The parameter estimation of conditional intensity function temporal point process as renewal process using Bayesian method and its application on the data of earthquake in East Nusa Tenggara

L Jatiningsih¹, Respatiwulan², Y Susanti², S S Handayani², Hartatik²
¹)Program Study of Mathematics, Universitas Sebelas Maret,
Jl. Ir. Sutami 36A Kentingan Jebres Surakarta 57126, INDONESIA
²) Program Study of Statistics, Universitas Sebelas Maret,
Jl. Ir. Sutami 36A Kentingan Jebres Surakarta 57126, INDONESIA
E-mail: ljatiningsih@student.uns.ac.id

Abstract. A temporal point process is a one-dimensional point process where events are observed by time. A temporal point process can be viewed as a renewal process with time between events as random variables. The conditional intensity function of the process represents the expected rate dependent on time between events so that we can predict earthquakes activities. This research will be discussed about parameters estimation of conditional intensity function using Bayesian method and be applied to earthquake data in East Nusa Tenggara (ENT) from January 2015 to April 2018. The ENT is categorized as earthquake prone, because it is flanked by two zones of earthquakes. In southern region there are the Eurasian and Indo-Australian plates sub ducted, and in the north there is a back arc thrust. Based on the research results, it was obtained the conditional intensity estimation. The application of conditional intensity estimation represents the expected rate of earthquake occurrence in East Nusa Tenggara at a certain time interval. The time intervals are in units of days. The greatest conditional intensity is at the time interval (day 1096th to 11897th) that is 0.058798617. It means that the probability earthquake occurrence in East Nusa Tenggara at that time period is very low with the total number of occurrences are 28 earthquakes.

1. Introduction

According to Schoenberg [1], the point process is a set of random points that located in a particular area. The temporal point process is defined as a one-dimensional point process that is related to the events observed based on the time. In the point process, a conditional intensity function can be determined which represents the expected rate of the event that depends on past events before time \( t \) (Laub et al. [2]). If the conditional intensity function depends only on the inter-events times, then the corresponding point process is called a renewal process (Naser dkk. [3]). So, the temporal point process can be called a renewal process. The studies of temporal point process as a renewal process have been carried out by Sunusi et al. [4], where the parameter estimation by using the maximum likelihood method. In this paper, we will discuss parameter estimation of the temporal point process as renewal process by using Bayesian method to find the conditional intensity function.

Meanwhile, the temporal point process as a renewal process and the parameters estimation of the conditional intensity function of the process can be applied on earthquakes data. An earthquake is
an event of shaking earth caused by collisions between plates of the earth, active faults of volcanic activity or rock debris. Earthquakes that caused by volcanic activity and rock debris have relatively small damage compared to earthquake by collisions between earth plates and active faults (Kementerian ESDM [5]). The East Nusa Tenggara (ENT) is one of the regions in Indonesia with high seismicity levels. The ENT’s earthquakes is mainly caused by the collision of the Indo-Australian plate and the Eurasian Plate. It causes a subduction zone along the southern region of ENT (Hasanudin dkk. [6]). Therefore, this paper will estimate parameter of the conditional intensity function of the temporal point process as a renewal process with Bayesian method for earthquake data in ENT to know the probability of earthquake occurrence in ENT.

2. Temporal Point Process and Conditional Intensity Function
Baddeley [7] states that the temporal point process is a one-dimensional point process that has natural ordering properties in which the properties are not present in a higher dimensional process. The temporal point process can be defined through the inter-event times \( \tau_i = t_i - t_{i-1} \).

The conditional intensity function \( \lambda(t_i|\mathcal{F}_t) \) of the temporal point process represents the expected rate of event that depends on events before time \( t \) (Laub et. al. [2]). According to Ogata [8], the conditional intensity function \( \lambda(t_i|\mathcal{F}_t) \) can be written as follows.

\[
\lambda(t_i|\mathcal{F}_t) = \lim_{\Delta t \to 0} \frac{P[N(t_i+\Delta t)=1|\mathcal{F}_t]}{\Delta t}
\]

if the limit exists. The \( \mathcal{F}_t \) notation represents past events before time \( t \).

However, the conditional intensity function does not depend on past events before, but only depends on the events at the time \( t_i \) and \( t_{i-1} \). So, it can be determined the conditional intensity function of the temporal point process is considered as a renewal process, which represents the expectation of the event rate which depends on the inter-events times.

3. Result and Discussion
3.1. Conditional Intensity Function Temporal Point Process that fulfills the Renewal Process
In the temporal point process can be determined conditional intensity function which states the expected event rate that depend on past events before time \( t \) (Laub et. al. [2]). The temporal point process is considered as a renewal process so that it can determine the intensity function conditional on the temporal point process that fulfills the renewal process. The conditional intensity function that is determined depends on the time \( t_i \) and \( t_{i-1} \) fulfills the renewal process. The conditional intensity functions that fulfills the renewal process is

\[
\lambda(t_i|\mathcal{H}_t) = \frac{f(t_i|\mathcal{H}_t)}{1-F(t_i|\mathcal{H}_t)} = \frac{f(t_i)}{1-F(t_i)}.
\]

Furthermore, the inter-events times \( \tau_i \) is assumed to be exponentially distributed so the conditional intensity function temporal point process as the renewal process with the inter-events times having an exponential distribution is

\[
\lambda(t_i|\mathcal{H}_t) = \frac{f(t_i)}{1-F(t_i)} = \frac{\theta e^{-\theta \tau_i}}{1-(1-e^{-\theta \tau_i})} = \frac{\theta e^{-\theta \tau_i}}{e^{-\theta \tau_i}} = \theta.
\]

3.2. The Parameter Estimation of Conditional Intensity Function Temporal Point Process as Renewal Process using Bayesian Method
A Bayesian method is used to estimate the parameters of the conditional intensity function temporal point process as a renewal process. The Bayesian method needs to determine a prior distribution, a posterior distribution, and Squared Error Loss Function (SELF) Bayesian estimation (Larson [9]). A gamma distribution is used as a prior distribution for \( \theta \), that is \( f(\theta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} \theta^{\alpha-1} e^{-\frac{\theta}{\beta}}, \alpha > 0, \beta > 0 \) dan \( 0 < \theta < \infty \).
Then a posterior distribution can be formulated using the prior distribution and the sample data distribution. The posterior distribution is obtained as $f(\theta | \mathbf{x}) = \frac{\theta^{n+\alpha-1}e^{-\theta(\sum_{i=1}^{n} x_i + \frac{n}{\beta})}}{\Gamma(n+\alpha)(\sum_{i=1}^{n} x_i + \frac{n}{\beta})^{n+\alpha}}$ or it calls Gamma $\left(n + \alpha, \frac{1}{\sum_{i=1}^{n} x_i + \frac{n}{\beta}}\right)$.

After the posterior distribution is obtained, then the parameter estimation is carried out using the Bayesian method through the SELF approach by determining the expected value of the posterior distribution (Larson [9]). It is obtained the parameter estimation of $\theta$,

$$\hat{\theta} = \frac{(n+\alpha)}{\left(\sum_{i=1}^{n} x_i + \frac{n}{\beta}\right)} \quad (4)$$

By substituting equation (4) to equation (3) is obtained the parameter estimation of conditional intensity function temporal point processes that fulfills the renewal process, that is

$$\lambda(t_i | H_{t_i}) = \frac{(n+\alpha)}{\left(\sum_{i=1}^{n} x_i + \frac{n}{\beta}\right)}, \quad (5)$$

where $\lambda(t_i | H_{t_i})$ is conditional intensity function that depend on inter-events times, $n$ is the number of events at certain time intervals, $\sum_{i=1}^{n} x_i$ is the sum of inter-events times, $\alpha = 1$, and $\beta = \bar{x}$.

3.3. Application of the Conditional Intensity Estimation

The parameter estimation in equation (5) is applied to earthquake data. The data present event time (date, month and year) of the earthquakes on January 2015 until April 2018 in ENT. The data is obtained from the United Stated Geology Survey (USGS).

Based on the obtained data, the event time of earthquake occurrence in the form of date, month and year can be converted into days i.e. January 1st 2015 to 1st day, January 2nd 2015 to 2nd day, and so on up to the last data of the event time earthquake was April 6th 2018 to 1924th day. Next, a quarterly time interval is formed. So, we get the time interval (0; 90] is the 1st day to the 90th day (it means the time interval is January 2015 until March 2015), the time interval (90; 181] is the 91st day to the 181st day, and so on. It is clear that the time intervals are in units of days.

In this paper, the inter-events time data must fulfill the assumption of exponential distribution. Therefore, Kolmogorov-Smirnov test was conducted to know whether the inter-events time earthquake data had an exponential distribution. Based on data, there are 28 earthquakes from January 2015 to April 2018. The test statistic value is $D^* = 0.895707$ and the value $D^* < D(\alpha) = 1.094$, so $H_0$ does not rejected. It means that the data is exponentially distributed.

The some calculations of the parameter estimation uses equation (5) is shown in Table 1.

| Table 1. Results of estimation conditional intensity on earthquake data in ENT |
|-----------------------------|-----------------------------|-----------------------------|
| Time Intervals | The number of earthquake events | Conditional Intensity |
| (days) | | |
| (0,90] | 1 | 0.023526156 |
| (90,181] | 0 | 0 |
| (181,273] | 0 | 0 |
| (273,365] | 2 | 0.012657777 |
| (365,456] | 1 | 0.02380615 |
| (456,547] | 1 | 0.018180316 |
| (547,639] | 0 | 0 |
| (639,731] | 1 | 0.011560307 |
| (731,821] | 0 | 0 |
| Interval               | Events | Intensity   |
|-----------------------|--------|-------------|
| (821,912]            | 2      | 0.014777608 |
| (912,1004]           | 1      | 0.029405405 |
| (1004,1096]          | 1      | 0.026661927 |
| (1096,1187]          | 6      | 0.058798617 |

Table 1 shows that conditional intensity for each time interval varies depending on the number of events and the time between events in a certain time interval. The conditional intensity represents the expected rate of earthquake events on the January 2015 until April 2018 in ENT. At time intervals (0; 90] the number of events is 1, the inter-events times are 85 days, and the average inter-events times is 85 days so that the expected rate of earthquake event at the time interval (0; 90] is $\lambda(t_i|H_{t_i}) = \frac{(1+1)}{(85+\frac{1}{85})} = 0.023526156$. The conditional intensity at the time interval (90; 181] and (181, 273] are $\lambda(t_i|H_{t_i}) = 0$, because an earthquake does not occurs at these time interval. The conditional intensity at the time interval (273; 365] is $\lambda(t_i|H_{t_i}) = \frac{(2+1)}{(237+\frac{1}{237})} = 0.012657777$. The conditional intensity at the time interval (273; 365] is smaller than conditional intensity at the time interval (0; 90], because the inter-events times at the time interval (237; 365] is greater than the inter-events times at the time interval (0; 90] (237 days > 85 days). The greatest conditional intensity is at the time interval (1096, 1187) that is 0.058798617. It means that the probability earthquake occurrence in East Nusa Tenggara at January 2016 until March 2016 is very low. The conditional intensity value is obtained between 0 and 0.058798617 with the total number of occurrences are 28 earthquakes.

4. Conclusion
Based on the results and discussion, there are two conclusions.

1. The parameter estimation of the conditional intensity function temporal point process as a renewal process is
   
   $\lambda(t_i|H_{t_i}) = \frac{(n + \alpha)}{(\sum_{i=1}^{n} x_i + \frac{1}{P})}$

2. Conditional intensity estimation on earthquake data in ENT for each time interval varies depending on the number of events and the inter-events time at certain time interval. The conditional intensity represents the expected rate of earthquake events on the January 2015 until April 2018 in ENT. The greatest conditional intensity is 0.058798617 at the time interval (day 1096th to 11897th). It means that the probability earthquake occurrence in East Nusa Tenggara is very low with the total number of occurrences are 28 earthquakes.

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