Insight into nucleon structure from lattice calculations of moments of parton and generalized parton distributions

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This talk presents recent calculations in full QCD of the lowest three moments of generalized parton distributions and the insight they provide into the behavior of nucleon electromagnetic form factors, the origin of the nucleon spin, and the transverse structure of the nucleon. In addition, new exploratory calculations in the chiral regime of full QCD are discussed.

1. INTRODUCTION

Lattice field theory not only offers the prospect of precisely calculating the experimentally observable properties of the nucleon from first principles, but also offers the deeper opportunity of obtaining insight into how QCD actually works in producing the rich and complex structure of hadrons. Beyond simply calculating numbers, we would like to answer basic questions of hadron structure. For example, how does the nucleon quark and gluon structure produce the observed scaling behavior of form factors? How does the total spin of the nucleon arise from the spin and orbital angular momentum of its quark and gluon constituents? What is the transverse, as well as longitudinal structure of the nucleon light-cone wave function? As the quark mass is continuously decreased from a world in which the pion mass is 1 GeV to the physical world of light pions, how does the physics of the quark model and adiabatic flux tube potentials evolve into the physics of chiral symmetry breaking, where instantons, quark zero modes, and the associated pion cloud play a dominant role? As discussed below, contemporary lattice calculation are beginning to provide insight into these and other fundamental questions in hadron structure.

Because of asymptotic freedom, high energy lepton scattering provides precise measurements of matrix elements of the light-cone operator
\[
O(x) = \int \frac{d^4 \lambda}{(2\pi)^4} e^{i \lambda \cdot x} \bar{\psi}(\frac{\lambda^2}{2}) \not\! p e^{-i g} \int_{\lambda/2}^{\lambda/2} d \alpha n \cdot A(\alpha n) \psi(\frac{\lambda^2}{2})
\]
where \(n\) is a unit vector along the light-cone.

The familiar quark distribution \(q(x)\) specifying the probability of finding a quark carrying a fraction \(x\) of the nucleon’s momentum in the light cone frame is measured by the diagonal nucleon matrix element, \(\langle P|O(x)|P\rangle = q(x)\). Expanding \(O(x)\) in local operators via the operator product expansion generates the tower of twist-two operators,
\[
O_q^{(\mu_1 \mu_2 \cdots \mu_n)} = \bar{\psi}_q (\not\! x^n D \cdots i \not\! \gamma D) \psi_q,
\]
and the diagonal matrix element \(\langle P|O_q^{(\mu_1 \mu_2 \cdots \mu_n)}|P\rangle\) specifies the \((n - 1)\)th moment of the quark distribution \(\int dx x^{n-1} q(x)\).
The generalized parton distributions $H(x,ξ,t)$ and $E(x,ξ,t)$ are measured by off-diagonal matrix elements of the light-cone operator

$$\langle P'|O(x)|P \rangle = \langle \langle \tilde{\alpha} \rangle H(x,ξ,t) + \frac{i\Delta^\mu}{2m} \langle \sigma^{\mu\nu} n_\nu \rangle E(x,ξ,t) \rangle,$$

where $\Delta^\mu = P'^\mu - P^\mu$, $t = \Delta^2$, $ξ = -n \cdot \Delta/2$, and $\langle \langle \Gamma \rangle \rangle = U(P')\Gamma U(P)$ for Dirac spinor $U$. Off-diagonal matrix elements of the tower of twist-two operators $\langle P'|O_3^{(μ_1μ_2...μ_n)}|P \rangle$ yield moments of the generalized parton distributions, which in the special case of $ξ = 0$, are

$$\int dx x^{n-1} H(x,0,t) = A_{n,0}(t)$$

$$\int dx x^{n-1} E(x,0,t) = B_{n,0}(t),$$

where $A_{n,i}(t)$ and $B_{n,i}(t)$ are referred to as generalized form factors (GFF’s). Analogous expressions in which the light-cone operator $O(x)$ and twist-two operators contain an additional $γ_5$ measure the longitudinal spin density, $Δq(x)$ and spin-dependent generalized parton distributions $H(x,ξ,t)$ and $E(x,ξ,t)$ with moments $A_{n,i}(t)$ and $B_{n,i}(t)$.

In this talk, I will discuss recent calculations of the generalized form factors $A_{(n=1,2,3),0}(t)$ and $\tilde{A}_{(n=1,2,3),0}(t)$ in full, unquenched QCD in the currently computationally accessible domain that I refer to as the “heavy pion world” and discuss their physical significance. In addition, I will discuss initial efforts to explore the chiral regime in which the pion mass is sufficiently light that one can use chiral perturbation theory to extrapolate to the physical pion mass. Although results in the heavy pion world cannot be directly compared with experiment, they nevertheless provide important insight into how QCD works, and provide the first step in the ultimate program of studying how hadronic physics evolves from the heavy pion world to our physical world.

2. LATTICE CALCULATION

The lowest three moments of spin-independent GFF’s considered in this talk are

$$\langle P'|O^{μ_1}|P \rangle = \langle \langle γ^{μ_1} \rangle \rangle A_{10}(t)$$

$$+ \frac{i}{2m} \langle \langle σ^{μ_1α} \rangle \rangle Δ_α B_{10}(t),$$

$$\langle P'|O^{(μ_1μ_2)}|P \rangle = B^{(μ_1 \langle γ^{μ_2} \rangle} A_{20}(t)$$

$$+ \frac{i}{2m} \langle \langle σ^{μ_2α} \rangle \rangle Δ_α B_{20}(t)$$

$$+ \frac{1}{m} Δ^{(μ_1 μ_2)} C_{20}(t),$$

$$\langle P'|O^{(μ_1 μ_2 μ_3)}|P \rangle = \tilde{P}^{(μ_1 \tilde{μ}_2 \langle γ^{μ_3} \rangle} A_{30}(t)$$

$$+ \frac{i}{2m} \langle \langle σ^{μ_2 μ_3} \rangle \rangle Δ_α B_{30}(t)$$

$$+ Δ^{(μ_1 μ_2)} \langle γ^{μ_3} \rangle A_{32}(t)$$

$$+ \frac{i}{2m} \langle \langle σ^{μ_3α} \rangle \rangle Δ_α B_{32}(t).$$

Generalized form factors $A_{(n=1,2,3),0}(t)$ and $\tilde{A}_{(n=1,2,3),0}(t)$ were calculated using the new method introduced in Ref. 5. We considered all the combinations of $\tilde{P}$ and $\tilde{P}'$ that could produce the same four-momentum transfer $t = (P' - P)^2$, subject to the conditions that $\tilde{P} = \frac{2π}{\sqrt{n_x n_y n_z}} (-n_x, n_y, n_z)$ and $\tilde{P}' = (0, 0, 0)$ or $\frac{2π}{\sqrt{n_x n_y n_z}} (-1, 0, 0)$. Using all these momentum combinations for a given $t$ below 3.5 GeV, we calculated all the $H(4)$ cubic group lattice operators and index combinations producing the same continuum GFF’s, obtaining an overdetermined set of equations from which we extracted the most statistically accurate measurement of the desired GFF’s the available lattice data can provide. As discussed in connection with Fig. 3, the errors are substantially smaller than obtained by the common practice of measuring a single operator with a single momentum combination.

We calculated connected diagram contributions using approximately 200 SESAM full QCD configurations with Wilson fermions at $β = 5.6$ on $16^3 \times 32$ lattices. These calculations in the heavy pion world were performed at each of three quark masses, $κ = 0.1570, 0.1565, \text{and} 0.1560$, cor-
Figure 1. Electromagnetic form factor ratio $F_2/Q^2 F_1(Q^2)$, with dipole fits denoted by dashed curves. Experimentally, it has recently been shown that the next to leading order light cone wave function yields $F_2 \sim F_1 \log^2(Q^2/\Lambda^2)/Q^2$, and the agreement between this prediction and the JLab data is shown in the lower portion of Fig. 1.

Since the short range quark structure dominates this physics, it is reasonable to expect that omission of the pion cloud in the heavy pion world should not destroy the qualitative behavior. Indeed, our lattice results plotted in the top portion of Fig. 1 for the value $\Lambda = 0.3$ GeV yields excellent agreement with the $Q^2$ behavior of the experimental data.

3. ELECTROMAGNETIC FORM FACTORS

One of the early successes of perturbative QCD was the understanding of how the short range quark structure of a hadron governs the behavior of exclusive processes at large momentum transfer. However, whereas simple counting rules suggested that $F_2 \sim F_1/Q^2$, experimental data from JLab shows that $F_2$ falls off much more slowly. Theoretically, it has recently been shown that the next to leading order light cone wave function yields $F_2 \sim F_1 \log^2(Q^2/\Lambda^2)/Q^2$, and the agreement between this prediction and the JLab data is shown in the lower portion of Fig. 1.

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4. ORIGIN OF THE NUCLEON’S SPIN

In the nonrelativistic quark model with three quarks in angular momentum zero single particle states, the total proton spin of $1/2$ arises trivially from adding the spins of the three quarks. The so-called spin crisis arose when deep inelastic scattering measurements of the lowest moment of the spin-dependent structure function, $\Delta \Sigma = \langle 1 \rangle_{\Delta u} + \langle 1 \rangle_{\Delta d}$, indicated that only of the order of 30% of the nucleon spin arises from quark spins. Physically, in the heavy pion world where the quarks become less relativistic and are described by the quark model, it is reasonable to expect most of the nucleon spin to arise from the valence quark spin. As the quarks become lighter,
one expects this fraction to decrease as the relativistic quarks acquire more angular momentum and instanton effects, for example, remove helicity from the valence quarks and transfer it to the gluons and to quark-antiquark pairs. We would hope to observe this behavior clearly on the lattice.

The total quark contribution to the nucleon spin is given by the extrapolation to \( t = 0 \) of \( A_{20}^{u+d}(t) \) and \( B_{20}^{u+d}(t) \) shown in Figure 2. Since \( A_{20}^{u+d}(t) \) is calculated directly at \( t = 0 \) and \( B_{20}^{u+d}(t) \) is well fit by a constant that is measured to be nearly zero with small errors, the connected contribution to the angular momentum is measured to within a few percent.

The calculation of the generalized form factors \( A_{20}(t), B_{20}(t), \) and \( C_{20}(t) \) provides an excellent example of the power of the new method introduced in Ref. [5]. Figure 3 shows how adding additional operators and external momentum combinations reduces the error in \( C_{20}(t) \) by a factor of 5 from the standard minimally determined case to our highly overdetermined case.

Combined with the results of \( \Sigma \) from reference [12], we obtain the connected diagram contributions to the decomposition of nucleon spin shown in Tab. [4] and plotted in Fig. [4]. Similar results have been obtained in Refs. [13,14]. To the extent that the disconnected diagrams do not change the qualitative behavior, we conclude that of the order of 70% of the spin of the nucleon arises from the quark spin and a negligible fraction arises from the quark orbital angular momentum in a heavy pion world where \( m_\pi \sim 700 - 900 \) MeV. This behavior is just as expected from the arguments above. As lattice calculations approach the chiral limit, it will be interesting to fill in this graph and observe the quark spin contribution decrease to \( \sim 30\% \) to agree with experiment.

5. TRANSVERSE STRUCTURE OF THE NUCLEON

In general, \( H(x, \xi, t) \) is complicated to interpret physically because it combines features of
both parton distributions and form factors, and depends on three kinematical variables: the
momentum fraction \(x\), the longitudinal component of the momentum transfer \(\xi\), and the total
momentum transfer squared, \(t\). In the particular case in which \(\xi = 0\), however, Burkardt [5]
has shown that \(H(x, 0, t)\), as well as its spin-dependent counterpart \(\tilde{H}(x, 0, t)\), has a simple
and revealing physical interpretation.

It is useful to consider a mixed representation in which transverse coordinates are specified in
coordinate space, the longitudinal coordinate is specified in momentum space and one uses light-
cone coordinates for the longitudinal and time directions: \(x^\pm = (x^0 \pm x^3)/\sqrt{2},
\ p^\pm = (p^0 \pm p^3)/\sqrt{2}\). Using these variables, letting \(x\) denote the momentum fraction
and \(b_\perp\) denote the transverse displacement (or impact parameter) of the light
cone operator relative to the proton state, one may define an impact parameter dependent parton
distribution
\[
q(x, b_\perp) \equiv \langle P^+, R_\perp = 0, \lambda | \tilde{O}(x, b_\perp) | P^+, R_\perp = 0, \lambda \rangle,
\]
where
\[
O_q(x, b_\perp) = \int \frac{dx^-}{4\pi} e^{ix^+p^+} \tilde{q}(-\frac{x^-}{2}, b_\perp) \gamma^\perp q(\frac{x^-}{2}, b_\perp).
\]

Burkardt shows that the generalized parton distribution \(H(x, 0, t)\) is the Fourier transform of
the impact parameter dependent parton distribution, so that
\[
H(x, 0, -\Delta^2) = \int d^2b_\perp q(x, b_\perp) e^{i\vec{b}_\perp \cdot \vec{\Delta}_\perp}
\]
\[
A_{n,0}(\Delta^2) = \int d^2b_\perp \int dx^n q(x, b_\perp) e^{i\vec{b}_\perp \cdot \vec{\Delta}_\perp}, \quad (5)
\]
fraction nucleon to depend significantly on

\[ \text{momentum of partons varies with momentum fraction} \]

one could determine how the transverse distribution

of partons gives a negligible contribution. In this case the active parton represents the (transverse) parton carries all the momentum and the spectator partons gives zero spatial extent. Indeed, explicit light cone wave functions [16,17] bear out this expectation, with the result [18]

\[ q(x, b_\perp) = (4\pi)^{N-1} \sum_{n,c} \sum_{a=1}^{N} \int \prod_{j=1}^{n} dx_{j} d^{2}r_{\perp j} \]

\[ \times \delta \left( 1 - \sum_{j=1}^{n} x_{j} \right) \delta^{2} \left( \sum_{j=1}^{n} x_{j} r_{\perp j} \right) \delta (x - x_{a}) \]

\[ \times \delta^{2} \left( b_{\perp} + (1 - x)r_{\perp a} - \sum_{j \neq a}^{n} x_{j} r_{\perp j} \right) \]

\[ \times \Psi_{n, c}(x_{1}, \ldots; r_{1}, \ldots) \Psi_{n, c}(x_{1}, \ldots; r_{1}, \ldots), \]

where \( a \) denotes the index of the active parton, \( N \) is the number of partons in the Fock state and the sum over \( c \) represents the sum over all additional quantum numbers characterizing the Fock state.

Here, one explicitly observes \( \lim_{x \to 1} q(x, b_\perp) \propto \delta(b_\perp) \). Since \( H(x, 0, t) \) is the Fourier transform of the transverse distribution, the slope in \(-t = \Delta^{2}\) at the origin measures the rms transverse radius. As a result, we expect the substantial change in slope with \( x \). In particular, as \( x \to 1 \), the slope should approach zero. Hence, when we calculate moments of \( H(x, 0, t) \), the higher the power of \( x \), the more strongly large \( x \) is weighted and the smaller the slope should become.

Therefore, this argument makes the qualitative prediction that the slope of the generalized form factors \( A_{n, 0}(t) \) and \( \tilde{A}_{n, 0}(t) \) should decrease with increasing \( n \), and we expect that this effect should be strong enough to be clearly visible in lattice calculations of these form factors.

Figures [15] and [16] show the generalized form factors \( A_{n, 0}(t) \) and \( \tilde{A}_{n, 0}(t) \) for the lowest three moments, \( n = 1, 2, \) and \( 3 \). The form factors have been normalized to unity at \( t = 0 \) to make the dependence of the shape on \( n \) more obvious. Note that \( A_{1,0}, \tilde{A}_{3,0}, \) and \( \tilde{A}_{2,0} \) depend on the difference between the quark and antiquark distributions whereas \( \tilde{A}_{1,0}, \tilde{A}_{3,0}, \) and \( A_{2,0} \) depend on the sum. Hence only comparisons between moments differing by \( n = 2 \) compare the same physical quantity with different weighting in \( x \). To facilitate determination of the slope of the form factors and to guide the eye, the data have been fit using a

\[ \chi_{2}^{2} \text{/ fm}^{2} > 0.2 \]

\[ 0 \leq x_{av} \leq 1, \]
dipole form factor

\[ A_{n,0}^{\text{dipole}} = \frac{a}{(1 - m^2_q)^2} \]  

(6)

The solid line denotes the least-squares fit and the shaded error band shows the error in the slope \(\Delta m_d\) given by the fit. Although the dipole fit is purely phenomenological, we note that it is statistically very well separated, and differ dramatically for the three moments. Indeed, as discussed quantitatively below, the slope at the origin decreases by more than a factor of 2 between \(n = 1\) and \(n = 3\), indicating that the transverse size decreases by more than a factor of 2. The top panel of Fig. 6 shows analogous results for lighter quarks, \(m_x = 0.74\) GeV, where we observe the same qualitative behavior but slightly weaker dependence on the moment. The second panel of Fig. 6 shows the flavor singlet combination \(A^u + A^d\), for which we have had to omit the disconnected diagram because of its significantly greater computational cost. Comparing this figure with the top panel calculated at the same quark mass, we note that while the connected contributions to \(A^u - A^d\) are qualitatively similar, there is significant quark flavor dependence that can be used to explore the nucleon wave function. The bottom panel of Fig. 6 shows the spin-dependent flavor non-singlet form factors \(A^u - A^d\) at the heaviest quark mass. Comparison with the top of Fig. 6 displays the difference between the spin averaged and spin dependent densities. We observe a striking difference, in that the change between \(n = 1\) and \(n = 3\) form factors for \(q(x, b_1)_1 - q(x, b_1)_i\) is roughly 6 times smaller than for \(\frac{1}{2}(q(x, b_1)_1 + q(x, b_1)_i)\).

Finally, it is useful to use the slope of the form factors at \(t = 0\) to determine the transverse rms radius,

\[ \langle r^2_{\perp} \rangle^{(n)} = \frac{\int d^2b_1 b_1^2 \int dx \, x^{n-1} q(x, b_1)}{\int d^2b_1 \int dx \, x^{n-1} q(x, b_1)} \]  

(7)

Transverse rms radii calculated in this way for the first three moments are plotted in Fig. 6 for \(m_x = 0.870\) GeV. To set the scale, the transverse charge radius at this mass is \(\langle r^2_{\perp} \rangle_{\text{charge}} = 0.48\) fm, which is two-thirds the experimental transverse size 0.72 fm, reflecting the effect of the absence of a significant pion cloud. The nonsinglet transverse size \(\langle r^2_{\perp} \rangle_{u-d} = 0.38\) fm is slightly smaller than the rms charge radius, but drops 62% to 0.14 fm for \(n=3\). The singlet size \(\langle r^2_{\perp} \rangle_{u+d}\) is 0.46 fm, and drops 43% to 0.27 for \(n=3\). This is a truly dramatic change in rms radius arising from changing the weighting by \(x^2\).

To display the change in transverse size with \(x\) in Fig. 6 the rms radii for each moment are plotted at the average value of \(x\) corresponding to that moment. If one neglects the fact that the antiquark distribution contributes to even and odd moments with opposite sign, the mean value of \(x\) in the distributions \(q(x, xq(x), and x^2q(x)\) are determined directly from the moments of structure functions measured on the lattice. Applying a small correction for the effect of the alternating antiquark contributions yields the mean values of \(x\) for each moment plotted in the Figure. The \(x\) dependence shown in this Figure is quite striking, with the nonsinglet transverse size dropping 62% as the mean value of \(x\) increases from 0.2 to 0.4, and going to zero when \(x\) reaches 1.0.

6. EXPLORATORY CALCULATION IN THE CHIRAL REGIME

Full QCD calculations with light quark masses are notoriously expensive, so significant compromises are required to begin to explore the chiral regime. Our initial exploration of this regime is a hybrid calculation using MILC configurations with staggered sea quarks and domain wall valence quarks. The MILC configurations on a \(20^3 \times 64\) lattice use strange quark masses \(am_s = 0.05\) and light quark masses \(am_{u+d} = 0.01\) and 0.05 with the Asqtad action corresponding to lattice spacing 0.13 fm. HYP-smearing was used to reduce the effect of dislocations. Chiral valence quarks were calculated using domain wall fermions with \(L_5 = 16\) and \(M = 1.7\). The lat-
tice size of 2.6 fm can sustain pions as light as 300 MeV, and our initial calculations were carried out for pion masses of approximately 343 and 635 MeV.

In the long term, our plan is to attain high statistics on several lattice volumes and two coupling constants for a range of quark masses. Hybrid partially quenched chiral perturbation theory will be used to correct for the inconsistency between chiral valence quarks and staggered sea quarks, and perturbative renormalization is expected to be adequate because of the improved convergence arising from HYP-smearing. However, the initial explorations shown in Figs. 8 and 9 are based on roughly 100 configurations, only have tree-level renormalization, and are not corrected using chiral perturbation theory. Figure 8 shows the well-known case of the momentum fraction, \( \langle x \rangle \), for which the chiral extrapolation formula [21] is roughly constant in the heavy pion regime of the three SESAM points, and then decreases sharply in the region of \( m^2_{\pi} \sim 0.2 \text{ GeV}^2 \) by 50% to agree with the experimental point. The two MILC points, which unlike the renormalized SESAM results must still be renormalized, do not yet give any indication of decreasing in the chiral regime, and it is an open question as to whether the lowest point is subject to large finite volume corrections. The axial charge shown in the upper panel of Fig. 9 is also presently unsatisfactory at the light quark point. In this case, it is well known that finite volume effects produce large discrepancies, so calculations on a larger lattice are clearly needed in this case. Only for the case of the first moment of the spin distribution, \( \langle x \rangle_{\Delta u-\Delta d} \), is the qualitative behavior roughly consistent with experiment.

Whereas these initial results are still far from being quantitatively controlled, they clearly demonstrate feasibility of hybrid calculations in the chiral regime. Improvement of statistics, careful study of finite volume effects, and calculation of hybrid chiral perturbation corrections and renormalization factors offer the potential for a first glimpse at hadron structure in the chiral regime.

Figure 8. Hybrid chiral calculation of \( \langle x \rangle_{u-d} \).

Figure 9. Hybrid chiral calculation of \( \langle 1 \rangle_{\Delta u-\Delta d} \) and \( \langle x \rangle_{\Delta u-\Delta d} \).
7. SUMMARY AND OUTLOOK

In the heavy pion world presently accessible to unquenched lattice QCD, we have calculated the lowest 3 generalized form factors $A_{n,0}$ and $\tilde{A}_{n,0}$ to $-t = 3$ GeV$^2$ and shown that they provide insight into several important aspects of hadron structure. We note that in cases that are comparable, our results are consistent with the calculations of the lowest two moments in Ref. [13]. The overdetermined method for measuring generalized form factors produces good statistics up to 3 GeV$^2$, and enables meaningful study of electromagnetic form factors, calculation of the origin of the nucleon spin, and study of the transverse structure of light cone wave functions. A particularly striking result is the dramatic 62% decrease in the transverse size $\langle r^2 \rangle_{u-d}$ between the first and third moment. We also observed clear dependence of the transverse distribution on flavor and spin and have shown that the commonly used factorization Ansatz $H(x,0,t) = Q(x)F(t)$ is fundamentally wrong.

The most immediate challenges are to extend these calculations to the chiral regime of realistic quark masses, and to extend techniques for evaluating disconnected diagrams [22] to these observables. When precise, controlled extrapolations to the physical pion mass are finally achieved, they will play a special role in our understanding of hadron structure.

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