Static Hedging of Freight Risk under Model Uncertainty

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Abstract

Freight rate derivatives constitute a very popular financial tool in shipping industry, that allows to the market participants and the individuals operating in the field, to reassure their financial positions against the risk occurred by the volatility of the freight rates. The special structure of the shipping market attracted the interest of both academics and practitioners, since pricing of the related traded options which are written on non-storable assets (i.e. the freight service) is not a trivial task. Management of freight risk is of major importance to preserve the viability of shipping operations, especially in periods where shocks appear in the world economy, which introduces uncertainty in the freight rate prices. In practice, the reduction of freight risk is almost exclusively performed by constructing hedging portfolios relying on freight rate options. These portfolios needs to be robust to the market uncertainties, i.e. to choose the portfolio which returns will be as less as it gets affected by the market changes. Especially, for time periods where the future states of the market (even in short term) are extremely ambiguous, i.e. there are a number of different scenarios that can occur, it is of great importance for the firms to decide robustly to these uncertainties. In this work, a framework for the robust treatment of model uncertainty in (a) modeling the freight rates dynamics employing the notion of Wasserstein barycenter and (b) in choosing the optimal hedging strategy for freight risk management, is proposed. A carefully designed simulation study in the discussed hedging problem, employing standard modelling approaches in freight rates literature, illustrates the capabilities of the proposed method with very satisfactory results in approximating the optimal strategy even in high noise cases.

Keywords: freight rate derivatives; freight risk; model uncertainty; static hedging; Wasserstein barycenter;

1 Introduction

Seaborn trade has been developed rapidly the last fifty years since the majority of international trade transportations and supply chain activities are performed almost exclusively through the shipping industry. Valuable and necessary commodities for the growth and functionality of the economies like crude oil, oil products, iron ore, coal grain and others, are primarily chartered between ports through cargo ships using the international sea trade routes. Alongside with the increasing activity in the shipping industry concerning worldwide transportations, it came up the need for all parties involved, to constitute a separate market equipped with financial products that will allow them to perform their operations at acceptable levels of financial risk [Alizadeh & Nomikos, 2009]. The most important source of risk that appears in shipping operations, is the so called freight risk, which concerns the volatility in the freight rate prices, on which mostly depends the cost of freight services. Freight rates could significantly differ across various sea trade routes and time periods and induce significant operational hazards that

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need to be taken into account by shipping firms or firms that use freight services in order to remain viable.

The involved parties in seaborne trade operations (typically the shipowners and the charterers) always seek for hedging strategies that will allow them to control the induced freight risks. The latter, resulted in the constitution of the Forward contract Freight Agreement (FFA) market, which acts in parallel to the physical market (i.e. the physical world where contracts are conducted according to the spot freight rates), allowing for the trade of financial derivatives able to reduce the exposure to freight risk. Being a relatively new market, and due to the special nature of the freight service as a commodity (non-storable like electricity), the completeness assumption may not always hold (Adland, Anestad, & Abrahamsen, 2021), since the deals are typically conducted over the counter (OTC) through brokers agencies (e.g. Forward Freight Agreement Brokers’ Association (FFABA)). FFA market is characterized by very high volatilities, similar to those observed in the electricity markets, and the freight rate prices strongly depend on the demand for freight services. Despite the aforementioned inconsistencies and singularities, FFA market traded products are the most preferable (and appropriate) hedging instruments for the operating parties in the shipping sector, being by construction strongly related to the physical market’s assets (spot freight rates). Although FFA market offers the financial mechanism for managing freight risk, the uncertainty that characterizes several aspects of the freight risk management task needs to be carefully assessed and treated.

Forecasting of the freight rate prices is a well studied problem indicating several directions in successfully predicting and modelling different phenomena displayed by the freight rates when the stationarity assumption holds (see e.g. Batchelor, Alizadeh, & Visvikis, 2007; Chen, Meersman, & Voord, 2012; Katris & Kavussanos, 2021; Munim & Schramm, 2021). However, there is not just one model that is universally accepted for modeling the random behaviour of the freight rates in general, since different characteristics may be displayed depending on the market condition. In the literature various approaches have been examined so far, capturing various characteristics displayed occasionally by the freight rates processes, e.g. mean-reversion effects (Benth, Koekbakker, & Tai, 2015), seasonality patterns (Kavussanos & Alizadeh-M, 2001), rapid changes in prices that could be interpreted as jumps (Kyriakou, Pouliasis, Papapostolou, & Andriou, 2017), etc. In periods where the market is not in a stable state, there is a growing ambiguity on which is the most plausible model to describe the situation. Information collected by market agents may be used to derive estimates for the forthcoming situation, however depending on the states of the market and other factors, the received information could be quite inhomogeneous. Under such circumstances, the major priority of the risk manager in charge is to properly discount the market information input and derive financial decisions/strategies that will remain robust to the scenario that will actually occur.

Robustness property in modern decision making was formally introduced in a quantitatively framework a decade ago in (L. Hansen & Sargent, 2001; L. P. Hansen & Sargent, 2011; Maccheroni, Marinacci, & Rustichini, 2006). Clearly, the meaning of remaining robust to uncertainty when making a financial decision, refers to the low degree of sensitivity of the deduced optimal decision with respect to the scenario that materializes. Under the framework of model uncertainty, the robustness of the decision making process is twofold and refers to (a) the optimal action/decision/strategy to be chosen, and (b) aggregation/combinations of the provided information (models) on which strongly depends the optimal strategy to be selected.

In this paper, a framework that handles the model uncertainty issues that appear in shipping markets is proposed, bringing to the first place the appropriate design of effective and robust to model uncertainty hedging schemes for managing freight risk. The discussed approach is based on the aggregation of various information sources, identified by certain probability models, through barycentric approaches that have been successfully introduced in decision theory (Petracou, Xepapadeas, & Yannacopoulos, 2022) and in financial risk management (G. Papayiannis & Yannacopoulos, 2018). The proposed framework is directly implemented in
designing the decision rule for selecting the optimal (static) hedging strategy, that will remain robust to model uncertainty, for reducing the exposure of a position to freight risk. Moreover, a detailed simulation study is performed, employing standard models that are used in describing freight rate dynamics, to illustrate the capabilities of the proposed methodology and testing its sensitivity with respect to different levels of information homogeneity.

2 The Framework of Freight Derivatives and Model Uncertainty

2.1 Freight Derivatives Market and Freight Options

In 1985, the Baltic Exchange\(^1\), supported by shipping market brokers, has undertaken a central role in constructing independently prices for the freight service of the ocean-going cargo carrying vessels, the so called freight rates, that could be used as a base to be written futures and option contracts. This was succeeded by collecting information from a group of designated brokers, referred to as the panelists, who are the middle persons between charterers and shipowners being aware of the prevailing market freight rates in individual routes. The Baltic Exchange started collecting these freight rates, calculated weighted averages of them and reported them in the form of indices for freight services. These indices are published on a daily basis, for each sector of the shipping industry, i.e. dry-bulk, tanker and container sector, for the major trade routes that the shipping operations take place. Over the years, more appropriate indices are constructed and revised for the better and more accurate monitoring of the freight rate prices. For instance, in the dry-bulk sector, the Baltic Exchange Dry Bulk Index (BDI) was launched in 1985, the Baltic Panamax Index (BPI) was launched in 1998, the Baltic Capesize Index (BCI) appeared in 1999 and was revised in May 2014 (BCI 2014) and the Baltic Supramax Index (BSI) was launched in 2005 (for more details on the subject please see the excellent references [Alizadeh & Nomikos, 2009] and [Kavussanos, Tsouknidis, & Visvikis, 2021]). The individual dry-bulk and tanker routes or baskets of routes of the Baltic Exchange Freight Indices serve as the underlying assets of freight derivative contracts.

Forward freight agreements (FFAs) introduced in early 90’s as over-the-counter (OTC) derivatives contracts. Typically, such agreements constitute private contracts between a seller and a buyer for settling a freight rate, for a specified cargo or type of vessel, for either one or a combination of the major trade routes (Alexandridis, Kavussanos, Kim, Tsouknidis, & Visvikis, 2018). Freight rate options are path-dependent contingent claims, and in particular, Asian type derivatives written on the spot freight rates which are non-traded in the market, depending on a number of predefined dates through their arithmetic average, to avoid market manipulation phenomena especially close to the maturity of the options. An FFA is a cash-settled financial contract that provides to the owner of the contract the difference between the average of the spot freight rate prices \(S(t)\) at a number of predefined time instants (dates) \(T_1, T_2, ..., T_N\) and the future price \(F(t, T_1, T_N)\) multiplied by a factor \(D\) (cargo size in tonnes or number of days for the charter). The value of an FFA can be calculated by discounting the cash flow received at the maturity time \(T_N\) taking conditional expectation under the pricing measure \(Q\) (however, according to [Adland et al., 2021] there exist statistical arbitrage profits in the freight option markets, suggesting a degree of market inefficiency). Since there is no cost in entering to an FFA, the expectation can be taken equal to zero, i.e.

\[
E_Q \left[ e^{-r(T_N-t)} D \left( \frac{1}{N} \sum_{i=1}^{N} S(T_i) - F(t, T_1, T_N) \right) \right] | \mathcal{F}_t ] = 0
\]

where \(r > 0\) denotes the risk-free interest rate and \(\mathcal{F}_t\) the information for the spot prices up to

\(^1\)https://balticexchange.com/en/index.html
time $t$. Solving the above with respect to $F(t,T_1,T_N)$ we obtain the representation

$$F(t,T_1,T_N) = \frac{1}{N} \sum_{i=1}^{N} E_Q[S(T_i)|\mathcal{F}_t]$$

where $Q$ denotes the equivalent martingale measure satisfying the market completeness assumption. The latter relation is useful for determining the dynamics of the $F(t,T_1,T_N)$ by specifying the stochastic model which describes the spot freight rates dynamics.

An Asian option of this type can be interpreted as a Europe option on the forward contract value $F(t,T_1,T_N)$. For any strike price $K$ and maturity time $T = T_N$, we obtain either a call or a put option. A call option on the FFA contract referred to as a caplet, provides freight rate protection for the buyer (typically a charterer) above a predetermined level (the cap rate) and its price is calculated by the formula

$$C(t,T) = e^{-r(T-t)} D E_Q \left[ (F(T_N,T_1,T) - K)^+ | \mathcal{F}_t \right].$$

A put option on the FFA contract referred to as a floorlet, guarantees downside protection on the freight rates for the buyer (typically the shipowner) at a predetermined level (the floor rate) and its value is derived by the formula

$$P(t,T_N) = e^{-r(T_N-t)} D E_Q \left[ (K - F(T_N,T_1,T_N))^+ | \mathcal{F}_t \right].$$

Unfortunately, closed-form pricing formulas for these options cannot be in general derived, even in the typical setting of a geometric Brownian motion model for the spot dynamics, due to the arithmetic average term. However, in [Koekebakker, Adland, & Sødal, 2007] an analytic expression is derived under some assumptions leading to a lognormal approximation.

### 2.2 Modelling freight rate dynamics

Several approaches have been proposed in the literature so far for modelling commodities and in particular freight rate prices. Here we briefly discuss some well celebrated models that are employed in practice. The first model appeared in the literature for modelling commodities prices is the famous Black’s model (Black, 1976). According to this approach, the spot freight-rate prices $S(t)$ are described by the classical geometric Brownian motion (GBM) model

$$dS(t) = \mu S(t) dt + \sigma S(t) dW(t)$$

where $\mu, \sigma$ denote the drift and the volatility coefficients while $W(t)$ is the standard Brownian motion process. In this perspective, the price of the modelled commodity (in our case the rate for the freight service) is treated as any other risky asset in the market. Although for certain time periods this model could be adequate to describe the evolution of the freight rate prices, it cannot efficiently capture typical phenomena that occur in this high volatile market, e.g. rapid changes, seasonality trends, etc. A second attempt in modelling commodity prices appeared in [Schwartz, 1997], where the Ornstein-Uhlenbeck model (OU) was employed to capture mean-reversion effects. In this case, freight rate prices are described by the model

$$dS(t) = \alpha (\mu - S(t)) dt + \sigma dW(t)$$

where $\mu$ denotes the long-term average of the process, $\alpha > 0$ denotes the mean-reversion rate towards the mean level $\mu$, $\sigma$ being the volatility parameter and $W(t)$ the typical Brownian motion. This approach has been proved very successful in capturing the typical mean-reversion behavior that appears in many commodities. A more flexible two-factor model framework has also appeared in the literature (both GBM and OU model are cases of one-factor models) where the freight rate prices (or the log-prices) are modelled by the sum of two separate processes with
possibly different characteristics, e.g. a GBM model and a OU model, or combinations of them or other stochastic models (see e.g. [Prokopczuk, 2011] and references therein). Even more interesting approaches have been examined the latter years where jump diffusion processes are employed to capture the phenomena of rapid changes in freight rate prices. Such models were investigated either using as a basis GBM models or OU models, enriched with jump terms, providing a very flexible framework in capturing different characteristics of the market. Some recent attempts in this direction with applications in pricing freight rate options can be found in [Gómez-Valle & Martínez-Rodríguez, 2021] [Kyriakou et al., 2017].

2.3 The issue of model uncertainty in freight markets

FFA market presents high and many times extreme volatilities in freight rates since prices are directly affected by the demand and supply levels for freight services that are observed in the market (similarly to the behaviour of electricity markets). Analysis of historical data establishes the effect of seasonality in the freight rates which this effect can be captured by employing GARCH-type models [Kavussanos & Alizadeh-M, 2001]. However, there are several characteristics that are occasionally displayed by the freight rate dynamics for which there is not a single universally acceptable model that can capture them. For example, a model with mean-reversion terms should generally be acceptable for the description of the freight service prices. However, depending on the world economy situation, it could appear large periods where the mean reversion behaviour is not observed at all and employing such a term would be misleading and disastrous for financial decisions. A recent example is the world economic crisis in 2008, where all markets, including the shipping market, were on the downside for a very large time period [Samitas, Tsakalos, et al., 2010]. More recently, the COVID-19 pandemics significantly affected the market operations [Gavalas, Syriopoulos, & Tsatsaronis, 2022; Kamal, Chowdhury, & Hosain, 2021; Notteboom, Pallis, & Rodrigue, 2021]. Such events seriously disturb the economic activity at all levels and introduce long-term effects in the market which may enter different states for a large period of time with much different patterns than the usual ones to be observed [Ishizaka, Tezuka, & Ishii, 2018]. Under such circumstances, due to the high level of ambiguity, the operational risk is highly increased, and it is not clear which is the most appropriate model to be used for discounting the decisions. As part of the confusion that governs the market, incoming information from market sources concerning the forthcoming situation may be quite divergent leading to the classical model uncertainty setting, where there is a number of provided models (predictions) however it is not clear which one should be trusted. Obviously, models that have been calibrated using historical data when market was in a steady state, are practically useless to discount decisions when the market is in shock. Even in case where certainty for the occurrence of specific phenomena in the near future that will affect the price dynamics exists (e.g. certain drift effects, rapid fluctuations like downward or upward price shocks), there is ambiguity concerning their intensity. For instance, consider the case that upward jumps are expected to the freight rates due to a rapid increase in the demand of freight service at a certain trade route [Nomikos & Tsouknidis, 2022]. In such a case, the magnitude of the jump in the freight service prices cannot be precisely estimated and there may be several scenarios on that, depending on the evolution of the demand for the freight service in the route.

All the aforementioned cases need a special treatment during the decision making process. Practically, a shipping firm that desires to hedge the induced freight risk from its shipping operations under such conditions, faces the problem of model uncertainty, i.e. several scenarios could happen in the near future but it is extremely difficult, if not impossible, to distinguish which scenario will actually occur. In such a case, the various possible realizations (scenarios) must be handled in a robust manner, to reduce the sensitivity of the hedging decision that is chosen with respect to the scenario that actually materializes. A framework that is appropriate for handling model uncertainty in problems of this type was recently proposed in
and (Petracou et al., 2022) under the perspectives of financial risk quantification and group decision making, respectively. In both approaches, various scenarios or opinions are identified by certain probability models, and the notion of the Wasserstein barycenter (roughly the sense of median in the space of probability models) is employed to estimate an aggregate model that robustly represents the received information and which is then used to derive a robust decision rule. In what follows, these ideas are carefully employed under the context of hedging freight risk under model ambiguity.

3 Static hedging of freight risk under model uncertainty

There is an increasing trend among academics and industry practitioners towards abandoning dynamic hedging and turning attention more and more on static hedging which is easier and cheaper to implement (see e.g. (Carr, Ellis, & Gupta, 1998), (Carr & Wu, 2014), (Leung & Lorig, 2016)) and in many cases have been proved a better option than the commonly used delta hedging (Adland, Ameln, & Børnes, 2020). Static hedging is suggested in hedging of exotic type derivatives that are path-dependent, like the ones used in shipping industry, and in many cases is possibly the only option due to the market nature (e.g. market incompleteness issues like the case of freight rate market which does not allow for dynamic hedging). Static hedging can be either exact or approximate, the latter term referring to the construction of a static portfolio approaching a required target, e.g. the payoff of a financial obligation or a contingent claim to be hedged as close as possible with the concept of closeness being quantified by an appropriate measure of distance. In this section, static hedging strategies are derived under the model uncertainty framework for reducing the freight risk of shipping operations.

3.1 A robust principle for choosing the pricing measure

Consider the case where an owner of a forward freight contract (FFA) desires to construct a hedging strategy for this contract. Since the value of the contract depends on the condition of the market, the final outcome of this deal may significantly vary, depending on the situation in both markets (physical and FFA market). As a result, the contract owner needs to secure her/his position against to market volatilities by appropriately selecting her/his hedging strategy. Due to the discussed limitations in the shipping freight rates market, the static hedging approach seems to be the most prominent choice, especially when the time period to be covered is short. Otherwise, a static hedging approach could be also applied in a multi-stage framework (i.e. approximating delta hedging through multi-stage static hedging). The main issue with freight options is that depend on underlying assets (the spot freight rates) which are non-tradable, therefore many kind of uncertainties arise. For example, following the discussion in Section 2 when the market is not on equilibrium or is perturbed by certain events, it is not straightforward how to choose a plausible model for the spot price and the FFA dynamics. Different choices may arise depending on the situation, e.g. a period with typical market behaviour situation may require a model with mean-reversion effects, a period with higher volatilities may be best described by using a more volatile model like geometric Brownian motion, inside information by various market agents/sources might also be taken into account, etc. Therefore, an appropriate choice of model should be robust to these uncertainties, i.e. a robust model should lead to a decision that whatever scenario is realized, the optimal derived decision should be scenario-insensitive.

Let us introduce a framework to treat model uncertainty through a simple but very appropriate example for the typical situation in the freight market. Assume tha the spot freight rates of a certain cargo vessel (non-traded asset) is denoted by $S(t)$ and a related market index for the freight rates at the specific trade route that the vessel operates is denoted by $\tilde{S}(t)$. Given
that the market risk-free rate is \( r > 0 \), the processes \( S(t), \tilde{S}(t) \) are either assumed to follow (a) a two-dimensional Black’s model (GBM) described by the dynamics

\[
\begin{align*}
    dS(t) &= \mu S(t)dt + \sigma S(t)dW(t) \\
    d\tilde{S}(t) &= \tilde{\mu} \tilde{S}(t)dt + \tilde{\sigma} \tilde{S}(t)d\tilde{W}(t) \\
    \mathbb{E}_Q[dW(t)d\tilde{W}(t)] &= \rho dt
\end{align*}
\]

where \( \mu, \tilde{\mu} \) denote the respective drifts, \( \sigma, \tilde{\sigma} \) the respective volatilities, and \( W(t), \tilde{W}(t) \) the respective correlated Brownian motions (with \( \rho \in (-1, 1) \) denoting the correlation coefficient), or (b) a two-dimensional Schwarz’s model (OU) given by the dynamics

\[
\begin{align*}
    dS(t) &= \alpha (\mu - S(t))dt + \sigma dW(t) \\
    d\tilde{S}(t) &= \tilde{\alpha} (\tilde{\mu} - \tilde{S}(t))dt + \tilde{\sigma} d\tilde{W}(t) \\
    \mathbb{E}_Q[dW(t)d\tilde{W}(t)] &= \rho dt
\end{align*}
\]

where \( \mu, \tilde{\mu} \) denote the respective long-term average rates, \( \alpha, \tilde{\alpha} \) the mean-reversion intensity parameters, \( \sigma, \tilde{\sigma} \) the respective volatilities and \( W(t), \tilde{W}(t) \) as above. Let us denote by \( Q \) the probability measure (model) that describes the joint dynamics of \( S, \tilde{S} \). An investor holds a FFA contract (let us say a charterer) depending on the spot prices \( S(t) \) and wishes to cover her/his exposure through a hedging portfolio depending on the index \( \tilde{S}(t) \) (e.g. a portfolio of derivatives and forward contracts written on \( \tilde{S}(t) \)). Since, the spot freight rates \( S(t) \) is a non-tradable asset, and due to the lack of sufficient statistical data, various uncertainties are introduced as to the exact values of the parameters of the true model. The investor to counter the uncertainty concerning the true (but unknown) model \( Q \), is provided with a collection of \( m \) models \( \Omega = \{ Q_1, Q_2, \ldots, Q_m \} \) by her/his network of market agents/sources. Each model possibly contains different fragments of reality and therefore the investor cannot fully allocate her/his trust only to one of these models. The issue arising here, is how to efficiently aggregate the available information into a single model which will be used to robustly discount the outcomes of the financial decisions to be made.

A robust and efficient approach in combining different beliefs into a single aggregate model has been proposed in \cite{Papaviasannis2018} and \cite{Petracou2022}, where the notion of the Wasserstein barycenter (derived from the generalized mean sense offered by the notion of Fréchet mean \cite{Frechet1948}) is combined within the frameworks of convex risk measures and expected utility to properly discount decisions under model ambiguity. The Wasserstein distance is a proper metric in the space of probability models (see \cite{Santambrogio2015} and \cite{Villani2008}) and between two probability models \( Q_1, Q_2 \in \mathcal{P}(\Omega) \) is computed (according to Kantorovich formulation) as the minimal cost of the problem

\[
W_2^2(Q_1, Q_2) = \min_{\Pi \in \Pi(Q_1, Q_2)} \int_{\Omega \times \Omega} ||x - y||_2^2 d\Pi(x, y)
\]

where \( \Pi(Q_1, Q_2) \) is the set of transport plans with marginals the measures \( Q_1, Q_2 \in \mathcal{P}(\Omega) \). Since Wasserstein distance is fully compatible with the geometry of the space of probability models, constitutes a very effective instrument for the quantification of discrepancies between various opinions which can be indentified by certain probability models. On the examined framework where a prior set with a multitude of models is provided, the notion of Wasserstein barycenter is employed (see e.g. \cite{Agueh2011} for technical discussion), i.e. an appropriate analog of the mean sense in the space of probability models (opinions), to derive a single aggregate model comprising all the available information from the set \( \Omega \). Given a weighting
vector $w \in \Delta^{m-1}$ which can be considered as each model’s contribution to the final aggregate, the Wasserstein barycenter is defined as the minimizer of the respective Fréchet variance

$$Q^*(w) = \arg \min_{Q \in \mathcal{P}(\Omega)} \sum_{i=1}^{m} w_i W_2^2(Q, Q_i).$$

In general, problem (10) does not admit a closed-form solution, however for the case that all models in $\Omega$ are members of a Location-Scatter family (e.g. Gaussian), which is the case in the discussed setting with GBM or OU models, the barycenter of the set $\Omega$ can be characterized through its parameters in a semi-closed form. In particular, the location of $Q^*(w)$ is represented as the weighting average

$$m_B(w) = \sum_{i=1}^{m} w_i m_i$$

with $m_i$, $i = 1, 2, ..., m$ denoting the location parameters of models $Q_1, Q_2, ..., Q_m$, while the dispersion characteristics of $Q^*(w)$ are represented by the covariance matrix $C_B$ which satisfies the matrix equation

$$C_B = \sum_{i=1}^{m} w_i \left( C_i^{1/2} C_B C_i^{1/2} \right)^{1/2}$$

with $C_i$ for $i = 1, 2, ..., m$ denoting the covariance matrices of models $Q_1, Q_2, ..., Q_m$. The matrix $C_B$ can be obtained numerically by the fixed-point scheme proposed in (Alvarez-Esteban, Del Barrio, Cuesta-Albertos, & Matrán, 2016). For the case of non Location-Scatter models, numerical schemes based on the entropic regularization of the main problem to achieve higher convergence rate have been proposed in the literature (see e.g. Carlier, Duval, Peyré, & Schmitzer, 2017; Clason, Lorenz, Mahler, & Wirth, 2021; Cuturi, 2013).

The barycenter stated in (10) serves as the aggregate model of the set $\Omega$ depending on the weighting vector $w$. The latter can be realized as a sensitivity parameter chosen by the investor, determining the level of contribution/influence of each provided model in $\Omega$ to the model that will be actually used to discount the decisions. For instance, if $w_i = 0$ for a certain $i$ is set, then this particular model is omitted as unrealistic or untrustworthy. Similarly, if $w_i = 1$ is chosen, then the investor fully allocates her/his interest to this particular model and omits the rest. Equal weighting, i.e. $w_1 = w_2 = ... = w_m = 1/m$, corresponds to the typical barycenter case where no extra information about the credibility of each model is available. In the case that these models concern different scenarios that could be realized (and they do not represent just different opinions), the choice of the discounting probability measure in this way allows for determining a robust strategy across scenarios avoiding to concentrate to just one of them. In this perspective, $w$ could be understood as the probability (possibly in a subjective view) for each one of the scenarios to occur. Weighting the various scenarios and not concentrating to just one of them, should lead to optimal decisions/strategies which financial output will be less affected by the scenario that actually occurs. This is the essence of robustness, however the choice of weighting vector $w$ and the quality of the information provided by the prior set $\Omega$ (does it contains the “true” scenario?) can act as sensitivity parameters and should be carefully chosen and assessed. However, hybrid schemes combining aversion preferences with data-driven approaches in updating the weighting vector whereas new information batches are available can be developed by applying appropriate scoring rules (see, e.g. G. Papayiannis, Galanis, & Yannacopoulos, 2018; G. I. Papayiannis & Yannacopoulos, 2018).

Clearly, there is always the chance that no model in $\Omega$ is realized or the investor does not fully trust none of these models. In that case, the investor may quantify her/his aversion variationally in the spirit of Hansen & Sargent (2001; Maccheroni et al., 2006) and the deduced

3 denoting the unit simplex, i.e. $\Delta^{m-1} := \{ w \in \mathbb{R}^m : \sum_{i=1}^{m} w_i = 1, \ w_i \geq 0, \ \forall i = 1, 2, ..., m \}$
model would be a distorted version of the barycenter model as computed by the set $Q$ (please see (G. Papayiannis & Yannacopoulos, 2018) and the related results therein). The level of distortion depends on the level of reliability that the investor allocates to the provided information with the special cases corresponding to the barycenter model (fully trust in $Q$) and the most distorted model (no trust in $Q$) which corresponds to the deep uncertainty case. In this work, we focus on the case that the investor is fully confident regarding the plausibility of the provided models and we refer to the other very interesting cases for future work in the subject.

3.2 Optimal static hedging strategies under the model uncertainty framework

Let us state in a mathematical formulation the static hedging problem that the investor faces. Consider that the investor has the obligation (as a charterer) to cover the cost of carrying for a cargo of size $D$ (in tonnes) at a future time $t = T$ which depends on the spot freight rate (per tonne) that will hold at this time in the physical market, $S(T)$. As a result, the obligation that has to be covered by the charterer is

\begin{equation}
\Phi(S) = DS(T)
\end{equation}

or if an FFA contract has been conducted on the spot prices $S$ depending on some dates $T_1, T_2, ..., T_N$, the obligation is expressed through the relation

\begin{equation}
\Phi(S) = D \frac{1}{N} \sum_{i=1}^{N} S(T_i).
\end{equation}

The second type of deals is preferred when both participants desire to avoid market volatility effects and therefore the freight service total price is determined through the arithmetic average of the spot prices at days $T_1, T_2, ..., T_N$. Since the obligator is afraid of an uprising trend of the freight rates, he/she decides to build a hedging strategy to partly finance this forward contract. Due to the fact that the spot prices $S(t)$ are not traded, there is available to her/him a library of hedging instruments with underlying asset the route index $\tilde{S}(t)$, i.e. European type call/put options, FFAs, etc. For simplicity, let us assume that the mapping referring to this library is

\begin{equation}
\Psi(\tilde{S}) = \left( \begin{array}{c} \Psi_1(\tilde{S}) \\ \Psi_2(\tilde{S}) \end{array} \right) = \left( \begin{array}{c} D(K_1 - \tilde{S}(T))^+ \\ D \left(K_2 - \frac{1}{N} \sum_{i=1}^{N} \tilde{S}(T_i) \right)^+ \end{array} \right)
\end{equation}

containing a European call option and a FFA-type call option on the freight rate prices in the related route with strike prices $K_1$ and $K_2$, respectively. The investor builds her/his hedging portfolio with respect to an allocation vector $\theta = (\theta_1, \theta_2)^T \in \mathbb{R}^2$, and the portfolio value at time $t = T$ will be

\begin{equation}
V_H(T; \theta) = \Psi(\tilde{S}, T) \theta = \theta_1 \Psi_1(\tilde{S}, T) + \theta_2 \Psi_2(\tilde{S}, T)
\end{equation}

After assuming such a position, the remaining risk at maturity $T$, using a quadratic loss function, can be represented by

\begin{equation}
L(\theta; S, \tilde{S}) := (\Phi(S) - V_H(T; \theta))^2
\end{equation}

where $\Phi(S)$ is the obligation determined by (13) or (14). The goal of the proposed static hedging mechanism, is to hedge the paying obligation $\Phi$ depending on $S$, with the portfolio $V_H$ returns which is built on a collection of derivatives written on $\tilde{S}$, in order to reduce the freight risk of the position in $\Phi$. The use of the quadratic loss function can be interpreted as a penalization for diverging from the exact hedge position and corresponds to cases where both super- or sub-hedging may lead to losses for the investor. Note that in place of $\Phi$ could be used and other
type of derivatives that are preferred in similar operations, e.g. exchange options like Margrabe option \cite{Margrabe1978} or other type of exotic derivatives.

From relation (17) is evident that the possible outcomes of the loss function depend on the probability model describing the evolution of the processes $S(t)$ and $\tilde{S}(t)$, therefore the problem needs to restated in a stochastic formulation. In particular, the optimal (static) hedging strategy could be determined as the minimizer of the problem

\begin{equation}
\min_{\theta \in \Theta} \mathbb{E}_Q[L(\theta; S, \tilde{S})]
\end{equation}

where $\Theta$ denotes the set of all admissible hedging strategies and $Q$ is the probability law describing the dynamics of $S, \tilde{S}$. However, since there is uncertainty regarding the true model $Q$, and given the multitude of models $Q$ that the investor have been provided with, a robust version of the problem (18) is considered with respect to the probability model that is selected. In particular, the investor has to solve the minimax problem

\begin{equation}
\min_{\theta \in \Theta} \max_{Q \in \mathcal{Q}_\eta} \mathbb{E}_Q[L(\theta; S, \tilde{S})]
\end{equation}

where $\mathcal{Q}_\eta$ denotes the set of plausible probability models according to the aversion preferences of the investor from the set of the provided models $\mathcal{Q}$. The set $\mathcal{Q}_\eta$ can be expressed as (using the Wasserstein metric sense)

\begin{equation}
\mathcal{Q}_\eta := \left\{Q \in \mathcal{P}(\Omega) \mid \sum_{i=1}^{m} w_i W_2^2(Q, Q_i) \leq \eta, \forall w \in \Delta^{m-1} \right\}
\end{equation}

where $\eta > 0$ denotes the sensitivity parameter quantifying the aversion intensity and $w$ the weighting vector for the models in prior set $\mathcal{Q}$. Setting the minimal value $\eta_*$ that can be attained for the Fréchet variance of a model $Q$ from the set $\mathcal{Q}$, is equivalent to employ the Wasserstein barycenter of the set (as defined in (10)) as the discounting measure. Clearly, taking values $\eta > \eta_*$ will lead to deformations of the barycentric model providing even worse estimates for expected loss compared to the ones that can be derived by trusting only the information provided in $\mathcal{Q}$. Problem (19) essentially means that the investor chooses the hedging strategy so as to minimize the maximum expected loss over the set of possible probability laws for $L$, thus making a hedging decision under the worst-case scenario (since the robust version of the problem is expressed as a maximization problem of the expected loss with respect to the admissible set of probability models). This problem can also be realized as a game where nature plays against the decision maker (investor), where the first player (nature) chooses the model that will produce the worst outcomes (losses) for the other side while the second player (investor) seeks to be protected against worst-case outcomes.

Given that the probability law that is chosen as the discounting measure for the financial decision is the barycenter of the set $\mathcal{Q}$ (following the previous discussion), i.e. the model $Q_* := Q_*(w)$ as determined in (10) for a certain choice of $w \in \Delta^{m-1}$, the solution to the robust version of the static hedging problem stated in (19) is obtained in analytic form as a least-square estimate. In particular, the minimizer of the problem

\begin{equation}
\min_{\theta \in \Theta} \mathbb{E}_{Q_*}[L(\theta; S, \tilde{S})]
\end{equation}

is obtained as

\begin{equation}
\theta_* = \mathbb{E}_{Q_*} \left[ (\Psi(\tilde{S})^T \Psi(\tilde{S}))^{-1} \right] \mathbb{E}_{Q_*} \left[ \Psi(\tilde{S})^T \Phi(S) \right]
\end{equation}

which is a robust solution to the typical quadratic hedging problem. Note that the calculation of the robust solution stated in (22) relies on our capability to produce samples from the pricing
measure $Q_\ast$. Therefore it is a crucial step in the whole procedure the characterization of $Q_\ast$ or at least to have the ability to simulate samples from this model. For the case discussed in this paper (please see Section 4) this task is straight-forward. However, for more complex models one has to be aware of the computational complexity that may face in obtaining $Q_\ast$. Possibly, in some cases, it would be a more profitable strategy to treat directly the minimax problem stated in (19) in terms of required computational time.

Depending on the available hedging instruments in $\Psi(\tilde{S})$, problem (21) might need to be enhanced with some regularization terms with respect to the hedging strategy $\theta$ to simultaneously perform selection of the hedging instruments that are more effective (see e.g. LASSO type regularization schemes (Hastie, Tibshirani, & Friedman, 2009)). For example, library $\Psi(\tilde{S})$ may contain both put and call options on various maturities, so employing penalties like LASSO, will provide hedging schemes that will completely omit budget allocation to financial instruments that cannot have a significant impact in reducing risk. In this direction, the penalized version of the problem might be re-written as

$$\min_{\theta \in \Theta} \left\{ E_{Q_\ast} [L(\theta; S, \tilde{S})] + R(\theta) \right\}$$

where $R(\theta)$ denotes the employed regularization term (e.g. for LASSO regularization term we have $R(\theta) = \lambda \|\theta\|_1$ with $\lambda > 0$ denoting the sensitivity parameter of the regularization).

4 Application in choosing robust hedging strategies for freight risk reduction

In this section, simulation experiments based on the models discussed in Section 3 are performed to illustrate the robustness properties of the proposed method in choosing hedging strategies to eliminate freight risk. A number of synthetic examples are created using the GBM model, the OU model and a combination of them, as standard modeling approaches for describing the underlying assets dynamics (the spot freight rates and the FFA rates), while information sets of varying homogeneity are considered to test the sensitivity of the method to the information heterogeneity.

4.1 Determination of the distribution for the aggregate model

A standard step before the selection of the optimal hedging strategy, is the determination of the (aggregate) model under which the optimal strategy (hedging ratios) will be derived. Following the discussion in Section 3 this model is obtained by combining the information provided through the prior set of models $Q$ and weighting them, according to the trust that the risk manager allocates to each information source. Setting these preferences through the determination of the weighting vector $w \in \Delta^{m-1}$, the aggregate model $Q_\ast$ is obtained through the solution of the problem stated in (10). Attempting to provide some examples where closed-form solutions or at least semi-analytic expressions can be derived, we constrain ourselves to work with probability models that are members of some Location-Scatter family for which the aggregate model can be explicitly characterized by applying the equations (11) and (12). As a result, the considered prior sets of models for the freight rates related to the hedging problem (18), are of the same type but with different parameters. The model settings that are considered are briefly presented below.
Case I: Black’s model (GBM)

As a first example we assume the typical GBM model for both processes (spot freight rates and FFA rates) with correlated Brownian motions described by the dynamics:

\[
\begin{aligned}
    dS(t) &= \mu S(t)dt + \sigma S(t)dW(t) \quad \text{(spot dynamics)} \\
    d\tilde{S}(t) &= \tilde{\mu}\tilde{S}(t)dt + \tilde{\sigma}\tilde{S}(t)d\tilde{W}(t) \quad \text{(FFA-rate dynamics)} \\
    \mathbb{E}[dW(t)d\tilde{W}(t)] &= \rho dt
\end{aligned}
\]

(24)

Employing the logarithmic prices in both cases, the bivariate random variable

\[ X(t) := (X_1(t), X_2(t))^\prime = (\log(S(t)), \log(\tilde{S}(t)))^\prime \]

is normally distributed for fixed \( t \). In particular, for a fixed time instant \( t \) and given the information collected up to time instant \( s < t, F_s \), the random variable \( X(t) \) is distributed according to \( N(\mathbf{m}_{t|s}, C_{t|s}) \) where the location parameter vector is

\[
\mathbf{m}_{t|s} := \left( \begin{array}{c} m_{t|s} \\ \tilde{m}_{t|s} \end{array} \right) = \left( \begin{array}{c} \mathbb{E}[\log(S(t))|F_s] \\ \mathbb{E}[\log(\tilde{S}(t))|F_s] \end{array} \right) = \left( \begin{array}{c} \log(S(s)) + \left( \mu - \frac{\sigma^2}{2} \right)(t-s) \\ \log(\tilde{S}(s)) + \left( \tilde{\mu} - \frac{\tilde{\sigma}^2}{2} \right)(t-s) \end{array} \right)
\]

(25)

and the dispersion matrix is defined as

\[
C_{t|s} := \left( \begin{array}{cc} v_{t|s} & \gamma_{t|s} \\ \gamma_{t|s} & \tilde{v}_{t|s} \end{array} \right) = \left( \begin{array}{cc} \sigma^2(t-s) & \rho \sigma \tilde{\sigma}(t-s) \\ \rho \sigma \tilde{\sigma}(t-s) & \tilde{\sigma}^2(t-s) \end{array} \right)
\]

(26)

with \( v_{t|s} := \text{Var}(\log(S(t))|F_s) \), \( \tilde{v}_{t|s} := \text{Var}(\log(\tilde{S}(t))|F_s) \) and \( \gamma_{t|s} := \text{Cov}(\log(S(t)), \log(\tilde{S}(t))|F_s) \).

Case II: Schwarz’s model (OU)

As a second interesting case, the OU model is assumed for both processes (spot freight rates and FFA rates) with correlated Brownian motions described by the dynamics:

\[
\begin{aligned}
    dS(t) &= \alpha(\mu - S(t))dt + \sigma dW(t) \quad \text{(spot dynamics)} \\
    d\tilde{S}(t) &= \tilde{\alpha}(\tilde{\mu} - \tilde{S}(t))dt + \tilde{\sigma}d\tilde{W}(t) \quad \text{(FFA-rate dynamics)} \\
    \mathbb{E}[dW(t)d\tilde{W}(t)] &= \rho dt
\end{aligned}
\]

(27)

In this case, there is no need to perform some transformation to obtain an elliptical probability model. In fact, the bivariate random variable

\[ X(t) := (X_1(t), X_2(t))^\prime = (S(t), \tilde{S}(t))^\prime \]

is distributed for fixed \( t \) according to the Gaussian \( N(\mathbf{m}_{t|s}, C_{t|s}) \) where the location parameter vector is determined by

\[
\mathbf{m}_{t|s} := \left( \begin{array}{c} m_{t|s} \\ \tilde{m}_{t|s} \end{array} \right) = \left( \begin{array}{c} \mathbb{E}[S(t)|F_s] \\ \mathbb{E}[\tilde{S}(t)|F_s] \end{array} \right) = \left( \begin{array}{c} e^{-\alpha(t-s)}S(s) + \mu (1 - e^{-\alpha(t-s)}) \\ e^{-\tilde{\alpha}(t-s)}\tilde{S}(s) + \tilde{\mu} (1 - e^{-\tilde{\alpha}(t-s)}) \end{array} \right)
\]

(28)

and the dispersion matrix is

\[
C_{t|s} := \left( \begin{array}{c} \frac{\sigma^2}{2\alpha}(1 - e^{-2\alpha(t-s)}) \\ \frac{\rho \sigma \tilde{\sigma}}{2\alpha} \sqrt{\frac{[1-e^{-2\alpha(t-s)}]/[1-e^{-2\tilde{\alpha}(t-s)}]^{\alpha/\alpha}}{\text{Var}(\tilde{S}(t), \tilde{S}(t))} \frac{\rho \sigma \tilde{\sigma}}{2\alpha} (1 - e^{-2\tilde{\alpha}(t-s)}) \end{array} \right)
\]

(29)

with \( v_{t|s} := \text{Var}(S(t)|F_s) \), \( \tilde{v}_{t|s} := \text{Var}(\tilde{S}(t)|F_s) \) and \( \gamma_{t|s} := \text{Cov}(S(t), \tilde{S}(t)|F_s) \).
Case III: Mixed Black-Schwarz model (GBM-OU)

A mixed model version is considered as a third illustrative example, where the spot freight rates are assumed to follow a GBM model, and the FFA-rates are assumed to follow the OU model with their stochastic dynamics described by

\[
\begin{align*}
    dS(t) &= \mu S(t)dt + \sigma S(t)dW(t) \quad \text{(spot dynamics)} \\
    dS(t) &= \bar{\alpha}(\bar{\mu} - \tilde{S}(t))dt + \tilde{\sigma}d\tilde{W}(t) \quad \text{(FFA-rate dynamics)} \\
    \mathbb{E}[dW(t)d\tilde{W}(t)] &= \rho dt
\end{align*}
\]

For convenience and in order to be able to characterize the probability model through a Location-Scatter family member, let us consider the random variable \(X(t) = (X_1(t), X_2(t))' = (\log(S(t)), \tilde{S}(t))'\) for any fixed \(t\). In this case, the bivariate random variable \(X(t)\) is normally distributed with location determined by the vector

\[
\begin{pmatrix}
    m_{t|s} \\
    \tilde{m}_{t|s}
\end{pmatrix} := \begin{pmatrix}
    \mathbb{E}[^{\log(S(t))}|F_s] \\
    \mathbb{E}[^{\tilde{S}(t)}|F_s]
\end{pmatrix} = \begin{pmatrix}
    \log(S(s)) + \left(\mu - \frac{\sigma^2}{2}\right)(t - s) \\
    e^{-\bar{\alpha}(t-s)}\tilde{S}(s) + \bar{\mu} (1 - e^{-\bar{\alpha}(t-s)})
\end{pmatrix}
\]

while the respective dispersion matrix is determined as

\[
\begin{pmatrix}
    C_{t|s} \\
    \tilde{C}_{t|s}
\end{pmatrix} = \begin{pmatrix}
    \sigma^2(t - s) & \rho\sigma\tilde{\sigma}\sqrt{(t-s)\frac{1-e^{-2\bar{\alpha}(t-s)}}{2\bar{\alpha}}} \\
    \rho\sigma\tilde{\sigma}\sqrt{(t-s)\frac{1-e^{-2\bar{\alpha}(t-s)}}{2\bar{\alpha}}} & \frac{\sigma^2}{2\bar{\alpha}}(1 - e^{-2\bar{\alpha}(t-s)})
\end{pmatrix}
\]

where \(v_{t|s} := \text{Var}(\log(S(t)|F_s)), \tilde{v}_{t|s} := \text{Var}(\tilde{S}(t)|F_s)\) and \(\gamma_{t|s} := \text{Cov}(\log(S(t)), \tilde{S}(t)|F_s)\).

In all the above cases, the aggregate model \(Q_s\) is characterized in semi-closed form by applying the equations (13)–(12). The obtained model is then used for the derivation of the optimal (static) hedging strategy, i.e. for the calculation of the hedging ratios displayed in (22) which are expressed in terms of expectations with respect to the aggregate probability model \(Q_s\).

4.2 Design of the simulation study and numerical results

For the three different model settings, numerical experiments are conducted under prior sets of varying homogeneity to determine the best hedging strategy for covering the open position of the problem stated in (13). To be able to perform a comparison, we assume that a true model exists for the bivariate random variable \((S, \tilde{S})\), which is known to the experimenter but unknown to the risk manager. The risk manager, on account of incomplete information, has been provided with a number of plausible models describing the evolution of the processes \((S, \tilde{S})\), in certain aspects may be diverging, comprising the prior set \(\Omega\). The risk manager wishes to static hedge her/his future obligations that have to be covered at time \(T = T_N\) following the robust static hedging approach to counter the uncertainty concerning the exact financial amount that has to be covered at time \(T\).

For assessing the performance of the method, the obtained hedging strategy is compared to the optimal hedging strategy under the true model (if the latter was known). To check for the robustness of the findings, a number of experiments are performed for each one of the three models discussed in Section 4.1. These experiments are divided in three groups depending on the homogeneity levels of the models that the investor are provided with. We will refer to these three groups as three different scenarios of information homogeneity: the scenario of (a) High Homogeneity (HH), (b) Medium Homogeneity (MH) and (c) Low Homogeneity (LH), determined by the degree of divergence of the models in \(\Omega\). For each experiment, \(m = 10\) different models that constitute the prior set \(\Omega\) are generated as perturbations of the true
model (by generating perturbations on the true model’s parameters). Specifically, the various scenarios for the prior set $\Omega$ are generated by choosing random values for the model parameters by adding a noise term $\xi$ simulated from an appropriate Uniform distribution $U(\alpha, \beta)$ where $\alpha, \beta$ are chosen according to the parameter scaling and the homogeneity scenario that is considered.

\[
\text{Model} \mu \tilde{\mu} \alpha \tilde{\alpha} \sigma \tilde{\sigma} \rho
\]

| Model                  | $\mu$ | $\tilde{\mu}$ | $\alpha$ | $\tilde{\alpha}$ | $\sigma$ | $\tilde{\sigma}$ | $\rho$ |
|------------------------|-------|---------------|----------|-------------------|---------|-----------------|-------|
| Black’s (GBM)          | 0.03  | 0.01          | -        | -                 | 0.55    | 0.40            | 0.75  |
| Schwarz’s (OU)         | 60    | 60            | 0.25     | 0.40              | 15      | 10              | 0.75  |
| Black-Schwarz (GBM-OU)| 0.03  | 60            | -0.60    | 0.55              | 8       | 0.75            |

Table 1: Model parameters used for the true models in the three experiments

| Black’s model          | Investing horizon | 3 months | 6 months | 9 months | 12 months |
|------------------------|-------------------|----------|----------|----------|-----------|
| $\theta_1$ (%)         | 0.00              | 0.00     | 0.00     | 0.00     |
| $\theta_2$ (%)         | 88.72             | 83.36    | 75.62    | 72.45    |
| Expected Profit (%)    | 15.48             | 14.47    | 12.86    | 11.82    |

| Schwarz’s model        | Investing horizon | 3 months | 6 months | 9 months | 12 months |
|------------------------|-------------------|----------|----------|----------|-----------|
| $\theta_1$ (%)         | 84.69             | 62.62    | 44.16    | 40.09    |
| $\theta_2$ (%)         | 15.31             | 37.38    | 55.84    | 59.91    |
| Expected Profit (%)    | 16.68             | 15.59    | 16.62    |

| Black-Schwarz model    | Investing horizon | 3 months | 6 months | 9 months | 12 months |
|------------------------|-------------------|----------|----------|----------|-----------|
| $\theta_1$ (%)         | 100.00            | 73.94    | 50.90    | 41.58    |
| $\theta_2$ (%)         | 0.00              | 26.06    | 49.10    | 50.42    |
| Expected Profit (%)    | 27.90             | 35.42    | 43.02    | 50.79    |

Table 2: True model optimal static hedging ratios for each model case and all investing horizons considered

As a specific example we consider the hedging problem stated in (19), with initial prices for the spot freight rate $S(0) = 50$ (in USD) and the FFA freight rate $\tilde{S}(0) = 45$ (in USD), while the strike prices concerning the options $\Psi_1$ and $\Psi_2$ are selected as $K_1 = \tilde{S}(0) \cdot 120\%$ and $K_2 = \tilde{S}(0) \cdot 110\%$, respectively. Four different investing horizons are considered for all models and homogeneity levels of information: 3, 6, 9 and 12 months (in trading days). Simulations of model paths are performed using the typical step $dt = 1/252$, corresponding to the daily trading frequency. The true model parameters for each considered case are illustrated in Table 1 and the optimal strategies obtained per case under the true model are displayed in Table 2. For all experiments $B = 100000$ paths are simulated using the distributions characterizations in Section 4.1. The obtained results are compared to the optimal decision obtained by the true model. The performance of the method is assessed through the typical statistical error indices: Mean Absolute Error (MAE) and Root of Mean Square Error (RMSE), calculated with respect to the optimal results obtained by the true model. Expected profit is expressed as the percentage of the amount (per unit of charge) that should be covered for the freight service if the obligation should be paid at time $t = 0$.

In Table 2 is shown that the three model settings provide quite different optimal hedging ratios, with respect to the hedging instruments considered, along the various investing horizons. In Tables 3, 4 and 5 are displayed the results obtained for each model case under all homogeneity scenarios and for all investing horizons considered. Comparing the results to the optimal solutions obtained under the true model, it seems that the applied method displays a remarkable performance in retrieving the true hedging ratios and the expected profits, even in the case of low homogeneity in the provided information set. In all three model settings considered (GBM, OU and GBM-OU), the error indices seems to be at the same levels and not to be seriously affected by the various information homogeneity levels indicating a robust to information heterogeneity behaviour of the method. When more distant investing horizons...
barycenter. A number of numerical experiments employing standard models used in freight risk characterization of the pricing measure in semi-closed form using the notion of Wasserstein which optimal hedging strategies are derived. A key intermediate step in the procedure is the was studied. A framework for treating robustly the model ambiguity has been proposed, under In this paper, the problem of hedging freight risk in shipping operations under model uncertainty are used, the error indices seems to naturally increase due to the growing uncertainty of the long-term states. However, the error rates remain almost at the same levels for all information homogeneity scenarios. In general, the proposed method provides in most of the cases, estimates very close to the ones obtained by the true models, displaying quite a robust behaviour to the various information heterogeneity levels.

### 5 Conclusions

In this paper, the problem of hedging freight risk in shipping operations under model uncertainty was studied. A framework for treating robustly the model ambiguity has been proposed, under which optimal hedging strategies are derived. A key intermediate step in the procedure is the characterization of the pricing measure in semi-closed form using the notion of Wasserstein barycenter. A number of numerical experiments employing standard models used in freight risk
Table 5: Static hedging performance results from 100,000 simulations employing mixed Black-Schwarz model (GBM-OU) for varying homogeneity levels (high, medium and low) in the set of priors $\Omega$.

management practice were performed. The results justify the ability of the proposed method in choosing effective (static) hedging strategies while treating robustly the various levels of information heterogeneity.

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