Quantum effects across dynamical horizons

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Abstract
We present a generalization of the Hawking effect for dynamical trapping horizons by calculating the tunneling rate in the Hamilton–Jacobi formalism. It turns out that all horizons classified by Hayward are subjected to thermal quantum effects. While the Hawking effect for future outer and past inner trapping horizons is given as a particle emission, we show that the Hawking effect for future inner and past outer trapping horizons translates to an absorption. The universality of the treatment allows a natural transfer to the static case.

Keywords: Hawking effect, dynamical horizons, tunneling method, Hamilton–Jacobi formalism

1. Introduction

In its original version, the Hawking effect describes particle production by black holes [1]. The most common illustration uses Hawking pairs, that is to say, close to the black hole, a pair of particles is created and one particle falls into the black hole while the other escapes to future infinity, where it is measured. Energy budget considerations lead to the conclusion that the black hole will lose energy because of this process which results in a shrinking radius. Hence, the in-falling particle could in principle be interpreted as a particle with negative energy tunneling inside the black hole [2]. Hawking particles will potentially cause the black hole to disappear. Those calculations have led to plenty of follow-up articles, mostly discussing information loss due to the Hawking effect involving a vast variety of physical and mathematical ideas about the fate of the information inside. A central issue in all those considerations, however, is the nature of the Hawking process. Its occurrence presumes necessarily—but not sufficiently—the presence of a horizon which acts as a separation principle for the Hawking
This is because horizons are boundaries between two space-time regions which are to some extent causally disjoint. In other words, they provide a suitable gravitational separation between the two Hawking partners.

The connection between particle production and the presence of a horizon suggests that black holes are just one example where this process occurs. In principle, any kind of horizon might be furnished with the Hawking effect. A direct consequence [5] is the belief that horizons could be seen as thermodynamic objects with a specific temperature. In the case of black holes, this temperature is defined through the spectrum of the emitted Hawking quanta. Although thermodynamic quantities are characterized with respect to an averaging process, the Hawking temperature is sharply defined. Nevertheless, it fulfills the property of a temperature by being semi-positive and obeying a law analogous to the first law of thermodynamics [6].

Physical considerations demand to formulate all predictions such that they are in principle measurable. For the initial Hawking proposal, the observer needs to be infinitely far away and has to wait infinitely long. There the black hole is characterised by an event horizon: well-defined in the asymptotic regions and static. However, this setup is unsuitable for practical purposes because the event horizon itself is not physically observable [7] by local experiments. Hawking and Ellis [8] have revisited the definition of horizons to describe local and dynamical black holes. However, this idea also involves the global assumption that the space-time is asymptotically flat which is a severe restriction since our Universe is not considered to fulfill this assumption. Hayward [9] generalized Hawking and Ellis’s idea to formulate a quasi-local definition of dynamical horizons resulting in the definition of trapping horizons which does not require asymptotic flatness. These are classified concerning their causal structure by using the area change along ingoing and outgoing light rays which yields four basic types of trapping horizons. The black hole horizon, for example, is categorized as future outer trapping horizon (FOTH) (for the definition cf section 2, or [9]). In short, trapping horizons can be physically understood as the horizon a specific observer perceives by testing the spacetime with light-torches: chosen a point in spacetime, the observer switches the torch on and measures how light rays evolve in the nearby. Hayward’s notion is then clearly local and time-dependent. This is of paramount importance for dynamical spacetimes but also when we intend to study the Hawking effect itself. In general, Hawking particles might cause the horizon to change and therefore trapping horizons provide a more realistic framework.

While Hawking’s analysis incorporates the Bogolubov coefficients, we want to work in the tunneling picture [10–14] to study the Hawking effect for dynamical horizons. Especially, the Hamilton–Jacobi method [15] has been proven to be a powerful tool, reproducing for static black holes that the horizon emits a thermal spectrum of particles. This method supports a strict generalization beyond the black hole case and gives a similar result for Hubble horizons in expanding cosmologies [15], that is, the observer will detect a leakage of Hawking particles into the Hubble sphere. Besides past inner (Hubble horizon type) and future outer (black hole type) trapping horizons, two other types emerge: future inner and past outer trapping horizons. Examples for future inner horizons occur in collapsing cosmologies with a big crunch while for a past outer the most prominent representative is a white hole. Nevertheless, there are systems with more than one horizon, e.g. electrically charged black holes, like the Reissner–Nordström solution, have future inner and past outer horizons while cosmologies dominated by stiff matter acquire past inner and past outer ones.

In the following we will present the outline of this article: the second section provides first an introduction to Hayward’s horizon classification, then we restrict ourselves to spherically symmetric cases in order to provide explanatory examples. The third section reviews the tunneling picture in the Hamilton–Jacobi method, and the fourth section starts by applying this to future outer (black hole-type) trapping horizons. From these considerations, we will inves-
igate quantum effects for the remaining classes of trapping horizons to find a generalization of the Hawking effect. While we observe particle emission by future outer and past inner trapping horizons, we find as a main result that the Hawking effect for past outer and future inner trapping horizons turns out to be an absorption. In the fifth section, we give a summary and discuss implications of this work as well as a possibility to extend our analysis to more general spacetimes.

2. Dynamical horizons

The relevant geometrical objects in the setup are horizons, which are a boundary between observable and unobservable events.

A black hole, for example, is defined in an asymptotically flat spacetime through its event horizon. The exact localisation of the event horizon’s position, however, presumes the knowledge about the whole evolution of the spacetime. Therefore, the notion of an event horizon is purely teleological which turns this concept practically useless for observations made by local experiments. If one wants to locate the event horizon, one needs to solve the Cauchy problem for the full future development of a set of Cauchy data.

In order to perform realistic experiments (locally and in finite time), we need to abandon the mathematically handy notion of event horizons for a more operational definition of horizons which eventually can be measured by local experiments. These requirements culminated in the notion of dynamical trapping horizons, which are quasi-local concepts based on the local evolution of light rays involving only a limited amount of global properties while being in-principle detectable. Let us investigate the different definitions in detail before going into specific examples.

General relativity defines the causal structure of a space-time \((M, g)\) with pseudo-Riemannian manifold \(M\) and metric tensor \(g\) by introducing a variety of different sets which allow to define an event horizon. We assume all manifolds to be time-orientable such that the terms ‘future’ and ‘past’ can be assigned globally and unambiguously. To develop an intuition for the structure of event horizons, we consider the case of a Schwarzschild black hole. Following [8], the naïve idea that horizons are separating boundaries can be recast into mathematically robust terms by introducing the concept of the causal past \(J^-\). The set \(J^- (\mathscr{I}, M)\) is the region of space-time which causally affects events on a compact spacelike or null set \(\mathscr{I}\). Taking the asymptotically flat exterior region of a Schwarzschild black hole, we specify the observer to be at future null-infinity \(\mathscr{I}^+\) and look at its causal past. Then we can define the black hole event horizon to be the boundary \(\mathcal{J}^- (\mathscr{I}^+, M)\) where \(\mathcal{J}\) is the closure of the (asymptotically simple) manifold \(M\). Analogously, a past event horizon can be defined as \(\mathcal{J}^+ (\mathscr{I}^-, M)\) which describes the white hole event horizon in Kruskal space-time.

In the formalism of Hawking and Ellis, a trapped region of a manifold \(M\) is the set \(M - \mathcal{J}^- (\mathscr{I}^+, M)\) that is the portion of \(M\) which cannot communicate with any observer at \(\mathscr{I}^+\); intuitively all light rays from events in the trapped region are destined to stay inside. The Schwarzschild black-hole interior is such a region. Note, an analogous definition can be found by investigating \(\mathcal{J}^+ (\mathscr{I}^-, M)\). In this procedure, the leftover portion would correspond to a past- or anti-trapped region which describes, for example, the interior of an eternal white hole. As already said, these definitions are non-local and teleological since the observer rests at \(\mathscr{I}^+\) or \(\mathscr{I}^-\), i.e. they require the knowledge about the entire causal past/future \(J^\pm\) and are non-dynamical.

A remedy was found by Hayward [9] who constructed a general notion of dynamical horizons by introducing the double-null foliation: let \((M, g)\) be a four-dimensional, globally time-
orientable space-time which is foliated into spacelike hypersurfaces $\Sigma_t$ along the temporal coordinate $t$. Then, consider a two-dimensional, compact, orientable, space-like surface $S$ on $\Sigma_t$ [16] and two half-open intervals $\mathcal{I}^\pm = [0, l^\pm)$ such that we can define a smooth embedding $i: \mathcal{I}^+ \times \mathcal{I}^- \times S \rightarrow M$. We can construct two null congruences $l^\pm$ (outgoing ‘+’ and ingoing ‘−’) which are orthogonal to $S$ and allow for a definition of $\mathcal{L}_\pm$, the Lie derivative with respect to a normal direction to $\Sigma$. To formulate a quasilocal description of a horizon, we will use the expansion $\theta^\pm = h^{-1}(\mathcal{L}_\pm h)$ of $S$ along the null congruences $l^\pm$ which describes whether $S$ will expand ($\theta^+ > 0$) or shrink ($\theta^- < 0$) if infinitesimally dragged along $l^\pm$; $h$ is the induced two-dimensional metric on $S$.

By definition, a compact two-dimensional spatial surface $S$ is a trapped surface, if $\theta^+ \theta^- > 0$ everywhere on $S$. In particular, if both $\theta^+ < 0$ everywhere on $S$ we call $S$ a future trapped surface while if $\theta^+ > 0$ everywhere on $S$, then $S$ is called a past-trapped surface, later referred to as anti-trapped surface. The portion $\mathcal{F}$ of $\Sigma_t$ that is foliated by (anti-)trapped surfaces, is known as the (anti-)trapped region of $\Sigma_t$. We have seen that with the help of light rays, we can study the causal structure of spacetimes and detect trapped regions by a local experiment using torches. In a flat spacetime, future-directed outgoing light rays diverge ($\theta^+ > 0$) while ingoing light rays converge ($\theta^- < 0$). This behavior characterizes normal regions. On the other hand, in a trapped region $\mathcal{F}$ both, ingoing and outgoing light rays are converging, i.e. $\theta^+ < 0$ and $\theta^- < 0$. In other words, everywhere in $\mathcal{F}$ the light-cone is so bent that all future-directed signals are trapped in $\mathcal{F}$ and cannot escape.

Black holes can therefore be defined by their future trapped regions because the light which has fallen inside is destined to stay in the interior. An expanding Universe, in contrast, admits an anti-trapped region ($\theta^+ > 0$) beyond the Hubble sphere with radius $R = R_H$. In a homogeneous and isotropic slicing, the expansion will apparently reach superluminal speed at $R > R_H$ [17], such that the effective speed along the ingoing direction is not fast enough to counteract the expansion, ending up in a net outgoing motion. Indeed, the future light cone in the anti-trapped region is so bent toward the outward direction that signals coming from sources outside ($R > R_H$) are excluded from the normal region inside ($R < R_H$).

A crucial concept in the definition of trapping horizons involve marginally trapped surfaces which are spatial two-dimensional surfaces $\Sigma$ on which either $\theta^+ = 0$ (future marginally trapped) or $\theta^- = 0$ (past marginally trapped)\(^4\). A future trapping horizon is the closure $\mathcal{H}$ of a three-dimensional surface $\Sigma$ which is foliated by marginally trapped surfaces with the condition $\theta^+ = 0$ and $\theta^- < 0$ and respectively $\theta^+ = 0$ and $\theta^- > 0$ for past trapping horizons. In other words, the union of marginally trapped surfaces along the time flow constructs a dynamical notion for horizons [16]. It should be noted, that the occurrence of the trapping horizon is independent of the chosen foliation [15].

Dynamical horizons could be either space-like or time-like, however, if for a considerable amount of time no matter has crossed, the horizon will stabilize and form a static null horizon eventually approaching an event horizon [19–21] in asymptotically flat spacetimes. Therefore, the treatment of dynamical horizons clearly covers static horizons but it should be mentioned that trapping horizons can occur even in systems which never form an event horizon [16].

\(^3\) Similar concepts were used by Hawking and Ellis [8] to define apparent horizons. However, this description imposes two global assumptions, i.e. asymptotic flatness and regular predictability, absent in Hayward’s notion. Asymptotic flatness, for example, cannot be realized for a de Sitter Universe filled with a positive cosmological constant.

\(^4\) The definition in this article exchanges the null congruences $l^\pm$ for past and future which deviates from the original definition in [9] where $l^\pm$ are fixed. In the framework used in [9], $\theta^+ = 0$, while the distinction between future and past depends on the sign of $\theta^-$. Except for peculiar cases, e.g. plane-wave spacetimes, both notions coincide (see [18] for more details).
We can study the behavior of the null expansions across the marginally trapped surface by involving the Lie derivative $\mathcal{L}_\pm \theta^\pm$ which shows in which direction specific light rays (outgoing or ingoing) are trapped. We call a future horizon outer if $\mathcal{L}_- \theta^+ < 0$ and inner if $\mathcal{L}_- \theta^+ > 0$ at $\mathcal{H}$. For past horizons, we can define similarly an outer horizon as $\mathcal{L}_+ \theta^- < 0$ and the corresponding inner horizon $\mathcal{L}_+ \theta^- > 0$ to hold at $\mathcal{H}$. Note, the expressions $\theta^\pm \theta^\mp$ as well as $\mathcal{L}_\pm \theta^\pm$ are geometrical invariants in the sense of [9].

For illustrative reasons, we will change to the spherically symmetric framework and discuss some examples. It should be noted that the areal radius $R$ is fully characterising the surface of symmetry. Consider the horizon to be at $R_H$, the normal region can be either outside the horizon ($R_{\text{normal}} > R_H$) like for a black hole or inside ($R_{\text{normal}} < R_H$) like for a Hubble sphere in an expanding cosmology. Mathematically, this property is reflected by the Lie derivative of the expansion $\theta^\pm$ along in- or outgoing light rays.

Hence, trapping horizons can be classified into four types: future horizons are defined using $\mathcal{L}_-$, the Lie derivative along ingoing light rays at the horizon, because $\theta^\pm < 0$ in the trapped region indicates that the classically allowed direction is ingoing, i.e. this is the null geodesic unchanged by geometry:

(a) **Future outer trapping horizon** (FOTH): $\mathcal{L}_- \theta^+ |_{R=R_H} < 0$

FOTHs are the most prominent type since they cover black hole horizons: the horizon lies in the future of the observer who can cross it from the normal region, outside at $R > R_H$, into the trapped region inside the horizon.

(b) **Future inner trapping horizon** (FITH): $\mathcal{L}_- \theta^+ |_{R=R_H} > 0$

Examples for FITHs are big crunch scenarios of contracting cosmologies: the horizon is the surface where a homogeneous and isotropic collapse, as seen from an observer at $R = 0$, is happening at the speed of light $c$. Beyond the horizon, the contraction appears to be faster than $c$ [17], hence, there is no chance to escape the collapse. For FITHs, the normal region is inside, at $R < R_H$, and signals from the observer will remain inside the horizon.

In contrast to future horizons, the Lie derivative along outgoing light rays $\mathcal{L}_+$ is used to characterize past horizons. Note that in this setup the outgoing direction is classically favored:

(c) **Past inner trapping horizon** (PITH): $\mathcal{L}_+ \theta^- |_{R=R_H} > 0$

This type describes, for example, an observer in an expanding Universe surrounded by a Hubble sphere. The normal region is inside the Hubble horizon and signals from events beyond, i.e. at $R > R_H$, will not enter the normal region (as long as the time evolution does not collect them naturally). Indeed, in the observer’s reference frame, the expansion will push signals apart faster than the speed of light [17] at $R > R_H$. Another notable example of PITHs is the horizon of the de Sitter spacetime [5, 22].

(d) **Past outer trapping horizon** (POTH): $\mathcal{L}_+ \theta^- |_{R=R_H} < 0$

An observer in the normal region at $R > R_H$, i.e. outside the horizon, will localize the horizon in the past light cone. Examples like white holes have the feature that radial geodesics lead away from this object and make it impossible to enter, consequently, all observers in the interior will be released into the normal region.

Reissner–Nordström black holes are solutions of Einstein’s equations where future inner and future outer horizons occur. If we assume they were suffering from an evaporation like Schwarzschild black holes, we must incorporate the Hawking effect for future outer and future inner horizons; even in the case of black hole formation, we can encounter the presence of both horizons (see [16]) such that a general description for (dynamical future) horizons is of paramount importance to analyze gravitational collapses concisely.
In contrast to static spacetimes, dynamical spacetimes do not admit a global timelike Killing vector field. Therefore, it seems that there is no preferred time direction along which a notion of energy could be defined. However, Kodama [23] found that in spherically symmetric spacetimes exists a vector field $K$ which is divergence-free and generates a preferred time-flow together with an associated energy flux [24]. The Kodama vector is defined by [25] $K = g^{-1}(\star dR)$ with $\star$ being the Hodge star in the space perpendicular to the spheres of symmetry, or in components $K^\nu = e^\mu_\nu \nabla_\mu R$ where $e^\mu_\nu$ is the $(1 + 1)$-dimensional Levi-Civita tensor in the temporal-radial plane. In particular, this definition involves that $\mathcal{L}_K R = 0$ which can be interpreted such that $K$ is always orthogonal to the spheres of symmetry [26]. In the normal region, $K$ is time-like, on the horizon null, and in the trapped region space-like; for static spacetimes, $K$ is parallel to the Killing vector field.

By the use of the Clebsch decomposition in $(1+1)$-dimension, one can show that the Kodama vector $K$ naturally induces a temporal coordinate, the Kodama time $t$, in the regions where $K$ is timelike. Hence, $K$ defines a preferred class of fiducial observers which move along integral curves associated to $K$ with velocity $V = K/\|K\|$ for which $K \propto \partial_t$ [24]. These properties of the Kodama vector allow to introduce an invariant (see [27] for details) surface gravity notion, namely the Hayward–Kodama surface gravity which is defined through $\kappa = \nabla_\nu (\star K^\nu)$ at $R = R_\text{H}$. Hayward [9] provided an operational definition by $\kappa_{\text{H}} = \frac{1}{2} \ast d \star dR = \frac{1}{2} \Omega, R$ at the horizon. The expression $\ast d$ is a divergence, and $d$ the gradient which allows to rewrite $\ast d = \Box = \gamma^{-1}(d, d)$. The $\Box$, operator is constructed with respect to the metric $\gamma$ of the two-dimensional space normal to the spheres of symmetry [28]. This astronomical quantity describes for static black holes the (gravitational) acceleration experienced by a test particle at the horizon. For generic spacetimes exists no such interpretation [29] and the surface gravity can only be understood as a parameter connected to the temperature.

Additionally, a fiducial observer in the normal region, can exploit the proportionality between $K$ and $\partial_t$ and deduce the invariant Kodama-energy along the flow of $K$, similar to the energy associated to the Killing vector in static spacetimes. For a particle with action $S_0$, the Kodama energy is defined as $\omega = -K^\nu \nabla_\nu S_0$. Although one can choose a Kodama foliation and refer to a fiducial observer, it should be stressed that this remains a choice. One can always refer to different local observers thanks to the individual covariance of the energy $\omega$ and the surface gravity $\kappa_{\text{H}}$.

In the following, we will restrict ourselves to spherically symmetric spacetimes to illustrate how a generalisation of the Hawking effect can be achieved employing a certain symmetry. Generally, a four-dimensional spherically symmetric spacetime can be locally coordinatised by the metric $g = g_{ij}(x)dx^i dx^j + R^2(x)dt^2 \Omega$ with signature $(-, +, +, +)$ [30] where $\gamma$, defined as above, covers the temporal-radial plane and $R(x)$ depends on the $x^i$ explicitly. The solid angle is given by $d^2 \Omega = d\Omega \otimes d\Omega + \sin^2(\varphi) d\varphi \otimes d\varphi$. Notice that in a Kodama foliation, corresponding to a Kodama observer, any metric in the Kodama coordinates $t$ and $R$ would acquire a diagonal form [26]

$$
g = g_{tt}(t, R)dt \otimes dt + g_{RR}(t, R)dR \otimes dR + R^2 d^2 \Omega,
$$

due to the properties $\mathcal{L}_K R = 0$ and $K \propto \partial_t$. To properly describe quantum processes across the horizon, we refer to a coordinate system which covers both regions, trapped and normal and is regular at the horizon. Therefore, the concrete calculations in this article will be carried out in the Eddington–Finkelstein–Bardeen metric [31]

$$
g = -e^{2\psi(x, R)}C(R)d\xi \otimes d\xi \pm 2 \ e^{\psi(x, R)}dR \otimes dR + R^2 d^2 \Omega
$$

with $d^2 \Omega$ denoting the solid angle, $R$ the areal radius, and $C$ and $\psi$ functions which stipulate the
properties of the horizon. At the horizon $R = R_H$, the derivative of $C(R)$ is assumed to be finite and different from zero. The variable $\xi = \{v, u\}$ covers the retarded or advanced light cone coordinates with $u = t - R^*$ being the outgoing, $v = t + R^*$ the ingoing light cone coordinate, and $R^* = \int_R^{R_H} \frac{dR}{C(R)}$. One choice, $\xi = v$, is particularly used to treat future trapping horizons, since $v$ is well-behaving across the horizon, that is to say, $v$ remains ingoing even in the trapped region; for the same reason, the other choice, $\xi = u$, is best suitable for past trapping horizons.

The off-diagonal term in (2) acquires a plus sign for $\xi = v$ and a minus sign for $\xi = u$.

Note, if at the horizon $L^\pm \theta^\pm = 0$ then the horizon is degenerate, i.e. it is an inner and outer horizon at the same time and $s_H \equiv 0$ [28]. This type occurs for example in a radiation dominated Universe.

3. Hamilton–Jacobi formalism

In the previous section, we noticed that trapping horizons could behave as semipermeable boundaries. Then, the Hawking effect is proposed to be the process acting against the geometrically preferred direction of horizon crossing; this heuristic idea will be the guiding principle throughout this article and the following calculations. We will introduce the Hamilton–Jacobi tunneling method, which enables us to study the change of the particle number in the presence of dynamical horizons through tunneling and provides an intuitive understanding of the ongoing effects. This picture was first suggested by Parikh and Wilczek [10], Massar and Parentani [32], and has been connected to the Hamilton–Jacobi formalism by Padmanabhan [33], and thoroughly reviewed by Vanzo et al [15, 34, 35]. Basically, in the normal region, near $R_H$, a particle–antiparticle pair is created due to the excitation of the vacuum by the strong gravitational field; the horizon itself is interpreted as sort of barrier [36] inducing a pole in the propagator of the particle. Once the pair is created, the tunneling of one particle through the horizon could separate them, preventing the pair to recombine. The remaining particle will eventually propagate to the observer where it is measured as Hawking particle.

In this article, the gedankenexperiment guiding our analysis, independently of the horizon type, will be the following: in the normal region, where the Kodama vector is timelike, the Kodama-observer prepares a state at an initial (Kodama-) time $t_{\text{start}}$ and counts a certain number of particles. Then the initial conditions are evolved to a later time $t_{\text{end}} > t_{\text{start}}$ and the observer will count again. If the detection counts more particles we will say the Hawking effect is an emission of particles by the horizon, if fewer particles are counted, we will interpret the Hawking effect as an absorption of particles by the horizon. The particle count can be realised by a clicking event of an Unruh–DeWitt detector [27] which moves along the Kodama flow such that the energy of the particle can be related to the invariant Kodama energy.

To implement these tunneling phenomena mathematically, the appropriate description would be to find emission and absorption rates in quantum field theory on curved spacetimes. However, a quantum mechanical formulation by a WKB approximation and the quantum field theoretic description agree for quasi-local states: the field-theoretical analog of a tunneling would be the propagation across $R = R_H$, given by the two-point function $\langle \phi(R_1)\phi(R_2) \rangle$ with $R_1$ and $R_2$ on opposite sides with respect to the horizon. It should be mentioned, the choice of the vacuum is crucial to define the energy spectrum seen by the observer. Since the number operator is state-dependent, we will argue that there is a special or preferred vacuum which describes the energy spectrum of the two-point function seen by an observer which moves along trajectories generated by a vector field reflecting the symmetries. In spherical symmetric spacetimes this is given by the Kodama vector and hence the choice of a Kodama foliation [23]. For quasi-local states, the two-point function can be related to the WKB rate [37, 38], which is an observable [35] as well. Therefore, we are allowed to use techniques from the
WKB framework of a particle in a potential as long as our states are suitably localized. The WKB approximation is valid in regimes where the effective potential, induced by the geometry, varies slowly in time compared to the frequency of the particle. This approach allows us to describe even slowly varying, time-dependent, spacetimes. To be consistent with the WKB condition, we assume the Kodama energy of the tunneling particles $\omega$ to be small compared to the energy scale set by the classical geometric background which in terms of (2) translates to
\[
\partial_R C(R) \ll 2\omega.
\]

While the Hawking effect itself gives dynamics to the horizon, the effect is, however, too small to violate the slow-evolution assumption set by the WKB approximation. For a black hole this can be justified by a back-of-the-envelope calculation: consider a stellar black hole with mass $M \approx 10^{30}$ kg and radius $r_s = 2G_N M / c^2 \approx 1500$ m where $c$ is the speed of light and $G_N$ Newton’s constant. An emission of an extremely heavy particle with almost Planck mass $M_P \approx 10^{-8}$ kg changes the Schwarzschild radius and consequently the scale induced by the geometry by $\delta r_s \approx 10^{-35}$ m which is negligible even in the case of numerous particle emissions.

The basic idea of the Hamilton–Jacobi method connects the tunneling probability to an imaginary contribution in the classical action of the particle $S_0$. For our analysis we consider a scalar field $\phi$ with mass $m$ satisfying the Klein–Gordon equation
\[
\left( \Box - \frac{m^2}{\hbar^2} \right) \phi = 0
\]
where $\Box$ is constructed with respect to (2). Note that the description of tunneling processes through black hole horizons is state-independent for scalar fields [37]. Within the validity of the WKB approximation, we take
\[
\phi = e^{i\hbar S} = e^{i\hbar S_0 + S_1 + O(\hbar)}
\]
as ansatz for the solution to (3), where we included $\hbar$ explicitly which serves as smallness parameter in the second step. Furthermore, we expanded the complex action $S$ to incorporate quantum effects to first order in $\hbar$. After substituting this approximation into (3) and splitting the resulting equation into real and imaginary part, we take the semi-classical limit, $\hbar \to 0$, and obtain the relativistic Hamilton–Jacobi equation for the classical action
\[
g^{-1}(dS_0, dS_0) + m^2 = 0.
\]
In [15], it has been argued that the tunneling paths are most likely given by null-like trajectories and the mass term in the Hamilton–Jacobi equation can be neglected. Even if considering timelike trajectories one can show that the result does not substantially deviate from the massless case [34]. Taking this into account, the above equation can be solved with the following ansatz for $S_0$:
\[
S_0 = \int \partial_\xi S_0 \, d\xi + \int \partial_R S_0 \, dR.
\]
We perform the integration along the dynamical path of the particle but we should be aware that we could in principle encounter poles which can be avoided by a complexification of the manifold, resulting in an imaginary part in the classical action $S_0$. All integration ranges will be specified when we analyze the specific horizons. The $\xi$-integral is vanishing along null tunneling paths and therefore, does not contribute to $\text{Im}(S_0)$ [15]. Instead, the horizon induces a pole in the radial integration at $R_H$ (see next section or [39]), which exactly describes the classically
forbidden path. We suppressed the angular part because, there will be no pole in the $\vartheta$- and $\varphi$-integration due to the underlying spherical symmetry [40]. Analogous to quantum mechanics, where tunneling processes through a barrier result from imaginary momenta, gravitational tunneling is facilitated by an imaginary contribution to $S_0$. The WKB wave-function experiences a discontinuity at the pole but will be able to pass around on a complex path. Within this framework, the rate for an individual particle to tunnel is given by

$$\Gamma \propto e^{-\frac{1}{\hbar} \text{Im}(S_0)},$$

(7)

which shows the immediate relation to the imaginary part of the classical action [41]. Expression (7) describes always the classically forbidden process whenever the rate is exponentially suppressed. This is the case for gravitational tunneling, hence, we claim for all realizations of the Hawking effect that $\text{Im}(S_0) > 0$. Furthermore, the rate in (7) adopts the exponent’s diffeomorphism invariance which ensures that the occurrence of the Hawking effect is a gauge invariant statement [35].

4. Hawking effect

In this section, we develop the idea that the Hawking effect counteracts the classical direction of horizon crossing such that it can be applied to all types of trapping horizons. We start with the analysis of FOTHs and recover Hawking’s result via the Hamilton–Jacobi method before we consider other horizon types. Our treatment will be restricted to scalar degrees of freedom but in principle higher spins would be possible. As an example, tunneling of fermions by dynamical black holes has been analyzed with the outcome that the dominant contribution matches the results for scalar fields [42].

For a causal propagation, the equation of motion (3) will be equipped with the Feynman prescription $+i\epsilon$, that is to say, positive frequency modes propagate to the future and negative to the past. Solutions to the Klein–Gordon equation with anti-Feynman prescription $-i\epsilon$ describe processes where positive frequency modes propagate to the past and negative to the future, i.e. events are ordered complementary to the Feynman propagator. Although we could perform all calculations with the Feynman prescription, it will turn out that in some setups the anti-Feynman propagator provides a clearer physical picture.

In the following subsections, we derive the Hamilton–Jacobi equation from $(\Box - m^2/\hbar^2 \pm i\epsilon)\phi = 0$ by applying it to (4) and taking the semi-classical limit. Then, the prescription in the Klein–Gordon equation will be transferred to the integral for $S_0$ as a complexification of the radial coordinate $R$ allowing the particle to pass around the pole at the horizon. Even though there is no classically allowed path across the horizon, quantum particles could escape through complex paths.

4.1. Future outer trapping horizon

Characteristic for FOTHs is that an in-falling observer could easily enter but never escape. It is illustrative to use the black hole as an example whenever we would like to clarify the physical idea behind our calculation and compare the results with the existing literature. A comprehensive analysis of dynamical black holes was performed in [15], where the special case of FOTH was widely analyzed and several results for dynamical black holes were collected. As more explicit examples, the FOTHs ($\theta^+ = 0$) of the Vaidya and the McVittie solution have been studied in [43]. The McVittie metric describes a static black hole in an expanding Universe, hence, it has a future outer black-hole horizon and a past inner cosmological horizon. By using
the cosmological expansion to give fiducial dynamics to the black hole horizon, the Hawking radiation for this specific dynamical FOTH is calculated via the tunneling method.

Nevertheless, our calculations account for spherically symmetric FOTHs. Consider an FOTH at radius $R_H$, the corresponding Hawking effect, as classically forbidden propagation, is a tunneling from the trapped interior to the normal exterior region. In the Klein–Gordon equation, we use the Feynman prescription $+i\varepsilon$ because we treat a causal emission toward the future. We start to derive the relevant quantities along the Kodama vector $K = (e^{-\psi(v,R)}, 0)$:

the surface gravity

$$\kappa_H = \frac{1}{2} \partial_R C(R)|_{R=R_H},$$  \hspace{1cm} (8)

which has to be evaluated at the horizon and the Kodama energy for a particle in motion [15]

$$\omega = -K^\mu \partial_\mu S_0 = -e^{-\psi(v,R)} \partial_v S_0.$$  \hspace{1cm} (9)

We could rewrite (6) by using (8), (9), and the identification with the momentum $k = \partial_R S_0$ to get

$$S_0 = -\int e^{\psi(v,R)} \omega \, dv + \int k \, dR.$$  \hspace{1cm} (10)

Again, the angular parts are suppressed because of symmetry reasons. Plugging this into the Hamilton–Jacobi equation with Feynman prescription, yields a quadratic equation for $k$ which can be solved to first order in $\varepsilon$. We get two solutions

$$k_1 = -\frac{i\varepsilon}{2\omega},$$  \hspace{1cm} (11)

$$k_2 = \frac{2\omega}{C(R)} + \frac{i\varepsilon}{2\omega}.$$  \hspace{1cm} (12)

These solutions represent the ingoing and outgoing directions of motion. The momentum $k_2$ describes solutions which are going from inside to outside, while $k_1$ covers solutions which are falling into the black hole. We will see that only $k_2$ corresponds to a tunneling path, because the roots of $C(R)$ cause this solution to have a pole in the radial coordinate, which can be bypassed on a complex path. Hence, $k_2$ generates an imaginary contribution in the action

$$\text{Im}(S_0) = \text{Im}\left( \int \frac{2\omega \, dR}{C(R) \left( 1 - \frac{1}{4\omega^2} \right)} \right).$$  \hspace{1cm} (13)

Via a Taylor expansion of the function $C(R)$ around $R_H$ we can impose a near horizon approximation

$$C(R) = (\partial_R C)|_{R=R_H}(R - R_H) + O\left((R - R_H)^2\right).$$  \hspace{1cm} (14)

We would like to mention that $C(R)$ is zero at $R_H$ while its derivative is finite and non-zero. As already pointed out, the Hawking effect here is an emission and we could describe it as a particle going from the trapped region into the normal region where it escapes to the observer. However, the definition of particles and observers in the trapped region is a touchy business. Therefore,

\[5\text{The function } C(R) \text{ changes sign for } R > R_H \text{ and } R < R_H. \text{ Therefore, and because } C(R) \text{ is a smooth function, the zero should be at the horizon.}\]
we would like to use an equivalent picture such that we could perform our experiment in the normal region, exploiting the features of the Kodama vector. Mathematically, an emission can be described similarly as [10]: after pair-creation in the vicinity of the horizon, a negative energy particle ($\omega \rightarrow -|\omega|$), or identically a hole, tunnels from the normal region into the black hole interior. The resulting tunneling path of this process would be the one of a negative energy particle traveling from outside to inside along a complexified path. Together with (8), expansion (14) can be plugged into (13) yielding

$$\text{Im}(S_0) = \text{Im} \left( \int_{R_1}^{R_2} \frac{-|\omega| dR}{\kappa_H \left( R - R_H - \frac{\omega}{\kappa_H^2} \right)} \right),$$

(15)

where $R_1 < R_H < R_2$ and $\varepsilon'$ denotes the rescaled smallness parameter. Note, the $\varepsilon$-prescription in the Klein–Gordon equation has become a complexification of the radial coordinate. After some manipulations

$$\text{Im}(S_0) = \lim_{\varepsilon'' \to 0} - |\omega| \kappa_H^{-1} \int_{\delta/\varepsilon''}^{1 - \delta/\varepsilon''} \frac{1}{\left( \frac{R - R_H}{\varepsilon''} \right)^2 + 1} d \left( \frac{R - R_H}{\varepsilon''} \right),$$

(16)

where we perform the integration across the horizon with one coordinate in the trapped region and one in the normal region such that $\delta = |R_1 - R_2|$ is small and we find

$$\text{Im}(S_0) = \lim_{\varepsilon'' \to 0} - |\omega| \kappa_H^{-1} \left[ \arctan \left( -\frac{\delta}{\varepsilon''} \right) - \arctan \left( \frac{\delta}{\varepsilon''} \right) \right].$$

(17)

The integration yields an additional minus sign which would not have been present for the positive energy particle tunneling outwards. However, the final result will stay unaffected because of the sign change of $\omega$. After taking the limit $\varepsilon'' \to 0$

$$\text{Im}(S_0) = \frac{\pi |\omega|}{\kappa_H},$$

(18)

The imaginary part is a positive number because the surface gravity $\kappa_H > 0$ for outer horizons. Equation (18) can then be used to calculate the tunneling rate (7)

$$\Gamma \propto \exp \left( -\frac{2\pi |\omega|}{\hbar \kappa_H} \right).$$

(19)

As expected the Hawking effect for black holes obeys $\text{Im}(S_0) > 0$ which mathematically reflects the classically prohibited process leading to an exponential suppression. To match the quantum mechanical tunneling rate and the temperature, we follow the procedure outlined in [44]. The first step involves the Boltzmann distribution which gives the probability to emit a particle, or in other words we compare the probability of having $n$ particles to the probability of having $n + 1$ particles, at a fixed energy $E$

$$P_{\text{emission}} = e^{-\frac{E}{k_B T}} P_{\text{absorption}},$$

(20)

with Boltzmann constant $k_B$. It turns out that the emission probability given by (20) is exponentially suppressed and can be associated to the tunneling rate in (19). We can easily read off the corresponding parameter

$$T = \frac{\hbar \kappa_H}{2\pi k_B},$$

(21)
which coincides exactly with the temperature found by Hawking. We should note that the relation between the quantity $T$ and the temperature of black hole thermodynamics would have strictly applied if we had sent the observer to the asymptotic regions, where other thermodynamic quantities, e.g. the mass, are well-defined [6]. Specifically, the parameter $T$ in the tunneling picture is generically a non-equilibrium temperature, since it is plagued by gravitational modifications $T(t,R) = T_E/\Omega(t,R) + F(\Omega(t,R))$ with $T_E$ the equilibrium temperature and $F$ a complicated function of the geometric factor $\Omega(t,R)$ (see [45] for explicit calculations). Considering metric (2), Hayward et al [25] showed that for slowly evolving setups $\Omega(t,R) \equiv \sqrt{C(R)}$ while $F(\Omega(t,R))$ becomes subdominant with respect to the first term. Since the WKB approximation requires the spacetime, and hence the horizon, to vary slowly, this constitutes just a small deviation from the thermal equilibrium and approximates a full thermal equilibrium only in asymptotically flat regions.

Hence, the tunneling description reproduces that an observer at future infinity measures a thermal spectrum coming from the FOTH. From now on we will call the quantity $T$ (Hayward–Kodama) temperature with the warning that this identification is only valid in an appropriate approximation such as in the asymptotic regions of spacetime [36, 46].

### 4.2. Future inner trapping horizon

The following subsection shows that for the FITH a different sort of Hawking effect exists. In particular, we will verify that the FITH is the absorptive partner of the FOTH with respect to quantum effects. FITHs appear, for example, in contracting cosmologies with a big crunch in the future, as well as in more general black hole solutions, like the Reissner–Nordström black hole, in combination with an FOTH. Hence, to understand the Hawking effect for general black hole solutions it is, as a first step, important to understand the Hawking effect for FITHs.

Before we explore the actual calculation, we analyze the setup from the tunneling perspective. FITHs enclose the normal region inside while the trapped region lies beyond the horizon. In the example of a big crunch, the Universe contracts such that, at a distance $R_H$ to the observer, the contraction reaches apparently the speed of light. Classically, all signals sent by the observer at $R = 0$ will be bound to distances $R \leq R_H$, i.e. the classically forbidden path would be to cross the FITH from the normal into the trapped region. As quantum effect, we therefore expect to see an absorption of particles by the horizon; referring to our gedanken-experiment, the observer would count less particles in the final than in the initial state. In the normal region, we could portray an absorption in two ways: either we describe the system by a causal Feynman propagation and allow the horizon to emit negative energy particles/holes; or alternatively we illustrate this process through an emission of positive energy particles by the horizon toward the past. In the first case, the negative energy particles emitted by the horizon annihilate some of the initially prepared particles while the latter case could be regarded as adding particles to the initial state which then get absorbed during a causal time evolution. In both descriptions, the net process is a detection of less particles in the observer's final state.

To connect FITHs to thermodynamics, the easiest way would be to choose the second description using the anti-Feynman prescription, i.e. $-i\varepsilon$, and count particles emitted toward the past. Then, we observe a spectrum which we can directly relate to the temperature of the horizon. In the tunneling picture, emission into the past coincides with a Hawking pair where the negative energy particle crosses the horizon while the positive energy particle reaches the observer’s initial state, traveling both backwards in time. According to this, we analyze for the FITH scenario negative energy particles which path goes from the normal region inside ($R_1 < R_H$) into the trapped region outside ($R_2 > R_H$) but backwards in time. The
Hamilton–Jacobi equation yields similar solutions for $k$ as in section 4.1

\begin{equation}
k_1 = \pm \frac{i\varepsilon}{2\omega},
\end{equation}

\begin{equation}
k_2 = \frac{2\omega}{C(R)} - \frac{i\varepsilon}{2\omega},
\end{equation}

but now with the anti-Feynman prescription. Again, $k_2$ admits a pole at the horizon leading to a non-zero tunneling probability. With this in mind, we get as imaginary part of $S_0$

\begin{equation}
\text{Im}(S_0) = \text{Im} \left( \int_{R_1}^{R_2} \frac{-|\omega|dR}{-|\kappa_H| \left( R - R_H + \frac{\varepsilon'}{|\kappa_H|} \right)} \right).
\end{equation}

Due to the fact that $\kappa_H < 0$ for inner horizons, the imaginary part acquires the same value as in section 4.1. This results in a non-zero tunneling rate

\begin{equation}
\Gamma \propto \exp \left( -\frac{2\pi|\omega|}{\hbar|\kappa_H|} \right)
\end{equation}

showing that there is a non-zero probability for the particle to get absorbed by the horizon. Comparison with the Boltzmann distribution (20) yields as temperature of the emitted spectrum

\begin{equation}
T = \frac{\hbar|\kappa_H|}{2\pi k_B}.
\end{equation}

With the tunneling picture we could transfer our principle idea about the Hawking effect and find that FITHs are subjected to an absorptive Hawking effect. With respect to the definition, FITHs are the absorptive partners to the emissive FOTHs.

Note, we could have calculated everything in the Feynman prescription; in that case we would have to look at the correct process and change (20) accordingly to be

\begin{equation}
P_{\text{absorption}} = e^{-\frac{\hbar}{k_B} P_{\text{emission}}}
\end{equation}

because now the less probable process is given by the absorption of a positive energy particle. For an emission of a negative energy particle we have to invert (27) but since we compare negative energy emission, we need to replace $E \rightarrow -|E|$. As we mentioned before we compare the tunneling rate with the probability (27) of going from $n$ positive energy particles to $n - 1$, or equivalently from $n$ negative energy particles to $n + 1$, resulting in a positive temperature like (26). In this light, the formalism gives a consistent result and supports the proposed interpretation. Nevertheless, one could worry about measuring negative energies in the system, but the physical interpretation is still consistent because the emitted negative energy particles deplete the initially prepared state and lower the number of particles in the normal region. In order to be more illustrative, we could exploit the big crunch a bit more considering now a photon test field in our contracting Universe. We would like to do the same gedankenexperiment as before and measure particles arriving at our detector. In this cosmology, an observer would experience a constant influx through the horizon caused by the presence of the photons. This is predicted by (27) telling us that the emission process is geometrically favored while the absorption is exponentially suppressed. If we now switch on quantum effects, we will observe a reduction of the natural emission spectrum by the absorption spectrum associated to $T$. 

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4.3. Past inner trapping horizon

In the next two subsections, we show that an emissive/absorptive horizon pair exists also for past horizons. Let us take an expanding cosmology with big bang as example where the Universe expands with a Hubble law from the initial singularity. Since the expansion is homogeneous and isotropic, the observer at \( R = 0 \) can only see events inside the Hubble sphere with radius \( R < R_H \) and will not be able to see beyond because all events at \( R > R_H \) are trapped outside the horizon [17]. For Friedmann–Lemaître–Robertson–Walker spacetimes, which admit a PITH, the Hawking effect has been investigated in [15] and found, similar to the black hole case, to be an emission with a thermal spectrum.

The Hamilton–Jacobi formalism requires to identify the classically forbidden process which is now an emission of particles from the anti-trapped region outside the Hubble sphere into the normal region inside. Again, this is a causal emission into the future and we adopt the Feynman prescription \(+ i \varepsilon\) in the Klein–Gordon equation. In the tunneling picture this could be recasted as a negative energy particle crossing the horizon from the normal region at \( R_1 < R_H \) to \( R_2 > R_H \). Therefore, we can perform the steps similar to section 4.1 and solve the Klein–Gordon equation with a Feynman prescription. Whenever we treat a past trapping horizon, we use (2) with outgoing lightcone coordinate \( \xi = u \). Altogether, the Hamilton–Jacobi equation yields two different solutions for the momentum

\[
\begin{align*}
  k_1 &= -\frac{i \varepsilon}{2 \omega}, \\
  k_2 &= -\frac{2 \omega}{C(R)} + \frac{i \varepsilon}{2 \omega},
\end{align*}
\]

with \( k_1 \) now being the outgoing and \( k_2 \) the ingoing solution to first order in \( \varepsilon \). Carrying out the same procedure as in the previous sections we find for the negative energy particle

\[
\text{Im}(S_0) = \text{Im} \left( \int_{R_1}^{R_2} \frac{-|\omega|dR}{-|\kappa_H| \left( R - R_H - \frac{i \varepsilon}{|\kappa_H|} \right)} \right)
\]

where the surface gravity is now negative \( \kappa_H \rightarrow -|\kappa_H| \). As before, the non-zero imaginary part appears because of a pole at the horizon and we could interpret the resulting process as a net particle flux into the observer’s Hubble sphere. The imaginary part equals (18), the resulting rate equals (19), and the emitted thermal spectrum peaks at a temperature

\[
T = \frac{\hbar |\kappa_H|}{2\pi k_B}
\]

Note that this is in perfect agreement with the temperatures found for PITHs in an FLRW [15, 47, 48] as well as in a de Sitter background [5].

The two horizons, PITH and FOTH, are both subjected to a Hawking effect which is described by an emission of particles (toward the future). Therefore, from the perspective of quantum field theory both horizons are emissive horizons.

4.4. Past outer trapping horizon

POTHs can occur in a fully extended black-hole spacetime describing white hole horizons or appear in cosmologies dominated by a stiff fluid, i.e. equation of state parameter \( w = 1 \). The latter example has both a past inner and a past outer trapping horizon. For the sake of developing intuition, we will consider the white hole as the main example. Since white holes can
classically just emit, the Hawking effect, as counter-effect, is expected to act as an absorption. By definition, POTHs enclose the trapped region inside at \( R < R_H \) while the normal region is located outside. We want to specify that the trapped region is, in fact, anti-trapped, i.e. classical trajectories depart from each other in the future development such that the geometrically favored direction is outgoing. Then, the process of interest is the path entering the white hole or, in other words, traverses the POTH from the normal into the anti-trapped region. Following the arguments in section 4.2, we employ the anti-Feynman prescription to describe absorption, that is, we prepare the initial state in the future and perform a backward-in-time measurement. To assign a temperature to this emitted spectrum, we collect the particles which are emitted toward the past. The calculations in sections 4.1–4.3 change as follows: the solutions for \( k \) are given by (28) and (29) but with the anti-Feynman prescription

\[
\begin{align*}
k_1 &= + \frac{i \epsilon}{2 \omega}, \\
k_2 &= - \frac{2 \omega}{C(R)} \pm \frac{i \epsilon}{2 \omega}.
\end{align*}
\]

Again, only the ingoing momentum \( k_2 \) contributes to the tunneling rate because of the pole induced by the zero in \( C(R) \). The corresponding imaginary part reads

\[
\text{Im}(S_0) = \text{Im} \left( \int_{R_1}^{R_2} \frac{-|\omega| dR}{\kappa_H (R - R_H + \frac{\omega}{\kappa_H})} \right)
\]

where \( \kappa_H \) is now positive. The above integral (34) describes a negative energy particle tunneling backwards in time into the anti-trapped region. This accounts for a net particle flux which the horizon emits toward the past [10]. The resulting imaginary part

\[
\text{Im}(S_0) = + \frac{\pi |\omega|}{\kappa_H}
\]

induces a non-zero tunneling rate. In other words, the non-removable pole at \( R_H \) furnishes the POTH with an absorptive Hawking effect. We see again that the anti-Feynman prescription leads to a thermal spectrum with temperature

\[
T = \frac{\hbar \kappa_H}{2 \pi k_B},
\]

emitted toward the past. When we consider the example of an emitting white hole, we would conclude that the Hawking effect reduces the spectrum seen by a future observer, who could interpret this as emission of holes or an absorption of particles, like in section 4.2. POTHs are therefore the absorptive partner of PITHs and our results for POTHs are in accordance to the idea of [15] about the white hole event horizon, which is a static null version of POTHs.

5. Discussion

Hayward classified trapping horizons with respect to null expansions along in- and outgoing light rays into four types. In this article, we analyzed quantum effects associated with these horizons and verified for spherically symmetric spacetimes that all these trapping horizons are subjected to a Hawking effect by applying the tunneling picture via the Hamilton–Jacobi method. The idea is to formulate the Hawking effect as the propagation opposite to the path
preferred by general relativity. In the tunneling picture, the horizon might induce a pole in the action which excludes classical paths. However, a remedy can be found by looking at quantum processes: the consistency of the theory stays unharmed as long as avoiding poles on complexified paths happens on time scales that do not violate macrocausality. According to this, we found that past outer and future inner trapping horizons are subjected to an absorptive Hawking effect.

Within our framework, quantum field theory on curved spacetime suggests consequently two types of horizons: absorptive horizons (FITH and POTH) for which the Hawking effect lowers the number of detected particles and emissive horizons (FOTH and PITH) for which radiation from the horizon occurs. To strengthen the physical intuition related to the absorption effect, we consider a black hole which admits a future inner and future outer trapping horizon together with a suitably stable inner normal region. By energy budget considerations the Hawking effect induces an evaporation of black holes. Reducing the black hole mass implies to extract energy out of the black hole interior. Tunneling paths for black holes admitting an inner and outer horizon have therefore to cross both. According to our gedankenexperiment, an observer in the interior, surrounded by the inner horizon, i.e. in the normal region, will prepare particles and send them toward the horizon. After some time has elapsed, the observer counts the particles and will measure a depletion of particles because of the Hawking absorption. From this, the observer would infer some particles have crossed the future inner horizon into the trapped region. To get the energy finally out of the black hole, the particles have to cross the outer horizon as well. An outside observer would then perceive an ordinary Hawking emission. From this perspective, the result that one realization of the Hawking effect corresponds to an absorption seems reasonable. Because the probability for traveling fully through the trapped region is strongly restricted by the evanescence of the wave-function, we could estimate the lifespan of black holes with two horizons compared to Schwarzschild black holes from the tunneling perspective of Hawking radiation. Intuitively, we hypothesize that crossing two trapping horizons through the trapped region reduces the overall tunneling probability which will enhance the life-time accordingly. Additionally, charged black holes suffer from the problem of mass inflation. A full consideration of the Hawking effect might be helpful to address this issue. However, this scenario needs further investigations in the future.

On the other hand, our results for FITHs and POTHs need further discussions. Considering a radiation filled contracting cosmology or white hole, an absorptive Hawking effect is the equivalent of a thermal emission of holes which reduce the ejection caused by geometry. However, there is an important difference between emissive and absorptive Hawking effect: in case of FOTHs and PITHs the horizon crossing occurs from the trapped (or anti-trapped respectively) to the normal region while for FITHs and POTHs the Hawking effect describes an absorption into the trapped (or anti-trapped) region. The normal region supports particle to stay there, but the trapped (or anti-trapped) region of FITHs and POTHs will force the particles to leave as soon as possible. Hence, the Hawking process gets revoked. While this takes place, already new Hawking particles will have entered the (anti-)trapped region, possibly slowing down the cosmic contraction in big crunch scenarios or increasing a white hole’s lifetime. The observable contribution from the Hawking effect will not result in the abundance but in the absence of particles. This raises the question of whether a Hawking effect for FITHs and POTHs occurs as an in-principle detectable thermal spectrum. We doubt that this is the case because the released particle might recombine with its Hawking partner in the normal region.

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6 Recent publications show that the innermost normal region of Reissner–Nordström black holes are highly unstable under small perturbations while in Kerr black holes this region is stable. Due to the axisymmetry this needs further investigation from the perspective of the tunneling method.
and the effective particle number stays unchanged. The only detectable thermal spectrum will occur in the case when the absorbing horizons are cascaded with emitting horizons. A penetration of FOTHs and PITHs is likely but a total, eternal absorption seems to be unlikely unless there is a horizon providing an additional exit from the trapped region.

Possible applications of these results exist in gravitational collapse scenarios which lead to the formation of black holes. Inclusion of the Hawking effect could regularize the collapse such that trapped surfaces similar to black holes will form but without ever forming a singularity [16, 52, 53]. In some classes of collapsing solutions such as the Vaidya spacetime, a future inner and a future outer trapping horizon are present. The evolution of the inner trapping horizon, predicted by general relativity, finally forms a singularity while the outer asymptotically approaches the black hole event horizon. In this case, the Hawking effect leads the outer horizon to shrink and our results predict a slowdown in the evolution of the inner horizon such that both horizons could approach each other. In particular, the possibility for the occurrence of a closed trapped surface without the formation of a singularity opens up when both horizons meet. In this sense, the Hawking effect could facilitate a stellar collapse without singularity.

Sebastiani, Vanzo, and Zerbini [54] showed that traversable static wormhole horizons are not subjected to a Hawking process. Their result does not raise a contradiction to ours, instead, it fits into the presented physical idea: the wormholes in [54] are in principle traversable and an observer could always enter and exit. Such static wormhole horizon lacks the property of being semipermeable. Mathematically, this is explained by the presence of a removable pole at the horizon with the consequence that the action has no imaginary part causing a Hawking effect.

Finally, it should be noticed that our argumentation might be generalisable beyond spherically symmetric spacetimes. In [14], a generalisation of the Kodama vector has been proposed. This dual expansion vector, for which similar quantities as the energy along the flow as well as the surface gravity can be defined, serves as a possible path toward an application of the presented formalism to more general spacetimes. Since only FOTHs and PITHs have been studied by the authors, its application to FITHs and POTHs and the possibility of a Hawking absorption needs to be investigated in the future.

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