CRITICAL ANALYSIS OF THEORETICAL ESTIMATES FOR B TO LIGHT MESON FORM FACTORS AND THE B → ψK(K*) DATA

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Abstract

We point out that current estimates of form factors fail to explain the non-leptonic decays B → ψK(K*) and that the combination of data on the semi-leptonic decays D → K(K*)ℓν and on the non-leptonic decays B → ψK(K*) (in particular recent polarization data) severely constrain the form (normalization and q² dependence) of the heavy-to-light meson form factors, if we assume the factorization hypothesis for the latter. From a simultaneous fit to B → K(K*)ψ and D → K(K*)ℓν data we find that strict heavy quark limit scaling laws do not hold when going from D to B and must have large corrections that make softer the dependence on the masses. We find that A₁(q²) should increase slower with q² than A₂, V, f+. We propose a simple parametrization of these corrections based on a quark model or on an extension of the heavy-to-heavy scaling laws to the heavy-to-light case, complemented with an approximately constant A₁(q²). We analyze in the light of these data and theoretical input various theoretical approaches (lattice calculations, QCD sum rules, quark models) and point out the origin of the difficulties encountered by most of these schemes. In particular we check the compatibility of several quark models with the heavy quark scaling relations.

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1 Introduction.

Heavy-to-heavy meson form factors like \( B \to D^{(*)} \) obey a very constraining principle, namely the heavy quark symmetry that relates all form factors to the Isgur-Wise function \([1]\). However, even in this case, large corrections of order \( 1/m_c \) can occur and many uncertainties remain \([2]\), and most importantly the scaling function \( \xi \) remains unknown. In the case of heavy-to-light meson form factors like \( D \to K^{(*)} \) or \( B \to K^{(*)} \) there are also rigorous results in the asymptotic heavy quark limit, but which are much weaker, namely relations between the form factors \( D \to K^{(*)} \) and \( B \to K^{(*)} \) at fixed \( \vec{q} \) near the zero recoil point \( \vec{q} = 0 \), i.e. \( q^2 = q^2_{\text{max}} \). This is a small kinematical region, and furthermore, no relation is obtained between the various form factors of a given hadron decay, unlike the heavy-to-heavy case. From semi-leptonic \( D \to K^{(*)} l \nu \) decays \([13]\) we have data for the form factors, although with large errors, in a completely different kinematic region, namely at small \( q^2 \), and we cannot from these data extract information on the \( B \to K^{(*)} \) form factors without knowledge of the \( q^2 \) dependence, for which we do not have any rigorous result in the heavy-to-light case. In both cases, one must unavoidably appeal to models, like the quark model, or theory, or just make phenomenological Ansätze, for example assume a pole or a dipole \( q^2 \) dependence.

On the other hand, we have very interesting data for the decays \( B \to \psi K^{(*)} \), in particular recent polarization data. If we assume factorization, these data can give us precious informations on the \( B \to K^{(*)} \) form factors at a different kinematic point \( (q^2 = m_{\psi}^2) \) than the data on semi-leptonic D decays (mainly at \( q^2 = 0 \)) or the heavy quark limit QCD scaling laws \( (q^2 = q^2_{\text{max}}) \). The combination of these D semi-leptonic and B non-leptonic data plus the rigorous scaling law in the asymptotic heavy quark limit can severely constrain the corrections to asymptotic scaling laws and provide rich information on the gross features of the \( q^2 \) dependence of the form factors, as we will see below. This is the main object of this paper.

Present models of non-leptonic B decays are in trouble to describe the \( B \to \psi K^{(*)} \) decays, namely the ratio of decay rates \( \psi K/\psi K^* \) and the \( \psi K^* \) polarization data simultaneously. Moreover, the scaling law in the heavy quark limit is not always verified by current models of heavy-to-light form factors, and it is important
to consider this matter to gauge the theoretical consistency of models and not only their phenomenological description of data.

This paper is organized as follows. In section 2 we address the simplest question, namely the comparison of the different models of non-leptonic B decays with the data on $B \to \psi K^{(*)}$, to see that there is a serious difficulty. In section 3 we discuss the theoretical constraints for the heavy-to-light form factors at the light of the data to set a general Ansatz for the form factors. We use these results in section 4 where we compare and discuss the different theoretical schemes. In particular, we make the distinction between quark models with ad hoc $q^2$ dependence and quark models which derive this dependence from the wave function overlap. We discuss also the QCD sum rules and the lattice QCD results. In subsection 4.3.6 we propose a quark model that fulfills the theoretical and some phenomenological requirements stated in section 3. Finally, our conclusions are given in section 5. A short overview of this work has already been given at the Beauty 94 conference.

2 $B \to \psi K^{(*)}$ data are hardly compatible with current estimates.

To be definite, let us write the form factors:

$$
< P_f | V_{\mu} | P_i > = \left( p^i_{\mu} + p^f_{\mu} - \frac{m_i^2 - m_f^2}{q^2} q_{\mu} \right) f_+(q^2) + \frac{m_i^2 - m_f^2}{q^2} q_{\mu} f_0(q^2)
$$

$$
< V_f | A_{\mu} | P_i > = (m_f + m_i) A_1(q^2) \left( \frac{\varepsilon^* \cdot q}{q^2} q_{\mu} \right)
$$

$$
- A_2(q^2) \frac{\varepsilon^* \cdot q}{m_f + m_i} \left( p^i_{\mu} + p^f_{\mu} - \frac{m_i^2 - m_f^2}{q^2} q_{\mu} \right) + 2m_f A_0(q^2) \frac{\varepsilon^* \cdot q}{q^2} q_{\mu}
$$

$$
< V_f | V_{\mu} | P_i > = i \frac{2V(q^2)}{m_f + m_i} \varepsilon_{\mu\nu\rho\sigma} p_i^\nu p_f^\rho \varepsilon^{*\sigma}.
$$

(1)

where we use the convention $\varepsilon^{0123} = 1$.

The point that we want to emphasize in this paper is that the non-leptonic decays $B \to \psi K, \psi K^*$ can help to get hints about two questions concerning these form factors, namely: i) the sign and size of the $1/m_Q$ corrections to the asymptotic
heavy-to-light scaling relations as well as ii) the gross features of the \(q^2\) dependence of the form factors. Of course, we must assume factorization to relate these decays to the form factors.

In the standard Shifman, Vainshtein and Zakharov (SVZ) \(^{[4]}\) factorization assumption, that we will call standard SVZ factorization, one deduces the non leptonic amplitudes from form factors and annihilation constants. There are two types of two-body decays corresponding to the two different color topologies, the so-called classes I and II of Bauer, Stech and Wirbel (BSW) \(^{[5]}\), \(^{[6]}\), respectively proportional to the effective color factors

\[
a_1 = \frac{1}{2} \left[ c_+ \left( 1 + \frac{1}{N_c} \right) + c_- \left( 1 - \frac{1}{N_c} \right) \right] \quad a_2 = \frac{1}{2} \left[ c_+ \left( \frac{1}{N_c} + 1 \right) + c_- \left( \frac{1}{N_c} - 1 \right) \right].
\]

(2)

where \(c_\pm\) are QCD short distance factors.

The decays we are interested in here, \(B \rightarrow \psi K, \psi K^*\), are of class II. This standard SVZ factorization, that applies literally with expression (2) using \(N_c = 3\), is known to fail definitely in class II decays. On the other hand there is a distinct phenomenological factorization prescription proposed by BSW which derives \(a_1\) and \(a_2\) by fitting the observed \(B_d\) decays. We call this factorization prescription phenomenological in the sense that \(a_1\) and \(a_2\) are fitted from the data and not obtained through theoretical relations (2). It must be stressed that these fitted coefficients have no intrinsic meaning in the sense that they are depending on the models used to estimate the form factors and annihilation constants. The model used has been traditionally chosen to be the BSW model, later modified by Neubert, Rieckert, Stech and Xu \(^{[6]}\). These authors found, from a fit to the two-body \(B\) decays:

\[
|a_1| = 1.11 \quad |a_2| = 0.21.
\]

(3)

The magnitude of \(|a_2|\) is incompatible with the expectation from (2) and the short distance QCD factors for \(N_c = 3\): \(a_2 \sim 0.1\). More recently, the sign of \(a_2/a_1\) has been unambiguously found positive by considering class III decays \(^{[7]}\) that depend on the interference between \(a_1\) and \(a_2\). This sign is inconsistent with the once proposed
prescription\[\text{[5]}\] of taking the limit $N_c \to \infty$ in eq. (2) since one has $c_+ < c_-$ for the short distance QCD factors $c_+, c_-$. We obtain, within the factorization assumption, the following amplitudes in the B meson rest frame:

\[ A\left(\bar{B}_d^0 \to \psi K\right) = -\frac{G}{\sqrt{2}} V_{cb} V_{cs}^* 2 f_\psi m_B f_+(m_\psi^2) a_2 p \]\( (4)\)

\[ A^{pv}\left(\bar{B}_d^0 \to \psi(\lambda = 0) K^*(\lambda = 0)\right) = -\frac{G}{\sqrt{2}} V_{cb} V_{cs}^* m_\psi f_\psi \left[ \left( m_B + m_{K^*}\right) \left( \frac{p^2 + E_{K^*}E_\psi}{m_{K^*}m_\psi}\right) A_1(m_\psi^2) - \frac{2p^2}{m_B + m_{K^*}m_{K^*}m_\psi} A_2(m_\psi^2) \right] a_2 \]\( (5)\)

\[ A^{pv}\left(\bar{B}_d^0 \to \psi(\lambda = \pm 1) K^*(\lambda = \pm 1)\right) = -\frac{G}{\sqrt{2}} V_{cb} V_{cs}^* m_\psi f_\psi \left( m_B + m_{K^*}\right) A_1(m_\psi^2) a_2 \]\( (6)\)

\[ A^{pc}\left(\bar{B}_d^0 \to \psi(\lambda = \pm 1) K^*(\lambda = \pm 1)\right) = \pm \frac{G}{\sqrt{2}} V_{cb} V_{cs}^* m_\psi f_\psi \frac{m_B}{m_B + m_{K^*}} 2V(m_\psi^2) a_2 p . \]\( (7)\)

We see that the non-leptonic data plus the factorization hypothesis can give us information on the form factors at a different kinematic point ($q^2 = m_\psi^2$) than the data on semi-leptonic $D$ decays (small $q^2$) or the heavy quark limit QCD scaling laws (at $q^2_{max}$).

The data for the total rates\[\text{[7]}\] are:

\[ BR\left(\bar{B}_d^0 \to \psi K^0\right) = (7.5 \pm 2.4 \pm 0.8) \times 10^{-4} \]

\[ BR\left(\bar{B}_d^0 \to \psi K^{*0}\right) = (16.9 \pm 3.1 \pm 1.8) \times 10^{-4} \]

\[ BR\left(B^- \to \psi K^-\right) = (11.0 \pm 1.5 \pm 0.9) \times 10^{-4} \]

\[ BR\left(B^- \to \psi K^{*-}\right) = (17.8 \pm 5.1 \pm 2.3) \times 10^{-4} \]

and the recent results of ARGUS\[\text{[8]}\], CLEO\[\text{[8]}\] and CDF\[\text{[9]}\] concerning the $K^*$ polarization in the $\bar{B}_d \to \psi K^{*0}$ decay, are:
\[
\begin{align*}
\frac{\Gamma_L}{\Gamma_{tot}} &> 0.78 \ (95\% \ C.L.) \ \text{ARGUS} \\
\frac{\Gamma_L}{\Gamma_{tot}} &= 0.80 \pm 0.08 \pm 0.05 \ \text{CLEO} \\
\frac{\Gamma_L}{\Gamma_{tot}} &= 0.66 \pm 0.10^{+0.10}_{-0.08} \ \text{CDF} \\
\end{align*}
\] (8)

where \(\Gamma_L\) is the partial width for the longitudinal polarization whose amplitude is given by (3).

As we have pointed out these decays are affected by the phenomenological factor \(a_2\) which is not well known from other sources. To avoid this uncertainty, we will consider the ratio of the total rates

\[
R \equiv \frac{\Gamma \left( \bar{B}_d^0 \rightarrow \psi K^{*0} \right)}{\Gamma \left( \bar{B}_d^0 \rightarrow \psi K^0 \right)} = 1.64 \pm 0.34 \ \text{CLEO II [10] (9)}
\] (9)

and the polarization ratio for \(\psi K^{*0}\):

\[
R_L \equiv \frac{\Gamma_L \left( \bar{B}_d^0 \rightarrow \psi K^{*0} \right)}{\Gamma_{tot} \left( \bar{B}_d^0 \rightarrow \psi K^{*0} \right)}
\] (10)

that are independent of \(a_2\).

Assuming factorization, any model or Ansatz on the heavy-to-light meson form factors will give a prediction for these ratios that can be compared to experiment. These ratios in terms of the form factors are given by eqs. (16) and (35).

From these formulae one can already conclude qualitatively that:

i) To get \(R_L\) sufficiently large, one needs \(V/A_1\) and \(A_2/A_1\) to be small enough.

ii) To get \(R\) not too large \(f_+/A_1\) must not be too small.

We will consider the predictions for these ratios from the following theoretical schemes:

1. Pole model of Bauer, Stech and Wirbel (BSWI) [5].
2. Pole-dipole model of Neubert et al. (BSWII) [6].
3. Quark model of Isgur, Scora, Grinstein and Wise (ISGW) [11].
\[ \frac{\Gamma(K^*)}{\Gamma(K)} \quad \frac{\Gamma_L}{\Gamma_{tot}} \quad \frac{A^{\psi^0}(m_{\psi}^2)}{A^{\psi^0}(m_{\psi}^2)} \quad \frac{V^{\psi^0}(m_{\psi}^2)}{V^{\psi^0}(m_{\psi}^2)} \quad \frac{I^{\psi^0}(m_{\psi}^2)}{I^{\psi^0}(m_{\psi}^2)} \]

| Model         | \(\frac{\Gamma(K^*)}{\Gamma(K)}\) | \(\frac{\Gamma_L}{\Gamma_{tot}}\) | \(\frac{A^{\psi^0}(m_{\psi}^2)}{A^{\psi^0}(m_{\psi}^2)}\) | \(\frac{V^{\psi^0}(m_{\psi}^2)}{V^{\psi^0}(m_{\psi}^2)}\) | \(\frac{I^{\psi^0}(m_{\psi}^2)}{I^{\psi^0}(m_{\psi}^2)}\) |
|---------------|-------------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| BSWI [5]      | 4.23                                | 0.57                            | 1.01                            | 1.20                            | 1.23                            |
| BSWII [6]     | 1.61                                | 0.36                            | 1.41                            | 1.77                            | 1.82                            |
| ISGW [11]     | 1.71                                | 0.07                            | 2.00                            | 2.58                            | 2.30                            |
| QCDSR [23]    | 7.60                                | 0.36                            | 1.19                            | 2.66                            | 1.77                            |
| Soft-Pole (CDF) | 2.15                                | 0.45                            | 1.08                            | 2.16                            | 1.86                            |
| CLEO II [8, 10]| 1.64 ± 0.34                         | 0.8 ± 0.1                       | 0.66 ± 0.1                      |                                 |                                 |
| CDF [9]       | -                                   |                                 |                                 |                                 |                                 |

Table 1: Comparison of different models, a QCD Sum Rules calculation and our preferred Ansatz (Soft-Pole as defined in table 3) to experiment. In the fifth line CDF means fit to this data as explained in table 3.

- 4. QCD sum rules (QCDSR) [23].

The results are given in Table 1. We do not include here the lattice results on the form factors [20] because they are still affected by large errors; we will discuss these results in section 4. The conclusion of the table is that there is a problem for all known theoretical schemes since both ratios \(R\) and \(R_L\) cannot be described at the same time. A priori there are three possible explanations:

i) The theoretical schemes for form factors are to be blamed for the failure.

ii) The experimental numbers are not to be trusted too much.

iii) The basic BSW factorization assumption, which allows to relate \(B \to K^{(*)}\psi\) to the form factors, is wrong for class II decays.

In section 3, we will explore the first possibility by trying to formulate form factors satisfying the relevant theoretical principles and being able to describe the experimental situation. In the light of our discussion in the latter section we will return, in section 4, to these models and try an analysis of their theoretical difficulties.

### 3 Phenomenological heavy-to-light scaling form factors confronted to \(B\) and \(D\) experiments.
3.1 Setting the problem.

Our aim will be to perform a combined experimental and theoretical study of form factors, simultaneously for both of \(D \rightarrow K^{(*)}\ell\nu\) and \(B \rightarrow K^{(*)}\psi\) decays, assuming BSW factorization for the latter. We would like to proceed as much independently as possible of the detailed theoretical approaches, using general Ansätze that respect the heavy-to-light asymptotic scaling laws, some of them being complemented by ideas derived from heavy-to-heavy scaling law formulae. Only guided by rigorous theoretical laws and some commonly admitted theoretical prejudices, we will try to display general trends suggested by the experiment. Finally we will underline that experiment, as it stays today, is not easy to account for in a theoretically reasonable manner. We will also advocate the use of a Quark Model inspired prescription, that we call the “QMI” Ansatz, an extension of some heavy-to-heavy scaling relations to the heavy-to-light system. Although not fully successful, this model is able to account roughly for a large set of data.

First, let us review available data. Besides the indirect indications coming from the above \(B \rightarrow \psi K(K^*)\) non leptonic data complemented by the BSW factorization assumption, there are data on the \(D \rightarrow K(K^*)\ell\nu\) form factors, mainly around \(q^2 = 0\). We shall use the world average \(^{13}\):

\[
\begin{align*}
 f_{+}^{sc}(0) & = 0.77 \pm 0.08 \\
 V_{sc}(0) & = 1.16 \pm 0.16 \\
 A_{1}^{sc}(0) & = 0.61 \pm 0.05 \\
 A_{2}^{sc}(0) & = 0.45 \pm 0.09
\end{align*}
\]

(11)

\[
\begin{align*}
 V_{sc}(0) / A_{1}^{sc}(0) & = 1.9 \pm 0.25 \\
 A_{2}^{sc}(0) / A_{1}^{sc}(0) & = 0.74 \pm 0.15
\end{align*}
\]

(12)

As to the \(q^2\) dependence the indications are poor except for the \(f_+\) form factor where good indications seem to support the relevant vector meson pole dominance.
We will use these indications for the $q^2$ dependence only in a second stage.

To organize the discussion which must handle a great number of possibilities, we will often first concentrate on the evolution from $D$ to $B$ of the ratios between different form factors ($A_2/A_1$, $V/A_1$, etc.) for which we can formulate more general statements, and then consider the values and evolutions of form factors themselves ($A_1, f_+$, etc.) which involve additional assumptions. The advantage of discussing first the ratios is that we can then draw more direct conclusions from the $B \rightarrow \psi K(K^*)$ data which depend only on the ratios of form factors and already exclude a number of possibilities before considering absolute branching ratios which also involve the unknown $a_2$ defined in eq. (2).

3.2 Asymptotic scaling laws for the heavy-to-light form factors.

What can be learned from the theory? The only exact results take the form of asymptotic theorems [16] valid for the initial quark mass $m_Q$ large with respect to a typical scale $\Lambda$, of QCD, to the final meson mass, $m_f$, and to the final momentum, $\vec{q}$:

$$\hat{f}_+(\vec{q}^2), \quad \hat{V}(\vec{q}^2), \quad \hat{A}_2(\vec{q}^2) = m_Q \hat{f}_+(\vec{q}^2) \left( 1 + O\left( \frac{\Lambda}{m_Q} \right) + O\left( \frac{|\vec{q}|}{m_Q} \right) + O\left( \frac{m_f}{m_Q} \right) \right)$$

$$\hat{A}_1(\vec{q}^2) = (m_Q)^{-\frac{1}{2}} \left( 1 + O\left( \frac{\Lambda}{m_Q} \right) + O\left( \frac{|\vec{q}|}{m_Q} \right) + O\left( \frac{m_f}{m_Q} \right) \right)$$

where we have used “hats” on form factors to indicate that they depend three-momentum, the natural variable in the heavy-to-light case:

$$\hat{f}(\vec{q}^2) = f(q^2), \quad \text{with} \quad \vec{q}^2 = \left( \frac{m_i^2 + m_f^2 - q^2}{2m_i} \right)^2 - m_f^2$$

The asymptotic scaling law (13) allows to relate the form factors, say $D \rightarrow K$ and $B \rightarrow K$, at small recoil $|\vec{q}| \ll m_D$ (i.e. close to $q_{\text{max}}^2$ for each process):

$$\frac{\hat{f}_{+}^{sb}(\vec{q}^2)}{\hat{f}_{+}^{sc}(\vec{q}^2)}, \quad \frac{\hat{V}_{sb}(\vec{q}^2)}{V_{sc}(\vec{q}^2)}, \quad \frac{\hat{A}_2^{sb}(\vec{q}^2)}{A_2^{sc}(\vec{q}^2)} = \left( \frac{m_B}{m_D} \right)^{\frac{1}{2}} \left( 1 + O\left( \frac{\Lambda}{m_D} \right) + O\left( \frac{|\vec{q}|}{m_D} \right) + O\left( \frac{m_f}{m_D} \right) \right)$$

1Unless specified otherwise, we use the initial meson rest frame.
\[
\frac{A_{sb}^b(q^2)}{A_{sc}^c(q^2)} = \left( \frac{m_D}{m_B} \right)^{\frac{1}{2}} \left( 1 + O \left( \frac{\Lambda}{m_D} \right) + O \left( \frac{|\vec{q}|}{m_B} \right) + O \left( \frac{m_f}{m_D} \right) \right),
\]

which hold for \(m_B\) and \(m_D\) much larger than \(\Lambda\), the spectator quark and final meson masses as well as the final meson momentum.

### 3.3 Failure of the simple-minded extrapolation from \(D\) to \(B\) according to the asymptotic scaling law.

In this subsection we will stress qualitatively that \(B \to K(\ast)\psi\) data seem to exclude a simple-minded extrapolation from \(D \to K(\ast)l\nu\) data at \(q^2 = 0\) according to the heavy-to-light asymptotic scaling law. Since the momentum \(\vec{q}\) is different in the two above-mentioned sets of data, an hypothesis on the \(q^2\) dependence is needed. However, before coming to more quantitative discussions (see \textsection 3.5), we will simply assume that this dependence does not change the qualitative consequences of the scaling laws.

The ratio \(\Gamma_L/\Gamma_{tot}\) is given by:

\[
\frac{\Gamma_L(B \to K^*\psi)}{\Gamma_{tot}(B \to K^*\psi)} = \frac{\left( 3.162 - 1.306 \frac{A_{sb}^b(m_\psi^2)}{A_{sc}^c(m_\psi^2)} \right)^2}{2 \left[ 1 + 0.189 \left( \frac{V^{sb}(m_\psi^2)}{A_{sc}^c(m_\psi^2)} \right)^2 \right] + \left( 3.162 - 1.306 \frac{A_{sb}^b(m_\psi^2)}{A_{sc}^c(m_\psi^2)} \right)^2},
\]

From this expression it is apparent that \(A_2/A_1\) must not be too large\(^4\) in view of the large experimental value of \(R_L\) \(\text{(8)}\), all the more if \(V/A_1\) is large. For example, setting \(V = 0\) we get the very conservative upper bound \(A_2/A_1 \leq 1.3\) for \(R_L > 0.5\). For a more realistic value of \(V/A_1 \simeq 2\), the upper bound becomes \(A_2/A_1 \leq 1\).

Now, according to strict application of the asymptotic scaling laws described above, \(A_2/A_1\) \((V/A_1)\) would be multiplied at fixed \(\vec{q}\) by \(m_B/m_D = 2.83\). From the central experimental \(D\) value, \(A_{sc}^c/A_1^c = 0.74\) \((V^{sc}/A_1^c = 1.9)\), one gets \(A_{sb}^b/A_1^b = 2.09\) \((V^{sb}/A_1^b = 5.38)\) at \(q^2 = 16.56\) GeV (corresponding in \(B\) decay to the same \(\vec{q}^2\) as

\(^4\)Strictly speaking very large values, \(A_2/A_1 \geq 3.9\), could also account for a large \(R_L\), but these are unrealistic.
$q^2 = 0$ in $D$ decay). This is in drastic contradiction with experiment unless there is an unexpectedly strong $q^2$ variation down to $q^2 = m_W^2$. A naive insertion of these values in eq. (13) would indeed give $R_L = 0.014$ which is 4 to 5 sigmas away from the most favorable CDF value. Clearly the message is that a softening of the increase with respect to the asymptotic scaling law is required.

### 3.4 Extending the heavy-to-heavy Isgur-Wise scaling laws into a heavy-to-light class of Ansätze.

At this point it is useful to notice that there is an overlap between the domains of validity of the heavy-to-heavy and heavy-to-light scaling laws, namely when heavy-to-heavy scaling laws are applied when both masses are large but the final quark mass is sensibly smaller than the initial one: $m_{Q_f} \ll m_{Q_i}$. In this domain the heavy-to-heavy scaling law provides corrections to the asymptotic heavy-to-light scaling law of order $m_{Q_f}/m_{Q_i}$ that go in the desired direction of a softening. This will be explained in the next subsection.

#### 3.4.1 Reminder about asymptotic scaling laws for heavy-to-heavy transitions.

It is well known that a much stronger set of relations than the one in subsection 3.2 comes from the Isgur-Wise scaling laws [1] for transition form factors between two heavy quarks. Using the notations in [1]:

$$\frac{\sqrt{4m_P m_P}}{m_P + m_P} f_+(q^2) = \frac{\sqrt{4m_P m_P}}{m_P + m_P} \frac{f_0(q^2)}{1 - \frac{q^2}{(m_P + m_P)^2}} = \frac{\sqrt{4m_P m_V}}{m_P + m_V} V(q^2) =$$

$$= \frac{\sqrt{4m_P m_V}}{m_P + m_V} A_0(q^2) = \frac{\sqrt{4m_P m_V}}{m_P + m_V} A_2(q^2) = \frac{\sqrt{4m_P m_V}}{m_P + m_V} \frac{A_1(q^2)}{1 - \frac{q^2}{(m_P + m_V)^2}} = \xi(v_i, v_f) \quad (17)$$

for $m_P$, $m_P$, and $m_V$ much larger than the typical scale $\Lambda$ of QCD. It must be added that in the same limit $m_P$ and $m_V$ are in fact equal so that our writing of different masses is only meant for later use in the real subasymptotic regime, where they are very different ($m_K \neq m_K^*$).
The denominator that divides $A_1(q^2)$ is a straightforward consequence of the heavy quark symmetry and of the definition of the different form factors. It has not the meaning of a dynamical pole related to some intermediate state. It is still in the mathematical sense a pole of the ratio $A_2(q^2)/A_1(q^2)$ etc, and we shall call it for simplicity the “kinematical pole”.

When eq. (17) may be applied, it is much stronger than the heavy-to-light constraint (13). Using this heavy-to-heavy relation (17) for two different values ($m_P = m_B, m_D$) we automatically obtain the heavy-to-light one (13) when one makes $m_P$ much larger than $m_f$.

Indeed, at fixed $\vec{q}$, $v_i.v_f = \sqrt{1 + \vec{q}^2/m_f^2}$ (in the rest frame of the initial meson) is fixed. It is also simple to show that:

$$\frac{4m_P m_f}{(m_P + m_f)^2} \left( 1 - \frac{q^2}{(m_P + m_f)^2} \right) = \frac{2}{1 + v_i.v_f}$$

so that the preceding equations write finally in terms of masses and the fixed $\vec{q}$, with $m_P \gg m_f$:

$$2 \left( \frac{m_P}{m_P} \right)^{1/2} \left( 1 - \frac{m_P}{m_P} + .. \right) \hat{f}_+(q^2) = \frac{\sqrt{m_P m_f}}{m_f + E_P} (1 + \frac{m_P}{m_f}) \hat{f}_0(q^2) =$$

$$2 \left( \frac{m_L}{m_P} \right)^{1/2} \left( 1 - \frac{m_L}{m_P} + .. \right) \hat{V}(q^2) = 2 \left( \frac{m_V}{m_P} \right)^{1/2} \left( 1 - \frac{m_V}{m_P} + .. \right) \hat{A}_0(q^2) =$$

$$2 \left( \frac{m_V}{m_P} \right)^{1/2} \left( 1 - \frac{m_V}{m_P} + .. \right) \hat{A}_2(q^2) = \frac{\sqrt{m_P m_V}}{m_V + E_P} (1 + \frac{m_P}{m_V}) \hat{A}_1(q^2) = \xi(E_f/m_f)$$

where $E_f = E_{V_f} \simeq E_P$ are the final energies in the initial rest frame, $E_f = \sqrt{m_f^2 + \vec{q}^2}$.

The “hats” on form factors have been defined in eq (14), and we have used

$$1 - \frac{q^2}{(m_P + m_f)^2} = \frac{m_P (m_f + E_f)}{(m_P + m_f)^2}$$

We see that eq. (19) includes specific values for the $O(m_f/m_P)$ corrections to the heavy-to-light scaling law (13). Of course these corrections are in principle

---

*Although the singularity happens to fall at the branching point of a t-channel cut.

*In our notations $m_f$ represents generically $m_P$ and $m_V$. 

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only valid if \(m_f\) is heavy. Any specific model claiming to handle the domain of mass \(\Lambda \ll m_f \ll m_P\), should obviously satisfy the relations (19).

An essential effect displayed by formula (19) is that it softens the asymptotic scaling relation (13), i.e. it leads to a slower increase (decrease) of \(A_2, V, f_+ (A_1)\) when the initial mass \(m_P_i\) increases at fixed \(\vec{q}\).

Another very important aspect of the Isgur-Wise relations (17) is that all the ratios of form factors for the same quark masses and at the same transfer \(q^2\) (or equivalently \(\vec{q}\)) are completely fixed by the theory: quite strikingly, the form factors must be all asymptotically equal at \(q^2 = 0\), and therefore their ratios are completely independent of the masses; this extends to any fixed \(q^2\) provided that \(q^2\) is small with respect to \(m_B^2\):

\[
\begin{align*}
  f_+(0) = f_0(0) = V(0) = A_0(0) = A_2(0) = A_1(0) = \xi ((v_i \cdot v_f)_{q^2=0})
\end{align*}
\]  

(21)

where

\[
(v_i \cdot v_f)_{q^2=0} = \frac{m_{P_i}^2 + m_f^2}{2m_{P_i}m_f}
\]

This is to be contrasted with the heavy-to-light scaling (13) which leaves these ratios at the same \(q^2\) undetermined, because the dependence in \(\vec{q}\) is undetermined. Even when non asymptotic effects are included, we expect from eq. (21) the ratios \(f_+/A_1, V/A_1\) and \(A_2/A_1\) to behave very softly as a function of the heavy initial mass at \(q^2 = 0\).

### 3.4.2 The basic Soft-scaling-Pole Ansatz

We now formulate our model based on an extension of the heavy-to-heavy scaling relations (17). Let us first assume that we are in a situation described in the preceding section with two heavy quarks but \(m_i \gg m_f \gg \Lambda\). As we have argued, the form factors obey the heavy-to-light scaling relations (13) with specific form factor ratios and specific \(O(m_f/m_i)\) corrections, eq. (19). To these, one should also add the unknown \(O(\Lambda/m_f)\) corrections to the heavy quark symmetry.

Let us now consider the intermediate region where the final quark ceases to be heavy. Our ignorance comes from the fact that the \(O(\Lambda/m_f)\) corrections become
large and may totally modify the above mentioned specific relations. Our hypothesis will be that it is not so, i.e. that using some of the features of eq. (17) is indeed a good approximation. This hypothesis, although admittedly arbitrary, may be empirically justified by the fact (see section 3.3) that data demand a softened heavy-to-light scaling, and that formula (17) or equivalently (19) does present such a behaviour. Theoretical arguments in favor of the present Ansatz will come below and in section 3.4.3.

It is obvious that an unrestricted extension of Isgur Wise formulae (17) cannot describe quantitatively the form factors for a simple reason: the $D \to K^{(*)} l\nu$ form factors at $q^2 = 0$, eq. (11), are obviously not equal to each other, contrary to what is predicted by eq. (21); and the formula will also completely fail for $B \to K^{(*)} \psi$, since it would predict from formula (35) a much too large ratio $\Gamma(K^*)/\Gamma(K) \simeq 4$. This is after all expected because we do not believe that the $D$ is heavy, not to speak about the $K$ or $K^*$. But notwithstanding this failure we will try to apply to the heavy-to-light case the $q^2$ and mass dependence implied by formulae (17). The problem of $D$ decays can be trivially cured by assuming, as we shall do, a “rescaling” of each form factor to put it in agreement with the $D$ data at $q^2 = 0$. We assume also that these rescaling factors $(r_+, r_V, r_1, r_2)$ are independent of the initial heavy quark mass and of $q^2$. In other words, we assume the $O(\Lambda/m_f)$ corrections to be properly taken into account by these constant rescaling factors. Let us thus start from eq. (17), multiply for conveniency the l.h.s. and the rhs by $(1 + v_i \cdot v_f)/2$, and rescale the form factors as mentioned above. We obtain:

\[
\frac{m_{P_i} + m_{P_f}}{\sqrt{4m_{P_i}m_{P_f}}} \left[ 1 - \frac{q^2}{(m_{P_i} + m_{P_f})^2} \right] \frac{f_+(q^2)}{r_+} = \frac{m_{P_i} + m_{V_f}}{\sqrt{4m_{P_i}m_{V_f}}} \left[ 1 - \frac{q^2}{(m_{P_i} + m_{V_f})^2} \right] \frac{V(q^2)}{r_V} =
\]

\[
= \frac{m_{P_i} + m_{V_f}}{\sqrt{4m_{P_i}m_{V_f}}} \left[ 1 - \frac{q^2}{(m_{P_i} + m_{V_f})^2} \right] \frac{A_2(q^2)}{r_2} = \frac{m_{P_i} + m_{V_f}}{\sqrt{4m_{P_i}m_{V_f}}} \left[ \frac{A_1(q^2)}{r_1} = \eta(\vec{q}, m_f) \right]
\]

where $m_f$ is as usual the final meson mass: $m_{P_f}$ or $m_{V_f}$. In fact, to conform with the asymptotic Isgur-Wise heavy-to-heavy scaling, the rescaling parameters $r_+, r_V, r_2, r_1 = 1 + O(\Lambda/m_f)$ should depend on the final active quark mass $m_{q_f}$ and
tend to one when it goes to infinity, but this does not matter here since the final quark will remain the s-quark all over this study. In formula (22) we have introduced a function $\eta(\vec{q}, m_f)$, since $\vec{q}$ is the natural variable in the heavy-to-light scaling case.

It should be repeated that at fixed $q^2$ formula (22) corrects the asymptotic scaling (13) by the replacement $m_{P_i}^{1/2} \rightarrow (m_{P_i} + m_f)/m_{P_i}^{1/2}$. This correction to the asymptotic scaling induces a softer behaviour than the asymptotic scaling, eq. (13), i.e. a slower increase/decrease, when $m_{P_i}$ increases. Consequently eq (13) becomes specified into:

$$\hat{f}_{sb}(\vec{q}^2) / \hat{f}_{sc}(\vec{q}^2), \quad \hat{V}_{sb}(\vec{q}^2) / \hat{V}_{sc}(\vec{q}^2), \quad \hat{A}_{sb}(\vec{q}^2) / \hat{A}_{sc}(\vec{q}^2) = \left( \frac{m_B + m_f}{m_{D} + m_f} \right) \left( \frac{m_B}{m_D} \right)^{-\frac{1}{2}},$$

$$\hat{A}_{sb}(\vec{q}^2) / \hat{A}_{sc}(\vec{q}^2) = \left( \frac{m_D + m_f}{m_B + m_f} \right) \left( \frac{m_D}{m_B} \right)^{-\frac{1}{2}}, \quad (23)$$

Another important consequence of formula (22) is that at $q^2 = 0$ the form factor ratios stay constant for fixed final masses when $m_{P_i}$ varies, except for a small variation of $f_+/A_1$ of the order $O((m_{V_i} - m_{P_i})/m_{P_i})$:

$$\frac{A_2(0)}{A_1(0)} = \frac{r_2}{r_1}, \quad \frac{V(0)}{A_1(0)} = \frac{r_V}{r_1},$$

$$\frac{f_+(0)}{A_1(0)} = \frac{r_+ \left( \frac{m_{P_i}}{m_{V_i}} \right) \frac{m_{P_i} + m_{V_i}}{m_{P_i} + m_{P_i}} \frac{\eta(\vec{q}_1, m_{P_i})}{\eta(\vec{q}_2, m_{V_i})}}{r_1} \left( \frac{m_{P_i}}{m_{V_i}} \right) \frac{m_{P_i} + m_{V_i}}{m_{P_i} + m_{P_i}} \frac{\eta(\vec{q}_1, m_{P_i})}{\eta(\vec{q}_2, m_{V_i})} \right)^{\frac{1}{2}}, \quad (24)$$

where $|\vec{q}_1| = (m_{P_i}^2 - m_{P_i}^2)/2m_{P_i}$ and $|\vec{q}_2| = (m_{P_i}^2 - m_{V_i}^2)/2m_{P_i}$ correspond to $q^2 = 0$ for a pseudoscalar and a vector final meson respectively.

The formula (22) is of course a purely phenomenological assumption, although suggested by our quark model analysis in the weak binding limit (see below); it still presents a large arbitrariness, corresponding to the freedom of $\eta(\vec{q}, m_f)$.

Until section 3.6 we shall not need to specify the function $\eta(\vec{q}, m_f)$ as we shall be concerned only with form factor ratios. For completeness we shall now simply state our choice, referring to section 3.6 for justifications:

$$\eta(\vec{q}, m_f) = 1 \quad (25)$$
3.4.3 Theoretical justifications

a) Quark model: How do we justify this Ansatz and in particular this “rescaling” procedure? We are mainly motivated by the fact that the general structure of the Isgur-Wise relations (17) will be shown to appear also in the heavy to light case in a quark model with weak binding treatment described below, the Orsay Quark Model (OQM). It gives the kinematical pole factor, differentiating $f_+, A_2, V$ from $A_1$. It also displays the $O(m_f/m_i)$ corrections predicted by the heavy-to-heavy scaling laws. On the other hand the quark model analysis leads to expect two types of $O(\Lambda/m_f)$ corrections:

i) Corrections taking into account the finite mass of the spectator quark, which are present in the weak binding treatment.

ii) Corrections to the weak binding limit, not included in the OQM.

In this model the dominant correction to asymptotic scaling and the dominant features of $q^2$ dependence are represented by the Ansatz (22), while additional corrections are present but are small.

b) $B \to K^*\gamma$: An amusing example that exhibits the same trends as we advocate is provided by the $B \to K^*\gamma$ form factors. Defining the $T_i$ form factors as follows,

$$<K^*, k, \epsilon|s\sigma^{\mu\nu}q_\mu \frac{1 + \gamma_5}{2}b|B, p> = -2\epsilon_{\mu\nu\lambda\sigma}p^\lambda k^\sigma T_1(q^2) - i \left[ \epsilon^*_\mu (m_B^2 - m_{K^*}^2) - \epsilon^* \cdot q(p + k)\mu \right] T_2(q^2) - i \epsilon^* \cdot q \left[ q_\mu - \frac{q^2}{m_B^2 - m_{K^*}^2}(p + k)\mu \right] T_3(q^2)$$

(26)

it is well known that, for $q^2 = 0$, using the identity $\sigma_{\mu\nu}\gamma_5 = \frac{i}{2}\epsilon_{\mu\nu\lambda\sigma}\sigma^{\lambda\sigma}$, one obtains the exact relation:

$$T_1(0) = T_2(0)$$

(27)

It has also been shown [21] that

$$T_1(q^2) = \sqrt{m_Q} \left( 1 + O\left( \frac{\Lambda}{m_Q} \right) + O\left( \frac{|q|}{m_Q} \right) \right)$$

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\[ T_2(q^2) = \frac{1}{\sqrt{m_Q}} \left( 1 + O\left( \frac{\Lambda}{m_Q} \right) + O\left( \frac{q}{m_Q} \right) \right) \]  

(28)

In the heavy-to-heavy case one may also show that

\[
\sqrt{4m_P m_{Vf}} T_1(q^2) = \sqrt{4m_P m_{Vf}} \frac{T_2(q^2)}{1 - \frac{q^2}{(m_P + m_{Vf})^2}} = \frac{1}{2} \xi (v \cdot v')
\]  

(29)

which is of course fully compatible with the relation (27).

But the new thing here is that the relation (27) remains exact when the final quark becomes light. Since the scaling behaviours of the \( T_1 \) and \( T_2 \) differ in the vicinity of \( q_{max}^2 \), (28), the equality (27) is a clear indication that the \( q^2 \) behaviour of both form factors differs sensibly. For example a pole dominance hypothesis for both form factors is totally excluded by these relations. Furthermore, an extension of relation (29) to the heavy-to-light domain, as we have suggested in the section 3.4.2, would directly comply with both relations (27) and (28). Finally let us insist that this is by no means a proof of our Ansatz, it is simply a hint that it may point towards the right direction.

c) Matrix elements: Some light can be shed on our Ansatz (22), as far as the mass dependence is concerned, by noting that it amounts to assume that the matrix elements satisfy an uncorrected asymptotic scaling. To illustrate this point let us consider a final vector meson \( V_f \) with a polarization \( \epsilon^T \) orthogonal to the initial and final meson momenta.

From eqs. (1) and (22) the matrix elements scale as follows:

\[
\langle V_f, \epsilon^T, q| A_\mu| P_i \rangle = r_1 \eta(q, m_{Vf}) \epsilon_\mu
\]

\[
\langle V_f, \epsilon^T, q| V_\mu| P_i \rangle = iv \eta(q, m_{Vf}) \left( u_f \times \epsilon^T \right)_\mu
\]

(30)

where \( v_f^\mu = p_f^\mu / m_f \) and eq. (20) has been used.

In this example it is clear that the matrix elements scale exactly like \( \sqrt{m_P m_f} \), which is their asymptotic heavy-to-light scaling behaviour. Our claim in favor of
the softened scaling eq (22) is equivalent to the statement that the matrix element
asymptotic scaling laws are not corrected at non-asymptotic masses. In other words
our softened scaling Ansatz is equivalent to a precocious asymptotic scaling of the
matrix elements.

d) QCD sum rules, lattice calculations: We will argue in section 4.2 that QCD
sum rules qualitatively favor the $q^2$ dependence of the form factor ratios as depicted in
eq. (22), i.e. they generally show an increase of the ratios $A_2/A_1, V/A_1, f_+/A_1$ with $q^2$
not very different from the increase due to the kinematical pole $1/(1-q^2/(m_P + m_f)^2)$.

Lattice calculations [20] on their side, favor a softened heavy-to-light scaling,
as a function of the heavy masses, for the leptonic decay constant $F_P$ and for the form
factors except $A_2$. However, if this result seems strongly established for the leptonic
decay constants, the $m_Q$ dependence of the form factors, and particularly $A_2$, are not
yet known with enough precision to be conclusive.

3.5 Confronting the form factors to experimental ratios.

We now turn to experiment in order to fix the remaining free parameters and to know
whether the data can be really understood in the above phenomenological framework.
We begin by the discussion of the ratios.

Although we would like to stick to our theoretical prejudices, eqs (22) and
(23), we still feel that the situation is very uncertain in the subasymptotic regime.
Therefore we shall also test other competing schemes in order to know by compar-
ison whether experiment is actually giving definite indications on the $q^2$ and mass
dependence of the form factors. A few definite lessons will be drawn but the exercise
will not prove as conclusive as we would have wished.

3.5.1 Some simple prescriptions for mass and $q^2$ dependence

We intend to perform several $\chi^2$ fits of the ratios of form factors according to the
following method. We assume a given evolution prescription for the dependence of the
ratios of form factors as a function of the heavy mass at $q^2_{\text{max}}$, and a given prescription
for the behaviour of these ratios as a function of $q^2$ for fixed masses. Next we combine
in one $\chi^2$ fit the experimental results for $D \to K^{(*)} l \nu$ at $q^2 = 0$ and the experimental results for the ratios $R$ and $R_L$ for $B \to K^{(*)} \psi$. The evolution prescriptions we have used are now described:

1) **Soft-scaling-Pole** corresponds exactly to the assumption (22), i.e. the heavy-to-heavy inspired scaling which implies a softened scaling in heavy quark mass at fixed $\vec{q}$, and a ratio to $A_1(q^2)$ that exhibits the kinematical pole similar to the heavy-to-heavy scaling formulae.

2) **Soft-scaling-Constant** assumes the same softened scaling at $q^2_{\text{max}}$ as before, but a ratio of form factors which stays constant as $q^2$ varies for fixed masses:

$$\frac{\sqrt{4m_P m_P f_+(q^2)}}{m_P + m_f} \frac{f_+(q^2)}{r_+} = \frac{\sqrt{4m_P m_{V_f}} V(q^2)}{m_P + m_{V_f}} \frac{V(q^2)}{r_V} =$$

$$= \frac{\sqrt{4m_P m_{V_f}} A_2(q^2)}{m_P + m_{V_f}} \frac{A_2(q^2)}{r_2} = \frac{(m_P + m_{V_f}) A_1(q^2)}{\sqrt{4m_P m_{V_f}}} \frac{A_1(q^2)}{r_1} = \eta(\vec{q}, m_f)$$

(31)

3) **Hard-scaling-Pole** assumes hard scaling, i.e. the asymptotic laws without $1/m_Q$ corrections. This is achieved by replacing in the “soft-pole” prescriptions at $q^2_{\text{max}}$:

$$\sqrt{m_P/(m_P + m_f)} \to 1/\sqrt{m_P},$$

but keeping the $q^2$ dependence at fixed masses as in (22). It corresponds to:

$$2 \left( \frac{m_P}{m_{V_f}} \right)^{1/2} \frac{(m_P + m_f)^2}{4m_P m_{V_f}} \left[ 1 - \frac{q^2}{(m_P + m_f)^2} \right] \frac{f_+(q^2)}{r_+} =$$

$$2 \left( \frac{m_{V_f}}{m_P} \right)^{1/2} \frac{(m_P + m_{V_f})^2}{4m_P m_{V_f}} \left[ 1 - \frac{q^2}{(m_P + m_{V_f})^2} \right] \frac{V(q^2)}{r_V} =$$

$$= 2 \left( \frac{m_{V_f}}{m_P} \right)^{1/2} \frac{(m_P + m_{V_f})^2}{4m_P m_{V_f}} \left[ 1 - \frac{q^2}{(m_P + m_{V_f})^2} \right] \frac{A_2(q^2)}{r_2} =$$

$$\frac{1}{2} \left( \frac{m_P}{m_{V_f}} \right)^{1/2} \frac{A_1(q^2)}{r_1} = \eta(\vec{q}, m_f)$$

(32)
4) **Hard-scaling-Constant** assumes the same hard scaling as above and assumes that the ratios do not depend on $q^2$. It corresponds to:

$$2 \left( \frac{m_{P_1}}{m_{P_i}} \right)^{\frac{1}{2}} \frac{f_+(q^2)}{r_+} = 2 \left( \frac{m_{V_i}}{m_{P_i}} \right)^{\frac{1}{2}} \frac{V(q^2)}{r_V} =$$

$$= 2 \left( \frac{m_{V_i}}{m_{P_i}} \right)^{\frac{1}{2}} \frac{A_2(q^2)}{r_2} = \frac{1}{2} \left( \frac{m_{P_i}}{m_{V_i}} \right)^{\frac{1}{2}} \frac{A_1(q^2)}{r_1} = \eta(\vec{q}, m_f) \quad (33)$$

In the prescriptions 2) and 4), the assumption that the form factors have a constant ratio in $q^2$ is inspired from the popular pole dominance approximation\[\parallel\].

### 3.5.2 Lessons from our global $\chi^2$ fits.

We now describe the results of our $\chi^2$ fits. For the experimental results on $D \to K^{(*)}l\nu$ at $q^2 = 0$ we take the world average estimated by Witherell [13], and for $R_L$ we have used in two different fits the results from CLEO II [8] and from CDF [9]. For $R$ we have used $1.64 \pm 0.34$ from CLEO II [10].

Before discussing the outcomes of these fits, let us notice that in this exercise, the result only depends on the double ratios

$$R_2 = \frac{A_2^{sb}(m_\psi^2)/A_1^{sb}(m_\psi^2)}{(A_2^{sc}(0)/A_1^{sc}(0))}$$

$$R_+ = \frac{(f_+^{sb}(m_\psi^2))/A_1^{sb}(m_\psi^2))}{(f_+^{sc}(0)/A_1^{sc}(0))} \quad (34)$$

since within our Ansatz, the double ratio $R_V$ satisfies

$$R_V = \frac{\left(V^{sb}(m_\psi^2)/A_1^{sb}(m_\psi^2)\right)}{(V^{sc}(0)/A_1^{sc}(0))} = R_2.$$

The values of these double ratios depend both on the mass dependence of the form factor ratios, i.e. the corrections to asymptotic scaling, and on the $q^2$ dependence of these ratios.

\[\parallel\] In fact in the nearest pole dominance hypothesis, the form factor ratios are not exactly constants since the position of the pole depends on the form factor considered. But our aim here is simply to exhibit the main trends. Therefore we have used the “constant ratio” hypothesis for the sake of simplicity.
Table 2: The extrapolation procedures are explained in the text. In each case, the values of the ratios of form factors $V_{\text{sc}}(0)/A_{1}\text{c}(0)$ and $A_{2}\text{c}(0)/A_{1}\text{c}(0)$ have been fitted to minimize the $\chi^2$ relative to the experimental numbers in the last line. A two parameter fit for three constraints leaves one degree of freedom (dof). The experimental value for $R_L = \Gamma_L/\Gamma_{\text{tot}}$ is taken from CDF or CLEO according to what is indicated in the first column. “AVE” refers to the world average for the form factor ratios in $D \rightarrow K^{(*)}\nu$. Whenever it was needed to combine statistical and systematic errors, we have combined them in quadrature.

| extrapolation | $\frac{A_{2\text{c}}(m_{B}^2)}{A_{1\text{c}}(0)}$ | $\frac{V_{\text{sc}}(0)}{A_{1\text{c}}(0)}$ | $\frac{A_{2\text{c}}(0)}{A_{1\text{c}}(0)}$ | $\frac{\Gamma_L}{\Gamma_{\text{tot}}}$ | $\chi^2$/dof |
|---------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|---------------|
| Soft-Pole: CDF| 1.34                             | 1.82                             | 0.656                            | 0.490                            | 1.9           |
| Soft-Pole: CLEO| 1.34                             | 1.67                             | 0.520                            | 0.567                            | 9.0           |
| Soft-Cons: CDF| 1.77                             | 1.75                             | 0.556                            | 0.386                            | 5.7           |
| Soft-Cons: CLEO| 1.77                             | 1.53                             | 0.385                            | 0.520                            | 16.5          |
| Hard-Pole: CDF| 2.14                             | 1.70                             | 0.480                            | 0.322                            | 9.5           |
| Hard-Pole: CLEO| 2.14                             | 1.43                             | 0.292                            | 0.498                            | 22.5          |
| Hard-Cons: CDF| 2.83                             | 1.66                             | 0.375                            | 0.233                            | 16.1          |
| Hard-Cons: CLEO| 2.83                             | 1.29                             | 0.165                            | 0.481                            | 31.9          |
| exp: AVE + CDF| -                                | $1.9 \pm .25$                   | $0.74 \pm .15$                   | $0.66 \pm .14$                   | -             |
| exp: CLEO II | -                                | -                                | -                                | $0.80 \pm .10$                   | -             |
We have first performed a fit restricted to the $K^*$ final state, leaving aside the $f_+$ form factors and the ratio $\Gamma(B \to K^*\psi)/\Gamma(B \to K\psi)$. The results are displayed in table 2. As a first conclusion from table 2 one sees that the best fit is Soft-scaling-Pole. The reason for that is that the large experimental values of $\Gamma_L/\Gamma_{tot}$ and $A_2^{sc}(0)/A_1^{sc}(0)$ impose the double ratio $R_2$ to be rather small, as argued in section 3.3, thus suggesting either that the asymptotic scaling law ($A_2/A_1 \propto m_Q$) is strongly softened, or that the $A_2/A_1$ ratio decreases dramatically with decreasing $q^2$, or some compromise between both effects. Indeed the Soft-scaling-Pole Ansatz has both a softened scaling at $q^2_{max}$ and a decrease of the $A_2/A_1$ ratio with decreasing $q^2$. Therefore, this Ansatz yields the smallest ratio $R_2$, and thus a larger $\Gamma_L/\Gamma_{tot}$. A roughly acceptable fit is thus obtained for CDF data, with a confidence level of $\sim 20\%$, but not for CLEO. To obtain an acceptable fit with CLEO data, one would need a value of $R_2$ sensibly smaller than 0.8. Such a low value seems very difficult to obtain in any natural way. Table 2 also shows that Soft-scaling is generally favored and Hard-scaling is in very strong disagreement with data.

In a second step we have added to our fits the data concerning the $K$ final state. The results are displayed in table 3. Qualitatively there is no big change. Looking in more detail, it appears that the best fits now include on the same level the Soft-scaling-Pole and the Soft-scaling-Constant cases, with a worst confidence level (around 2\%). The reason for that is that all our Ansätze correspond to $R_+ \simeq R_2$, and, while the data on $\Gamma_L/\Gamma_{tot}$ require a small $R_2$ as just argued, the data on $\Gamma(B \to K^*\psi)/\Gamma(B \to K\psi)$ and $f_+^{sc}(0)/A_1^{sc}(0)$ on the contrary require a not too small double ratio $R_+$. This can be understood as follows.

The ratio $R$ (9) is given by:

$$R = 1.081 \left( \frac{A_1^{sb}(m_\psi^2)}{f_+^{sb}(m_\psi^2)} \right)^2 \left\{ 2 \left[ 1 + 0.189 \left( \frac{V^{sb}(m_\psi^2)}{A_1^{sb}(m_\psi^2)} \right)^2 \right] + \left( 3.162 - 1.306 \frac{A_2^{sb}(m_\psi^2)}{A_1^{sb}(m_\psi^2)} \right)^2 \right\}$$

Multiplying the l.h.s. of eqs (16) and (35) we get:
Table 3: The extrapolation procedures are explained in the text. In each case, the values of the ratios of form factors $V_{sc}^c(0)/A_{sc}^c(0)$, $A_{sc}^c(0)/A_{sc}^c(0)$ and $f_{sc}^c(0)/A_{sc}^c(0)$ have been fitted to minimize the $\chi^2$ relative to the experimental numbers in the last line. A three parameter fit for five constraints leaves two degrees of freedom (dof). The experimental value for $R_L = \Gamma_L/\Gamma_{tot}$ is taken from CDF or CLEO according to what is indicated in the first column. “AVE” refers to the world averages for form factor ratios in $D \rightarrow K^{(*)}\nu \nu$. Whenever it was needed to combine statistical and systematic errors, we have combined them in quadrature.
\[ R(1 - R_L) = 2.162 \left( \frac{A_{1}^{sh}(m_{\psi}^2)}{f_{+}^{sh}(m_{\psi}^2)} \right)^2 \left[ 1 + 0.189 \left( \frac{V^{sh}(m_{\psi}^2)}{A_{1}^{sh}(m_{\psi}^2)} \right)^2 \right] \] (36)

This gives obviously a lower bound on \( f_+/A_1 \). For the conservative upper bounds of \( R \leq 2.5 \) and \( 1 - R_L \leq 0.5 \) and setting still more conservatively \( V \) to zero we get \( f_+/A_1 \geq 1.32 \). For a more realistic estimate, \( V/A_1 \simeq 2 \), and \( R \leq 2.0 \) we get \( R_+ \geq 1.60 \). Contrarily to our discussion in section 3.3, we find here a lower bound which in itself is compatible with the hard scaling behaviour but not with such a soft scaling as required for \( A_2/A_1 \) (remember we had \( A_2/A_1 \leq 1.3 \) for \( V = 0 \) and for a more realistic \( V/A_1 \), \( A_2/A_1 \leq 1 \)). Clearly the trend for \( f_+/A_1 \) is somewhat opposite to the one for \( A_2/A_1 \).

The \( \chi^2 \) fits tried a compromise between these two opposite trends, and this is why the Soft-scaling-Constant Ansatz now fits as well as the Soft-scaling-Pole one: although the prediction for \( \Gamma_L/\Gamma_{tot} \) is worst for the former, it gives a better \( \Gamma(B \to K^*(\psi))/\Gamma(B \to K\psi) \) ratio, since it corresponds to larger double ratios. Again the Hard-scaling cases are rejected. The two best fits are hardly acceptable in the case of the CDF value \( R_L = 0.66 \pm 0.14 \) and fail with CLEO II’s much more restrictive value \( R_L = 0.80 \pm 0.095 \) (as it would have failed with the Argus bound \( R_L > 0.78, 95\% \) confidence level). There is a real difficulty, as noted in [29], to account for the data on \( D \to K^{(*)}l\nu \) and \( B \to K^{(*)}\psi \).

**What could be the way out of this dilemma?** One may of course question the factorization hypothesis which is the basic hypothesis in all this paper. Although we are thoroughly convinced that the factorization hypothesis may very reasonably be doubted in a \( 1/N_c \) subdominant channel as is \( B \to K^{(*)}\psi \), we decided to leave all this discussion outside the present paper. We may also hope that more precise experiments will evolve in a direction that will make the problem not so acute. Although the \( \chi^2 \) may seem horrific when the CLEO II value for \( R_L \) is used, it should be kept in mind that a small variation of the experimental value may lead to a dramatic decrease of the \( \chi^2 \). A comparison with CDF gives a first example of that.

Finally our hypothesis, displayed in eqs (31), (31)-(33), leading \( V(q^2)/A_1(q^2) \propto \)
$A_2(q^2)/A_1(q^2)$ and $f_+(q^2)/A_1(q^2) \propto A_2(q^2)/A_1(q^2)$ may also be criticised. It would of course be meaningless to relax these constraints in the above-described $\chi^2$ fits, since we would have too many free parameters. Qualitatively it is obvious that any prescription with $R_+$ sensibly larger than $R_2$ would lessen the $\chi^2$. But we see no sign of such a trend in the models we have considered. Another lessening of the $\chi^2$ would happen if $R_V = (V^{sb}(m_\psi^2)/A_1^{sb}(m_\psi^2))/(V^{sc}(0)/A_1^{sc}(0))$ was sensibly smaller than $R_2$. This happens to be the case, although in a quantitatively insufficient amount, in the Orsay Quark Model (see section 4.3.6): in eq. (54) it appears that $V(q^2)$ contains a relatively large corrective factor $Y$, (58), which decreases with the initial mass. This term tends to decrease all the $\chi^2$ in table [6] but not enough. It has the effect of providing for the form factor $V(q^2)$ an even larger softening of the increase predicted by the asymptotic scaling law (13). It is interesting to notice that such a large correction to the asymptotic scaling law for $V(q^2)$ in the direction of a softening has been found in lattice calculations [20], although the large statistical errors in these calculations do not allow to draw yet a final conclusion. Finally, our difficulties to get small $\chi^2$ is not too surprising since the $\chi^2$ tends anyhow to become large when experimental errors decrease, unless a very accurate model is available, which is certainly not the case in the present attempts.

To summarize, we have found, using our simple phenomenological Ansätze for the dependence of the form factors ratios in $q^2$ and in masses, that:

- The experiment comparison between $B \to K^{(*)}\psi$ and $D \to K^{(*)}\ell\nu$ favors a soft heavy-to-light scaling, eqs. (22) and (31).

- The best $q^2$ dependence cannot be selected from this analysis alone, although the separated phenomenological study of the $K^*$ final states (table [4]), as well as several theoretical considerations, tend to favor the existence of the “kinematical pole” as in eq. (22).

- There remains a difficulty to reconcile experimental results in $B \to K^{(*)}\psi$ and

\*\*The latter equation is only approximately valid in eqs. (22) and (31) due to the $K - K^*$ mass difference.
$D \rightarrow K^{(*)}\ell\nu$ when taking CDF results for $R_L (\chi^2/dof \simeq 3)$ which grows even worst when using CLEO or ARGUS values for $R_L$. There seems to be also a particular difficulty to fit simultaneously $R$ and $R_L$. Only fragile indications of possible ways out these difficulties are known today.

### 3.6 $q^2$ dependence of $A_1(q^2), f_+(q^2)$, etc. from experiment.

Up to now we have mainly considered the ratios of form factors, $A_2/A_1, V/A_1$ and $f_+/A_1$. In this subsection we try to go beyond and consider how the form factors themselves depend on $q^2$. We shall now gather from different sources information about $A_1(q^2), f_+(q^2)$, and we shall see that these combined informations are rather compatible with what we already know about the ratios.

Although pure phenomenology using combined final $K$ and $K^*$ data did not allow us to choose between Soft-scaling-Pole (22) and Soft-scaling-Constant (31) Ansätze, the separated phenomenology for the $K^*$ final state and also several theoretical arguments lead us to chose the Soft-scaling-Pole.

Of course the above rough agreement of Soft-scaling-Pole for $\Gamma_L/\Gamma_{tot}$ and for $\Gamma(B \rightarrow K^*\psi)/\Gamma(B \rightarrow K\psi)$ does not depend of the value of $\eta(\vec{q}, m_f)$ in eq. (22). This subsection is devoted to argue in favor of our choice in eq. (25) for $\eta(\vec{q}, m_f)$. The meaning of eq. (25) is in fact that we choose $A_1(q^2)$ to be a constant:

$$A_1(q^2) = r_1 \sqrt{\frac{4m_P m_{V_f}}{m_P + m_{V_f}}}. \quad (37)$$

Of course, only the product $r_1 \eta(\vec{q}, m_f)$ is relevant, not the separate values of $r_1$ and $\eta(\vec{q}, m_f)$. Next, let us stress that our QMI Ansatz, eqs (22) and (25), does not mean that we believe $A_1(q^2)$ to be a constant. We are indeed sure, from its analytic properties (the axial current singularities in the $t$-channel), that $A_1(q^2)$ is not a constant. Our Ansatz really means that we believe $A_1(q^2)$ to vary slowly with $q^2$ in the physically relevant region, at least slower than predicted from pole dominance. Eq. (23) is only the simplest possible Ansatz to express this feature of a slow variation. Let us now summarize a few arguments in favor of the slow variation of $A_1(q^2)$. 

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i) Pole-like behavior of $D \to Kl\nu$. In [13] it is argued that $f_+(q^2)$ in $D \to K^{(*)}l\nu$ decay may well be fitted by a vector meson pole, and the fitted pole mass is $M^* = (2.00 \pm 0.11 \pm 0.16)$ GeV, in good agreement with the value of 2.1 GeV expected for the mass of the $D^*_s$ meson. This fit does not establish the detailed analytic form of $f_+(q^2)$ since an exponential fit is told to agree as well. But it certainly conveys the message of a $f_+(q^2)$ increasing with $q^2$ like a pole term rather than, say, a dipole or a constant. Combined with both eq. (22) this $q^2$ dependence points toward a constant $\eta(q^2, m_K)$ ††.

ii) The phenomenological factorization coefficient $a_2$. Although there is no theoretical principle to fix $a_2$ in the phenomenological BSW factorization prescription, it seems reasonable that it cannot be too different from its value, $a_2^{SVZ} \simeq 0.1$, in the standard SVZ factorization, eq. (2). The results for our favored Soft-scaling-Pole are displayed in table 4. It appears that the double pole assumption gives very large values for $a_2$, while the constant behaviour for $A_1$ is favored as it gives the smallest $a_2$ (remember that this corresponds to a pole behaviour of $f_+$). This confirms our choice (25). Still our preferred fitted $a_2$, ranging from 0.22 to 0.28, might be considered as rather large compared to the SVZ value.

iii) Orsay Quark Model. OQM will be detailed in section 4.3.6. It seems to suggest a weak $q^2$ dependence for $A_1(q^2)$, related to the slow variation of a Lorentz contracted overlap near $q^2 = 0$, and an approximate pole behavior for the rest of the form factors.

iv) Lattice calculations and QCD Sum Rules. Lattice and QCD sum rule results will be commented in sections 4.1 and 4.2. The situation is not so clear, errors are still large in lattice calculations, different QCD Sum Rules calculations ††One may object that the pole of $f_+(q^2)$ in our QMI model is at $m_D + m_K \simeq 2.3$ GeV, sensibly larger than the fitted pole above-mentioned. This objection is obviously valid, it is obvious that eq. (22) and (25) do not provide the correct analytic properties to the form factors, they do not have the wanted poles and cuts, etc. For sure, our Ansätze are only approximations that we hope to be good enough in the physical region.
Table 4: The fitted values of the phenomenological factorization parameter $a_2$ from $B \to \psi K$ branching ratios are given in column four, the one fitted from $B \to \psi K^*$ in the last column. The first two lines use BSW models. The starting point for the other lines are the Soft-Pole form factor ratios in table 3, both with CLEO and CDF values for $R_L$. Given the ratios, either we take $A_1$ or $f_+$ from $D \to K^{(*)}l\nu$ experiment. In the last two columns we report the range of fitted $a_2$ obtained with four different choices: $A_1$ or $f_+$ from experiment, CLEO or CDF for $R_L$. It appears that these ranges are narrow enough. An additional prescription is used for the $A_1$ dependence on $q^2$: pole dominance or constant. The corresponding indication is given in a transparent way in the first column.

show discrepancies. However there is a converging set of indications against a pole dominance in the case of $A_1$, generally pointing towards a flatter $q^2$ dependence.

3.7 Mass dependence of form factors at $q^2 = 0$.

We have already noticed, eq. (24), that our Ansatz (22) for the form factor ratios implies a very simple mass dependence at $q^2 = 0$. In this section we will draw the consequences of our different Ansätze on the mass dependence of the form factors at $q^2 = 0$.

Hard-scaling-Constant with a pole for $A_1$. This prescription, using eq (33) and

$$A_1(q^2) = \frac{A_1(0)}{1 - \frac{q^2}{m_B^2}},$$

is equivalent to assuming a pole dominance for all form factors, as was done in [5] and [19]. At $q^2 = 0$ one obtains:

$$\frac{f_+^{sb}(0)}{f_+^{sc}(0)}, \quad \frac{V^{sb}(0)}{V^{sc}(0)}, \quad \frac{A_2^{sb}(0)}{A_2^{sc}(0)} = \left(\frac{m_D}{m_B}\right)^{\frac{1}{2}} \left(1 + O\left(\frac{1}{m_D}\right)\right)$$

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\[ \frac{A_{1}^{sb}(0)}{A_{1}^{sc}(0)} = \left( \frac{m_D}{m_B} \right)^{\frac{3}{2}} \left( 1 + O \left( \frac{\Lambda}{m_D} \right) \right). \] (39)

**Hard-scaling-Pole with a pole for** \( A_1 \). This prescription starts from (32) also with (38). It yields double poles for the other form factors than \( A_1 \). It was used in [6]. At \( q^2 = 0 \) it gives the same ratio for all form factors:

\[ \frac{f_+^{sb}(0)}{f_+^{sc}(0)}, \quad \frac{V^{sb}(0)}{V^{sc}(0)}, \quad \frac{A_1^{sb}(0)}{A_1^{sc}(0)}, \quad \frac{A_2^{sb}(0)}{A_2^{sc}(0)} = \left( \frac{m_D}{m_B} \right)^{\frac{3}{2}} \left( 1 + O \left( \frac{\Lambda}{m_D} \right) \right). \] (40)

**Hard-scaling-Pole with a constant for** \( A_1 \). Eq. (32) with (25). It gives at \( q^2 = 0 \) also the same ratio for all form factors, with a different power:

\[ \frac{f_+^{sb}(0)}{f_+^{sc}(0)}, \quad \frac{V^{sb}(0)}{V^{sc}(0)}, \quad \frac{A_1^{sb}(0)}{A_1^{sc}(0)}, \quad \frac{A_2^{sb}(0)}{A_2^{sc}(0)} = \left( \frac{m_D}{m_B} \right)^{\frac{3}{2}} \left( 1 + O \left( \frac{\Lambda}{m_D} \right) \right). \] (41)

The three preceding cases have only an asymptotic validity, for \( m_B, m_D \to \infty \). In eqs (32) and (33), the corrections have been retained, which explains an \( O(1/m_D) \) difference between the latter and eqs. (39)-(41). On the contrary, the next equation retains the non-asymptotic corrections.

**Soft-scaling-Pole with a constant for** \( A_1 \). Eq. (22) with (25). As we have already told, this is our prefered Ansatz. It gives at \( q^2 = 0 \)

\[ \frac{f_+^{sb}(0)}{f_+^{sc}(0)}, \quad \frac{V^{sb}(0)}{V^{sc}(0)}, \quad \frac{A_1^{sb}(0)}{A_1^{sc}(0)}, \quad \frac{A_2^{sb}(0)}{A_2^{sc}(0)} = \left( \frac{m_B}{m_D} \right)^{\frac{3}{2}} \left( \frac{m_D + m_K}{m_B + m_K} \right). \] (42)

which reduce of course to (41) in the asymptotic regime.

The results (39) to (42) will be useful in section 4 to discuss the models.
4 Discussion of theoretical approaches.

The discussion in section 3 provides us with some tools to look further into the theoretical schemes considered in the literature:

1. Lattice calculations (LC).
2. QCD sum rules (QCDSR).
3. Pole model of Bauer, Stech and Wirbel [5].
4. Pole-dipole model of Neubert et al. (NRSX) [6].
5. VMD model of Altomari and Wolfenstein [19].
6. Quark model of Isgur, Scora, Grinstein and Wise (ISGW) [11].
7. Orsay quark model (OQM) [31].

In examining all these approaches, we will pay attention to two main aspects:

i) To what extent are they fulfilling the asymptotic theorems, including the heavy-to-heavy scaling when both masses are heavy?

ii) Why do they fail at explaining the $B \to \psi K(K^*)$ data?

4.1 Lattice complemented with $q^2$ Ansatz.

We have used the lattice Monte Carlo calculations of the form factors performed by the European Lattice Collaboration at $\beta = 6.4$. The details on the lattice parameters can be found in ref. [20]. What is relevant here is that the lattice spacing is large enough to allow relatively large quark masses, from which to extrapolate up to the $B$ meson. Reversely, the statistics is not too high, leading to large errors. For the light quark we have used the value $\kappa = 0.1495$ which happens to be very close to the "physical" strange quark: $\kappa_s = 0.1495 \pm 0.0001$ [20]. The description of the extrapolation in the heavy quark mass up to the $b$ quark is to be found in [20]. But we have modified the extrapolation procedure in $q^2$. In [20] a pole dominance
approximation was used for all form factors. Since we have strong reasons exposed in this paper to doubt this hypothesis, we have done the following:

The lattice calculations have been performed for the $A_1$ form factors at five different values of $q^2$. However, due to the statistical noise, we have only used the three closest to $q_{max}^2$. We then perform a two parameter fit for $A_1$:

$$A_1(q^2) = a + \frac{b}{q^2 - M_p^2}$$

or equivalently

$$A_1(q^2) = \frac{aq^2 + c}{q^2 - M_p^2}, \quad c = b - aM_p^2$$

where $M_p$ is the lattice mass of the lightest t-channel axial meson pole. The justification of such a form is twofold: i) the form factor must indeed present a pole at $q^2 = M_p^2$; ii) the constant $a$ mimics the subtraction constant of the dispersion relation.

To present the results of our $q^2$ dependence fit of $A_1(q^2)$ we will define a ratio

$$P_1 = \left( \frac{M_p^2 - q_{max}^2}{A_1(q_{max}^2)} \right) \frac{\partial A_1(q^2)}{\partial q^2} \bigg|_{q_{max}^2}$$

such that $P_1 = 1$ in the pole dominance hypothesis. Using two different analysis methods explained in [20] we find $P_1 = 0.38 \pm 0.50 (0.92 \pm 0.41)$ for the “analytic” (“ratio”) method. One may see an indication in the direction of a flatter behaviour of $A_1$ than predicted by the pole dominance, but the errors prevent any firm statement.

Concerning the other form factors, $A_2, f_+, V$, the lattice calculations do not give a direct estimate at $q_{max}^2$, and the same fitting procedure is not possible. We then have chosen to assume that the $q^2$ dependence of the ratios $A_2(q^2)/A_1(q^2), f_+(q^2)/A_1(q^2)$ and $V(q^2)/A_1(q^2)$ are given by eq. (22) or by eq. (31).

Our results are reported in table 5. Clearly the errors are overwhelming for the ratio $R$, but the results for the ratio $R_L$ delivers a clear message in favor of the “Soft-scaling-Pole” prescription for the $q^2$ dependence of form factors ratios. With

\[\text{Let us repeat that only the } q^2 \text{ dependence is taken from eqs. (22) and (31), the mass dependence has been fitted from the lattice as explained in [20].} \]
Table 5: The results from lattice calculations at $\beta = 6.4$. The mass dependence of the form factors and the $q^2$ dependence of $A_1(q^2)$ has been fitted as explained in the text. The $q^2$ dependence of the ratios $A_2/A_1$, $f_+/A_1$ and $V/A_1$ have been taken according to a prescription indicated in the first column. The experimental number have been taken from CDF, those from CLEO II being indicated in brackets.

| form factors ratios | $\frac{\Gamma(K^*)}{\Gamma(K)}$ | $\frac{\Gamma_L}{\Gamma_{\text{tot}}}$ |
|---------------------|----------------------------------|----------------------------------|
| Soft-Pole eq.(22)   | 3.5 ± 2.5                        | 0.47 ± 0.11                      |
| Soft-Cons. eq.(31)  | 1.9 ± 1.4                        | 0.27 ± 0.16                      |
| exp.                | 1.64 ± 0.34                      | 0.64 ± 0.14 (0.80 ± 0.10)        |

the latter prescription, the lattice results is within 1σ from CDF, but 3σ from CLEO II.

Preliminary results from APE lattice collaboration [18] on $B \to K^*\gamma$ seem to indicate also an increase of $T_2(q^2)$ with $q^2$ much slower than expected from pole dominance, i.e. the analogous of parameter $P_1$ defined in eq (13) looks much smaller than 1. This is interesting in view of the fact that $T_2$ is asymptotically equal to $A_1$. Similarly, $T_1$ is asymptotically equal to $V$. Now, APE collaboration finds a much faster increase of $T_1$ with $q^2$ than $T_2$, as would be predicted from an extension of eq (29) to the heavy-to-light system.

4.2 QCD sum rules.

There has been several studies [22]-[26] of the $q^2$ dependence of heavy-to-light form factors using different varieties of QCD sum rules: Laplace sum rules, hybrid sum rules and light-cone sum rules. Some kind of consensus seems to have emerged, that we could characterize by saying that these authors find an agreement with vector meson dominance for vector current form factors, and a more gentle slope for $A_1$. Still, when one looks in more detail, the different predictions for $A_1$ differ somehow. Ali et al. [24] find a softly increasing $A_1$ for all $q^2$, while Ball [23] finds $A_1$ decreasing with $q^2$ for $q^2 \leq 15 \text{GeV}^2$. Narison [26] finds a decreasing $A_1$ in the $q^2 \leq 0$ region, and catches up with Ball’s result. Ball finds an increasing $A_2$ in the same region $q^2 \leq 15 \text{GeV}^2$ where $A_1$ decreases, and interestingly enough, at a first glance the
plots show that the in this $q^2$ region, the ratios $A_2(q^2)/A_1(q^2)$ and $V(q^2)/A_1(q^2)$ are not very different from the “kinematical pole” term $1/(1 - q^2/(m_P + m_{V_i}))^2$ (see eq. (22)). Unhappily, for the limited $q^2 \geq 15$ GeV$^2$ region, the trend is reversed and the ratio $A_2(q^2)/A_1(q^2)$ even starts decreasing. Ali et al. also find a $V(q^2)/A_1(q^2)$ ratio that has some analogy with the “kinematical pole” in the whole region they plot: $q^2 \leq 17$ GeV$^2$. Finally, comparing Belyaev at al \cite{23} to Ali et al. \cite{24} we see that $f_+(q^2)/A_1(q^2)$ also increases with $q^2$, although maybe in a milder way than $V(q^2)/A_1(q^2)$. Finally, let us mention that Narison \cite{26} finds asymptotically, when the mass really goes to infinity ($m_Q \gg m_b$), an analytic evidence for a pole behaviour of the form factor ratios $A_2/A_1$, $V/A_1$ and $f_+/A_1$, but this happens through a polynomial decrease of $A_1$ and a constancy of the other form factors.

To summarize, notwithstanding sensibly different predictions for $A_1$, there is an almost general agreement (except for a small domain near $q^2_{max}$ in \cite{23}) on an increase of the form factor ratios, rather similar to the “kinematical pole” behaviour in \cite{22}.

Using the results obtained by Patricia Ball, \cite{23} and \cite{27} for $B \to \pi, \rho$, and assuming they are also valid for $B \to K^{(*)}$, we have computed the ratios $R$ and $R_L$ that are reported in table \ref{tab:1}. $R_L$ comes out rather small due to a too large value: $A_2(m_\psi^2)/A_1(m_\psi^2) \approx 1.2$: indeed, given the value $V(m_\psi^2)/A_1(m_\psi^2) \approx 2.8$ in \cite{27}, the constraint $R_L \geq 0.5$ translates into $A_2(m_\psi^2)/A_1(m_\psi^2) \leq 0.8$. The ratio $R$ comes out too large due to a too small value $f_+(m_\psi^2)/A_1(m_\psi^2) \approx 1$, while a reasonable lower bound of $\sim 2$ may be derived from eq. (30). We have neglected $SU(3)$ breaking which could change the results, but we doubt this change could be large enough to recover an agreement with $B \to K^{(*)}\psi$ data. Once more we see how difficult a challenge these data are for all known theoretical approaches.

\subsection{Quark Models.}

Under this heading are included very different approaches. This wide range of methods reflects primarily the inability of the simple-minded non relativistic model to describe the form factors: this inability consists in two main facts: i) Away from

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The model is highly ambiguous, as one soon reaches relativistic velocities. ii) The slope $\rho^2$ of the Isgur-Wise function is definitely too small. Various attempts have been made to cure these defects, either by appealing to ideas connected with vector meson dominance (VMD) or with the relativistic effects \[14\], \[15\]. We have restricted ourselves to the more extensively used models in literature, leaving aside very interesting ones such as the work by Jaus and Wyler \[28\].

As a general remark, it must be emphasized that all quark models except the OQM, section 4.3.6, do not satisfy the heavy-to-heavy scaling when both $m_{P_i}$ and $m_f$ are large, and in some cases (BSW models) violate also heavy-to-light scaling, as we shall argue in subsection 4.3.1. Also, on the empirical side, all models fail to explain $D \to K^*$ decays. The axial form factors are too large, resulting in a too large $\Gamma(D \to K^*\ell\nu)/\Gamma(D \to Kl\nu)$ ratio. This is easily understandable by the fact that no attempt has been made to incorporate in them the binding effects which are crucial in obtaining a relative reduction of axial with respect to vector form factors (remember $D \to K$ is a purely vector transition). A similar situation was discussed in the past about the nucleon axial vector coupling, the $G_A/G_V$ (sometimes noted $g_1/f_1$) ratio \[30\].

4.3.1 BSW models.

The BSW models have been used extensively to analyze non-leptonic $D$ and $B$ decays with the help of the additional BSW factorization assumption. There are two main and very different ingredients in these models, which should not be confused:

i) A standard quark model, which is used only at $q^2 = 0$. The values at $q^2 = 0$ are found to be approximately the same for all $c \to s$ and for all $b \to s$ form factors:

$$f^c_{+}(0) \simeq V^{sc}(0) \simeq A^{sc}_{1}(0) \simeq A^{sc}_{2}(0) \simeq 0.8$$

$$f^{sb}_{+}(0) \simeq V^{sb}(0) \simeq A^{sb}_{1}(0) \simeq A^{sb}_{2}(0) \simeq 0.35.$$  \hspace{1cm} (46)

One notes that the values of the form factors $V$ and $A_2$ in (46) are not consistent with what is now known from experimental $D$ semi-leptonic decays \[11\] and
(2).

From eq. (35) in [4], one can deduce the common asymptotic behavior of all the form factors at \( q^2 = 0 \) as function of \( m_P \):

\[
h \propto \frac{1}{m_{P_i}}
\]

(48)

ii) Different possible Ansätze about the \( q^2 \) dependence away from 0, which lead to two distinct phenomenological models: either a pole for each form factor (BSW I), [5], or a dipole for some and a pole for others (BSW II), [6]. These Ansätze are probably motivated by the feeling that naïve application of the quark model would fail, and also by the general idea of pole dominance; the latter reason is why one may speak of “hybrid” models. It must be said that in the case of BSW II, this VMD idea is in addition mixed with still another idea perhaps contradictory to it, the squaring of the pole, which is inspired from Isgur-Wise heavy-to-heavy scaling.

In both cases the asymptotic behavior at \( q^2 = 0 \) as function of the heavy mass will be found below to be contradictory with what could be deduced from heavy-to-light scaling and the assumed \( q^2 \) dependence. This implies that the models do not fulfill the heavy-to-light scaling properties.

4.3.2 First BSW model.

The BSW single pole model [5] for all the form factors has been used to analyze the non-leptonic \( D \) and \( B \) decays. Note that, independently of the precise value of the ratio between the numbers, the ratio between \( B \) and \( D \) form factors is roughly identical for all form factors. This seems hardly compatible with the relations (39), which result from the combination of hard scaling and the \( q^2 \) dependence assumed in the BSW I Model, and would imply asymptotically a different \( B \) to \( D \) ratio for \( f_+ \), \( V \), \( A_2 \), and \( A_1 \) respectively. This suggests that the model does not fulfill the heavy-to-light scaling properties. One can indeed prove rigorously this fact in the asymptotic limit \( m_{P_i} \gg m_f \), by observing that the asymptotic behavior (48) is in contradiction with the asymptotic relations (39) deduced from heavy-to-light scaling.

The BSW I model gives \( R_L = 0.59 \); this is not too bad, but the ratio \( R = 4.23 \)
is much too large (see table [1]). This is due to the fact that the ratio \( f_+/A_1 \) is too small as seen in the table [1].

4.3.3 Second BSW or NRSX model.

The pole-dipole model of Neubert, Rieckert, Stech and Xu (NRSX) [6] uses, to our knowledge, the same values (46) and (47) at \( q^2 = 0 \). It obviously results that all form factors have the same ratio \( 0.35/0.8 = 0.44 \) for their \( q^2 = 0 \) value at \( B \) versus \( D \). The equality of these ratios is now in agreement with what is expected, in the \( m_P \gg m_f \) limit, from heavy-to-light scaling and the assumed \( q^2 \) dependence: (48). But the value of the ratio, 0.44, is larger than expected from the same relations. Although the latter should hold only asymptotically, this suggests somewhat that the model violates heavy-to-light scaling. This can be proven rigorously in the same manner as above by noting that the asymptotic behavior (48) at \( q^2 = 0 \) contradicts the relations (40).

The model gives a reasonable value \( R = 1.61 \), but \( R_L = 0.36 \) is too low, which seems to have escaped notice, with the recent exception of [19]. Overall one could estimate that this model is not faring too badly. This is however obtained by form factors (table [1]) rather different from the form factors we advocate by appealing to asymptotic principles (Soft-scaling-Pole solution in table [1]): \( A_2/A_1 \) is sensibly higher, and \( V/A_1 \) is sensibly lower.

4.3.4 Altomari Wolfenstein Model.

We quote this model [19] as an interesting proposal although it has not been applied to the \( D \to K^{(*)} \ell \nu \) and \( B \to K^{(*)} \psi \) phenomenology. In this model one assumes that the non relativistic quark model is valid at \( q^2_{\text{max}} \), completed by a vector dominance assumption for the \( q^2 \) dependence. With such assumptions, it is easy to see that heavy-to-light scaling is satisfied asymptotically. On the other hand, it is obvious that the model has not the \( q^2 \) dependence required to satisfy heavy-to-heavy scaling away from \( q^2_{\text{max}} \) when both hadrons are made heavy.
4.3.5 ISGW Quark Model.

Although one is tempted to classify it among the non-relativistic models, it results from a modification of the NR model form factors which is no more the one predicted by the wave functions; this has then some common “hybrid” spirit with the previous A-W model. The justification given is however different: to cure the failure of the NR approximation, an ad hoc adjustment of the slope is made to take into account relativistic effects which are indeed expected to enlarge the slope.

More precisely this adjustment consists in making in the NR formulae the replacement:

\[ \vec{q}^2 \to \frac{1}{\kappa^2} \frac{m_f}{m_i} (q_{\text{max}}^2 - q^2) \]  

(49)

where is a $\kappa^2$ phenomenological factor $\simeq 0.5$. However we do not think that this prescription suffices to account for the variety of the expected relativistic effects that will be discussed in section 4.3.6.

One feature of the model is that in the transition to $0^-$ and $1^-$ all the form factors are equal to their $q_{\text{max}}^2$ value times a common exponential function of $q^2$; the form factors are then easily found to respect exactly the asymptotic heavy-to-light scaling; on the contrary they obviously violate the heavy-to-heavy scaling except at $q_{\text{max}}^2$, due to an inappropriate $q^2$ dependence. This is bothering since the model should apply without any change to the heavy-to-heavy case. This failure is easily understandable since the relativistic boost of spin, which is necessary to obtain the heavy-to-heavy scaling away from $q_{\text{max}}^2$, is missing in the model.

The failure of the model for $\Gamma_L/\Gamma_{\text{tot}}$ corresponds to the fact that $A_2/A_1$ is much too large (table 1). This in turn is related to the fact that the form factor ratios $A_2/A_1$, already too large in $D \to K^{(*)}l\nu$, is still increased up to $B \to K^{(*)}\psi$. Indeed, although this increase is soften at $q_{\text{max}}^2$ by the light final masses, being independent of $q^2$, this ratio is not further depressed by a faster decrease of $A_2$ with respect to $A_1$ as would be obtained by our Ansatz.

Finally, it should be noticed that in some decays where the final state has a large velocity ($B \to \pi l\nu$) there is a dramatic suppression of the form factor because
of the exponential fall-off. We attribute this drawback to the absence of Lorentz contraction of the wave function.

4.3.6 Orsay Quark Model for form factors.

The Orsay Quark Model (OQM) for form factors is a semi-relativistic weak binding model that has been presented in the lectures [14] (see older references therein) and that will be described in detail elsewhere [31]. In this section, $M, P, E$ refer to hadron masses, energies momenta, while $m, \vec{p}, \epsilon$ to quark masses, momenta, energies.

The model incorporates two main relativistic effects of the center of mass motion: the Lorentz contraction of the wave function and the Lorentz boost of the spinors. On the other hand, we adopt a weak-binding treatment. One makes everywhere the approximation of retaining only linear terms in internal momenta and one sets $M_i$ and $M_f$ equal to the sum of corresponding quark constituent masses. Therefore, as we explain in detail below, we do not consider it as a truly phenomenological model; it is rather an analytical instrument to discuss the specific effects of center-of-mass motion. We give now only the general principles behind it and write down the explicit form of the form factors. The total wave function writes

$$\psi^\text{tot}_{\vec{P}}(\{\vec{p}_i\}) = \delta\left(\sum_i \vec{p}_i - \vec{P}\right) \psi_{\vec{P}}(\{\vec{p}_i\})$$

where the internal wave function is given by

$$\psi_{\vec{P}}(\{\vec{p}_i\}) = N \left[ \prod_i S_i(\vec{p}_i) \right] \psi_{\vec{P}=0}(\{\tilde{\vec{p}}_i\})$$

with

$$\tilde{\vec{p}}_iT \equiv \vec{p}_iT$$

$$\tilde{\vec{p}}_iz \equiv \frac{E}{M} p_{iz} - \frac{P}{M} \epsilon_i$$

$$\tilde{\epsilon}_i \equiv \frac{E}{M} \epsilon_i - \frac{P}{M} p_{iz} \simeq m_i$$

and where
\[ S_i(\vec{P}) = \sqrt{\frac{E + M}{2M}} \left( 1 + \frac{\vec{\alpha}_i \cdot \vec{P}}{E + M} \right) \]

is a the Lorentz boost acting on the spinors

\[ u_{\vec{P} = 0} \simeq \left( \begin{array}{c} \chi \\ \frac{\vec{\pi}}{2m} \chi \end{array} \right) \]

the normalization being global for the internal wave function:

\[
\int \psi^+_P(\{\vec{p}_i\}) \psi_P(\{\vec{p}_i\}) \delta \left( \sum_i \vec{p}_i - \vec{P} \right) \prod_i d\vec{p}_i = 2E
\]

giving

\[ N = \sqrt{2M \sqrt{1 - \beta^2}} \]

The matrix element of an operator acting on the quark 2 will read, in the equal velocity frame (a collinear frame where the velocities are equal in magnitude and opposite in direction), after some algebra:

\[
\int \psi^+_P(\{\vec{p}_i\}) O(2) \psi_P(\{\vec{p}_i\}) \delta \left( \sum_i \vec{p}_i - \vec{P}_f \right) \prod_i d\vec{p}_i d\vec{p}'_i \delta (\vec{p}_1 - \vec{p}'_i) \delta (\vec{p}_2 - \vec{p}'_2 - \vec{q})
\]

\[
= \frac{N_i N_f}{\sqrt{1 - \beta^2}} \delta \left( \vec{P}_i - \vec{P}_f - \vec{q} \right) \int \psi^+_P(\{\vec{p}_i\}) \delta \left( \vec{p}_1 - \frac{m_1}{M_f} \vec{p}_i + \frac{m_i}{M_i} \vec{P}_i \right) S^+_2(\vec{P}_f) O(2) S_2(\vec{P}_i)
\]

\[
\psi_{\vec{P}_i = 0} \left( \vec{p}_1 - \frac{m_1}{M_i} \vec{P}_i, \vec{p}_2 + \frac{m_i}{M_i} \vec{P}_i \right) \delta \left( \sum_i \vec{p}_i \right) \prod_i d\vec{p}_i \quad . \quad (52)
\]

The wave function at rest is assumed to be given by the harmonic oscillator potential.

With these ingredients one can compute all the form factors we are interested in. Calling \( M_i, M_f \) the initial and final hadron masses, \( m_i, m_f \) the initial and final active quark masses, we find:

39
\[ f_+(q^2) = \frac{\sqrt{4M_iM_f}}{M_i + M_f} \left[ 1 - \frac{q^2}{(M_i + M_f)^2} \right] I(q^2) \left[ 1 + \frac{M_i - M_f}{M_i + M_f} X \right] \]  
(53)

\[ V(q^2) = \frac{\sqrt{4M_iM_f}}{M_i + M_f} \left[ 1 - \frac{q^2}{(M_i + M_f)^2} \right] I(q^2)(1 + Y) \]  
(54)

\[ A_1(q^2) = \frac{\sqrt{4M_iM_f}}{M_i + M_f} I(q^2) \left[ 1 + \frac{(M_i - M_f)^2 - q^2}{(M_f + M_i)^2 - q^2} Y \right] \]  
(55)

\[ A_2(q^2) = \frac{\sqrt{4M_iM_f}}{M_i + M_f} \left[ 1 - \frac{q^2}{(M_i + M_f)^2} \right] I(q^2) \left[ 1 + \frac{(M_i - M_f)^2 - q^2}{(M_f + M_i)^2 - q^2} Y - \frac{2M_f(M_i + M_f)}{(M_f + M_i)^2 - q^2} X \right] \]  
(56)

where, for the harmonic oscillator potential:

\[ I(q^2) = \left( \frac{2R_iR_f}{R_f^2 + R_i^2} \right)^{\frac{3}{2}} \exp \left( -\frac{2m^2R_i^2R_f^2}{R_f^2 + R_i^2} \left[ \frac{(M_f - M_i)^2 - q^2}{(M_f + M_i)^2 - q^2} \right] \right) \]  
(57)

and

\[ X = \frac{-m}{R_f^2 + R_i^2} \left( \frac{R_f^2}{m_i} - \frac{R_i^2}{m_f} \right) \quad \text{Y} = \frac{m}{R_f^2 + R_i^2} \left( \frac{R_f^2}{m_i} + \frac{R_i^2}{m_f} \right) \]  
(58)

parametrize corrections to the scaling limit, proportional to the spectator quark mass \( m \) and \( R_i \) and \( R_f \) are the radii of the initial and final mesons.

**Isgur Wise scaling.** It is important to realize that, with such a model, in the limit where both \( m_i \) and \( m_f \) are made heavy, one obtains exactly the whole set of scaling relations of Isgur-Wise, eq. \([17]\). The scaling function \( \xi(v_i,v_f) \) depends of course on the potential except for the relation \( \xi(1) = 1 \). In the case of the harmonic oscillator potential:

\[ \xi(v_i,v_f) = \frac{2}{1 + v_i v_f} \exp \left[ -\frac{m^2 R^2}{\sqrt{2}} \left( \frac{v_i v_f - 1}{v_i v_f + 1} \right) \right]. \]

The corresponding slope at the origin, within the weak binding and linear approximation, is :
\[ \rho^2 = -\xi'(1) = \frac{1}{2} + \frac{m^2 R^2}{2\sqrt{2}} \sim 0.9 \]

where \( R \) is the radius of a light-light meson.

\( q^2 \) Dependence. The expressions for the form factors above show that the \( q^2 \) dependence of \( A_1(q^2) \) is very weak especially near \( q^2 = 0 \) and that the \( q^2 \) dependence of the form factors \( f_+(q^2), V(q^2) \) and \( A_2(q^2) \) is dominated by the same kinematic pole that appears in the Isgur-Wise relations in the heavy-heavy case (3). In the model this kinematic pole comes simply from the Lorentz factor

\[ 1 - \beta^2 = \frac{4M_i M_f}{(M_i + M_f)^2} \frac{1}{1 - \frac{q^2}{(M_i + M_f)^2}} \]

that does not affect \( A_1(q^2) \) because this form factor is related to a purely transverse component of the axial current. The \( q^2 \) dependence of \( A_1 \) comes essentially from the exponential and it becomes rather weak when \( \vec{q}^2 \) is large, i.e. near \( q^2 = 0 \); indeed it tends to a constant. This would be true for any potential. It is a simple effect of the Lorentz contraction.

This model’s prediction is therefore similar to the QMI Ansatz, eqs. (22) and (25). The \( q^2 \) dependence of form factor ratios is almost the same in both models.

Corrections to scaling. For finite \( m_i, m_f \), the heavy-to-heavy scaling laws are broken in OQM by various effects

i) the radii in \( I(q^2) \) depend on the flavour

ii) the terms containing \( X \) and \( Y \) are of order \( m/m_{i,f} \). The scaling is broken because the spectator mass is no longer negligible.

On the whole, these scaling violations are rather small except for \( f_+ \) and especially for \( V \). That the latter is large is easily understandable even in the most naive non relativistic approximation. Indeed, the \( 1 + Y \) coefficient in (54) varies from 2 for \( m_i = m_f = m \) to 1 when \( m_i, m_f \gg m \).
Phenomenological shortcomings. Although the model is unique in obeying the full set of asymptotic scaling laws in contrast to previous models, it is not a satisfactory phenomenological model, because it lacks essential effects. First, due to the weak-binding approximation, the axial to vector matrix elements at $q_{\text{max}}^2$ are reduced to their static SU(6) value. For instance, one finds in this model for the nucleon axial to vector current coupling ratio: $G_A/G_V = 5/3$ (sometimes written $-g_1/f_1$) [30], which is too large by $\sim \sqrt{2}$. The same thing happens in $D \rightarrow K^*l\nu$ and could also explain why $\Gamma(D \rightarrow K^*l\nu)/\Gamma(D \rightarrow Kl\nu)$ is predicted too large by a factor 2 in most quark models. Indeed, the axial form factors $A_1, A_2$ are found too large with respect to the vector ones. In the OQM:

$$A_1^{sc}(0) = 0.89, \quad A_2^{sc}(0) = 0.87, \quad V^{sc}(0) = 1.15, \quad f_+^{sc}(0) = 0.74$$

$$\frac{\Gamma_L}{\Gamma_{\text{tot}}} = 0.41 \quad \frac{\Gamma(B \rightarrow K^*)}{\Gamma(B \rightarrow K)} = 4.75 \quad (59)$$

This is quite bad. To cure the problem with $G_A/G_V = 5/3$, we have adopted in the past the old recipe of multiplying the axial current by an adhoc factor $g_A = 0.7$ [32]. This gives, multiplying all axial form factors by $g_A$:

$$\frac{\Gamma_L}{\Gamma_{\text{tot}}} = 0.34 \quad \frac{\Gamma(B \rightarrow K^*)}{\Gamma(B \rightarrow K)} = 2.8 \quad (60)$$

Of course this is still unsatisfactory. Besides, such a recipe has no theoretical grounding. One should include systematically the binding corrections, which are known to correct the discrepancy for $G_A/G_V$ [33], [34].

Second, there is another difficulty concerning $f_+$. It tends to be too small by itself at large recoil for the pion final state and this cannot be easily explained by binding corrections which should have a reductive effect. This smallness is connected with the small mass of the $\pi$ as observed in [14]. We suspect that this could be connected with the Goldstone character of the $0^-$. Indeed, in the approximations of the present Quark Model the $0^-$ and $1^-$ should be degenerate.
4.4 The phenomenological analysis by Gourdin, Kamal and Pham.

While we were in the process of writing this paper we received a paper by Gourdin, Kamal and Pham which also study the relation between $B \to K^{(*)}\psi$ and $D \to K^{(*)}l\nu$ experiments with the help of heavy-to-light scaling rules, and also confront some theoretical approaches with $B \to K^{(*)}\psi$ experiments. We agree with these authors on their main conclusion that the current models do not fit the $B \to K^{(*)}\psi$ data. We differ with them in two respects. They have used all over the pole dominance for the $q^2$ dependence of all form factors, while we conclude to a different $q^2$ behaviour of the different form factors. Second, their eq. (29) which they present as the heavy-to-light scaling relation is in our opinion one possible form of the $m_Q$ dependence. The detailed dependence on masses contained in these eqs. is an arbitrary Ansatz, though an admissible one, since it is compatible with the asymptotic scaling relations. This Ansatz contains non-asymptotic corrections which amount to a softening of the asymptotic scaling law. Therefore we do not the least object to its use as a model, but only to the claim that it should be taken as the heavy-to-light Isgur Wise scaling law. Notice that their eq. (29) differs from the mass dependence in our Ansatz and. Finally we agree with the claim by these authors that $D \to K^{(*)}l\nu$ and $B \to K^{(*)}\psi$ data are difficult to reconcile within the heavy-to-light scaling laws. This difficulty, beyond the $\Gamma_L/\Gamma_{tot}$ problem stressed by the authors, shows up in $\Gamma(K^*)/\Gamma(K) = 2.86$ predicted from their eq. (33).

5 Conclusions.

All the approaches we have considered in this paper encounter difficulties in accounting for the $B \to K^{(*)}\psi$ data, particularly with the large $\Gamma_L/\Gamma_{tot}$ (CLEO and ARGUS data). At present it seems safer to keep open three possibilities to get out of this problem.

- Experiment may not have yet delivered its ultimate word, as the variation between different experiments seem to indicate, and it might evolve towards
data easier to account for.

- Although we did not discuss the factorization assumption, it should be kept in mind that it rests on no theoretical ground for color suppressed decay channels, as is the case for $B \to K(\ast)\psi$.

- Finally, models may be wrong. This will now be discussed in more details.

Of course, the first requirement for any model is to fulfill the heavy-to-light scaling relations. This has been seen not to be the case for the most popular BSW I and BSW II models, notwithstanding their relatively good empirical successes.

Our analysis has allowed to extricate from data some general trends, namely “softened” scaling, a sensibly different $q^2$ behaviour of $A_1$ versus $A_2, V, f_+$, and $A_1$ slowly varying with $q^2$. Ansätze that take these indications as a guide, obtain better values for $\Gamma_L/\Gamma_{tot}$, and a more reasonable $a_2$, although there remains a general tendency to underestimate $\Gamma_L/\Gamma_{tot}$ with respect to present data. Let us now comment on these general trends.

Data definitely exclude “hard scaling” i.e. the strict application of asymptotic heavy-to-light scaling formulae in the finite mass domain. We have proposed a “softened” Ansatz which is based on an extension of heavy-to-heavy scaling relations down to the light final meson case, with some rescaling. In fact this is equivalent to assuming a precocious scaling for the axial and vector current matrix elements. Consequently, the ratio $A_2/A_1$ does not increase too fast with the heavy mass.

There are indications from lattice calculations, form Quark Model, and to some degree from phenomenology, that $V$ should undergo an even softer scaling.

Another consequence of the above Ansatz, as well as of the Orsay Quark Model is that $A_2/A_1, V/A_1$ and $f_+/A_1$ should have a pole like behaviour in $q^2$, leading to an increase with $q^2$. This improves the agreement with $B \to K(\ast)\psi$ data, and seems to be corroborated by QCD Sum Rules calculations.

$D \to Kl\nu$ experiments seem to show a pole like behaviour for $f_+(q^2)$. Combined with our preceding Ansatz for the ratios, this implies an approximately constant $A_1(q^2)$. This particular $q^2$ behaviour is corroborated by the Orsay Quark Model, while
QCD Sum Rules give $q^2$ dependence of $A_1$ that never increases very fast, although different detailed shapes are proposed. Lattice calculations, within large errors, might give the same indication.

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