Optimal Energy Storage Operation Chart and Output Distribution of Cascade Reservoirs Based on Operating Rules Derivation

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Received: 20 July 2022 / Accepted: 19 September 2022 / Published online: 11 October 2022
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Abstract

An energy storage operation chart (ESOC) is one of the most popular methods for conventional cascade reservoir operation. However, the problem of distributing the total output obtained from the ESOC has not yet been reasonably solved. The discriminant coefficient method is a traditional method for guiding the output distribution by determining the order of reservoir supply or storage; however, it cannot quantify the water used in operation. Thus, this study develops a new output distribution model using a polynomial fitting method and an artificial neural network to express the functional relationship derived from the deterministic optimization results of long-term runoff series to maximize power generation. Cascade reservoirs of the lower reaches of the Jinsha River in China were selected for the case study. Compared to the discriminant coefficient method, the proposed method can rationally distribute the total output, thus avoiding the problem of concentrated deserting water in downstream reservoirs that occurs in the discriminant coefficient method. In general, this study proposes an effective alternative method to guide cascade reservoir operation.

Keywords Hydropower operation · Energy storage operation chart · Output distribution · Polynomial fitting method · Artificial neural network · Operating rules derivation

1 Introduction

As a sustainable clean energy source, hydropower has the unique advantages of being flexible, renewable, and rich, and plays an important role in flood control, water supply, power generation, and some social and economic benefits (Labadie 2004). Hydropower operation is a complex nonlinear problem because of the existence of complementary and hydraulic...
head effects (Cheng et al. 2008; Zeng et al. 2014). The reservoir operation problem is expressed by an optimization model in the form of standard mathematical programming and solved by optimization algorithms. Many researchers have studied different optimization algorithms for long-term power generation scheduling, which can be divided into deterministic and stochastic optimization methods (Labadie 2004). Deterministic optimization methods regard the runoff process as known, and the dispatching process is past-oriented. Unfortunately, because of the limitation of runoff forecast accuracy, deterministic optimization methods cannot be used to directly guide actual reservoir operation (Chen 2021; Wang et al. 2022). Thus, stochastic optimization, which involves operations under the probabilistic descriptions of stochastic streamflow processes without the presumption of perfect inflow forecasting, has been proposed and has received considerable attention (Labadie 2004).

As a popular method for guiding actual reservoir operation when future runoff is unknown, the reservoir operation chart is effective and simple; thus, it has been widely used (Li et al. 2022; Liu et al. 2011). A conventional reservoir operation chart is drawn by inverse calculation using hydrological inflows in typical years; however, the optimal reservoir operation chart is optimized with one or more objectives, which effectively utilizes streamflow data and operates without perfect inflow forecasting (Wang et al. 2010). With the rapid development of the reservoir group scale in recent decades, the main line of research on reservoir operation charts has been developed from a single reservoir to a multiple reservoir system. In this way, a few studies optimized the single reservoir operation chart for the objective of maximizing the cascade benefits; however, it is still an application on a single reservoir and is limited (Wang et al. 2010). Based on the concept of reservoir state described by energy storage, an energy storage operation chart (ESOC) was proposed (Ji et al. 2014; Liu et al. 2019).

The ESOC is similar to a single operation chart but determines the total output of the reservoir system according to the energy storage and current period of the cascade system. There are several ESOC studies about drawing, application, and optimization. Liu et al. (2019) studied the key issue of typical dry year selection and drawdown level optimization and then optimized the output coefficients of ESOC using the progressive optimality algorithm (POA). Cheng et al. (2010) derived the ESOC from single-reservoir operation charts and distributed the output of each reservoir according to the gap value of ESOC and single-reservoir operation chart results. Ji et al. (2014) drew the ESOC using the discriminant coefficient method and established two models with the maximum guarantee rate objective and the maximum power generation objective, respectively. Although various degrees of success have been achieved, current studies still adopt the traditional discriminant coefficient method to distribute the output. Given the total output obtained by the ESOC, the discriminant coefficient method determines the order of the reservoir supply or storage; thus, the output of each reservoir can be confirmed. However, in the cascade system guided by the discriminant coefficient method, the upstream reservoirs always supply water before the downstream reservoirs, which leads to a lack of water in the upstream reservoirs and much desert water in the downstream reservoirs. In addition, the output distribution by the discriminant coefficient method assesses the information from the current stage and ignores the utilization of historical hydrologic inflow records and deterministic optimization results (Ji et al. 2014; Jiang et al. 2014).

This study aims to develop a new output distribution model for the optimized ESOC to overcome the defects of the discriminant coefficient method, which is derived from the deterministic optimization results of long-term runoff series to maximize power generation. The remainder of this paper is organized as follows. Section 2 introduces the primary
methods used in this study, including the method of drawing the ESOC, deterministic optimization model, ruler derivation method for output distribution, and the optimization method of the ESOC. The case study is given, and some analyses are extracted in Sect. 3. The conclusions are drawn in Sect. 4.

2 Methodology

2.1 Drawing and Simulation of ESOC

The ESOC is an effective method for medium- and long-term joint dispatching problems, which can make full use of cascade benefits (Jiang et al. 2016; Liu et al. 2019). In the traditional operation chart, the output curves divide the chart into several zones, including the guaranteed, reduced, and increased output zones. The ESOC adopts the same output zone division method as the traditional operation chart. To accurately describe the current state of cascade reservoirs, the ESOC determines the total output of the reservoir system according to the energy storage and current period to guide the reservoir operation decision (Jiang et al. 2016).

The discriminant coefficient method is a traditional method for determining the order of storage and water supply and is always used to draw the ESOC (Ji et al. 2014; Jiang et al. 2019). However, the discriminant coefficient method has some shortcomings in drawing ESOC, which are as follows. (1) It is difficult to select typical dry years because of the influence of human factors. (2) Determining the guaranteed output value is difficult. Therefore, this study draws the ESOC based on the single-reservoir operation chart of each reservoir.

The single-reservoir operation chart of each reservoir could be drawn as the steps which are described by Cheng et al. (2010). Then, each operation line in each simulated single-reservoir operation chart operates in turn to obtain the effective storage capacity and accumulated water head of downstream reservoirs. The corresponding energy storage value of the reservoir is then calculated according to the effective water storage and accumulated water head, and the total energy storage value of the cascade reservoirs can be obtained by summing the energy storage value of the operation line of each reservoir. The formula for cascade energy storage can be expressed as follows:

\[ E = \lambda \cdot \sum_{i=1}^{n} V_i \cdot \sum_{j=i}^{n} \frac{(H_j + H_d)}{2} \]

where \( H_j \) is the current water head of the \( j \)th reservoir in the cascade reservoirs, \( H_d \) is the corresponding water head to the dead-water level, \( V_i \) is the current regulating capacity of the \( i \)th reservoir in the cascade reservoirs, \( \lambda \) is a coefficient, and \( n \) is the reservoir number.

2.2 Deterministic Reservoir Operation Optimization Model

2.2.1 Objective Function

The total power generation and the total deserted outflow in cascade reservoirs are the two indexes that people care about most. There is a synergistic and competitive relationship between the two objectives. The two objectives are as follows:
1. Maximizing the total power generation

$$\max E = \sum_{i=1}^{T} \sum_{i=1}^{M} N_{i,t} \Delta T_i = \sum_{i=1}^{T} \sum_{i=1}^{M} K_{i,t} O_{i,t} H_{i,t} \Delta T_i$$  \hspace{1cm} (2)$$

where $E$ is the total hydropower generation produced in the $T$ periods and $M$ reservoirs; $N_{i,t}$ is the output of reservoir $i$ in period $t$; $K_{i,t}$, $H_{i,t}$, and $O_{i,t}$ are the power production coefficient, water head, and outflow of reservoir $i$ in period $t$, respectively; $\Delta T_i$ is the length of period $t$.

2. Minimizing the total deserted outflow

$$\min S_i = \sum_{t=1}^{T} \sum_{i=1}^{M} \left( O_{i,t} - Q_{i,t} \right)$$  \hspace{1cm} (3)$$

where $S_i$ is the total deserted outflow in the $T$ periods and $M$ reservoirs and $Q_i$ is the generating discharge.

### 2.2.2 Operation Constraints

The optimal operation of a multi-reservoir system is a complex constrained optimization problem. The following constraints on the stations should be considered.

1. Water head equation

$$H_t = \frac{Z_{t-1} + Z_t}{2} - Z_{down}$$  \hspace{1cm} (4)$$

where $H_t$ is the water head at period $t$, $Z_{t-1}$ and $Z_t$ are the initial water levels at periods $t-1$ and $t$, respectively, and $Z_{down}$ is the tail water level at period $t$.

2. Water balance constraint

$$V_t = V_{t-1} + (I_t - O_t) \Delta T_t$$ \hspace{1cm} (5)$$

$$O_t = S_i + Q_t$$ \hspace{1cm} (6)$$

where $V_{t-1}$ and $V_t$ are the initial reservoir storage at periods $t-1$ and $t$, respectively, and $I_t$ is the inflow at period $t$.

3. Water level constraint

$$Z_{min} \leq Z_t \leq Z_{max}$$ \hspace{1cm} (7)$$

$$|Z_t - Z_{t-1}| \leq \Delta Z$$ \hspace{1cm} (8)$$

where $Z_{min}$ and $Z_{max}$ are the minimum and maximum water levels at period $t$, respectively, and $\Delta Z$ is the maximum amplitude of water level variation.

4. Discharge constraint

$$O_{min} \leq O_t \leq O_{max}$$ \hspace{1cm} (9)$$

where $O_{min}$ and $O_{max}$ are the minimum and maximum outflow at period $t$, respectively.
5. Output constraint

\[ N_{\text{min}}^t \leq N_t \leq N_{\text{max}}^t \]  

(10)

where \( N_{\text{min}}^t \) is the minimum output at period t, and \( N_{\text{max}}^t \) is the maximum output at period t, which is determined by the expected output limit and the installed capacity limit.

6. Boundary condition

\[ Z_0 = Z_{\text{begin}}, Z_T = Z_{\text{end}} \]  

(11)

where \( Z_{\text{begin}} \) and \( Z_{\text{end}} \) are the water levels at the beginning and end of the calculation period, respectively.

This model aims to maximize the total power generation benefits and minimize the total deserted outflow under the defined constraints and boundary conditions. This means that the water levels at the beginning and end of the entire calculation period were known. These results show that the long-term power generation scheduling of cascade reservoirs is a deterministic optimization problem.

### 2.2.3 Optimization Method

The joint scheduling of a hydropower station group is a typical high-dimensional, non-linear, and strongly coupled complex constraint optimization problem with a close relationship between the hydraulic and electric power between adjacent reservoirs (Wan et al. 2020, 2021). Traditional deterministic optimization intelligent algorithms, such as DP, cannot resolve the problem of the so-called curse of dimensionality, particularly in cascade reservoirs (Wang et al. 2022). As a popular method in recent years, there are also some inevitable defects in intelligent algorithms, such as the narrow scope of application, difficulty in determining parameters, and local optimum problems. Thus, successive linear programming is appealing because of the availability of efficient linear programming solvers (Labadie 2004).

In this study, the Gurobi solver was adopted to establish a nonlinear model for the cascade reservoir optimization dispatching problem. Gurobi is a large-scale mathematical programming optimization solver developed by the Gurobi company for solving linear programming, integer programming, and nonlinear programming, and is widely used in many fields (Anand et al. 2017; Meng et al. 2019). Gurobi has many advantages in reservoir-dispatching problems such as rapid, stable, and multi-objective functions.

### 2.3 Output Distribution Model by Rules Derivation

#### 2.3.1 Polynomial Fitting

The polynomial fitting method has been widely used in many fields owing to its advantages, such as a clear structure, strong fitting ability, and reliable epitaxial accuracy. Thus, a polynomial fitting method was applied to express the functional relationship between the total output of the reservoir system \( N_{\text{total}} \) and the output distribution of the corresponding \( N \). Considering the obvious difference between the storage and drawdown periods, the time parameter \( t \) was introduced into this model to express
the difference. To avoid the problem of imbalanced fitting ability and generalization induced by the complex structure, the maximum number of fitting times was limited to five. Thus, the functional relationship can be expressed as follows.

\[ N = f(N_{total}, t) \]  

(12)

**2.3.2 Artificial Neural Network**

An artificial neural network (ANN) is a flexible mathematical structure that can learn and generalize from experiences inspired by the study of biological systems (Buyukyildiz et al. 2014). In recent years, ANN has been used for a wide variety of tasks in many different fields, including forecasting (Coulibaly et al. 1999; Sharghi et al. 2018). The universal approximation theorem has proven that a multi-layer ANN can describe any given continuous function when there are enough neuron nodes (Csáji 2001).

To maximize the fitting performance of the ANN and avoid the overfitting problem, this study used the mean square error of the fivefold cross-validation as the objective function to optimize the neural network. The sample data were standardized before being input into the model. The standardized formula is as follows:

\[ X'_i = \frac{X_i - \bar{X}}{\sigma} \]  

(13)

where \( X_i \) is the \( i \)th value before standardization, \( \bar{X} \) is the average value of \( X \), \( \sigma \) is the standard deviation of \( X \), and \( X'_i \) is the standardized data.

**2.3.3 Evolution Index**

To fully assess the performance of the curve fitting model and ANN, the following indexes are introduced for comparison: the correlation coefficient (\( R \)) and root mean square error (\( RMSE \)).

\( R \), an index ranging from \(-1\) to \(1\), describes the linear relationship between the forecasted data and observed data. The closer the absolute value of \( R \) is to \(1\), the closer the linear relationship between the forecasted and observed data. The definition of \( R \) is given as

\[ R = \frac{\sum_{i=1}^{n} (X_i - \bar{X})(X'_i - \bar{X}')} {\sqrt{\sum_{i=1}^{n} (X_i - \bar{X})^2(X'_i - \bar{X}')^2}} \]  

(14)

where \( X_i \) and \( \bar{X} \) are the \( i \)th value and average of the observed data, respectively; \( X'_i \) and \( \bar{X}'_i \) are the \( i \)th value and average of the forecasted data, respectively; and \( n \) is the size of the dataset for evaluation.

The \( RMSE \) is a reliability estimate index that can effectively measure the total differences between the forecasted data and observed data. The smaller the \( RMSE \), the more reliable the prediction. The \( RMSE \) is defined as
RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X}_i)^2}. \quad (15)

2.4 Optimization of Output Coefficients for Energy Storage Operation Chart

The progressive optimality algorithm (POA) is an improved dynamic programming algorithm proposed by Howson and Sancho (1975). In this study, the initial ESOC was optimized using a POA to improve the utilization efficiency of cascade water resources. POA transforms a complex multistage decision-making problem into a series of two-stage decision-making problems, which matches the problem of reservoir operation chart optimization (Chen 2021; Zhang et al. 2016b). The specific steps of the optimization of ESOC with POA are described by Jiang et al. (2014).

The panoramic view of the methodology used in this study is shown in Fig. 1.

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**Fig. 1** Panoramic view of the methodology used in this study
3 Case Study

3.1 Basin Introduction and Basic Data

The Jinsha River, which is the most important part of the upper Yangtze River in China, flows through the Qinghai, Tibet, Sichuan, and Yunnan provinces (Zhang et al. 2016a). The length of the Jinsha River is 3500 km, and the natural drop is approximately 5100 m. The average annual flow is 4920 m$^3$/s, with an annual runoff volume of 155 billion m$^3$, accounting for 16% of the total water in the Yangtze River. The lower reaches of the Jinsha River have the most abundant hydropower resources in the Jinsha River Basin and even the Yangtze River Basin, with a total length of 783 m and a natural drop of 729 m. In 2021, the Baihetan hydropower station began to operate, and giant cascade reservoirs were formed (Zhang and Huang 2020; Zhu et al. 2021).

At present, there are four hydropower stations in the lower reaches of the Jinsha River: Wudongde, Baihetan, Xiluodu, and Xiangjiaba, with a total installed capacity of approximately 40 million kW and an annual power generation of approximately 180 billion kWh. In this case, cascade reservoirs in the lower reaches of the Jinsha River were considered as the research object. Their geographical locations are shown in Fig. 2, and the characteristic data for each reservoir are listed in Table 1.

3.2 Results and Discussion

3.2.1 Deterministic Optimization Results

To avoid the influence of station operation in the upper reaches, this study calculated the reduction in runoff from July 1959 to June 2013 for a total of 54 years. In the cascade reservoirs, consisting of seasonal regulating reservoirs, the flood control level was determined as the end-of-year drawdown level of the reservoir for more electricity generation.
A nonlinear optimal operation model for cascade reservoirs was established and resolved using Gurobi.

Figure 3 shows the output distribution of the four reservoirs as a percentage of the total output in different months for the 54 years of deterministic optimization results. It can be observed that the percentages of output distribution in Wudongde and Baihetan increase after the flood season and gradually decrease before the flood season of the next year, and the percentages of output distribution in Xiluodu and Xiangjiaba have an inverse expression. This is because the upstream reservoirs have already dropped the water level below the flood control level at the end of the non-flood season, and a low head leads to a low power generation efficiency.

### 3.2.2 Output Distribution Models

The above constructed deterministic optimal operation model with the cascade hydropower reservoir drawdown law is employed in this section. The total output of the reservoir system and the output distribution of the corresponding single reservoir were derived from a

| Item                  | Unit | Wudongde | Baihetan | Xiluodu | Xiangjiaba |
|-----------------------|------|----------|----------|---------|------------|
| Normal level          | m    | 975      | 825      | 600     | 380        |
| Dead level            | m    | 945      | 765      | 540     | 370        |
| Flood control level   | m    | 952      | 785      | 560     | 370        |
| Installed capacity    | MW   | 10,200   | 16,000   | 12,600  | 6000       |
| Guaranteed output     | MW   | 2290     | 5075     | 3795    | 2009       |
| Regulation ability    |       | Seasonal | Seasonal | Seasonal | Seasonal   |

![Fig. 3](image-url) Output distribution of the four reservoirs as a percentage of the total output in different months
long series of simulation results. Considering the obvious difference between the storage and drawdown periods, the time parameter was introduced into this model to express the difference. Thus, the relationship between the output distribution, total output, and time parameter is described by the polynomial fitting method and the ANN as follows.

In this study, 928 probable functions were used for polynomial fitting, which was optimized using the Levenberg–Marquardt method. The polynomial fitting formula for each reservoir was determined, as shown in Table 2, where \( N \) is the output distribution of the corresponding reservoir, \( N_{total} \) is the total output of the reservoir system, \( t \) is the number of periods, and \( p_{1-9} \) are the parameters to be optimized. Figure 4 shows the fitting results for each reservoir using the polynomial fitting method.

Approximately 70% of the dataset was used for training and validation, while the rest was used for testing. This study used the mean square error of the fivefold cross-validation as the objective function to optimize the neural network. In addition, the number of neurons in the hidden layer ranges was determined to be 100, and the optional activation function of the neurons was determined as a rectified linear unit (ReLU). To improve the fitting performance and avoid numerical problems, the sample data were standardized in the optimization process. Figure 4 shows the fitting results of each reservoir using the ANN.

For a full comparison, Table 3 lists the evaluation indexes of the fitting results obtained by the two methods. It can be observed that the ANN outperforms the polynomial fitting method in various evaluation indexes at almost every reservoir, demonstrating the superiority of the ANN.

### 3.2.3 Optimization of Energy Storage Operation Chart

The ESOC based on the optimization method of the cascade energy storage scheduling diagram above is shown in Fig. 5. Table 4 shows the results of the deterministic optimization and optimal ESOCs, where ESOC-1 denotes the ESOC model allocating output by polynomial fitting, ESOC-2 denotes the ESOC model allocating output by ANN, and ESOC-3 denotes the ESOC model allocating output by the discriminant coefficient method. The average annual power generation in the cascade reservoirs of ESOC-1 and ESOC-2 are 1994.12 and 1992.98 kWh, respectively, which are greater than those of ESOC-3. This result indicates that the proposed output distribution method is valid in most cases with different inflows.

In addition, Table 4 shows the gap in power generation between the deterministic optimization and ESOCs results. The gap in total power generation and total deserting water

### Table 2 Functional expression obtained by the polynomial fitting method

| Reservoirs | Fitting formula |
|------------|----------------|
| Wudongde   | \( N = \frac{p_1 + p_2 \cdot N_{total} + p_3 \cdot N_{total}^2 + p_4 \cdot t + p_5 \cdot t^2}{1 + p_6 \cdot N_{total} + p_7 \cdot t + p_9 \cdot t^3} \) |
| Baihetan   | \( N = p_1 + p_2 \cdot N_{total} + p_3 \cdot N_{total}^2 + p_4 \cdot N_{total}^3 + p_5 \cdot t + p_6 \cdot t^2 + p_7 \cdot t^3 + p_8 \cdot t^4 + p_9 \cdot t^5 \) |
| Xiluodu    | \( N = p_1 + p_2 \cdot N_{total} + p_3 \cdot N_{total}^2 + p_4 \cdot N_{total}^3 + p_5 \cdot t + p_6 \cdot t^2 + p_7 \cdot t^3 + p_8 \cdot t^4 + p_9 \cdot t^5 \) |
| Xiangjiaba | \( N = p_1 + p_2 \cdot N_{total} + p_3 \cdot N_{total}^2 + p_4 \cdot N_{total}^3 + p_5 \cdot t + p_6 \cdot t^2 + p_7 \cdot t^3 + p_8 \cdot t^4 + p_9 \cdot t^5 \) |
between ESOC-2 and deterministic optimization (i.e., −0.96% and 8.41%, respectively) is better than the gap between ESOC-1 and deterministic optimization (i.e., −1.02% and 9.24%, respectively). Considering that the ANN outperforms the polynomial fitting method.

Fig. 4 Fitting results of each reservoir by (a) polynomial fitting method and (b) ANN
in various evaluation indexes, the better the evolution indexes of the fitting method, the better the result of the operation simulation.

In addition, it was found that the gap in power generation between the deterministic optimization result and the ESOC result obtained by the discriminant coefficient method increased in the downstream reservoirs. This is because the discriminant coefficient method causes the reservoir with a smaller value of the discriminant coefficient to be prematurely empty and generate less power. The proposed output distribution method can effectively solve this problem by determining the drawdown sequence and output value to avoid concentrated deserting water in downstream reservoirs.

For further analysis, Fig. 6 shows a box plot of the two models for different months. “diff-1” and “diff-2” represent the difference between ESOC-1 and the deterministic optimization and the difference between ESOC-2 and the deterministic optimization in terms of monthly power generation, respectively. It can be noted that in both methods, the median of the difference between the ESOCs and the deterministic optimization has a peak in the period of drawdown (August) and the period of water storage (April, May, and June). Meanwhile, the months with a large median had a large box. In particular, uncertainty is the main factor that influences the difference between ESOCs and deterministic optimization.

![Fig. 5 Optimal total output operation chart of the reservoir system](image_url)
### Table 4  Comparison of the deterministic optimization and ESOC results of power generation and deserting water by different output distribution methods

| Assessment Index                  | Reservoir | Deterministic optimization | ESOC-1 | ESOC-2 | ESOC-3 |
|-----------------------------------|-----------|---------------------------|--------|--------|--------|
|                                   |           | Value                     | gap (%)| Value  | Gap (%)| Value  | Gap (%)|
| mean power generation (10⁸ kWh)   | Wudongde  | 387.96                    | 0.17%  | 389.58 | 0.42%  | 384.69 | –0.84%|
|                                   | Baihetan  | 631.76                    | –1.09% | 623.26 | –1.35% | 625.22 | –1.04%|
|                                   | Xiluodu   | 647.4                     | –0.98% | 642.32 | –0.78% | 621.32 | –4.03%|
|                                   | Xiangjiaba| 346.31                    | –2.27% | 338.96 | –2.12% | 323.57 | –6.57%|
|                                   | cascade   | 2013.43                   | –1.02% | 1994.12| –0.96% | 1954.8 | –2.91%|
| mean deserting water (10⁸ m³)     | Wudongde  | 97.51                     | 10.79% | 108.03 | 10.79% | 107.23 | 9.97% |
|                                   | Baihetan  | 101.82                    | –9.17% | 92.88  | –8.78% | 99.77  | –2.01%|
|                                   | Xiluodu   | 170.55                    | 25.07% | 207.18 | 21.48% | 243.97 | 43.05%|
|                                   | Xiangjiaba| 245.44                    | 5.26%  | 258.96 | 5.51%  | 303.06 | 23.48%|
|                                   | cascade   | 615.32                    | 9.24%  | 667.05 | 8.41%  | 754.03 | 22.54%|
Considering that the nonlinear model and ESOC adopted in this study have good stability, the uncertainty should be mainly composed of the amount and shape of the inflow. Thus, an effective way to reduce the difference is to consider the uncertainty of inflow.

4 Conclusions

This study presents an ESOC with an optimal output distribution method for a cascade reservoir hydropower system by deriving distribution rules from the deterministic optimization results. Cascade reservoirs in the lower reaches of the Jinsha River of China (Wudongde, Baihetan, Xiluodu, and Xiangjiaba) were selected as a case study, and the major findings are summarized as follows.

1. The ANN outperformed the polynomial fitting in the evaluation indexes at each reservoir. The optimal ESOCs adopted the methods of polynomial fitting and ANN to distribute the output obtained from the average annual power generation of 1992.98 and 1994.12 kWh, respectively.
2. The proposed method can effectively optimize the ESOC by guiding the order of drawdown and determining the value of the output to avoid the problem of concentrated deserting water in downstream reservoirs, which usually occurs in the discriminant coefficient method.
3. The uncertainty of runoff is the main factor influencing the difference between ESOCs and deterministic optimization; thus, an effective way to reduce the difference should consider the forecasted runoff.
**Author Contribution** Yuxin Zhu, Jianzhong Zhou, and Yongchuan Zhang: conceptualization, methodology, supervision, writing, investigation, funding acquisition, and programming. Zhiqiang Jiang, Benjun Jia, and Wei Fang: review, formal analysis, and visualization. Shuai Liu: data curation and programming.

**Funding** This study was financially supported by the Natural Science Foundation of China (52179016), the Natural Science Foundation of Hubei Province (2021CFB597), and the Key Program of the National Natural Science Foundation of China (U1865202).

**Availability of Data and Materials** All data and codes are available from the corresponding author.

**Declarations**

**Ethics Approval** The author promises to comply with Ethical Standards.

**Consent to Participate** Not applicable.

**Consent to Publish** Not applicable.

**Competing Interests** The authors declare no competing interests.

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