Estimation of gas-dynamic characteristics of supersonic plasma jets based on spectroscopy results

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Abstract. A method for estimating the gas-dynamic characteristics of supersonic plasma jets based on the minimum number of plasma parameters (electron number density and temperature) that are easily measured by spectral diagnostic methods is considered. The course of the most important parameters characterizing the gas dynamics of the nonequilibrium plasma flow in the shock-wave region of a supersonic plasma jet created by a pulsed capillary discharge is revealed, and their values are estimated. A significant difference between the electron and gas temperatures was found in the vicinity of the central shock wave.

1. Introduction

Optical spectroscopy is often the only available plasma research tool. High-speed plasma flows, characterized by the presence of small-scale inhomogeneities, such as shock waves, whose study requires the use of non-contact diagnostic methods with high spatial and temporal resolution, are not the exception. However, the use of spectral methods does not always allow us to establish a complete set of parameters characterizing the plasma flow, whose state is rarely equilibrium. A typical example is spatially inhomogeneous flows, such as supersonic plasma jets [1], whose inherent attributes - large gradients of pressure, density and temperature - are the main factors of plasma nonequilibrium (thermal, ionization, Boltzmann). The use of spectral diagnostic under these conditions makes it possible to determine reliably only the electron number density and, in some cases, electron temperature. In rare cases, for example, by electron-vibrational-rotational transitions of molecules, it is possible to estimate gas temperature. However, the molecular components are not always present in the studied flow section, and the method itself is not very accurate.

Nevertheless, in some practically important cases, even a limited number of parameters obtained during spectral diagnostics are enough to restore the most plasma flow characteristics, if its gas dynamics is known. Such a possibility, in particular, exists for underexpanded supersonic plasma jets with a pronounced shock-wave structure of the initial section. As will be shown further, to evaluate the flow characteristics of the shock-wave structure, it is sufficient to be able to synchronously measure the electron number density and temperature on the jet axis in three cross-sections corresponding to the stagnation state (M = 0), critical state (M = 1), and the state behind the shock wave (Mach disk). In this case, it is sufficient to determine the electron temperature only in the critical cross-section, where its value does not much differ from the gas temperature. Using the listed parameters as the initial data of the gas-dynamic calculation allows us to restore not only the gas temperature, which is difficult to determine by spectral diagnostic methods, but also a number of the most important parameters characterizing the gas dynamics of the supersonic plasma flow: effective adiabatic index, critical
sound velocity, ratios of densities and temperatures in the shock wave, the speed, density and temperature of the flow before and after the shock wave, that is especially important in conditions of a limited choice of diagnostic methods and tools. In this paper, we consider the possibility of determining the above mentioned parameters using an example of a plasma jet created by a pulsed capillary discharge with an ablating wall [2].

2. The object and the methods of research
A capillary arrester, whose detailed description is given elsewhere [3], was used for plasma jets obtaining. The discharge ignition inside the capillary is accompanied by the wall evaporation, ionization of the evaporated substance and the subsequent plasma outflow into the flooded space (quiescent air under normal conditions). The following parameters of a capillary discharge are typical for conducted experiments: the capillary material is polymethylmethacrylate (PMMA, chemical formula C₅H₈O₂), the capillary diameter - d=1-2 mm, the capillary depth - h=5 mm, the peak value of discharge current - Iₘ=350-400 A, the discharge pulse duration - τ=1 ms, average mass flow rate of the ablating substance - G=0.3-0.4 g/s, mass-averaged stagnation enthalpy - H=100-200 MJ/kg, the pressure at the capillary outlet - pₐ=5-10 bar. Under these conditions, the jet contains the only one shock-wave section, which includes a barrel, central and reflected shock waves (Figure 1). The outflow conditions corresponded to Reynolds numbers Re=ρₐνₐd/μ=100-150 (ρₐ, νₐ, μ are the density, speed of sound, and viscosity at the capillary outlet, which were estimated using data [4]), the plasma ionization degree is close to unity, α≈1.

To determine the plasma parameters, high spatial (30-50 μm) and temporal (1-50 μs) resolution spectral tools were used. The spectra were recorded on the CCD matrix of the Andor iStar camera installed in the output focal plane of the MS-257 spectrograph (1800 and 2400 grating/mm, entrance slit size δ=20 μm). The emission spectra of the axial zone of the discharge inside the capillary and along the initial section of the plasma jet were registered in experiments. For this, a sharp image of the discharge was projected onto the spectrograph entrance slit, and the image of the near-axis zone was maximally aligned with the entrance slit.

3. Methods and results of spectral diagnostics
Since the capillary erodes during discharge (its diameter increases by approximately 0.02 mm per one discharge pulse), it is highly desirable to be able to synchronously measure both plasma parameters — temperature Tₑ and electron number density nₑ. Given the limitation of the spectral range of the recording equipment with an interval of 30-50 nm, this possibility was realized in the spectral range...
Δλ=650-685 nm. This interval includes the Hα line contour used to determine the electron number density based on the linear Stark effect, several singlet and multiplet lines of the carbon ion C II 657.8 nm, C II 658.3 nm, C II 678-682 nm, used to estimate the electron temperature by the Boltzmann plot method, as well as a rather intense Kramers–Unzold continuum in the region inside the capillary, which makes it possible to estimate $T_e$ using the relative intensities of the Hα line and the continuum [5]. Additionally, emission spectra were registered in the wavelength range Δλ=360-395 nm, containing lines of singly ionized copper atoms Cu II 368.65 nm ($E$=11.84 eV) and Cu II 392.06 nm ($E$=18.41 eV), which were used for the electron temperature estimation by relative intensities of the named lines.

In the range of the studied discharge parameters, the maximum emission intensity of the aforementioned lines of carbon and copper ions is achieved in the discharge axis, which makes it possible to accurately determine the electron temperature in this zone from the spectra recorded along the line of sight (along the chord passing through the discharge axis). However, for a number of reasons, reliable estimates of the electron temperature using the C II and Cu II lines can be obtained only in a limited flow region in the vicinity of the Mach disk. In the rarefaction region, this is prevented by the low emission intensity of these lines, and in the region inside the capillary the above lines merge with the continuum. Therefore, the electron temperature in these regions, as well as inside the capillary, was estimated by the method of relative intensities of the Hα line and continuum. An assessment of the emission reabsorption of both the Hα line and the continuum under experimental conditions indicates the suitability of this method for estimating the electron temperature inside the capillary with an error of less than 20% increased with discharge power.

The results of estimates of temperature and electron number density are presented in Figure 2. It should be noted that the electron number density belongs to the region of predominant emission of the Hα line, which does not coincide with the discharge axis. The difference between the measured and true values of the electron number density on the discharge axis is maximal in the non-isobaric jet section directly in front of the Mach disk, where the maximum pressure difference between the axis and the jet periphery is achieved. However, in the cross-sections used for calculating the gas-dynamic parameters (the bottom and outlet of the capillary, as well as the zone behind the Mach disk), which can be considered isobaric with a sufficient degree of accuracy, the difference between the measured and true values of electron number density does not exceed 20%.

![Figure 2. Longitudinal profiles of electron number density, $n_e$, and electron temperature, $T_e$, in the regions inside and outside the capillary: (1) position of the capillary outlet, (2) position of the central shock wave, (3) electron number density, (4), (5), (6) - electron temperature measured (4) by the method of relative intensities of the Hα line and continuum, (5) by the method of relative intensities of the lines of carbon ions C II and (6) of the lines of copper ions Cu II. The capillary diameter is $d=1$ mm.](image-url)
It can be seen (Figure 2) that the electron number density and temperature profiles vary along the jet axis in accordance with local values of the flow pressure and mass density, whose minima and maxima are located respectively before and after the central shock wave (Mach disk). Noteworthy is the good quantitative agreement between the estimates of the electron temperature in the vicinity of the Mach disk based on the C II and Cu II lines, which is additional confirmation of the predominant emission of these lines in the discharge axis. The estimates accuracy of the electron temperature obtained on the basis of the method of relative intensities of the Hα line and the continuum in the nonisobaric flow region and in the vicinity of the Mach disk is noticeably lower due to the continuum low intensity, as well as the mismatch between the zones of the predominant emission of the continuum and the Hα line. These obstacles are absent in the region inside the capillary, where the main emission fraction belongs to the continuum. The longitudinal temperature profile in this spatial domain agrees with good accuracy with the model of expendable nozzle [6,7].

4. Estimation of gas-dynamic parameters

Before proceeding to estimates of gas-dynamic parameters, we note the factors simplifying the calculation procedure. A specific feature of a capillary discharge is the spatial heterogeneity of the plasma chemical composition associated with the entry of the evaporated substance of the capillary wall into the discharge volume. In the case when a hydrocarbon polymer (PMMA, in particular) is used as the wall material, the composition of the axial discharge zone is determined primarily by hydrogen [2,3]. This circumstance allows us to consider the plasma of the axial zone to be single-ionized, \( \alpha = n_e / (n_i + n_e) \approx 1 \), and its chemical composition unchangeable over the entire considered flow section. In addition, in the region inside the capillary, its state is close to the thermal \( (T_e \approx T_i, a, a) \) and thermodynamic (Saha-Boltzmann) equilibrium. The plasma is isothermal due to the small equalization time of the electron and ion temperatures \( (\tau_{ei} \approx 3 \text{ ns}) \) compared with the gas-dynamic time \( (\tau_g \geq 50 \text{ ns}) \).

Plasma flow outside the capillary is characterized by a violation of both thermal and ionization equilibrium \([1,8]\). In the range of investigated discharge parameters, the ionization composition of the plasma is frozen starting from the vicinity of the capillary outlet, since the ionization and recombination times become comparable with the gas-dynamic time, \( \tau_e, \tau_r \approx \tau_g \). This allows us to extend the previously accepted assumption \( \alpha \approx 1 \) to the initial section of the plasma jet. Taking into account this approximation, as well as the condition of quasi-neutrality, \( n_e = n_i = n \), the equation of state of the plasma under conditions of the thermal equilibrium violation can be represented in the form

\[
p = n_e k T_e + n_i k T_i = 2nk \tilde{T}
\]

where \( \tilde{T} = \frac{T_e + T_i}{2} \) is the plasma characteristic temperature.

The initial system of equations used for estimation of the plasma jet parameters on the shock-wave section includes two relations between characteristic temperatures and mass densities before and after the central shock wave \([8]\]

\[
\tau = \frac{\rho_2}{\rho_1} = 2(y - 1) \left( \frac{1 + \frac{1}{2}M_i^2}{1 - \frac{1}{2}M_i^2} \right) \left( \frac{2y}{y-1}M_i^2 \right) \left( \frac{T_e}{T_i} \right)
\]

\[
\rho = \frac{\rho_2}{\rho_1} = \frac{n_2}{n_1} = \frac{y+1}{2} \left( \frac{M_i^2}{1 + \frac{1}{2}M_i^2} \right)
\]

and two equations for the critical speed of sound, whose value is determined from the one hand by the plasma parameters in the central shock wave

\[
a^2_{cr} = \frac{p_2 - p_1}{\rho_2 - \rho_1}
\]

and from the another hand by the plasma parameters in the critical cross-section (at the capillary outlet)

\[
a^2_{cr} = y \frac{p_a}{\rho_a}
\]
Here \( \rho \) - the mass density \((\rho \approx \mu m_p n)\), \( \mu \) - the effective molar mass of the plasma in the axial zone of the discharge \((\text{in the calculations, } \mu = 3 \text{ is taken})\), \( m_p \) - the proton mass, \( n \) - the electron number density \((\text{for simplicity we omit the index } 'e')\), \( \gamma \) - the adiabatic index, \( M \) - the Mach number, \( p \) - the static pressure. The indices “1” and “2” refer respectively to the zones before and after the shock wave, the index “a” refers to the cross-section at the capillary outlet.

Solving system (1) - (5), we obtain the dependence for determining the Mach number immediately before the shock wave \((\text{from the measured values of the electron number density on the capillary outlet and behind the shock wave})\)

\[
\rho_a = \frac{n_a}{n_2} = \frac{1}{M_1^2} \left( \frac{1 + \frac{1}{2} \frac{M_1^2}{\gamma - 1}}{\frac{1}{2} \frac{M_1^2}{\gamma - 1}} \right)^{\frac{\gamma}{\gamma - 1}}. \tag{6}
\]

The obtained value of \( M_1 \) is used to determine the relations of the characteristic temperatures (2) and mass densities (3) in the shock wave, of the Mach number \( M_2 \) behind the shock wave

\[
M_2^2 = \frac{1 + \frac{1}{2} \frac{M_1^2}{\gamma - 1}}{\frac{1}{2} \frac{M_1^2}{\gamma - 1}}, \tag{7}
\]

of the relation between the characteristic temperatures at the capillary outlet and behind the shock wave

\[
\tau_a = \frac{T_a}{T_2} = \frac{\gamma + 1}{\gamma - 1} \frac{M_2^2}{2 \gamma - 2 \gamma M_1^2 - 1}, \tag{8}
\]

and other parameters — the gas temperatures \((T_1 \text{ and } T_2, \text{ taking into account our definition in (1) of the characteristic temperature})\), the axial value of the electron number density \( n_1 \) before the shock wave, the critical speed of sound, the flow velocity before and after the shock wave.

The effective adiabatic index can be determined from the measured values of the electron number density or temperature in the stagnation zone \((\text{at the bottom of the capillary})\) and the critical cross-section \((\text{at the capillary outlet})\) using the relations for the expendable nozzle \([7]\), whose validity for capillary discharge experimentally confirmed in \([6]\):

\[
\frac{n_a}{n_0} = (\gamma + 1)^{1/\gamma} \tag{9}
\]

\[
\frac{T_a}{T_0} = (\gamma + 1)^{-\frac{\gamma - 1}{\gamma}} \tag{10}
\]

where the index “0” corresponds to the parameters measured in the stagnation zone \((\text{capillary bottom})\). Estimation of the effective adiabatic index gives a value \( \gamma = 1.25 \pm 0.05 \).

The results of estimates of the plasma jet parameters in the shock-wave region are summarized in Table 1. There is also given the value of the Mach number \( M_1 \) before the shock wave, calculated from the distance between the central shock wave and capillary edge according to relation \([9,10]\)

\[
\frac{x_c}{d} = M_1^{\frac{1}{\gamma - 1}} \times \left( \frac{\gamma - 1}{2} \right)^{\frac{1}{\gamma - 1}} \frac{y + 1}{4.8y} \tag{11}
\]

where \( x_c \) - the distance between the Mach disk and the capillary edge, \( d \) - the capillary diameter. It can be seen that estimates of the Mach number \( M_1 \) obtained in various ways are numerically consistent with each other.
Table 1. The estimates of the flow parameters.

| Position | $n_e$ | $T_e$ | $T_1$ | $p$ | $M(\rho_a)$ | $M(x_C/d)$ | $w$ |
|----------|-------|-------|-------|-----|-------------|-------------|-----|
| 0        | $23^a$ | 3.3$^a$ | 3.3 | 24.2 | 0 | - | 0 |
| a        | $10^5$ | 2.9$^a$ | 2.9 | 10.6 | 1 | 1 | 13.6 |
| 1        | 0.34 | 1.6 | 1.1 | 0.6 | 3.8$^b$ | 4.3$^c$ | 31.3 |
| 2        | 1.8$^a$ | 2.5 | 3.2 | 3.9 | 1.8 | 0.41 | 0.39 | 5.9 |

$^a$ - experimental values of the parameters used as initial data in the calculations
$^b$ – calculation by formula (6)
$^c$ – calculation by formula (11)

In the shock-wave region of the flow, the electron temperature differs significantly from the gas temperature, $T_e \neq T_{i,a}$, which is consistent with the results of theoretical studies [8,11]. In the expansion section, the electron temperature exceeds the temperature of the heavy component, $T_{i,e} > T_{i,a}$, whose effective cooling is due to the flow expansion. Directly before the Mach disk, the temperature difference reaches $T_{i,e} - T_{i,a} \approx 1$ eV. Since the heavy component undergoes rapid heating in the shock wave, the reverse inequality $T_{2,e} < T_{2,i,a}$ takes place immediately after the Mach disk, with $T_{2,i,a} - T_{2,e} \approx 1.4$ eV. A heating of the electrons in this region takes place with a time delay, during collision with ions and atoms. In this case, the characteristic temperature immediately after the shock wave is slightly lower than the stagnation temperature measured at the bottom of the capillary, $T_2 < T_{0,e,i}$. The pressure before the shock wave is significantly lower than atmospheric pressure, and immediately after the shock it noticeable exceeds atmospheric pressure. The last result shows that the approximate equality of atmospheric pressure and pressure behind the shock wave, adopted in [9] when obtaining dependence (10), does not hold in the general case, which, apparently, is one of the reasons for the difference in the estimates of the Mach numbers performed according to dependences (5) and (10).

5. Conclusions

Thus, knowledge of the minimum set of plasma parameters (electron number density and temperature), which are easily measured using spectral diagnostic methods, makes it possible to reveal the course of the most important parameters characterizing the gas dynamics of the nonequilibrium plasma flow in the initial section of a supersonic jet and to evaluate their magnitude. A fundamentally important point is the possibility of estimating the gas temperature and pressure — the plasma parameters that are difficult to measure in the absence of thermodynamic equilibrium and are of primary interest for many scientific and technical applications of nonequilibrium plasma flows.

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