical current, while a field having Lorentz algebra spin \( s, s \geq 1 \) and conformal dimension \( \Delta > s + d - 2 \) is referred to as an anomalous current. Accordingly, a field having Lorentz algebra spin \( s, s \geq 1 \) and conformal dimension \( \Delta = 2 - s \) is referred to as a canonical shadow, while a field having Lorentz algebra spin \( s, s \geq 1 \) and conformal dimension \( \Delta < 2 - s \) is referred to as an anomalous shadow.

In [20, 21], we developed the gauge invariant formulation of the canonical currents and shadows, while, in [22, 23], we generalized our gauge invariant approach to the case of anomalous currents and shadows\(^8\). Our gauge invariant approach to currents and shadows can be summarized as follows.

(i) Starting with a field content of currents (shadows) appearing in the standard CFT, we introduce auxiliary fields and Stueckelberg fields, i.e., we extend the space of fields entering the standard CFT.

(ii) On the extended space of fields, we introduce differential constraints, gauge transformations and \( so(d, 2) \) algebra transformations. The differential constraints are required to be invariant under the gauge transformations and the \( so(d, 2) \) algebra transformations\(^9\).

(iii) The gauge symmetries and the differential constraints allow us to match our approach and the standard CFT, i.e., by gauging away the Stueckelberg fields and by solving differential constraints to exclude the auxiliary fields, we get formulation of currents and shadows in the standard CFT.

We now start our brief review of our gauge invariant approach to currents and shadows with the discussion of field contents.

Field content of spin-\( s \) canonical current and spin-\( s \) canonical shadow. To discuss the gauge invariant formulation of arbitrary spin-\( s \) canonical current and spin-\( s \) canonical shadow we use the respective totally symmetric \( so(d - 1, 1) \) Lorentz algebra tensor fields which are assumed to be double-traceless,

\[
\phi^{a_1 \ldots a_{s'}}_{\text{cur}}, \quad \phi^{a_1 \ldots a_{s'}}_{\text{sh}}, \quad s' = 0, 1, \ldots, s, \quad (3.1)
\]

\[
\phi^{a_1 b_1 a_{s'}}_{\text{cur}} = 0, \quad \phi^{a_1 b_1 a_{s'}}_{\text{sh}} = 0, \quad \text{for } s' \geq 4. \quad (3.2)
\]

\(^8\) Before our discussions in [22, 23], the gauge invariant approach to anomalous currents was studied in [55]. In [55], the gauge invariant approach was developed by using the tractor approach. A discussion of various aspects of the tractor approach may be found in [56].

\(^9\) For a discussion of the differential constraints for conformal currents in the framework of standard CFT see, e.g., [57].
Conformal dimensions of the fields in (3.1) are given by

\[ \Delta(\phi^{\bar{a}_{1}...\bar{a}_{s'}}_{\text{cur}}) = s' + d - 2, \quad \Delta(\phi^{a_{1}...a_{s'}}_{\text{sh}}) = 2 - s'. \]  

(3.3)

**Field content of spin-s anomalous current and spin-s anomalous shadow.** To discuss the gauge invariant formulation of arbitrary spin-s anomalous current and spin-s anomalous shadow we use the respective totally symmetric \( so(d - 1, 1) \) Lorentz algebra tensor fields which are assumed to be double-traceless,

\[ \phi^{\bar{a}_{1}...\bar{a}_{s'}}_{\text{cur}, \lambda}, \quad \phi^{a_{1}...a_{s'}}_{\text{sh}, \lambda}, \quad s' = 0, 1, \ldots, s, \quad \lambda \in [s - s']_{2}. \]  

(3.4)

Conformal dimensions of the fields in (3.4) are given by

\[ \Delta\left(\phi^{\bar{a}_{1}...\bar{a}_{s'}}_{\text{cur}, \lambda}\right) = \frac{d}{2} + \kappa + \lambda, \quad \Delta\left(\phi^{a_{1}...a_{s'}}_{\text{sh}, \lambda}\right) = \frac{d}{2} - \kappa + \lambda. \]  

(3.6)

A summary of the field contents we use and appropriate ket-vectors are given in table 1. As shown in [22, 23], in the framework of AdS/CFT correspondence, the parameter \( \kappa \) in (3.6) is related to the mass parameter of the spin-s massive field in AdS\(_{d+1}\) as in table 2.

**Differential constraints for current and shadow.** For the ket-vectors of current and shadow given in table 1, we introduce the following differential constraints:

\[ \mathcal{L}\phi = 0, \]  

(3.7)

\[ \bar{\mathcal{L}} = \bar{\alpha} \bar{\partial} - \frac{1}{2} \alpha \bar{\alpha} \bar{\alpha}^2 - \bar{e}_{I} \mathbf{H}^{[1,2]} + \frac{1}{2} e_{I} \bar{\alpha}^2, \]  

(3.8)

where the operators \( e_{I} \) and \( \bar{e}_{I} \) are given in table 2, while the operator \( \mathbf{H}^{[1,2]} \) is given in (A.11). We note that constraint (3.7) is invariant under the gauge transformation and \( so(d, 2) \) algebra transformations which we discuss below.

**Two-point gauge invariant vertices.** For currents and shadows, one can construct two gauge invariant 2-point vertices. The first 2-point vertex, denoted by \( \Gamma^{\text{cur-sh}} \), is a local functional of current and shadow, while the second 2-point vertex, denoted by \( \Gamma^{\text{sh-sh}} \), is a non-local functional of shadows. Using notation \( \phi^{\text{cur}} \) and \( \phi^{\text{sh}} \) for the respective ket-vectors of currents and shadows given in table 1, we note the following expressions for the vertices:

\[ \Gamma^{\text{cur-sh}} = \int d^{d+1}x \mathcal{L}^{\text{cur-sh}}, \quad \mathcal{L}^{\text{cur-sh}} = \langle \phi^{\text{cur}} \mid \mu \mid \phi^{\text{sh}} \rangle, \]  

(3.9)

\[ \Gamma^{\text{sh-sh}} = \int d^{d+1}x_{1} d^{d+1}x_{2} \mathcal{L}^{\text{sh-sh}}, \]  

(3.10)

\[ \mathcal{L}^{\text{sh-sh}}_{12} \equiv \frac{1}{2} \left\langle \phi^{\text{sh}}(x_{1}) \mid \mu_{\nu} \right| \phi^{\text{sh}}(x_{2}) \rangle, \]  

(3.11)

where \( \mu_{\nu} \equiv \frac{\Gamma(\nu + d/2) \Gamma(\nu + 1)}{4^{\nu-d} \Gamma(\nu + d/2) \Gamma(\nu + 1)} \)

\[ \frac{\Gamma\left(\nu + d/2\right) \Gamma(\nu + 1)}{4^{\nu-d} \Gamma(\nu + d/2) \Gamma(\nu + 1)} \]

\[ |x_{12}|^{2} \equiv x_{12}^{a} x_{12}^{a}, \quad x_{12}^{a} = x_{1}^{a} - x_{2}^{a}, \]  

(3.13)
\[ \nu \equiv s + \frac{d - 4}{2} - N_c, \quad \kappa \equiv s + \frac{d - 4}{2}, \quad \text{for canonical shadow}, \quad (3.14) \]
\[ \nu \equiv \kappa + N_c - N_c, \quad \kappa \equiv \kappa, \quad \text{for anomalous shadow}, \quad (3.15) \]
where operators \( N_c, N_c, \mu \) are defined in the appendix.

**Gauge transformation parameter for canonical current and shadow.** To discuss gauge symmetries of spin-\( s \) canonical current and spin-\( s \) canonical shadow we use the respective gauge transformation parameters

\[ \xi^{a_1 \ldots a_{s'}}_{\text{cur},, \lambda}, \quad \xi^{a_1 \ldots a_{s'}}_{\text{sh},, \lambda}, \quad s' = 0, 1, \ldots, s - 1, \quad (3.16) \]
which are totally symmetric fields of the \( so(d-1,1) \) Lorentz algebra. The parameters are assumed to be traceless,

\[ \xi^{a_1 \ldots a_{s'}}_{\text{cur},, \lambda} = 0, \quad \xi^{a_1 \ldots a_{s'}}_{\text{sh},, \lambda} = 0, \quad \text{for } s' \geq 2. \quad (3.17) \]

Conformal dimensions of the gauge transformation parameters in (3.16) are given by

\[ \Delta \left( \xi^{a_1 \ldots a_{s'}}_{\text{cur},, \lambda} \right) = s' + d - 3, \quad \Delta \left( \xi^{a_1 \ldots a_{s'}}_{\text{sh},, \lambda} \right) = 1 - s'. \quad (3.18) \]

**Gauge transformation parameter for anomalous current and shadow.** To discuss the gauge symmetries of spin-\( s \) anomalous current and spin-\( s \) anomalous shadow we use the respective gauge transformation parameters

\[ \xi^{a_1 \ldots a_{s'}}_{\text{cur},, \lambda}, \quad \xi^{a_1 \ldots a_{s'}}_{\text{sh},, \lambda}, \quad s' = 0, 1, \ldots, s - 1, \quad \lambda \in \left[ s - 1 - s' \right]_{2}, \quad (3.19) \]
which are totally symmetric fields of the \( so(d-1,1) \) Lorentz algebra. The parameters are assumed to be traceless,

\[ \xi^{a_1 \ldots a_{s'}}_{\text{cur},, \lambda} = 0, \quad \xi^{a_1 \ldots a_{s'}}_{\text{sh},, \lambda} = 0, \quad \text{for } s' \geq 2. \quad (3.20) \]

Conformal dimensions of the gauge transformation parameters in (3.19) are given by

\[ \Delta \left( \xi^{a_1 \ldots a_{s'}}_{\text{cur},, \lambda} \right) \equiv \frac{d}{2} + \kappa + \lambda - 1, \quad \Delta \left( \xi^{a_1 \ldots a_{s'}}_{\text{sh},, \lambda} \right) \equiv \frac{d}{2} - \kappa + \lambda - 1. \quad (3.21) \]

The gauge transformation parameters and the corresponding ket-vectors are given in table 3.

Using a representation of the currents, shadows and gauge transformation parameters in terms of the ket-vectors \( |\varphi\rangle, |\xi\rangle \) given in tables 1 and 3, we now note that the gauge transformations can entirely be presented in terms of these ket-vectors. This is to say that the gauge transformations found in [20–23] take the form

\[ \delta |\varphi\rangle = G(\xi), \quad G \equiv a d - e_1 - \alpha^2 - \frac{1}{2N_\mu + d - 2} \tilde{e}_1, \quad (3.22) \]
where the ladder operators \( e_1 \) and \( \tilde{e}_1 \) are given in table 2. The following remarks are in order.

(i) Gauge transformation (3.22) takes the same form as the one in (2.6), i.e., gauge transformation for fields in (2.6) is distinguished from that for currents and shadows (3.22) only by the ladder operators \( e_1 \) and \( \tilde{e}_1 \).

(ii) Differential constraint (3.7) is invariant under gauge transformation (3.22).

(iii) The relation of our approach to the standard CFT is achieved by using the Stueckelberg gauge frame. This frame implies gauging away Stueckelberg fields and solving auxiliary
fields via the differential constraint. For the case of the spin-$s$ canonical current, the use of the Stueckelberg gauge frame leads to divergence-free and tracelessness constraints for $\phi_{\text{cur}}^{a \ldots a}$, 

$$\partial \phi_{\text{cur}}^{a_2 \ldots a_i} = 0, \quad \phi_{\text{cur}}^{a_{i+1} \ldots a_n} = 0,$$  \hspace{1cm} (3.23)

i.e., we see that, in the Stueckelberg gauge frame, our field $\phi_{\text{cur}}^{a_1 \ldots a_n}$ can be identified with the conserved current in the standard CFT. For the case of the spin-$s$ anomalous current, the use of the Stueckelberg gauge frame leads to a tracelessness constraint for $\phi_{\text{cur}}^{a_1 \ldots a_n}$. For the spin-$s$ shadow, the use of the Stueckelberg gauge frame leads to a tracelessness constraint for the field $\phi_{\text{sh}}^{a_1 \ldots a_n}$, which can be identified with the shadow in the standard approach to CFT. For a more detailed study of the relation of our approach to the standard CFT, see [20–23].

4. Relativistic symmetries of fields, currents and shadows

Relativistic symmetries of massless and massive fields in $R^{d-1,1}$ are described by the Poincaré symmetries, while relativistic symmetries of fields in AdS$_{d+1}$, conformal fields, currents and shadows in $R^{d-1,1}$ are described by the $so(d, 2)$ symmetries. In our approach, in order to treat fields, currents and shadows on equal footings, only $so(d - 1, 1)$ symmetries are realized manifestly. Therefore it is reasonable to represent the $so(d, 2)$ algebra so that to respect the manifest $so(d - 1, 1)$ symmetries$^{10}$. This is to say that the $so(d, 2)$ algebra consists of translation generators $P^a$, conformal boost generators $K^a$, the dilatation generator $D$, and generators $J^{ab}$ which span $so(d - 1, 1)$ algebra. Commutation relations of the $so(d, 2)$ algebra generators take the form

$$[D, P^a] = -P^a, \quad [P^a, J^{bc}] = \eta^{ab} P^c - \eta^{ac} P^b, \hspace{1cm} (4.1)$$

$$[D, K^a] = K^a, \quad [K^a, J^{bc}] = \eta^{ab} K^c - \eta^{ac} K^b, \hspace{1cm} (4.2)$$

$$[P^a, K^b] = \eta^{ab} D - J^{ab}, \quad [J^{ab}, J^{ce}] = \eta^{bc} J^{ae} + 3 \text{ terms} \hspace{1cm} (4.3)$$

For the case of fields in AdS$_{d+1}$ and conformal fields in $R^{d-1,1}$, requiring the $so(d, 2)$ symmetries implies that the actions of AdS fields and conformal fields (2.3) are invariant under the transformation $\delta \phi = G_{\text{diff}} [\phi]$, while for the case of currents and shadows, requiring the $so(d, 2)$ symmetries implies that differential constraints (3.7) are invariant under the transformation $\delta \phi = G_{\text{diff}} [\phi]$. For all these cases, the realization of $so(d, 2)$ algebra generators $G_{\text{diff}}$ in terms of differential operators acting on the respective ket-vectors $|\phi\rangle$ takes the form

$$P^a = \partial^a, \quad J^{ab} = x^a \partial^b - x^b \partial^a + M^{ab}, \hspace{1cm} (4.4)$$

$$D = x^a \partial^a + \Delta, \hspace{1cm} (4.5)$$

$$K^a = -\frac{1}{2} x^b x^a \partial^b + x^a D + M^{ab} x^b + R^a, \hspace{1cm} (4.6)$$

$^{10}$ For the case of AdS fields, the $so(d, 2)$ symmetries can manifestly be realized by using an ambient space approach (see e.g. [58–62]).
\[ M^{ab} = \alpha^a \bar{\alpha}^b - \alpha^b \bar{\alpha}^a. \]  

In (4.4)–(4.7), \( M^{ab} \) is a spin operator of the \( so(d-1,1) \) algebra, while \( \Delta \) in (4.5) is an operator which, for the case of conformal field, currents and shadows, is the operator of the conformal dimension. Operator \( R' \) in (4.6) is independent of coordinates \( x^\mu \). Explicit expressions for the operators \( \Delta \) and \( R' \) are given in table 4.

5. Light-cone gauge action of fields and 2-point vertices of currents and shadows

To develop a light-cone gauge formulation of fields, currents and shadows we use totally symmetric traceless tensor fields of the \( so(d-2) \) algebra. The field contents we use are presented in table 5. To simplify the presentation we use oscillators \( \alpha^i, \alpha^\zeta, \zeta, \nu^\oplus, \nu^\ominus, i = 1, \ldots, d-2 \), and introduce the corresponding ket-vectors which are also presented in table 5. Note that, in terms of the ket-vectors given in table 5, the tracelessness constraint takes the form

\[ \bar{\alpha}^i \alpha^i |\phi\rangle = 0. \]  

Also, note that, in the light-cone gauge approach, ket-vectors \( |\phi\rangle \) of fields, currents and shadows are not subject to a differential constraint and should satisfy only an algebraic tracelessness constraint (5.1).

**Light-cone gauge action of fields.** The light-cone gauge action and Lagrangian for fields in flat and AdS spaces we found take the form

\[ S = \int dx^+ d^{d-1}x \mathcal{L}, \quad \text{for fields in } R^{d-1,1}, \]

\[ S = \int dx^+ d^{d-1}x dz \mathcal{L}, \quad \text{for fields in } AdS_{d+1}, \]  

(5.2)

\( d^{d-1}x \equiv dx^- d^{d-2}x, \) where the coordinate \( x^+ \) is considered as an evolution parameter and light-cone gauge Lagrangian \( \mathcal{L} \) is given by

\[ \mathcal{L} = \frac{1}{2} \langle \phi | \left( \Box - M^2 \right) |\phi\rangle, \quad \Box = 2 \partial^+ \partial^- + \partial^i \partial^i. \]  

(5.3)

Operator \( M^2 \) in (5.3) takes the same form as the one entering the gauge invariant approach in table 2. From (5.3) and table 2, we see that, for the case of AdS fields, our light-cone gauge Lagrangian (5.3) leads to decoupled equations of motion which are easily solved in terms of the Bessel function. Our light-cone gauge notation may be found in the appendix 11.

Lagrangian (5.3) implies the standard equal-time Poisson–Dirac brackets,

\[ [\langle \phi(x), \langle \phi(x') \rangle |_{\text{equal } x^+} = -\frac{1}{2\partial^+} \delta^{d-1}(x - x') |\langle \phi \rangle |, \quad \text{for fields in } R^{d-1,1}, \]

\[ [\langle \phi(x), \langle \phi(x', z') \rangle |_{\text{equal } x^+} = -\frac{1}{2\partial^+} \delta^{d-1}(x - x') \delta(z - z') |\langle \phi \rangle |, \quad \text{for fields in } AdS_{d+1}, \]  

(5.4)

where we show the explicit dependence of the ket-vector on the space-time coordinates and the notation \(|\langle \rangle |\langle \rangle \) stands for the corresponding unit operator on space of the traceless ket-vectors (5.1).

11 Methods for solving equations of motion for higher-spin fields without gauge fixing are discussed in [63].
To summarize, starting with the gauge invariant Lagrangian for the double-traceless arbitrary spin fields (2.4) we obtained the light-cone gauge action in terms of fields which are traceless tensor fields of the $\mathfrak{so}(d - 2)$ algebra.

**Light-cone gauge 2-point vertices of currents and shadows.** In a light-cone gauge, the gauge invariant 2-point vertices given in (3.9), (3.10) take the form

\[
\Gamma_{\text{cur-sh}} = \int d^d x \, \mathcal{L}_{\text{cur-sh}} \equiv \left( \Phi_{\text{cur}}(x) \right) \Phi_{\text{sh}}(x),
\]

(5.5)

\[
\Gamma_{\text{sh-sh}} = \int d^d x_1 d^d x_2 \, \mathcal{L}_{\text{sh-sh}} \equiv \left( \Psi_{\text{sh}}(x_1) \right) \Psi_{\text{sh}}(x_2),
\]

(5.6)

\[
\mathcal{L}_{\text{12}} \equiv \frac{1}{2} \left( \Phi_{\text{sh}}(x_1) \right) \mathcal{L}_{\text{sh-sh}} \left( \Phi_{\text{sh}}(x_2) \right),
\]

(5.7)

\[
\mathcal{L}_{\text{sh-sh}} \equiv \frac{1}{2} \left( \Psi_{\text{sh}}(x_1) \right) \mathcal{L}_{\text{12}} \left( \Psi_{\text{sh}}(x_2) \right),
\]

(5.8)

where the operators $f_{\mu, \nu}$ take the same forms as in (3.12), (3.14), (3.15).

**Massless arbitrary spin field in AdS$_d$.** We now discuss some simplification of Lagrangian (5.3) for the case of massless field in AdS$_d$. For the case of AdS$_d$, we have $d = 3$.
Table 5. Field contents and the corresponding ket-vectors entering the light-cone gauge formulation of fields, currents and shadows. Fields with $s' \geq 2$ are totally symmetric traceless tensor fields of the $so(d-2)$ algebra, $\phi^{h_{-i'_e}\cdots} = 0$, i.e., $\bar{a}^2 |\psi\rangle = 0$. For the notion of $\lambda \in [s-s']_2$ and oscillator algebra see the Appendix. In the table, algebraic constraints express the homogeneity properties of ket-vectors $|\psi\rangle$ with respect to the oscillators.

| Field content | Ket-vector $|\psi\rangle$ and algebraic constraints |
|---------------|-----------------------------------------------|
| Massless spin-s Field in $R^{d-1,1}$ | $\phi^{h_{-i'_e}\cdots} = \frac{1}{s!}a^{i'_e}\cdots a^{i'_{s'}} |\psi\rangle = 0$ |
| Massive spin-s Field in $R^{d-1,1}$ | $\phi^{h_{-i'_e}\cdots} = \sum_{s'=0}^{s} \frac{\alpha^{i_e}\cdots \alpha^{i_{s'}}}{s'!} \phi^{h_{-i'_e}\cdots} |\psi\rangle = 0$ |
| Massless spin-s Field in $AdS_{d+1}$ | $\phi^{h_{-i'_e}\cdots} = \sum_{s'=0}^{s} \frac{\alpha^{i_e}\cdots \alpha^{i_{s'}}}{s'!} \phi^{h_{-i'_e}\cdots} |\psi\rangle = 0$ |
| Massive spin-s Field in $AdS_{d+1}$ | $\phi^{h_{-i'_e}\cdots} = \sum_{s'=0}^{s} \sum_{\lambda=0}^{d-2} \frac{\alpha^{i_e}\cdots \alpha^{i_{s'}}}{s'!} \phi^{h_{-i'_e}\cdots} |\psi\rangle = 0$ |
| Conformal spin-s Field in $R^{d-1,1}$ | $\phi^{h_{-i'_e}\cdots} = \sum_{s'=0}^{s} \frac{\alpha^{i_e}\cdots \alpha^{i_{s'}}}{s'!} \phi^{h_{-i'_e}\cdots} |\psi\rangle = 0$ |

$|\psi\rangle$ and algebraic constraints

$$(N_{\mu} - s) |\psi\rangle = 0$$

$$(N_{\mu} + N_{\nu} - s) |\psi\rangle = 0$$

$$(N_{\mu} + N_{\nu} - s) |\psi\rangle = 0$$
Table 5. (Continued.)

| Field content | Ket-vector $|\phi\rangle$ and algebraic constraints |
|---------------|------------------------------------------------|
| Canonical spin- $\phi_{c_{1_1}}$ | $s' = 0, 1, \ldots, s$  
Current in $R^{d-1,1}$
| | $\sum_{s'=0}^{s} \frac{\alpha_{c}^{s-s'} \alpha_{c}^{1} \ldots \alpha_{c}^{1}}{(s-s')!} \phi_{c_{1_1}}^{s-s'} |0\rangle$, $(N_{a} + N_{c} - s) |\phi\rangle = 0$ |
| Canonical spin- $\phi_{s_{1_1}}$ | $s' = 0, 1, \ldots, s$  
Shadow in $R^{d-1,1}$
| | $\sum_{s'=0}^{s} \frac{\alpha_{s}^{s-s'} \alpha_{s}^{1} \ldots \alpha_{s}^{1}}{(s-s')!} \phi_{s_{1_1}}^{s-s'} |0\rangle$, $(N_{a} + N_{c} - s) |\phi\rangle = 0$ |
| Anomalous spin- $\phi_{a_{1_1}}$ | $s' = 0, 1, \ldots, s$  
Current in $R^{d-1,1}$
| | $\sum_{s'=0}^{s} \sum_{\lambda \in[s-s']_{2}} \frac{\zeta_{s-s'\lambda}}{2} \alpha_{c}^{s-s'\lambda} \phi_{a_{1_1}}^{s-s'\lambda} |0\rangle$, $(N_{a} + N_{c} + N_{c} - s) |\phi\rangle = 0$ |
| Anomalous spin- $\phi_{s_{1_1}}$ | $s' = 0, 1, \ldots, s$  
Shadow in $R^{d-1,1}$
| | $\sum_{s'=0}^{s} \sum_{\lambda \in[s-s']_{2}} \frac{\zeta_{s-s'\lambda}}{2} \alpha_{s}^{s-s'\lambda} \phi_{s_{1_1}}^{s-s'\lambda} |0\rangle$, $(N_{a} + N_{c} + N_{c} - s) |\phi\rangle = 0$ |
and therefore the vector index of \( so(d - 2) \) algebra \( i \) takes only one value, \( i = 1 \). Therefore the traceless constraint (5.1) implies that \( |\phi\rangle \) can be presented as

\[
|\phi\rangle = |\phi^i\rangle + \alpha^1|\phi^i_{s-1}\rangle ,
\]

(5.9)

\[
\mathcal{N}_i |\phi^i\rangle = \lambda |\phi^i\rangle ,
\]

\[
\mathcal{N}_i |\phi^i_{s-1}\rangle = (s - 1)|\phi^i_{s-1}\rangle ,
\]

(5.10)

where new ket-vectors \( |\phi^i\rangle \), \( |\phi^i_{s-1}\rangle \) are independent of the oscillator \( \alpha^1 \) and depend only on the oscillator \( \alpha^s \). Relations (5.10) tell us that the ket-vectors \( |\phi^i\rangle \) and \( |\phi^i_{s-1}\rangle \) are the respective degree-\( s \) and degree-(\( s - 1 \)) homogeneous monomials in the oscillator \( \alpha^s \). Note that relations (5.10) are obtained from (5.9) and the fact that the ket-vector \( |\phi\rangle \) is homogeneous degree-\( s \) polynomial in the oscillators \( \alpha^1, \alpha^s \) (see table 5). In turn, relations (5.10) and expressions for the operator \( \mathcal{M}^2 \) for massless fields in \( AdS_4 \) given in table 2 imply that the operator \( \mathcal{M}^2 \) is simplified as

\[
-\mathcal{M}^2|\phi\rangle = \partial_+^2|\phi\rangle .
\]

(5.11)

Using (5.11) in (5.3), we get, as in [9], the following simple light-cone gauge Lagrangian for massless arbitrary spin field in \( AdS_4 \),

\[
\mathcal{L} = \frac{1}{2} \left\langle |\phi\rangle \left( \Box + \partial_+^2 \right) |\phi\rangle \right\rangle ,
\]

(5.12)

for massless field in \( AdS_4 \).

The following remarks are in order.

(i) For the derivation of our light-cone gauge formulation, we use the gauge invariant approach discussed in sections 2, 3. The gauge invariant approach is formulated in terms of the Lorentz algebra ket-vector \( |\phi\rangle \). To develop a light-cone gauge formulation of fields, currents and shadows we impose the following standard light-cone gauge condition on the ket-vector \( |\phi\rangle \):

\[
\alpha^+ \mathcal{H}^{(1,2)}|\phi\rangle = 0 ,
\]

(5.13)

where \( \mathcal{H}^{(1,2)} \) is given in (A.11). Using (5.13) and constraints obtained from action (5.2), we get the following relations for a gauge-fixed ket-vector \( |\phi\rangle \) and a light-cone ket-vector \( |\phi\rangle \),

\[
|\phi\rangle = \exp \left( -\frac{\alpha^+}{\alpha^+} \partial^i \partial^j + \frac{\alpha^+}{\partial^i} \partial^j e_l \right) |\phi\rangle ,
\]

\[
|\phi\rangle = |\phi\rangle \left|_{\alpha^+ = 0, \alpha^- = 0} \right. \).
\]

(5.14)

Note that, for the case of currents and shadows, in order to get relations in (5.14) we should use differential constraint (3.7). The method for the derivation of the light-cone gauge formulation from the gauge invariant formulation is the same as the one discussed in section 3.6 in [9]. Note that, in [9], we dealt with massless fields in \( AdS \) space. A remarkable feature of the gauge invariant approach we reviewed in sections 2, 3 in this paper is that our approach allows us to treat all fields (massless, massive and conformal) in flat and \( AdS \) spaces on an equal footing. This is the reason why the method in [9] is generalized to all the fields considered in this paper in a rather straightforward way.

(ii) For massless fields in \( AdS_{d+1} \), \( d > 3 \), and massive fields in \( AdS_{d+1} \), \( d \geq 3 \), results in this paper provide an alternative light-cone gauge formulation as compared to the one in [9, 13]. The advantage of the light-cone formulation of \( AdS \) field dynamics we develop in this paper as compared the one in [9, 13] is that the light-cone gauge action obtained in this paper leads to decoupled equations of motion which are easily solved in terms of Bessel functions. Light-cone gauge formulation of conformal field dynamics developed in this paper has not been discussed in earlier literature.
(iii) In the framework of AdS/CFT correspondence, Euclidean signature light-cone gauge Lagrangian, which involves a proper boundary term, and solution to Dirichlet problem for equations of motion of the light-cone gauge AdS field $\phi$ with boundary conditions corresponding to the light-cone gauge boundary shadow $|\phi_{sh}\rangle$ can be presented as

$$\mathcal{L}^E = \frac{1}{2} \langle \partial^\mu \phi | \partial^\mu \phi \rangle + \frac{1}{2} \left\langle \mathcal{T}_{\nu} \phi | \mathcal{T}_{\nu} \phi \right\rangle,$$

(5.15)

$$|\phi(x, z)| = \sigma_\nu \int d^d y \, G_\nu(x - y, z)|\phi_{sh}(y)|,$$

(5.16)

$$G_\nu(x, z) = \frac{c_\nu z^{\nu + \frac{d}{2}}}{(z^2 + |x|^2)^{\nu + \frac{d}{2}}}, \quad c_\nu \equiv \frac{\Gamma\left(\nu + \frac{d}{2}\right)}{\pi^{d/2} \Gamma(\nu)}, \quad \sigma_\nu \equiv \frac{2\Gamma(\nu)}{2\Gamma(\nu)(-\nu)^{\nu}}.$$ (5.17)

In (5.16), boundary canonical shadow enters the solution of the Dirichlet problem for the massless AdS field, while the boundary anomalous shadow enters the solution of the Dirichlet problem for massive AdS field. The $\nu$ and $\kappa$ are defined in (3.14), (3.15). Plugging (5.15), (5.16) in (5.2) gives action of AdS fields evaluated on the solution of the Dirichlet problem

$$-S_{\text{eff}} = 2\kappa \epsilon \Gamma^{\text{sh}-\text{sh}},$$

(5.18)

where the $\Gamma^{\text{sh}-\text{sh}}$ is defined in (5.6), (5.7).

6. Light-cone gauge realization of relativistic symmetries

Algebras of relativistic symmetries of fields, currents and shadows in $R^{d-1,1}$ and fields in AdS$_{d+1}$ contain the Lorentz subalgebra $so(d - 1, 1)$. In the light-cone approach, the Lorentz symmetries $so(d - 1, 1)$ are not realized manifestly. Therefore, in the framework of the light-cone approach, a complete description of field dynamics implies that we have to work out the explicit realization of the Lorentz symmetries $so(d - 1, 1)$ and the remaining relativistic symmetries as well. We now discuss the realization of relativistic symmetries in the framework of the light-cone gauge approach.

In the light-cone approach, the Poincaré algebra generators can be separated into two groups:

$$P^i, \quad P^+, \quad J^+, \quad J^-, \quad J^i, \quad \text{kinematical generators;}$$

(6.1)

$$P^-, \quad J^{-i}, \quad \text{dynamical generators.}$$

(6.2)

In order to discuss the relativistic symmetries of AdS and conformal fields, the Poincaré symmetries should be supplemented by the dilatation symmetry and conformal boost symmetries which also can be separated into two groups$^{12}$,

$$D, \quad K^i, \quad K^+, \quad \text{kinematical generators;}$$

(6.3)

$$K^-, \quad \text{dynamical generators.}$$

(6.4)

We now consider the light-cone gauge realization of relativistic symmetries for fields, currents and shadows in turn.

$^{12}$ Note that, for AdS fields, generators in (6.3), (6.4) realize isometry symmetries of AdS$_{d+1}$, while for conformal fields, these generators realize conformal symmetries of $R^{d-1,1}$.  


6.1. Light-cone gauge realization of relativistic symmetries for fields

The field theoretical representation of relativistic symmetry generators $G_{\text{field}}$ takes the form

$$G_{\text{field}} = \int dx^a d^a \partial x^d \delta^d \phi \bigg| G_{\text{diff}} \phi \bigg), \text{ for fields in } R^{d-1,1},$$

$$G_{\text{field}} = \int dx d^a \partial x^d \delta^d \phi \bigg| G_{\text{diff}} \phi \bigg), \text{ for fields in } AdS_{d+1}, \quad (6.5)$$

where $G_{\text{diff}}$ stands for the realization of the generators in terms of the differential operators acting on the $l(\phi)$. We now present the realization of the generators in terms of the differential operators $G_{\text{field}}$ acting on the $l(\phi)$.

**Kinematical generators:**

$$P^i = \partial^i, \quad P^+ = \partial^+, \quad (6.6)$$

$$J^{+-} = x^+ \partial^- - x^- \partial^+, \quad (6.7)$$

$$J^{+i} = x^+ \partial^i - x^i \partial^+, \quad (6.8)$$

$$J^{ij} = x^i \partial^j - x^j \partial^i + M^{ij}, \quad (6.9)$$

$$D = x^+ \partial^- + x^- \partial^+ + x^i \partial^i + \Delta, \quad (6.10)$$

$$K^+ = K_+^+ + n_1 \partial^+, \quad (6.11)$$

$$K^i = K_+^i + n_1 \partial^i + M^{ij} \partial^j + M^{i-} \partial^+ + M^{\theta^i}, \quad (6.12)$$

$$K_\alpha^\beta \equiv - \frac{1}{2} \left( 2x^+ x^- + x^i x^i \right) \partial^\alpha + x^a D, \quad a = +, i; \quad (6.13)$$

**Dynamical generators:**

$$P^- = - \partial^\alpha \partial^\beta + \mathcal{M} \frac{\partial^\alpha}{\partial^+}, \quad (6.14)$$

$$J^{-i} = x^- \partial^i - x^i \partial^+ + M^{-i}, \quad (6.15)$$

$$K^- = K^-_+ + n_1 \partial^- + x^i M^{-i} - M^{\theta^j} \frac{\partial^j}{\partial^+} + \frac{1}{\partial^+} B, \quad (6.16)$$

$$K_\alpha\beta \equiv - \frac{1}{2} \left( 2x^+ x^- + x^i x^i \right) P^- + x^- \partial^\alpha, \quad (6.17)$$

$$M^{-i} \equiv M^{\alpha} \frac{\partial^\alpha}{\partial^+} + \frac{1}{\partial^+} M^i, \quad M^{i-} = - M_-^i, \quad (6.18)$$

$$M^i \equiv e_i \alpha \partial^i - A^i \bar{\epsilon}^i, \quad \left[ M^i, \ M^j \right] = \mathcal{M}^2 \ M^{ij}, \quad (6.19)$$

$$M^{\theta^i} \equiv n_{0,i} \alpha^i + A^i \bar{\eta}_{0,i}, \quad B \equiv e_i \bar{n}_{0,i} + n_{0,i} \bar{\epsilon}_i - N_a, \quad (6.20)$$

where operators $A^i, M^i, N_a$ are given in (A.16). Operators $\mathcal{M}^2, e_i, \bar{\epsilon}_i, n_{0,i}, \bar{\eta}_{0,i}$ appearing in (6.10)–(6.16) take the same form as the ones in the gauge invariant approach for the fields we discussed in sections 2 and 4 (see tables 2 and 4). The following remarks are in order.
(i) The appearance of the ladder operators $\mathcal{M}^2$, $e_1$, $\bar{e}_1$, $n_{0,1}$, $\bar{n}_{0,1}$ in light-cone gauge operators (6.14)–(6.20), in gauge invariant Lagrangian (2.4), and in gauge transformations (2.6) demonstrates explicitly how the ladder operators entering the gauge invariant formulation of fields manifest themselves in the light-cone gauge formulation for those fields. We note that it is the use of the ladder operators that allows us to treat light-cone gauge fields in AdS space and conformal fields in flat space on an equal footing.

(ii) Taking into account equal-time commutation relations given in (5.4), we make sure that the equal time commutator of field with the field theoretical generators (6.5) takes the form

$$[\{\phi\}, G_{\text{field}}] = G_{\text{diff}}[\phi],$$

as it should be. Also we check that action (5.2) is invariant under transformation (6.21).

(iii) Using the operators $e_1$, $\bar{e}_1$ and $n_{0,1}$, $\bar{n}_{0,1}$ given in tables 2 and 4, we get the following alternative form for the operator $B$ (6.20):

$$B = -s - N_\kappa \left(2 s + d - 4 - N_\zeta\right), \quad \text{for massless spin–s field in } AdS_{d+1},$$

$$B = -s + N_\kappa \left(N_\zeta + \kappa - s - \frac{d - 4}{2}\right)$$

$$+ N_\zeta \left(N_\zeta - \kappa - s - \frac{d - 4}{2}\right), \quad \text{for massive spin–s field in } AdS_{d+1},$$

$$B = -s - N_\zeta \left(2 s + d - 4 - N_\zeta\right), \quad \text{for conformal spin–s field in } R^{d-1,1}.$$ (6.22)–(6.24)

(iv) Using generators (6.6)–(6.20), we compute the second order Casimir operator for conformal and AdS fields. Our expressions for eigenvalues of the Casimir operator in table 6 coincide with the ones given in textbooks. This provides an additional check for the generators given in (6.6)–(6.20).

### 6.2. Light-cone gauge realization of relativistic symmetries for currents and shadows

The representation for generators of $so(d, 2)$ algebra in terms of differential operators acting on light-cone ket-vectors of currents and shadows in table 5 is given by

$$P^+ = \partial^+, \quad P^- = \partial^-,$$

$$J^{+-} = x^+ \partial^- - x^- \partial^+, \quad J^{\pm i} = x^+ \partial^i - x^i \partial^+, \quad J^{ij} = x^i \partial^j - x^j \partial^i + M^{ij},$$

$$J^{-i} = x^- \partial^i - x^i \partial^- + M^{-i}, \quad D = x^+ \partial^- + x^- \partial^+ + x^i \partial^i + \Delta,$$

$$K^+ = K^+_\Delta,$$

$$K^i = K^i_{\Delta} + M^{ij}x^j + \frac{1}{2} \left\{ M^{-i}, x^+ \right\} + M^\Theta i,$$ (6.25)–(6.32)
Table 6. Second order \(so(d, 2)\) algebra Casimir operator \(C_2\) and its eigenvalues \((C^2)\) for light-cone gauge fields, currents and shadows. The parameter \(\kappa\) is given in table 2.

| Fields                              | Casimir operator \(C^2\) | Eigenvalues of \(C^2\) |
|-------------------------------------|--------------------------|------------------------|
| Massless spin-\(s\) Field in AdS\(_{d+1}\) | \(C_2 \equiv D(D - d) - \frac{1}{2} J^{ab} J^{ab} - 2 K^a P^a\) | \(2(s - 1)(s + d - 2)\) |
| Massive spin-\(s\) Field in AdS\(_{d+1}\) | \(= \Delta(\Delta - d) - 2B - \frac{1}{2} M^a M^a - 2n_{1,1} M^2\) | \(\kappa^2 + s(s + d - 2) - \frac{d^2}{4}\) |
| Conformal spin-\(s\) Field in \(R^{d-1,1}\) | \(2(s - 1)(s + d - 2)\) | \(2(s - 1)(s + d - 2)\) |
| Canonical spin-\(s\) Current \(R^{d-1,1}\) | \(2(s - 1)(s + d - 2)\) | \(2(s - 1)(s + d - 2)\) |
| Canonical spin-\(s\) Shadow \(R^{d-1,1}\) | \(2(s - 1)(s + d - 2)\) | \(\kappa^2 + s(s + d - 2) - \frac{d^2}{4}\) |
| Anomalous spin-\(s\) Current \(R^{d-1,1}\) | \(C_2 \equiv D(D - d) - \frac{1}{2} J^{ab} J^{ab} - 2 K^a P^a\) | \(\kappa^2 + s(s + d - 2) - \frac{d^2}{4}\) |
| Anomalous spin-\(s\) Shadow in \(R^{d-1,1}\) | \(\kappa^2 + s(s + d - 2) - \frac{d^2}{4}\) | \(\kappa^2 + s(s + d - 2) - \frac{d^2}{4}\) |

\(C_2 = D(D - d) + J^+(J^+ - d + 2) - 2J^a J^a - \frac{1}{2} J^{ab} J^{ab} - 2(K^a P^a + K^a P^a + K^a P^a)\)

\(\langle C_2 \rangle = e_0 (e_0 - d) + s(s + d - 2),\)

\(e_0 = s + d - 2\) for massless spin-\(s\) field in AdS\(_{d+1}\) and for canonical spin-\(s\) current in \(R^{d-1,1}\)

\(e_0 = 2 - s\) for conformal spin-\(s\) field in \(R^{d-1,1}\) and for spin-\(s\) shadow in \(R^{d-1,1}\)

\(e_0 = \frac{d}{2} + \kappa\) for massive spin-\(s\) field in AdS\(_{d+1}\) and for anomalous spin-\(s\) current in \(R^{d-1,1}\)

\(e_0 = \frac{d}{2} - \kappa\) for anomalous spin-\(s\) shadow in \(R^{d-1,1}\)

\(\kappa^2 = \frac{d^2}{4}\) for massless spin-\(s\) field in AdS\(_{d+1}\) and for canonical spin-\(s\) current in \(R^{d-1,1}\)

\(\kappa^2 = \frac{d^2}{4}\) for conformal spin-\(s\) field in \(R^{d-1,1}\) and for spin-\(s\) shadow in \(R^{d-1,1}\)

\(\kappa^2 = \frac{d^2}{4}\) for massive spin-\(s\) field in AdS\(_{d+1}\) and for anomalous spin-\(s\) current in \(R^{d-1,1}\)

\(\kappa^2 = \frac{d^2}{4}\) for anomalous spin-\(s\) shadow in \(R^{d-1,1}\)

\(K^2 = K_\Delta^2 + \frac{1}{2} \left\{ M^{\alpha i}, x^i \right\} - M^{i a} \frac{\partial^i}{\partial^a} + \frac{1}{\partial^a} B , \quad (6.33)\)

\(K_\Delta^2 \equiv -\frac{1}{2} \left( 2x^+ x - x^i x^i \right) \partial^a + \chi^a D , \quad a = +, - , i , \quad (6.34)\)

\(M^{-a} \equiv M_{ij} \frac{\partial^j}{\partial^+} + \frac{1}{\partial^+} M^i , \quad M^{-i} = -M^{\alpha i} , \quad (6.35)\)

\(M^i \equiv e_i \alpha^j - A^i \bar{e}_j , \quad \left[ M^i , M^j \right] = \Box M^{ij} , \quad (6.36)\)

where the operators \(\Delta\) and \(e_1, \bar{e}_i\) appearing in (6.30) and (6.36) respectively coincide with the ones appearing in gauge invariant formulation of currents and shadows in tables 2 and 4.

Expressions for operators \(M^i \Box \) and \(B\) appearing in (6.32), (6.33) are summarized in table 7.

The following remarks are in order.

(i) Using notation for \(G_{\text{diff}}\) for differential operators given in (6.25)–(6.33) and introducing transformation rules for currents and shadows
\[ \delta \phi = G_{\text{diff}} \phi \]  

(6.37)

we note that light-cone gauge vertices (5.5), (5.6) are invariant under transformation (6.37).

(ii) The generators in (6.25)–(6.33) are polynomial (i.e. local) with respect to derivatives $\partial^i, \partial^{-i}$. This property of generators in (6.25)–(6.33) is the main advantage of the light-cone gauge approach to currents and shadows in this paper against the one developed in [9, 10].

(iii) The appearance of the ladder operators $e_1, \bar{e}_1$ in light-cone operators (6.36) and in the covariant differential constraint in (3.7) demonstrates explicitly how the ladder operators entering the gauge invariant formulation of currents and shadows manifest themselves in the light-cone gauge formulation for currents and shadows.

(iv) Using generators (6.25)–(6.36), we compute the second order Casimir operator for currents and shadows. Our expressions for the eigenvalues of the Casimir operator in table 6 coincide with the ones given in textbooks. This provides an additional check for the generators given in (6.25)–(6.36).

7. Conclusions

In conclusion, let us briefly discuss a number of the potentially interesting applications and generalizations of our light-cone approach. One potentially interesting application is related to the problem of interaction vertices in higher-spin field theories. Although many methods for building interaction vertices for higher-spin gauge fields are known in the literature (see e.g. [65–76]), building interaction vertices for concrete models of higher-spin field theories is still a challenging problem. The light-cone gauge approach provides interesting possibilities for the study of interaction vertices of higher-spin field theories. This is to say that, on the one hand, some systematic methods for building light-cone gauge interaction vertices were developed in [6, 7, 64]. On the other hand, in this paper, we have demonstrated that the use of the ladder operators allows us to treat free AdS fields and conformal fields on an equal
footing. Therefore one can expect that results in this paper and in \[6, 7, 64\] will provide new interesting possibilities for studying the interaction vertices of AdS and conformal fields on an equal footing.

In this paper, we have considered the light-cone gauge action for the bosonic totally symmetric fields. Generalization of our approach to the case of fermionic fields \[77–79\] and mixed-symmetry fields \[80–82\] could also be of interest. We believe also that the light-cone gauge approach we have discussed in this paper might be useful for the study of light-cone gauge AdS string theory \[83, 84\].

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Appendix. Notation

Throughout the paper the notation $\lambda \in [k]_2$ implies that $\lambda = -k, -k + 2, -k + 4, \ldots, k - 4, k - 2, k$:

$$\lambda \in [k]_2 \quad \Rightarrow \quad \lambda = -k, -k + 2, -k + 4, \ldots, k - 4, k - 2, k.$$  \hfill (A.1)

**Notation in basis of Lorentz algebra** $so(d - 1, 1)$. Throughout the paper, $x^a$ denotes coordinates in $d$-dimensional flat space-time $\mathbb{R}^{d-1,1}$, while $\partial_a$ denotes derivatives with respect to $x^a$, $\partial_a \equiv \partial / \partial x^a$. The vector indices of the Lorentz algebra $so(d - 1, 1)$ take the following values: $a, b, c, e = 0, 1, \ldots, d - 1$. We use the mostly positive flat metric tensor $\eta_{ab}$. To simplify our expressions, we drop $\eta_{ab}$ in scalar products, i.e., we use $\eta_{ab} \equiv \eta^{aa} \eta_{bb}$. We use the creation operators $\alpha_a, \alpha^\dagger_a, \zeta, \nu^{\Phi}, \nu^{\bar{\Phi}}$ and the respective annihilation operators $\bar{\alpha}_a, \bar{\alpha}^\dagger_a, \bar{\zeta}, \bar{\nu}^{\bar{\Phi}}, \bar{\nu}^{\Phi}$. These operators, to be referred to as oscillators, satisfy the commutation relations\(^\text{13}\)

$$\begin{align*}
[\bar{\alpha}_a, \alpha^b] &= \eta^{ab}, & [\bar{\alpha}^\dagger_a, \alpha_c] &= 1, & [\zeta, \zeta] &= 1, \\
[\bar{\nu}^\Phi, \nu^{\bar{\Phi}}] &= 1, & [\bar{\nu}^{\bar{\Phi}}, \nu^{\Phi}] &= 1,
\end{align*}$$  \hfill (A.2)

$$\bar{\alpha}[0] = 0, \quad \bar{\alpha}^\dagger[0] = 0, \quad \zeta[0] = 0, \quad \bar{\nu}^{\bar{\Phi}}[0] = 0, \quad \bar{\nu}^{\Phi}[0] = 0.$$  \hfill (A.3)

We use the following hermitian conjugation rules for derivatives and oscillators:

$$\partial^a \dagger = -\partial_a, \quad \alpha^a \dagger = \bar{\alpha}_a, \quad \alpha_a \dagger = \bar{\alpha}^\dagger_a, \quad \zeta^\dagger = \zeta, \quad \nu^{\Phi \dagger} = \bar{\nu}^{\Phi}, \quad \nu^{\bar{\Phi} \dagger} = \bar{\nu}^{\bar{\Phi}}.$$  \hfill (A.4)

Throughout this paper we use operators constructed out of the derivatives and oscillators,:

$$\Box = \partial^a \partial_a, \quad \alpha \partial \equiv \alpha^a \partial_a, \quad \bar{\alpha} \partial \equiv \bar{\alpha}_a \partial^a,$$  \hfill (A.5)

$$\alpha^2 \equiv \alpha^a \alpha_a, \quad \bar{\alpha}^2 \equiv \bar{\alpha}_a \bar{\alpha}^a,$$  \hfill (A.6)

$$N_a \equiv \alpha^a \alpha_a, \quad N_\zeta \equiv \alpha^\zeta \zeta, \quad N_\zeta \equiv \zeta^\dagger \zeta,$$  \hfill (A.7)

$$N_{\nu^\Phi} \equiv \nu^{\Phi} \nu^{\bar{\Phi}}, \quad N_{\nu^{\bar{\Phi}}} \equiv \nu^{\bar{\Phi}} \nu^{\Phi}, \quad N_{\nu} \equiv N_{\nu^\Phi} + N_{\nu^{\bar{\Phi}}},$$  \hfill (A.8)

\(^\text{13}\) Applications and extensive study of the oscillator formalism may be found in \[85\].
\[ \tilde{\alpha}_a \equiv \alpha^a - \alpha^2 \frac{1}{2N_a + d - 2} \tilde{\alpha}_a, \]  
\[ \tilde{\alpha}_{\perp} \equiv \tilde{\alpha}_a - \frac{1}{2} \alpha^a \tilde{\alpha}_a, \]  
\[ \Pi^{[1,2]} \equiv 1 - \alpha^2 \frac{1}{2(2N_a + d)} \tilde{\alpha}_a^2, \quad \mu \equiv 1 - \frac{1}{4} \alpha^2 \tilde{\alpha}_a^2. \]  

**Notation in basis of \( so(d - 2) \) algebra.** To discuss light-cone gauge formulation we use basis of \( so(d - 2) \) algebra and decompose \( so(d - 1) \) algebra vector \( X^a \) as \( X^+ \), \( X^- \), \( X^i \), \( i = 1, \ldots, d - 2 \), where \( X^\pm \equiv (X^{d-1} \pm X^0)/\sqrt{2} \). This implies the following decomposition of the space-time coordinates, derivatives, oscillators, and scalar product:

\[ x^a = x^+ \times, x^- \times^i, \quad \partial^a = \partial^+, \partial^-, \partial^i, \quad \partial^\pm \equiv \partial/\partial x^\mp, \quad \partial^j \equiv \partial/\partial x^j, \]  
\[ \tilde{\alpha}^a = \alpha^a - \alpha^2 \frac{1}{2N_a + d - 2} \tilde{\alpha}^a, \]  
\[ \tilde{\alpha}^i = \tilde{\alpha}^a - \frac{1}{2} \alpha^a \tilde{\alpha}^i, \]  
\[ \tilde{\alpha}^a, \tilde{\alpha}^i \]  
\[ X^a Y^a = X^+ Y^- + X^+ Y^+ + X^1 Y^1. \]  

Vector indices of the algebra \( so(d - 2) \) take the values \( i, j = 1, \ldots, d - 2 \). We use operators constructed out of the spatial derivative and oscillators,

\[ a \partial \equiv \alpha^a \partial^a, \quad \tilde{a} \partial \equiv \tilde{\alpha}^a \partial^a, \quad a^2 \equiv \alpha^a \alpha^a, \quad \tilde{a}^2 \equiv \tilde{\alpha}^a \tilde{\alpha}^a, \]  
\[ M^a = \alpha^a \tilde{\alpha}^a, \quad a_1 \equiv \alpha^a - \alpha^2 \frac{1}{2N_a + d - 2} \alpha^a, \quad N_a \equiv \alpha^a \tilde{\alpha}^a. \]  

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