The puzzle of the quark model: Weak radiative hyperon decays *

Report No. 1702/PH

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November 19, 2018

Abstract

Weak radiative hyperon decays present us with a long-standing puzzle, namely the question of validity of a hadron-level theorem proved by Hara. We briefly discuss the conflict between expectations based on Hara’s theorem and experiment as well as the way in which the quark model evades the theorem. Violation of Hara’s theorem in the quark model is traced back to the issue of hadron compositeness and the nonequivalence of standard ways of imposing gauge-invariance condition at quark and hadron levels. This suggests that our understanding of nonlocal composite nature of hadrons may require some important change.

*To appear in the Proceedings of the conference ”Production and Decay of Hyperons, Charm and Beauty Hadrons”, Strasbourg, France, September 5-8, 1995
1 INTRODUCTION

Weak radiative hyperon decays (WRHD’s) have proved to be a challenge to both theorists and experimenters. Experimental difficulties result from small branching ratios (≈ $10^{-3}$) of WRHD’s and their copious photon backgrounds. Long history of unsuccessful theoretical approaches has led theorists to view the problem of WRHD’s as “a long-standing discrepancy” [1], an “unsolved puzzle” [2] or “the long-standing $\Sigma^+ \rightarrow p\gamma$ puzzle” [3]. Recently, their actual status has been extensively reviewed by J. Lach and the author [4]. It is presented here in brief.

2 THE CONFLICT

WRHD’s are rare strangeness-changing decays of hyperons into other ground-state baryons plus a photon. There are five experimentally observed WRD of ground-state octet baryons: $\Sigma^+ \rightarrow p\gamma$, $\Lambda \rightarrow n\gamma$, $\Xi^0 \rightarrow \Sigma^0\gamma$, $\Xi^0 \rightarrow \Lambda\gamma$, $\Xi^- \rightarrow \Sigma^-\gamma$.

Theoretical problems manifest themselves most clearly in the description of the $\Sigma^+ \rightarrow p\gamma$ decay. This particular decay should satisfy a fairly fundamental theorem proved by Hara [5]. It is therefore extremely interesting that

- there exists a conflict between experiment and expectations based on Hara’s theorem
- the quark model evades Hara’s theorem in a strange and thought-provoking way.

Hara’s theorem, proved at hadron level, reads:

*Parity-violating amplitude $A$ of the $\Sigma^+ \rightarrow p\gamma$ decay vanishes in exact $SU(3)$-flavour symmetry.*

For a nonzero parity-conserving amplitude $B$ one then expects decay asymmetry

$$\alpha = \frac{2\text{Re}(A^*B)}{|A|^2 + |B|^2} \tag{1}$$

to be small since $SU(3)$ is usually broken weakly.

Current experimental evidence, summarized in Fig.1, shows beyond any doubt that asymmetry in question is *large* (and *negative*). The most recent number, coming from the E761 experiment performed at Fermilab [6], is based on nearly 35 thousand events.
A standard first reaction to the above disagreement between experiment and theoretical expectations is to say that in this case SU(3)-breaking is perhaps stronger than elsewhere. In reality the situation is much more involved and delicate: in 1983 Kamal and Riazuddin showed [8] that Hara’s theorem is violated in the quark model also in the SU(3) limit. Explanation of this astonishing result was proposed in 1989 by the author [7].

Since the quark model violates Hara’s theorem even in the SU(3) limit, our attention must be focussed on other assumptions needed in its proof. However, the only apparent other assumptions are:

1. gauge invariance,
2. CP-invariance.

Gauge invariance requires that in the most general hadron-photon parity violating coupling

\[ \Psi_p \gamma_5 (\gamma_\mu F_1(q^2) + q_\mu F_2(q^2) + F_3(q^2)\sigma_{\mu\nu} q'') \Psi_{\Sigma^+} A^\mu \]  

one has \( F_1(0) = 0 \) and, consequently, for real, transverse, final photons \( (q^2 = q_\mu A^\mu = 0) \) only the \( F_3 \) term contributes.

CP-invariance (which relates \( p \leftrightarrow \bar{p}, \Sigma^+ \leftrightarrow \Sigma^- \)) requires that full coupling of the \( p, \Sigma^+ \) initial baryons and the \( \Sigma^+, p \) final baryons to real photons is

\[ F_3(q^2)(\Psi_p \gamma_5 \sigma_{\mu\nu} \Psi_{\Sigma^+} - \Psi_{\Sigma^+} \gamma_5 \sigma_{\mu\nu} \Psi_p) q'' A^\mu \]  

which is antisymmetric under \( \Sigma^+ \leftrightarrow p \) interchange. Since the weak Hamiltonian is symmetric under \( s \leftrightarrow d \) (\( \Sigma^+ \leftrightarrow p \)) interchange (SU(3) limit) we must have \( F_3 = 0 \) and, consequently, the parity-violating \( \Sigma^+ \rightarrow p \gamma \) amplitude vanishes.

One might therefore expect that the quark-model violation of Hara’s theorem results from breaking either gauge- or CP-invariance in quark-level calculations. Quark-model calculations are, however, explicitly gauge- and CP-
invariant, whether one uses the potential model \cite{8} or the bag model \cite{9}. The emerging question is thus: How can the quark model satisfy gauge- and CP-invariance, and yet violate the theorem?

3 AWAY FROM $SU(3)$

In the past an additional problem was caused by the sign of the $\Sigma^+ \rightarrow p\gamma$ asymmetry. Namely, assuming that the $\Sigma^+ \rightarrow p\gamma$ decay is dominated by the single-quark diagram of Fig. 2a, one can show \cite{4, 10} that asymmetry in question is

$$\alpha(\Sigma^+ \rightarrow p\gamma) = \frac{m_a^2 - m_d^2}{m_a^2 + m_d^2}$$

(4)

which is positive (+0.4 or +1.0 for constituent or current quark masses respectively) and thus in disagreement with experiment. Recent precise measurements of the $\Xi^- \rightarrow \Sigma^-\gamma$ branching ratio \cite{11} (which proceeds through diagram (2a) only) prove, however, that there is no way of reproducing the $\Sigma^+ \rightarrow p\gamma$ branching ratio by assuming the dominance of diagram (2a): the predicted branching ratio is then too small by a factor of one hundred.

4 QUARK DIAGRAMS

Out of all topologically possible quark diagrams shown in Fig. 2, contribution from diagrams (c) vanishes in the $SU(3)$ limit and is negligible in explicit calculations with broken $SU(3)$ \cite{4, 12}. Diagrams (d) are suppressed by the presence of two $W$ propagators. Thus, it is contribution from diagrams (b1) and (b2) only that may be significant. Violation of Hara’s theorem results from this very set of quark diagrams \cite{8}.

4.1 Hadron-level way

At the hadron level diagrams (b1) and (b2) correspond to the contribution from intermediate $\frac{1}{2}^-$ excited baryons. Using the quark model one can calculate the $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$ weak transition elements and the $\frac{1}{2}^- \rightarrow \frac{1}{2}^+ + \gamma$ electromagnetic couplings. Their relative size is governed by group-theoretical spin-flavour factors, the products of which are given in Table 1. When one identifies the results of these quark model calculations with those hadron-level expressions that are allowed by gauge invariance, one finds that contributions from diagrams (b1) and (b2) must enter with a relative minus sign, thus ensuring cancellation (in the $SU(3)$ limit) of the corresponding contributions to the $\Sigma^+ \rightarrow p\gamma$ decay \cite{12}.
Fig. 2. Quark diagrams for weak radiative hyperon decays.

In explicit models $SU(3)$ is broken in energy denominators resulting from propagation of the intermediate excited $1^-_2$ baryon. Since $m_{N^*} - m_{\Sigma^+} = \Delta \omega - \delta s$, $m_{\Sigma^*} - m_p = \Delta \omega + \delta s$ (where $\Delta \omega \approx 0.57 GeV$ is the energy difference between excited and ground-state baryons, and $\delta s = m_s - m_{u,d} \approx 0.19 GeV$ is the strange-nonstrange quark mass difference), diagrams (b1) and (b2) - having different energy denominators - do not cancel exactly \[12\]. The corresponding formulae (up to an uninteresting normalization factor) are given in column 2 of Table 2, where $x \equiv \frac{\delta s}{\Delta \omega} \approx \frac{1}{3}$. By construction the obtained $\Sigma^+ \rightarrow p \gamma$ parity violating amplitude vanishes in the $SU(3)$ limit ($x \rightarrow 0$).

Table 1. Group-theoretical factors for diagrams (b1) and (b2)

| process   | diag. (b1) | diag. (b2) |
|-----------|------------|------------|
| $\Sigma^+ \rightarrow p \gamma$ | $-\frac{1}{\sqrt{2}}$ | $-\frac{1}{3\sqrt{2}}$ |
| $\Lambda \rightarrow n \gamma$ | $+\frac{1}{2\sqrt{3}}$ | $+\frac{1}{\sqrt{3}}$ |
| $\Xi^0 \rightarrow \Lambda \gamma$ | 0 | $-\frac{1}{3\sqrt{3}}$ |
| $\Xi^0 \rightarrow \Sigma^0 \gamma$ | $+\frac{1}{3}$ | 0 |
4.2 Quark-level way

There is, however, no reason to identify quark model calculations of the $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$ weak transition elements and the $\frac{1}{2}^- \rightarrow \frac{1}{2}^+ + \gamma$ electromagnetic couplings with hadron-level expressions. One can perform all calculations at the strict quark level and only eventually evaluate the resulting expression in between the initial and final hadronic states. These direct quark-model calculations (potential model [8], bag model [9]) yield amplitudes proportional to the sum of spin-flavour factors corresponding to diagrams (b1) and (b2). In a consistent quark-level calculation the relative sign of spin-flavour factors of diagrams (b1) and (b2) is obviously fixed and it turns out to be positive: Energy denominators corresponding to diagrams (b1) and (b2) are of the same sign. With a relative positive sign the contributions of diagrams (b1) and (b2) add rather than cancel resulting in the violation of Hara’s theorem (see column 3, Table 2). Therefore, it is through insistence on identifying quark-level expressions with the hadron-level gauge-invariance-allowed amplitudes that the relative negative sign was previously generated.

Table 2. Parity violating amplitudes with SU(3) breaking:

| process           | (b1)-(b2) | (b1)+(b2) |
|-------------------|-----------|-----------|
| $\Sigma^+ \to p\gamma$ | $\frac{2}{3\sqrt{2}} - \frac{2}{3\sqrt{2}}$ | $\frac{2}{3\sqrt{2}} + \frac{2}{3\sqrt{2}}$ |
| $\Lambda \to n\gamma$  | $\frac{2-x-1}{3\sqrt{3}} + \frac{2-x}{3\sqrt{3}}$ | $\frac{2-x}{3\sqrt{3}} - \frac{1-x}{3\sqrt{3}}$ |
| $\Xi^0 \to \Lambda\gamma$ | $\frac{1+x}{3\sqrt{3}} - \frac{1+x}{3\sqrt{3}}$ | $\frac{1+x}{3\sqrt{3}} + \frac{1+x}{3\sqrt{3}}$ |
| $\Xi^0 \to \Sigma^0\gamma$ | $\frac{1+x}{3}$ | $\frac{1+x}{3}$ |

5 A CLOSER LOOK

The problem is thus as follows:

- if we apply gauge invariance at hadron level (original proof of Hara’s theorem, or pole model with $\frac{1}{2}^-$ intermediate baryons) - Hara’s theorem is satisfied

- if we apply gauge invariance at quark level - Hara’s theorem is violated.

In order to understand this one needs a way to translate the gauge-invariance condition from the quark to the hadron level (instead of using an ad hoc identi-
fication prescription). The way to do it is called the Kroll-Lee-Zumino scheme [13]. According to the KLZ scheme, translation of quark-level interactions with a photon to the hadron-level language is provided by the vector dominance model (VDM).

Standard VDM prescription is formulated at the hadron level and consists in:

1. calculating vector meson ($V^\mu$) couplings to hadrons ($H_1, H_2$) through $<H_2|J^V_\mu|H_1> V^\mu$ where $J^V_\mu$ are quark currents

2. replacing vector mesons by photons through $V^\mu \rightarrow \frac{e}{g_V} A^\mu$ (where $g_\rho = 5.0$).

The latter step may be obtained at a theoretical level by introducing a gauge-invariance-violating coupling $e_m^2 V \cdot A$ that induces photon mass. In the KLZ scheme one adds additional terms to cancel this photon mass so that gauge invariance is restored. Then, after redefining photon and vector-meson fields as well as electric charge, the VDM prescription turns out to be just a good approximation to the quark-level prescription in which photons couple to quarks directly and in an explicitly gauge-invariant way: $<H_2|J^V_\mu|H_1> A^\mu$ (for details see [4, 13, 14]).

The KLZ scheme permits an understanding of the origin of the violation of Hara’s theorem in the quark model [7]. Namely, explicit calculations of diagrams (b1) and (b2) with photon replaced by vector meson show that the coupling $\Sigma^+ \rightarrow p + (U$-spin singlet vector meson) does not vanish. Since no gauge-invariance condition is imposed in vector-meson case it is clear that the obtained coupling may be identified with the $F_1(q^2) \Psi_p \gamma_5 \gamma_\mu \Psi_{\Sigma^+} V^\mu$ term with a nonvanishing $F_1(0)$. This is in fact the standard identification (see eg. references contained in ref. [3]). Thus, according to the KLZ scheme and ref. [6] the quark-model result corresponds to the VDM-generated effective coupling $F_1(0) \Psi_p \gamma_5 \gamma_\mu \Psi_{\Sigma^+} A^\mu$ that does not vanish at $q^2 = 0$. This coupling was absent in the original derivation of Hara’s theorem in which, therefore, contribution from pointlike quarks was simply not taken into account.

6 OBSERVABLE CONSEQUENCES

When parity-violating amplitudes of Table 2 are supplemented with standard description of parity-conserving amplitudes one obtains different signatures for hadron- and quark- level predictions (see Table 3). Namely, if Hara’s theorem is satisfied (as in hadron-level approaches) all four asymmetries are of the same sign. On the other hand, if the quark-model route is strictly followed, Hara’s theorem is violated and the asymmetries of the $\Lambda \rightarrow n \gamma$ and the $\Xi^0 \rightarrow \Lambda \gamma$ decays are opposite to those of $\Sigma^+ \rightarrow p \gamma$ and $\Xi^0 \rightarrow \Sigma^0 \gamma$. Phenomenologically,
the $\Xi^0 \to \Lambda\gamma$ decay is a much cleaner case than $\Lambda \to n\gamma$ (see ref. [4]). It is therefore extremely important that the asymmetry of the $\Xi^0 \to \Lambda\gamma$ be precisely measured. Current data (Table 3) on the $\Xi^0 \to \Lambda\gamma$ asymmetry reject Hara’s theorem at an almost $3\sigma$ level. When other asymmetries and branching ratios are taken into account the disagreement with Hara’s theorem is even more significant (Table 3, for full account see ref. [4]).

We are therefore eagerly awaiting the results of the hyperon decay program in the E832 KTeV experiment at Fermilab, where the expected number of $\Xi^0 \to \Lambda\gamma$ events is 900, a factor of 10 greater than the number of events observed thus far. Measurements of the $\Xi^0 \to \Sigma^0\gamma$ asymmetry, planned in the same experiment, are also important: for this decay all models predict negative (and often large) asymmetries while the only experiment performed so far does not support a large negative asymmetry.

Table 3. Asymmetries and branching ratios - comparison of two selected conflicting models and experiment

| Asymmetries   | Hara th. satisfied | exp. | Hara th. violated |
|---------------|--------------------|------|-------------------|
| $\Sigma^+ \to p\gamma$ | $-0.80^{+0.32}_{-0.19}$ | $-0.76 \pm 0.08$ | $-0.95$ |
| $\Lambda \to n\gamma$ | $-0.49$ | $+0.80$ | |
| $\Xi^0 \to \Lambda\gamma$ | $-0.78$ | $+0.43 \pm 0.44$ | $+0.80$ |
| $\Xi^0 \to \Sigma^0\gamma$ | $-0.96$ | $+0.20 \pm 0.32$ | $-0.45$ |

Branching ratios (in units of $10^{-3}$)

| Asymmetries   | ref. [12] | exp. | ref. [4] |
|---------------|-----------|------|---------|
| $\Sigma^+ \to p\gamma$ | $0.99^{+0.26}_{-0.14}$ | $1.23 \pm 0.06$ | $1.3 - 1.4$ |
| $\Lambda \to n\gamma$ | $0.62$ | $1.63 \pm 0.14$ | $1.4 - 1.7$ |
| $\Xi^0 \to \Lambda\gamma$ | $3.0$ | $1.06 \pm 0.16$ | $0.9 - 1.0$ |
| $\Xi^0 \to \Sigma^0\gamma$ | $7.2$ | $3.56 \pm 0.43$ | $4.0 - 4.1$ |

7 LOOKING DEEPER

I believe that in a few years’ time predictions of the quark and vector-dominance models will be better confirmed experimentally. The problem will then be to understand this result at a deeper theoretical level.

Technical reasons for the difference between the original hadron-level predictions and the quark or vector-dominance models are already obvious. Namely, in the most naive quark-level calculations quarks are treated as free particles subject to proper (anti)symmetrization of their total wave function. Clearly, the gauge-invariance condition imposed in this language (with gauge transformations on quark fields located at $x_1, x_2, x_3$) is not equivalent to the gauge-invariance condition imposed in the hadron-level language (where gauge transformations are performed on an effective hadron field located at a different
point \( x \)). When such free quarks are confined by phenomenological tools (as in eg. potential model, bag model etc.) the difference in question does not vanish. In particular, unless artificially tailored to satisfy the standard hadron-level gauge-invariance condition, all QCD-inspired approaches with built-in contribution of free quarks must also yield violation of Hara’s theorem.

The physical origin of problems with Hara’s theorem is therefore related to the issue of unobservability of apparently free quarks. Violation of Hara’s theorem by the quark and vector-dominance models indicates that our present understanding of this point is very unsatisfactory. This question has been with us since the beginnings of the quark model (cf. the dubious assumption of additivity of magnetic moments of Dirac quarks which are free and yet always grouped into hadrons). Since the quark model was so tremendously successful, ways of maintaining the contribution from free quarks have been proposed that keep in line with the apparent unobservability of quarks in asymptotic states. With the advent of precise measurements of WRHD’s the original questions reappear with greatly increased strength. I do not think one can answer them in the traditional way: these questions appear in any QCD-inspired quark-confining framework with built-in contribution from free quarks. Ways of representing the freedom of quarks, different from the current ones, would have to be devised should Hara’s theorem be satisfied and quark freedom maintained. Such attempts would then have to confront the ultimate judge - the experiment. The latter favours the violation of Hara’s theorem, though.

Problems with Hara’s theorem are clearly related to a space-time description of composite hadronic states. It is therefore very interesting to note that the case of composite quantum states is beset with conceptual problems at the quantum/special relativity interface, problems that appear at any distance scale. In the opinion of many physicists working on the foundations of physics these problems require a profound change in our understanding of the nature of space. Thus it is very intriguing to note that the KLZ scheme may be viewed as connecting alternative ”space representations” of the underlying physics: the descriptions in terms of constituents (quarks located at points \( x_1, x_2, x_3 \)) and those in terms of composites (hadron located at \( x \)). In my opinion, therefore, hadron physics is more intimately related to the nature of space than it is generally acknowledged.

8 SUMMARY

In summary, WRHD’s probe the very basic assumptions of the quark model. These assumptions are in direct conflict with the standard way of imposing gauge-invariance condition at hadron level.

One cannot have both. One must either drastically modify the basic assumptions of the quark model or admit that the standard way of imposing the gauge-invariance condition at hadron level does not have much to do with
what happens in Nature. Is the quark-model way correct indeed? And - if yes - what does it mean?

I believe that WRHD’s provide us with an important clue to a deeper understanding of the question of how apparently free quarks combine to form hadrons as the only observable asymptotic states.

This work is supported in part by the KBN grant No 2P0B23108.

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