Analysis and Optimization of Cellular Network with Burst Traffic

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Abstract

In this paper, we analyze the performance of cellular networks and study the optimal base station (BS) density to reduce the network power consumption. In contrast to previous works with similar purpose, we consider Poisson traffic for users’ traffic model. In such situation, each BS can be viewed as M/G/1 queuing model. Based on theory of stochastic geometry, we analyze users’ signal-to-interference-plus-noise-ratio (SINR) and obtain the average transmission time of each packet. While most of the previous works on SINR analysis in academia considered full buffer traffic, our analysis provides a basic framework to estimate the performance of cellular networks with burst traffic. We find that the users’ SINR depends on the average transmission probability of BSs, which is defined by a nonlinear equation. As it is difficult to obtain the closed-form solution, we solve this nonlinear equation by bisection method. Besides, we formulate the optimization problem to minimize the area power consumption. An iteration algorithm is proposed to derive the local optimal BS density, and the numerical result shows that the proposed algorithm can converge to the global optimal BS density. At the end, the impact of BS density on users’ SINR and average packet delay will be discussed.

Index Terms

Cellular networks, M/G/1, Poisson traffic, stochastic geometry, optimal BS density.

I. INTRODUCTION

With the popularity of smart phones and new types of mobile terminals, Qualcomm has expected a thousand fold increase in the data traffic in this decade [1]. As a result, the wireless

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industry is facing an enormous challenge to provide higher end-user throughput. Deploying more base stations (BSs) is one of the possible approaches to meet this challenge \[2\]. For network planning, how many BSs we should deploy is an essential issue.

Some works focusing on the optimal BS density have been processed in academia. For example, \[3\] assumed BSs located on a regular triangular grid, and determined the minimal BS density such that the outage probability does not exceed a certain threshold. As we know, grid models similar with \[3\] have been used extensively in system-level simulations to evaluate the performance of cellular networks. However, grid models may be too idealized because of the irregularity of future BS deployment. Moreover, these models are mathematically intractable. We can hardly obtain concise results based on grid models.

On the other hand, stochastic geometry has been introduced by Andrews et al. as a tractable and accurate tool for performance analysis of cellular networks \[4\]. Some basic analysis results can be found in \[5\]-\[12\]. While \[5\] studied the performance of a typical user in homogeneous networks, \[6\]-\[9\] focused on extensive analysis in heterogeneous networks. Especially, cell range expansion for load balancing was considered in \[6\]. To improve the performance of range expansion users, the interference avoidance technology of almost blank subframe (ABS) was estimated in \[9\]. And the optimal ABS number was discussed in \[10\]. Besides the downlink communication, the uplink analysis considering power control has also been processed in \[11\]-\[12\].

Based on theory of stochastic geometry, the optimal BS density has been discussed in academia. For example, \[13\] studied the joint BS density and transmit power optimization problem to maximize the coverage probability in noise-limited wireless networks. \[14\] analyzed the optimal BS density to minimize the network energy consumption in both homogeneous and heterogeneous networks under users’ QoS (Quality of Service) constraint. The optimal BS density to minimize the overall cost including BS hardware, energy consumption and backhaul cables is discussed in \[15\].

However, more of the previous works like \[13\]-\[15\] assumed the BSs have full queues, i.e., the BSs would transmit consistently and cause interference to other users. In reality, BSs are usually deployed and operated on the basis of peak traffic volume, and the BSs will keep dormant when there is no traffic to deliver. That is to say, BSs are not always transmitting and causing inter-cell interference. Some SINR analysis works capturing this feature have been processed in \[16\]-\[18\]. \[16\] \[17\] incorporated a notion of BS load (i.e., the transmission probability of BSs)
and analyzed the performance of users. But they haven’t studied the BS transmission probability under different system parameters (i.e., BS density, user density, the users’ QoS constraints). \cite{18} assumed each user would occupy a fixed fraction of bandwidth resource, and derived the SINR distribution and average rate of users. Nevertheless, as we know, the fraction of bandwidth or time resource a user required depends on the user’s SINR and the user’s data rate demand. However, \cite{18} didn’t consider these two factors. The major difference between this paper and \cite{16-18} is that we consider burst traffic (i.e., the users’ traffic model is different from \cite{16-18}) and study the average transmission probability of BSs (It has not been done in \cite{16-17}). And the optimal BS density is also studied to provide system design insight.

In this paper, we analyze the performance of cellular networks with burst traffic and study the optimal BS density to minimize the power consumption of the network. The position of BSs and mobile users is modeled by independent Poisson Point Process (PPP). The contributions of this paper can be summarized as:

- We assume the traffic model of users is Poisson traffic stream. Therefore, every BS can be viewed as M/G/1 queuing model. And the transmission probability of BSs is equivalent to the utilization of M/G/1.
- We analyze the SINR distribution of a typical user located at the origin and derive the average transmission time of each users’ packet. We find the SINR distribution of users depends on the average transmission probability of BSs. However, the average transmission probability is defined in a nonlinear equation. It is difficult to obtain a closed-form solution. Therefore, we use bisection method to solve the nonlinear equation.
- Furthermore, we will apply our analysis results to study the optimization problem to minimize the network power consumption. And we propose an algorithm to derive the local optimal BS density. Fortunately, the numerical result shows that the proposed algorithm can converge to the global optimal BS density due to the convex-like property of the objective function.
- At the end, we will discuss the influence of BS density on users’ SINR and packet delay. Especially, we find the users’ SINR can be improved by increasing the BS density, which is different from the result of full buffer scenario.

The rest of this paper is organized as follows. In Section \ref{sec:system_model}, we describe the system model.
We analyze the typical user’s performance in Section III. The optimal BS density is studied in Section IV. Numerical results are provided in Section V and we also discuss the influence of increasing BS density on users’ QoS. Finally, Section VI concludes this paper.

II. System Model

We consider the downlink cellular network which consists of BSs and users located according to independent homogeneous PPP $\Phi_b$, $\Phi_u$ in the Euclidean plane (Fig. 1). The intensity of $\Phi_b$ and $\Phi_u$ are $\lambda_b$, $\lambda_u$. The transmit power of BSs is $P$. We apply standard path loss propagation model with path loss exponent $\alpha > 2$. The small scale fading on each link is i.i.d Rayleigh fading. The cell association is based on the average received power, i.e., each user is associated with the nearest BS. Under above assumption, the received power of a typical user from a BS is $Phr^{-\alpha}$, where the variable $h$ indicates the small scale fading, $r$ is the distance between the user and the tagged BS. Since cellular networks are usually interference-limited, we neglect the effect of thermal noise in this paper. Without loss of generality, we assume the bandwidth of the system is unit (i.e., 1Hz).

The traffic model of each user is Poisson traffic, i.e., discrete packets with arrival instants according to Poisson process [19]. The packet arrival rate of each user’s traffic is $\lambda$. Without
loss of generality, the packet size is unit, i.e., 1nat. (However, our framework can be easily extended to the scenario where the packet size is variable.) Thus, the transmission time of each packet can be expressed as $T = 1/R$, where $R$ is instantaneous achievable rate of the user. Because of the superposition property of Poisson process, the total traffic at a BS is still Poisson traffic. As a result, every BS can be viewed as M/G/1 queuing model. And the transmission probability $\rho$ of a BS is equivalent to the utilization of M/G/1 [20].

We assume an infinite buffer at each BS. The scheduling algorithm of BSs is FCFS (First Come First Serve). In order to avoid the users suffering from low SINR occupying too much time resource, we have a SINR target $\beta$. When a BS schedules a user, only if the user’s SINR achieves this target, the BS will deliver the user’s packet, i.e. success transmission. Otherwise, this packet will be dropped. For the seek of simplicity and tractability, more intelligent scheduling algorithms are beyond the scope of this paper.

III. PERFORMANCE ANALYSIS

In this section, we will analyze the SINR of the typical user and study the average transmission probability of BSs. This section provides a basic framework to estimate the performance of cellular network with burst traffic.

A. SINR Analysis of The Typical User

Without loss of generality, we will analyze the SINR of a typical user located at the origin. Based on Slivnyak’s Theorem [21], the SINR distribution of arbitrary user is the same with the typical user.

1) The Interfering BSs: Because we consider burst traffic for users, the BSs only transmit when they have packets to deliver (i.e., transmission mode). If there is no users’ packet in the buffer, the BS will keep dormant (i.e. idle mode). The BSs interfering the typical user are the BSs in transmission mode. To indicate the transmission/idle state of BSs at any moment, we expand $\Phi_b$ into independent marked PPP $\Phi_b^M = \sum \delta_{(x_i, (\rho_i, t_i))}$, where $\delta_{(x_i, (\rho_i, t_i))}$ is the Dirac measure. The mark $\rho_i$ is the transmission probability of BS $i$, which is related to the number of users that

\[\text{Due to the coupling of the numbers of users in adjacent cells, the marks } (\rho_i, t_i) \text{ of different BS is dependent. In the following part of this paper, we make the similar independence approximation as [15] [22] to simplify the analysis and derive more insightful results.}\]
BS \( i \) covers, the users’ SINR distribution and the traffic arrival rate\(^2\). The mark \( t_i \) is independent random variables uniformly distributed in \([0, 1]\), and it indicates which mode the BS \( i \) is in. If \( t_i \leq \rho_i \), it indicates that the BS \( i \) is in transmission mode. Otherwise, the BS \( i \) is in idle mode. Therefore, the BSs which cause interference to the typical user can be modeled by collection of points \( \Phi^I_b = \sum \delta_{(x_i, (\rho_i, t_i))} 1(t_i \leq \rho_i) \). And the SINR of the typical user can be expressed as

\[
\text{SINR} = \frac{P h_0 |x_0|^{-\alpha}}{\sum_{x_i \in \Phi_b \setminus \{x_0\}} P h_i |x_i|^{-\alpha},}
\]

where \( x_0 \) indicates the position of the BS which the typical user associated with. Without loss of generality, we assume the transmit power of BSs is unit, i.e., \( P = 1 \).

The readers who are familiar with performance analysis using stochastic geometry will know that the probability generating functional (PGFL) of \( \Phi^I_b \) is crucial in calculating the inter-cell interference. However, \( \Phi^I_b \) depends on the \( \rho_i \) of every BS in the network, whose distribution function is difficult to calculate. Fortunately, we find that the PGFL of \( \Phi^I_b \) is the same as \( \Phi^{thin}_b \), where \( \Phi^{thin}_b \) is PPP with intensity \( \lambda_b \mathbb{E}[\rho] \), i.e., the thinning of \( \Phi_b \). That is to say, we only have to calculate expectation \( \mathbb{E}[\rho] \) (i.e., average transmission probability) instead of the distribution function of \( \rho \) for SINR analysis.

**Theorem 1:** The Laplace functional of \( \Phi^I_b \) is the same as \( \Phi^{thin}_b \), i.e., \( \mathcal{L}_{\Phi^I_b}(f) = \mathcal{L}_{\Phi^{thin}_b}(f) \).

**Proof:** Based on the definition of Laplace functional of PPP and marked PPP, Theorem 1 can be proved as follows.

\[
\mathcal{L}_{\Phi^I_b}(f) = \mathbb{E} \left[ \exp \left( - \sum_{x_i \in \Phi^I_b} f(x_i) 1(t_i \leq \rho_i) \right) \right] \\
\quad \overset{(a)}{=} \exp \left\{ - \int_D \left( 1 - \mathbb{E}_{t, \rho} \left[ e^{-f(x) 1(t \leq \rho)} \right] \right) \Lambda(dx) \right\} \\
\quad \overset{(b)}{=} \exp \left\{ - \int_D \left( 1 - \mathbb{E}_{\rho} \left[ \rho e^{-f(x)} + 1 - \rho \right] \right) \Lambda(dx) \right\} \\
\quad = \exp \left\{ - \int_D \left( 1 - \mathbb{E}[\rho] e^{-f(x)} - 1 + \mathbb{E}[\rho] \right) \Lambda(dx) \right\} \\
\quad = \mathcal{L}_{\Phi^{thin}_b}(f),
\]

\(^2\)In this paper, we only consider the scenario the BSs are steady in the long term, i.e., \( \mathbb{E}_{\Phi_u}[\rho_i|\Phi_b] < 1 \).
where \((a)\) follows from the Laplace transform of marked PPP \([21]\), \((b)\) follows from that \(t\) is uniformly distributed in \([0, 1]\).

**Corollary 1:** The PGFL of \(\Phi^t_b\) is the same with \(\Phi^{thin}_b\), i.e., \(E\left[\prod_{x \in \Phi^t_b} f(x)\right] = E\left[\prod_{x \in \Phi^{thin}_b} f(x)\right]\).

**Proof:** For point process, the PGFL is equivalent to the Laplace functional. By substituting \(f(x) = e^{-g(x)}\) in the PGFL of \(\Phi^t_b\), this can be obtained directly from Theorem 1.

2) **SINR of A Typical User:** We will first provide some analysis results under the assumption that we are given \(E[\rho]\). And these results are necessary for calculating \(E[\rho]\) in part B.

**Lemma 1:** The complementary cumulative distribution function (CCDF) of SINR for a typical user is

\[
P[SINR > \eta] = \frac{1}{1 + E[\rho]Z(\eta)},
\]

where \(Z(y) = y^{2/\alpha} \int_{y^{-2/\alpha}}^{\infty} \frac{1}{1 + u^\alpha} du\).

**Proof:** See Appendix A.

We can find that the CCDF of SINR for a typical user is only related to the average transmission probability \(E[\rho]\). We can conclude that a smaller \(E[\rho]\) will lead to a higher probability that the SINR of the typical user achieves \(\eta\), i.e., the SINR of the typical user is generally higher with smaller \(E[\rho]\). The reason for this is that, there will be less BSs interfering the typical user, so the inter-cell interference is less with smaller \(E[\rho]\). For full buffer traffic scenario, the BSs are transmitting and causing interference all the time, i.e., \(E[\rho] = 1\). By substituting \(E[\rho] = 1\), the SINR of users in full buffer scenario can be derived, which is consistent with the result in \([5]\).

Furthermore, we can infer that the SINR of users in burst traffic scenario is higher than the full buffer scenario, our numerical results will also demonstrate this.

**Corollary 2:** Conditioning on success transmission, i.e., the typical user achieves the SINR target \(\beta\), the CCDF of SINR for the typical user is

\[
P_{ST}[SINR > \eta] = \frac{1 + E[\rho]Z(\beta)}{1 + E[\rho]Z(\eta)}, (\eta \geq \beta).
\]

**Proof:** Conditioning on success transmission, we have conditional probability:

\[
P_{ST}[SINR > \eta] = P[SINR > \eta | SINR > \beta]
= \frac{1 + E[\rho]Z(\beta)}{1 + E[\rho]Z(\eta)}.
\]
B. Average Transmission Probability

In part A, we assume we are given $E[\rho]$, and derive the typical user’s SINR distribution. We will derive $E[\rho]$ to complete the overall analysis in this part.

Because each BS can be viewed as M/G/1, and the transmission probability $E[\rho]$ is equivalent to the utilization of M/G/1. Therefore, we have

$$E[\rho] = E[\lambda_{tot} E[T]] = E[\lambda_{tot}] E[T],$$

(6)

where $\lambda_{tot}$ is the total arrival rate of users’ traffic at each BS, and $E[T]$ is average transmission time of each packet.

1) Average Service Time: From Corollary 2 we can derive the average transmission time of users’ packets in Corollary 3 which can be viewed as the average service time in M/G/1. Since the utilization of M/G/1 has a linear relationship with the average service time, this result is crucial to derive $E[\rho]$.

**Corollary 3:** The average transmission time of a packet is

$$E[T] = \int_{0}^{\log(1+\beta)} \left( 1 - \frac{1 + E[\rho] Z(\beta)}{1 + E[\rho] Z(e^{1/t} - 1)} \right) dt.$$  

(7)

**Proof:**

$$E[T] = \int_{0}^{\infty} P[T > t] dt$$

$$\overset{(a)}{=} \int_{0}^{\infty} P_{ST} \left[ \log (1 + SINR) < 1/t \right] dt$$

$$= \int_{0}^{\log(1+\beta)} P_{ST} \left[ SINR < e^{1/t} - 1 \right] dt$$

$$= \int_{0}^{\log(1+\beta)} \left( 1 - P_{ST} \left[ SINR \geq e^{1/t} - 1 \right] \right) dt,$$

(8)

where $(a)$ follows from the fact $T = 1/R$. By substituting (7) into (8), we derive the result in (7).
2) Average Transmission Probability: Since we have obtained average transmission time of each packet, base on (6), the average transmission probability is defined in a nonlinear equation.

**Theorem 2:** The average transmission probability $E[\rho]$ satisfies

$$\frac{\lambda_b}{\lambda \lambda_u} = \frac{1}{1 + E[\rho]} \int_{0}^{1} \frac{Z(e^{1/t} - 1) - Z(\beta)}{1 + E[\rho] Z(e^{1/t} - 1)} dt. \quad (9)$$

*Proof:* See Appendix B.

The nonlinear equation in (9) is complex, it is difficult to derive a closed-form expression for $E[\rho]$. However, we can get $E[\rho]$ by efficient numerical searching algorithms. It’s obvious that the right part of the nonlinear equation (9) is a decreasing function of $E[\rho]$. Besides, for a steady M/G/1, the utilization $E[\rho] \in (0, 1)$. Therefore, we can derive $E[\rho]$ from (9) using bisection method.

Based on Theorem 2, we can find that $E[\rho]$ depends only on the ratio $\frac{\lambda_b}{\lambda \lambda_u}$. Besides, $E[\rho]$ increases with the traffic arrival rate $\lambda$, the user density $\lambda_u$, decreases with the BS density $\lambda_b$. That is intuitive, more users, higher intensity traffic or less BSs will make each BS carry more traffic (i.e., larger $E[\lambda_{tot}]$), which leads to a higher probability that the BS is in transmission mode. Moreover, the inter-cell interference is more serious in such situation, i.e., the users are suffering from lower SINR. Thus, the BSs need more time resource to deliver users’ packets (i.e., larger average transmission time $E[T]$). This will further increase the value of $E[\rho]$.

By substituting $E[\rho] = 1$, we can get $\lambda_{max}$ in (10), which can be viewed as the maximum traffic arrival rate the cellular system can support for each user. If $\lambda \geq \lambda_{max}$, then we have $E[\rho] \geq 1$. That is to say the M/G/1 will not be steady any more, i.e., the BSs can’t meet the demand of the users’ traffic. We can find that $\lambda_{max}$ is proportional to the density of base stations but inversely proportional to the density of users. That demonstrates the fact that, deploying more BSs is an efficient approach to support the emerging new type data-hungry traffics in wireless networks.

$$\lambda_{max} = \frac{\lambda_b (1 + Z(\beta))}{\lambda_u \int_{0}^{1} \frac{Z(e^{1/t} - 1) - Z(\beta)}{1 + Z(e^{1/t} - 1)} dt}. \quad (10)$$

**Remark 1:** Combining the results in part A and part B, we can derive the typical user’s SINR distribution. As we have mentioned, the SINR distribution depends on the average transmission probability $E[\rho]$, and a lower $E[\rho]$ will lead a higher SINR for users. Recalling the fact that
\( \mathbb{E}[\rho] \) increases with \( \lambda, \lambda_u \) but decreases with \( \lambda_b \), we can conclude that the SINR of the typical user is higher with smaller \( \lambda, \lambda_u \) or larger \( \lambda_b \).

IV. BS Density Optimization

Since it is crucial to design energy efficient wireless network inspired by environmental awareness and the cost of energy. We focus on the optimal BS density \( \lambda_b^* \) to minimize the area power network consumption with fixed user density and traffic arrival rate. This will provide system design insight into the question how many BSs we should deploy.

A. BS Density Optimization Problem

Each BS may be in transmission mode or idle mode, this depends on whether there are users’ packets in the buffer or not. As we know, the power consumption of BSs is typically different in transmission/idle mode. We assume the power consumption in transmission mode is \( P_t \), and \( P_i \) for idle mode (\( P_t > P_i \)). Therefore, the average power consumption per unit area is \( \lambda_b (\mathbb{E}[\rho] P_t + (1 - \mathbb{E}[\rho]) P_i) \). Therefore, we formulate the optimization problem as follows

\[
P_0 : \min_{\lambda_b} \lambda_b (\mathbb{E}[\rho] P_t + (1 - \mathbb{E}[\rho]) P_i)
\]

s.t. \( \mathbb{E}[\rho] < 1 \). (11)

The constraint insures the BSs can meet the users’ demand, i.e., the BSs will not be over-loaded.

B. Local Optimal BS Density

As we have analyzed, the average transmission probability \( \mathbb{E}[\rho] \) is a implicit function of \( \lambda_b \), which is defined in (9). For convenience, we use the notation \( \mathcal{P}(\cdot) \) to indicate the mapping from \( \lambda_b \) to \( \mathbb{E}[\rho] \), i.e., the average transmission probability \( \mathbb{E}[\rho] \) is \( \mathcal{P}(\lambda_b) \) when the BS density is \( \lambda_b \). And \( \mathcal{P}(\lambda_b) \) decreases with \( \lambda_b \), thus the constraint is equivalent to the linear inequality

\[
\lambda_b > \lambda_b^{min} = \frac{\lambda \lambda_u \int_{\log(1+ \beta)}^{1} \frac{e^{\frac{1}{\tau}} - 1 - Z(\beta)}{1 + Z(\frac{e^{\frac{1}{\tau}} - 1})} dt}{1 + Z(\beta)}.
\] (12)

Therefore, \( P_0 \) is equivalent to
\[ P1: \]

\[
\begin{align*}
\min_{\lambda_b} & \quad \lambda_b \left( \overline{\rho}(\lambda_b) P_t + (1 - \overline{\rho}(\lambda_b)) P_i \right) \\
\text{s.t. } & \quad \lambda_b > \frac{\lambda \lambda_u \int_0^{\frac{1}{\log(1+\beta)}} \frac{Z(e^{1/t} - 1) - Z(\beta)}{1 + Z(e^{1/t} - 1)} dt}{1 + Z(\beta)}.
\end{align*}
\]

(13)

Due to the complicated functional relationship between \( \overline{\rho}(\lambda_b) \) and \( \lambda_b \) in (9), it’s very difficult to verify whether the objective function is convex theoretically. However, we can still use convex optimization method to derive the local optimal BS density \( \tilde{\lambda}_b^* \).

**Theorem 3:** The local optimal BS density \( \tilde{\lambda}_b^* \) to minimize the average power consumption can be derived by iteration

\[
\lambda_b^{n+1} = \max \left( \lambda_b^n - \epsilon \left( (P_t - P_i) \frac{\partial f(\lambda_b^n)}{\partial \lambda_b^n} + P_i \right), \lambda_b^{\min} \right),
\]

(14)

where

\[
\frac{\partial f(\lambda_b^n)}{\partial \lambda_b^n} = \overline{\rho}(\lambda_b^n) + \lambda_b^n \frac{\partial \overline{\rho}(\lambda_b^n)}{\partial \lambda_b^n},
\]

\[
\frac{\partial \overline{\rho}(\lambda_b^n)}{\partial \lambda_b^n} = - \left( \frac{Z(\beta)}{1 + \overline{\rho}(\lambda_b^n) Z(\beta)} \lambda_b^n + \frac{\lambda \lambda_u}{1 + \overline{\rho}(\lambda_b^n) Z(\beta)} \right) \int_0^{\frac{1}{\log(1+\beta)}} \left( \frac{Z(e^{1/t} - 1) - Z(\beta)}{1 + \overline{\rho}(\lambda_b^n) Z(e^{1/t} - 1)} \right) dt \right)^{-1}
\]

(15)

and \( \epsilon \) is a sufficiently small step-size.

**Proof:** See Appendix C.

Although we can not prove the objective function of \( P1 \) is convex theoretically, but our numerical results in Section V show that the proposed algorithm in Theorem 3 will converge to the global optimal \( \lambda_b^* \) because of the convex-like property of the objective function.

**V. Numerical Results**

In this section, we will provide numerical results through Monte Carlo simulation. We set \( \lambda_u = 10^{-5} m^{-2} \), the path loss exponent \( \alpha = 4 \). The SINR target \( \beta = -5dB \).
A. **SINR with Different Traffic Arrival Rate**

We first set BS density \( \lambda_b = 10^{-6} m^{-2} \). The SINR distribution of users with different traffic arrival rate is presented in Fig. 2. Comparing the result in full buffer scenario, the SINR is generally higher in burst traffic scenario because of less inter-cell interference. We can find that the SINR degrades with larger traffic arrival rate. The reason is that, when increasing the traffic arrival rate, the average transmission probability of BSs will be higher. As a result, the users will suffer from heavier inter-cell interference and have lower SINR. With the increase of traffic arrival rate, the SINR distribution converges to the result of full buffer scenario. The reason is that, the average transmission probability of BSs will converge to 1 with the increase of traffic arrival rate.

B. **Area Power Consumption**

Next, we will evaluate the network power consumption with different BS density. The purpose of this part is similar with the optimization of homogeneous network in [14], where full buffer traffic is considered. However, we will have a quite different result from [14]. Without loss of generality, we assume the power consumption of BS in transmission mode is unit, i.e., \( P_t = 1 \), and we consider different power consumption in idle mode, i.e., \( P_i = 0.08, 0.1, 0.12P_t \). Fig. 3 shows the average power consumption with different BS density. In [14], the optimal BS
density in homogeneous network is the minimum BS density that can meet the demand of users, deploying more BSs will consume more power. However, as shown in Fig. 3, it’s different from [14] in Poisson traffic scenario. The optimal BS density $\lambda_b^*$ would be larger than $\lambda_b^{min}$, i.e., more BSs may reduce the total power consumption. The reason is that, the power consumption is lower in idle mode, deploying more BSs can make each BS stay in idle mode more frequently. And we find that the optimal BS density is larger when the power consumption in idle mode is lower.

Through the numerical result, we can find the proposed algorithms in Theorem 3 can converge to the global optimal BS density because of the convex-like probability of the objective function. In this paper, the main goal is to find the optimal BS density to minimize the power consumption of the networks. However, we can also take other CAPEX (Capital Expenditures) and OPEX (Operational Expenditures) into consideration when determining the optimal BS density.

C. Discussion of BS Density

Deploying more BSs to improve the users’ QoS is a tendency of future cellular networks. We will discuss the influence of BS density on users’ SINR and packet delay in this part.

1) SINR of Users: As we have mentioned in Remark 1, the SINR of users will be generally higher with larger $\lambda_b$, which is depicted in Fig. 4. Some readers may be confused by the result of full buffer scenario in [5], where the SINR of users does not depend on BS density.
In full buffer scenario, because the increase in signal power by making users closer to BS is counter-balanced by the increase of interference, deploying more BSs (i.e., large $\lambda_b$) does not affect the SINR distribution of the typical user. However, as we just analyzed, the SINR distribution is related to $\lambda_b$ in Poisson traffic scenario, and a larger BS density will lead to a higher SINR. The reason is that deploying more BSs will make the average transmission probability $E[\rho]$ smaller, thus, the increase of inter-cell interference will not be as strong as the full buffer scenario. As a result, the increase in useful signal power will not be overwhelmed by the increase of interference in burst traffic scenario. Therefore, we have a different result from [5].

Since the users will have a higher SINR in denser networks, higher order modulation technology is needed to achieve the channel capacity. Moreover, the number of users covered by the same BS will be smaller, i.e., each user can get more wireless resource (e.g., number of subcarriers). Therefore, each user can get a sufficiently higher average data rate. Consequently, data-hungry application (e.g., FTP) can be easily supported by increasing the BS density.

2) Delay of Users’ Packet: As we know, burst traffic (e.g., VoIP, video streams) is often sensitive to delay. Therefore, we will also evaluate this performance indicator.

Since each BS can be viewed as M/G/1, we can derive the average delay of users’ packets based on P-K formula [20].
Theorem 4: The average delay of users’ packet is

$$E[D] = E[T] + \frac{\lambda \lambda_u E[T^2]}{2 \lambda_b (1 + E[\rho] Z(\beta)) (1 - E[\rho])},$$

(16)

where

$$E[T^2] = \int_0^{(\log(1 + \beta))^2} \left( 1 - \frac{1 + E[\rho] Z(\beta)}{1 + E[\rho] Z(e^{1/\sqrt{t}} - 1)} \right) dt.$$  

(17)

Proof: See Appendix D.

The first item of (16) indicates the transmission delay, the second item is the queuing delay. It is intuitive that both the transmission delay and queuing delay will decrease with BS density. There are two reasons for this proposition. First, the instantaneous data rate of users will be higher as we have discussed. Second, the total traffic arrival rate at each BS will be smaller for there will be fewer users covered by the same BS. Fig. 5 shows the average delay with different BS density. We can find that the average delay of users’ packet will first drop sharply with the increase of BS density. However, the impact of increasing BS density on average delay will be weaker when the BS density is larger. Above all, we can conclude that the performance of average packet delay will be improved by increasing the BS density.

Remark 2: Both the performance of SINR and the average packet delay will be improved by deploying more BSs. We can conclude that the QoS of both data-hungry and delay-sensitive applications can be guaranteed by increasing the BS density. However, the power consumption
of the network will be larger when the BS density is not appropriate. Therefore, the tradeoff of users’ QoS and power consumption of the network needs further study, which is beyond the scope of this paper.

VI. CONCLUSION

This paper analyzed the performance of cellular networks with Poisson traffic streams and discuss the optimal BS density to minimize the average network power consumption. Different from the result of full buffer scenario, the users’ SINR distribution depends on the average transmission probability of BSs, which is related to user density, BS density and traffic arrival rate. The average transmission probability is defined in a nonlinear equation, which is solved through simple bisection method. With the increase of traffic arrival rate, our SINR analysis result will converge to the result of full buffer scenario. We formulate the optimization problem to minimize the area power consumption, and the optimal BS density can be derived through the proposed iteration algorithm. And we also have a different conclusion on the optimal BS density from previous works in full buffer scenario. At the end, the influence of BS density on users’ QoS has been discussed. Both the performance of users’ SINR and packet delay can be improved by increasing the BS density. Extensive work of our framework could include heterogeneous network and the research on offloading schemes in cellular networks.

APPENDIX A

Proof of Lemma

$$P [SINR > \eta]$$

$$= \int_0^\infty \mathbb{E}_{\Phi'_b} \left[ \prod_{x_i \in \Phi'_b \cap B^c (r)} \frac{1}{1 + \eta r^\alpha |x_i|^{-\alpha}} \right] f_r (r) \, dr,$$

where (a) follows from similar procedure in [5], and $$f_r (r) = 2\pi \lambda_b r e^{-\pi \lambda_b r^2}$$ is the distribution of the distance between the typical user and the associated BS. The PGFL of $$\Phi'_b$$ can be derived as follows.

Some useful results on distances of PPP can be found in [23].
\[
E_{\Phi_b} \left[ \prod_{x_i \in \Phi_b \cap B^c(r)} \frac{1}{1 + \eta r^\alpha |x_i|^{-\alpha}} \right]
\]

\[= (b) \ E_{\Phi_b}^{thin} \left[ \prod_{x_i \in \Phi_b \cap B^c(r)} \frac{1}{1 + \eta r^\alpha |x_i|^{-\alpha}} \right]
\]

\[= (c) \ exp \left( -E[\rho] \lambda_b \int_r^\infty \frac{1}{1 + \eta^{-1} r^{-\alpha} x^\alpha} 2\pi x dx \right)
\]

\[= \ exp \left( -E[\rho] \lambda_b \pi r^2 \eta^{2/\alpha} \int_{\eta^{-2/\alpha}}^{\infty} \frac{1}{1 + u^{\alpha/2}} du \right), \quad (19)
\]

where \((b)\) follows from Corollary \[\Pi\] \((c)\) follows from the property of PGFL of PPP. Substituting \((19)\) into \((18)\), we get the SINR distribution:

\[
P[SINR > \eta] = \int_0^\infty \exp \left( -E[\rho] \lambda_b \pi r^2 Z(\eta) \right) 2\pi \lambda_b r e^{-\pi \lambda_b r^2} dr
\]

\[
= \int_0^\infty \exp \left( -\lambda_b \pi r^2 (1 + E[\rho] Z(\eta)) \right) 2\pi \lambda_b r e^{-\pi \lambda_b r^2} dr
\]

\[
= \frac{1}{1 + E[\rho] Z(\eta)}. \quad (20)
\]

\textbf{APPENDIX B}

\textit{Proof of Theorem 2}: The average total traffic arrival rate at each BS that have to be delivered is \(E[\lambda_{tot}] = \lambda E[N] P[SINR \geq \beta]\), where \(E[N]\) is the average number of users covered by each BS. Therefore, we have

\[
E[\rho] = E[\lambda_{tot}] E[T]
\]

\[
= \lambda E[N] P[SINR \geq \beta] E[T]
\]

\[= \lambda E[N] P[SINR \geq \beta] E[T]
\]

\[= \lambda \frac{\lambda_u}{\lambda_b} P[SINR \geq \beta] E[T]
\]

\[= \lambda \frac{\lambda_u}{\lambda_b} \frac{1}{1 + E[\rho] Z(\beta)} \int_{\log(1+\beta)}^{1} \frac{E[\rho] \left( Z(e^{1/t} - 1) - Z(\beta) \right)}{1 + E[\rho] Z(e^{1/t} - 1)} dt, \quad (21)
\]
where (a) follows from the fact that the mean number of users per BS covering $\mathbb{E}[N] = \frac{\lambda u}{\lambda_b}$, which is a direct result of the Neveu exchange formula \[21\]. After some algebraic manipulation, we derive (9) from (21).

**APPENDIX C**

*Proof of Theorem 3* We can use gradient descent method to derive the local optimal BS density $\tilde{\lambda}_b^\ast$ \[24\]. Since the objective function is approximately convex in feasible set, the iteration algorithm can converge to the global optimal BS density.

The gradient of objective function of $P1$ is

\[ (P_t - P_i) \left( \overline{p}(\lambda_b) + \lambda_b \frac{\partial \overline{p}(\lambda_b)}{\partial \lambda_b} \right) + P_i. \]  

We derive $\frac{\partial \overline{p}(\lambda_b)}{\partial \lambda_b}$ from the implicit function \[9\]. Replacing $\mathbb{E}[\beta]$ with $\overline{p}(\cdot)$ in \[9\], we have

\[ F(\lambda_b, \overline{p}) = \frac{1}{1 + \overline{p}Z(\beta)} \int_0^{\log(1+\beta)} \frac{Z(e^{1/t} - 1) - Z(\beta)}{1 + \overline{p}Z(e^{1/t} - 1)} dt - \lambda_b \lambda u \]

\[ = 0. \]  

(23)

We first need to derive the derivative $\frac{\partial F}{\partial \lambda_b}, \frac{\partial F}{\partial \overline{p}}$:

\[ \frac{\partial F}{\partial \lambda_b} = -\frac{1}{\lambda \lambda_u}, \]

\[ \frac{\partial F}{\partial \overline{p}} = -\frac{Z(\beta)}{1 + \overline{p}Z(\beta)} \frac{\lambda_b}{\lambda \lambda_u} - \frac{1}{1 + \overline{p}Z(\beta)} \int_0^{\log(1+\beta)} \frac{(Z(e^{1/t} - 1) - Z(\beta)) Z(e^{1/t} - 1)}{(1 + \overline{p}Z(e^{1/t} - 1))^2} dt. \]  

(24)

\[ ^4\text{This will be demonstrated in the following numerical results.} \]
Therefore, we have
\[
\frac{\partial \rho}{\partial \lambda_b} = -\frac{\partial F}{\partial \lambda_b} = \frac{\lambda \lambda_u}{1 + \bar{\rho}Z(\beta)} \int_0^{1/\beta} \frac{Z(e^{1/t} - 1)Z(e^{1/t} - 1)}{(1 + \bar{\rho}Z(e^{1/t} - 1))^2} dt \]

(25)

Above all, combining with the constraint \( \lambda_b > \lambda_b^{min} \), we derive the iteration algorithm.

**APPENDIX D**

**Proof of Theorem 4** Since every BS can be viewed as M/G/1, we can derive the average delay based on P-K formula approximately.

\[
E[D] = E[T] + \frac{E[\lambda_{tot}] E[T^2]}{2(1 - E[\rho])}.
\]

(26)

\( E[\lambda_{tot}] = \lambda_u \frac{1}{\lambda_b + E[\rho] Z(\beta)} \) is derived in Appendix B. The secondary moment \( E[T^2] \) can be calculated as follows.

\[
E[T^2] = \int_0^\infty P[T^2 > t] dt = \int_0^\infty P_{ST}[\log (1 + SINR) < 1/\sqrt{t}] dt = \int_0^{(1/\log(1+\beta))^2} P_{ST}[SINR < e^{1/\sqrt{t}} - 1] dt = \int_0^{(1/\log(1+\beta))^2} \left(1 - \frac{1 + E[\rho] Z(\beta)}{1 + E[\rho] Z(e^{1/\sqrt{t}} - 1)}\right) dt.
\]

(27)

Substituting \( E[\lambda_{tot}] \) and \( E[T^2] \), we can derive the average delay in Theorem 4.

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