Two-dimensional mathematical modeling of interaction of a shock wave with moving cylinders

D A Sidorenko and P S Utkin
Institute for Computer-Aided Design of the Russian Academy of Sciences, Vtoraya Brestskaya 19/18, Moscow 123056, Russia
E-mail: pavel_utk@mail.ru

Abstract. The problem of interaction of a planar shock wave with cylinders of different masses is considered. The cylinder can move translationally under the action of the pressure force. Mathematical model is based on two-dimensional Euler equations. Numerical algorithm is based on the Cartesian grid method for the calculations of flows in the areas with varying geometry. The algorithm and its program realization are tested on the problem about the lifting of the cylinder behind the transmitted shock wave. The curves of the cylinder speed variation in time are plotted. The explanations about the qualitative view of the curves for the different cylinders masses are given. For one mass the analysis of the dynamics of the cylinder motion is carried out from the point of view of the non-stationary shock waves patterns that are realized as a result of interaction of the shock wave with the cylinder.

1. Introduction
The problem of interaction of a shock wave (SW) with a cloud of particles is canonical in order to investigate dense high speed flows of multiphase media [1]. For the clarification of the details of the process multidimensional gas dynamic mathematical modeling can be used. In our previous investigation [2] the problem of interaction of planar SW with array of stationary cylinders was considered. The results are compared with results of modeling under Baer–Nunziato type equations. This paper is an extension of [2] in terms of cylinders motion.

From the purpose of clarification of heterogeneous media mechanics in recent investigation [3], the dynamics of motion of two or three spheres [two-dimensional (2D) axisymmetric statement is considered] behind transmitted SW using Navier–Stokes equations is considered. It was shown that relaxation model for single particle gives inadequate results and collective effects are needs to be considered.

Experimental and theoretical methods of modeling of SW interaction with stationary object of complex shape are well-known and used for many years. The basic numerical study of the interaction of SW with stationary cylinder using Navier–Stokes and Euler equations is paper [4]. In case of moving bodies situation is more complex. Even for the case of a single moving cylinder in the 2D case or sphere in the three-dimensional case, description of flow and pressure forces, acting on bodies, is still actual problem both in numerical [5] and natural experiments [6]. The purpose of this work is developing the numerical algorithm for calculation of SW propagation in domains with moving boundaries as well as parametric investigation of SW moving cylinder interaction.
2. Mathematical model, numerical algorithm, verification

2D Euler equations in computational domain with moving boundaries are solved:

\[
U_t + \mathbf{F}_x + G_y = 0,
\]

\[
U = \begin{bmatrix}
\rho \\
\rho u \\
\rho v \\
\rho e
\end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix}
\rho u \\
\rho u^2 + p \\
\rho uv \\
u(e + p)
\end{bmatrix}, \quad G = \begin{bmatrix}
\rho v \\
\rho v^2 + p \\
\rho uv \\
v(e + p)
\end{bmatrix}.
\]

Here and further the notations are standard. On each time step for each computational cell the status is assigned. The following statuses are possible—the outer cell (the whole cell is inside the body or is intersected by the contour of the body) and the inner cell (the whole cell is inside the computational domain). Solution is built in the inner cells only. The finite volume scheme with Godunov type numerical flux of the second approximation order [7] is used:

\[
U_{i,j}^{n+1} = U_{i,j}^n - \frac{\Delta t}{\Delta x} \left( F_{i+1/2,j} - F_{i-1/2,j} + G_{i,j+1/2} - G_{i,j-1/2} \right).
\]  

(1)

For the edges between outer cells intersected by the contours of the bodies and the inner cells the special procedure of the parameters definition from the side of intersected cell for the subsequent numerical flux calculation is used [8]. As an example we will describe the algorithm of calculation of the numerical flux \( F_{i+1/2,j} \) through the vertical edge which is common for the outer cell \((i, j)\) and the inner cell \((i+1, j)\). Let us the outer cell corresponds to the cell intersected by the body contour which moves with the velocity \(v_b\). Denote the vector of conservative variables in cell \((i + 1, j)\) as \( U_{i+1,j} = [\rho, \rho u, \rho v, \rho e] \). For the flux \( F_{i+1/2,j} \) calculation the solution of the Riemann problem \( U_R(., .) \) with the left state \( U_{\text{ghost}} = [\rho, \rho(2v_{bx} - u), \rho v, \rho v(2v_{bx} - u, v, p)] \) and the right state \( U_{i+1,j} \) should be found. The solution of the formulated problem \( U_{i+1/2,j}^* = U_R(U_{\text{ghost}}, U_{i+1,j}) \) in the local one-dimensional (1D) case along the x-axis contains two SWs or two rarefaction waves connected via contact surface without density gap with the gas velocity \(v_{bx}\). Denote the speed of sound at the contact surface as \( c_{i+1/2,j}^* \). The numerical flux is determined as

\[
F_{i+1/2,j} = \begin{cases}
F[U_{i+1/2,j}^*], \\
F[U_R(U_{\text{ghost}}, U_{i+1/2,j}^*)], \\
F[U_{i+1,j}]
\end{cases}
\]

if \( v_{bx} < -c_{i+1/2,j}^* \),

if \( -c_{i+1/2,j}^* < v_{bx} < c_{i+1/2,j}^* \),

if \( v_{bx} > c_{i+1/2,j}^* \),

The other fluxes are calculated in the same manner.

After the fluxes are known the equations of movement of bodies are integrated. Then for each cell both its status and the vector of conservative variables are updated. If after the bodies movement the cell becomes an outer one the solution is not built in it. If the cell is still the inner one the solution is found in accordance with (1). If the cell becomes the inner one but it was the outer one the vector of conservative variables is determined by the formula:

\[
U_{i,j}^{n+1} = \frac{\alpha_x}{\alpha_x + \alpha_y} U_{i+1/2,j}^* + \frac{\alpha_y}{\alpha_x + \alpha_y} U_{i-1/2,j}^* + \frac{\alpha_{i,j+1}}{\alpha_{i,j+1} + \alpha_{i,j-1}} U_{i,j+1/2}^* + \frac{\alpha_{i,j-1}}{\alpha_{i,j+1} + \alpha_{i,j-1}} U_{i,j-1/2}^*.
\]

\[
\alpha_{i,\pm1,j} = \begin{cases}
0, & \text{if the cell } (i \pm 1, j) \text{ was an inner one at the previous time step,} \\
1, & \text{otherwise},
\end{cases}
\]

\[
\alpha_{i,j\pm1} = \begin{cases}
0, & \text{if the cell } (i, j \pm 1) \text{ was an inner one at the previous time step,} \\
1, & \text{otherwise},
\end{cases}
\]

\[
\alpha_x = \max(\alpha_{i+1,j}, \alpha_{i-1,j}), \quad \alpha_y = \max(\alpha_{i,j+1}, \alpha_{i,j-1}).
\]

Here \( U^* \) is the solution of the Riemann problem on the correspondent edge.
Figure 1. The dynamics of (a) abscissa and (b) ordinate of the cylinder center motion in the test problem about cylinder lift off by the SW: 1—[9]; 2—[10] (for calculation in [10] three different approaches were used); 3—[11]; 4—[12]; 5—[13].

The described algorithm is relatively simple to implement and unlike the methodology from [2] does not require solving the problem of calculating the shape or the square of the intersection of body contour with regular computational cell. The main difference of the described algorithm from the work [8] is the usage of the exact solution of the Riemann problem for $U^*$ calculation and correspondent numerical fluxes and the method of approximation order increase.

The numerical algorithm and its program realization were tested with the use of the 1D and 2D problems from [8] about defined bodies motion as well as bodies motion under pressure forces. Frequently occurred in literature test about cylinder lift off behind SW was also considered. At initial time moment in rectangular channel with length 1.0 and height 0.2 at a distance of 0.15 from the left boundary the cylinder with the diameter 0.1 and from the material with density 0.77 is situated at the bottom boundary. The channel is filled with the quiescent air with the density 1.0 and under the pressure 1.0. All the values are given in the non-dimensional form. From left to right planar SW with Mach number 3.0 propagates. Initial coordinate of SW front is 0.08. The type of the left boundary condition of the computational domain is inflow with the parameters behind SW, the type of the others is non-penetrating condition. Calculation time is 0.33. Calculation grid include $2000 \times 400$ cells.

Figure 1 illustrates the dynamics of the cylinder center motion caused by pressure forces. The computational results from the papers [9–13] are also plotted. It should be noted the deviation of all computational results in dynamics of cylinder abscissa and ordinate. This results show the necessity of further development of the numerical methods for the simulations of flows in the areas with varying boundaries and the investigations of the bounds of its applicability.

Deviations of the computational results obtained by different approaches affect also on the flow field. Comparison of figure 2 with analogous spatial distributions from [9–13] shows, that wave pattern in left part, which includes reflected from the cylinder SWs, is almost the same in all calculations. At the same time vortexes structures near the cylinder are different. The possible reason is the usage of the inviscid gas model.

Modeling of motion of multiple bodies in a flow leads to the necessity of taking into account the interaction of bodies as with gas as with each other. Kinematic impact theory as model of interaction was chosen [15]. The model considers the eccentric inelastic collision of two arbitrary shape bodies with friction and restitution coefficients taking into account. Based on momentum
Figure 2. Density gradient magnitude distribution in lift off problem at final stage of process.

Figure 3. Density isolines at the time moment 11.6 $\mu$s in problem of two cylinders interaction behind transmitted SW: (a) results from [14]; (b) the present paper.

and angular momentum conservation laws under the instantaneous impact approximation and absence of deformation assumptions, bodies velocities and angular velocities after the impact are defined. The model was realized and tested on the problem of interaction of a pair of initially resting cylinders with SW with Mach number 5.0 [14]. As a result, the cylinders start moving, later collide and move away each other. Predicted density isolines as well as the data from [14] are depicted in figure 3.

3. Interaction of a shock wave with moving cylinder
Consider the problem about of motion of circular cylinder far away from computational domain boundaries. Cylinder moves under the action of the planar SW with Mach number 1.5. SW propagates from left to right. The gas in front of the SW is at rest, initial pressure and density are equal to 1.0. Specific heat ratio is equal to 1.4. Cylinders with masses $m$ from 2.0 to 0.005 are considered. The height of the cylinder is taken equal to unit of length. Cylinder diameter is equal to 0.25, grid resolution is 50 cells per the diameter. Computational area is the rectangular 6.0 (24 diameters of the cylinder) $\times$ 2.0 (8 diameters of the cylinder). Coordinates of the lower left node are (0, 0). Coordinates of the center of the cylinder are (0.625, 1.0). The initial abscissa of SW is 0.5, its location corresponds to the left boundary of the cylinder. The calculations last up to the moment of the arrival of the disturbances to the boundaries of the computational area to avoid its influence.
Gas flow behind the SW induces the translational motion of the cylinder due to the symmetry of the problem. The motion of cylinder is described by the second Newton law:

\[ \ddot{x}_c(t) = \frac{F}{m} = \frac{1}{m} \int p(x, y, t) dS(x, y, t), \quad \dot{x}_c(0) = 0, \quad \dot{x}_c(0) = 0, \quad x_c(0) = x_{c0}, \]

where \( m \) is the mass of the cylinder, \( p(x, y, t) \) is the gas pressure, the integration is carried out over the cylinder surface.

In the case of heavy cylinder (\( m = 1 \) to 2), its acceleration is quite small and there are small differences between the flow patterns in comparison with the stationary cylinder. The pressure distribution on the surface is so that the force acting on the cylinder increases while the SW do not bend the cylinder. Then it decreases monotonically and with constant sign, see, e.g., [4]. This leads to monotone increase of cylinder velocity, figure 4(a). Figure 4(a) demonstrates the dynamics of cylinders of different masses. It can be seen that character of curves depends from the cylinder mass.

In the case of lightweight cylinder (\( m = 0.005 \)), the pressure force accelerates the cylinder to velocity larger than the ambient gas velocity. This leads to the formation of high pressure area at the right side of cylinder and low pressure at the left. After that pressure force will change the sign and velocity will decrease. This leads to the pressure increase at left from cylinder and decrease at the right with a delay in time. So the wave-like decreasing of the cylinder velocity and the oscillating regime for pressure force dynamics are realized.

The next interesting case corresponds to the intermediate value of cylinder mass [the case \( m = 0.2 \) in figure 4(a)]. In this case the pressure force reaches a maximum, after that decreases to 0 and finally slowly monotonically increases, figure 4(b). Describe the flow which is realized in this case. In papers [5, 16, 17] the SW configurations at the initial stage of the interaction of the SW with moving cylinder are described. To explain the non-monotone dynamics of cylinder motion consider the configurations which are realized in later times.

The interaction of the SW with the cylinder leads to the Mach reflection with the reflected wave, Mach wave and the contact surface that couples in the triple point \( T_1 \). The situation is the same for both moving and stationary cylinders, figure 5. But in case of moving cylinder there are areas of rarefaction and compression at the left and at the right of the cylinder. After some time the compression waves form the curvilinear SW which starts to interact with Mach waves.
Figure 5. Wave pattern in the case of (a) moving ($m = 0.2$, time 0.23) and (b) stationary (time 0.18) cylinder at the initial stage. Density gradient magnitude distribution, $T_1$ denotes the first pair of triple points.

Figure 6. (a) Wave pattern on the stage of the second pair of triple point appearance, time moment 0.29; (b) wave pattern on the stage when the secondary triple points move away from each other, time moment 0.58. Density gradient magnitude distribution, $m = 0.2$, the case of moving cylinder, $T_1$ and $T_2$ are the first and the second pairs of triple points.

That move towards each other, see figure 5(a). As far as transmitted SW moves to the right and Mach waves moves towards each other, pressure force acting on cylinder starts to decrease.

The reason is the compression of gas in the new SW and Mach waves on the right boundary of the cylinder and rarefaction at the left boundary of the cylinder. The second pair of triple points appears, namely $T_2$—the points of intersection of Mach waves with new SW, figure 6(a). Then Mach waves interact with each other near the point C. Approximately at the moment when the point of contact of the upper and lower Mach waves reaches the lower and the upper point of the cylinder correspondingly, figure 6(b), the pressure force get the minimal value and the curve of the velocity of the cylinder demonstrates the flex point.
4. Conclusions
The computational algorithm of the Cartesian grid method of the second approximation order for the mathematical modeling of SWs propagation in the areas with variable boundaries is developed. The algorithm and its program realization are tested on the problem of cylinder lift off behind the transmitted SW. The analysis of the numerical solutions of this problem using other methods for the simulations of the motion of bodies in gas shows the possibility of significant difference in flow structure, so further work on construction and analysis of such class of methods is of topicality.

The modeling of motion of cylinder with different mass under the action of pressure forces behind the transmitted SW is carried out. In each case, a curve of the velocity of cylinder in time is plotted; explanations of qualitative view of curves are given. For one of the considered masses the analysis of cylinder dynamics from point of view of non stationary wave patterns, which are realized as a result of interaction of SW and cylinder, is carried out. The obtained results relates to the clarification of the mechanisms of particles relaxation behind the transmitted SW.

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