A Graphical Model of A Flywheel Energy Storage System based on Causal Ordering Graph and Energetic Macroscopic Representation Methods

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Abstract. Flywheel energy storage system is becoming more and more popular for its high efficiency, no pollution and low cost during the whole life-time. To study its characteristics and control methods, the model of a flywheel system is required to be established firstly. This paper built the graphical model of a flywheel system based on the Causal Ordering Graph (COG) and Energetic Macroscopic Representative (EMR) methods, which enhance the visualization and clarify the inside energy flow of the model. A typical control method of the flywheel is also derived and realized by the graphical methods. Simulation results based on Matlab/Simulink verify the effectiveness of the proposed model and its control method.

1. Introduction

Flywheel energy storage system is attracting more and more attention for its high efficiency, no pollution and low cost during the whole life-time[1,2]. The mathematical model of a flywheel system is very clarified nowadays, however the graphical model of a flywheel has not been related too much. Comparing to the pure mathematical model[3], the graphical model is considered to be more suitable for a complex system because it can enhance the visualization, clarify the inside energy flow and achieve the control method of the system directly[4].

This paper proposed a graphical model of a flywheel system based on the Causal Ordering Graph (COG) and Energetic Macroscopic Representation (EMR) methods, and achieved the control method of the flywheel system according to its Maximum Control Structure (MCS). Though a flywheel model may not be very complex, the work here will still be the foundation of further research on a flywheel array, which is composed of lots of flywheel systems and definitely considered as a complicated system. Simulation results based on Matlab/Simulink verify the effectiveness of the proposed model and its control method.

2. The Mathematical Model of a Flywheel System

A typical flywheel system is illustrated as Figure 1.

Figure 1. The structure of a typical flywheel energy storage system

Then the mathematical model of the flywheel system here can be described as following.
Flywheel equation:

\[ R_1 : \quad J \frac{d\Omega}{dt} = T_i \]  
\[ R_2 : \quad T_i = T_e - T_{loss} \]  
\[ R_3 : \quad T_{loss} = f \Omega \]  

Where \( J \) indicates the rotational inertia of the flywheel, \( \Omega \) indicates the mechanical speed of the flywheel, \( T_i \) indicates the total torque imposed on the flywheel, \( T_e \) indicates the electrical torque of the permanent magnet synchronous motor (PMSM), \( T_{loss} \) indicates the equivalent resistant torque and \( f \) indicates the equivalent damping ratio.

PMSM equation:

\[ R_4 : \quad \frac{dv_{sd}}{dt} = v_{sd} - R_s i_{sd} + \omega v_{sq} \]  
\[ R_5 : \quad \frac{dv_{sq}}{dt} = v_{sq} - R_s i_{sq} - \omega v_{sd} \]  
\[ R_6 : \quad \frac{dv_{rd}}{dt} = -R_s i_{rd} + \omega v_{rq} \]  
\[ R_7 : \quad \frac{dv_{r}}{dt} = -R_s i_{r} - \omega v_{rd} \]  
\[ R_8 : \quad \begin{bmatrix} i_{sd} \\ i_{rd} \end{bmatrix} = \begin{bmatrix} L_s & M \\ M & L_r \end{bmatrix}^{-1} \begin{bmatrix} v_{sd} \\ v_{rd} \end{bmatrix} \]  
\[ R_9 : \quad \begin{bmatrix} i_{sq} \\ i_{rq} \end{bmatrix} = \begin{bmatrix} L_s & M \\ M & L_r \end{bmatrix}^{-1} \begin{bmatrix} v_{sq} \\ v_{rq} \end{bmatrix} \]  
\[ R_{10} : \quad \omega_s = \omega_r - p\Omega \]  
\[ R_{11} : \quad T_e = p(v_{sd} i_{sq} - v_{sq} i_{sd}) \]  

Where \( R_s \) indicates the stator resistance, \( L_s \) indicates the stator inductance, \( v_{sd} \) indicates the d-axis stator flux, \( v_{sq} \) indicates the q-axis stator flux, \( i_{sd} \) indicates the d-axis stator current, \( i_{sq} \) indicates the q-axis stator current, \( R_r \) indicates the rotor resistance, \( L_r \) indicates the rotor inductance, \( v_{rd} \) indicates the d-axis rotor flux, \( v_{rq} \) indicates the q-axis rotor flux, \( i_{rd} \) indicates the d-axis rotor current, \( i_{rq} \) indicates the q-axis rotor current, \( M \) indicates the mutual inductance, \( \omega_s \) indicates the synchronous speed, \( \omega_r \) indicates the rotor electrical speed, \( P \) indicates the pair of poles.

Converter equation (including the MSC, GSC and the DC-Link)
\( R_{12}: \quad m_s = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} s_s \)  
\( R_{13}: \quad u_s = m_j u \)  
\( R_{14}: \quad i_{gm} = m_s^T i_s \)  
\( R_{15}: \quad v_s = \frac{1}{3} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} u_s \)  
\( R_{16}: \quad m_g = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} s_g \)  
\( R_{17}: \quad u_t = m_g u \)  
\( R_{18}: \quad i_{gm} = m_g^T i_t \)  
\( R_{19}: \quad v_t = \frac{1}{3} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} u_t \)  
\( R_{20}: \quad C \frac{du}{dt} = i_e \)  
\( R_{21}: \quad i_e = i_{re} - i_{gm} \)  
\( R_{22}: \quad L_t \frac{di_t}{dt} = v_f \)  
\( R_{23}: \quad v_f = v_t - v_{rt} - v_g \)  
\( R_{24}: \quad v_{rt} = R_i g \)  

Where \( m_s \) indicates the modulation function of motor side converter (MSC), \( u_s = [u_{s1}, u_{s2}]^T \) indicates stator line voltage, \( i_s = [i_{s1}, i_{s2}]^T \) indicates motor line current, \( v_s = [v_{s1}, v_{s2}]^T \) indicates motor phase voltage, \( i_{gm} \) indicates the dc-side current, \( m_g \) indicates the modulation function of grid side converter (GSC), \( u_t = [u_{t1}, u_{t2}]^T \) indicates grid line voltage, \( i_t = [i_{t1}, i_{t2}]^T \) indicates grid-side filter current, \( v_t = [v_{t1}, v_{t2}]^T \) indicates grid phase voltage, \( C \) indicates the capacity of DC-Link, \( i_e \) indicates the charging current of DC-Link, \( L_t \) indicates the inductance of grid-side filter, \( R_i \) indicates the resistance of grid-side filter, \( v_f \) indicates the voltage imposed on inductance, \( v_g \) indicates the grid voltage, \( v_{rt} \) indicates the voltage dip on resistance.

3. The Modelling based on COG and EMR methods
   The COG, proposed by Professor Huatier in L2EP Lab in France, is a graphical modelling method especially for electromechanical energy conversion system\(^5\). The COG is based on the natural casual relation, which is considered as an integral one, and can achieve the control method by a special kind of inversion calculation. The basic units are illustrated as Figure 2.
In Figure 2, $R_a$ is used to describe the static relation with formation like $y(t)=ax(t)+b$, and $R_b$ is used to describe the dynamic relation with formation like $\frac{d[y(t)]}{dt}=ax(t)+b$. The inversion calculation principles are illustrated in Figure 3.

![Figure 2. The basic units in COG method](image)

![Figure 3. The inversion calculation in COG method](image)

The EMR method, proposed by Professor Bouscayrol, is a graphical modelling method for the complex electromechanical conversion system which is composed of many electromechanical units\cite{6}. Like COG, only integral casual relations are considered in the method. And the basic unit in EMR can be found in Table 1.

**Table 1. Basic Units in EMR Method**

| unit      | flag | instructions                  | example               |
|-----------|------|--------------------------------|-----------------------|
| Energy    | ![Energy](image) | The source or load of the energy | Power source         |
| Storage   | ![Storage](image) | The energy storage system      | Inductance/capacitance |

\[\text{Inductance/capacitance}\]
Table 1. cont.

Conversion

Electrical conversion  Power converter

Mechanical conversion  Gear box

Electromechanical conversion  DC motor

Electrical couple  Series connection

Mechanical couple  Belt pulley

Electromechanical couple  Motor

According to the COG and EMR discussed above, the mathematical model in Section 2 can be converted into the graphical model as Figure 4 and Figure 5.
4. Simulation Results and Analysis

A 400 kW flywheel energy storage system has been built up in the Matlab/Simulink system. The simulation results are illustrated as Figure 6.

In Figure 6, it can be noticed that the active power and reactive power of the flywheel system can be controlled decoupled, which indicates the proposed modelling can control method can achieve the desired performance.
5. Conclusion
In this paper, a graphical model of a flywheel system is proposed to enhance the visualization, clarify the inside energy flow and achieve the control strategy by graphical calculation. The simulation results verify the effectiveness of the proposed model.

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