Revealing Nuclear Pions Using Electron Scattering

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Abstract

A model for the pionic components of nuclear wave functions is obtained from light front dynamical calculations of binding energies and densities. The pionic effects are small enough to be consistent with measured nuclear dimuon production data and with the nucleon sea. But the pion effects are large enough to predict substantial nuclear enhancement of the cross section for longitudinally polarized virtual photons for the kinematics accessible at Jefferson Laboratory.
The belief that pions are the carrier of the nuclear force and hence the dominant agent responsible for the binding of nuclei has been a basic premise of nuclear physics since the time of Yukawa’s original theory. This tenet was strengthened via the observation of very significant effects of pion exchange currents in a variety reactions involving electromagnetic probes of nuclei [1]. Furthermore, chiral Lagrangians involving pions and nucleons are believed to be the correct low-energy representations of QCD [2].

All of this basic physics was brought into question by lepton-nucleus deep inelastic scattering experiments and related measurements of the production of Drell-Yan $\mu^+\mu^-$ pairs in high energy proton-nucleus interactions. The lepton-nucleus deep inelastic scattering observed by the EMC experiment [3,4] showed that there is a significant difference between the parton distributions of free nucleons and nucleons in a nucleus, including the substantial result that the nuclear structure function is suppressed in the region of the Bjorken variable $x \sim 0.5$. This means that the valence quarks of bound nucleons carry less plus-momentum than those of free nucleons. But partons must carry the total momentum of the nucleus so the plus-momentum of non-nucleonic constituents must be enhanced. One useful postulate was that it is nuclear pions which carry a larger fraction of the plus-momentum in the nucleus than in free space [5,6]. Such models [4] explain the shift in the valence distribution and provide also an enhanced nuclear anti-quark distribution, observable in Drell-Yan experiments [7]. However, no such enhancement was observed [8], and no substantial pionic enhancement was found in $(p,n)$ reactions [9]. These failures to observe the influence of nuclear pions were termed a severe crisis for nuclear theory [10].

But the magnitude of the crisis depends on the size of the pionic effects expected from nuclear theories. The earliest calculations [6] included enhancements via the effects of $\pi N \leftrightarrow \Delta$ transitions incorporated by summing the RPA series. Another calculation, involving significant $\Delta$ effects, evaluated the expectation value of the pionic number density operator in a variational wave function [11] and the result was characterized as infinite nuclear matter containing about 0.18 excess pions per nucleon. Both sets of calculations [6,11] led to significant nuclear pion content and a corresponding enhancement of the nuclear anti-quark distribution, which was not observed experimentally [5]. However, it was recognized almost immediately that one could reduce [12] the predicted sea enhancement by a very small increase in the parameter $g'$ used by [6] to represent the short-ranged nucleon-nucleon correlations. Thus it seems desirable to make further calculations of nuclear wave functions in a manner which includes pionic components along with nuclear saturation properties.

Furthermore, it is natural to expect that calculations of high energy observables would benefit by using light front dynamics [13]. These considerations led us to use light front dynamics to compute nuclear wave functions [14]. One approach was to use a new light-front one boson exchange potential [15] to compute the saturation properties of infinite nuclear matter in a Bruckner theory calculation which obtained very reasonable values of the binding energy density, and compressibility. This calculation goes beyond mean field theory, as is necessary to obtain a non-zero pion content for nuclei with equal numbers of neutrons and protons. Another result is that the number of excess pions per nucleon is 0.05 which seems to be in the range shown [12] as allowed by the Drell-Yan data. This smaller value of the pion excess arises mainly from the use of one-boson exchange, in which the $\Delta$ is absent [16]. Thus a reasonable nuclear theory exists which could coexist with the di-muon production data, but it is highly desirable to verify this set of dynamics by computing the
di-muon production cross section and by observing a non-zero signal of pionic effects. These are the aims of the present work.

One possibility for verification arises from measurements \[17\] of the ratio \( R \), of scattering of virtual photons in a longitudinal (L) or transverse polarization state (T), \( R \equiv \sigma_L/\sigma_T \) from nuclei. A large value of \( \sigma_L \), and the corresponding violation of the Callan-Gross relation \[18\], indicates the presence of nuclear bosons as fundamental constituents of nuclei \[19\]. Indeed, the HERMES collaboration found a factor of five enhancement of \( R \), but this was for values of \( x \) and \( Q^2 \) at which the pionic effects are too small to be relevant \[19\].

Here we study the possible effects of nuclear pions on \( \sigma_L \) as a function of \( x \) and \( Q^2 \) and therefore to determine if such effects are detectable. We shall also compute those effects for the nucleon target to predict its the pionic content. These effects are also relevant to experiments aimed at determining the pion elastic form factor \[20\].

The start is to confront the deep inelastic scattering data on the nucleon and the nuclear di-muon production data which place severe constraints on the pionic content of the nucleon \[21\] and nuclei, and ensure coexistence of the dynamical model with those data. The necessary phenomenology has been described often, see the reviews \[4\]. The main point is that the quark and anti-quark distributions can be given as convolution of the \( q, \bar{q} \) distributions in a given nuclear hadronic constituent with the light cone distribution functions: \( f_{\pi/A}(y) \), \( f_{\pi/N}(y) \), the probability to find an excess pion in the nucleus (A) or nucleon (N), with a plus-momentum given by \( y m_N \). These quantities are obtained from the square of the ground state nuclear of nucleon wave function, computed using light front dynamics, as:

\[
f_{\pi/N}(y) = m_N \int d^2k_\perp \langle N|n_\pi(k)|N \rangle \tag{1}
\]

\[
f_{\pi/A}(y) = \frac{1}{A} m_N \int d^2k_\perp \langle A|n_\pi(k)|A \rangle - f_{\pi/N}(y) \tag{2}
\]

where \( n_\pi(k = (k_\perp, k^+)) \) is the pion number operator. The function \( f_{\pi/N}(y) \) is constrained by the data on the nucleon sea which restrict the \( \bar{u} \) and \( d \) distributions to be similar \[21\]. The important input is the \( \pi N \) form factor, which we take to be of the Cloudy Bag Model form \[22\] with a bag radius, 0.9 fm, large enough to be consistent with the constraints.

The quantity \( f_{\pi/A}(y) \) controls the nuclear pionic effects. We would like to obtain this directly from our latest light front calculation \[15\], of nuclear matter properties. This calculation provides the integral quantity for the excess number of pions as \( \int_0^\infty dy \ f_{\pi/A}(y) = 0.05 \), but not the function \( f_{\pi/A} \) itself. However in all models this function is expected to be vanish for very large and very small values of \( y \) and smoothly rise to a maximum, at about \( y = 0.2, 0.3 \). Thus the overall strength of the effects in Drell-Yan scattering and \( \sigma_L \) we wish to examine here are not expected to be very sensitive to details or have much model dependence as long as the overall normalization is constrained. Therefore we revive our old calculation \[12\], but now using parameters which yield 0.05 for the number of excess pions in nuclear matter. The simplest way to achieve this value is to vary the value of \( g' \). The relation \( g' = 0.795 \) gives the desired value for nuclear matter and also that excess nuclear pions carry a fraction 0.016 of the nuclear plus momentum. We account for the difference between \( Fe \) and nuclear matter by reducing the Fermi momentum from 1.37 fm\(^{-1}\) to 1.3 fm\(^{-1}\) \[23\]. This reduces the excess pion number to 0.04 and the momentum fraction to 0.135. The results for the pionic distribution functions are shown in Fig. \[1\].
We see that $f_{\pi/N,A}$ differ from zero for $y \sim 0.2 - 0.5$, but $f_{\pi/A}$ vanishes rapidly as $y$ approaches 0. This means that pionic effects are irrelevant to understanding the HERMES effect, which occurs for $x_{Bj} \sim 0.01$.
which a quark from the incident proton is annihilated by a nuclear anti-quark (from either a nucleon or a pion) to form a virtual photon which emerges as a di-muon–$\mu^+\mu^-$ pair. The pionic contributions must be added to the contributions of the nucleons. The nucleonic effects are computed using the formalism of \cite{24} and \cite{25}. The key parameter is the average separation energy, which is taken as 56 MeV, as obtained from an extrapolation of the nuclear matter value of 71 MeV. Such parameters allow a reasonable qualitative representation of the nuclear deep inelastic scattering data \cite{4,25,26}.

With the present model, it is the nucleonic effects which control the computed Drell-Yan results. These are shown in Fig. 2 for values of the target plus momentum fraction, $x_t$ large enough so that the effects of shadowing are absent. The computations are made using the experimental acceptance \cite{8}. We see that it is possible to reproduce the experimental data with a theory which is based on detailed nuclear dynamics such as light front Bruckner theory \cite{15}.

The question remaining is, “Can there be a signal to observe these pions?” Thus we turn to the role of pionic effects on $\sigma_L$ for nuclear and nucleon targets. The pionic contribution to the hadronic tensor is given by:

$$\delta\pi W^{\mu\nu} = \frac{1}{4\pi M_A} \int d^4\xi \ e^{i\mathbf{q} \cdot \mathbf{\xi}} \langle P | J_\pi^\mu(\xi) J_\pi^\nu(0) | P \rangle,$$

where $|P\rangle$ represents the nucleon or nuclear target ground state of total momentum $P$, and the momentum of the virtual photon is $q = (\nu, \mathbf{0}, -\sqrt{\nu^2 + Q^2})$. The current operator which accounts for the effects arising from the nuclear pions is $J_\pi^\mu(\xi)$:

$$J_\pi^\mu = i(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*),$$

with $\phi$ as the complex pion field operator. The effects of the pion charge form factor $F_\pi(Q^2)$ are included below.

We are concerned with $\sigma_L$, and use the standard general formula \cite{27,28};

$$\sigma_L = \frac{Q^2 4\pi^2\alpha}{(Q^2 + \nu^2)^{3/2}} W^{00}. $$

We now evaluate $\delta\pi W^{00}$ by using $J_\pi^0$ in Eq. (3). Then insert a set of states which asymptotically contain a nucleus and a free pion of momentum $p^\mu$. The sum over nuclear states can be performed using closure if one uses light front variables ($q^\pm = q^0 \pm q^3$) to describe the momentum of the photon and of the nuclear pion ($k_\perp, k^+$). For large enough values of $\nu$ the interactions of the struck pion with the residual nucleus may be ignored, the energy of the outgoing pion may be taken as $\nu$, and the effects of non-zero values of nuclear excitations energies (proportional to $q^+/q^-$) appearing in the energy-momentum conserving delta function inherent in Eq. (3) may be ignored. Then the result of a straightforward evaluation leads to the result:

$$\delta\pi W^{00}(N, A) = \frac{\nu^2}{2Q^2} \frac{2 f_{\pi/(N,A)}(Q)}{M_N} F_\pi^2(Q^2),$$

where $F_\pi(Q^2)$ is taken as a monopole form consistent with the observed mean square radius, and the factor of $2/3$ accounts for the fact that only charged pions enter in electron scattering.
(with N=Z). Note that energy momentum conservation gives the relation \( k^+ = M_N \xi = \frac{Q^2}{\nu} \), and \( \xi \) is the Nachtmann variable.

\[ k^+ + \sigma = M_N \xi = Q^2/\nu, \]

and \( \xi \) is the Nachtmann variable.

FIG. 3. Pionic contribution to the photon longitudinal cross section on the proton, as a function of \( Q^2 \) and \( x \).

The pionic contribution to the longitudinal cross section for a proton (p=N) target, \( \sigma_L^\pi(p) \), is obtained from Eqs. (5) and (6), and is displayed in Fig (3). Cross sections of this size are routinely observable. The decrease with increase of \( Q^2 \) is caused by the pion form factor. The value of \( \sigma_L^\pi(p) \) for \( x = 0.2 \), \( Q^2 = 0.7 \text{ GeV}^2 \) corresponds to a contribution to \( R \) of the proton of about 0.04, small enough to avoid any contradiction with existing data.

We now turn to the case of nuclear targets. The standard general formula (5) allows us to relate \( \delta^\pi W^{00} \) to the corresponding contribution (per nucleon) \( \delta^\pi \sigma_L \) to the longitudinal cross section, so that the nuclear longitudinal cross section (per nucleon) is given by \( \sigma_L(A) = \sigma_L(D) + \delta^\pi \sigma_L \). It is desirable to present a ratio \( \sigma_L(A)/\sigma_L(D) \), or equivalently \( \sigma_L(A)/\sigma_T(D) R_D \), which allows the use of parameterizations for \( F_2(D) \) and \( R(D) \). Thus we obtain the result:

\[
\frac{\sigma_L(A)}{\sigma_L(D)} = 1 + \frac{Q^4}{(Q^2 + \nu^2) \nu A F_2^D R_D} \frac{\delta^\pi W^{00}}{(1 + R_D)},
\]

which may be evaluated using Eq. (6) to be:

\[
\frac{\sigma_L(A)}{\sigma_L(D)} = 1 + x \frac{f^\pi(\xi) \nu^2}{Q^2} \frac{F_2^\pi(Q^2)}{F_2^D R_D} (1 + R_D),
\]

where as usual \( x = Q^2/2M_N \nu \). Note that only the pionic effects are included here. The effects of nuclear vector and scalar mesons, so important in understanding the HERMES effect [19], are not included because these appear at much lower values of \( x \) than we shall consider.
The results of our calculation of $\sigma_L$ on $^{56}\text{Fe}$ are shown in Fig. 4 which shows the difference between the left hand side of Eq. (8) and unity. Results for other nuclei can be obtained by using the different pion distribution functions of Fig. 1.

![FIG. 4. Enhancement of longitudinal cross sections, as a function of $Q^2$ and $x$.](image)

Large effects are predicted for a quantity which is readily measurable at Jefferson Laboratory. The largest effects obtained with the lowest value of $Q^2 = 0.3 \text{ GeV}^2$, correspond to very low values of $\nu \sim 0.4 \text{ GeV}$, where there may be substantial corrections to the light-front closure approximation because $q^+$ is not vanishingly small. While there seems to be no reason why the effects of such corrections would cancel the effects shown here, the interpretation of the data would be simplified by observing effects at larger values of $Q^2$. In this case one obtains smaller values of $q^+$, and larger values of $\nu$, but the outgoing pion energies are in the vicinity of the $\pi$-nucleon resonance region, so that final state interaction effects could change the shape of the curve. However, since the variable $\sigma_L$ involves an inclusive measurement the overall strength as represented by, e.g. the area under the curve, should not be affected very much.

The substantial 15-30% effects, for values of $x \sim 0.2 - 0.3$, present an opportunity to observe a significant nuclear enhancement of $\sigma_L$. This is an excellent opportunity to unravel a significant long-standing mystery involving the absence of nuclear pionic effects.

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