Symmetric Peaks-Based Spectrum Sensing Algorithm for Detecting Modulated Signals

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ABSTRACT Cognitive radio is considered an effective solution to spectrum shortages, which has been a significant issue in the next generation of wireless communications. In this paper, we focus on spectrum sensing under a low signal-to-noise-ratio (SNR) and noise fluctuation and propose a symmetric peaks-based spectrum sensing algorithm for modulated signals. First, we analyze the characteristics of the cyclic autocorrelation function of modulated signals, and construct a detection domain for detecting primary users based on the characteristics of the cyclic autocorrelation function of primary signals. Then, we introduce the significance level factor into the spectrum sensing, and develop a symmetric peaks criterion. Following this criterion, we propose a symmetric peaks-based spectrum sensing algorithm. Finally, we give the probabilities of detection and false alarm of the spectrum sensing algorithm, discuss the effect of the significance level factor on the spectrum sensing performance, and compare the complexity of the algorithm with that of other algorithms. The spectrum sensing algorithm proposed does not require any prior knowledge of primary user signals or noise in the systems, and can sense modulated signals under very low SNR.

INDEX TERMS Wireless communication, cognitive radio, spectrum sensing, cyclic autocorrelation function, symmetric peak.

I. INTRODUCTION

Due to the explosive growth of wireless devices and services, the scarcity of spectrum resources has become more and more serious. On the other hand, the current fixed spectrum allocation policy has some idles in the time domain, frequency domain and space domain. It leads to low utilization efficiency of the spectrum. The research work has found that the spectrum utilization rate of most licensed wireless bands is approximately 0.1 [1]. Cognitive radio (CR) is considered a promising approach to improve the spectrum utilization and relieve the spectrum shortage [2].

In cognitive radio networks, there are two types of users: primary users (PUs) and secondary users (SUs). SUs should be able to detect very weak PUs’ signals continuously to find the idle spectrum to transmit their data without the interference with the PUs in the licensed band.

There are varieties of spectrum detection algorithms. The first one is the non-correlated detection algorithm. The energy detection algorithm [3] is a widely used non-correlation detection algorithm. It does not require any prior knowledge of the PU signal, and has low computational complexity. To improve the poor spectrum detection performance, some modified energy detection algorithms have been proposed [4], [5]. However, the spectrum sensing performance of the energy detection algorithm is poor under a low SNR and noise fluctuation.

The second type of algorithm is the correlation detection algorithm, which includes matched filter (MF) detection, waveform detection, correlation characteristic detection, and
cyclic prefix autocorrelation detection [6]. Due to the robust performance under a low SNR, MF detection has been selected to assist the basic energy detector for observing very weak signals [7]. Waveform detection uses the known signal patterns to detect PU signals. It does not require any silence period of SUs [8]. The cyclic prefix autocorrelation detection can provide accurate detection of orthogonal frequency-division multiplexing (OFDM) based PU signals [9]. However, the correlation detection algorithms often require the prior information of the primary users to detect the primary signals.

The third type of algorithm is the eigenvalue detection algorithm, such as the maximum-minimum eigenvalues detection, maximum eigenvalue detection, and energy minimum eigenvalue detection [10], in which the maximum-minimum eigenvalues detection (MME) is best one and performs well under a low SNR and can be used to detect any signals [11]. In addition, Li et al. proposed other spectrum detection algorithms based on the generalized likelihood ratio [12] and the signal information entropy [13], which improve the effect of noise uncertainty on spectrum sensing performance. Recently, Ly, et al. proposed some spectrum detection algorithms for specific types of PU signals, for instance, binary phase shift keying (BPSK), OFDM and generalized frequency division multiplexing (GFDM) [14]–[16].

Cognitive radios often work under a low SNR and noise fluctuation, e.g., next-generation cognitive cellular networks, cognitive wireless broadband access and cognitive femtocells [17]. The noise uncertainty and low SNR are the two main factors that affect the spectrum sensing decision [18], [19]. To that end, we focus on spectrum sensing under the low SNR and noise fluctuation and propose a symmetric peaks-based spectrum sensing (SPS) algorithm for modulated signals.

This paper is organized as follows. In Section II, some relevant works are summarized. The system model is presented in Section III. Section IV analyzes the cycle autocorrelation characteristics of the modulated signals and mathematical features of the cyclic autocorrelation function. Section V derives the symmetric peaks criterion and propose the SPS spectrum sensing algorithm. The probability of false alarm and probability of detection of the SPS algorithm are given in Section VI. Simulation results and analysis are described in Section VII. The effect of the significance level factor on the spectrum sensing performance and the computational complexities are discussed in Section VIII. Finally, some conclusions are drawn in Section IX.

II. RELATED WORK

As is well known, most of the signals in wireless networks are modulated signals (PUs’ signals) and have the cyclic autocorrelation property, while noise in the channel has little cyclic autocorrelation [20]. This property can be used to decide whether modulated signals are present [21], [22].

In [23], a cyclostationary spectrum energy-based spectrum sensing algorithm with the adaptive decision threshold is proposed, it reduces the effect of noise uncertainty on the spectrum sensing. In [24], Gouldieff et al. derive the stochastic behavior of the cyclic autocorrelation power and use it to detect the spectrum. It has robustness to noise uncertainty and low computational cost. In [25], an algorithm based on the cyclic autocorrelation characteristic feature and the theory of the Hilbert transformation is proposed. Compared with the conventional cyclic autocorrelation characteristic feature detection algorithm, it can flexibly change the computational complexity according to the current electromagnetic environment by choosing its sampling times and the step size of the cyclic frequency. Reference [26] proposes a new cyclic autocorrelation characteristic-based spectrum sensing method with uncertain arrivals of primary users. It first divides the spectrum sensing period of an SU into several intervals according to the reciprocal of the cyclic frequency, then computes the test statistic from these intervals based on the likelihood ratio test, subsequently calculates the threshold based on the Newman-Pearson criteria and finally makes a decision. In [20], the PU signals in colored Gaussian noise are detected by looking for single or multiple cycle frequencies at single or multiple time lags in the cyclic autocorrelation function (CAF) of the noisy PU signal. It explicitly exploits the knowledge under the null hypothesis of PU signals being absent. Reference [27] proves that the cyclic autocorrelation property of an OFDM signal exhibits time and frequency overlap due to the guard period. Moreover, the peak characteristics is used to detect whether the PU is present. Reference [28] proposes an efficient frequency domain cyclic prefix autocorrelation-based wideband spectrum sensing (FD-AC) algorithm, which provides accurate detection of the OFDM-based primary user signals. Unlike any current autocorrelation algorithms, the statistical knowledge autocorrelation-based (SKAB) algorithm uses the first order statistical knowledge to detect OFDM signals [29], which is invariant to phase distortion caused by the frequency offset. The FD-AC and SKAB algorithms are especially used to detect OFDM signals.

In this paper, we propose a new method based on the symmetric peaks of the cyclic autocorrelation function of primary signals. It is shown that the symmetric peaks of the cyclic autocorrelation function of the signal received can be used to detect the presence of the signal. Based on the significance level factor, we quantify the distributions of these symmetric peaks and find the detection thresholds for the proposed detection algorithms. The probability of detection and probability of false alarm are also derived. The proposed method overcomes the noise uncertainty problem and can perform well even under a very low SNR. It can be used for modulated signal detection without knowledge of the signal, channel and noise power. The major contributions of this work are as follows:

- Demonstrate the symmetry of the cyclic autocorrelation peaks of modulated signals, especially single-carrier modulated signals (taking the BPSK signal as an example) and multi-carrier modulated signals (taking OFDM signal as an example), and construct the
detection domain for detecting primary users based on the symmetry of the cyclic autocorrelation peaks of the modulated signals. This reduces the effect of noise on the spectrum sensing.

- Introduce the significance level factor into the spectrum sensing algorithm to find a trade off between the probability of detection and the probability of false alarm.
- Develop a symmetric peaks criterion and propose the SPS spectrum sensing algorithm, which does not require any prior knowledge of primary user signals or channel noise. It can sense very weak modulated signals and reduce the effect of noise fluctuation on the spectrum sensing.
- Analyze the probability of false alarm and the probability of detection of the SPS algorithm.

### III. SYSTEM MODEL

Consider a cognitive network with $J$ PUs and one SU. The signal received by the SU in the spectrum sensing period $T$ can be given by

$$r(t) = \sum_{i=1}^{l} s_i(t) + n(t), \quad 0 \leq t \leq T,$$

where $s_i(t)$ ($i = 1, 2, \ldots, l$) is the $i$th PU’s signal, $n(t)$ is the additive white Gaussian noise (AWGN) in the channel, and $n(t) \sim N(0, \sigma_n^2)$.

The spectrum detection can be expressed as a binary hypothesis as follows

$$\begin{cases} H_0 : r(t) = n(t), & 0 \leq t \leq T, \\ H_1 : r(t) = \sum_{i=1}^{l} s_i(t) + n(t), & 0 \leq t \leq T, \end{cases}$$

where $H_0$ is the hypothesis that there is no PU signal present and $H_1$ is the hypothesis that there is a PU signal present. The spectrum detection is to find an appropriate characteristic statistic $\epsilon$ from the signal received, derive the corresponding criterion, and determine whether the PU signal is present or absent.

### IV. CHARACTERISTICS OF MODULATED SIGNALS

#### A. CYCLIC AUTOCORRELATION OF MODULATED SIGNALS

For a modulated signal $s(t)$ with the cyclostationary period $T_0$, the time-varying autocorrelation function $R_s(t, \tau)$ is defined as

$$R_s(t, \tau) = \mathbb{E}\left\{s(t + \tau/2)s^*(t - \tau/2)\right\},$$

where the $\mathbb{E}\{\cdot\}$ denotes the mathematical expectation operation, the $*$ denotes the conjugate operation. Suppose the spectrum detecting duration $T = (2N+1)T_0$, it can be rewritten as

$$R_s(t, \tau) \overset{\Delta}{=} \lim_{T \to \infty} R_s(t, \tau)_T = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} R_s(nT_0, \tau),$$

where $R_s(nT_0, \tau)$ is the time-varying autocorrelation function for the signals received, we can say that the PU signal is present or absent.

Because $R_s(t, \tau)$ is a periodic function with a period $T_0$, it can be expanded by the Fourier series as follows

$$R_s(t, \tau) = \sum_{m=-\infty}^{\infty} R_s^{m/T_0}(\tau)e^{j2\pi mt/T_0},$$

and the Fourier coefficient is given by

$$R_s^{m/T_0}(\tau) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} s(t + \tau/2)s^*(t - \tau/2)e^{-j2\pi mt/T_0} dt,$$

where the second-order cycle frequency $\alpha = m/T_0$. The Fourier coefficient is also known as the cyclic autocorrelation function of $s(t)$ [20]. However, the cyclic autocorrelation function with Gaussian white noise exists only at $\tau = 0$. Yet, the cyclic autocorrelation function of AWGN still appears at other values of $\tau$ in practice, although the amplitude of the cyclic autocorrelation function at $\tau = 0$ is much larger than that at the other values.

Fig. 1 and Fig. 2 show the cyclic autocorrelation functions of BPSK signals with a 320 MHz carrier frequency and Gaussian white noise respectively. From Fig. 1, we know that there are three groups of peaks for BPSK signals, i.e., at $\alpha = \pm 640$ MHz and at $\alpha = 0$, two of which are symmetrical. From Fig. 2, we see that there is only one group of peaks for the AWGN at $\alpha = 0$ around $\tau = 0$. Obviously, if we find the pair of symmetric peak groups in the cyclic autocorrelation function of the signals received, we can say that the PU signal is present. Otherwise, the PU signal is absent. Hence, the pair of symmetric peak groups in the cyclic autocorrelation function can be used as the characteristic statistic to detect whether the PU signal is present or not.

#### B. CHARACTERISTICS OF SINGLE-CARRIER MODULATED SIGNALS

Without loss of generality, we take the BPSK as the example to analyze the characteristics of the cyclic autocorrelation.
A BPSK signal is defined as

\[ s(t) = a(t) \cos(2\pi f_c t), \tag{7} \]

where \( f_c \) denotes the carrier frequency and

\[ a(t) = \sum_{n=-\infty}^{\infty} a_n q(t - nT_b), \tag{8} \]

where \( a_n \) is an independent and identically distributed data sequence, \( q(t) \) denotes the modulation pulse, and \( T_b \) denotes the symbol period. Then, the cyclic autocorrelation function of BPSK signals can be formulated as follows [30]

\[
R^c_{s}(\tau) = \frac{1}{2} R^c_{a}(\tau) \cos(2\pi f_c \tau) + \frac{1}{4} R^{c2}_a(\tau) e^{2\pi i}, \\
+ \frac{1}{4} R^{c-2}_a(\tau) e^{-2\pi i}, \\
+ \frac{\sigma_a^2}{T_b} \sin(\pi(\alpha - 2\beta)) e^{j2\pi a t_0}, \\
+ \frac{\sigma_a^2}{T_b} \sin(\pi(\alpha + 2\beta)) e^{-j2\pi a t_0}, \\
+ \frac{\sigma_a^2}{T_b} \sin(\pi(\alpha - 2\beta)) e^{j2\pi a t_0}, \\
+ \frac{\sigma_a^2}{T_b} \sin(\pi(\alpha + 2\beta)) e^{-j2\pi a t_0}, \\
= 0, \text{ otherwise}, \tag{9} \]

where \( R^c_{a}(\tau) \) is the cyclic autocorrelation function of \( a(t) \)

\[
R^c_{a}(\tau) = \left\{ \begin{array}{ll} 
\frac{\sigma_a^2}{T_b} \sin(\pi(\alpha - 2\beta)) e^{j2\pi a t_0}, & \alpha = \frac{m}{T_b}, \\
0, & \text{otherwise}, 
\end{array} \right. \tag{10} \]

\( \sigma_a^2 \) is the variance of \( a_n \), and \( m = 0, 1, 2, \ldots \) It can be concluded from equation (10) that at \( \tau = 0 \), \( R^c_{a}(0) \) obtains a cyclic autocorrelation peak at \( \alpha = 0 \). Clearly, substituting \( R^c_{a}(0) \) into equation (9), there is one pair of symmetric peak groups

\[
\begin{align*}
R^c_{y}(0) &= R^c_{a}(0) e^{2\pi i f_c t_0}, \\
&= \left\{ \begin{array}{ll} 
\frac{\sigma_a^2}{T_b} \sin(\pi(\alpha - 2\beta)) e^{j2\pi a t_0}, & \alpha = \frac{m}{T_b}, \\
0, & \text{otherwise}, 
\end{array} \right. 
\end{align*} \tag{11} \]

Fig. 3 shows the cyclic autocorrelation function of the BPSK signal with a 250 KHz bit rate, 320 MHz carrier frequency and \( -10 \) dB SNR when \( m = 0 \). It shows that there is one pair of symmetric peak groups at 640 MHz around \( \alpha = 0 \). This characteristic of the cyclic autocorrelation function can be used as statistics for spectrum detection.

C. CHARACTERISTICS OF MULTI-CARRIER MODULATED SIGNALS

OFDM is a typical multi-carrier signal. To overcome multi-path fading, the OFDM signal is supplemented with a guard interval in the form of a cyclic prefix. It can be described as follows

\[ s(t) = \sum_{k=0}^{K-1} \sum_{l=-\infty}^{\infty} c_k l q(t - lT_0) e^{2\pi i k (t - lT_0) / T_0}, \tag{12} \]

where \( c_k, l \) is an independent and identically distributed data sequence, \( f_k = f_0 + k\Delta f = f_0 + k/T_b, k = 1, 2, \ldots, K \) is subcarrier frequency, \( f_0 \) is the carrier frequency, \( K \) is the number of subcarriers, \( \Delta f \) is the subcarrier spacing, \( T_b \) is the valid OFDM symbol length, \( T_0 = T_b + T_G \) is the duration interval of the OFDM symbol, and \( T_G \) is the guard interval of the OFDM symbol. The cyclic autocorrelation function of an OFDM signal can be calculated by [31]

\[
R^c_{y}(\tau) = \frac{\sigma_y^2}{T_0} \sin(\pi N \Delta f \tau) e^{2\pi i [f_0 + N \Delta f \tau]}, \\
\int_{-\infty}^{\infty} e^{2\pi i (\alpha - f) \tau} q(f) q(\alpha - f) df \tag{13} \]

where \( \sigma_y^2 \) is the variance of the data sequence \( c_k, l, \alpha = nT_0 \), and \( q(f) \) is the Fourier transform of the rectangular pulse \( q(t) \).

Fig. 4 shows the cyclic autocorrelation functions of the OFDM signal with \( T_0 = 0.9765 \times 10^{-5} \text{ s}, T_b = 0.7812 \times 10^{-5} \text{ s}, \Delta f = 1/T_b = 1.28\text{ KHz} \) and \( K = 64 \). Some peak group signals are exhibited at \( \alpha = \pm m T_0, \pm 1024 \text{ Hz}, \pm 2048 \text{ Hz}, \ldots \), and \( \tau = \pm T_b = 0.7812 \times 10^{-5} \text{ s} \).

Fig. 5 and Fig. 6 show slices of Fig. 4 at \( \tau = 0 \) and \( \alpha = 0 \), respectively. There are symmetric peaks that are distributed around \( \alpha = 0 \) or \( \tau = 0 \). Just as for single-carrier modulated
signals, this characteristic of the cyclic autocorrelation function, e.g., the symmetric peaks, can be used as statistics for spectrum detection.

In summary, each modulated signal has pairs of symmetric peaks in its cyclic autocorrelation functions as showed in Fig. 7, where the yellow ‘x’ denotes the maximum peak of the cyclic autocorrelation function and the red ‘#’ and white ‘o’ denote the peaks on the cyclic frequency axis $\alpha$ and the delay time axis $\tau$, respectively. However, the cyclic autocorrelation function of AWGN also has peaks at $\alpha = 0$ around $\tau = 0$. For a given signal $r(t)$, if we construct a detection domain function, which consists of all $R_{\alpha}^\tau(r)$ except for $\tau = 0$ and $\alpha = 0$ (denoted by a yellow ‘x’ in Fig. 7) as follows

$$\left| R_k^\tau(r) \right| > \mu + \beta \times \sigma, \quad R_k \subset R^\tau_{\alpha}(r), \quad k = 1, 2,$$  \hspace{1cm} (19)

where $R^\tau_{\alpha}(r)$ is the cyclic autocorrelation function of $r(t)$, we can use the symmetric peaks in the detection domain $R^\tau_{\alpha}(r)$ as the statistical characteristics to detect whether the modulated signal is present.

V. SYMMETRIC PEAKS-BASED SPECTRUM SENSING ALGORITHM

In this section, we describe the spectrum sensing algorithm, which is based on the symmetric peaks of the cyclic autocorrelation function.

According to the central limit theorem, the cyclic autocorrelation function $R^\tau_{\alpha}(r)$ obeys the Gaussian distribution [32]. Thus, we can derive the symmetric peaks criterion.

**Definition 1:** For a given probability of false alarm $P_{fa}$, the significance level factor $\beta$ is defined as

$$\sqrt{\frac{2}{\pi}} \int_{\beta}^{\infty} \exp \left(-\frac{z^2}{2}\right) dz \leq P_{fa}, \quad (15)$$

where “$z$” is the independent variable of the exponential function, which represents the value of the cyclic autocorrelation function.

It is clear that different probabilities of false alarm require different significance level factors. For example, if the probability of false alarm required is equal to 0.1, we need $\beta \approx 1.64$ from

$$\sqrt{\frac{2}{\pi}} \int_{\beta}^{\infty} \exp \left(-\frac{z^2}{2}\right) dz \leq 0.1; \quad (16)$$

if the false alarm probability is 0.01, we need $\beta \approx 2.57$ from

$$\sqrt{\frac{2}{\pi}} \int_{\beta}^{\infty} \exp \left(-\frac{z^2}{2}\right) dz \leq 0.01; \quad (17)$$

and if the false alarm probability is 0.001, we need $\beta \approx 3.29$ from

$$\sqrt{\frac{2}{\pi}} \int_{\beta}^{\infty} \exp \left(-\frac{z^2}{2}\right) dz \leq 0.001. \quad (18)$$

As the significance level factor $\beta$ increases, the probability of false alarm will decrease. However, as $\beta$ increases, the probability of detection will also decrease. Generally, we choose a smaller significance level factor $\beta$ as long as it satisfies (15).

**Symmetric peaks criterion:** If we find a pair of symmetric peaks $R_1$ and $R_2$ from cyclic autocorrelation function $R^\tau_{\alpha}(r)$ as follows

$$\left| R_k \right| > \mu + \beta \times \sigma, \quad R_k \subset R^\tau_{\alpha}(r), \quad k = 1, 2,$$  \hspace{1cm} (19)
where \( \mu \) and \( \sigma^2 \) are the mean and variance of \( \mathcal{R}_s^a(\tau) \) with respect to \( \alpha \) or \( \tau \) respectively, we say that there is a PU signal present with a significance level \( \beta \).

Thus, we can obtain a spectrum sensing algorithm that is based on the pairs of symmetric peaks of the cyclic autocorrelation function, as shown in Algorithm 1.

**Algorithm 1: SPS Algorithm**

1. Initialization: set the significance level factor \( \beta \) according to the false alarm probability;
2. Calculate the cyclic autocorrelation function \( \mathcal{R}_s^a(\tau) \) according to equation (6);
3. Construct the detection domain \( \mathcal{R}_s^a(\tau) \);
4. Calculate the means \( \mu_1, \mu_2 \) and variances \( \sigma_1^2, \sigma_2^2 \) of \( \mathcal{R}_s^a(\tau) \) for \( \alpha \) and \( \tau \), respectively;
5. Find the pair of symmetric peaks \( |R_1| \) and \( |R_2| \) from \( \mathcal{R}_s^a(\tau) \);
6. Decision: if \( |R_k| > \mu_1 + \beta \times \sigma_1 \) or \( |R_k| > \mu_2 + \beta \times \sigma_2 \), \( k = 1, 2 \), the PU is present; otherwise, there is no PU present;
7. End.

**VI. PERFORMANCE ANALYSIS**

In this section, we analyze the probabilities of false alarm and probabilities of detection of the SPS spectrum sensing algorithm.

**A. PROBABILITY OF FALSE ALARM**

When there is no signal, the mean of \( \mathcal{R}_s^a(\tau) \) is

\[
E[\mathcal{R}_s^a(\tau)|H_0] = \frac{1}{T} \int_{-T/2}^{T/2} E[n(t + \frac{\tau}{2})n^*(t - \frac{\tau}{2})e^{-j2\pi\alpha t}]dt,
\]

\( \forall (\alpha, \tau) \neq (0, 0) = 0. \) \hspace{1cm} (20)

and the variance of \( \mathcal{R}_s^a(\tau) \) is

\[
\text{Var}(\mathcal{R}_s^a(\tau)|H_0) = \frac{1}{T^2} \int_{-T/2}^{T/2} \int_{-T/2}^{T/2} E[n(t_1 + \frac{\tau}{2})n^*(t_1 - \frac{\tau}{2})n(t_2 + \frac{\tau}{2})n^*(t_2 - \frac{\tau}{2})e^{-j2\pi\alpha(t_1-t_2)}]dt_1dt_2,
\]

\( \forall (\alpha, \tau) \neq (0, 0) = 0. \) \hspace{1cm} (21)

Hence the probability of false alarm of the SPS algorithm is 0 in theory.

**B. PROBABILITY OF DETECTION**

When there is a signal, the mean of \( \mathcal{R}_s^a(\tau) \) is

\[
E[\mathcal{R}_s^a(\tau)|H_1] = \frac{1}{T} \int_{-T/2}^{T/2} E[r(t + \frac{\tau}{2})r^*(t - \frac{\tau}{2})e^{-j2\pi\alpha t}]dt,
\]

\[\forall (\alpha, \tau) \neq (0, 0) = 0\]

\[= \frac{1}{T} \int_{-T/2}^{T/2} E[R_r(t, \tau)e^{-j2\pi\alpha t}]dt, \forall (\alpha, \tau) \neq (0, 0) \]

\[= \frac{2}{T} \int_{0}^{T/2} R_r(t, \tau)\cos(2\pi\alpha t)dt, \forall (\alpha, \tau) \neq (0, 0) = 0. \] \hspace{1cm} (22)

The second moment of \( \mathcal{R}_s^a(\tau) \) in this case can be calculated by

\[
E[|\mathcal{R}_s^a(\tau)|^2|H_1] = E[\frac{1}{T} \int_{-T/2}^{T/2} r(t + \frac{\tau}{2})r^*(t - \frac{\tau}{2})e^{-j2\pi\alpha t}dt]^2,
\]

\[
\forall (\alpha, \tau) \neq (0, 0) = \frac{1}{T^2} \int_{-T/2}^{T/2} \int_{-T/2}^{T/2} E[r(t_1 + \frac{\tau}{2})r^*(t_1 - \frac{\tau}{2})]dt_1dt_2,
\]

\[
E[r(t_2 + \frac{\tau}{2})r^*(t_2 - \frac{\tau}{2})e^{-j2\pi\alpha(t_1-t_2)}]dt_1dt_2,
\]

\[\forall (\alpha, \tau) \neq (0, 0) = (\sigma_1^2 + \sigma_2^2)^2 + 3(\sigma_1^2)^2. \] \hspace{1cm} (23)

Therefore, the variance of \( \mathcal{R}_s^a(\tau) \) under \( H_1 \) is

\[\text{Var}(\mathcal{R}_s^a(\tau)|H_1) = E[|\mathcal{R}_s^a(\tau)|^2|H_1] - |E[\mathcal{R}_s^a(\tau)|H_1]|^2 = (\sigma_1^2 + \sigma_2^2)^2 + 3(\sigma_1^2)^2. \] \hspace{1cm} (24)

Thus, the cyclic autocorrelation function \( \mathcal{R}_s^a(\tau) \) obeys the Gaussian distribution as follows

\[H_1 : \mathcal{R}_s^a(\tau) \sim N(0, (\sigma_1^2 + \sigma_2^2)^2 + 3(\sigma_1^2)^2). \] \hspace{1cm} (25)

Since the Gaussian distribution function is a symmetric distribution function, we can find a pair of symmetric peaks \( R_1 \) and \( R_2 \) from \( \mathcal{R}_s^a(\tau) \) in theory, which satisfy (19). This means that the probability of detection of the SPS algorithm is statistically 1.

However, it is only statistically that the probability of false alarm is 0 and the probability of detection is 1. As well known, the noise in the wireless communication systems is not an ideal AWGN, no matter in practical system or in simulating system, as shown in Fig. 2. If the AWGN is ideal, the all of values of cyclic autocorrelation function should be equal to 0 except \( \tau = 0 \). Fig. 8 shows the slice of Fig. 2 at \( \alpha = 0 \). It is obvious that the values of cyclic autocorrelation function of simulating AWGN at \( \tau \neq 0 \) are not equal to 0, despite the amplitudes of values at \( \tau \neq 0 \) is much smaller than that at \( \tau = 0 \). Therefore, the probability of false alarm cannot be 0 and the probability of detection cannot be 1 in practice.

**VII. SIMULATIONS AND ANALYSIS**

In this section, we present some simulation results of the SPS algorithm and compare them with those of other algorithms. In the extensive simulations we execute in MATLAB,
we take the BPSK signal (single carrier signal) and OFDM signal (multi carrier signal) as the example PU signals. The bit rate is 256 kbit/s, the carrier frequency of the BPSK signal is 320 MHz, the carrier frequency of the OFDM signal is 512 MHz, the subcarrier space is 256 KHz, the number of subcarriers is 64, the duration interval of the OFDM symbol is 80 µs, and the valid symbol length is 64 µs. To ensure that the probability of false alarms is approximately 0.001, we set the significance level factor $\beta = 3$ in the SPS algorithm.

Fig. 9 compares the probabilities of detection of the SPS algorithm with those of the MME algorithm [10], FD-AC algorithm [28] and SKAB algorithm [29]. Obviously, the SPS algorithm has the highest detection probability among all algorithms. This result occurs because the decision statistic, i.e., the peak of the cyclic autocorrelation function of the signal received, concentrates more PU signal energy by using the cyclic autocorrelation. If the probability of detection required is 0.9, the SNRs in the SPS algorithm will be only $-14.32$ dB to detect the BPSK signal and $-14.2$ dB to detect the OFDM signal. The SNRs in the MME algorithm should be $-10.4$ dB to detect the BPSK signal and $-9.76$ dB to detect the OFDM signal. The SNR in the FD-AC algorithm should be $-10.4$ dB to detect the OFDM signal. The SNR in the SKAB algorithm should be $-7.08$ dB to detect the OFDM signal. The SPS algorithm has a margin of approximately 4 dB over the MME algorithm, 3.5 dB over the FD-AC algorithm and 7 dB over the SKAB algorithm. As the SNR increases, the superiority of the SPS algorithm becomes increasingly remarkable.

Fig. 10 illustrates the probabilities of false alarm of the four algorithms. Although we set $\beta = 3$ to ensure that the probability of a false alarm is less than 0.01, the simulated probability of a false alarm of the SPS algorithm is approximately 0.002, much lower than 0.01. The probabilities of false alarm of the other three algorithms are approximately 0.01. In the SPS algorithm, the decision statistic is the peak of the cyclic autocorrelation function of the signal received. It smooths the fluctuation of noise. Therefore, the SPS algorithm has the lowest probability of false alarm.

It can be seen from Fig. 9 and Fig. 10 that although the probabilities of detection and false alarm of the SPS algorithm are better than those of the others, there is still a gap relative to the theoretical values. This gap exists because the actual channel noise cannot be an ideal white Gaussian noise. However, the theoretical probabilities of detection and false alarm of the SPS algorithm show that the SPS algorithm would perform well in cognitive networks.

Fig. 11 shows the effect of noise fluctuation on the probabilities of detection of the four algorithms. All of the probabilities of detection with 4 dB of noise fluctuation are lower than those with 0 dB of noise fluctuation (no noise fluctuation), but the degrees of decline are different. Taking the detection of an OFDM signal as example, if the probability of detection required is 0.9, the degree of decline of the SPS algorithm is 0.24 dB, the degree of decline of the MME algorithm is 0.88 dB, the degree of decline of the FD-AC algorithm is 2.96 dB, and the degree of decline of the SKAB algorithm is 2.92 dB. Evidently, the SPS algorithm has more robustness to noise fluctuations than do the other three algorithms due to the decision statistics and decision threshold in the SPS algorithm being independent on the noise.

Fig. 12 gives a comparison of the probabilities of detection between multiple primary users and a single primary user. When we combine the OFDM signal and BPSK signal to form the PU signals, the probability of detection is apparently higher than that when only the OFDM signal or BPSK signal...
is present. The greater the number of primary users present in a cognitive radio network, the stronger the assumption (19) will be, the higher the probability of detection will be, and the closer it will be to the theoretical value.

In summary, the simulation results show that the proposed SPS algorithm is the most effective algorithm under a low SNR and unknown noise power variation. Whether a single carrier signal or a multicarrier signal is present, the SPS algorithm outperforms the MME, FD-AC, and SKAB algorithms, with a margin of approximately 4 dB.

VIII. DISCUSSIONS

A. EFFECT OF THE SIGNIFICANCE LEVEL FACTOR

In this section, we discuss the effect of the significance level factor $\beta$ on the spectrum sensing performance in the proposed algorithm.

Fig. 13 and Fig. 14 present the performance with respect to the detection of a BPSK signal when the significance level factor $\beta = 2$, 3, and 4. For a given PU signal, the smaller the value of $\beta$ is, the larger the probability $P( |R_k| > \mu_j + \beta \times \sigma_j, j, k = 1, 2 )$. It means that pairs of symmetric peaks occur more frequently, the detection probability is higher, and the probability of false alarm is higher.

From Fig. 13 and Fig. 14, we know that a smaller value of $\beta$ will result in a higher probability of detection and probability of false alarm simultaneously. Generally, we choose $\beta$ according to the probability of false alarm required.

B. COMPUTATIONAL COMPLEXITY

The computational complexities of the sensing algorithms are dependent on the multiplication operations of the algorithms. Table 1 gives a comparison of the MME, FD-AC, SKAB and SPS algorithms. It is shown that the computational complexity of the SPS algorithm is much lower than that of the FD-AC and SKAB algorithms and a little higher than that of the MME algorithm. However, the spectrum sensing performance of the SPS algorithm is better than that of the
TABLE 1. Computational complexities of the sensing algorithms.

| Algorithm | Multiplications | Multiplications with $M = 2048$, $N_r = 1$, $L = 6$, $K_{comp} = 242$, $M' < N_0 < M$ |
|-----------|-----------------|-----------------------------------------------------------------|
| MME       | $M^2 N_r M_l (N_r + 1)$ | 12248                                                                 |
| PD-AC     | $3 + 2 N_r K_{comp}$ | 41444                                                               |
| SKAB      | $2M' + 4M + 4N_0 + 11$ | 20299                                                              |
| SPS       | $8M_r + 16$        | 16400                                                              |

Note: $M$ is the number of samples used in the calculation, $N_r$ is the number of receivers, $L$ is the smoothing factor [10], $K_{comp}$ is the number of subcarriers used in the autocorrelation calculation [29], $M'$ is the number of samples recorded, and $N_0$ is the number of multiplication operations carried out in the conventional autocorrelation-based algorithm [30].

the probability of detection and probability of false alarm we will optimize the significance level factor according to the significance level factor would affect the probability of having considerable robustness to noise uncertainty. However, is insensitive to the noise power and is suitable for spectrum sensing. We introduce the significance level factor into the spectrum sensing to ensure that the required probability of false alarm is satisfied, and develop the symmetric peaks test criterion. The algorithm does not require any prior knowledge of the primary user signals or channel noise, is insensitive to the noise power and is suitable for spectrum sensing under a low SNR. The simulation results show that the proposed algorithm performs well under a low SNR and has considerable robustness to noise uncertainty. However, the significance level factor would affect the probability of detection and probability of false alarm. In our future work, we will optimize the significance level factor according to the probability of detection and probability of false alarm required.

IX. CONCLUSIONS
According to the cyclic autocorrelation property of the modulated signal, a novel spectrum sensing algorithm based on symmetric peaks is proposed. We determine the symmetric peaks from the characteristics of the cyclic autocorrelation function of modulated signals and use them as the statistics in spectrum sensing. We introduce the significance level factor into the spectrum sensing to ensure that the required probability of false alarm is satisfied, and develop the symmetric peaks test criterion. The algorithm does not require any prior knowledge of the primary user signals or channel noise, is insensitive to the noise power and is suitable for spectrum sensing under a low SNR. The simulation results show that the proposed algorithm performs well under a low SNR and has considerable robustness to noise uncertainty. However, the significance level factor would affect the probability of detection and probability of false alarm. In our future work, we will optimize the significance level factor according to the probability of detection and probability of false alarm required.

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