Abstract. Harmonics generated with fs laser pulses are not only among the most intense sources in the wavelength range of a few tens of nanometers but also a far better source than X-ray lasers from the coherence point of view. It is anticipated, that in the near future, the harmonics issued from femtosecond laser-produced plasmas will represent a coherent source much brighter than all other XUV sources apart from the XFEL. These particular aspects, correlated with the fact that high order harmonics produced by reflection of an ultrashort and intense laser pulse on an overdense plasma layer impose virtually no restrictions on the intensity of the laser employed, lead the way to a series of potential applications in plasma diagnostics and nonlinear optics. This work presents a theoretical study on the high harmonics generation mechanism occurring at the reflection of a femtosecond Ti-Sapphire terawatt laser pulse (λ₀ = 0.8 μm and pulse durations around 100-150 fs) on an overdense steep Al plasma layer. Insights on how different laser parameters influence this generation mechanism and its efficiency are provided.
When talking about HHG, it is important to consider the type of laser used as the properties of the high order harmonics are strongly influenced by the incident radiation. Furthermore, the conversion efficiency of the laser energy into harmonics is also dependent on the incident pulse. For a \( p \)-polarized femtosecond laser pulse, Tarasevitch et al. \[1\] established a conversion efficiency comparable to that of gas harmonics, its order being of \( 10^{-6} \) for the 10\(^{th}\) order harmonic in case of an incident laser wavelength \( \lambda_0 = 0.8 \mu m \) with an intensity ranging within \( 10^{17} - 10^{18} \text{Wcm}^{-2} \).

Using a femtosecond laser pulse imposes several restrictions such as a pre-pulse free laser and a high contrast pulse. A pre-pulse leads to significantly less intense harmonics and decreases the highest order of observable peaks while a poor pulse contrast may lead to the suppression of the generation of high harmonics.

Other notable factors that have been accounted for, are the steep density gradient and the polarization of the laser beam. Up to present, HHG has been achieved almost exclusively with \( p \)-polarized incident laser pulses. However, recently, Tarasevitch et al. \[2\] have obtained high order harmonics with an \( s \)-polarized relativistic intensity laser pulse.

Taking into consideration that high harmonics generated with a femtosecond laser lead to a power per solid angle of at least an order of magnitude higher than the one resulted from those generated with a picosecond laser, this paper aims to provide insights on the influence of different parameters of a \( fs \) Ti-sapphire terawatt laser (\( \lambda_0 = 0.8 \mu m \) and pulse durations around 100-150 \( fs \)) on the high harmonics generation mechanism and its efficiency.

### 2. Using the Particle-in-Cell approach to model high harmonics generation. Results.

Theoretical or experimental investigations of high harmonics generation have reported considerable improvement by using particle-in-cell simulations \[3,4,5,6\].

This present work uses the LPIC++, a 1D3V fully relativistic code with high spectral resolution in order to describe the laser-plasma interaction. The plasma, placed in the centre of the simulation box, is considered as a steep layer, plane, preionized and collision-less in a 1D description. The laser is interacting either at normal, either at oblique incidence, tapping on the plasma layer from the left side. Practically, this code treats the plasma kinetically, solving the relativistic equations of motion for a set of electrons and ions incorporated within a macroparticle. Each macroparticle obeys a distribution law that satisfies a kinetic Vlasov equation. The 1D3V PIC code accounts for the three components of the particle velocities. Densities, currents and electromagnetic fields are defined on a 1D grid and updated after each particle push. The electromagnetic fields are calculated using Maxwell’s equations. The longitudinal electric field is obtained from Poisson’s equation while the transverse electromagnetic fields yield from the corresponding current density after having solved the wave equation. The basic algorithm for solving these equations is that of Birdsall and Langdon \[7\] and it includes the “leap-frog” scheme for resolving particle pushing.

The code further defines the laser pulse by introducing a time-dependent condition for the transverse fields at the front side of the simulation box. The oblique incidence is treated by using the Bourdier method \[8\].

This study assumes a Ti-sapphire femtosecond laser system. Its intensity is one of the parameters that were adjusted, going as far as the relativistic regime. In other words this intensity is varied between \( 10^{17} \text{Wcm}^{-2} \) and \( 10^{19} \text{Wcm}^{-2} \). The pulse shape was chosen a sine function and the durations of the pulse are 100, respectively 150 \( fs \). The emission of the reflected light and, therefore, of the harmonics is restricted to the specular direction, all other effects leading to an emission into a broader angular space being neglected. However, it is known that, in the case of tightly focused pulses, on near-critical plasmas, the electron density is pushed with tremendous force and fast, so that the reflection surface changes orientation during each optical cycle, leading to emission away from specular direction \[9\].

The plasma layer was chosen to be aluminium, one time ionized. The ions are considered as an immobile background in which electrons move. Femtosecond laser-produced plasmas have little or no
time to expand. During the interaction, the plasma surface can thus be represented with a good approximation by a simple step profile, overdense, so that it acts like a mirror reflecting incident light specularly. This is the oscillating mirror model introduced by Bulanov et al. [10].

The density of the plasma layer was varied in the range between $n_e = 2n_c$ and $n_e = 15n_c$.

Although the 1D approach of this study might seem simplistic and limited, it has the advantage that it allows for a large number of time steps per laser cycle, along with more particles per cell such that high orders of harmonics can be resolved. Realistic plasma densities demand a high number of particles per cell in order to tackle all important non-linear surface dynamics. Relativistic surface oscillations are responsible for high harmonics generation along with resonance absorption and accounting for the dynamics of this surface is challenging, numerically as well as in terms of computational resources and simulation time.

The number of macroparticles per cell was set to 200 and the temperature adjusted accordingly, taking into consideration both the real physical conditions as well as the numerical ones. The total number of cells in the simulation box is 4096 out of which the plasma occupies 1500 cells, right in the middle of the box. A total of 1500 of vacuum cells can be found in front of the plasma layer in order to avoid the escape of particles. The number of cells per laser wavelength is 1000 and the number of simulation steps is 100 steps per laser cycle.

Simulation results reveal strong oscillations of the critical plasma surface driven by the normal component of the laser field and by the ponderomotive force. Figure 1 and 2 depict such oscillating plasma surfaces, driven by field strengths of $a_0 = 0.7$ (corresponding to $I\lambda^2 = 6.76 \cdot 10^{17} \text{Wcm}^{-2} \text{µm}^{-2}$), respectively by $a_0 = 1$ ($I\lambda^2 = 1.38 \cdot 10^{18} \text{Wcm}^{-2} \text{µm}^{-2}$), with the laser either normally incident, either reaching the surface at an incidence angle of $30^\circ$ relative to the normal. The laser is considered $p$ polarized.

Figure 1 a). Normal incidence, laser field strength $a_0 = 0.7$, pulse duration 150fs, electron density relative to the critical plasma density $n/n_c = 4$.

Figure 1 b). Oblique incidence with $\alpha = 30^\circ$, laser field strength $a_0 = 0.7$, pulse duration 150fs, electron density relative to the critical plasma density $n/n_c = 4$, polarization $p$. 

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At intensities above $10^{18}$ W/cm$^2$, electrons swing in the laser pulse with relativistic energies. A significant change in the interaction between the electrons and the electromagnetic field occurs in the relativistic regime. The dipole approximation is no longer valid as the amplitude of the electron oscillations is not small with respect to the laser wavelength. Even a harmonically oscillating electron can be a source of harmonics [11]. The plasma electron dynamics does not depend on $a_0$ and the plasma electron density $n_e$ separately but rather both factors influence its behaviour. This can be expressed by introducing a similarity parameter $S = (n_e/n_0)/a_0$. By observing the yield spectra and the behaviour of plasma in certain density and laser pulse characteristics, one can derive assumptions about the desired plasma density when having a particular laser intensity or about the required laser intensity when knowing the density ratio in order to achieve a desired harmonics spectra.

Figures 3 and 4 illustrate the harmonics spectra obtained by reflection on the oscillating critical plasma surfaces pictured in Figures 1 a) and 2 b). The harmonics spectra obtained here are power spectra of the electric field.
3. Conclusion
Harmonic emission increases with the laser intensity but it also depends on the plasma density. A high ratio between the electron density and the critical density leads to a nearly flat plasma surface, thus affecting the harmonics yield. A too low ratio causes an extreme disturbance to the plasma layer, consequently completely altering the nature of the reflected pulse. At sufficiently high intensities and moderate densities, surface deformities can be caused by Rayleigh-Taylor like instabilities driven by the imbalance between the ponderomotive laser pressure and the thermal plasma pressure, resulting in spectra rich in high order harmonics.

Shortening the pulse duration as well as using pulses interacting with plasma at oblique incidence were found to produce more harmonics and of higher amplitudes.

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