Using Rasch Analysis to Identify Difficult Course Outcomes in Linear Algebra

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Abstract

Linear Algebra, the second Engineering Mathematics subject for all the engineering students in Universiti Kebangsaan Malaysia contains abstract concepts and difficult for students to understand the subject. This study aimed to identify the tough Course Outcome or topics for the Linear Algebra subject. A total of 282 engineering students from electrical, mechanical, chemical and civil engineering participated in the pre-test. The study based on a set of pre-test questions which covers the entire Course Outcomes of Linear Algebra subject. The content of pre-test questions consists of items based on the level of Bloom Taxanomy (knowledge, comprehension, analysis, synthesis and evaluation). Rasch Measurement Model is used to analyze the result. Summary statistics for questions, summary statistics for persons, fit statistics and item dimensionality test are the output achieved from Rasch analysis. Summary statistics for item indicated a good item difficulty of spread. Summary statistics for person indicated a low person reliability value. This means students provided irregular answers for the pre-test questions. Fit statistics identified two questions as misfit. These questions need to be restructured in future. The item dimensionality test indicates the pre-test questions are within the scope of measuring students’ problem-solving ability. Analysis showed that two Course Outcomes identified as the difficult Course Outcomes in Linear Algebra subject. They are the concepts of vector space, diagonalization, quadratic forms and power series. Questions related to comprehension and application level of Bloom Taxanomy are difficult for students to be solved. This study provides an insight view of the Linear Algebra subject by under-pinning the difficult area of the subject. Engineering faculty students should be given more emphasize on the difficult Course Outcome of Linear Algebra subjects. Efforts should be taken by educators to illustrate those topics in a much more simple way.

Keywords: Course Outcome, Difficult, Engineering Mathematics, Linear Algebra, Rasch Measurement Model

1. Introduction

Engineering Mathematics is one important foundation subject to all departments in engineering. The essence of Engineering Mathematics will be applied in many other engineering subjects throughout the engineering programme. Teaching for understanding means students will be able to see the connections between mathematical ideas in various situations. This situation includes either the mathematics problem-solving scenarios or real-life scenarios¹.

The assessment and documentation on how students are learning need to be included in a tertiary institution². This will help to improve the level of education. In³ supports that a faculty or department must formulate the Programme Outcomes and this should be followed by the Course Outcomes. Course Outcomes are designed according to the syllabus of the course offered. Upon deriving the Programme Outcomes and Course Outcomes, student’s achievement can be obtained at the end of the semester.
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In\textsuperscript{4} supports that the evaluation of the Course Outcomes and Programme Outcomes depends on students' performance in quizzes, final examinations, capstone projects and submissions of assignments which contribute to their learning achievements.

Since there is no specific method to measure Course Outcomes, it is quite difficult to measure the performance of each Course Outcome. A modern measurement method, which is the Rasch measurement method, was introduced to measure the Course Outcomes performance of each student. This is well documented in literature\textsuperscript{5}.

The Rasch analysis provides a reliable and reputable measurement rather than establishing the 'best fit line'. Rasch results provide the lecturers with more precise data on students' learning ability achievement. This in fact due to the fact that Rasch focuses on constructing the measurement instrument accurately. This is well documented in the literature\textsuperscript{6}.

The Rasch output will be used as guideline for lecturers to monitor students' performance in each Course Outcome as to gauge the degree of effectiveness of the teaching and learning plans for any course based on the Course Outcomes\textsuperscript{3}.

This study concentrates on outlining the difficult Course Outcomes in Linear Algebra subject for engineering students. The Rasch Measurement Model was used to analyse the output. This study will help lecturers to identify which areas that the engineering students are weak in and it will help the former to take remedial actions to help the latter to widen their knowledge in the Linear Algebra subject.

### 2. Methodology

In Universiti Kebangsaan Malaysia, Linear Algebra (KKKQ 1223) which is known as Engineering Mathematics II is a 4 credit hour subject taken by the Electrical, Mechanical, Chemical and Civil Engineering students in the second semester. A pre-test is a test conducted before the final examination. The pre-test covers all The Course Outcomes of a subject. The pre-test for Linear Algebra was conducted in semester II 2015 / 2016. This two-hour test comprises of five questions and the questions are subjective.

282 students from Electrical, Mechanical, Civil and Chemical departments had sat for this pre-test. The pre-test was validated by two lecturers who teach the subject. The Course Outcome and Programme Outcome are used as the basis to construct the pre-test questions. In addition, in each question, one level of Bloom Taxonomy (Knowledge, Comprehension, Application, Analysis, Evaluation and Creation) was examined.

The pre-test question numbers together with the Course Outcome, Bloom Taxonomy description are labeled in Table 1.

| Question Outcome | Course | Bloom Taxonomy Description |
|------------------|--------|---------------------------|
| 1(i)             | 1      | Knowledge                 |
| 1(ii)            | 1      | Comprehension             |
| 1(iii)           | 1      | Application               |
| 2                | 2      | Comprehension             |
| 3(i)             | 3      | Application               |
| 3(ii)            | 3      | Comprehension             |
| 4(i)             | 4      | Application               |
| 4(ii)            | 4      | Comprehension             |
| 5                | 5      | Comprehension             |

### 3. Results and Discussion

Grades which are compiled in the Excel \textquoteleft prn format was transferred using Bond & Box Steps which is a customized WINSTEPS\textsuperscript{7}. The WINSTEPS program provides detailed statistics on the Summary Statistics, Fit Statistics and Item Dimensionality Test.

#### 3.1 Summary Statistics for Items

Figure 1 shows the measurement of items involved in this study. 'Item' represents the questions tested on the pre-test of the Linear Algebra subject. Item separation is the distance in logits between items of different levels of difficulty\textsuperscript{8}. The higher values of separation represent the spread of items along the continuum and lower values indicate item redundancy\textsuperscript{9}.

The item reliability is 0.98 which indicates good item difficulty spread. It can be noted that Item Mean is 0 and it also gives a good statistics summary for the item with the Item Separation noted at 7.84 logit. Logit is a unit derived from transforming ordinal data into an interval scale\textsuperscript{10}. From Figure 1, the maximum item or the highest location of the item on the logit ruler is +0.86 logit and the mini-
3.2 Summary Statistics for Individuals

The summary of the statistical results for the measurement of individuals is given in Figure 2. Individuals represent the students who took the Linear Algebra pre-test. The Cronbach $\alpha$ shows that the test reliability is 0.49. The person reliability is 0.22. A reliability value below 0.7 indicates that the students provide irregular responses to the items. We assume that they can answer the question, but actually the students had failed to do so. Some pre-test questions are less likely answerable, but some others can also be answered successfully.

If the separation value is 1.0 logit or below, the students cannot be divided into any group. This means that they have the same ability. Figure 2, the person separation is 0.53 logit. Therefore, the students cannot be divided into any group. Individual mean is at -0.70 logit. This means that students’ performance is lower than the expected performance. The negative value indicates that students failed to achieve the level of Bloom Taxanism in each Course Outcome. This also reflects the fact that the students are less competent.

The maximum or the highest item on the difficulty logit ruler is at +0.98 logit while the minimum or the lowest item is located at -1.84 logit. The difference between the highest and lowest measures is average. This indicates that the measurement ruler of individuals’ ability is similar or homogeneous. This implies that the students are of the same ability.

Summary of 282 Measured (Extreme and Non-Extreme) Person

3.3 Fit Statistics

According to Bond and Fox, the fit statistics result will determine whether or not the data fit a construct. Rasch experts examine the item fit by both the infit and outfit. The acceptable region for the Point-Measure Correlation, MNSQ and $z$-Standard are $0.4 < x < 0.8$, $0.5 < MNSQ < 1.5$ and $-2 < z < 2$.

Figure 3, the point measure correlation for item 2 and item 5 is 0. The outfit MNSQ and outfit ZSTD shows maximum measure. Therefore both item 2 and item 5 are considered misfit. The Course Outcomes related to these questions are considered as difficult Course Outcomes in Linear Algebra subject.

| Total | Count | Measure | Model Error | Infit | Outfit |
|-------|-------|---------|-------------|-------|--------|
| Score |       |         |             | MNSQ  | ZSTD   |
|       |       |         |             | MNSQ  | ZSTD   |
| MEAN  | 16.0  | .70     | .41         |       |        |
| S.D.  | 5.0   | .57     | .26         |       |        |
| MAX.  | 34.0  | .98     | 1.13        | .27   | -2.1   |
| MIN.  | 9.0   | -1.84   | .27         | .12   | .09    |

The maximum or the lowest item on the difficulty logit ruler is at +0.98 logit while the minimum or the lowest item is located at -1.84 logit. The difference between the highest and lowest measures is average. This indicates that the measurement ruler of individuals’ ability is similar or homogeneous. This implies that the students are of the same ability.

Summary of 282 Measured (Extreme and Non-Extreme) Person

Figure 1. Summary statistics for items (questions).

Figure 2. Summary statistics for individual (person).
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3.4 Item Dimensionality Test

The dimensionality of the instruments is determined by the standard residual variance of the eigenvalues which indicate the ‘direction’ of the measurement. Figure 4 shows that the raw variance unexplained by measures is 50.2% where it is higher than the 40% minimum. Therefore, the instrument is within the acceptable dimensionality and it showed enough reliability in measuring the students’ problem-solving skills in pre-test questions. The unexplained variance of the 1st contrast is 11.3%. This value is within 5% to 15%. This item dimensionality is unidimensionality.

Table of Standardized Residual variance (in Eigenvalue units)

| Entry Number | Total Count | Total Score | Measure | Model S.E. | INFIT MNSQ | ZSTD | Outfit MNSQ | ZSTD | PT-Measure CORR. | EXP. | Exact Match OBS% | EXP% |
|--------------|-------------|-------------|---------|------------|------------|------|-------------|------|------------------|------|-----------------|------|
| 4            | 65          | 65          | .324    | 1.77       | Maximum Measure | .00  | .00         | 100.0 | 100.0            |      | 2               |      |
| 9            | 65          | 65          | .324    | 1.77       | Maximum Measure | .00  | .00         | 100.0 | 100.0            |      | 5               |      |
| 5            | 83          | 65          | .91     | .16        | 1.31 .9    | 1.07 | .4          | .33   | .37              | 82.8 | 82.3            | 3(i) |
| 2            | 116         | 65          | .41     | .10        | 1.00 .1    | .87  | 1           | .49   | .53              | 51.6 | 54.8            | 1(ii)|
| 7            | 121         | 65          | .35     | .10        | .91 -.4    | .53  | -1.0        | .62   | .54              | 56.3 | 54.1            | 4(i) |
| 8            | 124         | 65          | .32     | .10        | 1.02 .2    | 1.19 | .5          | .53   | .55              | 39.1 | 45.5            | 4(ii)|
| 3            | 214         | 65          | -.38    | .08        | 1.22 1.5   | 1.29 | 1.0         | .48   | .59              | 25.0 | 22.5            | 1(iii)|
| 6            | 258         | 65          | -.72    | .09        | .74 -1.5   | .51  | -1.3        | .63   | .53              | 42.2 | 36.3            | 3(iii)|
| 1            | 277         | 65          | -.90    | .11        | 1.15 .7    | 1.06 | .3          | .50   | .50              | 51.6 | 52.2            | 1(i) |
| MEAN         | 147.0       | 65.0        | .72     | .48        | 1.05 .2    | .93  | .0          |       |                  | 49.8 | 49.7            |      |
| S.D.         | 77.0        | .0          | 1.45    | .69        | .18 .9     | .29  | .8          |       |                  | 16.6 | 17.1            |      |

Figure 3. Item measure for fit statistics.

| Total raw variance in observations | --Empirical-- | Modeled |
|------------------------------------|---------------|---------|
| Total variance explained by measures | = 14.0        | 100.0%  |
| Raw variance explained by persons  | = 7.0         | 50.2%   |
| Raw variance explained by persons  | = 1.3         | 9.2%    |
| Raw variance explained by persons  | = 5.8         | 40.9%   |
| Raw variance explained by persons  | = 7.0         | 49.8%   |
| Unexplained variance in 1st contrast | = 1.6         | 11.3%   |
| Unexplained variance in 2nd contrast | = 1.5         | 10.5%   |
| Unexplained variance in 3rd contrast | = 1.1         | 7.6%    |
| Unexplained variance in 4th contrast | = 1.0         | 7.1%    |
| Unexplained variance in 5th contrast | = .9          | 6.3%    |

Figure 4. Item dimensionality test.

4. Conclusion

The pre-test questions for Linear Algebra (KKKQ 1223) subjects were valid except question 2 and question 5 as both questions identified as misfit. The eigenvalue of the raw variable explained by the measure is 50.2% which is more than 40%. This indicates that the sample and questions are reliable in measuring the students’ ability to answer questions.

The person reliability is 0.22. This means that the students provide irregular responses for the questions. The mean person -0.70 indicates that’s students’ performance in pre-test is below the expectation. This implies that students fail to answer all the questions based on the Course Outcome. This can be seen from the high value of MNSQ.
and ZSTD which states that students’ answers are not predictable.

The Rasch analysis identified that the students are weak in Course Outcome 2 and Course Outcome 5. More tutorials should emphasize these areas. In conclusion, the Linear Algebra pre-test questions cover well all the Course Outcome, Programme Outcome and level of Bloom Taxonomy. The Rasch measurement Model identifies the misfit questions and tests the validity and reliability of questions.

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6. References

1. Van de Walle JA, Karp KS, Bay–Williams JM. Elementary and Middle School Mathematics: Teaching Developmentally. Boston: Allyn and Bacon; 2009. PMcid: PMC2606062.
2. Cartwright R, Weiner K, Streamer-Veneruso S. Student Learning Outcomes Assessment Handbook, Montgomery College, Montgomery Country, Maryland; 2009.
3. Shahir A, Riza AAOKR, Azami Z, Baba MD, Noorhisham TK, Mardina A, Mazlan MT, Andanastuti M, Che HA. Implementing Continual Review of Programme Educational Objectives and Outcomes for OBE Curriculum Based on Stakeholder's Input. Proceeding of the 7th WSEAS International Conference on Education and Educational Technology (Edu’08); 2008.
4. Rozeha AR, Azami Z, Saifudin M. Application of Rasch Measurement in Evaluation of Learning Outcomes: A Case Study in Electrical Engineering. Regional Conference on Engineering Mathematics, Mechanics, Manufacturing and Architecture (EM3ARC); 2007.
5. Kamsuria A, Nazlena MA, Suhaila Z. An Improved Course Assessment for Analyzing Learning Outcomes Performance using Rasch Model, Procedia Social and Behavioral Sciences, 2011; 18:442–49. Crossref
6. Azrilah AA, Azlinah M, Azami Z, Sohaimi Z, Hamzah AG, Mohd SM. Evaluation of Information Professionals Competency Face Validity Test using Rasch Model. 5th WSEAS / IASME International Conference on Engineering Education (EE’08); 2008. p. 22-24.
7. Bond TG, Fox CM. Bonds and Fox Steps (customized version of WINSTEPS), Computer Software. 2006.
8. Draugalis JT, Jackson TR. Objective Curricular Evaluation: Applying the Rasch Model to a Cumulative Examination, American Journal of Pharmaceutical. 2004; 68(2):1−12.
9. Green KE. Survey Development and Validation with the Rasch Model. International Conference on Questionnaire Development, Evaluation and Testing. South Carolina, USA; 2002.
10. Bond TG, Fox CM. Applying the Rasch Model: Fundamental Measurement in the Human Science. Mahwah, New Jersey: Lawrence Erlbaum Associates; 2001.
11. Pomeranz JL, Byers KL, Moorhouse MD, Velozo CA, Spitzngel RJ. Rasch Analysis as a Technique to Examine the Psychometric Properties of a Career Ability Placement Survey Subtest, Rehabilitation Counselling Bulletin. 2008; 51(4):251–59. Crossref.