AZIMUTHAL ASYMMETRIES IN INCLUSIVE SINGLE PION ELECTROPRODUCTION

K.A. Oganessyan \(†\)

INFN-Laboratori Nazionali di Frascati I-00044 Frascati, via Enrico Fermi 40, Italy
DESY, Deutsches Elektronen Synchrotron Notkestrasse 85, 22603 Hamburg, Germany
\(†\) E-mail: kogan@mail.desy.de

Abstract

The leading and sub-leading order results for pion electroproduction in polarized and unpolarized semi-inclusive deep-inelastic scattering, are considered putting emphasis on transverse momentum dependent effects appearing in azimuthal asymmetries. In particular the spin-dependent (single, double) and spin-independent asymmetries of the distributions in the azimuthal angle \(\phi\) of the pion related to the lepton scattering plane are discussed.

1 Introduction

Semi-inclusive deep inelastic scattering (SIDIS) of leptons off a nucleon is an important process to study the internal structure of the nucleon and its spin properties. In particular, measurements of azimuthal distributions of the detected hadron provide valuable information on hadron structure functions, quark-gluon correlations and parton fragmentation functions.

The kinematics of SIDIS in the case of longitudinally polarized target is illustrated in Fig.1: \(k_1\) (\(k_2\)) is the 4-momentum of the incoming (outgoing) charged lepton, \(Q^2 = -q^2\), where \(q = k_1 - k_2\) is the 4-momentum of the virtual photon. \(P (P_h)\) is the momentum of the target (observed) hadron, \(x = Q^2/2(Pq), y = (Pq)/(P_k_1), z = (PP_h)/(Pq)\), \(k_{1T}\) is the incoming lepton transverse momentum with respect to the virtual photon momentum direction, and \(\phi\) is the azimuthal angle between \(P_{hT}\) and \(k_{1T}\) around the virtual photon direction, angle \(\theta_\gamma\) is a virtual photon emission angle and \(S_{lab}\) is the target polarization parallel to the incoming lepton momentum, \(S_L\) and \(S_{Tx}\) are the longitudinal and transverse spin in the virtual photon frame, respectively [1]:

\[
S_L = S_{lab} \cos \theta_\gamma, \quad S_{Tx} = S_{lab} \sin \theta_\gamma, \quad (1)
\]

\[
\sin \theta_\gamma = \sqrt{\frac{4M^2 x^2}{Q^2 + 4M^2 x^2} \left(1 - y - \frac{M^2 x^2 y^2}{Q^2}\right)}. \quad (2)
\]

In SIDIS one assumes the factorization of the cross section, schematically

\[
d\sigma^{lN\to l' \pi X} = \sum_q f^{H\to q} \otimes d\sigma^{eq\to eq} \otimes D^{q\to h}, \quad (3)
\]

\(^1\)Invited talk at Spin-01, August 2 – 7, JINR Dubna, Russia.
where the soft parts, the distribution function $f$ and the fragmentation function $D$ depend not only on $x$ and $z$, respectively, but also on quark’s transverse momenta; $d\sigma^{eq\rightarrow eq}$ describes the scattering among elementary constituents and can be calculated perturbatively in the framework of quantum chromodynamics (QCD).

Due to the non-zero parton intrinsic transverse momentum, in the SIDIS cross section besides the conventional twist-2 non-perturbative blocks, there are different combinations of twist-two and twist-three structures, which could be probed in azimuthal asymmetries of hard scattering processes. The complete tree-level description expression containing contributions from twist-two and twist-three distribution and fragmentation functions in SIDIS has been given in Ref. [2]. The full result for the SIDIS cross section contains a large number of terms. In this respect it is more practical to split up it in the parts involving the lepton polarizations, unpolarized (U) or longitudinally (L) polarized keeping only the $\phi$-independent and $\phi$-dependent terms, relevant in following. It can be presented in the following way\[2:\]

\[
d\sigma^{eN\rightarrow ehX} \propto \sigma^{(0)}_{UU} + \frac{1}{Q} \cos \phi \sigma^{(1)}_{UU} + \frac{1}{Q} \sin \phi \sigma^{(2)}_{LU} + \frac{1}{Q} \sin \phi \sigma^{(3)}_{UL} + \sin 2\phi \sigma^{(4)}_{UL} + \sin(\phi + \phi_S)S_T \sigma^{(5)}_{UT} + \frac{1}{Q} \sin(2\phi - \phi_S)S_T \sigma^{(6)}_{UT} + \sin(3\phi - \phi_S)S_T \sigma^{(7)}_{UT} + \lambda_S \sigma^{(8)}_{LL} + \frac{1}{Q} \lambda_S \cos \phi \sigma^{(9)}_{LL} + \lambda_S \cos(\phi - \phi_S) \sigma^{(10)}_{LT} + \frac{1}{Q} \lambda_S \cos(2\phi - \phi_S) \sigma^{(11)}_{LT} ,
\]

where the first subscript corresponds to beam polarization and the second one to the target polarization. Here the terms proportional to $1/Q$ indicate the “kinematical” or dynamical twist-3 contributions.

In inclusive processes, at leading $1/Q$ order, besides the well-known parton distribution $f_1(x)$, often denoted as $q(x)$, the longitudinal spin distribution $g_1(x)$, often denoted as $\Delta q(x)$, there is a third twist-two distribution function, the transversity distribution function $h_1(x)$, also often denoted as $\delta q(x)$. It was first discussed by Ralston and Soper\[3\] in double transverse polarized Drell-Yan scattering. The transversity distribution $h_1(x)$

\[2\] Up to the $1/Q$ order.
measures the probability to find a transversely polarized quark in a transversely polarized nucleon. It is equally important for the description of the spin structure of nucleons as the more familiar function $g_1(x)$; their information being complementary. In the non-relativistic limit, where boosts and rotations commute, $h_1(x) = g_1(x)$; then difference between these two functions may turn out to be a measure for the relativistic effects within nucleons. On the other hand, there is no gluon analog on $h_1(x)$. This may have interesting consequences for ratios of transverse to longitudinal asymmetries in polarized hard scattering processes (see e.g. Ref. [4]).

The transversity remains still unmeasured, contrary to the case for spin-average and helicity structure functions, which are known over large ranges of $Q^2$ and $x$ (see Fig.2) The reason is that it is a chiral odd function, and consequently it is suppressed in inclusive deep inelastic scattering (DIS) [6, 7]. Since electroweak and strong interactions conserve chirality, $h_1(x)$ cannot occur alone, but has to be accompanied by a second chiral odd quantity. It is illustrated in Fig. 3 from Ref. [8].

In principle, transversity distributions can be extracted from cross section asymmetries in polarized processes involving a transversely polarized nucleon. In the case of hadron-hadron scattering these asymmetries can be expressed through a flavor sum involving a product of two chiral-odd transversity distributions. This is one of the main goals of the spin program at RHIC [9] (Fig. 3b).
In the case of SIDIS off transversely polarized nucleons (Fig.3c) there exist several methods to access transversity distributions. One of them, the twist-3 pion production \( [10] \), uses longitudinally polarized leptons and measures a double spin asymmetry. Other methods do not require a polarized beam, and rely on the polarimetry of the scattered transversely polarized quark. They consist on:

- the measurement of the transverse polarization of \( \Lambda \)'s in the current fragmentation region \( [11, 7] \),
- the observation of a correlation between the transverse spin vector of the target nucleon and the normal to the two-meson plane \( [12, 13] \),
- the observation of the “Collins effect” in quark fragmentation through the measurement of pion single target-spin asymmetries \( [14, 15, 2] \).

In Sec. 2 I will focus on the last method – Collins fragmentation function, \( H_1^\perp \) which can be simply interpreted as the production probability of an unpolarized hadron from a transversely polarized quark \( [14] \). A first indication of a nonzero \( H_1^\perp \) comes from analysis of the 91-95 LEP1 data (DELPHI) \( [16, 3] \). In numerical calculations the Collins ansatz \( [14] \) for the analyzing power of transversely polarized quark fragmentation was used:

\[
A_C(z, k_T) \equiv \frac{|k_T| H_1^\perp(z, k_T^2)}{M_h D_1(z, k_T^2)} = \frac{\eta M_C |k_T|}{M_C^2 + k_T^2}, \tag{5}
\]

where \( \eta \) is taken as a constant, although, in principle it could be \( z \) dependent. In this case the \( z \) dependence of the single-spin azimuthal asymmetries (SSAA) conditioned by strong correlations between kinematical variables.

Due to non-zero quark transverse momentum in semi-inclusive processes, at leading \( 1/Q \) order, in addition to the above discussed distribution functions, there are three non-vanishing distribution functions, \( g_{1T}, h_1^\perp_{L}, h_1^\perp_{T} \). These functions relate the transverse (longitudinally) polarization of the quark to the longitudinally (transverse) polarization of the nucleon.

## 2 Single-spin azimuthal asymmetries

In recent years significant SSAA have been observed in experiments with transversely polarized proton and anti-proton beams, respectively\( [18] \).

Very recently a significant target-spin asymmetry of the distributions in the azimuthal angle \( \phi \) of the pion related to the lepton scattering plane for \( \pi^+ \) electroproduction in a longitudinally polarized hydrogen target has been observed by the HERMES collaboration \( [17, 20] \) (Fig.4). At the same time the SMC collaboration has studied the azimuthal distributions of pions produced in deep inelastic scattering off transversely polarized protons and deuterons \( [21] \).

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3Possibilities of measuring \( H_1^\perp \) at BELLE \( [17] \) are currently being examined.
These non-zero asymmetries may originate from multi-parton correlations in initial or final states and non-zero parton transverse momentum. They have initiated a number of phenomenological approaches to evaluate these asymmetries using different input distribution and fragmentation functions. An analysis of different approximations, which aim at explaining the experimental data, have been provided in Ref. \cite{22}. The approximation where the twist-2 transverse quark spin distribution in the longitudinally polarized nucleon, $h_{1L}^{+(-)}(x)$, is considered small enough to be neglected \cite{23,24} with the assumption of the $u$-quark dominance are in good agreement with the Bjorken-$x$, $z$, and $P_{hT}$ behaviors of the $\sin\phi$ asymmetry for charged and neutral pion production observed at HERMES (see Figs. 5,6) \cite{4}. Note, that it does not require the twist-3 interaction-dependent part of the fragmentation function, $\bar{H}(z)$, to be zero, which leads to the inconsistency that all T-odd fragmentation functions would be required to vanish \cite{2,25}.

Results on SSAA provide evidence in support of the existence of non-zero chiral-odd structures that describe the transverse polarization of quarks. New data are expected from future HERMES, COMPASS measurements on a transversely polarized target, which will give direct access to the transversity \cite{26}.

### 3 Spin-independent azimuthal asymmetries

The nonzero transverse momenta of the partons as a consequence of being confined by the strong interactions inside hadrons, generates significant azimuthal asymmetries, which show up as a $\cos\phi$ and $\cos2\phi$ terms in the unpolarized SIDIS \cite{27}. The experimental results of the EMC \cite{28} and E665 \cite{29} are described well by the simple parton model \cite{30}, where both first-order perturbative QCD effects \cite{31} and non-perturbative intrinsic transverse momentum effects \cite{27} were taken into account. The most important contribution to azimuthal dependence of unpolarized SIDIS comes from kinematical effect of the intrinsic transverse momentum. However, to get a complete behavior of azimuthal distributions one has to take into account also higher twist effects \cite{32,2}. In the Ref. \cite{33} authors tried to isolate the higher twist effects. A self-consistent study of unpolarized azimuthal asymmetries, i.e. taking into account all effects that generate these asymmetries is still an open issue. To make an analogy between spin-independent and double-spin $\cos\phi$ asymmetries I would like to present the tree level contributions to spin-independent $\cos\phi$ asymmetry.

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Footnote 4: For $\pi^-$ production the data are consistent with zero in agreement with the result of the approximation.
Figure 5: $A_{UL}^{\sin \phi}$ for $\pi^+$ electroproduction as a function of $z$, the Bjorken-$x$, and of the $P_{hT}$ from Ref. [19]. Error bars include the statistical uncertainties only. The open bands at the bottom of the panel represent the systematic uncertainty. The curves show the range of predictions of a model calculation [24].

Figure 6: The same as in Fig.6 for neutral pion electroproduction. Data are from Ref. [20]. The curves show the range of predictions of a model calculation [24].
 asymmetry from Ref. [2]:
\[ d\sigma \propto d\sigma_{UU}^{(0)} + \frac{1}{Q} \cos \phi d\sigma_{UU}^{(1)} \]
being
\[ d\sigma_{UU}^{(0)} \propto \sum_a e_a^2 f_1^a(x) D_1^a(z), \]
\[ d\sigma_{UU}^{(1)} \propto -\sum_a e_a^2 \left( f_\perp^{a(1)}(x) D_1^a(z) - f_1^a(x) \tilde{D}_\perp^{a(1)}(z) \right), \]
where
\[ f_\perp^{a(1)}(x) = f_1^{a(1)}(x)/x + \tilde{f}_\perp^{a(1)}, \]
\[ \tilde{D}_\perp^{a(1)}(z) = D_\perp^{a(1)}(z) - zD_1^{a(1)}(z). \]

When in the Eq.(4) the interaction dependent parts of DF’s and FF’s are set to zero, the asymmetry reduces to a kinematical effect conditioned by intrinsic transverse momentum of partons in the nucleon as was calculated by Cahn [27].

4 Double-spin azimuthal asymmetries

Here I will focus my attention on the \( \cos \phi \) moment of the double-spin azimuthal asymmetry (DSAA) for pion electroproduction in semi-inclusive deep inelastic scattering of longitudinally polarized leptons off longitudinally polarized protons (for details see Ref. [34]). The contribution to double-spin \( \cos \phi \) asymmetry with the combinations of different leading and sub-leading distribution and fragmentation function can be symbolically presented in the following way:

\[ d\sigma \propto \lambda_S L d\sigma_{LL}^{(8)} + \frac{1}{Q} \lambda_S L \cos \phi d\sigma_{LL}^{(9)} + \lambda_S T \cos \phi \sin \theta_\gamma d\sigma_{LT}^{(10)}, \]

where
\[ d\sigma_{LL}^{(8)} \propto \sum_a e_a^2 g_1^a(x) D_1^a(z), \]
\[ d\sigma_{LL}^{(9)} \propto \sum_a e_a^2 \left( g_1^a(x) \tilde{D}_\perp^{a(1)}(z) - g_1^{a(1)}(x) D_1^a(z) - h_1^{a(1)}(x) \tilde{E}_a(x) \right), \]
\[ d\sigma_{LT}^{(10)} \propto \sum_a e_a^2 g_1^a(x) D_1^a(z). \]

The \( \cos \phi \) DSAA in the SIDIS cross-section can be defined as appropriately weighted integral of the cross section asymmetry:

\[ < |P_{hT}| \cos \phi >_{LL} = \frac{\int d^2 P_{hT} |P_{hT}| \cos \phi \left( d\sigma^{++} + d\sigma^{-} - d\sigma^{+-} - d\sigma^{-+} \right)}{\frac{1}{4} \int d^2 P_{hT} \left( d\sigma^{++} + d\sigma^{-} + d\sigma^{+-} + d\sigma^{-+} \right)}. \]
Here $++, -- (+-, ++)$ denote the anti-parallel (parallel) polarization of the beam and target and $M_h$ is the mass of the final hadron. The above defined weighted asymmetry is related to the experimentally observable asymmetry through the following relation

$$A_{LL}^{\cos \phi} \approx \frac{1}{\langle P_{hT} \rangle} \langle |P_{hT}| \cos \phi \rangle_{LL}.$$  

(13)

Using the Eqs. (4), (8) one can get

$$A_{LL}^{\cos \phi} = 4 \frac{d\sigma_{(9)}^{LL} + \sin \theta \gamma d\sigma_{(10)}^{LT}}{d\sigma_{UU}^{(0)}}.$$  

(14)

To estimate that asymmetry we take into account only the $1/Q$ order contribution to the DSAA which arises from intrinsic $p_T$ effects similar to the $\cos \phi$ asymmetry in unpolarized SIDIS, i.e. all twist-3 interaction dependent distribution and fragmentation functions are set to zero. In that approximation assuming a Gaussian parameterization for the distribution of the initial parton’s intrinsic transverse momentum, $p_T$, in the helicity distribution function $g_1(z, p_T^2)$ one obtains

$$g_1^{(1)}(x) = \frac{\pi < p_T^2 >}{2M^2} g_1(x).$$  

(15)

To estimate the transverse asymmetry contribution ($d\sigma_{LT}^{(10)}$) into the $A_{LL}^{\cos \phi}$, one can act as in the Ref. [35].

In Fig.7, the asymmetry $A_{LL}^{\cos \phi}(x)$ of Eq.(14) for $\pi^+$ production on a proton target is presented as a function of $x$-Bjorken. The curves are calculated by integrating over the HERMES kinematic ranges, taking $\langle P_{hT} \rangle = 0.365$ GeV as input (for more details see Ref. [34]).

From Fig.7 one can see that the approximation where all twist-3 DF’s and FF’s are set to zero gives the large negative double-spin $\cos \phi$ asymmetry at HERMES energies. Note that the ‘kinematic’ contribution to $A_{LL}^{\cos \phi}(x)$ coming from the transverse component of the target polarization, is negligible.

5 Conclusions

The leading and sub-leading order results for pion electroproduction in polarized and unpolarized semi-inclusive deep-inelastic scattering, putting emphasis on transverse momentum dependent effects appearing in azimuthal asymmetries is considered. In particular, $^5$this leads to positive $g_1(x)$.

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\[ \text{Figure 7: } A_{LL}^{\cos \phi} \text{ for } \pi^+ \text{ production as a function of Bjorken } x. \]
the spin-dependent (single, double) and spin-independent asymmetries of the distributions in the azimuthal angle $\phi$ of the pion related to the lepton scattering plane are discussed.

It is shown that the approximation where the twist-2 *transverse* quark spin distribution in the *longitudinally* polarized nucleon, $h_{1L}^{1(1)}(x) \approx 0$ with the assumption of the u-quark dominance, gives a consistent description of recent HERMES data on SSAA.

In *spin-independent* SIDIS the different mechanisms which generate $\cos \phi$ asymmetry is discussed. At moderate $Q^2$ and small $P_h T$ the main contribution to the asymmetry comes from kinematical $p_T$ effects describing well the existing experimental results.

At HERMES kinematics a sizable negative $\cos \phi$ double-spin asymmetry for $\pi^+$ electroproduction in SIDIS is predicted taking into account only the $1/Q$ order contribution to the DSAA which arises from intrinsic $p_T$ effects similar to the $\cos \phi$ asymmetry in unpolarized SIDIS: all twist-3 interaction dependent distribution and fragmentation functions are set to zero. The “kinematical” contribution from target transverse component ($S_{T,x}$) is well defined and it is shown that its contribution to $A_{LL}^{\cos \phi}$ is negligible. The double-spin $\cos \phi$ asymmetry is a good observable to investigate the weights of twist-2 and twist-3 contributions at moderate $Q^2$. It may give a possibility to estimate $<p_T^2>$ of partons in the nucleon. The complete behavior of azimuthal distributions may be predicted only after inclusion of higher-twist and pQCD contributions, nevertheless, if one consider the kinematics with $P_h T < 1$ GeV and $z < 0.8$, one can isolate the non-perturbative effects conditioned by the intrinsic transverse momentum of partons in the nucleon.

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