Moduli stabilization and the pattern of soft SUSY breaking terms

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(Dated: March 26, 2022)

Abstract

In string compactification preserving $N = 1$ SUSY, moduli fields (including the string dilaton) are plausible candidates for the messenger of SUSY breaking at low energy scales. In a scenario that moduli-mediated SUSY breaking is significant, the pattern of soft SUSY breaking terms depends crucially on how the light moduli with mass $m \lesssim O(8\pi^2 m_{3/2})$ are stabilized. We discuss the correspondence between the pattern of soft terms and the stabilization mechanism of light moduli within the framework of 4D effective supergravity which is generalized to include a SUSY-breaking uplifting potential which might be necessary to get the phenomenologically viable de-Sitter (or Minkowski) vacuum. In some special case, light moduli can be stabilized by controllably small perturbative corrections to the Kähler potential, yielding the soft terms dominated by the moduli-mediated contribution. In more generic situation, light moduli are stabilized by non-perturbative effects encoded in the superpotential and a quite different pattern of soft terms emerges: the anomaly-mediated soft terms become comparable to the moduli-mediated ones. Such mixed moduli-anomaly mediated soft terms can be described by a mirage messenger scale hierarchically lower than $M_{\text{Planck}}$, and lead to low energy superparticle masses qualitatively different from those of other mediation models such as mSUGRA scenario, gauge-mediation, and anomaly-mediation.

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I. INTRODUCTION

Low energy supersymmetry (SUSY) is one of the prime candidates for physics beyond the standard model at TeV scale \[1\]. One of the key questions on low energy SUSY is the origin of soft SUSY breaking terms of the visible gauge/matter superfields in the low energy effective lagrangian \[2\]. Most of the phenomenological aspects of low energy SUSY are determined by those soft terms which are presumed to be induced by the auxiliary components of some messenger fields. In string theory, moduli fields including the string dilaton are plausible candidates for the messenger of SUSY breaking. In addition to string moduli, the 4-dimensional supergravity (SUGRA) multiplet provides a model-independent source of SUSY breaking, i.e. the anomaly mediation \[3\], which induces a soft mass \( m_{\text{soft}} \sim m_{3/2}/8\pi^2 \). To identify the dominant source of soft terms, one needs to compute the relative ratios between different auxiliary components including the auxiliary component of the 4D SUGRA multiplet, which requires an understanding of how the messenger moduli are stabilized at a nearly 4D Poincare invariant vacuum. If some moduli \( X \) are stabilized with a heavy mass \( m_X \gg 8\pi^2 m_{3/2} \), their auxiliary components are typically suppressed as \( F_X/X \sim m_{3/2}^2/m_X \ll m_{3/2}/8\pi^2 \). As a result, the soft terms mediated by such heavy moduli are negligible compared to the anomaly-mediated ones. On the other hand, light moduli \( T \) with \( m_T \lesssim 8\pi^2 m_{3/2} \) generically have \( F_T/T \gtrsim m_{3/2}/8\pi^2 \), and then the soft terms mediated by light moduli can be comparable to or dominate over the anomaly-mediated contributions. As we will see, different values of the anomaly to moduli-mediation ratio lead to qualitatively distinguishable patterns of low energy superparticle spectrum. This means that one can probe the mechanism of moduli stabilization through the low energy superparticle masses which might be observable at future collider experiments.

In this talk, I discuss the correspondence between the soft terms and the stabilization mechanism of light moduli for string compactifications which can realize the low energy SUSY at TeV scale together with the high scale gauge coupling unification at \( M_{\text{GUT}} \sim 2 \times 10^{16} \) GeV \[4\]. Theoretical framework of the discussion is the 4D effective SUGRA which allows the computation of moduli \( F \)-components and the resulting soft terms in a controllable approximation scheme. In order to obtain a phenomenologically viable de-Sitter (or Minkowski) vacuum, the effective SUGRA is generalized to include a SUSY-breaking uplifting potential as in the recent KKLT construction of de-Sitter vacua in Type IIB string
in section II, I briefly discuss 4D effective SUGRA which contains an uplifting sector. I then present in section III several different examples of light moduli stabilization, including the case that the light moduli are stabilized by controllably small perturbative corrections to the Kähler potential [6] as well as the KKLT stabilization [5] achieved by non-perturbative superpotential [7]. The perturbative Kähler stabilization is possible in a rather limited situation and leads to the soft terms dominated by moduli-mediation, while the stabilization by non-perturbative superpotential can be implemented in more generic situation and leads to the soft terms receiving comparable contributions from both the moduli mediation and the anomaly mediation [4]. Section IV is devoted to the discussion of the low energy superparticle spectrum for generic values of the anomaly to moduli-mediation ratio [8, 9, 10].

II. 4D EFFECTIVE SUGRA WITH UPLIFTING SECTOR

Quite often, moduli stabilization with low energy SUSY yields hierarchical moduli masses: some moduli $X$ get large masses $m_X \gg 8\pi^2 m_{3/2}$, while the other moduli $T$ remain to be light with $m_T \lesssim 8\pi^2 m_{3/2}$. For instance, the moduli stabilized by quantized fluxes [11] are typically heavy if the compactification scale $M_{\text{com}}$ and the string scale $M_{\text{string}}$ are hierarchically higher than $m_{3/2}$, e.g. $m_X \sim M_{\text{com}}^n/M_{\text{string}}^{n-1} \gg 8\pi^2 m_{3/2}$ for the moduli masses induced by $n$-form flux. The heavy moduli masses are dominated by the SUSY-preserving part: $m_X \simeq \langle e^{K/2} \partial_X^2 W/\partial_X \partial_X^* K \rangle$ where $K$ and $W$ are the Kähler potential and the superpotential of 4D SUGRA, and then the resulting auxiliary components are suppressed as $F^X/X = \langle e^{K/2} D_X W^* / X \partial_X \partial_X^* K \rangle \sim m_{3/2}^2/m_X$ [4]. Such heavy moduli are decoupled from low energy SUSY breaking, thus can be safely integrated out, leaving an effective 4D SUGRA which describes the stabilization of light moduli. Our theoretical framework is such an effective SUGRA of light moduli which contains also an uplifting sector which might be necessary to get a phenomenologically viable de-Sitter (or Minkowski) vacuum [5]. Here we will be focusing on the compactifications with $M_{\text{com}} \gtrsim M_{\text{GUT}} = 2 \times 10^{16}$ GeV which can accomodate the successful 4D gauge coupling unification at $M_{\text{GUT}}$. The uplifting sector might originate from a SUSY-breaking anti-brane in the underlying string compactification. Although it appears to break SUSY explicitly, the uplifting sector can be consistently accomodated in 4D $N = 1$ SUGRA through a Goldstino operator in $N = 1$ superspace [4].
The effective action of light moduli $T_i$ and the visible gauge and matter superfields, $W_\alpha^I$ and $Q^I$, can be written as (in the unit with the 4D Planck scale $M_{Pl} = 1$):

$$S_{\text{eff}} = \int d^4x \sqrt{g^C} \left[ \int d^4\theta \left\{ CC^\ast \left( -3 \exp(-K/3) \right) - C^2 C^2 \theta^2 \bar{\theta}^2 P_{\text{lift}} \right\} \right. \left. + \left\{ \int d^2\theta \left( \frac{1}{4} f_a W^{\alpha\alpha} W_\alpha^a + C^3 W \right) + \text{h.c.} \right\} \right],$$  

where

$$K = K_0(T_i, T_i^*) + Z_I(T_i, T_i^*) Q^I Q^{I*},$$

$$P_{\text{lift}} = P_0(T_i, T_i^*) + X_I(T_i, T_i^*) Q^I Q^{I*},$$

$$W = W_0(T_i) + \frac{1}{6} \lambda_{IJK} Q^I Q^J Q^K, \quad f_a = f_a(T_i).$$

Here we are using the superconformal formulation of 4D SUGRA with chiral compensator superfield $C$, and $g^C_{\mu\nu}$ is the 4D metric in superconformal frame which is related to the Einstein frame metric $g^E_{\mu\nu}$ as $g^C_{\mu\nu} = (CC^\ast)^{-1/2} g^E_{\mu\nu}$. For simplicity, we choose the superconformal gauge in which both the fermionic component of $C$ and the scalar auxiliary component of SUGRA multiplet are vanishing, and then ignore the dependence of SUGRA multiplets other than the spacetime metric-dependence. There still remains a residual super Weyl invariance under the transformation

$$C \to e^{-2\tau} C, \quad g^C_{\mu\nu} \to e^{2(\tau + \tau^*)} g^C_{\mu\nu}, \quad \theta^\alpha \to e^{-\tau + 2\tau^*} \theta^\alpha,$$

where $\tau$ is a complex constant, and the uplifting spurion operator $\theta^2 \bar{\theta}^2 P_{\text{lift}}$ should be invariant under this super Weyl transformation in order to keep the consistency of formulation. For the consistency with the full $N = 1$ local SUSY, one might replace the Grassmann coordinate $\theta^\alpha$ in the spurion operator by the Goldstino superfield $\Lambda^\alpha = \theta^\alpha + \text{Goldstino-terms}$:

$$\theta^2 \bar{\theta}^2 P_{\text{lift}} \rightarrow \Lambda^2 \bar{\Lambda}^2 P_{\text{lift}},$$

which will not affect our subsequent discussion.

The $D$-type uplifting spurion operator in (1) does not affect the standard on-shell relations for the SUSY breaking auxiliary components (in the Einstein frame):

$$\frac{F^C}{C} = \frac{1}{3} \partial_i K_0 F^i + m_{3/2}^*, \quad F^i = -e^{K_0/2} \left( \partial_i \partial_j K_0 \right)^{-1} (D_j W_0)^*,$$

where $F^C$ and $F^i$ denote the auxiliary $F$-components of $C$ and $T_i$, respectively, and $m_{3/2} = e^{K_0/2} W_0$. On the other hand, the moduli potential is modified to include the uplifting
potential from spurion operator:

\[ V_{TOT} = V_F + V_{\text{lift}}, \tag{6} \]

where

\[ V_F = \left( \partial_i \partial_j K_0 \right) F^i F^j - 3e^{K_0} |W_0|^2 \]

is the standard \( F \)-term potential of the \( N = 1 \) SUGRA, and the uplifting potential is given by

\[ V_{\text{lift}} = e^{2K_0/3} P_0(T_i, T^*_i). \]

It should be stressed that this uplifting potential is fundamentally different from the \( D \)-term potential of the \( N = 1 \) SUGRA which is associated with 4D gauge symmetry. To see the difference explicitly, let us consider a 4D SUGRA with an anomalous \( U(1) \) gauge symmetry (but without the uplifting sector) \([13]\) under which

\[ \delta T = i\delta_{GS}, \quad \delta \Phi^I = i q_I \Phi^I, \tag{7} \]

where the non-linear transformation of \( T \) (\( \delta_{GS} = \text{real constant} \)) is introduced to realize the Green-Schwarz (GS) anomaly cancellation mechanism, and \( q_I \) is the \( U(1) \) charge of generic chiral superfield \( \Phi^I \) other than \( T \). The resulting \( N = 1 \) SUGRA potential is given by

\[ V_{N=1} = V_F + V_D = \left[ (\partial_A \partial_B K) F^A F^{B*} - 3e^K |W|^2 \right] + \left[ \frac{1}{2} g^2 D^2 \right], \tag{8} \]

where \( F^A \) is the \( F \)-component of \( \Phi^A = (T, \Phi^I) \), \( g \) is the \( U(1) \) gauge coupling, and the auxiliary \( D \)-component of the \( U(1) \) vector multiplet contains the \( T \)-dependent Fayet-Iliopoulos (FI) term \( \xi_{FI} = \delta_{GS} \partial_T K \) arising from the non-linear \( U(1) \) transformation of \( T \):

\[ D = -i\delta \Phi^A \partial_A K = \xi_{FI} + q_I \Phi^I \partial_I K = i \frac{(\partial_A \partial_B K) \delta \Phi^A F^{B*}}{m_{3/2}}. \tag{9} \]

Here the last expression of \( D \) is based on the \( U(1) \) invariance of \( W \) and is an identity which is valid as long as \( W \neq 0 \). The above expression of \( D \) already shows a fundamental difference between \( V_D = g^2 D^2/2 \) and \( V_{\text{lift}} \). The \( D \)-term potential \( V_D \) can never shift the supersymmetric AdS minimum (\( \langle F^A \rangle = 0 \) and \( \langle W \rangle \neq 0 \)) of \( V_F \) since \( \langle V_D \rangle_{\text{SUSY-AdS}} = 0 \) is a global minimum of \( V_D \). On the other hand, \( V_{\text{lift}} \) generically shifts the supersymmetric AdS minimum of \( V_F \) to a SUSY-breaking true vacuum of \( V_{TOT} = V_F + V_{\text{lift}} \).
Even for a SUSY-breaking solution with $F^A \neq 0$, the possible value of $V_D$ is severely constrained. Applying the stationary condition

$$\langle \partial_A V_{N=1} \rangle = 0$$

(10)

together with the $U(1)$ invariance of $K$ and $W$, one finds the following relation on the VEVs of the $F$ and $D$-components [14]:

$$i\left( \frac{1}{M_{Pl}^2} K_{AB} F^A F^{B*} - m_{3/2}^2 + M_V^2 \right) D = \frac{g^2}{2} \delta \Phi^A [\partial_A \ln(\text{Re}(f))] D^2$$

$$+ F^A F^{B*} [K_{CB} \partial_A \delta \Phi^C + \delta \Phi^C \partial_C K_{AB}],$$

(11)

where $f$ is the $U(1)$ gauge kinetic function, and

$$M_V^2 = g^2 K_{AB} \delta \Phi^A \delta \Phi^{B*} = g^2 M_{Pl}^2 \delta_{GS} \partial_T \partial_T \partial_K + ...$$

(12)

denotes the $U(1)$ gauge boson mass. This shows that $M_V^2 \gg m_{3/2}^2 M_{Pl}$ as long as $T$ is stabilized at a value which does not give a hierarchically small $\langle \partial_T \partial_T \partial_K \rangle$. Then the constraint (11) implies $\langle D \rangle \ll m_{3/2}^2 M_{Pl}$ since $\langle F^A \rangle = O(m_{3/2}^2 M_{Pl})$ or smaller, finally leading to $\langle V_D \rangle \ll m_{3/2}^2 M_{Pl}^2$. In fact, if the modulus $T$ corresponds to the string dilaton or the volume modulus, the corresponding $M_V^2$ is rather close to $M_{Pl}^2$ for the range of $\langle T \rangle$ which can be compatible with the high compactification scale $M_{\text{com}} \gtrsim M_{GUT}$. Then $\langle V_D \rangle = O(m_{3/2}^4)$ which is too small to be useful for uplifting the negative vacuum energy density $-3m_{3/2}^2 M_{Pl}^2$ in $V_{N=1}$. On the other hand, there is no constraint on $V_{\text{lift}}$ such as (11) since there is no associated gauge symmetry. As a result, one can freely adjust the size of $\langle V_{\text{lift}} \rangle$ to be $O(m_{3/2}^2 M_{Pl}^2)$ in order to get the desired nearly flat de-Sitter vacuum.

Once $K_0$, $W_0$ and $P_0$ are given, one can compute $F^C$ and $F^i$ by minimizing $V_{TOT} = V_F + V_{\text{lift}}$ under the fine tuning condition for $\langle V_{TOT} \rangle = 0$. The resulting soft terms of canonically normalized visible fields can be written as

$$L_{\text{soft}} = -\frac{1}{2} M_a \lambda^a \lambda^a - \frac{1}{2} m_I^2 |\phi^I|^2 - \frac{1}{6} A_{IJK} y_I y_J y_K \phi^I \phi^J \phi^K + \text{h.c.},$$

(13)

where $\lambda^a$ are gauginos, $\phi^I$ are the scalar components of the matter superfields $Q^I$, and $y_{IJK} = \lambda_{IJK} / \sqrt{c^{-K_a} Z_I Z_J Z_K}$ are the canonically normalized Yukawa couplings. Since the anomaly-mediated contributions to soft masses can be comparable to the moduli-mediated contributions, i.e. $F^C / 8 \pi^2 C \sim F^i / T_i$, we need to include both contributions in a consistent
manner, yielding the following form of soft parameters at energy scales just below $M_{GUT}$:

$$M_a = F^i \partial_i \ln \left( \text{Re}(f_a) \right) + \frac{b_a g_a^2 F^C}{8\pi^2 C},$$

$$A_{IJK} = -F^i \partial_i \ln \left( \frac{\lambda_{IJK}}{e^{-K_0^2}Z_I Z_J Z_K} \right) - \frac{1}{16\pi^2} (\gamma_I + \gamma_J + \gamma_K) \frac{F^C}{C},$$

$$m_i^2 = \frac{2}{3} (V_F + V_{\text{lift}}) + e^{2K_0^2/3} Z_I^{-1} X_I - F^i F^j \partial_i \partial_j \ln \left( e^{-K_0/3} Z_I \right)$$

$$- \frac{1}{32\pi^2} d \gamma_i \left| \frac{F^C}{C} \right|^2 + \frac{1}{16\pi^2} \left\{ (\partial_\gamma_i) F^i \left( \frac{F^C}{C_0} \right)^* + \text{h.c.} \right\},$$

where $b_a$ and $\gamma_I$ are the one-loop beta function coefficients and the anomalous dimension of $Q^I$ which are given by

$$b_a = -\frac{3}{2} \text{tr} \left( T_a (\text{Adj}) \right) + \frac{1}{2} \sum_i \text{tr} \left( T_a^2 (Q^I) \right),$$

$$\gamma_I = 2C_2(Q^I) - \frac{1}{2} \sum_{JK} \left| y_{IJK} \right|^2 \left( \sum_a g_a^2 T_a^2 (Q^I) \equiv C_2(Q^I) 1 \right),$$

$$\partial_\gamma_I = -\frac{1}{2} \sum_{JK} \left| y_{IJK} \right|^2 \partial_i \ln \left( \frac{\lambda_{IJK}}{e^{-K_0^2}Z_I Z_J Z_K} \right) - 2C_2(Q^I) \partial_i \ln \left( \text{Re}(f_a) \right),$$

where $\omega_{IJK} = \sum_{KL} y_{IJK} y_{JLK}^*$ is assumed to be nearly diagonal.

Note that the uplifting operator $\Lambda^2 \bar{\Lambda}^2 P_{\text{lift}} = \Lambda^2 \bar{\Lambda}^2 [P_0 + X_I Q^I Q^I*]$ affects the soft scalar mass as

$$\Delta m_i^2 = e^{2K_0/3} \left[ \frac{2}{3} P_0 + Z_I^{-1} X_I \right] = \frac{2}{3} V_{\text{lift}} M_{Pl}^2 + e^{2K_0/3} Z_I^{-1} X_I,$$

where the first term is the consequence of lifting the vacuum energy density by $V_{\text{lift}}$, while the second term originates from the matter-Goldstino contact term $X_I \Lambda^2 \bar{\Lambda}^2 Q^I Q^I*$ in the uplifting operator. In the KKLT-type moduli stabilization, one needs $V_{\text{lift}} = O(m_{3/2}^2 M_{Pl}^2)$ in order to get a phenomenologically viable de-Sitter (or Minkowski) vacuum. Still $X_I$ can be negligibly small compared to $m_{3/2}^2$ if the uplifting sector is sequestered from the visible sector.

In the KKLT compactification of Type IIB string theory, the uplifting operator originates from anti-$D3$ brane ($\bar{D}_3$) which is stabilized at the end of an warped throat. On the other hand, if one wishes to accommodate the gauge coupling unification at $M_{GUT} \sim 2 \times 10^{16}$ GeV, the visible sector should be assumed to live on $D$-branes located at the un-warped region of the Calabi-Yau space. As the $D$-branes of $Q^I$ are separated from the $\bar{D}_3$ of $\Lambda^a$ by a long throat, the coefficient $X_I$ of the superspace contact interaction between $Q^I$ and $\Lambda^a$ is expected to be highly suppressed. Unfortunately, a reliable calculation of $X_I$ requires a
detailed knowledge of the underlying string compactification, which is not available at this moment.

III. SOME EXAMPLES

A. Stabilization by perturbative Kähler corrections

If the light moduli have a flat potential when the leading order form of the Kähler potential is used, some of the light moduli might be stabilized by (controllably small) perturbative corrections to the Kähler potential. A simple example is the stabilization of the volume modulus \( T = V_{\text{CY}}^{2/3} + ic_4 \) in Type IIB string compactification on a Calabi-Yau (CY) orientifold with positive Euler number \([6]\). (Here \( V_{\text{CY}} \) is the volume of the CY orientifold, and \( c_4 \) is a zero mode of the RR 4-form field.) After the dilaton and complex structure moduli get heavy masses \( m \sim M^3_{\text{com}} / M^2_{\text{st}} \gg 8\pi m_{3/2} \) by 3-form fluxes, the 4D effective SUGRA of the Kähler moduli takes the no-scale form at leading order in \( \alpha' \) and string loop expansions, thus gives a flat potential of Kähler moduli. Considering only the overall volume modulus while including the higher order corrections to the Kähler potential, one finds \([6]\)

\[
K_0 = -3 \ln (T + T^*) + \frac{\xi_1}{(T + T^*)^{3/2}} - \frac{\xi_2}{(T + T^*)^2},
\]

\[
W_0 = \omega_0, \quad V_{\text{lift}} = \frac{D_0}{(T + T^*)^2}, \tag{16}
\]

where \( \xi_1 \) is the coefficient of \( \alpha' \) correction which is positive for a positive Euler number, \( \xi_2 \) is the coefficient of string loop correction, \( \omega_0 \sim m_{3/2} \) (in the unit with \( M_{\text{Pl}} = 1 \)) is a constant induced by 3-form fluxes, and \( V_{\text{lift}} \) is the uplifting potential generated by anti-D3 brane. It is then straightforward to see that, if \( \xi_{1,2} > 0 \), \( \text{Re}(T) \) is stabilized by the competition between the \( \alpha' \) and string loop corrections at a vacuum value

\[
\langle \text{Re}(T) \rangle \simeq 5.1 \frac{\xi_2}{\xi_1}, \tag{17}
\]

for which

\[
\frac{F^T}{(T + T^*)} \simeq m_{3/2}, \quad \frac{1}{8\pi^2} \frac{F^C}{C} \simeq 6.3 \times 10^{-3} \frac{\xi_1}{(T + T^*)^{3/2}} m_{3/2},
\]

\[
m_T \simeq 0.8 \frac{\xi_1^{1/2}}{(T + T^*)^{3/4}} m_{3/2}, \quad V_{\text{lift}} \simeq 0.12 \frac{\xi_1}{(T + T^*)^{3/2}} m_{3/2}^2 M_{\text{Pl}}^2, \tag{18}
\]
In order for the above volume modulus stabilization to be a reliable approximation, one needs the Kähler corrections to be significantly smaller than the leading order term:

\[
\left\langle \frac{\xi_1}{(T + T^*)^{3/2}} \right\rangle \simeq 3.1 \times 10^{-2} \frac{\xi_1^4}{\xi_2^3} \ll 1,
\]

Then the anomaly to modulus mediation ratio in this Kähler stabilization scheme is negligibly small:

\[
\left( \frac{1}{8\pi^2} \frac{F^C}{C} \right) \left( \frac{F^T}{T + T^*} \right)^{-1} \ll 6.3 \times 10^{-3},
\]

and the soft terms are dominated by the modulus-mediated contributions of \( \mathcal{O}(m_{3/2}) \). Note that the size of the uplifting potential which is required to get a nearly flat dS vacuum is not significant compared to \( m_{3/2}^2 M_{Pl}^2 \), thereby the effects of the uplifting sector on soft terms can be ignored also. An interesting feature of this Kähler stabilization is that the RR axion \( \text{Im}(T) \) can be identified as the QCD axion solving the strong CP problem, though it might suffer from the cosmological difficulty since its decay constant \( f_a \sim M_{GUT} \). [13]

One difficulty of the above Kähler stabilization of the volume modulus is that it works only for \( \xi_1 > 0 \) which requires the Euler number \( \chi = 2(h_{1,1} - h_{1,2}) > 0 \). In order to get \( \omega_0 \) hierarchically lower than \( M_{Pl} \), one might need many independent NS and RR 3-form fluxes, i.e. need a large value of \( h_{1,2} \). In such case, the number of Kähler moduli is also large, \( h_{1,1} > h_{1,2} \), thus there remains many unstabilized Kähler moduli. To avoid this difficulty, one might stabilize some of the Kähler moduli by non-perturbative superpotential. However then the Kähler moduli unfixed by non-perturbative superpotential do not have a flat potential, thereby can not be stabilized by a controllably small Kähler correction. Another potential difficulty of the Kähler stabilization is that it generically predicts \( m_T \lesssim m_{\text{soft}} \), thus might suffer from the cosmological moduli problem when the soft masses of visible fields have the weak scale size.

### B. KKLT-type stabilization by non-perturbative superpotential

A more interesting possibility is to stabilize the light moduli by non-perturbative superpotential. The simplest example would be the KKLT stabilization described by the following effective SUGRA

\[
K_0 = -n_0 \ln(T + T^*), \quad W_0 = \omega_0 - A e^{-aT}, \quad V_{\text{lift}} = \frac{D_0}{(T + T^*)^\ell},
\]
where $A$ is a constant of order unity (in the unit with $M_{Pl} = 1$). Stabilization of the Calabi-Yau volume modulus in Type IIB can be described by this form of effective SUGRA with $n_0 = 3$ and $\ell = 2$ after (i) the heavy dilaton and complex structure moduli are integrated out, and (ii) the hidden gaugino condensation and anti-$D3$ brane are introduced to generate $A e^{-aT}$ and $V_{\text{lift}}$, respectively. Stabilization of the heterotic dilaton might be described also by this form of effective SUGRA with $n_0 = 1$ after (i) the Kähler and complex structure moduli get heavy masses by the geometric tortion and NS fluxes, and (ii) the hidden gaugino condensation and an anti-NS5 brane are introduced. At any rate, one easily finds that $T$ is stabilized at

$$a(T) \simeq \ln(A/\omega_0) \simeq \ln(M_{Pl}/m_{3/2})$$

(22)

for which

$$F_T (T + T^*) \simeq \frac{3\ell}{2n_0 \ln(M_{Pl}/m_{3/2})}, \quad \frac{1}{8\pi^2} F_C \simeq \frac{m_{3/2}}{8\pi^2},$$

$$m_T \simeq m_{3/2} \ln(M_{Pl}/m_{3/2}), \quad V_{\text{lift}} \simeq 3m_{3/2}^2 M_{Pl}^2.$$  

(23)

If the low energy SUSY is realized within this KKLT stabilization scheme, one would have $\ln(M_{Pl}/m_{3/2}) \sim 4\pi^2$, and then the corresponding anomaly to modulus mediation ratio is essentially of order unity:

$$\left( \frac{1}{8\pi^2} F_C \right) \left( \frac{F_T}{T + T^*} \right)^{-1} \simeq \frac{n_0 \ln(M_{Pl}/m_{3/2})}{3\ell \pi^2} = O(1).$$

(24)

As will be discussed in the next section, such mixed modulus-anomaly mediation can give a highly distinctive pattern of superparticle masses at TeV scale. An interesting feature of the KKLT stabilization is the little hierarchy structure:

$$m_T \sim m_{3/2} \ln(M_{Pl}/m_{3/2}) \sim m_{\text{soft}} \left[ \ln(M_{Pl}/m_{3/2}) \right]^2,$$

(25)

for which the cosmological moduli and gravitino problems can be avoided even when the soft masses have the weak scale size.

Unlike the Kähler stabilization, the KKLT stabilization can be easily generalized to the multi Kähler moduli case. For each Kähler modulus $T_i$, there can be a stack of $D7$ branes warpping the corresponding 4-cycle. Then the gaugino condensation on those $D7$ branes will generate the non-perturbative superpotential $A_i e^{-a_i T_i}$ stabilizing $T_i$. As an explicit example, let us consider a Calabi-Yau compactification with two Kähler moduli whose effective
SUGRA is given by \[17\]:

\[
K_0 = -2 \ln \left[ (T_1 + T_1^*)^{3/2} - (T_2 + T_2^*)^{3/2} \right]
\]

\[
W_0 = \omega_0 - A_1 e^{-a_1 T_1} - A_2 e^{-a_2 T_2},
\]

\[
V_{\text{lift}} = \frac{D_0}{[ (T_1 + T_1^*)^{3/2} - (T_2 + T_2^*)^{3/2} ]^{2/3}},
\]

where \( A_{1,2} \) are constants of order unity (in the unit with \( M_{Pl} = 1 \)). We then find

\[
a_i \langle T_i \rangle \simeq \ln(A_i/\omega_0) \simeq \ln(M_{Pl}/m_{3/2}) \quad (i = 1, 2),
\]

and

\[
\frac{F^1}{(T_1 + T_1^*)} \simeq \frac{F^2}{(T_2 + T_2^*)} \simeq \frac{3\ell}{2n_0 \ln(M_{Pl}/m_{3/2})},
\]

\[
\frac{1}{8\pi^2} \frac{F^C}{C} \simeq \frac{m_{3/2}}{8\pi^2}, \quad m_{T,1,2} \simeq m_{3/2} \ln(M_{Pl}/m_{3/2}), \quad V_{\text{lift}} \simeq 3m_{3/2}^2 M_{Pl}^2,
\]

which are essentially same as the case of single Kähler modulus.

If the light moduli are stabilized by a race-track form of superpotential, the anomaly to moduli mediation ratio can be significantly enhanced, leading to the anomaly-dominated SUSY breaking. For instance, for the effective SUGRA

\[
K_0 = -3 \ln(T + T^*), \quad W_0 = A_1 e^{-a_1 T} - A_2 e^{-a_2 T}, \quad V_{\text{lift}} = \frac{D_0}{(T + T^*)^{\ell}}
\]

with \( A_{1,2} = O(1) \) and the race-track fine-tuning \( |a_1 - a_2| \simeq (a_1 + a_2)/\ln(M_{Pl}/m_{3/2}) \), one finds \[4\]

\[
\frac{F^T}{(T + T^*)} \simeq \frac{3\ell}{4} \frac{m_{3/2}}{[\ln(M_{Pl}/m_{3/2})]^2}, \quad \frac{1}{8\pi^2} \frac{F^C}{C} \simeq \frac{m_{3/2}}{8\pi^2},
\]

\[
m_T \simeq \frac{4}{3} m_{3/2} [\ln(M_{Pl}/m_{3/2})]^2, \quad V_{\text{lift}} \simeq 3m_{3/2}^2 M_{Pl}^2,
\]

\[
\left( \frac{1}{8\pi^2} \frac{F^C}{C} \right) \left( \frac{F^T}{T + T^*} \right)^{-1} \simeq \frac{2}{3\ell} \frac{[\ln(M_{Pl}/m_{3/2})]^2}{4\pi^2} = O(4\pi^2).
\]

IV. LOW ENERGY PHENOMENOLOGY OF THE MIRAGE MEDIATION

Let us discuss how the low energy superparticle spectrum behaves as the anomaly to moduli mediation ratio varies. The soft terms of \[14\] renormalized at the scale just below
where

\[ M_0 \equiv F^i \partial_i \ln (\text{Re}(f_a)), \]
\[ a_{IJK} M_0 \equiv -F^i \partial_i \left( \frac{\lambda_{IJK}}{e^{-K_0/3}Z_I Z_J Z_K} \right), \]
\[ c_I |M_0|^2 \equiv -F^i F^j \partial_i \partial_j \ln \left( e^{-K_0/3}Z_I + e^{2K_0/3}Z_I^{-1}X_I \right) \]

represent the moduli-mediated soft terms (including the soft scalar mass from the matter-Goldstino contact term), and

\[ \alpha \equiv \left( \frac{4\pi^2}{\ln(M_{Pl}/m_{3/2})} \right) \left( \frac{1}{M_0} \right) \left( \frac{1}{4\pi^2} \frac{F^C}{C} \right) \]

parameterizes the anomaly to moduli mediation ratio. Note that \( \alpha \) is of order unity when the two mediations give comparable contributions to the gaugino masses.

Taking into account 1-loop RG evolution, the low energy gaugino masses at the renormalization point \( \mu \) are given by

\[ M_a(\mu) = M_0 \left[ 1 - \frac{1}{4\pi^2} b_a g_a^2(\mu) \ln \left( \frac{M_{GUT}}{(M_{Pl}/m_{3/2})^{\alpha/2}} \right) \right], \]

where \( g_a(\mu) \) are the running gauge couplings at scale \( \mu \). As for the low energy values of \( A_{IJK} \) and \( m_i^2 \), if \( y_{IJK} \lesssim 1/\sqrt{8\pi^2} \) or \( a_{IJK} = c_I + c_J + c_K = 1 \) for the \( I-J-K \) combination with \( y_{IJK} \gtrsim 1/\sqrt{8\pi^2} \), they are given by

\[ A_{IJK}(\mu) = M_0 \left[ a_{IJK} + \frac{1}{8\pi^2}(\gamma_I(\mu) + \gamma_J(\mu) + \gamma_K(\mu)) \ln \left( \frac{M_{GUT}}{(M_{Pl}/m_{3/2})^{\alpha/2}} \right) \right], \]
\[ m_i^2(\mu) = |M_0|^2 \left[ c_I - \frac{1}{8\pi^2} Y_I \left( \sum_j c_J Y_J \right) g_a^2(\mu) \ln \left( \frac{M_{GUT}}{\mu} \right) \right. \]
\[ + \left. \frac{1}{4\pi^2} \left( \gamma_I(\mu) - \frac{1}{2} \frac{d\gamma_I(\mu)}{d\ln \mu} \ln \left( \frac{M_{GUT}}{(M_{Pl}/m_{3/2})^{\alpha/2}} \right) \right) \ln \left( \frac{M_{GUT}}{(M_{Pl}/m_{3/2})^{\alpha/2}} \right) \right], \]
where $\gamma_I(\mu)$ denote the running anomalous dimensions at $\mu$ and $Y_I$ is the $U(1)_Y$ hypercharge of $Q^I$.

The above results of low energy soft masses show an interesting feature: the low energy gaugino and (1st and 2nd generation) scalar masses in the mixed modulus-anomaly mediation with the messenger scale $M_{GUT}$ are same as those in the pure moduli-mediation with a mirage messenger scale

$$M_{\text{mirage}} \equiv (m_{3/2}/M_{Pl})^{\alpha/2}M_{GUT}. \quad (36)$$

As an immediate consequence of this mirage mediation, when $\alpha$ increases from $\alpha = 0$ (pure moduli-mediation) to $\alpha \simeq 2$ (mixed moduli-anomaly mediation), the superparticle masses at TeV scale get closer to each other as the mass splitting due to the RG running (over the scales between $M_{GUT}$ and $M_{\text{mirage}}$) is cancelled. Note that the mirage messenger scale does not correspond to a physical threshold scale. Still the physical gauge coupling unification scale is $M_{GUT}$, and the Kaluza-Klein and string threshold scales are a little above $M_{GUT}$.

In Fig.1, we depict how the pattern of low energy superparticle masses varies as a function of $\alpha$. Fig.2 shows the low energy superparticle masses for $0 \leq \alpha \leq 3$, which contains the range of $\alpha$ suggested by the KKLT-type stabilization. Particularly interesting values of $\alpha$ would be $\alpha = 1$ which is predicted by the KKLT-type models (21) and (26) with $n_0 = 3$, $\ell = 2$ and $f_a = T$, and also $\alpha = 2$ which gives nearly unified gaugino (and sfermion) masses at TeV scale. Note that this pattern of low energy superparticle masses in the mirage mediation are qualitatively different from those predicted by other SUSY breaking scenarios such as the mSUGRA, gauge-mediation and anomaly-mediation.

We finally remark that the moduli stabilization schemes discussed in section III can avoid dangerous SUSY flavor and CP violations in a natural manner even when all superparticle masses are close to the weak scale [4]. The resulting (mixed moduli-anomaly mediated) soft terms preserve the quark and lepton flavors if the modular weights of the matter fields are flavor universal, which would be achieved if the matter fields with common gauge charge originate from the same geometric structure. They also preserve CP since the relative CP phases between $F^i$ and $F^C$ can be rotated away by the shift of the axion-component of $T_i$ [4, 9, 18].
FIG. 1: The pattern of superparticle masses at $M_{SUSY} = 1$ TeV for the entire range of the anomaly to moduli mediation ratio $\alpha$. Note that the sign convention of the gaugino masses for $0 \leq \tan(\alpha/4) \leq \pi/2$ is different from the convention for $\pi/2 \leq \tan(\alpha/4) \leq \pi$. Here we choose $a_{IJK} = 3$ and $n_I \equiv 1 - c_I = 0$. The shaded region indicates the range $2/3 \leq \alpha \leq 2$ and the short-dashed curves denote the 3rd generation squarks/sleptons.

Acknowledgments

This work is supported by the KRF Grant funded by the Korean Government (KRF-2005-201-C00006), the KOSEF Grant (KOSEF R01-2005-000-10404-0), and the Center for High Energy Physics of Kyungpook National University. I thank the organizers of the Summer Institute 2005 for the hospitality during the workshop, and A. Falkowski, K. S. Jeong, L. Kallosh, T. Kobayashi, A. Linde, H. P. Nilles, K. Okumura and M. Yamaguchi for the collaborations and/or useful discussions.

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FIG. 2: Superparticle masses at $M_{SUSY} = 1$ TeV for $0 \leq \alpha \leq 3$. The shaded regions correspond to $\alpha = 2/3, 1, 2$, taking into account 10% uncertainty. Again the short-dashed curves denote the 3rd generation squarks/sleptons.

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