First study of the gluon-quark-antiquark static potential in SU(3) Lattice QCD

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We study the long distance interaction for hybrid hadrons, with a static gluon, a quark and an antiquark with lattice QCD techniques. A Wilson loop adequate to the static hybrid three-body system is developed and, using a $24^3 \times 48$ periodic lattice with $\beta = 6.2$ and $a \sim 0.075$ fm, two different geometries for the gluon-quark segment and the gluon-antiquark segment are investigated. When these segments are perpendicular, the static potential is compatible with confinement realized with a pair of fundamental strings, one linking the gluon to the quark and another linking the same gluon to the antiquark. When the segments are parallel and superposed, the total string tension is larger and agrees with the Casimir Scaling measured by Bali. This can be interpreted with a type-II superconductor analogy for the confinement in QCD, with repulsion of the fundamental strings and with the string tension of the first topological excitation of the string (the adjoint string) larger than the double of the fundamental string tension.

I. INTRODUCTION

Here we explore the static potential of the hybrid three-body system composed of a gluon, a quark and an antiquark using lattice QCD methods. The Wilson loop method was devised to extract from pure-gauge QCD the static potential for constituent quarks and to provide detailed information on the confinement in QCD. In what concerns gluon interactions, the first lattice studies were performed by Michael [1, 2] and Bali extended them to other SU(3) representations [3]. Recently Okiharu and colleagues [4, 5] extended the Wilson loop for tree-quark baryons to tetaquarks and to pentaquarks. Our study of hybrids continues the lattice QCD mapping of the static potentials for exotic hadrons.

The interest in hybrid three-body gluon-quark-antiquark systems is increasing in anticipation to the future experiments BESIII at IHEP in Beijin, GLUEX at JLab and PANDA at GSI in Darmstadt, dedicated to study the mass range of the charmonium, with a focus in its plausible hybrid excitations. Moreover, several evidences of a gluon effective mass of 600-1000 MeV from the Lattice QCD gluon propagator in Landau gauge, [6, 7], from Schwinger-Dyson and Bogoliubov-Valatin solutions for the gluon propagator in Landau gauge [8], from the analogy of confinement in QCD to superconductivity [9], from the lattice QCD breaking of the adjoint string [10], from the lattice QCD gluonic excitations of the fundamental string [11], from constituent gluon models [11, 12, 13] compatible with the lattice QCD glueball spectra [14, 15, 16, 17], and with the Pomeron trajectory for high energy scattering [18, 19] may be suggesting that the static interaction for gluons is relevant.

Importantly, an open question has been residing in the potential for hybrid system, where the gluon is a colour octet, and where the quark and antiquark are combined to produce a second colour octet. While the constituent quark (antiquark) is usually assumed to couple to a fundamental string, in constituent gluon models the constituent gluon is usually assumed to couple to an adjoint string. Notice that in lattice QCD, using the adjoint representation of SU(3), Bali [3] found that the adjoint string is compatible with the Casimir scaling, were the Casimir invariant $\lambda_i \cdot \lambda_j$ produces a factor of $9/4$ from the $qq$ interaction to the $gg$ interaction. Thus we already know that the string tension, or energy per unit lenght, of the adjoint string is 1.125 times larger than the sum of the string tension of two fundamental strings. How can these two pictures, of one adjoint string and of two fundamental strings, with different total string tensions, match? This question is also related to the superconductivity model for confinement, is QCD similar to a Type-I or Type-II superconductor? Notice that in type Type-II superconductors the flux tubes repel each other while in Type-I superconductors they attract each other and tend to fuse in excited vortices. This is sketched in Fig. 1. The understanding of the hybrid potential will answer these questions.

In Section II we produce a Wilson Loop adequate to study the static hybrid potential. In Section III we present the results of our Monte-Carlo simulation, in a $24^3 \times 48$ pure gauge lattice for $\beta = 6.2$, corresponding to a lattice size of $(1.74 \text{ fm})^3 \times (3.48 \text{ fm})$, assuming a string tension $\sqrt{\sigma} = 440$ MeV. In Section IV we interpret the
results and conclude.

II. HYBRID WILSON LOOP

In principle any Wilson loop with a geometry similar to the one in Fig. 2 and describing correctly the quantum numbers of the hybrid is adequate, although the signal to noise ratio may depend in the choice of the Wilson loop. A correct Wilson loop must include a SU(3) octet, the gluon, a SU(3) triplet, the quark and a SU(3) anti-triplet, the antiquark. It must also include the connection between the three links of the gluon, the quark and the antiquark.

In the limit of infinite quark mass, a nonrelativistic potential $V$ can be derived from the large time behaviour of euclidean time propagators. Typically, one has a meson potential $V$ between the three links of the gluon, the quark and the antiquark. It must also include the connection to the one in Fig. 2 and describing correctly the quantum potentials, respectively.

The potentials, respectively, come that when the potentials parallel to the antiquark-gluon segment. We denote to study the glue lump are the real 8 matrix SU(3) representation. If one compares the Wilson loop in eq. (4) with eq. (6), it comes that when $t$ corresponds to a single lattice spacing, then the gluonic time-like links used by Michael and colleagues \cite{22} to study the glue lump are the real $8 \times 8$ matrices, $U_{\alpha}^{\lambda}$, whereas in the investigation of Casimir scaling by Bali \cite{24}, the author worked directly with adjoint links, i.e. with the $8 \times 8$ matrix SU(3) representation. If one now compares the Wilson loop in eq. (4) with eq. (6), it is a “Michael link”.

III. THE STATIC HYBRID POTENTIAL

In this paper we consider two possible hybrid geometries: $\perp$ with the quark-gluon segment perpendicular to the antiquark-gluon segment; $\parallel$ with the quark-gluon segment parallel to the antiquark-gluon segment. We denote the potentials, respectively, $V_{\perp}(r_1, r_2)$ and $V_{\parallel}(r_1, r_2)$, where $r_1$ ($r_2$) is the quark-gluon (antiquark-gluon) distance in lattice units, defined in eq. (4).

Here we discuss the results of our simulation with 142 $24^4 \times 48$, $\beta = 6.2$ pure-gauge Wilson action SU(3) configurations. The configurations are generated with the version 6 of the MILC code \cite{21}, via a combination of Cabbibo-Mariani and overrelaxed updates. In order to improve the signal to noise ratio, the links are replaced by “fat links” \cite{22}

\[
U_{\mu}(s) \rightarrow \frac{1}{1 + 6w} \left( U_{\mu}(s) + w \sum_{\mu \neq \nu} U_{\nu}(s)U_{\mu}(s + \nu)U_{\nu}^\dagger(s + \mu) \right)
\]
followed by a projection into SU(3). We use $w = 0.2$ and iterate this ”fat link” 25 times both in the spatial direction and in the temporal one. The temporal smearing reduces the short-range Coulomb potential but produces a clearer signal for the long-range potential, the one we are interested in. Furthermore, to improve the quality of the signal, we explore the symmetry $r_1 \leftrightarrow r_2$ when computing $V_\perp (r_1, r_2)$ and $V_\parallel (r_1, r_2)$.

Using eq. (1), the static potentials are extracted from the fit of minus the log of the Wilson loop, $-\log W$, for large euclidian time $t$. This fit provides us with the potential, and we estimate the respective error bar with the jackknife method.

We are essentially interested in the largest possible distances, to compare the different possible string tensions. With $24^3 \times 48$ periodic lattices, the maximum distance we reach is $10a$. In this way we avoid possible finite volume effects. At $12a$, due to our periodic boundary conditions, we already could approach a maximum, deviating from the linear one. Notice that we calibrate our lattice spacing $a \sim 0.075$ fm as in Bali and Schilling [23]. Thus our maximal distance is still comfortably shorter than the string breaking distance, larger than 1 fm, and comfortably longer than the perturbative distance of say, 0.3 fm. We also start measuring the potentials at the distance of $6a$ because we are interested in studying the long-distance, non-perturbative part of the static potentials. Our results for the static hybrid potentials $V_\perp$ and $V_\parallel$ are displayed in Fig. 3.

Again, to get the string tensions $\sigma$, we fit the large distance part of the potentials with a linear potential, with the same method we used for the temporal fit to extract the static potentials. We admitting a singlet quark-antiquark singlet string tension of $440$ MeV, which corresponds to an inverse lattice spacing of $a^{-1} = 2718 \pm 32$ MeV, according to Bali and Schilling [22].

To study the onset of two fundamental strings, we plot in Fig. 4 the perpendicular geometry potential $V_\perp$ as a function of the sum of the two distances in lattice spacing units, $r_1$ between the quark and the gluon and $r_2$ between the antiquark and the gluon, as in eq. (1). Indeed the potential is linear in the sum of the distances. We further fit the potential to $V_\perp(r_1, r_2) = c_0 + \sigma(r_1 + r_2)$, and get

$$\sqrt{\sigma} = 441 \pm 6 \text{ MeV} = (1.00 \pm 0.01)\sqrt{\sigma_0}$$  (8)

($\chi^2/dof = 1.34802$) which is consistent with $\sigma = \sigma_0$, reinforcing the picture that, at long distances we have two fundamental strings one linking the quark and the gluon and, the other, linking the antiquark to the gluon.

To study if the double fundamental string picture can also be compatible with the Casimir scaling result found by Bali [3], we also consider the case where the quark and antiquark are superposed. In this case the static quark and antiquark are equivalent to a static gluon, and therefore our potential is equivalent to a static gluon-gluon potential. This is the case of the parallel geometry potential $V_\parallel$ when the two distances $r_1$ between the quark and the gluon and $r_2$ between the antiquark and the gluon, as in eq. (1), are identical, $r_1 = r_2$. This is plotted in Fig. 4 and indeed we find a linear behaviour. We further fit the
the fundamental strings and with the string tension of the first topological excitation of the string (the adjoint string) larger than the double of the fundamental string tension. Nevertheless, because the energy of two fundamental strings plus the repulsive energy measured here is quite similar to the energy of the adjoint string measured by Bali, this shows that the pure gauge QCD is similar to a Type-II superconductor quite close to the phase transition to a Type-I superconductor.

Our results are important for constituent models for hybrids and glueballs. In the three-body hybrid, with one quark, one antiquark and one gluon, our results suggest that the best potential model has only two fundamental strings, plus a repulsion acting only when the two fundamental strings are close. In the two body gluon-gluon glueball, our results suggest that the string tension is similar to the one of the Casimir Scaling model, with a factor of the order of $\frac{1}{10}$ when compared with the quark-antiquark potential. We can also extrapolate our result for three-body glueballs, relevant for the odderon problem. With three gluons, a triangle formed by three fundamental strings costs less energy than three adjoint strings with a starfish-like geometry. Thus we anticipate that the three-gluon potential is similar to a sum of three mesonic quark-antiquark interactions, plus a repulsion acting only when there is superposition of the fundamental strings.

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