FORMULATION OF MAGNETIC MONOPOLES IN HOT GAUGE THEORIES

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In this talk, I discuss the formation of magnetic monopoles in a phase transition from the confining SU(2) phase to the Coulomb phase in a hot Georgi-Glashow model. I argue that monopoles are formed from long-wavelength thermal fluctuations, which freeze out after the phase transition.

1. Introduction

In the non-inflationary Big Bang scenario, magnetic monopoles formed at the phase transition of the Grand Unified Theory (GUT) lead to a severe cosmological problem, because their energy density would be so high that the universe would have collapsed under its own weight a long time ago. This problem is famously solved by inflation, provided that the temperature of the universe never reaches the GUT scale after inflation ended. On the other hand, this also gives a constraint for possible inflationary models.

Most of the work done on the monopole problem deals with the subsequent annihilations of monopole-antimonopole pairs, but it has been generally assumed that the monopoles were formed by the Kibble mechanism.\(^3\) In that case, causality would give a lower bound for the number density of one monopole per Hubble volume at the time of the GUT transition. However, the Kibble mechanism implicitly assumes that the symmetry broken in the transition is a global one, and its validity is gauge field theories, such as GUTs, is therefore not obvious.

There is also a possibility in many inflationary models that a broken symmetry is temporarily restored by non-thermal fluctuations\(^5\) even if the equilibrium temperature never exceeds the critical temperature. This would impose even stronger constraints on inflationary models, but calculating what they really are requires an understanding of the principles that govern monopole formation more generally.
In this talk, I will discuss monopole formation in gauge field theories and review the basic idea of the scenario presented in Ref. 1. This scenario is different from the Kibble mechanism, and the difference is reflected both in the number density and in the spatial distribution of monopoles. Therefore, we are not simply using different language to describe the same physics.

2. Monopole formation

For simplicity, let us consider an SU(2) gauge field coupled to an adjoint Higgs field \( \Phi \) at a non-zero temperature. Although realistic GUTs are much more complicated, this toy model has all the properties that are important for monopole formation. We imagine that the Higgs potential has the form

\[
V(\Phi) = m^2(t)\text{Tr}\Phi^2 + \lambda \text{Tr}\Phi^4,
\]

where \( \lambda \) is high enough so that the transition is continuous and the mass parameter \( m^2(t) \) varies with time. We start from thermal equilibrium at a high enough value of \( m^2 \) so that the SU(2) symmetry is unbroken, and then gradually decrease \( m^2 \) through the critical point into the broken phase.

If the symmetry were global, one could simply say that in the broken phase the ground state corresponds to a non-zero \( \Phi \), and because of the SU(2) symmetry, there is a two-sphere of different vacua. Ideally, \( \Phi \) should point into the same direction everywhere, but that would mean that it is correlated over an infinite distance, and that cannot be achieved in a finite time. When the critical point is approached, the equilibrium correlation length \( \xi \) diverges, but in reality it can only reach some finite value \( \hat{\xi} \) before the system falls out of equilibrium. Following Kibble, we can then argue that if we consider two points separated by more than \( \hat{\xi} \), the choice of the vacuum at these points must be uncorrelated. This leads to the formation of magnetic monopoles.

In gauge theories, there are extra complications. Because \( \Phi \) is not gauge invariant, neither it nor its correlators are physical observables. Monopoles do, of course, still exist but we cannot use the above argument to describe their formation. Furthermore, \( \Phi \) cannot be used as an order parameter, either. There is strictly speaking no phase transition between the “symmetric” and the “broken” phase in the gauge theory. This means that all correlation lengths remain finite at the transition point. This is another reason why Kibble’s argument cannot be used in the gauge theory.

It has been suggested that these problems could be avoided by fixing a gauge so that only a global SU(2) symmetry remains. The problem with this idea is that in a typical fixed gauge, there is no reason why \( \Phi \) should
be constant in space even in the broken phase, since the gauge field can be non-zero. On the other hand, one could also fix the unitary gauge by rotating $\Phi$ everywhere to the same direction, and then a naive application of the Kibble scenario would lead to the wrong result that no monopoles are formed. Obviously, the gauge field plays an important role in the problem.

The first step in understanding the dynamics of the phase transition is understanding the nature of the two phases in equilibrium. All we actually need to know about the symmetric phase is that it is confining and that the longest correlation lengths are of order $1/g^2 T$. In perturbation theory, there is a massless photon in the broken phase, but non-perturbative effects make it massive, giving it a magnetic screening mass $m_B \propto \exp(-m_M/2T)$, where $m_M$ is the monopole mass. At zero temperature, we would then reach the true Coulomb phase.

One can now see that although there are no diverging correlation lengths at the transition point, the magnetic screening length grows faster and faster as we go deeper into the broken phase. If we keep on decreasing $m^2$ at a constant rate, the screening length $\xi_B = 1/m_B$ would eventually have to grow faster than the speed of light. This is impossible, and therefore system must fall out of equilibrium.

The fact that the system falls out of equilibrium does not, of course, necessarily mean that monopoles are formed. However, the diverging correlation length here is the magnetic screening length, defined by

$$\langle B_i(\mathbf{x}) B_j(\mathbf{y}) \rangle \sim \exp(-m_B|\mathbf{x} - \mathbf{y}|).$$

(2)

Here $B_i$ could, in principle, be any operator that couples to the photon, but for our purposes it is most convenient to use the 't Hooft operator for the magnetic field or its discretized analogue. That has the advantage that $B_i$ only has delta function sources, which can be interpreted as magnetic monopoles.

Using this interpretation, we can define the monopole density $\rho_M$ as $\rho_M = \nabla \cdot \mathbf{B}$, and it follows that the magnetic charge-charge correlator will also fall exponentially with the same decay rate

$$\langle \rho_M(\mathbf{x}) \rho_M(\mathbf{y}) \rangle \sim \exp(-m_B|\mathbf{x} - \mathbf{y}|).$$

(3)

This exponential tail will become longer as we go deeper into the broken phase and $m_B$ decreases. There will therefore be long-wavelength charge fluctuations, which will persist and stop the monopole density from falling to zero.

If we assume that the system stays in equilibrium long enough after the perturbative transition point, we can actually calculate the charge-charge
correlator

$$\langle \rho_M(\vec{x})\rho_M(\vec{y}) \rangle \approx q_M^2 n_M \left( \delta(\vec{x}-\vec{y}) - \frac{m_B^2}{4\pi|\vec{x}-\vec{y}|} e^{-m_B|\vec{x}-\vec{y}|} \right).$$

(4)

Here $q_M = 4\pi/g$ is the magnetic charge and

$$n_M \approx g^2 T/\xi_B^2$$

(5)

is the thermal monopole density.

Deeper into the broken phase, the monopoles cost more energy and the monopole density $n_M$ must therefore decrease. Because monopoles are stable, this can only take place through annihilations of monopole-antimonopole pairs. At first, when $\xi_B$ is small enough, the charge-charge correlation can keep the form (4) and the system can stay in equilibrium, because monopoles can travel the distance $\xi_B$ to be annihilated.

However, sooner or later the distance $\xi_B$ becomes too long for this, and the monopoles will instead try to find antimonopoles closer by. The long-distance part of the correlator (4) will then freeze and $\xi_B$ stop to some value $\hat{\xi}_B$. In the ideal case, all monopole-antimonopole pairs of size less than $\hat{\xi}_B$ are annihilated, but larger pairs survive.

Basically, this means that the charge distribution at the time of the freeze-out gets smoothed so that all details at scales shorter than $\hat{\xi}_B$ disappear, and the remaining charge distribution is divided into quantized unit charges, which are the monopoles.

To obtain a rough estimate of the number density of monopoles, we can assume that the typical charge inside a given sphere of radius $\hat{\xi}_B$ cannot change but gets frozen to the value it had when the system fell out of equilibrium,

$$Q_M(\hat{\xi}_B) = \sqrt{\left\langle \left( \int_{\xi_B}^{\hat{\xi}_B} d^3x \rho_M(\vec{x}) \right)^2 \right\rangle} \approx \sqrt{T\hat{\xi}_B}.$$

(6)

This means that in the final state, there will be $Q_M(\hat{\xi}_B)/q_M$ monopoles inside the sphere, and most strikingly, they would all have the same sign. If $Q_M(\xi_B)/q_M$ is large, there will be regions of many monopoles but no antimonopoles (and, of course, others with many antimonopoles but no monopoles). In other words, this scenario predicts positive correlations between monopoles at short distances, whereas the Kibble mechanism would predict negative correlations.\(^1\)
We can also see that because there are roughly \( Q_M(\hat{\xi}_B)/q_M \) monopoles inside any sphere of volume \( \hat{\xi}_B^3 \), the monopole number density will be

\[
n_M \approx \frac{Q_M(\hat{\xi}_B)}{q_M \hat{\xi}_B^3} \approx q^{-1}_M \sqrt{\frac{T}{\hat{\xi}_B^5}} \approx g \sqrt{\frac{T}{\hat{\xi}_B^5}}.
\]

(7)

3. Conclusions

In this talk, I have only tried to present the scenario in its simplicity and not to do any quantitative calculations. Some very simple estimates can be done easily,\(^1\) but more precise calculations are going to need a much better knowledge of the equilibrium and non-equilibrium properties of hot gauge theories.

Firstly, static equilibrium simulations can be used to measure the screening length \( \xi_B \) or, even better, the charge-charge correlator as a function of \( m^2 \). Real-time simulations such as those used to measure the sphaleron rate can be used to find out when the freeze-out takes place and determine \( \hat{\xi}_B \). And finally, simulations of the whole phase transition could be used to test this scenario. Because the phenomenon is sensitive to non-perturbative dynamics of a non-Abelian gauge field, simulations like that would be a great challenge to any methods based on, say, Hartree or 2PI formalisms, but should be relatively straightforward to do using the classical approximation.

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