Radiative Transfer in Clumpy and Fractal Media

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Abstract

A Monte Carlo model of radiative transfer in multi-phase dusty media is applied to the situation of stars and clumpy dust in a sphere or a disk. The distribution of escaping and absorbed photons are shown for various filling factors and densities. Analytical methods of approximating the escaping fraction of radiation, based on the Mega-Grains approach [4], are discussed. Comparison with the Monte Carlo results shows that the escape probability formulae provide a reasonable approximation of the escaping/absorbed fractions, for a wide range of parameters characterizing a clumpy dusty medium. A possibly more realistic model of the interstellar medium is one in which clouds have a self-similar hierarchical structure of denser and denser clumps within clumps [3], resulting in a fractal distribution of gas and dust. Monte Carlo simulations of radiative transfer in such multi-phase fractal media are compared with the two-phase clumpy case.

1 Introduction

Radiative transfer plays an important role in the evolution of the spectral appearance of galaxies, and at wavelengths longer than the Lyman limit, scattering by dust in the interstellar medium (ISM) is the major factor. The ISM is known to be composed of at least three phases: diffuse clouds, dense molecular clouds, and a low density inter-cloud medium (ICM). Most likely the ISM has a spectrum of densities and temperatures, as proposed by theoretical models [7] [2], with correlated multi-scale spatial structure, as evidenced by sky surveys such as IRAS and HI radio surveys. The transfer of radiation becomes complicated in such an inhomogeneous medium, however in most cases the effective optical depth is less than that of the homogeneous medium with equal mass of dust, allowing relatively more photons to escape.
The simplest model of an inhomogeneous medium is two phases (densities): dense clumps of dust in a less dense ICM. Radiative transfer through such a clumpy plane-parallel medium was investigated by Boissé [1], and then by Hobson & Scheuer [3] for two and three phase media. Their results for a three phase medium were found to be significantly different than the two phase case. Recently Witt & Gordon [10] performed Monte Carlo simulations of radiative transfer from a central source in a two phase clumpy medium within a sphere. All these investigations verify the expectation that the medium becomes more transparent as the degree of clumpiness is increased.

2 Monte Carlo Simulations

To further explore these effects we have developed a Monte Carlo code for simulating radiative transfer with multiple scattering in an inhomogeneous dusty medium [9] [6]. The geometry and density of the medium can be specified by a continuous functional $\rho(x, y, z)$, or on a three dimensional grid. For each wavelength, the number of photons absorbed by the dust in each element of the 3D grid is saved, allowing computation of the dust temperatures and resulting infrared emission spectrum. The grid resolution is limited only by the available computer memory: increasing the number of grid elements does not affect the computation time. This is achieved by employing the Monte Carlo method of imaginary/real scatterings and rejections [6] in selecting the random distances each photon travels between interactions, instead of numerical integration across volume elements of the grid. Our Monte Carlo simulations agree exactly with the radiative transfer results of Witt & Gordon for the situation they considered, that of cubic clumps on a body centered cubic percolation lattice. However, we find that randomly located spherical clumps create a more natural two phase medium, and the radiative transfer properties can then be approximated by the Mega-Grains approach of Hobson & Padman [4].

Since the ISM has a wide spectrum of densities controlled by both compressible turbulence and gravitational collapse [7] [2], the structure of gas and dust clouds is more likely a self-similar hierarchy of denser clumps within clumps. A fractal distribution of matter has exactly such properties. We construct fractal cloud models using a modification of the algorithm described by Elmegreen [3], then create density maps on a 3D grid, considering everything outside of the fractal cloud to be the low density ICM. The maximum density contrast is determined by the resolution of the density map grid. Any subset which is much larger than the resolution limit is also a fractal cloud, by the self-similar construction.

Figure 1 shows an array of images, each one being a map of the photons from a central source that are absorbed by dust in a 2D slice through the center of two types of inhomogeneous media. Blue coloring indicates the min-
Figure 1: Maps of the photons from a central source that are absorbed by dust in a 2D slice through the center of two types of inhomogeneous media, as simulated by Monte Carlo methods. For each type of media (each column), simulated maps are shown at two values of the equivalent homogeneous optical depth $\tau_{\text{hom}}$, which is the radial optical depth of absorption and scattering that would result if the dust was distributed uniformly in the sphere. The volume filling factor of the clumps and fractal cloud is $f_c = 0.1$. Blue coloring indicates the minimum absorption, red indicates more absorption, yellow is maximum, and the logarithmic scaling over five orders of magnitude is the same for each map. See text for more information.
imum absorption, red indicates more absorption, yellow is maximum, and the logarithmic scaling over five orders of magnitude is the same for each image. Simulations of the left column are in a two-phase clumpy medium, in which the clumps are spherical, 30 times denser than the ICM, and have a volume filling factor $f_c = 0.1$. Simulations of the right column are in a medium with a fractal cloud of dimension $D = 2.7$, filling factor $f_c = 0.1$, and densities having an exponential distribution (tending toward lognormal) with an average that is 30 times the ICM density, and maximum that is 260 times the ICM density. In all cases the dust is characterized by a scattering albedo $\omega = 0.6$ and an angular scattering phase function parameter $g = \langle \cos \theta \rangle = 0.6$, which are typical values for UV photons scattering off dust grains.

Each row of images is at the same homogeneous optical depth $\tau_{\text{hom}}$, which is the radial optical depth of absorption and scattering that would result if the dust was distributed uniformly in the sphere instead of in clumps. The upper row has $\tau_{\text{hom}} = 1$ and lower row is for $\tau_{\text{hom}} = 10$. Increasing $\tau_{\text{hom}}$ can be viewed as either increasing the dust abundance or decreasing the wavelength of the photons, resulting in more absorption. As $\tau_{\text{hom}}$ increases the clumps become opaque, creating the apparent shadows behind the clumps. However scattering by the dust causes photons to go behind the clumps and become absorbed, thus diminishing the effect of what would otherwise be completely dark shadows in the case of no scattering. As the clumps become opaque absorption occurs more at the clump surfaces.

By observing the fraction of photons that escape in the case of a central source, an effective optical depth is defined as $\tau_{\text{eff}} = -\ln(L_{\text{escape}}/L_{\text{emit}})$. Usually $\tau_{\text{eff}} < \tau_{\text{hom}}$ in a clumpy medium and the functional relationship is nonlinear. This can be seen in Fig. 2(a), were the effective optical depths of 20 randomly created fractal dust clouds (with $D = 2.5$ and $f_c = 0.065$) are plotted versus the equivalent homogeneous optical depth. The upper graph is for no scattering and the lower graph is with scattering. Also plotted for comparison are the homogeneous optical depth, and effective optical depth of related two-phase clumpy medium (solid curve) having the same $f_c$, and clump densities equal to the average density of fractal clouds. Most of the fractal clouds have $\tau_{\text{eff}}$ that varies as a function of $\tau_{\text{hom}}$ similar to the two-phase clumpy medium. However, many of the fractal clouds modeled have $\tau_{\text{eff}}$ that exceeds $\tau_{\text{hom}}$ for low values of $\tau_{\text{hom}}$. This is because the position of the source relative to the fractal cloud is more important than in the case of two-phase clumpy medium. The spherical clumps are small and their distribution is uniformly random so the relative position of the source is not so important, whereas the fractal cloud is a more connected set, and even though $f_c$ is small, if the source is near to any part of the cloud, it is near to a lot of the cloud and thus its emission is more likely to be absorbed or scattered.
Figure 2: (a) Monte Carlo computations of the effective optical depths of 20 random fractal dust clouds (diamonds), plotted versus and compared with equivalent homogeneous optical depth (dashed curve), and effective optical depth of related two-phase clumpy medium (solid curve). The upper graph is for no scattering and the lower graph is with scattering. (b) Comparison of Monte Carlo simulations of radiative transfer in two-phase clumpy media with Mega-Grains approximation. Horizontal axis is the volume filling factor of spherical clumps, upper graph is for central source and no scattering, lower graph is for uniformly distributed emitters with scattering. See text for further explanation.
3 Analytic Approximations

Since Monte Carlo simulations can require a large amount of computer time, it is useful to have analytical approximations for the basic results of radiative transfer: the fraction of photons escaping and the fraction of photons absorbed in each phase of the medium. In the case of a spherical homogeneous medium with uniformly distributed emitters, the escape probability formula of Osterbrock is an exact solution when there is absorption only.

\[
P_0(\tau) = \frac{3}{4\tau} \left[ 1 - \frac{1}{2\tau^2} + \left( \frac{1}{\tau} + \frac{1}{2\tau^2} \right) e^{-2\tau} \right]
\]

Lucy et al. suggested a formula that extends any absorption only escape probability to approximately include the effects of scattering,

\[
P(\tau, \omega) = P_0(\tau) \frac{1}{1 - \omega[1 - P_0(\tau)]}
\]

where \( \omega > 0 \) is the scattering albedo, the optical depth \( \tau \) includes both absorption and scattering, and \( P_0 \) is any escape probability for \( \omega = 0 \). Lucy’s formula is based on the assumption that the scattered photons mimic the photons emitted uniformly by the sources so that the \( \omega = 0 \) escape probability formula applies recursively.

The combination of Eqs. (1) and (2), which we call the Osterbrock-Lucy formulae, was tested extensively against Monte Carlo radiative transfer simulations and was found to be a reasonable approximation of the fraction of photons escaping from a homogeneous medium. However, since the angular distribution of the scattered photons is ignored in Lucy’s approximation, the formula is exact for only a single value of the angular scattering parameter \( g = \langle \cos \theta \rangle \), where \( \theta \) is the deflection angle, and this value also depends \( (\tau, \omega) \). Coincidentally, the \( g \) dependence of escape probability validity follows the scattering properties of silicate and graphite dust, that is, for low optical depths the escape probability agrees with the isotropic scattering \( (g = 0) \) case, and as optical depth increases the agreement shifts toward more forward scattering cases \( (g \rightarrow 1) \).

For the case of a two-phase clumpy medium, Hobson & Padman provide formulae approximating the effective radiative transfer properties by assuming spherical clumps and treating them as “Mega-Grains”. The upper graph in Fig. 2(b) compares the effective radial optical depth of a spherical clumpy medium obtained from the Mega-Grains approximation (dashed curve) to \( \tau_{eff} \) derived from Monte Carlo simulations (diamonds), over the full range of clump filling factor \( f_c \), in the case of \( \tau_{hom} = 5.0 \) and absorption only \( (\omega = 0) \). Each clump has a radius which is 5% of the radius of the spherical region, and density which is 100 times the ICM density. The graph shows that the Mega-Grains approximation is valid for \( f_c < 0.25 \), but overpredicts \( \tau_{eff} \) at larger
filling factors. By introducing another dependence on $f_c$, the Mega-Grains approximation can be extended to the full range of $0 \leq f_c \leq 1$ (solid curve), fitting the Monte Carlo results better and reproducing the correct asymptotic value of $\tau_{\text{eff}} = \tau_{\text{hom}}$ in the $f_c \to 1$ limit.

The Mega-Grains (MG) approximation gives the effective optical depth $\tau_{\text{eff}}$ and effective albedo $\omega_{\text{eff}}$ of the clumpy medium. Using these parameters, the escaping fraction of photons for the case of a uniform source can be computed by substituting $\tau_{\text{eff}}$ and $\omega_{\text{eff}}$ directly into the Osterbrock-Lucy escape probability formulae, Eqs.(1) and (2). The lower graph in Fig.2(b) compares this analytically computed escaping fraction to the Monte Carlo simulations of uniformly distributed emitters, including scattering. The standard MG approximation gives agreement with Monte Carlo only for $f_c < 0.15$. Introducing another dependence on $f_c$ in the MG formulae (scaling the clump radius by $1 - f_c$) and using the escape probability formulae to get the effective albedo of each clump, improves the agreement with Monte Carlo results for the full range of $f_c$. The escaping fractions determined by the combination of the extended MG and escape probability approximations are found to be in reasonable agreement with Monte Carlo results for $0 < \tau_{\text{hom}} \leq 40$ and $0 \leq f_c \leq 1$. These approximations can be applied to the case of a uniformly distributed source in a disk by using an effective radius: $R_{\text{eff}} = 3Rh/(R+2h)$, where $R$ is the actual radius and $h$ is the half-thickness of the disk.

We have also formulated equations for estimating what fraction of photons get absorbed in clumps and what fraction in the ICM, for the cases of central and uniform source, and find the equations to be reasonable approximations of the Monte Carlo results. These absorbed fractions are necessary for computing the dust temperatures and the resulting infrared emission. A test of the approximations that needs to be performed is to check how well the dust temperatures thus calculated match the distribution of dust temperatures from the corresponding Monte Carlo simulation.

4 Summary

The degree of clumpiness of a medium is as important as the total dust mass and scattering albedo for the transfer of radiation. For example, Fig.3 compares the effective optical depth (absorption and scattering) and effective albedo of spherical regions of dust with different degrees of clumpiness, computed using the MG approximation, as a function of photon wavelength. The dust is composed of equal amounts of graphite and silicates by mass, having the $a^{-3.5}$ grain size distribution for $0.001\mu m < a < 0.25\mu m$. The solid curves are the homogeneous case ($f_c = 0$) of no clumps, with dust mass density of $1.6 \times 10^{-23}gm/cm^3$. The dotted curves are for the case $f_c = 0.05$ with $\rho_c/\rho_{\text{icm}} = 100$, and the dashed curves are for the extreme case of $f_c = 0.01$. 

Figure 3: Radiative transfer properties of graphite and silicate dust distributed in a sphere three different ways. Solid curves are $\tau_{\text{hom}}$ and albedo $\omega$ from the center of sphere for the case of a homogeneous medium. Dotted curves are $\tau_{\text{eff}}$ and $\omega_{\text{eff}}$ for a two-phase clumpy medium with $f_c = 0.05$ and clump to ICM density ratio of $\rho_c/\rho_{\text{icm}} = 10^2$. Dashed curves are for a clumpy medium with $f_c = 0.01$ and $\rho_c/\rho_{\text{icm}} = 10^4$. 
with $\rho_c/\rho_{icm} = 10^4$. Each clump has a radius of 0.01 pc, and the radius of the spherical region is 0.6 pc. All cases have the same total mass of dust. From the graphs it is evident that the effective radiative transfer properties of the dusty medium can be radically affected by the degree of clumpiness.

Simulations of radiative transfer in a fractal distribution of dust indicate that the medium becomes more transparent as the fractal dimension decreases. Since the filling factor follows the fractal dimension, this behavior is similar to that of a two-phase clumpy medium, but there can be significant qualitative and quantitative differences, as seen in Figs.1 and 2(a). However, it may be possible to use the Mega-Grains approach to approximate the effective optical depth of a random fractal cloud when the sources are not correlated with the cloud, as shown by the solid curves in Fig.2(a). Those smooth solid curves are actually created by the MG approximation for the case of spherical clumps with radii $R_c = 0.05$ of the medium radius (MG agrees with Monte Carlo). The parameters $f_c = 0.065$ and $\rho_c/\rho_{icm} = 16.2$ used in the MG approximation match the filling factor and average density of the fractal cloud, but the radii of the clumps is a free parameter, and the value $R_c = 0.05$ happens work in this case. However, for the case shown in Fig.1, $R_c = 0.05$ does not work, as seen by the different values of $\tau_{eff}$ resulting when $\tau_{hom}$ increases. So either an effective $R_c$ needs to be determined as a function of cloud fractal dimension, or some other generalization of the MG approximation is needed.
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