Clustering-based extensions of the common age effect multi-population mortality model

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Over the past decades, life expectancy has risen...

Figure: Development of West German period life expectancy at birth from 1968 to 2017 (data source: Human Mortality Database (2019)).
... more than expected.

"[U]nderestimation has been widespread and persistent: 20-year forecasts of longevity made in recent decades in Australia, Canada, Japan, New Zealand, and the United States have been too low by an average of three years."

Michaelson and Mulholland (2014)

"Each additional year of unanticipated life expectancy [...] can increase pension liabilities by as much as 4% to 5%.

Michaelson and Mulholland (2014)"
Aims of this talk

Outline methods to cluster populations based on their mortality

Extend the CAE model using clustering methods

Evaluate the new model on real data
Data

We use the same data as Kleinow (2015), which we retrieve from the Human Mortality Database (2019). They cover
- the years $t$ from 1948 to 2007,
- the ages $x$ from 53 to 87,
- the male populations $i = 1, \ldots, P$ of $P = 10$ chosen countries.

We consider the number of deaths $D_{x,t}^i$, exposure $E_{x,t}^i$ and death rate

$$m_{x,t}^i := \frac{D_{x,t}^i}{E_{x,t}^i}.$$
Modeling one population

Lee-Carter (LC) model

Lee and Carter (1992) propose the following stochastic model:

\[ \log m_{x,t} = \alpha_x + \beta_x \kappa_t, \]

with

- basic age structure of mortality \( \alpha_x \),
- period effect \( \kappa_t \) (projected by a random walk with drift),
- age effect \( \beta_x \).

Assuming a Poisson distribution of \( D_{x,t} \), the model can be fit by maximum likelihood estimation (Brouhns et al. (2002)).
Modeling **multiple** populations

### Individual Lee-Carter (ILC) model
Ignore the multi-population structure and use one Lee-Carter model per population:

\[
\log m_{x,t}^i = \alpha_x^i + \beta_x^i \kappa_t^i.
\]

### Common age effect (CAE) model
Kleinow (2015) proposes

\[
\log m_{x,t}^i = \alpha_x^i + \beta_x^i \kappa_t^i.
\]
Further research could focus on developing techniques that can identify age effects which are only common to some countries but not others.

Kleinow (2015)

CAE\((k, C)\) model

Perform a clustering of the populations, i.e., determine \(k \in \{1, \ldots, P\}\) and a surjective function \(C : \{1, \ldots, P\} \rightarrow \{1, \ldots, k\}\) such that \(C(i) = C(j)\) if and only if populations \(i\) and \(j\) exhibit similar age effects. Fit a CAE model on each cluster, i.e.,

\[
\log m_{x,t}^i = \alpha_x^i + \beta_x^{C(i)}\kappa_t^i.
\]
Method 1: \textit{k}-Means clustering

Fit ILC model and cluster the individual age effect vectors \((\beta^1_x)_x, \ldots, (\beta^p_x)_x\) with the \textit{k}-Means algorithm. Choose \textit{k} by minimizing the Bayesian Information Criterion

\[
\text{BIC} := -2L_{\text{max}} + \log(n_{\text{obs}}) \cdot n_{\text{par}}
\]

of the resulting \text{CAE} (\textit{k}, \text{C}) model.

Figure: Age effects by \textit{k}-Means cluster for males aged 53 to 87 in ten countries between 1948 and 1987.

(1) AUT, (2) FRA, SWE, CH, (3) AUS, CAN, NZL, UK, USA, (4) DNK
Methods 2 and 3: ACF clustering and likelihood-ratio-based clustering

- Li and Lee (2005) propose the augmented common factor (ACF) model
  \[ \log m_{x,t}^i = \alpha_x^i + \beta_x \kappa_t + \beta_x \kappa_l^i. \]
  → Perform a clustering based on how well populations can be described by the common factor.

- Using the likelihood ratio test, assess the hypothesis that two populations have significantly different age effects.
  → Use hierarchical clustering to partition the set of populations.

Figure: Dendrogram for between-cluster average linkage likelihood-ratio distances based on mortality data for males aged 53 to 87 in ten countries between 1948 and 1987.
Method 4: Fuzzy maximum likelihood clustering

We focus on a simple, yet important special case:

CAE Fuzzy model, $k = 2$

\[
\log m_{x,t}^i = \alpha_x^i + (\omega^i \beta_x^1 + (1 - \omega^i) \beta_x^2) \kappa_t^i
\]

with $0 \leq \omega^i \leq 1$. 
Fuzzy clustering weights

Figure: Weights $\omega^j$ (or $1 - \omega^j$, if $\omega^j < 0.5$) of the fuzzy maximum likelihood clustering algorithm for males aged 53 to 87 in ten countries between 1948 and 1987.

Figure: Age effects by cluster for males aged 53 to 87 in ten countries between 1948 and 1987 obtained by the CAE Fuzzy model ($k = 2$).
Empirical model comparison

- quantity of interest: death rates
- split data into training set (1948 to 1987) and test set (1988 to 2007)
- measure goodness of fit and forecasting performance by
  - bias := \( \frac{1}{n} \sum_{j=1}^{n} (\hat{y}_j - y_j) \),
  - mean absolute error, MAE := \( \frac{1}{n} \sum_{j=1}^{n} |\hat{y}_j - y_j| \),
  - mean absolute percentage error, MAPE := \( \frac{1}{n} \sum_{j=1}^{n} \frac{|\hat{y}_j - y_j|}{y_j} \),
  - root-mean-square error, RMSE := \( \sqrt{\frac{1}{n} \sum_{j=1}^{n} (\hat{y}_j - y_j)^2} \),
  - BIC (only in-sample).
### Goodness of fit

**Table:** In-sample error measures for males aged 53 to 87 in ten countries between 1948 and 1987.

| Model                                      | Bias   | MAE    | MAPE  | RMSE  | BIC   |
|--------------------------------------------|--------|--------|-------|-------|-------|
| ILC                                        | $-2.67 \cdot 10^{-5}$ | 2.20%  | 3.25% | 4.06% | 180555|
| CAE                                        | $-11.50 \cdot 10^{-5}$ | 2.42%  | 3.51% | 4.56% | 186649|
| CAE($k$, C), k-Means clustering             | $-3.52 \cdot 10^{-5}$ | 2.25%  | 3.33% | 4.17% | 180191|
| CAE($k$, C), ACF clustering                 | $-4.77 \cdot 10^{-5}$ | 2.33%  | 3.44% | 4.26% | 186669|
| CAE($k$, C), LR clustering                  | $-11.84 \cdot 10^{-5}$ | 2.37%  | 3.42% | 4.48% | 182468|
| CAE Fuzzy, $k = 2$                         | $-8.56 \cdot 10^{-5}$ | 2.37%  | 3.41% | 4.49% | 180719|
| CAE Fuzzy, $k = 3$ (chosen via BIC)        | $-3.48 \cdot 10^{-5}$ | 2.24%  | 3.31% | 4.15% | 180265|
Table: Out-of-sample error measures for males aged 53 to 87 in ten countries between 1988 and 2007 (models fit on data from 1948 to 1987).

| Model                                    | Bias  | MAE   | MAPE  | RMSE  |
|------------------------------------------|-------|-------|-------|-------|
| ILC                                      | 6.01% | 6.75% | 20.38%| 10.05%|
| CAE                                      | 6.15% | 6.75% | 19.64%| 9.82% |
| CAE($k$, $C$), k-Means clustering         | 6.04% | 6.68% | 20.31%| 9.94% |
| CAE($k$, $C$), ACF clustering             | 6.05% | 6.68% | 19.97%| 9.65% |
| CAE($k$, $C$), LR clustering              | 6.06% | 6.64% | 19.51%| 9.68% |
| CAE Fuzzy, $k = 2$                       | 5.86% | 6.43% | 19.57%| 9.37% |
| CAE Fuzzy, $k = 3$ (chosen via BIC)      | 6.13% | 6.77% | 20.22%| 10.23%|
Conclusion

- We have proposed four methods to group populations based on their mortality/age effects.
- We have seen that the resulting clustering-based variants of the CAE model
  - let us gain some insights into historical mortality data,
  - potentially allow more accurate mortality projections.
- Possible extensions and improvement ideas for our model include
  - employing different time series projection methods (e.g., accounting for correlation),
  - using common additive age effects $\alpha_i^j$ as well,
  - evaluating other selection criteria than BIC for the number of clusters $k$,
  - evaluating other data sets (more/different populations).
References

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