Modeling and Analysis of Delta Kinematics FDM Printer

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Abstract. Additive manufacturing makes it possible to speed up the process of manufacturing a product using a CAD model many times over. This advantage is effectively used in the manufacture of small batches of products with complex surfaces in the automotive and aviation industries. Improvements in printers are needed to improve accuracy and productivity. Traditionally, delta kinematics are considered to have advantages over sequential linear kinematics due to their high travel speed and relatively low cost. However, delta kinematics has received limited application, mainly for personal FDM printers. The article attempts to understand the advantages and disadvantages of delta kinematics for use in 3D printers. For this, the displacements and velocities were simulated for four examples of motion trajectories. The analysis showed that the average speed of movement of the extruder is approximately equal to, and in some cases less than the linear speeds of movements along the rods. At the same time, to ensure a uniform speed of the extruder, significant accelerations are required along the individual rods. This leads to vibrations and ultimately limits the maximum speeds.

1. Introduction
In recent years, there has been a significant demand for additive technologies in industry, in particular in mechanical engineering. A key advantage of additive technologies is the ability to increase design complexity without proportionally increasing the cost of manufacturing a part, which is impossible in traditional manufacturing. Much research has been devoted to the development of additive manufacturing methods [1-5]. For example, in article [6], the features of multi-axis deposition technology are considered and studies are performed to improve the generation of the deposition trajectory. There are a number of studies devoted to the development of techniques and instruments for measuring the geometric specifications and roughness of additive manufacturing products [7-11]. For mechanical engineering products, as a rule, an additional technological operation is required to reduce the roughness. For this, abrasive processing is traditionally used [12-16].

Currently used 3D printers can be divided into three groups according to their kinematic structure. The article [19] provides a review of publications on additive manufacturing and 3D printer designs depending on the number of degrees of freedom. The first group has traditional kinematics with successive linear displacements along the axes of the Cartesian coordinate system [17, 18]. The second group uses a kinematic diagram of robots with successive rotary links. The third group of printers is...
2. Mathematical Model of Inverse Kinematics

In the general case, two problems of kinematics are solved: forward and inverse. These tasks were first solved for the delta robot. The delta kinematics of the robot was originally described in [27]. Another more modern study of kinematics is the work [28]. Then similar mathematical models were obtained for the delta 3D printer.

The mechanism (robot or 3D printer) consists of fixed and small moving platforms, connected by three pairs of passive arms (Fig. 1). Each pair of levers forms a parallelogram, the tops of which are connected by spherical joints. One of the arms of each lever moves along the rods from a separate engine. The other arm of the lever is pivotally connected to the movable platform. Structurally, the parallelogram moves in such a way that one side is always parallel to the plane of the base. Therefore, the movable platform will also always be parallel to the base. In a delta robot, three motors are located at the top and each of them rotates one of three levers. In a delta 3D printer, the motors are located at the bottom and linearly move the arms along the rods through the belt drives.

In the direct problem, the coordinates of the position of the three rods determine the Cartesian coordinates of the extruder in the fixed platform system. This task is more difficult, than inverse kinematics. The solution to this problem is necessary for modeling trajectories and calculating the working area of a 3D printer. We described such a problem in [29]. Inverse kinematics consists in determining
the z coordinate of the three rods at a known position of the extruder. This article uses inverse kinematics for modeling.

Consider the solution to the inverse kinematics problem. It is required to find the distances \( h_1, h_2, h_3 \) of points \( A_1, A_2, A_3 \) of the attachment of the levers on the rods. Cartesian coordinates \( X_c, Y_c, Z_c \) of the center of the circle \( C \) of the movable platform are given. The geometrical dimensions of the platforms (radii \( r, R \)) and the length of the rods \( l \) are known.

The distances of the attachment points of the levers on the rods are determined by the dependencies:

\[
\begin{align*}
    h_1 &= \sqrt{l^2 - (X_{A1} - X_{B1})^2 + (Y_{A1} - Y_{B1})^2 + Z_c}; \\
    h_2 &= \sqrt{l^2 - (X_{A2} - X_{B2})^2 + (Y_{A2} - Y_{B2})^2 + Z_c}; \\
    h_3 &= \sqrt{l^2 - (X_{A3} - X_{B3})^2 + (Y_{A3} - Y_{B3})^2 + Z_c},
\end{align*}
\]

where \( l \) is the length of the rods; \( X_{A1}, X_{A2}, X_{A3}, Y_{A1}, Y_{A2}, Y_{A3} \) are Cartesian coordinates of the attachment points of the levers on the rods at points \( A_1, A_2, A_3 \); \( X_{B1}, X_{B2}, X_{B3}, Y_{B1}, Y_{B2}, Y_{B3} \) are Cartesian coordinates of the levers attachment points at points \( B_1, B_2, B_3 \) on the movable platform.

It is possible to determine the Cartesian coordinates of points \( A_1, A_2, A_3, B_1, B_2, B_3 \), based on the assumption that the movable platform does not have the ability to rotate. Then the Cartesian coordinates of points \( A_1, A_2, A_3 \) of the attachment of the levers on the rods are found from the following geometric relations:

\[
\begin{align*}
    X_{A1} &= 0; & Y_{A1} &= R; \\
    X_{A2} &= R\cos(\pi/6); & Y_{A2} &= -R\sin(\pi/6); \\
    X_{A3} &= -R\cos(\pi/6); & Y_{A3} &= -R\sin(\pi/6);
\end{align*}
\]

where \( r \) is the radius of the moving platform; \( R \) is the radius of the fixed platform.

The Cartesian coordinates of the points \( B_1, B_2, B_3 \) of the attachment of the levers on the movable platform are found from the following geometric relationships:

\[
\begin{align*}
    X_{B1} &= X_c; & Y_{B1} &= Y_c + r; \\
    X_{B2} &= X_c + r\cos(\pi/6); & Y_{B2} &= Y_c - r\sin(\pi/6); \\
    X_{B3} &= X_c - r\cos(\pi/6); & Y_{B3} &= Y_c - r\sin(\pi/6).
\end{align*}
\]

Substituting the obtained expressions for \( X_{A1}, X_{A2}, X_{A3}, Y_{A1}, Y_{A2}, Y_{A3}, X_{B1}, X_{B2}, X_{B3}, Y_{B1}, Y_{B2}, Y_{B3} \) into the equations for \( h \), you can simply calculate inverse kinematics.

The considered mathematical model is implemented in the MATLAB (Fig. 2).

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**Figure 1.** Mathematical model of 3D printer.  
**Figure 2.** Simulation model of 3D printer.
3. Kinematics simulation

The basis for modeling the linear velocities of the levers and the extruder is the considered mathematical model of inverse kinematics. The movements and speeds of the extruder, fixed on the movable platform of the printer, and the arms along the three rods are investigated. Of all the variety of possible extruder trajectories, four are singled out the most characteristic. Modeling was performed for four examples (Fig. 3): a – straight line on the horizontal plane XOY, b – vertical straight line parallel to Z, c – circular arc on the XOY plane, d – an arbitrary straight line in the printer’s working area. Black represents the extruder path, and red, green, and blue represent the paths of the individual arms. Most often, when printing, the trajectories of Fig. 3 a and b, which ensure the formation of one layer over the entire area. Trajectories in Fig. 3 c and d are usually used to move the extruder to the next position to form a new layer in height.

![Extruder trajectories: a – straight line on the XOY plane, b – straight line parallel to Z, c – circular arc on the XOY plane, d – an arbitrary straight line in space.](image)

The calculated displacements of the three levers are given in Fig. 4 depending on the time $t$. It was set, that displacements along the trajectories in Fig. 3 are performed in 10 s. Used the same colors for the three levers. As can be seen from Fig. 4, the displacements have a non-linear form. In addition to nonlinearity, the movements are characterized by a sign reversal, that is, in one cycle, movements are carried out both up and down the rods. For the given examples, one of the three levers has alternating signs. An exception is option b (vertical movement of the extruder). In this case, the movements of all levers are linear.

The design speeds of the extruder and three arms are shown in Fig. 5 depending on the time $t$. The velocities are obtained by numerical differentiation of displacements in time. Same color coding used for extruder and arms. In Fig. 5 b all speeds are straight lines and are superimposed on each other.

Analysis of Fig. 5 and other simulation results showed the following. Lever speeds vary significantly over time up to 30 times. The average speed of movement of the extruder is commensurate, and in some cases less than the linear speed of movement along the arms. For only one variant (Fig. 5 b), the speeds are equal. Significant acceleration of the individual arms is required to maintain a uniform extruder speed. This leads to uneven friction conditions on the individual rods and ultimately to vibrations. It is not possible to provide uniformity of speeds and accelerations for arbitrary trajectories with delta kinematics only at the design level. Structural solutions of a 3D printer are required that provide vibration resistance at significant accelerations on one or two supports.
Figure 4. Extruder trajectories: a – straight line on the X0Y plane, b – straight line parallel to Z, c – circular arc on the X0Y plane, d – an arbitrary straight line in space.

Figure 5. Speeds of the extruder and arms for options a, b, c, d.
4. Summary
The article presents the results of modeling a delta 3D printer based on the developed mathematical model of inverse kinematics. The calculation was performed for four examples of trajectories in the form of a straight line and an arc of a circle within the working area of the printer. The movements and speeds of the extruder and the rods were calculated. The analysis showed that the average speed of movement of the extruder is comparable, and in some cases less than the linear speed of movement along the rods. Significant accelerations on individual rods are required to ensure uniform extruder speed. Given the low rigidity of the delta printer, this leads to vibrations. Therefore, despite the small masses of the moving parts of the printer, it is not possible to use the maximum speeds of the motors. Thus, in terms of performance, no advantages of delta kinematics over sequential kinematics have been identified. To realize high speeds of the extruder, constructive solutions of a 3D printer are required to increase rigidity or vibration resistance.

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