Numerical and statistical modeling based investigation of the detection efficiency of high-spectral-resolution lidars at different laser radiation wavelengths

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Abstract. In the present work, a numerical and statistical modeling approach is developed for estimating the operation efficiency of high-spectral-resolution lidars (HSRLs), when determining the atmospheric aerosol extinction profiles along the lidar line of sight (LOS) at different sensing (laser) radiation wavelengths from the UV, VIS and NIR spectral ranges. The efficiency estimation is based on numerical modeling of the lidar signal profile along the LOS and the corresponding profile of the signal-to-noise ratio (SNR) of its measurement. The optimally efficient wavelength (at certain distances along the LOS) is that ensuring maximum signal strength and SNR and, consequently, brighter and clearer lidar images of specific aerosol objects and background. The results about the optimum wavelengths obtained under different atmospheric-turbidity conditions (clear or hazy atmosphere), in the presence of cirrus clouds, are proved by statistical modeling and processing of realistic (noisy) lidar profiles and recovering the atmospheric extinction profiles along the LOS. It is shown that up to certain characteristic altitudes (that are lower at higher turbidity) the UV wavelengths are advantageous in the above sense. Above the mentioned altitudes, successively, first the VIS and then the NIR wavelengths become more efficient compared to the UV wavelengths.

1. Introduction

An express and vast-scale active optical sensing of the gaseous and aerosol atmospheric components is achievable by the so-called elastic lidars (laser radars), based on Mie and Rayleigh scattering processes, where the detected scattered light has the same wavelength as the laser pulses irradiating the atmosphere [1, 2]. Using elastic lidars is a way to determine the spatial distributions (and their evolution) of the aerosol extinction and backscattering coefficients that in turn are proportional to the mass concentration of the aerosol matter [3]. The lidar return signal in this case is sufficiently intensive to ensure a high signal-to-noise ratio (SNR) of the measurements and, correspondingly, accurate lidar signal profiles at relatively long distances along the line of sight (LOS) [1, 2]. A disadvantage of these lidars is an ambiguity in the determination of the LOS profiles of the aerosol extinction and backscattering coefficients, due to the unknown exact relation (lidar ratio) between them [4].
The disadvantage of the elastic lidars is avoided in the so-called Raman and high-spectral-resolution lidars (HSRLs) whose performance is based on combined analysis of the elastic lidar return from the atmosphere and the molecular only, Raman or Rayleigh lidar return, respectively [4, chapters 5 and 9]. These lidars allow one to determine unambiguously the LOS profiles of the atmospheric extinction and backscattering coefficients.

The elastic-lidar sensing efficiency, depending on the laser wavelength at equal other factors of importance, was studied by us earlier [5-7]. Specific numerical and statistical modeling was performed to find the optimum (UV, VIS, or NIR) laser wavelengths providing maximum lidar return signal strength and SNR and, correspondingly, maximum bright and accurate lidar signal profiles. Appropriate models were used of the vertical distributions of the absorption and scattering coefficients of cirrus clouds (CCs) and Saharan dust layers (SDLs), and of the background gaseous and aerosol atmospheric components. The measurement fluctuations were generated as due to the prevailing Poisson shot noise.

The main purpose of the present work is to estimate, using a similar modeling approach, the wavelength-dependent efficiency of the HSRLs when recovering the atmospheric-extinction vertical profiles in the presence of, e.g., CCs in clear or hazy atmosphere.

2. Modeling approach

2.1. HSRL equation

The HSRL equation describes the Rayleigh-backscattering-due lidar return signal from the atmospheric molecular component, depending on the characteristics of the atmosphere and the lidar facilities. It is convenient to be written in the form [4, chapter 4, 7]

\[ N_s(\lambda, z) = (A/\pi^2)T \eta(z)C_L \beta_m(\lambda, z) \exp \left\{ -2 \int_0^z \mu(\lambda, z')dz' \right\}, \]

where \( \lambda \) is the laser radiation wavelength; \( z = ct/2 \) is the coordinate of the scattering volume along the LOS, and \( t \) is the time period after the pulse emission; \( A \) is the receiving aperture area; \( \eta(z) = \Omega/(\Omega + A/z^2) \) is the receiving efficiency function, where \( \Omega \) is the solid angle of view of the receiving optical system; \( C_L = cE_0ec^2/2e \), \( c \) is the speed of light, \( E_0 \) is the laser pulse energy, \( S \) [A/W] is current-to-(light)power photodetection sensitivity, \( \tau_c \) is the integration (response) time of the receiving electronics, and \( e \) is the electron charge; \( \beta_m(\lambda, z) = b_m\mu_m(\lambda, z) \), and \( \mu_m(\lambda, z) = \mu_m(\lambda, z) + \mu_m(\lambda, z) + \mu_m(\lambda, z) + \mu_m(\lambda, z) \) are the LOS profiles of the atmospheric molecular Rayleigh backscattering coefficient and overall extinction coefficient, respectively, \( b_m(\lambda, z) = 3/8\pi \approx 0.119 \text{ sr}^{-1} \) [1], \( \mu_m(\lambda, z) \) is the molecular Rayleigh scattering coefficient, and \( \mu_m(\lambda, z) \) and \( \mu_m(\lambda, z) \) are the extinction coefficient profiles of the molecular (index m) and aerosol (index a) atmospheric components and of strongly scattering compact aerosol objects (index o); and \( N_s(\lambda, z) \) is the number of photoelectrons produced in a photon detector by Rayleigh-backscattered photons during a \( \tau_c \) – long interval. In the numerical calculations we assume that \( A = 25\pi10^{-2} \text{ m}^2, \Omega = 10^6 \text{ sr}, E_0 = 750 \text{ mJ}, S = 0.5 \text{ A/W}, \text{ and } \tau_c = 1 \text{ ns} \).

2.2. Signal-to-Noise Ratio (SNR)

The SNR of measuring the lidar signal profiles using photomultipliers (PMs) is expressible as [1]

\[ \text{SNR}(\lambda, z) = \xi N_s(\lambda, z) \left\{ \xi F \left[ N_s(\lambda, z) + N_b + N_d \right] \right\}^{-1/2}, \]

where \( \xi \) is the PM photoelectron collection ability, and \( F \) is the noise excess factor. Since \( \xi \) and \( F \) are usually near (below and above) unity, respectively, we shall assume for simplicity that \( \xi \approx F \approx 1 \). \( N_b(\lambda) \) and \( N_d \) are the means of background-due photoelectrons and dark current-due electrons, respectively, assumed to be known and subtracted from the overall (signal+background) measured...
signal. Equation (2) is derived [1], assuming Poisson statistics of the photoelectron and dark electron fluctuations. At a dark current of 5.10^{-10} A [8], N_d = 3 for \tau_e = 1 ns. Also, using experimental data for the sky background [9], we obtain that at a 10 nm interference filter, N_d = 302, 437, and 72 for \lambda = 337.1, 514.5, and 1060 nm, respectively.

2.3. Determination of the LOS extinction profile

The LOS profiles of \mu_\text{m} (\lambda, z) and \beta_\text{m} (\lambda, z) are independently determinable on the basis of the corresponding air density profiles measured, e.g., by radiosondes [4]. Then, the extinction profile \mu_e (\lambda, z) is directly obtainable from equation (1) in the form

$$
\mu_e(\lambda, z) = -0.5 \frac{d}{dz} [\ln \Phi(z)],
$$

where \Phi(z) = N_e(\lambda, z) / [(A/z^2)\eta(z)C_1\beta_\text{m}(\lambda, z)].

2.4. Models of the extinction and backscattering coefficient profiles \mu_e (\lambda, z) and \beta_\text{m} (\lambda, z)

2.4.1. Clear and hazy atmospheres.

In both cases, we have \mu_e (\lambda, z) = \mu_\text{m} (\lambda, z) + \mu_\text{a} (\lambda, z), where \mu_\text{m} (\lambda, z) = \mu_\text{m} (\lambda, z) + \mu_\text{a} (\lambda, z), \mu_\text{a} (\lambda, z) = \mu_\text{a} (\lambda, z) + \mu_\text{a} (\lambda, z). \mu_\text{m} (\lambda, z), \mu_\text{a} (\lambda, z), \mu_\text{a} (\lambda, z), \mu_\text{a} (\lambda, z) are the corresponding molecular and aerosol absorption coefficients, and \mu_\text{m} (\lambda, z), \mu_\text{a} (\lambda, z) are the corresponding molecular and aerosol scattering coefficients. Realistic data concerning the profiles of the absorption and scattering coefficients and of the extinction coefficients as a whole are taken here from a report of McClatchey et al. [10], where the vertical sampling interval is 1 km and more. The cases of midlatitude clear and hazy atmospheres are considered corresponding to visibilities of 23 and 5 km [10]. For convenience, the data profiles are approximated here by simple integrable analytical expressions:

$$
\mu_p(\lambda, z) = A_p(\lambda) / \left[ 1 + \exp \left[ (z - z_{\text{ref}}(\lambda)) / w_p(\lambda) \right] \right],
$$

where p = e or s when q = m and p = e when q = a; \lambda_p [m^(-1)], z_{\text{ref}}[m], and w_p [m] are best-fit least-squares approximation parameters. The values obtained of the parameters A_em, z_0_em, w_em, A_sm, z_0_sm, w_sm, A_m, z_0_m, and w_m, are given in tables in [6]. Model data [10] for \mu_\text{m} (\lambda, z), \mu_\text{a} (\lambda, z), and \mu_\text{a} (\lambda, z) along with the corresponding approximating curves in the case of hazy atmosphere and wavelength 514.5 nm are illustrated in [6].

2.4.2. Compact larger-particle-made aerosol objects in the atmosphere.

In the presence of such objects (cirrus clouds or Saharan dust layers), \mu_e (\lambda, z) = \mu_\text{m} (\lambda, z) + \mu_\text{a} (\lambda, z) + \mu_\text{a} (\lambda, z). \mu_\text{m} (\lambda, z), \mu_\text{a} (\lambda, z), \mu_\text{a} (\lambda, z), \mu_\text{a} (\lambda, z) describing their contribution may be modeled [6] as superpositions, \mu_\text{a} (\lambda, z) = \sum \mu_\text{a} (\lambda, z), of bell-shaped modes:

$$
\mu_\text{a} (\lambda, z) / \mu_\text{a} (\lambda, z) = \beta_{\text{el}} (\lambda, z) / \beta_{\text{el}} (\lambda, z) = [1 + (z - z_{\text{ref}}) / w_{\text{el}},
$$

that are symmetric with respect to the corresponding positions z_{\text{ref}} of their peak values \mu_\text{a} (\lambda, z) and have characteristic widths w_{\text{el}}. \beta_{\text{el}} is a single-mode model \mu_\text{a} (\lambda, z) of a cirrus cloud with mode parameters: z_0 = 15 km, \mu_\text{a} (\lambda, z_0) = \mu_\text{a} (z_0) = 0.5 \times 10^3 \text{m}^{-1}, and w_0 = 600 m. The value chosen of p is 4. The extinction coefficient \mu_\text{a} is assumed to be practically independent of \lambda within the UV, VIS and NIR spectral ranges [1] because of the large size parameters of the scattering particles [11].

2.5. Statistical modeling of realistic (noisy) lidar return signal profiles

The statistical-modeling procedure includes first the determination of the mean return signal profiles, for different sensing-radiation wavelengths and atmospheric visibilities, and the corresponding mean
stationary background levels due to dark current and optical background. The mean signal profiles $N_s(\lambda, z)$ are obtainable from equation (1) using the above-described models of $\mu_{\text{em}}(\lambda, z)$, $\mu_{\text{sc}}(\lambda, z)$, $\mu_{\text{ea}}(\lambda, z)$, and $\beta_{\text{ea}}(\lambda, z)$. Further, using routine program codes based on the knowledge of the mean lidar and background profiles, we have reproduced the main measurement operations, that is, the measurement of the Poisson-fluctuating response to the lidar return plus the background, and subtraction of the mean background level. Ensemble averaging procedures have also been performed and analyzed along with the retrieval of the extinction coefficient profiles $\mu_A(\lambda, z)$.

3. Results and discussion
The numerical modeling of the mean lidar-signal and SNR profiles was performed using equations (1) and (2), considering different laser wavelengths and atmospheric turbidities according to the atmospheric and lidar characteristics and models accepted above. The results obtained show that up to altitudes of about 25 km, in clear atmosphere, and 9 km, in hazy atmosphere, the maximum signal strength is obtained at $\lambda = 337.1$ nm with a slight advantage over the case of $\lambda = 514.5$ nm (figure 1a, c). At higher altitudes, the signal is maximum at $\lambda = 514.5$ nm. At altitudes above 100 km and 60 km, respectively, even the wavelength $\lambda = 1060$ nm becomes advantageous over $\lambda = 337.1$ nm. Similar is the picture concerning the SNR profiles (figure 1b, d), where the corresponding characteristic altitudes are 30 km, 12 km, 82 km and 35 km (instead of 25, 9, 100 and 60 km).

![Figure 1](image-url)

**Figure 1.** Mean vertical HSRL profiles (a, c) and corresponding SNR profiles (b, d) at different laser wavelengths for clear and hazy atmospheres, respectively, with a cirrus cloud at 15 km altitude.

Noisy return signal profile estimates corresponding to figure 1c, for $\lambda = 337.1$ nm, 514.5 nm and 1060 nm, are represented in figure 2a-c. They are obtained by averaging over 1000 signal realizations.
(laser shots) and subtraction of the mean background level. It is seen that the range of the noise correlates with the SNR behavior outlined in figure 1d. Up to about 12 km altitude, the relative signal fluctuations at \( \lambda = 337.1 \) nm are comparable but lower than those at \( \lambda = 514.5 \) nm. They are noticeably lower than in the case when \( \lambda = 1060 \) nm. Above 12 km, however, the relative noise range is minimum at \( \lambda = 514.5 \) nm.

![Figure 2](image1.png)

**Figure 2.** Noisy HSRL signal profile estimates given in appropriate scales, at different altitudes and wavelengths, obtained by averaging over 1000 signal realizations created using the mean profiles shown in figure 1c (hazy atmosphere).

Extinction coefficient profiles, retrieved using equation (3) and noisy profiles averaged over 10 000 laser shots, are shown in figure 3. There, the image of the cloud disposed at an altitude of 15 km (above 12 km) seems clearest (most accurately retrieved) at \( \lambda = 514.5 \) nm. At altitudes below 12 km, the extinction profiles seem more accurately recovered at \( \lambda = 337.1 \) nm. Thus, Saharan dust layers that are usually located at altitudes of 3–4 km would be better imaged by 337.1 nm laser wavelength.

![Figure 3](image2.png)

**Figure 3.** Extinction coefficient profiles in hazy atmosphere retrieved using realistic HSRL profiles like those in figure 2, but averaged over 10 000 laser shots.
4. Conclusion
As a whole, the modeling approach developed in this work is of importance in the selection of optimal HSRL or rotational Raman lidar parameters (the sensing laser wavelength, in particular) ensuring maximum bright and accurate images (vertical extinction distributions) of the aerosol stratification and compact aerosol objects at different altitudes in the atmosphere. Some initial results obtained here using the approach under consideration concern the wavelength-dependent efficiency of sensing the vertical extinction distribution in clear and hazy atmospheres containing cirrus clouds. It is shown that up to certain altitudes, due to stronger (back)scattering of the sensing radiation, the UV wavelengths ensure better (more accurate) imaging of the aerosol field and objects. At higher altitudes, however, the VIS sensing wavelengths become maximum efficient because of relatively strong (back)scattering and lower attenuation. With increasing the atmospheric turbidity and, respectively, the attenuation of light, the relative advantages of the shorter (UV) wavelengths decrease while those of the longer (VIS and NIR) wavelengths increase. A further development of the approach includes investigating, analytically and by statistical modeling, the accuracy of recovering the profiles of the aerosol backscatter coefficient and the lidar ratio.

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