VTEC Reconstruction of the Ionospheric Grid with Kriging Interpolation

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Abstract: In order to solve the ionospheric delay error. In this paper, IGS station data is used for global ionospheric delay modeling by using Kriging interpolation for grid ionospheric VTEC reconstruction. Then the accuracy of the above method is compared with that of CODE product and inverse distance weighted method for reconstructing the ionosphere. The results show that the variance of difference between the above method and the CODE product is within 4TECU and the mean value of the difference is 0.65 TECU. Using Kriging interpolation for grid ionospheric VTEC reconstruction has high precision and the accuracy of the grid value is very close to the final product of the CODE Analysis Center. Compared with the inverse distance weighting method, the Kriging reconstructed ionosphere tends to be truer and the modeling is more accurate in high latitudes. In the mid-latitude and low latitudes, the station is densely distributed, and the Kriging interpolation is more accurate than the inverse distance-weighted interpolation. The accuracy is 0.2 TECU; and in the high latitude area, there are fewer stations, and a part of Kriging interpolation accuracy is improved by 0.3TECU.

1. Introduction

With the application of GPS technology and the extensive use of GPS receiving equipment, it is possible to obtain a wide range of long-term continuous ionospheric TEC data. Therefore, people are paying more and more attention to the study of various phenomena of the ionosphere using GPS data. At present, foreign countries have made significant research progress in establishing regional ionospheric models using GPS observation data. In China, some surveying, astronomy and other related schools and units have also conducted a lot of research, such as the School of Geodesy and Geomatics of Wuhan University, Shanghai Astronomical Observatory of Chinese Academy of Sciences and Chinese Academy of Surveying and mapping and other institutions, which have made great contributions to China's ionospheric research\(^1\).

Interpolation algorithms are often used in GPS data processing. For example, the tropospheric and ionospheric delay correction of the roving station in the CORS network uses a linear combination model (LCM), a distance correlation model (DIM), a boundary surface fitting model, and linear interpolation model (LIM)\(^2\). The Kriging is widely used in geostatistics and has evolved in many forms. Domestic and foreign scholars have also begun to apply this method to the study of ionospheric delay correction, and there are currently relatively few studies. In the literature\(^3\), the VTEC information of the uniformly distributed base station data extracted from the Qingdao CORS network is modeled by grid ionospheric interpolation and the various interpolation precisions are compared and analyzed. And the results show that the Kriging interpolation and minimum curvature interpolation methods have the highest accuracy. In the literature\(^4\), using the IGS station data to calculate the
vertical electron content of the puncture point for regional grid interpolation and GIM grid interpolation experiment, and it is finally proved that the effect of the Kriging method in studying the electronic content of the region is closer to the real situation. In the literature [5], by processing the observation data of five base stations of Chongqing CORS network, and using the Kriging interpolation theory to insert VTEC over another station, and the results show that Kriging interpolation is more accurate and more stable. This paper draws on the methods and experience of predecessors and performs grid ionospheric VTEC reconstruction based on Kriging interpolation for global ionospheric delay modeling. A variety of methods is compared for accuracy analysis and useful conclusions is obtained.

2. Interpolation calculation method for ionospheric grid VTEC

2.1 Ionospheric delay extraction

Using the dual-frequency receiver, the observation values of different frequencies of the same station star at the same time can be combined to make difference, and the ionospheric delay related observation quantity can be obtained [6-7].

The observation equation for pseudorange measurement can be expressed as [8]:

\[ P_{m,k}^i = \rho_k^i + c(\delta_k^i - \delta_s^i) + I_k^i + \Delta \rho_k^i + \epsilon \]  

(1)

Where \( i \) is the satellite number; \( k \) is the station number; \( m \) is the frequency of the satellite signal; \( \rho_k^i \) represents the geometric distance from the satellite to the station; \( \delta_k^i \) is the receiver clock difference; \( \delta_s^i \) is the satellite clock error; \( I_k^i \) represents the ionospheric delay; \( \Delta \rho_k^i \) represents the tropospheric delay; \( \epsilon \) represents observation noise. In the case of observations with multiple frequencies, and there are multiple observations for each station relative to the satellite. The observation equation for the other frequency is:

\[ P_{n,k}^i = \rho_k^i + c(\delta_k^i - \delta_s^i) + I_k^i + \Delta \rho_k^i + \epsilon \]  

(2)

The difference between the two formulas is available:

\[ P_{k,4}^i = P_{k,2}^i - P_{k,m}^i = \frac{A}{f_n^2} - \frac{A}{f_m^2} (b_n - b_m) \]  

(3)

Where \( b_n \) is the hardware delay of the code observations at frequency \( n \).

The ionospheric corrections that should be added when determining the satellite-to-receiver distance using carrier phase measurements and ranging code measurements [9-11]:

\[ V_{ion} = \frac{40.28 \times TEC}{f^2} \]  

(4)

From formula (3) and formula (4):

\[ P_1 - P_2 = \frac{40.28(f_2^2 - f_1^2)}{f_1^2 f_2^2} TEC + c(DCB_r + DCB_s) \]  

(5)

Where \( f_1 \) and \( f_2 \) are the frequencies of the dual-frequency carrier phase, \( TEC \) is the total ionization content of the oblique path, \( DCB_r \) is the receiver differential code deviation, and \( DCB_s \) is the satellite differential code deviation.

2.2 Global ionospheric delay modeling

The function model mainly refers to the use of a certain mathematical function to express the changes of the ionospheric TEC parameters with time and space, and then using the optimal estimation method to estimate the model coefficients. The common ionospheric models include polynomial model, low-order spherical harmonic function model, spherical crown harmonic function model, and
triangular series model\cite{12-13}.

In this paper, the spherical harmonic function model is used for single-layer modeling of the ionosphere, and the specific function model is as follows\cite{14-16}: 

$$VETC(\beta,S) = \sum_{n=0}^{n_{\text{max}}} \sum_{m=0}^n \tilde{P}_{nm}(\sin \beta)(C_{nm}\cos(mS) + S_{nm}\sin(mS))$$  \hspace{1cm} (6)

Where $\beta$ is the geographic latitude or geomagnetic latitude of the puncture point; $S = \lambda - \lambda_0$ is the daily solidity of the puncture point; $\lambda$ is the puncture point longitude; $\lambda_0$ is the solar longitude; $n_{\text{max}}$ is the highest order of the spherical harmonic expansion; $\tilde{P}_{nm}$ is complete The normalized n-order m-order Legendre function; $P_{nm} = N_{nm} P_{nm}$, $N_{nm}$ is the planning functions, $P_{nm}$ is the classical incompletely normalized Legendre function; $C_{nm}$ and $S_{nm}$ are the unknown spherical harmonic coefficients, which are the parameters of the global ionospheric model.

2.3 Kriging reconfigurable ionospheric grid VTEC

Since the spherical harmonic model function produces negative and minimum values during the modeling of the global ionosphere, and this phenomenon does not correspond to the actual distribution of the ionosphere. The Kriging reconstructed ionospheric TEC map can eliminate the gross error in the deformed region and make the ionospheric model closer to the true value. The Kriging method gives different weights to each observation value according to the different spatial positions of samples and the degree of correlation between samples, and carries out sliding weighted averaging to estimate the value of unknown points\cite{17}.

Assume that for the regionalized variable $Z(x_i)$, and its observation value at a series of sampling points $x_1, x_2, \cdots, x_n$ is $Z(x_1), Z(x_2), \cdots, Z(x_n)$. The estimated value $Z(x_0)$ of a certain grid point $x_0$ in the region can be estimated by a linear combination of $n$ sample point attribute values, and that is:

$$Z(x_0) = \sum_{i=1}^{n} \lambda_i Z(x_i)$$  \hspace{1cm} (7)

Where $\lambda_i$ is the weighting factor. The equations are solved to obtain $n$ weight coefficients under the premise of guaranteeing the estimator and minimizing the estimated variance according to the principle of the Kriging method, and the equations are \cite{18-19}: 

$$\begin{align*}
\sum_{i=1}^{n} \lambda_i \gamma(x_i, x_j) + \mu = \gamma(x_0, x_j) \hspace{1cm} j = 1, 2, 3, 4, \cdots, n \\
\sum_{i=1}^{n} \lambda_i = 1
\end{align*}$$  \hspace{1cm} (8)

Where $\gamma(x_i, x_j)$ is the value of the variation function between sampling points $x_i$ and $x_j$.

3. Data processing and analysis

3.1 Data selection

In this paper, the global IGS station of 105-107 days in 2016 was downloaded and the site distribution map was drawn based on the distribution of the sites downloaded during the three days. It can be seen that the stations are evenly distributed from Figure 1.
3.2 Accuracy Analysis of ionospheric grid reconstruction

Ionospheric solution calculation using the downloaded three-day IGS station data. Firstly, the ionization delay at the puncture point is calculated by the dual-frequency receiver, then the global ionospheric model is established by using the 15th-order spherical harmonic function, and finally the ionospheric grid VTEC reconstruction is performed. The results of the experiment on the 106th day of 2016 were analyzed as shown in Figure 2 and Figure 3.

Due to space limitations, Figure 2 shows the global ionosphere distribution at 2:00 pm and 8:00 on the 106th day of 2016, and Figure 3 shows the difference between global ionospheric results and CODE at 2:00 and 8:00 on the 106th day of 2016. It can be seen from Fig. 3 that the variance of the CODE product is within 4TECU, the mean value of the difference is 0.65 TECU, and the precision is high. The accuracy of the grid value is very close to the final product of the CODE analysis center. However, there are also a small range of low latitude areas and the CODE products with a large difference, up to 12TECU. The reason is that the area is an active area of the ionosphere, and the
global ionospheric model cannot be well fitted.

In order to better reflect the ionospheric accuracy solved by the IGS station, and the ionospheric accuracy is evaluated by bias and RMS. The bias and RMS are the mean deviation and root mean square between the ionospheric TEC grid and the measured ionosphere, respectively. The formula is as follows:

\[
\text{bias} = \frac{\sum_{n=1}^{N} (TEC_{m,n} - TEC_{g,n}) \cdot MF_{n}}{N} 
\]  

\[
\text{RMS} = \sqrt{\frac{\sum_{n=1}^{N} (TEC_{m,n} - TEC_{g,n}) \cdot MF_{n}}{N}^2} 
\]  

Where \( TEC_{m,n} \) and \( TEC_{g,n} \) are results of the calculation of the point ionosphere and CODE ionosphere; \( MF_{n} \) is the corresponding ionospheric projection function value; \( N \) is the number of calculation points. Using the data of 340-344 days in 2016, the experiment was carried out to analyze the consistency between the solved ionospheric TEC grid and the IGS ionospheric TEC grid product. The test records are shown in Table 1.

| Tab 1. New strategy Kriging reconfigurable ionosphere products and CODE products comparison results record |
| area | product | 341 | 342 | 343 |
| high-latitude area | average | 0.02 | -0.22 | -0.08 |
| middle latitude area | variance | 1.32 | 1.19 | 1.24 |
| low latitude area | average | 0.39 | 0.36 | 0.19 |
| middle latitude area | variance | 1.32 | 1.31 | 1.17 |
| Southern Hemisphere | high-latitude area | average | 1.22 | 0.89 | 1.20 |
| middle latitude area | variance | 2.48 | 2.74 | 2.20 |
| low latitude area | average | 1.29 | 0.89 | 1.11 |
| middle latitude area | variance | 2.49 | 2.74 | 1.05 |
| high-latitude area | average | 0.48 | 1.85 | 1.31 |
| middle latitude area | variance | 1.35 | 2.20 | 1.36 |
| Southern Hemisphere | high-latitude area | average | 0.32 | 0.38 | 0.21 |
| middle latitude area | variance | 1.36 | 1.04 | 1.53 |

Traditional ionospheric modeling methods do not truly reflect the global ionospheric situation. The corresponding puncture point could not be obtained in the area with few stations, the ionospheric modeling was incomplete, and the deformation was common. In order to make the ionospheric model closer to the true value, and the ionospheric modeling uses the Kriging method for reconstruction, and the ionospheric accuracy is compared with the inverse distance weighting result.

Inverse distance weighted interpolation algorithm is a commonly used local interpolation algorithm, which is based on similar principle, also known as weight. Each interpolation point is affected by all sampling points. The weight decreases with the increase of the distance between the sampling point and the interpolation point, and the closer the sampling point is to the interpolation point, the greater the weight. Moreover, when the sampling point is beyond a certain range of the interpolation point, the weight can be neglected. The general fixed weight is the reciprocal of the K square of the distance, and the K usually takes 1 or 2. The value at any interpolation point is the sum of the weights of each sampling point, which is expressed as:

\[
z_p = \left( \sum_{i=1}^{n} \frac{z_i}{d_i^2} \right) \left( \sum_{i=1}^{n} \frac{1}{d_i^2} \right)^{-1} 
\]  

Where \( d_i \) is the distance from the i-th sample point to the interpolation points.
Tab 2. New strategy inverse distance weighted reconstruction ionosphere product and CODE product comparison result record

| area                  | product            | 341   | 342   | 343   |
|-----------------------|--------------------|-------|-------|-------|
|                       | average           | variance | average | variance | average | variance |
| high-latitude area    | -0.22             | 1.72   | -0.42  | 1.89   | -0.38   | 1.74     |
| Middle latitude area  | 1.59              | 1.35   | 1.06   | 1.91   | 1.19    | 1.17     |
| Low latitude area     | 1.42              | 2.98   | 1.44   | 2.74   | 1.15    | 2.98     |
| Low latitude area     | 1.29              | 2.99   | 0.89   | 2.80   | 1.06    | 2.68     |
| High latitude area    | 0.48              | 2.96   | 1.20   | 2.41   | 1.41    | 2.46     |
| High latitude area    | 0.52              | 2.86   | 0.48   | 2.64   | 0.26    | 2.33     |

It can be seen from Table 1 that the average accuracy of the Kriging reconfigurable ionospheric delay grid is within 4 TECU, and the mean value of the interpolation is 0.72 TECU. Comparing Table 1 with Table 2, it can be found that the Kriging reconstructed ionosphere tends to be truer and modeled more accurately in high latitudes. In the mid-latitude and low latitudes, the stations are densely distributed, and the Kriging interpolation is 0.2TECU more accurate than the inverse distance weighted interpolation. In the high latitude area, there are fewer stations, and the accuracy of partial Kriging interpolation is increased by 0.3TECU.

4. Conclusion
In this paper, IGS station data is used for global ionospheric delay modeling by using Kriging interpolation for grid ionospheric VTEC reconstruction. Then make a Comparison of the accuracy between the above method and the CODE product and the inverse distance weighting method to reconstruct the ionospheric. The results show that:

1) The variance of the CODE product is within 4TECU, the mean value of the difference is 0.65 TECU, and the precision is high. The accuracy of the grid value is very close to the final product of the CODE analysis center. However, there are also a small range of low latitude areas and the CODE products with a large difference, up to 12TECU.

2) The average accuracy of the Kriging reconfigurable ionospheric delay grid is within 4 TECU, and the mean value of the interpolation is 0.72 TECU. Comparing Table 1 with Table 2, it can be found that the Kriging reconstructed ionosphere tends to be truer and modeled more accurately in high latitudes. In the mid-latitude and low latitudes, the stations are densely distributed, and the Kriging interpolation is 0.2TECU more accurate than the inverse distance weighted interpolation. In the high latitude area, there are fewer stations, and the accuracy of partial Kriging interpolation is increased by 0.3TECU.

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