Thermo piezoelectric sound waves in a nanofiber using Timoshenko beam theory incorporated with surface effect

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Abstract. This study investigates that the sound wave propagation of Nanofibers under the influence of surface effect via piezo thermo elasticity using Timoshenko form of beam equation. The equation of analytical model is obtained for Nanofiber through shear and rotation effect. Curves are drawn for frequency, phase velocity, piezoelectric strain and dynamic displacement at different vibration modes of Nanofibers. From the result obtained, it is seen that the surface effect gives significant contribution to the physical variables. The presented study is expected to be more helpful for the design of piezo-thermo-mechanical Nanofiber-based devices.

1. Introduction
Smart composite like thermo-piezoelectric Nano materials are used for smart applications as fiber sensors and actuators and structural components of self monitoring structures. Nowadays, Nano structures such as Nanofibers, Nano beams and Nano membranes are very attractive field for many researchers due to their improvement of the quality properties. The invent of Nanotubes (CNTs) in the early periods gave much attraction in many areas of industrial engineering and Nano sciences. Many applications of CNTs have been reported in the area like biomedical engineering, water and air purification field and electronic devices in Nano electro mechanical systems. Wu and Dzenis[1] developed a method to find the wave propagation in Nanofibers in the context of continuum mechanics. The wave propagation in prestretched polymer Nanofibers is read from Wu[2], they obtained result that the size effect of polymer Nanofibers have more influence with either decreasing fiber radius or increasing wave number. Guang Li et al.[3] reported the microwave absorption enhancement of porous carbon fibers compared with carbon Nanofibers and they concluded that the content of the absorbents influences the microwave absorption properties of the composites.

Thermo elastic Nanofibers are more important structural components in thermo sensors and their modified engineering components. Ahmed Elgafy and Khalid Lafdi[4] studied the effect of carbon Nanofiber additives on thermal behaviour of phase change materials and they found that the heat transfer phenomenon at Nanoscale seems to be more sensitive in surface area. Xinpeng Zhao et al.[5] studied the thermal conductivity model for Nanofiber networks. They constructed the thermal conductivity of the network by both the inter-fiber contact resistance and intrinsic thermal resistance of the Nanofibers. Abhinav Malhotra and Martin Maldovan[6] developed the thermal transport in semiconductor Nanotubes. Also, they concluded that the thermal conductivity of the Nanotube
depends on the outer diameter even for the same shell thickness. For the energy conversion in a smart way piezoelectric materials are commonly used. Piezoelectric nano fibers are plays an important role in butane lighters and improvised potato cannons. Jiyoung Chang et al. [7] analyzed the piezoelectric Nanofibers for energy scavenging applications. Arash Tourki Samaei et al. [8] investigated the frequency analysis of piezoelectric Nanowires with surface effects and they found that the surface effects tend to increase the natural frequency in lower modes.

Sound waves with higher frequency have been used in modern engineering and bio mechanics research field. Heireche et al. [9] discussed the sound wave propagation in single-walled carbon Nanotubes with initial axial stress and they showed that the phase velocities decrease with increasing compressive initial stresses. Berrabal et al. [10] investigated the comparative study of sound wave propagation in single-walled carbon Nanotubes using nonlocal elasticity for Aluminium and Nickel.

Here, we concentrated the sound wave propagation of Nanofibers under the effect of surface related force in the presence of thermo piezoelectric forces using Timoshenko form of beam equation. Analytical derivation of motion of Nanofiber is derived via Lorentz’s force, thermal and piezoelectric terms in the presence of surface effect. The graphs are presented for the physical quantities.

2. Thermo electro elastic model of Nanofiber

The geometry of Nanofiber in the basic form is displayed in figure 1. via shear force and rotation effect.

![Figure 1. Geometry of Nanofiber in transverse direction.](image)

The dynamic equation of fiber in vertical direction is given as

$$ S - \left( S + \frac{\partial S}{\partial x} dx \right) + \left( E \varepsilon_0 A + 2 \pi R T_i \right) \frac{\partial^2 w}{\partial x^2} dx = \rho A \frac{\partial^2 w}{\partial x^2} dx $$

(1)

where $E$ is Young’s modulus, $S$ is the shear force, $A$ is cross-sectional area, $\varepsilon_0$ represent prestrain, $R$ radius of Nanofiber, $\gamma$ surface tension or stress, $\rho$ mass density and $w$ deflection of the Nanofiber. The thermal force $T_i$ is defined as

$$ T_i = -\alpha \gamma A E T $$

(2)

where $\alpha$ is expansion constant of thermal expansion, $T$ is temperature variation. Processing the moments at centre about an axis to perpendicular to $x, y$ plane, we get
\[-M + \left( M + \frac{\partial M}{\partial x} \right) - \frac{S}{2} dx - \left( \frac{S}{2} + \frac{1}{2} \frac{\partial S}{\partial x} \right) dx + E \varepsilon_0 A \frac{\partial w}{\partial x} + 2 \pi \gamma R \frac{\partial w}{\partial x} = J \frac{\partial^2 \phi}{\partial t^2} \]

where \( M \) is the moment of bending of Nanofiber and \( \phi \) is the angle produced only by bending moment, \( J = \rho ld \).

The following basic equation of flexural mode of the Nanofiber is derived from Equations (1) - (3) in the presence of thermal force

\[ \rho A \frac{\partial^2 w}{\partial x^2} + \frac{\partial S}{\partial x} - \left( E \varepsilon_0 A + 2 \pi \gamma R + T_i \right) \frac{\partial^2 w}{\partial x^2} = 0 \]  

(4)

\[ \rho I \frac{\partial^2 \phi}{\partial t^2} + S - \left( E \varepsilon_0 A + 2 \pi \gamma R \right) \frac{\partial w}{\partial x} = 0 \]  

(5)

The bending and shear force is given in terms of cross sectional area by

\[ M = \int_A w \Pi dA + \int_A \Pi^p dA \]  

(6)

\[ S = \int_A \tau dA \]  

(7)

where \( \Pi \) and \( \tau \) indicates the normal and shear stress of the fiber. \( \Pi^p \) represents the piezoelectric normal stress.

Then the stress –strain relation in the context of Hooke’s one-dimensional case are given by

\[ \Pi = E \varepsilon \]  

(8)

\[ \Pi^p = E \varepsilon - d_{31} E_s \]  

(9)

\[ \tau = G \gamma_0 \]  

(10)

where \( E_s \) is the electric field and \( \phi \) is the electric potential, \( G = E/2(1+\nu) \) denotes shear modulus, \( d_{31} \) denote the piezoelectric strain constant. The normal and shear strain component \( \varepsilon \) and \( \gamma_0 \) can be expressed by

\[ \varepsilon = w \frac{\partial \phi}{\partial x} \]  

(11)

\[ \gamma_0 = \phi - \frac{\partial w}{\partial x} \]  

(12)

In one dimensional case, the expression connecting the electric field and electric potential is presented in the following form

\[ E_s = - \frac{\partial \phi}{\partial x} \]  

(13)

Using Equations (11)-(13) upon Equations (8) - (10) and employing Equations (6) and (7) in the remaining relations, we can obtain the bending force as follows

\[ M = 2EI \frac{\partial \phi}{\partial x} - d_{31} I_s E_s \]  

(14)

where \( I = \int_A w^2 dA \), \( I_s = \int_A wdA \) are the inertial force through area of the cross section of the Nanofiber and shear force is given as

\[ S = G \left( \phi - \frac{\partial w}{\partial x} \right) A \alpha \]  

(15)

where \( \alpha = \frac{6(1+2\nu+6\nu^2)}{7(1+2\nu+4\nu^2)} \) is the coefficient of shear. Applying the Equations (14) - (15) upon Equations (4) – (5), we get
\[ \rho \frac{\partial^2 w}{\partial t^2} + 3G \left( \frac{\partial \phi}{\partial x} - \frac{\partial^2 w}{\partial x^2} \right) - \left( E\varepsilon_0 + \frac{2\gamma}{R} - \alpha ET \right) \frac{\partial^2 w}{\partial x^2} = 0 \]  \hspace{1cm} (16)

\[ \rho \frac{\partial^2 \phi}{\partial x^2} + 3GA \left( \phi - \frac{\partial w}{\partial x} \right) - \left( EA\varepsilon_0 \frac{\partial \phi}{\partial x} + 2\pi R \frac{\partial w}{\partial x} \right) - 2EI \frac{\partial^2 \phi}{\partial x^2} - d_iI \frac{\partial^2 \phi}{\partial x^2} = 0 \]  \hspace{1cm} (17)

3. Analytical solution of the problem

To analyze the effect of thermo electro elastic effects and other forces on the vibration of the Nanofibers, we solve the coupled Equations (16) and (17) for the time harmonic case of vibrations. We seek for the flexural wave motion as

\[ w(x,t) = \tilde{W}e^{i\omega t}\sin\left(\frac{n\pi x}{L}\right) \]

\[ \phi(x,t) = \tilde{\phi}e^{i\omega t}\cos\left(\frac{n\pi x}{L}\right) \]

where \( \tilde{W} \) is the amplitude of deflection and \( \tilde{\phi} \) is the amplitude of the slope due to bending deformation of the fiber. Also, the frequency of the vibration of Nanofiber is denoted by \( \omega \).

Substituting Equations (18) in Equations (16) – (17), We get

\[ \begin{bmatrix} \rho \omega^2 - 3G \left( \frac{n\pi}{L} \right)^2 - (E\varepsilon_0 + \frac{2\gamma}{R} - \alpha ET) \left( \frac{n\pi}{L} \right)^2 \\ - \left( 3GA + (EA\varepsilon_0 + 2\pi R\gamma) \frac{n\pi}{L} \right) \end{bmatrix} \tilde{W} = \begin{bmatrix} -3G \left( \frac{n\pi}{L} \right) \\ -(\rho I \omega^2 + 3GA + 2EI \left( \frac{n\pi}{L} \right)^2 + d_iI \left( \frac{n\pi}{L} \right)^2) \end{bmatrix} \tilde{\phi} = 0 \]  \hspace{1cm} (19)

\[ \begin{bmatrix} \rho \omega^2 - 3G \left( \frac{n\pi}{L} \right)^2 - (E\varepsilon_0 + \frac{2\gamma}{R} - \alpha ET) \left( \frac{n\pi}{L} \right)^2 \\ - \left( 3GA + (EA\varepsilon_0 + 2\pi R\gamma) \frac{n\pi}{L} \right) \end{bmatrix} \tilde{W} = \begin{bmatrix} \tilde{W} \\ \tilde{\phi} \end{bmatrix} = 0 \]  \hspace{1cm} (20)

By solving the homogeneous Equations (19) and (20), we can get the trivial solution. The non-trivial solution of the above equations can be obtained as

\[ \begin{vmatrix} \rho \omega^2 - 3G \left( \frac{n\pi}{L} \right)^2 - (E\varepsilon_0 + \frac{2\gamma}{R} - \alpha ET) \left( \frac{n\pi}{L} \right)^2 \\ - \left( 3GA + (EA\varepsilon_0 + 2\pi R\gamma) \frac{n\pi}{L} \right) \end{vmatrix} = \begin{vmatrix} -3G \left( \frac{n\pi}{L} \right) \\ -(\rho I \omega^2 + 3GA + 2EI \left( \frac{n\pi}{L} \right)^2 + d_iI \left( \frac{n\pi}{L} \right)^2) \end{vmatrix} = 0 \]  \hspace{1cm} (21)

The dispersion equation is computed by solving the determinant in Equation (21) as follows

\[ \beta_1 \omega^2 + \beta_2 \omega^2 + \beta_3 = 0 \]  \hspace{1cm} (22)

where

\[ \beta_1 = \rho^2 I \]

\[ \beta_2 = -\rho \left[ 3GA + d_iI \left( \frac{n\pi}{L} \right)^2 \right] + \left( 3G + 2E + E\varepsilon_0 + \frac{2\gamma}{R} - \alpha ET \right) \left( \frac{n\pi}{L} \right)^2 \]

\[ \beta_3 = 3G^2 A \left( \frac{n\pi}{L} \right)^2 + 2EI \left( \frac{n\pi}{L} \right)^4 + \left( \frac{2\gamma}{R} - \alpha ET \right) \left( \frac{n\pi}{L} \right)^2 + \left( E\varepsilon_0 + \frac{2\gamma}{R} - \alpha ET \right) \left( 2EI + d_iI \right) \left( \frac{n\pi}{L} \right)^2 \]

Equation (22) expresses the two natural frequencies. Lower frequency represents the flexural wave and higher frequency shows the transverse motion of the fiber.
4. Numerical discussions

In this section the numerical computation of thermo piezo electric Nanofiber with surface is presented. The mechanical and other interacted material properties are considered from [1] as 

\[ E = 100\text{MPa}, \]
\[ v = 0.5, \rho = 2000\text{kg/m}^3, T = 40\text{K}, d_{31} = 2.54 \times 10^{-10}\text{m/V}, \alpha_s = -1.6 \times 10^{-6} \text{K}^{-1}, H_s = 10^7 \text{Am}^{-1} \]

and \( \eta = \frac{4\pi}{10^7} \text{Hm}^{-1} \). The prestrain value is taken as zero. The influence of the frequency of the Nanofiber is defined by the relation \( \Gamma = \frac{\theta}{\omega} \) where \( \theta \) is the frequency and \( \omega \) is the fundamental frequency of the Nanofiber.

![Figure 2. Frequency distribution with wave number via \( \gamma = 0.05, \Gamma = 0.5 \)](image2)

![Figure 3. Frequency distribution with wave number via \( \gamma = 0.10, \Gamma = 1.0 \)](image3)
Figures 2-3 represent the frequency scale with varying wave number of the thermo electro elastic Nano fiber with fixed surface effect $\gamma = 0.05, \Gamma = 0.5$ and $\gamma = 0.1, \Gamma = 1.0$ for different vibration modes. From figure 2-3, it is observed that the wave frequency increases with respect to its wave number for the different values of surface effect. Figure 3 indicates the high in magnitude of frequency due to the increase in surface effect value. A comparative plot is given in figure 4 and 5 between the phase velocity and wave number of the thermo electro elastic Nanofiber for the values mode number. It seen from figure 4 and 5 that, with the effect of surface force, the phase velocity values decreases in the negative direction with an increasing wave number. This may happen due to the effect of surface effect and other interacted forces.

Figures 6-8 displays the comparison between the dynamic displacement and the parameter x/L of the Nanofiber for the different values surface effects and temperature. From figures 6-8, it appears that the dynamic displacement values first increases and then decreases in the considered values of the parameter x/L for the different modes of deflection. At the higher values of surface effect and
Figure 6. Variation of displacement versus $(x/L)$ with $\gamma = 0.05, \Gamma = 0.5$ & $T = 30^\circ C$

Figure 7. Variation of displacement versus $(x/L)$ with $\gamma = 0.10, \Gamma = 1.0$ & $T = 40^\circ C$

(at $\gamma = 0.1, 0.15, \Gamma = 0.5, 1.0$ & $T = 40, 50$), it is noted some crossing over lines which may denote the transport of energy from one mode to another due to the effect of surface effect and other interacted forces. The 3D plots in figures 9-10, clarifies the dependence of mode shape with distance and time. It is concluded that the increase in distance and time parameter values will decreases the mode shape of the Nanofibers.

5. Conclusions
Investigation is made to study the effect of longitudinal thermal and piezoelectric on propagation of sound waves of Nanofiber via surface force using Timoshenko form of beam theory. Dispersion equations of Nanofiber are obtained in flexural direction. Vibration characteristics of physical variables are studied from graphs. The observation is concluded that

- The values of frequency increases as wave number increases via surface effect.
Figure 8. Variation of displacement versus (x/L) with $\gamma = 0.15, \Gamma = 1.5 & T = 50^\circ C$

Figure 9. Variation of mode shape with x and t via $\gamma = 0.05, \Gamma = 0.5 & T = 30^\circ C$

- Phase velocity decreases as wave number increases in the presence of surface effect of the Nanofiber.
- The values of piezoelectric strain increases and in wave propagation trend when the wave number increases with increasing values of surface effect and temperature
- The surface effect and temperature field vectors have an important role on the distribution of dynamic displacement.
- The effect of distance and time also influences the mode shape of the Nanofiber while bending. Deformation of a Nanofiber depends on the nature of force applied and also by the thermo-piezo effect.
Figure 10. Variation of mode shape with x and t via $\gamma = 0.15, \Gamma = 1.0$ & $T = 40^\circ C$

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