A general analysis of Wtb anomalous couplings

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Abstract: We investigate new physics effects on the Wtb effective couplings in a model-independent framework. The new physics effects can be parametrized by four independent couplings, \( f_1^R, f_2^L, f_2^R \) and \( f_2^R \). We further introduce a set of parameters \( x_0, x_m, x_p \) and \( x_2 \) which exhibit a linear relation to the single top production cross sections. Using recent data for the \( t \)-channel single top production cross section \( \sigma_t \), \( tW \) associated production cross section \( \sigma_W \), \( s \)-channel single top production cross section \( \sigma_s \), and W-helicity fractions \( F_0, F_L \) and \( F_R \) collected at the 8 TeV LHC and Tevatron, we perform a global fit to impose constraints on the top quark effective couplings. Our global fitting results show that the top quark effective couplings are strongly correlated. We show that (i) improving the measurements of \( \sigma_t \) and \( \sigma_W \) is important in constraining the correlation of \( (f_1^R, f_2^R) \) and \( (f_2^L, f_2^R) \); (ii) \( f_1^L \) and \( f_2^R \) are anti-correlated, and are sensitive to all the four experiments; (iii) \( f_2^L \) and \( f_2^R \) are also anti-correlated, and are sensitive to the \( F_0 \) and \( F_L \) measurements; (iv) the correlation between \( f_2^L \) and \( f_2^R \) is sensitive to the precision of the \( \sigma_t \), \( \sigma_W \) and \( F_0 \) measurements. The effective Wtb couplings are studied in three kinds of new physics models: the \( G(221) = SU(2)_L \otimes SU(2)_R \otimes U(1)_X \) models, the vector-like quark models and the Littlest Higgs model with and without \( T \)-parity. We show that the Wtb couplings in the left-right model and the un-unified model are sensitive to the ratio of gauge couplings when the new heavy gauge boson’s mass \( (M_{W'}) \) is less than several hundred GeV, but the constraint is loose if \( M_{W'} > 1 \) TeV. Furthermore, the Wtb couplings in vector-like quark models and the Littlest Higgs models are sensitive to the mixing angles of new heavy particles and SM particles.

Keywords: top quark, TeV physics, anomalous coupling

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1 Introduction

The discovery of the Higgs boson at the Large Hadron Collider (LHC) [1, 2] completes the Standard Model (SM) of particle physics. So far, no evidence of new physics (NP) has been observed at the LHC. It is possible that the scale of new physics \( \Lambda_{NP} \) is much higher than the electroweak symmetry breaking (EWSB) scale \( (v = 246 \text{ GeV}) \). If so, we expect to look for indirect effects of NP with the SM particles which we know. The top quark, discovered at the Tevatron [3, 4], is the heaviest particle in the SM. With its mass around the EWSB scale, the top quark is believed to play an important role in connecting the SM and NP. The Wtb coupling plays a pivotal role in top quark physics. The top quark decay and single top quark production processes are sensitive to the Wtb coupling. In addition, it offers a very promising way to probe for NP at the LHC. For instance, the production rate of single top quarks and the polarization of the top quark can be modified by NP beyond the SM, such as new gauge boson W’s, vector-like fermions, etc. Those heavy particles are predicted in many NP models, such as \( SU(2)_L \otimes SU(2)_R \otimes U(1)_X \), often denoted as \( G(221) \) models \([5, 6]\), vector-like quark models \([7–10]\), Little Higgs models \([11–13]\), warped/composite simpli-
fied models [14] and many others. It is convenient to classify the underlying theories according to different approaches that modify the Wtb coupling. A simple case is that the new heavy particles mix with SM particles at tree-level, like a new gauge boson $W'$ or a new fermion $T(B)$. Another way to generate the anomalous Wtb coupling is through the loop-level. Typical examples are supersymmetric models [15, 16] and two-Higgs-doublet models [17].

The top quark decays before hadronization as its lifetime is much smaller than the typical hadronization time scale. In the SM, the dominant decay mode of top quark is $t \to W^+ b$, so the Wtb coupling governs the top quark decay process. One way to probe the structure of the Wtb coupling is to study the helicity fractions of the W boson in top quark decay. The W-boson helicity fractions are defined as the partial rate for a given helicity state divided by the total decay rate: $F_L, F_R, F_0, F_{L,R,O}$, where $F_L, F_R, F_0$ are the left-handed, right-handed and longitudinal helicity fractions, respectively. The W helicity fractions, calculated to the accuracy of the next-to-leading order (NNLO) QCD corrections in the SM, are $F_0 = 0.687 \pm 0.005$, $F_L = 0.311 \pm 0.005$, $F_R = 0.0017 \pm 0.0001$ for a top quark mass of $m_t = 172.8 \pm 1.3$ GeV [18, 19]. Recently, the ATLAS and CMS collaborations have measured the W helicity fractions in top quark decays at $\sqrt{s} = 7$ TeV, and CMS 8 TeV results are also available. These results are consistent with the SM predictions [19, 20].

At hadron colliders, the top quark can be produced singly in three channels: the $t$-channel process ($ub \to td$), the $W$-associated production process ($bg \to tW^-$), and the $s$-channel process ($ud \to W^+ \to tb$). The $t$-channel cross section is predicted at next-to-next-to-leading order plus the contribution due to the resummation of soft-gluon bremsstrahlung (NNLO+NNLL) to be $\sigma_t = 87.8^{+3.4}_{-1.9}$ pb [21, 22] at the LHC with $\sqrt{s} = 8$ TeV. The theoretical predictions for the $tW$-channel and $s$-channel at $\sqrt{s} = 8$ TeV at NLO+NNLL precision in QCD are $\sigma_{tW} = 22.4 \pm 1.5$ pb [23] and $\sigma_s = 5.6 \pm 0.2$ pb [24], respectively. The cross sections at $\sqrt{s} = 13$ TeV are $\sigma_t = 221^{+5}_{-3}$ pb, $\sigma_{tW} = 72.6 \pm 1.3 \pm 1.3$ pb and $\sigma_s = 11.29 \pm 0.18 \pm 0.26$ pb [25]. Recently, both ATLAS and CMS collaborations have measured the single top quark production cross sections at $\sqrt{s} = 7$ TeV, $\sqrt{s} = 8$ TeV and $\sqrt{s} = 13$ TeV. These are summarized in Sec. 3.

In this work, we use the effective field theory (EFT) to calculate the single top quark production cross sections and W helicity fractions, and use the recent experimental data (denoted as Wtb measurements) to determine the general Wtb couplings. Model-independent analyses of the Wtb couplings have already been performed using the EFT approach, for example in Refs. [26–49]. In our work, we compute the deviation from the SM by including the dimension-6 operators for top quark decay and single top quark production processes. We incorporate the quadratic terms of dimension-6 operators to obtain the correlations among different operators in the single top quark cross section and W helicity fraction calculations. The constraints on Wtb couplings based on some of the recent experimental data were studied in Refs. [46, 47]. We perform a global fit of the general Wtb couplings by analysing the correlations among different couplings and discuss the implication of the top quark effective theory for several NP models.

The paper is organized as follows. In Section 2 we calculate the single top quark production cross sections via an effective field theory approach. In Section 3 we present the allowed parameter space of the general Wtb couplings after incorporating the most recent ATLAS and CMS results. In Section 4 we discuss the constraints on various new physics models from the Wtb couplings. Finally, we conclude in Section 5.

## 2 Top quark effective field theory

### 2.1 Dimension-6 operators and effective Wtb couplings

Using the EFT approach to explore possible NP effects has been discussed widely, see for example Refs. [27, 29, 30, 32, 40, 50–57]. A model-independent way to parametrize the low energy effects of NP theories is the linearly realized effective Lagrangian, which incorporates the particle content and symmetries of the SM. The nonlinearly realized mechanism of the electroweak symmetry is studied in Refs. [27, 51, 53, 54]. In this paper, we assume the new scalar particle observed at the LHC is the SM Higgs boson, and use the linear realization to parametrize the NP effects [50, 55]. The effective Lagrangian before electroweak symmetry breaking is

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{C_i}{A^2} O_i + \mathcal{O} \left( \frac{1}{A^3} \right),$$

where $\mathcal{L}_{\text{SM}}$ is the SM Lagrangian, $A$ is the characteristic scale of new physics, $O_i$ denotes $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge invariant dimension-6 operators, and $C_i$ corresponds to the Wilson coefficient which represents the strength of the effective operator $O_i$. The dimension-5 operator violates lepton number conservation and is not considered in this work. The close agreement between experimental measurements and SM predictions indicates that the NP effects should be small. Hence, we restrict ourselves to dimension-6 operators in this work. The complete set of dimension-6 effective operators generating anomalous Wtb couplings is [50, 55]

$$O^{(3)}_{\psi} = i(\bar{\psi} \tau^i D_\mu \phi)(\bar{q}_L \gamma^\mu \gamma^5 q_L), \quad O_{\phi\phi} = i(\bar{\phi} \gamma^\mu D_\mu \phi)(\bar{t}_R \gamma^\mu b_R),$$

where $\phi$ is the new scalar field and $\psi$ is the top quark field.
Three types of dimension-6 operators contribute to the Wtb couplings. The first type is operators involving a scalar field carrying one covariant derivative, the second is operators involving fermion and scalar fields both carrying one covariant derivative, and the third is operators involving field strength tensors.

The operators \( O_{DT} \) and \( O_{DB} \) are of the first type. They can be generated at tree-level after integrating out the new heavy particles, such as a heavy charged vector boson \( W^\pm \) that mixes with SM gauge bosons \( W^\pm \) [5, 6] or a heavy quark that mixes with top quark or bottom quark [9, 10]. A pictorial illustration of the relation between the tree-level effective operators and the possible NP models is shown in Fig. 1(a) and Fig. 1(b). We use the bold-red line to denote the NP particles. The anomalous Wtb couplings arise at tree-level after spontaneous symmetry breaking with \( \langle \phi \rangle = \frac{v}{\sqrt{2}} \).

The operators \( O_{DV}, O_{DB}, O_{Dq} \) fall into the second category. Those operators give a contribution of order \( p^2/A^2 \), where \( p \) is the typical momentum scale in the process. Such operators, corresponding to the vertices involving three external lines, can be induced only at loop-level after integrating out the heavy particles [58].

The operators \( O_{qW}, O_{bW}, O_{bW} \) are of the third type. Usually, effective operators which involve the field strength tensor are generated only at loop level if the complete theory is a gauge theory [58]. Typical examples include supersymmetric models [15, 16, 59], two-Higgs-doublet models [17], etc.

A pictorial illustration of the relation between loop-induced effective operators and the possible underlying theories is shown in Fig. 1(c–e). The Wtb coupling can be induced in the NP models with extended gauge structure, which yields extra gauge bosons (see Fig. 1(c)), or new scalar particles which consist of a new charge doublet models [17], etc. or new fermions which carry one discrete quantum number to avoid mixing with the top quark and bottom quark at tree-level (see Fig. 1(e)). We will comment on the impact of Wtb measurements on several NP models in Section 4.
As usual, the effective Lagrangians consist of redundant terms, which can be removed by the classical equations of motion [60]. This is based on the equivalence theorem of the $S$ matrix. After we use the equations of motion to remove redundant operators, the relevant operators reduce to

\[ O_{\phi \phi}^{(3)} = i (\phi^\dagger \tau^i D_\mu \phi) (\bar{q}_L \gamma_\mu \tau^i q_L) \]
\[ O_{tW} = (\bar{q}_L \sigma^{\mu \nu} \tau^i t_R) \phi W^\mu_W \]
\[ O_{bW} = (\bar{q}_L \sigma^{\mu \nu} \tau^i b_R) \phi W^\mu_W. \]

Assuming new anomalous couplings arise from the above dimension-6 operators, we can parametrize the general effective Wtb couplings as [36]

\[ \mathcal{L}_{Wtb} = \frac{g}{\sqrt{2} m_W} W^-_\mu \tilde{b} \gamma^\mu \left[(1 + f_1^L) P_L + f_1^R P_R\right] t \]
\[ - \frac{g}{\sqrt{2} m_W} \partial_\mu W^-_\mu \tilde{b} \sigma^{\mu \nu} \left(f_2^L P_L + f_2^R P_R\right) t + \text{h.c.}, \] (4)

where $P_{L,R} = (1 \mp \gamma_5)/2$ are the usual chirality projectors, $m_W$ is the W-boson mass and the Cabibbo-Kobayashi-Maskawa matrix element $V_{tb}$ is taken to be 1 in our analysis. In the SM, the values of the coefficients $f_i$ vanish at tree level. Those couplings could be generated if NP exists. Although those couplings can in general be complex quantities, we assume the four coefficients are real in our calculation\(^1\). The coefficients of the effective Wtb couplings are related to the Wilson coefficients of the dimension-6 operators as follows [29]:

\[ f_1^L = \frac{C_{\phi \phi}^{(3)} v^2}{A^2}, \quad f_1^R = \frac{1}{2} C_{\phi \phi}^{\ast} v^2 \bar{A}^2, \]
\[ f_2^L = \sqrt{2} C_{tW}^{\ast} \frac{v^2}{\bar{A}^2}, \quad f_2^R = \sqrt{2} C_{tW} \frac{v^2}{\bar{A}^2}. \] (5)

2.2 Single top production

In this section we discuss the contribution of general effective Wtb couplings to the total cross sections of single top production and the W helicity fractions in top quark decay. In this work, we focus on the four independent operators, see Eq. (3). The top quark can be produced singly through the electroweak interaction. Depending on the kinematics of the W boson involved, single top production is usually sub-categorized into three channels: $s$-channel production ($q_W^2 > 0$), $t$-channel production ($q_W^2 < 0$), and $tW$ associated production ($q_W^2 = m_W^2$), where $q_W$ denotes the four momentum of the W boson.

We separate the total cross section of the single top production into the SM contribution plus the contributions from anomalous Wtb couplings

\[ \sigma_i = \sigma_i^{\text{SM}} + K \Delta \sigma_i, \]

where $\sigma_i^{\text{SM}}$ denotes the cross section of the $i$-channel ($i = s,t,tW$) single top production in the SM with the approximate NNLO QCD correction, and $\Delta \sigma_i$ denotes the variation from the SM prediction induced by the anomalous couplings at tree level. The $K$-factor, defined as $K \equiv \sigma_i^{\text{NNLO}}/\sigma_i^{\text{LO}}$, describes the approximate NNLO QCD corrections in the SM. We assume the anomalous couplings receive exactly the same corrections as the SM processes. Following Ref. [36] we write the contributions of anomalous couplings to the single top productions as

\[ \Delta \sigma_s = a_1 x_0 + a_2 x_m + a_3 x_p + a_5 x_5, \]
\[ \Delta \sigma_t = b_0 x_0 + b_1 x_m + b_2 x_p + b_5 x_5, \]
\[ \Delta \sigma_{tW} = c_0 x_0 + c_1 x_m + c_2 x_p + c_5 x_5, \] (7)

where we reparametrize the four coefficients $f_1^{L,R}$ as $x_0, x_m, x_p, x_5$ [36]

\[ x_0 = \left(1 + f_1^L + \frac{f_2^L}{a_1}\right)^2 + \left(f_1^R + \frac{f_2^R}{a_1}\right)^2 - 1, \]
\[ x_m = \left(1 + f_1^L + a_1 f_2^L\right)^2 - 1, \]
\[ x_p = \left(f_1^R + a_1 f_2^R\right)^2, \]
\[ x_5 = a_1^2 \left[f_2^L (f_2^L)^2 + (f_2^R)^2\right], \] (8)

with $a_1 \equiv m_t/m_W$. Note that the terms proportional to the bottom quark mass have been ignored in our calculation due to the suppression factor $(m_b/m_W)^2$\(^2\).

The coefficients ($a_i, b_i, c_i$) depend on the collider type and energy and have to be calculated numerically. Ref. [36] calculated the $a_i$’s and $b_i$’s at the Tevatron Run II and 14 TeV LHC. In this work we update both the $a_i$’s and $b_i$’s using the CTEQ6L parton distribution functions (PDFs) [62] at the 7 TeV, 8 TeV and 13 TeV LHC. The numerical values of the coefficients are shown in Table 1.

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1\) The complex anomalous couplings have been studied in Ref. [47], and the ATLAS collaboration also give a constraint on the imaginary part $f_2^R$ in Ref. [61].

2\) We deem that neglecting the bottom quark mass in the present work is reasonable because inclusion of the bottom mass only induces a tiny asymmetry of about 0.01 $\sim$ 0.02 [46, 47] in the allowed region of $f_1^{L,F}$, whereas the length of the marginal allowed region of one variable in the global fit is typically $\mathcal{O}(1)$, see Fig. 2. Even if only one parameter such as $f_1^L$ is allowed to vary and all the other parameters are turned off, the length of the allowed region reaches about 0.2 (see Fig. 5) which is still larger than the amount of asymmetry induced by the bottom mass.
To obtain the coefficients \((c_0, c_m, c_p, c_5)\) of the tW-channel single top production, we rewrite the cross section in terms of different combinations of the effective couplings

\[
\Delta \sigma_{tW} = \left[(1 + f_1^L)^2 - 1 + (f_2^R)^2\right] \sigma_0 + \left[(1 + f_1^R) f_2^R\right] \sigma_{N1} + \left[(f_1^L)^2 + (f_2^R)^2\right] \sigma_{N2},
\]

where \(\sigma_0\) denotes the SM LO prediction while \(\sigma_{N1}\) and \(\sigma_{N2}\) represent the partial cross section that is proportional to \(f_2^R\) and \((f_2^R)^2\), respectively. The cross sections \(\sigma_0, \sigma_{N1}\) and \(\sigma_{N2}\) are obtained by integrating out the final state phase space, and then convoluting with the initial state PDFs. The coefficients \((c_0, c_m, c_p, c_5)\) are then determined from \(\sigma_0, \sigma_{N1}\) and \(\sigma_{N2}\) as follows:

\[
\begin{align*}
    c_0 &= \frac{\sigma_{N1} a_t - 2 \sigma_0 a_t^2}{2(1 - a_t^2)}, \\
    c_m &= \frac{2 \sigma_0 - \sigma_{N1} a_t}{2(1 - a_t^2)}, \\
    c_p &= \frac{2 \sigma_0 - \sigma_{N1} a_t}{2(1 - a_t^2)}, \\
    c_5 &= \frac{\sigma_{N1} - 2 \sigma_0 a_t}{2(1 - a_t^2) a_t^2} - \frac{2 \sigma_0 - \sigma_{N1} a_t}{2(1 - a_t^2)}.
\end{align*}
\]

The numerical values of the coefficients are given in Table 1 at the 7 TeV, 8 TeV and 13 TeV LHC.

With the help of the coefficients \(a_t, b_t\) and \(c_t\), the contributions of the Wtb anomalous couplings can be written as:

\[
\Delta \sigma = 2 \left(\beta_0 + \beta_m\right) f_1^L + 2 \left(\frac{1}{a_t \beta_0 + a_t \beta_m}\right) f_2^R + \left(\beta_0 + \beta_m\right) \left(f_1^L\right)^2 + \left(\beta_0 + \beta_p\right) \left(f_2^R\right)^2 + 2 \left(\frac{1}{a_t \beta_0 + a_t \beta_m}\right) f_1^L f_2^R + 2 \left(\frac{1}{a_t \beta_0 + a_t \beta_p}\right) f_1^L f_2^L \right)
\]

\[
\left(\frac{1}{a_t \beta_0 + a_t \beta_p}\right) \left(\beta_0 + \beta_m\right) \left(f_1^L\right)^2 + \left(\frac{1}{a_t \beta_0 + a_t \beta_p}\right) \left(\beta_0 + \beta_p\right) \left(f_2^R\right)^2,
\]

where \(\beta_0, \beta_1, \beta_2\) denote the single top quark cross section coefficients of different channels.

### 2.3 W helicity fractions in top decay

For completeness, we also list the fractions of the W helicity in the top quark decay in terms of \(x_i\) [36],

\[
\begin{align*}
    F_0 &= \frac{a_t^2 (1 + x_0)}{a_t^2 (1 + x_0) + 2 (1 + x_m + x_p)}, \\
    F_L &= \frac{2 (1 + x_m)}{a_t^2 (1 + x_0) + 2 (1 + x_m + x_p)}, \\
    F_R &= \frac{2 x_p}{a_t^2 (1 + x_0) + 2 (1 + x_m + x_p)},
\end{align*}
\]

where \(F_0, F_L, F_R\) represent the fractions of W bosons with longitudinal polarization \((W_0)\), left-handed polarization \((W_L)\) and right-handed polarization \((W_R)\). \(x_0\) represents the contribution from the W boson longitudinal polarization, and \(x_m\) and \(x_p\) denote the contributions from W boson left and right handed polarization, respectively.

Neglecting terms which are proportional to the bottom quark mass, the tree level results of the W helicity
fractions in the SM are
\[ F_0^{\text{SM}} = \frac{a_t^2}{a_t^2 + 2} = 0.70, \quad F_L^{\text{SM}} = \frac{2}{a_t^2 + 2} = 0.30, \]
\[ F_R^{\text{SM}} = 0. \] (13)

In the SM the top quark decays predominantly into the longitudinal W boson because the coupling of the top quark to the longitudinal W boson is similar to the Yukawa coupling, which is proportional to the top quark mass. The top quark cannot decay into a right-handed W boson owing to the purely left-handed Wtb coupling in the SM. When the bottom quark mass is ignored, the right-handed W boson is forbidden by angular momentum conservation.

However, the anomalous coupling \( f_L^R \) or \( f_L^L \) can yield a right-handed W boson in top quark decay. Different from the \( f_L^L \) coupling, the \( f_L^R \) contribution to a \( W_R \) involves flipping the chirality of the top quark, which gives rise to a factor of \( a_t \). Therefore, \( F_R \) is proportional to \( x_t = (f_L^R + a_t f_L^L)^2 \). Similarly, the \( f_L^L \) coupling can also produce a \( W_L \) in the top quark decay by flipping the top quark’s chirality. As a result, \( F_L \) is proportional to \( 1 + x_t = (1 + f_L^R + a_t f_L^L)^2 \). All of the four effective Wtb couplings can generate a \( W_0 \) in the top quark decay. \( F_0 \) is proportional to \( a_t^2(1 + x_t) = (a_t^2+f_L^R)^2+(a_t^2+f_L^L)^2 \).

3 Global fit of the effective Wtb couplings

3.1 Experimental data and statistical analysis

The single top production cross sections and the W helicity fractions have been measured at the Tevatron and the LHC. The best measurement of the cross section of the s-channel single-top production is given at the Tevatron at \( \sqrt{s} = 1.96 \) TeV with luminosity 9.7 fb\(^{-1} \) [75]. We also consider the updated experimental results of the t-channel and tW-channel cross sections and W-helicity measurements at both the CMS and ATLAS collaborations. All the experimental data are summarized in Table 2.

### Table 2. Recent measurements of the cross sections for the single top quark productions and the W helicity fractions at the Tevatron and LHC.

|                      | CMS                        | ATLAS                        | Tevatron                   |
|----------------------|----------------------------|----------------------------|---------------------------|
| s-channel            | -                          | -                           | 1.29^{+0.20}_{-0.15} pb   |
| (1.96 TeV)           |                            |                              |                           |
| t-channel            | 83.6 ± 2.3 ± 7.4 pb [77]   | 82.6 ± 1.2 ± 11.4 ± 3.1 ± 2.3 pb [78] | -                         |
| (8 TeV)              | (value ± stat ± sys)       | (value ± stat ± syst ± PDF ± lumi) |                           |
| tW-channel           | 227.9 ± 9.1 ± 14.0^{+27.7}_{-27.7} ± 3.8 pb [79] | 247 ± 6.4 ± 32.5 ± 3.1 ± 3.6 pb [80] | -                         |
| (13 TeV)             | (value ± stat ± sys ± exp ± theo ± lumi) | (value ± stat ± syst ± PDF ± lumi) |                           |
| tW-channel           | 25.0 ± 4.7 pb [81]         |                              |                           |
| (8 TeV)              |                            |                              |                           |
| W-helicity (7 TeV)   | \( F_0 = 0.626 ± 0.034\) (stat.) ± 0.048 (syst.) | \( F_L = 0.359 ± 0.021\) (stat.) ± 0.028 (syst.) [20] | -                         |
| W-helicity (8 TeV)   | \( F_L = 0.015 ± 0.034 \) |                              | \( F_R = 0.059 ± 0.015\) (stat.) ± 0.023 (syst.) |

We perform a global \( \chi^2 \) test to obtain the present constraints on the effective Wtb couplings. In the statistical analysis, the \( \chi^2 \) is defined as
\[ \chi^2 = \sum_i \frac{(O_i^{\exp} - O_i^{\text{th}})^2}{\delta \sigma_i^2}, \] (14)
where \( O_i^{\exp} \) and \( O_i^{\text{th}} \) are the experimental values and the theoretical predictions for the experimental observable \( i \), respectively. \( \delta \sigma_i \) represents the total error of the experimental measurement, which is defined as \( \delta \sigma_i = \sqrt{(\delta \sigma_i^{\text{exp}})^2 + (\delta \sigma_i^{\text{theo}})^2} \). The CERN library MINUIT [83] is used in our analysis to obtain the best-fit values of the effective Wtb couplings and the contours at different confidence levels (C.L.).

In this work we consider both direct and indirect constraints on the effective Wtb couplings. The direct constraints arise from the experimental measurements of top quark productions and decays, while the indirect constraints arise from precision measurements of flavor physics. For example, the anomalous Wtb cou-
plings can contribute to flavor changing neutral current (FCNC) processes through quantum effects involving top quarks inside the loop. In particular, the inclusive decay $B \to X_s \gamma$ provides very stringent bounds on the anomalous Wtb couplings [84].

3.2 Constraints on the effective couplings and operators

Below we present the allowed regions of the effective Wtb couplings based on a global-fit analysis of all the four effective Wtb couplings $f_{1,2}^{L,R}$. We plot in Fig. 2 the allowed parameter space for the effective Wtb couplings at 68%, 90% and 95% C.L., and investigate the correlations among the effective Wtb couplings. We note that the allowed parameter space on the plane of the $(f_L^1, f_R^1)$ and $(f_L^1, f_L^2)$ contours is mainly driven by the $t$-channel and $tW$-channel cross sections. Figure 2(a) shows the allowed parameter space of $(f_L^1, f_R^1)$. The cross sections of the single top quark production processes are proportional to $(1 + f_L^1)^2$ and $(f_R^1)^2$; see Eq. (11). It yields a circular contour region on the plane of $f_L^1$ and $f_R^1$ and favors a negative $f_L^1$. Figure 2(b) displays the contour on the plane of $(f_L^1, f_L^2)$, which is elliptical. Even though the cross sections are proportional to $(1 + f_L^1)^2$ and $(f_L^2)^2$, the $(f_L^2)^2$ term contributes less to the cross section than the $(1 + f_L^1)^2$ term. This generates the ellipse shape. We also note that the differences among the parameter spaces at the three confidence levels are very small. This is because the contours of the effective couplings on the plane of two effective Wtb couplings are a projection from the four dimension parameter space down to a two dimension subspace. That projection leads to the small differences.

![Fig. 2. (color online) Allowed parameter space on the plane of the effective Wtb couplings at confidence levels of 68% (red region), 90% (black region) and 95% (blue region). $|f_L^1| \leq 1$ is required in our analysis.](image)

The $(f_L^1, f_L^2)$ contour shown in Fig. 2(c) clearly indicates a strong anti-correlation between $f_L^1$ and $f_L^2$. Such a behavior can be understood from the single top production cross sections, which are approximately proportional to $(1 + f_L^1 + a_t f_L^2)^2$. As a result, the two islands of the allowed parameter space are symmetric around the point $(f_L^1 = -1, f_L^2 = 0)$. Note that the region of $f_L^1 < -1$ is also possible. The correlation in the $(f_R^1, f_L^2)$ contour originates from the relation $(f_R^1 + a_t f_L^2)^2$ in the cross sections, yielding the anti-correlation band in Fig. 2(d) which is centered around the point $(f_R^1 = 0, f_L^2 = 0)$. The tightest constraints on both the $(f_L^1, f_R^1)$ and $(f_L^2, f_L^2)$
contours come from the tW-channel cross section and the W helicity fractions.

Figure 2(c) shows a positive $f_2^R$ is preferred when demanding $|f_1^t| \leq 1$. Therefore, both the $(f_1^t, f_2^R)$ and $(f_2^t, f_2^R)$ contours only allow positive $f_2^R$. Due to the interference effect between the anomalous Wtb couplings and the SM, the linear term which is proportional to $f_2^R$ is valid in the cross sections. Therefore the relation of $f_2^R$ with $f_2^R$ or $f_2^R$ is like $f_2^R \sim a_1(f_2^R)^2$ or $b_1(f_2^R)^2$, as shown in Figs. 2(e) and (f). The tW channel cross section still determines the shapes of the $(f_1^t, f_2^R)$ and $(f_2^t, f_2^R)$ contours.

The current experimental data of $\sigma_s$ and $\sigma_{tW}$ exhibit $\sim 20\%$ uncertainties. The accuracy of $\sigma_{tW}$ is expected to be improved at the forthcoming LHC Run 2, but the precise measurement of $\sigma_s$ is still challenging. We thus consider the $\sigma_s$, $\sigma_{tW}$ and the W-helicity fractions in our global analysis to constrain the effective Wtb couplings. Using the $x_i$'s while neglecting $O(x_i^2)$ and higher order contributions, we can extract the correlation between different channels:

$$
\begin{pmatrix}
    \sigma_0^{tot}/\sigma_t^{SM} \\
    \sigma_0^{tot}/\sigma_{tW}^{SM} \\
    \sigma_0^{tot}/\sigma_{tW}^{NP} \\
    \sigma_0^{tot}/\sigma_{tW}^{LEP} \\
    \sigma_0^{tot}/\sigma_{tW}^{LHC} \\
\end{pmatrix} = \begin{pmatrix}
    a_0^{tot} \\
    a_0^{tot} \\
    a_0^{tot} \\
    a_0^{tot} \\
    a_0^{tot} \\
\end{pmatrix} = \begin{pmatrix}
    \Delta \sigma_t^{NP}/\sigma_t^{SM} \\
    \Delta \sigma_{tW}^{NP}/\sigma_{tW}^{SM} \\
    \Delta \sigma_{tW}^{NP}/\sigma_{tW}^{LEP} \\
    \Delta F_0^{NP}/F_0^{SM} \\
    \Delta F_L^{NP}/F_L^{SM} \\
\end{pmatrix},
$$

where $\Delta \sigma_t^{NP} = (\sigma_t^{exp} - \sigma_t^{SM})$ and $\Delta \sigma_{tW}^{NP} = (\sigma_{tW}^{exp} - \sigma_{tW}^{SM})$ are the variations from the SM prediction. $\Delta F_0^{NP} = (F_0^{exp} - F_0^{SM})$ and $\Delta F_L^{NP} = (F_L^{exp} - F_L^{SM})$ denote the variation of $W_0$ and $W_L$ helicity fractions from the SM theory prediction. The factor $a_0^{tot}$ and $c_0^{tot}$ are given by:

$$
a_0^{tot} = K(t) a_t(t) + K(\bar{t}) a_{\bar{t}}(\bar{t}),
$$

$$
c_0^{tot} = 2K(tW) c_t(tW),
$$

where the coefficients $a_t$ and $c_t$ are given in Table 1. $K(t)$ denotes the K-factor of the t-channel single top quark production, $K(\bar{t})$ denotes the K-factor of the t-channel single anti-top quark production and the $K(tW)$ is the K-factor of the tW associated production. Solving the systems of linear equations shown in Eq. 15 gives rise to the following relations

$$
\begin{pmatrix}
    x_0 \\
    x_m \\
    x_p \\
    x_5 \\
\end{pmatrix} = \begin{pmatrix}
    1.756 & -0.755 & 0.267 & -0.068 \\
    1.756 & -0.755 & -0.733 & 0.932 \\
    0.000 & 0.000 & -2.315 & -1.000 \\
    -1.547 & 1.545 & -1.918 & 0.606 \\
\end{pmatrix}
$$

where $\sigma_t^{exp}$ and $\sigma_{tW}^{exp}$ denote central values of the experimental data of $t$-channel and tW-channel cross sections, respectively. Similarly, $F_0^{exp}$ and $F_L^{exp}$ are experimental central values of the longitudinal and left-handed helicity fraction. Note that the central values of the experimental data determine the $x_i$'s and the correlations of effective couplings, while the experimental errors are translated into the errors of $x_i$'s, which yield the allowed parameter spaces of those effective couplings. Below we employ the error propagation equation of the weighted sums functions to study the dependence of the $\delta x_i$'s on experimental errors, which are labelled as $\delta \sigma_t^{exp}$, $\delta \sigma_{tW}^{exp}$, $\delta F_0^{exp}$ and $\delta F_L^{exp}$.

The variance of $x_0$ is

$$
(\delta x_0)^2 = 3.084 \left( \frac{\delta \sigma_t^{exp}}{\sigma_t^{SM}} \right)^2 + 0.570 \left( \frac{\delta \sigma_{tW}^{exp}}{\sigma_{tW}^{SM}} \right)^2 + 0.071 \left( \frac{\delta F_0^{exp}}{F_0^{SM}} \right)^2 + 0.005 \left( \frac{\delta F_L^{exp}}{F_L^{SM}} \right)^2,
$$

in which the $\sigma_t$ and $\sigma_{tW}$ measurements dominate over the W-helicity measurements. Improving the measurements of $\sigma_t$ and $\sigma_{tW}$ is important to test the correlations of $(f_1^t, f_2^R)$ and $(f_2^t, f_2^R)$.

The variance of $x_m$ is

$$
(\delta x_m)^2 = 3.084 \left( \frac{\delta \sigma_t^{exp}}{\sigma_t^{SM}} \right)^2 + 0.570 \left( \frac{\delta \sigma_{tW}^{exp}}{\sigma_{tW}^{SM}} \right)^2 + 0.537 \left( \frac{\delta F_0^{exp}}{F_0^{SM}} \right)^2 + 0.869 \left( \frac{\delta F_L^{exp}}{F_L^{SM}} \right)^2.
$$

All coefficients are comparable, such that one has to consider all the four experiments to determine $\delta x_m$. As $f_1^t$ and $f_2^R$ are anti-correlated in $x_m = (1 + f_1^t + a_1 f_2^R)^2 - 1$, improving $\delta x_m$ would further constrain the correlation between $f_1^t$ and $f_2^R$, i.e. the band in Fig. 2(c) tends to be narrower.

The $x_p = (f_1^t + a_1 f_2^R)^2$ is directly linked to the right-handed W-helicity fraction $F_R$, which is inferred from $F_0$ and $F_L$ measurements. As a result, $\delta x_p$ depends only on the W-helicity measurement as following:

$$
(\delta x_p)^2 = 5.359 \left( \frac{\delta F_0^{exp}}{F_0^{SM}} \right)^2 + \left( \frac{\delta F_L^{exp}}{F_L^{SM}} \right)^2.
$$

As a result, a strong anti-correlation between $f_1^t$ and $f_2^R$ can be obtained from the $F_0$ and $F_L$ measurements.
The variance of $x_5$ is given by

$$
(\delta x_5)^2 = 2.393 \left( \frac{\delta \sigma _{\exp }^{tL}}{\sigma _{\text{SM}}^{tL}} \right)^2 + 2.387 \left( \frac{\delta \sigma _{\exp }^{tW}}{\sigma _{\text{SM}}^{tW}} \right)^2 + 3.679 \left( \frac{\delta F_0^{\exp }}{F_{0\text{SM}}} \right)^2 + 0.004 \left( \frac{\delta F_L^{\exp }}{F_{L\text{SM}}} \right)^2.
$$

(22)

It is sensitive to the precision of $\sigma_t$, $\sigma_{tW}$ and $F_0$ measurements.

From the precision measurement of the Wtb couplings, one can also derive conservative bounds on the NP scales when no deviation is seen compared to SM predictions. Though we expect $C_i = O(1)$, their precise values are unknown. Measurements such as the ones described above can be used to obtain the ratios of these coefficients, but the values of $A_t$ cannot be obtained separately. Therefore, we define dimensionless parameters $\tilde{C}_i$ according to the Wilson coefficient $C_i$ and normalize to 1 TeV,

$$
\tilde{C}_i \equiv C_i \left( \frac{1 \text{ TeV}}{A_t} \right)^2.
$$

(23)

The allowed parameter contours for the parameters $\tilde{C}_{\phi q}^{(3)}$, $\tilde{C}_{bW}$, $\tilde{C}_{tW}$ are shown in Fig. 3. Although the values of effective couplings $f_{1L}^\phi$, $f_{1L}^b$, $f_{L}^L$ and $f_{L}^R$ are of the same order of magnitude, each individual parameter $\tilde{C}_i$ is different. For example, the $\tilde{C}_{bW}$ equal to 27 is also allowed at 95% C.L., while the maximal value for $\tilde{C}_{bW}$ and $\tilde{C}_{tW}$ is 5 at the same C.L. The difference comes from the relation between the effective couplings $(f_{1L/R}^\phi)$ and the Wilson coefficients $C_i$’s in Eq. (5). In principle, after we know the range of the Wilson coefficients at low energy, we can obtain the values at NP scale $A_t$ by renormalization group equations, and further determine the NP parameter space. However, in this paper, we focus on the model independent approach to search for NP effects, and will not calculate the mixing of the different operators.

Figure 4 displays the lower bound on the NP scale $A_t$ obtained from the Wtb measurements at 95% C.L. with $|C_i| = 1$. The red bars represent those limits when all the four operators contribute simultaneously, while the blue bars display those limits obtained when we consider one parameter at a time. The current bound implies $A_{\phi q} > 246$ GeV, $A_{\phi b} > 188.6$ GeV, $A_{bW} > 443.2$ GeV, $A_{tW} > 561.7$ GeV when all the four operators contribute simultaneously, while $A_{\phi q} > 1000$ GeV, $A_{\phi b} > 440.6$ GeV when we consider one parameter at a time.
3.3 Constraints on the effective Wtb couplings and operators in 2-dimensional subspaces

So far we have considered the full correlations among the four effective couplings and explored the allowed parameter space of the effective Wtb couplings. However, not all but only some of the effective couplings are non-zero in many NP models.

In those cases, we limit the parameter space to subspaces and redo the global analysis in those subspaces. Based on the NP models to be discussed, we perform the following parameter scans:

1) We scan the 1-dim parameter space by considering either $f^L_1$ or $f^R_1$ only. For example, in the $G(221)$ models and vector-like quark models, the dominant correction is in the coupling $f^L_1$ or $f^R_1$, depending on the detail of the models (see Tables 4 and 6).

2) We also consider the case where both $f^L_1$ and $f^R_1$ are modified, e.g., the vector-like quark doublet ($T;B$) model and triplet cases, shown in Table 6.
3) Another case is where both $f_L^1$ and $f_R^2$ are modified. A typical example is MSSM. Ref. [85] has shown that the anomalous $Wtb$ couplings have the following features $f_L^1 > f_R^2 > f_R^1 > f_L^2$ in the MSSM.

The results of the parameter scan are plotted in Fig. 5. In our analysis, we include the constraints from $b \to s\gamma$ [84] and update the limits of those anomalous couplings using the updated experimental data [86], see Table 3. From Table 3, we note that $b \to s\gamma$ imposes strong constraints on $f_L^1$ and $f_R^2$, which can be viewed as the results of $m_b$ suppression for the right-handed bottom quark in the pure left-handed $Wtb$ vertex, while the amplitude is enhanced by $m_t$ if the right-handed $Wtb$ vertex exists [33, 84–90]. We also note that the central value of $f_L^1$ is negative, while $f_R^1$ is zero in that case (see Fig. 5(a) and Fig. 5(b)). This is because the cross section is proportional to $(1+f_L^1)^2$, and the minimal $\chi^2$ is dominant, determined by the most precise experiment. In this case, the most precise experiment is the measurement of the $t$ channel cross section (see Table 2). However, the central value of the experiment is smaller than the approximate NNLO SM prediction (see Table 1). Therefore the best-fitted value of $f_L^1$ is negative. For the $f_L^1$ case, the cross section is proportional to $(1+(f_L^1)^2)$, thus the minimal $\chi^2$ corresponds to the SM case, and the central value of $f_L^1$ is zero. As shown in Fig. 5(a) and Fig. 5(b), $f_L^1$ and $f_R^1$ are constrained to be $-0.05 < f_L^1 < 0.025$ and $|f_R^1| < 0.13$ at 90% C.L. by direct experimental measurements of the top quark. Similarly, we translate the allowed region of the effective couplings to the coefficients $C_i$ of dimension-6 operators, with the results shown in Fig. 6.

Table 3. The current 95% C.L. bound on the structure of the $Wtb$ vertices from $B \to X_s\gamma$ with $\mu_\sigma = 160$ GeV, where $\mu_\sigma$ is the top quark and electroweak gauge boson decoupling scale [84]. The branching ratio $\mathcal{B}(B \to X_s\gamma)_{E_\gamma > 1.6}$ GeV = $(3.43 \pm 0.21 \pm 0.07) \times 10^{-4}$ is used [86].

| bound | $f_L^1$ | $f_R^1$ | $f_L^2$ | $f_R^2$ |
|-------|---------|---------|---------|---------|
| upper | 0.04    | 0.0021  | 0.0011  | 0.19    |
| lower | $-0.11$ | $-0.0008$ | $-0.0004$ | $-0.48$ |

![Figure 6](image-url)
4 Top couplings in new physics models

As mentioned in Section 1, the Wtb coupling can be modified by many kinds of NP models. In this section, we discuss the constraints from the effective Wtb couplings in several NP models.

4.1 G(221) models

The G(221) models [5, 91–102] represent a class of NP models with SU(2)_1 \times SU(2)_2 \times U(1)_X gauge symmetry. There are two breaking patterns: SU(2)_L \times SU(2)_R \times U(1)_X \rightarrow SU(2)_L \times U(1)_Y \times U(1)_X \rightarrow SU(2)_L (BP-I) or SU(2)_1 \times SU(2)_2 \times U(1)_Y \times U(1)_X \rightarrow SU(2)_L \times U(1)_Y \times U(1)_X (BP-II). In BP-I, a scalar doublet field \Phi \sim (1, 2)_{1/2} or a triplet field \Sigma \sim (1, 3), with a vacuum expectation value (VEV) v is introduced to induce the symmetry breaking of SU(2)_L \times SU(2)_R \times U(1)_X \rightarrow SU(2)_L \times U(1)_Y at the TeV scale, where the numbers in the parentheses are the quantum numbers of SU(2)_1 and SU(2)_2, respectively, and the numbers in the subscripts of the parentheses are the U(1)_X charges.

\begin{align*}
M_{W^\pm}^2 &= \frac{e^2 v^2}{4 s_W^2} \left(1 - \frac{s_\beta^2}{x}\right), \\
M_{W^\pm}^2 &= \frac{e^2 v^2}{4 s_W^2} \left(1 - \frac{s_\phi^2}{x}\right),
\end{align*}

where e denotes the electron charge and x = u^2/v^2.

We also abbreviate the trigonometric functions as c_{\phi} \equiv \cos \phi, s_{\phi} \equiv \sin \phi, s_{2\beta} \equiv \sin 2\beta, c_W \equiv \cos \theta_W and s_W \equiv \sin \theta_W where \theta_W is the SM weak mixing angle.

The third generation quarks play a special role in several G(221) models, and the Wtb couplings are modified through the mixing effects between the new gauge boson W and SM gauge boson W at tree-level. In this work, we will use the left-right model [91–93], un-unified model [94, 95] and top-flavor model [96–98, 103] as examples to discuss the impact of the Wtb measurements on those NP models. The charge assignments of the third generation quark fields under the G(221) gauge groups and the detailed expressions of \hat{f}_1^L and \hat{f}_1^R of those NP models are listed in Table 4.

Table 4. The charge assignments of the third generation quark fields under the G(221) gauge groups and the gauge couplings of the third generation quarks with W boson in several G(221) models.

| model   | SU(2)_1 | SU(2)_2 | U(1)_X | \hat{f}_1^L | \hat{f}_1^R |
|---------|---------|---------|--------|-------------|-------------|
| left-right | (t_L)  | (t_R)  | 1       | 0           | sin 2\beta  |
|          | (b_L)  | (b_R)  | \frac{6}{x} | 0          | \frac{x}{s_\phi^2} |
| un-unified | (t_L)  | (t_R)  | -1      | -1          | 0           |
|          | (b_L)  | (b_R)  | \frac{6}{x} | 0          | \frac{x}{s_\phi^2} |
| top-flavor | (t_L)  | (t_R)  | 1       | 0           | \frac{x}{s_\phi^2} |
|          | (b_L)  | (b_R)  | \frac{6}{x} | 0          | \frac{x}{s_\phi^2} |

At the electroweak scale, the symmetry is further broken by a bi-doublet scalar field \nu \sim (2, 2)_0 with two VEVs \nu_1 and \nu_2. We introduce \nu = \sqrt{\nu_1^2 + \nu_2^2} and a mixing angle tan \beta = \nu_1/\nu_2 for convenience. In BP-II, the breaking of SU(2)_1 \times SU(2)_2 \rightarrow SU(2)_L is induced by a scalar bi-doublet \Phi \sim (2, 2)_0 with one VEV v at TeV scale, and the electroweak symmetry breaking is induced by a Higgs doublet H \sim (2, 1)_{1/2} with VEV \nu. After symmetry breaking, the new gauge boson W' obtains mass and mixes with the SM gauge boson W. For simplicity, we define a new mixing angle \phi,

\begin{align*}
\tan \phi &= \frac{g_1}{g_2}, \\
\tan \phi &= \frac{g_1}{g_2},
\end{align*}

where g_1, g_2 and g_{\phi} are the gauge couplings of SU(2)_1, SU(2)_2 and U(1)_X, respectively. The gauge bosons' masses are

\begin{align*}
M_{W^\pm}^2 &= \frac{e^2 v^2}{4 s_W^2} \left(x + 1 + \frac{s_{2\beta}^2 M_W^2}{x s_W^2}\right), \\
M_{W^\pm}^2 &= \frac{e^2 v^2}{4 s_W^2} \left(x + s_{\phi}^2 + \frac{s_{2\beta}^2 M_W^2}{x s_W^2}\right).
\end{align*}

We discuss the impact of the effective Wtb coupling measurements on the parameter space of the G(221) models. Figure 7 presents the allowed parameter space at 68%, 90% and 95% C.L., respectively in the left-right model in BP-I and the un-unified and top-flavor models in BP-II. In the left-right model, we show the allowed parameter space in the (M_{W'}, s_{\phi}) plane for s_{\phi} = 0.1 and s_{\phi} = 0.9 in Fig. 7(a) and Fig. 7(b), respectively. Figure 7(a) shows that, for s_{\phi} = 0.1, a vast parameter space of moderate s_{\beta} and smaller M_{W'} is not allowed by the highly constrained right-handed Wtb coupling, see Table 4. For a larger s_{\phi} = 0.9, the constraint for M_{W'} tends to be looser, see Fig. 7(b). The reason is that the gauge coupling of the SM U(1)_Y is related to the gauge couplings g_2 and g_\phi as 1/g_2^2 = 1/g_\phi^2 + 1/g_X^2. Thus, g_Y is approximately equal to g_X in the limit s_{\phi} \rightarrow 0, which corresponds to the decoupling region. The shape of the allowed parameter space for s_{\phi} = 0.9 is the same as that of s_{\phi} = 0.1 in the small M_{W'} region.

Figures 7(c–e) display the allowed parameter space in the plane of (M_{W'}, s_{\phi}) for s_{\beta} = 0.1, 0.5, 0.9, which shows that M_{W'} is not sensitive to s_{\beta}. This is because s_{\beta} only induces mixing between the gauge boson W and W' (see Eq. (26)), and the primary source of the mass of the W' is from the first step breaking, which is proportional to the VEV u [5].
In the un-unified and top-flavor models, only the left-handed Wtb coupling is modified. The allowed parameter space in the $(M_{W'}, s_\phi)$ plane is shown in Fig. 7(f) (un-unified model) and Fig. 7(g) (top-flavor model). We note that the shape in the large $s_\phi$ region is different between the un-unified model and top-flavor model. The difference can be understood from the effective Wtb couplings shown in Table 4. The effect consists of two factors: one is $Wtb$ coupling, the other is $W-W'$ mixing. The former is different in the two models, but the latter is the same. The gauge coupling of the heavy gauge boson $W'$ with the top and bottom quarks is proportional to $\tan\phi$ in the un-unified model, while it is proportional to $\cot\phi$ in the top-flavor model. Thus, larger $s_\phi$ is allowed in the top flavor model compared with the un-unified model. Another important feature in the top-flavor model is that the constraint on $M_{W'}$ is not sensitive to the parameter $s_\phi$, as shown in Fig. 7(g). In the model, $f_1^L$ is modified by the $W'$ mass, which is proportional to $x/(s_\phi^2 x_0^2)$.

In order to validate our EFT prescription of the NP effects, $W'$ should be heavy. Figure 7 shows the constraints of $G(221)$ models from Wtb measurements are weak when new gauge bosons are heavier than 800 GeV. The constraints from low energy precision measurements and direct searches at the Tevatron and LHC have pushed the new heavy gauge bosons to several TeV [6]. Therefore, it is difficult to further constrain the parameter space by the Wtb measurements.

### 4.2 Vector-like quark models: $T$ and $B$

The vector-like quark (VLQ) is a common ingredient of many NP models. In order to keep the discussion general, we employ an effective Lagrangian approach to parametrize the effects of vector-like quarks. The quantum numbers of the new VLQ with respect to the $SU(2)_L \otimes U(1)_Y$ gauge group are summarized in Table 5 [7–10].

| model | $T$ | $B$ | $X$ | $T$ | $B$ | $Y$ | $X$ | $T$ | $B$ | $Y$ |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $SU(2)_L$ | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 3 |
| $U(1)_Y$ | 2 | 3 | 1 | 6 | 1 | 2 | 1 | 3 | 3 |

The Wtb coupling is modified by the mixing between the top quark or bottom quark with their corresponding vector-quark partner, e.g. the top partner $T$ or bottom partner $B$. After spontaneous symmetry breaking, the mass terms of the top quark and top partner $T$ in the
singlet or triplet models are [9]
\[ \mathcal{L}_{\text{mass}}^* = -\frac{y t}{\sqrt{2}} \bar{t}_L t_R - x_i \bar{t}_i t_R - M T_L T_R + \text{h.c.,} \]  
(28)
where \( y_i \) is the Yukawa coupling of top quark in the SM, \( x_i \) represents the mixing parameter between \( t \) and \( T \), and \( M \) denotes the VLQ’s mass.

In the case of doublet VLQ models, the mass terms are [9]
\[ \mathcal{L}_{\text{mass}}^* = -\frac{y t}{\sqrt{2}} \bar{t}_L t_R - x_i \bar{t}_i t_R - M T_L T_R + \text{h.c..} \]  
(29)
The weak and mass eigenstates can be related by 2 \( \times \) 2 unitary matrices,
\[ \begin{pmatrix} t_{L,R} \\ T_{L,R} \end{pmatrix} = \begin{pmatrix} c_{L,R} & s_{L,R} \\ -s_{L,R} & c_{L,R} \end{pmatrix} \begin{pmatrix} t'_{L,R} \\ T'_{L,R} \end{pmatrix}, \]  
(30)
where \( c_{L,R} \) and \( s_{L,R} \) denote the cosine and sine of the mixing angles between left-handed and right-handed top quark \( t_{L,R} \) with the top partner \( T_{L,R} \). In this section, we use the prime in the superscript on mass eigenstates to distinguish from the weak eigenstates. After diagonalizing the mass matrices, we can rewrite the mixing angles according to the parameters in Eqs. (28) and (29).

In the case of singlet and triplet VLQ models
\[ s'_L = \frac{M x_t}{\sqrt{(M^2 - m^2_t)^2 + M^2 x_t^2}}, \quad s'_R = \frac{m_t}{M} s'_L. \]  
(31)
For the doublet VLQ models
\[ s'_R = \frac{M x_t}{\sqrt{(M^2 - m^2_t)^2 + M^2 x_t^2}}, \quad s'_L = \frac{m_t}{M} s'_R. \]  
(32)

Table 6. The Wtb and Zbb couplings in 7 models, where \( c_{L,R} \) and \( s_{L,R} \) denote the cosine (sine) of the mixing angles of left-handed top quark and bottom quark with their heavy partners, respectively. Similarly, \( s'_R \) and \( s'_L \) are the sine of the mixing angles of right-handed top quark and bottom quark with their heavy partners, respectively.

| model | (T) | (B) | (X,T) | (T,B) | (B,Y) | (X,T,B) | (T,B,Y) |
|-------|-----|-----|-------|-------|-------|---------|---------|
| \( f_L^{1} \) | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| \( f_R^{1} \) | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \( f_L^{2} \) | 1 | 1 | (c_{L,R}^2) | 1 | (c_{L,R}^2) | (c_{L,R}^2) | (c_{L,R}^2) |
| \( f_R^{2} \) | 0 | 0 | 0 | 0 | (s_{R,L}^2) | (s_{R,L}^2) | (s_{R,L}^2) |

4.2.1 Mixing angles
Using the results of Section 3, we translate the allowed region of the effective Wtb couplings to the parameter space of VLQ models at 68%, 90% and 95% C.L., respectively; see Fig. 8. In the triplet and singlet models, VLQs mainly couple to the left-handed top or bottom quark, while in the doublet models they couple to the right-handed top or bottom quark. As a result, \( s_{L,R}^{t/b} \gg s_{R,L}^{t/b} \) in the triplet model, and \( s_{L,R}^{t/b} \gg s_{R,L}^{t/b} \) in the doublet model. It is, therefore, convenient to neglect the smaller mixing angles in our parameter scan. Figure 8(a) shows the allowed parameter space of \( (s_{R,L}^{t/b}, s_{L,R}^{t/b}) \) in the doublet \( (T,B) \) model. It yields the hyperbola contour region on the plane, which is determined by the right-handed Wtb coupling \( f_R^{1} = s_{R,L}^{t/b} \). Figure 8(b) displays the contour on the plane of \( (s_{R,L}^{t/b}, s_{L,R}^{t/b}) \) in the triplet models, and the allowed parameter space is symmetric about \( s_{L,R}^{t/b} \) and \( s_{L,R}^{t/b} \). Such behavior can be understood from
the deviation of the left-handed Wtb coupling between the triplet models and the SM

\[ f^L_t = \sqrt{1 - (s^L_L)^2} \sqrt{1 - (s^L_R)^2} + \sqrt{2} s^L_L s^L_R - 1. \]  

Furthermore, the upper limit of \( f^L_t \) determines the shape of the parameter contour in the top-right and bottom-left regions, and the lower limit of \( f^L_t \) determines the boundary of the parameter space in the top-left and bottom-right regions. Note that only one mixing angle exists in the singlet models \( T \) and \( B \), and in the doublet models \( (X,T) \) and \( (B,Y) \). Figure 8(e) shows the allowed parameter space of the sole mixing angle, \( s^L_L \) or \( s^L_R \), which indicates \( |s^L_L| < 0.34 \) at 95% C.L..

4.2.2 Mixing angles versus \( M_{T,B} \)

We note that the constraints from the Wtb measurements on the parameter space of the VLQ models are weak. However, the left-handed top quark and bottom quark form a \( SU(2)_L \) weak doublet, and the left-handed WtLa coupling is always related to the left-handed ZtLa and ZbLa couplings. The \( R_b \) and \( A^b_{FB} \) measurements at LEP-II [104] impose severe constraints on the ZbLa couplings, which yields a correlation between ZtLa and WtLa [40, 105]. To fully constrain the allowed parameter space of the VLQ models, it is necessary to include the Zbb and Ztt couplings in the analysis. The complete study will be presented elsewhere. In this work, we will allow a variation of \(-0.2\%\) and \(+1\%\) for the left-handed ZbLa coupling, while in the right-handed case, \(-5\%\) and \(+20\%\) is used [9].

Another important constraint comes from the Peskin-Takeuchi parameters \( S, T \) and \( U \) [106]. The contribution of an arbitrary number of vector-like singlet and doublet quarks to the \( S, T \) and \( U \) parameters have been calculated in Ref. [107], and is generalised to arbitrary couplings in Ref. [108]. In our work, we calculate the \( T \) parameter of all possible VLQ models. Our analytical results are consistent with Ref. [109]. The definition of the \( T \) parameter is

\[ \alpha T = \frac{\Pi_{WW}(0)}{\alpha_{WW}^{\text{SM}}} - \frac{\Pi_{ZZ}(0)}{\alpha_{ZZ}^{\text{SM}}}, \]

where the notation \( \Pi_{WW}(0) \) and \( \Pi_{ZZ}(0) \) denotes the vacuum polarization amplitudes of the W loop and Z loop at zero momentum, respectively, \( \alpha \) is the fine-structure constant and \( m_{WW}^{\text{SM}} \) is the mass of the W or Z boson. In the NP model, the contribution of the \( U \)-parameter is usually very small and can be neglected. Fixing \( U = 0 \), the \( T \) parameter is obtained [110]

\[ \Delta T = T - T^{\text{SM}} = 0.10 \pm 0.07, \]

where the reference top quark and Higgs boson masses are \( m_t = 173 \) GeV and \( m_H = 125 \) GeV.

We present the allowed regions of the VLQ models in Figs. 9–12. In order to better understand the impact of various bounds, we separate the constraints into different categories: the Wtb coupling constraint at 95% C.L. (green region), \( T \) parameter constraint at 95% C.L. (red region) and Zbb coupling constraint (black line). The top and bottom quark partners must be heavy in order to validate our EFT prescription of the NP effects. Currently, both the ATLAS and CMS collaborations have searched for various VLQs and imposed bounds on the heavy quark’s mass [111, 112]. The current mass limit of top partners are \( m_T > 660 \) GeV for singlet \( T \) and triplet \( X,T,B \), \( m_T > 855 \) GeV for doublet \( X,T \) and triplet \( T,B \), and \( m_T > 878 \) GeV for triplet \( T,B,Y \). Bounds on the bottom quark partner’s mass are \( m_B > 735 \) GeV for singlet \( B \), \( m_B > 450 \) GeV for doublets \( T,B \), \( (B,Y) \) and triplet \( T,B,Y \), and \( m_B > 408 \) GeV for triplet \( X,T,B \) (see the purple lines).

We plot in Figs. 9–12 the effects of the Wtb mea-
measurements on the parameter space of the VLQ models. We note the following common features when comparing different experimental constraints. There is no direct constraint on the heavy VLQ’s mass from the Wtb measurements, as $f^L_t$ and $f^R_t$ depend only on the quark mixing angles (see Table 6). For most of the parameter space of the VLQ models, the bounds from the Wtb couplings are weak. We also consider the measurements of the Zbb coupling if a bottom-quark partner is present. The Zbb coupling was measured very precisely at LEP II [104], such that it leads to a much tighter bound than the Wtb coupling measurements at the Tevatron and LHC.

In the singlet T model, the light T quark contributes largely to the $\tilde{T}$ parameter, thus a large quark mixing angle is needed to respect the $\tilde{T}$ parameter constraints [113], and the bound from the Wtb measurements plays an important role for a light T quark, say $M_T \sim 300$ GeV,

$$\frac{\Delta T_T}{T_{SM}^F} = (s^L_t)^2 \left[ -1 + (c^L_t)^2 \right] + (s^L_t)^2 \frac{M^2_{T}}{m_t^2} + (c^L_t)^2 \frac{2M^2_{W}}{M^2_{T} - m_t^2} \ln \frac{M^2_{T}}{m_t^2},$$

(40)

where $T_{SM}^F \equiv 3/(16\pi s_w^2 c_w^2) (m_t^2/m_b^2)$ denotes the contributions from the third generation SM quarks. For a heavy T quark, the $\tilde{T}$ parameter constraint dominates over the Wtb coupling. Fig. 9 also shows the allowed parameter space of the singlet B model. We note that, different from the singlet T model, a two-fold contour of the $\tilde{T}$ parameter occurs. The reason is that $\Delta T_T > 0$ in the singlet T model, and there is no constraint from the lower limit of the $\tilde{T}$ parameter. That result is obvious in the heavy mass region $M_T \gg m_t$. In the small mass region ($M_T \sim m_t$), Eq. (40) can be written as

$$\frac{\Delta T_T}{T_{SM}^F} = (s^L_t)^2 \left( \frac{M^2_{T}}{m_t^2} - 1 \right) \left( 1 + (c^L_t)^2 \right),$$

(41)

which shows the $\Delta T_T$ is positive if $M_T > m_t$. But the $\Delta T_T$ is not always positive in the singlet B, thus both the upper and lower limits of the $\tilde{T}$ parameter give a constraint on the parameter space of the singlet B [107],

$$\frac{\Delta T_B}{T_{SM}^B} = (s^L_b)^2 \left( s^L_t \right)^2 \frac{M^2_{B}}{m_t^2} - \frac{2M^2_{W}}{M^2_{B} - m_t^2} \ln \frac{M^2_{B}}{m_t^2}. \quad (42)$$

We also note that the two-fold contours of the $\tilde{T}$ parameter in the singlet B model tend to overlap each other when $M_B > 5.5$ TeV due to the decoupling of the heavy VLQ.

Figure 9 shows the allowed region of the singlet T quark model (left) and the singlet B quark model (right). In the singlet T model, the light T quark contributes largely to the $\tilde{T}$ parameter, thus a large quark mixing angle is needed to respect the $\tilde{T}$ parameter constraints [113], and the bound from the Wtb measurements plays an important role for a light T quark, say $M_T \sim 300$ GeV,

$$\frac{\Delta T_T}{T_{SM}^F} = (s^L_t)^2 \left[ -1 + (c^L_t)^2 \right] + (s^L_t)^2 \frac{M^2_{T}}{m_t^2} + (c^L_t)^2 \frac{2M^2_{W}}{M^2_{T} - m_t^2} \ln \frac{M^2_{T}}{m_t^2},$$

(40)

where $T_{SM}^F \equiv 3/(16\pi s_w^2 c_w^2) (m_t^2/m_b^2)$ denotes the contributions from the third generation SM quarks. For a heavy T quark, the $\tilde{T}$ parameter constraint dominates over the Wtb coupling. Fig. 9 also shows the allowed parameter space of the singlet B model. We note that, different from the singlet T model, a two-fold contour of the $\tilde{T}$ parameter occurs. The reason is that $\Delta T_T > 0$ in the singlet T model, and there is no constraint from the lower limit of the $\tilde{T}$ parameter. That result is obvious in the heavy mass region $M_T \gg m_t$. In the small mass region ($M_T \sim m_t$), Eq. (40) can be written as

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$$\frac{\Delta T_B}{T_{SM}^B} = (s^L_b)^2 \left( s^L_t \right)^2 \frac{M^2_{B}}{m_t^2} - \frac{2M^2_{W}}{M^2_{B} - m_t^2} \ln \frac{M^2_{B}}{m_t^2}. \quad (42)$$

We also note that the two-fold contours of the $\tilde{T}$ parameter in the singlet B model tend to overlap each other when $M_B > 5.5$ TeV due to the decoupling of the heavy VLQ.

Figure 9 shows the allowed region of the singlet T quark model (left) and the singlet B quark model (right). In the singlet T model, the light T quark contributes largely to the $\tilde{T}$ parameter, thus a large quark mixing angle is needed to respect the $\tilde{T}$ parameter constraints [113], and the bound from the Wtb measurements plays an important role for a light T quark, say $M_T \sim 300$ GeV,

$$\frac{\Delta T_T}{T_{SM}^F} = (s^L_t)^2 \left[ -1 + (c^L_t)^2 \right] + (s^L_t)^2 \frac{M^2_{T}}{m_t^2} + (c^L_t)^2 \frac{2M^2_{W}}{M^2_{T} - m_t^2} \ln \frac{M^2_{T}}{m_t^2},$$

(40)

where $T_{SM}^F \equiv 3/(16\pi s_w^2 c_w^2) (m_t^2/m_b^2)$ denotes the contributions from the third generation SM quarks. For a heavy T quark, the $\tilde{T}$ parameter constraint dominates over the Wtb coupling. Fig. 9 also shows the allowed parameter space of the singlet B model. We note that, different from the singlet T model, a two-fold contour of the $\tilde{T}$ parameter occurs. The reason is that $\Delta T_T > 0$ in the singlet T model, and there is no constraint from the lower limit of the $\tilde{T}$ parameter. That result is obvious in the heavy mass region $M_T \gg m_t$. In the small mass region ($M_T \sim m_t$), Eq. (40) can be written as

$$\frac{\Delta T_T}{T_{SM}^F} = (s^L_t)^2 \left( \frac{M^2_{T}}{m_t^2} - 1 \right) \left( 1 + (c^L_t)^2 \right),$$

(41)

which shows the $\Delta T_T$ is positive if $M_T > m_t$. But the $\Delta T_T$ is not always positive in the singlet B, thus both the upper and lower limits of the $\tilde{T}$ parameter give a constraint on the parameter space of the singlet B [107],

$$\frac{\Delta T_B}{T_{SM}^B} = (s^L_b)^2 \left( s^L_t \right)^2 \frac{M^2_{B}}{m_t^2} - \frac{2M^2_{W}}{M^2_{B} - m_t^2} \ln \frac{M^2_{B}}{m_t^2}. \quad (42)$$

We also note that the two-fold contours of the $\tilde{T}$ parameter in the singlet B model tend to overlap each other when $M_B > 5.5$ TeV due to the decoupling of the heavy VLQ.

Figure 10 shows the allowed parameter space of the doublet $(X,T)$ model (left) and the doublet $(B,Y)$ model (right), respectively. The bound on $s_b$ is dominated by the $\tilde{T}$ parameter in the heavy VLQ’s mass region. It mainly stems from the fact that the top partner or bottom partner of doublet models mainly have right-handed couplings to SM particles, and the left-handed mixing angle is highly suppressed by factor $m_{(1,0)}/M$ in the heavy mass region, see Eq. (32).

The $(X,T)$ quark doublet contributes to the $\tilde{T}$ parameter as follows:

$$\frac{\Delta T_{(X,T)}}{T_{SM}^F} = \frac{2}{m_t^2} \left[ M^2_X + M^2_T - \frac{2M^2_{M}M^2_{T}}{M^2_{X} - M^2_{T}} \ln \frac{M^2_{X}}{M^2_{T}} \right] + 2M_T M_X \left( \frac{M^2_M}{M^2_{T}} - \frac{M^2_T}{M^2_{X}} \right). \quad (43)$$
The quark mixing yields a mass splitting between the two vector-quarks in the same doublet, which breaks the SU(2) symmetry. Such a breaking effect leads to a non-zero contribution to the $\hat{T}$ parameter.

$$\hat{T}_{(X,T,B)} = \frac{\hat{T}_{(X,T,B)}}{T_{SM}} = \frac{4}{m_t^2} \left[ M_T^2 + M_B^2 - \frac{2M_T^2M_B^2}{M_T^2 - M_B^2} \ln \frac{M_T^2}{M_B^2} \right. $$

$$+ 2M_TM_B \left( \frac{M_T^2 + M_B^2}{M_T^2 - M_B^2} \ln \frac{M_T^2}{M_B^2} - 2 \right) $$

$$+ M_T^2 + M_X^2 - \frac{2M_T^2M_X^2}{M_T^2 - M_X^2} \ln \frac{M_T^2}{M_X^2} $$

$$+ 2M_TM_X \left( \frac{M_T^2 + M_X^2}{M_T^2 - M_X^2} \ln \frac{M_T^2}{M_X^2} - 2 \right), \quad (44)$$

Figure 10. (color online) Allowed parameter space at 95% C.L. of the (X,T) doublet model in the plane of $(M_T, s_{h_T}^T)$ and of the (B,Y) doublet model in the plane of $(M_B, s_{h_B}^T)$ after including indirect and direct constraints. The convention is the same as in Fig. 9.

**Fig. 10.** (color online) Allowed parameter space at 95% C.L. of the (X,T) doublet model in the plane of $(M_T, s_{h_T}^T)$ and of the (B,Y) doublet model in the plane of $(M_B, s_{h_B}^T)$ after including indirect and direct constraints. The convention is the same as in Fig. 9.

Figure 11 shows the allowed region of a doublet (T,B) model. We note that the Zbb constraint is different between the $(s_{h_R}^T, M_B)$ and $(s_{h_R}^T, M_T)$ plane. The reason is that the dangerous bound of the Zbb coupling is sensitive to the mixing angle of the bottom quark sector, whereas for the top quark sector, the constraint is indirect and comes from the allowed range of the mass parameter $M$. However, the mixing parameter $x_t$ is not fixed in the $(s_{h_R}^T, M_T)$ plane, therefore the constraint is very weak in the $(s_{h_R}^T, M_T)$ plane, see Eq. (32). The constraint from the $T$ parameter is sensitive to the parameter $x_t$, and the constraint is weaker for the smaller $x_t$. This is because $x_t$ represents the mixing between the top quark and top partner, and the smaller $x_t$ corresponds to the decoupling limit. Note that, even in the case of $s_{h_R}^T = 0$, the $T$ parameter constraint demands $M_B$ to be larger than several hundred GeV for $x_t = 100$ GeV; see the left-hand figure in Fig. 11. This arises from the non-decoupling effect of the top-quark partner.

The (X,T,B) quark triplet contributes to the $\hat{T}$ parameter as follows:

$$\hat{T}_{(X,T,B)} = \frac{4}{m_t^2} \left[ M_T^2 + M_B^2 - \frac{2M_T^2M_B^2}{M_T^2 - M_B^2} \ln \frac{M_T^2}{M_B^2} \right. $$

$$+ 2M_TM_B \left( \frac{M_T^2 + M_B^2}{M_T^2 - M_B^2} \ln \frac{M_T^2}{M_B^2} - 2 \right) $$

$$+ M_T^2 + M_X^2 - \frac{2M_T^2M_X^2}{M_T^2 - M_X^2} \ln \frac{M_T^2}{M_X^2} $$

$$+ 2M_TM_X \left( \frac{M_T^2 + M_X^2}{M_T^2 - M_X^2} \ln \frac{M_T^2}{M_X^2} - 2 \right), \quad (44)$$

**Fig. 11.** (color online) Allowed parameter space at 95% C.L. of the SM-like (T,B) doublet model in the plane of $(M_T, s_{h_T}^T)$ and (M_T, s_{h_B}^T) after including indirect and direct constraints. In the plane $(M_B, s_{h_B}^T)$, we fix the top quark sector’s mixing parameter $x_t = 100, 50$ GeV, and in the $(M_T, s_{h_T}^T)$ plane, the bottom quark sector’s mixing parameter $x_b$ is fixed to 50 GeV. The convention is the same as in Fig. 9.
The allowed regions of the triplet \((X,T,B)\) and \((T,B,Y)\) model are shown in Fig. 12. It shows that the allowed parameter space of \(s_L^t\) is smaller than that of \(s_L^b\) in the heavy mass region. The reason is that the mixing parameter of the bottom quark sector \((\sqrt{2}x_b)\) is larger than the top quark sector \((x_t)\), see Eq. (34). A similar result holds for the triplet \((T,B,Y)\) model except that \(s_L^b < s_L^t\) in the heavy mass region and the sign of the mixing parameter is opposite \((x_b \to -x_b, x_t \to \sqrt{2}x_b)\); see Eq. (35).

Fig. 12. (color online) Allowed parameter space at 95\% C.L. of the \((X,T,B)\) triplet model in the \((M_T, s_L^t)\) plane and of the \((T,B,Y)\) triplet model in the \((M_B, s_L^b)\) plane after including indirect and direct constraints. The convention is the same as in Fig. 9.

4.3 Little Higgs models

Little Higgs models have been proposed to solve the hierarchy problem [114]. The Higgs boson in the Little Higgs models is a pseudo-Goldstone boson arising from the global symmetry breaking at TeV scale and is light due to the collective symmetry breaking mechanism. The one-loop divergence to the Higgs boson mass is cancelled between top-quark and SM gauge bosons and their partners. The Littlest Higgs model was proposed in Ref. [114], based on the \(SU(5)/SO(5)\) nonlinear sigma model, with a locally gauged subgroup \(G_1 \otimes G_2 = [SU(2)_L \otimes U(1)_Y] \times [SU(2)_R \otimes U(1)_Y]\).

The global symmetry \(SU(5)\) is spontaneously broken down to the subgroup \(SO(5)\) at the scale of \(f\). At the same time, the gauge symmetry \(G_1 \otimes G_2\) is broken down to the diagonal subgroup \(SU(2)_L \otimes U(1)_Y\), which is identified as the SM electroweak symmetry.

In the Littlest Higgs model, a vector-like T quark and new heavy gauge boson \(W^\prime\) mix with SM particles and modify the Wtb vertex [12],

\[
\mathcal{L}_{\text{Wtb}} = \frac{g}{\sqrt{2}} \left[ c_L - \frac{v^2}{2f^2} c_\phi^2 (c_\phi^2 - s_\phi^2) \right] \bar{t} \gamma^\mu P_L b W_{\mu}^+ + \text{h.c.}, \tag{45}
\]

where \(c_\phi = g_1/\sqrt{g_1^2 + g_2^2}\) with \(g_1\) and \(g_2\) are the gauge couplings of the \(SU(2)_1\) and \(SU(2)_2\), respectively, and \(c_L\) is the cosine of the mixing angle of the top quark and T quark.

After considering the Wtb measurements, we plot the allowed parameter space of the Littlest Higgs model in Fig. 13, which shows the allowed parameter space at 68\% (blue region), 90\% (purple region) and 95\% (red region).
C.L., respectively for \( f = 1 \) TeV. It shows that the constraint on the mixing angle \( s_\phi \) is weak but the constraint on \( s_L \) is much tighter. This can be understood from Eq. (45), as the contribution of \( s_\phi \) to the Wtb coupling is suppressed by the scale \( f \), while \( s_L \) directly modifies the Wtb coupling.

Fig. 13. (color online) Allowed parameter space at 68%, 90% and 95% confidence levels of the Littlest Higgs model (a) and the Littlest Higgs with T-parity model (b). \( s_L \) is the sine of the mixing angle of the top quark and T quark.

The W’ mixing with the W boson at tree-level modifies the oblique \( \hat{T} \) parameter significantly. In order to respect the electroweak precision tests (EWPT), the scale \( f \) has to be above several TeV. The current lower bound of \( f \) in the Littlest Higgs model at 95% C.L. is 5.1 TeV [115]. Refs. [116–118] introduce a new discrete symmetry to forbid tree-level mixing between W and W’ and relax \( f \) down to hundreds of GeV [119]. A simple case is the Littlest Higgs with T-parity (LHT) model, in which the SM particles are even and the new heavy gauge bosons and scalars are odd under T-parity. The only T-parity even non-SM particle is the top partner T and it will contribute to the Wtb coupling through its mixing with the top quark. In the LHT model, the Wtb coupling is [11],

\[
\mathcal{L}_{\text{Wtb}} = \frac{g}{\sqrt{2}} c_L \bar{t}_R \gamma^\mu P_L b_R W^\mu + \text{h.c.} \quad (46)
\]

Figure 13(b) shows that the effective Wtb coupling measurements require \(|s_L| < 0.34\) at 95% C.L.

In order to further constrain the mixing angle, we include the Zbb coupling and oblique parameter constraints. To leading order and in the limit \( M_T \gg m_t \gg m_W \), the variation of Zbb is given by [120],

\[
\delta g_L^b = \frac{g^3}{32\pi^2} \frac{m_t^4}{m_W^2 M_T^2} R^2 \log \frac{M_T^2}{m_t^2}, \quad (47)
\]

where \( M_T \) denotes the T quark mass, and \( R = \lambda_1/\lambda_2 = s_L M_T/m_t \). The correction to the right-handed Zbb vertex is negligible in this case. The electroweak precision constraints on the LHT model have been calculated in Ref. [120], which shows that the \( \hat{T} \) parameter induced by the T quark loop is much larger than the \( \hat{S} \) and \( \hat{U} \) parameters for the same model parameters. Therefore we will only include the \( \hat{T} \) parameter bound in our analysis. The contribution from the heavy T quark is,

\[
\frac{\Delta \hat{T}_F}{T_{\text{SM}}} = s_L^2 \left[ \frac{s_L^2}{x_t} - 1 - c_L^2 - \frac{2c_L^2}{1 - x_t} \log x_t \right], \quad (48)
\]

where \( x_t = m_t^2/M_T^2 \).

Fig. 14. (color online) Allowed parameter space at 95% C.L. of the Littlest Higgs with T-parity model with \( f = 1 \) TeV after including indirect and direct constraints. The convention is the same as in Fig. 9.
After considering the Wtb, Zbb and oblique \( \hat{T} \) parameter experimental data, we plot in Fig. 14 the allowed parameter space of LHT model at 95% C.L. with \( f = 1 \) TeV in the \((s_L, M_f)\) plane.

We separate the constraints into different categories: the Wtb coupling constraint at 95% C.L. (green region), \( \hat{T} \)-parameter constraint at 95% C.L. (red region) and Zbb coupling constraint (black line).

From Fig. 14, we note that the bounds from \( \hat{T} \) parameter and Zbb coupling are almost identical and are tighter than the bound from the Wtb coupling. None of the constraints are sensitive to the heavy T quark mass.

5 Discussion and conclusions

We have studied the top quark effective couplings using a set of higher dimensional operators made out of the SM fields. The leading contributions from NP can be captured by the dimension-six operators that are related to the top quark anomalous couplings \( f_{1,2}^{L,R} \). Using the recent data of t-channel single top production \( \sigma_t, tW \) associated production \( \sigma_W \), s-channel single top production \( \sigma_s \) and W-helicity fractions \( F_0, F_L \) and \( F_R \) collected at the 7 TeV, 8 TeV and 13 TeV LHC as well as at the Tevatron, we performed a global fit to impose constraints on the anomalous couplings \( f_{1,2}^{L,R} \). The current data is sensitive to the top quark effective couplings and yields strong correlations among the top quark anomalous couplings.

We introduced \( x_0, x_p, x_m \) and \( x_5 \) (see Eq. (8)) to study the correlations among the top quark effective couplings. The variables \( x_i \)'s are sensitive to the correlations among the top quark anomalous couplings; for example, \( x_m \) probes the correlation between \( f_1^L \) and \( f_2^R \), \( x_p \) tests the correlation between \( f_1^R \) and \( f_2^L \), \( x_5 \) is sensitive to the relation between \( f_2^L \) and \( f_2^R \), and \( x_0 \) knows about all the four anomalous couplings. Precisely measuring the \( x_i \)'s can probe the correlations among the top anomalous couplings, which may shed light on new physics models. We note that

(i) Improving the measurements of \( \sigma_t \) and \( \sigma_W \) is important for constraining \( x_0 \), which can be translated into the correlation of \( (f_1^R, f_2^R) \) and \( (f_1^L, f_2^L) \); see Fig. 2(e) and Fig. 2(f).

(ii) \( x_m \) is sensitive to all the four experiments. As \( f_1^L \) and \( f_2^R \) are anti-correlated in \( x_m \), reducing the uncertainty of \( x_m \) would tighten the correlation between \( f_1^L \) and \( f_2^L \); see Fig. 2(c).

(iii) \( x_p \) is directly linked to the right-handed W-helicity fraction \( F_R \), which is inferred from \( F_L \) and \( F_0 \) measurements. The strong anti-correlation between \( f_1^R \) and \( f_2^L \) is sensitive to the \( F_0 \) and \( F_L \) measurements; see Fig. 2(d).

(iv) \( x_5 \) is sensitive to the precision of \( \sigma_t, \sigma_W \) and \( F_0 \) measurements.

At the 13 or 14 TeV LHC, the single top production cross sections increase and further affect the dependence of \( x_i \)'s on the single top production and W-helicity fraction measurements. For example, the coefficients of \( \sigma_t \) and \( \sigma_W \) decrease when the collider energy increases from 8 TeV to 13 or 14 TeV. The W-helicity fractions are measured in the top quark decay process, however, so the coefficients of \( F_0 \) and \( F_L \) remain almost the same as those at the 8 TeV LHC. Therefore, at the 13 or 14 TeV machine, the W-helicity fraction measurements play a more important role in determining \( x_i \)'s. The \( x_i \)'s can be better measured at the 13 or 14 TeV machine. More specifically, the \( F_0 \) measurement is important for the precision of the \( x_i \)'s.

After exploring the allowed parameter space of those top quark effective couplings, we discussed their impact on the following three new physics models: the \( G(221) = SU(2)_1 \otimes SU(2)_2 \otimes U(1)_X \), vector-like quark models and Littlest Higgs models with and without T-parity. These NP models modify the W-tb coupling through gauge boson mixing or quark mixing if such a mixing is not forbidden by symmetry. For example, the W in the so-called \( G(221) \) model can mix with the SM W-boson to affect the Wtb coupling; in the vector quark models new heavy quarks mix with the SM top quark or bottom quark to shift the Wtb coupling; in the Littlest Higgs model both gauge boson mixing and quark mixing are present. We translated our model-independent constraints of the top quark effective couplings into the parameter space of each new physics model.

Of the \( G(221) \) models, we considered three typical models: the left-right model, the un-unified model and the top-flavor model. The structure of the Wtb coupling highly depends on the quantum number of top and bottom quarks under the \( G(221) \) group and the symmetry breaking of \( G(221) \rightarrow SU(2)_L \otimes U(1)_Y \). We showed that the Wtb coupling is sensitive to \( \sin \phi \), which describes the ratio of gauge couplings in all \( G(221) \) models except the top-flavour model when \( M_W \) is less than several hundred GeV. In the left-right model we define \( \tan \beta \) as the ratio of two vacuum expectation values. We note that the Wtb coupling is not sensitive to \( \sin \beta \). The current experimental limits can be evaded if \( M_W \) is larger than 2 TeV.

We considered seven kinds of vector quarks in this work: (i) weak singlet model: \( (T) \) quark and \( (B) \) quark; (ii) weak doublet model: \( (X, T), (T, B) \) and \( (B, Y) \); (iii) weak triplet models: \( (X, T, B) \) and \( (T, B, Y) \) model. The structure of the Wtb coupling depends strongly on the weak quantum number of vector quarks. We showed that the Wtb coupling is sensitive to the mixing angle \( s_L^i \) or \( s_L^i \).
in all the vector quark models except the ($T,B$) doublet model. To fully constrain the allowed parameter space of vector quark models, we included the constraints of oblique $T$ parameter and Zbb couplings. We note that the $T$ parameter and Zbb couplings impose much tighter constraints on the mixing angles than the effective Wtb couplings obtained from $\sigma_{s,t,W}$ and $F_{0/L/R}$.

The Wtb coupling in the Littlest Higgs model is modified by both quark and gauge boson mixing. The Littlest Higgs model is severely constrained by W-W' mixing at tree-level. We then considered the Littlest Higgs model with T-parity, which forbids the tree-level mixing of W and W' bosons. The Wtb coupling is modified by the mixing of the top quark and top quark partner, which gives rise to constraints similar to the vector-like quark models.

The forthcoming LHC Run-II will collect more top quark pairs and single top quark events, which will improve the measurements of single top production and the W-helicity fraction. We expect tighter limits will be made on the effective top quark couplings and the dimension-six operators when new data is available. That will help us to probe new physics beyond the standard model.

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