On the Topology of J-Groups

A topological J-group is a topological group which contains an element $w$ and admits a continuous self-map $f$ such that $f(x \cdot w) = f(x) \cdot x$ holds for all $x$. We determine for many important examples of topological groups if they are topological J-groups or not. Besides other results, we show that the underlying topological space of a pathwise connected topological J-group is weakly contractible which is a strong and unexpected obstruction that depends only on the homotopy type of the underlying space.

Keywords: Topological group, J-group, homotopy group, compact group, Lie group.

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