A Signal Reconstruction Method Based on Quadratic Spline Interpolation Applied in Power Quality Analysis for SCM-TTU

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Abstract. Aiming at the accurate and quick analysis of power quality especially of harmonic problem in power system, interpolation approaches should be applied if the samples collected by hardware are not matched with Fast Fourier Transform (FFT) calculation. A quadratic spline interpolation algorithm is proposed on Single Chip Micyoco-Transformer Terminal Unit (SCM-TTU) in this paper. This algorithm considers the continuous of interpolation function and also the first derivative in order to improve the accuracy. It could be applied before FFT calculation to avoid the mismatch of sampling rates. A simulation with mathematic example and also an application with SCM-TTU contains the harmonics from 2nd to 50th orders are experimented, results are presented to show the effectiveness.

Keywords: Harmonic analysis; Power quality; Quadratic spline interpolation; SCM-TTU.

1. Introduction

With the progress of power system, the number of non-linear components applied in power grid increases rapidly. The usage of these devices improves the convenience of daily life, however, involves lots of power quality problems as well. Harmonic pollution is one of the typical problems, which not only deteriorates the power quality, but also adversely affects the safety and stability of power grid and even the economic operations [1-7]. Therefore, in order to take available measures to reduce the harmonics and improve the power quality, it is necessary to guarantee the validity of harmonic measurement firstly.

However, in previous Transformer Terminal Unit (TTU) designs, as AC sampling board and process unit were separated, when the size of memory or the operation speed of process unit was insufficient, the data would be discontinuous. Thus, harmonic analysis could not be monitored as Fast Fourier Transform (FFT) was not available due to the spectrum leakage [8,9].

A new design called SCM-TTU is proposed in which a microcontroller unit is added. This part could handle the sampling and processing together. Therefore, the problem of data discontinuity can be avoided. Whereas, one more question is that the sampling frequency may not be matched with FFT. For example, in power quality analysis, the calculation needs 4096 points for ten cycles while the sample rate of signal captured by the chips may be 16000 or 32000 per second, as a result, samples cannot be applied directly.

Other techniques, like interpolation, should be added before the FFT calculation in order to reconstruct the signal. Interpolation methods could change the sample rate to meet the different demands, so that the signals resampled could match the requirement of FFT calculation. Various interpolation methods are available in the literature to solve the signal reconstruction problem, quadratic spline can be the one which can balance the accuracy and the calculating speed. This
interpolation algorithm is performed with a simple mathematical model and then applied in SCM-TTU in this article. The paper is organized into six parts. Section 2 presents the process of power system harmonic measurement. In the third section, the method of quadratic spline interpolation is introduced. A mathematic example is proposed in Section 4 to compare with other methods. SCM-TTU is performed with this algorithm in Section 5 to show the effectiveness. And the last section is the conclusion.

2. Process of Power System Harmonic Measurement

In power grid, harmonics mainly come from the distributed power sources and the electronic components in the distributed generation systems. Suppose that the continuous time-domain signal containing $M$ harmonics can be expressed as:

$$x(t) = \sum_{m=0}^{M} A_m \cos(2\pi f_m t + \varphi_m)$$

(1)

where $M$ is the harmonic order components in the signal, $A_m$, $\varphi_m$ and $f_m$ present the amplitude, phase and frequency of the $m$-th harmonic respectively.

To measure the harmonics with SCM-TTU, the process is defined as shown in Figure 1.

![Figure 1. Process of harmonic analysis with interpolation.](image)

The first step is sampling, for which signal is extracted by the SCM-TTU with a rate of $f_s$. Limited by the frequency of data acquisition of microcontroller, $f_s$ is fixed and may not match the frequency required by FFT. The mismatch of sampling rates prevents these samples from being used directly in the FFT step. Consequently, in order to improve the accuracy of harmonic analysis, it would be necessary to adjust the sampling frequency according to FFT, thus the signal obtained by the initial sampling points must be reconstructed.

During the reconstruction in the second step, interpolation algorithm based on the given samples could be used to determine the value of the signal at the specified point defined by FFT. As shown in Figure 2, the signal $f(t)$ is a function of time $t$, $x_1, x_2, \cdots, x_n$ are the samples selected by the frequency $f_s$ in discrete time, the value of each initial sample is marked by the dotted line. And the dash-dotted curve is the reconstruction of $F(x)$ composed by the set of initial samples. The points marked by solid lines $x'_1, x'_2, \cdots, n$ are interpolated by the actual sampling frequency $f'_s$ which determined by FFT based on the reconstruction [10].

After $x'_1, x'_2, \cdots, n$ are chosen, FFT is available to transfer the signal into frequency domain, then analysis could be carried out.
3. Quadratic Spline Interpolation Approach

Commonly, deterministic interpolation methods can be divided into two categories, namely global methods and local ones. Global methods like global polynomial interpolation approach are based on the similarity among the entire dataset of sampling points and/or the smoothness of the entire surface to create a fitted curve. On the contrary, local methods such as piecewise polynomial interpolations calculate predictions from the measured points within neighbourhoods. The characteristic of the local deterministic interpolation methods is that the basis function would not be high degree, and the local accuracy could be guaranteed only if the intervals between samples are small enough [11-14].

In the case of harmonic analysis, the signal is an oscillation function with high frequency as all harmonics up to 50th are superposed together. The local accuracy is the most important during analysis especially for high degree harmonics. Thus, the piecewise methods are more suitable as they can reflect the oscillations by rebuilding the signal segment by segment. However, the piecewise methods which do not considering the deviations like linear interpolation may lead in errors especially around the inflection points. Moreover, cubic spline interpolation method is smooth enough but time consuming for SCM-TTU in real time calculation. For the sake of avoiding these inconveniences, quadratic spline interpolation method is used in this article.

Suppose that there is a set of sampling points \( [x_i, y_i] \) where \( i = 0, 1, \ldots, n \) for the function \( y = F(x) \), which makes \( n+1 \) points and \( n \) intervals between them. The quadratic spline interpolation is a piecewise continuous curve, passing through each of the values in the set. Each interval has an independent quadratic polynomial with its own coefficients:

\[
F_i(x) = a_i (x-x_i)^2 + b_i (x-x_i) + c_i
\]  

where \( x \in [x_i, x_{i+1}] \). These polynomial segments are denoted as \( F(x) \) together. Since each of the \( n \) intervals has 3 coefficients, a total of \( 3n \) parameters is required to defined the spline \( F(x) \), thus \( 3n \) independent conditions are needed to fix them.

Result in the continuousness, two conditions can be got from the requirement that each of the polynomial segments should match the values at both ends of its interval:

\[
F_i(x_i) = y_i, \quad F_i(x_{i+1}) = F_{i+1}(x_{i+1}) = y_{i+1}
\]  

Then since the interpolation should be smooth, the first derivatives are also taken into consideration and required to be continuous:
\[ F_i^r (x_{i+1}) = F_{i+1}^r (x_{i+1}) = y_{i+1} \]  

Equations (3) and (4) propose \( 3n - 1 \) conditions, to solve the problem a boundary condition is still needed. The first derivative at the endpoint \( x_n \) is required and normally set to be zero:

\[ F_0^r (x_n) = 0 \]  

With these conditions, the coefficients in equation (2) can be written as:

\[
\begin{align*}
    a_i (x_i - x_j)^2 + b_i (x_i - x_j) + c_i &= y_i \\
    a_i (x_{i+1} - x_j)^2 + b_i (x_{i+1} - x_j) + c_i &= y_{i+1} \\
    2a_i (x_i - x_j) + b_i &= y_j
\end{align*}
\]

The differential expression is as follow:

\[ F_i^r (x_i) = y_i = 2a_i (x_i - x_j) + b_i \]  

Define \( h_i = x_{i+1} - x_i \) as the step size, equation (6) it can be obtained that:

\[
\begin{align*}
    a_i &= \frac{y_{i+1} - y_i - h_i y_j}{h_i^2} \\
    b_i &= y_j \\
    c_i &= y_i
\end{align*}
\]

then from equation (8), the polynomials can be deducted as:

\[ F_i^r (x) = \frac{y_{i+1} - y_i - h_i y_j}{h_i^2} (x_i - x_i)^2 + y_i (x_i - x_j) + y_j \]  

Quadratic spline interpolation which is convenient for practical applications has continuity at the initial sampling points, and the burden of calculation is much smaller than the cubic spline interpolation [15].

4. Mathematical Example Simulation

For the tests shown in this section, a signal created by equation (1) is used for the purpose of understanding the implications of the choice of different interpolation methods. The amplitude and the frequency of fundamental wave are set as 220V and 50Hz respectively, harmonic ratios are set as 5% of all harmonics up to 50th order and the phase of each harmonic is 0 rad.

| Methods                | time required for 10 cycles (s) | Maximum relative error | Error of effective value of 50th harmonic |
|------------------------|-------------------------------|------------------------|-----------------------------------------|
| Linear                 | 0.00621                       | 18.087%                | -8.561%                                 |
| Nearest                | 0.0241                        | 140.309%               | -4.866%                                 |
| Hermite                | 0.0110                        | 16.285%                | -2.843%                                 |
| Whittaker-Shannon      | 0.846                         | 35.139%                | -0.856%                                 |
| Cubic spline           | 0.0467                        | 0.535%                 | -1.066%                                 |
| Quadratic spline       | 0.0119                        | 3.844%                 | -0.490%                                 |

Table 1. Results for different interpolation methods.

For simulation, initial sampling points are selected by \( f_s = 16000 \) as in reality. And for FFT calculation, the frequency of resample \( f_r \) is 4096 for 10 cycles, so 20480 point per second based on fundamental wave of 50Hz. To quantify the impact of quadratic spline method on time requiring and the accuracy,
the results of other approaches like linear, nearest, Hermite, Whittaker-Shannon and cubic spline interpolations are also presented here for comparison. The purpose is to ensure that errors must be less than 5% with real values and the running time must be as short as possible since long running time would block the following data in real time.

Results of these different interpolation approaches are shown in Table 1 and the following figures, and the horizontal coordinates are selected from $2.6 \times 10^{-3}$ s to $3.5 \times 10^{-3}$ s in order to display the details. The results contain running time required for 10 cycles and maximum error between interpolation and the real signal, while the error between effective value of 50th harmonic calculated by FFT using resampled points and the real signal is presented.

![Figure 4. The comparison of linear interpolation with real function.](image)

![Figure 5. The comparison of Hermite interpolation with real function.](image)

![Figure 6. The comparison of nearest interpolation with real function.](image)

![Figure 7. The comparison of Whittaker-Shannon interpolation with real function.](image)
Figure 8. The comparison of cubic spline interpolation with real function. It can be seen that linear interpolation is the fastest while its maximum relative error and the error of effective value of 50th harmonic are both over the threshold of 5%. Hermite interpolation has almost the same conclusion, this is because these two algorithms only consider the continuous of interpolation function. Nearest interpolation is the worst as the signal reconstructed is ladder-shaped, this is owing to its strategy, the values of the interpolation function are depended on the value of nearest samples. And Whittaker-Shannon is the most time-consuming method, the actual running time increases sharply with the increase of sample size. Cubic spline and quadratic spline are the most accurate among all approaches. They both achieve the requirement of accuracy as they get an interpolation formula that is continuous in the first or even the second derivatives. Moreover, in some parts cubic spline performs better, as shown in Table 1, cubic is more accurate than quadratic spline interpolation in most cases, but for higher harmonics like the 50th, the latter one performs better. Furthermore, the former method spends much more time than the latter one while the time consuming could not be ignored.

5. Application in SCM-TTU

SCM-TTU is a device that can be applied in low-pressure side of the transformer. It supervises the actions and power qualities in order to guarantee the security of distribution automation system by sampling AC inputs (such as voltage, current etc.), states (e.g. switch state) and executes remote control command.

Table 2 shows time required and errors of SCM-TTU with quadratic spline interpolation method. For comparison, the linear interpolation, Whittaker-Shannon method and cubic spline method are also presented. The results present the same consequence as those in last section, among those approaches, Whittaker-Shannon takes so much time that the following data is blocked, the linear interpolation is the fastest but also the most imprecise. In general, the quadratic spline could be the most efficient considering both the accuracy and the time requiring.

| Methods           | time required for 10 cycles (ms) | Error of effective value of 50th harmonic |
|-------------------|----------------------------------|------------------------------------------|
| Linear            | 0.613                            | -3.259%                                  |
| Whittaker-Shannon | Out of range                      | --                                       |
| Cubic spline      | 4.612                            | -2.059%                                  |
| Quadratic spline  | 1.868                            | -1.320%                                  |
6. Conclusion
Considering the non-synchronous frequency of hardware and FFT method, it is difficult to analyse the harmonic directly. To solve this problem, this paper presented a SCM-TTU power quality analysis method based on quadratic spline interpolation algorithm. The advantage of this technique is that the signal reconstruction is continuous, this condition guarantees the accuracy of reconstruction. The results in parts 4 and 5 show that both maximum relative error and the error of effective value of 50th harmonic could be under 4% by quadratic spline interpolation, and the time consumption is acceptable. It proves the effectiveness of the method, which can balance the analytical accuracy and time required.

Acknowledgements
This work is supported by a grant from Grid State science and technology program: Research, development and application of low delay, safe and reliable intelligent agent device (No. 5700-202041163A-0-0-00).

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