R/S analysis based study on long memory about CODMn in Poyang Lake Inlet and Outlet

Lili Wang

Jiangxi Province Key Laboratory for Water Information Cooperative Sensing and Intelligent Processing, Nanchang Institute of Technology, Nanchang Jiangxi 330099, China;
School of Science, Nanchang Institute of Technology, Nanchang Jiangxi 330099, China.

wllzh2008@126.com

Abstract. Rescaled range analysis (R/S) is applied to the long memory behavior analysis of water CODMn series in Poyang Lake Inlet and Outlet in China. The results show that these CODMn series are characterized by long memory, and the characteristics have obvious differences between the Lake Inlet and Outlet. Our findings suggest that there was an obvious scale invariance, namely CODMn series in Lake Inlet for 13 weeks and CODMn in Lake Outlet for 17 weeks. Both displayed a two-power-law distribution and a similar high long memory. We made a preliminary explanation for the existence of the boundary point $t_c$, using self-organized criticality. This work can be helpful to improvement of modelling of lake water quality.

1. Introduction

Lake water pollution is one of the most important environmental problems in the world. Its impact on human health has been the focus of the public as well as governments [1]. Thus, the monitoring, analysis and prediction of the lake water quality come into being. The monitoring, analysis and prediction of lake water quality, the information about the environmental quality of lakes in the future can be provided timely for the society and government, which is conducive to timely and targeted taking measures to control the pollution of lakes. In particular, when a heavy pollution events was predicted, we can inform the related factories will be limited, and remind people of the water-drinking safety surrounding the lake area so as to reduce the damage of water pollution to their body [2].

On the basis of the importance of monitoring, analysis and prediction of lake water quality, many scholars from all over the world have carried out a lot of researches and put forward various forecasting methods [3]. However, at present, it seems difficult to achieve an ideal prediction target. Because there are not only the distribution and emission of lake water pollution in the main influencing factors but many complex factors, such as the weather condition, the pollutant migration and transformation, and the complex terrain of lakes [4]. These factors are not independent mutually, but they can make complex nonlinear interactions in various spatial and temporal scales, which makes there be complicated nonlinear features of lake pollutant concentration as the time goes by. If we can study some important dynamic characteristics of the time evolution of lake water pollutant in a proper nonlinear mathematical physics method, we can deeply understand the inherent law and mechanism of
the time evolution of lake water pollutant. It can play an important role in the future research and development of more effective lake water quality prediction methods as well as the development of new environmental governance measures [5].

So far, a variety of nonlinear analysis methods have been used to study the time evolution process of lake water pollution. For example, various multivariate method [6,7], principal component analysis (PCA) [8], fractal method [9] and fuzzy mathematical method [10, 11]. Among them, R/S analysis method, as one of the nonlinear methods to deal with complex phenomena, has been applied in many research fields, such as graphic language [12], hydrology [13], earthquake research[14, 15] and climatology [16]. It can make people deal with many complex problems related to time scales in nature from a new point of view, and more profoundly reflect the internal dynamic characteristics of the objects, among which a very important characteristics is long memory behavior. There are many ways to study long memory, such as DFA [17], spectral analysis [18], DCCA [19] etc., but R/S analysis has obvious advantage, see Shi et al. (2008) [20]. Recently, a few scholars from abroad studied the scale characteristics of the change of the lake pollutant through R/S analysis. Therefore, taking Poyang Lake's chemical oxygen demand (CODMn) in Poyang Lake Inlet and Outlet as the research object, we try to provide a new way to study the time revolution of lake water quality in this paper, which will play an important role in exploring the evolution of the lake water pollution.

2. Study area and data

In this paper, Poyang Lake Inlet and Outlet water CODMn series are chosen as research objects. Poyang Lake (115°47′~116°45′E, 28°22′~29°45′N) is the largest freshwater lake in China [21]. Poyang Lake Inlet and Outlet water quality data are provided freely on the Internet by Ministry of Environmental Protection the People's Republic of China web site: http://www.mep.gov.cn/. We have used weekly average CODMn monitoring data of Poyang Lake Inlet and Outlet during 2016. The study area and monitoring sections are shown in Figure 1. In this study, we investigate the long memory characteristics of CODMn between Poyang Lake Inlet and Outlet in China by the rescaled range analysis (R/S). The study area and monitoring sections are shown in Figure 2.

3. Method

The ratio of the range (R) to standard deviation (S) of this integrated and detrended time series (R/S) behaves as the length of time series t, between which satisfy a following relationship: (R/S)t ∝ (bt)H. The Hurst exponent (H) is defined as the slope of the regression line for all points[log(t), log(R/S)] [22]. For white noise, the series corresponds to a random walk and H=0.5, indicates each data in the
sequence is independent, unrelated, or entirely random; that is to say, the trend of the previous period will not affect the latter. \( H > 0.5 \), shows the data is positively correlated (namely positive long-memory); in other words, the growth (decrease) trend over the past few weeks will mean a growth (decrease) trend at the same time interval; the closer the \( H \) value to 1, the stronger the (positive) long memory. \( 0 < H < 0.5 \), demonstrates the data has negative correlation (namely negative long-memory); in other words, the growth (decrease) trend over the past few weeks will mean a decrease (growth) trend at the same time interval; the closer the \( H \) value to 0, the stronger the (negative) long memory.

4. Results and discussion

![Figure 2](image-url)  
**Figure 2.** Location of the study area and monitoring sections. (A) China. (B) Poyang Lake and its vicinity.

![Figure 3](image-url)  
**Figure 3.** R/S analysis performed on the CODMn series of Poyang lake inlet and outlet.
In order to investigate the long memory behavior of water quality in Poyang Lake, we analyzed the time series of the CODMn in Poyang Lake Inlet and Outlet. Figure 3 is their log(t)–log(R/S). There are both two distinct intervals in Figure, where \( t_c \) is the boundary point. The corresponding time of \( t_c \) is 13 weeks and 17 weeks respectively, which reflect the influence of natural monthly pollution on lake water. For the CODMn in Lake Inlet, there is a situation in the 13 weeks (\( t<13 \)), \( H_I=0.9822 \) with \( r^2=0.9944 \), which indicates there is a high positive long-memory behavior in the CODMn time series within the scaling region of 13 weeks. It is namely that, in the scaling region in the future, CODMn series would strongly maintain its original variation trend. When the scaling region is more than 13 weeks (\( t>13 \)), \( H_I=0.7970 \) with \( r^2=0.9878 \). As compared with \( H_I \), it is close to 0.5, which shows there is still a high positive long-memory behavior for the CODMn time series in Lake Inlet. However, the long memory is significantly weaker than that of the 13 weeks scaling region, and its random process is more and more significant. It is namely that the correlation between CODMn change in the future and past will decrease gradually.

As to the CODMn in Lake Outlet, its long memory behavior is opposite to that of the CODMn series of Lake Inlet. Specifically, in the 13 weeks (\( t<13 \)), \( H_I=0.7502 \) with \( r^2=0.9939 \), there is a high positive long-memory behavior of the CODMn time series of Lake Outlet; while as \( t>13 \), \( H_I=0.9901 \) with \( r^2=0.9924 \), there is a growing stronger positive long-memory.

For the existence of the boundary point \( t_c \), the self-organized criticality posed by Bak and Chen [23] could help explain why. It was well known that the self-organized criticality was the theory which could be used to explain the universal organization principle of a large number of extend dissipative dynamical systems [24, 25]. Environmental experts pointed out that water pollution system is a kind of critical state with a unstable local dynamics and a macro stable dynamics, which is spontaneously formed by an open complex dynamical system composed of a plurality of components under the driving of the external input energy and material. That is the self-organized critical state of CODMn change. In the state, a small local disturbance could trigger a chain reaction and lead to a disaster; its synergy may be extended to the entire system and form a large avalanche, so as to output matter and energy into the outside. For example, the spread of aquatic plants, and the sudden discharge of organic matter. The critical state is characterized by a various sizes of collapse time, and there was a negative power-law relation between the size of the collapse (namely time scales) and the distribution probability. The state is stable and robust in dynamics, which is namely that the system is stable against external disturbances and inner fluctuations as well as the variations in initial conditions. In the evolution of the critical state, there is no special provision on the initial state of the system. That means the critical state is a dynamical attractor. Meanwhile, the critical state is robust to disturbances. In other words, when the system is deviated from the critical state, it would automatically return to the critical state, and there is no equilibrium in the system forever. The point of time corresponding to the critical state is exactly the demarcation point \( t_c \).

But why is \( t_c \) obviously different for CODMn series in Poyang Lake Inlet and Outlet, which was required to be further studied.

5. Conclusions
The long memory behaviors of CODMn in Poyang Lake Inlet and Outlet was analyzed using \( R/S \) analysis method. The experimental result indicated that there was an obvious scale invariance, namely CODMn in Lake Inlet for 13 weeks and CODMn in Lake Outlet for 17 weeks. Both displayed a two-power-law distribution and a similar high long memory.

The cause that there was \( t_c \) boundary point in the CODMn series in Poyang Lake Inlet and Outlet were preliminarily explained. When CODMn change went into the self-organized critical state, a small local disturbance could trigger a chain reaction and lead to a disaster, which is characterized by a various sizes of collapse time, and there was a negative power-law relation between the size of the collapse (namely time scales) and the distribution probability. Moreover, the state is stable and robust.
in dynamics, and there is no special provision on the initial state of the system. The point of time corresponding to the critical state is exactly the demarcation point $t_c$.

Acknowledgment
This work was supported by the Open Research Fund of Jiangxi Province Key Laboratory of Water Information Cooperative Sensing and Intelligent Processing (2016WICSIP014), the Science and Technology Project of Jiangxi Provincial Department of Education (GJJ151109), the Opening Fund of Key Laboratory of Poyang Lake Wetland and Watershed Research (Jiangxi Normal University), Ministry of Education (PK2017002), the China Postdoctoral Science Foundation (2016M600515), the Jiangxi Province Postdoctoral Science Foundation (2017KY48), the Key Project of Jiangxi Provincial Department of Science and Technology (2016BBF60061) and the National Science Foundation Emergency Management of China (41641023).

References
[1] Beyhan M and Kaçıkoc M 2014 Water Air Soil Poll. 225 1
[2] Stidson RT, Gray CA and Mepham CD 2012 Water Environ. J 6 7
[3] Koponen S, Pulliainen J, Kallio K and Hallikainen M. 2002 Remote Sens. Environ. 79 51
[4] Chen MQ, Chen MJ, Lu YF, Wang LG and Huang YJ 2016 Water Sci. Technol. 73 1591
[5] Ozmen M, Gungorli A, Kucukbay FZ and Guler RE 2006 Ecotoxicology 151 57
[6] Chen Y, Zhao KP, Wu YY, Gao SS, Cao W, Bo Y, Shang ZY, Wu J and Zhou F 2016 Water 8 86.
[7] Gu Q, Zhang Y, Ma LG, Li JD, Wang K, Zheng KF, Zhang XB and Li S 2016 Sustainability 8 243
[8] Gong QL, Liu Y and Tang BB 2016 Environ. Sci. Technol. 39 111
[9] Shi K, Liu CQ, Huang ZW and Su Y 2010 Water Sci. Technol. 61 2113
[10] Wang ZX, Zhang X X, Liu N, Liu B, Li WX and Cheng SP 2011 Fresen. Environ. Bull. 20 1381
[11] Tao L, Cai SM, Yang HD, Wang XL, Wu SJ and Ren XY 2009 Environ. Eng. Sci. 26 451
[12] Dominguez AC, Camilo OV and Martínez EH 2016 IEEE International Engineering Summit, II Cumbre Internacional De Las Ingenierias. (pp.1-6). IEEE.
[13] Zhao XH, Chen X and Huang Q 2017 Water Resour. 44 31
[14] Lana X, Martínez MD, Hosseini SA and Serra C 2017 Geosci. J. 21 355
[15] Gkarlaouni C, Lasocki S, Papadimitriou E, Tsaklidis G 2017 Chaos Soliton. Fract. 96 30
[16] Liu ZH, Sun LN, Wang JY, Wang LL, Shi K and Liu CQ 2017 Fresen. Environ. Bull. 26 7681
[17] Liu ZH, Wang LL and Zhu HS 2015 Atmos. Pollut. Res. 6 457
[18] Liu ZH, Xu JH, Chen ZS, Nie Q and Wei CM 2014 Stoch. Env. Res. Risk. A. 28 1383
[19] Liu ZH, Wang LL, Yu X, Wang SQ, Deng CZ, Xu JH, Chen ZS and Bai L 2017 Atmos. Sci. Lett. 8 230
[20] Shi K, Liu CQ, Ai NS and Zhang XH 2008 Nonlinear Anal.-Real. 9 693
[21] Wang H, Zhao YJ, Liang DF, Deng YQ and Pang Y 2017 Chemosphere 168 1604
[22] Hurst HE 1951 T. Am. Soc.Civ. Eng. 116 776
[23] Bak P and Chen K 1991 Sci. Am. 264 26
[24] Liu ZH, Xu JH and Li WH 2017 Sci. Cold. Arid Reg. 9 476
[25] Liu ZH, Xu JH and Shi K 2014 Theor. Appl. Climatol. 115 685