DETECTION OF $\ell = 4$ AND $\ell = 5$ MODES IN 12 YEARS OF SOLAR VIRGO-SPM DATA—TESTS ON KEPLER OBSERVATIONS OF 16 Cyg A AND B

Mikkel Nørup Lund1, Hans Kjeldsen1, Jørgen Christensen-Dalsgaard1, Rasmus Handberg1,2, and Victor Silva Aguirre1

1 Stellar Astrophysics Centre, Department of Physics and Astronomy, Aarhus University, Ny Munkegade 120, DK-8000 Aarhus C, Denmark; mikkeln@phys.au.dk
2 School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham B15 2TT, UK

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ABSTRACT

We present the detection of $\ell = 4$ and $\ell = 5$ modes in power spectra of the Sun, constructed from 12 yr full-disk VIRGO-SPM data sets. A method for enhancing the detectability of these modes in asteroseismic targets is presented and applied to Kepler data of the two solar analogues 16 Cyg A and B. For these targets, we see indications of a signal from $\ell = 4$ modes, while nothing is yet seen for $\ell = 5$ modes. We further simulate the power spectra of these stars and from this we estimate that it should indeed be possible to see such indications of $\ell = 4$ modes at the present length of the data sets. In the simulation process, we briefly look into the apparent misfit between observed and calculated mode visibilities. We predict that firm detections of at least $\ell = 4$ should be possible in any case at the end of the Kepler mission. For $\ell = 5$, we do not predict any firm detections from Kepler data.

Keywords: asteroseismology – methods: data analysis – stars: individual (16 Cyg A, 16 Cyg B) – stars: oscillations – stars: solar-type – Sun: oscillations

Online-only material: color figures

1. INTRODUCTION

Stars observed for the purpose of asteroseismology are measured in full-disk integrated light. In such observations, there will be an unavoidable geometrical cancellation effect between bright and dark patches on the stellar surface from the standing harmonic oscillations excited in the star. The NASA Kepler satellite, dedicated to finding planets using the transit method (Koch et al. 2010), delivers very high quality photometric data that are ideal for asteroseismic studies (Gilliland et al. 2010a). However, even with such exquisite data it has so far only been possible to detect modes of degree $\ell = 3$ (octupole) in two main-sequence (MS) stars, 16 Cyg A and B (Metcalfe et al. 2012), while detections in subgiants and red giants have been seen for some time (see, e.g., Bedding et al. 2010; Huber et al. 2010; Mosser et al. 2012).

Turning to our own star, the Sun has been studied extensively using helioseismology. Here the surface can be resolved whereby the concern of cancellation effects is avoided and very high degree modes can be studied. As mentioned above, we do not have this luxury when studying other stars using asteroseismology; here instead the global low-$\ell$ modes can be studied. To learn about other stars from our Sun, the unresolved surface has been studied in so-called Sun-as-a-star observations, and this has been ongoing for more than a decade with velocity observations from ground (e.g., BiSON; see Chaplin et al. 1996b) and space (e.g., GOLF; see Gabriel et al. 1995) and space-born photometric observations (e.g., VIRGO; see Fröhlich et al. 1995; Frohlich et al. 1997). These observations suffer from the same cancellation effects as experienced for distant stars. Owing to the difference in the stellar noise properties between velocity and photometric measurements, the highest $\ell = 4$ modes (hexadecapole) have been seen in, e.g., BiSON (Chaplin et al. 1996a) and GOLF data (Roca Cortés et al. 1998) for a long time—with the detection of these already predicted by Christensen-Dalsgaard & Gough (1989). In full irradiance observations of the Sun, on the other hand, only modes up to $\ell = 3$ have so far been directly observed (see, e.g., García 2009).

It is clear that the detection of higher-degree modes would be of great importance for stellar modeling with the extra constraints added. It would to a greater extent become possible to perform stellar structure inversions (e.g., Basu et al. 1997), albeit not with very high precision. In the ideal case where these higher degree modes could actually be resolved, a wealth of information could be extracted on, for instance, the rotational properties of the star (e.g., Gizon & Solanki 2004).

In this study, we take a closer look at the photometric data from VIRGO as these data sets hint at what might be possible with long time series from the Kepler satellite, and the results of this will be presented in Section 2. In Section 3, we describe the method we propose to use for other stars in the search for these higher degree modes. We test in Section 4 this method on two of the most promising targets in the Kepler field, viz., 16 Cyg A and B, with results presented in Section 5. Furthermore, we simulate the power spectra of these two targets, described in Section 6, and present the results from these in Section 7. In Section 8, we test the signal from $\ell = 4$ in the solar data when using data lengths corresponding to the amount of data available for 16 Cyg A and B. Finally, we will discuss our findings in Section 9.

2. SOLAR ANALYSIS

2.1. Data

For the solar analysis, we used the data from Fröhlich (2009) with the corrections described therein. More specifically we use 12 yr data sets from the three Sun photometers (SPM)

3 Birmingham Solar Oscillation Network.

4 Global Oscillations at Low Frequencies.

5 Variability of Solar Irradiance and Gravity Oscillations.
Figure 1. Power spectra of VIRGO-SPM data, smoothed with a 1.8 μHz boxcar filter. The color of the respective power spectra corresponds to color of the SPM-filter used. As seen the power levels are clearly highest in the blue band, followed by the green band and lastly the red band. In order to make the distinction between the power spectra easier, circles have been added on the peaks of the central \( \ell = 0 \), 1 modes for the red and the green power spectra. See also Figure 12 for the position in wavelength of the different filters.

(A color version of this figure is available in the online journal.)

Figure 2. Zoom-in on the power spectra of the VIRGO-SPM data, binned in segments covering 1.8 μHz. The color of the respective power spectra corresponds to color of the respective SPM-filter used. Note the different vertical axes. The positions of surface-corrected Model S frequencies have been indicated by vertical lines, showing a clear correspondence to the observed peaks in the power spectra. The legend gives the colors and line styles used for the different mode degrees.

(A color version of this figure is available in the online journal.)

of the VIRGO instrument on board the ESA/NASA SOHO spacecraft. The mid-wavelengths of these photometers are at 402 nm (blue), 500 nm (green), and 862 nm (red). The power spectra computed from the corrected time series are shown in Figure 1. These were calculated using a sine-wave fitting method (see, e.g., Kjeldsen 1992; Frandsen et al. 1995), normalized according to the amplitude-scaled version of Parseval’s theorem (see Kjeldsen & Frandsen 1992), in which a sine wave of peak amplitude, A, will have a corresponding peak in the power spectrum of A².

The peak seen at 5555 μHz is an artifact and stems from the Data Acquisition System (DAS) of VIRGO, which has a corresponding reference period of 3 minutes (Jiménez et al. 2005).

2.2. \( \ell = 4 \) and \( \ell = 5 \) Modes

In Figure 2, we show a zoom-in around the frequency of maximum power, \( \nu_{\text{max}} \), of the power spectra after a 1.8 μHz binning has been applied.

In this figure, we have indicated frequencies computed from the solar structure model Model S (Christensen-Dalsgaard et al. 1996). These frequencies have been corrected for near-surface effects as described by Christensen-Dalsgaard (2012, Equation (14)).

From a mere visual inspection of this figure, it is quite clear that the prominent peaks in the power spectrum agree with the frequencies from Model S. This is of course to be expected considering that Model S is calibrated to match the Sun. However, the match also includes many of the Model S \( \ell = 4 \) (hexadecapole) and \( \ell = 5 \) (dotriacontapole; Ellis & Gough 1988) modes, confirming that it is indeed signal from \( \ell = 4 \) and \( \ell = 5 \) modes that is seen. The strong peak \( \sim 2777 \mu \text{Hz} \), which coincides with the frequency of an \( \ell = 5 \) mode, is not of stellar nature but is an artifact, having half the frequency of the signal from the DAS. The observed frequencies also match up with values reported from, e.g., LOI7 (Appourchaux et al. 1995; Appourchaux & Virgo Team 1998), BiSON (Broomhall et al. 1998), and others.

\( \ell = 0 \) modes are characterized by the oscillation of the entire star with no preferred direction, \( \ell = 1 \) modes oscillate in a single meridional plane, \( \ell = 2 \) modes oscillate in two meridional planes, \( \ell = 3 \) modes oscillate in three meridional planes, \( \ell = 4 \) modes oscillate in four meridional planes, and so on. The number of meridional planes in which an \( \ell \) mode oscillates is given by \( \ell \), and the number of times the star oscillates out of phase is given by \( \ell \). For example, a \( \ell = 3 \) mode oscillates out of phase three times, and a \( \ell = 4 \) mode oscillates out of phase four times.

6 Solar and Heliospheric Observatory.

7 The Luminosity Oscillations Imager (on SOHO).
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Figure 3. Zoom-ins on two central orders ($n = 19, 20$) of the power spectrum from VIRGO data of the red SPM-filter. A 1.8 $\mu$Hz boxcar smoothing has been applied. The vertical lines (two drawn for each degree) show the range in the mode frequency estimates (including errorbars) from LOI (Appourchaux & Virgo Team 1998), BiSON (Broonhall et al. 2009) (only $\ell = 4$), MDI (Larson & Schou 2008), BiSON+LOWL (Basu et al. 1997), and GONG (only $\ell = 5$) (Komm et al. 2000). As seen, the correspondence between these estimates and the VIRGO data is unequivocal.

We can further visualize the power excess from the $\ell = 4$ and $\ell = 5$ modes in the well-known échelle diagram (Grec et al. 1983), where the power spectrum is first divided in segments corresponding to the so-called large separation given by the frequency difference between modes of same degree and consecutive radial order. Subsequently these segments are stacked on top of each other. In practice, the frequency is plotted against its modulo to the large separation—we have on the ordinate plotted the mid-frequency of the respective segments. In order to obtain a nice representation of the ridges, it is customary to add an arbitrary value to the frequencies before taking the modulo, thereby allowing a shift of the ridges on the abscissa (see, e.g., Bedding 2011). In Figure 4, we show the échelle diagram of the power spectrum from the red band, here also with the Model S frequencies over-plotted. In Figure 5, we give the same échelle diagram, though in a narrower frequency range. To make clearer the excess from the $\ell = 4$ and $\ell = 5$ modes, the power spectrum has been divided by a linear representation of the stellar background, and the colors have been truncated such that the maximum corresponds roughly to the highest $\ell = 3$ mode.

3. ENHANCING THE DETECTABILITY

To make easier the detection of the $\ell = 4$ and $\ell = 5$ modes in stars other than the Sun, we propose here a simple three step method that should accomplish this. Firstly, the power spectrum is stretched; this is described in Sections 3.1 and 3.2.

Secondly, the power spectrum is smoothed in Section 3.3. Thirdly, the stretched, smoothed power spectrum is collapsed in Section 3.4. The final spectrum we tentatively name the SC-spectrum (Straightened-Collapsed).

3.1. Parameterization of the Power Spectrum

In the first step, we parameterize the power spectrum in order to account for the possible departures from the asymptotic description of the constituent modes. The motivation for this step is that in the end we need to correct for such departures in order to facilitate the co-addition (collapsing) of power from many radial orders of a specific degree. This is inspired by a step in the Octave code (Hekker et al. 2010), where the frequency scale of the power spectrum is modified to account for departures in

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8 Michelson Doppler Imager (on SOHO).
9 Global Oscillation Network Group.
the large separation originate, e.g., from sharp changes in the
degree. There exist departures from the asymptotic description
frequency relation (Tassoul1980), which to a good approximation
the power-spectrum-of-power-spectrum (PS)
the asymptotic description and thereby obtain a higher signal in
the power-spectrum-of-power-spectrum (PS ⊗ PS).

We start from the following version of the asymptotic fre-
quency relation (Tassoul 1980), which to a good approximation is
applicable to acoustic modes of high radial order \( n \) and low
angular degree \( \ell \):

\[
v_{nl} = \Delta \nu(n + \ell/2 + \epsilon) - \ell(\ell + 1)D_0. \tag{3.1}
\]

In this equation, \( \Delta \nu \) is the large separation, \( \epsilon \) is a dimension-
less offset sensitive to the surface layers, and \( D_0 \) is a quantity
sensitive to the sound-speed gradient near the core of the star
(see, e.g., Scherrer et al. 1983; Christensen-Dalsgaard 1993;
Bedding & Kjeldsen 2010).

It is well known from both theory and observations (see,
e.g., Mosser et al. 2011, 2013; Kallinger et al. 2012) that
Equation (3.1) is in fact only an approximate description as \( \Delta \nu \),
\( D_0 \), and \( \epsilon \) all have small dependencies on both frequency and
degree. There exist departures from the asymptotic description
on both small and large scales. Small-scale oscillations in
the large separation originate, e.g., from sharp changes in the
sound-speed profile (one source being the He II ionization zone;
Houdek & Gough 2007)—we will not account for this in the
following. The larger scale departures manifest themselves as
overall curvatures or tilts of the ridges in the échelle diagram.
With the larger scale departures in mind, Hekker et al. (2010)
described one way to find the variation in the large separation
as a function of the radial order, \( d \Delta \nu/dn \) (see also Mosser &
Appourchaux 2009; Roxburgh 2009). This variation in the large
separation was used in Campante et al. (2010), where it was
introduced in the asymptotic relation as a term quadratic in
\( n \)—we will follow the same procedure for this term. The small
separation \( \delta \nu_{\ell 2} \equiv v_{n \ell 0} - v_{n-1 \ell 2} = 6D_0 \), and in turn \( D_0 \), is
for the Sun found to decrease almost linearly with frequency
(Elsworth et al. 1990). With this in mind, we introduce to our
modified asymptotic relation a term \( dD/dn \), which is linear
in \( n \). We end up with the following modified version of the
asymptotic relation (see also Mosser et al. 2011, 2013):

\[
\tilde{v}_{nl} = \Delta \nu_{\text{pivot}}(n + \ell/2 + \epsilon) - \ell(\ell + 1)D_0 - \ell(\ell + 1)\frac{dD_0}{dn}(n - n_{\text{pivot}}) + (n - n_{\text{pivot}})^2 \frac{d\Delta \nu/dn}{2}.
\tag{3.2}
\]

We have denoted this model frequency by \( \tilde{v}_{nl} \) to indicate that
this is a predicted value only. \( \Delta \nu_{\text{pivot}} \) denotes the value of the
large separation at \( n = n_{\text{pivot}} \) (various formulations exist for
finding \( \Delta \nu \); see, e.g., Kallinger et al. 2012). Here \( n_{\text{pivot}} \) (which
is not constrained to be of integer value) can be seen as the
pivot point for the variations in both the large separation and
\( D_0 \)—the frequency at which \( n \sim n_{\text{pivot}} \) is generally coinciding
with the frequency of maximum power of the modes, \( v_{\text{max}} \).
In the description of Equation (3.2), we have assumed that the
various parameters are frequency-dependent only and neglected
any potential dependencies on angular degree. In the following,
we will not touch upon the physical meaning behind the
frequency dependencies in \( \Delta \nu \) and \( D_0 \) but merely use them in
our parameterization of the frequencies in the power spectrum.
The interested reader is referred for instance to Tassoul
(1980), Houdek & Gough (2007), and most recently Mosser
et al. (2013) and references therein for more on the physics
behind the departures.

3.2. Modifying the Frequency Scale

The modification or straightening of the frequency scale in
the power spectrum comes down to three steps:

1. Fitting of the parameterization given by Equation (3.2).
The fitting of the modes is best illustrated in the échelle
diagram described in Section 2.2, and we will do so from
now onward.

If only observed frequencies are available, e.g., from peak-
bagging (e.g., Appourchaux 2003), the above parameteriza-
tion is simply fitted to these frequencies, with the free
parameters being \( \Psi = \{d\Delta \nu/dn, D_0, dD_0/dn, \epsilon, n_{\text{pivot}}\} \).

Note that even if the identification of the modes with respect
to the radial order is wrong, a good fit can still be obtained
as \( n_{\text{pivot}} \) is a free parameter. The large separation \( \Delta \nu \) is not
included in the optimization as this can be determined rela-
tively easily, and any small deviations from \( \Delta \nu_{\text{pivot at } n_{\text{pivot}}} \) can be accounted for by \( \epsilon \).

If on the other hand a stellar model with computed frequencies
is available, and which after applying a surface correction (e.g.,
Kjeldsen et al. 2008b) matches the observed data well, one can apply the fitting to these modeled
frequencies. Here it is then possible to use also the model
calculated frequencies for the \( \ell = 4 \) and \( \ell = 5 \) modes in
the fitting.

With this first step, we obtain estimates for the parameters
entering Equation (3.2).

2. Estimate frequencies for a targeted degree.
The reason for selecting a specific degree (the “targeted”
degree) is that as we have included a variation in \( D_0 \)
in our parameterization, we cannot modify the frequency
scale (as in Hekker et al. 2010) such that all degrees fulfil
Equation (3.1). The reason for this is the factor of \( \ell(\ell + 1) \) on
the term describing the variation depending on \( D_0 \), making
this term \( \ell \)-dependent.

From the estimated parameters (\( \Psi \)) found above one can now use Equation (3.2) to estimate the frequencies \( \tilde{v}_{nl} \) of a
specific or targeted degree. This could for instance be the \( \ell = 4 \) or \( \ell = 5 \) modes. One should be aware that small errors in \( \epsilon \) and \( D_0 \) can cause a big offset in the estimate of, e.g., the \( \ell = 4 \) or \( \ell = 5 \) mode frequencies. The biggest issue is \( D_0 \) due to the factor of \( \ell(\ell + 1) \) on this parameter; furthermore, this value is one of the most difficult ones to constrain as many modes of different degree are needed. In contrast, \( \epsilon \) (and \( \Delta \nu \) for that matter) can be fairly well constrained by modes of the same degree. This issue is mainly a concern if the fit of the parameterization in step (1) is made to observed frequencies only, e.g., of peak-bagged modes up to \( \ell = 2 \), and the desire is to identify power from modes of a higher degree, e.g., \( \ell = 4 \) or \( \ell = 5 \). The concern becomes irrelevant if a well-matching model exists.

3. Correct for deviation from Equation (3.1) for a specific degree.

We now wish to take out the dependencies on frequency in the modes spacings. This is to get a power spectrum that for modes of a targeted degree follows Equation (3.1) more strictly, i.e., with equidistance between mode frequencies—equivalent to a straight ridge in the échelle diagram. This is accomplished by changing the frequency scale in the power spectrum using frequencies computed in step (2) for the modes of the targeted degree.

An interpolation is made between the computed mode frequencies from step (2) as the independent variable and their modulo with the large separation as the dependent variable—this would correspond to a flipped échelle diagram; see Figure 6. We denote the obtained interpolation by “I.” This interpolation now follows the ridge in the échelle diagram from the computed frequencies of the targeted degree. We can now for every frequency \( (\nu_{\text{old}}) \) in the frequency scale of the power spectrum compute a new modified frequency \( (\nu_{\text{new}}) \) by adding a value \( \delta \nu \):

\[
\nu_{\text{new}} = \nu_{\text{old}} + \delta \nu.
\]  

(3.3)

This added value is obtained from the interpolation as (see Figure 6)

\[
\delta \nu = I(\nu_{\text{old}}) - I(\nu_{\text{ad}}).
\]  

(3.4)

With this value of \( \delta \nu \) the reference frequency sets the position on the abscissa of the targeted ridge in the échelle diagram—by choice we set the reference frequency equal to the value of \( \nu_{\text{max}} \). As we wish to co-add the segmented power spectrum, we need in the end to map the modified power spectrum onto a frequency scale with a regular step size.

In Figure 7, the method of straightening has been applied to Model S frequencies. In the left panel the fit of Equation (3.2) is made to Model S frequencies (squares) with degrees \( \ell = 0 \) (blue), \( \ell = 1 \) (red), \( \ell = 2 \) (green), \( \ell = 3 \) (magenta), \( \ell = 4 \) (cyan), and \( \ell = 5 \) (yellow). The solid black lines illustrate the interpolation to the frequencies estimated from the obtained fit of Equation (3.2). The dashed black lines give the behavior of the fit after the straightening procedure, with the straightened Model S frequencies now given as circles. The straightening is targeted at the \( \ell = 4 \) modes, and they now form a vertical line. As seen, the position on the abscissa of this vertical line is given by the value of the fit at the chosen reference frequency \( \nu_0 \). The reference frequency is given by the horizontal dashed black line and was here set equal to \( \nu_{\text{max,}} = 3150 \mu \text{Hz} \). In the right panel the same procedure is followed, but here only the modes having \( \ell = 0 \)–2 were used. The use of less modes leads to a fit (step 1) with a poorly determined value of \( D_0 \) (mainly), which in turn results in a bad estimation of \( \ell = 3 \)–5 modes (step 2). When using the badly estimated values of \( \ell = 4 \) modes in the straightening (step 3), an offset between the actual (straightened Model S frequencies) and predicted (straightened estimated frequencies) value on the abscissa of the straightened \( \ell = 4 \) modes is seen.

3.3. Smoothing

For every mode of degree \( \ell \), there are \( 2\ell + 1 \) degenerate \( m \)-components ranging in value from \(-\ell\) to \(+\ell\), with \( m \) being the so-called azimuthal order. The degeneracy of these modes is lifted when the star rotates, with the \( m \)-components being spread out in frequency and thereby taking away power from the central frequency at \( m = 0 \). In the straightening procedure, it is the position of this central \( m = 0 \) component that is found.

For this reason, we apply a smoothing to the straightened power spectrum in order to boost the detectability, as it is desirable to merge the power contained in the rotationally split \( m \)-components. The effect from this merging of power to the central frequency will depend on the size of the rotational splitting compared to the mean mode width. If the splitting is very large compared to the mode width, the smoothing will not merge much of the power from different \( m \)-components. However, the smoothing will in any case decrease the point-to-point scatter in the power spectrum from the \( \chi^2 \) noise, allowing any underlying structures to stand out more clearly. In addition, the smoothing takes out the impact of small wiggles that unavoidably will be present in the ridge for the degree of interest, even after the straightening procedure. The wiggles from for
instance the He\textsc{II} ionization zone will also still be present in the straightened ridge as we did not account for these. It is difficult to determine the smoothing level that optimizes a visual detection, as this indeed is a very qualitative measure, and the impact of a given smoothing level will depend on the rotational splitting, mode width, and inclination angle of the specific star. Also, a “too high” smoothing can result in a significant smoothing of the distance from this frequency. This is done in order to not simply add noise to the collapsed spectrum when using frequencies far away from $v_{\text{max}}$.

The envelope of the $p$-modes is most often described by a Gaussian function, and we chose this as our weighting function:

$$G(v) = \exp \left( -\frac{(v - v_{\text{max}})^2}{2\sigma_{\text{env}}^2} \right).$$

Mosser et al. (2012a) give the following relationship between $v_{\text{max}}$ and the FWHM, and thereby the spread of the Gaussian envelope:

$$\text{FWHM} \approx 0.66 \frac{v_{\text{max}}^{0.88}}{\sigma_{\text{env}}} \Rightarrow$$

$$\sigma_{\text{env}} \approx 0.66 \frac{v_{\text{max}}^{0.88}}{2\sqrt{\ln(2)}}.$$

With this, we can construct the new weighted power spectrum to collapse:

$$P_{\text{new}}(v) = P_{\text{old}}(v) G(v).$$

The power spectrum that now has a modified frequency scale and weighted power values is then collapsed to form the SC-spectrum, and the steps taken should ensure that the signal from the degree of interest should be better defined in frequency and easier to identify in power. Notice that the application of Equation (3.3) on the power spectrum results as wanted in frequency equidistance of the modes for the targeted degree, but for modes of another degree it will generally have the opposite effect; equivalently, when the power spectrum is collapsed in general, only the targeted degree will have a more well-defined signal peak, while the peaks of other degrees will tend to be smeared out to some extent.

Even with the above weightening, modes far from $v_{\text{max}}$ will mainly contribute noise to the SC-spectrum. Therefore, one should preferably only include a relatively small number of central orders. The optimum number of overtones to add for a specific target will depend on the sbr as a function of frequency for that specific target, in addition to the large separation, and is therefore not easily generalized.
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3.5. Remarks

The choice of modes in the fitting of Equation (3.2) is important. As seen in the échelle diagram of the Sun in Figure 4, the ridges have in general an “S”-shape from the variation in the large separation. The section of the ridge that is well fitted by a quadratic term in \( n \) is only the top part of this “S” (or, equivalently, only the lower part), i.e., from about 2400 \( \mu \)Hz and up. As seen the curvature of this upper part of the “S” is quite well centered around \( \nu_{\text{max}} \) at around 3150 \( \mu \)Hz. So, using modes that lie far away from \( \nu_{\text{max}} \) can result in a bad fit of Equation (3.2).

For more evolved stars, mixed modes should as far as possible be excluded from the fit to the modes—so, e.g., \( \ell = 1 \) modes experiencing the largest effect of the mode bumping should be left out.

Another aspect to be aware of is that the collapsed power from the modes \( \ell = 0–5 \) will to some extent be contaminated with power from even higher degree modes. As also mentioned in, e.g., Appourchaux & Virgo Team (1998), \( \ell = 1 \) modes are polluted by \( \ell = 6 \) and \( \ell = 9 \) modes, while \( \ell = 7 \) modes fall in the proximity of the \( \ell = 4 \) modes, \( \ell = 8 \) near the \( \ell = 5 \) modes, and so forth. For this reason, it will furthermore be very unlikely to pick up isolated signals from, e.g., \( \ell = 6–8 \) modes.

If hypothesis testing is desired on the collapsed spectrum, in the form of for instance an \( H_0 \) test, the last step of weighting could be left out, and a binning of points rather than a smoothing would be more suitable.

4. ANALYSIS OF 16 Cyg A AND B

We will now apply the method of the SC-spectrum to the two solar analogues 16 Cyg A\(^{10}\) (G1.5V) and B\(^{11}\) (G3V). These two very similar stars are in fact part of a hierarchal triple system (16 Cygni), with a faint M dwarf as the third component. The most striking difference between the two stars is that measurements in radial velocity show 16 Cyg B to host a Jupiter-mass planet in a highly elliptical orbit (Cochran et al. 1997). The second major difference between the two stars is that measurements—so, e.g., Gizon & Solanki (2003). The vertical dashed lines show the positions of the split \( m \)-components.

(A color version of this figure is available in the online journal.)

Figure 8. Top two panels give the functional form of \( \xi_{i,0,0}(i) \) (first) and \( \xi_{i,3,0}(i) \) (second). The \( m = 0 \) component (blue) goes to a value of 1 at \( i = 0^\circ \).

Bottom two panels render the same information, only here a top view of the rotationally split multiplets (\( \nu_i = 0.37 \mu \)Hz) is shown, with the visibilities of different azimuthal components given by the gray scale, ranging from black (high visibility) to white (low visibility) (Gizon & Solanki 2003). The vertical dashed lines show the positions of the split \( m \)-components.

For both stars, we used short cadence (SC, \( \Delta t = 58.85 \) s; Gilliland et al. 2010a) data from quarters Q7–Q13, corresponding in the end to ~643 days, all downloaded from the KASOC webpage.\(^{13}\) The stars were not observed in SC in Q0–Q5, as both stars are highly saturated.\(^{14}\) These data have therefore not been included in our data sets. The data type used is the uncorrected simple aperture photometry (SAP). The correction of the time series was done by high-pass filtering individual sub-quarters (~1 month) by a 1 day moving median filter. Bad data

4.1. Kepler Data

For both stars, we used short cadence (SC, \( \Delta t = 58.85 \) s; Gilliland et al. 2010a) data from quarters Q7–Q13, corresponding in the end to ~643 days, all downloaded from the KASOC webpage.\(^{13}\) The stars were not observed in SC in Q0–Q5, as both stars are highly saturated.\(^{14}\) These data have therefore not been included in our data sets. The data type used is the uncorrected simple aperture photometry (SAP). The correction of the time series was done by high-pass filtering individual sub-quarters (~1 month) by a 1 day moving median filter. Bad data

\(^{10}\) KIC 12069424, HR 7503, HD 186408.

\(^{11}\) KIC 12069449, HR 7504, HD 186427.

\(^{12}\) Kepler magnitudes are nearly equivalent to \( R \)-band magnitudes (Koch et al. 2010).

\(^{13}\) Kepler Asteroseismic Science Operations Center: kasoc.phys.au.dk.

\(^{14}\) The saturation limit for Kepler is about \( K_p \sim 11.5 \) (Gilliland et al. 2010b); large custom aperture masks are needed in order to capture as much flux as possible. However, the SC observations made in parts of Q6 did not make use of custom masks on the CCD, which resulted in a rather poor data quality.
points (or outliers) were then removed, with a bad datum being identified as one having a point-to-point flux difference falling outside $3\sigma$—with $\sigma$ found as the standard deviation of the point-to-point flux differences of the entire time series (García et al. 2011). We did not estimate this standard deviation (STD) directly but used instead the more robust median-absolute-deviation (MAD), and from this estimated the STD via the scaling $\text{STD} = 1.4826 \times \text{MAD}$. The filtering of bad data points was performed iteratively four times. In addition, we used the “Quality” entry in the FITS files for the SAP to remove points with known artifacts. The final time series had duty cycles of 85.7% (16 Cyg A) and 82.2% (16 Cyg B). With these duty cycles in mind the spectral windows for the two stars were checked, and we found no significant leakage of power into side-lobes. In any case, the smoothing of the power spectrum should ensure than any potentially leaked power is still accounted for.

The power spectra (see Figure 9) were calculated in the same manner as for the solar data, except for the use of statistical weights in the computation. Weights were computed as $w_i = 1/\sigma_i^2$, with $\sigma_i$ found from a 3 day windowed MAD of the corrected time series.

For the modeling of the stellar background signal, we use a sum of power laws (Harvey 1985), here in the version proposed by Karoff (2008) (see also Huber et al. 2009), and in addition, we add a Gaussian function to account for the excess $p$-mode power:

$$B(\nu) = \sum_{i=0}^{3} \frac{4\sigma_i^2 \tau_i}{1 + (2\pi \nu \tau_i)^2 + (2\pi \nu \tau_i)^4} + A \exp \left( -\frac{(\nu - \nu_{\text{max}})^2}{2\sigma_g^2} \right) + B_N,$$

where $B_N$ is the white shot-noise component, $\sigma_i$ is the rms intensity of the $i$th noise component, $\tau_i$ is the corresponding timescale of the noise component, and $A$ and $\sigma_g$ are the amplitude and spread, respectively, of the $p$-mode envelope. The three background components included, and shown in Figure 9, account for the activity (magenta), granulation (green), and faculae (blue) signals, respectively. When correcting the power spectrum for the stellar background, we of course leave out the Gaussian $p$-mode envelope (dashed white) from the full fit (thick red).

### 5. RESULTS ON DETECTABILITY FROM BONA FIDE DATA

We applied the method presented in Section 3 to the $Kepler$ data of 16 Cyg A and B. Figure 10 gives the échelle diagrams of the two stars, with the same color convention for the different degrees as used in Figure 7 for the Sun. In the figure, observed mode frequencies are given as white stars with associated errorbars. Model frequencies (see Section 6.1) are given as circles, with the distinction that filled circles were used in the fitting of Equation (3.2), while white circles were left out—see Section 3.5 for the considerations made in the selection of modes to include in the fit. In the plot, we have indicated the radial order $n$ of the $\ell = 0$ modes. The fit to the model frequencies was made with the straightening of the $\ell = 4$ ridge in mind, and the black line depicts the obtained fit (similar curves of course exist for the other degrees; see Figure 7). The dashed cyan line gives the position of the ridge after the straightening, with the reference frequency $\nu_0$ (dashed black line) set equal to the value of $\nu_{\text{max}}$ obtained from fitting Equation (4.1) to the power spectra, viz., $2101 \, \mu Hz$ (16 Cyg A) and $2552 \, \mu Hz$ (16 Cyg B). The regions between the two thin horizontal lines give the range chosen in the collapsing of the straightened power spectra, i.e., the $\sim 7$ orders closest to $\nu_{\text{max}}$. This region largely coincides with the region wherein $\ell = 3$ modes are readily observed.

In Figure 11 the $SC$-spectra are shown and rendered in a multitude of different amounts of applied smoothing. The top two panels give the full $SC$-spectra, while the bottom two panels show zoom-in versions. The dashed cyan line indicates as in Figure 10 the position of the straightened $\ell = 4$ ridge and thereby gives the expected position of the possible excess from $\ell = 4$ modes. The horizontal lines give the median value of the

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15 The multiplicative factor of 1.4826 converts (approximately) the MAD for a normal distribution to a consistent measure of the STD and can be found as $1/\Phi^{-1}(3/4)$, with $\Phi^{-1}$ representing the inverse cumulative distribution function or quantile function.

16 The bit values of various known artifacts can be found in Fraquelli & Thompson (2012).

17 Percentage of final number of points to the total expected number of points given the length and cadence of the time series.
Figure 10. Method from Section 3 applied to Kepler data of 16 Cyg A (left) and B (right). The gray scale in these échelle diagrams ranges from white at low power to black at high power. Observed mode frequencies are given as white stars with associated errorbars. Model frequencies (see Section 6.1) are given as circles, with filled circles used in the fitting of Equation (3.2), while white circles were left out. The black line following the $\ell = 4$ ridge illustrates the obtained fit to estimated frequencies. The radial order, $n$, of the $\ell = 0$ modes is indicated by the numbers. The two horizontal black lines give the range in frequency that was collapsed in making the SC-spectrum, while the black dashed line indicates the frequency of $\nu_{\text{max}}$. The vertical dashed cyan line gives the positions of the $\ell = 4$ ridge after the straightening.

(A color version of this figure is available in the online journal.)

Figure 11. SC-spectra for 16 Cyg A (left) and B (right) with different amounts of applied smoothing. The top two panels give the full SC-spectra, while the bottom two panels show zoom-in versions. The dashed cyan line indicates as in Figure 10 the position of the straightened $\ell = 4$ ridge and hence the expected position of a possible excess from $\ell = 4$ modes. The horizontal lines give the median values of the SC-spectra in the minimum and maximum smoothing cases.

(A color version of this figure is available in the online journal.)
for the straightening. However, we note that the small separation described in Section 6.1 below, and no other model was tested here no noticeable rise was seen in the collapsed power. The ℓ = 5 ridge is in any case more difficult to capture due to its close proximity to the strong ℓ = 0 modes.

Model frequencies were obtained from the stellar model described in Section 6.1 below, and no other model was tested for the straightening. However, we note that the small separation δν₀₂ seems to be correctly reproduced as the model frequencies match well with the observed values for both the ℓ = 0 and ℓ = 2 ridges. From the definition of δν₀₂ as 6D₀ (see Section 3.1) this means that D₀ is approximately correct and the ℓ = 4 ridge should therefore be located at nearly the same place in the echelle diagrams for all the models that correctly reproduce δν₀₂.

6. SIMULATED POWER SPECTRA

In order to check if a potential excess power in the SC-spectrum is in line with what can be expected for ℓ = 4 or ℓ = 5 from the current amount of observed data, we simulated the power spectra of 16 Cyg A and B. This also enables a test on the impact of longer observing times on the detectability of the ℓ = 4 and ℓ = 5 modes, and thereby hints to the needed amount of data for a solid detection. Furthermore, we can investigate the role of the currently unknown stellar inclination angles and test the effect of applying different amounts of smoothing. In the following, the various aspects underlying the simulations will be described, to wit, the stellar modeling, the synthetic power spectrum, visibilities, and noise properties.

6.1. Stellar Modeling

For the stellar models needed to calculate the pulsation frequencies we use pre-existing18 from the asteroseismic modeling portal (AMP; Metcalfe et al. 2009; Woitaszek et al. 2010)—see Table 1 for the parameters of the adopted models. The models were in AMP found by optimizing (using a parallel genetic algorithm (GA); see Metcalfe & Charbonneau 2003) the fit of the observed oscillation frequencies (published in MC12) and observational constraints to modeled oscillation frequencies. The spectroscopic constraints listed in Table 1 are from Ramírez et al. (2009). We refer the reader to MC12 and references therein for the input physics and specific details of the modeling. Notice that the final AMP parameters quoted in MC12 were found after using a localized Levenberg–Marquardt optimization algorithm (utilizing singular-value-decomposition) to adjust the stellar parameters from the values found by the GA, viz., the values given in Table 1. For this reason the values in Table 1 will differ slightly from the ones given in MC12. With the best-fitting AMP models, we use the “Aarhus adiabatic pulsation package” (Christensen-Dalsgaard 2008) to compute the oscillation frequencies including ℓ = 4 and ℓ = 5 modes. Finally, we apply an empirical correction for surface effects in order to have model frequencies that resemble the observed frequencies of 16 Cyg A and B

| Parameter | Model A | Model B |
|-----------|---------|---------|
| M (M☉)    | 1.10    | 1.07    |
| R (R☉)    | 1.2361  | 1.1256  |
| L (L☉)    | 1.5669  | 1.2616  |
| τ (Gyr)   | 6.5425  | 5.8162  |
| 8M⊙       | 2.06    | 2.00    |
| ν₀₂       | 0.2510  | 0.2430  |
| Z         | 0.02239 | 0.02032 |
| Tₐₐ (K)   | 5814.01 | 5771.32 |

Note. See MC12 and references therein for the calculation of the luminosity.

6.2. The Synthetic Limit Spectrum

Even though it has not yet been possible to extract mean rotation periods from the power spectra of 16 Cyg A and B, we can with gyrochronology make a rough estimate for the rotation period. The mean rotation period is found using the expression of Barnes (2007)

\[ P_{rot} = t^n a \left[ (B - V)_{0} - c \right]^{b}, \]

with parameters \( n = 0.519, a = 0.773, b = 0.601, \) and \( c = 0.4. \) Here \( r \) is the stellar age in Myr, for which we use the model values given in Table 1. The values for \( (B - V)_{0} \) are 0.64 (16 Cyg A) and 0.66 (16 Cyg B), both adopted from Johnson & Morgan (1953). Using Equation (6.2) on the model results gives for both models a rotation period of \( P_{rot} \approx 31 \) days, equivalent to a rotation frequency of \( ν_{rot} \approx 0.37 \mu Hz. \) Using instead the finally adopted common age of 6.8 ± 0.4 Gyr from MC12 results only in minor differences. In setting up the model power spectrum,

18 16 Cyg A: amp.phys.au.dk/browse/simulation/191; 16 Cyg B: amp.phys.au.dk/browse/simulation/189.

19 Using \( r = 1. \)
the effect of rotation on a mode of radial order \( n \), degree \( \ell \), and azimuthal order \( m \) is included to first order as (Ledoux 1951)

\[
v_{n\ell m} = v_{n\ell} + m(1 - C_{n\ell})v_\ell , \tag{6.3}
\]

where the effect of the Coriolis force is represented by the dimensionless parameter \( C_{n\ell} \). For high-order, low-degree acoustic modes, as the ones seen in the solar analogues 16 Cyg A and B, the rotational frequency splitting is dominated by advection and the parameter \( C_{n\ell} \) is set equal to 0.

In the simulated power spectra, we describe individual modes as a standard Lorentzian (see, e.g., Anderson et al. 1990) given by

\[
L_{n\ell m}(\nu) = S_{n\ell m} \left[ 1 + \left( \frac{\nu - \nu_{n\ell m}}{\Gamma_{n\ell}/2} \right)^2 \right]^{-1} . \tag{6.4}
\]

This shape is appropriate for describing a damped-driven oscillation such as the stochastically excited \( p \)-modes. In Equation (6.4) \( \Gamma_{n\ell} \) is the damping rate for the mode and gives the FWHM value of \( L_{n\ell m}(\nu) \). Furthermore, the mode lifetime is given by \( \tau_{n\ell} = (\pi \Gamma_{n\ell})^{-1} \). No mode asymmetries have been included in our description.

For the width of the individual modes, there is a general consensus in the field of a high temperature dependence, commonly given as a power law \( \Gamma_{\text{max}} \propto T_\text{eff}^{n} \) with \( \Gamma_{\text{max}} \) as the mode linewidth at \( \nu_{\text{max}} \). Also, there is a frequency dependence to \( \Gamma \), with a local minimum at \( \nu_{\text{max}} \) and with decreasing widths toward lower frequency and increasing width toward higher frequencies (see, e.g., Isaak 1986; Libbrecht 1988; Chaplin et al. 1997). Typically, no dependence is assumed for the degree \( \ell \) of the mode (Libbrecht 1988; Houdek et al. 1999). There is, however, not an unequivocal value for the exact size of the dependence (given by \( n \)), and this is indeed still a matter of great debate in the literature. For MS stars Chaplin et al. (2009) found a temperature exponent of \( n \approx 4 \), Baudin et al. (2011a, 2011b) found using CoRoT\textsuperscript{20} observations \( n \approx 16 \pm 2 \), while Appourchaux et al. (2012) from Kepler observations found a value of \( n \approx 15.5 \pm 1.6 \) if the measurement was made at maximum mode amplitude and \( n \approx 13.0 \pm 1.4 \) if it was made at maximum mode height. Belkacem et al. (2012) found from a theoretical approach an exponent of \( n \approx 10.8 \), and with the expression for \( \Gamma \) including a small dependence on surface gravity (see also Belkacem et al. 2013). See also Corsaro et al. (2012), who adopt an exponential scaling as a function of \( T_\text{eff} \).

In this work, we have chosen to scale the width of the individual modes from solar values\textsuperscript{21} and use a temperature exponent of \( n = 7.5 \) (G. Houdek 2012, private communication), whereby \( \Gamma(\nu) \) is found according to

\[
\Gamma(\nu_{n\ell,*}) = \left( \frac{T_\text{eff}}{5777} \right)^{7.5} \times \tilde{\Gamma}_0(v_{n\ell,*}/v_{\text{max,*}}) . \tag{6.5}
\]

Here \( \tilde{\Gamma}_0 \) gives the solar values for the linewidth on a frequency scale of \( v_{n\ell,*}/v_{\text{max,*}} \). The asterisk subscript denotes the stellar values in the equation. Note that this equation is not the same as the linewidth relation given in, e.g., Appourchaux et al. (2012), which instead gives the mode linewidth at maximum mode height/amplitude as a function of effective temperature.

\textsuperscript{20} CONvection ROtation and planetary Transits.

\textsuperscript{21} From BiSON observations.

The exponent on the temperature dependence is in general a very important input parameter as it impacts the height and by extension the detectability of a given mode in the power spectrum (see Equation (6.8)). However, as we are here dealing with solar analogues having effective temperature comparable to the Sun, the difference in mode linewidths from different exponents of the temperature dependence is negligible. As an example, with the temperature of \( T_\text{eff} = 5825 \text{ K} \) for 16 Cyg B the difference in mode linewidth at \( \nu_{s,*} = \nu_{\text{max,*}} \) between using an exponent of \( n = 7.5 \) and one of \( n = 15.5 \) (Appourchaux et al. 2012) amounts to a difference in linewidth of \( \sim 0.073 \mu\text{Hz} \). Furthermore, the smoothing applied in the making of the \( \Delta C \)-spectrum ensures that any small differences in mode linewidth will be rendered unimportant.

In assuming equipartition of energy between the components of an \((n\ell m)\)-multiplet, the height \( S_{n\ell m} \) in Equation (6.4) can be written as

\[
S_{n\ell m} = \mathcal{E}_{\ell m}(i) S_n = \mathcal{E}_{\ell m}(i) \tilde{V}_\ell^2 \alpha_{n\ell} , \tag{6.6}
\]

where \( \mathcal{E}_{\ell m}(i) \) is the geometrical function given in Equation (3.5). \( \tilde{V}_\ell \) is the square of the so-called relative mode visibility, i.e., the squared amplitude (power) ratio between different \( \ell \)-components normalized to the radial modes—we calculate our own values for the visibilities in Section 6.3, as tabulated values in the literature seldom include calculations for \( \ell = 4 \) and \( \ell = 5 \) (see Christensen-Dalsgaard & Gough 1982, for calculations of visibilities for velocity measurements). The last factor in Equation (6.6) represents a frequency-dependent amplitude modulation. We describe this modulation by a Gaussian function, \( G(\nu) \), such as the one given in Equation (3.6):

\[
\alpha_{n\ell}(\nu) = H_{\text{max}} G(\nu) . \tag{6.7}
\]

In this formula, \( H_{\text{max}} \) is the maximum height of the radial \((\ell = 0)\) modes found at the corresponding frequency \( \nu_{\text{max}} \).

The height (in power density) is found as (Fletcher et al. 2006; Chaplin et al. 2008)

\[
H(T) = \frac{2A^2/\pi \Gamma}{\left[ 1 + (2/\pi \Gamma T)^2 \right]} . \tag{6.8}
\]

where \( A \) is the mode amplitude and \( T \) is the observing length.

The maximum amplitude \( A_{\text{max}} \) is for both models estimated from the power spectra of 16 Cyg A and B. This is done following the prescription given in Kjeldsen et al. (2008a), which is a revised treatment of the Kjeldsen & Bedding (1995) procedure (see also Michel et al. 2009; Mosser et al. 2012a). In brief, the power spectrum is first heavily smoothed to produce a single power bump. For the smoothing, we used a boxcar filter with a width of \( 4 \Delta \nu \), with estimated values for \( \Delta \nu \) of 103.4 \( \mu\text{Hz} \) (16 Cyg A) and 116.97 \( \mu\text{Hz} \) (16 Cyg B). The same smoothing is done on the previously fitted background function in Equation (4.1) (without Gaussian envelope). Power (ppm\textsuperscript{2}) is then converted to power spectral density (ppm\textsuperscript{2} \( \mu\text{Hz}^{-1} \)) by multiplying the power spectrum with the effective observation length (equivalent to the area under the spectral window). Now the smoothed background function is subtracted from the smoothed power spectrum, leaving ideally only the oscillation bump left. The maximum value of the power bump is now multiplied by \( \Delta \nu \), thereby giving the total power contained in all modes within one large separation. As we would like to determine the amplitude per oscillation mode at the peak in power (\( \nu_{\text{max}} \)), we divide by the sum \( \tilde{V}_\ell^2 \) of squared relative
visibilities \((c\text{ factor in Kjeldsen et al. 2008a})\) as this gives the effective number of modes per order. This can be written as

\[
\langle A_{\ell=0,\text{max}} \rangle = \frac{P_{\text{max}} \Delta v}{V_{\text{tot}}^2},
\]

where \(P_{\text{max}}\) gives the peak value of the smoothed power bump, in units of power density.

In the fit of Equation (4.1) to the power spectra we obtain direct values for \(P_{\text{max}}\) from the value of \(A\), and we tested that this value indeed corresponds to the value obtained from the above procedure. The fit also directly gives us the ingredients for Gaussian function \(G(v)\), specifically \(\sigma_v\) and \(v_{\text{max}}\).

The final limit spectrum is now constructed as the sum of Lorentzian functions from the individual modes:

\[
P(v_j) = \sum_{n=-\infty}^{\infty} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{E_{\ell n}(i) S_{\ell m}}{1 + (v_j - v_{\ell m})^2} + N(v_j).
\]

In this equation, \(N(v_j)\) gives the added noise term (see Section 6.4 below) comprising both the instrumental and stellar noise contributions.

### 6.3. Visibilities

The visibilities used in the simulations of the power spectra are naturally of great importance and deserve special attention.

We calculate theoretical visibilities following the method described in Ballot et al. (2011, hereinafter BL11). To test this method, we first calculate the solar visibilities and then compare to values measured directly from the power spectra of the Sun.

The visibility of a mode of degree \(\ell\) can be written as (Dziembowski 1977; Gizon & Solanki 2003)

\[
V_\ell = \sqrt{\pi(2\ell + 1)} \int_0^1 P_\ell^0(\mu) \frac{W(\mu) d\mu}{\mu}.
\]

In this equation \(P_\ell^0\) is the \(\ell\)th order Legendre polynomial, and \(\mu\) is a measure of the projected distance of a surface element to the stellar limb given by \(\mu = \cos \phi\), with \(\phi\) being the angle between the line of sight and the normal to the surface at the position of the element. Thus, \(\mu\) varies from 0 at the limb to 1 at the center. \(W(\mu)\) is linked to the limb-darkening (LD) function for the star, given by the relative intensity at a specific wavelength \(\lambda\) to the center, i.e., \(g_\lambda(\mu) = I_\lambda(\mu)/I_\lambda(1)\).

When the observation is performed over a wavelength band, \(W(\mu)\) can be approximated by the factor \(W_K(\mu)\) given by (see also Berthomieu & Provost 1990; Michel et al. 2009)

\[
W_K(\mu) = \frac{\int T_K(\lambda) T_{\text{eff}}(\lambda) g_\lambda(\mu) d\lambda}{\int T_K(\lambda) B(\lambda, T_{\text{eff}}) H_\lambda G_\lambda d\lambda},
\]

where

\[
G_\lambda = \int_0^1 g_\lambda(\mu) d\mu \quad \text{and} \quad H_\lambda = \left( \int_0^1 g_\lambda(\mu) d\mu \right)^{-1}.
\]

In Equation (6.12) \(B\) is the Planck function. \(T_K(\lambda)\) is the transfer function and is given by

\[
T_K(\lambda) = \mathcal{E}_K(\lambda)/E_\nu = \mathcal{E}_K(\lambda)\lambda/\hbar c,
\]

where \(\mathcal{E}_K(\lambda)\) is the spectral response of the detector with which the observations are obtained as a function of wavelength.
subtract the power contained under this fitted function in the frequency ranges used for the respective modes.

To get the squared visibilities \( \tilde{V}_2^2 \), we now interpolate the \( \ell = 0 \) estimated power values in frequency. The values obtained for other modes are now simply divided by the value of the \( \ell = 0 \) modes at the interpolated frequency of the mode. Finally, the median of the estimated single-mode visibilities for a given degree is adopted as the final visibility, with an error bar estimated by the standard deviation of these values around the median value.

The visibilities obtained from this approach are also given in Figure 13 and Table 2. As seen, the observations do not match the calculated values very convincingly, with the highest deviation seen for the \( \ell = 3 \) modes. Salabert et al. (2011) perform the same test for the Sun but estimated the visibilities by extracting the heights (Salabert et al. 2004) of all \( 2\ell + 1 \) components of multiplets up to \( \ell = 3 \). We also give their results in Figure 13 and note that our estimates are generally in line with these within the quoted errors. Clearly, the method of fitting the modes directly should give more accurate estimates of the visibilities, but at the cost of a much higher computational effort, especially if methods such as MCMC are used; we find that the simple method described above serves well the purpose of this analysis.

From the theoretical values plotted in Figure 13, it is evident that there are differences between the different SPM detectors—especially for the relative difference between \( \tilde{V}_2^2 \) and \( \tilde{V}_3^2 \) in the red band as compared to the blue and green bands. The fact that there are differences between the filters is confirmed in the data; see Figure 14. However, we see much smaller relative differences for instance \( \tilde{V}_2^2 \) and \( \tilde{V}_3^2 \), and the largest relative difference is in fact seen between \( \tilde{V}_2^2 \) and \( \tilde{V}_4^2 \). In Figure 14, we compare the three filters in a very simple manner by dividing one of the central orders in the 1.8 \( \mu \)Hz boxcar smoothed power spectrum by the peak in power of the \( \ell = 3 \) mode. For the \( \ell = 0 \) mode, we simply get the inverse of the relative squared \( \ell = 3 \) visibility, i.e., \( \tilde{V}_0^2 / \tilde{V}_3^2 = 1 / \tilde{V}_3^2 \). The comparison can, however, be made for the \( \ell = 1 \) and 2 modes. As seen, the greatest

| Star        | \( \tilde{V}_{\text{fit}}^2 \) | \( \tilde{V}_1^2 \) | \( \tilde{V}_2^2 \) | \( \tilde{V}_3^2 \) | \( \tilde{V}_4^2 \) | \( \tilde{V}_5^2 \) |
|-------------|-------------------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| Theoretical Values |
| 16 Cyg A    | 3.04                          | 1.50               | 5.13E-1            | 2.17E-2            | 6.48E-3            | 1.23E-3            |
| 16 Cyg B    | 3.04                          | 1.50               | 5.15E-1            | 2.22E-2            | 6.36E-3            | 1.24E-3            |
| Sun (red)   | 2.92                          | 1.45               | 4.50E-1            | 1.08E-2            | 9.12E-3            | 6.71E-4            |
| Sun (green) | 3.15                          | 1.54               | 5.70E-1            | 3.45E-2            | 4.25E-3            | 1.57E-3            |
| Sun (blue)  | 3.33                          | 1.60               | 6.63E-1            | 6.13E-2            | 1.43E-3            | 2.08E-3            |
| Estimated Values |
| 16 Cyg A    | 3.34 ± 0.38                   | 1.53 ± 0.25        | 0.69 ± 0.11        | 0.11 ± 0.03        | –                  | –                  |
| 16 Cyg B    | 3.42 ± 0.21                   | 1.59 ± 0.10        | 0.72 ± 0.09        | 0.10 ± 0.02        | –                  | –                  |
| Sun (red)   | 2.92 ± 0.07                   | 1.46 ± 0.03        | 0.49 ± 0.02        | 0.05 ± 0.02        | 0.01 ± 0.003       | –                  |
| Sun (green) | 3.15 ± 0.08                   | 1.56 ± 0.03        | 0.63 ± 0.03        | 0.08 ± 0.02        | 0.004 ± 0.008      | –                  |
| Sun (blue)  | 3.33 ± 0.07                   | 1.60 ± 0.02        | 0.70 ± 0.02        | 0.11 ± 0.02        | 0.005 ± 0.007      | –                  |
difference is evident for the \( \ell = 1 \) modes, while the \( \ell = 2 \) modes show little difference—unlike what is expected from theory.

This mismatch in visibilities is likely caused, in part at least, by an erroneous treatment of the LD close to the limb of the star. The \( \ell = 3 \) modes are relatively more sensitive to the LD, as compared to, e.g., \( \ell = 1 \) modes, due to the fact that the symmetry of the spherical harmonic function of \( \ell = 3 \) modes results in total cancellation in the absence of LD. To test if a small change in the solar LD function can enable a sufficiently high value of the \( \ell = 3 \) modes, we add an exponent \( \alpha \) on the solar LD function in Equation (6.16), i.e., we replace \( g_3(\mu) \) by \( g_3(\mu)^\alpha \) in Equations (6.12) and (6.13), and increase \( \alpha \) until a value of \( \tilde{V}_\ell^2 = 0.1 \) is reached. To obtain this increase, values of \( \alpha = 3.1 \) (red band), \( \alpha = 1.7 \) (green band), and \( \alpha = 1.35 \) (blue band) were needed. The result of this procedure is shown in Figure 15 for the case of the green band. In all cases, the shape of the LD function changes greatly in appearance, ending in a nearly straight line for high values of \( \alpha \). Furthermore, the change results in much too high values of the squared relative visibilities for these might be significantly underestimated when following the approach above.

As for the Sun, we tested the impact of the LD law, here by comparing the visibilities obtained by using different laws, namely, a linear law, a quadratic law, and a three-parameter law. See Figure 16 for this procedure applied to 16 Cyg B. For all of these we used the LD parameters from Sing (2010) and found that only the linear LD law deviated significantly from the others. So, even though the quadratic and nonlinear LD laws differ near the limb, they still give very similar results for the visibilities.

In Figure 17, we show both the theoretically computed values using the four-parameter LD law and the values obtained from the power spectra of 16 Cyg A and B. As we have no good knowledge of the mode linewidths, we here simply summed power in a range of \([\ell \nu_s + 2.5 \mu \text{Hz}]\) and using \( \nu_s = 0.37 \mu \text{Hz} \). From Figure 17, it is clear that, as for the Sun, the theoretical predictions do not agree well with the observations. Again the largest relative deviation is seen for \( \ell = 3 \) modes, but also for \( \ell = 2 \) modes a non-negligible deviation is seen. We have also illustrated the values that would be obtained for the two stars from the tabulated values in BL11.

Deheuvels et al. (2010) also found deviations between measured and theoretically predicted values in the analysis of the solar-like CoRoT target HD 49385, where visibilities were estimated in the same manner as for the Sun in Salabert et al. (2011). We have included the estimated values for this star in Figure 17, and also show the theoretical values obtained when using the four-parameter LD law together with the CoRoT calibrated LD parameters in Sing (2010). Mathur et al. (2013) recently analyzed the binary system HD 169392, also observed.
Figure 16. Appearance of different LD laws for 16 Cyg B (left). As seen, the greatest difference between the different laws is in general found near the stellar limb ($\mu = 0$). For each LD law, the computed squared relative visibilities are given (right) for $\ell$-values up to $\ell = 5$.

Figure 17. Visibilities up to $\ell = 3$ for 16 Cyg A, 16 Cyg B, HD 169392A (Mathur et al. 2013), and HD 49385 (Deheuvels et al. 2010). Illustrated are values estimated from the power spectrum with their corresponding error bars, theoretical values from BL11, and theoretical values obtained from using the four-parameter LD law, with parameter values from Sing (2010).

(A color version of this figure is available in the online journal.)

with CoRoT, and were able to estimate the mode visibilities for the A-component of the system. These values have also been included in Figure 17, and again accompanied by theoretical predictions. Both of these stars are very similar to 16 Cyg A and B in terms of effective temperature and metallicity but are likely a bit more evolved having slightly lower surface gravities. The similarities can also be seen in the visibilities predicted from theory. For both these stars, the same trend is observed in the deviations as for 16 Cyg A and B.

Unfortunately, the trend in the deviation is not smooth, and we are therefore not in a position to estimate the expected deviation for the $\ell = 4$ modes from simple extrapolation of the deviation. Even though an estimate of the $\ell = 4$ visibilities was obtained for the Sun (see Figure 13), the error estimates on these values make them unfit for any inference on the trend in the deviations. We are furthermore not convinced of the validity of these estimates as the $\ell = 4$ modes are very close to the background noise level. Because of this lack of predictive power, we err on the side of caution and choose for the $\ell = 4$ and $\ell = 5$ visibilities the theoretically predicted values, keeping in mind that these are likely underestimated. For the lower-degree modes, we adopt the values obtained from the power spectrum.

6.4. Noise Properties

The noise level in our simulations is of course of great importance as it, given a certain limit spectrum, sets the signal-to-noise level in the power spectrum, which ultimately determines if the signal of a given mode will stand out from the noise.

For the noise in the synthetic spectrum, we first tested the prescription by Gilliland et al. (2010b; see also Chaplin et al. 2011; Gilliland et al. 2011) for the instrumental “shot” noise in Kepler SC observations as a function of Kepler magnitude $K_p$, with the noise level in the amplitude spectrum given as

$$\sigma_{\text{amp}} = \frac{10^6}{cN^{1/2}} \left( c + 9.5 \times 10^5 (14/K_p)^5 \right)^{1/2} \text{ ppm}$$

$$c = 1.28 \times 10^{0.4(12-K_p)+7},$$

(6.17)

with $N$ being the number of data points in the time series. This will result in a noise level in the power spectrum of (Kjeldsen & Bedding 1995)

$$N_{\text{instr}} = \frac{4\sigma_{\text{amp}}^2}{\pi} \text{ ppm}^2.$$  

(6.18)

When comparing the noise obtained from this description with the value estimate from Equation (4.1), we found that Equation (6.17) underestimated the shot noise by a factor $\sim 8$. It should be noted that Equation (6.17) is only intended to give a minimal noise term. Furthermore, as 16 Cyg A and B are both highly saturated targets with flux collected from a large custom aperture, it is not guaranteed that Equation (6.17) is at all applicable. For this reason, we have chosen to use the noise estimated from Equation (4.1), which we scale to the considered observing length $T_{\text{obs}}$ by dividing by the factor $\sqrt{T_{\text{obs}}/6437}$. For the background component, we add the signal extracted for the two stars in fitting Equation (4.1) to the power spectra.
In calculating the realization noise in the power spectrum, we follow the method given in Anderson et al. (1990) and Gizon & Solanki (2003) where the Box–Muller transform is used, and a realization of the power spectrum is given as

\[ P(v_j) = -\ln(U_j) \mathcal{P}(v_j). \] (6.19)

Here \( U_j \) is a uniform distribution on \([0, 1]\) and \( \mathcal{P}(v_j; \Theta) \) is the limit spectrum given in Equation (6.10). This approach ensures a power spectrum obeying the generally assumed \( \chi^2 \) 2 dof (degrees-of-freedom) statistic (Woodard 1984).

7. RESULTS ON DETECTABILITY FROM SIMULATED DATA

Because of the high similarity of the two stars and their power spectra, we chose to only simulate the power spectrum of 16 Cyg B. In Figure 18, an example of a simulated power spectrum can be seen, computed following the description of Section 6.2. The top panel shows the simulation when using an inclination of \( i = 50^\circ \) and a frequency resolution corresponding to the length of the observed data, i.e., 643 days. Also shown is the 1 \( \mu \)Hz smoothed version, and as seen the two spectra are very similar. The non-smoothed spectra naturally look somewhat different due to the \( \chi^2 \)-noise. A likely contribution to the difference between the simulation and the observation comes from the inclination angle, which might be different from \( i = 50^\circ \).

From the simulated power spectra, we are in a position to test if the \( \ell = 4 \) modes are likely to be found in Kepler data. To address this question, we made a Monte Carlo (MC) set of simulated power spectra with a frequency resolution corresponding to an observing length of 643 days.

For an inclination angle of \( i = 50^\circ \), we simulated 100 power spectra that included both \( \ell = 4 \) and \( \ell = 5 \) modes in addition to 100 power spectra including only degrees up to \( \ell = 3 \). For all power spectra, we computed the \( SC \)-spectrum with smoothing levels from 1–5 \( \mu \)Hz and found for each smoothing level the mean \( SC \)-spectrum from the 100 simulations. The result of this can be seen in the left panel of Figure 19. From this MC set, it is found that the power spectra including \( \ell = 4 \) and \( \ell = 5 \) modes in mean have a noticeable excess power in the \( SC \)-spectrum at the predicted position from the straightening when compared to power spectra not including these modes. The choice of optimum smoothing level is again difficult to estimate as the smoothing in general smears out the signal over a larger frequency range while at the same time reducing the noise. We note than an excess is seen for all included inclination angles and all smoothing levels.

To quantify the detectability of the \( \ell = 4 \) modes further, we simulated 2000 power spectra for each of the inclinations \( i = 10^\circ, 30^\circ, 50^\circ, 70^\circ, 90^\circ \). Half of these included modes of degree \( \ell = 0–5 \), while half only included modes of degree \( \ell = 0–3 \). For all of these simulated power spectra, we computed the collapsed spectrum and chose a single smoothing level of 2 \( \mu \)Hz. This smoothing level will encompass all \( m \)-components of \( \ell = 4 \) assuming that the modes are split by less than 0.5 \( \mu \)Hz.

The maximum value was then found in each \( SC \)-spectrum in a \( \pm 2 \mu \)Hz window around the expected position of the collapsed \( \ell = 4 \) power. Two distributions for this maximum value were obtained for each of the inclinations used—one for the \( SC \)-spectra including only \( \ell = 0–3 \) modes and one for the \( SC \)-spectra that include \( \ell = 0–5 \). Before the maximum values were found, we divided the individual \( SC \)-spectra with the ratio of their median value to the median of the \( SC \)-spectrum of 16 Cyg B (Figure 11). This is done such that the maximum value found in the \( SC \)-spectrum of 16 Cyg B can be more readily compared to the distributions of maximum values from the simulations. The mean kernel density estimations (KDEs) from the different inclination angles are given by the black (\( \ell = 0–3 \)) and red (\( \ell = 0–5 \)) curves in the right panel of Figure 19. The gray region around each mean curve gives the range for the KDEs of the different inclinations used. With the obtained KDE, we can now better test the significance of the signal seen for 16 Cyg B and estimate how often one would be in a position to see such a signal. We take as our null hypothesis (\( H_0 \)) that only noise is present and thus that the black KDE holds for our maximum value. The \( H_0 \) hypothesis will only be rejected in favor of the alternative \( H_1 \) hypothesis (not only noise is present) if the \( p \)-value of a given observed maximum value falls below a given significance level \( \alpha \). The lower the value of \( \alpha \), the less likely one is to make so-called Type I errors where \( H_0 \) is erroneously rejected in favor of \( H_1 \). The significance level \( \alpha \) is often set to 0.05 (5% chance of Type I errors), giving an observation “significant at the 5% level” if \( H_0 \) can be rejected. The sparsely hatched region below the black curve in Figure 19 indicates the region where \( H_0 \) can be rejected in favor of \( H_1 \) at the 5% level. Assuming now that \( H_1 \) is true and that there is additional power present, we can find the probability of correctly rejecting \( H_0 \) in favor of \( H_1 \) at the 5% level. This probability is given by the combined area of the densely and sparsely hatched regions under the red curve. From this, we find that in the case where additional power (here from \( \ell = 4 \) modes) is present, there is a \( \sim 21\% \) chance of correctly rejecting \( H_0 \) in favor of \( H_1 \) at the 5% level. Correspondingly, there is a \( \sim 79\% \) chance that \( H_0 \) will not be rejected even though \( H_1 \) is true (a Type II error), and one will not be able to claim a significant detection. The black vertical dashed line gives the value obtained from 16 Cyg B.

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22 Using Silverman’s rule of thumb (Silverman 1986) for determining the kernel bandwidth.

23 The probability of obtaining a result equal to or more extreme than what was actually observed under the assumption of the null hypothesis.
Figure 19. Left: SC-spectra for an MC set of simulated power spectra, here with an adopted inclination of $i = 50^\circ$. Two hundred power spectra were simulated, and half of these only included modes of degree $\ell = 0$–3, while half included modes of degree $\ell = 0$–5. The five sets of curves correspond to five smoothing levels applied, going from $1 \mu$Hz at the bottom to $5 \mu$Hz at the top. The dashed cyan line shows the position of the straightened $\ell = 4$ ridge, as in Figure 10, and hence the expected position of a possible excess from the $\ell = 4$ modes. From the simulated power spectra the mean SC-spectrum was found for each smoothing level, given by the solid curves, in addition to the mean absolute deviation from this curve, given by the gray regions around the respective mean curves. For each set, the bottom curve is obtained from the simulated data only including modes of degree $\ell = 0$–3, while the top curve is for data including $\ell = 0$–5 modes. For clarity offsets have been added to separate the curves. Right: distribution (kernel density) for the maximum value in a $2 \mu$Hz smoothed collapsed simulated power spectra. The black curve gives the mean distribution from 5000 spectra with inclinations of $i = 10^\circ$, $30^\circ$, $50^\circ$, $70^\circ$, $90^\circ$ (1000 spectra for each) when including modes of degree $\ell = 0$–3, while modes of degree $\ell = 0$–5 were included for the red curve. The gray regions around these mean curves give the range in the individual distributions from the different values of the inclination. The sparsely hatched region gives the $>95\%$ area under the black curve, and thus the region where the null hypothesis ($H_0$) can be rejected. The densely hatched region gives the corresponding region under the red curve and indicates how often one will be in a position to correctly reject $H_0$. The dashed black line gives the value measured for 16 Cyg B.

(A color version of this figure is available in the online journal.)

when following the same procedure as for the simulated SC-spectra. Under the assumption that our simulations indeed give an accurate description of the observations for 16 Cyg B, we can from the maximum value of 16 Cyg B reject $H_0$ at the $5\%$ level. In fact, the maximum value comes, with a p-value of 0.01041, very close to the $1\%$ significance level. However, we note that such a direct comparison between observations and simulations should be done very cautiously, as we indeed have a rather poor handle on some of the important input parameters such as relative visibilities and mode linewidths. Also, we note that the maximum value for 16 Cyg B is in the high end of what would be expected from the simulations with a p-value of 0.14 with respect to the red curve. This could possibly indicate that the value used for the relative visibility of $\ell = 4$ is underestimated.

We can complement the above estimate of the significance of the detection (or rather the rejection of $H_0$) in a Bayesian manner using Bayes’s theorem to calculate the posterior probability. A proper Bayesian analysis, using, e.g., MCMC or Multi-Nest (Feroz et al. 2009) to approximate the posterior, would require assumed priors for the visibilities and the parameters entering Equation (6.10). Assuming that these parameters are known and correctly set in the models, the posterior probability for the hypothesis $H_i$ given the measured peak value of the SC-spectrum, $x$, can be readily estimated from the MC simulations. For $H_0$, and equivalently for $H_1$, the posterior is given as (see, e.g., Berger & Sellke 1987; Cowan 1998; Appourchaux et al. 2009; Broomhall et al. 2010)

$$P(H_0|x) = \frac{\pi_0 P(x|H_0)}{\pi_0 P(x|H_0) + \pi_1 P(x|H_1)},$$

(7.1)

where $\pi_i$ is the prior probability that a given hypothesis $H_i$ is true, while $P(x|H_i)$ (the likelihood) is the probability of observing the data obtained under the assumption of the hypothesis $H_i$. Assuming no prior preference for $H_0$ over $H_1$ or vice versa (i.e., $\pi_0/\pi_1 = 1$), the posterior probabilities, i.e., the probabilities in favor of a given hypothesis after actually making an observation, then give $P(H_0|x) \approx 14\%$ and $P(H_1|x) \approx 86\%$. We can quantify the meaning of these values further by the posterior odds ratio in favor of $H_0$ against $H_1$ given the measured value $x$ as

$$O_{0,1} = \frac{P(H_0|x)}{P(H_1|x)} = \frac{\pi_0 P(x|H_0)}{\pi_1 P(x|H_1)} = \frac{\pi_0}{\pi_1} B_{0,1}.$$  

(7.2)

Here $B_{0,1}$ is the so-called Bayes factor. Assuming again no prior preference for $H_0$ over $H_1$, the posterior odds ratio is simply given by the Bayes factor. From the simulated distributions and the observed peak value of the SC-spectrum ($x$), we obtain a posterior odds ratio in favor of $H_1$ against $H_0$ of $O_{1,0} = 6.2$ (note that $O_{1,0} = 1/O_{0,1}$). The evidence for $H_1$ over $H_0$ can be judged from the obtained odds ratio using the Jeffreys scale (Jeffreys 1961). According to this scale, a value for $O_{1,0}$ between 3 and 10 can be interpreted as “substantial” evidence in favor of $H_1$ over $H_0$. An odds ratio between 10 and 30 is needed for a “strong” evidence in favor of $H_1$ over $H_0$, while a ratio $>100$ is needed for “decisive” evidence.

In the top panel of Figure 20 we give a subset of the SC-spectra from the simulated 16 Cyg B data, with a frequency resolution corresponding to an observing length of 643 days and using the same range in frequency in the collapsing of the power spectra as for the real data. The different panels of this figure show the impact of a change in the adopted stellar inclination angle, where we have chosen a minimal set of six inclination angles: $i = 0^\circ$, $10^\circ$, $30^\circ$, $50^\circ$, $70^\circ$, $90^\circ$. Again the dashed cyan lines give the position of the straightened $\ell = 4$ ridge. Comparing these SC-spectra with the mean profiles from the MC set, it is clear that the noise realization has a rather large impact.
In the bottom panel of Figure 20, we give the SC-spectra as in the top panel, but here using a frequency resolution corresponding to an observing length twice the current length, i.e., $\sim 1286$ days. As the noise has only been reduced by a factor $\sqrt{2}$ in doubling the observing length there is still much variation in the SC-spectra as compared to Figure 19. In Figure 21, we used a simulated observing length four times the current observing length, i.e., $\sim 2572$ days, whereby the shot noise is reduced by a factor two. Note that this corresponds roughly to the amount of data that would have been available had the Kepler mission continued uninterrupted until its eighth year of operation. However, in light of the loss of a second reaction wheel, needed for the hitherto fine-pointing stability of the spacecraft (Koch et al. 1996), and the planned continuation of the mission (dubbed “K2”), this will not be possible. In the top panel, we show the SC-spectrum targeted at the $\ell = 4$ modes. The signal from these $\ell = 4$ modes here stands out very clearly and should be readily observable in the power spectrum. In the bottom panel, we targeted the SC-spectrum to the $\ell = 5$ modes. Here small indications of excess are seen from the $\ell = 5$ modes and a detection could be possible, but still the $\ell = 0$ modes overshadow the signal.

The impact of the noise realization hinted above is further illustrated in Figure 22, where the SC-spectra from six realizations of simulated data having an observation length of 643 days and in inclination of $i = 50^\circ$ are shown. The noise realization has a non-negligible impact on the SC-spectrum, where some cases (e.g., the bottom-right panel) show an excess comparable to what is seen in 16 Cyg A and B, and then again some show no signs of an excess (e.g., the middle-left panel). It can
also be seen that the noise in some cases aids in the making of rather spiked features; see, e.g., the middle-left panel at around 65 \( \mu \)Hz. However, the fact that the peak is very narrow speaks against the origin being that of \( \ell = 4 \) modes. The reason for this is first of all that the straightening of modes is not perfect; there will still be small deviation and consequently a small broadening in the SC-spectrum. Secondly, the \( \ell = 4 \) modes will in most cases have some of the power placed in \( m \)-components that lie away from the central \( m = 0 \) component; this too will result in a broadening of the \( \ell = 4 \) excess in the SC-spectrum.

8. INFERENCE FROM SEGMENTED SOLAR DATA

To further test the validity of the observed signal for 16 Cyg A and B, we investigate how the signal from \( \ell = 4 \) modes is seen in the solar data. The solar time series from the blue SPM filter was first divided into segments of 643 days length, resulting in separate small time series. To each of these, we added normally distributed noise by an amount that results in a shot noise level in the power spectrum equal to the level observed in 16 Cyg B. For the six power spectra computed, we calculate the corresponding SC-spectrum, here using the fit to the Model S mode shown in the left panel of Figure 7. In the collapsing of the spectrum, we used the central 12 modes from about 2400 \( \mu \)Hz to 4022 \( \mu \)Hz. In the left panel of Figure 23 the six SC-spectra are shown after applying a boxcar smoothing of 4 \( \mu \)Hz, with the vertical dashed line giving the expected position of the \( \ell = 4 \) ridge. As seen, the mean level of the SC-spectrum increases with time. The reason for this increase in the mean level can be seen in the right panel of Figure 22, which shows one of the central orders in the power spectrum. Here the relatively large variation in the power of the modes along with clear frequency shifts are seen, owing to
both the stochastic nature of the excitation and the solar activity cycle (see, e.g., Gelly et al. 2002). In the SC-spectra, we see in all cases an excess from the $\ell = 4$ modes comparable in width and general appearance to the possible signal seen in 16 Cyg A and B. Furthermore, the amount of excess is seen to follow the increase in the mean level of the spectra. The relatively strong signal seen at the position of $\ell = 5$ is greatly dominated by the instrumental peak described in Section 2.

9. DISCUSSION

We have found clear evidence for $\ell = 4$ and $\ell = 5$ modes in the Sun from all VIRGO-SPM filters using a time series of 12 yr. Furthermore, we find indications, albeit no conclusive proof, for the $\ell = 4$ modes in the Kepler data of the solar analogues 16 Cyg A and B. The credibility of our findings is supported by our simulations, in which a qualitatively similar signal from the $\ell = 4$ modes is found using a simulation length equal to the length of analyzed Kepler data. Under the assumption that our simulations accurately describe the observed SC-spectrum of 16 Cyg B, we can reject the null hypothesis at the 5% significance level. Furthermore, we obtain a posterior odds ratio of $O_{1,0} = 6.2$, judged as “substantial” evidence in favor of $H_1$ over $H_0$. Both tests are in favor of a detection of additional power from $\ell = 5$ modes. From our simulations we also find that if additional power is indeed present there will only be a $\sim 21\%$ chance of actually being able to reject $H_0$ at the 5% significance level. Our simulations further suggest that in any case, a solid detection of the modes should be possible with four times the amount of data currently available. Also, we see that in using
only subsets of the solar data with a length equal to the time series of 16 Cyg A and B, we are still able to see an excess signal from the $\ell = 4$ modes. We find at this time no indications for the $\ell = 5$ modes in the Kepler data, and a solid detection of these modes for 16 Cyg A and B will be very difficult even with very long time series, mainly due to the strength of the $\ell = 0$ modes. A detection of these modes might have been possible at the end of the nominal length of the extended Kepler mission if continued observations could have been made.

The validity of our simulations is of course conditioned by the ingredients used being correct. The major uncertainty in the simulations concerns the mode visibilities, where a discrepancy was found between values estimated from the power spectra and from theory. This issue clearly deserves some extra attention, not only due to the fact that it affects the reliability of our simulations, especially for the $\ell = 4$ and $\ell = 5$ modes where direct measurements are very difficult even for the Sun, but also because these values are most often fixed a priori in the process of peak-bagging, which clearly will affect the validity of the returned results if these are in fact fixed to wrong values.

Another less important aspect is the uncertainty of the temperature dependence on the mode linewidths, which is currently a property with little consensus among different groups. However, we see the scaling of linewidths from the solar values as solid due to the similarity of 16 Cyg A and B to the Sun.

The shot noise added in the simulations is set from data with an observing length of 643 days; we note that there is no guarantee that the instrumental noise will remain at the same level if the Kepler mission progresses. Additionally, colored noise sources, other than the fitted stellar noise, were neglected in the simulations.

We have shown the ability of the introduced method, i.e., the $SC$-spectrum, to modify the power spectrum in a way that should increase the signal in the collapsed spectrum. It is clear that the gain from using the $SC$-spectrum depends mainly on the amount of deviation from the general asymptotic description given by Equation (3.1). Nonetheless, the simplicity of the method and the fact that it provides a well-defined position in frequency in the collapsed spectrum for the potential excess power makes it worthwhile to implement and use the $SC$-spectrum.

The method will be applied again once more data become available, to test the validity of both our findings and the simulations. Here possibly also the inclinations and rotational splittings of the 16 Cyg A and B stars will be better constrained, and models better fitting the low degree modes are likely available.

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