Study on High Frequency Vibrations using Statistical Energy Analysis for Different Composite Materials

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Abstract. Statistical Energy Analysis developed in early 1960s is used to determine the vibration response and energy levels in the structures subjected to high frequency vibrations by using statistical approach. In this work, Statistical Energy Analysis parameters like coupling loss factor and velocity response for composite plate like structures are analytically determined. Velocity response and coupling loss factor for L-shaped composite plate like structure subjected to point load have been computed for three different composite materials. Velocity response and coupling loss factors were determined and compared for the materials considered. The results show that composite materials having high specific modulus had low velocities and high coupling loss factor.

Keywords: Coupling loss factor, High frequency vibration, Velocity response.

1. Introduction
Matters with high specific stiffness find wide application in aerospace structures where minimum weight is required. Specific stiffness or Stiffness to weight ratio is the ratio of elastic modulus to mass density of a material. In the field of automotive and aerospace industries, advanced composite laminates having high-strength-weight ratio are widely used. A composite laminate is a combination of fibrous materials that are bonded together layer by layer to obtain the required engineering properties - bending stiffness, strength, coefficient of thermal expansion, and in-plane stiffness. The individual layers consist of high-modulus, high-strength fibres in a polymeric, metallic, or ceramic matrix material.

In the design of aerospace structures and machineries operating at high speeds, study of vibration response at high frequencies is of paramount importance to arrive at methods to control vibrations in high frequency range. Finite Element Analysis (FEA) become less practical in the area of high frequency vibrations. Statistical Energy Analysis (SEA) was developed by Lyon [1,2] in early 1960's to determine the vibration response and energy levels in the structures subjected to high frequency vibrations by using statistical approach.

SEA predicts vibration response in structures by dividing it into subsystems and measuring the energy level and energy transmitted across the subsystems. Vibrations encountered in engineering applications are characterised based on the frequency range: low, mid and high frequency region [3]. Low frequency region can be defined as an area where resonance peak dominates. In this region one can clearly observe the effect of end conditions and change in geometrical parameters. In high frequency region, unlike in
low frequency region, study of individual modes and frequencies becomes difficult due to high modal overlap. The mid frequency region exhibits transient effect of low and high frequencies. SEA involves predicting the vibration response of a complex structure by dividing it into a number of subsystems and is characterized by mean energy per mode and the change in energy level between subsystems. Experimental SEA helps in analysing the plate and beam like structures with different configurations subjected to bending wave transmission. SEA is characterized by loss factor comprising of internal loss factor and coupling loss factor. Internal loss factor corresponds to damping in the subsystem itself and coupling loss factor (CLF) corresponds to the energy dissipation during flow across the subsystems. The change in energy level between subsystems is defined by the loss factor. SEA has been widely applied for high frequency vibration analysis of isotropic materials. Different configurations like L-shaped, T-shaped, ribbed plates, have been analysed [4]. The coupling loss varies with the configuration employed. The parameters for the SEA have been computed using analytical, experimental, FEA [5], TSEA [6] and other hybrid methods [7-8]. The main drawback in experimental technique lies in measuring the response at source and receiver at a large number of points [9] for composite materials which are heterogeneous. FEA, used as an auxiliary method for SEA, has limitations in high frequency range [15,17].

In this work, SEA parameters have been computed for L-shaped configuration using classical wave approach. Classical wave approach predicts reliably the response in high frequency domain when compared with other experimental and numerical techniques. Analysis was carried out using three different composite materials for L-shaped configuration. The influence of specific modulus on CLF and velocity response was investigated. CLF varied linearly with the increase in frequency. The results obtained using classical wave approach employed in this work are in close agreement with the results obtained by Abdullah [15].

2. Statistical Energy Analysis

In SEA, a complex structure is divided into number of coupled subsystems. Exchange of energy takes place between the coupled subsystems along the coupling junctions, where each subsystem represents mode type. During the transfer of energy between coupled subsystems, loss of energy level is due to coupling loss factor and internal loss factor. The relation for energy flow between coupled subsystems (Figure 1) is established by using power balance equation.

\[ \bar{P}_{i,\text{in}} = \bar{P}_{i,\text{diss}} = \omega \eta \langle E \rangle \]  

where, \( \bar{P}_{i,\text{in}} \) - power injected in subsystem \( i \),
\( \langle E \rangle \) - Frequency averaged energy in subsystem \( i \),
\( \eta \) - Structural damping loss factor,
\( \langle \cdot \rangle \) - indicates spatial averaging, and bar indicates frequency averaging.

The power balance equation is given by [10, 11],

\[
\bar{P}_{i,\text{in}} = \bar{P}_{i,\text{diss}} + \bar{P}_{ij} \\
\bar{P}_{j,\text{in}} = \bar{P}_{j,\text{diss}} + \bar{P}_{ji}
\]

(2)

(3)

where, the power transmitted between subsystem \( i \) and \( j \) is given by,

\[
\bar{P}_{ij} = \omega \eta_{ij} \langle E_i \rangle - \omega \eta_{ji} \langle E_j \rangle, \\
\bar{P}_{ji} = \omega \eta_{ji} \langle E_j \rangle - \omega \eta_{ij} \langle E_i \rangle.
\]

(4)

The power balance equation for \( n \) systems is given by [12, 13],

\[
\omega \begin{bmatrix}
\eta_1 + \sum_{i=1}^{N} \eta_{ii} n_1 & -\eta_2 n_1 & \cdots & -\eta_N n_1 \\
-\eta_2 n_2 & \eta_2 + \sum_{i=2}^{N} \eta_{ii} n_2 & \cdots & -\eta_2 n_2 \\
\vdots & \vdots & \ddots & \vdots \\
-\eta_N n_N & \cdots & \cdots & \eta_N + \sum_{i=N}^{N-1} \eta_{ii} n_N
\end{bmatrix}
\begin{bmatrix}
\langle E_1 \rangle \\
\langle E_2 \rangle \\
\vdots \\
\langle E_N \rangle
\end{bmatrix}
= \begin{bmatrix}
\bar{P}_{i,1} \\
\bar{P}_{i,2} \\
\vdots \\
\bar{P}_{i,N}
\end{bmatrix}
\]

(5)

3. Analysis and Procedure
SEA was carried on L-Plate like structure made of composite materials. Glass/Epoxy, Boron/Epoxy and Graphite/Epoxy were considered for determining coupling loss factors and velocity response for the configuration considered subjected to a point load. Two laminates of a composite having four layers oriented at zero degree with L-configuration (Figure 2) were considered for the analysis. Classical SEA technique was used to determine SEA parameters.

![Figure 2. L-Plate Configuration](image)
The stacking sequence of the laminate is given in table 1. Length of coupling ‘L’ and the width of plate ‘W’ were considered as 1m. A unit transverse harmonic load with a frequency range from 1000 Hz to 13000 Hz was applied on one of the plates and responses on both the plates are calculated. The CLF and velocity responses have been computed using the analytical wave approach through a program developed using MATLAB. The properties of the different composite plates are given in Table 1.

| Material & Physical properties | Glass/Epoxy | Boron/Epoxy | Graphite/Epoxy |
|-------------------------------|----------------|-------------|----------------|
| Elasticity modulus of plies    | $E_x$ 39 GPa  | $E_x$ 204 GPa | $E_x$ 181 GPa  |
|                               | $E_y$ 8.6 GPa | $E_y$ 18.5 GPa | $E_y$ 10.3 GPa |
| Poisson’s ratio of plies       | $\nu_{xy}$ 0.28 | $\nu_{xy}$ 0.23 | $\nu_{xy}$ 0.28 |
|                               | $\nu_{yx}$ 0.0617 | $\nu_{yx}$ 0.0208 | $\nu_{yx}$ 0.0159 |
| Average density                | $\rho$ 2100 kg/m$^3$ | $\rho$ 2500 kg/m$^3$ | $\rho$ 2200 kg/m$^3$ |
| Specific Modulus               | $(E/\rho)$ 18.57 MPa/kg/m$^3$ | $(E/\rho)$ 81.6 MPa/kg/m$^3$ | $(E/\rho)$ 82.27 MPa/kg/m$^3$ |
| Number of plies                | $n$ 4 | 4 | 4 |
| Orientation angle              | $\theta$ 0 | 0 | 0 |
| Plate size                     | $L = 1$ m | $L = 1$ m | $L = 1$ m |
|                               | $W_1 = W_2 = 1$ m | $W_1 = W_2 = 1$ m | $W_1 = W_2 = 1$ m |
| Total thickness                | $t$ 4X10^{-3} m | 4X10^{-3} m | 4X10^{-3} m |

3.1. Analytical formulation for orthotropic plate

SEA for orthotropic plates can be applied only when modal density is high. Analytical classical wave approach for determining CLF and input power for semi-infinite orthotropic plates subjected to force impedance are discussed in the following section. The CLF for a line junction is given by [14]:

$$\eta_j = \frac{\delta f_j \tau_{ij}}{\pi f \left( 2 - \tau_{ij} \right)}$$  \hspace{1cm} (6)

where, $\delta f_j$ is average modal frequency spacing, $\tau_{ij}$ is the transmission co-efficient.

Average modal frequency spacing is given by,

$$\delta f_j = \frac{h}{\sqrt{3A}} \sqrt{c_{Lx} c_{Ly}}$$  \hspace{1cm} (7)

where, $h$ is thickness of the plate, $A$ is the surface area of the plate, $c_{Lx}$ and $c_{Ly}$ are the longitudinal wave speed on X and Y directions respectively.

Longitudinal wave speed for orthotropic plate is given by
\[ c_{lx} = \sqrt{\frac{E_x}{\rho (1-\nu_{xy}^2)}} \quad \text{and} \quad c_{ly} = \sqrt{\frac{E_y}{\rho (1-\nu_{yx}^2)}} \]  

where, \( E_x \) and \( E_y \) are Elastic modulus in X and Y directions respectively, \( \rho \) is the density of the orthotropic material considered, and \( \nu_{xy} \), \( \nu_{yx} \) are the Poisson’s ratios. Transmission co-efficient for orthotropic plate is given by,

\[ \tau_{ij} = \frac{4 \text{Re}(Z_i) \text{Re}(Z_j)}{\sum_{s=1}^{S} |Z_s|^2} \]

where \( Z_i \) is the force impedance in Z-direction and here, \( S \) is the number of subsystems connected at a common joint.

Force impedance is given by,

\[ Z = \frac{4}{\sqrt{3}} \rho h^2 \sqrt{c_{lx}^{-2} c_{ly}^{-2}} \]

4. Results and discussion
The variation of CLF and velocity response against the variation of frequency for different composite materials considered has been plotted. The CLF of the composite material plotted for analytical approach seems to be good agreement with results given by Abdullah [15].

Figure 3 and figure 4 show the variation of velocity response in plate 1 and plate 2 with respect to increase in frequency respectively. A harmonic load was applied on plate 1 and velocity response was computed on both the plates and plotted. The energies of both the plates were computed and inversed to determine the velocities of plates using inverse method [16]. The velocity of plate 1 and plate 2 against frequency in the range of 1000 Hz to 13000 Hz has been plotted for three materials Glass/Epoxy, Boron/Epoxy and Graphite/Epoxy.

![Figure 3. Velocity vs Frequency (Plate 1)](image_url)

The velocity of plate1 and plate 2 decreases with the increase in frequency. It can be seen that the Glass/Epoxy had higher velocity response compared to that of Boron/Epoxy and Graphite/Epoxy. The velocity response of Glass/Epoxy was higher because of lower longitudinal wave speed. Longitudinal
wave speed was highest in Boron/Epoxy among the three materials. Velocity of plate 1 was higher compared to that of plate 2 since load was applied on plate 1.

![Figure 4. Velocity vs Frequency (Plate 2)](image)

CLF was plotted against frequency for all the three materials as shown in figure 5. Figure shows a semi log plot of CLF vs frequencies. It can be seen that the CLF in Glass/Epoxy was least among the three materials. CLF can also be viewed as function of specific modulus from equation 7. The specific modulus of Glass/Epoxy was least among the three materials while the magnitude of specific moduliof Boron/Epoxy and Graphite/Epoxy were nearly equal. The CLF is independent of internal loss factor at high frequencies and depends more on specific modulus. The CLF of Boron/Epoxy and Graphite/Epoxy is in close match since the specific moduli of both the materials are nearly same. Coupling loss is observed to vary linearly with increase in frequency.

![Figure 5. Coupling loss factor vs Frequency](image)

SEA parameters were determined by a modal based method for symmetrically laminated composite plates by Abdullah [15]. The figure 6 shows the effect of CLF on fibre orientation with frequency. The results obtained by modal based approach showed considerable enhancement of CLF with change in fibre orientation while the conventional SEA was independent of fibre orientation. The enhanced result
of CLF was observed only in mid-frequency region while both the methods had close results in high frequency region.

![Figure 6](image)

**Figure 6.** Coupling loss factor vs Frequency: o: θ=0°, x: θ=15°, □: θ=30°, + θ=45°, __: Analytical

Comparing figure 5 and 6, it can be inferred that there is a good correlation between the results obtained by modal based method and classical wave approach in the high frequency domain.

5. Conclusion
The SEA studies on different composite materials were carried out and velocity responses and coupling loss factors were compared. The velocity response and coupling loss factors were computed using analytical classical wave approach. The coupling loss factor, which is independent of internal damping, was found varying linearly with frequency. It can be seen that the material having high specific modulus has high coupling loss factor. This work demonstrates that for heterogeneous materials the CLF depends on specific modulus. It is observed that higher the specific modulus lower will be the velocity response in the plates. A novel way of looking at the CLF has been shown in this paper specially for composite materials.

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