Oscillations Beyond Three-Neutrino Mixing

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Abstract. The LSND, Gallium and reactor neutrino anomalies can be explained by short-baseline neutrino oscillations due to the mixing of the active neutrinos with sterile neutrinos at the eV scale. I review the results of a 3+1 global fit of short-baseline neutrino oscillation data that includes the recent measurements of the MINOS, IceCube, and NEOS experiments, and I discuss the implications for neutrinoless double-beta decay.

The LSND [1, 2], Gallium [3–7] and reactor [8–10] anomalies are intriguing indications in favor of short-baseline neutrino oscillations due to sterile neutrinos at the eV scale. Here, I consider 3+1 active-sterile neutrino mixing (see Ref. [11]), in which there are three sub-eV massive neutrinos $\nu_1, \nu_2,$ and $\nu_3$ which are the main constituents of the three standard active neutrinos $\nu_e, \nu_\mu,$ and $\nu_\tau,$ and there is a fourth massive neutrino $\nu_4$ at the eV scale which is mainly sterile ($\nu_4 \simeq \nu_s$).

In the framework of 3+1 active-sterile mixing, the effective oscillation probabilities of the flavor neutrinos in short-baseline experiments are given by [12]

$$P^{(SBL)}_{\alpha\beta} \simeq \left| \delta_{\alpha\beta} - \sin^2 2\theta_{\alpha\beta} \sin^2 \left( \frac{\Delta m_{41}^2 L}{4E} \right) \right|,$$

where $\alpha, \beta = e, \mu, \tau, s, L$ is the source-detector distance and $E$ is the neutrino energy. The short-baseline oscillation amplitudes depend only on the absolute values of the elements in the fourth column of the mixing matrix:

$$\sin^2 2\theta_{\alpha\beta} = 4|U_{\alpha4}|^2 \left| \delta_{\alpha\beta} - |U_{\beta4}|^2 \right|.$$

Here I review the results of the 3+1 global fit of short-baseline neutrino oscillation data presented in Ref. [13], which includes the recent measurements of the MINOS [14], IceCube [15], and NEOS [16] experiments, and I discuss the implications for neutrinoless double-beta decay.

It is well known (see Ref. [11]) that the global fits of short-baseline data are affected by the so-called “appearance-disappearance” tension, that is present [17] for any number $N_s$ of sterile neutrinos in $3+N_s$ mixing schemes which are perturbations of the standard three-neutrino mixing required for the explanation of the observation of solar, atmospheric and long-baseline neutrino oscillations. In Ref. [18], we proposed a “pragmatic approach” in which the appearance-disappearance tension is alleviated by excluding from the global fit the low-energy bins of the MiniBooNE experiment [19, 20], which have an anomalous excess of $\bar{\nu}_e$-like events that is widely considered to be suspicious and is under investigation in the MicroBooNE experiment at Fermilab. In this paper I assume the pragmatic approach from the beginning.
Table 1. Results of the pragmatic 3+1 global PrGlo16A, PrGlo16B, and PrGlo17 fits of SBL data. The first group of rows gives: the minimum $\chi^2 (\chi^2_{\text{min}})$, the number of degrees of freedom (NDF), the goodness of fit (GoF), the best fit values of the mixing parameters $\Delta m^2_{41}$, $|U_{e4}|^2$, $|U_{\mu4}|^2$, and of the oscillation amplitudes $\sin^2 2\theta_{e\mu}$, $\sin^2 2\theta_{ee}$, $\sin^2 2\theta_{\mu\mu}$. The second group of rows gives the $\chi^2$ difference $\Delta \chi^2_{\text{NO}}$ between the $\chi^2$ of no oscillations and $\chi^2_{\text{min}}$ and the resulting number of $\sigma$'s ($n\sigma_{\text{NO}}$) for NDF$_{\text{NO}}$ degrees of freedom corresponding to the number of fitted parameters. The third group of rows gives the results for the appearance-disappearance parameter goodness of fit [21]: the $\chi^2$ difference $\Delta \chi^2_{\text{PG}}$ and the resulting goodness of fit GoF$_{\text{PG}}$ for NDF$_{\text{PG}}$ degrees of freedom.

|                | PrGlo16A | PrGlo16B | PrGlo17 |
|----------------|----------|----------|---------|
| $\chi^2_{\text{min}}$ | 262.0    | 530.3    | 595.1   |
| NDF            | 244      | 519      | 579     |
| GoF            | 20%      | 36%      | 31%     |
| $\Delta m^2_{41}$ | 1.6      | 1.6      | 1.7     |
| $|U_{e4}|^2$   | 0.026    | 0.030    | 0.020   |
| $|U_{\mu4}|^2$ | 0.013    | 0.011    | 0.015   |
| $\sin^2 2\theta_{e\mu}$ | 0.0014   | 0.0013   | 0.0012  |
| $\sin^2 2\theta_{ee}$ | 0.10     | 0.12     | 0.079   |
| $\sin^2 2\theta_{\mu\mu}$ | 0.053    | 0.042    | 0.058   |
| $\Delta \chi^2_{\text{NO}}$ | 48.3     | 47.3     | 47.4    |
| NDF$_{\text{NO}}$ | 3        | 4        | 4       |
| $n\sigma_{\text{NO}}$ | 6.4      | 6.1      | 6.1     |
| $\Delta \chi^2_{\text{PG}}$ | 3.8      | 4.7      | 7.2     |
| NDF$_{\text{PG}}$ | 2        | 2        | 2       |
| GoF$_{\text{PG}}$ | 15%      | 9.7%     | 2.7%    |

Figure 1. Allowed regions in the $\sin^2 2\theta_{e\mu}$$-\Delta m^2_{41}$, $\sin^2 2\theta_{ee}$$-\Delta m^2_{41}$, and $\sin^2 2\theta_{e\mu}$$-\Delta m^2_{41}$ planes obtained in the pragmatic 3+1 global PrGlo16A, PrGlo16B, and PrGlo17 fits of SBL data.
The pragmatic global fit of short-baseline neutrino oscillation data [13] considers the following three groups of experiments:

(A) The ($\nu_\mu \rightarrow \nu_e$) appearance data of the LSND [2], MiniBooNE [19, 20] (without the anomalous low-energy bins), BNL-E776 [22], KARMEN [23], NOMAD [24], ICARUS [25] and OPERA [26] experiments.

(B) The following ($\nu_e$) disappearance data: 1) The ratios of measured and predicted [8–10] $\bar{\nu}_e$ rates of the short-baseline reactor experiments listed in Table 1 of Ref. [13]); 2) The $\bar{\nu}_e$ spectra measured in the Bugey-3 [27] and NEOS [16] short-baseline reactor experiments; 3) the data of the GALLEX [28–30] and SAGE [31–33] Gallium radioactive source experiments with the statistical method discussed in Ref. [6]; 4) the solar neutrino constraint on $\sin^2 2\theta_{ee}$ [13]; 5) the KARMEN [34,35] and LSND [36] $\nu_e + {^{12}C} \rightarrow {^{12}N}_{g.s.} + e^-$ scattering data [37], with the method discussed in Ref. [38].

(C) The constraints on ($\nu_\mu$, $\nu_\tau$) disappearance obtained from the data of the CDHSW experiment [39], from the analysis [40] of the data of atmospheric neutrino oscillation experiments, from the analysis of the SciBooNE-MiniBooNE neutrino [41] and antineutrino [42] data, and the recent constraints of the MINOS [14] and IceCube [15] experiments.

Table 1 and Figure 1 summarize the results of the following three global fits:

**PrGlo16A.** In this analysis we considered all the appearance and disappearance SBL data available in 2016 (see Ref. [13]), except MINOS [14] and IceCube [15]. The PrGlo16A fit is an update of the PrGLO fit presented in Ref. [11], with a similar set of data.

**PrGlo16B.** In this analysis we added the MINOS [14] and IceCube [15] data to the data considered in the PrGlo16A fit, in order to clarify their effects on the results of the analysis.

**PrGlo17.** In this analysis we added the NEOS [16] data, which have been available to us in the beginning of 2017.

From Table 1 one can see that all the three fits have an acceptable goodness of fit and the case of no oscillations is excluded at the level of about 6$\sigma$. On the other hand, the parameter goodness of fit, decreases from 15% in the PrGlo16A fit to 9.7% in the PrGlo16B fit to 2.7% in the PrGlo17 fit. This is a symptom of the increase of the appearance-disappearance tension caused by the addition of the MINOS and IceCube data in the PrGlo16B fit and the addition of NEOS data in the PrGlo17 fit.

The left panel in Fig. 1 shows the allowed regions in the $\sin^2 2\theta_{\mu\mu} - \Delta m^2_{41}$ plane, which is relevant for ($\nu_\mu$) disappearance. One can see that the constraints on ($\nu_\mu$) disappearance given by the MINOS and IceCube data disfavor the low-$\Delta m^2_{41}$–high-$\sin^2 2\theta_{\mu\mu}$ part of the region allowed by the PrGlo16A fit. This effect was expected [43] and is consistent with the results of the 3+1 global fit presented in Ref. [44], which updated Ref. [45] with the addition of the IceCube data. There is an increase of the appearance-disappearance tension caused by the shrinking if the allowed range of $|U_{e4}|^2$ from 0.0050 – 0.033 at 3$\sigma$ in the PrGlo16A fit to 0.0048 – 0.023 in the PrGlo16B fit, which is quantified by the decrease of the parameter goodness of fit from 15% in the PrGlo16A fit to 9.7% in the PrGlo16B.

The center panel in Fig. 1 shows the allowed regions in the $\sin^2 2\theta_{ee} - \Delta m^2_{41}$ plane, which is relevant for ($\nu_e$) disappearance. One can see that the inclusion of the NEOS constraints has the dramatic effect of fragmenting the allowed region in three islands with narrow $\Delta m^2_{41}$ widths. The best-fit island is at $\Delta m^2_{41} \approx 1.7 eV^2$. There is an island allowed at $2\sigma$ at $\Delta m^2_{41} \approx 1.3 eV^2$, and an island allowed at $3\sigma$ at $\Delta m^2_{41} \approx 2.4 eV^2$. Moreover, the NEOS constraints shifts the allowed range of $|U_{e4}|^2$ from 0.013 – 0.050 at 3$\sigma$ in the PrGlo16B fit to 0.0098 – 0.031 in the
Figure 2. Sensitivities of future experiments compared with the PrGlo17 allowed regions in Fig. 1.

PrGlo17 fit. The corresponding increase of the appearance-disappearance tension is quantified by the decrease of the parameter goodness of fit from 9.7% in the PrGlo16B fit to 2.7% in the PrGlo17 fit.

Figure 2 shows a comparison of the sensitivities of future experiments with the PrGlo17 allowed regions for: (a) $\nu_\mu \rightarrow \nu_e$ transitions; (b) $\nu_\mu$ disappearance; (c),(d) $\nu_e$ disappearance. It is clear that these experiments will give definitive information on the existence of active-sterile short-baseline oscillations connected with the LSND, Gallium and reactor anomalies.

The determination of active-sterile neutrino mixing is of interest also for the phenomenology of neutrinoless double-$\beta$ decay experiments [7,46–57].
If massive neutrinos are Majorana particles (see the recent reviews in Refs. [56, 58]), in the case of 3+1 mixing the rate of neutrinoless double-β decay is proportional to the square of the effective Majorana mass

$$|m_{\beta\beta}| = |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_2} m_2 + |U_{e3}|^2 e^{i\alpha_3} m_3 + |U_{e4}|^2 e^{i\alpha_4} m_4.$$  (3)

In this expression there are three completely unknown complex phases $\alpha_2, \alpha_3, \alpha_4$ which depend on the Majorana phases in the neutrino mixing matrix. These unknown complex phases can generate cancellations between the different mass contributions. Figure 3 shows the range of allowed values of $|m_{\beta\beta}|$ as a function of the lightest neutrino mass in the cases of 3ν and 3+1 mixing with normal and inverted ordering of the three lightest neutrinos [55].

From Fig. 3 one can see that the presence of an additional massive neutrinos at the eV scale can change dramatically the predictions for the possible range of values of $|m_{\beta\beta}|$. In the case of a normal 3ν mass hierarchy ($m_1 \ll m_2 \ll m_3$) the value of $|m_{\beta\beta}|$ is dominated by the contribution of $\nu_4$, which implies that $1 \times 10^{-2} \lesssim |m_{\beta\beta}| \lesssim 7 \times 10^{-2}$ eV. This range of values of $|m_{\beta\beta}|$ is larger than that predicted by the standard 3ν mixing in the case of a normal hierarchy and similar to that predicted in the case of an inverted hierarchy in the standard 3ν mixing scheme. On the other hand, in the case of an inverted 3ν mass ordering there can be a complete cancellation between the contribution of $\nu_4$ and those of the three standard light neutrinos, leading to the disappearance of the lower limit for $|m_{\beta\beta}|$ predicted by the standard 3ν mixing scheme.

The next generation of neutrinoless double-beta decay experiments (see Refs. [59–64]) is planned to explore the range of $|m_{\beta\beta}|$ between about $1 \times 10^{-2}$ and $5 \times 10^{-2}$ eV predicted by the standard 3ν mixing in the case of an inverted hierarchy. They are not expected to reach the range of $|m_{\beta\beta}|$ between about $8 \times 10^{-4}$ and $5 \times 10^{-3}$ eV predicted by the standard 3ν mixing in the case of a normal hierarchy. From Fig. 3 it is clear that the predictions are dramatically changed in the 3+1 neutrino mixing scheme and a positive result in these experiments is guaranteed in the case of a normal mass hierarchy, whereas in the case of an inverted mass hierarchy the allowed
range of \(|m_{3}\rangle\) goes from zero to about 0.1 eV.

Let me finally emphasize that the confirmation of the existence of sterile neutrinos would be a major discovery which would have a profound impact not only on neutrino physics, but on our whole view of fundamental physics, because it would prove that there is new physics beyond the Standard Model at the low-energies accessible in laboratory experiments. The measurement of the properties of the sterile neutrinos can give important information on this new physics.

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