Heat transfer through combined thermal insulation of cylindrical vacuum resistive furnace

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Abstract. The paper is devoted to simulation problem of heat transfer in vacuum shaft electric resistive furnace with combined insulation. Concept of critical diameter of cylindrical combined thermal insulation is explained. Physical-and-mathematical model for investigation of interconnection of geometrical and thermophysical parameters of the combined thermal insulation with heat flow through it is proposed. Dependences of effective heat transfer coefficient of screen partition of the combined thermal insulation and effective critical diameter of the insulation partition of the combined insulation upon its geometrical and thermophysical parameters are uncovered.

1. Introduction
Vacuum electric furnaces are universal furnaces used in various branches of industry and science. They often have screen insulation made of metallic screens. Application of the screen insulation in practice does not introduce into the working space any contaminants which can affect treated workpiece. Main disadvantage of such installations is comparatively low efficiency that is about 25-30%. The screen material is chosen with regards to the working temperature. As usual, for screen temperature more than 1100°C refractory metals, such as wolfram, molybdenum, tantalum, niobium and their alloys, are used; for temperatures less than 1100°C – double nichrome X20H80-H, for less than 900°C – stainless steel 12X18H10T [1]. Life time of such insulation is determined by life time of the hottest screen. That is because replacement of only one screen is impossible. The whole insulation set must be replaced.

Alternative to the screen insulation for high-temperature resistive furnaces with working space temperature more than 1100°C is using combined thermal insulation. It allows significantly reducing heat losses of the electric furnace and employing not so expensive materials for screens.

Application of combined thermal insulation for cylindrical vacuum electric resistive furnaces (shaft, elevator) has been discussed for a long time: theoretical graphoanalytical method for their parameters calculation have been proposed in [2]. But wide usage such insulation has found for designs with square and rectangular cross-section of work-space, that is with flat side walls [3-5]. This insulation design conventionally includes two layers: the first layer, formed by liner (high-temperature filling of metal oxides granules, fibrous material and so on, placed between two flat or coaxial sheets made from high-temperature metal), and the second one, that is package of metal screens. In such insulation a nonlinear behavior of heat transfer is observed due to, first of all, radiative heat transfer in screen layer of the insulation, where heat flow is proportional to fourth power of temperature, and, secondly, temperature dependence of thermal conductivity coefficient of the lining layer, that is function of temperature in power 1 or 2. In usage of combined cylindrical insulation
essential role plays nonlinear dependence of heat flow through the lining layer, connected with its curved form.

Possible designs of the combined insulation that have different interleaving schemes are the following:

1) Heating elements – liner layer – package of screens – water-cooled case;
2) Heating elements – package of screens – liner layer – water-cooled case.

The first design is preferable because it is few times less expensive, than the second one [1].

2. Mathematical model

2.1. Problem definition

The aim of the work is to study influence of geometrical and thermophysical parameters of lining and screen layers of the combined insulation on heat flow through it, as well as on effective heat transfer coefficient of the screen part of the combined thermal insulation and on effective critical diameter of the lining part of the combined thermal insulation.

Figure 1 shows scheme of the combined thermal insulation of a vacuum shaft electric resistive furnace designed according to first option. Number of screens, chosen with regard to recommendations in [1, 6], equals to 6. Temperature of heating element set to be 1200°C as this value is closest and exceeding 1100°C, that requires application of refractory metals for screens. The heating element is made from molybdenum as a hollow cylinder, having two narrow vertical slices – for two-phased feeding, and three slices – for three-phased feeding. So the model heater is represented by thin continuous molybdenum sheet.

As lining a bulk granules of zirconium dioxide have been chosen. This material has found application in such systems due to low effective thermal conductivity coefficient for high temperatures. Table 1 contains properties of the material depending on temperature and porosity.

2.2. The assumptions made when creating a model

- Combined thermal insulation represents a cylindrical one-dimensional system so that temperature changes only in the radial direction.
- In lining part of the combined insulation, heat is transferred by conductivity and depends on

| Porosity, % | 100°C | 600°C | 1000°C | 1200°C | 1400°C | 1600°C |
|------------|-------|-------|--------|--------|--------|--------|
| 0          | 1,92  | 2,09  | 2,21   | 2,21   | 2,27   | 2,29   |
| 13         | 1,69  | 1,8   | 1,98   | 2,04   | 2,09   |        |
| 23,5-32    | 0,465 | 0,64  | 0,815  | 0,99   | 1,22   | 1,63   |
| 53         | 0,326 | 0,36  | 0,372  | 0,396  | 0,418  |        |

Figure 1. Model of vacuum resistive furnace with combined thermal insulation: 1 – heating element, 2 – liner, 3 – package of screens, 4 – water-cooled case

Table 1. The effective thermal conductivity coefficient of zirconium dioxide depending on temperature for different porosities [6].
temperature.

- The lining part is metallic case filled with granules of insulative material; the case forming metal can be chosen from wide range (molybdenum, wolfram, double nichrome and so on) with regard to working temperature; so, emissivity of the lining part varied within a wide range of $\varepsilon_{11}=\varepsilon_{12}=0.2; 0.5; 0.8$.

- Outside of lined part of the insulation heat transfer carried out by radiation; environment in the vacuum furnace is transparent for radiation; emissivity of surfaces of lining and screens depends on temperature; screens are thermally thin bodies; furnace case is water-cooled with a surface temperature $t_k=35^\circ C$.

- The complex heat transfer problem must be solved for thermal calculation of vacuum electric resistive furnace to find heat flow through its insulation. So, system of nonlinear equations has been composed. As an initial approximation for the system iterative solution, a linear distribution of temperature over lining and screens ranged from $1200^\circ C$ (heater temperature) to $35^\circ C$ (water-cooled case temperature).

The equations system, describing heat transfer from heater to water-cooled furnace case, includes three equations: radiative heat flow from heater to lining, conductive heat flow through lining, and radiative heat flow through set of screens towards furnace case. In steady state this equation system reads as:

$$Q = \left( t_{11} - t_{12} \right) \pi \frac{1}{2 \cdot \lambda \ln \left( \frac{r_{12}}{r_{11}} \right)} + 2 \cdot \pi \cdot r_{12} \cdot \sigma \cdot \left[ \left( t_{12} \right)^4 - \left( t_{sh} \right)^4 \right]$$

$$Q = \frac{1}{\varepsilon_{11}} + \frac{r_{sh}}{r_{11} \cdot \varepsilon_{sh}} - \frac{1}{\varepsilon_{12}} + \frac{r_{11}}{r_2} \frac{1}{\varepsilon_2} - 1 + \frac{r_{12}}{r_3} \frac{1}{\varepsilon_3} - 1 + \frac{r_{12}}{r_4} \frac{1}{\varepsilon_4} - 1 + \frac{r_{12}}{r_5} \frac{1}{\varepsilon_5} - 1 + \frac{r_{12}}{r_6} \frac{1}{\varepsilon_6} - 1$$

$$Q = \left[ \left( \frac{T_0}{100} \right)^4 - \frac{t_{11}}{100} \right]^{4} - 2 \cdot 5.67 \cdot \pi \cdot r_{00}$$

where $T_0$ is the heater temperature, $K$; $t_{11}, t_{12}$ are temperatures of inner and outer surfaces of lined part, respectively, $^\circ C$; $T_{sh}$ is the case temperature, $K$; $r_{00}$ is the heater radius, $m$; $r_{11}, r_{12}$ are inner and outer radii of lined part, respectively, $m$; $r_{11}, \ldots, r_{6}$ are radii of $1 \ldots 6$-th screen, respectively, $m$; $r_{sh}$ is the radius of furnace case, $m$; $\varepsilon_{00}$ is the emissivity of heater; $\varepsilon_{11} = \varepsilon_{12}$ are emissivities of inner and outer surfaces of lining part of the insulation; $\varepsilon_{1}, \ldots, \varepsilon_{6}$ are emissivities of $1 \ldots 6$-th screen, respectively; $\lambda = f(t)$ is the thermal conductivity coefficient of liner insulation, W/(m·K); $\sigma$ is the Stefan-Boltzmann constant, W/(m²·K⁴).

Concept of critical diameter is introduced in theory of heat transfer due to characteristic of heat transfer through cylindrical wall for third kind boundary condition at the outer surface. The critical diameter is value of the inner diameter of thermal insulation, made from lining and fibrous materials, that corresponds to its minimal thermal resistivity and maximal heat flow for conductive heat transfer [7]. Similarly to heat transfer through such cylindrical thermally insulative wall, for heat transfer through combined thermal insulation for its lined part the effective critical diameter can be introduced as follows:

$$Q = \left( \frac{t_{11} - t_{12}}{\lambda \ln \left( \frac{r_{12}}{r_{11}} \right)} \right) + 2 \cdot \pi \cdot r_{12} \cdot \sigma \cdot \left[ \left( t_{12} \right)^4 - \left( t_{sh} \right)^4 \right]$$

$$Q = \frac{1}{\varepsilon_{11}} + \frac{r_{sh}}{r_{11} \cdot \varepsilon_{sh}} - \frac{1}{\varepsilon_{12}} + \frac{r_{11}}{r_2} \frac{1}{\varepsilon_2} - 1 + \frac{r_{12}}{r_3} \frac{1}{\varepsilon_3} - 1 + \frac{r_{12}}{r_4} \frac{1}{\varepsilon_4} - 1 + \frac{r_{12}}{r_5} \frac{1}{\varepsilon_5} - 1 + \frac{r_{12}}{r_6} \frac{1}{\varepsilon_6} - 1$$

$$Q = \left[ \left( \frac{T_0}{100} \right)^4 - \frac{t_{11}}{100} \right]^{4} - 2 \cdot 5.67 \cdot \pi \cdot r_{00}$$
The lining critical diameter in the considered model can be calculated by formula (2), where heat transfer coefficient is defined on the basis of thermal similarity of radiative heat transfer through screen layer of the insulation and convective heat transfer on outer surface of the lined part, which is described by Newton's law of cooling [7]:

$$Q = k_{eff} (t_{12} - t_{sh}),$$

(3)

where $Q$ is the heat flow through combined insulation calculated by solving equations system (1); $t_{sh}$ is the furnace case temperature, °C.

Since the system is considered to be in a steady state, heat flux has the same value as for heat transfer from heater to lining, through lining and set of screens. So, we can equate right sides of the second equation of system (1) and expression (3) and derive formula for the effective heat transfer coefficient for lined part of the combined insulation:

$$k_{eff} = \frac{2 \cdot \pi \cdot \eta_2 \cdot \alpha \cdot \left[ (t_{12})^4 - (t_{sh})^4 \right]}{1 + \frac{\eta_2}{\epsilon_{sh}} \left[ \frac{1}{\epsilon_{sh}} - 1 \right] + \frac{\eta_2}{\eta_1} \left[ \frac{2}{\epsilon_0} - 1 \right] + \frac{\eta_2}{\eta_3} \left[ \frac{2}{\epsilon_0} - 1 \right] + \frac{\eta_2}{\eta_4} \left[ \frac{2}{\epsilon_0} - 1 \right] + \frac{\eta_2}{\eta_5} \left[ \frac{2}{\epsilon_0} - 1 \right] + \frac{\eta_2}{\eta_6} \left[ \frac{2}{\epsilon_0} - 1 \right] \left( t_{12} - t_{sh} \right)}.$$

(4)

Values of heat flow through combined insulation, effective heat transfer coefficient of the screen part and critical diameter of lining part have been calculated with MathCad-program. Firstly, input data and initial temperature approximation must be defined. After calculation of heat flux, temperatures at all surfaces were calculated. Then the calculated temperatures were used as an initial approximation for new heat flux calculation. This procedure was applied for every value of variable parameters.

3. Analysis of the results

Figure 2 presents diagrams of maximal heat flux $Q$, corresponding to critical diameter of lining layer of the insulation on its thickness for different surface emissivities $\epsilon_{11} = \epsilon_{12} = 0.2; \epsilon_{11} = \epsilon_{12} = 0.5; \epsilon_{11} = \epsilon_{12} = 0.8$. Diagrams on figure 2 shows that emissivity value of insulation lining part insignificantly affects change of the heat flux of the system.
Figure 3 depicts dependency diagrams of heat flux $Q$ upon insulation thickness for different inner diameters of the insulation lining part $d_{11} = 2r_{11}$:

1 – 110 mm; 2 – 100 mm; 3 – 90 mm; 4 – 70 mm; 5 – 60 mm; 6 – 50 mm; 7 – 40 mm; 8 – 30 mm.

Figure 4 presents dependence of the critical diameter of combined insulation lining upon the thermal conductivity coefficient and lining thickness. In common industrial resistive furnaces the critical diameter, as in heat engineering theory for insulation of heating pipes, is determined by just thermophysical parameters such as the thermal conductivity coefficient of furnace lining material and the heat transfer coefficient on its outer surface and, in practice, does not depend on geometrical parameters of the lining. However for combined thermal insulation of vacuum resistive furnace, the critical diameter of lining depends not only on its thermal conductivity coefficient, but also on its thickness and inner diameter. Similarly to the lining critical diameter in common industrial furnaces, the lining critical diameter of combined insulation of vacuum electric resistive furnace depends on its thermal conductivity coefficient almost linearly. With increase of thermal insulation lining thickness its critical diameter decreases, and, besides, the dependence gets weaker with growth of the lining inner diameter. So, for the inner diameter $d_{11} \geq 500$ mm it becomes practically independent on lining thickness.
4. Conclusion
Findings of performed work are as follows:

1) In vacuum resistive furnace with combined insulation, lining critical diameter phenomenon was observed, similarly as in other lined furnaces. Condition (5) must be considered when choosing thermal insulation material.

2) The dependences of the critical diameter of lining part of combined insulation of vacuum resistive furnace upon its thermal conductivity coefficient, thickness and inner diameter were uncovered.

5. References
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