Prescribed Performance Voltage Control Strategy of LC Inverter

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Abstract. Aiming at the voltage tracking problem of the LC inverter, a new prescribed performance controller is proposed. In order to make the voltage tracking error converge entirely according to the trajectory described by the performance function, a new state variable is introduced referring to the integral sliding surface. The specific form of the voltage controller is obtained according to the voltage equation in the $dq$ coordinate frame and the system stability requirement. The adopted control law is easy to implement. It does not need to adjust the parameters repeatedly so that the inverter's steady-state and dynamic performance indicators can be close to the prescribed values. The MATLAB/Simulink platform simulation experiment verifies the feasibility and effectiveness of the proposed new prescribed performance control strategy.

1. Introduction
The voltage source inverter filtered by the LC low-pass filter has been widely used in uninterruptible power supplies, induction motor drives, solid-state transformers, AC microgrids due to the advantages of low cost, high reliability, and strong adaptability [1-4]. The control goal of the LC inverter is to quickly respond to load changes while achieving accurate tracking of the voltage command value. To this end, experts and scholars have proposed many control strategies, such as proportional-integral control, sliding mode variable structure control, active disturbance rejection control, and model predictive control [5-8].

The design elements of a typical control system mainly include the stability, stable and transient characteristics of the system. However, whether it is a complex frequency-domain design method based on classical control theory or a time-domain state-space method in modern control theory, their point of departure is to meet the stability requirements of the system and obtain the range of control parameters according to the stability criterion. Afterward, the required performance is obtained through the posterior optimization of the control parameters, and the steady-state and transient indicators of the system cannot be prescribed [9]. In response to the above problems, prescribed performance control was first proposed in [10]. The so-called prescribed performance control ensures that the tracking error is always within the boundary determined by the performance function during the convergence process. The system's steady-state and transient performance indicators meet the prescribed conditions. A nonlinear PID controller based on the prescribed performance control performance function is designed, which has been successfully applied to a four-rotor UAV system in [11]. A robotic arm motion controller which combines PD control with prescribed performance is designed to improve the dynamic response performance of the system in [12]. In order to solve the trajectory tracking problem of the underwater vehicle, a new controller which combines the prescribed...
performance control with the terminal sliding mode is proposed to suppress the jitter caused by the sliding mode function and improve the control accuracy in [13].

Although prescribed performance control has been widely used in many fields, there is almost no report in the literature about its use to improve the performance of LC inverters. This article improves the prescribed performance control for the voltage loop control of the LC inverter. When designing the controller, the system's performance indicators can be prescribed without multiple parameter adjustments, which reduces the difficulty of design and has better stable and transient performance at the same time.

2. Main Circuit Structure and Mathematical Model of LC Inverter

The main circuit structure of the LC-type inverter is shown in figure 1, and the AC side output is connected to the load through an LC filter. In the figure, \( L \) is the filter inductance; \( C \) is the filter capacitor; \( R \) is the resistance of the filter inductance.

![Figure 1. Proposed topology of inverter.](image)

According to figure 1 and Kirchhoff’s rules, the mathematical model of the inverter in the dq coordinate frame is established, and the current equation is

\[
\begin{align*}
L \frac{d i_d}{dt} &= u_d - u_{od} + \omega L i_q - R i_d \\
L \frac{d i_q}{dt} &= u_q - u_{oq} - \omega L i_d - R i_q
\end{align*}
\]

(1)

where \( u_d \) and \( u_q \) are the \( dq \) axis components of the output voltage of the three-phase bridge, \( u_{od} \) and \( u_{oq} \) are the \( dq \) axis components of the capacitor voltage, \( i_d \) and \( i_q \) are the \( dq \) axis components of the inductor current.

The voltage equation of the LC inverter is

\[
\begin{align*}
C \frac{du_{od}}{dt} &= i_d - i_{od} + \omega C u_{oq} \\
C \frac{du_{oq}}{dt} &= i_q - i_{oq} - \omega C u_{od}
\end{align*}
\]

(2)

where \( i_{od} \) and \( i_{oq} \) are the \( dq \) axis components of the load current respectively.

3. Design of Voltage Controller Based on Prescribed Performance

Since the \( d \)-axis and the \( q \)-axis are symmetrical, the voltage controller is designed with the \( d \)-axis as an example. Assuming that the current is fully tracked, that is, \( i_{dref} = i_d \), the \( d \)-axis voltage equation can be obtained according to equation (2) as
\[ u_{ad} = \frac{1}{C} i_{dref} + z \]  

(3)

where \( z \) is disturbance, \( z = o u_{dref} - i_{dref} C \).

In order to make the transient and stable performance indicators of the system prescribed, the performance function shown in equation (4) is introduced as follows [10].

\[ \rho(t) = (\rho_0 - \rho_\infty)e^{-t/T_0} + \rho_\infty \]  

(4)

where \( \rho_0 > \rho_\infty > 0 \), \( \rho_0 \) represents the prescribed upper limit of overshoot, \( \rho_\infty \) represents the prescribed upper limit of the steady-state error, and the time constant \( T_0 \) represents the convergence speed of the error, the smaller the \( T_0 \), the faster the convergence speed.

On this basis, define the system state variable \( w \) referring to the integral sliding surface [14].

\[ w = s + h \int_{-\infty}^{\infty} s dt \]  

(5)

where \( s \) is the trajectory error, \( s = \rho - e_{ud} \), \( \rho \) is the performance function, \( e_{ud} \) is the \( d \)-axis voltage tracking error, \( e_{ud} = u_{adref} - u_{ad} \), \( u_{adref} \) is the set value of \( u_{ad} \), and \( h \) is the integral coefficient.

In formula (5), let \( w = 0 \) and take the derivative of \( t \) to get

\[ \dot{s} = -hs \]  

(6)

Equation (6) shows that the trajectory error approaches zero with the time constant \( 1/h \) as the exponent. For reference from the analysis of cascade control, \( \rho \) is the outer loop, and its time constant is \( T_0 \), \( w \) is the inner loop, and the time constant is \( 1/h \). If \( h \) can be ensured to be much larger than \( 1/T_0 \), the tracking error of the system can converge according to the expected trajectory of equation (4). However, if \( h \) is too large, it will cause jitter. In this article, \( h = 100/T_0 \). The derivative of the error signal and with respect to time, and using equation (3) to get

\[ e_{ad} = \dot{u}_{adref} - \dot{i}_{dref} = -z - \frac{1}{C} i_{dref} \]  

(7)

Furthermore, after deriving equation (5), substituting equation (7) into it, we can get

\[ \dot{w} = \dot{\rho} + z + \frac{1}{C} i_{dref} + h(\rho - e_{ad}) \]  

(8)

Let \( \dot{w} = -kw \), among them, \( k \) is the proportional coefficient, \( k > 0 \), and the \( d \)-axis command current \( i_{dref} \) can be obtained from equation (5) as

\[ i_{dref} = -C(kw + \dot{\rho} + h\rho - he_{ad} + z) \]  

(9)

The Lyapunov method is used to analyze the stability of the new prescribed performance controller, and the system Lyapunov function is defined as

\[ V = \frac{1}{2} w^2 \]  

(10)

Take the derivative of equation (10) to get

\[ \dot{V} = w\dot{w} \]  

(11)

From the above analysis, we can see that when the voltage controller shown in equation (9) is used, the system is asymptotically stable.

Substituting equation (5) into equation (9), we can get
\[ i_{d_{ref}} = -C[(h + k)(\rho - e_{ad}) + h k \int_{-\infty}^{t}(\rho - e_{ad}) dt + \dot{\rho}] - \omega C u_{eq} + i_{ad} \]  

\[(12)\]

4. Simulation Results

Build a model in the MATLAB/Simulink platform to compare and analyze the voltage tracking error convergence performance of the system under different performance indicators. The inverter system parameters are shown in Table 1. Among them, the voltage loop controller is shown in equation (12), and the current loop is controlled by the hypo-time-optimal current loop [15]. The control strategy is shown in Table 2.

| Parameters | Descriptions | Values |
|------------|--------------|--------|
| \( u_{dc}/V \) | DC bus voltage | 800 |
| \( f_s/kHz \) | operating frequency | 10 |
| \( L/mH \) | Filter inductance | 2.6 |
| \( R/\Omega \) | resistance | 0.1 |
| \( C/\mu F \) | Filter capacitor | 19 |
| \( u_{ref}/V \) | Reference voltage | 311 |
| \( f_r/Hz \) | Rated frequency | 50 |
| \( P_r/kW \) | rated power | 10 |
| \( k \) | Voltage loop proportional coefficient | 0.1 |
| \( k_{ip} \) | Current loop proportional coefficient | 16.3363 |
| \( k_{ii} \) | Current loop integral coefficient | 628.3185 |
| \( i_c/A \) | Current error band | 2 |

| Control strategy | \( \rho_0 \) | \( \rho_\infty \) | \( T_0 \) |
|------------------|----------|----------|----------|
| Strategy 1       | 311      | 3.11     | 0.01     |
| Strategy 2       | 311      | 0.311    | 0.005    |

In order to verify the effectiveness of the new prescribed performance control strategy proposed in this paper, the inverter was started with a 10kW load during the simulation. Figures 2~3 show the simulation waveforms when using two control strategies under this working condition. Among them, since the dual-loop control is performed in the \( dq \) coordinate system, and \( u_{eqref} = 0 \), the tracking performance of the system is analyzed according to the \( d \)-axis voltage error and the \( a \)-phase voltage.

Observing figure 2(a) and figure 3(a), we can see that the \( d \)-axis voltage error can converge according to the expected trajectory of the performance function under the two control strategies. At the same time, the above figure also records the system adjustment time under different strategies. In figure 2(a), when the control strategy 1 is adopted, the time required for the \( d \)-axis voltage tracking error to converge to the steady-state value is 0.055 s, and the system adjustment time for the control strategy 2 is as shown in figure 2(a) 0.028s.
After the system is stable, a fast Fourier transform (FFT) analysis of 5 cycles is performed on the a-phase voltage data during 0.1~0.2s. At the same time, calculate the average value of the d-axis voltage tracking error during the calculation, and obtain the steady-state performance indicators of the system under different control strategies as shown in Table 3. It can be seen from table 3 that during steady-state operation, the d-axis voltage tracking error and the a-phase voltage effective value under the two control strategies are close to the expected value specified by the prescribed performance function, and the voltage total harmonic distortion (THD) is also very small. The new prescribed performance controller has good steady-state performance.

**Table 3.** Steady-state performance under different control strategies.

| Control strategy | $e_{mi}$/V | $e_{mr}$/V | $u_{ri}$/V | $u_{rr}$/V | THD/% |
|------------------|------------|------------|------------|------------|-------|
| Strategy 1       | 3.11       | 3.7        | 307.89     | 307.9      | 0.32  |
| Strategy 2       | 0.31       | 0.73       | 310.69     | 310.7      | 0.49  |

Where $e_{mi}$ is the expected average value of $e_{ub}$, $e_{mr}$ is the actual average value of $e_{ub}$, $u_{ri}$ is the expected effective value of phase a voltage, and $u_{rr}$ is the actual effective value of $u_r$.

5. Conclusion
For the LC inverter, this paper proposes a voltage control method based on a new prescribed performance controller. This method takes the voltage tracking error as the goal to converge according to the expected trajectory of the performance function to design the controller, which can be seen through the MATLAB/Simulink platform simulation experiment is that...
The indicators of the new inverter control system can be preset through the performance function, and better control effects can be obtained without multiple adjustments when setting the controller parameters, which reduces the design complexity.

Under the action of the step response, the voltage error may converge according to the preset trajectory, and it has higher control accuracy in the steady-state, and the stable and transient performance of the system are improved.

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