A classical perspective on nonlocality in quantum field theory

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A classical statistical field theory hidden variable model for the quantized Klein-Gordon model is constructed that preserves relativistic signal locality and is relativistically covariant, but is at the same time relativistically nonlocal, paralleling the Hegerfeldt nonlocality of quantum theory. It is argued that the relativistic nonlocality of this model is acceptable to classical physics, but in any case the approach taken here characterizes the nonlocality of the quantized Klein-Gordon model in terms of concepts from classical statistical field theory.

1 Introduction

This paper takes a relativistically local classical model for quantum field theory not to be possible. Obviously we then have the choice of abandoning classical models or considering what relativistically nonlocal classical models are possible. We take here a classical statistical field theory approach, which leads to a model of classical probability measures over classical fields for the quantized Klein-Gordon model that preserves relativistic signal locality and is relativistically covariant despite being relativistically nonlocal (that is, there are causal dynamical relationships between classical field values at space-like separated points).

We will pursue infinite dimensional classical statistical field theory models for quantum field theory because I take it that ways of introducing the necessary relativistic nonlocality into finite dimensional classical models for non-relativistic quantum theory are not acceptable (and I can’t see any better way), and because models of quantum field theory are empirically superior to finite dimensional models of non-relativistic quantum mechanics. If relativistic nonlocality is
acceptably introduced into classical statistical field theory in a way that adequately matches
the Hegerfeldt nonlocality of quantum field theory, then an approximate reduction of both a
classical statistical field theory model and the corresponding quantum field theory model to
finite dimensional models will result in a finite dimensional classical model that in general will
not perfectly describe the relativistic nonlocality present in either the quantum field theory
model or its finite dimensional reduction. The relativistic nonlocality in such finite dimensional
classical models may not be acceptable as a fundamental approach, but it may nonetheless be
useful as the consequence of an approximate reduction of an acceptable classical statistical field
theory.

Although classical models can be taken realistically, I take them, in a post-empiricist way, to
be ‘as if’ real. The construction of classical models is often motivated by a naïve classical realism,
but a post-empiricist view is simply that it is useful to have available models of many different
types. Visualizability of a model is a pragmatic virtue, and does not prove that the world is the
same as such mathematics. Equally, although the possibility of constructing models of great
accuracy is an important pragmatic virtue, it also does not prove that the world is ultimately
very like a model of a given theory. It may be only marginally useful to have models that are
visualizable in competition with the numerous existing interpretations of quantum field theory,
but I take it to be at least a little useful.

In the absence of a convincing post-empiricist interpretation of classical and quantum physics,
section 2 is a brief outline of an approach to classical and quantum models that is largely a
polemic, but does have some implications for the subsequent development, because it describes
the attitude that will be taken to the Kochen-Specker paradox. It can safely be ignored by
those who wish to rush to the mathematical construction of a classical statistical field theory
model for the quantized Klein-Gordon model, which is contained in section 3. Finally, section 4
argues that the relativistic nonlocality of this classical model is acceptable to classical physics.

2 On the interpretation of classical and quantum theory

A classical statistical field theory model does not describe measurement directly. If we introduce
a device to measure the field somewhere, then in principle we have to construct a bigger model
which includes a model of the device. This requires another device to measure properties of
the first device, etc., etc. To ‘avoid’ this regress (although it remains as a formal problem),
measurement in a classical statistical field theory model requires a feature in the model to be
pragmatically identifiable in the world. The way in which we make this pragmatic identification
depends on the level of approximation we require. At a quite direct level, a meter pointer in
a model will be in some configuration relative to other parts of the model, and we can see
objects in the world which we take to be in approximately the same relative configuration. We assume that the meter pointer is not affected much by us seeing it, so that we do not have to include ourselves in the model. Less directly, if an electric current is large enough in a model to be measured without significant perturbation by an ammeter, we do not need to model a particular ammeter. Still less directly, and most usually (and as we shall do in section 3), we may construct a classical statistical field theory model quite abstractly, without any concessions to measurement[1], and know that there are classes of functions on the phase space that correspond with moderate inaccuracy and perturbation to measurements of real finite systems by devices we have available to us. In a classical statistical field theory model, the inaccuracy of and perturbation caused by a measurement device is in principle supposed to be arbitrarily reducible, but it is in principle as well as in practice that we have only a finite array of imperfect devices, so not all details of a classical model are verifiable.

We will take quantum theory in a similar, quite pragmatic way. Quantum theory is a formalism for summarizing the results of multiple experimental arrangements, which we can largely take to consist of different arrangements of carefully calibrated preparation devices and measurement devices. Our description of a measurement device is quite pragmatic: we might describe a device in a first approximation as measuring momentum, but refine our description to a positive operator valued measure, which we can construct partly from approximate classical measurements of the device and partly through a finite process of calibration. We never know precisely what observable we measure with a given real device. The regress of measurement devices described above for classical models is of course very well known in quantum theory, and can be ‘avoided’ in a similarly pragmatic way.

Following the Copenhagen interpretation, each arrangement of preparation and measurement devices is described classically, as are the results we obtain. We take a quantum theory description of multiple experimental arrangements to be a theoretical construction from many classical descriptions. Consequently, we will require a classical statistical field theory model only to give an account for a classical description of an individual experiment, and not to give an account for the multiple classical descriptions that are summarized by a quantum theory description.

We will find that Planck’s constant can be considered to be a measure of fluctuations of a classical field: we cannot improve on the Heisenberg uncertainty principle because all our measurement devices are subject to the same fluctuations. This is a very old understanding of quantum theory in classical terms, which can only work adequately, however, if relativistically nonlocal models are accepted. We take it that insofar as a classical statistical field theory model

[1] Very often, classical statistical field theory models occupy all of the space-time of the model; there is no room for a measurement device (or for us) without making the model mathematically quite intractable.
has properties that can only be measured using a device that improves upon a classical version of this limit, those properties are in fact not verifiable, at least at present. Unless we can find a way of reducing the fluctuations of the field locally, we will of course never be able to improve on the Heisenberg uncertainty principle.

There is a little used approach to understanding quantum mechanics, due to Mackey, that is more-or-less consistent with the classical statistical field theory approach to quantum field theory taken here. For Mackey, quantum mechanics is “basically a revision of statistical mechanics in that one studies the change in time of probability measures but no longer supposes that the motion of these measures is that induced by a motion of points in phase space” (quoted by Jammer(1966, p. 156)). This is quite a conventional statement for Mackey, which just asserts that there are no trajectories, but we will apply it to quantum field theory as a starting point for a modal interpretation. We will regard momentum information in both classical and quantum mechanics as the additional information required to describe the evolution of a probability measure over a configuration space of classical fields, even though the additional information required is very different in the two cases. Momentum information in quantum field theory is then a concept particular to quantum field theory, which is not ‘real’ in a classical statistical field theory model, even though probability measures over configuration space are taken to be a common conceptual foundation. When we say in a quantum theory model that we measure quantum momentum (or any other observable that is not compatible with the field observable) with a given device, we can in principle construct a (relativistically nonlocal) classical model of the whole apparatus in which the position of a classical pointer (say) is correlated with some function of the classical state.

Although ideal classical measurement is noncontextual, each combination of a classical statistical field theory model with a model of a real measurement device changes the details of the classical statistical field theory model in a different way, so the properties of the field, when considered in combination with multiple measurement devices, are essentially contextual. Both this contextuality and the approach of the last paragraph, taking quantum momentum as not a classical observable, are natural ways to evade the strictures of the Kochen-Specker paradox (see, for example, Redhead(1987, ch. 5)). The contextuality of real measurement, in particular, is very natural in a classical statistical field theory.

Mackey’s approach leads to a modal interpretation of quantum field theory that is different from a classical statistical field theory only because of the way in which the evolution of the common probability measure over a common sample space of classical field configurations is specified. Quantum mechanical momentum describes the evolution of the probability measure over classical field configurations directly, whereas classical momentum describes the evolution of single classical field configurations, indirectly inducing the evolution of the probability mea-
sure. A classical statistical field theory, however, in general makes no attempt to compute individual trajectories, but manipulates classical probability measures in a way that is quite comparable to that of quantum field theory. Mackey’s approach applied to non-relativistic, finite-dimensional quantum mechanics is not particularly helpful, largely because the violation of Bell inequalities remains problematic in its implication of relativistic nonlocality, but the classical relativistic nonlocality implicit in the evolution of probability measures over a configuration space of classical fields described by a quantum field theory can at least potentially be reasonable as a classical thermal physics. Such an interpretation of quantum field theory is non-relativistic, but of course the same interpretation is possible for any observer with their own choice of configuration space; it is to a Lorentz covariant interpretation as a Hamiltonian formalism is to a Lagrangian formalism — it is not manifestly covariant, but it is nonetheless applicable.

As a modal interpretation of quantum field theory, the approach we have taken is similar to Kaloyerou’s Bohmian approach to the interpretation of quantum field theory. At least for the observables of boson fields, and in principle for the observables of fermion fields, Kaloyerou(1996) takes quantum field theory to describe a probability measure over classical fields. Mackey’s approach differs from a Bohmian approach because it is not committed to a particular classical evolution, and the Hamiltonian ansatz introduced below is also quite different to introducing a Bohmian nonlocal evolution. From the point of view both of classical statistical field theory and of quantum field theory, it is in principle not possible from experimental evidence to identify a Hamiltonian with a Bohmian degree of precision, even if we believe or act as if there might be one.

3 A classical relativistically nonlocal model for the quantized Klein-Gordon model

In simple-minded terms, a classical statistical field theory at equilibrium describes the consequences of a Gibbs probability measure over a sample space of classical fields. For the simplest possible model, the Gaussian model, with a Hamiltonian

\[ H[\phi] = \int \frac{1}{2} \{(\nabla \phi)^2 + m^2 \phi^2\} d^3x, \]

and probability density \( P[\phi] \) proportional to \( \exp(-H[\phi]/kT) \), the fluctuations of the field are Gaussian distributed, with variance

\[ \sigma^2 = kT \int \frac{d^3k}{(2\pi)^3} \frac{|\tilde{f}(k)|^2}{k^2 + m^2}. \]
where \( f(x) \) is a test function used to construct a classical observable

\[
\phi_f = \int \phi(x)f(x)d^3(x)
\]

(see, for example, Binney et al.(1993, §8.1)). We have had to introduce the test function \( f(x) \) because the variance of the fluctuations ‘at a point’ are infinite, as we see if we take \( f(x) \) to be a delta function, for which \(|\tilde{f}(k)|^2 = 1\). Provided we choose \( \tilde{f}(k) \) appropriately, \( \sigma^2 \) will be finite.

The Hamiltonian of the Gaussian model can be written in fourier transform terms as

\[
H[\phi] = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2}(k^2 + m^2)|\tilde{\phi}(k)|^2.
\]

If we consider a more general ansatz for a non-interacting Hamiltonian,

\[
H_\xi[\phi] = \int \frac{d^3k}{(2\pi)^3} \xi(k)|\tilde{\phi}(k)|^2,
\]

generalizing the argument in Binney et al.(1993, Appendix L.3.2), the fluctuations of the field are again Gaussian distributed, with variance

\[
\sigma_\xi^2 = kT \int \frac{d^3k}{(2\pi)^3} \frac{|\tilde{f}(k)|^2}{2\xi(k)}.
\]

Note that if \( \xi(k) \) is chosen appropriately, the fluctuations at a point are finite, and we can properly talk of a probability measure over a sample space of classical fields. The simplest regularization of applying a frequency cutoff corresponds to taking \( \xi(k) = \frac{1}{2}(k^2 + m^2) \) when \( k \leq \Lambda \), \( \xi(k) = \infty \) when \( k > \Lambda \), but smooth dynamical regularizations can also be adopted, such as \( \xi(k) = \frac{1}{2}(k^2 + m^2e^{\frac{|k|}{\Lambda}}) \) or \( \xi(k) = \frac{1}{2}(k^2 + m^2)^\alpha, \alpha > \frac{3}{2} \) (which is largely equivalent to dimensional regularization).

We can compare the vacuum state of a quantized Klein-Gordon field, with Hamiltonian

\[
\int \frac{1}{2} \{ \tilde{\pi}^2 + (\nabla \tilde{\phi})^2 + m^2 \tilde{\phi}^2 \} d^3x,
\]

with the equilibrium state of the Hamiltonian ansatz \( H_\xi \) in classical statistical field theory. The formalisms of quantum field theory and of classical statistical field theory are obviously very different, and equilibrium in the Gaussian model is in general 3-dimensional Euclidean invariant rather than 4-dimensional Poincaré invariant. Nonetheless, fluctuations of the quantum field are Gaussian distributed in the quantized Klein-Gordon vacuum state, just as they are in the Gaussian model, with variance

\[
\sigma_{QFT}^2 = \hbar \int \frac{d^3k}{(2\pi)^3} \frac{|\tilde{f}(k)|^2}{2\sqrt{k^2 + m^2}} = \hbar \int \frac{d^4k}{(2\pi)^2} 2\pi\delta(k^\mu k_\mu - m^2)\theta(k_0)|\tilde{f}(k)|^2
\]
(see, for example, Itzykson and Zuber(1980, p. 119)), where again \( f(x) \) is a test function used to construct a quantum observable

\[
\hat{\phi}_f = \int \hat{\phi}(x)f(x)d^4(x).
\]

Although \( f(x) \) can be any function on the whole of the 4-dimensional Minkowski space, only components of the fourier transform \( \tilde{f}(k) \) that are on-shell, satisfying \( k^\mu k_\mu = m^2, \ k_0 > 0 \), contribute to the variance. Consequently, for this purpose any function is equivalent to any other function that has the same on-shell components. In particular, \( f(x) \) can be a product \( f(3)\delta(t-t_0) \) of a function on a timelike 3-dimensional hyperplane with the delta function \( \delta(t-t_0) \) which picks out that hyperplane, allowing a direct comparison with a 3-dimensional classical statistical field theory equilibrium state.

There is nothing absolutely quantum mechanical about the vacuum state of the quantized Klein-Gordon field, because we can generate the same fluctuations on a space-like hyperplane in a classical way by taking \( \xi(k) = kT\sqrt{k^2 + m^2}/\hbar \) (of course the \( kT \) just cancels with the \( kT \) in the Gibbs distribution). These classical and quantum models agree for all correlation functions that are restricted to a timelike 3-dimensional hyperspace: in both models, all connected correlation functions are zero, except for

\[
\langle \phi_f \phi_g \rangle = \frac{\hbar}{(2\pi)^3} \int \frac{d^3k}{2\sqrt{k^2 + m^2}} \tilde{f}^*(k)\tilde{g}(k).
\]

Within the Hamiltonian ansatz we have adopted, only \( \xi(k) = kT\sqrt{k^2 + m^2}/\hbar \) results in the same fluctuations as the quantized Klein-Gordon vacuum state. Furthermore, a Gibbs probability measure constructed using a simple perturbation of the Hamiltonian ansatz does not result in the same fluctuations as the quantized Klein-Gordon state, because in general such perturbations generate a non-Gaussian distributed probability measure. So this relativistically nonlocal classical Hamiltonian is the only candidate for a classical statistical field theory hidden-variable model for the quantized Klein-Gordon model, unless a significantly more complicated and less conventional ansatz is adopted.

In a classical statistical field theory framework, we can understand Planck’s constant of action as a Poincaré invariant measure of field fluctuations that is a 4-dimensional analogue of the thermal energy \( kT \), which can itself be understood as a 3-dimensional Euclidean invariant measure of field fluctuations. That is, Planck’s constant can be regarded as no longer a fundamental constant, but in a loose sense as a measure of “4-dimensional temperature”.

In classical terms, a state in which there are fluctuations everywhere is clearly not a minimum of energy. What is at a minimum for the vacuum state compared to states in the Fock space, which are asymptotically identical to the vacuum state, is the Helmholtz free energy, which, in
a statistical field theory, is the appropriate measure of energy to determine the possibility of transitions between states.

The function \( \xi(k) = kT\sqrt{k^2 + m^2}/\hbar \) is a nonlocal operator; indeed, Segal and Goodman (1965) prove that it is anti-local for functions in \( L^2(\mathbb{R}^3) \): a function and its transform, defined by \( \tilde{g}(k) \rightarrow \xi(k)\tilde{g}(k) \), can both vanish in a given region only if the function is identically zero. Segal and Goodman relate this directly to the Reeh-Schlieder property of a quantum field theoretic vacuum (see, for example, Haag (1996, §II.5.3)). As a simple multiplication of fourier transforms, this operator can be understood as a convolution of \( g(x) \) with \( \frac{1}{r^2}K_2(mr) \), the inverse fourier transform, up to a constant, of \( \xi(k) \). \( K_2(mr) \) is a modified Bessel function which approaches zero faster than exponentially as \( r \) approaches infinity. The departure from locality is therefore exponentially small at distances large compared to \( m^{-1} \). There is no reason to expect precisely the same behaviour as is given by the classical heat equation, since that is a different mathematics, but the qualitative behaviour is classically familiar. This straightforward classical field approach characterizes the relativistic nonlocality of the quantized Klein-Gordon field in classical terms by producing a particular model, but it reflects Hegerfeldt’s very general proof of the classical relativistic nonlocality of quantum theory (see Hegerfeldt (1998)); if a quantum system with a Hamiltonian that is bounded below is strictly localized in a finite region, it immediately develops infinite tails.\footnote{But note that if the Hamiltonian is bounded above (i.e. the energy is not infinite) as well as below, the quantum system cannot be strictly localized, which is enough to preserve relativistic signal locality (see Buchholz and Yngvason (1994)).} To reproduce such behaviour, a classical Hamiltonian must certainly be relativistically nonlocal. The interesting question will be whether this particular nonlocality is actually very unpleasant in classical terms.

If we construct a 4-vector \( k^\mu = (\omega, k) \), where \( \omega = \sqrt{k^2 + m^2} \), we can write the Hamiltonian

\[
H'[\phi] = \frac{kT}{\hbar} \int \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + m^2} |\tilde{\phi}(k)|^2
\]

as

\[
H'[\phi] = \frac{kT}{\hbar} \int \frac{d^3k}{2\omega(2\pi)^3} 2\omega |\tilde{\phi}(k)|^2 = \frac{kT}{\hbar} \int \frac{d^4k}{(2\pi)^4} 2\pi \delta(k^\mu k_\mu - m^2) \theta(k^0) 2k^0 k^0 |\tilde{\phi}(k)|^2,
\]

with the additional constraint that off-shell wavenumbers have probability zero, which enforces \( \omega = \sqrt{k^2 + m^2} \) as a constraint.\footnote{It is clear that we can extend the 3-dimensional Gaussian model into four dimensions by an arbitrary choice of \( \omega \) as a function of \( k \), but the choice we have made uniquely results in a Lorentz covariant formalism (up to a sign, of course).} That is, \( H'[\phi] \) is the 00-component of a Lorentz covariant energy-momentum tensor,

\[
T^{\mu\nu}[\phi] = \frac{kT}{\hbar} \int \frac{d^4k}{(2\pi)^4} 2\pi \delta(k^\mu k_\mu - m^2) \theta(k^0) 2k^\mu k^\nu |\tilde{\phi}(k)|^2,
\]
which, in momentum space and for mass $m$, is the simplest possible Lorentz covariant energy-momentum tensor. We have constructed a relativistically covariant formalism for this classical statistical field theory, with no preferred rest frame, despite its relativistic nonlocality. This minimalist Lorentz covariant energy momentum tensor, as a classical equivalent of the quantized Klein-Gordon model, deserves consideration equal to the classical Klein-Gordon model, provided only that we are prepared to forget prejudices against at least some forms of nonlocality.

In this covariant formulation, the probability measure over the phase at a given wavenumber is uniform, with no correlations between the phases at different wavenumbers, so the classical equilibrium state in one frame is also the classical equilibrium state in another frame, and we can consider it equivalent to the Lorentz invariant Klein-Gordon vacuum state.

When formulated in this way, the nature of the relativistic nonlocality is not immediately clear, since there is a non-zero probability density only for on-shell fourier modes of the classical field, none of which propagates faster than light. In this classical statistical field theory model, the effective nonlocality has to be taken to be due to the analytic nature of the initial conditions.

This classical Hamiltonian can also reproduce the action of the quantized Klein-Gordon evolution on non-vacuum states. Both for the classical model and for the quantized Klein-Gordon model, a non-vacuum state describes a system of probability measures different from that of the vacuum state. A creation operator

$$a^\dagger_g = \int \frac{d^4k}{(2\pi)^4} a^\dagger(k)\tilde{g}(k)$$

in quantum field theory acts on the vacuum state $|0\rangle$, with the Gaussian probability density

$$\rho_0(q) = \frac{1}{\sqrt{2\pi(f,f)}} \exp\left[\frac{-q^2}{2(f,f)}\right],$$

where, taking $\hbar = 1$, the variance $(f,f) = \sigma_{QFT}^2$ has been expressed using the relativistically invariant inner product

$$(f,g) = \int \frac{d^3k}{(2\pi)^3} \frac{\tilde{f}^*(k)\tilde{g}(k)}{2\sqrt{k^2 + m^2}} = \int \frac{d^4k}{(2\pi)^4} \frac{2\pi\delta(k^\mu k_\mu - m^2)\theta(k_0)\tilde{f}^*(k)\tilde{g}(k),}{2(k^2 + m^2)}$$

to give the state $a^\dagger_g |0\rangle$, with the non-Gaussian probability density

$$\rho_1(q) = \frac{1}{\sqrt{2\pi(f,f)}} \left[1 - \frac{|(f,g)|^2}{(f,f)(g,g)} + \frac{q^2}{(f,f)(f,f)(g,g)}\right] \exp\left[\frac{-q^2}{2(f,f)}\right].$$

If we choose different functions for $f$ and leave $g$ fixed, then, as the inner product $(f,g)$ changes, so the probability density varies between $\rho_0(q)$, when $(f,g) = 0$, and $\frac{q^2}{(f,f)\rho_0(q)}$, when $|(f,g)|^2 = (f,f)(g,g)$. Note, again, that these inner products depend only on the on-shell
The inner product \((f, g)\) should be carefully understood to be nonlocal: if \(f(x)\) and \(g(x)\) have disjoint supports, their simple inner product \(\int f(x)g(x)dx\) will of course be zero, but the inner product \((f, g)\) is equivalent to the simple inner product of \(f\) with \(g\) in convolution with the inverse fourier transform of \((k^2 + m^2)^{-\frac{1}{2}}\), which will not generally be zero, however separated the supports of \(f(x)\) and \(g(x)\) are, although it \textit{will} decrease faster than exponentially as the separation increases.

The probability measure \(\rho_1(q)\) can be reproduced at any given time \(t_0\) within a classical statistical field theory model just as an initial condition. \(a^\dagger_b \vert 0\rangle\) determines a probability measure over real functions at a time \(t_0\), which we can take as a classical initial condition. We can extend this 3-dimensional model to four dimensions by the functional methods of Morgan(2001), which can be applied to general states in the Fock space of the quantized Klein-Gordon model.

As further examples, the probability densities for the states \((a^b_b)^2 \vert 0\rangle\) and \((a^3_b) \vert 0\rangle\) are

\[
\rho_2(q) = \frac{1}{\sqrt{2\pi (f, f)}} \left[ 2 - 4\theta + 3\theta^2 + (4\theta - 6\theta^2) \frac{q^2}{(f, f)} + \theta^2 \frac{q^4}{(f, f)^2} \right] \exp \left[ -\frac{q^2}{2(f, f)} \right]
\]

and

\[
\rho_3(q) = \frac{1}{\sqrt{2\pi (f, f)}} \left[ \frac{6 - 18\theta + 27\theta^2 - 15\theta^3}{(f, f)} + \frac{(18\theta - 54\theta^2 + 45\theta^3)}{(f, f)^2} \frac{q^2}{(f, f)} + \frac{(9\theta^2 - 15\theta^3)}{(f, f)^3} \frac{q^4}{(f, f)^2} \right] \exp \left[ -\frac{q^2}{2(f, f)} \right],
\]

where we have written \(\theta\) for \(\frac{(|f, g|)^2}{(f, f)(g, g)}\). The above probability densities are all even in \(q\), but for the coherent superposition \(\exp(a^b_b) \vert 0\rangle\) and for the simple linear superposition \((ua^b_b + v) \vert 0\rangle\), for example, terms that are odd in \(q\) appear,

\[
\rho_c(q) = \frac{1}{\sqrt{2\pi (f, f)}} \exp \left[ -\frac{(q - [(f, g) + (g, f)])^2}{2(f, f)} \right]
\]

and

\[
\rho_{1,0}(q) = \frac{1}{\sqrt{2\pi (f, f)}} \left[ \frac{1 - \frac{|u|^2 (f, g)^2}{(f, f)(|u|^2 (g, g) + |v|^2)} + \frac{q^2}{(f, f)^2} \frac{|u|^2 (g, g) + |v|^2}{(f, f)(|u|^2 (g, g) + |v|^2)} \frac{v^* u (f, g) + u^* v (g, f)}{|u|^2 (g, g) + |v|^2} + \frac{q^4}{(f, f)^4} \frac{|u|^2 (g, g) + |v|^2}{|u|^2 (g, g) + |v|^2} \frac{|u|^2 (g, g) + |v|^2}{|u|^2 (g, g) + |v|^2}}}{(f, f)(|u|^2 (g, g) + |v|^2)} \right] \exp \left[ -\frac{q^2}{2(f, f)} \right].
\]

The closeness of this relativistically nonlocal classical statistical field theory model to the quantized Klein-Gordon model means that we can preserve relativistic signal locality just by restricting classical states to those which model quantum states of bounded energy. General classical states, with the necessary relativistically nonlocal evolution, would of course allow relativistic signal locality to be violated, but this is no more than the Hegerfeldt nonlocality present in quantum theory.
A quantum state, as a summary of all the correlation functions that can be constructed for the quantized Klein-Gordon model, is not completely reproduced by the corresponding relativistically nonlocal classical statistical field theory model. Reflecting the modal approach to field observables discussed in section 2, correlation functions of the form $\langle \phi f_1 \phi f_2 \ldots \phi f_n \rangle$ are equal in the two models only when the supports of $f_i(x)$ are mutually spacelike separated. When the supports of $f_1(x)$ and $f_2(x)$ are not spacelike separated, for example, $[\phi f_1, \phi f_2] \neq 0$ in the quantum model, so there is no possibility of observing a correlation function $\langle \phi f_1 \phi f_2 \rangle$ using quantum theoretically ideal measurement devices. Classically ideal measurement devices can observe this correlation function, but such devices would have to exhibit no quantum fluctuations, and we have no means to achieve this. We have to explicitly model measurement devices in a classical statistical field theory if they are classically nonideal because they do significantly perturb the system we wish to observe, and this ensures that measurement must be contextual in a classical statistical field theory. We can nonetheless imagine the results we would obtain if we did have classically ideal measurement devices, just as we only imagine the results we would obtain if we did have quantum theoretically ideal devices for measuring position or quantum momentum precisely.

The quantized real Klein-Gordon model has no invariant discrete structure, because there is no separation of the quantum state space into superselection sectors, so the vacuum state can be continuously deformed into the state $a^\dagger |0\rangle$ by taking a path from $(u, v) = (0, 1)$ to $(u, v) = (1, 0)$. Without discrete superselection sectors, a quantum field theory is hardly a “quantum” theory at all, because there is then no invariant discrete structure in the theory. The quantized Klein-Gordon model is better regarded as being about fields than about particles. The superselection structures that appear in gauge theories require, however, that some such superpositions are not allowed, so that the vacuum cannot be continuously deformed into states that are not in the same superselection sector. Correspondingly, a classical statistical field theory model for a gauge theory will have to have a discrete topological structure, which will complicate matters considerably.

4 Is this relativistic nonlocality classically acceptable?

A principal claim of this paper is that the classical statistical field theory model we have constructed for the quantized Klein-Gordon model is perfectly acceptable as classical thermal physics. Inevitably there are differences of detail, since properties of the operators we have introduced are not mathematically identical to properties of the heat equation, but there is a broad similarity of faster than exponential decrease with increasing distance.

As an approximate analogy, we can consider a detailed classical model of sound waves in
classical materials\textsuperscript{4} at finite temperature, taking the speed of sound as analogous to the speed of light. In such a model, it seems that nonlocality, relative to the ‘sound-cone’, is largely expected. The classical wave equation is not an adequate description of any real classical material; there are always thermal effects, which are more-or-less described by the heat equation. In a more adequate model, thermal effects which are outside our control are transmitted faster than the speed of sound, reflecting the very high thermal energy of a small number of atoms, but they allow us to send faster-than-sound messages only under the extreme conditions of blast waves, for example\textsuperscript{5}. If electromagnetic effects were not so accessible, the speed of sound would have been as strong a limitation on the development of physics as the speed of light has been. Note that if we describe the local propagation of sound using a 4-dimensional metric tensor the conditions of a blast wave would correspond to the simultaneous propagation of a light wave and a gravitational wave, which lies outside our current theoretical scope in quantum physics\textsuperscript{6}.

The lack of localization in quantum field theory that appears very unreasonable from the perspective of classical particle physics seems very reasonable from the perspective of a classical thermal physics of a continuum. We would not expect in a continuum classical thermal physics, for example, to be able to prepare a state corresponding to a high temperature within a finite region, while a lower temperature prevailed everywhere else, without a thermal gradient between them, because doing so would require us to coordinate thermal fluctuations everywhere in the universe at some earlier time, so as to result in a thermodynamically very unlikely state indeed. Furthermore, if we did achieve such a localized thermal state, the heat equation would ensure that the localization would be lost instantly.

Relativistic nonlocalities of the EPR kind in the quantized Klein-Gordon model should emerge as a consequence of the Hegerfeldt-type nonlocality we have introduced in this classical statistical field theory model, without any more dramatic nonlocality having to be introduced. We can perhaps expect that the violation of Bell-type inequalities by quantized massive spin-1 models will also emerge from a similar classical model, although at a greater theoretical distance

\textsuperscript{4}For a close analogy, we should consider the classical material to be in a solid phase, so that it supports transverse waves. Quantum field theory cannot correspond exactly to any finite lattice (translation symmetry cannot be broken in the vacuum state; see, for example, Haag(1996, Theorem 3.2.4)), but can be an effective field theory if a lattice is sufficiently small (which would be the case for a Planck scale lattice).

\textsuperscript{5}Alternatively, consider the difficulty of sending a message through rock at greater than the (local) speed of sound, using only bullet-like pieces of rock.

\textsuperscript{6}Taken seriously, the analogy with sound has much to suggest to quantum gravity, because the ‘sound-cone’ is not microscopically defined for a lattice, and depends, for example, on the local lattice temperature and flow. The light metric potentially becomes an effective field rather than a description of a fundamental geometry. Note, however, that the relativistic nonlocality of the classical model of section 3 sits badly with the local structure of general relativity, so that, despite the common classicality, unification will not be helped as much as we would like by the methods of this paper.
the details of quantized massless spin-1 models will have to be checked more carefully. Further papers will consider the situation for quantized spin-\( \frac{1}{2} \) models and for the standard model of particle physics in particular. It will also be interesting to investigate in detail whether and how the violation of Bell inequalities emerges from just the more elementary Hegerfeldt nonlocality.

In formal terms, the classical statistical field theory model we have constructed satisfies the relativistic principles of relativistic covariance and relativistic signal locality, so it is acceptable as classical relativistic physics. More as a matter of intuitive acceptability, the (unobservable) relativistic nonlocality that is present in the theory is no different in kind from the nonlocality that emerges in classical thermal models that are approximately described by the heat equation, so it should not worry us too much from a classical point of view, and it is no different from Hegerfeldt nonlocality, so it should also not worry us too much from a quantum point of view.

5 Conclusion

What is remarkable about quantum field theory from a classical perspective is its combination of thermal and wave equations, what we can loosely call the nonlocal and the local in classical field equations, in a single formalism, despite the systematic, empirically motivated assertion of relativistic signal locality. Since a relativistically covariant formalism is possible and relativistic signal locality can be maintained despite the relativistic nonlocality, and the relativistic nonlocality is no more than might be expected of a classical thermal physics, it seems quite reasonable for a classical physicist. It is perhaps enough, however, to know that such a relativistically nonlocal classical formalism is possible, for it may well not be mathematically more effective than the existing formalisms of quantum field theory.

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