The Formation and Early Evolution of Protostellar Accretion Disks

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Abstract. Newly formed stars are often observed to possess circumstellar disks, from which mass continues to be accreted onto the star and fed into outflowing jets, and which eventually may evolve into dusty debris disks and planetary systems. Recent modeling developments have made it possible, for the first time, to study the formation and early evolution of rotationally supported protostellar disks in the context of a realistic scenario of star formation in weakly ionized, magnetic, molecular cloud cores. The derived semianalytic solutions incorporate ambipolar diffusion and magnetic braking and may be extended to include centrifugally driven disk winds; they can be used to examine the full range of expected behaviors of real systems and their dependence on physical parameters.

1. Introduction

Protostellar disks are of much interest in the study of star formation since it is likely that most of the mass assembled in a typical low-mass young stellar object (YSO) is accreted through a disk. Furthermore, such disks are the incubators of planetary systems, so their properties are also directly relevant to the process of planet formation. Circumstellar disks have been detected in $\sim 25 - 50\%$ of pre-main-sequence stars in nearby dark clouds (e.g., Beckwith & Sargent 1993). At the time when the protostars become visible as classical T Tauri stars, their disk masses (as inferred from observations of dust emission and from spectroscopic measurements) are usually $\lesssim 10\%$ of the central mass.

Rotationally supported circumstellar disks evidently originate in the collapse of self-gravitating, rotating, molecular cloud cores. Molecular line observations (e.g., Goodman et al. 1993; Kane & Clemens 1997) have established that a majority of dense ($\gtrsim 10^4 \text{ cm}^{-3}$) cloud cores show evidence of rotation, with angular velocities $\sim 3 \times 10^{-15} - 10^{-13} \text{ s}^{-1}$ that tend to be uniform on scales of $\sim 0.1 \text{ pc}$, and with specific angular momenta in the range $\sim 4 \times 10^{20} - 3 \times 10^{22} \text{ cm}^2 \text{ s}^{-1}$. The cores can transfer angular momentum to the ambient gas through torsional Alfvén waves (by the process of magnetic braking), and this mechanism also acts to align their angular momentum vectors with the local large-scale magnetic field (e.g., Mouschovias & Ciolek 1999). This alignment can occur on the dynamical timescale and hence could be achieved even in cores whose lifetimes are of that order (as in the hierarchical-ISM scenario of Elmegreen 2000). Once dynamical collapse is initiated and a core goes into a near–free-fall state, the specific angular
momentum is expected to be approximately conserved, resulting in a progressive increase in the centrifugal force that eventually halts the collapse and gives rise to a rotationally supported disk on scales $\sim 10^2$ AU. These expectations are consistent with the results of molecular-line interferometric observations of contracting cloud cores (e.g., Ohashi et al. 1997; Belloche et al. 2002).

This contribution summarizes recent advances in semianalytic modeling of the formation and early evolution of protostellar accretion disks. It has now become possible to study these processes in the context of a realistic scenario of star formation in molecular cloud cores that are threaded by a dynamically significant magnetic field. A more detailed account of this work is given in Krasnopolsky & Königl (2002; hereafter KK02).

2. Modeling Framework

Numerical simulations of magnetically supported clouds have demonstrated that the gas rapidly contracts along the field lines and maintains force equilibrium along the field even during the collapse phase (e.g., Fiedler & Mouschovias 1993; Galli & Shu 1993), including in cases where the clouds are initially elongated in the field direction (e.g., Nakamura, Hanawa, & Nakano 1995; Tomisaka 1996). This motivates treating the collapse as being quasi one-dimensional.\(^1\)

To obtain semianalytic solutions, KK02 adopted the assumption of self-similarity in space and time, with a similarity variable $x \equiv r/Ct$ (where $r$ is the distance from the origin, $C$ is the isothermal speed of sound, and $t$ is the time).\(^2\) This assumption is motivated by the fact that core collapse is a multiscale problem, which is expected to assume a self-similar form away from the outer and inner boundaries and not too close to the onset time (e.g., Shu 1977; Hunter 1977). This behavior has been verified by previous numerical and semianalytic treatments of restricted core-collapse problems – with/without rotation and with/without magnetic fields. The assumption of isothermality, which underlies the $C = \text{const}$ ansatz, is justified mainly by the fact that thermal stresses do not play a major role in the dynamics of the collapsing core.

Molecular cloud cores are known to be weakly ionized. Therefore, even though they also carried out reference calculations under the assumption of ideal MHD, KK02 incorporated ambipolar diffusion into the model: the magnetic field lines are frozen into the charged particle component (ions, electrons, grains) and couple to the dominant neutral component through ion–neutral drift. Although the drift velocity is negligible during the early phase of the dynamical collapse, ambipolar diffusion becomes important within the gravitational “sphere of influence” of the YSO once the central mass begins to grow (Ciolek & Königl 1998; Contopoulos, Ciolek, & Königl 1998). When the incoming matter enters this region, it decouples from the field and continues moving inward. The decoupling

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\(^1\) The complementary approach of axisymmetric, 2D collapse simulations in which new mass is added to the system only from above and below the disk plane tends to produce disk-to-YSO mass ratios $\sim 1$, much higher than typically observed.

\(^2\) For a typical sound speed $C = 0.19 \text{ km s}^{-1}$, $x = 1 \Leftrightarrow \{400, 4000\}$ AU at $t = \{10^4, 10^5\}$ yr.
front, in turn, moves outward and steepens into a C-type ambipolar diffusion (AD) shock (the existence of which was first predicted by Li & McKee 1996).

To incorporate ambipolar diffusion into the self-similarity formulation, it is necessary to assume that the ion density scales as the square root of the neutral density: \( \rho_i = K \rho_{\text{neutral}}^{1/2} \). As discussed in KK02, this is a good approximation for the core-collapse problem: it applies on both ends of a density range spanning \( \sim 8 \) orders of magnitude, which applies roughly on radial scales \( \sim 10^{14} \) AU, with \( K \) varying by only 1 order of magnitude across this interval.

The transition from a nearly freely falling, collapsing core to a quasi-stationary, rotationally supported disk involves a strong deceleration in a centrifugal shock. This shock is distinct from the ambipolar-diffusion shock mentioned above: it typically occurs at a different radius and is hydrodynamic, rather than hydromagnetic, in nature.

To allow mass to accumulate at the center in a 1D, rotating-core collapse, an angular momentum transport mechanism must be present. In their basic model, KK02 assumed that vertical transport through magnetic braking continues to operate also during the collapse phase of the core evolution. To incorporate this mechanism into the self-similar model, it is necessary to assume that \( V_{A, \text{ext}} \), the Alfvén speed in the external medium, is a constant. KK02 verified that, in their derived solutions, magnetic braking indeed dominates the most likely alternative angular-momentum transport mechanisms — magnetorotational instability-induced turbulence and gravitational torques. However, they also found that angular momentum transport by a centrifugally driven magnetic disk wind arises naturally (and may dominate) in their fiducial disk solutions. They went on to show that the latter mechanism may be incorporated into the model without significantly modifying the basic formulation.

3. Self-Similar Model

KK02 employed a set of vertically-integrated thin-disk equations. In their formulation (using cylindrical coordinates \( r, \phi, z \)), they took the vertical magnetic field component to be constant (except when evaluating \( \partial B_z / \partial z \)) and assumed that \( B_r \) and \( B_\phi \) increase \( \propto z \) in the disk; all terms \( \mathcal{O}(H/r) \) (where \( H \) is the disk scale height) were neglected except in the azimuthal current-density term \( [B_{r,s} - H(\partial B_z / \partial r)] \) (where the subscript \( s \) denotes a surface value). Furthermore, a monopole approximation was employed for the radial gravity \( g_r \) and for \( B_{r,s} \) (e.g., Li & Shu 1997): \( g_r = GM(r,t)/r^2 \), \( B_{r,s} = \Psi(r,t)/2\pi r^2 \), relating these quantities to, respectively, the mass and magnetic flux enclosed within \( r \).

In the self-similar formulation, the various physical quantities are expressed as dimensionless functions of the similarity variable \( x \) in the following fashion:

\[
H(r,t) = C t h(x), \quad \Sigma(r,t) = (C/2\pi G t) \sigma(x), \quad V_r(r,t) = C u(x), \quad V_\phi(r,t) = C v(x), \quad g_r(r,t) = (C/t) g(x), \quad J(r,t) = C^2 t j(x),
\]

A nearly constant value \( V_{A, \text{ext}} \approx 1 \) km s\(^{-1}\) is, in fact, indicated in molecular clouds in the density range \( \sim 10^3 - 10^7 \) cm\(^{-3}\) (e.g., Crutcher 1999).
where \( \Sigma \) is the surface mass density, \( V \) is the velocity, \( J \) is the specific angular momentum, and \( \dot{M} \) is the mass accretion rate. From the assumption of vertical hydrostatic equilibrium, one can derive a quadratic equation for \( h(x) \), whose solution is

\[
h = \frac{\hat{\sigma} x^3}{2\hat{m}_c} \left[ -1 \left( 1 + \frac{8\hat{m}_c}{x^3\hat{\sigma}^2} \right)^{1/2} \right],
\]

where \( \hat{m}_c \equiv m_c - x^3 b_{r,s}(db_z/dx)/\sigma \) (with \( m_c \) being the central-mass eigenvalue) and \( \hat{\sigma} \equiv \sigma + (b_{r,s}^2 + b_{\phi,s}^2)/\sigma \).

The adopted initial conditions were \( \sigma \to A \), \( b_z \to \sigma/\mu_0 \), \( u \to u_0 \), \( v \to v_0 \) as \( x \to \infty \), with the parameter values \( A = 3 \), \( \mu_0 = 2.9 \), \( u_0 = -1 \) based on numerical simulations of nonrotating magnetic cores and with the choice of the value of \( v_0 \) motivated by the measured angular velocities in molecular cloud cores.

The behavior in the limit \( x \to 0 \) (corresponding to \( r \to 0 \) at a fixed \( t \)) can be derived from the constituent equations. In particular, the asymptotic behavior of an ambipolar diffusion-dominated circumstellar disk is given by

\[
\begin{align*}
\dot{m} &= m = m_c, \\
j &= m_c^{1/2} x^{1/2}, \\
-u &= w = (m_c/\sigma_1)x^{1/2}, \\
\sigma &= \frac{(2\eta/3\delta)(2m_c)^{1/2}}{[1 + (2\eta/3\delta)^{-2}]^{1/2}} x^{-3/2} \\
 &= \sigma_1 x^{-3/2}, \\
b_z &= -b_{\phi,s}/\delta = [m_c^{3/4}/(2\delta)^{1/2}] x^{-5/4}, \\
b_{r,s} &= \psi/x^2 = (4/3)b_z, \\
h &= \{2/[1 + (2\eta/3\delta)^2]m_c\}^{1/2} x^{3/2}.
\end{align*}
\]

There are 4 model parameters that can be varied to explore the solution space: \( \delta \equiv |B_{\phi,s}|/B_z \) (the adopted cap on the azimuthal field strength), \( \alpha \equiv C/V_{\text{A,ext}} \), \( v_0 \equiv V_{\phi,0}/C \), and \( \eta \equiv \tau_{ni}(4\pi G\rho)^{1/2} \) (where \( \tau_{ni} \propto 1/\rho_i \) is the neutral–ion momentum-exchange time).

4. Results

4.1. Fiducial Solution

This solution corresponds to \( \eta = 1 \), \( v_0 = 0.73 \), \( \alpha = 0.08 \), and \( \delta = 1 \) (which yield \( m_c = 4.7 \)). In this case the initial rotation is not very fast and the braking is moderate, leading to the formation of a disk (with outer boundary at the centrifugal shock radius \( x_c = 1.3 \times 10^{-2} \)) within the ambipolar-diffusion region (enclosed by the AD shock radius \( x_a = 0.41 \approx 30 x_c \)). One can distinguish the following main flow regimes (see Fig. 1):
Figure 1. Behavior of normalized flow variables in the fiducial solution.

- Outer region \((x > x_a)\): ideal-MHD infall.

- AD shock: resolved as a continuous transition (but may in some cases contain a viscous subshock); KK02 estimated \(x_a \approx \sqrt{2}\eta/\mu_0\).

- Ambipolar diffusion-dominated infall \((x_c < x < x_a)\): near free-fall controlled by the central YSO’s gravity.

- Centrifugal shock: its location depends sensitively on the diffusivity parameter \(\eta\), which affects the amount of magnetic braking for \(x < x_a\); KK02 estimated \(x_c \approx (m_c v_0^2/A^2) \exp[-(2^{3/2} m_c/\mu_0)^{1/2} \eta^{-3/2}]\).

- Keplerian disk \((x < x_c)\): asymptotic behavior is approached after a transition zone representing a massive ring (of width \(\sim 0.1 x_c\) and mass \(\sim 8\%\) of the disk mass within \(x_c\), which in turn is \(\lesssim 5\%\) of \(m_c\)).
The asymptotic $x \to 0$ solution (see § 3) implies that the angle between the meridional projection of $\mathbf{B}$ and the rotation axis is radially constant and equal to $\sim 53^\circ$, which exceeds the minimum value of $30^\circ$ for launching a centrifugally driven wind from a “cold” Keplerian disk (Blandford & Payne 1982). This feature of the solution is attractive in view of the fact that centrifugally driven disk winds are a leading candidate for the origin of the bipolar outflows that are frequently observed to emanate from YSOs (e.g., Königl & Pudritz 2000).

### 4.2. Limiting Cases: Fast Rotation and Strong Braking

By modifying the model parameters, one can study the range of possible behaviors in collapsing cores. Figure 2 shows two limiting cases, which bracket the fiducial solution.

![Figure 2](image)

**Figure 2.** Fast-rotation (left) and strong-braking (right) solutions.

The fast rotation case differs from the fiducial solution primarily in having a large initial-rotation parameter ($v_0 = 1.5$). It has the following distinguishing features:

- The centrifugal shock is located within the self-gravity–dominated (and ideal-MHD) region; a back-flowing region is present just behind the shock.
- The central mass is comparatively small ($m_c = 0.5$), giving rise to a non-Keplerian outer disk region.
- The ideal-MHD/ambipolar-diffusion transition occurs behind the centrifugal shock and is gradual rather than sharp.

The strong braking case ($\eta = 0.5$, $v_0 = 1$, $\alpha = 10$, and $\delta = 10$; yielding $m_c = 5.9$) is characterized by large values of the braking parameters $\alpha$ and $\delta$. It is distinguished by having

- no centrifugal shock (or circumstellar disk); the $x \to 0$ behavior resembles that of the nonrotating collapse solution of Contopoulos et al. (1998).
5. Some Implications of the Model

The formation of rotationally supported circumstellar accretion disks basically resolves the angular momentum problem in star formation (although the exact value of the YSO angular momentum is determined by processes near the stellar surface that are not included in this model). In particular, the derived solutions demonstrate that angular momentum transport can be sufficiently efficient to allow most of the inflowing mass to end up (with effectively no angular momentum) at the center, with the central mass dominating the dynamics well beyond the outer edge of the disk even as the inflow is still in progress. These solutions reveal that the ambipolar-diffusion shock, even though it is usually located well outside the region where the centrifugal force becomes important, helps to enhance the efficiency of angular momentum transport through the magnetic field amplification that it induces. The revitalization of ambipolar diffusion behind the AD shock in turn goes a long way toward resolving the magnetic flux problem in star formation (as already pointed out by Ciolek & Königl 1998 and Contopoulos et al. 1998).

To the extent that self-similarity is a good approximation to the situation in collapsing cloud cores, it is conceivable that T Tauri (Class II) protostellar systems, whose disk masses are typically inferred to be \( \lesssim 10\% \) of the central mass, have had a similarly low disk-to-star mass ratio also during their earlier (Class-0 and Class-I) evolutionary phases. It would be interesting to test this possibility by observations. The model also predicts that, in cases where magnetic braking is particularly strong, essentially all the angular momentum is removed well before the inflowing gas reaches the center. Such systems may correspond to slowly rotating YSOs that show no evidence of a circumstellar disk (e.g., Stassun et al. 1999; 2001). Another distinguishing characteristic of the solutions is the appearance of an ambipolar-diffusion and/or a centrifugal shock. The implied processing of the disk material in these shocks (particularly the latter one) may have implications to the composition of protoplanetary disks (e.g., the annealing of silicate dust; see Harker & Desch 2002).

The diffusive Keplerian disk models are by and large magnetorotationally stable, basically because the matter/field coupling is generally too weak to allow the instability to grow. (The well-coupled surface layers should typically also be stable because of strong magnetic squeezing; e.g., Wardle & Königl 1993.) Furthermore, the Toomre stability criterion to fragmentation \( (Q_{\text{Toomre}} > 1) \) is well satisfied for the rotationally supported disk solutions (except in the outer layers of fast-rotation models). Angular momentum transport by gravitational torques is unlikely to be important under these conditions (e.g., Lin & Pringle 1987). However, as noted in § 2 (see also § 4.1), angular momentum transport by a centrifugally driven wind may play a key role. KK02 found that the steady-state, radially self-similar disk-wind solution of Blandford & Payne (1982) can be naturally incorporated into the asymptotic ambipolar-diffusion disk solution given in § 3, making it possible to study the effects of wind angular-momentum and mass removal from the disk and to better constrain the relevant parameters of a combined disk/wind model. In a preliminary analysis, they inferred that the asymptotic solution evidently corresponds to the weakly coupled disk/wind configurations discussed by Li (1996).
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