Interaction of Electromagnetic P–Wave with Metal Films Located Between Two Dielectric mediums

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Abstract

Generalization of the theory of interaction of the electromagnetic $P$–waves with a metal film on a case of the film concluded between two dielectric environments is carried out.

Key words: degenerate plasma, dielectric permeability, metal films, dielectric media, transmittance, reflectance, absorptance.

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1. Introduction

The problem of interaction of an electromagnetic wave with a metal film long time draws to itself attention \cite{1} – \cite{9}. It is connected as with theoretical interest to this problem, and with numerous practical applications.

Nowadays there is the theory of interaction of an electromagnetic wave with metal film in the case when electron reflection from a film surface has a specular character \cite{1} – \cite{5}. In these works it was considered the freely hanging films in the air. In other words it was considered the case, when dielectric permeability of the environments surrounding a film is equal to unity.

However in overwhelming majority of cases it is not so \cite{6}. As a rule in practice one deals with the films located on some dielectric substrate. Meet also cases when the metal film is located between two dielectric environments. Generalization of the available theory of electromagnetic radiation interaction with metal film on a such situation will be the purpose of our work.

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Thus, we consider a situation, in which halfspace from which the electromagnetic wave falls on film, has dielectric permeability $\varepsilon_1$. We will consider that halfspace, in which electromagnetic wave gets, leaving a film, has the dielectric permeability $\varepsilon_2$. Last halfspace is called as a substrate.

2. Statement of problem

Let us consider the thin layer of metal located between two dielectric environments. We will assume, that these environments are not magnetic. Their dielectric permeability we will designate through $\varepsilon_1$ and $\varepsilon_2$. Let us designate these environments the first and the second media accordingly. We will assume that the first media are not absorbing. Let on a film from the first media the electromagnetic wave falls. Incidence angle we will designate as $\theta$. Let us assume, that a vector of magnetic field of the electromagnetic waves is parallel to a layer surface. Such wave is called E – wave \[3\] (or P – wave \[1\]).

We take the Cartesian system of coordinates with the beginning of coordinates on the surface of a layer adjoining to the first media. Axis $x$ we will direct into the metal layer. Axis $y$ we will direct parallel to magnetic field vector of electromagnetic wave.

The components of electric and magnetic field vectors we will search in the form

$$H_y(x, z, t) = H_y(x) e^{-i\omega t + ik_z z},$$

and

$$E_x(x, z, t) = E_x(x) e^{-i\omega t + ik_z z}, \quad E_z(x, z, t) = E_z(x) e^{-i\omega t + ik_z z}.$$

We denote the thickness of the layer by $d$.

The values $Z^{(1)}$ and $Z^{(2)}$ are designations of impedancies on the bottom layer surfaces to antisymmetric on electric field configurations of external fields (case 1) and symmetric configurations (case 2) accordingly.

In the case 1 we have the following relations on components of electric and magnetic field and on derivative of electric field

$$E_z(0) = -E_z(d), \quad H_y(0) = H_y(d), \quad \frac{dE_z(+0)}{dx} = \frac{dE_z(d-0)}{dx}. \quad (1)$$

Accordingly in case 2 we have the following relations

$$E_z(0) = E_z(d), \quad H_y(0) = -H_y(d), \quad \frac{dE_z(+0)}{dx} = -\frac{dE_z(d-0)}{dx}. \quad (2)$$

Let’s notice, that last two relations on derivatives of electric field from the first (1) and second (2) cases are consequences of uniformity of the film.

Out of the layer it is possible to present electric fields in the following form

$$E^{(j)}_z(x) = \begin{cases} a_j h_j e^{ik_z(x-d)} + b_j h_j e^{-ik_z(x-d)}, & x > d, \\ h_j e^{ik_1 x} + p_j h_j e^{-ik_1 x}, & x < 0, \quad j = 1, 2. \end{cases} \quad (3)$$
Indexes ”1” and ”2” at factors $a_j, b_j, h_j, p_j$ and field projections $E_y$ and $H_y$ correspond to the first and to the second cases accordingly.

The impedance thus in both cases is defined as follows (see, for example, [12] and [11])

$$Z^{(j)} = \frac{E_j^{(j)}(+0)}{H_y^{(j)}(+0)}, \quad j = 1, 2. \quad (4)$$

3. Surface impedance (wave interaction with film)

From Maxwell equations we have following relations for impedancies [3]

$$\frac{dE_z}{dx} - i\frac{\omega}{c} \sin \theta E_x + i\frac{\omega}{c} H_y = 0. \quad (5)$$

Here $c$ is the speed of light.

For dielectric environments from Maxwell equations follows the following relations [3]

$$i\frac{\omega}{c} \varepsilon_j E_x - i\frac{\omega}{c} \sin \theta H_y = 0, \quad j = 1, 2. \quad (6)$$

From last equations (6) we obtain on boundary of plasma

$$E_x(-0) = \frac{\sin \theta}{\varepsilon_1} H_y(0), \quad E_x(d + 0) = \frac{\sin \theta}{\varepsilon_2} H_y(d). \quad (7)$$

Indexes ”1” and ”2” at dielectric permeability correspond to the cases of two dielectric environments.

From the equation (5) and relations (7) we have

$$\frac{dE_z}{dx}(-0) = -i\frac{\omega}{c} \beta_1 H_y(0), \quad \frac{dE_z}{dx}(d + 0) = -i\frac{\omega}{c} \beta_2 H_y(d). \quad (8)$$

Here

$$\beta_j = 1 - \frac{\sin^2 \theta}{\varepsilon_j}, \quad j = 1, 2.$$

Considering a continuity on border of plasma of quantities $E_z$ and $H_y$ thus for quantity of an impedance (4) we receive [11]

$$Z^{(j)} = -i\frac{\beta_j \omega}{c} \frac{E_z^{(j)}(-0)}{\frac{dE_z^{(j)}(-0)}{dx}}, \quad j = 1, 2.$$

The account of symmetry of the electric field for the first case according to (1) and (3) leads to the following relations

$$-a_1 - b_1 = 1 + p_1,$$

$$\beta_1 k_{2x}(a_1 - b_1) = \beta_2 k_{1x}(1 - p_1).$$
Solving this system, we get
\[ a_1 = \frac{\beta_2 k_1 x - \beta_1 k_2 x}{2\beta_1 k_2 x} - \frac{\beta_2 k_1 x + \beta_1 k_2 x}{2\beta_1 k_2 x} p_1, \]
\[ b_1 = -\frac{\beta_2 k_1 x + \beta_1 k_2 x}{2\beta_1 k_2 x} + \frac{\beta_2 k_1 x - \beta_1 k_2 x}{2\beta_1 k_2 x} p_1. \]

The account of symmetry of a field for the second case according to (2) and (3) leads to the relations
\[ a_2 + b_2 = 1 + p_2, \]
\[ \beta_1 k_2 x (a_2 - b_2) = -\beta_2 k_1 x (1 - p_2). \]

The solution of last system has the following form
\[ a_2 = -\frac{\beta_2 k_1 x - \beta_1 k_2 x}{2\beta_1 k_2 x} + \frac{\beta_2 k_1 x + \beta_1 k_2 x}{2\beta_1 k_2 x} p_2, \]
\[ b_2 = \frac{\beta_2 k_1 x + \beta_1 k_2 x}{2\beta_1 k_2 x} - \frac{\beta_2 k_1 x - \beta_1 k_2 x}{2\beta_1 k_2 x} p_2. \]

Let us consider the following configuration of the field
\[ E_y(x) = b_2 h_2 E_y^{(1)}(x) - b_1 h_1 E_y^{(2)}(x). \]

Then the field \( E_y(x) \) has the following structure
\[ E_y(x) = \begin{cases} (a_1 b_2 - a_2 b_1) h_1 h_2 e^{ik_2 x(x-d)}, & x > d, \\ (b_2 - b_1) h_1 h_2 e^{ik_1 x} + (p_1 b_2 - p_2 b_1) h_1 h_2 e^{-ik_1 x}, & x < 0. \end{cases} \]

Thus, the field corresponds to the electromagnetic wave falling on the film from negative halfspace. A wave partially passes through the film, and it is partially reflected.

Thus
\[ a_1 b_2 - a_2 b_1 = \frac{\beta_2 k_1 x}{\beta_1 k_2 x} (p_2 - p_1), \]
\[ b_2 - b_1 = \frac{\beta_2 k_1 x + \beta_1 k_2 x}{\beta_1 k_2 x} + \frac{\beta_1 k_2 x - \beta_2 k_1 x}{2\beta_1 k_2 x} (p_1 + p_2), \]
\[ p_1 b_2 - p_2 b_1 = \frac{(p_1 + p_2)(\beta_2 k_1 x + \beta_1 k_2 x)}{2\beta_1 k_2 x} + \frac{\beta_1 k_2 x - \beta_2 k_1 x}{\beta_1 k_2 x} p_1 p_2. \]

4. Transmittance, reflectance and absorptance

The quantities \( p_1 \) and \( p_2 \) may be to presented as
\[ p_j = \frac{ck_1 x Z^{(j)} + \beta_1 \omega}{ck_1 x Z^{(j)} - \beta_1 \omega}, \quad j = 1, 2. \]
For reflection coefficient we obtain the following expression

\[ R = \frac{(p_1 + p_2)(\beta_2 k_{1x} + \beta_1 k_{2x}) + 2(\beta_1 k_{2x} - \beta_2 k_{1x})p_1 p_2}{2(\beta_2 k_{1x} + \beta_1 k_{2x}) + (\beta_1 k_{2x} - \beta_2 k_{1x})(p_1 + p_2)} \].

The quantity \( k_{2x} \) we can express as follows [11]

\[ k_{2x} = \frac{\omega}{c} \sqrt{\varepsilon_2 - \varepsilon_1 \sin^2 \theta}, \quad k_{1x} = \frac{\omega}{c} \sqrt{\varepsilon_1 \cos \theta}. \]

Taking into account expressions for \( p_1 \) and \( p_2 \) we can present them in the forms

\[ p_j = \sqrt{\varepsilon_1 \cos \theta} Z(j) + \beta_1 \sqrt{\varepsilon_1 \cos \theta} Z(j), \quad j = 1, 2. \]

Time average value of a flux of the energy of electromagnetic fields \( \langle S \rangle \) is equal to [7]

\[ \langle S \rangle = \frac{c}{16\pi} \{ [EH^*] + [E^*H] \}. \]

Here the asterisk designates complex conjugation.

Using relation (8) for quantity \( \langle S_x \rangle \) we get

\[ \langle S_x \rangle = -\frac{c}{16\pi}(E_z H_y^* + E_y^* H_z) = \frac{c^2}{8\pi \omega} |E_y|^2 \text{Re} \left( \frac{k_{jx}}{\beta_j} \right), \quad j = 1, 2. \]

Let us enter a designation

\[ \bar{p} = \frac{p_1 + p_2}{2}. \]

We obtain for reflection coefficient the following result

\[ R = \left| \frac{\beta_{12} \sqrt{\varepsilon_2 - \varepsilon_1 \sin^2 \theta} (\bar{p} + p_1 p_2) + \sqrt{\varepsilon_1 \cos \theta} (\bar{p} - p_1 p_2)}{\beta_{12} \sqrt{\varepsilon_2 - \varepsilon_1 \sin^2 \theta} (1 + \bar{p}) + \sqrt{\varepsilon_1 \cos \theta} (1 - \bar{p})} \right|^2. \] (9)

Here

\[ \beta_{12} = \frac{\beta_1}{\beta_2} = \frac{\varepsilon_2 (\varepsilon_1 - \sin^2 \theta)}{\varepsilon_1 (\varepsilon_2 - \sin^2 \theta)} \]

We can present the coefficient \( T \) in the form

\[ T = \text{Re} \left( \beta_{12} \frac{k_{2x}}{k_{1x}} \right) |a_1 b_2 - a_2 b_1|^2. \]

We get now with using the obtaining relations

\[ T = 4k_{1x} |a_1 b_2 - a_2 b_1|^2 \frac{p_2 - p_1}{2(k_{1x} + \beta_{12} k_{2x}) + (\beta_{12} k_{2x} - k_{1x})(p_1 + p_2)}, \]

or

\[ T = \text{Re} \left\{ \beta_{12} \sqrt{\varepsilon_1 (\varepsilon_2 - \varepsilon_1 \sin^2 \theta)} \right\} \cos \theta \]

Here

\[ \beta_{12} = \frac{\beta_1}{\beta_2} = \frac{\varepsilon_2 (\varepsilon_1 - \sin^2 \theta)}{\varepsilon_1 (\varepsilon_2 - \sin^2 \theta)} \]
For transparent environments the quantities $\varepsilon_1$ and $\varepsilon_2$ are real. From the obtained formula for the transmission coefficient is clear, that in this case at
\[
\sin^2 \theta \geq \frac{\varepsilon_2}{\varepsilon_1}
\]
the transmission coefficient is equal to zero, as
\[
\text{Re} \left\{ \beta_{12} \sqrt{\varepsilon_1 (\varepsilon_2 - \varepsilon_1 \sin^2 \theta)} \right\} = 0.
\]

It corresponds to full internal reflection.

We note that by $\sin^2 \theta \to \varepsilon_2/\varepsilon_1$ the transmission coefficient $T \to 0$. The reflection coefficient $R$ by $\theta \to \pi/2$ tends to 1.

Now we can find the absorption coefficient $A$ according to the formula
\[
A = 1 - T - R. \quad (10)
\]

Coefficients of transmission $T$ and reflection $R$ of the electromagnetic waves by layer at $\varepsilon_1 \to 1, \varepsilon_2 \to 1$ transform in earlier known expressions [1], [8]
\[
T = \frac{1}{4} |p_1 - p_2|^2, \quad R = \frac{1}{4} |p_1 + p_2|^2.
\]

Let us consider a case when electrons are specular reflected from a film surface. Then for quantities $Z^{(j)}$ ($j = 1, 2$) are satisfied the following relations [1], [8]
\[
Z^{(j)} = -\frac{2it\Omega}{W} \sum_{n=\infty}^{n=-\infty} \frac{1}{Q^2} \left( \frac{Q_x^2}{\Omega^2 \varepsilon_l} + \frac{Q_x^2}{\Omega^2 \varepsilon_tr - Q^2} \right), \quad j = 1, 2,
\]
where
\[
W = W(d) = \frac{\omega_p d}{c} \cdot 10^{-7},
\]
and the thickness of a film $d$ is measured in nanometers, for $Z^{(1)}$ summation is made on odd $n$, and for $Z^{(2)}$ on the even.

Here $\varepsilon_tr$ and $\varepsilon_l$ are transverse and longitudinal dielectric permeability accordingly, $\omega_p$ is the plasma (Langmuir) frequency,
\[
\varepsilon_tr = \varepsilon_tr(q_1, \Omega), \quad \varepsilon_l = \varepsilon_l(q_1, \Omega), \quad \Omega = \frac{\omega}{\omega_p},
\]
\[
\mathbf{q}_1 = \frac{v_F}{c} \mathbf{Q}, \quad \mathbf{Q} = (Q_x, 0, Q_z), \quad Q_x = \frac{\pi n}{W(d)}, \quad Q_z = \sqrt{\varepsilon_1 \Omega \sin \theta},
\]
module of vector $q_1$ is equal to

$$q_1 = \frac{v_F}{c} \sqrt{\frac{n^2 n^2}{W^2(d)} + \varepsilon_1 \Omega^2 \sin^2 \theta},$$

$$\varepsilon_{tr}(q_1, \Omega) = 1 - \frac{3}{4 \Omega q_1^2} \left[2(\Omega + i \varepsilon)q_1 + \left[(\Omega + i \varepsilon)^2 - q_1^2\right] \ln \frac{\Omega + i \varepsilon - q_1}{\Omega + i \varepsilon + q_1}\right],$$

$$\varepsilon_i(q_1, \Omega) = 1 + \frac{3}{q_1^2} \left[\frac{1}{2} + \frac{\Omega + i \varepsilon}{2q_1} \ln \frac{\Omega + i \varepsilon - q_1}{\Omega + i \varepsilon + q_1}\right],$$

$q_1$ is the dimensionless wave vector, $q = \frac{\omega_p}{v_F} q_1$ is the dimensional wave vector, $\varepsilon = \frac{\nu}{\omega_p}$, $\nu$ is the effective electron collision frequency.

We transform now these functions $Z^{(1)}$ and $Z^{(2)}$

$$Z^{(1)} = -\frac{4i \Omega}{W(d)} \sum_{n=1}^{\infty} \frac{1}{\Omega^2 \varepsilon_{tr}(\Omega, \varepsilon, d, 2n - 1, \theta, \varepsilon_1) - Q(\Omega, d, 2n - 1, \theta, \varepsilon_1)},$$

$$Z^{(2)} = -\frac{2i \Omega}{W(d)} \left[\Omega^2 \varepsilon_{tr}(\Omega, \varepsilon, d, 0, \theta, \varepsilon_1) - Q(\Omega, d, 0, \theta, \varepsilon_1)\right] - \frac{4i \Omega}{W(d)} \sum_{n=1}^{\infty} \frac{1}{\Omega^2 \varepsilon_{tr}(\Omega, \varepsilon, d, 2n, \theta, \varepsilon_1) - Q(\Omega, d, 2n, \theta, \varepsilon_1)}.$$

We will consider the important special case of formula for transmission coefficient. We will assume that $\varepsilon_1$ and $\varepsilon_2$ are real. Then expression for transmission coefficient we can present in the following form

$$T = \cos \theta \beta_{12} \Re \sqrt{\varepsilon_1(\varepsilon_2 - \varepsilon_1 \sin^2 \theta)} \left|\frac{p_1 - p_2}{\beta_{12} \sqrt{\varepsilon_2 - \varepsilon_1 \sin^2 \theta}(1 + \bar{p}) + \sqrt{\varepsilon_1} \cos \theta(1 - \bar{p})}\right|^2. \quad (11)$$

We will introduce the new quantity $\varepsilon_{12} = \frac{\varepsilon_2}{\varepsilon_1}$. Now the formulas (11) and (9) transform to the form

$$T = \beta_{12} \cos \theta \Re \sqrt{\varepsilon_{12} - \sin^2 \theta} \left|\frac{p_1 - p_2}{\beta_{12} \sqrt{\varepsilon_{12} - \sin^2 \theta}(1 + \bar{p}) + \cos \theta(1 - \bar{p})}\right|^2, \quad (12)$$

and

$$R = \left|\frac{\beta_{12} \sqrt{\varepsilon_{12} - \sin^2 \theta(\bar{p} + p_1 p_2)} + \cos \theta(\bar{p} - p_1 p_2)}{\beta_{12} \sqrt{\varepsilon_{12} - \sin^2 \theta(1 + \bar{p}) + \cos \theta(1 - \bar{p})}}\right|^2. \quad (13)$$

5. Analysis of results
Let us consider the case of a thin film of sodium. Then \( \omega_p = 6.5 \times 10^{15} \text{sec}^{-1} \), \( v_F = 8.52 \times 10^7 \text{cm/sec} \).

We will carry out graphic analysis of coefficients of transmission, reflections and absorption. We will use formulas (12) and (13) for coefficients of transmission and reflections, and coefficient of absorption we will analyze using the formula (10).

On fig. 1–6 we represent the behavior of transmission coefficient (Fig. 1–3) and reflection coefficient (Fig. 4–6) as functions of dimensionless frequency of an electromagnetic wave \( \Omega \). The incidence angle of an electromagnetic wave is equal to \( \theta = 75^\circ \), dimensionless frequency of electron collisions is equal to \( \varepsilon = 10^{-3} \), i.e. dimensional frequency of collisions is equal to \( \nu = 0.001\omega_p \).

Comparison with similar result \[4\] for interaction of electromagnetic \( p \)-waves with freely hanging thin film shows, that substrate influence leads to some reduction of transmittance and increase of reflectance. In the region of resonant frequencies the behavior of coefficients of transmission and reflection somewhat varies.

We consider now the system glass – metal film – air. We take then \( \varepsilon_1 = 4, \varepsilon_2 = 1 \). On Fig. 7 we represent behavior of all coefficients of transmission, reflection and absorption as functions of a incidence angle of the electromagnetic wave. For the film thickness 10 nanometers the transmittance is the monotonously decreasing function, the reflectance is the monotonously increasing function, and the absorptance has one maximum.

On Fig. 8 we represent the behavior of coefficients \( T, R \) and \( A \) as functions of dimensionless frequency \( \Omega \) \( (0 \leq \Omega \leq 1.5) \). We consider the system mica – film – air \( (\varepsilon_1 = 8, \varepsilon_2 = 1) \). The film thickness is equal to 100 nanometers. We take \( \nu = 0.001\omega_p, \theta = 15^\circ \). It is interesting to notice, that the coefficients of transmission and absorption have minimum in the point \( \Omega = 1 \), i.e. at \( \omega = \omega_p \), and the reflection coefficient has the sharp maximum in this point.

On Fig. 9 we represent character of ”comb teeth” of transmission coefficient for the film 10 nanometers thickness in region of the resonant frequencies \( (1.4 \leq \Omega \leq 1.53) \) for collision electron frequency \( \nu = 0.001\omega_p \) and for a incidence angle of electromagnetic wave \( \theta = 15^\circ \). Thus the system mica – film – air, i.e. \( \varepsilon_1 = 8, \varepsilon_2 = 1 \) is considered.

6. Conclusion

In the present work generalization of the theory of interaction electromagnetic radiation with a metal film on the case of a film concluded between two various dielectric environments is produced. Formulas for coefficients of transmission, reflection and absorption of an electromagnetic wave are deduced. The graphic analysis of these coefficients depending on oscillation frequency of an electromagnetic field, incidence angle of an electromagnetic waves, thickness of the film and character of the second dielectric environment (substrate).
Figure 1: Transmittance, air–film–glass, $d = 1$ nm, $0 \leq \Omega \leq 1.5$, $\nu = 0.001\omega_p$, $\theta = 75^\circ$.

Figure 2: Transmittance, air–film–glass, $d = 2$ nm, $0 \leq \Omega \leq 1.5$, $\nu = 0.001\omega_p$, $\theta = 75^\circ$.

Figure 3: Transmittance, air–film–glass, $d = 5$ nm, $0 \leq \Omega \leq 1.5$, $\nu = 0.001\omega_p$, $\theta = 75^\circ$. 
Figure 4: Reflectance, air–film–glass, $d = 1$ nm, $0 \leq \Omega \leq 1.5$, $\nu = 0.001 \omega_p$, $\theta = 75^\circ$.

Figure 5: Reflectance, air–film–glass, $d = 2$ nm, $0 \leq \Omega \leq 1.5$, $\nu = 0.001 \omega_p$, $\theta = 75^\circ$.

Figure 6: Reflectance, air–film–glass, $d = 5$ nm, $0 \leq \Omega \leq 1.5$, $\nu = 0.001 \omega_p$, $\theta = 75^\circ$. 
Figure 7: Transmittance, Reflectance, Absorptance, glass–film–air, $d = 10$ nm, $\Omega = 1$, $\varepsilon_1 = 4, \varepsilon_2 = 1, \nu = 0.001\omega_p, \theta = 15^\circ$.

Figure 8: Transmittance, Reflectance, Absorptance, mica–film–air, $d = 100$ nm, $\varepsilon_1 = 8, \varepsilon_2 = 1, 0 \leq \Omega \leq 1.5, \nu = 0.001\omega_p, \theta = 15^\circ$.

Figure 9: Character of "comb teeth", glass–film–air, $d = 10$ nm, $1.4 \leq \Omega \leq 1.53, \nu = 0.001\omega_p, \theta = 15^\circ$. 
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