Simultaneous quantum state exchange or transfer between two sets of cavities and generation of multiple Einstein-Podolsky-Rosen pairs via a superconducting coupler qubit

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We propose an approach to simultaneously perform quantum state exchange or transfer between two sets of cavities, each containing \( N \) cavities, by using only one superconducting coupler qubit. The quantum states to be exchanged or transferred can be arbitrary pure or mixed states and entangled or nonentangled. The operation time does not increase with the number of cavities, and there is no need of applying classic pulses during the entire operation. Moreover, the approach can be also applied to realize quantum state exchange or transfer between two sets of qubits, such as that between two multi-qubit quantum registers. We further show that the present proposal can be used to simultaneously generate multiple Einstein-Podolsky-Rosen pairs of photons or qubits, which are important in quantum communication. The method can be generalized to other systems by using different types of physical qubit as a coupler to accomplish the same task.

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I. INTRODUCTION

Physical systems composed of cavities (a.k.a. resonators) and superconducting qubits such as charge, flux, transmon and phase qubits are considered as one of the most promising candidates for quantum information processing (QIP) [1]. Reasons for this are as follows. First, the use of conventional microfabrication techniques allows straightforward scaling to large numbers of stationary qubits. Second, circuit cavities can be used to engineer a variety of qubit types and interactions [2-6]. Last, the strong coupling limit can readily be achieved with superconducting qubits coupled to circuit cavities, which was earlier predicted theoretically [4,7] and has been experimentally demonstrated [8,9]. During the past decade, much progress has been made in quantum state engineering and quantum logic operations with superconducting qubits coupled to a single superconducting cavity. For instance, theoretical proposals have been presented for generating various quantum states of a single superconducting cavity (e.g., Fock states, coherent states, squeezed states, the Schrödinger Cat state, and an arbitrary superposition of Fock states) [10-12]; and experimental preparation of a Fock state and a superposition of Fock states of a superconducting cavity has been reported [13,14]. On the other hand, many schemes have been proposed for realizing quantum logical gates and generating quantum entanglement with two or more superconducting qubits coupled to a cavity (usually in the form of coplanar waveguide resonator) [2-4,7,14-19]. So far, quantum logic operations [6, 20-23] and entanglement [20,25-28] involving two [6,20] or three qubits [21-23] have been experimentally demonstrated with superconducting qubits coupled to a single cavity. Furthermore, quantum information transfer between two qubits [6], three-qubit quantum error correction [22], two-qubit Grover search and Deutsch-Jozsa quantum algorithms [24], and three-qubit Shor’s algorithm to factor the number 15 [28] have also been implemented in such a system.

Attention is now shifting to quantum state engineering with larger systems composed of multiple cavities (and qubits) instead of a single cavity. Within the circuit QED architecture, several theoretical schemes for generating entangled photon Fock states of two resonators have been proposed [29,30]. By coupling two resonators via a phase qubit, Wang et al. recently demonstrated an entangled NOON state of photons in two superconducting microwave resonators [31]. Moreover, proposals for generating entanglement of photons and superconducting qubits in multiple cavities coupled by a superconducting qutrit have been presented recently [32,33].

In this paper, we focus on another interesting aspect, i.e., quantum state exchange and transfer among multiple cavities. The former is necessary to perform QIP involving qubits (photons or other matter qubits) located in different cavities, and the latter is needed for efficient transfer of quantum information from solid state quantum registers specialized in processing quantum information to quantum memory cells (consisting of long lifetime qubits) for long time storage. Besides their significance in QIP, it is also known that both quantum state exchange and transfer are important in quantum (networked) communication. In addition, we will consider generation of multiple Einstein-Podolsky-Rosen (EPR) pairs of photons or qubits, which are important in quantum communication.
classical microwave pulse is needed for the operation, and (v) only one superconducting qubit is used as a coupler.

Any quantum state exchange or transfer task can be implemented in circuit QED by using only one superconducting coupler qubit. Because only one superconducting coupler qubit is needed, the circuit is greatly simplified and the engineering complexity and cost is much reduced. The method presented here is quite general and can be applied to other systems by using different types of physical qubit (e.g., a quantum dot) as a coupler to accomplish the same task.

Furthermore, the present method can be applied to exchanging or transferring quantum states between two sets of cavities (e.g., two multi-qubit quantum registers). To see this, let us look at Fig. 1(b), which shows a set of $N$ cavities ($a_1, a_2, ... , a_N$), and the other set of $N$ cavities ($b_1, b_2, ... , b_N$), respectively. The qubits embedded in cavities can be solid-state qubits, atoms, etc. Exchanging or transferring quantum states between the two sets of qubits can be done as follows. First, by performing local operations (i.e., each local operation is performed on one qubit in each cavity such that the state of the qubit in each cavity is transferred onto the field of each cavity), one can map the state of each qubit to the cavity in which it is located. Second, one can use the present method to exchange or transfer quantum states between these two sets of cavities. Last, map the state of each cavity back to the respective qubit through local operations. In this way, quantum states can be exchanged or transferred between these two sets of qubits. In a similar manner, $N$ EPR pairs of photons simultaneously generated can be transferred onto qubits distributed in $2N$ cavities, by performing local operations within each cavity.

The paper is organized as follows. In Sec. II, we show how to exchange quantum states between two sets of cavities by using a superconducting coupler qubit interacting with these cavities. Under the large detuning condition, the coupler qubit does not exchange energy with the cavities, but it can mediate coupling between any pair of cavities whose quantum states are to be exchanged. We further discuss the fidelity of operation by considering the factors that may reduce fidelity, and then estimate the fidelity numerically for several initial states of four cavities coupled to a phase qubit as an example. In Sec. III, we show how to simultaneously generate multiple EPR pairs of photons or qubits by using the present proposal. A concluding summary is given in Sec. IV.

II. MULTI-CAVITY QUANTUM STATE EXCHANGE AND TRANSFER

Consider $2N$ cavities coupled to a superconducting qubit $A$ [Fig. 1(a)]. The first set of $N$ cavities are labeled as

\[ a_1, a_2, ... , a_N \], while the second set of $N$ cavities are denoted as \( b_1, b_2, ... , b_N \). Each cavity could be a one-dimensional coplanar waveguide resonator which is capacitively coupled to the coupler qubit $A$. (b) The diagram of a superconducting qubit $A$ and $2N$ cavities, each cavity hosting a qubit (labelled by a dark dot), which can be a solid-state qubit, an atom, etc. Quantum states can be exchanged or transferred between $N$ qubits (in one set of $N$ cavities) and another $N$ qubits (in the other set of $N$ cavities). In addition, $N$ EPR pairs can be simultaneously generated using photons or qubits distributed in $2N$ cavities. For details, see the discussion given in the text.
cavities $a_1, a_2, ..., a_N$ while the second set of $N$ cavities are labeled as cavities $b_1, b_2, ..., b_N$. In addition, the two levels of the qubit $A$ are denoted as $|g\rangle$ and $|e\rangle$ (Fig. 2). Suppose that cavity $a_j$ ($b_j$) with $j = 1, 2, ..., N$ is coupled to the $|g\rangle \leftrightarrow |e\rangle$ transition with coupling strength $g_j$ ($\mu_j$) and detuning $\Delta_j = \omega_{eg} - \omega_{aj} = \omega_{eg} - \omega_{bj}$ (Fig. 2). Here, $\omega_{aj}$ ($\omega_{bj}$) is the frequency of cavity $a_j$ ($b_j$). In the interaction picture, the Hamiltonian of the whole system is given by

$$H = \sum_{j=1}^{N} \left( g_j e^{i \Delta_j t} \hat{a}_j \hat{S}_+ + \mu_j e^{i \Delta_j t} \hat{b}_j \hat{S}_+ + \text{H.c.} \right),$$

where $\hat{S}_+ = |e\rangle \langle g|$, and $\hat{a}_j$ ($\hat{b}_j$) is the photon annihilation operator of cavity $a_j$ ($b_j$).

Under the large detuning condition $\Delta_j \gg g_j, \mu_j$, the cavities do not exchange energy with the qubit. However, the qubit can mediate dispersive coupling between the cavities. Cavity $a_j$ is coupled to $b_j$ only when the detunings satisfy the following conditions

$$\frac{|\Delta_j - \Delta_k|}{\Delta_j^{-1} + \Delta_k^{-1}} \gg g_j g_k, \mu_j \mu_k; \ j \neq k.$$  

Then we can obtain the following effective Hamiltonian

$$H_{eff} = H_0 + H_I,$$

with

$$H_0 = \sum_{j=1}^{N} \left( g_j^2 \frac{\Delta_j}{\Delta_j} \hat{a}_j^+ \hat{a}_j + \frac{\mu_j^2}{\Delta_j} \hat{b}_j^+ \hat{b}_j \right) |e\rangle \langle e|$$

$$- \sum_{j=1}^{N} \left( g_j^2 \frac{\Delta_j}{\Delta_j} \hat{a}_j^+ \hat{a}_j + \frac{\mu_j^2}{\Delta_j} \hat{b}_j^+ \hat{b}_j \right) |g\rangle \langle g|,$$

$$H_I = \sum_{j=1}^{N} \lambda_j (\hat{a}_j^+ \hat{b}_j + \hat{a}_j \hat{b}_j^+) (|e\rangle \langle e| - |g\rangle \langle g|),$$

where $\lambda_j = g_j \mu_j / \Delta_j$, and the two terms in the first (second) bracket of $H_0$ are ac-Stark shifts of the level $|e\rangle$ ($|g\rangle$) induced by the cavity modes $a_j$ and $b_j$, respectively.
In the following, we set $g_j = \mu_j$ (achievable by tuning the coupling capacitance between the qubit and cavity $a_j$ as well as the coupling capacitance between the qubit and cavity $b_j$). In a new interaction picture under the Hamiltonian $H_0$, we have

$$\tilde{H}_I = e^{iH_0t} H_I e^{-iH_0t} = H_I.$$  \hspace{1cm} (5)

When the qubit is initially in the lower level $|g\rangle$, it will remain in this state throughout the interaction. Thus, based on Eqs. (4) and (5), one can see that the effective Hamiltonian governing the field dynamics is then given by

$$H_e = -\sum_{j=1}^{N} \lambda_j (\hat{a}_j^+ \hat{b}_j^+ + \hat{a}_j^+ \hat{b}_j).$$ \hspace{1cm} (6)

which leads to the transformations

$$e^{-iH_e t} \hat{a}_j^+ e^{iH_e t} = \cos(\lambda_j t) \hat{a}_j^+ + i \sin(\lambda_j t) \hat{b}_j^{+},$$
$$e^{-iH_e t} \hat{b}_j^+ e^{iH_e t} = \cos(\lambda_j t) \hat{b}_j^+ + i \sin(\lambda_j t) \hat{a}_j^+.$$ \hspace{1cm} (7)

In the following, we set $\lambda_j = \lambda$, i.e., $g_j \mu_j / \Delta_j = \lambda$ (independent of $j$), which can be met by adjusting the frequencies of cavities $a_j$ and $b_j$ such that $\Delta_j = g_j \mu_j / \lambda$. For $\lambda t = \pi/2$, we obtain the following transforms

$$e^{-iH_e t} \hat{a}_j^+ e^{iH_e t} = \hat{b}_j^+, \quad e^{-iH_e t} \hat{a}_j e^{iH_e t} = -i \hat{b}_j,$$
$$e^{-iH_e t} \hat{b}_j^+ e^{iH_e t} = i \hat{a}_j^+, \quad e^{-iH_e t} \hat{b}_j e^{iH_e t} = -i \hat{a}_j.$$ \hspace{1cm} (8)

Any initially unentangled field state of the first set of $N$ cavities $(a_1, a_2, \ldots, a_N)$ and the second set of $N$ cavities $(b_1, b_2, \ldots, b_N)$ can be described by the density operator $\rho^a(0) \otimes \rho^b(0)$, where the first (second) part of the product is the initial density operator of the first (second) set of $N$ cavities, taking a general form of

$$\rho^a(0) = \sum_{n_j, m_j = 0}^{\infty} P_{n_j, m_j} \prod_{j=1}^{N} |n_j\rangle_{a_j} \langle m_j|,$$ \hspace{1cm} (9)
$$\rho^b(0) = \sum_{s_k, t_k = 0}^{\infty} P_{s_k, t_k} \prod_{k=1}^{N} |s_k\rangle_{b_k} \langle t_k|,$$ \hspace{1cm} (10)

where the subscript $a_j$ ($b_k$) represents cavity $a_j$ ($b_k$) as mentioned above, $P_{n_j, m_j}$ is the coefficient of the component $\prod_{j=1}^{N} |n_j\rangle_{a_j} \langle m_j|$ of the initial density operator for the cavities $(a_1, a_2, \ldots, a_N)$, and the same notation applies to $P_{s_k, t_k}$ for the cavities $(b_1, b_2, \ldots, b_N)$. In terms of $|n_j\rangle_{a_j} = \frac{\hat{a}_j^{+} \hat{b}_j^{+}}{\sqrt{n_j!}} |0\rangle_{a_j}$ and $|s_k\rangle_{b_k} = \frac{\hat{b}_k^{+} \hat{a}_k^{+}}{\sqrt{s_k!}} |0\rangle_{b_k}$, we can write down the initial state as

$$\rho^a(0) \otimes \rho^b(0) = \sum_{n_j, m_j = 0}^{\infty} P_{n_j, m_j} \sum_{s_k, t_k = 0}^{\infty} P_{s_k, t_k} \prod_{j=1}^{N} \prod_{k=1}^{N} \left( \frac{\hat{a}_j^{+} \hat{b}_j^{+}}{\sqrt{n_j!}} |0\rangle_a \otimes |0\rangle_b \frac{\hat{b}_k^{+} \hat{a}_k^{+}}{\sqrt{s_k!}} |0\rangle_b \right).$$ \hspace{1cm} (11)

where $|0\rangle_a = |0\rangle_{a_1} \ldots |0\rangle_{a_N}$ and $|0\rangle_b = |0\rangle_{b_1} \ldots |0\rangle_{b_N}$.

Based on Eq. (11), one can easily find that under the Hamiltonian $H_e$, the state of the cavity system after an
evolution time \( t = \pi / (2\lambda) \) is given by

\[
\rho_c (t) = e^{-iH_c t} \rho^a (0) \rho^b (0) e^{iH_c t}
\]

\[
= \sum_{n_j, m_j=0}^{\infty} \sum_{s_k, t_k=0}^{\infty} \rho \{n_j, m_j\} \rho \{s_k, t_k\} \prod_{j=1}^{N} \prod_{k=1}^{N} \left[ \left( i\hat{b}_j \right)^{n_j} \left( i\hat{a}_k \right)^{s_k} \right]
\]

\[
\otimes |0\rangle_{t_k} \left( -i\hat{b}_j \right)^{m_j} \left( -i\hat{a}_j \right)^{t_k} \left[ \sqrt{n_j \lambda k^j} \right] \right]
\]

\[
= \sum_{s_k, t_k=0}^{\infty} \rho \{s_k, t_k\} \prod_{k=1}^{N} \left[ i^{s_k} (-i) \left[ \langle s_k \rangle_{a_k} \langle t_k \rangle \right] \right]
\]

\[
\otimes \sum_{n_j, m_j=0}^{\infty} \rho \{n_j, m_j\} \prod_{j=1}^{N} \left[ i^{n_j} (-i) \left[ \langle n_j \rangle_{b_j} \langle m_j \rangle \right] \right].
\]

where we have used the unitary transformations described by Eq. (8). Note that in the last two lines of Eq. (12), the first part of the product represents the \( N \)-cavity state of \( (a_1, a_2, ..., a_N) \) while the second part is that of \( (b_1, b_2, ..., b_N) \).

After returning to the original interaction picture, the state of the whole system is given by

\[
\rho'_{cA} (t) = e^{-iH_{0t}} \rho_c (t) \rho_A (t) e^{iH_{0t}} = \rho'_{c} (t) \otimes \rho_A (t),
\]

where \( \rho_A (t) = \rho_A (0) = |g\rangle \langle g| \), and

\[
\rho'_{c} (t) = \rho^a (t) \otimes \rho^b (t)
\]

with

\[
\rho^a (t) = \sum_{s_k, t_k=0}^{\infty} \rho^a \{s_k, t_k\} \prod_{k=1}^{N} \left[ e^{i\phi_k (s_k-t_k) \pi} \langle s_k \rangle_{a_k} \langle t_k \rangle \right],
\]

\[
\rho^b (t) = \sum_{n_j, m_j=0}^{\infty} \rho^b \{n_j, m_j\} \prod_{j=1}^{N} \left[ e^{i\theta_j (n_j-m_j) \pi} \langle n_j \rangle_{b_j} \langle m_j \rangle \right],
\]

where \( \phi_k = 1/2 + \left(g_k^2 / \Delta_k \right) / (2\lambda) \) and \( \theta_j = 1/2 + \left(\mu_j^2 / \Delta_j \right) / (2\lambda) \). This is equivalent to the quantum state swap operation plus additional photon-number-dependent phase shifts on the respective cavities. For implementation of a quantum information processing task, these phase shifts can be absorbed into the corresponding local operations.

In the above, we have shown how to implement quantum state exchange between one set of \( N \) cavities \( (a_1, a_2, ..., a_N) \) and another set of \( N \) cavities \( (b_1, b_2, ..., b_N) \). It should be mentioned here that when each of the second set of cavities is initially in the vacuum state, the quantum state exchange protocol described above becomes quantum state transfer from the first to the second set of cavities.

The method presented above can in principle be used to implement \( N \)-cavity arbitrary quantum state exchange and transfer. However, as the number of cavities increases it may become more difficult to satisfy condition (2). In addition, beyond a certain number the coupling strength of the coupler qubit \( A \) with each cavity decreases as the number of cavities increases, which leads to a longer operation time. Thus, decoherence, caused by relaxation and dephasing of qubit and/or cavity decay, may become a bigger problem. Nevertheless, we remark that when the number of cavities coupled to qubit \( A \) is limited to about 4 to 6, condition (2) can be readily satisfied and the qubit-cavity couplings can remain sufficiently strong.

We should mention that the Hamiltonian (6) with \( j = 1 \), i.e., \( \lambda(\hat{a}\hat{b}^+ + \hat{a}^+\hat{b}) \), was previously used in quantum conversion between two cavity fields or two coupled resonators, quantum conversion between light and a macroscopic oscillator, quantum conversion between light and the motion of a trapped atom (or the center-of-mass motion of an
ion), and quantum state transfer from light to matter [34-40]. However, it is worth noting that the main purpose of this work is to construct the Hamiltonian (6) with \( j > 1 \), the main result of this paper, which is nontrivial because it can be used to achieve simultaneous quantum state exchange or transfer among multiple cavities, as shown above.

Furthermore, the present proposal can be used to transfer a multi-cavity entangled state from \( N \) cavities to another \( N \) cavities. This result is remarkable when compared with the previous proposals [34-40]. It is noted that the latter can only be used to transfer a single cavity quantum state (not entangled) from one cavity to another cavity, because they are based on the Hamiltonian \( \lambda(\hat{a}b^+ + \hat{a}^+b) \) only for two cavities.

Finally, we should point out that a Hamiltonian \( \sum_{j=1}^{N} \lambda_j(\hat{a}_j\hat{a}_{j+1}^+ + \hat{a}_{j+1}\hat{a}_j^+) \), describing the interaction between two neighbor cavities in an array of cavities, has been discussed previously (e.g., see [41,42]). It is obvious that this Hamiltonian is different from ours given in Eq. (6).

We stress that this work is of interest in at least two aspects. First, the protocol can be used to swap or transfer any type of multipartite entanglement (e.g., the GHZ state \( |000...0\rangle + |111...1\rangle \), the W state \( \frac{1}{\sqrt{N}}(|000...01\rangle + |001...01\rangle + \ldots + |100...00\rangle) \), the cluster state, etc.) from one set of \( N \) cavities to another set of \( N \) cavities via a single intermediate coupler qubit only. Second, the protocol can be used to simultaneously generate multiple EPR pairs of photons or qubits, as shown below.

As an example, let us consider four cavities coupled via a superconducting qubit. In the following, we will first give a general discussion on the fidelity of the operation. To quantify how well the proposed protocol works out, we then estimate the fidelity numerically for several initial states of the four-cavity (\( N = 2 \)) system.

The dynamics of the system, with finite qubit relaxation and dephasing and photon lifetime included, is determined by

\[
\frac{d\rho}{dt} = -i[H,\rho] + \sum_{j=1}^{2} \kappa_j L [\hat{a}_j] + \sum_{j=1}^{2} \kappa'_j L [\hat{b}_j]
+ \gamma_S (S_z \rho S_z - \rho) + \gamma L [S_-],
\]

where \( H \) is Hamiltonian (1), \( L [\hat{a}_j] = \hat{a}_j \rho \hat{a}_j - \hat{a}_j^+ \hat{a}_j \rho / 2 - \rho \hat{a}_j \hat{a}_j / 2 \), \( L [\hat{b}_j] = \hat{b}_j \rho \hat{b}_j - \hat{b}_j^+ \hat{b}_j \rho / 2 - \rho \hat{b}_j \hat{b}_j / 2 \), and \( L [S_-] = S_- \rho S_- - S_+ S_- / 2 - \rho S_+ S_- / 2 \). In addition, \( \kappa_j (\kappa'_j) \) is the photon decay rate of cavity \( a_j (b_j) \), \( \gamma_S \) and \( \gamma \) are the dephasing rate and the energy relaxation rate of the level \( |e\rangle \) of the qubit, respectively.

The fidelity of the operation is given by

\[
F = \sqrt{\rho_{\text{id}} \rho_{\text{id}}^*},
\]

where \( \rho_{\text{id}} \) is the state of an ideal system (i.e., without relaxation, dephasing, photon decay, crosstalks, etc.) and \( \rho \) is the final density operator of the system when the operation is performed in a realistic physical system. From the description given in the previous section, one can see that

\[
\rho_{\text{id}} = \rho^a(t) \rho^b(t) \otimes |g\rangle \langle g|,
\]

where \( \rho^a(t) \) and \( \rho^b(t) \) are, respectively, the states of cavities \( (a_1, a_2) \) and \( (b_1, b_2) \) given in Eqs. (15) and (16) for \( N = 2 \).

According to the previous section, we have \( g_1 = \mu_1 \), \( g_2 = \mu_2 \), \( g_2 = \sqrt{\frac{\Delta_2}{\Delta_1}} g_1 \), and \( \mu_2 = \sqrt{\frac{\Delta_1}{\Delta_2}} \mu_1 \). Choose \( \gamma_S^{-1} = 5 \mu s \), \( \gamma^{-1} = 50 \mu s \), and \( \kappa_{a1}^{-1} = \kappa_{a2}^{-1} = \kappa_{b1}^{-1} = \kappa_{b2}^{-1} = 20 \mu s \). In addition, we set \( \Delta_1 / (2\pi) = 1 \) GHz and \( \Delta_2 / (2\pi) = 0.5 \) GHz. For the parameters chosen here, the fidelity versus \( b = \Delta_1 / g_1 \) is plotted in Fig. 3, from which one can see that for \( b = 21 \) a high fidelity of \( \sim 99.5\% \), \( 99.5\% \), \( 99.0\% \), and \( 98.7\% \) can be, respectively, achieved for the four cavities initially in the following states: (i) \( |\psi^a(0)\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \) and \( |\psi^b(0)\rangle = |00\rangle \), (ii) \( |\psi^a(0)\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \) and \( |\psi^b(0)\rangle = \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle) \), (iii) \( |\rho^a(0)\rangle = \frac{1}{2} (|00\rangle |00\rangle + |11\rangle |11\rangle) \) and \( |\rho^b(0)\rangle = \frac{1}{2} (|00\rangle |00\rangle + |11\rangle |11\rangle) \), and (iv) \( |\rho^a(0)\rangle = \frac{1}{2} (|00\rangle |00\rangle + |11\rangle |11\rangle) \) and \( |\rho^b(0)\rangle = \frac{1}{2} (|00\rangle |00\rangle + |11\rangle |11\rangle) \).

For \( b = 21 \), we have \( g_1 / (2\pi) = \mu_1 / (2\pi) \approx 47.6 \) MHz and \( g_2 / (2\pi) = \mu_2 / (2\pi) \approx 33.7 \) MHz. Note that a coupling constant \( g / (2\pi) \approx 220 \) MHz has been reported for a superconducting qubit coupled to a one-dimensional CPW (coplanar waveguide) resonator [27], and that \( T_1 \) and \( T_2 \) can be made to be on the order of \( 10 \) – \( 100 \) \( \mu s \) for state-of-the-art superconducting devices at the present time [43]. For superconducting qubits, the typical qubit transition frequency is between 4 and 10 GHz. As an example, let us consider the four cavities with frequency \( \nu_{a1} = \nu_{a2} = 6 \) GHz and \( \nu_{b2} = \nu_{b2} = 6.5 \) GHz. For the cavity frequencies chosen above and the values of \( \kappa_{a1}^{-1}, \kappa_{a2}^{-1}, \kappa_{b1}^{-1}, \) and \( \kappa_{b2}^{-1} \) used in the numerical calculation, the required quality factors for the four cavities are \( Q_{a1} = Q_{b1} \approx 7.5 \times 10^5 \), and \( Q_{a2} = Q_{b2} \approx 8.2 \times 10^5 \), respectively. Note that superconducting CPW resonators with a (loaded) quality factor \( Q \sim 10^6 \) have been experimentally demonstrated [44-46]. Our analysis given here demonstrates that exchange or transfer of the states of photons in up to four cavities is feasible within the present circuit QED technique.
Before ending this section, it should be mentioned that the impact of higher qubit levels (above the level $|e\rangle$) on the fidelity is negligible at the optimal point of Fig. 3 (where the large detuning condition meet well). The reason for this is as follows. As long as the large detuning condition is met, the coupler qubit $A$ remains in the ground state and thus the excited level $|e\rangle$ is not populated. Since this level $|e\rangle$ is not excited, all other higher-energy levels would not be occupied during the operation. In this sense, one can expect that the affect of the higher energy levels of the qubit on the fidelity at the optimal point $b = 21$ is negligible.

III. GENERATION OF MULTIPLE EPR PAIRS

Assume that the first set of $N$ cavities is initially in the state $|\psi(0)\rangle_a = \otimes_{j=1}^N |1\rangle_{a_j}$ (i.e., each cavity is in a single-photon state) while the second set of $N$ cavities $(b_1, b_2, ..., b_N)$ is initially in the vacuum state $|\psi(0)\rangle_b = \otimes_{j=1}^N |0\rangle_{b_j}$. Based on Eq. (7), one can easily find that under the Hamiltonian $H_e$, the state of the cavity system after an evolution time $t$ is given by

$$|\psi(t)\rangle_{ab} = e^{-iH_\mu t} |\psi(0)\rangle_a |\psi(0)\rangle_b$$

$$= \otimes_{j=1}^N \left[ (e^{-iH_\mu t}a_j^+ e^{iH_\mu t}) e^{-iH_\mu t} |0\rangle_{a_j} |0\rangle_{b_j} \right]$$

$$= \otimes_{j=1}^N \left[ \cos(\lambda_j t) |1\rangle_{a_j} |0\rangle_{b_j} + i \sin(\lambda_j t) |0\rangle_{a_j} |1\rangle_{b_j} \right], \quad (20)$$

where we have used $e^{-iH_\mu t} |0\rangle_{a_j} |0\rangle_{b_j} = |0\rangle_{a_j} |0\rangle_{b_j}$ because of $\hat{a}_j^+ \hat{b}_j^+ |0\rangle_{a_j} |0\rangle_{b_j} = 0$ and $\hat{a}_j^+ \hat{b}_j |0\rangle_{a_j} |0\rangle_{b_j} = 0$.

After returning to the original interaction picture by performing a unitary transformation $e^{-iH_\mu t}$, it is easy to find that the state of the cavity system is given by

$$|\psi(t)\rangle_{ab} = e^{iN\lambda t} \otimes_{j=1}^N \left[ \cos(\lambda t) |1\rangle_{a_j} |0\rangle_{b_j} + i \sin(\lambda t) |0\rangle_{a_j} |1\rangle_{b_j} \right], \quad (21)$$

where we have used $g_j^2/\Delta_j = \mu_j^2/\Delta_j = \lambda_j = \lambda$ (as set above). This result (21) shows that for $\lambda t = \pi/4$, every two corresponding cavities $a_j$ and $b_j$ are prepared in an entangled EPR pair of photons, i.e., $|EPR\rangle_{a_j,b_j} = 1/\sqrt{2} \left( |1\rangle_{a_j} |0\rangle_{b_j} + i |0\rangle_{a_j} |1\rangle_{b_j} \right)$. Namely, the $N$ EPR pairs of photons distributed in the $2N$ cavities are simultaneously generated after the operation. By performing a local operation to transfer the state of each cavity to the qubit located in the respective cavity, the prepared $N$ EPR pairs of photons can be transferred onto the qubits in the $2N$ cavities.

IV. CONCLUSION
We have proposed a method to simultaneously perform quantum state exchange or transfer between two sets of cavities, by using only one superconducting coupler qubit. By transferring quantum information between each qubit and the respective cavity, the present method can be also extended to implement quantum state exchange or transfer between two sets of $N$-qubit quantum registers. As shown above, this work is of interest because the procedure for implementing quantum state exchange or transfer does not depend on the initial states of cavities (either pure or mixed states). The quantum state exchange or transfer between multiple pairs of cavities can be performed simultaneously, the operation time does not increase with the number of cavities [38], and there is no need for applying classical microwave pulses during the entire operation so that it greatly simplifies the operation. In addition, our analysis shows that exchanging or transferring quantum states of photons in four ($N = 2$) cavities by a coupler superconducting qubit is achievable with the present experimental capability. Furthermore, we have shown that this proposal can be used to simultaneously generate multiple EPR pairs of photons or qubits. Finally, it is noted that the superconducting coupler qubit in this proposal can be replaced by a different type of physical qubit, such as a quantum dot, to accomplish the same task.

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