Mapping the train model for earthquakes onto the stochastic sandpile model

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Abstract. We perform a computational study of a variant of the “train” model for earthquakes [Phys. Rev. A 46, 6288 (1992)], where we assume a static friction that is a stochastic function of position rather than being velocity dependent. The model consists of an array of blocks coupled by springs, with the forces between neighbouring blocks balanced by static friction. We calculate the probability, \( P(s) \), of the occurrence of avalanches with a size \( s \) or greater, finding that our results are consistent with the phenomenology and also with previous models which exhibit a power law over a wide range. We show that the train model may be mapped onto a stochastic sandpile model and study a variant of the latter for non-spherical grains. We show that, in this case, the model has critical behaviour only for grains with large aspect ratio, as was already shown in experiments with real ricepiles. We also demonstrate a way to introduce randomness in a physically motivated manner into the model.

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1 Introduction

Earthquakes are the subject of intensive study. The interest in the study of these phenomena can be either academic, triggering the development of models which help to unveil their basic characteristics, or highly practical, with the objective of minimising the effects of massive destruction caused by extreme events. In line with practical thinking, modelling can be of importance when one tries to understand the dynamical causes of earthquakes, with the eventual goal being that of forecasting. The successful achievement of this goal would have obvious economic consequences, especially for some of the heavily populated regions which are subject to seismic activity. One difficulty which is present in the study of earthquakes is the impossibility of direct observational access to the microscopic dynamics of tectonic faults, leaving us without a complete physical description of their occurrence. The only data that can be obtained are the result of measurements of crust displacements and of seismic waves, rather than of the close-up dynamics of the fault movements.

Any model or theory that intends to explain the occurrence of earthquakes should also give a satisfactory explanation of the large quantity of statistics which have been gathered. It is well known that real earthquakes follow empirical power laws for the frequency distribution of their magnitudes and for the temporal decay of aftershock rates. These laws are known as the Gutenberg-Richter scaling law and Omori’s law [1, 2].

Various models based on the observed evidence for large scale motions of the earth’s crust, that is, on the dynamics of tectonic plates sliding slowly against each other, have been proposed to investigate earthquakes [3–7]. The treatment of these problems from the point of view of friction leads to a nonlinear description of the phenomena. To describe the mechanisms of continuous media we generally would solve a set of partial differential equations, but these solutions can become extremely expensive in terms of computational time. It is usual to simplify the problem by picturing locations on opposite sides of a fault as a two-dimensional network of masses connected by springs, which model the elastic interactions, and pinned down by static friction. Once we have discretised the problem, we are also free to use cellular automata techniques, which allow us to work with larger systems at less computational cost.

In 1967, Burridge and Knopoff [3] proposed a model based on the picture described above, which imitates the “stick-slip” dynamics of real events [8, 9], and whose dynamics could be solved numerically, referred to in this manuscript as the BK model. The system they studied consisted of a chain of masses coupled by springs and in contact with a rough surface. Each connecting spring represents a continuous section of a fault line, giving a linear...
elastic coupling between the blocks that represent the irregularities on the fault interface. This was the precursor of the train model, where the spring connected to the first block generates the instability. The results for the BK model and others developed subsequently use a velocity dependent friction force to produce a dynamical instability, which then leads to complex dynamical behaviours [10,11]. Another model which is phenomenologically similar to real seismic faults was proposed by Olami et al [4], and is known as the OFC model. This model uses a dynamical field, which is thought to represent local forcing, over a regular lattice. The value of the field is updated synchronously over the whole lattice at discrete time intervals, rising monotonically and uniformly up to a threshold value. Once this threshold is reached at one site, the site is said to become unstable and the value of the field at this site is reset to some residual value, often taken as zero, while a fraction of the value by which it is decreased is distributed among its neighbours. If this fraction is smaller than unity, the model is said to be non-conservative. Complex behaviour can arise due to clustering and synchronisation of the field variable, leading to a cascade of sites becoming unstable in sequence. The choice of neighbours of a site can either be performed once, obeying the regular metric of the lattice, or randomly, a new list of neighbours being chosen at each updating. The latter method of redistributing neighbours destroys any spatial correlations [12]. Even when this model uses only short range local interactions, long range correlations appear due to criticality, and it still exhibits a behaviour similar to the empirical laws. We note that models with long range interactions have also been analysed and show behaviours which follow the empirical phenomenology [7]. In general, when one treats sliding blocks, the overdamped regime is considered, although the underdamped regime has also been studied [13]. By operating in the overdamped regime, inertia is neglected, so that it plays no role in the dynamics. By so doing, any wave mechanisms for the relaxation of energy and stress are not taken into account. Because the energy carried by wave movements does not exceed 10% of the total liberated in an event, this approximation is usually considered to be acceptable.

In line with the ideas discussed above, this paper develops and uses a spring-block model where the equilibrium position of each block on the surface depends on the balance between the local stochastic maximum static friction and the elastic forces. We find numerical evidence that our model has similar descriptive powers to the Burridge-Knopoff model. We also show how our model can be mapped onto a sandpile model which has the Oslo model [14] as limiting case, reinforcing the conjecture that both these models belong to the same universality class.

2 Description of a train model variant

The train model, introduced in 1992 by Souza-Vieira [6], is a mechanical model with blocks and springs, inspired by the Burridge-Knopoff experiment, for dynamic earthquake simulation. The model consists of a one-dimensional chain of blocks connected by springs. Each block is in contact with a lower rough surface, and the chain is pulled at one end with a small constant velocity. This model is completely deterministic. In order to study the effect of velocity independent friction we develop here a discrete stochastic version of the train model. A model with stochastic friction such as ours has previously been proposed to study the characteristic dynamics of a block sliding on a rough inclined plane [15]. We will focus on the displacement between points where the elastic forces are balanced by static friction between the blocks and the surface. The blocks are joined by ideal and equal springs. The stability is established solely by the balance of the three forces acting on each block – exerted by the two springs and the local static friction. Thus, whenever \((\sum F)_i > 0\), block \(i\) moves until the next point where \((\sum F)_i = 0\), with \((\sum F)_i < 0\) never occurring. Fundamentally, an earthquake is a fracture that does not proceed instantaneously, but is initiated locally and propagates rapidly across the surface of the fault. It eventually stops, either due to energy dissipation or when it encounters a stronger asperity, or point of contact. We model these asperities as random points of contact between the two surfaces. The maximum static friction force, \(M_i\), is due to these randomly scattered contact points between the block and the surface. We represent the roughness of both surfaces by means of binary strings of 0 s and 1 s. Each bit can be thought of as representing the average properties of the surface over an arbitrarily small length. If, for instance, a certain region is more prominent than the average, the corresponding bit is set to 1, and to 0 in the opposite case [15]. Thus, when the block is put in contact with the surface, the only regions which contribute to the frictional forces are those in which both have 1 at the corresponding locations of their bit strings, and \(M_i\) is the number of such coincidences.

The dynamics of the system are as follows: a chain of blocks is placed over a surface in some initial configuration. The rightmost block, or block 0, nicknamed the “engine”, moves at a constant speed to the right, and pulls its left neighbour. The engine’s motion is the generator of instability in the system. The simulation starts with global equilibrium: the total force on each block is 0. At each step, the engine moves one unit distance to the right and a time counter is incremented. After some steps, its left neighbour is driven into an unstable state and an avalanche event sets in. For speeding up the actual simulation, the engine is moved right the actual number of steps needed to unbalance its left neighbour, and the time counter updated accordingly. During the avalanche, the positions of the blocks are updated as follows: Block \(i\) sits over some entry of this chain. Block \(i - 1\) is \(X_i\) positions to the right and block \(i + 1\) is \(X_{i+1}\) positions to the left of block \(i\), pulling block \(i\), which will move a number of unit steps to the right until \(\Delta X_i = X_i - X_{i+1} \leq M_i\), where \(M_i\) is the position-dependent maximum friction between block \(i\) and the surface currently below it. Successive blocks are