Rolling Bearings Fault Diagnosis Using VMD and Multi-tree Mahalanobis Taguchi System

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Abstract. In order to effectively identify the various faults of rolling bearings and the severity of the faults, an intelligent fault diagnosis system based on variational mode decomposition and multi-tree Mahalanobis Taguchi system with multiple mahalanobis distance was proposed. Vibration signals were decomposed into multiple BLIMFs by variational mode decomposition, and time domain and frequency domain features were extracted. The multiple mahalanobis distance method was used to solve the problem of many features in the diagnosis system. By using the advantages of Mahalanobis Taguchi system in features optimization, sensitive modal components for diagnosis and recognition were selected. Multi-tree Mahalanobis Taguchi system was constructed for intelligent identification of multiple fault states. Finally, rolling bearings fault data was tested to verify the accuracy of the algorithm and compared with other algorithms. The results show that the algorithm can simplify the diagnosis system, reduce training time and improve the recognition accuracy.

1. Introduction
Rolling bearings are key components of rotating machinery. Their running state directly affect performance, running efficiency and useful life of equipment. The failure of rolling bearing usually occurs in the components of the bearing, including inner race, outer race and rolling elements. Mechanical equipment vibration signals are mostly non-stationary and nonlinear signals. Early fault signals are very weak, which may encounter attenuation and be easily interfered by background noise in the transmission path. Therefore, extracting fault information from such signals is the key to accurate diagnosis of mechanical equipment [1].

Empirical mode decomposition (EMD) proposed by Huang et al. [2] can adaptively decompose a nonstationary signal into multiple intrinsic mode functions (IMFs). However, EMD has some disadvantages like mode mixing problem and sensitivity to noise. Ensemble Empirical Mode Decomposition (EEMD) eliminates the noise of the original signal by adding Gaussian random white noise, and restrain mode mixing problem in EMD [3]. However, they still have defects in anti-noise, end effect, and incorrect separation of signals with similar frequencies.

Variational modal decomposition (VMD) has been proposed by Dragomiretskiy and Zosso [4]. It can adaptively decompose vibration signals into a number of band-limited intrinsic mode functions (BLIMFs) and get rid of a series of problems caused by recursive decomposition methods such as EMD. Many scholars have successfully applied VMD to signal decomposition of rolling bearing fault diagnosis [5-6]. VMD needs to set the number $k$ of BLIMFs, and many scholars have discussed the setting of $k$ value [7-8]. If we setup initial $k$ values firstly, then sensitive modal components in every
fault condition can be intelligently selected based on data drive. It not only won’t cause excessive decomposition, but also be favourable for removing redundant modal components and accelerating the identification efficiency. It is the point of this study.

On the other hand, selecting an appropriate classifier to intelligently identify the bearing fault is important. Many scholars have tried various intelligent algorithms to identify rolling bearing fault, including support vector machine (SVM), artificial neural network (ANN) and other intelligent algorithms. However, all kinds of algorithms also have their own defects. For example, SVM has low classification efficiency when solving multi-classification problems. In ANN, network structures are difficult to determine and easy to fall into local minima when training data.

Mahalanobis Taguchi system (MTS) has been proposed by quality engineering expert Dr.Genichi Taguchi in 1990s, which is a multi-system pattern recognition method [9]. MTS is successful because it has many advantages. MTS is based on data driven without relying on the data distribution assumption. It can realize dimension reduction based on OAs and SN ratios. Diagnosis and prediction are simple and fast. In recent years, many researchers have used MTS into rolling bearing fault diagnosis.

The paper decomposed the vibration signal into multiple BLIMFs through VMD, and extracted multiple fault features in each component. At this time, for each vibration signal, the number of fault features is huge. Due to the limitation of OAs in designing orthogonal experiments, MTS is difficult to optimize the classification system containing many variables. Therefore, firstly, this paper adopted Mahalanobis Taguchi system based on multiple mahalanobis distance. Feature subsets replaced individual features in participating in the construction of the classifier to solve the problem of large number of features. At the same time, OAs and SN ratios were used to select important feature subsets in each signal. Thus, VMD was combined with MMD-MTS classifier. Secondly, due to MTS is designed to solve binary classification problem, this paper built Multi-tree MTS (MT-MTS) based on training data to solve the fault diagnosis of rolling bearings. And the diagnosis results using other algorithms were compared with improved MT-MTS based on VMD and MMD to verify this algorithm’s rationality and effectiveness.

2. Variational mode decomposition and features extraction

2.1. Variational mode decomposition

VMD obtains adaptive signal decomposition thought searching optimal solution of constrained variational model. Accordingly, the input signal is decomposed into a series of modal component with sparse properties.

In VMD, the square norm $L^2$ of the gradient of the demodulated signal is calculated, and the bandwidth of each mode component is estimated. The expression for the corresponding constrained variation model is as follows:

$$\min_{\{u_k,\{\omega_k\}\}} \left\{ \sum_k \left| \hat{\chi}_k((\omega(t) + \frac{j}{\pi t})u_k(t)e^{-j\omega_k t}) \right|^2 \right\},$$

s.t. $\sum_k u_k = f$

Where, $\{u_k\} = \{u_1, \ldots, u_k\}$ represent the $k$ BLIMF after decomposition, and $\{\omega_k\} = \{\omega_1, \ldots, \omega_k\}$ represent central frequency of each component.

The second-order penalty factor $\hat{\chi}$ and Lagrange multiplying operator $\lambda(t)$ are introduced to solve the optimal solution of the constrained variational problem. The extended Lagrange expression is as follows:

$$L(\{u_k\},\{\omega_k\}, \lambda) = a \sum_k \left| \hat{\chi}_k((\omega(t) + \frac{j}{\pi t})u_k(t)e^{-j\omega_k t}) \right|^2 + \left| f(t) - \sum_k u_k(t) \right|^2 + \lambda < f(t) - \sum_k u_k(t) >$$
The “saddle point” of extended Lagrange expression is calculated via the alternating direction method of multiplier algorithm. The detail algorithm can be found in [4].

2.2. Choosing sensitive BLIMFs in the diagnostic process

When signals are decomposed using VMD, $K$ value need to be determined. Many scholars have studied on determining $K$ value based on the principle of similar center frequency, kurtosis selection criteria, particle swarm optimization, etc. However, in order to unify measurement in all states, the number $K$ of modal components in each state are required to be equal to participate in the diagnosis and identification of the classifier.

Firstly, the initial $K$ value is given to decompose the vibration signal by VMD base on the principle of similar center frequency. Then, because MTS can optimize features, invalid modal components are eliminated in MTS’ diagnosis process. It can reduce computational complexity and improve the diagnosis accuracy. See the following section for specific elimination methods. This method can not only obtain modal components that affect the diagnosis result and achieve the best diagnosis effect, but also eliminate the invalid modal components based on the diagnosis process, making the signal decomposition method closely combined with the intelligent classifier.

2.3. BLIMFs’ features extraction

Vibration signals contain a large amount of information during mechanical operation. The success of fault diagnosis depends on the selection of BLIMFs’ features. Time domain features are intuitive and easy to understand, which is the original basis for signal fault diagnosis is. Extracting time domain features can judge the running state. Therefore, this paper extracts 10 time domain features of each modal component to reflect the weak change of vibration signal in time domain.

However, time domain features are difficult to determine the location, type and severity of the faults. Therefore, this paper continues to extract two frequency domain features from each modal component through fast Fourier transform. Frequency domain features can distinguish different faults. To sum up, a total of 12 features in time domain and frequency domain are extracted for each modal component. Then the total number of features in the system should be $12 \times K$. $K$ is the number of components in modal decomposition. All selected features are presented in table 1.

| Time domain features | Frequency domain features |
|----------------------|--------------------------|
| Mean                 | Wave index               |
| Root mean square     | Peak index               |
| Standard deviations  | Impulsion index          |
| Skewness             | Tolerance index          |
| Kurtosis             | Kurtosis index           |

3. The improved multi-tree Mahalanobis Taguchi system

3.1. Mahalanobis Taguchi system

MTS includes Mahalanobis space (MS) and Taguchi method. MS is constructed as a reference space by using normal samples of standardized data. Its core is Mahalanobis distance (MD), which is used to measure the similarity between unknown samples and known samples. In Taguchi method, orthogonal arrays (OAs) and signal-to-noise (SN) ratios are used to measure the contribution of each variable in system. Selecting important variables from the original variables is essential to MTS. Finally, unknown samples are identified by threshold. More details of the MTS algorithm can be found in [9]. In this paper, use the threshold $T$ [10] to differentiate between normal and abnormal samples:

$$T = \mu + 2.66\bar{R}$$

(3)
Where $\mu$ is mean of the normal samples MDs, $\overline{R}$ is mean of normal samples MD moving ranges.

3.2. Multiple Mahalanobis distance

In the process of multiple pattern recognition, the selection of features not only determines the recognition ability of the system, but also determines the recognition speed of the system. However, as the number of features increases, classifiers become very difficult to process large data sets. Also, MTS is more difficult to choose the important features through OAs and SN ratio. Therefore Dr. Genichi Taguchi designed MMD method.

In MMD method, a large number of features are divided into feature subsets. MD of each feature subset is calculated to obtain a new measurement for measuring the huge features, which is recorded as MMD. MMD is introduced to compute SN ratios and thereby select suitable feature subsets. MMD method is suitable for processing a large number of features, and it can effectively reduce the complexity of the problem. The specific steps of MMD-MTS method are as follows:

**Step 1:** Defines feature subsets of the original features. These feature subsets may be extracted in different modes. Meanwhile, the number of variables in these feature subsets can be inconsistent.

**Step 2:** MD corresponding to each feature subset of normal samples and abnormal samples is calculated and recorded as $MD_{nor}$ and $MD_{abn}$. Then the square root of the $MD_{nor}$ and $MD_{abn}$ are calculated and recorded as $MMD_{nor}$ and $MMD_{abn}$.

**Step 3:** These feature subsets are regarded as feature variables (control factors). The $MD_{nor}$ and $MD_{abn}$ would provide required data for these subsets. If there are $k$ feature subsets, the problem will be transformed into a new multi-system pattern recognition problem with $k$ variables. At this time, $MMD_{nor}$ and $MMD_{abn}$ calculated according to $MD_{nor}$ or $MD_{abn}$ will be used as the new measurement to participate in the construction and validity analysis of MTS.

**Step 4:** For each experiment of OA, the SN ratio is calculated by $MMD_{abn}$ and the important feature subsets are identified.

**Step 5:** According to the optimized feature subsets of the system, the unknown samples are identified and predicted thought the new measurement and threshold.

3.3. MMD-MT-MTS

MTS is used to solve the problem of binary classification in multivariate pattern recognition. However, there are a lot of multi-classification problems in practice. Therefore, constructing reasonable multi-classification rules are an important means to improve the classification accuracy of classifiers.

In computer science, a multi-tree is an ordered tree in which each node contains multiple subtrees. Different hierarchical structures of multi-tree have great influence on classification accuracy. If a classification error occurs at a certain node, the error will be extended to the next node, and the error rate of subsequent classification will be higher and higher. Therefore, it is necessary to select the appropriate multi-tree structure. Firstly, we divide the classes that are easy to be divided (which are not easy to generate misclassification), and then divide the classes that are not easy to be divided. Thus the possible errors can be kept away from the roots as far as possible. MMD-MT-MTS is divided into the following four steps:

**Step 1:** Each class executes MMD-MTS. Calculate recognition accuracy of each class as reference space for the same testing set. The class with the highest recognition accuracy is selected as the reference space of root node. The root node samples data are removed in training set and testing set. The MMD-MTS is executed for every remaining category again and the recognition accuracy is calculated. The class with the highest recognition accuracy is taken as the reference space of the node in the second level. And so on until every level node is found.

**Step 2:** If a class contains many small classes, it is first regarded as a large class to execute step 1. When the class is divided as "leaf" node, the small categories begin to be divided. Calculate recognition accuracy of each small categories as reference space for the same testing set. The class
with the highest recognition accuracy is selected as the reference space of the node. Chen [11] introduces MTS to identify Fisher’s irises data based on MD range and demonstrates the effectiveness of MTS in multi-classification. Therefore, choose a special category as the reference space and the MMDs range are used as the selection path. Then the nodes are divided into endpoints of small categories. Finally, the multi-tree structure is constructed.

**Step 3:** Once the multi-tree is established, the important feature sets are selected using OAs and SN ratios for each node. At the same time, threshold and MMD range is calculated as branch path.

**Step 4:** The testing samples are identified thought the MMD-MT-MTS model.

### 4. Fault diagnosis of rolling bearings based on MMD-MT-MTS

The vibration signal of rolling bearing is decomposed into $K$ modal components by VMD, and features are extracted for each component. The features are extracted from this $K$ modal components are regarded as $K$ feature subsets and used to construct the initial feature subsets of MMD-MT-MTS. Then a multi-tree structure is constructed and MMD-MT-MTS is executed to realize intelligent diagnosis of multiple types of faults. The rolling bearing fault diagnosis method based on VMD and MMD-MT-MTS includes three stages. The specific steps and flow chart are shown as follows:

**Step 1:** For rolling bearing vibration signals with multiple states, VMD is used to decompose the original vibration signals, and the time domain and frequency domain features of each component are extracted to construct the initial feature subsets of MMD-MT-MTS.

**Step 2:** The collected features data are divided into training sets and testing sets according to cross-validation rules. The training samples are used to construct a proper multi-tree, and the reference space of each node and the branch paths based on threshold or MMD ranges are determined to construct an MMD-MT-MTS classifier.

**Step 3:** Through MMD-MT-MTS classifier, fault diagnosis is carried out on testing samples to identify fault types.

The algorithm flow chart is shown in figure 1.

![Figure 1. The flow chart of rolling bearing fault diagnosis.](image)

### 5. Results and discussion

In order to verify the validity of VMD and MMD-MT-MTS in mechanical fault diagnosis, bearing data were collected from the experimental rig of rolling bearing failure simulation in the Electrical Engineering Laboratory of Case Western Reserve University. The experimental equipment is shown in Figure 2.
Motor bearings were seeded with faults using electro-discharge machining (EDM). Faults ranging from 0.007 inches in diameter to 0.021 inches in diameter were introduced separately at the inner raceway (IR), rolling element (i.e. ball) (B) and outer raceway (OR). In this paper, the diameter of damage is 0.007 inches for early failures, 0.014 inches for moderate failures and 0.021 inches for severe failures. According to this classification, bearing failure types are determined to be 10 categories, and the specific category labels are shown in Table 2.

| Bearing state | Label | Fault diameter (inches) |
|---------------|-------|-------------------------|
| N             | 1     | 0                       |
| IR            | 2/3/4 | 0.007/0.014/0.021       |
| B             | 5/6/7 | 0.007/0.014/0.021       |
| OR            | 8/9/10| 0.007/0.014/0.021       |

The original vibration signal is divided into a plurality of sample signals by a window of the same length. The vibration signal images in the normal state and the other three early fault states are plotted, as shown in Figure 3. The 100 vibration signals data in every state were selected, then the sample size is 1000. The validity of VMD and MMD-MT-MTS in rolling bearing fault diagnosis was verified by 5-fold cross-validation, that is, 80 groups of samples in every state were taken as training set and the remaining 20 groups were taken as testing set in each experiment.

5.1. Signal decomposition and feature subsets construction
1000 signals in the sample were decompose by VMD. Based on the principle of centre frequency approximation [7], the number \( K = 6 \) of BLIMFs in VMD was preset. Figure 4 shows the BLIMFs of a signal decomposed by VMD in the normal state. At the same time, the time domain and frequency domain characteristics of each modal component in the signal are extracted based on Matlab software, and the Mahalanobis distance of each modal component is calculated as characteristic parameter of this components.

5.2. Construction and fault identification of MMD-MT-MTS classifier
This section takes a cross-validation as an example to illustrate the experiment.

**Step 1:** Based on the principle of 5-fold cross-validation, the training samples were selected, totalling 800 samples. The features of each modal component are extracted as a feature subset of each
sample. The MDs of these feature subsets are calculated and the root mean square of these MDs is calculated as a new measurement scale to participate in the construction of the classifier.

**Step 2:** Build a multi-tree structure. Firstly, the different damage degree of each state as a class participates in the selection of root nodes. Calculate the accuracy of the testing set with the four states as the reference space, as shown in Table 3. As can be seen from Table 3, in the first level, the classifier has the highest accuracy when the normal state as the normal sample and the other three fault states as the abnormal sample. Therefore, the root node selects the MMD-MTS classifier with the normal state as the reference space.

After the root node is determined, the normal state samples are removed. The training set only has samples in IR, B and OR fault state. Continue with the MMD-MTS for these three states and calculate the accuracy of the testing set, as shown in Table 3. The level 2 node selects the MMD-MTS classifier with IR fault sample as the reference space. By analogy, the level 3 node selects the MMD-MTS classifier with B failure sample as the reference space.

After each fault state is divided into node, the different damage degree of each state began to be divided. It is interesting to find that the recognition accuracy is highest when the early failure as the reference space. Thus, the reference space is constructed based on the samples of the early failure, and MMD-MTS is executed to classify the damage degree and form leaf nodes, so as to construct a complete multi-tree structure, as shown in Figure 6.

**Table 3.** Executed accuracy of MMD-MTS algorithm.

| Multi-tree levels | N       | IR      | B       | OR      |
|-------------------|---------|---------|---------|---------|
| The level 1       | 100%    | 97.5%   | 97.5%   | 97%     |
| The level 2       | removed | 97.7%   | 96.7%   | 96.1%   |
| The level 3       | removed | removed | 99.2%   | 97.5%   |

After confirming the structure of multi-tree, each node executes MMD-MTS. Calculate the MMDs of normal and abnormal samples in training samples, and verifies the validity of the reference space of each node. The calculated MMDs are shown in Figure 5.

![Figure 5](image1.png)

**Figure 5.** MMD calculated with N, IR, B, IR2, B5 and OR8 samples as reference space respectively.

According to the constructed multi-tree, MMD-MT-MTS is performed to select the feature subsets of the four states. The OAs and SN ratios response values of each component in the four states are calculated by MMD, and the sensitive modal components are selected as shown in Table 4. At the same time, the path conditions of each branch of the multi-tree are determined according to the thresholds or MMD ranges calculated by the MMD-MT-MTS. The results and multi-tree construction are shown in Figure 6.
Table 4. The selected BLIMFs of fault signals.

| State | The selected BLIMFs       |
|-------|--------------------------|
| N     | IMF1, IMF2, IMF4, IMF6    |
| IR    | IMF1, IMF2, IMF4, IMF5, IMF6 |
| B     | IMF2, IMF3, IMF5          |
| OR    | IMF4, IMF5, IMF6          |

Table 5. The result of MMD-MMTS fault diagnosis.

| Actually class | Testing results | Accuracy |
|----------------|-----------------|----------|
|               | N1 | IR2 | IR3 | IR4 | B5 | B6 | B7 | OR8 | OR9 | OR10 |         |
| N1             | 20 | 0   | 0   | 0   | 0  | 0  | 0  | 0   | 0   | 0     | 100%    |
| IR2            | 0  | 20  | 0   | 0   | 0  | 0  | 0  | 0   | 0   | 0     | 100%    |
| IR3            | 0  | 0   | 20  | 0   | 0  | 0  | 0  | 0   | 0   | 0     | 100%    |
| IR4            | 0  | 0   | 1   | 19  | 0  | 0  | 0  | 0   | 0   | 0     | 95%     |
| B5             | 0  | 0   | 0   | 0   | 0  | 20 | 0  | 0   | 0   | 0     | 100%    |
| B6             | 0  | 0   | 0   | 2   | 0  | 18 | 0  | 0   | 0   | 0     | 90%     |
| B7             | 0  | 0   | 0   | 0   | 0  | 0  | 18 | 1   | 0   | 0     | 90%     |
| OR8            | 0  | 0   | 0   | 0   | 0  | 0  | 0  | 20  | 0   | 0     | 100%    |
| OR9            | 0  | 0   | 2   | 0   | 0  | 0  | 0  | 0   | 18  | 0     | 90%     |
| OR10           | 0  | 0   | 0   | 0   | 0  | 0  | 0  | 0   | 0   | 20    | 100%    |

5.3. Discussion

After 5-fold cross-validation, the identification results of various states of rolling bearings are shown in Table 4. The accuracy of the 200 test data predictions was 96.5%. The experimental results show that the rolling bearing fault diagnosis algorithm based on VMD and MMD-MMTS is effective and accurate.

In the MMD-MMTS algorithm, the effective feature subsets of each state can be effectively identified by OAs and SN ratios. The system selected four sensitive BLIMFs in the normal state and 3-5 sensitive BLIMFs in other states. It realizes the components reduction based on the diagnosis.
process. The effective modal components obtained through the screening can be used to assist the subsequent fault diagnosis work.

In addition, Table 6 lists the accuracy of the diagnostic recognition based on the same database with other algorithms. As can be seen from Table 6, the VMD and MMD-MMTS algorithms have higher recognition accuracy than other algorithms. This also shows that the improvement of the method in this paper has practical significance.

Table 6. The accuracy of algorithms.

| Signal decomposition and classification algorithm | Accuracy(%) | The number of features |
|--------------------------------------------------|-------------|------------------------|
| VMD+MMD+MT+MTS                                   | 96.5        | 3-5                    |
| EEMD+AMTS                                        | 94          | 9                      |
| VMD+SVM                                          | 95.5        | 3-5                    |

6. Conclusion
Based on VMD and MMD-MT-MTS method, an intelligent fault diagnosis method is presented in this paper. The vibration signal is decomposed into several modal components by VMD, and the extracted features of each component construct the feature subsets. The MDs of the subsets are regarded as the characteristic value of the signal. Taking the root mean square of the MDs as a new measurement scale, we participate in the construction of the MMD-MTS. At the same time, a multi-tree is constructed for multi-class fault identification, so as to realize the intelligent fault diagnosis of rolling bearings. Compared with other research methods, the advantages of this paper are shown as follows:

(1) Based on the diagnosis process, the sensitive modal components in each state are selected reasonably through OAs and SN ratios of the MMD-MTS. The redundant modal components are eliminated, and the diagnosis efficiency is improved. The VMD signal decomposition method is closely combined with the MMD-MT-MTS classifier based on finding the key modal components in the diagnosis process.

(2) In order to solve the problem of the large number of features, MMD method is used to construct MMD-MT-MTS classifier in the form of feature subsets and new measurement scale. It not only fully reflects the fault characteristics, but also optimizes the subsets of features based on the advantages of the classifier itself. It realizes the real dimension reduction of the system, and the recognition effects are very good.

(3) A multi-classification MTS based on multi-tree is proposed. By constructing MMD-MT-MTS for multi-classification fault diagnosis of rolling bearings, the severity of rolling bearings under different fault states can be identified effectively. This method not only solves the limitation that MTS is binary classifier, but also makes the multi-classification recognition based on multi-tree, which is easy to operate and has high classification accuracy.

In the future, MMD-MT-MTS will be applied to more equipment for mechanical fault diagnosis. At that same time, the method can be extended to more computer technology field, just like speech recognition and face recognition.

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