The Identification Method of Transformer Excitation Inrush Current Based on the Decay Characteristics of Aperiodic Component

Jianqin Feng¹, Pei Zhang¹*, Zhifei Chen²

¹School of Electrical and Information Engineering, Zhengzhou University of Light Industry, Dongfeng Rd 5, Zhengzhou, China
²School of Management and Engineering, Zhengzhou University, Science Avenue 100, Zhengzhou, China

Email: zhangpei_zzuli@163.com

Abstract. Focusing on the optical current transformer secondary side current aperiodic component decay characteristics, a novel identification method of excitation inrush current based on aperiodic component dynamic decay characteristics was proposed, and the traditional transformer excitation inrush current identification method based on the various static characteristics were break through. First, considering the transformer losses were exist and the truth of characteristics of excitation inrush current was not a periodic function, the aperiodic component was calculated by taking different cycles of the excitation inrush current derivation. Second, taking the (k+1)th cycle of excitation inrush current integration as an example, the five different forms excitation inrush current integration were found, and the five forms aperiodic component function expression were written. Then, taking derivation of five-stage aperiodic component and the aperiodic component decay speed expression was got. Compared with the normal operation and short circuit faults occurred, using the aperiodic component decay dynamic time constant to identity the excitation inrush current. Combined with the related simulation, not only verified this novel identification method was effective, but also provides a new way for the application of dynamic characteristics in excitation inrush current identification and relay protection.

1. Introduction

The inrush current produced when transformer no-load switching, its value could up to the 4~8 times of rated current. The inrush current could cause the winding deformation of the transformer, harm the insulation of the transformer, reduce the reliability of the power supply and cause the unwanted operation of transformer differential protection[1], which has been widely concerned for a long time.

At present, many recognition methods of transformer inrush current have been proposed [1-7]. Reference [2] proposed a method of multi feature about identifying inrush current in the feature space based on the summary characteristics of each feature value of differential current. Reference[1] obtained the imaginary part of the fundamental wave according to the full wave Fourier algorithm and half wave Fourier algorithm, recognized inrush current by using the correlation coefficient. Reference
[3] proposed a classification algorithm based on virtual equivalent inductance to identify transformer inrush current.

These identification methods focused on the use of many static characteristics, such as the existence of discontinuous angle, the contents of second harmonic or third harmonic, the symmetrical of waveform.

Any method which takes the static characters as the distinguishing feature is easily disturbed by noise factors. When considering the various static characters, the inrush current was assumed as constant in the sampling process, which is contradictory to the actual attenuation process of inrush current. The static characters existed both in fault process and normal operation. The only different is the contents [4]. In a word, the present methods could not solve the problem of identifying inrush current because the complexity of inrush current.

A novel method to identify the inrush current is proposed based on the dynamic characteristics of the aperiodic component of the inrush current in this paper.

2. The Aperiodic Component of Inrush Current

2.1. The Definition of Aperiodic Component of Inrush Current Considering Attenuation

In the case of ideal transformer (ignoring the transformer loss), the inrush current is a complete periodic function and does not decay. So, the DC component Iμ0 could be obtained by integrating the inrush current in any period, such as formula 1[5].

\[
I_{\mu 0} = \frac{1}{2\pi} \int_{0}^{2\pi} i(\xi)d\xi
\]

Where i is the inrush current of the transformer, ξ is the integral variable.

The aperiodic component of the inrush current of the ideal transformer is constant in this time.

Considering the actual attenuation of inrush current, the inrush current is no longer a simple periodic function. Therefore, different aperiodic components could be calculated according to the integral of the inrush current in different periods[6].

According to the traditional integral method, this research proposed a novel method to calculate the aperiodic component considering the decay situation as shown in formula (2).

\[
i_{ic}(t)=\frac{1}{T} \int_{t}^{t+T} i(\xi)d\xi
\]

Where t is the time variable, T is the power frequency period.

2.2. The Expression of Excitation Inrush Current

The inrush current is calculated by the relation between the flux and the current of the transformer. Assuming \(t_j\) is the alternating time between the duration period and the intermittent period, \(t_0\) is the moment of the transformer no-load closing(the initial moment of the first intermittent period). \(t_1\) is the initial moment of the first duration period. \(t_2\) is the initial moment of the second intermittent period. \(t_3\) is the initial moment of the second duration period. \(t_m\) is the final moment of the last duration period(the final moment of the whole inrush current).

Therefore, \(t_{2k} \leq t < t_{2k+1}\) is the intermittent period, \(t_{2k+1} \leq t < t_{2(k+1)}\) is the duration period. \((k=0, 1, ..., \frac{m}{2} - 1)\).
According to the relationship between the flux and the current of the transformer, the method Fourier decomposition method is used to decompose the inrush current into the periodic component and the free component in the reference.

The inrush current is due to the transformer core saturation, the relation between the magnetic flux and voltage could be described as formula (3) when the amplitude of rated voltage and the amplitude of rated flux as the base value to represent the voltage of \( u \) and \( \Phi \).

\[
\frac{d\Phi}{dt} = u
\]

Assuming the transformer no-load closing in the \( t=0 \), and the voltage in the transformer equals \( u = u_m \sin(\omega t + \alpha) \). Where \( u_m \) is the peak voltage.

The mathematical description of inrush current \( i(t) \) is formula (4).

\[
i(t) = i_p(t) + i_{np}(t)
\]

Where \( i_p \) and \( i_{np} \) are the forced component and free component of inrush current respectively, as shown in formula (5) and (6).

\[
i_p(t) = \begin{cases} 
0, & t_{2k} \leq t < t_{2k+1} \\
I_{mp} \sin(\omega t + \alpha - \beta]\mu), & t_{2k+1} \leq t < t_{2(k+1)}
\end{cases}
\]

Where \( I_{mp} \) is the amplitude of the forced component, \( \alpha-\beta]\mu \) is the phase angle of the inrush current during the duration period.

\[
i_{np}(t) = \begin{cases} 
0, & t_{2k} \leq t < t_{2k+1} \\
I_{np0}(t_{2k+1})e^{-(t-t_{2k+1})/\tau]\mu}, & t_{2k+1} \leq t < t_{2(k+1)}
\end{cases}
\]

Where \( I_{np0} \) is the initial value of the free component, \( \tau]\mu \) is the decay time constant of aperiodic component.

2.3. The Aperiodic Component Expression
Taking the integral of the inrush current in the k+1 period as an example, the aperiodic component of the inrush current obtained.

1) When \( t \) was in \( [kT, t_{2k+1}] \), because \( t_{2k+1} - kT < t_{2k+3} - (k + 1)T \), the range of \( t+T \) was \( (k + 1)T \leq t+T < t_{2k+1} + T \), the integral value of aperiodic component in one cycle was the integral value of aperiodic component in \( k+1 \) cycle. The integral region is shown in Figure 1(a).

2) When \( t \) was in \( [t_{2k+1}, t_{2k+3} + T] \), the range of \( t+T \) was \( t_{2k+1} + T \leq t+T < t_{2k+3} \), the integral region is shown in Figure 1(b).

3) When \( t \) was in \( [t_{2k+3} + T, t_{2k+4}] \), the range of \( t+T \) was \( t_{2k+3} \leq t+T < t_{2k+4} \), the integral region was shown in Figure 1(C).

4) When \( t \) was in \( [t_{2k+4} + T, t_{2k+5}] \), the range of \( t+T \) was \( t_{2k+4} \leq t+T < t_{2k+5} \), the integral region is shown in Figure 1(d).

5) When \( t \) was in \( [t_{2k+5}, (k+1)T] \), the integral region is all of the inrush current of next period. The integral region is shown in Figure 1(e).
Based on the 5 cases above, the aperiodic component $i_{DC}$ was obtained according to formula (2)–(6), as shown in formula (7).
The characters of aperiodic component variation of inrush current

In order to have an intuitive description of aperiodic components, the aperiodic component change curve was shown in Figure 2 according to the formula (6). Two periods was elected for detail analysis at initial decay period and the end of decay period.

According to Figure 2, the aperiodic component decay rule is close to an exponential function and the decay time is more than 2 seconds. Based on the statistical data, the inrush current of transformer aperiodic components could last tens of seconds or even a few minutes. The aperiodic component of inrush current decays very slow.

According to the local curve of Figure 2, the aperiodic component does not decay in the intermittent period. The aperiodic component decays in the duration period.

According to the two sampling curves in the beginning of period and the end of period, the duration period is longer than the intermittent period at the beginning, but become shorter and shorter. Therefore, the aperiodic component attenuation velocity decreased. Namely, the decay rate of the aperiodic component is the fastest at the beginning of the inrush current.

In summary, the inrush current aperiodic components decay speed is slower than the aperiodic component of short-circuit current decay time, which means the dynamic time constant of inrush current aperiodic component is greater than the dynamic time constant of short-circuit current aperiodic component.
3. The Attenuation Velocity of Aperiodic Component of Inrush Current

Taking derivation of aperiodic components (7) analysis, the attenuation velocity of aperiodic component expressions was obtained as shown in formula (8).

\[\dot{I}_{dc}(t) = \begin{cases} 0; & 0 \leq t < t_{1}\text{th duration period} \\ \frac{1}{T}\left[-I_{ap}(t_{j+1})e^{-\alpha t_{j+1}} - I_{ap}(t_{j+2})e^{-\alpha t_{j+2}}\right]; & \left(1 + \frac{1}{T}\right) t_{j+1} \leq t < (k+1)T \\ \frac{1}{T}\left[-I_{ap}(t_{j+1})e^{-\alpha t_{j+1}} - I_{ap}(t_{j+2})e^{-\alpha t_{j+2}}\right]; & \left(1 + \frac{1}{T}\right) t_{j+1} \leq T \leq t_{j+1} - T \\ \frac{1}{T}\left[-I_{ap}(t_{j+1})e^{-\alpha t_{j+1}} - I_{ap}(t_{j+2})e^{-\alpha t_{j+2}}\right]; & 0 \leq t < t_{j+1} - T \\ 0; & t_{j+1} \leq t < (k+1)T \end{cases} \]

The attenuation velocity of aperiodic component of inrush current obtained according to formula (8), as shown in Figure 3.

![Figure 3](image-url)  
**Figure 3.** The attenuation velocity of inrush current aperiodic component.

According to Figure 3, the change rate of aperiodic component is negative. The negative sign represents the aperiodic component is decay.

The whole attenuation velocity decreased, but in the intermittent period, the aperiodic component attenuation velocity is zero, which means the aperiodic component does not decay. The aperiodic component decay rate continued to decrease in the duration period. In the two adjacent cycles, the initial decay rate of the next cycle is equal to the end decay rate of the previous cycle[7].

4. Simulation Analysis

MATLAB was used to simulate the electromagnetic calculation of power system in this research. The parameters of the system were shown in Figure 4. The power supply was represented by a lumped parameter, and the lines were represented by Bergeron models.
(1) The decay rate of aperiodic component of short circuit current
The peak current of primary side was 89.2A when the transformer was operation normally. Because the circuit impedance tends to zero when the secondary side of the transformer occurred short-circuit fault, while the transformer primary side waveform decreased, the ultimately ended of the attenuation peak was 798.5A, which was the 8.95 times than the normal operation of the current peak value.

The component of the short circuit current aperiodic component was obtained by using the Fourier decomposition method, as shown in Figure 5.

The short-circuit fault occurred in 0.5s in Figure 5. According to the flux conservation principle, the transformer primary side current increased rapidly in 5 cycles because the existence of inductance. After reaching the peak value, the waveform shows attenuation trend because the existence of the loop impedance. The component of aperiodic component in the current of the primary side is exponentially decaying.

In this research, the data of the aperiodic component of short circuit current is used to exponential fitting. The fitting curve is shown in formula (9).

\[ i_p = 9.794e^{-0.011t} \]  

Where the time constant \( r = 0.011 \)

![Figure 4. Model and parameters of simulation system.](image)

(2) The decay rate of aperiodic component of inrush current
In this research, the least square method is used to fit the aperiodic components in each period and the quantitative changes of aperiodic components of dynamic time constant obtained[8]. The dynamic time constant fitting curve of aperiodic component is shown in Table 1.
**Table 1.** The inrush current dynamic time constant of the aperiodic component fitting curve

|   | 1   | 2   | 3   | 4   | 5   | 6   |
|---|-----|-----|-----|-----|-----|-----|
| T | 0.228 | 0.232 | 0.237 | 0.241 | 0.247 | 0.253 |
| T | 7 | 8 | … | 99 | 100 | … |
| T | 0.257 | 0.261 | … | 0.635 | 0.698 | … |
| T | 0.257 | 0.261 | … | 0.635 | 0.698 | … |

According to Table 1, the time constant of the aperiodic component of inrush current value increased gradually from the first cycle and the minimum time constant of the initial period is 0.228.

Because the aperiodic component does not decay in the discontinuous period and only decay in the duration period, the average decay rate of aperiodic component in one cycle is slower and the time constant is also greater.

According to the time constant in Table 1 and formula (9), it is found that the minimum time constant 0.228 of the aperiodic component of the inrush current is greater than the time constant 0.011 of the aperiodic component of the short-circuit current.

The aperiodic component of the inrush current of the transformer has the characteristics of slow attenuation and long duration. It provides a contrast with the characteristics of fast attenuation and short duration of the short circuit current aperiodic component.

![Time constant of aperiodic component of inrush current and short circuit current](image)

**Figure 6.** Variation rules of time constant of aperiodic component of inrush current and short circuit current.

**5. The Identification Method of Inrush Current**

Based on the above analysis, it is possible to distinguish the inrush current by using the different dynamic time constant of the aperiodic component of the inrush current and the time constant of the aperiodic component of the short circuit current. The flow chart is shown in Figure 7.
According to Figure 7, the transformer relay protection device sampling current first, extracting the aperiodic components by using the least square method second, and judging whether there is an aperiodic sampling current. There is no inrush current if there is no discrimination of aperiodic components. If there is discrimination of aperiodic components, then go to further recognition according to the time constants. If the time constant is greater than the setting value, the case is inrush current. If the time constant is less than the setting value, the case is not inrush current.

The lasting time of aperiodic component of short-circuit current was small, usually lasting tens of milliseconds\[^9\] \[^10\]. But the lasting time of aperiodic component of inrush current was long, usually lasting several seconds or even minutes \[^11\]. Therefore, the range of could adopt 0.15–0.5 according to the different transformer capacities.

6. Conclusions
This research proposed a novel method to identify inrush current based on attenuation characteristics of aperiodic components according to the attenuation characteristics of aperiodic component. Some conclusions are as follow.

1. The aperiodic component does not decay in the intermittent period of inrush current. The aperiodic component does decay in the duration period of inrush current. The velocity attenuation of \( k^{th} \) cycle at the end of the period is equal to the decay rate of \((k+1)^{th}\) cycle at the beginning of the period.

2. The decay rate of aperiodic component of the inrush current is slower than that of the short circuit current, that is, the time constant of the dynamic component of the aperiodic component of the inrush current is greater than that of the short circuit current.

3. The inrush current occurred according to judging the dynamic attenuation time constant of the extracted aperiodic component is greater than the setting value.

7. Acknowledgements
The work here is supported by the National Natural Science Foundation of China(51507157), and the work here is supported by the National Natural Science Foundation of China(51607158). Supported by the Graduate’s Scientific Research Foundation of Zhengzhou University of Light Industry.

8. References
[1] Yuan An, Lulin Tian and Jiajun Liu 2008 Transformer excitation inrush identification based on morphology-wavelet theory *J. Electric Power Automation Equipment*. 7 41-45
[2] Xue and Wang 2012 Synthetic transformer inrush identification based on characteristic space *J. Electric Power Automation Equipment*. 11 83-86
[3] Tao Zheng, Geye Lu and Yanjie Zhao 2016 A discriminating algorithm for identifying inrush of UHV voltage-regulating transformer based on virtual equivalent inductance *J. Transactions of China Electrotechnical Society*. 7 118-125
[4] Zhengqing Han and Shuping Liu 2007 Magnetizing inrush identification based on differential current characteristics *J. Electric Power Automation Equipment*. 9 51-4
[5] Jianping Jiang and Yiguang Jiang 2013 Detection method of weak signals via stochastic resonance *J. Ship Electronic Engineering*. 5 143-145
[6] Hongquan Ji, Yihan Yang and Yansong Li 2007 Study on line aperiodic component current comparison pilot protection based on optical current transducers *J. Proceedings of the CSEE*. 19 45-49
[7] Lin Teng and Qingquan Jia 2002 Optical current transducer and its application in protective relaying *J. Power System Technology*. 1 31-33
[8] Lihong Xue and Dongping Li 2015 Research on piecewise linear fitting method based on least square method in 3D space points *J. The Open Automation and Control Systems Journal*. 7 1575-1579
[9] Jianqin Feng, Sifang Huang and Hailong Song 2014 Short-Circuit current aperiodic components and their application in relay protection *J. Electrotechnics Electric*. 12 35-8
[10] Yong Feng, Shiming Liu and Wei Chen, etc 1999 Study on the new algorithm of decaying DC components *J. RELAY*. 3 14-15
[11] Xidong and Xu 2005 Research on power transformer longitudinal differential protection *J. Doctoral Dissertation of Zhejiang University*. 2 54-55