Quantum critical phenomena of excitonic insulating transition in two dimensions

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We study the quantum criticality of the phase transition between Dirac semimetal and excitonic insulator in two dimensions. Even though the system has a semimetallic ground state, there are observable effects of excitonic pairing at finite temperatures and/or finite energies, provided that the system is in proximity to excitonic insulating transition. To determine the quantum critical behavior, we consider three potentially important interactions, including the Yukawa coupling between Dirac fermions and excitonic order parameter fluctuation, the long-range Coulomb interaction, and the disorder scattering. We employ the renormalization group technique to study how these interactions affect quantum criticality and also how they influence each other. We first investigate the Yukawa coupling in the clean limit, and show that it gives rise to typical non-Fermi liquid behavior. Adding random scalar potential to the system always turns such a non-Fermi liquid into a compressible diffusive metal. In comparison, the non-Fermi liquid behavior is further enhanced by random vector potential, but is nearly unaffected by random mass. Incorporating the Coulomb interaction may change the results qualitatively. In particular, the non-Fermi liquid state is protected by the Coulomb interaction for weak random scalar potential, and it becomes a diffusive metal only when random scalar potential becomes sufficiently strong. When random vector potential or random mass coexists with Yukawa coupling and Coulomb interaction, the system is stable non-Fermi liquid state, with fermion velocities flowing to constants in the former case and being singularly renormalized in the latter case. These quantum critical phenomena can be probed by measuring observable quantities. We also find that, while the fermion velocity anisotropy is not altered by the excitonic quantum fluctuation, it may be driven by the Coulomb interaction to flow to the isotropic limit.

I. INTRODUCTION

In the past decade, the unconventional properties of various Dirac/Weyl semimetal (SM) materials have been investigated extensively. Many of the unconventional properties are related to the existence of isolated Dirac/Weyl points, at which the conduction and valence bands touch. When the chemical potential is tuned to exactly the Dirac points, the fermion density of states (DOS) vanishes at the Fermi level. As a result, the Coulomb interaction is long-ranged due to the absence of static screening. Extensive previous studies have revealed that the Coulomb interaction leads to a variety of unconventional low-energy behaviors.

Among all the known SM materials, two-dimensional Dirac SM, abbreviated as 2D DSM hereafter, has been studied most extensively, usually in the context of graphene. Renormalization group (RG) analysis has revealed that the long-range Coulomb interaction is marginally irrelevant in the weak-coupling regime. When the Coulomb interaction is strong enough, the originally massless fermions can acquire a dynamical mass gap via the formation of stable particle-hole pairs. This gap generating scenario is non-perturbative, and has the same picture as excitonic pairing, a notion proposed decades ago. In the special case of 2D DSM, such an excitonic gap dynamically breaks a continuous chiral (sublattice) symmetry, which can be regarded as a condensed-matter realization of the dynamical chiral symmetry breaking. The finite gap opened at the Dirac point drives the SM to undergo a quantum phase transition (QPT) into an excitonic insulator (EI). The EI is induced only when the effective interaction strength, denoted by $\alpha$, exceeds some critical value $\alpha_c$, which defines SM-EI quantum critical point (QCP).

In recent years, the possibility of SM-EI transition in graphene has been investigated by means of various analytical and numerical techniques. Early calculations predicted that the Coulomb interaction in suspended graphene is strong enough to open an excitonic gap at zero temperature. Specifically, the critical value $\alpha_c$ was claimed to be smaller than the physical value $\alpha = 2.16$. However, no visible experimental evidence for excitonic gap has been observed at low temperatures. More careful numerical calculations revealed that the critical value $\alpha_c$ is actually larger than 2.16, which implies that the Coulomb interaction cannot generate a finite excitonic gap. Owing to the conceptual importance and also the potential technical applications, theorists are still searching for possible approaches to promote excitonic pairing in various SM materials. For instance, it was proposed that excitonic pairing may be promoted by an additional short-range repulsive interaction or by certain extrinsic effects, such as strain.

Most previous works on SM-EI QPT have focused on the precise calculation of $\alpha_c$ at zero temperature ($T = 0$)
by means of various techniques. In this paper, we propose to explore the signatures of excitonic pairing at finite $T$ and/or finite $\omega$. Here is our logic: even though the exact zero-$T$ ground state of suspended graphene (or other 2D DSMs) is gapless, the quantum fluctuation of excitonic pairs still have observable effects at finite $T$ and/or $\omega$ if the system is in the quantum critical regime around the putative SM-EI QCP. Recent Monte Carlo simulations and Dyson-Schwinger equation study both suggest that the value $\alpha_c$ is not far from the physical value of suspended graphene. As illustrated in the schematic phase diagram Fig. 1, if $\alpha$ is slightly smaller than $\alpha_c$, no excitonic gap is opened at $T = 0$ and the excitonic order parameter has a vanishing mean-value. However, the quantum fluctuation of excitonic order parameter is not negligible at finite $T$ and/or $\omega$ and may lead to considerable corrections to observable quantities. For instance, the nuclear-magnetic-resonance measurements performed by Hirata et al. indicate that the compound $\alpha$-(BEDT-TTF)$_2$I$_3$ is close to an SM-EI QCP and that the excitonic fluctuation results in singular corrections to the nuclear magnetic resonance relaxation rate.

We study the quantum critical phenomena emerging in the broad quantum critical regime around SM-EI QCP, with the aim to explore observable effects of excitonic pairing. For this purpose, we take suspended graphene (typical 2D DSM) as our starting model, and calculate the interaction corrections to some observable quantities of Dirac fermions. In this regime, the gapless fermions interact with the quantum critical fluctuation of excitonic order parameter, which is described by a Yukawa coupling term. The long-range Coulomb interaction is still present and needs to be properly taken into account. Moreover, there is always certain amount of quenched disorder in realistic materials, and the fermion-disorder coupling might play a vital role. The actual quantum critical phenomena cannot be accurately determined if one or more of these interactions are naively ignored or improperly treated. We emphasize that, these three kinds of interaction may have very complicated mutual influence. To make a generic analysis, we will treat all the three kinds of interaction on equal footing and study their interplay carefully.

As the first step, we treat the Yukawa coupling in the clean limit, and demonstrate that this coupling leads to strong violation of Fermi liquid (FL) theory. Indeed, the quasiparticle residue $Z_f$ vanishes at low energies, and the fermion DOS $\rho(\omega)$ receives power-law corrections from the excitonic fluctuation. Both of these two features are typical non-Fermi-liquid (NFL) behaviors. If the fermion dispersion is originally anisotropic, the ratio between two fermion velocities is unrenormalized.

The next step is to incorporate quenched disorder and analyze its interplay with the Yukawa coupling. We find that the resultant low-energy properties depend sensitively on the nature of the disorder. Adding random scalar potential (RSP) to the system always turns the NFL caused by Yukawa coupling in the clean limit into a compressible diffusive metal (CDM). The CDM state is characterized by the generation of a finite zero-energy fermion DOS and a constant zero-$T$ disorder scattering rate. Different from RSP, random vector potential (RVP) tends to further enhance the NFL behavior, whereas random mass (RM) has negligible effects on the system.

We finally incorporate the Coulomb interaction, and find that it changes the above results qualitatively. In the case of weak RSP, the Coulomb interaction suppresses disorder scattering and as such renders the stability of the NFL state caused by Yukawa coupling. However, such a NFL is converted into CDM once RSP becomes sufficiently strong. The combination of Yukawa coupling, Coulomb interaction, and RVP produces a stable NFL state in which the two fermion velocities flow to constant values in the zero energy limit. When the Yukawa coupling, Coulomb interaction, and RM are considered simultaneously, we show that the Coulomb interaction is marginally irrelevant and RM is irrelevant. These results indicate that the true quantum critical phenomena are determined by a delicate interplay of excitonic fluctuation, Coulomb interaction, and disorder scattering.

Our results might be applied to understand some 2D DSM materials, such as uniaxially strained graphene and organic compound $\alpha$-(BEDT-TTF)$_2$I$_3$. In these systems, the fermion velocities along different directions may be unequal. It is thus necessary to examine how interactions change the anisotropy. According to our RG analysis, the fermion velocity anisotropy is unaffected by the excitonic fluctuation, but could be significantly suppressed by the Coulomb interaction.

The rest of the paper will be arranged as following.
The model is presented in Sec. II. The RG equations for the corresponding parameters are shown in Sec. III. The numerical results for different conditions are given and analyzed in Sec. IV. The main results are summarized in Sec. V. The detailed derivation of the RG equations can be found in Appendices.

II. THE MODEL

The fermion energy dispersion in intrinsic graphene is isotropic. It becomes anisotropic when graphene is deformed. Generically, the action of free 2D Dirac fermions with anisotropic dispersion is given by

$$S_f = \sum_{\sigma=1}^{N} \int_{\tau, \mathbf{x}} \bar{\Psi}_{\sigma}(\tau, \mathbf{x}) \left[ \partial_{\tau} \gamma_0 + \mathcal{H}_f \right] \Psi_{\sigma}(\tau, \mathbf{x}),$$

where \( \int_{\tau, \mathbf{x}} \equiv \int d\tau \int d^{2}\mathbf{x} \) and \( \mathcal{H}_f = -iv_1 \nabla_1 \gamma_1 - iv_2 \nabla_2 \gamma_2 \). Here, \( \Psi \) is a four-component spinor, and \( \bar{\Psi} = \Psi^\dagger \). The matrices \( \gamma_{0,1,2} \) are defined as \( \gamma_{0,1,2} = (\tau_3, -i\tau_2, i\tau_1) \otimes \tau_3 \) in terms of Pauli matrices \( \tau_i \) with \( i = 1, 2, 3 \). The gamma matrices satisfy the anti-commutative rule \( \{ \gamma_\mu, \gamma_\nu \} = 2\text{diag}(1, -1, -1) \). The fermion species is denoted by \( \sigma \), which sums from 1 to \( N \). Fermion flavor \( N \) is assumed to be a general large integer. We use \( v_1 \) and \( v_2 \) to represent the fermion velocities along two orthogonal directions.

The action of the quantum fluctuation of excitonic order parameter can be written as

$$S_b = \int_{\tau, \mathbf{x}} \left[ \frac{1}{2} (\partial_{\tau} \phi)^2 + \frac{c^2}{2} (\nabla \phi)^2 + \frac{r}{2} \phi^2 + \frac{u}{24} \phi^4 \right],$$

where \( c \) is the boson velocity. Varying boson mass \( r \) tunes the QPT between SM and EI phases. At the QCP, the mass vanishes, i.e., \( r = 0 \), and the boson field \( \phi \) describes the quantum critical fluctuation of excitonic order parameter. The quartic self-interacting term has a coupling constant \( u \). The Yukawa coupling between fermions and excitonic order parameter is given by

$$S_{fb} = \lambda \sum_{\sigma=1}^{N} \int_{\tau, \mathbf{x}} \bar{\Psi}_{\sigma} \Psi_{\sigma},$$

where \( \lambda \) is the corresponding coupling constant.

The excitonic pairing originates from the Coulomb interaction between fermions and their anti-fermions (holes). Inside the EI phase, a finite gap is opened at the Fermi level and strongly suppresses the low-energy fermion DOS. In this case, the Coulomb interaction and even the fermionic degrees of freedom can be neglected, and the low-energy properties of the EI phase is mainly governed by the dynamics of neutral excitons. In contrast, the fermions remain gapless at the SM-EI QCP. The Coulomb interaction between gapless fermions may play an important role at low energies. The action for Coulomb interaction is described by

$$S_{cc} = \frac{1}{4\pi} \sum_{\sigma, \sigma'=1}^{N} \int_{\tau, \mathbf{x}, \mathbf{x}'} \rho_{\sigma}(\tau, \mathbf{x}) \frac{e^2}{|\mathbf{x} - \mathbf{x}'|} \rho_{\sigma}(\tau, \mathbf{x}'),$$

where \( \int_{\tau, \mathbf{x}, \mathbf{x}'} \equiv \int d\tau \int d^{2}\mathbf{x} \int d^{2}\mathbf{x}' \). The fermion density operator is defined as \( \rho_{\sigma}(\tau, \mathbf{x}) = \bar{\Psi}_{\sigma}(\tau, \mathbf{x}) \gamma_0 \Psi_{\sigma}(\tau, \mathbf{x}) \). In addition, \( e \) is electric charge and \( \epsilon \) dielectric constant.

Disorder exists in almost all realistic materials. Many of the low-energy behaviors of fermions are heavily affected by disorder scattering, especially at low \( T \). The fermion-disorder coupling is formally described by

$$S_{\text{dis}} = v_1 \int d\tau d^{2}\mathbf{x} \bar{\Psi}_{\sigma}(\mathbf{x}) \Gamma \Psi_{\sigma}(\mathbf{x}) A(\mathbf{x}).$$

The random field \( A(\mathbf{x}) \) is assumed to be a Gaussian white noise, i.e., \( \langle A(\mathbf{x}) \rangle = 0 \) and \( \langle A(\mathbf{x}) A(\mathbf{x}') \rangle = \Delta \delta^2(\mathbf{x} - \mathbf{x}') \). Here, \( \Delta \) is the impurity concentration, and \( v_1 \) measures the strength of a single impurity. The disorders are classified by the expression of \( \Gamma \) matrix [51]. For \( \Gamma_0 = \gamma_0 \), \( A(\mathbf{x}) \) is a RSP. For \( \Gamma_1 = \mathbb{1}_4 \), \( A(\mathbf{x}) \) serves as a RM. In comparison, RVP has two components \( A_{1,2}(\mathbf{x}) \), characterized by \( \Gamma = (\gamma_1, \gamma_2) \) and \( v_1 = (v_{11}, v_{12}) \).

The free fermion propagator has the form

$$G_0(\omega, \mathbf{k}) = \frac{1}{-i\omega \gamma_0 + v_1 k_1 \gamma_1 + v_2 k_2 \gamma_2}.$$  

The Yukawa coupling can be treated by the RG method in combination with the \( 1/N \) expansion. Following the scheme developed by Huh and Sachdev [57], we re-scale \( \phi \) and \( r \) as follows: \( \phi \rightarrow \phi / \lambda \) and \( r \rightarrow N r / \lambda^2 \). Accordingly, the bare propagator of \( \phi \) is expressed as

$$D_0^A(\Omega, \mathbf{q}) = \frac{1}{\Omega^2 + c^2 q^2} + \frac{1}{N r}.$$  

Near the QCP, we take \( r = 0 \) and then get

$$D_0^A(\Omega, \mathbf{q}) = \frac{\lambda^2}{\Omega^2 + c^2 q^2}.$$  

The free boson propagator is drastically altered by the polarization function, which, to the leading order of \( 1/N \) expansion, is

$$\Pi^A(\Omega, \mathbf{q}) = N \int \frac{d\omega}{2\pi} \frac{d^2 k}{(2\pi)^2} \times \text{Tr} [G_0(\omega, \mathbf{k}) G_0(\omega + \Omega, \mathbf{k} + \mathbf{q})] = -\frac{N}{4 v_1 v_2} \sqrt{\Omega^2 + v_1^2 q_1^2 + v_2^2 q_2^2}.$$  

Now the dressed boson propagator becomes

$$D^A(\Omega, \mathbf{q}) = \frac{1}{\Omega^2 + c^2 q^2} + \Pi^A(\Omega, \mathbf{q}).$$
It is obvious that $\Pi^A$ dominates over the free term in the low-energy regime. Thus, the above expression can be further simplified to

$$D^A(\Omega, q) \approx \frac{1}{\Pi^A(\Omega, q)}. \quad (11)$$

The bare Coulomb interaction is described by

$$D^B_0(q) = \frac{2\pi e^2}{c|q|} \cdot (12)$$

The dynamical screening is encoded in the polarization $\Pi^B(\Omega, q)$, whose leading order expression is given by

$$\Pi^B(\Omega, q) = -N \int \frac{d\omega}{2\pi (2\pi)^2} \text{Tr} \left[ \gamma_0 G_0(\omega, k) \gamma_0 \times G_0(\omega + \Omega, k + q) \right] = \frac{N}{8v_1v_2} \frac{\pi \epsilon^2}{\Omega^2 + \pi^2 q^2 + v_1^2 q_1^2 + v_2^2 q_2^2}. \quad (13)$$

The dressed Coulomb interaction can be written as

$$D^B(\Omega, q) = \frac{1}{\frac{4\pi e^2}{c|q|} + \Pi^B(\Omega, q)}. \quad (14)$$

In previous works on the quantum criticality of SM-EI transition, the interplay of Yukawa coupling, Coulomb interaction, and disorder has never been systematically studied. Here, we emphasize that all the three interactions could be very important at low energies and thus should be treated equally.

### III. Renormalization Group Equations

The interplay of distinct interactions can be handled by means of perturbative RG approach. The detailed RG calculations are presented in the Appendices. In this section, we only list the coupled RG equations of a number of model parameters and then analyze their low-energy properties. The effective model contains several independent parameters, such as $v_1$, $v_2$, and $v_\tau$. These parameters are renormalized by interactions. To specify how the interactions alter the fermion dispersion anisotropy, we need to determine the flow of the ratio $v_2/v_1$. Moreover, to judge whether FL theory is applicable, we should compute the flow equation of the residue $Z_f$.

After incorporating three types of interaction in a self-consistent way, we find that the coupled RG equations for $Z_f$, $v_1$, $v_2$, and $v_2/v_1$ are given by

$$\frac{dZ_f}{d\ell} = (C^A_0 + C^B_0 - C^g_0) Z_f, \quad (15)$$

$$\frac{dv_1}{d\ell} = (C^A_0 + C^B_0 - C^A_1 - C^B_1 - C^g_0) v_1, \quad (16)$$

$$\frac{dv_2}{d\ell} = (C^A_0 + C^B_0 - C^A_2 - C^B_2 - C^g_0) v_2, \quad (17)$$

$$\frac{d(v_2/v_1)}{d\ell} = (C^A_1 - C^A_2 + C^B_1 - C^B_2) \frac{v_2}{v_1}. \quad (18)$$

RG analysis is performed by integrating out the modes defined within the momentum shell $e^{-\delta} \Lambda < |k| < \Lambda$, where $\Lambda$ is an UV cutoff and $\ell$ is a running parameter. The lowest energy limit is reached as $\ell \to \infty$. For RSP, the flow equation of $v_\tau$ takes the form

$$\frac{dv_\tau}{d\ell} = 0. \quad (19)$$

For the two components of RVP, the flow equations for $v_{\tau 1}$ and $v_{\tau 2}$ are

$$\frac{dv_{\tau 1}}{d\ell} = (C^A_0 + C^B_0 - C^A_1 - C^B_1 - C^g_0) v_{\tau 1}, \quad (20)$$

$$\frac{dv_{\tau 2}}{d\ell} = (C^A_0 + C^B_0 - C^A_2 - C^B_2 - C^g_0) v_{\tau 2}. \quad (21)$$

For RM, the flow equation of $v_\tau$ is

$$\frac{dv_\tau}{d\ell} = (2C^A_0 + C^A_1 + C^A_2 + 2C^B_0 - C^B_1 - C^B_2 \quad -2C^g_0) v_\tau. \quad (22)$$

Here, we introduce a new parameter $C^g_0$ to characterize the effective strength of disorder. For RSP and RM, it is

$$C^g_0 = \frac{v_\tau^2 \Delta}{2\pi v_1 v_2}. \quad (23)$$

For RVP, we have

$$C^g_0 = \frac{(v^2_{\tau 1} + v^2_{\tau 2}) \Delta}{2\pi v_1 v_2}. \quad (24)$$

The three coefficients $C^A_0$, $C^A_1$, and $C^A_2$ appearing in the coupled RG equations are

$$C^A_0 = \frac{1}{8\pi^3} \int_{-\infty}^{+\infty} dx \int_0^{2\pi} d\theta \times \frac{x^2 - \cos^2 \theta}{(x^2 + \cos^2 \theta + (v_2/v_1)^2 \sin^2 \theta)^2} G^A(x, \theta) \quad (25)$$

$$C^A_1 = \frac{1}{8\pi^3} \int_{-\infty}^{+\infty} dx \int_0^{2\pi} d\theta \times \frac{-x^2 + \cos^2 \theta - (v_2/v_1)^2 \sin^2 \theta}{(x^2 + \cos^2 \theta + (v_2/v_1)^2 \sin^2 \theta)^2} G^A(x, \theta) \quad (26)$$

$$C^A_2 = \frac{1}{8\pi^3} \int_{-\infty}^{+\infty} dx \int_0^{2\pi} d\theta \times \frac{-x^2 - \cos^2 \theta + (v_2/v_1)^2 \sin^2 \theta}{(x^2 + \cos^2 \theta + (v_2/v_1)^2 \sin^2 \theta)^2} G^A(x, \theta) \quad (27)$$

where

$$G^A(x, \theta) = \frac{1}{\sqrt{x^2 + \cos^2 \theta + (v_2/v_1)^2 \sin^2 \theta}}. \quad (28)$$
The coefficients \( C_0^B, C_1^B \), and \( C_2^B \) are

\[
C_0^B = \frac{1}{8\pi^3} \int_{-\infty}^{\infty} dx \int_0^{2\pi} d\theta \left( -x^2 + \cos^2 \theta + (v_2/v_1)^2 \sin^2 \theta \right) \frac{G^B(x, \theta)}{(x^2 + \cos^2 \theta + (v_2/v_1)^2 \sin^2 \theta)^2}, \quad (29)
\]

\[
C_1^B = \frac{1}{8\pi^3} \int_{-\infty}^{\infty} dx \int_0^{2\pi} d\theta \left( -x^2 + \cos^2 \theta - (v_2/v_1)^2 \sin^2 \theta \right) \frac{G^B(x, \theta)}{(x^2 + \cos^2 \theta + (v_2/v_1)^2 \sin^2 \theta)^2}, \quad (30)
\]

\[
C_2^B = \frac{1}{8\pi^3} \int_{-\infty}^{\infty} dx \int_0^{2\pi} d\theta \left( -x^2 - \cos^2 \theta + (v_2/v_1)^2 \sin^2 \theta \right) \frac{G^B(x, \theta)}{(x^2 + \cos^2 \theta + (v_2/v_1)^2 \sin^2 \theta)^2}, \quad (31)
\]

with

\[
G^B(x, \theta) = \frac{1}{2\pi \alpha_1} + \frac{1}{8\pi v_1 v_2} \cos^2 \theta + (v_2/v_1)^2 \sin^2 \theta \sqrt{x^2 + \cos^2 \theta + (v_2/v_1)^2 \sin^2 \theta}. \quad (32)
\]

An effective parameter

\[
\alpha_1 = \frac{e^2}{\epsilon v_1} \quad (33)
\]

is defined to represent the Coulomb interaction strength. The electric charge \( e \) is not renormalized due to the absence of logarithmic term in the polarization \( \Pi^B \), and \( \epsilon \) takes a constant value in any given sample. The value of \( \alpha_1 \) is determined by the renormalization of velocity \( v_1 \).

The coupled flow equations can be simplified. According to Eq. (19), we know that

\[
v_1 = v_{10} \quad (34)
\]

is independent of \( \ell \) for RSP. Thus we re-write \( C_g \) as

\[
C_g = \frac{v_{10}^2 \Delta}{2\pi v_1 v_2}. \quad (35)
\]

The flow equation for \( C_g \) is given by

\[
\frac{dC_g}{d\ell} = (-2C_0^A - 2C_0^B + C_1^A + C_1^B + C_2^A + C_2^B + 2C_g) C_g. \quad (36)
\]

For RVP, from Eqs. (16), (17), and (22), one gets

\[
\frac{d(v_{11}/v_1)}{d\ell} = 0, \quad \frac{d(v_{12}/v_2)}{d\ell} = 0, \quad (37)
\]

which indicate that

\[
\frac{v_{11}}{v_1} = \frac{v_{10}}{v_1}, \quad \frac{v_{12}}{v_2} = \frac{v_{20}}{v_2}. \quad (38)
\]

Accordingly, \( C_g \) now can be written as

\[
C_g = \frac{\Delta}{2\pi} \left( \frac{v_{10}^2 v_1}{v_1^2 v_1} + \frac{v_{20}^2 v_2}{v_2^2 v_2} \right). \quad (39)
\]

The corresponding RG equation is

\[
\frac{dC_g}{d\ell} = \frac{(v_{11}^2 - v_{12}^2)}{2\pi v_1 v_2} \left( -C_1^A - C_1^B + C_2^A + C_2^B \right). \quad (40)
\]

For RM, through Eqs. (10), (17), and (22), we obtain the following flow equation

\[
\frac{dC_g}{d\ell} = (2C_1^A + 3C_1^A + 3C_2^A + 2C_0^B - C_2^B - 2C_g) C_g. \quad (41)
\]

IV. QUANTUM CRITICAL PHENOMENA

In this section, we will solve the RG equations and then apply the solutions to analyze the quantum critical phenomena. We adopt the following steps: first, examine the low-energy behaviors induced solely by the quantum critical fluctuation of excitonic order parameter; second, introduce quenched disorder into the system and study its interplay with the Yukawa coupling; finally, investigate the impact of Coulomb interaction on the results.

Although the RG calculations are carried out at \( T = 0 \), it is possible to extract the \( T \)-dependence of observable quantities from RG results. We can regard \( k_B T \), where \( k_B \) is Boltzmann constant, as a free parameter that tunes the energy scale: increasing (decreasing) \( T \) amounts to increasing (decreasing) the energy \( \omega \). The dependence of observable quantities on \( \omega \) and/or \( T \) can be computed from the solutions of RG equations as follows. One solves the flow equations at \( T = 0 \) and gets the \( \ell \)-dependence of model parameters, such as fermion velocities, which leads to the \( \ell \)-dependence of various observable quantities. On the basis of these results, one converts the \( \ell \)-dependence of an observable quantity into the \( \omega \)-dependence of the same quantity at \( T = 0 \) by using the transformation \( \omega = \omega_0 e^{-\ell} \), where \( \omega_0 \) is some high energy, or into the \( T \)-dependence of the same quantity by using the transformation \( T = T_0 e^{-\ell} \), where \( T_0 \) takes a large value. For examples, the low-energy DOS \( \rho(\omega) \) can be directly obtained from \( \rho(\ell) \), and the \( T \)-dependent specific heat \( C_v(T) \) can be obtained from \( C_v(\ell) \). This approach has been extensively employed to calculate the \( \omega \)- and/or \( T \)-dependence of many observable quantities of Dirac/Weyl fermions subject to the Coulomb interaction [14, 15, 23, 29, 34, 50, 73] and gapless nodal fermions coupled to the nematic quantum fluctuation [57, 53].

A. Non-Fermi liquid behavior induced by excitonic fluctuation

If 2D DSM is far from SM-EI transition, the ground state is a robust SM. While the Coulomb interaction is long-ranged, it can only produce normal FL behavior [4, 11, 12, 16]. As the system approaches to the SM-EI QCP, the excitonic fluctuation becomes stronger and eventually invalidates the FL description at \( T = 0 \). Now
we illustrate how FL theory breaks down at the QCP by analyzing the solutions of RG equations.

In the clean limit, the excitonic fluctuation leads to the following RG equations

\[
\frac{dZ_f}{d\ell} = C_0^A Z_f, \quad \frac{dv_1}{d\ell} = (C_0^A - C_1^A) v_1, \quad \frac{dv_2}{d\ell} = (C_0^A - C_2^A) v_2, \quad \frac{d(v_2/v_1)}{d\ell} = (C_1^A - C_2^A) \frac{v_2}{v_1},
\]

These equations will be solved in the isotropic and anisotropic cases respectively.

1. Isotropic limit

We first consider the isotropic limit, i.e., \( v_1 = v_2 = v \).

In this case, we have

\[
C_0^A = C_1^A = C_2^A = -\frac{2}{3\pi^2 N} = -\eta_A^A.
\]

Accordingly, the RG equations can be simplified to

\[
\frac{dZ_f}{d\ell} = -\eta_A^A Z_f, \quad \frac{dv}{d\ell} = 0.
\]

The velocity is a constant, i.e., \( v = v_0 \). Thus, the fermion dispersion is unrenormalized, and the dynamical exponents is \( z = 1 \). The specific heat behaves as

\[
C_v(T) \sim T^{d/z} \sim T^2.
\]

The residue is given by \( 6 \)

\[
Z_f = Z_{f0} e^{-\eta_A^A \ell} = e^{-\eta_A^A \ell},
\]

which flows to zero quickly in the limit \( \ell \to \infty \). \( Z_f \) is connected to the real part of retarded self-energy \( \text{Re}\Sigma^R(\omega) \) via the definition

\[
Z_f = \frac{1}{1 - \frac{d}{d\omega} \text{Re}\Sigma^R(\omega)}.
\]

Employing the transformation \( \omega = \omega_0 e^{-\ell} \), we get the following expression

\[
\text{Re}\Sigma^R(\omega) \sim \omega^{1-\eta_A^A}.
\]

Using the Kramers-Kronig relation, we can easily obtain the imaginary part

\[
\text{Im}\Sigma^R(\omega) \sim \omega^{1-\eta_A^A},
\]

which exhibits typical NFL behavior. The renormalized DOS depends on \( \omega \) as follows

\[
\rho(\omega) \sim \omega^{1+\eta_A^A}.
\]

2. Anisotropic case

In the generic anisotropic case, namely \( v_1 \neq v_2 \), we integrate over variable \( x \) in Eqs. (23)-(27) and find

\[
C_0^A = -\frac{v_2/v_1}{3\pi^2 N} \int_0^{2\pi} d\theta \frac{1}{(\cos^2 \theta + (v_2/v_1)^2 \sin^2 \theta)}
\]

\[
= -\frac{v_2/v_1}{3\pi^2 N} \frac{2\pi}{v_2/v_1} = -\eta_A^A,
\]

\[
C_1^A = \frac{v_2/v_1}{3\pi^2 N} \int_0^{2\pi} d\theta \frac{\cos^2 \theta - 3(v_2/v_1)^2 \sin^2 \theta}{(\cos^2 \theta + (v_2/v_1)^2 \sin^2 \theta)^2}
\]

\[
= \frac{v_2/v_1}{3\pi^2 N} \left( -\frac{2\pi}{v_2/v_1} \right) = -\eta_A^A,
\]

\[
C_2^A = \frac{v_2/v_1}{3\pi^2 N} \int_0^{2\pi} d\theta \frac{-3\cos^2 \theta + (v_2/v_1)^2 \sin^2 \theta}{(\cos^2 \theta + (v_2/v_1)^2 \sin^2 \theta)^2}
\]

\[
= \frac{v_2/v_1}{3\pi^2 N} \left( -\frac{2\pi}{v_2/v_1} \right) = -\eta_A^A,
\]

which are exactly the same as the isotropic case. Accordingly, the RG equations for \( v_1 \) and \( v_2 \) are

\[
\frac{dv_1}{d\ell} = \frac{dv_2}{d\ell} = 0,
\]

which implies that

\[
v_1 = v_{10}, \quad v_2 = v_{20}.
\]

Thus, the fermion velocities are not renormalized, and the anisotropy is not changed by the Yukawa coupling. The low-energy properties of specific heat \( C_v(T) \), residue \( Z_f \), fermion damping rate \( \text{Im}\Sigma^R(\omega) \), and DOS \( \rho(\omega) \) are the same as those obtained in the isotropic case.

B. Excitonic fluctuation and disorder

We then include disorder and examine how it affects the above results. Now the coupled RG equations of \( Z_f, v_1, v_2, \) and \( v_2/v_1 \) are

\[
\frac{dZ_f}{d\ell} = (C_0^A - C_g) Z_f = -(\eta_A^A + C_g) Z_f, \quad \frac{dv_1}{d\ell} = (C_0^A - C_1^A - C_g) v_1 = -C_g v_1, \quad \frac{dv_2}{d\ell} = (C_0^A - C_2^A - C_g) v_2 = -C_g v_2, \quad \frac{d(v_2/v_1)}{d\ell} = (C_1^A - C_2^A) \frac{v_2}{v_1} = 0.
\]

For RSP, \( C_g \) satisfies

\[
\frac{dC_g}{d\ell} = 2C_g^2,
\]

whose solution is

\[
C_g = \frac{C_{g0}}{1 - 2C_{g0} \ell}.
\]
It is clear that this $C_g$ diverges as $\ell \to \ell_c$, where $\ell_c = 1/2C_{g0}$. Substituting Eq. (65) into Eqs. (60)-(62), we obtain

$$Z_f = e^{-\eta^4 \ell} \sqrt{1 - 2C_{g0} \ell}, \quad v_1 = v_{10} \sqrt{1 - 2C_{g0} \ell}, \quad v_2 = v_{20} \sqrt{1 - 2C_{g0} \ell}. \quad (66)$$

$$v_1 = v_{10} \sqrt{1 - 2C_{g0} \ell}, \quad v_2 = v_{20} \sqrt{1 - 2C_{g0} \ell}. \quad (67)$$

We can see that, $Z_f$, $v_1$, and $v_2$ all flow to zero as $\ell \to \ell_c$. Such singular behaviors are generally believed to indicate the instability of the system: RSP drives the system into a disorder-dominated CDM. The characteristic feature of CDM is that, the fermions acquire a finite disorder scattering rate

$$\gamma_{\text{imp}} = |\text{Im}\Sigma^R(0)|. \quad (69)$$

In the meantime, the zero-energy DOS $\rho(0)$ also becomes finite, being a function of $\gamma_{\text{imp}}$. According to the calculations given in Refs. [51, 93], the specific heat displays a linear-in-$T$ behavior, namely

$$C_v(T) \sim T. \quad (70)$$

The NFL quantum critical state realized in the clean limit is turned into a CDM once RSP is added to the system, even when RSP is very weak. The fermion damping effect, the low-energy DOS, and the specific heat of CDM phase are all distinct from those of the NFL phase.

For RVP, the RG equation for $C_g$ is

$$\frac{dC_g}{d\ell} = \left(\frac{v_1^2 - v_2^2}{2\pi v_1 v_2}\right) \frac{\Delta}{(-C_A^1 + C_A^2)} = 0, \quad (71)$$

implying that

$$C_g = C_{g0}. \quad (72)$$

Substituting Eq. (72) into Eqs. (60)-(62) yields

$$Z_f = e^{-\eta^4 \ell \sqrt{C_{g0} + 4\eta^4 \ell}}, \quad v_1 = v_{10} e^{-C_{g0} \ell}, \quad v_2 = v_{20} e^{-C_{g0} \ell}. \quad (73)$$

The real and imaginary parts of retarded fermion self-energy are

$$\text{Re}\Sigma^R(\omega) \sim \omega^{1-(\eta^4+C_{g0})}, \quad (74)$$

$$\text{Im}\Sigma^R(\omega) \sim \omega^{1-(\eta^4+C_{g0})}, \quad (75)$$

which are still NFL-like behaviors. Comparing to the clean limit, $Z_f$ approaches to zero more quickly and the fermion damping becomes stronger. The velocity $v$ goes to zero rapidly with growing $\ell$, thus the fermion dispersion is substantially altered. In addition, the dynamical exponent $z$ becomes $z = 1 + C_{g0}$. It is easy to find that, the specific heat is

$$C_v(T) \sim T^{d/z} \sim T^{2/(1+C_{g0})}, \quad (76)$$

and the low-energy DOS is

$$\rho(\omega) \sim \omega^{(1-C_{g0})/(1+C_{g0})+\eta^4}. \quad (77)$$

An apparent conclusion is that both DOS and specific heat are enhanced by RVP at low energies.

For RM, the RG equation for $C_g$ becomes

$$\frac{dC_g}{d\ell} = -8\eta^4 C_g - 2C_g^2. \quad (78)$$

Its solution is

$$C_g(\ell) = \frac{4\eta^4 C_{g0}}{(C_{g0} + 4\eta^4) e^{8\eta^4 \ell} - C_{g0}}, \quad (79)$$

which vanishes in the limit $\ell \to \infty$. Substituting Eq. (80) into Eqs. (60)-(62), we get

$$Z_f = e^{-\eta^4 \ell \sqrt{C_{g0} + 4\eta^4 \ell}}, \quad (81)$$

$$v_1 = v_{10} e^{-C_{g0} \ell}, \quad (82)$$

$$v_2 = v_{20} e^{-C_{g0} \ell}. \quad (83)$$

In the low-energy regime, the residue still behaves as $Z_f \sim e^{-\eta^4 \ell}$. From the $\ell$-dependence of $Z_f$, we obtain

$$\text{Re}\Sigma^R(\omega) \sim \omega^{1-\eta^4}, \quad (84)$$

$$\text{Im}\Sigma^R(\omega) \sim \omega^{1-\eta^4}, \quad (85)$$

which are the same as the clean case. As shown by Eqs. (83) and (84), $v_1$ and $v_2$ approach to finite values in the lowest energy limit. Accordingly, the fermion DOS still exhibits the behavior $\rho(\omega) \sim \omega^{1+\eta^4}$, and the specific heat is still of the form $C_v(T) \sim T^2$. We thus see that RM does not qualitatively change the low-energy properties of observable quantities.

The above RG results indicate that, the low-energy properties of the SM-EI QCP depend heavily on the disorder type. Such properties can be experimentally probed by measuring observable quantities, such as DOS and specific heat. However, we should remember that the long-range Coulomb interaction is entirely ignored in the above RG analysis. This might miss important quantum many-body effects. In the next subsection, we will study whether or not the above results are substantially altered when the Coulomb interaction is incorporated.

## C. Interplay of three kinds of interaction

We now analyze the physical consequence of the interplay of all the three kinds of interaction, first in the isotropic limit and then in the more generic anisotropic case. We will see that the Coulomb interaction tends to suppress the fermion velocity anisotropy.
In the clean limit, $Z_f$ and $v$ flow as follows
\begin{align}
\frac{dZ_f}{d\ell} &= (-\eta^A + C_0^B) Z_f, \\
\frac{dv}{d\ell} &= C^B v.
\end{align}

The numerical solutions are shown in Fig. 2. The velocity $v$ increases as the energy is lowered. The Coulomb interaction is marginally irrelevant since its strength parameter $\alpha = e^2/\nu_e$ flows to zero slowly in the lowest energy limit. Both $C_0^B$ and $C^B$ vanish as $\alpha \to 0$. The velocity renormalization produces logarithmic-like correction to the temperature or energy dependence of some observable quantities, including specific heat and compressibility \[^2\]. The singular renormalization of fermion velocities has been observed by various experimental tools \[^80, 95–97\]. At low energies, $C_0^B$ is much smaller than $\eta^A$. Thus, the Coulomb interaction only slightly alters the low-energy behavior of $Z_f$ induced by the excitonic fluctuation.

For RSP, the RG equation of $C_g$ is
\begin{equation}
\frac{dC_g}{d\ell} = (-2C^B + 2C_g) C_g.
\end{equation}

For a given $\alpha_0$, there exists a critical value $C^B(\alpha_0)$. The system exhibits entirely different low-energy properties when $C_{g0}$ is greater and smaller than $C^B(\alpha_0)$. To illustrate this, we show the $\ell$-dependence of $Z_f$, $v$, $\alpha$, and $C_g$ in Fig. 3. If $C_{g0} < C^B(\alpha_0)$, $Z_f$, $\alpha$, and $C_g$ all flow to zero as $\ell \to \infty$, but $v$ increases with growing $\ell$. These results indicate that weak RSP is suppressed by the Coulomb interaction. If $C_{g0} > C^B(\alpha_0)$, both $C_g$ and $\alpha$ formally diverge at some finite energy scale, whereas both $Z_f$ and $v$ decrease rapidly down to zero at the same energy scale. Thus, strong RSP still drives a NFL-to-CDM transition. As can be seen from the flow diagram presented in Fig. 4(a), the $(\alpha, C_g)$ plane is divided by the critical line $C_{g0} = C^B(\alpha_0)$ into two distinct phases: the NFL phase and the CDM phase.

For RVP, the $\ell$-dependence of $Z_f$, $v$, $\alpha$, and $C_g$ are shown in Fig. 5. The parameter $C_g$ does not flow at all, namely
\begin{equation}
\frac{dC_g}{d\ell} = 0.
\end{equation}

We fix $C_g$ at a constant: $C_g = C_{g0}$. For a given $C_{g0}$, $v$ approaches to a constant value $v^*$ in the zero energy limit. The value of $v^*$ is obtained from
\begin{equation}
C^B(\alpha^*) = C_{g0},
\end{equation}

where $\alpha^* = \epsilon^2/v^*\epsilon$. RG analysis indicates that the system always flows to a stable infrared fixed point for any two given initial values of $\alpha$ and $C_g$. Connecting all of these fixed points forms a critical line on the $\alpha$-$C_g$ plane, as shown in Fig. 4(b). Near the critical line, the specific heat behaves as
\begin{equation}
C_v(T) \sim \frac{1}{v^{*2}} T^2 \sim T^2.
\end{equation}
The residue is
\[ Z_f \sim e^{(-\eta^A + C_g^B(\alpha^*) - C_{g0})} \sim e^{(-\eta^A + C_g^B(\alpha^*))}, \]  
(98)
where \( C_g^B(\alpha^*) \) is negative. This \( Z_f \) flows to zero more quickly than that induced purely by excitonic fluctuation. The retarded fermion self-energy is
\[
\begin{align*}
\text{Re} \Sigma^R(\omega) &\sim \omega^{1-(\eta^A - C_g^B(\alpha^*))}, \\
\text{Im} \Sigma^R(\omega) &\sim \omega^{1-(\eta^A - C_g^B(\alpha^*))}.
\end{align*}
\]  
(99, 100)
The DOS takes the form
\[ \rho(\omega) \sim \omega^{1+\eta^A - C_g^B(\alpha^*)}. \]  
(101)
For RM, the RG equation for \( C_g \) is given by
\[ \frac{dC_g}{d\ell} = (-8\eta^A + 2C^B - 2C_0)C_g, \]  
(102)
The numerical results are plotted in Fig. 6. We observe that \( C_g \) always approaches to zero quickly, which indicates that RM is irrelevant in the low-energy regime. The Coulomb interaction is marginally irrelevant and leads to singular renormalization of fermion velocity. Accordingly, the DOS and specific heat are
\[
\begin{align*}
\rho(\omega) &\sim \frac{\omega^{1+\eta^A}}{\ln^2(\omega_0/\omega)}, \\
C_v(T) &\sim \frac{T^2}{\ln^2(T_0/T)}.
\end{align*}
\]  
(103, 104)
In the presence of RM, the two parameters \( (\alpha, C_g) \) always flow to the stable infrared fixed point \( (0, 0) \).
We now compare the quantum critical phenomena to the physical properties of the SM phase. Deep in the SM phase, the excitonic fluctuation can be completely ignored. The low-energy behavior is governed by the interplay of Coulomb interaction and disorder, which has already been extensively investigated \[50, 98, 103\]. When
In the SM-EI quantum critical regime, increasing the effective strength of RSP drives a FL-CDM phase transition. If RVP is added to the system, it is irrelevant around the SM-EI QCP, but is marginal and results in a stable critical line on the SM-EI QCP. Therefore, RVP produces the same qualitative low-energy behaviors in the SM phase and around the SM-EI QCP.

We learn from the above analysis that, even if 2D DSM has a gapless SM ground state, the fluctuation of excitonic order parameter gives rise to observable effects at finite $T$ and/or $\omega$. The quantum critical regime can be distinguished from the pure SM phase by measuring the $\omega$-dependence of fermion damping rate and/or the $T$-dependence of specific heat.

To provide a complete analysis of the quantum critical phenomena, we summarize in Table I the low-energy properties induced by all the possible combinations of three types of interaction. The quantities presented in Table I include the residue $Z_f(\ell)$, damping rate $\Im \Sigma^R(\omega)$, fermion DOS $\rho(\omega)$, and specific heat $C_v(T)$. We can see that distinct interactions affect each other significantly. The critical phenomena cannot be reliably determined if their mutual influence is not carefully handled.

| Interaction | $Z_f(\ell)$ | $\Im \Sigma^R(\omega)$ | $\rho(\omega)$ | $C_v(T)$ |
|------------|-------------|------------------------|---------------|----------|
| QFEO       | $e^{-\eta_1 \ell}$ | $\omega^{1+\eta_1}$ | $\omega^{1+\eta_1}$ | $T^2$ |
| QFEO+RSP   | $e^{-\eta_1 \ell} \sqrt{1-2C_{g\ell}}$ | $\gamma_{\text{imp}}$ | $\gamma_{\text{imp}} \ln(v_{\text{FEO}}/v_{\text{imp}})$ | $\rho(0)T$ |
| QFEO+RVP   | $e^{-\eta_1 \ell} \gamma_{\text{imp}} C_{g\ell}$ | $\omega^{1-(\eta_1+C_{g\ell})}$ | $\omega^{1-(\eta_1+C_{g\ell})}$ | $T^2/(1+\gamma_{\text{imp}})$ |
| QFEO+RM    | $e^{-\eta_1 \ell}$ | $\omega^{1-\eta_1}$ | $\omega^{1-\eta_1}$ | $T^2$ |
| QFEO+CI    | $e^{-\eta_1 \ell}$ | $\omega^{1-\eta_1}$ | $\omega^{1+\eta_1}/\ln^2(\omega_{\text{imp}}/\omega)$ | $T^2/\ln^2(T_0/T)$ |
| QFEO+CI+RSP | $C_{g\ell} < C_B(\alpha_0)$ | $e^{-\eta_1 \ell}$ | $\omega^{1-\eta_1}$ | $\omega^{1+\eta_1}/\ln^2(\omega_{\text{imp}}/\omega)$ | $T^2/\ln^2(T_0/T)$ |
| QFEO+CI+RVP | $C_{g\ell} > C_B(\alpha_0)$ | $\lim_{\ell \to \infty} Z_f(\ell) \to 0$ | $\Im \Sigma^R(\omega)$ | $\gamma_{\text{imp}} \ln(v_{\text{FEO}}/v_{\text{imp}})$ | $\rho(0)T$ |
| CI+RVP     | $e^{-\eta_1 \ell}$ | $\omega^{1-\eta_1}$ | $\omega^{1+\eta_1}/\ln^2(\omega_{\text{imp}}/\omega)$ | $T^2/\ln^2(T_0/T)$ |
| CI+RM      | $e^{-\eta_1 \ell}$ | $\omega^{1-\eta_1}$ | $\omega^{1+\eta_1}/\ln^2(\omega_{\text{imp}}/\omega)$ | $T^2/\ln^2(T_0/T)$ |
action strength should be attributed to the fact that the Coulomb interaction in the anisotropic case. Blue, red, green, black, and magenta lines correspond to \( v_{20}/v_{10} = 10, 5, 1, 0.5, 0.1 \). We choose \( \alpha_{10} = 1.0 \). As \( \ell \to \infty \), the system flows to the isotropic limit.

First, we consider the case of RSP. As shown in Fig. 8, for given values of \( \alpha_{10} \) and \( C_{g0}, C_g \) becomes divergent at some finite energy scale if the bare velocity ratio \( v_{20}/v_{10} \) exceeds a critical value. Both \( Z_f \) and fermion velocities flow to zero at the same energy scale. The anisotropy is suppressed, but the ratio does not flow to the isotropic limit. If the bare value \( v_{20}/v_{10} \) is small, \( C_{g0} \) flows to zero quickly as the energy is lowered. Meanwhile, the fermion velocities increase, and the ratio \( v_2/v_1 \to 1 \). Apparently, the isotropic limit is mainly driven by the Coulomb interaction. The residue \( Z_f \) still vanishes, owing to the excitonic fluctuation. For given values of \( \alpha_{10} \) and \( C_{g0} \), varying the velocity ratio \( v_{20}/v_{10} \) leads to QPT between CDM phase and NFL phase.

In the case of RVP, we show the evolution of \( Z_f, v_1, v_2/v_1, \) and \( C_g \) in Fig. 9. Comparing to the clean limit, the ratio \( v_2/v_1 \) approaches to unity more quickly. This should be attributed to the fact that the Coulomb interaction strength \( \alpha \) flows to certain finite value in the presence of RVP but vanishes in the clean limit. Therefore, the suppression of velocity anisotropy is more significant once RVP is introduced.

We finally turn to the impact of RM. According to Fig. 10, the disorder parameter \( C_g \) of RM always flows to zero quickly with decreasing energy. The low-energy behaviors of \( Z_f \) and \( v_1 \) are nearly the same as those obtained in the clean limit, and the velocity ratio \( v_2/v_1 \to 1 \) as the energy is lowered down to zero.

V. SUMMARY AND DISCUSSION

In summary, we have presented a systematic study of the quantum critical phenomena around the SM-EI QCP in 2D DSM. The Yukawa coupling between Dirac fermions and excitonic quantum fluctuation, the long-range Coulomb interaction, and the disorder scattering are treated on equal footing, focusing on their mutual in-
fluence and the consequent low-energy properties of the quantum critical regime. We first studied the influence of quantum critical fluctuation of excitonic order parameter, and showed that it invalidates the FL description. We further demonstrated that, adding RSP always drives a NFL-to-CDM transition, and adding RVP further reinforces the NFL behaviors. Nevertheless, adding RM does not change the qualitative results obtained in the clean limit. Once Coulomb interaction is also incorporated, the above results are altered. In particular, the NFL state is protected by the Coulomb interaction for weak RSP, but is eventually replaced by CDM state if RSP is strong enough. When RVP or RM coexist with excitonic fluctuation and Coulomb interaction, the system is in a NFL state. To characterize the NFL and CDM phases, we have calculated several quantities, including the residue, damping rate, fermion DOS, and specific heat. The predicted quantum critical phenomena can be directly probed by experiments.

The results obtained in this paper might be applied to judge whether or not a 2D DSM is close to the SM-EI QCP. Deep in the gapless SM phase, the properties of the system are determined by the combination of Coulomb interaction and disorder. As the system approaches the SM-EI QCP, i.e., \( \alpha \rightarrow \alpha_c \), the excitonic quantum fluctuation becomes progressively more important, driving the system to enter into the quantum critical regime. Even when the zero-\( T \) ground state is gapless, the system could exhibit nontrivial quantum critical behaviors in the \( \omega \)- and/or \( T \)-dependence of observable quantities, as illustrated in Fig. 1 and Table I.

We finally give a brief remark on the existence of the excitonic QCP in realistic graphene. For a 2D DSM, all the previous analytical and numerical calculations \[53–73\] have confirmed that an excitonic gap is generated only when \( \alpha > \alpha_c \), where \( \alpha_c \) is a nonzero critical value. Recent theoretical studies revealed that the physical value of \( \alpha \) in suspandent graphene is not far from the critical value \( \alpha_c \). The system would become even closer to the excitonic QCP when strain is applied \[53, 54, 52\]. The organic conductor \( \alpha \)-(BEDT-TTF)\(_3\)I\(_3\), an anisotropic 2D DSM, may also be close to the excitonic QCP \[83\]. The theoretical results obtained in this work could be utilized to explore the quantum critical phenomena around the putative excitonic QCP in 2D DSM materials.

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### Appendix A: Polarization functions

We now calculate the polarization functions caused by the particle-hole collective excitations. There are two polarization functions, corresponding to the dynamical screening effects of the quantum critical fluctuation of excitonic order parameter and the long-range Coulomb interaction, respectively.

#### 1. Polarization function for excitonic fluctuation

For the quantum excitonic fluctuation, the polarization function is defined as

\[
\Pi^4(\Omega, q) = N \int \frac{d\omega}{2\pi} \frac{d^2k}{(2\pi)^2} \text{Tr} \left[ G_0(\omega, k) \times G_0(\omega + \Omega, k + q) \right].
\]

(A1)

Substituting the free fermion propagator into Eq. (A1), we obtain

\[
\Pi^4(\Omega, q) = -\frac{4N}{v_1 v_2} \int_0^1 dx \int \frac{d^3k}{(2\pi)^3} \frac{k \cdot (k + q)}{k^2(k + q)^2}.
\]

(A2)

where \( k = (\omega, k) \). Here, we have employed the following transformations

\[
v_1 k_1 \rightarrow k_1, \quad v_2 k_2 \rightarrow k_2, \quad v_1 q_1 \rightarrow q_1, \quad v_2 q_2 \rightarrow q_2.
\]

(A3)

Using the Feynman parametrization formula

\[
\frac{1}{AB} = \int_0^1 dx \frac{1}{[Ax + (1 - x)B]^2},
\]

one gets

\[
\Pi^4(\Omega, q) = -\frac{4N}{v_1 v_2} \int_0^1 dx \left\{ \int \frac{d^3k}{(2\pi)^3} \frac{k^2}{k^2 + x(1-x)q^2} \right\}.
\]

(A5)

Let \( k + xq \rightarrow k, \Pi_A \) can be further written as

\[
\Pi^4(\Omega, q) = -\frac{4N}{v_1 v_2} \int_0^1 dx \left\{ \int \frac{d^3k}{(2\pi)^3} \frac{k^2}{k^2 + x(1-x)q^2} \right\}.
\]

(A6)

Performing integration over \( k \) by using the standard formula of dimensional regularization

\[
\int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 + \Delta)^n} = \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(n - \frac{d}{2})}{\Gamma(n) \Delta^{n - \frac{d}{2}}} \cdot
\]

(A7)

\[
\int \frac{d^d k}{(2\pi)^d} \frac{k^2}{(k^2 + \Delta)^n} = \frac{1}{(4\pi)^{d/2}} \frac{d \Gamma(n - \frac{d}{2} - 1)}{\Gamma(n) \Delta^{n - \frac{d}{2}} - 1}
\]

(A8)
we find that

$$\Pi^A(\Omega, q) = \frac{2N}{v_1 v_2} \sqrt{q^2} \int_0^1 dx \sqrt{x(1-x)}$$

$$= \frac{N}{4v_1 v_2} \sqrt{\Omega^2 + q_1^2 + q_2^2}. \quad (A9)$$

By taking $q_1 \to v_1 q_1$ and $q_2 \to v_2 q_2$, we get

$$\Pi^A(\Omega, q) = \frac{N}{4v_1 v_2} \sqrt{\Omega^2 + v_1^2 q_1^2 + v_2^2 q_2^2}. \quad (A10)$$

2. Polarization function for Coulomb interaction

For the Coulomb interaction, the polarization function is given by

$$\Pi^B(\Omega, q) = -N \int \frac{d\omega}{2\pi} \frac{d^3 k}{(2\pi)^3} \text{Tr} \left[ \gamma_0 G_0(\omega, \mathbf{k}) \gamma_0 \right.$$

$$\times G_0(\omega + \Omega, \mathbf{k} + \mathbf{q}). \right] \quad (A11)$$

Substituting Eq. (A9) into Eq. (A11) leads to

$$\Pi^B(\Omega, q) = \frac{4N}{v_1 v_2} \int \frac{d^3 k}{(2\pi)^3} \frac{2k_0(k_0 + q_0) - k \cdot (k + q)}{k^2(k + q)^2}. \quad (A12)$$

Repeating the calculational steps that lead to Eq. (A10), we finally obtain

$$\Pi^B(\Omega, q) = \frac{N}{8v_1 v_2} \frac{v_1^2 q_1^2 + v_2^2 q_2^2}{\sqrt{\Omega^2 + v_1^2 q_1^2 + v_2^2 q_2^2}}. \quad (A13)$$

Appendix B: Fermion self-energy

The fermion self-energy corrections come from three sorts of interaction, namely the Yukawa coupling, Coulomb interaction, and disorder scattering. The former two interactions are inelastic, and the third one is elastic. We now calculate them in order.

1. Contribution from Yukawa coupling

The fermion self-energy induced by the Yukawa coupling takes the form

$$\Sigma^A(\omega, \mathbf{k}) = \int' \frac{d\Omega}{2\pi} \frac{d^2 \mathbf{q}}{(2\pi)^2} G_0(\Omega + \omega, \mathbf{q} + \mathbf{k}) D^A(\Omega, \mathbf{q})$$

$$= -\int' \frac{d\Omega}{2\pi} \frac{d^2 \mathbf{q}}{(2\pi)^2} \left[ -i(\Omega + \omega) \gamma_0 + v_1(q_1 + k_1) \gamma_1 + v_2(q_2 + k_2) \gamma_2 \right] D^A(\Omega, \mathbf{q}). \quad (B1)$$

This self-energy can be expanded in powers of $i\omega$, $v_1 k_1$, and $v_2 k_2$. To the leading order, we get

$$\Sigma^A(\omega, \mathbf{k}) = i\omega \gamma_0 \int' \frac{d\Omega}{2\pi} \frac{d^2 \mathbf{q}}{(2\pi)^2} \frac{-\Omega^2 + v_1^2 q_1^2 + v_2^2 q_2^2}{\Omega^2 + v_1^2 q_1^2 + v_2^2 q_2^2} \frac{1}{\sqrt{\Omega^2 + v_1^2 q_1^2 + v_2^2 q_2^2}}. \quad (B2)$$

$$-v_1 k_1 \gamma_1 \int' \frac{d\Omega}{2\pi} \frac{d^2 \mathbf{q}}{(2\pi)^2} \frac{-\Omega^2 + v_1^2 q_1^2 + v_2^2 q_2^2}{\Omega^2 + v_1^2 q_1^2 + v_2^2 q_2^2} \frac{1}{\sqrt{\Omega^2 + v_1^2 q_1^2 + v_2^2 q_2^2}}. \quad (B3)$$

$$-v_2 k_2 \gamma_2 \int' \frac{d\Omega}{2\pi} \frac{d^2 \mathbf{q}}{(2\pi)^2} \frac{-\Omega^2 + v_1^2 q_1^2 + v_2^2 q_2^2}{\Omega^2 + v_1^2 q_1^2 + v_2^2 q_2^2} \frac{1}{\sqrt{\Omega^2 + v_1^2 q_1^2 + v_2^2 q_2^2}}. \quad (B4)$$

To carry out RG calculation, we choose to integrate over the integral variables within the range

$$\int' \frac{d\Omega}{2\pi} \frac{d^2 \mathbf{q}}{(2\pi)^2} = \frac{1}{8\pi^3} \int_{-\infty}^{\infty} d\Omega \int_0^{2\pi} d\theta \int_{|\mathbf{q}|}^\infty d|\mathbf{q}| |\mathbf{q}|. \quad (B5)$$
where \( b = e^{-\ell} \). It is then easy to obtain
\[
\Sigma^A(\omega, \mathbf{k}) = (-i\omega\gamma_0 C_0^A + v_1 k_1 \gamma_1 C_1^A + v_2 k_2 \gamma_2 C_2^A) \ell. \tag{B6}
\]

The expressions of \( C_i^A \) are given by Eqs. (25)–(28).

### 2. Contribution from Coulomb interaction

The fermion self-energy induced by the Coulomb interaction is
\[
\Sigma^B(\omega, \mathbf{k}) = -\int' \frac{d\Omega}{2\pi} \frac{d^2 q}{(2\pi)^2} \gamma_0 G(\Omega + \omega, q + \mathbf{k}) \gamma_0 D^B(\Omega, q) = \int' \frac{d\Omega}{2\pi} \frac{d^2 q}{(2\pi)^2} \frac{[-i(\Omega + \omega)\gamma_0 + v_1(q_1 + k_1)\gamma_1 + v_2(q_2 + k_2)\gamma_2]}{(\Omega + \omega)^2 + v_1^2(q_1 + k_1)^2 + v_2^2(q_2 + k_2)^2} \gamma_0 D^B(\Omega, q). \tag{B7}
\]

Performing integrations according to Eq. (B5), we obtain
\[
\Sigma^B(\omega, \mathbf{k}) = (-i\omega\gamma_0 C_0^B + v_1 k_1 \gamma_1 C_1^B + v_2 k_2 \gamma_2 C_2^B) \ell. \tag{B9}
\]

The expressions of \( C_i^B \) can be found in Eqs. (29)–(32).

### 3. Contribution from disorder scattering

The fermion self-energy generated by disorder is
\[
\Sigma_{\text{dis}}(\omega) = \Delta v_l^2 \int' \frac{d^2 k}{(2\pi)^2} \Gamma G_0(\omega, \mathbf{k}) \Gamma = i\omega v_l^2 \Delta \int' \frac{d^2 k}{(2\pi)^2} \frac{\Gamma \gamma_0 \Gamma}{(\omega^2 + v_1^2 k_1^2 + v_2^2 k_2^2)} \approx i\omega\gamma_0 C_g \ell, \tag{B10}
\]

where
\[
C_g = \frac{v_l^2 \Delta}{2\pi v_1 v_2}. \tag{B11}
\]

for both RSP and RM, and
\[
C_g = \frac{(v_{l1}^2 + v_{l2}^2) \Delta}{2\pi v_1 v_2}. \tag{B12}
\]

for RVP.

### Appendix C: Corrections to fermion-disorder coupling

The fermion-disorder coupling receives vertex corrections from three sorts of interaction, including the Yukawa coupling, the Coulomb interaction, and the fermion-disorder interaction, which will be studied below.

#### 1. Vertex correction due to Yukawa coupling

The vertex correction due to Yukawa coupling is
\[
V^A = -\int' \frac{d\Omega}{2\pi} \frac{d^2 q}{(2\pi)^2} G_0(\Omega, q) v_l \Gamma G_0(\Omega, q) D^A(\Omega, q). \tag{C1}
\]

For RSP, \( \Gamma = \gamma_0 \) and we get
\[
V^A = v_l \gamma_0 (-C_0^A) \ell. \tag{C2}
\]
For the two components of RVP defined by $\Gamma = \gamma_1$ and $\Gamma = \gamma_2$, $V_A$ is given by

$$V^A = v_1 \gamma_1 \left( -C^A_1 \right) \ell, \quad (C3)$$

and

$$V^A = v_\gamma \gamma_2 \left( -C^A_2 \right) \ell, \quad (C4)$$

respectively. For RM with $\Gamma = 1$, $V_A$ is

$$V^A = v_1 \Gamma \left( C^0_0 + C^A_1 + C^A_2 \right) \ell. \quad (C5)$$

2. Vertex correction due to Coulomb interaction

The vertex correction due to Coulomb interaction is

$$V^B = - \int \frac{d^2 q}{(2\pi)^2} G_0(\Omega, q) v_\Gamma \Gamma G_0(\Omega, q) \gamma_0 \times D^B(\Omega, q). \quad (C6)$$

For RSP with $\Gamma = \gamma_0$, $V_B$ is

$$V^B = v_\gamma \gamma_0 \left( -C^B_0 \right) \ell. \quad (C7)$$

For the two components of RVP defined by $\Gamma = \gamma_1$ and $\Gamma = \gamma_2$, we obtain

$$V^B = v_1 \gamma_1 \left( -C^B_1 \right) \ell, \quad (C8)$$

and

$$V^B = v_\gamma \gamma_2 \left( -C^B_2 \right) \ell, \quad (C9)$$

3. Vertex correction from disorder

The vertex correction due to disorder has the form

$$V^{\text{dis}} = \Delta v_B^2 \int \frac{d^2 p}{(2\pi)^2} \Gamma G_0(0, k) v_\Gamma \Gamma G_0(0, k) \Gamma$$

$$= v_\Gamma \Delta v_B^2 \int \frac{d^2 p}{(2\pi)^2} \frac{1}{(v_1 k_1^2 + v_2 k_2^2)^2} \times \Gamma(v_1 k_1 \gamma_1 + v_2 k_2 \gamma_2) \Gamma(v_1 k_1 \gamma_1 + v_2 k_2 \gamma_2) \Gamma. \quad (C11)$$

For RSP with $\gamma = \gamma_0$, $V^{\text{dis}}$ is

$$V^{\text{dis}} = v_\gamma \gamma_0 C g \ell. \quad (C12)$$

For the two components of RVP defined by $\gamma = \gamma_1$ and $\gamma_2$, $V^{\text{dis}}$ is

$$V^{\text{dis}} = 0. \quad (C13)$$

For RM with $\Gamma = 1$, $V^{\text{dis}}$ is

$$V^{\text{dis}} = -v_\Gamma \Gamma C g \ell. \quad (C14)$$

Appendix D: Derivation of the coupled RG equations

The action for the free fermions is given by

$$S_{\Psi} = \sum_{\sigma=1}^N \int \frac{d\omega}{2\pi} \frac{d^2 k}{(2\pi)^2} \bar{\Psi}_\sigma(\omega, k) \left( -i\omega + v_1 k_1 \gamma_1 + v_2 k_2 \gamma_2 \right) \Psi_\sigma(\omega, k). \quad (D1)$$

Including the fermion self-energies induced by excitonic quantum fluctuation, Coulomb interaction, and disorder scattering, the action of fermions becomes

$$S_{\Psi} = \sum_{\sigma=1}^N \int \frac{d\omega}{2\pi} \frac{d^2 k}{(2\pi)^2} \bar{\Psi}_\sigma(\omega, k) \left[ -i\omega \gamma_0 + v_1 k_1 \gamma_1 + v_2 k_2 \gamma_2 - \Sigma^A(\omega, k) - \Sigma^B(\omega, k) - \Sigma^{\text{dis}}(\omega) \right] \Psi_\sigma(\omega, k)$$

$$\approx \sum_{\sigma=1}^N \int \frac{d\omega}{2\pi} \frac{d^2 k}{(2\pi)^2} \bar{\Psi}_\sigma(\omega, k) \left[ -i\omega \gamma_0 e^{-C^A_0 - C^B_0 \ell} + v_1 k_1 \gamma_1 e^{\left( C^A_1 + C^B_1 \ell \right)} + v_2 k_2 \gamma_2 e^{\left( C^B_2 + C^A_2 \ell \right)} \right] \Psi_\sigma(\omega, k). \quad (D2)$$

Making the following re-scaling transformations:

$$\omega = \omega' e^{-\ell}, \quad (D3)$$
$$k_1 = k_1' e^{-\ell}, \quad (D4)$$
$$k_2 = k_2' e^{-\ell}, \quad (D5)$$
$$\Psi = \Psi' e^{\frac{C^A_0 + C^B_0 - C^A_1 e^{\ell}}{2}}.$$. 

$$\ell$$.
\[ v_1 = v'_1 e^{-(c^A_0 - c^B_0 + c^A_1 + c^B_1 + c_s) \ell}, \]  
\[ v_2 = v'_2 e^{-(c^A_0 - c^B_0 + c^A_2 + c^B_2 + c_s) \ell}, \]

the fermion action is re-written as

\[ S_{\psi'} = \sum_{\sigma=1}^{N} \int \frac{d\omega'}{2\pi} \frac{d^2 k'}{(2\pi)^2} \bar{\psi}'_\sigma(\omega', k') \left[ -i\omega' \gamma_0 + v'_1 k'_1 \gamma_1 + v'_2 k'_2 \gamma_2 \right] \psi'_\sigma(\omega', k'), \]

which recovers the form of the original action.

The action for the fermion-disorder coupling is

\[ S_{\text{dis}} = \sum_{\sigma=1}^{N} \int \frac{d\omega}{2\pi} \frac{d^2 k}{(2\pi)^2} \frac{d^2 k_1}{(2\pi)^2} \bar{\psi}_\sigma(\omega, k) \psi_\sigma(\omega, k_1) A(k - k_1), \]

After taking into account the quantum corrections, it becomes

\[ S_{\text{dis}} = \sum_{\sigma=1}^{N} \int \frac{d\omega}{2\pi} \frac{d^2 k}{(2\pi)^2} \frac{d^2 k_1}{(2\pi)^2} \bar{\psi}_\sigma(\omega, k) \left( v_1 \Gamma + V^A + V^B + V_{\text{dis}} \right) \psi_\sigma(\omega, k_1) A(k - k_1). \]

In the case of RSP, we obtain

\[ S_{\text{dis}} = \sum_{\sigma=1}^{N} \int \frac{d\omega}{2\pi} \frac{d^2 k}{(2\pi)^2} \frac{d^2 k_1}{(2\pi)^2} \bar{\psi}_\sigma(\omega, k) \left( v_1 \Gamma + (C^A_0 - C^B_0 + c_s) \ell \right) \psi_\sigma(\omega, k_1) A(k - k_1), \]

For the two components of RVP, \( S_{\text{dis}} \) is expressed as

\[ S_{\text{dis}} = \sum_{\sigma=1}^{N} \int \frac{d\omega}{2\pi} \frac{d^2 k}{(2\pi)^2} \frac{d^2 k_1}{(2\pi)^2} \bar{\psi}_\sigma(\omega, k) \left[ v_1 \gamma_1 + v_1 \gamma_1 \left( C^A_1 \right) \ell + v_1 \gamma_1 \left( C^B_1 \right) \ell \right] \psi_\sigma(\omega, k_1) A(k - k_1), \]

and

\[ S_{\text{dis}} = \sum_{\sigma=1}^{N} \int \frac{d\omega}{2\pi} \frac{d^2 k}{(2\pi)^2} \frac{d^2 k_1}{(2\pi)^2} \bar{\psi}_\sigma(\omega, k) \left[ v_2 \gamma_2 + v_2 \gamma_2 \left( C^A_2 \right) \ell + v_2 \gamma_2 \left( C^B_2 \right) \ell \right] \psi_\sigma(\omega, k_1) A(k - k_1), \]

respectively. For RM, \( S_{\text{dis}} \) is cast in the form

\[ S_{\text{dis}} = \sum_{\sigma=1}^{N} \int \frac{d\omega}{2\pi} \frac{d^2 k}{(2\pi)^2} \frac{d^2 k_1}{(2\pi)^2} \bar{\psi}_\sigma(\omega, k) \left[ v_1 \mathbb{I} + v_1 \mathbb{I} \left( C^A_0 + C^A_1 + C^A_2 \right) \ell + v_1 \mathbb{I} \left( C^B_0 - C^B_1 - C^B_2 \right) \ell - v_1 \mathbb{I} C_y \ell \right] \times \psi_\sigma(\omega, k_1) A(k - k_1), \]

\[ = \sum_{\sigma=1}^{N} \int \frac{d\omega}{2\pi} \frac{d^2 k}{(2\pi)^2} \bar{\psi}_\sigma(\omega, k) v_1 \mathbb{I} e^{(C^A_0 + C^A_1 + C^A_2 + C^B_0 - C^B_1 - C^B_2 - c_s) \ell} \psi_\sigma(\omega, k_1) A(k - k_1). \]

We then employ the re-scaling transformations given by Eqs. (D3)–(D6). The random potential \( A(k) \) should be re-scaled as follows

\[ A(k) = A'(k') e^{\ell}. \]
The parameter $v_\Gamma$ is re-scaled as

$$v_\Gamma = v'_\Gamma,$$

for Eq. (D12).

$$v_\Gamma = v'_\Gamma e^{(-C^A_0 - C^B_0 + C^A_1 + C^B_1) \ell}$$

(D18)

for Eq. (D13),

$$v_\Gamma = v'_\Gamma e^{(-C^A_0 - C^B_0 + C^A_2 + C^B_2) \ell}$$

(D19)

for Eq. (D14), and

$$v_\Gamma = v'_\Gamma e^{(-2C^A_0 - C^A_1 - C^A_2 - 2C^B_0 + C^B_1 + C^B_2 + 2C^g) \ell}$$

(D20)

for Eq. (D15). After carrying out the above manipulations, we re-write the action for fermion-disorder coupling as follows

$$S_{\text{dis}} = \sum_{\sigma=1}^{N} \int \frac{d\omega'}{2\pi} \frac{d^2k'}{(2\pi)^2} \int \frac{d^2k''}{(2\pi)^2} \bar{\Psi}^{\dagger}_\sigma(\omega', k') v'_\Gamma \Psi^{\dagger}_\sigma(\omega', k''_1) A'(k' - k''_1),$$

(D21)

which restores the form of the original action.

From Eqs. (D6), we obtain the RG equation for the quasiparticle $Z_f$

$$\frac{dZ_f}{d\ell} = (C^A_0 + C^B_0 - C^g) Z_f.$$

(D22)

According to Eqs. (D7) and (D8), the RG equations for $v_1$ and $v_2$ are given by

$$\frac{dv_1}{d\ell} = (C^A_0 + C^B_0 - C^A_1 - C^B_1 - C^g) v_1,$$

(D23)

$$\frac{dv_2}{d\ell} = (C^A_0 + C^B_0 - C^A_2 - C^B_2 - C^g) v_2.$$

(D24)

The RG equation for the velocity ratio $v_2/v_1$ can be readily derived:

$$\frac{d}{d\ell} \left( \frac{v_2}{v_1} \right) = \frac{dv_2}{dv_1} \frac{v_2}{v_1} = \frac{(C^A_1 - C^A_2 + C^B_1 - C^B_2) v_2}{v_1}.$$

(D25)

Based on Eqs. (D17)-(D20), we obtain the RG equation for the parameter $v_\Gamma$

$$\begin{cases} 
\frac{dv_1}{d\ell} = 0 & \text{RSP,} \\
\frac{dv_1}{d\ell} = (C^A_0 + C^B_0 - C^A_1 - C^B_1 - C^g) v_\Gamma & \gamma_1 \text{ component of RVP} \\
\frac{dv_1}{d\ell} = (C^A_0 + C^B_0 - C^B_2 - C^g) v_\Gamma & \gamma_2 \text{ component of RVP} \\
\frac{dv_1}{d\ell} = (2C^A_0 + C^A_1 + C^A_2 + 2C^B_0 - C^B_1 - 2C^B_2 - 2C^g) v_\Gamma & \text{RM}
\end{cases}$$

(D26)

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