Renormalization Group Improved Exponentiation of Soft Gluons in QCD

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ABSTRACT

We extend the methods of Yennie, Frautschi and Suura to QCD for the summation of soft gluon effects in which infrared singularities are cancelled to all orders in $\alpha_s$. An explicit formula for the respective renormalization group improved exponentiated cross section is obtained for $q + (\bar{q}') \to q + (\bar{q}') + n(G)$ at SSC energies. Possible applications are discussed.

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As the SSC is now being constructed, it is an important problem to compute reliably the higher order radiative corrections to the relevant physics processes which it will probe either as signal or as background. Recently \cite{1}, we have shown that the higher order radiative corrections to SSC processes generated by multiple photon effects can be handled by the Yennie-Frautschi-Suura (YFS) \cite{2} methods which two of us (S.J. and B.F.L.W.) have realized via Monte Carlo methods in the context of high precision \(Z^0\) physics. The outstanding issue is to extend these YFS Monte Carlo exponentiation methods to QCD multiple gluon radiative effects. In this Letter, we prove that such an extension exists. Its realization in an explicit Monte Carlo event generator will appear elsewhere \cite{4}.

Specifically, we want to compute the QCD analogues of the YFS infrared functions \(\tilde{B}\) and \(B\), which represent the real and virtual infrared singularities in QED, respectively: at order \(\alpha\), the cross sections

\[
d\sigma^{(1\text{-}\text{loop})} - 2\alpha \text{Re}B \ d\sigma_B
\]

and

\[
\int_{k \leq K_{\text{max}}} dk \ \tilde{\beta}_1(k) = d\sigma^{B_1} - 2\alpha \tilde{B}(K_{\text{max}}) \ d\sigma_B
\]

are both infrared finite by definition and in addition

\[
\text{SUM}_{\text{IR}} \equiv 2\alpha \text{Re}B + 2\alpha \tilde{B}(K_{\text{max}})
\]

is also infrared finite, so that the cancellation of infrared singularities is realized to all orders in \(\alpha\) via the exponentiation of (3) as a consequence of the independent emission of arbitrarily soft photons in QED \cite{2}. Here, \(K_{\text{max}}\) is the detector resolution soft photon parameter (a photon is soft if its energy is less than \(K_{\text{max}}\)); \(d\sigma^{(1\text{-}\text{loop})}\) is the relevant generic cross section with one-loop virtual corrections; and \(d\sigma^{B_1}\) is the corresponding cross section with one real \(\gamma\) emission.

In QCD, consider our prototypical SSC process \(q + (\bar{q}') \rightarrow q + (\bar{q}') + (G)\) at order \(\alpha_s\). The relevant Feynman graphs and kinematics are shown in Fig. 1. Following the definitions of \(\tilde{B}\) and \(B\) in Ref. \cite{2}, we get for QCD the results

\[
B_{\text{QCD}} = \frac{i}{(8\pi^3)} \int \frac{d^4k}{(k^2 - m_G^2 + i\epsilon)} \left[ C_F \left( \frac{2p_1 + k}{k^2 + 2k \cdot p_1 + i\epsilon} + \frac{2p_2 - k}{k^2 - 2k \cdot p_2 + i\epsilon} \right)^2 \right]
\]
and we always assume an SDC/GEM trigger such that
standard infrared regulator mass and

\[ C_{QCD} = \frac{N_c}{2\pi^2} \]

with

\[ \Delta C_s = \left\{ \begin{array}{ll} -1 & , \quad qq' \text{ incoming} \\ -1/6 & , \quad q\bar{q}' \text{ incoming} \end{array} \right., \quad \Delta C_t = -3/2, \quad \text{and} \]

\[ \Delta C_u = \left\{ \begin{array}{ll} -5/2 & , \quad qq' \text{ incoming} \\ -5/3 & , \quad q\bar{q}' \text{ incoming} \end{array} \right. \]

and we always assume an SDC/GEM trigger such that \(|q^2| = |(p_1 - q_1)^2| \gg \Lambda_{QCD}^2\) and \(|q'^2| = |(p_2 - q_2)^2| \gg \Lambda_{QCD}^2\).

The important fact is that graphs (v)-(vii) in Fig. (1b) and (v) in Fig. (1c) do not contribute to the infrared singularities in the respective cross sections. More important is the fact that

\[ \text{SUM}_{\text{IR}}(\text{QCD}) = 2\alpha_s \text{Re}B_{\text{QCD}} + 2\alpha_s \tilde{B}_{\text{QCD}}(K_{\text{max}}) \]
is also infrared finite: for \( m_q = m_{q'} = m \), for example, we get

\[
\text{SUM}_{\text{IR}}(\text{QCD}) = \frac{\alpha_s}{\pi} \sum_{A=\{s,t,u,s',t',u'\}} (-1)^{\rho(A)} (C_F B_{\text{tot}}(A) + \Delta C_A B'_{\text{tot}}(A)) \tag{9}
\]

where

\[
B_{\text{tot}}(A) = \log(2K_{\text{max}}/\sqrt{|A|})^2 (\ln(|A|/m^2) - 1) + \frac{1}{2} \ln(|A|/m^2) - 1
- \pi^2/6 + \theta(A) \pi^2/2,
\]

\[
B'_{\text{tot}}(A) = \log(2K_{\text{max}}/\sqrt{|A|})^2 \ln(|A|/m^2) + \frac{1}{2} \ln(|A|/m^2) - \pi^2/6 + \theta(A) \pi^2/2 \tag{10}
\]

and

\[
\rho(A) = \begin{cases} 
0, & A = s, s', t, t' \\
1, & A = u, u' 
\end{cases}.
\tag{11}
\]

(The general expression for (9) with \( m_q \neq m_{q'} \) can be inferred from Refs. [2], [3] and [8] and it is in agreement with the infrared cancellations in (9).) Thus, for soft gluons with wavelengths \( \gg 1/\Lambda_{\text{QCD}} \), we find that we may carry through the methods of YFS to QCD [5].

Specifically, our exponentiated multiple gluon cross section takes the form

\[
d\sigma_{\text{exp}} = \exp[\text{SUM}_{\text{IR}}(\text{QCD})] \sum_{n=0}^{\infty} \prod_{j=1}^{n} \frac{d^3k_j}{k_j} \int \frac{d^4y}{(2\pi)^4} e^{iy\cdot(p_1+p_2-q_1-q_2-\sum k_j)+D_{\text{QCD}}}
* \tilde{\beta}_n(k_1, \ldots, k_n) \frac{d^3q_1}{q_1^0} \frac{d^3q_2}{q_2^0}
\tag{12}
\]

and

\[
D_{\text{QCD}} = \int \frac{d^3k}{k} \bar{S}_{\text{QCD}}(k) \left[ e^{-iy\cdot k} - \theta(K_{\text{max}} - k) \right],
\tag{13}
\]

\[
\bar{\beta}_0 = d\sigma^{(1-\text{loop})} - 2\alpha_s \text{Re} B_{\text{QCD}} d\sigma_B,
\]

\[
\bar{\beta}_1 = d\sigma^{B_1} - \bar{S}_{\text{QCD}}(k)d\sigma_B, \ldots
\tag{14}
\]

where \( d\sigma^{(1-\text{loop})} \) is the \( O(\alpha_s) \) cross section for Figs. (1a) and (1b) [2], \( d\sigma^{B_1} \) is the cross section for Fig. (1c) [3], and \( d\sigma_B \) is the Born cross section for Fig. (1a). Formulas for the remaining \( \bar{\beta}_n \) can be inferred from Ref. [2]-[4] and [8]; we show only the \( \bar{\beta}_i \) relevant to exact \( O(\alpha_s) \) exponentiation for the sake of pedagogy. Note that the dummy parameter \( K_{\text{max}} \) may correspond
to an experimental detector resolution for soft gluon jet energies; we emphasize that (12) is
independent of $K_{\text{max}}$, in complete analogy with the corresponding circumstance in QED.

The hard gluon residuals $\tilde{\beta}_n$ in (12) can be improved via the usual renormalization group
methods in complete analogy with the renormalization group improvement of the QED YFS
hard photon residuals $\tilde{\beta}_n$ in Ref. [8]. This follows from the fact that QCD, like QED, is a
renormalizable quantum field theory, which is perturbatively calculable so long as the relevant
momentum transfer squared is large compared to $\Lambda_{\text{QCD}}^2$. This latter requirement is satisfied
for (12), where we have in mind the SDC/GEM acceptances at the SSC for $\sqrt{s} = 40 \text{ TeV}$ pp
collisions. Following the arguments in Ref. [8], we conclude that we get the renormalization
group improved version of (12) by making the substitutions in (12) (here we use the standard
notation [9] for the running masses $m_i(\lambda)$ and the scaled external momenta $\{\lambda \vec{p}_i\}$)

$$
\tilde{\beta}_n(\lambda \{\vec{p}_i\}; m_i, \alpha_s) \rightarrow \lambda^{2D_{\Gamma}} \tilde{\beta}_n(\{\vec{p}_i\}; m_i(\lambda), \alpha_s(\lambda)) \exp \left[- \int_{\lambda_1}^{\lambda} d\lambda' \frac{2\gamma_{\Gamma}(\lambda')}{\lambda'} \right]
$$

(15)

where $D_{\Gamma}$ is the respective engineering dimension of the amplitude from which $\tilde{\beta}_n$ is constructed
and $\gamma_{\Gamma}$ is the anomalous dimension of that amplitude [9]. (We allow that a finite renormalization
group transformation may have been used to implement Weinberg’s renormalization group
operator at a convenient off-shell point and to return our amplitude to the mass shell thereafter.)

In this way, we arrive at the renormalization group improved exponentiated multiple gluon
theory which is entirely analogous to our renormalization group improved YFS theory in Ref. [8]
for QED.

The results (12) and (15) lend themselves to the same kind of Monte Carlo event generator
realization as did the theory in Ref. [8]. We have constructed two such event generators from
(12) and (15), one of which, SSCBHLG, treats multiple gluon radiation from both initial and
final states, and the other of which, SSCYFSG, treats multiple gluon radiation from the initial
state only. These two Monte Carlo event generators will be discussed in detail elsewhere [4].

In conclusion, in this Letter, we have shown that the infrared singularities in QCD allow,
for our typical SSC process, the same kind of exponentiation as one can do in QED. This opens
the way for the first time for the simulation, via Monte Carlo event generators, of quantum
amplitude based multiple gluon radiation on an event-by-event basis to all orders in $\alpha_s$. Such
results in the context of SSC/LHC physics will appear elsewhere [3].

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Figure 1: The process $q + \bar{q}' \rightarrow q + \bar{q}'' + (G)$ to $O(\alpha_s)$: (a) Born approximation; (b) $O(\alpha_s)$ virtual correction; (c) $O(\alpha_s)$ bremsstrahlung process.
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