Simulation of Heat Transfer in Cylinder Husks Furnace with Finite Difference Method

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Abstract. Simulation of heat transfer on cylinder husk furnace with FDM (Finite Difference Method) has been performed. This simulation aimed to study the heat transfer based on conduction and convection mechanism on the cylinder husk furnace. Fluid dynamics inside the cylinder of husk furnace was obtained by solving the Navier-Stokes equations with laminar flow approach. The initial temperature in all parts of the cylinder is room temperature, except at the bottom of the cylinder is 491 °C. Through numerical calculation of heat conduction and convection equation by FDM method, we got that the velocity of fluid flow is ranged between 3.32 - 13.75 m/s and 2.00 - 13.75 m/s for fixed and unfixed temperature of the cylinder blanket, respectively.

Introduction

Husk furnace is a stove which uses fuel from rice husk. The husk furnace components consist of cone part and cylinder part (see Fig 1). The cylinder part of husk furnace is the important part of the furnace that will ensure the heat is delivered well. Heat distribution in the cylinder husk furnace can be determined theoretically by studying heat transfer based on conduction and convection process. As we knew that the Graetz problem is the heat transfer problem of laminar fluid flow in ducts [1]. Hsu [2] studied the Graetz problem with axial diffusion in circular tube, using a semi- infinite domain formulation with a specified inlet condition, while Michelsen and Villadsen [3] analyzed the effects of axial diffusion in a infinite domain formed by an insulated preparation region followed by an isothermal wall. Both studies used numerical schemes to complete the solution of the problem. Barros and Sphaier [4] proposed analytical approximations for solving an extended version of the Graetz problem with axial diffusion in an infinite domain. The Graetz problem for non-Newtonian fluids with constant wall temperature and heat flux was studied numerically by Shih and Tsou [5]. Fluid dynamics inside the cylinder husk furnace determines how convection works, that can be obtained by solving the Navier-Stokes equations [6]. The pressure gradient of the fluid inside the cylinder can be derived from temperature gradient or fluid density gradient. The aim of this research is to study the heat transfer on

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the cylinder husk furnace based on conduction and convection mechanism with laminar fluid flow approach.

**Figure 1. Husk Furnace and Cylinder**

**Result and Discussion**

2.1 Mathematical Modelling of Heat Transfer Cylinder

Heat transfer on the cylinder husk furnace is associated with the phenomenon of conduction and convection. The governing equation for the heat transfer can be expressed as

\[ \rho C_p \frac{\partial T}{\partial t} + \frac{\mathbf{U} \cdot \nabla T}{\mathbf{g}} = k \nabla^2 T \]  

(1)

With \( \mathbf{U} \) is fluid flow velocity, \( \rho \) is air density, \( C_p \) is constant of material specific heat, \( k \) is constant of conductivity thermal matter, \( \alpha \) is constant of diffusivity thermal matter, and \( T \) is temperature on cylinder [7].

The term on the right side of Equation (1) corresponds to the heat conduction. The Laplacian on the equation in cylindrical coordinate is written as:

\[ \nabla^2 T = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \]  

(2)

While, the second term on left side of the equation corresponds to the heat convection and in cylindrical coordinate the term can be written as:

\[ \mathbf{U} \cdot \nabla T = U_r \frac{\partial T}{\partial r} + U_z \frac{\partial T}{\partial z} + U_\theta \frac{\partial T}{\partial \theta} \]  

(3)

With \( U_r \) is fluid flow velocity for \( r \) radius direction and \( U_z \) is fluid flow velocity for \( z \) height direction. \( U_\theta \) is fluid flow velocity for \( \theta \) degrees direction. From equation (2) and (3), we combine it to equation (1), so the equation be:

\[ \frac{\partial T}{\partial t} = \alpha \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) - \frac{1}{\rho C_p} \left( U_r \frac{\partial T}{\partial r} + U_z \frac{\partial T}{\partial z} + U_\theta \frac{\partial T}{\partial \theta} \right) \]  

(4)

Because of cylindrical is symmetry, there is no gradient temperature respect to \( \theta \) angles.
So, the heat transfer equation on cylinder husk furnace can be simplified to:

$$\frac{\partial T}{\partial t} = \alpha \left( \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2} \right) - \frac{1}{\rho c_p} \left( \frac{1}{r} \frac{\partial T}{\partial r} + U \frac{\partial T}{\partial z} \right)$$  \hspace{1cm} (5)

2.2 Finite Difference Methods (FDM)

Finite difference method is one of numerical technique to solved differential equation problem [8]. Here we used FDM to solve equation (7) based on forward and center formulas that are given by [9]:

$$\frac{df(x_i)}{dx} \approx \frac{f_{i+1} - f_i}{\Delta x}$$

$$\frac{d^2 f(x_i)}{dx^2} \approx \frac{f_{i+1} - 2f_i + f_{i-1}}{\Delta x^2}$$

Based on the relations, we transform the continuous differential equation problem for heat distribution on cylinder to discrete differential equation problem:

$$\frac{\partial T}{\partial t} = \frac{T(z_{j+1}, r_i, t_k) - T(z_j, r_i, t_k)}{\Delta t}$$  \hspace{1cm} (6)

$$\frac{\partial T}{\partial r} = \frac{T(z_j, r_{i+1}, t_k) - 2T(z_j, r_i, t_k) + T(z_j, r_{i-1}, t_k)}{\Delta r}$$  \hspace{1cm} (7)

$$\frac{\partial^2 T}{\partial r^2} = \frac{T(z_j, r_{i+1}, t_k) - 2T(z_j, r_i, t_k) + T(z_j, r_{i-1}, t_k)}{\Delta r^2}$$  \hspace{1cm} (8)

$$\frac{\partial^2 T}{\partial z^2} = \frac{T(z_{j+1}, r_i, t_k) - 2T(z_j, r_i, t_k) + T(z_{j-1}, r_i, t_k)}{\Delta z^2}$$  \hspace{1cm} (9)

To simplify those equations, we introduce new index notation as follows:

$$T(z_{j+1}, r_i, t_k) = T_{j+1, i, k}$$  \hspace{1cm} (10)

$$T(z_j, r_{i+1}, t_k) = T_{j, i+1, k}$$  \hspace{1cm} (11)

$$T(z_j, r_{i-1}, t_k) = T_{j, i-1, k}$$  \hspace{1cm} (12)

$$T(z_{j+1}, r_i, t_k) = T_{j+1, i, k}$$  \hspace{1cm} (13)

$$T(z_{j-1}, r_i, t_k) = T_{j-1, i, k}$$  \hspace{1cm} (14)

Where \( j \) index for height cylinder, \( i \) index for radius cylinder, \( k \) index for time. So, the governing numerical equation for heat distribution conduction and convection on cylinder is:

$$T_{j, i, k+1} = T_{j, i, k} + \alpha \left( \frac{T_{j+1, i, k} + T_{j, i+1, k} - 2T_{j, i, k}}{\Delta r} + \frac{T_{j+1, i, k} + T_{j, i-1, k} - 2T_{j, i, k}}{\Delta r^2} + \frac{T_{j+1, i, k} + T_{j, i+1, k} - 2T_{j, i, k}}{\Delta z^2} \right) - \frac{1}{\rho c_p} \left( \frac{T_{j+1, i, k} - T_{j, i, k}}{\Delta r} + \frac{T_{j, i+1, k} - T_{j, i, k}}{\Delta z} \right)$$  \hspace{1cm} (15)
2.3 Fluid Dynamics
Fluid dynamics inside the cylinder husk furnace was obtained by solving the Navier-Stokes equations with laminar flow approach. Analytical solution of the equation for laminar flow case is given by [10]:

\[ u(x) = \frac{1}{2} \frac{h^2}{\mu} \frac{dP}{dz} \left[ 1 - \left( \frac{x}{h} \right)^2 \right] \]  

(16)

Where boundary condition at the wall is:

\[ u = 0 \text{ at } x = \pm R \]

The pressure gradient on equation (27) as function of temperature can be written as follows:

\[ \frac{dP}{dz} = \left( \frac{\rho}{1 + \beta(T - T_0)} \right) gh \]  

(17)

2.4 Numerical Calculation of Heat Conduction and Convection Equation
The initial temperature in all parts of the cylinder is at room temperature, except the bottom of the cylinder is at 491 °C. There are two simulation case namely fixed and unfixed cylinder blanket temperature. For fixed cylinder blanket temperature assumed that the cylinder walls in contact with heat reservoir and unfixed temperature assumed that the cylinder walls in contact with low heat conductivity medium.

Figure 2. Heat distribution in the cylinder by conduction for fixed (a) and unfixed (b) blanket temperature

Figure 2 show the simulation of heat distribution on the cross section of cylinder furnace caused by conduction process for one minute. It is appears that the rate of heat transfer in the fixed temperature case is lower than unfixed temperature case. In fixed temperature case, the heat transfer only occurs in the fluid, there is no contribution from the wall heat conduction. While in unfixed temperature case, there is a contribution from the wall heat conduction with higher conductivity than fluid, so the temperature of cylinder edge is higher than its center.
Figure 3. Heat distribution in the cylinder by conduction and convection for fixed (a) and unfixed (b) blanket temperature

Figure 3 show the simulation of heat distribution on the cross section of cylinder furnace caused by conduction and convection process simultaneously, for one minute. By comparing Fig 2 and Fig 3, it can be concluded that the heat transfer in the cylinder husk furnace is dominated by convection mechanism. On the mechanism of convection, the heat transfers through the movement of air particles due to the pressure gradient caused by the temperature gradient. While in conduction, there is only vibration between atoms of air due to a temperature gradient. As with the previous simulation, the rate of heat transfer in the fixed temperature case is lower than unfixed temperature case.

Figure 4. Fluid velocity profile for unfixed (a) and fixed (b) temperature of the cylinder blanket, respectively (m/s).

Figure 4 above shows the fluid velocity profile in the middle of cylinder for unfixed and fixed temperature case. The velocity profile is parabolic with maximum speed at the center of the cylinder. The velocity of fluid flow inside the cylinder is ranged between 2.00 - 13.75 m/s and 3.32 - 13.75 m/s.
for unfixed and fixed temperature case, respectively. The different result is due to the pressure gradient near the wall in fixed temperature is bigger than unfixed temperature case.

3. Conclusion

We have performed simulation of heat transfer on cylinder husk furnace with FDM (Finite Difference Method). Based on numerical calculation with FDM for heat conduction and convection, heat transfer in the cylinder husk furnace is dominated by convection mechanism. On convection mechanism, the heat transfer through the movement of air particles due to the pressure gradient caused by the temperature gradient. While in conduction, there is only vibration between atoms of air due to a temperature gradient. The velocity of fluid flow inside the cylinder is ranged between 2:00 – 13.75 m/s and 3:32 – 13.75 m/s for unfixed and fixed temperature case, respectively. The different result is due to the pressure gradient near the wall in fixed temperature is bigger than unfixed temperature case. The velocity profile is parabolic with maximum speed at the center of the cylinder.

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