ERG and Reggeon Field Theory

Carlos Contreras H.
Departamento de Física, Universidad Tecnica Federico Santa María, Valparaíso, Chile
E-mail: carlos.contreras@usm.cl

Abstract. In this paper we present our recent non perturbative functional renormalization group analysis of Reggeon Field Theory to the interactions of Pomeron and Odderon fields. We establish the existence of a fixed point and its universal properties, which exhibits a novel symmetry structure in the space of Odderon-Pomeron interactions. We briefly discuss the implications of our findings for the existence of an Odderon in high energy scattering.

1. Introduction
In recent papers [1, 2] we have started an analysis of the flow equations of reggeon field theory (RFT) [3, 4, 5, 6, 7, 8], following the idea that RFT might provide a good description of strong interactions in the Regge limit and infrared region: rapidity \( Y \to \infty \) and transverse distances (impact parameter) \( |x_\perp| \to \infty \). Using the Wetterich formulation of the functional renormalization group equations [9, 10] we can study the problem in two transverse dimensions. As our main result [1] we have established the existence of a critical theory with one relevant direction: in the multidimensional space of the parameters of the effective potential, there exists one direction which is UV attractive (IR repulsive), whereas all other directions are IR attractive. We have verified the properties that RFT is in the same universal class with the simplest directed percolation model in statistical physics [11].

This investigation of RFT should be seen as a first step in searching for an effective theory which describes the high energy Regge limit of QCD. Regge theory is being used for analyzing the nonperturbative \( pp \) scattering at the Tevatron (FermiLab), \( pp \) scattering at the ISR (CERN), RHIC (BNL) and at the LHC (CERN), and \( \gamma p \)-scattering at HERA (DESY). On the perturbative side, high energy QCD has been analyzed using Regge theory (in particular, the BFKL Pomeron with various applications in \( e^+ e^- \) scattering, forward jets in \( \gamma^* p \) scattering, and Mueller-Navelet jets in \( pp \) or \( p\bar{p} \)). Whereas the first group of high energy scattering processes is characterized by transverse distances of hadronic sizes, the second one addresses scattering processes of small transverse diameters. This suggests to search, in the space of 2 + 1 - dimensional reggeon field theories, for an interpolation between the two domains: for long transverse distances the Pomeron field has intercept very close to unity and a nonzero \( t \)-slope, for short transverse distances the BFKL intercept is significantly above one, and the slope is very small.

Within such a program in [1] we have restricted our analysis to one reggeon field, the Pomeron field. However, there exists one other Regge singularity the Odderon with intercept at or near one. In the nonperturbative region, the search for the Odderon has stimulated a long-lasting debate: the strongest evidence for its existence comes from the observed difference in the dip structure in the \( t \)-dependence of elastic cross section of \( pp \) or \( p\bar{p} \) scattering. In contrast, in the perturbative region the existence of the Odderon is well-established [12]: in nonabelian \( SU(3) \) gauge theory bound states of reggeized gluons exist: the BFKL Pomeron [13] and the Odderon. These two states represent the two equally important fundamental bound states of the \( SU(3) \)
gauge theory. Whereas the BFKL intercept is well above one, the odderon intercept has been found to be exactly at one [14, 15, 16, 17]. Self interactions of the Pomeron [18, 19, 20, 21] as well as interactions between Pomeron and Odderon naturally appear in perturbative QCD analysis [22, 23]. Analogous results are obtained also in the Color Glass Condensate, dipole and Wilson line approach [24, 25, 26].

The existence of the perturbative region motivates interest in the question whether the Odderon exists also in the nonperturbative region. To explore the connection between the UV region and the nonperturbative IR region we have to consider the Odderon, the IR fixed point structure should confirm whether the Odderon survives the flow from UV to IR. Also, such an analysis should provide information on the interaction between Odderon and Pomeron. In this paper we therefore extend our previous analysis to interactions of two fields, Pomeron and Odderon and it is organized as follows. In section 2 we describe our setup and in the next section we discuss first implication for real physics. In the last section we present numerical results. In a concluding section we discuss first implication for real physics.

2. The setup

In the following we consider interactions between Pomeron and Odderon fields. As before, $\psi, \psi^\dagger$ denote the Pomeron field, and for the Odderon we introduce the field $\chi, \chi^\dagger$. The effective action $\Gamma_k(\psi, \psi^\dagger, \chi, \chi^\dagger)$ has the form:

$$\Gamma_k = \int d^2x d\tau \left( Z_P \left( \frac{1}{2} \dot{\psi}^\dagger \dot{\psi} - \alpha_P \psi^\dagger \nabla^2 \psi \right) + Z_O \left( \frac{1}{2} \dot{\chi}^\dagger \dot{\chi} - \alpha_O \chi^\dagger \nabla^2 \chi \right) + V_k(\psi, \psi^\dagger, \chi, \chi^\dagger) \right),$$

(1)

where $D$ denotes the number of spatial dimension. $D = 2$ is the physical case of our interest, but for our analytic study we find it convenient to keep $D$ as a continuous parameter. For the lowest truncation the effective action takes the form:

$$V_3 = -\mu_P \psi^\dagger \psi + i\lambda (\psi^\dagger \psi) \psi - \mu_O \chi^\dagger \chi + i\lambda_2 \chi^\dagger (\psi + \psi^\dagger) \chi + \lambda_3 (\psi^\dagger \chi^2 + \chi^\dagger \psi^2).$$

(2)

but if we will study high order truncation we can consider the quartic truncation:

$$V_4 = \lambda_{41} (\psi \psi^\dagger)^2 + \lambda_{42} \psi \psi^\dagger (\psi^2 + \psi^\dagger)^2 + \lambda_{43} (\chi \chi^\dagger)^2 + i\lambda_{44} \chi \chi^\dagger (\chi^2 + \chi^\dagger)^2$$

$$+ i\lambda_{45} \psi \psi^\dagger (\chi^2 + \chi^\dagger)^2 + \lambda_{46} \psi \psi^\dagger \chi \chi^\dagger + \lambda_{47} \chi \chi^\dagger (\psi^2 + \psi^\dagger)^2$$

(3)

and the quintic truncation has the following eleven terms:

$$V_5 = i \left( \lambda_{51} (\psi \psi^\dagger)^2 (\psi + \psi^\dagger) + \lambda_{52} \psi \psi^\dagger (\psi^3 + \psi^\dagger^3) + \lambda_{53} \chi \chi^\dagger (\psi^3 + \psi^\dagger^3) + \lambda_{54} \psi \psi^\dagger \chi \chi^\dagger (\psi + \psi^\dagger) \right)$$

$$+ \lambda_{55} (\chi^2 \psi^\dagger)^2 + \lambda_{56} (\chi^2 \psi^\dagger \psi + \chi^\dagger \psi^\dagger \psi^2) + i\lambda_{57} (\chi^2 \psi^\dagger \psi^2 + \chi^\dagger \psi^\dagger \psi^2)$$

$$+ i \left( \lambda_{58} (\chi^4 \psi^\dagger + \chi^\dagger \psi^\dagger) + \lambda_{59} (\chi \chi^\dagger)^2 (\psi + \psi^\dagger) \right)$$

$$+ \lambda_{510} \chi \chi^\dagger (\chi^2 \psi + \chi^\dagger \psi^\dagger) + \lambda_{511} \chi \chi^\dagger (\chi^2 \psi^\dagger + \chi^\dagger \psi).$$

(4)

Note the differences in the structure of the effective potential compared to the pure Pomeron case. As described in [1, 2], for the pure Pomeron case the couplings are real-valued for even powers of the Pomeron fields, whereas odd powers require imaginary couplings. This is a consequence of the even-signature of the Pomeron exchange. Since the Odderon has negative signature the situation is different. Considering signature conservation, $t$-channel states with an odd number of Odderons never mix with pure Pomeron channels and the transition $P \to OO$ is real valued:

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the two-Odderon cut is positive (in contrast to the two Pomeron cut), and there is no need for an imaginary coupling. On the other hand, the transition $O \rightarrow OP$ has to be imaginary, since the Odderon-Pomeron cut carries a minus sign. As a result, all triple couplings are imaginary, except for the real-valued transition $P \rightarrow OO$.

In the sector of quartic couplings, all couplings involving Pomerons only are real-valued. Once the Odderon is included, again most quartic couplings remain real, but there are two exceptions: the transitions $O \rightarrow OOO$ and $P \rightarrow P + OO$ are imaginary. This can be easily understood considering a contribution to such quartic vertices coming by the composition of two triple ones. For the quintic part the ‘exceptional’ terms are in the second and fourth lines: in all these terms we either create or annihilate a pair of Odderons.

The signature-conservation rule, together with the structure of the quartic and quintic couplings of the effective potential suggests the following transition rules:

(i) states with even and odd numbers of Odderon never mix.
(ii) states will be labelled by the number of Odderon pairs, $n$. We assign a quantum number $O_n$. Signature rules imply that transitions changing $n$ by odd numbers come with ‘exceptional’ couplings (e.g. the transitions $P \rightarrow OO$, $O \rightarrow OOO$, or $P \rightarrow P + OO$), whereas transitions changing $n$ by even numbers are ‘normal’ and have the same structure as pure Pomeron couplings (e.g., the transition: Pomeron to four Odderons is imaginary).

In the perturbative region, the transition $P \rightarrow OO$ has been computed [22, 23] and found to be nonzero. As one of our task is to verify this result.

Another important element is the introduction of dimensionless variables. The field variables are rescaled as: $\psi = Z^{1/2}_P k^{-D/2} \psi$, $\tilde{\chi} = Z^{1/2}_O k^{-D/2} \chi$ and for the potential we introduce $\tilde{V} = \frac{V}{\alpha'_p k^{D/2}}$. This choice implies that we introduce the dimensionless ratio $r = \frac{\alpha'_O}{\alpha'_p}$, and the Odderon slope $\alpha'_O$ will be written as

$$\alpha'_O = r \alpha'_p.$$ (5)

Finally, the couplings are rescaled in the following way:

$$\bar{\mu}_P = \frac{\mu_p}{Z_P \alpha'_p k^2}, \quad \bar{\mu}_O = \frac{\mu_O}{Z_O \alpha'_p k^2}, \quad \bar{\lambda} = \frac{\lambda}{Z_P^{3/2} \alpha'_p k^2} k^{D/2}, \quad \bar{\lambda}_{2,3} = \frac{\lambda_{2,3}}{Z_O Z_P^{1/2} \alpha'_p k^2} k^{D/2}. \quad (6)$$

With these definitions the classical scaling (canonical) of the potential which would result by neglecting the quantum fluctuations in the flow equation becomes:

$$\left( - (D + 2) + \zeta_P \right) \tilde{V} + \left( \frac{D}{2} + \frac{\eta_p}{2} \right) \left( \tilde{\psi} \frac{\partial \tilde{V}}{\partial \tilde{\psi}} + \tilde{\psi} \frac{\partial \tilde{V}}{\partial \tilde{\psi}^1} \right) + \left( \frac{D}{2} + \frac{\eta_O}{2} \right) \left( \tilde{\chi} \frac{\partial \tilde{V}}{\partial \tilde{\chi}} + \tilde{\chi} \frac{\partial \tilde{V}}{\partial \tilde{\chi}^1} \right). \quad (7)$$

The scale $k$ dependent regulator functions are chosen as follows:

$$R_p(q^2) = Z_P \alpha'_p (k^2 - q^2) \Theta(k^2 - q^2),$$
$$R_O(q^2) = Z_O \alpha'_O (k^2 - q^2) \Theta(k^2 - q^2) = r Z_O \alpha'_p (k^2 - q^2) \Theta(k^2 - q^2). \quad (8)$$

This optimized regulator [27] allows for a simple analytic integration in a closed form. Moreover, the anomalous dimensions are defined by:

$$\eta_P = - \frac{1}{Z_P} \partial_t Z_P, \quad \eta_O = - \frac{1}{Z_O} \partial_t Z_O; \quad \zeta_P = - \frac{1}{\alpha'_p} \partial_t \alpha'_p, \quad \zeta_O = - \frac{1}{\alpha'_O} \partial_t \alpha'_O. \quad (9)$$
3. RG flow

In order to find the flow equation of the potential we need to compute the rhs of the dimensionful flow equations which result from scale $k$ controlled contributions from quantum fluctuations:

$$\partial_t \Gamma = \frac{1}{2} Tr[\Gamma^{(2)} + R]^{-1} \partial_t R.$$  \hfill (10)

The trace on the rhs extends over a $4 \times 4$ matrix. The propagator matrix can be written the following form:

$$\Gamma^{(2)} + R = \begin{pmatrix} \Gamma^{(2)}_P & \Gamma^{(2)}_{PO} \\ \Gamma_{OP} & \Gamma^{(2)}_O \end{pmatrix},$$  \hfill (11)

where the $2 \times 2$ block matrices are:

$$\Gamma^{(2)}_P = \begin{pmatrix} V_{\psi \psi} & Z_P(-i\omega + \alpha_p'q^2) + R_P + V_{\psi \psi}^{\dagger} \\ Z_P(i\omega + \alpha_p'q^2) + R_P + V_{\psi \psi}^{\dagger} & V_{\psi \psi}^{\dagger \psi \psi} \end{pmatrix},$$  \hfill (12)

$$\Gamma^{(2)}_O = \begin{pmatrix} V_{\chi \chi} & Z_O(-i\omega + \alpha_O'q^2) + R_O + V_{\chi \chi}^{\dagger} \\ Z_O(i\omega + \alpha_O'q^2) + R_O + V_{\chi \chi}^{\dagger} & V_{\chi \chi}^{\dagger \chi \chi} \end{pmatrix},$$  \hfill (13)

and

$$\Gamma_{PO} = \begin{pmatrix} V_{\psi \chi} & V_{\psi \chi}^{\dagger} \\ V_{\psi \chi}^{\dagger \chi \chi} & V_{\psi \chi}^{\dagger \psi \psi} \end{pmatrix} \Gamma_{OP} = \begin{pmatrix} V_{\chi \psi} & V_{\chi \psi}^{\dagger} \\ V_{\chi \psi}^{\dagger \psi \psi} & V_{\chi \psi}^{\dagger \chi \chi} \end{pmatrix}.$$  \hfill (14)

The momentum integral contained in the trace can be done as described in [1]. The energy integral will be performed by complex integration. Unfortunately the analytic expression for the full flow of the potential is quite involved and difficult to handle. Since we are interested in an analysis based on polynomial expansions of the potential in terms of the Pomeron and Odderon fields, we find it more convenient to derive directly the flow equations for the polynomial coefficients (couplings).

In this work we shall limit ourself in analyzing the flow of the potential expanded around the origin (zero fields), i.e. we shall employ a weak field expansion. Therefore, for the derivation of the beta-functions we find it convenient to expand the inverse of (11) in the following way:

$$[\Gamma^{(2)} + R]^{-1} = [\Gamma^{(2)}_{\text{free}} - V_{\text{int}}]^{-1} = G(\omega, q) + G(\omega, q)V_{\text{int}}G(\omega, q) + G(\omega, q)V_{\text{int}}G(\omega, q)V_{\text{int}}G(\omega, q) + ...$$  \hfill (15)

Here we absorb the masses (intercepts minus one) into the free propagators:

$$G(\omega, q) = \begin{pmatrix} G_P(\omega, q) & 0 \\ 0 & G_O(\omega, q) \end{pmatrix},$$  \hfill (16)

where

$$G_P(\omega, q) = \begin{pmatrix} 0 \\ (Z_P(i\omega + \alpha_p'q^2) + R_P - \mu_P)^{-1} \end{pmatrix} \begin{pmatrix} Z_P(-i\omega + \alpha_p'q^2) + R_P - \mu_P \end{pmatrix}^{-1}$$  \hfill (17)

and

$$G_O(\omega, q) = \begin{pmatrix} 0 \\ (Z_O(i\omega + \alpha_O'q^2) + R_O - \mu_O)^{-1} \end{pmatrix} \begin{pmatrix} Z_O(-i\omega + \alpha_O'q^2) + R_O - \mu_O \end{pmatrix}^{-1}.$$  \hfill (18)
The interaction matrix $V_{\text{int}}$ is derived from the effective potential:

$$V_{\text{int}} = - \begin{pmatrix}
V_{\psi \psi}^r & V_{\psi \psi}^{r \dagger} & V_{\psi \chi}^r & V_{\psi \chi}^{r \dagger} \\
V_{\psi \psi}^{r \dagger} & V_{\psi \psi}^{r \dagger \dagger} & V_{\psi \chi}^{r \dagger} & V_{\psi \chi}^{r \dagger \dagger} \\
V_{\chi \psi}^r & V_{\chi \psi}^{r \dagger} & V_{\chi \chi}^r & V_{\chi \chi}^{r \dagger} \\
V_{\chi \psi}^{r \dagger} & V_{\chi \psi}^{r \dagger \dagger} & V_{\chi \chi}^{r \dagger} & V_{\chi \chi}^{r \dagger \dagger}
\end{pmatrix},$$  \hspace{1cm} (19)

where the upper script 'r' reminds that the reggeon masses have been removed.

Finally $\dot{R}$ consisting of two block matrices.

$$\dot{R} = \begin{pmatrix}
\dot{R}_P & 0 \\
0 & \dot{R}_O
\end{pmatrix},$$  \hspace{1cm} (20)

where

$$\dot{R}_P = \partial_t R_P(q^2)O_+, \quad \dot{R}_O = \partial_t R_O(q^2)O_+. \hspace{1cm} (21)$$

and

$$O_\pm = \begin{pmatrix}
0 & 1 \\
\pm 1 & 0
\end{pmatrix}. \hspace{1cm} (22)$$

After the momentum integrals and after the use of dimensionless variables, the factors of the block matrices can be replaced by

$$\dot{R}_P \rightarrow N_DA_D(\eta_P, \zeta_P)O_+, \quad \dot{R}_O \rightarrow rN_DA_D(\eta_O, \zeta_O)O_+, \hspace{1cm} (23)$$

where the factors $N_D$ and $A_D$ are defined in [1], and $\eta_P, \zeta_P$ and $\eta_O, \zeta_O$ are the anomalous dimensions of the Pomeron and Odderon fields, respectively.

We are this left with the energy integrals in the expansion:

$$\int \frac{dz'}{2\pi} Tr \left[ \dot{R} G(z') (1 - V_{\text{int}} G(z') + V_{\text{int}} G(z') V_{\text{int}} G(z') - V_{\text{int}} G(z') V_{\text{int}} G(z') V_{\text{int}} G(z') + ...) \right] \hspace{1cm} (24)$$

where $z' = i\omega'/(\alpha'_p k^2)$, and the free propagators in (16), as a result of the momentum integration and the use of dimensionless variables, have become:

$$G_P(\omega, q) \rightarrow G_P(\omega) = \begin{pmatrix}
0 & (-z + 1 - \tilde{\mu}_P)^{-1} \\
(z + 1 - \tilde{\mu}_P)^{-1} & 0
\end{pmatrix}, \hspace{1cm} (25)$$

$$G_O(\omega, q) \rightarrow G_O(\omega) = \begin{pmatrix}
0 & (-z + r - \tilde{\mu}_O)^{-1} \\
(z + r - \tilde{\mu}_O)^{-1} & 0
\end{pmatrix}. \hspace{1cm} (26)$$

For the derivation of the beta functions, we take derivatives of (24) with respect to the field variables and subsequently set the fields equal to zero.
3.1. $\beta$ functions for cubic truncation

For this approximation of the effective potential, we keep on the rhs of (24) the terms with two and three V's. The $z$-integral is done by complex integration and we use the condition $r - \mu_O > 0$ which is the physical relevant region. The flow equations we find:

\[
\begin{align*}
\dot{\mu}_P &= (-2 + \eta_P + \zeta_P)\mu_P + 2A_P \frac{\lambda^2}{(1 - \mu_P)^2} - 2A_{O\mu} \frac{\lambda^2}{(r - \mu_O)^2}, \\
\dot{\mu}_O &= (-2 + \eta_O + \zeta_P)\mu_O + 2(A_P + A_{O\mu}) \frac{\lambda^2}{(1 + r - \mu_P - \mu_O)^2}, \\
\dot{\lambda} &= (-2 + D/2 + \zeta_P + \frac{3}{2}\eta_P)\lambda + 8A_P \frac{\lambda^3}{(1 - \mu_P)^3} - 4A_{O\mu} \frac{\lambda_3^3}{(r - \mu_O)^3}, \\
\dot{\lambda}_2 &= (-2 + D/2 + \zeta_P + \frac{1}{2}\eta_P + \eta_O)\lambda_2 \\
&\quad + \frac{2\lambda^2(6A_P + 5A_{O\mu}) + 4\lambda^3(A_P + A_{O\mu}) - 4\lambda_2\lambda_3^3(2A_P + 2A_{O\mu})}{(1 + r - \mu_P - \mu_O)^3} \\
&\quad + \frac{2A_P\lambda_2^2(r - \mu_O)^2}{(1 - \mu_P)^2(1 + r - \mu_P - \mu_O)^3} - \frac{4A_{O\mu}\lambda_2\lambda_3^3(1 - \mu_P)^2}{(1 - \mu_O)^2(1 + r - \mu_P - \mu_O)^3} \\
&\quad + \frac{2\lambda_2^2(3A_P + A_{O\mu})(r - \mu_O)}{(1 + r - \mu_P - \mu_O)^3} - \frac{4\lambda_2\lambda_3^3(A_P + 3A_{O\mu})(r - \mu_O)}{(r - \mu_O)(1 + r - \mu_P - \mu_O)^3} \\
\dot{\lambda}_3 &= (-2 + D/2 + \zeta_P + \frac{1}{2}\eta_P + \eta_O)\lambda_3 \\
&\quad + \frac{2\lambda_2^2\lambda_3^3(A_P + 2A_{O\mu})}{(r - \mu_O)(1 + r - \mu_P - \mu_O)^2} + \frac{4\lambda_2\lambda_3^3(2A_P + A_{O\mu})}{(1 + r - \mu_P - \mu_O)^2} \\
&\quad + \frac{2\lambda_2^2\lambda_3^3A_{O\mu}(1 - \mu_P)}{(r - \mu_O)^2(1 + r - \mu_P - \mu_O)^2} + \frac{4\lambda_2\lambda_3^3A_{O\mu}(r - \mu_O)}{(1 - \mu_P)^2(1 + r - \mu_P - \mu_O)^2},
\end{align*}
\]

where we have defined

\[
A_P = N_D A_D(\eta_P, \zeta_P), \quad A_O = N_D A_D(\eta_O, \zeta_O).
\]

For the truncations of fourth order and beyond, one can use Mathematica and the results for the beta functions are already lengthy and we will not be listed here.

3.2. Anomalous dimensions

In order to study the critical properties, we need not only the beta functions for the coupling constants of the effective potential but also the anomalous dimensions. Using (9), the evolution equation for $r$ then becomes:

\[
\dot{r} = r(-\zeta_O + \zeta_P),
\]

which tells that at criticality the Pomeron and Odderon transverse space scaling laws do coincide.

To obtain the anomalous dimensions we define the two-point vertex functions:

\[
\Gamma_P^{(1,1)}(\omega, q) = \frac{\delta^2 \Gamma}{\delta \psi(\omega, q) \delta \psi^\dagger(\omega, q)}|_{\psi = \psi^\dagger = \chi = \chi^\dagger = 0}
\]

and

\[
\Gamma_O^{(1,1)}(\omega, q) = \frac{\delta^2 \Gamma}{\delta \chi(\omega, q) \delta \chi^\dagger(\omega, q)}|_{\psi = \psi^\dagger = \chi = \chi^\dagger = 0}.
\]
From the flow equations we obtain:

\[ \dot{\Gamma}_P^{(1,1)}(\omega, q) = \alpha'_P \int \frac{dz' dz' q'}{(2\pi)^{D+1}} Tr \left[ \hat{R} G(z', q') \frac{\delta V_{\text{int}}}{\delta \eta} G(z + z', q + q') \frac{\delta V_{\text{int}}}{\delta \eta} G(z', q') \right] |_O + \ldots \]

\[ \dot{\Gamma}_O^{(1,1)}(\omega, q) = \alpha'_P \int \frac{dz' dz' q'}{(2\pi)^{D+1}} Tr \left[ \hat{R} G(z', q') \frac{\delta V_{\text{int}}}{\delta \chi} G(z + z', q + q') \frac{\delta V_{\text{int}}}{\delta \chi} G(z', q') \right] |_O + \ldots \]

(32)

(33)

where the subscript 'O' indicates that, after differentiation, we have set all field variables inside the trace equal to zero and the dots indicate that there are more terms containing second derivatives of \( V_{\text{int}} \) with respect to the field variables which will not contribute. The anomalous dimensions are obtained by taking derivatives with respect to energy and momentum:

\[ Z_P = \lim_{\omega \rightarrow 0, q \rightarrow 0} \frac{\partial}{\partial (i\omega)} \Gamma_P^{(1,1)}(\omega, q); \quad Z_O = \lim_{\omega \rightarrow 0, q \rightarrow 0} \frac{\partial}{\partial (i\omega)} \Gamma_O^{(1,1)}(\omega, q) \]

(34)

and

\[ Z_P \alpha'_P = \lim_{\omega \rightarrow 0, q \rightarrow 0} \frac{\partial}{\partial q^2} \Gamma_P^{(1,1)}(\omega, q); \quad Z_O \alpha'_O = \lim_{\omega \rightarrow 0, q \rightarrow 0} \frac{\partial}{\partial q^2} \Gamma_O^{(1,1)}(\omega, q). \]

(35)

Introducing

\[ \dot{\Gamma}_P^{(1,1)} = I_P^{(1,1)}(\omega, q), \quad \dot{\Gamma}_O^{(1,1)} = I_O^{(1,1)}(\omega, q), \]

the anomalous dimensions are then given by:

\[ -\eta_P = \frac{1}{Z_P \alpha'_P} \lim_{\omega \rightarrow 0, q \rightarrow 0} \frac{\partial}{\partial (i\omega)} I_P^{(1,1)}(\omega, q); \quad -\eta_O = \frac{1}{Z_O \alpha'_O} \lim_{\omega \rightarrow 0, q \rightarrow 0} \frac{\partial}{\partial (i\omega)} I_O^{(1,1)}(\omega, q) \]

(36)

(37)

and

\[ -\eta_P - \xi_P = \frac{1}{Z_P \alpha'_P} \lim_{\omega \rightarrow 0, q \rightarrow 0} \frac{\partial}{\partial q^2} I_P^{(1,1)}(\omega, q), \quad -\eta_O - \xi_O = \frac{1}{Z_O \alpha'_O} \lim_{\omega \rightarrow 0, q \rightarrow 0} \frac{\partial}{\partial q^2} I_O^{(1,1)}(\omega, q). \]

(38)

We obtain after the momentum integral and considering the expansion around zero fields:

\[ \eta_P = -\frac{2A_P \lambda^2}{(1 - \mu_P)^3} + \frac{2A_O r \lambda^3_2}{(r - \mu_O)^3}; \quad \eta_O = -\frac{4(A_P + A_O r) \lambda^3_2}{(1 + r - \mu_P - \mu_O)^3} \]

(39)

and

\[ \eta_P + \xi_P = -\frac{N_D \lambda^2}{D(1 - \mu_P)^3} + \frac{N_D r \lambda^3_2}{D(1 - \mu_P)^3}; \quad \eta_O + \xi_O = -\frac{4N_D \lambda^3_2}{D(1 + r - \mu_P - \mu_O)^3}. \]

(40)

(41)

3.3. \textit{Analysis} \( \epsilon \)-\textit{expansion to the cubic truncation}

We show the results of an analysis of the theory close to the critical dimension \( D = 4 - \epsilon \) at one loop. Such an analysis can help to identify a possible critical behavior of the system which may survive, at a qualitative level, down to \( D = 2 \).

Evaluating the Eq. (27), (28) and (39)-(41) in the \( \epsilon \)-expansion we find only one fixed point:

\[ \mu_P = \frac{\epsilon}{12}, \quad \lambda^2 = \frac{8\pi^2}{3} \epsilon, \quad \eta_P = -\frac{\epsilon}{6}, \quad \xi_P = \xi_O = \frac{\epsilon}{12}, \]

\[ \mu_O = \frac{95 + 17\sqrt{33}}{2304} \epsilon, \quad \lambda^2_2 = \frac{23\sqrt{6} + 11\sqrt{22}}{48} \epsilon, \quad \lambda_3 = 0, \quad \eta_O = -\frac{7 + \sqrt{33}}{72} \epsilon, \quad r = \frac{3}{16} (\sqrt{33} - 1). \]

(42)
Finally, the spectral analysis of the stability matrix show the other universal quantities of the system, apart from the anomalous dimensions. In particular we find two negative eigenvalues, associated to two relevant directions, and the corresponding critical exponents:

\[
\lambda^{(1)} = -2 + \frac{\epsilon}{4} \quad \nu_P = \frac{1}{2} + \frac{\epsilon}{16}; \quad \lambda^{(2)} = -2 + \frac{\epsilon}{12} \quad \nu_O = \frac{1}{2} + \frac{\epsilon}{48}.
\]

We have not found other solutions with all real couplings and $\lambda_3 \neq 0$ and we also note that the values of the couplings, the critical exponents and anomalous dimensions in the Pomeron sector are exactly the same as in the pure Pomeron case [1]. This seems to favour, at least in the vicinity of the critical dimension $D = 4$, the existence of just two non trivial fixed points, one with the pure pomeron content, and another one with both interacting fields, where the interaction responsible for creating the odderon fields is turned off. That is the scaling solution of Eq. (42) is a theory conserving the Odderon number.

4. Numerical results

We perform an analysis in the physical $D = 2$ transverse space searching the fixed point and the critical properties using polynomial truncation for the potential (LPA) with increasing order up to order 9.

Our analysis is essentially in the LPA approximation with the addition of an extra coupling $r$, depending on the anomalous dimensions $\zeta_P$ and $\zeta_O$, which we have evaluated at the lowest order. In all the other beta functions the anomalous dimensions have been completely neglected. Such a strategy is based on our previous experience with the Pomeron theory. Indeed, the polynomial expansion around the origin without the inclusion of the anomalous dimensions was giving, a very good estimate of the critical exponent $\nu$.

The fixed point for the cubic truncation is: $(\mu_P = 0.111111, \mu_O = 0.110753, \lambda = 1.05034, \lambda_2 = 1.44665, \lambda_3 = 0, r = 0.921810)$, which has three negative and three positive eigenvalues, i.e. there are three relevant directions. Two of the negative eigenvalues $\lambda_O = -1.9398$ and $\lambda_P = -1.8860$ are associated to the $\nu_P$ and $\nu_O$ critical exponents, respectively. The third negative eigenvalue $\lambda^{(3)} = -0.0916$ is close to zero and with an eigenvector mainly associated to the $r$ coupling.

This solution is also the one found in the $\epsilon$-expansion analysis. We observe a 'decoupling' of the two sectors: compared to the pure Pomeron case, the Pomeron is not affected by the presence of the Odderon, whereas the Odderon 'feels' the Pomeron. This decoupling is due to the vanishing of the triple coupling $\lambda_3$.

All these features also appear in the solution obtained in the quartic truncation: $(\mu_P = 0.274381, \mu_O = 0.26979, \lambda = 1.34738, \lambda_2 = 1.79401, \lambda_3 = 0, \lambda_{41} = -2.88712, \lambda_{42} = -1.27076, \lambda_{43} = -0.83228, \lambda_{44} = 0, \lambda_{45} = 0, \lambda_{46} = -5.2784, \lambda_{47} = -2.2078, r = 0.88018)$. The stability properties are the same as in the cubic case: three negative eigenvalues ($-1.8159, -1.6751$ and $-0.20957$). The Pomeron and Odderon sectors are decoupled, since the couplings $\lambda_3, \lambda_{44}, \lambda_{45}$ vanish.

We extend our analysis up to order 9 in the polynomial expansion. We collect the results found in Fig. 1 (the values for $\mu_P$ and $\mu_O$ and the the non zero couplings) and Fig. 2 (the critical exponents $\nu_P$ and $\nu_O$ and the third negative eigenvalue) in order to show the convergence with respect to the order of the truncation. We note that $\mu_P > \mu_O$ in all truncations and we see how at order 9 a good stability is reached.

Finally, we have the following estimates: for the critical exponents ($\nu_P \simeq 0.73, \nu_O \simeq 0.6$), for the anomalous dimensions ($\eta_P \simeq -0.33, \eta_O \simeq -0.35$) and $\zeta_P = \zeta_O \simeq +0.17$. Numerical studies of the flow of the dimensionless and dimensionful parameters, for the Pomeron-Odderon system remains a task for future analysis.

We observe that this special fixed point solution is associated to a critical theory conserving the Odderon number. We do not find any other physical critical solution with all couplings

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Figure 1. Values of the parameters of the fixed point solution of the LPA truncations for different orders \( n \) of the polynomial \( (3 \leq n \leq 9) \). The masses (which equal intercept minus one) \( \mu_P \) (red curve) and \( \mu_O \) (blue dotted curve) for the Pomeron and Odderon fields are in the left panel. The first non zero couplings \( \lambda, \lambda_2, \lambda_{41}, \lambda_{42}, \lambda_{43}, \lambda_{46}, \lambda_{47}, r \) are reported on the right panel.

Figure 2. Values of the critical exponents of the fixed point solution of the LPA truncations for different orders \( n \) of the polynomial \( (3 \leq n \leq 9) \). The two negative leading eigenvalues define the two critical exponents \( \nu_P \) (red curve) and \( \nu_O \) (blue dotted curve) for the Pomeron and Odderon fields (left panel). We report also the value of a third negative eigenvalue found in our approximation (right panel).

5. Discussion and outlook

In this paper we have extended our previous analysis of Pomeron RFT to a system of interacting Pomeron and Odderon fields in the infrared limit. Where the main motivation for including the Odderon comes from the observation that, in the UV region where perturbative QCD applies, there exist two fundamental composite states of reggeized gluons, the BFKL Pomeron with intercept well above one and a very small slope, and the Odderon with intercept at (or very close to) one and a small slope. This raises the question, when moving towards the nonperturbative IR region, to what extent the interactions between these fundamental fields lead to serious modifications, e.g. a suppression of the Odderon exchange at high energies.

From (6) we see that near the fixed point both intercepts, \( \alpha_P(0) - 1 = \mu_P/Z_P \sim k^{2-\zeta}\tilde{\mu}_P \) and \( \alpha_O(0) - 1 = \mu_O/Z_O \sim k^{2-\zeta}\tilde{\mu}_O \) go to zero as \( k \) becomes small. Since the fixed point value of \( \tilde{\mu}_P \) is slightly larger than \( \tilde{\mu}_O \) we conclude that, for small but nonzero values of \( k \), the Pomeron intercept is larger than the Odderon intercept. However, the most important conclusion to be drawn from this fixed point analysis is that the Odderon exists in the IR limit and does not die out with energy. Future study will be devoted to understand the QCD-RFT transition.
There are several questions to be addressed by future studies. Phenomenologically, not much is known about the Odderon slope [28, 29], and our result might be seen as an asymptotic prediction. Finally, the possibility that in the deep IR region the POO vertex is suppressed may also have phenomenological consequences. Processes involving a simple Odderon exchange, like hadron scattering $pp - pp$ or meson photo-production [30] would be allowed in asymptotic IR [31].

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