1. Introduction

Quality control is a process employed to ensure a certain level of quality in a product or service. The basic goal of quality control is to determine whether the monitored process or product will fulfill these characteristics or not to ensure that the products, services, or processes provided meet specific requirements. Acceptance sampling is one of the essential statistical tools, which is used for quality control and improvement, especially when product testing is destructive, very expensive, and very time consuming (Starbird, 1994). There are a lot of different kinds to classify acceptance sampling plans. One of basic classification of the acceptance sampling plans is by variables and attributes (Montgomery et al., 2011). Sampling plans by variables are shown to be superior to attribute sampling plans because they require less sampling but providing the same protection to the producer and the consumer (Montgomery, 2009). In recent years, a lot of variable sampling plans have been developed for a wide variety of situations. One can refer to Yen and Chang (2009), Wu and Liu (2014), Aslam et al. (2015), Sheu et al. (2014), and Wu et al. (2015).

The coefficient of variation (CV), which is defined as the ratio of the standard deviation to the mean, can better reflect the overall degree of dispersion. It is considered as an important metric to quantify and compare variations. Using the CV to describe variability is more meaningful than standard deviation. In a steel factory, the mean of each batch of steel may be different because of different techniques and different proportion of ingredients or other factors. Usually steel users are more concerned about just what batch of steel of which tensile strength is more stable (that is less relative variability), but do not care about the average tensile strength. Because the stability of tensile strength is very important for the enterprises who use the steel in batches to complete remanufacturing production (Wang and Xiao, 2001). In the foundry industry, quality of Magnesium chloride demands stability. If the product quality is unstable, it is easy to make the mold deformation and fracture. To assess whether the stability of product quality has reached the required quality level, an acceptance sampling inspection can be carried out.

The coefficient of variation (CV), which is defined as the ratio of the standard deviation to the mean, can better reflect the overall degree of dispersion. It is considered as an important metric to quantify and compare variations. Using the CV to describe variability is more meaningful than standard deviation. In a steel factory, the mean of each batch of steel may be different because of different techniques and different proportion of ingredients or other factors. Usually steel users are more concerned about just what batch of steel of which tensile strength is more stable (that is less relative variability), but do not care about the average tensile strength. Because the stability of tensile strength is very important for the enterprises who use the steel in batches to complete remanufacturing production (Wang and Xiao, 2001). In the foundry industry, quality of Magnesium chloride demands stability. If the product quality is unstable, it is easy to make the mold deformation and fracture. To assess whether the stability of product quality has reached the required quality level, an acceptance sampling inspection can be carried out.

CV is a good measure of the reliability of experiments, which is, the smaller the CV value, the higher the reliability (see for Gomez and Gomez (1984); Steel and Torrie (1980); Taye and Njuho (2008)). Tong and Chen (1991), Wang and Xiao (2001) have pointed that the CV can be used as a quality parameter. As stated by Castagliola et al. (2015), the CV can also be used in the fields of materials engineering and manufacturing where some quality characteristics related to
the physical properties of products constituted by metal alloys or composite materials often have a standard deviation which is proportional to their population mean. Tool cutting life and several properties of sintered materials are typical examples from this setting. More recently, it has received increasing attention in statistical quality control, one can refer to Kang et al. (2007), Calzada and Scariano (2013), and Castagliola et al. (2015). For instance, Tong and Chen (1991) proposed the variable single sampling plan considering the CV as the quality parameter for normal distribution.

During the inspection of the products, the producers care more about the inspection cost which is directly related to the sample size. So the researchers want to propose a more efficient sampling plan to lower the inspection cost, time and efforts. The single sampling plan is a basic to all acceptance sampling plans and is the most widely used plan due to the simplicity in its administration. However, Liu et al. (2014) pointed that the decision to reject the lot based only on the first sample may lead to disruption of good relations between the producer and the consumer in some cases. A new sampling scheme by variables inspection, named variables two-stage sampling plan, is the extension of the variables single sampling plan and used to examine the situations where the lots are not accepted on the first inspection. This type of sampling plans provides a better operating characteristics (OC) curve than the single sampling and is considered favorable for the producers. Moreover, Aslam et al. (2013) have stated that the two stage sampling plan is more efficient than the single sampling plan because it could reduce the average sample number (ASN) required as compared with the single sampling plan and provide the protection to the producer when the product under inspection is questionable. That is, the use of the two stage sampling scheme can maximize the profit of the industry.

In this paper, we will utilize the CV to evaluate the lot quality on the assumption that product lots submitted for inspection from a process having a constant CV value. The goal of this paper is to develop a variable two stage sampling plan using CV as the quality parameter for the normal distribution, with expectation that it will provide the same protection with a smaller sample size than the single sampling plan proposed by Tong and Chen (1991) so as to save the inspection cost. The rest of the paper is organized as follows: The designing of the variable two stage sampling plan based on the CV is presented in Section 2 and the optimal plan parameters are determined and tabulated in Section 3. Section 4 discusses the efficiency of the proposed two stage sampling plan over the existing single sampling plan. A practical example is considered in Section 5 and Section 6 gives some concluding remarks.

2. Variable two stage sampling plan based on CV

Assuming the quality of interest $X$ obeys the normal distribution $N(\mu, \sigma^2)$ where $\mu$ and $\sigma^2$ are unknown, the CV of $X$ is defined as follows

$$V = \frac{\sigma}{\mu}$$  \hspace{1cm} (1)

Suppose that $X_1, X_2, \ldots, X_n$ is a sample of size $n$, the natural estimate of $V$ is

$$\hat{V} = \frac{\hat{S}}{\hat{X}}$$  \hspace{1cm} (2)

where $\hat{S} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2}$ is the sample standard deviation, $\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$ is the sample mean.

Iglewicz et al. (1968) have noticed that the statistic $\sqrt{n}/\hat{V}$ follows the noncentral $t$ distribution, i.e.

$$\sqrt{n}/\hat{V} \sim t(n-1, \delta),$$

where $n-1$ is the degrees of freedom, and $\delta = \sqrt{n}/\hat{V}$ is the noncentrality parameter. Denote the cumulative distribution function (cdf) of $\hat{V}$ as
\[ P(V \leq u) = 1 - G_{n-1,\delta} \left( \frac{\sqrt{n}}{u} \right) \]  

(3)

where \( G(\cdot) \) is the cdf of the \( t(n-1, \delta^2) \) distribution, \( \delta = \sqrt{n}/V \).

Steel and Torrie (1980), Taye and Njuho (2008) have pointed out that the CV is a good measure of the reliability of the experiment. Here we use the CV as the quality metric to decide whether to accept a lot. Let \( V_{\text{AQL}} \) and \( V_{\text{LQL}} \) denote the quality level of acceptable quality level (AQL) and limiting quality level (LQL) based on CV, respectively. Then the proposed two stage sampling plan using the CV can be described as follows:

Step 1: Choose the values of \( (V_{\text{AQL}}, V_{\text{LQL}}) \) based on the CV at producer’s risk \( \alpha \) and consumer’s risk \( \beta \).

Step 2: Select a first random sample of size \( n_1 \), \((X_1, X_2, \ldots, X_{n_1})\), from the lot, then compute

\[ \hat{V}^1 = \sqrt{\frac{\sum_{i=1}^{n_1} (X_i - \bar{X})^2 / n_1 - 1}{\sum_{i=1}^{n_1} X_i / n_1}}. \]

Accept the entire lot if \( \hat{V}^1 \leq k_{a1} \) and reject the lot if \( \hat{V}^1 > k_{a1} \). Go to Step 3 if \( k_{a1} < \hat{V}^1 \leq k_{r} \).

Step 3: Select a second random sample of size \( n_2 \), and compute

\[ \hat{V}^2 = \hat{V}^1 + \sqrt{\frac{\sum_{i=1}^{n_2} (X_i - \bar{X})^2 / n_2 - 1}{\sum_{i=1}^{n_2} X_i / n_2}}. \]

Accept the entire lot if \( \hat{V}^2 \leq k_{a2} \), otherwise, reject the lot.

Obviously, there are five parameters \( n_1, n_2, k_{a1}, k_{a2}, \) and \( k_r \) in the above proposed sampling plan. It reduces to Tong and Chen’s (1991) single sampling plan when \( k_{a1} = k_r \).

The probability of lot acceptance and rejection at the first stage can be calculated as, respectively

\[ P_A(v) = P(V^1 \leq k_{a1} | V = v) = 1 - G_{n-1,\delta} \left( \frac{\sqrt{n_1}}{k_{a1}} \right) \]  

(4)

\[ P_R(v) = P(V^1 > k_{r} | V = v) = G_{n-1,\delta} \left( \frac{\sqrt{n_1}}{k_{r}} \right) \]  

(5)

So the probability of making a decision at the first stage can be calculated as

\[ P(v) = P_A(v) + P_R(v) = P(V^1 \leq k_{a1} | V = v) + P(V^1 > k_{r} | V = v) \]

\[ = 1 - G_{n-1,\delta} \left( \frac{\sqrt{n_1}}{k_{a1}} \right) + G_{n-1,\delta} \left( \frac{\sqrt{n_1}}{k_{r}} \right) \]  

(6)
The probability of lot acceptance at the second stage is

\[ P_a^2(v) = P(k_{a1} < V^1 \leq k_r | V = v)P(V^2 \leq k_{a2} | V = v) \]

\[ = \left( G_{n-a1}^{\frac{n}{k_{a1}}} - G_{n-a1}^{\frac{n}{k_r}} \right) \cdot \left( 1 - G_{n-a2}^{\frac{n}{k_{a2}}} \right) \]

(7)

where \( \delta_i = \sqrt{n_i} / v, i = 1, 2 \).

Therefore, the probability of lot acceptance for the proposed two stage sampling plan based on the CV is given by

\[ P_a(v) = P\{\text{Accepting the lot } | V = v\} = P_a^1(v) + P_a^2(v) \]

\[ = 1 - G_{n-a1}^{\frac{n}{k_{a1}}} + \left( G_{n-a1}^{\frac{n}{k_{a1}}} - G_{n-a1}^{\frac{n}{k_r}} \right) \cdot \left( 1 - G_{n-a2}^{\frac{n}{k_{a2}}} \right) \]

(8)

The average sample number (ASN) which is the number of items inspected in a lot until the final decision is made can be obtained as follows:

\[ \text{ASN}(v) = n_1P_a(v) + (n_1 + n_2)(1 - P_a(v)) \]

\[ = n_1 + n_2 - n_2 \left( 1 - G_{n-a1}^{\frac{n}{k_{a1}}} + G_{n-a1}^{\frac{n}{k_r}} \right) \]

(9)

3. Determination of parameters for the proposed plan

Yen et al. (2009) stated “A well-designed sampling plan must provide a probability of at least \( 1 - \alpha \) of accepting a lot if the product quality level is \( v_{AQL} \) and a probability of no more than \( \beta \) of accepting a lot if the level of the product quality is \( v_{LQL} \).” That is, the desirable OC curve of a sampling plan should pass through \((1 - \alpha, v_{AQL})\) and \((\beta, v_{LQL})\).

Thus for the specified \( \alpha, \beta, v_{AQL}, \) and \( v_{LQL} \), the proposed two stage sampling plan parameters must satisfy the following two inequalities:

\[ P_a(v_{AQL}) = P\{\text{Accepting the lot } | V = v_{AQL}\} \geq 1 - \alpha \]

(10)

\[ P_a(v_{LQL}) = P\{\text{Accepting the lot } | V = v_{LQL}\} \leq \beta \]

(11)

Assuming \( n = n_1 = n_2 \), we note that there might be multiple solutions of the proposed plan parameters \((n, k_{a1}, k_{a2}, k_r)\) for satisfying above Eqs. (10) and (11) based on given quality and risk requirements because the plan parameters are needed to be determined simultaneously. Balamurali and Jun (2009) stated “a sampling plan would be desirable if the sample size required for inspection is minimal and offers the same protection to both the producer and the consumer.”

Here we consider the average of ASN at both AQL and LQL as the objective function and make it to be minimized with the above two constraints. Then the proposed plan parameters \((n, k_{a1}, k_{a2}, k_r)\) can be determined by solving the following optimization problem:
Min \( ASN = \frac{1}{2} (ASN(v_{AQL}) + ASN(v_{LQL})) \)

\[
= \frac{1}{2} \left[ n + n(G_{x-1,\delta_{AQ}} \sqrt{\frac{n}{k_{a1}}} - G_{n-1,\delta_{AQ}} \sqrt{\frac{n}{k_{a2}}}) + n + n(G_{x-1,\delta_{AQ}} \sqrt{\frac{n}{k_{a1}}} - G_{n-1,\delta_{AQ}} \sqrt{\frac{n}{k_{a2}}}) \right]
\]

s.t.

\[
1 - G_{x-1,\delta_{AQ}} \sqrt{\frac{n}{k_{a1}}} + (G_{x-1,\delta_{AQ}} \sqrt{\frac{n}{k_{a1}}} - G_{n-1,\delta_{AQ}} \sqrt{\frac{n}{k_{a2}}}) \cdot (1 - G_{n-1,\delta_{AQ}} \sqrt{\frac{n}{k_{a2}}}) \geq 1 - \alpha \quad (13)
\]

\[
1 - G_{x-1,\delta_{AQ}} \sqrt{\frac{n}{k_{a1}}} + (G_{x-1,\delta_{AQ}} \sqrt{\frac{n}{k_{a1}}} - G_{n-1,\delta_{AQ}} \sqrt{\frac{n}{k_{a2}}}) \cdot (1 - G_{n-1,\delta_{AQ}} \sqrt{\frac{n}{k_{a2}}}) \leq \beta \quad (14)
\]

where \( \delta_{AQ} = \sqrt{n} / \sqrt{\frac{v_{AQ}}{2}}, \delta_{LQ} = \sqrt{n} / \sqrt{\frac{v_{LQ}}{2}}, \ k_{a1} > k_{a2} > k_{a3} \geq 0 \).

In order to determine the proposed two stage sampling plan, we use the Monte Carlo simulation to solve this minimization problem. In each criterion, 10,000 samples are randomly chosen using replacement criterion and this process is repeated 2000 times to search for the minimum value of ASN and the corresponding plan parameters (The \( R \) algorithm procedure is given in the appendix). Tables 1-3 summarize the proposed plan parameters \((n, k_{a1}, k_{a2}, k_{r}, ASN)\) for various quality levels \((v_{AQL}, v_{LQL})\) under \((\alpha, \beta) = (0.05, 0.10), (0.10, 0.10)\) and \((0.10,0.05)\), and some commonly used combinations of \((v_{AQL}, v_{LQL})\). For instance, if \((\alpha, \beta) = (0.10, 0.10)\) under the quality level \((v_{AQL}, v_{LQL}) = (0.06, 0.09)\), we can obtain \((n, k_{a1}, k_{a2}, k_{r}, \text{ASN}) = (15, 0.06328, 0.069782, 0.077903, 18.767)\) from Table 2. That is, 15 inspected measurements are taken from the lot randomly. If \( \hat{V} \leq 0.06328 \), the entire lot will be accepted; if \( \hat{V} > 0.077903 \), the lot will be rejected. If 0.06328 < \( \hat{V} \leq 0.077903 \), we should take a second random sample of size 15 from the lot and recalculate \( \hat{V} \). If \( \hat{V} \leq 0.069782 \), accept the lot; otherwise, reject the lot.

Table 1  The proposed plan parameters when \( \alpha=0.05, \beta=0.10 \)

| \( v_{AQL} \) | \( v_{LQL} \) | \( n \) | \( k_{a1} \) | \( k_{a2} \) | \( k_{r} \) | \( ASN \) |
|---|---|---|---|---|---|---|
| 0.05 | 0.06 | 90 | 0.053462 | 0.054047 | 0.056916 | 104.34 |
| 0.07 | 28 | 0.055771 | 0.05702 | 0.062489 | 32.346 |
| 0.08 | 15 | 0.056126 | 0.065814 | 0.065843 | 17.416 |
| 0.09 | 10 | 0.055618 | 0.070324 | 0.073498 | 12.319 |
| 0.10 | 7 | 0.053342 | 0.072335 | 0.075093 | 8.7812 |
| 0.11 | 6 | 0.054689 | 0.073221 | 0.080471 | 7.4941 |
| 0.12 | 5 | 0.050515 | 0.07682 | 0.085344 | 6.5564 |
| 0.06 | 0.07 | 133 | 0.063809 | 0.065089 | 0.066498 | 149.3 |
| 0.08 | 36 | 0.061927 | 0.070546 | 0.074369 | 47.104 |
| 0.09 | 19 | 0.064987 | 0.076516 | 0.077182 | 22.883 |
| 0.10 | 11 | 0.058363 | 0.081347 | 0.084397 | 15.034 |
| 0.11 | 10 | 0.071659 | 0.081712 | 0.0846 | 11.298 |
| $v_{AQL}$ | $v_{LQL}$ | $n$ | $k_{a1}$ | $k_{a2}$ | $k_r$ | ASN |
|----------|----------|-----|----------|----------|-------|------|
| 0.05     | 0.06     | 67  | 0.050894 | 0.055574 | 0.057083 | 87.534 |
| 0.07     | 0.07     | 21  | 0.050214 | 0.058929 | 0.063517 | 28.173 |
| 0.08     | 0.08     | 12  | 0.05348  | 0.056564 | 0.069346 | 15.202 |
| 0.09     | 0.09     | 8   | 0.051014 | 0.060956 | 0.076536 | 10.769 |
| 0.10     | 0.10     | 7   | 0.058101 | 0.062189 | 0.073797 | 8.2135 |
| 0.11     | 0.11     | 6   | 0.058569 | 0.059884 | 0.076463 | 7.0261 |
| 0.12     | 0.12     | 4   | 0.037399 | 0.070011 | 0.086956 | 5.8602 |
| 0.08     | 0.09     | 15  | 0.06328  | 0.069782 | 0.077903 | 18.767 |
| 0.07     | 0.10     | 10  | 0.063415 | 0.069479 | 0.087069 | 13.087 |
| 0.11     | 0.11     | 7   | 0.046997 | 0.084276 | 0.085895 | 10.275 |
| 0.12     | 0.12     | 7   | 0.069328 | 0.078629 | 0.085872 | 8.1081 |
| 0.07     | 0.08     | 129 | 0.071527 | 0.075311 | 0.0765  | 160.67 |
| 0.09     | 0.09     | 41  | 0.075211 | 0.075886 | 0.08177 | 47.57 |
| 0.10     | 0.10     | 20  | 0.073147 | 0.083166 | 0.086908 | 24.781 |
| 0.11     | 0.11     | 13  | 0.073796 | 0.085616 | 0.094474 | 16.571 |
| 0.12     | 0.12     | 9   | 0.067059 | 0.08861  | 0.098495 | 12.255 |
| 0.08     | 0.09     | 154 | 0.081542 | 0.085518 | 0.086824 | 193.87 |
| 0.10     | 0.10     | 51  | 0.085373 | 0.086872 | 0.091923 | 59.144 |
| 0.11     | 0.11     | 29  | 0.088975 | 0.09024  | 0.094155 | 31.469 |
| 0.12     | 0.12     | 14  | 0.077662 | 0.097109 | 0.10772 | 19.582 |
| 0.09     | 0.10     | 220 | 0.092771 | 0.093825 | 0.096575 | 260.49 |
| 0.11     | 0.11     | 66  | 0.096285 | 0.097679 | 0.10099 | 73.525 |
| 0.12     | 0.12     | 30  | 0.092513 | 0.10499  | 0.10707 | 37.547 |
| 0.10     | 0.11     | 260 | 0.10255 | 0.10416  | 0.10725 | 319.06 |
| 0.12     | 0.12     | 69  | 0.10311 | 0.11024  | 0.11367 | 86.942 |

Table 2  The proposed plan parameters when $\alpha=0.10, \beta=0.10$
Table 3 The proposed plan parameters when \( \alpha = 0.10 \), \( \beta = 0.05 \)

| \( v_{AQL} \) | \( v_{LQL} \) | \( n \) | \( k_{a1} \) | \( k_{a2} \) | \( k_r \) | ASN     |
|-----------|-----------|------|-------|-------|------|--------|
| 0.05      | 0.06      | 88   | 0.0514| 0.054237| 0.055625| 106.65 |
| 0.07      | 0.05      | 27   | 0.049058| 0.060458| 0.060668| 35.644 |
| 0.08      | 0.045237  | 14   | 0.044623| 0.064039| 0.065504| 19.712 |
| 0.09      | 0.063029  | 11   | 0.053834| 0.063029| 0.067243| 13.152 |
| 0.10      | 0.051142  | 8    | 0.051142| 0.064052| 0.072668| 10.138 |
| 0.11      | 0.050165  | 7    | 0.050165| 0.069981| 0.070967| 8.7108 |
| 0.12      | 0.054402  | 6    | 0.054402| 0.061926| 0.076198| 7.2309 |
| 0.06      | 0.07      | 121  | 0.061279| 0.063178| 0.067547| 157.97 |
| 0.08      | 0.06       | 39   | 0.062625| 0.067064| 0.070604| 47.413 |
| 0.09      | 0.056423  | 19   | 0.056423| 0.074083| 0.076605| 26.228 |
| 0.10      | 0.06395   | 14   | 0.06395 | 0.071188| 0.077396| 16.698 |
| 0.11      | 0.056417  | 9    | 0.056417| 0.080577| 0.08119  | 11.868 |
| 0.12      | 0.059472  | 8    | 0.059472| 0.079175| 0.088949| 10.442 |
| 0.07      | 0.08      | 153  | 0.06926 | 0.075271| 0.076412| 208.45 |
| 0.09      | 0.068146  | 46   | 0.068146| 0.080724| 0.081344| 62.221 |
| 0.10      | 0.073904  | 28   | 0.073904| 0.08274 | 0.085553| 33.609 |
| 0.11      | 0.072369  | 17   | 0.072369| 0.0853  | 0.087603| 20.744 |
| 0.12      | 0.066252  | 11   | 0.066252| 0.085211| 0.099411| 15.232 |
| 0.08      | 0.09      | 209  | 0.081737| 0.083907| 0.08645 | 256.93 |
| 0.10      | 0.08027   | 65   | 0.084027| 0.086509| 0.091261| 76.482 |
| 0.11      | 0.082742  | 32   | 0.082742| 0.092546| 0.09599 | 39.416 |
| 0.12      | 0.082717  | 20   | 0.082717| 0.097561| 0.099441| 24.653 |
| 0.09      | 0.10      | 275  | 0.091922| 0.095246| 0.095618| 324.45 |
| 0.11      | 0.093818  | 81   | 0.093818| 0.099608| 0.10002 | 93.507 |
| 0.12      | 0.093179  | 39   | 0.093179| 0.099065| 0.10774 | 48.628 |
| 0.10      | 0.099008  | 299  | 0.10537 | 0.10705 | 0.11046 | 375.89 |
| 0.12      | 0.10225   | 96   | 0.10225 | 0.11090 | 0.11406 | 116.06 |

From Tables 1-3, we can see that for the fixed values of \( \alpha \), \( \beta \) and \( v_{AQL} \), the value of ASN decreases as the value of \( v_{LQL} \) increases. For example, \( v_{AQL}=0.05, (\alpha, \beta)=(0.05,0.10) \), when \( v_{LQL}=0.07 \), ASN=32.346 and for all other same values, ASN=12.319 when \( v_{LQL}=0.09 \), and ASN=6.5564 when \( v_{LQL}=0.12 \).

4. Comparative analysis

There are several sampling plans available for a specific protection to the producer and the consumer. The OC curves and the required sampling size are two commonly criteria used to evaluate the performance of the sampling plans. In this section, we will use these two criteria to demonstrate the advantages of the proposed two-stage sampling plan over the single plan proposed by Tong and Chen (1991).

4.1 Sample sizes required for inspection

In order to compare the sample sizes required for inspection in the two stage sampling plan and the single plan with
different values of $AQL_v$ and $LQL_v$, the $AQL_v$ value is fixed at 0.06 and $LQL_v$ value increases from 0.07 to 0.12. The results are showed in Figure 1 ($\alpha =0.05$, $\beta =0.10$). From Fig.1, the required sample size $n$ of these two sampling plans both decreases as the value of $LQL_v$ rises from 0.07 to 0.12. Clearly, the required sample size $n$ is larger as the value of $LQL_v$ is closer to the value of $AQL_v$. Moreover, we also find that the single sampling plan requires larger sample size than the two stage sampling plan when $LQL_v$ takes any value between 0.07 and 0.12. Therefore, the two stage sampling plan is a more cost-effective plan while the single plan is relatively uneconomical.

![Figure 1](image)

**Fig. 1** Required sample size of two stage sampling plan and single sampling plan with ($\alpha , \beta$)=(0.05, 0.10).

On the other hand, we also list the sample sizes required for the two stage sampling plan and the single plan in Table 4 with commonly used risks ($\alpha , \beta$)=(0.05, 0.10), (0.10, 0.10), (0.10, 0.05) and various combinations of quality levels ($AQL_v$, $LQL_v$). From Table 4, it is obvious that the sample size required for the proposed two stage sampling plan is smaller than the single plan for all cases when the values of all specified parameters are the same for two sampling plans. For example, if ($\alpha , \beta$)=(0.05,0.10) and ($AQL_v$, $LQL_v$)=(0.05, 0.06), the sample size $n=131$ required for the single sampling plan, while ASN=104.34 for the two stage sampling plan.
Table 4  The comparison of sample size for three sampling plans based on the CV

| $v_{AQL}$ | $v_{LQL}$ | $n$ | $ASN$ | $n$ | $ASN$ | $n$ | $ASN$ |
|---------|----------|-----|------|-----|------|-----|------|
| 0.05    | 0.06     | 131 | 104.34 | 101 | 87.534 | 134 | 106.65 |
| 0.07    | 39       | 32.346 | 31 | 28.173 | 41 | 35.644 |
| 0.08    | 20       | 17.416 | 17 | 15.202 | 23 | 19.712 |
| 0.09    | 14       | 12.319 | 12 | 10.769 | 15 | 13.152 |
| 0.10    | 11       | 8.7812 | 9 | 8.2135 | 12 | 10.138 |
| 0.11    | 9        | 7.4941 | 8 | 7.0261 | 9 | 8.7108 |
| 0.12    | 7        | 6.5564 | 7 | 5.8602 | 8 | 7.2309 |
| 0.06    | 0.07     | 182 | 149.3 | 142 | 120.88 | 186 | 157.97 |
| 0.08    | 53       | 47.104 | 42 | 35.335 | 56 | 47.413 |
| 0.09    | 28       | 22.883 | 22 | 18.767 | 28 | 26.228 |
| 0.10    | 17       | 15.034 | 15 | 13.087 | 20 | 16.698 |
| 0.11    | 14       | 11.298 | 11 | 10.275 | 15 | 11.868 |
| 0.12    | 11       | 9.1222 | 9 | 8.1081 | 12 | 10.442 |
| 0.07    | 0.08     | 242 | 201.85 | 188 | 160.67 | 248 | 208.45 |
| 0.09    | 69       | 57.355 | 55 | 47.57 | 72 | 62.221 |
| 0.10    | 35       | 28.411 | 28 | 24.781 | 36 | 33.609 |
| 0.11    | 23       | 19.711 | 19 | 16.571 | 25 | 20.744 |
| 0.12    | 17       | 14.042 | 14 | 12.255 | 17 | 15.232 |
| 0.08    | 0.09     | 311 | 271.33 | 242 | 193.87 | 318 | 256.93 |
| 0.10    | 88       | 71.478 | 69 | 59.144 | 91 | 76.482 |
| 0.11    | 44       | 38.552 | 35 | 31.469 | 46 | 39.416 |
| 0.12    | 28       | 23.281 | 23 | 19.582 | 30 | 24.653 |
| 0.09    | 0.10     | 327 | 317.43 | 303 | 260.49 | 336 | 324.45 |
| 0.11    | 109      | 92.619 | 85 | 73.525 | 112 | 93.507 |
| 0.12    | 55       | 43.41 | 44 | 37.547 | 56 | 48.628 |
| 0.10    | 0.11     | 335 | 379.5 | 335 | 319.06 | 343 | 375.89 |
| 0.12    | 132      | 112.11 | 103 | 86.942 | 136 | 116.06 |

Note: $n$ and $ASN$ denote the sample size of the single sampling plan and two-stage sampling plan respectively.

4.2 OC curves

The OC curve shows the probability of accepting a lot under different quality levels of the submitted lots. It can also show the discriminatory power for sampling plans, from which the producer and the consumer can judge whether the sampling plan is appropriate.

In order to show the efficiency of the proposed sampling plan, Fig.2 displays the OC curves of the two stage sampling plan and the single sampling plan for two cases: (a) $(v_{AQL}, v_{LQL}) = (0.06, 0.08)$ and (b) $(v_{AQL}, v_{LQL}) = (0.09, 0.12)$ with $(\alpha, \beta) = (0.05, 0.10)$. Since the greater the OC curve's slope, the better is the discriminatory power of the sampling plan. From Fig.2, we can find that the OC curves of the proposed two stage sampling plan are more
discriminating than that of the single sampling plan in both case (a) and case (b). Therefore, it is reasonable to conclude the two stage sampling plan has a better performance.

In addition, from Fig. 2 we can see that the curves of these two sampling plans have similar shape in case (a) or in case (b), but the sample size required by the two stage sampling plan is much fewer. For example, the single sampling plan requires \( n = 55 \) while the two stage sampling plan requires \( n = 37 \), ASN = 43.41 in case (b). Since the two stage sampling plan requires fewer sample size to give the desired protection, the cost of inspection will greatly be reduced.

**Fig. 2** OC curves of the two sampling plans for different quality levels:

(a) \((v_{AQL}, v_{LQL}) = (0.06, 0.08)\), (b) \((v_{AQL}, v_{LQL}) = (0.09, 0.12)\).

From the analysis of the above, the proposed two stage sampling plan is better than the single sampling plan because it requires a smaller sample size under the same level of protection. That is, it can save the time and the cost of the inspection. Therefore, it is reasonable to conclude our proposed two-stage sampling plan has a better performance.

5. **An application example**

In this section, we will consider the actual data collected from an electronic component manufacturer discussed by Wu et al. (2012) to illustrate how the proposed two stage sampling plan can be used in practice. This factory produces various types of resistors. Here, we have investigated a type of resistors whose quality characteristics is the thickness. In the business contract formulated from the producer and the buyer, suppose \( v_{AQL} = 0.05, v_{LQL} = 0.06 \) with producer’s risk \( \alpha = 0.10 \) and buyer’s risk \( \beta = 0.10 \). This implies that the producers require a probability of at least 90% of accepting the entire lot if the CV of the thickness is less than 0.05, and the consumers require the acceptance probability would be at most 10% if the CV of the thickness is larger than 0.06.

Based on the above values, we can obtain the plan parameters as \((n, k_{a1}, k_{a2}, k_r, \text{ASN}) = (67, 0.050894, 0.055574, 0.057083, 87.534)\) from Table 2. Hence, we take 67 samples randomly from the entire lot and the observed measurements (unit in mil) are displayed in the following Table 5. The collected data are very close to the normal distribution, which is showed by Wu et al. (2012) using the p-p chart.
From Eq. (2) and the observations in Table 5, the estimation of the CV is calculated as follows

\[ \hat{V} = \frac{S}{\bar{X}} = 0.04962, \]

where \( \bar{X} = 9.83866, \) \( S = 0.48816. \)

Because \( \hat{V} = 0.04962 < k_{0.1}, \) we conclude that the lot should be accepted.

6. Conclusions

This paper develops a variable two stage sampling plan based on the coefficient of variation (CV) for lot sentencing when the quality characteristic of the products follows a normal distribution. Several tables for the proposed plan parameters under various conditions are given for practical use. We use two criteria (the required sample size and OC curve) to compare the efficiency of the proposed plan with the single plan proposed by Tong and Chen (1991). The results show that our proposed plan is more efficient than the other one. Hence, the proposed plan would be useful for product acceptance determination because it can save the cost and time of inspection. For future research, the proposed method may be considered to non-normal distribution.

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Appendix
Algorithm procedure for minimizing ASN using two-stage sampling plan based on CV:

1. Specify the values of risks $\alpha, \beta$ and the quality levels $v_{AQL}, v_{LQL}$.

2. Select the sample size $n$ and the initial values of $k_{a1}, k_{a2}$ and $kr$.

3 Calculate $P_a(v_{AQL}), P_a(v_{LQL})$ and ASN.

4. Select those combination of $P_a(v_{AQL})$ and $P_a(v_{LQL})$, where $(P_a(v_{AQL}) \geq 1 - \alpha$ and $P_a(v_{LQL}) \leq \beta)$.

5. Repeat Step 2 through Step 4 a sufficient number (10,000 say) of times to calculate the minimum value of ASN and
the corresponding plan parameters $n, k_{a1}, k_{a2}$ and $k_r$ satisfying the above constraint.