Nature of Chiral Transition in QCD and Sigma Meson

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Abstract

After showing the characteristic features of the chiral transition in QCD at finite temperature $T$, we note the significance of the sigma-mesonic mode in the chiral transition: Sigma meson may be regarded as the “Higgs particle” in QCD, though the existence of such a mode in the real world is still unclear. We show the properties of the sigma meson in hot and/or hadronic matter, from which we propose experiments to reveal the existence the sigma meson clearly; the experiments include relativistic heavy-ion collisions to create hot hadronic matter, and electro-production in heavy nuclei.
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Abstract

We summarize the present status of the theoretical understanding of the characteristic features of the chiral transition in QCD at finite temperature $T$. We emphasize the significance of the sigma-mesonic mode in the chiral transition: Sigma meson may be regarded as the “Higgs particle” in QCD, though the existence of such a mode in the real world is still unclear. We show the properties of the sigma meson in hot and/or hadronic matter, from which we propose experiments to reveal the existence the sigma meson clearly; the experiments include relativistic heavy-ion collisions to create hot hadronic matter, and electro-production in heavy nuclei.

1 Introduction

The basic viewpoint which underlies the present report is that a change in the ground state (vacuum) as caused by that of the environment may reflect in changes of properties of elementary excitations, hadrons in the case of QCD. The salient features of QCD vacuum are (1) absence of free quarks and colored gluons, (2) dynamical breaking of chiral symmetry, axial anomaly, approximate flavor-$SU(3)$ symmetry and so on. In this report, we shall examine how, if any, properties of Nambu-Goldstone bosons ($\pi, K, \eta$) and the scalar meson $\sigma$ would change when the chiral symmetry is getting restored. The present report is based on Ref. [1], done mostly in collaboration with T. Hatsuda.

1.1 Significance of Sigma meson in the chiral symmetry breaking in QCD

What is sigma meson? Sigma meson is iso-scalar and scalar meson with a low mass $\sim 600 - 700$ MeV. More specifically, sigma meson we refer to is the quantum fluctuation of the order parameter $<\bar{q}q>$ of the chiral transition. Thus in a sense, sigma meson is the Higgs particle in the chiral symmetry of QCD: In the standard electro-weak theory, the expectation value of a scalar field called Higgs field is the order parameter for the spontaneous breaking of the $SU(2) \otimes U(1)$ symmetry, and the quantum fluctuation of the field in the new vacuum is the so called Higgs particle. So we should seek sigma meson to demonstrate that the real world is realized due to the dynamical breaking of chiral symmetry, as eagerly as high-energy experimentalists search the Higgs particle.
Actually, the expected mass (the real part of the mass) of sigma meson is about $600 \sim 700 \text{ MeV}$, which could be seen in the phase shift of the $\pi-\pi$ scattering in the $I = J = 0$ channel. However, the experimental phase shift does not show the expected resonance behaviour for the center-of-mass energy below 900 MeV; this is the main source of a skeptical view about the existence of sigma meson. The resolution of the skepticism is rather simple, though; the strong coupling of sigma with $2\pi$ gives rise a large imaginary mass or a large width $\Gamma_{\sigma} \sim 450 \text{ MeV}$ of $\sigma$, because of which the energy where the phase shift cut $90^\circ$ will be shifted up to about 1 GeV where the experimental phase shift shows a resonance behavior with a so complicated structure that the actual particle content contained in this region are not well understood yet. Thus in the real world at $T = 0$, an experimental demonstration of sigma meson might be very difficult. Our point is, however, that once one can create a high-temperature and/or high-density system by relativistic heavy-ion collisions, for example, one would have a good chance to see sigma meson as a sharp! resonance.

### 1.2 Effective lagrangian approach

When one approaches these problems, some effective lagrangian or model would be desirable in which the salient features of QCD are embodied. Simulations of lattice QCD bear so strong constraints on the computing ability that it is still awkward to compute various quantities freely, especially the dynamical aspects of the system. One of the merits of the studies based on effective theories lies in the fact that a physical idea or view can be easily put in a calculation and thereby helps us getting physical insight into the problems under consideration: These will be helpful in future simulations on lattice QCD with forthcoming super computers. Of course, it is also desirable if one can obtain a ‘BCS’ theory for chiral symmetry breaking from QCD directly.

Our calculation is based on the generalized Nambu-Jona-Lasinio model which embodies the explicit breaking of $SU_f(3)$ symmetry as given by the flavor-dependent current quark masses, and the $U_A(1)$ anomaly given by the determinantal six-fermion interaction.

The lagrangian we take is the following:

\[
\mathcal{L}_{NJL} = \bar{q}(i\gamma \cdot \partial - m)q + \sum_{a=0}^{8} \frac{g_5}{2} \left[ (\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5\lambda_a q)^2 \right] + g_0 [\text{det}\bar{q}_i(1 - \gamma_5)q_j + \text{h.c.}],
\]

\[
\equiv \mathcal{L}_0 + \mathcal{L}_S + \mathcal{L}_{SB} + \mathcal{L}_D,
\]

where the quark field $q_i$ has three colors ($N_c = 3$) and three flavors ($N_f = 3$), $\lambda_a$ ($a=0\sim8$) are the Gell-Mann matrices with $\lambda_0 = \sqrt{\frac{2}{3}} \mathbf{1}$. The second term is the explicit $SU_f(3)$-breaking part with $m = \text{diag}(m_u, m_d, m_s)$ being the current quark mass matrix. The last term is a reflection of the axial anomaly of QCD, which has the $SU_L(3) \otimes SU_R(3)$-invariance but breaks the $U_A(1)$-symmetry. This term gives rise to mixings of the different flavors both in the scalar and pseudo-scalar channels in the mean field approximation.
It is noteworthy that the NJL model can be cast into a form of a linear sigma model by integrating out the quark fields, and hence a non-linear sigma model with the parameters in the Lagrangian fixed by the few parameters of the underlying NJL model. The relation among QCD, NJL model and sigma models for chiral transition has a good analogy with QED for electrons and ions, BCS theory, and phenomenological Ginzburg-Landau model, as shown below:

QED $\rightarrow$ BCS $\rightarrow$ Ginzburg-Landau model

$\uparrow$ (effective theory) $\uparrow$ (integrating out fermions) $\uparrow$

QCD $\rightarrow$ NJL $\rightarrow$ $\sigma$ models

2 Static Properties

Recent lattice simulations show that the order and even the existence of the phase transition(s) are largely dependent on the number of the flavors especially when the physical current quark masses are used\cite{8}: For $m_u \sim m_d \sim 10 \text{MeV} \ll 100 \text{MeV} \sim m_s$, the phase transition may be weak 1st order or 2nd order or not exist.

The gross feature of the $T$ dependence and the striking difference between the condensates of $u$ ($d$) quark and the $s$ quark can be well described by the NJL model\cite{9}. It is noteworthy that at high temperatures, the flavor $SU_f(3)$-symmetry gets worse badly\cite{2}, which may reflect in the baryon and the vector meson spectra, because they are well described by the constituent quark models.

The calculations based on the NJL model show \cite{3,2} that the variation of the non-strange condensate with temperature is very large, while that of the strange quark is moderate. This contrast between the non-strange and the strange sectors is also reflected in the change of the constituent quark masses. Hence one sees that the $SU_f(3)$-symmetry is no more a good symmetry even approximately at temperatures larger than 150 MeV because the restoration of the chiral symmetry in the different sectors is achieved quite differently. One may also recognize that the approximate $SU_f(3)$-symmetry seen at $T = \mu_i = 0$ is rather accidentally realized by the spontaneous breaking of the chiral symmetry\cite{4}.

Even away from the problem of the $SU_f(3)$-symmetry, the possible change of the spectra of baryons and vector mesons such as $\rho, \omega$ and $\phi$ mesons might provide us with a good signature of the formation of hot hadronic matter: For instance, if one applies the naive quark model to the vector mesons, our results tell us that these mesons would decrease their masses as $T$ is raised, and the rate of the change are larger in the non-strange vector mesons than in $\phi$ meson. However the manifestation of the change might be more drastic for $\phi$ meson because with a small change of the mass ($\sim 30$ MeV), $m_\phi$ gets into the subthreshold to the process of $\phi \rightarrow 2K$.

\footnote{In discussing the flavor symmetry in terms of the constituent quark masses, we are clearly taking not only the constituent quark picture of hadrons but also the view that the constituent quark masses may be identified with the masses generated by the spontaneous breaking of the chiral symmetry.}
3 Dynamic Properties — Collective Excitations —

Lattice simulations[3] and effective theories [1, 10] predict the existence of \( \sigma \) meson, the mass of which decreases as \( T \) is raised till \( T_c \), a “critical temperature”: \( m_\pi \) is found to be constant as long as \( T < T_c \). It should be noted here that the lattice simulations only give the screening masses, i.e., the mass-like parameters of the space-correlations of the hadronic composite operators, while the effective theories such as the NJL model can give real masses as well as screening ones.

The calculations with the NJL model [1, 2, 10] show that pion hardly changes its mass but only starts become heavy near \( T \sim 200 \text{ MeV} \), while sigma meson, which is found to be dominantly composed of the nonstrange sigma meson \( \sigma_{NS} \sim (\bar{u}u + \bar{d}d)/\sqrt{2} \), decreases the mass \( m_\sigma(T, \rho_B) \) as the chiral symmetry gets restored and eventually \( m_\sigma(T) \) becomes smaller than twice of pion mass \( m_\pi \) at a temperature \( T_\sigma \sim 190 \text{ MeV} \). This means that the width \( \Gamma_\sigma \) of \( \sigma \) meson due to the process \( \sigma \to \pi\pi \) vanishes at \( T_\sigma \). It means that \( \sigma \) meson would appear as a sharp resonance at high temperature, though the large width \( \Gamma_\sigma \sim 500 \text{ MeV} \) at \( T = 0 \) prevent us from seeing \( \sigma \) meson clearly at \( T = 0 \) [1]. Thus one sees that the results obtained in the two-flavor case [1] are confirmed in the three-flavor case with the axial anomaly incorporated.

The behavior of kaon at finite temperature was examined by the present author with use of the \( SU(3) \)-NJL model including the anomaly term[2]. It was found that as long as the system is in the NG phase, the mass of kaon \( m_K(T) \) keeps almost a constant, the value at \( T = 0 \).

Then how about hadronic excitations at \( T > T_c \). It is remarkable that there seem exist colorless hadronic excitations even in the high-\( T \) phase[10] contrary to the naive picture of it. There should exist precursory soft modes in the high temperature phase prior to the phase transition if the chiral transition is of second order or weak first order: The soft modes are actually fluctuations of the order parameter of the phase transition, \( \langle\langle (\bar{q}i\gamma_5q)^2\rangle\rangle \) and hence \( \langle\langle (\bar{q}i\gamma_5\tau q)^2\rangle\rangle \) due to the chiral symmetry. We demonstrated these using an effective theory of QCD.[10]

The lattice simulations [11] showed that the screening masses of pion and sigma meson are both well below \( 2\pi T \), which indicates that the interactions between q-q in the pseudo-scalar and the scalar channels are still rather strong even in the high-\( T \) phase, as suggested in the NJL model.

As for the correlations in the vector channel, see [12].

\( T_c \) may be defined as the temperature at which \( m_\pi \) starts to go high.
4 Implication to Experiment

sigma meson I The sigma meson would decrease the mass while $m_\pi$ keeps constant at high temperatures. This suggests that at high temperatures the decay $\sigma \rightarrow 2\pi$ would get suppressed and finally hindered, and then only the electro-magnetic process $\sigma \rightarrow 2\gamma$ is allowable. It means that sigma meson may show up as a sharp resonance with the mass $m_\sigma \sim 2m_\pi$. Thus we propose to observe $\pi^+\pi^-$, $2\pi^0$, $2\gamma$ and construct the invariant mass and examine whether there is a bump in the mass region 300 to 400 MeV.

sigma meson II Recently, Weldon find that in the charged system, the process $\sigma \rightarrow \gamma \rightarrow 2\text{leptons}$ is possible, because $\pi^+$ and $\pi^-$ have different chemical potentials, respectively. The detection of lepton pairs would be hopeful because they interact with the matter only weakly in comparison with hadrons.

5 Summary and concluding remarks

We have discussed possible character change of several hadrons especially sigma meson. The change is associated with the chiral restoration. We have seen that sigma meson would appear as a sharp resonance, decaying only by the electromagnetic process $\sigma \rightarrow \gamma\gamma$ with an only tiny width and a low mass. Therefore it would be interesting to detect $2\gamma$'s with invariant masses of several hundred MeV in relativistic heavy ion collisions.

Recently there are some suggestions that the effects of chiral transition might be more significant at finite baryonic density than at finite T. At finite baryon density, there arises a vector-scalar coupling as is well-known in the $\sigma$-$\omega$ model. On account of the coupling, it is possible to create sigma meson in a nucleus by electron-nucleus scattering, due to the process $\gamma^* (\text{virtual}) \rightarrow \sigma$. By measuring the decay products from $\sigma$ such as $2\pi(\pi^\pm, 2\pi^0)$ or $2\ell^\pm$, one would be able to see rather sharp resonance of the sigma meson. To make the experiment meaningful, one should examine the processes with a nucleon being emitted simultaneously for momentum matching. Furthermore, to avoid the large decay product from $\rho$ meson, the detection of neutral pions would give clearer data for sigma meson. Such experiment should be feasible in CEBAF.

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