A multi-objective optimization model for reservoir operation during droughts

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Abstract. General guidance for reservoir operation can be referred to as the Standard Operation Procedure (SOP). Real circumstances such as drought, however, do not necessarily comply with these ideal assumptions, which then trigger the studies on optimal hedging rules. More frequent mild droughts are preferred to less frequent severe droughts. In the present work, the ε-constrained-based multi-objective optimization model was developed to optimize hedging rules for a reservoir located in Lombok Island, Central Indonesia. The rules were aimed to minimize two objective functions namely maximum Single Shortage ($S_S$) and Total Deficit ($T_D$). The former is a measure of reservoir operation during drought in short-term, whereas the latter is for long-term evaluation. Results show that these two measures are conflicting in nature, that is, the greater the $S_S$, the smaller the $T_D$ and vice versa. The trade-offs between $S_S$ and $T_D$ can be a helpful assist for water operators in executing their routine daily tasks by providing water allocation alternatives.

1. Introduction

Droughts are kinds of water-related disasters [1], whose impact may involve safety, health, aesthetics and agricultural productivity. Appropriate water supply during droughts therefore becomes a necessity, including that in reservoir operation. Reservoirs’ releases need to be rationed in a proper way to avoid worse impact on water users. In general, more frequent but smaller water shortages are preferred than single but severe abrupt water shortage. The former is called “hedging” in reservoir operation, which is trickier than implementing the Standard Operating Policy (SOP). One should bear in mind, however, that while severe water shortage can effectively be undermined by hedging, less reliable water supply can also be resulted by unnecessary hedging.

Little attention has been given to hedging until up to late 1970s, a decade after the Harvard Water Program was held [2]. Up to date, significant number of studies have been conducted in deriving hedging rules which vary in terms of, among others, reservoirs’ performance measures during drought, the numbers of objective functions, and the optimization modelling purposes.

Studies by [3] and [4] provide a basis for reservoir performance measures adopted in considerable amount of studies on hedging. In their study, [3] introduce the terms of reliability, resilience and vulnerability to assess reservoirs’ performance. These three criteria were combined into single shortage index by imposing various weights in later studies such as in [5], [6], [7] and [8].

Apart from shortage index, reservoirs’ performance measures were also assesses using single-objective functions which took various forms of maximizing economic benefit such as [9], [10] and...
Another measure which takes form of minimizing the squared difference between water demand and the reservoir releases was employed in [12].

Most previous studies mentioned above presented hedging rules optimization in a single objective function, and the main goal is to seek an appropriate combination of the weights \( w_1, w_2 \) and \( w_3 \) which in some cases may be difficult enough. Recently, multi-objective optimization of hedging rules for reservoir operation during drought has been emerging. Studies such as [8,13,14] utilized two reservoir performance criteria namely Total Shortage Ratio (TSR) and Maximum Single-period Shortage Ratio (MSR).

The present study employs multi-objective function optimization model including a short-term objective and a long-term objective. Reservoir performance measure in short-term and long-term are measured by Single Shortage (\( S_S \)) and Total Deficit (\( T_D \)), respectively. In addition, the model was enhanced by shortening the operational timescales of agriculture by using bi-monthly periods (14-16 days, depending on the months).

2. Study Location and methodology

2.1. Study location

Batujai reservoir is located about 20 km to the southeast of Mataram, the capital city of West Nusa Tenggara Province. The reservoir can be reached about 30-minute by car.

![Study reservoir](Source: [14])

2.2. Methodology

The objective functions are to minimize two measures namely Single Maximum Shortage (\( S_S \)) and Total Deficit (\( T_D \)). The constraints involve water balance equation, and reservoir’s storage at any period should be between the reservoir’s minimum and maximum capacity. These constraints are given in Equation (1) and (2).

\[
S_{t+1} = S_t + I_t - R_t - W_t - E_t
\]

\[
C_{\text{min}} \leq S_{t+1} \leq C_{\text{max}}
\]
Where:
\( S_{t+1} \): storage at time \( t+1 \) (million cubic metre, MCM)
\( S_t \): storage at time \( t \) (MCM)
\( I_t \): inflow at time \( t \) (MCM)
\( R_t \): irrigation water supply at time \( t \) (MCM)
\( W_t \): raw water supply at time \( t \) (MCM)
\( E_t \): evaporation at time \( t \) (MCM)
\( C_{\text{min}} \): reservoir minimum capacity (1.40 MCM)
\( C_{\text{max}} \): reservoir maximum capacity (23.00 MCM)

One year of bi-monthly inflow data was utilized here, spanning from November 2000 to October 2001. Irrigation water requirement values were obtained by multiplying the Net Field Requirement (NFR in litre/sec/ha) with the designated irrigation area i.e. 4,000 hectares. Raw water must be supplied at the rate of 200 litre/sec, whereas free water surface evaporation values were calculated by multiplying the daily evaporation rate (mm/day) with free water surface area (m²). The trade-offs between reservoir’s storage and its free water surface area is represented by a quadratic function depicted in Figure 2 below.

![Figure 2. Trade-offs between Batujai reservoir volume and water surface area.](image)

Optimization was run during three cropping seasons (CS) in one year, namely CS-I, CS-II and CS-III. These cropping seasons ranging from November-February, March-June and July-October, respectively. The method of behavioural analysis was adopted here to assess the storages at every bi-monthly period, which the study reservoir is assumed full at the beginning of the optimization. Another optimization variable is spill which took place if the storages at a bi-monthly period exceeded the reservoir maximum capacity.

The decision variables were the actual watered irrigation areas in three cropping seasons which could be less than or equal to the designated irrigation area i.e. 4,000 hectares. These actual irrigation areas could be different from one to another cropping season, but should be exactly the same within one cropping season. \( T_D \) were minimized while keeping \( S_S \) fixed at selected values varying from 725,000 to 775,000 m³, and one more SS at 1,000,000 m³ was added. Deficit occurs whenever actual irrigation water supplies are less than the designated irrigation water requirement, or otherwise deficit would be zero. Maximum deficit in a single bi-monthly period equals to \( S_S \), whereas \( T_D \) can be calculated by aggregating all deficits within one-year optimization. The \( \epsilon \)-constraint method was adopted in the present work to generate the trade-offs between the two drought indices by Microsoft Excel’s Solver add-in. Finally, the drought indices at no hedging rules would be compared to those with hedging rules.
3. Results and discussions

3.1. Results
Four pairs of SS and TD were obtained after running the multi-objective optimization model, these pair of drought indices are depicted in Figure 3.

![Figure 3. Trade-offs between SS and TD.](image)

Water shortage values within the entire optimization period at selected SS are depicted in Figure 4. It can be seen that SS without hedging rules is \(1,282.68 \times 10^3\) m\(^3\), much greater than those with implementation of the rules. This maximum single deficit occurred at Aug-II, whereas those with hedging rules implementation occurred twice i.e. in May-I and Aug-II.

For the sake of brevity, only optimization and simulation results for SS of 775,000 m\(^3\) is given here in Table 1, which corresponds to TD of 5,551,465 m\(^3\).

3.2. Discussions
Four values of maximum SS and their corresponding values of TD have successfully been generated by the multi-objective optimization model. Perhaps the most important result is the understanding of the trade-offs between the selected objective functions represented by the drought indices. It is clear that these indices are conflicting in nature which means that the greater one index, the lesser the other one and so forth. By perceiving this trade-offs well, one can select the best water allocation alternative based on predefined criteria of SS and TD.

Implementation of hedging rules yielded in more frequent and milder water shortages than that of no hedging rules (i.e. no constraint of SS scenario). Without hedging rules, the maximum single deficit occurred at the second half-month of August (Aug-II) at 1,282.68 \times 10^3 m. In contrast, implementation of hedging rules yielded two milder droughts i.e. at May-I and Aug-II which reduced the SS of no hedging rules by 66-77\%. 
Figure 4. Water shortage at bi-monthly periods at selected $S_S$ and no hedging rule

4. Conclusions and recommendations

4.1. Conclusions

A simple yet useful multi-objective optimization model was developed to avoid single and severe drought impact as well as to generate more frequent and milder drought. The trade-offs between two objective functions were traversed by adopting the $\varepsilon$-constraint method. One objective function is to minimize the maximum single deficit ($S_S$) and the other is to minimize total deficit ($T_D$) during one year optimization. These drought indices were minimized by utilizing Microsoft Excel’s Solver add-in. The model has successfully implemented to obtain the water allocation alternatives for Batujai reservoir operation during drought.

It can be concluded that these two measures are conflicting in nature, i.e. the greater one measure the lesser the other and so forth. By perceiving this understanding well, one can select the best water allocation alternative based on these drought criteria. In addition, by implementing the hedging rule twice milder drought were obtained in order to avoid single severe drought.

Although the optimization model in this study was applied to Batujai reservoir located in Lombok sland, the model developed conceptually transferable to any other reservoir system by using a simple yet robust tool.

4.2. Recommendations

Significant amount of studies has been accomplished on water management during drought, which results in development of considerable number of drought indices. Two of these indices namely Single Shortage ($S_S$) and Total Deficit ($T_D$) were adopted in the present work in order to understand the trade-offs between the study reservoir’s performance in both short-term and long-term, respectively. Other drought indices mentioned in the previous sections are worth investigating because they may offer different insight about drought management in general and hedging rules in particular.
Shorter operational timescales such as daily or even hourly may be a great help in real-time water management during drought. Last but not least, longer management timescales such as yearly have an advantage in capturing forecasted inflow data.

### Table 1. Optimization results for maximum $S_t$ of 775,000 m$^3$

| Cropping Seasons | Days       | $S_t$ (m$^3$) | $I_t$ (m$^3$) | Designated releases (m$^3$) | $R_t$ (m$^3$) | $W_t$ (m$^3$) | $E_t$ (m$^3$) | $S_{t+1}$ (m$^3$) | Spill (m$^3$) | Deficit (m$^3$) |
|------------------|------------|---------------|---------------|-----------------------------|--------------|--------------|--------------|-----------------|-------------|---------------|
| Nov-I            | 23,000,000 | 45,751,010    | 7,776,000     | 7,776,000                   | 194,400      | 1,135        | 23,000,000   | 37,779,475      | 0           |               |
| Nov-II           | 23,000,000 | 32,132,923    | 6,272,640     | 6,272,640                   | 194,400      | 975          | 23,000,000   | 25,664,908      | 0           |               |
| Dec-I            | 23,000,000 | 4,067,465     | 4,147,200     | 4,147,200                   | 194,400      | 827          | 23,000,000   | 38,068,473      | 0           |               |
| Jan-I            | 19,504,848 | 11,407,834    | 6,946,560     | 6,946,560                   | 194,400      | 747          | 23,000,000   | 770,975         | 0           |               |
| Jan-II           | 23,000,000 | 10,836,302    | 6,746,112     | 6,746,112                   | 207,360      | 926          | 23,000,000   | 3,881,904       | 0           |               |
| Feb-I            | 23,000,000 | 14,267,644    | 6,193,152     | 6,193,152                   | 194,400      | 853          | 23,000,000   | 7,892,234       | 0           |               |
| Feb-II           | 23,000,000 | 2,805,904     | 0             | 0                           | 194,400      | 747          | 23,000,000   | 2,623,572       | 0           |               |
| Mar-I            | 23,000,000 | 3,832,640     | 0             | 0                           | 194,400      | 816          | 23,000,000   | 3,637,424       | 0           |               |
| Apr-I            | 23,000,000 | 15,037,302    | 881,280       | 768,531                     | 194,400      | 853          | 23,000,000   | 14,073,518      | 112,749      |
| Apr-II           | 23,000,000 | 2,455,283     | 4,043,520     | 3,526,201                   | 194,400      | 893          | 21,733,789   | 517,319         | 0           |
| May-I            | 21,733,789 | 4,769,280     | 3,159,109     | 2,900,000                   | 194,400      | 787          | 20,299,663   | 610,171         | 0           |
| May-II           | 20,299,663 | 3,879,355     | 4,644,864     | 4,050,611                   | 207,360      | 788          | 19,920,259   | 594,253         | 0           |
| Jun-I            | 19,920,259 | 5,349,401     | 3,369,600     | 2,938,501                   | 194,400      | 671          | 22,136,088   | 431,099         | 0           |
| Jun-II           | 22,136,088 | 2,540,160     | 2,250,716     | 2,250,716                   | 194,400      | 706          | 21,422,066   | 289,444         | 0           |
| Jul-I            | 21,422,066 | 826,330       | 3,307,175     | 3,307,175                   | 194,400      | 732          | 18,746,089   | 425,305         | 0           |
| Jul-II           | 18,746,089 | 2,817,970     | 5,193,490     | 5,193,490                   | 207,360      | 745          | 16,162,464   | 670,886         | 0           |
| Aug-I            | 16,162,464 | 203,529       | 6,687,360     | 5,925,355                   | 194,400      | 721          | 10,245,516   | 762,005         | 0           |
| Aug-II           | 10,245,516 | 203,687       | 6,801,408     | 6,026,408                   | 207,360      | 518          | 4,214,918    | 775,000         | 0           |
| Sep-I            | 4,214,918  | 227,587       | 3,214,080     | 2,847,845                   | 194,400      | 259          | 1,400,000    | 366,235         | 0           |
| Sep-II           | 1,400,000  | 232,143       | 0             | 0                           | 194,400      | 124          | 1,437,619    | 0               | 0           |
| Oct-I            | 1,437,619  | 265,359       | 0             | 0                           | 194,400      | 150          | 1,508,429    | 0               | 0           |
| Oct-II           | 1,508,429  | 190,080       | 0             | 0                           | 207,360      | 161          | 1,490,988    | 0               | 0           |

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