I. INTRODUCTION

Experimental and theoretical studies of high-energy giant resonances (GRs) have been undertaken in recent years to understand better how the different characteristics associated with GR formation (concentration of the particle-hole strength, coupling to the continuum, the spreading effect) are affected with increasing the GR energy. Many of known high-energy GRs are the next vibration modes (the overtones) relative to the respective low-energy GRs (the main tones). The lowest energy overtone is the isoscalar giant dipole resonance (ISGDR) experimentally studied in a few medium-heavy and heavy nuclei via the $(\alpha,\alpha')$ reaction \[1, 2, 3\]. The ISGDR is the overtone of the $1^-$ zero-energy spurious state (SS), associated with center-of-mass motion. The experimental results of Refs. \[1, 2, 3\] are concerned with distribution of the respective dipole strength. Only recently direct nucleon decays of the ISGDR have been firstly observed using the $(\alpha,\alpha'N)$ reactions \[4, 5\]. These studies are planned to be continued \[6\]. The isovector giant charge-exchange (in the $\beta^-$ channel) monopole and spin-monopole resonances are the overtones of the isobaric analogue and Gamow-Teller resonances, respectively. These overtones have been studied via charge-exchange reactions \[7, 8\]. Direct proton decays of the giant spin-monopole resonance have been recently observed using the $(^3\text{He},tp)$ reaction \[9\]. Other candidates for studies of high-energy GRs are the overtones of the isoscalar giant monopole and quadrupole resonances (ISGMR2 and ISGQR2, respectively). The corresponding main tones, having relatively low energy, have been much experimentally studied \[10, 11, 12\]. However, the evidence for existence of the ISGQR2 has apparently been found only recently \[13, 14\].

Theoretical studies of the isoscalar overtones deal primarily with the ISGDR. Microscopically, this GR is mainly due to $3\hbar\omega$ particle-hole-type excitations, while the overtones of the isoscalar quadrupole and monopole GRs are mainly due to $4\hbar\omega$ excitations. First microscopic calculations of the strength distribution for the ISGDR and for all the above-mentioned isoscalar overtones have apparently been done in the eighties (Refs. \[11\] and \[12\], respectively). In recent years Hartree-Fock + RPA calculations with the use of the Skyrme interactions have been done to specify the ISGDR energy and, after comparing with experimental data, to make a conclusion about the nuclear incompressibility \[13, 14\]. Similar goals are pursued in approaches based on relativistic version of the RPA and on the semi-classical treatment of nuclear vibrations (see, e.g., Refs. \[15\] and \[16\], respectively). However, the microscopic structure and “differential” properties of high-energy GRs, corresponding to collective excitations of nuclei as finite-size open Fermi-systems, are of particular interest. Attempts to describe the main properties (the strength distribution, energy-dependent transition density, partial direct-nucleon-decay branching ratios) of the ISGDR have been undertaken in Refs. \[17, 18\] within a semi-microscopic approach, based on both the CRPA method and a phenomenological treatment of the spreading effect. In these references the gross properties (parameters of the strength distribution, transition density) have been satisfactorily described for the ISGDR in a few medium-heavy and heavy nuclei. Contrary to the gross properties, the partial branching ratios for direct nucleon decay of a particular GR carry information on its microscopic structure and also on coupling to the continuum and the spreading effect. The attempts (undertaken in Refs. \[17, 18\] for the ISGDR in $^{208}$Pb) to evaluate the partial direct-nucleon decay branching ratios met with difficulties concerned with taking the spreading effect into account. The way to overcome these difficulties has been drawn in Ref. \[19\]. The overtone of the isoscalar monopole resonance was theoretically studied within a CRPA-based approach in Ref. \[20\] mainly for searching for narrow
pursue the following goals in the present work:

(i) extension of the CRPA-based semi-microscopic approach to describe direct nucleon decay of high-energy GRs;

(ii) description of the gross properties of the isoscalar main-tone resonances to check abilities of the approach and to find out probing operators appropriate for overtone studies;

(iii) calculation of the partial direct-nucleon-decay branching ratios for the high-energy component of the ISGDR;

(iv) description of the main properties of the second isoscalar giant monopole and quadrupole resonances; and

(v) comparison of the calculation results, obtained for singly- and doubly-closed-shell nuclei $^{58}\text{Ni}$, $^{90}\text{Zr}$, $^{116}\text{Sn}$, $^{144}\text{Sm}$, and $^{208}\text{Pb}$, with available experimental data.

The paper is organized as follows. In Sect. II the basic elements, ingredients and new points of the approach are presented. Sect. III contains calculation results and available experimental data on properties of the isoscalar giant main-tone and overtone resonances. Discussion of the results and concluding remarks are given in Sect. IV.

II. BASIC ELEMENTS AND INGREDIENTS OF THE APPROACH

The continuum-RPA method and a phenomenological treatment of the spreading effect are the basic elements of the semi-microscopic approach. In implementations of the approach the following phenomenological input quantities are used: a realistic nuclear mean field and the Landau-Migdal partical-hole interaction bound together by selfconsistency conditions; an energy- and radial-dependent smearing parameter.

A. CRPA equations

The CRPA equations are taken in the form accepted within the Migdal’s finite Fermi-system theory [22]. As applied to description of isoscalar particle-hole-type excitations in closed-shell nuclei, these equations are given in detail in Refs. 17, 18 and not shown here. (For reader’s convenience we use (in the main) the notations of Refs. 17, 18 and sometimes refer to equations, tables and figures from these references). The nuclear polarization $P_\lambda(\omega)$, strength function $S_\lambda(\omega)$, and the energy-dependent transition density $\rho_\lambda(r,\omega) = \rho(L(r,\omega)Y_{LM}(\hat{n}))$ ($\omega$ is the excitation energy), corresponding to an isoscalar probing operator $V_\lambda(r) = V_L(r)Y_{LM}(\hat{n})$, can be calculated to describe gross properties of the respective isoscalar GR within the CRPA. The above-listed quantities are expressed via the radial part of the effective probing operators $\hat{V}_\lambda^{(r)}(r,\omega)$, which are different from $V_L(r)$ due to core-polarization effects caused by the particle-hole interaction. The $\omega$-dependence of the above quantities comes from that of the free particle-hole propagator. The latter can be expressed in terms of occupation numbers $n^{\alpha}_\mu$, radial bound-state single-particle wave functions $\chi^\alpha_\mu$, and radial Green functions $g^\alpha_\mu(r,r',\epsilon_\mu \pm \omega)$ to take exactly the single-particle continuum into account (see Eqs. (1)–(3) of Ref. 17 and Eq. (1) of Ref. 18).

When compared with Ref. 22, the new element of the CRPA equations is used within the approach — the direct-nucleon-escape amplitude $M^\lambda_{\mu}(\omega)$ (see Eqs. (4), (5) of Ref. 17)). This amplitude is proportional to the product of $(n^{\alpha}_\mu)^{1/2}$ and the matrix element of the respective radial effective operator taken with the use of the radial bound-state wave function $\chi^\alpha_\mu$ and the radial continuum-state wave function $\chi^\lambda_{(\mu)}(\mu$ is the set of quantum numbers for an occupied single-particle level; $\epsilon = \epsilon_\mu + \omega$ and $(\lambda)$ are the energy and quantum numbers of an escaped nucleon, respectively), $c = \mu, (\lambda), \alpha$ is the set of nucleon-decay-channel quantum numbers compatible with the respective selection rules). The $\omega$-dependence of the direct-nucleon-escape amplitude comes not only from the effective probing operator but also from the continuum-state wave function.

The partial direct-nucleon-decay branching ratio $b^\lambda_{\mu}(\delta)$ can be reasonably defined as the ratio of the squared nucleon-escape amplitude integrated over a certain excitation-energy interval $\delta = \omega_1 - \omega_2$ to the respective strength function integrated over the same interval. The total branching ratio $b^{\lambda}_{\mu \text{tot}} = \sum b^\lambda_{\mu}$ is equal to unity for arbitrary interval $\delta$, as it follows from the unitary condition, which is valid within the CRPA (Eq. (4) of Ref. 17).

For comparison with the experimental branching ratios we sometimes replace the occupation numbers $n_\mu$ in the expression for $b^\lambda_{\mu} = \sum (\lambda) b^\lambda_{\mu}$ with the respective experimental spectroscopic factors $S^\mu_\lambda$. In such a way we take phenomenologically into account coupling of single-hole states $\mu^{-1}$ populated after direct nucleon decay of GRs to low-energy collective states.

In description of the gross properties of high-energy GRs, nucleon pairing in open-shell subsystems can be neglected with a high accuracy. To take into account
the effect of nucleon pairing on the direct-nucleon-decay branching ratios in the CRPA equations we replace the occupation numbers $n_{\mu}$ with the respective Bogolyubov’s factors $v_{\mu}^2$. The latter can be calculated in an isospin-selfconsistent way with the use of experimental pairing energies \cite{23}. To take phenomenologically into account coupling of single-quasi-particle states populated after direct nucleon decay to low-energy collective states the calculated $b_{\mu}$ value is divided by $v_{\mu}^2$ and multiplied by $S_{\mu}$ \cite{23}.

An important aspect of theoretical studies of GR overtones within the RPA is the choice of an appropriate probing operator. It is convenient to choose this operator with the condition that the main tone is not being excited. In this case the overtone exhausts most of the respective particle-hole strength. As applied to description of the ISGDR, charge-exchange giant monopole, and spin-monopole resonances the choice of appropriate probing operators is discussed in Refs. \cite{17}, \cite{23}, and \cite{24}, respectively. In particular, the radial part of the isoscalar second-order dipole probing operator $V_{L=1}^{(2)}(r) = r^3 - \eta_{L=1} r$ is used for description of the ISGDR. Parameter $\eta_{L=1}$ is defined by the condition: $\int p_{L=1}^{SS}(r) V_{L=1}^{(2)}(r) r^2 dr = 0$, where $\rho_{L=1}^{SS}(r)$ is the spurious-state transition density. To describe the properties of the second isoscalar giant monopole and quadrupole resonances the radial part of the respective second-order probing operators is taken in the form: $V_{L=1}^{(2)}(r) = r^4 - \eta_{L} r^2$. The parameters $\eta_{L}$ in this expression are determined by the condition:

$$\int p_{L}(r, \omega_{peak}) V_{L=1}^{(2)}(r) r^2 dr = 0,$$

where $p_{L}(r, \omega_{peak})$ is the radial part of the energy-dependent main-tone transition density taken at the peak energy of the main-tone strength function.

### B. Smearing procedure

Within the approach, the smearing effect, i.e. coupling of particle-hole-type doorway states to many-quasi-particle configurations, is phenomenologically taken into account in terms of a proper smearing parameter. Somewhat different smearing procedures are used for description of low- and high-energy GRs. The above-mentioned $\omega$-dependent quantities calculated within the CRPA for a low-energy (“subbarrier”) GR can be expanded in terms of the Breit–Wigner-type doorway-state resonances nonoverlapped on their total escape widths $\Gamma^\uparrow$. To take the spreading effect in evaluation of the energy-averaged $\omega$-dependent quantities into account, the doorway-state resonances are smeared independently of one another. The smearing procedure comprises the replacement of $\Gamma^\uparrow$ by $\Gamma^\uparrow + I$, or (what is the same) the replacement of $\omega$ by $\omega + iI/2$. The parameter $I$, the mean doorway-state spreading width, is fitted to reproduce the experimental total width of the considered GR in calculations of the energy-averaged strength function. To calculate the energy-averaged direct-nucleon-escape amplitudes it is also necessary to average the potential-barrier penetrability in a proper way because the directly escaped nucleons have relatively low (“subbarrier”) energies. Thus, the energy-averaged transition density and partial direct-nucleon-decay branching ratios can be evaluated within the approach without the use of any free parameters.

**TABLE I:** The peak energy, total width (both in MeV), and parameters $\eta_{L}$ (in fm$^2$) calculated for the $L = 0, 1, 2$ isoscalar GRs in nuclei under consideration. The respective experimental values (given with errors) are taken from Ref. \cite{23} ($L = 0, 1$) and Ref. \cite{23} ($L = 2$).

| Nucl. | $\eta_{L=1}$ | $\eta_{L=0}$ | $\eta_{L=2}$ | $\omega_{peak}^{HE}$ | $\omega_{peak}^{LE}$ |
|-------|--------------|--------------|--------------|-------------------|-------------------|
| $^{58}$Ni | 12.31 | 18.21 | 4.86 | 90-10 | 12.49 |
| $^{90}$Zr | 10-25 | 16.47 | 2.64 | 87 | 13.07 |
| $^{116}$Sn | 10-20 | 15.38 | 2.13 | 79 | 12.51 |
| $^{144}$Sm | 10-20 | 14.85 | 2.05 | 80 | 11.91 |
| $^{208}$Pb | 10-20 | 13.89 | 2.05 | 79 | 10.51 |

The LE-component has additional maximum at 12.1 MeV

The HE-component has additional maximum at 11.4 MeV

**TABLE II:** Parameters of the ISGMR and ISGQR calculated for a certain excitation-energy interval. All the parameters are given in MeV except for $x$, which is given in %.

| Nucl. | $\omega_{1} - \omega_{2}$ | $\bar{\omega}$ | $\Delta$ | $x$ | $\omega_{1} - \omega_{2}$ | $\bar{\omega}$ | $\Delta$ | $x$ |
|-------|-----------------|---------|--------|-----|-----------------|---------|--------|-----|
| $^{58}$Ni | 12-31 | 18.21 | 3.06 | 89 | 10-20 | 14.89 | 1.90 | 70 |
| $^{90}$Zr | 10-25 | 16.47 | 2.64 | 87 | 9-18 | 13.07 | 1.78 | 70 |
| $^{116}$Sn | 10-20 | 15.38 | 2.13 | 79 | 8-19 | 12.51 | 2.13 | 72 |
| $^{144}$Sm | 10-20 | 14.85 | 2.05 | 80 | 7-19 | 11.91 | 2.31 | 74 |
| $^{208}$Pb | 10-20 | 13.89 | 2.05 | 79 | 6-17 | 10.51 | 2.18 | 71 |
This method has been employed in Ref. [17] to decribe quantitatively direct nucleon decay of the ISGMR in a few nuclei. As applied to direct nucleon decay of the ISGDR in $^{208}$Pb, the method results only in qualitative description because the respective doorway-state resonances are significantly overlapped [17]. The above-outlined smearing procedure is based on a statistical assumption: after energy-averaging the doorway states “decay” into many-quasi-particle configurations independently of one another [24]. Such an assumption seems to be reasonable in view of the complexity of many-quasi-particle configurations at high excitation energies.

As follows from CRPA calculations of the high-energy GR strength function, the doorway-state resonances are overlapped. In such a case, the smearing procedure comprises the replacement of $\omega$ by $\omega + i/2$ in the CRPA equations. This replacement implies, in fact, the use of the imaginary part of the single-particle potential, $\mp(i/2)\delta (r, \omega)$, when the radial Green functions and continuum-state wave functions are calculated. As a result, the energy-averaged $\omega$-dependent quantities (strength function $S_{L}(\omega)$, transition density $\bar{\rho}_{L}(r, \omega)$, direct-nucleon-escape amplitude $M^{L}_{ex}(\omega)$) can be calculated at once with the use of a radial- and energy-dependent smearing parameter in the CRPA equations. In accordance with the statistical assumption, the following parameterization for $I$ is used: $I(r, \omega) = I(\omega) f_{ws}(r, R^{*}, a)$, where $f_{ws}$ is the Woods-Saxon function taken with $R^{*} = R$ ($R$ and $a$ are the radius and diffuseness of the isoscalar part of the nuclear mean field, respectively). The calculated strength distribution and transition density of high-energy GRs are found to be almost independent of the “cut off” radius at $R^{*} > (1.7 - 1.9)R$.

The energy-dependent part of the smearing parameter $I(\omega)$ is in the form:

$$I(\omega) = \frac{\alpha (\omega - \Delta)^{2}}{1 + (\omega - \Delta)^{2}/B^{2}}, \quad \omega > \Delta;$$

$$I(\omega) = 0, \quad \omega < \Delta$$

with universal parameters. Such an energy dependence is used for the absorption potential in some versions of the optical model for nucleon-nucleus scattering [25].

The above-outlined smearing procedure has been used in Ref. [18] to describe quantitatively the gross properties of the ISGMR and ISGDR in a few nuclei. However, this procedure can not be directly applied to evaluation of the energy-averaged escape amplitudes $M^{L}_{ex}(\omega)$ (and, therefore, the respective branching ratios) because of nonphysical absorption of escaped nucleons outside the nucleus. For this reason, in the present work all the energy-averaged $\omega$-dependent quantities are calculated using the “cut off” radius $R^{*} = R$ together with

| nucl. | $\omega_{1} - \omega_{2}$ | $\bar{\omega}$ | $\Delta$ | $x$ | $\omega_{1} - \omega_{2}$ | $\bar{\omega}$ | $\Delta$ | $x$ |
|-------|------------------------|----------------|---------|-----|------------------------|----------------|---------|-----|
| $^{58}$Ni | 5-16 | 11.56 | 2.08 | 12 | 16-36 | 26.62 | 4.55 | 75 |
| $^{90}$Zr | 11-18 | 13.78 | 2.10 | 10 | 18-32 | 25.42 | 3.48 | 68 |
| $^{116}$Sn | 11-18 | 13.91 | 1.99 | 11 | 18-32 | 25.09 | 3.23 | 67 |
| $^{144}$Sm | 5-15 | 10.50 | 2.36 | 13 | 15-35 | 24.78 | 4.22 | 75 |
| $^{208}$Pb | 8-15 | 11.18 | 1.89 | 14 | 15-24 | 20.73 | 2.39 | 42 |
| 5-15 | 9.90 | 2.55 | 17 | 15-35 | 23.02 | 3.90 | 74 |
| 15-60 | 23.96 | 5.83 | 81 |

FIG. 1a: The calculated relative energy-weighted strength function for the ISGMR. The thick, thin, dashed, dotted, and dash-dotted lines are for $^{58}$Ni, $^{90}$Zr, $^{116}$Sn, $^{144}$Sm, and $^{208}$Pb, respectively.

FIG. 1b: The reduced energy-dependent transition density calculated at the peak energy of the ISGMR (Table III). The notations are the same as in FIG. 1a.
TABLE IV: Comparison of calculated and experimental parameters of the ISGMR and ISGDR. The experimental data (given with errors) are taken from Refs. [29] and [2], respectively. All the parameters are given in MeV.

|       | $^{90}$Zr | $^{116}$Sn | $^{208}$Pb |
|-------|-----------|------------|------------|
| ISGMR |           |            |            |
| $\bar{\omega}$ | 17.89 ± 0.20 | 16.5 | 16.07 ± 0.12 | 15.4 | 14.17 ± 0.28 | 13.9 |
| $\Delta$ | 3.14 ± 0.09 | 2.6 | 2.16 ± 0.08 | 2.1 | 1.93 ± 0.15 | 2.1 |
| LE-ISGDR |           |            |            |
| $\bar{\omega}$ | 16.2 ± 0.8 | 13.8 | 14.7 ± 0.5 | 13.9 | 12.2 ± 0.6 | 11.2 |
| $\Delta$ | 1.9 ± 0.7 | 2.1 | 1.6 ± 0.5 | 2.0 | 1.9 ± 0.5 | 1.9 |
| HE-ISGDR |           |            |            |
| $\bar{\omega}$ | 25.7 ± 0.7 | 25.4 | 23.0 ± 0.6 | 25.1 | 19.9 ± 0.8 | 20.7 |
| $\Delta$ | 3.5 ± 0.6 | 3.5 | 3.7 ± 0.5 | 3.2 | 2.5 ± 0.6 | 2.4 |

The transition densities related to different energy re-

ditions [18, 23, 26] the branching ratios cal-

gross properties of a high-energy GR are found

tative single-particle resonances in the energy dependence

Such a choice allows us to describe correctly the respec-

tions [18]. The squared and properly normalized energy-

FIG. 2a: The calculated relative energy-weighted strength

function for the ISGQR. The notations are the same as in

FIG. 1a.

the properly increased value of the intensity $\alpha$ in Eq. 2.

Such a choice allows us to describe correctly the respec-
tive single-particle resonances in the energy dependence

of the continuum-state wave function, while the calculated

gross properties of a high-energy GR are found almost the same within both versions of the smearing

procedures. In Refs. [18, 23, 26] the branching ratios cal-
culated for a few overtones in $^{208}$Pb are overestimated

because of ignoring absorption of escaped nucleons inside

the nucleus. This shortcoming is partially eliminated in Ref. [19], where all $\omega$-dependent quantities are calculated

using the smearing parameter with intermediate radius

$R^* = 1.3R$ to take the spreading effect into account in

evaluation of the branching ratios for the ISGDR in a few

nuclei.

In conclusion of this Subsect., we define the energy-

averaged quantities suitable for description of the GR

main properties. The relative energy-weighted strength

function $y_L(\omega) = \omega S_L(\omega)/(EWSR)_L$ is used to

show exhaustion of the respective energy-weighted

sum rule (EWSR) by a particular GR. The use of

the reduced energy-dependent transition density

$R_L(r, \omega) = r^2 \rho_L(r, \omega)S_L^{-1/2}(\omega)$ normalized by the condition $\int R_L(r, \omega)V_L(r)dr = 1$ is convenient to compare

the transition densities related to different energy re-

tions [18]. The squared and properly normalized energy-

averaged direct-nucleon-escape amplitude determines the

respective differential partial branching ratio:

$$\frac{d\bar{b}_L^T(\omega)}{d\omega} = \frac{\sum_{\lambda} |\bar{M}_\lambda^T(\omega)|^2}{\int_{\omega_1}^{\omega_2} S_L(\omega)d\omega}.$$  

These quantities are explicitly given in Ref. [17]. Within the RPA, the isovector symmetry of the model Hamiltonian can be restored. As a result, the isovector part of the mean field is calculated selfconsistently via the Landau-Migdal

isovector parameter $f'$ and the neutron-excess density

C. Ingredients of the approach

Within the approach, a phenomenological isoscalar

part of the nuclear mean field (including the spin-orbit term) and the (momentum-independent) Landau-Migdal

particle-hole interaction are used as the input quantities

for CRPA calculations. Parameterization of these quantities is explicitly given in Ref. [17]. Within the RPA, the isovector symmetry of the model Hamiltonian can be restored. As a result, the isovector part of the mean field is calculated selfconsistently via the Landau-Migdal

isovector parameter $f'$ and the neutron-excess density
We start with description of the main-tone resonances. This description allows us

- to check the quality of restoration of the translation invariance of the model within the CRPA;
- to check abilities of the semi-microscopic approach because the gross properties of the isoscalar giant monopole and quadrupole resonances are extensively studied experimentally;
- to evaluate the parameters $\eta_L$ in the expression for the radial part of the second-order probing operators appropriate for microscopic studies of the overtones.

The $1^-$ spurious state associated with the center-of-mass motion is the lowest energy main-tone state. The method to find out properties of the SS within the CRPA is described in detail in Ref. [18] (see also Subsect. II.C). The method allows us to specify the value of the Landau-Migdal parameter $f^{\text{ex}}$ and to calculate the characteristics of the SS, relative isoscalar dipole strength $x^{SS}_{L=1}$ and the transition density $\rho^{SS}_{L=1}(r)$. The calculated values of $f^{\text{ex}}$ and $x^{SS}_{L=1}$ for nuclei under consideration (except for $^{58}$Ni) are given in Table I of Ref. [18]. For $^{58}$Ni we obtain $f^{\text{ex}} = -2.646$ and $x^{SS}_{L=1} = 96\%$. The values

The 1− spurious state described way for each nucleus under consideration. The universal parameters of Eq. 2 used for description of the spreading effect $\alpha = 0.125$ MeV$^{-1}$, $\Delta = 3$ MeV, $B = 7$ MeV are taken the same, as in Ref. [18], except for the $\alpha$ value ($\alpha = 0.085$ MeV$^{-1}$ is used in Ref. [18] together with $R^* = 1.8R$).

III. PROPERTIES OF THE ISOSCALAR GIANT MULTIPOLAR RESONANCES

A. Gross properties of the main-tone resonances

The method to find out properties of the SS within the CRPA is described in detail in Ref. [18] (see also Subsect. II.C). The method allows us to specify the value of the Landau-Migdal parameter $f^{\text{ex}}$ and to calculate the characteristics of the SS, relative isoscalar dipole strength $x^{SS}_{L=1}$ and the transition density $\rho^{SS}_{L=1}(r)$. The calculated values of $f^{\text{ex}}$ and $x^{SS}_{L=1}$ for nuclei under consideration (except for $^{58}$Ni) are given in Table I of Ref. [18]. For $^{58}$Ni we obtain $f^{\text{ex}} = -2.646$ and $x^{SS}_{L=1} = 96\%$. The values

![FIG. 3a: The calculated energy-averaged strength function for the ISGDR. The notations are the same as in FIG. 1a.](image1)

![FIG. 3b: The calculated relative energy-weighted strength function for the ISGDR in $^{58}$Ni.](image2)
The isoscalar monopole and quadrupole GRs in nuclei under consideration are experimentally studied in many works [1, 2, 28, 29, 30]. The deduced strength distributions are presented either in terms of the peak energy \( \omega_{\text{peak}} \) and total width \( \Gamma_L \) (obtained by the Lorentzian or Gaussian fit), or in terms of the mean energy \( \bar{\omega}_L \) and RMS energy dispersion \( \Delta_L \) (obtained for a certain excitation energy interval). These data are used below for comparison with the results, obtained within the microscopic approach. The energy-averaged strength functions \( S_L(\omega) \) calculated with use of the radial part \( V_L(r) = r^2 \) \( (L = 0, 2) \) of the first-order probing operators exhibit well prominent resonances in their energy dependence. The respective peak energies and total widths are given in Table I together with the corresponding experimental values. The relative energy-weighted strength functions \( y_L(\omega) = \omega S_L(\omega)/(EW\, SR)_L \) are shown in Fig. 1a \( (L = 0) \) and in Fig. 2a \( (L = 2) \) for nuclei under consideration. Microscopically the ISGMR and ISGQR are mainly due to 2\( \omega \) particle-hole excitations. The low-energy component of the ISGQR (Fig. 2a) is due to 0\( \hbar \omega \) excitations corresponding to single-particle transitions with changing both the radial and orbital quantum numbers and having, therefore, a small relative strength. The calculated strength functions are used to evaluate (for a certain energy interval) the parameters of the ISGMR and ISGQR: mean energy \( \bar{\omega}_L \), RMS energy dispersion \( \Delta_L \), relative strength \( x_L \). These values are listed in Table II. Comparison with the available experimental data is given in Tables IV and V. The reduced transition densities \( R_{L}(\omega_{\text{peak}}) \) calculated for the main-tone isoscalar monopole and quadrupole

\[ f^{ex} = -2.789 \text{ and } x_{L=1}^{SS} = 94\% \text{ are obtained for } ^{116}\text{Sn with taking neutron pairing into account. The radial dependence of the calculated transition density } \rho_{L=1}^{SS}(r) \text{ is found to be close to } d\rho(r)/dr, \text{ where the ground-state density } \rho(r) \text{ is determined by the bound-state wave functions for the occupied levels:} \]

\[
\rho(r) = \sum_{\alpha=n,p} \sum_{\mu} \frac{(2j_\mu + 1)}{4\pi r^2} \eta_\alpha \langle \chi_\mu^\alpha(r) \rangle^2. \tag{4}
\]

This result is quite acceptable, because the calculated relative strength \( x_{L=1}^{SS} \) is rather close to 100\%. For the same reason, the parameter \( \eta_{L=1} \) in the expression for the second-order dipole operator (Subsect. II A) is almost equal to the value \( 5 < r^2 > /3 \) (averaging is performed over the ground-state density of Eq. 4). This value is employed in many works (see, e.g. Refs. II 13, 14, 17, 18). The \( \eta_{L=1} \) values used in calculations of the main properties of the ISGDR in nuclei under consideration are given in Table I.

The isoscalar monopole and quadrupole GRs in nuclei under consideration are experimentally studied in many works [1, 2, 28, 29, 30]. The deduced strength distributions are presented either in terms of the peak energy \( \omega_{\text{peak}} \) and total width \( \Gamma_L \) (obtained by the Lorentzian or Gaussian fit), or in terms of the mean energy \( \bar{\omega}_L \) and RMS energy dispersion \( \Delta_L \) (obtained for a certain excitation energy interval). These data are used below for comparison with the results, obtained within the microscopic approach. The energy-averaged strength functions \( S_L(\omega) \) calculated with use of the radial part \( V_L(r) = r^2 \) \( (L = 0, 2) \) of the first-order probing operators exhibit well prominent resonances in their energy dependence. The respective peak energies and total widths are given in Table I together with the corresponding experimental values. The relative energy-weighted strength functions \( y_L(\omega) = \omega S_L(\omega)/(EW\, SR)_L \) are shown in Fig. 1a \( (L = 0) \) and in Fig. 2a \( (L = 2) \) for nuclei under consideration. Microscopically the ISGMR and ISGQR are mainly due to 2\( \omega \) particle-hole excitations. The low-energy component of the ISGQR (Fig. 2a) is due to 0\( \hbar \omega \) excitations corresponding to single-particle transitions with changing both the radial and orbital quantum numbers and having, therefore, a small relative strength. The calculated strength functions are used to evaluate (for a certain energy interval) the parameters of the ISGMR and ISGQR: mean energy \( \bar{\omega}_L \), RMS energy dispersion \( \Delta_L \), relative strength \( x_L \). These values are listed in Table II. Comparison with the available experimental data is given in Tables IV and V. The reduced transition densities \( R_{L}(\omega_{\text{peak}}) \) calculated for the main-tone isoscalar monopole and quadrupole
GRs in nuclei under consideration are shown in Fig. 1b and Fig. 2b, respectively. The transition densities are used to evaluate parameters $\eta_L$ accordingly to Eq. (1). These values are given in Table I.

### B. Properties of the ISGDR

The gross properties of the ISGDR in nuclei under consideration are described within the approach using the isoscalar second-order dipole probing operator with parameters $\eta_L=1$ taken from Table I. The calculated energy-averaged strength function $S_{L=1}^{(2)}(\omega)$ (Fig. 3a) exhibits a “bi-modal” energy dependence, corresponding to the low- and high-energy components of the ISGDR (LE- and HE-ISGDR, respectively). As a resonance in the strength function energy dependence, the HE-ISGDR is well prominent and can be described in terms of the peak energy and total width, while the LE-ISGDR is less prominent. The peak energies of both components are given in Table I together with the latest experimental data. The relative energy-weighted strength functions $y_{L=1}^{(2)}(\omega)$ are very close to those, shown in Fig. 1 of Ref. [18] for nuclei under consideration (except for $^{58}$Ni) and are not shown here. The parameters of both components calculated with the use of the above strength functions are given in Table II. A part of these results related to certain excitation-energy intervals are compared with the available experimental data deduced for the same intervals (Table IV). The strength function $y_{L=1}^{(2)}(\omega)$ calculated for $^{58}$Ni is shown in Fig. 3b. Because the main-tone transition density is nodeless, the energy-dependent transition density $R_{L=1}^{(2)}(r, \omega_{\text{peak}})$ calculated at the peak energy of each component exhibits one-node radial dependence (Figs. 3c, 3d).

Direct nucleon decay of giant resonances is closely related to their microscopic structure. For this reason, the decay probabilities belong to the main properties of GRs together with the energy, total width, and transition density. The use of the CRPA method together with the phenomenological treatment of the spreading effect allows us to evaluate the partial direct-nucleon-decay branching ratios for various GRs within the semi-microscopic approach. Turning to the HE-ISGDR in nuclei under consideration, we present the direct-nucleon-decay branching ratios $b_{\text{tot}}^{L=1}$ calculated with the use of unit spectroscopic factors $S_{\mu}$ for single-hole states in closed-shell subsystems and of unit ratio $S_{\mu}/v^{2}_{\text{eff}}$ for single-quasiparticle states in open-shell subsystems (Tables VI and VII). Some partial branching ratios corresponding to population of deep-hole states are not shown, but they are included in the values of the respective total branching ratios also given in the tables. The recent experimental data of Refs. [6, 7] on the partial direct-proton-decay branching ratios for the HE-ISGDR in $^{208}$Pb are given in Table XI together with the respective calculated values obtained with the use of the experimental spectroscopic factors. Calculated for the same resonance, the differential partial proton branching ratios of Eq. (5) are shown in Fig. 4. One can...
FIG. 6a: The reduced energy-dependent transition density calculated at the peak energy of the ISGMR2 (Table I). The notations are the same as in FIG. 1a.

FIG. 6b: The reduced energy-dependent transition density calculated at the peak energy of the ISGQR2 (Table I). The notations are the same as in FIG. 1a.

see from this figure the role of the penetrability factor in formation of the HE-ISGDR in different proton decay channels.

C. Main properties of the ISGMR2 and ISGQR2

We describe the gross properties of the isoscalar monopole and quadrupole overtones in nuclei under consideration, using the respective second-order probing operators with parameters $\eta_L$ ($L = 0, 2$) taken from Table I. The overtone strength is distributed over a wide energy interval exhibiting the main peak at a high excitation energy. The $\omega_{\text{peak}}$ values are given in Table II.

The relative energy-weighted strength functions $y^{(2)}_L(\omega)$ are shown in Figs. 5a ($L = 0$) and 5b ($L = 2$), while the parameters of the high-energy components are given in Table XII. Because the main-tone transition density has one node inside of the nucleus ($L = 0$, Fig. 1a) or is nodeless ($L = 2$, Fig. 1b), the overtone transition density has two nodes ($L = 0$, Fig. 6a) or one node ($L = 2$, Fig. 6b), respectively. The direct-nucleon-decay branching ratios $b^{a}_{L}\mu$, calculated for the high-energy components of both overtones with the use of unit spectroscopic factors, are given in Tables VI-XI. Partial branching ratios for direct proton decay of the above resonances in $^{208}$Pb are also calculated with the use of experimental spectroscopic factors (Table XI).

IV. DISCUSSION OF RESULTS AND SUMMARY

Within the CRPA-based semi-microscopic approach, all the main properties of a given GR can be described in a transparent and rather simple way using universal parameters for medium-heavy and heavy (spherical) nuclei. The symmetries of the model Hamiltonian are restored via the respective selfconsistency conditions. In the present version of the approach the spin-orbit part of the nuclear mean field is mainly responsible for incomplete restoration of translation invariance of the model. If this part was taken equal to zero, the SS would exhaust more than 99.5% of the respective sum rule, while the gross properties of isoscalar GRs are practically not changed. The selfconsistency of the present version of the approach can apparently be improved, provided that the isoscalar spin-dependent part of the Landau-Migdal particle-hole interaction is taken into account. The respective study is outside the scope of the present work and will be addressed in a future publication.

Discussing the results, we start from the gross properties of the isoscalar GRs. Taking $^{208}$Pb as an example, one can see from the results of the semi-microscopic cal-
TABLE VII: Calculated partial branching ratios for direct nucleon decay of the HE-ISGRs in $^{90}$Zr ($S_\mu = 1$). Branching ratios are given in %.

| $\mu^{-1}$ | $b_{\mu}^{L=1}$ | $b_{\mu}^{L=0}$ | $b_{\mu}^{L=2}$ |
|------------|----------------|----------------|----------------|
| neutron    | (18-32 MeV)    | (23-39 MeV)    | (25-40 MeV)    |
| (9/2)$^+$  | 10.2           | 7.6            | 14.1           |
| (1/2)$^-$  | 2.8            | 3.6            | 2.1            |
| (5/2)$^-$  | 4.6            | 5.8            | 5.0            |
| (3/2)$^-$  | 5.7            | 7.3            | 4.8            |
| (7/2)$^-$  | 4.5            | 7.2            | 7.2            |
| (1/2)$^+$  | 0.9            | 2.4            | 1.8            |
| $b_{\mu}^{tot}$ | 29.6          | 40.0           | 39.9           |

TABLE VIII: Calculated partial branching ratios for direct nucleon decay of the HE-ISGRs in $^{116}$Sn ($S_\mu = 1$ and $S_\mu = v_\mu$ are used in calculations of proton and neutron branching ratios, respectively). Branching ratios are given in %.

| $\mu^{-1}$ | $S_\mu$ | $b_{\mu}^{L=1}$ | $b_{\mu}^{L=0}$ | $b_{\mu}^{L=2}$ |
|------------|---------|----------------|----------------|----------------|
| neutron    | (15-35 MeV) | (22-38 MeV)    | (25-37 MeV)    |
| (1/2)$^-$  | 0.006   | 0.04           | 0.20           | 0.06           |
| (3/2)$^-$  | 0.007   | 0.07           | 0.27           | 0.07           |
| (7/2)$^-$  | 0.011   | 0.11           | 0.16           | 0.06           |
| (11/2)$^-$ | 0.188   | 1.9            | 0.83           | 2.7            |
| (3/2)$^+$  | 0.195   | 0.84           | 0.76           | 0.45           |
| (1/2)$^+$  | 0.362   | 0.89           | 1.0            | 0.46           |
| (7/2)$^+$  | 0.862   | 4.65           | 3.7            | 4.75           |
| (5/2)$^+$  | 0.896   | 6.7            | 5.9            | 4.4            |
| (9/2)$^+$  | 0.992   | 4.9            | 5.8            | 8.7            |
| (1/2)$^-$  | 0.994   | 1.8            | 2.3            | 1.4            |
| (3/2)$^-$  | 0.996   | 3.45           | 4.8            | 3.2            |
| $b_{\mu}^{tot}$ | 28.8      | 25.7           | 34.7           |

We try to elucidate the universal phenomenological description of the total width of an arbitrary GR using an appropriate smearing parameter with the saturation-like energy dependence of Eq. (22). In applying to isovector GRs, a similar attempt was found to be satisfactory [20]. According to this description, the total width of low-energy GRs (except for the isobaric analog resonance) is mainly due to the spreading effect. Such a case is realized for the main-tone isoscalar monopole and quadrupole resonances. As follows from CRPA calculations, a significant part of the total width of high-energy GRs is due to the particle-hole strength distribution and coupling to the continuum. The rest is due to the spreading effect, which leads also to averaging the strength distribution over the energy. Regarding the experimental total widths of the isoscalar GRs in nuclei under consideration, we note that the widths of the ISGMR and ISGQR, the RMS energy dispersion for both components are reasonably described within the approach. The experimental total width of the HE-ISGDR in $^{208}$Pb is $26.9 \pm 0.7$ MeV [22, 23] and also the scattering overtones is not so regular, as it takes place for more collective GRs (Table II). As a result, the A-dependence of $\sigma_{HE}$ for these overtones is not so regular, as it takes place for more collective GRs (Table II). According to the data shown in Tables II, IV, V, the experimental energies of the ISGMR, ISGQR, and HE-ISGDR in nuclei from a wide mass interval are reasonably described within the approach. The energy of the LE-ISGDR in the same nuclei is described satisfactorily (Table II, IV, V). The energy of the LE-ISGDR in the same nuclei is described satisfactorily (Table II, IV, V). The energy of the recently found ISGQR2 in $^{208}$Pb is $6.0 \pm 1.3$ MeV [22, 23].

Thus, the calculated energies of the second isoscalar giant quadrupole and monopole resonances in nuclei under consideration can be used as a guide in search of these GRs experimentally.

As expected, the radial dependence of the one-node

couplings of the relative energy-weighted strength function $y_L(\omega)$ (Fig. 7) the general tendency for changing isoscalar strength distribution with increasing excitation energy. The main components of the ISGMR2 and ISGQR2 are not well-collectivized exhausting a not-too-large part of the respective total strength (Table XI). As a result, the A-dependence of $\rho_{HE}^{\alpha \alpha'}$ for these overtones is not so regular, as it takes place for more collective GRs (Table II). According to the data shown in Tables II, IV, V, the experimental energies of the ISGMR, ISGQR, and HE-ISGDR in nuclei from a wide mass interval are reasonably described within the approach. The energy of the LE-ISGDR in the same nuclei is described satisfactorily (Table II, IV, V). The energy of the recently found ISGQR2 in $^{208}$Pb is $26.9 \pm 0.7$ MeV [22, 23] and also the scattering overtones is not so regular, as it takes place for more collective GRs (Table II). As a result, the A-dependence of $\rho_{HE}$ for these overtones is not so regular, as it takes place for more collective GRs (Table II). According to the data shown in Tables II, IV, V, the experimental energies of the ISGMR, ISGQR, and HE-ISGDR in nuclei from a wide mass interval are reasonably described within the approach. The energy of the LE-ISGDR in the same nuclei is described satisfactorily (Table II, IV, V). The energy of the recently found ISGQR2 in $^{208}$Pb is $6.0 \pm 1.3$ MeV [22, 23].

Thus, the calculated energies of the second isoscalar giant quadrupole and monopole resonances in nuclei under consideration can be used as a guide in search of these GRs experimentally.

We try to elucidate the universal phenomenological description of the total width of an arbitrary GR using an appropriate smearing parameter with the saturation-like energy dependence of Eq. (22). In applying to isovector GRs, a similar attempt was found to be satisfactory [20]. According to this description, the total width of low-energy GRs (except for the isobaric analog resonance) is mainly due to the spreading effect. Such a case is realized for the main-tone isoscalar monopole and quadrupole resonances. As follows from CRPA calculations, a significant part of the total width of high-energy GRs is due to the particle-hole strength distribution and coupling to the continuum. The rest is due to the spreading effect, which leads also to averaging the strength distribution over the energy. Regarding the experimental total widths of the isoscalar GRs in nuclei under consideration, we note that the widths of the ISGMR and ISGQR, the RMS energy dispersion for both components of the ISGQR are well described within the approach (Tables II, IV, V). The experimental total width of the ISGQR2 in $^{208}$Pb is $6.0 \pm 1.3$ MeV [22, 23].
transition density $R_L(r, \omega_{\text{peak}})$ for the ISGMR and HE-ISGDR (Figs. 1b and 3c) is rather close to that of the respective transition density calculated within the scaling model \cite{22}, provided the ground-state density of Eq. 1 is used in the calculations. It can be seen, for instance, from Fig. 2 of Ref. 18. We note also, that the difference of the transition densities calculated at the peak energy of each ISGDR component is not so large (Figs. 3b and 3c) to allow for searching for exotic explanations of LE-ISGDR nature. As it is also expected, the overtone transition density has one extra node inside the nucleus relative to the main-tone transition density (Figs. 1b, 2b, 3c, 5b, 6b). Concluding consideration of the gross properties of the isoscalar GRs, we note that the use of the modified smearing procedure (as compared with that of Ref. 18) leads practically to the same results. It can be seen from comparison of the data from Tables III and IV with those from Tables I and II of Ref. 18, respectively. The use of the modified smearing procedure (Subsect. II B) allows us to describe within the present approach the direct-nucleon-decay branching ratios for high-energy GRs. The respective calculation results obtained for the isoscalar overtones in nuclei under consideration, are given in Tables VI-IX. For the ISGDR, the total direct-nucleon-decay branching ratio decreases with decrease of the peak energy (and with increase of the mass number). The relative change is larger for the total proton branching ratios due to the difference in the penetrability factors. We note, that the results of Ref. 18, where the main shortcoming in evaluation of the direct-nucleon-decay branching ratios for high-energy GRs was eliminated (Subsect. II B), are close to those of Tables VI-IX.

The recent experimental data of Refs. 6, 7 on partial direct-proton-decay branching ratios for the HE-ISGDR in $^{208}$Pb are satisfactorily described within the present approach, provided that experimental spectroscopic factors for the final single-hole states of $^{207}$Tl (Table XI) are taken into account. In Ref. 6 direct neutron decay of the same resonance into the final states of $^{207}$Pb from excitation-energy interval 0–6 MeV has been also observed. The deduced branching ratio $23 \pm 5\%$ apparently agrees with the value 15.2% obtained with the use of unit spectroscopic factor for the respective one-hole states (Table XI). Reasonable description of the above data allows us to infer that the calculation results shown in Tables VI-IX will be useful for the analysis of forthcoming experimental data on direct nucleon decays of the HE-ISGDR in several medium-heavy and heavy nuclei \cite{8}. Some evidence for direct proton decay of the ISGQR2 in $^{208}$Pb in the coincidence ($\alpha, \alpha'$) experiments have been recently found \cite{8, 9}. It allows us to hope that the calculated parameters of the ISGQR2 and ISGMR2 (including the energy, total width, transition density, and branching ratios for direct-nucleon-decay) would be also agreeable.

### Table IX: Calculated partial branching ratios for direct neutron decay of the HE-ISGDR in $^{144}$Sm ($S_\mu = 1$). Branching ratios are given in %.

| $\mu^{-1}$ | $b_{\mu}^{L=1}$ (15-35 MeV) | $b_{\mu}^{L=0}$ (21-38 MeV) | $b_{\mu}^{L=2}$ (24-36 MeV) |
|------------|--------------------------|--------------------------|--------------------------|
| (1/2)$^+$  | 1.9                      | 2.0                      | 1.0                      |
| (3/2)$^+$  | 3.4                      | 3.3                      | 2.2                      |
| (11/2)$^-$ | 5.2                      | 4.9                      | 9.9                      |
| (5/2)$^+$  | 5.1                      | 5.7                      | 4.4                      |
| (7/2)$^+$  | 2.6                      | 4.1                      | 3.7                      |
| (9/2)$^+$  | 1.8                      | 4.3                      | 5.2                      |
| $b_{\mu}^{\text{tot}}$ | 23.1                      | 35.1                      | 35.3                      |

### Table X: Calculated partial branching ratios for direct nucleon decay of the HE-ISGDR in $^{208}$Pb ($S_\mu = 1$). Branching ratios are given in %.

| $\mu^{-1}$ | $b_{\mu}^{L=1}$ (15-35 MeV) | $b_{\mu}^{L=0}$ (25-35 MeV) | $b_{\mu}^{L=2}$ (25-35 MeV) |
|------------|--------------------------|--------------------------|--------------------------|
| neutron    |                          |                          |                          |
| (1/2)$^-$  | 0.7                      | 2.0                      | 0.2                      |
| (5/2)$^-$  | 2.5                      | 0.7                      | 1.3                      |
| (3/2)$^-$  | 1.8                      | 0.6                      | 0.6                      |
| (13/2)$^+$ | 3.8                      | 1.3                      | 7.5                      |
| (7/2)$^-$  | 4.3                      | 1.9                      | 2.7                      |
| (9/2)$^-$  | 2.1                      | 1.1                      | 3.0                      |
| $b_{\mu}^{\text{tot}}$ | 22.4                      | 24.7                      | 32.5                      |

### Table XI: Partial branching ratios for direct proton decay of the HE-ISGDRs in $^{208}$Pb into some one-hole states of $^{207}$Tl. Experimental spectroscopic factors $S_\mu$ taken from Ref. 31 are used in calculations. Excitation-energy intervals are taken the same as in Table X. Calculation results for decays of the HE-ISGDR are compared with the experimental data of Refs. 4, 5. Branching ratios are given in %.

| $\mu^{-1}$ | $S_\mu$ | $b_{\mu}^{L=1}$ | $b_{\mu}^{L=0}$ | $b_{\mu}^{L=2}$ |
|------------|---------|----------------|----------------|----------------|
| (1/2)$^+$  | 0.55    | 0.65           | 2.3 $\pm$ 1.1  | 0.34 $\pm$ 0.06 | 1.43 $\pm$ 0.66 |
| (3/2)$^+$  | 0.57    | 0.80           | 1.2 $\pm$ 0.7  | 0.31 $\pm$ 0.05 | 3.60 $\pm$ 1.57 |
| (11/2)$^-$ | 0.58    | 0.29           | 1.07 $\pm$ 0.17| 3.89 $\pm$ 1.89|
| (5/2)$^+$  | 0.54    | 0.75           |                |                |
| (7/2)$^+$  | 0.26    | 0.02           |                |                |
| $\sum b_{\mu}^{L}$ | 2.51                      | 12.24                      | 5.58                      |

*Preliminary results*
TABLE XII: Parameters of the ISGMR2 and ISGQR2 calculated for a certain excitation-energy interval. All the parameters are given in MeV except for $x$, which is given in $\%$.

| nucl. | $\omega_1 - \omega_2$ | $\bar{\omega}$ | $\Delta$ | $x$ | $\omega_1 - \omega_2$ | $\bar{\omega}$ | $\Delta$ | $x$ |
|-------|----------------------|----------------|----------|----|----------------------|----------------|----------|----|
| $^{58}$Ni | 23-40 | 31.1 | 4.7 | 52 | 25-40 | 32.5 | 4.1 | 45 |
| $^{90}$Zr | 23-39 | 30.6 | 4.4 | 51 | 25-40 | 32.2 | 4.1 | 48 |
| $^{116}$Sn | 22-38 | 29.6 | 4.6 | 50 | 25-37 | 31.3 | 3.3 | 40 |
| $^{144}$Sm | 21-38 | 29.2 | 4.7 | 56 | 24-36 | 30.4 | 3.3 | 43 |
| $^{206}$Pb | 25-35 | 30.2 | 2.8 | 34 | 25-35 | 30.0 | 2.6 | 38 |

useful for experimental search of these resonances.

In conclusion, we extend a CRPA-based partially self-consistent semi-microscopic approach to describe direct nucleon decay of high-energy giant resonances. The main properties of the isoscalar overtones (ISGDR, ISGMR2, ISGQR2) in a few singly- and doubly-closed-shell nuclei are described within the approach and found to be in reasonable agreement with available experimental data including the latest ones. Abilities of the approach for description of the main-tone resonances are successfully checked. Predictions concerning forthcoming experimental data on the isoscalar overtones are also presented.

Acknowledgments

The authors are indebted to M. Fujiwara, U. Garg, and M.N. Harakeh for providing the latest experimental data. The authors are thankful to M.N. Harakeh for many interesting discussions and valuable remarks concerning the manuscript. Two authors (I.V.S. and M.H.U.) are grateful to U. Garg for hospitality during their stay at University of Notre Dame and acknowledge support from the National Science Foundation under grant No. PHY-0140324. M.H.U. is grateful to M.N. Harakeh for hospitality during the long-term visit at KVI and acknowledges support from the “Nederlandse organisatie voor wetenschappelijk onderzoek” (NWO).