A Novel Method for Sorting Radar Signal Based on Welch Power Spectrum characteristics

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Abstract—The accuracy rate of current sorting methods of radar emitter signal is not high and they are sensitive to the signal-noise-rate (SNR). To solve these problems, a novel sorting method is proposed. Firstly, Welch power spectrum of the received signal is analyzed and its energy is normalized. Then its complexity characteristics are extracted to reduce dimension and sort easily, and are used as the sorting parameters. The last sorting is accomplished by KFCM algorithm. According to simulation results, the Complexity characteristics of Welch Power Spectrum have excellent separability and stability in low SNR.

1. INTRODUCTION
Radar emitter signal sorting is one of the key technologies in EW system. It refers to the separation of pulses belonging to different emitters from the intercepted dense radar pulse stream. With the increasing proportion of new complex radar, the sorting of unknown radar emitter signal plays an increasingly important role in EW, and there are more and more problems to be solved. The current radar emitter signal sorting algorithm is mainly based on the analysis of various conventional parameters of intercepted signals, such as arrival time, angle of arrival, carrier frequency, pulse width, etc. Among them, time of arrival sorting is a common method, such as sequence difference histogram, PRI transform and improved PRI transform algorithm. However, these algorithms have some defects and are difficult to apply to the current complex electromagnetic environment. Four regular parameters, DOA, RF, PW and PA, are often used to sort conventional radar emitter signals accurately [1], but when the parameters of signals change sharply and quickly, the sorting right rate will be lowered greatly.

In-pulse characteristic is one of the most special characteristic for radar emitter signal, although current regular characteristics of radar emitter signal vary greatly, the in-pulse characteristics have certain stability. At present, some scholars introduce in-pulse characteristics such as entropy value, wavelet feature and resemblance coefficient to radar emitter signal sorting and recognition and acquire some effect [2-6]. The sorting methods through adding in-pulse characteristics are sensitive to the noise and suitable for limited signal modulated modes. For solving above problems, this paper proposes a novel method. Firstly, Welch power spectrum of the received signal is analyzed and its energy is normalized. Then its complexity characteristics are extracted to reduce dimension and sort easily, and are use as the sorting parameters of radar emitter signal. At last, the sorting is accomplished by KFCM algorithm.

2. WELCH POWER SPECTRUM
Welch power spectrum is an improvement on the basis of traditional power spectrum. There are two
main points: (1) in order to increase the number of segments, partial overlap is allowed to improve the variance performance; (2) non rectangular window function is used to eliminate the spectral distortion by removing the correlation of adjacent sequence segments, so as to better control the performance of Welch spectrum estimation resolving power. Therefore, Welch power spectrum has good performance and has good application in the classification and recognition of communication and lung sound signals [7][8]. The calculation process of Welch is as follows:

If one random signal sequence is \( \{x(t)\}_{t=0}^{N-1} \), it is segmented as \( L \) parts. The length of each segment is \( M \). The overlap between segments is \( M - k \). Signal sequence of segment \( i \) is:

\[
x^i(t) = x[t + (i-1)k] \quad 0 \leq t \leq M - 1, 1 \leq i \leq L
\]

(1)

Where, \( 0 < k < M \), \( L \) and \( M \) meet: \( (L-1)k + M \leq N \). The segmentation diagram is shown in Figure 1.

The power spectrum estimation of segment \( i \) is:

\[
\hat{P}^i(\omega) = \frac{1}{MU} \left| \sum_{t=0}^{M-1} x^i(t) w(t) e^{-j\omega t} \right|^2
\]

(2)

Where, \( w(*) \) is one non-rectangular window function; \( U \) is a normalization factor in order to make Welch power spectrum asymptotic unbiased, namely:

\[
U = \frac{1}{M} \sum_{t=0}^{M-1} w^2(t)
\]

(3)

Welch power spectrum estimation is defined as:

\[
\hat{P}(\omega) = \frac{1}{L} \sum_{i=1}^{L} \hat{P}^i(\omega)
\]

(4)

In order to reflect the difference of Welch power spectrum of different radar emitter signals, the Welch power spectrum of eight typical radar emitter signals, including CW, LFM, FSK, BPSK, QPSK, lfm-bpsk, fsk-bpsk and NLFM, is taken as an example, as shown in Figure 2, where the horizontal axis denotes frequency (MHz) and the vertical axis denotes normalized amplitude.
Figure 2 Welch power spectrum of eight kinds of radar emitter signals

It can be seen from Figure 2 that the Welch power spectrum of different radar emitter signals has certain differences, which can be used to characterize the characteristics of signals and distinguish each other. When the signal is superimposed with noise, Welch power spectrum also has good variance performance. The theoretical analysis is as follows:

According to [7], the variance of spectrum estimation defined by Welch is:

$$\text{var}[\hat{P}(\omega)] = \frac{1}{L^2} \sum_{i=1}^{L} \sum_{j=1}^{L} \text{cov}[\hat{P}^i(\omega), \hat{P}^j(\omega)]$$

(5)

If the signal contained noise $x(n)$ is a stationary stochastic process, the variance of (5) only depends on $r = i - j$, and Equation (5) can be written as simple summation expression, namely:

$$\text{var}[\hat{P}(\omega)] = \frac{1}{L} \Gamma(\omega) \sum_{r=-(L-1)}^{L-1} \left[(1 - \frac{|r|}{L}) \Gamma_r(\omega) \right]$$

(6)

where, $\Gamma_r(\omega) = \text{cov}[\hat{P}^i(\omega), \hat{P}^j(\omega)]$; $\Gamma_0(\omega) = \text{var}[\hat{P}^i(\omega)](i = 1, 2, \cdots, L)$, $\frac{\Gamma_r(\omega)}{\Gamma_0(\omega)}$ is the normalized variance of $\hat{P}^i(\omega)$ and $\hat{P}^{i-r}(\omega)$; when the correlation between segments is small, (6) can be written approximately as follows:
\[
\text{var}[\hat{P}(\omega)] \approx \frac{1}{L} \Gamma_{0}(\omega)
\]

(7)

It can be seen from equation (7) that the variance of Welch power spectrum is approximately reduced to \(1/L\) of the variance of each segment, but the actual variance of Welch power spectrum is larger than that of formula (7) due to the fact that there is correlation between each sequence segment. In order to intuitively reflect the influence of noise on Welch power spectrum, taking LFM as an example, Welch power spectrum and spectrum under 0, 5, 10 and 15 dB SNR environment are given, as shown in Figure 3 (horizontal axis: frequency / MHz; vertical axis: normalized amplitude; red: Welch power spectrum; blue: spectrum).

![Figure 3 Welch power spectrum and frequency spectrum of LFM](image)

It can be seen from Figure 3 that when the signal is superimposed with noise, the Welch power spectrum basically keeps its original shape, and compared with the spectrum, Welch power spectrum is more insensitive to noise. The main reason is that the frequency spectrum directly carries out Fourier transform on the signal sequence, while the Welch power spectrum calculates the mean value after segmenting (overlapping) Fourier transform on the signal sequence. The results in Fig. 3 further verify the conclusion of equation (7), which shows that segmentation of signal sequence can improve the variance performance and make Welch power spectrum more conducive to extracting signal features from noise than spectrum.

3. Complexity Characteristics

Although the influence of noise has been improved and the difference between different radar signals has been highlighted after getting the Welch power spectrum of radar emitter signal, its dimension is large, which is not convenient to characterize signal features and subsequent sorting and recognition. It is necessary to consider using some features with simple extraction and strong applicability for dimensionality reduction. It can be seen from Fig. 2 that the Welch power spectrum of different radar emitters has different geometric scales and sparsity, so it is considered to select the complexity feature to describe it.

3.1 Box Dimension

Box dimension is an important parameter to describe the signal’s geometrical scale in fractal theory, and it has the advantage of easy engineering implementation [9].


\((F, d)\) is a metric space, \(\epsilon\) is a nonnegative real number, \(\tilde{B}(f, \epsilon)\) is a closed ball whose center is \(f\) and radius is \(\epsilon\). \(A\) is a nonempty subset of \(F\), as for every positive number \(s\), \(M(A, \epsilon)\) is the number of the smallest closed ball that covers \(A\).

\[
M(A, \epsilon) = \{N : A \subset \bigcup_{i=1}^{N} \tilde{B}(f_i, \epsilon)\}
\]

(8)

Where \(f_1, f_2, \ldots, f_N\) are different points of \(F\).

Set \(A\) is a compact set and is a nonnegative real number, if exist

\[
D_f = \lim_{\epsilon \to 0} \frac{\ln M(A, \epsilon)}{\ln(1/\epsilon)}
\]

(9)

we can regard \(D_f\) as the fractal dimension of set \(A\), that is \(D_f = D_f(A)\), and regard \(A\) have fractal dimension \(D_f\), this kind of dimension we call box dimension.

Use the following formulas to calculate Welch Power Spectrum’s box dimension \(D_f\)

\[
d(\Delta) = \sum_{i=1}^{N} |f(i) - f(i + 1)|
\]

(10)

\[
d(2\Delta) = \sum_{i=1}^{N/2} \left\{ \max\left[ f(2i - 1), f(2i), f(2i + 1) \right] - \min\left[ f(2i - 1), f(2i), f(2i + 1) \right] \right\}
\]

(11)

\[
D_f = 1 + \log_2 \frac{d(\Delta)}{d(2\Delta)}
\]

(12)

3.2 Information Dimension

Box dimension reflects the geometric scale information of Welch Power Spectrum. If the characteristic parameters want to fully reflect complexity characteristics, we also need information dimension to describe the density feature of Welch Power Spectrum\(^{[10]}\).

\(X\) is the set of \(R^n\), \(\{A_i\}_{i=1}^{N}\) is a limited \(\delta\) cover of \(X\), \(P_i\) stands for the probability set \(X\) falls in set \(A_i\), its value is

\[
P_i = \frac{N(X)}{N(X \cap A_i)} \quad i = 1, 2, \ldots, N
\]

(13)

Where \(N(X)_i\) and \(N(X \cap A_i)\) present the number of elements separately. Make entropy of information

\[
S_f = -\sum_{i=1}^{N} P_i \log P_i
\]

(14)

As the configuration entropy of \(X\). If entropy of information satisfies the following relation

\[
S_f(\delta) \sim \log \delta^D
\]

(15)

Then define the information dimension of \(X\) as
In order to analyze the performance of complexity features, the complexity characteristics of Welch power spectrum of 8 types of radar emitter signals are calculated respectively when the signal-to-noise ratio is 0, 5, 10 and 15 dB, and 200 signals are generated for each type, and the mean and variance of the complexity characteristics of Welch power spectrum of 8 types of radar emitter signals under the corresponding SNR environment are obtained, as shown in Figure 4. number one to eight separately stands for the signal of CW, LFM, FSK, BPSK, QPSK, LFM-BPSK, FSK-BPSK and NLFM.

\[
D_I = \frac{\sum_{i=1}^{N} g(i)}{G} \log \frac{g(i)}{G}
\]

Figure 4 Complexity characteristic mean of eight kinds of radar emitter signals
It can be seen from Figure 4 that when one of the complexity features is not enough to distinguish the 8 types of radar emitter signals, the analysis can be carried out in combination with another feature. Generally speaking, the complexity characteristics of Welch power spectrum of 8 types of radar emitter signals are different, that is, they have good separation. It can be seen from the variance of box dimension and information dimension in Figure 5 that the complexity characteristics of Welch power spectrum of 8 radar emitter signals are less affected by noise, that is, it has good stability, which lays a good foundation for subsequent sorting.

4. IMULATION ANALYSIS
In order to analyze the performance of Welch Power Spectrum characteristic, KFCM is used for the sorting experiments. Eight kinds of radar emitter signals are used in the simulation, which are CW, LFM, FSK, BPSK, QPSK, LFM-BPSK, and NLFM. The signal of FSK has two frequency points of 20MHz and 40MHz, the signal of FSK-BPSK has two frequency points of 20MHz and 40MHz, the rest have the carrier frequency of 30MHz, pulse width of 10us, and sampling frequency is 120MHz. The band width of LFM signal is 2MHz; the encoding law of FSK signal is [100110]; the encoding
law of BPSK signal is [11100010010]; the encoding law of QPSK signal is [01230312211300112012]; the band width of LFM-BPSK signal is 5MHz and encoding law is [11100010010]; both the carrier frequency and encoding law of the FSK-BPSK signal are [11100010010]; signal of NLFM is sine signal. In the SNR of 5dB, 10dB, 15dB and 20dB, create 100 signals of each kind.

KFCM algorithm is used to sort the eight kinds of radar emitter signals. A simulation experiment is made in each SNR. The characteristics used are box dimension and information dimension of Welch power spectrum. Oriental cluster number is c=2, largest cluster number is Cmax=8, iteration number T is 50, stop condition is ≤0.001, kernel function is Guassian Radial Basis. Table 1 shows the eight signals' sorting accuracy in different SNR conditions. From Table 1, we can know that when SNR is 20dB or 15 dB, eight signals' sorting accuracy is 100%; with the SNR goes low, sorting accuracy is decreasing at the same time. When the SNR is 10dB, we can know that the characteristics of QPSK and LFM-BPSK are partly overlapped; as a result the sorting accuracy decreases, while the rest are still 100%. When the SNR is 5dB, overlapping degree of the eight kinds of signal's characteristics complexity increases, sorting accuracies decrease, lowest of them is 90%, but the degree of separation of some signals are still excellent and sorting accuracy is 100%. From the analysis above, we know the accuracies are still satisfactory in Low SNR.

| SNR  | CW  | LFM | FSK | LFM-BPSK | BPSK | QPSK | FSK-BPSK | NLFM |
|------|-----|-----|-----|---------|------|------|---------|-----|
| 5dB  | 100 | 100 | 94  | 91      | 100  | 90  | 93      | 100 |
| 10dB | 100 | 100 | 100 | 96      | 100  | 94  | 100     | 100 |
| 15dB | 100 | 100 | 100 | 100     | 100  | 100 | 100     | 100 |
| 20dB | 100 | 100 | 100 | 100     | 100  | 100 | 100     | 100 |

5. CONCLUSION
It is an effective method to select the in pulse feature as the sorting parameter of radar emitter signal. The existing algorithm has a high SNR threshold. To solve this problem, this paper proposes a new sorting method to extract the Welch power spectrum features of the received radar emitter signal, which can effectively enhance the stability of the algorithm under the condition of low signal-to-noise ratio. The dimension feature extraction reduces the computational complexity of the algorithm. Simulation results show that the algorithm is effective.

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