Frequency Plan Design for Multibeam Satellite Constellations Using Linear Programming

Juan Jose Garau-Luis, Sergi Aliaga, Guillem Casadesus, Nils Pachler, Edward Crawley, and Bruce Cameron

Abstract

Upcoming large satellite constellations and the advent of tighter steerable beams will offer unprecedented flexibility. Consequently, this will require resource management strategies to be operated in high-dimensional and dynamic environments, as existing satellite operators are unaccustomed to operational flexibility and automation. Frequency assignment policies have the potential to drive constellations' performance in this new context, but are no exception to real-time and scalability requirements. Most of existing frequency assignment methods fail to fulfill these requirements, or are unable to meet them without falling short on efficiency. In this paper we propose a new frequency assignment method that prioritizes operational requirements. We present an algorithm based on Integer Linear Programming that fully defines a frequency plan while respecting key system constraints such as handovers, interference, and gateway dimensioning. We can encode goals such as bandwidth maximization or power reduction and produce optimal or quasi-optimal plans according to such objectives. In our experiments, we find this method allocates at least 50% more bandwidth and reduces power consumption by 40% compared to previous operational benchmarks. Compared to previous solutions, the performance advantage increases with the dimensionality of the constellation; in an experiment with a 5,000-beam MEO constellation we allocate three times more bandwidth.

Index Terms

Satellite communications, frequency assignment, multibeam constellations, resource management

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I. Introduction

Although satellite operators have been in the market for many years, their operations still tend to be relatively static—in time. This is especially the case for spectrum allocation policies. When frequency plans do vary, the changes are generally manually-operated, in an effort to find margin. This process entails dealing with a complex optimization problem that has been long studied. However, upcoming changes in the satellite communications industry and the need for automation at scale pose additional challenges to spectrum optimization which previous methods might not be able to address; new algorithmic approaches are required instead.

A. Motivation

Three trends define the new satellite communications landscape. First, the dimensionality of some of the upcoming constellations draws new operation challenges. Examples such as SpaceX’s 4,408-satellite LEO constellation with up to 32 beams per satellite [1], [2] and SES’s O3b mPOWER MEO constellation, consisting of 11 satellites able to power thousands of beams each [3]. As a result, any future frequency assignment strategy must account for scalability, which is challenging for some methods used in the past.

Second, new types of users are entering the market. Spaceborn data services are expected to grow substantially in the coming years [4], and mobile users will constitute a significant fraction of this new demand [5]. Moving away from a market dominated by fixed terminals means that spectrum management policies must adapt to dynamic and uncertain environments. In that context, real-time frequency assignment is crucial to address the behavior of new users.

Finally, the introduction of highly flexible payloads is constituting a market disruption in terms of resource management. In the next generation of communication satellites, operators will be able to reconfigure frequency plans in orbit, intermittently, and on a beam level. Efficiently exploiting this flexibility to such degree of granularity is almost impossible without the use of optimization algorithms.

This new context calls for revisiting the strategies and algorithms previously utilized to make frequency assignment decisions. This problem is not only NP-hard and highly constrained [6], but now it also demands algorithmic solutions that are fast and scale well in order to be successfully operated in upcoming high-dimensional and dynamic environments.
B. Literature Review

The frequency assignment problem can generally be decomposed into two sub-problems: central frequency assignment (i.e., which central frequency) and bandwidth allocation (i.e., how much bandwidth), with previous existing studies proposing solutions for one of the two or both—we cover them in this section. In Table I we summarize the related work for the three different sub-problems (central frequency assignment, bandwidth allocation, and joint frequency and bandwidth assignment) and describe the experimental setups that have been used in the literature. We can observe a tradeoff between the performance and the scalability of previous solutions: more complex algorithms involving metaheuristics, linear programming or Artificial intelligence (AI), show good results in simple scenarios (usually single-satellite with a small number of beams), whereas rule-based heuristic solutions are validated in larger-scale scenarios (NGSO constellations with a greater number of beams) but do not guarantee an optimal assignment.

1) Central Frequency Assignment problem: Reference [7] formulates this problem as a graph coloring problem and uses local search and Simulated Annealing after an initial greedy combinatorial optimization approach. In [8], the authors identify that classical Integer Linear Programming (ILP) optimizers present scalability issues and thus propose a greedy algorithm. Other authors explore Artificial Intelligence (AI) algorithms to try to meet the requirements of future scenarios. In [9], a Deep Reinforcement Learning (DRL) model is proposed to solve the Dynamic Channel Allocation (DCA) problem, and it is shown to closely match the performance of state-of-the-art DCA algorithms. While the online operation of such models consists of simple forward passes of a neural network—substantially faster than other methods—the robustness and stability of DRL models are still disputed in many real world scenarios [27]. The works mentioned above are tested in scenarios with less than 500 beams. Scalability is further limited when including frequency reuse (FR) variables as part of the optimization [7], [9]. Authors in [10] propose a DRL-based method that accounts for FR variables and is validated for 2,000 beams. However, despite its scalability, this algorithms does not always satisfy the complete set of operational constraints.

2) Bandwidth Allocation problem: The work presented in [11] successfully addresses bandwidth allocation in a LEO constellation with 70 satellites and 350 beams, but the proposed solution is limited to finding a feasible heuristic allocation. In [12], a binary search-based method is proposed as a fairer alternative to the water-filling approach considered in [13]. Later studies
TABLE I: Reviewed literature summary. Frequency Reuse (FR) refers to whether the proposed solutions allow for all beams to use the all available spectrum and polarizations (✓) or the frequency reuse strategy (beam color and polarization) is fixed (e.g., four color frequency reuse).

| Problem                          | Method                                | Satellites | Beams | FR |
|---------------------------------|---------------------------------------|------------|-------|----|
| Central Frequency               | Greedy algorithm, simulated annealing | 1 GEO      | 150   | ✓  |
| Camino et al. 2014 [7]          | Integer linear programming, greedy algorithm | 1 GEO      | 200   |    |
| Kiatmanaroj et al. 2012 [8]     | Deep reinforcement learning           | 1 GEO      | 37    | ✓  |
| Hu et al. 2018 [9]              | Deep reinforcement learning           | 7 MEO      | 2,000 | ✓  |
| Garau-Luis et al. 2021 [10]     |                                        |            |       |    |
| Bandwidth Allocation            |                                        |            |       |    |
| Kisseleff et al. 2019 [11]      | Heuristic                             | 70 LEO     | 347   | –  |
| Park et al. 2012 [12]           | Lagrange multipliers, iterative method | 1 GEO      | 20    | –  |
| Choi et al. 2005 [13]           | Water filling                         | 1 GEO      | 100   | –  |
| Wang et al. 2013 [14]           | Lagrange multipliers, golden selection method | 1 GEO      | 10    | –  |
| Abdu et al. 2022 [15]           | Successive Convex Approximation       | 1 GEO      | 67    | –  |
| Paris et al. 2019 [16]          | Genetic algorithm                     | 1 GEO      | 63    | –  |
| Pachler et al. [17]             | Particle Swarm Optimization           | 1 GEO      | 200   | –  |
| Ferreira et al. 2018 [18]       | Deep reinforcement learning           | 1 LEO      | 1     | –  |
| Ortiz-Gomez et al. 2021 [19]    | Convolutional Neural Networks         | 1 GEO      | 82    | –  |
| Central Frequency + Bandwidth Allocation | Linear programming       | 1 GEO      | 8     | ✓  |
| Kibria et al. 2020 [20]         | Linear programming                    | 1 GEO      | 27    | ✓  |
| Abdu et al. 2021 [21]           | Deep Reinforcement Learning           | 1 LEO      | 40    | ✓  |
| Zheng et al. 2020 [22]          | Deep Reinforcement Learning           | 1 GEO      | 37    | ✓  |
| Hu et al. 2020 [23]             |                                        | 1 GEO      | 200   |    |
| Salcedo et al. 2005 [24]        | Hopfield neural network, genetic algorithm | 4,400 LEO  | 10,000| ✓  |
| Cocco et al. 2018 [25]          | Simulated annealing                   | 1 GEO      | 200   | ✓  |
| Pachler et al. 2020 [26]        | Greedy algorithm                      | 7 MEO      | 5,000 | ✓  |

[14], [15] also use the same fairness-centered optimization objective but allocate bandwidth using convex optimization instead. However, both studies test their methods on single-satellite and single-gateway use cases with no more than 70 beams. Concerning more advanced methods, metaheuristics [16], [28] and AI methods [18], [19] have also been used to allocate bandwidth in single-satellite systems. All of the cited studies consider simultaneously optimizing other variables, such as power or the roll-off factor, but the solution quality and robustness of these methods are not tested for larger systems with more than 200 beams.

3) Joint Central Frequency Assignment and Bandwidth Allocation: Both classic optimization and AI approaches have been proposed for the joint problem, with most of the works accounting for FR optimization. Some studies propose linear programming, with [29] addressing the problem
of multiuser aggregation and access control design for carrier aggregation, and [21] also optimizing for power. Both works are tested in a single GEO satellite setup with less than 30 beams. DRL is also explored as a candidate solution in [22] and [23], where it is emphasized how traditional optimization methods impose a hard constraint on real-time use cases. Both test their solutions in scenarios with a single satellite and up to 40 beams. In [24], a neural network combined with a genetic algorithm is used to address the problem through an interference minimization lens; it is tested on multiple scenarios, with a maximum of 36 beams and 128 bandwidth channels. The same problem is addressed in [25] also optimizing for power, using a capacity-oriented objective function and the Simulated Annealing algorithm. Tests on a single satellite with 200 beams and 16 bandwidth channels show that this method reduces both unmet and excess capacity compared to traditional approaches based on conventional payloads.

The constraint satisfaction algorithm presented in [26] is the only solution identified to address scenarios entailing satellite constellations with more than 500 beams. The proposed algorithm uses a greedy assignment to produce valid frequency plans in short amounts of time. However, given the greedy approach and the absence of an objective function, it remains unclear how optimal its solution is when a certain metric must be prioritized (e.g., bandwidth maximization).

Despite the wide variety of studies addressing the frequency assignment problem, to the best of our knowledge there is not method that is fully aligned with the requirements of the future landscape of satellite communications: 1) feasibly and optimally addressing both the central frequency assignment and bandwidth allocation subproblems, especially accounting for FR and multiple satellites, 2) scaling up to thousands of beams, and 3) being able to perform in dynamic environments with near-real-time constraints. The latter point has been a motivating factor only in studies proposing greedy algorithms or DRL.

C. Paper Objectives and Contributions

To try to close the research gap identified in the literature review, we propose a frequency plan design algorithm based on Integer Linear Programming (ILP) that optimizes for central frequency assignment, bandwidth allocation, and frequency reuse on the beam level in multibeam satellite constellations. The algorithm produces optimal or quasi-optimal plans according to an objective function, and these plans respect interference, handover, gateway dimensioning constraints present in NGSO. The objective function can reflect multiple goals, such as maximizing
Fig. 1: A constellation with \( N_S \) identical satellites in the same orbit and \( N_B \) beams is considered. Gateways, fixed terminals, and mobile users are connected to the network.

bandwidth, minimizing frequency reuse, and minimizing power. In this paper’s experiments we show that:

1) When modeling a NGSO constellation with approximately 100 beams, we are able to obtain an optimal frequency plan according to the objective function in minimal computing time.

2) When modeling a NGSO constellation with approximately 1,000 beams, we propose a way to scale the presented method and produce, in short amounts of time, feasible frequency plans that show significant gains compared to the greedy method presented in [26] (this method is the only one that has proven both this level of scalability and flexibility as per Table I). The scaled version eventually converges to an optimal solution.

3) The scaled version is able to design fully-specified and feasible frequency plans in a real-world-based simulation using the O3b mPOWER constellation [30] with approximately 5,000 beams.

D. Paper Structure

The remainder of this paper is structured as follows: Section II presents the frequency plan design problem, its assumptions, and the constraints involved; Section III introduces the optimization method based on Integer Linear Programming, together with the requirements that can be encoded in the objective function; Section IV discusses the results of applying our optimization algorithm to different scenarios; and finally Section V remarks the conclusions of the paper.
II. Problem Statement

The goal is to design a frequency plan for a constellation with $N_S$ identical multibeam satellites, all of them in the same orbital plane (see Figure 1). To that end, we have to completely define frequency utilization for all of the constellation’s $N_B$ beams. Each beam can serve one or more users, or gateways that connect to the ground segment. We assume all beams constantly serve their gateway or their group of users, which might be mobile—the beam then “follows” the users. An input to the problem is how much throughput each user requires, which might correspond to real-time demands or committed rates defined by user contracts. At any point in time, each beam is powered from one—and just one—of the satellites of the constellation. If the orbit of the satellites is NGSO, then handover operations take place and the satellite powering each beam changes over time.

In terms of frequency resources, we assume all satellites are allowed to use the exact same part of the spectrum, which is divided into $N_{BW}$ equal bandwidth chunks or slots. Likewise, all satellites have identical frequency reuse mechanisms: there are $N_{FR}$ frequency reuses available, as well as $N_P$ polarizations for each reuse. Polarizations allow to use more spectrum in a concentrated area without incuring additional interference. For example, $N_P = 2$ when using right-handed and left-handed circular polarizations (RHCP and LHCP, respectively).

To define a full frequency plan, the operator must decide, for each beam, how many and which bandwidth slots are assigned, and which reuse group and polarization should be used. Formally, this corresponds to selecting, for every beam $b \in \{1, ..., N_B\}$:

- A discrete number of bandwidth slots $b_b$, which can’t be greater than $N_{BW}$. This number might have a lower bound required to satisfy the link budget equation.
- A positive integer $f_b$, that indicates the first bandwidth slot used. Beam $b$ then uses slots $f_b, f_b + 1, ..., f_b + b_b - 1$, all of them part of the available spectrum.
- A positive integer $r_b$ representing a frequency reuse out of the $N_{FR}$ available.
- In case $N_P = 2$, a binary variable representing the chosen polarization.

Figure 2 introduces a representation of this decision space in the form of a grid, with $N_{FR} \cdot N_P$ rows and $N_{BW}$ columns. Each column represents a frequency slot, whereas each row corresponds to a combination of a frequency reuse and a polarization. As shown in the figure, rows are sorted, first, by frequency reuse, and second, by polarization. With this representation, making a frequency assignment for a beam turns into picking a specific cell in the grid corresponding to
Fig. 2: Frequency assignment representation in the form of a grid with $N_{FR} \cdot N_P$ rows and $N_{BW}$ columns. In this example, $N_{FR} = 3$, $N_P = 2$, and $N_{BW} = 13$. Each of the 3 beams being powered by the satellite is assigned to a cell in the grid representing the first slot (black squares) and to a certain number of consecutive slots (colored cells). For example, the beam depicted in green is assigned to reuse group 2, left polarization, and is using slots 8 and 9.

Fig. 3: Handover operation example between two satellites at time instants $t_1$ and $t_2$.

the first slot (black squares in the figure) and choosing a valid number of slots (colored cells).

**Problem scope and restrictions**

We assume the constellation’s orbit can be NGSO. Consequently, frequency plans must account for handover operations. Although new digital payloads will allow the reassignment of frequency resources during a handover, it might not be possible or preferred to do so due to multiple reasons. In that case, beams continue utilizing the same frequency resources when switching satellites. This is depicted in Figure 3, in which the green beam switches from satellite 1 to satellite 2 and preserves the same frequency reuse, polarization, and bandwidth slots.
Fig. 4: Inter-group restriction example between beam 1 and beam 2. Diagram shows the moment beam 2 is to be assigned and beam 1 has already been assigned.

Therefore, when a handover occurs, it is critical that the resources to be used on the new satellite are not being used by any other beam already. This constitutes an important restriction when making frequency assignment decisions for a NGSO constellation and applies to any pair of beams being powered by the same satellite at any point in time. We define this constraint for a pair of beams as an *intra-group* restriction between those beams. Then, the set $\mathcal{R}_A$ represents all pairs $(i, j)$ such that beams $i$ and $j$ hold an intra-group restriction. We assume that this set is known in advance and is externally updated during operation if needed.

A second type of restriction to consider is that, if two beams whose footprints are close use the same polarization, they might interfere with each other if their assigned frequency slots overlap. This event, which we define as an *inter-group* restriction, is represented in Figure 4. Similarly to the handover constraint case, the set $\mathcal{R}_E$ encodes all pairs of beams $(i, j)$ that hold an inter-group restriction. There are different interference criteria that could be used to construct this set (e.g., SINR level, angular distance threshold); we assume the concrete choice and therefore the set $\mathcal{R}_E$ are provided by the operator.

The last type of restriction concerns gateway dimensioning. We might want to decrease bandwidth use to a pair of beams not only when they overlap in the handover schedule or when they might interfere with each other, but also if they consume resources from the same gateway at some point in time. The goal is to design the frequency plan such that it accounts for the gateway dimensioning as well. We assume the usable spectrum at the gateway corresponds to the same usable spectrum at the satellite. In addition, the gateway uses the same number of polarizations $N_P$ but can not reuse frequency. Available to us we have the set $\mathcal{R}_G$, which encodes all pairs of beams $(i, j)$ that share a gateway at some point in time.
Finally, the optimization scope of this paper is frequency assignment at the beam level, which, to our best knowledge, remains an unsolved problem in high-dimensional and dynamic constellations with frequency reuse mechanisms. As a further level of decomposition, the beams could potentially be split into multiple carriers to serve each user. However, we leave frequency assignment at the carrier level out of the scope of this paper. This problem can be addressed in multiple ways that can coexist with our optimization framework (e.g., TDMA, FDMA, CDMA). The method presented in this work, by restricting interference among beams, ensures that carriers within one beam do not interfere with carriers from other beams and therefore frequency assignment at the carrier level could be solved locally for each beam.

III. PROPOSED OPTIMIZATION METHOD

In this section we encode each decision and restriction presented in the previous section as variables and constraints of an optimization method based on Integer Linear Programming (ILP). In the context of satellite resource allocation, methods such as ILP allow operators to easily encode problem needs, objectives, and constraints with low granularity with respect to the decision variables. Then, there exist multiple commercially-available tools that automatically produce the optimal solution of each program.

In each subsection we outline how a specific feature of the frequency assignment problem is encoded in the ILP formulation using linear operators. We also give an overview of possible objective functions to guide the search to select the best feasible frequency plan. We assume the constraint sets $R_A$, $R_E$, and $R_G$ that have been introduced in the previous section are an input to the model. We frame the algorithm in the context of optimizing the downlink frequency plan. However, this method is also compatible with uplink frequency plan design, independently or jointly with the downlink plan (see Appendix D).

A. Frequency Plan Decisions

On the beam level, frequency assignment decisions consist of choosing how many bandwidth slots, which ones, and which frequency reuse and polarization to use. From the perspective of the grid representation introduced in Figure 2, this means selecting 1) a column and 2) a row in the grid, and then 3) a number of consecutive slots. We encode the column (i.e., the first slot) as an integer variable $f_i$, with domain \{1, ..., $N_{BW}$\}, for each beam $i \in \{1, ..., N_B\}$. Then we encode the row (i.e., frequency reuse and polarization) as an integer variable $g_i$, with domain...
Finally, the number of consecutive slots is encoded as an integer variable \( b_i \), with the same domain as \( f_i \). We introduce these variables in the ILP formulation as follows:

\[
f_i \in \{1, \ldots, N_{BW}\}, \quad \forall i \in \{1, \ldots, N_B\} \tag{1}
\]

\[
g_i \in \{1, \ldots, N_{FR} \cdot N_P\}, \quad \forall i \in \{1, \ldots, N_B\} \tag{2}
\]

\[
b_i \in \{1, \ldots, N_{BW}\}, \quad \forall i \in \{1, \ldots, N_B\} \tag{3}
\]

Any frequency plan can then be decoded using these three variables per beam. The optimization method returns these values to the operator.

We have defined the bandwidth variable (3) with domain \( \{1, \ldots, N_{BW}\} \). However, we might be interested in specifying higher lower bounds, for contractual reasons or since using a single bandwidth slot might not be enough to satisfy the link budget equation for certain beams [31]. This way, we could redefine variable (3) as

\[
b_i \in \{c_i, \ldots, N_{BW}\}, \quad \forall i \in \{1, \ldots, N_B\} \tag{3}
\]

where \( c_i \) is the minimum number of slots that beam \( i \) requires. Similarly, the domain of variables (1) and (2) could also be changed to split frequency resources following criteria specified by the operator or the users’ contracts.

**B. Constraint: Limited spectrum**

We now start accounting for the constraints of the problem. We first encode the constraint that all bandwidth slots used must be within the spectrum limits imposed by the system (i.e., only the \( N_{BW} \) considered slots can be used). This is encoded as follows:

\[
f_i + b_i - 1 \leq N_{BW}, \quad \forall i \in \{1, \ldots, N_B\} \tag{4}
\]

**C. Constraint: Intra-group or handover restrictions**

We start by considering the intra-group restrictions, given by the set \( \mathcal{R}_A \). This type of restrictions are caused by handover operations and are relevant if and only if constrained beams that use the same frequency reuse and polarization overlap in their bandwidth slots. We first introduce the constraints that encode the intra-group restrictions and then describe each of the elements involved. A pair of constraints is defined per restriction:

\[
f_i + b_i \leq f_j + M(2 - y_{ij} - z_{ij}), \quad \forall i, j \text{ s.t. } (i, j) \in \mathcal{R}_A \tag{5}
\]

\[
f_j + b_j \leq f_i + M(1 - y_{ij} + z_{ij}), \quad \forall i, j \text{ s.t. } (i, j) \in \mathcal{R}_A \tag{6}
\]
where $M$ is a “sufficiently large” number. Both constraints make use of two types of auxiliary binary variables: $y_{ij}$ and $z_{ij}$, which we now explain. Constraints (5) and (6) enforce a non-overlapping frequency assignment between beams $i$ and $j$ holding an intra-group restriction if and only if both beams use the same reuse group and polarization. In case they don’t, these constraints should not make any effect. To that end, we use variable $y_{ij}$ and the following constraints:

$$y_{ij} \in \{0, 1\}, \quad \forall i, j \text{ s.t. } (i, j) \in R_A$$

$$g_i \geq g_j - M(1 - y_{ij}), \quad \forall i, j \text{ s.t. } (i, j) \in R_A$$

$$g_i \leq g_j + M(1 - y_{ij}), \quad \forall i, j \text{ s.t. } (i, j) \in R_A$$

If $y_{ij} = 1$, the method enforces that $g_i = g_j$, since both (8) and (9) are active. If $y_{ij} = 0$, the opposite should occur; however, this cannot be achieved solely with variable $y_{ij}$. To enforce strict inequality when $y_{ij} = 0$, we introduce binary variables $p_{ij}$ to account for both cases $g_i > g_j$ and $g_j > g_i$. Hence, we add the following constraints:

$$p_{ij} \in \{0, 1\}, \quad \forall i, j \text{ s.t. } (i, j) \in R_A$$

$$g_i - g_j \geq \epsilon - M(1 - p_{ij} + y_{ij}), \quad \forall i, j \text{ s.t. } (i, j) \in R_A$$

$$g_i - g_j \leq -\epsilon + M(p_{ij} + y_{ij}), \quad \forall i, j \text{ s.t. } (i, j) \in R_A$$

which are active if and only if $y_{ij} = 0$. In that case, and when $p_{ij} = 1$, (11) is active and we have $g_i > g_j$. On the contrary, if $p_{ij} = 0$, (12) is active and $g_j > g_i$ holds.

The effect of binary variable $z_{ij}$ comes into play if beams $i$ and $j$ hold a restriction and are assigned to the same frequency reuse and polarization (i.e., $g_i = g_j$). In this case, we want to make sure that $f_i + b_i \leq f_j \text{ or } f_j + b_j \leq f_i$, i.e., we want to ensure that beam $i$ has allocated spectrum either to the left or to the right of beam $j$, but without overlapping. These two possible scenarios are taken into account with variable $z_{ij}$ and the following constraints:

$$z_{ij} \in \{0, 1\}, \quad \forall i, j \text{ s.t. } (i, j) \in R$$

$$f_j - f_i \geq 0 - M(1 - z_{ij}), \quad \forall i, j \text{ s.t. } (i, j) \in R$$

$$f_i - f_j \geq \epsilon - Mz_{ij}, \quad \forall i, j \text{ s.t. } (i, j) \in R$$

where $M$ is again a “sufficiently large” number, $\epsilon$ is a very small positive number, and $R$ represents the union $R_A \cup R_E \cup R_G$ (we define these constraints for all intra-group, inter-group,
TABLE II: Encoding of different frequency assignment cases by means of auxiliary variables when beams $i$ and $j$ share an intra-group constraint.

| Auxiliary variables | Frequency assignment case |
|---------------------|---------------------------|
| $z_{ij} = 1$        | $f_i \leq f_j$            |
| $z_{ij} = 0$        | $f_i > f_j$               |
| $y_{ij} = 1$        | $g_i = g_j$ (same freq. reuse and polarization) |
| $y_{ij} = 0$ and $p_{ij} = 1$ | $g_i > g_j$ |
| $y_{ij} = 0$ and $p_{ij} = 0$ | $g_i < g_j$ |

and gateway restrictions. Given a restriction $(i, j)$, if $z_{ij} = 1$, (14) is active and enforces that $f_j \geq f_i$ (i.e., beam $j$ can not use lower frequencies than beam $i$’s). On the contrary, if $z_{ij} = 0$, (15) is active and the effect is the opposite.

Note that at most one of the intra-group constraints can be active at a given time. Constraint (5) does so for the case in which $z_{ij} = 1$ ($f_i \leq f_j$), whereas constraint (6) is active when $z_{ij} = 0$ ($f_i > f_j$). To summarize the effect of the auxiliary variables introduced so far, Table II shows how the different frequency assignment cases between two beams holding an intra-group restriction are encoded by means of variables $z_{ij}$, $y_{ij}$, and $p_{ij}$.

D. Constraint: Inter-group or interference restrictions

The inter-group restrictions are given by set $\mathcal{R}_E$ and concern all pairs of beams with close footprints, which might interfere with each other during operations. Setting a threshold for how close two interfering beams can be is up to the operator’s policy, which might prefer to trade additional inter-group restrictions for further interference mitigation. One way to define this set might be to impose an angular distance threshold between beams; we leave this decision out of the scope of this paper.

As in the intra-group case, a pair of constraints is defined per restriction:

$$f_i + b_i \leq f_j + M(1 + s_{ij} - z_{ij}), \quad \forall i, j \text{ s.t. } (i, j) \in \mathcal{R}_E$$  \hfill (16)

$$f_j + b_j \leq f_i + M(s_{ij} + z_{ij}), \quad \forall i, j \text{ s.t. } (i, j) \in \mathcal{R}_E$$  \hfill (17)

where $M$ is a sufficiently large number. Inter-group constraints rely on auxiliary binary variables $s_{ij}$ and $z_{ij}$. The effect of variable $z_{ij}$ is given by constraints (14) and (15), which have been introduced previously, and considers the cases in which beam $i$’s spectrum is to the left or right of beam $j$’s.
As introduced in Figure 4, inter-group restrictions can negatively impact the performance of the system when both beams are using the same polarization, regardless of their frequency reuse. To specifically focus on polarization, we first introduce the following variables and constraints:

\[ k_i \in \{1, ..., N_{FR}\}, \quad \forall i \in \{1, ..., N_B\} \]  
(18)

\[ m_i \in \{0, N_P - 1\}, \quad \forall i \in \{1, ..., N_B\} \]  
(19)

\[ g_i = N_P k_i - m_i, \quad \forall i \in \{1, ..., N_B\} \]  
(20)

Variable \( k_i \) encodes the frequency reuse assigned to beam \( i \) whereas variable \( m_i \) encodes its polarization, if any.

Similar to constraints (7) - (9), we introduce binary variable \( s_{ij} \) for each pair of beams holding an inter-group constraint. This variable encodes whether beams \( i \) and \( j \) use the same polarization, by means of the following constraints:

\[ s_{ij} \in \{0, N_P - 1\}, \quad \forall i, j \text{ s.t. } (i, j) \in \mathcal{R}_E \]  
(21)

\[ m_i \geq m_j - M s_{ij}, \quad \forall i, j \text{ s.t. } (i, j) \in \mathcal{R}_E \]  
(22)

\[ m_i \leq m_j + M s_{ij}, \quad \forall i, j \text{ s.t. } (i, j) \in \mathcal{R}_E \]  
(23)

If \( s_{ij} = 0 \) then both (22) and (23) are active, and \( m_i \) and \( m_j \) are enforced to be equal. Note that in case \( N_P = 1 \), then \( s_{ij} \) is always zero, since there is only one polarization.

Then, following the same idea behind constraints (10) - (12), we use binary variable \( d_{ij} \) to help encoding whether \( m_i > m_j \) or \( m_j > m_i \), i.e., enforcing \( m_i \) and \( m_j \) to be different in case there is more than one polarization and \( s_{ij} = 1 \). The following constraints explain this idea:

\[ d_{ij} \in \{0, 1\}, \quad \forall i, j \text{ s.t. } (i, j) \in \mathcal{R}_E \]  
(24)

\[ m_i - m_j \leq -\epsilon + M (1 + d_{ij} - s_{ij}), \quad \forall i, j \text{ s.t. } (i, j) \in \mathcal{R}_E \]  
(25)

\[ m_i - m_j \geq \epsilon - M (2 - d_{ij} + s_{ij}), \quad \forall i, j \text{ s.t. } (i, j) \in \mathcal{R}_E \]  
(26)

which can only be active if \( s_{ij} = 1 \) (more than one polarization). Then, if \( d_{ij} = 0 \), (25) is active and therefore \( m_j > m_i \). On the other hand, if \( d_{ij} = 1 \), (26) is active and \( m_i > m_j \). Table III summarizes all cases that inter-group restrictions-related auxiliary variables encode.

E. Constraint: Gateway dimensioning

The gateway dimensioning constraints are given by the set \( \mathcal{R}_G \) and correspond to beams that can not overlap in frequency when they connect to the same gateway. Since we assume gateways
TABLE III: Encoding of different frequency assignment cases by means of auxiliary variables when beams \(i\) and \(j\) share an *inter-group* constraint.

| Auxiliary variables | Frequency assignment case |
|---------------------|---------------------------|
| \(z_{ij} = 1\)     | \(f_i \leq f_j\)          |
| \(z_{ij} = 0\)     | \(f_i > f_j\)             |
| \(s_{ij} = 0\)     | \(m_i = m_j\) (same polarization) |
| \(s_{ij} = 1\) and \(d_{ij} = 0\) | \(m_i < m_j\)          |
| \(s_{ij} = 1\) and \(d_{ij} = 1\) | \(m_i > m_j\)          |

do not reuse frequency but share the same number of polarizations with the satellites, we can encode these constraints by replicating the constraints introduced for inter-group restrictions, i.e., constraints (16) and (17) and auxiliary constraints (18)-(26). Some pairs of beams \((i, j)\) might be in both \(R_E\) and \(R_G\), in that case we only need to define the constraints once.

**F. Objective Function**

So far, we have discussed the decisions and constraints that need to be encoded to define valid frequency plans, i.e., plans that do not violate any constraint. However, we have not addressed how to prioritize different valid frequency plans according to the operator’s preferences and goals. In certain occasions, operators might prefer plans that maximize bandwidth allocation or plans that use as few frequency reuses as possible. To encode these preferences into the ILP formulation, we propose the following objective function to be maximized:

\[
\max \sum_{i=1}^{N_B} (\beta_{1,i}b_i - |\beta_{2,i}|g_i - |\beta_{3,i}|f_i - |\beta_{4,i}|P_i(f_i, b_i)) \tag{27}
\]

where \(\beta_{1,i}, \beta_{2,i}, \beta_{3,i}, \text{ and } \beta_{4,i}\) are weighting parameters for beam \(i\), with \(\beta_{k,i} \in \mathbb{R}\). This function combines four different objectives:

1) \(\beta_{1,i}b_i\) maximizes allocated bandwidth for beam \(i\), or minimizes it when \(\beta_{1,i} < 0\). A better control over used bandwidth can lead to a more efficient power consumption.

2) \(-|\beta_{2,i}|g_i\) attempts to use as few frequency reuses as possible, which is the case if \(|\beta_{2,i}| > 0\).

Setting \(\beta_{2,i} = 0\) for all beams enables a uniform use of frequency reuses and polarizations. The coefficient \(\beta_{2,i}\) uses absolute terms because the effects of maximizing and minimizing \(g_i\) are symmetric according to the formulation introduced in the previous section.

3) \(-|\beta_{3,i}|f_i\) seeks to have the same effect on the number of frequency slots used. If \(-|\beta_{3,i}| < 0\), lower parts of the spectrum are prioritized. If \(\beta_{3,i} = 0\), then the spectrum is used uniformly.
We specifically subtract this term given lower frequencies may require less power consumption, and therefore are sometimes preferred by operators.

4) If available, $-|\beta_{4,i}|P_i(f_i, b_i)$ represents a proxy for the RF power consumed by beam $i$ when using slots $f_i, f_i + 1, \ldots, f_i + b_i - 1$. Reducing RF power when possible is reflected by this operand when $\beta_{4,i} > 0$. In the experiments, this metric is precomputed for all possible $f_i$ and $b_i$. In the following section we provide additional details on its computation.

The weighting parameters are used to define a priority hierarchy over these objectives. While these parameters can be identical for all beams, operators might be interested in prioritizing additional bandwidth or certain bands for specific beams, for example.

**G. Computational Complexity**

The frequency plan problem is NP-hard; we can prove this by assimilating the formulation to the graph coloring problem studied in graph theory [32]. Specifically, when assuming that all beams have equal bandwidth, the frequency assignment problem is equivalent to the NP-hard graph coloring problem, as has been shown in prior spectrum management research [7]. This entails that the problem might be computationally intractable for large numbers of beams. In the next section we prove that the presented method finds the optimal solution for scenarios with up to 182 beams. For scenarios with a larger number of beams, we propose an iteration-based procedure (full details provided in Appendix A) in which we only optimize a subset of the beams in every iteration. We achieve affordable computing times at the cost of switching from global search to local search, which might lead us to converge to local optima. We apply this method to cases with 1,000 to 5,000 beams.

**IV. Results**

In this section we present and discuss the results and performance of the proposed frequency plan optimization method. Specifically, we carry out three different experiments (see Table IV) to validate it: 1) first, we test the algorithm on three scenarios in the 100-beam range in which spectrum usage maximization is required. 2) Then, we study the scalability of this method by optimizing frequency assignment in 1,000-beam systems. To that end, we introduce an iteration-based optimization procedure that reduces the impact of NP-hardness and allows us to decrease the overall optimization time. 3) Finally, we consider a satellite constellation based on O3b
mPower (5,000 beams) and real-world-based data to assess the benefits of using this approach to reduce on-board power consumption.

TABLE IV: Dimensionality of the satellite system and frequency assignment parameters used in each experiment. In all cases we consider a seven-satellite constellation ($N_S = 7$) and two polarizations ($N_P = 2$). †The parameter $N_{ch}$ corresponds to the number of changes per iteration when the iteration-based adaptation is used (see Section IV-C).

| Experiment | Freq. reuses $N_{FR}$ | Bandwidth slots $N_{BW}$ | Users $N_U$ | Beams $N_B$ | Changes $N_{ch}$ † |
|------------|-----------------------|---------------------------|-------------|-------------|-----------------|
| 1          | 8                     | 40                        | 50, 60, 100 | 96, 118, 182 | -               |
| 2          | 20                    | 200                       | 1,000       | 1,060       | 25, 50, 100     |
| 3          | 20                    | 200                       | 20,000      | 5,000       | 50              |

A. Experimental setup

1) Constellation parameters: In all cases, the experiments make use of the O3b mPOWER constellation filed by O3b limited [30], a representative of a MEO constellation system. The constellation has $N_S = 7$ satellites, each capable of using $N_P = 2$ polarizations, and we adjust the number of users, the number of beams, and the rest of spectrum parameters for each experiment. User distributions as well as gateway locations are provided by SES S.A. based on realistic configurations. In the analyses, we focus on optimizing the downlink frequency assignment (see Appendix D for uplink considerations), and vary the number of beams $N_B$, frequency reuses $N_{FR}$, and bandwidth slots $N_{BW}$ to represent scenarios with different spectrum availability. The specific parameters configured for each of the three experiments are summarized in Table IV. The variable $N_{ch}$ is used within the iteration-based procedure described in Section IV-C. Given a license is available to the authors, we use the commercial solver Gurobi [33] for the experiments. Other solvers could also be used, the algorithm is agnostic to that choice.

2) Beam placement: Before the method begins the optimization, given a particular user distribution, we use the beam placement algorithm described in [26] to determine the required number of beams and their positions. Based on this beam placement, we consider two beams will have an inter-group or interference restriction if the center of their footprints is closer than four times the half-cone angle.

3) Benchmarks: We also make use of the heuristic greedy-based frequency assignment algorithm described in [26] as a baseline benchmark throughout the remainder of the paper. This
The heuristic algorithm sorts the beams based on their number of constraints and sequentially assigns a central frequency and bandwidth for each. In case the algorithm fails to converge, it reduces the bandwidth of each beam according to a constant factor and retries the assignment process for all beams. As opposed to this method, the heuristic algorithm does not allow to target specific beams and all beams are penalized equally.

The same frequency plan provided by the heuristic algorithm [26] serves also as a warm-start when using the iteration-based procedure detailed in Section IV-C. In that section and in Section IV-D, we also compare the results with a water-filling baseline, which assigns a central frequency to the beams based on an estimate of the required bandwidth.

4) Figures of merit: Given this problem imposes constraints that limit spectrum utilization [26], we use the total amount of utilized bandwidth, i.e., \( \sum_{i=1}^{N_B} b_i \), as a figure of merit in order to compare the performance of this method against others. However, we normalize it and report it as the fraction of used bandwidth over the total available capacity, defined as follows:

\[
BW = \frac{1}{C_{tot}} \sum_{i=1}^{N_B} b_i
\]  

(28)

where \( C_{tot} \) is the total system capacity, defined as the total number of frequency slots available in the constellation:

\[
C_{tot} = N_S \cdot N_{BW} \cdot N_{FR} \cdot N_P
\]  

(29)

We normalize it to be able to compare the performance of this method in scenarios with different values for \( N_S, N_{BW}, N_{FR}, \) and \( N_P \). Across this section, we use \( BW_{WF}, BW_{HEU}, \) and \( BW_{ILP} \) to refer to the normalized total assigned bandwidth for the water-filling, the heuristic, and the optimized plans, respectively.

B. Experiment 1: Maximizing Bandwidth Allocation

In this experiment, the goal is to verify the behavior of the optimization method by maximizing bandwidth allocation in three scenarios with an increasing number of users and, in turn, an increasing number of beams, as described in Table IV. We encode bandwidth allocation maximization in the proposed objective function (27) by adjusting the weighting parameters as follows:

\[
\beta_{1,i} = 1, \quad \beta_{2,i} = \beta_{3,i} = \beta_{4,i} = 0, \quad \forall i \in \{1, \ldots, N_B\}
\]
Fig. 5: Comparison for one specific satellite in the constellation between the frequency assignment produced by the heuristic algorithm serving as baseline (left) and the ILP method (right) when bandwidth maximization is prioritized. The scenario is defined with $N_S = 7$, $N_P = 2$, $N_{FR} = 8$, $N_{BW} = 40$, and $N_B = 182$. The horizontal axis represents the frequency slots $f_i \in \{1, \ldots, N_{BW}\}$, whereas the vertical axis represents the frequency reuse and polarizations $g_i \in \{1, \ldots, N_{FR} \cdot N_P\}$. Gold coloring is used for gateway beams, while a different hue is used depending on the central frequency assigned to each user beam.

As mentioned, the rationale behind improving spectrum utilization is that it generally results in lower power consumption, without explicitly resorting to the introduction of power considerations in the formulation by means of parameters $\beta_{4,i}$. We explore this latter case in Section IV-D. While we do not restrict the maximum bandwidth that can be assigned per beam, in real operations this might be necessary due to spectral efficiency considerations [31]. If needed, a bandwidth upper bound can be also encoded in our formulation by means of equation (3).

Figure 5 shows the frequency assignment at a specific instant for one of the satellites when using the heuristic algorithm and when using the presented method. Each colored region represents the frequency slots assigned to a beam. As observed, the colored region of the plot is substantially larger when using this method. This corresponds to a 73% bandwidth increase with respect to the heuristic baseline solution, while satisfying all frequency and handover constraints (see Table V). Since the plot represents the frequency assignment for a specific time instant corresponding to a specific routing of beams to satellites, unassigned slots might be reserved for
TABLE V: Results for Experiment 1, focused on maximizing bandwidth allocation.

| Scenario | Users $N_U$ | Beams $N_B$ | $BW_{HEU}$ | $BW_{ILP}$ | $BW$ Increase |
|----------|-------------|-------------|-------------|-------------|---------------|
| 1.1      | 50          | 96          | 0.18        | 0.27        | 51%           |
| 1.2      | 60          | 118         | 0.25        | 0.41        | 61%           |
| 1.3      | 100         | 182         | 0.21        | 0.37        | 73%           |

beams that are soon to undergo or have recently undergone handover operations in the simulation.

An increase in the number of beams might not be proportional to an increase in the total number of frequency slots that we are able to allocate. This is clearly reflected by the values of $BW_{ILP}$ presented in Table V, since moving from 96 beams to 118 beams entails a total assigned bandwidth increase, but the effect of moving from 118 beams to 182 beams is the opposite. In the latter case, when increasing the number of beams, the number of intra- and inter-group constraints also increases, and therefore frequency assignments can not exploit more frequency slots in order to avoid frequency overlaps.

C. Experiment 2: Iteration-based extension and computing time tradeoffs

In this experiment, we test the scalability of the ILP method and develop an iteration-based procedure to reduce the computing time for high-dimensional scenarios, given the problem is NP-hard. As the dimensionality of both the constellation and user base increase, making frequency decisions in real time becomes more challenging, since the number of constraints scales quadratically with respect to the number of beams. When computing time is a constraint, there is a point in which every algorithm addressing NP-hardness must make tradeoffs in order to meet time requirements. We explore the tradeoffs of the method in this section.

In Appendix A, we present a method to speed up the optimization when dimensionality is large. In this modified approach, we turn the prior formulation into an iteration-based method where the search space is restricted to “interesting” regions. For each beam, we select a few variable assignments for each decision variable and let the solver choose the best combinations across beams. In each iteration, the optimization algorithm changes the frequency assignment of $N_{ch}$ beams, while the rest are kept fixed. Although the algorithm can make frequency decisions without any prior input, we find that using a complete —but suboptimal— frequency plan as a warm-start substantially decreases runtime. There is no requirement of validity for the warm-start frequency plan, since the ILP method can amend violated constraints. In the experiments, we
use the frequency plan provided by the heuristic algorithm presented in [26] as a warm-start. In general, we achieve a significant reduction in the number of variables and constraints that are considered during each call to the optimizer.

To test the iterative optimization method, we use a scenario with higher dimensionality, with $N_{FR} = 20$ frequency reuses, $N_{BW} = 200$ bandwidth slots, and $N_U = 1,000$ users. The users and their corresponding gateways are connected using $N_B = 1,060$ beams. We therefore increase both the search space —more resources— and the constraint set —more beams— compared to the first experiments. We still optimize the frequency plan to maximize the allocated bandwidth. We run the algorithm using three different configurations: $N_{ch} = 25, 50,$ and $100$ changes at each iteration. The beams assigned at every iteration are chosen randomly and we do not impose any restriction on how many times a single beam can undergo assignment. Given the nature of the procedure, we might require specific beams to be reassigned in different iterations before converging to a stable solution. In the experiments we run the algorithm until getting this stable solution; we judge that the algorithm has converged when the total allocated bandwidth does not increase during 50 consecutive iterations. Since we introduce stochastic elements, we run the algorithm 100 times for each of the selected $N_{ch}$ and report statistics on the 100 runs.

Table VI presents the results of using the iterative approach to reach a stable solution in less time when using $N_{ch} = 25, 50,$ and $100$. We compare the total assigned bandwidth increase with respect to the warm-start and the computing time (wall-clock time). The value of $N_{ch}$ is directly related to the tradeoff between computing time and number of iterations. In all cases, we are able to find a global frequency assignment that more than triples the total assigned bandwidth without violating any constraints of the system. When using $N_{ch} = 25$ we do so in less than one third the amount of time required for $N_{ch} = 100$, although, since we are assigning fewer beams at a time, we observe that the improvement per iteration is lower, as graphically represented in Figure 6. In particular, this Figure represents the convergence analysis of the algorithm under different values of $N_{ch}$. In combination with Table VI, it can be seen that more changes per iteration allow for a better usage of bandwidth at the convergence point, at the cost of a higher convergence time. The average total number of assignments, i.e. $N_{ch} \cdot N_{it}$, is approximately $1.1 \times 10^5$, $1.6 \times 10^5$, and $2.0 \times 10^5$ for $N_{ch} = 25, 50,$ and $100$, respectively. Note that this is about two orders of magnitude higher than the number of beams ($N_B = 1,060$ in these experiments). Table VII provides additional insights on the time and iterations it takes to increase the bandwidth by $100\%$, $200\%$, and $300\%$. Generally, we can observe the ILP method achieves $80\%$ of the final
TABLE VI: Results for Experiment 2 at the convergence point, reported over 100 different runs and focused on maximizing bandwidth allocation using the iterative optimization method. All simulations run on a server with 20 cores of an Intel 8160 processor and 192 GB of RAM. SEM = Standard Error of the Mean.

| Changes $N_{ch}$ | $BW_{WF}$ | $BW_{HEU}$ | $BW_{ILP}$ | $BW$ Increase (%) | Num. iterations $N_{it}$ | Comp. time (min) |
|------------------|------------|------------|------------|-------------------|--------------------------|------------------|
|                  | Mean±SEM   | Mean±SEM   | Mean±SEM   | Mean±SEM          | Mean±SEM                 | Mean±SEM        |
| 25               | 0.05       | 0.13       | 0.61±0.00  | 359±0             | 5082±45                  | 13.5±0.1        |
| 50               | 0.62±0.00  | 364±0      | 124±3.1    | 5082±44.6         | 3090±36                  | 27.9±0.3        |
| 100              | 0.63±0.00  | 367±0      | 131±0.2    | 1866±32           | 1866±32                  | 90.1±1.2        |

TABLE VII: Results for Experiment 2, number of iterations and the computing time required to reach an increase in allocated bandwidth of 100%, 200%, and 300% for three different algorithm configurations: $N_{ch} = 25, 50,$ and 100. Number of iterations and computing time required to converge are also included. All simulations run on a server with 20 cores of an Intel 8160 processor and 192 GB of RAM. SEM = Standard Error of the Mean.

| Changes $N_{ch}$ | Num. iterations $N_{it}$ | Comp. time (min) |
|------------------|--------------------------|------------------|
|                  | Mean±SEM                 | Mean±SEM         |
| $BW$ Increase    | 100%                     | 200%             |
|                  |                           | 300%             | Convergence       | 100%                     | 200%             | 300%             | Convergence       |
| 25               | 176±0.2                  | 476±0.5          | 1248±3.1        | 5082±44.6           | 2.5±0.0          | 4.1±0.0          | 6.4±0.0          | 13.5±0.1          |
| 50               | 90±0.1                   | 242±0.3          | 626±1.3         | 3090±35.7           | 3.7±0.0          | 6.6±0.0          | 10.7±0.0         | 27.9±0.3          |
| 100              | 49±0.1                   | 131±0.2          | 336±0.8         | 1866±32.0           | 9.0±0.0          | 16.3±0.1         | 27.9±0.1         | 90.1±1.2          |

bandwidth improvement in less than 20% of the convergence time.

With these experiments, we assess that the iteration-based procedure enables the scalability of this optimization method despite the problem being NP-hard, which is an important requirement for future high-dimensional constellation operations. Although limiting the number of beams that the optimization algorithm can modify significantly reduces the dimensionality of the solution space, the performance improvement comes at the expense of converging to suboptimal solutions and, therefore, needing to do around one hundred more assignments in total. Nonetheless, robustness and fast reaction times are generally preferred over optimality when operating in highly dynamic and time-dependent environments, where the feasibility of the frequency assignment might be only temporary. Appropriately setting hyperparameters such as $N_{ch}$ is key in those cases [34].
TABLE VIII: Results for Experiment 3, minimizing power consumption in a real-world scenario.
†Power decrease compared to heuristic baseline.

| Changes $N_{ch}$ | $BW_{WF}$ | $BW_{HEU}$ | $BW_{ILP}$ | $BW$ Increase | Power Decrease† | Iterations $N_{it}$ |
|------------------|-----------|------------|------------|---------------|-----------------|-------------------|
| 50               | 0.113     | 0.175      | 0.614      | 251%          | 39.8%           | 2464              |

D. Experiment 3: Minimizing Power Consumption in a Real-World Scenario

Finally, in this third experiment, we demonstrate the benefits of using this method to optimize the frequency assignment in a realistic scenario with a large-scale constellation and user base. The user data is provided by SES S.A. and includes throughput demand for 20,000 users located around the globe which, after running the beam placement algorithm, are grouped into approximately 5,000 beams (see Appendix B). In this experiment we prioritize directly reducing power consumption by leveraging the fourth term of the objective function equation (27), $-|\beta_{1,i}|P_i(f_i, b_i)$. For the purposes of this section, we understand $P_i$ as a linear function that returns the consumed power for beam $i$ given its $f_i$ and $b_i$ assignments. Its full implementation details into the formulation are described in Appendix C.

Figure 7 shows the warm-start (heuristic baseline algorithm) and the optimized frequency plan (ILP method) for the seven satellites of the constellation at one moment of the simulation. For this experiment, we run the iterative implementation of the optimization algorithm with $N_{ch} = 50$.
Fig. 7: Comparison between the warm-start and the optimized frequency assignment in a scenario with $N_S = 7$, $N_P = 2$, $N_{FR} = 20$, $N_{BW} = 200$, and $N_B = 5000$.

until a stable solution under real-time operational constraints is reached. The results presented in Table VIII show that using the presented method reduces the power consumption by up to a 40% after fewer than 2,500 iterations. This is achieved by increasing the total allocated bandwidth by more than 250%, which can be observed in Figure 7: the optimized plan has significantly larger colored regions, i.e., assigned frequency slots, as well as several unused combinations of a frequency reuse and polarization.

The results of this experiment demonstrate that the framework with the iteration-based procedure is able to substantially improve the frequency assignment in a high-dimensional, real-world-based system. The algorithm is able to navigate the search space of this highly-constrained problem and change the frequency allocation of beams to decrease the overall power consumption. The number of changes per iteration $N_{ch}$ and the total number of iterations can be modified to adjust the system’s operation to meet any possible time requirements. Although we used
a specific representative constellation for the experiments, this framework can be adapted to constellations with different flexibilities: number of satellites and planes, number of frequency reuses and slots, additional operational constraints, etc.

V. CONCLUSION

In this paper, we have proposed a novel frequency assignment method that addresses both the central frequency assignment and bandwidth allocation problems and meets the scalability requirements of the upcoming dynamic satellite environments. The optimization framework is based on Integer Linear Programming and provides optimal or quasi-optimal frequency plans based on an objective function while respecting handover, interference, and gateway dimensioning constraints. In addition, the framework adapts to the presence of frequency reuse mechanisms and multiple polarizations. This whole set of requirements has not been previously addressed altogether. The objective function is flexible and allows encoding multiple goals such as maximizing bandwidth, minimizing the number of active frequency reuses, and minimizing RF power consumption. We have also presented an iteration-based implementation of the framework that not only enables its operation in high-dimensional use cases but also introduces degrees of flexibility to configure it depending on the scenario and computing time constraints.

We have carried out three different experiments to validate the performance, scalability, and potential real-world operability of this method, respectively. The results of the first experiment prove the ability of the framework to efficiently solve the frequency plan design problem, achieving a 73% improvement on the total allocated bandwidth in 100-beam scenarios. The iterative optimization method analyzed in the second experiment is able to optimize the frequency assignment in higher-dimensional scenarios (1,000-beam range), achieving over a 300% improvement on spectrum usage in a time-efficient manner. Finally, the results from the third experiment demonstrate the benefits of using the framework in a real-world scenario with approximately 20,000 users and 5,000 beams. In this case, the framework increases the allocated bandwidth but prioritizes making an efficient use of frequency reuses and polarizations in order to achieve a 40% reduction in on-board power consumption.
APPENDIX A

ITERATION-BASED OPTIMIZATION FORMULATION

The following lines describe the iteration-based ILP formulation used in Sections IV-C and IV-D, which aims to reduce the number of decision variables and the dimensionality of the search space during every call to the solver. As opposed to the initial formulation, the proposed speed-ups do not guarantee finding the global optimum once the algorithm converges, but substantially reduce the amount of computing time needed to obtain a good enough result.

A. From full-optimization to an iteration-based approach

Optimizing a large number of beams at the same time entails a high-dimensional combinatorial problem. Without accounting for interference restrictions, each beam has an order of $N_{FR}N_{P}N_{BW}^2$ assignment options. Then, the total number of possible —not necessarily feasible— frequency plan combinations is in the order of $(N_{FR}N_{P}N_{BW}^2)^N_B$. While this number is still tractable for low dimensional scenarios, optimizing for a large number of beams is infeasible from a time perspective. A way to reduce the dimensionality of the problem is to tune only certain beams at a time, while keeping the rest fixed, and iterate over the optimization procedure while selecting different sets of beams so that all beams are optimized. The number of beams that are allowed to change at every iteration is denoted by $N_{ch}$.

B. Reducing the search space by ranking

Although we have a search space of size $N_{FR}N_{P}N_{BW}^2$ for each beam, only a small region of the space is interesting from an optimization point of view. Each beam $i$ contributes to the objective function in the form of:

$$\beta_{1,i}b_i - |\beta_{2,i}|g_i - |\beta_{3,i}|f_i - |\beta_{4,i}|P_i(f_i,b_i)$$

(27)

By looking at the contribution we can rank the different assignment options based on their benefit to the total objective value. Let us denote $x_{f,g,b,i}$ as a binary decision variable that represents choosing the option for beam $i$ which has initial frequency $f$, frequency reuse $g$, and bandwidth $b$. We can rewrite the contribution of each beam as:

$$\sum_{f,g,b} (\beta_{1,i}b - |\beta_{2,i}|g - |\beta_{3,i}|f - |\beta_{4,i}|P_i(f,b)) x_{f,g,b,i} = \sum_{f,g,b} l_{f,g,b} x_{f,g,b,i}$$

(30)
Where $l_{f,g,b,i}$ is the contribution of the option denoted by $x_{f,g,b,i}$. Now, instead of considering all the feasible options, we consider only a subset of alternatives for each beam, denoted by the symbol $\mathcal{V}_i$. By considering a strict subset, we are limiting the options of the algorithm and reducing the search space of solutions, with the expectation that the optimal or close to optimal solution lies within the selected subset. To decide which assignments belong to the subset $\mathcal{V}_i$, we pre-compute a ranking for each possible bandwidth $b$, and include the options in the top of each ranking in the set. In our experiments we select the top 10 candidates per bandwidth assignment. We also check that none of these options at the top of each ranking violates restrictions with beams that are not being changed in the current iteration. The new problem formulation can be stated as:

$$\max \sum_{i=1}^{N_B} \sum_{f,g,b\in \mathcal{V}_i} l_{f,g,b,i} x_{f,g,b,i}$$

(31)

Additionally, we need to impose that at most one option can be active at a time:

$$\sum_{f,g,b\in \mathcal{V}_i} x_{f,g,b,i} = 1 \ \forall i$$

(32)

### C. Including intra- and inter-group restrictions

The final aspect to address in this new approach is how to include intra- and inter-group constraints. As mentioned, interference with fixed beams is passively included by disregarding the infeasible options. The only restrictions to be included are the ones between beams that are allowed to change in the same iteration. Since the decision variables are now just binary activation variables with implicit frequency reuse, initial frequency, and bandwidth, we can easily precompute if two options from two different beams collide or not. If they do, we only need to add a restriction of the type:

$$x_{f_i,g_i,b_i,i} + x_{f_j,g_j,b_j,j} \leq 1$$

(33)

Which ensures that at most one of the two options will be active, guaranteeing constraint satisfaction in the final solution.

### D. Computational Complexity

Now, the complexity of the problem is solely determined by the number of beams that are allowed to change at each point, and the number of options considered for each beam, which are hyperparameter tuning decisions that can be assessed independently of the problem
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characteristics. The complexity of the problem is still NP-hard due to the combinatorial nature of the solution. However, modern mathematical solvers allow us to solve this type of problems fast up to a certain dimensionality, which can be controlled by changing number of changes and the number of options per beam. Note that if we set $N_{ch} = N_B$ and consider all valid options, the formulation is equivalent to the original formulation presented in this work.

Appendix B

Beam Placement Distribution

Figure 8 shows the beam placement distribution that results after using the beam placement algorithm from [26] on a set of approximately 20,000 users. 5,000 beams are used to serve these users.

Fig. 8: Beam placement for scenario with approximately 20,000 users and 5,000 beams.

Appendix C

Power Computing Algorithm

One of the elements in the objective function equation (27) corresponds to a metric for power consumption. As power equations are not linear [31], we can not directly use them in our
framework. As a solution, we precompute, for each of the $N_B$ beams, the consumed power for each possible frequency assignment.

To that end, we use the satellite communications models described in [16]. For simplicity, we describe the procedure to compute the necessary power $P_b$ for one beam $b$ given its data rate demand $D_b$ and a certain allocated bandwidth $BW_b$. We assume the satellites use the MODCOD schemes defined in the standards DVB-S2 and DVB-S2X [35]. Given a certain roll-off factor $\alpha_b$, we can compute the lower bound of the required spectral efficiency as

$$\Gamma_{req} = \frac{D_b(1 + \alpha_b)}{BW_b}$$

(34)

We select the MODCOD whose spectral efficiency is the lowest such that $\Gamma \geq \Gamma_{req}$. If no such MODCOD exists, it means that the frequency assigned to the beam is too low to serve the required demand only by increasing power. As a mathematical representation, we would need a power of infinity to cover that demand, which translates to setting a power value of $P_b = M$ in the formulation, where $M$ is a sufficiently large number.

Otherwise, we can get the appropriate value for $E_b/N$ from the MODCOD scheme. Since in our work we have considered interference mitigation mechanisms by means of the inter-group constraints, we assume interference is negligible. We then compute the necessary $C/N_0$ as

$$\left. \frac{C}{N_0} \right|_b = \left. \frac{E_b}{N} \right|_b \cdot \frac{D_b}{BW_b}$$

(35)

With $C/N_0$ in dB, we can then compute the power as:

$$P_b = \left. \frac{C}{N_0} \right|_b + OBO - G_{Tx} - G_{Rx} + FSPL + 10 \log_{10}(kT_{sys})$$

(36)

where OBO is the power-amplifier output back-off, $G_{Tx}$ and $G_{Rx}$ are the transmitting and receiving antenna gains, respectively, $k$ is the Boltzmann constant, and $T_{sys}$ is the system temperature (assumed to be 290K). FSPL and $L_{atm}$ account for the free-space path losses, respectively. We assume FSPL are significantly larger than atmospheric losses, and losses at the transmitting and receiving antennas, so we neglect all the latter.

We compute a power value $P_b$ for each possible assignment of $BW_b$, and repeat the process for all beams in the constellation.

APPENDIX D

UPLINK FREQUENCY PLAN

We have presented our optimization method in the context of optimizing the downlink frequency plan. However, both uplink and downlink plans need to be fully defined during op-
erations. Here we address how our method adapts to the uplink case, which depends on the presence of on-board computing capabilities in the satellites.

In systems with on-board computing capabilities, the electromagnetic signals can be decoded and recoded using different MODCODs. This enables the use of different bandwidths for uplink and downlink and, therefore, decouples both problems. In this context, our algorithm as described in this paper can be used to obtain the uplink and downlink frequency plans independently.

In cases without on-board processing, both uplink and downlink frequency plans are inevitably coupled, since the bandwidth for the forward and return links must be constant. Consequently, both plans must be computed together, thus increasing the number of beams $N_B$ the algorithm deals with. The only required additional constraints correspond to imposing matching uplink and downlink beams to have the same bandwidth (i.e., $b_{i,\text{up}} = b_{i,\text{down}}$). Then, the same algorithm can be used. Note that uplink and downlink beams will not interfere with each other and will not present handover or gateway dimensioning constraints.

ACKNOWLEDGMENTS

This work was supported by SES S.A.. The authors would like to thank SES S.A. for their input to this paper and their financial support. The project that produced these results also received the support of a fellowship from “la Caixa” Foundation (ID 100010434). The fellowship code is LCF/BQ/AA19/11720036. The authors also want to thank the anonymous reviewers who provided insightful feedback, which has greatly contributed to improve the overall quality of the paper.

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