Candidate bus selection for dynamic VAR planning towards voltage stability enhancement considering copula-based correlation of wind and load uncertainties

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Abstract
As a critical step of VAR planning, candidate bus selection can significantly reduce the scale of the planning problem without impairing the optimality of the planning decisions. A novel candidate bus selection method is proposed considering not only the capacity sensitivity of candidate buses, but also the correlation among uncertainties of wind power generation and load demands. First, two metrics are proposed for different sensitivity analysis purposes to evaluate voltage performance in the post-contingency stage comprehensively, considering the extent of the voltage deviation and voltage recovery time simultaneously. Then, D-vine copulas are constructed systematically and employed to generated correlated operational scenarios for the dependent variable sensitivity analysis based on Kucherenko indices. Morris screening method is used for the independent capacity sensitivity analysis of the candidate buses. Finally, a 2-dimensional candidate bus selection process is proposed for the decision-makers to determine the candidate buses flexibly. The proposed method is tested on a modified Nordic 74-bus system with an industry-standard complex load model. Simulation results verify the effectiveness and accuracy of the proposed method.

1 INTRODUCTION

Escalating environmental pressure leads to increasing grid integration of renewable energy resources, like wind power. However, the inertia of power systems is decreased remarkably due to the high penetration level of wind energy in the power system and the associated prevalence of power electronic converters. The low-inertia power system is thus becoming vulnerable when exposed to equipment failures or weather-triggered voltage disturbances [1, 2]. Besides, the increasing portion of induction motor loads, which will absorb a large amount of reactive power during a disturbance, further exacerbates the dynamic voltage response of the power system.

The deployment of dynamic VAR compensation devices with short response time (milliseconds), like STATCOM (static synchronous compensator), is effective to address this voltage stability issue. The performance of STATCOM depends on installation sites and capacity [3, 4]. Generally, there are two approaches for the VAR planning problem: The one-step approach and the two-step approach. The one-step approach optimizes the capacities and locations of reactive power sources simultaneously. Since all the buses are considered as candidate buses, this approach may have an extremely high computational burden and is also difficult to solve. As for the two-step approach, first, only the most influential buses are selected as the candidate buses for the installation. Then, the optimization of the capacity is carried out. By reducing the number of buses in the capacity optimization process, the two-step approach significantly reduces the scale and complexity of the VAR planning problem. So, identifying the influential bus, namely candidate bus selection, is a critical step of VAR planning. Conventionally, metrics for steady-state [5, 6] and short-term [7, 8] voltage stability evaluation are used to select the buses. Considering the dynamic feature of STATCOMs, steady-state indices using load margin of branches or voltage violations are no longer adequate. The short-term voltage stability metrics mainly based on post-contingency voltage trajectories can better describe the dynamic process after a contingency. Conventionally, only the extent of...
voltage deviation is considered in the evaluation [7, 8]. Alternatively, industrial standards [9, 10] can be applied for the assessment, but their binary judgment is not as helpful as a quantitative one. In recent years, new short-term voltage stability metrics [11–13] are proposed to analyze details of voltage recovery. Ref. [11] proposes an index to evaluate the recovery speed of voltage, based on the time that the voltage magnitude takes to recover to an acceptable threshold. Ref. [12] uses the bus with the largest deviation to represent the overall performance of the system. A continuous metric considering both the time and extent of the deviation is proposed in ref. [13]. Although these indices can satisfy their specific research goals, they are inadequate for a candidate bus selection problem in a wind power-penetrated power system. This is because a wind turbine is highly sensitive to both the extent and the length of voltage deviation [1] and the delayed voltage recovery process should be further analyzed and evaluated. Furthermore, none of the studies considers the adaptability of the metrics in different contexts, such as the difference between the evaluation of operational scenarios and the evaluation of capacity sensitivity.

With appropriate metrics, sensitivity analysis techniques are employed in previous studies [11, 14–16] to select the candidate buses for the installation. Both global [11] and local [14–16] sensitivity techniques are used. However, in these studies, sensitivity analysis only focuses on the capacity of the VAR sources and the contingencies, while the uncertainties of wind power generation and the load variations are not considered in the candidate bus selection stage. In this context, there is a high risk of inappropriately selecting a specific load and wind power generation level as a typical operation profile. Furthermore, given the varying load demand and the stochasticity of wind power, results from these methods are not robust and their optimality will be compromised when subjected to the unconsidered operational uncertainties.

On the other hand, there is a high level of stochastic dependence/correlation between different loads and wind power generators which can also significantly affect the VAR planning decisions, and these dependencies are non-linear and non-Gaussian [17]. Conventional random sampling methods considering the operational uncertainty separately, such as Monte Carlo method with independent distribution, suffers from the ignorance of correlation between these uncertainties. Even for a method considering a joint distribution of wind and load [18], the linear correlation may not be adequate to reveal the dependence between uncertainties correctly. Although not directly implemented in a candidate bus selection problem, copula theory [19] has been demonstrated to be effective in representing the correlation between different uncertainties in power system planning and voltage stability problems [17, 21, 22]. Multivariate Gaussian copula is used in these works as an effective and accurate copula family to represent correlations between uncertainties of load demand, PV generation and wind power generation. However, the choice of copula families in these power system applications heavily depends on the uncertainty data set and a case-by-case trial and error method, the efficiency/accuracy of different copula families are yet to be verified and the copula families should be selected systematically.

To overcome the inadequacies of the previous works, this paper proposes a new candidate bus selection method for dynamic VAR planning, considering not only the capacity of STATCOMs, but also the load demand uncertainty and the randomness of wind generation as correlated input parameters. The contributions of this paper are as follows:

(i) New metrics are proposed to evaluate the short-term voltage stability in two application scenarios, considering the extent and the duration of voltage deviation simultaneously. One metric is designed for the evaluation under different operational scenarios while the other one is used in the capacity sensitivity analysis stage which flexibly assigns high priorities to the worst buses and thus improves the accuracy of capacity sensitivity analysis.

(ii) D-vine copulas are implemented to generate correlated load-wind scenarios. Pair copula selection for D-vine copulas is optimally carried out to accurately represent the dependence of the real power system data to ensure the effectiveness and accuracy of the selected copula families.

(iii) Sensitivity analyses for dependent (load wind speed) and independent variables (VAR capacity) are conducted with affordable computational cost.

(iv) A novel candidate bus selection method combining D-vine copulas, Kucherenko indices and Morris screening method is proposed. The most influential buses, in terms of voltage stability enhancement, are identified considering the independent variables and dependent variables simultaneously.

2 SHORT-TERM VOLTAGE STABILITY INDICES

Two novel indices are developed to evaluate the voltage stability performance of power systems for 2 evaluation cases. TVSI (transient voltage stability index) is mainly focusing on the evaluation of operational scenarios. TVSI further considers the continuity of capacity and the impact of the capacity increase on the voltage performance in the capacity sensitivity analysis process.

2.1 Index for dependent uncertainties sensitivity analysis

In the literature, short-term voltage stability indices are either defined only based on the deviation of the post-contingency voltage trajectory [7, 8, 15] or based on the duration and the extent of voltage deviation separately [11, 13]. However, since wind turbine is highly sensitive to minor but frequent voltage deviations [1], both the extent and the duration of the voltage deviation have a significant impact on the performance and LVRT/HVRT (low/high voltage ride-through) of the wind turbines. This paper proposes TVSI to evaluate the voltage recovery process during the whole post-contingency stage under
different operational scenarios. It is defined by: 1) The extent of voltage deviation $t^d$ (for assessment of the direct impact of voltage deviation), 2) the corresponding maximum acceptable duration of voltage deviation $t^{d_u}$ and $t^{d_l}$ (for assessment of voltage recovery speed). $S_1$, $S_2$, $S_3$ and $S_4$ are defined as voltage deviation areas formed by the post-contingency voltage trajectories, upper and lower voltage limit, and the specific time span ($t^{d_u}$ and $t^{d_l}$).

1. $S_1$: Deviation below lower voltage threshold within $t^{d_l}$
2. $S_2$: Deviation below lower voltage threshold beyond $t^{d_l}$
3. $S_3$: Deviation above upper voltage threshold within $t^{d_u}$
4. $S_4$: Deviation above upper voltage threshold beyond $t^{d_u}$

Based on $S_1$, $S_2$, $S_3$ and $S_4$, a general definition of $TVSI^j_i$ for each bus is formulated as (1). It consists of two parts: normal voltage deviation evaluation and penalized voltage deviation evaluation. Penalty parameters ($\alpha^l$ and $\alpha^u$) are introduced to assess the detrimental impact of delayed voltage recovery, which are usually ignored by conventional indices. A typical scenario is illustrated in Figure 1. So, 1) $S_2$ and $S_3$ will be zero if there is no delayed voltage recovery, and 2) $S_1$ and $S_4$ will be zero if there is no upper voltage deviation.

Normal deviation penalization defines a threshold to determine the significance of a bus when compared with the average performance of the whole system. Penalties of critical contingencies:

$$TVSI^j_i = \frac{1}{N^{b}} \sum_{i \in B} \left( TVSI^j_i / N^{b} \right)^2 \times p_k$$

With the evaluation results of each bus, the voltage performance of the whole system can be expressed in a root-mean-square way as (2), which can fully reveal the delayed voltage recovery of a specific bus even if other buses experience a relatively faster recovery. Finally, a risk-based index is proposed to quantify the short-term voltage performance based on probabilities of critical contingencies:

In the candidate bus selection stage, the sensitivity of the indices to the capacity of the VAR resource is very important. The metrics for evaluation of short-term voltage stability are usually assigned a fixed priority to a bus or even without a priority for a bus. When the buses with inferior voltage performance (with a higher priority) have been adequately compensated, even though further improvement is difficult and costly, these buses still account for a large portion of the evaluation results due to the fixed priorities. This will lead to inaccurate results and inefficient compensation. Therefore, $TVSI^r$ is proposed in this paper using an adaptive strategy to assign a higher priority to the current worst bus (in terms of short-term voltage performance) dynamically throughout the whole process of capacity sensitivity analysis as illustrated in Figure 2. For instance, bus 3 is in high priority group in the first capacity step in the sensitivity analysis and it will be moved to the average priority group due to the compensation of STATCOMs. As the capacity step increases, bus 3 is actually adequately compensated and it should be moved to lower priority group. The system-level voltage performance evaluation can be calculated as (3), where $\beta_j$ is the adaptive priority for each bus defined as (4). $N^{b}$ is the number of buses, $B$ is the set of all buses and $\sigma$ is a threshold to determine the significance of a bus when compared with the average voltage performance of other buses in the power system. Definition of $TVSI^r_i$ is same as (1).

$$TVSI^r_i = \frac{1}{N^{b}} \sum_{i \in B} TVSI^r_i \times \beta_i / N^{b}$$

$$\beta_j = \begin{cases} p_{low}, & TVSI^r_j \leq (1 - \sigma) \times \sum_{i \in B} TVSI^r_i / N^{b} \\ p_{high}, & TVSI^r_j \geq (1 + \sigma) \times \sum_{i \in B} TVSI^r_i / N^{b} \\ p_{ave}, & other \end{cases}$$
Compared with conventional indices that assign constant priorities to buses, the proposed TVSI is more suitable for capacity sensitivity analysis. For instance, at the beginning of the optimization process, the adaptive priorities will reinforce the impacts of the buses with inferior voltage performance. As the VAR compensation for these buses increases, their impacts on the system as a whole will decrease accordingly and other buses might become the new most influential ones. So, this adaptive strategy can increase the accuracy of capacity sensitivity analysis and avoid inefficient compensation.

3 METHODOLOGY

The proposed candidate bus selection method consists of three major steps: 1) Dependent operational variable sensitivity analysis, 2) independent capacity sensitivity analysis and 3) selection of candidate buses.

3.1 Dependent operational variable sensitivity analysis

3.1.1 Statistical inference based on D-vine copulas

Copula theory [18] has been demonstrated as an effective approach to describe the stochastic dependence between random variables. Different copulas families, such as Clayton copula, Frank copula and Gaussian copula can be used to model the dependence between two or multiple random variables. The multivariate Gaussian copula is reported to be an effective and accurate option for the applications in power system planning and voltage stability problems [20–22]. However, for a candidate bus selection problem considering correlated wind power generation and loads, the complex patterns of these dependent multivariate data make the analysis and determination of the best suitable pair copulas difficult. In this paper, instead of arbitrarily applying one specific copula family or using a non-systematic method, like trial and error, D-vine copula [23] is employed to optimally determine the suitable parameters of the pair-copula in a hierarchical way. Basically, D-vine copula is a Statistical Inference process that generally consists of two steps: 1) Fitting a number of probabilistic models to the available data, and 2) comparing them to select the best fitting one. The benefits of applying D-vine copula in the candidate bus selection problem are twofold: 1) Select the copula family and the corresponding parameters systematically, and 2) easy to expand the structure to incorporate more correlated variables.

Let us take an \( M \)-dimensional copula as an example. It is defined as a multivariate distribution over \([0, 1]^M\) with uniform marginals in the unit interval. Sklar’s theorem [18], which is the basis of copula theory, states that, for any \( M \)-variate distribution \( F_X \) of an \( M \)-dimensional input random vector \( X = \{ X_1, \ldots, X_M \} \) with marginals \( F_{X_1}, \ldots, F_{X_M} \), there is a copula \( C \) linking \( F_X \) of a random vector to its joint cumulative distribution function \( F_X \):

\[
F_X (x) = C \left( F_{X_1} (x_1), F_{X_2} (x_2), \ldots, F_{X_M} (x_M) \right) \quad (5)
\]

Based on Equation (5), the joint PDF (probability distribution function) of \( X \) can be obtained by differentiation as Equation (6).

\[
f_X (x) = \epsilon \left( F_{X_1} (x_1), F_{X_2} (x_2), \ldots, F_{X_M} (x_M) \right) \prod_{i=1}^{M} f_{X_i} (x_i) \quad (6)
\]

where the copula density function \( \epsilon (\cdot) \) is expressed as Equation (7). The conditional densities can be obtained as Equation (8).

\[
f_{X_{1|2,\ldots,M}} (x_1 | x_2, \ldots, x_M) = f_X (x) / \prod_{i=2}^{M} f_{X_i} (x_i) \quad (8)
\]

For high-dimensional problems, there will be many possible pair copulas. For example, for a 5-dimensional \( X \), 240 different pair copulas should be constructed. To organizing the pair copulas more efficiently, D-vine copula construction [23] is employed to describe multivariate copulas (\( M > 2 \)) and decompose the copula density function in a specific way. According to the chain rule of probability, \( f_X (x) \) can be expressed as Equation (9).

\[
f_X (x) = \prod_{j=1}^{M} f_{X_{j|j+1,\ldots,M}} (x_{j+1}, \ldots, x_M) \quad (9)
\]

With Equations (6) and (9), we have:

\[
\epsilon \left( F_{X_1} (x_1), F_{X_2} (x_2), \ldots, F_{X_M} (x_M) \right) \prod_{i=1}^{M} f_{X_i} (x_i) = \prod_{j=1}^{M} f_{X_{j|j+1,\ldots,M}} (x_{j+1}, \ldots, x_M) \quad (10)
\]

Each conditional PDF (on the right-hand side of Equation (10)) can be obtained in the form of a pair copula density (a differentiated pair copula). Eventually, a D-vine copula can be obtained as a tensor product of \( M(M-1)/2 \) pair copulas:

- \( M-1 \) pair copulas (unconditioned) between variable \( X_i \) and \( X_{i+1} \), \( i = 1, \ldots, M-1 \)
- \( M-2 \) pair copulas between variable \( X_i \) and \( X_{i+2} \) conditioned on \( X_{i+1} \), \( i = 1, \ldots, M-2 \)
- one pair copula between variable \( X_1 \) and \( X_M \) conditioned on \( X_1, \ldots, X_{M-1} \)
3.1.2 Pair copula families

| Name            | Cumulative distribution function $C(u, v; \theta)$ | Parameter range |
|-----------------|---------------------------------------------------|-----------------|
| Frank           | $\frac{1}{2} \log \left( \frac{1 - e^{-\theta (u - v)}}{1 - e^{-\theta v}} \right)$ | $\theta \in \mathbb{R}\setminus\{0\}$ |
| Gaussian        | $\Phi_{2,\theta}(\Phi^{-1}(u), \Phi^{-1}(v))$ | $\theta \in (-1, 1)$ |
| Clayton         | $(u^{-\theta} + v^{-\theta} - 1)/\theta$ | $\theta > 0$ |
| Gumbel          | $\exp(-((\log u)^\theta + (\log v)^\theta)^{1/\theta})$ | $\theta \in [1, \infty)$ |
| Student-$t$     | $t_{2,\theta}(\Phi^{-1}(u), \Phi^{-1}(v))$ | $r > 1$, $\theta \in (-1, 1)$ |

Thus, an $M$-dimensional D-vine copula is defined as Equation (11).

$$C(u) = \prod_{j=1}^{M-1} \prod_{i=1}^{M-j} C_{ij}[i+1,...,i+j-1]$$

$$\left( \frac{1}{(i+1,...,i+j-1)} \right)$$

(Figure 3 shows an example of a 5-dimensional D-vine with 10 edges and 4 trees (T). No node in any tree is connected to more than 2 edges. Each edge can be associated with a pair copula. For instance, the edge in $T_4$ corresponds to the pair copula $C_{15234}$. Through iterative construction of pair copulas, D-vine copula is an effective way to reveal the complex and hidden dependence patterns in a multivariate dataset.

3.1.3 Selection of pair copulas for D-vine copulas

For a given data set (input variables), there exists a set of pair copulas that best fits the data set. Table 1 lists 5 most common pair copula families. $Φ$ is the univariate standard normal distribution, and $Φ_{2,\theta}$ is the bivariate normal distribution with “0” means, unit variance and correlation parameter $\theta$. $t_{,}$ and $t_{,\theta}$ are the univariate and bivariate $t$ distribution (both with $r$ degrees of freedom and a correlation parameter $\theta$).

In principle, the best-fitted pair copulas can be determined by iterating maximum likelihood fitting over all vine structures and all possible parametric copula families. However, for a candidate bus selection problem with a large number of independent variables (the number of potential candidate buses), the prohibitive computation cost makes this approach impractical due to a large number of possible vine structures and the different possible pair copulas compromising the D-vine structure. Therefore, an alternative estimation approach with an affordable computation complexity is employed in this paper. This approach takes advantage of the specific structure of D-vine copulas to find an estimated solution. The optimal structure of D-vine copula is determined heuristically so as to first select the pairs with the strongest dependence and then put these pairs in the upper trees of the D-vine. Specifically, the dependence of pairs with the strongest dependence and then put these pairs in the upper trees of the D-vine. Specifically, the dependence of the pairs (for instance, $X_i$ and $X_j$) is measured through Kendall’s $τ$ [24], which is defined by Equation (12). $(X_i, X_j)$ is a pair of the independent copy of $(X_i, X_j)$.

$$τ_{ij} = \Pr (X_i - \tilde{X}_i) (X_j - \tilde{X}_j) > 0$$

$$-\Pr (X_i - \tilde{X}_i) (X_j - \tilde{X}_j) < 0$$

(12)

Let the $C_{ij}$ denotes the copula of $(X_i, X_j)$, then we have:

$$τ_K (X_i, X_j) = \frac{1}{4} \int \int_{[0,1]^2} C_{ij} (u, v) du dv - 1$$

(13)

where $K \in [1, \ldots, M]$. The variables $X_{ij}$, $X_{i1}, \ldots, X_{ij}$ should be optimally arranged to get the maximum $\sum_{j=1}^{M} \tau_{ik}$. For a large $M$, it can be solved by genetic algorithm or by a more general method proposed in [25].

3.1.3 Global sensitivity analysis based on Kucherenko indices

Kucherenko indices, which is first proposed in [26], is used in this paper as a global sensitivity analysis technique for the dependent variables (the load and wind power uncertainties). They can be regarded as a generalization of Sobol’ sensitivity indices [27] which is designed for models with independent variables. First, let us divide the input variables $X$ into 2 complementary subsets $X_1$ and $X_2$. So, the total variance of the model, $Y = \mathcal{M}(X)$, can be expressed as Equation (14).

$$\text{Var}[Y] = \text{Var}[\mathbb{E}[Y|X_1]] + \mathbb{E} [\text{Var}[Y|X_2]]$$

(14)

After the normalization by $\text{Var}[Y]$, the first summand on the right-hand side of Equation (14) becomes the closed indices of Sobol’s index with respect to $X_i$ [27], which accounts for the sum of all univariate and interaction effects between these.
variables.

\[
S_v = \frac{\text{Var} [E(Y | X_v = x_v)]}{\text{Var}[Y]}
\]  

(15)

The second summand is written as Equation (16), which represents the total effect of the variables \(X_i\), includes all univariate and interaction effects between \(X_i\), and the interaction effects between \(X_i\) and \(X_j\).

\[
S_w^T = \frac{E[\text{Var}[Y | X_v = x_v]]}{\text{Var}[Y]}, w = \sim v
\]  

(16)

Equation (15) can also be expressed in terms of sample covariance as Equation (17). Thus, the indices can be estimated as (17).

\[
S_v = \text{Cov} [Y, Y_v] / \text{Var}[Y]
\]  

(17)

However, if the variables are correlated with each other, Equation (17) cannot be used to estimate the indices. To address this deficiency, sample-based Kucherenko indices are employed: Approximate conditioning on intervals \(a_i < x_i < a_i'\) replaces the exact conditioning on specific values \(X_i = x_i\), where \(a_i / a_i'\) are the multi-dimensional lower/upper bounds. Specifically, the approximate conditioning is realized by selecting subsets of the sample \(X\) belonging to the interval. For instance, with an input sample set \(X\) and a corresponding output \(Y\) (of size \(N\)), the estimated total effect can be written as the weighted mean of conditional variance as follows:

\[
S_v^T = \frac{1}{D} \sum_{j=1}^{B} \frac{N_{hj}}{N} \text{Var}[Y|X_{hj}]
\]  

(18)

\[
D = \frac{1}{N} \sum_{j=1}^{N} \mathcal{M}^2(x_j) - \mathcal{M}^2_0
\]  

(19)

\[
\mathcal{M}_0 = \frac{1}{N} \sum_{j=1}^{N} \mathcal{M}(x_j)
\]  

(20)

where \(B \) and \(N_{hj}\) are the total number of histogram bins and the \(j\)-th bin, respectively, and \(\text{Var}[Y|X_{hj}]\) is the sample variance of \(Y\) that correspond to \(X_{hj}\) (in \(j\)-th bin). Obviously, in order to calculate \(S_v^T\) in (18), \(X\) should be firstly divided into \(B\) hyper-rectangles (bins) in the space.

Accordingly, the first-order index is the weighted variance of conditional mean estimates, which can be written as Equation (21).

\[
S_v = \frac{1}{D} \sum_{j=1}^{B} \frac{N_{hj}}{N} \left( \bar{E}[Y|X_{hj}] - \bar{\mu}_E \right)^2
\]  

(21)

where \(\bar{E}[Y|X_{hj}]\) is the sample mean of \(Y\) with \(X_{hj}\), and \(\bar{\mu}_E\) is the weighted sample mean of \(\bar{E}[Y|X_{hj}]\) as Equation (22).

\[
\bar{\mu}_E = \frac{1}{D} \sum_{j=1}^{B} \frac{N_{hj}}{N} \bar{E}[Y|X_{hj}]
\]  

(22)

3.2 Independent capacity sensitivity analysis

Different from the dependent uncertainties analysed in the previous sub-section, the capacities of STATCOMs are independent with each other. On the other hand, the interaction of different STATCOM buses still exists. So, an LHS-based (Latin hypercube sampling) Morris screening method is used to analyse the independent capacity sensitivity, considering the interaction between different STATCOM buses. Morris screening method [28] can efficiently identify the influential buses by creating a multi-dimensional semi-global trajectory within its searching space. It tries to cover more exhaustively the input space. Instead of focusing on a single global perturbation, Morris screening makes multiple different perturbations (in this study, it is the capacities of different STATCOMs) possible by creating a grid. Then, the sensitivity measure is built based on the mean and standard deviation of the input variables. For a better representative of the real variability, LHS is used in this paper to generate random samples following the normal distribution. The range of STATCOM capacity is modelled as an uncertainty space in this paper. One variable at a time will be changed by a magnitude of \(\Delta_w\) which is determined by the LHS results. So, the elementary effect of a \(\Delta_w\) change is defined as follows (\(f(\cdot)\) is the evaluation result of a specific capacity):

\[
\text{EE}_{i,m} = \frac{f(\alpha_{i,m} + \Delta_w) - f(\alpha_{i,m})}{\Delta_w}, \forall i \in B, m = 1, ..., N^w
\]  

(23)

Then, the mean and the standard deviation of the elementary effects, are defined as Equations (24) and (25). \(\mu^*\) describes the sensitivity strength between the input variable and the output while the \(\sigma^*\) denotes the non-linearity (interactions) of the input variables.

\[
\mu^*_i = \frac{\left( \sum_{m=1}^{N^w} |\text{EE}_{i,m}| \right) / N^w}{|N^w|}
\]  

(24)

\[
\sigma^*_i = \sqrt{\left( \frac{\sum_{m=1}^{N^w} \left( |\text{EE}_{i,m}| - \mu^*_i \right)^2 }{|N^w|} \right) / N^w}
\]  

(25)

A smaller \(\mu^*_i\) indicates that the installation of STATCOMs at \(i\)th bus has a relatively lower effect on the system. On the other hand, a larger \(\sigma^*_i\) means it has higher non-linear interactions with other buses. So, an ideal candidate bus is the bus with a large \(\mu^*_i\) and a small \(\sigma^*_i\).

3.3 Selection of candidate buses

As shown in Figure 4, there are 4 typical cases according to the results of the dependent uncertainties sensitivity.
analysis and the independent capacity sensitivity analysis at the ith bus:

1. Large values of both analysis results (red) indicate that the bus is sensitive to the variations of load and wind power generation, and the deployment of a STATCOM at the bus has a high impact on the voltage stability.
2. Small values of both analysis results (green) indicate that the bus is insensitive to the variations of load and wind power generation, and the deployment of a STATCOM at the bus has a low impact on the voltage stability.
3. If the independent analysis result is of small values and the dependent analysis result is of large value (blue), it indicates that the bus is insensitive to the variations of load and wind power generation, but the deployment of a STATCOM at the bus has a high impact on the voltage stability.
4. If the independent analysis result is of large values and the dependent analysis result is of small value (purple), it indicates that the bus is sensitive to the variations of load and wind power generation, but the deployment of a STATCOM at the bus has a low impact on the voltage stability.

Therefore, the ideal candidate buses should be sensitive and high impact buses (red ones). However, in practice, there might be no such bus, or these buses are too few. In this case, according to the practical needs and the priority of the decision-makers, blue and purple ones in the figure can be selected as candidate buses as well. This two-dimensional selection approach (capacity sensitivity and uncertainty sensitivity) provides the decision-makers with flexible solutions to incorporate more realistic consideration rather than a simple deterministic set of candidate buses.

### 3.4 Computation steps

Figure 5 shows the computational flowchart and detailed computation steps are as follows:

#### 3.4.1 Initialization

Algorithm parameters, and the probabilities of contingencies are set up in this stage.

#### 3.4.2 Construction of D-vine structure

The pair copulas for the D-vine structure is optimally selected based on Kendall’s τ. Then, the D-vine structure (like Figure 3) is constructed by place the current strongest dependence pair in the upper tree level. The various dependences among different load buses and wind farm buses are represented by the high-dimensional copula.

#### 3.4.3 Dependent uncertainties sensitivity analysis

Based on the input samples (loads and wind power output at each bus) generated by the D-vine copulas, both steady-state power flow (PF) calculations and time-domain simulations are carried out. TVSI_r is calculated and serves as “Y” in the calculations of Kucherenko indices. Candidate buses are selected based on the Kucherenko indices (first-order estimates, Equation (21)).

#### 3.4.4 Independent capacity sensitivity analysis

LHS-based random capacity samples are generated for time-domain simulations with a set of contingencies. Then, TVSI_a is
TABLE 2 Parameters for simulation analysis

| Parameters | Value                  |
|------------|------------------------|
| $\omega$   | $\pi$                  |
| $\delta$   | 0.1                    |
| $f_{dl}/f_{du}$ | 0.6 s/0.6 s          |
| $v_{dl}/v_{du}$ | 0.9 pu/1.1 pu       |
| $\alpha_l/\alpha_u$ | 2.0/2.0          |
| $p_{low}, p_{avg}, p_{high}$ | 0.8, 1.0, 1.2 |
| LHS levels | 20 (0–100 MVar)        |
| LM, SM, DL, TS, CP, $K_p$ | 25%, 15%, 10%, 10%, 10%, 2 |
| Fault setup | 3-phase short-circuit (cleared after 0.1 s) |

calculated to evaluate the dynamic voltage performance of the system for each contingency with all capacity samples. Rotor angle constraint [29] is used to check the dynamic stability of the system. Samples failed to satisfy the constraint are penalized to make sure their TVSI are inferior to other stable cases. The mean and the standard deviation of the elementary effects are calculated, and the candidate buses are selected according to the principle of Morris screening method as described in Sections 3.2 and 3.3.

3.4.5 Selection of candidate buses

After the dependent and independent sensitivity analysis results are obtained, the final candidate buses can be selected (as illustrated in Figure 4): An ideal candidate bus is a bus that is sensitive and with high capacity impact (located among the red triangles), while buses located among blue circles and purple diamonds can also be selected according to the practical needs of the decision-makers if there is no bus near the red area.

4 NUMERICAL RESULTS

4.1 Test system and parameters

The proposed method is tested on a modified Nordic 74-bus system [30] as illustrated in Figure 6. Generators g11, g14 and g17 are replaced with double-fed induction generators modelled in PSSE [31]. SVSMO3 model is used for STATCOMs. An industry-standard complex load model “CLOD” [31] is used to represent the time-varying load dynamics. It consists of: 1) 3-phase small and large induction motor load (SM and LM), 2) discharge lighting load (DL), 3) saturation of distribution transformers (TS), 4) static load, consisting of a constant MVA component (CP) and a load whose real and imaginary components varies with an exponent $K_p$ and a voltage squared, respectively. A vector [LM, SM, DL, TS, CP] represents the proportions of different load types. The values of simulation parameters are listed in Table 2. Contingency selection (line 4031–4041, 4032–4044 and 4042–4044) is based on our previous work [15]. The steady-state analysis and time-domain simulations are carried out in Matlab and PSSE, respectively.

The load variations and wind speed are obtained from [32, 33] over 1 year (8760 hours). Pearson correlation matrix [28] is used to form a 25*25 matrix to show the correlation patterns between these 25 uncertainties as presented in Figure 7. The column 1–22 and row 1–22 are the load buses, and column 23–25 and row 23–25 represent the wind speed data. The dependences between different uncertainties, including not only load-load and wind-wind dependence, but also load-wind dependence) can be identified in Figure 7. Three load zones can be clearly identified because of their relatively high correlation. Therefore, instead of considering load and wind uncertainties independently, it is critical to model the correlated uncertainties correctly for the evaluation of dynamic voltage performance.

4.2 Simulation results and discussions

The overall computation time is 50168 s, consisting of 28337 s for the dependent uncertainty sensitivity analysis and 21831 s for the independent capacity sensitivity analysis. As for the computation time of multivariate Gaussian copula approach, it is 27901 s for the dependent uncertainty sensitivity analysis. Considering the copula pair and the structure of D-vine are
optimally determined in the proposed method, the increase of the computation time is acceptable. Figure 8 shows the optimized 25-dimensional D-vine structure, which consists of 300 (25*24/2) edges and 24(25–1) trees. In the figure, the numbers 1–22 represent the load buses (from bus 41 to bus 4072) and 23–25 denote the three buses with wind farms. Instead of applying one copula family to all the pairs in the D-vine copulas, the proposed method flexibly assigns different copula families to each pair and thus the correlation between the variables of loads and wind speed can be better revealed.

Table 3 lists the sensitivity analysis results for three methods: 1) No correlation considered, 2) statistical inference based on D-vine copula (the proposed method) and 3) statistical inference based on multivariate Gaussian copula. Figure 9 show the sensitivity analysis results when the correlated uncertainties (load level and wind power generation) are modelled as uncorrelated ones (conventional sensitivity analysis method). In this case, the dependencies between uncertainties as illustrated in Figure 7 are not reflected in the sensitivity analysis results. On the contrary, Figure 10 illustrated the corresponding sensitivity analysis result based on the proposed method, where the correlation between the uncertainties (as illustrated in Figure 7) is clearly revealed (highly dependent uncertainties are also appeared in groups as in Figure 7). On the other hand, Figure 11 demonstrates that multivariate Gaussian copula also fails to reveal the dependencies between the uncertainties correctly. For instance, as listed in Table 3, the results of bus 62 and bus 43 are even negative.
The sensitivity analysis results also demonstrate that some variables might be insignificant when they are considered as independent uncertainties, but their importance will be revealed when their correlation with other variables being considered. For instance, as listed in Table 3, the load variations at bus 42 and 51 are relatively insignificant (rank 17th and 12th among all the load buses in Figure 9) when considered as an independent variable. However, when the correlations between the uncertainties are considered, their ranks climb to the 4th and 1st, respectively. On the other hand, the load variations at bus 1042 are very significant (rank 1st among all the load buses in Figure 9) when considered as an independent variable. When the correlations between the uncertainties are considered, its rank drops to the 13th.

Figure 12 shows the independent capacity sensitivity analysis result based on Morris screening method. The advantage of this two-dimensional results is that, although $\mu^*$ is the main determinant in selecting the candidate bus, the $\sigma^*$ can provide valuable information about the interaction between these buses (in terms of their voltage response with different capacities of STATCOMs) to the decision-makers. For instance, bus 4042 and bus 2031 have very close values of $\mu^*$, but the bus 2031 is still preferred because of its relatively smaller $\sigma^*$.

With results for the dependent uncertainties sensitivity analysis and the independent capacity sensitivity analysis, the decision-maker can select the candidate buses based on Figure 13. As explained in Figure 4, wind farm buses (g11 and g14) are ideal candidate buses because their impacts on the system are larger than other buses in both sensitivity analyses. Instead of a set of deterministic candidate bus selection results, this 2-dimensional selection method can provide flexible options to the decision-makers. If the planners are more concerned with the voltage stability enhancement brought by the deployment of STATCOMs, the buses with high sensitivity to the STATCOM capacity (large value at y-axis, like bus 42 and 2031) can be selected. If the robustness against load variation and wind power uncertainties is a high priority, the decision-maker can select the buses with large values at x-axis (like bus 51 and g17). Alternatively, Fuzzy membership function can also be employed to select the candidate buses.

For a conventional trajectory-based voltage stability index with a fixed priority throughout the whole process, the change rate is very small even the capacity changes significantly (from

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**FIGURE 10** Dependent uncertainties sensitivity analysis results (the proposed method)

**FIGURE 11** Dependent uncertainties sensitivity analysis results (Gaussian copulas)

**FIGURE 12** Independent capacity sensitivity analysis results

**FIGURE 13** Two-dimensional candidate bus selection

**FIGURE 14** Comparison between TVSP* and conventional index
TABLE 4  Optimized capacity of STATCOM (in MVar)

| Methods          | Bus no. | Capacity |
|------------------|---------|----------|
| Proposed method  | 41, 42  | 39.92, 101.9 |
| Gaussian copula  | 51, 101  | 1013, 1041 |
| No correlation   | 41, 42  | 49.25, 93.43 |

TABLE 5  Results comparison (in million US dollar)

| Methods     | Total cost | TVSI\textsuperscript{a} | Robust index |
|-------------|------------|----------------|--------------|
| Proposed method | 32.856     | 0.3024          | 1.3598       |
| Gaussian copula | 36.165     | 0.3209          | 1.4587       |
| No correlation | 31.797     | 0.3044          | 1.4998       |

10–120 MVar) as shown in Figure 14. However, this indifference will lead to inaccurate sensitivity analysis results because as the capacity increases, more and more buses are no longer in the deficiency of reactive power and the voltage performance of the power system mainly depends on other buses still in the deficiency. In contrast, the change rate of TVSI\textsuperscript{a} is changing adaptively: At the beginning, TVSI\textsuperscript{a} is very sensitive to the capacity change and is becoming less sensitive as the installed capacity increases.

To further demonstrate the superiority of the proposed method, the selected candidate buses are used for a STATCOM capacity optimization problem (the details of the solving method for the problem can be found in [15]) in Nordic 74-bus test system and the results are compared with Gaussian copula method and the conventional method (correlation is not considered). The final optimized capacity of STATCOM for different methods are listed in Table 4. In order to demonstrate the advantage of the proposed method, a Robust index [34], which is designed to be the smaller the better, is introduced to quantitatively evaluate the robustness of the optimized results when subjected to operational uncertainties (load level and wind power generation). As listed in Table 5, the proposed method obtains the best robustness among the three methods. At the same time, it also achieves a better TVSI\textsuperscript{a} than Gaussian copula method and conventional method. Although compared with the conventional method there is a slight investment increase (3.22%), the proposed method improves the robustness of the optimization results on a larger scale (10.29%), making the optimization results less sensitive to the operational uncertainties.

5  CONCLUSION AND FUTURE WORK

This paper proposes a novel candidate bus selection method for the dynamic VAR planning in a wind-penetrated power system. It is the first candidate bus selection study to employ the sensitivity analyses for dependent operational uncertainties and independent capacity simultaneously. Case studies on a modified Nordic 74-bus test system show the effectiveness and advantages of the proposed method. The key conclusions are summarized as follows:

(i) D-vine copula, constructed through optimal pair copulas selection, can effectively and correctly reveal the correlation between dependent uncertainties. The proposed method can systematically address this statistical inference problem rather than using a case-by-case trial and error method.

(ii) Some variables might be less significant as an independent uncertainty, but their impact on the system can be significant when their correlation with other variables are considered. So, modelling of the uncertainties independently or inaccurately (such as with inappropriate copulas) will compromise the accuracy of sensitivity analysis.

(iii) A set of two-dimensional results, rather than one deterministic result, enables the decision-maker to select candidate buses flexibly based on their priorities and the engineering practices.

(iv) Different voltage stability metrics are developed for operational scenario evaluation (TVSI\textsuperscript{a}) and capacity impact evaluation (TVSI\textsuperscript{b}). The adaptive TVSI\textsuperscript{a}, which assigns the high priorities to the worst buses flexibly, can improve the accuracy of capacity sensitivity analysis.

For future work, it would be interesting to incorporate the uncertainty of the fault clearing time into the model to further improve the robustness of the proposed method.

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