Stability prediction maps in turning of difficult-to-cut materials

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Abstract

This paper faces the problem of defining stability charts when cutting Inconel 718. The method predicts the free-chatter zones in longitudinal chatter when the tool vibrates in the tangential direction. This case may occur in longitudinal turning and boring cases when the toolholder must overhang long distances. The study proposes a 1/2DOF dynamic model to implement the effect of the tangential mode on chip regeneration in the regenerative plane based on obtaining experimentally a dynamic displacement factor. On sight of simulations and experimental results, the use of the model provides a reliable approach to obtain chatter free conditions.

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1. Introduction

The performance demands for parts in gas turbine engines require the utilization of materials that maintain their mechanical properties at high temperatures. Heat resistant super alloys (HRSA) are suitable for the hottest parts of a gas turbine withstanding operating conditions between 600-1000\textdegree{}C, with no damaging reduction in mechanical strength, wear and corrosion. In aerospace machining, nickel-based alloys constitute over 50\% of the weight in advanced aircraft engines. Common types include the commercial trademarks Inconel, Waspalloy or Rene, which are used in disks, cases, rings or shafts (see Figure 1a).

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Unfortunately, such materials present poor machinability and rapid damage of the tool due to: (1) abrasives carbides content, (2) tendency to work hardening, (3) chemical affinity and adhesion onto the cutting tool, (4) low thermal conductivity. In addition, the machining behaviour will be different depending on the prior treatment of the material. Typical turning operations in these kinds of components involve longitudinal turning and slotting with large tool overhangs where tangential modes in the cutting speed direction may be source of chatter (see Figure 1b). Table 1 shows the typical HRSA components and their relative operations.

Concerning to chatter modelling, the available literature has primarily focused on chatter due to flexible modes in radial or axial directions [Chiou and Liang (1998), Rao and Shin (1999), Clancy and Shin (2002), Ozlu and Budak (2007a), Ozlu and Budak (2007b)]. However, in certain cases where the toolholder overhang must comprise long distances, chatter may arise due to dynamic oscillations of the tool in the cutting speed direction. In such cases, the ram covers profound distances in Z-axis and dynamic displacements of the tool tip may result in chatter on the regenerative plane (plane XZ). This paper presents a numerical approach to deal with this particular type of vibration. The model has been applied to longitudinal turning, slender tools and has been validated for different types of tool geometry.

2. Collocation method for solving DDE

2.1 Chebyshev collocation method

A huge amount of problems in engineering are time-delayed systems in nature which require solving delay differential equations (DDE). As long as chatter is due to a closed loop where the present state of the cutting system t depends on a previous state t-τ, such type of equations has been of particular interest in chatter theory. The simultaneous utilization of Floquet's theory [Floquet (1883)] and Chebyshev polynomials [Butcher (1997), Butcher et al. (2005), Butcher et al. (2009), Elías et al. (2009)] has been useful to solve the stability of periodic systems. The problem is initiated starting from the expression of a DDE in space state form:

\[ \dot{x}(t) = [A(t)] \cdot x(t) + [B(t)] \cdot x(t-T), \]

which solution is: \( x(t) = (t) \) in \(-\tau \leq t \leq 0\). T represents the delay time, \( t \) is the current time, \( x(t) \) is a nx1 state vector in the interval \([0, \tau]\) and \( (t) \) is a nx1 state vector in the previous interval \([-\tau, 0]\). Using \( N \) collocation points, Eq. 1 is numerically approximated as:

\[ \left[D_N\right] \cdot v = \left[M_A\right] \cdot v + \left[M_B\right] \cdot w, \] (2)

where \( v \) and \( w \) are the Chebyshev polynomial discretizations of the vectors \( x(t) \) and \( x(t-\tau) \) at the collocation points \( \left(t_j(t) = \cos(j\pi/(N-1))\right) \), \( j = 0, 1, 2, ..., N-1 \). \( [M_A] \) and \( [M_B] \) are the approximation matrices, extended at the same
collocation points, which depend on the modal parameters and specific cutting forces and \([D_N]\) represents the differentiation matrix which relates the discretized vector \(x(t)\) and its derivative, and is built from essential differentiation matrix \([D]\) whose elements are described as:

\[
d_{0,0} = \frac{2n^2 + 1}{6}, \quad d_{n,n} = -\frac{2n^2 + 1}{6}, \quad d_{j,j} = \frac{-t_j}{2(1-t_j)}, \quad j = 1, 2, ... N - 1, \quad d_{i,j} = \frac{c_i (-1)^{i+j}}{c_j (t_i - t_j)}, \quad i \neq j, \quad j = 0, 1, ... N, \quad c_i = \begin{cases} 2, & \text{if } i = 0, N \\ 1, & \text{if } i \neq 0, N \end{cases}
\]

Finally, the stability is studied calculating the most dominant eigenvalues of the monodromy matrix \([U]\) built from:

\[
[U] = [D_N - M_A]^{-1} \cdot [M_B],
\]

If the maximum eigenvalues have modulus equal to unity, the system is critically stable.

2.2 2-DOF dynamic model for turning

2.2.1 Chip thickness model

When the tool overhangs long distances, the weakest direction of the tool is in the cutting direction \(Y\). In this case, modification of the chip thickness occurs when the cutting edge moves away from the plane of the cutting speed direction. This deformation is accompanied by a horizontal displacement defined by \(\delta x = v \cdot \delta y\), where \(v\) is the dynamic displacement coefficient [Tobias (1965), Endres et al. (1990)].

The model assumes the following hypotheses: 1.) rigid workpiece-slender tool; 2.) 2 dominant modes in the tangential direction; 3.) variable cutting coefficients of the cutting parameters; and 4.) process damping and wear are not considered.

For a 2DOF model with two modes in the same direction the chip thickness can be obtained from:

\[
h(t) = -\sum_{i=1}^{2} v_i \left( y_i (t) - y_i (t - T) \right),
\]

where \(v_i\) is the dynamic displacement coefficient and \(y_i(t) - y_i(t - \tau)\) the corresponding actual and prior displacements of mode \(i\).

2.2.2 Dynamic force model

For a 2DOF system, the equation of motion is expressed as:

\[
m_1 \cdot \ddot{y}_1 (t) + c_1 \cdot (\dot{y}_1 (t) - \dot{y}_2 (t)) + k_1 \cdot (y_1 (t) - y_2 (t)) = F_{y_1},
\]

\[
m_2 \cdot \ddot{y}_2 (t) + c_2 \cdot (\dot{y}_2 (t) - \dot{y}_1 (t)) + k_2 \cdot (y_2 (t) - y_1 (t)) = F_{y_2},
\]

\[
m_3 \cdot \ddot{y}_3 (t) + c_3 \cdot (\dot{y}_3 (t) - \dot{y}_4 (t)) + k_3 \cdot (y_3 (t) - y_4 (t)) = F_{y_3},
\]

\[
m_4 \cdot \ddot{y}_4 (t) + c_4 \cdot (\dot{y}_4 (t) - \dot{y}_3 (t)) + k_4 \cdot (y_4 (t) - y_3 (t)) = F_{y_4},
\]

\[
m_5 \cdot \ddot{y}_5 (t) + c_5 \cdot (\dot{y}_5 (t) - \dot{y}_6 (t)) + k_5 \cdot (y_5 (t) - y_6 (t)) = F_{y_5},
\]

\[
m_6 \cdot \ddot{y}_6 (t) + c_6 \cdot (\dot{y}_6 (t) - \dot{y}_5 (t)) + k_6 \cdot (y_6 (t) - y_5 (t)) = F_{y_6},
\]
\[ m_2 \cdot \ddot{y}_2(t) + c_2 \cdot \dot{y}_2(t) + k_2 \cdot y_2(t) - c_1 \left( \ddot{y}_1(t) - \dot{y}_2(t) \right) - k_1 \left( y_1(t) - y_2(t) \right) = F_{y,2}, \quad (6b) \]

where \( m, c \) and \( k \) are the modal parameters of each mode obtained by hammer impact tests and \( F_y \) is the dynamic force, which depends on the relative tool displacement. For mode \( i \):

\[ F_{y,i} = K_{cy} \cdot l \cdot v_i \left[ -\left( y_i(t) - y_i(t-T) \right) \right], \quad (7) \]

Introducing this relation into the dynamic equation of motion and rearranging it yields to:

\[ [M] \cdot \{\ddot{y}(t)\} + [C] \cdot \{\dot{y}(t)\} + [K^*] \cdot \{y(t)\} = [K_c] \cdot \{y(t-T)\}, \quad (8) \]

where:

\[ [M] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}; \quad [C] = \begin{bmatrix} c_1 & -c_1 \\ -c_1 & c_1 + c_2 \end{bmatrix}; \quad (9a) \]

\[ [K^*] = \begin{bmatrix} k_1 + K_{cy} \cdot l \cdot v_i & -k_i \\ -k_i & k_1 + k_2 + K_{cy} \cdot l \cdot v_i \end{bmatrix}; \quad [K_c] = K_{cy} \cdot l \cdot \begin{bmatrix} v_i & 0 \\ 0 & v_2 \end{bmatrix}, \quad (9b) \]

The system is uncoupled by using transformation:

\[ \{p(t)\} = [M] \cdot \{\dot{y}(t)\} + 0.5 \cdot [C] \cdot \{y(t)\}, \quad (10) \]

Rearranging the terms and using the state space gives:

\[ \begin{bmatrix} \{\ddot{y}(t)\} \\ \{\dot{p}(t)\} \end{bmatrix} = \begin{bmatrix} A^* \end{bmatrix} \cdot \begin{bmatrix} \{\dot{y}(t)\} \\ \{p(t)\} \end{bmatrix} + \begin{bmatrix} B^* \end{bmatrix} \cdot \begin{bmatrix} \{y(t-T)\} \\ \{p(t-T)\} \end{bmatrix}, \quad (11) \]

where \([A]\) and \([B]\), which are necessary in Eq. [4] to obtain the surface function of the eigenvalues are identified as:

\[ A = \begin{bmatrix} -0.5 [M]^{-1} [C] & [M]^{-1} \\ 0.25 [C] [M]^{-1} [C] - [K^*] & -0.5 [C][M]^{-1} \end{bmatrix}, \quad B = \begin{bmatrix} [0] & [0] \end{bmatrix}, \quad (12) \]

Therefore, the model is built based on: 1.) the assembly of a cutting model, which assigns specific cutting coefficients for any combination \((V_c, a_p, f_z)\); 2.) the modal parameters of the system; and 3.) the dynamic displacement coefficient, which describes the dynamic behaviour of the system and depends on the vibration frequency.
3. Cutting geometry modeling

The stability limit of common orthogonal turning processes can be accurately predicted by a 1D stability model [Ozlu and Budak (2007)]. However, general geometrical parameters such as inclination angle, rake angle or nose radius cannot be neglected in some cases. The more nonlinear the force-to-chip area ratio is, the greater is the need to find a precise value for the specific cutting energy.

The cutting geometry model is based on the typical mechanistic model. Having each tool its own nose radius and characteristic positioning, the corresponding specific cutting forces are calculated at different depths of cut and cutting speeds (see Table 1).

| Tool       | Nose radius $r_c$ [mm] | Approach angle $\kappa$ [º] | Rake angle $\gamma$ [º] | Inclination angle $\lambda$ [º] | Workpiece material | Cutting conditions |
|------------|------------------------|-----------------------------|-------------------------|--------------------------------|-------------------|-------------------|
| VBMT 160408 PR | 0.8                    | 93                          | 0                       | 0                               | Inconel 718       | $V_c$ [m/min] 50-125 | ap [mm] 0.2-1 | £.10 / 0.20 |
| SNMG 120408 PM | 0.8                    | 45                          | -8                      | 0                               | Inconel 718       | $V_c$ [m/min] 50-125 | ap [mm] 0.2-1 | £.10 / 0.20 |

The cutting tests were carried out in a multi-tasking turning centre (CMZ TC25BTY) using a 9257B Kistler® dynamometer (8192 samples/s) and a multi-channel analyzer (Oros35 NV-GATE by Oros®) for force acquisition. Before each cut, the worn edge was removed by a new one to avoid any influence of wear. Then, the experimental forces were filtered and post-processed with Matlab®. Finally, multiple regression analysis was applied on the historical data using the software Design Expert® to obtain the surface functions of the cutting coefficients for each cutter. Table 2 shows such functions.

| Tool       | Feed [mm/rev] | $K_{c_y}$ [N/m²]         |
|------------|--------------|--------------------------|
| VBMT 160408 PR | 0.10             | 6.181e9-1.829e9$V_c$⁻¹.967e12$a_p$⁻¹.923e11$V_c$a_p⁻¹.5041e8$V_c$²⁻¹.527e15$a_p^2$ |
| VBMT 160408 PR | 0.20             | 4.382e9-4.451e8$V_c$⁻¹.4189e11$a_p$ |
| SNMG 120408 PM | 0.10             | 5.929e9+2.567e8$V_c$⁻¹.699e12$a_p$⁻¹.251e12$V_c$a_p⁻¹.3459e7$V_c$²⁻¹.4328e15$a_p^2$ |
| SNMG 120408 PM | 0.20             | 4.572e9-3.494e8$V_c$⁻¹.167e12$a_p$ |

4. Frequency response functions

Before the tests, the toolholder was machined twice for each insert to increase the flexibility. Two different feedrates were analyzed for each tool. The frequency response functions (FRF) were obtained by hammer impact tests using an instrumented hammer (086C03, PCB Piezotronics®), a teardrop accelerometer (352C22, PCB Piezotronics®) and a data acquisition system (Oros35 NV-GATE). Figure 2a shows the FRF for both tools and Figure 2b presents the modal parameters.
5. Tool dynamic displacement

Chatter regeneration in plane XZ is due to chip thickness variation in X direction which is directly dependent on tangential deformation ($\delta_t$) as seen before. The dependence ratio $v$ is a complex factor that depends on the vibration frequency and dynamics tests are necessary to account for this vibration. Table 3 shows the general conditions of such tests which were done scanning increasing depths of cut.

Table 3. Dynamic tests for frequency vibration measurements.

| Tool          | Cutting parameters |
|---------------|--------------------|
|               | $f_z$ [mm/rev] | $a_p$ [mm] | $V_c$ [m/min] |
| VBMT 160408 PR| 0.10 / 0.20      | 0.20-1.00  | 50-150        |
| SNMG 120408 PM| 0.10 / 0.20      | 0.20-1.00  | 50-150        |

The cutting speed should receive attention due to its influence on vibration frequency and factor $v$. Obviously, increasing the depth of cut tends to instability moving away the vibration frequency from the natural frequency. Finally, the effect of the feed is more delicate. In a nutshell, it can be said that using high feedrates at high cutting speeds is recommended the lower is the natural frequency of the system.

While the static coefficient is easily derived making use of the formulation for a cantilever beam given by Strength of Materials [Arnold (1946)], the dynamic factor is the key to construct stability charts. In chatter due to a tangential mode of the tool, the stability of the system is strongly determined by the dynamic displacement coefficient which has decisive influence on the chip thickness. This non-dimensional parameter depends on the dynamic features of the system but also on the cutting geometry of the tool. Indeed, this parameter is built from the modal stiffness and natural frequency of the system, the cutting coefficient, the cutting edge engaged during cutting as well as from vibration frequency:

$$v = \frac{k}{K_{cy} \cdot b \left( \frac{\alpha_k}{\omega_n} \right)^2 - 1} \quad (13)$$

If $v<0$, the cutting edge moves against the workpiece, the chip thickness increases (see Figure 3) and the vibration frequency reduces itself. On the contrary, if $v>0$, the cutting edge moves away from the workpiece and the system tends to stability. Note that sign of $v$ depends on the tool vibration frequency during cutting, which is responsible of the stability/instability of the system, while the specific cutting energy and modal stiffness tend to decrease or increase this absolute state.
To simulate the stability boundaries, a value of the $v$ factor is needed. Dynamic tests (see Table 3) will supply useful information about the dynamic behaviour of the system for a particular combination of modal parameters, cutting conditions and tool geometry. The vibration frequency during each of the cuts was obtained and then introduced in Eq. 13 to calculate $v$. Figures 3a and 3b show the dynamic displacement coefficients corresponding to the second mode of VBMT insert when $f_z=0.1$ and $0.2$ [mm/rev], respectively:

![Fig. 3. $v$ functions of mode 2645 Hz (VBMT). (Left) $f_z=0.1$ [mm/rev]; (right) $f_z=0.2$ [mm/rev].](image)

### 6. Experimental validation

This section shows the correlation between the analytical predictions and the experimental data. The experiments cover two different modal properties and tool geometries. As chatter criterion, the cutting forces were recorded and transformed to frequency spectrum in order to visualize the dominant peak and its magnitude and relative location with respect to the natural frequency and to the cutting frequency and its multiples. All the cutting tests were carried out with new edges.

The simulations were performed with 1200 collocation points and mesh density of 100x100 points for rectangles of 100 [rpm] x 1 [mm]. To obtain the stability charts as function of cutting speed an average workpiece diameter was chosen from all the cutting passes. Table 4 shows the cutting conditions of the validation tests.

| Tool          | Cutting parameters | Vc [m/min] | D [mm] |
|---------------|--------------------|------------|--------|
| VBMT 160408 PR| $f_z=0.1$ [mm/rev] | 50-125     | 82.95  |
| VBMT 160408 PR| $f_z=0.2$ [mm/rev] | 50-125     | 90.05  |
| SNMG 120408 PM| $f_z=0.1$ [mm/rev] | 50-125     | 75.60  |
| SNMG 120408 PM| $f_z=0.2$ [mm/rev] | 50-125     | 68.85  |

6.1 Simulations vs. experimental results for VBMT insert

The comparison of experimental and simulation results can be seen in Figures 4a and 4b. A reasonable agreement is found between the analytical and experimental results. Taking a look to these figures, note the influence of the feedrate over the boundary limits. For example, point $V_c=100$ [m/min]/$a_p=0.80$ [mm] showed vibration frequencies of 1870 [Hz] for $f_z=0.1$ [mm/rev] (Figure 5a, stable) and 1777 [Hz] for $f_z=0.2$ [mm/rev] (Figure 5b, unstable). The difference in vibration amplitudes is also observed. For the first feedrate, the system benefits from wider free chatter zones at high cutting speeds. Feed affects to $K_{cy}$ but more important, to $\alpha_c$ (see Eq.13). In this case, its effect is definitive causing higher vibration frequencies in comparison with the second feed tested. These values lead to increased dynamic displacement coefficient and thus, wider free-chatter zones.
5.2 Simulations vs. experimental results for VBMT insert

Figures 4a and 4b show the corresponding graphs in the case of the VBMT insert. On one hand, similar tendencies and absolute values are found. So for this tool, increasing the feedrate does not penalize excessively the maximum allowable depth of cut. However, it must be highlighted that introducing high feedrates reduces the transition zone between stable cases and chatter and so, the depth of cut must be carefully programmed. For example, point $V_c=75$ [m/min]/$a_p=0.80$ [mm] showed a nearly stable cutting ($f_c=1600$ [Hz]) for $f_z=0.1$ [mm/rev] while $f_z=0.2$ [mm/rev] lead to clear chatter ($f_c=1482$ [Hz]).

6. Conclusions

In this work, a new approach has been used to simulate the instability in longitudinal turning due to a non-rigid tool in the tangential direction. This type of chatter is not usually studied but may appear in external and internal turning of deep cylindrical parts. The stability boundaries profit from the fact that the collocation algorithm allows
to include the influence of various cutting parameters. The method has been validated for Inconel 718 using different combinations of tool geometries and cutting parameters. Some of the contributions are:

- A methodology has been presented to simulate precise stability charts in longitudinal turning including 1-DOF/2-DOF dynamic modelling which accounts for tangential modes and dynamic characterization of the tool by means of the dynamic displacement factor $v$, necessary to perform the simulations.

- In chatter cases due to modes in tangential direction, the effect of cutting speed is even more pronounced than in other turning cases where the relative work-tool displacement takes place in chip thickness regenerative plane XZ. In fact, it may lead to differences up to 500% in the admissible depth of cut. The envelope of the stability diagrams is no longer a horizontal line. However, cutting at high cutting speeds will lead to increased wear of the tool. So, a compromise must be kept between the couple cutting speed-depth of cut.

- It has been noted that despite the SNMG tool had higher values of stiffness and damping ratios than VBMT tool, the critical limit at low cutting speeds does not take any benefit. Also, using high feedrates at high cutting speeds is recommended the lower is the natural frequency of the system.

- The method has proved to be accurate since 90% of the tested points were in touch with reality. The accuracy has been manifested in all cases below 15-20% of the depth of cut.

- The main limitations are: on one hand, the simulations need information about the frequency vibrations. Due to this, dynamic tests are required not only for model validation but also to account for the vibration frequencies. On the other hand, computational times are a drawback of the method, even more at low cutting speeds and/or high natural frequencies.

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