Estimation of Critical Current of HTS RF-SQUID

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Abstract. The operation of the RF SQUID is restricted by the condition that the inductance parameter $\beta_L$ must be in the range of 1–3. However, since both ends of the Josephson junction (JJ) of RF-SQUID are shorted, it is difficult to non-destructively estimate the critical current ($I_C$). Thus, we proposed a technique for the non-destructive measurement of the $I_C$ of a high-temperature superconducting (HTS) RF-SQUID ring by evaluating the behaviour of the flux in superconducting thin films using a SQUID magnetometer. A superconducting ring sample with JJ was placed below the HTS SQUID magnetometer and cooled down to 77 K. The change in the SQUID output was monitored on application of the magnetic field. When increasing the field, the waveforms indicated that the screening current of the ring sample exceeded the $I_C$ of the JJ, and the JJ became a normal-conducting state. As a result, we estimated the $I_C$ of the JJ of this sample as 134 $\mu$A using the values of mutual inductance and the coupling coefficient $\alpha$ between the coil and the sample.

1. Introduction
One advantage of the high-temperature superconducting (HTS) RF-SQUID, which consists of a superconducting loop and a Josephson junction (JJ) is that it is easier to fabricate than an HTS DC-SQUID, which has two JJs. Another advantage is that it is resistant to static electricity because there is no direct electric wiring. Thus, the RF-SQUID has been used in an extensive range of applications [1–3].

The operation of the RF-SQUID is restricted by the condition that the inductance parameter $\beta_L = 2\pi I_C L_S/\Phi_0$ must be in the range of 1–3 [4], where $I_C$ is the critical current of JJ, $L_S$ is the inductance of the SQUID, and $\Phi_0$ is the flux quantum. Thus, the $I_C$ of the JJ should be precisely controlled when it is fabricated. However, it is difficult to evaluate the $I_C$ of the JJ by a four-probe method without breaking the superconducting loop of the RF-SQUID because both ends of the JJ are shorted.

Therefore, we focused on the change in the total magnetic flux in the superconducting loop to analyze the $I_C$. We expected that the amount of magnetic flux in a superconducting loop (ring sample) can be observed by evaluating the superconducting thin films using a SQUID magnetometer [5]. We proposed a method to non-destructively evaluate the $I_C$ of the RF-SQUID ring sample in this study.

2. Experimental details
2.1. Principle of Critical Current Estimation
The schematic of the experimental principle is presented in Figure 1. As illustrated in Figure 1(a), the coil generating the magnetic flux, superconducting ring sample to be measured, and RF-SQUID used to detect the magnetic flux are coaxially located. When a coil current $I_{coil}$ flows through the coil, magnetic flux (represented by a solid arrow) is applied to the sample and a screening current $I_{Sc}$ is induced in the
sample that generates a shielding flux (represented by a dotted arrow) in the direction that cancels the applied flux. In the range where \( I_{\text{SL}} \) is less than the \( I_C \) of the JJ of the sample, the applied flux from the coil is cancelled by the screening flux. As the \( I_{\text{coil}} \) increases, the applied flux increases, as indicated in Figure 1(b). The moment \( I_{\text{SL}} \) exceeds \( I_C \) of the JJ, the JJ becomes a normal-conducting state, and a number of fluxes close to \( L_S I_C / \Phi_0 \) threads to the ring sample. After entry of the fluxes, the \( I_{\text{SL}} \) decreases to zero and JJ is restored to a superconducting state. To understand the behaviour, the change in the \( I_{\text{SL}} \) and the SQUID output voltage for an increase in the \( I_{\text{coil}} \) is depicted in Figure 1(c) [6]. By detecting the magnetic flux entry into the ring sample with the RF-SQUID magnetometer, which is located above the ring sample, the \( I_C \) of the sample JJ can be estimated.

\[ I_{\text{SL}} < I_C \]
\[ I_{\text{SL}} = I_C \]

**Figure 1.** Schematic of the experimental principle. (a) \( I_{\text{SL}} < I_C \), (b) \( I_{\text{SL}} = I_C \), (c) Change of \( I_{\text{SL}} \) and \( V_{\text{out}} \) for \( I_{\text{coil}} \).

2.2. Sample preparation

The schematic of the superconducting ring sample for the evaluation is depicted in Figure 2. A YBCO thin film, having 200 nm thickness, was fabricated by an RF-magnetron sputtering on the SrTiO\(_3\) bicrystal substrate (10 mm \( \times \) 10 mm \( \times \) 0.5 mm, 30 degrees). The RF-SQUID pattern with a hole (100 \( \mu \)m \( \times \) 100 \( \mu \)m) and a JJ (4 \( \mu \)m width) was formed by photolithography and Ar ion-milling. This was employed as the superconducting ring sample for the experiment.
2.3. Experimental setup

The experimental setup is presented in Figure 3. The ring sample was sealed in a plastic capsule. A rectangular one-turn coil for magnetic field generation, with internal dimensions of 3.3 mm × 3.3 mm and a width of 0.35 mm, was adhered to the YBCO surface with grease. The YBCO HTS RF-SQUID magnetometer with a flux-focuser made by Jülicher SQUID GmbH was employed for the detection of the flux. The RF-SQUID was also capsuled and fixed coaxially, where the gap between the surface of the SQUID magnetometer and the surface of the ring sample was 4.6 mm, as exhibited in Figure 3(a). The RF-SQUID was driven by the SQUID electronics (JSQ Tiger Controller and HTSL-RF-SQUID-ELEKTRONIL Version 4.0) made by Jülicher SQUID GmbH. The capsule was cooled with liquid nitrogen in a Dewar vessel placed in a three-layered magnetically shielded cylinder (Figure 3(b)). A triangular wave current with a frequency of 100 Hz was made to flow in the coil for flux generation. The coil current via a shunt resistor and the SQUID output voltage were simultaneously observed on an oscilloscope and recorded.

3. Results and Discussions

Figure 4(a) displays the waveforms of the coil current and output of the RF SQUID. While the $I_{\text{coil}}$ increases monotonically, some flat areas can be observed in the SQUID output voltage $V_{\text{out}}$. This phenomenon is expected because the applied flux from the coil is cancelled out by the magnetic flux due to the screening current induced in the ring sample.
Figure 4. Experimental results. (a) Waveforms of Coil Current $I_{\text{coil}}$ and SQUID Output $V_{\text{out}}$, (b) Coil Current versus SQUID output.

Figure 4(b) depicts the waveform of the coil current versus the SQUID output converted from the time trace, illustrated in Figure 4(a). A Lowess smoothing filter with a span of 0.1 was applied to remove the effect of noise. The constant voltage steps can be observed in the range of $I_{\text{coil}}$ of ±271 μA and 767–1015 μA, whereas there were variations in $V_{\text{out}}$ to follow $I_{\text{coil}}$ in the rest of the region. The region of constant $V_{\text{out}}$ indicates that the screening flux generated in the ring sample cancelled out the applied flux. Furthermore, it is considered that when the $I_{\text{SL}}$ induced in the sample exceeded the $I_{\text{C}}$ of JJ, the flux threaded into the sample. Here, the $I_{\text{C}}$ of the JJ of the sample could be estimated by calculating the $I_{\text{SL}}$ using $I_{\text{coil}}$ (271 μA) and mutual inductance $M_i$ between the coil and the sample [5].

Figure 5 highlights each parameter used in the calculation and presents the relational expression of each parameter. Equation (1) displays the relation between the $I_{\text{coil}}$ and the $I_{\text{SL}}$.

$$I_{\text{SL}} = \frac{\Phi_{\text{SL}}}{L_{\text{SL}}} = \frac{M_i \times I_{\text{coil}}}{L_{\text{SL}}} = \frac{a \sqrt{I_{\text{coil}} \times L_{\text{SL}}}}{L_{\text{SL}}} \times I_{\text{coil}}$$ (1)

- $I_{\text{SL}}$: Screening current induced in the ring sample.
- $\Phi_{\text{SL}}$: Magnetic flux penetrated to the sample.
- $L_{\text{SL}}$: Inductance of the sample.
- $L_{\text{coil}}$: Inductance of the coil.
- $M_i$: Mutual inductance between the sample and the coil.
- $\alpha$: Coupling coefficient between the sample and the coil.
- $I_{\text{coil}}$: Coil current.

Figure 5. Parameters for calculation of $I_{\text{SL}}$.

The pattern of the superconducting ring sample has a square hole (100 μm × 100 μm) in the center (see Figure 2). The inductance of the hole can be described as $L_{\text{SL}} = 1.25 \mu_l D^2$, where $\mu_l$ represents the vacuum permeability and $D$ denotes the length of one side of a square. Using this formula, $L_{\text{SL}}$ was calculated as 157 pH. The inductance of the coil was found to be 3.84 nH, by assuming that it is a...
circular one turn coil having the same area. The $\alpha$ represents a coupling coefficient of the flux threading into the sample from the coil. In this calculation, we assumed $\alpha = 0.1$. Equation (2) shows the calculation of $I_C$ by substituting $I_{\text{coil}}$ of 271 $\mu$A.

$$I_C = \frac{a\sqrt{I_{\text{coil}} \times I_{\text{SL}}}}{I_{\text{SL}}} \times I_{\text{coil}} = \frac{0.1 \times \sqrt{\left(3.84 \times 10^{-9}\right) \times \left(157 \times 10^{-12}\right)}}{157 \times 10^{-12}} \times (271 \times 10^{-6}) = 134 \text{ [\muA]}$$  \hspace{2cm} (2)

The $I_C$ of the JJ of this sample was estimated to be 134 $\mu$A. To calculate the accurate $I_C$, the $\alpha$ must be determined by the experiment. We may determine the accurate $I_C$ by defining the $\alpha$ by cutting the superconducting loop of the sample and measuring the $I_C$.

4. Conclusions
We proposed a technique for the non-destructive measurement of the $I_C$ of an RF-SQUID ring by evaluating the behaviour of the flux in superconducting thin films using a SQUID magnetometer. The RF SQUID magnetometer was placed above the superconducting ring sample, which has one JJ, and the magnetic field was applied to the sample. At a lower field, a constant voltage region was observed on the waveform of the coil current versus the SQUID output curve, which suggested that the screening current $I_{\text{SL}}$ was induced in the ring sample and cancelled the applied flux. When the field was increased, the resulting waveforms suggested that the $I_{\text{SL}}$ exceeded the critical current $I_C$ of the JJ, and the JJ became a normal-conducting state. Finally, the $I_C$ of the JJ of this sample was estimated as 134 $\mu$A after considering the mutual inductance and the coupling coefficient $\alpha$ between the coil and the sample. In the future, we plan to establish the non-destructive estimation method for the $I_C$ of RF-SQUID by defining the $\alpha$ precisely.

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