Flavor violation and extra dimensions

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Abstract

We analyze new sources of flavor violation in models with extra dimensions. We focus on three major classes of five dimensional models: models with universal extra dimension, models with split fermions, and models with warped extra dimension. We study the implications of these new sources on the associate CP violating asymmetries to the rare $B$-decays. We show that among these models only the split fermions scenario may accommodate the recent experimental deviation between the CP asymmetry of $B_d \to \phi K_S$ and $\sin 2\beta$.

1 Introduction

It is a common belief held by many theorists for a long time that our world is not limited to just four dimensions but it can extend to as many as eleven. More recently, extra dimensions have been proposed as an alternative way to address the origin of the TeV scale [1–3]. This has led to a new surge of activities to look for their implications for particle phenomenology.

There are many variations on the basic theme of extra dimensions in order to discuss different puzzles of the standard model; for instance, there have been attempts to understand geometrically the origin of the fermion mass hierarchy [4, 5], CP violation [6] etc. A generic feature of this class of models is that they contain new sources of flavor violation, due to the coupling of zero-mode fermions to the Kaluza-Klien (KK) excitations of the gauge and Higgs bosons, which might imply severe constraints on the string scale. This raises the possibility of a conflict between the new physics scale preferred by the solution of the hierarchy problem ($M_S \sim \mathcal{O}(1)$ TeV) and the one needed to satisfy the flavor constraints. Thus the issue of flavor violation in extra dimension models is an important one.
and has only recently begun to be addressed. In particular, in several recent papers [7] this has been discussed in the context of the universal extra dimension models [8]. Our goal in this paper is to pursue this in the context of different extra dimension models and to isolate possible flavor violation signals.

We are specially motivated by the recent experimental measurements [9, 10] of the CP asymmetries of $B_d \to \phi K_S$ ($S_{\phi K_S}$) and $B_d \to \eta' K_S$ ($S_{\eta' K_S}$), which seem to indicate a deviation from standard model predictions by more than two standard deviations. This has been considered as perhaps a hint for new CP and/or flavor violating sources beyond the SM. There have been attempts to interpret this within the framework of supersymmetric theories with a new flavor structure beyond the Yukawa matrices i.e. in the squark mass matrices. These results can be easily accommodated [11, 12] even if the phase in the CKM mixing matrix is the only source of CP violation [13]. If this anomaly is confirmed it would be natural to look for the type of new physics that can generate such deviation between $S_{\phi K_S}$ and $S_{J/\psi K_S}$ and to ask if models with extra dimensions can accommodate this result.

In this paper we study the implications of models with extra dimensions on these processes. We analyze three major classes of five dimensional models. We start with the models with universal extra dimension, then we consider the models with split fermions, and finally we deal with scenario of warped extra dimension. We discuss possible new sources of flavor and CP violations that these models could have beyond those in the CKM. We also study the impact of these new sources on the associated CP violating asymmetries to the rare $B$ decays. We show that in the case of universal extra dimension, the flavor and the CP violation are given by the CKM matrix as in the SM and no significant deviations in the results of the CP asymmetries of the $B$ decays are expected. We also emphasize that with split fermions and models with warped extra dimensions, where fermions and gauge fields live in the bulk, the KK contributions to the $B_d - \bar{B}_d$ and $B_d \to \phi K_S$ occur at the tree level. The $\Delta M_{B_d}$ experimental limit then implies that the compactification scale is of order $10^4$ GeV. With such a large scale, we find that only the split fermion models can lead to a significant deviation between the CP asymmetries $S_{\phi K_S}$ and $S_{J/\psi K_S}$ and the recent results by Belle can be accommodated.

The paper is organized as follows. In the next section, we present model independent expressions for the flavor violating interactions due to the KK-excitations of the gauge fields in models with large extra dimensions. In section 3, we study the effect of the new flavor on the rare $B$-processes in the models with universal extra dimension. Section 4 is devoted for analyzing the impact of the new sources of flavor in split fermions scenarios in $B_d - \bar{B}_d$ mixing and CP asymmetry of $B_d \to \phi K_S$. In section 5, we carry the same analysis in the models with warped extra dimension. Our conclusions are presented in section 6.
2 New source of flavor violation

We start our analysis by examining the impact of the infinite towers of the Kaluza-Klein (KK) modes that occur in extra dimensions on the $\Delta B = 2$ processes ($B$ here stands not for baryon number but the $b$ quark number); in particular we study these effects for $B_d - \bar{B}_d$ mixing and the CP asymmetry $S_{J/\psi K_S}$, and the $\Delta B = 1$ processes, like $B_d \to \phi K_S$. Generally $\Delta M_{B_d}$ and $S_{J/\psi K_S}$ can be calculated via

$$\Delta M_{B_d} = 2|\langle B_d|H^{\Delta B=2}_{\text{eff}}|\bar{B}_d \rangle|,$$

$$S_{J/\psi K_S} = \sin 2\beta_{\text{eff}}, \quad \beta_{\text{eff}} = \frac{1}{2}\arg\langle B_d|H^{\Delta B=2}_{\text{eff}}|\bar{B}_d \rangle,$$

where $H^{\Delta B=2}_{\text{eff}}$ is the effective Hamiltonian for the transition $\Delta B = 2$. In the framework of the SM,

$$S_{J/\psi K_S}^{\text{SM}} = \sin 2\beta, \quad \beta = \arg\left(-\frac{(V_{CKM})_{cd}(V^*_{CKM})_{cb}}{(V_{CKM})_{td}(V^*_{CKM})_{tb}}\right).$$

The effect of the KK contribution can be described by a dimensionless parameter $r_{KK}^2$ and a phase $2\theta_{KK}$ defined as

$$r_{KK}^2 e^{2i\theta_{KK}} = \frac{M_{12}(B_d)}{M_{12}^{\text{SM}}(B_d)},$$

where $M_{12}(B_d) = M_{12}^{\text{SM}}(B_d) + M_{12}^{KK}(B_d)$, and $M_{12}^{KK}$ is the new contribution arising from the exchange of KK-modes. Thus $\Delta M_{B_d}$ is given by $\Delta M_{B_d} = 2|M_{12}^{\text{SM}}(B_d)| r_{KK}^2$ and the CP asymmetry of $B_d \to J/\psi K_S$ is given as

$$S_{J/\psi K_S} = \sin 2\beta_{\text{eff}} = \sin(2\beta + 2\theta_{KK}),$$

where $2\theta_{KK} = \arg\left(1 + M_{12}^{KK}/M_{12}^{\text{SM}}\right)$. The SM contribution is known at NLO accuracy in QCD and it is given by

$$M_{12}^{\text{SM}} = \left(\frac{G_F}{4\pi}\right)^2 M_W^2 (V_{td}V_{tb})^2 S_0(x_t)\eta_B [\alpha_s(\mu_b)]^{-6/23} \left[1 + \frac{\alpha_s(\mu_b)}{4\pi} J_5\right] \left(\frac{4}{3}m_{B_d} f_{B_d}^2 B_1(\mu)\right),$$

where $x_t = (m_t/m_W)^2$. The renormalization group evolution factors $\eta_B$ and $J_5$ are given by $\eta_B = 0.551$ and $J_5 = 1.627$. The Inami-Lim function $S_0(x)$ is given by

$$S_0(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x^3 \ln x}{2(1-x)^3}.$$

Now we turn to the effect of the KK contributions to the CP asymmetries of the $B_d \to \phi K_S$. The direct and the mixing CP asymmetry are respectively given by [11]

$$C_{\phi K_S} = \frac{|\mathcal{P}(\phi K_S)|^2 - 1}{|\mathcal{P}(\phi K_S)|^2 + 1}, \quad S_{\phi K_S} = \frac{2Im\left[\mathcal{P}(\phi K_S)\right]}{|\mathcal{P}(\phi K_S)|^2 + 1},$$

where $\mathcal{P}(\phi K_S)$ is the polarized matrix element for the transition $B_d \to \phi K_S$. The SM contribution is known at NLO accuracy in QCD and it is given by

$$M_{12}^{\text{SM}} = \left(\frac{G_F}{4\pi}\right)^2 M_W^2 (V_{td}V_{tb})^2 S_0(x_t)\eta_B [\alpha_s(\mu_b)]^{-6/23} \left[1 + \frac{\alpha_s(\mu_b)}{4\pi} J_5\right] \left(\frac{4}{3}m_{B_d} f_{B_d}^2 B_1(\mu)\right),$$

where $x_t = (m_t/m_W)^2$. The renormalization group evolution factors $\eta_B$ and $J_5$ are given by $\eta_B = 0.551$ and $J_5 = 1.627$. The Inami-Lim function $S_0(x)$ is given by

$$S_0(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x^3 \ln x}{2(1-x)^3}.$$
where $\mathcal{P}(\phi K_S) = \frac{\bar{A}(\phi K_S)}{A(\phi K_S)}$. The $\bar{A}(\phi K_S)$ and $A(\phi K_S)$ are the decay amplitudes of $\bar{B}_d^0$ and $B_d^0$ mesons, which can be written as

$$\bar{A}(\phi K_S) = \langle \phi K_S | \mathcal{H}_{\text{eff}}^{\Delta B=1} | \bar{B}_d^0 \rangle, \quad A(\phi K_S) = \langle \phi K_S | (\mathcal{H}_{\text{eff}}^{\Delta B=1})^\dagger | B_d^0 \rangle.$$ (8)

The mixing parameters $q/p$ is given by

$$\frac{q}{p} = \sqrt{\frac{M_{12} - \frac{i}{2} \Gamma_{12}}{M_{12} - \frac{i}{2} \Gamma_{12}}}.$$ (9)

The $\Delta B = 1$ effective Hamiltonian, including the KK-mediation, is given by

$$\mathcal{H}_{\text{eff}}^{\Delta B=1} = \sum_i (C_i^{SM} + C_i^{KK}) Q_i + (L \leftrightarrow R),$$ (11)

where $Q_i$ are the operators represent the $b \to s\bar{s}s$ transition. Thus, one obtains

$$A(\phi K_S) = A^{SM}(\phi K_S) + A^{KK}(\phi K_S).$$ (12)

In this respect and following the parametrization of the SM and the KK as in Ref.[11], the CP asymmetries of $B_d \to \phi K_S$ are given by

$$S_{\phi K_S} = \frac{\sin 2\beta + 2R_{KK} \cos \delta_{KK} \sin(\theta'_{KK} + 2\beta) + R_{KK}^2 \sin(2\theta'_{KK} + 2\beta)}{1 + 2R_{KK} \cos \delta_{KK} \cos \theta'_{KK} + R_{KK}^2},$$ (13)

$$C_{\phi K_S} = -\frac{2R_{KK} \sin \delta_{KK} \sin \theta'_{KK}}{1 + 2R_{KK} \cos \delta_{KK} \cos \theta'_{KK} + R_{KK}^2},$$ (14)

where $R_{KK} = |A^{KK}/A^{SM}|$, $\theta'_{KK} = \arg(A^{KK}/A^{SM})$, and $\delta_{KK}$ is the strong phase.

### 3 Flavor violation in universal extra dimension Scenario

Recently there has been a growing interest concerning the models with one universal extra dimension (UED), as an alternative view of gauge hierarchy problem of the SM [8]. In this class of models, the compactification scale is the only additional free parameter relative to the SM. Also, the tree level KK contributions to low energy processes are absent, however, their one loop contributions are important as have emphasized in Ref. [7, 8]. It was shown that for $1/R \lesssim 400$ GeV, the KK impacts could be very significant. Nonetheless, the electroweak precision impose a lower bound on $1/R$ to be larger than 300 GeV.

Concerning the flavor violation and CP violation in this model, it is, as in the SM, given by the CKM matrix only. Therefore, one would not expect a significant deviation
from the SM results in the CP asymmetries of the $B$ decays. In fact, within the UED scenario, the main effect of the KK modes on these processes is the modification of the Inami-Lim one loop functions, as was found for other processes in Ref.[7]. We will show that this modification is quite limited and cannot explain the $2.7\sigma$ deviation from $\sin 2\beta$ in the process $B_d \rightarrow \phi K_S$ announced by Belle and BaBar Collaborations [9, 10].

In the models with UED, the fifth dimension is compactified on the orbifold $S_1/Z_2$ to produce chiral fermion in four dimensions. There are infinite KK modes of the SM particle with universal masses

$$m^2_{(n)} = m^2_0 + \frac{n^2}{R^2},$$

(15)

where $m_0$ is the mass of the zero mode, which is the ordinary SM particles. It was noticed that the ratio $x_{i(n)} = m^2_{i(n)}/M^2_W(n)$ is a natural variable that enter the Inami-Lim functions [7], where $m_{i(n)}$ are the masses of the fermionic KK modes and $m_W(n)$ are the masses of the $W$ boson KK modes. The effective Hamiltonian for the $\Delta B = 2$ transition in the UED is given by [7]

$$H_{\Delta B=2}^{\text{eff}} = \frac{G_F^2}{16\pi^2} M^2_W (V_{tb}^* V_{td})^2 \eta_B S(x_t, 1/R) \left[ \alpha_s^{(5)}(\mu_b) \right]^{-6/23} \left[ 1 + \frac{\alpha_s^{(5)}(\mu_b)}{4\pi} J_5 \right] Q (\Delta B = 2) + h.c. \tag{16}$$

where $S(x_t, 1/R)$ is defined as

$$S(x_t, 1/R) = S_0(x_t) + \sum_{n=1}^{\infty} S_n(x_t, x_n). \tag{17}$$

The KK contributions are represented by the function $S_n(x_t, x_n)$. A lower bound on the compactification scale $1/R > 165$ GeV has been obtained by requiring the KK contribution does not exceed the experimental central value $\Delta M_{B_d} < 0.484 (ps)^{-1}$ [18].

It is worth mentioning that with the above expression of $H_{\Delta B=2}^{\text{eff}}$, the phase $\theta_{KK}$ defined in the previous section is identically zero, hence there is no new source of CP violation in this class of models and the CP asymmetry of $B_d \rightarrow J/\psi K_S$ would be given by the SM value, $\sin 2\beta$, which is consistent with the experimental measurements.

Now we analyze the KK contributions to the CP asymmetry $B_d \rightarrow \phi K_S$ decay in this class of models with UED. The effective Hamiltonian for the $\Delta B = 1$ transitions through the dominant gluon and chromomagnetic penguins is given by

$$H_{\Delta B=1}^{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts} \sum_{i=3}^{6} C_i O_i + C_g O_g + (L \leftrightarrow R), \tag{18}$$

where $O_i$ and $O_g$ are the relevant local operators, which are given in Ref.[19]. The corresponding Wilson coefficients are given as follows:

$$C_3(M_W) = -3 C_4(M_W) = C_5(M_W) = -3 C_6(M_W) = -\frac{\alpha_s}{24\pi} \tilde{E}(x_t, 1/R), \tag{19}$$

5
and

\[ C_g(M_W) = -\frac{1}{2} E'(x_t, 1/R). \]  

(20)

The functions \( \tilde{E}(x_t, 1/R) \) and \( E'(x_t, 1/R) \) are given by

\[ F(x_t, 1/R) = F_0(x_t) + \sum_{n=1}^{\infty} F_n(x_t, x_n), \quad F \equiv \tilde{E}, E', \]

where \( \tilde{E}_{0,n} \) and \( E'_{0,n} \) are given in Ref.[7]. Taking into account the lower bound obtained on the compactification scale from \( \Delta M_{B_d} \) measurements, one finds that the ratio \( R_{KK} \lesssim 0.05 \) and the phase \( \theta'_{KK} \sim -0.02 \). Clearly these values are much smaller than the required values mentioned in Ref.[11] in order to deviate \( S_{\phi K_S} \) from \( \sin 2\beta \). Indeed we find that, in this case, the total \( S_{\phi K_S} \) is given by \( S_{\phi K_S} \simeq 0.72 \). Thus, one can conclude that an experimental confirmation of a deviation of \( S_{\phi K_S} \) from \( \sin 2\beta \) will disfavor models with UED.

We point out that the original UED models do not address the neutrino mass problem and an extended version of this model based on the gauge groups \( SU(2)_L \times U(1)_{\epsilon_R} \times U(1)_{B-L} \) and \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) which solves the neutrino mass problem has been discussed in [14]. In the four dimensional left-right models [15], there is the well known \( W_L - W_R \) exchange box graph [16] which makes a large contribution to flavor changing hadronic processes such as \( K - \bar{K} \) mixing. One might therefore suspect that there will be similar contributions in the 5D left-right model which will then lead to tighter constraints on the compactification scale. However it turns out that since the lightest \( W_R \) mode in 5D LR models of the type discussed in ref.[14] is a KK mode with a specific \( Z_2 \times Z_2' \) quantum number \((+,-)\), it only connects the known quarks and leptons (which are zero modes of the 5D fermion field) to KK excitations of the quarks with \( Z_2 \times Z_2' \) quantum numbers \((+,-)\). Therefore there is no box graph with \( W_R \) and \( W_L \) exchange that contributes to FCNC processes. So only flavor changing diagrams arise from exchange of KK modes of two \( W_L \)'s in a manner very similar to the UED models with \( SU(2)_L \times U(1)_Y \) gauge group. This implies that the standard model prediction for \( B_d \to \phi K_S \) CP asymmetry is unaffected in the 5D LR models. Furthermore, there are no \( W_L - W_R \) mixing contributions to penguin type graphs since the same \( Z_2 \times Z_2' \) symmetry forbids \( W_L - W_R \) mixing. This is to be contrasted with the 4-dimensional left-right model case, where the \( W_L - W_R \) loop gluon penguin is indeed present and gives a large new CP violating contribution to \( B_d \to \phi K_S \) and lead to significant deviation from the standard model predictions [17].

Thus our first conclusion is that in all the UED type models constructed so far, no significant deviation from the standard model prediction for \( B_d \to \phi K_S \) CP asymmetry is to be expected.
4 Flavor violation in split fermions scenario

Another class of extra dimensions which have drawn a lot of attention is the so called split fermion scenario [4]. The idea of split fermion is that having different fermion localization along the fifth direction may provide a geometrical way to look at the fermion mass hierarchy. As in the UED type models, in these models, all particles are considered to be in all five dimensions; but in contrast with the UED models, the SM fermions $\psi_i$ are localized at different points in the fifth dimension with Gaussian profiles $\psi_i \sim e^{-\left(y-y_i\right)^2/\sigma^2}$. Here $y_i$ is the position of the quark in the fifth dimension and $\sigma$ is the width of the fermion wave functions with $\sigma \ll R$. In order to avoid introducing another hierarchy, one takes $\sigma/R \sim 0.1$. The quark mass matrices arise from the interaction of fermions and the vacuum expectation value (vev) of Higgs zero mode and are given by

$$
(M_u)_{ij} = \frac{v_0(\lambda_u)_{ij}}{\sqrt{2}} e^{-\frac{(\Delta_{ij})^2}{4\sigma^2}},
$$

$$
(M_d)_{ij} = \frac{v_0(\lambda_d)_{ij}}{\sqrt{2}} e^{-\frac{(\Delta_{ij})^2}{4\sigma^2}},
$$

where $\Delta_{ij} = |y_i - y_j|$ is the distance between flavor $i$ and $j$. The parameters $(\lambda_{u,d})_{ij}$ are the 5D Yukawa couplings, which are in general arbitrary matrices. In order to relate the hierarchy of the fermion masses to the locations of different fermion families, one assumes that these couplings are of order unity. However, in order to avoid the factorizable form of the quark mass matrices which has always two vanishing eigenvalues, the $\lambda_{u,d}$ matrices can not be unit matrices. Thus, in this class of models the number of free parameters is larger than the number of the observed fermion masses and mixings and it is very easy to accommodate different types of Yukawa textures with hierarchical or non-hierarchical features. Also with complex $\lambda_{ij}$ one can get the SM phase in the CKM of order $\pi/2$.

Examples of hierarchical Yukawa couplings have been obtained in Ref. [20–22], which fit all the quark and lepton masses and mixing angles. For instance, the solution of Ref.[21] leads to the following position for the up and down quarks in the fifth dimension:

$$
y_{Q_L} \sim \sigma \begin{pmatrix} 0 \\ 14.2349 \\ 8.20333 \end{pmatrix}, \quad y_{d_R} \sim \sigma \begin{pmatrix} 19.4523 \\ 5.15818 \\ 10.1992 \end{pmatrix}, \quad y_{u_R} \sim \sigma \begin{pmatrix} 6.13244 \\ 20.092 \\ 9.64483 \end{pmatrix},
$$

As we will explain below, in these models and due to the non-universal couplings with KK-gluon, both left- and right-handed rotations $V_{L,R}$ that diagonalize the mass matrix are observable. This is unlike the case in the SM where only $V_L$ rotations are physical ($V_{CKM} = V_L^{d^c}V_L^u$) and $V_R$ is completely decoupled. In general, $V_R$ matrix has six phases, these new phases might play an important rule in the CP violation of the $B$ system and this what we are going to examine below. It is also worth mentioning that for a hierarchical $V_R$, this new KK effect is suppressed, and therefore, in our analysis we choose the parameters $\lambda_{ij}$ such that $V_R$ is a non-hierarchical matrix.
The fields living in the bulk can be defined to be even or odd under the $Z_2$ parity. Thus we will assume that the gauge boson in 5D bulk are even under the $Z_2$ and have the following Fourier expansion

$$A_\mu(x,y) = \frac{1}{\sqrt{R}} A_\mu^{(0)} + \sqrt{\frac{2}{R}} \sum_{n=1}^{\infty} \cos \frac{ny}{R} A_\mu^{(n)}(x).$$

(23)

where $R$ is the size of the extra dimension. The relevant terms of the effective 4D Lagrangian as far as flavor is concerned are given by

$$\mathcal{L} = \bar{d}_R M_d^{\text{diag}} d_L + \bar{u}_R M_u^{\text{diag}} u_L + \frac{g}{\sqrt{2}} W_\mu \bar{u}_L \gamma^\mu V_{CKM} d_L$$

$$+ \sum_{n=1}^{\infty} \left[ \sqrt{2} g_s G_\mu^{A(n)} \left( \bar{d}_L \gamma^\mu T^A U_L^{d(n)} d_L + \bar{d}_R \gamma^\mu T^A U_R^{d(n)} d_R \right) + (d \leftrightarrow u) \right] + \text{h.c.}$$

(24)

In obtaining Eq.(24), the fifth dimension has been integrated out and due to the Gaussian nature of the quark wave functions, only the interaction of the KK-gluon, $G_\mu^{A(n)}$, with zero mode mass eigenstate quarks at the points $y_i$, given in Eq.(22), are picked. The matrices $U^{d(n)}_{L,R}$ are defined as

$$U^{d(n)}_{L(R)} = V^{d}_{L(R)} C^{d(n)}_{L(R)} V^{d}_{L(R)}$$

(25)

where $V^{d}_{L,R}$ are the unitary matrices that diagonalize the down quark mass matrix. The non-universal couplings $C^{d(n)}_{L(R)}$ are given by

$$C^{d(n)}_{L(R)} = \text{diag} \left\{ \cos \left( \frac{ny_{dL(R)}}{R} \right), \cos \left( \frac{ny_{sL(R)}}{R} \right), \cos \left( \frac{ny_{bL(R)}}{R} \right) \right\}.$$ 

(26)

The non-universality in the position of the different families at $y$-direction leads to a new source of flavor in $U^{d(n)}_{L,R}$ in addition to the usual $V_{CKM}$. It is clear that this flavor violation can be mediated at tree level by the KK modes of the gluon which makes it quite dangerous and strong bounds on the compactification scale have been obtained [23]. Of course, there are similar contributions from the KK modes of the $\gamma$, $Z$ and $W$ bosons; however, one expects that the largest effect is due to the KK gluon.

Unlike the SM and UED, the contribution from the KK-modes in split fermion models to the $B_d - \bar{B}_d$ mixing is at the tree level. The KK-gluon, shown in Fig.1, gives the dominant contributions to this process. However, since the KK-modes of the gluino has no coupling between the $b$ and $c$ quarks at tree level, its contribution to the $B_d \rightarrow J/\psi K_S$ decay takes place at loop level. Therefore, the main source of CP violation in this process is through the oscillation as in the SM.

In the models with split fermions, the $\Delta B = 2$ effective Lagrangian for the KK-gluon exchange is given by

$$\mathcal{L}^{\Delta B=2}_{KK} = \frac{2}{3} g_s^2 \sum_{n=1}^{\infty} \frac{1}{M_n^2} \sum_{i,j=L,R} U_{i(bd)}^{d(n)\ast} U_{j(bb)}^{d(n)} (\bar{b}_i \gamma^\mu d_i)(\bar{b}_j \gamma^\mu d_j).$$

(27)
Figure 1: The KK-modes gluon contributions to the $B_d - \bar{B}_d$ mixing.

The hadronic matrix elements $\langle \bar{B}_d | (\bar{b}_i \gamma^\mu d_i)(\bar{b}_j \gamma^\mu d_j) | B_d \rangle$, $i, j = L, R$ are given by

$$
\langle \bar{B}_d | (\bar{b}_L \gamma^\mu d_L)(\bar{b}_L \gamma^\mu d_L) | B_d \rangle = \frac{1}{3} m_{B_d} f_{B_d}^2 B_1(\mu),
$$

(28)

$$
\langle \bar{B}_d | (\bar{b}_L \gamma^\mu d_L)(\bar{b}_R \gamma^\mu d_R) | B_d \rangle = \frac{1}{4} \left( \frac{m_{B_d}}{m_b(\mu) + m_d(\mu)} \right)^2 m_{B_d} f_{B_d}^2 B_4(\mu),
$$

(29)

where $m_{B_d}$ is the mass of the $B_d$ meson and $m_b$ and $m_d$ are the masses of masses of the $b$ and $d$ quarks at the scale $\mu$. In our analysis, we assume that $m_b(m_b) = 4.6$ GeV and $m_d(m_b) = 0.0054$ GeV. The B-parameters are given by: $B_1(m_b) = 0.87$ and $B_4(m_b) = 1.16$. The other operators which obtained by exchanging $L \leftrightarrow R$ from the above ones, have the same matrix elements, since the strong interaction preserve parity. Thus, $M_{12}^{KK} = \langle \bar{B}_d | H_2^{\Delta B=2} | B_d \rangle$ can be written as

$$
M_{12}^{KK} = \frac{2}{3} \frac{g_s^2}{M_c^2} \sum_{k,m=1}^3 \sum_{i,j=L,R} \langle \bar{B}_d | (\bar{b}_i \gamma^\mu d_i)(\bar{b}_j \gamma^\mu d_j) | B_d \rangle (V^d_{ik})_3 (V^d_{kj})_3 (V^d_{jk})_3 (V^d_{im})_3 (V^d_{jm})_3 \sum_n \frac{\cos(\frac{n(y)}{R}) \cos(\frac{n(u)}{R})}{n^2}.
$$

(30)

For the example of 5D split fermion discussed above, the experimental limit $\Delta M_{B_d} < 3.2 \times 10^{-13}$ GeV leads to a lower bound on the compactification scale of order $10^4$ GeV. As can be seen from table 1, the bounds on $M_S$, for $\sigma/R = 0.1, 0.05, 0.01$, derived from the experimental measurement of $\Delta M_K \simeq 3.5 \times 10^{-15}$ GeV are about one order of magnitude larger than those obtained from the experimental limit on $\Delta M_{B_d}$. However in the $K - \bar{K}$ mixing, there is a significant uncertainty due to the low QCD corrections which make this constraint is unreliable. Therefore, we will follow the conservative approach and will consider, through our analysis, the $B_d - \bar{B}_d$ mixing to constrain the compactification scale.

Applying these constraints on the compactification scale, one finds that the values of the phase $\theta_{KK}$ are very small ($\lesssim 10^{-2}$), hence the CP asymmetry $S_{J/\psi K_S}$ remains equal
Table 1: The lower bounds on the compactification scale from the experimental measurements of $\Delta M_{B_d}$ and $\Delta M_K$ as function of the parameter $\sigma/R$. The unit of the mass $M_S$ is in GeV.

| $\sigma/R$ | Lower bounds on $M_S$ from Exp. $\Delta M_{B_d}$ | Lower bounds on $M_S$ from Exp. $\Delta M_K$ |
|------------|---------------------------------|---------------------------------|
| 0.1        | $10^4$                          | $8 \times 10^5$                |
| 0.05       | $8 \times 10^3$                 | $3 \times 10^5$                |
| 0.01       | $5 \times 10^3$                 | $8 \times 10^4$                |

Figure 2: The SM and the KK-modes gluon contributions to the $B_d \to \phi K_S$ decay.

to the SM prediction. It is worth stressing that, the KK contribution to $B_d - \bar{B}_d$ mixing (30) is proportional to the transition factor between the first and third generations which is typically very small. This counts as an extra suppression factor for the phase $\theta_{KK}$, in addition to the large compactification scale. Thus, the chance of having a significant effect on the CP asymmetry $S_{J/\psi K_S}$ is reduced.

Now we consider the KK contribution to the CP asymmetry $S_{\phi K_S}$. The tree level KK-modes gluon contributions to the $B_d \to \phi K_S$ decay is shown in Fig. 2. The decay amplitude of $\bar{B}_d^0 \to \phi K_S$ (employing the naive factorization approximation) is given by

$$\bar{A}(\phi K_S) = \langle \phi K_S | H_{\text{eff}}^{B=1} | \bar{B}_d^0 \rangle$$

$$= \frac{2}{3} g_s^2 \sum_{n=1}^{\infty} \frac{1}{M_n^2} \sum_{i,j=L,R} U_{i(bs)}^d U_{j(ss)}^d \langle \phi K_S | (\bar{b}_i \gamma^\mu s_i)(\bar{s}_j \gamma^\mu s_j) | \bar{B}_d^0 \rangle.$$  

(31)

The matrix elements $\langle \phi K_S | (\bar{b}_i \gamma^\mu s_i)(\bar{s}_j \gamma^\mu s_j) | \bar{B}_d^0 \rangle$ can be found in Ref.[11]. It is remarkable that the coupling $U_{(bs)}^d U_{(ss)}^d$ in $\bar{A}(\phi K_S)$ is typically larger than the coupling $U_{(bd)}^d U_{(db)}^d$ of the $B_d - \bar{B}_d$ mixing, with one or two orders of magnitude. The size of this deviation strongly depends on the non-universality among the parameters $C_{LL,RR}^{(n)}$ and also on the flavor structure of the rotational matrices $V_{L,R}^d$ that diagonalize the down quark mass matrix. This implies that it is quite natural to have significant $KK$ contributions to
the CP asymmetry of $B_d \to \phi K_S$ process and negligible one to the CP asymmetry of $B_d \to J/\psi K_S$.

As shown in Eq.(13), the deviation of $S_{\phi K_S}$ from $\sin 2\beta$ is governed by the size of the ratio $R_{KK}$ and the phase $\theta'_{KK}$. In this class of models, there is a new source of flavor and CP violation due to the impact of the right handed rotation $V_{dR}^d$. We find that the size of the $R_{KK}$ and $\theta'_{KK}$ strongly depend on the favor structure of $V_{dR}^d$ and the non-hierarchical form of $V_{dR}^d$ is favored to enhance their size and hence increase deviation between $S_{\phi K_S}$ and $S_{J/\psi K_S}$. It is also worth noting that the required form of $V_{dR}^d$ can be obtained by tuning the arbitrary 5D Yukawa couplings $\lambda_{ij}$. We have explicitly studied different examples and found that $R_{KK}$ can vary from $O(0.01)$ with $V_{dR}^d \sim O(V_{CKM})$ to $R_{KK} \sim 0.3$ for non-hierarchical $V_{dR}^d$. Moreover, the phase $\theta'_{KK}$ is also quite sensitive to the structure of $V_{dR}^d$ and it could be of order $O(1)$.

In Fig.3 we present the predicted values of the CP asymmetry of $S_{\phi K_S}$ as function of the position of the right-handed bottom quark in the fifth dimension (which is one of the relevant parameters for $S_{\phi K_S}$). The other quark positions have been fixed as in Eq.(22). Also the matrix $V_R$ has been chosen to be non-hierarchical and three values of $\sigma/R$ have used as in Table 1. As can be seen from this figure, it is possible to deviate $S_{\phi K_S}$ significantly from $S_{J/\psi K_S}$. Moreover, with a larger mixing in $V_R$ (specially between the 2$^{nd}$ and 3$^{rd}$ generations) or with different set of positions from those given in Eq.(22), one gets smaller and even negative values for $S_{\phi K_S}$.

Thus in the split fermions scenario, the free parameters can be easily adjusted in order to accommodate the anomaly $S_{\phi K_S}$ and $\sin 2\beta$. 

Figure 3: The figure shows the predictions for $S_{\phi K_S}$ as a function of $y_{bR}/\sigma$ for various choice of the ratio $\sigma/R$. The various values of $\sigma/R$ are 0.1 for the solid line, 0.05 for dashed line and 0.01 for dash-dotted line.
5 Flavor violation in warped extra dimensions

Finally we consider the five dimensional models with warped geometry where the SM fermions and gauge bosons correspond to bulk fields [24]. The warped geometry has been proposed as solution of the hierarchy problem. In the original model, the SM fields were localized to one of the boundaries and gravity is allowed to propagate in the bulk. However, it was realized that the scenarios of SM gauge bosons and fermions in the bulk may lead to a new flavor and possible geometrical interpretation for the hierarchy of quark and lepton masses. The Higgs field has to be confined to the TeV brane in order to obtain the observable masses of the $W$ and $Z$ gauge bosons.

We will consider the scenario of Ref.[3], based on the metric

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^{\mu}dx^{\nu} + dy^2,$$

where $\sigma(y) = \kappa|y|$ with $\kappa \sim M_P$ is the curvature scale determined by the negative cosmological constant in the five dimensional bulk. The fermion fields reside in the bulk of this non-factorizable geometry can be decomposed as

$$\Psi(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \psi^{(n)}(x)e^{2\sigma(y)}f_n(y).$$

Here $R$ is the radius of the compactified fifth dimension on an orbifold $S_1/Z_2$ so that the bulk is a slice of $AdS_5$ space between two four dimensional boundaries. The zero mode wave function is given by [5]

$$f_0(y) = \frac{e^{-\sigma(y)}}{N_0}.$$

where $c = m_\psi/\kappa$ and $m_\psi$ is the bulk mass term. Using the orthonormal condition:

$$\frac{1}{2\pi R} \int_{-\pi R}^{\pi R} dy e^{\sigma}f_0(y)f_0(y) = 1,$$

one finds that $N_0$ is given by

$$N_0 = \sqrt{\frac{e^{\pi R\kappa(1-2c)} - 1}{\pi \kappa R(1-2c)}}.$$

The tower of fermion KK excited states is not relevant to our discussion here. Also the massless gauge fields that propagate in this curved background can be decomposed as [5]

$$A_\mu(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} A^{(n)}_\mu(x)f_n^A(y),$$

with $f_n^A$ is given as

$$f_n^A(y) = \frac{\sigma(y)}{N_n} \left[ J_1 \left( \frac{m_n}{\kappa} e^{\sigma(y)} \right) + b^A(m_n)Y_1 \left( \frac{m_n}{\kappa} e^{\sigma(y)} \right) \right],$$

12
where $J_1$ and $Y_1$ are the Bessel function of order one and $b^A(m_n) = -J_0(m_n/\kappa)/Y_0(m_n/\kappa)$. The coupling of the gauge KK modes to the fermion is given by

$$g_{ijn} = \frac{g^{(5)}}{2\pi R^{3/2}} \int_{-\pi R}^{\pi R} e^{\sigma(y)} f_i(y) f_j(y) f_n^A(y) dy.$$  \hspace{1cm} (38)

As in the split fermion scenario, the non-universal parameters $c_i$ lead to non-universal couplings to the KK state of the gluon. In the basis of mass eigenstates we have the following flavor dependent couplings

$$U_{L(R)}^{d(n)} = V_{L(R)}^{d+} g_{00n} V_{L(R)}^d,$$  \hspace{1cm} (39)

where the $g_{00n}$ is given by

$$g_{00n} = g \left( \frac{1 - 2c}{e^{\pi n R(1-2c)} - 1} \right) \frac{\kappa}{N_n} \int_0^{\pi R} e^{(1-2c)\kappa y} \left[ J_1 \left( \frac{m_n}{\kappa} e^{\kappa y} \right) + b^A(m_n) Y_1 \left( \frac{m_n}{\kappa} e^{\kappa y} \right) \right].$$  \hspace{1cm} (40)

The gauge coupling $g$ is defined as $g = g_5/\sqrt{2\pi R}$, where $g_5$ is the 5D gauge coupling. Finally, the unitary matrices $V_{L,R}^d$ diagonalize the down quark mass matrix which is given in this model as

$$Y_{ij}^d = \frac{l_{ij}}{\pi \kappa R} f_{10iL}^d(\pi R) f_{0jR}^d(\pi R).$$  \hspace{1cm} (41)

The dimensionless parameters $l_{ij}$ are defined as $\lambda_{ij}^{(5)} \sqrt{\kappa}$ where $\lambda_{ij}^{(5)}$ are the 5D Yukawa couplings which are free parameters to be fixed by the observable masses and mixing. Therefore, this class of model with warped geometry is similar to the models with split fermions on large extra dimensions where the number of free parameters is larger than the number of the quark masses and mixings. In this respect, any type of Yukawa textures can be obtained in this models by tuning the free parameters $c_i$ and $l_{ij}$. For instance, in order to get the non-hierarchical Yukawa textures that we have used in the previous section:

$$|Y_d| \simeq \begin{pmatrix} 0.006 & 0.018 & 0.018 \\ 0.012 & 0.056 & 0.037 \\ 0.0189 & 0.005 & 0.99 \end{pmatrix}, \quad |Y_u| \simeq \begin{pmatrix} 0.001 & 0.0017 & 0.019 \\ 0.0037 & 0.0069 & 0.0002 \\ 0.012 & 0.041 & 0.976 \end{pmatrix},$$  \hspace{1cm} (42)

one can start with any reasonable choice of $c_i$ which is consistent with the observed hierarchy between different generations and between up and down type quarks and find the corresponding couplings $\lambda_{ij}^{(5)}$ that lead to these textures. This choice is not unique, for example, we can use the parameters $c_i$ listed in Ref.[26]:

$$c_{Q_1} = 0.643, \quad c_{Q_2} = 0.583, \quad c_{Q_3} = 0.317,$$
$$c_{D_1} = 0.643, \quad c_{D_2} = 0.601, \quad c_{D_3} = 0.601,$$
$$c_{U_1} = 0.671, \quad c_{U_2} = 0.528, \quad c_{U_3} = -0.460.$$  \hspace{1cm} (43)
with the following values of $\lambda_{ij}^{(5)}$:

$$
|l^{d}_{ij}| \simeq \begin{pmatrix}
1.36 & 1.08 & 0.72 \\
0.45 & 0.58 & 0.39 \\
0.03 & 0.002 & 0.42
\end{pmatrix}, \\
|l^{u}_{ij}| = \begin{pmatrix}
28 & 0.8 & 0.66 \\
17.7 & 0.57 & 0.0014 \\
2.19 & 0.13 & 0.23
\end{pmatrix}
$$

(44)
to get the Yukawa couplings in Eq. (42).

As advocated above, this class of model leads to a flavor violation at the tree level similar to the split fermion scenario. The effective Hamiltonians for the $\Delta B = 2$ and $\Delta B = 1$ processes can be expressed as in Eqs. 26 and 30 but with $U_{d(n)}^{L(R)}$ as given in Eq. 38. Having fixed the Yukawa couplings in both cases, thus the difference between the flavor predictions of these two models is mainly due to the difference between the non-universal couplings: $c_{L,R}^{d(n)}$ (in split fermions) and $g_{00n}$ (in warped geometry). In fact, it is easy to note that the non-universality of $c_{L,R}^{d(n)}$ is much stronger than the non-universality of $g_{00n}$. For instance with our above assumption, the $g_{001}$ which gives the dominant contribution is given by $g_{001} \simeq g \text{ diag}\{0.04, 0.03, 4.7 \times 10^{-7}\}$ while $C_{L}^{d(1)}$ is given by $C_{L}^{d(1)} \simeq \text{ diag}\{1, 0.14, 0.68\}$. Such large non-universality in split fermions scenario has been proved to be very useful in order to get a significant effect on the CP asymmetry of $B_d \rightarrow \phi K_S$ process, even with compactification scale of order $10^4$ GeV. This makes the chance of saturated the new results of the CP asymmetry is possible within this class of model.

In warped geometry, the compactification scale is constrained by the electroweak corrections to be $M_S \gtrsim 10$ TeV [25]. It was shown that with this value of $M_S$ most of the flavor violating processes induced by the KK excitations of the gauge boson are suppressed and quite below their experimental limits [26]. We have also verified that the KK contribution to $\Delta M_{B_d}$ is quite negligible respect to the SM value. The same is true for the CP asymmetry $S_{J/\psi K_S}$ which is essentially given by the SM value $\sin 2\beta$. Finally, we comment on the KK contributions to $B_d \rightarrow \phi K_S$ in the warped extra dimensions. We find that the ratio of the KK amplitude to the SM amplitude is very small ($R_{KK} \sim \mathcal{O}(10^{-2})$), which implies that it is not possible in this class of models to deviate the value of $S_{\phi K_S}$ from the value of $S_{J/\psi K_S}$.

6 Conclusion

To summarize, we have searched for new flavor violating CP asymmetric effects in three different classes of extra dimension models that could significantly alter the predictions of the standard model and have found that the only models where measurable deviations can occur are those based on the split fermion hypothesis. In other models such CP violating effects are suppressed because of the constraints from the CP conserving sector. While experimental confirmation of such deviations will not necessarily be an evidence
for such models, any lack of deviation from standard model will impose constraints on
the parameters of the model. For instance, agreement with standard model would mean
that the Yukawa couplings in the split fermion models have specific patterns. It may also
impose constraints on the location of the flavors in the fifth dimension in specific models.

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