INVESTIGATE THE DEPENDENCE OF THE LIGHT REFRACTIVE INDEX OF AN IDEAL GAS ON ITS PRESSURE USING INTERFEROMETERS

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Abstract
This article analyzes the working principle of the Michelson interferometer and the ability to measure some physical quantities. Using the Michelson Interferometer, the ability to detect not only the light wave but also the full wavelength of radio waves was analyzed. Using the Mach-Zehnder and Michelson interferometers, it was determined that the refractive index of air depends on its pressure. The results obtained in two different interferometers were compared comparatively in the graphs.

Introduction:
Today, not only in science, but also in technology, various methods of measuring the size of objects require high accuracy. Therefore, today interferometers are widely used in scientific research and production in the detection of very small displacements, refractive index of the environment and uneven surfaces [1].

It is known that the refractive index of a vacuum is 1, while the refractive index of air is very little different from it. However, by varying the pressure in the chamber, it was investigated that such small differences could also be detected by changing the interference pattern on the Mach-Zehnder and Michelson interferometers.

In this paper, a comparative analysis of the results obtained on two different interferometers was performed, experimentally determining that the refractive index of air depends on its pressure using Mach-Zehnder and Michelson interferometers. Using the Michelson interferometer, the ability to measure not only the wavelength of optical waves, but also the wavelength of high-frequency electromagnetic waves was investigated.

Basic equations of light interference
Interference of light - the addition in space of two or more coherent waves, in which at different points there is an increase or decrease in the amplitude of the resulting wave.

The equation of coherent waves joining at a point \( M \) in space is as follows:

\[
x_1 = A_1 \cos \left( t - \frac{s_1}{v_1} \right) \quad x_2 = A_2 \cos \left( t - \frac{s_2}{v_2} \right)
\]

resulting oscillation amplitude is

\[
A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos \delta
\]
the phase difference \( \delta \) of oscillations excited at point \( M \) is equal to
\[
\delta = \omega \left( \frac{s_2 - s_1}{v_2 - v_1} \right) = \omega \left( \frac{s_2}{c/n_2} - \frac{s_1}{c/n_1} \right) \\
= \frac{\omega}{c} \left( s_2 n_2 - s_1 n_1 \right) = \frac{2\pi\nu}{c} \left( L_2 - L_1 \right) = \frac{2\pi}{\lambda_0} \Delta
\]
(3)

Where
\[
\nu = c / n, \quad \omega = 2\pi\nu, \quad \lambda_0 = c / \nu
\]
(4)

the multiplication of the geometric path length \( s \) of the light wave in a given medium by the refractive index of this medium \( n \) is called the optical path length [2].
\[
L = s \cdot n
\]
(5)

if the optical path difference \( \Delta \) is equal to an integer number of wavelengths in vacuum (an even number of half-waves)
\[
\Delta = \pm m\lambda_0 = \pm 2m\frac{\lambda_0}{2} \quad (m = 0, 1, 2, \ldots)
\]
(6)

Then \( \delta = \pm 2m\pi \)

and oscillations excited at the point \( M \) will occur in the same phase, and the resulting amplitude of the attached waves increases. The (6) expression is called the maximum condition.

if the optical path difference \( \Delta \) is equal to an odd number of half-waves
\[
\Delta = \pm (2m + 1)\frac{\lambda_0}{2} \quad (m = 0, 1, 2, \ldots)
\]
(7)

Then
\[
\delta = \pm (2m + 1)\pi
\]

and oscillations excited at the point \( M \) will occur in antiphase and the resulting amplitude of the attached waves decreases. The (7) expression is called the minimum condition.

Using the Mach-Zehnder interferometer to determine the light refractive index of air as a function of its pressure.
The Mach-Zehnder interferometer operates on the following principle (in Fig. 1). The coherent light beam supplied by a suitable source (1) is split into two by an optical component (2). These partial beams move along different paths, deflected by mirrors (4-4') and directed to another optical component, where they are combined and superimposed (2'). The result is an interference pattern. If the path length of one of these partial rays, the refractive index of the medium and the geometric path, changes, this produces a phase shift relative to the unperturbed ray. This in turn causes a change in the interference pattern on the screen (5), which allows us to draw conclusions about changes in the optical path.

Unlike a Michelson interferometer, light rays do not reflect from each other after separation, but rather travel along separate paths until they are re-converged. The result of measurements on transparent materials, for example, a measurement of the refractive index, is easier to understand and therefore can be investigated. However, there is no way to define the length of the geometric path. To determine the refractive index of air, the evacuated chamber (5) is
placed in the path of one of the partial beams of the interferometer. The optical path length of this partial beam changes during the experiment by pumping air out of this chamber. We can then determine the refractive index of the air based on the change in the interference pattern and the corresponding change can be made using a Michelson interferometer; however, then we would have to take into account that the beam passes through the camera twice. It follows from Maxwell's equations that the speed of electromagnetic waves is:

\[ v = \frac{1}{\sqrt{\varepsilon \mu}} \cdot \frac{1}{\sqrt{\varepsilon \mu}} = \frac{c}{n} \]  

(8)

Where \( c = 1/\sqrt{\varepsilon \mu} = 2,9979 \cdot 10^8 \text{ m/s} \) - speed of light in vacuum, \( n = 1/\sqrt{\varepsilon \mu} \) - refractive index of medium. \( \varepsilon = 8,85 \cdot 10^{-12} \Phi / \text{m} \) - electrical constant, \( \mu = 4\pi \cdot 10^{-7} \text{Gn/m} \) - magnetic constant, \( \varepsilon \) and \( \mu \) - dielectric and magnetic permeability of the medium, respectively.

For air \( \mu = 1 \), then the refractive index \( n = \sqrt{\varepsilon} \). In accordance with the classical theory of dispersion, a molecule of a medium can be regarded as a system that includes electrons in an equilibrium position. Under the action of an external periodic field of the wave, electrons are displaced from the equilibrium position, while the atom acquires an electric moment. The electrical displacement (electrical induction) of the medium is determined by the ratio:

\[ D = \varepsilon_0 E + P \]  

(9)

Where \( P = \alpha \varepsilon_0 NE \) - electric moment acquired by a unit volume of a medium under the action of an external field \( E \); \( \alpha \) - polarizability coefficient, characterizes the structure of the molecule; \( N \) - number of molecules per unit volume.

The electrical displacement \( D \) and the field strength \( E \) are related:

\[ D = \varepsilon \varepsilon_0 E \]  

(10)

Then (2) can be written as

\[ \varepsilon \varepsilon_0 E = \varepsilon_0 E + \alpha \varepsilon_0 NE \]  

(11)

Wherefrom \( \varepsilon = 1 + \alpha N \) and the refractive index:

\[ n = \sqrt{\varepsilon} = \sqrt{1 + \alpha N} \approx 1 + \frac{\alpha N}{2} \]  

(12)

From molecular kinetic theory, pressure \( p \) is related to temperature \( T \):

\[ p = N k T \]  

(13)

Where \( k \) - Boltzmann's constant. Then the refractive index for ideal gases is:

\[ n = 1 + \frac{\alpha p}{2 k T} \]  

(14)

Now by determining what the coefficient \( \alpha \) is equal to for the Michelson interferometer, we can determine the expression of the refractive index of air as a function of its pressure. A constant pressure at the formula (7) will take as:
\[ n(p) = 1 + A \cdot p \]  

(15)

where we used the following definition \( A = \alpha / 2kT \).

If an air chamber is not placed in the laser light path, then there will be a difference in the optical path

\[ \Delta = L_2 - L_1 = 0. \]

If an air chamber 3 (Fig.1) is placed in the path of the laser light and a certain amount of air is sucked out of it, the refractive index of the air also changes due to the pressure change then the difference between the optical paths of the two rays

\[ \Delta = L_1 + d \cdot (n_1 - n_2) = d \cdot \Delta n = \Delta p \cdot A \cdot d \]  

(16)

Where \( \Delta p = p_2 - p_1 \); \( n_1 \) and \( n_2 \) refractive indices at pressures \( p_1 \) and \( p_2 \), respectively.

In this case, the interference pattern will shift by \( \Delta m \) lines, and the stroke difference becomes:

\[ \Delta = \Delta m \cdot \lambda = A \cdot \Delta p \cdot d \]  

(17)

The relationship between the number of lines and pressure:

\[ \Delta m = \frac{A \cdot d}{\lambda} \Delta p \]  

(18)

Where \( \tan \phi = A \cdot d / \lambda = \Delta m / \Delta p \) the tangent of the slope of the dependence of the number of new interference fringes on pressure [3]. Having determined the tangent from the graph of this dependence, we can find the constant \( A \):

\[ A = \frac{\lambda}{d} \frac{\Delta m}{\Delta p} \]  

(19)

Using the expression for \( A \), it can be found that the refractive index of air depends on its pressure [15-19]:

\[ n(p) = 1 + A \cdot p = 1 + \frac{\lambda}{d} \frac{\Delta m}{\Delta p} \cdot p \]  

(20)

Using formula (20), it is possible to determine experimentally that the refractive index of air depends on its pressure. It is known that the refractive index of a vacuum is equal to 1 and the refractive index of air also differs very little from 1. In Fig.2 shows the pressure dependence of the light refractive index of the air on a Mach-Zehnder interferometer Fig.2

![The dependence of air pressure on its light refractive index on a Mach-Zehnder interferometer](image)

Measurement the wavelength of electromagnetic waves using a Michelson interferometer
An interferometer is a measuring device whose operation is based on the phenomenon of light interference. There are many types of interferometers, in this section we use a Michelson interferometer. The schematic diagram of the Michelson interferometer is shown in Fig. 3. The laser beam, passing through the diffusing lens 5, is divided into two using a translucent mirror 3. After passing a certain path, one ray is reflected from the unmovable mirror 1, the second from the mirror 2, the position of which can be changed. The beams return along their trajectories to the translucent mirror. From the mirror, the rays go to screen 4, where, as a result of their addition, an interference pattern is observed - alternating light and dark rings [4]. Looking at the change in the interference pattern, can be measure some physical quantities.

In this paper, we will study how to determine the wavelength of an electromagnetic wave and the refractive index of air depends on its pressure, using a Michelson interferometer.

Measuring the wavelength of an electromagnetic wave using a Michelson interferometer

Interferometers are devices that are used to accurately measure the length of electromagnetic waves not only in the visible, but also in other ranges. The principle of operation of the interferometer is as follows: a beam of electromagnetic radiation (light, radio waves, etc.) is spatially divided into two or more coherent beams using one or another device. Each of them passes different optical paths and is directed to the screen, creating an interference pattern, according to which it is possible to establish the phase difference of the interfering beams at a given point in the pattern [4].

The Michelson interferometer for measurement wavelength electromagnetic wave is shown in Fig. 4. Electromagnetic wave propagates from source 1. Part of the incident flow is reflected from the semitransparent plate 2 in the direction of the mirror 3, the other part passes through the plate 2 and propagates in the direction of the mirror 4. Having reflected from the mirrors, the rays again reach plate 2 and pass in the direction of the receiver 5. Subject to the conditions of spatial and temporal coherence, these rays will interfere. The result of the interference depends on the optical path difference $\Delta$ from plate 2 to the mirrors. The amplification or weakening of the vibration amplitude, recorded by the receiver 5, occurs when the position of one of the mirrors, for example, mirror 4, changes.

The condition for maxima (or minima) has the form:

$$\Delta = n\Delta r = m \cdot \lambda \ (21)$$

Where $n$ - refractive index of medium, $\Delta r$ - geometric difference of path, $\lambda$ - wavelength, $m$ - order of interference maxima.

Then, for two neighboring maxima (minima) of $m$ and $m+1$ orders, we can write the equality

$$n\Delta r_1 = m \cdot \lambda \text{ and } n\Delta r_2 = (m + 1) \cdot \lambda$$

Hence we get

$$\lambda = n(\Delta r_2 - \Delta r_1) = 2n(x_2 - x_1) \ (22)$$

Where $x_2 - x_1$ - the minimum distance by which it is necessary to move the movable mirror 4 in order to obtain amplification (attenuation) of oscillations again.

Determination the dependence of the refractive index of air
from its pressure using a Michelson interferometer

We place on the path of one of the rays, for example, going to the movable mirror, a chamber of length $d$ (Fig. 5), filled with air at a pressure $p_1$, and then pump air into it to a pressure $p_2$.

Let us represent the optical path length of this ray at the pressure $p_1$ in the chamber as:

$$S_1 = L_1 + 2d \cdot n_1$$

Where $L_1$ - the entire optical path of the beam outside the camera, $2d \cdot n_1$ - optical path in the chamber [5].

At pressure $p_2$ ($L_1$ does not change):

$$S_1' = L_1 + 2d \cdot n_2.$$

Thus, when the pressure changes from $p_1$ to $p_2$, an additional path difference is introduced into the path difference of the interfering rays $S_1' - S_1$:

$$\Delta = S_1' - S_1 = 2d(n_2 - n_1) = 2A \cdot \Delta p \cdot d \quad (23)$$

Where $\Delta p = p_2 - p_1$; $n_1$ and $n_2$ refractive indices at pressures $p_1$ and $p_2$, respectively.

In this case, the interference pattern will shift by $\Delta m$ lines, and the stroke difference becomes:

$$\Delta = \Delta m \cdot \lambda = 2A \cdot \Delta p \cdot d \quad (24)$$

The relationship between the number of lines and pressure:

$$\Delta m = \frac{2A \cdot d}{\lambda} \Delta p \quad (25)$$

Where $\tan \varphi = 2A \cdot d / \lambda = \Delta m / \Delta p$ the tangent of the slope of the dependence of the number of new interference fringes on pressure. Having determined the tangent from the graph of this dependence, we can find the constant $A$:

$$A = \frac{\lambda}{2d} \frac{\Delta m}{\Delta p} \quad (26)$$

Using the expression for $A$, it can be found that the refractive index of air depends on its pressure [15]:

$$n(p) = 1 + Ap = 1 + \frac{\lambda}{2d} \frac{\Delta m}{\Delta p} = 1 + \frac{\Delta m}{\Delta p} \quad (27)$$

Using the formula (27) it is possible to determine the dependence of the refractive index of air on its pressure using the Michelson interferometer. In Fig. 5 shows the pressure dependence of the light refractive index of the air on a Michelson interferometer.

Since the light refractive index of air differed very little from that of vacuum, it was obtained for suddenly different values of refractive index to compare the results obtained on two different interferometers on a graph. (Fig 6).
Conclusion:
In the this article, the Michelson interferometer explored a methodology for measuring not only the light wavelength but also the wavelength of an electromagnetic wave. The next section of the article analyzes the formula for the refractive index of an ideal gas depending on its pressure. It is known that the refractive index of vacuum is equal to 1. The refractive index of air very little differs from it. However, by varying the pressure in the chamber, it was verified that even such small differences can be detected by shifting the interference pattern on the Mach-Zehnder and Michelson interferometer. The results obtained in two different interferometer were compared using a graph.

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