Current-induced dynamics of composite free layer with antiferromagnetic interlayer exchange coupling

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Current-induced dynamics in spin valves including composite free layer with antiferromagnetic interlayer exchange coupling is studied theoretically within the diffusive transport regime. We show that current-induced dynamics of a synthetic antiferromagnet is significantly different from dynamics of a synthetic ferrimagnet. From macroscopic simulations we obtain conditions for switching the composite free layer, as well as for appearance of various self-sustained dynamical modes. Numerical simulations are compared with simple analytical models of critical current based on linearized Landau-Lifshitz-Gilbert equation.

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I. INTRODUCTION

After the effect of spin transfer torque (STT) in thin magnetic films had been predicted\textsuperscript{11,12} and then experimentally proven\textsuperscript{13,14}, it was generally believed that current-controlled spin valve devices would replace soon the memory cells operated by external magnetic field. Such a technological progress, if realized, would certainly offer higher data storage density and faster manipulation with the information stored on a hard drive memory. However, it became clear soon that some important issues must be solved before devices based on spin torque could be used in practice. The most important is the reduction of current density needed for magnetic excitation (switching) in thin films, as well as enhancement of switching efficiency and thermal stability. Some progress has been made by using more complex spin valve structures and/or various subtle switching schemes based on optimized current and field pulse\textsuperscript{15,16}.

A significant enhancement of thermal stability can be achieved by replacing a simple free layer (single homogeneous layer) with a system of two magnetic films separated by a thin nonmagnetic spacer, known as composite free layer (CFL). The spacer layer is usually thin so there is a strong RKKY exchange coupling between magnetic layers\textsuperscript{9,10}. In practice, antiferromagnetic configuration is preferred as it reduces the overall magnetic moment of the CFL structure and makes the system less vulnerable to external magnetic fields and thermal agitation. When the antiferromagnetically coupled magnetic layers are identical, we call the structure synthetic antiferromagnet (SyAF). If they are different, then the CFL has uncompensated magnetic moment and such a system will be referred to as synthetic ferrimagnet (SyF).

Current and/or field induced dynamics of CFLs is currently a subject of both experimental and theoretical investigations\textsuperscript{11–14}. Switching scheme of SyAF by magnetic field pulses has been proposed in Ref.\textsuperscript{11}, and then the possibility of current-induced switching of SyAF was demonstrated experimentally\textsuperscript{15}. In turn, the possibility of critical current reduction has been shown for a CFL with ferromagnetically coupled magnetic layers\textsuperscript{15}. However, the reduction of critical current in the case of antiferromagnetically coupled CFLs still remains an open problem. In a recent numerical study on switching a SyAF free layer\textsuperscript{16} it has been shown that the corresponding critical current in most cases is higher than the current required for switching of a simple free layer, and only in a narrow range of relevant parameters (exchange coupling, layer thickness, etc.) the critical current is reduced. Hence, proper understanding of current-induced dynamics of CFLs is highly required. We also note, that CFL can be used as a polarizer, too. Indeed, it has been shown recently\textsuperscript{17} that SyAF used as a reference layer (with magnetic moment fixed fixed to adjacent antiferromagnetic layer due to exchange anisotropy) might be excited due to dynamical coupling\textsuperscript{18} with a simple sensing layer\textsuperscript{19,20}.

The main objective of this paper is to study current-induced dynamics of a CFL with antiferromagnetic RKKY coupling in metallic spin valve pillars. We consider a system AF/F\textsubscript{0}/N\textsubscript{1}/F\textsubscript{1}/N\textsubscript{2}/F\textsubscript{2}, shown in Fig. 1, where AF is an antiferromagnetic layer (used to bias magnetization of the reference magnetic layer F\textsubscript{0}), F\textsubscript{1} and F\textsubscript{2} are two magnetic layers, while N\textsubscript{1} and N\textsubscript{2} are non-magnetic spacers. The part F\textsubscript{1}/N\textsubscript{2}/F\textsubscript{2} constitutes the CFL structure with antiferromagnetic interlayer exchange coupling. We examine current-induced dynamics of both SyAF and SyF free layers. These two structures differ only in the thickness of F\textsubscript{1} layer, while RKKY coupling and other pillar parameters remain the same.

We assume that spin-dependent electron transport is diffusive in nature, and employ the model described in Refs.\textsuperscript{21,22}. An important advantage of this model is the fact that it enables calculating spin current components and spin accumulation consistently in all magnetic and nonmagnetic layers, as well as current-induced torques exerted on all magnetic components. The torques acting at the internal interfaces of CFL introduce additional dynamical coupling between the corresponding...
The paper is organized as follows. In section 2 we derive the assumed models for spin dynamics and STT calculations. In section 3 we analyze STT acting on CFL and present results from numerical simulations on current-induced switching and magnetic dynamics. Some additional information on STT calculation can be found in the Appendix. Critical currents are derived and discussed in section 4. Finally, summary and general conclusions are in section 5.

II. MAGNETIZATION DYNAMICS

In the macrospin approximation, magnetization dynamics of the CFL is described by two coupled Landau-Lifshitz-Gilbert (LLG) equations,

$$\frac{d\mathbf{S}_i}{dt} + \alpha \mathbf{S}_i \times \frac{d\mathbf{S}_i}{dt} = \mathbf{\Gamma}_i,$$

$$\mathbf{\Gamma}_i = -|\gamma_i|/\mu_0 \mathbf{S}_i \times \mathbf{H}_{\text{eff}} + \frac{|\gamma_i|}{M_s d_i} \mathbf{\tau}_i,$$  \hspace{1cm} (1)

for $i = 1, 2$, where $\mathbf{S}_i$ stands for a unit vector along the net spin moment of the $i$-th layer, whereas $\mathbf{H}_{\text{eff}}$ and $\mathbf{\tau}_i$ are the effective field and current-induced torque, respectively, both acting on $\mathbf{S}_i$. The damping parameter $\alpha$ and the saturation magnetization $M_s$ are assumed the same for both magnetic layers of the CFL. Furthermore, $|\gamma|$ is the gyromagnetic ratio, $\mu_0$ is the vacuum permeability, and $d_i$ stands for thickness of the $i$-th layer.

The effective magnetic field for the $i$-th layer is

$$\mathbf{H}_{\text{eff}} = -\mathbf{H}_{\text{app}} e_z - \mathbf{H}_{\text{ani}} (\mathbf{S}_i \cdot e_z) e_z + \mathbf{H}_{\text{dem}}(\mathbf{S}_i) + \mathbf{H}_{\text{int}}(\mathbf{S}_0, \mathbf{S}_j) + \mathbf{H}_{\text{RKKY}}(\mathbf{S}_i, \mathbf{S}_j),$$  \hspace{1cm} (2)

where $i, j = 1, 2$ and $i \neq j$. In the latter equation, $\mathbf{H}_{\text{app}}$ is the external magnetic field applied along the easy axis in the layers’ plane (and oriented opposite to the axis $z$), $\mathbf{H}_{\text{ani}}$ is the uniaxial anisotropy field (the same for both magnetic layers), and $\mathbf{H}_{\text{dem}} = (\mathbf{N}_i \cdot \mathbf{S}_i) M_s$ is the self-demagnetization field of the $F_i$ layer with $\mathbf{N}_i$ being the corresponding demagnetization tensor. Similarly, $\mathbf{H}_{\text{int}} = (\mathbf{N}_{ij} \cdot \mathbf{S}_j) M_s + (\mathbf{N}_{ji} \cdot \mathbf{S}_i) M_s$ describes the magnetostatic influence of the layers $F_i$ and $F_j$ on the layer $F_i$, respectively. Here, $M_s$ is the saturation magnetization of the layer $F_0$, which might be generally different from $M_s$. Components of the tensors $\mathbf{N}_i$, $\mathbf{N}_{0i}$ and $\mathbf{N}_{ij}$ used in our simulations have been determined by the numerical method introduced by Newell et al.\textsuperscript{24} This method was originally developed for magnetic systems with non-uniform magnetization. To implement it into a macrospin model we considered discretized magnetic layers with uniform magnetizations, calculated tensors in each cell of the layer, and then averaged them along the whole layer. Since these tensors are diagonal, the magnetization and magnetostatic fields can be expressed as $\mathbf{H}_{\text{dem}} = (H^\parallel_{\text{dem}} S_{\text{dem}}^x, H^\perp_{\text{dem}} S_{\text{dem}}^y, H^\perp_{\text{dem}} S_{\text{dem}}^z)$, and $\mathbf{H}_{\text{int}} = (H^\parallel_{\text{int}} S_{\text{int}}^x, H^\perp_{\text{int}} S_{\text{int}}^y, H^\perp_{\text{int}} S_{\text{int}}^z) + (H^\parallel_{\text{RKKY}} S^x, H^\perp_{\text{RKKY}} S^y, H^\perp_{\text{RKKY}} S^z)$, with $S_{\text{dem}}$, $S_{\text{int}}$, and $S_{\text{RKKY}}$ denoting the components of the vector $\mathbf{S}_i$ ($i = 0, 1, 2$) in the coordinate system shown in Fig.1. Finally, $H_{\text{RKKY}}$ stands for the RKKY exchange field acting on $\mathbf{S}_i$, which is related to the RKKY coupling constant as $H_{\text{RKKY}} = -J_{\text{RKKY}}/(\mu_0 M_s d_i)$. To include thermal effects we add to the effective field $\mathbf{H}_{\text{eff}}$ a stochastic thermal field $\mathbf{H}_{\text{th}} = (H_{\text{th,}\parallel}, H_{\text{th,}\perp}, H_{\text{th,}\text{RKKY}})$. For both spins its components obey the rules for Gaussian random processes $\langle H_{\text{th,}\parallel}(t) \rangle = 0$ and $\langle H_{\text{th,}\perp}(t) H_{\text{th,}\perp}(t') \rangle = 2D\delta_{\parallel,\perp}\delta_{\perp,\parallel}(t - t')$, where $i, j = 1, 2$ and $\xi, \eta, \zeta = x, y, z$. Here, $D$ is the noise amplitude, which is related to the effective temperature, $T_{\text{eff}}$, as

$$D = \frac{\alpha k_B T_{\text{eff}}}{|\gamma_i|/\mu_0 M_s V_i},$$  \hspace{1cm} (3)

where $k_B$ is the Boltzmann constant, and $V_i$ is the volume of $F_i$ layer.

In general, the current-induced torques acting on $\mathbf{S}_1$ and $\mathbf{S}_2$ can be expressed as a sum of their in-plane and out-of-plane components $\mathbf{\tau}_1 = \mathbf{\tau}_{\parallel,1} + \mathbf{\tau}_{\perp,1}$ and $\mathbf{\tau}_2 = \mathbf{\tau}_{\parallel,2} + \mathbf{\tau}_{\perp,2}$, respectively. In a CFL structure, the layer $F_1$ is influenced by STT induced by the polarizer $F_0$, as well as by STT due to the layer $F_2$. In turn, the layer $F_2$ is influenced by the torques from the layer $F_1$. Hence we can write

$$\mathbf{\tau}_{\parallel,1} = I \mathbf{S}_1 \times \left[ \mathbf{S}_1 \times \left( a_1^{(0)} \mathbf{S}_0 + a_1^{(2)} \mathbf{S}_2 \right) \right],$$  \hspace{1cm} (4a)

$$\mathbf{\tau}_{\perp,1} = I \mathbf{S}_1 \times \left[ b_1^{(0)} \mathbf{S}_0 + b_1^{(2)} \mathbf{S}_2 \right],$$  \hspace{1cm} (4b)

$$\mathbf{\tau}_{\parallel,2} = I a_2^{(1)} \mathbf{S}_2 \times \left( \mathbf{S}_2 \times \mathbf{S}_1 \right),$$  \hspace{1cm} (4c)

$$\mathbf{\tau}_{\perp,2} = I b_2^{(1)} \mathbf{S}_2 \times \mathbf{S}_1,$$  \hspace{1cm} (4d)

where $I$ is the charge current density, which is positive when electrons flow from the layer $F_2$ towards $F_0$ (see Fig. 1), while the parameters $a_i^{(j)}$ and $b_i^{(j)}$ ($i, j = 1, 2$) are independent of current $I$, but generally depend on magnetic configuration.
We write the current density in the spin space as \( \mathbf{j} = j_0 \hat{\mathbf{1}} + \mathbf{j} \cdot \mathbf{\sigma} \), where \( j_0 \) is the particle current density (\( I = e j_0 \)), \( \mathbf{j} \) is the spin current density (in the units of \( \hbar/2 \)), \( \mathbf{\sigma} \) is the vector of Pauli matrices, and \( \hat{\mathbf{1}} \) is a \( 2 \times 2 \) unit matrix. In frame of the diffusive transport model described in the introduction, the considered pillars ferromagnetic interlayer exchange coupling. As described in the plane and perpendicularly to the plane configuration of the CFL. The STT acting at N/F interfaces as a function of the angle \( \theta_2 \) between \( \hat{\mathbf{z}}_1 \) and \( \hat{\mathbf{z}}_2 \). Figure 2 shows all three cartesian components \( \mathbf{J}_{1y} \) and \( \mathbf{J}_{1x} \) are transversal to the plane defined by \( \hat{\mathbf{S}}_0 \) and \( \hat{\mathbf{S}}_1 \). Analogically, \( \mathbf{J}_{2y} \) and \( \mathbf{J}_{2x} \) are transversal to the plane defined by \( \hat{\mathbf{S}}_0 \) and \( \hat{\mathbf{S}}_2 \). The relevant spin current transformations are given in Appendix A. Note, that the spin current components depend linearly on \( I \), so the parameters \( a_1^{(j)} \) and \( b_1^{(j)} \) are independent of the current density \( I \).

III. NUMERICAL SIMULATIONS

In this section we present results on our numerical simulations of current-induced dynamics for two metallic pillar structures including CFL with antiferromagnetic interlayer exchange coupling. As described in the introduction, the considered pillars have the general structure AF/F0/N1/F1/N2/F2 (see Fig.1). More specifically, we consider spin valves Cu - IrMn(10)/Py(8)/Cu(8)/Co(d1)/Ru(1)/Co(d2) - Cu, where the numbers in brackets stand for the layer thicknesses in nanometers. The layer Py(8) is the Permalloy polarizing layer with its magnetization fixed due to exchange coupling to IrMn. In turn, Co(d1)/Ru(1)/Co(d2) is the CFL (F1/N2/F2 structure) with antiferromagnetic RKKY exchange coupling via the thin ruthenium layer. The coupling constant between Co layers has been assumed as \( J_{\text{RKKY}} \approx -0.6 \text{mJ/m}^2 \), which is close to experimentally observed values. Here, we shall analyze two different geometries of CFL. The first one is a SyAF structure with \( d_1 = d_2 = 2 \text{nm} \), while the second one is a synthetic ferrimagnet (SyF) with \( d_1 = 2d_2 = 4 \text{nm} \).

Simulations have been based on numerical integration of the two coupled LLG equations with simultaneous calculations of STT, see Eq. (4). We have assumed typical values of the relevant parameters, i.e., the damping parameter has been set to \( \alpha = 0.01 \), while the uniaxial anisotropy field \( H_{\text{ani}} = 45 \text{kA/m} \) in both magnetic layers of the CFL. In turn, saturation magnetization of cobalt has been assumed as \( M_s(Co) = 1.42 \times 10^6 \text{Am}^{-1} \), and for permalloy \( M_s(Py) = 6.92 \times 10^6 \text{Am}^{-1} \). The demagnetization field and magnetostatic interaction of magnetic layers have been calculated for layers of elliptical cross-section, with the major and minor axes equal to 130 nm and 60 nm, respectively.

For both structures under consideration we have analyzed the current-induced dynamics as a function of current density and external magnetic field. The results have been presented in the form of diagrams displaying time-averaged values of the pillar resistance. Numerical integration of Eq. (4) has been performed using corrector-predictor Heun scheme, and the results have been verified for integration steps in the range from 10^{-4} ns up to 10^{-6} ns. The STT components acting on CFL spins have been calculated at each integration step from the spin currents, which have been numerically calculated from the appropriate boundary conditions. Similarly, resistance of the studied pillars has been calculated from spin accumulation in frame of the model used also for the STT description (for details see also Ref. 27).

A. Spin transfer torque

Let us analyze first the angular dependence of STT components in the structures under consideration. Although the thicknesses of magnetic layers in the studied SyAF and SyF structures are different, the angular dependence of STT components as well as their amplitudes are very similar. Thus, the analysis of STT in SyAF applies also qualitatively to the studied SyF free layer.

First, we analyze STT components in the case when SyAF is rotated as a rigid structure, i.e. the antiparallel configuration of \( \hat{\mathbf{S}}_1 \) and \( \hat{\mathbf{S}}_2 \) is maintained. To have a nonzero torque between F1 and F2 layers, \( \hat{\mathbf{S}}_2 \) has been tilted away from the antiparallel configuration by an angle of 1°. Figure (2) shows all three cartesian components (see Fig.1 for definition of the coordinate system) of STT acting at N/F interfaces as a function of the angle \( \theta \) between \( \hat{\mathbf{z}}_1 \) and \( \hat{\mathbf{S}}_1 \). While the \( y \) and \( z \) components are in the plane of the layers (the spins of CFL are rotated in the layer plane), the component \( x \) is normal to the layer plane. However, \( \tau_x \) remains negligible at all interfaces of the CFL. The STT acting at \( N_1/F_1 \) reveals a standard (non-wavy) angular dependence, and vanishes when \( \hat{\mathbf{S}}_1 \) is collinear with \( \hat{\mathbf{S}}_0 \). Its amplitude is comparable to STT in standard spin valves with a simple free layer. The STT at \( F_1/N_2 \) and \( N_2/F_2 \) interfaces also depends on the an-
tilted away from the antiparallel configuration with $\hat{S}_1$ by an angle of $1^\circ$. 

As will be shown in the following, CFL is usually not switched as a rigid structure, but generally forms a configuration which deviates from the antiparallel one. Figure 2 shows how the STT components at the $F_1/N_2$ and $N_2/F_2$ interfaces vary when $\hat{S}_2$ is rotated by an angle $\theta'$, while $\hat{S}_1$ remains fixed and is parallel to $\hat{S}_0 = \hat{e}_z$. In such a case, the torque acting at $N_1/F_1$ interface remains zero, as $\hat{S}_1$ stays collinear to $\hat{S}_0$. As before, the out-of-plane components are also negligible in comparison to the in-plane ones. The in-plane components of STT reveal standard angular dependence at both interfaces. The amplitude of STT at the internal interfaces of CFL is comparable to that acting at the $N_1/F_1$ interface in the case of noncollinear configuration of $\hat{S}_0$ and $\hat{S}_1$, when $\hat{S}_2$ is fixed in the direction antiparallel to $\hat{S}_0$. 

When $\hat{S}_1$ is noncollinear to $\hat{S}_0$, the spin accumulation in $N_1$ layer increases and consequently the amplitude of STT at $F_1/N_2$ and $N_2/F_2$ decreases. In turn, when $\hat{S}_1$ is antiparallel to $\hat{S}_0$, the STT inside the CFL structure is reduced by more than a factor of 2. Nonetheless, the STT acting at the internal interfaces of the studied CFL layers might have a significant effect on their current-induced dynamics and switching process, provided the magnetic configuration of CFL might deviate remarkably from its initial antiparallel configuration.

**B. Synthetic antiferromagnet**

First, we examine dynamics of the SyAF free layer. From symmetry we have $H_{RKKY}\approx H_{RKKY_2} = H_{RKKY}$, and we have set $H_{RKKY} = 2\, kOe$, which corresponds to $J_{RKKY} \approx -0.6\, mJ/m^2$. We have performed a number of independent numerical simulations modelling SyAF dynamics induced by constant current and constant in-plane external magnetic field. The latter is assumed to be smaller than the critical field for transition to spin-flop phase of SyAF. Accordingly, each simulation started from an initial state close to $\hat{S}_1 = -\hat{S}_2 = -\hat{e}_z$. To have a non-zero initial STT for $\hat{S}_1$, both spins of the SyAF have been tilted by $1^\circ$ in the layer plane so that they remained collinear.

From the results of numerical simulations we have constructed a map of time-averaged resistance, shown in Fig. 4(a). The resistance has been averaged in the time interval of 30 ns following initial 50 ns equilibration time of the dynamics. The diagram shows only that part of the resistance, which depends on magnetic configuration, and hence varies with CFL dynamics. The constant part of resistance, due to bulk and interfacial resistances of the studied structure, has been calculated to be as large as $R_{\text{sp}} = 19.74\, \Omega m^2$. For the assumed initial configuration, magnetic dynamics has been observed only for negative current density. When the current is small, no dynamics is observed since the spin motion is damped into the closest collinear state ($\hat{S}_1 = -\hat{S}_2 = -\hat{e}_z$, marked as $\uparrow\downarrow$) of high resistance. After exceeding a certain threshold value of current density, there is a drop in the averaged resistance, which indicates current induced dynamics of the SyAF free layer. Figures 4(b) and (f) show that this drop is associated with switching of the whole SyAF structure into an opposite state ($\hat{S}_1 = -\hat{S}_2 = \hat{e}_z$, marked as $\downarrow\uparrow$). 

From Fig. 4(a) follows that the threshold current for dynamics onset markedly depends on the applied field and reaches maximum at a certain value of $H_{\text{app}}$, $H_{\text{app}} = H_0$. Furthermore, it appears that mechanisms of the switching process for $H_{\text{app}} < H_0$ and $H_{\text{app}} > H_0$ are qualitatively different. To distinguish these two mechanisms, we present in Figs. 4(b – i) basic characteristics of switching, calculated for $I = 1.0 \times 10^8\, \text{A/cm}^{-2}$ and for fields $H_{\text{app}} = -400\, \text{Oe}$, which is below $H_0$ [Figs. 4(b – e)], and $H_{\text{app}} = 400\, \text{Oe}$, which lies above $H_0$ [Figs. 4(f – i)]. Figs. 4(b) and (f) present time evolution of the z-components of both spins. To better understand...
and applied magnetic field. Examples of switching processes
of the corresponding variation of pillar resistance, (e) and (i)
the overall magnetization of the free layer, (d) and (h) show
the amplitude of overall SyAF magnetization, defined as
SyAF dynamics, in Figs. 4(c) and (g) we plotted
the $\hat{S}_1$ (red solid line) and $\hat{S}_2$ (black dashed line) in the time interval from 0 to 10 ns, where switching takes place.

the SyAF dynamics, in Figs. 4(c) and (g) we plotted
the amplitude of overall SyAF magnetization, defined as
$m = |\hat{S}_1 + \hat{S}_2|$. This parameter vanishes for antiparallel alignment of both spins of CFL, but becomes nonzero when the configuration deviates from the antiparallel one. Magnetization of SyAF is also a measure of the CFL coupling to external magnetic field. Futhermore, Figs. 4(d) and (h) show the corresponding time variation of the resistance, $R$, which might be directly extracted from experimental measurements as well. In addition, in Figs. 4(e) and (i) we show the trajectories of $\hat{S}_1$ and $\hat{S}_2$ in the real space taken from the time interval from $t = 0$ to 10 ns. In addition, from Figs. 4(a) it has been found that the point where the threshold current reaches its maximum is located at $H_0 \simeq H_{\text{app}}^\text{crit}$, which indicates its relation to magnetostatic interaction of F2 and fixed polarizer. This also has been confirmed by analogical simulations disregarding the magnetostatic coupling between magnetic layers, which resulted in similar diagram, but with $H_0 = 0$ (not shown). This fact significantly facilitates understanding the mechanism of SyAF switching.

The initial configuration assumed above was $\hat{S}_1 = \hat{S}_2 \simeq \hat{S}_0$ with $\hat{S}_0 = e_z \left(\downarrow\downarrow\right)$. When the magnitude of current density is large enough and $I < 0$, orientation of $\hat{S}_1$ becomes unstable and $\hat{S}_1$ starts to precess with small angle around $-e_z$. Initial precession of $\hat{S}_1$ induces precession of $\hat{S}_2$ mainly via the RKKY coupling. Generally, response to the exchange field is slower than current-induced dynamics. Therefore, a difference in precession phase of $\hat{S}_2$ and $\hat{S}_1$ appears, and configuration of SyAF deviates from the initial antiparallel one. This in turn enhances the STT acting on F2, which tends to switch $\hat{S}_2$. Its amplitude, however, is small in comparison to the strong RKKY coupling. Further scenario of the dynamics depends then on the external magnetic field. When $H_{\text{app}} < H_0$ [Figs. 4(b – e)] the Zeeman energy of $\hat{S}_2$ has a maximum in the initial state and external magnetic field tends to switch $\hat{S}_2$ to the opposite orientation. Competition between the torques acting on SyAF results in out-of-plane precessions of both spins. After several precessions $\hat{S}_1$ reaches the opposite static state, which is stable due to STT. In turn, $\hat{S}_2$ is only slightly affected by STT, and its dynamics is damped in the external magnetic and RKKY exchange fields. In contrast, when $H_{\text{app}} > H_0$ [Figs. 4(f – i)], Zeeman energy of F2 has a local minimum in the initial state, which stabilizes $\hat{S}_2$. Therefore, in a certain range of current density, SyAF does not switch but remains in self-sustained coherent in-plane precessions (red area in the upper part of Fig. 4(a)). For a sufficient current density, the SyAF structure becomes destabilized and the precessional angle increases until the spins pass the $(x, y)$-plane. Consequently, the precessional angle decreases and spins of the SyAF are stabilized in the opposite state $(\uparrow\downarrow)$. Moreover, as shown in Fig. 4(c), the switching process for $H_{\text{app}} < H_0$ is connected with high distortion of SyAF configuration, while $m$ in a certain point reaches its maximum value (corresponding to parallel orientation of both spins). Contrary, the $m$ remains small for $H_{\text{app}} > H_0$ [Figs. 4(g)], and the effective magnetic moment of the free layer stays smaller than magnetic moment of a single layer. This might play an important role in applications of spin-torque devices based on CFLs.

The two switching mechanisms described above dominate the current-induced dynamics when the current density is close to the dynamics threshold. For higher current densities, the nonlinearities in SyAF dynamics become more pronounced, which results in bistable behavior of the dynamics, especially for $H_{\text{app}} < H_0$ and $I \gtrsim 10^8\text{Acm}^{-2}$. In that region, the number of out-of-plane precessions before SyAF switching increases with the current density. However, their precessional angle
increases in time and consequently $\hat{S}_1$ might reach an out-of-plane static point slightly tilted away from the $\hat{e}_x$ direction while $\hat{S}_2 = \hat{e}_x$ remains in the layers plane. The out-of-plane static states (marked as $\leftarrow\uparrow$) have small resistance and appear as dark red spots in the diagram shown in Fig. 4(a).

In addition, from the analysis of the dispersion of pillar resistance (not shown) one finds that except of a narrow region close to the dynamics threshold with persistent in-plane precessions, no significant steady-state dynamics of SyAF element appears. As will be shown below, such a behavior might be observed when CFL becomes asymmetric (SyF free layer).

C. Synthetic ferrimagnet

Let us study now spin valve with SyF as a free layer, assuming $d_1 = 4$ nm and $d_2 = 2$ nm. Accordingly, $H_{\text{RKKY}}$ remains 2 kOe while $H_{\text{RKKY}}$ is reduced to 1 kOe. As in the case of SyAF, from the averaged time-dependent part of the pillar resistance we have constructed a diagram presenting current-induced dynamics, see Fig. 4(a). The static part of resistance is now $R_{\text{sp}} = 19.80 \, \Omega \, \text{m}^2$. The diagram has some features similar to those studied in the previous subsection. However, the maximum critical current is shifted towards negative values of $H_{\text{app}}$, even if magnetostatic interaction between magnetic layers is neglected. This asymmetry is caused by the difference in exchange and demagnetization fields acting on layers $F_1$ and $F_2$. Moreover, this difference leads to more complex dynamics of the CFL’s spins than that in the case of SyAF.

Generally, there are several dynamic regimes to be distinguished. The first one is the region of switching from $\uparrow\uparrow$ configuration to the opposite one, $\downarrow\downarrow$, which is located at largest values of $H_{\text{app}}$ in the diagram. Mechanism of the switching is similar to that of SyAF shown in Figs. 4(f – i), where CFL changes its configuration just via in-plane precessional states with a small value of $m$ (weak distortion of the antiparallel alignment of $\hat{S}_1$ and $\hat{S}_2$). Furthermore, the darker area above $H_0$ indicates one of the possible self-sustained dynamic regimes of SyF, i.e. the in-plane precessions (IPP); see Figs. 4(b – e). This precessional regime starts directly after the SyF switching, and $\hat{S}_1$ and $\hat{S}_2$ precess around $\hat{e}_x$ and $-\hat{e}_x$, respectively. Due to different effective fields in $F_1$ and $F_2$, and energy gains due to STT, the spins precess with different precessional angles [Fig. 4(e)] and consequently different frequencies. Because of the strong interlayer coupling and spin transfer between the layers, amplitudes of their precessions are periodically modulated in time. This modulation appears also in the time dependence of pillar resistance. Conversely, below $H_0$ the dynamic is dominated by large angle out-of-plane precessions (OPP) of both spins, as shown in Figs. 4(f – i). This dynamic state is connected with a strong distortion of the antiparallel CFL configuration, i.e. large value of $m$, and large variation of the resistance. From Fig. 4(i) one can see that trajectories of $\hat{S}_1$ and $\hat{S}_2$ are rather complicated including both IPP and OPP regimes with dominant OPP.

D. Power spectral density

From the analysis of current-induced dynamics we found that self-sustained dynamics in structures with SyF free layer is much richer than that in systems with SyAF free layer [see Figs. 5(b – i)]. Therefore, in this section we restrict ourselves to dynamic regimes of the SyF free layer only. More specifically, we shall examine the power spectral density (PSD) as a function of current density and external magnetic field.
The text describes the simulation of coupled magnetic layers and the effect of applied current on the spin dynamics. It explains how the system's response to an applied field changes with current density. The figures show power spectral density (PSD) plots for different configurations, illustrating how the system's response evolves over time. The text highlights the importance of considering the thermal fluctuations and the competition between STT and RKKY coupling effects. The figures depict the trajectories of the spins in a two-dimensional plane, showing how they precess and change direction in response to the applied current. The PSD plots are used to analyze the frequency components of the system's response, with a particular emphasis on the frequency at which the resistance varies significantly.
The in-plane precessional angle increases with current density and hence the peak frequency in PSD decreases and becomes broader. In real systems, however, one might expect the peaks narrower than those obtained in the macrospin simulations, as observed in standard spin valves with a simple free layer.\textsuperscript{30,31}

IV. CRITICAL CURRENTS

First, we derive some approximate expressions for critical current density needed to induce dynamics of CFL, derived from linearized LLG equation. In metallic structures, the out-of-plane torque components are generally much smaller than the in-plane ones, and therefore will be omitted in the analytical considerations of this section \((\hat{b}_I^{(0)}, \hat{b}_I^{(2)}, \hat{b}_I^{(1)} \rightarrow 0)\).

The coupled LLG equations in spherical coordinates can be then written as

\[
\frac{d\mathbf{S}}{dt} = \frac{1}{1 + \alpha^2} \mathbf{M} \cdot \mathbf{\hat{v}},
\]

where \(\mathbf{S} = (\theta_1, \phi_1, \theta_2, \phi_2)^T\) is a 4-dimensional column vector which describes spin orientation in both layers constituting the CFL, and \(\mathbf{\hat{v}} = (v_{\theta}, v_{\phi}, v_{\theta}, v_{\phi})^T\) stands for the torque components, \(v_{\theta} = \Gamma_{\theta} \cdot \hat{e}_{\theta}\) and \(v_{\phi} = \Gamma_{\phi} \cdot \hat{e}_{\phi}\), with \(\hat{e}_{\theta} = (\hat{e}_\theta \times \hat{S}_i)/\sin \theta_i\) and \(\hat{e}_{\phi} = (\hat{S}_i \times \hat{e}_{\phi})/\sin \theta_i\) denoting unit vectors in local spherical coordinates associated with \(\hat{S}_i\).

In turn, the \(4 \times 4\) matrix \(\mathbf{M}\) takes the form

\[
\mathbf{M} = \begin{pmatrix}
1 & \alpha & 0 & 0 \\
-\alpha/\sin \theta_1 & 1/\sin \theta_1 & 0 & 0 \\
0 & 0 & 1 & \alpha \\
0 & -\alpha/\sin \theta_2 & 1/\sin \theta_2 & 0
\end{pmatrix}.
\]

Static points of the CFL dynamics have to satisfy \(v_{\theta} = 0\) and \(v_{\phi} = 0\) for both \(i = 1\) and \(i = 2\). These conditions are obeyed in all collinear configurations, i.e. \(\theta_i = 0, \pi\). Additional four trivial static points can be found in the out-of-plane configurations with \(\theta_i = \pi/2\) and \(\phi_i = 0, \pi\).

Following Ref. \textsuperscript{23} we linearize Eq. (6) by expanding \(\mathbf{v}\) into a series around the static points, which leads to

\[
\frac{d\mathbf{\bar{S}}}{dt} = \frac{1}{1 + \alpha^2} \mathbf{M} \cdot \mathbf{\bar{v}},
\]

where \(\mathbf{\bar{S}}\) is a Jacobian matrix of \(\partial \mathbf{v}/\partial \mathbf{\bar{S}}\) components. The matrix product \(\mathbf{M} \cdot \mathbf{J}\) defines here the dynamic matrix \(\mathbf{D} = \mathbf{M} \cdot \mathbf{J}\). This matrix allows one to study stability of CFL’s spins in their static points. If \(\text{Tr} \{ \mathbf{D} \} \) is negative, the static point is stable, otherwise it is unstable. Hence, the condition for critical current is\textsuperscript{22} \(\text{Tr} \{ \mathbf{D} \} = 0\).

To obtain threshold critical current for dynamics onset of individual spins in the CFL, we assume first that one of the spins is fixed in its initial position and investigate stability of the second spin. The dynamic matrix \(\mathbf{D}\) becomes then reduced to a \(2 \times 2\) matrix. Considering initial position of SAF with \(\hat{S}_1 = -\hat{S}_2 = -\hat{e}_z\) (i.e. \(\theta_1 = \pi\) and \(\theta_2 = 0\), marked as \(\uparrow\uparrow\), and polarizer \(\hat{S}_0 = \hat{e}_z\), the stability condition leads to the following critical currents \(I_{c1}^{\uparrow\uparrow}\) and \(I_{c2}^{\uparrow\uparrow}\) for \(\hat{S}_1\) and \(\hat{S}_2\), respectively:

\[
I_{c1}^{\uparrow\uparrow} = -\alpha \frac{\mu_0 M_s d_1 d_2}{a_1^{(1)} + a_2^{(1)}} \left[ -H_{\text{ext}}^{1\uparrow} + H_{\text{ani}} - H_{\text{RKKY}} 1 + H_1^3 \right],
\]

\[\text{with } H_{\text{ext}}^{1\uparrow} = H_{\text{app}} - H_{z^1}^0 - H_{z^2}^{21},\]

\[
I_{c2}^{\uparrow\uparrow} = -\alpha \frac{\mu_0 M_s d_1 d_2}{a_1^{(1)} + a_2^{(1)}} \left[ H_{\text{ext}}^{2\uparrow} + H_{\text{ani}} - H_{\text{RKKY}} 2 + H_2^3 \right],
\]

\[\text{with } H_{\text{ext}}^{2\uparrow} = H_{\text{app}} - H_{z^2}^0 - H_{z^1}^{22}.\]  

In both above expressions \(a_1^{(0)}, a_1^{(2)}, a_2^{(1)}\) are taken in the considered static point, while the demagnetization field for the \(i\)-th layer is given by

\[
H_{z}^i = \frac{H_{z}^{ani} + H_{z}^{ext}}{2}.
\]

Analogically, one can derive similar formulas for critical currents in the opposite (\(\uparrow\downarrow\)) magnetic configuration of the CFL.

Now we relax the assumption that one of the spins is fixed, and consider both spins of the CFL as free. Then, we calculate the trace of the whole \(4 \times 4\) matrix, which leads to the following expression for critical current destabilizing the whole CFL:

\[
I_{c,\text{CFL}}^{\uparrow\uparrow} = -\alpha \frac{\mu_0 M_s d_1 d_2}{d_1 (a_1^{(0)} + a_2^{(2)}) + d_2 a_1^{(1)}} \left[ -H_{\text{ext}}^{1\uparrow} + 2 H_{\text{ani}} - H_{\text{RKKY}} 1 + H_1^3 + H_2^4 \right],
\]

\[\text{with } H_{\text{ext}}^{1\uparrow} = H_{z^1}^0 - H_{z^1}^{21} + H_{z}^{21} + H_{z}^{12}.\]

Since the spins of CFL are antiparallel in the considered static point, \(I_{c,\text{CFL}}^{\uparrow\uparrow}\) is independent of external magnetic field. The above equation describes the critical current at which the CFL is destabilized as a rigid structure (unaffected by external magnetic field along the \(z\)-axis).

Numerical calculations presented below show that critical current is usually smaller than that given by Eq. (12). Apparently, as shown by numerical simulations, there is a phase shift in initial precessions of \(\hat{S}_1\) and \(\hat{S}_2\). Such a phase shift slightly perturbs initial antiparallel configuration and might reduce the critical current for the dynamics onset.

Similar formula also holds for the opposite configuration (\(\uparrow\downarrow\)), where the critical current is given by

\[
I_{c,\text{CFL}}^{\uparrow\downarrow} = -\alpha \frac{\mu_0 M_s d_1 d_2}{d_2 (a_1^{(0)} + a_2^{(2)}) - d_1 a_1^{(2)}} \left[ -H_{\text{ext}}^{2\downarrow} + 2 H_{\text{ani}} - H_{\text{RKKY}} 1 - H_{\text{RKKY}} 2 + H_1^3 + H_2^4 \right],
\]

\[\text{with } H_{\text{ext}}^{2\downarrow} = H_{z^1}^0 - H_{z^2}^0 - H_{z}^{21} + H_{z}^{12}.\]
with $H_{ax}^{\downarrow \uparrow} = H_0^{\downarrow \uparrow} - H_0^{\uparrow \downarrow} - H_0^{\downarrow \downarrow} - H_0^{\uparrow \uparrow}$.

Now we discuss the theoretical results on critical currents in the context of those following from numerical simulations. Let us consider first the critical currents for individual spins of the SyAF free layer, assuming that the second spin remains stable in its initial position, Eqs. (9) and (10). The corresponding results obtained from the formula derived above are presented in Table I, where we have omitted a weak dependence on $H_{app}$. For the studied structure with SyAF free layer, $I_{c1}^{\downarrow \uparrow}$ is negative while $I_{c2}^{\downarrow \uparrow}$ is positive. From our analysis follows, that the current density at which dynamics appears in the simulations (Fig. 4(a)) is higher than that given by $I_{c1}^{\downarrow \uparrow}$. However, we checked numerically that the critical value $I_{c1}^{\downarrow \uparrow}$ agrees with the critical current obtained from simulations when assuming $\hat{S}_1$ as free and fixing $\hat{S}_2$ along $\hat{e}_z$.

|       | SyAF | SyF |
|-------|------|-----|
| $I_{c1}$ | -0.31 | 0.43 |
| $I_{c2}$ | 0.98  | -0.46 |
| $I_{CFL}$ | -0.86 | 0.87 |

TABLE I. Critical current densities in the units of $10^8$ Acm$^{-2}$, calculated according to equations (9), (10), and (12) for both SyAF and SyF free layers.

Following the above discussion of the CFL dynamics, one can understand the shift of the threshold current as follows. Initially, when the current density exceeds $I_{c1}^{\downarrow \uparrow}$, $\hat{S}_1$ becomes destabilized. Then, $\hat{S}_2$ responds to the initial dynamics of $\hat{S}_1$ with similar coherent precession. However, $\hat{S}_2$ should still be stable in its initial position at this current density and common precessions of the two coupled spin moments damps the initial dynamics. Accordingly, SyAF ends up in the closest static state ($\uparrow \downarrow$). However, as the current density increases, the initial precessions of $\hat{S}_1$ become more pronounced, which in turn means that the initial antiparallel configuration becomes distorted and $\hat{S}_2$ becomes destabilized. This results in coupled dynamics of both spins and finally leads to switching of the SyAF structure.

On the other hand, we have also calculated the critical current for the whole SyAF structure according to Eq. (12), and for the given structure we got $I_{c}^{\downarrow \uparrow}$ shown in Fig. 4(a) by the dashed vertical line (see also Table I). Equation (12) describes stability of the whole CFL, and since the interlayer coupling is strong, $I_{c}^{\downarrow \uparrow}$ corresponds to the current density at which both spins become destabilized simultaneously preserving their antiparallel orientation. As can be seen in Fig. 4(a), this is the maximum threshold current density for current-induced dynamics. Because the rigid structure consisting of two antiparallel spins is not influenced by an external homogeneous magnetic field, there is no dependence of $I_{c}^{\downarrow \uparrow}$ on $H_{app}$. Nevertheless, from our numerical simulation follows that the threshold current for the SyAF dynamics, $I_{thr}$, obeys the condition $|I_{c1}^{\downarrow \uparrow}| < |I_{thr}| < |I_{c}^{\downarrow \uparrow}|$, provided that $|I_{c1}^{\downarrow \uparrow}| < |I_{c2}^{\downarrow \uparrow}|$ or (as in our case) $I_{c2}^{\downarrow \uparrow}$ has different sign.

When the SyAF is in the $\uparrow \downarrow$ configuration, the spin accumulation and spin current are different from those in the $\downarrow \uparrow$ configuration (at the same voltage). This in turn leads to different spin torques, which is the reason why the critical currents destabilizing $\uparrow \downarrow$ state are different from those for $\downarrow \uparrow$, as shown in Table I. From the critical currents one can expect relatively symmetric hysteresis with applied current in structures with SyAF. In contrast, $I_{CFL}^{\downarrow \uparrow}$ for the SyF is negative, similarly as $I_{CFL}^{\downarrow \uparrow}$, but it is significantly larger, which indicates lack of hysteresis. To compare switching of the SyAF and SyF free layers from the $\downarrow \uparrow$ to $\uparrow \downarrow$ configurations with the opposite one ($\uparrow \downarrow$ to $\downarrow \uparrow$), we have simulated dynamics of the corresponding CFLs assuming $H_{app} = 0$ and varying current density. The simulations have been performed in the quasistatic regime, i.e., for each value of current density the spin dynamics was first equilibrated for 50 ns and then averaged values of spin components and pillar resistance were calculated from the data taken for the next 30 ns of dynamics. In order to prevent the system from collapsing into a static state with zero torque, we have included a thermal stochastic field corresponding to $T_{th} = 5 K$ [see Eq. (3)]. Starting from $I = 0$ and going first towards negative currents we have constructed the current dependence of the averaged resistance and related $z$-components of both spins, as shown in Fig. 4(b) for both SyAF [Figs. 4(a–c)] and SyF [Figs. 4(d–f)] free layers, one can see relatively symmetric hysteresis with the current density. In both cases direct switching from $\downarrow \uparrow$ to $\uparrow \downarrow$ state occurs at a current density comparable to $I_{CFL}^{\downarrow \uparrow}$. In contrast, in the case of SyF free layer, the second transition ($\uparrow \downarrow$ to $\downarrow \uparrow$) appears at a current density which is very different from that predicted by the linearized LLG model. Moreover, in both cases switching from $\uparrow \downarrow$ to $\downarrow \uparrow$ state does not appear directly, but through some precessional states. More precisely, as the positive current density increases, both spins start precessing in the layers' plane prior to switching. The in-plane precessions are connected with a significant drop in the resistance and with a reduction of the $s_z$-components. The range of IPP regime is particularly large in the case of SyF. From the analysis of spins’ trajectories one may conclude that the angle of IPPs increases with increasing current density, and after exceeding a certain threshold angle CFL switches to the $\downarrow \uparrow$ configuration.

The other factor giving rise to the the difference in switching from $\uparrow \downarrow$ to $\downarrow \uparrow$ and from $\uparrow \downarrow$ to $\uparrow \downarrow$ follows from the fact that the magnetostatic interaction of the CFL’s layers with the polarizer is different in the $\uparrow \downarrow$ and $\downarrow \uparrow$ states. To prove this we have constructed analogical hysteresis loops for SyAF and SyF free layers disregarding magnetostatic interaction with the $F_0$ layer; see Figs. 4(g) and

\[
|H_{app}| = 9.65 \text{ Acm}^{-2} \quad \text{and} \quad |H_{app}| = 9.65 \text{ Acm}^{-2}.\]
loop is due to a significant asymmetry of STT in SyF becomes highly asymmetric. The asymmetry of SyF dynamics via the corresponding magnetostatic field. While the both switchings are realized via large decrease in contrast to the case when F

FIG. 7. Hysteresis loops of the resistance for the studied pillars with SyAF (a) and SyF (d) free layers. Panel (b) and (c) depict spin dynamics of \( \vec{S}_1 \) and \( \vec{S}_2 \) in SyAF, respectively, corresponding to resistance loop (a). Panels (e) and (f) show dynamics of \( \vec{S}_1 \) and \( \vec{S}_2 \) in SyF, respectively, corresponding to resistance loop (d). The initial point of each hysteresis loop is marked with a dot. The arrows indicate direction of the current change. Figures (g) and (h) correspond to the upper parts of (a) and (d), in which however the effects due to magnetostatic field of the reference layer to the CFL spins have been omitted.

(h). For both SyAF and SyF free layers we observe now large decrease in \( R \) for both switchings. This implies that both switchings are realized via in-plane precessions, in contrast to the case when F₂ influences the CFL dynamics via the corresponding magnetostatic field. While the hysteresis loop for SyAF remains symmetric, the one for SyF becomes highly asymmetric. The asymmetry of SyF loop is due to a significant asymmetry of STT in \( \uparrow \) and \( \uparrow \uparrow \) states, which was previously shaded by the magnetostatic coupling with the layer F₀.

V. DISCUSSION AND CONCLUSIONS

We have studied current-induced dynamics of SyAF and SyF composite free layers. By means of numerical simulations we identified variety of dynamical regimes. The most significant difference between dynamics of SyAF and SyF free layers concerns the evidence of self-sustained dynamics of both CFL spins. While in the case of SyAF only coupled in-plane precessions in a narrow window of external parameters (\( H_{\text{app}} \) and \( I \)) are observed, SyF free layer reveals more complex and richer dynamics, with the possibility of coupled out-of-plane precessions which might be interesting from the application point of view. Furthermore, as shown by numerical simulations, both SyAF and SyF are switchable back and forth without the need of external magnetic field. For SyAF element two possible ways of switching have been identified. Since they lead to different switching times, their identification might be crucial for optimization of switching in real devices with SyAF free layers. However, one has to note that the diagrams shown in Figs. 4 and 5 may be changed when magnetization in CFL becomes non-homogeneous.

A disadvantage of the studied structures is their relatively low efficiency of switching, i.e. high amplitude of critical current and long switching time. In order to show more sophisticated ways of tuning the CFL devices, we have analyzed critical currents derived from the linearized LLG equation. The formula (12) has been identified as the maximum value of critical current at which dynamics of the CFL structure should be observed. This formula reveals some basic dependence of critical current on spin valve parameters, and therefore might be useful as an initial tool for its tuning. However, in some cases non-linear effects in CFL dynamics might completely change the process of CFL switching, as shown by the presented numerical simulations. But the effects of non-linear dynamics go beyond the simple approach of linearized LLG equation, and their study requires more sophisticated nontrivial methods and/or numerical simulations.

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Appendix A: Transformations of spin current

Torque acting on the left interface of F₁ is calculated from x and y components of \( j'_{x} = \mathbf{T}(\theta_1, \phi_1)j_1 \), where \( j_1 \) is spin current vector in N₁ layer (written in global frame; shown in Fig. 1), and \( \mathbf{T}(\theta_1, \phi_1) = \mathbf{R}_{x}(\theta_1)\mathbf{R}_{z}(\phi_1 - \pi/2) \), where \( \mathbf{R}_{q}(\alpha) \) is the matrix of rotation by angle \( \alpha \) along the axis \( q \) in the counterclockwise direction when looking towards origin of the coordinate system. Hence \( j'_{x} \) components can be written as

\[
\begin{align*}
\hat{j}'_{1x} &= j_{1x} \sin \phi_1 - j_{1y} \cos \phi_1, \\
\hat{j}'_{1y} &= j_{1x} \cos \phi_1 + j_{1y} \sin \phi_1 \cos \theta_1 - j_{1z} \sin \theta_1, \\
\hat{j}'_{1z} &= j_{1x} \cos \phi_1 + j_{1y} \sin \phi_1 \sin \theta_1 + j_{1z} \cos \theta_1,
\end{align*}
\]

where \( (\theta_1, \phi_1) \) are spherical coordinates of \( \hat{S}_1 \) in the global frame. Similarly, we define torques’ amplitudes...
on the left interface of $F_2$ from the components of transformed spin current vector $j''_2 = T(\theta_2, \phi_2)j_2$. In this case, however, $j_2$ is not written in the global frame, but in the local coordination system coordinate system connected with $\hat{S}_1$. To rotate local coordinate system of $\hat{S}_1$ to local coordinate system of $\hat{S}_2$ we need to know spherical angles $\theta_2$ and $\phi_2$ of vector $\hat{S}_2$ in the local coordinate system of $\hat{S}_1$. This might be done by transforming first $\hat{S}_2$ vector to local coordinate system of $\hat{S}_1$ as $\hat{S}_2' = T(\theta_1, \phi_1) \cdot \hat{S}_2$ and calculate its angles $\theta_2$ and $\phi_2$. Then we can calculate components of $j''_2$ similarly as for the left interface

$$j''_{2x} = j_{2x} \sin \phi_2 - j_{2y} \cos \phi_2, \quad (A2a)$$

$$j''_{2y} = (j_{2x} \cos \phi_2 + j_{2y} \sin \phi_2) \cos \theta_2 - j_{2z} \sin \theta_2, \quad (A2b)$$

$$j''_{2z} = (j_{2x} \cos \phi_2 + j_{2y} \sin \phi_2) \sin \theta_2 + j_{2z} \cos \theta_2, \quad (A2c)$$

Equation in $N_2$, which is adjacent non-magnetic interface from the right-hand side of $F_1$, are written in local coordinate system of $\hat{S}_1$. To apply the definition of $a_{12}$ and $b_{12}$ we need to rotate the local coordinate system so, that its $y$-axis will lie in the layer given by vectors $\hat{S}_1$ and $\hat{S}_2$. This might be done by single rotation of local coordinate system around its $z$-axis by angle $\phi_2 - \pi/2$, $j''_2 = R_z(\phi_2 - \pi/2) j_2$, where

$$j''_{2x} = j_{2x} \sin \phi_2 - j_{2y} \cos \phi_2, \quad (A3a)$$

$$j''_{2y} = j_{2x} \cos \phi_2 + j_{2y} \sin \phi_2, \quad (A3b)$$

$$j''_{2z} = j_{2z}. \quad (A3c)$$

Note, angle $\phi_2$ is calculated for vector $\hat{S}_2$ transformed into coordinate system of $\hat{S}_1$ as in previous case.

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1. J. C. Slonczewski, J. Magn. Magn. Mater. 159, L1 (1996).
2. L. Berger, Phys. Rev. B 54, 9353 (1996).
3. M. Tsoi, A. G. M. Hansen, K. D. Bass, W.-C. Chiang, M. Seck, V. Tsoi, and P. Wyder, Phys. Rev. Lett. 80, 4281 (1998).
4. J. A. Katine, F. J. Albert, R. A. Buhrman, E. B. Myers, and D. C. Ralph, Phys. Rev. Lett. 84, 3149 (2000).
5. S. Serrano-Guisan, K. Rott, G. Reiss, J. Langer, B. Ocker, and H. W. Schumacher, Phys. Rev. Lett. 101, 087201 (2008).
6. D. E. Nikov, G. I. Bourianoff, G. Rowlands, and I. N. Krivorotov, J. Appl. Phys. 107, 113910 (2010).
7. L. Liu, T. Moriyama, D. C. Ralph, and R. A. Buhrman, Appl. Phys. Lett. 94, 122508 (2009).
8. P. Baláž, M. Gmitra, and J. Barnaš, Phys. Rev. B 79, 144301 (2009).
9. S. P. Parkin and D. Mauri, Phys. Rev. B 44, 1731 (1991).
10. P. Grünberg, R. Schreiber, Y. Pang, M. B. Brodsky, and H. Sowers, Phys. Rev. Lett. 57, 2442 (1986).
11. J.-V. Kim, T. Devolder, C. Chappert, C. Maunfour, and R. Fournel, Appl. Phys. Lett. 85, 4094 (2004).
12. T. Ochiai, Y. Jiang, A. Hirohata, N. Tzukia, S. Sugimoto, and K. Inomata, Appl. Phys. Lett. 86, 242506 (2005).
13. N. Smith, S. Maat, M. J. Carey, and J. R. Childress, Phys. Rev. Lett. 101, 247205 (2008).
14. S.-W. Lee and K.-J. Lee, Journal of Magnetics 15, 149 (2010).
15. C.-T. Yen, W.-C. Chen, D.-Y. Wang, Y.-J. Lee, C.-T. Shen, S.-Y. Yang, C.-H. Tsai, C.-C. Hung, K.-H. Shen, M.-J. Tsai, et al., Appl. Phys. Lett. 93, 092504 (2008).
16. C.-Y. You, J. Appl. Phys. 107, 073911 (2010).
17. S. Mao, A. Mack, E. Singleton, J. Chen, S. S. Xue, H. Wang, Z. Gao, J. Li, and E. Murdock, J. Appl. Phys. 87, 5720 (2000).
18. S. Urazhdin, Phys. Rev. B 78, 060405(R) (2008).
19. D. Gusakova, D. Houssameddine, U. Ebels, B. Dieny, L. Buda-Prejbeanu, M. C. Cyrille, and B. Delaët, Phys. Rev. B 79, 104406 (2009).
20. D. Houssameddine, J. F. Sierra, D. Gusakova, B. Delaët, U. Ebels, L. D. Buda-Prejbeanu, M.-C. Cyrille, B. Dieny, B. Ocker, J. Langer, et al., Appl. Phys. Lett. 96, 072511 (2010).
21. J. Barnaš, A. Fert, M. Gmitra, I. Weymann, and V. K. Dugaev, Phys. Rev. B 72, 024426 (2005).
22. M. Gmitra and J. Barnaš, in Toward Functional Nanomaterials, edited by Z. Wang (Springer, 2009), pp. 285 – 322.
23. N. C. Emley, R. A. Buhrman, and D. C. Ralph, Phys. Rev. B 69, 094421 (2004).
24. U. Ebels, D. Houssameddine, I. Firastrau, D. Gusakova, C. Thirion, B. Dieny, and L. D. Buda-Prejbeanu, Phys. Rev. B 78, 024436 (2008).
25. A. J. Newell, W. Williams, and D. J. Dunlop, J. Geophys. Res. 98, 9551 (1993).
26. M. D. Stiles and A. Zangwill, Phys. Rev. B 66, 014407 (2002).
27. M. Gmitra and J. Barnaš, Phys. Rev. B 79, 012403 (2009).
28. M. Gmitra and J. Barnaš, Phys. Rev. Lett. 96, 207205 (2006).
29. S. Urazhdin, W. L. Lim, and A. Higgins, Phys. Rev. B 80, 144411 (2009).
30. J. C. Sankey, I. N. Krivorotov, S. I. Kiselev, P. M. Bragacna, N. C. Emley, R. A. Buhrman, and D. C. Ralph, Phys. Rev. B 72, 224427 (2005).
31. J.-V. Kim, Phys. Rev. B 73, 174412 (2006).
32. S. Wiggins, Introduction to Applied Nonlinear dynamical Systems and Chaos (Springer-Verlag, 1990).