I. INTRODUCTION

The direct detections of gravitational waves\cite{1,2} provide the evidence of the existence of black hole. With the increasing of these detections, one may examine whether there are some characteristics which is different from general relativity. For black hole case, there may have some fundamental scalar fields interacting with black holes. However, no-hair theorems dictate that asymptotically flat black holes in general relativity cannot support spatially regular static scalar field configurations\cite{4,10}. This become an obstacle to consider new fundamental scalar field.

Recently, the no-hair theorem is violated by considering a non-minimally coupling of scalar field and Gauss-Bonnet invariant\cite{11,12,13}. In particular, they show Gauss-Bonnet term may lead the Schwarzschild black hole become unstable and the corresponding black hole with hair is more stable. Therefore, the Schwarzschild black hole will evolve into the corresponding hair black hole. This physical phenomenon is called spontaneous scalarization which was first found in neutron stars\cite{15}. Furthermore, spontaneous scalarization was also found in Kerr black hole\cite{16,17}. Since then, various black hole with non-minimally coupling of scalar field and Gauss-Bonnet invariant has been studied\cite{18,23}. Interestingly, spontaneous scalarization can also be induced by Maxwell fields when one consider non-minimal couplings between scalar fields and Maxwell fields\cite{24,25}. At present, spontaneous scalarization phenomenons was also studied in various gravity theory\cite{26,32}. In addition, analytic treatment of spontaneous scalarization were studied in\cite{37,41}.

When studying spontaneous scalarization phenomenons, it is important to find the ranges of coupling parameter which allow black hole solutions. In\cite{12,13}, they consider non-minimally coupling of scalar field and Gauss-Bonnet invariant in Schwarzschild black and the numerical computations show the spontaneous scalarization is characterized by discrete sets $\eta \in \{\eta_n^- , \eta_n^+ \}_{n=0}^{+\infty}$. In particular, the discrete eigenvalues $\{\eta_n^\pm \}_{n=0}^{+\infty}$ correspond to composed black-hole-linearized-scalar-field cloudy configurations. In other words, this correspond to test field limit where the background is fixed. Interestingly, in\cite{37}, by examining the numerical data that are available in the physics literature, they find the coupling parameter $\eta_n^\pm$ has an asymptotic universal behavior $\sqrt { \eta_{n+1}^\pm } - \sqrt { \eta_n^\pm } \simeq 2.72$. Motivated by this interesting observation, they use an analytic method to study the $\eta_n^\pm$’s asymptotic behavior. Surprisingly, the numerical data are described extremely well by the analytic method (the deviations are smaller than 1% in the large $n$ limit). Motivated by it, it is interesting to investigate whether there is a similar asymptotic universal behavior in other black holes. Therefore, in this paper, we consider the RN black hole with a non-minimally coupling of scalar field and Gauss-Bonnet invariant. This model has been numerical studied in\cite{23}. Different from Schwarzschild black
hole where the coupling parameter can only take positive value, in this case, the coupling parameter can take positive or negative value. Therefore, we will use the analytic method to study the coupling parameter’s asymptotic behavior for both positive and negative cases. To check the validity of this analytic method, we will also compare analytic results with the numerical data.

The rest of this paper is as follows. In Sec. II, we introduce the model with a scalar field coupled to Gauss-Bonnet invariant. In Sec. III, we use the analytic method to study the coupling parameter’s asymptotic behavior. In Sec. IV, we compare analytic results with the numerical data. In Sec. V, we give a brief conclusion.

II. DESCRIPTION OF THE MODEL

In this paper, we consider the model contains a non-minimal coupling between a real scalar field and the Gauss-Bonnet term. The model reads

\[ S = \int d^4x \sqrt{-g} \left[ \frac{R}{2} + \gamma \phi^2 R_{GB}^2 - \partial_a \phi \partial^a \phi - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} \right] \]  

(1)

Where \( \gamma \) is the coupling parameter and the Gauss-Bonnet term \( R_{GB}^2 \) is given by

\[ R_{GB}^2 = R_{abcd} R^{abcd} - 4 R_{ab} R^{ab} + R^2 \]  

(2)

We use the units in [23] where \( 8\pi G = 1 \). Using the conventions in [23], the RN solution reads

\[ ds^2 = -N dt^2 + \frac{1}{N} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \]  

(3)

\[ N(r) = 1 - \frac{2M}{r} + \frac{Q^2}{2r^2} \]  

(4)

The event horizon is located at \( r_h = M + \sqrt{M^2 - Q^2/2} \) with the extremal limit at \( r_h = M = Q/\sqrt{2} \). Variation with respect to the scalar field \( \phi \) leads to the following equations

\[ \partial_a \partial^a \phi + \gamma \phi R_{GB}^2 = 0 \]  

(5)

Using the field decomposition

\[ \phi(r, \theta, \varphi) = \sum \frac{\psi_{lm}}{r} Y_{lm}(\theta, \varphi) \]  

(6)

Eqs. (5) can be written as

\[ N' d^2 \psi \over dr'^2 + N' d\psi \over dr' = \left( \frac{l(l+1)}{r^2} + \frac{N'}{r} - \gamma R_{GB}^2 \right) \psi \]  

(7)

We use the tortoise coordinate \( y \) defined by

\[ \frac{dr}{dy} = N(r) \]  

(8)

We obtain a Schrödinger-like differential equation

\[ \frac{d^2 \psi}{dy^2} - V \psi = 0 \]  

(9)

The effective radial potential in the Schrödinger-like differential equation is given by

\[ V(r) = N \left( \frac{l(l+1)}{r^2} + \frac{N'}{r} - \gamma R_{GB}^2 \right) \]  

(10)

In [23], they solve this problems with including backreaction of scalar field and the solutions of coupling parameter which allow hairy black hole are many discrete intervals. For \( \gamma > 0 \), the hairy black hole is characterized by a discrete set \( \gamma \in [\gamma_{cr(n)}^{+}, \gamma_{0(n)}^{+}]_{n=0}^{\infty} \). For \( \gamma < 0 \), the hairy black hole is characterized by a discrete set \( \gamma \in [\gamma_{0(n)}^{-}, \gamma_{cr(n)}^{-}]_{n=0}^{\infty} \). In particular, the endpoints \( \gamma_{0(n)} \) correspond to the static bound-state scalar field configurations and it can be determined by the differential equation (6) with the boundary condition \( \psi(r_h < \infty) \) and \( \psi(\infty) \to 0 \). In Schwarzschild black hole, they show the coupling parameter exists an asymptotic universal behavior. Therefore, in next section, we will use the analytic method to explore whether the discrete resonant spectrum \( \{\gamma_{0(n)}\}_{n=0}^{\infty} \) has a similar asymptotic universal behavior.
III. ANALYTIC METHOD

In this section, we use the analytic method to solve the Schrödinger-like differential equation (9). The Schrödinger-like differential equation (9) can be studied by a standard WKB analysis. For bound-state field configurations, there is a well-known quantization condition

$$\int_{y_-}^{y_+} \sqrt{-V(y, \gamma)} dy = (n - \delta)\pi; \quad n = 1, 2, 3, \ldots,$$

(11)

where \{y_-, y_+\} are the turning point of \(V(y)\), \(\delta\) is phase depends on the number of turning points. Using the relation between tortoise coordinate \(y\) and radial coordinate \(r\), Eqs. (11) can be written as

$$\int_{y_-}^{y_+} \frac{-V(y, \gamma)}{N(r)} dr = (n - \delta)\pi; \quad n = 1, 2, 3, \ldots,$$

(12)

It is easy to see horizon radius \(r_h\) leads \(V(r_h) = 0\). Therefore, the horizon radius \(r_h\) is a turning point. The other turning points are determined by

$$\frac{l(l + 1)}{r^2} + \frac{N'}{r} - \gamma R_{GB}^2 = 0$$

(13)

In the eikonal large \(\gamma\) regime, the Eqs. (13) can be approximated by

$$R_{GB}^2 = 0$$

(14)

Using this approximation, Eqs. (12) can be written as

$$\int_{r_-}^{r_+} \sqrt{R_{GB}^2 N(r)} dr = (n - \delta)\pi; \quad n = 1, 2, 3, \ldots,$$

(15)

In order to find the turning points, we need to study the sign of \(R_{GB}^2\) at the interval \([r_h, \infty]\). This has been done in [23]. We will briefly review the conclusion. The expression of \(R_{GB}^2\) reads [23]

$$R_{GB}^2 = \frac{12}{r^6} + \frac{12(r - 2)Q^2}{r^4} + \frac{(3r^2 - 12r + 10)Q^4}{r^3}$$

(16)

where they set \(r_h = 1\) and use \(Q\) to replace \(M\) in expression of \(R_{GB}^2\).

For \(Q < \hat{Q} = \sqrt{2(3 - \sqrt{6})} \approx 1.05\). The \(R_{GB}^2\) is positive at the interval \([r_h, \infty]\). Therefore, the turning point are \(r_- = r_h\) and \(r_+ = +\infty\). From Eqs. (16), we find the \(\gamma\) only can take positive value in this situation. Eqs. (16) becomes

$$\sqrt{-\gamma_0(n)} = (n - \frac{1}{4})\pi / \int_{r_h}^{+\infty} \sqrt{R_{GB}^2 N(r)} dr; \quad n = 1, 2, 3, \ldots,$$

(17)

For \(\sqrt{2} > Q > \hat{Q}\). There is a turning point \(r_0\) where \(R_{GB}^2(r_0) = 0\) and \(r_h < r_0 < \infty\). Therefore, the \(R_{GB}^2\) can be divided into two parts: \(R_{GB}^2(r) < 0\) at \((r_0, r_h)\) and \(R_{GB}^2(r) > 0\) at \((r_0, +\infty)\). Therefore, from Eqs. (16), we see the \(\gamma\) can take positive or negative value. This is different from Schwarzschild case where \(\gamma\) can only take positive value.

For positive \(\gamma\), we take \(r_- = r_0, r_+ = +\infty\) and Eqs. (16) becomes

$$\sqrt{\gamma_0(n)} = (n - \frac{1}{4})\pi / \int_{r_0}^{+\infty} \sqrt{R_{GB}^2 N(r)} dr; \quad n = 1, 2, 3, \ldots,$$

(18)

For negative \(\gamma\), we take \(r_- = r_h, r_+ = r_0\) and Eqs. (16) becomes

$$\sqrt{-\gamma_0(n)} = (n - \frac{1}{2})\pi / \int_{r_h}^{r_0} \sqrt{R_{GB}^2 N(r)} dr; \quad n = 1, 2, 3, \ldots,$$

(19)

Therefore, we get three expressions of \(\gamma_0(n)\) in different situations. Using their discrete spectrum, we can find quantity relations of their behavior.
For \( Q < \bar{Q} = \sqrt{2(3 - \sqrt{6})} \approx 1.05 \), from Eqs. (17), we find
\[
\sqrt{\gamma_0(n+1)} - \sqrt{\gamma_0(n)} = \pi/\int_{r_h}^{r_\infty} \sqrt{\frac{R_{GB}^2}{N(r)}} dr
\]  
(20)
where the integration in the right side of Eqs. (20) is a constant for a fixed \( Q \). Therefore we analytically derive a relation of \( \gamma \).

For \( \sqrt{2} > Q > \bar{Q} \) and \( \gamma > 0 \), we find
\[
\sqrt{\gamma_0(n+1)} - \sqrt{\gamma_0(n)} = \pi/\int_{r_h}^{r_\infty} \sqrt{\frac{R_{GB}^2}{N(r)}} dr
\]  
(21)

For \( \sqrt{2} > Q > \bar{Q} \) and \( \gamma < 0 \), we find
\[
\sqrt{-\gamma_0(n+1)} - \sqrt{-\gamma_0(n)} = \pi/\int_{r_h}^{r_0} \sqrt{\frac{R_{GB}^2}{N(r)}} dr
\]  
(22)

In this section, we derive an analytical relation of \( \gamma_0(n) \). We will compare the analytical result with numerical data in next section.

IV. ANALYTICAL RESULTS VERSUS NUMERICAL DATA

It is interesting to compare the analytical result with numerical data. The numerical solutions can be found by integrating equation (17) from horizon to infinity. Using the transformation \( \psi \rightarrow \lambda \psi \), we can set \( \psi(r_h) = 1 \) and following the convention in [23], we set \( r_h = 1 \). The infinity boundary condition is \( \psi(10000) = 0 \). In Sec. III we derive three different relations (20) (21) and (22) in different situations. We will examine them separately and suppress subscript "0" in \( \gamma_0(n) \) for convenience. We use the ratio \( R_n \equiv \sqrt{\frac{\gamma_n^{\text{analytic}}}{\gamma_n^{\text{numerical}}}} \) to examine the difference between the analytical result with numerical data.

For \( Q < \bar{Q} = \sqrt{2(3 - \sqrt{6})} \approx 1.05 \). The coupling parameter can only take positive value. We can use Eqs. (17) to get the analytical result. We consider three cases: \( Q = 0, Q = 0.5 \) and \( Q = 1.0 \). The results can be seen from Table I

| \( n \) | 1 | 2 | 3 | 4 | 17 | 18 | 19 | 20 |
|------|---|---|---|---|----|----|----|----|
| \( R_{n}(Q = 0) \) | 1.1977 | 1.0781 | 1.0483 | 1.0350 | 1.0076 | 1.0071 | 1.0067 | 1.0064 |
| \( R_{n}(Q = 0.5) \) | 1.1553 | 1.0711 | 1.0455 | 1.0334 | 1.0075 | 1.0071 | 1.0067 | 1.0063 |
| \( R_{n}(Q = 1) \) | 1.0261 | 1.0315 | 1.0275 | 1.0240 | 1.0071 | 1.0067 | 1.0064 | 1.0060 |

From the Table I, we see analytical results agree well with numerical data and the agreement between the analytical result and numerical data improves with increasing of \( n \). In fact, the WKB is believed to be valid in the large \( n \) limit. However, at this time, the difference between the analytical result and numerical data is already less than 1% when \( n = 20 \).

Let us check the behavior of \( \sqrt{\gamma_n^{\text{numerical}}} - \sqrt{\gamma_n^{\text{numerical}}} \). In order to examine this behavior, we define \( \Delta_n \equiv \sqrt{\gamma_n^{\text{numerical}}} - \sqrt{\gamma_n^{\text{numerical}}} \). The results can be seen from Table II.

| \( n \) | 1 | 2 | 3 | 4 | 16 | 17 | 18 | 19 |
|------|---|---|---|---|----|----|----|----|
| \( \Delta_{n}(Q = 0) \) | 0.6783 | 0.6802 | 0.6803 | 0.6803 | 0.6802 | 0.6802 | 0.6802 | 0.6802 |
| \( \Delta_{n}(Q = 0.5) \) | 0.6902 | 0.6985 | 0.6999 | 0.7004 | 0.7000 | 0.7010 | 0.7010 | 0.7010 |
| \( \Delta_{n}(Q = 1) \) | 0.8914 | 0.9039 | 0.9107 | 0.9152 | 0.9227 | 0.9228 | 0.9228 | 0.9229 |

The corresponding analytical results can be obtained from Eqs. (20). For \( Q = 0, Q = 0.5 \) and \( Q = 1.0 \), the corresponding analytical result are 0.6802 0.7010 and 0.9232. Comparing them with the numerical data in table II, we find analytical result coincide well with numerical data.
Now, we consider the cases where \( Q > \tilde{Q} = \sqrt{2(3 - \sqrt{6})} \approx 1.05 \). At this time, the coupling parameter \( \gamma \) can take positive or negative value. For the positive branch, we use \( R^+ = \sqrt{\gamma^\text{analytically}/\gamma^\text{numerical}} \) to compare analytically result and numerical data. For the negative branch, we use \( R^- = \sqrt{\gamma^\text{analytically}/\gamma^\text{numerical}} \). We consider three cases: \( Q = 1.1, Q = 1.2 \) and \( Q = 1.4 \). The results can be seen from Table III. It is easy to see the difference between analytical results and the numerical data becomes smaller with the increasing of \( n \). The difference is already smaller than 1% at \( n = 20 \). These results confirm the validity of the analytic method.

Now, we examine the asymptotic behavior of \( \gamma \). For positive branch, we use \( \Delta^+_n = \sqrt{\gamma^\text{analytically}/\gamma^\text{numerical}} \) to investigate the asymptotic behavior of \( \gamma \). The results are shown in Table IV

| \( n \) | \( R^+ (Q = 1.1) \) | \( R^+ (Q = 1.2) \) | \( R^+ (Q = 1.4) \) | \( R^+ (Q = 1.1) \) | \( R^+ (Q = 1.2) \) | \( R^+ (Q = 1.4) \) | \( R^+ (Q = 1.1) \) | \( R^+ (Q = 1.2) \) | \( R^+ (Q = 1.4) \) |
|---|---|---|---|---|---|---|---|---|---|
| 0.6798 and 2.1380. For negative branch, we use \( \Delta^-_n = \sqrt{\gamma^\text{analytically}/\gamma^\text{numerical}} \) to investigate the asymptotic behavior of \( \gamma \). The results are shown in Table IV

| \( n \) | \( \Delta^+_n (Q = 1.1) \) | \( \Delta^+_n (Q = 1.2) \) | \( \Delta^+_n (Q = 1.4) \) | \( \Delta^+_n (Q = 1.1) \) | \( \Delta^+_n (Q = 1.2) \) | \( \Delta^+_n (Q = 1.4) \) | \( \Delta^-_n (Q = 1.1) \) | \( \Delta^-_n (Q = 1.2) \) | \( \Delta^-_n (Q = 1.4) \) |
|---|---|---|---|---|---|---|---|---|---|
| 1.3433 | 1.3445 | 1.3456 | 1.3464 | 1.3478 | 1.3478 | 1.3483 | 1.3483 | 1.3483 | 1.3483 |
| 1.3431 | 1.3441 | 1.3451 | 1.3461 | 1.3474 | 1.3474 | 1.3478 | 1.3478 | 1.3478 | 1.3478 |
| 1.3430 | 1.3440 | 1.3450 | 1.3460 | 1.3473 | 1.3473 | 1.3477 | 1.3477 | 1.3477 | 1.3477 |
| 1.3430 | 1.3440 | 1.3450 | 1.3460 | 1.3473 | 1.3473 | 1.3477 | 1.3477 | 1.3477 | 1.3477 |
| 1.3430 | 1.3440 | 1.3450 | 1.3460 | 1.3473 | 1.3473 | 1.3477 | 1.3477 | 1.3477 | 1.3477 |
| 1.3430 | 1.3440 | 1.3450 | 1.3460 | 1.3473 | 1.3473 | 1.3477 | 1.3477 | 1.3477 | 1.3477 |
| 1.3430 | 1.3440 | 1.3450 | 1.3460 | 1.3473 | 1.3473 | 1.3477 | 1.3477 | 1.3477 | 1.3477 |
| 1.3430 | 1.3440 | 1.3450 | 1.3460 | 1.3473 | 1.3473 | 1.3477 | 1.3477 | 1.3477 | 1.3477 |
| 1.3430 | 1.3440 | 1.3450 | 1.3460 | 1.3473 | 1.3473 | 1.3477 | 1.3477 | 1.3477 | 1.3477 |
| 1.3430 | 1.3440 | 1.3450 | 1.3460 | 1.3473 | 1.3473 | 1.3477 | 1.3477 | 1.3477 | 1.3477 |

For positive branch, the corresponding analytical result which can be obtained separately from Eqs. (21) are 1.3396 1.6798 and 2.1380. For negative branch, the corresponding analytical results which can be obtained separately from Eqs. (22) are 5.1761 1.6172 and 0.3270. It is easy to see the analytical results agree well with the numerical data.

V. CONCLUSION

Recently, by considering the non-minimally coupling of scalar field and Gauss-Bonnet invariant \cite{12, 13}, they show the black hole can support bound-state hairy scalar configurations. Using numerical method, they show the black hole with scalar hair is characterized by a discrete set \( \tilde{\eta} \in [\tilde{\eta}, \tilde{\eta}]_{n=0} \), where \( \tilde{\eta} \) is the dimensionless coupling parameter between scalar field and Gauss-Bonnet invariant. In the interesting paper \cite{37}, they find the \( \tilde{\eta}^+ \) which correspond the hairy black-hole-linearized-scalar-field configurations has an asymptotic universal behavior \( \sqrt{\tilde{\eta}^+_{n+1} - \sqrt{\tilde{\eta}^+_n}} \). By using an analytical methods, they provide a compact explanation for this asymptotic universal behavior and the analytical results agree well with the numerical observed.

Motivated by this interesting asymptotic behavior, in this paper, we study the RN black hole with the non-minimally coupling of scalar field and Gauss-Bonnet invariant. We investigate whether the coupling parameter has a similar asymptotic behavior in this situation. What’s more, it is also interesting to examine the validity of the analytical method in this situation. Different from Schwarzschild black hole where the coupling parameter can only take positive value, in this case, the coupling parameter \( \gamma \) can take positive or negativive value. By using analytical method, in Sec. \( \text{III} \) we derive three different relations (20) (21) and (22) in different situations. In Sec. \( \text{IV} \) we compare analytically results with the numerical data (see the data in Table II and IV). We find the analytical results agree well with the numerical data. Therefore, we conclude that the coupling parameter in RN black hole has a similar asymptotic behavior and analytical method give an excellent explanation for this asymptotic universal behavior.
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