Fracture analysis of center cracked laminated composite plate subjected to bi-axial loading using extended finite element method

Achchhe Lal¹, Khushbu Jain² and S P Palekar³

¹Assistant Professor, Department of Mechanical Engineering, SVNIT, Surat, GJ, India
²Research Scholar, Department of Mechanical Engineering, SVNIT, Surat, GJ, India
³Research Scholar, Department of Mechanical Engineering, SVNIT, Surat, GJ, India

E-mail: lalachchhe@yahoo.co.in

Abstract: Among various numerical method, finite element method (FEM) is extensively used but handling the fracture problem with conventional FEM is time-consuming and costly. So, extended finite element method (XFEM) which is based on partition of unity method and enrichment functions is used to handle material with discontinuities. In the present study, fracture analysis of center cracked layered composite plate under different biaxial loadings (tensile, shear and combined) using XFEM is investigated and mixed mode stress intensity factor (MMSIF) is calculated for different parameters.

1. Introduction
In recent year applications of fibre reinforced composites (FRCP) have increased in aerospace structure, automotive, marine, biomedical engineering due to its several advantages as high strength to weight ratio, low fabrication cost, better wear, corrosion, thermal resistance and high impact resistance. In most of the practical application, FRCP is subjected to complex state of stress and the presence of crack under complex loadings, accurate prediction of fracture parameter as stress intensity factor (SIF) is essential to understand the behaviour of fracture of materials. During manufacturing and fabrications, due to improper design of the constituent components, error in manufacturing and production techniques are the major cause of discontinuity like cracks, inclusions, voids and holes. The structural safety and reliability of FRCP structures can be increased by predicting fracture failure up to significant level.

When a crack exists in the structures, accurate predictions of fracture parameters like stress intensity factors (SIFs) and their effect on unstable fracture propagation under the action of biaxially acting tensile, shear or even the combination of both these stresses becomes necessary. In last decades many numerical methods are suggested and utilised by researchers. In conventional FEM every time remeshing is required when crack advances that leads to more computational time and cost. Significant improvement in conventional FEM are relised and now XFEM is used for modeling the discontinuities like crack, where extra enrichment functions are used for crack face and tip. In this direction Asadpoure and Mohammadi [1] proposed XFEM to understand the fracture analysis of orthotropic materials, where extra enrichment functions is used near crack tip. In this present paper, partition of unity based enrichment function is used to model crack geometry, where re-meshing is not required.
The accuracy of the present results is checked by other numerical methods. Meek and Ainsworth [2] presented finite element analysis with lower and upper bond theory of center cracked plate under biaxial loading. In this present study, by considering wide range of biaxial stress ratio limit load can be investigated. Lim et al. [3] presented fracture analysis of cracked orthotropic plate under biaxial loading. In the present study, it is confirmed that only singular term is not sufficient to predict the crack extension. Mostafavi et al. [4] carried out investigation based on experimental and FEM both to show the effect of stress multi-axiality on various fracture parameters of Aluminium alloy. Wang et al. [5] proposed a new approach called as modified dugdale model, which finds out the coupled effect at the crack tip, which was projected earlier for the crack tip opening displacement. Lal et al. [6] investigated XFEM based stochastic fracture analysis of composite plate with central crack under uniaxial loading, where, second-order perturbation technique (SOPT) and Monte Carlo simulation (MCS) are used to evaluate MMSIF. The present results are compared with analytical results from literature. Tabarraei and Sukumar [7] introduced two-dimensional crack growth modelling by using Laplace interpolant and by adopting mean value co-ordinates for non-convex elements in the framework of XFEM. Yan [8] presented fracture analysis of square plate with emanating cracks from elliptical hole under bi-axial loadings by displacement discontinuity method. It is observed that the present numerical is quite efficient and accurate for the fracture analysis of complicated crack under biaxial loadings. Goldstein and Shifrin [9] solved first mode crack deviation of orthotropic plane under bi-axial loading where, crack geometry is represented as a thin elliptical hole. In this present work, condition for crack stability and crack deviation is obtained and the present results are compared with the results of the conventional model of the crack, where crack is considered as perfect cut. Lal and Palekar [10] presented XFEM based fracture analysis of laminated composite plate with through thickness cracks of different types like line, semi circular, semi elliptical and arbitrary curves under tensile and shear loading. The effect of randomness in different fracture parameters is studied by SOPT and MCS. It is observed that random variables like length of crack, crack depth, loading and angle of laminate with effect more on fracture behaviour of the plate. Lal et al. [11] presented stochastic fracture analysis of laminated composite plate with edge crack underneath tensile, shear and combined loadings. In this present work XFEM with M-interaction, SOPT and MCS are used to study the effect of variation of fracture parameter on MMSIF. The result obtained is also compared with other published results.

In this present work XFEM based fracture analysis of center cracked composite plate underneath biaxial tensile, shear and combined loading is investigated MATLAB environment. The effect of crack angle and orthotropic angle on MMSIF is also investigated.

2. Mathematical formulation
The stress- strain relationship by the form of Hooke’s law for an orthotropic body is represented as

$$\sigma_i = R_{ij} \varepsilon_j \quad (i, j = 1, 2, 6)$$  \hspace{1cm} (1)

where $R_{ij}$ ($i, j = 1, 2, 6$) are the compliance coefficients of the orthotropic material along the local Cartesian coordinates axes $(x_i, y_i)$ as shown in figure 1.

At any point $x$ displacement field located in the domain of cracked body by XFEM is represented as

$$u = u_{\text{FEM}} + u_{\text{XFEM}}$$  \hspace{1cm} (2)

with

$$u_{\text{XFEM}} = u_{\text{Crack Face}} + u_{\text{Crack tip}}$$  \hspace{1cm} (3)

Therefore, the Eq. (2) can take the form

$$u^b(x) = \sum_{j=1}^{n} L_j(x) u_j + \sum_{k=1}^{n} L_k(x) \phi(x) a_k$$  \hspace{1cm} (4)

where $u_j$ is the nodal degrees of freedom (DOF) in conventional FEM, $a_k$ the extra of DOF to CFEM model, $L_j$ and $L_k$ are the shape functions and $\phi(x)$ is the irregular enrichment function. The displacement field can be approximated by rearranging Eq. (4) as below
\[ u^h(x) = \left[ \sum_{i=1}^{n} L_i(x) u_j \right] + \left[ \sum_{i=1}^{d_I} L_i(x) \left( H(\xi(x)) - H(\xi(x_i)) \right) a_i \right] + \left[ \sum_{i=1}^{n} L_i(x) \sum_{j=1}^{d_f} \left( F_i^1(x) - F_i^1(x_i) \right) b_{ij}^1 \right] + \left[ \sum_{i=1}^{n} L_i(x) \sum_{j=1}^{d_f} \left( F_i^2(x) - F_i^2(x_i) \right) b_{ij}^2 \right] \]  

(5)

Where, the three parentheses [ ] represent the linear, discontinuous and tip enrichment parts respectively. Nodes that belong to two crack tips are enriched with the two enrichment functions \( F_i^1(x) \), \( F_i^2(x) \) respectively for each crack tip, and \( H(\xi) \) is denoted by Heaviside function.

In Eq. (5) the \( c_f \) are the set of nodes that have the crack face in their support domain, \( t_1 \) and \( t_2 \) are the sets of nodes for tips 1 and 2 respectively. \( u_j \) are the nodal displacements (standard degrees of freedom), \( a_i, b_{ij}^1, b_{ij}^2 \) are the vectors for extra DOF for the nodes located on the crack face and the two crack tips respectively. The functions \( F_i^1, F_i^2 \) are the crack tip enrichment functions for tip 1 and 2. \( H(\xi(x)) \) is the Heaviside function is the step function for modelling strong discontinuity/crack face.

\[ H(\xi) = \begin{cases} +1 & \xi \in \Omega^+ \\ -1 & \xi \in \Omega^- \end{cases} \]  

(6)

Here, the value of +1 and -1 is considered for the positive side and other side of crack face.

In the XFEM the global form of linear equations for discrete system is written as

\[ F = KU \]  

(7)

Where \( F \) is the external force vector, \( K \) is the stiffness matrix and \( U \) is the DOF. \( K \) and \( F \) for each element are defined as

\[ K_{ij} = \begin{bmatrix} K_{xx} & K_{xy} & K_{yx} & K_{yy} \\ K_{xy} & K_{yy} & K_{yx} & K_{xx} \\ K_{yx} & K_{xy} & K_{xx} & K_{yy} \\ K_{yy} & K_{xx} & K_{yx} & K_{xy} \end{bmatrix} \]  

(8)

Where

\[ F_i^e = \{ F_i^1, F_i^2, F_i^{b1}, F_i^{b2}, F_i^{b3}, F_i^{b4} \} \]

(9)

\[ U = \{ u, a, b_1, b_2, b_3, b_4 \} \]

(10)

\[ K_{ij}^{ss} = \int_{t_1} t_2 (B^e) \nabla DB^e d\Omega(r,s = u,a,b) \]  

(11)

Here, B and D are shape function and material stiffness matrix.

For orthotropic material

\[ D = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{12} & C_{22} & C_{23} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} = \begin{bmatrix} E_{11} & 0 & 0 \\ 0 & E_{22} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \]  

(12)

(a) For laminated composite material [6]
\begin{equation}
D = \sum_{k=1}^{NL} \int_{z_{k-1}}^{z_k} f(z) \, dz \tag{13}
\end{equation}

where the number of layers in layered composite is represented by \( NL \).

The general form of \( J \)-integral method for the calculation of MMSIF can be written as

\begin{equation}
J = \int_A \left( \frac{1}{2} W - \sigma_{ij} \frac{\partial u_i}{\partial x_j} \right) n_j \, dA \tag{14}
\end{equation}

Where \( A \) and \( W \) are the area surrounded by crack tip and strain energy density,

\begin{equation}
W = \frac{1}{2} \sigma_{ij} \epsilon_{ij} = \frac{1}{2} R_{ijkl} \epsilon_{ij} \epsilon_{kl} \tag{15}
\end{equation}

Using the equivalent domain integral, Eq. (14) can be transformed to

\begin{equation}
J = \int_A \left( \sigma_{ij} \frac{\partial u_i}{\partial x_j} - W \Delta_{ij} \right) \frac{\partial q}{\partial x_j} \, dA \tag{16}
\end{equation}

Where, \( \Delta_{ij} \) is the Kronecker delta and \( q \) is a linearly varying function, \( q=1 \) at the crack tip and \( q=0 \) at the outer boundary as shown in figure 1. The \( J \)-integral is the combination of the actual and auxiliary states of \( J \) integrals as:

\begin{equation}
J = J^{\text{act}} + J^{\text{aux}} + M \tag{17}
\end{equation}

Where \( M \) is the interaction integral and it is represented as

\begin{equation}
M = \int_A \left[ \sigma_{ij} \frac{\partial u^{\text{act}}_{ij}}{\partial x_j} + \sigma_{ij}^{\text{aux}} \frac{\partial u_{ij}}{\partial x_j} - W \Delta_{ij} \right] \frac{\partial q}{\partial x_j} \, dA \tag{18}
\end{equation}

for linear elastic conditions

\begin{equation}
W^{\text{m}} = \frac{1}{2} (\sigma_{ij}^{\text{act}} \epsilon_{ij} + \sigma_{ij}^{\text{aux}} \epsilon_{ij}) \tag{19}
\end{equation}

After some arrangement the following equation is obtained [6]

\begin{equation}
M = 2m_1 K_I K_I^{\text{act}} + m_2 (K_I K_II^{\text{aux}} + K_{II}^{\text{aux}} K_II) + 2m_3 K_II K_{II}^{\text{aux}} \tag{20}
\end{equation}

By setting states as \( K_I^{\text{act}} = 1, K_I^{\text{aux}} = 0 \) and \( K_II^{\text{act}} = 1, K_{II}^{\text{aux}} = 0 \), MMSIF can be calculated

The MSIFs can be evaluated by calculating \( M \) and solving linear algebraic equations as.

\begin{equation}
M^{(1)} = 2m_1 K_I + m_2 \tag{21}
\end{equation}

\begin{equation}
M^{(2)} = m_2 K_I + 2m_3 K_II \tag{22}
\end{equation}
3. Result and discussion
A square plate with \(H=W=10\) mm and unit thickness with inclined center crack of crack angle \(\alpha\) and semi crack length \((a=1\) mm\) is considered. The plate is under uniform tensile stress and shear stress in the \(X\) and \(Y\) directions respectively is shown in figure 2(a). The factor \(K\) is the biaxial load factor, which is defined as the ratio of the load applied parallel to the crack to that applied perpendicular to it, where the crack is aligned with the \(X\) or \(Y\) coordinate axis. The meshing and enrichment of crack tip and face are shown in figure 2 (b-c). The material properties utilised for this study are \(E_{11}=144.80\) Gpa, \(E_{22}=11.70\) Gpa, \(G_{12}=9.66\) Gpa and \(\nu_{12}=0.21\), the crack angle \((\alpha)=45^\circ\) and \(K=2\).

Figure 3 shows the comparative study, where MMSIF of \(K_I\) and \(K_{II}\) are calculated for different crack angle and are compared with the result from literature. This comparative study shows that the current approach is quite efficient for this type of study.

Figure 4 shows the effect of orthotropic angle on MMSIF of \(K_I\) and \(K_{II}\) for layered composite plate under the action of different loadings. From the figure 4 (a-b) it is observed that bi-axially applied tensile, shear and combined stresses follow the same trend for \(K_I\) and \(K_{II}\). The values of \(K_I\) are decreasing up to \(\beta=\pm 30^\circ\), and then they start to increase and are maximum at \(\beta=\pm 40^\circ\) and \(90^\circ\) the values are same. The values of \(K_{II}\) are very small as compared to \(K_I\) which represents there is no response of fibre angle on \(K_{II}\) which is also observed in uniaxial loading. The MSIFs \(K_I\) and \(K_{II}\) follow almost sinusoidal variation about 40° fibre angles for all types of biaxial loads considered in this study.
4. Conclusions
In the present work analysis of MSIFs of a central crack, inclined crack of orthotropic and symmetric angle-ply laminated composite plates under different biaxial loadings is presented. Present results show significant effects of variation of fibre angle of an inclined centre crack of laminated composite plates. For all types of stresses, the following conclusions are written

- There is a variation in the value of MSIFs with the increase in fibre orientation angle $\beta^\circ$, which reveals that fibre orientation angle determines fracture axis in the case of orthotropic and laminated composite structures.
- Central inclined crack shows a mode-I failure under tensile stress, mode-II failure when subjected to shear stress and mixed mode failure under combine stress. While in the case of an eccentric crack, as the distance of crack from the plate edge increases the crack tips follow mixed mode fracture failure when subjected to shear and combine stress, and shows a mode-I failure under tensile stress.

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