Properties of tunnel Josephson junctions with a ferromagnetic interlayer

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We investigate superconductor/insulator/ferromagnet/superconductor (SIFS) tunnel Josephson junctions in the dirty limit, using the quasiclassical theory. We formulate a quantitative model describing the oscillations of critical current as a function of thickness of the ferromagnetic layer and use this model to fit recent experimental data. We also calculate quantitatively the density of states (DOS) in this type of junctions and compare DOS oscillations with those of the critical current.

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I. INTRODUCTION

It is well known that superconductivity and ferromagnetism are two competing orders, however their interplay can be realized when the two interactions are spatially separated. In this case the coexistence of the two orderings is due to the proximity effect.1,2,3 Experimentally this situation can be realized in superconductor/ferromagnet (S/F) hybrid structures. The main manifestation of the proximity effect in S/F structures is quasiclassical equations can be used for analysis.4,5,6,7,8,9,10,11,12,13,14,15

SIFS junctions, i.e. S/F/S trilayers with one transparent interface and one tunnel barrier between S and F layers, represent practically interesting case of π junctions. SIFS structure offers the freedom to tune the critical current density over a wide range and at the same time to realize high values of a product of the junction critical current $I_c$ and its normal state resistance $R_N$.2,3 In addition, Nb based tunnel junctions are usually underdamped, which is desired for many applications. SIFS π junctions have been proposed as potential logic elements in superconducting logic circuits.4,5 SIFS junctions are also interesting from the fundamental point of view since they provide a convenient model system for a comparative study between 0-π transitions observed from the critical current and from the density of states (DOS). At the same time, despite such an interest, there is no complete theory yet of SIFS junctions which could provide quantitative predictions for critical current and DOS in such structures. All existing theories dealt only with a number of limiting cases, when either linearized quasiclassical equations can be used for analysis.4,5,6,7,8,9,10,11,12,13,14,15

SIFS junctions are characterized by the dimensionless parameters $\gamma_{B1}$ and $\gamma_{B2}$, respectively, where $\gamma_{B1,B2} = R_{B1,B2}n_0/\xi_f$, $R_{B1,B2}$ are the resistances of left and right S/F interfaces, respectively, $n_0$ is the conductivity of the F layer, $\xi_f$, $D_f$ is the diffusion coefficient in the ferromagnetic metal and $T_c$ is the critical temperature of the superconductor (we assume $h = k_B = 1$). We also assume that the S/F interfaces are not magnetically active. We will consider diffusive limit, in which the elastic scattering length $l$ is much smaller than the decay characteristic length $\xi_f$. In this paper we concentrate on the case of a SIFS tunnel Josephson junction, when $\gamma_{B1} \gg 1$ (tunnel barrier) and $\gamma_{B2} = 0$ (fully transparent interface). For comparison, we also consider two other limiting cases: an SFS...
FIG. 1: Geometry of the considered system. The thickness of the ferromagnetic interlayer is \(d_f\). The transparency of the left S/F interface is characterized by the \(\gamma_{B1}\) coefficient, and the transparency of the right F/S interface is characterized by \(\gamma_{B2}\).

Under conditions described above, the calculation of the Josephson current requires solution of the one-dimensional Usadel equations. In the F layer the equations have the form:

\[
\begin{align*}
D_s \frac{\partial}{\partial x} \left[ (\omega \pm i h) \sigma_z + \frac{1}{2\tau_m} \sigma_z \hat{G}_{f(1)} \sigma_z \right] &= \left[ \begin{array}{c} \partial_x \hat{G}_{f(1)} \end{array} \right],
\end{align*}
\]

where \(D_s\) is the diffusion coefficient in the superconductor. In Eq. (2) \(\hat{G}_s \equiv \hat{G}_{s(1)}\) and we omit subscripts \('\uparrow') because equations in superconductor look identical for spin up and spin down electron states.

In Eqs. (1)-(2) we use following matrix notations (we omit \(\gamma\)):

\[
\hat{G}(x, \omega) = \begin{pmatrix} G & F \\ F^* & -G \end{pmatrix}, \quad \hat{\Delta}(x) = \begin{pmatrix} 0 & \Delta(x) \\ \Delta(x)^\ast & 0 \end{pmatrix},
\]

where \(G\) and \(F\) are normal and anomalous Green’s functions, respectively, and \(\Delta(x)\) is the superconducting pair potential. The matrix Green’s function \(\hat{G}\) satisfies the normalization condition,

\[
\hat{G}^2 = 1, \quad \hat{F}^2 + FF^* = 1,
\]

and the pair potential \(\Delta(x)\) is determined by the self-consistency equation

\[
\Delta(x) \frac{T_c}{T} = \frac{\pi T_c}{\omega} \sum_{\omega > 0} \left( \frac{2\Delta(x)}{\omega} - F_{f\uparrow} - F_{f\downarrow} \right). \tag{5}
\]

The boundary conditions for the Usadel equations at the left and right sides of each S/F interface are given by relations:

\[
\begin{align*}
\xi_n \gamma f \left( \hat{G}_f \hat{D}_f \hat{G}_f \right)_{\pm d_f/2} &= \xi_s \left( \hat{G}_s \hat{D}_s \hat{G}_s \right)_{\pm d_f/2}, \tag{6a} \\
2\xi_n \gamma_{B1} \left( \hat{G}_f \hat{D}_f \hat{G}_f \right)_{-d_f} &= \left[ \hat{G}_s, \hat{G}_s \right]_{d_f/2}, \tag{6b} \\
2\xi_n \gamma_{B2} \left( \hat{G}_f \hat{D}_f \hat{G}_f \right)_{d_f/2} &= \left[ \hat{G}_f, \hat{G}_s \right]_{d_f/2}, \tag{6c}
\end{align*}
\]

where \(\gamma = \xi_n \sigma_n / \xi_s \sigma_s\), \(\sigma_s\) is the conductivity of the S layer and \(\xi_s = \sqrt{D_s / 2\pi T_c}\).

To complete the boundary problem we also set boundary conditions at \(x = \pm \infty\):

\[
\hat{G}_s(\pm \infty) = \frac{\omega}{\sqrt{|\Delta|^2 + \omega^2}}, \tag{7a}
\]

\[
F_{f\uparrow}\left(-\infty\right) = \frac{|\Delta|e^{-\phi/2}}{\sqrt{|\Delta|^2 + \omega^2}}, \quad F_{f\downarrow}\left(+\infty\right) = \frac{|\Delta|e^{\phi/2}}{\sqrt{|\Delta|^2 + \omega^2}}, \tag{7b}
\]

where \(\phi\) is the superconducting phase difference between S electrodes.

In Matsubara technique it is convenient to parameterize the Green’s function in the following way, making use of the normalization condition, Eq. (4):

\[
\hat{G} = \begin{pmatrix} \cos \theta & \sin \theta e^{i\chi} \\ \sin \theta e^{-i\chi} & -\cos \theta \end{pmatrix}.
\]

Solving a system of nonlinear differential equations, Eqs. (1)-(7), generally can be fulfilled only numerically. We present full numerical calculation in Sec. IV. The analytical solution can be constructed in case of one S/F bilayer, when \(d_f > \xi_{f1}\). In that case, the decay of the Cooper pair wave function in first approximation occurs independently near each interface. Therefore we can consider the behavior of the anomalous Green’s function near each S/F interface, assuming that the ferromagnetic interlayer is infinite. This analytical calculation for an S/F/S trilayer with long ferromagnetic interlayer is performed in the next section.

The general expression for the supercurrent is given by

\[
J_s = \frac{i\pi T_c \sigma_n}{2e} \sum_{n=-\infty}^{\infty} \sum_{s=\uparrow,\downarrow} \left( \bar{F}_{f\sigma} \frac{\partial}{\partial x} F_{f\sigma} - F_{f\sigma} \frac{\partial}{\partial x} \bar{F}_{f\sigma} \right), \tag{9}
\]

where \(\bar{F}_{f(1)}(x, \omega) = F_{f(1)}(x, -\omega)\) are the anomalous Green’s functions in the ferromagnet.
III. CRITICAL CURRENT OF JUNCTIONS WITH LONG FERROMAGNETIC INTERLAYER

We need to solve the complete nonlinear Usadel equations in the ferromagnet, Eqs. (1). For SIF junctions, an analytical solution may be found if $d_f \gg \xi_{f1}$ and we can set the phase of the anomalous Green’s function to zero (see discussion in Sec. [1].

Setting $\chi_s = \chi_f = 0$ we have the following $\theta$-parameterizations of the normal and anomalous Green’s functions, Eq. (8), $G = \cos \theta$ and $F = \sin \theta$. In this case we can write Eqs. (1) in the F layer as

$$\frac{D_f}{2} \frac{\partial^2 \theta_{f1(\xi)}}{\partial x^2} = \left( \omega \pm i h + \frac{\cos \theta_{f1(\xi)}}{\tau_m} \right) \sin \theta_{f1(\xi)}. \quad (10)$$

In the S layer the Usadel equation, Eq. (2), may be written as

$$\frac{D_s}{2} \frac{\partial^2 \theta_x}{\partial x^2} = \omega \sin \theta_x - \Delta(x) \cos \theta_x. \quad (11)$$

The self-consistency equation in the S layer acquires the form

$$\Delta(x) \ln \frac{T_c}{T} = \pi T \sum_{\omega > 0} \left( \frac{2 \Delta(x)}{\omega} - \sin \theta_{s1} - \sin \theta_{s1} \right). \quad (12)$$

In the case of $\chi_s = \chi_f = 0$, the boundary conditions, Eqs. (3), for the functions $\theta_{f,s}$ at each S/F interface can be written as

$$\xi_n \gamma \frac{\partial \theta_x}{\partial x} \pm d_f/2 = \xi_s \frac{\partial \theta_x}{\partial x} \pm d_f/2, \quad (13a)$$

$$\xi_n \gamma_1 \frac{\partial \theta_x}{\partial x} \pm d_f/2 = \sin (\theta_x - \theta_s) \pm d_f/2, \quad (13b)$$

$$\xi_n \gamma_2 \frac{\partial \theta_x}{\partial x} \pm d_f/2 = \sin (\theta_x - \theta_f) \pm d_f/2. \quad (13c)$$

The boundary conditions at $x = \pm \infty$ are

$$\theta_x(\pm \infty) = \arctan \frac{\Delta}{\omega}. \quad (14)$$

In the equation for the supercurrent, Eq. (9), the summation goes over all Matsubara frequencies. It is possible to rewrite the sum only over positive Matsubara frequencies due to the symmetry relation,

$$\theta_{f(s)}(\omega) = \theta_{f(s)}(-\omega). \quad (15)$$

In what follows, we will use only $\omega > 0$ in equations containing $\omega$.

For the left interface (tunnel barrier at $x = -d_f/2$), a first integral of Eq. (10) leads to

$$\frac{\xi_f}{2} \frac{\partial \theta_x}{\partial x} = -q \sin \frac{\theta_x}{2} \sqrt{1 - \varepsilon^2 \sin^2 \frac{\theta_x}{2}}. \quad (16)$$

where $\xi_f = \sqrt{D_f/h}$ and the boundary condition $\theta_f(x \to \infty) = 0$ has been used. In Eq. (10) we use the following notations

$$q = \sqrt{2/\hbar} \sqrt{\omega \pm i h + 1/\tau_m}, \quad (17a)$$

$$\varepsilon^2 = \left( 1/\tau_m \right) (\omega \pm i h + 1/\tau_m)^{-1}. \quad (17b)$$

Here we again adopt convention that positive sign ahead of $\hbar$ corresponds to the spin up state ($\uparrow$) and negative sign to the spin down state ($\downarrow$). Here and below we did not write spin labels ‘$\uparrow$ ($\downarrow$)’ explicitly but imply them everywhere they needed.

For the right interface ($x = d_f/2$), a first integral of Eq. (10) leads to a similar equation,

$$\frac{\xi_f}{2} \frac{\partial \theta_f}{\partial x} = q \sin \frac{\theta_f}{2} \sqrt{1 - \varepsilon^2 \sin^2 \frac{\theta_f}{2}}. \quad (18)$$

Following Faure et al.\(^{23}\) we integrate Eq. (16), which gives

$$\sqrt{1 - \varepsilon^2 \sin^2 \frac{\theta_f}{2}} - \cos \frac{\theta_f}{2} = g_1 \exp \left( -2q \frac{d_f/2 + x}{\xi_f} \right). \quad (19)$$

The integration constant $g_1$ in Eq. (19) should be determined from the boundary condition at the left S/F interface, Eq. (13b). Since we consider the tunnel limit ($\gamma_{21} \gg 1$), we can neglect small $\theta_f$ in the right hand side of Eq. (13b) and also assume, neglecting the inverse proximity effect,

$$\theta_f(-d_f/2) = \arctan \frac{\Delta}{\omega}. \quad (20)$$

Then Eq. (13b) becomes

$$\xi_n \gamma_1 \frac{\partial \theta_x}{\partial x} \pm d_f/2 = -G(n), \quad G(n) = \frac{\Delta}{\sqrt{\omega^2 + \Delta^2}}. \quad (21)$$

From Eqs. (15) and (21) we obtain the boundary value of $\theta_f$ at $x = -d_f/2$ and substituting it into Eq. (19) we finally get

$$g_1 = \frac{G^2(n)}{16 \gamma_{21}} \frac{1 - \varepsilon^2}{q^2} \left( \frac{\xi_f}{\xi_n} \right)^2. \quad (22)$$

Linearizing Eq. (19), we can now obtain the anomalous Green’s function in the ferromagnetic layer of the SIF tunnel junction with infinite F layer thickness. Similar formula for the FS bilayer with a transparent interface ($\gamma_{21} = 0$) was developed by Faure et al.\(^{23}\) [to obtain it one should integrate Eq. (19) and then linearize the resulting equation]. The anomalous Green’s function at the center of the F layer in a SIFS junction may be taken as the superposition of the two decaying functions, taking into account the phase difference in each superconducting electrode,

$$\theta_f = \frac{4}{\sqrt{1 - \varepsilon^2}} \left[ \sqrt{g_1} \exp \left( -q \frac{d_f/2 + x}{\xi_f} - i \frac{\theta_f}{2} \right) \right] + \sqrt{g_2} \exp \left( q \frac{x - d_f/2}{\xi_f} + i \frac{\theta_f}{2} \right). \quad (23)$$
The expression for $g_2$ was obtained in Ref. [23] for the rigid boundary conditions at the transparent FS interface, $\theta_f(d_f/2) = \arctan(|\Delta|/\omega)$ and reads

$$g_2 = \frac{(1 - \epsilon^2)F^2(n)}{\sqrt{(1 - \epsilon^2)F^2(n) + 1 + 1}}.$$

The expression for $g_2$ obtained in Ref. [23] for the rigid boundary conditions at the transparent FS interface, $\theta_f(d_f/2) = \arctan(|\Delta|/\omega)$ and reads

$$F(n) = \frac{|\Delta|}{\omega + \sqrt{\omega^2 + |\Delta|^2}}.$$

Using the above solutions and Eqs. (9), (15) we arrive at sinusoidal current-phase relation in a SIFS tunnel Josephson junction with the critical current

$$I_cR_N = \frac{16\pi T}{e}|\text{Re} \sum_{n=0}^{\infty} G(n)F(n) \exp(-qd_f/\xi_f)|.$$

Here and below we fix positive sign in the definition of $q$, $\epsilon^2$ in Eqs. (17): $q = \sqrt{2/\hbar}\sqrt{|\omega + i\hbar + 1/\tau_m|}$. $\epsilon^2 = (1/\tau_m)(\omega + i\hbar + 1/\tau_m)^{-1}$. It is possible since we already performed summation over spin states and have to define now spin-independent values. In Eq. (25) and below $R_N$ is a full resistance of an S/F/S trilayer, which include both interface resistances of left and right interfaces and the resistance of the ferromagnetic interlayer. In case of SIFS and SIFIS junctions the F layer resistance can be neglected compared to large resistance of the tunnel barrier.

At this point we define the characteristic lengths of the decay and oscillations $\xi_{f1,2}$ as,

$$\frac{1}{\xi_{f1,2}} = \frac{1}{\xi_f} \sqrt{1 + \left(\frac{\omega}{\hbar} + \frac{1}{\hbar \tau_m}\right)^2 \pm \left(\frac{\omega}{\hbar} + \frac{1}{\hbar \tau_m}\right)}.$$

The critical current in Eq. (25) is proportional to the small exponent $\exp(-d_f/\xi_f)$. The terms neglected in our approach are of the order of $\exp(-2d_f/\xi_f)$ and they give a tiny second-harmonic term in the current-phase relation.

The critical current equation (25) can be simplified in the limit of vanishing magnetic scattering, $\tau_m^{-1} \ll \pi T_c$,

$$I_cR_N = \frac{16\pi T}{e} \sum_{n=0}^{\infty} G(n)F(n) \exp(-\frac{qd_f}{\xi_f}) \cos\left(\frac{qd_f}{\xi_f}\right).$$

The damped oscillatory behavior of the critical current can be clearly seen from this equation. With increasing $d_f$ the junction undergoes the sequence of $0-\pi$ transitions when positive values of the $I_cR_N$ product correspond to a zero state and negative values correspond to a $\pi$ state.

Eq. (25) in the absence of spin-flip scattering coincides with the corresponding equation, Eq. (37), from the Ref. [17] taken in the limit of long $d_f \gg \xi_f$ in case of $\gamma_{f1} \gg 1$, $\gamma_{f2} = 0$.

Using the same approach we can obtain the equation for the critical current in a SIFS structure with two strong tunnel barriers between the ferromagnet and both superconducting layers ($\gamma_{f1,2} \gg 1$),

$$I_cR_N = \frac{4\pi T \xi_f}{e\xi_f} \frac{\gamma_{f1} + \gamma_{f2}}{\gamma_{f1}\gamma_{f2}} \text{Re} \left[ \sum_{n=0}^{\infty} G(n) \exp\left(-\frac{qd_f}{\xi_f}\right) q \right].$$

This formula coincides with corresponding expression Eq. (39) for the critical current in a SIFIS structure in Ref. [23] for $\gamma_{f1,2} = \gamma_f \gg 1$ and $d_f \gg \xi_f$. Eq. (29) near $T_c$ may be
written as (for \( T_c \ll h \))

\[
I_c R_N = \frac{\pi |\Delta|^2 \xi_{f_2}}{2 e T_c \xi_n} \frac{\gamma_0 + \gamma_2}{\gamma_0 \gamma_2} \times \cos(\Psi) \exp \left( -\frac{d_f}{\xi_{f_1}} \right) \sin \left( \Psi - \frac{d_f}{\xi_{f_2}} \right),
\]

where \( \Psi \) is defined by \( \tan(\Psi) = \frac{\xi_{f_2}}{\xi_{f_1}} \). Eq. (30) in the absence of spin-flip scattering coincides with the corresponding equation, Eq. (35), from the Ref. 17, taken in the limit of long \( d_f \gg \xi_{f_1} \).

We also provide here equation for the critical current in an SFS junction [see Ref. [23, Eq. (74)]], written in our notations,

\[
I_c R_N = \frac{64 \pi T_c d_f}{e \xi_f} \text{Re} \sum_{n=0}^{\infty} \sqrt{\left( 1 - e^2 \right) F^2(n) + 1} \left[ F^2(n) + 1 \right]^2.
\]

We compare critical current dependencies over \( d_f \) for SFS [Eq. (31)], SIFS [Eq. (25)] and SIFIS [Eq. (29)] structures in Fig. 2. Each of above junction types undergoes the sequence of 0-\( \pi \) transitions with increasing thickness of the F layer. From the figure we see that the transition from 0 to \( \pi \) state occurs in SIFS tunnel junctions at shorter \( d_f \) than in SFS junctions with transparent interfaces, but at longer \( d_f \) than in SIFIS junctions with two strong tunnel barriers. This tendency can be qualitatively explained by the fact that in structures with barriers (SIFS, SIFIS) part of the \( \pi \) phase shift occurs across the barriers. Therefore a thinner F layer in a SIFS junction compared to an SFS one is needed to provide the total shift of \( \pi \) due to the order parameter oscillation. For the same reason, 0-\( \pi \) transition in a SIFIS junction occurs at a smaller thickness than in a SIFS junction. We note that in Fig. 2 we plot both analytical and numerical calculated \( I_c(d_f) \) dependencies, where numerical calculation was performed for full boundary problem Eqs. (1)-(7) [see further discussion in Sec. IV].

In Fig. 4 we plot the F layer thickness dependence of the critical current in a SIFS junction for different values of spin-flip scattering time. For stronger spin-flip scattering the period of supercurrent oscillations increases and the point of 0-\( \pi \) transition shifts to the region of larger \( d_f \). The same tendency exists for SFS and SIFIS junctions.

In Fig. 4 we plot the F layer thickness dependence of the critical current in a SIFS junction for different values of the exchange field \( h \). We see that for large exchange fields \( h \gg \pi T_c \), the critical current scales with the ferromagnetic coherence length \( \xi_f \).

From comparison with numerical results presented in Fig. 2, we conclude that the results for the critical current in SIFS junctions presented in Figs. 3 and 4 give correct magnitude of the \( I_c R_N \) product for \( d_f \geq \xi_{f_2}/2 \).

As an application of the developed formalism, we present in Fig. 5 the theoretical fit of the experimental data for a Nb/Al₂O₃/Nio₆₆Cu₀₄d/Nb junction by Weides et al. making use of Eq. (25). We used following values of parameters: \( R_B = 3.9 \text{ m} \Omega, D_T = 3.9 \text{ cm}^2 \text{s}^{-1}, T = 4.2 \text{ K}, T_c = 7.2 \text{ K} \) (damped critical temperature in Nb). Good agreement was obtained with the following parameters: \( h/k_B = 950 \text{ K}, 1/\tau_m = 1.6 \text{ h} \) (see Fig. 5). These parameters can be compared with parameters obtained by Oboznov et al. for similar ferromagnetic material, Ni₀.₅₃Cu₀.₄₇: \( h/k_B = 850 \text{ K}, 1/\tau_m = 1.3 \text{ h} \). Higher Ni concentration in the NiCu alloy in the experiment of Weides et al. results in higher exchange field.

In Ref. 13 it was suggested that a “dead” layer exists in the ferromagnet near each SF interface, which does not take part in the “oscillating” superconductivity. Other authors also include into consideration the existence of nonmagnetic layers at the interface of the ferromagnet and the superconductor or normal metal. Thickness of the “dead” layer cannot be calculated quantitatively in the framework of our model and also can not be directly estimated from the experiment. In the experiment of Weides et al. the range of F layer thicknesses was rather narrow and only the first 0-\( \pi \) transition was
observed. Due to these reasons we did not take into account the existence of a nonmagnetic layer in our fit. This question deserves separate detailed experimental and theoretical study.

We should mention that the above estimates of exchange field and spin-flip scattering time could be different if we consider magnetically active S/F interfaces. It was shown in Ref. 28 that the effect of spin-dependent boundary conditions deserves separate detailed experimental and theoretical study.

IV. CRITICAL CURRENT OF JUNCTIONS WITH ARBITRARY LENGTH OF THE FERROMAGNETIC INTERLAYER

In the previous section we derived the expression for the critical current of a SIFS junction in case of considerably long F layer thickness, \( \xi_f \gg \xi_1 \). For arbitrary F layer thickness in the absence of spin-flip scattering, general boundary problem (1)-(7) was solved numerically using the iterative procedure.\(^\text{29}\) Starting from trial values of the complex pair potential \( \Delta(x) \) and the Green’s functions \( G_{s,f} \), we solve the resulting boundary problem. After this we recalculate \( G_{s,f} \) and \( \Delta(x) \). We repeat the iterations until convergence is reached. The self-consistency of calculations is checked by the condition of conservation of the supercurrent across the junction.

In Fig. 2 we compare numerically and analytically calculated \( I_c(d_f) \) dependencies in case of SFS, SIFS and SIFIS junctions. We see that, as expected, the numerical method provides correction only for small length of ferromagnetic layer.

We note that for SFS and SIFS junctions analytical curves (31) and (32) practically coincide with numerical results in the region of the first 0-\(\pi\) transition. For a SIFIS junction this transition occurs at smaller \( d_f \), where the assumptions of the section \( \text{III} \) are not valid. However, in presence of strong spin-flip scattering the first 0-\(\pi\) transition peak in a SIFIS junction shifts to the region of larger \( d_f \) and Eq. (29) describes the transition accurately.

The main result of this section is that Eq. (25) for the critical current of a SIFS junction can be used as a tool to fit experimental data in SIFS junctions with good accuracy.

V. DENSITY OF STATES OSCILLATIONS IN THE FERROMAGNETIC INTERLAYER

It is known that in a ferromagnetic metal attached to the superconductor the quasiparticle DOS at energies close to the Fermi energy has a damped oscillatory behavior.\(^\text{31,34,35}\) Experimental evidence for such behavior was provided by Koutos et al.\(^\text{26}\) In SIFS junctions we can compare the DOS oscillations with the critical current oscillations.

We are interested in the quasiparticle DOS in the F layer in the vicinity of the tunnel barrier (\( x = -d_f/2 + 0 \) in Fig. 1). Below we will refer to the local DOS at this point. For the case
of strong tunnel barrier ($\gamma_{bi} \gg 1$) left S layer and right FS bilayer in Fig. 1 are uncoupled. Therefore we need to calculate the DOS in the FS bilayer at the free boundary of the ferromagnet. Solving numerically Eqs. (10)-(14), we set to zero the $\theta_{f}$ derivative at the free edge of the FS bilayer, $x = -d_f/2$, $(\partial \theta_{f}/\partial x)_{-d_f/2} = 0$.

We use the self-consistent two step iterative procedure. In the first step we calculate the pair potential coordinate dependence $\Delta(x)$ using the self-consistency equation in the S layer, Eq. (12). Then, by proceeding to the analytical continuation in Eqs. (10), (11) over the quasiparticle energy $i\omega \to E + i0$ and using the $\Delta(x)$ dependence obtained in the previous step, we find the Green’s functions by repeating the iterations until convergence is reached. We define the full DOS $N(E)$ and the spin resolved DOS $N_{\uparrow\downarrow}(E)$, normalized to the DOS in the normal state, as

$$N(E) = \frac{[N_{\uparrow}(E) + N_{\downarrow}(E)]}{2}, \quad (32a)$$

$$N_{\uparrow\downarrow}(E) = \text{Re} \left[ \cos \theta_{\uparrow\downarrow}(i\omega \to E + i0) \right]. \quad (32b)$$

The numerically obtained energy dependencies of the DOS at the free F boundary of the FS bilayer are presented in Figs. 6 and 7. Fig. 6 demonstrates the DOS energy dependence for different $d_f$. At small $d_f$ full DOS turns to zero inside a miniband, which vanishes with the increase of $d_f$. Then the DOS at the Fermi energy $N(0)$ rapidly increases to the values larger than unity and with further increase of $d_f$ it oscillates around unity while it’s absolute value exponentially approaches unity (see also Fig. 8). In Fig. 7 we also plot the spin resolved DOS energy dependencies $N_{\uparrow}(E)$ and $N_{\downarrow}(E)$. Fig. 7 demonstrates full DOS energy dependence for different values of spin-flip scattering time. For stronger spin-flip scattering the miniband closes at smaller $d_f$, the period of the DOS oscillations at the Fermi energy increases and the damped exponential decay occurs faster.

In case of long F layer ($d_f \gg \xi_f$) it is also possible to obtain an analytical expression for the DOS at the free boundary of the ferromagnet,

$$N_{\uparrow\downarrow}(E) = \text{Re} \left[ \cos \theta_{\uparrow\downarrow}(E) \right] \approx 1 - \frac{1}{2} \text{Re} \theta_{\uparrow\downarrow}^2(1), \quad (33)$$

where $\theta_{\uparrow\downarrow}(E)$ is a boundary value of $\theta_{f}$ at $x = -d_f/2$. It can be obtained by the mapping method, similar to the one used in the electrostatic problems. We consider the FS bilayer where $x \in [-d_f/2, d_f/2]$ stands for the ferromagnetic metal and $x > d_f/2$ stands for the superconductor; the point $x = -d_f/2$ corresponds to the free F layer boundary. For infinite F layer ($d_f \to \infty$) the solution for $\theta_{f}(x)$ far from the interface is given by the exponential term in Eq. (23), written in the real energy space,

$$\theta_{f}(x) = \frac{4}{\sqrt{1 - \eta^2}} \sqrt{2} \exp \left( -\frac{p}{\xi_f} \right),$$

where

$$p = \sqrt{\frac{2}{\hbar} \sqrt{1 - iE_R \pm i\omega + 1/\tau_m}}, \quad (35a)$$

$$\eta^2 = (1/\tau_m)(-iE_R \pm i\omega + 1/\tau_m)^{-1}, \quad (35b)$$

$$g_2 = \frac{\sqrt{1 - \eta^2}^2 F(E)}{[\sqrt{1 - \eta^2}^2 F(E) + 1 + 1]^2}, \quad (35c)$$

$$F(E) = \frac{|\Delta|}{-iE_R + \sqrt{|\Delta|^2 - E_R^2}} = E_R = E + i0. \quad (35d)$$

Here, as above, positive sign ahead of $\hbar$ corresponds to the spin up state in Eq. (34) and negative sign for the spin down state. By using the arrow ‘from right to left’ in $\theta_{f}(x)$ we want to stress that this solution is induced in the ferromagnet from the right FS interface.

In the case of finite ferromagnet length the boundary conditions at the free F layer boundary, $x = -d_f/2$, become

$$\theta_{f}(x = -d_f/2) = \theta_{\uparrow\downarrow}(E), \quad \left( \frac{\partial \theta_{f}(x)}{\partial x} \right)_{-d_f/2} = 0. \quad (36)$$

To ensure these conditions we add another exponential solution,

$$\theta_{f}(x) = \frac{4}{\sqrt{1 - \eta^2}} \sqrt{2} \exp \left( -\frac{3d_f/2 + x}{\xi_f} \right). \quad (37)$$

resulting from the mirror image of the F layer with respect to the point $x = -d_f/2$. At $x = -d_f/2$ both exponential terms are equal to each other and the final solution, $\theta_{\uparrow\downarrow}(E) = \theta_{f}(x = -d_f/2) + \theta_{f}(x = -d_f/2)$, is two times larger than the solution for infinite ferromagnetic layer at this point and reads

$$\theta_{\uparrow\downarrow}(E) = \frac{8F(E)}{\sqrt{1 - \eta^2}^2 F(E) + 1 + 1} \exp \left( -\frac{d_f}{\xi_f} \right). \quad (38)$$
This equation coincides with the result obtained in Ref. 32 by direct integration of the Usadel equation.

In Fig. [8] we plot analytically and numerically calculated function
\[ \delta N(d_f) = |1 - N_0|, \quad N_0 = N(E = 0), \tag{39} \]

together with the \( I_c(d_f) \) dependence for a SIFS junction. We see that the point of 0-\( \pi \) transition on the \( I_c(d_f) \) plot does not coincide with the first minimum of \( \delta N(d_f) \) corresponding to sign change of \( 1 - N_0 \). This difference can be qualitatively explained as follows. The transition from 0 to \( \pi \) state in a junction, seen as sign change of \( I_c(d_f) \), is the result of interference of solutions for \( \theta_f \) originating from two S electrodes. 0-\( \pi \) transition in \( I_c(d_f) \) occurs approximately at such thickness \( d_f \) when the boundary value of \( \theta_f \) in Eq. (23) at \( x = -d_f/2 \) becomes negative, i.e. when \( \theta_f \) acquires the phase shift \( \pi \). On the other hand, sign change of \( 1 - N_0 \) occurs at such \( d_f \) when the boundary value \( \theta_b \) in Eq. (38) becomes an imaginary number, i.e. when \( \theta_f \) acquires the phase shift \( \pi/2 \). It occurs at smaller \( d_f \) compared to 0-\( \pi \) transition in the critical current. Corresponding 0 and \( \pi \) states defined from \( I_c \) and from the DOS are indicated in Fig. [8].

It is also seen from Fig. [8] that the DOS oscillations have the period approximately twice smaller than those of the critical current. This fact is easy to see from the analytical expression for \( \delta N(d_f) \). Using Eqs. (32) - (39) we obtain
\[ \delta N(d_f) = 32 \left| \frac{1}{\sqrt{2 - \eta_0^2}} \exp \left( -p_0 \frac{2d_f}{\xi_f} \right) \right|, \tag{40} \]

where \( \eta_0 = \eta(E = 0) \) and \( p_0 = p(E = 0) \) in Eqs. (35a) - (35b). At vanishing magnetic scattering, \( \tau_m^{-1} \ll \pi T_C \), this equation can be simplified,
\[ \delta N(d_f) = \frac{32}{3 + 2\sqrt{2}} \left| \frac{2d_f}{\xi_{f1}} \cos \left( \frac{2d_f}{\xi_{f2}} \right) \right|, \tag{41} \]

where characteristic lengths of decay and oscillations \( \xi_{f1/2} \) are given by Eq. (26b) with the substitution \( i\omega \rightarrow E + i0 \). This equation can be compared with Eq. (27). We see that the period of the DOS oscillations is approximately twice smaller than the period of the critical current oscillations and the exponential decay is approximately twice faster than the decay of the critical current.

VI. CONCLUSION

We have developed a quantitative model, which describes the oscillations of the critical current as a function of the F layer thickness in a SIFS tunnel junctions with thick ferromagnetic interlayer, \( d_f \gg \xi_{f1} \), in the dirty limit. We justified this model by numerical calculations in general case of arbitrary \( d_f \): for all values of parameters characterizing material properties of the ferromagnetic metal numerical and analytical results coincide in physically important region of the first 0-\( \pi \) transition. Thus the derived analytical expression for the critical current can be used as a tool to fit experimental data in various types of SIFS junctions. We have discussed the details of the damped oscillatory behavior of the critical current for different values of the F layer parameters.

We also studied the superconducting DOS induced in a ferromagnet by the proximity effect. We showed that the oscillation pattern of DOS at the Fermi energy in the ferromagnet (at location of the tunnel junction) does not coincide with that of the critical current in a SIFS junction and it’s period is approximately twice smaller. Therefore the DOS oscillations do not reflect the 0-\( \pi \) transition in \( I_c(d_f) \). We calculated the quasiparticle DOS in the F layer in close vicinity of the tunnel barrier which can be used to obtain current-voltage characteristics for a SIFS junction. These calculations will be presented elsewhere.

Finally, we used our results to fit recent experimental data for SIFS tunnel junctions and extracted important parameters of the ferromagnetic interlayer.

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