New unfolded higher spin systems in AdS$_3$

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Abstract
We investigate the unfolded formulation of bosonic Lorentz tensor fields of arbitrary spin in AdS$_3$ containing a parity breaking mass parameter. They include deformations of the linearizations of the Prokushkin–Vasiliev higher spin theory around its critical points. They also provide unfolded formulations of linearized topologically massive higher spin fields including their critical versions. The gauge invariant degrees of freedom are captured by infinite towers of zero forms. We also introduce two inequivalent sets of gauge potentials given by trace constrained Fronsdal fields and trace unconstrained metric-like fields.

Keywords: higher spin gravities, free differential algebras, topological massive gravity, new massive gravity

1. Introduction

Higher spin gravity theories in 3D have provided a fruitful ground for exploring phenomena that have proven to be difficult to tackle so far in higher dimensions, such as the nature of black holes. The existing 3D higher spin theories are essentially of two types. The simpler and most studied one is purely topological, i.e. without any local bulk degrees of freedom, admitting a Chern–Simons off-shell formulation [1]. The second one, which was constructed by Prokushkin and Vasiliev [2], has a finite number of propagating scalar fields and is more akin to Vasiliev’s 4D higher spin gravity. What has been notably lacking, however, is an acceptable fully nonlinear higher spin extension of topologically massive gravity, which
provides a richer 3D toy model for quantum gravity as it propagates a massive graviton and has interesting black hole solutions of its own. Proposals exist at the linearized [3] and the fully nonlinear level [4], but they are problematic as we shall comment on below, and furthermore made within the standard approach to field theory that has not yet proven to be suitable for building up any fully nonlinear completion.

Indeed, all known fully nonlinear formulations of higher spin theories containing local bulk degrees of freedom have been obtained within the unfolded approach to field theory [5–7]. It has the characteristic feature of introducing one infinite tower of auxiliary zero-form fields for each local bulk degree of freedom. These fields obey a chain of first-order equations and as a result they encode all higher derivatives of the dynamical field (in the metric-like formulation) that are non-vanishing on-shell. In this chain, the dynamical equations reside at the lowest levels while the rest do not give any new information on-shell. More precisely, in the case of a scalar field, the combination of the two lowest levels yields the Klein–Gordon equation, while for spins \( s \geq 1 \) the dynamical equations reside at the first level, in the form of Maxwell’s equations for \( s = 1 \) and their generalizations to spins \( s \geq 2 \) involving the generalized spin-\( s \) Weyl tensors.

While seemingly uneconomic for lower-spin theories with equations of motion that are second order in derivatives, unfolded dynamics becomes indispensable in the formulation of higher spin theories, as their as Lorentz covariant interactions necessarily involve higher derivatives. Remarkably, as found by Vasiliev [5–7], unfolded dynamics not only provides a systematic approach to higher spin interactions but also exhibits an underlying hidden symmetry in the form of associative differential algebras. This enabled Vasiliev to write down simple and elegant fully nonlinear equations in 4D in the form of a covariant constancy condition on a master zero-form and a constant-curvature condition on a master one-form. Indeed, the 3D Prokushkin–Vasiliev (PV) theory [2] was formulated essentially along the aforementioned lines. Its deformation to yield topologically massive fields with spins \( s \geq 1 \), however, has turned out to be a challenging and still open problem.

Motivated by this, we here present an unfolded formulation of free topologically massive higher spin fields in \( \text{AdS}_3 \), that we expect will play a role in the fully nonlinear context, and that may also service as a first step toward a classification of linearized such systems. In particular, our construction gives a higher spin generalization of various higher derivative extensions of gravity, including critical versions which harbor logarithmic modes; in the case of gravity, see [8] for a review and references.

As noted above, a basic property of unfolded dynamics [6, 7, 9–12] is that the local bulk degrees of freedom (but not boundary states) are represented by infinite towers of gauge invariant Lorentz tensorial zero-forms that furnish infinite-dimensional representations of the isometry algebra of the spacetime background. Thus, the unfolded analysis facilitates the characterization of the propagating bulk modes using group theoretical methods without any reference to gauge potentials.

In \( \text{AdS}_3 \), the propagating bulk modes can thus be identified via the eigenvalues of the spin operators in the isometry algebra \( \text{so}(2, 2) \cong \text{so}(1, 2)_{h=1} \oplus \text{so}(1, 2)_{h=2} \), or equivalently, in terms of the mass-square operator and the quadratic Casimir of \( \text{so}(2, 2) \). Although our main focus will be on unitary representations of lowest-energy type, we shall also exhibit finite-dimensional representations and non-unitary higher-order spin-\( s \) singletons as well as unitary representations in which the energy is unbounded; for similar more generalized spectral analyses in \( D \geq 4 \), see [13]. We shall also extend the zero-form sector by elevating the parity

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5 In [7], linearized unfolded equations for tensor and spinor fields in \( \text{AdS}_3 \) were constructed using spinor oscillators. In the present note, which is limited to tensor fields, we shall use vector oscillators.
breaking mass parameter to an arbitrary matrix. As we shall see, the extended system provides a unified description of a number of higher spin theories, including theories that are topological, topologically massive and general massive and their critical versions (harboring logarithmic modes).

While the zero-form sector captures the gauge invariant local bulk degrees of freedom, their interactions require gauge potentials, whose introduction at the linearized level is the topic of study in section 2, where we make contact with two different proposals given in the literature based on either trace unconstrained metric-like potentials \( h_{a_1...a_s} \) or Fronsdal potentials \( q_{a_1...a_s} \). Thus, the primary curvature tensor of the zero-form sector, which is a divergence free traceless and symmetric Lorentz tensor \( \Phi_{a_1...a_s} \), is equated to either the dual of a de Wit–Freedman-type higher-spin curvature \( R_{a_1...a_s} \) (given by \( s \) curls of \( h_{a_1...a_s} \)) or the Fronsdal curvature \( F_{a_1...a_s} \) (built from up to two derivatives of \( q_{a_1...a_s} \)) [14–16]. In other words, one and the same set of local degrees of freedom, carried by \( h_{a_1...a_s} \) or \( q_{a_1...a_s} \), can be integrated and carried by either \( h_{a_1...a_s} \) or \( q_{a_1...a_s} \), possibly leading to inequivalent nonlinear extensions.

Turning to the existing proposals for topologically massive higher spins, a quadratic action has been given in [3] and a fully nonlinear Chern–Simons action has been given in [4]. Both yield linearized equations for spin \( s \geq 2 \) Fronsdal fields, which, however, lack trace conditions on the Fronsdal curvatures. The resulting traceless spin-\( s \) modes and spin-\((s - 2)\) trace modes come with opposite signs in the action, indicating the presence of ghosts. In the unfolded formulation, however, the tracelessness of the curvatures is built into the equations of motion by construction. Another difference is that the critical values for the mass parameter \( \mu \) at which the massive modes become singletons are spin independent in [3, 4], while they do depend on the spin in our formalism. Moreover, in both [3] and [4] the aforementioned spin independence implies that the relative coefficient between the two terms in the equation of motion (4.27) below is given by \( 1/\mu(s - 1) \), which leads to decoupling of spin-1 modes. In the unfolded approach, this coefficient can be chosen freely, and the natural choice is \( 1/\mu(s) \), allowing for spin-1 modes (but leading to spin dependent critical points).

As for the fully nonlinear proposal in [4], it has the additional problem (besides the presence of ghosts) that the Cartan–Frobenius integrability conditions lead to nonlinear algebraic constraints on the fields on-shell. These constraints are satisfied at the linearized level due to properties of the AdS3 vacuum solution, but not at the second order and beyond, with the exception of special field configurations such as those considered in [17].

The plan of the paper is as follows: in section 2, we construct the linear unfolded zero-form system in the case of a single propagating degree of freedom by using Cartan–Frobenius integrability to fix all free parameters in terms of the inverse AdS radius \( \lambda \) and a parity breaking mass-parameter, that we shall denote by \( \mu \). We also exhibit an indecomposable structure, that arises for critical values for \( \mu \) as well as various dual representations parameterized by two real numbers \( \alpha, \beta \in [0, 1] \), and analyze the spectrum using harmonic expansion methods. In section 3, we describe an extended zero-form system consisting of \( N \) unfolded towers mixed together via an arbitrary mass matrix \( \mu_{ij} \) \((i, j = 1, ..., N)\). In section 4, we introduce the one-form gauge potentials in the two aforementioned inequivalent fashions. In the conclusions, besides summarizing our results, we also provide further comments on existing proposals for quadratic [3] and nonlinear [4] actions for topologically massive higher spin gravities, pointing out their problematic features. Our conventions and some elements of \( so(2, 2) \) algebra are given in appendix A. Appendix B contains technical details related to the integrability of the unfolded equations in the zero-form sector.
2. Unfolded zero-form system: single tower

In what follows we shall construct a general unfolded zero-form system that describes a single spin-$s$ degree of freedom on an AdS$_3$ background (including the topological case which arises in the limit where the mass parameter goes to infinity). To this end, we write down an ansatz for the unfolded equations for an infinite set of totally-symmetric and traceless Lorentz tensors$^6$ \( \{ \Phi_{a(n)} \}_{n \geq s} \). All constants are fixed by the requirement of Cartan–Frobenius integrability. As a result, the couplings to the background frame field \( h^n \) and the Lorentz connection in the Lorentz covariant derivative \( V \) define a Lorentz covariant \( so(2, 2) \) representation matrix for an infinite-dimensional \( so(2, 2) \) module, that we shall denote by \( \mathcal{T}_{(s)} \) (see appendix A for definitions and conventions). We shall then examine the spectrum of the system by expanding the fields in terms of harmonic functions on AdS$_3$, which is equivalent to expanding the Lorentz covariant basis elements of \( \mathcal{T}_{(s)} \) in terms of compact basis elements on which the two \( so(2) \) subalgebras of \( so(2, 2) \) act diagonally.

2.1. The unfolded equations

Following Lopatin and Vasiliev [18], we introduce vector oscillators obeying \((a = 0, 1, 2)\)
\[
\begin{bmatrix}
\alpha_a, \bar{\alpha}^a
\end{bmatrix} = \delta^b_a, \quad (2.1)
\]
and collect the zero-forms in the master field$^7$
\[
|\Phi^{(i)}\rangle = \sum_{n=1}^{\infty} \frac{1}{n!} \Phi_{a(n)} \bar{\alpha}^{a_1} \ldots \bar{\alpha}^{a_n} |0\rangle. \quad (2.2)
\]
Our ansatz for the unfolded zero-form system reads \((n = s, s + 1, \ldots)\)
\[
\mathcal{V} \Phi_{a(n)} = h^a \Phi_{a(n+1)} + \mu_a h^b \epsilon_{bac} \Phi_{a(n-1)c} + \lambda_a \left[ h_a \Phi_{a(n-1)} - \frac{n-1}{2n+1} h^b \Phi_{ba(n-2)} \eta_{ab} \right], \quad (2.3)
\]
where \( \mu_a \) and \( \lambda_a \) are parameters of dimension mass and mass-squared, respectively, and the last term on the right-hand side removes the trace of \( h_a \Phi_{a(n-1)} \) (so that the right-hand side is symmetric and traceless). In terms of the master-one form, the above equations assume the more compact form
\[
D \Phi^{(i)} = 0, \quad D := \mathcal{V} - i h^a \bar{\rho}(P_a) = \mathcal{V} - i \left( \sigma^- + \sigma^0 + \sigma^+ \right), \quad (2.4)
\]
where the operators \( \sigma^-, \sigma^0 \) and \( \sigma^+ \) are given in appendix B. Equation (2.4) defines, via (2.3), the action of the AdS$_3$ transvections \( P_a \) in the Lorentz covariant basis for the zero-form module \( \mathcal{T}_{(s)} \) (in which the Lorentz generators \( M_{ab} \) act canonically). As a result, the integrability condition \( D^2 \Phi^{(i)} = 0 \) endows \( \mathcal{T}_{(s)} \) with the structure of \( so(2, 2) \) module and fixes the parameters \( \mu_a \) and \( \lambda_a \).

Before proceeding, we would like to remark that for \( \mu_a = 0 \) and \( s = 0 \), unfolded equations of the form (2.3) were considered in [7] using (undeformed) spinor oscillators and shown to generically describe a massless scalar on AdS$_3$. In [7] also spinor fields on AdS$_3$

$^6$ In 3D, the Lorentz algebra admits only such tensorial representations, since any two anti-symmetrized indices can be dualized to a (co)vector index using the epsilon tensor. We use a notation wherein \( \Phi_{a(n)} \equiv \Phi_{a_1 \ldots a_n} \) and repeated indices denoted by the same letter are implicitly symmetrized. Symmetrization as well as anti-symmetrization is performed with unit strength.

$^7$ The operator \( \bar{\alpha} \) has dimension of length and hence the operator \( \alpha \) has dimension of mass, if no dimensionful constant is introduced on the right-hand side of (2.1).
were considered, for which $\mu_n$ is non-zero. In [19], these systems were generalized by employing deformed oscillator algebra so as to generically describe a massive scalars and fermions (except at critical points). In [19], the significance of the restriction to $n \geq s$ at critical points was also discussed. Subsequently, a nonlinear extension of this unfolded system, using spinor oscillators, was obtained by Prokushkin and Vasiliev [2]. Unfolded equations of the form (2.3) were studied by Lopatin and Vasiliev in any dimension $D \geq 4$ [18], using vector oscillators, and in the absence of the $\mu_n$ term since these are tailored to $D = 3$.

Analysis of Cartan–Frobenius integrability. The details of the analysis of the integrability condition are given in appendix B. The required expressions for $\mu_n$ and $\lambda_n$ are

\begin{align}
  s = 0: & \quad \mu_n = 0, \quad \forall n \in \mathbb{N}, \\
  s > 0: & \quad \mu_n = \frac{\mu}{n + 1}, \quad n \in \mathbb{N}, \quad n \geq s,
\end{align}

and

\begin{align}
  s = 0: & \quad \lambda_n = \frac{n \lambda^2}{2n + 1} \left[ \frac{M_0^2}{\lambda^2} + 1 - n^2 \right], \quad n \in \mathbb{N}, \\
  s > 0: & \quad \lambda_n = \frac{n^2 - s^2}{n(2n + 1)} \left[ \frac{\mu^2}{s^2} - \lambda^2 n^2 \right], \quad n \in \mathbb{N}, \quad n \geq s,
\end{align}

where $\mu$ and $M_0$ are arbitrary real parameters with dimension of mass. Thus the unfolded equations in $\mathcal{T}_{(0)}$ take the form ($n \geq 0$)

\begin{equation}
  V_b \Phi_{a_1 \ldots a_n} = \Phi_{ba_1 \ldots a_n} + \frac{n \lambda^2}{2n + 1} \left( \frac{M_0^2}{\lambda^2} + 1 - n^2 \right) \left( \eta_{b(a_1} \Phi_{a_2 \ldots a_n)} - \frac{n - 1}{2n - 1} \eta_{(a_1a_2} \Phi_{a_3 \ldots a_n)b} \right),
\end{equation}

while for $s > 0$ they take the following form in $\mathcal{T}_{(s)}$ ($n \geq s$):

\begin{equation}
  V_b \Phi_{a_1 \ldots a_n} = \Phi_{ba_1 \ldots a_n} + \frac{\mu}{n + 1} \epsilon_{b(a_1} \Phi_{a_2 \ldots a_n)c} \\
  + \frac{n^2 - s^2}{n(2n + 1)} \left[ \frac{\mu^2}{s^2} - \lambda^2 n^2 \right] \left( \eta_{b(a_1} \Phi_{a_2 \ldots a_n)} - \frac{n - 1}{2n - 1} \eta_{(a_1a_2} \Phi_{a_3 \ldots a_n)b} \right).
\end{equation}

For $\mu = 0$, the latter equation system coincides with the linearized equations in the zero-form sector of the PV theory at the critical point where the deformation parameter $\nu = 2s + 1$ [2, 19] (see (2.28) below).

Critical points. From (2.8) it follows that if $\mu = \pm \lambda s'$ for some integer $s' > s$, then $\lambda_{s'}$ vanishes and hence the set $\{ \Phi_{(a_1(0)} \}_{a_2 \ldots a_s'}$ remains invariant under covariant differentiation. Thus, at these critical points, the so(2, 2) module $\mathcal{T}_{(s)}$ exhibits an indecomposable structure.

8 Such a phenomenon also appears for massive scalars [9] and higher spin fields [20] for special values of the mass. More precisely, in the limit when the mass of a spin-$s$ field in AdS$_d$ tends to a specific value $m_t$, it decomposes into a direct sum of a partially massless field of spin $s$ and depth $t$ and a massive field of spin $t$. The zero-form module of the latter field contains a finite-dimensional submodule. Such modules are also employed in unfolding as modules where gauge potentials take their values.
As we shall see in section 2.2, the finite-dimensional representations $\{ \Phi_{(n)} \}_{n=s', ..., s-1}$ show up in the spectral analysis.

The Casimir operators. Using the unfolded equations, we can evaluate the Casimir operator $C_2[so(2, 2)]$ in $\mathcal{T}_{(s)}$. From (2.4), (2.6) and (2.8) it follows that the value of the mass-squared operator $M^2 := -\eta_{ab} \rho^a \rho^b$ on the primary zero-form $\Phi_{(s)}$ in $\mathcal{T}_{(s)}$ is given by

$$\tilde{\rho} \left( -p^b p_b \right) \Phi_{(s)} = \left( \frac{\mu^2}{s^2} - \lambda^2 (s + 1) \right) \Phi_{(s)}, \quad s > 0. \quad (2.11)$$

Thus $\Phi_{(s)}$ obeys

$$\left[ \Box - \left( \frac{\mu^2}{s^2} - \lambda^2 (s + 1) \right) \right] \Phi_{(s)} = 0. \quad (2.12)$$

Using also the fact that

$$\tilde{\rho} \left( \frac{1}{2} M^{ab} M_{ab} \right) \Phi_{(s)} = s (s + 1) \Phi_{(s)}, \quad (2.13)$$

it follows that for $s > 0$

$$C_2[so(2, 2)] \Phi_{(s)} = s^2 - 1 + \frac{\mu^2}{\lambda^2 s^2}. \quad (2.14)$$

In the case of $s = 0$, we find

$$C_2[so(2, 2)] \Phi_{(0)} = \lambda^2 M_0^2. \quad (2.15)$$

The algebra $so(2, 2) = so(1, 2) \oplus so(1, 2)$ is generated by

$$J_a^{(s)} := \frac{1}{2} \left( M_a + \frac{\epsilon}{\lambda} \rho_a \right), \quad M^a := \frac{1}{2} \epsilon^{abc} M_{bc}, \quad \epsilon^{012} = 1,$$

$$\left[ J_a^{(s)}, J_b^{(s)} \right] = (-i) \epsilon_{abc} J_c^{(s)}, \quad \left[ J_a^{(s)}, J_b^{(s)} \right] = 0. \quad (2.16)$$

The corresponding Casimir operators

$$C_2[so(1, 2)_{h(0)}] = -\frac{1}{4} \left( C_2[so(1, 2)_{h(0)}] - \frac{\rho^a \rho_a}{\lambda^2} \right) + \frac{\epsilon}{2\lambda} M^a P_a, \quad (2.17)$$

assume the following values in $\mathcal{T}_{(s)}$ for $s > 0$:

$$C_2[so(1, 2)_{h(0)}] \Phi_{(s)} = -\frac{1}{4} \left( s^2 - 1 + \frac{\mu^2}{\lambda^2 s^2} \right) + \frac{\epsilon \mu}{2\lambda}, \quad (2.18)$$

where we have used (2.3), (2.11) and (A.4).

Primary equations of motion. Combining the equations (2.9) for $n = 0$ and $n = 1$ yields the scalar field equation

$$\left( \Box - M_0^2 \right) \Phi = 0. \quad (2.19)$$

9 Note that this equation can also be obtained by acting with $\epsilon_{\mu \nu} \partial_\nu$ on both sides of the first order equation (2.22).
The first equation in the chain (2.10), on the other hand, reads
\[ V_\nu \Phi_{\nu (s)} = \Phi_{\mu \nu (s)} + \frac{\mu}{s + 1} e_{\mu \nu}^\rho \Phi_{\rho (s-1) \mu}. \] (2.20)
Taking a trace yields
\[ V^\rho \Phi_{\mu \nu (s-1)} = 0. \] (2.21)
Contracting with \( e^{\mu \nu} \) produces
\[ \Phi_{\nu (s)} + \frac{s}{\mu} e_{\nu}^{\rho \sigma} V_\rho \Phi_{\mu \nu (s-1)} = 0, \quad s > 0, \] (2.22)
where the second term is totally symmetric by virtue of (2.21). For \( s = 2 \) and appropriate identification of \( \mu \), equation (2.22) is the linearized field equation of topologically massive gravity [21]. Upon introducing gauge potentials (see in section 4), the tensor \( \Phi_{\nu (s)} \) is identified with either the dual of the de Wit–Freedman-like curvature or the Fronsdal tensor. In the first case, (2.22) becomes an \((s + 1)\)-derivative field equation for a metric-like gauge potentials, while in the latter case it takes the form of a three-derivative field equation for a Fronsdal gauge potential.

Limits. The field equation (2.22) has the following two noteworthy limits:
\[ \mu \to \infty : \quad \Phi_{\nu (s)} = 0, \quad s > 0, \] (2.23)
\[ \mu \to 0 : \quad \epsilon_{\nu}^{\rho \sigma} V_\rho \Phi_{\mu \nu (s-1)} = 0, \quad s > 0. \] (2.24)
In the \( \mu \to \infty \) limit, all the spin \( s > 0 \) fields become nonpropagating. Upon the introduction of Fronsdal gauge potentials, this case corresponds to the linearized Blencowe theory [1], or, equivalently, the spin \( s > 0 \) sector of the linearized PV system for generic values of the deformation parameter \( \nu \) [2, 22].

The \( \mu \to 0 \) limit corresponds to the zero-form field equations of the linearized critical PV system at \( \nu = 2s + 1 \). At these critical points, the linearization of the PV curvature constraint, however, becomes a more subtle matter due to the fact that the source term in this equation becomes singular in the critical limits. It remains to be seen if a suitable regularization of this term can lead to the equations that we shall propose in section 4. If so, then equation (2.24), with \( \Phi_{\nu (s)} \) now expressed in terms of potentials, becomes the linearized field equation for conformal Chern–Simons gravity in 3D, which has been studied in detail in [24]. For \( s > 2 \), however, it describes either an \( s + 1 \) or three derivative field equation depending on the nature of the gauge potential introduced. We also note that such a tentative equation would differ from the vanishing of the spin-\( s \) Cotton tensor, which is expected to involve \( 2s - 1 \) derivatives [16, 25, 26]. Nonetheless the prospects of their exhibiting similar behavior, owing to higher-spin symmetries (as opposed to Weyl symmetry) has been noted in [24].

Indecomposable structures. The unfolded system (2.4) remains integrable after one performs the following rescaling:
\[ \sigma^{\text{new}} = \mathcal{N}(N + 1) \sigma^{\text{old}}, \quad \sigma^{\text{+new}} = \Theta(N - s) \mathcal{N}^{-1}(N) \sigma^{\text{+old}}, \]
\[ \Theta(x) = 0 \quad \text{for} \quad x \leq 0 \quad \text{and} \quad \Theta(x) = 1 \quad \text{for} \quad x > 0, \] (2.25)
provided \( \mathcal{N}(n) \) and \( \mathcal{N}^{-1}(N) \Lambda(n) \) are non-vanishing. In other words, the system (2.3) remains integrable upon multiplication of the first term on the right-hand side by \( \mathcal{N}_{n+1} \) and sending \( \lambda_n \) to \( \lambda_n / \mathcal{N}_n \). These rescalings can be used to alter the indecomposable structure of \( \mathcal{T}_{ij} \) at the critical points \( \mu = \lambda x \). Letting \( \alpha, \beta \in [0, 1] \), the general (in)decomposable structures for \( s > 0 \) can be obtained from
\[ V_{\alpha} \Phi_{\beta_1 \ldots \beta_n} = \left[ \frac{(n + 1)^2 - s^2}{(2n + 1)(2n + 3)} \right] \Phi_{\beta_1 a_1 \ldots a_n} + \frac{n}{n + 1} \epsilon_{\beta_1 a_1 \ldots a_n} \epsilon_{\alpha_1 a_1 \ldots a_n} \Phi_{\beta_1 a_1 \ldots a_n} \]

where we have chosen a nonsingular \( n \) dependent factor in \( \mathcal{N}_n \) for the convenience of comparing our result with that of [2, 19]. Indeed, for \( \mu = 0 \) and using the terminology employed in [19] (see (2.29) below), we see that \( \alpha = \beta = 0 \) gives the co-twisted representation, while \( \alpha = 1, \beta = 0 \) gives the (dual) twisted representation [2, 19]. For \( \mu \neq 0 \), the choice \( \alpha = 0, \beta = 1 \) gives our original unfolded system (2.3) whose indecomposable structure at the critical values for \( \mu \) was spelled out above, while other choices yield other inequivalent indecomposable structures at these critical points. Thus, for \( \beta = 0 \) or \( \beta = 1 \), the ideals closed under exterior covariant differentiation are given by \( \{ \Phi_{\alpha}(0) \} \) for \( n \geq s' \) and \( n \leq s' - 1 \), respectively.

**Multi-spinor notation.** We conclude this section by converting the unfolded equations from vector indices to two-component Majorana spinor indices, as to make more explicit contact with the linearized critical PV system [2, 19]. To this end, we employ \( s_0 (1, 2) \) Dirac \( \gamma \)-matrices to define

\[ V_{\alpha} = \frac{i}{\sqrt{2}} \left( \gamma_{\alpha} \right)^{\alpha \beta} V_{\beta}, \quad V_{\alpha \beta} = \frac{i}{\sqrt{2}} V^\alpha \left( \gamma_{\alpha} \right)_{\alpha \beta}, \quad \gamma_{\alpha \beta} \gamma_{\gamma \delta} \eta_{\alpha \beta} = 0. \]

Thus, the \( \mu \)-deformed unfolded system (2.10) in spinor notation takes the form

\[ V_{\alpha} \Phi_{\beta_1 \ldots \beta_n} = \frac{i}{\sqrt{2}} \left( \gamma_{\alpha} \right)^{\alpha \beta} \Phi_{\beta_\beta a_1 \ldots a_n} - \frac{i\mu}{4\sqrt{2} (n + 1)} \left( \gamma_{\alpha} \right)_{\alpha \beta} \Phi_{\beta \beta a_1 \ldots a_n} \]

while the dual unfolded system (2.26) with \( \alpha = 1 \) and \( \beta = 0 \) reads

\[ V_{\alpha} \Phi_{\alpha_1 \ldots \alpha_n} = \frac{i}{\sqrt{2}} \left[ \frac{(n + 1)^2 - s^2}{(2n + 1)(2n + 3)} \right] \Phi_{\alpha_\beta a_1 \ldots a_n} - \frac{i\mu}{4\sqrt{2} (n + 1)} \left( \gamma_{\alpha} \right)_{\alpha \beta} \Phi_{\alpha_\beta a_1 \ldots a_n} \]

2.2. Spectrum and unitarity

The question of spectrum depends on which representations of \( s_0 (2, 2) \) we employ in analyzing the field equations (2.19) and (2.22). It is an easy matter to determine the spectrum if we employ the lowest-weight UIRs that have the property of being finite at the origin and having finite norm with respect to the group invariant Haar measure (see, for example, [27, 28]). They are denoted by \( D(E_0, s_0) \), where \( E_0 \) is the lowest energy and \( s_0 \) is the helicity.
of the lowest energy state, which must be an integer. In this subsection we shall take \( \mu \geq 0 \), without any loss generality, and we shall set the inverse \( \operatorname{AdS}_3 \) radius \( \lambda = 1 \).

**Unitary representations.** In the case of the scalar field, its lowest-energy representations are \( D(1 \pm \sqrt{1 + M_0^2}, 0) \), where reality of \( E_0 \) requires the Breitenlohner–Freedman bound \( M_0^2 \geq -1 \). The irrep with the upper sign is unitary, while the lower sign case is unitary provided that \( -1 \leq M_0^2 \leq 0 \).

Turning to (2.22), we expand the primary zero-form as

\[
\Phi_{\alpha(\tau)}(x) = \sum_{q} \Phi_{q}^{(E_0; s_0)}(x), \quad E_0 \geq |s_0|, 
\]

where \( \Phi_{q}^{(E_0; s_0)} \) are constants and \( Y_{a(i,q)}^{(E_0; s_0)}(x) \) are totally symmetric and divergence-free \( so(2, 2) \) representation functions in \( D(E_0, s_0) \) obeying10

\[
e_{a}^{bc} \nabla_b Y_{a(i,q)}^{(E_0; s_0)} = -\text{sign}(s_0)(E_0 - 1) Y_{a(i,q)}^{(E_0; s_0)},
\]

and the sum includes all lowest-weight representations that contains a symmetric and traceless Lorentz spin-\( s \) tensor, which implies \( s_0 = \pm s \). Substituting this expansion into (2.22), one finds for \( \mu > 0 \) that11

\[
(E_0, s_0) = \left( 1 + \frac{\mu}{s}, s \right), \quad s \geq 1, \quad \mu \geq s(s - 1),
\]

where the unitarity bound is saturated by singleton representations. For \( \mu = 0 \), there are two possibilities, namely the singleton representations \( D(1, \pm 1) \).

**Compact basis.** In order to exhibit the \( so(1, 2)_c \oplus so(1, 2)_t \) content of the spectrum, we use (A.12) and introduce compact basis vectors \( \{|m_{\pm}; m_{0} \}\) for \( so(1, 2)_c \) obeying12

\[
(J_{0}^{(c)} - m_{0}) |m_{\pm}; m_{0}\rangle = 0.
\]

It is important to note that \( m_{\pm} + m_{0} \) must be integer in order for Lorentz tensors to be expandable in this basis. As can be seen from

\[
2i J_{0}^{(c)} = -i L_{\tau}^{c} + L_{\tau}^{*}, \quad 2i J_{L}^{(c)} = -i L_{\tau}^{c} - L_{\tau}^{*},
\]

the lowest energy representations are given by direct products of highest weight representations of \( so(1, 2)_r \) with lowest weight representations of \( so(1, 2)_c \). Using

\[
C_{2}[so(1, 2)_c] = J_{L}^{(c)} J_{L}^{(c)} - J_{0}^{(c)}(J_{0}^{(c)} - \varepsilon),
\]

and (2.18), one finds the characteristic equation \((s > 0)\)

\[
-j^{(c)}(j^{(c)} - \varepsilon) = -\frac{1}{4} \left( s^{2} - 1 + \frac{\mu^{2}}{s^{2}} \right) + \frac{1}{2} \varepsilon \mu,
\]

where \( j^{(c)} \) are the eigenvalues of \( J_{0}^{(c)} \), with roots

\[
j_{\pm}^{(c)} = \pm \left( \frac{s \pm \sqrt{s^{2} - 4}}{2} \right).
\]

10 Lorentz invariance dictates this form of the result, and the coefficient is easily obtained by acting with \( \epsilon_{\mu
u} \nabla_{\nu} \) on both sides of the equation, then using (A.23).

11 Choosing \( \mu < 0 \) gives the opposite helicity solution with lowest weight \((1 - \frac{s}{s}, -s)\).

12 We assume that the multiplicity for each \((m_{\pm}, m_{0})\) vector to be 0 or 1.
The condition of integer spin means that the allowed roots are \((j_j^+)^{\pm}_{\pm}\) corresponding to \(D(1 + \frac{\mu}{2}, s)\) which is unitary for \(\mu \geq s(s - 1)\), and \((j_j^+)^{\pm}_{\mp}\) corresponding to \(D(1 + \frac{\mu}{2}, -s)\) which is unitary for \(\mu \leq s(1 - s)\). These unitarity conditions are clearly seen from those on the representations of the \(so(1, 2)\) subalgebras\(^{13}\).

Finite dimensional representations. At the critical points

\[ \mu = \pm ss', \quad s' = s + 1, \quad s + 2, \ldots \]  

finite dimensional representations arise. These are given by direct products of finite dimensional \(so(1, 2)_{\pm}\) representations with negative lowest weights \(j_j^+ = \frac{1}{2}(1 + s - s')\) with finite dimensional \(so(1, 2)_{\pm}\) representations with positive weights \(j_j^- = \frac{1}{2}(-1 + s + s')\). The finite dimensions are given by \(s^2 - s^2\), which are indeed equal to the sum of the dimensions of the Lorentz tensors \(\{\Phi_{\alpha(\sigma)}\}_{\alpha,s,\sigma+1,...,s'-1}\).

Higher-order singletons. At the degenerate points (2.38), there also arise interesting infinite dimensional lowest energy representations. They are given by direct products of infinite-dimensional \(so(1, 2)_{\pm}\) representations with negative highest weights \(j_j^+ = -(s + s' + 1)/2\) with finite-dimensional \(so(1, 2)_{\pm}\) representations with negative lowest weights \(j_j^- = (-s' + s + 1)/2\). Correspondingly, we have

\[ (E_0, s_0) = (s + 1, s'), \]  

which is unitary only for \(s' = s + 1\), yielding the singleton \(D(s + 1, s + 1)\)\(^{14}\).

Representations with unbounded energy. Finally, let us mention that we could relax the condition of lowest-energy for a unitarizable \(so(2, 2)\) module by building 3D analogs of the higher-dimensional UIRs found in [13]. These involve the principal and complementary series of \(so(1, 2)_{\pm}\), and they are built by taking direct products of the representations of \(so(1, 2)_{\pm}\) both or which are lowest-weight or highest-weight type. At the classical level, and in \(D \geq 4\), it was argued [13] that they give fully nonlinear solutions that are higher spin analogs of the soliton in 5D [32], but it remains an open question whether they actually give rise to unitary states in the quantum theory.

3. Unfolded zero-form system: multiple towers

Let us extend the previous construction to a more general setting. From now on we will use a set of zero forms \(\{\Phi^i_{\alpha(\sigma)}\}_{\alpha,s,\sigma+1,...,N}\) enumerated by an index \(i = 1, \ldots, N\). All previous considerations remain valid with the sole difference that \(\mu_i\) and \(\lambda_i\) become matrix valued functions of \(n\): \(\mu_i \to (\mu_i^\mu)_n\) and \(\lambda_i \to (\lambda_i^\lambda)_n\). Integrability of the unfolded equations requires the \(\mu\) matrix to commute with the \(\lambda\) matrix, and the general solution of the integrability condition is given by

\[ s = 0: \quad (\mu^i_i^\mu)_n = 0 \quad \forall \, n \in \mathbb{N}, \]  

\(^{13}\) The lowest or highest-weight unitary and irreducible representations of \(so(1, 2)\) are the infinite-dimensional discrete series \(D^+(j^+)^{\pm}_{\pm}\) and \(D^-(j^-)^{\pm}_{\mp}\) that are respectively lowest-weight and highest-weight and where \(j^+ > 0, j^- < 0\), plus the trivial, one-dimensional representation \(j = 0\).

\(^{14}\) See [29] for a recent work where tensor products of higher-order singletons were considered via character formulae. Some earlier works involving higher order singletons include [13, 30] and [31].
\[
\begin{aligned}
&\text{s > 0: } (\mu^i)_{n} = \frac{\mu^i_j}{n + 1}, \quad n \in \mathbb{N}, \quad n \geq s, \\
&\text{s = 0: } (\lambda^i_j)_{n} = \frac{2n^2}{n - 1} \left[ (M^2)^{1/2}_i + \left( 1 - n^2 \right) \delta^i_j \right], \quad n \in \mathbb{N}, \\
&\text{s > 0: } (\lambda^i_j)_{n} = \frac{n^2 - s^2}{n(2n + 1)} \left[ (\mu^2)^{1/2}_j - \lambda^2 n^2 \delta^i_j \right], \quad n \in \mathbb{N}, \quad n \geq s.
\end{aligned}
\]

It follows that
\[
\rho \left( -P^i P^i_{(b)} \right) \Phi^i_{a(i)} = \left( \frac{(\mu^2)^{1/2}_j}{s^2} - \lambda^2 (s + 1) \delta^i_j \right) \Phi^i_{a(i)},
\]

which means that the mass-squared operator is not necessarily diagonal. Only the matrix \(\mu^i_j\) arises in the linearized field equations of the primary fields, which take the form
\[
\left( \square \delta^i_j - (M^2)^{1/2} \right) \Phi^j = 0,
\]

\[
\nabla^i \Phi^i_{(i)} = \Phi^i_{(i)} + \frac{\mu^i_j}{s + 1} \epsilon_{i,j}^{\alpha} \Phi^i_{(i)}.
\]

As before, it follows that
\[
\frac{1}{s} \mu^i_j \Phi^i_{(i)} + \epsilon_{\alpha}^{\beta} \nabla^i \Phi_{(i)} = 0, \quad \nabla^i \Phi_{(i)} = 0, \quad s > 0.
\]

Generally the matrix \(\mu^i_j\) can always be brought to Jordan form, which we shall consider shortly. As a simple example, however, let us take the matrix \(\mu^i_j\) to be
\[
\begin{pmatrix}
0 & m \\
m & 0
\end{pmatrix},
\]

where \(m\) is an arbitrary constant with dimension of mass. Substitution into (3.8) gives
\[
\epsilon_{\alpha}^{\beta} \nabla^i \Phi_{(i)}^{(1)} + \frac{m}{s} \Phi_{(i)}^{(2)} = 0,
\]

\[
\epsilon_{\alpha}^{\beta} \nabla^i \Phi_{(i)}^{(2)} + \frac{m}{s} \Phi_{(i)}^{(1)} = 0,
\]

Expressing \(\Phi^{(2)}\) in terms of \(\Phi^{(1)} \equiv \Phi\) from (3.10) and plugging the result into (4.14) yields
\[
\square \Phi_{(i)} - \left( \frac{m^2}{s^2} - \lambda^2 (s + 1) \right) \Phi_{(i)} = 0.
\]

For \(s = 2\), this equation, with appropriate identification of the mass parameters, is the linearized field equation of new massive gravity [33]. The equation can be factorized as
\[
\left[ D \left( \frac{\partial}{\partial x} \right) D \left( -\frac{\partial}{\partial x} \right) \Phi^{(s)} \right]_{\mu(\eta)} = 0, \tag{3.13}
\]

where \( D(\eta) \) are first-order linear differential operators defined by [8]
\[
\left[ D(\eta) \Phi^{(s)} \right]_{\mu(\eta)} = [D(\eta)]_{\mu} \delta_{\nu} + \frac{\eta}{\sqrt{8}} \epsilon_{\mu}^{\nu} \eta \left( 12 \right). \tag{3.14}
\]

The rank-s tensor \( D(\eta) \Phi^{(s)} \) is traceless, totally symmetric and divergence-free provided \( \Phi \) satisfies the same constraints. From (3.13) we see that the eigenfunctions of \( \square \) acting on spin-s fields are linear combinations of solutions to the equations
\[
D \left( \frac{\partial}{\partial x} \right) \Phi^{(s)}_{\mu(\eta)} = 0, \quad D \left( -\frac{\partial}{\partial x} \right) \Phi^{(s)}_{\mu(\eta)} = 0. \tag{3.15}
\]

These are, of course, the equations (3.10) and (3.11) for the combinations \( \Phi^{(s)}_{\mu(\eta)} = \Phi^{(s)}_{\mu(\eta)} + \Phi^{(s)}_{\mu(\eta)}. \)

Let us now consider the case in which \( \epsilon_{\mu(\eta)} \) has \( N \) distinct non-vanishing eigenvalues, viz.
\[
\epsilon_{\mu(\eta)} = \text{diag} (m_1, m_2, \ldots, m_N), \quad m_i \neq 0, \tag{3.16}
\]
where \( m_i \neq m_j \) if \( i \neq j \). This results in \( N \) first order equations
\[
D \left( \frac{\partial}{\partial x} \right) \Phi^{(s)}_{\mu(\eta)} = 0, \quad i = 1, \ldots, N, \quad \text{(no sum)}. \tag{3.17}
\]

Since all \( m_i \) are assumed to be distinct, the solutions of these equations are those of
\[
D \left( \frac{\partial}{\partial x} \right) D \left( \frac{\partial}{\partial x} \right) \cdots D \left( \frac{\partial}{\partial x} \right) \Phi^{(s)}_{\mu(\eta)} = 0. \tag{3.18}
\]

The case of \( N = 2 \) and \( m_1 = -m_2 \equiv m \) gives (3.13) discussed above. Taking \( N = 2 \) and \( m_1 \neq -m_2 \) gives a higher spin generalization of generalized massive gravity.

Finally, if the mass matrix cannot be diagonalized, it can be brought into a direct sum of Jordan blocks, each given by \( r \times r \) blocks of the form
\[
\mu_{(r)} = \begin{pmatrix}
m & \lambda & m & \lambda & \cdots & m & \lambda \\
m & \lambda & m & \lambda & \cdots & m & \lambda \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
m & \lambda & m & \lambda & \cdots & m & \lambda \\
\end{pmatrix}. \tag{3.19}
\]

In this block, equations (3.8) take the form
\[
D \left( \frac{\partial}{\partial x} \right) \Phi^{(s)}_{\mu(\eta)} = -\frac{1}{2} \lambda^{r-1} \Phi^{(s)}_{\mu(\eta)}, \quad i = 1, \ldots, r - 1, \tag{3.20}
\]
\[
D \left( \frac{\partial}{\partial x} \right) \Phi^{(s)}_{\mu(\eta)} = 0. \tag{3.21}
\]

Eliminating the fields \( \Phi^{(s)}_{\mu(\eta)} \) in terms of \( \Phi^{(s)}_{\mu(\eta)} \) successively, and denoting \( \Phi^{(s)}_{\mu(\eta)} \equiv \Phi^{(s)}_{\mu(\eta)} \), one finds
\[
D \left( \frac{\partial}{\partial x} \right) \Phi^{(s)}_{\mu(\eta)} = 0. \tag{3.22}
\]

These are higher spin generalizations of 3D critical massive gravities. In addition to the standard spin-s solution with lowest AdS energy \( E_0 = 1 + \frac{m}{8} \), they have \( p \)-fold logarithmic solutions for \( p = 1, \ldots, r - 1 \). For \( s = 2 \) they have been studied extensively, see for example, [34, 35] where their holographic duals have been shown to be Logarithmic CFTs. The issue of unitarity of these models has also been investigated. In particular, if \( r \) is odd, it has been argued in [36] by means of a toy model for free scalar fields that there exists a unitary truncation of the theory that maintains some of the logarithmic modes.
4. Introduction of gauge potentials

It is a general property of unfolded equations that the gauge invariant local degrees of freedom are fully encoded in the sector of zero forms. The description of their interactions, however, requires the introduction of gauge potentials. Sometime ago, Prokushkin and Vasiliev [2] constructed fully nonlinear higher spin gravity theory in three dimensions in terms of a master zero-form field and a master one-form gauge field. For generic values of a deformation parameter given by an expectation value, these equations describe only massive scalar degrees of freedom, while gauge fields with spin $s \geq 1$ are topological. At critical values of the PV deformation parameter, one recovers (2.24) in the zero-form sector. At present, it is not known how to include the one-form in the linearization of the critical PV system, as we shall discuss further in the conclusions. Moreover, the deformation of the full PV equations to include our deformation parameter $\mu$ is not known.

Our aim here is to introduce gauge potentials for the $\mu$-deformed system, in a fashion that utilizes the unfolding techniques. As we are working at the linearized level without an underlying nonabelian higher spin algebra, we will study one spin at a time rather than packaging them all in a one-form master field.

While the unfolded formulation is indispensable for the description of higher spin gravities with local degrees of freedom, there is no unique way of introducing the required gauge potentials [5, 10, 37, 38] 15. Indeed, as we shall see, there are two natural ways to let the zero-form source properly define linearized higher spin curvatures. In one approach, we shall use trace-unconstrained metric-like potentials which lead to primary field equations generalizing the results of [15] in Minkowski space, and for special values of the spin, to arbitrary spins in AdS$_3$. This approach leads to an important single trace condition on the dual of the generalized Riemann tensor (equivalent to double-trace condition on the generalized Riemann tensor itself). In the second approach, we shall employ Fronsdal-type trace-constrained gauge potentials and the above mentioned trace-conditions appear naturally here as well. Thus, in both of these approaches, the unfolded formalism that we are using makes it far more convenient to demonstrate the decoupling of redundant gauge degrees of freedom contained in the gauge potentials.

4.1. Trace-unconstrained gauge potentials

We introduce a set of one-forms

$$w = \{ \omega_{m(t-1),n(t)} \}, \quad t = 0, 1, \ldots, s - 1,$$

(4.1)

taking their values in two-row-shaped Young tableaux of $gl(3)$, with the first row of length $s - 1$ and the second of length $t$. The structure group is $so(1, 2)$. We use $m, n, \ldots$ to denote indices on the trace-unconstrained objects and we use $h^m_n$ to convert between flat and curved indices. The unfolded equations that we propose read

$$\nabla \omega_{m(t-1),n(t)} - h^p \omega_{m(t-1),n(t)p} = \rho_1 \left( h_n \omega_{m(t-1),n(t-1)} - \frac{t-1}{s-1} h_m \omega_{m(t-2),n(t-1)} \right) = 0,$$

(4.2)

15 A similar issue was discussed recently in the context of AdS$_2$ in [39], where the gauge potentials introduced therein transform like maximal depth partially-massless fields. The Weyl module of that theory is similar to the AdS$_3$ $\mu$-deformed Weyl modules we constructed here, so one can expect that gauge modules of partially-massless fields can be consistently used for 3D $\mu$-deformed systems as well.
for $t < s - 1$ and
\[
\mathcal{V}_t \omega_m(n-1,n-1) - \rho_{s-1}(h_n \omega_m(n-1,n-2) - (s - 1) h_m \omega_m(s-2)n,n(s-2)) = h^p h^q \epsilon_{pq} \epsilon_{m_1n_1} \cdots \epsilon_{m_{s-1}n_{s-1}} \epsilon_{s-1} \Phi_{s-2}.
\] (4.3)

for $t = s - 1$ and where $\Phi_{s-1}$ is the primary zero-form discussed in section 2, obeying (2.20).

In establishing the integrability of the above equations we use (2.20) and it is required that
\[
\rho_t = \lambda^2 (s - t). \tag{4.4}
\]

Equations (4.2) and (4.3) possess the following gauge invariances:

$t < s - 1$:\quad $\delta_t \omega_m(n-1,n(t)) = \nabla \epsilon_{m(n-1),n(t)} - h^p \epsilon_{m(n-1),n(t)p}$
\[= \rho_t \left(h_n \epsilon_{m(n-1),n(t-1)} - \frac{s - 1}{s - t} h_m \epsilon_{m(s-2)n,n(t-1)}\right), \tag{4.5}\]

$t = s - 1$:\quad $\delta_t \omega_m(n-1,n(s-1)) = \nabla \epsilon_{m(n-1),n(s-1)} - \rho_{s-1}(h_n \epsilon_{m(s-2)n,n(s-2)})$.
\[= \rho_{s-1}(h_n \epsilon_{m(s-2)n,n(s-2)}) - (s - 1) h_m \epsilon_{m(s-2)n,n(s-2)}). \tag{4.6}\]

where the parameters $\{\epsilon_{m(n-1),n(t)}\}_{t=0,1,\ldots,s-1}$ are zero-forms that possess the same $g(3)$ irreducible symmetries as their corresponding one-form connections. The one-forms $\omega_m(n-1)$ and $\omega_m(n-1,n)$ provide possible higher spin analogs of the gravitational dreibein and spin connection, respectively, while $\omega_m(n-1,n(t))$ for $t > 1$ are auxiliary fields. From (4.2) and (4.3) it follows that all the components of the connections can either be set to zero by using the algebraic gauge transformations contained in (4.5) and (4.6), or be expressed as derivatives of the unconstrained metric-like field
\[
h_{\mu_1\cdots\mu_s} := sh_{[\mu_2\cdots\mu_s]} \omega_{\mu_1]m_2\cdots m_s}. \tag{4.7}\]

One can thus write (4.3) as
\[
R_{\mu_1\cdots\mu_s} = \left(\begin{array}{c} \frac{1}{2} \\
\end{array}\right)^{\mu_1} \epsilon_{\mu_1\mu_2} \cdots \epsilon_{\mu_1\mu_s} \Phi_{\mu_1\cdots\mu_s}, \tag{4.8}\]

where the linearized generalized Riemann tensor is defined by
\[
R_{\mu_1\cdots\mu_s} := \delta^p_{[\mu_1} \epsilon_{\mu_2\cdots\mu_s]} \epsilon^m \epsilon_{m} \epsilon_{m} \Phi_{\mu_1\cdots\mu_s} + O(\lambda^2). \tag{4.9}\]

where the order $\lambda^2$ terms are determined by the requirement of invariance under the abelian gauge transformations $\delta h_{\mu_1\cdots\mu_s} = s \epsilon_{\mu_1} \epsilon_{\mu_2} \cdots \epsilon_{\mu_s}$. Its precise form can be found in [14]. It is well-known that such a tensor is unique up to Hodge dualizations and it is a generalization of the de Wit–Freedman curvature [40] to AdS space. The Hodge dual of this curvature, which we shall denote by $\bar{R}$, is defined as
\[
\bar{R}_{\mu_1\cdots\mu_s} = \epsilon_{\mu_1} \epsilon_{\mu_2} \cdots \epsilon_{\mu_s} R_{\mu_1\cdots\mu_s}. \tag{4.10}\]

Thus, equation (4.3) takes the simple form
\[
\bar{R}_{\mu_1\cdots\mu_s} = \Phi_{\mu_1\cdots\mu_s}, \tag{4.11}\]

which imposes the tracelessness condition
\[
\bar{R}_{\mu_1\cdots\mu_s} = 0. \tag{4.12}\]
Combining (4.11) with (2.22) gives
\[ R_{\mu_1 \ldots \mu_s} + \frac{s}{\mu} \epsilon(\mu) \epsilon^{\mu \nu} \epsilon^{\rho \sigma} R_{\nu \rho \sigma} = 0. \] (4.13)

Thus, the full set of linearized field equations are given by (4.13) and (4.12). The spectral analysis has been performed in 3D Minkowski space in [15, 16] for \( s = 3 \) and \( s = 4 \), where thus the equations are of order 4 and 5, respectively. Indeed, they propagate a single degree of freedom, which is consistent with the fact that the gauge invariant primary zero-form obeys a first-order equation.

In the case of several families of zero-forms discussed in section 3, the unfolded system can be extended with gauge potentials in an analogous way. Namely, in (4.3) \( \Phi_{\gamma \ldots c} \) has to be replaced with the primary tensor of the first family of zero-forms \( \Phi^1_{\gamma \ldots c} \). This leads to
\[ \tilde{R}_{m_1 \ldots m_s} = \Phi^1_{m_1 \ldots m_s}. \] (4.14)

The primary Weyl tensor \( \Phi^1_{\gamma_1 \ldots \gamma_s} \) is in turn subjected to equations of motion such as (3.18) or (3.22). Together with (4.14) they give the equations of motion in terms of gauge potentials. For example, for the higher spin extension of new massive gravity from (3.12) one gets
\[ \square \tilde{R}_{\gamma (t)} - \left( \frac{m^2}{s^2} - \lambda^2 (s + 1) \right) \tilde{R}_{\gamma (t)} = 0. \] (4.15)

Let us note once again that (4.14) implies the tracelessness condition (4.12).

Another possible extension of the presented construction is to consider several families of one-forms. Let them be enumerated by an index \( \mu = 1, \ldots, M \). Then instead of (4.2) and (4.3) one has
\[ V_{\rho (m(t-1),t)} - h^\rho \omega_{(m(t-1),t)} = \rho_{(t-1)} \left( R_{m(s-1)} + \omega_{(m(s-1),t)} - \frac{r-1}{s-1} h_m \omega_{(m(s-2),n(t-1))} \right) = 0, \] (4.16)

\[ V_{\rho (m(t-1),t-1)} = \rho_{(t-1)} \left( h_m \omega_{(m(t-1),n(t-2))} - (s-1) h_m \omega_{(m(t-2),n(t-2))} \right), \] (4.17)

where \( B^\rho \) is some constant matrix intertwining zero- and one-forms, and \( \Phi^1_{\gamma (t)} \) are the primary zero-forms discussed in section 3, obeying (3.7). Even though the modifications of the gauge sector of the unfolded equations do not affect the dynamical content of the theory, they may turn out to be necessary to construct interactions. In particular, the \( M = N \) case may play a special role since it might as the linearization of the critical PV system for matrix-valued fields [2].

The following additional remarks are in order. Firstly, the condition (4.12) simplifies (4.9) so that it can be written simply as
\[ \tilde{R}_{m_1 \ldots m_s} = \epsilon_{m_1} \epsilon_{m_2} \ldots \epsilon_{m_s} \epsilon_{q_1} \ldots \epsilon_{q_s} V_{(p_1 \ldots p_s)} V_{(q_1 \ldots q_s)} - \text{traces}. \] (4.18)

Second, working with metric-like gauge fields in \( D \geq 4 \), the linearized spin-\( s \) Riemann tensor is given by \( s \) curls of the gauge potential (just like in 3D). However, unlike 3D, it decomposes into a traceless part given by the spin-\( s \) Weyl tensor (which has no 3D analog) and a trace part given by \( s - 2 \) curls of the Fronsdal tensor. Hence, equating Riemann and Weyl tensors in \( D \geq 4 \), yields the Fronsdal equation, putting the theory completely on-shell. Moreover, the integrability of this curvature constraint implies the complete unfolding system of the Weyl zero-form. In 3D, however, the Weyl tensor vanishes identically, and instead we equate the
rank-$s$ primary zero-form to the Hodge-dualized Riemann tensor as in (4.11). Tracing this constraint sets to zero the doubly traced generalized Riemann tensor, which removes an unphysical spin-$(s - 2)$ mode. The equation of motion (2.20) then supplies the dynamical equation of motion for the trace-free spin-$s$ field.

Finally, the consistency of the combined system of equations (4.17) and (3.7) can be seen as follows. The unfolded equation for the primary Weyl tensors $\Phi^i_{(s)}$ compatible with the curl of (4.17) is

$$B^\mu \left[ \nabla \Phi^i_{(s)} - h^b \left( \Phi^j_{(s)} + \epsilon_{ba} \Psi^i_{(s-1)b} \right) \right] = 0,$$

where $\Phi^i_{(s)}$ and $\Psi^i_{(s-1)b}$ are symmetric and traceless but otherwise arbitrary tensors. Thus, setting $\Psi^i_{(s)} = \mu^i_j \Phi^j_{(s)}/(s + 1)$, where $\mu^i_j$ are arbitrary real constants, we observe that this equation is satisfied for any $B^\mu$ in view of (3.7), which we have already established in section 3 to describe a consistent set of equations.

4.2. Trace constrained gauge potentials

Another natural possibility is to use a set of traceless one-forms

$$\nu = \{v_{a(s-1),b(t)}\}, \quad t = 0, 1,$$

taking values in one and two-row-shaped tensors of $so(1, 2)$. Focusing on the case of a single zero-form tower ($N = 1$), the unfolded equations then read

$$\nabla v_{a(s-1),b} = h^b v_{a(s-1),b},$$

$$\nabla v_{a(s-1),b} = h^b \left( h^e v_{a(s-2)b} - h_a \Phi_{(s-2)bce} \right)
+ \tau \left( h^b v_{a(s-1),b} - h_a v_{a(s-2)b} + \frac{s-2}{s-1} \left[ h_c v^c_{ba(s-3)} \eta_{ab} - h_c v^c_{a(s-2)} \eta_{ab} \right] \right),$$

where $\Phi_{(s)}$ is the primary zero-form obeying (2.20). The Cartan–Frobenius integrability of these equations requires

$$\tau = (s - 1)^2.$$  

Equation (4.21) is the zero-torsion condition for the unfolded Lopatin–Vasiliev system [18] in AdS$_3$. Using this condition to eliminate $v_{a(s-1),b}$, equation (4.22) takes the form

$$F_{\mu_1...\mu_s} = \Phi_{\mu_1...\mu_s},$$

with Fronsdal tensor defined as [41]

$$F_{\mu_1...\mu_s} = [\Box - s(s - 3)] \Psi_{\mu_1...\mu_s} - s V_{(\mu_1} \nabla^\nu \Psi_{\mu_2...\mu_s)\nu}
+ \frac{s(s - 1)}{2} \left[ V_{(\mu_1} V_{\mu_2} - 2 g_{(\mu_1} \mu_2)} \Psi_{\mu_3...\mu_s)},
$$

where we have set $\lambda = 1$ and $\Psi_{(s)}$ is the the doubly traceless gauge field

$$\Psi_{a(s)} = s v_{a(s-1)}, \quad \Psi_{a(s-4)} = 0.$$

Equation (4.24) imposes the tracelessness condition on the Fronsdal tensor. Thus, combining (4.24) with (2.22) we end up with the dynamical field equations
For $s = 1$, equation (4.27) describes a topologically massive photon. For $s = 2$, equations (4.27) and (4.28) describe topologically massive gravity, and for $s \geq 3$ they provide a generalization thereof to higher spins.

In comparison, in $D \geq 4$, the formulations in terms of trace constrained potentials, the corresponding set of linearized curvatures consists of the Fronsdal tensor, generalized torsions and the generalized spin-$s$ Weyl tensor. The linearized curvature constraints set the Fronsdal tensor, and not just its $(s - 2)$th curl, and the generalized torsions to zero, leaving the spin-$s$ Weyl tensor given by the traceless projection of $s$ curls of the trace constrained gauge potential. The latter curvature constraint then induces the complete unfolding system of the Weyl zero-form. In 3D, however, as we have just seen, the use of trace constrained potentials leads to an identification of the primary zero-form with the (two-derivative) Fronsdal tensor, and the fields go on-shell as the result of the first level of equations in the Weyl zero-form module, which we have imposed by hand, as noted earlier.

Finally, in order to exhibit the extra boundary states that arise in the gauge potentials in AdS$_3$, we impose the de Donder gauge and removing the trace mode by using (4.28) and on-shell gauge transformations. From (4.27) and (4.28) we thus find

\[
\Box - s(s - 3) \left( \phi_{\mu_1 \cdots \mu_s} + \frac{s}{\mu} \varepsilon^{(\mu_1} V_{\mu_2} F_{\mu_3 \cdots \mu_s)} = 0, 
\phi_{\mu_1 \cdots \mu_{s-2}} = 0, 
\phi_{\mu \nu_{s-1}} = 0. 
\]

Expanding $\phi$ in lowest-energy UIRs, the resulting lowest weights are

\[
(E_0, s_0): \quad (s, s), \quad (s, -s), \quad \left(1 + \frac{\mu}{s}, s\right). 
\]  (4.30)

The first two states are boundary singleton states that arise in the gauge potential but not in the primary zero-form, which only contains the last state. If $\mu = s(s - 1)$ then the latter mode disappears but a logarithmic mode appears due to degeneracy with the singletons in the gauge field, see e.g. the discussions in the case of critical gravity in in [8]. In the $\mu \to 0$ limit, the massive spin-$s$ state turns into a state with lowest weight $(1, s)$, which is the spin-$s$ analog of the $(1, 2)$ state that arises in conformal Chern–Simons gravity as a partially massless state [24].

5. Conclusions

We have constructed unfolded formulations providing a higher spin generalization in flat spacetime as well as AdS$_3$ of linearized theories of massive gravity with higher derivatives such as topologically massive gravity, general massive gravity and their critical versions. We have also analyzed their spectrum and exhibited unitary as well as interesting non-unitary representations, including higher order singletons and finite-dimensional representations of $so(2, 2)$ arising at critical points. The latter leads to indecomposable structures that are similar to those in the linearized PV model found in [2, 19], though there are two quite different parameters that govern the critical limits. We have also glued two different types of gauge potentials to the zero-form sector. Although these obey quite different equations of motion, in one case being of third order in derivatives and in the other case being of order $s + 1$ in
derivatives, the unfolded formulation guarantees that they propagate the same local bulk degrees of freedom, though boundary states may differ, which we leave for future studies.

It seems natural to conjecture that the ultimate construction of a fully nonlinear massive higher spin gravity in 3D will require elements of the existing Prokushin–Vasiliev higher spin theory [2], which in its present and generic form describes massive scalars coupled to non-propagating topological higher spin fields. Indeed one of the main motivations for the present paper has been to provide a convenient framework for such a construction by first tackling the desired linearized field equations. At that level, we have seen that our unfolded equations for the zero-form module correspond to deformation of the linearized PV system at its critical point by a parity breaking mass parameter $\mu$. The full $\mu$-deformed PV theory, however, remains to be constructed. It may involve a generalization of the PV system, or possibly, and more simply, the expansion around a new vacuum solution such that the resulting linearized field equations accommodate our deformation parameter $\mu$. Possibly, progress can also be made by a closer examination of the curvature constraint in the critical PV system, and seeking a finite result for the source term that depends on the zero-form master field and that naively diverges at the critical point.

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Appendix A. Conventions and decomposition of $so(2, 2)$

Conventions. The anti-de Sitter $AdS_{d+1}$ algebra $so(2, d)$ is presented by hermitian generators $M_{AB}$ obeying the commutation relations

$$[M_{AB}, M_{CD}] = i \eta_{BC} M_{AD} - i \eta_{AC} M_{BD} - i \eta_{BD} M_{AC} + i \eta_{AD} M_{BC},$$  \hspace{1cm} \text{(A.1)}$$

where $(\eta_{Aa})= \text{diag}(-, -, +, +)$ and the $so(2, 2)$ vector index $A = 0', 0, 1, 2$, also decomposed as $A = 0', a$ in terms of the Lorentz $so(1, 2) \subset so(2, 2)$ index $a = 0, 1, 2$. Hence, $(\eta_{ab})= \text{diag}(-, +, +)$. The $AdS_3$ transvection operators are defined by

$$P_a := \lambda M_{0a},$$  \hspace{1cm} \text{(A.2)}$$

so that the $AdS_3$ algebra is presented by

$$[M_{ab}, M_{cd}] = 2i \eta_{[c} M_{a]d} - 2i \eta_{d[b} M_{a]c}, \quad [P_a, P_b] = i \lambda^2 M_{ab}, \quad [M_{ab}, P_c] = 2i \eta_{c[b} P_{a]},$$  \hspace{1cm} \text{(A.3)}$$
The action of the Lorentz generator in the vector representation is
\[ \rho_\lambda(M_{ab})V = -2i \eta_{\lambda[}V_{\alpha]} \] (A.4)
and the Lorentz-covariant differential operator is
\[ V := d - \frac{i}{\sqrt{2}} \omega^{\mu\nu} \rho(M_{ab}). \] (A.5)

The AdS$_3$ differential $D$ is defined by
\[ D := V - i \mathcal{H}^{\alpha} \rho(P_\alpha) = d + \frac{1}{2} \Omega^{AB} \rho(M_{AB}) = d + \Omega, \] (A.6)
where the vielbein one-forms $\mathcal{H}^\alpha := \lambda^{-1} \Omega^{0\alpha}$ are required to have invertible components. Explicitly, the flatness condition on the connection $D$ gives
\[ V^2 \equiv \frac{1}{2} \mathcal{R}^{\mu\nu\lambda\rho} \rho(M_{ab}) = \frac{1}{2} \lambda^2 \mathcal{H}^{\alpha} \rho(M_{ab}), \] (A.7)
so that, in particular,
\[ V^2V^a = -\lambda^2 \mathcal{H}^{\alpha} \rho \mathcal{H}^\beta \rho \mathcal{V}^{\beta}. \] (A.8)

In the metric formulation where $g_{\mu\nu}$ denote the components of the AdS$_3$ metric, this implies that the components of the Riemann tensor are given by
\[ R_{\mu\nu\lambda\rho} = -\lambda^2 \left( g_{\mu\rho} g_{\nu\beta} - g_{\mu\beta} g_{\nu\rho} \right). \] (A.9)

Decomposition. The $so(2, 2)$ algebra has two irreducible subalgebras $so(1, 2)_{\pm\pm}$ of $so(2, 2)$ generated by $J^{(\pm)}_\alpha$ where $\varepsilon = \pm$. These subalgebras are defined as
\[ J^{(\pm)}_\alpha := \frac{1}{2} \left( M_\alpha + \frac{\varepsilon}{\lambda} \rho \right), \quad M^\alpha := \frac{1}{2} \varepsilon^{\lambda\mu \nu \rho} M_{\lambda \mu \nu \rho}, \quad \varepsilon^{012} = 1, \] (A.10)
\[ \left[ J^{(+)\alpha}_a, J^{(+)\beta}_b \right] = (-i) \epsilon_{abc} \eta^{cd} J^{(+)\delta}_b, \quad \left[ J^{(+\alpha}_a, J^{(-\beta)}_b \right] = 0. \] (A.11)

In the compact basis the algebra $so(2, 2)$ takes the form
\[ J^{(0)}_\alpha := J^{(0)}_1 \pm i J^{(0)}_2, \quad \left[ J^{(0)}_1, J^{(0)}_1 \right] = \pm J^{(0)}_2, \quad \left[ J^{(0)}_2, J^{(0)}_2 \right] = \pm 2 J^{(0)}_1. \] (A.12)

We normalize the quadratic Casimirs as follows (A, B = 0, a with $a = 0, j$ and $j = 1, 2$)
\[ C_2[so(2, 2)] = \frac{1}{2} M^{AB} M_{AB}, \quad C_2[so(1, 2)_{\pm\pm}] = \frac{1}{2} M^{ab} M_{ab}. \] (A.13)
\[ C_2[so(1, 2)_{\pm\pm}] = \eta^{\mu\nu} J^{(\mu)}_a J^{(\nu)}_b. \] (A.14)

It follows that
\[ C_2[so(2, 2)] = C_2[so(1, 2)_{\pm\pm}] - P_a P^a, \] (A.15)
\[ C_2[so(1, 2)_{+}] + C_2[so(1, 2)_{-}] = -\frac{1}{2} C_2[so(2, 2)], \] (A.16)
\[ C_2[so(1, 2)_{+}] - C_2[so(1, 2)_{-}] = M^a P_a. \] (A.17)

From the last two expressions we get
\[ C_2[so(1, 2)_{\pm\pm}] = -\frac{1}{2} C_2[so(2, 2)] + \frac{\varepsilon}{2} M^a P_a, \] (A.18)
which plays an important role in section 3.
In constructing the representations of the algebra $so(2, 2)$, it is useful to split its generators $M_{AB}$ into
\[ M_{12}, \quad L_j^\pm := M_{0j} \mp i M_{j0}, \quad E := \lambda M_{00} \equiv R_0, \quad j = 1, 2, \] (A.19)
which obey the algebra
\[ [E, L_j^\pm] = \pm \lambda L_j^\pm, \quad [L_j^+, L_j^-] = 2i M_{0j} - 2\lambda^{-1} \delta_j E. \] (A.20)

The operators of spin and energy are given by
\[ E := R_0 = J_0^{(+) -} - J_0^{(--)}, \quad S := M_{12} = \left( J_0^{(+) +} + J_0^{(--) -} \right). \] (A.21)

In a lowest-weight irreducible representation of $so(2, 2)$ with vacuum $|E_0, s_0\rangle$, one has
\[ L_j^- |E_0, s_0\rangle = 0 = \left( \lambda^{-1} E - E_0 \right) |E_0, s_0\rangle, \] (A.22)
so that
\[ C_2[so(2, 2)] |E_0, s_0\rangle = \left( C_2[so(2)] - \partial_0 L_j^+ L_j^- - E_0 \left( -E_0 + 2 \right) \right) |E_0, s_0\rangle, \]
\[ = \left( -E_0 \left( -E_0 + 2 \right) + C_2[so(2)] \right) |E_0, s_0\rangle, \] (A.23)
where $C_2[so(2)] = \frac{1}{2} M^{ij} M_{ij} = S^2$.

**Appendix B. Oscillator formulation of the zero form module**

We define the one-forms
\[ \sigma^- := (-i) E^-, \quad \sigma^0 := (-i) \frac{M(N)}{N} E, \quad \sigma^+ := (-i) \frac{\Lambda(N)}{N} \pi E+, \]
\[ E^- := h^a \alpha_a, \quad E := h^a \epsilon_{abc} \bar{\alpha}^b \alpha_c, \quad E^+ := h_a \bar{\alpha}^a, \]
where $\Lambda(N)$ and $M(N)$ are functions of the number operator. $N := \bar{\alpha}^a \alpha_a$ and
\[ \pi := \left( 1 - \frac{1}{2(N - 1)} \bar{T} T \right), \quad T := \eta^{ab} \alpha_a \alpha_b, \quad \bar{T} := \eta_{ab} \bar{\alpha}^a \bar{\alpha}^b. \] (B.1)

Introducing $H_a := \epsilon_{abc} h^b \bar{h}^c$, one has
\[ \{ E, \pi \} = 0, \quad \{ E, E^\pm \} = -2 E^\pm E^-, \quad \{ E, E^- \} = -H^a \alpha_a, \quad \{ E, E^+ \} = -H_a \bar{\alpha}^a. \] (B.2)

We define the zero-form $so(2, 2)$-module $\tilde{\theta}(P_0)$ via the representation $\tilde{\rho}(P_0)$ of the AdS$_3$ transvection generators, the Lorentz generators acting tensorially, i.e. according to (A.4). Therefore, when acting in $\tilde{T}$, the AdS$_3$ connection $D$ is
\[ D := V - i h^a \bar{\rho}(P_0) = V - i \left( \sigma^- + \sigma^0 + \sigma^+ \right), \] (B.3)
and the unfolded, linear equations (2.3) for the propagation of fields in AdS$_3$ are given by
\[ D |\Phi^{(0)}\rangle = 0. \] (B.4)

In comparing this equation with (2.3), we note that $M(n) := \mu_a$ and $\Lambda(n) := \lambda_n$. The Cartan–Frobenius integrability of these equations guarantees that the set of zero-forms in $|\Phi^{(0)}\rangle$ forms a representation of $so(2, 2)$, where each of the zero-form field $\Phi^{(0)}$ in $|\Phi^{(0)}\rangle$ transforms in the totally-symmetric rank-$n$ representation of the Lorentz subalgebra $so(1, 2)_{Lor} \subset so(2, 2)$. 

Decomposing the integrability condition into linearly independent pieces gives the five a priori independent conditions
\[
(\sigma^-)^2 = 0, \quad \{\sigma^-, \sigma^0\} = 0, \quad V^2 + \{\sigma^-, \sigma^+\} + (\sigma^0)^2 = 0,
\]
\[
\{\sigma^0, \sigma^+\} = 0, \quad (\sigma^+)^2 = 0. \tag{B.5}
\]

The first and last consistency equations above are identically satisfied, as can be seen from the definition of \(\sigma^2\). In order to evaluate the left-hand sides of the remaining three equations, we use (B.2). This yields
\[
\{\sigma^-, \sigma^0\} = \frac{1}{2} H^a \alpha_a \left[ M (N - 1) - \frac{(N + 1)}{N} M (N) \right]. \tag{B.6}
\]

The consistency equation \(\{\sigma^-, \sigma^0\} = 0\) provides us with a recursion relation
\[
n \mu_{n+1} - (n + 1) \mu_n = 0, \tag{B.7}
\]
whose solution is given in (2.5) and (2.6). Next, one finds that the consistency of the equation \(V^2 + \{\sigma^-, \sigma^+\} + (\sigma^0)^2 = 0\) gives the recursion relation
\[
-n^2 \lambda^2 = \frac{n^2}{(n + 1)} \left( \frac{n + \frac{1}{2}}{n + \frac{1}{2}} \right) \lambda_{n+1} - n \lambda_n, \quad \lambda_0 = 0, \quad \lambda_1 = \frac{M^2}{3}, \quad n \geq s \geq 0, \tag{B.8}
\]
whose solution is given in (2.7) and (2.8). Finally, the equation \(\{\sigma^+, \sigma^0\} = 0\) is found to be satisfied as a consequence of the others, so that the integrability conditions \(D^2 = 0\) have all been treated.

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