The Formation Abstract Representations in the Product Quality Management

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Abstract. The study is devoted to the theoretical justification of procedures and algorithms for managing the quality of products. The methods to control the abstract object are used. Since the object itself, for example, quality is a reflection from the material carrier: products and the production process, control and disturbing influences are attached to them. Control actions are divided into three types: parametric, structural and organizational. The first two types are associated with the flow of technological processes, and due to the application of managing organizational influences, the necessary external and internal conditions are achieved that increase the quality of products. It is shown that even a simple linear formulation allows us to pose and solve the optimal control problem and obtain practically important results.

1. Introduction

Product quality is the main competitive advantage of any enterprise and managing it will always be an urgent task [1]. To ensure it, structures are created specially that control its level [2-5]. Moreover, the control object is an abstract object and direct control, as well as an assessment of its level in this case, is impossible [6]. On the other hand, behind any abstract concept is a material carrier, in this case it is a really manufactured product and a manufacturing process that ensures its manufacture - quality reflects their condition. And it is already possible to exert real impacts on products and on the production process that ensure the demand for quality and really evaluate specific indicators. At the same time, the formation algorithms of both control actions and the algorithm for the integral assessment of the quality level are important. Let’s consider these questions in more detail.

2. Theory

Although the quality is associated with the manufactured products, it is nevertheless generated by production, therefore it is necessary to apply a systematic approach to its research. Figure 1 shows the control system of an abstract object [6]. It is based on the production process, the input of which receives the necessary raw materials, materials, energy, and the finished product appears at the output. The concept of product quality is displayed at the management level. The quality level is formed as an integral quantity, estimated by the output (and intermediate) indicators. Control actions can be divided into three types: parametric, structural and organizational. Parametric effects are due to controlled
changes in the parameters of the process - technological modes. Structural changes are associated with the restructuring of the system structure, the introduction of additional elements that improve product

quality, or the removal of unnecessary, poorly functioning. Finally, organizational influences are associated with the creation of numerous conditions ensuring high quality products.

Disturbing effects include fluctuations in the input quality of raw materials, materials, energy, external influences on each of the elements of the system, as well as internal changes in the parameters of the system, for example, tool wear, personnel fatigue, and equipment aging.

The integrated assessment of the product quality level can be additive, that is, the values of indicators can add up with certain weight coefficients $\alpha_i$

$$Y = \sum_{i=1}^{N} \alpha_i Z_i$$  \hspace{1cm} (1)

However, this method of evaluation does not give zero, even if any element of the product or production equipment is out of order and the product in this case is not suitable for use. In this case, the multiplicative method helps, in which even zero of one indicator resets the quality level

$$Y = \beta \prod_{i=1}^{N} Z_i$$  \hspace{1cm} (2)

But in these two cases, the assessment is made on the principle of black / white. If the level of performance of each element in the product is important, you can mitigate the assessment using a method that can be called averaging

$$Y = \gamma_1 \sum_{i=1}^{N} \alpha_i Z_i + \gamma_2 \prod_{i=1}^{N} Z_i$$  \hspace{1cm} (3)

Finally, if the product is complex enough, and all assessment methods are used for different product elements, we use the combined method

$$Y = \gamma_1 \sum_{i=1}^{N} \alpha_i Z_i \cdot \prod_{i=N+1}^{M} Z_i \cdot \left(\gamma_2 \sum_{i=M+1}^{P} \beta_i Z_i + \gamma_3 \prod_{i=M+1}^{P} Z_i\right)$$  \hspace{1cm} (4)

3. Model and methods
We restrict ourselves to the case of linear systems and describe the transformations occurring in the control object. We associate the vector of the output quantity in the form of its components with control and disturbing influences, assuming that the rate of its change is proportional to the influences

![Diagram](https://example.com/diagram.png)

**Figure 1.** Abstract object management scheme - product quality.
\[ \dot{Y}_i + GY_i = \frac{\partial Y_i}{\partial u_i} u_i + \frac{\partial Y_i}{\partial f_i} f_i, \quad i = 1, \ldots, N \]  

where \( Y_i \) - is the component of the integral indicator of the quality level, evaluated in accordance with (4) \( u_i \) - is the control, and \( f_i \) - is the disturbing effect, \( \frac{dY_i}{du_i}, \frac{dY_i}{df_i} \) are the corresponding influence functions [7], \( N \) is the number of quality components and, at the same time, the dimension of the control object.

Organizational influences are allocated \( v_i \) resources, which are in the denominator in formula (5), on the basis of the hypothesis that the organizational measures that condition them only reduce disturbances, not completely compensating them. Limit external impacts on quality through organizational measures

\[ F_i \leq \frac{\partial Y_i}{\partial f_i} f_i \]  

here \( F_i \) are the corresponding constants.

Now we can pose the problem of optimal quality management [8]. For this we use the method of Professor A.M. Letova - ADOC (analytical design of optimal controllers) [9], according to which the optimal control functional is:

\[ \int_{t_0}^{t_e} \left[ \sum_{i=1}^{N} (u_i^2 + v_i^2) - \varepsilon \sum_{i=1}^{N} Y_i^2 \right] dt \rightarrow \min \]  

here \( t_e \) is the final control time, \( \varepsilon \) is the weight coefficient.

We solve the problem by the Euler-Lagrange method. Then the Lagrangian is equal to

\[ L = \sum_{i=1}^{N} (u_i^2 + v_i^2) - \varepsilon \sum_{i=1}^{N} Y_i^2 + \sum_{i=1}^{N} \psi_i \left( \frac{\partial Y_i}{\partial u_i} u_i + \frac{\partial Y_i}{\partial f_i} f_i - \dot{Y}_i - GY_i \right) + \sum_{i=1}^{N} \lambda_i \left( \frac{\partial Y_i}{\partial f_i} f_i - F_i \right) \]

We compose Euler equations in all variables

\[ \frac{\partial L}{\partial Y_i} - \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{Y}_i} \right) = 0 \quad -2\varepsilon Y_i - \psi_i G + \frac{d\psi_i}{dt} = 0 \quad i = 1, \ldots, N \]  

\[ \frac{\partial L}{\partial u_i} = 2u_i + \psi_i \frac{\partial Y_i}{\partial u_i} = 0 \quad i = 1, \ldots, N \]  

\[ \frac{\partial L}{\partial v_i} - \frac{\partial L}{\partial \dot{v}_i} = 2v_i - \lambda_i \frac{\partial Y_i}{\partial f_i} f_i = 0 \quad i = 1, \ldots, N \]

\[ \frac{\partial L}{\partial \dot{u}_i} + \frac{\partial L}{\partial \dot{v}_i} = \frac{\partial Y_i}{\partial u_i} u_i + \frac{\partial Y_i}{\partial f_i} f_i - \dot{Y}_i - GY_i = 0 \quad i = 1, \ldots, N \]

\[ \frac{\partial L}{\partial \dot{v}_i} = \frac{\partial Y_i}{\partial f_i} + \frac{\partial Y_i}{\partial \lambda_i} \lambda_i = 0 \quad i = 1, \ldots, N \]

\[ \frac{\partial L}{\partial f_i} = \frac{\partial Y_i}{\partial f_i} f_i - F_i = 0 \quad i = 1, \ldots, N \]

4. Data and solution

From the last equation, we can determine the relationship between perturbing and organizational-controlling influences

\[ \frac{\partial Y_i}{\partial f_i} f_i = F_i \quad i = 1, \ldots, N \]  

From (10) the equation we have

\[ \psi_i = -\frac{2v_i}{2u_i} \quad i = 1, \ldots, N \]

We differentiate this equality with respect to time and substitute it and the resulting derivative of the Lagrange multipliers into equation (9), we obtain

\[ -\varepsilon Y_i + \frac{G}{\psi_i} u_i - \frac{1}{\psi_i} \frac{d u_i}{d t} = 0 \quad i = 1, \ldots, N \]
We substitute the obtained results into equation (12)

\[ \frac{1}{\varepsilon} \frac{d^2u_i}{dt^2} + \left( \frac{\partial Y_i}{\partial u_i} - \frac{G^2}{\varepsilon} \frac{\partial^2 Y_i}{\partial u_i^2} \right) u_i + F_i = 0 \quad i = 1, \ldots, N \]  

(18)

The solution to this equation is found in the reference [10]

\[ u_i = C_1 ch \left( t \sqrt{\frac{G^2}{\varepsilon} \frac{\partial Y_i}{\partial u_i}} \right) + C_2 sh \left( t \sqrt{\frac{G^2}{\varepsilon} \frac{\partial Y_i}{\partial u_i}} \right) + \frac{F_i}{\frac{\partial Y_i}{\partial u_i}} \]  

(19)

Substituting this solution into equation (17), we can obtain the dependence of improving product quality over time. Solving together (11) and (13) we obtain the relationship of control actions aimed at the production process and organizational control actions

\[ v_i = \sqrt{\frac{F_i}{\frac{\partial Y_i}{\partial u_i}}} u_i \]  

(20)

We illustrate the solution with a practical example, for example, an assessment of the quality of a car. Imagine a car consisting of three parts: body I, engine II and transmission III. Let each part be evaluated by two indicators: the first is additive, the second is multiplicative, and the third is averaged, then the total score is

\[ Y = \gamma_1 \left( \alpha_1 Z_1 + \alpha_2 Z_2 \right) \cdot Z_3 Z_4 \cdot \left( \gamma_2 \left( \alpha_5 Z_5 + \alpha_6 Z_6 \right) + \gamma_3 Z_5 Z_6 \right) \]  

(21)

The data determined by equations (13) - (15), as well as the source data used to assess the level of quality are summarized in table 1.

| Table 1. The Source data. |
|--------------------------|
| Parameter | I | II | III |
| \( \varepsilon \) | - | 1 | - |
| \( \alpha_i \) | 0.1, 0.07 | - | 0.03, 0.05 |
| \( Y_i \) | 1, 1, 0.1 | - | - |
| \( \frac{\partial Y_i}{\partial Z_i} \) | 0.211, 0.297 | 0.339, 0.2 | 0.432, 0.45 |
| \( \frac{dZ_i}{du_i} \) | 0.1 | 0.3 | 0.2 |
| \( F_k \) | 0.01 | 0.01 | 0.01 |
| \( t_k \) | 480 min | - | - |
| \( Y_0 \) | 5 | 6 | 4 |

5. Results and discussion
The schedule of changes in control actions and improving the quality of products is presented in Figure 2. Green shows the change in control actions, red shows an increase in quality.

The obtained analytical dependences of the control actions and improving the quality of products from time to time allow us to draw the following conclusions:

1) improving product quality is proportional not only to the magnitude of the control actions, but also to the rate of their change over time;
Figure 2. Dependence of changes in product quality (red) and optimal control effects on time (green).

2) the organization of the conditions necessary to ensure a high level of product quality, with optimal management, sends control actions to the root of two times less than the management of the production itself;

3) the methodology for improving the quality of products may contain an assessment of the constituent parts of products in additive, multiplicative and averaging ways.

6. Conclusion
Thus, the formation of abstract representations in the management of product quality allows you to develop specific algorithms for assessing key indicators, identify integrated control and disturbing influences, set and solve problems of optimal control of the quality level of products.

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