Case-Based Merging Techniques in OAKPLAN

FUB

Technical Report

Abstract

Case-based planning can take advantage of former problem-solving experiences by storing in a plan library previously generated plans that can be reused to solve similar planning problems in the future. Although comparative worst-case complexity analyses of plan generation and reuse techniques reveal that it is not possible to achieve provable efficiency gain of reuse over generation, we show that the case-based planning approach can be an effective alternative to plan generation when similar reuse candidates can be chosen.

Keywords: Case-Based Planning, Domain-Independent Planning, Merging Techniques, Case-Based Reasoning, Heuristic Search for Planning, Kernel Functions.
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1 Introduction
This report describes a case-based planning system called OAKPLAN, with a special emphasis on newly implemented merging techniques. This extension was motivated by the observation that, especially in some domains, the performance of the system can be greatly improved by remembering some elementary solutions for a simple problems, which can be combined to address significantly more complex scenarios.

2 Preliminaries
In the following we present some notations that will be used in the paper with an analysis of the computational complexity of the problems considered.

2.1 Planning formalism
Similarly to Bylander’s work [9], we define an instance of propositional planning problem as:

Definition 1 An instance of propositional STRIPS planning problem is a tuple $\Pi = (P_r, I, G, Op)$ where:

- $P_r$ is a finite set of ground atomic propositional formulae;
- $I \subseteq P_r$ is the initial state;
- $G \subseteq P_r$ is the goal specification;
- $Op$ is a finite set of operators, where each operator $o \in Op$ has the form $o^p \Rightarrow o_+, o_-$ such that
  - $o^p \subseteq P_r$ are the propositional preconditions,
  - $o_+ \subseteq P_r$ are the positive postconditions (add list),
  - $o_- \subseteq P_r$ are the negative postconditions (delete list)

and $o_+ \cap o_- = \emptyset$.

We assume that the set of propositions $P_r$ has a particular structure. Let $O$ be a set of typed constants $c_i$, with the understanding that distinct constants denote distinct objects (corresponding to individual entities following the Conceptual Graphs notation [12]). Let $P$ be a set of predicate symbols, then $P_r(O, P)$ is the set of all ground atomic formulae over this signature. Note that we use a many-sorted logic formalisation since it can significantly increase the efficiency of a deductive inference system by eliminating useless branches of the search space of a domain [13, 14, 61]. A fact is an assertion that some individual entities exist and that these entities are related by some relationships.

A plan $\pi$ is a partially ordered sequence of actions $\pi = (a_1, \ldots, a_m, C)$, where $a_i$ is an action (completely instantiated operator) of $\pi$ and $C$ defines the ordering relations between the actions of $\pi$. A linearisation of a partially ordered plan is a total order over the actions of the plan that is consistent with the existing partial order. In a totally ordered plan $\pi = (a'_1, \ldots, a'_m)$, a precondition $f$ of a plan action $a'_j$ is supported if (i) $f$ is added by an earlier action $a'_j$ and not deleted by an intervening action $a'_k$ with $j < k < i$ or (ii) $f$ is true in the initial state and $\neg a'_k$. 

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with \( k < i \) s.t. \( f \in \text{del}(a'_k) \). In a partially ordered plan, a precondition of an action is \textit{possibly supported} if there exists a linearisation in which it is supported, while an action precondition is \textit{necessarily supported} if it is supported in all linearisations. An action precondition is \textit{necessarily unsupported} if it is not possibly supported. A \textit{valid plan} is a plan in which all action preconditions are necessarily supported.

The complexity of STRIPS planning problems has been studied extensively in the literature. Bylander \[9\] has defined PLANSAT as the decision problem of determining whether an instance \( \Pi \) of propositional STRIPS planning has a solution or not. PLANMIN is defined as the problem of determining if there exists a solution of length \( k \) or less, i.e., it is the decision problem corresponding to the problem of generating plans with minimal length. Based on this framework, he has analysed the computational complexity of a general propositional planning problem and a number of generalisations and restricted problems. In its most general form, both PLANSAT and PLANMIN are PSPACE-complete. Severe restrictions on the form of the operators are necessary to guarantee polynomial time or even NP-completeness \[9\].

To address the high complexity of the planning problems, different heuristical approaches arise. The case based approach relies on encountering problems similar to each other and tries to reuse previously found plans to solve new problems. If successful, this can save a considerable amount of resources. Clearly, to apply such a technique, we need similar problems to have similar solution — in other words, we need the world to be \textit{regular}. When solving a new problem, the planner searches previously solved problems and retrieves the most suitable one which is then adapted to solve the current problem.

In general, solved problems are stored in a case base, which is a collection of cases; a case is a pair consisting of a problem description and its solution. Following the formalisation of Liberatore \[37\], we define a case base as follows:

\[ \text{Definition 2} \quad \text{A case base, or a plan library, is a set } \{\Pi_i, \pi_i\} \text{ where each } \{\Pi_i, \pi_i\} \text{ is a planning case with } \Pi_i \text{ being an instance of propositional STRIPS planning problem and } \pi_i \text{ a solution to } \Pi_i. \]

Note that different planners implement cases differently — several solution plans may be stored instead of just one, justifications may be added, some planners even avoid storing a plan as a set of actions and store its derivational trace instead.

In order to realise the benefits of remembering and reusing past solutions, a case-based system needs to efficiently implement several procedures, such as those for retrieving analogous cases (Retrieve), adapting them to suit the new problem (Reuse, Revise), and building and maintaining a case base of sufficient size and coverage to yield useful analogues (Retain). When a case-based system faces a new problem, it performs these procedures in a sequence that starts by querying the case base and ends by (possibly) updating it (Fig. 1).

Regardless of the underlying formalisation, there are two main approaches to case-based planning, whose fundamental difference is in the way they adapt the cases to solve the current problem. A \textit{conservative} or a \textit{transformational} plan adaptation insists on reusing as much of the starting plan \( \pi_i \) as possible. It turns out that such adaptation may be very expensive. Moreover, the quality of the solution strongly depends on the correspondence between the retrieved case and the current problem, which is influenced by the way the case is retrieved from the case base as well as by the case base itself, or rather by its competence. Intuitively, a \textit{competence} is a feature of the case base referring to the variety of problems for which the case
The system queries the case base to retrieve case(s) useful for solving the problem \( \Pi \) (retriever); the solution(s) from the retrieved case(s) are applied to \( \Pi \) in order to find its solution \( \pi \) (reuse); the new solution is revised to identify and correct possible flaws (revise); the verified solution \( \pi \) and the problem \( \Pi \) are used to create a new case that may be introduced to the case base, in addition the system may be capable of auto-analysis of the case base when reaching some limit conditions (retain).

Essential to this trivial result is that, similarly to most modern plan adaptation and case-based planning approaches [3, 24, 56, 57], we do not enforce plan adaptation to be conservative, in the sense that we do not require to reuse as much of the starting plan \( \pi^* \) to solve the new plan. The computational complexity of plan Reuse and Modification for STRIPS planning problems has been analysed in a number of papers [9, 10, 35, 37, 44].

Moreover empirical analyses show that plan modification for similar planning instances is somewhat more efficient than plan generation in the average case [8, 18, 23, 24, 44, 54, 57].
It is crucial that the system has at its disposal a competent case base. Intuitively, the competence of the case base measures how often the case base provides a good reuse candidate. Of course, the number of successful reuses also depends on other components of the system (how exactly is the case base queried, what is the reuse strategy, when do we consider the solution a “successful reuse”, etc.).

The competence of the case base grows as we keep adding solutions to new diverse problems, but only to a certain point when the query over the case base takes too much time to be answered. If the system works in the environment where the problems tend to be very complex, it is unfortunately very unlikely that the case base could contain sufficient number of cases to cover majority or at least a significant part of the kinds of problems to be solved. One of the promising directions for such scenarios is to attempt to build a case base of “elementary” cases which address only simple sub-problems and which are combined together to address also complex problems. The number of such elementary problems is lower and hence it is easier to achieve a competent case base.

Clearly such modification needs to be accompanied by the change of the retrieval policy — we no longer look for the most similar case, but for a suitable set of partial solutions. The partial solutions are then put together; depending on the interaction between the subgoals a simple concatenation may be sufficient, but in case the subgoals interfere the concatenation may be very inefficient (e.g. in the logistics domain).

2.2 Graphs

Graphs provide a rich means for modelling structured objects and they are widely used in real-life applications to represent molecules, images, or networks. On a very basic level, a graph can be defined by a set of entities and a set of connections between these entities. Due to their universal definition, graphs have found widespread application for many years as tools for the representation of complex relations. Furthermore, it is often useful to compare objects represented as graphs to determine the degree of similarity between the objects. More formally:

**Definition 3** A labeled graph $G$ is a 3-tuple $G = (V, E, \lambda)$, in which

- $V$ is the set of vertices,
- $E \subseteq V \times V$ is the set of directed edges or arcs,
- $\lambda : V \cup E \to \mathcal{P}(L_\lambda)$ is a function assigning labels to vertices and edges;

where $L_\lambda$ is a finite set of symbolic labels and $\mathcal{P}(L_\lambda)$ represents the set of all the multisets on $L_\lambda$. Note that our label function considers multisets of symbolic labels, with the corresponding operations of union, intersection and join [6], since in our context they are more suitable than standard sets of symbolic labels in order to compare vertices or edges accurately as described later. The above definition corresponds to the case of directed graphs; undirected graphs are obtained if we require for each edge $[v_1, v_2] \in E$ the existence of an edge $[v_2, v_1] \in E$ with the same label. $|G| = |V| + |E|$ denotes the size of the graph $G$, while an empty graph such that $|G| = 0$ will be denoted by $\emptyset$. An arc $e = [v, u] \in E$ is considered to be directed from $v$ to $u$; $v$ is called the source node and $u$ is called the target node of the arc; $u$ is said to be a direct successor of $v$, $v$ is said to be a direct predecessor of $u$, while $v$ is said to be adjacent to the vertex $u$ and vice versa.

Here we present the notion of graph union which is essential for the definition of the graphs used by our matching functions:
Definition 4 The union of two graphs $G_1 = (V_1, E_1, \lambda_1)$ and $G_2 = (V_2, E_2, \lambda_2)$, denoted by $G_1 \cup G_2$, is the graph $G = (V, E, \lambda)$ defined by

- $V = V_1 \cup V_2$.
- $E = E_1 \cup E_2$.
- $\lambda(x) = \begin{cases} 
\lambda_1(x) & \text{if } x \in (V_1 \setminus V_2) \cup (E_1 \setminus E_2) \\
\lambda_2(x) & \text{if } x \in (V_2 \setminus V_1) \cup (E_2 \setminus E_1) \\
\lambda_1(x) \lor \lambda_2(x) & \text{otherwise}
\end{cases}$

where $\lor$ indicates the join, sometimes called sum, of two multisets [6], while $\lambda(\cdot)$ associates a multiset of symbolic labels to a vertex or to an edge.

In many applications it is necessary to compare objects represented as graphs and determine the similarity among these objects. This is often accomplished by using graph matching, or isomorphism techniques. Graph isomorphism can be formulated as the problem of identifying a one-to-one correspondence between the vertices of two graphs such that an edge only exists between two vertices in one graph if an edge exists between the two corresponding vertices in the other graph. Graph matching can be formulated as the problem involving the maximum common subgraph (MCS) between the collection of graphs being considered. This is often referred to as the maximum common substructure problem and denotes the largest substructure common to the collection of graphs under consideration. More precisely:

Definition 5 Two labeled graphs $G = (V, E, \lambda)$ and $G' = (V', E', \lambda')$ are isomorphic if there exists a bijective function $f : V \to V'$ such that

- $\forall v \in V, \lambda(v) = \lambda'(f(v))$.
- $\forall [v_1, v_2] \in E, \lambda([v_1, v_2]) = \lambda'([f(v_1), f(v_2)])$.
- $[u, v] \in E$ if and only if $[f(u), f(v)] \in E'$.

We shall say that $f$ is an isomorphism function.

Definition 6 An Induced Subgraph of a graph $G = (V, E, \lambda)$ is a graph $S = (V', E', \lambda')$ such that

- $V' \subseteq V$ and $\forall v \in V', \lambda'(v) \in \lambda(v)$.
- $E' \subseteq E$ and $\forall e \in E', \lambda'(e) \subseteq \lambda(e)$.
- $\forall v, u \in V', [v, u] \in E'$ if and only if $[v, u] \in E$.

A graph $G$ is a Common Induced Subgraph (CIS) of graphs $G_1$ and $G_2$ if $G$ is isomorphic to induced subgraphs of $G_1$ and $G_2$. A common induced subgraph $G = (V, E, \lambda)$ of $G_1$ and $G_2$ is called Maximum Common Induced Subgraph (MCIS) if there exists no other common induced subgraph of $G_1$ and $G_2$ with $\sum_{v \in V} |\lambda(v)|$ greater than $G$. Similarly, a common induced subgraph $G = (V, E, \lambda)$ of $G_1$ and $G_2$ is called Maximum Common Edge Subgraph (MCES), if there exists no other common induced subgraph of $G_1$ and $G_2$ with $\sum_{e \in E} |\lambda(e)|$ greater than $G$. Note that, since we are considering multiset labeled graphs, we require a stronger condition than standard MCIS and MCES for labeled graph, in fact we want to maximise the
total cardinality of the multiset labels of vertices/edges involved instead of the simple number of vertices/edges.

As it is well known, subgraph isomorphism and MCS between two or among more graphs are NP-complete problems [19], while it is still an open question if also graph isomorphism is an NP-complete problem. As a consequence, worst-case time requirements of matching algorithms increase exponentially with the size of the input graphs, restricting the applicability of many graph based techniques to very small graphs.

2.3 Kernel Functions for Labeled Graphs

In recent years, a large number of graph matching methods based on different matching paradigms have been proposed, ranging from the spectral decomposition of graph matrices to the training of artificial neural networks and from continuous optimisation algorithms to optimal tree search procedures.

The basic limitation of graph matching is due to the lack of any mathematical structure in the space of graphs. Kernel machines, a new class of algorithms for pattern analysis and classification, offer an elegant solution to this problem [48]. The basic idea of kernel machines is to address a pattern recognition problem in a related vector space instead of the original pattern space. That is, rather than defining mathematical operations in the space of graphs, all graphs are mapped into a vector space where these operations are readily available. Obviously, the difficulty is to find a mapping that preserves the structural similarity of graphs, at least to a certain extent. In other words, if two graphs are structurally similar, the two vectors representing these graphs should be similar as well, since the objective is to obtain a vector space embedding that preserves the characteristics of the original space of graphs.

A key result from the theory underlying kernel machines states that an explicit mapping from the pattern space into a vector space is not required. Instead, from the definition of a kernel function it follows that there exists such a vector space embedding and that the kernel function can be used to extract the information from vectors that is relevant for recognition. As a matter of fact, the family of kernel machines consists of all the algorithms that can be formulated in terms of such a kernel function, including standard methods for pattern analysis and classification such as principal component analysis and nearest-neighbour classification. Hence, from the definition of a graph similarity measure, we obtain an implicit embedding of the entire space of graphs into a vector space.

A kernel function can be thought of as a special similarity measure with well defined mathematical properties [48]. Considering the graph formalism, it is possible to define a kernel function which measures the degree of similarity between two graphs. Each structure could be represented by means of its similarity to all the other structures in the graph space. Moreover a kernel function implicitly defines a dot product in some space [48]; i.e., by defining a kernel function between two graphs we implicitly define a vector representation of them without the need to explicitly know about it.

From a technical point of view a kernel function is a special similarity measure $k : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$ between patterns lying in some arbitrary domain $\mathcal{X}$, which represents a dot product, denoted by $\langle \cdot, \cdot \rangle$, in some Hilbert space $\mathcal{H}$ [48]; thus, for two arbitrary patterns $x, x' \in \mathcal{X}$ it holds that $k(x, x') = \langle \phi(x), \phi(x') \rangle$, where $\phi : \mathcal{X} \to \mathcal{H}$ is an arbitrary mapping of patterns from the domain $\mathcal{X}$ into the feature space $\mathcal{H}$. In principle the patterns in domain $\mathcal{X}$ do not necessarily have to be vectors; they could be strings, graphs, trees, text documents or other objects. The vector representation of these objects is then given by the map $\phi$. Instead of performing the expensive transformation step explicitly, the kernel can be calculated directly, thus performing
the feature transformation only implicitly: this is known as kernel trick. This means that any set, whether a linear space or not, that admits a positive definite kernel can be embedded into a linear space.

More specifically, kernel methods manage non-linear complex tasks making use of linear methods in a new space. For instance, take into consideration a classification problem with a training set $S = \{(u_1, y_1), \ldots, (u_n, y_n)\}$, $(u_i, y_i) \in \mathcal{X} \times Y$, for $i = 1, \ldots, n$, where $\mathcal{X}$ is an inner-product space (i.e. $\mathbb{R}^d$) and $Y = \{-1, +1\}$. In this case, the learning phase corresponds to building a function $f \in Y^{\mathcal{X}}$ from the training set $S$ by associating a class $y \in Y$ to a pattern $u \in \mathcal{X}$ so that the generalisation error of $f$ is as low as possible.

A functional form for $f$ consists in the hyperplane $f(u) = \text{sign}(\langle w, u \rangle + b)$, where $\text{sign}(\cdot)$ refers to the function returning the sign of its argument. The decision function $f$ produces a prediction that depends on which side of the hyperplane $\langle w, u \rangle + b = 0$ the input pattern $u$ lies. The individuation of the best hyperplane corresponds to a convex quadratic optimisation problem in which the solution vector $w$ is a linear combination of the training vectors:

$$w = \sum_{i=1}^{n} \alpha_i y_i u_i,$$

for some $\alpha_i \in \mathbb{R}^+$, $i = 1, \ldots, n$.

In this way the linear classifier $f$ may be rewritten as

$$f(u) = \text{sign}\left(\sum_{i=1}^{n} \alpha_i y_i \langle u_i, u \rangle + b\right)$$

As regards complex classification problems, the set of all possible linear decision surfaces might not be rich enough in order to provide a good classification, independently from the values of the parameters $w \in \mathcal{X}$ and $b \in \mathbb{R}$ (see Figure 2). The aim of the kernel trick is that of overcoming this limitation by adopting a linear approach to transformed data $\phi(u_1), \ldots, \phi(u_n)$ rather than raw data. Here $\phi$ indicates an embedding function from the input space $\mathcal{X}$ to a feature space $\mathcal{H}$, provided with a dot product. This transformation enables us to give an alternative kernel representation of the data which is equivalent to a mapping into a high-dimensional space where the two classes of data are more readily separable. The mapping is achieved through a replacement of the inner product:

$$\langle u_i, u \rangle \rightarrow \langle \phi(u_i), \phi(u) \rangle$$

![Figure 2: The kernel approach for classification. Left: non-linearly separable input provided by dots and crosses. Middle: perfect or approximate linear-separability can be achieved in feature space via the mapping $\phi$. Right: linear decision surface in feature space defines a complex decision surface in input space.](image-url)
and the separating function can be rewritten as:

\[ f(u) = \text{sign} \left( \sum_{i=1}^{n} \alpha_i y_i \langle \phi(u_i), \phi(u) \rangle + b \right) \]  

(1)

The main idea behind the kernel approach consists in replacing the dot product in the feature space using a kernel \( k(u, v) = \langle \phi(v), \phi(u) \rangle \); the functional form of the mapping \( \phi(\cdot) \) does not actually need to be known since it is implicitly defined by the choice of the kernel. A positive definite kernel [20] is:

**Definition 7** Let \( \mathcal{X} \) be a set. A symmetric function \( k: \mathcal{X} \times \mathcal{X} \to \mathbb{R} \) is a positive definite kernel function on \( \mathcal{X} \) iff \( \forall n \in \mathbb{N}, \forall x_1, \ldots, x_n \in \mathcal{X}, \) and \( \forall c_1, \ldots, c_n \in \mathbb{R} \)

\[ \sum_{i,j \in \{1,\ldots,n\}} c_i c_j k(x_i, x_j) \geq 0 \]

where \( \mathbb{N} \) is the set of positive integers. For a given set \( S_u = \{u_1, \ldots, u_n\} \), the matrix \( K = (k(u_i, u_j))_{i,j} \) is known as Gram matrix of \( k \) with respect to \( S_u \). Positive definite kernels are also called Mercer kernels.

**Theorem 1 (Mercer's property [41])** For any positive definite kernel function \( k \in \mathbb{R}^{\mathcal{X} \times \mathcal{X}} \), there exists a mapping \( \phi \in \mathcal{H}^\mathcal{X} \) into the feature space \( \mathcal{H} \) equipped with the inner product \( \langle \cdot, \cdot \rangle_{\mathcal{H}} \), such that:

\[ \forall u, v \in \mathcal{X}, \quad k(u, v) = \langle \phi(u), \phi(v) \rangle_{\mathcal{H}} \]

The kernel approach replaces all inner products in Equation 1 and all related expressions to compute the real coefficients \( \alpha_i \) and \( b \), by means of a Mercer kernel \( k \). For any input pattern \( u \), the relating decision function \( f \) is given by:

\[ f(u) = \text{sign} \left( \sum_{i=1}^{n} \alpha_i y_i k(u_i, u) + b \right) \]  

(2)

This approach transforms the input patterns \( u_1, \ldots, u_n \) into the corresponding vectors \( \phi(u_1), \ldots, \phi(u_n) \) \( \in \mathcal{H} \) through the mapping \( \phi \in \mathcal{H}^\mathcal{X} \) (cf. Mercer’s property, Theorem 1), and uses hyperplanes in the feature space \( \mathcal{H} \) for the purpose of classification (see Figure 2). The dot product \( \langle u, v \rangle = \sum_{i=1}^{d} u_i v_i \) of \( \mathbb{R}^d \) is actually a Mercer kernel, while other commonly used Mercer kernels, like polynomial and Gaussian kernels, generally correspond to nonlinear mappings \( \phi \) into high-dimensional feature spaces \( \mathcal{H} \). On the other hand the Gram matrix implicitly defines the geometry of the embedding space and permits the use of nonlinear techniques in the feature space so as to derive complex decision surfaces in the input space \( \mathcal{X} \).

While it is not always easy to prove positive definiteness for a given kernel, positive definite kernels are characterised by interesting closure properties. More precisely, they are closed under sum, direct sum, multiplication by a scalar, tensor product, zero extension, pointwise limits, and exponentiation [48]. Well-known examples of kernel functions are:

- **Radial Basis Functions**
  \[ k_{RBF}(x, x') = \exp \left( -\frac{\|x-x'\|^2}{2\sigma^2} \right); \]

- **Homogeneous polynomial kernels**
  \[ k_{poly}(x, x') = \langle x, x' \rangle^d \ (d \in \mathbb{N}); \]

- **Sigmoidal kernels**
  \[ k_{Sig}(x, x') = \tanh \left( \kappa(x \cdot x') + \theta \right) \]

- **Inv. multiquadratic kernels**
  \[ k_{inv}(x, x') = \frac{1}{\sqrt{\|x-x'\|^2 + \epsilon^2}} \]
A remarkable contribution to graph kernels is the work on convolution kernels, that provides a general framework to deal with complex objects consisting of simpler parts [30]. Convolution kernels derive the similarity of complex objects from the similarity of their parts. Given two kernels $k_1$ and $k_2$ over the same set of objects, new kernels may be built by using operations such as convex linear combinations and convolutions. The convolution of $k_1$ and $k_2$ is a new kernel $k$ with the form

$$k_1 \ast k_2(u, v) = \sum_{u_1, v_1; u_2, v_2} k_1(u_1, v_1)k_2(u_2, v_2)$$

where $u = \{u_1, u_2\}$ refers to a partition of $u$ into two substructures $u_1$ and $u_2$ [30, 48]. The kind of substructures depends on the domain of course and could be, for instance, subgraphs or subsets or substrings in the case of kernels defined over graphs, sets or strings, respectively. Different kernels can be obtained by considering different classes of subgraphs (e.g. directed/undirected, labeled/unlabeled, paths/trees/cycles, deterministic/random walks) and various ways of listing and counting them [34, 45, 46]. The consideration of space and time complexity so as to compute convolution/spectral kernels is important, owing to the combinatorial explosion linked to variable-size substructures.

In the following section we present our Optimal Assignment Kernel as a symmetric and positive definite similarity measure for directed graph structures and it will be used in order to define the correspondence between the vertices of two directed graphs. For an introduction to kernel functions related concepts and notation, the reader is referred to Scholkopf and Smola’s book [48].

### 3 Case-Based Planning and OAKPLAN

Here we provide a detailed description of the case-based approach to planning and its implementation in the state-of-the-art CBP system OAKPLAN.

A case-based planning system solves planning problems by making use of stored plans that were used to solve analogous problems. CBP is a type of case-based reasoning, which involves the use of stored experiences (cases); moreover there is strong evidence that people frequently employ this kind of analogical reasoning [21, 47, 60]. When a CBP system solves a new planning problem, the new plan is added to its case base for potential reuse in the future. Thus we can say that the system learns from experience.

In general the following steps are executed when a new planning problem must be solved by a CBP system:

1. **Plan Retrieval** to retrieve cases from memory that are analogous to the current (target) problem (see section 3.1 for a description of our approach).

2. **Plan Evaluation** to evaluate the new plans by execution, simulated execution, or analysis and choose one of them (see section 3.2).

3. **Plan Adaptation** to repair any faults found in the new plan (see section 3.3).

4. **Plan Revision** to test the solution new plan $\pi$ for success and repair it if a failure occurs during execution (see section 3.4).

5. **Plan Storage** to eventually store $\pi$ as a new case in the case base (see section 3.4).
In order to realise the benefits of remembering and reusing past plans, a CBP system needs efficient methods for retrieving analogous cases and for adapting retrieved plans together with a case base of sufficient size and coverage to yield useful analogues. The ability of the system to search in the library for a plan suitable to adaptation\(^1\) depends both on the efficiency/accuracy of the implemented retrieval algorithm and on the data structures used to represent the elements of the case base.

A planning case of the case base corresponds to a planning problem \(\Pi\) (defined by an initial state \(I\), a goal state \(G\) and a set of operators \(O\)) a solution \(\pi\) of \(\Pi\) and additional data structures derived by \(\Pi\) and stored in the case base so as to avoid their recomputation. The case base competence should increase progressively during the use of the case base system itself, every time solution plans enhancing the competence of the case base are produced.

The possibility of solving a large number of problems depends both on the size and on the competence of the library with respect to the agent activity. Furthermore this competence could be increased during the agent activity, in fact the solution plans of new problems could be added to the library.

Similarly to the Aamodt & Plaza’s classic model of the problem solving cycle in CBR [1], Figure 3 shows the main steps of our case-based planning cycle and the interactions of the different steps with the case base. In the following we illustrate the main steps of our case-based planning approach, examining the different implementation choices adopted.

### 3.1 Plan Retrieval

Although the plan adaptation phase is the central component of a CBP system, the retrieval phase critically affects the system performance too. As a matter of fact the retrieval time is a component of the total adaptation time and the quality of the retrieved plan is fundamental for the performance of the successive adaptation phase. With OAKPLAN a number of functions for the management of the plan library and matching functions for the selection of the candidate plan for adaptation have been implemented.

\(^{1}\)A plan suitable to adaptation has an adaptation cost that is lower with respect to the other candidates of the case base and with respect to plan generation.
The retrieval phase has to consider all the elements of the plan library in order to choose a good one that will allow the system to solve the new problem easily. Hence it is necessary to design a similarity metric and reduce the number of cases that must be evaluated accurately so as to improve the efficiency of the retrieval phase. Anyway the efficiency of a plan adaptation system is undoubtedly linked to the distance between the problem to solve and the plan to adapt. In order to find a plan which is useful for adaptation we have to reach the following objectives:

- The retrieval phase must identify the candidates for adaptation. The retrieval time should be as small as possible as it will be added to the adaptation time and so particular attention has been given to the creation of efficient data structures for this phase.
- The selected cases should actually contain the plans that are easier to adapt; since we assume that the world is regular, i.e. that similar problems have similar solutions, we look for the cases that are the most similar to the problem to solve (with respect to all the other candidates of the case base). In this sense, it is important to define a metric able to give an accurate measure of the similarity between the planning problem to solve and the cases of the plan library.

To the end of applying the reuse technique, it is necessary to provide a plan library from which “sufficiently similar” reuse candidates can be chosen. In this case, “sufficiently similar” means that reuse candidates have a large number of initial and goal facts in common with the new instance. However, one may also want to consider the reuse candidates that are similar to the new instance after the objects of the selected candidates have been systematically renamed. As a matter of fact, every plan reuse system should contain a matching component that tries to find a mapping between the objects of the reuse candidate and the objects of the new instance such that the number of common goal facts is maximised and the additional planning effort to achieve the initial state of the plan library is minimised. Following Nebel & Koehler’s formalisation [44], we will have a closer look at this matching problem.

### 3.1.1 Object Matching

As previously said we use a many-sorted logic in order to reduce the search space for the matching process; moreover we assume that the operators are ordinary STRIPS operators using variables, i.e. we require that if there exists an operator \( o_k \) mentioning the typed constants \( \{c_1 : t_1, ..., c_n : t_n\} \subseteq O \), then there also exists an operator \( o_l \) over the arbitrary set of typed constants \( \{d_1 : t_1, ..., d_n : t_n\} \subseteq O \) such that \( o_l \) becomes identical to \( o_k \) if the \( d_i \)'s are replaced by \( c_i \)'s. If there are two instances

\[
\Pi' = \langle \mathcal{P}(O', P'), \mathcal{I}', \mathcal{G}', \mathcal{O}_P \rangle
\]

\[
\Pi = \langle \mathcal{P}(O, P), \mathcal{I}, \mathcal{G}, \mathcal{O}_P \rangle
\]

such that (without loss of generality)

\[
O' \subseteq O
\]

\[
P' = P
\]

\[
\mathcal{O}_P \subseteq \mathcal{O}_P
\]
then a mapping, or matching function, from \( \Pi' \) to \( \Pi \) is a function

\[
\mu : O' \to O
\]

The mapping is extended to ground atomic formulae and sets of such formulae in the canonical way, i.e.,

\[
\mu(p(c_1 : t_1, ..., c_n : t_n)) = p(\mu(c_1) : t_1, ..., \mu(c_n) : t_n)
\]

\[
\mu(\{p_1(\ldots), ..., p_m(\ldots)\}) = \{\mu(p_1(\ldots)), ..., \mu(p_m(\ldots))\}
\]

If there exists a bijective matching function \( \mu \) from \( \Pi' \) to \( \Pi \) such that \( \mu(G') = G \) and \( \mu(I') = I \), then it is obvious that a solution plan \( \pi' \) for \( \Pi' \) can be directly reused for solving \( \Pi \) since \( \Pi' \) and \( \Pi \) are identical within a renaming of constant symbols, i.e., \( \mu(\pi') \) solves \( \Pi \). Even if \( \mu \) does not match all goal and initial-state facts, \( \mu(\pi') \) can still be used as a starting point for the adaptation process that can solve \( \Pi \).

In order to measure the similarity between two objects, it is intuitive and usual to compare the features which are common to both objects [39]. The Jaccard similarity coefficient used in information retrieval is particularly interesting. Here we examine an extended version that considers two pairs of disjoint sets:

\[
\text{complete_simil}_\mu(\Pi', \Pi) = \frac{|\mu(G') \cap G| + |\mu(I') \cap I|}{|\mu(G') \cup G| + |\mu(I') \cup I|}
\]

(3)

In the following we present a variant of the previous function so as to overcome the problems related to the presence of irrelevant facts in the initial state description of the current planning problem \( \Pi \) and additional goals that are present in \( \Pi' \). In fact while the irrelevant facts can be filtered out from the initial state description of the case-based planning problem \( \Pi' \) using the corresponding solution plan \( \pi' \), this is not possible for the initial state description of the current planning problem \( \Pi \). Similarly, we do not want to consider possible “irrelevant” additional goals of \( G' \); this could happen when \( \Pi' \) solves a more difficult planning problem with respect to \( \Pi \). We define the following similarity function so as to address these issues:

\[
simil_\mu(\Pi', \Pi) = \frac{|\mu(G') \cap G| + |\mu(I') \cap I|}{|G| + |\mu(I')|}.
\]

(4)

Using \( \text{simil}_\mu \) we obtain a value equal to 1 when there exists a mapping \( \mu \) s.t. \( \forall f \in I', \ \mu(f) \in I \) (to guarantee the applicability of \( \pi' \)) and \( \forall g \in G, \ \exists g' \in G' \) s.t. \( g = \mu(g') \) (to guarantee the achievement of the goals of the current planning problem). Note that these similarity functions are not metric functions, although we could define a distance function in terms of the similarity as \( \text{Dist}_\mu(\Pi', \Pi) = 1 - \text{simil}_\mu(\Pi', \Pi) \). and it easy to show that this distance function is indeed a metric.

Finally we define the following optimisation problem, which we call \text{obj_match}:

**Instance:** Two planning instances, \( \Pi' \) and \( \Pi \), and a real number \( k \in [0, 1] \).

**Question:** Does a mapping \( \mu \) from \( \Pi' \) to \( \Pi \) such that \( \text{simil}_\mu(\Pi', \Pi) = k \) exist and there is no mapping \( \mu' \) from \( \Pi' \) to \( \Pi \) with \( \text{simil}_{\mu'}(\Pi', \Pi) > k \)?

It should be noted that this matching problem has to be solved for each potentially relevant candidate in the plan library to select the corresponding best reuse candidate. Of course, one may use structuring and indexing techniques to avoid considering all plans in the library. Nevertheless, it seems unavoidable solving this problem a considerable number of times before.
an appropriate reuse candidate is identified. For this reason, the efficiency of the matching component is crucial for the overall system performance. Unfortunately, similarly to Nebel & Koehler’s analysis [44], it is quite easy to show that this matching problem is an NP-hard problem.

**Theorem 2** \( \text{obj_match} \) is NP-hard.

The proof of this theorem and of the following ones can be found in Appendix ???. This NP-hardness result implies that matching may be indeed a bottleneck for plan reuse systems. As a matter of fact, it seems to be the case that planning instances with complex goal or initial-state descriptions may not benefit from plan-reuse techniques because matching and retrieval are too expensive. In fact existing similarity metrics address the problem heuristically, considering approximations of it [43, 58]. However, this theorem is interesting because it captures the limit case for such approximations.

**Planning Encoding Graph.** We define a particular labeled graph data structure called *Planning Encoding Graph* which encodes the initial and goal facts of a single planning problem \( \Pi \) to perform an efficient matching between the objects of a planning case and the objects of the current planning problem. The vertices of this graph belong to a set \( V_\Pi \) whose elements are the representation of the objects \( O \) of the current planning problem \( \Pi \) and of the predicate symbols \( P \) of \( \Pi \):

\[
V_\Pi = O \cup \bigcup_{p \in P} I_p \cup \bigcup_{q \in P} G_q
\]

i.e. for each predicate we define two additional nodes, one associated to the corresponding initial fact predicate called \( I_p \) and the other associated to the corresponding goal fact predicate called \( G_q \). The labels of this graph are derived from the predicates of our facts and the sorts of our many-sorted logic. The representation of an entity (an object using planning terminology) of the application domain is traditionally called a *concept* in the conceptual graph community [12]. Following this notation a *Planning Encoding Graph* is composed of three kinds of nodes: *concept* nodes representing entities (objects) that occur in the application domain, *initial fact relation* nodes representing relationships that hold between the objects of the initial facts and *goal fact relation* nodes representing relationships that hold between the objects of the goal facts.

The *Planning Encoding Graph* of a planning problem \( \Pi(I,G) \) is built using the corresponding initial and goal facts. In particular for each propositional initial fact \( p = p(c_1 : t_1, ..., c_n : t_n) \in I \) we define a data structure called *Initial Fact Encoding Graph* which corresponds to a graph that represents \( p \). More precisely:

**Definition 8** Given a propositional typed initial fact \( p = p(c_1 : t_1, ..., c_n : t_n) \in I \) of \( \Pi \), the *Initial Fact Encoding Graph* \( E^I(p) = (V_p, E_p, \lambda_p) \) of fact \( p \) is a directed labeled graph where

- \( V_p = \{I_p, c_1, ..., c_n\} \subseteq V_\Pi \);
- \( E_p = \{[I_p, c_1], [c_1, c_2], [c_1, c_3], ..., [c_1, c_n], [c_2, c_3], [c_2, c_4], ..., [c_{n-1}, c_n]\} = \bigcup_{i=1, ..., n} \bigcup_{j=i+1, ..., n} [c_i, c_j]\)
- \( \lambda_p(I_p) = \{I_p\}, \lambda_p(c_i) = \{t_i\} \) with \( i = 1, ..., n \);
Figure 4: Initial Fact Encoding Graph $E^I(p)$ of the propositional initial fact $p = p(c_1 : t_1, ..., c_n : t_n)$

Initial fact: 

\[
\begin{align*}
\lambda(I_p) &= \{(I_p, 1)\} = \{I_p\}, \quad \lambda(c_1) = \{(t_1, 1)\} = \{t_1\}, ..., \lambda(c_n) = \{(t_n, 1)\} = \{t_n\} \\
\end{align*}
\]

\[
\text{Figure 5: Initial Fact Encoding Graph } E^I(\text{on } A \text{ B}) \text{ of the propositional initial fact } (\text{on } A \text{ B}).
\]

\[
\begin{align*}
\lambda(I_{on}) &= \{I_{on}\}, \quad \lambda(A) = \{Obj\}, \quad \lambda(B) = \{Obj\}
\end{align*}
\]

- $\lambda_p([I_p, c_1]) = \{I_p^{0,1}\}; \quad \forall [c_i, c_j] \in E_p, \lambda_p([c_i, c_j]) = \{I_p^{i,j}\};$

  i.e. the first node of the graph $E^I(p)$, see Figure 4, is the initial fact relation node $I_p$ labeled with the multiset $\lambda_p(I_p) = \{(I_p, 1)\} = \{I_p\}$, it is connected to a direct edge to the second node of the graph, the concept node $c_1$, which is labeled by sort $t_1$ (i.e. $\lambda_p(c_1) = \{(t_1, 1)\} = \{t_1\}$); the node $c_1$ is connected with the third node of the graph $c_2$ which is labeled by sort $t_2$ (i.e. $\lambda_p(c_2) = \{(t_2, 1)\} = \{t_2\}$) and with all the remaining concept nodes, the third node of the graph $c_2$ is connected with $c_3$, $c_4$, ..., $c_n$ and so on. The first edge of the graph $[I_p, c_1]$ is labeled by the multiset $\{I_p^{0,1}\}$. Similarly a generic edge $[c_i, c_j] \in E_p$ is labeled by the multiset $\{I_p^{i,j}\}$.

For example, in Figure 5 we can see the Initial Fact Encoding Graph of the fact “$p = (\text{on A B})$” of the BlocksWorld domain. The first node is named as “$I_{on}$” and its label is the multiset $\lambda_p(I_{on}) = \{(I_{on}, 1)\} = \{I_{on}\}$, the second node represents the object “A” with label $\lambda_p(A) = \{(Obj, 1)\} = \{Obj\}$ and finally the third node represents the object “B” and its label is $\lambda_p(B) = \{Obj\}$; the label of the $[I_{on}, A]$ arc is the multiset $\{I_{on}^{0,1}\} = \{I_{on}^{0,1}\}$ and the label of the $[A, B]$ arc is the multiset $\{I_{on}^{1,2}\} = \{I_{on}^{1,2}\}$.

Similarly to Definition 8 we define the Goal Fact Encoding Graph $E^G(q)$ of the fact $q = q(c'_1 : t'_1, ..., c'_m : t'_m) \in G$ using $\{G_q\}$ for the labeling procedure.

Given a planning problem $\Pi$ with initial and goal states $I$ and $G$, the Planning Encoding Graph of $\Pi$, that we indicate as $E_{\Pi}$, is a directed labeled graph derived by the encoding graphs of the initial and goal facts:

\[
E_{\Pi}(I, G) = \bigcup_{p \in I} E^I(p) \cup \bigcup_{q \in G} E^G(q)
\]

\[
\text{(5)}
\]

\[\text{In the following we indicate the multiset } \{(x, 1)\} \text{ as } \{x\} \text{ for sake of simplicity.}\]
Figure 6: Planning Encoding Graph for the Sussman Anomaly planning problem in the BlocksWorld domain.

\[ \lambda(A) = \{(\text{Obj}, 3)\}, \lambda(B) = \{(\text{Obj}, 4)\} \text{ and } \lambda(C) = \{(\text{Obj}, 3)\} \]

Moreover it could be useful to point out that if an object \( c \) appears more than once in an initial (goal) fact \( p(c_1, \ldots, c_n) \) of a planning problem \( \Pi \), then the corresponding Initial (Goal) Fact Encoding Graph is built as usual (instantiating \( n \) nodes, one each \( c_i \)), while during the construction of the Planning Encoding Graph obtained by merging the Initial and Goal Encoding Graphs of \( \Pi \), the nodes that correspond to the same object are merged into a single vertex node.

\(^3\)Following the conceptual graph notation, the first and third level nodes correspond to initial and goal fact relation nodes, while the nodes of the second level correspond to concept nodes representing the objects of the initial and goal states.
Algorithm RELAXEDPLAN

Input: a set of goal facts \( G \), the set of facts that are true in the current state \( INIT \), a possibly empty relaxed plan \( A \)

Output: a relaxed plan \( ACTS \) estimating a minimal set of actions required to achieve \( G \)

1. \( G = G - INIT \); \( ACTS = A \)
2. \( F = \bigcup_{a \in ACTS} Add(a) \)
3. while \( G - F \neq \emptyset \)
4. \( g = \) “a fact in \( G - F \)”
5. \( bestact = \text{Bestaction}(g) \)
6. \( Rplan = \text{RelaxedPlan}(Pre(bestact), INIT, ACTS) \)
7. \( ACTS = \text{Asel}(Rplan) \cup \{ bestact \} \)
8. \( F = \bigcup_{a \in ACTS} Add(a) \)
9. return \( ACTS \)

Figure 7: Algorithm for computing a relaxed plan estimating a minimal set of actions required to achieve a set of facts \( G \) from the state \( INIT \). \text{Bestaction}(g) \) is the action that is heuristically chosen to support \( g \) as described in [22].

This graph representation can give us a detailed description of the “topology” of a planning problem without requiring any a priori assumptions on the relevance of certain problem descriptors for the whole graph. Furthermore it allows us to use Graph Theory based techniques in order to define effective matching functions. In fact a matching function from II’ to II can be derived by solving the Maximum Common Subgraph problem on the corresponding Planning Encoding Graphs. A number of exact and approximate algorithms have been proposed in the literature so as to solve this graph problem efficiently. With respect to normal conceptual graphs [12] used for Graph-based Knowledge Representation, we use a richer label representation based on multisets. A single relation node is used to represent each predicate of the initial and goal facts which reduces the total number of nodes in the graphs considerably. This is extremely important from a computational point of view since, as we will see in the following sections, the matching process must be repeated several times and it directly influences the total retrieval time.

In the following we examine a procedure based on graph degree sequences that is useful to derive an upper bound on the size of the MCES of two graphs in an efficient way. Then we present an algorithm based on Kernel Functions that allows to compute an approximate matching of two graphs in polynomial time.

3.2 Plan Evaluation Phase

The purpose of plan evaluation is that of defining the capacity of a plan \( \pi \) to resolve a particular planning problem. It is performed by simulating the execution of \( \pi \) and identifying the unsupported preconditions of its actions; in the same way the presence of unsupported goals is identified. The plan evaluation function could be easily defined as the number of inconsistencies in the current planning problem. Unfortunately this kind of evaluation considers a uniform cost in order to resolve the different inconsistencies and this assumption is generally too restrictive. Then our system considers a more accurate inconsistency evaluation criterion so as to improve the plan evaluation metric. The inconsistencies related to unsupported facts are evaluated by computing a relaxed plan starting from the corresponding state and using the RELAXEDPLAN algorithm in LPG [22]. The number of actions in the relaxed plan determines
Algorithm EvaluatePlan

Input: a planning problem $\Pi = (I, G)$, an input plan $\pi$ and an adaptation cost limit $C_{\text{limit}}$

Output: a relaxed plan to adapt $\pi$ in order to resolve $\Pi$

1. $C_{\text{State}} = I$; $R_{\text{plan}} = \emptyset$
2. forall $a \in \pi_i$ do
3.   if $\exists f \in \text{Pre}(a) \setminus f \notin C_{\text{State}}$ then
4.      $R_{\text{plan}} = \text{RELAXEDPLAN}(\text{Pre}(a), C_{\text{State}}, R_{\text{plan}})$
5.   if $|R_{\text{plan}}| > C_{\text{limit}}$ then
6.      return $R_{\text{plan}}$
7. $C_{\text{State}} = (C_{\text{State}} \setminus \text{Del}(a)) \cup \text{Add}(a)$
8.   if $\exists g \in G \setminus g \notin C_{\text{State}}$ then
9.      $R_{\text{plan}} = \text{RELAXEDPLAN}(G, C_{\text{State}}, R_{\text{plan}})$
10. return $R_{\text{plan}}$

---

Figure 8: Algorithm to evaluate the ability of $\pi$ to solve the planning problem $\Pi$

the difficulty to make the selected inconsistencies supported; the number of actions in the final relaxed plan determines the accuracy of the input plan $\pi$ to solve the corresponding planning problem.

Figure 7 describes the main steps of the RELAXEDPLAN function. It constructs a relaxed plan through a backward process where $\text{Bestaction}(g)$ is the action $a'$ chosen to achieve a (sub)goal $g$, and such that: (i) $g$ is an effect of $a'$; (ii) all preconditions of $a'$ are reachable from the current state $\text{INIT}$; (iii) the reachability of the preconditions of $a'$ requires a minimum number of actions, evaluated as the maximum of the heuristically estimated minimum number of actions required to support each precondition $p$ of $a'$ from $\text{INIT}$; (iv) $a'$ subverts a minimum number of supported precondition nodes in $A$ (i.e., the size of the set $\text{Threats}(a')$ is minimal).

Figure 8 describes the main steps of the EVALUATEPLAN function. For all actions of $\pi$ (if any), it checks if at least one precondition is not supported. In this case it uses the RELAXEDPLAN algorithm (step 4) so as to identify the additional actions required to satisfy the unsupported preconditions. If $R_{\text{plan}}$ contains a number of actions greater than $C_{\text{limit}}$ we can stop the evaluation, otherwise we update the current state $C_{\text{State}}$ (step 7). Finally we examine the goal facts $G$ (step 8) to identify the additional actions required to satisfy them, if necessary.

In order to improve the efficiency of the system and reuse as many possible parts of previously executed plans we have adopted plan merging techniques [?], which are based on the well-known divide and conquer strategy.

In order to apply this strategy, our system must accomplish two further subtasks: problem decomposition and plan merging. The problem decomposition is performed identifying the set of actions and the initial facts needed for a single goal and storing them in the case base as a new problem instance (if not already present); moreover these new instances remain related to the original solution plan in order to maintain a statistic of their effective usage. The stored (sub)cases are then used in the merging phase in order to identify a single global plan that satisfies all goals. We progressively identify the unsatisfied goals and the corresponding (sub)cases that allow to satisfy them, giving the preference to the (sub)plans that allow us to improve the plan metric and that have been successful in a greater number of times in analogous situations.

---

4RELAXEDPLAN is described in detail in [22]. It also computes an estimation of the earliest time when all facts in $G$ can be achieved, which is not described in this paper for sake of simplicity.
Algorithm MERGE SUBPLANS
Input: A planning problem \( \Pi(I, G) \), a plan library \( \mathcal{L} = (\Pi_i, \pi_i) \);
Output: A (quasi) solution plan \( \pi \) for \( \Pi \);
1. \( \pi = \arg\min_{\pi_i \in \mathcal{L}} \text{EvPlan}(I, \pi_i, G) \);
2. repeat
3. \forall \text{unsatisfied facts } f \in \{G \cup \text{prec}(\pi)\} \text{ do}
   3.1 Let \( \pi_f \in \mathcal{L} \) be the best plan that satisfies \( f \) s.t.
      \( \text{EvPlan}(I, \text{merge}(\pi, \pi_f), G) < \text{EvPlan}(I, \pi, G) \);
   3.2 if \( \pi_f \neq \emptyset \) then
   3.3 \( \pi = \text{merge}(\pi, \pi_f) \); break;
4. until \( \pi_f \neq \emptyset \);
5. return \( \pi \);

Figure 9: Algorithm for merging the elements in the library in order to solve a planning problem \( \Pi \).

3.2.1 Application of plan merging techniques

We have used case-based plan merging techniques to store plans. Moreover, in order to reuse as many as possible parts of previously executed plans, we decompose the solution plans into subparts that allow us to satisfy every single goal or a set of interrelated goals and we store these subparts in the case base, if they are not already present.

When a new e-learning planning problem must be solved, we search in the case base if a plan that already solves all goals exists. If such a plan does not exist we apply plan merging techniques that progressively identify (sub)plans in the case base that can satisfy the goals. This phase consists in reusing parts of the retrieved plans to complete a new one. Figure 9 describes the process for merging plans of the library in order to find a plan \( \pi \) that solves the current planning problem \( \Pi \) or that represents a quasi-solution \( \pi^{?} \) for it. At step 1 we search in the library the plan that satisfies all the goals with the lowest heuristic adaptation cost, where the function \( \text{EvPlan}(I, \pi, G) \) determines the adaptation effort by estimating the number of actions that are necessary to transform \( \pi \) into a solution of the problem \( \Pi(I, G) \).\(^5\) This step corresponds to the extraction of the best plan of the library (if it exists) as proposed by the standard OAKPLAN system. At steps 3.x, we progressively analyse the unsatisfied goals and the unsatisfied preconditions of the current plan \( \pi \), trying to identify in the library a subplan \( \pi_f \) that can be merged with \( \pi \) in order to satisfy \( f \) (and other unsatisfied facts if possible) and reduce, at the same time, the global heuristic adaptation cost, where \( \text{merge} \) identifies the best part of \( \pi \) where the actions of \( \pi_f \) can be inserted in producing a new global plan.\(^6\) If such a plan exists, we merge it with \( \pi \) at step 3.3 and we restart from step 3 reconsidering all the unsatisfied facts. The repeat loop halts when all the goals and preconditions are satisfied, i.e. when we have found a solution plan, or when there is not a suitable plan that can be extracted from the library that satisfies the remaining unsupported facts. In this case, the plan \( \pi \) does not represent a solution plan. However, it can be used as a starting point for a local search process to find a solution plan for the current planning problem.

Figure 10 describes the main steps of the retrieval phase. We initially compute a relaxed plan \( \pi_R \) for \( \Pi \) (step 1.1) using the \text{EVALUATEPLAN} function on the empty plan which is needed so as to define the generation cost of the current planning problem \( \Pi \) (step 4.1)\(^7\) and an estimate of the initial state relevant facts (step 1.2). In fact we use the relaxed plan \( \pi_R \) so as to filter out

---

\(^5\)See \text{EVALUATEPLAN} for a more detailed description.
\(^6\)In our tests we have considered the earliest and the latest part of \( \pi \) where \( f \) can be satisfied.
\(^7\)The \( \alpha_G \) coefficient gives more or less importance to plan adaptation vs plan generation; if \( \alpha_G > 1 \) then it is more likely to perform plan adaptation than plan generation.
1.2. Define the set of initial relevant facts of $\Pi$ using $\pi$: $I_{\pi_R} = I \cap \bigcup_{a \in \pi_R} \text{pre}(a)$.

Then in step 1.3 the Planning Encoding Graph of the current planning problem $\Pi$ and the degree sequences that will be used in the screening procedure are precomputed. Note that the degree sequences are computed considering the Planning Encoding Graph $\mathcal{E}_{\Pi_R}$ of the planning problem $\Pi_R(I_{\pi_R}, G)$ which uses $I_{\pi_R}$ instead of $I$ as initial state. This could be extremely useful in practical applications when automated tools are used to define the initial state description without distinguishing among relevant and irrelevant initial facts.

Steps 1.4 – 1.7 examine all the planning cases of the case base so as to reduce the set of candidate plans to a suitable number. It is important to point out that in this phase it is not necessary to retrieve the complete planning encoding graphs of the case base candidates $G_{IV}$ but only their sorted degree sequences $L^1_{IV}$ which are precomputed and stored in the case base.

---

Algorithm RETRIEVEPLAN

Input: a planning problem $\Pi$, a case base $C = (\Pi_i, \pi_i)$

Output: candidate plan for the adaptation phase

1.1. $\pi_R = \text{EVALUATE_PLAN}(\Pi, \text{EMPTY_PLAN}, \infty)$

1.2. Define the set of initial relevant facts of $\Pi$ using $\pi$: $I_{\pi_R} = I \cap \bigcup_{a \in \pi_R} \text{pre}(a)$

1.3. Compute the Planning Encoding Graphs $\mathcal{E}_{\Pi}$ and $\mathcal{E}_{\Pi_R}$ of $\Pi(I, G)$ and $\Pi_R(I_{\pi_R}, G)$ respectively, and the degree sequences $L^1_{\Pi_R}$

1.4. For all $\Pi_i \in C$ do

1.5. $\text{simil}_i = \text{simil}^\Pi(\mathcal{E}_{\Pi_i}, \mathcal{E}_{\Pi_R})$

1.6. $\text{push}((\Pi_i, \text{simil}_i), \text{queue})$

1.7. $\text{best_DS_simil} = \max(\text{best_DS_simil}, \text{simil}_i)$

2.1. Load the Planning Encoding Graph $\mathcal{E}_{\Pi}$, and compute the matching function $\mu_{\text{base}}$ using $K_{\text{base}}(\mathcal{E}_{\Pi}, \mathcal{E}_{\Pi})$

2.2. For all $(\Pi_i, \mu_{\text{base}}) \in \text{queue}_1$ s.t. $\text{best_DS_simil} - \text{simil}_i \leq \text{limit}$ do

3.1. If $\text{simil}_{\mu'}(\Pi_i, \Pi) \geq \text{simil}_{\mu_{\text{base}}}(\Pi_i, \Pi)$ then $\mu_i = \mu'$

3.2. For all $(\Pi_i, \mu_i) \in \text{queue}_2$ s.t. $\text{best_DS_simil} - \text{simil}_{\mu}(\Pi_i, \Pi) \leq \text{limit}$ do

4.1. $\text{best_cost} = \alpha \cdot |\pi_R| \cdot \text{best_plan} = \text{EMPTY_PLAN}$

4.2. For all $(\Pi_i, \mu_i) \in \text{queue}_2$ s.t. $\text{best_DS_simil} - \text{simil}_{\mu_i}(\Pi_i, \Pi) \leq \text{limit}$ do

5.1. Return $\text{best_plan}$

* We limited this evaluation to the best 700 cases of $\text{queue}$.

---

Figure 10: Algorithm to find a suitable plan for the adaptation phase from a set of candidate cases or the empty plan (in case the “generative” approach is considered more suitable).

In the relaxed planning graph analysis the negative effects of the domain operators are not considered and a solution plan $\pi_R$ of a relaxed planning problem can be computed in polynomial time [31].
On the contrary the planning encoding graph and the degree sequences of the input planning problem are only computed in the initial preprocessing phase (step 1.3).

All the cases with a similarity value sufficiently close\(^9\) to the best degree sequences similarity value \((\text{best}_\text{ds}_\text{simil})\) are examined further on (steps 2.1–2.4) using the \(\mathcal{K}_{\text{base}}\) kernel function. Then all the cases selected at steps 2.x with a similarity value sufficiently close to the best \(\text{simil}_{\mu_{\text{base}}}\) similarity value \((\text{best}_\mu_{\text{base}}\text{simil})\) (step 3.1) are accurately evaluated using the \(\mathcal{K}_\mathcal{N}\) kernel function, while the corresponding \(\mu_{\mathcal{N}}\) function is defined at step 3.2. In steps 3.3–3.5 we select the best matching function found for \(\Pi_i\) and the best similarity value found until now.

We use the relaxed plan \(\pi_R\) in order to define an estimate of the generation cost of the current planning problem \(\Pi\) (step 4.1). The \text{best}_\text{cost} value allows to select a good candidate plan for adaptation (which could also be the empty plan). This value is also useful during the computation of the adaptation cost through \textsc{EvaluatePlan}, in fact if such a limit is exceeded then it is wasteful to use CPU time and memory to carry out the estimate and the current evaluation could be terminated. The computation of the adaptation cost of the empty plan allows to choose between an adaptive approach and a generative approach, if no plan gives an adaptation cost smaller than the empty plan.

For all the cases previously selected with a similarity value sufficiently close to \(\text{best}_\text{simil}\) (step 4.2) the adaptation cost is determined (step 4.4). If a case of the case base determines an adaptation cost which is lower than \(\text{best}_\text{cost}\cdot\text{simil}_{\mu_i}(\Pi_i,\Pi)\) then it is selected as the current best case and also the \text{best}_\text{cost} and the \text{best}_\text{plan} are updated (steps 4.5–4.7). Note that we store the encoded plan \(\mu_i(\pi_i)\) in \text{best}_\text{plan} since this is the plan that can be used by the adaptation phase for solving the current planning problem \(\Pi\). Moreover we use the \(\text{simil}_{\mu_i}(\Pi_i,\Pi)\) value in steps 4.4–4.6 as an indicator of the effective ability of the selected plan to solve the current planning problem maintaining the original plan structure and at the same time obtaining low distance values.

### 3.3 Plan Adaptation

As previously exposed, the plan adaptation system is a fundamental component of a case-based planner. It consists in reusing and modifying previously generated plans to solve a new problem and overcome the limitation of planning from scratch. As a matter of fact, in planning from scratch if a planner receives exactly the same planning problem it will repeat the very same planning operations. In our context the input plan is provided by the plan retrieval phase previously described; but the applicability of a plan adaption system is more general. For example the need for adapting a precomputed plan can arise in a dynamic environment when the execution of a planned action fails, when the new information changing the description of the world prevents the applicability of some planned actions, or when the goal state is modified by adding new goals or removing existing ones [18, 22].

Different approaches have been considered in the literature for plan adaptation; strategies vary from attempting to reuse the structure of an existing plan by constructing bridges that link together the fragments of the plan that fail in the face of new initial conditions [27, 28, 29, 32], to more dynamic plan modification approaches that use a series of plan modification operators to attempt to repair a plan [38, 57]. From a theoretical point of view, in the worst case, plan adaptation is not more efficient than a complete regeneration of the plan [44] when a conservative adaptation strategy is adopted. However adapting an existing plan can be in practice more

\(^9\)In our experiments we used \text{limit} = 0.1.
Algorithm INSERT_CASE(\textit{Case}, \pi, \Pi(I, G))

\textbf{Input:} a case base \textit{Case}, a solution plan \pi for planning problem \Pi with initial state \textit{I} and goal state \textit{G}.

\textbf{Output:} insert the planning case in \textit{Case} if not present.

1. Define the set of initial state relevant facts \textit{I}_\pi of \Pi using the input plan \pi
2. Compute the Planning Encoding Graph \mathcal{E}_\pi of \Pi(I_\pi, G)
3. for each case \((\Pi_i, \pi_i) \in \text{Case}\)
4. Compute the matching function \mu_i using \mathcal{K}_N(\mathcal{E}_{\Pi_i}, \mathcal{E}_\pi)
5. if complete_simil\mu_i(\Pi_i, \Pi_\pi) = 1 \land |\pi_i| \leq |\pi| \text{ then}
6. return FALSE;
7. enfor
8. Insert the planning problem \Pi_\pi(I_\pi, G), its solution plan \pi, the Planning Encoding Graph \mathcal{E}_\pi and the data structures for the screening procedure in \textit{Case}
9. return TRUE;

Figure 11: High-level description of INSERT_CASE.

efficient than generating a new one from scratch, and, in addition, this worst case scenario does not always hold, as exposed in [3] for the Derivation Analogy adaptation approach. Plan adaptation can also be more convenient when the new plan has to be as “similar” as possible to the original one.

Our work uses the LPG-adapt system given its good performance in many planning domains but other plan adaptation systems could be used as well. LPG-adapt is a local-search-based planner that modifies plan candidates incrementally in a search for a flawless candidate. We describe the main components of the LPG-adapt system in the following section. It is important to point out that this paper relates to the description of a new efficient case-based planner and in particular to the definition of effective plan matching functions, no significant changes were made to the plan adaptation component (for a detailed description of it see [18]).

### 3.4 Plan Revision & Case Base Update

Any kind of planning system that works in dynamic environments has to take into account failures that may arise during plan generation and execution. In this respect case-based planning is not an exception; this capability is called plan revision and it is divided in two subtasks: evaluation and repair. The evaluation step verifies the presence of failures that may occur during plan execution when the plan does not produce the expected result. When a failure is discovered, the system may react by looking for a repair or aborting the plan. In this first hypothesis the LPG-adapt system is invoked on the remaining part of the plan; in the latter hypothesis the system repeats the CBP cycle so as to search a new solution.

After finding the plan from the library and after repairing it with the LPG-adapt techniques the solution plan can be inserted into the library or be discarded. The case base maintenance is clearly important for the performance of the system and different strategies have been proposed in the literature [51, 56]. Furthermore our attention has been oriented towards the improvement of the competence of the case base; a solved planning problem is not added to the case base only if there is a case that solves the same planning problem with a solution of a better quality.\(^{10}\)

Such a check has been introduced to the end of keeping only the best solution plans for certain

\(^{10}\)In our experiments we have considered only the number of actions for distinguishing between two plans that solve the same planning problem but other and more accurate metrics could be easily added, i.e. consider for example actions with not unary costs.
**Algorithm UPDATELIBRARY**

*Input:* A solution plan \( \pi \) for \( \Pi = (I, G) \) and a set of facts \( F \);

*Output:* Update the plan library inserting new elements obtained considering subplans of \( \pi \);

1. compute the set of causal links \( C_\pi \) in \( \pi \);
2. \( S = G \cup F \cup \{G_j \subseteq G \mid \bigcap_{g_j \in G_j} \pi_{g_j} \neq \emptyset \} \);
3. **forall** \( G_i \in S \) do 
   3.1 **CHECK** & **INSERT**(\( \Pi_{G_i}, \pi_{G_i} \));

---

Figure 12: Algorithm for updating the plan library inserting subplans of a given input plan \( \pi \).

kinds of problems in the library as there can be different plans that can solve the same problems with different sets of actions.

Figure 11 describes the main steps of the function used to evaluate the insertion of a planning problem \( \Pi \) solved in the case base. First of all we compute the set of initial state relevant facts \( I_\pi \) using the input plan \( \pi \); this set corresponds to a subset of the facts of \( I \) relevant for the execution of \( \pi \). It can be easily computed, as described in section 3.2, using the preconditions of the actions in \( \pi \):

\[
I_\pi = I \cap \bigcup_{a \in \pi} \text{pre}(a).
\]

Note that \( I_\pi \) identifies all the facts required for the execution of the plan \( \pi \) and that this definition is consistent with the procedure used in the RETRIEVEPLAN algorithm for the relaxed plan \( \pi_R \). Then we compute the Planning Encoding Graph \( E_\pi \) of the new planning problem \( \Pi_\pi(I_\pi, G) \) having \( I_\pi \) as initial state instead of \( I \). At steps 3–6 the algorithm examines all the cases of the case base and if it finds a case that solves \( \Pi \) with a plan of a better quality with respect to \( \pi \) then it stops and exits. In order to do so we use the similarity function \( \text{complete\_simil}_\mu \), described in section 3.1.1, which compares all the initial and goal facts of two planning problems. Otherwise if there is no case that can solve \( \Pi_\pi \) with a plan of a better quality with respect to \( \pi \) then we insert the solved problem in the case base. As we can observe at step 8, a planning case is made up not only by \( \Pi_\pi \) and \( \pi \), but also other additional data structures are precomputed and added to the case base so that their recomputation during the Retrieval Phase can be avoided.

Recently, the system was extended with a set of maintenance policies guided by the cases’ similarity, as is described in [25, 26].

Figure 12 describes the algorithm for updating the plan library with parts of an input plan \( \pi \). In short, UPDATELIBRARY identifies the subplans of \( \pi \) that can be inserted in the plan library to increase the competence of the library in itself [52, 56]. Here \( \pi_{g_j} \) represents the subplan of \( \pi \) that satisfies \( g_j \) starting from \( I \). Note that it can be easily identified considering the set of causal links \( C_\pi \) of \( \pi \) computed at step 1. In a similar way, it is possible to compute the set of facts \( I_{g_j} \) that are necessary to apply the actions of \( \pi_{g_j} \).

At step 2 we identify the set of facts that will be examined for the insertion in the library. In particular we consider all the goals \( G \), the elements of \( F \) and the subsets of interacting goals \( G_i \). The \( F \) set represents a set of facts, different by the input goals, that could be useful for the following merging phase such as unsupported facts of a previous adaptation phase. Moreover, the sets of interacting goals \( G_i \) can be easily computed considering the actions in the subplans \( \pi_{g_j} \) that are in common to the different goals.

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[11] We have used this simple definition instead of using the causal links in \( \pi \) in order to compute the set of relevant facts since it allows to obtain slightly better performance than the corresponding version based on causal links.
| Planner                  | Num. | Speed  | Quality     | Distance     |
|-------------------------|------|--------|-------------|--------------|
| OAKPLAN                 | 100  | 16.31  | -72098 (99) | 18.76 (12)   |
| small CB                | 50   | 13.79  | -38973 (47.4) | 139 (0.63)  |
| small CB & incr tech.   | 100  | 13.5   | -71048 (97.3) | 233 (0.91)  |
| OAKPLAN-merge           | 100  | 30.66  | -70444 (95.9) | 2.06 (80.8) |
| small CB                | 100  | 40.3   | -68709 (93.5) | 2.18 (86.4) |
| small CB & incr tech.   | 100  | 35.8   | -69607 (94.5) | 2.21 (85.7) |
| SGPLAN5                 | 100  | 13.73  | -69883 (95.4) | 231.5 (0.76) |
| LPG                     | 66   | 67.46  | -48315 (63.2) | 163 (0.87)  |

Table 1: Number of problems solved, average CPU-time (seconds), average Quality and average Distance of OAKPLAN, OAKPLAN-merge, SGPLAN5 and LPG.

The `CHECK&INSERT(((I_{G_i}, G_i), π)` function (step 3.1) searches if there not exists a case-base element (\(Π_j, π_j\)) whose goals and initial state perfectly match with the current goals and initial state, respectively. In this case, we insert the current planning problem \(Π_{G_i} = (I_{G_i}, G_i)\) and its solution plan \(π_{G_i}\) in the library. Otherwise, we have to decide whether to insert \((Π_{G_i}, π_{G_i})\) and remove \((Π_j, π_j)\), or simply skip the insertion of \((Π_{G_i}, π_{G_i})\). In our tests we have used an update policy that maintains the plan with the lowest number of actions, but other policies could be used as well considering, for example, the plan qualities, their makespan, or the robustness to exogenous events. Moreover, `CHECK&INSERT` ignores too small and too big plans \(π\); in fact, a small plan \(π\) could determine the inclusion in the library of a high number of very small plan fragments that have to be considered in the merging phase, while a big plan \(π\) could determine the insertion in the library of very big subplans that are difficult to merge.\(^{12}\)

4 Experimental Results

In this section, we present an experimental study aimed at testing the effectiveness of OAKPLAN in a number of standard benchmark domains.

We have experimented with several courses, but here focus on a real, large-size Moodle course of around 90 LOs on Natural Sciences inspired on http://www.profesorenlinea.cl. We have created nine initial configurations (with 10, 20... 90 fictitious students, respectively), and defined 10 variants per configuration (plus an additional variant for the 90 problem), thus considering 100 planning problems in total (the 91 variants plus the 9 initial configurations). Each variant artificially simulates the changes that may occur during the route execution in an incremental way. That is, in the first variant some equipment is no longer available. The second variant maintains these changes and includes restrictions on the students’ availability; and so on for the other variants.

In addition to OAKPLAN and our case base planner OAKPLAN-merge, we have used two state of the art planners, SGPLAN5 and LPG\(^{13}\). All tests were performed on an Intel(R) Xeon(TM) CPU 2.40GHz with 2GB of RAM, and censored after 10 minutes. In our tests, the solution plans (i.e. the learning routes) inserted in the case base were obtained by using the best quality plans generated by LPG and SGPLAN5 on the initial-configuration planning problems used to create the corresponding variants.

Figure 13 depicts the results: the time taken to produce a solution — the first one for LPG, OAKPLAN and OAKPLAN-merge — (top); the quality of the generated routes (middle); and

\(^{12}\)In our tests we have used \(5 \leq |π| \leq 200\).

\(^{13}\)For a further description of these planners see http://ipc.icaps-conference.org.
Figure 13: Respectively per rows, CPU time (on a logarithmic scale), plan quality, and number of different LOs w.r.t. the plan retrieved from the library (for OAKPLAN and OAKPLAN-merge) or the best solution available of the base problem (for LPG and SGPLAN6) — small values are preferable except for quality. In the first column we compare the case-based approaches (OAKPLAN and OAKPLAN-merge) vs. replanning (LPG and SGPLAN6). In the second column we compare the case-based approaches considering different input case bases.
the stability, in terms of distance of the new routes to the original ones (bottom). We show the best distance and plan quality across all plans produced in the entire optimization phase\footnote{14}. In the plots on the left we compare OAKPLAN (with and without merging techniques) vs. LPG and SGPLAN6 using a “complete” case-base that contains all the base problems and the corresponding solutions (the case-base for the merging variants contains also the selected subplans of the base problems). Here we can observe the general good behaviour of the case-based techniques, which are comparable in terms of CPU-time to SGPLAN6 and slightly better than the other planners in terms of plan quality for the bigger instances. OAKPLAN-merge is slightly slower than OAKPLAN since it tries to reuse not only the base problems but also their subplans. The results demonstrate that the case-based approach is at least as fast as replanning, and sometimes faster. Obviously, plan retrieval techniques show less useful when the changes are significant and fixing the route requires more effort than simply discarding it and rebuilding a new one from scratch. But the benefits for investing this effort can be seen in terms of stability. On the other hand, the retrieval and adaptation process sometimes comes at a price in terms of quality, as the route is adapted to fit a new configuration rather than constructed expressly for it. But our experiments show that the quality for the case-based approach can be better than for replanning, particularly in the most complex problems (see Figure 13).

Finally, the best values for stability are achieved in OAKPLAN-merge. While replanning generates routes that are consistently very different to the original ones, the differences between the retrieved plan and the solution plans are very small. This high value is not particularly surprising since LPG and SGPLAN6 do not know the target plans used for this comparison. These distance values are interesting since they are a clear indicator of the good behaviour of case-based techniques and show that the generative approach is not feasible when we want to preserve the stability of the plans produced. Moreover, we can observe the extremely good behaviour of OAKPLAN-merge; its distance values are obtained considering the number of different actions w.r.t. the matching plan provided by the retrieval phase, which is not necessarily obtained directly by the solution plan stored in the case base (as in OAKPLAN), but also using the different subplans (highlighted to the teachers) obtained by the analysis of the case-based elements that best fit the current initial state and goals. This indicator is very appealing in an e-learning setting as the students/teachers do not want to deal with an entirely new learning route after a little change happens during execution. Quite the contrary, students and teachers prefer a kind of inertia in the learning routes that facilitates the learning process.

In the plots on the right of Figure 13 we analyse the behaviour of OAKPLAN and OAKPLAN-merge to study the impact of using a case base considering: i) a case base created using only the smallest base problem (with 10 students), ii) a case base where the base problems are progressively inserted after the corresponding variants have been evaluated (it initially contains only the smallest base problem). In the first case, we primarily want to evaluate the ability of the merging techniques to reuse the solutions available in the case base at the increase of the “differences” (in terms of number of students) among the current situation and the elements stored in the case base. In particular, we want to examine the scalability in terms of students which is extremely important in our context where a teacher could decide to evaluate the effectiveness of an e-learning course considering a limited number of students before using it for the whole class.

\footnote{14}Note that the first plan generated by LPG, OAKPLAN and OAKPLAN-merge, the best quality plan and the best distance plan could be different plans. It depends on the teacher’s preferences to give more importance to the plan quality or to the plan stability by selecting the most appropriate solution plan during the validation process.
Here we can observe the general good behaviour of OAKPLAN-merge, while OAKPLAN without merging techniques is able to solve only 50 variants with high distance values w.r.t. the solution plan of the base problems. In fact, OAKPLAN can retrieve from the case base only the base problem with 10 students, whose solution plan is very different w.r.t. the final solution plan, so we have not plotted the distance w.r.t. the solution plan provided by the retrieval phase since it corresponds to the solution plan of the base problem with 10 students which is obviously extremely different w.r.t. the final solution plan.

Regarding the tests with the incremental case base, we want to analyse the behaviour of OAKPLAN considering a case base that contains elements which are structurally not to much different w.r.t. the current situation. For example, considering the solution of the variants with 50 students, the case base does not contain the base problem with 50 students but it contains the base problems with 10, 20, 30 and 40 students. Here we want to examine the situation where a teacher has already used a course in different classes and wants to reuse the stored experiences in a new (slightly bigger) class. As expected, the behaviour of OAKPLAN-merge does not change significantly neither in term of CPU-time nor in term of distance and plan quality; on the contrary, the CPU-time of OAKPLAN without merging techniques decreases significantly since it can replan starting from a case base element with a slightly lower number of students w.r.t. the current situation; moreover it is now able to solve all the variants considered. Regarding the plan differences w.r.t. the solution of the base problems we can observe values which are similar to the ones obtained for replanning. This is not surprising since the elements stored in the case base are different w.r.t. the current situation and OAKPLAN does not know the target solution plan; on the contrary the performances of OAKPLAN-merge are extremely good, both considering the case base with only the smallest base problems and the incremental case base. However, it is important to point out that in this case the comparison in terms of plan distances are performed considering directly the plan provided by the retrieval phase. It is up to the teacher to decide if (s)he wants to validate elements that considers only previously executed courses or also subparts of them.

Globally we can observe that the use of plan merging techniques are potentially very effective in our execution context, allowing to obtain efficiently new e-learning routes which are very similar to the combination of previously executed ones (or subpart of them). This is extremely important since it allows to highlight to the teachers the changes w.r.t. already executed learning routes, facilitating the validation process. Moreover, the stored plans can also contain some notes, regarding for example the pedagogical motivations associated to the selection or combination of specific LOs, annotated by the teacher during the creation of the original learning route or during previously executions of the learning route; in this case, these notes can be easily reexamined by the teachers facilitating the learning route validation process.

5 Related Work

In the following section we examine the most relevant case-based planners considering their retrieval, adaptation and storage capabilities. Moreover, we present an empirical comparison of the performance of OAKPLAN vs. the FAR-OFF system and some comments on the advantages of OAKPLAN with respect to other case-based planners.

Some CBP systems designed in the past do not consider any generative planning in their structure, and find a solution only by the cases stored in the case base. These CBP systems are called reuse-only systems. As reuse-only systems cannot find any planning solution from scratch, they cannot find a solution unless they find a proper case in the case base that can
be adapted through single rules. An alternative approach to reuse-only systems is the reuse-optimal approach, which uses a generative planning system that is responsible to adapt the retrieved cases. This feature allows a CBP system to solve problems that cannot be solved only by using stored cases and simple rules in the adaptation phase. Empirically, a great number of reuse-optimal CBP systems has shown that the use of a case base can permit them to perform better in processing time and in a number of planning solutions than the generative planning that they incorporate.

Obviously the retrieval phase critically affects the systems performance; it must search in a space of cases in order to choose a good one that will allow the system to solve a new problem easily. In order to improve efficiency in the retrieval phase, it is necessary either to reduce the search space or to design an accurate similarity metric. Reducing the search space, only a suitable subset of cases will be available for the search process and an accurate similarity metric will choose the most similar case to decrease the adaptation phase effort. In the literature there are different domain dependent and a few domain independent plan adaptation and case-based planning systems, which mostly use a search engine based on a space of states [24, 29, 56, 57]. An alternative approach to planning with states is that of plan-space planning or hierarchical systems [4] that search in a space of plans and have no goals, but only tasks to be achieved. Since tasks are semantically different from goals, the similarity metric designed for these CBP systems is also different from the similarity rules designed for state-space based CBP systems. For a detailed analysis of case-based and plan adaptation techniques see the papers of Spalazzi [53] and Munoz-Avila & Cox [42].

The CHEF system [27] is the first application of CBR in planning and it is a reuse-only system which is important especially from a historical point of view. It solves problems in the domain of Szechwan cooking and is equipped with a set of plan repair rules that describe how a specific failure can be repaired. Given a goal to produce a dish with particular properties, CHEF first tries to anticipate any problems or conflicts that may arise from the new goal and repairs problems that did not arise in the baseline scenario. It then executes the plan and, if execution results in a failure, a repair algorithm analyses the failure and builds an explanation of the reason why the failure has occurred. This explanation includes a description of the steps and states leading towards the failure as well as the goals that these steps tried to realise. Based on the explanation, a set of plan repair strategies is selected and instantiated to the specific situation of the failure. After choosing the best of these instantiated repair strategies, CHEF implements it and uses the result of the failure analysis to improve the index of this solution so that it will not be retrieved in situations where it will fail again.

Much attention has been given to research that designs suitable similarity metrics. It focuses on choosing the most adaptable case as the most similar one, such as the DIAL [36] and DÉJÀVU [50] systems. The DIAL system is a case-based planner that works in disaster domains where cases are schema-based episodes and uses a similarity assessment approach, called RCR, which considers an adaptability estimate to choose cases in the retrieval phase. Our similarity functions differ from the RCR method since they are based on a domain knowledge that is available in action definitions, while the RCR method uses the experience learned from the adaptation of previous utilisation of cases. They also differ in their applicability because the RCR method considers specifically disaster domains while our approach is suitable for domain independent planning.

Similarly, the DÉJÀVU system operates in design domains and uses an adaptation-guided retrieval (AGR) procedure to choose cases that are easier to be adapted. The AGR approach in the DÉJÀVU system uses additional domain knowledge, called capability knowledge, which
is similar to that used to solve conflicts in partial-order planning systems. This additional knowledge allows to identify the type and the functionality of a set of transformations, which are performed by actions, through a collection of agents called specialists and strategies. It must be well specified so as to maximise the AGR performance. Our similarity functions differ from the AGR approach because we do not use any domain knowledge besides that obtained from actions and states, which is the minimal knowledge required to define a domain for planning systems.

The PLEXUS system [2] confronts with the problem of “adaptive planning”, but also addresses the problem of runtime adaptation to plan failure. PLEXUS approaches plan adaptation with a combination of tactical control and situation matching. When plan failure is detected it is classified as either beginning a failing precondition, a failing outcome, a case of differing goals or an out-of-order step. If we ignore how to manage incomplete knowledge, the repair strategy involves the fact of replacing a failed plan step with one that might achieve the same purpose. It uses a semantic network to represent abstraction classes of actions that achieve the same purpose.

The GORDIUS [49] system is a transformational planner that combines small plan fragments for different (hopefully independent) aspects of the current problem. It does not perform an anticipation analysis on the plan, on the contrary it accepts the fact that the retrieved plan will be flawed and counts on its repair heuristics to patch it; in fact, much of the GORDIUS work is devoted to developing a set of repair operators for quantified and metric variables. The previous approaches differ with respect to OAKPLAN fundamentally because they are domain dependent planners; on the contrary OAKPLAN uses only the domain and planning problems descriptions.

Three interesting works developed at the same time adopt similar assumptions: the PRIAR system [33], the SPA system [29] and the Prodigy/Analogy system [58, 59]. PRIAR uses a variant of Nonlin [55], a hierarchical planner, whereas SPA uses a constraint posting technique similar to Chapman’s Tweak [11] as modified by McAllester and Rosenblitt [40]. PRIAR’s plan representation and thus its algorithms are more complicated than those of SPA. There are three different types of validations (filter condition, precondition, and phantom goal) as well as different reduction levels for the plan that represents a hierarchical decomposition of its structure, along with five different strategies for repairing validation failures. In contrast to this representation the plan representation of SPA consists of causal links and step order constraints. The main idea behind the SPA system that separates it from the systems mentioned above is that the process of plan adaptation is a fairly simple extension of the process of plan generation. In the SPA view, plan generation is just a special case of plan adaptation (one in which there is no retrieved structure to exploit). With respect to our approach that defines a matching function $\mu$ from $\Pi$ to $\Pi'$ that maximises the similarity function $\text{simil}_\mu$, it should be noted that in PRIAR and SPA the conditions for the initial state match are slightly more complicated. In PRIAR the number of inconsistencies in the validation structure of the plan library is minimised; in SPA the number of violations of preconditions in the plan library is maximised. Moreover the problem-independent matching strategy implemented in SPA runs in exponential time in the number of objects since it simply evaluates all possible mappings. On the contrary we compute an approximate matching function in polynomial time and use an accurate plan evaluation function on a subset of the plans in the library.

The Prodigy/Analogy system also uses a search oriented approach to planning. A library plan (case) in a transformational or case-based planning framework stores a solution to a prior problem along with a summary of the new problems for which it would be a suitable solution,
but it contains little information on the process that generates the solution. On the other hand derivational analogy stores substantial descriptions of the adaptation process decisions in the solution, whereas Veloso’s system records more information at each choice point than SPA does, like a list of failed alternatives. An interesting similarity rule in the plan-space approach is presented in the CAPLAN/CBC system [43] which extends the similarity rule introduced by the Prodigy/Analogy system [58, 59] by using feature weights in order to reduce the errors in the retrieval phase. These feature weights are learned and recomputed according to the performance of the previous retrieved cases and we can note that this approach is similar to the RCR method used by the DIAL system in disaster domains. There are two important differences between our approach and the similarity rules of CAPLAN/CBC, one of which is that the former is designed for state-space planning and the latter for plan-space planning. Another difference is that our retrieval function does not need to learn any knowledge to present an accurate estimate: our retrieval method only needs the knowledge that can be extracted from the problem description and the actions of the planning cases.

O-Plan [15, 16] is based on the strategy of using plan repair rules as well. The effects of every action are confirmed while execution is performed. A repair plan formed by additional actions is added to the plan every time a failing effect is necessary in order to execute some other actions. We call repair plans the prebuilt ones which are in a position to repair a series of failure conditions. For instance, we can have repair plans including a plan to replace either a flat tyre or a broken engine. When an erroneous condition is met, the plan is no longer executed but a repair plan is inserted and executed. When the repair plan is complete, the regular plan is executed once more. Failures are repaired by O-Plan by adding actions. It follows that it does not use either refinements or requires a history. However it is not complete and there are some failures which cannot be repaired.

MLR [44] is another case-based system and it is based on a proof system. While retrieving a plan from the library that has to be adapted to the current world state, it makes an effort to employ the retrieval plan as if it were a proof to set the goal conditions from the start. Should this happen, there is no need for any iteration to use the plan, otherwise, the outcome is a failed proof that can provide refitting information. On the basis of the failed proof, a plan skeleton is built through a modification strategy and it makes use of the failed proof to obtain the parts of the plan that are useful and removes the useless parts. After the computation of this skeleton, gaps are filled through a refinement strategy which makes use of the proof system. Although our object matching function is inspired to the Nebel & Koehler’s formalisation, our approach significantly differs from theirs since they do not present an effective domain independent matching function. In fact, their experiments exhibit an exponential run time behaviour for the matching algorithm they use, instead we show that the retrieval and matching processes can be performed efficiently also for huge plan libraries. The matching function formalisation proposed by Nebel & Koehler also tries to maximise first the cardinality of the common goal facts set and second the cardinality of the common initial facts set. On the contrary we try to identify the matching function \( \mu \) that maximise the \( \text{simil}_\mu \) similarity value which considers both the initial and goal relevant facts and an accurate evaluation function based on a simulated execution of the candidate plans is used to select the best plan that has to be adapted.

Nebel & Koehler [44] present an interesting comparison of the MLR, SPA and PRIAR performance in the BlocksWorld domain considering planning instances with up to 8 blocks. They show that also for these small sized instances and using a single reuse candidate the matching costs are already greater than adaptation costs. When the modification tasks become more difficult, since the reuse candidate and the new planning instance are structurally less similar, the
savings of plan modification become less predictable and the matching and adaptation effort is higher than the generation from scratch. On the contrary OAKPLAN shows good performance with respect to plan generation and our tests in the BlocksWorld domain consider instances with up to 140 blocks and a plan library with ten thousands cases.

The LPA* algorithm is used by the SHERPA replanner [38]. This algorithm was originally bound to repair path plan and backtrack to a partial plan having the same heuristic value as before the unexpected changes did in the world using the unrefinement step once. SHERPA is not useful to solve every repair problem, owing to the unrefinement strategy and the single application thereof. Its use is restricted to those problems whose actions are no longer present in the domain description. It follows that through the unrefinement step unavailable actions are removed.

The Replan [7] model of plans is similar to the plans used in the hierarchical task network (HTN) formalism [17]. A task network is a description of a possible way to fulfil a task by doing some subtasks, or, eventually (primitive) actions. For each task at least one of such task networks exists. A plan is created by choosing the right task networks for each (abstract) task chosen, until each network consists of only (primitive) actions. Throughout this planning process, Replan constructs a derivation tree that includes all tasks chosen, and shows how a plan is derived. Plan repair within Replan is called partialisation. For each invalidated leaf node of the derivation tree, the (smallest) subtree that contains this node is removed. Initially, such an invalid leave node is a primitive action and the root of the corresponding subtree is the task containing this action. Subsequently a new refinement is generated for this task. If the refinement fails, a new round is started in which task subtrees that are higher in the hierarchy are removed and regenerated. In the worst case, this process continues until the whole derivation tree is discarded.

A very interesting case-based planner is the FAR-OFF\footnote{FAR-OFF is available at http://www.fei.edu.br/~flaviot/faroff.} (Fast and Accurate Retrieval on Fast Forward) system [56]. It uses a generative planning system based on the FF planner [31] to adapt similar cases and a similarity metric, called ADG (Action Distance-Guided), which, like EVALUATEPLAN, determines the adaptation effort by estimating the number of actions that is necessary to transform a case into a solution of the problem. The ADG similarity metric calculates two estimate values of the distance between states. The first value, called \textit{initial similarity value}, estimates the distance between the current initial state $I$ and the initial state of the case $I_\pi$ building a relaxed plan having $I$ as initial state and $I_\pi$ as goal state. Similarly the second value, called \textit{goal similarity value}, estimates the distance between the final state of the case and the goals of the current planning problem. Our EVALUATEPLAN procedure evaluates instead every single inconsistency that a case base solution plan determines in the current world state $I$.

The FAR-OFF system uses a new competence-based method, called Footprint-based Retrieval [51], to reduce the space of cases that will be evaluated by ADG. The Footprint-based Retrieval is a competence-based method for determining groups of footprint cases that represent a smaller case base with the same competence of the original one. Each footprint case has a set of similar cases called Related Set [51]. The union of footprint cases and Related Set is the original case base. On the contrary OAKPLAN uses a much more simple procedure based on the $\text{sim}_{lds}$ function to filter out irrelevant cases. The use of Footprint-based Retrieval techniques and case base maintenance policies in OAKPLAN is left for future work. It is important to point out that the retrieval phase of FAR-OFF does not use any kind of abstraction to match cases and problems.
Figure 14: CPU-time, number of different actions and plan qualities for the Logistics variants. Here we examine OAKPLAN vs. FAR-OFF.
The FAR-OFF system retrieves the most similar case, or the ordered $k$ most similar cases, and shifts to the adaptation phase. Its adaptation process does not modify the retrieved case, but only completes it; it will only find a plan that begins from the current initial state and then goes to the initial state of the case, and another plan that begins from the state obtained by applying all the actions of the case and goes to a state that satisfies the current goals $G$. Obviously, the completing of cases leads the FAR-OFF system to find longer solution plans than generative planners, but it avoids wasting time in manipulating case actions in order to find shorter solutions length. To complete cases, the FAR-OFF system uses a FF-based generative planning system, where the solution is obtained by merging both plans that are found by the FF-based generative planning and the solution plan of the planning case selected. On the contrary OAKPLAN uses the LPG-adapt adaptation system, which uses a local search approach and works on the whole input plan so as to adapt and find a solution to the current planning problem.

In Figure 14 we can observe the behaviour of OAKPLAN vs FAR-OFF considering different variants of the greater case bases provided with the FAR-OFF system in the Logistics domain; similar results have been obtained in the BlocksWorld, DriverLog and ZenoTravel domains. Globally, we can observe that FAR-OFF is always faster than OAKPLAN both considering the retrieval and the total adaptation time although also the OAKPLAN CPU-time is always lower than 0.6 seconds. Considering OAKPLAN, most of the CPU-time is devoted to the computation of the matching functions which are not computed by FAR-OFF since it simply considers the identity matching function that directly assigns the objects of the case base to those of the current planning problem with the same name. In fact, it does not consider objects which are not already present in the case base and, to overcome this limitation, the variants used in this test are directly obtained by the problems stored in the case bases.

Regarding the plan qualities and the plan distances, it is important to point out that for each variant solved by OAKPLAN we consider only the first solution produced since FAR-OFF does not perform a plan optimisation process. However OAKPLAN is able to obtain better plans both considering the plan quality and the plan distance values. Globally, OAKPLAN is able to find plans with 20% better quality and 24% better plan distances. Moreover further improvements on plan qualities and distance values of OAKPLAN could be obtained by performing the optimisation process of LPG-adapt.

Finally, note that in this experiment we have used the case bases provided by FAR-OFF which contain 700 elements each and the corresponding cases are generated by creating randomly planning problems all with the same configuration: same objects, trucks and airplanes simply disposed in different ways. This kind of experiment is highly unfavourable to OAKPLAN since our first screening procedure cannot filter out a significant number of cases as they all have the same structure. On the contrary, in the experiments described in the previous sections the case bases used by OAKPLAN in the standard configuration (not the “small” versions) are not constrained to a particular planning problem but they have been generated by considering all the different planning problems configurations used in the International Planning Competitions. This is a much more realistic situation, where the cases are added to the case base when the planning problems provided by the users are resolved as time goes by.

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16We have used the case bases for the logistics-16-0, logistics-17-0 and logistics-18-0 Logistics IPC2 problems. For each problem considered the FAR-OFF system must have a case base with the same structure to perform tests. More than 700 cases belong to each case base and for each case base we have selected two planning cases and randomly generated 36 variants.

17In STRIPS domains the plan quality is obtained by considering the number of actions in the solution plan.
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