Boundary CFT from Holography

Mohsen Alishahiha and Reza Fareghbal

School of physics, Institute for Research in Fundamental Sciences (IPM)
P.O. Box 19395-5531, Tehran, Iran

E-mails: alishah@ipm.ir, fareghbal@theory.ipm.ac.ir

Abstract

We explore some aspects of holographic dual of Boundary Conformal Field Theory (BCFT). In particular we study asymptotic symmetry of geometries which provide holographic dual of BCFTs. We also compute two-point functions of certain bosonic and fermionic operators in the dual BCFT by making use of AdS/BCFT correspondence.
1 Introduction

Extension of AdS/CFT correspondence \cite{1,2,3} to the case of boundary conformal field theories (BCFTs) has recently been addressed in \cite{4,5}. The idea was to start with the standard AdS/CFT correspondence and to seek for a modification of the bulk gravitational theory such that the dual theory becomes a CFT defined on a space with a boundary.

Actually, the main idea of the AdS/BCFT construction of \cite{4} is as follows: One may start with an asymptotically locally AdS geometry where we typically impose Dirichlet boundary on the metric as we approach the boundary. It is, however, possible to modify the geometry by imposing two different boundary conditions on the metric as one approaches the boundary. This procedure automatically implies that the boundary is divided into two parts. While in the first part, where the BCFT is supposed to live, we still impose the Dirichlet boundary condition; on the other part, the metric would satisfy the Neumann boundary condition. The interface between two parts is, indeed, the boundary of the space where the BCFT is defined.

More precisely, consider a gravitational model which admits an AdS vacuum solution. The simplest action contains Einstein-Hilbert action with a negative cosmological constant

$$S = \frac{1}{16\pi G_N} \int_M d^{d+1}x \sqrt{-g} (R - 2\Lambda) + \frac{1}{8\pi G_N} \int_{\partial M} d^d x \sqrt{-h}K + S_{\text{matter}},$$

(1.1)

where \(g\) and \(h\) are the bulk and boundary metrics, respectively. The second term is the Gibbons-Hawking boundary term \cite{6} which is given by the trace of extrinsic curvature \(K = h^{ab}K_{ab}\). \(S_{\text{matter}}\) is a possible matter field one may add on the boundary. Following the above construction, the boundary is divided into two parts \(\partial M = N \cup Q\) such that \(\partial Q = \partial N\). The metric satisfies the Neumann boundary condition on \(Q\). With this boundary condition the bulk geometry would also be modified in such a way that the gravitational theory lives only in a portion of the whole AdS space (see, for example, figure\cite{7}). The modified geometry would provide a holographic dual for BCFT.

The variation of the action with respect to the metric leads to the following boundary terms

$$\delta S = \frac{1}{16\pi G_N} \int_N d^{d+1}x \sqrt{-h}(K_{ab} - Kh_{ab})\delta h^{ab} + \frac{1}{16\pi G_N} \int_Q d^d x \sqrt{-h}(K_{ab} - Kh_{ab})\delta h^{ab}$$

$$+ \frac{1}{2} \int_Q d^d x \sqrt{-h} T_{ab} \delta h^{ab}.$$  

(1.2)

Here the last term came from the variation of the matter action defined on the boundary \(Q\). While the metric satisfies the Dirichlet boundary condition on the subboundary \(N\), we impose the Neumann boundary condition on subspace \(Q\), which leads to the following constraint on the metric \cite{4}:

$$K_{ab} - Kh_{ab} = 8\pi G_N T_{ab}.$$  

(1.3)
When we add a constant matter field, the above equation reads

\[ K_{ab} = (K - T)h_{ab}, \quad (1.4) \]

where the constant \( T \) may be interpreted as the tension of the boundary surface \( Q \).

In practice, one may start with an asymptotically locally AdS\( d+1 \) solution parameterized by \((z, y, x_1 \cdots, x_{d-1})\) and then using the boundary condition \((1.4)\), the boundary surface \( Q \) may be described by a hypersurface given by a curve \( f(z, y, \cdots, x_{d-1}) = 0 \). Although we could proceed with a complicated boundary, in what follows we will consider the simplest case where the CFT lives in a half space. More precisely we will consider the case where the BCFT lives on a \( d \) dimensional space parameterized by \((y, x_1, \cdots, x_{d-1})\) for \( y \geq 0 \). In other words \( y \) denotes the perpendicular distance from the boundary defined at \( y = 0 \).

Let us consider an AdS\( d+1 \) geometry in the Poincaré coordinates

\[ ds^2 = \frac{dz^2 + dy^2 - dx_1^2 - \cdots - dx_{d-1}^2}{z^2}. \quad (1.5) \]

Then the boundary \( Q \) is given by the following curve which is, indeed, a solution of the boundary condition \((1.4)\)

\[ y(z) = \frac{Tz}{\sqrt{(d - 1)^2 - T^2}}. \quad (1.6) \]

With this boundary the AdS geometry is divided into two parts where the gravitational theory lives in the upper half of the geometry as depicted in figure\( 1 \). This geometry provides a holographic description of a BCFT on the half space defined by \((y, x_1, \cdots, x_{d-1})\) with \( y > 0 \). This is the model we will be mostly studying in this letter.

It is the aim of this letter to further explore some aspects of AdS/BCFT correspondence. In particular we study asymptotic symmetry of geometries which provide holographic dual of BCFT. We also compute the two-point function of certain bosonic and fermionic operators in the BCFT, using AdS/BCFT correspondence. The resultant two-point functions agree with those in the literature of BCFT. Therefore this may be thought of as a nontrivial check for recently conjectured AdS/BCFT correspondence.

The letter is organized as follows: In the next section we shall study the asymptotic symmetry of geometries which provide holographic dual of BCFTs. In section three utilizing the holographic description, we compute the two-point function of a scalar operator in the dual BCFT. In section four we will redo the same computations for a fermionic operator. The last section is devoted to discussion.

\[ ^1 \text{The radius of the AdS is set to one.} \]
Figure 1: AdS geometry with a new boundary $Q$, which divides the space into two parts. While we impose the Dirichlet boundary condition on $N$, on $Q$ the metric satisfies the Neumann boundary condition. The geometry provides holographic dual of a BCFT in half space. The boundary $Q$ is given by equations (1.6).

2 Asymptotic symmetry

In this section we would like to study asymptotic symmetry of geometries which provide holographic dual of BCFTs. Following the general sprite of the AdS/CFT correspondence, one expects that the corresponding asymptotic symmetry will be the symmetry of BCFTs.

Although we can explore the asymptotic symmetry of a generic BCFT, we will address the question for the case where the dual BCFT lives in a half space. The corresponding geometry is, indeed, given in the previous section. Moreover for simplicity we will also assume that the boundary $Q$ has zero tension, i.e. $T = 0$. In this case, the boundary $Q$ is defined by the hypersurface $y = 0$ as shown in figure 2.

We note that, although the asymptotic symmetry can be worked out for any dimensions, the interesting case would be a two dimensional BCFT where one expects that the symmetry enhances to a Virasoro algebra. Therefore, in what follows we will consider the two dimensional BCFTs.

To proceed, it is useful to define coordinates $x^\pm = x_1 \pm y$ in which the metric (1.5) becomes

$$ds^2 = \frac{dz^2 - dx^+ dx^-}{z^2},$$

and the boundary is defined by $x^+ = x^-$. This would provide a holographic dual for a BCFT defined in the upper half plane.

To study the asymptotic symmetry, we perturb the above metric such that the fall off of the components of the resultant geometry satisfy the following boundary
conditions \[ \delta g_{zz} = \delta g_{z+} = \delta g_{z-} = \mathcal{O}(z), \quad \delta g_{+-} = \mathcal{O}(1), \] as one approaches the boundary at \( z = 0 \). The asymptotic Killing vectors which leave the above conditions unchanged are given by \[ \zeta^z = \frac{z}{2} \left[ \frac{dT^+(x^+)}{dx^+} + \frac{dT^-(x^-)}{dx^-} \right] + \cdots, \]
\[
\zeta^+ = T^+(x^+) + \frac{z^2}{2} \frac{d^2T^-(x^-)}{dx^-} + \cdots, \]
\[
\zeta^- = T^-(x^-) + \frac{z^2}{2} \frac{d^2T^+(x^+)}{dx^+} + \cdots, \] (2.3)
where \( T^+, T^- \) are arbitrary functions of \( x^+ \) and \( x^- \), respectively. Expanding these functions in the form of
\[
T^\pm(x^\pm) = \sum_n L_n^\pm e^{inx^\pm}, \] (2.4)
results in two Virasoro algebras generated by \( L_n^+ \) and \( L_n^- \).

Actually, since the above considerations are based on the tensorial relations, imposing the Neumann condition \[ (1.3) \] does not impose any constraint on the generators given in \[ (2.3) \]. We note, however, that in general the above asymptotic Killing vectors may change the location of the boundary. More precisely, under the action of the above asymptotic Killing vectors, in leading order, \( x^\pm \) maps into \( \tilde{x}^\pm \) as follows
\[
\tilde{x}^+ = x^+ + T^+(x^+) + \frac{z^2}{2} \frac{d^2T^-(x^-)}{dx^-2}, \quad \tilde{x}^- = x^- + T^-(x^-) + \frac{z^2}{2} \frac{d^2T^+(x^+)}{dx^+2}, \] (2.5)
It is clear from \[ (2.5) \] that the boundary defined by \( x^+ = x^- \), will not remain fixed by the generators of the asymptotic symmetry group. In order to keep it fixed, one
needs to impose the following condition on the generators of the asymptotic Killing vectors

\[ T^+(x^+)\big|_{x^+=x^-} = T^-(x^-)\big|_{x^+=x^-}. \]  

By making use of (2.4) one finds that \( L_n^+ = L_n^- \) and thus the functions \( T^\pm(x^\pm) \) are no longer independent. As a result, we find that the asymptotic symmetry of geometries which provides a holographic dual of BCFTs is effectively a copy of the Virasoro algebra. More precisely, the left and right moving modes are related, as expected.

We should mention that asymptotic symmetries of a theory of gravity in a background consisting of two patches of AdS3 spacetime glued together along an AdS2 brane has been studied in [8]. The corresponding symmetry is generated by a single Virasoro algebra. We note that this is essentially the asymptotic symmetry of a geometry which could provide holographic dual for BCFT.

It is worth mentioning that the simple relation we have found between \( L_n^+ \) and \( L_n^- \), \( (L_n^+ = L_n^-) \), is a consequence of the simple boundary we have chosen, i.e. \( y = 0 \). Had we considered a more complicated boundary, \( L_n^+ \) would be related to \( L_n^- \) in a more involved relation.

It is easy to generalize the above consideration to higher dimensions. Doing so, one finds that the asymptotic symmetry group is \( SO(1, d) \) which is the symmetry group of Euclidean d-dimensional BCFT.

## 3 Correlation functions of scalar field

Having discussed the holographic dual of BCFTs, it would be interesting to study correlation functions of different operators in the BCFT by making use of its holographic dual. Indeed, it is the aim of this section to evaluate the two-point function of a bosonic operator in an Euclidean BCFT in half space using its holographic dual. The corresponding bulk geometry, given in the previous section, is

\[ ds^2 = \frac{1}{z^2} \left( dz^2 + dy^2 + d\vec{x}^2 \right) \quad y, z \geq 0 \]  

which provides a holographic dual for the \( d \) dimensional BCFT in the half space parameterized by \( (y, x_1, \cdots, x_{d-1}) \) with \( y \geq 0 \). We recall that by construction, the above metric has two boundaries at \( z = 0 \) and \( y = 0 \); while we impose the Dirichlet boundary condition on the metric at \( z = 0 \), the metric satisfies the Neumann condition on \( y = 0 \).

Let us consider a massive free scalar field in the background (3.1)

\[ S = \frac{1}{2} \int d^{d+1}x \sqrt{g} \left( \partial_\mu \Phi \partial^\mu \Phi + m^2 \Phi^2 \right). \]  

\(^2\)We use a notation in which \( \vec{X} = (y, \vec{x}) \equiv (y, x_i) = (y, x_1, \cdots, x_{d-1}). \)
We would like to calculate the two-point function of the corresponding dual operator using the general rules of AdS/CFT correspondence. To do so, we will mostly follow the notation of [9].

The linear variation of the action \( S(3.2) \) leads to the following boundary term

\[
\delta S_{\text{bdy}} = \int_{\partial M} d^d x \sqrt{h} n^\mu \delta \Phi \partial_\mu \Phi,
\]

where \( n^\mu \) is a unit vector normal to the boundary. Since in our case the boundary is made of two parts \( \partial M = N \cup Q \) defined by \( z = 0 \) and \( y = 0 \) surfaces, respectively, one arrives at

\[
\delta S_{\text{bdy}} = \int_N d^d x \sqrt{g_{zz}} \delta \Phi \partial_z \Phi + \int_Q d^d x \sqrt{g_{yy}} \delta \Phi \partial_y \Phi.
\]

The boundary condition at \( z = 0 \) follows from the standard dictionary of AdS/CFT correspondence. Usually we impose the Dirichlet boundary condition at \( z = 0 \). Actually, in general, the asymptotic behavior of the scalar field as one approaches \( z = 0 \) may be recast to the following form

\[
\Phi(z, \vec{x}) = z^{d-\Delta} [\phi(0) + z^2 \phi(2) + \cdots + z^{2\Delta-d} (\phi(2\Delta-d) + 2\psi(2\Delta-d) \ln(z)) + \cdots],
\]

where \( \Delta \) is determined through equation \( m^2 - \Delta(\Delta - d) = 0 \).

According to the dictionary of the AdS/CFT correspondence, \( \phi_0 \) is interpreted as the source of a dual scalar operator, \( \mathcal{O} \), with scaling dimension \( \Delta \) in the dual conformal field theory. Moreover, the expectation value of the dual operator, \( \langle \mathcal{O} \rangle \), is determined by \( \phi_0(2\Delta-d) \) up to local counterterms. The corresponding two-point function is also given by

\[
\langle \mathcal{O} \mathcal{O} \rangle = -\frac{\delta \phi(2\Delta-d)}{\delta \phi(0)} \bigg|_{\phi(0)=0}.
\]

On the other hand, one should also impose a proper boundary condition on the other boundary, \( Q \), at \( y = 0 \). In this case, one may impose either Dirichlet, \( \Phi|_{y=0} = 0 \), or Neumann, \( \partial_y \Phi|_{y=0} = 0 \), boundary conditions. Therefore, in order to find the two-point function of scalar operator \( \mathcal{O} \), one needs to solve the equation of motion with the above boundary conditions.

To proceed, we start with an ansatz

\[
\Phi(\vec{x}, y, z) = z^{\frac{d}{2}} f(z) h(y) \exp(-i \vec{\omega} \cdot \vec{x}),
\]

We note that in the present case where we have two boundaries, \( Q \) and \( N \), it might be necessary to add another boundary term on \( Q \) in order to maintain conformal symmetry on the boundary \( N \) (to have a BCFT on \( N \)). Indeed a boundary term might be crucial to prevent energy flow on the boundary \( Q \). We would like to thank M. M. Sheikh Jabbari for a comment on this point.
by which the equation of motion reads
\[
\frac{1}{z^2 f(z)} \left( z^2 \frac{d^2 f(z)}{dz^2} + z \frac{df(z)}{dz} - (\nu^2 + \bar{\omega}^2 z^2) f(z) \right) + \frac{1}{h(y)} \frac{d^2 h(y)}{dy^2} = 0,
\]
(3.8)
where \( \nu = \Delta - \frac{d^2}{2} \). Therefore, one finds
\[
z^2 \frac{d^2 f(z)}{dz^2} + z \frac{df(z)}{dz} = (\nu^2 + k^2 z^2) f(z)
\]
(3.9)
and
\[
\frac{d^2 h(y)}{dy^2} = -q^2 h(y)
\]
(3.10)
where \( q \) is a constant and \( k^2 = \bar{\omega}^2 + q^2 \).

It is easy to solve these equations. In particular, for the Dirichlet or Neumann boundary conditions the solutions of the equation (3.10) are
\[
h(y) = c_0 (e^{-iqy} \pm e^{iqy}),
\]
(3.11)
where \( c_0 \) is a constant and the plus sign is for the Neumann boundary condition while the minus sign is for the Dirichlet one.

Therefore, the most general solution for \( \phi \) may be written as follows
\[
\Phi(\vec{x}, y, z) = \frac{z^{d/2}}{(2\pi)^d 2\nu \Gamma(\nu)} \int_{-\infty}^{+\infty} d^d k \, k^\nu K_\nu(kz) e^{-i\bar{\omega}z} (e^{-iqy} \pm e^{iqy}) \phi(0)(\vec{\omega}, q),
\]
(3.12)
where \( K_\nu(kz) \) is modified Bessel function and \( \phi(0)(\vec{\omega}, q) \) is the source of the dual operator. Note that in our notation \( d^d k \) stands for \( d^{d-1} \omega dq \). Moreover, in order to normalize the source we have explicitly put a factor of \( k^\nu \).

An immediate consequence of the above expression for a general solution of the equation of motion is that in order for the solution to satisfy the desired boundary conditions, the source \( \phi(0)(\vec{\omega}, q) \) should be either an even or odd function with respect to \( q \). More precisely, for Dirichlet condition \( \Phi|_{y=0} = 0 \) one finds
\[
\phi(0)(\vec{\omega}, -q) = -\phi(0)(\vec{\omega}, q)
\]
(3.13)
while for Neumann boundary condition \( \partial_y \Phi|_{y=0} = 0 \) we get
\[
\phi(0)(\vec{\omega}, -q) = \phi(0)(\vec{\omega}, q).
\]
(3.14)
Therefore we have
\[
\phi(0)(\vec{\omega}, q) = \frac{1}{2(2\pi)^d} \int d^{d-1} x' dy' e^{i\vec{\omega} \cdot \vec{x}'} (e^{iqy'} \pm e^{-iqy'}) \phi(0)(\vec{x}', y'),
\]
(3.15)
where \( \phi(0)(\vec{x}', y') \) is the Fourier transform of the source \( \phi(0)(\vec{\omega}, q) \). Plugging this expression into the equation (3.12) we arrive at
\[
\Phi(\vec{x}, y, z) = \int d^{d-1} x' dy' \phi(0)(\vec{x}', y') G(\vec{x}, y; \vec{x}', y', z)
\]
(3.16)
where $G(\vec{x}; \vec{y}', y', z)$ is the bulk-to-boundary propagator of the scalar field in the
presence of the boundary at $y$ which can be expressed in terms of the bulk-to-
boundary propagator of the scalar field when there is no boundary, $G_0(\vec{x}; \vec{y}', y', z)$,
as follows:

$$
G(\vec{x}; \vec{y}', y', z) = \frac{1}{4}
\left(G_0(\vec{x}; \vec{y}', y', z) \pm G_0(\vec{x}; -y; \vec{y}', y', z) \pm G_0(\vec{x}; \vec{y}', -y', z)
+ G_0(\vec{x}; -y; \vec{y}', -y', z)\right)
$$

(3.17)

where the plus and minus signs are for the Neumann and Dirichlet boundary condi-
tions, respectively. In our notation, the bulk-to-boundary propagator for the scalar
field on the $AdS$ space without a boundary, $G_0(\vec{x}; \vec{y}', y', z)$, is

$$
G_0(\vec{x}; \vec{y}', y', z) = \frac{z^{d/2}}{(2\pi)^d \Gamma(\nu)} \int d^d k \ k^\nu K_\nu(kz) e^{i\vec{\omega}.(\vec{x}-\vec{x}')} e^{iq(y-y')}.
$$

(3.18)

From this expression and utilizing the asymptotic behavior of modified Bessel func-
tion, one can read the two-point function by making use of the general rule given
by the equation (3.6). The resultant two-point function is

$$
\langle O(\vec{X}_1)O(\vec{X}_2) \rangle_{CFT} \sim \frac{1}{|\vec{X}_1 - \vec{X}_2|^{2\Delta}}.
$$

(3.19)

Therefore the corresponding two function of the boundary CFT is found as follows

$$
\langle O(\vec{X}_1)O(\vec{X}_2) \rangle_{BCFT} = \frac{1}{4}
\left(\langle O(\vec{X}_1)O(\vec{X}_2) \rangle_{CFT} \pm \langle O(\vec{X}_1)O(\vec{X}_2^*) \rangle_{CFT}
\pm \langle O(\vec{X}_1^*)O(\vec{X}_2) \rangle_{CFT} + \langle O(\vec{X}_1^*)O(\vec{X}_2^*) \rangle_{CFT}\right)
$$

(3.20)

that is

$$
\langle O(\vec{X}_1)O(\vec{X}_2) \rangle_{BCFT} \sim \frac{1}{4}
\left(\frac{1}{|\vec{X}_1 - \vec{X}_2|^{2\Delta} \pm \frac{1}{|\vec{X}_1 - \vec{X}_2^*|^{2\Delta}} \pm \frac{1}{|\vec{X}_1^* - \vec{X}_2|^{2\Delta}} + \frac{1}{|\vec{X}_1^* - \vec{X}_2^*|^{2\Delta}}\right)
$$

(3.21)

where $\vec{X}_{1,2}^*$ are images of $\vec{X}_{1,2}$ with respect to the boundary, i.e. if $\vec{X} = (y, \vec{x})$ then
$\vec{X}^* = (-y, \vec{x})$. Defining

$$
\zeta = \frac{|\vec{X}_1 - \vec{X}_2||\vec{X}_1^* - \vec{X}_2^*|}{|\vec{X}_1^* - \vec{X}_1||\vec{X}_2 - \vec{X}_2^*|},
$$

(3.22)

it is notable that the expression (3.20) may be written in the following form

$$
\langle O(\vec{X}_1)O(\vec{X}_2) \rangle_{BCFT} = \left[\frac{|\vec{X}_1 - \vec{X}_2||\vec{X}_1^* - \vec{X}_2^*|}{|\vec{X}_1^* - \vec{X}_1||\vec{X}_2 - \vec{X}_2^*||\vec{X}_1^* - \vec{X}_2^*|\vec{X}_1^* - \vec{X}_2^*|}\right]^\Delta F(\zeta),
$$

(3.23)

8
where
\[
F(\zeta) = (constant) \left[ (\zeta + 1)^{\Delta} \pm \zeta^{\Delta} \right]
\] (3.24)

Again the plus and minus signs correspond to the Neumann and Dirichlet boundary conditions, respectively. We note that the resultant two-point function has the expected form of the correlation function of scalar operators in BCFTs \[10\]. Therefore, one may want to conclude that reproducing the expected results for the correlation function from holographic dual could be thought of as a nontrivial check for the recently proposed AdS/BCFT correspondence \[4,5\].

4 Correlation functions of fermions

In this section, we study the two point function of a fermionic operator in the BCFT considered in the previous sections, using its gravity dual. To do so, we start with a fermionic field in the bulk whose action is
\[
S = \int_M d^{d+1}x \sqrt{g} \bar{\psi} (D - m) \psi + \int_{\partial M} d^d x \sqrt{h} \bar{\psi} \psi
\] (4.1)

As it was shown in \[11\] the present of the boundary term is crucial to get a well-defined variational principle. Actually, this boundary term is also necessary for AdS/CFT to work \[12, 13\]. In what follows, we will mostly follow the notation of \[13\].

The corresponding equations of motion are
\[
(D - m) \psi = \left( z \gamma_\mu \partial_\mu - \frac{d}{2} \gamma_z - m \right) \psi = 0,
\]
\[
\bar{\psi} (\bar{D} + m) = \bar{\psi} \left( \bar{\partial}_\mu \gamma_\mu z - \frac{d}{2} \gamma_z + m \right) = 0,
\] (4.2)

where \(\gamma_\mu\) are the Dirac matrices of \(d + 1\) dimensional Euclidean space.

To solve the equations of motion, it is useful to decompose the spinor as \(\psi = \psi^+ + \psi^-\) where \(\psi^\pm = \frac{1}{2}(1 \pm \gamma_z)\psi\). Using the equation of motion one finds \[13\]
\[
\psi^- = \int \frac{d^d k}{(2\pi)^d} e^{-i k \cdot \vec{X}} z^{(d+1)/2} K_{m+\frac{1}{2}}(k z) a^- (k),
\]
\[
\psi^+ = \int \frac{d^d k}{(2\pi)^d} e^{-i k \cdot \vec{X}} z^{(d+1)/2} K_{m-\frac{1}{2}}(k z) a^+ (k),
\] (4.3)

where \(a^\pm(k)\) are arbitrary spinors satisfying \(\gamma_z a^\pm(k) = \pm a^\pm(k)\). Moreover,
\[
a^-(k) = \frac{i \vec{k} \cdot \gamma}{k} a^+ (k).
\] (4.4)

\[4\]See \[14\] for a nice description of possible boundary terms and boundary conditions.
\[5\]To be specific we assume \(m > 0\).
To proceed, we will have to impose proper boundary conditions on the spinors at the boundaries $Q$ and $N$. For boundary $N$, we will follow the standard dictionary of AdS/CFT correspondence. More precisely, for $m > 0$, using the asymptotic behavior of the modified Bessel function near $z = 0$, one finds that $\psi^-$ diverges leading to the source term for the dual operator

$$\psi^-(z, \vec{X}) \sim z^{\frac{d}{2} - m - \frac{3}{2}} \Gamma(m + \frac{1}{2}) \int \frac{d^dk}{(2\pi)^d} e^{-i\vec{k} \cdot \vec{X}} k^{-m - \frac{1}{2}} a^-(k) + \ldots$$

(4.5)

where $\psi_0^-$ is the source of the dual fermionic operator. By making use of this notation the solutions (4.3) read

$$\psi^- = \frac{1}{2m - \frac{1}{2}} \Gamma(m + \frac{1}{2}) \int \frac{d^dk}{(2\pi)^d} e^{-i\vec{k} \cdot \vec{X}} z^{(d+1)/2} k^{m + \frac{1}{2}} K_{m + \frac{1}{2}}(kz) \psi_0^-(k),$$

$$\psi^+ = \frac{1}{2m - \frac{1}{2}} \Gamma(m + \frac{1}{2}) \int \frac{d^dk}{(2\pi)^d} e^{-i\vec{k} \cdot \vec{X}} z^{(d+1)/2} k^{m - \frac{1}{2}} K_{m - \frac{1}{2}}(kz)(-i\vec{k} \cdot \gamma) \psi_0^-(k).$$

(4.6)

So far we have considered the boundary condition at $z = 0$. Now we should also impose a proper boundary condition on the boundary $Q$ at $y = 0$. Following the boundary condition on boundary $N$, it is natural to impose the Dirichlet boundary condition on $\psi^-$ at $y = 0$. Doing so one finds

$$\psi^-(z, y, x_i) = \int \frac{d^dk}{2(2\pi)^d} e^{-i\omega x_i} \frac{(e^{-iqy} - e^{iqy})}{2m - \frac{1}{2}} \Gamma(m + \frac{1}{2}) z^{(d+1)/2} k^{m + \frac{1}{2}} K_{m + \frac{1}{2}}(kz) \psi_0^-(\omega_i, q),$$

$$\psi^+(z, y, x_i) = \frac{1}{2m - \frac{1}{2}} \Gamma(m + \frac{1}{2}) \int \frac{d^dk}{(2\pi)^d} e^{-i\omega x_i} z^{(d+1)/2} k^{m - \frac{1}{2}} K_{m - \frac{1}{2}}(kz) \psi_0^-(\omega_i, q) \times [-i\omega_i \gamma_i (e^{-iqy} - e^{iqy}) - iq\gamma_y (e^{-iqy} + e^{iqy})] \psi_0^-(\omega_i, q).$$

(4.7)

Moreover, it turns out that the source would be an odd function with respect to $q$, i.e. $\psi_0^-(\omega_i, -q) = -\psi_0^-(\omega_i, q)$. Here, $d^dk$ stands for $d^{d-1}\omega dq$.

Similarly, one can solve the equation of motion for the conjugate spinor $\bar{\psi}$. Again, it is useful to decompose the conjugate spinor as $\bar{\psi} = \bar{\psi}^+ + \bar{\psi}^-$, where $\psi^\pm = \frac{1}{2} \psi^1(1 \pm \gamma_z)$. Imposing the proper boundary condition the equation of motion can be solved leading to the following solutions:

$$\bar{\psi}^+(z, y, x_i) = \int \frac{d^dk}{2(2\pi)^d} e^{-i\omega x_i} \frac{(e^{-iqy} - e^{iqy})}{2m - \frac{1}{2}} \Gamma(m + \frac{1}{2}) z^{(d+1)/2} k^{m + \frac{1}{2}} K_{m + \frac{1}{2}}(kz) \bar{\psi}_0^+(\omega_i, q),$$

$$\bar{\psi}^-(z, y, x_i) = \frac{1}{2m - \frac{1}{2}} \Gamma(m + \frac{1}{2}) \int \frac{d^dk}{(2\pi)^d} e^{-i\omega x_i} z^{(d+1)/2} k^{m - \frac{1}{2}} K_{m - \frac{1}{2}}(kz) \bar{\psi}_0^+(\omega_i, q) \times \bar{\psi}_0^+(\omega_i, q) [i\omega_i \gamma_i (e^{-iqy} - e^{iqy}) + iq\gamma_y (e^{-iqy} + e^{iqy})].$$

(4.8)
with the condition $\bar{\psi}_0^+(\omega_i, -q) = -\bar{\psi}_0^+(\omega_i, q)$.

Now we have all the ingredients to compute the on-shell action. Indeed, plugging
the solution into the action, the bulk term vanishes while from the boundary term,
after adding a proper counterterm to subtract the divergent term, one finds

$$S_{\text{on shell}} = 2A_0 \int \frac{d^dk}{(2\pi)^d} k^{2m} \bar{\psi}_0^+(\omega_i, q) \frac{i\omega_i \gamma_i}{k} \psi_0^-(\omega_i, -q),$$

(4.9)
where $A_0$ is a numerical constant. Note that in comparison with the case where there
is no boundary, in the above expression, the term $q \gamma_y$ is absent. We note, however,
that it is possible to add this term to the equation, though the contribution of this
term vanishes due to the fact that the source is an odd function with respect to $q$.
Taking this comment into account one can Fourier transform the source to find

$$\frac{2S_{\text{on shell}}}{A_0} = \int d^{d-1}x_1 d^{d-1}x_2 dy_1 dy_2 \int \frac{d^dk}{(2\pi)^d} e^{i\omega_i(x_1-x_2)}(e^{iqy_1} - e^{-iqy_1})(e^{-iqy_2} - e^{iqy_2})$$

$$\times k^{2m} \bar{\psi}_0^+(x_1, y_1) \frac{i(\omega_i \gamma_i + q \gamma_y)}{k} \psi_0^-(x_2, y_2).$$

(4.10)
which may be written as

$$S_{\text{on shell}} = \frac{1}{4} \left[ I^{(0)}(y_1, y_2) - I^{(0)}(y_1, -y_2) - I^{(0)}(-y_1, y_2) + I^{(0)}(-y_1, -y_2) \right].$$

(4.11)
Here $I^{(0)}(y_1, y_2)$ is the on shell action of the fermion for the case where the space
has no boundary and is given by [13]:

$$I^{(0)}(y_1, y_2) = 2A_0 \int d^{d-1}x_1 d^{d-1}x_2 dy_1 dy_2 \ e^{i\omega_i(x_1-x_2) + iy_1 - y_2}$$

$$\times \bar{\psi}_0^+(x_1, y_1) \frac{i\gamma_i(x_1 - x_2) + i\gamma_y(y_1 - y_2)}{|(x_1 - x_2)^2 + (y_1 - y_2)^2|^{d+2m+1}} \psi_0^- (x_2, y_2).$$

(4.12)
From this result, we conclude that if we assume a coupling between the source and
the dual operator as

$$\int d^d x (\bar{\psi}_0^+ \chi^+ + \bar{\chi}^- \psi_0^-),$$

(4.13)
then the two-point function of the dual operator in the boundary CFT can be written
as a summation of four two-point functions of a CFT defined in the whole space
without the boundary as follows:

$$\langle \chi^+(\bar{X}_1) \bar{\chi}^- (\bar{X}_2) \rangle_{BCFT} = \frac{1}{4} \left[ (\chi^+(\bar{X}_1) \bar{\chi}^- (\bar{X}_2))_{CFT} - (\chi^+(\bar{X}_1) \bar{\chi}^- (\bar{X}_2))_{CFT} \right.$$

$$- \langle \chi^+(\bar{X}_1) \bar{\chi}^- (\bar{X}_2) \rangle_{CFT} + \langle \chi^+(\bar{X}_1) \bar{\chi}^- (\bar{X}_2) \rangle_{CFT} \right].$$

(4.14)
Using the explicit expression for the two-point function of the CFT theory

\[
\langle \chi^+ (\vec{X}_1) \bar{\chi}^- (\vec{X}_2) \rangle_{CFT} \sim \frac{\gamma \cdot (\vec{X}_1 - \vec{X}_2)}{|\vec{X}_1 - \vec{X}_2|^{d+2m+1}} \tag{4.15}
\]

and taking into account that \( \gamma_y q \) factor drops out of the expression of two-point function, one arrives at

\[
\langle \chi^+ (\vec{X}_1) \bar{\chi}^- (\vec{X}_2) \rangle_{BCFT} = 2 A_0 \gamma_i (x_1 - x_2)_i \left( \frac{1}{|\vec{X}_1 - \vec{X}_2|^{d+2m+1}} - \frac{1}{|\vec{X}_1 - \vec{X}^*_2|^{d+2m+1}} \right) - \frac{1}{|\vec{X}_1 - \vec{X}_2|^{d+2m+1}} + \frac{1}{|\vec{X}_1 - \vec{X}^*_2|^{d+2m+1}} \right), \tag{4.16}
\]

Utilizing the notation we have used in the previous section the above expression can be recast into the following form

\[
\langle \chi^+ (\vec{X}_1) \bar{\chi}^- (\vec{X}_2) \rangle_{BCFT} = \gamma_i (x_1 - x_2)_i F(\zeta)
\times \left[ \frac{|\vec{X}_1 - \vec{X}^*_1||\vec{X}_2 - \vec{X}^*_2|}{|\vec{X}_1 - \vec{X}_2||\vec{X}^*_1 - \vec{X}^*_2||\vec{X}_1 - \vec{X}^*_2||\vec{X}_1 - \vec{X}_2|} \right]^{\frac{d+1}{2} + m} \tag{4.17}
\]

where

\[
F(\zeta) = (\text{constant}) \left[ (\zeta + 1)^{\frac{d+1}{2} + m} - \zeta^{\frac{d+1}{2} + m} \right]. \tag{4.18}
\]

5 Conclusions

In this letter we have explored some aspects of AdS/BCFT correspondence, including the asymptotic symmetry of geometries which are conjectured to provide the holographic dual of BCFTs. In particular, we have demonstrated that in the two-dimensional BCFT, the corresponding asymptotic symmetry of the dual geometry is indeed two copies of the Virasoro algebra subject to a constraint relating the left and right moving generators.

Using the general dictionary of AdS/CFT correspondence, we have also computed two-point functions of certain bosonic and fermionic operators in a BCFT, by making use of its holographic dual. The resultant correlation functions are in agreement with those in the literature of BCFT. Therefore, our results may be considered as a check for the newly proposed AdS/BCFT correspondence.

It should be mentioned that in our study we have considered the simplest examples in which the BCFT lives in half space (upper half plane in two dimensions). From the bulk theory point of view, the corresponding gravitational theory lives in a portion of AdS geometry space separated from other parts by a simple hypersurface given by \( y = 0 \). This is the hypersurface where the metric satisfies the Neumann boundary condition.
We have observed that in this simple holographic model, the two-point functions of the operators have a symmetric structure reminiscent of method of image in electrostatic. More precisely, the two-point function of the BCFT in half space can be written in terms of four two-point functions of the operators and their images in a CFT which is defined in whole space without any boundary.

Generalization to more complicated boundaries is straightforward, though a little bit tedious. In particular, one may consider a BCFT on a disc or strip. One would expect that in these cases, the method of image can be also used to write the corresponding two-point function though, the procedure is more involved.

Acknowledgments

We would like to thank Ali Naseh for collaboration in the early stage of this project, as well as useful discussions. We would also like to thank Davod Allahbakhshi, Mohammad R. Mohammadi, Ali Mollabashi and M. M Sheikh Jabbari for useful discussions.

References

[1] J. M. Maldacena, “The Large N limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. 2, 231 (1998) [Int. J. Theor. Phys. 38, 1113 (1999)] arXiv:hep-th/9711200.

[2] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “Gauge theory correlators from noncritical string theory,” Phys. Lett. B 428, 105 (1998) arXiv:hep-th/9802109.

[3] E. Witten, “Anti-de Sitter space and holography,” Adv. Theor. Math. Phys. 2, 253 (1998) arXiv:hep-th/9802150.

[4] T. Takayanagi, “Holographic Dual of BCFT,” arXiv:1105.5165 [hep-th].

[5] M. Fujita, T. Takayanagi and E. Tonni, “Aspects of AdS/BCFT,” arXiv:1108.5152 [hep-th].

[6] G. W. Gibbons and S. W. Hawking, “Action Integrals and Partition Functions in Quantum Gravity,” Phys. Rev. D 15, 2752 (1977).

[7] J. D. Brown, M. Henneaux, “Central Charges in the Canonical Realization of Asymptotic Symmetries: An Example from Three-Dimensional Gravity,” Commun. Math. Phys. 104, 207-226 (1986).

[8] C. Bachas, “Asymptotic symmetries of AdS2-branes,” arXiv:hep-th/0205115.
[9] K. Skenderis, “Lecture notes on holographic renormalization,” Class. Quant. Grav. 19, 5849 (2002) [arXiv:hep-th/0209067].

[10] J. L. Cardy, “Conformal Invariance and Surface Critical Behavior,” Nucl. Phys. B 240, 514 (1984).

[11] M. Henneaux, “Boundary terms in the AdS / CFT correspondence for spinor fields,” arXiv:hep-th/9902137.

[12] M. Henningson and K. Sfetsos, “Spinors and the AdS / CFT correspondence,” Phys. Lett. B 431, 63 (1998) [arXiv:hep-th/9803251].

[13] W. Mueck and K. S. Viswanathan, “Conformal field theory correlators from classical field theory on anti-de Sitter space. 2. Vector and spinor fields,” Phys. Rev. D 58, 106006 (1998) [arXiv:hep-th/9805145].

[14] J. N. Laia and D. Tong, “A Holographic Flat Band,” arXiv:1108.1381 [hep-th].