Creating two-dimensional bright solitons in dipolar Bose-Einstein condensates

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We propose a realistic experimental setup for creating quasi-two-dimensional (2D) bright solitons in dipolar Bose-Einstein condensates (BECs), the existence of which was proposed in Phys. Rev. Lett. 100, 090406 (2008). A challenging feature of the expected solitons is their strong inherent anisotropy, due to the necessary in-plane orientation of the local moments in the dipolar gas. This may be the first chance of making multidimensional matter-wave solitons, as well as solitons featuring the anisotropy due to their intrinsic dynamics. Our analysis is based on the extended Gross-Pitaevskii equation, which includes three-body losses and noise in the scattering length, induced by fluctuations of currents inducing the necessary magnetic fields, which are factors crucial to the adequate description of experimental conditions. By means of systematic 3D simulations, we find a ramping scenario for the change of the scattering length and trap frequencies which results in the creation of robust solitons, that readily withstand the concomitant excitation of the condensate.

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Bose-Einstein condensation (BEC) of magnetic atoms, or molecules carrying electric moments, has attracted a great deal of attention, starting from the pioneering theoretical analyses of such condensates [1], which was followed by the creation of BEC in the ultracold gas of 52Cr atoms, using magnetic [2] and all-optical [3] techniques. Very recently, a condensate of 164Dy atoms, with a much larger magnetic moment, was created too [4]. The making of magnetic- and electric-dipolar BEC may also be expected, respectively, in erbium [5] and in molecular gases [6]. An updated review of this rapidly progressing field was given in Ref. [7].

One of the promising possibilities, which has been, thus far, investigated only theoretically, is the creation of bright solitons in dipolar condensates. This can be easily predicted in various one-dimensional (1D) settings, using, in particular, periodic potentials induced by optical lattices as the stabilizing factor [8]. In the limit of a very deep lattice, the BEC wave function becomes nearly discrete, which makes it possible to predict stable 2D dipolar solitons [9] and vortices [10], as well as 3D solitons [11], in the (quasi-)discrete form.

As concerns solitons, the most challenging issue is the creation of such stable matter-wave modes in quasi-2D (pancake-shaped) condensates, as well as their counterparts in the form of spatiotemporal solitons in optics. In spite of intensive theoretical discussions [12], no experimental results in this area have been reported thus far, the most essential obstacle being the inherent instability of 2D solitons to the collapse, driven by the self-focusing cubic nonlinearity (very recently, it was proposed to create stable bright solitons by means of self-defocusing nonlinearity, which is possible in a rather exotic situation with the strength of the nonlinearity growing at $r \to \infty$ faster than $r^D$, where $D$ is the spatial dimension [13]).

The dipolar condensates suggest new challenges and possibilities for achieving this fundamental purpose, by making use of the competition between local interactions and long-range dipole-dipole interactions (DDIs). A quasi-2D isotropic configuration implies that the local moments are polarized perpendicular to the pancake’s plane, in which case the DDI is repulsive. In that case, the creation of (bright) 2D solitons may be possible if the sign of the DDI is effectively reversed by means of a rapidly oscillating magnetic field [14]; in the same setting, stable isotropic solitons with embedded vorticity were predicted too [15], and various 2D localized structures may be stabilized by trapping potentials acting in the plane [16]. The very fact of the existence of multidimensional solitons in the dipolar BEC can be proven in a rigorous mathematical form [17]. It is also relevant to mention that stable isotropic solitons are possible in optical media with nonlocal (thermal or orientational) nonlinearities, as was demonstrated experimentally [18], and studied in detail theoretically [19].

The most challenging issue, which is unique to the multidimensional dipolar media, is the possibility of the creation of a new species of anisotropic solitons, based on the in-plane polarization of local moments (obviously, the anisotropy cannot manifest itself in 1D solitons). The stationary form of such quasi-2D solitons, and some of their dynamical properties, were investigated in Refs. [20, 21], but the problem of the actual creation of the solitons under experimentally relevant conditions has not been addressed before. The complex anisotropic structure expected in the solitons makes this problem crucially important, as the mode may be considered physically robust only if it is capable to self-trap from experimentally feasible initial configurations. In this work, we show, by means of systematic simulations of a realistic model, that
anisotropic solitons in dipolar BEC can be created, using available experimental techniques. This result may have more general implications, suggesting the realizability of various complex modes predicted in multidimensional quantum gases, such as skyrmions.

As mentioned above, there are currently two atomic species available for creating magnetic dipolar BEC, viz., $^{52}\text{Cr}$ and $^{164}\text{Dy}$. Since the latter has been condensed very lately, our main results refer to chromium, but at the end of this work we address dysprosium too.

The making of the anisotropic solitons is possible in a strongly dipolar regime, which may be defined by the dimensionless ratio of the DDI and local interaction, $\epsilon_{\text{DD}} = m\rho_0\mu^2/(12\pi ah^2)$. Here, $m$ is the atom's mass, $\mu$ the free-space permeability, $\mu$ the atom's magnetic moment, and $a$ the s-wave scattering length. As shown in Ref. [20], the necessary condition for the existence of solitons is $\epsilon_{\text{DD}} > 1$. In the case of $^{52}\text{Cr}$, the natural value of this ratio is $\epsilon_{\text{DD}} = 0.16$ [23]. Therefore, it is necessary to tune the scattering length to enhance the relative strength of the DDI, which may be accomplished by using the Feshbach resonance [21, 22]. For $^{52}\text{Cr}$, we thus find that the critical scattering length below which the 2D solitons can exist is $15a_B$. However, this condition is not sufficient, as it is only an upper bound, and the soliton may actually be found at still smaller values of $a$ [21]. On the other hand, if $a$ is too small, i.e. $\epsilon_{\text{DD}}$ exceeds a certain critical value, the condensate will collapse. Therefore, solitons only exist within a certain range of values of the scattering length. For typical trap frequencies and numbers of atoms in the condensate, the relevant range for chromium is in a ballpark of $4a_B$ [21].

To explore conditions for the creation of the anisotropic solitons, we consider BEC of $N$ atoms with magnetic dipole moment $\mu$, which are fully polarized by means of an external uniform magnetic field. At a sufficiently low temperature, the dynamics of the condensate obey the extended Gross-Pitaevskii equation (GPE):

$$\dot{h}_0(t) + \dot{h}_{\text{int}}(t) + \dot{h}_{\text{loss}}(t) = i\hbar \partial_t \Psi(r,t),$$

with the single-particle Hamiltonian

$$\dot{h}_0(t) = -\frac{\hbar^2}{2m} \Delta + \frac{m}{2} \left[ \omega_x^2(t)x^2 + \omega_y^2(t)y^2 + \omega_z^2(t)z^2 \right],$$

the mean-field interaction Hamiltonian

$$\dot{h}_{\text{int}}(t) = \frac{4\pi a(t)\hbar^2}{m} |\Psi(r,t)|^2$$

$$+ \frac{\mu_B\hbar^2}{4\pi} \int d^3r' \frac{1 - 3\cos^2 \theta'}{|r - r'|^3} |\Psi(r',t)|^2,$$

and the loss term

$$\dot{h}_{\text{loss}}(t) = - \frac{i\hbar L_3}{2} |\Psi(r,t)|^4.$$

Here, $\omega_{x,y,z}$ are the trap frequencies in the respective directions, $L_3$ determines the rate of three-body losses, and $\Psi$ is the mean-field wave function. The dipoles are polarized along the $z$-axis, and it is assumed that the trap frequencies in $x$- and $z$-directions may be time-dependent, along with the scattering length of the contact interaction, $a$. We aim to consider the following scenario: Start with the dipolar BEC at parameters for which the condensate is stable in the extended state filling the entire 3D trap, and ramp down the scattering length and frequencies $\omega_x, \omega_z$ simultaneously to values at which a stable anisotropic soliton is expected. In the course of doing this, the trap will be switched off completely in the $x$- and $z$-directions. Obviously, this condensate-steering scenario will excite the condensate. We will exploit this fact for proving the self-trapping nature of the DDIs, since stable oscillations without the external trap are only possible if the long-range interactions provide for the self-confinement. To make our model realistic, we include losses via the imaginary-potential term [3], with an empirically known loss coefficient, $L_3 = 2 \times 10^{-40}$ m$^9$/s [21, 27], and random noise in the scattering length, which stems from the current in the magnetic coils inducing the Feshbach resonance. The latter is of special significance, because a typical root-mean-square value of the noise, $\sim 1 - 2 a_B$, is not much smaller than the range of $a$ within which the soliton exists. The noise that we used in the simulations is a uniform distribution of all frequencies up to 500 Hz. We do not expect higher frequencies to significantly affect the dynamics of the soliton because the time scale of these dynamics is $\sim 1$ s, as shown below. We have analyzed the noise signal of the Feshbach-inducing current in the experimental setup employed at the laboratory in Stuttgart [28], observing close agreement with the uniform distribution adopted in our scheme.

The ramping sequence is shown in Fig. 1. The initial condensate contains 20 000 atoms, and is prepared in the trap with $(\omega_x, \omega_y, \omega_z) = 2\pi \times (72, 426, 72)$ Hz and $a = 27a_B$. The scattering length is then ramped down to $11.5a_B$ within 70 ms. The trap is first held constant for $t_{\text{hold}} = 4$ ms and is then ramped to $\int f_{x,z} = f_1 = 6.5$ Hz within 58 ms. Finally, it is switched off within subsequent 33 ms. We emphasize the necessity for eliminating the trap. Even a very weak trapping potential, with $f \sim$ a few Hz, affects the behavior of the soliton, making it very difficult to distinguish the resulting dynamics from that of an ordinary trapped condensate. For the simulations, the 3D spatial domain was discretized into $512 \times 128 \times 512$ grid points, and marching in time is performed using the Split-Operator method. The DDI integral is evaluated with the aid of the Fourier convolution formula. Since this numerical scheme is very demanding, the code was implemented using the parallel computing architecture CUDA, and it was run on a modern graphics card, enabling a very high degree of parallelization.
Results of two such simulation runs are displayed in Fig. 2. The left column shows absorption images of the condensate ($|\Psi|^2$ integrated over the $y$-direction and normalized to the maximum value) in the absence of the noise in the scattering length, while, in the right column, the noise with the root-mean-square value of $0.5 \, a_B$ was added. In the first two frames, the soliton is being shaped. It is observed that few atoms escape the core condensate in the $x$-direction, along which the DDI is repulsive. The condensate then grows further in the $x$- and $z$-directions, but eventually inverts its dynamics and refocuses. Without the noise, this occurs at $t \approx 303$ ms. Since this is long after the trap has been switched off, the self-confining potential needed for this outcome can only be provided by the DDIs. Thus, we find a clear evidence for the self-trapping of the robust soliton. Remarkably, the condensate refocuses as well when the noise is added to the scattering length, as is shown in the right column of Fig. 2, i.e., the buildup of the soliton is robust against this experimentally inevitable noise.

To gain more insight into the dynamics, we monitored the root-mean-square extensions of the condensate in the $x$- and $z$-directions during the self-trapping process, as shown in Fig. 3. It is observed that the expansion in these directions is roughly synchronous, which implies that a breathing mode has been excited.

When the noise is added to the scattering length, the condensate reaches a greater extension, and the oscillation frequency decreases. Above a critical amplitude of the noise, for which we can give the lower bound of $1 \, a_B$, we expect the soliton to decay. This behavior can be understood in terms of a simple picture: One can replace the noise by an effectively larger scattering length, which also lowers the binding energy of the system and therefore lowers the binding energy of the system and there-
fore slows down its dynamics. Consistent with Ref. [21], the increase of the scattering length, $a$, also leads to a larger extension of the soliton. Above the critical value of $a$, the soliton decays into a freely expanding condensate. From the experimental point of view, the noise may even help to watch the soliton dynamics in situ because of the comparably large observable change in the condensate extensions.

The small dip in the $x$-extension at around 100 ms is noteworthy, as it is caused by the still present external trap with a small frequency, $f_1$, which decelerates the expansion. In line with this trend, increasing $f_1$ from 6.5 Hz to 8 Hz we have observed a more pronounced dip and a smaller value of the maximum extension, together with an increased oscillation frequency. For smaller holding times $t_{\text{hold}}$, meaning that the ramping sequence for the trap begins earlier, we find a shallower dip, leading to slower dynamics and larger extensions of the condensate.

The simulations show that the time scale governing the dynamics of the soliton is on the order of 1 s, which is relatively large for trapped condensates. The main limitation for holding times is imposed by losses caused by three-particle collisions, which is modeled in Eq. (1) by imaginary potential [3], proportional to the squared number density. From Fig. 3, we observe that, in the case of the ideal ramp, the soliton is larger in the $x$- and $z$-directions by a factor of $4-5$ than the initial condensate (in the $y$-direction, the size is approximately constant). This results in the density reduced by factor $\approx 15-25$, hence the loss rate will rapidly drop when the trap is switched off. Indeed, Fig. 4 where the evolution of the number of atoms in the condensate during the simulation is plotted, demonstrates that most atoms are lost within the first 50 ms. Without the noise in the scattering length, the atom number then stays nearly constant for subsequent 300 – 400 ms, and decreases at the end of the simulation, when the soliton has reached its smallest extension, i.e., it has returned to the largest density. With the noise present, the mean extensions are even larger, therefore the losses are reduced even more. For the $^{52}\text{Cr}$ condensate with the scattering length around 11$a_B$, we thus conclude that the atom losses will not cause decay of the soliton.

Finally, for parameters of the $^{164}\text{Dy}$ condensate we find that the threshold scattering length necessary for the existence of the anisotropic solitons is 133$a_B$. Since the actual scattering length of Dy is estimated to be smaller than this value [4], anisotropic solitons may be created in this condensate by means of a simpler procedure, gradually switching off the trap only and keeping the scattering length constant. Applying a simple Gaussian trial wave function to evaluate the mean-field energy of a condensate which is only trapped in $y$-direction, as was done in Refs. [20, 21], we can estimate the binding energies of the soliton in the $(x,z)$ plane for various scattering lengths. This energy scale determines the frequency of oscillation of the excited soliton, showing a crucial dependence on the scattering length. For the confinement of 20000 atoms in the $y$-direction, we choose a trap frequency of 200 Hz, and conclude that the relevant time scale varies from 0.5 s to 2 s for the scattering length between 126$a_B$ and 122$a_B$. At 128$a_B$, the time scale already reaches a value $\approx 10$ s.

In conclusion, we have performed systematic simulations of the realistic model describing the evolution of the dipolar condensate en route to the formation of the recently predicted new species of localized multidimensional solitary modes, in the form of quasi-2D anisotropic solitons. The model includes factors which are crucially important to the adequate description of the experiment, such as the three-body losses and Feshbach-induced noise in the scattering length. The scenario for the formation of the solitons was elaborated, by gradually switching off the trap and decreasing the scattering length. It is demonstrated that, in the chromium and dysprosium BEC alike, the solitons may be safely formed, surviving the inevitable excitation of the condensate. Further challenging problems may be the making of soliton clusters, and the study of interactions between anisotropic solitons. A more general implication of the reported results is the possibility to elaborate similar dynamical scenarios for the creation, under realistic conditions, of various complex multidimensional patterns predicted by the analysis in quantum gases.

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