Electron teleportation with quantum dot arrays

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An electron teleportation protocol, inspired by the scenario by Bennett et al., is proposed in a mesoscopic set-up. A superconducting circuit allows to both inject and measure entangled singlet electron pairs in an array of three normal quantum dots. The selection of the teleportation process is achieved in the steady state with the help of two superconducting dots and appropriate gating.

Teleportation of the electron spin is detected by measuring the spin-polarized current through the normal dot array. This current is perfectly correlated to the pair current flowing inside the superconducting circuit. The classical channel required by Bennett’s protocol, which signals the completion of a teleportation cycle, is identified with the detection of an electron charge in the superconducting circuit.

PACS 74.50+r, 73.23.Hk, 03.65.Ud

Teleportation (TP) recently entered the realm of quantum physics when Bennett et al.\textsuperscript{[1]} proposed a protocol to reconstruct the unknown state of a given particle at a different location. The sender, Alice, and the receiver, Bob, share an entangled pair \( |\Psi^-\rangle \), and Alice performs a joint measurement on the “source” particle and her part of the pair. The result of the measurement is communicated through a classical channel to Bob, allowing him to reconstruct the initial state on his part of the pair. This protocol has since been experimentally demonstrated with polarized photons\textsuperscript{[2]}, as well as proposed in atomic physics\textsuperscript{[3]} and solid state optics\textsuperscript{[4]}. Besides its fundamental character, TP is likely to become an essential element of future information processing schemes\textsuperscript{[5]}. It is certainly relevant to test these manifestations of non-locality\textsuperscript{[6]} with massive particles in nanostructured devices, with the advantage that these can be integrated in (quantum) electronic circuitry. Similar analogies between photon propagation and phase-coherent electron transport in nanostructures were illustrated by the fermion version of the Hanbury-Brown and Twiss experiment\textsuperscript{[7]}.

The general principle of the present mesoscopic scheme for TP – an array of quantum dots with superconductors – is inspired of Ref\textsuperscript{[1]}, but follows more closely its optical implementation\textsuperscript{[4]}. Alice’s measuring device for entangled (singlet) electron pairs is an s-wave superconductor, as is the generator of the entangled electron pairs\textsuperscript{[8,9,10,11,12]}. Similarly to the the optics experiment only one of the four Bell states is measured.

However photons interact weakly (except during their generation and detection process). On the contrary, electrons in nanostructures experience strong Coulomb interactions, which can be used to ensure that electrons be injected one by one from/to a quantum dot through tunnel barriers\textsuperscript{[13]}. Indeed, further control can be obtained in a multidot array, by means of intradot and interdot Coulomb correlations: here, the “correct” TP sequence (injection, pair creation, measurement, classical channel and detection) can be precisely selected, while operating in the steady state, by an appropriate initial choice of gate voltages.

The “device” is depicted in Fig.\textsuperscript{1}[a]: three normal (N) dots, 1, 2 and 3, and two superconducting (S) dots a, b, are placed in alternation: N-dots can only communicate via the S-dots. Dots 1 and 2 are coupled to dot a – Alice’s measuring device – by tunnel junctions, while 2 and 3 are coupled to b – the source of entangled pairs. Furthermore, dots a, b are connected by tunnel junctions to a superconducting (S) circuit where Cooper pairs only are transferred. Reservoir L emits in dot 1 the electron to be teleported, and reservoir R (“Bob”) collects the teleported state from dot 3.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{a) The TP cell contains: i) NN junctions between reservoirs L, R and dots 1 and 3; ii) N-S junctions between (1, a), (a, 2), (2, b) and (b, 3), and S-S junctions between a (b) and the bulk superconductor S. Capacitive couplings select the correct sequence. Detector D (i.e. capacitive coupling between a and S) signals the passage of a Cooper pair in the superconducting circuit. b) Sketch of the energy level configuration of dots 1, 2 and 3 (electron energies \( \pm \varepsilon \) are symmetric with respect to the superconductor chemical potential \( \mu_S \)).}
\end{figure}

First, focus on the isolated system of 5 dots. Following\textsuperscript{[14]}, an entangled singlet pair of particles \(|\Psi^S\rangle_{23} = \ldots\rangle\)
2^{-1/2}(|↑↓⟩_{23}−|↓↑⟩_{23}) is produced by b. Coulomb blockade [13] prohibits double occupancy in each dot (the same is true for Cooper pair occupancy in the superconducting dots). Bringing together the singlet \(|\Psi^S⟩_{23}\) with the state \(|σ⟩_1\) to be teleported, the resulting state with dots 1, 2, 3 occupied leaves the spin in 3 unspecified. This three-particle wave function is now decomposed among the 4 Bell states for electron spins in dots (1, 2, 3) \(1/4 \: \left[ |\Psi⟩_{123} = -(1/2)|\Psi^S⟩_{12}|σ⟩_3 + \sum |Ψ^T_{αβ}⟩_{12}|σ⟩_3 \right]\) where \(|σ⟩_3\) are unitary transforms of \(|σ⟩\) \((i = 0, ±)\) and \(|Ψ^T_{αβ}⟩_{12}\) the three triplet states. a acts as a detector for the singlet state of electrons in 1 and 2; absorption of a Cooper pair only occurs if (1, 2) contain a singlet.

The remaining spin in dot 3 necessarily acquires the same state \(|σ⟩\) as the initial spin in dot 1, as required by TP. The absorption of the singlet electron pair from (1, 2) also destroys the initial spin state in dot 1, therefore satisfying the “non-cloning theorem” [14], and this transition is made irreversible because it is followed by the (irreversible) injection of a “new” electron from reservoir \(L\).

A microscopic model supports this TP protocol. N-dots are assumed to have a discrete spectrum, with level spacing comparable to the gaps \(Δ_{a,b,S} \sim Δ\) of the S-dots and S-circuit. The S-dots have a continuous quasi-particle spectrum, and \(Δ_{a,b} > E_{C_a} \equiv e^2/C_{S_a}\). Only two occupation numbers are kept for each dot. N-dots (S-dots) have “empty” states with an even number \(N^0\) of electrons, and have “filled” states with \(N^0 + 1\) \((N^0 + 2)\) electrons. The Hamiltonian which describes the TP cell reads \(H = H_0 + H_t + H_C\) where \(H_0\) describes the isolated elements (dots and reservoirs). The single electron hopping term \(H_t\) has amplitudes \(t_{αβ}\) \((α, β = \{L, R, S, 1, 2, 3, a, b\})\). Only one level is relevant in each N-dot, and next nearest neighbor hoppings are neglected. The Coulomb contribution has the standard form: \(H_C = (1/2)\sum_{μ, ν = 1, 2, a, b} U_{μν}\delta N_μ\delta N_ν\), where \(δN_μ = N_μ − N_μ^0\) is the deviation from the effective number of electrons imposed by the gates (voltage \(V_{Gr}\)). The coefficients \(U_{μν}\) form the inverse capacitance matrix of this five dot system, and are computed [13] from the individual capacitances \(C, C'\), \(C_s\) and \(C_g\) of the NN, NS, SS and gate junctions respectively (see Fig. 1a).

The dots are coupled to the N/S reservoirs with energy line widths \(Γ_{L,R} = 2π\sum_{l, R} |ν_{l,R}(0)|^2 \) with density of states \(N_{l,R}(0)\) (and similarly \(Γ_{Sa} = Γ_{Sb}\)). The chemical potential \(μ_S\) of the superconductor is located in the middle of the left/right reservoir potentials \(μ_S ± eV/2\). Dot configurations are identified by the occupation numbers of dots 1, 2, 3: 0 or 1 \((0, 1, 0\) or 2\) for the N-dots (S-dots). Charging energy differences \(ΔE_{μν}^{P}\) between the initial and final configurations of the five dot system enter the \(O(H_t^2)\) calculation of the pair tunneling amplitude from b to 2, 3 (and similarly from a to (1, 2)): \(A_{μν}^{P} \simeq 2∑_{k, x} u_k^x v_k t_{23} t_{2b})/(iη − E_k^0 − ΔE_{00020}^{P})\), with \(η\) an infinitesimal, \(u_k, v_k\) the usual BCS parameters, \(x = 0, 1\) \((x = 1, 0\)\) \(E_k\) is the quasiparticle energy involved in the creation of a quasiparticle. The amplitude \(A_{μν}^{P(b)}\) is at most comparable to \(Γ_{Sa,b}\), and decreases with the distance between the two junctions involved in cross Andreev reflection [16]. The transition amplitude \(A_S\) between a and \(b\) is the S-circuit is in general larger [17]. Consider the system in the absence of connections with the N,S leads. Dot gate voltages are adjusted so that the pair transitions \(A_{P}^{b,a, S}\) are resonant. Discarding virtual processes with more than one quasiparticle in \(a\) or \(b\), one obtains the effective pair Hamiltonian

\[
H_{eff} = A_{P}^{b,a}Ψ_{12}^† Ψ_a + A_{P}^{b}Ψ_{23}^† Ψ_b + A_S(Ψ_a^† + Ψ_b^†)Ψ_S + H.c.
\]  

The TP sequence is now illustrated (Fig. 2) in a steady state operation of the whole circuit (“TP cell”), by applying a constant bias between reservoirs \(L\) and \(R\). Circuit parameters and gate voltages are chosen such that the TP cell is symmetric in changing \(1\) \((a)\) into \(3\) \((b)\), thus \(A_{P} = A_{P}^{b,a}\) (no phase difference exists in the S part of the cell). The TP sequence repeats itself cycle after cycle, each one achieving teleportation of an electron injected in \(1\) from \(L\), and detection in \(R\) of the teleported electron in \(3\). Start with dots \(1, 3\) and \(b\) occupied (upper right in Fig. 2). The teleportation process is triggered by the escape of the electron in \(3\) in reservoir \(R\). Doing so, the energy level in \(b\) is lowered, thus interrupting the previously resonant Cooper pair transfer. Now (lower right) the pair in \(b\) resonates with \(2, 3\), building with \(1\) the aforementioned state \(|Ψ⟩_{12}\). Measurement of the singlet state in \((1, 2)\) by \(a\) (Alice) is achieved when a new electron is injected into \(1\), yet also raising the energy level in \(a\) in the process. The remaining electron in \(3\) thus irreversibly acquires the state of the previous one in \(1\), while the new electron waits in \(1\) to be teleported in the next sequence.

Note that: i) Incoherent processes are brought by the reservoirs and the applied bias. The latter bias also determines the direction of TP (right or left) in an otherwise symmetric TP cell. This allows pair production from \(b\) and pair measurement in \(a\) to be both irreversible. ii) Successive TP cycles are linked together in such a way that a detection event triggers pair production for the next cycle, and an injection event triggers pair measurement for the previous cycle. iii) The classical channel corresponds to the detection of an extra Cooper pair in the superconducting circuit \((a + S + b)\). In Fig. 1a this detection is schematized by the presence of a detector \(D\), positioned between \(a\) and \(S\). It conveys the information about the (classical) charge in dot \(a\), this signals the completion of the pair transition 1, 2 → a, the destruction of
the original (quantum) spin state in dot 1 and the instantaneous reconstruction of this state in dot 3. This is enabled by the entangled pair, in full agreement with the TP principle [1]. iv) This classical information should in principle be transmitted to Bob, in order to distinguish whether the electron he receives from 3 is the result of a TP or any other transport process. Here, this would require a time resolved correlation measurement between the current in the S-circuit and that injected in R, i.e. the analog of coincidence measurements performed in the optical implementation [3]. Yet, the present mechanism has the merit of automatically implementing TP in the five-dot cell; v) As in optics [3], measurement of the sole singlet state reduces to 1/4 the efficiency of TP, but not the fidelity, equal to 1 in the ideal sequence depicted above.

FIG. 2. TP sequence: upper dots (white) are S-dots a and b, lower dots (shaded) are N-dots 1, 2, 3. Horizontal transitions only are resonant. Starting from the framed configuration (upper right), an electron in 3 escapes in R; next, a pair (from b) creates an entangled state 2, 3 (wiggly line) with rate $A_P$, leaving all N-dots filled. A pair 1, 2 then escapes in a. The electron in 3 acquires the spin state of dot 1, as confirmed by the absorption of a signet state in a and the subsequent injection of an electron from L.

The sequence reads: $\ldots 10021 \rightarrow [10020 \leftrightarrow 10101 \leftrightarrow 02001] \rightarrow [12001 \leftrightarrow 10001 \leftrightarrow 10021] \ldots$ Close inspection of the energy balance $\Delta E^{a}_{ij}$ of all the electronic transitions in the TP cell reveals that it is indeed possible to force this sequence with the help of constant gate voltages only [3]. As an example, let us assume that $C = C' = C_S = 100 C_g$. First, the resonance condition for pair transitions implies $\Delta E^{10101}_{10020} = \Delta E^{02001}_{10101} = \Delta E^{12001}_{10001} = \Delta E^{10201}_{10001} = 0$. One finds that this fixes $\tilde{N}_a, \tilde{N}_b = 0.97$, and $\tilde{N}_1, \tilde{N}_2 = 0.67$. Second, injection and detection are ensured (with $\mu_{LR} = \pm e V/2$) by $\Delta E^{12001}_{10020} < e V/2$, $\Delta E^{10201}_{10001} < e V/2$, therefore $V > (\tilde{N}_1 - 0.9)e V/C$. Third, the transfer of an electron from 3 to R is allowed from state 10021 but, among other unwanted transitions, not from 10101 or 02001: this can be achieved in a certain range of $V$ because $\Delta E^{10101}_{10100} - \Delta E^{10200}_{10200} = 2 U_{33} - U_{23} = 11 e^2/30 C > 0$ and $\Delta E^{12001}_{10020} - \Delta E^{10201}_{10020} = U_{13} + 2 U_{33} - 2 U_{a3} = 13 e^2/30 C > 0$.

TP fidelity is reduced by other transport processes, yet which are suppressed by our choice of resonant Cooper pair transfers. First, a direct electron transfer can result from two consecutive cotunneling transitions from dot 1 to dot 2, and from dot 2 to 3, while generating virtual quasiparticles $1|021⟩$. Cotunneling can be avoided by maximizing the energy differences for transitions from dot 1 to dot 2, by tuning the parameter $\tilde{N}_1, \tilde{N}_2$. Positive (negative) gate voltages applied to dots 1, 3 (dot 2) guarantee that cotunneling involves a positive energy $2 \epsilon$ (Fig. 1), with $A_P < \epsilon < \Delta$. The amplitude for cotunneling from dot 1 to 3 is reduced as it scales like $A_P^2/\epsilon \ll A_P$. Cotunneling is quenched by maximizing $\Delta E^{10121}_{12001}, \Delta E^{10221}_{10001}, \Delta E^{00121}_{10001} = (\tilde{N}_1 - \Delta 8/15) e^2 / C \approx 2 \epsilon$. At $T = 0$, optimal operation is obtained with $\tilde{N}_1 = 0.9 \sim 1, \tilde{N}_2 \sim 1/3$ and finite bias $0 < \epsilon < 3C$. A second process is Josephson tunneling between a and b, independently of the pair current involved in the TP sequence: Cooper pairs can be transmitted by cotunneling through dot 2 only. However, this process [22] involves quasiparticle excitations in a or b, contrary to the TP process. Note that TP involves a (normal) spin-conserving current between L and R, perfectly correlated to a pair current in the S circuit. This signature of the coupled quantum and classical channels allows to distinguish TP from the other two processes: the pair current is missing in cotunneling, while the normal current is missing in the Josephson process.

Each TP cycle may involve a new spin state, independent of the previous one. Yet it is convenient, to test TP, to fully spin polarize both L and R, in order to measure the spin correlation between the incoming and outgoing electrons (similarly to the optics experiment [3]). Assuming the TP cell to be weakly coupled to the reservoirs, transport across the dot array can be described by a master equation. Defining states $|\uparrow, 2, 0, 0, \uparrow⟩ = |a⟩$, $|\uparrow, 0, 0, 0, \uparrow⟩ = |e⟩$, $|\uparrow, 0, 0, 2, \uparrow⟩ = |b⟩$, $|\uparrow, 0, 2, 0, \downarrow⟩ = |1⟩$, $|0, 2, 0, \uparrow⟩ = |3⟩$, and $|S, T⟩$ the states $|10101⟩$ with wave functions $|\Psi^S⟩_{12} |σ⟩_{3}$ and $(1/\sqrt{3}) \sum_{\nu} |Ψ^T⟩_{12} |σ⟩_{3}$, the Bloch equations for the reduced density matrix, describing both the populations and the coherences $\sigma_{\mu ν}$ ($\mu, \nu = a, b, c, 1, 2, 3$) can be written in the general form [23] at zero temperature:

$$\dot{σ}_{\mu ν} = i \sum_\nu \Omega_{\mu ν} (σ_{\mu ν} - σ_{ν μ}) - \sum_\lambda (Γ_{μ λ} σ_{μ λ} - Γ_{λ μ} σ_{λ ν})$$

and

$$\dot{σ}_{\mu ν} = i \sum_\nu (σ_{μ λ} Ω_{ν λ} - σ_{λ ν} Ω_{μ λ}) - \frac{σ_{μ ν}}{2} \sum_\lambda (Γ_{μ λ} + Γ_{λ ν})$$

with $Ω_{\nu a} = Ω_{ac} = Ω_{bc} = Ω_{ab} = A_S$, the tunneling rate for Cooper pairs from a to S (to b). $Ω_{1S} = Ω_{S1} = - A_P / 2$, $Ω_{1T} = Ω_{TF} = - √2 A_P / 2$, $Ω_{3S} = Ω_{S3} = A_P$, $Γ_{1} = Γ_{R}$, $Γ_{3} = Γ_{L}$, all the other $Ω_{\mu ν}$’s and $Γ_{μ ν}$’s are zero. The steady state TP current $I_{tel} = e ⟨Γ_R σ_{bb}⟩$ (from L to R) then reads:

$$I_{tel} = \frac{Γ_L Γ_R}{(Γ'_L + 4 Γ_R)} |A_P^2 + 2Γ'_L Γ_R/(Γ'_L + 4 Γ_R)|$$

with $Γ'_L = Γ_L (3 + Γ'_L/2 A_P^2)$. Note that the above analysis does not depend on the incident polarization as depicted in Fig. 2 as the two spin channels are totally decoupled. Aside from corrections due to cotunneling, the
only transport channel through the dot array is the TP process. Unless direct evidence comes from individual electron coincidence counting (as for photons), a signature of TP is already provided by the equality of the TP current and the pair current \( I_P = 2I_{tel} \).

As in quantum optics, a proof for nonclassical spin correlations requires to check the above equality for parallel spin polarizations of reservoirs \( L \) and \( R \), taking successively two values corresponding to non-orthogonal quantum states. Refined diagnosis for TP can be searched in noise correlations measurements or with future time-resolved measurements.

Limiting factors are now considered. First, it is crucial to maintain spin coherence during the TP sequence (on a time scale \( \sim h/R_{KL} \), which turns out to be “short” in practice). This coherence can be destroyed by spin-orbit coupling, or by collisions with the other electrons within the dot. Such spin-flip processes can be minimized provided that the level spacing in the dots is larger than the temperature and the resonance width of the dots: “empty” dot states of \( 1, 2, 3 \) should preferably have even filling \( N_{\mu} \). Second, the present scheme is clearly optimized if Cooper pair transfer from the N-dots pairs to each S-dots has an maximal amplitude \( A_P \). This amplitude is strongly reduced by a geometrical factor in two and three dimensions when the two N-S tunnel barriers as spaced farther than a few nanometers. On the other hand, the size of the S-dots is large enough so that \( E_{C_{ab}} < \Delta \), thus precise lithography bringing N-dot pairs close together (however avoiding direct tunneling between N-dots) is required. An alternative would be to define the dots with quasi one-dimensional conductors (nanotubes) placed in contact with the superconductor, as the geometrical constraint is relaxed.

To sum up, an electron spin teleportation scheme which employs N-S hybrid nanodevices for electrons is proposed. It relies on current nanofabrication techniques and operates in the steady state, using Coulomb correlations in the dot array. The TP current which flows between the N-reservoirs is locked with the pair current in the S-circuit. The device could be implemented using bent, gated, contacted carbon nanotubes next to superconductors. The feasibility of this proposal relies on the equality of the TP current and the pair current, using Coulomb correlations in the dot array. The device could be implemented using bent, gated, contacted carbon nanotubes next to superconductors. The feasibility of this proposal relies on the equality of the TP current and the pair current, using Coulomb correlations in the dot array. The device could be implemented using bent, gated, contacted carbon nanotubes next to superconductors.

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Stimulating discussions with V. Bouchiat are gratefully acknowledged. LEPES is under contract with Grenoble universities, UJF and INPG.