Baryogenesis Constraints on the Minimal Supersymmetric Model

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Abstract

Requirement that the vacuum expectation values of Higgs fields immediately after the phase transition be large enough imposes constraints upon the parameters of the minimal supersymmetric model. In particular, one obtains the upper bounds on the lighter CP-even Higgs mass and the soft supersymmetry breaking scale for different values of the top quark mass.

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1 Introduction

Several recent papers have studied the possibility of baryogenesis in the Standard Model and its minimal extensions. For example, references [1] - [6] impose constraints on the parameters of the Standard Model, minimal supersymmetric model (MSUSY), models with additional bosons or singlet Majoron model. In particular, authors of ref. [1] estimate the upper limit on the lighter CP-even Higgs in MSUSY to be equal to that in the Standard Model which they calculate to be 55 GeV. However, they have restricted their analysis to the case of the top quark lighter than about 115 GeV. The present paper includes the corrections due to the heavy top and its supersymmetric partners.

The obvious question is: are these corrections important? Contributions of the heavy top and stop to the effective potential have been calculated in many papers ( [7] - [13] ) and turn out to be a dominant part of the one loop effective potential . These results have been used in ref. [14] in order to impose limits on the parameters of MSUSY from existing data at LEP and in ref. [15] to analyze Higgs signals in the future hadron supercolliders.

In this paper we consider the one loop effective potential including contributions from W and Z gauge bosons, top and stop quarks and two CP-even Higgs bosons of the MSUSY in the region of parameter space where they play a non-negligible role.

Next we numerically calculate the zero and nonzero temperature VEV’s and impose the requirement that the latter be large enough immediately after the phase transition to sufficiently suppress baryon number violating processes. This constraint gives upper bounds on the lighter CP-even Higgs mass and on the soft SUSY breaking scale $\mu$.

In section 2, we briefly review how baryogenesis imposes constraints on parameters of the Standard Model and in section 3 we give a short description of the Higgs sector of MSUSY. Our calculation is presented in section 4 and the results are analyzed in section 5.

2 Constraints from Electroweak Baryogenesis

Much attention has been devoted recently to the possibility of creating the baryon asymmetry during the electroweak phase transition. For this to be true one has to satisfy certain basic requirements. First, the amount of CP violation inherent in the model has to be large enough to account for the observed baryon to photon ratio of about $10^{-8}$. Standard Model does not satisfy this requirement but it is possible to extend it to have enough CP violation. Secondly, the phase transition has to be first order to provide for departure from thermal equilibrium.

Once these requirements are met it is possible to construct a mechanism which shows how the baryon asymmetry was created during the electroweak phase transition, ( see for example ref. [3] ) instead of at much higher energies. This lower energy scale will allow for predictions of baryogenesis to be experimentally verifiable.

Regardless of the details of the particular mechanism of creating the baryon asymmetry, it is important to make sure that baryon number violating processes after the phase transition do not erase any previously created symmetry. To be more specific: after the phase transition at temperature $T_c$, the Higgs field acquires a vacuum expectation value $v(T_c)$. The rate of baryon number violation by thermal fluctuations is proportional to the Boltzmann factor $\exp(-M_{sph}/T_c)$, where $M_{sph}$ is the mass of the sphaleron field configuration or equivalently the
height of the barrier separating the gauge field configurations with different baryon numbers. \( M_{\text{sph}} \) was calculated at zero \( T \) in ref.\[16]\:

\[
M_{\text{sph}}(T_c) = 4\pi B\left(\frac{\lambda}{g^2}\right)\frac{v}{g_w}
\]

with: \( B(\frac{\Delta}{g^2}) \in [1.52, 2.70] \), for \( \Delta \in [0, \infty) \), where the \( \lambda \) is Higgs self coupling.

These baryon number violating processes have to proceed at a rate much smaller than the expansion rate of the universe after the transition, therefore the exponent of the Boltzmann factor has to be sufficiently large. Shaposhnikov \[17\] showed that:

\[
\frac{M_{\text{sph}}(T_c)}{T_c} \geq 45
\]

This equation was derived under the assumption \[22\] that at \( T = T_c \) (1) can be approximated by the same expression with \( v, \lambda, g^2 \) evaluated at \( T = T_c \).

How does this constrain parameters of the model? For instance, in the Standard Model, the one loop effective potential \( V_T(\phi) \) of the Higgs field at a temperature \( T \) much higher than the masses of the particles is given by:

\[
V_T(\phi) = \gamma(T^2 - T_2^2)\phi^2 - ET\phi^3 + \frac{\lambda_T}{4}\phi^4
\]

where the coefficients \( \gamma, E, \lambda_T \) are positive and are determined by the parameters of the Standard Model. In particular, \( \lambda_T \) is the temperature dependent effective quartic self coupling of the Higgs field.

The phase transition occurs near the point where the temperature dependent effective mass of the Higgs field vanishes:

\[
\gamma(T^2 - T_2^2) \approx 0
\]

Then, the VEV is given by:

\[
v(T_c) \approx \frac{3ET}{\lambda_T}
\]

If we want to have \( v(T_c) \) sufficiently large to satisfy (1) and (2), we must have \( \lambda_T \) sufficiently small. This in turn imposes the upper limit on the Higgs self coupling \( \lambda \) and therefore on the Higgs mass.

The similar thing will happen in MSUSY as our results will show.

3 Higgs Sector of MSUSY

Higgs sector of MSUSY (see for instance ref. [18]) contains two complex Higgs doublets with the following \( SU(3) \times SU(2) \times U(1) \) quantum numbers:

\[
H_1 = \left( \begin{array}{c} H^0_1 \\ H^-_1 \end{array} \right) \in (1, 2, -1/2), \quad H_2 = \left( \begin{array}{c} H^+_2 \\ H^0_2 \end{array} \right) \in (1, 2, +1/2)
\]
From these eight real fields spontaneous symmetry breaking decouples three unphysical Goldstone bosons and one is left with five physical Higgs bosons, namely: two CP-even scalars, one CP-odd scalar and a pair of charged scalars. The tree level Higgs potential:

\[ V = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - m_3^2 (H_1 H_2 + h.c.) + \frac{g_1^2}{8} (H_1^+ \bar{\sigma} H_1 + H_2^+ \bar{\sigma} H_2)^2 \]
\[ + \frac{g_2^2}{8} (|H_1|^2 - |H_2|^2)^2 \]  

(7)

can be restricted to the real components of the neutral Higgs fields, \( \phi_1 = Re H_1 \), \( \phi_2 = Re H_2 \).

\[ V_{tree} = m_1^2 |\phi_1|^2 + m_2^2 |\phi_2|^2 - m_3^2 \phi_1 \phi_2 + \frac{g_1^2 + g_2^2}{8} (\phi_1^2 - \phi_2^2)^2 \]

(8)

One can always choose such a field basis that \( m_3, v_1, v_2 \) are real and positive. Constants \( m_1, m_2 \) and \( m_3 \) have to satisfy certain conditions. Requiring that the potential be bounded from below gives:

\[ \frac{m_1^2 + m_2^2}{2} \geq m_3^2 \]  

(9)

and the spontaneous symmetry breaking condition is:

\[ m_3^4 \geq m_1^2 m_2^2 \]  

(10)

Here \( v_1 \) and \( v_2 \) are proportional to vacuum expectation values of \( \phi_1 \) and \( \phi_2 \):

\[ < \phi_1 > \equiv \frac{v_1}{\sqrt{2}} \quad < \phi_2 > \equiv \frac{v_2}{\sqrt{2}} \]

(11)

and \( \tan \beta \) is defined to be their ratio: \( \tan \beta \equiv v_2/v_1 \). Here:

\[ \sqrt{v_1^2 + v_2^2} = 246 \text{ GeV} \]  

(12)

to reproduce the measured values of gauge boson masses:

\[ m_w^2 = \frac{g_1^2}{4} (v_1^2 + v_2^2) \quad , \quad m_z^2 = \frac{g_1^2 + g_2^2}{4} (v_1^2 + v_2^2) \]

(13)

Fields \( \phi_1 \) and \( \phi_2 \) couple to down and up type quarks respectively. For example, after the spontaneous symmetry breaking top quark gets the mass:

\[ m_t^2 = h_t^2 < \phi_2 >^2 \]  

(14)

However, if the supersymmetry is softly broken, its scalar superpartner “stop” will have the different mass:

\[ m_{\tilde{t}}^2 = h_t^2 < \phi_2 >^2 + \mu^2 \]  

(15)

where we consider only the case of common soft supersymmetry-breaking mass \( \mu \) for \( \tilde{t}_L \) and \( \tilde{t}_R \) and vanishing off-diagonal elements of the \( 2 \times 2 \) stop mass matrix. (The following analysis can be easily generalized to include all of these terms.)
Finally CP-even physical eigenstates $h$ and $H$, with masses $m_h < m_H$ are obtained by diagonalizing the mass matrix:

$$
\mathcal{M} = \frac{1}{2} \begin{pmatrix}
\partial^2 V_{\text{tree}} \\
\partial \phi_i \partial \phi_j
\end{pmatrix}_{\text{min}} = \begin{pmatrix}
A & C \\
C & B
\end{pmatrix}
$$

and their masses are given by:

$$
m^2_{h,H} = \frac{1}{2}(A + B \mp \sqrt{(A - B)^2 + 4C^2})
$$

All this was at the tree level, but, as was already mentioned, in order to obtain limits on the Higgs mass it is paramount to include one-loop corrections. In the effective potential approach used in ref. [7] and [8], masses of the Higgs bosons are approximated with the eigenvalues of the matrix of second derivatives of the one-loop effective potential evaluated at its minimum.

The one-loop effective potential at zero temperature is given by the expression:

$$
V^0 = V_{\text{tree}}(Q) + \frac{1}{64\pi^2} \text{Str} \left\{ \mathcal{M}^4(\phi) \left[ \ln \frac{\mathcal{M}^2(\phi)}{Q^2} - \frac{3}{2} \right] \right\}
$$

Here $\mathcal{M}^2(\phi)$ is the field dependent squared mass matrix, $Q$ is the renormalization scale and the supertrace is given by:

$$
\text{Str} \ f(\mathcal{M}^2) = \sum_i (-1)^{2J_i} g_i f(m_i^2)
$$

The sum runs over all the physical particles $i$ of spin $J_i$, field dependent mass eigenvalue $m_i$ and multiplicity $g_i$ that couple to fields $\phi_1$ and $\phi_2$. In our case these are W and Z bosons, top and stop quarks and $h$ and $H$ bosons with multiplicities:

$$
g_w = 6, \ g_z = 3, \ g_t = g_{\tilde{t}} = 12, \ g_h = g_H = 1
$$

We have neglected the contributions due to other quark-squark flavors. This is justifiable insofar as their masses are small. As was pointed out in ref. [7], also the bottom-sbottom contributions can be non negligible for very large values of $\tan \beta$. This case will not be relevant for us.

### 4 Nonzero Temperature Effective Potential

When the temperature is nonzero, effective potential gets a contribution [19]:

$$
\Delta V_T = \frac{T^4}{2\pi^2} \sum_i g_i I_{\pm} \left[ \frac{m_i(\phi)}{T} \right]
$$

where $g_i$ are multiplicities as before, $m_i(\phi)$ are field dependent masses and $I_-(I_+)$ which are to be used for bosons (fermions) are given by:

$$
I_{\pm}(y) = \mp \int_0^\infty x^2 \ln \left( 1 \pm e^{-\sqrt{x^2 + y^2}} \right) dx
$$
This contribution to the effective potential describes the interactions of the Higgs bosons with the thermal bath surrounding them. Expressions (22) are rather difficult to operate with, especially when the masses depend on fields in a complicated way. Fortunately, as shown in ref. [2], one can always use either high temperature (small $y$) or low temperature expansion (high $y$), so that the mistake in determining $\Delta V_T$ is never bigger than 10 percent.

High temperature expansions of (4.2) are given by:

$$h_-(y) = -\frac{\pi^4}{45} + \frac{\pi^2 y^2}{12} - \frac{\pi}{6} y^3 - \frac{y^4}{32} \ln \left( \frac{y^2}{c_b} \right), \quad h_+(y) = -\frac{7\pi^4}{360} + \frac{\pi^2 y^2}{24} + \frac{y^4}{32} \ln \left( \frac{y^2}{c_f} \right).$$

$$\ln c_b \approx 5.41, \quad \ln c_f \approx 2.64,$$

(23)

Whereas the low temperature expansion is:

$$l(y) = -\sqrt{\frac{\pi}{2}} y^{3/2} e^{-y} \left( 1 + \frac{15}{8y} \right)$$

(24)

In this calculation we will always use one of these expansions or linear interpolation between them. By substituting field dependent values of $m_w$, $m_z$, $m_t$, $m_\tilde{t}$, $m_h$ and $m_H$ from equations (13) - (17) into the expressions for the effective potential (18) and (21) one obtains the full zero and nonzero temperature one-loop effective potentials. The critical temperature in this system is close to the point where the temperature dependent effective mass matrix has a zero eigenvalue.

What can we get out of this? If we take $m_t$ and $\mu$ to be our input parameters we have: 3 SUSY parameters $m_1$, $m_2$ and $m_3$; 2 zero-temperature VEV’s $<\phi_1>$ and $<\phi_2>$; 2 nonzero-temperature VEV’s $<\phi_1>_T$ and $<\phi_2>_T$ and the critical temperature – altogether eight unknowns.

How many conditions do we have? First the fixed magnitude of the zero T VEV (11) and (12), then 2 zero-T minima:

$$\frac{\partial V^0}{\partial \phi_1} = \frac{\partial V^0}{\partial \phi_2} = 0$$

(25)

next 2 nonzero-T minima:

$$\frac{\partial V^T}{\partial \phi_1} = \frac{\partial V^T}{\partial \phi_2} = 0$$

(26)

the critical temperature condition:

$$\det \left( \frac{\partial^2 V^T}{\partial \phi_i \partial \phi_j} \right)_{\phi_1=\phi_2=0,T_c} \approx 0$$

(27)

and, finally, from (1) and (2), the condition that nonzero-T VEV be sufficiently large:

$$v(T_c) = \sqrt{v_1^2(T_c) + v_2^2(T_c)} \geq v_{crit}(T_c) = \frac{45g_w T_c}{4\pi B(\lambda/g_w^2)}$$

(28)

If we take the equality in (28) (which corresponds to the upper limit on the $m_h$), we have seven equations in eight unknowns, therefore we can impose one relation between them. This
was done numerically in the form: \( m_h = m_h(tan \beta) \) for different values of parameters \( m_t \) and \( \mu \). Here, \( m_h \) is the upper limit on the mass of the lighter CP-even Higgs field.

This program was realized in MATHEMATICA and maxima of curves \( m_h(tan \beta) \) are given in figures 1, 2 and 3 for \( m_t = 115 \text{ GeV} \), 150 GeV and 200 GeV respectively.

The argument \( \lambda^{eff}/g^2 \) in the function \( B(\lambda^{eff}/g^2) \) was determined in ref. [5] for the general case of a two doublet model:

\[
\lambda^{eff} = \lambda_1 \cos^4 \beta_T + \lambda_2 \sin^4 \beta_T + 2h \cos^2 \beta_T \sin^2 \beta_T
\]  

Here \( \beta_T \) is the nonzero-T “mixing angle of VEV’s”:

\[
\tan \beta_T = \frac{v_2(T_c)}{v_1(T_c)}
\]  

In the MSUSY case:

\[
\lambda_1 = \lambda_2 = \frac{g_1^2 + g_2^2}{4}, \quad h = -\frac{g_1^2 + g_2^2}{4}
\]

therefore:

\[
\lambda^{eff} = \frac{g_1^2 + g_2^2}{4} \cos^2(2\beta_T)
\]

There are several causes of uncertainty in this calculation. First, we have assumed that the phase transition happens at point \( T_c \) where (27) is satisfied. This is not true. It was already noticed in ref. [1] that since the phase transition happens earlier, when the vacuum expectation value is smaller than at \( T_c \), the actual bound is stronger than the one we take. In other words, if we were able to calculate the phase transition temperature exactly the upper limit on the Higgs mass would be lower than the one we impose. Unfortunately, at the phase transition the one loop effective potential is not accurate at the origin due to infrared divergences and therefore we can just estimate the critical temperature.

Secondly, when using eq. (28) for the MSUSY we used the fact derived in [5] that the upper bound on the sphaleron mass in MSUSY is the sphaleron mass of the Standard Model. Therefore, our bound is again weaker than the actual one but still it will turn out to be very strong. However, the more precise calculation would require calculating the sphaleron mass in MSUSY at the critical temperature.

Finally, as is usual in the study of phase transitions in early universe, we are using effective potential which is a static quantity for a system which not only evolves but evolves out of equilibrium.

One should keep all of these caveats in mind when interpreting the results which are given in the next section.

5 Results and Discussion

Figures 1, 2 and 3 show upper limits on the Higgs mass for \( m_t = 115, 150 \) and 200 GeV respectively. Points are obtained as maxima of curves \( m_h(tan \beta) \) for different values of \( \mu \). For easier visibility they have been connected by straight lines. One can draw two conclusions from these results.
First, for values of $\mu = 150\text{GeV}$ (which is the asymptotic lower experimental mass limit at 90% c.l. for a gluino mass lower than 400 GeV - see ref. [20]), we get the upper limit on the Higgs mass to be 51 GeV, 54.5 GeV and 63 GeV for 3 different values of the top mass. One should compare this with experimental lower limit of 41 GeV (see [14] and [21]).

Second, for the considered region of $\tan\beta > 1$, there was always a maximal value of $\mu$ above which $v(T_c)$ was never big enough to satisfy requirement (28). This gives the upper limit on the soft-SUSY breaking scale of about 750 GeV, 250 GeV and 170 GeV for three top masses considered. One should compare this with the previously mentioned asymptotic lower mass limit of 150 GeV.

As a conclusion one can establish the following “no loose theorem” from these results: either the top quark is heavy (fig 3.) in which case Higgs can be as heavy as 63 GeV but SUSY breaking scale is very close to its experimental lower limit or top is lighter than 150 GeV (fig 1. and 2.) but then the Higgs mass is lighter than 55 GeV and thus close to its experimental lower limit.

As with all other calculations in supersymmetric models this one has the trouble that there are simply too many unknown parameters. We have considered that region in parameter space which has been searched by experiments [14], [20], [21] (i.e. for a common soft supersymmetry-breaking mass for $\tilde{t}_L$ and $\tilde{t}_R$ and vanishing off-diagonal mass elements for the stop mass matrix). The limits that we obtain complement the experimental results and severely limit the parameter space of MSUSY.

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