MHD SLIP FLOW OVER AN EXPONENTIALLY STRETCHING SHEET IN THE PRESENCE OF ENGINE OIL

Fauzia Raza¹ and Dr. A. K. Tiwari²

¹Department of Mathematics, Shri Venkateshwara University, Gajraula, U.P., India
²Department of Mathematics, Birla Institute of Technology (Mesra), Patna Campus, Bihar, India

Abstract-The problem of MHD slip flow over an exponentially sheet in the presence of engine oil through a porous medium is considered. Engine oils are made from a heavier, thicker petroleum hydrocarbon base stock derived from crude oil, with additives to get better certain properties. The main part of a certain sort of engine oil is made up of hydrocarbons with between 18 and 34 carbon atoms per molecule. Two types of CNTs, namely, single- and multi-wall CNTs are used with water, engine oil as base fluids. The empirical correlations are used for the thermo-physical properties of CNTs in terms of the solid volume fraction of CNTs. The boundary layer partial differential equations are first converted to an ordinary equation. The consequences of variant parameters such as radiation parameter Nr, solid volume fraction φ of the nanoparticles, Prandtl number Pr, Lewis number Le and combined magnetic and porosity parameter M on velocity, temperature and concentration profile. The obtained results are perceived through plots.

Keywords: Nanofluid, Porous Media, Stretching sheet, MHD, Engine oil, Thermal radiation, “bvp4c”.

I. INTRODUCTION

Nanofluids are liquid suspensions of nanoparticles whose critical measures are less than 100 nm[1]. Due to their excellent properties associated with heat transfer performance, mass transfer and chemical stability [2-5] and so on. Nanofluids have become greatly pleasing and exhibit a great number of potential applications in many fields like having transport, refrigeration and making somewhat cold processes. Sakiadis[6] analyzed the problem of boundary layer flow of Newtonian and non-Newtonian fluids. After the pioneering work by Sakiadis[6], many researcher worked on boundary layer flow over linear and non-linear stretching surface[7-15]. The consequences of variant parameters governing the flow of a viscous fluid past a nonlinearity stretching sheet was analyzed by Vajravelu[16], Cortell[17-19] and Afzal [20]. The problem of flow over a quadratic stretching sheet was studied by Kumaran and Ramaniah[21]. Despite all these studies, not too many analyses focused on an exponentially stretching sheet. Nadeem and Lee[22] has taken into consideration the problem of boundary layer flow of nanofluid over an exponentially stretching surface and he made observations that the boundary layer thickness reduces with increase in thermophoresis parameter. Bidin and Nazar[23], Nadeem et al[24], Magyari and Keller[25], Sanjayanand and Khan[26], Sajid and Hyat [27], Partha et al[28] and Elbashbeshy[29] analyzed heat transfer temperament past an exponentially stretching sheet.

The suspension of nanoparticles contained by the base fluid is not enough to improve the thermal conductivity alone, since the size and shape of particles also matter. Murshed et al[30] considered the effects of carbon nanotubes (CNT) and he perceived that the CNT provide around six times improved thermal conductivity as equated to other materials at the room temperature. The CNT are the allotropes of carbon with a cylindrical nanostructure. Carbon Nanotubes, long, thin cylinders of carbon, were revealed in 1991 by Sumio Iijima. These are huge macromolecules that are distinctive for their size, shape, and significant physical properties. They can be supposed of as a sheet of graphite (a hexagonal
lattice of carbon) rolled into a cylinder. These interesting structures have sparked much excitement in recent years and a large amount of research has been devoted to their understanding. Currently, the physical properties are still being discovered and questioned had arguments about.

Nanotubes have a very wide range of electronic, thermal, and to do with the structure properties that change depending on the different kind of nanotube (formed by its distance across circle, length, and chirality, or twist). The three main types of CNT are, single wall carbon nanotubes (SWCNT), double wall carbon nanotubes (DWCNT) and multiple wall carbon nanotubes (MWCNT). Carbon nanotubes have an undergone growth tensile strength than steel and Kevlar. Their strength comes from the sp² bonds between the specific carbon atoms. This bond is even durable than the sp³ bond found in diamond. Under high pressure, individual nanotubes can bond together, trading some sp² bonds for sp³ bonds.

This provides the chance of producing long nanotube wires. Carbon nanotubes are not merely strong, they are also elastic.

It of a nanotube and cause it to bend without damaging to the nanotube and the nanotube will give to its first form when the force is took off. Nadeem et al[31] analysed convective heat transfer in MHD slip flow over a stretching surface in the presence of CNT and they found that the engine oil based CNT have higher skin friction and heat transfer rate as compared to water based CNT. The present paper is motivated to understand the second law of thermodynamics analysis for engine oil induced by an exponentially stretching surface with convective surface boundary conditions in presence of thermal radiation in a porous media.

II. FORMULATION OF THE PROBLEM

Consider the two-dimensional steady flow of engine oil past an exponentially stretching sheet. Let the x-axis is taken along the stretching surface in the direction of motion and y-axis is normal to it. The plate is stretched along the x-direction with a velocity \( U_w = U_0 e^{x/t} \) defined at \( y = 0 \). A variable magnetic field \( B(x) = B_0 e^{x/t} \) is applied normal to the sheet, \( B_0 \) being a constant. The thermo-physical properties of regular fluid and nanoparticles are given in Table 1 and Table 2.

| Base fluids | Nanoparticles |
|-------------|---------------|
| water       | Engine oil    |
| \( \rho \)  | SWCNT         |
| 997         | 884           |
| \( \rho Cp (\times 10^6) \) | 2600 |
| 4179        | 1910          |
| \( k \)     | 6600          |
| 0.613       | 3000          |

| SWCNT | \( \phi \) | \( \rho \) | \( \rho Cp (\times 10^6) \) | \( k \) | MWCNT | \( \phi \) | \( \rho \) | \( \rho Cp (\times 10^6) \) | \( k \) |
|-------|-------------|--------|-----------------|-----|-------|-------------|--------|-----------------|-----|
| Water | 0.00        | 997    | 4.167           | 0.613 | Water | 0.00        | 997    | 4.167           | 0.613 |
|       | 0.04        | 1061   | 4.044           | 1.051 |       | 0.04        | 1021   | 4.051           | 1.011 |
|       | 0.08        | 1125   | 3.921           | 1.528 |       | 0.08        | 1045   | 3.935           | 1.444 |
|       | 0.12        | 1189   | 3.799           | 2.048 |       | 0.12        | 1069   | 3.819           | 1.916 |
|       | 0.16        | 1253   | 3.676           | 2.618 |       | 0.16        | 1093   | 3.703           | 2.434 |
|       | 0.2         | 1317   | 3.554           | 3.245 |       | 0.2         | 1117   | 3.588           | 3.002 |
| Engine| 0.00        | 884    | 1.688           | 0.144 | Engine| 0.00        | 884    | 1.688           | 0.144 |
The continuity, momentum, energy and concentration equations governing such type of flow can be written as

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  
\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2} - \frac{\nu}{k} u - \frac{\sigma B^2(x)}{\rho_{nf}} u \]  
\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho_c p)_{nf}} \frac{\partial \theta}{\partial y} \]  
\[ u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} - \gamma_0 (C - C_\infty) \]  

Here, \( q_r \) is the radiative heat flux, \( C \) is the nanoparticle fluid concentration \( \nu_{nf} \) is the kinematic viscosity, \( \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \) is the dynamic viscosity of the nanofluid, \( \rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s \) is the density of the nanofluid, 
\( \alpha_{nf} = \frac{k_{nf}}{(\rho_c p)_{nf}} \) is the thermal diffusivity with \( k_{nf} \) is the thermal conductivity of the fluid,

Where \( k_{nf} = k_f (k_s + 2k_f - \phi(k_s - k_f)) \)\( (k_s + 2k_f - \phi(k_s - k_f)) \), \( c_p \) is the heat capacity at constant pressure and
\( (\rho c_p)_{nf} = (\rho c_p)_f (1 - \phi) + (\rho c_p)_s \phi \)

The corresponding boundary conditions are:

\[ u = U_w(x) + V_0 \frac{\partial u}{\partial y}, \quad v = 0, \]
\[ -k \frac{\partial T}{\partial y} = (T_W - T), \quad C = C_w(x) \text{ at } y = 0 \]
\[ u \to 0, v \to 0, \quad T \to T_\infty, \quad C \to C_\infty \text{ at } y \to \infty \]  

The sheet of the temperature is
\[ T_w = T_\infty + T_0 e^\frac{x}{\delta} \]  

where \( T_0 \) is the reference temperature, \( T_w \) is the surface temperature and \( T_\infty \) is the temperature of the fluid outside the boundary layer. The wall surface concentration \( C_w(x, t) \) is given by the expression
\[ C_w = C_\infty + C_0 e^\frac{x}{\delta} \]  

\( C_\infty \) is the wall surface concentration and \( C_\infty \) is the concentration of the fluid outside the boundary layer and
\( \gamma_0(x) = \gamma e^{x} \tag{8} \)

where \( \gamma_0(x) \) is the variable reaction rate, \( L \) is the reference length and \( \gamma \) is a constant.

The radiative heat flux under Rosseland approximation [37] has the form:

\[ q_r = -\frac{\sigma}{3k_1} \tag{9} \]

where \( k_1 \) and \( \sigma \) are the mean absorption coefficient and the Stefan-Boltzmann constant.

We assume that the temperature difference within the flow is sufficiently small such that \( T^4 \) can be expressed as a linear function of temperature. Hence expanding \( T^4 \) in Taylor series about \( T_\infty \) and neglecting higher order terms, we get:

\[ T^4 \approx 4T_\infty^3 - 3T_\infty^4 \tag{10} \]

Using equations (5) and (6), equation (3) reduces to:

\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_n f \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3k_1 \gamma} \frac{\partial^2 T}{\partial y^2} \tag{11} \]

Now introducing the following similarity transformations:

\[ \eta = \frac{u_0}{2vt} e^{x/2} y, \tag{12} \]

\[ u = u_0 e^{x/2} f'(\eta), v = -\sqrt{\frac{u_0}{2l}} e^{x/2} f(\eta) + \eta f'(\eta) \tag{13} \]

\[ \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \quad h(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{14} \]

Using equations (12)-(14), the governing equations becomes:

\[ f''' - (1 - \phi)^2 \{ 1 - \phi + \phi \frac{\rho_s}{\rho_f} \} \left[ 2f'^2 - ff'' + Mf' \right] = 0 \tag{15} \]

\[ \theta'' + \frac{1}{(1+3\frac{N_r}{Pr})} \frac{k_f}{k_{nf}} \left( 1 - \phi + \phi \frac{(\rho Cp)_s}{(\rho Cp)_f} \right) \left( f \theta' - f' \theta \right) = 0 \tag{16} \]

\[ h'' + Le \left( f h' - f' h - \zeta h \right)' = 0 \tag{17} \]

And the transformed boundary conditions are:

\[ f(0) = 0, f'(0) = 1 + Vf''(0), \theta'(0) = -Bi(1 - \theta(0)), \phi(0) = 1 \]

\[ f'(\infty) \rightarrow 0, \theta(\infty) \rightarrow 0, \phi(\infty) \rightarrow 0 \tag{18} \]

Where \( Pr = \frac{\nu_f}{\alpha_f} \) is the Prandtl number, \( Nr = \frac{4\sigma T_\infty^3}{k k_1} \) is the parameter of radiation, \( Le = \frac{v_f}{D_B} \) is the Lewis number, \( M = \frac{\sigma B_0^2}{u_0 \rho H_f} + \frac{\nu_{nf}}{k_0 u_0} \) is the combined magnetic and porosity parameter. \( \zeta \) is the instantaneous
reaction rate parameter. \( Bi = h_0 \sqrt{\frac{\mu_0}{2\nu l} e^{\frac{x}{2l}}} \) is the Biot number \([38]\). Where \( h_0 \) is convective heat transfer coefficient.

The physical quantities of interest are the skin friction coefficient, the local Nusselt number and Sherwood number which are defined as

\[
\begin{align*}
C_f &= \frac{\mu}{\rho f \tau u_s} \left( \frac{\partial u}{\partial y} \right)_{y=0} \\
Nu &= -\frac{x}{(T_W - T_\infty)} \left( \frac{\partial T}{\partial y} \right)_{y=0} \\
Sh &= -\frac{x}{(C_W - C_\infty)} \left( \frac{\partial C}{\partial y} \right)_{y=0}
\end{align*}
\]

With \( \mu \) and \( k \) are the dynamic viscosity and thermal conductivity, respectively. Using non-dimensional variables, we have

\[
\begin{align*}
\sqrt{2} Re_x^{1/2} C_f &= f''(0) \\
\frac{Nu}{\sqrt{2} Re_x^{1/2}} &= -(1 + \frac{4}{3} Nr) \frac{x}{2l} \theta'(0) \\
\frac{Sh}{\sqrt{2} Re_x^{1/2}} &= -\sqrt{\frac{x}{2l}} h'(0)
\end{align*}
\]

### III. METHODS OF SOLUTION

The set of non-linear differential equations (15)-(17) with boundary conditions (18) constitute a two-point boundary value problem. These highly non-linear differential equations cannot be solved analytically. As a result, these equations are solved by the software MATLAB function “bvp4c”. The function has three vital variables: the name of the M-file enumerating an ordinary differential equation system of the design, the term of the M-file computing the boundary values, and an initial approximation of the outcome arranged with the MATLAB function “bvpinit”. The variables are defined as:

\[
\begin{align*}
y_1 &= f, y_2 = f', y_3 = f'', y_3' = f'''
\end{align*}
\]

\[
\begin{align*}
y_4 &= \theta, y_5 = \theta', y_5' = \theta''
\end{align*}
\]

\[
\begin{align*}
y_6 &= \phi, y_7 = \phi', y_7' = \phi''
\end{align*}
\]

Using (25), (26) and (27), the equations (15) - (17) can be written as

\[
y_3' - (1 - \phi)^{2.5} \left\{ 1 - \phi + \phi \frac{\rho_s}{\rho_f} \right\} (2 y_2^2 - y_1 y_3 + M y_2) = 0
\]
Bvp4c implements a collocation method for the solution of BVPs subject to general nonlinear, two-point boundary condition. The approximate solution is a continuous function that is a cubic polynomial on each subinterval of a mesh. It satisfies the differential equations at both ends and the midpoint of each subinterval and also its boundary conditions. The solver then estimates the error of the numerical solution on each subinterval. If the solution does not satisfy the tolerance criteria, the solver adapts the mesh and repeats the procedure.

IV. RESULTS AND DISCUSSION

Table 3: Pr= Nr= Le=1, M=0.1, y=0.2, Bi=0.1, ζ=0.1

| φ    | -f''(0) | -θ'(0) | -h'(0) |
|------|---------|--------|--------|
|      | SWCNT   | MWCNT  | SWCNT  | MWCNT  | SWCNT  | MWCNT  |
| 0.00 | 0.9789  | 0.9789 | 0.0876 | 0.0876 | 0.8068 | 0.8068 |
| 0.08 | 0.8895  | 0.8893 | 0.0872 | 0.0872 | 0.9058 | 0.9059 |
| 0.16 | 0.8135  | 0.8130 | 0.0873 | 0.0873 | 0.9356 | 0.9389 |

To achieve prime characteristic for engine oil flow induced by an exponentially stretching surface with radiation effect and MHD effects on a number of parameters namely, Prandtl number, Lewis number, Biot number, united magnetic and porosity parameter on velocity, temperature and concentration profile is depicted in fig1–fig7. Table3 shows computational values of -f''(0) and -θ'(0) and -h'(0) with different values for φ.

Fig1 is for variation in combined magnetic and porosity parameter M on velocity profile. There is a declination in velocity profile with increase in M, this exhibits a reduction of the thickness of the momentum boundary layer. Fig2 is for variation in velocity slip parameter. It is obvious from the figure that the velocity profile decreases with increase in V. Hence, less amount of flow is drawn and pushed away in the velocity direction, as the slip gets stronger.

Fig3 is for variation in radiation parameter Nr on temperature as well as on concentration profile. The temperature profile increases with the increase in radiation parameter Nr. The figure reveals that the profile of temperature is increased with the increase in thermal radiation Nr. This reality disclosed the result that the increase in value of Nr for given k and T∞ means a declination in Rosseland absorptivity k₁. According to equations (3) and (5), the divergence of the radiative heat flux increases as k₁ decreases which in turn increase the rate of heat transferred to
Figure 1: $\gamma = 0.2$, $\phi = 0.08$. Velocity profile for different values of combined magnetic and porosity parameter $M$.  

Figure 2: $M = 0.1$, $\phi = 0.08$. Velocity profile for different values of velocity slip parameter.  

Figure 3: $Pr = 6.25$, $\phi = 0.08$, $Bi = 0.1$. Temperature profile for different values of radiation parameter $Nr$.  

Figure 4: $\phi = 0.08$, $Nr = 5$, $Bi = 0.1$. Temperature profile for different values of Prandtl number $Pr$.  

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The fluid and therefore the fluid temperature rises and thermal layer also rises with rise in $Nr$. It is apparent from the figure that the concentration profile declines with increase in $Nr$ and hence the thickness of species boundary layer is also declined.

Fig 4 is plotted for variation in Prandtl number $Pr$ on temperature as well as on concentration profiles. As an actual important thermo-physical property of a fluid, Prandtl number expresses the ratio of momentum diffusivity to thermal diffusivity in the regime. For $Pr < 1$, thermal diffusivity exceeds momentum diffusivity, hence heat will diffuse at a quicker rate than momentum. $Pr > 1$, momentum diffusivity will exceed the thermal diffusivity. For $Pr = 1$, both the viscous and energy diffusion rates will be same. Engine oils have low thermal conductivity, therefore they are characterized by large values of Prandtl number $Pr$. It is apparent from the figure that the thermal boundary layer thickness decreases as $Pr$ increases. Also it is observed that, there is an enhancement in concentration profile with increase in Prandtl number $Pr$, hence the thickness of species boundary layer is also enhanced.

Fig 5 exhibits for variation in Biot number $Bi$ on temperature and concentration profiles. Biot number $Bi$ is a dimensionless quantity used in heat transfer calculations. It is named following the French physicist Jean-Baptiste Biot (1774–1862), and gives a simple index of the ratio of the heat transfer resistances surrounded by and at the surface of a body. This ratio determines whether or not the temperatures within a body will vary significantly in space, while the body heats or cools over time, from a thermal gradient applied to its surface. The rise in heat transfer coefficient has been observed with the increase in Biot number $Bi$ which in turn shows the increase in temperature. Moreover, the
concentration profile is increased with increase in Biot number $Bi$, the thermal and concentration boundary layer thickness are increasing function of Biot number $Bi$. Fig6 exhibits the variation in Lewis number $Le$. Lewis number signifies the relative contribution of thermal diffusion rate to species diffusion rate in the boundary layer regime and it is evident from the figure that the concentration profile decreases with increasing $\eta$ and boundary layer for $h(\eta)$ is decreased. Fig7 is depicted the variation in chemical reaction parameter on concentration profiles. It is witnessed that the concentration boundary layer thickness reduces with increase in chemical reaction parameter.

All figures are depicted for SWCNT’s.

V. CONCLUSION

In this paper, the consequences of Convective heat transfer in MHD slip flow over an exponentially stretching sheet in the presence of engine oil through a porous media with thermal radiation has been analysed. The investigation is performed for variant mentioned parameters and some conclusions are summarized as follow:

- The thermal boundary layer thickness decreased as Prandtl number increased.
- The Sherwood number is greatest at $\phi = 0.16$.
- The local Nusselt number is highest at $\phi = 0.0$.
- The temperature profile decreases as $Pr$ increases.

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