Stability Analysis of 3D flow over a deforming surface with suction effect: A Buongiorno’s model

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Abstract: In this article, three-dimensional boundary layer flow is considered with zero flux boundary condition. The finalized system of differential equation are solved using bvp4c function. Dual solutions are explored and stability analysis is performed to know the stability of the outcoming solutions. The regressions analysis is carried out for the local Nusselt number. The impact of various pertinent parameter are observed graphically for the skin friction coefficient, the local Nusselt number and for some sample graph of velocity, temperature and concentration profiles. It is found that the first solution is stable and the impact of Brownian motion parameter is almost negligible on the heat transfer rate.

Keywords: Realistic approach, Permeable shrinking/stretching sheet, Stability and Regression analyses.

1. Introduction

The analysis of the heat and mass transfer for a permeable stretching surface was investigated by Gupta and Gupta [1], while Miklavčič and Wang [2] discussed the shrinking flow for the viscous model whereas three-dimensional flow has been explored first time by Wang [3] for the flat stretching surface. Devi et al. [4] then examined for unsteady 3D flow. Gorla et al. [5] discussed the 3D flow for the nanofluid model, while dual solutions have been obtained for three-dimensional MHD flow using the nanofluid model by Raju et al. [6]. Kuznetsov and Nield [7] revised their model using passive control of nanoparticles. Later on, Yadav et al. [8] used the revised model approach by considering no particle flux on the boundaries for nanofluid model in a rotating porous medium with various physical aspects. Recently, Jahan et al. [9] studied the nanofluid revised model for permeable deforming sheet. Since we have explored dual solutions, so it is important to check the reliability of outcoming solutions. In this regard, Merkin [10] was the first scholar who discussed the stability of solutions. Later, Weidman et al. [11] did the stability analysis and reported that first is solution is stable as compared to the second solution. Ishak [12] considered stability analysis for the shrinking sheet. Recently, Jahan et al. [9] effectively applied the stability analysis utilizing nanofluid model.

The aim of this article is to explore the physical reliability of outcoming solutions and to know the effect of Brownian movement on heat transfer rate utilizing sensible approach for Buongiorno’s model. The governing equations are transformed into ordinary differential equations (ODEs) using appropriate similarity transformations and afterward tackled numerically. The stability of the multiple solutions is checked through the stability inspection. In last, the impact of various governing parameters on the flow and heat transfer characteristics are done.
2. Mathematical formulation

Consider a 3D flow of a nanofluid past a permeable deforming flat sheet. It is presumed that the flat surface is stretched/shrinking consistently in the both x- and y-directions with the velocities 
\[ u(x,t) = u_w(x,t) \] and 
\[ v(y,t) = v_w(y,t), \]
respectively. It is additionally expected that mass flux velocity is \( w = w_0 \). Beneath these situations, the boundary layer equations are (Devi et al. [4]):

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = v \frac{\partial^2 u}{\partial z^2}, \tag{2}
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = v \frac{\partial^2 v}{\partial z^2}, \tag{3}
\]

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha \frac{\partial^2 T}{\partial z^2} + \beta \left( D_B \frac{\partial C}{\partial z} + \left( \frac{DT}{T_e} \right) \right)^2, \tag{4}
\]

\[
\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + w \frac{\partial C}{\partial z} = D_B \frac{\partial^2 C}{\partial z^2} + \left( \frac{DT}{T_e} \right) \frac{\partial^2 T}{\partial z^2}, \tag{5}
\]

along with the initial and boundary conditions

\[
t < 0: \begin{cases} v = 0, \quad u = 0, \quad w = 0, \text{ for any } x, y, z \\ T = T_e, \quad C = C_{\infty} \end{cases} \tag{6}
\]

\[
t \geq 0: \begin{cases} v = v_w(y,t) = a \lambda x, \quad u = u_w(x,t) = a \lambda y, \\ w = w_0 \text{ at } z = 0, \\ T = T_e, \quad D_B \frac{\partial C}{\partial z} + \frac{D_T}{T_e} \frac{\partial T}{\partial z} = 0 \\ w = 0, \quad u = 0, \quad T = T_e, \quad C = C_{\infty} \text{ as } z \to \infty \end{cases} \tag{7}
\]

Here \( u, v \) and \( w \) are the velocity components alongside \( x-, y-, \) and \( z- \) directions, respectively. \( t \) is the time, \( \nu \) is the kinematic viscosity, \( a \) is positive constant, \( \lambda \) is the stretching \((\lambda > 0)\) or shrinking \((\lambda < 0)\) parameter. Here the boundary condition \( C = C_{\infty} \) is replaced with \( D_B \frac{\partial C}{\partial z} + \frac{D_T}{T_e} \frac{\partial T}{\partial z} = 0 \) by following Kuznetsov and Nield [7].

In order to get the ordinary differential equations, used the accompanying similarity variables:

\[
\eta = \left( \frac{a}{\nu} \right) \frac{1}{2} z, \quad \theta(\eta) = \frac{T - T_e}{T_e - T_{\infty}}, \quad \phi(\eta) = \frac{C - C_{\infty}}{C_{\infty}} \tag{8}
\]

where primes denote differentiation with respect to \( \eta \). Substituting equation (8) into equations (1) to (5), we get:

\[
f''' + (f + g) f'' - f' = 0, \tag{9}
\]

\[
g''' + (f + g) g'' - g' = 0, \tag{10}
\]

\[
\frac{1}{\Pr} \theta''' + (f + g) \theta'' + Nb \phi \theta' + Nt \theta'^2 = 0, \tag{11}
\]
subject to the boundary conditions

\[ f(0) = S, g(0) = 0, \quad f'(0) = h_0, g'(0) = h_1, \quad \theta(0) = 1, \quad N_t \phi(0) + N_t \theta(0) = 0 \]

\[ f'(\eta) \rightarrow 0, \quad g'(\eta) \rightarrow 0, \quad \theta(\eta) \rightarrow 0, \quad \phi(\eta) \rightarrow 0 \quad \text{as} \ \eta \rightarrow \infty \]

Here \( S \) is the surface mass transfer parameter with \( S > 0 \) for suction, \( \Pr \) is the Prandtl number, \( \Sc \) is the Schmidt number, \( Nt \) is the thermophoresis parameter and \( Nb \) is the Brownian motion parameter, which are defined as:

\[ S = -\frac{w_0}{\sqrt{\text{av}}}, \ \Pr = \frac{v}{\alpha}, \ \Sc = \frac{v}{D_g}, \ Nb = \frac{\beta D_g(C_a)}{v}, \ \Nt = \frac{\beta D_b(T_u - T_a)}{T_v}. \]

The skin friction coefficients \( C_{f_x} \) and \( C_{f_y} \) and the local Nusselt number \( N_{ux} \) in finalized form are:

\[ C_{f_x} \Re_x^{1/2} = 2f'(0), \quad C_{f_y} \Re_y^{1/2} = 2g'(0), \quad N_{ux} \Re_x^{1/2} = -\theta'(0) \]

in which \( \Re_x = u_w(x,t)x/v \) and \( \Re_y = v_w(x,t)y/v \) represent the local Reynolds numbers.

3. Flow Stability

To test the stability of outcoming solutions, we present the new dimensionless time variable \( \tau = at \).

Here \( \tau \) is related to a preliminary value problem and is consistent with the query of which solution may be acquired in practice (physically realizable). So redefine the equation (8) as:

\[ \phi(\eta, \tau) = \frac{C - C_a}{C_a}, \ \eta = (vt)^{1/2}, \ \tau = at \]

with the goal that Eqs. (2) - (5) can be composed as

\[ \frac{\partial^3 f}{\partial \eta^3} + (f + g) \frac{\partial^3 f}{\partial \eta^2} \left( \frac{\partial f}{\partial \eta} \right)^2 - \frac{\partial^3 f}{\partial \eta^2 \partial \tau} = 0 \]

\[ \frac{\partial^3 g}{\partial \eta^3} + (f + g) \frac{\partial^3 g}{\partial \eta^2} \left( \frac{\partial g}{\partial \eta} \right)^2 - \frac{\partial^3 g}{\partial \eta^2 \partial \tau} = 0 \]

\[ \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial \eta^2} + (f + g) \frac{\partial^2 \theta}{\partial \eta} + Nb \frac{\partial \theta}{\partial \eta} + \Nt \frac{\partial^2 \theta}{\partial \eta^2} + \Nt \frac{\partial \theta}{\partial \tau} = 0 \]

\[ \frac{\partial^2 \phi}{\partial \eta^2} + \Sc (f + g) \frac{\partial \phi}{\partial \eta} + \Nt \frac{\partial \phi}{\partial \eta} - \Sc \frac{\partial \phi}{\partial \tau} = 0 \]

corresponding to boundary conditions

\[ f(0, \tau) = S, g(0, \tau) = 0, \ \frac{\partial f(0, \tau)}{\partial \eta} = h_0, \ \frac{\partial g(0, \tau)}{\partial \eta} = h_1, \ \theta(0, \tau) = 1, \ \Nt \frac{\partial \phi(0, \tau)}{\partial \eta} + \Nt \frac{\partial \theta(0, \tau)}{\partial \eta} = 0 \]

\[ \frac{\partial f(\eta, \tau)}{\partial \eta} \rightarrow 0, \ \frac{\partial g(\eta, \tau)}{\partial \eta} \rightarrow 0, \ \theta(\eta, \tau) \rightarrow 0, \ \phi(\eta, \tau) \rightarrow 0 \quad \text{as} \ \eta \rightarrow \infty \]

Introducing the following terms as follow (see Weidman and Sprague [13]):

\[ f(\eta, \tau) = f_0(\eta) + e^{\tau \sigma} F(\eta), \ g(\eta, \tau) = g_0(\eta) + e^{\tau \sigma} G(\eta), \ \theta(\eta, \tau) = \theta_0(\eta) + e^{\tau \sigma} H(\eta), \ \phi(\eta, \tau) = \phi_0(\eta) + e^{\tau \sigma} M(\eta) \]
in which $\gamma$ is the obscure eigenvalue parameter, $F(\eta), G(\eta), H(\eta), M(\eta)$ are small relative to $f_0(\eta), g_0(\eta), \theta_0(\eta)$ and $\phi_0(\eta)$. Switching equation (22) into Eqs. (17)-(21) and then setting $\tau = 0$, we attain the succeeding linearized problem:

\begin{align*}
F_0''' + (f_0 + g_0)F_0' + (F_0 + G_0)f_0 + (-2f_0 + \gamma)F_0' &= 0 \\
G_0''' + (f_0 + g_0)G_0' + (F_0 + G_0)g_0 + (-2g_0 + \gamma)G_0' &= 0 \\
\frac{1}{Pr} H_0''' + (f_0 + g_0)H_0' + (F_0 + G_0)\theta_0' + Nb\theta_0'M_0' + Nb\phi_0'H_0' + 2Nt\theta_0'H_0' + \gamma H_0 &= 0 \\
M_0'' + Sc[(f_0 + g_0)M_0' + (F_0 + G_0)\phi_0'] + \frac{Nt}{Nb} H_0'' + \gamma ScM_0 &= 0
\end{align*}

subject to boundary conditions

\begin{align*}
F_0(0) &= 0, \\
G_0(0) &= 0, \\
F_0'(0) &= 0, \\
G_0'(0) &= 0, \\
H_0(0) &= 0, \\
NbM_0'(0) + NtH_0'(0) &= 0 \\
F_0'(\eta) &\to 0, G_0'(\eta) &\to 0, \\
H_0(\eta) &\to 0, \\
M_0(\eta) &\to 0
\end{align*}

as $\eta \to \infty$ (27)

For a set value of $\gamma$, we solve the system (23)-(26) subject to (27) alongside new boundary circumstances $F_0''(0) = 1$ (see Harris et al. [14]).

4. Results and Discussion

The system of nonlinear ODEs (9)-(12) along with the boundary conditions (13) are comprehended numerically with the bvp4c function. Flow parameters that will be discussed are the Brownian motion parameters ($Nb$), thermophoresis parameters ($Nt$), Schmidt number ($Sc$) and suction /injection parameters ($S$). Tables 1-3 are drawn for smallest eigenvalues, critical values and for the validity of the calculated results, respectively. Figures 1-4 are drawn for the physical parameter of the skin friction coefficient and the local Nusselt number. We have seen that for higher values of the suction parameters, the skin friction coefficient increases monotonically as depicted in Fig. 1. It is important to note the effect of Brownian motion parameter on the local Nusselt number. It is observed that either we increase or decrease the value of Brownian motion parameter, the heat exchange rate does not get influenced as shown in Fig. 2. It means that the reduced Nusselt number is relatively autonomous of the Brownian motion parameter. In Fig. 3, the heat transfer rate decreases monotonically for thermophoresis parameter. Figure 4 shows that Schmidt number reduces the heat transfer rate but its effect is not high.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{image.png}
\caption{Variation of the skin friction coefficient $Re^{1/2} C_f$ for some values of $S$ when $Pr = 6.2, Nb = 0.5 = Nt, Sc = 2$.}
\end{figure}
Fig. 2. Variation of local Nusselt number with $\lambda = \lambda_1$ for different estimation of $Nb$ at the point when $S = 1, Nt = 0.5, \text{Pr} = 6.2, Sc = 2$.

Fig. 3. Variation of local Nusselt number with $\lambda = \lambda_1$ for various values of $Nt$ when $S = 1, Nb = 0.5, \text{Pr} = 6.2, Sc = 2$.

Fig. 4. Variation of local Nusselt number for various values of $Sc$ when $S = 1, Nt = 0.5 = Nt$, $\text{Pr} = 6.2$.

As dual solutions exist so it is imperative to recognize the stability of the more than one solutions. Imperative For this reason, stability analysis is performed for the upper (first) and lower (second) branch solutions. We calculated the smallest eigenvalues and it is observed that positive eigenvalues produce an initial decay of disturbances, and that is why the flow is more stable, while negative smallest eigenvalues result in an initial increase of disturbances, because of that reality that the flow ends up flimsy. The turning from positive to negative values can be observed in the Fig. 5 that has been drawn for the skin friction coefficient towards the suction parameter. Further, Table 1 depicts the smallest eigenvalues for numerous values of $\lambda$ and we has seen that smallest eigenvalues are
positive for the primary (first) solution and negative for the second one solution, so physically it means that the first solution is stable while the second solution is unstable. Actually, equations (23)-(27) give an infinite set of eigenvalues $\gamma_1 < \gamma_2 < \ldots$; so if the littlest eigenvalue is positive then there occur an initial decay and if the smallest eigenvalue is negative, then growth in the disturbances will take place and flow will be unstable. Table 2 is calculated to find the critical values for the stretching/shrinking parameter and the suction parameter. To validate our calculated results, the comparison is made using different numerical techniques in Table 3. It is found that for numerous values of $S, \lambda, \lambda_t$, bvp4c function and shooting technique give the same results that increase our confident about the validity and correctness of the solutions.

![Graph](image)

**Fig. 5.** Variation in the skin friction coefficient against $S$ when $Pr = 6.2, \lambda = \lambda_t = -0.1, Nt = 0.5 = Nr, Sc = 2$.

**Table 1.** Smallest eigenvalues $\gamma$ at numerous values of $S$ and $\lambda$.

| $S$ | $\lambda = \lambda_t$ | Upper branch (first solution) | Lower branch (second solution) |
|-----|------------------------|-------------------------------|--------------------------------|
| 1   | 0.5                    | 0.7785                        | -0.1068                        |
| 1   | -0.1                   | 0.1957                        | -0.025                         |
| 1   | -0.15                  | 0.0134                        | -0.013                         |
| 2   | 0                      | 1.0987                        | -0.1061                        |
| 2   | -0.1                   | 0.9915                        | -0.1001                        |
| 2   | -0.3                   | 0.7487                        | -0.0789                        |
| 2   | -0.5                   | 0.3954                        | 0.0065                         |
| 2.5 | -0.3                   | 1.3461                        | -0.1094                        |
| 2.5 | -0.5                   | 1.0896                        | -0.094                         |

**Table 2.** Values of $\lambda$, $S$, at different values of $\lambda$ and $S$.

| $S$ | $\lambda = \lambda_t$ | $S_\gamma$ | $\lambda_t$ |
|-----|------------------------|-------------|-------------|
|     |                        |             |             |
| $S$ | $\lambda$ | $\lambda_i$ | $f''(0)$ | $g''(0)$ | $-\theta^*(0)$ | $f''(0)$ | $g''(0)$ | $-\theta^*(0)$ |
|-----|---------|---------|---------|---------|-------------|---------|---------|-------------|
| 1   | 1       | 0.5     | -1.71424| -0.79468| 5.136454    | -1.71424| -0.79468| 5.136454    |
| 3   | 1       | 0.5     | -3.36892| -1.64892| 12.95847   | -3.36892| -1.64892| 12.95847   |
| -1  | 1       | 0.5     | -0.68017| -0.24907| 0.217793   | -0.68017| -0.24907| 0.217793   |
| 1   | -0.12   | -0.1    | 0.09284 | 0.078995 | 3.952426   | 0.09284 | 0.078995 | 3.952426   |
| 3   | -0.12   | -0.1    | 0.353054| 0.294555 | 12.46706   | 0.353054| 0.294555 | 12.46706   |

Table 3. Numerical values of $f''(0), g''(0)$ and $\theta^*(0)$ for comparison purpose

The temperature distribution is almost independent of the Brownian motion parameter effect. This can be seen in Figs. 6 whereas distribution of nanoparticles decrease for $Nb$ as shown in Fig. 7. Figure 8 shows that as we increase suction, the temperature profile starts to decrease.

Fig. 6. Temperature distribution $\theta(\eta)$ for some values of $Nb$ when $S = 1, \text{Nt} = 0.5, \lambda = \lambda_i = -0.1, \text{Pr} = 6.2.$
Fig. 7. Influence of $Nb$ on the concentration profile $\phi(\eta)$ when
$$\lambda = \lambda_1 = -0.1, Nt = 0.5, S = 1, Pr = 6.2.$$ 

Fig. 8. Influence of $S$ on the Temperature distribution $\theta(\eta)$ when
$$\lambda = \lambda_1 = -0.1, Nb = 0.5 = Nt, Pr = 6.2.$$ 

### 4.1 Nusselt number estimation

The assessed equation for the local Nusselt number is (Nield and Kuznetsov [15])

$$Nu_{est} = Nu_r + C_b Nb + C_i Nt + C_{bb} Nb^2 + C_{rt} Nt^2 + C_{bt} Nb Nt$$

(28)

Here $C_b$, $C_i$, $C_{bb}$, $C_{rt}$, and $C_{bt}$ are the coefficients in quadratic regression. At various values of $Pr$ and $Sc$, we performed the quadratic regression for 30 sets of values of $Nb$ and $Nt$ inside the range of $[0.1,0.2,0.3,0.4, 0.5]$. The Adjusted $R^2$ is also calculated where the most extreme relative error is observed through this expression $\varepsilon = \frac{|Nu - Nu_{est}|}{Nu}$. This regression is valid in the range $[0, 0.5]$ only. From Table 4, it is obvious that Brownian movement parameter has no impact on the local Nusselt number and it coincides with Fig. 2. It is also determined that heat transfer rate diminishes with the boom of thermophoresis parameter. This conduct matches with graphical outcome displayed in Fig. 3.

**Table 4.** Detail of quadratic regression coefficients with adjusted $R^2$ and maximum relative error when $\lambda = \lambda_1 = 0.5$.

| $Pr$ | $Sc$ | $Nu_r$ | $C_b$   | $C_i$   | $C_{bb}$ | $C_{rt}$ | $C_{bt}$ | Adjusted $R^2$ | $\varepsilon$ |
|------|------|--------|---------|---------|----------|----------|----------|----------------|-------------|
| 1    | 10   | 1.335446 | 0.00000 | -0.56919 | 0.00000  | 0.134385 | 0.00000  | 1              | 6.84E-05    |
| 10   | 10   | 10.40924 | 0.00000 | -20.7363 | 0.00000  | 15.78091 | 0.00000  | 0.999806      | 0.010501    |
5. Conclusion
In this paper, we have investigated the heat transfer in a nanofluid for 3D flow past a deforming sheet with suction effect using passive control of the nanoparticles fraction on the boundary. The system of nonlinear ODEs is solved with an efficient bvp4c function, numerically. The heat transfer rate does not vary with the variation of Brownian motion parameter. While the local Nusselt number abatements with increment in $N_t$. The same behavior is seen through quadratic regression for $Nb$ and $Nt$ for local Nusselt number. Stability of the solutions obtained, is checked through the method of stability analysis. It is observed that positive eigenvalues create an initial decay of disturbances, while negative eigenvalues generate an initial growth of disturbances. Thus that is why the first branch solution is stable whereas the second branch solution is unstable.

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