Properties of 8,979 nuclei ranging from $^{16}\text{O}$ to $^{339}\text{136}$ and extending from the proton drip line to the neutron drip line have been calculated by use of the 1992 version of the finite-range droplet model. The calculated quantities include the ground-state mass, deformation, microscopic correction, odd-proton and odd-neutron spins and parities, proton and neutron pairing gaps, binding energy, one- and two-neutron separation energies, quantities related to $\beta$-delayed one- and two-neutron emission probabilities, $\beta$-decay energy release and half-life with respect to Gamow-Teller decay, one- and two-proton separation energies, and $\alpha$-decay energy release and half-life. For 1,654 nuclei heavier than $^{16}\text{O}$ whose masses were known experimentally in 1989 and which were included in the adjustment of model constants, the theoretical error is 0.669 MeV. For 371 additional nuclei heavier than $^{16}\text{O}$ whose masses have been measured between 1989 and 1996 and which were not used in the adjustment of the model constants, the theoretical error is 0.570 MeV. We also discuss the extrapolateability of two other recent global models of the macroscopic-microscopic type, and conclude with a brief discussion of the recently discovered rock of metastable superheavy nuclei near $^{272}\text{110}$ that had been correctly predicted by macroscopic-microscopic models.

1 Introduction

For purposes ranging from nuclear astrophysics to the quest for superheavy elements, one needs to be able to predict accurately the masses and other properties of nuclei that lie far from the shores of the narrow peninsula corresponding to previously known nuclei. Approaches developed over the years to achieve this difficult yet all-important goal include (1) microscopic theories starting with an underlying nucleon-nucleon interaction, (2) macroscopic-microscopic models utilizing calculated shell and pairing corrections, (3) mass formulas with empirical shell terms whose parameters are extracted from adjustments to experimental masses, (4) algebraic expressions based on the nuclear shell model, and (5) neural networks.

At the most fundamental of the above levels, fully selfconsistent microscopic theories have seen progress in both the nonrelativistic Hartree-Fock approximation and more recently the relativistic mean-field approximation. Although microscopic theories offer great promise for the future, their current
accuracies are typically a few MeV, which is insufficient for most practical applications. At the second level of fundamentality, the macroscopic-microscopic method—where the smooth trends are obtained from a macroscopic model and the local fluctuations from a microscopic model—has been used in several recent global calculations that are useful for a broad range of applications. We will concentrate here on the 1992 version of the finite-range droplet model, with particular emphasis on how well it extrapolates to new regions of nuclei, but will also briefly discuss two other models of this type.

2 Finite-Range Droplet Model

In the finite-range droplet model, which takes its name from the macroscopic model that is used, the microscopic shell and pairing corrections are calculated from a realistic, diffuse-surface, folded-Yukawa single-particle potential by use of Strutinsky’s method. In 1992 we made a new adjustment of the constants of an improved version of this model to 28 fission-barrier heights and to 1,654 nuclei with \( N, Z \geq 8 \) ranging from \(^{16}\text{O}\) to \(^{263}\text{Th}\) whose masses were known experimentally in 1989. The improvements include minimization of the nuclear potential energy of deformation with respect to \( \epsilon_3 \) and \( \epsilon_6 \) shape degrees of freedom in addition to the usual \( \epsilon_2 \) and \( \epsilon_4 \) deformations, use of the Lipkin-Nogami extension of the BCS method for calculating the pairing correction, use of a new functional form and optimized constant for the effective-interaction pairing gap, use of an eighth-order Strutinsky shell correction, and inclusion of a zero-point energy in the quadrupole degree of freedom only.

This model has been used to calculate the ground-state mass, deformation, microscopic correction, odd-proton and odd-neutron spins and parities, proton and neutron pairing gaps, binding energy, one- and two-neutron separation energies, quantities related to \( \beta \)-delayed one- and two-neutron emission probabilities, \( \beta \)-decay energy release and half-life with respect to Gamow-Teller decay, one- and two-proton separation energies, and \( \alpha \)-decay energy release and half-life for 8,979 nuclei with \( N, Z \geq 8 \) ranging from \(^{16}\text{O}\) to \(^{339}\text{U}\) and extending from the proton drip line to the neutron drip line. These tabulated quantities are now available electronically on the World Wide Web at the Uniform Resource Locator [http://t2.lanl.gov/publications/publications.html](http://t2.lanl.gov/publications/publications.html).

For the original 1,654 nuclei included in the adjustment, the theoretical error, determined by use of the maximum-likelihood method with no contributions from experimental errors, is 0.669 MeV. Although some large systematic errors exist for light nuclei they decrease significantly for heavier nuclei.
3 Extrapolateability to New Regions of Nuclei

Between 1989 and 1996, the masses of 371 additional nuclei heavier than $^{16}\text{O}$ have been measured, which provides an ideal opportunity to test the ability of mass models to extrapolate to new regions of nuclei whose masses were not included in the original adjustment. Figure 1 shows as a function of the number of neutrons from $\beta$-stability the individual deviations between these newly measured masses and those predicted by the 1992 finite-range droplet model. The new nuclei fall into three categories, with the first category corresponding to 273 nuclei lying on both sides of the valley of $\beta$-stability. The second category corresponds to 91 proton-rich nuclei produced by fragmentation of a $^{238}\text{U}$ projectile in the storage-ring experiment (ESR) at the GSI. The third category corresponds to seven proton-rich superheavy nuclei discovered in the separator for heavy-ion reaction products (SHIP) at GSI whose masses are estimated by adding the highest $\alpha$-decay energy release at each step in the decay chain to known masses. This procedure could seriously overestimate the experimental masses of some of the heavier nuclei because different energy releases have been observed in some cases. To account for this uncertainty, we have assigned a mass error of 0.5 MeV for each of these seven nuclei. Also, to account for errors of unknown origin, we have included an additional
0.076 MeV contribution to the mass errors for each of the 91 nuclei in the second category. The theoretical error of the 1992 finite-range droplet model for all of the 371 newly measured masses is 0.570 MeV. The reduction in error arises partly because most of the new nuclei are located in the heavy region, where the model is more accurate.

Analogous deviations are shown in Fig. 2 for version 1 of the 1992 extended-Thomas-Fermi Strutinsky-integral model of Aboussir, Pearson, Dutta, and Tondeur. In this model, the macroscopic energy is calculated for a Skyrme-like nucleon-nucleon interaction by use of an extended Thomas-Fermi approximation. The shell correction is calculated from single-particle levels corresponding to this same interaction by use of a Strutinsky-integral method, and the pairing correction is calculated for a $\delta$-function pairing interaction by use of the conventional BCS approximation. The constants of the model were determined by adjustments to the ground-state masses of 1,492 nuclei with mass number $A \geq 36$, which excludes the troublesome region from $^{16}$O to mass number $A = 35$. The theoretical error corresponding to 1,540 nuclei whose masses were known experimentally at the time of the original adjustment is 0.733 MeV. The theoretical error for 366 newly measured masses for nuclei with $A \geq 36$ is 0.739 MeV.
Similar results are shown in Fig. 3 for the 1994 Thomas-Fermi model of Myers and Swiatecki. In this model, the macroscopic energy is calculated for a generalized Seyler-Blanchard nucleon-nucleon interaction by use of the original Thomas-Fermi approximation. For $N, Z \geq 30$ the shell and pairing corrections were taken from the 1992 finite-range droplet model, and for $N, Z \leq 29$ a semi-empirical expression was used. The constants of the model were determined by adjustments to the ground-state masses of the same 1,654 nuclei with $N, Z \geq 8$ ranging from $^{16}$O to $^{263}$106 whose masses were known experimentally in 1989 that were used in the 1992 finite-range droplet model. The theoretical error corresponding to these 1,654 nuclei is 0.640 MeV. The reduced theoretical error relative to that in the 1992 finite-range droplet model arises primarily from the use of semi-empirical microscopic corrections in the extended troublesome region $N, Z \leq 29$ rather than microscopic corrections calculated more fundamentally. The theoretical error for 371 newly measured masses is 0.620 MeV.

Close examination of Figs. 1–3 reveals that for the nuclei in the first category, the theoretical error decreases in the FRDM (1992) but increases in the other two models. For the 91 nuclei in the second category, the theoretical error decreases in all three models, although the decrease is much less in the ETFSI-1 (1992) model than in the other two models. For the seven nuclei in the third category, whose experimental masses may be severely overestimated, the
Table 1: Extrapolateability to New Regions of Nuclei.

| Model       | Original nuclei | New nuclei | Error ratio |
|-------------|-----------------|------------|-------------|
|             | $N_{\text{nuc}}$ | $N_{\text{nuc}}$ | Error (MeV) | Error (MeV) |        |
| FRDM (1992) | 1654 0.669      | 371 0.570   | 0.85        |
| ETFSI-1 (1992) | 1540 0.733     | 366 0.739   | 1.01        |
| TF (1994)   | 1654 0.640      | 371 0.620   | 0.97        |

Theoretical error decreases in the TF (1994) model but increases in the other two models. Table summarizes the overall situation.

4 Future Progress

The disparity between the predictions of the above three models for the seven proton-rich superheavy nuclei provides an opportunity for future progress. If the estimates for the experimental masses of these nuclei were to be taken at face value, the results could be telling us that the droplet-model expansion for the Coulomb redistribution energy in the FRDM (1992) is overestimating the lowering in ground-state mass through the development of a central depression in the nuclear charge density. The magnitude of this energy, which is several MeV for a heavy nucleus, increases strongly with increasing proton number.

Another possibility that deserves further exploration is the need for an isospin-dependent curvature energy in the FRDM (1992). A simple resolution of the long-standing nuclear-curvature-energy anomaly has been offered in the TF (1994) model. This model is characterized by a curvature-energy constant $a_3 = 12.1$ MeV but nevertheless adequately reproduces nuclear ground-state masses through the counteraction of terms that are of still higher order in $A^{-1/3}$. The fission barriers of medium-mass nuclei calculated with such a large curvature-energy constant have in the past been significantly higher than experimental values, but the shape dependence of a new congruence energy arising from a greater-than-average overlap of neutron and proton wave functions could resolve this difficulty.

Because of the present open questions concerning $\alpha$-decay chains with different energy releases, the ultimate key to resolving these questions is more experimental data on the masses of superheavy nuclei, including especially $\alpha$-decay chains of even-even systems.
5 Rock of Metastable Superheavy Nuclei

The heaviest nucleus known to man, $^{277}112$, was discovered in February 1996 at the GSI by use of the gentle fusion reaction $^{70}\text{Zn} + ^{208}\text{Pb} \rightarrow ^{1}n + ^{277}112$. It is the latest in a series of about 10 recently discovered nuclei lying on a rock of deformed metastable superheavy nuclei predicted to exist near the deformed proton magic number at 110 and deformed neutron magic number at 162. Most of the metastable superheavy nuclei that have been discovered live for only about a thousandth of a second, after which they generally decay by emitting a series of alpha particles. However, the decay products of the most recently discovered nucleus $^{277}112$ show for the first time that nuclei at the center of the predicted rock of stability live longer than 10 seconds. The excellent agreement between these observations and theoretical predictions confirms the predictive power of current nuclear-structure models.
One possibility to reach the island of spherical superheavy nuclei near $^{290}_{110}$ that is predicted to lie beyond our present horizon involves the use of prolately deformed targets and projectiles that also possess large negative hexadecapole moments, which leads to large waistline indentations.

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$\sigma_{7} = 1.642 \text{ MeV (SHIP 1994−1996)}$

$\sigma_{91} = 0.379 \text{ MeV (ESR 1996)}$

$\sigma_{273} = 0.614 \text{ MeV (Audi-Wapstra 1995)}$

$\sigma_{1654} = 0.669 \text{ MeV (Original nuclei)}$

FRDM (1992)

$M_{\text{exp}} - M_{\text{calc}} \text{ (MeV)}$

Neutrons from $\beta$-stability
\[ \sigma_{1540} = 0.733 \text{ MeV (Original nuclei)} \]
\[ \sigma_{268} = 0.758 \text{ MeV (Audi-Wapstra 1995)} \]
\[ \sigma_{91} = 0.651 \text{ MeV (ESR 1996)} \]
\[ \sigma_{7} = 1.368 \text{ MeV (SHIP 1994–1996)} \]
$\sigma_{1654} = 0.640$ MeV (Original nuclei)

- $\sigma_{273} = 0.706$ MeV (Audi-Wapstra 1995)
- $\sigma_{91} = 0.352$ MeV (ESR 1996)
- $\sigma_7 = 0.372$ MeV (SHIP 1994–1996)