Low-Temperature Dephasing in Disordered Conductors: the Effect of “1/f” Fluctuations

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Electronic quantum effects in disordered conductors are controlled by the dephasing rate of conduction electrons. This rate is expected to vanish with the temperature. We consider the very intriguing recently reported apparent saturation of this dephasing rate in several systems at very low temperatures. We show that the “standard model” of a conductor with static defects can not have such an effect. However, allowing some dynamics of the defects may produce it.

I. INTRODUCTION

Electronic transport in real conducting materials at low temperatures is now known \cite{1} to exhibit important quantum phenomena. Even in the macroscopic limit, weak localization and the associated magnetoresistance \cite{2} are by now commonly observed. Sample-specific conductance fluctuations \cite{3} are relevant in mesoscopic samples at low temperatures. The phase coherence of the electrons must be preserved over the relevant length scales to maximize these effects. Thus, the precise understanding how the phase coherence of the electrons is lost, is a crucial issue in mesoscopic physics (as well as, obviously, in other branches of physics). One of the important insights \cite{4} gained from these studies is that elastic scattering does not destroy phase coherence. It takes inelastic scattering changing the quantum state of other degrees of freedom to do that. Thus, the dephasing (sometimes called ”decoherence”) of Quantum-Mechanical interference occurs by inelastic scattering \cite{4} off the degrees of freedom which do not directly participate and are not measured in the interference experiment \cite{5,6}. Such degrees of freedom are often referred to as the ”environment”. This dephasing is characterized by the “phase-breaking” time, $\tau_\phi$, for the electron. This phase-breaking time generically increases \cite{4} with decreasing temperatures and diverges when $T \rightarrow 0$, because of the decreasing phase space available for inelastic scattering. This paper is devoted to the problem of the apparent saturation of $\tau_\phi$ at low temperatures, recently observed in disordered metals \cite{4,5,6}. We show \cite{1} that the “standard model” of a conductor with static defects, $\tau_\phi$ must in fact diverge when $T \rightarrow 0$. However, taking into account the relaxational dynamics of the scattering centers may produce a finite, weakly temperature dependent $\tau_\phi$ at very low temperatures, but normally not in the strict $T \rightarrow 0$ limit. It is the same defect dynamics which produces the intimately related low-frequency (1/f) conductance noise.

In the standard picture, one considers electrons performing diffusive motion due to defects, just above the Fermi energy, and interacting via the Coulomb interaction with the other electrons. It is straightforward to obtain the dephasing rate from the strength of the inelastic scattering of the considered electron by the environment, i.e. using the trace left in the latter \cite{4}. A similar result was first obtained in ref. \cite{4} by using the effect of the electromagnetic fluctuations due to the electron gas on the considered electron. The equivalence \cite{4} of these two points of view is guaranteed by the fluctuation-dissipation theorem.

It is easy to manipulate these cited expressions in the semiclassical picture, for a general $V_q$, into:

$$1/\tau_\phi = \int \int dq d\omega |V_q|^2 S_p(q,\omega) S_{env}(-q,-\omega).$$

(1)

where $S_p(q,\omega)$ is the dynamic structure factor of the diffusing electron (a Lorentzian with width $Dq^2$) and $S_{env}(-q,-\omega)$ is the same for the environment. Evaluating these integrals with the environment being the electron gas, yields the well-known expressions found first by Altshuler et al. \cite{5}. For 2D and 1D (thin films and wires) some care is needed to eliminate the infrared (small $q$)-divergence by considering the phase difference of two paths \cite{6}. These results are in a quantitative agreement with experiments, except for the low-temperature limit which we discuss below. We remark that the theory described here for dephasing is perturbative in the electron-environment interaction. Since $\tau_\phi$ is much longer than the elastic scattering time, $\tau$, the dephasing is weak and there is no need to invoke nonperturbative ideas.

\begin{enumerate}
\item Here the term ”inelastic” implies just changing the quantum state of the environment. It is irrelevant how much energy is transferred in this process. This includes zero energy transfer – flipping the environment to a degenerate state, if that is possible.
\end{enumerate}
II. STATEMENT OF THE PROBLEM, PROOF THAT ZERO-POINT MOTION DOES NOT DEPHASE

Recently, Mohanty et al [8] have published extensive experimental data indicating that contrary to general theoretical expectations and to the results mentioned above, the dephasing rate in films and wires does not vanish as \( T \to 0 \). Serious precautions [9] were taken to eliminate experimental artifacts. It was speculated that such a saturation of the dephasing rate when \( T \to 0 \), might follow from interactions with the zero point motion of the environment. These speculations have received apparent support from calculations in ref. [11]. However, the latter were severely criticized in refs. [12,10,13] and were in disagreement with experiments in ref. [14]. In fact, it is clear that since dephasing must be associated with a change of the quantum state of the environment, it cannot happen as \( T \to 0 \) (a macroscopic degeneracy of the environment’s ground state is not considered here). In that limit neither the electron nor the environment has any energy to exchange. This qualitative argument can be made into a more formal proof [10]. While demonstrating that zero point motion does not dephase, this proof does show what further physical assumptions can in fact produce a finite dephasing rate at very low temperatures.

The proof [11] uses eq.4 and applies the very general detailed-balance relationship

\[
S(q, \omega) = S(-q, -\omega) e^{-\hbar \omega/k_B T}, \tag{2}
\]

to either \( S_p(q, \omega) \) or \( S_{env}(-q, -\omega) \). It is immediately seen that the integrand of eq.4 is a product of two factors one of which vanishes for \( \omega > 0 \) and the other for \( \omega < 0 \), as \( T \to 0 \). Thus the integral and the dephasing rate vanish in general when \( T \to 0 \). However, if, for example, \( S_{env}(-q, -\omega) \) has an approximate delta-function peak at small \( \omega \) due to an abundance of low-energy excitations, one may get a finite dephasing rate at temperatures higher than the width of that peak. Should \( S_{env}(-q, -\omega) \) have an \( \omega \) behavior at low \( \omega \), the dephasing rate would have a \( \log T \) term at the corresponding temperatures. Such near-degeneracies of the ground state are known to exist in disordered, glassy, systems. These follow from the many “mesoscopic” realizations of the disorder configuration. The system slowly fluctuates among these many states and it may in fact not be in full equilibrium. This may cause [16] the commonly observed low-frequency (often “1/f”) noise [17,18]. To include the effects of such fluctuations on the dephasing, one has, in principle, just to add their relevant contribution to the full \( S_{env}(-q, -\omega) \).

An extremely useful phenomenological model for such rearrangements that has been employed to explain the low-temperature properties of glasses [19], considers [20] two-level tunneling systems (TLS). For a review, see refs. [21,22].

One may consider the dephasing due to the TLS by either obtaining the appropriate inelastic scattering rate of the conduction electron off the TLS or by adopting a time-domain picture. According to the latter, one considers two interfering paths, and when enough motion of the TLS occurs during the time to execute the paths, the interference is lost. As mentioned above, these two descriptions are equivalent. We shall use the first for a simple calculation but the second may be useful in the physical argumentation below. It is the phase shift due to the motion of the scatterers which produces both the conductance change and the dephasing. These occur, respectively, when the defect motion is slower or faster than the time scales of the electronic motion [24].

III. GENERAL ”1/F” CONSIDERATIONS

We start with a rather generic picture, strongly related to 1/f noise [4]. We consider those defects that are rearranging, for example, by tunneling at low temperatures at a rate \( \nu \) satisfying:

\[
\hbar/\tau_\phi << h \nu << k_B T, \tag{3}
\]

where \( \tau_\phi \) is the dephasing time. The second inequality reflects the fact demonstrated above, that transfers of energies >> \( k_B T \) are cut off. The first inequality can be justified from an uncertainty principle argument.

It is even more conveniently understood from the time-domain point of view: as mentioned above, to decohere a given path, one needs motions that occur on time scales shorter than its duration. A discussion of the fact that fast fluctuations yield dephasing and slow ones yield conductance fluctuations can be found in ref. [2].

We denote the fraction of the defects that move on the time scales of eq.4 by \( p \ll 1 \). According to the standard theory for the 1/f noise, if \( \nu \) is an exponential of (minus) a large dimensionless number which is distributed more or less uniformly, the distribution of \( \nu \) is 1/(\( \nu \lambda \)) where \( \lambda \) is a normalization constant. The noise power per a decade of frequency [15] is therefore proportional to the fraction of defects participating in the tunneling motion, \( p_\nu \), divided by \( \lambda \). Therefore, \( \lambda \) is given, roughly, by the number of decades for which 1/f noise prevails. Thus, for the relevant frequency range of eq.4

\[
p = \frac{p_\nu}{\lambda} \log \left( \frac{\tau_\phi k_B T}{\hbar} \right). \tag{4}
\]

Their density is \( n_0 = n_1 p_\nu \) where \( n_1 \) is the total defect density. The density of active centers per frequency decade, \( n_1 \) is smaller by the logarithmic factor, i.e. \( n_1 = n_0 p_\nu / \lambda \).

We now briefly review the mesoscopic fluctuation model of Feng, Lee and Stone [16] for 1/f noise. One
considers the system as the temperature is lowered and the "standard-model" dephasing length, $L_\phi$, is increasing $[24]$. We consider the 3D case, which is the worse for our purposes. In refs. $[25,16]$ it was found that the change of the dimensionless conductance of a coherence volume due to moving a single defect with an effective microscopic scattering cross-section $g$ is given in 3D by:

$$\Delta g = \frac{1}{(k_F\ell)^2} \frac{\ell}{L_\phi} F.$$  

The factor $F$ is a product of two possible correction factors $F_1$ and $F_2$: when the impurity moves a distance $d << (1/k_F)$, $F_1$ becomes the factor $(k_Fd)^2$ and if the impurity scattering cross section $\sigma << (1/k_F)^2$, $F_2$ becomes $\sigma k_F^2$. We also remember that under 1/f conditions, the density of defects in any frequency window between $\nu$ and $\nu+h\nu$ is the same, $nI\log\nu$, independently of $\nu$, for a constant $\alpha$. For a given temperature, and $L_\phi$, we choose $\alpha$ so that the change of the dimensionless conductance $g$, for monoenergetic electrons due to all defects in the above frequency window and in the coherence volume $L_\phi^3$ attains its full value of order unity. This determines $\alpha$ to be given in 3D by:

$$1/\log\alpha = (\frac{L_\phi}{\ell})^2 \frac{p_t}{\lambda} F_1.$$  

From the above estimates and ref. $[18]$, one finds for the relative 1/f noise power per decade in a macroscopic bulk sample with N electrons, by averaging the noise over the many coherent volumes and over an energy band of $k_BT$:

$$\frac{(\Delta g)^2}{g^2} \sim \frac{L_\phi^3}{N^3} \frac{p_t}{\lambda k_F\ell} \frac{\hbar}{\tau_\phi k_BT} F_1,$$  

where the last factor, valid when the ratio $\frac{\tau_\phi k_BT}{\hbar} \gg 1$, is due to energy-averaging $[19]$. We shall see that typical values of the parameter $\frac{\lambda}{\hbar}$ can be on the order of $10^{-3}$. This implies that the Hooge parameter can reach the order of $10^{-2}$ in reasonably good metals ($k_F\ell \sim 100$) at low temperatures. Much larger values of the Hooge parameter are possible for dirtier metals. Interestingly, values of that parameter between one and ten have already been found experimentally $[18]$ for very dirty metals at low temperatures. Should $L_\phi$ saturate at very low temperatures, the temperature dependence of the Hooge parameter would then convert from that of eq. $[5]$ to a simple proportionality to $1/T$. The latter should be valid as long as $\frac{\tau_\phi k_BT}{\hbar} \gg 1$. This might offer another way to observe the saturation of $L_\phi$. The dependence of the Hooge parameter on $k_F\ell$ should also be modified upon saturation of the dephasing owing to losing the dependence of $L_\phi$ on purity.

We now consider the frequency range of eq. $[5]$. Our principal physical observation is that when $L_\phi$ becomes large enough so that the conductance fluctuation due to motions in all that range attains its full value of order $e^2/h$, an important crossover occurs in the system. This  

Adopting the physical path-integral picture described in ref. $[16]$, it is seen that below the crossover temperature, eq. $[5]$ essentially all paths crossing a coherent region (within the standard model) in the system, encounter and are significantly phase-shifted by, defects moving at the relevant rate. Therefore we expect that eq. $[5]$ should give the crossover to the regime where the dephasing is dominated by defect motion. Below $T_0$, $1/\tau_\phi$ is roughly saturated (apart from the logarithmic correction) and given by its standard-model value at the crossover temperature, eq. $[16]$. This statement, which is (see below) in order-of-magnitude agreement with the results of ref. $[31]$, is consistent with but more general than, particular models that can be considered and will be treated below. It substantiates the belief that the defect dynamics is indeed the explanation for the intriguing results of ref. $[31]$. The above is valid as long as $\hbar/\tau_\phi < k_BT$. $1/\tau_\phi$ should approach zero at temperatures below the saturated $\hbar/\tau_\phi$.

### IV. Model Calculations

We now perform a more detailed evaluation within the TLS model. The Born approximation scattering cross section from a particle in a double-minimum potential is easily calculated, see for example ref. $[20]$. The tunnel splitting of the lowest two levels in the double well is denoted by $\Omega$ and the asymmery between the wells by $B$. The important role of $B$. The elastic and inelastic scattering cross sections are given, up to the same constant, for momentum transfer $\mathbf{q}$ by:

$$\sigma_{el}(q,\omega) \sim \cos^2(q.d)\delta(\omega), \quad \sigma_{in}(q,\omega) \sim \sin^2(q.d)\delta(\omega \pm \Omega).$$  

Here $\mathbf{d}$ is the vector separating the two minima. The $\pm$ signs in $\sigma_{in}$ reflect energy gain and loss of the tunneling energy by the scattered particle, corresponding to situations where the tunneling particle was in the initial symmetric or antisymmetric states. For $B >> \Omega$, $\sigma_{in}$ acquires an additional factor of $(\Omega/B)^2$, as mentioned above. We write the dephasing rate $1/\tau_0$ by the TLS as the sum of the diffusive and the ballistic portions of the integral.
over $q$ in eq.\[8\]. We start with the diffusive ($q \ll 1/\ell$) contribution. We perform the integration in eq.\[8\], using equation \[8\] (times $n_0$) for the environment and the usual diffusive Lorentzian for the electron. We denote the defect scattering cross section by $a^2$, where $a$ is a microscopic length (of the order of or smaller than $1/k_F$, in metals). We remember that $1/\ell = n_0 a^2$. The result is:

$$\frac{1}{\tau_0}d \sim \frac{pF_1}{\tau} \left(\frac{k_F\ell}{\hbar}\right)^2. \tag{10}$$

The ballistic ($q \gg 1$) contribution to $1/\tau_0$ is easily obtained from the cross section, eq.\[8\] and the density of the relevant scatterers. For $k_F\ell \gg 1$:

$$\frac{1}{\tau_0}b \sim \frac{pF_1}{\tau}. \tag{11}$$

Perhaps surprisingly, the ballistic contribution is dominant for a good metal, $k_F\ell \gg 1$. Obviously, once the rate $1/\tau_0$ obtained from the TLS becomes comparable to the standard model $1/\tau_\phi$, the former will dominate and the effective dephasing rate will be $1/\tau_0$. It depends very weakly (logarithmically, by eq.\[8\]) on the temperature. And at even lower temperatures, once it becomes smaller than the temperature, it should vanish as well. The result of eq.\[11\] agrees with the more general eq.\[8\].

To get a low temperature dephasing rate of $10^{-2}/\tau$, we need a value of $p \sim 10^{-3}$. Thus, taking very roughly $\log(\tau_0 k_B T)/\lambda \cong 0.1$, we need that $p_1 \sim 10^{-2}$ of the defects participate in the $1/f$ noise. The impurity density, for impurities whose scattering length is atomic, is of the order of $k_F^2/\ell$. This typically corresponds to total impurity concentrations of the order of $10^{-2}$. Thus, we have to assume a $\sim 100$ ppm concentration of low-energy “two-level” defects at the relevant range, to get a low temperature dephasing rate comparable to that of ref.\[8\]. This is not a very unlikely possibility. The test whether it really exists would be, as we shall see later, by checking if appropriate levels of $1/f$ conductance noise exist in the system. Confirming that at microwave-frequencies would be a more stringent test. The cross-over of the $1/f$ noise behavior discussed above, and its correlation with the saturation of $\tau_\phi$ would also test our picture.

For completeness, we now treat the TLS model in some more detail, taking into account more seriously the double well asymmetry, $B$. This may give us some insight on, and a rough order of magnitude estimate for, the parameter $p_1$ introduced above. We take the distribution of $B$ to be flat between zero and $B_0$, where $B_0$ can be expected to be on the order of $10^{-2}$ eV, for atomic motion.\[2\]

As above, we take the distribution of $\Omega$ to be $1/\Omega$ starting from some very small $\Omega_{\min}$ up to some cutoff $\Omega_{\max}$. We shall see that for $\Omega_{\max} \ll k_B T$, the dephasing rate is constant but it is easy to see that it vanishes linearly with temperature once $\Omega_{\max} \gg k_B T$. We have to integrate the dominant ballistic contribution as in eq.\[10\] over the above distributions of $B$ and $\Omega$: $P(B,\Omega) = 1/(\Omega B_0 \lambda)$. For $h/\tau_\phi < \Omega_{\max} \ll k_B T$, the contributions of $\Omega < B$ and $\Omega > B$ are comparable and the result is:

$$\frac{\tau}{\tau_0} \sim \frac{\Omega_{\max}}{B_0 \lambda} F_1. \tag{12}$$

Thus the ratio $\frac{\Omega_{\max}}{B_0 \lambda}$ basically replaces (apart from the logarithmic correction) the unknown parameter $p_1$ above. The very reasonable estimate $\frac{\Omega_{\max}}{B_0 \lambda} \sim 10^{-4}$, yields an order of magnitude in agreement with the results of ref.\[8\]. For the different regime, $\Omega_{\max} < h/\tau_\phi \ll k_B T$, we find:

$$\frac{1}{\tau} \sim \Omega_{\max}(F_1 B_0 h \lambda \tau)^{-1/2}, \tag{13}$$

which has an encouraging order of magnitude as well.

V. CONCLUDING REMARKS

Checking experimentally the value of the Hooge parameter as a function of purity and temperature \[27\] would be very valuable. The crucial issue is whether our mechanism is consistent with the observed level of low-temperature (1/f) noise. The nontrivial requirement is that the roughly sufficient levels of such noise that occur at low frequencies can persist up to gigahertz frequencies. While this is not the standard regime for 1/f noise, it must be remembered that five orders of magnitude in frequency may correspond to a factor of two in barrier height. Experimental studies of the possible correlation between the saturation of $\tau_\phi$ and the 1/f noise (or associated two-level absorption) at $10^8$–$10^9$ Hz, are crucial.

We conclude that while the “standard model” of disordered metals (in which the defects are strictly frozen) gives of course an infinite $\tau_\phi$ as $T \to 0$, there may be other physical ingredients that can make $\tau_\phi$ finite at very low temperatures, without contradicting any basic law of physics. The TLS model used here is a particular example and its requirements may or may not be satisfied in the real samples. But, other models with similar dynamics might exist as well, as suggested by the qualitative argument following eq.\[8\]. We reemphasize that $\tau_\phi$ will in fact diverge in the $T \ll h/\tau_\phi$ limit, which is in fact a strong prediction that can be checked experimentally. Thus, our results do not imply dephasing by zero-point fluctuations, which has been repeatedly, and wrongly, claimed in the literature. The failure of the semiclassical approximation used in those considerations was clarified in ref.\[10\].
It should perhaps be emphasized that in a typical measurement, the fluctuations in the whole frequency range between $1/\tau_\phi$ and the inverse of the (macroscopic) measurement time are averaged upon.

It may be asked why the results reported in refs. [12,13] for dirtier samples do not exhibit saturation of $\tau_\phi$. One may speculate that this can be due not only to the fact that the diffusive regime scattering from the TLS is weaker, but, perhaps more importantly, to a possible crossover to a new regime where correlations among the relaxation of different defects become important. Such a crossover was recently invoked to explain the experimental results of ref. [28]. This should push the relaxation modes to lower frequencies and thus make them less effective for dephasing.

It may be asked whether the picture we have introduced is necessarily a nonequilibrium one. Clearly, on any time scale shorter than $1/\omega_{\min}$, the (unknown) lower cutoff of the $1/f$ noise, the system is not in full equilibrium. However, the modes having the relevant time-scales, of eq.(5) may well be in a partial equilibrium. At these frequencies the $S(q, \omega)$ of the system contains the contribution of these modes which do not appear in (and are actually a small correction to) the Drude contribution. This full $S(q, \omega)$, from eq.(5) is the only input necessary to determine $\tau_\phi$. We think that its portion at the relevant frequencies may well be the equilibrium one. Obviously, such a contribution will show up in saturable electromagnetic and ultrasonic absorption in the metal. This will become easier to observe for smaller $k_F\ell$.

We further mention that an apparent saturation of $L_\phi$ at low temperatures implies that the length-scale for quantum behavior remains finite when $T \to 0$. Thus, “quantum critical behavior” will be blocked at very low temperatures. One may speculate that this can explain several recently observed low temperature phenomena: the metallic 2D behavior and the effective rounding of the Quantum-Hall-to-insulator and the superconductor-to-insulator transitions.

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