Research on Compensation Algorithm for Residual Carrier Frequency Offset in Multi-tone Parallel System

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Abstract. The influence of residual carrier frequency offset on demodulation for multi-tone parallel signals is theoretically deduced and simulated, and two compensation algorithms for residual carrier frequency offset are proposed. Taking 16 tones for example, the simulation results in AWGN channel show that the influence of the residual carrier frequency offset is in agreement with the theoretical derivation, and the loss of error code performance is less than 0.1db when the frequency offset is less than 0.4% of the sub-carrier symbol rate. The thresholds of the SNR lossing less than 0.1 db are obtained: Eb/N0=5 db for short burst signals and Eb/N0=4db for continuous signals. The theoretical analyses and simulation results show that the proposed algorithms are applicable to multi-tone parallel signals.

1. Introduction
The HF multi-tone parallel signal makes the sub-carrier bandwidth tend to be smaller than the channel coherence bandwidth by extending the symbol period and increasing the number of carriers, so frequency selective fading can be avoided well in multi-tone parallel systems [1, 2]. However, due to the difference from transceivers and the Doppler shift, frequency deviations are introduced, resulting in changes in the spectral structure and phase of the signal, and disrupting the orthogonality of the sub-carriers. Multi-tone parallel signals generally have a Doppler tone and when the received signal has a Carrier Frequency Offset (CFO), this tone is used for frequency estimation and CFO correction. Next is synchronous extraction, including time and frequency synchronization [3]. Affected by the quality and length of the received signal, the frequency estimation often has a certain error, resulting in residual CFO in the signal before FFT transform.

At present, there are few studies on the influence of residual CFO on multi-tone parallel signal demodulation. The multi-tone parallel system is similar to the Orthogonal Frequency Division Multiplexing (OFDM) system and it may be referred to the related research of OFDM. In the literature [4, 5], the Cyclic Prefix (CP) of OFDM is used for frequency offset estimation, and CP-based maximum likelihood tracking algorithm can effectively track residual CFO, but multi-tone parallel system does not use such a mechanism. In the literature [6, 7], the algorithm can obtain better synchronization performance by inserting training sequences, belonging to the Data-Aided (DA) method, but it is not suitable for multi-tone parallel signals. In the literature [8], the residual CFO interference model of OFDM is deduced, and a Non-Data-Aided (NDA) tracking method based on decision feedback is proposed.

Based on the research ideas in literature [8], this paper establishes a residual CFO interference model for multi-tone parallel systems, and proposes two residual CFO compensation algorithms based
on the derivation results, and performs simulation verification and analysis based on 16 tones parallel signals.

2. Multi-tone parallel signal modulation and demodulation model

The multi-tone parallel signal has a similar modulation method to the OFDM, and can be demodulated by means of FFT. With \( N \) representing the number of sub-carriers and \( T \) representing the duration (period) of the parallel signal symbol and \( d_i(k), k=0,1,2,\ldots,N-1 \), representing the data symbol assigned to the \( i \)th frame \( k \)th sub-carrier, and the rectangular function \( \text{rect}(t)=1, |t| \leq T/2 \). The equivalent baseband signal of the parallel signal symbol starting from \( t=t_i \) can be expressed as follows [9].

\[
s(t)=\sum_{k=0}^{N-1} d_i(k) \cdot \text{rect}\left(t-t_i - \frac{T}{2}\right) \cdot \exp\left(j2\pi \frac{i}{T}(t-t_i)\right), \quad t_i \leq t \leq t_i + T \tag{1}\]

For the OFDM system, the sub-carrier spacing \( f_b \) is equal to the sub-carrier symbol rate \( R_b \) so that the symbol period of any sub-carrier is an integer multiple cycle in one OFDM symbol period. Sub-carriers satisfy the following formula in one OFDM symbol period:

\[
\frac{1}{T} \int_0^T \exp(j2\pi f_t) \cdot \exp(j2\pi f_j t) dt = \begin{cases} 1, & i=j \\ 0, & i \neq j \end{cases} \tag{2}
\]

So the sub-carriers are orthogonal to each other and the receiver can demodulate the sub-carriers by means of FFT, not affected by other sub-carriers. FFT is a fast and efficient implementation mode. For the multi-tone parallel signal, it is feasible to separately filter and demodulate each sub-carrier but not efficient. Because the sub-carrier spacing is greater than the sub-carrier symbol rate, the multi-tone signal can not be demodulated directly like OFDM. But if the multi-tone signal symbol is truncated in the time domain to make the symbol period \( T^* = 1/f_b \), it also satisfies the orthogonality to perform FFT demodulation. However, since the time domain is not continuous, the adjacent frames of each sub-carrier after the FFT transform have a fixed phase difference, which is determined by the frequency point of sub-carrier and called a linear phase increment. The mathematical expression of the demodulation process is given below.

The symbol of the \( i \)th frame \( k \)th sub-carrier of the multi-tone parallel signal is expressed as \( d_i(k) = d_i(k) \cdot \exp(j\theta_i(k)) \), then the equivalent baseband signal can be expressed as:

\[
s(i) = \sum_{k=0}^{N-1} d_i(k) \cdot \exp\left(j2\pi \frac{i}{N} k\right), \quad i=1,2,3,\ldots \tag{3}\]

Next to demodulate multi-tone signal by FFT transform and the received signal is firstly sampled to \( f_s = N \cdot f_b \). At this time, the FFT frequency resolution \( f_s / N \) is just the sub-carrier spacing \( f_b \), which satisfies the condition of quadrature demodulation by FFT transform. The samples of \( i \)th frame after the sample rate of \( f_s \) is \( M = f_s / R_b \), which can be expressed as \( s_i(n), n=0,1,2,\ldots,M-1 \). After the timing synchronization, the time domain signal of each frame is truncated, so that the sub-carrier symbol period \( T^* = 1/f_b \). In this case, each sub-carrier is orthogonal when the FFT is performed and the symbol data of the \( i \)th frame is \( s_i(n), n=0,1,2,\ldots,N-1 \). The FFT result of \( i \)th frame \( k \)th sub-carrier can be expressed as:

\[
S_i(k) = \sum_{n=0}^{M-1} s_i(n) \cdot \exp\left(-j2\pi n \frac{k}{N}\right) = d_i(k) \cdot \exp\left(j2\pi \frac{ik}{N}\right), \quad k=0,1,2,\ldots,N-1 \tag{4}\]

Where \( \exp\left(j2\pi \frac{ik}{N}\right) \) is the linear phase increment of \( i \)th frame \( k \)th sub-carrier, then the sub-carrier symbol can be expressed as:
\[ d_i(k) = |d_i(k)| \exp(j \theta_i(k)) = S_i(k) \cdot \exp\left(-j \frac{2\pi ik}{N}\right) \quad k = 0, 1, 2, \ldots, N-1 \] (5)

After the FFT is completed, the sub-carrier symbol \( d_i(k) \) is obtained, and then demodulated according to the sub-carrier modulation type. For the 16 tones, the sub-carrier is modulated by TDQPSK (Time Differential Quadrature Phase Shift Keying). The original modulation information of sub-carrier can be demodulated by performing difference between the preceding and succeeding frames.

3. Research on residual CFO compensation technology for multi-tone parallel signal

3.1. Influence of residual CFO on FFT quadrature demodulation

Since the accuracy of initial frequency estimation is affected by signal quality and length, there will be carrier frequency offset, which will affect the demodulation performance. This paper makes the following theoretical derivation and analysis.

Assuming that \( \Delta f \) is the normalized frequency shift compared to the stated frequency point \( f_k \) of \( k \)th sub-carrier. That means the real frequency of \( k \)th sub-carrier is \( f_k = f_k + \Delta f \). The equivalent baseband signal can be expressed as equation (6).

\[
\hat{s}(i) = \sum_{k=0}^{N-1} d_i(k) \cdot \exp\left(j 2\pi f_i \frac{\hat{f}_k}{f_s}\right)
\]

\[= \sum_{k=0}^{N-1} d_i(k) \cdot \exp\left(j 2\pi f_i \frac{f_k}{f_s}\right) \cdot \exp\left(j 2\pi i \frac{\Delta f}{f_s}\right) \] (6)

According to equation (3) and equation (6):

\[
\hat{s}(i) = s(i) \cdot \exp\left(j 2\pi i \frac{\Delta f}{f_s}\right)
\] (7)

Then the truncated time domain signal \( \hat{s}_i(n), n = 0, 1, 2, \ldots, N-1 \) of \( i \)th frame can be expressed as:

\[
\hat{s}_i(n) = s_i(n) \cdot \exp\left(j 2\pi \left(i + \frac{n}{M}\right) \frac{\Delta f}{f_s}\right)
\] (8)

Then the demodulation result of \( i \)th frame \( k \)th sub-carrier after N-point FFT transform can be expressed as:

\[
\hat{S}_i(k) = \sum_{n=0}^{N-1} \hat{s}_i(n) \cdot \exp\left(-j 2\pi n \frac{k}{N}\right)
\]

\[= \sum_{n=0}^{N-1} s_i(n) \cdot \exp\left(j 2\pi \left(i + \frac{n}{M}\right) \frac{\Delta f}{f_s}\right) \cdot \exp\left(-j 2\pi n \frac{k}{N}\right) \] (9)

\[= \sum_{n=0}^{N-1} s_i(n) \cdot \exp\left(j 2\pi i \frac{\Delta f}{f_s}\right) \cdot \exp\left(-j 2\pi n \frac{k}{N}\right) + \sum_{n=0}^{N-1} s_i(n) \cdot \exp\left(n \frac{\Delta f}{Mf_s}\right) \cdot \exp\left(-j 2\pi n \frac{k}{N}\right)
\]

According to equation (4) and (9):

\[
\hat{S}_i(k) = \hat{S}_i(k) \cdot \exp\left(j 2\pi i \frac{\Delta f}{f_s}\right) + \sum_{n=0}^{N-1} s_i(n) \cdot \exp\left(n \frac{\Delta f}{Mf_s}\right) \exp\left(-j 2\pi n \frac{k}{N}\right)
\] (10)

The first term on the right side of the equation (10) indicates that there is a frequency offset \( \Delta f \) to the unbiased result \( \hat{S}_i(k) \) after FFT transform, which is equal to the residual CFO. The second term on the right side of the equation (10) is an interference term to the output result \( \hat{S}_i(k) \) and it is related
to the symbol data \( s_i(n) \), the sub-carrier position \( k \), and the residual CFO \( \Delta f' \). When \( k \) and \( \Delta f' \) are fixed, the second term is equivalent to an additive interference to sub-carriers. Therefore, when there is a residual CFO before FFT demodulated, the effect is not only reflected in adding a fixed frequency offset to all sub-carriers, but also due to the destruction of the orthogonality between the sub-carriers, there will be an additive interference associated with multiple factors. That is ICI (Inter-Carrier Interference), which affects the demodulation BER performance. The interference form the second term is different on different frames of different sub-carriers, so that different sub-carriers have different initial phase deviations. In the actual receiving process, since the time domain data is not known, the interference term cannot be calculated by (10) and moved away from \( \hat{S}_i(k) \).

3.2. The compensation algorithm for residual CFO

The effect of residual CFO on FFT Quadrature demodulation is analyzed in Section 2.1. The conclusion is that the residual CFO existing before FFT transform will break the orthogonality between sub-carriers, causing ICI and affecting sub-carrier BER performance. Therefore, it is necessary to study a suitable compensation method for residual CFO in multi-tone parallel systems. Two algorithms are proposed for different applications in this paper.

The first compensation algorithm is for the short burst signal, as shown in figure 1. When the signal length is short, due to the lack of sufficient symbols to achieve stable tracking, the synchronous tracking method similar to the Phase Locked Loop (PLL) is not applicable. In this case, the residual CFO estimation can be performed on a single sub-carrier after FFT, and then the open loop method is used to correct the frequency offset in current burst signal. The specific method of frequency offset estimation depends on the situation, with NDA methods such as frequency doubling method and difference method, and DA methods such as correlation method.

![Figure 1. The first compensation algorithm for residual CFO](image)

The second residual CFO compensation algorithm is for the long burst signal and the continuous signal. As shown in figure 2, the algorithm adopts the implementation of synchronous tracking. When the signal length is long, the estimated accuracy is changed by the channel in time, and the single-estimation correction mode cannot achieve the best performance by single estimation and correction. In this case, continuous tracking is needed. It can be seen from the previous conclusion that the BER performance is affected by the residual CFO, and this part of the performance loss cannot be compensated after the FFT. So the phase adjustment is performed before the FFT, and after FFT transform, the Phase Detector (PD) of PLL needs to use the sub-carrier symbol to track phase error.
4. Experiments

4.1. The specification of simulated signal
The simulation experiment was carried out with 16 tones signal, and its specifications are as follows [10].
- 605 Hz is a Doppler tone for correcting the frequency deviation;
- 2915 Hz is a synchronous tone, used for signal synchronization in the first 5 frames, and used to transmit data after the first 5 frames;
- 935 Hz ~ 2365 Hz is 14 data tones, the sub-carrier spacing is 110 Hz;
- All sub-carriers are modulated by TDQPSK;
- The sub-carrier rate is 45Baud and 75Baud. In this paper, 75Baud is selected for simulation.
- Adopting Hamming coding technology.

4.2. The simulation of influence on demodulation
Taking the 16 tones in Section 3.1 as an example, the influence of residual CFO under Gaussian channel on the BER performance is evaluated. The sub-carriers of 16 tones signal adopt DQPSK modulation, and the differential demodulation can resist frequency offset. That is, the frequency offset of the first term on the right side of the equation (10) does not affect the BER performance.

Figure 3 shows the demodulation constellation comparison of 16 tones signals in noise-free channel. The 16 tones signal has 15 data tones, and the tone 5 and tone 14 are randomly chosen for observation. It can be seen that when there is a residual CFO, the constellation convergence after differential demodulation becomes worse, and the constellation point distributes a Gaussian distribution near the desired point. Indicates that the interference is related to the sub-carrier position $k$. As the residual CFO increases, the constellation tends to diverge, indicating that the interference magnitude is positively correlated with $\Delta f$.The above simulation results are consistent with the conclusions of Section 3.1.
Figure 3. Demodulation constellation comparison in noise-free channel

Figure 4 shows the demodulation BER curve of the 16 tones signal in AWGN channel. It can be seen that as $\Delta f$ increases, the BER performance loss decreases. When $\Delta f$ reaches 1.6% of the sub-carrier symbol rate (about 1.2 Hz), the performance loss is nearly 1db. The results in Figure 4 show that when $\Delta f$ is controlled within 0.4% of the sub-carrier symbol rate (about 0.3 Hz), the BER performance loss after the FFT transform is less than 0.1db so that the influence of the interference item of the equation (10) is negligible.

Figure 4. The BER performance curve in AWGN channel

4.3. The compensation algorithms simulation
The simulation experiment for the first algorithm is as follows. For each detected burst, the sub-carrier frequency offset estimation is performed by frequency doubling method after FFT transform, and the obtained result is fed back to the FFT for residual CFO correction. And then the second FFT transform is performed to get the output. It is assumed that $L$ sub-carrier symbols are selected for estimation, with symbol rate $R_s=75$Baud. The estimated accuracy is about $R_s/(4L)\approx 0.29Hz$ when $L=64$, which is about 0.4% of the symbol rate and the estimation accuracy increases as the estimated length $L$ increases. It can be seen from figure 4 that the performance loss is less than 0.1db in this case.
Figure 5 is the MSE (Mean Square Error) curve of residual CFO estimation when the estimated symbols $L=64$. The red broken line is determined by the result of figure 4, and the area below the broken line is the MSE range that satisfies the error performance loss of less than 0.1db. It can be seen that when the residual CFO range is within 1.6% of the sub-carrier symbol rate, the threshold whose error performance loss is less than 0.1db is $E_b/N_0=5$db.

![Figure 5](image)

**Figure 5.** The MSE compensating curve of the first algorithm

The simulation experiment for the second algorithm is as follows. After FFT demodulation, sub-carriers are phase-detected using a Costas loop, and the phase-detection results are fed back to the FFT for phase synchronization tracking. The speed of the residual CFO capturing and the jitter range are affected by the loop filter. By adjusting the loop parameters, better synchronization tracking performance can be obtained.

![Figure 6](image)

**Figure 6.** The MSE compensating curve of the second algorithm

Figure 6 shows the MSE curve of the locked frequency offset after the synchronization tracking is stabilized. The description of red broken line is the same as in figure 5. It can be seen that when the residual CFO range is within 1.6% of the sub-carrier symbol rate, the threshold whose BER performance loss is less than 0.1db is $E_b/N_0=4$db. At the same time, the performance of the second
The algorithm is related to the range of residual CFO. The smaller the residual CFO, the better the synchronization tracking performance. When the residual CFO range is within 0.8% of the sub-carrier symbol rate, the threshold whose BER performance loss is less than 0.1db is $E_b/N_0=1db$.

5. Conclusion
In this paper, the method of multi-tone parallel signal modulation and demodulation is studied, and the influence of residual CFO is theoretically derived. The conclusion shows that the residual CFO will destroy the orthogonality between sub-carriers, resulting in an additive noise interference and affecting the sub-carrier error performance. Based on the derivation results, two residual CFO compensation algorithms are proposed in this paper. The 16 tones signal is taken as an example to verify the simulation. The simulation results show that the influence of residual CFO on parallel signal demodulation is consistent with the theoretical derivation, and the BER performance loss is less than 0.1db when the residual CFO is within 0.4% of the sub-carrier symbol rate. At the same time, the threshold of the error performance loss less than 0.1db is obtained for the algorithm proposed in this paper. The threshold for the short burst signal is $E_b/N_0=5db$, and the threshold for the continuous signal is $E_b/N_0=4db$. Therefore the algorithm is suitable for multi-tone parallel signals.

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