Does a cloud of strings affect shear viscosity bound?

Mehdi Sadeghi¹,³ and Hadi Ranjbari²

¹ Department of Physics, School of Sciences, Ayatollah Boroujerdi University, Boroujerd, Iran
² Department of Physics, Payame Noor University (PNU), PO Box 19395-3697, Tehran, Iran
E-mail: mehdi.sadeghi@abru.ac.ir

Received 18 April 2019, revised 18 August 2019
Accepted for publication 11 September 2019
Published 27 September 2019

Abstract
The Einstein AdS black brane with a cloud of strings background in context of massive gravity is introduced. There is a momentum dissipation on the boundary because of graviton mass on the bulk. The ratio of shear viscosity to entropy density is calculated for this solution. This value violates the KSS bound if we apply the Dirichlet boundary and regularity on the horizon conditions. Our result shows that this value is independent of the cloud of strings.

Keywords: black brane, cloud of strings, massive gravity, fluid/gravity duality, shear viscosity

1. Introduction

Graviton is massless in general theory of relativity (GR) formulated by Einstein. However, hierarchy problem and the brane-world gravity solutions [1, 2] predict the existence of massive graviton. There are some problems such as the cosmological constant problem and the current acceleration that GR can not explain them. For these reasons GR must be modified. There are some modifications of GR theory such as massive gravity [3], bi-metric gravity [4], braneworld cosmology [5], scalar-tensor gravity [6], \( f(R) \) gravity [7] and Lovelock gravity [8].

Massive gravity formulated in flat spacetime by Pauli and Fierz [10] and it’s ghost-free and the generalization in the non-flat background was introduced by de Rham, Gabadadze and Tolley (dRGT) [3] in which the Boulware–Deser ghost [11] does not appear. The accelerated expansion of the universe without considering the dark energy could be explained by massive gravity. Massive gravity is also help us to understand the quantum gravity effects [9]. In this paper, our goal is further explore of a string cloud in the framework of massive theories of gravity.

Gauge-gravity duality relates two different theories and it is a useful tool for studying strongly coupled theories [12–15]. In this duality, gauge theory lives in the boundary of

³ Author to whom any correspondence should be addressed.
anti-de Sitter space-time and gravity is inhabited on the bulk. In long wavelength limit, gauge-gravity duality leads to fluid-gravity duality [16, 17]. It means fluid mechanics is an effective theory of field theory. The degree of freedoms of theory in boundary reduces to energy density $\epsilon$, thermodynamics pressure density $p$ and fluid velocity $u^\nu$. There is a constrained for fluid velocity as $u^2 = -1$. So the number of fluid variables is five and they can be found by equation of motion,

$$\nabla_\mu T^{\mu\nu} = 0,$$

$$T^{\mu\nu} = u^\mu u^\nu + p\eta^{\mu\nu},$$

$$\eta^{\mu\nu} = (-1, 1, 1, 1)$$

and equation of state $\epsilon = -p + Ts$ where $T$ is temperature and $s$ is entropy density. To illustrate the effects of dissipation we should add extra pieces to the $T^{\mu\nu}$,

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu} - \sigma^{\mu\nu},$$

$$\sigma^{\mu\nu} = \eta^{\mu\alpha}\eta^{\nu\beta}[\eta(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha) + (\zeta - \frac{2}{3}\eta)g_{\alpha\beta}\nabla \cdot u]$$

where $\eta$, $\zeta$, $\sigma^{\mu\nu}$ and $p^{\mu\nu}$ are shear viscosity, bulk viscosity, shear tensor and projection operator, respectively [18].

There are several ways to extract the transport coefficients: Kubo formula, pole method, membrane paradigm and the constitutive relation of energy-momentum stress tensor of the dual fluid and corresponding Navier–Stokes equations. Kubo formula and membrane paradigm are determined on horizon. In this work we are interested in calculating shear viscosity by Green–Kubo formula [19].

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int d\tau e^{i\omega\tau} \langle [T^x_y(x), T^x_y(0)] \rangle = - \lim_{\omega \to 0} \frac{1}{\omega} \sum G_{xy}^{\omega}(\omega, \vec{0})$$

where $T^x_y$ are spatial parts of energy-momentum tensor. The value of $\frac{2}{\pi}$ is one of the most interesting results from fluid-gravity duality which takes the value of $\frac{1}{4\pi}$ for all field theories which are dual to Einstein–Hilbert gravity, known as Kovtun, Son and Starinets (KSS) conjecture [19–22]. This lower bound is also satisfied for transverse shear viscosity per entropy density for anisotropic black brane [23] and five-dimensional $F(R)$-gravity considered as non-perturbative stringy effective action [24]. It is supported both by experimental data and theoretical analysis for quark-gluon-plasma. The ratio of shear viscosity to entropy density is proportional to the inverse squared of the coupling of quantum thermal gauge theory. It means the stronger the coupling, the weaker the shear viscosity per entropy density [25].

In this paper we consider massive gravity with a cloud of strings [26–36] and introduce the black brane solution. Finally, we study the effect of a cloud of strings on the value of $\frac{2}{\pi}$ and suggest some comments about the field dual to this gravity model.

2. The Einstein AdS black brane with a cloud of string background in context of massive gravity

The action is given by,

$$I = \frac{1}{2} \int d^4x \sqrt{-g} \left[ R - 2\Lambda + m^2 \sum_{i=1}^4 \epsilon_i \mathcal{L}_i(g, f) \right] + T_p \int_{\Sigma} \sqrt{-\gamma} d\lambda^0 d\lambda^1,$$

$$\text{(5)}$$
where $R$ is the scalar curvature, $\Lambda = \frac{-3}{l^2}$ is cosmological constant, $l$ is the radius of AdS spacetime, $f$ is a fixed rank-2 symmetric tensor known as reference metric and $m$ is the mass parameter. $c_i$’s are constants and $U_i$ are symmetric polynomials of the eigenvalues of the $4 \times 4$ matrix $K_{\mu \nu} = \sqrt{g} f_{\mu \alpha} f_{\nu \lambda} U_1 = [K], U_2 = [K]^2 - [K]^2, U_3 = [K]^3 - 3[K][K^2] + 2[K^3], U_4 = [K]^4 - 6[K^2][K]^2 + 8[K^3][K] + 3[K^2]^2 - 6[K^4].$ (6)

The square root in $K$ means $(\sqrt{A})_{\mu \nu} (\sqrt{A})^{\nu \lambda} = A_{\mu \lambda}$ and the rectangular brackets denote traces, where the last part is called the Nambu–Goto action of a string and $(\lambda^0, \lambda^1)$ is a parametrization of the worldsheet, $T_p$ is a positive quantity and is related to the tension of the string and $\gamma$ is the determinant of the induced metric $[33–36]$

$$\gamma_{ab} = 8_{\mu \nu} \frac{\partial x^\mu}{\partial \lambda^a} \frac{\partial x^\nu}{\partial \lambda^b}.$$ (7)

Nambu–Goto action can be written in terms of $\Sigma^{\mu \nu}$ as follows,

$$S_{NG} = T_p \int_{\Sigma} \sqrt{-\frac{1}{2} \Sigma^{\mu \nu} \Sigma^{\rho \sigma} d \Sigma^0 d \Sigma^1}.$$ (8)

where

$$\Sigma^{\mu \nu} = \epsilon^{ab} \frac{\partial x^\mu}{\partial \lambda^a} \frac{\partial x^\nu}{\partial \lambda^b}$$ (9)

is the space-time bi-vector. $\epsilon^{ab}$ is two-dimensional Levi-Civita tensor, $\epsilon^{01} = -\epsilon^{10} = 1$ and $\epsilon^{00} = \epsilon^{11} = 0$.

$\Sigma^{\mu \nu}$ must be satisfied in the following conditions to form a surface,

$$\Sigma^{\mu (\alpha} \Sigma^{\beta \gamma)} = 0$$ (10)

$$\nabla_\mu \Sigma^{\mu (\alpha} \Sigma^{\beta \gamma)} = 0.$$ (11)

Where the square brackets denote antisymmetrization in the enclosed indices. We get an useful identity by definition of $\gamma_{ab}$ equation (7) and $\Sigma^{\mu \nu}$ equation (9),

$$\Sigma^{\mu \sigma} \Sigma^{\sigma \tau} \Sigma^{\tau \nu} = \gamma \Sigma^{\mu \nu}.$$ (12)

The energy-momentum tensor for a cloud of strings is calculated by variation of the metric as follows,

$$T^{\mu \nu} = 2 \frac{\partial \mathcal{L}}{\partial g^{\mu \nu}} = \rho \Sigma^{\mu \sigma} \Sigma^{\nu \sigma} \sqrt{-\gamma}$$ (13)

where $\rho$ is the proper density of a string cloud and conservation of the energy-momentum tensor $\nabla_\nu T^{\mu \nu} = 0$ results in,

$$\nabla_\mu (\rho \Sigma^{\mu \sigma}) \Sigma^{\sigma \nu} \sqrt{-\gamma} + \rho \Sigma^{\mu \sigma} \nabla_\mu \left( \Sigma^{\sigma \nu} \sqrt{-\gamma} \right) = 0,$$ (14)
which multiplication of equation (14) by $\Sigma_{\nu\alpha}$ leads to $\nabla_\mu (\rho \Sigma^{\mu\sigma}) \Sigma_{\sigma\nu} / \gamma = 0$. Contracting the previous identity with $\Sigma_{\alpha\nu}$ and using equation (12), we obtain $\nabla_\mu (\rho \Sigma^{\mu\sigma}) \Sigma_{\sigma\nu} = 0$. Finally by using a system of coordinates adapted to the parametrization of the surface, we get,

$$\partial_\mu (\sqrt{-g} \rho \Sigma^{\mu\sigma}) = 0.$$  \hspace{1cm} (15)

By considering the following metric as an ansatz for a four-dimensional planar AdS black brane,

$$ds^2 = -f(r)dt^2 + dr^2 + h_{ij}d\xi^i d\xi^j.$$  \hspace{1cm} (16)

The equations of motion are given by,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} - m^2 \chi_{\mu\nu} = T^{S}_{\mu\nu}$$  \hspace{1cm} (17)

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ is the Einstein tensor, $T^{S}_{\mu\nu}$ is the energy-momentum tensor of matter that we consider as a cloud of string and $\chi_{\mu\nu}$ is massive term [37],

$$\chi_{\mu\nu} = \frac{c_1}{2} (4t_1 g_{\mu\nu} - K_{\mu\nu}) + \frac{c_2}{2} (4t_2 g_{\mu\nu} - 2t_1 K_{\mu\nu} + 2K^{2}_{\mu\nu}) + \frac{c_3}{2} (4t_3 g_{\mu\nu} - 3t_2 K_{\mu\nu}
+ 6t_1 K^{2}_{\mu\nu} - 9K^{3}_{\mu\nu}) + \frac{c_4}{2} (4t_4 g_{\mu\nu} - 4t_3 K_{\mu\nu} + 12t_2 K^{2}_{\mu\nu} - 24t_1 K^{3}_{\mu\nu} + 24K^{4}_{\mu\nu}).$$  \hspace{1cm} (18)

Using this metric ansatz, rr equation of motion equation (17) reduces to,

$$rf'(r) + f(r) - k + \Lambda r^2 - m^2 (c_0 c_1 r + c_0^2 c_2) = r^2 T^r_r.$$  \hspace{1cm} (19)

in which a prime denotes a differentiate with respect to the radial coordinate $r$. The density $\rho$ and the bivector $\Sigma$ are the function of $r$ for the spherically symmetric and static string cloud. The only non-vanishing component of the bivector $\Sigma$ is $\Sigma^{tr} = -\Sigma^{rt}$, since $\Sigma$ is a bivector. So the surviving components of bivector are given by [32],

$$\Sigma^{\sigma\mu} = A(r) \left( \delta^\sigma_0 \delta^\mu_1 - \delta^\sigma_1 \delta^\mu_0 \right).$$  \hspace{1cm} (20)

By plugging the non-zero components of $\Sigma^{\mu\nu}$ in equation (13) we have

$$T^{\sigma\mu} = -\rho \sqrt{-\gamma} \Sigma^{\nu\sigma} \Sigma^{r\nu}.$$  \hspace{1cm} (21)

By inserting the value of $\gamma = \Sigma^{\nu\sigma} \Sigma_{\nu\sigma} = -(\Sigma^r)^2$ in equation (21), we get

$$T^r_r = T^t_t = -\rho |A(r)|$$  \hspace{1cm} (22)

and from equation (15), we obtain $\partial_\mu (\sqrt{-g} T^\mu_r) = 0$ which implies,

$$T^\nu_r = -\frac{a}{r^2} \text{diag}[1, 1, 0, 0]$$  \hspace{1cm} (23)

where $a$ is a positive constant.

By substituting the value of $T^\nu_r$ in equation (19),

$$rf'(r) + f(r) - k + \Lambda r^2 - m^2 (c_0 c_1 r + c_0^2 c_2) = -a.$$  \hspace{1cm} (24)
\( f(r) \) is found as follows,
\[
f(r) = k - \frac{b}{r} - a - \frac{\Lambda}{3} r^2 + m^2 \left( \frac{c_0 c_1}{2} r + c_0^2 c_2 \right). \tag{25}
\]
Event horizon is at \( f(r_0) = 0 \) and we can find \( b \) by applying this condition,
\[
b = r_0 \left[ k - \frac{\Lambda}{3} r_0^2 - a + m^2 \left( \frac{c_0 c_1}{2} r_0 + c_0^2 c_2 \right) \right] \equiv r_0 \left( k - \frac{\Lambda}{3} r_0^2 - a + \Delta \right) \tag{26}
\]
where \( \Delta \) is,
\[
\Delta \equiv m^2 \left( \frac{c_0 c_1}{2} r_0 + c_0^2 c_2 \right). \tag{27}
\]
In our case \( k \) is zero. By substituting \( b \) in \( f(r) \) we have,
\[
f(r) = \frac{1}{r} \left[ -a(r - r_0) \right] + m^2 \left( \frac{c_0^2 c_2}{2} (r - r_0) + m^2 \frac{c_0 c_1}{2} (r_0 - r_0) \right]. \tag{28}
\]
The entropy density can be found by applying Hawking–Bekenstein formula,
\[
s = \frac{A}{4G V_2} = \frac{4\pi}{V_2} \int d^2 x \sqrt{-g} = \frac{4\pi}{V_2} \int d^2 x \sqrt{\chi} = 4\pi \sqrt{\chi(r_0)} = \frac{4\pi r_0^2}{l^2} \tag{29}
\]
where \( V_2 \) is the volume of the constant \( t \) and \( r \) hyper-surface with radius \( r_0 \), \( \chi(r_0) \) is the determinant of the spatial metric on the horizon and we used \( \frac{1}{\pi r_0^2} = 1 \) so \( \frac{1}{4\pi} = 4\pi \). The temperature is
\[
T = \frac{f'(r_0)}{4\pi} = \frac{1}{4\pi r_0} \left( \frac{3r_0^2}{l^2} - a + m^2 \left( r_0 c_0 c_1 + c_0^2 c_2 \right) \right). \tag{30}
\]

3. Holographic aspects of the solution

From the prescription of Green–Kubo formula (4) we can read the shear viscosity by 2-point function of energy-momentum tensor. According to AdS/CFT duality for calculating \( [T^x, y(x), T^x, y(0)] \) we should perturb the bulk metric by \( \delta g^{x y} \).

We consider the metric and energy-momentum tensor as the following,
\[
dx^2 = -g_{tt}(r) dt^2 + g_{rr}(r) dr^2 + g_{xx}(r) dx^i dx_i \tag{31}
\]
\[
T_{\mu \nu} = \text{diag} \left( T_t(t), T_r(r), T_x(r) \right) \tag{32}
\]
they are homogeneous and isotropic in the field theory directions. By applying the perturbation \( \delta g^{x y} = \phi(r) e^{-i \omega t} \) in the metric background (31) the decouple mode is,
\[
\frac{1}{\sqrt{-\delta g}} \partial_\mu \left( \sqrt{-g} g^{\mu \nu} \partial_\nu \phi \right) + [g^{xy} \omega^2 - m(r)^2] \phi = 0 \tag{33}
\]
Shear viscosity is calculated by equation (4),
\[
\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \Im \mathcal{G}_{\nu\nu}(\omega, k = 0) = \frac{\sqrt{\chi(r_0)}}{16\pi G_N} \phi_0(r_0)^2 = \frac{s}{4\pi} \phi_0(r_0)^2. \tag{34}
\]
Then, we will have,
\[
\eta = \frac{1}{4\pi} \frac{s}{4\pi} \phi_0(r_0)^2 \tag{35}
\]
where \(\phi_0\) is the solution of mode equation (33) at zero frequency \((\omega = 0)\).
\(\phi\) has these 2 conditions: (i) \(\phi\) is regular at horizon \(r = r_0\) and (ii) goes like to \(\phi = 1\) near the boundary as \(r \to \infty\).

For calculation of shear viscosity from Green–Kubo formula we should perturb the metric (16) according to above procedure,
\[
dx^2 = \frac{f_1(r)}{l^2} \frac{dr^2}{l^2} + \frac{\beta}{f_1(r)} dr^2 + \frac{r^2}{l^2} (dx^2 + dy^2 + 2\phi(r)dx dy) \tag{36}
\]
\[
f_1(r) = \frac{\beta}{r} \left[ -a(r - r_0) + \frac{\Lambda}{3} (r^3 - r_0^3) + m^2 c_0 c_2 (r - r_0)

+ m^2 c_1 c_1 \left( r^3 - r_0^3 \right) \right] = f^2(r). \tag{37}
\]
By choosing the metric perturbation as \(\delta g_{\nu\nu} = \frac{\beta}{l} \phi(r) e^{\pm \phi}\) and inserting it into the action equation (5) and keeping up to \(\phi^2\) [18–22, 25], we get:
\[
S_2 = -\frac{1}{2} \int d^4x \left( K_1 \phi'^2 - K_2 \phi^2 \right) \tag{38}
\]
we demand \(\omega = 0\), where
\[
K_1 = \frac{r^2 f_1(r)}{l^4} = \frac{r^2 f(r)}{l^4}

= \frac{r(r - r_0)}{2l^2} \left( -2a + 2c_0 c_2 l^2 m^2 + c_0 c_1 l^2 m^2 (r + r_0) \right) + \frac{r(r^3 - r_0^3)}{l^4}

K_2 = \frac{c_0 c_1 m^2 r}{2l^2} \tag{39}
\]
then the EoM is,
\[
(K_1 \phi')' + K_2 \phi = 0. \tag{40}
\]
We try to solve the mode equation (40) perturbatively in \(m^2\) and \(a\). So firstly consider \(m = a = 0\). Then the EoM will be given as follows,
\[
r(r^3 - r_0^3) \phi'' + (4r^3 - r_0^3) \phi' = 0. \tag{41}
\]
The solution is
\[
\phi(r) = C_2 + C_1 (\log r + \log(r - r_0)). \tag{42}
\]
Then applying the boundary conditions (regularity at horizon and $\phi = 1$ at the boundary) gives $C_2 = 0$ and $C_1 = 1$ which means that $\phi(r) = 1$ is a constant solution. In this case, according to equation (35),

$$\frac{\eta}{s} = \frac{1}{4\pi} \phi(r_0)^2 = \frac{1}{4\pi}.$$  \hspace{1cm} (43)

Now consider $m^2$ and $a$ to be a small parameter and try to solve equation (40). By Putting $\phi = \phi_0 + m^2\phi_1(r) + a\phi_2(r)$ where $\phi_0 = 1$ and expanding EoM in terms of powers of $m^2$ and $a$, we will find,

$$m^2 \left( c_{0c} c^2 r^2 + 2r(r^3 - r_0^3)\phi''_1(r) + 2(4r^3 - r_0^3)\phi'_1(r) \right) = 0$$  \hspace{1cm} (44)

$$a \left( r(r^3 - r_0^3)\phi''_2 + (4r^3 - r_0^3)\phi'_2 \right) = 0.$$  \hspace{1cm} (45)

Thus we find the solutions,

$$\phi_1(r) = C_2 - \frac{m^2}{24r_0^4} \left( 2\sqrt{3}c_{0c} c^2 r_0^2 \text{ArcTan} \left( \frac{2r + r_0}{\sqrt{3}r_0} \right) \right.$$

$$+ 24C_1 \log r + 2c_{0c} c^2 r_0 \log(r_0 - r) - c_{0c} c^2 r_0 \log(r^2 + rr_0 + r_0^2) - 8C_1 \log(r - r_0) - 8C_1 \log(r^2 + rr_0 + r_0^2) \left) \right)$$

$$\phi_2(r) = C_3 + \frac{C_4}{3r_0} \left( \log(r - r_0) + \log(r^2 + rr_0 + r_0^2) - 3 \log r \right).$$  \hspace{1cm} (46)

$\phi(r)$ is as follows,

$$\phi(r) = \phi_0 - \frac{m^2}{24r_0^4} \left( 2\sqrt{3}c_{0c} c^2 r_0^2 \text{ArcTan} \left( \frac{2r + r_0}{\sqrt{3}r_0} \right) \right.$$

$$+ 24C_1 \log r + 2c_{0c} c^2 r_0 \log(r_0 - r) - c_{0c} c^2 r_0 \log(r^2 + rr_0 + r_0^2) - 8C_1 \log(r - r_0) - 8C_1 \log(r^2 + rr_0 + r_0^2) \left) \right)$$

$$+ \frac{aC_4}{3r_0} \left( \log(r - r_0) + \log(r^2 + rr_0 + r_0^2) - 3 \log r \right)$$  \hspace{1cm} (47)

where $\Phi_0 = \phi_0 + aC_3 + m^2C_2.$

By applying regularity on horizon condition, we will get rid of $\log(r - r_0)$ if we choose $C_1 = \frac{1}{4}c_{0c} c^2 c_0$. The second boundary condition is at $\phi(r = \infty) = 1$, which gives,

$$\Phi_0 = 1 + \frac{m^2 \sqrt{3}c_{0c} c_0}{24r_0} (\sqrt{3} + 2i).$$  \hspace{1cm} (48)

Thus we have found $\phi(r)$ from horizon to boundary. The solution is as follows,

$$\phi(r) = 1 + \frac{m^2 \sqrt{3}c_{0c} c_0}{24r_0} \left( \sqrt{3} \pi - 2\sqrt{3} \text{ArcTan} \left( \frac{2r + r_0}{\sqrt{3}r_0} \right) - 6 \log r + 3 \log(r^2 + rr_0 + r_0^2) \right).$$  \hspace{1cm} (49)

So we have solved the mode equation (40) up to the first order in $a$ and $m^2$. In order to calculate $\Phi_0$ we should find the value of $\phi(r)$ at $r = r_0$.\[7]
\[
\frac{\eta}{s} = \frac{1}{4\pi} \phi(r_0)^2 = \frac{1}{4\pi} \left(1 + \frac{m_c^2 c^2 l^2}{36r_0} (\sqrt{3\pi} + 9 \log 3)\right).
\] (51)

KSS bound is violated by applying regularity of the solution of mode equation at horizon and \(\phi(r) = 1\) on the boundary of AdS.

The violation of KSS in higher derivative gravity is not arbitrary. Causality sets a different and lower bound but \(\frac{\eta}{s}\) cannot vanishes to 0 continuously. In massive gravity the case is different, at low temperature the \(\frac{\eta}{s}\) ratio goes to zero in a power law fashion [38]. From a technical point of view there is another important difference. Massive gravity theories violate the KSS bound due to the appearance of a mass term in the equation for the shear component of the graviton (33). This is not true in the higher derivative gravity where the reason of the violation is very different.

Momentum dissipation not always implies the violation of the KSS bound [39]. There are fluids theory, which are introduced in [40] and studied in detail in [41], which dissipate momentum (like your dRGT theory) but they do not violate the KSS bound. Similar results are also presented in [42] from both analytical and numerical aspects.

4. Conclusion

We study the aspects of field theory sector of massive gravity with a cloud of strings. \(\frac{\eta}{s}\) is calculated by applying the Dirichlet boundary and regularity on the horizon conditions [38]. This value is an important quantity in fluid-gravity duality and proportional to the inverse squared of the coupling of the field theory sector, so it means that the field theory side of massive gravity with a cloud of strings is the same as massive gravity theory. There is a conjecture that states \(\frac{\eta}{s} = \frac{1}{4\pi}\) for Einstein–Hilbert gravity, known as KSS bound [43] which it is violated for higher derivative gravity [43–48]. Our result shows massive gravity with a cloud of string behaves like pure AdS massive gravity [49]. Our outcome also shows that a cloud of strings acts like a charge comparison to [49, 50] results.

Acknowledgments

The authors would like to thank Shahrokh Parvizi and Matteo Baggioli for valuable suggestions and comments.

ORCID iDs

Mehdi Sadeghi ☞ https://orcid.org/0000-0001-9869-751X

References

[1] Dvali G, Gabadadze G and Porrati M 2000 Phys. Lett. B 484 112
[2] Dvali G, Gabadadze G and Porrati M 2000 Phys. Lett. B 485 208
[3] de Rham C, Gabadadze G and Tolley A J 2011 Phys. Rev. Lett. 106 231101
[4] Hassan S F and Rosen R A 2012 Confirmation of the secondary constraint, absence of Ghost in massive gravity and bimetric gravity J. High Energy Phys. JHEP04(2012)123
[5] Gergely L A 2006 Brane-world cosmology with black strings Phys. Rev. D 74 024002
[6] Brans C and Dicke R H 1961 Mach’s principle and a relativistic theory of gravitation Phys. Rev. 124 925
Akbar M and Cai R G 2007 Thermodynamic behavior of field equations for f(R) gravity Phys. Lett. B 648 243

Lovelock D 1971 The Einstein tensor and its generalizations J. Math. Phys. 12 498

Vasiliev M A 1996 Int. J. Mod. Phys. D 5 763

Fierz M and Pauli W 1939 On relativistic wave equations for particles of arbitrary spin in an electromagnetic field Proc. R. Soc. A 173 211

Boulware D G and Deser S 1972 Phys. Rev. D 6 3368

Maldacena J M 1999 The Large N limit of superconformal field theories and supergravity Int. J. Theor. Phys. 38 1113

Kovtun P, Son D T and Starinets A O 2005 Viscosity in strongly interacting quantum field theories Int. J. Phys. D 45 473601

Casalderrey-Solana J, Liu H, Mateos D, Rajagopal K and Wiedemann U A 2014 Gauge/String Duality, Hot QCD and Heavy Ion Collisions (Cambridge, UK: Cambridge University Press)

Mateos D 2007 String theory and quantum chromodynamics Class. Quantum Grav. 24 S713

Bhattacharyya S, Hubeny V E, Minwalla S and Rangamani M 2008 Nonlinear Fluid Dynamics from Gravity J. High Energy Phys. JHEP05(2014)147

Kovtun P 2012 Lectures on hydrodynamic fluctuations in relativistic theories J. Phys. A: Math. Theor. 45 473001

Son D T and Starinets A O 2007 Viscosity, black holes and quantum field theory Ann. Rev. Nucl. Part. Sci. 57 95

Policastro G, Son D T and Starinets A O 2001 The shear viscosity of strongly coupled $N = 4$ supersymmetric Yang–Mills plasma Phys. Rev. Lett. 87 081601

Kovtun P, Son D T and Starinets A O 2005 Viscosity in strongly interacting quantum field theories from black hole physics Phys. Rev. Lett. 94 111601

Policastro G, Son D T and Starinets A O 2002 From AdS/CFT correspondence to hydrodynamics J. High Energy Phys. JHEP09(2002)043

Sadeghi M 2019 Transverse shear viscosity to entropy density for the general anisotropic black brane in Horava–Lifshitz gravity Indian J. Phys. (https://doi.org/10.1007/s12648-019-01523-6)

Nojiri S and Odintsov S D 2011 Non-singular modified gravity unifying inflation with late-time acceleration and universality of viscous ratio bound in f(R) theory Prog. Theor. Phys. Suppl. 190 155

Kovtun P, Son D T and Starinets A O 2003 Holography and hydrodynamics: diffusion on stretched horizons J. High Energy Phys. JHEP10(2003)064

Letelier P S 1979 Clouds Of strings in general relativity Phys. Rev. D 20 1294

Richarte M G and Simeone C 2008 Traversable wormholes in a string cloud Class. Quantum Grav. 25 195006

Barbosa D and Bezerra V B 2016 On the rotating Letelier spacetime Gen. Rel. Grav. 48 149

Herscovici E and Richarte M G 2010 Black holes in Einstein–Gauss–Bonnet gravity with a string cloud background Phys. Lett. B 689 192

Ganguly A, Ghosh S G and Maharaj S D 2014 Accretion onto a black hole in a string cloud background Phys. Rev. D 90 064037

Bronsikov K A, Kim S W and Skvortsova M V 2016 The Birkhoff theorem and string clouds Class. Quantum Grav. 33 195006

Lee T H, Baboolal D and Ghosh S G 2015 Lovelock black holes in a string cloud background Eur. Phys. J. C 75 297

Mazharimosavati S H and Halilsoy M 2016 Cloud of strings as source in 2 + 1-dimensional $f (R) = R^n$ gravity Eur. Phys. J. C 76 95

Morais Graa J P, Salako G I and Bezerra V B 2017 Quasinormal modes of a black hole with a cloud of strings in Einstein–Gauss–Bonnet gravity Int. J. Mod. Phys. D 26 1750113

Cai R G, Hu Y P, Pan Q Y and Zhang Y L 2015 Thermodynamics of Black Holes in Massive Gravity Phys. Rev. D 91 024032
[38] Hartnoll S A, Ramirez D M and Santos J E 2016 Entropy production, viscosity bounds and bumpy black holes J. High Energy Phys. JHEP03(2016)170
[39] Alberte L, Baggioli M and Pujolas O 2016 Viscosity bound violation in holographic solids and the viscoelastic response J. High Energy Phys. JHEP07(2016)074
[40] Alberte L, Baggioli M, Khmelnitsky A and Pujolas O 2016 Solid Holography and Massive Gravity J. High Energy Phys. JHEP02(2016)114
[41] Baggioli M and Grieninger S 2019 Zoology of solid and fluid holography: goldstone modes and phase relaxation (arXiv:1905.09488 [hep-th])
[42] Baggioli M and Buchel A 2019 Holographic viscoelastic hydrodynamics J. High Energy Phys. JHEP03(2019)146
[43] Brigante M, Liu H, Myers R C, Shenker S and Yaida S 2008 Viscosity bound violation in higher derivative gravity Phys. Rev. D 77 126006
[44] Brigante M, Liu H, Myers R C, Shenker S and Yaida S 2008 The Viscosity bound and causality violation Phys. Rev. Lett. 100 191601
[45] Neupane I P and Dadhich N 2009 Entropy bound and causality violation in higher curvature gravity Class. Quantum Grav. 26 015013
[46] Sadeghi M and Parvizi S 2016 Hydrodynamics of a black brane in Gauss–Bonnet massive gravity Class. Quantum Grav. 33 035005
[47] Parvizi S and Sadeghi M 2019 Holographic aspects of a higher curvature massive gravity Eur. Phys. J. C 79 113
[48] Sadeghi M 2018 Black brane solution in rastall AdS massive gravity and viscosity bound Mod. Phys. Lett. A 33 1850220
[49] Sadeghi M 2018 Einstein–Yang–Mills AdS black brane solution in massive gravity and viscosity bound Eur. Phys. J. C 78 875
[50] Wu B and Zou D C 2018 Viscosity/entropy ratio in the context of dRGT massive gravity Europhys. Lett. 124 20002