The Adoption of Blockchain-based Decentralized Exchanges: A Market Microstructure Analysis of the Automated Market Maker

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Abstract

We analyze the market microstructure of Automated Market Maker (AMM) with constant product function, the most prominent type of blockchain-based decentralized crypto exchange. We show that, even without information asymmetries, the order execution mechanism of the blockchain-based exchange induces adverse selection problems for liquidity providers if token prices are volatile. AMM is more likely to be adopted for pairs of coins which are stable or of high personal use for investors. For high volatility tokens, there exists a market breakdown such that rational liquidity providers do not deposit their tokens in the first place. The adoption of AMM leads to a surge of transaction fees on the underlying blockchain if token prices are subject to high fluctuations.

Keywords: Cryptocurrency; FinTech; Decentralized Finance; Market Microstructure.

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1 Introduction

Since the emergence of Bitcoin in 2008, both practitioners and academics have argued that financial innovations such as tokenization of assets and decentralized ledgers, along with the backbone blockchain technology, will disrupt traditional financial services (see, e.g., Cong and He (2019), Abadi and Brunnermeier (2018), Chiu and Koeppl (2019), Cong, Li, and Wang (2020), Gan, Tsoukalas, and Netessine (2021)). However, even though thousands of crypto tokens have been created, and the total capitalization of cryptocurrencies has exceeded 1.7 trillions as of early 2021, no blockchain-based financial service providers has yet truly challenged traditional financial intermediaries, such as banks, brokerage, and exchanges. Ironically, most transactions of crypto tokens still rely on unregulated centralized intermediaries that expose investors to the risk of thefts and exit scams (see, e.g., Pennec, Fiedler, and Ante (2021), Cong et al. (2020), Gandal et al. (2018)).

In the mid of 2020, a new type of blockchain application called decentralized finance (commonly referred to as DeFi) has finally emerged and started challenging traditional financial intermediaries (see, for instance, Harvey, Ramachandran, and Santoro (2021)). DeFi utilizes open-source smart contracts on blockchains to provide financial services which typically rely on centralized financial intermediaries, including lending, borrowing, and trading transactions. The decentralized exchange that runs on the Ethereum blockchain has invented a new form of exchange that no longer relies on central intermediaries—Automated Market Maker (AMM). The biggest of such decentralized exchanges, Uniswap, has become the fourth-largest cryptocurrency exchange by daily trading volume, just a few months after its launch (Kharif (2020)).

Since the mechanism underlying decentralized exchanges is fundamentally different from that of traditional exchanges, the classical market microstructure literature has little to say about promises and pitfalls of this new form of exchanges. As a result, many important questions remain unanswered, such as: Can AMM provides sufficient incentives for provision of liquidity? Will there be any transaction breakdown where the liquidity reserve in the
AMM is drained? What kind of tokens are this new type of exchange suitable for? How do investors behave in this market? What is the execution cost of orders when there is competition on execution between arbitrageurs and liquidity providers?

In this paper, we develop a game theoretical model to answer the above questions. To the best of our knowledge, our paper is the first to analyze the economic incentives behind this novel blockchain-based financial service provider. We start by analyzing the adverse selection problem faced by the liquidity providers in the AMM. We show that even under complete information, liquidity providers can still be adversely selected from arbitrageurs when the prices of tokens in the exchange fluctuate. This stands in contrast with the adverse selection problem the market makers face in a centralized limit-order-book exchange, which arises because of asymmetric information (see, for instance, Glosten and Milgrom (1985)). Moreover, unlike market makers in a centralized limit-order-book exchange who can optimally charge bid-ask spreads to alleviate adverse selection, the liquidity providers in the AMM can neither have a bid-ask spread nor front-run due to the unique order execution structure of blockchain-based exchange.

We then analyze the investors’ decision in the AMM. We show that the trading volume increases if there is higher heterogeneity in investors’ preferences on tokens, and decreases in the fee charged by liquidity providers. This set of results are reminiscent the centralized limit-order-book exchange where heterogeneous preferences among agents lead to trade, and transaction cost decreases trades (see, for example, Duffie, Gâteanu, and Pedersen (2005) and Glosten and Milgrom (1985)).

We analyze the incentives of liquidity providers as well as the adoption of the decentralized exchange. We first illustrate that if prices of tokens are sufficient volatile, liquidity providers are not willing to provide liquidity in equilibrium, leading to a market shutdown. Indeed, when price fluctuations are too large, the adverse selection becomes too severe, and liquidity providers may earn a negative payoff for provision of liquidity. Moreover, market breakdown is less likely to occur if the token holder can extract large private benefits from using the
token on its corresponding platform, or if the heterogeneity of preferences among agents is high. This analysis sheds insights on what type of tokens are most suitable to be traded on the AMM: (1) stable coins that do not experience large price fluctuations, (2) tokens for which investors have high personal use.

We conclude by analyzing the spillover effects from the large adoption of AMM on the underlying blockchain. We show that large adoption leads to a surge of transaction fees on the blockchain underlying the exchange. This imposes negative externalities on other platforms operating on the same blockchain because high transaction fees may prevent consumers from using the platform or cause large delays in the operations of the platform. Hence, our results highlight an often overlooked mechanism: not only does the blockchain affect the DeFi applications built on it, but the DeFi applications also affect the underlying blockchain as well as the other applications that operate on it.

The rest of the paper is organized as follows. Section 2 introduces the institutional details of crypto exchanges and mechanism of AMM. Section 3.1 introduces the setup of our model, section 3.2 analyzes the adverse selection liquidity providers face, section 3.3 studies the behaviors of investors, and section 3.4 examines market breakdown and the spillover effect of the adoption of the exchange to its underlying blockchain.

**Literature Review.** Our paper contributes to the so-far scarce literature studying DeFi projects operating on blockchains. In particular, there are few studies on decentralized crypto exchanges, primarily from practitioners and the Computer Science community. Angeris et al. (2021) shows that the AMM can track the market price closely under no-arbitrage conditions; Bartoletti, Chiang, and Lluch-Lafuente (2021) abstracts the AMM away from the actual underlying economic mechanisms and studies its mathematical properties; Daian et al. (2020) provides empirical evidence for the existence of arbitrage at AMM. Harvey, Ramachandran, and Santoro (2021) provides a good survey of the DeFi applications. Our paper is the first to explore the fundamental market micro-structure problems concerning AMM, such
as adverse selection, market breakdown, execution cost, investor decisions, and liquidity provider incentives (adoption problem). Our results also complement existing literature on centralized crypto exchanges and prices of cryptocurrencies. (e.g., Griffins and Shams (2020), Gandal et al. (2018), Liu and Tsyvinski (2019), Li, Shin, and Wang (2018)).

Our paper adds to the growing literature on blockchains and tokenization in economics and finance (e.g., Yermack (2017), Harvey (2016), Abadi and Brunnermeier (2018), Cong and He (2019), Cong, Li, and Wang (2020), Sockin and Xiong (2020), Biais et al. (2019b), Halaburda et al. (2020), Biais et al. (2019a), Prat and Walter (2018)). Apart from illuminating the market microstructure of blockchain-based exchanges, our study also suggests a novel economic channel: the adoption of DeFi has spillover effects on other platforms built on the same blockchain.

2 Decentralized Crypto Exchanges

In this section, we introduce the institutional details of crypto exchanges, with a focus on the AMM.

2.1 Crypto Exchanges: Centralized vs. Decentralized

Since the introduction of bitcoin in 2008 by Nakamoto (2008), the idea of tokenization increased its popularity. As of January 2021, there are over 4000 crypto tokens created, distributed, and circulated (Bagshaw (2020)).

With the increasing adoption of cryptocurrencies by investors, many exchanges have been created specifically for the exchange of crypto tokens. As of early 2021, there exist more than 500 crypto exchanges. Those exchanges usually fall into two categories: centralized exchanges and decentralized exchanges (often called DEX).

A centralized cryptocurrency exchange is a trusted intermediary who monitors and facilitates crypto trades as well as securely stores tokens and fiat currencies (such as US dollars).
Similar to the equity market, the centralized cryptocurrency exchanges are often in the form of limit order books, and many of them also provide leverage trading and derivative trading. However, different from the equity market, most of the centralized cryptocurrency exchanges are unregulated and lack of proper insurance for the assets stored in the exchanges. This presents concerns for their safety, trustworthiness, and potential manipulations. Importantly, breaches, thefts and exit scams are not uncommon for crypto centralized exchanges. For example, Gandal et al. (2018) reveals price manipulation behavior on the crypto exchanges including Mt.Gox which, at its peak, was responsible for more than 70% of bitcoin trading. In early 2014, Mt.Gox suddenly closed its platform and filed for bankruptcy as it claimed that the platform’s wallet was hacked and a great amount of the assets were stolen. Other centralized exchanges subject to thefts and exit scams include Binance, BitKRX, BitMarket, PonziCoin, and so on.

Because of the concerns presented by centralized crypto exchanges, decentralized exchanges are becoming critical platforms for purchasing and selling crypto tokens. As of August 2020, decentralized crypto exchanges have already accounted for more than 5% of the total crypto trading, and their market shares have been increasing (McSweeney (2020)). Different from centralized exchanges, decentralized exchanges are blockchain-based smart contracts which can operate without a trusted central authority. Most of them are in the form of AMM that tracks a constant production function, such as Uniswap and Sushiswap. In particular, Uniswap makes up around 50% of the trading volume of decentralized crypto exchanges, and Sushiswap accounts for more than 20% of the trading activities. (Smith and Das (2021))

Unlike centralized exchanges which facilitate trades and manage all the orders with their own infrastructure, decentralized exchanges take in, manage and execute orders through a blockchain (typically Ethereum). Instead of sending the order directly to the exchange, the users have to submit their orders to the blockchain network and specify a transaction cost (the gas price and gas limit in the Ethereum network). The blockchain miner that finds the
very next block will execute the orders according to the attached gas price (from highest to lowest), and append them to the blockchain. However, even though the blockchain guarantees decentralization of the exchange, it also limits the speed of order execution because the block generation speed is typically 15s/block for Ethereum networks. This means that an order submitted to a decentralized exchange needs to wait for at least few seconds before its execution. In comparison, an order can be executed in milliseconds in a centralized exchange (Sedgwick (2018)).

2.2 AMM with Constant Product Function

We provide a brief description of the mechanism of AMM, which uses a constant product function. We limit our discussion to the elements that are essential for our study of market microstructure, and refer to Adams (2020) for a more detailed introduction.

AMM does not rely on a limit-order book for transactions. Rather, it develops a new market structure called liquidity pool. A liquidity pool allows to directly exchange two crypto tokens, let us say, token A and token B, instead of first selling token A for fiat currency, such as USD, and then purchasing token B using the proceeds from the asset sale. Each liquidity pool manages a pair of tokens.

The liquidity pool essentially works by incentivizing token owners to deposit their tokens into the smart contract. Assume a liquidity pool manages the exchange of two tokens, token A and token B, and each token A is worth $p_A$ and each token B is worth $p_B$. Anyone who owns both token A and token B can choose to be a liquidity provider by depositing an equivalent value of each underlying token in the AMM and in return, receiving a pool token which proves her share of the AMM. For example, if the current reserve in the liquidity pool contains 10 tokens A and 5 tokens B, and the current price is 1 for token A and 2 for token B, then the liquidity provider must deposit token A and token B in the ratio of 2 : 1 to ensure equivalent value. After the liquidity provider deposits 10 token A and 5 token B,

\[ \text{It is also possible to pool multiple tokens together, but pools with only a pair are most common. The pricing dynamics of pools with more than two tokens is almost identical to pools with just a pair of tokens.} \]
the liquidity provider can claim pool tokens that account for half of the total tokens in the current liquidity reserve. The current reserve of the liquidity pool is then 20 token A and 10 token B. The liquidity provider can exit the liquidity pool by trading in her pool tokens, and receive her share of the liquidity reserve in the AMM. For instance, if the liquidity reserve contains 25 tokens A and 9 tokens B when the liquidity provider exits, then the liquidity provider receives 12.5 token A and 4.5 token B.

Suppose a new investor arrives and wants to exchange an amount \( \Delta_A \) of token A, for an amount \( \Delta_B \) of token B. Then \( \Delta_A \) and \( \Delta_B \) must satisfy \((20 + \Delta_A)(10 - \Delta_B) = 200 \times 10 = 2000\). That is, the product of the amount of both tokens must be a constant. Formally, assume the initial liquidity reserve in the AMM contains \( y_A \) token A and \( y_B \) token B. Then the trade needs to satisfy \((y_A + \Delta_A)(y_B - \Delta_B) = y_A y_B\). On top of the amount \( \Delta_A \) of tokens A already exchanged by the investor, the investors must pay an additional amount \( f\Delta_A \) of token A as transaction fee, and most of the transaction fee is then added to the liquidity pool. Hence, the fee increases the total liquidity reserve of the AMM as well as the AMM share of the liquidity providers. The fee incentivizes the liquidity providers to deposit in the first place. The transaction fee is typically very small (for instance, it is 0.3% for Uniswap). It is worth mentioning that if \( \Delta_A \to 0 \), then \( \frac{\Delta_A}{\Delta_B} \to \frac{y_A}{y_B} \). This suggests that the ratio between the amount of two tokens in the liquidity pool is exactly the spot exchange rate when the trading size is infinitesimally small.

3 Analysis of the AMM

We develop our model in section 3.1. In Section 3.2 we analyze arbitrage opportunities and adverse selection problems ex-post, i.e., after the asset shock realizes. In Section 3.3 we study investors’ decisions. Finally, we analyze the incentive of liquidity providers, the execution cost at AMM, and their implication for AMM adoption in Section 3.4.
3.1 Model Setup

Our model timeline consists of three periods indexed by $t, t = 0, 1, 2$. There are three kinds of agents: liquidity providers, arbitrageurs, and traders. Every agent has a discount factor equal to 1.

The agents have access to two tokens, that is, token A and token B. The tokens can be used directly on the corresponding platforms, platform A and platform B. Alternatively, they can be exchanged for a single nonstorable consumption good used as a numeraire at prices which are public information for all agents. There is a smart contract that functions as the AMM for token A and token B. The fee for using the AMM is $f \times q$ token $i, i = A, B$ where $f$ is a constant and $q$ is the amount of token $i$ exchanged for the other token in the AMM. The fees are collected by the liquidity providers.

At date $t = 0$, a single token A is valued at $p_A$, and a single token B has value $p_B$. There are $n > 1$ liquidity providers who are endowed with positive, equal value of token A and token B. The initial endowments of the liquidity provider $i, i = 1, 2, 3, ..., n$, are $w_i x_A$ and $w_i x_B$ respectively for token A and token B, where $\sum_1^n w_i = 1, w_i > 0$. The total initial endowment for the liquidity providers are then $x_A$ and $x_B$. At $t = 0$, the liquidity providers decide whether to deposit their tokens in the AMM and how much to deposit.

We denote the amount of tokens A and B deposited in the AMM at $t = 0$, respectively by $y_A^{(0)}$ and $y_B^{(0)}$. Since the AMM requires the deposited tokens A and B to have equal value, we have $y_A^{(0)} p_A = y_B^{(0)} p_B$ at $t = 0$. The liquidity provider $i$ deposits $w'_i y_A^{(0)}$ and $w'_i y_B^{(0)}$ tokens into the AMM, where $\sum_1^n w'_i = 1, w'_i > 0$.

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2In practice, a small portion of the fees may be collected by the platform that designs the AMM. However, this would not qualitatively change our results and can be incorporated by multiplying the fee collected by the liquidity provider with a constant term.

3The assumption of equal value endowment does not affect the results. Assume the liquidity provider is endowed with value $x_A p_A$ of token A and value $x_B p_B > x_A p_A$ of token B. Because the liquidity providers are required to deposit equal value of token A and token B to the AMM, i.e., the AMM, the maximum value of tokens a liquidity provider can deposit are $x_A p_A$ for token A and $x_A p_A$ for token B. In this way, the extra value of token B, $x_B p_B - x_A p_A$ does not affect the liquidity provider’s decision, and the set of feasible actions to the liquidity providers (how much value to deposit) is identical to the case that the liquidity provider is endowed with value $x_A p_A$ of token A and value $x_A p_A$ of token B.
At date $t = 1$, an investor arrives to the AMM to trade. An investor is characterized by an intrinsic type, that is “type A” or “type B.” A “type A” investor is willing to use token A on its corresponding platform, and extracts a private benefit of $(1 + \alpha)p_A$ from using one token A. A “type A” investor does not use platform B, so she only receives $p_B$ for each token B. Similarly, a “type B” investor receives $(1 + \alpha)p_B$ for each token B and $p_A$ for each token A. The investor arriving to the AMM is a “type A” or a “type B” with equal probability, and the investor chooses the quantity she trades via the AMM to maximize its total utility, that is, $(1 + \alpha)p_Aq_A + p_Bq_B$ for “type A” investors, and $(1 + \alpha)p_Bq_B + p_Aq_A$ for “type B” investors, where $q_A$ and $q_B$ are the amount of token A and token B the investor holds. Following classical assumptions in market microstructure literature (see, for instance, Glosten and Milgrom (1985)), we assume that investors are not liquidity constrained and have sufficient tokens to trade.

After the investor arrives and trades, the ratio between the amount of token A and token B at the AMM deviates from $\frac{y_A^{(0)}}{y_B^{(0)}}$. Since the spot exchange rate is solely decided by the ratio between the amount of tokens, this deviation may present arbitrage opportunities.

We assume that there are $m > 1$ risk-neutral arbitrageurs who do not use tokens on either platforms and only exchange tokens for consumption good. Hence, their utility function is $p_Aq_A + p_Bq_B$ where $q_A$ and $q_B$ are the amount of tokens the arbitrageur holds. The arbitrageurs take advantage of the price deviation at the AMM and trade to maximize their utility. Since all $m$ arbitrageurs are aware of this arbitrage opportunity, they need to compete to get their order executed: the first arbitrage order that arrives at the AMM gets executed and exploits the arbitrage opportunity while the rest of orders fail. The arbitrageurs compete by adding a transaction fee $4g$ to their order. We will also refer to such fee as “gas fee” interchangeably throughout the paper, following its institutional counterpart. The order with the highest fee will be the first to be included on the underlying blockchain and executed. The arbitrageurs can observe the transaction fee attached to their orders by

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4We assume that the fee is zero if there is no competition, such as when the investor arrives at $t = 1$. 

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other arbitrageurs. Due to competition, the arbitrageurs end up earning zero profit from each trade, and the transaction fee g they attach to their order is exactly their profit from arbitrage.

At date $t = 2$, an exogenous asset shock which hits either token A or token B occurs. The probability that it hits token A is the same as the probability that it hits token B, and is equal to $\frac{1}{2}$. The asset shock that hits token A increases the value of token A. Formally, a single token A can be exchanged for $(1 + \beta)p_A$ consumption good after being hit by the asset shock instead of the initial amount $p_A$. Similarly, if the asset shock hits token B, then it increases the value of token B by $(1 + \beta)$.

The liquidity provider can withdraw their tokens at the end of period $t = 2$ (after the shock is realized at the beginning of this period) by submitting a withdraw order and attaching a gas fee. When the withdraw order is executed, the liquidity provider $i$ receives $y_A'w_i'$ token A and $y_B'w_i'$ token B where $y_A', y_B'$ are the total reserves in the AMM before the first withdrawal and $w_i'$ is the share of reserves that liquidity provider $i$ initially deposited. At the end of period $t = 2$, the liquidity provider exchanges her tokens for the consumption good.

Upon realization of the asset shock, the spot exchange rate in the AMM remains unchanged, which creates an arbitrage opportunity. The arbitrageurs are incentivized to trade in the token not hit by the shock for the token which becomes more valuable after being hit by the shock. This, in turn, leads to a loss for the liquidity providers who then have strong incentives to withdraw their token ahead of the arbitrageurs.

The timeline of the model is depicted in Figure 1. In the following sections, we formally analyze the model backward. We first show that at $t = 2$, even without asymmetric information, the liquidity providers at the AMM will be subject to adverse selection from the arbitrageurs. Moreover, we show that liquidity providers are unable to offset the adverse selection by exiting the contract early. We illustrate that adverse selection is due to price fluctuation of the tokens and the unique microstructure of the AMM built on the blockchain.
Then, we analyze the trading activity of investors at $t = 1$. Lastly, we examine everything from an ex-ante standpoint. We provide conditions under which the liquidity providers do not provide liquidity at period $t = 0$, which thus results in a liquidity freeze at the decentralized exchange. We also show the unexpected negative spillover effect from the adoption of the AMM to its underlying blockchain.

### 3.2 Asset Shock and Arbitrage Opportunities

At $t = 2$, after the asset shock is realized, the liquidity providers and the arbitrageurs move simultaneously and submit their orders to the blockchain. A liquidity provider needs to decide the amount of token A or token B that she wants to trade in as well as the gas fee she attaches to the order. A liquidity provider decides the gas fee she attaches to her exit order only. A liquidity provider decides the gas fee she attaches to her exit order. We show that the liquidity provider will be subject to adverse selection and token value loss, if the price fluctuation of the token is sufficiently large.

Denote the amount of tokens A and B in the AMM at the beginning of period $t = 2$,
respectively by \( y_A^{(2)} \) and \( y_B^{(2)} \). Without loss of generality\(^5\) we first assume the asset shock hits token B, and the consumption good a token B can exchange rises from \( p_B \) to \((1 + b)p_B\).

Upon realization of the asset shock, the spot price at the AMM \( \frac{y_A^{(2)}}{y_B^{(2)}} \) may deviate from the fair value price \( \frac{(1+b)p_B}{p_A} \). To take advantage of this deviation, the arbitrageur submits an order to the AMM and exchange \( x \) tokens A for token B. An arbitrageur aims for the optimal arbitrage, i.e., chooses the buy order which solves the following optimization problem.

\[
\max_{x>0} -p_A(1 + f)x + p_B(1 + \beta) \left( y_A^{(2)} - y_A^{(2)} \frac{y_B^{(2)}}{y_A^{(2)} + x} \right)
\]

where \(-p_A(1 + f)x\) is the value of token A traded in plus the fee paid to the liquidity providers, \( p_B(1 + \beta)(y_A^{(2)} - y_A^{(2)} \frac{y_B^{(2)}}{y_A^{(2)} + x})\) is the value of token B received by the arbitrageur from the order, and \((y_A^{(2)} - y_A^{(2)} \frac{y_B^{(2)}}{y_A^{(2)} + x})\) is the amount of tokens B received. Solving for the optimal arbitrage yields the following result:

**Proposition 1.** If the price change is greater than a certain threshold, \( \beta > \beta_1 \), then an arbitrageur earns a positive payoff \( \pi > 0 \) from the optimal arbitrage. Moreover, the threshold \( \beta_1 \) is increasing in the fee charged \( f \); the payoff \( \pi \) and optimal trading size \( x^* \) are increasing in \( \beta \) and decreasing in \( f \).

The above proposition indicates that when the price fluctuation \( \beta \) is sufficiently large, it is profitable for arbitrageurs to trade in the token not hit by the positive asset shock and exchange it for the other, more valuable token. The larger the price fluctuation, the more severe the adverse selection, and the higher the payoff attained from the arbitrage. Moreover, the higher the fee charged by the AMM, the higher the trading cost for the arbitrageur, which in turn reduces the arbitrage profit as well as the order size.

The profit of arbitrageurs equals to the loss of token value of the liquidity providers. Formally, if an optimal arbitrage order gets executed, it yields a loss of token value in the amount \( w'_i \pi \) to the liquidity provider \( i \). Then, the liquidity provider has an incentive to

\(^5\)The case where asset shock hits token A can be solved using symmetrical arguments.
submit a withdrawal order to the AMM. To avoid the token value loss, the withdrawal order must be executed and included in the underlying blockchain before the arbitrage order submitted by arbitrageurs. This means that the liquidity provider must pay a gas fee higher than the one attached to the arbitrage order. Because of competition\textsuperscript{6}, the gas fee the arbitrageurs pay for their arbitrage order is exactly their gain $\pi$ from the optimal arbitrage order, so the arbitrageurs have a zero net profit. However, the liquidity provider $i$ never pays a gas fee higher than $w'_i\pi$ since they would otherwise incur a cost even higher than the loss from adverse selection $w'_i\pi$. Hence, for liquidity providers, the adverse selection problem is not avoidable by exiting the contract before the arbitrage is exploited, and we have the following result:

\textbf{Proposition 2.} If $\beta > \beta_1$, an optimal arbitrage order is the first order executed after the realization of asset shock. The arbitrage leads to a token value loss of $w'_i\pi$ for the liquidity provider $i$, and the gas fee attached to the arbitrage order is exactly $\pi$.

It worth noticing that the adverse selection problem in the AMM is fundamentally different from the adverse selection problem arising in typical open limit-order book markets. In a limit-order book market, studied for instance in Glosten and Milgrom (1985) and Glosten (1994), market markers can be adversely selected by other investors who have private information about the future realization of asset returns. However, in the AMM, the adverse selection exists even if there is complete information. This is because the adverse selection problem here is due to price movement of tokens and to the order execution mechanism of decentralized exchanges built on blockchains. Moreover, the market makers in traditional open limit-order book markets can offset the adverse selection problem by placing a bid-ask spread, or they can even front-run the orders when they are able to predict the direction of order flow. In contrast, the AMM does not charge a bid-ask spread, and the liquidity

\textsuperscript{6}One can think the competition for the first execution as a second-price auction with complete information. For all arbitrageurs, the first execution has value $\pi$, and for the liquidity provider $i$, the first execution has a value $w'_i\pi$. At the equilibrium of this subgame, the gas fee that the arbitrageurs pay for the first execution is $\pi$. 

14
providers are also unable to front-run the arbitrage order because the priority of order execution is decided by the gas fee attached to the order. As a result, the liquidity providers participating in the AMM are subject to more severe adverse selection problem, and they must be compensated by enough transaction fees from trading activities. However, this creates another tension that we will analyze in the next section: trading volume from the investors decreases in the fee $f$ charged by the AMM.

3.3 Investors’ Decisions

In this section, we analyze the trading activity of investors in period $t = 1$. At $t = 1$, the investor arrives to the exchange and decides the amount of token A and token B she trades in. After the trade is finished, the arbitrageurs exploit any arbitrage opportunities available.

For illustration purposes, we first assume a “type A” investor arrives at $t = 1$. The corresponding result for the other type follow from symmetry arguments. Since a “type A” investor has personal use for token A, she uses token B to exchange for token A. Formally, when the type A investor decides the amount $q$ of token A to exchange for, she solves:

$$\max_{q>0} p_A(1 + \alpha)q - p_B(1 + f) \left( \frac{y_A(0) y_B(0)}{y_A(2) - q} - y_B(0) \right),$$

where $p_A(1 + \alpha)q$ is the utility the “type A” investor receives from using the token A she obtains from trading, $\left( \frac{y_A(0) y_B(0)}{y_A(2) - q} - y_B(0) \right)$ is the amount of token B the investor has to trade in, and $p_B(1 + f) \left( \frac{y_A(0) y_B(0)}{y_A(2) - q} - y_B(0) \right)$ represents the total value of token B the investor trades in.

Lemma 3. The amount of tokens obtained from the trade $q$ is $(1 - \sqrt{\frac{f+1}{\alpha+1}}) y_A(0)$ if the arriving investor is of “type A” and $(1 - \sqrt{\frac{f+1}{\alpha+1}}) y_B(0)$ if the arriving investor is of “type B”.

The above lemma shows that the trading volume is increasing in $\alpha$, the extra utility investors receive from using the tokens on their corresponding platforms. There will be a trade as long as $\alpha > f$. Moreover, the higher the trading cost $f$, the lower the trading volume.
Trading generates fees for the liquidity providers, which is a compensation for the adverse selection problem they face at \( t = 2 \). Denote the amount of tokens A and tokens B in the AMM after the investor trades as \( y_A^{(1)} \) and \( y_B^{(1)} \) respectively. Since the trades by the investors alter the ratio of token A and token B in the AMM, that is, \( \frac{y_A^{(1)}}{y_B^{(1)}} \neq \frac{y_A^{(0)}}{y_B^{(0)}} = \frac{p_B}{p_A} \), there exists an arbitrage opportunity. As for the arbitrage strategy in period \( t = 2 \), the arbitrage yields token value loss for the liquidity providers.

### 3.4 Liquidity Freeze and Negative Spillovers

In this section, we examine whether the liquidity providers are willing to deposit their tokens into the decentralized exchange at \( t = 0 \). If none of the liquidity deposits their tokens, the AMM contains no tokens, and thus there is no trading activities at \( t = 1, 2 \). We denote this scenario as a “liquidity freeze”.

From an ex-ante perspective, the liquidity providers have two choices: hold the tokens or deposit them into the exchange. As shown in the previous sections, liquidity providers earn transaction fees by depositing but they are subject to adverse selection from the arbitrageurs after asset shocks. The liquidity providers will deposit their tokens if and only if they can earn higher payoff than just holding the tokens. Formally, the liquidity providers deposit the tokens if the following payoff is positive:

\[
 w_i' \mathbb{E}[p_A^{(2)} (y_A^{(2)} - y_A^{(0)}) + p_B^{(2)} (y_B^{(2)} - y_B^{(0)}) - \pi],
\]

where \( p_A^{(2)} \) and \( p_B^{(2)} \) are the amount of consumption good a single token A and token B can exchange at the end of period \( t = 2 \). If the liquidity provider \( i \) deposits, then \( w_i' y_A^{(2)} \) and \( w_i' y_B^{(2)} \) are respectively the amounts of token A and token B that the liquidity provider \( i \) deposits at the AMM at the beginning of period \( t = 2 \); if instead, the liquidity provider \( i \) just hold those tokens, the corresponding amount will be \( w_i' y_A^{(0)} \) and \( w_i' y_B^{(0)} \). Hence, \( w_i' (p_A^{(2)} (y_A^{(2)} - y_A^{(0)}) + p_B^{(2)} (y_B^{(2)} - y_B^{(0)})) \) is the difference of token values between choosing to deposit and choosing to...
hold at the beginning of period $t = 2$. The last term, $w'_t \pi$, is the loss of token value due to adverse selection at $t = 2$, which will only be incurred if the liquidity providers deposit their tokens. If the above expression is positive, the liquidity providers can earn a higher return by depositing their tokens at $t = 0$, and thus they choose to supply liquidity to the AMM.

The strategy profile of the game described in 3.1 can be defined as a tuple

$$(w', q_A, q_B, x_1, g_1, x_2, g_2),$$

where $w' = (w'_1, w'_2, ..., w'_n)$ is the portion of tokens liquidity providers deposit at $t = 0$, $q_A, q_B$ are the amount of tokens a “type A” investor and a “type B” investor respectively trades for at $t = 1$, and $(x_1, g_1), (x_2, g_2)$ are the arbitrage orders and attached gas fees arbitrageurs submit to the AMM at $t = 1$ and $t = 2$ respectively. We then solve for the subgame perfect equilibrium of the game, and we define a subgame perfect equilibrium as ”liquidity freeze equilibrium” if $w' = 0$, that is, no liquidity providers deposit at $t = 0$.

The following proposition characterizes the condition under which ”liquidity freeze” arise in equilibrium.

**Proposition 4.** For given $\alpha, f$, if the price fluctuation is sufficiently large, that is, $\beta > \beta_2$, then a ”liquidity freeze” occurs in equilibrium. Moreover, the threshold $\beta_2$ is increasing in $\alpha$.

The above proposition states that when the tokens are sufficiently volatile, adverse selection is severe, and a liquidity freeze occurs. This implies that the AMM may face adoption problems for tokens that face large price fluctuations. Liquidity provided for those pairs are likely to be low. Conversely, it suggests that the AMM may be more suitable for pairs of stable coins which are pegged to stable asset and designed to have low volatility. Moreover, the comparative statics analysis suggests that the AMM is more likely to be adopted for popular tokens whose corresponding platforms attract investors by providing them with high personal use value. Because those tokens attract investors and lead to high trading volumes, liquidity providers can earn large transaction fees and be compensated for the adverse
selection problem they face.

**Proposition 5.** If the “liquidity freeze” is not an equilibrium outcome, that is, $\beta < \beta_2$, then the gas fees $g_1$ and $g_2$ are increasing in $\beta$ and in the amount of tokens $y_A$ and $y_B$ deposited by liquidity providers.

The above proposition captures an important, yet undesirable, consequence of the AMM. Intuitively, if there is large fluctuation of asset prices, or a large amount of tokens deposited at the AMM, we expect higher return for arbitrageurs. This is because of a gas fee surge due to competition between arbitrageurs. This suggests that as the AMM becomes more popular and widely adopted, the instability of its underlying infrastructure—blockchain—may increase. Surge of gas fees means that many other platforms built on the same blockchain may lose their costumers due to high transaction fees (barriers to entry), or significantly delay the submission of orders by their costumers. This means that the large adoption of the AMM may impose negative externalities on its underlying blockchain platform as well as on other platforms relying on the same blockchain. These results highlight that the platforms on a blockchain are not independent, and that the usage of one platform may significantly affect others.
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Appendix A  Technical Calculations and Proofs

Proof of Proposition 1 The first-order condition (FOC) of the optimization problem stated in (1) is

\[
\frac{(\beta + 1)p_B y_A^{(2)} y_B^{(2)}}{(x + y_A^{(2)})^2} = (f + 1)p_A.
\]

We solve the FOC and pick the larger root because the smaller root is infeasible:

\[
x^* = \sqrt{\frac{(\beta + 1)p_B y_A^{(2)} y_B^{(2)}}{(f + 1)p_A}} - y_A^{(2)}.
\]

We have that \(x^*\) feasible (only positive amount is feasible) if and only if \(\beta > \beta_1 = \frac{(1+f)y_B^{(2)} p_A}{y_B^{(2)} p_B} - 1\). If \(x^*\) is feasible, then it is easy to verify that it attains the global maximum on \([0, \infty)\).

Since \(x = 0\) yields zero payoff, it is sufficient to verify that \(x^* > 0\) to guarantee that \(x^*\) yields a positive payoff. This is because the payoff function achieves its global maximum if \(x = x^*\) and thus higher than the payoff at \(x = 0\). If \(x^* < 0\), then the payoff monotonically decreases on the interval \([0, \infty)\), which means that the arbitrageur does not trade.

Since \(\beta_1\) is a positive linear function respect to \(f\) and the slope is positive, it is obvious that \(\beta_1\) increases in \(f\).

\[\square\]

Proof of Lemma 3 We first consider the case where the investor that arrives is “type A”. We solve for the FOC and pick the larger root. This yields

\[
q^* = y_A^{(0)} - \sqrt{\frac{(\alpha + 1)p_B y_A^{(0)} y_B^{(0)}}{(f + 1)p_A}}.
\]

Using the identity \(y_A^{(0)} p_A = y_B^{(0)} p_B\) which results from the equal value of the two tokens at \(t = 0\), we have

\[
q^* = y_A^{(0)} - y_A^{(0)} \sqrt{\frac{(\alpha + 1)}{(f + 1)}}.
\]
If $q^* > 0$, then the investor trades and chooses to exchange for $q^*$ token A because $q^*$ attains the global maximal of the investor’s payoff function. If instead, $q^* \leq 0$, then the investor does not trade because 0 is optimal.

The case where the investor that arrives is “type B” can be calculated by symmetry argument and replace A with B.

Proof of Proposition 4. We denote the event where a “type $i$” investor arrives and the shock hits token $j$ as event $S_{ij}$, where $i,j \in \{A, B\}$. Then we can write (2) as:

$$\sum_{i,j \in \{A, B\}} P[S_{ij}] E[p_A(y_A^{(2)} - y_A^{(0)}) + p_B(y_B^{(2)} - y_B^{(0)}) - \pi|S_{ij}],$$

(3)

where $P[S_{ij}] = \frac{1}{4}$, and we need to calculate path-dependent variables $y_A^{(2)}, y_B^{(2)}, \pi$. We first calculate those variables conditional on event $S_{ij}$ and plug them into (3). We then simplify the expression by using the identity $y_A^{(0)} p_A = y_B^{(0)} p_B$, and obtain

$$U(\alpha, \beta, f) = \frac{1}{2} p_A y_A^{(0)} \left( \sqrt{\frac{f+1}{\alpha+1} f^2} - 2(f+1)f \sqrt{\frac{f}{\alpha+1} \sqrt{f+1}} - 2\beta - 4 \right)$$

$$+4(f+1) \sqrt{(\beta + 1) \left( 1 - \frac{f}{\sqrt{\alpha+1} \sqrt{f+1}} \right) \left( (f+1) \sqrt{1 - \frac{f}{\sqrt{\alpha+1} \sqrt{f+1}}} - \frac{f \sqrt{f+1}}{\sqrt{\alpha+1}} \right) }.$$  

As $\beta \to \beta_1$, there is no adverse selection at $t = 2$, and the liquidity provider earns a positive payoff. As $\beta \to \infty$, the payoff of the liquidity provider converges to $-\infty$.

We then take the partial derivative of the above expression respect to $\beta$. We have

$$\frac{\partial U}{\partial \beta} = (-1 + \frac{1+f}{\sqrt{1+\beta}}) p_A y_A^{(0)}.$$

As $\beta$ increases from $\beta_1$ to $\infty$, $\frac{\partial U}{\partial \beta}$ is either negative on $(\beta_1, \infty)$, or positive on $(\beta, s)$ and then negative on $(s, \infty)$. As $\beta \to \infty$, $\frac{\partial U}{\partial \beta}$ converges to $-1$.

For the first case where the partial derivative is negative in $(\beta_1, \infty)$, for given $\alpha, f$ the payoff of the liquidity provider is monotonically decreasing in $\beta$. By the limit argument
above, the continuity of the payoff function respect to $\beta$, and the Intermediate Value Theorem (IVT), there exits $\beta_2$ such that $U(\alpha, \beta_2, f) = 0$. By monotonicity, $U > 0$ if $\beta < \beta_2$, and $U < 0$ if $\beta > \beta_2$.

We then consider the second case where the partial derivative $\frac{\partial U}{\partial \beta}$ is positive on $(\beta, s)$ and then negative on $(s, \infty)$. $U$ increases on $(\beta, s)$, and thus $U(s) > U(\beta_1) > 0$ (we omit $\alpha, f$ here as we assume that they are fixed). Using the same argument as in the first case, we deduce the existence of $\beta_2 \in (s, \infty)$ such that $U(\alpha, \beta_2, f) = 0$, and $U > 0$ when $\beta < \beta_2$, and $U < 0$ when $\beta > \beta_2$.

Combining the two cases, we conclude that for given $f, \alpha$, there exists $\beta_2$ such that the payoff from providing liquidity is positive if $\beta < \beta_2$, and negative if $\beta > \beta_2$. In this way, the liquidity providers do not deposit if $\beta > \beta_2$, and this leads to a “liquidity freeze” equilibrium.

We then apply the Implicit Function Theorem (IFT) to explore how $\beta_2$ changes as $\alpha$ increases. From the above analysis, we already know that $\frac{\partial U}{\partial \beta} < 0$ at $\beta_2$. All we need to check is the sign of $\frac{\partial U}{\partial \alpha}$ at $\beta_2$.

For the case where $\alpha \leq f$, there is no trade for the investors, and thus $\alpha$ is irrelevant.

We then consider the case where $\alpha > f$. We first take the partial derivative of $U$ respect to $\alpha$. Since the denominator of the partial derivative is always positive, we only need to analyze the sign of the numerator:

$$\sqrt{\beta + 1} \left( 2\sqrt{\alpha + 1}(f + 1) + 2\sqrt{\alpha + 1}(f + 1) \sqrt{1 - \frac{f}{\sqrt{\alpha + 1}\sqrt{f + 1}}} - 3(f + 1) \right)$$

$$- \sqrt{\alpha + 1} f \left( \sqrt{1 - \frac{f}{\sqrt{\alpha + 1}\sqrt{f + 1}}} + 1 \right)$$

$$\ast \sqrt{1 - \frac{f}{\sqrt{\alpha + 1}\sqrt{f + 1}}} \left( (f + 1) \sqrt{1 - \frac{f}{\sqrt{\alpha + 1}\sqrt{f + 1}}} - \frac{f\sqrt{f + 1}}{\sqrt{\alpha + 1}} \right)$$

We then plug in the equality $U(\alpha, \beta_2, f) = 0$ and multiply it by a positive value $4(\beta + 1)(f + 1)$. We then have
\[
\sqrt{\alpha + 1}f \left( \sqrt{1 - \frac{f}{\sqrt{\alpha + 1}f + 1}} + 1 \right) \left( \frac{\sqrt{f + 1}^2}{\sqrt{\alpha + 1}} - 2(f + 1)f \sqrt{1 - \frac{f}{\sqrt{\alpha + 1}f + 1}} - 2\beta - 4 \right)
+ 4(\beta + 1)(f + 1) \left( 2\sqrt{\alpha + 1}(f + 1) \left( \sqrt{1 - \frac{f}{\sqrt{\alpha + 1}f + 1}} + 1 \right) - 3f\sqrt{f + 1} \right).
\]

Expanding the expression above, we obtain

\[
8\sqrt{\alpha + 1}\beta^2 + 8\sqrt{\alpha + 1}\beta f \sqrt{1 - \frac{f}{\sqrt{\alpha + 1}f + 1}} + 14\sqrt{\alpha + 1}\beta f + 14\sqrt{\alpha + 1}\beta f \sqrt{1 - \frac{f}{\sqrt{\alpha + 1}f + 1}} + 8\sqrt{\alpha + 1}\beta - 2\sqrt{\alpha + 1}f^3
\]
\[
- 2\sqrt{\alpha + 1}f^3 \sqrt{1 - \frac{f}{\sqrt{\alpha + 1}f + 1}} + \sqrt{f + 1}f^3 \sqrt{1 - \frac{f}{\sqrt{\alpha + 1}f + 1}} + 6\sqrt{\alpha + 1}f^2 + 6\sqrt{\alpha + 1}f^2 \sqrt{1 - \frac{f}{\sqrt{\alpha + 1}f + 1}} + 12\sqrt{\alpha + 1}f
\]
\[
+ 12\sqrt{\alpha + 1}f \sqrt{1 - \frac{f}{\sqrt{\alpha + 1}f + 1}} + 8\sqrt{\alpha + 1} \sqrt{1 - \frac{f}{\sqrt{\alpha + 1}f + 1}} + 8\sqrt{\alpha + 1} - 12\beta \sqrt{f + 1}f^2 - 12\beta \sqrt{f + 1}f + \frac{2f^4}{\sqrt{f + 1}}
\]
\[
+ \sqrt{f + 1}f^3 + \frac{2f^3}{\sqrt{f + 1}} - 12\sqrt{f + 1}f^2 - 12\sqrt{f + 1}f.
\]

Using the inequality \(\alpha > f\), we conclude that the above expression is positive at \(\beta_2\). By the implicit function theorem, we deduce that

\[
\frac{\partial \beta_2}{\partial \alpha} = -\left. \frac{\partial U}{\partial \beta} \right|_{\beta_2} \left. \frac{\partial U}{\partial \alpha} \right|_{\beta_2} \geq 0,
\]

that is, \(\beta_2\) increases in \(a\).

\[
\square
\]

**Proof of Proposition** As we have argued, the gas fee is equal to the profit from arbitrage order.
We first consider \( g_2 \). For the case where \( \beta \leq \beta_1 \), there is no adverse selection and arbitrage, and the gas fee is constant.

We then solve the case where \( \beta_1 < \beta < \beta_2 \). For the case where the event \( S_{AB} \) occurs, we plug the optimal arbitrage order’s buy size \( x^* \) into the payoff function of the arbitrageur, and have

\[
-2(f + 1)\sqrt{(\beta + 1)p_By_A^{(2)}y_B^{(2)}} + (\beta + 1)p_By_B^{(2)} + (f + 1)p_Ay_A^{(2)}.
\]

We take the partial derivative respect to \( \beta \) and obtain

\[
p_By_B^{(2)}\left(\frac{(\beta + 1)p_By_A^{(2)}y_B^{(2)}}{(f + 1)p_A} - y_A^{(2)}\right) = p_By_B^{(2)}\left(\frac{x^*}{\sqrt{(\beta + 1)p_By_A^{(2)}y_B^{(2)}}} - \frac{x^*}{(f + 1)p_A}\right) > 0.
\]

This means that the payoff of arbitrageurs increase in \( \beta \), and so does the gas fee.

We first write \( y_A^{(2)}, y_B^{(2)} \) in \( y_A^{(0)}, y_B^{(0)}, f, \alpha, \beta \). We then plug these expressions into the payoff functions and use the identity \( y_A^{(0)}p_A = y_B^{(0)}p_B \). This leads to

\[
-2(f + 1)\sqrt{(\beta + 1)p_By_A^{(2)}y_B^{(2)}} + (\beta + 1)p_By_B^{(2)} + (f + 1)p_Ay_A^{(2)}
\]

\[
= (f + 1)\sqrt{(f + 1)\left(f\left(-\sqrt{\frac{f + 1}{\alpha + 1}} + f + 1\right) - \sqrt{\frac{f + 1}{\alpha + 1}} + 1 - \frac{f}{\sqrt{\alpha + 1}\sqrt{f + 1}}\right) - \sqrt{\frac{f + 1}{\alpha + 1}} + 1 - \frac{f}{\sqrt{\alpha + 1}\sqrt{f + 1}}}
\]

\[
+ (f + 1)p_Ay_A^{(0)}\sqrt{(f + 1)\left(f\left(-\sqrt{\frac{f + 1}{\alpha + 1}} + f + 1\right) + f + 1\right)}
\]

\[
+ (\beta + 1)p_Ay_A^{(0)}\sqrt{(f + 1)\left(f\left(-\sqrt{\frac{f + 1}{\alpha + 1}} + f + 1\right) + f + 1\right)}
\]

It is easy to see that the payoff (which is positive) is proportional to \( y_A^{(0)} \) (and also to \( y_B^{(0)} \) as \( y_A^{(0)}p_A = y_B^{(0)}p_B \)). This means that the fee increases in \( y_A^{(0)}, y_B^{(0)} \).

The analysis for \( g_1 \) follows a similar pattern to the one above, and we can solve it using the same procedure. \( \square \)