

SU(3) mass-splittings of heavy-baryons in QCD

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We extract directly (for the first time) the charmed (C = 1) and bottom (B = –1) heavy-baryons (spin 1/2 and 3/2) mass-splittings due to SU(3) breaking using double ratios of QCD spectral sum rules (QSSR) in full QCD, which are less sensitive to the exact value and definition of the heavy quark mass, to the perturbative radiative corrections and to the QCD continuum contributions than the simple ratios commonly used for determining the heavy baryon masses. Noticing that most of the mass-splittings are mainly controlled by the ratio \( \kappa \equiv \langle \bar{s}s \rangle/\langle d\bar{d} \rangle \) of the condensate, we extract this ratio, by allowing 1\( \sigma \) deviation from the observed masses of the \( \Xi_{cc} \) and of the \( \Omega_{cc} \). We obtain: \( \kappa = 0.74(3) \), which improves the existing estimates: \( \kappa = 0.70(10) \) from light hadrons. Using this value, we deduce \( M_{\Omega_{cc}} = 6078.5(27.4) \) MeV which agrees with the recent CDF data but disagrees by 2.4\( \sigma \) with the one from D0. Predictions of the \( \Omega_{cc} \) and of the spectra of spin 3/2 baryons containing one or two strange quark are given in Table 2. Predictions of the hyperfine splittings \( \Omega_{cc} - \Omega_{c} \) and \( \Xi_{cc} - \Xi_{c} \) are also given in Table 3. Starting for a general choice of the interpolating currents for the spin 1/2 baryons, our analysis favours the optimal value of the mixing angle \( b \approx (−1/5 \sim 0) \) found from light and non-strange heavy baryons.

1. Introduction

QSSR \[12\] à la SVZ \[3\] has been used earlier in full QCD \[4,5\] and in HQET \[7\] for understanding heavy baryons [charmed (ccq), bottom (bbq), double charm (ccq), double bottom (bbq) and (bcq)] masses. Recent observations at Tevatron of families of b-baryons \[8,9\] and of the \( \Omega_{c} \) baryon by Babar and Belle \[10\] have stimulated different theoretical activities for understanding their nature \[11,12,13,14,15,16,17,18,19\]. QSSR results are in quite good agreement with recent experimental findings but with relatively large uncertainties. The inaccuracy of these results is mainly due to the value of the heavy quark mass and of its ambiguous definition when working to lowest order (LO) in the radiative \( \alpha_s \) corrections in full QCD and HQET\(^1\), where the heavy quark mass is the main driving term in the QCD expression of the baryon two-point correlator used in the QSSR analysis. Another source of uncertainty is the effect of the QCD continuum which parametrizes the higher baryon masses contributions to the spectral function and the \textit{ad hoc} choices of interpolating baryon currents used in different literatures. In this paper, we shall concentrate on the analysis of the heavy baryon mass-splittings due to \( SU(3) \) breaking using double ratios (DR) of QCD spectral sum rules (QSSR), which are less sensitive to the exact value and definition of the heavy quark mass and to the QCD continuum contributions than the simple ratios used in the literature to determine the absolute value of heavy baryon masses.

In this letter, we extend the previous analysis in \[4,5\] by including the new \( SU(3) \) breaking terms: \( m_s \) and the ratio of the condensate \( \kappa \equiv \langle \bar{s}s \rangle/\langle d\bar{d} \rangle \).

For the spin 1/2 baryons, and following Ref. \[4\], we work with the lowest dimension general currents:

\[
\begin{align*}
\eta_{\Xi_Q} &= \epsilon_{abc} \left[ (q^T C \gamma_5 s_b) + b(q^T C s_b) \gamma_5 \right] Q_c, \\
\eta_{\Lambda_Q} &= \eta_{\Xi_Q} (s \to q), \\
\eta_{\Omega_Q} &= \epsilon_{abc} \left[ (s^T C \gamma_5 q_b) + b(s^T C q_b) \gamma_5 \right] s_c, \\
\eta_{\Xi_Q} &= \eta_{\Lambda_Q} (s \to q), \\
\eta_{\Xi_Q} &= \frac{1}{\sqrt{2}} \epsilon_{abc} \left[ (s^T C \gamma_5 q_b) q_c + (q^T C \gamma_5 q_b) s_c \right. \\
&\left. + b \left( (s^T C q_b) \gamma_5 q_c + (q^T C q_b) \gamma_5 s_c \right) \right],
\end{align*}
\]

where we use standard notations; \( b \) is \textit{a priori} an arbitrary mixing parameter. Its value has been found to be:

\( b = -1/5 \),

in the case of light baryons \[20\] and in the range \[4,5,6\]:

\( -0.5 \leq b \leq 0.5 \),

for non-strange heavy baryons, which do not favour the Ioffe choice \( b = -1 \) \[21\]. The corresponding two-point correlator reads:

\[
S(q) = i \int d^4 x \, e^{i q x} \langle 0 | T \bar{\pi}(x) \pi(0) | 0 \rangle \\
\equiv \vec{q} F_1 + F_2,
\]

where \( F_1 \) and \( F_2 \) are two invariant functions.

For the spin 3/2 baryons, we follow Ref. \[5\] and work with the interpolating currents:

\[
\begin{align*}
\eta_{\Xi_Q}^\mu &= \sqrt{\frac{2}{3}} \left[ (q^T C \gamma_5 q) s + (s^T C \gamma_5 q) q + (q^T C \gamma_5 s) q \right], \\
\eta_{\Lambda_Q}^\mu &= \frac{1}{\sqrt{2}} \eta_{\Xi_Q}^\mu (q \to s), \\
\eta_{\Xi_Q}^\mu &= \frac{1}{\sqrt{2}} \eta_{\Xi_Q}^\mu (s \to q),
\end{align*}
\]

where an anti-symmetrization over colour indices is understood. The normalization in Eq. (3) is chosen in such
a way that in all cases one gets the same perturbative contribution. The corresponding two-point correlator reads:

\[ S_{\mu
u}(q) = i \int d^4x \, e^{i qx} \langle 0 | T \pi^\mu(x) \pi^\nu(0) | 0 \rangle \]

\[ \equiv g^{\mu
u} \langle qF_1 + F_2 + \ldots \rangle \]  

(6)

In the following, the contribution of the heavy quark condensate \( mQ \langle QQ \rangle \) will not appear [5] as it is cancelled by a part of the gluon condensate contribution due to the heavy quark relation \( mQ \langle QQ \rangle + (1/12 \pi) (\alpha_s G^2) \approx 0 \) [3]. Again due to this relation, the one due to \( m_s \langle gg \rangle \) is numerically negligible because of the extra \((1/12 \pi)\) loop factor. The same loop factor also numerically suppresses the contributions of \( (\alpha_s G^2) \) and \( m_s (\alpha_s G^2) \) compared to the other terms. These negligible \( SU(3) \) breaking contributions will not be considered in the following.

2. The spin 1/2 two-point correlator in QCD

The \( \Sigma_Q(Qqq) \) and \( \Xi_Q(Qss) \) baryons

The expression for \( \Lambda_Q \) has been (first) obtained in [4]. The additional \( SU(3) \) breaking terms for the \( \Omega_Q \) are:

- \( F_1 \):

\[ \text{Im} F_1^{m_s} |_{pert} = \frac{3m_s m_Q^3}{2 \pi^3} (1 - b^2) \times \]

\[ \left[ \frac{2}{x} + 3 - 6x + x^2 + 6 \ln x \right], \]

\[ \text{Im} F_1^{m_s} |_{is} = \frac{3m_s m_Q^3}{2 \pi^3} (1 + b^2) (1 - x^2), \]

\[ F_1^{m_s} |_{mix} = \frac{m_s m_Q^3}{2 \pi^3} \left[ \frac{1}{m_Q^2 - q^2} (7 + 22b + 7b^2) \right] \]

\[ - (1 + b^2) \int_0^1 \frac{da (1 - a)}{m_Q^2 - (1 - a) q^2} \]

\[ \text{Im} F_1^{m_s} |_{D=6} = \frac{m_s m_Q^3 m_G^2}{2 \pi^3} (1 - b^2) \frac{1}{8(m_Q^2 - q^2)^2}. \]  

(10)

- \( F_2 \):

\[ \text{Im} F_2^{m_s} |_{pert} = \frac{3m_s m_Q^3}{2 \pi^3} (1 - b^2) \times \]

\[ \left[ \frac{1}{x^2} + \frac{6}{x} + 3 + 2x - 6 \ln x \right], \]

\[ \text{Im} F_2^{m_s} |_{is} = - \frac{3m_s m_Q^3}{2 \pi^3} (3 + 2b + 3b^2) (1 - x), \]

\[ F_2^{m_s} |_{mix} = \frac{m_s m_Q^3 m_G^2}{2 \pi^3} \times \]

\[ \left[ \frac{1}{m_Q^2 - q^2} (25 + 22b + 25b^2) \right] \]

\[ -3(5 + 6b + 5b^2) \times \]

\[ \int_0^1 \frac{da}{m_Q^2 - (1 - a) q^2} \]

\[ \text{Im} F_2^{m_s} |_{D=6} = \frac{m_s m_Q^3 m_G^2}{2 \pi^3} (1 - b^2) \frac{1}{8(m_Q^2 - q^2)^2} \left[ 1 + \frac{m_Q^2}{m_Q^2 - q^2} \right]. \]  

(11)

The \( \Sigma_Q(Qqq) \) and \( \Xi_Q(Qss) \) baryons

The expression for \( \Xi_Q \) tends to the one of the \( \Sigma_Q \) in the chiral limit \( m_q \rightarrow 0 \) and is very similar with one of the \( \Omega_Q \). The \( SU(3) \) breaking corrections read:

- \( F_1 \):

\[ \text{Im} F_1^{m_s} |_{pert} = \frac{3m_s m_Q^3}{2 \pi^3} (1 - b^2) \times \]

\[ \left[ \frac{2}{x} + 3 - 6x + x^2 + 6 \ln x \right], \]

\[ \text{Im} F_1 |_{is} = \frac{-3m_Q^3 m_G^2}{2 \pi^3} (\langle ss \rangle + \langle qg \rangle) (1 - b^2)(1 - x)^2, \]

\[ \text{Im} F_1^{m_s} |_{mix} = \frac{m_s m_Q^3}{2 \pi^3} (1 - x^2) \left[ -2(1 - b^2) \langle qg \rangle \right. \]

\[ + (5 + 2b + 5b^2) \langle ss \rangle \].
Im $F_1|_{mix} = \frac{(\langle \bar{s}G_s \rangle + \langle \bar{q}G_q \rangle)}{m_Q 2s^2 \pi} (1 - b^2) (13x^2 - 6x)$, 

$F_1^{m_s}|_{mix} = -\frac{m_s}{2^8 3^2 \pi^2} \left[ \frac{1}{m_Q^2 - q^2} \right] \left[ 13 + 10b + 13b^2 \right] \times \langle \bar{s}G_s \rangle - 6(1 - b^2) \langle \bar{q}G_q \rangle + \left[ 3(1 - b^2) \langle \bar{q}G_q \rangle \right. \\
-3(3 + 2b + 3b^2) \langle \bar{s}G_s \rangle \left. \right] \times \int_0^1 \frac{da}{m_Q^2 - (1 - a)q^2}, \\
F_{1|D=6} = \frac{\rho(\bar{s}s)(\bar{q}q)}{24} \frac{(1 - b^2)}{m_Q^2 - q^2}, \\
F_1^{m_s}|_{D=6} = -\frac{m_s m_Q \rho(\bar{s}s)(\bar{q}q)}{16} \frac{1 - b^2}{(m_Q^2 - q^2)^2} 
(12) \\
F_2 : \\
Im F_2^{m_s}|_{pert} = \frac{3m_s m_Q}{2^9 \pi^3} (1 - b^2) \times \\
\left[ \frac{1}{x^2} - \frac{6}{x} + 3 + 2x - 6 \ln x \right], \\
Im F_{2|ss} = -\frac{3m_Q^2((\bar{s}s) + \langle \bar{q}q \rangle)}{2^9 \pi \left( \frac{1}{x} - \frac{1}{x} \right)^2 \times (1 - b^2)x \left( 1 - \frac{1}{x} \right)^2, \\
Im F_{2|ss}^{m_s} = \frac{m_s m_Q}{2^9 \pi} (1 - x) \left( 1 - b^2 \right)(\bar{s}s) - 2(5 + 2b + 5b^2)\langle \bar{q}q \rangle, \\
Im F_{2|mix} = \frac{(\bar{s}G_s) + \langle \bar{q}G_q \rangle}{2^8 \pi} (1 - b^2) (6 + x), \\
F_2^{m_s}\left|_{mix} \right. = \frac{m_s m_Q}{2^8 3^2 \pi^2} \left[ \frac{1}{m_Q^2 - q^2} \right] \left[ 5(1 - b^2) \langle \bar{s}G_s \rangle - 6(5 + 2b + 5b^2) \langle \bar{q}G_q \rangle \right] + \\
\left[ -3(1 - b^2) \langle \bar{s}G_s \rangle + 6(3 + 2b + 3b^2) \times \langle \bar{q}G_q \rangle \right] \int_0^1 \frac{da}{m_Q^2 - (1 - a)q^2}, \\
F_{2|D=6} = \frac{m_Q \rho(\bar{s}s)(\bar{q}q)}{24} \frac{(5 + 2b + 5b^2)}{m_Q^2 - q^2} \frac{1}{m_Q^2 - q^2}, \\
F_2^{m_s}|_{D=6} = -\frac{m_s \rho(\bar{s}s)(\bar{q}q)}{16} \frac{1 - b^2}{m_Q^2 - q^2} \times \\
\left[ 1 + \frac{m_Q^2}{m_Q^2 - q^2} \right]. 
(13) \\
We have checked the existing results in [4] obtained in the chiral limit and all our previous results agree with these ones.

3. The spin 3/2 two-point correlator in QCD

The QCD expression of the two-point correlator for the $\Sigma^*_Q(Qqq)$ has been (first) obtained in the chiral limit $m_{u,d} = 0$, to LO in $\alpha_s$ and up to the contributions of the $D = 6$ condensates in [4]. In this letter, we extend the previous analysis by including the new SU(3) breaking $m_s$ correction terms and consider the SU(3) breaking of the ratio of quark condensates $\langle \bar{s}s \rangle \neq \langle \bar{q}q \rangle$ like we did for the spin 1/2 case.

The $\Sigma^*_Q(Qqq)$ and $\Xi^*_Q(Qsq)$ baryons

The additional terms and replacement due to SU(3) breaking for the $\Xi^*_Q$ compared with the one of the $\Sigma^*_Q(Qqq)$ in [3] are:

- $F_1$:

$$Im F_1^{m_s}|_{pert} = \frac{m_s m_Q}{48 \pi^3} \left[ \frac{2}{x} + 3 - 6x + x^2 + 6 \ln x \right],$$

$$Im F_{1|ss} = \frac{m_Q}{6 \pi} \left[ \langle \bar{q}q \rangle + \langle \bar{s}s \rangle \right] (1 - x)^2,$$

$$Im F_1^{m_s}|_{ss} = \frac{m_s}{12 \pi} \left[ 2(1 - x^2) \langle \bar{q}q \rangle - (1 - x^2) \langle \bar{s}s \rangle \right],$$

$$Im F_{1|mix} = \frac{7M_Q^2}{3^2 2^4 \pi} \left[ \langle \bar{q}q \rangle + \langle \bar{s}s \rangle \right] \frac{x^2}{m_Q^2},$$

$$F_2^{m_s}|_{mix} = \frac{m_s M_0^2}{144 \pi^3} \left[ 12 \langle \bar{q}q \rangle - 9 \langle \bar{s}s \rangle \right] +$$

$$\frac{1}{2} \int_0^1 \frac{da}{m_Q^2 - (1 - \alpha)a^2} \times \\
\left[ (1 - 3\alpha) \langle \bar{s}s \rangle + \langle \bar{q}q \rangle \right],$$

$$F_{2|D=6} = \frac{4}{9} \frac{\rho(\bar{s}s)(\bar{q}q)}{m_Q^2 - q^2},$$

$$F_2^{m_s}|_{D=6} = \frac{2}{9} \frac{m_Q m_s \rho(\bar{s}s)(\bar{q}q)}{(m_Q^2 - q^2)^2}. 
(14)$$

- $F_2$:

$$Im F_2^{m_s}|_{pert} = \frac{m_s m_Q}{192 \pi^3} \left[ \frac{3}{x^2} - \frac{16}{x} + 12 + x^2 - 12 \ln x \right],$$

$$Im F_{2|ss} = \frac{m_Q^2}{18 \pi} \left[ \langle \bar{q}q \rangle + \langle \bar{s}s \rangle \right] \left( \frac{2}{x} - 3 + x^2 \right),$$

$$Im F_2^{m_s}|_{ss} = \frac{m_s m_Q}{12 \pi} (1 - x) \left[ 6 \langle \bar{q}q \rangle - (1 + x) \langle \bar{s}s \rangle \right],$$

$$Im F_{2|mix} = \frac{M_Q^2}{18 \pi} \left[ \langle \bar{q}q \rangle + \langle \bar{s}s \rangle \right] \left( 1 + \frac{3}{4} x^2 \right),$$

$$F_2^{m_s}|_{mix} = \frac{m_s m_Q M_0^2}{72 \pi^3} \left[ 3 \langle \bar{q}q \rangle - (\langle \bar{s}s \rangle) \right] +$$

$$\langle \bar{q}q \rangle \int_0^1 \frac{da}{m_Q^2 - (1 - \alpha)a^2} \times \\
\left[ 1 + \frac{m_Q^2}{m_Q^2 - q^2} \right],$$

$$F_{2|D=6} = \frac{2 m_s}{3} \frac{m_Q \rho(\bar{s}s)(\bar{q}q)}{m_Q^2 - q^2},$$

$$F_2^{m_s}|_{D=6} = \frac{2 m_s^2 \rho(\bar{s}s)(\bar{q}q)}{9 (m_Q^2 - q^2)^2}. 
(15)$$

where $x \equiv m_Q^2/s$ and $\langle \bar{s}G_s \rangle \equiv g(\bar{s}\sigma_{\mu\nu}A_\mu A_\nu/s) \equiv M_0^2(\bar{s}s)$. 

The $\Omega_Q^0(Qss)$ baryons

Compared with the expression of the $\Sigma_Q^0(Qqq)$ in [5], the additional $SU(3)$ breaking terms for the $\Omega_Q^0$ are:

- $F_1$:

\[
\text{Im} F_{1s}^{\text{mix}}|_{\text{pert}} = \frac{m_s m_Q^3}{24 \pi^3} \left[ \frac{2}{x} + 3 - 6x + x^2 + 6 \ln x \right],
\]

\[
\text{Im} F_{1s}^{\text{mix}}|_{2s} = -\frac{m_s \langle \bar{s}s \rangle}{6 \pi} \left( 1 - 2x^2 + x^3 \right),
\]

\[
F_{1s}^{\text{mix}}|_{\text{mix}} = \frac{m_s M_{Qs}^2 \langle \bar{s}s \rangle}{72 \pi^2} \left[ \frac{3}{m_Q^2 - q^2} \right. + 2 \int_0^1 \frac{d\alpha (1-\alpha)(2-3\alpha)}{m_Q^2 - (1-\alpha)q^2},
\]

\[
F_{1s}^{\text{mix}}|_{D=6} = -\frac{4 m_s m_p \rho(\bar{s}s)^2}{9 (m_Q^2 - q^2)^2}.
\]

- $F_2$:

\[
\text{Im} F_{2s}^{\text{mix}}|_{\text{pert}} = \frac{m_s m_Q^4}{96 \pi^3} \left[ \frac{3}{x^2} - \frac{16}{x} + 12 + x^2 - 12 \ln x \right],
\]

\[
\text{Im} F_{2s}^{\text{mix}}|_{2s} = -\frac{m_s m_Q \langle \bar{s}s \rangle}{6 \pi} \left( 5 - 6x + x^2 \right),
\]

\[
F_{2s}^{\text{mix}}|_{\text{mix}} = \frac{m_s m_Q M_{Qs}^2 \langle \bar{s}s \rangle}{36 \pi^2} \left[ \frac{6}{m_Q^2 - q^2} \right. + \int_0^1 \frac{d\alpha}{m_Q^2 - (1-\alpha)q^2},
\]

\[
F_{2s}^{\text{mix}}|_{D=6} = -\frac{4 m_s m_p \rho(\bar{s}s)^2}{9 (m_Q^2 - q^2)^2}.
\]

We have checked the existing results in [5] obtained in the chiral and $SU(2)$ limits and agree with these ones.

4. Form of the sum rules and QCD inputs

We parametrize the spectral function using the standard duality ansatz: “one resonance” + “QCD continuum”. The QCD continuum starts from a threshold $t_c$ and comes from the discontinuity of the QCD diagrams. Transferring its contribution to the QCD side of the sum rule, one obtains the finite energy Laplace/Borel sum rules:

\[
|\lambda_{B_i^0}|^2 \frac{M_{B_i^0}}{\rho} e^{-M_{B_i^0}^2/\tau} = \int_{t_q}^{t_c} ds e^{-s/\tau} \frac{1}{\pi} \text{Im} F_2(s),
\]

\[
|\lambda_{B_i^0}|^2 \frac{M_{B_i^0}}{\rho} e^{-M_{B_i^0}^2/\tau} = \int_{t_q}^{t_c} ds e^{-s/\tau} \frac{1}{\pi} \text{Im} F_1(s),
\]

where $\tau \equiv 1/M^2$ is the sum rule variable and $\lambda_{B_i^0}$ and $M_{B_i^0}$ are the heavy baryon residue and mass. Notice that, within our choice of the interpolating currents, we may have a contamination due to the negative parity states contribution, which we can quantify from the ratio of sum rules associated to $(M_Q F_1 - F_2)$ and $(M_Q F_1 + F_2)$ [7]. We shall check (a posteriori) that this ratio is negligible at the stability regions. On the other, the observed negative parity states are systematically heavier than the positive ones such that they can be legitimately included into the QCD continuum contribution. Consistently, we also take into account the $SU(3)$ breaking at the quark and continuum threshold:

\[
\sqrt{t_c/\tau}(SU(3)) \simeq \left( \sqrt{t_c/\tau}(SU(2)) \equiv m_Q + \hat{m}_{q_1} + \hat{m}_{q_2} \right),
\]

\[
\sqrt{t_c/\tau}(SU(3)) \simeq \left( \sqrt{t_c/\tau}(SU(2)) \equiv \sqrt{\tau} \right) + \hat{m}_{q_1} + \hat{m}_{q_2},
\]

where $q_{1,2} \equiv q$ or/and $s$ depending on the channel. $\hat{m}_{q_i}$ are the running light quark masses. $m_Q$ is the heavy quark mass, which we shall take in the range covered by the running and on-shell mass (see Table 1) because of its ambiguous definition when working to lowest order perturbative QCD. One can estimate the baryon masses from the following ratios:

\[
R_{i}^q = \frac{\int_{t_q}^{t_c} ds s e^{-s/\tau} \text{Im} F_1(s)}{\int_{t_q}^{t_c} ds e^{-s/\tau} \text{Im} F_1(s)},
\]

\[
R_{21}^q = \frac{\int_{t_q}^{t_c} ds e^{-s/\tau} \text{Im} F_2(s)}{\int_{t_q}^{t_c} ds e^{-s/\tau} \text{Im} F_1(s)},
\]

where at the $\tau$-stability point :

\[
M_{B_i^0}^\tau \simeq \sqrt{R_{i}^q} \simeq \sqrt{R_{21}^q}.
\]

These quantities have been used in the literature for getting the baryon masses and lead to a typical uncertainty of 15-20% [15]. In order to circumvent these problems, we work with the double ratio of sum rules (DR) [23]:

\[
\hat{R}_{i}^{sd} = \frac{\sqrt{R_{i}^q}}{\sqrt{R_{21}^q}}, \quad \hat{R}_{21}^{sd} \equiv \frac{\sqrt{R_{21}^q}}{\sqrt{R_{21}^q}},
\]

which take directly into account the $SU(3)$ breaking effects. These quantities are obviously less sensitive to the choice of the heavy quark masses, to the perturbative radiative corrections and to the value of the continuum threshold than the simple ratios $R_i$ and $R_{21}$ [3]. Analogous DR quantities have been used successfully (for the first time) in [23] for studying the mass ratio of the $0^{++}/0^{+}$ and $1^{++}/1^{-}$ B-mesons, in [24] for extracting $f_B/f_B$, in [25] for estimating the $D \to K/D \to \pi$ semi-leptonic form factors and in [26] for extracting the strange quark mass from the $e^+e^- \to I = 1,0$ data. For the numerical analysis who shall introduce the RGI quantities $\hat{m}_q$ and $\hat{m}_{q_i}$ [27]:

\[
\hat{m}_q(\tau) = \frac{\hat{m}_q}{(\log \sqrt{\tau}/\Lambda)^{2/\beta_1}},
\]

\[
\hat{q}(\tau) = \frac{\hat{q}}{(\log \sqrt{\tau}/\Lambda)^{2/\beta_1}}.
\]

\[
\hat{q}(\tau) = \frac{\hat{q}}{(\log \sqrt{\tau}/\Lambda)^{1/3\beta_1}},
\]

\[
\hat{q}_q(\tau) = \frac{\hat{q}_q}{(\log \sqrt{\tau}/\Lambda)^{1/3\beta_1}},
\]

\[
\hat{q}_q(\tau) = \frac{\hat{q}_q}{(\log \sqrt{\tau}/\Lambda)^{1/3\beta_1}}.
\]
where \( \beta_1 = -(1/2)(11 - 2n/3) \) is the first coefficient of the \( \beta \) function for \( n \) flavours. We have used the quark mass and condensate anomalous dimensions reviewed in [1]. We shall use the QCD parameters in Table 1. At the scale where we shall work, and using the parameters in the table, we deduce:

\[
\rho = 2.1 \pm 0.2 ,
\]

which controls the deviation from the factorization of the four-quark condensates. We shall not include the \( 1/q^2 \) term discussed in [28,29], which is consistent with the LO approximation used here as the latter has been motivated for a phenomenological parametrization of the larger order terms of the QCD series.

Table 1
QCD input parameters. For the heavy quark masses, we use the range spanned by the running \( \overline{MS} \) mass \( m_q(M_Z) \) and the on-shell mass from QSSR compiled in page 602,603 of the book in [1].

| Parameters | Values                  | Ref. |
|------------|-------------------------|------|
| \( \Lambda \) | \((353 \pm 15) \) MeV   | [30,1] |
| \( \bar{m}_d \) | \((6.1 \pm 0.5) \) MeV   | [311,9] |
| \( \bar{m}_s \) | \((114.5 \pm 20.8) \) MeV | [311,9] |
| \( \bar{\mu}_d \) | \((263 \pm 7) \) MeV     | [311] |
| \( \kappa \equiv \langle \bar{s}s \rangle/\langle \bar{d}d \rangle \) | \((0.7 \pm 0.1) \) | [311,20] |
| \( M_0^2 \) | \((0.8 \pm 0.1) \) GeV\(^2\) | [30,123] |
| \( \langle (\alpha_s G^2) \rangle \) | \((6.8 \pm 1.3) \times 10^{-2} \) GeV\(^4\) | [30,1,22,23,35,34,35,2] |
| \( \rho_{Q_s}(dd)\rangle \) | \((4.5 \pm 0.3) \times 10^{-4} \) GeV\(^6\) | [30,122] |
| \( m_c \) | \((1.18 \sim 1.47) \) GeV  | [139,1,35] |
| \( m_b \) | \((4.18 \sim 4.72) \) GeV  | [139,1,35] |

5. \( \kappa \equiv \langle \bar{s}s \rangle/\langle \bar{d}d \rangle \) from the spin 1/2 baryons

As a preliminary step of the analysis, we check the different results obtained in full QCD and in the chiral limit [31]:

\[
\begin{align*}
M_{\Sigma^0} &= (2.45 \sim 2.94) \ \text{GeV} , \\
M_{\Sigma^+} &= (5.70 \sim 6.62) \ \text{GeV} , \\
M_{\Sigma^0} - M_{\Lambda^0} &\leq 207 \ \text{MeV} , \\
M_{\Sigma^+} - M_{\Lambda^0} &\leq 163 \ \text{MeV} ,
\end{align*}
\]

which we confirm. However, we have not tried to improve these results for the absolute values of the masses due to the ambiguity in the definition of the heavy quark mass input mentioned earlier at LO, which induces large errors.

\( \Xi_c(\bar{c}sq)/\Lambda_c(\bar{c}qq) \)

- **Optimal choice of the currents:** We start from the general choice of interpolating currents given in Eq. [1]. We study the \( b \)-behaviour of the predictions in Fig. [1b]), by fixing \( \tau = 0.35 \) GeV\(^{-2}\) and \( t_c = 15 \) GeV\(^2\). The result presents \( b \)-stability around \( b = 0 \), which is in the range given in Eq. [4] obtained from light baryons and heavy non-strange baryons. We consider this value as the optimal choice of the interpolating currents. However, this generous range does not favour the \textit{ad hoc} choice around

\[
b \simeq 0 .
\]

- **\( \tau \) stabilities:** we show in Fig. [1b]) the \( \tau \) behaviour of the different DR at fixed \( t_c = 15 \) GeV\(^{-2}\) and \( b = 0 \).

- **\( t_c \) stabilities and choice of the sum rules:** fixing \( b = 0 \) from the previous analysis, we study in Fig. [1c]) the \( t_c \)-behaviour of the predictions. Among the three sum rules, we retain \( r_{21}^{sd} \) which is the most stable in \( t_c \) and then less affected by the higher state contributions.

- **Results:** From this \( r_{21}^{sd} \) sum rule, we can deduce the DR:

\[
r_{21}^{sd} = 1.080(10)(2)(6)(2)(1.5) ,
\]

where the value \( \kappa = 0.7 \) has been taken. We have considered the mean value of \( r_{21}^{sd} \) from \( t_c = 10 \) to 20 GeV\(^2\). The errors are due respectively to the values of \( t_c , \tau = (0.35 \pm 0.05) \) GeV\(^{-2}\), \( m_c , m_b \) and the factorization of the four-quark condensate \( \rho \). The errors due to \( b \) and some other SU(3) symmetric QCD parameters are negligible. The large error due to \( \kappa \) compiled in Table 1 is not included in Eqs. [27]. Using as input the data [9]:

\[
M_{\Lambda^0}^{exp} = (2286.46 \pm 0.14) \ \text{MeV} ,
\]

and adding the different errors quadratically, one can deduce:

\[
M_{\Xi_c} = (2469.4 \pm 26.6) \ \text{MeV} ,
\]
which agrees nicely with the data [9]:
\[ M_{\Xi_c}^{exp} = (2467.9 \pm 0.4) \text{ MeV} \, . \] (30)

For improving the existing value of \( \kappa \), we allow a 1\( \sigma \) deviation of the DR prediction from the experimental value. In this way, we deduce:
\[ \kappa = 0.700(50) \, . \] (31)

\[ \Xi_b(bq)/\Lambda_b(bq) \]
We repeat the previous analysis in the case of the \( b \)-quark. The analysis of the ratio of sum rules shows similar curves than for the charm case except the obvious change of scale. Using \( r_{\Xi_b}^{sd} \), we illustrate the analysis using \( \kappa = 0.74 \). In this way, we obtain:
\[ r_{\Xi_b}^{sd} = 1.030(2.5)(0.5)(1.5)(0.5)(0.5) \, , \] (32)
where the errors come from \( t_c \) from 45 to 80 GeV\(^2\), \( \tau = (0.18 \pm 0.05) \) GeV\(^{-2}\), \( m_b \), \( m_s \) and the factorization of the four-quark condensate \( \rho \). From this result, and using:
\[ M_{\Lambda_b}^{exp} = (5620.2 \pm 1.6) \text{ MeV} \, , \] (33)
we deduce:
\[ M_{\Xi_b} = (5789 \pm 16) \text{ MeV} \, , \] (34)
which agrees quite well with the data [9]:
\[ M_{\Xi_b}^{exp} = (5792.4 \pm 3.0) \text{ MeV} \, . \] (35)

Allowing a 1\( \sigma \) deviation from the data, we deduce:
\[ \kappa = 0.738(23) \, . \] (36)

\[ \Omega_c(css)/\Sigma_c(cqq) \]
We do an analysis similar to the one in the previous section. The result for the \( c \)-quark is shown in Fig 2. One can notice that the optimal choice of the current is the same as in Eq. (2) which we fix to the value \( b=0 \). One can notice from Fig (2b) and Fig (2c), that only \( r_{\Omega_c}^{sd} \) presents simultaneously \( \tau \) and \( b \) stabilities from which we extract the optimal result. Using \( \kappa = 0.78 \), the final result from \( r_{\Omega_c}^{sd} \) is:
\[ r_{\Omega_c}^{sd} = 1.111(1.4)(1.3)(16.4)(0.2) \, . \] (37)

The errors are due respectively to the values of \( \tau = (1.0 \sim 1.2) \) GeV\(^{-2}\), \( m_c \), \( m_s \) and the factorization of the four-quark condensate \( \rho \). The one due to \( t_c \) from 10 to 20 GeV\(^2\) is negligible. Using this previous result together with the experimental averaged value [9]:
\[ M_{\Xi_c}^{exp} = (2453.6 \pm 0.25) \text{ MeV} \, , \] (38)
one can deduce:
\[ M_{\Omega_c} = 2726.9(40.5) \text{ MeV} \, , \] (39)
in good agreement with the data:
\[ M_{\Omega_c}^{exp} = (2697.5 \pm 2.6) \text{ MeV} \, . \] (40)

Now, we study the influence of \( \kappa \) on the mass prediction. Allowing a 1\( \sigma \) deviation from the experimental mass, we deduce the estimate:
\[ \kappa = 0.775(15) \, . \] (41)

![Figure 2](image.png)

**Final value of \( \kappa \)**

Taking the (arithmetic) mean value of \( \kappa \equiv \langle \bar{s}s \rangle / \langle \bar{d}d \rangle \) from the different channels \( \Xi_c,b \) [Eqs. (31) and (36)] and \( \Omega_c \) [Eq. (31)], we deduce:
\[ \kappa = 0.738(29) \, , \] (42)
which we can consider as an improved estimate of this quantity compared with the existing one from the light mesons [31] and baryons [20]:
\[ \kappa = 0.700(100) \, . \] (43)

**6. The mass of the \( \Omega_b(bss) \)**

We repeat the previous analysis of the \( \Omega_c(css) \) in the case of the \( b \)-quark. The curves present the same qualitative behaviour as in the case of the charm, where, only \( r_{\Omega_c}^{sd} \) survives the different tests of stabilities. The optimal value is taken at the extremum \( \tau = (0.25 \pm 0.05) \) GeV\(^{-2}\) and in the \( t_c \) stability region. Then, we obtain:
\[ r_{\Omega_b}^{sd} = 1.0455(20)(22)(41)(13)(37) \, . \] (44)

The errors are due respectively to the values of \( \tau \), \( m_b \), \( m_s \) and the factorization of the four-quark condensate \( \rho \). The last error is due to \( \kappa \). The one due to \( t_c \) in the \( t_c \) stability region is negligible. Using this value together with the experimental averaged value [9]:
\[ M_{\Xi_b}^{exp} = 5811.2 \text{ MeV} \, , \] (45)
one can deduce the result:
\[ M_{\Omega_b} = 6075.6(37.2) \text{ MeV} \, , \] (46)
which we compile in Table 2. This result agrees within the errors with the one from the CDF collaboration \[37\]: 6054.4(6.9) MeV but disagrees by about 2.4 \(\sigma\) with the D0 value \[38\]: \(M_{\Xi_c^{D0}} = 6165.0(13.0)\) MeV given in the same table.

**Figure 3.** \( \Omega_0/\Sigma_c \): a) \(\tau\)-behaviour of DR given \(b = 0\) and \(tc = 60\) GeV\(^2\); \(r_1^{ds}\) dashed-dotted (blue), \(r_2^{ds}\) dotted (green), \(r_1^{\bar{d}s}\) continuous (red); b) \(b\)-behaviour of the DR given \(\tau = 0.25\) GeV\(^{-2}\) (\(\tau\)-stability of \(r_1^{ds}\)) and \(tc = 60\) GeV\(^2\); c) \(t_c\)-behaviour of \(r_2^{ds}\) given \(b = 0\) and \(\tau = 0.25\) GeV\(^{-2}\). We have used \(\kappa = 0.738\).

7. **The mass of the \( \Xi'_{c,b}(Qsq) \)**

We do a similar analysis for the \( \Xi'_{c,b} \), which we show in Fig. 4 where we have only retained the \(r_2^{ds}\) which satisfies all stability tests. We obtain:

\[ M_{\Xi_c} = 2559(25) \text{ MeV}, \]

which we compile in Table 2. Our prediction is in good agreement (1\(\sigma\)) with the data \[9\]:

\[ M_{\Xi_c}^{exp} = 2576(3.1) \text{ MeV}. \]

A similar analysis is done for \( \Xi_b' \), which is summarized in Fig. 5. One can notice that here the \(b\)-stability occurs at 0 in this channel and there is no sharp selection between \(r_1^{ds}\) and \(r_2^{ds}\). We obtain the mean from \(r_1^{ds}\) and \(r_2^{ds}\):

\[ M_{\Xi_b} = 1.014(3.4)(5)(1.7)(0.5)(2)(0.5)(3), \]

where the sources of the errors are \(\tau = (0.5 \pm 0.1)\) GeV\(^{-2}\), \(m_b\), \(m_s\), \(\rho\), \(b = (0. \pm 0.2)\), and \(\kappa\). The last error comes from the choice of the sum rules. Using the experimental value of \(M_{\Sigma_c}\), we predict:

\[ M_{\Xi_c} = 5893(42) \text{ MeV}, \]

which we compile in Table 2.

8. **The masses of the spin 3/2 baryons**

As a preliminary step of the analysis, we check the different results obtained in \[5\]:

\[ M_{\Sigma_c} = (2.15 \sim 2.92) \text{ GeV}, \]

\[ M_{\Sigma_c^*} - M_{\Sigma_c} = 3.3 \text{ GeV}, \]

and confirm them. However, like in the spin 1/2 case, we have not tried to improve these (old) results.

\[ \Xi_b'(csq)/\Sigma_c'(cq), \]

We repeat the previous DR analysis for the case of the \( \Xi_b' \). We show in Fig. 6a) the \(\tau\)-behaviour of the mass predictions for \(t_c = 14\) GeV\(^2\). From this analysis, we do not retain \(r_2^{ad}\) which differs completely from \(r_1^{ds}\) and \(r_2^{ds}\), while we do not consider \(r_1^{ds}\) which is \(\tau\)-instable. We show in Fig. 6b) the \(t_c\)-behaviour of \(r_2^{ds}\) given \(\tau = 0.9\) GeV\(^{-2}\).

We deduce the optimal value:

\[ r_2^{\Xi_c} = 1.049(10)(4)(4)(17.5)(1). \]

The errors are due respectively to the values of \(\tau = (0.9 \pm 0.1)\) GeV\(^{-2}\), \(m_b\), \(m_s\), \(\rho\) (factorization of the four-quark condensate) and \(\kappa \equiv \langle \bar{s}s \rangle/\langle \bar{d}d \rangle = 0.74 \pm 0.03\). The
From this figure, we shall not retain ones due to some other parameters are negligible. Using the data \[ M_{\Sigma_c^{*}}(bsq)/\Sigma_b^{*}(bqq) \]

We extend the analysis to the case of the bottom quark. The curves are qualitatively similar to the charm case. We deduce:

\[ r_{2}^{sd} = 1.022(2)(2)(0.5)(1)(2) \].

The sources of the errors are the same as for the \( \Sigma_c^{*} \), where here \( \tau = (0.25 \pm 0.05) \) GeV\(^{-2} \) and \( m_c \) replaced by \( m_b \). The ones due to some other parameters are negligible. Using the averaged data \[ M_{\Sigma_b^{*}} = (5832.7 \pm 6.5) \text{ MeV} \],

and adding the different errors quadratically, we deduce:

\[ M_{\Sigma_b^{*}} = (5961 \pm 21) \text{ MeV} \],

which we report in Table 2.

\[ \Omega_b^{*}(css)/\Sigma_b^{*}(cqq) \]

We pursue the analysis to the case of the \( \Omega_b^{*}(css) \). We show the \( \tau \)-behaviour of the different DR in Fig. 7b). From this figure, we shall not retain \( r_{2}^{sd} \) which differs completely from \( r_{1}^{sd} \) and \( r_{2}^{ds} \). Given the optimal value of \( \tau = 1 \) GeV\(^{-2} \), we show the \( \tau \)-behaviour of the DR \( r_{1}^{ds} \) and \( r_{2}^{ds} \) in Fig. 7b). The final result is the mean from \( r_{1}^{ds} \) and \( r_{2}^{ds} \):

\[ r_{21}^{rd} = 1.109(3)(10)(10)(4)(0.5)(8) \].

The 1st error is due to the choice of \( r_{1}^{sd} \). The other ones are due to \( \tau = (1.0 \pm 0.2) \) GeV\(^{-2} \), \( m_c, m_s, \rho \) and \( \kappa \). The other QCD parameters give negligible errors. Using the averaged data in Eq. (56), and adding the different errors quadratically, one can deduce the result in Table 2.

\[ \Omega_b^{*}(bss)/\Sigma_b^{*}(bqq) \]

We repeat the previous analysis in the \( b \)-channel. The curves are qualitatively analogue to the ones of the charm. We shall not consider \( r_{2}^{sd} \) because of its incompatibility with the other ones. From the mean of \( r_{1}^{ds} \) and \( r_{2}^{ds} \), we deduce:

\[ r_{21}^{rd} = 1.040(4)(2)(4.6)(0.2)(6) \],

where the sources of the errors are the same as for \( \Omega_c^{*} \), where \( \tau = (0.30 \pm 0.05) \) GeV\(^{-2} \) here and \( m_c \) replaced by \( m_b \). Using the averaged data in Eq. (56), and adding the different errors quadratically, one can deduce:

\[ M_{\Omega_b^{*}} = (6066 \pm 49) \text{ MeV} \],

which we report in Table 2.

9. Hyperfine mass-splitting

Combining the results for spin 1/2 and spin 3/2 given in Table 2 we deduce in Table 3 the values of the hyperfine mass-splitting. From our analysis, one expects that the \( \Omega_Q^{*} \) can only decay electromagnetically to \( \Omega_Q^* + \gamma \) due to phase space, while the \( \Xi_Q^{*} \) can, in addition, decay hadronically to \( \Xi_Q^* + \pi \). On one hand, our result for the
Table 3

| Hyperfine Splittings | Data |
|----------------------|------|
| \(M_{\Omega_c} - M_{\Xi_c} = 173(21)\) | 179(1) |
| \(M_{\Xi_c} - M_{\Xi_b} = 82(33)\) | 70(3) |
| \(M_{\Xi_b} - M_{\Omega_c} = 95(38)\) | 70(3) |
| \(M_{\Xi_b} - M_{\Xi_b} = 169(21)\) | - |
| \(M_{\Xi} - M_{\Xi} = 68(47)\) | - |
| \(M_{\Omega} - M_{\Omega} = -10(61)\) | \(M_{\Omega} - M_{\Omega} = 22\) |

heavy quark mass, to the perturbative radiative corrections and to the QCD continuum contributions than the simple ratios commonly used in the current literature for determining the heavy baryon masses:

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