Three-body model calculations for $N = Z$ odd-odd nuclei with $T = 0$ and $T = 1$ pairing correlations

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We study the interplay between the isoscalar ($T = 0$) and isovector ($T = 1$) pairing correlations in $N = Z$ odd-odd nuclei from $^{14}$N to $^{30}$Cu by using three-body model calculations. The strong spin-triplet $T = 0$ pairing correlation dominates in the ground state of $^{14}$N, $^{18}$F, $^{30}$P, and $^{30}$Cu with the spin-parity $J^p = 1^+$, which can be well reproduced by the present calculations. The magnetic dipole and Gamow-Teller transitions are found to be strong in $^{18}$F and $^{42}$Sc as a manifestation of SU(4) symmetry in the spin-isospin space. We also discuss the spin-quadrupole transitions in these nuclei.

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I. INTRODUCTION

The pairing correlation is one of the most remarkable effects in nuclear physics. It appears in many properties of nuclei, including odd-even mass staggering, as well as the large energy gap between the first excited and the ground states in even-even nuclei compared to odd-even nuclei. In literature, the spin-singlet $T = 1$ pairing has been mainly discussed in nuclear physics, since the large spin-orbit splitting prevents to couple a spin-triplet ($T = 0, S = 1$) pair in the ground state [1, 2]. Another reason for this is that the large neutron-excess along the stability line of the nuclear chart suppresses the proton-neutron pairing. A recent availability of radioactive beams has opened up an opportunity to measure structure properties of unstable nuclei along the $N = Z$ line, strongly enhancing a possibility to measure new properties of nuclei such as pairing correlations related with the spin-triplet $T = 0$ pairing. It is thus quite interesting and important to study the competition between the spin-singlet $T = 1$ and the spin-triplet $T = 0$ pairing interactions in odd-odd $N = Z$ nuclei and seek an experimental evidence for the competition in the spins of low-lying states. In this paper, we focus our study in sd- and pf- shell nuclei, in which the ground state spins and spin-isospin transitions are observed. In order to study the ground state and the low-lying excited states in odd-odd $N = Z$ nuclei in these mass regions, we apply a three-body model with a density-dependent contact interaction between the valence neutron and proton.

The paper is organized as follows. In Sec. II, we explain the three-body model employed in the present study. In Sec. III, we present the results of the calculations and discuss the ground state properties of odd-odd $N = Z$ nuclei. We also discuss the magnetic moments, the magnetic dipole transitions, the isovector spin-quadrupole transitions, and the Gamow-Teller transitions in these nuclei. We summarize the paper in Sec. IV.
a uniformly charged sphere of radius $R$ and charge $Z_C e$, where $Z_C$ is the atomic number of the core nucleus. We use a contact interaction between the valence neutron and proton, $V_{np}$, given as

$$V_{np}(r_1, r_2) = \hat{P}_s v_s \delta(r_1 - r_2) \left[ 1 + x_s \left( \frac{\rho(r)}{\rho_0} \right)^\alpha \right]$$

$$+ \hat{P}_t v_t \delta(r_1 - r_2) \left[ 1 + x_t \left( \frac{\rho(r)}{\rho_0} \right)^\alpha \right]$$ (4)

where $\hat{P}_s$ and $\hat{P}_t$ are the projectors onto the spin-singlet and spin-triplet channels, respectively:

$$\hat{P}_s = \frac{1}{4} - \frac{1}{4} \sigma_p \cdot \sigma_n, \quad \hat{P}_t = \frac{3}{4} + \frac{1}{4} \sigma_p \cdot \sigma_n.$$ (5)

In each channel in Eq. (4), the first term corresponds to the interaction in vacuum while the second term takes into account the medium effect through the density dependence. Here, the core density is assumed to be a Fermi distribution of the same radius and diffuseness as in the core-valence particle interaction, Eq. (3). The strength parameters, $v_s$ and $v_t$, are determined from the proton-neutron scattering length as

$$v_s = \frac{2\pi^2 \hbar^2}{m} \frac{a_{pn}^{(s)}}{\pi - 2a_{pn}^{(s)} k_{cut}},$$ (6)

$$v_t = \frac{2\pi^2 \hbar^2}{m} \frac{a_{pn}^{(t)}}{\pi - 2a_{pn}^{(t)} k_{cut}},$$ (7)

where $a_{pn}^{(s)} = -23.749$ fm and $a_{pn}^{(t)} = 5.424$ fm are the empirical p-n scattering lengths in the spin-singlet and spin-triplet channels, respectively. $k_{cut}$ is the momentum cut-off introduced in treating the delta function, which is related with the cutoff energy as $E_{cut} = \hbar^2 k_{cut}^2 / m$. The strengths $v_s$ and $v_t$ determined from the scattering lengths depend on the cutoff energy, $E_{cut}$, as will be discussed in Sec. III. The three parameters $x_s$, $x_t$, and $\alpha$ in the density-dependent terms in Eq. (4) are determined so as to reproduce energies of the ground ($J^\pi = 1^+$), the first excited ($J^\pi = 3^+$), and the second excited ($J^\pi = 0^+$) states in $^{18}$F with respect to the three-body threshold (See also Ref. 3). The density $\rho(r)/\rho_0$ is replaced by a Fermi function $f(r)$ hereafter.

The Hamiltonian (1) is diagonalized in the valence two-particle model space. The basis states for this are given by a product of proton and neutron single particle states with the single particle energy $\epsilon(\tau)$, which are obtained with the single-particle potential $V_{pc}$ in Eq. (1) ($\tau = p$ or $n$). To this end, the single-particle continuum states are discretized in a large box. We include only those states satisfying $\epsilon_p^{(p)} + \epsilon_n^{(n)} \leq E_{cut}$. We use the proton-neutron formalism without antisymmetrization in order to take into account the breaking of the isospin symmetry due to the Coulomb interaction.

FIG. 1: (Color online) The excitation energies of the first $0^+$ and the first $3^+_1$ states in $^{18}$F obtained with the three-body model as a function of the spin-orbit strength $v_{ls}$ in the mean-field potential. The excitation energies are measured from the energy of the first $1^+_2$ state.

FIG. 2: (Color online) The energies of the first $0^+_1$ and the first $1^+_2$ states in $N = Z$ nuclei. The upper panel (a) shows experimental data and the lower panel (b) corresponds to calculated results. The values with the arrows show the transition probabilities for the magnetic dipole transitions, $B(M1)$ (the calculated values are shown in the brackets for $^{14}$N and $^{30}$P). The experimental data are taken from Ref. 10.

III. RESULTS

The spin-orbit potential in the mean-field potentials plays a crucial role in determining the properties of $T = 0$ pairing as discussed in Refs. 1, 8, 9. In Fig. 1 we plot the energy differences between the first $0^+$ and $1^+$ states and between the first $3^+$ and $1^+$ states in $^{18}$F as a function of the spin-orbit coupling strength $v_{ls}$. We use the
cutoff energy of $E_{\text{cut}} = 20$ MeV. It is clearly seen in Fig. [1] that the $T = 0$ pairing correlations decreases as the spin-orbit interaction increases. That is, the energy difference $E_{0^{-}} - E_{1^{+}}$ decreases and eventually the spectrum is reversed so that the $0^{+}$ state becomes the ground state, where the $T = 1$ pairing overcomes the $T = 0$ pairing.

The calculated spectra for $^{14}$N, $^{18}$F, $^{30}$P, $^{34}$Cl, $^{42}$Sc, and $^{58}$Cu nuclei are shown in Fig. [2] together with the experimental data. The spin-purity for the ground state of the nuclei in Fig. [2] are $J^s = 1^+$ except for $^{34}$Cl and $^{42}$Sc. This feature is entirely due to the interplay between the isoscalar spin-triplet and the isovector spin-singlet pairing interactions in these $N = Z$ nuclei. In the present calculations, the ratio between the isoscalar and the isovector pairing interactions is $v_t/v_s = 1.9$ for the energy cutoff of the model space, $E_{\text{cut}} = 20$ MeV. This ratio is somewhat larger than the value ~1.6 obtained in Ref. [9] from the shell model matrix elements in $p$- and $sd$-shell nuclei. For a larger model space with $E_{\text{cut}} = 30$ MeV, the ratio becomes 1.6, but the agreement between the experimental data and the calculations somewhat worsens quantitatively even though the general feature remains the same. It is remarkable that the energy differences $\Delta E = E(0^{1}_j) - E(1^{+}_j)$ are well reproduced in $^{34}$Cl and $^{42}$Sc both qualitatively (the inversion of the $1^{+}$ and $0^{+}$ states in the ground state) and quantitatively (the absolute value of the energy difference). The model description is somewhat poor in $^{14}$N and $^{30}$P because the cores of these two nuclei are deformed, although the ordering of the two lowest levels are correctly reproduced.

The probability of the total spin $S = 0$ and $S = 1$ components for the $0^{+}$ and the $1^{+}$ states, respectively, are listed in Table [I]. The total spin $S = 0$ and $S = 1$ components in two particle configurations can be calculated with a formula

$$
|j_\pi j_\nu J_{\pi} J_{\nu} J_j > = \sum_{L,S} \left\{ \begin{array}{ccc} l_\pi & l_\nu & L \\ s & s & S \end{array} \right\} \hat{L}\hat{S}j_\pi j_\nu ((l_\pi l_\nu)_{LS}; J_j) 
$$

(8)

with the $9j$ symbol and a factor $\hat{L} = \sqrt{2L+1}$. For a $j_\pi = j_\nu = j = l + 1/2$ configuration, the $S = 0$ and $S = 1$ components are given by the factors $(j + 1/2)/2j$ and $(j - 1/2)/2j$, respectively, for $J = 0$. For a $j_\pi = j_\nu = j = l - 1/2$ configuration, on the other hand, they are $(j + 1/2)/(2j + 2)$ and $(j - 3/2)/(2j + 2)$ for $S = 0$ and $S = 1$, respectively. Notice that $s_{3/2}$ configuration has only $S = 0$ component if $J = 0$. Otherwise, the two particle states have a large mixture of the $S = 0$ and $S = 1$ components. In general, the $S = 1$ and $S = 0$ components are thus largely mixed in the wave functions of both the ground and the excited states. An exception is $^{30}$P. In this nucleus, the dominant configuration in the $0^{+}$ state is $(2s_{1/2}^7 \otimes 2s_{1/2}^7)$, which can couple only to the total spin $S = 0$. On the other hand, in the $1^{+}$ state, the dominant configuration is $(2s_{1/2}^7 \otimes 1d_{3/2}^7) T = 0$ which
can couple only to the total spin $S = 1$ with the total angular momentum $L = 2$.

We next discuss the magnetic moment for the $1^{+}$ state, and the magnetic dipole transition strength $B(M1) \downarrow$ and the isovector spin-quadrupole transition strength $B(IVSQ) \uparrow$ between the $0^{+}$ and $1^{+}$ states. The symbol $\downarrow (\uparrow)$ means the transition from the excited (ground) to the ground (excited) states. The magnetic operator is defined as

$$
\mu = \langle 1^{+} | \sum_i (g_s(i) s_i + g_l(i) l_i) | 1^{+} \rangle , 
$$

(9)

where $g_s(i)$ and $g_l(i)$ are the spin and the orbital $g$ factors, respectively. The reduced magnetic dipole transition probability is given by

$$
B(M1 : J_i \rightarrow J_f) = \frac{3}{4\pi} \frac{1}{2J_i + 1} \left| \sum_i \langle J_f | \mathbf{g}_s(i) \mathbf{s}_i + \mathbf{g}_l(i) \mathbf{l}_i | J_i \rangle \right|^2 . 
$$

(10)

where the double bar means the reduced matrix element in the spin space. We take the bare $g$ factors $g_s(\pi) = 5.58$, $g_s(\nu) = -3.82$, $g_l(\pi) = 1$, and $g_l(\nu) = 0$ for the magnetic moment and the magnetic dipole transitions in the unit of the nuclear magneton $\mu_N = e\hbar/2mc$. The spin-quadrupole transition is defined by

$$
B(IVSQ : J_i \rightarrow J_f) = \frac{1}{2J_i + 1} \left| \sum_i \langle J_f | \mathbf{\tau}_2(i) \mathbf{r}_i^2 [\mathbf{\sigma}(i) \mathbf{Y}_2(i)]^{(\lambda = 1)} | J_i \rangle \right|^2 . 
$$

(11)

The calculated magnetic moments and the magnetic dipole transitions are listed in Table [II] together with the spin quadrupole transitions. The calculated magnetic moment in $^{14}$N reproduces well the observed one,
while the agreement is worse in $^{58}\text{Cu}$. This is due to the fact that the core of $^{58}\text{Ni}$ might be largely broken and the $f_{7/2}$-hole configuration is mixed in the ground state of $^{58}\text{Cu}$. The values for $B(M1)$ are also shown in Table II that is, in $^{18}\text{F}$ and $^{42}\text{Sc}$. The $B(M1)$ value from 0$^+$ to 1$^+$ in $^{18}\text{F}$ is the largest one so far observed in the entire region of nuclear chart. We notice that our three-body calculations provide remarkable agreements not only for these strong transitions in $^{18}\text{F}$ and $^{42}\text{Sc}$ but also quenched transitions in the other $N = Z$ nuclei such as in $^{14}\text{N}$ and $^{34}\text{Cl}$.

In the case of $^{18}\text{F}$, the 0$^+$ and 1$^+$ states are largely dominated by the $S = 0$ and $S = 1$ spin components, respectively, with the orbital angular momentum $l = 2$ (see Table I). Therefore, the two states can be considered as members of SU(4) multiplet in the spin-isospin space. This is the main reason why the $B(M1)$ value is so large in this nucleus, since the spin-isospin operator $g_s$ stt connects between two states in the same SU(4) multiplet, that is, the transition is allowed, and the isovector factor is the dominant term in Eq. (10) with $g_s^{IV} = (g_s(\nu) - g_s(\pi))/2 = -4.70$. The configurations in $^{42}\text{Sc}$ are also similar to those in $^{18}\text{F}$ in terms of SU(4) multiplets, although they are dominated by $l = 3$ wave functions. For $^{14}\text{N}$ and $^{34}\text{Cl}$, the $B(M1)$ transitions do not acquire any enhancement, since the $S = 0$ component in the 0$^+$ state is suppressed due to the $j = l - 1/2$ coupling (both the 0$^+$ and 1$^+$ states have very large $1p_{1/2}^2(1d_{3/2}^2)$ configurations in $^{14}\text{N}$ ($^{34}\text{Cl}$)). These indications for the SU(4) symmetry in $^{18}\text{F}$ and $^{42}\text{Sc}$ are consistent with the results obtained in Refs. [13, 14]. In nuclei $^{30}\text{P}$ and $^{58}\text{Cu}$, the 1$^+$ state is dominated by $1d_{3/2}^2s_{1/2}$ and $2p_{3/2}^2f_{5/2}$ configurations, respectively, while the 0$^+$ state is governed by the $2s_{1/2}^2$ and $2p_{3/2}^2$ configurations, respectively. Therefore the isovector spin-quadrupole transitions are largely enhanced in the two nuclei even though the $B(M1)$ is much quenched.

We also calculate the Gamow-Teller (GT) strength

$$B(\text{GT}) : 0^+ \rightarrow 1^+ = \frac{g_A^2}{4\pi} \left| \langle 1^+ | \sum_i t_{-}(i)\sigma(i)|0^+ \rangle \right|^2 , \quad (12)$$

where $g_A$ is the axial-vector strength, and summarize the results in Table III. One can again see the strong GT transition between the lowest 0$^+$ and 1$^+$ states in $A = 18$ and 42 systems, which exhaust a large portion of the GT sum rule value. This can also be interpreted as a manifestation of SU(4) symmetry in the wave functions of these nuclei. We note here again that the result obtained in Ref. [13] by an analysis of GT transition also implies a good SU(4) symmetry in the $A = 18$ system. On the other hand, for $^{58}\text{Cu}$, the GT strength is largely fragmented and no strong state in $B(\text{GT})$ is seen near the ground state. The experimental data are consistent with the calculated results as can be seen in Table III.

### IV. SUMMARY

We have studied the properties of the lowest 0$^+$ and 1$^+$ states in the odd-odd $N = Z$ nuclei in the sd- and pf- shell region with the three-body model with valence proton and neutron and a core. The ratio between the spin-triplet isoscalar and the spin-triplet isovector pairing...
interactions, $v_s/v_t$, is determined to be 1.9 based on the neutron-proton scattering lengths and the energy cut-off of the model space. It was pointed out that the energy ordering of the $0^+$ and $1^+$ states is very sensitive to the strength of spin-orbit coupling, i.e., the spin-orbit splitting prevents the strong spin-triplet pairing interactions and makes the ground states of $^{34}$Cl and $^{42}$Sc to have $J^\pi = 0^+$. The energy differences between the lowest $0^+$ and $1^+$ states are well reproduced by our model qualitatively (that is, the inversion of the level ordering between the two states) and quantitatively (that is, the excitation energy). It was shown that the calculated wave functions of the lowest $0^+$ and $1^+$ states in $^{18}$F and $^{42}$Sc have typical features of the SU(4) multiplets in the spin-isospin space and give the strong magnetic dipole transitions strength between the $0^+$ and $1^+$ states. The GT transitions from the neighboring even-even $T = 1, T_z = 1$ nuclei $^{18}$O and $^{42}$Ca with the $J^\pi = 0^+$ to the $1^+$ states in the odd-odd $T = 0$ nuclei $^{18}$F and $^{42}$Sc are also shown to be very strong, exhausting a substantial amount of the GT sum rule. The calculated transitions give quantitatively good accounts of the observed strong $B(M1)$ and $B(GT)$ values in the two nuclei. In the other $N = Z$ nuclei, $B(M1)$ transitions are rather hindered, while the spin-quadrupole transitions are found to be rather strong.

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